Chern-Simons Field Theory and Completely Integrable Systems

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Abstract

We show that the classical non-abelian pure Chern-Simons action is related in a natural way to completely integrable systems of the Davey-Stewartson hierarchy, via reductions of the gauge connection in Hermitian spaces and by performing certain gauge choices. The Bäcklund Transformations are interpreted in terms of Chern-Simons equations of motion or, on the other hand, as a consistency condition on the gauge. A mapping with a nonlinear $\sigma$-model is discussed.

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In the last ten years a great effort has been devoted in the study of certain classes of nonlinear fields, namely the Chern-Simons (CS) \cite{1,2}, the nonlinear $\sigma$-model and certain completely integrable models, either at the quantum and in the classical level. There exists a very wide literature concerning these topics and several connections between them have been pointed out from long time \cite{3}. However, only recently a precise relationship has been established at the classical level \cite{4}.

In this letter we show that the (apparently trivial) classical non-abelian CS field theory provides nonlinear $\sigma$-models in a algorithmic way, including the basic structures for their integrability. In other words, we provide a recipe for supplying completely integrable models from solvable models and what is the interplay between the two concepts.

In doing so, for simplicity we restrict ourselves to consider the $SU(2)$-CS theory, given by the action

$$S[J] = \frac{k}{4\pi} \int_{\mathcal{M}} \text{Tr} \left( J \wedge dJ + \frac{2}{3} J \wedge J \wedge J \right),$$

where $J$ is the 1-form gauge connection with values in the Lie algebra $su(2)$ and $\mathcal{M}$ is a 3-manifold. The classical equations of motion for the action \cite{4} are given by the zero-curvature condition

$$F \equiv dJ + J \wedge J = 0,$$

whose solutions are easily found in terms of chiral currents in the Lie group $SU(2)$. The action \cite{4} is invariant under general coordinate transformations (preserving orientation and volumes). Moreover, under a generic gauge map $G : \mathcal{M} \to SU(2)$ the gauge connection transforms as usual by $J \to G^{-1} J G + G^{-1} dG$. Correspondingly, the action \cite{4} changes as $S[J] \to S[J] + 2\pi k W(G)$, where $W(G)$ is the winding number of the map $G$ taking integer values, as prescribed by the homotopy theory \cite{5}.

Now, locally we trivialize $\mathcal{M}$ in the form $\Sigma \times \mathbb{R}$, where $\Sigma$ is a Riemann surface and $\mathbb{R}$ is interpreted as the time. This operation breaks the general covariance of the theory.

On the other hand, we can introduce a $\mathbb{Z}_2$-decomposition in the algebra of the 1-forms $J$. In terms of the Lie algebra $\hat{g}$ of $G$ (\equiv $SU(2)$ in the case we are considering), this means that $\hat{g} = \hat{l}(0) \oplus \hat{l}(1)$, $[\hat{l}(i), \hat{l}(j)] \subset \hat{l}(i+j) \mod (2)$, where $\hat{l}(0)$ is the Lie algebra of a proper subgroup $\mathcal{H}$ of $G$ and $\hat{l}(1)$ is the
complement of \( \hat{l}(0) \) in \( \hat{g} \). The subgroup \( \mathcal{H} \) is chosen to be invariant under an involution over \( \mathcal{G} \). The group \( \mathcal{G} \) acts transitively on the coset space \( \mathcal{G} / \mathcal{H} \), which is a symmetric space \( [6] \). At any point \( p_0 \in \mathcal{G} / \mathcal{H} \), the tangent space \( T_{p_0} (\mathcal{G} / \mathcal{H}) \) is isomorphic to \( \hat{l}(1) \). The natural Riemann connection defined on these spaces is torsionless. Moreover, we require the existence of a complex structure on such a space, thus we deal with a Hermitian symmetric space \( [6] \).

The current \( J \) is decomposed in the form

\[
J = J^{(0)} + J^{(1)},
\]

where \( J^{(1)} \) and \( J^{(0)} \) are 1-forms taking values in the tangent space and in the isotropy algebra of the considered hermitian space, respectively. The two components of the current obtained by the previous decomposition will play different roles in the theory. Indeed, we will see that the model \([1]\) will become a non-relativistic theory for some matter fields minimally coupled to a residual CS gauge field (in the sense that it is associated with the isotropy group \( \mathcal{H} \)), plus some constraints expressing the torsionless character of the target space \( \mathcal{G} / \mathcal{H} \). Indeed, if for \( \mathcal{G} \equiv SU(2) \) we choose \( \mathcal{H} \equiv U(1) \), the related hermitian space is the sphere \( SU(2)/U(1) \cong S^2 \cong CP^1 \) and the action \([1]\) can be rewritten in the form

\[
S = -\frac{k}{\pi} \int_{\Sigma \times \mathbb{R}} \left\{ \frac{1}{2} \epsilon^{\lambda\mu\nu} v_\lambda \partial_\mu v_\nu \\
+ i \frac{1}{2} \left( \psi_+^* D_0 \psi_+ - \psi_+ (D_0 \psi_+)^* - \psi_-^* D_0 \psi_- + \psi_- (D_0 \psi_-)^* \right) \\
- iq_0 \left( D \psi_+ - D \psi_- \right) + iq_0 \left( D \psi_+ - D \psi_- \right)^* \right\} dx^0 dx^1 dx^2,
\]

where the fields \( \psi_{\pm} \) parametrize the space components of \( J^{(1)} \) (they can be considered as scalar complex matter fields), \( v_\mu \) parametrizes \( J^{(0)} \) and represents the abelian CS gauge field associated with the \( U(1) \) invariance. Finally, the field \( q_0 \), related to the time component of \( J^{(1)} \), plays the role of a Lagrange multiplier, enforcing a constraint, which is the remnant of the torsionless property of the chosen target space. \( D_0 = \partial_0 - 2iv_0 \), \( D = \partial_z - 2iv \) and \( \bar{D} = \partial_{\bar{z}} - 2iv^* \) denote covariant derivatives, where we have employed the usual complex variables \( z = x_1 + ix_2 \) and \( \bar{z} = x_1 - ix_2 \) and with \( v = \frac{1}{2} (v_1 - iv_2) \). By resorting to higher dimensional compact group \( \mathcal{G} \), we get analogous structures, in which we can embed several types of non-relativistic \( CP^n \) models.
Moreover, for the system described by (4), we studied the canonical structure, which turns out to be a completely constrained Hamiltonian system \[4\].

However, the previous approach contains also an unexpected structure. In fact, among the variations of the action (4) let us take for brevity only the equations

\[
\bar{D}\psi = D\psi, \\
\partial_z v^* - \partial_{\bar{z}} v = -i \left( |\psi|^2 - |\psi|^2 \right),
\]

which are the torsionless condition and the Gauss-Chern-Simons law (GCS), respectively. These equations are very important, because they are the unique equations which do not contain time derivatives and time-components of the currents \(J\).

We handle Eqs. (5 - 6) by the help of the new matrix fields

\[
\mathcal{V} = \begin{pmatrix} v^* \\ v \end{pmatrix}, \quad \hat{\Psi}_\pm = \begin{pmatrix} \psi_\pm - \psi_\pm^* \\ \psi_\pm \\ \psi_\mp \\ \psi_\mp^* \end{pmatrix}.
\]

Furthermore, let us introduce

\[
B^{(1)} = \frac{i}{2} \sigma_3 \left( \hat{\Psi}_- - \hat{\Psi}_+ \right)
\]

Combining the GCS law in Eq. (6) with its complex conjugate, we obtain

\[
\text{Tr} \left\{ \sigma_3 \left[ \left( \frac{\partial}{\partial z} \frac{\partial}{\partial \bar{z}} \right) \mathcal{V} + B^{(1)} \hat{\Psi}_- - \hat{\Psi}_+ B^{(1)} \right] \right\} = 0
\]

Since the quantity in the square brackets is a diagonal matrix and no information is supplied about the identity component, we have the relation

\[
\left( \frac{\partial}{\partial z} \frac{\partial}{\partial \bar{z}} \right) \mathcal{V} + B^{(1)} \hat{\Psi}_- - \hat{\Psi}_+ B^{(1)} = f \sigma_0
\]

where \(f = f(z, \bar{z})\) is an arbitrary function and \(\sigma_0\) is the identity matrix.

On the other hand, the torsionless condition can be written as

\[
\left( \frac{\partial}{\partial z} \frac{\partial}{\partial \bar{z}} \right) B^{(1)} + \frac{i}{2} \left( \frac{\partial}{\partial z} - \frac{\partial}{\partial \bar{z}} \right) \hat{N} - \hat{N} \hat{N} = 0
\]

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Equations (10) and (11) can be summed up to give the expression
\[
T_+ \left[ \frac{i}{2} (\partial_\tau - \partial_z) + V + B^{(1)} \right] - \left[ \frac{i}{2} (\partial_\tau - \partial_z) + V + B^{(1)} \right] T_- = f, \tag{12}
\]
where
\[
T_\pm = \begin{pmatrix} \partial_z \\ \partial_\tau \end{pmatrix} - \hat{\Psi}_\pm. \tag{13}
\]
Since the previous procedure is invertible (the summation is made over independent components), Eq. (12) is equivalent to the system (5 - 6) (modulo \(f\)).

Putting \(f \equiv 0\), Eq. (12) coincides with the space part of the Bäcklund transformation for the two-dimensional Zachkarov-Shabat problem [7], in which the principal spectral problem is given by Eq. (13), and the first order Bäcklund-gauge operator is
\[
B = \nabla + V + B^{(1)}, \tag{14}
\]
with \(\nabla = \frac{i}{2} (\partial_\tau - \partial_z)\). The operator \(B\) transforms an eigenfunction \(\phi_-\) of the linear problem \(T_- \phi_- = 0\) into \(\phi_+ = B\phi_-\), which is an eigenfunction of \(T_+ \phi_+ = 0\).

It is well known [7] that the triad \(B, T_\pm\) enables one to introduce a continuous extra-dependency on a parameter, say \(\tau\), and two operators of the form \(T_\pm^{(\tau)} = i \partial_\tau + \sum_{k=0}^N T_{\pm, k} \nabla^{N-k}\) such that: 1) \([T_\pm, T_\pm^{(\tau)}] = 0\) and 2) \(T_\pm^{(\tau)} B - BT_\pm^{(\tau)} = 0\). The lowest order operator of such a type, leading to a non-trivial equation, can be put in the form
\[
T_\pm^{(\tau)} = i \partial_\tau - 8 \sigma_3 \nabla^2 - 8i \hat{\Psi}_\pm \nabla + 4 \begin{pmatrix} \partial_z \\ -\partial_z \end{pmatrix} \hat{\Psi}_\pm - 8 \eta_\pm \sigma_0 + \frac{1}{4} \zeta_\pm \sigma_3, \tag{15}
\]
and the corresponding evolution equation reads
\[
i \partial_\tau \psi_\pm + 2 \left( \partial_z^2 + \partial_\tau^2 \right) \psi_\pm + \frac{1}{2} \zeta_\pm \psi_\pm = 0, \tag{16}
\]
\[
\partial_z \partial_\tau \zeta_\pm = 8 \left( \partial_z^2 + \partial_\tau^2 \right) |\psi_\pm|^2, \tag{16}
\]
\[
\partial_z \partial_\tau \eta_\pm = \left( \partial_z^2 - \partial_\tau^2 \right) |\psi_\pm|^2, \tag{16}
\]
which is the Davey-Stewartson II equation (DS II) [7]. We summarize the first result saying that when we embed the classical “trivial” CS theory into
a special geometric setting (we chose the special trivialization $\Sigma \times \mathbb{R}$ for the space-time and a hermitian space as target space), some topics related to the completely integrable models appear. However, at this stage the construction is not complete at all. Indeed if we look at the evolution equations arising from the action (14), we have

$$D_0 \psi_+ - \bar{D} q_0 = 0,$$

$$D_0 \psi_- - D q_0 = 0.$$  \hspace{1cm} (17)

In its turn the evolution of $q_0$ and of $v_0$ is completely arbitrary, since these quantities are Lagrange multipliers associated with the gauge degrees of freedom. This structure is quite different from the DS equation, unless we break the general gauge invariance of the CS theory, requiring a constraint on $q_0$. Which is the form of such a constraint?

The simplest choice (Weyl gauge) is provided by $q_0 = v_0 \equiv 0$, which has been widely exploited ([2], [8]). In this gauge the Lagrangian becomes quadratic and the quantization using the canonical formalism can be performed. However, here we want to explore other gauge choices and their consequences at the classical level.

For instance, let us put

$$q_0 = 2i \left[ \left( \mathcal{D} + \frac{i}{2} w_+ \right) \psi_+ + \left( D + \frac{i}{2} w_- \right) \psi_- \right],$$

where we assume that the complex function $w_+ = w_+$ is invariant under and a $U(1)$ gauge transformation acting on the CS-fields. Moreover, accordingly to them $(w_+, w_-)$ transforms controvariantly under space-time transformations. The constraint (18) admits $U(1)$ as residual gauge symmetry. At the same time the general covariance symmetry is broken and only special Lie-point symmetries are allowed. A detailed analysis of this aspect is skipped for the moment.

The substitution of Eq. (18) into the CS-field equations provides certain nonlinear evolution equations for $\psi_\pm$. In particular, Eqs. (17) become

$$D_0 \psi_+ - 2i \left( D^2 + \bar{D}^2 \right) \psi_+ + \psi_+ \partial_2 w_+ + w_+ D \psi_+ + w_- D \psi_+ =$$

$$\left[ 4i \left( |\psi_+|^2 - |\psi_-|^2 \right) - \partial_2 w_- \right] \psi_-,$$

$$D_0 \psi_- - 2i \left( D^2 + \bar{D}^2 \right) \psi_- + \psi_- \partial_2 w_- + w_- D \psi_- + w_- D \psi_- =$$

$$\left[ 4i \left( |\psi_+|^2 - |\psi_-|^2 \right) + \partial_2 w_+ \right] \psi_+. \hspace{1cm} (19)$$
Moreover, we have to take into account Eqs (5 - 6) and the equation involving the time-derivatives of \( v, \bar{v} \) (the “electric strenght” field), which we write for an arbitrary \( q_0 \)

\[
\begin{align*}
\partial_0 v - \partial_z v_0 &= i \left( q_0 \psi_+^* - q_0^* \psi_- \right), \\
\partial_0 \bar{v} - \partial_z \bar{v}_0 &= -i \left( q_0^* \psi_+ - q_0 \psi_-^* \right).
\end{align*}
\]  

(20)

We notice that in Eqs. (19) the coupling between the components \( \psi_\pm \) is nonlocal through the \( U(1) \) gauge fields, and local by the r.h.s.. The quantity \( w_\pm \) is a sort of external field.

However, inspired by the previous discussion on the DS equation, we can switch off the local coupling just by putting

\[
\partial_z w_+ = -4i \left( |\psi_+|^2 - |\psi_-|^2 \right),
\]

(21)

which implies a sort of Gauss law like Eq. (3) containing also the zero-divergence condition. Then, we keep \( v \) and \( w_- \) still distinct. On the other hand, exploiting this similarity, we can combine them into the irrotational field \( A = 4v - \frac{1}{2}w_+ \), with \( w_- = w_L + w_T = -\partial_z \omega - i\partial_z \chi \). The stream function \( \chi \) and the potential \( \omega \) satisfy the Poisson and the Laplace equations, respectively

\[
\begin{align*}
\partial_z \partial_\bar{z} \chi &= -4 \left( |\psi_+|^2 - |\psi_-|^2 \right) \\
\partial_z \partial_\bar{z} \omega &= 0.
\end{align*}
\]  

(22)

(23)

Exploiting the \( U(1) \) gauge invariance of Eqs. (19) and (20), we can make the substitution

\[
A = \partial_2 \Lambda \ (\Lambda \in \mathbb{R}), \quad \psi_\pm = \Psi_\pm e^{\pm i \Lambda}, \quad v_0 = \frac{1}{4} \left( A_0 + \partial_0 \Lambda \right),
\]

(24)

where \( A_0 \) is a new time component of the CS \( U(1) \)-scalar field. In this formalism the condition (3) reads

\[
\left( \partial_z + \frac{1}{4} \partial_\bar{z} \chi \right) \Psi_- = \left( \partial_z - \frac{1}{4} \partial_\bar{z} \chi \right) \Psi_+
\]

(25)

and the “electric” field equations (20) become a pair of compatible first order equations for \( A_0 \) in terms of \( \Psi_\pm \), their derivatives and derivatives of \( \chi \).
However, it is convenient to introduce the quantities

\[ A_0^{(\pm)} = A_0 \mp \left( \partial_z^2 \chi + \partial_{\bar{z}}^2 \chi \right) - \frac{1}{4} \left( (\partial_z \chi)^2 + (\partial_{\bar{z}} \chi)^2 \right) + \frac{i}{2} \left( \partial_z \omega \partial_{\bar{z}} \chi - \partial_{\bar{z}} \omega \partial_z \chi \right) - 2i \left( \frac{\partial^2}{\partial_z^2} \right) \omega, \]

which allows us to write the time evolution for \( \Psi_\pm \) in the form

\[ i \partial_0 \Psi_\pm + 2 \left( \partial_z^2 + \partial_{\bar{z}}^2 \right) \Psi_\pm + \frac{1}{2} A_0^{(\pm)} \Psi_\pm - i (\partial_z \omega \partial_{\bar{z}} + \partial_{\bar{z}} \omega \partial_z) \Psi_\pm = 0. \]  

(27)

From Eq. (26) we notice that the functions \( A_0^{(\pm)} \) are not independent, but they are related by

\[ A_0^+ - A_0^- = -2 \left( \partial_z^2 + \partial_{\bar{z}}^2 \right) \chi - 2 \left( \partial_z^2 - \partial_{\bar{z}}^2 \right) \omega. \]  

(28)

Moreover, they satisfy the equation

\[ \partial_z \partial_{\bar{z}} A_0^{(\pm)} = 8 \left( \partial_z^2 + \partial_{\bar{z}}^2 \right) |\Psi_\pm|^2, \]  

(29)

which is a consequence of the electric strength (20).

To summarize, by the specific gauge choices (18) and (24) we have obtained a formally decoupled pair of DS-like equations for the fields \( \left( \Psi_\pm, A_0^{(\pm)} \right) \).

A generalizing term, involving first order derivatives of \( \Psi_\pm \), has coefficients depending on the harmonic map \( \omega \). For \( \omega = \text{const} \) we recover the system (16). Actually, Eqs. (22), (25) and (28) close the system, introducing a non-local coupling. But Eqs. (22) and (25) are essentially the system (5-6) discussed at the beginning, providing the space part of the Bäcklund transformations for the DS system. In other words, we have obtained a pair of DSII systems coupled by the Bäcklund transformations. This result can be used in looking for classes of solutions for the CS theory in the special gauge (18), following the standard methods developed in the context of the completely integrable systems. For instance, for \( \Psi_+ \equiv 0 \) we can find \( \Psi_- \) in terms of solutions of the Liouville equation, to which our system of equations reduces. Such solutions of multivortex type are widely discussed in [4, 10].

The discussion above can be extended to the generalized DSII system (i.e. for non-constant harmonic background \( \omega \)). In fact the system given by Eqs.
(27 - 29) (for instance, let us consider the "-" case) admits as Lax pair the operator $T_-$ defined in (13) and the generalization of (15)

$$T_-(\tau) = i\partial_0 - 8\sigma_3 \nabla^2 - 8 \left( i\hat{\Psi} + R \right) \nabla + 4 \left( \frac{\partial_2}{\partial_2} - \partial_2 \right) \hat{\Psi} - 4i\sigma_3 \left[ R, \hat{\Psi} \right] - 2\eta_0 + \frac{1}{4} \left( A_0^{(-)} - 2i\partial_0^2 \omega \right) \sigma_3 , \quad (30)$$

where

$$R = \frac{1}{4} \begin{pmatrix} \partial_\omega & -\partial_\omega \\ -\partial_\omega & \partial_\omega \end{pmatrix} . \quad (31)$$

Furthermore, in analogy with the DS equation [11], one can look for a gauge transformation between the above system to a spin model. In fact, introducing the spin field $S$ ($S \in SU(2)/U(1)$) one can easily prove that the system (in real variables)

$$\begin{align*}
\partial_0 S + \Re (w_+) \partial_1 S - \Im (w_+) \partial_2 S + i S \left( \partial_1^2 - \partial_2^2 \right) S &= 0 , \\
\partial_1 \Re (w_+) + \partial_2 \Im (w_+) &= 0 , \\
\partial_1 \Re (w_+) - \partial_2 \Im (w_+) &= -i \frac{1}{2} Tr \left( S \left[ \partial_1 S, \partial_2 S \right] \right) .
\end{align*} \quad (32)$$

is equivalent to (22-23-27-29) (only one pair of fields, for instance $\left( \Psi_-, A_0^{(-)} \right)$, is kept). This can be seen by looking for a suitable non-degenerate matrix $g$, such that the Lax pair of the spin model

$$L = i\partial_2 + S\partial_1 , \quad M = \partial_0 + 2iS\partial_1^2 + (i\partial_1 S + S\partial_2 S - \Im (w_+) S + \Re (w_+)) \partial_1 \quad (33)$$

takes the form

$$T_- = g^{-1} L g , \quad T_-(\tau) = g^{-1} M g , \quad S = g\sigma_3 g^{-1} . \quad (34)$$

Then we can interprete this system as a two dimensional continuous spin field in a moving frame, determined by the incompressible velocity field $\Re (w_+), \Im (w_+)$ in a non-euclidean space metric $(+, -)$. The vorticity is determined by the density of the topological charge. These types of systems have been discussed in [9] - [12]. They can be considered as generalizations of the well-known Ishimori model [13]. In this context it is interesting to
observe that the diagonal element of the Bäcklund operator (14) takes a physical meaning. Finally, we notice that the system (33) can be treated by resorting to the tangent space representation approach [9]. This method allows to describe the spin model (33) in terms of a non-relativistic gauge theory. In the specific case one obtains the full system (22-23-27-29). If we consider a generalization of the above liquid spin model, by introducing an arbitrary non-vanishing coupling constant $\theta$ between the vorticity and the topological density in the third equation of (33), the resulting system is no longer integrable and can be analyzed only by the help of the tangent space representation approach. However, the main claim is that one cannot combine the velocity field and a suitable gauge field into into an irrotational field, like $A$. Such a “phenomenological” model could be related to the creation of vortices in the superfluid $^3He$ [14], but it is not reducible in the framework of the $SU(2)$-CS theory developed in this work.

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