Dynamical Correlations after a Quantum Quench

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We consider dynamic (non equal time) correlation functions of local observables after a quantum quench. We show that in the absence of long-range interactions in the final Hamiltonian, the dynamics is determined by the same ensemble that describes static (equal time) correlations. For many integrable models static correlation functions of local observables after a quantum quench relax to stationary values, which are described by a generalized Gibbs ensemble (GGE). The same GGE then determines dynamic correlation functions and the basic form of the fluctuation dissipation theorem holds, although the absorption and emission spectra are not simply related as in the thermal case. For quenches in the transverse field Ising chain (TFIC) we derive explicit expressions for the time evolution of dynamic order parameter correlators after a quench.

Introduction. By virtue of their weak coupling to the environment ultra-cold atomic gases provide ideal testing grounds for studying nonequilibrium dynamics in isolated many-particle quantum systems. Recent experiments \cite{Kinoshita06,Wenger10} have observed essentially unitary time evolution on long time scales. This has stimulated much theoretical research on fundamental questions such as whether observables generically relax to time independent values, and if they do, what principles determine their stationary properties. Relaxational behaviour at first may appear surprising, because unitary time evolution maintains the system in a pure state at all times. However, it can be understood intuitively as a property of a given finite subsystem in the thermodynamic limit, with the role of the bath being played by the rest of the system.

Dimensionality and conservation laws strongly affect the out-of-equilibrium dynamics. Ground breaking experiments by Kinoshita, Wenger and Weiss \cite{Kinoshita06} on trapped $^{87}$Rb atoms established that three dimensional condensates “thermalize” rapidly, i.e. relax quickly to a stationary state characterized by an effective temperature, whereas the relaxation of quasi one-dimensional systems is slow and towards an unusual non-thermal distribution. This difference has been attributed to the presence of approximate conservation laws in the quasi-1D case, which are argued to constrain the dynamics. The findings of Ref. \cite{Kinoshita06} sparked a tremendous theoretical effort aimed at clarifying the effects of quantum integrability on the nonequilibrium evolution in many-particle quantum systems, see e.g. Refs \cite{Gross10,Maldacena11} and references therein. A widely held view, that has emerged from these studies, is that the reduced density matrix of any subsystem (which determines correlation functions of all local observables within the subsystem) is described in terms of either an effective thermal (Gibbs) distribution or a so-called generalized Gibbs ensemble (GGE) \cite{GGE}. The former is believed to represent the generic case, while substantial evidence suggests that the latter arises for integrable models.

Theoretical research so far has focussed on static properties in the stationary state. A question of both great experimental relevance and theoretical interest is what characterizes the dynamical properties at late times after a quench. These can be accessed by experimental probes at finite energies, such as photoemission spectroscopy \cite{Proctor11}. In the first part of this letter we prove quite generally, that dynamical correlations of local operators acting within a given subsystem in the stationary state after a quantum quench are determined by the same distribution function as static correlations. In particular this means that whenever the GGE describes static correlations in the stationary state, it also applies to the dynamics.

Stationary State Dynamics after a Quantum Quench. We consider the following quench protocol. The system is prepared in the ground state $|\Psi_0\rangle$ of a lattice Hamiltonian $H(h_0)$ with local interactions, where $h_0$ is a system parameter such as a magnetic field. At time $t = 0$ we suddenly change $h_0$ to $h$ and the system time evolves unitarily with Hamiltonian $H(h)$ thereafter. We are interested in expectation values of the form $(t_1, \ldots, t_n > 0)$

$$
\langle \Psi_0(t)|O_{1}(t_1) \cdots O_{n}(t_n)|\Psi_0(t)\rangle,
$$

where $O_j$ are local observables. We wish to demonstrate the following. If the stationary state of a quantum many-body system after a quantum quench is described by a density matrix $\rho_{\text{stat}}$ such that for observables $O_j$ acting only within a subsystem $S$ one has

$$
\lim_{t \to \infty} \langle \Psi_0(t)|O_{1} \cdots O_{n}|\Psi_0(t)\rangle = \text{Tr}(\rho_{\text{stat}}O_{1} \cdots O_{n}),
$$

then dynamical correlations are described by the same density matrix, i.e. for $t_1, \ldots, t_n$ fixed we have

$$
\lim_{t \to \infty} \langle \Psi_0(t)|O_{1}(t_1) \cdots O_{n}(t_n)|\Psi_0(t)\rangle = \text{Tr}(\rho_{\text{stat}}O_{1}(t_1) \cdots O_{n}(t_n)).
$$

The proof of this statement is based on the Lieb-Robinson bound \cite{LR} and more specifically the following theorem by Bravyi, Hastings and Verstraete \cite{BH}: let $O_A$ be an operator that differs from the identity only within a local region $A$. Now define the projection of the (non-local) operator $O_A(t)$ to the subsystem $S \supset A$ by

$$
O_A^{(S)}(t) = \frac{\text{tr}_S[O_A(t)] \otimes I_S}{\text{tr}_S[I_S]},
$$

where $I_S$ is the identity in subsystem $S$. The proof is then straightforward: by using the fact that $O_A^{(S)}(t)$ is local in $S$, it follows that

$$
\rho_{\text{stat}}(O_A^{(S)}(t)) = \text{Tr}(\rho_{\text{stat}}O_A^{(S)}(t)) = \text{Tr}(\rho_{\text{stat}}O_A(t)),
$$

and hence

$$
\lim_{t \to \infty} \langle \Psi_0(t)|O_{1}(t_1) \cdots O_{n}(t_n)|\Psi_0(t)\rangle = \text{Tr}(\rho_{\text{stat}}O_{1}(t_1) \cdots O_{n}(t_n)).
$$

This completes the proof.
where $\bar{S}$ is the complement of $S$. If the time evolution is induced by a short-range lattice Hamiltonian, then

$$\|O_A(t) - O_A^{(S)}(t)\| \leq c|A|e^{-\frac{d_{1-t}|t|}{\xi}}, \quad (5)$$

where $\|\cdot\|$ is the operator norm, $v$ is the maximal velocity at which information propagates [24], $d$ is the (smallest) distance between $\bar{S}$ and $A$, $|A|$ is the number of vertices in set $A$, and $\xi$, $c$ positive constants. Assuming the operator $O_2$ to be bounded, $\|O_2\| \leq \kappa$, we therefore have

$$\left|\langle \delta O_1(t_1)O_2(t_2)\rangle_t\right| \leq \|\delta O_1(t_1)O_2(t_2)\| \leq c_1|A|\kappa e^{-\frac{d_{1-t}|t|}{\xi}}, \quad (6)$$

where $\langle \cdot \rangle_t$ denotes expectation value with respect to $|\Psi_0(t)\rangle$ and $\delta O_1(t) = O_1(t) - O_1^{(S)}(t)$.

The first inequality holds because the operator norm is an upper bound for the expectation value on any state, while in the last step we used [5]. Eqn (6) implies that

$$\langle \prod_{j=1}^2 O_j(t_j)\rangle_t = \langle O_1^{(S)}(t_1)O_2^{(S)}(t_2)\rangle_t + a_1(t_1,t_2,t)e^{-\frac{d_{1-t}|t|}{\xi}}, \quad (7)$$

where $a_1(t_1,t_2,t)$ is a bounded function. By repeating the steps leading to (7) for the operators $O_2(t_2)$ we arrive at

$$\langle O_1(t_1)O_2(t_2)\rangle_t = \langle O_1^{(S)}(t_1)O_2^{(S)}(t_2)\rangle_t + \sum_{i=1}^2 a_i(t_1,t_2,t) \exp\left(-\frac{d_{1-t}|t|}{\xi}\right),$$

where $a_2(t_1,t_2,t)$ is another bounded function. We may now use the assumption [2] for the expectation value on the right hand side since all operators act within subsystem $S$.

$$\lim_{t \to \infty} \langle O_1(t_1)O_2(t_2)\rangle_t = \text{Tr}(\rho_{\text{stat}}O_1^{S}(t_1)O_2^{S}(t_2)) + \sum_{i=1}^2 a_i(t_1,t_2)e^{-\frac{d_{1-t}|t|}{\xi}}, \quad (8)$$

where $\lim_{t \to \infty} a_i(t_1,t_2,t) = a_i(t_1,t_2)$ is assumed to exists for simplicity[37]. The chain of inequalities [4] also holds for the average with respect to the density matrix $\rho_{\text{stat}}$, i.e.

$$\text{Tr}(\rho_{\text{stat}}O_1(t_1)O_2(t_2)) = \text{Tr}(\rho_{\text{stat}}O_1^{S}(t_1)O_2^{S}(t_2)) + \sum_{i=1}^2 b_i(t_1,t_2)e^{-\frac{d_{1-t}|t|}{\xi}}, \quad (9)$$

where $b_i(t_1,t_2)$ are bounded functions of $t_{1,2}$.

Finally, combining (8) and (9) and then taking the size of the subsystem $S$ to be infinite we obtain (3) in the case $n = 2$. The generalization to arbitrary $n$ is straightforward.

**Generalized Gibbs ensemble.** We now concentrate on a quantum quench in an integrable model in one dimension with Hamiltonian $H(h) = H_1$ and local conservation laws $I_{n \geq 1}$, i.e. $[I_n, H_1] = 0$. The full (reduced) density matrix of the system (of a subsystem $A$) at time $t$ after the quench is

$$\rho(t) = |\Psi_0(t)\rangle\langle\Psi_0(t)|, \quad \rho_A(t) = \text{Tr}_A(\rho(t)),$$

where $\bar{A}$ is the complement of $A$. It is widely believed, and was shown for quenches of the transverse field in the TFIC in Refs [26, 27], that

$$\lim_{t \to \infty} \rho_A(t) = \text{Tr}_{\bar{A}}(\rho_{\text{GGE}}), \quad (11)$$

where

$$\rho_{\text{GGE}} = \frac{1}{Z_{\text{GGE}}} e^{-\sum_m \lambda_m I_m},$$

is the density matrix of the GGE and $Z_{\text{GGE}}$ ensures the normalization $\text{tr}(\rho_{\text{GGE}}) = 1$. Eqn (11) establishes that all local, equal time correlation functions of a given subsystem in the stationary state are determined by the GGE [12]. Applying our result (3) to the case at hand, we conclude that dynamic correlation functions are also given by the GGE, i.e.

$$\lim_{t \to \infty} \langle \rho_{\text{GGE}}O_1(t_1)\ldots O_n(t_n)\rangle = \text{Tr}(\rho_{\text{GGE}}O_1(t_1)\ldots O_n(t_n)). \quad (13)$$

**Fluctuation Dissipation Relation (FDR).** A key question regarding dynamical properties in the stationary state after a quench is whether a FDR holds [28]. Given the result (13), we can answer this question for cases where the stationary state is either described by a thermal distribution with effective temperature $T_{\text{eff}}$ or by a GGE. In the former case, the standard thermal FDR with temperature $T_{\text{eff}}$ applies. The GGE case is more involved and we turn to it next. The linear response function of observables $A_j$ and $B_l$ acting on sites $j$ and $l$ of a translationally invariant lattice of $L$ sites is

$$\chi_{AB}(\omega, Q) = -\frac{i}{L} \sum_{j,l} \int_0^{\infty} d\tau e^{i\omega\tau} \text{tr}[\rho_{\text{GGE}}[A_j(\tau), B_l]]. \quad (14)$$

On the other hand, the spectral function of the same two observables in the stationary state is given by

$$S_{AB}(\omega, Q) = \frac{1}{L} \sum_{j,l} \int_{-\infty}^{\infty} \frac{d\tau}{2\pi} e^{i\omega\tau} \text{tr}[\rho_{\text{GGE}}A_j(\tau)B_l]. \quad (15)$$

Using a Lehmann representation in terms of Hamiltonian eigenstates it is straightforward to show that

$$-\frac{1}{\pi} \text{Im} \chi_{AB}(\omega, Q) = S_{AB}(\omega, Q) - S_{BA}(\omega, -Q), \quad (16)$$

i.e. the basic form of the FDR holds. However, as was already noted in Ref. [28] for the TFIC, unlike in the thermal (Gibbs) case, the negative frequency part $S_{BA}(\omega, -Q)$ is not related to the positive frequency part by a simple relation of the form $S_{AB}(\omega, -Q) = f(\omega) S_{BA}(\omega, Q)$, where $f(\omega)$ is independent of $A$ and $B$. 


Transverse Field Ising Chain. We now focus on the dynamics after a quantum quench in a particular example, the TFIC described by the Hamiltonian

$$H(h) = -J \sum_{j=1}^{L} \left[ \sigma_j^x \sigma_{j+1}^x + h \sigma_j^z \right],$$

(17)

where $\sigma_j^x$ are the Pauli matrices at site $j$, $J > 0$ and we impose periodic boundary conditions $\sigma_{L+1}^x = \sigma_1^x$. The model (17) is a crucial paradigm of quantum critical behaviour and quantum phase transitions [29]. At zero temperature and in the thermodynamic limit it exhibits ferromagnetic ($h < 1$) and paramagnetic ($h > 1$) phases, separated by a quantum critical point at $h_c = 1$. For $h < 1$ and $L \to \infty$ there are two degenerate ground states. Spontaneous symmetry breaking selects a unique ground state, in which spins align along the $z$-direction. On the other hand, for magnetic fields $h > 1$ the ground state is non-degenerate and, as the magnetic field $h$ is increased, spins align more and more along the $z$-direction.

The order parameter for the quantum phase transition is the ground state expectation value $\langle \sigma_j^z \rangle$. We note that the model (17) is (approximately) realized in systems of cold Rb atoms confined in an optical lattice [30].

Two point dynamical correlation functions are of particular importance due to their relationships to response functions measured in photoemission and scattering experiments. The two-point function of transverse spins $\langle \Psi_0(t) | \sigma_j^x(\tau_1) \sigma_{j+1}^x(\tau_2) | \Psi_0(t) \rangle$ in the TFIC can be calculated by elementary means [31] as it is local in terms of Jordan-Wigner fermions. Our goal is to determine the dynamical order-parameter two-point function

$$\rho^{xx}(\ell, t + \tau_1, t + \tau_2) = \langle \Psi_0(t) | \sigma_{j+1}^x(\tau_1) \sigma_j^x(\tau_2) | \Psi_0(t) \rangle,$$

(18)

after quenching the transverse field at time $t = 0$ from $h_0$ to $h$ for times $\tau_1, \tau_2 \geq 0$. This can be achieved by employing a generalization of the form factor methods recently developed in Ref. [32] to the non-equal-time case, and augmenting the results obtained in this way by exploiting the knowledge of exact limiting behaviours derived in Refs [26, 32]. Our approach is outlined in [33]. For quenches within the ordered phase ($h_0, h < 1$) we obtain for large positive $\ell, t$

$$\rho^{xx}(\ell, t + \tau, t) \simeq C_{FF}(h_0, h) R(\ell, \tau, t),$$

(19)

where

$$R(\ell, \tau, t) = \exp \left[ \int_0^\tau \frac{dk}{\pi} \log \left( \cos \Delta_k \right) \right] \times \min \left\{ \max\{\epsilon'_{h}(k, \tau, \ell), \epsilon''_{h}(k, 2t + \tau)\} \right\},$$

and

$$C_{FF}(h_0, h) = \frac{1 - h_0 + \sqrt{1 - h_0^2}}{2\sqrt{1 - h_0^2}}. \left( \frac{1}{1 - h_0^2} \right).$$

(20)

Here $\epsilon_{h}(k) = 2J \sqrt{1 + h^2 - 2h \cos k}$ is the dispersion relation of elementary excitations of the Hamiltonian $H(h)$.

![FIG. 1: Non-equal-time two point function after a quench in the ordered phase from $h_0 = 1/3$ to $h = 2/3$. The distance and time $T$ are fixed at $\ell = 20$ and $T/T_F = 16/3$ respectively.](image)

For quenches within the disordered phase ($h_0, h > 1$), we obtain for $v_{max}(2t + \tau) > \ell$

$$\rho^{xx}(\ell, t + \tau, t) \simeq h C_{FF}(h_0^{-1}, h^{-1}) F(\ell, \tau, t) R(\ell, \tau, t),$$

(22)

where $R(\ell, \tau, t)$ and $C_{FF}$ are given by [20] and $F(\ell, \tau, t)$ is given by

$$F(\ell, \tau, t) = \int_{-\pi}^{\pi} \frac{dk}{\pi} J_{\ell} e^{itk} \left[ e^{-i\epsilon_{h}(k)} \right] + 2i \tan \left( \frac{\Delta_{h}}{2} \right) \cos \left( \epsilon_{h}(2t + \tau) \right) \text{sgn}(\ell - \epsilon'_{h}(k)).$$

(23)
In the complementary regime \( v_{\text{max}}(2t + \tau) < \ell \) the correlator is exponentially small and the expressions \((22, 23)\) no longer apply. Outside the “light-cone” \( v_{\text{max}} \tau < \ell \) the first contribution in \((23)\) is exponentially small, whereas the second one decays as a power-law. The result \((22)\) is obtained by a generalization of the form factor approach developed in Ref. [32] and is based on an expansion in the density of excitations of the critical state after the quench. Hence it is most accurate for quenches where this density is low and breaks down for quenches from/to the quantum critical point. In Figs 2 and 3 we compare the asymptotic result \((19)\) to numerics obtained in the way described above. The agreement for the chosen set of parameters (\( \ell = 30, h_0 = 2, h = 3 \) and \( T/t_F = 16/3 \)) is seen to be excellent. The value of \( \rho^{xx}(\ell, t + \tau, t - \tau) \) at \( \tau = 0 \) equals the known equal-time correlator at time \( T \) after the quench \((32)\), which is small in the case considered. The correlator remains largely unchanged up to a horizon at \( \tau = t_F/2 \) (corresponding to \( t = t_F/2 \) in \((19)\)), and for times \( \tau > t_F \) exhibits an oscillatory \( \tau^{-1/2} \) power-law decay. We note that the result \((19, 21)\) can be obtained in an alternative way by generalizing the semiclassical approach of Ref. [35] (see also [13, 29]) to the non-equal time case, and then elevating it using exact limiting results of Refs [26, 32]. While this method fails to reproduce the result for quenches in the disordered phase outside the light-cone, i.e. \( v_{\text{max}} \tau < \ell \), it provides a physical picture. The behaviour is similar to the finite temperature case \([29]\) and for \( h_0, h < 1 \) can be understood in terms of classical motion of domain walls. For \( h_0, h > 1 \) (and within the light-cone), quantum fluctuations (associated with the function \( F \) in \((23)\)) give rise to the oscillatory behaviour seen in Fig. 3 while relaxation occurs at longer scales and is again driven by classical motion of particles (spin flips) \([29]\). A simple picture emerges when we Fourier transform \( \rho^{xx}(\ell, t + \tau, t) \) at \( t \to \infty \) for small quenches. As a function of \( \omega \) for fixed \( q \) the resulting “dynamical structure factor” for quenches within the disordered phase is dominated by a narrow, asymmetric peak around \( \omega = \varepsilon_h(q) \), while for \( h_0, h < 1 \) we observe a broadening of the \( \delta \)-function peak associated with the ferromagnetic order in the initial state. Both of these are qualitatively similar to the finite-T equilibrium response \([20, 35]\). Having established in the first part of this work that the \( t \to \infty \) limit of \( \rho^{xx}(\ell, t + \tau_1, t + \tau_2) \) is described by the GGE, an important question is how quickly this limiting behaviour is approached. It follows from \((19, 21)\) that for quenches within the ordered (disordered) phase the limiting value for fixed \( \tau_{1,2} \) and \( \ell \) is approached as a \( t^{-3} (t^{-3/2}) \) power law.

**Conclusions.** We have considered dynamical correlation functions of local observables after a quantum quench. We have shown, that dynamical correlators of local observables in the stationary state are governed by the same ensemble that describes static correlations. For quenches in the TFIC this implies that they are given by a GGE, for which the basic form of the fluctuation dissipation theorem holds. We have obtained explicit expressions for the time evolution of dynamic order parameter correlators after a quench in the TFIC.

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Supplementary Material

The dynamical two-point functions \[ H(h)|k_1,\ldots,k_n\rangle_a = \left[ \sum_{j=1}^{n} \varepsilon_h(k_j) \right]|k_1,\ldots,k_n\rangle_a, \]
\[ P|k_1,\ldots,k_n\rangle_a = \left[ \sum_{j=1}^{n} k_j \right]|k_1,\ldots,k_n\rangle_a, \]
are determined by a generalization of the form factor approach recently developed in Ref. \[22\]. The latter is based on a Lehmann representation of two-point functions in terms of simultaneous eigenstates of the momentum operator \( P \) and the post-quench Hamiltonian \( H(h) \)

\[ H(h)|k_1,\ldots,k_n\rangle_a = \left[ \sum_{j=1}^{n} \varepsilon_h(k_j) \right]|k_1,\ldots,k_n\rangle_a, \]
\[ P|k_1,\ldots,k_n\rangle_a = \left[ \sum_{j=1}^{n} k_j \right]|k_1,\ldots,k_n\rangle_a, \]
where \( a = \mathrm{R}, \mathrm{NS} \) correspond to periodic/antiperiodic boundary conditions on the Jordan-Wigner fermions \[22\]. For a quench within the disordered phase the state \( |\Psi(t)\rangle \) has the following representation in a large, but finite volume \( L \)

\[ |\Psi(t)\rangle = \frac{|B(t)\rangle_{\mathrm{NS}}}{\sqrt{\langle B(t)|B(t)\rangle_{\mathrm{NS}}}}, \]

where

\[ |B(t)\rangle_{\mathrm{NS}} = \sum_{n=0}^{\infty} \frac{i^n}{n!} \sum_{p_1 \ldots p_n \in \mathrm{NS}} \prod_{j=1}^{n} K(p_j)e^{-2\varepsilon_h(p_j)t} \times | - p_1, p_1, \ldots, - p_n, p_n \rangle_{\mathrm{NS}}, \]

\[ K(k) = \frac{\sin(k)(h_0 - h)}{2\sqrt{\xi_h(k)\xi_h(k)}} + 1 + hh_0 - (h + h_0)\cos(k). \]

The function \( K(p) \) is related to the quantum cos(\(\Delta_p\)) defined in the main text by \( K(p) = \tan(\Delta_p/2) \). The dynamical order parameter two-point function

\[ \rho^{\alpha\beta}(t, t + \tau_1, t + \tau_2) = \frac{\langle B(t)\sigma_m^x(t_1)\sigma_m^x(t_2)|B(t)\rangle_{\mathrm{NS}}}{\langle B|B\rangle_{\mathrm{NS}}}. \]
has the following Lehmann representation

\[
\begin{align*}
\text{NS}(B(t)|\sigma^x_{\ell+m}(\tau_1)\sigma^x_{m}(\tau_2)|B(t))_{\text{NS}} &= \sum_{m,n=0}^{\infty} \frac{i^{n-m}}{n!m!} \sum_{0<q_1,\ldots,q_n \in \mathbb{NS}} \prod_{j=1}^{n} K(p_j) e^{-2i(t+\tau_2)\epsilon_h(p_j)} \left[ \prod_{l=1}^{m} K(k_l) e^{2i(t+\tau_1)\epsilon_h(k_l)} \right] \\
&\times \sum_{s=0}^{\infty} \sum_{p_1,\ldots,p_s \in \mathbb{R}} \prod_{r=1}^{s} e^{i(\tau_2-\tau_1)\epsilon_h(q_r)+iq_r\ell} s! \langle k_m,-k_m,\ldots,k_1-k_1|\sigma^x_{m}|q_1,\ldots,q_s \rangle \\
&\times \langle q_s,\ldots,q_1|\sigma^x_{m}|-p_1,p_1,\ldots,-p_n,p_n \rangle,
\end{align*}
\]

The form factors

\[
\langle k_m,-k_m,\ldots,k_1-k_1|\sigma^x_{m}|q_1,\ldots,q_s \rangle
\]

are known exactly [34], see eqns (109)-(111) of Ref. [32]. The leading behaviour of (29) is evaluated by considering it as a formal expansion in powers of the function \( K(p) \). As shown in Ref. [32] this corresponds to an expansion, where the small parameter is the density of excitations of the post-quench Hamiltonian \( H(h) \) in the initial state \( |\Psi_0(0)\rangle \). We determine the dominant contributions at large \( \ell, t \) and \( |\tau_1-\tau_2| \) to (29) for a given order in the formal expansion in powers of \( K(p) \), and then sum these to all orders. The structure of this calculation is similar to the equal time case \((\tau_1 = \tau_2)\) considered in Ref. [32], but the details differ substantially and will be reported elsewhere. The result of the form factor calculation for quenches within the disordered phase is

\[
\rho^{xx}(\ell, t+\tau, t) \simeq 2J\sqrt{h(h^2-1)} \frac{i}{2} F(\ell, \tau, t) R_0(\ell, \tau, t),
\]

where the function \( F \) is given in (23) and

\[
R_0(\ell, \tau, t) = \exp\left[-2 \int_0^{\pi} \frac{dk}{\pi} K^2(k) \times \min\left\{ \max\{\epsilon'_h(k)\tau, \ell\}, \epsilon'_h(k)(2t+\tau) \right\} \right].
\]

We now use that the general structure of the resummation for \( \tau_{1,2} \neq 0 \) is the same as for \( \tau_{1,2} = 0 \). This allows us to go beyond the low-density expansion by exploiting results obtained in Refs [26, 32] for \( \tau_1 = \tau_2 = 0 \) by means of determinant techniques. In this way we arrive at eqn (22). Quenches within the ordered phase are analyzed in the same way.