A New Data-Driven Approach of Reference Shaping

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Abstract: In this paper, we propose a data-driven method of reference shaping to improve the tracking performance of uncertain linear systems. The proposed method can be implemented on a system based on its input-output data; the finite-time $L_2$-norm of the tracking error, estimated using the data, can be minimized. Moreover, the proposed algorithm can be extended for cases where the control inputs are constrained owing to actuation limits. The effectiveness of the proposed method is demonstrated through a numerical simulation and an experiment conducted using the cart system.

Keywords: Data-Driven Control, Reference Shaping, Input Constraints, Reference Governor

1. INTRODUCTION

Most of the real control systems, such as chemical plants and power grids, can undergo variations in their dynamic characteristics owing to failures or equipment modifications; see Gertler (1988); Mei et al. (2011). When the control objectives cannot be achieved due to such changes, one of the solutions is to appropriately redesign the controllers of the plant. However, this redesign requires stopping the operations of the plant, and hence, this approach is not very practical in terms of operating costs. Therefore, operators need another approach to avoid the stoppage of operations while improving the control performance.

One way to achieve this objective is to shape an external reference signal such that the control performance is improved. This approach is called reference shaping in literature such as Boettcher et al. (2011); Suzuki and Sugie (2008). In Boettcher et al. (2011), a method has been proposed for designing an optimal reference signal by solving a convex optimization problem. In Suzuki and Sugie (2008), an offline shaping algorithm was proposed based on model predictive control theory in literature, e.g., Allgöwer and Zheng (2012). Instead of offline designs of reference signals, for shaping a reference to a desired one in real-time, the design of a reference governor, which is a dynamical compensator added at the top of the control system, has been proposed in literature Angeli et al. (2001); Bemporad (1998); Gilbert et al. (1995); Gilbert and Kolmanovskiy (2002). These designs require full knowledge of the system dynamics. Thus, owing to the massive complexity of real control systems, such model-based approaches are not very practical.

Recently, data-driven control in which the controllers are redesigned using only operational data, without utilizing any system models, is receiving attention. Many studies on reinforcement learning (RL)-based control are emerging, e.g., Vrabie et al. (2009); Jiang and Jiang (2014). However, these RL-based approaches are limited to state-feedback control. In contrast, in literature Campi et al. (2002); Campi and Savaresi (2006); Kaneko (2013), methods for redesigning an output-feedback controller using a single input-output data have been proposed. These approaches enable one-shot learning of a feedback controller. However, to the best of our knowledge, data-driven approaches of reference shaping have not yet been proposed.

In this paper, we propose a new data-driven method of reference shaping for improving the tracking performance of unknown linear systems. The proposed method comprises two steps. First, by solving an optimization problem formulated by the single input-output data, we hypothetically redesign an output-feedback controller that can improve the tracking performance. Note that this redesign is hypothetical; the designed controller is not implemented in the plant. Next, we shape the reference signal offline, such that the control input when the reference is applied coincides with the ideal input generated by the hypothetical controller. This proposed method can be conducted without knowing the system model, and the finite-time $L_2$-norm of the tracking error, estimated using the data, can be minimized. Moreover, the proposed algorithm can be extended for cases where the control inputs are constrained owing to actuation limits. The proposed methods are expected to be useful for real, complex control systems that facilitate modifications of system components. The effectiveness of the proposed methods is investigated through a numerical simulation and an experiment conducted using a cart system.

Notation: Given a transfer function $G(s)$ we omit the term $(s)$ when no confusion occurs. Given an input signal $u(t)$ and a system whose transfer function is $G$, we denote the output in time-domain by $Gu(t)$. The set of proper and stable transfer functions is denoted by $\mathcal{RH}_\infty$. Given $\tau > 0$, the finite-time $L_2$-norm of a signal $x(t)$ is defined as
Given a reference signal $r$ in Fig. 1 and a desired closed-loop transfer function $T_d \in \mathcal{RH}_\infty$, let the output $y$ follow a desired trajectory $T_d r$, i.e., $y \approx T_d r$. Now, we suppose that the dynamics of $G$ is drastically changed from the previous one owing to failures or equipment modifications. As a result, the output $y$ no longer follows $T_d r$. One way to improve the tracking performance is to redesign $C$ in Fig. 1 such that $y \approx T_d r$ is satisfied again. However, when control systems are in operation, this redesign requires stopping the operations of the plant, and hence, this approach is not very practical in terms of operating costs. Therefore, instead of redesigning $C$, we aim to shape the reference signal $r$ in Fig. 1 so that $y$ follows $T_d r$ as much as possible. This problem is formulated as follows.

**Problem 1.** Consider a closed-loop system in Fig. 1. Let Assumptions 1-3 be satisfied. Given $T_d \in \mathcal{RH}_\infty$ and $r$, find $r^*$ such that

$$y^* := \frac{GC}{1 + GC}r^*$$

is close to $T_d r$ as much as possible.

If the model of $G$ was known, one can construct $r^*$ by taking $r^* = T_d [(GC/(1 + GC))^{-1} r]$. However, since $G$ is completely unknown, this model-based approach no longer applies. Instead a data-driven approach needs to be developed, which is shown in the next section.

### 3. PROPOSED METHOD

We show a method to solve Problem 1 by using one pair of input-output data of $r$ and $y$, denoted by $\{r_0, y_0\}$. Throughout this paper, we assume that $r_0, y_0$ and the desired output trajectory, denoted by $y_d$, are finite-time and have an identical time length denoted by $\tau$. Then, $r_0, y_0, y_d$ satisfy

$$y_0 = \frac{GC}{1 + GC}r_0, \quad y_d = T_d r_0$$

where $T_d$ is a desired closed-loop transfer function. The proposed method consists of the following two steps:

i) Design a new feedback controller $C^*$ virtually so that the closed-loop dynamics is close to $T_d$.

ii) Shape the reference signal such that the actual control input coincides with the one generated by the virtual controller.

Note that the controller $C^*$ is not implemented to the actual plant. We will use this as an intermediate component for the reference shaping. Before stating the detail of the step i), in the next subsection we show how the step ii) can be carried out.

#### 3.1 Reference Shaping based on Virtual Controller

For our derivation, we hypothetically assume that the virtual controller design is done perfectly, i.e., we have $C^*$ satisfying

$$T_d = \frac{GC^*}{1 + GC^*}.$$  (3)

The control input generated by this $C^*$ and the original controller $C$ are defined as

$$u^* := \frac{C^*}{1 + GC^*}r_0, \quad u_0 := \frac{C}{1 + GC}r_0.$$  (4)

respectively. Hence, we have

$$u^* - u_0 = \frac{C^* - C}{(1 + GC^*)(1 + GC)}r_0.$$  (5)

On the other hand, it follows from (1) that

$$u^* = \frac{C}{1 + GC^*}r^*.$$  (6)

The equation (6) implies that $u^*$, which is the control input generated by the virtual controller, coincides with the input when $r^*$ is applied to the closed-loop system in Fig. 1. By substituting (6) and the second equation in (4) into (5), the relation (5) can be equivalently written as

$$r^* - r_0 = \frac{C^* - C}{(1 + GC^*)(1 + GC)}C^{-1} r_0 = \frac{C^* - C}{(1 + GC^*)C}r_0.$$  (7)

By substituting (3) into this equation, we have

$$r^* = r_0 + (1 - T_d)\frac{C^* - C}{C}r_0.$$  (8)

In (8), the second term in the right-hand side represents the compensation signal determined by the virtual controller $C^*$. The following theorem straightforwardly follows the above discussion.

**Theorem 1.** Consider Problem 1 and let $r^*$ be given by (8). Then, $y^*$ in (1) satisfies $y^*(t) \equiv y_d(t)$ for $t \in [0, \tau]$.

**Proof:** The claim follows from the above discussion.

Theorem 1 implies that the shaped reference signal $r^*$ in (8) is an ideal solution of Problem 1. It should be noted that the unknown system dynamics $G$ does not appear in (8). Thus, we can compute $r^*$ without knowing the system model.
3.2 Data-Driven Design of Virtual Controller

We show how to design the virtual controller $C^*$ in (8) by using the measured data $\{r_0, y_0\}$. In the previous subsection, we have assumed that $C^*$ satisfies (3). In reality, however, it may be very difficult to find $C^*$ which satisfies this relation exactly. Thus, for designing $C^*$, we consider an evaluation function

$$J := \|y_d - \frac{GC}{1 + GC} r^*\|_{L_2, \tau}. \quad (9)$$

From simple calculation by using (2), $J$ can be written as

$$J = \|y_d - \frac{GC}{1 + GC} \left(1 + (1 - T_d) \frac{C^* - C}{C}\right) r_0\|_{L_2, \tau}$$

$$= \|y_d - \left(T_d y_0 + (1 - T_d) \frac{C^*}{C} y_0\right)\|_{L_2, \tau}. \quad (10)$$

It should be emphasized that all of $y_d, T_d, C$ and $y_0$ in (10) are known quantities. Thus, we can carry out the minimization of $J$ over $C^*$. To solve this minimization problem in a tractable manner, a parametrization of $C^*$ described below would be effective. Let $\rho \in \mathbb{R}^n$ and $C^*$ be parametrized as

$$C^*(s; \rho) = \sum_{k=1}^n \rho_k c_k(s) \quad (11)$$

where $\rho_k$ is the $k$-th element of $\rho$ and $c_k(s)$ is a given stable transfer function. Note that $c_k(s)$ can be non-proper because the signal shaping (8) can be performed in offline. Using the notations

$$q_0 := y_d - T_d y_0, \quad q_k := -(1 - T_d) \frac{c_k}{C} y_0. \quad (12)$$

the equation (10) can be rewritten as

$$J = \|q_0 + \sum_{k=1}^n \rho_k q_k\|_{L_2, \tau}. \quad (13)$$

Clearly, the minimization problem $\min_\rho J$ can be easily solved by a standard least square method.

The proposed algorithm is summarized as follows.

**Algorithm 1: Reference shaping**

1. Collect input-output data $\{r_0, y_0\}$.
2. Find $C^*$ minimizing $J$ in (10).
3. Compute $r^*$ in (8).

**Remark 1.** From a different viewpoint, the evaluation function $J$ in (10) is derived in the third author’s recent paper; Ikezaki and Kaneko (2019).

3.3 Reference Governor Design

Algorithm 1 can be computationally heavy when $r^*$ is very long (i.e. $\tau$ is long). An alternative way instead of this step is to implement a dynamical compensator so-called reference governor at the top of the control system,\footnote{When $c_k(s)$ is non-proper, the computation of $r^*$ in (8) is carried out for $t \in [0, \tau - \epsilon]$ where $\epsilon > 0$ is a given small value.}

![Fig. 2. The closed-loop system with a reference governor as shown in Fig. 2. From (8), one can expect that the reference governor designed as $R = (1 - T_d) \frac{C^*}{C} + T_d$ improves tracking performance. This can be shown as follows.

Let the transfer function from $r$ to $y$ in Fig. 2 be denoted as $T^* := R \frac{GC}{1 + GC}$. Note that if $C^*$ is stable, then $T^*$ is stable under Assumptions 1-3. The stability of $C^*$ can be easily guaranteed by parametrizing $C^*$ with stable functions, as described in Section 3.2. In view of this, we assume that $C^*$ is stable. Let $C_d$ be a desired feedback controller satisfying

$$T_d = \frac{GC_d}{1 + GC_d}. \quad (16)$$

Note that this ideal controller is, in general, different from $C^*$ obtained by minimizing $J$ in (10). Let

$$\Delta := C^* - C_d. \quad (17)$$

Using these notations, $T^*$ in (15) can be written as

$$T^* := \frac{GC}{1 + GC} \left(1 + (1 - T_d) \frac{C_d + \Delta}{C}\right)$$

$$= \frac{GC}{1 + GC} T_d \left(1 + \frac{1}{GC_d} \frac{C_d + \Delta}{C}\right)$$

$$= \frac{GC}{1 + GC} T_d \left(1 + \frac{1}{GC} + \frac{1}{GCC_d} \Delta\right)$$

$$= T_d + (1 - T_d) S G C_d \Delta$$ \quad (18)$$

where $S := 1/(1 + GC)$. From this error analysis, we can see that, as $C^*$ is closer to $C_d$, the tracking performance of the entire control system with $R$ in (14) is theoretically shown to improve. Later, we will demonstrate this through a numerical simulation.

4. EXTENSION TO INPUT-CONSTRAINED SYSTEMS

In this section, we propose a reference shaping for input-constrained systems. We consider a closed-loop system as shown in Fig. 1. Let Assumptions 1-3 be satisfied. Moreover, we assume that $u$ in Fig. 1 must satisfy

$$u \leq u(t) \leq \overline{u}, \quad \forall t \geq 0 \quad (19)$$

where $u$ and $\overline{u}$ are given and known constant values. In these settings, we design $r^*$ so that $y^*$ in (1) is close to $y_d$ in (2) as much as possible while satisfying the input constraint (19).

The control input when the updated reference signal $r^*$ in (8) is applied can be written as

$$u^* = \left(T_d + (1 - T_d) \frac{C^*}{C}\right) u_0. \quad (20)$$
Hence, the constraint (19) on $u^*$ over a finite-time horizon $t \in [0, \tau]$, where $\tau$ is defined as the time length of the measured data, can be equivalently written as

$$u^* \leq \left( T_d + (1 - T_d) \frac{C^*}{C} \right) u_0 \leq \bar{u}. \tag{21}$$

As long as $y^*(\tau)$ is close to $y_d(\tau)$, one can expect that $u^*(t)$ for $t > \tau$ satisfies the original input constraint (19). It should be emphasized that the constraint (21) is convex with respect to $C^*$. When the controller $C^*$ is parametrized as (11), the constraint (21) can be equivalently written as a linear constraint

$$A\rho \leq b \tag{22}$$

where $A := [-1, 1]^T \otimes [a_1, \ldots, a_n]$, $b := [u^T, \bar{u}^T]^T$, $\otimes$ is the Kronecker product, and

$$a_k := (1 - T_d)\frac{c_k}{C}u_0, \quad \bar{b} := -\bar{u} + T_d u_0, \quad \bar{b} := \bar{u} - T_d u_0.$$

The proposed reference shaping under input constraints can be summarized as follows.

**Algorithm 2: Reference shaping under input constraints**

1. Collect input-output data $(r_0, y_0)$.
2. Find $C^*$ minimizing $J$ in (10) while satisfying (21).
3. Compute $r^*$ in (8).

We end this section by shortly describing the reference governor design under the input constraint (19). Even though the control input is constrained, the reference governor $R$ in Fig. 2 can be constructed as (14) as long as $C^*$ is designed.

**Remark 2.** While the proposed method above can handle a more generic input constraint such as $T_d u(t) \leq \nu(t)$ where $T$ and $\nu(t)$ are a known linear dynamical system and signal respectively, for simplicity, we have considered (19) as an input constraint to be imposed. Similarly, we can also consider an output constraint.

5. INVESTIGATION OF PROPOSED ALGORITHMS

5.1 Numerical Simulation

We show the effectiveness of the proposed methods through a numerical simulation. Consider a control system in Fig. 1 with

$$G = \frac{12s + 8}{20s^4 + 113s^3 + 147s^2 + 62s + 8}, \quad C = 3 + \frac{1}{2s}.$$  

Let

$$T_d = \frac{27}{s^4 + 9s^2 + 27s + 27}$$

and the original reference signal $r_0$ be a step signal. In Fig. 3, the red dotted and blue solid lines show the actual output $y_d$ and the desired trajectory $y_d$ in (2), respectively. Clearly, $y_0$ does not follow $y_d$.

To improve the tracking performance, we apply Algorithm 1 to the control system. At Step 2 in the algorithm, we parametrize $C^*$ as a standard PID controller described as

$$C^*(s; \rho) = \rho_1 + \frac{\rho_2}{s} + \rho_3 s.$$  \tag{23}

Clearly, this controller can be written as (11) with $n = 3$. Next, we compute $q_k$ in (12), and subsequently, we minimize $J$ in (13) over $\rho \in \mathbb{R}^3$ by a standard least square method. Then $\rho^* = [6.1288, 0.9878, 6.5459]$. Using the virtual controller $C^*$ with this optimized parameter,
we design \( r^* \) in (8). In Fig. 3, the green chained line shows the output when \( r^* \) is applied to the system. By comparing the lines in this figure, we can see that the tracking performance is improved by shaping the reference signal.

To compare the tracking performance in the frequency domain, we design a reference governor \( R \) in (14), where \( C^* \) is chosen as (23) with the aforementioned optimized parameter \( \rho^* \). In Fig. 4, blue solid, red dotted, and green chained lines show the Bode diagrams of \( T_d, GC/(1+GC) \), and \( T^* \) in (15), respectively. We can see that the addition of the reference governor \( R \) makes the whole control system dynamics close to the desired one.

In Fig. 5 the green chained line shows \( u^* \) in (6), which is the control input when \( r^* \) is applied to the system in Fig. 1. The peak value of \( u^* \) is 71.6. One may need to suppress this peak to protect the actuator. To this end, we consider an input constraint (19) with \( \bar{u} = -25 \) and \( \underline{u} = 25 \). Since the virtual controller is parametrized as (23), this input constraint can be described as a linear constraint (21). Then the problem of minimizing \( J \) in (10) while satisfying (21) can be solved by the quadratic programming. We denote the obtained virtual controller and reference signal by \( u^*_{\text{inp}} \) and \( r^*_{\text{inp}} \). Let \( u_{\text{inp}} \) and \( y_{\text{inp}} \) denote the control input and output when \( r^*_{\text{inp}} \) is applied to the system, respectively. The control input \( u_{\text{inp}} \) is depicted by the purple solid line in Fig. 5. Moreover, in Fig. 6 we show \( y_0 \) in (2), \( y^* \), and \( y^*_{\text{inp}} \) by the red dotted, green chained, and purple solid lines, respectively. From these figures we can see that \( r^*_{\text{inp}} \) achieves a better tracking performance than \( r_0 \) while satisfying the input constraint.

5.2 Experimental Investigation

Next we investigate the effectiveness of Algorithm 2 through experiment. We use a cart control system as shown in Fig. 7. The cart on the belt can be moved by applying a voltage to the motor. The position of the cart can be measured by the potentiometer. Let \( u \) (V) and \( y \) (m) denote the applied voltage and the measured position, respectively. Let the sampling interval for both of the control and measurement be 0.01 (sec). A discrete-time controller, which is implemented to the motor driver, controls the motor by feeding back the measured position through the DSP board, so that the trajectory of the cart position follows a desired one. Let the controller be given as

\[
C_{\text{dis}}(z; \rho) = \rho_1 + \rho_2 \frac{0.01}{z - 1} + \rho_3 \frac{z - 1}{0.01 + 10(z - 1)}. \tag{24}
\]

We regard the triple of this \( C_{\text{dis}}(z; \rho) \), a sampler, and a zero-order holder as the continuous-time controller \( C(s; \rho) \) in Fig. 1.

Let \( r_0 \) be a step signal so that the cart moves ahead by 0.15(m), and \( \rho = [0.1, 0.1, 0.08] \) in (24). In Fig. 8, the purple solid and red dotted lines show the reference signal.
Fig. 10. Input signal generated by $C^*$. $r_0$ and the measured output $y_0$, respectively. This figure shows a poor tracking performance despite the position controller $C$ is actuated.

To improve the performance we consider shaping the reference signal by using the data $\{r_0, y_0\}$. Let

$$T_d = \frac{4}{s + 4}. \quad (25)$$

To protect the motor, we need to make the magnitude of the applied voltage be less than 1.5(V), i.e., (19) with $\rho = -1.5(\text{V})$ and $\pi = 1.5(\text{V})$. We parametrize $C^*$ as the form of (24). In these settings, by applying Algorithm 2 to the cart control system, we obtain an optimal parameter $\rho^* = [1.47, -0.0957, -9.57]$. Based on this, we design $r^*$ in (8). In Fig. 9, the blue solid, red dotted, and green chained lines show the desired trajectory $y_d$, the initial output $y_0$, and the shaped output $y^*$ in (1), respectively. Furthermore, Fig. 10 shows the control input $u$ when $r^*$ is applied to the cart control system. These results imply that the proposed algorithm can be effective for real physical systems under input constraints.

6. CONCLUSION

In this paper, we have proposed a data-driven approach of reference shaping to improve tracking performance of uncertain linear systems. The proposed method can be implemented on a system based on its input-output data; the finite-time $L_2$-norm of the tracking error, estimated by using the data, can be minimized. Moreover, the proposed algorithm can be extended for cases where the control inputs are constrained owing to actuation limits. The effectiveness of the proposed methods has been shown through a numerical simulation and an experiment of a cart system. The proposed methods are expected to be useful for real, complex control systems that facilitate modifications of system components. The experimental result implies that the proposed approaches are to what extent robust against noises. Extension to noisy cases is one of future works.

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