Comments on “Ohm’s Law Survives to the Atomic Scale” by Weber et al.

Mukunda P. Das1* and Frederick Green2

1 Department of Theoretical Physics, Research School of Physics and Engineering, The Australian National University, Canberra, ACT 0200, Australia.
2 School of Physics, The University of New South Wales, Sydney, NSW 2052, Australia.
* Corresponding author: mukunda.das@anu.edu.au

Abstract. The recent article "Ohm’s Law Survives to the Atomic Scale" by Weber et al. 1 reveals ohmic transport in quantized P-in-Si wires. We argue that their results have two main deficiencies: (a) the interpretation of conductance data is inadequate for serious systematics; (b) metallic-like structures hold few implications for quantum computing 2.

In their recent paper Weber et al. 1 claim novel observations on the persistence of ohmic behavior in embedded P-in-Si wires, effectively fabricated at atomic scales. To further promote the sophisticated and quite delicate materials processing achieved in this work, it would be important to retain certain theoretical issues – issues of principle – clearly to the fore. To that end we offer below several comments on the authors’ theoretical interpretation of their observed results.

1. The statement that “The unambiguous demonstration of Ohmic scaling is a constant resistivity $\rho_W = R_W(A_e/L)$ independent of geometric variables, such as wire length or width” does not appear thoroughly conclusive from the published data. Table 1 and Fig 1E contain the main results of this work. In Table 1, $\rho_W$, which by convention defines an intrinsic material constant, varies between 0.10 and 0.43 m$\Omega$cm. Fig. 1E presents quite sparse and scattered data for the product $\rho_W w$, with $w$ the nominal wire width, in which the linear interpolation fails to cut the stated error bars. The plot is log-log; once presented on the linear scale, the real magnitudes of these discrepancies are much more evident. It is hard to reasonably describe the claimed ohmic relationship as “unambiguous”.

2. The STM-patterned P structure in a Si matrix constitutes a quasi-one-dimensional (1D) wire of high (metallic) carrier density. In Fig. 2C, the presented data for $\rho_W$ are said to correspond to bulk electron densities of $10^{21}$ cm$^{-3}$. The physics in question here concerns one-dimensional transport. Thus, what is the physical relevance of a bulk electronic density in the 1D context? The current-carrying capacity of P-in-Si nanowires is further compared with a nominal Cu equivalent, scaled to comparable dimensions. For Cu at $T = 4.2$K, we have $\rho_W \sim 1 \mu \Omega$cm while, for the P-in-Si wires, $\rho_W$ comes out $\sim 0.1 \mu \Omega$cm This is a difference of two orders of magnitude.

3. The (finite) resistance of these P-in-Si wires is in the highly diffusive regime and, hence, far from ballistic ideality. The mean free path is $l \sim 81$nm. According to the Landauer formula in its diffusive elastic regime $3$, $R_{\text{calc}} = \frac{h}{2eN}(1 + L/l)$. 


This is a two-terminal formula, which consequently includes both the true wire and its contact resistances \[3\]. To cite just one example: wire W1 has \(L = 312\text{nm}, w = 11.0\text{nm}\) and \(N \approx 20\), yielding \(R_{\text{calc}} = 25.8\text{k}\Omega\). This presents a large resistance leading to correspondingly large values of local Joule heating, at least in the voltage ranges adopted for industrial-standard Si microelectronics.

4. The measured resistance, as stated in the caption of Fig.1E, is a four-terminal result; that is, it should be the intrinsic wire resistance, sans any contact resistance at all. On the other hand the formula as applied for calculated resistance, quoted from Weber et al. in our preceding Point 3, is a two-terminal result that includes the contact resistance \[3\].

The difference between these two distinct resistances in the ideal metallic limit can be drastic, as attested by the definitive 1D wire measurements of dePicciotto et al. \[4\]. In the work of Weber et al. the contact resistance works out at \(12.9/N\text{k}\Omega\). This is much smaller than for the wire itself, which is highly resistive. One can safely conclude that the P-in-Si wire dominates the overall resistive behaviour. In other words, it behaves ohmically because it is an unexceptionally ohmic structure from the start; from the operational standpoint, its “atomic” nature is irrelevant (all metallic resistance is the direct outcome of strong processes at the quantum level).

Aside from the above, there is a technical confusion. The experimental \(R_W\) is a four-terminal result; but \(R_{\text{calc}}\) follows the two-terminal formula (see their Table 1). The authors claim good agreement (including wires 4 and 5) between these two structurally very different theoretical quantities. This means that the influence of the contacts (within the Landauer formula) ought to be negligible. But for samples 4 and 5, there is a large mismatch; thus the effect of the contact resistances becomes hard to quantify.

Another feature to note, and hopefully understand, is that the measured wire resistances are higher than the calculated resistances for wire samples 1, 4 and 5 but lower than those for wires 2 and 3. In our view the models used for analysis of the observations are inconsistent and inadequate.

5. Finite resistance necessarily entails power dissipation. Here, the heat production would be \(\sim 100\) times that of a Cu channel of equivalent geometry. The paper offers no analysis of the practical implications for quantum-computer architectures, where the extreme feature densities of such P-in-Si metallization would exacerbate (as a notorious side-effect of Moore’s Law) the problem of highly localized heat production and its removal. An ohmic wire – however made – remains highly dissipative and thus a problematic destroyer of quantum entanglement were it to be seriously considered, as envisaged, for interconnects in quantum computing.

References

[1] B. Weber et al., Science 335, 64 (2012).
[2] This Comment has been released on the Science web site at http://comments.sciencemag.org/content/10.1126/science.1214319#comments
[3] M. J. M. de Jong and C. W. J. Beenakker, Phys. Rev. B 51, 16867 (1995).
[4] R. dePicciotto, H. L. Stormer, L. N. Pfeiffer, K. W. Baldwin and K. W. West, Nature 411, 51 (2001).