Dynamical violation of scale invariance and the dilaton in a cold Fermi gas

Gordon W. Semenoff and Fei Zhou

Department of Physics and Astronomy, University of British Columbia, 6224 Agricultural Road, Vancouver, British Columbia, Canada V6T 1Z1

We use the large N approximation to find an exotic phase of a cold, two-dimensional, N-component Fermi gas which exhibits dynamically broken approximate scale symmetry. We identify a particular weakly damped collective excitation as the dilaton, the pseudo-Goldstone boson associated with the broken approximate scale symmetry. We argue that the symmetry breaking phase is stable for a range of parameters of the theory and there is a fluctuation induced first order quantum phase transition between the normal and the scale symmetry breaking phases which can be driven by tuning the chemical potential. We find that the compressibility of the gas at its lower critical density is anomalously large, 2N times that of a perfect gas at the same density.

There has been some interest in approximately scale invariant relativistic quantum field theories where the scale symmetry can be spontaneously broken, resulting in the appearance of a light dilaton as a pseudo-Goldstone boson. This idea has been a theme in walking technicolor theories for dynamical breaking of the symmetry in the standard model [1]-[2] as well as in the idea that the standard model Higgs field could itself be a light dilaton [3]-[4]. It also appears in the tricritical O(N) model in three dimensions where the large N limit has the Bardeen-Moshe-Bander phase [5] which has spontaneously broken scale invariance and a massless dilaton. The latter is unstable when the infinite N limit is relaxed. The dilaton becomes a tachyon [6]. It is interesting to ask whether there are other physically plausible contexts where approximate scale symmetry is dynamically broken, the symmetry breaking phase is stable and where theoretical analysis would be under good analytic control. In the following we shall point out that the large N limit of a cold, non-relativistic, N-component, two-dimensional Fermi gas with an attractive delta-function two-body interaction could be such a system. We shall study its quantum critical behaviour at the phase transition between a phase where it acquires a density of particles, and in which the density itself acts as an order parameter, and a phase where the density goes to zero. This phase boundary becomes a line of first order phase transitions. This model exhibits an example of the Coleman-Weinberg mechanism for dynamical symmetry breaking [7]. What is more, it has approximate scale symmetry and we shall identify the pseudo-goldstone mode corresponding to the dilaton.

Normally, a cold Fermi gas with an attractive interaction would be expected to be a superfluid with a condensate of Cooper pairs. Indeed, this should be the fate of the system that we discuss. However, we shall find that, at large N, the pairing instability is exceedingly weak, with the Cooper pair binding energy $m_B$ exponentially small in N and also exponentially smaller than the dynamically generated scale, $\rho_{\text{crit}}$, of the density, $m_B \sim e^{-2N\rho_{\text{crit}}}$ [10]. (We use units where $\hbar = 1 = 2m$.)

As an open system, where Fermions are allowed to flow in and out to a reservoir, the cold Fermi gas is described by the non-relativistic quantum field theory with (imaginary time) action, $S = \int dt d^2x \mathcal{L}$, with

$$\mathcal{L} = \psi_a^* \dot{\psi}_a + \vec{\nabla} \psi_a^* \cdot \vec{\nabla} \psi_a - \frac{g}{2N} (\psi_a^* \psi_a)^2 - \mu \psi_a^* \psi_a \quad (1)$$

and the grand canonical thermodynamic potential defined by the functional integral

$$e^{-\nu \phi} = \int [d\psi_a(x) d\psi_a^*(x)] e^{-S[\psi, \psi^*]} \quad (2)$$

where $\nu$ is the volume of space-time, $\psi_a(x)$ are anticommuting functions, the paired indices $a$ run from 1 to N and are implicitly summed where they appear in the Lagrangian density and $\mu$ is the chemical potential. In the classical theory described by (1), the chemical potential is the only parameter with a non-zero scaling dimension. It has dimension of energy or 1/distance$^2$. Setting it to zero would yield a scale invariant classical field theory. However, it is well-known that quantization of the theory described by (1) and (2) breaks the scale invariance with a scale anomaly [11]-[16]. The coupling constant $g$ obtains a beta-function [12] from the renormalization of a logarithmic ultraviolet divergence which occurs in Fermion-Fermion two-body scattering,

$$\beta(g(\Lambda^2)) = \Lambda^2 \frac{d}{d\Lambda^2} g(\Lambda^2) = -\frac{g^2}{8\pi N} + \mathcal{O}(\frac{1}{N^2}) \quad (3)$$

and it becomes a scale-dependent running coupling

$$\frac{1}{g(\Lambda_1^2)} - \frac{1}{g(\Lambda_2^2)} = \frac{1}{8\pi N} \ln \frac{\Lambda_1^2}{\Lambda_2^2} + \mathcal{O}(\frac{1}{N^2}) \quad (4)$$

where $\Lambda_i$ are scales with dimensions of 1/distance. We will call the ultraviolet cutoff $\Lambda$. The beta function is small and the scale dependence is weak in the large N limit where $N \rightarrow \infty$ with $g$ fixed. In the strict infinite N limit, the model is scale invariant. In the following, we shall study this model, both in the scale invariant infinite N limit and at large but finite N where the scale symmetry is weakly broken.

At the infinite N limit, we shall find a state of this system which can have a non-zero density per species,

$$\rho = \frac{1}{N} \sum_{a=1}^{N} \langle \psi_a^*(x) \psi_a(x) \rangle \quad (5)$$
when the coupling constant is tuned to a strong coupling fixed point and the chemical potential is simultaneously tuned to zero. At large but finite \(N\), we shall find that this phase remains stable and that, within a wide band of renormalization group trajectories, the running-coupling self-tunes to the fixed point.

We shall find that the large \(N\) limit of the theory described by \([1]\) has a thermodynamic potential

\[
\Phi = \frac{N}{2} \left[ g \rho^2 - \frac{(\mu + g\rho)^2}{8\pi} \right] + \ldots
\]  

(6)

In this simple equation, the first term is the mean field potential energy and the second term is the Fermi pressure with the mean field effective chemical potential \(\mu + g\rho\). The ellipses denote corrections to the large \(N\) limit, which we shall ignore for the moment. This potential should be minimized as a function of \(\rho\) while holding \(\mu\) at a fixed value. The value of \(\rho\) at the minimum is the physical value of the density. There is a solution only if \(0 < g \leq 4\pi\) and the result is an equation which relates the density and the chemical potential,

\[
\mu = (4\pi - g)\rho + \ldots, \quad g = 4\pi - \frac{\mu}{\rho}
\]  

(7)

and, upon substituting into (6), determines the pressure

\[
P = N\frac{\rho^2}{2} (4\pi - g) + \ldots
\]  

(8)

In Eq. (7) we see that there are two ways to set the chemical potential to zero. One is to put the density to zero, so that there are no particles. The other is to tune the coupling constant so that \(g = 4\pi\) which we could do while keeping \(\rho\) non-zero. In this case, both the chemical potential and the pressure vanish for any value of the density. The scale symmetry is spontaneously broken with the density playing the role of the order parameter. The compressibility

\[
\kappa = \frac{1}{(N\rho)^2} \left( \frac{d(N\rho)}{d\mu} \right) = \kappa_0 \frac{4\pi}{4\pi - g}
\]  

(9)

diverges and the critical point (where \(\kappa_0\) is the compressibility of a perfect gas at the same density \(\kappa_0 = \frac{1}{N\rho^2} \frac{1}{4\pi}\)).

We shall find that a non-propagating, non-oscillating collective mode emerges at this critical point. We call this mode the dilaton. It is a close relative of the breathing mode of cold atoms in a harmonic trap which is predicted by a dynamical conformal algebra that appears there \([19]\). In that case the frequency of the breathing is related to the harmonic potential of the trap. We can view our system as the limit where the trapping potential is removed (while keeping the density fixed and simultaneously tuning the chemical potential to zero and the two-body coupling to the fixed point) so that the frequency of oscillation of the breathing mode goes to zero.

A crucial question is that of stability of the phase with finite density per Fermion species, particularly when scale symmetry violation is turned on by relaxing the infinite \(N\) limit. We will find that including order \(1/N\) corrections helps in this regard. When \(N\) is not strictly infinite, the coupling \(g\) has the beta-function in Eq. (3) which results in a order \(1/N\) violation of scale invariance. The renormalization group flow of the coupling constant will make it a function of the density and the equilibrium density will be determined by the equation \(g(4\pi\rho) = g^\ast\) where \(g^\ast = 4\pi + \mathcal{O}(1/N)\). As illustrated in Fig. 1, this equation always has a solution on the attractive branch of the running coupling.

To find the grand canonical potential to the next-to-leading order in \(1/N\), it is convenient to begin with the

![FIG. 1: The flow of \(g\) as the energy scale \(\epsilon\) decreases. The critical density is the scale at which the running coupling reaches \(g^\ast\), i.e. \(g(4\pi\rho_{\text{crit}}) = g^\ast\). See Eqs. (24) and (23). The dashed line is the position of the Landau pole which is exponentially smaller than \(\rho\) in the large \(N\) limit (Eq. (30)).](image)

![FIG. 2: The solid line is the critical density \(\rho_{\text{crit}}\) which forms the phase boundary at a line of first order phase transitions. The homogenous Fermi liquid phase is above the line and what is likely a clumped inhomogeneous phase is below it. The figure is schematic. The space between the lower critical density and \(\rho = 0\) axis is exaggerated. The phase in this region is likely a mixed phase with clumps of fermions and the first order transition is very similar to a liquid-gas transition. See also Fig. 3.](image)
action \( \tilde{S} = \int dt d^2 x \tilde{L} \)

\[ \tilde{L} = \psi_a^* \dot{\psi}_a + \nabla \psi_a - (\mu + g\sigma) \psi_a^* \psi_a + \frac{Ng\sigma^2}{2} \]  
(10)

and the functional integral

\[ e^{-\Phi} = \int [d\psi_a(x)d\psi_a^*(x)d\sigma(x)] e^{-\tilde{S}[\psi,\psi^*,\sigma]} \]
(11)

The Hubbard-Stratonovich \( \sigma \)-field is related to the Fermion density per species, for example \( \langle \sigma \rangle = \rho \) and \( < \sigma \sigma > \gg < \rho \sigma > \sim -1/g \). If we first do the Gaussian integral over \( < \sigma \sigma > \), we recover the purely Fermionic theory \([1]\). If, instead of integrating \( \sigma \) (x), we first perform the Gaussian integral over \( \psi_a(x) \), we obtain the effective action for the \( \sigma \)(x)-field

\[ S_{\text{eff}} = \int dt d^2 x \frac{Ng\sigma^2}{2} - N \text{Tr} \ln(D - \nabla^2 - (\mu + g\sigma)) \]
(12)

This effective action is proportional to \( N \) and the dynamics of \( \sigma(x) \) are therefore weakly coupled and semi-classical when \( N \) is large. The remaining functional integration over the variable \( \sigma(x) \) can be computed in the saddle point approximation. To proceed, we denote \( \sigma(x) = \rho + \delta \sigma(x) \). We expand \( S_{\text{eff}} \) to quadratic order in \( \delta \sigma(x) \), drop the linear terms, and we do the Gaussian integral over \( \delta \sigma(x) \) \([20]\). The result is the potential density

\[ \Phi = \frac{Ng\sigma^2}{2} - N \frac{\text{Tr} \ln(D - \nabla^2 - (\mu + g\rho))}{V} \]
(13)

where the ellipses denote terms of order \( 1/N \) and

\[ \Delta^{-1}(x, y) = N \left[ g\delta(x, y) + g^2 \Pi(x, y) \right] \]
(14)

\[ \Pi(x, y) = D(x, y)D(y, x) \]
(15)

\[ D(x, y) = (x - y) \frac{1}{\nabla^2 - (\mu + g\rho)} \]
(16)

The density \( \rho \) is to be determined so that it minimizes \( \Phi \). If we keep only the leading order in \( N \) (the first two terms in \([13]\)) and cancel divergent terms which are quadratic and quartic in the cutoff with counter-terms (there are no logarithmic divergences at this order), the potential is as in Eq. \([6]\) whose implications we have already discussed in the text following Eq. \([6]\).

Before we go on to discuss the next-to-leading order, let us consider the collective behaviour at the leading order at large \( N \). The \( \sigma \)-field propagator is \( \Delta \) in \([14]\). We search for collective modes by studying the singularities of \( \Delta(i\omega, k) \) analytically continued to real frequency \( i\omega \rightarrow \omega \). The elementary one-loop integral yields

\[ \Pi(\omega, k) = \frac{(\omega + k^2 + i\epsilon)\sqrt{1 - \frac{4(\mu + g\rho)k^2}{(\omega + k^2 + i\epsilon)^2} - k^2}}{8\pi k^2} \]

\[ \quad + \frac{(-\omega + k^2 - i\epsilon)\sqrt{1 - \frac{4(\mu + g\rho)k^2}{(-\omega + k^2 + i\epsilon)^2} - k^2}}{8\pi k^2} \]

When the interaction is attractive and subcritical, that is, when \( 0 < g < 4\pi \), the equation \( \Delta^{-1} = N[g + g^2\Pi(x, k)] = 0 \) has no real solution for \( \omega \). This is consistent with the fact that zero sound is strongly damped in a Fermi liquid with an attractive interaction. However, if we tune the coupling to \( g = 4\pi \), \( \Delta^{-1} = N[g + g^2\Pi(x, k)] \) has a zero, and \( \Delta \) a pole, at \( \omega = 0 \). Near the critical point, and in the \( \omega << p\sqrt{4\pi\rho} \) regime,

\[ \Delta(\omega, k) = \frac{4\pi}{Ng\rho} \sqrt{\frac{\pi}{4}} |k| \]

(17)

This collective mode is the dilaton. It is damped unless \( g = 4\pi \), where the scale symmetry of the underlying model becomes exact. At that point, \( \Gamma = 0 \) and the pole is at \( \omega = 0 \).

To proceed to the next order of the \( 1/N \) expansion, consider the integral which evaluates the last term on the right-hand-side of Eq. \([13]\).

\[ \frac{1}{2V} \text{Tr} \ln \Delta^{-1} = \frac{1}{2} \int \frac{d\omega}{2\pi} \int_0^\Lambda k^2 \frac{2}{4\pi} \ln \Delta^{-1}(\omega, k^2) \]

\[ = \left[ -g^2(\mu + g\rho)^2 \ln \frac{\Lambda^2}{\mu + g\rho} + \frac{(\mu + g\rho)^2 \varphi(\rho)}{8\pi} + \ldots \right] \]
(18)

where we have isolated the quartic, quadratic and logarithmically divergent terms, dropped the quartic and quadratically divergent ones, assuming that they are canceled by counter-terms that we would add to the original Lagrangian density and we name the remaining finite integral \( \frac{g^2(\mu + g\rho)^2 \varphi(\rho)}{4\pi} \). Since it is finite, its \( \mu + g\rho \)-dependence is given by dimensional analysis. Our results will not depend on the precise form of this function. The renormalization of this theory is well-known and in the leading order it is confined to coupling constant renormalization which leads to the beta function \([3]\). What is more, the logarithmic terms spoil the large \( N \) expansion. When evaluated on the solutions for \( \rho \), the logarithms are large and they compensate the small factors of \( 1/N \). This is a well-known phenomenon which already occurs in the first example in the paper by Coleman and Weinberg \([9]\) on dynamical symmetry breaking. Its solution is to use the renormalization group to re-sum the logarithmically singular terms to all orders. The result in our case is simple: it is given by replacing \( g \) in the potential by the running coupling constant \( g(\bar{\mu}) \) at the scale of the effective chemical potential, \( \bar{\mu} = \mu + g\rho \). Then, the renormalization group improved potential at the next-to-leading order in the large \( N \) expansion has the form

\[ \Phi = N \left[ g(\bar{\mu})\rho^2 - \frac{\bar{\mu}^2}{8\pi} \left( 1 - \frac{\varphi(g(\bar{\mu}))}{4\pi} \right) \right] \]

\[ + g(\bar{\mu}) \left( \rho - \frac{\bar{\mu}}{4\pi} \right)^2 \frac{g(\bar{\mu})^2}{16\pi N} \ln \frac{\Lambda^2}{\bar{\mu}} + \ldots \]
(19)

The cutoff dependence has not entirely canceled. However, the cutoff-dependent term is equal to the square of
the derivative by \( \rho \) of the leading order terms and it and its derivative will vanish to the order in \( 1/N \) to which we are working when it is evaluated on the solution of the equation for \( \rho \). The equation \( \frac{d\Phi}{d\rho} = 0 \) yields
\[
\mu = \rho \left[ 4\pi - g(\tilde{\mu}) + \frac{\varphi(g(\tilde{\mu}))}{N} - \frac{2\pi \beta(g(\tilde{\mu}))}{\tilde{\mu}} \right] + \ldots \tag{20}
\]
\[
\tilde{\mu} = \mu + g(\tilde{\mu})\rho \; , \; \tilde{\mu} \frac{d\rho}{d\tilde{\mu}} g(\tilde{\mu}) = \beta(g) = -\frac{g^2}{8\pi N} + \ldots \tag{21}
\]
Eqs. (20) and (21) determine \( \rho \), and therefore \( \tilde{\mu} \) as functions of \( \mu \). Eq. (20) is identical to \( (7) \) with \( 1/N \) corrections. When \( \mu = 0 \), there are two solutions of \( (20) \) and \( (21) \), either \( \rho = 0 \), or \( \rho \neq 0 \) with the latter determined so that the running coupling constant obeys the equation \( 0 = 4\pi - g(\tilde{\mu}) + \frac{\varphi(g(\tilde{\mu}))}{N} - \frac{2\pi \beta(g(\tilde{\mu}))}{g(\tilde{\mu})} \) with \( \tilde{\mu} = g(\tilde{\mu})\rho \). Which of these is the physical solution is obtained by comparing their thermodynamic potentials, \( \Phi \). We see that \( \Phi = 0 \) when \( \rho = 0 \) and \( \Phi < 0 \) when \( \rho \) is the other, non-zero solution. Thus, when \( \mu = 0 \), the solution with \( \rho \neq 0 \) is energetically preferred. Now, let us further decrease \( \mu \) from zero to small negative values. The phase with \( \rho = 0 \) remains at \( \Phi = 0 \) and the phase with \( \rho \neq 0 \) has \( \Phi \) increasing and eventually coming to \( \Phi = 0 \) to compete with the \( \rho = 0 \) phase. This is the point of first order phase transition, where both phases have the same (zero) value of the thermodynamic potential. By plugging the solution for \( \rho \) into \( (19) \), we see that \( \Phi \) vanishes when the running coupling constant comes to the value \( g^* \) which obeys the equation
\[
4\pi - g^* + \frac{\varphi(g^*)}{N} - \frac{4\pi \beta(g^*)}{g^*} + \mathcal{O}(1/N^2) = 0 \tag{22}
\]
This equation has solution
\[
g^* = 4\pi \left( 1 + \frac{\varphi(4\pi)}{4\pi N} - \frac{\beta(4\pi)}{4\pi N} + \mathcal{O}(1/N^2) \right) \tag{23}
\]
and, the lower critical density is given by
\[
g(4\pi \rho_{\text{crit}}) = g^* \; , \; \rho_{\text{crit}} = \frac{\Lambda^2}{4\pi} e^{-8\pi N \left( \frac{1}{\sqrt{\pi \kappa N}} - \frac{1}{\pi} \right)} \tag{24}
\]
The pressure, is
\[
P = N \rho^2 \left[ 4\pi - g(\tilde{\mu}) + \frac{\varphi(g(\tilde{\mu}))}{N} + \frac{g(\tilde{\mu})}{2N} \right] + \ldots \tag{25}
\]
\[
\tilde{\mu} = 4\pi \rho + \mathcal{O}(1/N) \tag{26}
\]
Eq. (25) is equal to the pressure in \( (6) \) plus \( 1/N \) corrections. The pressure vanishes when \( \rho = \rho_{\text{crit}} \). The critical chemical potential is of order \( 1/N \) and negative,
\[
\mu_{\text{crit}} = 2\pi \rho_{\text{crit}} \frac{\beta(g^*)}{g^*} = -\frac{\pi}{N} \rho_{\text{crit}} \tag{27}
\]
The compressibility no longer diverges at the critical point,
\[
\kappa = \kappa_0 \frac{4\pi}{g^* - g(\tilde{\mu}) + \frac{\varphi(g(\tilde{\mu}))}{2\pi g(\tilde{\mu})}} = \frac{2N + \ln \frac{-\rho}{\rho_{\text{crit}}}}{1 + \ln \frac{-\rho}{\rho_{\text{crit}}}} \tag{28}
\]
where, as in \( (9) \), \( \kappa_0 \) is the compressibility of a perfect Fermi gas at the same density. At the critical density, rather than diverging, the compressibility is anomalously large, \( 2N \) times \( \kappa_0 \). The damping constant of the dilaton at the critical density is obtained from the curvature of the potential, \( \frac{d^2 \Phi}{d\rho^2} \) at \( \rho = \rho_{\text{crit}} \), where it no longer goes to zero but it is suppressed by a factor of \( 1/N \). Eq. (17) is replaced by
\[
\Delta(\omega, k) = \frac{\sqrt{4\pi \rho_{\text{crit}}}}{\Gamma - i\omega} \; , \; \Gamma = \frac{1}{N} \sqrt{4\pi \rho_{\text{crit}} |\vec{k}|} \tag{29}
\]
The pressure and compressibility are positive, indicating stability of this phase for values of \( \mu \) larger than \( \mu_{\text{crit}} \).

The first order behaviour arises from the fact that, as seen in Fig. 1, the density is always larger than the scale of the Landau pole which occurs at
\[
m_B = \Lambda^2 e^{-\frac{8\pi N}{\pi \kappa_0 N}} \sim e^{-2N \rho_{\text{crit}}} \tag{30}
\]
This is the binding energy of a Cooper pair which appears in the Fermion-Fermion channel. This weakly bound pair should Bose condense at zero temperature so that, in the strict mathematical sense, the final state of this system
should be a superfluid. This Bose condensation is only possible if there is a density of Fermions to pair and it should therefore aid the stability of the symmetry breaking phase by further lowering its free energy. However, the Cooper pair binding energy is exceedingly small and even a small temperature in the range $n_B k_B T < \rho$ or other small randomizing effects would destabilize the superfluidity but not the spontaneous density.

Although the system that we have described is largely paradigmatic it could conceivably be experimentally realizable, for example, using Fermionic atoms with high atomic or nuclear spin and approximately spin independent attractive interactions. Such systems have been realized for $N$ up to $N=10$, which should be large enough to create a hierarchy of scales in Eq. (30). In Fig. 3 we plot the upper critical value of the coupling $g(\Lambda^2)/g^*$ versus $N$ in the interval $5 \leq N \leq 10$ for a fixed and reasonable value of the density $\rho/\Lambda^2 = 10^{-7}$. The compressibility is plotted versus the ratio of the density to the critical density for that range of $N$ is plotted in Fig. 4. The large enhancement of the compressibility, by a factor of $2N$, at $\rho \to \rho_{crit}$ is clearly seen. The pressure and compressibility of the two-dimensional Fermi gas with attractive interactions, nonzero temperature and $N=2$ have been studied numerically in Ref. [22]. If such a study is extendable to the large $N$ regime, it would be interesting as it would, for example, be possible to explore whether the phenomenon which we describe would be visible for small or intermediate values of $N$.

Acknowledgments

This work is supported in part by NSERC Discovery grants 288179 and 1197908. GWS thanks L.C.R. Wijewardhana for discussions and participation in an early state of this work and Igor Herbut, Lev Lipatov and Arkady Vainshtein for discussions.

[1] T. Appelquist, Y. Bai, Phys. Rev. D 82, 071701 (2010). [arXiv:1006.4375 [hep-ph]].
[2] M. Hashimoto, K. Yamawaki, Phys. Rev. D 83, 015008 (2011). [arXiv:1009.5482 [hep-ph]].
[3] W. D. Goldberger, B. Grinstein, W. Skiba, Phys. Rev. Lett. 100, 111802 (2008). [arXiv:0708.1463 [hep-ph]].
[4] B. Grinstein, P. Uttayarat, JHEP 1107, 038 (2011). [arXiv:1105.2370 [hep-ph]].
[5] B. Bellazzini, C. Csaki, J. Hubisz, J. Serra, J. Terning, Eur. Phys. J. C 73, no. 2, 2333 (2013). [arXiv:1209.3290 [hep-ph]].
[6] C. Csaki, C. Grojean, J. Terning, [arXiv:1512.00468 [hep-ph]].
[7] W. A. Bardeen, M. Moshe, M. Bander, Phys. Rev. Lett. 52, 1188 (1984).
[8] H. Omid, G. W. Semenoff, L. C. R. Wijewardhana, Phys. Rev. D 94, 125017 (2016). [arXiv:1605.00790 [hep-th]].
[9] S. Coleman, E. Weinberg, Phys. Rev. D 7, 1888 (1973).
[10] Note that the large $N$ limit which we shall use is different from the one which is used in P. Nikolic and S. Sachdev, Phys. Rev. A 75, 033608 (2007). The latter sought to emphasize the fermion pairing channel and study Cooper pair condensation, whereas our large $N$ limit is designed to suppress Cooper pair binding by making the interaction in the Fermion-Fermion channel and the beta function weak, of order $1/N$.
[11] R. Jackiw, Phys. Today 25 (11), 23 (1972); C. Hagen, Phys. Rev. D5, 377 (1972); U. Niederer, Helv. Phys. Acta 45, 802 (1972); R. Jackiw, M. A. Beg Memorial Volume, A. Ali, P. Hoodbhoy eds. World Scientific, 1991.
[12] O. Bergman, Phys. Rev. D 46, 5474 (1992).
[13] B. Holstein, Am. J. Phys. 61, 142 (1993).
[14] M. Olshanii, H. Perrin, V. Lorent, Phys. Rev. Lett. 105, 095302 (2010).
[15] J. Hofmann, Phys. Rev. Lett., 108:185303, (2012).
[16] J. Levinsen, M. M. Parish, Annual Review of Cold Atoms and Molecules, vol. 3, ch. 1, 1-75 (2015).
[17] Y. Nishida, D. T. Son, Phys. Rev. D 76, 086004 (2007). [arXiv:0706.3746 [hep-th]].
[18] S. Golkar and D. T. Son, JHEP 1412, 063 (2014). [arXiv:1408.3629 [hep-th]].
[19] L. F. Pitaevskii, A. Rosch, Phys. Rev. A 55, R853 (1997).
[20] R. Jackiw, Phys. Rev. D 9, 1686 (1974).
[21] B. J. DeSalvo, M. Yan, P. G. Mickelson, Y. N. Martinez de Escobar, and T. C. Killian, Phys. Rev. Lett. 105, 030402 (2010); S. Taie, Y. Takasu, S. Sugawa, R. Yamazaki, T. Tsujimoto, R. Murakami, and Y. Takahashi, Phys. Rev. Lett. 105, 190401 (2010).
[22] E. R. Anderson and J. E. Drut, Phys. Rev. Lett. 115, 115301 (2015).
[23] As a result of the first order phase transition in large-$N$ fermion gases discussed here, we anticipate a phase separation phenomena in a trap if the trap potential is slowly varying and the local density near the centre is higher than the critical one discussed in the article. Away from the centre of a trap with relatively higher densities, the density profile along the radial direction becomes sharply clipped at the point where the density decreases to the critical value; and beyond that point, the fermions likely form a clumped liquid state but spreading over a very narrow region. In 2D traps, this implies that there shall an interface which separates an inner disk of stable fermions near the centre and a thin surrounding ring of liquid at the outer boundary. This distinct singular density profile can be the potential smoking gun of the large $N$ physics discussed in this Letter.