State-feedback based adaptive control algorithm of the active electromagnetic bearing

I S Dymov¹,², D A Kotin¹, V A Elanakova¹

¹ Novosibirsk State Technical University, 20, Karla Marksa ave., Novosibirsk, 630073, Russia
² E-mail: dymov.2010@corp.nstu.ru

Abstract. The paper is devoted to the development and investigation of automatic adaptive control system of electric machine rotor radial displacement stabilization with active magnetic bearings. A new approach of solving the problem of stabilizing the rotating element of motor, based on active current updating of the spatial position is offered. The development of automatic control algorithm is done using the Lyapunov function method. The paper describes the way of constructive implementation of this method, structural synthesis methodology of control law, as well as the results of simulation, confirming potential of conducted research. Practical implementation of the developed system will increase the parametric reliability of electrical equipment for various purposes.

1. Introduction

The development of modern electrical equipment is inextricably associated with the increasing requirements in terms of eliminating any occurring mechanical vibrations that reduce accuracy of this equipment and are unacceptable. The problem of precision equipment which includes DC or AC electric motors is particularly acute. The main source of mechanical vibrations in electrical plants is a rotating rotor of the electric machine used [1]. The developments of fundamentally new types of supports using magnetic and electric fields to create appropriate reactions, could improve operational capabilities and reduce the number of defects. These suspensions among others include active magnetic bearings (AMB) designed for non-contact retention and centring of the rotating part of electric machines for various purposes during their operation. According to the principle of operation, an active magnetic bearing is a closed automatic control system and is a set of magnetic rotor suspension with electromagnets and a block of rotor position sensors installed directly on a machine, as well as a control system that provides stability of the radial position of the rotation axis [2].

Today existing active magnetic bearings are divided into axial and radial ones. An axial electromagnetic bearing consists of two electromagnets secured in the frame of the machine that are two fixed cylinders with a laid excitation winding located on different sides of a rotating solid disk mounted directly on the motor shaft. Radial electromagnetic bearings consist of a stationary part fixed in the frame of the machine and are made in the form of a multi-pole package with excitation windings at the poles, that generally form four zones of the electromagnet with mutually perpendicular axes, and a rotating part mounted directly on the shaft of the electric machine [3]. It is advisable to install the latter in the mechanisms to which ultra-precision requirements for the accuracy of the technological process are imposed.

Improving the performance of high-precision electrical installations through the use of new methods
of synthesis of control systems is becoming a priority requirement for the development of laser devices, lathes and milling machines, orientation systems and spacecraft stabilization. Therefore, the main task will be to study the adaptive controller of the active stabilization system according to the state of the spatial position of the electric machine rotor.

2. Materials and methods

2.1. Method of active stabilization of the radial displacement

The application of the method of radial displacement active stabilization makes it possible to influence the position of the rotor of the electric motor by introducing compensating links into the control circuit that change the stiffness and damping coefficients. The movement of the rotor of the electric machine in the gap between the movable and stationary parts of active bearings is estimated by means of position sensors located along the axes of radial electromagnets. Position sensors generate signals proportional to the displacement of the rotor from the central position. The control system operating with position sensor signals sends reference input signals to the relevant control objects (CO) that are presented by field magnets. When currents flow in the windings of electromagnets, magnetic forces arise that prevent the displacement of the rotor of the motor and return it to the central position [4].

Function diagram of design and process control system of active stabilization is presented in Figure 1.

![Function diagram showing considering method of active stabilization of the rotor.](image)

The legend of Fig. 1 is as follows:

- $U^x_{ref}, U^y_{ref}$ – the reference signals on the desired radial displacement in the plane X and Y respectively, V;
- PC – the spatial position controllers;
- Con – the converter;
- $U^u_{ref}$ – the reference signal for the converter voltage, V;
- $U_{ex}$ – the output voltage of the converter (excitation voltage of the electromagnet), V;
- EM – the electromagnet;
- PS – the spatial position sensors of the rotor, registering the radial displacement in the plane X and Y;
- $K_{pf}$ – the coefficient for radial displacement feedback in planes X and Y.

2.2. State-feedback based adaptive control algorithm

The mathematical model of the CO in general form:
\[ a_n y^{(n)}(t) + a_{n-1} y^{(n-1)}(t) + \ldots + a_1 y(t) = u(t) \]  

(1)

where \( y(t) \) – the output coordinate; \( u(t) \) – the control action; \( a_i (i = 0,1,\ldots,n) \) – unknown parameters, but the sign of \( a_i \) is known.

Then the reference model is the following equation:

\[ y_m^{(n)}(t) + a_{n-1} y_m^{(n-1)}(t) + \ldots + a_1 y_m(t) = \beta_0 g(t) \]  

(2)

where \( y_m(t) \) – the output coordinate; \( a_i (i = 0,1,\ldots,n) \) and \( \beta_0 \) – known positive constants; \( g(t) \) – the reference action.

Adaptive control systems are nonlinear, so the main method of their study is the Lyapunov function method. Let’s write the Lyapunov equation:

\[ PA + A^T P = -Q \]  

(3)

where \( Q \) – the positive definite matrix; \( A \) – \( n \times n \) the matrix in the form of:

\[
A = \begin{bmatrix}
0 & 1 & 0 & \ldots & 0 \\
0 & 0 & 1 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \ldots & 1 \\
-a_n & -a_{n-1} & -a_{n-2} & \ldots & -a_1
\end{bmatrix}
\]

where in the last line of the matrix \( A \) there are the coefficients of the reference model equation.

If we will take the matrix \( qI_n \) instead of \( Q \) (where \( q \) – the positive coefficient, \( I_n \) – the unit matrix of the \( n \) order), then without violation of generality, it is possible to write the equation (3), to choose \( q = 1 \), thereby, transforming into an equation of the form:

\[ PA + A^T P = -I_n \]  

(4)

The adaptive control algorithm that provides global stability and convergence of the control error \( e(t) = y(t) - y_m(t) \) and tends to zero at \( t \to \infty \) [5] is:

\[ u(t) = \hat{k}_0 g(t) + \hat{k}_{01} y_m^{(n-1)} + \ldots + \hat{k}_{0n} y = \hat{k}^T v \]

(5)

\[ \hat{k} = -\text{sign}(a_0) \hat{A} v B^T P x, \]

where \( \hat{k} = (\hat{k}_{00}, \hat{k}_{01}, \ldots, \hat{k}_{bn})^T \) – the vector of controller variable parameters;

\[ v = \begin{bmatrix} g^{(n-1)} & \ldots & y \end{bmatrix}^T \] – the signal vector;

\( \hat{A} \) – the determined arbitrary positive \((n+1) \times (n+1)\) matrix; \( P \) – \( n \times n \) matrix that is the solution of (4); \( B \) – \((n-1) \times 1\) – the matrix that has the following form:
3. Results and Discussion

Let’s carry out mathematical simulation of the system for elimination of radial displacement of the rotor of the electric machine. The general block diagram of the state-feedback based adaptive control system with only one electromagnet is shown in Figure 2.

![General flow diagram of the state-feedback based adaptive control system.](image)

The legend of Fig. 2 is as follows:

- \( g \) – the reference signal to the required radial displacement, m;
- \( W_{PC}(p) \) – the transfer function of the radial displacement controller;
- \( W_{EM}(p) \) – the control object transfer function;
- \( W_m(p) \) – the transfer function of the reference model;
- \( U \) – voltage supplied to the control object, V.
- \( k_{00}, k_{01} \) – varied parameters.

The mathematical model of electromagnetic processes in the control coil of the electromagnet is presented as the following differential equation:

\[
U = \left( Ri - L \frac{di}{dt} \right) K_1 - f
\]  

(6)

where \( i \) – the electromagnet exciting current, A; \( R \) – the electromagnet active resistance, Ohm; \( L \) – the electromagnet inductance, H.

For synthesis, the flow diagram of the control object subsystem is presented in Figure 3.
The flow diagram of the control object subsystem.

The legend of Fig. 3 is as follows:

\( f \) – the disturbance effect, m.

Let the equation of the mathematical model have the following form:

\[
\frac{dx}{dt} = g(K_2 x - \frac{dx}{dt})K_3
\]

where \( K_1, K_2, K_3 \) – the coefficients of the reference model.

The flow diagram of the reference model subsystem is presented in Figure 4.

Based on the order of the equation of the considered reference model, the matrix \( A \) from the expression (4) takes the form:

\[
A = -K_{2m}
\]

Then, the solution of Lyapunov equation (4) will be:

\[
P = \frac{1}{-K_{2m}}
\]

Hence, for the adaptive control algorithm according to (5) we obtain:

\[
U = \hat{\hat{K}}_{00} g + \hat{\hat{K}}_{01} y
\]

\[
\begin{pmatrix}
\hat{\hat{K}}_{00} \\
\hat{\hat{K}}_{01}
\end{pmatrix} = -\gamma e \begin{pmatrix}
g \\
y
\end{pmatrix}
\]

where \( \gamma \) – an arbitrary positive constant.

The solution of the problem of finding the transfer function of the adaptive controller has a feature due to the need to ensure ideal tracking of the output of the reference model. Therefore, for the existing transfer function of the CO and the transfer function of the reference model, the equality shall be performed:
\[ W_m(p) = \frac{W_{PC}(p)W_{EM}(p)}{1+W_{PC}(p)W_{EM}(p)} \]  \hspace{1cm} (8)

Having solved this equation with respect to the transfer function of the adaptive position controller, we obtain:

\[ W_{PC}(p) = \frac{1}{W_{EM}(p)} \frac{W_m(p)}{1-W_m(p)} \]  \hspace{1cm} (9)

The flow diagram of the adaptive controller subsystem is presented in Figure 5.

**Figure 5.** The flow diagram of the adaptive position controller subsystem.

The legend of Fig. 5 is as follows:

- \( K_{1PC}, K_{2PC}, K_{3PC}, K_{4PC} \) – coefficients of the flow diagram, at that
  - \( K_{1PC} = L \cdot K_{3m} \);
  - \( K_{2PC} = R \cdot K_{3m} \);
  - \( K_{3PC} = \frac{1}{K_1(K_{2m} - K_{3m})} \);
  - \( K_{4PC} = \frac{1}{K_{1m}(K_{2m} - K_{3m})} \).

The results of the digital simulation are transient displacement processes under the mandatory condition of a non-zero reference signal, and the disturbing influence that simulates a step-by-step application of the load resistant torque on the motor shaft, accompanied by a micrometre displacement from the central axis of rotation, at each simulation is supplied at the time of \( t = 3 \) seconds (Figures 6 and 7).

Analysing the results of the digital simulation of the obtained algorithm of adaptive control of the active electromagnetic bearing, it can be concluded that the synthesized automatic control system due to the operation of the adaptation algorithm has astatism to both external and parametric displacements.
Figure 6. Transient displacement process when changing the physical parameters of the CO: 1 – Constant physical parameters of the CO; 2 – The physical parameters of the CO increased in 100 times; 3 – The physical parameters of the CO reduced in 100 times.

Figure 7. The state-feedback based transient process at different displacements: 1 – Nanometre displacement from the central axis; 2 – Micrometer displacement from the central axis; 3 – Tenfold micrometer displacement from the central axis.
4. Conclusion
The simulation results prove that the obtained transient processes correspond to the nature of the ultra-precision process equipment, since the compensation of radial displacement is performed in the micrometer accuracy range.

References
[1] Dymov I S, Kotin D A 2016 Method for implementation of active stabilization the spatial position of mechatronic device rotor Retrieved from: http://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=7756721.
[2] Yuanping X, Long D, Jin Z, Chaowu J, Qintao G 2016 Active magnetic bearings used as exciters for rolling element bearing outer race defect diagnosis ISA Transactions 61 221-228.
[3] Filatov A, Hawkins L, McMullen P 2016 Homopolar Permanent-Magnet-Biased Actuators and Their Application in Rotational Active Magnetic Bearing Systems Actuators 5 5-26
[4] Dymov I S, Kotin D A 2017 Signal-Adaptive Controller for Micro Electric Drive Rotor Radial Displacement Updating 18th International Conference of Young Specialists on Micro/Nanotechnologies and Electron Devices EDM 2017: conference proceedings ed. A I Khristolyubova (Novosibirsk: NSTU) pp 572 – 578.
[5] Kim D P 2004 The theory of automatic control Multidimensional, nonlinear, optimal and adaptive systems: Proc. Allowance vol 2 (Moscow: FIZMATLIT) p 464