Femtosecond to picosecond electron-energy relaxation and Fröhlich coupling in quantum dots

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Electron relaxation in quantum dots is studied theoretically in polar semiconductor materials, with an emphasis put on the phonon-bottleneck problem and the electron-LO-phonon coupling. The theory is based on multiphonon states of the electron-phonon system and the self-consistent Tamm-Dancoff approximation is used for the electronic self-energy. Electronic relaxation rate is shown numerically to be on the scale from hundreds fs to tens ps, for electron energy-level separations being in the broad range from about one LO-phonon energy to about three or four optical-phonon energies. Despite of displaying some resonance features, the electronic relaxation rate does not appear to be crucially dependent on the quantum dot size.

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I. INTRODUCTION

The semiconductor quantum dots (QD) have attracted attention due to the earlier expectations concerning their use in high efficiency semiconductor lasers. These expectations of the high efficiency were based mainly on the δ−function like electronic density of states of such zero-dimensional (0D) structures and on an expected narrow gain region. In later experiments the luminescence of these 0D structures was found to decrease with decreasing the lateral size of quantum dots. Besides attributing this luminescence decrease to technological difficulties in the process of realization of the lateral confinement for electrons and holes, a different explanation was suggested, ascribing the luminescence decrease to the so called 'phonon−bottleneck effect'. The phonon-bottleneck hypothesis was supported by an earlier detailed analysis of the electron-phonon interaction in quantum dots based on Born approximation to electron-phonon scattering.

In experiments, the relaxation of electronic energy was often measured on the scale of picoseconds, being therefore fast enough to express doubts about the existence of the bottleneck effect. A significant dependence of the electronic relaxation rate on the quantum-dot size was not reported in papers and the relaxation efficiency was reported to be rather independent of the relation between the electronic energy-level separations and the optical-phonon energy. Basing on the experimental observations, it has been pointed out that the relaxation process of electrons in quantum dots should be considered as a multiphonon process, and that the multiphonon transitions should be expected to be the main mechanism providing the electron-energy relaxation.

The quantum dot system appears to be relatively simple because the electronic structure of the bound states, unperturbed by the lattice motion, may consist of only several discrete energy levels. It is well known that the electron-phonon system of the quantum dot can be exactly diagonalized when the coupling of the electrons to the lattice vibrations is restricted to the transverse coupling (see below) terms only. Another simple and exactly soluble model, with no dissipation of the electronic energy, is the system with two electronic energy levels coupled to a single mode of the lattice vibrations, with the Hamiltonian formally equivalent to Jaynes Cummings model. A similar non-dissipating electron-phonon system was studied in a one-dimensional system. Although we are aware of several simplified systems in which the electronic subsystem does not relax the energy, it is not yet clear which of their properties can be generalized to a little more complicated realistic systems which we likely meet in the real quantum dots, namely in the systems, in which the carriers, confined in the dots, interact with a large number of the lattice vibrational modes, and in which the electron-phonon coupling is more general than that of the Jaynes Cummings model.

The role of the multiphonon states in the formulation of the electron transport problem in quantum dots was emphasized in papers. The need for the self-consistent treatment of the effect of the "collision broadening" of the electronic energies in quantum dots was emphasized in references. These two requirements were recently taken into account in the study of the electronic spectral density. Very sharp spectral density peaks were obtained. This sharpness was interpreted as an indicator of a very long electronic lifetime, without paying a sufficient attention to the lineshape of the spectral density features. In our previous paper it was shown that the spectral
density peaks may diverge to infinity as the inverse of the square root of the energy variable. Although such maxima in the electronic spectral density function are integrable, the relation of the "width" of such spectral features to the electronic lifetime may not be as simple as it may be in the case of the Lorentzian peak shape. The relation of the electronic spectral densities to the rate of relaxation of the electronic energy, reported preliminarily in references 31, 32, is treated in the present paper.

Besides the electron-LO-phonon coupling, other mechanisms were considered recently as possible candidates for the explanation of the experimental data on the fast electronic relaxation in quantum dots. So, the finite lifetime of the optical phonon was shown to provide an efficient mechanism of relaxing the severe restrictions imposed by the energy conservation in the Born approximation upon the electron-LO-phonon scattering 34,35. Also, the ultrafast electron energy relaxation in quantum dots has been recently suggested to be explained by the interaction of the carriers with the defect states in the quantum dot, taking into account the lattice relaxation mechanism 32.

It has been shown recently, that the electronic scattering in the low-dimensional structures should be treated with caution. Namely, the Born approximation to the electronic scattering, giving good results in the Monte Carlo semiclassical simulations of the electronic transport properties of the bulk semiconductor samples, appears sometimes to be rather insufficient in the quasi-two and quasi-one dimensional structures like quantum wells and quantum wires 37,38. These observations provide additional arguments in favor of going beyond the Born approximation in the electron-phonon scattering in quantum dots. Understanding the failure of the Born approximation as an implication of the multiple reflections of the charge carrier from the boundaries of the low-dimensional structure, it can be expected that such dimensionality effects should be present in the zero-dimensional structures like quantum dots. In such a case, the multiple scattering of electrons should be taken into account and the multiphonon states of the electron-phonon system of the quantum dot should be of importance.

The purpose of this paper is to develop further the theory of the electronic relaxation in quantum dots, basing on the multiple phonon scattering of the electrons, considering the interaction of the quantum dot electrons with the longitudinal optical phonons of the bulk matrix sample into which the quantum dot is built, and to present the numerical results of the electronic relaxation rate in dependence on the separation of the electronic energies and on the temperature of the lattice.

II. THE MODEL AND HAMILTONIAN

Generally, there may be several electrons and several holes in a quantum dot structure. Assuming the charge neutrality of the system one can speak then about single-exciton, bi-exciton, etc., states, which are strongly influenced by the carrier-carrier coupling, expected to be significant in quantum dots 39,40. The inclusion of the carrier-carrier coupling may complicate the analysis of the carrier relaxation in quantum dots, although it may be rather significant from the point of view of a quantitative comparison with experimental data. It is the purpose of this paper to concentrate the attention on the electron-phonon mechanism of the carrier relaxation. Therefore, the simplest model system is chosen to be considered, namely that consisting of a single electron in the quantum dot, while the presence of holes in the valence-band states is completely ignored. The authors believe that the basic features of the fast relaxation mechanism of the excited carriers in quantum dots, based on the electron-LO-phonon Fröhlich interaction, are contained already in this relatively simple single-electron model. The inclusion of the carrier-carrier correlation is therefore left to a future work.

In order to simplify the numerical part of the work, the quantum well is assumed to be of cubic shape, with infinitely high electronic potential energy outside the well. The penetration of the electronic stationary wave functions into the potential barriers is therefore neglected. Confining the electronic motion to the lowest lying electronic states is believed to minimize the effect of this infinitely-deep quantum-well approximation. The Schrödinger equation with the electronic effective mass inside the well gives a set of solutions. From these electronic states we consider only two states in the present model: the ground state \( \psi_0 \), with the energy \( E_0 \), which will be put equal zero here, and one excited state \( \psi_1 \) belonging to the triply degenerate first excited energy \( E_1 \). This set of two nondegenerate electronic unperturbed eigenstates provides a minimum basis set for considering the electron-energy relaxation due to electronic transitions between two electronic states coupled via the electron-phonon interaction. The complete electron-phonon Hamiltonian \( H \) then reads 33

\[
H = H_0 + H_1,
\]

where

\[
H_0 = \sum_{n=0}^{1} E_n c_n^+ c_n + \sum_q E_{LO} b_q^+ b_q,
\]

\[
H_1 = \sum_q E_{LO} b_q^+ b_q + V_{LO} \psi_0^+ \psi_1.
\]
\[ H_1 = \sum_{m,n=0}^{1} \sum_{q} A_q \Phi(n,m,q) (b_q - b^+_q) c_n c_m \]  

III. RELAXATION RATE

In photoluminescence experiments an electron may be excited by a light pulse from the valence-band states into the conduction band states and can be finally prepared in an electronic excited state in the quantum dot. In the case of a time-resolved detection of the luminescence, corresponding to the process of annihilation of the electron with a hole in the valence-band states, with the simultaneous emission of a light photon, the rise time of the luminescence can be determined experimentally \[4\] \[5\]. The value of the rise time is limited from below by the time which is needed for the transport process in which the electron is transferred from the excited state to the lower energy state, with emitting or absorbing simultaneously the phonon, both the longitudinal and the transverse terms will be taken into account in this work. The interaction operator \( H_1 \) considered in this work does not have the form of the Jaynes Cummings operator and the electronic system is coupled to a large number of the phonon modes, so that approximative theoretical methods will be used in the following analysis.

The process of the electronic relaxation can be theoretically formulated within the theory of the nonequilibrium statistical operator \[47\]. The rapidity of the relaxation of the electronic energy can be expressed with the help of the time derivative of the population, \( d < c^+_1 c_1 > /dt \), of the electronic excited state (\( n = 1 \)). In the nonequilibrium statistical operator theory, assuming not too short time scale, the state of the system is assumed to be described by a set of quantities \( < P_k > \), with the corresponding set \( \{ P_1, P_2, \ldots, P_k, \ldots \} \) of operators, suitable for the description of the system. The averaging in \( < P_k > \) is performed with the nonequilibrium statistical operator. In the case presently considered the electronic subsystem will be characterized by the mean values \( N_1 = < c^+_1 c_1 > \) and \( N_0 = < c^+_0 c_0 > \) giving the average population of the electronic excited state and the ground state, respectively.

The state of the
phonon system, assumed to be permanently at the thermodynamic equilibrium, will be characterized by the mean value \(< b_{q}^{\dagger} b_{q} >\) given by the Bose-Einstein distribution function at the lattice temperature \(T_L\), independent of time. Confining our choice of \(\{P_{m}\}\) to \(< c_{1}^{\dagger} c_{1} >\), \(< c_{0}^{\dagger} c_{0} >\) and \(< b_{q}^{\dagger} b_{q} >\), the kinetic period of the evolution of the nonequilibrium system under study is presumed.

The well-known expansion of the nonequilibrium statistical operator in powers of the interaction leads to the well-known expansion of the "collision integral" in the generalized kinetic equation giving the time evolution of the mean value \(< P_{k} >\). The lowest-order (in \(H_1\)) terms of the collision term of this equation then read [47]:

\[
\frac{\partial < P_{k} >}{\partial t} = S_{k}^{(0)} + S_{k}^{(1)} + S_{k}^{(2)} + \ldots ,
\]

where

\[
S_{k}^{(0)} = \frac{1}{i\hbar} < [P_{k}, H_0] >
\]

and

\[
S_{k}^{(1)} = \frac{1}{i\hbar} < [P_{k}, H_1] >
\]

The term \(S_{k}^{(2)}\), which is at least of the second order in \(H_1\), contains generally a factor expressing the memory properties of the system [47]. In the markovian approximation [47], considered here, it simplifies to the following expression:

\[
S_{k}^{(2)} = -\frac{1}{\hbar^2} \int_{-\infty}^{0} dt_{1} e^{\varepsilon t_{1}} < [H_1(t_1), [H_1, P_{k}]] >
\]

The averaging \(< \ldots >_{q} = Tr(\ldots \rho_{q}(t))\) is performed with the so-called quasi-equilibrium statistical operator \(\rho_{q}\) defined as

\[
\rho_{q}(t) = Q_{q}^{-1} \exp\{-\sum_{k} F_{k}(t)P_{k}\},
\]

\[
Q_{q} = Tr \exp\{-\sum_{k} F_{k}(t)P_{k}\},
\]

in which the quantities \(F_{k}(t)\), playing the role of the intensive quantities describing the system at equilibrium (e. g. temperature), are determined from the conditions

\[
< P_{k} >_{q} = < P_{k} >.
\]

Consistently with our choice of the operator set \(\{P_{1}, P_{2}, \ldots, P_{k}, \ldots\}\), the term \(S_{k}^{(0)}\) is zero. The coupling of the electronic subsystem to the laser light, which is not considered here, may lead to the appearance of the interband electronic polarization, which would be then a manifestation of the coherence between the electronic and light systems [48]. Similarly, the field of the LO phonons can be coherently coupled to the system of excitations of the electronic subsystem. These effects could be obtained already from the term \(S_{k}^{(1)}\). The right-hand side of eq. (7) gives then the average value of the product of two electronic and one phonon operators. Consistently with our choice of the operator set \(\{P_{k}\}\) the term \(S_{k}^{(1)}\) will be ignored. Extending however the operator set in a suitable way, the Rabi oscillations, studied recently by Inoshita et al. [29], could be considered simultaneously with the term \(S_{k}^{(2)}\). The coherent coupling between the electrons and the LO phonons may play a role especially in the range of short time after the light pulse. This effect will not be considered in this work. The lowest-order part of the time derivative of \(< P_{k} >\) is then given by the formula [3].

In the formula [8], giving the lowest-order markovian contribution to the relaxation rate \(\partial < P_{k} > /\partial t\), the operator \(H_1(t_1)\) is the interaction operator \(H_1\) expressed in the Heisenberg representation. When approximating this Heisenberg representation by the interaction representation, the formula [6] leads directly to the well-known Born approximation to the collision integral. On the other hand, the Heisenberg representation of the interaction operator \(H_1\) in [8] makes it possible to sum partially the terms contributing to \(S_{k}^{(2)}\) up to infinite orders in \(H_1\) [47].

The emphasis will be put now on the obtaining the kinetic equation for the time evolution of the population \(< c_{1}^{\dagger} c_{1} >\) of the upper electronic state. At first, the commutator \([H_1, c_{1}^{\dagger} c_{1}]\) is calculated. As a result, several terms
are obtained, each being a product of two electronic and one phonon operators. The average in the formula (8) then reads:

$$< [H_1(t_1), [H_1, c_{r}^+ c_{l}^-]] >_q = \sum_{r,s,n,p} A_{r} A_{p} \Phi(r, s, p) \Phi(n, 1, q) \times \{ \Phi(n, 1, q) < [b_{r}^+(t_1) - b_{s}^-(t_1) c_{r}^+ c_{l}^-] >_q + \Phi(1, n, q) < [b_{r}^+(t_1) c_{r}^+ c_{l}^-] >_q \}.$$  (12)

Writing down the commutators in the latter equation explicitly according to the definition, the average quantity $\Lambda = < [H_1(t_1), [H_1, c_{r}^+ c_{l}^-]] >_q$ appears to consist of a number of terms. Each of these terms is an average of the product of six particle operators, three of them being in the Heisenberg representation. An example of these terms contributing to $\Lambda$ is:

$$\sum_{r,s,n,p,q} A_{r} A_{p} \Phi(r, s, p) \Phi(1, n, q) < b_{r}^+(t_1) c_{r}^+ c_{l}^- b_{q} c_{q}^- c_{n} >_q.$$  (13)

The average value in the latter formula, and in the other terms contributing to $\Lambda$, will be decoupled according to the following scheme:

$$< b_{r}^+(t_1) b_{q} c_{r}^+ c_{l}^- c_{m} >_q \approx < b_{r}^+(t_1) b_{q} >_q < c_{r}^+ c_{l}^- c_{m} >_q < c_{s}(t_1) c_{l}^+ >_q,$$  (14)

in which only the particle operators belonging to different interaction operators are paired. All the anomalous averages of the type like $< b_{r}^+(t_1) b_{q} >$ and $< c_{r}^+ c_{l}^- c_{m} >$ are considered to be zero. The latter kind of the decoupling of the averages of the operator products in the relaxation rate formula was studied earlier in connection with the electron energy collision broadening and the electronic relaxation [9,14] in bulk samples. It was shown, that the above type of the decoupling leads to the relaxation rate formula with plausible thermodynamical properties. Although this decoupling scheme appears to work well in the bulk structures, it is not obvious whether it is suitable enough in the present case of the decoupling leads to the relaxation rate formula with plausible thermodynamical properties. Although this decoupling scheme appears to work well in the bulk structures, it is not obvious whether it is suitable enough in the presently considered two-level zero-dimensional system. This decoupling is only an approximation to the full correlation function (13). The question of the validity of this decoupling will not be studied in this work.

In the above obtained single-particle correlation functions, like $< c_{s}(t_1) c_{l}^+ >_q$, the diagonal approximation

$$< c_{s}(t_1) c_{l}^+ >_q \approx < c_{s}(t_1) c_{l}^+ >_q \delta_{n,r}$$  (15)

is assumed to hold. This assumption means that the quantum number $n$, indexing the unperturbed electronic states $\psi_n$, remains a good quantum number and that the effects of the quantum coherence, included eventually in terms like $< c_{l}^+ c_{n} >_q$, do not change considerably, in materials with a rather weak electron-phonon coupling like GaAs, the resulting picture provided by the diagonal approximation. The way of decoupling the average $\Lambda$ is consistent with the above-made assumption about the choice of the operators $P_k$ describing the system under study. Nevertheless, the possible influence of the non-diagonal terms like $< c_{r}(t_1) c_{s}^+ >_q$ remains to be verified in a future work.

The phonon correlation functions will be taken into consideration in the diagonal approximation too. This approximation would not be appropriate when considering the hot phonon effect with the LO phonons produced in the area of the quantum dot. The hot-phonon effect will be neglected here. Assuming that the phonon one-particle correlation functions like $< b_{r}^-(t_1) b_{q}(t_1) >$ are invariant under the operation of $q \rightarrow -q$, we get:

$$< [H_1(t_1), [H_1, c_{r}^+] >_q = 2 Re \sum_{q} A_{q} |^{2} \Phi(1, 0, q) |^{2} \{ < b_{q} b_{q}^+ >_q + < b_{q}^+ b_{q} >_q \}$$  (16)

$$\times ( < c_{r}^+ c_{l}^- >_q < c_{r}^+ c_{l}^- >_q - < c_{l}^+ c_{r}^- >_q < c_{l}^+ c_{r}^- >_q \},$$

where $Re$ denotes the real part of a complex number. The knowledge of the time $t_1$-dependence of the single-particle correlation function in the latter equation would make it possible to perform the integration over $t_1$ in (8) and to get in this way the rate of change of the population $< c_{r}^+ c_{l}^- >_q$ as a function of the time $t$.

In calculating the single-particle correlation function we shall proceed in the following way: The single-particle correlation function will be determined formally for the system with the Hamiltonian (1), in the case of the thermodynamic equilibrium at a temperature $T$ and the obtained functional dependence of the electronic and phonon quantities on the temperature will be formally transferred to the case of electrons and phonons having different respective temperatures $T_e$ and $T_L$.
In the following the electronic correlation functions will be expressed in terms of spectral densities (see eqs. (A12), (A13) and (A14) in Appendix A). The electrons and phonons will be assumed to have different temperatures. Namely, at time \( t = 0 \) with \( N_1 = 1 \), the phonon distribution function \( \nu_{LO} = \langle b_{\alpha q}^+ b_q \rangle \) will be given by the Bose-Einstein distribution function taken at a temperature \( T_L \). In the two-level electronic system, unperturbed by the electron-phonon interaction, with the energy levels \( E_0 \) and \( E_1 \), the chemical potential is \( \mu = (E_0 + E_1)/2 \). The population \( N_1 \) of the excited electronic state at a temperature \( T_e \) is

\[
N_1 = \frac{1}{1 + \exp(\frac{E_1 - E_0}{k_B T_e})},
\]

\((k_B \text{ is Boltzmann constant})\). For example, the state of the electronic subsystem with one electron in the excited state and with the empty ground state then corresponds to the limit of \( T_e \to 0 \). In this way the population of this two-level system can be formally expressed with the help of the electronic temperature \( T_e \). For the purpose of the present calculation it is suitable to describe the state of the two-level electronic system in terms of the population of the electronic state rather than in terms of an electronic temperature.

Considering the process of the electron-energy relaxation as a markovian process and taking into account the decoupling (16), the theory of the nonequilibrium statistical operator leads finally to the following formula for the rate of change \( dN_1/dt \) of the population of the electronic state with \( n = 1 \):

\[
\frac{dN_1}{dt} = -\frac{2\pi}{\hbar} \alpha_{101} \left[ N_1 (1 - N_0) \left( (1 + \nu_{LO}) \int_{-\infty}^{\infty} dE \sigma_1(E) \sigma_0(E - E_{LO}) + \nu_{LO} \int_{-\infty}^{\infty} dE \sigma_1(E) \sigma_0(E + E_{LO}) \right) \right. \\
\left. - N_0 (1 - N_1) \left( (1 + \nu_{LO}) \int_{-\infty}^{\infty} dE \sigma_0(E) \sigma_1(E - E_{LO}) + \nu_{LO} \int_{-\infty}^{\infty} dE \sigma_0(E) \sigma_1(E + E_{LO}) \right) \right],
\]

where \( \sigma_0 \) and \( \sigma_1 \) are electronic spectral densities (see eq. (A14)), \( \nu_{LO} \) is Bose-Einstein distribution of LO-phonons at temperature \( T_L \) of the lattice. The constant \( \alpha_{mn} \) is defined as

\[
\alpha_{mn} = \sum_q |A_q|^2 |\Phi(n, m, q)|^2, \quad \alpha_{mn} = \alpha_{nm}.
\]

The first two terms in the square brackets in eq. (18) are the only terms contributing at the initial moment of time \( (t = 0) \), at which \( N_0 = 0 \). The first term provides the production of phonons, while the second term has the meaning of decreasing the electronic population \( N_1 \) while absorbing an LO phonon. This second term, with the absorption of a phonon, appears in the formula from two reasons: First, the relaxation rate formula (18) does not contain the energy conservation \( \delta \)-function as it would appear in the first order of the time dependent perturbation calculation formula. Second, the term with the absorption of a phonon depends on the overlap of the two spectral densities, \( \sigma_1 \) and \( \sigma_0 \). This overlap of \( \sigma_1 \) and \( \sigma_0 \) is determined by the properties of the interacting electron-phonon system, namely, by the multiple-phonon nature of the eigenstates of this system, which may allow for the non-zero contribution of this phonon-absorption term to the relaxation rate at \( t = 0 \). In fact, the presently used approximation to the electronic self-energy (see below) assumes the multiple-phonon property of the eigenstates of the system. The numerical values of the contribution of the phonon-absorption term comes out nevertheless rather small in GaAs (see below). This is in agreement with the fact, that the electron-LO-phonon coupling in the bulk GaAs is not strong. The last two terms in the square brackets have the meaning of a transfer of the electron from the state \( n = 0 \) to the excited state \( n = 1 \). These terms would play a role at the later stages of the relaxation, namely at \( t > 0 \), which is however not treated here.

The electronic self-energy is determined in the self-consistent Tamm-Dancoff (or the self-consistent Born) approximation [31]. The use of the self-consistent Tamm-Dancoff approximation makes it possible to include the states in which the electron is coupled various numbers of phonons and to go beyond the Born approximation when determining the self-energy of this highly singular system with \( \delta \)-function type unperturbed electronic density of states. It is shown in the Appendix B that after a simplification, concerning the electronic distribution function \( N_m \), the Dyson equation of the (retarded) electronic self-energy for the electron in the state \( n \) can be brought to the following form [31]:
the Dyson equation (20) can be written in the form of the following set of equations for
of equation (20) gives the leading contribution, while the other term can be neglected. Under these approximations
Y and the optical phonon energy
E
M
α
the longitudinal interaction. The dependencies of
α
neglected. In fact, the constants
E
optical phonon with the energy
E
approximately that an electron with energy
E
numerical solution of eq. (20) can be performed in the present model.


An approximation can be made, under which the Dyson equation (20) can be solved analytically. This approximation
consists of several simplifications, as follows:

The first approximation step is the neglection of the real part of the self energy. The relaxation rate (18) is
proportional to the product of the spectral densities, which depend significantly on the imaginary part of the self-
energy. It will be seen later, that the direct comparison with the result of the numerical solution of the full equation
(21) shows, that neglecting the real part of self-energy does conserve important properties of (20).

On the basis of a simple Born approximation approach, it may be expected that for the relaxation process it holds
approximately that an electron with energy
E
1
will make a transition to the state with the energy
E
0
and emits an optical phonon with the energy
E
LO
and that the transverse part of the electron-LO-phonon coupling can therefore be neglected. In fact, the constants
α
00
and
α
11
characterizing the transverse coupling are larger than
α
01
characterizing the longitudinal interaction. The dependencies of
α
nm
on the detuning between the electronic excitation energy
E
1
and the optical phonon energy
E
LO
are shown in Fig. 1 for GaAs. It may also be expected that the imaginary part
of the self-energy, \( Y_n(E) \), is significantly nonzero near the energy of
E = E
n
. Therefore, one of the terms on the rhs of equation (21) gives the leading contribution, while the other term can be neglected. Under these approximations the Dyson equation (20) can be written in the form of the following set of equations for \( Y_n(E) \) (reminding that \( M_n(E) = R_n(E) - iY_n(E) \)):

\[
-Y_1(E) = \alpha_{01} \text{Im} \left\{ \frac{1 - N_0 + \nu_{LO}}{E - E_0 - E_{LO} + iY_0(E - E_{LO}) + 0^+} \right\},
\]

(21)

\[
-Y_0(E) = \alpha_{01} \text{Im} \left\{ \frac{N_1 + \nu_{LO}}{E - E_1 + E_{LO} + iY_1(E + E_{LO}) + 0^+} \right\}.
\]

(22)

The real part of the self-energy was neglected in this equation. The non-negative solutions of this equation set are:

\[
Y_0(E) = \sqrt{\frac{|E|}{|E - E_1 + E_{LO}|}} \sqrt{\xi - |E - E_1||E - E_{LO}|},
\]

(23)

\[
Y_1(E) = \sqrt{\frac{|E - E_1|}{|E - E_{LO}|}} \sqrt{\xi - |E - E_1||E - E_{LO}|},
\]

(24)

IV. AN ANALYTICAL SOLUTION

An approximation can be made, under which the Dyson equation (20) can be solved analytically. This approximation
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On the basis of a simple Born approximation approach, it may be expected that for the relaxation process it holds
approximately that an electron with energy \( E_1 \) will make a transition to the state with the energy \( E_0 \) and emits an optical phonon with the energy \( E_{LO} \) and that the transverse part of the electron-LO-phonon coupling can therefore be neglected. In fact, the constants \( \alpha_{00} \) and \( \alpha_{11} \) characterizing the transverse coupling are larger than \( \alpha_{01} \) characterizing the longitudinal interaction. The dependencies of \( \alpha_{nm} \) on the detuning between the electronic excitation energy \( E_1 \) and the optical phonon energy \( E_{LO} \) are shown in Fig. 1 for GaAs. It may also be expected that the imaginary part
of the self-energy, \( Y_n(E) \), is significantly nonzero near the energy of \( E = E_n \). Therefore, one of the terms on the rhs of equation (21) gives the leading contribution, while the other term can be neglected. Under these approximations the Dyson equation (20) can be written in the form of the following set of equations for \( Y_n(E) \) (reminding that \( M_n(E) = R_n(E) - iY_n(E) \)):
where \( \xi = \alpha_{01}(1 + \nu_{LO}) \) and we have put \( E_0 \) equal to zero. It is seen, that the imaginary part of the self-energy \( Y_n \) goes to zero as \( \sqrt{E} \) at the energy \( E = E_n \). Also, \( Y_1 \) diverges at \( E = E_{LO} \) and \( Y_0 \) diverges at \( E = E_1 - E_{LO} \), both like \( 1/\sqrt{E} \). The corresponding spectral densities (see (14)) then diverge at these points on the energy axis like \( \sqrt{E} \).

The plot of \( Y_0(E) \) and \( Y_1(E) \) was presented in the Rapid Communication [3] and will not be repeated here. It is demonstrated there, that some peaks in \( \sigma_n(E) \) of a state with a given \( n \) can be understood as being phonon satellites of the electronic state with \( m \neq n \). This property of the self-energy and of the spectral density guaranties, that the two spectral densities, as they occur in the formula (15) for the relaxation rate, have always a nonzero overlap, no matter how large the quantum dot is, or, in other words, how large the detuning between \( E_1 - E_0 \) and \( E_{LO} \) is. This property of the self-energy can be seen as a formal expression of the idea of the absence of the phonon bottleneck, even in the present case of electrons interacting with dispersionless phonons.

The relaxation rate \( dN_1/dt \) (at \( t > 0 \)) computed in the approximation of the equations (22) and (23) to the electronic self-energy, is shown in Fig. 3. In this Figure the relaxation rate reaches the order of \( 1/\text{ps} \) in a range of the dot size, in which \( E_{LO} - E_1 \) varies in the range of about 30 meV. The relaxation rate in Fig. 3 demonstrates, that the present mechanism of the electronic energy relaxation, with the transverse electron-LO-phonon interaction neglected, can give the relaxation times in the range of picoseconds or hundreds of femtoseconds in a rather broad range of the detuning. The double-maximum shape of the relaxation rate in Fig. 3 is probably due to the approximations introduced in the analytical solution and due to the satellite structure of the electronic spectral densities obtained in this approximation to the self-energy [31].

V. NUMERICAL SOLUTION

In this section the numerical solution of the full equation (20) is presented. That part of the electron-phonon coupling, which is proportional to the constant \( \alpha_{01} \), was shown in the previous section to give the effect of the absence of the phonon bottleneck and the relaxation times in the range of picoseconds. In the present section the coupling constants \( \alpha_{00} \) and \( \alpha_{11} \), characterizing the transverse coupling of electrons to the phonons, are taken into account and both the real and imaginary parts of the self-energy are included into the computation.

The numerical calculations are performed for GaAs, InAs and InP [24,33]. The Dyson equation (20) for the retarded self-energy is solved by iterations. In contrast to the previous section, the Eqs. (20) are solved numerically, substituting a positive finite number \( \Delta \) instead of \( \theta_+ \) in (20). We used \( \Delta = 10^{-3} \text{meV} \). The real and imaginary parts of the self-energy are presented in Fig. 3 for GaAs for the detuning chosen to be \( E_{LO} - E_{LO} = -8 \text{meV} \) at \( T_L = 77 \text{K} \). This Figure shows the rather complicated satellite structure of the electron self-energy obtained in the Tamm-Dancoff approximation. In the curves, displayed in the Figure, the phonon satellites of both the electronic ground and excited states are observed.

The spectral density of the electron in the excited state would be \( \delta(E - E_1) \) in the noninteracting case \( (H_1 = 0) \). In the interacting electron-phonon system the spectral densities \( \sigma_0(E) \) and \( \sigma_1(E) \) have a rather complex structure displayed in Fig. 4. In this Figure the numerical data are computed for the parameters of GaAs at the detuning \( E_{LO} - E_1 = -8 \text{meV} \) at \( T_L = 77 \text{K} \). Namely, the optical-phonon energy \( E_{LO} = 36.2 \text{meV} \) and the value of the energy of the excited unperturbed electronic state is chosen to be 44.2 meV. Both Figs. 4a and 4b show multiple peaks. It is seen that one of the characteristic energy separations among the individual graphs is 36.2 meV. This can be interpreted as follows: an electronic state in the quantum dot with the electron-phonon interaction appears to consist of such components, which correspond to the two unperturbed electronic states coupled to various numbers of LO-phonons. For example, the peak near \( E = 0 \) in Fig. 4a corresponds to the phonon-less \( n = 0 \) line, while the peak near 36 meV corresponds to a one-phonon satellite of the electronic ground state feature \( (n = 0) \). In the same Fig. 4a the feature near 8 meV can be seen as a one-phonon satellite of the excited state \( (n = 1) \), having the energy decreased from \( E_1 \) by \( E_{LO} \). Similar energy relations are observed in the Fig. 4b. The main peaks in Fig. 4 appear to be spread over an interval of several meV. The shape of the peaks was characterized in the previous paper [31], where it was shown, that the shape of the individual maxima in the pattern of the spectral density on the quantum dot described by the present model, do not have the form of a Lorentzian. Although the present results do not allow to compare with the known experimental data on luminescence, it may be said that the occurrence of the phonon satellites in the spectral densities is in a qualitative agreement with the experimental data on luminescence, in which luminescence maxima separated by one optical-phonon energy use to be observed (see e. g. [17,20,21]).

A considerable attention has been recently paid in experiment to the width of the luminescence lines [34–50] in connection with the coherent phonons in the nanocrystals and semiconductor quantum dots. The issue of the variation of the ”width” of a spectral line, whether it decreases or increases, with changing such parameters like the quantum dot size or the detuning, might help to judge the relevance of a particular theoretical model of the quantum dot or a nanocrystal. As it is seen in Fig. 1 and as it was discussed using analytical arguments in the reference [31], the
shape of a feature in the spectral density differs from a Lorentzian, so that it not straightforward to speak about the linewidth, at least in the case of the present model based on the LO phonons. An attention should be given to generalizing the present model in order to include the acoustic phonons and the electron-hole interaction. This question will not be further analyzed in this work.

A significant property of the spectral densities $\sigma_0(E)$ and $\sigma_1(E)$ is their mutual overlap. Namely, from Fig. 1 it is observed that despite of the 'nearly discrete' structure of these spectral densities, the function $\sigma_0(E \pm E_{LO})$ has a nonzero overlap with $\sigma_1(E)$, in a rather broad and continuous range of the detuning $E_{LO} - E_1$. This property of the spectral densities guarantees a nonzero electronic relaxation rate in the broad range of the detuning $E_{LO} - E_1$.

The numerical results giving the rate of the excited state depopulation, $dN_1/ dt$, as a function of the detuning $E_{LO} - E_1$, are displayed in Fig. 5 for GaAs. The relaxation rate is computed for the initial state with $N_1 = 1$ and $N_0 = 0$ and with two chosen temperatures of the lattice. The most intensive relaxation occurs, when the optical-phonon energy is at resonance with the excited-state energy $E_1$. In the region of small dots ($E_{LO} - E_1 < 0$) the rate displays an overall decrease with decreasing the detuning, at both temperatures. At such a detuning, which corresponds to $E_1$ equal to $2E_{LO}$, $3E_{LO}$ and $4E_{LO}$ ($E_{LO} - E_1$ equal to about $-36$ meV, $-72$ meV and $-108$ meV, respectively) further resonance maxima appear. Although the sharp peaks of the spectral densities, as they are displayed in Fig. 5, broaden when the lattice temperature is increased, the overall values of the relaxation rate decrease with increasing temperature. This decrease of the electronic relaxation rate with increasing the temperature of the lattice can be explained taking into account the nature of the electronic states determined in the self-consistent Tamm-Dancoff approximation to the electronic self-energy. The electron is coherently coupled to a number of the LO-phonons. This property of this electron-phonon system is responsible for the effect of a non-zero relaxation even at the condition of nonzero detuning $E_{LO} - E_1$. As the temperature of the lattice increases, this coherence weakens, which leads to a decrease of the relaxation rate.

It may be interesting to see, how much the process accompanied by the absorption of the LO-phonon (the term proportional to $N_1(1 - N_0)\nu_{LO}$ in eq. (18)) contributes to the total relaxation rate at $t = 0$. The contribution of the latter term is given by the dashed line in Fig. 5 while the contribution of the main term, the one proportional to $N_1(1 - N_0)(1 + \nu_{LO})$, is given by the full line. We see, that in accord with the fact that the electron-LO-phonon coupling in GaAs is rather weak, the phonon-absorption term gives only a rather small contribution to the total rate $dN_1/ dt$.

The relaxation rate reaches the maximum of the order of about 1 electron per 100 femtoseconds. This value has the order of magnitude of the experimental observations in some studied samples [18], in which the electronic relaxation rate reaches the maximum of the order of about 1 electron per 100 femtoseconds. This value has the order of magnitude of the experimental observations in some studied samples [18], in which the electronic relaxation rate reaches the maximum of the order of about 1 electron per 100 femtoseconds.

The overall efficiency of the presently considered mechanism of the electron energy relaxation depends on the strength of the Fröhlich coupling. This dependence is presented in Fig. 4, in which the relaxation rate is displayed for InAs, GaAs and InP at 77 K of the lattice temperature.

Summing up, using a single-electron model of quantum dot with two electron energy levels and with the electron coupled to dispersionless bulk optical phonons, this work presents the calculation of the electron-energy relaxation in quantum dots in polar semiconductors, at least in the case of small dots, such, in which the detuning is near to zero or negative. Let us remind, that the present theory is formulated for small dots (negative detuning). For large dots ($E_{LO} - E_1 > 0$) the present model has to be extended to include the effect of more unperturbed electron energy levels. This generalization of the model of the quantum dot is not done in this work.

The overall efficiency of the presently considered mechanism of the electron energy relaxation depends on the strength of the Fröhlich coupling. This dependence is presented in Fig. 4, in which the relaxation rate is displayed for InAs, GaAs and InP at 77 K of the lattice temperature.

Summing up, using a single-electron model of quantum dot with two electron energy levels and with the electron coupled to dispersionless bulk optical phonons, this work presents the calculation of the electron-energy relaxation in quantum dots in polar semiconductors. The relaxation time is found to be in the interval from hundreds of femtoseconds to tens of picoseconds in such quantum dots, in a broad range of the detuning, in which the energy of the electronic excited state is equals from about one LO-phonon energy to about three or four times the energy of the LO-phonon. This overall agreement of the present results with the theoretical data suggests, that the electron-LO-phonon interaction provides an effective multiphonon mechanism giving the femtosecond or picosecond electron-energy relaxation time in a rather broad range of the quantum dot size of small quantum dots. Such conclusions apply to the temperature range from low temperatures of the lattice to the room temperature.

APPENDIX A: EQUILIBRIUM CORRELATION FUNCTIONS

In this Appendix several formulas are reminded, connecting the correlation functions with the spectral density and the self-energy in an equilibrium system at nonzero temperature. In particular, an approximate formula expressing the time dependent correlation functions in terms of the self-energy, suitable for a generalization to the case of a
nonequilibrium system, is presented.

In the thermodynamic equilibrium at temperature \( T \) the time-dependent single-particle correlation functions can be spectrally decomposed in the well-known way \( [17] \) according to the formulas:

\[
< B(t') A(t) > = \frac{1}{2\pi} \int_{-\infty}^{\infty} J_{BA}(\omega) e^{i\omega(t'-t)} d\omega
\]  

(A1)

and

\[
< A(t) B(t') > = \frac{1}{2\pi} \int_{-\infty}^{\infty} J_{BA}(\omega) e^{\frac{\omega h}{\hbar} r} e^{i\omega(t'-t)} d\omega,
\]  

(A2)

where \( J_{BA}(\omega) \) is spectral intensity of the correlation function \( < B(t') A(t) > \). The spectral intensity can be obtained from the knowledge of the retarded thermodynamic Green’s function

\[
\langle A(t); B(t') \rangle_{r} = \frac{1}{i\hbar} \theta(t-t') < [A(t), B(t')]_{\eta} >,
\]  

(A3)

where the symbol \( \langle ... \rangle \) denotes the averaging performed over the grand-canonical ensemble, \( A(t) = \exp(i\bar{H}t/\hbar)A \exp(-i\bar{H}t/\hbar) \), \( \bar{H} = H - \mu N \), \( \mu \) is the chemical potential and \( N \) is operator of number of particles. Also, \( [A, B]_{\eta} = AB - \eta BA \), in which \( \eta = \pm 1 \) for Bose and Fermi particles, respectively. Defining the Fourier picture \( \langle A; B | \omega \rangle_{r} \) as

\[
\langle A(t); B(t') \rangle_{r} = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\omega(t-t')} d\omega \langle A; B | \omega \rangle_{r}
\]  

(A4)

and when the spectral intensity is real, then there is the relation between the Green’s function and the spectral intensity

\[
\text{Im} \langle A; B | \omega \rangle_{r} = -\frac{1}{2\hbar} (e^{\frac{\omega h}{\hbar} r} - \eta) J_{BA}(\omega).
\]  

(A5)

Assuming in the present work that the influence of the electron-phonon interaction on the characteristics of the phonons is only minor, the phonon correlation functions will be determined in the zero-order approximation. Realizing that \( [17] \)

\[
\langle b_{q}; b_{q}^{+} | \omega \rangle_{r} = \frac{1}{\hbar \omega - E_{LO} + i0_{r}},
\]  

(A6)

one obtains

\[
< b_{q}(t_{1}) b_{q}^{+} > + < b_{q}^{+}(t_{1}) b_{q} > = (\nu_{LO} + 1) e^{-i\omega_{LO} t_{1}} + \nu_{LO} e^{i\omega_{LO} t_{1}},
\]  

(A7)

with

\[
\nu_{LO} = \frac{1}{e^{\frac{\omega_{LO}}{T_{c}}} - 1}.
\]  

(A8)

The electronic time-dependent correlation functions will be calculated with nonzero electron-phonon coupling \( H_{1} \).

In this case the spectral intensity will be expressed in terms of the electronic self-energy. In the diagonal approximation neglecting the inter-level correlations due to the electron-phonon coupling, the retarded Green’s function of the \( n \)-th electronic state can be written as

\[
\langle c_{n}; c_{n}^{+} | \omega \rangle_{r} = \frac{1}{\hbar \omega - E_{n} + \mu - M_{n}^{(r)}(\hbar \omega) + i0_{r}},
\]  

(A9)

where \( M_{n}^{(r)}(\hbar \omega) \) is the retarded self-energy of the \( n \)-th electronic state. Working with the retarded self-energy only we shall drop the index \( (r) \) in the symbol \( M_{n}^{(r)}(\hbar \omega) \). Defining \( R_{n}(\hbar \omega) = \text{Re} M_{n}(\hbar \omega) \) we shall write \( M_{n}(\hbar \omega) = R_{n}(\hbar \omega) - iY_{n}(\hbar \omega) \). Such a decomposition of the self-energy into the real and imaginary parts allows one to work with the real and non-negative quantity \( Y_{n}(\hbar \omega) \) in what follows.

With \( (A2) \) the correlation function \( < c_{n}(t_{1}) c_{n}^{+} > \) reads:
\[ <c_n(t_1)c_n^+> = \frac{\hbar}{\pi} \int_{-\infty}^{\infty} \left( 1 - \frac{1}{e^{\frac{\hbar \omega}{T}} + 1} \right) \frac{Y_n(\hbar \omega) + 0_+}{(\hbar \omega - E_n + \mu - R_n(\hbar \omega))^2 + (Y_n(\hbar \omega) + 0_+)^2} e^{-i\omega_1 t_1} d\omega. \]  

(A10)

Let us notice, that in the zero-order approximation, in which \( R_n \) and \( Y_n \) go to zero, the expression \( \exp(\hbar \omega/k_B T) \) in the latter formula can be substituted by \( \exp((E_n + \mu)/k_B T) \). In formula (A10) we shall approximately substitute the expression \( 1/(\exp(\hbar \omega/k_B T) + 1) \) by the value of the Fermi-Dirac distribution function, giving the average number of electrons in the \( n \)-th electronic state. In this work therefore the electronic population will be approximated by

\[ N_n = \frac{1}{e^{\frac{\hbar \omega}{T}} + 1}. \]  

(A11)

Two correlation functions then are:

\[ <c_n(t_1)c_n^+> = \hbar (1 - N_n) \int_{-\infty}^{\infty} \sigma_n(\omega) e^{-i\omega t_1} d\omega, \]  

(A12)

\[ <c_n^+(t_1)c_n> = \hbar N_n \int_{-\infty}^{\infty} \sigma_n(\omega) e^{i\omega t_1} d\omega, \]  

(A13)

where the spectral density \( \sigma_n(E) \) is

\[ \sigma_n(\omega) = \frac{1}{\pi} \frac{Y_n(\hbar \omega) + 0_+}{(\hbar \omega - E_n + \mu - R_n(\hbar \omega))^2 + (Y_n(\hbar \omega) + 0_+)^2}, \]  

(A14)

going to the \( \delta \)-function in the limit of zero self-energy and fulfilling the rule

\[ 1 = \hbar \int_{-\infty}^{\infty} \sigma_n(\omega) d\omega, \]  

(A15)

for \( n = 0, 1 \).

The formulas (A7), (A12) and (A13) provide the dependence of the spectral densities on the temperatures, or on the populations, of the electrons and phonons, within the theory of the real time thermodynamic Green’s functions. These functional dependences will be assumed to be valid for the correlation functions, which we meet in the formula (13), in the general case of nonequilibrium state of the system. In this way, the electronic relaxation rate (18) can be expressed in terms of the electronic spectral density and in terms of the electronic self-energy.

The above given method of obtaining the single-particle correlation functions in the nonequilibrium system under study is accompanied by an inaccuracy, which may depend on the strength of the electron-phonon coupling. The reason for this is obvious upon comparing the statistical operators, with the help of which the averaging is performed. In the case of the thermodynamical equilibrium the averaging is performed in the grandcanonical ensemble of the system with the electron-phonon interaction included, while in the averages like \( <c_i^+(t)c_j>_q \), appearing in the formula (14), the quasi-equilibrium statistical operator does not depend on the electron-phonon interaction, at least in the approximation we assumed in this work. This question can be clarified with the use of a more systematic approach to this nonequilibrium system.

**APPENDIX B: ELECTRONIC SELF-ENERGY**

Using the language of the Feynman diagrams for the Matsubara Green’s functions [57, 58], the self-consistent Tamm-Dancoff approximation (or the self-consistent Born approximation) to the electronic self-energy can be expressed as a diagram containing two bare interaction vertexes connected with two lines, one of them corresponds to the full electronic Green’s function \( G \), while the other corresponds to the bare phonon Green’s function \( D^{(0)} \). The equation for the self-energy of the electron in the \( n \)-th state, \( M_n(i\hbar \omega_p) \), defined on the set of imaginary frequencies \( \omega_p = (2n + 1)\frac{\hbar \omega_1}{\hbar} \), \( n \) is integer, is

\[ M_n(i\hbar \omega_p) = k_B T \sum_q A_q |\Phi(m, n, q)|^2 \sum_r G_m(i\hbar \omega_p - i\hbar \omega_r) D^{(0)}(i\hbar \omega_r). \]  

(B1)
Using the Lehmann’s representation of the Green’s functions, the summation over the discrete imaginary frequencies \( \nu_r = 2r(\pi k_B T / \hbar) \), \( r \) being an integer, can be performed. In this way the equation for the retarded electronic self-energy in the state \( m \) of is obtained:

\[
M_n(E) = \sum_m \alpha_{nm} \left[ G_m^{ret}(E - E_{LO})(1 - n_F(E - E_{LO}) + \nu_{LO})
+ G_m^{ret}(E + E_{LO})(n_F(E + E_{LO}) + \nu_{LO}) \right],
\]

where \( n_F(E) = 1/(exp(E/K_B T_e) + 1) \) and \( \nu_{LO} = 1/(exp(E_{LO}/k_B T_L) - 1) \) Here \( G^{ret} \) denotes the retarded electronic Green’s function. Note, that in the latter equation the electronic Fermi-Dirac distribution function \( n_F(E) \) appears as a function of the energy variable \( E \). Assuming the knowledge of the electronic temperature, this function would present no difficulty. However, in this work we approximate these electronic distributions by the Fermi-Dirac distribution function \( N_m \), which is the value of the Fermi-Dirac distribution function in the state with the unperturbed electronic energy \( E_m \), at which the corresponding retarded Green’s function in the latter equation (B2) would have the pole in the case of no electron-phonon coupling. In this way the treatment of the electronic temperature can be avoided.

The equation for the retarded self-energy is \( (E_{LO} = \hbar \omega_{LO}) \):

\[
M_n(E) = \sum_m \alpha_{nm} \left[ \frac{1 - N_m + \nu_{LO}}{E - E_m - E_{LO} - M_m(E - E_{LO}) + i0^+}
+ \frac{N_m + \nu_{LO}}{E - E_m + E_{LO} - M_m(E + E_{LO}) + i0^+} \right].
\]

The equation (B3) closely resembles the similar equation for the self-energy of Frenkel excitons studied earlier. In the reference this equation is derived with help of the real-time thermodynamic Green’s functions.

In (B3) the chemical potential is not written. This form of the equation corresponds to the choice of the Hamiltonian \( H \) in the equation of motion for the Green’s function, instead of the Hamiltonian \( \hat{H} = H - \mu N \), although the statistical averaging is still performed with the grand-canonical ensemble. Consistently with this choice of Hamiltonian, the chemical potential \( \mu \) is dropped in eq. (A14).

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FIG. 1. The dependence of the coupling constants $\alpha_{ij}$ in GaAs on the detuning between the optical phonon energy $E_{LO}$ and the excited state energy $E_1$. 
FIG. 2. The dependence of the electronic relaxation rate ($-dN_1/dt$) on the detuning $E_{LO} - E_1$ as computed for the analytical solution of the simplified Dyson equation (with the transverse coupling terms neglected) at the temperatures of the lattice $T_L = 77$ K.
FIG. 3. Real and imaginary parts of electronic self-energy in the states with $n = 0, 1$ for the detuning $E_{LO} - E_1 = -8$ meV in GaAs at 77 K. The full and dashed lines denote, respectively, the imaginary part of the self-energy ($-\text{Im} M_0$) and ($-\text{Im} M_1$), while the dashed (dotted) line denotes $\text{Re} M_0$ ($\text{Re} M_1$).
FIG. 4. Spectral densities $\sigma_0(E)$ (a) and $\sigma_1(E)$ (b) in GaAs computed for the detuning $E_{LO} - E_1 = -8\text{meV}$, at the lattice temperature of 77 K (full line) and at the room temperature (dashed line). Note that the $E$–axis of graph (b) is shifted by the energy of optical phonon with respect to graph (a).
FIG. 5. Relaxation rate ($-dN_1/dt$) in GaAs as a function of the detuning $E_{LO}-E_1$ at two temperatures of the lattice.
FIG. 6. Two contributions to the total relaxation rate \((-dN_1/dt)\) in GaAs at \(T_L = 300\) K. The full line gives the contribution of the process accompanied by the emission of LO phonon, while the dashed line gives the one of the process with LO phonon absorption.
FIG. 7. A comparison of the relaxation rate ($-dN_1/dt$) at $T_L = 77$ K in three semiconducting materials.