Comment on “Phase-shift analysis of NN scattering below 160 MeV: Indication of a strong tensor force”

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Abstract

In his recent publication of a NN phase shift analysis below 160 MeV, Henneck reports relatively large values for the mixing parameter $\epsilon_1$. Based on these results, Henneck suggests that the strength of the $\rho$-meson tensor coupling to the nucleon may be weaker than used in present day NN interactions, like the Paris or Bonn potentials. We point out that at low energies ($\lesssim 100$ MeV) there is very little sensitivity to the strength of the $\rho$ coupling, due to the compensating effect
of the second order tensor term. In order to establish sensitivity, one has to go to energies \( \gtrsim 200 \text{ MeV} \), where the second order contribution has gone out. As it happens, the \( \epsilon_1 \) mixing parameter is well determined in the region of energies 200–300 MeV, and there is agreement with the predictions by the Paris and Bonn potentials; whereas the weak-\( \rho \) model is about 50\% above the data. This and additional considerations in triplet \( P \)-waves re-confirm that NN scattering requires the strong \( \rho \), consistent with the \( \pi\pi - N\bar{N} \) partial-wave analysis by Höhler and Pietarinen.

Following the measurement of the longitudinal spin correlation coefficient \( A_{zz} \) at 67.5 MeV by the Basel group \[1\], there has been a lot of controversy about the strength of the tensor force in the nucleon-nucleon (NN) interaction and about the strength of the \( \rho \)-meson tensor coupling.

Henneck has published a recent paper: “Phase-shift analysis of NN scattering below 160 MeV: Indication of a strong tensor force” \[2\]. In a fundamental sense, it is unreasonable to include “Indication of a strong tensor force” in a title. Since half a century, we all know from the deuteron that there is a strong tensor force.

In both Refs. \[1\] and \[2\] the implication is that the tensor coupling of the
\( \rho \)-meson must be weaker than used in present day NN interactions: Bonn [3, 4] or Paris [5]; more like the vector-dominance value.

In contradiction to these claims, Klomp et al. [6] and Machleidt and Slaus [7] show from phase shift fits, that the Bonn and Paris potentials do well in reproducing the \( S-D \) mixing parameter \( \epsilon_1 \), about which the controversy revolves, cf. Fig. 1.

Moreover, a recent paper by Wilburn et al. [12] shows Henneck’s value of \( \epsilon_1 \), arrived at from his phase shift analysis, to lie exactly on the Bonn curve at 25 MeV, and the low-energy measurements of \( \epsilon_1 \) to lie nicely (aside from a deviation the authors do not believe to be real) and to follow well the Bonn curve.

By weak and strong \( \rho NN \) coupling we shall mean \( \kappa_\rho = 3.7 \) and \( \kappa_\rho \approx 6 \), respectively; where \( \kappa_\rho \equiv f_\rho/g_\rho \), the ratio of the tensor to vector coupling constant. The weak \( \rho \)-coupling is arrived at by using vector dominance [13]; the strong \( \rho \)-coupling is obtained from a \( \pi\pi - N\bar{N} \) partial-wave analysis conducted by Höhler and Pietarinen [14]. Later work by Grein [15] basically confirmed the Höhler-Pietarinen result.

The usual argument about the connection of the strength of the tensor interaction and that of the \( \rho \)-meson coupling goes as follows:
The $\rho$-exchange tensor interaction has opposite sign to that from pion exchange [16, 4]. Therefore, summing up the $\rho$-exchange will decrease the tensor force. However, this argument is too simplistic as was realized long ago.

If the $\rho$-exchange tensor interaction is weak, of the vector dominance value, then the tensor force will be strong, because not much of the pion contribution is cancelled (see Fig. 8a of Ref. [16]). In this case, second-order effects of the tensor interaction will be strong [Eq. (2.10) of Ref. [16]] and one has an effective tensor contribution arising from these second order terms of

$$V_{\text{eff}}(r) = \frac{3 - 2\mathbf{\tau}_1 \cdot \mathbf{\tau}_2}{E} 2S_{12}(V_{\text{tensor}}^{(1)})^2$$  \hspace{1cm} (1)

where the average energy $\bar{E} \approx 200$ MeV. Note that the $\mathbf{\tau}_1 \cdot \mathbf{\tau}_2$ piece is negative, of opposite sign to the $\pi$-exchange tensor. This is, of course, easy to understand, because in the iterated exchange one has a box diagram, and the two pions in the crossed channel must, while the interaction is isovector, be in a $P$-wave because of Bose statistics. Consequently, a tensor interaction with weak-$\rho$ coupling builds up an effective $\rho$-meson type coupling, strengthening the $\rho$-channel. Once second-order effects are included, the net result is little different for weak- and strong-$\rho$ coupling, although different amounts of the
effective $\rho$ (iterated pion exchange) result in the two cases.

All potentials, with weak or strong $\rho$-tensor coupling, were constrained to fit the deuteron. Thus, it is no surprise that Henneck’s point at 25 MeV lies on the Bonn curve [12].

Our conclusion is that it is doubtful whether low-energy scattering experiments distinguish between weak and strong $\rho$-couplings. In going to higher energies, the second order term, Eq. (1), will tend to go out. How high in energy does one have to go?

In order to answer this question, let us remember that the main part of the symmetry energy in nuclei comes from the second order tensor interaction

$$V_{sym} \approx +\frac{12}{E}(V_{tensor}^{(1)})^2,$$

as one can deduce from Eq. (2.10) of Ref. [16]. Brown, Speth, and Wambach [17] showed that the $V_\tau$ so obtained dropped over a scale of energies 100 to 200 MeV, as the incident nucleon energy eats into the principal value integral. Of course, some of the symmetry energy comes from the vector coupling of the $\rho$-meson, but one can see directly from the work of Ref. [17] that this is small; very little of the interaction drops at the slow rate that $V_{\sigma\tau}$, which comes mainly from the Born term, drops.
The work of Ref. [17] employed a strong-\(\rho\) tensor coupling, giving a relatively weak \(V^{(1)}_{\text{tensor}}\). The calculations are straightforward, and would have given a much greater symmetry energy had a weak-\(\rho\) coupling been used.

Our first statement, which can be directly tested by calculations, is that the symmetry energy in nuclear matter calculations will be much too large if a weak-\(\rho\) coupling is used. Equivalently, the \(V_\tau\) used in investigations of isobaric analog states will be much too large.

Calculations of these matters introduce various many-body intermediate steps, which lead to suspicion in the views of experimentalists. However, the work of Ref. [17] points out that if one wants to see differences between the weak-\(\rho\) and strong-\(\rho\) scenarios, one should go to scattering energies \(E > 100\) MeV, by which time some of the second-order contribution of the tensor interaction is stripped off. Energies of 200 MeV would be better.

Our conclusion is that it would be safest to determine \(\epsilon_1\) at energies \(\gtrsim 200\) MeV where the second order contributions from the tensor interaction have been stripped off.

The Henneck \(\epsilon_1\) at 50 MeV is about 2.8\(^0\), what one would obtain with zero \(\rho\)-coupling (cf. Fig. 1). The Reid potential [8], with essentially vector dominance coupling, gives \(\approx 2.4^0\), whereas Paris [5] and Bonn [3, 4] (which
both use the strong \( \rho \) give \( \lesssim 2^0 \). Note that the square of the tensor coupling of the \( \rho \) has increased a factor of more than 2.5 in going from vector dominance to strong-\( \rho \) coupling, with a change of \( \approx 0.4^0 \) in \( \epsilon_1 \). This is what we mean by insensitivity at low energy. Quoted error in Henneck is \( \pm 0.25^0 \).

Since Reid fits the deuteron, we believe that this potential gives a good indication of what weak-\( \rho \) coupling will give in the 200-300 MeV region, once the second-order contribution to the effective-\( \rho \) exchange has gone out. Reid gives \( \epsilon_1 \approx 7^0 \) in this region; while Paris and Bonn predict \( \approx 4^0 \), in perfect agreement with the phase shift analyses by Arndt [9], Bugg and Bryan [10], and by the Nijmegen group [11] (cf. Fig 1). This confirms that NN scattering requires the strong \( \rho \).

We note that there are phase parameters which are even more suitable than \( \epsilon_1 \) to pin down the \( \rho \)-coupling strength, namely, the the triplet \( P \)-wave phase shifts. In contrast to \( \epsilon_1 \), the \( ^3 P_J \) phase shifts are reliably determined and there is no controversy among different researchers conducting phase shift analyses. Moreover, the effect of the \( \rho \)-meson is very large in \( P \)-waves. For \( ^3 P_0 \) and \( ^3 P_2 \), we demonstrate this in Fig. 2, where the predictions by the weak and the strong \( \rho \) are shown. It is clearly seen that these \( P \)-waves require by all means the strong \( \rho \). This is probably the best argument why NN scat-
tering needs the large $\rho$ tensor coupling, consistent with the determination by Höhler and Pietarinen \cite{14} in 1975.

Höhler \cite{19} has recently reviewed developments since the Höhler-Pietarinen work. He points out that a somewhat different method led to $\kappa_\rho = 6.1 \pm 0.6$ \cite{20}. Furthermore, the large Höhler-Pietarinen value agrees with an unpublished calculation by Gustafson, Nielsen, and Oades \cite{21}. We mentioned the work by Grein \cite{15} using NN forward dispersion relations, which is compatible with Höhler-Pietarinen. The chief point of Höhler \cite{19} is that the full information on the $\rho NN$ coupling is contained in the $\pi\pi - N\bar{N}$ $P$-wave helicity amplitudes and the direct way to extract coupling constants in an approximate description was employed in Ref. \cite{14}. These helicity amplitudes were obtained from $\pi N$ partial wave amplitudes with imposition of unitarity and analyticity by Höhler and Pietarinen. Work since that time has not consistently enforced these constraints \cite{19}.

In conclusion, we have shown that at low energies ($\lesssim 100$ MeV) there is very little sensitivity to the strength of the $\rho NN$ coupling. In order to establish sensitivity, one has to go to higher energies ($\gtrsim 200$ MeV). Phase shift analyses here clearly favor the strong $\rho NN$ coupling. Independently of this empirical argument, the strong $\rho NN$ coupling was firmly established in
1975 from detailed knowledge of the $\pi\pi - N\bar{N}$ helicity amplitudes [14].

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Figure Captions

Figure 1. The $\epsilon_1$ mixing parameter at low and intermediate energies. Predictions by the Bonn-B [4] (solid line), Paris [3] (solid line), and Reid potential [8] (dashed), as well as a model that does not include a $\rho$ meson ('No $\rho$', dotted line) are shown. The phase shift analysis by Henneck [2] is represented by the diamonds. Furthermore, the analyses by Arndt [9] (solid triangles), Bugg and Bryan [10] (solid dots), and the Nijmegen group [11] (solid squares) are displayed.

Figure 2. The $^3P_0$ and $^3P_2$ phase shifts of proton-proton scattering. The solid line gives the prediction by a meson model that includes the strong $\rho$, while the dashed line is obtained using the weak $\rho$. The solid dots represent the Nijmegen $pp$ multi-energy phase shift analysis [11]. (From Ref. [18].)

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