Stability derivatives of an oscillating wedge in viscous hypersonic flow

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Abstract. In this paper an oscillating wedge has been considered, and the fluid slabs are kept at 90° to the wedge surface. The solutions of the continuity, momentum, and energy equations are obtained. By using the Rankine-Hugoniot relations for shockwaves, we can find the conditions behind the shock. This theory is unsteady one because of the consideration of effect of secondary wave reflections. Solutions are obtained for hypersonic flow over the wedge by varying different wedge semi vertex angles. These results show extremely good consistency with Hu's predictions. When the effects of unsteadiness are considered then there is considerable change in the magnitude of the damping derivatives near the leading edge or initial 40 percent of the pivot positions and this difference is only marginal when we further down towards the trailing edge. However, this effect of unsteadiness is not visible in case of the stiffness derivatives. It is observed that the stiffness derivative increases with the increase in the wedge angle due to the increase in the plan form area of the wedge, resulting in the variation in the surface pressure distribution of the wedge. Further, due to the increment in the wedge angle the centre of pressure shifts towards the trailing edge. Keywords: Angle of attack, hypersonic flow, Mach number, oscillating wedge

1. Introduction
The knowledge of hypersonic flow over an oscillating wedge is a key to investigate the results related to spacecraft (shuttle) and re-entry body. Analytical study of unsteady flow with a small disturbance approach shows excellent results with the test done in the laboratory. So that we can avoid wastage of time and high investment for the experiment.

Appleton included secondary wave effects to analyse the problems connected to 2-D oscillating wedges. To improve an unsteady piston theory of an oscillating wedge, Appleton theory collaborated with Ghosh's large deflection similarity.

2. Literature Survey
Investigation of Tsien's [1] work was limited to conservative motion for unsteady hypersonic flow. Hayes and Probstein [5, 10] invented that, the equivalence principle proved by Hayes is applicable to both steady and unsteady flow. Hamaker and Wong [3] showed complete dynamic similarity for symmetric body. Light hill developed a theory on oscillating airfoils in hypersonic flow, which
includes pitching oscillations. Light hill [4] introduced a parameter $\delta$ to measure maximum inclination angle of Mach wave in the flow field. He assumed $M_1 \delta \leq 1$, which is of the order of maximum deflection of a streamline. Light hill [4] combined the two-dimensional unsteady flow problem to that of a gas flow in a tube driven by a piston which leads to “Piston Analogy”. In Lighthill piston theory, the concept of secondary wave effects has not been considered. McIntosh [8] and Appleton [7] have included the effects of secondary wave reflected from the shock and showed independently that, the neglect of these effects provides a reduction to the strong shock piston theory. Ghosh and Mistry [11] studied the flow over an oscillating two-dimensional wedge and developed a theory on quasi steady flow. Same concept has been used by Ghosh [12] to solve problems related to axis-symmetric flow. Rasta and Khan [13] extended Ghosh’s theory for attached shock.

3. Analysis

In the isentropic process, expression for the pressure on a piston as a power series in its velocity is given by,

$$\frac{P_2}{P_1} = 1 + \gamma \frac{U_p}{a_1} + \frac{\gamma (\gamma + 1)}{4} \left( \frac{U_p}{a_1} \right)^2 + \frac{\gamma (\gamma + 1)}{12} \left( \frac{U_p}{a_1} \right)^3 + \ldots$$

where $P_1$ & $P_2$ represents pressure ahead and after the shock in the fluid slab respectively.

Consider the governing equations:

Continuity equation: $\frac{\partial p}{\partial \tau} + \frac{\partial}{\partial y}(\rho \nu) = 0$

Momentum equation: $\rho \left( \frac{\partial \nu}{\partial \tau} + \nu \frac{\partial \nu}{\partial y} \right) + \frac{\partial p}{\partial y} = 0$

Energy equation: $\left( \frac{\partial}{\partial \tau} + \nu \frac{\partial}{\partial y} \right) \ln \left( \frac{p}{\rho_0} \right) = 0$

where $\rho$ is the density, $p$ is the pressure and, $\nu$ is the velocity component in the y direction and $\tau$ is the time which is measured from the instant the fluid slab reaches the leading edge of the wedge. The piston velocity of the wedge is given by

$$\nu_p = U_\infty \sin(\theta + \delta) + \dot{\theta}(x - hL)$$

where $L$ is the length of the flat plate, $hL$ is the position of the pivot point and $\theta$ is the angular displacement, which is positive for the nose down movement of the plate.

Pressure perturbation over the piston is given by,

$$\bar{P}_p = \rho_2 a_2 U_\infty \left[ \left( 1 + \frac{w}{1 - w} \right) \cos \delta + \left( 2 \frac{1 + w \delta'}{1 - w \delta'} - \frac{1 + w}{1 - w} \right) \frac{x}{L} - \left( \frac{1 + w}{1 - w} \right) h \frac{\dot{\theta}_L}{U_\infty} \right]$$

Restoring moment over the plate is given by

$$-M_r = \int_0^L (x - hL) \bar{P}_p dx$$

The stiffness derivative and damping derivatives of the plates in the transferred axes,
Non-dimensionalizing must be done with respect to $\frac{1}{2} \rho_\infty U_\infty^2 C^2$ and $\frac{1}{2} \rho_\infty U_\infty^2 C^3$.

The stiffness derivative and damping derivatives of the plates reduces to the following expressions.

\[
-C_{m_\theta} = \frac{1}{2} \rho_\infty U_\infty^2 C^2 \left( -\frac{\partial M_r}{\partial \theta} \right)
\]

\[
-C_{\dot{m}_\theta} = \frac{1}{2} \rho_\infty U_\infty^2 C^2 \left( -\frac{\partial \dot{M}_r}{\partial \theta} \right)
\]

Since the stiffness and damping derivatives for the wedges are equivalent to those of two flat plates, the RHS of above two equations are multiplied by 2. Therefore, the stiffness derivative of the wedge is

\[
-C_{m_\theta} = \frac{2 \rho_\infty a_2}{\rho_\infty U_\infty \cos \delta} \left[ \frac{1}{2} \left( \frac{1 + w}{1 - w} \right) - \left( \frac{1 + w}{1 - w} \right) h' \cos^2 \delta \right]
\]

And the damping derivative of the wedge is

\[
-C_{\dot{m}_\theta} = \frac{2 \rho_\infty a_2}{\rho_\infty U_\infty \cos^3 \delta} \left[ \frac{1}{3} \left( \frac{2}{1 - w} \frac{1 + w \delta'}{1 - w \delta'} - \frac{1 + w}{1 - w} \right) - \left( \frac{1 + w \delta'}{1 - w \delta'} \right) h' \cos^2 \delta + \left( \frac{1 + w}{1 - w} \right) h'^2 \cos^4 \delta \right]
\]

4. Results and Discussions

![Figure 1. Stiffness Derivative Vs pivot position at M=10.](image1)

![Figure 2. Stiffness Derivative Vs pivot position at M=12.](image2)
The stiffness derivatives of the oscillating wedge of different wedge semi vertex angles have been calculated by using Ghosh and Mistry (1980) and the present theory. Figures 1 to 3 shows the change in stiffness derivatives of oscillating wedge for various semi vertex angles at $\delta = 100, 120, 150, 200, 250, 300$ with respect to pivot point position by varying Mach numbers from $M = 10, M = 12, M = 14$. It is seen that increase in the value of semi vertex angle results in increase in the value of stiffness derivative up to $M = 14$. From Figures 4 to 7 it is observed that the magnitude of stiffness starts decreasing for $M = 15, M = 16, M = 18$ and $M = 20$, but there is no much variation. Thus, it satisfies Mach number independence principle. The stiffness value is maximum at the nose of the wedge in the initial stage and then it starts decrease linearly with respect to the pivot position.
Figures 8 to 14 shows the variation of damping derivatives with respect to pivot positions from 0 to 1 for Mach numbers $M = 10, 12, 14, 16, 18$ and $20$ respectively for different semi-vertex angles from $\delta = 10$ to $30$. In damping case, we can observe that magnitude increase with increase in wedge semi vertex angle. There is no much difference in the magnitude of damping derivative with pivot point position for Mach numbers $M = 15$ to $20$, thus it holds Mach number independence principle. Figure 15 compares the results for planar wedge with Ghosh and Mistry (1980) theory for $M = 100$, $\delta = 150$. Discrepancies in the present theory with that of Ghosh and Mistry (1980) theory for the plane wedge are attributed to the considerations of secondary wave reflections and the unsteady effects in the present theory. Also, the stiffness and damping derivatives estimated by the Ghosh and Mistry (1980) were quasi-steady whereas, the present theory evaluates unsteady stiffness and damping derivatives. One of the advantage of the present theory is that by subtracting the damping derivative by quasi-steady method from unsteady damping derivative to obtain damping derivatives due rate of change of angle of attack, which is the only way to evaluate the contribution to damping derivative in pitch due to rate of change of angle of attack. The effect of secondary waves used to increase the
damping for forward pivot position and decrease it for rearward position. This trend is attributed due to the variation in the surface pressure distribution of the wedge.

Figure 10. Damping derivative Vs pivot position at M=14.

Figure 11. Damping derivative Vs pivot position at M=15.

Figure 12. Damping derivative Vs pivot position at M=16.

Figure 13. Damping derivative Vs pivot position at M=18.
5. Conclusions

- The unsteady effect and secondary wave reflections has been considered in the present theory. When the effects of unsteadiness are considered then there is considerable change in the magnitude of the damping derivatives near the leading edge or initial 40 percent of the pivot positions and this difference is only marginal when we further down towards the trailing edge. However, this effect of unsteadiness is not visible in case of the stiffness derivatives.

- From the results it is observed that the stiffness derivative increases with the increase in the wedge angle due to the increase in the plan form area of the wedge, resulting in the variation in the surface pressure distribution of the wedge.

- Due to the increment in the wedge angle the center of pressure shifts towards the trailing edge.

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