Abstract

We obtain a (5+1)-dimensional global flat embedding of the Gibbons-Maeda-Garfinkle-Horowitz-Strominger spacetime in Einstein frame, and a (5+2)-dimensional global flat embedding in string frame. We show that the local free-fall temperatures for freely falling observers in each frames are finite at the event horizons, while the local temperatures for fiducial observers are divergent. We also observe that the local free-fall temperatures differ between the two frames.

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1 Introduction

Hawking discovered that a black hole emits thermal radiation with characteristic Hawking temperature $T_H$ as seen by asymptotic observers [1]. The local temperature measured by a fiducial observer at a finite distance from a black hole is then described by the Tolman temperature [2]

$$T_{\text{FID}} = \frac{T_H}{\sqrt{-g_{\mu\nu}\xi^\mu\xi^\nu}}, \quad (1.1)$$

where $\xi^\mu$ is a timelike Killing vector. Later, Unruh [3] showed that a uniformly accelerated observer in flat spacetime, with proper acceleration $a$, will detect thermal radiation at the Unruh temperature

$$T_U = \frac{a}{2\pi}. \quad (1.2)$$

These two effects are related, i.e., the Hawking effect for a fiducial observer in a black hole spacetime can be considered as the Unruh effect for a uniformly accelerated observer in a higher dimensional global embedding Minkowskian spacetime (GEMS). After confirming these ideas in an analysis of de Sitter (dS) [4] and anti-de Sitter (AdS) spacetimes [5] and their corresponding GEMSs, Deser and Levin [7] have shown that the GEMS approach provides a unified derivation of temperature for Bañados-Teitelboim-Zanelli, Schwarzschild-AdS(-dS), and Reissner-Nordström (RN) spacetimes. These results have since been extended to a wide variety of curved spacetimes [8, 9, 10, 11, 12, 13, 14, 15, 16], and also see references therein. Recently, Brynjolfsson and Thorlacius [17] have used the GEMS approach to define a local temperature for a freely falling observer outside Schwarzschild(-AdS) and RN spacetimes, and we have extended the results to RN-AdS spacetime [18]. In particular, here a local free-fall temperature is defined at special turning points of radial geodesics where a freely falling observer is momentarily at rest with respect to a black hole. It was shown that the local free-fall temperature approaches the Hawking temperature at spatial infinity, while it remains finite at the event horizon.

On the other hand, the spherically symmetric static charged black hole solution in low energy effective theory of heterotic string theory in four dimension was found by Gibbons, Maeda [19], and independently, by Garfinkle, Horowitz, Strominger [20], by turning antisymmetric tensor gauge fields off, from now on refereed to as the Gibbons-Maeda-Garfinkle-Horowitz-Strominger
(GMGHS) solution. See also [21, 22, 23, 24]. After these works, there were enormous interests in the spherically symmetric static charged black holes [25, 26, 27, 28, 29, 30, 31, 32, 33, 34] and see also references therein. In particular, this GMGHS black hole spacetime can be described by the solutions either in Einstein or string frames. In Einstein frame, the action is in the form of the Einstein-Hilbert action, while in string frame strings directly couple to the metric of $e^{2\phi}g_{\mu\nu}$ where $e^{2\phi}$ is a conformal factor and $g_{\mu\nu}$ is the Einstein metric. Even though the solutions in the two frames are related by a conformal transformation so that they are mathematically isomorphic to each other [35], however, there are differences in some of the physical properties of the black hole solutions in the two frames [36].

In this paper, we wish to study the GEMSs of the GMGHS black hole spacetimes both in the Einstein and string frames, and investigate how different GEMS embeddings are in the two frames. The GEMS approach is also important on its own, since it gives a powerful tool that simplifies the study of black hole physics by working instead, but equivalently, in an accelerated frame in a higher dimensional flat spacetime. In section 2, we briefly review the structure of the GEMS of the curved Schwarzschild and RN spacetimes, and their local free-fall temperatures. In section 3, we apply this GEMS approach to the charged GMGHS black hole spacetimes in the Einstein frame. In section 4, we also embed both the magnetically and electrically charged GMGHS black hole spacetimes in the string frame into higher-dimensional flat spacetimes, and find their local free-fall temperatures. Our conclusions are drawn in section 5.

2 Schwarzschild and RN spacetimes

2.1 Schwarzschild spacetime

The Schwarzschild spacetime described by the metric

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$ (2.1)

can be embedded into a (5+1)-dimensional Minkowskian spacetime

$$ds^2 = \eta_{IJ}z^Iz^J$$ (2.2)
with a metric $\eta_{IJ} = \text{diag}(-1, 1, 1, 1, 1, 1)$. Explicitly, the (5+1)-dimensional Minkowskian spacetime is given by the transformations [37]

$$z^0 = \frac{1}{k_H} \sqrt{1 - \frac{2M}{r}} \sinh(k_H t), \quad (2.3)$$

$$z^1 = \frac{1}{k_H} \sqrt{1 - \frac{2M}{r}} \cosh(k_H t), \quad (2.4)$$

$$z^2 = \int dr \sqrt{\frac{2M(r^2 + 2Mr + 4M^2)}{r^3}}, \quad (2.5)$$

$$z^3 = r \sin \theta \cos \phi, \quad (2.6)$$

$$z^4 = r \sin \theta \sin \phi, \quad (2.7)$$

$$z^5 = r \cos \theta. \quad (2.8)$$

Note that $k_H = \frac{1}{4M}$ is the surface gravity and the event horizon $r_H$ is $2M$. An observer, who is uniformly accelerated in the (5+1)-dimensional flat spacetime, follows a hyperbolic trajectory described by proper acceleration

$$a_6^{-2} = (z^1)^2 - (z^0)^2 = 16M^2 \left(1 - \frac{2M}{r}\right). \quad (2.9)$$

Thus, as was shown by Unruh [3], the Unruh temperature can be read as

$$T_U = \frac{a_6}{2\pi} = \frac{1}{8\pi M \sqrt{1 - \frac{2M}{r}}}. \quad (2.10)$$

In fact, this corresponds to the local temperature measured by a fiducial observer at a finite distance from the black hole, also called the fiducial temperature

$$T_{\text{FID}} = \frac{T_H}{\sqrt{-g_{00}}}, \quad (2.11)$$

where the Hawking temperature $T_H$ measured by an asymptotic observer is

$$T_H = \frac{1}{8\pi M}. \quad (2.12)$$

As results, the Hawking effect for a fiducial observer in the black hole spacetime can be said to be the Unruh effect for a uniformly accelerated observer in a higher-dimensional flat spacetime.
Now, consider a freely falling observer who is dropped from rest at $r = r_0$ and at $\tau = 0$. For a freely falling observer, there are turning points [17] of radial geodesics where a freely falling observer is momentarily at rest with respect to black holes. Making use of the orbit equations [18]

$$\frac{dt}{d\tau} = \sqrt{f(r_0)} \frac{f(r)}{f(r)}, \quad \frac{dr}{d\tau} = -\sqrt{f(r_0)} - f(r), \quad \text{with} \quad f(r) = 1 - \frac{2M}{r}, \quad (2.13)$$

one can obtain the $\tilde{a}_6$ acceleration given by

$$(\tilde{a}_6)^2 = \frac{r^3 + 2Mr^2 + 4M^2r + 8M^3}{16M^2r^3}. \quad (2.14)$$

Taking the local free-fall temperature

$$T_{\text{FFAR}} = \frac{\tilde{a}_6}{2\pi} \quad (2.15)$$

measured by the freely falling observer at rest (FFAR) to be the local Unruh temperature, one obtains

$$T_{\text{FFAR}} = \frac{1}{8\pi M} \sqrt{\frac{r^3 + 2Mr^2 + 4M^2r + 8M^3}{r^3}}, \quad (2.16)$$

which is reduced to the Hawking temperature $T_H$ at infinity. It is important to note that the local free-fall temperature at the event horizon is finite as $T_{\text{FFAR}} = 2T_H$, while the local temperature $T_{\text{FID}}$ (2.11) for the fiducial observer diverges as $r \to r_H$.

### 2.2 RN spacetime

In order to embed the charged RN black hole given by

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (2.17)$$

into the higher dimensional flat spacetime, one needs to introduce one more time dimension, compared with the GEMS embedding of the Schwarzschild spacetime. As a result, the embedded flat spacetime is described by the $(5+2)$-dimensional Minkowskian spacetime

$$ds^2 = \eta_{IJ}z^Iz^J \quad (2.18)$$
with a metric $\eta_{IJ} = \text{diag}(-1, 1, 1, 1, 1, -1)$ given by the transformations \[7, 8\]

\[
\begin{align*}
    z^0 &= \frac{1}{k_H} \sqrt{1 - \frac{2M}{r} + \frac{Q^2}{r^2}} \sinh(k_H t), \\
    z^1 &= \frac{1}{k_H} \sqrt{1 - \frac{2M}{r} + \frac{Q^2}{r^2}} \cosh(k_H t), \\
    z^2 &= \int dr \, \sqrt{\frac{2Mr^2 + r_H^2(r + r_H)}{r^2(r - M + \sqrt{M^2 - Q^2})}}, \\
    z^3 &= r \sin \theta \cos \phi, \\
    z^4 &= r \sin \theta \sin \phi, \\
    z^5 &= r \cos \theta, \\
    z^6 &= \int dr \, \sqrt{\frac{Q^2 r_H^4}{r^4(M^2 - Q^2)}},
\end{align*}
\]

where $r_H(= M + \sqrt{M^2 - Q^2})$ denotes the outer horizon and the surface gravity is $k_H = \sqrt{M^2 - Q^2}/r_H^2$. Note that since $z^6 \to 0$ in the $Q \to 0$ limit, the transformations are reduced to the Schwarzschild ones.

For an uniformly accelerating observer, the $a_7$-acceleration is given by

\[
a_7^{-2} = \frac{r_H^4}{M^2 - Q^2} \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right),
\]

and thus the fiducial temperature corresponding to the Unruh one is read as

\[
T_{\text{FID}} = \frac{\sqrt{M^2 - Q^2}}{2\pi r_H^2 \sqrt{1 - \frac{2M}{r} + \frac{Q^2}{r^2}}}. \tag{2.27}
\]

Then, one can obtain the Hawking temperature $T_H$ by taking $r \to \infty$ as

\[
T_H = \frac{\sqrt{M^2 - Q^2}}{2\pi r_H^2}. \tag{2.28}
\]

On the other hand, for the RN black hole, the local free-fall temperature seen by a freely falling observer is given by

\[
T_{\text{FFAR}} = \frac{1}{2\pi r_H^2} \sqrt{\frac{(M^2 - Q^2)(r^3 + r_H r^2 + r_H^2 r^2 + r_H^3)}{r^2(r - M + \sqrt{M^2 - Q^2})} - \frac{Q^2 r_H^4}{r^4}}, \tag{2.29}
\]

which is reduced to the local free-fall temperature $T_{\text{FFAR}}$ of the Schwarzschild black hole in Eq. (2.16) in the $Q = 0$ limit [17, 18].
3 GMGHS spacetime in Einstein frame

Now, let us start with the GMGHS action [20] in the Einstein frame

\[ S = \int d^4x \sqrt{-g} \left( R - 2(\nabla \phi)^2 - e^{-2\phi} F_{\mu\nu} F^{\mu\nu} \right), \]  

(3.1)

where \( R \) is the scalar curvature, \( \phi \) is a dilaton, and \( F_{\mu\nu} \) is the Maxwell field.

Spherically symmetric static charged solutions [20, 21, 22, 23, 24] to equations of motion of the action (3.1) are given by

\[ ds^2 = - \left( 1 - \frac{2M}{r} \right) dt^2 + \left( 1 - \frac{2M}{r} \right)^{-1} dr^2 + r \left( r - \frac{Q^2}{M} \right) (d\theta^2 + \sin^2 \theta d\phi^2), \]  

(3.2)

where \( Q \) is related to the magnetic and electric charges defined by

\[ F_{\theta\phi} = Q \sin \theta, \quad F_{tr} = \frac{Q e^{4\phi}}{r^2}, \]  

(3.3)

respectively. Moreover, they have the relation with the dilaton as

\[ e^{-2\phi} = 1 - \frac{Q^2}{Mr}, \quad e^{-2\phi} = 1 + \frac{Q^2}{Mr} \]  

(3.4)

for the magnetically and electrically charged black holes, respectively. This charged case seems to the string analog of the RN spacetime.

However, this (3+1)-dimensional curved spacetime (3.2) can be embedded into a (5+1)-dimensional Minkowskian spacetime

\[ ds^2 = \eta_{IJ} z^I z^J \]  

(3.5)

with a flat metric \( \eta_{IJ} = \text{diag}(-1, 1, 1, 1, 1, 1) \), similar to the Schwarzschild spacetime in contrast to the RN case. Here, the (5+1)-dimensional Minkowskian spacetime is explicitly given by the transformations

\[ z^0 = \frac{1}{k_H} \sqrt{1 - \frac{2M}{r} \sinh(k_H t)}, \]  

(3.6)

\[ z^1 = \frac{1}{k_H} \sqrt{1 - \frac{2M}{r} \cosh(k_H t)}, \]  

(3.7)

\[ z^2 = \int dr \sqrt{\frac{2M(r^2 + 2Mr + 4M^2)}{r^3}} + \frac{Q^2}{4M} \left( \frac{1}{r} - \frac{1}{r - \frac{Q^2}{M}} \right), \]  

(3.8)
\[ z^3 = \sqrt{r \left( r - \frac{Q^2}{M} \right)} \sin \theta \cos \phi, \]  
(3.9) 
\[ z^4 = \sqrt{r \left( r - \frac{Q^2}{M} \right)} \sin \theta \sin \phi, \]  
(3.10) 
\[ z^5 = \sqrt{r \left( r - \frac{Q^2}{M} \right)} \cos \theta, \]  
(3.11) 
which reduce to the transformations (2.3)-(2.8) in the \( Q \to 0 \) limit. However, they are very different from the embeddings of the RN spacetime (2.19)-(2.25). Note here that \( k_H = \frac{1}{4M} \) is the surface gravity and the event horizon \( r_H \) is given by \( 2M \) as like the Schwarzschild case. However, the area of the two sphere of constant \( t \) and \( r \) is smaller than the Schwarzschild one due to the presence of the dilaton.

Now, for an uniformly accelerating observer, the \( a_6 \)-acceleration is given by
\[ a_6 = \frac{1}{4M \sqrt{1 - \frac{2M}{r}}} \]  
(3.12) 
Then, the fiducial temperature corresponding to the Unruh one is simply written
\[ T_{\text{FID}} = \frac{1}{8\pi M \sqrt{1 - \frac{2M}{r}}}. \]  
(3.13) 
As results, we see that these are the same with the ones of the Schwarzschild spacetimes (2.10). Moreover, it is independent of the charge \( Q \).

On the other hand, making use of the orbit equations (2.13) with \( f(r) = 1 - \frac{2M}{r} \), we can obtain the \( \ddot{a}_6 \) acceleration for a freely falling observer given by
\[ (\ddot{a}_6)^2 = \frac{r^3 + 2Mr^2 + 4M^2r + 8M^3}{16M^2r^3}, \]  
(3.14) 
which is exactly the same with the 6-acceleration (2.14) for the freely falling observer in the Schwarzschild spacetime. As a result, we have the same local Unruh temperature \( T_{\text{FFAR}} \) (2.16) for the freely fall observer at rest.

In this respect, we know that even though the transformations of the embedding coordinates are differently given due to the charge of the GMGHS spacetime in the Einstein frame, it does not have any affects on the accelerations and the corresponding Unruh/free-fall temperatures.
4 GMGHS spacetime in string frame

In the string frame, the GMGHS action is described by

\[ S = \int d^4x \sqrt{-g} e^{-2\phi} \left( R + 4(\nabla \phi)^2 - F_{\mu \nu} F^{\mu \nu} \right), \]

which is related to the action (3.1) in the Einstein frame through the conformal transformation of \( g^{S \mu \nu} = e^{2\phi} g^{E \mu \nu} \).

4.1 Magnetically charged solution

Now, let us study the magnetically charged GMGHS solution in the string frame, which is given by

\[ ds^2 = \left( 1 - \frac{2M}{r} \right) dt^2 + \frac{dr^2}{\left( 1 - \frac{2M}{r} \right) \left( 1 - \frac{Q^2}{Mr} \right)} + r^2(d\theta^2 + \sin^2 \theta d\phi^2). \]

The (3+1)-dimensional curved spacetime can be embedded in a (5+2)-dimensional Minkowskian spacetime

\[ ds^2 = \eta_{IJ} z^I z^J \]

with a metric \( \eta_{IJ} = \text{diag}(-1, 1, 1, 1, 1, -1) \). The transformations are

\[ z^0 = \frac{1}{k_H} \int \frac{dr}{\sqrt{\frac{2M}{r} + \frac{4M^2}{r^2} + \frac{8M^3}{r^3} + \frac{Q^2}{Mr} + \frac{2Q^4}{M^2 r} + \frac{Q^6}{M^3 r^3}}} \left( 1 - \frac{Q^2}{Mr} \right)^3, \]

and

\[ z^1 = \frac{1}{k_H} \int \frac{dr}{\sqrt{\frac{2M}{r} + \frac{4M^2}{r^2} + \frac{8M^3}{r^3} + \frac{Q^2}{Mr} + \frac{2Q^4}{M^2 r} + \frac{Q^6}{M^3 r^3}}} \left( 1 - \frac{Q^2}{Mr} \right)^3. \]

8
Changing the frame from the Einstein to the string makes different the embeddings of the GMGHS spacetimes. Note that \( k_H = \frac{1}{4M} \) is the surface gravity and the event horizon \( r_H^m \) is also \( 2M \) as like the Schwarzschild case.

The \( a_7 \)-acceleration for an uniformly accelerating observer is given by

\[
a_7 = \frac{1}{4M} \sqrt{\frac{1 - \frac{Q^2}{Mr}}{1 - \frac{2M}{r}}}. \tag{4.11}
\]

Then, the fiducial temperature is given by

\[
T_{\text{FID}} = \frac{1}{8\pi M} \sqrt{\frac{1 - \frac{Q^2}{Mr}}{1 - \frac{2M}{r}}}. \tag{4.12}
\]

This becomes the Hawking temperature \( T_H \) at the asymptotic infinity, as expected.

On the other hand, for the case of the magnetically charged solution (4.2), we have orbit equations as

\[
\frac{dt}{d\tau} = \left( \frac{1 - \frac{2M}{r}}{1 - \frac{Q^2}{Mr}} \right)^{1/2} \left( \frac{1 - \frac{Q^2}{Mr}}{1 - \frac{2M}{r}} \right),
\]

\[
\frac{dr}{d\tau} = - \left[ \left( \frac{1 - \frac{2M}{r}}{1 - \frac{Q^2}{Mr}} \right) \left( 1 - \frac{Q^2}{Mr} \right) + \left( 1 - \frac{Q^2}{Mr} \right)^2 \left( \frac{1 - \frac{2M}{r_0}}{1 - \frac{Q^2}{Mr_0}} \right) \right]^{1/2}, \tag{4.13}
\]

which can be used to get the \( \tilde{a}_7 \) acceleration for a freely falling observer given by

\[
(\tilde{a}_7)^2 = \frac{1 + \frac{2M}{r} + \frac{4M^2}{r^2} + \frac{3M^3}{r^3} - \frac{2Q^2}{Mr} \left( 1 + \frac{2M}{r} + \frac{4M^2}{r^2} \right) + \frac{Q^2}{Mr^2} \left( 1 + \frac{2M}{r} \right)}{16M^2 \left( 1 - \frac{Q^2}{Mr} \right)}. \tag{4.14}
\]

As a result, we have the local free-fall temperature for the freely falling observer at rest as

\[
T_{\text{FFAR}} = \frac{\tilde{a}_7}{2\pi}. \tag{4.15}
\]

At the asymptotic infinity, the local free-fall temperature becomes the Hawking temperature \( T_H \). Note also that the local free-fall temperature in the string
frame depends on the charge, while the local free-fall temperature in the Einstein frame does not. Moreover, when \( Q \to 0 \), the temperature (4.15) reduces to the local free-fall temperature (2.16) for the Schwarzschild spacetime. At the event horizon, the local free-fall temperature remains finite as

\[
T_{F\text{FAR}} = \frac{\sqrt{1 - \frac{Q^2}{2M^2}}}{4\pi M}, \tag{4.16}
\]

while the fiducial temperature (4.12) diverges. It also depends on the charge, and has lower temperature, compared with the temperatures of \( T_{F\text{FAR}} = 2T_H \) in the Einstein frame and for the Schwarzschild spacetime.

On the other hand, in the extremal limit of \( 2M^2 = Q^2 \), the local free-fall temperature becomes

\[
T_{F\text{FAR}} = \frac{1}{8\pi M} \tag{4.17}
\]

so that the local free-fall temperature for the extremal black hole appears to be constant, depending on the mass of the black hole, for the freely falling observer.

### 4.2 Electrically charged solution

The electrically charged GMGHS solution in the string frame is given by

\[
ds^2 = -\left(1 + \frac{Q^2 - 2M^2}{Mr} \right) dt^2 + \frac{dr^2}{\left(1 + \frac{Q^2 - 2M^2}{Mr} \right)} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \tag{4.18}
\]

This spacetime can be also embedded in a (5+2)-dimensional Minkowskian spacetime

\[
ds^2 = \eta_{IJ} z^I z^J \tag{4.19}
\]

with a metric \( \eta_{IJ} = \text{diag}(-1,1,1,1,1,1,-1) \). The transformations are

\[
z^0 = \frac{1}{k_H} \sqrt{1 + \frac{Q^2 - 2M^2}{Mr}} \sinh(k_H t), \tag{4.20}
\]

\[
z^1 = \frac{1}{k_H} \sqrt{1 + \frac{Q^2 - 2M^2}{Mr}} \cosh(k_H t), \tag{4.21}
\]
\[ z^2 = \int dr \frac{\sqrt{\frac{2M}{r} \left[ 1 + \frac{2M}{r} + \frac{4M^2}{r^2} + \frac{Q^2}{Mr} \left( 2 + \frac{2M}{r} + \frac{Q^2}{Mr} + \frac{4MQ^2}{r^4} \right) \right]}}{\left( 1 + \frac{Q^2}{Mr} \right)^2}, \] (4.22)

\[ z^3 = r \sin \theta \cos \phi, \] (4.23)

\[ z^4 = r \sin \theta \sin \phi, \] (4.24)

\[ z^5 = r \cos \theta, \] (4.25)

\[ z^6 = \int dr \frac{\sqrt{\frac{Q^2}{Mr} \left[ 1 + \frac{16M^3}{r^3} + \frac{Q^2}{Mr} \left( 3 + \frac{4M^2}{r^2} + \frac{3Q^2}{Mr} + \frac{4MQ^2}{r^4} + \frac{Q^4}{Mr^2} \right) \right]}}{\left( 1 + \frac{Q^2}{Mr} \right)^2}, \] (4.26)

Note that \( k_H (= \frac{1}{4M}) \) is the surface gravity and the event horizon \( r^e_H \) is given by \( 2M - \frac{Q^2}{M} \), which is shifted from \( 2M \) by \( \frac{Q^2}{M} \) as compared with the event horizon \( r^m_H \) of the magnetically charged solution.

The \( a_7 \)-acceleration for an uniformly accelerating observer is given by

\[ a_7 = \frac{1 + \frac{Q^2}{Mr}}{4M \sqrt{1 + \frac{Q^2 - 2M^2}{Mr}}}, \] (4.27)

and thus the fiducial temperature is

\[ T_{\text{FID}} = \frac{1 + \frac{Q^2}{Mr}}{8\pi M \sqrt{1 + \frac{Q^2 - 2M^2}{Mr}}}. \] (4.28)

On the other hand, for the case of the electrically charged solution (4.18), we have orbit equations as

\[ \frac{dt}{d\tau} = \left( \frac{1 + \frac{Q^2 - 2M^2}{Mr_0}}{1 + \frac{Q^2}{Mr_0}} \right)^{1/2} \left( 1 + \frac{Q^2}{Mr} \right) \left( 1 + \frac{Q^2}{Mr_0} \right), \]

\[ \frac{dr}{d\tau} = - \left[ \left( 1 + \frac{Q^2}{Mr} \right) - \left( 1 + \frac{Q^2 - 2M^2}{Mr_0} \right) \left( 1 + \frac{Q^2}{Mr} \right)^2 \left( 1 + \frac{Q^2 - 2M^2}{Mr_0} \right)^2 \right]^{1/2}, \] (4.29)

which can be used to get the \( \tilde{a}_7 \) acceleration for a freely falling observer given
by
\[(\tilde{a}_T)^2 = \frac{h_e(r, M, Q)}{16M^2 \left(1 + \frac{Q^2}{M^2} \right)^2}, \quad (4.30)\]

where
\[
h_e(r, M, Q) = \left[ \left( 1 + \frac{2M}{r} + \frac{4M^2}{r^2} + \frac{8M^3}{r^3} \right) + \frac{Q^2}{2M} \left( 3 + \frac{4M}{r} + \frac{4M^2}{r^2} \right) \right. \]
\[
+ \frac{Q^4}{4M^2} \left( 3 + \frac{2M}{r} + \frac{8M^3}{r^3} + \frac{Q^2}{Mr} \right) \left] - \frac{4M^2Q^2}{r^3} \left( 2 + \frac{Q^2}{2M^2} + \frac{Q^4}{2Mr^3} \right). \quad (4.31)\]

As a result, we have the local free-fall temperature for a freely falling observer at rest as
\[T_{FFAR} = \frac{\tilde{a}_T}{2\pi} \quad (4.32)\]

At the asymptotic infinity the local free-fall temperature again becomes the Hawking temperature \(T_H\). Note that when \(Q \to 0\), the local \(FFAR\) temperature (4.32) reduces exactly again to the local free-fall temperature (2.16) for the Schwarzschild spacetime. At the event horizon, the local free-fall temperature also remains finite as
\[T_{FFAR} = \frac{\sqrt{1 - \frac{3Q^2}{4M^2}}}{4\pi M \left(1 - \frac{Q^2}{2M^2} \right)}, \quad (4.33)\]

while the fiducial temperature (4.28) diverges.

On the other hand, in the extremal limit of \(2M^2 = Q^2\), it becomes
\[T_{FFAR} = \frac{1}{8\pi M} \sqrt{1 + \frac{8M}{r} + \frac{2M^2}{r^2} + \frac{24M^3}{r^3} - \frac{48M^4}{r^4}} \left(1 + \frac{2M}{r} \right). \quad (4.34)\]

This contrasts with the magnetically charged extremal \(T_{FFAR}\) in Eq. (4.17), which is independent of \(r\), while the electrically charged extremal \(T_{FFAR}\) is a function of \(\tilde{r}\). However, as \(\tilde{r} \to \infty\), the temperature \(T_{FFAR}\) becomes the same as the local free-fall temperature \(T_{FFAR}\) (4.17) for the magnetically charge solution.

In short, we have plotted in Fig. 1 the local temperatures \(T_{FID}\) measured by the fiducial observers for the nonextremal GMGHS spacetime. All the fiducial
Figure 1: The local temperatures $T_{\text{FID}}$ measured by a fiducial observer for the nonextremal GMGHS spacetime with $M = 1$, $Q = 1$: the solid line for the solution in Einstein frame, the dotted line for magnetically charged solution in string frame, and the dashed line for electrically charged solution in string frame.

Figure 2: The local temperatures $T_{\text{FFAR}}$ and $T_{\text{FID}}$ for the nonextremal GMGHS spacetime with $M = 1$, $Q = 1$: the solid line for the solution in Einstein frame, the dotted line for magnetically charged solution in string frame, and the dashed line for electrically charged solution in string frame.

temperatures show the same behaviors that they blow up near the event horizons and become the Hawking temperatures at asymptotic infinities.
other hand, Fig. 2 shows that the local free-fall temperatures obtained in the string frames as well as in Einstein Einstein frame remain finite at the event horizons. Note that in the Einstein frame, the temperature is independent of the charge, while in the string frames the temperature depends on the charge so that the temperature differs between the two frames.

5 Conclusions

In this paper, we have obtained the (5+1)/(5+2)-dimensional global flat embeddings of the GMGHS spacetime according to the Einstein/string frames. In the Einstein frame, we need the (5+1)-dimensional embedding, which is similar to the embedding of the Schwarzschild spacetime. In some sense, it is expected since the metric is identical to the Schwarzschild black hole metric in the \((t - r)\) plane and thus the causal structure is the same. However, by changing the Einstein frame to the string frame, we have shown that regardless of the type of the charges the global flat embeddings of the GMGHS spacetime are needed one more time dimension as like the RN spacetime. In this respect, the GEMS of the GMGHS spacetime in the string frame is more like the ones of the RN spacetime.

We have also found the Unruh/fiducial temperatures in each frame and their corresponding Hawking temperatures. Moreover, by finding local free-fall temperatures for the freely falling observers, we have shown that the local free-fall temperature in the Einstein frame as well as in the string frame remains finite at the event horizon, while the Unruh/fiducial temperature is divergent. On the other hand, regardless of the frames, all the temperatures are reduced to the Hawking temperature \(T_H\) at infinity.

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References

[1] S. W. Hawking, Commun. Math. Phys. 43, 199 (1975) [Erratum-ibid. 46, 206 (1976)].

[2] R. C. Tolman, “Relativity, Thermodynamics and Cosmology”, New York, Dover Publication (1987).

[3] W. G. Unruh, Phys. Rev. D 14, 870 (1976).

[4] H. Narnhofer, I. Peter and W. E. Thirring, Int. J. Mod. Phys. B 10, 1507 (1996).

[5] S. Deser and O. Levin, Class. Quant. Grav. 14, L163 (1997) [gr-qc/9706018].

[6] S. Deser and O. Levin, Class. Quant. Grav. 15, L85 (1998) [hep-th/9806223].

[7] S. Deser and O. Levin, Phys. Rev. D 59, 064004 (1999) [hep-th/9809159].

[8] Y. -W. Kim, Y. -J. Park and K. -S. Soh, Phys. Rev. D 62, 104020 (2000) [gr-qc/0001045].

[9] S. -T. Hong, Y. -W. Kim and Y. -J. Park, Phys. Rev. D 62, 024024 (2000) [gr-qc/0003097].

[10] S. -T. Hong, Gen. Rel. Grav. 36, 1919 (2004) [gr-qc/0310118].

[11] H. -Z. Chen, Y. Tian, Y. -H. Gao and X. -C. Song, JHEP 0410, 011 (2004) [gr-qc/0409107].

[12] N. L. Santos, O. J. C. Dias and J. P. S. Lemos, Phys. Rev. D 70, 124033 (2004) [hep-th/0412076].

[13] R. Banerjee and B. R. Majhi, Phys. Lett. B 690, 83 (2010) [arXiv:1002.0985 [gr-qc]].

[14] R. -G. Cai and Y. S. Myung, Phys. Rev. D 83, 107502 (2011) [arXiv:1012.5709 [hep-th]].

[15] B. R. Majhi, arXiv:1110.6008 [gr-qc].
[16] B. Hu and H. -F. Li, Mod. Phys. Lett. A 27, 1250002 (2012) [arXiv:1101.4074 [hep-th]].

[17] E. J. Brynjolfsson and L. Thorlacius, JHEP 0809, 066 (2008) [arXiv:0805.1876 [hep-th]].

[18] Y. -W. Kim, J. Choi and Y. -J. Park, Int. J. Mod. Phys. A 25, 3107 (2010) [arXiv:0909.3176 [gr-qc]].

[19] G. W. Gibbons and K. Maeda, Nucl. Phys. B 298, 741 (1988).

[20] D. Garfinkle, G. T. Horowitz and A. Strominger, Phys. Rev. D 43, 3140 (1991) [Erratum-ibid. D 45, 3888 (1992)].

[21] G. T. Horowitz, In *Trieste 1992, Proceedings, String theory and quantum gravity '92* 55-99 [hep-th/9210119].

[22] M. Rakhmanov, Phys. Rev. D 50, 5155 (1994) [hep-th/9310174].

[23] K. C. K. Chan, J. D. E. Creighton and R. B. Mann, Phys. Rev. D 54, 3892 (1996) [gr-qc/9604055].

[24] S. Bose and D. Lohiya, Phys. Rev. D 59, 044019 (1999) [gr-qc/9810033].

[25] M. Natsuume, Phys. Rev. D 50, 3949 (1994) [hep-th/9406079].

[26] D. V. Galtsov and A. A. Garcia, Phys. Rev. D 52, 3432 (1995).

[27] S. S. Xulu, Int. J. Theor. Phys. 37, 1773 (1998) [gr-qc/9712100].

[28] F. -W. Shu and Y. -G. Shen, Phys. Rev. D 70, 084046 (2004) [gr-qc/0410108].

[29] S. Sur, S. Das and S. SenGupta, JHEP 0510, 064 (2005) [hep-th/0508150].

[30] A. Sheykhi, Phys. Rev. D 76, 124025 (2007) [arXiv:0709.3619 [hep-th]].

[31] Q. -Q. Jiang, Phys. Lett. B 666, 517 (2008).

[32] C. -M. Chen and D. -W. Pang, JHEP 1006, 093 (2010) [arXiv:1003.5064 [hep-th]].
[33] S. Fernando, Phys. Rev. D 85, 024033 (2012) [arXiv:1109.0254 [hep-th]].

[34] J. Choi, Y. -W. Kim and Y. -J. Park, Mod. Phys. Lett. A 28, 1350143 (2013) [arXiv:1306.3020 [gr-qc]].

[35] V. Faraoni, E. Gunzig and P. Nardone, Fund. Cosmic Phys. 20, 121 (1999) [gr-qc/9811047].

[36] R. Casadio and B. Harms, Mod. Phys. Lett. A 14, 1089 (1999) [gr-qc/9806032].

[37] C. Fronsdal, Phys. Rev. 116, 778 (1959).