Signature of strong atom–cavity interaction on critical coupling

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Abstract
We study a critically coupled cavity doped with resonant atoms with metamaterial slabs as mirrors. We show how resonant atom–cavity interaction can lead to a splitting of the critical coupling dip. The results are explained in terms of the frequency and lifetime splitting of the coupled system.

Keywords: critical coupling, metamaterials, normal mode splitting

1. Introduction
Metamaterials (MMs) are broadly defined as ‘artificial effectively homogeneous electromagnetic structures with unusual properties not readily found in nature’ [1]. They include but are not limited to materials having simultaneously negative values of permittivity and permeability (usually referred to as negative index materials or left-handed materials [2, 3]). Over the past decade many exotic applications of MMs have been suggested and tested. These range from superlensing to lasing spasers, optical nanocircuits to invisibility cloaks, cavity quantum electrodynamics (QED) to electromagnetically induced transparency [4–9]. In most of these applications the doubly negative response of the metamaterials (namely, negative permittivity ε and permeability μ) over a certain range of frequencies has been exploited. In fact, frequency dispersion and absorption have been identified as some of the crucial characteristics of the metamaterials, determining their figure of merit and suitability for device applications [2, 10–13]. Only recently it has been recognized that metamaterials, capable of exhibiting negative index, can have interesting applications in a different frequency domain (where it is not truly a negative index material).

Recall that in the weak interaction regime, the photon emitted by the excited atom can escape the cavity, before it can interact back with the atom. In case of strong interaction, due to high finesse of the cavity, the photon has a larger lifetime, and...
there is a periodic exchange of energy between the atom and the cavity mode. It is thus interesting to study atom–cavity interaction in the context of a critically coupled cavity. In this paper, we study the manifestations of this interaction when the intracavity medium is doped with atoms resonant with the CC frequency. It is clear that the experimentally realized metamaterials are mostly anisotropic and their response can be quite complex. We use the simplified and very often used Lorentz type response for modeling the MMs [14, 24–26] (assuming it to be isotropic and homogeneous). We show that like in usual vacuum field Rabi splitting [22, 23, 27, 28], here also we have the splitting of the critical coupling dip into two for large enough atom–cavity coupling (controllable by varying the dopant density), at frequencies higher and lower than the undoped CC resonance frequency. We will refer to these modes as the higher (lower) frequency split mode. We obtain and solve the dispersion relation for the layered medium. We show that the dispersion relation allows two distinct complex solutions for the eigenfrequencies at a given coupling strength. The real part of the eigenfrequency gives the approximate location of the modes, while the imaginary part gives the corresponding decay rates. Finally, we study the field distributions corresponding to the excitation of these split modes. We also compare the cases of doped and undoped cavity.

The structure of the paper is as follows. In section 2, we define our system and formulate the problem. Section 3 is devoted to numerical results pertaining to the frequency splitting in the total scattering data and also the roots of the dispersion equation. Finally, in section 4, we summarize the main results of this paper.

2. Formulation

Consider the system shown in figure 1 consisting of a spacer layer between two slabs of metamaterials. As mentioned in section 1, we use the following Lorentz type response for the permittivity and permeability of the MMs [24]:

\[
\begin{align*}
\epsilon(f) &= 1 - \frac{f_p^2 - f_{eo}^2}{f^2 - f_{eo}^2 + i\gamma f}, \\
\mu(f) &= 1 - \frac{f_p^2 - f_{mo}^2}{f^2 - f_{mo}^2 + i\gamma f},
\end{align*}
\]  

(1)

where \(f_p\) (\(f_{mp}\)) is the electric (magnetic) plasma frequency, \(f_{eo}\) (\(f_{mo}\)) is the electric (magnetic) resonance frequency and \(\gamma\) is the decay rate (we assumed the same decay rate for both electric and magnetic resonances). A slab of such a metamaterial was shown to exhibit a stopgap (like in one-dimensional photonic band gap structures) between its electric and magnetic plasma frequencies, characterized by a mostly imaginary refractive index [14]. We consider a frequency domain where such MMs do not possess negative refraction. We use two slabs of such MMs with distinct values of the plasma frequencies such that the stop band of one (\(M_t\), see figure 1) lies well inside that of the other (\(M_b\)). We assume the MM slabs to be isotropic and to have a 3D structure. It was shown recently that a dielectric cavity formed by two such MM slabs can exhibit critical coupling near the edges of the stop band of \(M_t\) [15]. The interplay of the FP resonances of the cavity with the stop band features of the MM layers resulted in the critical coupling phenomenon. Further, varying the angle of incidence, which controls the width of the MMs’ stop gap, was used to achieve critical coupling at other frequencies. Recall that there is practically no reflection or transmission from a critically coupled cavity resulting in an almost perfect absorption of the incident energy by the structure. In this study our goal is to investigate the effects of cavity–atom interaction when the intracavity medium (i.e., the dielectric layer) is doped with resonant atoms. We assume the atomic medium to be nonmagnetic and its response to be given by the following dielectric function

\[
\epsilon_2(f) = \epsilon_h + \frac{f_p^2}{f_{02}^2 - f^2 - i\gamma_2 f},
\]  

(2)

where \(\epsilon_h\) is the dielectric constant of the host material, \(f_p^2\) is proportional to the dopant density, \(f_{02}\) and \(\gamma_2\) are the resonance frequency and the decay rate of the atom, respectively. Furthermore, the atomic resonance frequency is assumed to be close to the CC frequency of the undoped cavity. In the following section we demonstrate that such structures can lead to splitting of the CC dip for sufficient atom–cavity coupling determined by the dopant density. Such normal mode splittings have been studied in detail in the context of FP, modulated FP or spherical cavities (supporting the whispering gallery modes) [29–32].

It is clear that the reflection and transmission profile for a system shown in figure 1 can be calculated using the standard characteristic matrix approach [33–35]. In particular, the amplitude reflection (\(r\)) and transmission (\(t\)) coefficients are given by

\[
\begin{align*}
&\quad r = \frac{(m_{11} + m_{12} t_{j}) p_i - (m_{21} + m_{22} t_{j}) p_f}{(m_{11} + m_{12} t_{j}) p_i + (m_{21} + m_{22} t_{j}) p_f}, \\
&\quad t = \frac{2p_t}{(m_{11} + m_{12} t_{j}) p_i + (m_{21} + m_{22} t_{j}) p_f}.
\end{align*}
\]  

(3)

where \(m_{ij}\) (\(i, j = 1, 2\)) are the elements of the total characteristic matrix of the structure (expressions for
$m_{ij}, i, j = 1, 2$ can be found, for example, in [33]), and

$$p_{i,f} = \begin{cases} \sqrt{\epsilon_{i,f}/\mu_{i,f}} \cos \theta_{i,f} & \text{(for TE polarization)} \\ \sqrt{\mu_{i,f}/\epsilon_{i,f}} \cos \theta_{i,f} & \text{(for TM polarization)} \end{cases}$$

(4)

and $\theta_{i,f}$ are the angles of incidence and emergence in the first and the last medium, respectively. The intensity reflection ($R$) and transmission ($T$) of the structure are given by $R = |r|^2$ and $T = |t|^2$ (for identical media of incidence and emergence).

Henceforth we shall consider only normally incident radiation (for which the two polarizations are indistinguishable) to bring out the essential features in the total scattering ($R + T$). Recall that we have $R + T + A = 1$ from energy conservation, where $A$ is the absorption in the structure. Thus $R + T = 0$ implies that all the incident energy is absorbed by the structure. This is what we refer to as critical coupling. Note that the reflection and transmission coefficients have a common denominator, the zeroes of which bear the information about the characteristic frequencies (eigenfrequencies) of the system. The corresponding equation (referred to as the dispersion relation) can be written as [30, 35]

$$D = (m_{11} + m_{12}p_f)p_1 + (m_{21} + m_{22}p_f) = 0.$$ 

(5)

The dispersion relation (5) can be solved only for complex frequencies, which carries all the information about the split modes and their associated decay rates. In fact, the real part of the roots gives the locations of the split modes, while the imaginary part corresponds to the width of these resonances. It is clear that equation (5) is transcendental in nature and there is no analytical method to solve it. We revert to a numerical scheme (‘fsolve’ in Matlab) to obtain the distinct branches for the split modes.

3. Results

In what follows we use scaled frequencies (in units of $f_0 = 10$ GHz = $c/\lambda_0$) and lengths (in units of $\lambda_0$) to enable us to deal with dimensionless quantities (as in [14]) and also for reference. The parameters for the response of the bottom MM slab ($M_b$ in figure 1) were taken from the experiment of Shelby et al [24]. The same set of parameters and the corresponding analytical expressions (again from the work of Shelby et al [24]) were used earlier to demonstrate stop band features of a MM slab [14] and also for CC [15], albeit without doping. For the top MM slab we use the same set of parameters except for the electric and magnetic plasma frequencies. This can be done since the plasma frequencies can be controlled by the filling fraction of the metal wires and split rings in the unit cell of the MM [25, 26]. Thus the parameters for the bottom MM layer ($M_b$) are as follows: $f_{np} = 1.095, f_{sp} = 1.28, f_{mo} = 1.005, f_{eo} = 1.03, \gamma = 0.001$ and $d_3 = 5$, while for the top MM layer ($M_t$) we take the same values, except for $f_{np} = 1.14, f_{sp} = 1.175$ and $d_1 = 3$. This choice of parameters ensures that the stop band of $M_t$ lies inside that of $M_b$. The host material for the spacer layer has $\epsilon_b = 3.8$. The entire layered structure is embedded in air ($\epsilon_i = \epsilon_f = 1, \mu_i = \mu_f = 1$). The spacer layer thickness ($d_2 \approx 7$) is fixed to attain CC for normal incidence near the right edge of the stop band of $M_t$ (solid curves in figure 2) without doping the cavity ($f_p = 0$) (see also [15]). The resonance frequency of the dopant atom is chosen to be near degenerate (not identical) with this CC frequency at $f_{t03} \approx 1.1738$ to ensure a strong interaction of the atoms with the cavity resonance at CC. We study the total scattering ($R + T$) of the structure at normal incidence using the dopant density as a parameter. We recall that the near-complete suppression of reflection and transmission from the structure ($R + T \equiv 0$) originates from the resonance of the incident radiation with one or more modes of the cavity. The calculations for total scattering have been performed for two dopant densities, namely, $f_p = 0.005, 0.01$. The results for two decay rates ($\gamma_2 = 0.0008, 0.001$) are shown in figures 2(a) and (b), respectively. These figures clearly demonstrate the splitting of the CC dip as a manifestation of the strong atom–cavity interaction (compare with the results of references [30, 32]). Unfortunately the critical coupling is lost at both the split frequencies. However, the higher frequency dip is closer to critical coupling for larger densities. Thus a larger dopant density can restore critical coupling at the right frequency component. This restoration is shown in figures 3(a) and (b) for both the values of $\gamma_2$. For example, for $\gamma_2 = 0.0008$ ($\gamma_2 = 0.001$), CC is restored at $f_p = 0.0265$ ($f_p = 0.0295$).
Figure 3. CC is achieved (solid curves) for the split resonance at higher frequency ($f$) for two dopant decay rates (a) $\gamma_2 = 0.0008$, $f_p = 0.0265$, $f = 1.179$ and (b) $\gamma_2 = 0.001$, $f_p = 0.0295$, $f = 1.18$. The split resonances at $f_p = 0.01$ (dashed curves) are produced for reference. The other parameters are the same as in figure 2.

Figure 4. The real ((a) and (c)) and imaginary ((b) and (d)) parts of the roots of the dispersion relation ($f_r$) of the layered structure as a function of the atomic density at $\gamma_2 = 0.0008$ (left panel) and 0.001 (right panel). The solid (dashed) curve corresponds to the split resonance at the lower (higher) frequency.

The dopant density may thus be used as a parameter to tune the CC frequency of the coupled system. A comparison of the left and right panels of figures 2 and 3 reveal that the splittings are stronger in case of lower atomic decay, which is quite expected. In fact the separation of the split resonances depends on the coupling strength and their resolution depends on how the coupling compares with the decay rates. The lower the decay rates, the easier it is to resolve the split modes. Another pertinent feature that should be noted from these figures is that the width of the higher (lower) frequency split resonance increases (decreases) with increasing $f_p$.

We now show that all the above features can easily be understood based on an analysis of the roots of the dispersion relation (5). The real and imaginary parts of the roots (as functions of the dopant density) for the two values of $\gamma_2$ ($\gamma_2 = 0.0008$, 0.001) are shown in the left and right panels of figure 4. The solid (dashed) lines give the branch for the lower (higher) frequency split mode. The well known frequency and lifetime splittings [30, 32] can easily be read from the top and bottom panels of figure 4. For extremely low coupling $f_p \approx 0$, the atomic and field modes are uncoupled, characterized by the near degenerate frequencies (top panel) while lifetimes are those of the bare atom and the bare cavity (bottom panel). As the coupling ($f_p$) increases the split mode frequencies move away from each other (top panel of figure 4) in conformity with the previous total scattering results. The split resonance at the higher frequency broadens (see figure 3(a)) since the corresponding decay rate magnitude increases (dashed curves in the bottom panels of figure 4). An opposite behavior is observed for the split mode at lower frequencies, for which the magnitude of the decay rate decreases.

Finally we look at the field intensity distribution inside the layered structure. Since the first and the last MM slabs are lossy, one expects an exponential decay of intensity in the first and the last layer. Whenever the modes of the cavity with the MM ‘mirrors’ are excited a corresponding field build-up is expected in the central dielectric layer. These features are shown in figure 5(a), where we have plotted the intensity distribution for excitation frequencies corresponding to those of higher (dashed) and lower (solid) split modes. Indeed, in the first MM slab, the electric field is a superposition of two exponential functions leading to a sharp drop of intensity. In the outermost MM layer the field is dominated by exponential decay. In the middle layer, the intensity depends on how close we are to the resonances of the system, and how strong these resonances are. For example, for the stronger resonance at the higher frequency split mode, the resonant enhancements are larger. The quickly oscillating field intensity in the dielectric
layer is due to its large optical width encompassing several wavelengths.

Doping can have very different effects on the field distribution depending on the working frequency. At the critical coupling frequency of the undoped cavity \( f \approx 1.174 \), doping splits the resonance resulting in high total scattering (close to 0.85) at \( f_p = 0.0265 \). Moreover, since the atoms are resonant at this frequency, losses are significant in the doped dielectric. This results in the reduction of the resonant field enhancement and leads to an exponential decay even inside the dielectric layer. This is shown by the solid curve in figure 5(b). An opposite effect of doping is seen at the right split mode frequency \( f = 1.179 \) at a doping concentration given by \( f_p = 0.0265 \). Here the doped system being at resonance exhibits field enhancement (dashed curve in figure 5(c)). Removal of the dopants again leads to a larger \( R + T \sim 0.84 \). However, now the atoms are off-resonant leading to low losses in the dielectric. Thus there is still build-up of the intensity (oscillatory) inside the dielectric layer, though it is less compared to the case of the critically coupled higher frequency split mode. It must be noted here that the top MM slab is nearly lossless at frequencies higher than its electric plasma frequency [14, 15]. Thus the field distribution in \( M_t \) at \( f = 1.179 \) deviates from the exponential decay (solid curve in figure 5(c)).

As mentioned earlier, the split modes carry the information about the couplings of the atom–cavity system. Examples of normal mode coupling are very many. For example in a thin metal film there can be a coupling of two interface surface plasmons resulting in symmetric and the antisymmetric field distributions across the film, with the antisymmetric mode having a node in the center of the film [28]. This particular mode has lower losses since most of its field profile resides outside the lossy metal film. These are sometimes referred to as long range surface plasmons (LRSP) while the symmetric mode is called short range surface plasmons (SRSP). LRSP (SRSP) mode occurs at lower (higher) frequencies. We have an analogous situation here where the mode mostly resides in the central dielectric layer with exponential tails extending into the MM layers. The parity of the mode can again be checked by looking at the zeroes of the intensity distribution. From the inset of figure 5(a) we can recognize the high frequency split mode (dashed curve) as the one with even parity, since the intensity distribution never touches zero, while the odd parity modes do touch null values at the nodal points. In our case also losses are lower for the odd parity modes since it is more tightly bound to the central dielectric layer with smaller exponential tails into the lossy MM slabs (see figure 5(a)).

4. Conclusion

In conclusion, we have studied a new kind of critically coupled cavity and shown that the critical coupling dip can exhibit a normal mode splitting when the intracavity medium is doped with resonant atoms. The intracavity medium with dopant atoms was modeled by a Lorentz type dielectric with a characteristic resonance frequency and decay rate. Numerical results for the total scattering \( R + T \) were obtained using a characteristic matrix formulation. Although we have considered normally incident radiation, we note that the results can be extended to the case of critical coupling at oblique incidence [15], yielding different results for TE and TM polarizations. The corresponding dispersion relations were solved for the location and width of the split resonances. Moreover, it was demonstrated that the atomic dopant density can be used as an additional handle to achieve critical coupling with respect to one of the split resonances.

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