Dynamical constraints on the quadrupole mass moment of the HD 209458 star

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Abstract

The aim of this paper is to dynamically constrain the quadrupole mass moment $J_2$ of the main-sequence HD 209458 star. The adopted method is the confrontation between the measured orbital period of its transiting planet Osiris and a model of it. Osiris is assumed to move along an equatorial and circular orbit. Our estimate, for given values of the stellar mass $M$ and radius $R$ and by assuming the validity of general relativity, is $J_2 = (3.5 \pm 3.85) \times 10^{-6}$. Previous fiducial evaluations based on indirect, spectroscopic measurements yielded $J_2 \sim 10^{-6}$; such a value is compatible with our result.

Key words: stars: rotation-stars: planetary systems – stars: individual, HD 209458 – extrasolar planets

1 Introduction

The quadrupole mass moment $J_2$ is an important astrophysical stellar parameter related to the inner structure and dynamics of a star (Paternò et al. 1996; Pijpers 1998).

In this paper we dynamically constrain the quadrupole of the main-sequence star HD 209458 from the measured orbital period $P$ of its transiting planet HD 209458b ‘Osiris’ (Charbonneau et al., 2000; Henry et al., 2000). It is the best-studied transiting planet to date, mainly due to its proximity and the resultant high apparent brightness of its parent star.

Until now, only a fiducial value $J_2 = 2 \times 10^{-6}$ by Miralda-Escudé (2002), based on spectroscopic measurements of rotational velocity (Queloz et al. 2000), exists for HD 209458. Also Winn et al. (2005) assumed $J_2 \sim 10^{-6}$. More generally, Miralda–Escudé (2002) investigated the possibility of dynamically measuring the quadrupole of a star from the node and periastron precessions of a transiting planet which cause a time variation of the duration of a transit; the periastron precession also induces a variation of the
transit period. Such orbital perturbations should be measured from an accurate photometric analysis of the light-curve of the transiting planet, but, until now, such a proposed strategy has not yet been implemented. Winn et al. (2005) pointed out that, for $J_2 \sim 10^{-6}$, the quadrupole node precession would amount to about 4 arcseconds per year; measuring such an effect would require high-precision photometry spanning several years (Winn et al. 2005).

2 The use of the orbital period of Osiris

The Newtonian gravitational potential $U$ of an oblate star of mass $M$ and equatorial radius $R$ can be written as

$$U = -\frac{G M}{r} + \frac{G M R^2 J_2}{2 r^3} (3 \cos^2 \theta - 1), \quad (1)$$

where $\theta$ is the co-latitude angle. For the orbital period of a planet of mass $m$ in a circular and equatorial ($\theta = \pi/2$) orbit of radius $a$ eq. (1) yields

$$P^{(N)} = P^{(0)} + P^{(J_2)} = 2\pi \sqrt{\frac{a^3}{G(M+m)}} - \frac{3\pi R^2 J_2}{2\sqrt{G(M+m)a}} \quad (2)$$

In fact, in addition to eq. (2), there is also a post-Newtonian, general relativistic part (Soffel 1989; Mashhoon et al. 2001; Iorio 2005; 2006) to be added; for circular orbits and $m \ll M$ (see Section A) it is

$$P^{(PN)} = \frac{3\pi \sqrt{G(M+m)a}}{c^2}, \quad (3)$$

where $c$ is the speed of light in vacuum, so that

$$P = P^{(0)} + P^{(J_2)} + P^{(PN)} \quad (4)$$

In the case of Osiris, eq. (2)-eq. (3) yield a reliable model of its orbital period because the eccentricity $e$ was recently evaluated to be $e = 0.014 \pm 0.009$ (Laughlin et al. 2005) and the inclination angle $\psi$ of the orbital plane to the star’s equator, determined by means of the Rossiter-McLaughlin effect (Rossiter 1924; McLaughlin 1924), should not be larger than about 5 deg, (Winn et al. 2005). Moreover, there are currently no observational evidences of the presence of other bodies around HD 209458 (Brown et al. 2001; Croll et al. 2005; Laughlin et al. 2005; Agol & Steffen 2006) which may require the introduction of additional perturbing terms in eq. (4); the inclusion of
the general relativistic correction of eq. (3) is required because it amounts to about 0.1 s, while the errors in the most recent measurements of the Osiris’ orbital period are 0.016 s (Wittenmyer et al. 2005) and 0.033 s (Knutson et al. 2006).

Thus, the HD 209458 quadrupole mass moment can be determined by comparing the model of eq. (2)-eq. (4) to the measured period $P_{\text{meas}}$, determined in a purely phenomenologically way from combined photometric transit and spectroscopic radial velocity techniques, independent of any gravitation theory, and solving for $J_2$

$$\frac{J_2}{2} = \frac{2P_{\text{meas}}}{3\pi R^2} \sqrt{G(M + m)a} + \frac{4}{3} \left(\frac{a}{R}\right)^2 + \frac{2G(M + m)a}{c^2 R^2}. \quad (5)$$

By using eq. (5) and the system parameters derived for $M = 1.07M_\odot$ and $R = 1.137R_\odot$ and $P_{\text{meas}} = 3.52474554$ d (Wittenmyer et al. 2005), we obtain

$$J_2 = 3.5 \times 10^{-5}. \quad (6)$$

It must be noted that the obtained result is free from any a priori, ‘imprinting’ effect by $J_2$ itself. Indeed, $M$ and $R$ are kept fixed, and $a$, determined from

$$\frac{K^3_M P^3}{8\pi^3} = \frac{a^3 m^3 \sin^3 i}{(m + M)^3} \quad (7)$$

which is independent of any model of the orbital period, is not affected by $J_2$ over timescales longer than one full orbital revolution.

Let us now evaluate the uncertainty in $J_2$ as

$$\delta J_2 \leq \delta J_2^{(P)} + \delta J_2^{(a)} + \delta J_2^{(m)}. \quad (8)$$

For the same values of $M$ and $R$ as before and $\delta P_{\text{meas}} = 0.016$ s (Wittenmyer et al. 2005) we have

$$\delta J_2^{(a)} = 2.404 \times 10^{-3},$$

$$\delta J_2^{(m)} = 1.437 \times 10^{-3},$$

$$\delta J_2^{(P)} = 5 \times 10^{-6}. \quad (9)$$

$1$ $K_M = \left(\frac{2\pi a}{P}\right) \sin i = \left(\frac{m}{m + M}\right) \left(\frac{2\pi a}{P}\right) \sin i$ is the projected semiamplitude of the star’s radial velocity.
The stellar mass was not included in the least-square solution by Wittenmyer et al. (2005) also because its determination is more model-dependent than the other parameters. The range of allowable values 1.06±0.13 solar masses (Cody & Sasselov 2002) was, instead, used; it comes from observational errors in temperature, luminosity, and metallicity as well as systematic errors in convection mixing-length and helium abundance. The resulting scattering in the determined values of \( J_2 \) is

\[
3.2 \times 10^{-5} < J_2 < 3.7 \times 10^{-5}.
\] (10)

Thus, we can state that the model-dependence of our estimate amounts to 5 \times 10^{-6}, so that a conservative estimate of the total uncertainty in \( J_2 \) is

\[
\delta J_2 \leq 3.851 \times 10^{-3}.
\] (11)

### 3 Discussion and conclusions

In this paper we dynamically constrained the quadrupole mass moment \( J_2 \) of the HD 209458 star from the orbital period \( P \) of its transiting planet Osiris, assumed to be in a circular and equatorial orbit. Its measured value—determined in a phenomenological way, independent of any gravitational theory—was compared to an analytical model including the Newtonian part, constituted by the usual Keplerian component and the term induced by \( J_2 \), and the post-Newtonian, general relativistic correction. The inclusion of the latter term, which is of the order of 0.1 s, is motivated by the ∼ 0.01 s level of accuracy reached in measuring \( P \). By keeping the stellar mass \( M \) and radius \( R \) fixed to values within a range determined from stellar evolution models and temperature/luminosity measurements, by assuming that general relativity is valid in the HD 209458 system as well and that Osiris is the only planet affecting the motion of its parent star in a detectable way, we obtain \( J_2 = (3.5 \pm 385.1) \times 10^{-5} \). While the uncertainty due to the error in the orbital period amounts to ∼ 10^{-6} only, the Osiris' mass and semimajor axis boost the total bias to ∼ 10^{-3}. Previous fiducial evaluations based on indirect, spectroscopic measurements giving \( J_2 \sim 10^{-6} \) are compatible with our result.

In order to make easier a comparison with our results, in Table 1 we quote the numerical values used for the relevant constants entering the calculation.
Table 1: Values used for the defining, primary and derived constants (http://ssd.jpl.nasa.gov/?constants#ref).

| constant | numerical value | units | reference |
|----------|-----------------|-------|-----------|
| \( c \)  | 299792458       | m s\(^{-1} \) | (Mohr & Taylor 2005) |
| \( GM_\odot \) | 1.32712440018 \times 10^{20} | m\(^3\) s\(^{-2} \) | (Standish 1995) |
| \( G \)  | \( (6.6742 \pm 0.0010) \times 10^{-11} \) | kg\(^{-1}\) m\(^3\) s\(^{-2} \) | (Mohr & Taylor 2005) |
| \( R_\odot \) | 6.95508 \times 10^8 | m | (Brown & C.-Dalsgaard 1998) |
| 1 mean sidereal day | 86164.09054 | s | (Standish 1995) |

A The post-Newtonian correction to the orbital period

In fact, the post-Newtonian gravito-electric correction to the orbital period does depend on both the eccentricity \( e \) and the initial value of the true anomaly \( f_0 \) according to

\[
P^{(PN)} = \left[\frac{3\pi}{c^2} \sqrt{G(M + m)a}\right] F(e, f_0),
\]

with (Soffel 1989; Mashhoon et al. 2001)

\[
F(e, f_0) = 3 - \frac{\nu}{3} - \frac{2\sqrt{1 - e^2}}{(1 + e \cos f_0)^2},
\]

and

\[
\nu = \frac{mM}{(M + m)^2}.
\]

Depending on \( e \) and \( \nu \), \( F \)–and \( P^{(PN)} \)–vanishes for those values of \( f_0 \) which satisfy the relation

\[
\cos f_0 = \frac{1}{e} \left[ \sqrt{\frac{6\sqrt{1 - e^2}}{9 - \nu} - 1} \right];
\]

it may also happen that the absolute value of the right-hand-side of eq. (15) is larger than 1, so that \( P^{(PN)} \neq 0 \).

In the case of the HD 209458 system, \( \nu \sim 10^{-4} \); recent refinements in the Osiris ephemerides yields \( e \leq 0.023 \) (Laughlin et al. 2005), so that

\[
\frac{1}{e} \left[ \sqrt{\frac{6\sqrt{1 - e^2}}{9 - \nu} - 1} \right] \sim -8 ;
\]
$F$ never vanishes, ranging from 0.90 to 1.09. Thus, the difference between the maximum and the minimum values of $P^{(PN)}$ is 0.019 s at the most: it just lies at the edge of the precision with which the orbital period is known, i.e. 0.016 s (Wittenmyer et al. 2005) and 0.033 s (Knutson et al. 2006), so that we can approximate $F$ to unity.

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