Terahertz wave emission from a long intrinsic Josephson junction: Numerical study on effect of in-plane fields in an experimental scale

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Abstract. Motivated by recent finding of terahertz wave emission from intrinsic Josephson junctions without external fields, field dependence of this behavior is investigated numerically in the junction with the width of experimental scale, 86μm. Assuming the dynamical boundary condition, a McCumber-like state with small spatial dependence of the electric field is stabilized without fields. As the in-plane field gradually increases, the frequency of emitted wave and applied current at the emission peak increase in proportional to the field, while intensity at the emission peak is almost independent of the field. These results suggest that the McCumber-like state without external fields may naturally connect to the emission state in strong fields.

1. Introduction
As a possible stable source of continuous terahertz electromagnetic wave, emission from intrinsic Josephson junctions (IJJs) such as Bi₂Sr₂CaCu₂O₈ (BSCCO) has been intensively studied. Historically, emission from IJJs in in-plane external magnetic fields [1] was studied first.[2, 3, 4, 5] In this case intensity of emission takes maximum when the distance between Josephson vortices is comparable to the wavelength of transverse plasma, and therefore frequency is tunable.

Recently emission from IJJs without external field was reported.[6] In spite of loss of tunableness of frequency, this emission looks more evident than those in in-plane fields, and two types of theoretical proposal appeared quite recently. One is based on a McCumber-like state which is Ohmic and translational invariant along the c axis,[7], and another is based on a novel state with phase kinks which is not Ohmic and has nontrivial periodic structure along the c axis.[8] In the present article we investigate the former state by means of field dependence.

2. Model and implementation
In the present article the external field $B_{\text{ext}}$ (including $B_{\text{ext}} = 0$) is applied along the $y$ axis, and Josephson vortices are driven by the dc bias current $J$ along the $c$ axis. Electromagnetic waves are emitted from both edges into the dielectrics. Thermal fluctuations are neglected, and Josephson vortices are straight along the $y$ axis. Then, the system can be reduced in two dimensions with $x$ and $c$ axes described by the following differential equations,[3]

$$
\partial_x^2 \psi_{l+1,l} = \left(1 - \zeta \Delta^{(2)}\right) \left(\partial_y E_{l+1,l}^y + \beta E_{l+1,l}^y + \sin \psi_{l+1,l} - J\right),
$$

(1)
where the superconducting layers are labeled by \(l\), and the electric field in the insulating layer and the gauge invariant phase difference between the \(l\)-th and \((l + 1)\)-th superconducting layers are chosen as basic quantities. The magnetic field in the insulating layer is obtained from

\[
\partial_{x'} \psi_{l+1, l} = (1 - \alpha \Delta^{(2)}) E_{l+1, l}',
\]

with the operator \(\Delta^{(2)}\) defined in \(\Delta^{(2)} X_{l+1, l} \equiv X_{l+2, l+1} - 2X_{l+1, l} + X_{l, l-1}\). In the above formulas the following scaled quantities in the MKSA unit are used:

\[
x' = \frac{x}{\lambda_c}, \quad t' = \omega_p t, \quad E_{l+1, l}' = \frac{\sigma_c}{\beta \lambda_c} E_{l+1, l}', \quad B' = \frac{2\pi \lambda_c d}{\phi_0} B, \quad J' = J / J_c,
\]

\[
\zeta = \frac{\lambda_{ab}}{sd}, \quad \alpha = \frac{\epsilon'_c \mu^2}{sd}, \quad \beta = \frac{\sqrt{\epsilon'_c} \sigma_c \lambda_c}{\epsilon_c c}, \quad \omega_p = \frac{c}{\sqrt{\epsilon'_c} \lambda_c}, \quad J_c = \frac{\phi_0}{2\pi \mu_0 \lambda_c d},
\]

with the penetration depths \(\lambda_{ab} = 0.4 \mu m\) and \(\lambda_c = 200 \mu m\), thickness of superconducting layers \(s = 3 \AA\) and that of insulating layers \(d = 12 \AA\), Debye length \(\mu = 0.6 \AA\), dielectric constant of the junction \(\epsilon'_c = \epsilon_c / \epsilon_0 = 10\) with permittivity of the junction \(\epsilon_c\) and that of vacuum \(\epsilon_0\), plasma frequency \(\omega_p\), conductivity \(\sigma_c\), flux quantum \(\phi_0\), critical current \(J_c\) and vacuum permeability \(\mu_0\), following the material parameters in Ref. [3]. They gives \(\alpha = 0.1\) and \(\beta = 0.02\) is chosen here.

Although it is difficult to treat several hundreds of layers taken in experiments directly, such large number of layers can effectively be introduced by the periodic boundary condition along the \(c\) axis, which corresponds to infinite layers. Simulated number of layers is \(N = 4\), and consistancy of numerical results is checked for \(N = 8\) and 12. Instead of considering dielectrics on edges, the dynamical boundary condition [9] is introduced. For infinite number of layers, this boundary condition is simplified [4] as the relation between dynamical part of boundary fields:

\[
\partial_{x'} \psi_{l+1, l} = B_{l+1, l}' + \tilde{B}_{l+1, l}', \quad \partial_{t'} \psi_{l+1, l} = (E_{l+1, l}') + \tilde{E}_{l+1, l}'; \quad \tilde{B}_{l+1, l}' = \pm \sqrt{\epsilon'_d / \epsilon'_c} \tilde{E}_{l+1, l}';
\]

with the dielectric constant of dielectrics \(\epsilon'_d = 10(= \epsilon'_c)\). The static part of electric field \(E_{l+1, l}'\) turns out to be \(J' / \beta\) numerically. Width of the junction \(L_x = 86 \mu m\) is divided into 80 numerical grids, and numerical calculations are performed on the basis of the RADAU5 ODE solver.[10]

3. Numerical results

First, emission without external fields is investigated. Current dependence of intensity, namely the strength of Poynting vector (symmetric on both edges), is shown in Fig. 1(a). Intensity takes maximum at the onset of emission, and a snapshot of dynamical part of electric and magnetic fields in the junction at current is displayed in the inset. Spatial dependence of the electric field is small, which is similar to the McCumber state,[11] and 4 layers are completely in-phase and equivalent. Then, time evolution of dynamical part of the fields in a single layer after the inset of Fig. 1(a) is shown in Figs. 1(b), 1(c) and 1(d) in chronological order. These behaviors were already reported by single-layer simulations assuming in-phase motion.[7] Now there arises a question if this McCumber-like state characterizes the emission without fields.[6]

Next, in-plane external field is gradually applied to this system. In Fig. 2(a), current dependence of intensity on the left edge (forward to vortex flow) is plotted for several weak fields ranging from 0.002T to 0.01T, where the averaged number of Josephson vortices in each layer is 0.5 for \(B_{ext} = 0.01 T\) and 2 for \(B_{ext} = 0.04 T\). In-phase behavior of the fields holds in these cases, and positions of Josephson vortices are the same in all the layers. For \(B_{ext} \geq 0.004 T\), intensity takes maximum at a certain current \(J_s(B_{ext})\), not at the onset current for emission. As the in-plane field increases,
Figure 1. (a) Current dependence of emission intensity without external fields. A snapshot of dynamical part of the electric field (solid line) and magnetic field (broken line) in the reduced unit (4) at the intensity peak is shown in the inset, which tells that 4 layers are in-phase and equivalent. Similar snapshots in a layer after the one in (a) are displayed in (b), (c) and (d).

Figure 2. Current dependence of emission intensity (a) for various weak in-plane fields and (b) for various stronger in-plane fields. Peak intensity is almost independent of applied fields. $J_s$ increases systematically, while the onset current for emission is almost unchanged. Then, current dependence of intensity shown in Fig. 1(a) may not be a specific curve without in-plane fields, but can be regarded as a part of general curves seen in Fig. 2(a) with the peak at $J_s = 0$, though screened by the cutoff of minimum current. Similar behavior is also observed for stronger fields ranging from 0.01T to 0.1T as shown in Fig. 2(b). These figures strongly suggest that the maximum intensity at $J = J_s$ is almost independent of in-plane fields.

Finally, theoretical background of the above results is discussed. If the intensity becomes maximum when the wavelength $\lambda$ of transverse plasma in the junction is equal to the distance between Josephson vortices, the frequency $f_*$ of the emitted wave at its maximum is given by

$$f_* = \frac{v_p}{\lambda} = \frac{c/\sqrt{\epsilon_c}}{\phi_0/d_{\text{ext}}} = \frac{cd}{\sqrt{\epsilon_c}\phi_0} B_{\text{ext}},$$

(7)

where the velocity of transverse plasma $v_p$ already reaches the maximum value $c/\sqrt{\epsilon_c}$ \cite{12} for infinite layers. On the other hand, the ac Josephson relation is given by $f = V/\phi_0 = d\langle E_{l+1,l}^z \rangle/\phi_0$, and the averaged value of the electric field is related with the current as $J = \sigma c \langle E_{l+1,l}^z \rangle$ or
Figure 3. Field dependence of (a) emission frequency and (b) bias current at the emission peak together with theoretical curves given in Eqs. (7) and (8), respectively.

\[ f_\star = \frac{d}{\phi_0} \cdot \frac{c \phi_0}{2\pi \sqrt{\epsilon \lambda_c d}} \cdot \frac{J'_c}{\beta} \]  

These relations between the quantities at the intensity peak and in-plane fields are in good agreement with numerical results as shown in Fig. 3. Owing to large velocity of transverse plasma, \( f_\star \) quickly increases as does the in-plane field, and exceeds 1THz for \( B_{\text{ext}} = 0.02 \)T. This frequency finally becomes \( \sim 3.8 \)THz when \( J_\star \) reaches the critical current.

4. Summary and conclusion

In the present article terahertz wave emission from intrinsic Josephson junctions is investigated numerically. For a junction with the width 86\,\mu m of experimental scale and effectively infinite number of layers, a McCumber-like state is stabilized without external fields with the dynamical boundary condition on edges. When the in-plane field is gradually applied, the frequency of emitted wave and current at the emission peak increase in proportional to the field, which are consistent with the ac Josephson relation and the Ohmic \( I-V \) characteristics, while intensity at the emission peak is almost independent of the field. These results suggest that the McCumber-like state without external fields may naturally connect to the emission state in strong fields and may not characterize the emission without external fields observed experimentally.

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