Hadronization via gravitational confinement

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Abstract. We analyze baryogenesis using a Bohr-type rotating neutrino model in which the strong force is modeled as the gravitational force between ultrarelativistic neutrinos. The kinetic energy plus rest energy of these neutrinos constitutes the rest energy of the gravitationally confined composite neutrino structure. In this work we derive a simple expression for the gravitational mass of the rotating neutrinos in terms of their rest mass and their total energy. It is found, both analytically and graphically, that when the gravitational mass reaches the Planck mass, then the relativistic mass of the neutrinos equals the effective mass of u and d quarks. The model contains no adjustable parameters and leads to semiquantitative (within 1%) agreement with the masses of baryons. It also shows that hadronization is easier to achieve with light particles such as neutrinos rather than with heavier particles. The model can also be extended by considering ultrarelativistic rotating positron or electron-neutrino pairs, triplets or quartets whose mass is found to match within 2% those of $W^\pm$, $Z^0$ and $H^0$ bosons.

1. Introduction

Since the pioneering 1905 special relativistic paper of Einstein [1] it is well known that the inertial mass, $m_i$, of any body moving on a straight line is larger than its rest mass, $m_o$, and the two are related via

$$m_i = \gamma^3 m_o,$$

where $\gamma$ is the Lorentz factor given by

$$\gamma = (1 - v^2/c^2)^{-1/2},$$

where $v$ is the speed of the body. Equation (1) is obtained [2] by combining the definition of force, $F$, i.e.

$$F = \frac{dp}{dt} = \frac{d(\gamma m_o v)}{dt} = \gamma^3 m_o \frac{dv}{dt},$$

where $p$ is the momentum, with Newton’s 2nd law of motion, i.e.

$$F = m_i \frac{dv}{dt}.$$

According to the equivalence principle [3], the inertial mass, $m_i$, equals the gravitational mass, $m_g$, and thus, from (1), (3) and (4), it follows

$$m_g = \gamma^3 m_o,$$
which is valid for any linear motion.

Using the approach of instantaneous reference frames [2] in which the arbitrary motion of a particle, $\mathbf{d}x$, at any point of the trajectory, is attributed to the action of a force, $\mathbf{dF}$, parallel to $\mathbf{d}x$, it is easy to show that equation (5) remains valid for arbitrary particle motion. Consequently, as shown in references [4] and [5] the inertial, and thus gravitational, mass of a body equals its so called longitudinal mass [1] which is invariant along the direction of motion, and thus is an inertial mass, while the so called transverse mass [1] is not invariant and thus is not an inertial mass [2]. Invariant means that the same force value is perceived by the laboratory observer and by an observer on the moving particle [2]. These are simple and well established concepts of special relativity [2], yet the importance of the resulting equation (5) has been only recently recognized. Starting from Newton’s universal gravitational law

$$F = \frac{G m_1 m_2}{d^2},$$

where $d$ is the distance, and using equation (5), one obtains

$$F = \frac{G m_{1,0} m_{2,0} \gamma_1^3 \gamma_2^3}{d^2}.$$  

(7)

It is interesting that more than 110 years after Einstein’s pioneering 1905 paper [1], we still use Newton’s universal gravitational law always with $\gamma_1 = \gamma_2 = 1$. However in doing so we neglect the important increase in gravitational mass and gravitational attraction with particle velocity as expressed by equation (5). In a book [4] and in several publications [5, 6, 7, 8, 9, 10, 11, 12] we have presented Bohr-type rotating lepton models for hadrons [4], and bosons [6, 7, 11], based on equation (7) in conjunction with the de Broglie wavelength equation, which is the basis of quantum mechanics. These models contain no adjustable parameters and allow for the computation of the masses of composite particles formed by the rotating leptons. An example is given in Table 1 for the case of three rotating neutrinos forming a composite rotational state with the mass of the proton. The electric charge of the latter, as well as the fractional charges of the quarks, induced by charge-induced dipole interactions [6, 7], can be explained if a positron is present at the center of the rotating neutrinos ring [6, 7]. The literature of neutrino electromagnetic interactions has been reviewed recently [13].

It is worth noting that the Bohr-type rotating lepton models for hadrons presented in [4, 5, 6, 7, 8, 9, 10, 11, 12] and further analyzed here, are fully consistent with the bagel shape of protons computed by Miller [14] via model proton wave functions constructed with Poincaré invariance for quark spin antiparallel to the proton spin and quark momenta of 1 GeV/c [14]. This value is in close agreement with the momenta (0.939 GeV/c) of the neutrinos in the rotating neutrino proton model computed in [4, 5], as shown in Table 1 using the value of 0.0437 eV/c$^2$ for the rest mass of neutrinos [4, 5, 15, 16, 17, 18] which is in good agreement with the recent results of the Kamiokande experiments [15, 17].

In the present work we first derive a simple equation and corresponding graphical method for determining the masses of gravitationally confined hadron structures and then we discuss the key advantageous property of neutrinos which makes them suitable for hadronization, i.e. for the generation of mass.

2. Results
2.1. Gravitational mass and its dependence on particle velocity and total energy

According to the weak equivalence principle of Einstein and Eötvös [3], which has been confirmed by the most modern torsion balance measurements to at least 1 part in 10$^{12}$ [3, 4], it is

$$m_g = m_t,$$

(8)
Figure 1. Schematic comparison and synthesis of the bagel shape of protons computed in [14] via model wave functions, constructed with Poincaré invariance, and of the three-rotating neutrino model of protons [4, 5, 6]. Particle size represents gravitational mass. The central particle is a positron of negligible speed, thus negligible gravitational mass [6].

i.e. gravitational mass equals inertial mass and therefore, as already stated, it follows from equation (1) that

$$m_g = \gamma^3 m_o,$$

where \( m_o \) is the rest mass.

Also, from Einstein’s special relativity [1, 2], the relativistic mass, \( m_r \), and the total (i.e. rest plus kinetic) energy, \( E \), of a particle are related to the rest mass \( m_o \) via

$$m_r = \gamma m_o \quad ; \quad E = m_r c^2 = \gamma m_o c^2.$$  \( (10) \)

Upon eliminating \( \gamma \) between (9) and (10) one obtains

$$m_g = \left(1/\gamma^2\right) m_r^3 \quad ; \quad m_g = \left(1/m_o^2 c^6\right) E^3.$$  \( (11) \)

These equations, and particularly the latter one, are found to be quite useful since they enable one to express directly the gravitational mass of a particle in terms of its total energy.

2.2. Forces between fast neutrinos

The potential importance of gravitational interactions between neutrinos and gravitational fields had been discussed already in 1957 by Dieter and Wheeler [19], long before it was established that neutrinos have mass [17].

Figure 2 is obtained from equations (11) and shows, in the left hand side axis, the dependence of the gravitational mass, \( m_{g,\nu} \), of a neutrino with rest mass \( m_{o,\nu}(= 0.0437 \text{ eV}/c^2) \) on its relativistic mass, \( m_{r,\nu} \), and on its total energy \( E(= m_{r,\nu} c^2) \). The slope of 3 on the log \( m_{g,\nu} \) vs log \( m_{r,\nu} \) and log \( m_{g,\nu} \) vs \( E \) plots of Figure 2 is the result of the 3d power in equations (11).

The right hand of axis of Figure 2 shows the dependence on \( m_{r,\nu} \) and \( E \) of the ratio of the gravitational force, \( F_G \), between two neutrinos with rest mass \( m_{o,\nu} \) each, computed from equation
Table 1. Comparison of the Bohr model for the H atom and of the rotating neutrino model for baryons [4, 5]

| Bohr model for the H atom | Bohr-Einstein model for baryons [4,5] |
|---------------------------|--------------------------------------|
| Electron as particle     | Neutrino as particle                |
| \( m_e V^2 / r = e^2 / qr^2 \) | \( \gamma \nu m_{\nu} V^2 / r = Gm_{\nu}^2 \nu^6 / \sqrt{3} r^2 \) |
| Newton’s law              | Relativistic equation of motion for \( \gamma m_{\nu} \) |
| Coulomb law              | for special relativity \( (m_i = \gamma m_0) \) |
|                          | for equivalence principle \( (m_{\nu} = m_i) \) |

Electron as wave
\[
\frac{\hbar}{m_e \nu} = r
\]
De Broglie (for \( n = 1 \))

Neutrino as wave
\[
\frac{\hbar}{\gamma \nu m_{\nu} \nu^2} \approx \frac{\hbar}{\gamma \nu m_{\nu} \nu^2} = r
\]
De Broglie (for \( n = 1 \))

**Results**
\[
\nu = \omega \epsilon \quad \gamma = 1
\]
\[
r = \frac{\hbar}{gm_0} = 0.5292 \times 10^{-10} \text{m}
\]
\[
V_e = -27.2 \text{ eV}
\]
\[
T = 13.6 \text{ eV}
\]
\[
\gamma = T + V_e = -13.6 \text{ eV}
\]
\[
\Delta (\text{RE}) = \Delta m^2 = T = (1/2)\epsilon^2 m_e^2 = 13.6 \text{ eV}
\]

**Results** (with \( m_{\nu} = 0.043723 \text{ eV}/c^2 \))
\[
\nu = \omega \epsilon \quad \gamma = 3^{1/2}(m_\nu / m_{\nu,\nu})^{1/2} = 7.163 \times 10^6
\]
\[
r = \frac{\hbar}{\gamma \nu m_\nu \nu^2} = 0.630 \times 10^{-13} \text{m}
\]
\[
V_0 = -(5/3)m_\nu \epsilon^2 = -1565.9 \text{ MeV}
\]
\[
T = m_\nu \epsilon^2 = 939.656 \text{ MeV}
\]
\[
p = m_\nu \epsilon = 939.656 \text{ MeV} / c
\]
\[
\gamma = T + V_0 = -(2/3)m_\nu \epsilon^2 = -626.4 \text{ MeV}
\]
\[
\Delta (\text{RE}) = \Delta m^2 = T = m_\nu \epsilon^2 = 3^{1/2}(m_{\nu,\nu} \epsilon)^{1/2} = -939.656 \text{ MeV}
\]

Per particle:
\[
T_\nu = 313.2 \text{ MeV}
\]
\[
\gamma_\nu = -208.8 \text{ MeV}
\]

(7), and of the Coulombic electrostatic force, \( F_e \), between a positron, \( e^+ \), and an electron, \( e^- \), at the same distance. This force is computed from Coulomb’s law, i.e. from
\[
F_e = \frac{e^2}{4\pi\epsilon_0 r^2},
\]
with \( \epsilon = 4\pi\epsilon_0 = 1.11 \times 10^{-10} \text{ C}^2/\text{Nm} \), where \( \epsilon_0 \) is the permittivity of vacuum.

Surprisingly, as shown in Figure 2, this ratio, which is computed directly from equations (7) and (12), i.e.
\[
\frac{F_G}{F_e} = \frac{\epsilon G m_{\nu,\nu} \gamma^6}{e^2} = \frac{\epsilon G m_{\nu,\nu}^2 \epsilon^2}{e^2 m_{\nu,\nu}^4 \epsilon^2} = \frac{\epsilon G E^6}{e^2 m_{\nu,\nu}^4 \epsilon^2},
\]

exceeds unity for relativistic neutrino masses above 130 MeV/c\(^2\) or momenta above 0.13 MeV/c., i.e. for total neutrino energies above 130 MeV. Consequently, for neutrino energies above 130 MeV, Newtonian gravity with gravitational masses is stronger than Coulombic attraction or repulsion. As shown in Figure 2, for a neutrino energy of 313 MeV (corresponding closely to the effective mass of u and d quarks [20, 21]), the \( F_G/F_e \) ratio takes the value \( 3^{1/2}/\alpha \approx 237 \).

Clearly, for such neutrino energies, their gravitational attraction is as strong as the Strong Force itself. As shown in the left axis of Figure 2, at the same neutrino energy of 313 MeV the
Figure 2. Plot of equations (11) for electron neutrinos \( (m_{\nu,0} = 0.0437 \text{ eV} \ [4, 5, 15, 16]) \) showing that baryons and mesons form from neutrinos when the gravitational mass, \( m_{g,\nu} \), of the latter reaches the Planck mass. (more precisely when it reaches \( 2m_{Pl} \) for mesons, \( 3^{1/4}m_{Pl} \) for baryons). At these points the \( F_G/F_e \) ratio reaches \( 4/\alpha(\approx 548) \) and \( 3^{1/2}/\alpha(\approx 237) \) respectively; \( \alpha = e^2/\hbar c \approx 1/137.035 \).

neutrino gravitational mass, \( \gamma^3m_0 \), reaches the value of \( 3^{1/4}m_{Pl} \), i.e. it exceeds the Planck mass (= \( \hbar c/G = 1.221 \times 10^{19} \text{ GeV}/c^2 \)) by 31.6%. This has been shown to be exactly the value of the gravitational mass of a neutrino rotating with two other partners around their gravitational mass center in circular motion [4, 5].

Indeed, upon combining the equation of motion

\[
\gamma_{\nu}m_{\nu,0}c^2/r = \frac{Gm_{\nu,0}^2c^4}{\sqrt{3}r^2}
\]

with the de Broglie equation

\[
r = \frac{\hbar}{\gamma_{\nu}m_{\nu,0}c}
\]

one obtains

\[
3\gamma_{\nu}m_{\nu,0} = 3^{13/12}(m_{Pl}m_{\nu,0}^2)^{1/3} = 939 \text{ MeV}/c^2,
\]

which is the proton mass and

\[
\gamma_{\nu}^3m_{\nu,0} = 3^{1/4}m_{Pl}
\]
which as presented in Figure 2 is the gravitational mass corresponding to the effective $u$ and $d$ quark mass of 313 MeV. This shows clearly that the effective mass of quarks is that of relativistic neutrinos.

It is worth noting in equation (13), that for fixed particle energy $E$, the ratio $F_G/F_e$ increases dramatically with decreasing neutrino rest mass $m_{\nu\nu}$. This is a direct consequence of equations (11) which show that, for fixed $E$, the gravitational mass of a particle increases significantly with decreasing rest mass $m_{\nu\nu}$. This in turn shows that gravitational confinement of neutrinos can take place with moderate (e.g. less than 500 MeV) energies (Fig. 2) because the rest mass of neutrinos is so small ($\sim 0.0437 \text{ eV}/c^2$) [4, 5, 15]. Indeed the condition for gravitational confinement, i.e.

$$m_g \geq m_{Pl},$$

(18)

implies in view of equations (11) that

$$E_{\nu} \geq (m_{Pl}m_{\nu\nu}^2)^{1/3}c^2 = 286 \text{ GeV},$$

(19)

or equivalently

$$m_{\nu\nu} \leq \frac{E^{3/2}}{m_{Pl}^{3/2}c^3},$$

(20)

which for $E_{\nu} = 0.313 \text{ GeV}$ gives

$$m_{\nu\nu} \leq 0.05 \text{ eV}/c^2.$$  

(21)

This limit practically coincides with the largest mass of electron neutrinos [4, 5, 15, 16].

Equation (19) written for electrons, which have a rest mass 1.17·10$^7$ times larger than that of neutrinos, leads to an $E_{\nu}$ value of 14.3 TeV. This shows why lighter particles such as neutrinos, are ideal for building composite particles such as hadrons.

3. Discussion and Conclusions

This paper presents a simple expression for the gravitational mass of particles in terms of their rest mass and their relativistic mass or their total energy. This expression, which is based entirely on the special relativistic definitions of rest, relativistic, inertial and gravitational mass, is then used in conjunction with the laws of Newton and Coulomb to show that the gravitational attraction between relativistic neutrinos with energies above 130 MeV is stronger than the electrostatic attraction between a positron and an electron at the same distance. This is because inertial and gravitational mass is accounted to increase with $\gamma^3$, as first derived by Einstein already in 1905 [1], whereas relativistic mass and total energy increase with $\gamma$ [1].

The same expression (equation (5)) shows that the energy (313 MeV) for which the neutrino gravitational mass reaches the Planck mass, corresponds to the effective mass of quarks, i.e. to one third the mass of protons or neutrons. This, in conjunction with the fact that the Newtonian gravitational attraction between two Planck masses equals $\hbar c/d^2$, shows that the Strong Force, which confines protons and neutrons, is the relativistic gravitational force. Thus, this same equation allows for the graphical determination of the mass of gravitationally confined rotating lepton composite particles as shown in Figure 2.

One interesting aspect of the rotating lepton model which is shown explicitly by equations (11) and by the derived equations (19) to (21) is that for any given particle energy, $E$, hadronization occurs easier with particles which have smaller rest mass. This explains why neutrinos are ideal as hadronization components.

The present results confirm that the use of the gravitational mass, $\gamma^3m_{\nu\nu}$, in Newton’s gravitational law leads to excellent agreement with experimental results in the microcosmos of hadrons. They show that gravity generates mass via gravitational confinement and concomitant trapping of neutrinos in ultrarelativisic orbits. The kinetic energy of these trapped neutrinos is
the rest energy of the composite rotational hadronic structures. The gravitational field generated by these ultrarelativistic neutrinos has the basic property of the Higgs field, i.e. it generates mass. The kinetics of this mass generation have been discussed in [9].

It is thus important to establish the consistency of the relativistic mass Newtonian equation (7) with the theory of general relativity (GR) since the latter is the most well known and generally accepted gravitational theory [22, 23]. Indeed the effective force computed from the effective potential of the Schwarzschild geodesics of neutrinos rotating around Planckian masses matches that computed via the present special relativistic approach [5, 8], which provides further support for the rotating neutrino model. In the GR treatment [5, 8] gravitational collapse of the composite particle to a singularity is prevented by the Heisenberg uncertainty principle. This is consistent with the hypothesis that protons and neutrons can be also viewed as microscopic black holes [8, 25, 26, 27] whose gravitational collapse to a singularity is prevented by the Heisenberg uncertainty principle, expressed via the de Broglie wavelength equation, leading to the generalized uncertainty principle and generalized event horizon concepts. Furthermore, equation (7) has been shown recently to give the same results with GR regarding the perihelion precession of Mercury [24], which was one of the key initial tests of GR. Consequently, equation (7) appears to be in good agreement with the GR treatment both in the microcosmos of quarks and in the macrocosmos of celestial motion.

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