Research Article

A Type-3 Fuzzy Approach for Stabilization and Synchronization of Chaotic Systems: Applicable for Financial and Physical Chaotic Systems

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In this paper, a new approach is presented for stabilizing and synchronizing financial chaotic systems. A new type-3 (T3) fuzzy-based system (FLS) with an online optimization scheme is designed to cope with chaotic behavior, high-level uncertainties, and unknown dynamics. An adaptive compensator also eliminates the effect of approximation errors (AEs) and perturbations. The stability of the dynamics of synchronization errors is guaranteed by the use of the Lyapunov method. Several simulations and comparisons demonstrate the superiority of the suggested control and synchronization scenarios.

1. Introduction

Chaos theory studies the mathematical formulation and behavior of dynamic systems sensitive to initial conditions. Chaos often has been seen as an interdisciplinary theory for exploring the randomness of complex chaotic systems to identify fundamental fractals, self-organization, basic patterns, fixed feedback loops, repetitions, and self-similarities. These special features have provided good potential applications for chaotic systems. Recently, various applications have been reported for chaotic systems such as image encryption [1], chaotic maps [2], time series [3], optimization algorithms [4], medical systems [5], and secure communications [6].

The control of chaotic systems (CSs) is a complex control problem because of their complex nonlinear dynamics, high senility to the initial condition, hard dynamic perturbation in most of their applications, and stochastic dynamical behavior such as symmetry and dissipation. The controllers in this field can be classified into three classes: classical methods, neuro-fuzzy controllers, and hybrid control methods.

For the first class, some model-based controllers have been presented. For example, in [7], the passivity-based approach is developed using a sliding-mode controller (SMC) and it is applied for unified CSs. In [8], bifurcation analyses are presented for a CS, and by the Lyapunov method, a robust synchronization scheme is proposed. The feedback controller is designed in [9], and the stability is investigated using the Barbashin–Krasovskii approach. The adaptive SMC is studied in [10], and its accuracy is evaluated on chameleon CSs. In [11], the behavior of CSs is analyzed...
and by the use of the finite-time stability theorem, the stability is studied in various conditions. In [12], the SMC scheme is developed and the bifurcation diagrams are analyzed. In [13], a fixed-time convergence scheme is presented for memductance-based CSs and the robustness is examined under bounded perturbations. In [14], the bifurcation of butterflyfish CSs is investigated and a linear control method is suggested for synchronization. The model-based adaptive controller is developed in [15] to investigate the stability of a generator of nuclear spin CSs. The projective control systems and synchronization of CSs are studied in [16].

The FLSs and neural networks (NNs) are widely used to cope with uncertainties. For chaotic systems, some neuro-fuzzy methods have also been studied. For example, in [17], NNs are used for prediction problems in Hyperjerk CSs and their physical circuit implementation is investigated. The finite-time synchronization of CSs is studied in [18] and, by analyzing the convergence time, an FLS-based controller is presented. In [19], the synchronization problem is studied by the use of NNs and the designed synchronized scheme is applied for a secure communication system. In [20], a Takagi–Sugeno FLS is used for modeling and a sampled-data control system is developed for synchronization. The FLS-based SMC is formulated in [21], and the better performance of FLS-based schemes is shown by applying a gyroscope CS. The backstepping SMC is suggested in [22] for stabilizing CSs in the presence of time delay, and FLSs are used to estimate some nonlinearities. In [23], based on reinforcement learning and FLSs, a synchronization scheme is developed. The performance of FLS-based synchronization methods is evaluated and analyzed in [24]. In [25], improvement of the synchronization accuracy and the convergence speed is studied by the use of FLSs. In [26, 27], the superiority of FLS-based controllers in robotic applications is studied.

The financial CSs are widely used in economic problems [28–30]. The dynamics of these classes of chaotic systems are much more complex because of the existence of various unpredictable factors. The stabilization and synchronization of financial CSs have been rarely studied. For example, the integral SMC is designed in [31] and the stabilizing conditions are studied. Similarly, the terminal SMC is developed in [32] and the control of a financial CS is analyzed. In [33], an $H_{\infty}$-based control system is designed and FLSs are used to investigate the robustness. The risk assessment of financial CSs is investigated in [34], and an FLS-based method is presented. In [35], an FLS-based system is proposed for forecasting applications. The finite-time control of hypercritic financial systems is investigated in [36], and some adaptation rules are suggested for stabilizing. A neuro-fuzzy-based controller is designed in [37] to guarantee the stability of the financial CS.

The literature reviewing show that

(i) In most studies, the controller is designed for a special case of CS and the designed controller cannot be applied for a CS with different parameters and dynamics [7–9]

(ii) The stability of most reviewed controllers is not guaranteed under perturbations and uncertainties [30, 38]

(iii) Some type-1 and type-2 neuro-fuzzy controllers have been developed for CSs, but most of them cannot handle the high uncertainties in CS dynamics [33–37]

(iv) Most of the previous studies are optimized in an offline scheme [33–37]

(v) The robustness against unknown perturbations in CS dynamics needs more studies [9, 36, 37]

Regarding the above discussion, a new type-3 fuzzy-based controller is developed for CSs. Most recently, the type-3 FLSs with strong uncertainty modeling capability have been developed. In various studies, it has been shown that the T3-FLSs result in much better performance in a high-noisy environment [39]. Some T3-FLS-based controllers have been developed MEMS gyroscopes and a class of fractional-order CSs [39–41]. In these studies, the T3-FLS-based controllers are developed for first-order fractional-order CSs and the special cases of MEMS gyroscopes. To the best of our knowledge, stable and robust controllers based on T3-FLSs have not been studied for financial CSs. As mentioned earlier, this class of CSs exhibits a complex stochastic behavior and their dynamics is perturbed in most applications. In this study, a new control scenario is suggested on the basis of T3-FLSs. The designed T3-FLSs are optimized by the Lyapunov learning rules. The robustness is analyzed in both stabilization and synchronization problems, and an adaptive compensator is developed to deal with AEs. The main contributions are

(i) The designed controller does not depend on the parameters and dynamics of the case study CS. It can be easily applied for the various cases of CSs.

(ii) The stability is ensured under perturbations and uncertainties.

(iii) A T3-FLS-based approach is developed for better handling the uncertainties.

(iv) The suggested controller is tuned in an online scheme. In other words, the free parameters of the T3-FLS are tuned at each sample time. This approach can handle unpredicted perturbations.

(v) The robustness is studied, and a compensator is developed.

2. Problem Formulation

The following financial CSs are considered [42]:

$$
\begin{align*}
\dot{x}_1 &= (-a + x_2)x_1 + x_3, \\
\dot{x}_2 &= -b\chi_2 - \chi_2^3 + 1, \\
\dot{x}_3 &= -\chi_1 - c\chi_3,
\end{align*}
$$

where $b$ denotes the investment cost, $a$ represents the saving amount, $c$ is elasticity of commercial demands, $x_1$ is the interest rate, $\chi_2$ represents investment demand, and $\chi_3$ is the price exponent. The bifurcation and the Lyapunov spectrum analysis have been studied in [43, 44]. The phase portraits are depicted in Figure 1 that shows the chaotic attractors.
The variation of the investment cost, the saving amount, and elasticity of commercial demands are assumed to be unknown, and also the dynamics is unknown. The suggested T3-FLS is used to cope with variations of parameters and dynamic uncertainties and perturbations. The suggested control diagram is shown in Figure 2. We see that T3-FLSs are optimized such that all states are stabilized and the outputs track the reader system. The compensators deal with the perturbations.

Remark 1. The case study CS 1 is considered in this paper because it is so popular in the field of economic plants, and it exhibits more complex chaotic behavior. However, the suggested controller does not depend on the dynamics, and it can be easily applied for the various cases of CSs.

3. Type-3 FLS

The neuro-fuzzy systems and learning methods are extensively used to tackle uncertainties. In this paper, a new stronger approach is formulated to cope with uncertainties in the complex dynamics of CSs. The suggested FLS scheme is depicted in Figure 3. All uncertainties are tackled using optimized T3-FLSs. The computations are given as follows:

(1) The inputs of the T3-FLS $\psi_i$ are $\chi_1, \chi_2$, and $\chi_3$.

(2) For inputs $\chi_1, \chi_2$, and $\chi_3$, the Gaussian MFs as shown in Figure 4 are considered. Unlike the type-2 MFs, in type-3 MFs, we have four membership values for each horizontal slice. Two of them are associated with the lower cut and the other two for the upper cut. The MFs for $\chi_i$ are defined as $\Omega_{\chi_i}$ and $\bar{\Omega}_{\chi_i}$ (see Figure 4). The centers of $\Omega_{\chi_i}$ and $\bar{\Omega}_{\chi_i}$ are determined considering the upper/lower bounds $\Omega_{\chi_i}$. For $\chi_i$, the upper memberships of $\bar{\Omega}_{\chi_i}$ and $\Omega_{\chi_i}$ considering the upper/lower slice $\bar{\eta}_i/\eta_i$ are written as

$$\bar{\mu}_{\Omega_{\chi_i}}(\chi_i) = \exp \left( -\frac{(\chi_i - C_{\Omega_{\chi_i}})^2}{\sigma_{\Omega_{\chi_i}}^2} \right).$$

$$\mu_{\Omega_{\chi_i}}(\chi_i) = \exp \left( -\frac{(\chi_i - C_{\Omega_{\chi_i}})^2}{\sigma_{\Omega_{\chi_i}}^2} \right).$$

$$\bar{\mu}_{\Omega_{\chi_i}}(\chi_i) = \exp \left( -\frac{(\chi_i - C_{\Omega_{\chi_i}})^2}{\sigma_{\Omega_{\chi_i}}^2} \right).$$

$$\mu_{\Omega_{\chi_i}}(\chi_i) = \exp \left( -\frac{(\chi_i - C_{\Omega_{\chi_i}})^2}{\sigma_{\Omega_{\chi_i}}^2} \right).$$

Figure 1: Phase portrait.
Figure 2: Control scheme.

Figure 3: A schematic view of the T3-FLS.

Figure 4: A schematic view of the type-3MF.

Complexity
Similarly, for $\Omega_i$, we have

$$
\mu_{\Omega_i}(x_i) = \exp \left( -\frac{(x_i - C_{\Omega_i})^2}{\sigma_{\Omega_i}^2} \right),
$$

where $C_{\Omega_i}$ and $\sigma_{\Omega_i}$ denote centers of $\Omega_i$ and $\Omega_i$, respectively. $\bar{x}_{\Omega_i}$ and $\bar{\sigma}_{\Omega_i}$ represent the upper width of $\Omega_i$ at the upper slice $\bar{y_i}$ and lower slice $y_i$.

In the next step, considering the memberships in step 2, the rule firings should be computed. We have $R$ rules. The $r$-th rule is written as

$$
\text{Rule } \#r: \quad \text{If } x_i \text{ is } \Omega_{r_i}, \text{ and } \bar{x}_r \text{ is } \Omega_{\bar{r}_i}, \quad \text{Then } \psi \in \left[ x_r, \bar{x}_r \right],
$$

where $x_i, \bar{x}_r$ denote the rule parameters.

By the use of the product T-norm and the simple type-reduction of [45], the output of $\psi_i$ is written as

$$
f_i = \left[ x^T_{i1}, \ldots, x^T_{iR}, \bar{x}_{i1}, \ldots, \bar{x}_{iR} \right],
$$

$$
\zeta_i = \left[ \zeta^T_{i1}, \ldots, \zeta^T_{iR}, \bar{\zeta}_{i1}, \ldots, \bar{\zeta}_{iR} \right],
$$

where $R$ is number of rules and $\bar{x}_{ir}$ and $x_{ir}$ represent the $r$-th rule parameters. $\zeta_{ir}$ and $\bar{\zeta}_{ir}$ are

$$
\bar{\zeta}_{ir} = \frac{\sum_{j=1}^{n} (\bar{v}_{jr} + \bar{\eta}_{jr})}{\sum_{j=1}^{n} (\bar{v}_j + \psi_j)}
$$

and

$$
\eta_{ir} = \frac{\sum_{j=1}^{n} (v_{jr} + \eta_{jr})}{\sum_{j=1}^{n} (v_j + \psi_j)},
$$

where $n_i$ denotes slice numbers and

$$
\zeta_i = \frac{\sum_{j=1}^{n} v_j \eta_{jr}}{\sum_{j=1}^{n} (v_j + \psi_j)}
$$

and

$$
\bar{\eta}_{ir} = \frac{\sum_{j=1}^{n} (\bar{v}_j + \bar{\eta}_{jr})}{\sum_{j=1}^{n} (\bar{v}_j + \psi_j)}
$$

and

$$
\eta_{ir} = \frac{\sum_{j=1}^{n} (v_j + \eta_{jr})}{\sum_{j=1}^{n} (v_j + \psi_j)},
$$

where $f_i$ and $\zeta_i$ are
4. Stabilization

In this section, the stabilizing controller is designed and its stability conditions are analyzed. The main results are presented in Theorem 1.

**Theorem 1.** The system (1) is stabilized by using controllers (17), adaptation rules (18), and compensators (19).

\[ u_i = -\psi_i(\chi_i f_i) - k_i x_i - u_{ci}, \]
\[ \dot{f_i} = u_i c_i X_i, \]
\[ u_{ci} = E_i \text{Sign}(\chi_i), \]

where \( i \) and \( k \) are constants, \( \psi_i \) denotes the T3-FLS, \( \chi \) and \( f \) are the input vector and the rule parameter vector of \( \psi_i \), respectively, and \( E_i \) is the upper bound of AE.

**Proof.** By applying the controllers \( u_i \), \( i = 1, 2, 3 \) (17) into (1), we have

\[
\begin{align*}
\dot{x}_1 &= (-a + x_2)x_1 + x_3 \\
&\quad - k_1 x_1 - \psi_1(\chi f_1) - u_{c1}, \\
\dot{x}_2 &= -b x_2 - x_1^2 + 1 \\
&\quad - k_2 x_2 - \psi_2(\chi f_2) - u_{c2}, \\
\dot{x}_3 &= -x_1 - c x_3 \\
&\quad - k_3 x_3 - \psi_3(\chi f_3) - u_{c3}.
\end{align*}
\]

From (31), \( \dot{\chi} \) is written as

\[
\begin{align*}
\dot{\chi}_1 &= (-a + x_2)x_1 + x_3 \\
&\quad - k_1 x_1 - \psi_1(\chi f_1) - u_{c1}, \\
\dot{\chi}_2 &= -b x_2 - x_1^2 + 1 \\
&\quad - k_2 x_2 - \psi_2(\chi f_2) - u_{c2}, \\
\dot{\chi}_3 &= -x_1 - c x_3 \\
&\quad - k_3 x_3 - \psi_3(\chi f_3) - u_{c3}.
\end{align*}
\]

The optimal T3-FLSs \( (\psi_i^+(\chi f_i^+)) \) are defined as T3-FLSs with optimal parameters \( (f_i^+) \). The optimal T3-FLSs are added and subtracted into (32); then, we have

\[
\begin{align*}
\dot{\chi}_1 &= (-a + x_2)x_1 + x_3 \\
&\quad - k_1 x_1 - \psi_1(\chi f_1) - u_{c1} \\
&\quad + \psi_1^+(\chi f_1^+) - \psi_1^+(\chi f_1^+), \\
\dot{\chi}_2 &= -b x_2 - x_1^2 + 1 \\
&\quad - k_2 x_2 - \psi_2(\chi f_2) - u_{c2} \\
&\quad + \psi_2^+(\chi f_2^+) - \psi_2^+(\chi f_2^+), \\
\dot{\chi}_3 &= -x_1 - c x_3 \\
&\quad - k_3 x_3 - \psi_3(\chi f_3) - u_{c3} \\
&\quad + \psi_3^+(\chi f_3^+) - \psi_3^+(\chi f_3^+).
\end{align*}
\]

By simplification, the dynamics of \( \chi_i, i = 1, 2, 3 \) in (33) is rewritten as

\[
\begin{align*}
\dot{\chi}_1 &= \psi_1^+(\chi f_1^+) - \psi_1(\chi f_1) \\
&\quad - k_1 x_1 - u_{c1}, \\
&\quad + (-a + x_2)x_1 + x_3 - \psi_1^+(\chi f_1^+), \\
\dot{\chi}_2 &= \psi_2^+(\chi f_2^+) - \psi_2(\chi f_2) \\
&\quad - k_2 x_2 - u_{c2} \\
&\quad - b x_2 - x_1^2 + 1 - \psi_2^+(\chi f_2^+), \\
\dot{\chi}_3 &= \psi_3^+(\chi f_3^+) - \psi_3(\chi f_3) \\
&\quad - k_3 x_3 - u_{c3} \\
&\quad - \chi - c x_3 - \psi_3^+(\chi f_3^+).
\end{align*}
\]

By the use of the vector from (5) we have

\[
\begin{align*}
\dot{\chi}_1 &= \frac{\gamma}{\gamma_1} \chi_1 \\
&\quad - k_1 x_1 - u_{c1} \\
&\quad + (-a + x_2)x_1 + x_3 - \psi_1^+(\chi f_1^+), \\
\dot{\chi}_2 &= \frac{\gamma}{\gamma_2} \chi_2 \\
&\quad - k_2 x_2 - u_{c2} \\
&\quad - b x_2 - x_1^2 + 1 - \psi_2^+(\chi f_2^+), \\
\dot{\chi}_3 &= \frac{\gamma}{\gamma_3} \chi_3 \\
&\quad - k_3 x_3 - u_{c3} \\
&\quad - \chi - c x_3 - \psi_3^+(\chi f_3^+),
\end{align*}
\]

where \( \bar{f}_i = f_i^+ - f_i \). For stability investigation, to prove the results of Theorem 1, a Lyapunov candidate is considered as

\[
V = \frac{1}{2} x_1^2 + \frac{1}{2} x_2^2 + \frac{1}{2} x_3^2 \\
+ \frac{1}{\gamma_1} \bar{f}_1 \bar{f}_1 + \frac{1}{\gamma_2} \bar{f}_2 \bar{f}_2 + \frac{1}{\gamma_3} \bar{f}_3 \bar{f}_3.
\]

The time derivative of (38) yields

\[
\begin{align*}
\dot{V} &= \frac{1}{\gamma_1} \bar{f}_1 \bar{f}_1 - \frac{1}{\gamma_1} \bar{f}_1 \bar{f}_1 - \frac{1}{\gamma_3} \bar{f}_3 \bar{f}_3.
\end{align*}
\]

Substituting from \( \dot{\chi}_i, i = 1, 2, 3 \) (26)–(28) into (39), we obtain
\[
\dot{V} = \begin{bmatrix} \dot{f}_1^T c_1 - k_1 x_1 - u_{c1} \\ + (a + \chi_2) x_1 + x_3 - \psi_1(x(f_1^*)) \\ \dot{f}_2^T c_2 - k_2 x_2 - u_{c2} \\ - b x_2 - x_1^2 + 1 - \psi_2(x(f_2^*)) \\ \dot{f}_3^T c_3 - k_3 x_3 - u_{c3} \\ - \chi_1 - c x_3 - \psi_3(x(f_3^*)) \end{bmatrix} x_1
\]
(19)

Equation (40) can be rewritten as
\[
\dot{V} = -k_1 x_1^2 + f_1^T c_1 x_1 - \frac{1}{t} f_1^T f_1 - u_{c1} x_1
\]
\[
- k_2 x_2^2 + f_2^T c_2 x_2 - \frac{1}{t} f_2^T f_2 - u_{c2} x_2
\]
\[
- k_3 x_3^2 + f_3^T c_3 x_3 - \frac{1}{t} f_3^T f_3 - u_{c3} x_3
\]
\[
+ [(-a + \chi_2) x_1 + x_3 - \psi_1(x(f_1^*))] x_1
\]
\[
+ [-b x_2 - x_1^2 + 1 - \psi_2(x(f_2^*))] x_2
\]
\[
+ [-\chi_1 - c x_3 - \psi_3(x(f_3^*))] x_3.
\]
(20)

We can write \(\dot{V} < 0\), and then, the proof is completed. \(\square\)

5. Synchronization

In this section, the synchronization controller is designed and its stability conditions are analyzed.

**Theorem 2.** System (1) tracks the leader states \((r_i)\) by using controller (28), adaptation rules (29), and compensator (30).

\[
u_i = \dot{r}_i - \psi_i(s_i f_i) - k_i s_i - u_{ci},
\]
(28)

\[
f_i = \frac{\epsilon}{E} s_i,
\]
(29)

\[
u_{ci} = E \text{sign}(s_i),
\]
(30)

where \(\epsilon\) and \(k\) are constants, \(\psi_i\) denotes the T3-FLS, \(s_i\) is defined as \(s_i = x_i - r_i\), the input vector \(s\) is defined as \(s = [s_1, \ldots, s_n]^T\) and \(n\) represents the state number, \(f\) is the rule parameter vector of \(\psi_i\), respectively, and \(E\) is the upper bound of AE.

**Proof.** By applying the controllers \(u_i, i = 1, 2, 3\) (28), the dynamics of (47) is written as

\[
\dot{s}_i = (-a + \chi_2) x_1 + x_3 - k_i s_i - \psi_i(s_i f_i) - u_{ci},
\]
(31)

\[
\dot{s}_2 = \epsilon x_2 - x_1^2 + 1 - k_2 s_2 - \psi_2(s_2 f_2) - u_{c2},
\]
\[
\dot{s}_3 = -\chi_1 - c x_3 - k_3 s_3 - \psi_3(s_3 f_3) - u_{c3}.
\]
From (31), \(\dot{s}\) is written as
\[ \dot{s}_1 = (-a + \chi_2)\chi_1 + \chi_3 - k_1 s_1 - \psi_1 (s(f_1)) - u_{c1}, \]
\[ \dot{s}_2 = -b\chi_2 - \chi_1^2 + 1 - k_2 s_2 - \psi_2 (s(f_2)) - u_{c2}, \]
\[ \dot{s}_3 = -\chi_1 - c\chi_3 - k_3 s_3 - \psi_3 (s(f_3)) - u_{c3}. \]

Similar to the proof of Theorem 2, the optimal T3-FLSs are added and subtracted into (32); then, we have
\[ \dot{s}_1 = (-a + \chi_2)\chi_1 + \chi_3 - k_1 s_1 - \psi_1 (s(f_1)) + \psi_1^* (s(f_1^*)) - \psi_1^* (s(f_1^*)), \]
\[ \dot{s}_2 = -b\chi_2 - \chi_1^2 + 1 - k_2 s_2 - \psi_2 (s(f_2)) - u_{c2} + \psi_2^* (s(f_2^*)) - \psi_2^* (s(f_2^*)), \]
\[ \dot{s}_3 = -\chi_1 - c\chi_3 - k_3 s_3 - \psi_3 (s(f_3)) - u_{c3} + \psi_3^* (s(f_3^*)) - \psi_3^* (s(f_3^*)). \]

The dynamics of (33) is rewritten as
\[ \dot{s}_1 = \psi_1^* (s(f_1^*)) - \psi_1 (s(f_1)) - k_1 s_1 - u_{c1} + (-a + \chi_2)\chi_1 + \chi_3 - \psi_1^* (s(f_1^*)), \]
\[ \dot{s}_2 = \psi_2^* (s(f_2^*)) - \psi_2 (s(f_2)) - k_2 s_2 - u_{c2} - b\chi_2 - \chi_1^2 + 1 - \psi_2^* (s(f_2^*)), \]
\[ \dot{s}_3 = \psi_3^* (s(f_3^*)) - \psi_3 (s(f_3)) - k_3 s_3 - u_{c3} - \chi_1 - c\chi_3 - \psi_3^* (s(f_3^*)). \]

By the use of the vector form (5), we have
\[ \dot{s}_1 = \tilde{f}_1^T \tilde{s}_1 - k_1 s_1 - u_{c1} + (-a + \chi_2)\chi_1 + \chi_3 - \psi_1^* (s(f_1^*)), \]
\[ \dot{s}_2 = \tilde{f}_2^T \tilde{s}_2 - k_2 s_2 - u_{c2} - b\chi_2 - \chi_1^2 + 1 - \psi_2^* (s(f_2^*)), \]
\[ \dot{s}_3 = \tilde{f}_3^T \tilde{s}_3 - k_3 s_3 - u_{c3} - \chi_1 - c\chi_3 - \psi_3^* (s(f_3^*)), \]
where \( \tilde{f}_i = f_i^* - f_i \). To prove the results of Theorem 2, a Lyapunov candidate is considered as
\[ \dot{V} = \frac{1}{2} \tilde{s}_1^2 + \frac{1}{2} \tilde{s}_2^2 + \frac{1}{2} \tilde{s}_3^2 + \frac{1}{2} - \tilde{f}_1 f_1 - \frac{1}{2} - \tilde{f}_2 f_2 - \frac{1}{2} - \tilde{f}_3 f_3, \]
\[ V = \frac{1}{2} \tilde{s}_1^2 + \frac{1}{2} \tilde{s}_2^2 + \frac{1}{2} \tilde{s}_3^2 + \frac{1}{2} - \tilde{f}_1 f_1 - \frac{1}{2} - \tilde{f}_2 f_2 - \frac{1}{2} - \tilde{f}_3 f_3. \]

Substituting \( \dot{s}_i, i = 1, 2, 3 \), from (35)–(37), we obtain
\[ \dot{V} = \left[ \begin{array}{c} \tilde{f}_1 \tilde{s}_1 - k_1 s_1 - u_{c1} \\ \tilde{f}_2 \tilde{s}_2 - k_2 s_2 - u_{c2} \\ 0 \end{array} \right] \tilde{s}_1 + \left[ \begin{array}{c} (-a + \chi_2)\chi_1 + \chi_3 - \psi_1^* (s(f_1^*)) \\ 0 \\ 0 \end{array} \right] \tilde{s}_2 + \left[ \begin{array}{c} 0 \\ (-a + \chi_2)\chi_1 + \chi_3 - \psi_1^* (s(f_1^*)) \\ 0 \end{array} \right] \tilde{s}_3 \]
\[ + \left[ \begin{array}{c} 0 \\ 0 \\ (-a + \chi_2)\chi_1 + \chi_3 - \psi_1^* (s(f_1^*)) \end{array} \right] \tilde{s}_3 \]
\[ + \left[ \begin{array}{c} -b\chi_2 - \chi_1^2 + 1 - \psi_2^* (s(f_2^*)) \\ 0 \\ 0 \end{array} \right] \tilde{s}_2 + \left[ \begin{array}{c} 0 \\ -b\chi_2 - \chi_1^2 + 1 - \psi_2^* (s(f_2^*)) \\ 0 \end{array} \right] \tilde{s}_3 \]
\[ + \left[ \begin{array}{c} 0 \\ 0 \\ -b\chi_2 - \chi_1^2 + 1 - \psi_2^* (s(f_2^*)) \end{array} \right] \tilde{s}_3. \]

Equation (40) can be rewritten as
\[ \dot{V} = -k_1 s_1^2 + \tilde{f}_1^T \tilde{s}_1 - \frac{1}{2} \tilde{f}_1^T \tilde{f}_1 - u_{c1} s_1 \]
\[ - k_2 s_2^2 + \tilde{f}_2^T \tilde{s}_2 - \frac{1}{2} \tilde{f}_2^T \tilde{f}_2 - u_{c2} s_2 \]
\[ - k_3 s_3^2 + \tilde{f}_3^T \tilde{s}_3 - \frac{1}{2} \tilde{f}_3^T \tilde{f}_3 - u_{c3} s_3 \]
\[ + [(-a + \chi_2)\chi_1 + \chi_3 - \psi_1^* (s(f_1^*))] s_1 \]
\[ + [-b\chi_2 - \chi_1^2 + 1 - \psi_2^* (s(f_2^*))] s_2 \]
\[ + [-\chi_1 - c\chi_3 - \psi_3^* (s(f_3^*))] s_3. \]

Equation (29) is simplified as
\[ \dot{V} = -k_1 s_1^2 + \tilde{f}_1^T \left( \chi_1 \chi_1 - \frac{1}{2} \tilde{f}_1 \right) - u_{c1} s_1 \]
\[ - k_2 s_2^2 + \tilde{f}_2^T \left( \chi_2 \chi_2 - \frac{1}{2} \tilde{f}_2 \right) - u_{c2} s_2 \]
\[ - k_3 s_3^2 + \tilde{f}_3^T \left( \chi_3 \chi_3 - \frac{1}{2} \tilde{f}_3 \right) - u_{c3} s_3 \]
\[ + [(-a + \chi_2)\chi_1 + \chi_3 - \psi_1^* (s(f_1^*))] s_1 \]
\[ + [-b\chi_2 - \chi_1^2 + 1 - \psi_2^* (s(f_2^*))] s_2 \]
\[ + [-\chi_1 - c\chi_3 - \psi_3^* (s(f_3^*))] s_3. \]

Considering the tuning rules (29), we obtain
\[ V = -k_1s_1^2 - k_2s_2^2 - k_3s_3^2 - u_1s_1 - u_2s_2 - u_3s_3 + \left[-c \psi_1 - c \psi_2 - c \psi_3\right] \]

From (43), we have
\[ V \leq -k_1s_1^2 - k_2s_2^2 - k_3s_3^2 - u_1s_1 - u_2s_2 - u_3s_3 + \left[\left(-a + \chi_2\right)\chi_1 + \chi_3 - \psi_1^s\left(s f_1^s\right)\right] |s_1| \]
\[ + \left[-b \chi_2 - \chi_3\left(s f_2^s\right)\right] |s_2| \]
\[ + \left[-c \chi_1 - \psi_1^s\left(s f_3^s\right)\right] |s_3| \]

By applying \( u_i \) from (9), we have
\[ V \leq -k_1s_1^2 - k_2s_2^2 - k_3s_3^2 - u_1s_1 - u_2s_2 - u_3s_3 + \left[\left(-a + \chi_2\right)\chi_1 + \chi_3 - \psi_1^s\left(s f_1^s\right)\right] |s_1| \]
\[ + \left[-b \chi_2 - \chi_3\left(s f_2^s\right)\right] |s_2| \]
\[ + \left[-c \chi_1 - \psi_1^s\left(s f_3^s\right)\right] |s_3| \]

Considering the fact that
\[ |\left(-a + s_2\right)s_1 + s_3 - \psi_1^s\left(s f_1^s\right)| < E_1, \]
\[ |\left(-b s_2 - s_1^2 + 1 - \psi_2^s\left(s f_2^s\right)| < E_2, \]
\[ |s_1 - c s_3 - \psi_3^s\left(s f_3^s\right)| < E_3, \]
we can write, \( V < 0 \), and then, the proof is completed. \( \square \)

6. Simulation

By various simulations, the feasibility and accuracy of the stabilizing and synchronizing scheme are verified.

6.1. Stabilization. In this section, the dynamics of (1) is stabilized by controller (17), tuning rules (18), and compensator (19). The simulation conditions are described in Table 1. The initial conditions are as follows: \( \chi_1 \left(0\right) = -0.20 \), \( \chi_2 \left(0\right) = 1.50 \), and \( \chi_3 \left(0\right) = 0.30 \). The dynamics is unknown. A general view on the simulation scheme is depicted in Figure 5. The trajectories of outputs \( \chi_1, \chi_2, \) and \( \chi_3 \) are depicted in Figure 6. We see that signals \( \chi_1, \chi_2, \) and \( \chi_3 \) are well converged to zero. The corresponding control signals \( u_1, u_2, \) and \( u_3 \) are given in Figure 7, and the outputs of T3-FLSs (\( \psi_1, \psi_2, \) and \( \psi_3 \)) are depicted in Figure 8. To better see the convergence to the zero point, the phase portrait is shown in Figure 9. We see that the phase trajectories have well reached the origin at a finite time.

6.2. Synchronization. In this section, the synchronization accuracy is examined. The chaotic system (1) is considered as a leader system, and the slave financial chaotic system is assumed as follows:
\[ \dot{r}_1 = 0.8\left(r_2(t) - r_1(t)\right) + u_1(t), \]
\[ \dot{r}_2 = 0.2r_1(t) + u_2(t), \]
\[ \dot{r}_3 = r_1(t) - 1.9r_3(t) + u_3(t). \]

System (47) is synchronized with (1) by using controller (40), tuning rules (41), and compensator (42). The simulation conditions are the same as those of the previous section. The initial conditions are as follows: \( r_1 \left(0\right) = -0.20, \) \( r_2 \left(0\right) = 1.50, \) \( r_3 \left(0\right) = 0.30, \) \( \chi_1 \left(0\right) = 1.0, \) \( \chi_2 \left(0\right) = -2.0, \) and \( \chi_3 \left(0\right) = 5.0. \) The trajectories of outputs \( \chi_1, r_1, \chi_2, r_2, \chi_3, r_3 \) are depicted in Figure 10. We see that signals \( \chi_1, \chi_2, \) and \( \chi_3 \) are well converged to \( r_1, r_2, \) and \( r_3. \) The synchronization errors in Figure 11 show a good synchronization performance. The corresponding control signals \( u_1, u_2, \) and \( u_3 \) are given in Figure 12, and the outputs of T3-FLSs (\( \psi_1, \psi_2, \) and \( \psi_3 \)) are depicted in Figure 13. To better see the synchronization, the phase portrait is shown in Figure 14. We see that the phase trajectories of the slave system have well reached the leader system at a finite time.

6.3. High Noisy Condition and Comparison. In this section, a high perturbation is applied to the system as shown in Figure 15. The simulation conditions are the same as those of the previous examination. The trajectories of outputs \( \chi_1, r_1, \chi_2, r_2, \) and \( \chi_3, r_3 \) are depicted in Figure 16. We see that the perturbations are well tackled, and the signals \( \chi_1, \chi_2, \) and \( \chi_3 \) are well converged to \( r_1, r_2, \) and \( r_3. \) The synchronization errors are shown in Figure 17 that shows a desired synchronization under high-level disturbances. The corresponding control signals \( u_1, u_2, \) and \( u_3 \) are given in Figure 18, and the phase portrait is shown in Figure 19. We see that similar to the previous examination, the phase trajectories of the slave system have well reached the leader system at a finite time in the presence of noisy conditions.
Figure 5: A general view of the simulation scheme.

Figure 6: Scenario 6.1: output signals.

Figure 7: Scenario 6.1: control signals.
Figure 8: Scenario 6.1: T3-FLS signals.

Figure 9: Scenario 6.1: a phase portrait.

Figure 10: Scenario 6.2: output signals.
To examine the superiority of the suggested control scenario versus other similar approaches, a comparison is presented. The root-mean-square of synchronization errors (RMSEs) for the designed controller are compared with those the type-1 FLS-based controller (T1-FLC) [46], type-2 FLS-based controller (T2-FLC) [47], and generalized FLS-based controller (GT2-FLC) [48]. The comparison results are shown in Table 2. We see that the designed controller gives better synchronization accuracy in high noisy conditions.
Figure 13: Scenario 6.2: T3-FLS signals.

Figure 14: Scenario 6.2: a phase portrait.

Figure 15: Scenario 6.3: random dynamic perturbation.
Remark 2. In this paper, a new T3-FLS-based controller is developed for both stabilization and synchronization of financial CSs. The simulations under various conditions show that the designed controller gives the desired efficiency. Even under the high noisy conditions and completely unknown dynamics, it is seen that the presented approach well regulates the target trajectories into reference. Also, by using the designed compensator, the robustness is well preserved under high-level perturbations. The designed controller does not depend on the model of CSs, and then, it can easily be applied to other cases of CSs.
7. Conclusion

In this paper, the synchronization and stabilization of financial CSs are studied and a new control system is presented. The dynamics is known and is approximated by using the designed T3-FLSs. The robustness is proved through the Lyapunov method. The accuracy of the designed controller is examined in three cases. In the first examination, the stabilization performance is investigated and it shows that all states are perfectly converged to zero at a finite time. For the second examination, the suggested controller is applied for a synchronization problem and it is shown that the case study financial chaotic system well tracks a non-identical chaotic system; the synchronization errors have

![Figure 18: Scenario 6.3: control signals.](image1)

![Figure 19: Scenario 6.3: a phase portrait.](image2)

| Method         | $s_1$   | $s_2$   | $s_3$   |
|---------------|---------|---------|---------|
| Proposed      | 0.0217  | 0.0266  | 0.0530  |
| T1-FLC [46]   | 0.8421  | 1.0871  | 1.2820  |
| T2-FLC [47]   | 0.4023  | 0.9142  | 1.0621  |
| GT2-FLC [48]  | 0.4128  | 0.8071  | 1.0278  |
reached zero. For the last examination, the accuracy is evaluated under a high-level noisy condition and it is shown that the suggested control scenario gives perfect results in the presence of high dynamic perturbations and unknown dynamics.

Data Availability
The data that support the findings of this study are available within the article.

Conflicts of Interest
The authors declare that they have no conflicts of interest.

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