Confinement by Monopoles in the Positive Plaquette Model of SU(2) Lattice
Gauge Theory

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Abstract

Confinement via ’t Hooft-Mandelstam monopoles is studied for the positive plaquette model in $SU(2)$ lattice gauge theory. Positive plaquette model configurations are projected into the maximum abelian gauge and the magnetic current extracted. The resulting magnetic current is used to compute monopole contributions to Wilson loops and extract a monopole contribution to the string tension. As was previously found for the Wilson action, the monopole contribution to the string tension agrees with the string tension calculated directly from the $SU(2)$ links. The fact that the positive plaquette model suppresses $Z_2$ monopoles and vortices is discussed.
This paper contains our results on confinement by monopoles in pure SU(2) lattice

gauge theory, for an action known as the positive plaquette model (PPM). This work

is part of our ongoing lattice gauge theory investigation of confinement by monopoles.

In our previous work we have obtained quantitative results for the string tension using

monopoles in $U(1)$ lattice gauge theory for both $d = 3$ and $d = 4$, [1, 2, 3] and for $SU(2)$

using the standard Wilson action (WA), in $d = 4$ [4].

The action for the PPM agrees with that of Wilson except that the PPM completely

suppresses plaquettes with negative trace, i.e. negative plaquettes are regarded as having

infinite action. In the weak coupling or small lattice spacing limit, the two actions are

equivalent. For calculations at finite lattice spacing $a$, the PPM represents an “improved”

action. A plaquette with negative trace is a clear lattice artifact. Writing the usual prod-

uct of links around a plaquette as $U_P = \exp(ia^2 F_{\mu\nu})$, a negative plaquette has $tr(U_P) < 0$,

which corresponds to a field-strength $F_{\mu\nu} \sim O(\pi/a^2)$. Such large field strengths play no

role in the continuum limit, and their suppression in the PPM should allow a clearer

view of the continuum limit to be obtained from calculations performed at finite lattice

spacing $a$. Recently the PPM has been subjected to a thorough study [5], which shows

it to be in the same universality class as the Wilson action. However, while the Wilson

action possesses a well-known dip in the step $\beta$-function, $\Delta \beta(\beta)$, no such dip occurs for

the PPM.

There is a more specific reason for exploring monopole confinement in the PPM. The

Wilson action, since it permits negative plaquettes, contains $Z_2$ monopoles and vortices.

($Z_2$ monopoles are associated with cubes whose faces contain an odd number of negative

plaquettes, and $Z_2$ vortices similarly require the presence of negative plaquettes.) These

$Z_2$ objects are associated with the center of the $SU(2)$ group, and there is a long history

of attempts to understand confinement in $SU(2)$ using them [7, 8, 9, 10, 11]. However, for

the PPM, there are no negative plaquettes, and therefore no lower bound on the $SU(2)$

string tension for the PPM can be obtained by considering $Z_2$ objects. (We assume the
latter are defined using single plaquettes.) In contrast, the ’t Hooft-Mandelstam or dual superconductor monopoles\cite{12, 13} will be shown below to give a quantitative explanation of the $SU(2)$ string tension for the PPM.

A correlation length large compared to the lattice spacing is desired for the continuum limit, but finite lattice size forces a compromise. In our previous work with the Wilson action on $16^4$ lattices, this compromise was struck with correlation lengths $\xi = 1/\sqrt{\sigma} \sim 5a$, corresponding to couplings $\beta_{WA} \sim 2.5$. In the PPM, a $16^4$ lattice was also used. To determine the values of $\beta_{PPM}$ which correspond to $\xi \sim 5a$, we assumed universality of the ratio $T_c/\sqrt{\sigma}$, which is known to be $0.69(2)$ for the Wilson action \cite{6}. The couplings for various deconfining temperatures have recently been determined very accurately for the PPM \cite{5}; in particular, for $T_c = 1/8a$, $\beta_{PPM}$=1.886(6). Assuming the PPM has the same value of $T_c/\sqrt{\sigma}$ as the Wilson action, we then obtain $\xi \sim 5.5a$ for $\beta_{PPM} = 1.886$.

Our runs in the PPM were carried out at $\beta_{PPM}$=1.886, and the smaller values 1.840 and 1.790.

The calculation proceeded in a similar manner to our previous work with the Wilson action. For each coupling mentioned above, 500 configurations were gathered, where every 20th configuration was saved after equilibrating for 1000 sweeps. An update of the lattice consisted of 1 Kennedy-Pendleton sweep \cite{15} and two overrelaxation sweeps \cite{16}. Any new links that resulted in a negative plaquette were rejected.

To locate monopoles, configurations are projected with high accuracy into the maximum abelian gauge. The gauge-fixed links are factored into a “charged” part, times a $U(1)$ link. The monopole location procedure of Toussaint and DeGrand is applied to the $U(1)$ links, and results in an integer-valued magnetic current $m_\mu(x)$\cite{14}. The procedure in effect locates a monopole by finding the end of its Dirac string.
The maximum abelian gauge is attained when

\[ X(y) \equiv \sum_\mu \left[ U_\mu(y)\sigma_3 U_\mu^\dagger(y) + U_\mu^\dagger(y - \hat{\mu})\sigma_3 U(y - \hat{\mu}) \right] \]

is diagonal \[\text{[4]}.\] Perfect diagonalization of \(X(y)\) is never achieved. An overrelaxation process is used to repeatedly sweep the lattice, stopping when the off-diagonal elements of \(X\) are sufficiently small. We used

\[ \langle |X^{ch}|^2 \rangle \equiv \frac{1}{L^4} \sum_x \left( |X^1(x)|^2 + |X^2(x)|^2 \right) \]

as a measure of the off-diagonal elements of \(X\), where \(X_1, X_2\) are the coefficients of \(\sigma_1, \sigma_2\) in a Pauli matrix expansion of \(X\). The overrelaxation process was stopped when \(\langle |X^{ch}|^2 \rangle \leq 10^{-10}\). This required approximately 1000 overrelaxation sweeps.

We now have two methods of extracting a potential; one directly from the \(SU(2)\) links, the other using the magnetic current to calculate monopole Wilson loops and a monopole potential \[\text{[4]}.\] In both cases, all Wilson loops \(W(R, T)\) were measured up to a maximum size of \(8 \times 12\). The full \(SU(2)\) and monopole potentials were determined by linear fits of \(\ln(W(R, T))\) vs \(T\). These fits were done for \(R \geq 2\) over the interval \(T = R + 1\) to 12. Having determined potentials \(V(R)\) for each \(R\), the string tensions \(\sigma\) were found by fitting \(V(R)\) to the form \(\alpha/R + \sigma \cdot R + V_0\), over the interval \(R = 2\) to \(R = 8\). The full \(SU(2)\) and monopole determinations of the string tension are shown in Table I, where it can be seen that the two are in excellent agreement. The agreement is of the same quality as in our previous work with the Wilson action, and is another piece of evidence in favor of confinement via \'t Hooft-Mandelstam monopoles.

The PPM monopole potentials are shown in Figure 1, where the solid curves are the results of the linear-plus-Coulomb fits. The Coulomb coefficients \(\alpha\) naturally differ for the monopole and full \(SU(2)\) potentials. The monopole contribution is purely non-perturbative and is supposed to be correct only in the large distance region. In particular the monopole potentials do not contain the Coulombic term coming from one
Table I

Table 1: The string tensions from monopoles and full $SU(2)$ for the PPM

gluon exchange present in the full $SU(2)$ potential. For the three couplings, $\beta_{PPM} = 1.886, 1.840, \text{and} 1.790$, the values of $\alpha$ from the monopole potentials are very small; $0.01(1), 0.02(1), 0.02(1)$ respectively. For the full $SU(2)$ PPM potentials the corresponding results for $\alpha$ are $-0.29(1), -0.30(1), -0.31(1)$. Both of these sets of results for $\alpha$ in the PPM are very similar to those obtained previously for the Wilson action.

The distributions of magnetic current for the Wilson action and positive plaquette models are qualitatively similar, but there are interesting quantitative differences. For the PPM coupling $\beta_{PPM} = 1.840$, the string tension is $0.036(1)$; within error bars of the string tension for the Wilson action at $\beta_{WA} = 2.50$, where our result was $0.034(1)$. These two couplings are thus approximately equivalent in terms of the physical string tension they produce. For the Wilson action at $\beta_{WA} = 2.50$, the fraction of links carrying magnetic current is $1.36(1) \times 10^{-2}$, whereas for the PPM coupling $\beta_{PPM} = 1.840$, the corresponding number is significantly smaller, $1.12(1) \times 10^{-2}$. In terms of the number of links with current, there are approximately 600 more links of magnetic current for the case of the $\beta_{WA} = 2.50$ Wilson action on a $16^4$ lattice. The nature of the difference becomes clear when the magnetic current is resolved into individual loops, each satisfying current conservation. For either action, a substantial fraction of the current resides in small loops of 4, 6, etc. links. These small loops of current have nothing to do with confinement.
Rather it is the large loops with numbers of links ranging from 50 up to several hundred which are responsible for the string tension. Specifically, if loops of size less than 50 links are eliminated from the magnetic current, the physical string tension is reproduced for both the $\beta_{WA} = 2.50$ Wilson action and the PPM at $\beta_{PPM} = 1.840$. Eliminating the contribution to the current coming from loops of less than 50 links, the fraction of lattice links occupied is $0.60(1) \times 10^{-2}$ for the Wilson action, very close to $0.58(1) \times 10^{-2}$, which is the result for the PPM action. So for that part of the magnetic current which is effective in producing the string tension, namely loops of current of 50 links and larger, the two actions have essentially the same fraction of links with current. The excess found for the Wilson action involves small loops of magnetic current, which play no role in confinement. This suggests that for successively improved actions, the fraction of magnetic current in small loops will steadily fall.

To summarize, we have demonstrated for SU(2) lattice gauge theory, that the quantitative explanation of confinement by 't Hooft-Mandelstam monopoles is the same for the PPM as it was for the standard Wilson action. This shows that the picture is robust. Further, the results for the PPM make clear that $Z_2$ monopoles and vortices are not responsible for the string tension.

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Figure 1: The potentials extracted from monopole Wilson loops for the PPM at $\beta_{PPM} = 1.790$ (triangles), 1.840 (squares), and 1.886 (diamonds). The solid lines are the linear-plus-Coulomb fits to each potential.