STRUCTURE OF DARK MATTER HALOS FROM HIERARCHICAL CLUSTERING

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Received 2000 August 5; accepted 2001 April 18

ABSTRACT

We investigate the structure of the dark matter halo formed in the cold dark matter scenario using N-body simulations. We simulated 12 halos with masses of \(6.6 \times 10^{11} - 8.0 \times 10^{14} \, M_{\odot}\). In all runs, the halos have density cusps proportional to \(r^{-1.5}\) developed at their centers, which is consistent with the results of recent high-resolution calculations. The density structure evolves in a self-similar way and is universal in the sense that it is independent of the halo mass and initial random realization of density fluctuation. The density profile is in good agreement with the profile proposed by Moore et al., which has a central slope proportional to \(r^{-1.5}\) and an outer slope proportional to \(r^{-3}\). The halo grows through repeated accretion of diffuse smaller halos. We argue that the cusp is understood as a convergence slope for the accretion of tidally disrupted matter.

Subject headings: cosmology: theory — dark matter — galaxies: formation — galaxies: kinematics and dynamics — methods: numerical

1. INTRODUCTION

In standard cosmological pictures such as cold dark matter (CDM) cosmology, dark matter halos are considered to be formed hierarchically; smaller halos formed first from initial density fluctuations and then merged with each other to become larger halos. In reality, the formation process of dark matter halos is rather complicated since a variety of processes, such as mergers between halos of various sizes and tidal disruption of small halos (satellites), proceed simultaneously.

One of the most influential works on dark matter halos is the "finding" of the universal profile by Navarro, Frenk, & White (NFW; 1996, 1997) although there were many analytical and numerical studies before NFW (see NFW 1996 or Bertschinger 1998 for reviews). NFW performed N-body simulations of the halo formation and found that the profile of the dark matter halo can be fitted by a simple formula,

\[
\rho(r) = \frac{\rho_0}{(r/r_s)(1 + r/r_s)^2},
\]

where \(\rho_0\) is a characteristic density and \(r_s\) is a scale radius. They also argued that the profile has the same shape, independent of halo mass, initial density fluctuation spectrum, or value of the cosmological parameters. It should be noted that before NFW, Dubinski & Carlberg (1991) also found in their high-resolution simulation that the halo can be well fitted by the Hernquist (1990) profile.

Many studies on the NFW "universal profile," both numerical and analytical, were done after their proposal. Many N-body simulations with similar resolution to those of NFW were performed, and results similar to those of NFW were obtained (Cole & Lacey 1996; Tormen, Bouchet, & White 1996; Brainerd, Goldberg, & Villumsen 1998; Thomas et al. 1998; Okamoto & Habe 1999; Huss, Jain, & Steinmetz 1999; Kravtov et al. 1998; Jing 2000). Analytical and semianalytical studies to explain the NFW universal profile were also done (Evans & Collet 1997; Syer & White 1998; Avila-Reese, Firmani, & Hernandez 1998; Nusser & Sheth 1999; Kulic 1999; Heriksen & Widrow 1999; Yano & Gouda 1999; Bullock et al. 2001; Subramanian, Cen, & Ostriker 2000; Lokas 2000). However, a clear understanding of the NFW profile has not yet been established. One of the reasons for this might be that all of these studies were trying to answer the wrong question.

Our previous study (Fukushige & Makino 1997, hereafter FM97) showed that the density profile obtained by high-resolution N-body simulation is different from the NFW universal profile. We performed simulations with 768,000 particles, while previous studies employed only \(\sim 20,000\). We found that the galaxy-sized halo has a cusp steeper than \(\rho \propto r^{-4}\).

This disagreement with the NFW universal profile was confirmed by other high-resolution simulations. Moore et al. (1998, 1999) and Ghiogna et al. (2000) performed simulations with up to \(4 \times 10^6\) particles and obtained results similar to ours. They found that cluster-sized halos also have steeper cusps than those in the NFW profile, and they proposed a modified universal profile: \(\rho = \rho_0/[r/r_s]^{1.5} \times [1 + (r/r_s)^{1.5}]\). On the other hand, Jing & Suto (2000) found that the density profile of dark matter is not universal. They performed a series of N-body simulations and concluded that the power of the cusp depends on mass. It varies from \(-1.5\) for galaxy-mass halos to \(-1.1\) for cluster-mass halos.

In this paper we again investigate the structure of dark matter halos using an N-body simulation. We performed N-body simulations of formation of 12 dark matter halos with masses of \(6.6 \times 10^{11} - 8.0 \times 10^{14} \, M_{\odot}\) using a special purpose computer GRAPE-5 (Kawai et al. 2000) and the Barnes-Hut (BH) tree code. In § 2 we describe the model of our N-body simulation; in § 3 we present the results of simulation; § 4 provides a summary and § 5 is a discussion of our results.

2. SIMULATION MODELS

We performed a total of 12 runs on four different mass scales that are summarized in Table 1. Initial conditions...
were constructed in a way similar to that in FM97. We assigned initial positions and velocities to particles in a spherical region with a radius of $R_{\text{Mpc}}$ surrounding a density peak selected from an unconstrained discrete realization of the standard CDM model $H_0 = 50$ km s$^{-1}$ Mpc$^{-1}$, $\Omega = 1$, and $\sigma_8 = 0.7$). The peak was chosen from an $R_{\text{box}}$ Mpc cube using a density field smoothed by a Gaussian filter of radius $(R_{\text{box}}/2)$ Mpc. The values of $R$ and $R_{\text{box}}$ in the comoving flame are summarized in Table 1. In order to generate the discrete realization of the CDM model, we used the COSMICS package.

We followed the evolution of the density peak by $N$-body simulation. We added the local Hubble flow and integrated the orbits directly in the physical space. We used the Plummer-softened potential with the softening length constant in physical space and a leapfrog integrator with

| Run   | $R$ (Mpc) | $R_{\text{box}}$ (Mpc) | $m$ ($M_\odot$) | $\epsilon$ (kpc) | $\Delta t$ (yr) | $z_{\text{start}}$ | $z_{\text{end}}$ |
|-------|-----------|------------------------|----------------|------------------|----------------|-------------------|----------------|
| 16M0  | 12.8      | 32                     | $3.0 \times 10^8$ | 0.56             | $1.6 \times 10^6$ | 18.8             | 0.0             |
| 16M1  | 16        | 32                     | $6.0 \times 10^8$ | 0.56             | $1.5 \times 10^6$ | 18.5             | 0.0             |
| 16M2  | 16        | 32                     | $6.1 \times 10^8$ | 0.56             | $1.6 \times 10^6$ | 20.4             | 0.0             |
| 8M0   | 6.4       | 16                     | $3.7 \times 10^7$ | 0.28             | $7.9 \times 10^3$ | 22.3             | 0.58            |
| 8M1   | 8         | 16                     | $7.6 \times 10^7$ | 0.28             | $7.5 \times 10^3$ | 22.2             | 0.63            |
| 8M2   | 8         | 16                     | $7.6 \times 10^7$ | 0.28             | $7.8 \times 10^3$ | 23.9             | 0.59            |
| 4M0   | 3.2       | 8                      | $4.7 \times 10^6$ | 0.14             | $7.8 \times 10^3$ | 25.9             | 1.6             |
| 4M1   | 4         | 8                      | $9.5 \times 10^6$ | 0.14             | $7.4 \times 10^3$ | 25.9             | 1.6             |
| 4M2   | 4         | 8                      | $9.5 \times 10^6$ | 0.14             | $7.6 \times 10^3$ | 27.4             | 1.2             |
| 2M0   | 1.6       | 4                      | $5.9 \times 10^5$ | 0.07             | $3.8 \times 10^4$ | 29.7             | 2.1             |
| 2M1   | 2         | 4                      | $1.2 \times 10^6$ | 0.07             | $3.6 \times 10^4$ | 29.7             | 2.2             |
| 2M2   | 2         | 4                      | $1.2 \times 10^6$ | 0.07             | $3.8 \times 10^4$ | 30.9             | 1.8             |

Fig. 1.—Snapshots from run 16M0 at 16 different redshifts, indicated by the number at the lower-left of each panel. The length of the side for each panel is equal to $4r_{200}$ at $z_{\text{end}}$. 
Fig. 2.—Snapshots from runs \{16, 8, 4, 2\}M1 at the final redshifts. The length of the side for each panel is equal to 4$r_{200}$ at $z_{\text{end}}$.

| Run    | $M_{200}$ ($M_\odot$) | $r_{200}$ (Mpc) | $N_{200}$ |
|--------|------------------------|-----------------|----------|
| 16M0   | $2.6 \times 10^{14}$   | 1.7             | 873170   |
| 16M1   | $7.8 \times 10^{14}$   | 2.4             | 1279383  |
| 16M2   | $8.0 \times 10^{14}$   | 2.4             | 1322251  |
| 8M0    | $2.8 \times 10^{13}$   | 0.48            | 745735   |
| 8M1    | $9.0 \times 10^{13}$   | 0.72            | 1186162  |
| 8M2    | $7.7 \times 10^{13}$   | 0.70            | 1015454  |
| 4M0    | $2.7 \times 10^{12}$   | 0.13            | 559563   |
| 4M1    | $8.0 \times 10^{12}$   | 0.20            | 846301   |
| 4M2    | $6.6 \times 10^{12}$   | 0.22            | 697504   |
| 2M0    | $6.6 \times 10^{11}$   | 0.062           | 643151   |
| 2M1    | $1.1 \times 10^{12}$   | 0.085           | 957365   |
| 2M2    | $1.0 \times 10^{12}$   | 0.096           | 923545   |

TABLE 2

HALO PROPERTIES AT $z = z_{\text{end}}$

shared and constant time steps. In Table 1 we summarize the individual particle mass $m$, softening length $\epsilon$, time step size $\Delta t$, and starting and ending redshifts $z_{\text{start}}$ and $z_{\text{end}}$. The particle masses are equal, and the total number of particles for each simulation is $(2.0\text{--}2.1) \times 10^6$.

We determined the radius $R$ Mpc for run 16M{0, 1, 2} using trial runs with smaller numbers of particles so that all particles lying inside of $r_{200}$ at $z_{\text{end}}$ are included. Here the radius $r_{200}$ is defined as the radius of the sphere at which the mean density $\rho$ is equal to 200$\rho_{\text{crit}}$, where $\rho_{\text{crit}}$ is the critical density. We did not include tidal effects from outside the R Mpc sphere. The regions of runs 8M{0, 1, 2}, 4M{0, 1, 2}, and 2M{0, 1, 2} are $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ times the size of run 16M{0, 1, 2}; the mass resolutions are times 2, 4, and 8, respectively. The ending redshifts $z_{\text{end}}$ for runs 8M{0, 1, 2}, 4M{0, 1, 2}, and 2M{0, 1, 2} are determined so that the
truncation outside the sphere did not influence the profile around \( r_{200} \).

The number preceding "M" in the run names (e.g., the "8" in run 8M1) indicates the length of the simulation box \( R_{\text{box}}/2 \) in Mpc. The number following "M" in the run names (e.g., the "1" in run 8M1) identifies the index for the random number seed used to generate initial density field. For example, runs \( \{16, 8, 4, 2\}M0 \) are a series of runs in which the amplitudes of the waves are different and the phases of initial density waves are the same.

For the force calculation, we used the BH tree code \((\theta = 0.75; \text{Barnes & Hut 1986; Barnes 1990; Makino 1991)}\), implemented on GRAPE-5 (Kawai et al. 2000), a special-purpose computer designed to accelerate \( N \)-body simulations. Using the tree code on two GRAPE-5 boards and a workstation whose CPU is 21264/677 MHz Alpha chip, one time step took 21 s. The total number of timesteps is about 6000–8000. Therefore we can complete one run in 35–50 CPU hr.

3. RESULT

3.1. Snapshot

Figure 1 shows the particle distributions for run 16M0 at 16 different redshifts. For these plots, we shifted the origin of coordinates to the position of the potential minimum so that the largest halo is at the center of the panel. Figure 2 shows the particle distribution for runs \( \{16, 8, 4, 2\}M1 \) together. The phases of waves for initial density field are the same for all these runs; only the amplitudes are different. In Table 2, we summarize the radius \( r_{200} \), mass \( M_{200} \), and number of particles \( N_{200} \) within \( r_{200} \) at \( z_{\text{end}} \).

3.2. Accuracy Criteria

In this study we plot the density only for the radii unaffected by numerical artifacts. We used the following two criteria: (1) \( t_{\text{rel}}(r)/t > 3 \) and (2) \( t_{\text{dy}}(r)/\Delta t > 40 \). We obtained criteria 1 and 2 experimentally, and the details of the experiments are discussed in §§ 3.2.1 and 3.2.2. We plot the densities only if both criteria are satisfied. Here \( t_{\text{rel}}(r) \) is the local two-body relaxation time, defined by

\[
t_{\text{rel}} = \frac{0.065v^3}{G^2\rho \ln (1/\epsilon)},
\]

(cf. Spitzer 1987) and \( t_{\text{dy}}(r) \) is the local dynamical time, defined by

\[
t_{\text{dy}} = (G\bar{\rho})^{-1/2},
\]

where \( \bar{\rho} \) is the average density within radius \( r \). Using these criteria, we judge whether the density profile is unaffected by numerical artifacts due to two-body relaxation (1) and step size for the time integration (2).

The inner limit of radii where the density is correctly calculated in our simulations is (0.01–0.02)\( r_{200} \) at the final redshifts. In most cases, criterion 1 for the two-body relaxation determines the limit radius for reliability.

3.2.1. Criterion for Two-Body Relaxation

In this subsection we evaluate the criterion \( t_{\text{rel}}(r)/t > 3 \) to distinguish the numerical artifact due to two-body relax-
In order to see the effect of two-body relaxation, we calculated the same model as run 16M0 but with several different values for total number of particles \( N \) and softening size \( \epsilon \). In Figure 3, we plot the final average density profiles \( \bar{\rho} \) for three simulations, with \( N/4, N/16, \) and \( N/16 \) and \( 4\epsilon \), where \( N \) and \( \epsilon \) indicate values used in run 16M0. Otherwise stated, we used the same simulation parameter as in run 16M0. We can see that the central density depends on \( N \) rather strongly and is lower for lower numbers of particles. For the same value of \( N \), larger softening has the small but clear effect of increasing the central density.

Figure 4 shows the ratio \( \bar{\rho}/\bar{\rho}_{\text{ref}} \), where \( \bar{\rho}_{\text{ref}} \) is the averaged density of the reference run in which the effect of the two-body relaxation is smallest. Here we used run 16M0 as the reference run. In Figure 5, we show the ratio \( \bar{\rho}/\bar{\rho}_{\text{ref}} \) plotted as a function of the ratio of the local two-body relaxation time \( t_{\text{rel}}(r) \) defined by equation (2) to simulation period \( t \). Note that \( t_{\text{rel}}(r) \) is monotonous and increasing function of \( r \). Therefore smaller \( t_{\text{rel}}(r) \) means smaller \( r \). We can see that the density tends to go below the reference value if \( t < 3t_{\text{rel}}(r) \). The difference between the reference run and runs with a smaller number of particles is insignificant if \( t < 3t_{\text{rel}}(r) \). From this result, we adopt \( t_{\text{rel}}(r)/t > 3 \) as the criterion for the two-body relaxation.

3.2.2. Criterion for Time Integration

In this subsection we evaluate the criterion \( t_{\text{dy}}(r)/\Delta t > 40 \) to distinguish the numerical artifact due to large step size of time integration. In order to see the effect of large step size, we calculated the same model as run 16M0 but with larger time step size \( \Delta t \). In Figure 6 we plot the final average density profiles \( \bar{\rho} \) for three simulations with \( 4\Delta t, 8\Delta t, \) and \( 16\Delta t \), where \( \Delta t \) indicates values used in run 16M0. Otherwise stated, we used the same simulation parameter as in run 16M0. We can see that the central density depends on \( \Delta t \) rather strongly and is lower for larger \( \Delta t \). Figure 7 shows

**Fig. 6.** Averaged density profile for run 16M0 (thick line) and for the same model as run 16M0 but with 4, 8, and 16 times larger and 4 times smaller time step size (thin lines). Unit of density is \( M_\odot \; \text{pc}^{-3} \).

**Fig. 7.** Ratio of the averaged density of the models in Fig. 6 to the reference run (run 16M0).

**Fig. 8.** Same as Fig. 7, but for a function of the ratio of the local dynamical time to time step size, \( t_{\text{dy}}(r)/\Delta t \). Arrow indicates the accuracy criterion 2.

**Fig. 9.** Averaged density profiles for run 16M0 (\( \epsilon = 0.56 \) kpc) and the same model as run 16M0 with \( \epsilon = 0.18, 1.7, 5, 15, \) and 30 kpc, where \( \epsilon \) is the softening length. Unit of density is \( M_\odot \; \text{pc}^{-3} \). The numbers beside the lines indicate softening length. Thick lines: Profiles for run 16M0 and the models with \( \epsilon = 1.7 \) kpc. Dashed lines: Density profile proportional to \( r^{-1.5} \). Arrow: Critical radius, defined by the accuracy criteria 1 and 2.
the ratio $\tilde{\rho}/\rho_{\text{ref}}$. Here we used run 16M0 as the reference run.

In Figure 8 we show the ratio $\tilde{\rho}/\rho_{\text{ref}}$ plotted as a function of the ratio of the local dynamical time $t_{\text{dy}}$, defined by equation (3), to the time step size $\Delta t$. Note that $t_{\text{dy}}(r)$ is monotonous and increasing function of $r$. Therefore smaller $t_{\text{dy}}(r)$ means smaller $r$. We can see that the density tends to go below the reference value if $t_{\text{dy}}(r)/\Delta t < 40$. From this result, we adapt $t_{\text{dy}}(r)/\Delta t > 40$ as the criterion for time integration.

Note that the number 40 is applicable only to the integration scheme we used: the leapfrog scheme integrated in physical space with constant time step size. This scheme has good characteristics such as time reversibility and symplecticity. The number 40 should increase when variable step size is used or the system is integrated in comoving space (where acceleration depends on velocity).

As a result of adapting this criterion, the total number of time steps to integrate in Hubble time becomes about 8000 in our simulation. The number is a little smaller than that reported in previous simulations (e.g., $\sim 50,000$ from Moore et al. 1999). We also calculated the same model as run 16M0, but with 4 times as many time steps. We confirmed that the density profile, shape, and anisotropy parameter do not change and that 8000 time steps is sufficient. In Figure 6 we show the density profile.

3.2.3. Other Numerical Effects

The potential softening also affects the density profile. In order to see the effect of potential softening, we calculated the same model as run 16M0 but with different softening lengths ($\epsilon$). In Figure 9, we plot the final average density profiles $\tilde{\rho}$ for six models, with $\epsilon = 0.18, 0.56, 1.7, 5, 15,$ and 30 kpc. Except for the softening length, we used the same simulation parameters as in run 16M0. In Figure 9, we can see that the central density is lower for both smaller and larger $\epsilon$. The former is because the two-body relaxation effect is stronger and the time integration is less accurate for smaller $\epsilon$. The latter is because the potential softening itself affects the density structure for larger $\epsilon$. The potential softening, therefore, should be set in the intermediate range.

The potential softening for run 16M0 ($\epsilon = 0.56$ kpc) is in the intermediate range, although it is not optimal. It does not affect the density profile outside of 20 kpc, which is the critical radius defined by the accuracy criteria 1 and 2. The ratios of the softening length to the critical radius in other runs are similar to that in run 16M0.

We made sure that the accuracy of the BH tree code did not influence the structure in the range where the above criteria are satisfied by resimulating the same initial model used in FM97, in which the direct summation is used. We found no systematic difference in the results. Therefore the accuracy of the BH tree code is satisfactory.

3.3. Density Profiles

Figure 10 shows the evolution of density profiles for run 16M0. The position of the center of the halo was determined using the potential minimum, and the density is averaged over each spherical shell whose width is $\log_{10} (\Delta r) = 0.0125$. Figure 11 shows run 2M0. For illustrative purposes, the densities are shifted vertically. Figure 12 show the density profiles at $z_{\text{end}}$ for all runs.

In all runs, we can see the central density cusps approximately proportional to $r^{-1.5}$. In other words, the power of
the cusp is $-1.5$ and is independent of halo mass, which is consistent with the result of Moore et al. (1999). In the outer region, the density profiles are very similar for all runs. The dependence of the power-law index of the inner cusp on halo mass observed by Jing & Suto (2000) was not reproduced in our simulations. In the following subsections we discuss the self-similar growth of the halo (§ 3.4), the universality of the profile (§ 3.5), and the mechanism for self-similar growth (§ 3.6).

3.4. Self-Similar Evolution

Figure 13 shows the growth of the halo without the vertical shift. In this figure it is clear that the halo grows in a self-similar way, keeping the density of the central cusp region constant.

If the evolution is self-similar, we can write the density as

$$\rho(r, t) = \rho_s(M) \rho_s(r_s),$$

$$r_s = r/r_s(M).$$

Here we write $\rho_s(M)$ and $r_s(M)$ as functions of the mass of the halo $M$ instead of as functions of time. The self-similar profile itself should have a central cusp of $\rho_s(r_s) \propto r_s^2$. The actual profile at the cusp region satisfies $\rho(r) = Cr^2$, with $C$ constant in time. Therefore $\rho_s$ and $r_s$ should satisfy $\rho_s \propto r_s^2$.

If we write $\rho_s$ and $r_s$ as functions of $M$, we have

$$\rho_s(M) = \rho_{00} \left( \frac{M}{M_{00}} \right)^{n/(3+n)},$$

$$r_s(M) = r_{00} \left( \frac{M}{M_{00}} \right)^{1/(3+n)},$$

where $\rho_{00}, r_{00},$ and $M_{00}$ are constants and $n$ is the power-law index of the cusp given by $\rho \propto r^n$. This self-similarity is illustrated in Figure 14. If we set $n = -1.5$ from the simulations, we obtain

$$\rho_s(M) \propto M^{-1},$$

$$r_s(M) \propto M^{2/3}.$$
In this subsection we discuss the universality of the density profile. Using a nondimensional free parameter $\delta$, we define new nondimensional variables expressed as

$$\rho_{**} = \rho_0 \delta^{-1},$$  

$$r_{**} = r_0 \delta^{1/3}.$$  

Figure 16 shows $\rho_{**}$ vs. $r_{**}$ of all runs at $z = z_{\text{end}}$. The values of $\delta^{-1}$ are 1.0, 0.4, and 0.6 for run $16M0$; 2.5, 1.0, and 3.0 for run $8M0$; 10.0, 3.0, and 6.0 for run $4M0$; and 35.0, 12.0, and 30.0 for run $2M0$. We can see that the 12 density structures agree nicely, which means that they are universal. In principle, any scaling on the $r$-$\rho$ plane can be expressed using two parameters. We used the total mass $M$ as one of two parameters to express the self-
Fig. 17.—Fitting of the density structure by the profile proposed by Moore et al. (1999; solid curve) and \( \rho \propto r^{-1.5} \) (dashed line).

Fig. 18.—Fitting of the density structure by the profile proposed by Navarro, Frenk, & White (1996, 1997).

Fig. 19.—Density profiles of smaller halos that are going to merge into the larger halo. Thick curve indicates the larger halo.
similarity discussed in § 3.4. The parameter \( d \) corresponds to the other freedom. The value of \( d \) is considered to reflect an amplitude of the density fluctuation at the collapse.

We attempted to fit the density structure to several profiles proposed in earlier studies. Figures 17 and 18 show the profile proposed by Moore et al. (1999) and by NFW (1996, 1997). The function forms are given by \( \rho_{\text{pp}} = r_{\text{pp}}^{-1.5} \times (1 + r_{\text{pp}})^{-1} \) and \( \rho_{\text{s}} = 100(0.5r_{\text{s}})^{-1}(1 + 0.5r_{\text{s}})^{-2} \), respectively. Our simulation results agree with the profile proposed by Moore et al. (1999) very well, while the agreement with the NFW profile is not as good.

In summary, the density structure of simulated halos is well expressed by

\[
\frac{\rho(r)}{\rho_0} (= \rho^{**}) = \frac{1}{(r/r_0)^{1.5}[1 + (r/r_0)^{1.5}]} \left( = \frac{1}{r^{1.5}[1 + r^{1.5}]} \right),
\]

(12)

where

\[
\rho_0 = 7 \times 10^{-4} \delta \left( \frac{M}{10^{14} M_\odot} \right)^{-1} M_\odot \text{ pc}^{-3},
\]

(13)

\[
r_0 = 0.2\delta^{-1/3} \left( \frac{M}{10^{14} M_\odot} \right)^{2/3} \text{ Mpc},
\]

(14)

where again \( M \) is the total mass of halos and \( \delta \) is a non-dimensional free parameter. The free parameter \( \delta \) is constant during evolution of a halo.

3.6. Formation Process of the Central Cusp

In this subsection we show that the central cusp grows through the accretion of the disrupted smaller halos by a larger halo. In the bottom-up structure formation a typical halo grows through repeated merging of smaller halos. In the CDM hierarchical clustering, the larger halo typically has a denser central region than the smaller halo. Therefore when two halos merge, the smaller halo is disrupted by the tidal field of the larger halo and the matter from the smaller halo is scattered. On the other hand, the central region of the larger halo survives the merging process more or less intact.

In Figure 13, we can clearly see that the cusp grows outward without changing the inner part. In Figure 19, we show density profiles of halos that will merge to the largest halo for runs 16M0 and 2M0. We plot six halos with more than 1000 particles that are nearest to the potential minimum together with the largest halo. It is clear that the central halo has the highest density.

The reason why larger halos have higher densities can be understood as follows. Let us consider the peaks of the fluctuation whose characteristic scale is \( \lambda \). If the total density field is composed only of the fluctuation whose scale is \( \lambda \), the peaks will collapse to halos with similar density almost simultaneously. Actually, there are contributions from fluctuations whose scale is larger than \( \lambda \) as well. A peak in high-density background would collapse to a halo with higher density than peaks in low-density background simply because of the difference in the background density. Later, the \"background,\" which is just a density peak of longer wavelength, would collapse. During this collapse, however, the high-density peak that collapsed earlier is not affected. Therefore larger halos tend to have higher central density than smaller halos.

In Figure 20 we show a one-dimensional trajectory of the particles for run 16M0. We randomly select 10 particles...
from the ones whose distances from the center of the halo at the end of the simulation are 0.02–0.03, 0.1–0.2, and 1–2 Mpc. Figure 20 shows that a large fraction of the particles in the inner region settles there early while those in the outer region tend to fall later. In other words, Figure 20 shows that the formation process discussed above actually takes place.

The cusp with the slope of $-1.5$ seems to be a "fixed point" or a "convergence point" for the growth of the halo through accretion of diffuse and small halos. Once the cusp with the slope of $-1.5$ forms, the density in the $r^{-1.5}$ cusp remains unchanged and the disrupted matter is accreted outside the $r^{-1.5}$ cusp, clearly shown in Figure 13.

Moreover, the power index of $-1.5$ seems to be a universal feature independent of the form of the initial power spectrum. The high-resolution simulations presented in this paper and those by Moore et al. (1999) show that the power of the cusp is $-1.5$, independent of mass scale. A preliminary result of another of our simulations from the initial power spectrum of $P(k) \propto k^{-1.7}$, which is shallower than that at cluster scale for standard CDM model, also shows that the power of the cusp is around $-1.5$.

Currently, we do not have a clear explanation for why the slope of the cusp is $-1.5$ when it forms through the accretion of disrupted small halos. We will discuss this topic more comprehensively elsewhere.

3.7. Origin of the Outer Profile

Figure 21 shows the distribution of particles on the $r-v_r$ plane, where $r$ is the distance from the center and $v_r$ is the radial velocity, at 16 different redshifts for run 16M0. We can see that the outer region consists of two components. The first component is infalling matter that is visible as a thick stream of particles in the lower right region of each panel; the vertical spreads visible in this stream are infalling smaller halos. The second component consists of the more scattered particles with an average velocity of nearly zero. As one can see from the time evolution, these stars gained energy in the central region when small halos accreted on the central halo. In Figure 22 we show density profiles of scattered and infalling particles separately. We separated the two components by defining appropriate boundaries in Figure 21. At around $r_{200}$, the contributions of the two components to the total profile are of the same order.
The profile in the outer region exhibits large fluctuations. The merging events occur intermittently, and the amount of scattered matter depends on earlier merging events. Nevertheless, the density profile fits the profile that is asymptotically proportional to $r^{-3}$.

Consequently, in the outer region, particle orbits show strong radial anisotropy. Figure 23 shows anisotropy in velocity distribution of the profile, together with simulation results for runs 16M$[0, 1, 2]$ and 2M$[0, 1, 2]$. The anisotropy is expressed by the anisotropy parameter $\beta$, defined as

$$\beta = 1 - \frac{\langle v^2_{\theta} \rangle}{\langle v^2_r \rangle},$$

where $\langle v^2_{\theta} \rangle$ and $\langle v^2_r \rangle$ are mean tangential and radial velocity dispersions. In this definition, $\beta = 0$ means that the velocity distribution is isotropic and $\beta = 1$ means that it is completely radial.

4. CONCLUSION

We performed $N$-body simulations of dark matter halo formation in the standard CDM model. We simulate 12 halos whose masses range from $6.6 \times 10^{11}$ to $8.0 \times 10^{14} M_\odot$. We introduced the accuracy criteria to guarantee that numerical artifacts due to two-body relaxation and time integration do not affect the result, and we obtained a density profile free from numerical artifacts down to radii of $(0.01\text{--}0.02)r_{200}$.

Our main conclusions are the following:

1. In all runs, the final halos have density cusps proportional to $r^{-1.5}$.
2. The density profile evolves self-similarly.
3. The density profile is universal, independent of halo mass, initial random realization of density fluctuation, and redshift. The density structure is in good agreement with the profile proposed by Moore et al. (1999).
4. The central cusp grows through the disruption and accretion processes of diffuse smaller halos. The slope of the central cusp seems to be a fixed point for the growth of the halo through accretion of tidally disrupted matter.

5. DISCUSSION

Here we discuss the relation between our results and those of the previous studies. We obtained a steeper inner cusp than that obtained by NFW, which was already found in high-resolution simulations (FM97; Moore et al. 1999; Ghigna et al. 2000; Jing & Suto 2000). The reason for this disagreement is that in low-resolution simulations, the central cusp is smoothed out by the two-body relaxation. If
the cusp is shallower than $-2$, the velocity dispersion decreases inward. The energy flows inward owing to the two-body relaxation, and the central region expands in what is called the granothermal expansion (Hachisu et al. 1978; Quinlan 1996; Heggie, Inagaki, & McMillan 1994; Endo, Fukushige, & Makino 1997). Therefore, the density in the cusp decreases and the cusp becomes shallower. Using the relations $t_{rel} \sim v^3 / \langle \rho \rangle m$, $\rho \sim r^{-1.5}$, and $v \sim r^{-0.25}$ and our simulation results, we can estimate the lower limit of the radius where the structure is free from the two-body relaxation effect as $\sim 0.01r_{200} / N(10^9)^{-0.44}(\rho_0/2.7 \times 10^{-4} M_\odot pc^{-3}0.44 \sigma_0/1300 \ km \ s^{-1})^{-1.33}$ for a cluster-sized halo at the present epoch, where $\sigma_0$ is velocity dispersion at the scale radius $r_0$. For simulations with $N = 10^4$ and $10^5$ within $r_{200}$, the limits are estimated as $\sim 0.08r_{200}$ and $\sim 0.03r_{200}$, respectively. Therefore, in simulations with $\sim 20,000$ particles, the central cusp would become significantly shallower owing to relaxation.

We could not reproduce the dependence of the slope observed by Jing & Suto (2000). This difference could be due to the smoothing by two-body relaxation in their cluster-sized halos. In this paper we show that the density profile within $0.01r_{200}$ is smoothed by the two-body relaxation. The density at $0.01r_{200}$ and the mass resolution obtained by Jing & Suto (2000) for their cluster-sized halo are similar to ours. The density profile in their simulations within $0.01r_{200}$, at which the profile begins to depart from $r^{-1.5}$ inward, could be affected by the two-body relaxation.

Our result is in good agreement with results of simulations by Moore et al. (1999) in which the tidal field was included. This agreement suggests that the neglect of the tidal field in our present study and in FM97 hardly affects the density profile. The formation mechanism we discussed in this paper also suggests that the tidal field from the mass outside the simulation sphere is not crucial.

Moore et al. (1999) argued that the merging process is not related to the structure. They simulated the halo formation using a power spectrum with a cutoff to suppress a merging event in smaller scale and showed that the profile does not change. However, in their simulation, several merging events took place since the cutoff wavelength is rather short. Therefore their conclusion that merging is not important is not really supported by their simulation. According to our explanation, several merging events in which the central large halo swallows smaller halos determine the structure of the halo. Such events did take place in Moore et al.’s simulations.

As discussed in §1, there are many analytical and semi-analytical studies that explain the density structure. However, no study succeeds in satisfactorily explaining the universality of the density profile. This is because none of these studies are based on the formation and growth processes of the halo that this paper discusses. Syer & White (1998) suggested that the cusp is a convergence point of a disrupted and sinking satellite and that the convergence slope depends on the initial power spectrum. In their model, some smaller halos were assumed to sink down to the center of the larger halo when two halos merged. However, as we discussed, smaller halos are always disrupted and never sink down to the center because the smaller halo is always less dense. Evans & Collet (1997) argued that the cusp is a steady-state solution of the Fokker-Plank equation. The solution is derived by assuming that many small clumps within a large halo evolve by two-body relaxation. In our simulations there are no such small clumps since they are disrupted before they reach the center.

We are grateful to Atsushi Kawai for his help in preparing the hardware and software environment of the GRAPE-5 system and to Yasushi Suto and Yoko Funato for many helpful discussions. To generate initial conditions, we used the COSMICS package developed by Edmund Bertschinger, to whom we express our thanks. Part of the numerical computations were carried out on the GRAPE system at the Astronomical Data Analysis Center of the National Astronomical Observatory, Japan. This research was partially supported by the Research for the Future Program of Japan Society for the Promotion of Science, grant JSPS-RFTP 97-P01102.

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