On extracting the $\rho-\omega$ mixing amplitude from the pion form-factor

A.G. Williams, a H.B. O’Connell, b and A.W. Thomas a

aCSSM and Department of Physics and Mathematical Physics, University of Adelaide, Australia 5005

bDepartment of Physics and Astronomy, University of Kentucky, Lexington, KY 40506, USA

In this paper we improve and extend a recent analysis which showed that the $\rho-\omega$ mixing amplitude cannot be unambiguously extracted from the pion electromagnetic form-factor in a model independent way. In particular, we focus on the argument that the extraction is sensitive to the presence of any intrinsic $\omega_I \rightarrow \pi\pi$ coupling. Our extended analysis confirms the original conclusion, with only minor, quantitative differences. The extracted mixing amplitude is shown to be sensitive to both the intrinsic coupling $\omega_I \rightarrow \pi\pi$ and to the value assumed for the mass of the $\rho^0$-meson.

ADP-97-19/T256
UK/97-15

1. Introduction

The $\rho-\omega$ mixing amplitude is traditionally extracted from the pion electromagnetic (EM) form-factor, $F_\pi(q^2)$, as measured in $e^+e^- \rightarrow \pi^+\pi^-$ [1]. The non-perturbative strong interaction effects that produce the significant enhancement in the interaction around $\sqrt{q^2} \approx 750\text{MeV}$ have been successfully parametrised using the vector meson dominance (VMD) model. Our interest lies in the appearance of the isoscalar $\omega$ meson in the isovector, $\rho^0$ meson dominated, pion form-factor. The traditional treatment neglects any intrinsic coupling of the $\omega$ to the two pion final state (i.e., that not proceeding through a $\rho^0$ meson) and hence assumes that the $\rho-\omega$ mixing amplitude is purely real. An argument due to Renard and others [2] shows that, within certain approximations, the imaginary part of the $\rho-\omega$ mixing amplitude is cancelled by the intrinsic $\omega$ decay contribution to the pion form-factor. This argument was critically questioned recently [3] and argued to be unjustified. In the following we discuss results from Ref. [4], which extend and improve these arguments. We confirm the central conclusion, finding, in addition, a considerable sensitivity to the value assumed for the $\rho$ mass.

*Talk given at the Int. Conf. on Quark Lepton Nuclear Physics, Osaka, May 20 - 23, 1997.*
2. The isospin pure basis

Analysis of $\rho - \omega$ mixing starts with the “isospin pure” fields, $\rho_I$ and $\omega_I$. These are by definition exact eigenstates of isospin. The photon to hadron interaction can be conveniently described using a formalism in which the renormalised vector meson propagator is a $2 \times 2$ diagonal matrix. For the pion EM form-factor, in the pure isospin limit, we have no $\rho - \omega$ mixing and no direct $\omega_I \rightarrow \pi \pi$ coupling and hence

$$F_\pi = \frac{1}{e}(f_{\gamma \rho_I}, f_{\gamma \omega_I}) \left( \begin{array}{cc} D_{\rho \rho}^I & 0 \\ 0 & D_{\omega \omega}^I \end{array} \right) \left( \begin{array}{c} g_{\rho_I \pi \pi} \\ 0 \end{array} \right),$$

where $e \equiv |e|$, $f_{\gamma V_i}$ is the coupling of the vector meson to the photon and $g_{\rho_I \pi \pi}$ is the coupling of the $\rho$ to the two pion final state. Here $D_{\rho \rho}^I$ and $D_{\omega \omega}^I$ are the scalar parts of the renormalised propagators for the isospin pure fields. As is standard in traditional treatments, coupling to conserved currents is assumed, allowing us to use $D_{\mu \nu}(q^2) = -g_{\mu \nu}D(q^2)$, where $D(q^2)$ is the “scalar propagator.”

The two eigenstates “mix” through the isospin violating mixing self-energy, $\Pi_{\rho \omega}(q^2)$, to allow the decay $\omega_I \rightarrow \rho_I \rightarrow \pi \pi$ through off-diagonal terms in the dressed, isospin-violating vector meson propagator matrix [5]. This introduces isospin violation (IV) in the vector meson propagator and we have (retaining only first order in isospin violation) [5]

$$D^I = \left( \begin{array}{cc} D_{\rho \rho}^I & 0 \\ 0 & D_{\omega \omega}^I \end{array} \right) \rightarrow D^I = \left( \begin{array}{cc} D_{\rho \rho}^I & D_{\rho \omega}^I(q^2) \\ D_{\rho \omega}^I(q^2) & D_{\omega \omega}^I \end{array} \right) = \left( \begin{array}{cc} D_{\rho \rho}^I \Pi_{\rho \omega}(q^2)D_{\omega \omega}^I \\ D_{\rho \omega}^I \Pi_{\rho \omega}(q^2)D_{\omega \omega}^I \end{array} \right) + \mathcal{O}((IV)^2).$$

However, a priori, in an effective Lagrangian model involving the vector mesons, all isospin violation sources are equally likely and there is no reason to exclude the possibility of the “intrinsic decay”, $\omega_I \rightarrow \pi \pi$, through the coupling $g_{\omega_I \pi \pi}$. The appropriate VMD-based expression for the pion form-factor in the isospin pure basis would then be given by

$$F_\pi(q^2) = \frac{1}{e}(f_{\gamma \rho_I}, f_{\gamma \omega_I}) \left( \begin{array}{cc} D_{\rho \rho}^I \Pi_{\rho \omega}D_{\omega \omega}^I \\ D_{\rho \omega}^I \Pi_{\rho \omega}D_{\omega \omega}^I \end{array} \right) \left( \begin{array}{c} g_{\rho_I \pi \pi} \\ g_{\omega_I \pi \pi} \end{array} \right)$$

$$= \frac{f_{\gamma \rho_I}}{e} \frac{1}{q^2 - m^2_\rho(q^2)}g_{\rho_I \pi \pi} + \frac{f_{\gamma \omega_I}}{e} \frac{1}{q^2 - m^2_\omega(q^2)}\Pi_{\rho \omega}(q^2) \frac{1}{q^2 - m^2_\rho(q^2)}g_{\rho_I \pi \pi}$$

$$+ \frac{f_{\gamma \omega_I}}{e} \frac{1}{q^2 - m^2_\omega(q^2)}g_{\omega_I \pi \pi},$$

where a fourth term on the RHS involving both $\Pi_{\rho \omega}$ and $g_{\omega_I \pi \pi}$ has been neglected since it is second order in isospin violation. One is always free to consider models where $g_{\omega_I \pi \pi}$ is strictly zero, but there is no model-independent requirement that this be so. For the renormalised, isospin-pure propagators, $D^I_{VV}$, we have used the physical $\rho$ and $\omega$ propagators, since $D_{VV} = D^I_{VV} + \mathcal{O}((IV)^2)$ [5] and since again we are working only to first order in isospin violation. For the physical $\rho$ and $\omega$ propagators we will use here the usual Breit Wigner form with a momentum-dependent width, i.e., $D_{VV} = \frac{1}{q^2 - m^2_V(q^2)}$, where $m^2_V(q^2) = m^2_V - i\Gamma_V \Gamma_V h_V(q^2)$ and where we have defined the momentum-dependent width $\Gamma_V(q^2) = \Gamma_V h_V(q^2)$ with $h_V(m^2_V) \equiv 1$ and where the mass parameter ($m_V$) and width parameter ($\Gamma_V$) are real.
3. The physical basis

The pion form-factor has two distinct resonance poles, yet Eq. (3) has a rather complicated pole structure. Therefore, one might change to a basis that has only a sum of simple poles. This can be done by transforming to what is referred to as the “physical” basis. It has been traditional to assume that $\Pi_{\rho\omega}$ is a constant which allows the off-diagonal terms in the propagator to be removed by a rotation of the fields. However, recent work has argued that, when $\rho - \omega$ mixing is generated only through vector mesons coupling to conserved currents, this amplitude is necessarily momentum dependent [3]. This result is consistent with an earlier study which concluded that momentum independent mixing was acceptable for scalar particles, but not for vectors coupling to conserved currents [6]. Various demonstrative model calculations support this [7]. To make allowance for this, Maltman, O’Connell and Williams (MOW) defined the physical basis through $\Pi_{\rho\omega}$ poles. This can be done by transforming to what is referred to as the “physical” basis. Therefore, one might change to a basis that has only a sum of simple resonance poles in $\rho - \omega$ [2].

It should be noted that $m$ is a constant which allows the off-diagonal terms in the usual way through

$$D_{\rho\omega}(q^2) = i \int d^4x e^{iq\cdot x} \langle 0 | T (\rho(x)\omega(0)) | 0 \rangle. \quad (5)$$

As the vector mesons couple to conserved currents, we can replace $D_{\rho\omega}(q^2)$ by $-g_{\mu\nu}D(q^2)$. Eq. (5) leads us to

$$D_{\rho\omega}(q^2) = D_{\rho\omega}^I(q^2) - \epsilon_1 D_{\omega\omega}^I(q^2) + \epsilon_2 D_{\rho\rho}^I(q^2)$$

$$= D_{\rho\rho}^I(q^2) [\Pi_{\rho\omega}(q^2) - \epsilon_1(D_{\rho\rho}^I(q^2))^{-1} + \epsilon_2(D_{\omega\omega}^I(q^2))^{-1}]D_{\omega\omega}^I(q^2). \quad (6)$$

Note that $D = CD^TC^T$. The physical basis is defined by requiring that there are no resonance poles in $D_{\rho\omega}(q^2)$, i.e., we choose $\epsilon_1$ and $\epsilon_2$ such that $D_{\rho\omega}(q^2)$ has no pole. The possible pole positions are the vector meson pole positions, $m_\rho^2$ and $m_\omega^2$. These singularities can be removed if the numerator of $D_{\rho\omega}(q^2)$ vanishes at these positions, leading us to,

$$\epsilon_1 = \frac{\Pi_{\rho\omega}(m_\omega^2)}{m_\omega^2 - m_\rho^2}, \quad \epsilon_2 = \frac{\Pi_{\rho\omega}(m_\rho^2)}{m_\rho^2 - m_\omega^2}. \quad (7)$$

It should be noted that $m_\rho^2$ and $m_\omega^2$ are the complex resonance pole positions. In the case where the mixing is momentum dependent we have $\epsilon_1 \neq \epsilon_2$ such that the matrix $C$ is not orthogonal and the transformation between bases is then not a simple rotation. In a similar fashion to Eq. (3), the coupling constants in the physical basis are defined via

$$f_{\gamma\omega} = f_{\gamma\omega_1} + \epsilon_2 f_{\gamma\rho_1}, \quad g_{\omega\pi\pi} = g_{\omega_1\pi\pi} + \epsilon_2 g_{\rho_1\pi\pi}$$

$$f_{\gamma\rho} = f_{\gamma\rho_1} - \epsilon_1 f_{\gamma\omega_1}, \quad g_{\rho\pi\pi} = g_{\rho_1\pi\pi}. \quad (8)$$

Note that $\epsilon_1 g_{\omega_1\pi\pi}$ is second order in isospin violation and so is not retained.
In this way, MOW defined the form-factor in the physical basis by

$$F_\pi(q^2) = \frac{1}{e} [g_{\omega\pi\pi} D_{\omega\omega} + g_{\rho\pi\pi} D_{\rho\rho} + g_{\rho\omega\pi} D_{\rho\omega} f_{\gamma\omega}] + \text{background.} \tag{9}$$

The non-resonant term, $D_{\rho\omega}$, was then included in the background, along with any other non-pole terms of the Laurent series expansions of the propagators, to arrive at an expression for the form-factor,

$$F_\pi(q^2) = H(\epsilon_1, \epsilon_2) \left[ P_\rho + A(\epsilon_1, \epsilon_2)e^{i\phi(\epsilon_1, \epsilon_2)} P_\omega \right] + \text{background.} \tag{10}$$

Here $P_{\rho,\omega}$ are simple poles and $H$ is an overall constant. The quantities $H, A$ and the Orsay phase, $\phi$, can be read off from Eq. (10) as we will show later. In terms of the isospin-pure basis and the transformation matrix, $C$, Eq. (10) can be written,

$$F_\pi = \frac{1}{e} (f_{\gamma\rho_1} f_{\gamma\omega_1}) C^T C D I C^T I \left( \begin{array}{c} g_{\rho_1\pi\pi} \\ g_{\omega_1\pi\pi} \end{array} \right). \tag{11}$$

Because of the closeness of the $\rho$ and $\omega$ pole positions, there is no practical distinction in their analysis between the two terms [i.e., $\Pi_{\rho\omega}(m_\rho^2)$ and $\Pi_{\rho\omega}(m_\omega^2)$], so we define a single parameter, $\epsilon \equiv \epsilon_2 = \Pi_{\rho\omega}(m_\rho^2)/(m_\omega^2 - m_\rho^2) \approx \epsilon_1$, where $\Pi_{\rho\omega}(m_\rho^2) \approx \Pi_{\rho\omega}(m_\omega^2) \approx \Pi_{\rho\omega}(m_\rho^2) \approx \Pi_{\rho\omega}(m_\omega^2)$. In other words, from this point on we will assume (as is done in all standard treatments) that the momentum dependence of $\Pi_{\rho\omega}(q^2)$ is negligible in the vector meson resonance region.

### 4. The Renard Argument

The major conclusion of the MOW analysis [3] concerns the competition between the two sources of isospin violation, namely, $\Pi_{\rho\omega}$ and $g_{\omega\pi\pi}$. All isospin violation in the pion form-factor is usually attributed to $\rho - \omega$ mixing [3], because the intrinsic decay of the $\omega$ is assumed to be cancelled [3]. Consider the $\omega$ pole term of Eq. (10). The coupling of the physical $\omega$ to the two pion final state is given by, $g_{\omega\pi\pi} = g_{\omega\pi\pi} + \epsilon g_{\rho\pi\pi}$. It is certainly a reasonable approximation to assume that the two pion intermediate state saturates the imaginary part of $\Pi_{\rho\omega}(q^2)$ around the $\rho$ resonance region and that this is proportional to the two pion piece of the $\rho$ self energy (which dominates the imaginary piece of the total $\rho$ self energy, due to the strong $\rho \rightarrow \pi\pi$ decay). We have then

$$\text{Im} \ \Pi_{\rho\omega}(m_\rho^2) = \text{Im} \ \Pi_{\rho\omega}(m_\rho^2) = \frac{g_{\omega\pi\pi}}{g_{\rho\pi\pi}} \text{Im} \ \Pi_{\rho\omega}(m_\rho^2) \equiv -G \hat{m}_\rho \Gamma_\rho(m_\rho^2) \approx -G \hat{m}_\rho \Gamma_\rho, \tag{12}$$

where we have defined $G \equiv \frac{g_{\omega\pi\pi}}{g_{\rho\pi\pi}}$, and assumed $h_\rho(m_\rho^2) \approx h_\rho(m_\rho^2) \equiv 1$. We hence define

$$\Pi_{\rho\omega}(m_\rho^2) \equiv \tilde{\Pi}_{\rho\omega}(m_\rho^2) - i G \hat{m}_\rho \Gamma_\rho. \tag{13}$$

Assuming saturation of the absorptive part by the two pion state, as described above, implies that $\tilde{\Pi}_{\rho\omega}(m_\rho^2)$ is real. Note that while the three pion state is kinematically accessible it also requires isospin violation and is, in addition, suppressed by the smaller phase space available. We follow standard practice and ignore it here.
Substituting Eq. (13) into the expression for $\epsilon$, we have

$$
\epsilon = \frac{\tilde{\Pi}_{\rho\omega}(m^2_\rho)}{m^2_\omega - m^2_\rho} - iG \frac{\hat{m}_\rho \Gamma_\rho}{m^2_\omega - m^2_\rho} \equiv -iz\tilde{T} - zG, \quad \text{where} \quad z \equiv \frac{i\hat{m}_\rho \Gamma_\rho}{m^2_\omega - m^2_\rho}, \quad \tilde{T} \equiv \frac{\tilde{\Pi}_{\rho\omega}(m^2_\rho)}{\hat{m}_\rho \Gamma_\rho}. \tag{14}
$$

Note that since in the resonance region we are neglecting momentum dependence of the resonance widths, i.e. we take $h_\omega(q^2) = h_\rho(q^2) = 1$, then we see that $m^2_\omega$, $m^2_\rho$, and $z$ are all constants. Recall that here, since we are using just the single parameter $\epsilon$, the mixing amplitude $\tilde{\Pi}_{\rho\omega}(m^2_\rho)$ should be understood to mean $\tilde{\Pi}_{\rho\omega}(q^2)$ with $q^2$ in the vector resonance region. Using this expression for $\epsilon$ in the $\omega$ pole term gives rise to Renard’s cancellation

$$
g_{\omega_1\pi\pi} + \epsilon g_{\rho_1\pi\pi} = g_{\omega_1\pi\pi}(1 - z) + \frac{\tilde{\Pi}_{\rho\omega}(m^2_\rho)}{m^2_\omega - m^2_\rho} g_{\rho_1\pi\pi} = \left[G(1 - z) + \frac{\tilde{\Pi}_{\rho\omega}(m^2_\rho)}{m^2_\omega - m^2_\rho}\right] g_{\rho_1\pi\pi} \tag{15}
$$

when one makes the approximation $z = 1$. However, MOW find that $z$ has a sizeable imaginary piece $[3]$ $z = 0.9324 + 0.3511i$, using the mass and width values from Bernicha et al. [3]. For comparison, one finds $z = 1.023 + 0.2038i$ using the values of Benayoun et al. [10]. The central observation of MOW was to point out that the deviation of $z$ from unity leads to a substantial contribution to $F_\pi(q^2)$ from $g_{\omega_1\pi\pi}$.

At this point MOW neglected the $\epsilon$ dependence of the constant, $H(\epsilon_1, \epsilon_2)$ in Eq. (10) and extracted $G$ and $\tilde{\Pi}_{\rho\omega}(m^2_\rho)$ by fitting $A$ and the Orsay phase $\phi$. In order to do a more careful analysis of the $G$ dependence of the extracted real part of the $\rho - \omega$ mixing amplitude, $\tilde{\Pi}_{\rho\omega}(m^2_\rho)$ we shall retain $H(\epsilon) \equiv H(\epsilon_1, \epsilon_2)$, which itself has a $G$ dependence through $\epsilon$ [via Eq. (14)]. Let us write Eqs. (3) and (10), [or equivalently Eq. (3)] as

$$
F_\pi(q^2) = \frac{1}{e} \left[ f_{\gamma_\rho_1} g_{\rho_1\pi\pi} P_\rho - f_{\omega_1} \epsilon(P_\rho - P_\omega) g_{\rho_1\pi\pi} + f_{\gamma_\omega_1} P_\omega g_{\omega_1\pi\pi} \right], \tag{16}
$$

where we have used the fact that $\epsilon(P_\rho - P_\omega) = -P_\rho \Pi_{\rho\omega} P_\omega$. Defining the ratio $r_I \equiv \frac{f_{\omega_1}}{f_{\gamma_\omega_1}}$, we can use Eqs. (14) and (14) to write Eq. (10) as

$$
F_\pi(q^2) = \frac{1}{e} f_{\gamma_\rho_1} g_{\rho_1\pi\pi} \left\{ P_\rho + r_I \left[ -\epsilon(P_\rho - P_\omega) + GP_\omega \right] \right\}
$$

$$= \frac{1}{e} f_{\gamma_\rho_1} g_{\rho_1\pi\pi} \left\{ P_\rho (1 + izr_I \tilde{T}) + r_I \left[ -iz\tilde{T} P_\omega + zGP_\omega + G(1 - z)P_\omega \right] \right\}. \tag{17}
$$

Comparing Eq. (17) to our earlier Eq. (14) it is seen to be a straightforward matter to identify the qualities $H$, $A$ and the Orsay phase, $\phi$. This is the central result.

Recall that to obtain this result, we followed standard treatments and neglected the momentum-dependence of the Breit-Wigner widths in the resonance region, $h_\rho(m^2_\rho) \approx h_\rho(\hat{m}_\rho^2) = 1$. We also neglected the three pion loop contribution to $\rho - \omega$ mixing, so that $\text{Im} \Pi_{\rho\omega}(q^2) = G \text{Im} \Pi_{\rho\rho}(q^2)$ and assumed a constant value for $\tilde{\Pi}_{\rho\omega}$ in the vector meson resonance region.

We arrive at a similar conclusion in a more straightforward way if we choose to stay in the isospin pure basis throughout and simply re-arrange Eq. (3) to give

$$
F_\pi(q^2) = \frac{1}{e} \left[ f_{\gamma_\rho_1} \frac{1}{q^2 - m^2_\rho(q^2)} + \frac{f_{\gamma_\omega_1}}{q^2 - m^2_\rho(q^2)} \tilde{\Pi}_{\rho\omega}(q^2) \frac{g_{\rho_1\pi\pi}}{q^2 - m^2_\omega(q^2)} \right]
$$

$$+ \frac{f_{\gamma_\omega_1}}{q^2 - m^2_\rho(q^2)} g_{\omega_1\pi\pi} \left( 1 - \frac{i\hat{m}_\rho \Gamma_\rho h(q^2)}{q^2 - \hat{m}_\rho^2 + i\hat{m}_\rho \Gamma_\rho h(q^2)} \right). \tag{18}
$$
We see immediately that in the isospin pure treatment the $G$ dependence (i.e., $g_{\omega \pi \pi}$ dependence) is cancelled at $q^2 = \hat{m}_\rho^2$ and somewhat suppressed around the pole region (where $\rho - \omega$ interference is most noticeable). Including the nonresonant off-diagonal propagator in the physical basis would make Eqs. (17) and (18) identical, but as we can only fit resonant terms plus an unknown non-resonant background to data, there is no practical difference between these two equations.

5. Results and conclusions

To determine values for $\tilde{\Pi}_{\rho \omega} (m_\rho^2)$ and $G$ we must choose an appropriate form factor and decide upon Eq. (18), which we re-write as

$$F_\pi = \frac{-a m_\rho^2}{q^2 - \hat{m}_\rho^2 + i \hat{m}_\rho \Gamma_\rho} \left[ 1 + r_I \frac{\tilde{\Pi}_{\rho \omega} (m_\rho^2) + G (q^2 - \hat{m}_\omega^2)}{q^2 - \hat{m}_\omega^2 + i \hat{m}_\omega \Gamma_\omega} \right].$$

(19)

We now perform a fit to pion form-factor data to extract values for $\hat{m}_\rho$, $\Gamma_\rho$, $\tilde{\Pi}_{\rho \omega} (m_\rho^2)$, $G$ and the normalisation constant, $a$, using the fitting routine Minuit [11] (case A in Table 1). We fix $m_\omega = 781.94$ MeV and $\Gamma_\omega = 8.43$ MeV, as given by the Particle Data Group [14].

It should be noted that we are here adopting this form to fit rather than using Eqs. (17) and (10), since we wish to isolate all of the $G$-dependence from the overall normalisation constant for the fit. It can be seen from Eq. (17) that $H$ in Eq. (10) is $G$-dependent and so we can not use the fitting procedure adopted by MOW. By construction, the constant $a$ in Eq. (19) is independent of $G$. For the analysis here we will assume the SU(3) value for $r_I \equiv g_{\gamma_\omega \gamma_\rho} / g_{\gamma_\pi \gamma_\rho}$, i.e., we will assume $r_I = 1/3$ [12]. However, from Eq. (19) we see that all that can really be extracted from the analysis is $r_I \tilde{\Pi}_{\rho \omega}$ and $r_I G$ and so if another value of $r_I$ is preferred our results can immediately be scaled in a straightforward way. Our data set consists of the 70 points listed in Ref. [13] in the region between 500 and 975 MeV. While the preferred value for $G$ is quite large – one usually expects isospin breaking at the few percent level, rather than 10% – the uncertainty is such that it lies only $2^{1/2}$ standard deviations from zero.

We can investigate the importance of direct isospin violation at the $\omega \to \pi \pi$ vertex by imposing the condition $G = 0$ (case B). However, when fitting the $\rho$ data, it is important to remember that the $\rho$ parameters should not be process-dependent. As noted by Benayoun et al., since the $\rho$ is a relatively broad resonance the value extracted for, say, the mass, can be greatly affected by the addition of other terms to a phenomenological form-factor [10]. To demonstrate that our conclusions concerning $G$ and $\tilde{\Pi}_{\rho \omega}$ do not depend on a particular choice of mass and width from one source, we have redone the fit using the Particle Data Group (PDG) values, which are averaged from a variety of processes [14].

In order to compare our results with the analysis of Maltman et al. [3], we perform a fit to the same data using Eq. (11) with $H(\epsilon)$ treated as a constant (thereby absorbing it into the normalisation constant $a$). To first order in isospin violation one has

$$A e^{i \phi} = \frac{r(G(1 - z) - iz \tilde{T})}{1 + izr \tilde{T} + zrG},$$

(20)
The resulting fit parameters are shown in Table 1 under the heading MOW. We see that they are very close to the fit using the full expression Eq. (17), shown in column A.

In conclusion, we have seen that, in agreement with the conclusions of Maltman et al., the pion form factor data supports equally well a large range of possible pairs of values for $G$ and $\tilde{\Pi}_{\rho\omega}$. In other words, it is not possible to extract the $\rho-\omega$ mixing amplitude in a model-independent way. The traditional method of extraction corresponds to assuming that there is no intrinsic $\omega \rightarrow \pi\pi$ coupling (i.e., that $G = 0$), which is highly unlikely. It should also be noted that these conclusions are entirely independent of what (if any) momentum-dependence is present in the $\rho-\omega$ mixing amplitude, since this was neglected in the resonance region in the usual way.

**REFERENCES**

1. H.B. O’Connell, B.C. Pearce, A.W. Thomas and A.G. Williams, [hep-ph/9501251](http://www.arxiv.org/abs/hep-ph/9501251). To appear in “Progress in Particle and Nuclear Physics,” ed. A. Faessler (Elsevier).
2. F.M. Renard in Springer Tracts in Modern Physics 63 (1972) 98 (Springer-Verlag); M. Gourdin, L. Stodolsky and F. Renard, Phys. Lett. 30B (1969) 347.
3. K. Maltman, H.B. O’Connell and A.G. Williams, Phys. Lett. B376 (1996) 19.
4. H.B. O’Connell, A.W. Thomas and A.G. Williams, preprint ADP-97-8/T246, [hep-ph/9705453](http://www.arxiv.org/abs/hep-ph/9705453) (to appear in Nucl. Phys. A).
5. H. O’Connell, B. Pearce, A. Thomas and A.Williams, Phys. Lett. B336 (1994) 1.
6. S. Coleman and H.J. Schnitzer, Phys. Rev. 134 (1964) B863.
7. T. Goldman, J. Henderson and A. Thomas, Few Body Systems 12 (1992) 123; J. Piekarzewicz and A.G. Williams, Phys. Rev. C 47 (1993) R2462; G. Krein, A. Thomas and A. Williams Phys. Lett. B317 (1993) 293; K. Mitchell, P. Tandy, C. Roberts and R. Cahill, Phys. Lett. B335 (1994) 282; R. Friedrich and H. Reinhardt, Nucl. Phys. A594 (1995) 406; S. Gao, C. Shakin and W. Sun, Phys. Rev. C 53 (1996) 1374; T. Hatsuda, E. Henley, T Meissner and G. Krein, Phys. Rev C 49 (1994) 452; K. Maltman, Phys. Rev. D 53 (1996) 2563, 2573; M.J. Iqbal, X. Jin, D.B. Leinweber, Phys. Lett. B367 (1996) 45, ibid B386 (1996) 55.
8. S.A. Coon and R.G. Barrett, Phys. Rev. C36 (1987) 2189.
9. A. Bernicha, J. Pestieau and G. Lopez Castro, Phys. Rev. D 50 (1994) 4454.
10. M. Benayoun et al., Zeit. Phys. C 58 (1993) 31.
11. F. James and M. Roos, Comput. Phys. Commun. 10 (1975) 343. Current version available from the CERN Program Library: D506, Minuit.
12. G. Dillon and G. Morpurgo, Z. Phys. C64 (1994) 467.
13. L.M. Barkov et al., Nucl. Phys. B256 (1985) 365.
14. R.M. Barnett et al. (Particle Data Group), Phys. Rev. D54 (1996) 1.