Grey Verhulst Power Load Forecasting Method Based on Background Value Optimization

Zonghong Huang¹, Dongsheng Dang¹, Chuncheng Gao², Lei Wang³∗

¹State Grid Ningxia Electric Power Eco-Tech Research Institute, Yinchuan, Ningxia Province, 750004, China
²Beijing Kedong Electric Power Control System CO., LTD, Beijing, 100192, China
³Corresponding author’s e-mail: 1250421169@qq.com

Abstract. According to the nonlinearity and uncertainty of the load sequence, the grey Verhulst model (GV) adapted to the "S" type growth is used to predict the future electricity consumption of Ningxia. This paper analyzes the application limitations of the traditional grey Verhulst model, and introduces the background value of the vector α modified GV model, thus constructing a more universal background value modified GV model, applying the global optimization ability of adaptive particle swarm optimization algorithm (APSO) to solve the optimal α value. A grey Verhulst model (APSO-GV) based on adaptive particle swarm optimization algorithm is proposed. The case study shows that the model has high prediction accuracy and universality.

1. Introduction

Power load forecasting is an important part of the power system [1]. At present, domestic and foreign scholars have conducted extensive explorations on load forecasting problems. Generally, it can be divided into intelligent prediction methods based on machine learning theory and classical prediction methods based on time series prediction principle. Among them, intelligent prediction methods include neural networks, extreme learning machines, expert systems and so on [2-5]. The prediction effect of this kind of method is good and has high theoretical value, but the actual operation is poor, the generalization ability is weak, and it is easy to be affected by factors such as data and experimental equipment. Different from the intelligent prediction method, the grey prediction method in the classical prediction method can be modeled by using the historical load data, and has the characteristics of less required data, complete modeling theory, and strong operability [6].

Many social and economic indicators in Ningxia are no longer exponentially growing, but have certain "S"-shaped curve characteristics, which have strong random volatility. The development will show a non-stationary random process of a certain trend. The Verhulst model is in good agreement with the description of stochastic processes with strong volatility and the "S" state [7]. The reference [8] uses the traditional grey Verhulst model for load forecasting, ignoring the influence of background values on parameters; reference [9] introduces a parameter to correct the background value, but takes the same parameter value for each time background value of different change trend, which cannot fully reduce the prediction error of the model; reference [10] adopts the background value cyclic correction method, but it is artificially determined that the value of the parameter α is unreliably and lacks theoretical basis. APSO has a good global search optimization ability, which can well optimize the background value parameters, so that it can better predict the overall accuracy of the model [11].
In view of this, the vector $\alpha$ is used to modify the background value of the GV model. That is to say, different correction parameters are taken for different time points. Therefore, the GV model is generalized to the GV model with background value correction. Since there is a significant nonlinear relationship between $\alpha$ and error, an adaptive particle swarm optimization algorithm with global search capability should be used to solve the $\alpha$ vector and a grey Verhulst model based on APSO optimization is constructed. After an example analysis, the results show that the optimization model is superior to the traditional GV model and extends the scope of application of the grey Verhulst model.

2. **Grey Verhulst model**

Set that there is a sequence of original samples $x^{(0)}$.

$$x^{(0)} = [x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)]$$

(1)

Generate a first-order cumulative generation sequence with 1-AGO

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i)$$

(2)

$z^{(1)}$ is the nearest mean sequence of $x^{(1)}$:

$$z^{(1)}(k) = \frac{1}{2} [x^{(1)}(k) + x^{(1)}(k-1)], k = 2, 3, \cdots, n$$

(3)

Then we call $x^{(0)}(k) + az^{(1)}(k) = b(z^{(1)}(k))^2$ as the grey Verhulst model, $a$ and $b$ are parameters, and the corresponding whitening equation is:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b(x^{(1)})^2$$

(4)

Perform a least squares estimation on the parameter column $[a, b]^T$:

$$[a, b]^T = (B^T B)^{-1} B^T Y$$

(5)

Where

$$B = \begin{bmatrix} -z^{(1)}(2) & (z^{(1)}(2))^2 \\ -z^{(1)}(3) & (z^{(1)}(3))^2 \\ \vdots & \vdots \\ -z^{(1)}(n) & (z^{(1)}(n))^2 \end{bmatrix}, Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}$$

Using the estimated values of parameters $a$ and $b$, the time response function of the grey Verhulst model can be solved with $x^{(1)}(1) = x^{(0)}(1)$ as the initial condition.

$$\hat{x}^{(1)}(k+1) = \frac{ax^{(0)}(0)}{bx^{(0)}(0) + (a-bx^{(0)}(0))e^{ak}}, k = 0, 1, \cdots, n$$

(6)

The predicted value of the original data sequence can be obtained after a cumulative subtraction and reduction.

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k), k = 1, 2, \cdots, n-1$$

(7)

When $k \to \infty$, if $a > 0$, then $\hat{x}^{(1)}(k) \to 0$; if $a = 0$, then $\hat{x}^{(1)}(k) \to a/b$, which means that there is a sufficiently large $k$ make $\hat{x}^{(1)}(k+1)$ and $\hat{x}^{(1)}(k)$ close enough. At this time, $\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \approx 0$, the system approaches death and the prediction accuracy is higher when dealing with "S" type or partial "S" type processes [12].

3. **Adaptive particle swarm optimization algorithm**

The particle swarm optimization algorithm is a global optimization evolution algorithm proposed by Kennedy and Eberhart, which is derived from the simulation of bird predation behavior. In PSO, the potential solution to each optimization problem is a particle in the search space. All particles have an appropriate value determined by the function being optimized. Each particle also has a velocity that
determines the direction and distance in which they fly, and then the particles follow the current optimal particle search in the solution space.

The PSO is initialized as a group of random particles (random solutions) and then iteratively finds the optimal solution. In each iteration, the particle updates itself by tracking two extreme value; the first extreme value is the optimal solution found by the particle itself, this solution is called the individual extreme value; the other extreme value is currently found by the entire population. The optimal solution, this extreme value is the global extreme value. In addition, it is also possible to use the part of the neighbor as the example, and the extreme value in all the neighbors is the local extreme value. Suppose that in a \( D \)-dimensional target search space, there are \( N \) particles forming a community, where the \( i \)-th particle position and velocity are:

\[
Y_i = (y_{i1}, y_{i2}, \ldots, y_{iD})
\]

\[
V_i = (v_{i1}, v_{i2}, \ldots, v_{iD})
\]

The optimal position that the \( i \)-th particle has searched so far is called the extreme value of the individual. The optimal position that the entire particle group has searched so far is called the global extreme value. Then:

\[
P_{\text{best}} = (p_{i1}, p_{i2}, \ldots, p_{iD})
\]

\[
P_{g\text{best}} = (p_{g1}, p_{g2}, \ldots, p_{gD})
\]

When you find these two optimal values, the particles update their speed and position according to the following formula:

\[
v_{id} = w \cdot v_{id} + c_1 r_1 (p_{id} - y_{id}) + c_2 r_2 (p_{g最佳} - y_{id})
\]

\[
y_{id} = y_{id} + v_{id}
\]

Where \( w \) is the inertia factor, \( c_1 \) and \( c_2 \) are learning factors, usually \( c_1 = c_2 = 2 \), \( r_1 \) and \( r_2 \) are uniform random numbers in the range \([0,1]\), \( v_{id} \) is the velocity of the particle, \( v_{id} \in [-v_{\text{max}}, v_{\text{max}}] \), \( v_{\text{max}} \) is a constant that is set by the user to limit the speed of the particles. In the standard particle swarm algorithm, the inertia factor \( w \) is generally set to 1. Since the inertia factor is a variable that affects the current particle velocity, a larger value is advantageous for global search, and a smaller value is advantageous for local search. In order to better balance the search ability of this algorithm, a linearly decreasing inertia weighting factor is proposed, as shown in equation (13).

\[
w(t) = w_{\text{max}} - \frac{(w_{\text{max}} - w_{\text{min}}) \times t}{k_{\text{max}}}
\]

Where \( w_{\text{max}} \) is the inertia weight maximum, generally set to 0.9; \( w_{\text{min}} \) is the inertia weight minimum, generally set to 0.4; \( t \) is the current iteration number; \( k_{\text{max}} \) is the maximum number of iterations.

In this paper, adaptive inertial factor is used to replace the linear inertia weight factor in PSO algorithm, and an APSO (adaptive particle swarm optimization algorithm) is proposed. This method is beneficial to the algorithm to generate better search ability in the global scope, which can make the particles tend to a better search space and improve the efficiency of the algorithm [13].

4. Grey Verhulst model based on APSO

In the process of prediction using the GV model, the trapezoidal approximation background value is used in the conventional method. The background value is a crucial factor for any prediction model. The traditional value of 0.5 is based on the average value, which makes the sequence more reasonable, but a single value does not completely optimize the sequence. Therefore, it is not universal. It can be known from equation (3) that the accuracy of the grey Verhulst model depends on the parameters \( a \) and \( b \), while the values of \( a \) and \( b \) depend on the construction of \( z^{(1)}(k) \), that is, the construction of \( z^{(1)}(k) \) is one of the key factors leading to the simulation error. The traditional background value formula \( z^{(1)}(k) = 0.5(x^{(1)}(k) + x^{(1)}(k-1)) \) will bring some error. Therefore, the optimization of the background value has a strong practical significance. Therefore, this paper introduces the vector \( \alpha = (\alpha_1, \alpha_2, \ldots, \alpha_d) \), and
constructs the corrected background value of the GV model containing the correction vector. In particular, when \( \alpha = (0.5, 0.5, \ldots, 0.5) \), it is the traditional GV model. Background value is:

\[
(14) \quad \alpha_{k} = \frac{1}{10} \left( x_k - x_{k-1} \right) + \left( 1 - \alpha_{k} \right) x^{(i)}_k
\]

In the specific operation, the background value can be changed by changing the parameter \( \alpha_k \). In the foregoing, by introducing the vector \( \alpha \), a GV model including a correction vector is proposed. It can be known from the formula (14) that as long as a set of \( \alpha_0 \) is given, there is:

\[
B_0 = \begin{bmatrix}
- \left( \alpha_{02} x^{(i)}(1) + (1 - \alpha_{02}) x^{(i)}(2) \right) & \left( \alpha_{02} x^{(i)}(1) + (1 - \alpha_{02}) x^{(i)}(2) \right)^2 \\
- \left( \alpha_{03} x^{(i)}(2) + (1 - \alpha_{03}) x^{(i)}(3) \right) & \left( \alpha_{03} x^{(i)}(2) + (1 - \alpha_{03}) x^{(i)}(3) \right)^2 \\
\vdots & \vdots \\
- \left( \alpha_{0n} x^{(i)}(n-1) + (1 - \alpha_{0n}) x^{(i)}(n) \right) & \left( \alpha_{0n} x^{(i)}(n-1) + (1 - \alpha_{0n}) x^{(i)}(n) \right)^2
\end{bmatrix}
\]

Substituting \( B_0 \) into equation (5) and solving the parameter \( [a \ b]^T \) can calculate the predicted value \( \hat{x}^{(0)}(k) \). Therefore, when the original sequence is determined, the only factor that affects the prediction accuracy of the model is that there is a highly nonlinear relationship between \( \alpha \) and the error known by equations (5) and (6) and it is difficult to solve the most good alpha vector. Based on this, this paper applies APSO to solve the GV model and constructs a grey Verhulst model based on adaptive particle swarm optimization algorithm. The process of particle swarm optimization algorithm for solving APSO-GV model is as follows:

**Step 1:** Initialization: randomly generate \( m \) particles whose initial position and initial velocity are:

\[
\alpha_i = [\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{in}], \quad (i = 1, 2, \ldots, m)
\]

\[
\alpha_i = [\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{in}], \quad (i = 1, 2, \ldots, m)
\]

To ensure that the APSO-GV model is superior to the traditional GV model, it can be assigned \( \alpha_1 = (0.5, 0.5, \ldots, 0.5) \);

**Step 2:** Calculate the fitness of each particle: set the individual optimal position of the particle to the current position \( P_{best} \), and set the global optimal position \( g_{best} \) to the position of the best particle in the initial particle population, aiming at the smallest square of the error, the objective function \( f(Y_i) \) is

\[
(17) \quad f(Y_i) = \sum_{k=1}^{n} (x^{(0)}(k) - \hat{x}^{(0)}(k))^2
\]

Where \( x^{(0)}(k) \) and \( \hat{x}^{(0)}(k) \) are the \( k \)th actual and predicted values of the load, respectively.

**Step 3:** Use equations (11) and (12) to update the velocity and position of each particle. If the particle fitness is better than the corresponding fitness of \( g_{best} \), \( g_{best} \) is set to the new position.

**Step 4:** Determine whether the convergence criterion of the algorithm is satisfied. If yes, perform step 5; otherwise, go to step 4 to iterate and continue to optimize.

**Step 5:** Output the global optimal position \( g_{best} \), obtain the global optimal solution of the vector \( \alpha \), and the algorithm runs.

The algorithm convergence criterion (iteration termination condition) is set to the maximum number of iterations or the objective function reaches the set ideal value.

5. **Example analysis**

In order to verify the validity and practicability of the adaptive particle swarm optimization grey Verhulst model, the annual electricity consumption of the second industry in Ningxia region from 2011 to 2017 was used as sample data. The original data is shown in Table 1. It can be seen from Table 1 that the annual electricity consumption in Ningxia region shows an increasing trend, but there is obvious volatility. The traditional GV model and the APSO-GV model proposed in this paper are compared respectively, and the model with the highest fitting accuracy is used to predict the annual electricity consumption of the secondary industry in 2018.

The adaptive particle swarm optimization algorithm is used to optimize the background value parameters of the GV model. The parameters in the algorithm are set as follows: 50 particles are
randomly generated, the maximum number of iterations is 200, $w_{\text{max}}=1.2$, $w_{\text{max}}=0.2$, $c_1=c_2=2$. The optimization results of the background value parameters are shown in Table 2. The predicted and actual values of the two models are fitted, and the fit is shown in Figure 1. Verify the prediction accuracy of the two models. The error test of the model is shown in Table 2.

Table 1. GV model test data (unit: TWH)

| Year number | 1     | 2     | 3     | 4     | 5     | 6     | 7     |
|-------------|-------|-------|-------|-------|-------|-------|-------|
| Electricity consumption | 673.2451 | 685.6061 | 747.8054 | 778.7450 | 803.3543 | 805.2383 | 888.4031 |

Table 2. Optimal parameters of adaptive particle swarm optimization

| Optimization value | $\alpha_1$ | $\alpha_2$ | $\alpha_3$ | $\alpha_4$ | $\alpha_5$ | $\alpha_6$ |
|--------------------|------------|------------|------------|------------|------------|------------|
|                    | 0.43       | 0.61       | 0.5        | 0.55       | 0.25       | 0.82       |

Table 3. Relative error comparison (unit: TWH)

| Years | Actual value | Grey Verhulst model | APSO-GV model | APSO-GV model |
|-------|--------------|----------------------|--------------|---------------|
|       | Predictive value | Relative error/% | Predictive value | Relative error/% |
| 1     | 673.2451 | 646.6814 | 3.9456 | 657.5634 | 2.3293 |
| 2     | 685.6061 | 710.3428 | 3.6080 | 700.8558 | 2.2243 |
| 3     | 747.8054 | 731.6739 | 2.1572 | 720.4780 | 3.6543 |
| 4     | 778.7450 | 735.7936 | 5.5155 | 755.5271 | 2.9815 |
| 5     | 803.3543 | 839.4983 | 4.4991 | 824.0673 | 2.5783 |
| 6     | 805.2383 | 829.7291 | 3.0414 | 836.1659 | 3.8408 |
| 7     | 888.4031 | 920.7692 | 3.6432 | 900.8934 | 1.4059 |
| Average relative error/% | 3.7729 | 2.7163 |

Through the comparison of the results of the above average relative error and the fitting of the actual value and the predicted value, it can be seen that although both models can perform power load forecasting, the prediction accuracy of the proposed APSO-GV model is better than that of the traditional GV model. The main reason is the reconstruction of the traditional background value construction method, and the establishment of a model more in line with the development of the original data. The model in this paper improves the fitting accuracy of the grey Verhulst model, and the optimization method has high application value.
6. Conclusion
This article has the following three main tasks:

a. According to the power system load growth rule and the application limitation of the grey Verhulst model, the background value correction vector $\alpha$ is introduced to correct the background value of the GV model, which theoretically improves the accuracy of the GV model and enhances the universality of the model.

b. Because there is obvious nonlinear relationship between vector $\alpha$ and error, it is difficult to solve with conventional algorithm, and APSO has global search ability and fast convergence speed. Therefore, this paper applies APSO to solve vector $\alpha$ and proposes grey Verhulst model (APSO-GV) based on the adaptive particle swarm optimization algorithm.

c. Apply the established APSO-GV model to power system load forecasting, and verify that the model has certain feasibility and high prediction accuracy.

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