A comparison of uncertainty estimation approaches for DNN-based camera localization.

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Abstract—Camera localization, i.e., camera pose regression, represents a very important task in computer vision, since it has many practical applications, such as autonomous driving. A reliable estimation of the uncertainties in camera localization is also important, as it would allow to intercept localization failures, which would be dangerous. Even though the literature presents some uncertainty estimation methods, to the best of our knowledge their effectiveness has not been thoroughly examined. This work compares the performances of three consolidated epistemic uncertainty estimation methods: Monte Carlo Dropout (MCD), Deep Ensemble (DE), and Deep Evidential Regression (DER), in the specific context of camera localization. We exploited CMRNet, a DNN approach for multi-modal image to LiDAR map registration, by modifying its internal configuration to allow for an extensive experimental activity with the three methods on the KITTI dataset. Particularly significant has been the application of DER. We achieve accurate camera localization and a calibrated uncertainty, to the point that some method can be used for detecting localization failures.

I. INTRODUCTION

Although DNN-based techniques achieve outstanding results in camera localization [1], [2], a main challenge is still unsolved: to determine when such models are providing a reliable output, i.e., to determine the uncertainty of each inference. This is a relevant problem, since inaccurate estimates could be dangerous, e.g., in the field of autonomous driving such mistakes could endanger other road users. Therefore, being able to assign a reliable degree of uncertainty to the models’ output increases their reliability as it allows to decide when they cannot be trusted for navigation [3], i.e., allows to avoid using wrong outputs.

Uncertainty on the output of the model can be divided into two different types: aleatoric and epistemic. "Aleatoric uncertainty represents the effect on the output given by variability in the input data that cannot be modeled: this uncertainty cannot be reduced even if more data were to be collected. Epistemic uncertainty, on the other hand, quantifies the lack of knowledge of a model, which arises from the limited amount of data used for tuning its parameters. This uncertainty can be mitigated with the usage of more data.” Adapted from [4].

Camera localization DNN-based proposals that also estimate uncertainty already exist in the literature, e.g., [5], [6]. However, only partial comparisons with the consolidated approaches are available, e.g., [5] just deals with MCD.

Given the importance of uncertainty estimation for Deep Neural Network (DNN)-based camera localization, in this work we propose a comparison of three state of the art methods for uncertainty estimation in Convolutional Neural Networks (CNNs), and show that are capable of providing calibrated uncertainties, and that some of them can also be used to detect localization failures. Although the comparison is intended to be generic w.r.t. camera localization, and not specific to any particular technique, in order to actually perform the experimental activity, we had to select a specific network, and adapt it to the uncertainty estimation approaches. We chose CMRNet [7], an approach for camera localization using a camera image and an available 3D map, typically built from LiDAR data. The reason is our familiarity with the model and its implementation. Moreover, while it has been a simple work to obtain data w.r.t. MCD and DE, we consider it significant to having developed a version of CMRNet that is able to estimate uncertainty by using DER.

II. RELATED WORK

In the last decade, many DNN-based approaches for camera localization emerged. In general, existing methods can be divided into two categories: camera pose regression [1], [2], [8]–[10] and place recognition [11]–[13] techniques. Using an image, the former category predicts the pose of a camera, while the latter finds a correspondence with a previously visited location, depicted in another image.

Multi-modal approaches, which employ image and Light Detection And Ranging (LiDAR) data, propose to jointly exploit visual information and the 3D geometry of a scene to achieve higher localization accuracy [14]–[16].

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Recently, DNN-based methods emerged also for image-to-LiDAR-map registration; an example is CMRNet [7], a Convolutional Neural Network (CNN) which performs direct regression of the camera pose parameters by implicitly matching a set of scene elements depicted in an RGB image with the same elements represented in a LiDAR image, i.e., an image synthesized from the LiDAR map. Such projection is performed using an initial rough camera pose estimate, which defines the point view from which we synthesise the view of the map. CMRNet is map-agnostic.

CMRNet++ [17] is a camera-to-LiDAR map registration method inspired by CMRNet, which is also camera-agnostic, and deals with the matching problem through an optical flow DNN-based approach. The pose regression is performed by feeding the set of correspondences to a Perspective-n-Points (PnP) [18] algorithm.

Feng et al. proposed another multi-modal approach [19], where a DNN is trained to extract descriptors from the 2D and the 3D patches by defining a shared feature space between heterogeneous data. Localization is then performed by exploiting the points for which 2D-3D correspondences have been found. Similarly, Cattaneo et al. [20] proposed a DNN-based method for learning a common feature space between images and LiDAR maps in order to produce global descriptors, used for place recognition.

Although the previous multi-modal pose regression techniques achieve outstanding results, they do not estimate the epistemic uncertainty of their predictions. This is an important limitation, especially considering the final goal: to deploy them in critical scenarios, where it is important to detect when the model is likely to fail.

Epistemic uncertainty estimation in Neural Networks (NNs) is a known problem. In the last years, different methods have been proposed to sample from the model posterior [21], [22] and, more recently, to provide a direct uncertainty estimate through evidential deep learning [23]–[25]. NNs uncertainty estimation gained popularity also in the computer vision field [4], [26], and different uncertainty-aware camera-only localization approaches have been proposed. For instance, Kendall et al. [5] introduced Bayesian PoseNet, a DNN that estimates the camera pose parameters and uncertainty by approximating the model posterior by means of dropout sampling [27]. Deng et al. [6] proposed another uncertainty-aware model, which relies on Bingham mixture models for estimating a 6DoF pose from an image. Recently, Petek et al. [28] proposed an approach to camera localization that exploits an object detection module, which is used to enable localization within sparse HD maps. In particular, their method learns to estimate uncertainty of the pose of the objects by means of a DER approach [24].

Another interesting approach is HydraNet [29], that is a neural network for estimating uncertainty on quaternions.

Although the mentioned techniques deal with the problem of camera localization, they learn to localize a camera in the environment represented in the training set. In contrast, CMRNet is map-agnostic, i.e., by being able to take in input a LiDAR-map, it can perform localization also in previously unseen environments. Furthermore, to the best of our knowledge, this is the first work to implement a DER-based approach for direct camera localization.

III. METHOD

In our analysis of the literature, we could single out three more significant methods for estimating uncertainty in a DNN: MCD [27], DE [22], and DER [24]. Although they all assume that epistemic uncertainty can be described by a normal distribution, they are different techniques, and require different interventions on the network to which they are applied. As briefly introduced earlier, we believe that CMRNet is a good candidate to highlight the differences between these methods. Therefore, in this section we first introduce it, and then describe the modifications required to estimate uncertainty using each of the three different methods.

A. Introduction to CMRNet

CMRNet is a regression CNN used to estimate the 6DoF pose of a camera mounted on-board a vehicle navigating within a LiDAR map [7]. In particular, this model takes two different images as input: an RGB image and a LiDAR image obtained by synthesizing the map view from the pose given as the initial rough camera pose estimate $H_{out}$. CMRNet performs localization by implicitly matching features extracted from both images, and estimates the misalignment $H_{out}$ between the initial and the camera pose.

In particular, $H_{out}$ is computed as: $tr(1,3) = (x, y, z)$ for translations, and unit quaternion $q(1,4) = (q_x, q_y, q_z, q_w)$ for rotations. We propose to estimate its epistemic uncertainty by providing a reliability value for each pose component. The estimation of possible cross-correlations between the pose components has not been considered in this paper.

B. Uncertainty-Aware CMRNet

We define an input camera image with $I_c$, an input LiDAR image as $I_l$, a set of trained weights with $W$ and an Uncertainty Aware (UA) version of CMRNet as a function $f(I_c, I_l, W)$.

Monte Carlo Dropout: The idea behind MCD is to sample from a posterior distribution by providing different output estimates given a single input, which are later used for computing mean and variance of a Gaussian distribution. This sampling is performed by randomly deactivating the weights of the fully-connected layers by mean of a random dropout function $d(W, p)$ multiple times during model inference, where $p$ represents the dropout probability. Therefore, for MCD there is no modification of the network architecture.

We applied the dropout to the regression part of the original CMRNet architecture. When many correlations between RGB and LiDAR features are found, we expect to obtain similar samples, despite the dropout application, that is, we expect our model to be more confident w.r.t. its predictions. For each pose parameter $\mu$, we compute the predicted value
and the corresponding epistemic uncertainty as follows:

\[
\begin{align*}
\mathbb{E}[\mu_i] &= \frac{1}{n} \sum_{j=1}^{n} f(I_c, I_l, W_j), \\
\text{Var}[\mu_i] &= \frac{1}{n} \sum_{j=1}^{n} (f(I_c, I_l, W_j) - \mathbb{E}[\mu_i])^2
\end{align*}
\]  

(1)

where \( n \) is the number of samples drawn for a given input. Please note that \( \mathbb{E}[\mu_i] \) and \( \text{Var}[\mu_i] \), for the orientation, are computed after the conversion from unit quaternion to Euler angles.

**Deep Ensemble**: DE-based approaches perform posterior sampling by exploiting different models trained using different initialization of the weights, but sharing the same architecture.

Using different parameterizations of the same model leads to the recognition of a wider range of data-patterns, and to an increment of the overall accuracy [30]. On the other hand, when receiving in input patterns not well-represented in the training set, all the Neural Network (NN)s in the ensemble would give out low-quality results, so leading to an increment of variance. In our case, we expect to obtain large epistemic uncertainty when each model identifies a different set of correspondences between RGB and LiDAR features, leading to significant different pose estimates.

By training CMRNet \( n \) times with different random initializations, we obtain a set of weights \( W_{set} = \{W_1, ..., W_n\} \), which describe different local minima of the model function \( f(\cdot) \). For each pose parameter \( \mu_i \), we compute the predicted expected value and the corresponding epistemic uncertainty as follows:

\[
\begin{align*}
\mathbb{E}[\mu_i] &= \frac{1}{n} \sum_{j=1}^{n} f(I_c, I_l, W_j), \\
\text{Var}[\mu_i] &= \frac{1}{n} \sum_{j=1}^{n} (f(I_c, I_l, W_j) - \mathbb{E}[\mu_i])^2
\end{align*}
\]  

(2)

where \( n \) represents the number of models of the ensemble. In this case too, \( \mathbb{E}[\mu_i] \) and \( \text{Var}[\mu_i] \) of rotations are computed after the conversion from unit quaternion to Euler angles.

**Deep Evidential Regression**: While adapting to MCD and DE methods does not require particular modifications of CMRNet, the technique proposed by Amini et al. [24] requires substantial changes both in the training procedure and in the final part of the architecture.

In Deep Evidential Regression, the main goal is to estimate the parameters of a Normal Inverse Gamma distribution \( NIG(\gamma, \nu, \alpha, \beta) \).

A neural network is trained to estimate the NIG parameters, which are then used to compute the expected value and the corresponding epistemic uncertainty, for each pose parameter:

\[
\begin{align*}
\mathbb{E}[\mu] &= \gamma, \\
\text{Var}[\mu] &= \frac{\beta}{\nu(\alpha - 1)}
\end{align*}
\]  

(3)

To train the model, the authors propose to exploit the Negative Log Likelihood \( \mathcal{L}^{\text{NLL}} \) and the Regularization \( \mathcal{L}^R \) loss functions, in order to maximize and regularize evidence:

\[
\mathcal{L}(W) = \mathcal{L}^{\text{NLL}}(W) + \lambda \cdot \mathcal{L}^R(W)
\]  

(4)

\[
\mathcal{L}^{\text{NLL}}(\gamma, \nu, \alpha, \beta) = -\log p(y|m) \quad \text{and} \quad \mathcal{L}^R = \Phi \cdot |y - \gamma|
\]  

(5)

where \( \Phi = 2\nu + \alpha \) is the amount of evidence, see [24] for details, and \( \lambda \) represents a manually-set parameter that affects the scale of uncertainty, \( p(y|m) \) represents the likelihood of the NIG. Note that, \( p(y|m) \) is a pdf that follows a t-Student distribution \( \text{St}(\gamma, \frac{\beta(1+\nu)}{\nu \alpha}, \nu) \) evaluated w.r.t. a target \( y \).

For a complete description of loss functions and theoretical aspects of DER, please refer to the work of Amini et al. [24].

To integrate DER within CMRNet, we need to deal with the following issues: how to apply DER for regressing multiple parameters, how to manage rotations, and how to aggregate the results when computing the final loss.
We changed the last FC-layers, which predict the rotation \( q_{(4,1)} = (q_x, q_y, q_z, q_w) \) and translation \( t_{(1,3)} = (x, y, z) \) components, in order to estimate the NIG distributions associated to each pose parameter. As it can be seen in Fig. 2, we modified CMRNet to regress euler angles instead of quaternions, then we changed the FC-layers to produce the matrices \( eul, t_{(4,3)} \), where each column \( [\gamma, \nu, \alpha, \beta]^T \) represents a specific NIG [24].

Since the original CMRNet model represents rotations using unit quaternions \( q_{(4,1)} \), we cannot compute the \( \mathcal{L}^{\text{NLL}} \) and \( \mathcal{L}^{\text{R}} \) loss functions directly, as addition and multiplication have a different behaviour on the \( S^3 \) manifold. As mentioned above, we modified the last FC-layer of CMRNet to directly estimate euler angles \( eul, t_{(1,3)} = (r, p, y) \). We also substitute the quaternion distance-based loss used in [7] with the smooth \( \mathcal{L}_1 \) loss [31], which will be later used also in \( \mathcal{L}^{\text{R}} \) and \( \mathcal{L}^{\text{D}} \), by also considering the discontinuities of euler angles. Although the euler angles representation is not optimal, it allows for an easier management of the training procedure and enables a direct comprehension of uncertainty for rotational components. As we will demonstrate in Sec. IV, this change does not produce a significant decrease of accuracy.

Since CMRNet performs multiple regressions, it is necessary to establish an aggregation rule for the \( \mathcal{L}^{\text{NLL}} \) and \( \mathcal{L}^{\text{R}} \) loss functions, which are computed for each predicted pose parameter. With the application of the original loss as in [24] we experienced unsatisfactory results. We under the impression that, in our task, \( \mathcal{L}^{\text{NLL}} \) presents an undesirable behaviour: since the negative logarithm function is calculated in the pdf of the t-Student distribution. Therefore, we decided to stop the training, change all models from scratch for a total of 400 epochs, by fixing a learning rate of 1e-4, by using the ADAM optimizer and a batch size of 24 on a single NVidia GTX1080ti. The code was implemented with the PyTorch library [32]. Concerning the DE models, random weights initialization was performed by defining a random seed before each training. For DER we initially fixed the scaling parameters \( (s_{rot}, s_{tr}, \lambda_{rot}, \lambda_{tr}) = (1, 1, 0.01, 0.1) \) and \( (s_{rot}^{\text{evd}}, s_{tr}^{\text{evd}}) = (0.1, 0.1) \). However, we experienced an increment of \( \mathcal{L}^{\text{evd}} \) after approximately 150 epochs. Therefore, we decided to stop the training, change \( (s_{rot}^{\text{evd}}, s_{tr}^{\text{evd}}) = (0.005, 0.005) \), and then proceed with the training. This modification mitigated overfitting. Deactivating \( \mathcal{L}^{\text{evd}} \) during the second training step led to uncalibrated uncertainties.

\[ \mathcal{L}^{\text{evd}} = \mathcal{L}^{\text{D}} + \lambda \mathcal{L}^{\text{R}} \] (7)

We noticed that the localization accuracy was decreasing, when employing only \( \mathcal{L}^{\text{evd}} \) during training. Therefore, we opted to also employ the original geometric loss function \( \mathcal{L}^{\text{G}} \) used in [7], and to employ the smooth \( L1 \) loss on rotations as geometric loss \( \mathcal{L}^{\text{rot}} \).

The overall loss is therefore computed as follows:

\[ \mathcal{L}_{\text{rot}} = \mathcal{L}_{\text{rot}}^{\text{G}} + s_{\text{rot}} \cdot \mathcal{L}_{\text{rot}}^{\text{evd}} \]
\[ \mathcal{L}_{\text{evd}} = \mathcal{L}_{\text{evd}}^{\text{G}} + s_{\text{evd}} \cdot \mathcal{L}_{\text{evd}} \]
\[ \mathcal{L}_{\text{final}} = s_{\text{rot}} \cdot \mathcal{L}_{\text{rot}} + s_{\text{tr}} \cdot \mathcal{L}_{\text{tr}} \] (8) (9)

where the \( s \) hyper-parameters represent scaling factors.

C. Training Details

For all the three methods (i.e., MCD, DE, DER), we followed a similar training procedure as in [7]. We trained all models from scratch for a total of 400 epochs, by fixing a learning rate of 1e-4, by using the ADAM optimizer and a batch size of 24 on a single NVidia GTX1080ti. The code was implemented with the PyTorch library [32]. Concerning the DE models, random weights initialization was performed by defining a random seed before each training. For DER we initially fixed the scaling parameters \( (s_{rot}, s_{tr}, \lambda_{rot}, \lambda_{tr}) = (1, 1, 0.01, 0.1) \) and \( (s_{rot}^{\text{evd}}, s_{tr}^{\text{evd}}) = (0.1, 0.1) \). However, we experienced an increment of \( \mathcal{L}^{\text{evd}} \) after approximately 150 epochs. Therefore, we decided to stop the training, change \( (s_{rot}^{\text{evd}}, s_{tr}^{\text{evd}}) = (0.005, 0.005) \), and then proceed with the training. This modification mitigated overfitting. Deactivating \( \mathcal{L}^{\text{evd}} \) during the second training step led to uncalibrated uncertainties.
This section presents the experimental activity performed to compare the three uncertainty estimation methods with CMRNet. Moreover, we demonstrate how DE and DER can be exploited for a setting up a diagnostic test for detecting un-reliable camera localization outcomes.

### A. Dataset

We used the KITTI odometry dataset [33] to train and validate our models, following the same experimental setting proposed in [7]. In particular, we used image and LiDAR data from KITTI sequences 03 to 09, and sequence 00 for the assessment of the estimated-uncertainty quality. We exploited the ground truth poses provided by [34] to create accurate LiDAR maps. To simulate the errors on the initial rough pose estimate, we added uniformly distributed noise both on translation \([-2m;+2m]\) and rotation components \([-10^\circ;+10^\circ]\). Please refer to [7] for more details about the preprocessing operations.

### B. Evaluation metrics

We evaluated the proposed methods by comparing both localization and uncertainty calibration accuracies. In particular, we assessed the localization by measuring the euclidean distance between the ground truth and the estimated translation/rotation components. Differently from the original approach, we did not use any iterative refinement of the final pose, that is, we did not exploit a sequence of CMR Nets (trained on increasingly reduced errors on the initial pose) to improve the accuracy; this has been done in order to provide a fair comparison. However, it would be possible to apply the same incremental refinement strategy to all methods. Furthermore, we verified the accuracy of the estimated uncertainty by using the calibration curves proposed by Kuleshov et al. [35], for each predicted pose component. This procedure allows us to reveal whether the trained model produces inflated or underestimated uncertainties, by comparing the observed and the ideal confidence level. From a technical perspective, a well-calibrated uncertainty-estimation curve should follow a \(y = x\) trend.

### C. Localization evaluation

Our experimental activities encompass the evaluation of the localization performances using all the methods presented in Section 3B, with respect to the original CMRNet proposal. With regard to CMRNet + MCD, we applied the dropout to the FC layers with a probability of 0.3 and obtained the approximated posterior by exploiting 30 samples. Our extensive experimental activity proves this setting provides the best trade-off between accuracy and uncertainty calibration, by considering also the computational costs of dropout sampling. We implemented a similar approach to identify the suitable number of networks as regards the CMRNet + DE approach. Here we identified the best performances in using 5 networks, not noticing any performance gain by adding more models to the ensemble.

Table I shows the results obtained, together with the statistics on the distribution of the initial pose in the first line. As it can be seen, CMRNet + MCD achieved results comparable with the original CMRNet, resulting the worst method among those evaluated. On the other hand, CMRNet + DER achieves slightly worse performances, compared to the original CMRNet implementation. However, some applications would appreciate the benefits that such an approach provides: a direct estimate of epistemic uncertainty, \(i.e., a\) reduced computational time and space required for inference, because of the absence of sampling (from the same NN for MCD, from the NNs in the ensemble for DE). Lastly, CMRNet + DE outperforms all the other approaches in terms of accuracy, at the expense of having to train, and execute during inference, \(n\) different networks, so reducing the standard deviation of the error, as expected from ensemble-based method.

Since we removed all dynamic objects (\(e.g., cars and pedestrians\)) from within the LiDAR maps, in order to mimic real-life usage, we had some mismatches between the RGB image and the LiDAR image. Therefore, we could observe a decrease of the overall accuracy \(w.r.t.\) to the work of Cattaneo et al. as the task is more difficult, since CMRNet has now to also implicitly learn how to discard incorrect matches.

### D. Uncertainty Calibration

To measure the quality of the uncertainty estimates, we used the calibration curves proposed by Kuleshov et al. [35].

### Table I: Localization Results

| Method                  | Translation Error (m) | Rotation Error (deg) |
|-------------------------|-----------------------|----------------------|
|                         | median | mean/std | median | mean/std |
| Initial Error           | 1.88   | 1.82 ± 0.56 | 9.8    | 9.6 ± 2.8 |
| CMRNet (no iter)        | 0.51   | 0.64 ± 0.46 | 1.3    | 1.6 ± 1.2 |
| CMRNet + MCD            | 0.58   | 0.68 ± 0.44 | 1.8    | 2.1 ± 1.3 |
| CMRNet + DE             | 0.47   | 0.57 ± 0.39 | 1.2    | 1.5 ± 1.1 |
| CMRNet + DER            | 0.54   | 0.65 ± 0.46 | 1.8    | 2.1 ± 1.4 |

Fig. 4: Qualitative comparison between original CMRNet and our uncertainty aware models on a slice of the kitti 00 run. While original CMRNet provides inaccurate estimates in proximity of the depicted curve, CMRNet + DE and CMRNet + DER are able to identify localization failures and finally to discard them.

IV. EXPERIMENTAL RESULTS

This section presents the experimental activity performed to compare the three uncertainty estimation methods with CMRNet. Moreover, we demonstrate how DE and DER can be exploited for a setting up a diagnostic test for detecting un-reliable camera localization outcomes.

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### D. Uncertainty Calibration

To measure the quality of the uncertainty estimates, we used the calibration curves proposed by Kuleshov et al. [35].
In Table II we report the mean calibration errors obtained for the translation and rotation component, measured as the distance between the ideal (i.e., $y = x$) and the observed calibration for each confidence interval. Furthermore, in Fig. [3] we show the calibration curves of the three relevant pose parameters when moving in 2D: $x$ and $y$ translation and yaw.

All three methods obtain good uncertainty calibration, but CMRNet + DE shows a better performance in terms of mean calibration errors, which are less than 0.1 for each pose component.

### E. Inaccurate Predictions Detection

Besides offering realistic uncertainty estimates, an uncertainty aware model should assign a larger uncertainty to an inaccurate localization prediction. An example of application of these estimates could be establishing whether our approach output may or may not be used by some higher level algorithm. In this line, a verification could consist in determining whether we see a decreasing error with decreasingly smaller uncertainty estimates [24]. To ensure that our model provides larger uncertainties in presence of inaccurate predictions, we propose a threshold-based strategy. For both translation and rotation we compute the trace of the covariance matrix, and compare them w.r.t. a threshold that allows to discard predictions with large uncertainty. Rather than deciding an arbitrary value for the thresholds, we use the value at the top 15% of the traces of the entire validation set, respectively for translation and rotation. The prediction is therefore discarded when both the trace of the covariance of the translation and of the covariance of the rotation are larger than their threshold.

In Table III we report the translation and rotation errors, together with the percentage of discarded predictions from a total of 4541 frames. As it can be seen, with CMRNet + DE we are able to detect inaccurate estimates and to improve the overall accuracy. With CMRNet + DER we obtain a large localization improvement, outperforming the original model. Furthermore, CMRNet + DER discards less predictions than the other methods, which means that it is able to produce more consistent uncertainties w.r.t. the different pose components. Although CMRNet + MCD provides good uncertainty calibration, this model is not able to produce uncertainty estimates that increase with the prediction accuracy. In fact, we obtain the same localization results reported in Table I even though CMRNet + MCD is the method that discards the largest amount of samples.

Another advantage of CMRNet + DE and CMRNet + DER is shown in Fig. [4] Each plot represents the same piece of path (125 frames) of the KITTI 00 run; in this curve all methods show large localization errors. However, by exploiting DE and DER we are able detect most localization failures. This is an interesting property, since both DE and DER can also be exploited as a tool to discover in which scenes the DNN is likely to fail, even for datasets without an accurate pose ground truth.

### CONCLUSIONS

We proposed a comparison of state of the art methods for uncertainty estimation in DNNs for camera localization. In particular, we considered two sampling-based methods, i.e., MCD and DE [22], [27], and a direct uncertainty estimation approach named DER [24].

To evaluate these methods, we proposed to integrate them within CMRNet [7], which performs map-agnostic localization by matching a camera observation with a LiDAR map, achieving high accuracy and ensuring robustness and scalability to new environments. To the best of our knowledge, this is the first work which integrates a DER-based approach within a DNN for direct camera pose regression.

We tested the proposed approaches on the KITTI dataset, evaluating localization accuracy and uncertainty calibration. Furthermore, we also assessed the relationship between the increase of accuracy and the decrease of the estimated uncertainty.

Although CMRNet + MCD showed good localization accuracy and uncertainty calibration, we could observe it not being able to guarantee that, in presence of large uncertainty, we also obtain large errors. Instead, for CMRNet + DE, we could observe an increase in the overall localization accuracy altogether with a decrease of the variance in the error distribution. CMRNet + DER also provides calibrated uncertainty, which can be used to remove inaccurate predictions. CMRNet + DER provides comparable results to the original model in terms of localization, and also enable the possibility of removing uncertain predictions to improve the final results. A notable advantage of CMRNet + DE w.r.t. sampling-based methods is its one-shot estimation of the uncertainty.

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