Spectra of Free Diquark in the Bethe-Salpeter Approach

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Abstract:
In this work, we employ the Bethe-Salpeter (B-S) equation to investigate the spectra of free diquarks and their B-S wave functions. We find that the B-S approach can be consistently applied to study the diquarks with two heavy quarks or one heavy and one light quarks, but for two light-quark systems, the results are not reliable. There are a few free parameters in the whole scenario which can only be fixed phenomenologically. Thus, to determine them, one has to study baryons which are composed of quarks and diquarks.

1 Introduction

The subject of diquarks has attracted attentions of theorists of high energy physics for decades. The reason is obvious. Since baryons are composed of three valence quarks, the three-body system is much more complicated than mesons which are two-body systems of quark and antiquark. If the diquark picture is applied, namely two of the constituent quarks constitute a color-anti-triplet sub-system, the three-body problem can be reduced to a much simpler two-body system. However, one would ask whether QCD, which is responsible for interaction between quarks or antiquarks, favors such diquark structure of two constituent quarks.

Recently, the topic on diquarks revives because it may bring up some direct phenomenological consequences. At relativistic heavy ion collisions (RHIC) with high temper-
ature and density, diquark production and even diquark condensation become hot topics [1]. Some authors suggest that pentaquarks are of a diquark-diquark-anti-quark structure [2, 3, 4, 5], and Zou et al. proposed that there is a pentaquark component in nucleons [6]. All the subjects concern a dynamics which results in the substantial diquark structure.

In 1964 Gell-Mann first proposed feasibility of the diquark structure [7], then Ida, Kabayashi [8], Tassie et al. [9] applied the concept to study baryons. Later many authors carefully analyzed the quantum numbers based on the group theory [10, 11, 12]. Anselmino [13] et al. indicated that two heavy quarks or one heavy and one light quarks may constitute stable scalar or axial vector diquark of color anti-triplet [3, 4]. However, for the two light quarks, it is not very clear if a substantial diquark can exist.

In this work we employ the Bethe-Salpeter (B-S) equation to study the spectra of free diquarks which are composed of two heavy quarks or one-heavy-one-light quarks, we also try to extend this approach to study the diquarks of two light quarks. Our results show that just as for the light pseudoscalar pions, this framework does not work well, unless one considers extra contributions to quark masses. Dai et al. [14] used the method to study the pion structure and recently, Wang et al. [15, 16] employed the same approach to study the diquarks which contain only two light quarks. We will come back to this issue in the last section. While numerically solving the B-S equation, we have employed the method developed by Chang et al. [17], which is proved to be powerful and efficient in the numerical computations.

This work is organized as follows. After this short introduction, we derive our formulations. In Sec.III, we present the numerical results, and then in the last section, we briefly discuss our results.

2 Formulation

In this work, we use the B-S equations to study the diquark structure. First we construct the Green’s function of the diquark in the 4 × 4 matrix form and derive the corresponding B-S equation.

2.1 Structures of diquarks

One can define the B-S wave function of a diquark as [18, 5]

$$\chi^k_P(x_1, x_2) = \frac{\epsilon_{ijk}}{\sqrt{6}} < 0|T(\psi_i(x_1)\bar{\psi}_j(x_2))|D > = e^{-iP \cdot X} \chi^k_P(x),$$  \hspace{1cm} (1)

where \(i, j, k\) are color indices which will be omitted later without misunderstanding. A Fourier transformation brings it into the momentum space as

$$\chi^k_P(x) = \frac{1}{(2\pi)^4} \int d^4q e^{iq \cdot x} \chi^k_P(p),$$  \hspace{1cm} (2)

where

$$\psi^c(x) = C\psi^T, C = i\gamma_0\gamma_2, X = \alpha_1 x_1 + \alpha_2 x_2, x = x_1 - x_2,$$
\[ \alpha_1 = \frac{m_1}{m_1 + m_2}, \alpha_2 = \frac{m_2}{m_1 + m_2}, P = p_1 + p_2, q = \alpha_2 p_1 - \alpha_1 p_2. \quad (3) \]

Thus the B-S wavefunction in the momentum space reads

\[ (\not{p}_1 - m_1)\chi_P(q)(\not{p}_2 + m_2) = \int d^4k \nabla(P, q; k)\chi_P(k), \quad (4) \]

where \( \nabla(P, q; k) \) is the kernel. Although the B-S equation has the same form as that for mesons, the kernels for the two cases are different. For the diquarks there are no annihilation diagrams which exist for meson case. In fact, the definition of the diquark B-S wavefunction automatically eliminates annihilation diagrams\(^1\). Greiner [19] indicates that the B-S equations for quark-quark system (diquark) and for quark-antiquark system (meson) have a formal symmetry.

According to the method given in Ref. [20], one can obtain an equation group which contains four independent equations.

\[ (M - \omega_1 P - \omega_2 P)\varphi_{P}^{\pm}(q_{P\perp}) = \Lambda_{1P}^{\pm}(q_{P\perp})\eta_P(q_{P\perp})\Lambda_{2P}^{\pm}(q_{P\perp}), \quad (5) \]
\[ (M + \omega_1 P + \omega_2 P)\varphi_{P}^{-}(q_{P\perp}) = -\Lambda_{1P}^{-}(q_{P\perp})\eta_P(q_{P\perp})\Lambda_{2P}^{-}(q_{P\perp}), \quad (6) \]

and

\[ \varphi_{P}^{\pm}(q_{P\perp}) = 0, \quad (7) \]
\[ \varphi_{P}^{-}(q_{P\perp}) = 0. \quad (8) \]

The B-S wavefunctions are normalized as following:

\[ \int \frac{d^3q_{P\perp}}{(2\pi)^3} \left| \varphi_{P}^{\pm}(q_{P\perp}) \right|^2 = 2M. \quad (9) \]

The kernel for the B-S equation for diquark has been widely discussed by many authors\(^2\)

\[ I(r) = V_s(r) + V_0 + \gamma^\mu \otimes \gamma_\mu V_v(r) = \beta \lambda r + V_0 - \gamma^\mu \otimes \gamma_\mu \frac{2\alpha_s}{3} \frac{e^{-\alpha r}}{r}. \quad (10) \]

To avoid the infrared divergence, one can introduce a convergence factor \( e^{-\alpha r} \). Thus the potentials are respectively\(^2\)

\[ V_s(r) = \frac{\beta \lambda}{\alpha}(1 - e^{-\alpha r}), \quad (11) \]
\[ V_v(r) = -\frac{2\alpha_s}{3} \frac{e^{-\alpha r}}{r}. \quad (12) \]

\(^1\)The four-point Green’s function for diquark is defined as \( \langle 0|T(\bar{\psi}(x_1)\bar{\psi}(x_2)\psi(x_3)\psi(x_4))|0 \rangle \) whereas for meson it is \( \langle 0|T(\bar{\psi}(x_1)\bar{\psi}(x_2)\psi(x_3)\psi(x_4))|0 \rangle \), since \( \psi(x) \) and \( \bar{\psi}(y) \) cannot contract, the annihilation diagrams do not exist in the diquark case.

\(^2\)In fact, for the Coulomb term, there is no infrared divergence, but the linear confinement, which is in the form of \( 1/k^4 \) in the momentum space, diverges. Introduction of a converging factor is necessary and is benign for the Coulomb term.
In the momentueme space the potential reads

\[ K(q) = I \otimes IV_s(q) + \gamma^\mu \otimes \gamma_\mu V_v(q) \]  

where

\[ V_s(q) = -(\frac{2\lambda}{\alpha} + V_0)\delta^3(q) + \frac{\beta \lambda}{\pi^2 (q^2 + \alpha^2)^2} \]  

and

\[ V_v(q) = -\frac{1}{3\pi^2 (q^2 + \alpha^2)} \frac{\alpha_s(q)}{\Lambda_{QCQ}}. \]  

The running coupling constant \( \alpha_s(q) \) is

\[ \alpha_s(q) = \frac{4\pi}{9} \frac{1}{\ln(a + \frac{q^2}{\Lambda_{QCQ}})}. \]  

where \( a \) is a constant which freezes the running coupling constant at low momentum.

Because diquark exists in the color-\( \bar{3} \) state, 
\[ \langle \bar{3}|\lambda^a\lambda^b|3 \rangle = \frac{1}{2} < 0|\lambda^a\lambda^b|0 >, \]  
where \( |0 > \) is the color singlet. Thus the coefficient of the Coulomb term, which originates from the one-gluon exchange, is \(-\frac{4\alpha_s}{3}\) for diquarks, whereas it is \(-\frac{4\alpha_s}{3}\) for mesons. The linear potential comes from the non-perturbative effects of QCD, it might have different form for diquark and meson, and so far we cannot determine its exact value from the QCD theory. Thus we introduce a phenomenological parameter \( \beta \). Indeed, some authors[15] argued that for diquarks, the linear confinement might not exist at all, if so, \( \beta = 0 \). On other aspect, if the dynamics for the bound state of diquark is similar to that for mesons, one should expect \( \beta = 0.5 \), in analog to the coefficient ratio in front of the Coulomb term. We set the parameter to be free within a range of \( 0 \leq \beta \leq 1.0 \).

### 2.2 Scalar diquark system

We carry out all the computations in the center-of-mass frame of the diquark, thus \( P = (M, 0) \) where \( P \) is the total four-momentum of the diquark and \( M \) is its mass.

For \( 0^+ \) diquark, the general form of its wavefunction is

\[ \varphi(q) = \gamma_0 b_1(q) + b_2(q) + \gamma_0 b_3(q) + b_4(q). \]  

Here we define \( q_\perp \) as \((0, q)\) which is perpendicular to \( P \). Substituting the wavefunction into the equations (7) and (8), we can obtain the constraint conditions

\[ b_1(q) = \frac{q^2(m_1 + m_2)^2b_4(q)}{(\omega_1 + \omega_2)(q^2 - m_1m_2 - \omega_1\omega_2)}, \]  

and

\[ b_2(q) = \frac{q^2(m_1 + m_2)b_3(q)}{q^2 - m_1m_2 - \omega_1\omega_2}. \]  

With these constraints, the B-S wavefunction of a \( 0^+ \) diquark has the following form

\[ \varphi(q) = \varphi(q) = \frac{q^2(\omega_1 - \omega_2)^2\gamma_0 b_4(q)}{(m_1 - m_2)(q^2 - m_1m_2 - \omega_1\omega_2)} + \frac{q^2(m_1 + m_2)b_3(q)}{q^2 - m_1m_2 - \omega_1\omega_2} + \gamma_0 b_3(q) + \gamma_0 \gamma_\perp b_4(q). \]
Substituting them into eqs. (5) and (6), we obtain an equation group for the component functions:

\[
(M - \omega_1 - \omega_2) \frac{2q^2((q^2 - m_1m_2 - \omega_1\omega_2)b_4(q) - (m_2\omega_1 + m_1\omega_2)b_5(q))}{q^2 - m_1m_2 - \omega_1\omega_2} = -\int \frac{d^3k}{(\omega_1\omega_2)(k^2 - m_1m_2 - \omega_1k\omega_2k)} \left\{ b_3(k)((V_s + 4V_v)(m_1 + m_2)(\omega_1 + \omega_2)q^2k^2 \\
+ (V_s - 2V_v)(m_1\omega_2 + m_2\omega_1)(k^2 - m_1m_2 - \omega_1k\omega_2k)q \cdot k) \right\} + b_4(k)((V_s - 2V_v)(\omega_1k - \omega_2k)q^2k^2 + V_s(q^2 + m_1m_2 + \omega_1\omega_2)(k^2 - m_1m_2 - \omega_1k\omega_2k)q \cdot k) \right\},
\]

(21)

\[
(M + \omega_1 + \omega_2) \frac{2q^2((q^2 - m_1m_2 - \omega_1\omega_2)b_4(q) + (m_2\omega_1 + m_1\omega_2)b_5(q))}{q^2 - m_1m_2 - \omega_1\omega_2} = \int \frac{d^3q}{(\omega_1\omega_2)(k^2 - m_1m_2 - \omega_1k\omega_2k)} \left\{ -b_3(k)((V_s + 4V_v)(m_1 + m_2)(\omega_1 + \omega_2)q^2k^2 \\
+ (V_s - 2V_v)(m_1\omega_2 + m_2\omega_1)(k^2 - m_1m_2 - \omega_1k\omega_2k)q \cdot k) \right\} + b_4(k)((V_s - 2V_v)(\omega_1k - \omega_2k)q^2k^2 + V_s(q^2 + m_1m_2 + \omega_1\omega_2)(k^2 - m_1m_2 - \omega_1k\omega_2k)q \cdot k) \right\}.
\]

(22)

The normalization of the component functions is set as

\[
\int \frac{d^3q}{(2\pi)^3} \frac{16\omega_1\omega_2(m_1 + m_2)b_3(q)b_4(q)}{-q^2 + m_1m_2 + \omega_1\omega_2} = 2M.
\]

(23)

### 2.3 Axial Vector diquark system

The general form of the wavefunction of 1+ diquark reads as

\[
\varphi(q) = \left\{ q_\perp \cdot \epsilon^\lambda_1 [f_1(q) + \gamma_0 f_2(q) + \frac{\gamma_0 f_3(q)}{M} + \frac{\gamma_0 \gamma_1 f_4(q)}{M}] \\
+ M f_5(q) + M f_6(q) + \gamma_0 f_7(q) + i\epsilon^{ijk} q_{\perp i} \gamma_j \gamma_k f_8(q) \right\} \gamma_5.
\]

(24)

The constraint conditions are

\[
f_2(q) = \frac{\omega_2 - \omega_1}{\omega_1 + \omega_2} f_6(q) + \frac{q^2 - m_1m_2 - \omega_1\omega_2}{M(m_1 + m_2)} f_4(q), \quad f_6(q) = \frac{m_1\omega_2 - m_2\omega_1}{M(\omega_1 + \omega_2)} f_8(q),
\]

\[
f_3(q) = \frac{M[2M\omega_1 f_5(q) + (m_2\omega_1 - m_1\omega_2)f_1(q)]}{q^2(\omega_1 + \omega_2)}, \quad f_7(q) = -\frac{M(m_1\omega_2 + m_2\omega_1)}{q^2(\omega_1 + \omega_2)} f_5(q).
\]

(25)

With these conditions the B-S wavefunction of 1+ diquark can be further written as

\[
\varphi(q) = \{ q_\perp \cdot \epsilon^\lambda_1 [f_1(q) + \gamma_0 \frac{\omega_2 - \omega_1}{\omega_1 + \omega_2} f_8(q) + \gamma_0 \frac{q^2 - m_1m_2 - \omega_1\omega_2}{M(m_1 + m_2)} f_4(q) \}
\]

(26)
The coupled equations are

\[
\frac{\dot{q}}{M} \left[ 2M \omega_1 f_5(q) + (m_2 \omega_1 - m_1 \omega_2) f_1(q) \right] + \frac{\gamma_0 \dot{q}}{M} f_4(q) = \]
\[+ M f_1^i \gamma_0 \frac{q^2 + m_1 m_2 - \omega_1 m_2}{M(m_2 - m_1)} f_3(q) + \gamma_0 f_3(q) + e^{\mu j k} \epsilon^i \gamma_5 f_3(q) \}
\]

\[\gamma_5. \]

(26)

The coupled equations are

\[2(M - \omega_1 - \omega_2) \frac{M q^2(\omega_1 + \omega_2)}{\omega_1 + \omega_2} f_1(q) - M^2 (m_1 \omega_2 + m_2 \omega_2) f_5(q) \]
\[- (q^2 + m_1 m_2 + \omega_1 \omega_2) q^2 f_4(q) + M (m_1 - m_2) q^2 f_6(q) \]
\[= \int \frac{d^3q}{k^2 \omega_1 \omega_2 (\omega_1 + \omega_2)} \{ f_1(q) M [(m_1 - m_2)(V_s + 2V_e)(m_1 \omega_2 - m_2 \omega_1)(q \cdot k)^2 \]
\[- (V_s - 4V_e)(q^2 + m_1 m_2 + \omega_1 \omega_2)(\omega_1 + \omega_2) k^2 q \cdot k \]
\[+ f_4(q) k^2 [(V_s + 2V_e)(m_1 \omega_2 + m_2 \omega_1)(m_1 \omega_2 + m_2 \omega_1)(q \cdot k - V_s(\omega_1 + \omega_2)(\omega_1 + \omega_2)(q \cdot k)^2) \]
\[+ f_5(q) M^2 [(m_1 - m_2)(V_s + 2V_e)(q^2 k^2 (\omega_1 + \omega_2) - 2 \omega_1 (q \cdot k)^2) \]
\[+ (V_s - 4V_e)(q^2 + m_1 m_2 + \omega_1 \omega_2)(m_1 \omega_2 + m_2 \omega_1)(q \cdot k) \]
\[+ f_6(q) M [(V_s(\omega_1 + \omega_2)(m_1 \omega_2 - m_2 \omega_1)(q^2 k^2 + (V_s + 2V_e)(m_1 \omega_2 + m_2 \omega_1)(\omega_1 - \omega_2) k^2 q \cdot k)] \}
\]

(27)

\[2(M + \omega_1 + \omega_2) \frac{M q^2(\omega_1 + \omega_2)}{\omega_1 + \omega_2} f_1(q) - M^2 (m_1 \omega_2 + m_2 \omega_2) f_5(q) \]
\[+ (q^2 + m_1 m_2 + \omega_1 \omega_2) q^2 f_4(q) + M (m_1 - m_2) q^2 f_6(q) \]
\[= \int \frac{d^3q}{k^2 \omega_1 \omega_2 (\omega_1 + \omega_2)} \{ - f_1(q) M [(m_1 - m_2)(V_s + 2V_e)(m_1 \omega_2 - m_2 \omega_1)(q \cdot k)^2 \]
\[- (V_s - 4V_e)(q^2 + m_1 m_2 + \omega_1 \omega_2)(\omega_1 + \omega_2) k^2 q \cdot k \]
\[+ f_4(q) k^2 [(V_s + 2V_e)(m_1 \omega_2 + m_2 \omega_1)(m_1 \omega_2 + m_2 \omega_1)(q \cdot k - V_s(\omega_1 + \omega_2)(\omega_1 + \omega_2)(q \cdot k)^2) \]
\[- f_5(q) M^2 [(m_1 - m_2)(V_s + 2V_e)(q^2 k^2 (\omega_1 + \omega_2) - 2 \omega_1 (q \cdot k)^2) \]
\[+ (V_s - 4V_e)(q^2 + m_1 m_2 + \omega_1 \omega_2)(m_1 \omega_2 + m_2 \omega_1)(q \cdot k) \]
\[+ f_6(q) M [(V_s(\omega_1 + \omega_2)(m_2 \omega_2 - m_1 \omega_2)(q^2 k^2 + (V_s + 2V_e)(m_1 \omega_2 + m_2 \omega_1)(\omega_1 - \omega_2) k^2 q \cdot k)] \}
\]

(28)

\[4(M - \omega_1 - \omega_2) \frac{M^2 (q^2 + m_1 m_2 + \omega_1 \omega_2) f_5(q) - M (\omega_1 + \omega_2) q^2 f_6(q)}{\omega_1 + \omega_2} \]
\[= \int \frac{d^3q}{k^2 \omega_1 \omega_2 (\omega_1 + \omega_2)} \{ - f_1(q) M [(V_s + 2V_e)(\omega_1 + \omega_2)(m_2 \omega_1 - m_1 \omega_2) q^2 k^2 - (q \cdot k)^2] \]

(29)
\[\begin{align*}
+f_4(q)k^2(m_1 - m_2)V_s(\omega_{1k} + \omega_{2k})[(q \cdot k)^2 - q^2k^2] \\
+2M^2f_5(q)[(V_s + 2V_t)(\omega_1 + \omega_2)/(\omega_{2k}q^2k^2 + \omega_{1k}(q \cdot k)^2) - V_s(m_1\omega_2 + m_2\omega_1)(m_1\omega_{1k} + m_2\omega_{2k})q \cdot k] \\
+2Mk^2f_8(q)[(V_s - 2V_t)(q^2 + m_1m_2 + \omega_1\omega_2)(\omega_{1k} + \omega_{2k})q \cdot k - V_s(m_1 - m_2)(m_1\omega_{2k} - m_2\omega_{1k})q^2].
\end{align*}\]

(29)

\[\begin{align*}
4(M + \omega_1 + \omega_2) &= M^2(q^2 + m_1m_2 + \omega_1\omega_2)f_5(q) - M(\omega_1 + \omega_2)q^2f_8(q) \\
&= \int \frac{d^3q}{4\pi^2} \left\{ -f_1(q)M(V_s + 2V_t)(\omega_1 + \omega_2)(m_2\omega_{1k} - m_1\omega_{2k})[q^2k^2 - (q \cdot k)^2] \\
&- f_4(q)k^2(m_1 - m_2)V_s(\omega_{1k} + \omega_{2k})[(q \cdot k)^2 - q^2k^2] \\
&+ 2M^2f_5(q)[(V_s + 2V_t)(\omega_1 + \omega_2)/(\omega_{2k}q^2k^2 + \omega_{1k}(q \cdot k)^2) - V_s(m_1\omega_2 + m_2\omega_1)(m_1\omega_{1k} + m_2\omega_{2k})q \cdot k] \\
&- 2Mk^2f_8(q)[(V_s - 2V_t)(q^2 + m_1m_2 + \omega_1\omega_2)(\omega_{1k} + \omega_{2k})q \cdot k - V_s(m_1 - m_2)(m_1\omega_{2k} - m_2\omega_{1k})q^2].
\end{align*}\]

(30)

The normalization is set as

\[\int \frac{d^3q}{(2\pi)^3M(\omega_1 + \omega_2)} \{ \omega_1\omega_2(q^2 + m_1m_2 + \omega_1\omega_2)f_1(q)f_4(q) \\
- \omega_1\omega_2(m_1\omega_2 + m_2\omega_1)Mf_4(q)f_5(q) + \omega_1\omega_2(m_1\omega_2 - m_2\omega_1)f_1(q)f_8(q) \\
+ M^2(m_2^2\omega_1 - m_1m_2\omega_1 - m_1m_2\omega_2 + \omega_1\omega_2^2 - \omega^2q^2)f_5(q)f_8(q) \} = 2M.\]

3 Numerical results and discussions

In our numerical computations, the input parameters[21] are \(m_u = 0.305\) GeV, \(m_d = 0.311\) GeV, \(m_s = 0.487\) GeV, \(m_c = 1.7553\) GeV, \(m_b = 5.224\) GeV, \(\lambda = 0.20\) GeV, \(\Lambda_{QCD} = 0.26\) GeV, \(\alpha = 2.7182\), and \(\alpha = 0.06\) GeV.

Besides the spectra of different free diquark states, we also evaluate their mean square-root radius which is defined as

\[\sqrt{<r^2>} = \sqrt{<r_x^2>} + <r_y^2> + <r_z^2> >,\]

(31)

with

\[<r_x^2> = \frac{\int d^3q Tr[<\varphi(q)| - \frac{\partial^2}{\partial q^2} |\varphi(q)>]}{\int d^3q Tr[<\varphi(q)|<\varphi(q)>]].\]

(32)

Our numerical results are tabulated below. In the following tables, we present the results corresponding to different values of \(\beta\) and the vacuum expectation value \(V_0\) (or the zero-point value).
| V_0(GeV) | 0.0 | -0.3 | -0.6 |
| --- | --- | --- | --- |
| | mass | msr | mass | msr | mass | msr |
| (ud)_0^+ | 0.6032 | 3.5287 | 0.3216 | 11.3828 | - | - |
| (us)_0^+ | 0.7839 | 3.6089 | 0.4962 | 11.3616 | 0.1982 | 11.3802 |
| (ds)_0^+ | 0.7898 | 3.6330 | 0.5021 | 11.3597 | 0.2041 | 11.3790 |
| (uc)_0^+ | 2.057 | 5.2610 | 1.763 | 11.3336 | 1.465 | 11.3652 |
| (dc)_0^+ | 2.063 | 5.5278 | 1.769 | 11.3293 | 1.471 | 11.3639 |
| (sc)_0^+ | 2.230 | 2.8423 | 1.943 | 10.0198 | 1.644 | 11.2610 |
| (ub)_0^+ | 5.527 | 5.2038 | 5.232 | 11.3260 | 4.933 | 11.3615 |
| (db)_0^+ | 5.532 | 6.4324 | 5.238 | 11.3217 | 4.939 | 11.3594 |
| (bs)_0^+ | 5.698 | 2.7435 | 5.410 | 8.8897 | 5.114 | 10.8857 |
| (bc)_0^+ | 6.918 | 1.1725 | 6.627 | 1.3093 | 6.334 | 1.4472 |
| (uu)_1^+ | 0.6127 | 11.2596 | 0.3156 | 11.3421 | - | - |
| (ud)_1^+ | 0.6186 | 11.2483 | 0.3215 | 11.3416 | - | - |
| (dd)_1^+ | 0.6246 | 11.2458 | 0.3275 | 11.3411 | - | - |
| (us)_1^+ | 0.7940 | 11.2386 | 0.4962 | 11.2577 | 0.1980 | 11.0937 |
| (dc)_1^+ | 0.8000 | 11.0985 | 0.5021 | 10.5140 | 0.2038 | 11.2139 |
| (ss)_1^+ | 0.9725 | 5.5968 | 0.6767 | 11.2590 | 0.3778 | 11.2756 |
| (uc)_1^+ | 2.060 | 5.4201 | 1.763 | 4.0055 | 1.465 | 3.8727 |
| (dc)_1^+ | 2.066 | 5.2305 | 1.769 | 3.9799 | 1.471 | 3.8273 |
| (sc)_1^+ | 2.232 | 2.9041 | 1.942 | 7.9565 | 1.644 | 3.5679 |
| (cc)_1^+ | 3.460 | 1.3865 | 3.169 | 1.5507 | 2.876 | 1.7133 |
| (ub)_1^+ | 5.528 | 4.3950 | 5.232 | 11.2767 | 4.933 | 4.8772 |
| (db)_1^+ | 5.534 | 4.2068 | 5.238 | 5.0016 | 4.939 | 4.7222 |
| (bs)_1^+ | 5.700 | 2.4584 | 5.411 | 4.9070 | 5.113 | 11.1654 |
| (bc)_1^+ | 6.912 | 1.1162 | 6.621 | 1.2287 | 6.328 | 1.3427 |
| (bb)_1^+ | 10.33 | 0.7254 | 10.04 | 0.7500 | 9.740 | 0.7742 |

Table 1: diquark mass(GeV) and mean square-root radius (fm) with $\beta = 0.0$
Table 2: diquark mass(GeV) and mean square root radius (fm) with $\beta = 0.25$
| $V_0$(GeV) | 0.0 | -0.3 | -0.6 |
|------------|-----|------|------|
|            | mass | msr  | mass | msr  | mass | msr  |
| (ud)$_{0+}$ | 0.7904 | 1.3035 | 0.7911 | 2.0982 | 0.7105 | 1.7727 |
| (us)$_{0+}$ | 1.028 | 2.3651 | 0.9731 | 1.5194 | 0.8582 | 1.5598 |
| (ds)$_{0+}$ | 1.036 | 2.2344 | 0.9789 | 1.5046 | 0.8623 | 1.5515 |
| (uc)$_{0+}$ | 2.394 | 0.9782 | 2.251 | 1.0897 | 2.086 | 1.2343 |
| (dc)$_{0+}$ | 2.400 | 0.9725 | 2.255 | 1.0838 | 2.088 | 1.2277 |
| (sc)$_{0+}$ | 2.575 | 0.8603 | 2.393 | 0.9666 | 2.192 | 1.0854 |
| (ub)$_{0+}$ | 5.888 | 0.9047 | 5.728 | 1.0192 | 5.553 | 1.1626 |
| (db)$_{0+}$ | 5.894 | 0.8995 | 5.733 | 1.0130 | 5.556 | 1.1560 |
| (bs)$_{0+}$ | 6.056 | 0.7977 | 5.859 | 0.8979 | 5.648 | 1.0088 |
| (bc)$_{0+}$ | 7.215 | 0.5893 | 6.946 | 0.6250 | 6.675 | 0.6574 |
| (ua)$_{1+}$ | 0.9900 | 0.4521 | 0.8590 | 1.3209 | 0.6549 | 1.2716 |
| (ud)$_{1+}$ | 0.9973 | 0.4806 | 0.8644 | 1.3176 | 0.6596 | 1.2697 |
| (dd)$_{1+}$ | 1.004 | 0.5075 | 0.8697 | 1.3115 | 0.6642 | 1.2664 |
| (us)$_{1+}$ | 1.192 | 0.7153 | 1.0222 | 1.2223 | 0.8099 | 1.2278 |
| (ds)$_{1+}$ | 1.199 | 0.7244 | 1.027 | 1.2185 | 0.8139 | 1.2239 |
| (ss)$_{1+}$ | 1.375 | 1.0575 | 1.178 | 1.1206 | 0.9470 | 1.1405 |
| (uc)$_{1+}$ | 2.428 | 0.9021 | 2.252 | 1.0265 | 2.064 | 1.1565 |
| (dc)$_{1+}$ | 2.434 | 0.9030 | 2.257 | 1.0251 | 2.067 | 1.1527 |
| (sc)$_{1+}$ | 2.603 | 0.8764 | 2.391 | 0.9605 | 2.168 | 1.0456 |
| (cc)$_{1+}$ | 3.789 | 0.6819 | 3.520 | 0.7152 | 3.247 | 0.7451 |
| (ub)$_{1+}$ | 5.884 | 0.7958 | 5.717 | 0.9110 | 5.535 | 1.0536 |
| (db)$_{1+}$ | 5.890 | 0.7996 | 5.721 | 0.9186 | 5.539 | 1.0403 |
| (bs)$_{1+}$ | 6.056 | 0.8078 | 5.852 | 0.8995 | 5.636 | 1.0172 |
| (bc)$_{1+}$ | 7.206 | 0.5962 | 6.933 | 0.6259 | 6.658 | 0.6527 |
| (bb)$_{1+}$ | 10.56 | 0.4669 | 10.28 | 0.4765 | 9.985 | 0.4854 |

Table 3: diquark mass(GeV) and mean square root radius (fm) with $\beta = 0.5$
| $V_0$(GeV) | 0.0  | -0.3 | -0.6 |
|-----------|------|------|------|
|           | mass | msr | mass | msr | mass | msr |
| $(ud)_{0^+}$ | -    | -    | 0.4383 | 9.1040 | 0.6124 | 1.7486 |
| $(us)_{0^+}$ | 1.069 | 9.1714 | 1.059 | 1.6103 | 0.9856 | 1.4060 |
| $(ds)_{0^+}$ | 1.083 | 3.2217 | 1.065 | 1.5805 | 0.9904 | 1.3955 |
| $(uc)_{0^+}$ | 2.485 | 0.8955 | 2.364 | 0.9580 | 2.223 | 1.0424 |
| $(dc)_{0^+}$ | 2.492 | 0.8894 | 2.368 | 0.9524 | 2.226 | 1.0372 |
| $(sc)_{0^+}$ | 2.677 | 0.7734 | 2.516 | 0.8427 | 2.337 | 0.9227 |
| $(ub)_{0^+}$ | 5.987 | 0.8029 | 5.846 | 0.8744 | 5.692 | 0.9620 |
| $(db)_{0^+}$ | 5.993 | 0.7978 | 5.850 | 0.8692 | 5.695 | 0.9565 |
| $(bs)_{0^+}$ | 6.163 | 0.7012 | 5.984 | 0.7688 | 5.792 | 0.8451 |
| $(bc)_{0^+}$ | 7.324 | 0.5236 | 7.063 | 0.4804 | 6.797 | 0.5805 |
| $(uu)_{1^+}$ | -    | -    | 0.3729 | 10.0319 | 0.8123 | 1.1460 |
| $(ud)_{1^+}$ | -    | -    | 0.4387 | 9.9951 | 0.8172 | 1.1435 |
| $(dd)_{1^+}$ | -    | -    | 0.4961 | 9.9615 | 0.8220 | 1.1391 |
| $(us)_{1^+}$ | 1.069 | 10.0515 | 1.143 | 1.0528 | 0.9662 | 1.0805 |
| $(ds)_{1^+}$ | 1.099 | 10.0266 | 1.149 | 1.0528 | 0.9707 | 1.0776 |
| $(ss)_{1^+}$ | 1.481 | 0.9391 | 1.311 | 0.9839 | 1.108 | 1.0013 |
| $(uc)_{1^+}$ | 2.525 | 0.7164 | 2.366 | 0.8233 | 2.195 | 0.9183 |
| $(dc)_{1^+}$ | 2.532 | 0.7193 | 2.371 | 0.8246 | 2.199 | 0.9182 |
| $(sc)_{1^+}$ | 2.714 | 0.7469 | 2.518 | 0.8146 | 2.310 | 0.8776 |
| $(cc)_{1^+}$ | 3.909 | 0.6086 | 3.648 | 0.6360 | 3.381 | 0.6597 |
| $(ub)_{1^+}$ | 5.975 | 0.6235 | 5.823 | 0.7122 | 5.662 | 0.7935 |
| $(db)_{1^+}$ | 5.980 | 0.6426 | 5.828 | 0.7176 | 5.666 | 0.7935 |
| $(bs)_{1^+}$ | 6.158 | 0.6628 | 5.970 | 0.7165 | 5.771 | 0.8260 |
| $(bc)_{1^+}$ | 7.315 | 0.5350 | 7.048 | 0.5573 | 6.778 | 0.5812 |
| $(bb)_{1^+}$ | 10.66 | 0.4299 | 10.37 | 0.4373 | 10.08 | 0.4445 |

Table 4: diquark mass(GeV) and mean square root radius (fm) with $\beta = 0.75$
| $V_0$(GeV) | 0.0 | -0.3 | -0.6 |
|-----------|------|------|------|
|           | mass | msr  | mass | msr  | mass | msr  |
| $(ud)_{0^+}$ | -    | -    | -    | -    | -    | -    |
| $(us)_{0^+}$ | -    | -    | -    | -    | 0.7448 | 9.8071 |
| $(ds)_{0^+}$ | -    | -    | -    | -    | 0.7903 | 9.8484 |
| $(uc)_{0^+}$ | 2.561 | 0.8500 | 2.454 | 0.8873 | 2.331 | 0.9410 |
| $(dc)_{0^+}$ | 2.568 | 0.8435 | 2.460 | 0.8814 | 2.335 | 0.9359 |
| $(sc)_{0^+}$ | 2.762 | 0.7208 | 2.617 | 0.7705 | 2.454 | 0.8285 |
| $(ub)_{0^+}$ | 6.070 | 0.7409 | 5.942 | 0.7918 | 5.803 | 0.8526 |
| $(db)_{0^+}$ | 6.077 | 0.7359 | 5.946 | 0.7869 | 5.806 | 0.8481 |
| $(bs)_{0^+}$ | 6.254 | 0.6418 | 6.087 | 0.6919 | 5.910 | 0.7490 |
| $(bc)_{0^+}$ | 7.417 | 0.4771 | 7.153 | 0.4883 | 6.906 | 0.5295 |
| $(uu)_{1^+}$ | -    | -    | -    | -    | -    | -    |
| $(ud)_{1^+}$ | -    | -    | -    | -    | -    | -    |
| $(dd)_{1^+}$ | -    | -    | -    | -    | -    | -    |
| $(us)_{1^+}$ | -    | -    | -    | -    | 0.750 | 10.5665 |
| $(ds)_{1^+}$ | -    | -    | -    | -    | 0.7905 | 10.6108 |
| $(ss)_{1^+}$ | 0.9872 | 10.4853 | 1.385 | 10.8219 | 1.236 | 0.9149 |
| $(uc)_{1^+}$ | 2.606 | 0.5984 | 2.458 | 0.6957 | 2.300 | 0.7763 |
| $(dc)_{1^+}$ | 2.613 | 0.6016 | 2.464 | 0.6991 | 2.304 | 0.7776 |
| $(sc)_{1^+}$ | 2.807 | 0.6531 | 2.623 | 0.7178 | 2.427 | 0.7731 |
| $(cc)_{1^+}$ | 4.015 | 0.5604 | 3.760 | 0.5831 | 3.500 | 0.6037 |
| $(ub)_{1^+}$ | 6.048 | 0.5339 | 5.909 | 0.5870 | 5.760 | 0.6630 |
| $(db)_{1^+}$ | 6.056 | 0.5290 | 5.914 | 0.5896 | 5.765 | 0.6567 |
| $(bs)_{1^+}$ | 6.244 | 0.5782 | 6.067 | 0.6251 | 5.881 | 0.6807 |
| $(bc)_{1^+}$ | 7.414 | 0.4878 | 7.151 | 0.5127 | 6.886 | 0.5403 |
| $(bb)_{1^+}$ | 10.75 | 0.4052 | 10.46 | 0.4104 | 10.18 | 0.4166 |

Table 5: diquark mass(GeV) and mean square root radius (fm) with $\beta = 1.0$

In the approach, we take the parameters which are obtained by fitting data for mesons [20, 21], then the only parameters which can be adjusted are $\beta$ and $V_0$. There are several remarks to make.

1. The masses of $0^+$ and $1^+$ diquarks which are composed of two heavy quarks or one-heavy-one-light quarks tend to be degenerate. This is understandable in the heavy quark effective theory (HQET) [22] and the results are consistent with that given in refs [4, 23].

2. With the same $\beta$ value, the estimated masses of diquarks increase as $V_0$ is larger.

3. The masses of diquarks increase with increment of the $\beta$ value.

It is also observed from the numerical results, that for lighter diquarks the axial diquark is 200 MeV heavier than the corresponding scalar diquark which is consistent with [24] and the lattice calculations [23]. As indicated, our approach might be suspicious for dealing with light diquarks, however, consistency of the numerical results with that obtained in
other approaches indicates its limited plausibility. We will discuss this issue in next section.

4 Discussion and conclusion

In this paper we systematically construct the wavefunctions of the scalar and axial vector diquarks and then numerically evaluate their spectra.

There are several free parameters which cannot be determined so far, because the diquarks are not experimentally measurable. If one tries to fix the parameters, he has to deal with the diquarks which reside in baryons. However in that case, the diquarks are no longer free. As the first step, in our work, we investigate the free diquarks which cannot independently exist in real world, and in our later works, we will take into account the effects due to existence of the extra quark in baryon. We employ the Bethe-Salpeter theory to study their spectra and some characteristics. One of the free parameters is $\beta$ which signifies the difference for the non-perturbative QCD confinement effect between diquark and meson and from a naive consideration, it should be within a range $0 \leq \beta \leq 1.0$. Our numerical results confirm that as $\beta > 0.75$, there is no solution. Another important parameter is $V_0$ which stands as the zero-point energy (in the momentum space, it has a form of $\propto V_0 \delta^3(q)$). In the case of mesons, we know well that such zero-point energy must be introduced to meet the data. In the case of diquark, it plays even more important role as it severely determines the characteristics of the two-quark system.

Moreover, as we apply the B-S approach to evaluate the diquarks of two heavy quarks or one-heavy-one-light quarks, everything works well, however, once we extend the same approach to the system of two light quarks, the solutions do not seem to be reasonable, or there are no solutions at all as the parameters take certain values. It is not surprising, because as well known, it is hard to use either the potential model or B-S equations to deal with pions which are supposed to contain $q$ and $q'$ with $q, q'$ being $u, d$ quarks. It is believed that the quark masses in this case are not simply a constant. The case about pions are carefully analyzed by Dai et al. [14] and the diquark system with two light quarks are studied by Wang et al. [15].

Indeed the diquark is not color-singlet and not a real physical state. It resides, generally, in baryons. Therefore, the estimated mass and mean square-root radius of a free diquark may be different from its practical value in baryons, because of the QCD interaction of the extra quark with this subject (diquark), if it indeed exists. That is the goal of our next work.

In the relativistic quark potential model, Ebert et al.[25] obtained the values of the masses and $< r^2 >^{1/2}$ as 3.226 GeV, 0.56 fm and 9.778 GeV, 0.37 fm for $1^3S_1$ of the axial diquarks $cc$ and $bb$ respectively, with inputs $m_c = 1.55$ GeV and $m_b = 4.88$ GeV. Comparing our results with theirs, deducing the rest mass contributions ($2m_c$ and $2m_b$) whereas we employ larger input values for $m_c$ and $m_b$ in the numerical computations, our spectra are consistent with theirs and the mean square-radius estimated in this work is about 1.2 times larger than that given in ref. [25]. Considering theoretical uncertainties, our results are qualitatively consistent with the values of ref.[25].

For lighter scalar diquarks, many authors estimated their spectra and obtained widely
diverging results. For example, the authors of [26, 27, 28] obtained $M_{(ud)_{0^+}} \leq 0.3\text{GeV}$, whereas the authors of [29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39] got $M_{(ud)_{0^+}} = 0.3 - 0.6\text{GeV}$, but $M_{(ud)_{0^+}} > 0.6\text{GeV}$ was suggested in Refs. [40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50]. For $M_{(us)_{0^+}}$, it is estimated as $0.63\text{GeV}$ [29], $0.88\text{GeV}$ [40], $0.948\text{GeV}$ [45] and $0.895\text{GeV}$ [51]. For $M_{(ud)_{1^+}}$, it is estimated as $0.614\text{GeV}$ [31], $0.806 \sim 0.95\text{GeV}$ [23, 41, 45, 51, 40], 1.05GeV $\sim 1.27\text{GeV}$ [34, 48, 47], $> 1.27\text{GeV}$ [52]. For $M_{(us)_{1^+}}$, 1.069GeV [45] and 1.5GeV [51]. For $M_{(as)_{1^+}}$, it is estimated as 1.203GeV [45] and 1.215 GeV [51]. The mass of $(cu)_{0^+}$ is given as $1.933\text{GeV}$ [53]. The various numbers indicate large theoretical uncertainties. To fix them, one needs to apply the results to baryons which is compose of diquarks and are observable physical states.

As suggested, there could be the 1/2-rule that all the corresponding values and probably $V_0$ employed for the diquarks, i.e. $\beta$ and $V_0$ should be 1/2 of the values for mesons. This rule follows the assumptions that the governing interaction is QCD, thus both short-distance (the Coulomb) and the long-distance (the confinement) are proportional to the expectation value of the Casimir operator $< \lambda^a \lambda^a >$ where $\lambda^a$ is the SU(3) Gell-Mann matrix. If so, the set of results with $\beta = 0.5$ should be more reasonable. However, the situation may be more complicated. Thus in this work, we keep it as an adjustable free parameter. As we indicated above, the diquark is not a physical state, so that to fix the parameter we need to put them into the baryons, or may be in the future, as hoped, in the high-energy heavy ion collisions, free diquark might be directly produced, and then we can have more information about its structure.

Indeed, until we know how the diquark interacts as a whole object with gluons, we cannot more reliably estimate the baryon case and evaluate the production rate and decay width of the baryon in the quark-diquark picture. Therefore, in our coming work, we are going to derive all the form factors at the effective interaction vertices as the diquarks are treated as a whole object.

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