Describing Hot and Dense Nuclear Matter with Gauged Linear $\sigma$-Model

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Abstract

To describe nuclear matter at high temperature and high baryon density appropriate for RHIC and LHC, an effective theory is proposed. Three developments underlie the effective theory: (1) relativistic mean field theory description of nuclear matter with mesons mediating interactions; (2) topological soliton description of the nucleon with hidden local symmetry; (3) phenomenological knowledge of nucleon-nucleon interaction and nucleon structure obtained from elastic NN scattering at c.m.energies of hundreds of GeV. When these developments are combined together, a gauged linear $\sigma$-model with anomalous action and condensed quark-antiquark ground state emerges as the effective theory.

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I. INTRODUCTION

A new and exciting frontier of physics is opening before us with the advent of the Relativistic Heavy Ion Collider (RHIC) at Brookhaven and the Large Hadron Collider (LHC) at CERN.\(^1\) At RHIC, a 20 TeV gold nucleus will hit head-on another 20 TeV gold nucleus, and the center of mass (c.m.) energy of two colliding nucleons will be about 200 GeV. This is far from the energy regime where nucleon-nucleon interaction has been extensively studied. Also, at RHIC temperatures of the order of 200 MeV and nuclear densities about ten times the normal nuclear density are expected to be reached. At LHC in lead-lead collision, the c.m. energy of two colliding nucleons will be 5.4 TeV — much greater than that at RHIC. Clearly, we have to confront questions such as these:

1. What kind of theoretical framework do we envisage to describe this new realm of physics, where nuclear matter reaches temperatures and densities comparable to that of the early universe and the interior of neutron stars?

2. How do we formulate the anticipated phase transition in heavy-ion collisions from the hadronic regime to the perturbative QCD regime?

Against this backdrop, I want to discuss an effective theory for describing nuclear matter at high density and high temperature. This effective theory emerges when the following three developments in three distinct but related areas are combined together:

1. Relativistic mean field theory (RMFT) based on quantum hadrodynamics (QHD) models can describe realistically nuclear many-body effects and properties of finite nuclei and can provide a suitable framework to study the density and temperature dependence of nuclear matter. This conclusion has been reached by Serot and Walecka after an extensive review of research in this field.\(^2\) We note that a QHD model is a field theory model based on an effective local Lagrangian density, that has hadronic degrees of freedom such as \(\pi, \sigma, \omega, \rho\) and nucleons as basic fermions.\(^3\)

2. Extensive studies by various groups have shown that the nucleon can be described as a topological soliton in the gauged nonlinear \(\sigma\)-model (NL\(\sigma\)M) that has \(\pi, \rho, \omega\) as the
dynamical degrees of freedom and that predicts successfully the low energy properties of the nucleon down to distances of the order of 0.5 F.\textsuperscript{4,5}

(3) Phenomenological analysis of high energy elastic $pp$ and $\bar{p}p$ scattering in the c.m. energy range $\sqrt{s} = 23 - 630$ GeV \textsuperscript{6} shows strong evidence in favor of the topological soliton model of the nucleon. The analysis, however, also indicates that the soliton is embedded in a condensed ground state of quarks and antiquarks.\textsuperscript{7,8} The $q\bar{q}$ ground state is analogous to a superconducting ground state and provides an outer cloud of the nucleon, which is responsible for diffraction scattering.

As we will see in the following sections, the effective theory we arrive at is the gauged linear $\sigma$-model with hidden symmetry and spontaneous breakdown of chiral symmetry. We note that effective theories — based on the Nambu-Jona-Lasinio model,\textsuperscript{9} the Lee-Wick-Friedberg model,\textsuperscript{10} the meson-quark-dilaton model \textsuperscript{11} of the nucleon— have been applied by various groups to investigate the density and temperature dependence of nuclear matter and chiral phase transition.\textsuperscript{12-14} Our approach brings in a new element to such investigations; namely, the phenomenological knowledge of nucleon-nucleon interaction and nucleon structure obtained from the analysis of high energy elastic scattering at NN c.m. energies of hundreds of GeV — an energy regime relevant to RHIC and LHC.

\section*{II. A GENERAL LAGRANGIAN FRAMEWORK}

To see how the three developments mentioned in the Introduction can be combined together, we first lay out a general Lagrangian framework. We begin with the linear $\sigma$-model of Gell-Mann and Levy that has SU(2)$_L \times$ SU(2)$_R$ global symmetry and spontaneous breakdown of chiral symmetry. The model is given by the Lagrangian density

\begin{equation}
\mathcal{L} = \bar{\psi} i \gamma^\mu \partial_\mu \psi + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \bar{\pi} \cdot \partial^\mu \bar{\pi}) - G \bar{\psi} \left[ \sigma + i \bar{\pi} \cdot \gamma^5 \bar{\pi} \right] \psi - \lambda \left( \sigma^2 + \bar{\pi}^2 - f_\pi^2 \right)^2.
\end{equation}

We next introduce a scalar-isoscalar field $\zeta(x)$ and a unitary field $U(x)$ in the following way:
\[
\sigma(x) + i \vec{\tau} \cdot \vec{\pi}(x) = \zeta(x) U(x). \tag{2}
\]

\(\zeta(x)\) is the magnitude of the fields \(\sigma(x)\) and \(\vec{\pi}(x)\): 
\(\zeta^2(x) = \sigma^2(x) + \vec{\pi}^2(x)\). \(U(x)\) is given by
\(U(x) = \exp \left(i \vec{\tau} \cdot \vec{\phi}(x) / f_\pi \right)\); here, \(\vec{\phi}(x)\) is the isovector pion field, and \(f_\pi\) is the pion decay constant (\(f_\pi \simeq 93\) MeV). The pions are the Goldstone bosons generated by the spontaneous breakdown of \(SU(2)_A\) symmetry. Let us next introduce right and left fermion fields:
\[
\psi_R = \frac{1}{2} \left( 1 + \gamma^5 \right) \psi, \quad \psi_L = \frac{1}{2} \left( 1 - \gamma^5 \right) \psi.
\]

In terms of these new fields, the Lagrangian density (1) can be written as
\[
\mathcal{L} = \bar{\psi}_R i \gamma^\mu \partial_\mu \psi_R + \bar{\psi}_L i \gamma^\mu \partial_\mu \psi_L + \frac{1}{2} \partial_\mu \zeta \partial^\mu \zeta
+ \frac{1}{4} \zeta^2 \text{tr} \left[ \partial_\mu U \partial^\mu U^\dagger \right] - G \zeta \left[ \bar{\psi}_L U \psi_R + \bar{\psi}_R U^\dagger \psi_L \right] - \lambda \left( \zeta^2 - f_\pi^2 \right)^2. \tag{3}
\]

Under right and left global transformations:
\[
\psi_R \to R \psi_R, \quad \psi_L \to L \psi_L; \tag{4}
\]
\[
U(x) \to L U(x) R^\dagger. \tag{5}
\]

Eq.\(^{(3)}\) makes it evident that the Lagrangian is invariant under these transformations and that the scalar-isoscalar field \(\zeta(x)\) is chiral invariant. When \(\zeta(x)\) is replaced by its vacuum value \(f_\pi\), the Lagrangian \(\mathcal{L}\) represents a nonlinear \(\sigma\)-model (NL\(\sigma\)M).

Following Bando et al., \(^{15}\) we now introduce the idea of a hidden local symmetry. Bando et al. took this symmetry to be \([SU(2)]_{\text{hidden}}\). We consider an extended version of this approach, and following Meissner et al. \(^{16}\) take the symmetry to be \([SU(2) \times U(1)]_{\text{hidden}}\). To implement the idea of a hidden local symmetry, we write \(U(x) = \xi^\dagger_L(x) \xi_R(x)\), where \(\xi_R(x), \xi_L(x)\) are \(SU(2)\)-valued fields which transform in the following way under \([SU(2)_L \times SU(2)_R]_{\text{global}} \times [SU(2) \times U(1)]_{\text{local}}\):
\[
\xi_R(x) \to h(x) \xi_R(x) R^\dagger, \quad \xi_L(x) \to h(x) \xi_L(x) L^\dagger, \tag{6}
\]
where \(h(x) \in [SU(2) \times U(1)]_{\text{local}}\). Eq\(^{(6)}\) shows that
\[ U(x) = \xi^\dagger_L(x) \xi_R(x) \rightarrow L U(x) R^\dagger \]  

(7)
as required, so that the global symmetry is maintained.

Let us focus on the pion sector of the Lagrangian (3):

\[ \mathcal{L}_\pi = \frac{1}{4} \zeta^2 \text{tr} \left[ \partial_\mu U \partial^\mu U^\dagger \right] \]
\[ = -\frac{1}{4} \zeta^2 \text{tr} \left[ \partial_\mu \xi_L \xi^\dagger_L - \partial_\mu \xi_R \xi^\dagger_R \right]^2. \]  

(8)

We gauge this symmetry by introducing the vector mesons \( \vec{\rho} \) and \( \omega \) as gauge bosons of the [SU(2)×U(1)]hidden symmetry. Gauging is done easily by replacing the ordinary derivative by the covariant derivative: \( \partial_\mu \rightarrow D_\mu = \partial_\mu + V_\mu \), where

\[ V_\mu = -i \frac{g}{2} [\vec{\tau} \cdot \vec{\rho} + \omega_\mu], \]  

(9)

and \( V_\mu \) transforms under the hidden symmetry in the following way:

\[ V_\mu \rightarrow h V_\mu h^\dagger + h \partial_\mu h^\dagger. \]  

(10)

The Lagrangian density of the gauged hidden-symmetry model is taken as \(^{15,16}\)

\[ \mathcal{L}_\pi = -\frac{1}{4} \zeta^2 \text{tr} \left[ (D_\mu \xi_L) \xi^\dagger_L - (D_\mu \xi_R) \xi^\dagger_R \right]^2 \]
\[ -\frac{1}{2} \zeta^2 \text{tr} \left[ (D_\mu \xi_L) \xi^\dagger_L + (D_\mu \xi_R) \xi^\dagger_R \right]^2 + \frac{1}{2g^2} \text{tr} \left[ F_{\mu\nu} F^{\mu\nu} \right], \]  

(11)

where the first term is exactly the same as the original term \( \frac{1}{4} \zeta^2 \text{tr} \left[ \partial_\mu U \partial^\mu U^\dagger \right] \). The second term is a gauge invariant term that generates the masses of the vector mesons. The third term is the Lagrangian density of the gauge field \( V_\mu(x) \), and \( F_{\mu\nu} \) is the nonabelian field tensor: \( F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + [V_\mu, V_\nu] \). The last term in (11) produces a term like \( \text{tr} \left[ L_\mu, L_\nu \right]^2 \), where \( L_\mu = U^\dagger \partial_\mu U \). This is the fourth-order derivative term introduced by Skyrme.

We next focus on the fermion-scalar-meson sector of the Lagrangian density (3) and introduce new fermionic variables:

\[ \psi^0_R(x) = \xi_R(x) \psi_R(x), \quad \psi^0_L(x) = \xi_L(x) \psi_L(x). \]  

(12)
These variables are invariant under the global right and left transformations. They transform only under the local hidden symmetry:

$$
\psi_0^R(x) \rightarrow h(x) \psi_0^R(x), \quad \psi_0^L(x) \rightarrow h(x) \psi_0^L(x)
$$

(13)

$$(h(x) \epsilon [SU(2) \times U(1)]_{\text{hidden}})$$

In terms of these new variables, the fermion-scalar-meson sector of (3) becomes

$$
\mathcal{L}_{F+S} = \bar{\psi}^0 i \gamma^\mu \left[ \partial_\mu + V_\mu + \tilde{v}_\mu + \tilde{a}_\mu \gamma^5 \right] \psi^0
$$

$$
+ \frac{1}{2} \partial_\mu \zeta \partial^\mu \zeta - G \zeta \bar{\psi}^0 \psi^0 - \lambda \left( \zeta^2 - f^2 \right)^2, \quad (14)
$$

where

$$
\psi^0 = \psi_0^R + \psi_0^L
$$

(15)

$$
\tilde{v}_\mu = \frac{1}{2} \left[ \xi_R \partial_\mu \xi_R^\dagger + \xi_L \partial_\mu \xi_L^\dagger \right] - V_\mu, \quad (16)
$$

$$
\tilde{a}_\mu = \frac{1}{2} \left[ \xi_R \partial_\mu \xi_R^\dagger - \xi_L \partial_\mu \xi_L^\dagger \right]. \quad (17)
$$

Both $\tilde{v}_\mu$ and $\tilde{a}_\mu$ transform covariantly under the hidden symmetry:

$$
\tilde{v}_\mu(x) \rightarrow h(x) \tilde{v}_\mu(x) h^\dagger(x),
$$

$$
\tilde{a}_\mu(x) \rightarrow h(x) \tilde{a}_\mu(x) h^\dagger(x).
$$

Since the covariant derivative also transforms covariantly,

$$
\partial_\mu + V_\mu \rightarrow h (\partial_\mu + V_\mu) h^\dagger,
$$

the Lagrangian density $\mathcal{L}_{F+S}$ is manifestly invariant under the hidden gauge symmetry, besides being invariant under the global symmetry.
III. EMERGENCE OF AN EFFECTIVE THEORY

We observe that the Lagrangian density in Eq.(14) is very similar to the Lagrangian density of the nucleon-scalar-meson sector of QHD-II, which has $\omega$, $\rho$, $\zeta$ (scalar-isoscalar meson) as the hadronic degrees of freedom besides $\pi$.\textsuperscript{2,3} This similarity, of course, suggests that we identify $\psi^0$ as the nucleon field and allow the potential energy density of the scalar field $V(\zeta) = \lambda (\zeta^2 - f_\pi^2)^2$ to be of more general form: $V(\zeta) = a \zeta^2 + b \zeta^3 + c \zeta^4$, so as to fit the bulk properties of finite nuclei and equilibrium properties of nuclear matter. Furthermore, if we take $\xi_L^\dagger = \xi_R = \xi$, so that $U = \xi^2$, then under the hidden local transformation:

$$\psi^0_R(x) \rightarrow h(x) \psi^0_R(x), \quad \psi^0_L(x) \rightarrow h(x) \psi^0_L(x),$$

(18)

and $h(x)$ has to satisfy the requirement

$$L \xi(x) h^\dagger(x) = h(x) \xi(x) R^\dagger,$$

(19)

i.e., the requirement for nonlinear realization of the chiral symmetry.\textsuperscript{2} Eqs.(18,19) are, indeed, the requirements imposed on the nucleon field both in QHD-II and in its extended version that incorporates the trace anomaly.\textsuperscript{17} We also note that the scalar field $\zeta$ in (14), which has a chiral invariant interaction $G \bar{\psi}^0 \psi^0$, can be identified with the scalar-isoscalar field of QHD. It is not the $\sigma$-field of single-boson exchange potentials with $m_\sigma \simeq 500$ MeV. In fact, if we write $\zeta(x) = f_\pi + \hat{\zeta}(x)$, then the fluctuation field $\hat{\zeta}(x)$ is to be identified with the $\sigma$-field. As Kalafatis and Vinh Mau have shown,\textsuperscript{18} it is the fluctuation field $\hat{\zeta}(x)$ that provides the NN medium range attraction. It is evident that the Lagrangian (14) with a generalized $V(\zeta)$ incorporates all the main features of nucleon-scalar-meson sector of QHD-II.

Turning to the Lagrangian density $\mathcal{L}_\pi$ in Eq.(11), let us consider that the scalar field $\zeta(x)$ can be replaced by its vacuum value $f_\pi$. Following Meissner et al.,\textsuperscript{16} we add to it the anomalous action, i.e., the Wess-Zumino-Witten (WZW) action:

$$\mathcal{L}_{WZW} = g_\omega \omega_\mu B^\mu,$$

(20)

where $B_\mu$ is the topological baryonic current:
\[ B^\mu = \frac{1}{24 \pi^2} \epsilon_{\mu\nu\rho\sigma} \operatorname{tr} \left[ U^\dagger \partial_\nu U U^\dagger \partial_\rho U U^\dagger \partial_\sigma U \right], \] (21)

and \( U = \exp \left[ i \vec{\tau} \cdot \vec{\phi} / f_\pi \right] \). The Lagrangian density we have now: \( \mathcal{L}_\pi + \mathcal{L}_{WZW} \) represents the gauged nonlinear \( \sigma \)-model with its anomalous action. This model has been extensively studied by many groups.\(^4,5,19\) It describes the nucleon as a topological soliton and predicts successfully the low energy properties of the nucleon. Also, the masses of \( \rho \) and \( \omega \) are generated dynamically from the gauge-invariant second term in Eq.\((\Pi)\) and leads to the KSFR relation \( m_\rho^2 = m_\omega^2 = 2 f_\pi^2 g^2 \). The last term in \((\Pi)\), as we noted earlier, reproduces Skyrme’s fourth order derivative term.

The discussion above shows that the Lagrangian density

\[ \mathcal{L} = \mathcal{L}_\pi + \mathcal{L}_{F+S} + \mathcal{L}_{WZW} \] (22)

with a generalized \( \mathcal{V}(\zeta) \) encompasses both QHD-II model for nuclear matter and NL\( \sigma \)M for the nucleon. If, in the above Lagrangian, we take the fermion to be the nucleon, drop \( \mathcal{L}_{WZW} \) and replace \( \zeta(x) \) in \( \mathcal{L}_\pi \) by its vacuum value \( f_\pi \), then we essentially obtain the QHD-II model.\(^2\) On the other hand, if we keep \( \mathcal{L}_\pi \) with \( \zeta(x) = f_\pi \) and \( \mathcal{L}_{WZW} \), but drop \( \mathcal{L}_{F+S} \), then we have the gauged NL\( \sigma \)M.\(^16,19\) We now examine how analysis of high energy elastic scattering impacts on these two developments.

High energy elastic scattering at the CERN ISR and SPS Collider in the c.m. energy range \( \sqrt{s} = 23 – 630 \) GeV has been analyzed by my collaborators and me over a number of years.\(^6\) From this analysis, we arrived at the following phenomenological description. The nucleon has a core and an outer cloud. High energy elastic scattering is primarily due to two processes: 1) a glancing collision where the outer cloud of a nucleon interacts with that of the other and gives rise to diffraction scattering; 2) a hard collision where one nucleon core scatters off the other core via \( \omega \) exchange, while their outer clouds overlap and interact independently. In the small momentum transfer region diffraction dominates, but as the momentum transfer increases, the hard scattering takes over.\(^8\)

Clearly, elastic scattering shows that the nucleon is a composite object with a core and a cloud. Hence, nucleons cannot be regarded as basic fermions as in QHD-II at c.m. energies of
hundreds of GeV. Instead, the basic fermions should be taken as more fundamental objects—quarks (perhaps, more precisely, effective quarks). Also, a change of fermionic variables is carried out in QHD-II to implement the nonlinear realization of chiral symmetry (Eq.(12)). Path integral formalism shows that, because of the fermion measure, change of fermionic variables induces an anomalous action, which in this case is the WZW action. Therefore, the WZW action has to be included in the effective model. However, as we discuss below, the dynamics of elastic scattering and nucleon structure indicate that $\pi$, $\zeta$, $\omega$ are appropriate effective degrees of freedom in the high energy region just as they are in the low energy region in QHD-II. Furthermore, the nonlinear realization of chiral symmetry implemented in the Lagrangian (22) has also been found to be more appropriate to describe properties of finite nuclei, such as charge density, spin-orbit splitting, etc. Hence, a relativistic mean field theory based on the Lagrangian (22) with quarks as the basic fermions should be a realistic framework for nuclear matter.

With regard to the gauged NL$\sigma$M, elastic scattering analysis supports the conclusion that the nucleon is a topological soliton. In Fig.1, the pion profile function calculated from high energy elastic scattering is compared with the pion profile functions obtained by Meissner et al. from low energy considerations. The profile functions are quite compatible even though they come from two totally different domains of physics. The gauged NL$\sigma$M, however, completely ignores the scalar field $\zeta$—replacing it from the very beginning by its vacuum value $f_\pi$ and neglecting the important interaction that $\zeta$ mediates between left and right quarks. The only contribution NL$\sigma$M keeps from the fermion sector is the anomalous action arising from the gauge dependence of the fermion measure. This implies that the soliton of the NL$\sigma$M lies in a noninteracting Dirac sea (Fig. 2a). If, on the other hand, we keep the scalar field $\zeta$, assume that replacing it by its vacuum value $f_\pi$ is reasonable in the pion sector, then in the fermion sector the left and right quarks still interact via the scalar field. In this case, we have a soliton that lies in an interacting Dirac sea (Fig. 2b). What one finds is that, if this scalar field has a critical behavior (by this I mean it is zero at small distances, but rises sharply at some distance $r = R$ to its vacuum value $f_\pi$ as in Fig.
2c), then the interacting Dirac sea can have considerably less energy than the noninteracting Dirac sea.\textsuperscript{7} The system, in this case, makes a phase transition to the interacting ground state and reduces its total energy substantially by the condensation energy. The phenomenon is analogous to superconductivity. It also solves a persistent problem of the NLσM; namely, its prediction of a large soliton mass ($\sim 1500$ MeV) compared to the actual mass (939 MeV) of the nucleon. Such a condensation phenomenon implies that the nucleon is a topological soliton embedded in a $q\bar{q}$ ground state, and the ground state provides an outer cloud. High energy elastic scattering analysis, as mentioned earlier, shows evidence of such a cloud that gives rise to diffraction scattering. A number of other consequences also follow from having a condensed $q\bar{q}$ ground state.\textsuperscript{8}

IV. SUMMARY AND CONCLUSION

We have discussed three independent but related developments:

(1) Relativistic mean field theory description of nuclear matter based on a QHD model with $\pi$, $\omega$, $\rho$, and $\zeta$ (scalar-isoscalar meson) as the hadronic degrees of freedom.

(2) Topological soliton description of the nucleon based on a gauged nonlinear $\sigma$-model with hidden local symmetry and anomalous action.

(3) Phenomenological knowledge of nucleon-nucleon interaction and nucleon structure obtained from analysis of high energy elastic scattering in the c.m. energy range $\sqrt{s} = 23 - 630$ GeV.

These three developments when combined together strongly suggest that the Lagrangian density

$$\mathcal{L} = \mathcal{L}_\pi + \mathcal{L}_{F+S} + \mathcal{L}_{WZW}$$

should provide a very realistic effective theory to investigate density and temperature dependence of nuclear matter over a wide range. Fermions in this effective theory are to be taken as “quarks” and not as nucleons. Furthermore, potential energy density $\mathcal{V}(\zeta)$ of the scalar
field should be taken to have a form more general than the Higgs potential in [3]. This can lead to a physical vacuum at $\zeta = f_\pi$ and a metastable or false vacuum at $\zeta = 0$.\textsuperscript{13} Since in mean field theory, meson fields are replaced by their constant vacuum expectation values, and only the fermion field is quantized, it is straightforward to obtain the thermodynamic potential $\Omega$ from the grand partition function. Quantities such as energy density, baryon density, and pressure can then be obtained from $\Omega$, and meson field equations can also be derived by extremizing it.\textsuperscript{3,14}

Investigation of high energy elastic scattering dynamics has shown that the scalar-isoscalar field $\zeta$, which is an essential element of quantum hadrodynamics models, plays the crucial role of an order parameter. Its critical behavior leads to a $q\bar{q}$ condensed ground state analogous to a superconducting ground state. Furthermore, vector meson $\omega$ behaves as a gauge boson coupled to the topological baryonic charge as in the gauged nonlinear $\sigma$-model with WZW action. Obviously, the fields $\pi$, $\zeta$, and $\omega$, which are appropriate degrees of freedom in the low energy region, also constitute appropriate degrees of freedom at NN c.m. energies of hundreds of GeV. Hence, a relativistic mean field theory based on the Lagrangian density (22) that has these degrees of freedom and quarks as basic fermions should provide a realistic framework to study nuclear matter at high densities and high temperatures—densities and temperatures that will be reached at RHIC and LHC.

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Fig. 1. The pion profile function $\theta(r)$ as a function of $r$ in fermi. The continuous curve represents $\theta(r)$ obtained from high energy elastic scattering. The dotted and the dashed curves represent $\theta(r)$ calculated from low energy in the minimal and in the complete soliton model by Meissner et. al. [Ref.19]

Noninteracting Dirac sea  Interacting Dirac sea  Critical behavior of $\zeta(r)$

Fig. 2a.  Fig. 2b.  Fig. 2c.