Abstract: The paper presents a few kinematic schemes of planar mechanisms with bars (linkages) used as part of the quadruped robot legs. The Dunshee linkage having only four elements as crank-rocker mechanism is analyzed. Further, the Klann linkage, which is accomplished by amplifying the crank-rocker mechanism with a dyadic kinematic chain, is also presented. More than that, the Jansen linkage, which is obtained by extending and amplifying the crank-rocker mechanism with two dyadic kinematic chains, is also analyzed. At the end of the paper, the authors present a novel linkage application consisting of a quadric kinematic chain.

1. General aspects

There are many websites that present a considerable number of planar mechanisms [1] in the structure of walking robots [2, 3, 4]. It is amazing how many ways of copying what animals can easily do exist, and how creative these are.

By analysing the mechanics of walking robots, we notice that the majority of robots with more than 2 legs use the pantograph type planar mechanism [1] with two mobile joints in order to walk.

We should notice that in nature, muscles / ligaments can be considered as extendible links, which corresponds to the pantograph mechanism. Pantograph mechanisms are so frequent in the structure of walking robots because they are extremely simple and versatile.

In figure 1 the trajectory of point E is determined by the horizontal motion of point A and by the vertical motion of point O. Obviously, this mechanism can be used effectively [7, 8] since it was successfully used in the Vehicle with Adaptable Suspension [8]. This operation requires two drive sources, which is regarded as a disadvantage since it increases the complexity and the energy consumption. This double mobility is common for most walking mechanisms that use pantographs.

Another disadvantage of this mechanism is that it requires a system to control the leg kinematics in order to determine the trajectory of the fulcrum. This system usually incorporates sensors for detecting the soil, and maintaining the position of the frame as to the soil, which requires a permanent control of the mechanism kinematics.
Another possibility of using a pantograph mechanism, in terms of a leg for a simple walking robot, is the one indicated in figure 2 [6, 8]. The articulated quadrangle includes the mobile elements \( A_0 A, AB, BB_0 \) and the fixed element \( A_0 B_0 \). The rocker \( BB_0 \) belongs both to the kinematic chain and to the pantograph.

The main advantage of this model is that the leg can be actuated by a single leg actuator. The double mobility pantograph is reduced to a mechanism with one degree of mobility [1], which is an advantage.

Although it is difficult to reach the efficiency of animals, it is still possible to build walking robots whose power consumption for the distance travelled is similar to that of wheeled vehicles off-road.

2. The Dunshee mechanism (quadrangle used for the quadruped walking robot)

This quadrangle mechanism has the smallest number of linkages (0, 1, 2, 3), having the possibility of changing the position of the fixed articulation \( C \) (figure 3) by means of the discreet rotation of bar \( CC_0 \) (noted with 0) around the fixed point \( C_0 \).

The interest point \( M \) is located in the plane of the reciprocating rod \( 2(AB) \), in the lower side, by means of the BM segment and the angle \( \alpha_2 \).

Crank \( 1(A_0 A) \) is joined with a gear (with the centre in \( A_0 \)) actuated by a central pinion with the centre in \( O \).

The cranks of the two quadrangle mechanisms, right (front) and left (back), are deferred by 180°, so that points \( M \) do not simultaneously touch the surface on which the walking robot moves.

If this complex mechanism is located on the right side of the mobile robot, then the same mechanism is located on the left, but the cranks are deferred accordingly.

The two central pinions are rigid on the central shaft \( O \), driven by the same electric motor, which has not been shown in the kinematic scheme (figure 3).

The mobility of the complex mechanism (figure 3) can be determined by means of the general formula [1, 2]:

\[
M_b = \sum_{m=1}^{5} mC_m - \sum_{r=2}^{6} rN_r
\]  

(1)
where: \( m \) is the mobility (the number of allowed movements) of a kinematic coupling; 
\( C_m \) is the number of class \( m \) kinematic couplings; 
\( r \) is the kinematic rank of the space associated to an independent closed kinematic contour; 
\( N_r \) is the number of independent closed kinematic contours.

![Figure 3. The Dunshee quadrangle mechanism (with a dyadic chain RRR) used for walking.](image)

By looking at the kinematic scheme (figure 3), we identify the following numerical values given as a matrix:

\[
\begin{bmatrix}
C_1 & C_2 & C_3 & C_4 & C_5 \\
N_2 & N_3 & N_4 & N_5 & N_6
\end{bmatrix} = \begin{bmatrix}
9 & 2 & 0 & 0 & 0 \\
0 & 4 & 0 & 0 & 0
\end{bmatrix}.
\]  

(2)

By replacing the data in formula (1), we obtain the mobility:

\[
M_b = (1 \times 9 + 2 \times 2) - 3 \times 4 = 1.
\]  

(3)

Using the Autodesk Inventor Professional program, we have obtained the building scheme of the Dunshee mechanism (figure 4).

![Figure 4. Building modelling of the Dunshee mechanism in two positions of the crank.](image)
3. The Klann mechanism (quadrangle amplified with a 3R dyade)

The Klann mechanism is obtained by amplifying the articulated quadrangle by means of an RRR type dyadic chain (figure 5), where the interest point M belongs to the plane of the kinematic element 4.

![Figure 5](image)

Figure 5. The Klann mechanism (with a dyadic chain), var. 1(a), var. 2(b).

The structural topological formula of this driving mechanism $MM$ (figure 5), having crank 1 as a leading element, forming the driving mechanism $MA(0,1)$, is expressed as follows:

$$MM = MA(0,1) + LD(2,3) + LD(4,5).$$

(4)

The Klann mechanism is presented in two variants (fig. 5). They are different especially because of the position of the fixed articulation $D_0$, more exactly the value of the abscissae of this point.

Using a simulation program, Invetor, we have obtained a representation of the Klann mechanism, variant 2 (figure 6).

Here are two print screen images.

![Figure 6](image)

Figure 6. The Klann mechanism var. 2 modelled in two different positions of the crank.

Using a MathCad program, the trajectory of point $M$ was inferred (figure 7 left), as well as the variation diagrams of the angular displacement of the points B, C, D and of the interest point $M$. 


Figure 7. Trajectories (left) and diagrams of the angular displacement (right) of the characteristic points B, C, D and M.

We shall notice that the trajectory of point M has been shown graphically (figure 7 left) by means of a closed reciprocating rod curve, which helps the robot to walk. The horizontal part of the curve is almost rectilinear (figure 7 left), which corresponds to the position where point M touches the horizontal plane on which the robot walks.

4. The Jansen mechanism (articulated quadrangle amplified with two 3R dyades)

This Jansen mechanism (figure 8) is obtained by amplifying the quadrangle mechanism crank - rocker A_0ABB_0(0,1,2,3), which includes a first dyadic chain LD(2,3), a second dyadic chain LD(4,5) in the same joints A (mobile) and B_0 (fixed), and then a third dyadic chain LD(6,7) in the mobile joints C and D.

\[ x_{B_0} = A_0B_0 = 21\text{mm}; \quad y_{B_0} = 0; \]
\[ A_0A = 5\text{mm}; \quad AB = 30\text{mm}; \quad BB_0 = 21\text{mm}; \]
\[ AD = 33\text{mm}; \quad B_0D = CE = 19\text{mm}; \]
\[ B_0C = DE = 20\text{mm}; \quad BC = 28.5\text{mm}; \]

Figure 8. The quadrangle Jansen mechanism (2 dyadic chains).

The structural – topological equation of this driving mechanism MM with crank 1 as driving element is expressed as follows:
\[ MM = MA(0,1) + LD(2,3) + LD(4,5) + LD(6,7). \] (5)

The mobility of the mechanism is checked using the formula (3) in which we introduce the numerical values obtained from inspecting the kinematic scheme analysed above (figure 8).

These values are included in the matrix:
\[
\begin{bmatrix}
C_1 & C_2 & C_3 & C_4 & C_5 \\
N_2 & N_3 & N_4 & N_5 & N_6
\end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\
0 & 3 & 0 & 0 & 0 \end{bmatrix}. \] (6)

Using this data in equation (1), we obtain the mobility:
\[ M_b = 1 \times 10 - 3 \times 3 = 1. \] (7)

Using the Inventor [10] program, the building scheme of the mechanism was drawn in two opposite positions of the leading crank (figure 9).

**Figure 9.** The Building scheme of the Jansen mechanism in two different positions of the crank.

5. A new walking mechanism with quadric chain

The new linkage has been proposed by the authors (figure 10), and it is obtained by connecting a complex kinematic chain of the tetradic type LTt(2,3,4,5) to the crank 1 and to the fixed element 0.

Both walking mechanisms (left and right) are actuated from a central pinion by means of the two cylindrical gears.

**Figure 10.** Linkage (with quadric chain).
The structural – topological equation of any of the two driving mechanisms is expressed as follows:

\[ MM = MA(0,1) + LTt(2,3,4,5). \]  

(8)

The mobility of the whole walking mechanism (left + right) can be checked using the formula (1) in which we replace the specific numerical parameters (figure 10):

\[
\begin{bmatrix}
C_1 & C_2 & C_3 & C_4 & C_5 \\
N_2 & N_3 & N_4 & N_5 & N_6
\end{bmatrix}
= \begin{bmatrix}
15 & 2 & 0 & 0 & 0 \\
0 & 6 & 0 & 0 & 0
\end{bmatrix}.
\]  

(9)

By replacing the numerical data from (9) in formula (1), we obtain:

\[ M_b = (1 \times 15 + 2 \times 2) - 3 \times 6 = 1. \]  

(10)

Using the Inventor [10] program, the building scheme (figure 11) of the new mechanism was developed for two positions of the leading crank (deferred by 90°).

**Figure 11.** Building scheme of the proposed tetradic chain mechanism in two positions.

6. Conclusions

Unlike the pantograph type walking mechanisms, the linkages (planar mechanisms with articulated bars) presented above as kinematic and building schemes are simpler, due to the number of kinematic elements and to the selected means of actuation. The Dunshee mechanism is the simplest kinematic chain, made up of only 4 kinematic elements of the crank – rocker type, where the contact point describes a grade 6 reciprocating rod curve. The Klann mechanism consists of 6 kinematic elements with a simple structure, using two dyadic
chains, but with a higher degree reciprocating rod curve (more than 6), which is more suitable for walking. The Jansen mechanism is made up of 8 kinematic elements with a relatively simple structure, having three dyadic chains, with a 4 degree reciprocating rod curve, which is closer to an ellipsis. The authors propose a new kinematic scheme for the walking mechanism based on 6 kinematic elements, but with a complex structure due to the existence of a quadric chain.

7. References
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