We present a nonlocal entanglement concentration scheme for reconstructing some maximally entangled multipartite states from partially entangled ones by exploiting cross-Kerr nonlinearities to distinguish the parity of two polarization photons. Compared with the entanglement concentration schemes based on two-particle collective unitary evolution, this scheme does not require the parties to know accurately information about the partially entangled states—i.e., their coefficients. Moreover, it does not require the parties to possess sophisticated single-photon detectors, which makes this protocol feasible with present techniques. By iteration of entanglement concentration processes, this scheme has a higher efficiency and yield than those with linear optical elements. All these advantages make this scheme more efficient and more convenient than others in practical applications.

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I. INTRODUCTION

Entanglement is a unique phenomenon in quantum mechanics and it plays an important role in quantum-information processing and transmission. For instance, quantum computers exploit entanglement to speed up the computation of problems in mathematics [1, 2]. The two legitimate users in quantum communication—say, the sender Alice and the receiver Bob—use an entangled quantum system to transmit a private key [3, 4, 5, 6, 7] or a secret message [8]. Also quantum teleportation [11], controlled teleportation [12], quantum dense coding [9, 10], quantum system to transmit a private key [3, 4, 5, 6, 7] or two legitimate users in quantum communication—say, quantum computers exploit entanglement to speed up the information processing and transmission. For instance, quantum mechanics and it plays an important role in quantum-information processing and transmission. For instance, where particles [9, 10], quantum system to transmit a private key [3, 4, 5, 6, 7] or two legitimate users in quantum communication—say, quantum computers exploit entanglement to speed up the information processing and transmission.

Here $H$ and $V$ represent the horizontal and vertical polarizations of photons, respectively. The Bell state $|\phi^+\rangle$ can also be degraded as a less pure entangled state like $|\Psi\rangle = |\alpha H\rangle_A |\beta V\rangle_B + |\beta H\rangle_A |\alpha V\rangle_B$, where $|\alpha|^2 + |\beta|^2 = 1$. Multiparticle entanglement states also suffer from channel noise. For instance, $|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|HH\cdots H\pm i|VV\cdots V\rangle)$ will become $|\Phi^\pm\rangle = |HH\cdots H\pm |VV\cdots V\rangle$. For three-particle quantum systems, their states with the form $|\Phi^\pm\rangle$ are called Greenberg-Horne-Zeilinger (GHZ) states. Now, the multipartite entangled states like $|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\cdots 0\rangle \pm |11\cdots 1\rangle)$ are also called multipartite GHZ states.

The method of distilling a mixed state into a maximally entangled state is named entanglement purification, which has been widely studied in recent years [15, 16, 17, 18, 19, 20, 21, 22]. Another way of distilling less entangled pure states into maximally entangled states that will be detailed here is called entanglement concentration. Several entanglement concentration protocols of pure nonmaximally entangled states have been proposed recently. The first entanglement concentration protocol was proposed by Bennett et al [23] in 1996, which is called Schmidt projection method. In their protocol [23], the two parties of quantum communication need some collective and nondestructive measurements of photons, which, however, are not easy to manipulate in experiment. Also the two parties should know accurately the coefficients $\alpha$ and $\beta$ of the partially entangled state $|\alpha|01\pm |\beta|10\rangle$ before entanglement concentration. That is, their protocol works under the condition that the two users obtain information about the coefficients and possess the collective and nondestructive measurement technique. Another similar scheme is called entanglement swapping [24, 25]. In these schemes [24, 25], two pairs of...
less entangled pairs belong to Alice and Bob. Then Alice sends one of her particles to Bob, and Bob performs a Bell-state measurement on one of his particle and Alice’s one. So Bob has to own three photons of two pairs, and they have to perform collective Bell-state measurements. Moreover, the parties should exploit a two-particle collective unitary evaluation of the quantum system and an auxiliary particle to project the partially entangled state into a maximally entangled one probabilistically.

Recently, two protocols of entanglement concentration based on a polarization beam splitter (PBS) were proposed independently by Yamamoto et al. [26] and Zhao et al. [27]. The experimental demonstration of the latter has been reported [28]. In their protocol, the parties exploit two PBSs to complete the parity-check measurement schemes of polarization photons. However, each of the two users Alice and Bob has to choose the instances in which each of the spatial modes contains exactly one photon. With current technology, sophisticated single-photon detectors are not likely to be available, which makes it such that these schemes can not be accomplished simply with linear optical elements.

Cross-Kerr nonlinearity is a powerful tool to construct a nondestructive quantum nondemolition detector (QND). It also has the function of constructing a controlled-not (CNOT) gate and a Bell-state analyzer [33]. Cross-Kerr nonlinearity was widely studied in the generation of qubits [29, 30, 31] and the discrimination of unknown optical qubits [32]. Cross-Kerr nonlinearities can be described with the Hamiltonian $H_{ck} = \hbar \chi a_s^+ a_s a_p^+ a_p$, where $a_s^+$ and $a_p^+$ are the creation operations and $a_s$ and $a_p$ are the destruction operations. If we consider a coherent beam in the state $|\alpha\rangle_p$ with a signal pulse in the Fock state $|\Psi\rangle_s = c_0|0\rangle_s + c_1|1\rangle_s$, the two photons combined to pick up a phase shift $\theta$ to if there is a photon in the mode. So the probe beam $|\alpha\rangle$ will pick up a phase shift of $\theta$ if the state is $|HH\rangle$ or $|VV\rangle$. Here $b_1$ and $b_2$ represent the up spatial mode and the down spatial mode, respectively.

![FIG. 1: The principle of our nondestructive quantum nondemolition detector (QND). Two cross-Kerr nonlinearities are used to distinguish superpositions and mixtures of $|HH\rangle$ and $|VV\rangle$ from $|HV\rangle$ and $|VH\rangle$. Each polarization beam splitter (PBS) is used to pass through $|H\rangle$ polarization photons and reflect $|V\rangle$ polarization photons. Cross-Kerr nonlinearity will cause the coherent beam to pick up a phase shift $\theta$ if there is a photon in the mode. The probe beam $|\alpha\rangle$ will pick up a phase shift of $\theta$ if the state is $|HH\rangle$ or $|VV\rangle$. Here $b_1$ and $b_2$ represent the up spatial mode and the down spatial mode, respectively.](image)

## II. ENTANGLEMENT CONCENTRATION OF PURE ENTANGLLED PHOTON PAIRS

### A. Primary entanglement concentration of less entangled photon pairs

The principle of our nondestructive QND is shown in Fig. 1. It is made up of four PBSs, two identical cross-Kerr nonlinear media, and an X homodyne measurement. If two polarization photons are initially prepared in the states $|\varphi\rangle_{b_1} = c_0|H\rangle_{b_1} + c_1|V\rangle_{b_1}$ and $|\varphi\rangle_{b_2} = d_0|H\rangle_{b_2} + d_1|V\rangle_{b_2}$, the two photons combined with a coherent beam whose initial state is $|\alpha\rangle_p$ interact with cross-Kerr nonlinearities, which will evolve the state of the composite quantum system from the original one $|\Psi\rangle_o = |\varphi\rangle_{b_1} \otimes |\varphi\rangle_{b_2} \otimes |\alpha\rangle_p$ to

$$|\Psi\rangle_T = [c_0d_0|HH\rangle + c_1d_1|VV\rangle]|\alpha e^{i\theta}\rangle_p + c_0d_1|HV\rangle|\alpha e^{i\theta}\rangle_p + c_1d_0|VH\rangle|\alpha\rangle_p. \tag{5}$$

One can find immediately that $|HH\rangle$ and $|VV\rangle$ cause the coherent beam $|\alpha\rangle_p$ to pick up a phase shift $2\theta$, and $|HV\rangle$ to pick up no phase shift. The different phase shifts can be distinguished by a general homodyne-heterodyne measurement ($X$ homodyne measurement). In this way, one can distinguish $|HH\rangle$ and $|VV\rangle$ from $|HV\rangle$ and $|VH\rangle$. This device is
also called a two-qubit polarization parity QND detector. Our QND shown in Fig.1 is a little different from the one proposed by Nemoto and Munro [33]. With the QND in [33], the $|HH\rangle$ and $|VV\rangle$ pick up no phase shift. However, it is well known that a vacuum state (zero-photon state) can also cause there to be no phase shift on the coherent beam. So one can not distinguish whether two photons or no photons pass through the two spatial modes. This modified QND can exactly check the number of photons if they have the same parity.

Suppose there are two identical photon pairs with less entanglement $a_1b_1$ and $a_2b_2$. The photons $a_1$ belong to Alice and photons $b$ to Bob. The photon pairs $a_1b_1$ and $a_2b_2$ are initially in the following unknown polarization entangled states:

$$
|\Phi\rangle_{a_1b_1} = \alpha |H\rangle_{a_1}|H\rangle_{b_1} + \beta |V\rangle_{a_1}|V\rangle_{b_1},
$$

$$
|\Phi\rangle_{a_2b_2} = \alpha |H\rangle_{a_2}|H\rangle_{b_2} + \beta |V\rangle_{a_2}|V\rangle_{b_2},
$$

where $|\alpha|^2 + |\beta|^2 = 1$. The original state of the four photons can be written as

$$
|\Psi\rangle = |\Phi\rangle_{a_1b_1} \otimes |\Phi\rangle_{a_2b_2} = \alpha^2 |H\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{a_2}|H\rangle_{b_2} + \alpha \beta |H\rangle_{a_1}|H\rangle_{b_1}|V\rangle_{a_2}|V\rangle_{b_2} + \alpha \beta |V\rangle_{a_1}|V\rangle_{b_1}|H\rangle_{a_2}|H\rangle_{b_2} + \beta^2 |V\rangle_{a_1}|V\rangle_{b_1}|V\rangle_{a_2}|V\rangle_{b_2}.
$$

After the two parties Alice and Bob rotate the polarization states of their second photons $a_2$ and $b_2$ by 90° with half-wave plates (i.e., $R_{90}$ shown in Fig.2), the state of the four photons can be written as

$$
|\Psi\rangle' = \alpha^2 |H\rangle_{a_1}|V\rangle_{a_2}|H\rangle_{b_1}|V\rangle_{b_2} + \alpha \beta |H\rangle_{a_1}|H\rangle_{a_2}|H\rangle_{b_1}|H\rangle_{b_2} + \alpha \beta |V\rangle_{a_1}|V\rangle_{a_2}|V\rangle_{b_1}|V\rangle_{b_2} + \beta^2 |V\rangle_{a_1}|H\rangle_{a_2}|V\rangle_{b_1}|H\rangle_{b_2}.
$$

Here $a_3$ ($b_3$) is used to label the photon $a_2$ ($b_2$) after the half-wave plate $R_{90}$.

From Eq. (6), one can see that the terms $|H\rangle_{a_1}|H\rangle_{a_2}|H\rangle_{b_1}|H\rangle_{b_2}$ and $|V\rangle_{a_1}|V\rangle_{a_2}|V\rangle_{b_1}|V\rangle_{b_2}$ have the same coefficient of $\alpha\beta$, but the other two terms are different. Now Bob lets the two photons $b_1$ and $b_3$ enter into the QND. With his homodyne measurement, Bob may get one of three different results: $|HH\rangle$ and $|VV\rangle$ lead to a phase shift of $\theta$ on the coherent beam, $|HV\rangle$ leads to $2\theta$, and the other is $|VH\rangle$, which leads to no phase shift. If the phase shift of homodyne measurement is $\theta$, Bob asks Alice to keep these two pairs; otherwise, both pairs are removed. After only this parity-check measurement, the state of the photons remaining becomes

$$
|\Psi''\rangle = \frac{1}{\sqrt{2}}(|H\rangle_{a_1}|H\rangle_{a_3}|H\rangle_{b_1}|H\rangle_{b_3} + |V\rangle_{a_1}|V\rangle_{a_3}|V\rangle_{b_1}|V\rangle_{b_3}).
$$

The probability that Alice and Bob get the above state is $P_{ab} = 2|\alpha\beta|^2$.

Now both pairs $a_1b_1$ and $a_3b_3$ are in the same polarizations. Alice and Bob use their $\lambda/4$-wave plates $R_{45}$ to rotate the photons $a_3$ and $b_3$ by 45°. The unitary transformation of 45° rotations can be described as

$$
|H\rangle_{a_3} \rightarrow \frac{1}{\sqrt{2}}(|H\rangle_{a_3} + |V\rangle_{a_3}),
$$

$$
|H\rangle_{b_3} \rightarrow \frac{1}{\sqrt{2}}(|H\rangle_{b_3} + |V\rangle_{b_3}),
$$

$$
|V\rangle_{a_3} \rightarrow \frac{1}{\sqrt{2}}(|H\rangle_{a_3} - |V\rangle_{a_3}),
$$

$$
|V\rangle_{b_3} \rightarrow \frac{1}{\sqrt{2}}(|H\rangle_{b_3} - |V\rangle_{b_3}).
$$

After the rotations, Eq. (9) will evolve into

$$
|\Psi'''\rangle = \frac{1}{2\sqrt{2}}(|H\rangle_{a_1}|H\rangle_{b_1} + |V\rangle_{a_1}|V\rangle_{b_1})(|H\rangle_{a_3}|H\rangle_{b_3} + |V\rangle_{a_3}|V\rangle_{b_3}) + \frac{1}{2\sqrt{2}}(|H\rangle_{a_1}|H\rangle_{b_1} - |V\rangle_{a_1}|V\rangle_{b_1})(|H\rangle_{a_3}|V\rangle_{b_3} + |V\rangle_{a_3}|H\rangle_{b_3}).
$$

The last step is to distinguish the photons $a_3$ and $b_3$ in different polarizations. Two PBSs are used to pass through $|H\rangle$ polarization photons and reflect $|V\rangle$ photons. From the Eq. (11), one can see that if the two detectors $D_1$ and $D_2$ or the two detectors $D_3$ and $D_4$ fire, the photon pair $a_1b_1$ is left in the state

$$
|\phi^+\rangle_{a_1b_1} = \frac{1}{\sqrt{2}}(|H\rangle_{a_1}|H\rangle_{b_1} + |V\rangle_{a_1}|V\rangle_{b_1}).
$$

If $D_1$ and $D_3$ or $D_2$ and $D_4$ fire, the photon pair $a_1b_1$ are left in the state

$$
|\phi^-\rangle_{a_1b_1} = \frac{1}{\sqrt{2}}(|H\rangle_{a_1}|H\rangle_{b_1} - |V\rangle_{a_1}|V\rangle_{b_1}).
$$
Both of these two states are the maximally entangled ones. In order to get the same state of $\Phi^\prime_2$, one of the two parties Alice and Bob should perform a simple local operation of phase rotation on her or his photon. The maximally entangled states are generated with above operations.

In our scheme, only one QND is used to detect the parity of the two polarization photons. If the two photons are in the same polarization $|HH\rangle$ or $|VV\rangle$, the phase shift of the coherent beam is $\theta$, which is easy to detect by the homodyne measurement. Furthermore, our scheme is not required to have sophisticated single-photon detectors, but only conventional photon detectors. This is a good feature of our scheme, compared with other schemes.

**B. Reusing resource-based entanglement concentration of partially entangled photon pairs**

With only one QND, our entanglement concentration has the same efficiency as that based on linear optics [26, 27]. The yield of maximally entangled states $Y$ is $|\alpha|^2$. Here the yield is defined as the ratio of the number of maximally entangled photon pairs, $N_m$, and the number of originally less entangled photon pairs, $N_l$. That is, the yield of our scheme discussed above is $Y_1 = \frac{N_m}{N_l} = |\alpha|^2$. In fact, $Y_1$ is not the maximal value of the yield of the entanglement concentration scheme with the QND.

In our entanglement concentration scheme above, the two parties Alice and Bob only pick up instances in which Bob gets the phase shift $\theta$ on his coherent beam and removes the other instances. In this way, the photon pairs kept are in the state $|\Psi\rangle'$. However, if Bob chooses a suitable cross-Kerr medium and controls accurately the interaction time $t$, he can make the phase shift $\theta = \chi t = \pi$. In this way, 20 and 0 represent the same phase shift 0. The two photon pairs removed by Alice and Bob in the scheme above are just in the state

$$|\Phi_1\rangle'' = \alpha^2 |H\rangle_{a_1} |V\rangle_{a_3} |H\rangle_{b_1} |V\rangle_{b_3} + \beta^2 |V\rangle_{a_1} |H\rangle_{a_3} |V\rangle_{b_1} |H\rangle_{b_3}. \tag{14}$$

This four-photon system is not in a maximally entangled state, but it can be used to get some maximally entangled state with entanglement concentration. In detail, Alice and Bob take a rotation by 90° on each photon of the second four-photon system and cause the state of this system to become

$$|\Phi_2\rangle'' = \beta^2 |H\rangle_{a_1} |V\rangle_{a_3} |H\rangle_{b_1} |V\rangle_{b_3} + \alpha^2 |V\rangle_{a_1} |H\rangle_{a_3} |V\rangle_{b_1} |H\rangle_{b_3}. \tag{15}$$

The state of the composite system composed of eight photons becomes

$$|\Phi_3\rangle'' = |\Phi_1\rangle'' \otimes |\Phi_2\rangle'' = \alpha^2 \beta^2 |H\rangle_{a_1} |V\rangle_{a_3} |H\rangle_{b_1} |V\rangle_{b_3} |H\rangle_{a_1} |V\rangle_{a_3} |H\rangle_{b_1} |V\rangle_{b_3} + \alpha^4 |H\rangle_{a_1} |V\rangle_{a_3} |H\rangle_{b_1} |V\rangle_{b_3} |H\rangle_{a_1} |V\rangle_{a_3} |H\rangle_{b_1} |V\rangle_{b_3} + \beta^4 |V\rangle_{a_1} |H\rangle_{a_3} |V\rangle_{b_1} |H\rangle_{b_3} |H\rangle_{a_1} |V\rangle_{a_3} |H\rangle_{b_1} |V\rangle_{b_3}. \tag{16}$$

For picking up the first two terms, Bob need only detect the parities of the two photons $b_1$ and $b'_3$ with the QND. As the two polarization photons $b_1$ and $b'_3$ in the first two terms have the same parity, they will cause the coherent beam $|\alpha\rangle_p$ to have a phase shift $\theta = \pi$. Those in the other two terms cause the coherent beam $|\alpha\rangle_p$ to have a phase shift 0.

When Bob gets the phase shift $\theta = \pi$, the eight photons collapse to the state

$$|\Phi_3\rangle'' = \frac{1}{2\sqrt{2}}(|H\rangle_{a_1} |V\rangle_{a_3} |H\rangle_{b_1} |V\rangle_{b_3} |H\rangle_{a_1} |V\rangle_{a_3} |H\rangle_{b_1} |V\rangle_{b_3} + |V\rangle_{a_1} |H\rangle_{a_3} |V\rangle_{b_1} |H\rangle_{b_3} |H\rangle_{a_1} |V\rangle_{a_3} |H\rangle_{b_1} |V\rangle_{b_3}. \tag{17}$$

The probability that Alice and Bob get this state is

$$P_{s_2} = \frac{2|\alpha\beta|^4}{(|\alpha|^2 + |\beta|^2)^2}. \tag{18}$$

They have the probability $P'_{s_2} = 1 - P_{s_2}$ to obtain the less entangled state

$$|\Phi_1\rangle'' = \alpha^4 |H\rangle_{a_1} |V\rangle_{a_3} |H\rangle_{b_1} |V\rangle_{b_3} |H\rangle_{a_1} |V\rangle_{a_3} |H\rangle_{b_1} |V\rangle_{b_3} + \beta^4 |V\rangle_{a_1} |H\rangle_{a_3} |V\rangle_{b_1} |H\rangle_{b_3} |H\rangle_{a_1} |V\rangle_{a_3} |H\rangle_{b_1} |V\rangle_{b_3} \tag{19}$$

which can be used to concentrate entanglement by iteration of the process discussed above. In this way, one can obtain easily the probability

$$P_{s_n} = \frac{2|\alpha\beta|^{2n}}{(|\alpha|^2 + |\beta|^2)^2}, \tag{20}$$

where $n$ is the iteration number of the entanglement concentration processes.

For the four photons in the state described by Eq. (17), Alice and Bob can obtain a maximally entangled photon pair with some single-photon measurements on the other six photons by choosing the basis $X = \{ |\pm x\rangle = \frac{1}{\sqrt{2}}(|H\rangle \pm |V\rangle) \}$. That is, Alice and Bob first rotate their polarization photons $a_3, b_1, a'_3, b'_1, a'_1$ and $b'_3$ by 45°, similar to the case discussed above (shown in Fig.2), and then measure these six photons. If the number of the antiparallel outcomes obtained by Alice and Bob is even, the photon pair $a_1b_1$ collapses to the state $|\phi^+\rangle_{a_1b_1} = \frac{1}{\sqrt{2}}(|H\rangle_{a_1} |H\rangle_{b_1} + |V\rangle_{a_1} |V\rangle_{b_1})$; otherwise the photon pair $a_3b_3$ collapses to the state $|\phi^-\rangle_{a_3b_3} = \frac{1}{\sqrt{2}}(|H\rangle_{a_3} |H\rangle_{b_3} - |V\rangle_{a_3} |V\rangle_{b_3})$.

With the iteration of the entanglement concentration process, the yield of our scheme is improved to be $Y$, i.e.,

$$Y = \sum_{i=1}^{n} Y_i. \tag{21}$$
where
\[
Y_1 = |\alpha\beta|^2, \\
Y_2 = \frac{1}{2}(1 - 2|\alpha\beta|^2)\frac{|\beta|^4}{(|\alpha|^4 + |\beta|^4)^2}, \\
Y_3 = \frac{1}{2^2}(1 - 2|\alpha\beta|^2)[1 - \frac{|\alpha\beta|^4}{(|\alpha|^4 + |\beta|^4)^2}], \\
Y_n = \frac{1}{2^{n-1}}(1 - 2|\alpha\beta|^2)\prod_{j=3}^{n-1}[1 - \frac{2|\alpha\beta|^{2j-1}}{(|\alpha|^{2^{j-1}} + |\beta|^{2^{j-1}})^2}].
\]
\[
|\alpha\beta|^{2^n} \quad (|\alpha|^{2^n} + |\beta|^{2^n})^2.
\]

The yield is shown in Fig. 3 with the change of the iteration number of entanglement concentration processes \(n\) and the coefficient \(\alpha \in [0, \sqrt{2}]\).

![Fig. 3](image)

**FIG. 3:** (Color online) The yield \((Y)\) is altered with the iteration number of entanglement concentration processes \(n\) and the coefficient \(\alpha \in [0, \sqrt{2}]\).

Certainly, Alice and Bob can also accomplish the iteration of the entanglement concentration by first measuring the two photons \(a_3\) and \(b_4\) in the state |\(\Phi_1\rangle\rangle^r\) described by Eq. \((14)\) with the basis \(X\) and then concentrating some maximally entangled states from the partially entangled quantum systems composed of the pairs \(a_1b_1\). In fact, after the measurements of the two photons with the basis \(X\), Alice and Bob can transfer the state of photon pair \(\alpha a_1b_1\) to \(\alpha a_1[H_{a_1}a_1]H_{b_1} + \beta[V_{a_1}b_1]|V_{b_1}\) with or without a unitary operation. Alice and Bob can accomplish the entanglement concentration with the same way discussed in Sec. II A.

The same as the entanglement concentration schemes with linear optical elements \([26, 27]\), the present scheme has the advantage that the two parties of quantum communication are not required to know the coefficients of the less entangled states in advance in order to reconstruct some maximally entangled states. Moreover, this scheme does not require sophisticated single-photon detectors and has a higher yield of maximally entangled states than those based on linear optical elements \([26, 27]\) as the efficiency in the latter is just \(|\alpha\beta|^2\) the probability that Alice and Bob get an Einstein-Podolsky-Rosen (EPR) pair from two partially entangled photon pairs is \(2|\alpha\beta|^2\) in Refs. \([26, 27]\). These good features make the present entanglement concentration scheme more efficient and more convenient than others in practical applications.

### III. ENTANGLEMENT CONCENTRATION OF LESS ENTANGLED MULTIPARTITE GHZ-CLASS STATES

It is straightforward to generalize our entanglement concentration scheme to reconstruct maximally entangled multipartite GHZ states from partially entangled GHZ-class states.

Suppose the partially entangled \(N\)-particle GHZ-class states are described as follows:
\[
|\Phi_1^\prime\rangle = \alpha|HH\cdots H\rangle + \beta|VV\cdots V\rangle,
\]
where \(|\alpha|^2 + |\beta|^2 = 1\). For two GHZ-class states, the composite state can be written as
\[
|\Psi^\prime\rangle = |\Phi_1^\prime\rangle_1 \otimes |\Phi_1^\prime\rangle_2 = (\alpha|HH\rangle_1|H\rangle_2\cdots |H\rangle_N
+ \beta|VV\rangle_1|V\rangle_2\cdots |V\rangle_N) \otimes
(\alpha|H\rangle_{N+1}|H\rangle_{N+2}\cdots |H\rangle_{2N}
+ \beta|V\rangle_{N+1}|V\rangle_{N+2}\cdots |V\rangle_{2N}.
\]

![Fig. 4](image)

**FIG. 4:** Schematic diagram of the multipartite entanglement concentration scheme. 2\(N\) particles in two partially entangled \(N\)-particle GHZ-class states are sent to \(N\) parties of quantum communication—say Alice, Bob, Charlie, etc. Photons 2 and \(N + 2\) are sent to Bob and enter into QND to complete a parity-check measurement. After the QND measurement, Bob asks the others to retain their photons if his two photons have the same parity (|\(HH\rangle\) or |\(VV\rangle\)) and remove them for next iteration if Bob gets an odd parity (|\(HV\rangle\) or |\(VH\rangle\)).
The principle of our entanglement concentration scheme for multipartite GHZ-class states is shown in Fig.4. 2N photons in two pairs of N-particle non-maximally entangled GHZ-class states are sent to Alice, Bob, Charlie, etc. (i.e., the N parties of quantum communication). Each party gets two photons. One comes from the state $|\Phi^+\rangle_1$ and the other comes from $|\Phi^+\rangle_2$, shown in Fig.4. Suppose Alice gets photon 1 and the photon $N+1$ and Bob gets photon 2 and photon $N+2$. Before entanglement concentration, each party rotates his second polarization photon by $90^\circ$, similar to the case for concentrating two-photon pairs. After the $90^\circ$ rotations, the state of the 2N photons becomes

$$
|\Psi^-\rangle = \alpha^2|H\rangle_1|H\rangle_2\cdots|H\rangle_N|V\rangle_{N+1}|V\rangle_{N+2}\cdots|V\rangle_{2N}
+ \alpha \beta |H\rangle_1|H\rangle_2\cdots|H\rangle_N|H\rangle_{N+1}|H\rangle_{N+2}\cdots|H\rangle_{2N}
+ \alpha \beta |V\rangle_1|V\rangle_2\cdots|V\rangle_N|V\rangle_{N+1}|V\rangle_{N+2}\cdots|V\rangle_{2N}
+ \beta^2|V\rangle_1|V\rangle_2\cdots|V\rangle_N|H\rangle_{N+1}|H\rangle_{N+2}\cdots|H\rangle_{2N}.
$$

(25)

Bob lets photons 2 and $N+2$ pass through his QND detector whose principle is shown in Fig.2. For $|HH\rangle$ and $|VV\rangle$, Bob gets the result with an $X$ homodyne measurement $\theta$; for $|HV\rangle$, the result is 20° and $|VH\rangle$ will make no phase shift. By choosing the phase shift $\theta$, Bob asks the others to retain their photons; otherwise, all the parties remove the photons. In this way, the whole state of the retained photons can be described as

$$
|\Psi^\prime\rangle = \frac{1}{\sqrt{2}}(|H\rangle_1|H\rangle_2\cdots|H\rangle_N|V\rangle_{N+1}|V\rangle_{N+2}\cdots|V\rangle_{2N}
+ |V\rangle_1|V\rangle_2\cdots|V\rangle_N|V\rangle_{N+1}|V\rangle_{N+2}\cdots|V\rangle_{2N}).
$$

(26)

The success probability is $2|\alpha \beta|^2$, the same as that for two-photon pairs $P_{s_1}$. The above state is a maximally entangled 2N-particle state. By measuring each of the photons coming from the second GHZ-class state with basis $X$, the parties will obtain a maximally entangled N-particle state, as after the photons $N+1$, $N+2$, ..., and 2N pass through the $R_{45}$ plates, which rotate the polarizations of photons by $45^\circ$, the state of the composite system becomes

$$
|\Psi^\prime\rangle = \frac{1}{\sqrt{2}}(|H\rangle_1|H\rangle_2\cdots|H\rangle_N\left(\frac{1}{\sqrt{2}}\right)^\otimes N(|H\rangle + |V\rangle)\otimes^N
+ |V\rangle_1|V\rangle_2\cdots|V\rangle_N\left(\frac{1}{\sqrt{2}}\right)^\otimes N(|H\rangle - |V\rangle)\otimes^N].
$$

(27)

By measuring the N photon with the conventional photon detectors, the N parties will obtain a maximally entangled state $|GHZ^+\rangle_{12\cdots N}$ if the number of parties who obtain a single-photon measurement outcome $|V\rangle$ is even; otherwise, they will obtain the maximally entangled state $|GHZ^-\rangle_{12\cdots N}$. Here

$$
|GHZ^+\rangle = \frac{1}{\sqrt{2}}(|H\rangle_1|H\rangle_2\cdots|H\rangle_N + |V\rangle_1|V\rangle_2\cdots|V\rangle_N),
$$

(28)

and

$$
|GHZ^-\rangle = \frac{1}{\sqrt{2}}(|H\rangle_1|H\rangle_2\cdots|H\rangle_N - |V\rangle_1|V\rangle_2\cdots|V\rangle_N).
$$

(29)

For the photons removed by the parties, the method discussed in Sec. II.B also works for improving the efficiency of a successful concentration of GHZ-class states and the yield. In this time, one need only replace $|HH\rangle$ and $|VV\rangle$ in Sec. II.B with $|HH\cdots H\rangle$ and $|VV\cdots V\rangle$, respectively.

IV. DISCUSSION AND SUMMARY

Compared with the entanglement concentration schemes by evolving the composite system and an auxiliary particle, the present scheme does not require the parties of quantum communication to know accurately information about the less entanglement states. This good feature makes the present scheme more efficient than those in Refs. 23, 24, 25 as the decoherence of entangled quantum systems depends on the noise of quantum channels or the interaction with the environment, which causes the two parties to be blind to the information about the state. With sophisticated single-photon detectors, entanglement concentration schemes with linear optical elements are efficient for concentrating some partially entangled states. With the development of technology, sophisticated single-photon detectors may be obtained in the future even though they are far beyond what is experimentally feasible at present. Cross-Kerr nonlinearity provides a good QND with which a parity-check measurement can be accomplished perfectly in principle 33. With the QND, our entanglement concentration scheme has a higher efficiency and yield than those with linear optical elements 26, 27.

In summary, we propose a different scheme for nonlocal entanglement concentration of partially entangled multipartite states. We exploit cross-Kerr nonlinearities to distinguish the parity of two polarization photons. Compared with other entanglement concentration schemes, this scheme does not require a collective measurement and does not require the parties of quantum communication to know the coefficients $\alpha$ and $\beta$ of the less entangled states. This advantage makes our scheme have the capability of distilling arbitrary multipartite GHZ-class states. Moreover, it does not require the parties to adopt sophisticated single-photon detectors, which makes this scheme feasible with present techniques. By iteration of entanglement concentration processes, this scheme has a higher efficiency than those with linear optical elements. All these advantages make this scheme more convenient in practical applications than others.
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[1] D. P. Divincenzo, Science 270, 255 (1995).
[2] C. H. Bennett and D. P. Divincenzo, Nature (London) 404, 247 (2000).
[3] A. K. Ekert, Phys. Rev. Lett. 67, 661 (1991).
[4] C. H. Bennett, G. Brassard, and N. D. Mermin, Phys. Rev. Lett. 68, 557 (1992).
[5] N. Gisin, G. Ribordy, W. Tittel, and H. Zbinden Rev. Mod. Phys. 74, 145 (2002).
[6] G. L. Long and X. S. Liu, Phys. Rev. A 65, 032302 (2002).
[7] F. G. Deng and G. L. Long, Phys. Rev. A 68, 042315 (2003).
[8] F. G. Deng, G. L. Long, and X. S. Liu, Phys. Rev. A 68, 042317 (2003); C. Wang, F. G. Deng, Y. S. Li, X. S. Liu, and G. L. Long, Phys. Rev. A 71, 044305 (2005); X. H. Li, F. G. Deng, and H. Y. Zhou, Phys. Rev. A 74, 054302 (2006).
[9] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. 69, 2881 (1992).
[10] X. S. Liu, G. L. Long, D. M. Tong, and F. Li, Phys. Rev. A 65, 022304 (2002).
[11] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. 70, 1895 (1993).
[12] A. Karlsson and M. Bourennane, Phys. Rev. A 58, 4394 (1998); F. G. Deng, C. Y. Li, Y. S. Li, H. Y. Zhou, and Y. Wang, Phys. Rev. A 72, 022338 (2004).
[13] F. G. Deng, X. H. Li, C. Y. Li, P. Zhou, and H. Y. Zhou, Phys. Rev. A 72, 044301 (2005); F. G. Deng, X. H. Li, C. Y. Li, P. Zhou, and H. Y. Zhou, Europ. Phys. J. D 39, 459 (2006); X. H. Li, P. Zhou, C. Y. Li, H. Y. Zhou, and F. G. Deng, J. Phys. B 39, 1975 (2006).
[14] R. F. Werner, Phys. Rev. A 40, 4277 (1989).
[15] C. H. Bennett, G. Brassard, S. Popescu, B. Schumacher, J. A. Smolin, and W. K. Wootters, Phys. Rev. Lett 76, 722 (1996).
[16] D. Deutsch, A. Ekert, R. Jozsa, C. Macchiavello, S. Popescu, and A. Sanpera, Phys. Rev. Lett 77, 2818 (1996).
[17] J. W. Pan, C. Simon, and A. Zellinger, Nature 410, 1067 (2001).
[18] J. W. Pan, S. Gasparoni, R. Ursin, G. Weihs, and A. Zellinger, Nature (London) 423, 417 (2003).
[19] M. Murao, M. B. Plenio, S. Popescu, V. Vedral, and P. L. Knight, Phys. Rev. A 57, R4075 (1998).
[20] M. Horodecki and P. Horodecki, Phys. Rev. A 59, 4206 (1999).
[21] Y. W. Cheong, S. W. Lee, J. Lee, and H. W. Lee, Phys. Rev. A 64, 042314 (2007).
[22] Y. B. Sheng, F. G. Deng, and H. Y. Zhou, Phys. Rev. A 77, 042308 (2008).
[23] C. H. Bennett, H. J. Bernstein, S. Popescu, and B. Schumacher, Phys. Rev. A 53, 2046 (1996).
[24] S. Bose, V. Vedral, and P. L. Knight, Phys. Rev A 60, 194 (1999).
[25] B. S. Shi, Y. K. Jiang, and G. C. Guo, Phys. Rev. A 62, 054301 (2000).
[26] T. Yamamoto, M. Koashi, and N. Imoto, Phys. Rev. A 64, 012304 (2001).
[27] Z. Zhao, J. W. Pan, and M. S. Zhan, Phys. Rev. A 64, 014301 (2001).
[28] Z. Zhao, T. Yang, Y. A. Chen, A. N. Zhang, and J. W. Pan, Phys. Rev. Lett 90, 207901 (2003).
[29] Y. M. Li, K. S. Zhang, and K. C. Peng, Phys. Rev. A 77, 015802 (2008).
[30] H. Jeong and N. B. An, Phys. Rev. A 74, 022104 (2006).
[31] G. S. Jin, Y. Lin, and B. Wu, Phys. Rev. A 75, 054302 (2007).
[32] B. He, J. A. Bergou, and Y. H. Ren, Phys. Rev. A 76, 032301 (2007).
[33] K. Nemoto and W. J. Munro, Phys. Rev. Lett 93, 250502 (2004).
[34] S. D. Barrett, P. Kok, K. Nemoto, R. G. Beausoleil, W. J. Munro, and T. P. Spiller, Phys. Rev. A 71, 060302 (2005).