Detection of Maximal Balanced Clique in Signed Networks Based on Improved Three-way Concept Lattice and Modified Formal Concept Analysis

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Abstract In the era of artificial intelligence including the fourth industrial revolution, social networks analyzing is a significant topic in big data analysis. Clique detection is a state-of-the-art technique in social network structure mining, which is widely used in a particular social network like signed network. There are positive and negative relationships in signed networks which detect not only cliques or maximal cliques but also maximal balanced cliques.

In this paper, two algorithms have been addressed to the problems. First, we modify three-way concept lattice algorithm using a modified formal context and supplement formal context to obtain an object-induced three-way concept lattice (OE-concept) to detect the maximal balanced cliques. Second, in order to improve the cost of memory and efficiency, we modify formal concept analysis algorithm by using modified formal context combine with supplement formal context to find the maximal balance cliques. Additionally, we utilized four real-world datasets to test our proposed approaches as well as the running time in the experimental section.

Keywords maximal balanced clique · modified three-way concept · Modified Formal Concept Lattice · signed networks

1 Introduction

With the development of information technology, many researchers are paying attention to social network structures analysis in computer science. Clique is a very popular research direction in social network structure mining. Vilakone & Park [1, 2] used clique detection and standardized cumulative gains to solve some problems in the movie recommendation system. However, the use of maximal clique detection and balanced clique in social network analysis are not applicable only on social media software but also can be applied on other domains, such as the Internet of Things (IoT), healthcare, biology, and more. Recently, several new issues combined social network with smart devices, smart cars, and medicines have been proposed.

Meanwhile, maximal clique detection can be used to detect the structure of proteins, which can prevent or control diseases. For this reason, we can find a group of the most suitable medical specialists for a target patient to get the best medical treatment through remote consultation. Besides, maximal clique can combine social network analysis with Social Internet of Things (SIoT) [3] detection and cyber-physical system (CPS), etc. to explore new topics to address other issues.

In many existing works, cliques are typically detected in undirected and unweight graphs [4–8]. However, some social networks contain positive and negative relationships between users in their real-life; it can be named as a signed social network. Significantly, there are some studies about mining cliques from the signed network [9–12]. Chen [13] used an enumeration framework to optimize the baseline enumeration algorithm for obtaining maximal balanced cliques in signed networks. Salminen [14] introduced a method for classifying hate in social networks using signed social net-
works. This method controlled and prevented spread promptly and prevented the proliferation of dangerous information. Shakeel [15] analyzed risk information through emotion analysis techniques to analyze signed social networks. However, few studies used formal concept analysis (FCA) to detect maximal balanced cliques in signed social networks. FCA is a novel approach applied in social network analysis. Furthermore, it gives a concept lattice which contains the structure of a social network and some relationships (objects and attributes) between the users.

In this paper, we propose an improved three-way concept lattice algorithm and a modified formal concept analysis algorithm to solve the maximal balanced clique (MBCP) detection problem. In our work, we design an improved three-way concept algorithm based on the traditional three-way concept lattice algorithm [16]. This method is most appropriate for the detection of maximal balanced cliques in a signed network. Then, we design a modified formal context analysis to optimize the first algorithm.

Specifically, signed network g is characterized by a graph structure with many nodes and some positive or negative edges. For example, Fig. 1 shows five nodes in signed network g. And there are some edges between them. In Fig. 1, the solid lines are positive edges, and the dotted lines are negative edges. In this paper, we will detect maximal balanced cliques in signed social networks.

Fig. 1 A signed network g

In this paper, there are some contributions as follow:

1) **Improved Three-way Concept Lattice:** According to the adjustment of the formal context and supplement formal context for a traditional three-way concept lattice method, we can obtain a modified OE-concept lattice (object-induced three-way concept lattice) and a modified AE-concept lattice (attribute-induced three-way concept lattice). Since we divide the positive and negative edges in a signed social network into two parts including a modified formal context and supplement formal context, the modified OE-concept lattice and modified AE-concept lattice can be used to detect maximal balanced cliques in a signed social network.

2) **Modified Formal Concept Analysis:** We have implemented the second approach to save the memory and improve the application efficiency of the first approach. This method is suitable for the symmetrical formal contexts, which is constructed by an undirected social network graph. Therefore, we maintain the modified formal context and supplement formal context of the first approach to obtain two modified concept lattices based on the traditional formal concept analysis method. Then we process these lattices to get the maximal balanced cliques.

3) **Experiment:** We apply four real-life datasets in this paper. One real-life dataset has been used as a case study to explain the application of the maximal balanced cliques. Also, we use the other three real-life datasets to test and compare the running time with the two approaches.

In this paper, Chapter 1 describes the background, related work and significances of research. Chapter 2 describes some preliminary knowledge. Chapter 3 shows the problem statement and the proposed method. Furthermore, we describe two proposed algorithms. Chapter 4 describes the results of the experiment, it was evaluated using four real-life datasets to find the maximal balance cliques in the signed social network. Chapter 5 describes conclusions and future research.

2 Basic notions

In this section, we introduce several basic concepts and properties for signed networks, maximal balanced cliques and three-way concept lattice.

2.1 Signed Networks and Balanced Clique

**Definition 1 (Signed networks [13])** Let \( G = (V, E^+, E^-) \) be an undirected and weighted signed network \( g \), where \( V \) denotes the set of nodes in \( g \), \( E^+ \) denotes the set of positive edges between each 2 nodes in \( g \), \( E^- \) denotes the set of negative edges between each 2 nodes in \( g \).

**Definition 2 (Balanced Network [13])** Given a signed network \( G = (V, E^+, E^-) \), it’s balanced network if it can be split into two subgraphs \( G_L \) and \( G_R \) so that \( \exists (u, v) \in E^+ \) which \( u, v \in G_L \) or \( u, v \in G_R \), and \( \exists (u, v) \in E^- \) which \( u \in G_L, v \in G_R \) or \( u \in G_R, v \in G_L \).
Definition 3 (Maximal Balanced Network [13])
Given a signed network \( G = (V, E^+, E^-) \), a maximal balanced clique \( MBC \) is a maximal subgraph of \( G \). \( MBC \) is a complete subgraph which \( \forall (u, v) \in MBC \rightarrow (u, v) \in E^+ \cup E^- \), and \( MBC \) can be divided into 2 subcliques \( C_1 \) and \( C_2 \), s.t. \( \forall u, v \in C_1 \) or \( u, v \in C_2 \rightarrow (u, v) \in E^+ \), and \( \forall u \in C_1, v \in C_2 \) or \( u \in C_2, v \in C_1 \rightarrow (u, v) \in E^- \). Given a signed network \( G = (V, E^+, E^-) \), a maximal balanced clique \( MBC \) is a maximal subgraph of \( G \) that satisfies the following constraints:

- Complete: \( C \) is complete, which \( \forall (u, v) \in E^+ \rightarrow (u, v) \in E^+ \cup E^- \).
- Balanced: \( C \) is balanced, i.e., it can be split into sub-cliques \( C_L \) and \( C_R \), such that \( \forall u, v \in C_L \) or \( u, v \in C_R \rightarrow (u, v) \in E^+ \), and \( \forall u \in C_L, v \in C_R \) or \( u \in C_R, v \in C_L \rightarrow (u, v) \in E^- \).

The main task of this paper is to detect all the maximal balanced cliques in the signed networks with an Improved three-way concept lattice. The definitions of the traditional three-way concept in the following section.

2.2 Formal concept lattice

Definition 4 (Formal context [17]) Let \( F = (O, A, I) \) be a formal context, where \( O = \{x_1, x_2, ..., x_n\} \) is the set of objects, \( A = \{a_1, a_2, ..., a_m\} \) is the set of attributes, and \( I \) is a binary relation between \( O \) and \( A \). Each \( x_i(i \leq n) \) is called an object, and each \( a_j(j \leq m) \) is called an attribute. If an object \( o \) has an attribute \( a \), we note \( o \uparrow a \) or \( \langle o, a \rangle \in I \).

Definition 5 (Operators \( \uparrow \) and \( \downarrow \)[17]) Let \( F = (O, A, I) \) be a formal context. The operator \( \uparrow \) and \( \downarrow \) on \( X \subseteq O \) and \( Y \subseteq A \) are respectively defined as:

\[
X^\uparrow = \{a \in A | \forall o \in O, (o, a) \in I\}
\]

\[
Y^\downarrow = \{o \in O | \forall a \in A, (o, a) \in I\}
\]

\( \forall o \in X, \text{ let } \{a\}^\uparrow = \alpha^\uparrow \). For \( \forall a \in A, \text{ let } \{a\}^\downarrow = \alpha^\downarrow \).

Definition 6 (Formal Concept [17]) For a formal context \( F = (O, A, I) \), if \( (O, A) \) satisfies \( O^\uparrow = A \) and \( A^\downarrow = O \), \( (O, A) \) is called a Formal Concept (FC), and \( O \) is the extent of the concept, \( A \) is the intent of the concept. FC(F) is an operation for obtaining a formal concept from formal context \( F \).

Definition 7 ( [17] Let \( F(K) \) denote the set of all formal concepts of the formal context \( F = (O, A, I) \). If \((O_1, A_1), (O_2, A_2) \in F(K)\), then let

\[
(O_1, A_1) \leq (O_2, A_2) \iff O_1 \subseteq O_2 \text{ and } A_1 \supseteq A_2
\]

Then “\( \leq \)” is a partial relation of \( F(K) \).

Example 1
Table 1 is a formal context \( F = (O, A, I) \).

| \( O \times A \) | \( a_1 \) | \( a_2 \) | \( a_3 \) | \( a_4 \) | \( a_5 \) |
|---|---|---|---|---|---|
| \( o_1 \) | 1 | 1 | 0 | 1 | 1 |
| \( o_2 \) | 1 | 1 | 1 | 0 | 0 |
| \( o_3 \) | 0 | 0 | 0 | 0 | 1 |
| \( o_4 \) | 1 | 1 | 1 | 0 | 0 |

\[
\{o_1, o_2, o_4\} \subseteq \{a_1, a_2\}
\]

\[
\{o_1, o_2\} \subseteq \{a_1\}
\]

\[
\{o_2\} \subseteq \{a_2\}
\]

\[
\{o_3\} \subseteq \{a_3\}
\]

\[
\{o_4\} \subseteq \{a_4\}
\]

\[
\{\}, \{a_1, a_2\}
\]

\[
\{\}, \{a_3\}
\]

\[
\{\}, \{a_4\}
\]

\[
\{\}, \{a_5\}
\]

Fig. 2 The Concept Lattice of A Formal Context \( F \).
2.3 Three-way concept lattice

Two special three-way operations are defined based on two operations (↑ and ↓) of traditional formal concept analysis.

Definition 9 (Operator ↑ + and ↓ + [16]) Let \( F = (O, A, I) \) be a formal context. For any \( X \subseteq O, Y \subseteq A \), a pair of operators \( \uparrow \) and \( \downarrow \) is defined by:

\[
X^{\uparrow+} = \{ a \in A | \forall o \in O, \neg (o \subseteq a) \} = \{ a \in A | \forall o \in O, o \not\subseteq a \}
\]

\[ (5) \]

\[
Y^{\downarrow+} = \{ x \in O | \forall a \in A, \neg (o \subseteq a) \} = \{ o \in O | \forall a \in A, o \not\subseteq a \}
\]

\[ (6) \]

And supplement formal context \( F^c = (O, A, I^c), I^c = (O \times A) - I \).

Definition 10 (Operator ≤ and ≥ [16]) Let \( F = (O, A, I) \) be a formal context. For any \( X, M \subseteq O, Y, N \subseteq A \), a pair of three-way operators \( \leq \) and \( \geq \) is defined by:

\[
X \leq (X^\uparrow, X^{\uparrow+})
\]

\[ (7) \]

\[
Y \leq (Y^\downarrow, Y^{\downarrow+})
\]

\[ (8) \]

\[
(Y, N) \geq = \{ x \in O | x \subseteq Y^\downarrow, x \subseteq N^{\downarrow+} \}
\]

\[ (9) \]

\[
(X, M) \geq = \{ a \in A | a \subseteq X^\uparrow, a \subseteq M^{\uparrow+} \}
\]

\[ (10) \]

Based on definition 10, three-way concept is defined as follows.

Definition 11 (Three-way concept [16]) The Three-way concept including two parts: AE-concept and OE-concept.

1) **AE-concept**: Let \( F = (O, A, I) \) be a formal context. For any \( X, M \subseteq O, Y \subseteq A \), \((X, M), Y\) is called attribute-induced three-way concept (AE-concept), if and only if \((X, M) \geq = Y \) and \( Y \leq = (X, M) \), where \((X, M)\) is called the extent and \( Y \) is called the intent of \((X, M), Y\). The set of all AE-concepts of \( F = (O, A, I) \) can form a lattice by some partial order structure, this lattice is called AE lattice. The partial order is defined by:

\[
((X_1, M_1), Y_1) \leq ((X_2, M_2), Y_2)
\]

\[ \iff (X_1, M_1) \subseteq (X_2, M_2) \iff Y_2 \subseteq Y_1 \]

\[ (11) \]

Similarly, object-induced three-way concepts is defined as follows.

2) **OE-concept**: Let \( F = (O, A, I) \) be a formal context. For any \( X \subseteq O, Y, N \subseteq A \), \((X, Y, N)\) is called object-induced three-way concept (OE-concept), if and only if \( X \leq = (Y, N) \) and \( (Y, N) \geq = X \), where \( X \) is called the extent and \( (Y, N) \) is called the intent of \((X, Y, N)\). The set of all OE-concepts of \( F = (O, A, I) \) can form a lattice by some partial order structure, this lattice is called OE lattice. The partial order is defined by:

\[
((X_1, Y_1, N_1), (X_2, Y_2, N_2)) \leq (X_1, Y_2, N_1) \iff (X_1, Y_1) \subseteq (X_2, Y_2) \iff (X_1, Y_1) \subseteq (X_2, Y_2)
\]

\[ (12) \]

We present an example to make it easier to understand three-way context and three-way concept lattice.

**Example 2** Table 2 shows a supplementary formal context \( F^C \) for Table 1. Fig. 3 illustrates the concept lattice for the context of Table 2. Fig. 4 shows the OE lattice for formal context \( F \) and \( F^C \).

| \( O \times A \) | \( a_1 \) | \( a_2 \) | \( a_3 \) | \( a_4 \) | \( a_5 \) |
|---|---|---|---|---|---|
| \( a_1 \) | 0 | 0 | 1 | 0 | 0 |
| \( a_2 \) | 0 | 0 | 0 | 1 | 1 |
| \( a_3 \) | 1 | 1 | 1 | 1 | 0 |
| \( a_4 \) | 0 | 0 | 0 | 1 | 1 |

**Table 2** A Supplement Formal Context \( F^C \).
3 Problem statement and proposed approach

In this section, we describe maximal balanced clique detection in signed networks and propose two modified and improved methods for maximal balanced clique detection in signed networks.

3.1 Problem statement

As shown in Fig. 1, there is a signed network $g$, we can detect and enumerate the maximal balanced cliques in this signed network. Fig.1 shows 5 nodes from $v_1$ to $v_5$, and there are positive relationships between each node in ($v_1, v_2, v_3$) and ($v_3, v_4$), respectively. They can be represented as ($v_1, v_2, v_3$) $\in E^+$ and ($v_3, v_4$) $\in E^+$. In addition, ($v_1, v_3$) $\in E^-$, ($v_2, v_3$) $\in E^-$, ($v_5, v_3$) $\in E^-$, ($v_1, v_4$) $\in E^-$, ($v_2, v_4$) $\in E^-$ and ($v_5, v_4$) $\in E^-$ has a negative relationship. Therefore, there is a maximal balanced clique set $MBC = \{\{v_1, v_2, v_3\}, \{v_3, v_4\}\}$ in Fig.1.

The purpose of this paper is to detect the maximal balanced cliques in the signed social network using modified formal context and supplement formal context.

In this paper, we proposed two solutions: the first method obtains the modified OE lattice through the existing three-concept lattice algorithm using the modified formal context and the supplement formal context. We detect all maximal balanced cliques in the OE lattice. The second method is to optimize the first method to improve the detection efficiency of the maximal balanced cliques.

3.2 Proposed approach

In signed social networks, we use the formal context and supplement formal context in a traditional three-way concept lattice to represent the positive and negative relationships between nodes. Because in Definition 9, there are no any positive or negative relation in formal context and supplement formal context, it cannot be used for a signed social network, in this section, we redefine the formal context and supplement formal context for signed social network.

**Definition 12 (Modified formal context)** If a signed network $g$ has nodes $\{v_1, v_2, \ldots, v_n\}$, each node $v_i$ represents an object and an attribute, and edge $e = (v_i, v_j)$ is the relationship between the objects and attributes, $I$ or $F^c$. If $e \in E^+$, notes $I(v_i, v_j) = 1$, if $e \in E^-$, notes $I^c(v_i, v_j) = 1$. Then, if $F = (O, A, I)$ and $F^c = (O, A, I^c)$ are modified formal contexts, they can be represented as two adjacency matrices containing only 0 and 1.

$$M(F) = \begin{cases} m_{ij} = 1, & \text{if } (v_i, v_j) \in E^+ \text{ and } i \neq j \\ m_{ij} = 1, & \text{if } i = j \\ m_{ij} = 0, & \text{if } (v_i, v_j) \not\in E^+ \text{ and } i \neq j \end{cases}$$

$$M(F^c) = \begin{cases} m_{ij} = 1, & \text{if } (v_i, v_j) \in E^- \text{ and } i \neq j \\ m_{ij} = 1, & \text{if } i = j \\ m_{ij} = 0, & \text{if } (v_i, v_j) \not\in E^- \text{ and } i \neq j \end{cases}$$

**Definition 13 (Modified OE and AE)** Given the modified formal context and supplement formal context to generate OE-concept and AE-concept in a three-way concepts algorithm, we can get a modified OE lattice (denoted MOE) and modified AE lattice (denoted MAE).

Maximal Cliques Detection Approach 1 (Improved Three-way Concept method): Modified formal context $F = (O, A, I)$ and supplement formal context $F^c = (O, A, I^c)$ are used to obtain modified three-way concept lattices, MOE and MAE. When there is a concept in MOE and $X = Y$, i.e. $(X, (X, N))$, $X$ and $N$ form the maximal balanced clique represented by $(X, N)$, and $(X, (X, N))$ is the maximal balanced OE-concept.

**Theorem 1** Let MOE be the OE-concepts lattice of a modified formal context $F$ and $F^c$. For any $(X, (Y, N)) \in MOE$, if $X = Y$, it notes $(X, (X, N))$, then the $(X, N)$ is a maximal balanced clique, s.t. $\forall u, v \in X$ or $u, v \in N \rightarrow (u, v) \in E^+$, and $\forall u \in X, v \in N$ or $u \in N, v \in X \rightarrow (u, v) \in E^-$. 

**Proof** There are $(X, Y) \in FC(F)$ and $(M, N) \in FC(F^c)$, and $(X, (Y, N)) \in MOE$. 

![Fig 4 The OE Lattice of Formal Context F and Supplement Formal Context Fc](image-url)
1) If $X = Y$, we can get $(X, X) = (X, Y) \in FC(F)$, $(X, X)$ is an equiconcept in $FC(F)$, $(X, X)$ is a maximal clique and $\forall u, v \in X$, $(u, v) \in E^\pm$.

2) Because of $(X, (Y, N)) \in MOE$, if $X = Y$, we can get $(X, (Y, N)) = (X, (X, N)) \in MOE$. According to Definition 10, for a modified OE-concept $(X, (X, N))$, $(X, N) = X^+ \cap N^\pm = X$. Based on Definition 5 and Definition 9, $X^+ = X = X$, $N^\pm = M$, then $X^+ \cap N^\pm = X \cap M = X$. Then, $X \subseteq M$. Because of $(M, N) \in FC(F^c)$, $\forall u \in M$, $v \in N$ or $u \in N$, $v \in M \rightarrow (u, v) \in E^-$, we can get that $\forall u \in X \subseteq M$, $v \in N$ or $u \in N$, $v \in X \subseteq M \rightarrow (u, v) \in E^-$. The working process of maximal cliques detection approach 1 is described in Algorithm 1. In Algorithm 1, Line 1 initializes a set of maximal balanced cliques and modified OE-concept. Line 3 constructs formal contexts $F$ and $F^c$ for signed social network $G$. The Construct_MFC is given by Algorithm 2. Line 4 follows the rules of traditional three-way concept algorithm according to the modified contexts $F$ and $F^c$ to get the modified OE lattice. Lines 5-8 use theorem 1 to detect the maximal balanced cliques from $MOE$. Finally, set $MBC$ returns from Line 9.

On Line 1 of algorithm 2, we initialize the matrices of both $F$ and $F^c$ to 0. Lines 3-10 construct two matrices, three-way formal context $F$ and supplement formal context $F^c$, according to definition 12. Line 11 return three-way contexts $F$ and $F^c$.

**Algorithm 1** The detection of Maximal Balanced Clique algorithms 1 $(G = (V, E^+, E^-))$

1: Initialize the set of MBC, MOE
   //MBC is the set of Maximal Balanced Clique
   //MOE is the set of Modified OE lattice
2: begin
3: $F, F^c \leftarrow$ Construct_MFC(G) // Construct the formal context $F$ and $F^c$ for signed social network $G$, Construct_MFC is Algorithm 2
4: $MOE \leftarrow$ Three-way concept $(V, F, F^c)$ // Using the traditional Three-way concept algorithm with modified context $F$ and $F^c$ to get a modified OE lattice MOE
5: for each $(x, (y, n)) \in MOE$
6: if $x = y$
7: if $\text{length}(x) > 1$ and $\text{length}(y) > 1$
8: $MBC \leftarrow (x, n)$
9: return $MBC$
10: end

Here, we provide an example to facilitate understanding of how to use a modified three-way concept method to detect maximal balanced cliques in signed social network.

**Example 3** Fig. 5 shows a signed social network graph, which includes 11 nodes, 8 positive edges, and 13 negative edges. The modified OE lattice of the signed social network $G$ is obtained using algorithms 1 and 2. Fig. 6 lists all OE-concepts of modified OE lattice. The extents and intents of each concept are separated by “#”. According to Theorem 1, we can find 4 suitable OE-concepts in this list: $(3, 4, 6) \# ((3, 4, 6), (2, 10, 11)); (5, 8) \# ((5, 8), (7, 9)); (2, 10, 11) \# ((2, 10, 11), (3, 4, 6)); (7, 9) \# ((7, 9), (5, 8))$. And we can get 2 maximal balanced cliques: $\{(3, 4, 6), (2, 10, 11)\}; \{(5, 8), (7, 9)\}$. In Fig. 7, the purple and orange parts are two maximal balanced cliques with vertex label numbers consistent with the above results.
Modified OE concepts list.

Due to the high time complexity of the improved three-way concept lattice method, if there are a large number of nodes in a signed social network, this method cannot quickly obtain the modified OE lattice, so it cannot quickly obtain the maximal balanced cliques. Therefore, we analyze the characteristics of OE-concepts based on a three-way concept lattices, and propose a detection algorithm of maximal balanced cliques that can be quickly obtained starting directly from the traditional concept lattices of formal context and supplement formal context.

For an undirected weighted signed network graph, vertices can be regarded as its objects and attributes of the formal context, so the corresponding formal context $F$ and supplement formal context $F_c$ are a symmetric matrix in undirected weighted signed network. The formal context $F$ represents a positive relationship between vertices, and the supplement formal context $F_c$ represents a negative relationship between vertices. Hao et. al [8] showed that when the extent and intent of a concept are equal, it is called an equiconcept, which is a maximal clique in the social network. Thus, for the modified formal context $F$, all equiconcepts are maximal cliques with positive relationships in signed social network graphs.

On the contrary, there should be a negative relationship between the extent and intent of the concept obtained from the supplement formal context $F_c$. If the extent and intent of the concept obtained from the supplement context $F_c$ are the same as the equiconcepts in the formal context $F$, it indicates that this concept of supplement context $F_c$ satisfies the definition 3. In the signed social networks, it is the maximal balanced clique.

Maximal Cliques Detection Approach 2 (Modified Formal Context Analysis): The modified formal context $F = (O, A, I)$ and supplement formal context $F^c = (O, A, F^c)$ are used as the formal contexts for the traditional formal concept analysis algorithm to obtain two modified formal concept lattices: $\Omega(F)$ and $\Omega(F^c)$. When there are 2 concepts $(X,Y)$ and $(M,N)$ in $\Omega(F)$ and $X = Y$, $M = N$, they are equiconcepts. At the same time, if $(X,N)$ in $\Omega(F^c)$, $X$ and $N$ form a maximal balanced clique, denoted $\{X,N\}$.

The above process is described in Algorithm 3. In algorithm 3, Line 1 initializes the sets of maximal balanced cliques, equiconcept, concept lattice $\Omega(F)$, and $\Omega(F^c)$. Line 3 constructs the formal context $F$ and $F^c$ for a signed social network $G$, while Construct_MFC is given Algorithm 2. Lines 4-5 obtain the concept lattice of formal $F$ and $F^c$. Lines 6-8 obtain the set of equiconcept. Lines 9-12 detect maximal balanced cliques through
Approach 2. Finally, the set of $MBC$ will return in Line 9.

The time complexity of the formal concept analysis with $F = (O, A, I)$ is $O(|Ω(K)||Ω(K^c)|)$, and the time complexity of the construction of the modified formal context and supplement formal context is $O(|O| |A|)$, the time complexity of detecting maximal balanced cliques is $O(|Equiconcept||Ω(k^c)|)$, the total time complexity of this method is $O(|Ω(K)||Ω(K^c)| + |O| |A| + |Equiconcept| \times |Ω(k^c)|)$. Obviously, in terms of time complexity, it is very better than original methods, which can reduce execution time significantly.

Algorithm 3 The detection of Maximal Balanced Cliques algorithm 2 ($G = (V,E^+,E^-)$)

1: Initialize the set of MBC, EC, $Ω(F), Ω(F^c)$
   // MBC is the set of Maximal Balanced Cliques
   // EC is the set of equiconcept
   // Ω($F$), $Ω(F^c)$ are the lattices of formal context $F$ and $F^c$
2: begin
3: $F, F^c \leftarrow$ Construct_MFC($G$) // Construct the formal context $F$ and $F^c$ for signed social network $G$
   Construct_MFC is Algorithm 2
4: $Ω(F) \leftarrow$ Formal concept analysis($F$) // Construct the lattice of formal context $F$ by a traditional formal concept analysis method
5: $Ω(F^c) \leftarrow$ Formal concept analysis($F^c$) // Construct the lattice of formal context $F^c$ by a traditional formal concept analysis method
6: for each concept $(x, y)$ in $Ω(F)$:
7:    if $x == y$:
8:       $EC \leftarrow$ $(x, y)$
9:    for each $(x, y) \in EC$:
10:       for each $(m, n) \in Ω(F^c)$:
11:          if $m == n$ or $x == n$:
12:             $MBC \leftarrow (x, n)$
13:       return $MBC$
14: end

Example 4 Fig. 8 shows the concept lattice of context $F$ and $F^c$, if we follow approach 2, we can get 4 equiconcept (the number of extent or intent is more than 1): $(3, 4, 6)$#, $(3, 4, 6)$; $(2, 10, 11)$#$(2, 10, 11)$, $(5, 8)$#, $(5, 8)$; $(7, 9)$#, $(7, 9)$#. We have the previous four equiconcepts to obtain the 4 concepts in lattice $Ω(F^c)$ and follow the rules in Approach 2: $(3, 4, 6)$#, $(5, 8)$#, $(7, 9)$#, $(7, 9)$#. Then, we obtain two maximal balanced cliques: $\{(3, 4, 6), \{2, 10, 11\}\}$, $\{(5, 8), \{7, 9\}\}$. As shown in Fig. 7, the purple and orange parts are two maximal balanced cliques with vertex labels numbers consistent with the above results.

4 Experiment

This chapter presents experimental results. The experimental environment is macOS Catalina operating system, two Intel 2.3 GHz Quad-Core Intel Core i5, RAM 16G, and the Programming language is Python 3.8. In this chapter, we use two approaches proposed in chapter 3 to perform experiments.

The modified three-way concept lattice method can deal with symmetric and asymmetric formal context, but the modified formal concept analysis can only handle symmetric formal context of maximal balanced detection. Therefore, we test approach 1(modified three-way concept) using three different real-life asymmetric datasets at first. Then, we use a large symmetric real-life datasets to compare approach 1 the approach 2 (modified formal concept analysis).

4.1 Modified Three-way Concept

Experiment 1: Case Study There is a real-life dataset AdjWordNet [18] downloaded from WordNet (http://wordnet.princeton.edu). This dataset contains 117 000 words for synonyms and antonyms. Synonyms have positive edges, antonyms have negative edges, and there is no edge between unrelated words.

In this case study, we selected 25 words (Table 3) from AdjWordNet and obtained three maximal balanced cliques, thus finding three sets of synonyms with opposite meanings (Table 4). In Table 4, each line in $S_1$ and $S_2$ represents a group of synonyms, and each column of $S_1$ and $S_2$ represents a group of two antonyms.
Table 3 25 words from AdjWordNet.

| raw       | rough     | rude       | relaxing   | refined   |
|-----------|-----------|------------|------------|-----------|
| interior  | smooth    | assumed    | false      | intimate  |
| uneasy    | ungratified | existent  | reposeful  | unsatisfied |
| outer     | undynamic | active     | undermentioned | sour    |
| participating |      |            |            |    |

Table 4 The result of AdjWordNet.

|            | $S_1$               | $S_2$               |
|------------|---------------------|---------------------|
| 1$^{st}$ maximal balanced clique | refined, smooth     | raw, rough, rude    |
| 2$^{nd}$ maximal balanced clique | assumed, false, sour | existent, veridical, actual |
| 3$^{rd}$ maximal balanced clique | relaxing, restful, reposeful | uneasy, ungratified, unsatisfied |

Experiment 2: Running time of three datasets

There are three real-life datasets (table 5), and the first dataset is 11 nodes from AdjWordNet [18], which is about synonyms and antonyms of words. The second dataset is a network of Intra-organization [19], which is a dataset of consulting company (46 employees) and records the frequency of requests for information or advice among employees. The third dataset is Freeman’s EIES networks [20], which includes some personal relationships among 46 researchers.

As shown in Fig. 9, the running time of the first data set is less than that of the other two datasets. Thus, although we can see that Approach 1 is highly sensitive by the number of nodes and edges, it has the advantage of being able to compute asymmetric formal context that are not limited to social network datasets.

Table 5 The asymmetric dataset.

|           | nodes | edges |
|-----------|-------|-------|
| AdjWordNet | 11    | 15    |
| Intra     | 46    | 879   |
| EIES      | 46    | 695   |

Fig. 9 The running time of modified three-way concept (Unit: ms).

4.2 Modified Formal Concept Analysis

In this session, we test approach 2 using the Slashdot dataset [21]. Slashdot is a technology-related news website known as a specific user community. The website has the capability for users to submit to the site about the latest news, which is mainly technology oriented, and for editors to evaluate the content. In addition, the Slashdot dataset contains friend and the relationships between Slashdot users. The Slashdot includes 77350 nodes and 516575 edges, but in this paper, we take only the first 2000 nodes of the dataset and experiment in groups of different sizes. We transform the relationships of a dataset into undirected relationships without considering a positive or negative relationship with one
direction to obtain a symmetric matrix that simulates an undirected weighted social network (Table 6).

Figure 10 shows the comparison of the run-time on dataset 1-9 with Approach 1 and Approach 2 methods. The blue column shows the runnigtime by Approach 1 method, and the orange polyline shows the runningtime by Approach 2 method. It can be seen that the running-time of the Approach 2 method is much shorter than the runningtime of the Approach 1 method. When the number of nodes increases to 900, that is, when running DataSet-9, measuring run-time using Approach 1 method takes a lot of time.

Figure 11 shows the runningtime of Approach 2 method for DataSet 10-14. Although the time is getting longer, the Approach 2 method can easily obtain execution results until 2000 nodes are reached. However, the limitation of Approach 2 method lies in that many real-life datasets are bidirectional graphs, while they only apply to undirected weighted signed social networks. Since both Approach 1 and 2 methods have advantages in each case, applying them to real-life social networks will allow us to choose a more appropriate method.

Table 6 The symmetric dataset.

| nodes  | nodes  |
|--------|--------|
| Dataset-1 | 100    | Dataset-8 | 800 |
| Dataset-2 | 200    | Dataset-9 | 900 |
| Dataset-3 | 300    | Dataset-10 | 1000 |
| Dataset-4 | 400    | Dataset-11 | 1300 |
| Dataset-5 | 500    | Dataset-12 | 1500 |
| Dataset-6 | 600    | Dataset-13 | 1700 |
| Dataset-7 | 700    | Dataset-14 | 2000 |

5 Conclusion

Based on the traditional formal concept analysis method and the three-way concept lattice algorithm, this paper proposes a modified three-way concept algorithm and a modified formal concept algorithm to detect the maximal balanced clique in the signed social network and reduce execution time. To demonstrate feasibility for the proposed method, in the performance evaluation, four real-life signed social networks are used to detect maximal balanced cliques, and synonyms and antonyms were used on the AdjWorldNet dataset. Also, we found that both two methods have advantages and disadvantages. Approach 1 method has a wider application range on asymmetric dataset, and Approach 2 method have a faster computation speed and higher efficiency. This proposed method can apply to detect of synonyms and antonyms in dictionary, the detection of social network user groups, and the analysis of protein molecular structure. It can be applied to more scenarios in the future. In addition, we will solve the problem of long running time in Approach 1 method.

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Compliance with ethical standards

Conflict of interest

Yixuan Yang, Doo-Soon Park, Fei Hao, Sony Peng, Min-Pyo Hong and Hyejung Lee have no conflict of interest.
Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Author contributions

All authors contributed to the study conception and design. Material preparation, data collection and analysis were performed by Yixuan Yang and Doo-Soon Park and Fei Hao. The first draft of the manuscript was written by Yixuan Yang and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

References

1. Vilakone, P., Xinchang, K. and Park, D.S., "Movie recommendation system based on users’ personal information and movies rated using the method of k-clique and normalized discounted cumulative gain." Journal of Information Processing Systems 16.2, 494-507, 2020.
2. Vilakone, P., Xinchang, K. and Park, D.S., "Personalized movie recommendation system combining data mining with the k-clique method." Journal of Information Processing Systems 15.5, 1141-1155, 2019.
3. Rad, M.M., Rahmani, A.M., Sahafi, A. and Qader, N.N., "Social Internet of Things: vision, challenges, and trends." Human-centric Computing and Information Sciences 10.1, 1-40, 2020.
4. Bron C, Kerbosch J., Finding all cliques of an undirected graph (algorithm 457). J. Commun. ACM, 16(9): 575-576, 1973.
5. Eppstein D, Löffler M, Strash D., Listing all maximal cliques in sparse graphs in near-optimal time. C. International Symposium on Algorithms and Computation. Springer, Berlin, Heidelberg, 403-414, 2010.
6. Eppstein D, Strash D., Listing all maximal cliques in large sparse real-world graphs. C. International Symposium on Experimental Algorithms. Springer, Berlin, Heidelberg, 364-375, 2011.
7. Yuan L, Qin L, Lin X, et al., Diversified top-k clique search. J. The VLDB Journal, 25(2): 171-196, 2016.
8. Hao F, Min G, Pei Z, et al., K-clique community detection in social networks based on formal concept analysis. J. IEEE Systems Journal, 11(1): 250-259, 2015.
9. Easley D, Kleinberg J., Networks, crowds, and markets: Reasoning about a highly connected world. J. Significance, 9: 43-44, 2012.
10. Kumar S, Spezzano F, Subrahmanian V S, et al., Edge weight prediction in weighted signed networks. C. 2016 IEEE 16th International Conference on Data Mining (ICDM). IEEE, 221-230, 2016.
11. Kunegis J, Lommatzsch A, Bauckhage C., The slashdot zoo: mining a social network with negative edges. C. Proceedings of the 18th international conference on World wide web., 741-750, 2019.
12. Leskovec J, Huttenlocher D, Kleinberg J., Signed networks in social media. C. Proceedings of the SIGCHI conference on human factors in computing systems., 1361-1370, 2010.