Modeling of Multi-Service Switching Networks With Multicast Connections

MACIEJ STASIAK, (Member, IEEE), MACIEJ SOBIERAJ, (Member, IEEE), AND PIOTR ZWIERZYKOWSKI, (Senior Member, IEEE)
Institute of Communication and Computer Networks, Poznań University of Technology, 60-965 Poznan, Poland
Corresponding author: Maciej Sobieraj (maciej.sobieraj@put.poznan.pl)
This work was supported in part by the Polish National Science Centre under Grant 0313/PNCN/3925.

ABSTRACT
This article presents an analytical model of a switching network that can be used in the nodes of a large number of modern multi-service networks. The model of a multi-service three-stage switching network proposed in this article has the advantage of providing the possibility to evaluate the point-to-point blocking probability for multicast connections. The information obtained in this way can then be used both at the stage of the estimation of the capacity of a new network and during the optimisation process of an existing network. In the calculations performed during the investigations, three scenarios for the fan out of multicast connections in the switching network are taken into consideration. The assumption is that the branching of a connection can occur in the first, second and the third stage of the switching network. The model proposed in the article is based on the concept of “effective” availability. The article discusses the methods for the determination of the effective availability parameter for successive connections that belong to a given multicast connection. The results of the analytical modelling are compared with the simulation data for three selected structures of multi-service switching networks to which different mixtures of multi-service BPP traffic are offered. The results of the calculations and the data provided by the simulation experiments validate and confirm high accuracy of the proposed model. This allows the model to be easily applied in practice to different environments to evaluate the capacities of the nodes in multi-service networks.

INDEX TERMS
Multicast connections, multi-service traffic, optical switches, switching network, traffic engineering.

I. INTRODUCTION
The rapid and dramatic increase in the capacity of computer networks is closely related with the increase in a variety of network services on offer, including those services that may require concurrent data delivery performance to multiple destinations (multicast). Concurrent delivery of data to a large number of receivers poses a real challenge to the network infrastructure. To guarantee effective data delivery to groups of recipients, network nodes (e.g. routers) have to support this type of communication with appropriate equipment and devices. The basic element of the structure of each network node is the switching network, that also has to be properly adjusted and well aligned to service this traffic [1]–[3].

From the point of view of the way in which traffic is serviced, networks can be divided into lossless networks (non-blocking, rearrangeable networks) [4]–[6] and networks with losses (blocking networks). Non-blocking networks are characterised by no occurrence of internal blocking, which could be attributed to the lack of the possibility to transfer data between a selected pair of free links: the input and output links. In practice, the application of non-blocking networks is often not financially viable, as networks of this type are too expensive to deploy and impractical, which is the result of the number of elements required for the network to make it a non-blocking network. In turn, for rearrangeable networks to operate an appropriate algorithm is required that would guarantee the possibility to change the structure of the internal connections in the network and thus provide the possibility to set up a next connection path in the network. The cost of the construction of such networks is lower than the corresponding non-blocking networks, but it is also necessary to provide additional control system that would provide effective operation of the algorithm that...
controls the network operation. This is particularly important for data streams with high bitrates for which the guarantee of a low delay is the critical parameter. The cost of these networks is also the decisive factor that considerably limits the possibilities for their practical application. Blocking networks can be then an accommodating solution to this problem. The architecture of these networks is a compromise between the acceptable - from the engineering point of view - level of losses/blocking and the involved cost of a network construction. However, networks of this type can be of significant practical importance. This article proposes a method that has the advantage of providing possibilities for modelling blocking networks that can service multicast traffic [7], [8].

Multi-service switching networks are modelled on the basis of the effective availability method. The concept of effective availability is based on a substitution (for the purposes of the process modelling) of the switching network by a model of a non-full-availability group. The basic parameter that describes the structure of such a group is its availability. The availability of a group that substitutes a switching network has to be equal to the effective availability of the network it replaces. The effective availability parameter of the network is determined on the basis of the structure and the load of a given switching network. The computational method is based on the assumption that the blocking probability in a given non-full-availability group will be the same as the blocking probability in the switching network it replaces. This assumption was used for the first time in an analysis of two-stage single-service switching networks [9], [10]. In the following works, in [11]–[16] among others, the concept of effective availability was expanded to include modelling of single-service switching networks with multiple number of stages. A large number of works discusses different methods to determine the value of the effective availability parameter in single-service switching networks, e.g. [12], [14]–[16], [19]. Currently, to analyse single-service switching networks, the formula proposed in [16] is most frequently used. The formula was derived from a modification of the formula proposed in [16]. This particular model is then expanded in [22] to include a model of multi-service switching network that services multicast traffic in networks with the point-to-group selection. [39] presents models of the switching network with the point-to-point selection that service multicast traffic, with the accompanying assumption that the output links for each successive component connection will be selected in succession.

This article presents a new model of a 3-stage switching network that services multicast traffic for the point-to-point selection and three instances of traffic branching: in the first, second and third stage. In each of the instances, two scenarios for the operation of the algorithm that controls setting up of the connection path for multicast traffic (communication tree construction) are taken into consideration. In the first scenario, the control algorithm sequentially chooses output links in the directions in which the recipients are located and then sets up a consecutive communication path after setting up the previous one. The assumption in the second scenario is that the control algorithm first chooses all the required output links, and then sets up connection paths for them one by one. The general assumption for all the algorithms was that the network serviced a mixture of Engset, Erlang and Pascal traffic. The basis for the proposed model rests on the appropriate modification of the computational method for multi-service switching networks without multicast connections, i.e. the PPBMT method (Point-to-Point Blocking for Multi-channel traffic) [21]. In the proposed model, a new method to determine effective availability for successive component connections of a multi-service connection under consideration is also used. Another assumption was that, as in [30], the blocking probability for multicast connections would be determined iteratively.

1 Such a mixture is often termed the BPP traffic in the literature (after the names of call streams, i.e. Bernoulli, Pascal and Poisson).
According to the authors’ knowledge, it is the first analytical model that allows to determine blocking probability in a multi-service switching networks with multicast connections where three instances of traffic branching were used. The main advantage of the proposed model is its universality. In addition to considering the branching in different stages of the switching network, different scenarios for setting up multicast connections are also taken into account.

The present article is further divided into 6 sections. Section 2 presents the structure of a multi-service Clos switching network and describes the way the multicast connections are executed (set up). Section 3 follows by giving a brief overview of the PPBMT method, which has the advantage of determining the blocking probability in switching networks with multicast connections. This section further discusses the said modification to the PPBMT method, i.e. the IPPBMT method (Iterative PPBMT). This method will then provide the basis for the considerations of switching networks with multicast connections. Section 4 presents the new modelling method which allows the point-to-point blocking probability in switching networks with multi-service traffic and multicast connections to be determined. This section also proposes a number of methods to determine effective availability for multicast connections. In the next section, Section 5, the results of the analytical modelling are compared with simulation data for a number of selected structures of multi-service three-stage Clos switching networks. Section 6 sums up the article.

II. POINT-TO-POINT SWITCHING NETWORK WITH MULTICAST CONNECTIONS

Figure 1 shows the structure of a multi-service, three-stage Clos network [4], [40]. The network under consideration is composed of \( k \) symmetrical switches with \( k \) input links and \( k \) output links. The assumption is that all links (input, output and inter-stage links) have the capacity equal to \( f \) AUs (Allocation Units). In traffic theory, AU is expressed in Kbps and is defined as the GCD (greatest common divisor) of bit rates of all traffic classes offered to the system [41]–[46]. The output links of the switching network are grouped into the so-called output directions. The assumption in the article is that the \( i \)-th output link in each of the switches of the last stage creates the \( i \)-th direction. Hence, the first output links of the switches of the last stage create the first direction, the second links construct the second direction, while the \( k \)-th links create the last direction. The switching network shown in Figure 1 has then \( k \) directions with \( k \) output links each. This switching network is offered \( M \) Engset, Erlang or Pascal traffic streams (Appendix A) that can be characterised by the following parameters:

- \( A(c) \) – the average traffic intensity of class \( c \),

\(^2\)The article introduces that the index \( i \) defines any Erlang traffic class, the index \( j \) any Engset traffic class and the index \( l \) any Pascal traffic class. Whereas the index \( c \) defines any traffic class, irrespectively of its type \( (1 \leq c \leq M) \), offered to the switching network.
• during setting up successive component connections in a given multicast connection, the control algorithm can make use of fragments of connecting paths that have been set up earlier, depending on the stage of the network in which branching occurs,
• in the control algorithm, the stage of the switching network that can perform branching the connection is clearly indicated. In the model proposed in the article, three variants of the execution of multicast connections in relation to the stages are considered, i.e. in the first, second and the third stage, respectively,
• a multicast connection of class \(c\) is rejected if only one, from among \(q_c\), component connections is in the blocking state due to either external or internal blocking.

III. METHODS FOR THE DETERMINATION OF THE BLOCKING PROBABILITY FOR UNICAST CONNECTIONS

This section provides a description of the PPBMT method [21] and the new IPPBMT method proposed for the determination of the point-to-point blocking probability in multi-service switching networks for unicast connections. This method will be used further on in the article to analyse switching networks with multicast connections.

A. THE PPBMT METHOD

The PPBMT method utilizes the principle, formulated in [12], that states that the internal blocking probability of a \(2\)-stage network with the point-to-point selection is exactly the same (identical) as the internal point-to-group blocking probability in a \((2 - 1)\)-stage network. The output group of a \((2 - 1)\)-stage switching network is composed of all inter-stage links of a \((2 - 1)\)-stage network that connect switches of the \((2 - 1)\)-stage with a selected switch of the last stage.

Let us consider the multi-service switching network from Figure 1. If this network operates in the point-to-point selection mode, then this network can be considered to be a two-stage network with the point-to-group selection, as it is shown in Figure 2. If the control algorithm has chosen the first output link of the first switch of the second stage to execute a given connection, then all the inter-stage links that connect this switch with the switches of the second stage (marked in Figure 2 by bold lines) will be considered to be the output direction of the two-stage network.

Let us assume that \(s\) output links (\(1 \leq s \leq k\)) in a given direction of the two-stage network (Figure 2) can serve a call of class \(c\). Our further assumption is that the second stage of the switching network has \(d_2(c)\) switches (\(0 \leq d_2(c) \leq k\)) that are available to a given switch of the first stage in the network. The available switch is such a switch of the second stage to which a free connecting path for a call of class \(c\) from a given switch of the first stage can be set up. Note that the notion of availability does not mean, at the same time, that the available switch has free output links for a call of class \(c\) in the demanded direction.

The internal point-to-point blocking state occurs when the output links of a given direction in a two-stage network, that belong to the group \(d_2(c)\) of the switches of the second stage in the switching network, are occupied and cannot serve a call of class \(c\) due to insufficient number of free AUs. In the PPBMT method [21], the internal point-to-point blocking probability can be determined by the following formula:

\[
E_{\text{int}}(c) = \sum_{s_1=0}^{k-d_2(c)} [P_2(c, s_1, s_2 \geq 1)]_{k \times k} \frac{(k - s_1)}{(d_2(c))}, \quad (1)
\]

where \(k\) is the capacity of a given direction of the two-stage network, whereas \([P_2(c, s_1, s_2 \geq 1)]_{k \times k}\) is the so-called distribution of free input links in the symmetrical switch of the third stage, with the capacity \(k \times k\) links, tasked with executing a given connection. This distribution defines the probability that \(s_1\) input links (that belong to a given direction of the two-stage network) and \(s_2\) output links of a switch of the third stage are free, with the assumption that at least one output link (in the demanded direction of the three-stage network) is free (Appendix B). The application of the distribution \([P_2(c, s_1, s_2 \geq 1)]_{k \times k}\) in (1) results from the fact that the internal blocking (determined by the occupancy of the input links to the designated switch of the third stage – these links construct the output direction of the two-stage network – Figure 2) can occur exclusively in the case of a free output link of the switch of the third stage, designated earlier to execute the connection. The lower index “2” under the symbol \(P\) in \([P_2(c, s_1, s_2 \geq 1)]_{k \times k}\) means that this distribution is related to the output direction of the two-stage network. Since the distribution \([P_2(c, s_1, s_2 \geq 1)]_{k \times k}\) can be determined on the basis of the so-called LAG model (Limited Availability Group) [47], presented in Appendix B, then it can be symbolically written as follows:

\[
[P_2(c, s_1, s_2 \geq 1)]_{k \times k} = \text{LAG}(k, f, A_{\text{dir}, 2}). \quad (2)
\]

where \(k\) is the number of links in LAG, each with the capacity \(f\) AUs. The set \(A_{\text{dir}, 2} = \{A_{\text{dir}, 2}(c) : 1 \leq c \leq M\}\) is a set of traffic offered to the output group of the two-stage network. External blocking occurs when all \(k\) output links in a given direction of the three-stage network cannot service a call of class \(c\) due to insufficient number of free

![Figure 2. Multi-service, three-stage Clos switching network with the point-to-point selection.](image-url)
AUs. To determine the external blocking probability, the distribution of free links \(P_3(c, s)\) in the output group of the three-stage network, composed of \(k\) links in a given direction, is used (also approximated by the LAG model (Appendix B). This distribution determines the probability that \(s\) output links in the demanded direction of the three-stage network can service a call of class \(c\). The lower index “3” in \([P_3(c, s)]_k\) means that this distribution refers to the output direction of the three-stage network:

\[
[P_3(c, s)]_k = \text{LAG}(k, f, A_{\text{dir}, 3}),
\]

where \(A_{\text{dir}, 3} = \{A_{\text{dir}, 3}(c) : 1 \leq c \leq M\}\) is a set of traffic offered to the output group of the three-stage network. Using the distribution of free output links \([P_3(c, s)]_k\), the external blocking probability can be written as follows:

\[
E_{\text{ext}}(c) = [P_3(c, 0)]_k.
\]

The total blocking probability \(E(c)\), for a stream of class \(c\), is the aggregated sum of the external and internal blocking probabilities. Assuming no possibility of a concurrent occurrence of external and internal blocking (this is excluded by the control algorithm that first checks if there are free output links in a given direction), we can get:

\[
E(c) = E_{\text{ext}}(c) + E_{\text{int}}(c)[1 - E_{\text{ext}}(c)].
\]

The determination of the blocking probability for calls of class \(c\) requires the parameter \(d_2(c)\) to be known. This parameter is called effective availability of the switching network [21].

**B. EFFECTIVE AVAILABILITY**

The phenomenon of internal point-to-point blocking in the switching network causes that, following the selection of a free output link in a given switch of the third stage by the algorithm used to set up connections, not all output links in a given direction in the two-stage network (created by all these inter-stage links between the second stage and the third stage that lead to a given switch of the third stage – Figure 2), are available for the call that arrives at the input of a given switch of the first stage. The event of internal blocking results then from the same occupancy state of the switching network, in which it is not possible to set up a free connecting path between a given switch of the first stage (at the input of which a new call arrived) and a switch of the second stage that has at least one link in the demanded direction, i.e., a link that leads to a given switch of the third stage in which a free output link to execute the connection has been chosen. If this connection path can be set up, then a given switch of the second stage is available to a switch of the first stage. Effective availability is defined as the average number of available switches of the last stage of the switching network (in this particular case, the second stage). Note that in the adopted definition of the available switch, the problem of the occupancy of the output link of this switch still remains unsolved. What is important here is the execution of a free connection path between the selected switches of the first and the last stage [16]. Effective availability \(d_2(c)\) for calls of class \(c\) in the multi-service, two-stage switching network can be determined on the basis of the following formula [21]:

\[
d_2(c) = [1 - E_{\text{link}, 1-2}(c)]^k + [E_{\text{link}, 1-2}(c)]^2,
\]

where \(E_{\text{link}, 1-2}(c)\) is the blocking probability for calls of class \(c\) in one inter-stage link, between stages 1 and 2, of the two-stage switching network. A single inter-stage link is approximated by the FAG model (Full Availability Group) that can be symbolically written as follows:

\[
E_{\text{link}}(c) = \text{FAG}(1, f, A_{\text{link}}),
\]

where \(A_{\text{link}} = A_{\text{link}}(c) : 1 \geq c \geq M\) is a set of traffic offered to a single inter-stage link of the switching network. A model of the inter-stage link is presented in Appendix B.

**C. THE IPPBMT METHOD**

In the PPBMT method, the problem of the exclusion of the possibility of concurrent occurrence of the events of external and internal blocking is solved by subtracting from the sum of the external blocking probability and internal blocking probability their product. The assumption in the IPPBMT method is the same as it is proposed in [30], according to which the output links of a three-stage network can be offered only this part of the total traffic offered to the switching network that is not lost at the inter-stage links due to internal blocking. In turn, the inter-stage links can be offered only this part of the total traffic offered to the switching network that is not lost at the output links due to external blocking.

Such an approach means that in an evaluation of the internal blocking probability for calls of class \(c\), only this part of traffic \(A_{\text{int}}(c)\) is involved that is not rejected due to the external blocking. While estimating the external blocking probability, in turn, we take into consideration only on this part of traffic \(A_{\text{ext}}(c)\) that is not rejected due to internal blocking:

\[
A_{\text{int}}(c) = A(c)[1 - E_{\text{ext}}(c)],
\]

\[
A_{\text{ext}}(c) = A(c)[1 - E_{\text{int}}(c)].
\]

With these assumptions, the blocking probability \(E_{\text{link}, 1-2}(c)\), and in consequence effective availability for calls of class \(c\), in one inter-stage link of the switching network (Formula (6)) is determined as the function of traffic \(A_{\text{int}}(c)\):

\[
E_{\text{link}, 1-2}(c) = \text{FAG}(1, f, A_{\text{link}, 1-2}),
\]

\[
d_2(c) = F(A_{\text{link}, 1-2}),
\]

where each element of the set \(A_{\text{link}, 1-2}\) can be determined in the following way:

\[
A_{\text{link}, 1-2} = \frac{1}{k} A_{\text{int}}(c).
\]

With the estimation of the internal blocking probability, the parameter \(A_{\text{link}, 1-2}(c)\) defines this part of traffic offered to the switching network \(A_{\text{int}}(c)\) that is offered to
one inter-stage link, between stage 1 and stage 2. The accompanying assumption is that traffic in the symmetrical switching network from Figure 1 flows symmetrically between individual inter-stage links. Similarly, the distribution $[P_2(c, s_1, s_2 \geq 1)]_{k \times k}$, required to determine the internal blocking probability, is determined on the basis of traffic $A_{\text{int}}(c)$:

$$[P_2(c, s_1, s_2 \geq 1)]_{k \times k} = \text{LAG}(k, f, A_{\text{dir},2}),$$

(13)

where each element of the set $A_{\text{dir},2}(c)$ can be determined as follows:

$$A_{\text{dir},2}(c) = \frac{1}{k} A_{\text{int}}(c).$$

(14)

With the estimation of the internal blocking probability, the parameter $A_{\text{dir},2}(c)$ is this part of traffic offered to the switching network $A_{\text{int}}(c)$ that is offered to one output direction of the two-stage network. The accompanying assumption is that traffic in the symmetrical switching network from Figure 1 flows (is distributed) symmetrically between the directions of the switching network. Distribution $[P_3(c, s)]_k$, required for the determination of the external blocking probability, can be determined on the basis of traffic:

$$[P_3(c, s)]_k = \text{LAG}(k, f, A_{\text{dir},3}),$$

(15)

where each element of the set $A_{\text{dir},3}$ can be determined in the following way:

$$A_{\text{dir},3}(c) = \frac{1}{k} A_{\text{ext}}(c).$$

(16)

With the evaluation of the external blocking probability, the parameter $A_{\text{dir},3}(c)$ defines this part of traffic offered to the switching network $A_{\text{ext}}(c)$ that is offered to one output direction of the three-stage network. In the proposed IPPBMT method, the exclusion of the concurrency of the events of internal and external blocking is performed on the basis of an appropriate division of traffic (Formulas (8) and (9)). As the consequence of this operation, the total blocking probability can be directly written as the sum of the external blocking probability and the internal blocking probability:

$$E(c) = E_{\text{ext}}(c) + E_{\text{int}}(c).$$

(17)

To determine the probabilities $E(c), E_{\text{ext}}(c)$ and $E_{\text{int}}(c)$ for calls of class $c$ it is necessary to construct an itinerary process, which is written below. The assumption in the method is that $X^{[n]}$ determines the value of the parameter $X$ in the $n$-th step of the iteration process.

**IPPBMT method**

1. Initiation of the iteration step $n = 0$.
2. Determination of the initial approximations of the external blocking probability and the internal blocking probability for all traffic classes:

$$\forall 1 \leq c \leq M E^{[0]}_{\text{int}}(c) = E^{[0]}_{\text{ext}}(c) = 0.$$  

(18)

3. Increase in the iteration step:

$$n = n + 1.$$  

(19)

4. Determination of the value of offered traffic $A_{\text{int}}^{[n]}(c)$ i $A_{\text{ext}}^{[n]}(c)$:

$$\forall 1 \leq c \leq M A_{\text{int}}^{[n]}(c) = A(c)[1 - E_{\text{ext}}^{[n-1]}(c)]$$  

(20)

$$\forall 1 \leq c \leq M A_{\text{ext}}^{[n]}(c) = A(c)[1 - E_{\text{int}}^{[n-1]}(c)].$$  

(21)

5. Determination of the blocking probability for each traffic class in a single inter-stage link (Formula (10)):

$$\forall 1 \leq c \leq M E^{[n]}_{\text{link}}(c) = \text{FAG}(1, f, \frac{1}{k^2} A_{\text{ext}}^{[n]}).$$  

(22)

6. Determination of effective availability for each traffic class offered to the switching network (Formula (6)):

$$d^{[n]}_2(c) = \left[1 - E^{[n]}_{\text{link},1-2}(c)\right] k + \left[E^{[n]}_{\text{link},1-2}(c)\right]^2,$$  

(23)

where $c \in \{1, \ldots, M\}$.

7. Determination of the distribution of free output links in the two-stage network (Formula (13))

$$\forall 1 \leq c \leq M \left[P_2(c, s_1, s_2 \geq 1)\right]_{k \times k} = \text{LAG}(k, f, \frac{1}{k} A_{\text{ext}}^{[n]}).$$  

(24)

8. Determination of the internal blocking probability for each class of traffic offered to the switching network (Formula (1)):

$$E^{[n]}_{\text{int}}(c) = \sum_{s_1=1}^{k-d^{[n]}_2(c)} \left[\left[\frac{k-s_1}{d^{[n]}_2(c)}\right] k\right].$$  

(25)

9. Determination of the distribution of free output links in the three-stage network (Formula (15))

$$\forall 1 \leq c \leq M \left[P_3(c, s)\right]_k = \text{LAG}(k, f, \frac{1}{k} A_{\text{int}}^{[n]}).$$  

(26)

10. Determination of the external blocking probability in the switching network for all traffic classes (Formula (4)):

$$\forall 1 \leq c \leq M E^{[n]}_{\text{ext}}(c) = [P_3^{[n]}(c, 0)]_k.$$  

(27)

11. Determination of the total blocking probability in the switching network for all classes of calls (Formula (17)):

$$\forall 1 \leq c \leq M E^{[n]}(c) = E^{[n]}_{\text{ext}}(c) + E^{[n]}_{\text{int}}(c).$$  

(28)
12: Determination of the relative error for the required accuracy of the iteration process in relation to each traffic class:

\[ \forall 1 \leq c \leq M \quad \left| E^{[n]}(c) - E^{[n-1]}(c) \right| \leq \varepsilon. \]  

(29)

if the condition is not satisfied, pass on to Step 3, if the condition satisfied, this terminates the iteration process:

\[ E(c) = E^{[n]}(c), \quad E_{c}(c) = E_{c}^{[n]}(c), \]  

(30)

where \( c = \{1, \ldots, M\} \) and \( \varepsilon = \{\text{int, ext}\} \).

In the proposed IPPBMT algorithm, in each step of the iteration \( n \) the values of the blocking probabilities \( E^{[n]}(c) \), \( E_{\text{int}}^{[n]}(c) \), \( E_{\text{ext}}^{[n]}(c) \) are determined on the basis of offered traffic \( A_{\text{int}}^{[n]}(c) \) and \( A_{\text{ext}}^{[n]}(c) \), which is, in turn, calculated on the basis of the blocking probability \( E_{\text{ext}}^{[n-1]}(c) \), \( E_{\text{int}}^{[n-1]}(c) \), determined in the preceding step of the iteration (Step 4).

IV. METHODS TO DETERMINE THE BLOCKING PROBABILITY IN MULTICAST CONNECTIONS

The assumption in the article is that the algorithm to control setting up multicast connections operates in the following way. For a call of class \( c \) registered at the input of a switch of the first stage, the algorithm successively sets up component connections between the registered switch of the first stage and the switches of the third stage, and in particular the links that have free links in the demanded directions. This article considers two scenarios for the choice of the switches of the third stage. In the first scenario, while executing a given component connection, the control algorithm chooses output links in the demanded direction, after the set-up of the preceding component connection. In the second scenario, the control algorithm first chooses all output links in the direction demanded by a multicast connection and then sets up successive component connections to the selected output links. After setting up the first component connection, to execute the remaining component connection the control algorithm can make use of this part of the connection path of the first connection that leads from the registered switch of the first stage to the switch of this stage in which branching takes place. Note that such an assumption is crucial in the case of those control algorithms that make use of the switches of the second and third stage in the switching network to replicate information. In the case of the application of branching in the first stage, i.e. in the switch at the input of which the new call has arrived, the control algorithm will set up connections outgoing from this switch to selected switches of the third stage. In this section we will discuss mathematical models for switching networks with the point-to-point selection and multicast connections service.

A. PRELIMINARY ASSUMPTIONS

The assumption in the proposed model is that a multicast connection of class \( c \) will be rejected if only one of the component connections is in the blocking state. Therefore, the blocking probability can be written in the following way:

\[ E(c) = 1 - \prod_{u=1}^{q_c} \left[ 1 - E_u(c) \right]. \]  

(31)

The probability is the total blocking probability of the \( u \)-th component connection. It defines the event that the setup of the \( u \)-th component connection \( 1 \leq u \leq q_c \) of a given multicast connection of class \( c \) will fail to be executed (will be unsuccessful), on condition that the preceding attempts to set up the connection were successfully terminated. The parameter \( E_u \) can be determined on the basis of the method developed for single-service switching networks with the parameters that take into consideration successful set-up of preceding \( u - 1 \) connections.

B. MODELLING OF BRANCHING CONNECTIONS IN THE FIRST STAGE

In this particular case, to model a component multicast connection an appropriately modified Formulas (1)-(5) can be used. These formulas take into consideration the number of preceding component connections. Effective availability \( d_{1}(c) \) in a two-stage network for the first component connection of class \( c \) can be determined in exactly the same way as for the unicast connection, and in consequence the determination of the internal blocking probability for the first component connection \( E_{\text{int},1}(c) \) of class \( c \) is reduced to the direct use of Formula (1). Let us consider the way the parameters \( d_{1}(c) \) and \( E_{\text{int},1}(c) \) are determined for the second component connection. The second component connection can only be set up provided the first connection has been successfully set up. This situation is shown in Figure 3. This means that one switch of the third stage is already busy, and in consequence the output group for the second component connection in a two-stage network will be decreased by the link that leads to this switch. This situation is presented in Figure 3. In the figure, all the links and switches that are...
used to set up the first component connection are represented by double line, including the first switch of the second stage. Therefore, in line with the assumptions adopted earlier, this switch cannot be used to set up the second component connection, and in consequence the output group of the two-stage network for the second component connection, indicated in Figure 3 by bold lines, has the capacity $f$ links. For the $u$-th component connection ($1 \leq u \leq q_c$), the capacity of the output group will then be equal. The assumption in the model under consideration is that effective availability for successive component connections will be determined on the basis of Formula (6), that in the adopted notation can be written as follows:

$$d_{2,u}(c) = [1 - E_{\text{link},1-2}(c)] x + [E_{\text{link},1-2}(c)]^2,$$  \hspace{1cm} (32)

where $x = k - u + 1$. After determining effective availability, it is possible, on the basis of (1), to determine the internal blocking probability for the $u$-th component connection:

$$E_{\text{int},u}(c) = \sum_{s_1=0}^{k-d_{2,u}(c)} [P_{2,u}(c, s_1, s_2 \geq 1)]_{x\times x} \left[ \frac{x - s_1}{d_{2,u}(c)} \right],$$

$$E_{\text{int},u}(c) = \sum_{s_1=0}^{k-d_{2,u}(c)} \left[ \frac{(x - s_1)}{d_{2,u}(c)} \right] [P_{2,u}(c, s_1, s_2 \geq 1)]_{x\times x},$$  \hspace{1cm} (33)

where $x = k - u + 1$.

In the considerations on modelling multicast connections it is necessary to take into account traffic multiplexing in those stages of the switching network in which branching takes place. In the variant under consideration, branching of the connection occurs as early as the first stage, hence traffic of class $c$ offered to the switching network between the first and the second stage, as well as traffic offered to the switching network between the second and the third stage, will be multiplexed $q_c$ times. Since the total number of the inter-stage links between the neighbouring stages and the number of output links in the switching network under consideration is $k^2$, then the average traffic of class $c$ per one inter-stage link between the stages 1-2 and 2-3 and the output link will be equal to:

$$A_{\text{link},1-2,1}(c) = A(c) \phi_{\text{link},1-2,1}(c),$$

$$A_{\text{link},2-3,1}(c) = A(c) \phi_{\text{link},2-3,1}(c),$$

$$A_{\text{link},3-\text{exit},1}(c) = A(c) \phi_{\text{link},3-\text{exit},1}(c),$$

where:

$$\phi_{\text{link},1-2,1}(c) = \phi_{\text{link},2-3,1}(c) = \phi_{\text{link},3-\text{exit},1}(c) = \frac{q_c}{k^2}.$$  \hspace{1cm} (37)

The index of the multiplexing coefficient $\phi_{\text{group,place,X}(c)}$ can be determined by three symbols. The first index relates to the type of group (in the considered case it is a single link, therefore the abbreviation “link”) is used, the second symbol relates to the location of a group in the structure of the switching network (e.g. the abbreviation “1-2” defines the inter-stage link between stages 1 and 2, the abbreviation “exit” a single output link of a three-stage switching network). The third index ($X$, represented by the Roman numerals, defines the stage of the network in which branching occurs (e.g. “1” denotes branching in the first stage of the switching network). The control algorithm treats all output links independently of one another and hence the external blocking probability for each of the component connections will be identical and equal to:

$$\forall_{1 \leq e \leq M} \forall_{1 \leq u \leq q_c} \quad E_{\text{ext},u}(c) = \{P_3(c, 0)\}_e.$$  \hspace{1cm} (38)

During the analysis of the output directions of a switching network it is also necessary to take into consideration traffic multiplexing at the output links of the switching network, both two-stage and three-stage:

$$A_{\text{dir},2,1}(c) = A(c) \phi_{\text{dir},2,1}(c),$$

$$A_{\text{dir},3,1}(c) = A(c) \phi_{\text{dir},3,1}(c),$$

where the traffic multiplexing coefficients are equal to:

$$\phi_{\text{dir},2,1}(c) = \frac{q_c (k - u + 1)}{k^2},$$

$$\phi_{\text{dir},3,1}(c) = \frac{q_c}{k}.$$  \hspace{1cm} (41)

In Formulas (39)-(42), the notation used earlier in Formulas (24)-(37) is adopted and the symbol denotes the intensity of traffic of class $c$ offered to the output group of the $h$-stage of the $z$-stage switching network (i.e. the group that leads to the only switch of the stage ($h + 1$) or to a group that creates the output if $h = z$) in which branching occurs in the first stage. Formulas (37), (41) and (42) take into consideration the assumption adopted for the model that traffic of class $c$ offered to the switching network spreads out symmetrically among all inter-stage links and the links of corresponding output directions. In the case of the execution of branching in the first stage of the network, the choice of either of the scenarios for the determination of appropriate output links has no influence on the execution of the model of a given multicast connection. The determination of the switch of the third stage with an output link in the demanded direction directly after the setup of the preceding component connection (the first scenario) or the choice of all output links (the second scenario) and then the execution of the successive component connections have no influence on any change in the parameters used in Formulas (33) and (38), since the choices of appropriate output directions of the three-stage network are independent of one another.

**C. MODELLING OF BRANCHING CONNECTIONS IN THE SECOND STAGE**

Let us consider the first scenario, in which the control algorithm always chooses an appropriate output link in the demanded direction after setting up the preceding component connection. In this case, effective availability and the blocking probability for the first component connection are modelled exactly as in the case of the unicast connection, i.e. on the basis of Formulas (1)-(5). To set up a connection, the
second component connection uses the same switch of the second stage that was chosen to set up the first component connection (Figure 4). The selected switch will be also used to set up next component connections. Therefore, for the second component connection (executed after the effective setup of the first connection) the selected switch of the second stage is the only available switch, and in consequence effective availability for the second component connection \( u \) and every one that follows \((2 \leq u \leq q_c)\) is equal to the unity:

\[
d_{2,u}(c) = 1. \tag{43}
\]

The internal blocking probability (1) for each following component connection \((2 \leq u \leq q_c)\), after taking into account (32), can therefore be rewritten in the following way:

\[
E_{\text{int},u}(c) = \sum_{s_1=0}^{k-1} [P_{2,u}(c, s_1, s_2 \geq 1)]_{k \times k} \frac{k - s_1}{k}. \tag{44}
\]

In the considerations on the modelling of multicast connections branched off in the second stage it is necessary to take into consideration multiplexing of traffic between the first and the second stage and between the second and the third stage, as well as at the output links of the three-stage switching network. Therefore, the average traffic of class \( c \) per one inter-stage link between the stages 1-2 and 2-3 and the output link is:

\[
A_{\text{dir},2,\Pi,1}(c) = A(c) \phi_{\text{dir},2,\Pi,1}(c), \tag{45}
\]

\[
A_{\text{dir},3,\Pi,1}(c) = A(c) \phi_{\text{dir},3,\Pi,1}(c), \tag{46}
\]

\[
A_{\text{dir},\text{exit},1}(c) = A(c) \phi_{\text{dir},\text{exit},1}(c), \tag{47}
\]

where:

\[
\phi_{\text{link},1,2,\Pi}(c) = \frac{1}{k^2}, \tag{48}
\]

\[
\phi_{\text{link},2,3,\Pi}(c) = \phi_{\text{link},\text{exit},\Pi}(c) = \frac{q_c}{k^2}. \tag{49}
\]

Traffic multiplexing also takes place at the output links of the switching network:

\[
A_{\text{dir},2,\Pi,1}(c) = A(c) \phi_{\text{dir},2,\Pi,1}(c), \tag{50}
\]

\[
A_{\text{dir},3,\Pi,1}(c) = A(c) \phi_{\text{dir},3,\Pi,1}. \tag{51}
\]

where:

\[
\phi_{\text{dir},2,\Pi,i}(c) = \phi_{\text{dir},3,\Pi,1}(c) = \frac{q_c}{k}. \tag{52}
\]

The notation adopted in Formulas (45) and (52) is the same as the notation described earlier in Section IV-C. Hence, according to the adopted notation, the symbol denotes the intensity of traffic of class \( c \) offered to the output group of the \( h \)-stage switching network, in which branching takes place in the second stage. As in the case of branching in the first stage, the control algorithm treats all output directions independently of one another and therefore the external blocking probability for each of the component connections will be the same and will be equal to:

\[
V_{\text{ext},u}(c) = [P_3(c, 0)]_{k \times k}, \tag{44}
\]

where multiplexing of traffic at the links of the output direction of the three-stage switching network will be identical as in the case of the branching off in the first stage and will be expressed by Formula (52). Similarly, as in the case of branching connections in the first stage, in the considerations on branching in the second stage of the network, the choice as to the scenario for the determination of appropriate output links has no influence on the execution of the model of a given multicast connection. The determination of the switch of the third stage with the output link in the demanded direction directly after the setup of the preceding component connection (the first scenario) or the choice of all output links (the second scenario) and the execution of successive component connections have no influence on any change in the parameters of the model, since the choice of the links of appropriate output directions in the three-stage switching network is independent of one another.

### D. Modelling of Connections Branched Off in the Third Stage

In the case of connection branching in the third stage, the blocking probability is determined in a different way than in the general scheme, presented in Section IV-B. This approach results from the fact that the control algorithm uses one connecting path (between the pair of switches of the first and the third stage) for all component connections. Therefore, the internal blocking probability can be determined exclusively by the internal blocking probability for the first component connection that can be estimated on the basis of Formula (1). Hence we have:

\[
E_{\text{int},u}(c) = \begin{cases} Z & \text{for } u = 1, \\ 0 & \text{for } 1 < u \leq q_c, \end{cases} \tag{44}
\]
The choice of component links. Let us consider first the blocking probability of each successive \( u \)-th component connection \((1 \leq u \leq q_c)\) will be determined as the blocking probability of one output link (in the selected switch):

\[
E_{\text{ext},u}(c) = \text{FAG}(1, f, A_{\text{link,exit,III}}),
\]

where \( A_{\text{link,exit,III}} = \{A_{\text{link,exit,III}}(c) : 1 \leq c \leq M \} \) is the set of traffic offered to one output link of the switching network in which each element is determined by Formula (60). Let us now consider the second scenario, in which the control algorithm first chooses all output links in the directed direction of a multicast connection, and then sets up successively component connections to the selected output links. In this case, the internal blocking probability relates only to the setup of the first component connection and is, as in the first scenario, determined by Formula (54). The external blocking probability in the second scenario occurs when none of the switches of the third stage have free output links necessary for a connection of class \( c \) to be executed in the demanded \( q_c \) directions. Let us treat the output links of one switch of the last stage, \( q_c \) demanded by a given call, as the LAG. In such a case, the distribution \( [P_{\text{switch}}(c, s)]_{q_c} \) is a distribution of free links in the selected set of output links of one switch of the third stage:

\[
[P_{\text{switch}}(c, s)]_{q_c} = \text{LAG}(q_c, f, A_{\text{switch,III}}),
\]

where \( A_{\text{switch,III}} = \{A_{\text{switch,III}}(c) : 1 \leq c \leq M \} \) is the set of traffic offered to the selected link group of one switch of the third stage, in which each element is determined as follows:

\[
A_{\text{switch,III}}(c) = A(c) \phi_{\text{switch,III}}(c),
\]

where:

\[
\phi_{\text{switch,III}}(c) = \frac{q_c}{k^2}.
\]

The traffic multiplexing coefficient per one output link of the switching network can be determined by Formula (62). By multiplying this result by \( q_c \) links, we get (71). The probability \( [P_{\text{switch}}(c, q_c)]_{q_c} \) determines the event that \( q_c \) selected output links of this switch are capable of servicing a call of class \( c \) that demands \( t_c \) AUs in each of the links. The complementary probability \( 1 - [P_{\text{switch}}(c, s)]_{q_c} \) determines the event that at least one link, from among \( q_c \) links, is not capable of servicing a call that demands \( t_c \) AUs. Therefore it can be assumed that the probability \( 1 - [P_{\text{switch}}(c, q_c)]_{q_c} \) will approximate the external blocking probability of the switching network. Thus, we have:

\[
E_{\text{ext}}(i) = \begin{cases} 
1 - [P_{\text{switch}}(c, q_c)]_{q_c} & \text{for } u = 1, \\
0 & \text{for } 1 < u \leq q_c.
\end{cases}
\]
Traffic multiplexing occurs only and exclusively in output links of the switching network and thus traffic of class \( c \) per one inter-stage link between stages 1-2 and 2-3 \( A_{\text{link},1-2,III}(c) \), \( A_{\text{link},2-3,III}(c) \), one output link \( A_{\text{link},\text{exit},III}(c) \) and traffic offered to the output direction of the two-stage \( A_{\text{dir},2,III}(c) \) and three-stage networks \( A_{\text{dir},3,III}(c) \) can be determined in exactly the same way as in Scenario 1, i.e. on the basis of Formulas (58)-(66).

**E. THE IPPBMT-MCAST METHOD**

In this section an extension to the IPPBMT method is proposed to model the blocking probability in switching networks that execute multicast connections. This new IPPBMT-MCAST method can be written in the form of the following algorithm:

**IPPBMT method**

1: Initiation of the iteration step \( n = 0 \).
2: Determination of the stage \( r \), in which branching takes place \( (r \in \{I, II, III\}) \).
3: Determination of initial approximations of the external and internal blocking probabilities for all traffic classes:

\[
E^{[0]}_{\text{ext}}(i) = 0, \quad E^{[0]}_{\text{int}}(i) = 0. \quad (73)
\]

4: Increase in the iteration step:

\[
n = n + 1. \quad (74)
\]

5: Determination of the value of offered traffic of class \( c \):

\[
\forall 1 \leq c \leq M \quad A^{[n]}_{\text{int}}(c) = A_c[1 - E^{[n-1]}_{\text{ext}}(c)],
\]

\[
\forall 1 \leq c \leq M \quad A^{[n]}_{\text{ext}}(c) = A_c[1 - E^{[n-1]}_{\text{int}}(c)]. \quad (75)
\]

6: Determination of traffic offered to appropriate links and groups of the switching network,

\[
\forall 1 \leq c \leq M \quad \left\{ \begin{array}{l}
A^{[n]}_{\text{link},1-2,r}(c) = A^{[n]}_{\text{int}}(c) \phi_{\text{link},1-2,r}(c), \\
A^{[n]}_{\text{link},2-3,r}(c) = A^{[n]}_{\text{int}}(c) \phi_{\text{link},2-3,r}(c), \\
A^{[n]}_{\text{link},\text{exit},r}(c) = A^{[n]}_{\text{ext}}(c) \phi_{\text{link},\text{exit},r}(c), \\
A^{[n]}_{\text{dir},2,r}(c) = A^{[n]}_{\text{int}}(c) \phi_{\text{dir},2,r}(c), \\
A^{[n]}_{\text{dir},3,r}(c) = A^{[n]}_{\text{ext}}(c) \phi_{\text{dir},3,r}(c),
\end{array} \right.
\]

for (branching in the third stage), we additionally determine:

\[
\forall 1 \leq c \leq M \quad A^{[n]}_{\text{switch},3,III}(c) = A^{[n]}_{\text{ext}}(c) \phi_{\text{switch},3,III}(c), \quad (79)
\]

7: Determination of the blocking probability for each traffic class \( c \) in a single inter-stage link (Formula (10)),

- branching in the first stage:

\[
E^{[n]}_{\text{link},1-2,1}(c) = E^{[n]}_{\text{link},2-3,1}(c) = \text{FAG}(1, f, A^{[n]}_{\text{link},1-2,1}). \quad (80)
\]

- branching in the second stage:

\[
\forall 1 \leq i \leq M \quad \left\{ \begin{array}{l}
E^{[n]}_{\text{link},1-2,II}(c) = \text{FAG}(1, f, A^{[n]}_{\text{link},1-2,II}), \\
E^{[n]}_{\text{link},2-3,II}(c) = \text{FAG}(1, f, A^{[n]}_{\text{link},2-3,II}),
\end{array} \right.
\]

(81)

- branching in the third stage:

\[
E^{[n]}_{\text{link},1-2,III}(c) = E^{[n]}_{\text{link},2-3,III}(c) = \text{FAG}(1, f, A^{[n]}_{\text{link},1-2,III}). \quad (82)
\]

8: Determination of effective availability for each traffic class offered to the switching network (Formula (6)),

- branching in the first stage:

\[
d^{[n]}_{2,1}(c) = \left[ 1 - E^{[n]}_{\text{link},1-2,1}(c) \right] x + \left[ E^{[n]}_{\text{link},1-2,1}(c) \right]^2, \quad (83)
\]

where \( c = \{1, \ldots, M\}, u = \{1, \ldots, q_c\} \) and \( x = k - u + 1 \).

- branching in the second stage:

\[
d^{[n]}_{2,2}(c) = \left\{ \begin{array}{ll}
Z & \text{for } u = 1, \\
1 & \text{for } 1 < u \leq q_c,
\end{array} \right.
\]

(84)

9: Determination of the distribution of free output links in the two-stage network (Formula (9))

\[
\forall 1 \leq i \leq M \quad [P^{[n]}_{2}(c, s_1, s_2 \geq 1)]_{k \times k} = \text{LAG}(k, f, A^{[n]}_{\text{dir},2,r}). \quad (88)
\]

10: Determination of the internal blocking probability for each traffic class of traffic offered to the switching network (Formulas (1), (33), (44) and (54)),

\[
E^{[n]}_{\text{int},1-2}(c) = E^{[n]}_{\text{int},2-3}(c) = \text{FAG}(1, f, A^{[n]}_{\text{int},1-2}).
\]
branching in the first stage:

\[ E_{\text{int},u}^{[n]}(c) = \sum_{s_1=0}^{k-d_2,2(c)} Z \times \left[ \frac{X}{d_{2,u}^{[n]}(c)} \right] . \]  

(89)

where \( c = \{1, \ldots, M\} \), \( u = \{1, \ldots, q_c\} \) and \( x = k - u + 1 \).

- branching in the second stage:

\[ E_{\text{int},u}^{[n]}(c) = \begin{cases} \sum_{s_1=0}^{k-d_2,2(c)} Z \times X & \text{for } u = 1, \\ \sum_{s_1=0}^{k-d_2,2(c)} Z & \text{for } 1 < u \leq q_c. \end{cases} \]  

(90)

where \( c = \{1, \ldots, M\} \) and \( u = \{1, \ldots, q_c\} \).

- branching in the third stage:

\[ E_{\text{int},u}^{[n]}(c) = \begin{cases} \sum_{s_1=0}^{k-d_2,1(c)} Z \times X & \text{for } u = 1, \\ 0 & \text{for } 1 < u \leq q_c. \end{cases} \]  

(91)

where \( c = \{1, \ldots, M\} \) and \( u = \{1, \ldots, q_c\} \).

In Formulas (90) and (91), the parameter \( X \) denotes:

\[ X = \left[ \frac{k-d_2,1(c)}{d_{2,u}^{[n]}(c)} \right] . \]  

(92)

whereas in Formulas (89), (90) and (91), the parameter \( Z \) denotes:

\[ Z = \left[ P_{2,u}(c, s_1, s_2 \geq 1) \right]_{(i) \times (s)}. \]  

(93)

11: Determination of the distribution of free output links in the three-stage network (Formula (15)):

\[ \forall 1 \leq c \leq M \quad \forall 1 \leq u \leq q_c \quad \left[ P_{3}^{[n]}(c, s) \right]_k = \text{LAG}(k, f, A_{\text{int},3,c}^{[n]}). \]  

(94)

12: For the determination of the distribution of free output links in a selected set of output links of one third-stage switch (Formula (69)):

\[ \forall 1 \leq c \leq M \quad \forall 1 \leq u \leq q_c \quad \left[ P_{\text{switch}}(c, s) \right]^{[n]}_q = \text{LAG}(q_c, f, A_{\text{switch},3,\text{III}}^{[n]}). \]  

(95)

13: Determination of the external blocking probability in the switching network for all traffic classes (Formula (4)):

- branching in the first or second stage:

\[ E_{\text{ext},u}^{[n]}(c) = \left[ P_{3}^{[n]}(c, 0) \right]_k \]  

(96)

where \( c = \{1, \ldots, M\} \) and \( u = \{1, \ldots, q_c\} \).

- branching in the third stage, Scenario 1:

\[ E_{\text{ext},u}^{[n]}(c) = \begin{cases} \left[ P_{3}^{[n]}(c, 0) \right]_k & \text{for } u = 1, \\ E_{\text{link,ext,III}}^{[n]}(c) & \text{for } 1 < u \leq q_c. \end{cases} \]  

(97)

where \( c = \{1, \ldots, M\} \).

- branching in the third stage, Scenario 2:

\[ E_{\text{ext},u}^{[n]}(c) = \begin{cases} 1 - \left[ P_{\text{switch}}(c, q_i) \right]^{[n]} & \text{for } u = 1, \\ 0 & \text{for } 1 < u \leq q_c. \end{cases} \]  

(98)

where \( c = \{1, \ldots, M\} \).

14: Determination of the total blocking probability in the switching network for all call classes (Formula (11), (19), (21)):

\[ \forall 1 \leq c \leq M \quad \forall 1 \leq u \leq q_c \quad E_u^{[n]}(c) = E_{\text{ext},u}^{[n]}(c) + E_{\text{int},u}^{[n]}(c). \]  

(99)

\[ E(c) = 1 - \prod_{u=1}^{q_c} \left[ 1 - E_u^{[n]}(c) \right]. \]  

(100)

15: Checking of the relative error of calculations for the required accuracy of the iteration process in relation to each traffic class:

\[ \forall 1 \leq c \leq M \quad \left| \frac{E(c) - E^{(n-1)}(c)}{E^{(n)}(c)} \right| \leq \varepsilon \]  

(101)

the condition is not satisfied, pass on to Step 4, the condition satisfied - termination of the iteration process:

\[ E(i) = E^{[n]}(c), \quad E_{\varepsilon}(c) = E_{\varepsilon}^{[n]}(c), \]  

(102)

where \( c = \{1, \ldots, M\} \) and \( z = (\text{int}, \text{ext}) \).

V. RESULTS

The method to determine the point-to-point blocking probability in switching networks with multicast connections proposed in this article is an approximate method. To evaluate the accuracy of the proposed method as well as the assumptions adopted for the method, the results of analytical calculations were compared with the data obtained in simulation experiments. The simulation experiments were carried out for 3-stage Clos networks. The switching networks under consideration were composed of square switches with \( k \times k \) links, each with the capacity of \( f \) AUs.

The study was performed for the traffic intensity of traffic \( a \) offered to a single AU from within the range from 0.4 to 1.2 Erl. The results of the simulation experiments are presented in the form of points plotted on the graph with the confidence intervals (Formula (104)) that were determined on the basis of the t-Student distribution (with 95-percent confidence level) for 5 series in the function of traffic offered.
to a single AU. Traffic $a$ offered to a single AU can be determined on the basis of the following equation:

$$a = \sum_{i=1}^{M_k} t_i a_{E_n(i)} \frac{1}{kkf} + \sum_{j=1}^{M_k} t_j a_{E_n(j)} \frac{S_{E_n(j)}}{kkf} + \sum_{l=1}^{M_k} t_l a_{E_n(l)} \frac{S_{E_n(l)}}{kkf}.$$  \quad (103)

The duration time for each of the series was determined on the basis of the time needed to generate 10,000,000 calls of the least active class. In each of the cases the confidence interval did not exceed the 5% of the average value of the simulation experiment result. The results of the calculations are presented in the graphs in the form of solid lines.

The confidence intervals are determined in the following way:

$$\left( \bar{x} - t_a \frac{\sigma}{\sqrt{r}}, \bar{x} + t_a \frac{\sigma}{\sqrt{r}} \right)$$  \quad (104)

where $\bar{x}$ is the arithmetic mean calculated from $r$ results (simulation runs), $t_a$ is the value of the $t$-Student distribution for $r-1$ degree of freedom. The parameter $\sigma$ that determines the standard deviation can be calculated from the following formula:

$$\sigma^2 = \frac{1}{1-r} \sum_{s=1}^{r} x_s^2 - \frac{r}{r-1} \bar{x}^2,$$  \quad (105)

where $x_s$ is the result obtained in the $s$-th simulation run.

The study was performed for four exemplary systems, and the following was to be determined for each of the systems:

- the structure and capacity of the network under scrutiny (parameters $k, f$, and $V$),
- the number of traffic classes ($M$),
- the number of allocation units demanded by individual traffic classes (the parameter $t_c$ for class $c$),
- the parameters of exponential service time (the parameter $\mu_c$ for class $c$),
- in the case of the second, third and fourth system, the number of traffic classes and the type of traffic are additionally given, e.g. $S_{E_0}(c) = 400$ denotes that the class $c$ is a Pascal traffic class and that this traffic is generated by 400 traffic sources, the parameter $S_{E_0/P_s}(c)$ missing means that a given class is Erlang traffic class and that traffic of this class is generated by infinite number of traffic sources,
- multicast traffic class with the number of directions required by calls of this class to set up a connection ($q_c$, where $c$ is the multicast traffic class).

In the case of the exemplary systems presented in the article, the execution time of the computational program for one given parameter value was less than 1 second (running the program on a computer with Intel Xeon X5670 processor and 32 GB RAM memory).

**System 1**

- Structure of switching network: $k = 4, f = 32$ AUs, $V = 128$ AUs;
- Traffic classes: $M = 4, t_1 = 1$ AU, $\mu_1^{-1} = 1, t_2 = 4$ AUs, $\mu_2^{-1} = 1, t_3 = 6$ AUs, $\mu_3^{-1} = 1, t_4 = 8$ AUs, $\mu_4^{-1} = 1$;
- Multicast: $q_4 = 2$.

**System 2**

- Structure of switching network: $k = 4, f = 30$ AUs, $V = 120$ AUs;
- Traffic classes: $M = 3, t_1 = 1$ AU, $\mu_1^{-1} = 1, t_2 = 3$ AUs, $\mu_2^{-1} = 1, S_{E_0}(2) = 600, t_3 = 5$ AUs, $\mu_3^{-1} = 1, S_{E_0}(3) = 800$;
- Multicast: $q_3 = 2$.

**System 3**

- Structure of switching network: $k = 4, f = 34$ AUs, $V = 136$ AUs;
- Traffic classes: $M = 4, t_1 = 2$ AUs, $\mu_1^{-1} = 1, t_2 = 4$ AUs, $\mu_2^{-1} = 1, t_3 = 8$ AUs, $\mu_3^{-1} = 1, S_{E_0}(3) = 600, t_4 = 10$ AUs, $\mu_4^{-1} = 1, S_{E_0}(4) = 600$;
- Multicast: $q_4 = 2$.

**System 4**

- Structure of switching network: $k = 4, f = 40$ AUs, $V = 160$ AUs;
- Traffic classes: $M = 4, t_1 = 2$ AUs, $\mu_1^{-1} = 1, t_2 = 5$ AUs, $\mu_2^{-1} = 1, t_3 = 7$ AUs, $\mu_3^{-1} = 1, t_4 = 9$ AUs, $\mu_4^{-1} = 1, S_{E_0}(4) = 750$;
- Multicast: $q_4 = 3$.

A comparison of the results obtained for different systems shows that, irrespectively of the system, the blocking probabilities for calls of the multicast class (class 4 in systems 1, 3 and classes 4 and 3 system 2) always take on the highest values of the blocking probabilities for all traffic classes studied in a given system. This results from the way multicast calls are serviced, as they require concurrent availability of resources in two or three links that belong to the selected directions outgoing from the network, while this event is statistically less likely to happen than the event of availability of resources only in one link that belongs to the selected direction outgoing from the switching network. It should also be added that the existence of multicast traffic does not influence significantly any decrease in the accuracy of the results of calculations obtained for the remaining traffic classes.$^3$

This section presents the selected results of the study performed for four systems and four algorithms for connection branching in the switching network.

The assumption in the article is that branching can occur in the first (Figs. 5, 6, 7 and 8); the second (Figs. 9 and 10) and the third (concurrent -Figs. 11, 12 and sequential -Figs. 13 and 14) stages of the switching network.

An analysis of the obtained results in terms of the values and accuracy of calculations obtained for different scenarios

$^3$Due to the high number of graphs that go beyond the space limitations of this analysis, the blocking probability graphs obtained for Systems 1-4 without branching traffic flow have not been included in the article.
for broadcast traffic execution shows that the least accurate results of the calculations of the blocking probability for calls of multicast calls were obtained in the scenarios where the branching off of the connection was performed in the second stage (Figs. 9 and 10), and in the algorithm for concurrent connection branching in the third stage (Figs. 11 and 12). In turn, branching in the first stage (Figs. 5, 6, 7 and 8), and in the third stage with the sequential algorithm (Figs. 13 and 14) makes it possible to obtain the most accurate results of the calculations of blocking probabilities for calls of the class with branching irrespectively of the system under consideration. The differences in the accuracy of the computational methods employed in different scenarios result from the assumptions and approximations adopted by the authors at the modelling stage. It should be stressed though that according to the best knowledge of the authors, the proposed methods have no...
A comparison of the results for the blocking probability of multicast calls in a given system after the application of different scenarios for branching of connections shows that, irrespectively of the system under investigation, the lowest values of the blocking probability can be obtained by the introduction of branching in the third stage of the network, but with the assumption that free links in the demanded output directions in one of the switches of the third stage will be selected first, and then a connection to this switch will be set up. This value of the blocking probability results from branching of the connection in the last stage of the network, meaning only one connection is set up between the first and third stages of the network (in exactly the same way as it is performed for unicast connections) and with the output link choice algorithm in the third stage. For this particular case, the assumption is that first the switch that has free links in all directions demanded by a given call is chosen, and then a connection is set up in the network. Such a selection of the switch guarantees the least possible external blocking, and, for this particular case, also the least total blocking.

A comparison of the results presented in Fig. 5-14 shows that the proposed method for the determination of the point-to-point blocking probability in switching networks with multicast connections is characterised by very high accuracy for the whole range of the load in the system, limited only by the number of iterations. The calculations were performed with the observance of the relative error in the iteration process at the level of $\epsilon = 10^{-4}$ (Formula (101)).

VI. CONCLUSION

This article presents an analytical model of the switching network that is the basic element of the node structure in a telecommunications network. The assumption in the network model under consideration is that the node can service a mixture of different traffic streams of multi-service traffic, including multicast traffic. Another assumption in the presented model is that the algorithm that controls the network operates according to the point-to-point selection. Yet another assumption is that the modelled network has a Clos network structure, i.e. the structure commonly used in network nodes with large capacities, including optical networks [53], [54]. The assumption is that the network is capable of servicing branching connections, while any connection fan out can take place in different stages of the network. It is adopted then that the switches that provide connection spreading can be located in the first, second...
or third stage of a 3-stage Clos switching network. The developed analytical model of the switching network takes into consideration four algorithms for setting up branching connections.

The model proposed in the article is an approximated model, and hence the results of the analytical calculations are compared with the simulation results. The simulation experiments confirm fair and satisfactory accuracy of the proposed analytical model. In conclusion, the proposed computational method can be employed at the stage of dimensioning and optimization of the nodes of multi-service networks that service branching connections.

**APPENDIX A**

**MODELLING OF THE FAG**

The FAG is a group that can always service a new call of a given class if it only has sufficient number of AUs necessary for this call to be set up. Assume that the FAG with the capacity of $f$ AUs services a mixture of multi-service BPP traffic (Bernoulli, Pascal, Poisson) that includes the set $M_{\text{En}}$ of Engset traffic class (Bernoulli call stream, exponential service time), the set $M_{\text{Er}}$ of Erlang traffic class (Poisson call stream, exponential service time) and the set $M_{\text{Pa}}$ of Pascal traffic classes (Pascal call stream, exponential service time), where $M = M_{\text{En}} \cup M_{\text{Er}} \cup M_{\text{Pa}}$ is the set of all traffic classes with the count $M$. Let us consider then the model FAG($1, f, A$).

Note that the mixture of BPP traffic includes all “Markovian” dependencies of the change in the traffic intensity on the occupancy state. In the case of Erlang traffic of class $i$, offered traffic $A_{\text{En}}(i, n)$ in the occupancy state $n$ AUs is independent of the occupancy state, therefore:

$$\forall \ i \in M_{\text{En}} \quad \forall \ 1 \leq n \leq f \quad A_{\text{En}}(i, n) = A_{\text{En}}(i). \quad (106)$$

For Engset traffic $A_{\text{En}}(j, n)$, the traffic intensity decreases, whereas for Pascal traffic $A_{\text{Pa}}(l, n)$ the traffic intensity increases along with the increase in the occupancy state $n$:

$$\forall \ j \in M_{\text{En}} \quad \forall \ 1 \leq n \leq f \quad A_{\text{En}}(j, n) = \alpha_{\text{En}}(j) [S_{\text{En}}(j) - y_{\text{En}}(j, n)]. \quad (107)$$

$$\forall \ l \in M_{\text{Pa}} \quad \forall \ 1 \leq n \leq f \quad A_{\text{Pa}}(l, n) = \alpha_{\text{Pa}}(l) [S_{\text{Pa}}(l) + y_{\text{Pa}}(l, n)]. \quad (108)$$

where $S_X(c)$ ($X \in \{\text{En}, \text{Pa}\}$) represents the number of traffic sources of class $i$ type $X$. The parameter $\alpha_X(c)$ is the average traffic intensity of traffic generated by one free source of class $c$ type $X$, whereas the parameter $y_{XY}(c, n)$ is the average number of calls of class $c$ type $X$ serviced in the state $n$ busy AUs.

The average Erlang traffic of class $i$ offered to FAG can be determined by Formula (106). The average Engset traffic of class $j$ and Pascal traffic of class $l$ can be determined by the following Formulas [48]:

$$\forall \ j \in M_{\text{En}} \quad A_{\text{En}}(j) = \frac{S_{\text{En}}(j) \alpha_{\text{En}}(j)}{1 + \alpha_{\text{En}}(j)}. \quad (109)$$

$$\forall \ l \in M_{\text{Pa}} \quad A_{\text{Pa}}(l) = \frac{S_{\text{Pa}}(l) \alpha_{\text{Pa}}(l)}{1 - \alpha_{\text{Pa}}(l)}. \quad (110)$$

The blocking probability for calls of any class $c$ type $X$ ($X \in \{\text{Er}, \text{En}, \text{Pa}\}$) results from the lack of a sufficient number of free AUs in FAG:

$$E_{\text{FAG}} = \sum_{n=f}^{f-1} [P(n)]_f \quad (114)$$

In the occupancy distribution (111) there are parameters (Formulas (112) and (113)) that can be determined on the basis of the occupancy distribution (111). Hence, to determine the distribution (111), an appropriate iterative algorithm was used. In each step of the iteration of this algorithm, the approximation of the distribution (111) can be determined on the basis of the average values of the number serviced calls of Engset and Pascal classes, that were in turn determined in the preceding step of the iteration. A detailed description of the algorithm can be found in [51].

**APPENDIX B**

**MODELLING OF LAG**

LAG is composed of $k$ separated links called component links, each with the capacity of $f$ AUs. The notion of separation results from the call admission algorithm. According to this algorithm, a call can be admitted for service only when there is at least one component link that can service this call. This means that the algorithm excludes the possibility of a “division” of the AUs demanded by a given call among a number of links. Our assumption is, as it is presented in Appendix VI, that LAG services a mixture of multi-service BPP traffic that includes the set $M_{\text{En}}$ of Engset traffic classes, the set $M_{\text{Er}}$ of Erlang traffic classes and the set $M_{\text{Pa}}$ of Pascal traffic classes, where $M = M_{\text{En}} \cup M_{\text{Er}} \cup M_{\text{Pa}}$ is the set of AUs.

FAG with BPP traffic is described, among others, in [49], [50], where the occupancy distribution in a group for BPP traffic is derived. This occupancy distribution can be written as follows:

$$[P(n)]_f = \frac{1}{n} \left\{ \sum_{M_{\text{En}}} A_{\text{En}}(t_i) [P(n - t_i)]_f + \sum_{M_{\text{Er}}} \alpha_{\text{Er}} [S_{\text{Er}}(j) - y_{\text{Er}}(j, n)] t_j [P(n - t_j)]_f + \sum_{M_{\text{Pa}}} \alpha_{\text{Pa}} [S_{\text{Pa}}(l) + y_{\text{Pa}}(l, n)] \eta [P(n - \eta)]_f \right\}. \quad (111)$$

where $[P(n)]_f$ is the occupancy probability of $n$ AUs in FAG with the capacity $f$ AUS. The parameters $y_{\text{En}}(j, n)$, $y_{\text{Pa}}(l, n)$ are the average values for the number serviced calls of class $j$ Engset type and class $l$ Pascal type in the occupancy state $n$:

$$y_{\text{En}}(j, n) = \frac{\alpha_{\text{En}}(j) [S_{\text{En}}(j) - y_{\text{En}}(j, n)] [P(n - t_j)]_f}{[P(n)]_f} \quad (112)$$

$$y_{\text{Pa}}(l, n) = \frac{\alpha_{\text{Pa}}(l) [S_{\text{Pa}}(l) + y_{\text{Pa}}(l, n - t_j)] [P(n - \eta)]_f}{[P(n)]_f} \quad (113)$$

The blocking probability for calls of any class $c$ type $X$ ($X \in \{\text{Er}, \text{En}, \text{Pa}\}$) results from the lack of a sufficient number of free AUs in FAG:

$$E_{\text{LAG}} = \sum_{n=f}^{f-1} [P(n)]_f \quad (114)$$

In the occupancy distribution (111) there are parameters (Formulas (112) and (113)) that can be determined on the basis of the occupancy distribution (111). Hence, to determine the distribution (111), an appropriate iterative algorithm was used. In each step of the iteration of this algorithm, the approximation of the distribution (111) can be determined on the basis of the average values of the number serviced calls of Engset and Pascal classes, that were in turn determined in the preceding step of the iteration. A detailed description of the algorithm can be found in [51].
of all traffic classes with the count $M$. The formulas that determine the average values of traffic of respective classes, presented in Appendix A remain as relevant as ever. In line with the adopted notation, the model under consideration can be written as $\text{LAG}(1, f, A)$.

The occupancy distribution in the LAG with the capacity $kf$ AUs can be approximated by the following recursive dependence [50]:

$$
[P(n)]_{kf} = \frac{1}{n} \left( \sum_{M \in \mathbb{M}} \alpha_{En} \sigma_i (n - t_j) \left[ P(n - t_i) \right]_{kf} + \sum_{M \in \mathbb{M}} \alpha_{En} [S_{En}(j) - y_{En}(j, n)] t_j \sigma_i (n - t_j) \times [P(n - t_i)]_{kf} \right. $$

$$
\left. + \sum_{M \in \mathbb{M}} \alpha_{Pa} [S_{Pa}(l) + y_{Pa}(l, n)] t_i \sigma_i (n - t_i) \times [P(n - t_i)]_{kf} \right),
$$

(115)

where $[P(n)]_{kf}$ is the occupancy probability $n$ AUs in LAG with the capacity $kf$ AUS. The parameters $\alpha_{i}(i)$ and $S_{X}(i)$, where $X, X \in \{\text{En}, \text{Pa}\}$, are represented analogously as in Appendix A. The parameters $y_{En}(j, n)$ and $y_{Pa}(l, n)$ are the average values of the number of serviced calls of the Engset and Pascal classes in the occupancy state $n$:

$$
y_{En}(j, n) = \frac{\alpha_{En}(j) \times Z}{[P(n)]_{kf}}
$$

(116)

In Formula (116), the parameter $Z$ denotes:

$$
Z = [S_{En}(j) - y_{En}(j, n - t_j)] \sigma_i (n - t_j) \left[ P(n - t_i) \right]_{kf}.
$$

(117)

$$
y_{Pa}(l, n) = \frac{\alpha_{Pa}(l) \times X}{[P(n)]_{kf}}
$$

(118)

In Formula (118), the parameter $X$ denotes:

$$
X = [S_{En}(l) - y_{Pa}(l, n - t_i)] \sigma_i (n - t_i) \left[ P(n - t_i) \right]_{kf}.
$$

(119)

The parameter $\sigma_i (n)$ w (115) is the so-called conditional transition probability between states $n$ and $n + t_c$ for calls of class $c$. This parameter, for the state $n$ busy AUSs, determiners the probability of such a distribution of free AUs in LAG in which it is possible to serve a new call of class $c$ [47]:

$$
\sigma_i (n) = \frac{F(kf - n, k, f, 0) - F(kf + n, k, t_c, 1, 0)}{F(kf - n, k, f, 0)}.
$$

(120)

The function $F(x, k, f, h)$ determines in a combinatorial way the number of arrangements of $x$ free AUs in the $k$ component links, each with the capacity $f$ AUs. The accompanying assumption is that in each component link there are $h$ free AUs:

$$
F(x, k, f, h) = \sum_{c=0}^{\left\lfloor \frac{x - h}{f - h} \right\rfloor} (-1)^{\left\lfloor \frac{x - h}{f - h} \right\rfloor} \binom{k}{c} \binom{w}{k - 1}.
$$

(121)

where $w = x - k(h - 1) - 1 - c(f - h + 1)$.

The occupancy distribution in LAG (Formula (115)) can be determined on the basis of the appropriate iteration algorithm that is described in [50], among others.

The distribution (115) provides the basis for the determination of the so-called distribution of free component links $[P(c, s)]_{k}$ [47] that describes the event that $s$ links in LAG are capable of servicing a call of class $c$:

$$
[P(c, s)]_{k} = \sum_{n=0}^{kf} [P(n)]_{kf} [P(c, s[kf - n])].
$$

(122)

where $[P(c, s[k])]_{k}$ is the conditional distribution of free links for a call of class $c$, that can be determined on condition that $x$ AUs in LAG is free:

$$
[P(c, s|k)]_{k} = \left( \prod_{z=0}^{k} \sum_{s \geq zf} F(z, s, f, 0) F(x - z, k - s, t_c - 1, 0) \right) \frac{1}{F(x, k, f, 0)},
$$

(123)

where $\Psi = sf$ for $x \geq sf$ and $\Psi = x$ for $x \leq sf$.

Note that the probability $[P(0|0)]_{k}$ determines the event in which none of $k$ component links can service a call of class $c$ due to the lack of sufficient number of AUs. This is then the blocking probability for calls of class $c$ in LAG:

$$
E_{\text{LAG}}(i) = [P(c, 0)]_{k}.
$$

(124)

The distribution $[P(c, s)]_{k}$ makes it possible to determine the distribution of free links in the switch $[P(c, s, 1) \geq 1]_{kk}$ [52]. In the case of the symmetrical switches with $k \times k$ links, the fact that at least one of the output links of the switch can service a call of class $i$ does not mean that at least one of its output links is free. The distribution of free links in the switch determines the probability of the event that $s_1$ free input links of the switch and $s_2 (s_2 \geq 1)$ free output links of this switch are free. The limitation $s_2 \geq 1$ results from the operation of the connection setup algorithm in the switching network. The algorithm makes an attempt to set up a connection between given switches of the first and third stages only when the latter one has a free output link in the demanded direction.

The input group and the output group of the switch with $kk$ links create groups with limited availabilities with the capacity equal to $kf$ AUs. While determining the distribution $[P(c, s_1, s_2 \geq 1)]_{kk}$, the assumption is that the distributions of free links in the input and output groups in the switch are independent of one another. Such an assumption makes it possible to determine the conditional distribution of free links in the switch $[P(c, s_1, s_2 \geq 1|x)]_{kk}$, provided that $x$ free AUs are in both the input group and the output group of the switch [21], [52]:

$$
[P(c, s_1, s_2 \geq 1|x)]_{kk} = [P(c, s_1)]_{k} \times Z.
$$

(125)

On the basis of the theorem on the total probability (the law of total probability), we can determine the unconditional
distribution of free links in the switch:

\[
[P(c, s_1, s_2 \geq 1|x)]_{ik} = \sum_{x=0}^{kf} [P(c, s_1|x)]_k Z[P(kf - x)]_{ik}.
\]  

(126)

The event space \((126)\) is determined by the pairs \((s_1, s_2)\). Since the pairs \((s_1, 0)\), following the adopted assumption \(s_2 \geq 1\), cannot occur, then the distribution \((126)\) was replaced by an appropriate truncated distribution:

\[
[P(c, s_1, s_2 \geq 1|x)]_{ik} = \frac{Z \times X}{1 - \sum_{x=0}^{kf} \sum_{s_1} Z \times X}.
\]  

(127)

In Formulas \((125)\) and \((126)\) and \((127)\), the parameter \(Z\) denotes:

\[
Z = \left[1 - \frac{P(c, s_2 = 0|x)]_i}{P}\right] \times P(c, s_1|x)_k \times P(kf - x)]_{ik}.
\]  

(128)

whereas the parameter \(X\) in Formula \((127)\) denotes:

\[
X = [P(c, s_1|x)]_k \times [P(kf - x)]_{ik}.
\]  

(129)

As results from the comparison \((126)\) and \((127)\), the distribution of free links in the switch, determined by Formula \((127)\), excludes the possibility of the occupancy of all output links of the switch for a call of class \(i\). Thus defined distribution takes into consideration then the operation of the algorithm for setting up connections in switching networks in which the controlling device makes an attempt to set up a connection exclusively for the case where there exists one free output in a given switch of the last stage.

REFERENCES

[1] L. Zhao, P. Shi, and H. Zhang, “Bi-directional Benes with large port-counts and low waveguide crossings for optical network-on-chip,” IEEE Access, vol. 9, pp. 115788–115800, 2021, doi: 10.1109/ACCESS.2021.3105137.

[2] M. K. Khattak, Y. Tang, H. Fahim, E. Rehman, and M. F. Majeed, “Effective routing technique: Augmenting data center switch fabric performance,” IEEE Access, vol. 8, pp. 37372–37382, 2020, doi: 10.1109/ACCESS.2020.2973932.

[3] C. Wu, L. Qiao, and Q. Chen, “Design of a 640-Gbps two-stage switch fabric for satellite on-board switches,” IEEE Access, vol. 8, pp. 68725–68735, 2020, doi: 10.1109/ACCESS.2020.2986300.

[4] C. Clos, “A study of non-blocking switching networks,” Bell Labs Tech. J., vol. 32, no. 2, pp. 406–424, 1953.

[5] K. Kabacinski, Nonblocking Electronic and Photonic Switching Fabrics., Boston, MA, USA: Springer, 2005, doi: 10.1007/b137691.

[6] K. Zahoor, K. Biélar, A. Erbad, A. Mohamed, and M. Guizani, “Multicast at edge: An edge network architecture for service-less crowdsourced live video multicast,” IEEE Access, vol. 9, pp. 59508–59526, 2021, doi: 10.1109/ACCESS.2021.3070814.

[7] W. Kabacinski, Nobilegging and Cloud Computing., Boston, MA, USA: Springer, 2005, doi: 10.1007/b137691.

[8] S. Hanczewski, M. Sobieraj, and M. D. Stasiak, “The direct method of effective availability for switching networks with multi-service traffic,” IEICE Trans. Commun., vol. E96-B, no. 6, pp. 1291–1301, 2016.

[9] M. Głąbowski, “Recurrent method for blocking probability calculation in multi-service switching networks with BPP traffic,” in Proc. 5th Eur. Perform. Eng. Workshop (CEPEW) (Lecture Notes in Computer Science), vol. 5261, N. Thomas and C. Juiz, Eds. Berlin, Germany: Springer, 2008, pp. 152–167, doi: 10.1007/978-3-540-87412-6_12.

[10] M. Głąbowski and M. D. Stasiak, “Modelling of multiservice switching networks with overflow links for any traffic class,” IET Circuits, Devices Syst., vol. 8, no. 5, pp. 358–366, 2014.

[11] M. Głąbowski and M. Sobieraj, “Analytical modelling of multiservice switching networks with multiservice source and resource management mechanisms,” Telecommun. Syst., vol. 66, no. 3, pp. 559–578, Nov. 2017.

[12] S. Hanczewski, M. Sobieraj, and M. D. Stasiak, “The direct method of effective availability for switching networks with multi-service traffic,” IEICE Trans. Commun., vol. E99-B, no. 6, pp. 1291–1301, 2016.

[13] M. Głąbowski, “Recurrent method for blocking probability calculation in multi-service switching networks with BPP traffic,” in Proc. 5th Eur. Perform. Eng. Workshop (CEPEW) (Lecture Notes in Computer Science), vol. 5261, N. Thomas and C. Juiz, Eds. Berlin, Germany: Springer, 2008, pp. 152–167, doi: 10.1007/978-3-540-87412-6_12.

[14] M. Głąbowski and M. D. Stasiak, “Point-to-point blocking in the switching networks with unicast and multicast switching,” Perform. Eval., vol. 48, nos. 1–4, pp. 249–267, 2002.

[15] L. Zhao, P. Shi, and H. Zhang, “Bi-directional Benes with large port-counts and low waveguide crossings for optical network-on-chip,” IEEE Access, vol. 9, pp. 115788–115800, 2021, doi: 10.1109/ACCESS.2021.3105137.

[16] M. K. Khattak, Y. Tang, H. Fahim, E. Rehman, and M. F. Majeed, “Effective routing technique: Augmenting data center switch fabric performance,” IEEE Access, vol. 8, pp. 37372–37382, 2020, doi: 10.1109/ACCESS.2020.2973932.

[17] C. Wu, L. Qiao, and Q. Chen, “Design of a 640-Gbps two-stage switch fabric for satellite on-board switches,” IEEE Access, vol. 8, pp. 68725–68735, 2020, doi: 10.1109/ACCESS.2020.2986300.

[18] C. Clos, “A study of non-blocking switching networks,” Bell Labs Tech. J., vol. 32, no. 2, pp. 406–424, 1953.

[19] W. Kabacinski and F. K. Lioutopoulos, “Multirate non-blocking generalized three-stage Clos switching networks,” IEEE Trans. Commun., vol. 50, no. 9, pp. 1486–1494, Sep. 2002.

[20] W. Kabacinski, Nonblocking Electronic and Photonic Switching Fabrics., Boston, MA, USA: Springer, 2005, doi: 10.1007/b137691.

[21] K. Zahoor, K. Biélar, A. Erbad, A. Mohamed, and M. Guizani, “Multicast at edge: An edge network architecture for service-less crowdsourced live video multicast,” IEEE Access, vol. 9, pp. 59508–59526, 2021, doi: 10.1109/ACCESS.2021.3070814.

[22] J. Alqahtani, H. H. Sinky, and B. Hamdaoui, “Clustered multicast and low waveguide crossings for optical network-on-chip,” IEEE Access, vol. 9, pp. 12693–12705, 2021, doi: 10.1109/ACCESS.2021.3051874.

[23] N. Bininda and W. Wendt, “Die effektive erreichbarkeit für abnehmerbundel hinter zwischenleitungsanlagen,” Nachrichtentechnische Zeitschrift, vol. 11, no. 12, pp. 579–585, 1959.

[24] A. D. Charkiewicz, “An approximate method for calculating the number of junctions in a crossbar system exchange,” Elektrotechnic, vol. 2, pp. 55–63, 1959.
MACIEJ STASIAK (Member, IEEE) received the M.Sc. and Ph.D. degrees in electrical engineering from the Institute of Communications Engineering, Moscow, Russia, in 1979 and 1984, respectively, and the D.Sc. degree in electrical engineering from the Poznań University of Technology, in 1996. In 2006, he was nominated as a Full Professor. From 1983 to 1992, he worked with the Polish Industry Sector as a Designer of electronic and microprocessor systems. In 1992, he joined the Poznań University of Technology, where he is currently the Director of the Faculty of Computing and Telecommunications, Institute of Communications and Computer Networks. He is the author or coauthor of more than 300 scientific papers and five books. His research interests include performance analysis and modeling of queuing systems, multi-service networks, and switching systems. Since 2004, he has been actively carrying out research on modeling and dimensioning of cellular networks.

MACIEJ SOBIERAJ received the M.Sc. and Ph.D. degrees in electronics and telecommunications from the Poznań University of Technology, Poland, in 2008 and 2014, respectively. Since 2007, he has been working at the Faculty of Computing and Telecommunications, Institute of Communications and Computer Networks, Poznań University of Technology. He is the coauthor of more than 40 scientific papers. His research interests include modeling multi-service cellular systems and switching networks and traffic engineering in TCP/IP networks.

PIOTR ZWIERZYKOWSKI (Senior Member, IEEE) received the M.Sc., Ph.D. (Hons.), and D.Sc. degrees in telecommunication from the Poznań University of Technology, Poland, in 1995, 2002, and 2015, respectively. Since 1995, he has been working at the Faculty of Computing and Telecommunications, Institute of Communications and Computer Networks, Poznań University of Technology. His research interests include performance analysis and modeling of multi-service networks and switching systems. He is the author or coauthor of four books, 33 book chapters, over 50 articles, and over 140 articles that have been published in communication magazines and presented at the national and international conferences. Recently, he has been also working as the guest/lead editor for numerous journals published by Elsevier, Hindawi, IEICE, IET, MDPI, and Wiley.