Masses and couplings of the negative parity excited baryons are studied in the QCD sum rule. Separation of the negative-parity spectrum is proposed and is applied to the flavor octet and singlet baryons. We find that the quark condensate is responsible for the mass splitting of the ground and the negative-parity excited states. This is expected from the chiral symmetry and supports the idea that the negative-parity baryon forms a parity doublet with the ground state. The meson-baryon coupling constants are also computed for the excited states in the QCD sum rule. It is found that the $\pi NN^*$ coupling vanishes in the chiral limit.

1 Introduction

The negative parity baryons have been successfully described by the (nonrelativistic) quark model as one-quark excited states belonging to the SU(6) $^{70}$ representation. The observed spectrum tends to agree with the prediction, although some of the states, such as $\Lambda(1405)$, $N(1535)$, have irregular masses and non-natural decay rates. Many refinements were proposed to achieve a quantitative agreement.

Yet, our understanding of the hadron physics insists that the role of the chiral symmetry must be important even in the baryon states. Indeed, the chiral symmetry suggests that the positive and negative parity states are paired into a parity doublet and the pair would be degenerate when the chiral symmetry is restored. Because the nonrelativistic quark model does not observe the chiral symmetry, a different approach is anticipated to understand the chiral structure of baryons, both the positive and negative parity states.

The technique of the QCD sum rule relates the hadron properties to the QCD parameters and is a powerful tool to extract the hadron properties from QCD, the first principle of the strong interaction. The QCD sum rule for the baryon, first proposed by Ioffe, considers a correlation function of an interpolating field (IF) that couples to the baryon state in question. The correlation function is computed in two ways, phenomenologically and theoretically, and their equality gives a sum rule. The nonperturbative effects, such as the quark condensate $\langle \bar{q}q \rangle$ and $\langle \alpha_s \pi GG \rangle$, are included as the power corrections of the operator product expansion of the correlator.

*e-mail address: oka@th.phys.titech.ac.jp
in the theoretical side. The quark condensate $\langle \bar{q}q \rangle$, which is the order parameter of the chiral symmetry breaking, gives effects of the chiral symmetry breaking to the hadron spectrum in the QCD sum rule.

We here study the masses and couplings of the negative-parity baryons in the QCD sum rule approach. Our aim is to study roles of the chiral symmetry in baryon excitations. We first present a technique to separate the positive and negative parity states in the sum rule, and then calculate their masses. It is important to separate contribution of the negative-parity baryon $(B^-)$ from that of the positive-parity baryon $(B^+)$. Since the IF for the baryon couples to both $B^-$ and $B^+$ simultaneously, in order to separate the $B^-$ contribution we use the “old-fashioned” correlation function defined as

$$\Pi(p) = i \int d^4 x e^{ip \cdot x} \theta(x_0) \langle 0 | J_B(x) \bar{J}_B(0) | 0 \rangle,$$

and construct sum rules in the rest frame ($\vec{p} = 0$). Our approach is suitable for investigating the mass splitting, because $B^+$ and $B^-$ can be treated simultaneously in this sum rule. We apply the technique not only to the flavor octet baryons (nucleons and hyperons) but also to the flavor singlet baryon $\Lambda_S$.

2 Negative-parity Baryons in the QCD Sum Rule

It is important to note that the interpolation field (IF) for a baryon does not specify its parity. In the mesonic case, the parity of the state is directly connected to the parity of the IF. For instance, the IF for the $\rho^+$-meson is a vector current, $\bar{d}\gamma_\mu u$, while that for $a_1^+$-meson, which is the chiral partner of $\rho$, is an axial vector current, $\bar{d}\gamma_\mu \gamma_5 u$. This is not the case for the baryon. The IF for the nucleon, for instance, is given by

$$J_N(x) = J^+(x) = \epsilon_{abc}[(u_a(x)C d_b(x))\gamma_5 u_c(x) + t(u_a(x)C \gamma_5 d_b(x))u_c(x)],$$

where $a$, $b$ and $c$ are color indices, $C = i\gamma_2 \gamma_0$ (standard notation) is for the charge conjugation and $t$ is a real parameter representing the mixing of two independent IFs. Then it may seem that the negative-parity baryon couples to the IF $J_\sim \equiv i\gamma_5 J^+$ because multiplying $i\gamma_5$ changes the “parity”. If one supposes that the correlation function of $J_N$ is given by

$$\Pi_+(p) = p_\mu \gamma^\mu \Pi_1(p^2) + \Pi_2(p^2),$$

then the correlation function for $J_\sim$ can be written as

$$\Pi_-(p) = -\gamma_5 \Pi_+(p) \gamma_5 = p_\mu \gamma^\mu \Pi_1(p^2) - \Pi_2(p^2).$$

The difference between $\Pi_+$ and $\Pi_-$ appears only in the sign in front of $\Pi_2(p^2)$. That is, they are given by the same functions $\Pi_1(p^2)$ and $\Pi_2(p^2)$. This means that the
information of the negative-parity nucleon is already included in $\Pi^+(p)$ since $J_N$ couples not only to the positive-parity nucleon but also to the negative-parity excited state $|B^-\rangle$. It is easy to see this from

$$
\langle 0|J_+|B^-\rangle\langle B^-|J_+|0\rangle = -\gamma_5\langle 0|J_-|B^-\rangle\langle B^-|J_-|0\rangle\gamma_5,
$$

where $|B^-\rangle$ denotes a single baryon state with negative parity. $J_-$ couples to the positive-parity states in the same way.

We have proposed a formulation for separating the negative-parity contribution from the sum rule (5). To do so, we use the “old-fashioned” correlation function (1). In the zero-width resonance approximation, we write the imaginary part in the rest frame $\vec{p} = 0$ as

$$
\text{Im } \Pi(p_0) = \sum_n \left[ (\lambda_n^+)^2 \frac{\gamma_0}{2} + \frac{1}{2} \delta(p_0 - m_n^+) + (\lambda_n^-)^2 \frac{\gamma_0 - 1}{2} \delta(p_0 - m_n^-) \right]
$$

$$
\equiv \gamma_0 A(p_0) + B(p_0),
$$

where $m_n^\pm$ denotes the mass of the $n$-th resonance and $\lambda_n^\pm$ the coupling strength of the IF to the resonance. Now, one sees that $A(p_0)$ and $B(p_0)$ are given by

$$
A(p_0) = \frac{1}{2} \sum_n [(\lambda_n^+)^2 \delta(p_0 - m_n^+) + (\lambda_n^-)^2 \delta(p_0 - m_n^-)],
$$

$$
B(p_0) = \frac{1}{2} \sum_n [(\lambda_n^+)^2 \delta(p_0 - m_n^+) - (\lambda_n^-)^2 \delta(p_0 - m_n^-)].
$$

and that the combination $A(p_0) + B(p_0)$ ($A(p_0) - B(p_0)$) contains only the positive-parity (negative-parity) states.

We, however, can no longer construct sum rules in $p^2$-space, since the “old-fashioned” correlation function is not analytic in $p^2$ space. Instead a dispersion relation can be written in the complex $p_0$ plane, because the correlation function (1) is analytic in the upper-half region of the complex $p_0$ plane. The theoretical side is given by the operator product expansion, which is valid at high energy i.e. $\Pi^{\text{OPE}}(p_0 = Q) \simeq \Pi^{\text{Phe}}(p_0 = Q)$ at large $|Q|$. Using the analyticity we obtain

$$
\int_0^Q [A^{\text{OPE}}(p_0) - A^{\text{Phe}}(p_0)] W(p_0) \, dp_0 = 0,
$$

$$
\int_0^Q [B^{\text{OPE}}(p_0) - B^{\text{Phe}}(p_0)] W(p_0) \, dp_0 = 0,
$$

where $W(p_0)$ is an arbitrary analytic function which is real on the real axis. Note that we use the fact that the imaginary part of the correlation vanishes for negative $p_0$. 


The standard choice of $W$ is the Borel weight $W(p_0) = \exp(-\frac{p_0^2}{M^2})$. We take the lowest mass pole and approximate others as continuum whose behavior above a threshold $s_0^t$ coincides with the theoretical side. Then we obtain two sum rules

$$\frac{1}{2}[\tilde{A}^{\text{OPE}}(M, s_0^+)) + \tilde{B}^{\text{OPE}}(M, s_0^+)) = (\lambda^+)^2 \exp[-\frac{(m^+)^2}{M^2}], \quad (9)$$
$$\frac{1}{2}[\tilde{A}^{\text{OPE}}(M, s_0^-) - \tilde{B}^{\text{OPE}}(M, s_0^-))] = (\lambda^-)^2 \exp[-\frac{(m^-)^2}{M^2}], \quad (10)$$

where

$$\tilde{A}^{\text{OPE}}(M, s_0^+) = \int_0^{s_0^+} dp_0 A^{\text{OPE}}(p_0) \exp(-\frac{p_0^2}{M^2}),$$
$$\tilde{B}^{\text{OPE}}(M, s_0^-) = \int_0^{s_0^-} dp_0 B^{\text{OPE}}(p_0) \exp(-\frac{p_0^2}{M^2}).$$

The first sum rule is for the baryons with positive parity and the second one is for negative-parity baryons. In these sum rules, we allow the threshold to be different for each parity.

### 3 Borel Sum Rules for Baryon Masses

It is important to choose an appropriate interpolating field (IF) in the QCD sum rule. The IF should have the same quantum numbers as the baryon in question, so that it creates or annihilates a single particle state of the baryon from the vacuum. For the spin $\frac{1}{2}$ octet baryon two independent IFs can be constructed without a derivative. The IF for the nucleon is given above, eq.(2). If we choose the parameter $t = -1$ and use the Fierz transformation, it is reduced to the Ioffe’s IF. This choice seems adjusted to the positive-parity baryon state. Instead, we found that $t = 0.8$ is appropriate for the negative-parity resonance. The IFs for the $\Sigma$ and $\Xi$ baryons are obtained by replacing a $d$-quark by an $s$-quark or $u$ by $s$ in eq.(2),

$$J_{\Sigma^+}(x) = \varepsilon_{abc}[(u_a(x)Cs_b(x))\gamma_5u_c(x) + t(u_a(x)C\gamma_5s_b(x))u_c(x)]. \quad (11)$$
$$J_{\Sigma^-}(x) = \varepsilon_{abc}[(s_a(x)Cd_b(x))\gamma_5s_c(x) + t(s_a(x)C\gamma_5d_b(x))s_c(x)]. \quad (12)$$

The IF for $\Lambda$ is given by

$$J_{\Lambda}(x) = \varepsilon_{abc}[(d_a(x)Cs_b(x))\gamma_5u_c(x) + (s_a(x)C\gamma_5s_b(x))\gamma_5d_c(x) + 2(u_a(x)C\gamma_5s_b(x))u_c(x) + (s_a(x)C\gamma_5u_b(x))u_c(x) + (s_a(x)C\gamma_5u_b(x))d_c(x) + 2(u_a(x)C\gamma_5s_b(x))s_c(x)]. \quad (13)$$
The parameter $t = -0.8$ is used also for the $\Sigma^*$, $\Lambda^*$ and $\Xi^*$ resonances.

The IF for the flavor singlet baryon $\Lambda_S$ is uniquely given by the flavor antisymmetric combination of the quark operators:

$$J_{\Lambda_S}(x) = \varepsilon_{abc}[(u_a(x)C\gamma_5 d_b(x))s_c(x) - (u_a(x)C d_b(x))\gamma_5 s_c(x) - (u_a(x)C\gamma_\mu d_b(x))\gamma^\mu s_c(x)].$$ (14)

The theoretical (OPE) sides (up to dimension 6) of the sum rules (7) and (8) for the nucleon are given by

$$\text{Im} A^{\text{OPE}}(p_0) = \frac{5 + 2t + 5t^2}{120 \pi^4} p_0^5 \theta(p_0) + \frac{5 + 2t + 5t^2}{2^9 \pi^2} p_0 \theta(p_0) \langle \frac{\alpha_s}{\pi} GG \rangle$$

$$- \frac{5 + 2t - 7t^2}{12} \delta(p_0) \langle \bar{q}q \rangle^2,$$ (15)

$$\text{Im} B^{\text{OPE}}(p_0) = - \frac{7t^2 - 2t - 5}{32 \pi^2} p_0^2 \theta(p_0) \langle \bar{q}q \rangle - \frac{3(1 - t^2)}{32 \pi^2} \theta(p_0) \langle \bar{q}g\sigma \cdot Gq \rangle.$$ (16)

In these expressions we neglect the up and down quark masses.

Note that the difference of the sum rules for $B_+$ and $B_-$ is the chiral odd term $B(p_0)$, which is proportional either to the quark condensate $\langle \bar{q}q \rangle$ or to the mixed condensate $\langle \bar{q}g\sigma \cdot Gq \rangle$. If the chiral symmetry is restored, for instance, at high temperature, the $B(p_0)$ term goes to zero (in the chiral limit). Then the sum rules (11) and (12) are identical, and will predict the same masses for the positive and negative parity baryons. This situation is similar to that in the linear sigma model for parity-doublet baryons proposed by DeTar and Kunihiro. There the positive and negative parity baryons are assumed to form a parity doublet and the Lagrangian has a chiral invariant mass term. Under the restoration of the chiral symmetry, $B_+$ and $B_-$ have the same mass, while in the spontaneous symmetry broken phase the mass splitting is proportional to the non vanishing vacuum expectation value of sigma.

In order to see the effect of the chiral symmetry breaking, we vary $\langle \bar{q}q \rangle$ and study its effects. $\langle \bar{q}g\sigma \cdot Gq \rangle$ is assumed to be proportional to $\langle \bar{q}q \rangle$ and therefore is varied together with $\langle \bar{q}q \rangle$. Fig.4 shows the masses of $N^+$ and $N^-$ for various values of $R = \langle \bar{q}q \rangle / \langle \bar{q}q \rangle_0$. One sees that both the masses of $N^+$ and $N^-$ tend to decrease for $R \to 0$ and become degenerate in the limit, although the $R$ dependencies are different. It should be noted that the behavior of the $N^+$ mass is different from the Ioffe’s formula

$$m^+ = [\langle \bar{q}q \rangle]^{1/3}.$$ (16)

We have three phenomenological parameters, the mass $m_{B_\pm}$, the threshold $s_\pm$ and the coupling strength $\lambda_{\pm}$ to be determined from the sum rules (11) and (12). These parameters are determined by solving the system of three equations, eq. (11) or (12), and its first and second derivatives with respect to the Borel mass.
Figure 1: Masses of $N^+$ and $N^-$ at $M = 2.5$ GeV for various values of the quark condensate. $R$ is the ratio of $\langle \bar{q}q \rangle$ to its standard value $\langle \bar{q}q \rangle_0$. The solid line is the Ioffe’s formula (16).

The theoretical side depends on the QCD parameters, such as the quark mass and the gauge coupling constant, and also on the other parameters that describe the properties of the nonperturbative vacuum of QCD, such as the quark and gluon condensates. We take the chiral limit for the up and down quarks, i.e. $m_q = 0$, where we use the symbol $q$ for the up and down quarks. We introduce the strange quark mass $m_s$, $\chi \equiv \langle \bar{s}s \rangle / \langle \bar{q}q \rangle$ and $\chi_5 \equiv \langle \bar{s}g_5 \cdot Gs \rangle / \langle \bar{q}g_5 \cdot Gq \rangle$ for the flavor SU(3) symmetry breaking. The gluon condensate is fixed to $\langle \chi^2 GG \rangle = (0.36 \text{ GeV})^4$. The vacuum saturation is assumed for evaluating the matrix element of the four-quark operators, i.e. $\langle (\bar{q}q)^2 \rangle = \langle \bar{q}q \rangle^2$. These parameters have some uncertainty, which depends on the truncation in the operator product expansion. In order to remove this uncertainty, we use our sum rules in the following way. The value of $\langle \bar{q}q \rangle$ and $m_0^2 \equiv \langle \bar{q}g_5 \cdot Gq \rangle / \langle \bar{q}q \rangle$ are determined so that the sum rules (9) and (10) for the nucleon reproduce the observed masses of $N^+$ and $N^-$. In doing so we require that the prediction of the sum rule at $M \simeq m_B$ coincides with the observed mass within 5% and also that for the Borel stability variation of the predicted mass against $M$ in the region $m_B \sim m_B + 0.5 \text{ GeV}$ is less than 10%. In the same way, the values of $m_s$, $\chi$ and $\chi_5$ are determined so that the sum rules (9) for the positive-parity hyperons give the observed masses of the $\Lambda_+$, $\Sigma_+$ and $\Xi_+$. The determined parameters are given in Table I. The masses of the positive-parity hyperons, $\Lambda_+$, $\Sigma_+$ and $\Xi_+$ are sensitive to the SU$_f$(3) breaking parameters $m_s$, $\chi$ and $\chi_5$. Therefore these parameters are determined well. We see that $\langle \bar{q}q \rangle$
determines the mass splitting of $B_+$ and $B_-$. 

The masses of the flavor octet and singlet baryons calculated in the QCD sum rule are shown in Table 2. The observed masses are reproduced fairly well. The masses of $\Lambda_\pi$, $\Sigma_\pi$, $\Xi_\pi$, $\Lambda_S-$ and $\Lambda_S+$ are the prediction without adjustable parameters. While these masses are evaluated at the Borel mass $M \simeq m_B$, they are almost stable against the Borel mass, that indicates that the sum rules work well in these cases. The excited $\Xi$ baryon with $J^P = \frac{1}{2}^-$ has not been identified by experiment, but resonances with unknown spin and parity are found at 1690 MeV and 1950 MeV. The sum rule prefers $\Xi(1690)$.

Our result suggests that the $B_-$ masses tend to be degenerate. This is the result of two different origins of the mass difference. The strange quark mass raises the hyperon masses, while the quark condensate widens the mass splitting of $B_+$ and $B_-$. Because the strange quark condensate is smaller than the up and down quark condensate, the effect of the strange quark mass is partly canceled in the negative parity baryons.

It is extremely interesting to observe that the QCD sum rule predicts the flavor-singlet $\Lambda_S$ spectrum in the reversed order. Namely, the baryon $\Lambda_S-$ is lighter than the positive parity $\Lambda_S+$. This is consistent with the quark model prediction that the Pauli principle forbids all quarks occupying the ground $s$-wave state. In the correlation function $B$ for the $\Lambda_S$, there is no dimension five term, $\langle \bar{q} g \sigma \cdot G q \rangle$. If we put the dimension five term in the $B$ correlation function by hand and calculate the masses of the $\Lambda_S+$ and $\Lambda_S-$, then we find that the increase of the dimension five term raises the mass of $\Lambda_S-$ and lowers that of $\Lambda_S+$.

We note several other approaches of the QCD sum rule for $B_-$.

| Table 1: The determined QCD parameters |
|----------------------------------------|
| $\langle \bar{q} g \rangle$ | $m_0$ | $m_s$ | $\chi$ | $\chi_5$ |
| (-0.244 GeV)$^3$ | 0.9 GeV | 0.1 GeV | 0.75 | 0.8 |
4 Meson Couplings to the Negative Parity Baryons

Concerning the meson-baryon couplings, the negative parity baryons, \( N(1535) \), \( \Lambda(1670) \) and \( \Sigma(1750) \) (denoted by \( B^* \)), possess interesting properties. The most distinguished feature is their relatively large decay widths of \( B^* \rightarrow \eta B^* \). Because of the smallness of the available phase space, this fact suggests relatively large coupling constants of \( \eta BB^* \) as compared to those of \( \pi BB^* \).

One may also look at the problem in the following way. Using the experimental decay widths of the resonances, we obtain, for example, \( g_{\pi NN^*} \sim 1 \) and \( g_{\eta NN^*} \sim 2 \). These values are in fact much smaller than those in the \( NN \) sector: \( g_{\pi NN} \sim 13 \) and \( g_{\eta NN} \leq 5 \). Furthermore, the pion couples weaker than the eta in the \( NN^* \) sector, as opposed to the \( NN \) sector. Thus, one may ask why the coupling \( g_{\pi NN^*} \) is suppressed so much as compared with other couplings.

In order to apply the QCD sum rule to the meson-baryon vertices, we follow the method used by Shiomi and Hatsuda. They studied the \( \pi NN \) coupling constant \( g_{\pi NN} \) by using the two point function between the vacuum and a one meson state in the soft meson limit \( (q^\mu \rightarrow 0) \). The relevant correlation function is

\[
\Pi^m(p) = i \int d^4x e^{ipx} \langle 0 | T J_N(x) \bar{J}_N(0) | m(q = 0) \rangle \\
= i\gamma_5 (\Pi_0^m(p^2) + \Pi_1^m(p^2)\not{\psi}),
\]

where \( J_N \) is defined in (2), and \( m \) denotes either \( \pi \) or \( \eta \). The parameter \( t \) will be chosen suitably depending on whether \( J_N \) should be coupled strongly to positive or
negative parity baryons.

Let us first look at the phenomenological side of the $\pi - N - N^*$ correlation function to see how this coupling strength can be extracted. We define the phenomenological $\pi NN^*$ interaction Lagrangian,

$$
\mathcal{L}_{\pi NN^*} = g_{\pi NN^*} \bar{N}^* \tau^i \pi^i N,
$$

(18)

where $N$ and $N^*$ are the field operators for the positive and negative parity nucleons, $\pi^i$ is the pion field, and $\tau^i (i = 1, 2, 3)$ are the Pauli matrices for isospin. From the Lagrangian (18), the $\pi NN^*$ contribution in the $\Pi^\pi(p)$ is given in the soft pion limit by

$$
g_{\pi NN^*} \lambda_N \lambda_{N^*} \left( \frac{p^2 + m_N m_{N^*}}{(p^2 - M_N^2)(p^2 - m_{N^*}^2)} + \frac{\not{p} (m_N + m_{N^*})}{(p^2 - M_N^2)(p^2 - m_{N^*}^2)} \right) i \gamma_5
$$

(19)

where $\lambda_N$ and $\lambda_{N^*}$ are defined by $\langle 0 | J_N | N \rangle = \lambda_N u_N$ and $\langle 0 | J_N | N^* \rangle = \lambda_{N^*} i \gamma_5 u_{N^*}$, respectively, with $u_B$ being the Dirac spinor for the baryon $B$. We note that there appear two terms in (19); one proportional to $\gamma_5$ and the other proportional to $\not{p} \gamma_5$. In contrast, the $\pi NN$ contribution has only one term,

$$
g_{\pi NN} \lambda_N^2 \frac{i \gamma_5}{p^2 - m_N^2},
$$

(20)

as is derived from the $\pi NN$ interaction Lagrangian

$$
\mathcal{L}_{\pi NN} = g_{\pi NN} \bar{N} i \gamma_5 \tau^i \pi^i N.
$$

(21)

We note that (20) is also obtained by replacing $M_{N^*}$ by $-M_N$ in (19).

In the soft pion limit, Shiomi and Hatsuda [14] studied the sum rule using the non-vanishing term of (20) and found that the resulting $\pi NN$ coupling constant satisfies the Goldberger-Treiman relation with $g_A = 1$. Recently, Birse and Krippa also studied the coupling constant $g_{\pi NN}$ at a non zero pion momentum [15]. For the $\pi NN^*$ coupling constant we study the term proportional to $\not{p} \gamma_5$, which is expected to have a dominant contribution from the $\pi NN^*$ coupling. In fact, there could be a contribution in the $i \not{p} \gamma_5$ term from positive parity resonances; a dominant part would be from the lowest resonance $N(1440)$. Such a term is, however, proportional to the mass difference $M_{N(1440)} - M_N$ unlike the sum as in the second term of (19). Thus the contribution from $N(1440)$ will be relatively suppressed as compared with that of $N(1535)$. Moreover since we choose the mixing parameter $t \sim 0.8$ such that the interpolating field (2) couples strongly to negative parity states, we expect least contamination from positive parity resonances.

The sum rule for the $\eta NN^*$ coupling is similarly constructed by replacing the isospin matrices $\tau$ in the $\pi NN^*$ coupling by the unit matrix.
The correlation function is now computed by the operator product expansion (OPE) perturbatively in the deep Euclidian region. The result for the terms of $i\psi \gamma_5$ takes the following form

$$\Pi_{\text{OPE}}(p) = i \int d^4x e^{ipx} \langle 0| T J(x) J_N(0) |m \rangle$$

$$\equiv i\gamma_5 \left[ C_4 \ln(-p^2) + C_6 \frac{1}{p^2} + \cdots \right] + i\gamma_5 \left[ C_8 p^2 \ln(-p^2) + \cdots \right],$$

where we allow to use the different mixing parameters in the interpolating fields such that $\bar{J}_N(0; t \sim 0.8)$ couples dominantly to the $N^*$ state, while $J_N(x; s = -1)$ to the $N$ state. Note that the terms of $i\psi \gamma_5$ are of even dimension. The correlation function (22) has been calculated up to dimension 8, ignoring higher order terms in $m_q$ and $\alpha_s$. The results are

$$C_4 \sim m_q \langle 0| \bar{q} i\gamma_5 q |m \rangle \xrightarrow{m_q \to 0} 0$$

$$C_6 = -\frac{s - t}{4} \left[ \langle dd \rangle \langle 0| \bar{u} i\gamma_5 u |m \rangle + \langle uu \rangle \langle 0| \bar{d} i\gamma_5 d |m \rangle \right]$$

$$C_8 = -\frac{s - t}{144} \left[ 25(\langle dG \cdot \sigma d \rangle \langle 0| \bar{u} i\gamma_5 u |m \rangle + \langle \bar{u} G \cdot \sigma u \rangle \langle 0| \bar{d} i\gamma_5 d |m \rangle) \right.$$  

$$\left. -7(\langle dd \rangle \langle 0| \bar{u} i\gamma_5 G \cdot \sigma u |m \rangle + \langle uu \rangle \langle 0| \bar{d} i\gamma_5 G \cdot \sigma d |m \rangle) \right]$$

where we assume the vacuum saturation for four-quark matrix elements. We evaluate the meson-vacuum matrix elements using the soft meson theorem, and obtain the following relations:

$$\langle 0| \bar{u} i\gamma_5 u |m \rangle = -\frac{\alpha_m}{f_m} \langle \bar{u} u \rangle,$$  

$$\langle 0| \bar{d} i\gamma_5 d |m \rangle = \pm \frac{\alpha_m}{f_m} \langle \bar{d} d \rangle,$$  

$$\langle 0| \bar{u} i\gamma_5 G \cdot \sigma u |m \rangle = -\frac{\alpha_m}{f_m} \langle \bar{u} G \cdot \sigma u \rangle,$$  

$$\langle 0| \bar{d} i\gamma_5 G \cdot \sigma d |m \rangle = \pm \frac{\alpha_m}{f_m} \langle \bar{d} G \cdot \sigma d \rangle,$$

where $\alpha_\pi = 1/\sqrt{2}$ and $\alpha_\eta = 1/\sqrt{6}$. Note that the sign difference between the pion and eta matrix elements comes from the isospin structures: $\pi^0 \sim \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d)$, and $\eta \sim \eta_8 \sim \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s)$ (by neglecting small mixing angle effects). We note that the $s\bar{s}$ component in $\eta$ is irrelevant up to dimension 8, since the interpolating field (2) does not contain strange quarks.

From (22) – (24), we find that the correlation function for the $\pi NN^*$ coupling vanishes in the chiral limit $m_q \to 0$, and therefore $g_{\pi NN^*} = 0$. In contrast, the correlation function for the $\eta NN^*$ coupling does not vanish, and so the coupling constant $g_{\eta NN^*}$ remains finite. We emphasize that the result from the OPE here...
is qualitatively consistent with the partial decay rates of \( N(1535) \), \( \Gamma_{N(1535) \rightarrow \pi N} \sim \Gamma_{N(1535) \rightarrow \eta N} \sim 70 \text{ MeV} \).

The vanishing correlation function for the \( \pi NN \) case is, in fact, a general consequence of chiral symmetry. We might have applied the soft meson theorem to the correlation function \( \langle 0 | J_N(x) \bar{J}_N(0) \rangle \) from the beginning. Using the transformation property \([Q^a_5, J_N] = i\gamma^5 \tau^a J_N\), we find

\[
\Pi^{\pi^a}(p) = \lim_{q \to 0} \int d^4xe^{ipx} \langle 0 | T J_N(x) \bar{J}_N(0) | \pi^a(q) \rangle = -\frac{i}{\sqrt{2}f_\pi} \int d^4xe^{ipx} \langle 0 | [Q^a_5, T J_N(x) \bar{J}_N(0)] | 0 \rangle = \frac{1}{\sqrt{2}f_\pi} \int d^4xe^{ipx} \{\gamma^5 \tau^a, \langle 0 | T J_N(x) \bar{J}_N(0) | 0 \rangle \}. \tag{30}
\]

In the last expression, we have recovered the vacuum to vacuum transition, which has a Lorentz structure \( \langle 0 | J_N(x) \bar{J}_N(0) | 0 \rangle \sim A\psi + B1 \). Thus in (30), the term of \( p/\gamma^5 \) disappears. We emphasize that this is a consequence of chiral symmetry. In order to achieve this result, the nucleon current has to transform appropriately under chiral transformations and also, the negative parity baryons are produced by the same nucleon current \( J_N \). These two facts determine the chiral properties of the positive and negative parity nucleons. As a consequence, the \( p/\gamma^5 \) term, which in general should exist in the two point function, is shown to disappear.

What are then implications of the vanishing matrix element of \( p/\gamma^5 \) term? By relating this with the sum rule, it is implied that the coherent sum over all coupling constants of the pion between various baryon excitations will vanish. One would, in fact, make a stronger statement by using properties of analytic functions. Namely, if an analytic function vanishes in a certain domain, it vanishes identically for whole analytic region. In the spirit of the QCD sum rule, we have found that the \( p/\gamma^5 \) term of the correlation function vanishes in the deep Euclidian region, which implies the identically vanishing \( p/\gamma^5 \) term for whole momentum space. There might be an exception case, if there are two (delta-function like) terms having equal strengths but with opposite signs, which sum up to be zero. In fact, the low-lying negative parity nucleons do look like that; there are two neighboring states of \( N(1535) \) and \( N(1650) \). Even in this case, one would say that a state which is coupled by the current \( J_N \) is a superposition of two (or more) degenerate states, and couplings with the pion is coherently added up and cancel.

One may wonder if such a suppression of \( g_{\pi NN^*} \) could be explained in some way by spontaneously broken chiral symmetry. From chiral symmetry point of view, it seems natural to put \( N \) and \( N^* \) in the same multiplet of chiral partner (or parity doublet). There have been several attempts to treat positive and negative parity baryons in this point of view.

DeTar and Kunihiro considered the parity doublet nucleons in the linear sigma
model of $SU(2) \times SU(2)$. In addition to the standard chiral invariant interaction terms, they introduced a chiral invariant mass term between the positive and negative
parity baryons. The strength $m_0$ for the non-standard mass term reduces to the mass of the would-be chiral doublet nucleons when the chiral symmetry restores. In the
spontaneously broken phase, the mass splitting is proportional to the non-zero value
of the quark condensate. In this model, it has been shown that $g_{\pi NN^*}$ is proportional
to $m_0$ to the leading order in $m_0$. Therefore, if $m_0 = 0$, the coupling constant $g_{\pi NN^*}$
vanishes. This is a rigorous consequence from chiral symmetry. The question, is
therefore, whether the non-standard mass term exists or not in the real world. This
can be examined by looking at baryon masses in the chiral symmetric phase. In the
QCD sum rule study\footnote{5}, the masses of $N$ and $N^*$ seem to be degenerate and decrease
as the quark condensate $\langle \bar{q}q \rangle$ is decreased. This implies a small (and possibly vanishing)
$m_0 \approx 0$ and so a small $g_{\pi NN^*}$.

The formulation of DeTar and Kunihiro may be extended to the chiral $U(1) \times
SU(2)$ model, where the eta is a unitary singlet. In this extension, there seems to exist
a natural explanation for the relation among the coupling constants: $g_{\pi NN} \gg g_{\eta NN}$,
while $g_{\pi NN^*} \ll g_{\eta NN^*}$.

5 Conclusion

We have applied the QCD sum rule method to the study of the negative-parity
baryons. The masses of the flavor octet and singlet baryon resonances are reproduced
fairly well. An interesting observation is near degeneracy of the octet members,
that seems to come from the interference between two different mechanisms of the
symmetry breaking. The flavor singlet $\Lambda_S$ has the negative-parity state as the lowest
mass, that is consistent with experiment. The formulation and the numerical results
suggest a picture that the positive and negative-parity baryons form a parity doublet,
which would be degenerate when the chiral symmetry is restored.

The calculation of the $\pi NN^*$ coupling has been performed and gives a new sur-
prise, that the coupling vanishes in the chiral limit. We suggest that the chiral
symmetry is responsible for this result. It seems consistent with the observation of
suppressed pion decay rate of $N(1535)$. We suggest that these properties of $N^*$
can be understood in a chiral effective theory for the parity doublet nucleons proposed
by Detar and Kunihiro\footnote{10}.

The authors would like to thank the organizers of the INT/CEBAF $N^*$ Workshop
for giving them chances to participate the exciting workshop and also acknowledge
INT and CEBAF for their support.

1. N. Isgur and G. Karl, Phys. Rev. D18 (1978) 4187; R. Koniuk and N. Isgur,
   Phys. Rev. D21 (1980) 1868.
2. M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B147 (1979) 385, 448.
3. L.J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rep. 127 (1985) 1.
4. B. L. Ioffe, Nucl. Phys. B188 (1981) 317, (E) B191 (1981) 591.
5. D. Jido, N. Kodama and M. Oka, Phys. Rev. D54 (1996) 4532.
6. D. Jido and M. Oka, hep-ph/9611322, “QCD sum rule for $\frac{1}{2}^-$ Baryons”.
7. D. Jido, M. Oka and A. Hosaka, hep-ph/9610520, “Suppression of $\pi NN(1535)$ Coupling in the QCD Sum Rule”.
8. Y. Chung, H.G. Dosch, M. Kremer and D. Schall, Nucl. Phys. B197 (1982) 55.
9. D. Espriu, P. Pascual and R. Tarrach, Nucl. Phys. B214 (1983) 285.
10. C. DeTar and T. Kunihiro, Phys. Rev. D39 (1989) 2805.
11. Particle Data Group, Phys. Rev. D54 (1996) 1.
12. S. H. Lee and H. Kim, Nucl. Phys. A612 (1997) 418; H. Kim and S. H. Lee, nucl-th/9610013.
13. J. P. Liu, Z. Phys. C 22 (1984) 171.
14. H. Shiozawa and T. Hatsuda, Nucl. Phys. A594 (1995) 294.
15. M. Birse and B. Krippa, Phys. Lett. B373 (1996) 9.
16. G. A. Christos, Z. Phys. C21 (1983) 83; Phys. Rev. D35 (1987) 330.