Theoretical analysis and simulation of fibre-top micro cantilever resonator excited optically

Jing Hua¹, Yueming Liu¹,² and Weijian Tian¹
¹College of Optical and Electronic Technology, China Jiliang University, Hangzhou 310018, China
²Corresponding Author, E-mail: liuym@cjlu.edu.cn

Abstract. Theoretical analysis and simulation of bi-layered optical fibre-end micro cantilever resonators was presented in this paper. First, corresponding theoretical model is setup and the resonating frequency is given in this case; Second, the typical characteristics of the micro resonator is simulated including deflection sensitivity, cantilever dimensions and metal coating optimization; Third, the working principle of this optimized micro cantilever is discussed based on optical interference theory of Fabry-Perot cavity. By detecting the optical output of Fabry-Perot cavity resulted from the cantilever deflection, the changes of environmental parameter (such as pressure or temperature) can be measured. Comparing with the traditional optical excited micro resonators, this fibre-top micro resonator has some favourable features, such as micro size, high sensitivity and optical integration and is more interesting and meaningful.

Keywords: micro cantilever, optical fibre, resonator sensors, simulation

1. Introduction
Micro cantilever resonator is widely used in micro-opto-electro-mechanical elements, such as micro-resonator sensors, micro-resonator switches, micro-modulators and etc. Normally the micro cantilever is fabricated on silicon wafer by micro machined process, and then should be excited firstly in related applications. The basic principle of these resonator sensors is that an environmental parameter (such as pressure or temperature) changes the first resonance frequency of the resonators, and then the environmental parameters can be detected through the change of the resonance frequency. Among all the exciting methods, the optical excitation is the most interesting method owing to its special advantages of non-contact and non-destructive. For the traditional optically excited silicon cantilever resonator, there are some difficulties that baffle its practical applications, such as the accurate light coupled position between the micro cantilever and the excited optical radiation.

In this paper, a novel cantilever resonator is presented based on the optical fibre end fabricated by carving tiny mechanical cantilever beams directly on the cleaved edge of an optical fibre. For this optical fibre-end micro cantilever resonators, there are more great advantages comparing with the traditional optical excited micro silicon resonators, such as a monolithic structure comprising micro cantilever resonator being positioned easily in such a way that it covers the core of the optical fibre. By this way, the optical excited light coupled to the fibre core is thus put on the micro cantilever accurately and then partially reflected by the cantilever resonator back into the same optical fibre to the signal detector. A metal layer can be deposited atop of the cantilever in order to enhance the reflection of the incoming excited light. From the research of modelling of bi-layered structure frequency and temperature distribution by optical excitation, a more accurate sensor can be obtained by optimizing with better sensitivity and reasonable sensing mechanism.
2. Frequency of bi-layered optical fibre-end micro cantilever resonators

To the bi-layered optical fibre-end micro cantilever resonators in figure 1, the frequency can be deduced as follow. The intrinsic frequency of micro cantilever can be expressed as

\[ f_{ci} = 0.046\varphi_i \frac{t}{L^2} \sqrt{\frac{E}{\rho}} \]  \hspace{1cm} (1)

\( L, \ t, \ \rho, \ E \) are respectively the length, thickness, density and modulus of the micro cantilever. \( f_{ci} \) is the frequencies of different resonance modes, and \( \varphi_i \) are corresponding coefficients. When \( \varphi_1 \) equals 3.52, we can get the first mode frequencies as

\[ f_{c1} = 0.162 \frac{t}{L^2} \sqrt{\frac{E}{\rho}} \]  \hspace{1cm} (2)

![Figure 1. Setup of bi-layered optical fibre-end micro cantilever resonators](image1)

A better view of the resonator Structure is given in figure 2 and figure 3. The length, width of the resonator are respectively \( L \) and \( W \), the thickness, modulus and density of the resonator and its coating are respectively \( t_2, E_2, \rho_2 \) and \( t_1, E_1, \rho_1 \). The excited light is guided onto its midpoint of the micro cantilever through the optic fibre. Based on the ‘Equivalent methods of parameters’ in vibration theory\(^{(3)}\), the frequency of the micro resonators can be expressed as

\[ f_c = 0.162 \frac{t_2}{L^2} \sqrt{\frac{E_2}{\rho_2 K'}} \]  \hspace{1cm} (3)

where \( K' \) is the equivalent coefficient which expressed as.

![Figure 2. Top view of resonators](image2)

![Figure 3. Simplified model of resonators](image3)
3. Sensitivity analysis of micro resonator deflection and coating optimization

Following the sensitivity of micro resonator shown in figure 4 must be discussed. Assuming that the thermal deflection of cantilever does not occur at the temperature $T_0$, but it appears when the temperature rises to $T_0 + \Delta T$. Taking thermal stress into account, the differential equation of deflection is setup by the thermal theory of elasticity as

$$K' = \frac{4 + 6 \left( \frac{t_1}{t_2} \right)^2 + 4 \left( \frac{t_1}{t_2} \right)^2 + \left( \frac{E_1}{E_2} \right) \left( \frac{t_1}{t_2} \right)^3 + \left( \frac{E_2}{E_1} \right) \left( \frac{t_2}{t_1} \right)}{1 + \left( \frac{t_2}{t_1} \right) \left( \frac{E_2}{E_1} \right) + \left( \frac{\rho_1 t_1}{\rho_2 t_2} \right)}$$

Figure 4. The comparison: (a) The normal structure (b) The deflection
\[ K = 4 + 6n + 4n^2 + \Phi \cdot n^3 + \frac{1}{\Phi n} \cdot \Phi = \frac{E_1}{E_2}, \beta = \frac{\alpha_1}{\alpha_2}, n = \frac{t_1}{t_2}, \gamma = \frac{\lambda_1}{\lambda_2} \]

### Table 1. Relative property parameters of materials

| The material | Density $\rho$ kg m$^{-3}$ | Young’s Modulus $E$ $10^{11}$ N m$^{-2}$ | Thermal conductivity $\lambda$ w m$^{-1}$k$^{-1}$ | Thermal expansion coefficient $\alpha$ $10^{-6}$ k$^{-1}$ | Specific heat $C$ J kg$^{-1}$ k$^{-1}$ |
|--------------|-------------------|-----------------|----------------|-----------------|----------------|
| SiO$_2$      | 2200              | 0.70            | 1.3            | 0.4             | 840            |
| Au           | 19300             | 0.73            | 296            | 14.2            | 129            |
| Cr           | 7100              | 0.25            | 96.5           | 8.5             | 438            |
| Al           | 2702              | 0.8             | 237            | 23.6            | 908            |

It is essential to choose the suitable metal coating and its thickness to improve the detection sensitivity of micro cantilever resonators, normally there are three coating selections which are Au, Al, or Cr. In order to do coating optimization, according to equation (8), we take parameters of micro cantilever resonators as follows: length $L=200\mu$m, width $W=30\mu$m, thickness $t_2=2\mu$m and absorption coefficient $\eta=0.1$, other material parameters were shown in table 1. Using the software MATLAB, the relationship between sensitivity and coating thickness ratio of three different metal materials are simulated in figure 5. From figure 5, it is clear that the best material as the metal coating is Aluminium, optimum coating thickness ratio is 0.039 and the maximum sensitivity is $1.43 \times 10^{-4} \mu$m/$\mu$W. Further simulation among the intensity of light, length and deflection of micro cantilever is shown in figure 6. We can know that when the light intensity changes from 0 to 400$\mu$W, the maximum deflection changes from 0 to 0.46$\mu$m.

**Figure 5.** Comparison of the sensitivity in case of three different coating materials

**Figure 6.** The relation among the intensity of light, length and deflection

### 4. Resonant sensing mechanism and theoretical analysis

When the periodically intensity-modulated light is guided onto the micro resonator, some of the optical energy is absorbed and the temperature distribution is formed along the resonator, then the resonator begins to vibrating via the photo thermal moment formed in the bi-layered materials which we named “bi-coating effect”\[8\]. If the frequency of the optical excitation is equal to the resonance frequency, the mechanical resonance occurs in the micro resonator.

A advantage in optical fibre-end micro resonators is that a Fabry-Perot Cavity formed between optical fibre end and micro cantilever which can be used to detect the resonant signals\[9\]. The reflected light signal is modulated by F-P interference cavity. Because micro cantilever resonators is made of bi-layer material, micro cantilever resonators will be deflected in response to changes of ambient...
temperature owing to different thermal expansion coefficient of two layer materials. Fabry-Perot cavity depth will be changed when the bi-layer material is deflected thermally, and this results in optical reflectance change of the Fabry-Perot basing on multiple optical interferences within the F-P cavity. The reflecting light from the Fabry-Perot cavity is carried back through the same fibre coupler and transfer to electricity by detector shown in figure 1.

The reflectance $I_R/I_0$ of Fabry-Perot cavity can be expressed as

$$I_R/I_0 = \frac{a^2 (r_1^2 + r_2^2 - 2r_1r_2\cos\Psi)}{1 + (r_1r_2)^2 - 2r_1r_2\cos\Psi}$$

(9)

where $r_1$ represents the reflectance of the fibre end face, $r_2$ represents the reflectance of micro cantilever resonators surface with coating, and $\Psi$, the round-trip propagation phase shift in the Fabry-Perot cavity, the value of $\Psi$ is given by

$$\Psi = 4\pi h/\lambda_0$$

(10)

where $h$ is the cavity depth and $\lambda_0$ is the optical wavelength. Combined equation(9) and equation (10) above, the output light intensity varies periodically as a function of the cavity depth $h$, but on the other hand, the variety of the cavity $h$ has particular relation with the environmental parameter. So when the output light intensity is measured, change of unknown parameter can be determined.

Next, we set some parameters and give the figure produced by software MATLAB to get a more clear understanding of relationship between cavity $h$ and optical intensity ratio $I_R/I_0$. When input light wavelength is equal to $1310\text{nm}$, $r_1=0.5$, and $r_2$ is respectively equal to $0.2$, $0.4$, $0.6$ or $0.8$, the performance curve of the cavity $h$ versus intensity ratio is shown in figure 8. From figure 8, we can know two key points, one is that reflected light intensity changes periodically as the cavity length varies, by detecting the change of light intensity, F-P cavity length variation $\Delta h$ can be calculated and then the environmental parameter can be determined; the other is that in the same circumstances, the higher the reflectivity $r_2$, the greater the reflectance $I_R/I_0$.

![Figure 8](image)

**Figure 8.** Performance curve of the cavity versus optical intensity ratio

5. Conclusions

Dimensions optimization and characteristics of bi-layered optical fibre-end micro cantilever resonators were described in this paper. The model of bi-layered structure frequency was setup and temperature distribution by excited light was analyzed firstly, then the sensitivity of the micro resonators was simulated and optimized by computer software MATLAB, in the end, resonant sensing mechanism and the character of Fabry-Perot cavity was concerned to detect the change of environmental parameters. The bi-layered optical fibre-end micro cantilever resonator which has greater sensitivity is a better choice to be used in further practical developments.

Acknowledgements

This paper was supported by Natural Science Foundation of Zhejiang Province of China (No. Y1091078) and Science Research Found of China Jiliang University.
References
[1] Hua qingan. Silicon micromachining technology, 1996. (Beijing : Science Press)
[2] Li shaohui. Optical fibre sensor, 1997. (Wuhan: Huazhong Technology University Press)
[3] Langdon R M. Resonator sensors—a review, 1985. J. Phys. E: Sci. Instrum. 18. 103–15
[4] Roard R J and Young W C. Formulas for stress and strain, 1975. (New York: McGraw-Hill Book Company). p 112
[5] Barnes J R, Stephenson R J and Woodburn C N, 1994. A femtojoule calorimeter using micromechanical sensors. Rev. Sci. Instrum. 65 (12). 3793–98
[6] Holman J P. Heat Transfer, 1986. (New York: McGraw-Hill Book Company)
[7] Liu yueming. Theoretical model and techniques of silicon micro resonator under photo-thermal excitation (D), 2001. Xi’an Jiaotong University.
[8] Chen G Y, Thundat T and Wachtter E A, 1995. Adsorptiob-induced surface stress and its effects on resonance frequency of micro cantilevers. J. Appl. Phys. 77(8). 3618–22
[9] Guo zhenhua. The inventors of the multibeam interferometer—C. Fabry and A. Perot, 2004. Physics. 4(33). 293–6
[10] Bi weihong. Mathematical model for fibre-optical non-symmetrical Fabry-Perot Interferometric cavity, 2000. ACTA OPTICA SINICA. 7(20). 874–6