REGULARIZED INVERSE REINFORCEMENT LEARNING

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ABSTRACT

Inverse Reinforcement Learning (IRL) aims to facilitate a learner’s ability to imitate expert behavior by acquiring reward functions that explain the expert’s decisions. Regularized IRL applies convex regularizers to the learner’s policy in order to avoid the expert’s behavior being rationalized by arbitrary constant rewards, also known as degenerate solutions. We propose analytical solutions, and practical methods to obtain them, for regularized IRL. Current methods are restricted to the maximum-entropy IRL framework, limiting them to Shannon-entropy regularizers, as well as proposing functional-form solutions that are generally intractable. We present theoretical backing for our proposed IRL method’s applicability to both discrete and continuous controls and empirically validate its performance on a variety of tasks.

1 INTRODUCTION

Reinforcement learning (RL) has been successfully applied to many challenging domains including games (Mnih et al., 2015; 2016) and robot control (Schulman et al., 2015; Fujimoto et al., 2018; Haarnoja et al., 2018). Advanced RL methods often employ policy regularization motivated by, e.g., boosting exploration (Haarnoja et al., 2018) or safe policy improvement (Schulman et al., 2015). While Shannon entropy is often used as a policy regularizer (Ziebart et al., 2008), Geist et al. (2019) recently proposed a theoretical foundation of regularized Markov decision processes (MDPs) – a framework that uses arbitrary convex functions as policy regularizers. Here, one crucial advantage is that an optimal policy is shown to uniquely exist, whereas multiple optimal policies may exist in the absence of policy regularization (and depending on the given reward structure).

Meanwhile, since RL requires a given or known reward function (which can often involve non-trivial reward engineering), Inverse Reinforcement Learning (IRL) (Russell, 1998; Ng et al., 2000) – the problem of acquiring a reward function that promotes expert-like behavior – is more generally adopted in practical scenarios like robotic manipulation (Finn et al., 2016b), autonomous driving (Sharifzadeh et al., 2016; Wu et al., 2020) and clinical motion analysis (Li et al., 2018). In these scenarios, defining a reward function beforehand is particularly challenging and IRL is simply more pragmatic. However, complications with IRL in unregularized MDPs are related to the issue of degeneracy, where any constant function can rationalize the expert’s behavior (Ng et al., 2000).

Fortunately, Geist et al. (2019) show that IRL in regularized MDPs – regularized IRL – does not contain such degenerate solutions due to the uniqueness of the optimal policy for regularized MDPs. Despite this, no analytical solutions of regularized IRL – other than maximum-Shannon-entropy IRL (MaxEntIRL) (Ziebart et al., 2008; Ziebart, 2010; Ho & Ermon, 2016; Finn et al., 2016a; Fu et al., 2018) – have been proposed.

Solutions of a functional form were proposed in Geist et al. (2019). However, they are generally intractable since a closed-form relation between the policy and optimal value function is needed to derive their solutions. This is not applicable with arbitrary policy regularization. Furthermore, practical algorithms for solving regularized IRL problems have not yet been proposed.

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We summarize our contributions as follows: unlike the solutions in Geist et al. (2019), we propose tractable solutions for regularized IRL problems (Section 3) that only require knowing the analytical form of policy regularization. We show that our solutions are valid in continuous control problems including solutions for Tsallis entropy regularization with multi-variate Gaussian policies (Section 3.2). We devise Regularized Adversarial Inverse Reinforcement Learning (RAIRL), a practical sample-based method for policy imitation and reward learning in regularized MDPs, which generalizes adversarial IRL (AIRL, Fu et al. 2018) (Section 4). Finally, we empirically validate our RAIRL method on both discrete and continuous control tasks, evaluating both episodic scores and divergence minimization perspective (Ghasemipour et al. 2019, Dadashi et al. 2020) (Section 5).

2 PRELIMINARIES

**Notation** For finite sets $X$ and $Y$, $Y^X$ is a set of functions from $X$ to $Y$. $\Delta^X (\Delta^X Y)$ is a set of (conditional) probabilities over $X$ (conditioned on $Y$). Especially for the conditional probabilities $p \in \Delta^X Y$, we say $p(\cdot | y) \in \Delta^X Y$ for $y \in Y$. $\mathbb{R}$ is the set of real numbers. For functions $f_1, f_2 \in \mathbb{R}^X$, the inner product between $f_1$ and $f_2$ on $X$ is defined as $(f_1, f_2)_X := \sum_{x \in X} f_1(x) f_2(x)$.

**Regularized Markov Decision Processes and Reinforcement Learning** We consider sequential decision making problems where an agent sequentially chooses its actions after observing the state of the environment, and the environment in turn emits a reward with state transition. Such an interaction between the agent and the environment is modeled as an infinite-horizon Markov Decision Process (MDP), $\mathcal{M} := (\mathcal{S}, \mathcal{A}, P_0, P, r, \gamma)$. The terms within the MDP are defined as follows: $\mathcal{S}$ is a finite state space, $\mathcal{A}$ is a finite action space, $P_0 \in \Delta^S$ is an initial state distribution, $P \in \Delta^S \times \mathcal{A}$ is a state transition probability, $r \in \mathbb{R}^S \times \mathcal{A}$ is a reward function, and $\gamma \in [0, 1)$ is the discount factor. We also define an MDP without reward as $\mathcal{M}^{-} := (\mathcal{S}, \mathcal{A}, P_0, P)$. The normalized state-action visitation distribution, $d_\pi \in \Delta^S \times \mathcal{A}$, associated with $\pi$ is defined as the expected discounted state-action visitation of $\pi$, i.e., $d_\pi(s, a) := (1 - \gamma) \cdot E_\pi[e^{\sum_{i=0}^{\infty} \gamma^i\mathbb{1}\{s_i = s, a_i = a\}}]$, where the subscript $\pi$ on $E$ means that a trajectory $(s_0, a_0, s_1, a_1, \ldots)$ is randomly generated from $\mathcal{M}^{-}$ and $\pi$, and $\mathbb{1}\{\cdot\}$ is an indicator function. Note that $d_\pi$ satisfies the transposed Bellman recurrence (Boularias & Chaib-Draa 2010; Zhang et al. 2019):

$$d_\pi(s, a) = (1 - \gamma) P_0(s) \pi(a | s) + \gamma \pi(a | s) \sum_{s', a'} P(s' | s, a) d_\pi(s', a').$$

We consider RL in regularized MDPs (Geist et al. 2019), where the policy is optimized with a causal convex regularizer. Mathematically for an MDP $\mathcal{M}^{-}$ and a strongly convex function $\Omega : \Delta^A \to \mathbb{R}$, the objective in regularized MDPs is to seek $\pi$ that maximizes the expected discounted sum of rewards, or return in short, with policy regularizer $\Omega$:

$$\arg \max_{\pi \in \Delta_\phi^A} J_\Omega (r, \pi) := E_\pi \left[ \sum_{i=0}^{\infty} \gamma^i \{r(s_i, a_i) - \Omega(\pi(\cdot | s_i))\} \right] = \frac{1}{1 - \gamma} E_{(s, a) \sim d_\pi} [r(s, a) - \Omega(\pi(\cdot | s))].$$

(1)

It turns out that the optimal solution of Eq. (1) is unique (Geist et al. 2019), whereas multiple optimal policies may exist in unregularized MDPs. In later work (Yang et al. 2019), $\Omega(\pi) = -\lambda E_{a \sim \pi} \phi(\pi(a))$, $\pi \in \Delta^A$ was considered for $\lambda > 0$ and $\phi : (0, 1) \to \mathbb{R}$ satisfying some mild conditions. For example, RL with Shannon entropy regularization (Haarnoja et al. 2018) can be recovered by $\phi(\pi(a)) = -\log \pi(a)$, while RL with Tsallis entropy regularization (Lee et al. 2020) can be recovered by $\phi(\pi(a)) = \frac{k}{q - 1} \left(1 - \pi(a)^{q - 1}\right)$ for $k > 0, q > 1$. The optimal policy $\pi^*$ for Eq. (1) with $\Omega$ from Yang et al. (2019) is shown to be

$$\pi^*(a | s) = \max \left\{ g_\phi \left( \mu^*(s) - Q^*(s, a) \right), 0 \right\},$$

(2)

$$Q^*(s, a) = r(s, a) + \gamma E_{s' \sim P(\cdot | s, a)} V^*(s'), V^*(s) = \mu^*(s) - \lambda \sum_{a \in A} \pi^*(a | s) \phi'(\pi^*(a | s)),$$

(3)

where $\phi'(x) = \frac{d}{dx} \phi(x)$, $g_\phi$ is an inverse function of $f_\phi$ for $f_\phi(x) := x \phi(x)$, $x \in [0, 1]$, and $\mu^*$ is a normalization term such that $\sum_{a \in A} \pi^*(a | s) = 1$. Note that we need to solve constraint optimization
We consider regularized IRL that generalizes MaxEntIRL in Eq. (4) to IRL with a general class of entropy regularizers and multi-variate Gaussian policies are used in continuous action spaces. Another commonly used IRL method is Adversarial Inverse Reinforcement Learning (AIRL) (Fu et al., 2019), which involves generative adversarial training (Goodfellow et al., 2014; Ho & Ermon, 2016) to acquire a solution of MaxEntIRL. AIRL considers the structured discriminator (Finn et al., 2016a) \( D(s, a) = \sigma(r(s, a) - \log \pi(a|s)) = \frac{e^{r(s, a)}}{e^{r(s, a)} + \pi(a|s)} \) for some distribution \( \sigma(x) := 1/(1 + e^{-x}) \) and iteratively optimizes the following objective:

\[
\max_{r \in \mathbb{R}^{S \times A}} \mathbb{E}_{(s, a) \sim d_\pi} \left[ \log D_{r, \pi}(s, a) \right] + \mathbb{E}_{(s, a) \sim d_{\pi}} \left[ \log (1 - D_{r, \pi}(s, a)) \right], \\
\max_{\pi \in \Delta_\pi} \mathbb{E}_{(s, a) \sim d_\pi} \left[ \log D_{r, \pi}(s, a) - \log (1 - D_{r, \pi}(s, a)) \right] = \max_{\pi \in \Delta_\pi} \mathbb{E}_{(s, a) \sim d_\pi} \left[ r(s, a) - \log \pi(a|s) \right].
\]

It turns out that AIRL minimizes the divergence between visitation distributions \( d_\pi \) and \( d_{\pi_E} \) by solving the following KL divergence KL (Ghasemipour et al., 2019) for Kullback-Leibler (KL) divergence KL (Ghasemipour et al., 2019).

3 Inverse Reinforcement Learning in Regularized MDPs

In this section, we propose the solution of IRL in regularized MDPs – regularized IRL – and relevant properties in Section 3.1. We then discuss a specific instance of our proposed solution where Tsallis entropy regularizers and multi-variate Gaussian policies are used in continuous action spaces.

3.1 Solutions of Regularized IRL

We consider regularized IRL that generalizes MaxEntIRL in Eq. (4) to IRL with a general class of convex policy regularizers:

\[
IRL(\pi_E; \Omega) := \arg \max_{r \in \mathbb{R}^{S \times A}} \left\{ J_\Omega(r, \pi_E) - \max_{\pi \in \Delta_\pi} J_\Omega(r, \pi) \right\}.
\]

For any convex policy regularizer \( \Omega \), regularized IRL does not suffer from degenerate solutions since there is a unique optimal policy in any regularized MDP (Geist et al., 2019). While Geist et al. (2019) proposed functional-form solutions of regularized IRL (Proposition 5 in Section D.4 of Geist et al., 2019), they are not analytical solutions since a closed-form relation between optimal policy and value function – which is generally intractable – is required. In the following lemma, we propose tractable and analytical solutions. Our solution is motivated from figuring out a reward function that is capable of converting regularized RL into equivalent divergence minimization problem associated with \( \pi \) and \( \pi_E \):

**Lemma 1.** For a policy regularizer \( \Omega : \Delta^A \to \mathbb{R} \), let us define

\[
t(s, a; \pi) := \Omega'(s, a; \pi) - \mathbb{E}_{a' \sim \pi(\cdot|s)} [\Omega'(s, a'; \pi) + \Omega(\pi(\cdot|s))] \\
\quad \Omega'(s, \cdot; \pi) := \nabla \Omega(\pi(\cdot|s)) := \left[ \frac{\partial \Omega(x)}{\partial x_{x \sim \pi(\cdot|s)}} \right]_{x \sim \pi(\cdot|s)} \in \mathbb{R}^A, s \in S.
\]

Then, \( t(s, a; \pi_E) \) for expert’s policy \( \pi_E \) is a solution of regularized IRL with \( \Omega \).
Proof. (Abbreviated. See Appendix A for full version.) With \( r(s, a) = t(s, a; \pi_e) \), the RL objective in Eq. 1 becomes equivalent to a problem of minimizing the discounted sum of Bregman divergences between \( \pi \) and \( \pi_e \)

\[
\arg\min_{\pi \in \Delta^A} \mathbb{E}_\pi \left[ \sum_{t=0}^{\infty} \gamma^t D^A_\Omega(\pi(s|t)||\pi_e(s|t)) \right],
\]

(8)

where \( D^A_\Omega \) is the Bregman divergence (Bregman 1967) defined by \( D^A_\Omega(\pi_1||\pi_2) = \Omega(\pi_1) - \Omega(\pi_2) - \langle \nabla \Omega(\pi_2), \pi_1 - \pi_2 \rangle \) for \( \pi_1, \pi_2 \in \Delta^A \). Due to the non-negativity of the Bregman divergence, \( \pi = \pi_e \) is a solution of Eq. 8 and is unique since Eq. 1 has the unique solution for arbitrary reward functions (Geist et al. 2019).

Especially for any policy regularizer \( \Omega \) represented by an expectation over the policy (Yang et al. 2019), Lemma 1 can be reduced to the following solution in Corollary 1.

Corollary 1. (See Appendix A for proof.) For \( \Omega(\pi) = -\lambda \mathbb{E}_{a \sim \pi} \phi(\pi(a)) \) with \( \pi \in \Delta^A \) (Yang et al. 2019), Eq. 7 becomes

\[
t(s, a; \pi) = -\lambda \cdot \{ f'_\phi(\pi(a|s)) - \mathbb{E}_{a' \sim \pi(s|a)} [f'_\phi(\pi(a'|s)) - \phi(\pi(a'|s))] \}
\]

(9)

for \( f'_\phi(x) = \frac{\partial}{\partial a} (x \phi(x)) \).

Throughout the paper, we denote reward baseline by the expectation \( \mathbb{E}_{a \sim \pi(s|a)} [f'_\phi(\pi(a|s)) - \phi(\pi(a|s))] \). Note that for continuous control tasks with \( \Omega(\pi) = -\lambda \mathbb{E}_{a \sim \pi} \phi(\pi(a)) \), we can obtain the same form of the reward in Eq. 9. When \( \lambda = 1 \) and \( \phi(x) = -\log x \), it can be shown that \( t(s, a; \pi) = -\log \pi(a|s) \), which was used as a reward objective in the previous work (Fu et al. 2018), and that the Bregman divergence in Eq. 8 becomes the KL divergence \( \text{KL}(\pi(s|a)||\pi_e(s|a)) \). Additionally, other solutions of IRL can be found by shaping \( t(s, a; \pi_e) \) as stated in the following lemma:

Lemma 2. Let \( \pi^* \) be the solution of Eq. 1 in a regularized MDP \( \mathcal{M}^r \) with a regularizer \( \Omega : \Delta^A \rightarrow \mathbb{R} \) and an arbitrary reward function \( r \in \mathbb{R}^{S \times A} \). Then, rewards \( r(s, a) - \Phi(s) \) with \( \Phi : S \rightarrow \mathbb{R} \) does not change the solution of Eq. 7. The proof is in Section D.4. of Geist et al. (2019).

From Lemma 1 and Lemma 2, we prove the sufficient condition of rewards being solutions of the IRL problem. However, the necessary condition – a set of those solutions are the only possible solutions – is not proved. In the following lemma, we check how the proposed solution is related to the normalized state-visitation distribution which can be discussed in the line of distribution matching perspective on imitation learning problems (Ho & Ermon 2016; Fu et al. 2018; Ghasemipour et al. 2019):

Lemma 3. Given the policy regularizer \( \Omega \), let us define \( \overline{\Omega}(d) := \mathbb{E}_{(s, a) \sim d} [\Omega(\bar{\pi}_d(s|a))] \) for an arbitrary normalized state-action visitation distribution \( d \in \Delta^{S \times A} \) and the policy \( \bar{\pi}_d(s|a) := \frac{d(s, a)}{\sum_{s', a'} d(s', a')} \) induced by \( d \). Then, Eq. 7 is equal to

\[
t(s, a; \bar{\pi}_d) = [\nabla \overline{\Omega}(d)](s, a).
\]

(10)

When \( \overline{\Omega}(d) \) is strictly convex and a solution \( t(s, a; \bar{\pi}_e) = [\nabla \overline{\Omega}(d_{\pi_e})] \) of IRL in Eq. 10 is used, the RL objective in Eq. 1 is equal to

\[
\arg\min_{\pi \in \Delta^A} D^A_{\overline{\Omega}}(d_{\pi}||d_{\pi_e}),
\]

where \( D^A_{\overline{\Omega}} \) is the Bregman divergence among visitation distributions defined by \( D^A_{\overline{\Omega}}(d_1||d_2) = \overline{\Omega}(d_1) - \overline{\Omega}(d_2) - \langle \nabla \overline{\Omega}(d_2), d_1 - d_2 \rangle \) for visitation distributions \( d_1 \) and \( d_2 \). The proof is in Appendix D.

In the above lemma, although the strict convexity of a policy regularizer \( \Omega \) does not guarantee the assumption on the strict convexity of \( \overline{\Omega} \), it has been shown to be true for Shannon entropy (Ho & Ermon 2016) and a specific instance of Tsallis entropy regularizers (Lee et al. 2018).
3.2 IRL with Tsallis entropy regularization and Gaussian policies

For continuous controls, multi-variate Gaussian policies are often used in practice (Schulman et al., 2015, 2017) and we consider IRL problems with those policies in this subsection. In particular, we consider IRL with Tsallis entropy regularizer \( \Omega(\pi) = - T^q_\pi(\pi) \) for a multi-variate Gaussian policy \( \pi(\cdot|s) = \mathcal{N}(\mu(s), \Sigma(s)) \) with \( \mu(s) = [\mu_1(s), \ldots, \mu_d(s)]^T \), \( \Sigma(s) = \text{diag}((\sigma_1(s))^2, \ldots, (\sigma_d(s))^2) \). In such a case, we can obtain analytical forms of the following quantities:

**Tsallis entropy.** The analytical form of Tsallis entropy for a multi-variate Gaussian policy is

\[
T^k_q(\pi(\cdot|s)) = \frac{k(1 - e^{(1-q)R_q(\pi(\cdot|s))})}{q - 1},
\]

for Renyi entropy \( R_q \). Its derivation is given in Appendix F.

**Reward baseline.** The reward baseline term \( \mathbb{E}_{a \sim \pi(\cdot|s)}[f^\phi(a|s) - \phi(\pi(a|s))] \) in Corollary 1 generally intractable except for either discrete control problems or when Shannon entropy regularization (where the reward baseline is equal to \(-1\)) is considered. Interestingly, as long as the analytical form of Tsallis entropy can be derived, that of the corresponding reward baseline can also be derived since the reward baseline satisfies

\[
\mathbb{E}_{a \sim \pi(\cdot|s)}[f^\phi(a|s) - \phi(\pi(a|s))] = (q - 1)\mathbb{E}_{a \sim \pi(\cdot|s)}[\phi(\pi(a|s))] - k = (q - 1)T^k_q(\pi(\cdot|s)) - k.
\]

Here, the first equality holds with \( f^\phi(x) = \frac{k}{q - 1}(1 - qx^{q-1}) = q\phi(x) - k \) for Tsallis entropy regularization.

**Bregman divergence associated with Tsallis entropy.** For two different multivariate Gaussian policies, we derive the analytical form of the Bregman divergence (associated with Tsallis entropy) between two policies. The resultant divergence has a complicated form, so we leave it in Appendix F.3 with its derivation.

For deeper understanding of Tsallis entropy and its associated Bregman divergence in regularized MDPs, we consider an example in Figure 1. We first assume that both learning agents’ and experts’ policies follow uni-variate Gaussian distributions \( \pi = \mathcal{N}(\mu, \sigma^2) \) and \( \pi^*_e = \mathcal{N}(0, (e^{-3})^2) \), respectively. We then evaluate the Bregman divergence in Figure 1 by using its analytical form and varying \( q \) from 1.0 – which corresponds to the KL divergence – to 2.0. We observe that the constant \( q \) from the Tsallis entropy affects the sensitivity of the associated Bregman divergence w.r.t. the mean and standard deviation of the learning agent’s policy \( \pi \). Specifically, as \( q \) increases, the size of the valley – relatively red regime in Figure 1 – across the \( \mu \)-axis and \( \log \sigma \)-axis decreases. This suggests that for larger \( q \), minimizing the Bregman divergence requires more tightly matching means and variances of \( \pi \) and \( \pi^*_e \).

Figure 1: Bregman divergence \( D^\Omega(\pi||\pi^*_e) \) associated with Tsallis entropy \( \Omega(\pi) = - T^4_\pi(\pi) \) between two uni-variate Gaussian distributions \( \pi = \mathcal{N}(\mu, \sigma^2) \) and \( \pi^*_e = \mathcal{N}(0, (e^{-3})^2) \) (green point in each subplot). In each subplot, we normalized the Bregman divergence so that the maximum value becomes 1. Note that for \( q = 1 \), \( D^\Omega(\pi||\pi^*_e) \) becomes the KL divergence \( \text{KL}(\pi||\pi^*_e) \).

4 ALGORITHMIC CONSIDERATION

Based on a solution for regularized IRL in the previous section, we focus on developing an IRL algorithm for \( \Omega(\pi) = - \lambda \mathbb{E}_{a \sim \pi} [\phi(\pi(a))] \) (Yang et al., 2019) in this section. Particularly to recover
The reward function \( t(s, a; \pi_e) \) in Corollary \( \Pi \) we design an adversarial training objective as follows. Motivated by Airl (Fu et al., 2018), we consider the following structured discriminator associated with \( \pi, r \) and \( t \) in Corollary \( \Pi \):

\[
D_{r,\pi}(s, a) = \sigma(r(s, a) - t(s, a; \pi)), \quad \sigma(z) = \frac{1}{1 + e^{-z}}, \quad z \in \mathbb{R}.
\]

Note that we can recover the discriminator of Airl in Eq. \( 5 \) when \( t(s, a) = \log \pi(a|s) (\phi(x) = \log x \text{ and } \lambda = 1) \). Then, we consider the following optimization objective of the discriminator which is the same as that of Airl:

\[
\hat{t}(s, a; \pi) := \arg \max_{r \in \mathbb{R}^{S \times A}} \mathbb{E}_{(s, a) \sim d_{\pi_E}} [\log D_{r,\pi}(s, a)] + \mathbb{E}_{(s, a) \sim d_{\pi}} [\log(1 - D_{r,\pi}(s, a))]. \tag{11}
\]

Since the function \( x \mapsto a \log \sigma(x) + b \log(1 - \sigma(x)) \) attains its maximum at \( \sigma(x) = \frac{a}{a+b} \), or equivalently at \( x = \log \frac{a}{b} \) (Goodfellow et al., 2014; Mescheder et al., 2017), it can be shown that

\[
\hat{t}(s, a; \pi) = t(s, a; \pi) + \log \frac{d_{\pi_E}(s, a)}{d_{\pi}(s, a)}. \tag{12}
\]

When \( \pi = \pi_E \) in Eq. \( 12 \), we have \( \hat{t}(s, a; \pi_E) = t(s, a; \pi_E) \) since \( d_{\pi} = d_{\pi_E} \), which means the maximizer \( \hat{t} \) becomes the solution of IRL after the agent successfully imitates the expert’s policy \( \pi_E \). To do so, we consider the following iterative algorithm. Assuming that we find out the optimal reward approximator \( \hat{t}(s, a; \pi^{(i)}) \) in Eq. \( 12 \) for the policy \( \pi^{(i)} \) of the \( i \)-th iteration, we get the policy \( \pi^{(i+1)} \) by optimizing the following objective:

\[
\pi^{(i+1)} := \arg \max_{\pi \in \Delta_{\pi_E}} \mathbb{E}_{(s, a) \sim d_{\pi}} \left[ \hat{t}(s, a; \pi^{(i)}) - \Omega(\pi(\cdot|s)) \right]. \tag{13}
\]

The above expectation in Eq. \( 13 \) can be decomposed into the following two terms

\[
\mathbb{E}_{(s, a) \sim d_{\pi}} \left[ \hat{t}(s, a; \pi^{(i)}) - \Omega(\pi(\cdot|s)) \right] = \mathbb{E}_{(s, a) \sim d_{\pi}} \left[ t(s, a; \pi^{(i)}) - \Omega(\pi(\cdot|s)) \right] - \text{KL}(d_{\pi}||d_{\pi_E})
\]

\[
= - \mathbb{E}_{(s, a) \sim d_{\pi}} \left[ D_{\Omega}(\pi(\cdot|s)||\pi^{(i)}(\cdot|s)) \right] - \text{KL}(d_{\pi}||d_{\pi_E}), \tag{14}
\]

where the second equality follows since Lemma \( \Pi \) implies that \( t(s, a; \pi^{(i)}) \) is a reward function that makes \( \pi^{(i)} \) an optimal policy in the \( \Omega \)-regularized MDP. Minimizing term (II) in Eq. \( 14 \) makes \( \pi^{(i+1)} \) close to \( \pi_E \) while minimizing term (I) can be regarded as a conservative policy optimization around the policy \( \pi^{(i)} \) (Schulman et al., 2015).

In practice, we parameterize our reward and policy approximations with neural networks and train them by using an off-policy Regularized Actor-Critic (RAC) (Yang et al., 2019) as described in Algorithm \( \Pi \). We evaluate our algorithm, Regularized Adversarial Inverse Reinforcement Learning (RAIRL), across various scenarios, below.
We consider an environment with a 2-dimensional continuous state space as described in Figure 3. At \( \pi \) we consider a 4-armed bandit environment as shown in Figure 2 (left). An expert’s policy \( \pi_e \) is considered.

We summarize the experimental setup as follows. In our experiments, we consider \( \Omega(\pi) = -\lambda \mathbb{E}_{s \sim \pi} [\phi(\pi(a))] \) with the following regularizers from Yang et al. (2019): (1) Shannon entropy \( \phi(x) = -\log x \), (2) Tsallis entropy regularizer \( \phi(x) = \frac{q}{q-1} (1 - x^{q-1}) \), (3) exp regularizer \( \phi(x) = e - e^x \), (4) cos regularizer \( \phi(x) = \cos(\frac{x}{2}) \), (5) sin regularizer \( \phi(x) = 1 - \sin(\frac{x}{2}) \).

In addition, we model the reward approximator of RAIRL as a neural network with either one of the following models: (1) Non-structured model (NSM) – a simple feed-forward neural network that outputs real values used in AIRL (Fu et al., 2018) – and (2) Density-based model (DBM) (Figure G.2 in Appendix) – a model using a neural network for \( \pi \) (softmax for discrete controls and multi-variate Gaussian model for continuous controls) of the solution in Eq. (1). For RL algorithm of RAIRL, we implement Regularized Actor Critic (RAC) (Yang et al., 2019) on top of SAC implementation of Rlpyt (Stooke & Abbeel, 2019). Other settings are summarized in Appendix G. For all experiments, we use 5 runs and report 95% confidence interval.

### 5.1 EXPERIMENT 1: MULTI-ARMED BANDIT (DISCRETE ACTION)

We consider a 4-armed bandit environment as shown in Figure 2 (left). An expert’s policy \( \pi_e \) is assumed to be either dense (with probability 0.1, 0.2, 0.3, 0.4 for \( a = 0, 1, 2, 3 \)) or sparse (with probability 0, 0, 1/3, 2/3 for \( a = 0, 1, 2, 3 \)). For those experts, we use RAIRL with actions sampled from \( \pi_e \) and compare learned rewards with the ground truth reward \( \ell(s, a; \pi_e) \) in Lemma 1. When \( \pi_e \) is dense, RAIRL successfully recovers the ground truth rewards irrespective of the reward model choices. When sparse \( \pi_e \) is used, however, RAIRL with a non-structured model (RAIRL-NSM) failed to recover the rewards for \( a = 0, 1 \) – where \( \pi_e(a) = 0 \) – due to the lack of samples at the end of the imitation. On the other hand, RAIRL with a density-based model (RAIRL-DBM) can recover the correct rewards due to the softmax layer which maintains the sum over the outputs equal to 1. Therefore, we argue that using DBM is necessary for correct reward acquisition since a set of demonstrations is generally sparse. In the following experiment, we show the choice of reward models indeed affects the performance of rewards.

### 5.2 EXPERIMENT 2: BERMUDA WORLD (CONTINUOUS OBSERVATION, DISCRETE ACTION)

We consider an environment with a 2-dimensional continuous state space as described in Figure 3. At each episode, the learning agent is initialized uniformly on the \( x \)-axis between \(-5 \) and 5, and there are 8 possible actions – an angle in \( \{-\pi, -\frac{3\pi}{2}, ..., \frac{3\pi}{2}, \pi\} \) that determines the direction of movement. An expert in Bermuda World considers 3 target positions \((-5, 10), (0, 10), (5, 10)\) and behaves stochastically as shown in Figure 3 (Left). With 1000 demonstrations sampled from the expert, the agent is evaluated during training with the Bregman divergences that correspond to each policy regularizer. RAIRL-DBM is shown to minimize the target divergence more effectively compared to RAIRL-NSM although both achieve comparable performances during training. Moreover, we substitute \( \lambda \) with 1, 5, 10 and observe that learning with \( \lambda \) larger than 1 returns better rewards – only \( \lambda = 1 \) was considered in AIRL (Fu et al., 2018). Note that in all cases, the minimum divergence
5.3 EXPERIMENT 3: MUJOCO (CONTINUOUS OBSERVATION AND ACTION)

We evaluate RAIRL on MuJoCo continuous control tasks (Hopper-v2, Walker-v2, HalfCheetah-v2, Ant-v2) as follows. We assume multivariate-Gaussian policies (with diagonal covariance matrices) for both learner’s policy $\pi$ and expert’s policy $\pi_E$. Instead of tanh-squashed policy in Soft-Actor Critic \cite{haarnoja2018soft}, we use hyperbolized environments – where tanh is regarded as a part of the environment – with additional engineering on the policy networks (See Appendix G.4 for details). We use 100 demonstrations from $\pi_E$ to evaluate RAIRL. In MuJoCo experiments, we focus on a set of Tsallis entropy regularizer ($\Omega(\pi) = -T_{q'}^{q}(\pi)$) with $q = 1, 1.5, 2$ – where Tsallis entropy becomes Shannon entropy for $q = 1$. We then exploit the analytical quantities for multi-variate Gaussian distributions in Section 3.2 to stabilize RAIRL and evaluate its performance in terms of various Bregman divergences.

The evaluation performances for RAIRL are depicted in Figure 4. From 30 rollout trajectories, we evaluate $\pi$ with an episodic score (Figure 4 (Left)) and averaged Bregman divergences between $\pi$ and $\pi_E$ (Figure 4 (Right)) associated with $\Omega(\pi) = -T_{q'}^{q}(\pi)$ with $q = 1, 1.5$ and 2. Note that the objective of RAIRL with $\Omega = -T_{q'}$ is to minimize the corresponding Bregman divergence with $q' = q$. In Figure 4 (Left), both RAIRL-DBM and RAIRL-NSM are shown to achieve the expert performance, irrespective of $q$, in Hopper-v2, Walker-v2, and HalfCheetah-v2. In contrast, RAIRL in Ant-v2 fails to achieve the expert’s performance within 2,000,000 steps and RAIRL-NSM highly outperforms RAIRL-DBM in our setting. Although the evaluation scores are comparable for all methods in
Hopper-v2, Walker-v2, and HalfCheetah-v2, respective divergences are shown to be highly different from one another as shown in Figure 4 (Right). RAIRL with \( q = 2 \) in most cases achieves the minimum Bregman divergence (over all three divergences with \( q' = 1, 1.5, 2 \)), whereas RAIRL with \( q = 1 \) – which corresponds to AIRL (Fu et al., 2018) – achieves the maximum divergence in most cases. This result is in alignment with our intuition from Section 3.2; as \( q \) increases, minimizing the Bregman divergence requires much tighter matching between \( \pi \) and \( \pi_E \). Therefore, we believe RAIRL is suitable for making imitation learning safer in practice compared to AIRL. Unfortunately, RAIRL fails to acquire a reward function that effectively minimizes the target divergence in continuous controls. We believe this is because \( \pi \) is a probability density function in continuous controls and causes large variance during training, while \( \pi \) is a mass function and is well-bounded in discrete control problems.

6 DISCUSSION AND FUTURE WORKS

We consider the problem of IRL in regularized MDPs (Geist et al., 2019), assuming a general class of convex policy regularizers. We theoretically derive its solution (a set of reward functions) and show that learning with these rewards is equivalent to a specific instance of imitation learning – i.e., one that minimizes the Bregman divergence associated with policy regularizers. We propose RAIRL – a practical sampled-based IRL algorithm in regularized MDPs – and evaluate its applicability on policy imitation (for discrete and continuous controls) and reward acquisition (for discrete control).

Finally, recent advances in imitation learning and IRL are built from the perspective of regarding imitation learning as statistical divergence minimization problems (Ghasemipour et al., 2019). Although Bregman divergence is known to cover various divergences, it does not include some divergence families such as \( f \)-divergence (Csiszár, 1963; Amari, 2009). Therefore, we believe that considering RL with policy regularization different from (Geist et al., 2019) and its inverse problem is a possible way of finding the links between imitation learning and various statistical distances.

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A PROOF OF LEMMA 1

Let us define \( \pi^a = \pi(\cdot|s) \). For \( r(s, a) = t(s, a; \pi_E) \), the RL objective Eq. (1) satisfies

\[
E_\pi \left[ \sum_{i=0}^{\infty} \gamma^i \left\{ r(s_i, a_i) - \Omega(\pi^a) \right\} \right] \overset{\text{(i)}}{=} E_\pi \left[ \sum_{i=0}^{\infty} \gamma^i \left\{ t(s_i, a_i; \pi_E) - \Omega(\pi^a) \right\} \right]
\]

\[
\overset{\text{(ii)}}{=} E_\pi \left[ \sum_{i=0}^{\infty} \gamma^i \left\{ \Omega'(s_i, a_i; \pi_E) - \Omega(a) \right\} \right]
\]

\[
\overset{\text{(iii)}}{=} E_\pi \left[ \sum_{i=0}^{\infty} \gamma^i \left\{ \Omega'(s_i, a_i; \pi_E) - \Omega(a) \right\} \right]
\]

\[
= E_\pi \left[ \sum_{i=0}^{\infty} \gamma^i \left\{ \Omega'(s_i, a_i; \pi_E) - \Omega(a) \right\} \right]
\]

where (i) follows from the assumption \( r(s, a) = t(s, a; \pi_E) \) in Lemma 1, (ii) follows from the definition of \( t(s, a) \) in Eq. (7), (iii) follows since taking the inner expectation first does not change the overall expectation, (iv) follows from the definition of \( \Omega' \) in Lemma 1 and \( \sum_{a \in \mathcal{A}} \pi(a) \{ \nabla \Omega'(\pi) \}(a) = \langle \nabla \Omega'(\pi), \pi \rangle \mathcal{A} \), and (v) follows from the definition of the Bregman divergence, i.e., \( D_{\Omega'}(\pi_1, \pi_2) = \Omega(\pi_1) - \Omega(\pi_2) - \langle \nabla \Omega(\pi_2), \pi_1 - \pi_2 \rangle \mathcal{A} \). Due to the non-negativity of \( D_{\Omega'} \), Eq. (15) is less than or equal to zero which can be achieved when \( \pi = \pi_E \).

B PROOF OF COROLLARY 1

Let \( a \in \{1, \ldots, |\mathcal{A}|\} \) and \( \pi_a = \pi(\cdot|a) \) for simplicity. For

\[
\Omega(\pi) = -\lambda \sum_{a \in \mathcal{A}} \pi_a \phi(\pi_a) = -\lambda \sum_{a \in \mathcal{A}} \pi_a \phi(\pi_a)
\]

with \( f_\phi(x) = x\phi(x) \), we have

\[
\nabla \Omega(\pi) = -\lambda \nabla \pi_{a=1}^{\ldots,|\mathcal{A}|} \sum_{a \in \mathcal{A}} f_\phi(\pi_a) = -\lambda [f'_\phi(\pi_1), \ldots, f'_\phi(\pi_{|\mathcal{A}|})]^T
\]

for \( f'_\phi(x) = \frac{\partial}{\partial \pi}(x\phi(x)) \). Therefore, for \( \pi^* = \pi(\cdot|s) \) we have

\[
t(s, a; \pi) = \langle \nabla \Omega(\pi^*), \pi^*, \pi^a \rangle + \Omega(\pi^a)
\]

\[
= -\lambda f'_\phi(\pi^a) - \sum_{a' \in \mathcal{A}} \pi_a' \phi(\pi_a') + \left( -\lambda \sum_{a' \in \mathcal{A}} \pi_{a'} \phi(\pi_{a'}) \right)
\]

\[
= -\lambda \left\{ f'_\phi(\pi_a) - \sum_{a' \in \mathcal{A}} \pi_a' f'_\phi(\pi_a') \right\} \mathcal{A} + \left( -\lambda \sum_{a' \in \mathcal{A}} \pi_{a'} \phi(\pi_{a'}) \right)
\]

\[
= -\lambda \left\{ f'_\phi(\pi_a) - \sum_{a' \in \mathcal{A}} \pi_a' f'_\phi(\pi_a') \right\}
\]

\[
= -\lambda \left\{ f'_\phi(\pi_a) - \sum_{a' \in \mathcal{A}} \pi_a' \phi(\pi_{a'}) \right\}
\]

\[
= -\lambda \left\{ f'_\phi(\pi_a) - \sum_{a' \in \mathcal{A}} \pi_a' \phi(\pi_{a'}) \right\}.
\]
C \textbf{Proof of Optimal Rewards on Continuous Controls}

Note that for two continuous distributions $\mathbb{P}_1$ and $\mathbb{P}_2$ having probability density functions $p_1(x)$ and $p_2(x)$, respectively, the Bregman divergence can be defined as (Guo et al., 2017; Jones & Byrne, 1990)

$$D^\mathcal{X}_\Omega(\mathbb{P}_1||\mathbb{P}_2) := \int_{\mathcal{X}} \{\omega(p_1(x)) - \omega(p_2(x)) - \omega'(p_2(x))(p_1(x) - p_2(x))\} \, dx,$$

(16)

where $\omega'(x) := \frac{\partial}{\partial x}\omega(x)$ and the divergence is measure point-wisely on $x \in \mathcal{X}$. Let us assume

$$\Omega(\pi) = \int_{\mathcal{A}} \omega(\pi(a)) \, da$$

(17)

for a probability density function $\pi$ on $\mathcal{A}$ and define

$$t(s, a; \pi) := \omega'(\pi^*(a)) - \int_{\mathcal{A}} [\pi^*(a') \omega'(\pi^*(a')) - \omega(\pi^*(a'))] \, da'.$$

(18)

for $\pi^* = \pi(\cdot|s)$. For $r(s, a) = t(s, a; \pi_E)$, the RL objective in Eq. (1) satisfies

$$\mathbb{E}_\pi \left[ \sum_{i=0}^{\infty} \gamma^i \{r(s_i, a_i) - \Omega(\pi^*)\} \right] = \mathbb{E}_\pi \left[ \sum_{i=0}^{\infty} \gamma^i \{t(s_i, a_i; \pi_E) - \Omega(\pi^*)\} \right]$$

$$= \mathbb{E}_\pi \left[ \sum_{i=0}^{\infty} \gamma^i \left\{ \omega'(\pi_E^*(a_i)) - \int_{\mathcal{A}} [\pi_E^*(a) \omega'(\pi_E^*(a)) - \omega(\pi_E^*(a))] \, da \right\} \right]$$

$$= \mathbb{E}_\pi \left[ \sum_{i=0}^{\infty} \gamma^i \left\{ \int_{\mathcal{A}} \pi_E^*(a) \omega'(\pi_E^*(a)) \, da - \int_{\mathcal{A}} [\pi_E^*(a) \omega'(\pi_E^*(a)) - \omega(\pi_E^*(a)) + \omega(\pi_E^*(a))] \, da \right\} \right]$$

$$= \mathbb{E}_\pi \left[ \sum_{i=0}^{\infty} \gamma^i \int_{\mathcal{A}} \left\{ \omega(\pi_E^*(a)) - \omega(\pi_E^*(a)) + \pi_E^*(a) \omega'(\pi_E^*(a)) - \pi_E^*(a) \omega'(\pi_E^*(a)) \right\} \, da \right]$$

$$= -\mathbb{E}_\pi \left[ \sum_{i=0}^{\infty} \gamma^i \int_{\mathcal{A}} \left\{ \omega(\pi_E^*(a)) - \omega(\pi_E^*(a)) - \pi_E^*(a) \omega'(\pi_E^*(a)) + \pi_E^*(a) \omega'(\pi_E^*(a)) \right\} \, da \right]$$

$$= -\mathbb{E}_\pi \left[ \sum_{i=0}^{\infty} \gamma^i \{\omega(\pi_E^*(a)) - \omega(\pi_E^*(a)) - \omega'(\pi_E^*(a)) \pi_E^*(a) + \pi_E^*(a) \omega'(\pi_E^*(a)) \} \, da \right]$$

$$= -\mathbb{E}_\pi \left[ \sum_{i=0}^{\infty} \gamma^i D^\mathcal{A}_\omega(\pi^*||\pi_E^*) \right],$$

(19)

where (i) follows from $r(s, a) = t(s, a; \pi_E)$, (ii) follows from Eq. (17) and Eq. (18), and (iii) follows from the definition of Bregman divergence in Eq. (16). Due to the non-negativity of $D_\omega$, Eq. (21) is less than or equal to zero which can be achieved when $\pi = \pi_E$. Also, $\pi = \pi_E$ is a unique solution since Eq. (1) has a unique solution for arbitrary reward functions.

D \textbf{Proof of Lemma 3}

\textbf{RL objective in Regularized MDPs w.r.t. normalized visitation distributions.} For a reward function $r \in \mathbb{R}^{\mathcal{S} \times \mathcal{A}}$ and a strongly convex function $\Omega : \Delta^{\mathcal{S}} \rightarrow \mathbb{R}$, the RL objective $J_\Omega(r, \pi)$ in Eq. (1) is equivalent to

$$\arg\max_\pi J_\Omega(r, d_\pi) := (r, d_\pi)_{\mathcal{S} \times \mathcal{A}} - \bar{\Omega}(d_\pi),$$

(20)

where for a set $\mathcal{D}$ of normalized visitation distributions [Syed et al., 2008]

$$\mathcal{D} := \left\{ d \in \Delta^{\mathcal{S} \times \mathcal{A}} : \sum_{a'} d(s', a') = (1 - \gamma)P_0(s') + \gamma \sum_{s,a} P(s'|s, a)d(s, a), \forall s' \in \mathcal{S} \right\},$$

13
we define $\bar{\Omega}(d) := \mathbb{E}_{(s,a) \sim d}[\Omega(\bar{\pi}_d(s))]$ and $\bar{\pi}_d(s) := \frac{d(s,a)}{\sum_{a'} d(s,a')} \in \Delta^d_S$ for $d \in \mathcal{D}$ and use $\bar{\pi}_a(s) = \pi(s)$ for all $s \in \mathcal{S}$. For $\bar{\Omega} : \mathcal{D} \rightarrow \mathbb{R}$, its convex conjugate $\bar{\Omega}^*$ is

$$\bar{\Omega}^*(r) := \max_{d \in \mathcal{D}} J_\Omega(r, d)$$

$$= \max_{d \in \mathcal{D}} (r, d)_{\mathcal{S} \times \mathcal{A}} - \bar{\Omega}(d)$$

$$(i) \quad \max_{\pi \in \Delta^d_S} (r, d_\pi)_{\mathcal{S} \times \mathcal{A}} - \bar{\Omega}(d_\pi)$$

$$= \max_{\pi \in \Delta^d_S} \sum_{s,a} d_\pi(s, a) [r(s, a) - \Omega(\pi(a|s))]$$

$$= (1 - \gamma) \cdot \max_{\pi \in \Delta^d_S} J_\Omega(r, \pi),$$

(21)

where $(i)$ follows from using the one-to-one correspondence between policies and visitation distributions (Syed et al., 2008; Ho & Ermon, 2016). Note that Eq. (21) is equal to the optimal discounted average return in regularized MDPs.

**IRL objective in Regularized MDPs w.r.t. normalized visitation distributions.** By using the RL objective in Eq. (20), we can rewrite the IRL objective in Eq. (6) w.r.t. the normalized visitation distributions as the maximization of the following objective over $r \in \mathbb{R}^{S \times A}$:

$$(1 - \gamma) \cdot \left\{ J_\Omega(r, \pi_E) - \max_{\pi \in \Delta^d_S} J_\Omega(r, \pi) \right\}$$

$$= J_\Omega(r, d_{\pi_E}) - \max_{d \in \mathcal{D}} J_\Omega(r, d)$$

$$= \min_{d \in \mathcal{D}} \left\{ J_\Omega(r, d_{\pi_E}) - J_\Omega(r, d) \right\}$$

$$= \min_{d \in \mathcal{D}} \left\{ (r, d_{\pi_E})_{\mathcal{S} \times \mathcal{A}} - \bar{\Omega}(d_{\pi_E}) \right\} - \bar{\Omega}(d)$$

$$= \min_{d \in \mathcal{D}} \left\{ \bar{\Omega}(d) - \bar{\Omega}(d_{\pi_E}) - \langle r, d - d_{\pi_E} \rangle_{\mathcal{S} \times \mathcal{A}} \right\}. \quad (22)$$

Note that if $\nabla \bar{\Omega}(d)$ is well-defined and $r = \nabla \bar{\Omega}(d_{\pi_E})$ for any strictly convex $\bar{\Omega}$, Eq. (22) is equal to

$$\min_{d \in \mathcal{D}} \left\{ \bar{\Omega}(d) - \bar{\Omega}(d_{\pi_E}) - \langle \nabla \bar{\Omega}(d_{\pi_E}), d - d_{\pi_E} \rangle_{\mathcal{S} \times \mathcal{A}} \right\} = \min_{d \in \mathcal{D}} D^S \times A(d||d_{\pi_E}),$$

where the equality comes from the definition of Bregman divergence.

**Proof of** $t(s, a; \pi_d) = \nabla \bar{\Omega}(d)(s, a)$. For simpler notation, we use matrix-vector notation for the proof when discrete state and action spaces $\mathcal{S} = \{1, \ldots, |S|\}$ and $\mathcal{A} = \{1, \ldots, |A|\}$ are considered. For a normalized visitation distribution $d \in \mathcal{D}$, let us define

$$d^s_a := d(s, a), \; s \in \mathcal{S}, a \in \mathcal{A},$$

$$d^s := [d^s_1, \ldots, d^s_{|A|}]^T \in \mathbb{R}^{|A|}, \; s \in \mathcal{S},$$

$$D := [d^1, \ldots, d^{|S|}]^T \in \mathbb{R}^{|S| \times |A|},$$

$$\bar{\pi}(x) := \frac{x}{1_{|A|}^T x} = \frac{1}{\sum_{a \in \mathcal{A}} d^s_a} \left[ x_1, \ldots, x_{|A|} \right]^T \in \mathbb{R}^{|A|}, \; x := \left[ x_1, \ldots, x_{|A|} \right]^T \in \mathbb{R}^{|A|},$$

where $1_{|A|} = [1, \ldots, 1]^T \in \mathbb{R}^{|A|}$ is an $|A|$-dimensional all-one vector. By using these notations, the original $\bar{\Omega}$ can be rewritten as

$$\bar{\Omega}(D) = \sum_{s,a} d^s_a \bar{\Omega}(d^s_a) = \sum_{s \in \mathcal{S}} 1_{|A|}^T d^s (\bar{\pi}(d^s_a)).$$
The gradient of $\Omega$ w.r.t. $D$ (using denominator-layout notation) is

$$\nabla_D \Omega(D) = \left[ \frac{\partial \Omega(D)}{\partial d_1}, \ldots, \frac{\partial \Omega(D)}{\partial d_{|S|}} \right]^T \in \mathbb{R}^{S \times A},$$

where each element of $\nabla_D \Omega(D)$ satisfies

$$\frac{\partial \Omega(D)}{\partial d_s} = \left[ \frac{\partial \Omega(D)}{\partial d_{s1}}, \ldots, \frac{\partial \Omega(D)}{\partial d_{s|A|}} \right]^T$$

$$= \frac{\partial}{\partial d_s} \left\{ \sum_{a \in S} I_{\pi_a}^T d_s^a \Omega(\pi(d^a)) \right\}$$

$$= \Omega(\pi(d^a)) I_{A} + I_{\pi_a}^T d_s \frac{\partial \Omega(\pi(d^a))}{\partial d_s}$$

$$= \Omega(\pi(d^a)) I_{A} + I_{\pi_a}^T d_s \frac{\partial \pi_a(d^a)}{\partial d_s} \frac{\partial \Omega(\pi(d^a))}{\partial \pi_a(d^a)} \frac{\partial \pi_a(d^a)}{\partial d_s}.$$ (23)

for

$$\frac{\partial \pi_a(d^a)}{\partial d_s} = \left[ \frac{\partial \pi_1(d^a)}{\partial d_s}, \ldots, \frac{\partial \pi_{|A|}(d^a)}{\partial d_s} \right],$$

$$\frac{\partial \pi_a(d^a)}{\partial d_s} = \frac{\partial}{\partial d_s} \left[ d_s \{ I_{\pi_a}^T d_s^a \} \right] = \frac{\partial d_s}{\partial d_s} \{ I_{\pi_a}^T d_s^a \}^{-1} + d_s \frac{\partial \{ I_{\pi_a}^T d_s^a \}^{-1}}{\partial d_s}.$$ (24)

Note that each element of $\frac{\partial \pi_a(d^a)}{\partial d_s}$ satisfies

$$\frac{\partial \pi_a(d^a)}{\partial d_s} = d_s \{ I_{\pi_a}^T d_s^a \}^{-1} + d_s \frac{\partial \{ I_{\pi_a}^T d_s^a \}^{-1}}{\partial d_s}$$

$$= \sum_{a' \in S} I_{a'} \{ I_{\pi_a}^T d_s^a \}^{-1} - d_s \{ I_{\pi_a}^T d_s^a \}^{-2} \pi_a(d^a) (I_{\pi_a}^T d_s^a)^{-1}$$

$$= \sum_{a' \in S} I_{a'} \{ I_{\pi_a}^T d_s^a \}^{-1} \{ I_{\pi_a}^T d_s^a \}^{-1}$$

$$= (I_{\pi_a}^T d_s^a)^{-1} \{ I_{\pi_a}^T d_s^a \}^{-1} \{ I_{\pi_a}^T d_s^a \}^{-1}$$

and thus,

$$\frac{\partial \pi_a(d^a)}{\partial d_s} = (I_{\pi_a}^T d_s^a)^{-1} \{ I_{A \times A} - I_{A \pi_a(d^a)} \} \{ I_{\pi_a}^T d_s^a \}^{-1}.$$ (25)

By substituting Eq. (24) into Eq. (23), we have

$$\frac{\partial \Omega(D)}{\partial d_s} = \Omega(\pi(d^a)) I_{A} + I_{\pi_a}^T d_s \frac{\partial \pi_a(d^a)}{\partial d_s} \frac{\partial \Omega(\pi(d^a))}{\partial \pi_a(d^a)} \frac{\partial \pi_a(d^a)}{\partial d_s}.$$ (25)

If we use the function notation, Eq. (25) can be written as

$$\nabla \Omega(d)(s, a) = \nabla \Omega(\pi_{d^a}(s))(a) - E_{a' \sim \pi_{d^a}(s)} \nabla \Omega(\pi_{d^a}(s))(a') + \Omega(\pi_{d^a}(s))$$

for $t$ of Eq. (7) in Lemma 1.
E DERIVATION OF BREGMAN-DIVERGENCE-BASED MEASURE IN CONTINUOUS CONTROLS

In Eq. (16), the Bregman divergence in the control task is defined as

$$D_{\Omega}^A(\mathbb{P}_1||\mathbb{P}_2) := \int_{X} \{\omega(p_1(x)) - \omega(p_2(x)) - \omega'(p_2(x))(p_1(x) - p_2(x))\} dx. \quad (26)$$

Note that we consider $\Omega(p) = \int_{X} \omega(p(x)) dx = \int_{X} [-f_\phi(p(x))] dx$ for $f_\phi(x) = x\phi(x)$, which makes Eq. (26) equal to

$$\int_{X} \{-p_1(x)\phi(p_1(x)) + p_2(x)\phi(p_2(x)) + f_\phi'(p_2(x))(p_1(x) - p_2(x))\} dx$$

$$= \int_{X} p_1(x) \{f_\phi'(p_2(x)) - \phi(p_1(x))\} dx - \int_{X} p_2(x) \{f_\phi'(p_2(x)) - \phi(p_2(x))\} dx$$

$$= E_{x \sim p_1} [f_\phi'(p_2(x)) - \phi(p_1(x))] - E_{x \sim p_2} [f_\phi'(p_2(x)) - \phi(p_2(x))].$$

Thus, by considering a learning agent’s policy $\pi^\ast = \pi(\cdot|s)$, expert’s policy $\pi^\ast_E = \pi_E(\cdot|s)$, and the objective in Eq. (8) characterized by the Bregman divergence, we can think of the following measure between expert and agent policies:

$$E_{s \sim d_s} \left[ D_{\Omega}^A(\pi^\ast||\pi^\ast_E) \right]$$

$$= E_{s \sim d_s} \left[ E_{a \sim \pi^\ast} \left[ f_\phi'(\pi^\ast_E(a)) - \phi(\pi^\ast(a)) \right] - E_{a \sim \pi^\ast_E} \left[ f_\phi'(\pi^\ast_E(a)) - \phi(\pi^\ast_E(a)) \right] \right]. \quad (27)$$

F TSALLIS ENTROPY AND ASSOCIATED BREGMAN DIVERGENCE AMONG MULTI-VARIATE GAUSSIAN DISTRIBUTIONS

Based on the derivation in [Nielsen & Nock, 2011], we derive the Tsallis entropy and associated Bregman divergence as follows. We first consider the distributions in the exponential family

$$\exp(\langle \theta, t(x) \rangle - F(\theta) + k(x)). \quad (28)$$

Note that for

$$\theta = \begin{bmatrix} \Sigma^{-1}\mu \\ -\frac{1}{2}\Sigma^{-1} \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix},$$

$$t(x) = \begin{bmatrix} x \\ x^T \end{bmatrix},$$

$$F(\theta) = -\frac{1}{4}\theta_1^T \theta_2^{-1} \theta_1 + \frac{1}{2} \log|\pi^{-1}| = \frac{1}{2}\mu^T \Sigma^{-1}\mu + \frac{1}{2} \log(2\pi)^d|\Sigma|,$$

$$k(x) = 0,$$

we can recover the multi-variate Gaussian distribution [Nielsen & Nock, 2011]:

$$\exp(\langle \theta, t(x) \rangle - F(\theta) + k(x))$$

$$= \exp \left( \mu^T \Sigma^{-1} x - \frac{1}{2} \text{tr}((\Sigma^{-1} xx^T) - \frac{1}{2} \mu^T \Sigma^{-1} \mu - \frac{1}{2} \log(2\pi)^d|\Sigma|) \right)$$

$$= \frac{1}{(2\pi)^d/2|\Sigma|^{1/2}} \exp \left( \mu^T \Sigma^{-1} x - \frac{1}{2} x^T \Sigma^{-1} x - \frac{1}{2} \mu^T \Sigma^{-1} \mu \right)$$

$$= \frac{1}{(2\pi)^d/2|\Sigma|^{1/2}} \exp \left( \frac{1}{2} (x - \mu)^T \Sigma^{-1} (x - \mu) \right).$$

For two distributions with $k(x) = 0$,

$$\pi(x) = \exp(\langle \theta, t(x) \rangle - F(\theta)), \hat{\pi}(x) = \exp(\langle \hat{\theta}, t(x) \rangle - F(\hat{\theta}))$$
Preprint

that share \( t, F, \) and \( k \), it can be shown that

\[
I(\pi, \hat{\pi}; \alpha, \beta) = \int \pi(x)^{\alpha} \hat{\pi}(x)^{\beta} dx
\]

\[
= \exp \left( F(\alpha\theta + \beta\hat{\theta}) - \alpha F(\theta) - \beta F(\hat{\theta}) \right)
\]

since

\[
\int \pi(x)^{\alpha} \hat{\pi}(x)^{\beta} dx
\]

\[
= \int \exp \left( \alpha \langle \theta, t(x) \rangle - \alpha F(\theta) - \beta \hat{\theta} \langle t(x) \rangle - \beta F(\hat{\theta}) \right) dx
\]

\[
= \exp \left( F(\alpha\theta + \beta\hat{\theta}) - \alpha F(\theta) - \beta F(\hat{\theta}) \right) \int \exp \left( \langle \alpha\theta + \beta\hat{\theta}, t(x) \rangle - F(\alpha\theta + \beta\hat{\theta}) \right) dx
\]

\[
= \exp \left( F(\alpha\theta + \beta\hat{\theta}) - \alpha F(\theta) - \beta F(\hat{\theta}) \right).
\]

F.1 Tsallis Entropy

For \( \phi(x) = \frac{k}{q-1} (1 - x^q - 1) \) and \( k = 1 \), the Tsallis entropy of \( \pi \) can be written as

\[
T_q(\pi) := E_{x \sim \pi} \phi(x) = \int \pi(x) \frac{1 - \pi(x)^q - 1}{q - 1} dx
\]

\[
= \frac{1}{q - 1} \int \pi(x)^q dx
\]

\[
= \frac{1}{q - 1} (1 - I(\pi, \pi; q, 0)) = \frac{1 - \exp(F(q\theta) - qF(\theta))}{q - 1}.
\]

If \( \pi \) is a multivariate Gaussian distribution, we have

\[
F(q\theta) = \frac{q}{2} \mu^T \Sigma^{-1} \mu + \frac{1}{2} \log(2\pi)^d |\Sigma| - \frac{1}{2} \log q^d,
\]

\[
qF(\theta) = \frac{q}{2} \mu^T \Sigma^{-1} \mu + \frac{q}{2} \log(2\pi)^d |\Sigma|,
\]

\[
F(q\theta) - qF(\theta) = \frac{1 - q}{2} \log(2\pi)^d |\Sigma| - \frac{1}{2} \log q^d
\]

\[
= (1 - q) \left\{ \frac{d}{2} \log 2\pi + \frac{1}{2} \log |\Sigma| - \frac{d \log q}{2(1 - q)} \right\}.
\]

For \( \Sigma = \text{diag} \{ \sigma_1^2, \ldots, \sigma_d^2 \} \), we have

\[
F(q\theta) - qF(\theta) = (1 - q) \left\{ \frac{d}{2} \log 2\pi + \frac{1}{2} \log |\Sigma| - \frac{d \log q}{2(1 - q)} \right\}
\]

\[
= (1 - q) \left\{ \frac{d}{2} \log 2\pi + \frac{1}{2} \log \prod_{i=1}^d \sigma_i^2 - \frac{d \log q}{2(1 - q)} \right\}
\]

\[
= (1 - q) \sum_{i=1}^d \left\{ \log \frac{2\pi}{2} + \log \sigma_i - \frac{d \log q}{2(1 - q)} \right\}.
\]
F.2 ANALYTICAL FORM OF BASELINE

For \( \phi(x) = \frac{k}{q-1} (1 - x^{q-1}) \), we have

\[
f'_\phi(x) = \frac{k}{q-1} (1 - qx^{q-1})
\]

\[
= \frac{k}{q-1} (q - qx^{q-1} - (q - 1))
\]

\[
= \frac{qk}{q-1} (1 - x^{q-1} - k)
\]

\[
= q\phi(x) - k.
\]

Therefore, the baseline can be rewritten as

\[
E_{x \sim \pi} [-f'_{\phi}(x) + \phi(x)] = E_{x \sim \pi} [k - q\phi(x) + \phi(x)] = (1 - q)T_q(\pi) + k.
\]

For a multivariate Gaussian distribution \( \pi \), the analytical form of \( E_{x \sim \pi} [-f'_{\phi}(x) + \phi(x)] \) can be derived by using that of Tsallis entropy \( T_q(\pi) \) of \( \pi \).

F.3 BREGMAN DIVERGENCE WITH TSALLIS ENTROPY REGULARIZATION

In Eq. (27), we consider the following form of the Bregman divergence:

\[
\int \pi(x) \{ f'_\phi(\hat{\pi}(x)) - \phi(\pi(x)) \} dx - \int \hat{\pi}(x) \{ f'_\phi(\hat{\pi}(x)) - \phi(\hat{\pi}(x)) \} dx.
\]

For \( \phi(x) = \frac{k}{q-1} (1 - x^{q-1}) \), \( f'_\phi(x) = \frac{k}{q-1} (1 - qx^{q-1}) = q\phi(x) - k \), and \( k = 1 \), the above form is equal to

\[
\int \pi(x) \left[ \frac{1 - q\hat{\pi}(x)^{q-1}}{q-1} \right] dx - T_q(\pi) - (q - 1)T_q(\hat{\pi}) + 1
\]

\[
= \frac{1}{q-1} - \frac{q}{q-1} \int \pi(x)\hat{\pi}(x)^{q-1} dx - T_q(\pi) - (q - 1)T_q(\hat{\pi}) + 1
\]

\[
= \frac{q}{q-1} - \frac{q}{q-1} \int \pi(x)\hat{\pi}(x)^{q-1} dx - T_q(\pi) - (q - 1)T_q(\hat{\pi}).
\]

For multivariate Gaussians

\[
\pi(x) = \mathcal{N}(x; \mu, \Sigma), \mu = [\nu_1, ..., \nu_d]^T, \Sigma = \text{diag}(\sigma_1^2, ..., \sigma_d^2),
\]

\[
\hat{\pi}(x) = \mathcal{N}(x; \hat{\mu}, \hat{\Sigma}), \hat{\mu} = [\hat{\nu}_1, ..., \hat{\nu}_d]^T, \hat{\Sigma} = \text{diag}(\hat{\sigma}_1^2, ..., \hat{\sigma}_d^2),
\]

we have

\[
\int \pi(x)\hat{\pi}(x)^{q-1} dx = I(\pi, \hat{\pi}; 1, q - 1) = \exp \left( F(\theta') - F(\theta) - (q - 1)F(\hat{\theta}) \right),
\]

where

\[
\theta = \left[ \begin{array}{c} \Sigma^{-1}\mu \\ \frac{1}{2}\Sigma^{-1} \end{array} \right],
\]

\[
\hat{\theta} = \left[ \begin{array}{c} \hat{\Sigma}^{-1}\hat{\mu} \\ \frac{1}{2}\hat{\Sigma}^{-1} \end{array} \right],
\]

\[
\theta' = \theta + (q - 1)\hat{\theta} = \left[ \begin{array}{c} \Sigma^{-1}\mu + (q - 1)\hat{\Sigma}^{-1}\hat{\mu} \\ \frac{1}{2}(\Sigma^{-1} + (q - 1)\hat{\Sigma}^{-1}) \end{array} \right] = \left[ \begin{array}{c} \theta'_1 \\ \theta'_2 \end{array} \right],
\]

\[
\theta'_1 = \left[ \begin{array}{c} \nu_1 + (q - 1)\hat{\nu}_1 \\ \frac{1}{\sigma_1^2} + (q - 1)\frac{1}{\hat{\sigma}_1^2} \end{array} ... \begin{array}{c} \nu_d + (q - 1)\hat{\nu}_d \\ \frac{1}{\sigma_d^2} + (q - 1)\frac{1}{\hat{\sigma}_d^2} \end{array} \right]^T,
\]

\[
\theta'_2 = \frac{1}{2} \text{diag} \left\{ \frac{1}{\sigma_1^2} + (q - 1)\frac{1}{\hat{\sigma}_1^2}, ..., \frac{1}{\sigma_d^2} + (q - 1)\frac{1}{\hat{\sigma}_d^2} \right\}.
\]
and

\[ F(\theta) = \frac{1}{2} \mu^T \Sigma^{-1} \mu + \frac{1}{2} \log(2\pi)^d |\Sigma| = \sum_{i=1}^{d} \left\{ \frac{\nu_i^2}{2\sigma_i^2} + \frac{\log 2\pi}{2} + \log \sigma_i \right\}, \]

\[ F(\hat{\theta}) = \frac{1}{2} \hat{\mu}^T \hat{\Sigma}^{-1} \hat{\mu} + \frac{1}{2} \log(2\pi)^d |\hat{\Sigma}| = \sum_{i=1}^{d} \left\{ \frac{\nu_i^2}{2\hat{\sigma}_i^2} + \frac{\log 2\pi}{2} + \log \hat{\sigma}_i \right\}, \]

\[ F(\theta + (q - 1)\hat{\theta}) = -\frac{1}{4} (\theta_1')^T (\theta_2')^{-1} \theta_1' + \frac{1}{2} \log | - \pi (\theta_2')^{-1}| \]

\[ = \sum_{i=1}^{d} \left\{ \frac{1}{2} \left( \frac{\nu_i}{\sigma_i^2} + (q - 1) \frac{\nu_i}{\hat{\sigma}_i^2} \right)^2 + \frac{\log 2\pi}{2} + \log \frac{1}{\frac{1}{\sigma_i^2} + (q - 1) \frac{1}{\hat{\sigma}_i^2}} \right\}. \]
G  EXPERIMENT SETTING

G.1  POLICY REGULARIZERS IN EXPERIMENTS

Table 1: Policy regularizers $\phi$ and their corresponding $f_\phi$ [Yang et al., 2019].

| reg. type | condition | $\phi(x)$ | $f_\phi'(x)$ |
|-----------|-----------|-----------|-------------|
| Shannon   | -         | $- \log x$ | $- \log x - 1$ |
| Tsallis   | $k > 0, q > 1$ | $\frac{k}{q-1} (1 - x^{q-1})$ | $\frac{k}{q-1} (1 - qx^{q-1})$ |
| Exp       | $k \geq 0, q \geq 1$ | $q - x^k q^e$ | $q - x^k q^e (k + 1 + x \log q)$ |
| Cos       | $0 < \theta \leq \pi/2$ | $\cos(\theta x) - \cos(\theta) - \cos(\theta) + \cos(\theta x) - \theta x \sin(\theta x)$ |
| Sin       | $0 < \theta \leq \pi/2$ | $\sin(\theta) - \sin(\theta x)$ | $\sin(\theta) - \sin(\theta x) - \theta x \cos(\theta x)$ |

G.2  DENSITY-BASED MODEL

\[ r_\theta(s, \cdot) \]

\[ \pi(\cdot|s) \]

\[ -f_\phi(\cdot) \]

G.3  EXPERT IN BERMUDA WORLD ENVIRONMENT

We assume a stochastic expert defined by

\[ \pi_\theta(a|s) = \frac{\sum_{t=1}^{3} (d(t))^{-1} \mathbb{1}[a = \text{Proj}(\theta(t))]}{\|s - s'\|_2^2 + \epsilon}, t = 1, 2, 3, \]

where $\theta(t) = \arctan2(\tilde{y}(t) - y, \tilde{x}(t) - x), d(t) = \|s(t) - s\|_2^2 + \epsilon$, for $s = (x, y)$, $\tilde{s}(1) = (-5, 10)$, $\tilde{s}(2) = (0, 10)$, $\tilde{s}(3) = (5, 10)$, $\epsilon = 10^{-4}$ and an operator $\text{Proj}(\theta) : \mathbb{R} \to \mathcal{A}$ that maps $\theta$ to the closest angle in $\mathcal{A}$.

G.4  MUJoCo EXPERIMENT SETTING

Instead of directly using MuJoCo environments with tanh-squashed policies proposed in Soft-Actor Critic (SAC) [Haarnoja et al., 2018], we move tanh to a part of environment – named hyperbolized environments in short – and assume Gaussian policies. Specifically, after an action $a$ is sampled from the policies, we pass $\tanh(a)$ to the environment. We then consider multi-variate Gaussian policy

\[ \pi(\cdot|s) = \mathcal{N}({\mu}(s), {\Sigma}(s)) \]

with $\mu(s) = [\mu_1(s), ..., \mu_d(s)]^T$, $\Sigma(s) = \text{diag}\{([\sigma_1(s)]^2, ..., ([\sigma_d(s)]^2)\}$, where

$-\arctanh(0.99) \leq \mu_i(s) \leq \arctanh(0.99), \log(0.01) \leq \log(\sigma_i(s) \leq \log(2)$
for all \( i = 1, ..., d \). Instead of using clipping, we use \( \tanh \)-activated outputs and scale them to be fit in the above ranges, which empirically improves the performance. Also, instead of using potential-based reward shaping used in AIRL (Fu et al., 2018), we update moving mean of intermediate reward values and update the value network with mean-subtracted rewards (so that the value network gets approximately mean-zero reward) to stabilize RL part of RAIRL. Note that this is motivated by Lemma 2 from which we can guarantee that any constant shift of reward functions does not change optimality.
G.5 Hyperparameters

Table 2, Table 3, and Table 4 list the parameters used in our Bandit, Bermuda World, and MuJoCo experiments, respectively.

Table 2: Hyperparameters for Bandit environments.

| Hyper-parameter          | Bandit               |
|-------------------------|----------------------|
| Batch size              | 500                  |
| Initial exploration steps | 10,000              |
| Replay size             | 500,000              |
| Target update rate (τ)  | 0.0005               |
| Learning rate           | 0.0005               |
| λ                       | 5                    |
| q (Tsallis entropy $T_q^k$) | 2.0                |
| k (Tsallis entropy $T_k^q$) | 1.0                |
| Number of trajectories  | 1,000                |
| Reward learning rate    | 0.0005               |
| Steps per update        | 50                   |
| Total environment steps | 500,000              |

Table 3: Hyperparameters for Bermuda World environment.

| Hyper-parameter          | Bermuda World         |
|-------------------------|-----------------------|
| Batch size              | 500                   |
| Initial exploration steps | 10,000              |
| Replay size             | 500,000               |
| Target update rate (τ)  | 0.0005               |
| Learning rate           | 0.0005               |
| q (Tsallis entropy $T_q^k$) | 2.0                |
| k (Tsallis entropy $T_k^q$) | 1.0                |
| Number of trajectories  | 1,000                |
| Reward learning rate    | 0.0005               |
| (For evaluation) λ      | 1                    |
| (For evaluation) Learning rate | 0.001            |
| (For evaluation) Target update rate (τ) | 0.0005         |
| Steps per update        | 50                   |
| Number of steps         | 500,000              |

Table 4: Hyperparameters for MuJoCo environments.

| Hyper-parameter          | Hopper | Walker2d | HalfCheetah | Ant    |
|-------------------------|--------|----------|-------------|--------|
| Batch size              | 256    | 256      | 256         | 256    |
| Initial exploration steps | 10,000 | 10,000   | 10,000      | 10,000 |
| Replay size             | 1,000,000 | 1,000,000 | 1,000,000  | 1,000,000 |
| Target update rate (τ)  | 0.005  | 0.005    | 0.005       | 0.005  |
| Learning rate           | 0.001  | 0.001    | 0.001       | 0.001  |
| λ                       | 0.0001 | 0.000001 | 0.00001    | 0.000001 |
| q (Tsallis entropy $T_q^k$) | 1.0    | 1.0      | 1.0         | 1.0    |
| Number of trajectories  | 100    | 100      | 100         | 100    |
| Reward learning rate    | 0.001  | 0.001    | 0.001       | 0.001  |
| Steps per update        | 1      | 1        | 1           | 1      |
| Number of steps         | 1,000,000 | 1,000,000 | 1,000,000  | 2,000,000 |