Mixed QCD–EW corrections to $pp \rightarrow \ell \nu + X$ at the LHC

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Abstract

We discuss about a recent computation of the mixed QCD–EW corrections to the hadroproduction of a massive charged lepton plus the corresponding neutrino through the Drell–Yan mechanism. The calculation, based on an extension of the $q_T$ subtraction formalism for heavy-quark production in next-to-next-to leading order QCD, includes for the first time all the real and virtual contributions due to initial- and final-state radiation, except for the finite part of the two-loop virtual correction, which is approximated in the pole approximation. We report results for the fiducial cross section and the transverse momentum spectrum of the charged lepton.

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1 Introduction

The production of a dilepton pair via the Drell-Yan (DY) mechanism has a special place in the precision phenomenology program at the LHC. Its relatively large production rates and clean signatures, given the presence of at least one charged lepton in the final state, make it a standard candle process for benchmarking and experimental calibrations. DY data are of great importance for PDF fits and for the (ultra)-precise measurement of fundamental electro-weak (EW) parameters as the $W$ mass. Furthermore, this process represents an important background for many New Physics searches.

Higher-order radiative corrections, both in strong and electro-weak interactions, are mandatory to match the accuracy of experimental results, expected to reach the (sub)percent level. Theoretically, the DY process is one of the most studied and well known processes: the state-of-the-art for fully differential predictions is represented by NNLO in QCD [1–5], and NLO in EW [6–15]. Recently, N$^3$LO QCD radiative corrections of the inclusive production of a virtual photon [16, 17] and of a $W$ boson [18] have been computed, and first estimates of fiducial cross sections for the neutral-current DY process at the same order have appeared [19].

In view of the level of precision attainable at the LHC, it becomes relevant to include $O(\alpha_S \alpha)$ mixed QCD–EW corrections. We discuss the case of the charged-current DY process

$$pp \rightarrow \ell^+ \nu_\ell + X.$$  

The computation of mixed corrections is a complicated task, the complexity being that of a NNLO calculation for a 2 $\rightarrow$ 2 process with many scales. One of the bottlenecks is the corresponding two-loop virtual amplitude. The evaluation of the 2 $\rightarrow$ 2 two-loop Feynman diagrams with internal masses is at the frontier of current computational techniques and the corresponding amplitude has not yet been fully worked out.

We present results for the mixed corrections to the charged current process, including, for the first time, all real and virtual contributions. Everything is exact but the two-loop virtual amplitude, which is approximated by its expansion around the resonant pole, applying the Pole Approximation of Ref. [20].

2 Structure of the computation

The differential cross section for the process in Eq. (1) can be written as

$$d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)},$$

where $d\sigma^{(0,0)} \equiv d\sigma_{LO}$ is the Born level contribution and $d\sigma^{(m,n)}$ the $O(\alpha_S^m \alpha^n)$ correction. The mixed QCD–EW corrections correspond to the term $m = n = 1$ in this expansion.

We achieve the cancellation of the infrared divergences by exploiting the $q_T$ subtraction formalism [21]. The extension of the method to the NLO EW and the mixed QCD–EW corrections have been worked out in Refs. [22] and [23], starting from the $q_T$ subtraction formalism for heavy quarks [24].

In the following we denote the transverse momentum of the system formed by the charged lepton and its corresponding neutrino with $q_T$. Focusing on the case of the mixed corrections, we
have schematically that $d\sigma^{(1,1)}$ can be evaluated as

$$d\sigma^{(1,1)} = \mathcal{H}^{(1,1)} \otimes d\sigma_{\text{LO}} + \left[ d\sigma_{R}^{(1,1)} - d\sigma_{\text{CT}}^{(1,1)} \right]$$  \hspace{1cm} (3)

where

- $d\sigma_{R}^{(1,1)}$ is the real contribution associated to configurations in which the charged lepton and the corresponding neutrino are accompanied by additional QCD and/or QED radiation that produces a recoil with finite transverse momentum $q_T$;
- the customary $q_T$ counterterm $d\sigma_{\text{CT}}^{(1,1)}$ cancels the singular behaviour in the limit $q_T \to 0$, rendering the cross section in Eq. (3) finite;
- $\mathcal{H}^{(1,1)}$ is a perturbatively computable function which encodes the contribution stemming from the two-loop virtual amplitude.

The symbol $\otimes$ in the first term of Eq. (3) denotes a convolution with respect to the longitudinal-momentum fractions $z_1$ and $z_2$ of the colliding partons.

In particular, the coefficient $\mathcal{H}^{(1,1)}$ can be decomposed as

$$\mathcal{H}^{(1,1)} = H^{(1,1)} \delta (1-z_1) \delta (1-z_2) + \delta \mathcal{H}^{(1,1)} ,$$  \hspace{1cm} (4)

where the hard contribution $H^{(1,1)}$ contains the 2-loop virtual corrections. More precisely, the finite contribution is defined as

$$H^{(1,1)} = \frac{2 \text{Re} \left( \mathcal{M}_{\text{fin}}^{(1,1)} \mathcal{M}^{(0,0)*} \right)}{|\mathcal{M}^{(0,0)}|^2}.$$  \hspace{1cm} (5)

in terms of the Born amplitude $\mathcal{M}^{(0,0)}$ and of the finite part, $\mathcal{M}_{\text{fin}}^{(1,1)}$, of the renormalised virtual amplitudes $\mathcal{M}^{(1,1)}$ entering the mixed QCD–EW calculations, after subtraction of the infrared poles in $d = 4 - 2\varepsilon$ dimensions with a customary subtraction operator [23].

For the computation $\mathcal{M}_{\text{fin}}^{(1,1)}$, we approximate the renormalised virtual amplitudes $\mathcal{M}^{(1,1)}$ in pole approximation including all the factorizable and non-factorizable contributions, see Fig.1. In particular, we include the initial-intial factorizable contribution, Fig.1a via the $W$ boson two-loop form factor of Ref. [25].

We remark that the renormalised amplitude in PA, $\mathcal{M}_{\text{PA}}^{(1,1)}$, presents the same structure of poles as the exact one; thereby, the finite part is consistently extracted applying the same subtraction operator.

The quality of the pole approximation for the charged current Drell-Yan process has been discussed in Ref. [23], based on the comparison with the exact results at the NLO level. Furthermore, the same work showed that the pole approximation can be improved beyond the resonant region by applying suitable reweighting factors. Recently, the exact mixed QCD–EW corrections to the neutral current process have been computed [26], providing a direct comparison between the exact two-loop amplitude and its pole approximation. In the following section, we will show result for the charged current process adopting the reweighting prescription of Ref. [23].

Before concluding this section, we mention that all the remaining real and virtual $\mathcal{O}(\alpha_s \alpha)$ contributions are evaluated without any approximation using OPENLOOPS [27–29] and RECOLA [30, 31]. The required phase space generation and integration is carried out within the MATRIX framework [32].


Figure 1: Factorizable and non factorizable contributions to the pole approximation.

3 Results

We consider the process $pp \rightarrow \mu^+ \nu_\mu + X$ at centre-of-mass energy $\sqrt{s} = 14$ TeV. We adopt a setup similar to Ref. [33], and, in particular, we work in the $G_\mu$ scheme with

$$G_F = 1.1663787 \times 10^{-5} \text{ GeV}^{-2} \quad \alpha(0) = 1/137.03599911$$

$$m_{W,OS} = 80.385 \text{ GeV} \quad m_{Z,OS} = 91.1876 \text{ GeV}$$

$$\Gamma_{W,OS} = 2.085 \text{ GeV} \quad \Gamma_{Z,OS} = 2.4952 \text{ GeV}$$

$$m_H = 173.07 \text{ GeV} \quad m_H = 125.9 \text{ GeV}$$

$$\mu_F = \mu_R = m_W.$$  

We use the complex-mass scheme [34] throughout and a diagonal CKM matrix. The on-shell values are translated to the corresponding pole values $m_V = m_{V,OS}/\sqrt{1 + \Gamma_{V,OS}^2/m_{V,OS}^2}$ and $\Gamma_V = \Gamma_{V,OS}/\sqrt{1 + \Gamma_{V,OS}^2/m_{V,OS}^2}$, $V = W, Z$, from which $\alpha = \sqrt{2} G_F m_W^2 (1 - m_W^2/m_Z^2)/\pi$ is derived. The muon mass is fixed to $m_\mu = 105.658369$ MeV. We use the following selection cuts,

$$p_{T,\mu} > 25 \text{ GeV}, \quad |y_\mu| < 2.5, \quad p_{T,\nu} > 25 \text{ GeV},$$

and work at the level of bare muons, i.e., no lepton recombination with close-by photons is carried out.

3.1 Fiducial cross section

In Tab. 1 we present our predictions for the fiducial cross section corresponding to the selection cuts in Eq. (11). We show the breakdown of the different contributions $\sigma^{(i,j)}$ into the various partonic channels. The contribution from the channels $u\bar{d}$, $c\bar{s}$ is denoted by $q\bar{q}$. The contributions from the channels $qg$, $gq$, and $q_Y$, $\bar{q}_Y$, which enter at NLO QCD and EW, are labelled by $qg$ and $q_Y$, respectively. The contribution from all the remaining quark–quark channels $qq'$, $\bar{q}\bar{q}'$, $q\bar{q}'$ (excluding $u\bar{d}$, $c\bar{s}$) to the NNLO QCD and mixed corrections is labelled by $q(\bar{q})q'$. Finally, the contributions
Table 1: The different perturbative contributions to the fiducial cross section and their breakdown into the various partonic channels is also shown. The numerical uncertainties are stated in brackets.

| \( \sigma \) [pb] | \( \sigma_{\text{LO}} \) | \( \sigma^{(1,0)} \) | \( \sigma^{(0,1)} \) | \( \sigma^{(2,0)} \) | \( \sigma^{(1,1)} \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( q\bar{q} \)  | 5029.2          | 970.5(3)        | -143.61(15)     | 251(4)          | -7.0(1.2)       |
| \( qg \)        | —               | -1079.86(12)    | —               | -377(3)         | 39.0(4)         |
| \( q(g)\gamma \)| —               | —               | 2.823(1)        | —               | 0.055(5)        |
| \( q(\bar{q})q' \)| —               | —               | —               | 44.2(7)         | 1.2382(3)       |
| \( gg \)        | —               | —               | —               | 100.8(8)        | —               |
| tot             | 5029.2          | -109.4(4)       | -140.8(2)       | 19(5)           | 33.3(1.3)       |
| \( \sigma/\sigma_{\text{LO}} \) | 1 | -2.2% | -2.8% | +0.4% | +0.6% |

3.2 Transverse momentum spectrum \( p_{T,\mu^+} \)

In Fig. 2, we show our results for the complete \( \mathcal{O}(\alpha_S^3) \) correction to the transverse momentum spectrum of the positively charged muon. We compare our results with those obtained by a multiplicative combination of the NLO QCD and NLO EW corrections. The latter approach is justified under the assumption of a completely factorisation of the two corrections. We define the multiplicative combination as follows: for each bin, the QCD correction, \( d\sigma^{(1,0)}/dp_T \), and the EW correction restricted to the \( q\bar{q} \) channel, \( d\sigma^{(0,1)}_{q\bar{q}}/dp_T \), are computed, and the factorised \( \mathcal{O}(\alpha_S^3) \) correction is calculated as

\[
\frac{d\sigma^{(1,1)}_{\text{fact}}}{dp_T} = \left( \frac{d\sigma^{(1,0)}}{dp_T} \right) \times \left( \frac{d\sigma^{(0,1)}_{q\bar{q}}}{dp_T} \right) \times \left( \frac{d\sigma^{(2,0)}}{dp_T} \right)^{-1}.
\]

We observe that the factorised approximation reproduces qualitatively well our result for the \( \mathcal{O}(\alpha_S^3) \). Beyond the Jacobian peak, it tends to overshoot the complete result. As \( p_T \) increases,
the negative impact of the mixed QCD–EW corrections increases and becomes rather sizeable, reaching at $p_T = 500 \text{ GeV}$ about $-140\%$ with respect to the LO prediction and $-20\%$ with respect to the NLO QCD result. This is not unexpected, since the high-$p_T$ region is dominated by $W + \text{jet}$ topologies, for which the $\mathcal{O}(\alpha_S \alpha)$ effects can be seen as NLO EW corrections.

4 Conclusion

Higher-order mixed QCD-EW corrections are relevant for processes that can be measured at the (sub)percent level, as the production of a lepton pair through the Drell-Yan mechanism. In this contribution, we presented new results for $\mathcal{O}(\alpha_S \alpha)$ corrections to the hadroproduction of a massive charged lepton plus the corresponding neutrino at the LHC, showing their impact on fiducial cross sections and on the transverse momentum spectrum of the charged lepton.
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