On optimal number of cognitive radios considering co-site electromagnetic compatibility

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Abstract

With the development and wide applications of wireless communication technology, the limited spectrum resources and the fixed spectrum allocation policy could no longer satisfy the demand for wireless communication. Just for this reason, many spectrum resources become spectrum holes because they are allocated but not used. Cognitive radio is now becoming one of the most important techniques for high utility of these spectrum holes. If the holes available to cognitive users are abundant over a certain time, it is worth consideration to increase network throughputs by orthogonal multiplexing as many as spectrum holes. A multi-transceiver configuration is one of the possible solutions for this purpose. With such a schema, all transceivers within a cognitive user work in a concurrent or parallel mode, by which the throughput of the network can be increased. However, co-site working cognitive radios may incur electromagnetic interference between each other. When more cognitive radios are equipped, much electromagnetic interference may be incurred. Many techniques are proposed to mitigate such co-siting interference; however, none of them have addressed the probability that the interference will happen. If the probability could be estimated in advance, the user will make a better planning on the configurations of the co-siting working radios. Based on an elaborated n-fold multiple integral model, we propose a novel method to decide how many cognitive radios can be installed for one cognitive user at most. This is our main contribution with this work, providing an enhanced ability to determine the optimal number of cognitive radios installed within each cognitive user. We make a strict deduction on electromagnetic compatibility probability with various parameters of cognitive radios. Simulations are performed and the results show that the electromagnetic compatibility of the simulated cognitive radio system meets the deducted probability by this method very well.

Keywords: Cognitive radio, Cognitive radio network, Electromagnetic compatibility, Co-channel/adjacent channel interference, Frequency difference

1 Introduction

With the development and wide applications of wireless communication technology, the limited spectrum resources and the fixed spectrum allocation policy could no longer satisfy the demand for wireless communication. The concept of cognitive radio (CR) was
firstly proposed by Joseph Mitola [1] as a new solution to this problem. By 2005, Simon
Haykin proposed a cognitive cycle model [2] as a guidance on the development of cog-
nitive radio system.

Based on CR techniques, a CR network (CRN) has wits to detect spectrum holes with-
out interference to the primary user and then to automatically configure the system
according to the current electromagnetic environment [3, 4]. Sometimes the spectrum
holes may be very abundant. In such cases, the secondary users have many spectrum
candidates for communication. This provides a good chance for cognitive users to
improve network throughputs. A simple but possible way is to integrate multiple trans-
ceivers into each cognitive user to transmit user data in a concurrent mode. These trans-
ceivers within one user work in a co-site mode. That is, they are located within short
distance between each other and use different spectrum holes for communication simulta-
nously [5].

When multiple transceivers work simultaneously, there may produce various kinds
of interference between each other [6–9], such as intermediate frequency interference,
Hermitian image interference, and co-channel/adjacent channel interference [10] caused
by transmitters. It may also produce harmonic interference, intermodulation interfer-
ence [11], and cross-modulation interference caused by the nonlinear mixing in either
transmitters or transceivers [12]. Generally, the more cognitive radios are equipped
cosite by a cognitive user, the more possibility of electromagnetic interference may
happen. For the system users, they expect that the possibility of electromagnetic inter-
ference should not exceed a threshold value. This implies that the number of transceivers
installed for one cognitive user should not exceed a certain number. To decide such an
optimum number of co-site transceivers, the implicit relation between the number of
transceivers and the possibility of electromagnetic compatibility (EMC) should be clari-
fied. Polak recently gave a good reference from an experimental view [13]. He empha-
sized the importance of frequency planning for radios located at the same object (co-site
work) and assignment of frequencies for co-site radios, and the model used by [13] and
our work are similar for the scenario where the distance between co-site antennas is less
than 1 km. The difference is the number of observed radios. Polak measured interfer-
ence on only two radios and did not discuss problems for scenarios with more than two
radios, which is the object by our work.

This paper deals with this problem in our cognitive radio network [14–17] which is
master-slave self-organized. The secondary user in the network is equipped with multi-
ple transceivers [18], allowing using multiple different channels to communicate. For the
cognitive radios in our system, intermediate-frequency interference could be suppressed
by the double conversion or increasing the quality factor of IF filter. Hermitian image
interference can be suppressed by choosing high IF frequency or increasing the quality
factor of transceiver-amplifier [19]. Therefore, both kinds of interference are not taken
into consideration in analysis of electromagnetic compatibility. Cross-modulation inter-
ference only occurs when the interference is an amplitude modulated signal [19] which
is not used in our system and thus such interference will not be discussed in this paper.

If the signal from a transmitter is near to the receiving frequency, the signal will reach
the receiver and generate interference to the receiver. This is so-called co-channel/adja-
cent channel interference while adjacent channel interference is related to the transmit
filter and the IF filter in a receiver [20], both of them need to be prevented because they cannot be avoided. From the results of the simulation for our system, we found that the co-channel/adjacent channel interference plays the most important role [21]. The possibility of co-channel/adjacent channel interference is much higher than that of harmonic interference and intermodulation interference, accounting for about 95% or more in all cases of interference. In addition, the harmonic interference can be effectively avoided or suppressed by algorithms [22, 23]. Therefore, this paper will focus on the analysis on the possibility of co-channel/adjacent channel interference to decide the optimum number of transceivers installed in a cognitive user. As far as we know, there have no such studies to address this cognitive radio planning problem based on electromagnetic compatibility probability analysis. Currently, increased studies are addressing co-site interference mitigation, such as how the cognitive engine-based dynamic spectrum access cloud services approach can include co-site interference mitigation as a subset of cloud services without an excessive increase in computational complexity [24], the approach for the mitigation of co-site interference in vehicular communication systems [25, 26], wideband co-site interference cancellation based on single-tap structure [27], suppression of the electromagnetic interference from satellite communication on-the-move system [28], and so on. Tokgoz proposed a method for the prediction of co-site interference between aperture antennas on a faceted convex surface [29]. The method by Salau, etc., may be useful towards prediction of co-site interference between monopole antennas [30]. As for frequency planning for radios located at the same object (co-site work) and assignment of frequencies for co-site radios, Polak gave an experimental but good reference [13].

Our contributions by this paper are summarized as follows:

1. We present a method to estimate EMC probability for a cognitive user working in a co-site multi-transceiver model for the first time.
2. We conduct complete simulations to evaluate the performance of the proposed method. The results show that our method can achieve high accuracy.
3. Our method provides the system manager of a CRN with a new ability to determine the optimal number of cognitive radios installed within each cognitive user.

The structure of this paper is as follows. Section II discusses the model of our system. Section III gives the analysis on EMC in detail. Section IV presents the simulations of EMC probability for our system in use. The conclusion is given in the last section.

2 Methods
We took a mathematical method in this work to build a model for estimating EMC probability for a cognitive user working in a co-site multi-transceiver mode. We introduced an $n$-fold multiple integral model into the work of the estimation of electromagnetic compatibility probability. This mathematical method meets the EMC model very well. Then we designed a simulation framework based on the parameters of radios we used. The detailed settings can be found in Section Simulations. The simulations framework consists both parts. The first part mainly focused the mathematical model, but with concrete radio related parameters. In the second part, we demonstrate what EMC
probability will be in different radio configurations. In both parts, we also give theoretic results to be compared. By comparing the results of the mathematical model to that by simulations, we checked the validness of our mathematical model.

3 The related models
3.1 The system model
Our system takes a topology in which some cognitive users are the parent nodes of other cognitive users. Taking Fig. 1 as an example, there are three cognitive users where one user, PSU (Parent Secondary User), is the parent of two users, CSU-1 (Child Secondary User 1) and CSU-2. Each user has two transceivers each of which uses a different spectrum for communication. Those transceivers using the same spectrum make a structure called a cluster, such as $C_1$ (the polygon with solid lines) and $C_2$ (the dotted polygon) in Fig. 1.

If one transceiver can contribute certain network throughputs, then two transceivers may double the throughputs if they can work simultaneously without interference between each other. Therefore, the multi-transceiver model may increase the throughputs for the network.

The system user may expect a level of electromagnetic compatibility for the system to work normally. We denote this expected EMC level by $P_{EMC}$ which is the probability of no electromagnetic interference among transceivers of one cognitive user. Therefore, the system model can be expressed by (1) where $EMC(TX_i)$ is the probability of no electromagnetic interference among all transceivers of one cognitive user, denoted by $\{TX_i\}$.

$$\Pr \{EMC(TX_i)\} \geq P_{EMC}$$

(1)

Certainly, we can find that the model by (1) works not only for the system presented by this paper, but also for other systems as long as multiple co-site radios are configured for the system.

3.2 Model of electromagnetic compatibility
Generally, the electromagnetic compatibility of the system is related not only to the interference signal strength, but also to the receiver’s ability to suppress the interference.

Suppose the frequency of an interference signal is $f_i$. The receiver will produce a certain suppression on the interference signal after it reaches the receiver, denoted by $R$, a discrete random variable with the range $\{r_1, \ldots, r_k\}$. Generally, $R$ is a step function of $\Delta f$, which is the absolute value of the difference between $f_i$ and receiving frequency $f_r$, denoted by $R(\Delta f)$, as shown in (2) where the intervals $(\delta_i, \delta_{i+1}]$ and each $r_i$ is determined by the receiver itself.

![Fig. 1 The network structure of our system consisting of two subnets (clusters) $C_1$ and $C_2$](image)
The probability distribution of \( R \) can be written as (3).

\[
\Pr \{ R = r_i \} = \Pr \{ \delta_i < \Delta f \leq \delta_{i+1} \}
\]

Suppose the power of interference signal reaching the receiver is \( P_t \), then the non-interference function is

\[
\hat{\delta}(P_t, L, R) = \begin{cases} 
1, & P_t - L - R \leq T_s \\
0, & \text{otherwise}
\end{cases}
\]

where \( T_s \) is the anti-interference threshold of receiver determined by the receiver itself and \( L \) is the attenuation of the interference signal through the antenna-feeder system, which will be addressed in the next subsection. If the value of the function is zero, the signal will interfere with the receiver thus having an effect on its normal work. Conversely, the interference signal will not affect the normal work of the receiver.

### 3.3 Model of attenuation of antenna-feeder system

The attenuation \( L \) of an antenna-feeder system is denoted by

\[
L = L_h + L_t + L_r
\]

where \( L_h \) is the horizontal isolation between transmitter and receiver antennas. \( L_t \) and \( L_r \) are the feeder attenuations of transmitter and receiver, respectively. We adopt a method in [31] to calculate \( L_h \) (dB) by

\[
L_h = 22 + 20 \log \left( \frac{f d_h}{C} \right) - (G_t + G_r) - (S_t + S_r)
\]

where \( f \) (Hz) is the frequency of the interference signal, \( C \) (m/s) is the speed of light, the \( d_h \) (m) is the horizontal distance between antennas, \( G_t \) (dBi) and \( G_r \) (dBi) are the direction gain of maximum radiation of transmitter antenna and receiver antenna, respectively, and \( S_t \) (dBp) and \( S_r \) (dBp) are the 90° to the direction of sidelobe level of transmitter antenna and receiver antenna, respectively. In this paper, we suppose that omnidirectional antennas are used, so \( S_t = 0 \) and \( S_r = 0 \).

The antenna-feeder system and related parameters are sketched in Fig. 2.
4 EMC probability analysis

4.1 Probability calculation of EMC

Based on the non-interference model by (4), we can calculate the probability of no co-channel/adjacent channel electromagnetic interference between $q$ co-site transceivers $\{TX_l|1 \leq l \leq q\}$ using

$$\Pr\{\text{EMC}(\{TX_l\})\} = \frac{1}{pn} \sum_{i=1}^{p} \sum_{j=1}^{n} \sum_{k=1}^{m} [P_{FD}(\delta_k, \delta_{k+1}, q) \ast \lambda(PT_i, L_j, r_k)].$$  \hspace{1cm} (7)

In (7), we suppose that the power of the interference signal is a uniform distributed discrete variable with the range $\{PT_1, PT_2, \ldots, PT_p\}$ where $p$ is the number of possible transmitting powers used by a transceiver.

Because the antenna-feeder attenuation is closely related to the frequency of a transmitting signal, we use $L_j$ to indicate such an attenuation of a signal with corresponding frequency $f_j$ and the total number of all possible frequencies is $n$. We assume that each transceiver uses the same antenna and feeder thus $L_j$ is independent of both the length of the feeder and the type of the antenna.

The most important item in (7) is $P_{FD}$ which is a probability distribution function of frequency difference among $q$ transceivers. This distribution function will be discussed in the next subsection.

4.2 Probability analysis on frequency difference

To define item $P_{FD}(\delta_k, \delta_{k+1}, q)$ in (7), we firstly introduce $P_{MF}(d_k, q)$ as

$$P_{MF}(d_k, q) = \Pr\left\{d_k = \min \left\{\delta_k \right\} \geq \frac{\Delta f}{f_n - f_1}\right\}$$  \hspace{1cm} (8)

where $d_k$ are normalized to a real number between 0 and 1, $f_n$ is the maximal working frequency and $f_1$ is the minimal one and thus $[f_1, f_n]$ form the spectrum scope of transceivers.

The meaning of $P_{MF}(d_k, q)$ can be explained by the meaning of its complementary form $(1 - P_{MF}(d_k, q))$ which is the volume of a defined polyhedron. When $q = 2$, it becomes the area of a defined $q$-dimension polygon which is shown by Fig. 3.

![Fig. 3](image) The 2D form of $(1 - P_{MF}(d_k, q))$ which is the area of hexagon ABCDEF.
Based on (8), $P_{FD}(\delta_k, \delta_{k+1}, q)$ is defined as (9) which means what probability of the minimum difference among $q$ random real numbers within $[0, 1]$ falls into the range $[\delta_k f_n, \delta_{k+1} f_n]$.

$$P_{FD}(\delta_k, \delta_{k+1}, q) = P_{MFD}(d_k, q) - P_{MFD}(d_{k+1}, q)$$ (9)

### 4.3 Deduction on minimum difference probability

Simply, we use $h(d, n)$ to denote $P_{MFD}(d_k, q)$ and thus as inspired by Fig. 3, we can calculate $h(d, n)$ by an integral expression:

$$h(d, n) = n! \int_0^1 \int_0^d \cdots \int_0^d dx \cdots dx$$ (10)

To calculate $h(d, n)$, we firstly prove a lemma below.

**Lemma 1**

$$f(x, n) = n! \int_0^x \int_0^x \cdots \int_0^x \underbrace{dx \cdots dx}_{n} = (x - d)[x - d(n + 1)]^n$$

for $n \in \mathbb{N}, 0 \leq d \leq 1$.

**Proof**

*with induction*

For the case of $n = 1$, we have

$$f(x, n) = n! \int_0^x \int_0^x \cdots \int_0^x \underbrace{dx \cdots dx}_{n} = \int_0^x dx = x - d$$ (11)

Therefore, this lemma holds for the case of $n = 1$.

Suppose that the lemma holds for $n$, and then with the method of integration by parts we have the following equation for the case of $(n + 1)$:

$$f(x, n + 1) = (n + 1)! \int_0^x \int_0^x \cdots \int_0^x \underbrace{dx \cdots dx}_{n}$$

$$= \frac{n+1}{n} \int_0^x (x - d)[x - d(n + 1)]^n$$

$$= (x - d)[x - d(n + 2)]^n$$

Based on Lemma 1, we can prove a theorem below to calculate $h(d, n)$.

$$h(d, n) = n! \int_0^1 \int_0^d \cdots \int_0^d \underbrace{dx \cdots dx}_{n} = (n - 1)^{n-1}(-d)^n + (1 - dn)^{n-1}$$ (12)

for $n \in \mathbb{N}, 0 \leq d \leq 1$.
5 Simulations

We performed two simulations, one for the minimum frequency difference probability and the other for the EMC probability. The parameters for simulations are introduced in the part ‘Parameters for Simulation,’ and the details of both simulations are given in following parts, respectively. Note that all the settings for simulation are specific for our system, and that all the settings can be changed whenever necessary. Nevertheless, the simulation process will be the same and the main conclusion based on simulations will be not changed. With the first simulation, our aim is to check the correctness of Lemma 1. The smaller the difference between the theoretic result by Lemma 1 and the simulated result is, the better the model by Lemma 1 is. For the second simulation, the theoretic model by (7) is evaluated. The performance metric is similar to that for the first simulation.

5.1 Parameters for simulation

The electromagnetic compatibility related parameters of the cognitive radio used for our system are listed in Table 1.

As shown in Table 1, the anti-interference threshold of the radio receiver $T_s$ is $-80$ dB. There are 5 candidates for the radio to select transmitting power, that is $p = 5$. The working frequency $f_i$ will be set from 30 to 300 MHz with a uniform spacing of 25 kHz, that is, the band of a channel is 25 kHz and there are 10800 channels in total. This spectrum section is widely used in mountainous areas for tactical networking [16]. The distance between radios is set to 10 m. The gain factor of the antenna used is assumed to 5 dB. With frequency difference $\Delta f$ varying from 25 kHz to 4 MHz, the receiver’s restraint on the interference varies from 0 to 160 dB. In particular, the restraint value 0 implies that the receiver has no any restraint on signals very near to the receiving signal. Thus, the minimal $\Delta f$ implies the minimal electromagnetic compatibility probability.

5.2 Simulation for minimum frequency difference

We performed a simulation for frequency difference distribution. The process is described in detail below.

Step 1: For $q = 2\ldots 15$ /* for different number of radios */
Step 2: For $k = 1\ldots 6$
   $p_i[k] \leftarrow 0$,$p_0[k] \leftarrow 0$ /*initialization*/
EndFor
Step 3: For pass=1..10,000 /*repeat 10,000 times*/
Step 4: For \( i = 1 \ldots q \) generate radio \( i \) with random frequency \( f_i \).

EndFor

Step 5: calculate the minimum frequency difference \( d \), s.t. \( d \leq |f_i - f_j|, i \neq j, 1 \leq i,j \leq q \)

Step 6: select \( k \), s.t. \( \frac{d}{n} \leq \rho[k] \leftarrow \text{delta}[k]+1 \)

Step 7: For \( k = 1 \ldots 6 \)
\[ \rho[k] \leftarrow \text{delta}[k]/10,000 \]
EndFor

Step 8: For \( k = 1 \ldots 6 \)
\[ \rho[k] \leftarrow \text{calculate} \ h(d_k, q) \text{with (12)}, d_k = \frac{\delta_k}{n} \]
EndFor

Step 9: For \( k = 1 \ldots 6 \)
\[ \text{calculate error and output: } |p_t[k] - p_s[k]| \]
EndFor

EndFor /*end of the program */

First, we generate \( q(q \in [2 \ldots 15]) \) cognitive radios each with a random frequency over the spectrum scope \( f_1 \sim f_n (n = 10,800) \).

Then the minimum difference between frequencies of these radios is calculated and normalized. Repeat both steps for 10,000 times and make a statistic analysis to get how many times the normalized minimum difference falls into \( \left[ 0, \frac{\delta_k}{fn-f_1} \right] \) (\( \delta_k \in \{\delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6\} \)).

After simulation, we calculated theoretic values \( P_{MFD} \) with (12) to be compared with simulated results (Step 9). The simulated results are shown in columns marked ‘S’ in Table 2 and the theoretic results are shown in adjacent ‘T’ columns. Specially, we chose data with \( \frac{\delta_k}{fn-f_1} \) varying from 1/270, 2/270 to 4/270 to be demonstrated in Fig. 4 for absolute errors between theoretically calculated probabilities and the simulations.

Finally, tuples of \((T,S)\) for each \( q \) and \( \delta_i \) are output to a file. We can obtain results from the file.

From the results in Table 2 and the absolute errors shown in Fig. 4, we can see that the data exhibit sound consistency between simulated values and deduced ones by

![Fig. 4 The absolute errors between theoretically minimum frequency difference probability and simulations](image-url)
Theorem 1, and that the errors between theoretically calculated probabilities and the simulations are very little (0.1–0.6%) for all cases of cognitive radio parameters and can nearly be negligible.

Note that the most frequently used values for $q$ are from 2 to 8 in our actual cognitive radio networks.

5.3 Simulation of EMC probability

This simulation is to examine what is the probability of no electromagnetic interference between $q$ co-site cognitive radios. The details are as follows.

**Step 1:** For $q = 2 \ldots 15$ /*for different number of radios*/
   $p_r \leftarrow 0$ /*zero interference probability*/

**Step 2:** For pass = 1 \ldots 100,000 /*repeat 100,000 times*/
   emc $\leftarrow 0$ /*Initialization for EMC counter*/

**Step 3:** For $i = 1 \ldots q$ /*construct a CRN system*/

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**Table 1** Parameters and settings for simulation

| Parameters | Definitions | Settings |
|------------|-------------|----------|
| $T_s$ | Anti-interference threshold of a radio receiver | $-80$ dB |
| $PT_1 \sim PT_p$ | Possible transmitting power used by a transceiver | 5, 10, 15, 25, 30dBm |
| $f_1 \sim f_n$ | The minimal working frequency and the maximal one | 30, 30.025, ..., 300MHz |
| $d_h$ | The horizontal distance between antennas (m) | 10m |
| $G_t, G_r$ | Direction gain of maximum radiation of tx, rx antenna (dBi) | 5dB |
| $R$ | Suppression of a receiver on the interference signal | 0dB if $\Delta f < \delta_1 = 25$kHz.
30 dB if 25 kHz $\leq \Delta f < \delta_2 = 50$ kHz.
50 dB if 50 kHz $\leq \delta_2 < \delta_3 = 100$ kHz.
90 dB if 100 kHz $\leq \delta_3 < \delta_4 = 1$ MHz.
100 dB if 1 MHz $\leq \delta_4 < \delta_5 = 2$ MHz.
120 dB if 2 MHz $\leq \delta_5 < \delta_6 = 4$ MHz.
1600 dB if $\Delta f \geq \delta_6 = 4$ MHz. |

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![Fig. 5 The probability of no interference between $q$ co-site cognitive radios](image-url)
| δ  | 0.025/270 | 0.05/270 | 0.1/270 | 1/270 | 2/270 | 4/270 |
|----|-----------|----------|---------|-------|-------|-------|
| q  | T  | S  | T  | S  | T  | S  | T  | S  | T  | S  | T  | S  |
| 2  | 0.000185 | 0.0002  | 0.00037 | 0.00043 | 0.00074 | 0.00077 | 0.007393 | 0.00719 | 0.014759 | 0.01499 | 0.02941 | 0.02919 |
| 3  | 0.000555 | 0.00053 | 0.00111 | 0.00102 | 0.00222 | 0.00198 | 0.022098 | 0.02184 | 0.043952 | 0.04446 | 0.08692 | 0.08715 |
| 4  | 0.001117 | 0.00128 | 0.00222 | 0.00209 | 0.00443 | 0.00458 | 0.043789 | 0.04383 | 0.086281 | 0.08606 | 0.16744 | 0.16586 |
| 5  | 0.00185  | 0.00172 | 0.00369 | 0.00389 | 0.00738 | 0.00767 | 0.072041 | 0.07152 | 0.140119 | 0.13812 | 0.26497 | 0.2627 |
| 6  | 0.002774 | 0.00263 | 0.00554 | 0.0056  | 0.01106 | 0.01086 | 0.106281 | 0.10591 | 0.203327 | 0.20307 | 0.37214 | 0.37046 |
| 7  | 0.003882 | 0.00379 | 0.00775 | 0.00799 | 0.01545 | 0.01602 | 0.145815 | 0.14603 | 0.273463 | 0.27183 | 0.48154 | 0.4765 |
| 8  | 0.005173 | 0.00534 | 0.01032 | 0.01059 | 0.02055 | 0.02065 | 0.189855 | 0.18875 | 0.347936 | 0.34636 | 0.58648 | 0.58436 |
| 9  | 0.006647 | 0.00661 | 0.01325 | 0.01326 | 0.02635 | 0.02685 | 0.237545 | 0.2383  | 0.424171 | 0.42298 | 0.68171 | 0.67848 |
| 10 | 0.008302 | 0.00832 | 0.01654 | 0.0164  | 0.03284 | 0.03221 | 0.287989 | 0.28544 | 0.499751 | 0.49854 | 0.7638  | 0.76037 |
| 11 | 0.010138 | 0.01006 | 0.02018 | 0.01982 | 0.04    | 0.03981 | 0.340279 | 0.33867 | 0.572556 | 0.56898 | 0.83117 | 0.82979 |
| 12 | 0.012154 | 0.01192 | 0.02417 | 0.02456 | 0.04781 | 0.04845 | 0.393521 | 0.39182 | 0.640842 | 0.636  | 0.88388 | 0.88256 |
| 13 | 0.014342 | 0.01446 | 0.02858 | 0.0282  | 0.05627 | 0.05566 | 0.446863 | 0.44915 | 0.703303 | 0.70047 | 0.92324 | 0.92307 |
| 14 | 0.016721 | 0.01648 | 0.03318 | 0.03355 | 0.06534 | 0.0656  | 0.499515 | 0.50076 | 0.759081 | 0.75741 | 0.95128 | 0.94923 |
| 15 | 0.019269 | 0.01936 | 0.03819 | 0.03874 | 0.07503 | 0.07507 | 0.550768 | 0.54946 | 0.80775 | 0.8066 | 0.97035 | 0.97072 |
generate radio $i$ with random frequency $f_i$ and random power $P_t(i)$

EndFor

Step 4: For $i = 1 \ldots q$

For $j = i + 1 \ldots q - 1$

calculate $\lambda(\mathcal{P}_i(L(i), R(|f_i - f_j|))$ with (4) where $L$ is defined by (6), and $R$ is defined by (2)

If $\lambda = 1$ Then $\text{emc} \leftarrow \text{emc} + 1$ and break

EndFor

EndFor

EndFor

Step 5: $\text{emc} \leftarrow \text{emc}/100000$

Step 6: For $i = 1 \ldots p$/* for all possible powers */

For $j = 1 \ldots n$/* all $n$ possible frequencies */

For $k = 1 \ldots 6$/* all possible restraints */

$P_{FD}(\delta_k, \delta_k+1, q) \leftarrow p_{FD}(d_k, q) - p_{FD}(d_{k+1}, q), d_k = \frac{\delta_k}{n-1}$

$pr \leftarrow pr + P_{FD}(\delta_k, \delta_k+1, q) \times \lambda(\mathcal{P}_i(L, R_k))$

EndFor

EndFor

EndFor

$pr \leftarrow pr/p/n$

Step 7: Output $pr$ and $\text{emc}$ for each $q$

EndFor /* End of for(q) */

In this simulation, we set parameters for each radio with random values listed in Table 1 and check whether they interfere with each other according to (4). Then we calculate such EMC probability defined by (7) with (12) (Step 6). While the program ends, the program outputs tuples of $(pr, \text{emc})$ for each number of radios $(q)$ to a file, by which we can obtain the simulation results.

Both deduced probability (denoted by $\text{emc}$) and simulated results (denoted by $pr$) are shown in Table 3. From the data, we see that the theoretic results are closely near to that

| Number of cognitive radios | Theoretical EMC probability | Simulated EMC possibility |
|----------------------------|------------------------------|--------------------------|
| 2                          | 0.99883                      | 0.99853                  |
| 3                          | 0.99606                      | 0.99561                  |
| 4                          | 0.99212                      | 0.99128                  |
| 5                          | 0.9864                       | 0.98558                  |
| 6                          | 0.97953                      | 0.97859                  |
| 7                          | 0.97193                      | 0.97039                  |
| 8                          | 0.96398                      | 0.96108                  |
| 9                          | 0.95273                      | 0.95076                  |
| 10                         | 0.9422                       | 0.93954                  |
| 11                         | 0.93115                      | 0.92754                  |
| 12                         | 0.91767                      | 0.91486                  |
| 13                         | 0.90357                      | 0.90161                  |
| 14                         | 0.88902                      | 0.88789                  |
| 15                         | 0.87581                      | 0.87379                  |
of simulation. We make Fig. 5 to show how the consistency reaches. Therefore, if the system user expects a probability of no interference between co-site cognitive radios, the user can use the data in Table 3 to decide the optimal number of co-site radios. For example, if 0.96 is expected, then at most 8 radios can be used for concurrent communication. With more than 8 co-site radios installed, the probability of electromagnetic interference may increase to an intolerable level. From the point of view of spectrum utility, 8 radios are an optimal configuration if only EMC-related factors need to be taken into consideration under the condition of abundant spectrum holes.

6 Results and discussion

The simulation section provides us both results. One is related to the probability of minimum frequency difference, which is listed in Table 2. In this table, the number of co-siting radios \( q \) is varying from 2 to 15. For each value of \( q \), the minimum frequency difference between the \( q \) radios will fall into a certain range \( \delta_i \). By setting the parameters of these radios with different parameters at random for some times, we get how much probability the minimum frequency difference will be in each range and compare it with the theoretic result. From the results in Table 2 and the absolute errors shown in Fig. 4, we can see that the errors between theoretically calculated probabilities by (12) and the simulations are within 0.1–0.6% for all cases of cognitive radio parameters. The other result is related to the probability of no interference between \( q \) co-site cognitive radios. From the deduced and simulated results shown in Table 3, we can see that the theoretic results are closely near to that of simulation. The errors between theoretically calculated probabilities and the simulations are still very little (0.1–0.5%) for all cases of cognitive radio numbers.

We can discuss the results as below. First of all, both absolute and relative errors in two simulations are very stable and are irrelevant to the number of co-site radios \( q \). This indicates us a sound belief in the proposed models. Most important is for the user. The system user can use the models to calculate the number of co-siting radios or directly use the results in Tables 2 and 3 if their system parameters are similar to ours. If the system user expects a probability of no interference between co-site cognitive radios, the user can use the formula (7) with (12) to decide the optimal number of co-site radios. For example, if 0.96 is expected, then at most 8 radios can be used for concurrent communication in our scenario. However, in a real world, other than EMC, we have many factors should be considered for installing multiple CRs in the same site, e.g., the power supply, the network management, and so on. Therefore, we suggest the method and simulation results related to EMC and optimal number of CRs be referred an important but not only guidance for CRN setup.

7 Conclusion

A cognitive radio network can increase data throughputs by configuring multiple transceivers for each radio to utilize as many as spectrum holes simultaneously. Such co-site transceivers may incur electromagnetic interference between each other. The requirement is to decide how many transceivers can be installed according to the level of acceptable electromagnetic interference. For this problem, we presented an elaborated \( n \)-fold multiple integral model to calculate the probability of frequency
difference distribution, and based on this model we proposed a method of electromagnetic probability estimation for multiple co-siting transceivers. The absolute and relative errors between the theoretically calculated probability and the simulated one, regardless of in frequency difference distribution and EMC estimation, are all at a negligible level of about 0.1–0.6%. Therefore, the electromagnetic interference level among multiple transceivers can be pre-determined with our methods proposed in this paper. Thus, our method can provide the system manager of a CRN with a new ability to determine the optimal number of cognitive radios installed within each cognitive user. This technique has made us get sound work achievements in our in-use cognitive radio system which is supporting 4 co-siting radios without obvious interference. As far as we know, it is the first time to address such cognitive radio planning problems based on electromagnetic compatibility probability analysis. As for our future work, we will extend our work to other scenarios with different parameters such as different distance between antennas, various feeder attenuations, and other spectrum bands. Furthermore, we will extend our approaches to thread collision prediction in computer science and other similar applications.

Abbreviations
CR: Cognitive radio; CRN: Cognitive radio network; EMC: Electromagnetic compatibility; PSU: Parent secondary user; CSU: Child secondary user.

List of symbols
$\delta$: The frequency of an interference signal; $P_{EMC}$: The probability of no electromagnetic interference among transceivers within one cognitive user; $\delta_i$: The $i$th transceiver; $R$: The suppression of a receiver on the interference signal; $\Delta f$: The absolute value of the difference between $f_i$ and receiving frequency $f_r$; $f_r$: The receiving frequency; $r_i$: A value of suppression of a receiver on an interference signal, in dB; $\delta_i$: A boundary value for $\Delta f$, e.g., $\delta_i < \Delta f \leq \delta_i + 1$; $P_t$: The power of interference signal reaching the receiver; $P_i$: The $i$th possible transmitting power used by a transceiver; $T$: The anti-interference threshold of a receiver; $L$: The attenuation of the interference signal through the antenna-feeder system; $\alpha(P_t, L, R)$: A binary function indicating whether interference exists with specified arguments; $L_h$: The horizontal isolation between transmitter and receiver antennas; $L_t$, $L_r$: The feeder attenuations of transmitter and receiver, respectively; $C$: The frequency of the interference signal, in Hz; $d_h$: The speed of light (m/s); $d_0$: The horizontal distance between antennas; $G_t$, $G_r$: The direction gain of maximum radiation of transmitter antenna and receiver antenna, respectively, dBi; $S_t$, $S_r$: The 90° to the direction of sidelobe level of transmitter antenna and receiver antenna, respectively, dBp; $f_1$, $f_n$: The minimal/maximal working frequency of a receiver; $P_{FD}(\delta_k, \delta_k + 1, q)$: A probability distribution function of frequency difference among $q$ transceivers.

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Authors’ contributions
Mingxue Liao carried out the whole theory modeling and reduction, simulation and programming, the design of the study and performed the statistical analysis. The author read and approved the final manuscript.

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Availability of data and materials
We use simulated data for this paper to make a deduction on what is the optimal number of cognitive radios to be installed on one site. Therefore, data sharing is not applicable to this article. We guarantee that anyone can repeat the simulation process and can get similar results with the simulation method and theoretic model.

Declarations
Competing interests
The author declares that they have no competing interests.
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