Use of indentation velocity derivative for estimation of envelope points when the wraparound points are known method

S Yu Lebedev¹, D A Babichev² and D T Babichev¹

¹Industrial University of Tyumen, Institute of Transport, Melnikayte Street 72, 625039 Tyumen, Russia
²Sibur Holding, ZapSibNeftekhim LLC, East Industrial District, Block 1, No. 6, Building 30, 626150 Tobolsk, Russia

E-mail: lebedevsergey1995@gmail.com

Abstract. There is a modification to the undifferentiated surface points’ estimation methods that are formed by envelope methods. This method: doesn’t need a lot of additional calculations; easily fits the existing analysis systems of forming processes; essentially reduces the overall total of calculations at adequate accuracy.

1. Task definition
Reference paper [1] describes a kinematic method of envelope points estimation when the wraparound points [2] are known. The method is based on the usage of terms “indention velocity and acceleration”, and as shown in [1] it lets reduce the value of the calculated points deflection from theoretical envelope by 1 to 3 orders of magnitude. Computer calculations which are partially provided in [1] showed that deflection behavior of the calculated points on Σ₃ from envelope Σ₂ (see figure 1) follows well defined laws: a) in the direction of envelope Σ₂ curvature radius growth, points Σ₃ are situated inside of the envelope surface Σ₂ field; b) deflections of points Σ₃ from Σ₂ are power low dependent. All this complies with physical sense and points at computational errors in the example under analysis being caused by third derivatives effect. Since indentation velocity runs over first derivatives; indentation acceleration - over second derivatives; and kinematic method [1] does not take account of higher order derivatives. It shall be noted that also earlier it was proposed to use high derivatives for analysis and synthesis of engagements. In such a way G.I. Shevelyova has developed the formal power series method [3]. D.T. Babichev wrote [4] about utility value of indentation velocity high derivatives, but up to now this idea has neither been implemented in methods, nor in calculations.

2. Research objective
To develop envelope points estimation method over wraparound points coordinates using indentation acceleration derivative for calculations. And to assess accuracy of the kinematic method provided in [1] when third derivatives are used in it.
Derivative of indention acceleration. It is found by differentiation of indention acceleration \( a_{BH} \) [1, formula (3)] with respect to time \( t \):

\[
\dot{a}_{BH} = \frac{da_{BH}}{dt} = \frac{d}{dt} (a_{12} \cdot \mathbf{n} + V_{12} \cdot \dot{\mathbf{n}}) = a_{12} \cdot \mathbf{n} + 2 \cdot a_{12} \cdot \mathbf{n} + V_{12} \cdot \ddot{\mathbf{n}}
\]

(1)

where \( V_{12} \) – is relative velocity vector; \( \mathbf{n} \) – unit normal vector to the surface, directed out of the generating element field; \( a_{12} \) – acceleration of the point situated on the generating surface \( \Sigma_1 \) and riding on it at a velocity “\(-V_{12}\)”; \( \dot{\mathbf{n}} \) – derivative of unit normal vector \( \mathbf{n} \).

For flat rack-and-gear drive where generating profile is a straight line, using dependencies (3)-(5) and (6b)-(8b) from [1] and assuming in generic engagement [1, figure 1]:

\[
\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = a_1 = a_2 = a_3 = 0, \gamma = 90^\circ, \omega_2 = -a_w, \omega_1 = \omega_3 = V_2 = V_3 = 0, \Sigma_3 = a_w, \text{ following simple formula was deduced for calculation of derivative } \dot{a}_{BH} \text{ of indention acceleration } a_{BH}:
\]

\[
\dot{a}_{BH} = \omega_1^3 \cdot r_b = -\omega_2^3 \cdot r_b = -r_b
\]

(2)

where \( r_b \) – base radius of the gear blank.

Using work formulae [1], along with dependence (2), plots were constructed (see figure 2) illustrating changes in: indention velocity \( V_{BH} \), indention acceleration \( a_{BH} \) and its derivative \( \dot{a}_{BH} \) along cutting edges in the rack gearing under consideration.

**Indention velocity** \( V_{BH} \) – see figures 2a and 2b – is equal to 0 at the action points K and K* of generating rack with envelope involute (not shown in the figure 2). At non-action points \( V_{BH} \) is numerically equal (under \( \omega_{12} = 1 \)) to the arm \( h \) of the unit normal vector relative to the pitch point P. Here \( V_{BH} > 0 \), \( \mathbf{n} \) creates moment relative to the pitch point P, co-directing normalized angular velocity \( \omega_{12} \) (reminder: \( \omega_{12} = \omega_{\text{TOOL}} - \omega_{\text{BLANK}} \)). When the rack is moving, action points K and K* are shifted along the rack profile and due to it indention velocities \( V_{BH} \) are changed at all the profile points. Three material circumstances shall be noted: Firstly, thicknesses of the layers cut by the cutting edge are shown in the sketches of \( V_{BH} \) (under \( V_{BH} > 0 \)), figure 2b. It is one of the key ideas of V.A. Shishkov [5], who introduced the term \( V_{BH} \). Secondly, in the points B and C of profiles’ jag following rule is in action: If direction of \( V_{BH} \) is changed at the jag - this jag is shape-generating, i.e. the point is situated on the envelope (see point C, figure 2b); if direction of \( V_{BH} \) is unchanged at the jag - the jag is not shape-generating, i.e. the point is situated out of envelope (see point B, figure 2b). It is one of the ideas of the paper [5], where shape generation by jags was studied carefully by the author. Thirdly, when the generating motion direction is changed, velocity \( V_{BH} \) reverses the direction at all the points of the generating profile.
Figure 2. Indention parameters for straight intervals of a rack profile: a) engagement scheme; b) velocity $V_{BH}$; c) acceleration $a_{BH}$; d) its derivative.

Indention acceleration $a_{BH}$ – see figures 2a и 2b – is equal to 0 at points $M$ и $M^*$. At other points of the side profiles $AB$ and $CD$ acceleration $a_{BH}$ is in linear dependence on the distance $e$ between current point and points $M$ or $M^*$: $a_{BH} = \alpha_1 \cdot e \cdot \cot g \alpha$, where $\alpha$ – is a rack pressure angle. In BC interval acceleration $a_{BH} = \alpha_2 \cdot r_f$, where $r_f$ – is a distance between BC interval and gear center - point O. Two material circumstances are to be noted. Firstly, value and direction of acceleration $a_{BH}$ depend neither on rack movements during generation, nor on direction of this motion. Secondly, at the points under $a_{BH} > 0$ envelope is formed inside of the generating element field (see intervals $MB$ and $CM^*$), i.e. gearwheel tooth undercutting takes place. [6]

Derivative $a_{BH}$ of indention acceleration $a_{BH}$ – see figure 2d: equal to $\ddot{a}_{BH} = -\alpha_2^2 \cdot r_b$; similar at all the points of side portions $AB$ and $CD$, and does not depend on rack movements. If the generating motion direction is changed, then direction $\ddot{a}_{BH}$ is changed as well. On the interval BC acceleration $\ddot{a}_{BH} = 0$.

Principles of the kinematic method where indention acceleration derivative is used. Below is envelope points’ estimation algorithm for the case of one-parameter envelope.

Stage 1 is similar to the stage 1 described in [1]: estimate coordinates $x_2, y_2, z_2$ on the wraparound $\Sigma_1$. Meanwhile calculate and keep in mind: a) projections $n_{x2}, n_{y2}, n_{z2}$ of unit normal vector $n_1$ to the surface $\Sigma_0$; b) relative velocities vectors: angular $\omega_{12}$ and linear $V_{12}$; c) indention velocity $V_{BH}$ and acceleration $a_{BH}$ Additionally derivative of indention acceleration $\ddot{a}_{BH}$ is to be found – for the involute gear teeth generation by a rack the formula is to be used (2).

Stage 2 is very alike with stage 2 of [1]:

- Calculate distances $\delta$ from the point on wraparound $\Sigma_1$ to envelope $\Sigma_2$ (along normals $n_1$ to $\Sigma_1$) and time $At$ of their transit by formulae:
\[
\Delta t = \min \left\{ \left| a_{BH} \pm \sqrt{a_{BH}^2 + 2 \cdot a_{BH} \cdot V_{BH}} \right| \right\} \\
\delta = a_{BH} \left( V_{BH} \cdot \Delta t + \frac{a_{BH} \cdot \Delta t^2}{2} - \text{sign}(V_{BH}) \cdot \frac{a_{BH} \cdot \Delta t^3}{6} \right)
\] (3)

- Estimate normal line \( \mathbf{N} \) and its unit vector \( \mathbf{n}^{(\Sigma_2)} \) to the envelope surface \([1, \text{formulae (13)}]\).
- Calculate normal distance \( \delta_n \) to the surface \( \Sigma_2 \) \([1, \text{formulae (14)}]\).
- Estimate coordinates of the wraparound points \( \Sigma_2 \) \([1, \text{formulae (15)}]\).

3. Evaluation of the method offered and accuracy estimation

The software used for the research article \([1]\) has been updated in accordance with the above mentioned algorithms and formulae. Using this software an involute spur gear forming process by counterpart rack has been simulated. Just like in \([1]\), gear reference diameter was set to \( d=1000 \) mm, i.e. \( a_w=500 \) mm. See figure 3 for the results of this simulation. It shows graphically the deflections of calculated surface \( \Sigma_3 \) points (see figure 1) from the envelope \( \Sigma_2 \), i.e. from the involute. It is apparent that: a) deflections almost do not depend on point situation on the involute; b) deflections tend to reduce in a stable and significant way in case if the number of teeth \( z \) and number of cuts \( k \) increase when the gear blank is rotated by one tooth (if \( z \) or \( k \) is doubled, deflection is reduced by a factor of more than 30); c) deflection value is negligible: under \( z=10 \) and \( k=10 \) maximum involute deflection for the gear with reference diameter \( d=1,000 \) is equal to only 0.000 1 micrometer; d) under \( z^*k>1,500 \) method error \( \Delta h<10^{-9} \) \( \mu \)m becomes commensurable with computer calculation accuracy which made up 15 to 16 decimal signs (wavy curves under \( z^*k=1,600 \) and \( z^*k=3,200 \) witness it clearly).

Figure 3. Kinematic method accuracy when indention acceleration derivatives are used.
Figure 4. Maximum errors of the three envelope calculation methods.

Figure 4 shows accuracy comparison for three methods of envelope calculation depending upon the number of teeth \( z \) and number of cuts \( k \) (by the example of rack treatment of an involute gear). Three top curves (method 1) – maximum wraparound \( \Sigma_1 \) deflection from involute \( \Sigma_2 \) on the circles (from top to bottom): tip circle, reference circle, close to base circle. It is obvious that on the reference surface under \( z^*k=100 \) deflection makes up about 100 \( \mu \)m. Second-from-the-bottom curve (method 2) – maximum deflection for kinematic method described in [1]. One can see that on the reference circle under \( z^*k=100 \) deflection makes up more than 1 \( \mu \)m; i.e. reduced by almost two orders of magnitude. Bottom curve (method 3) – shows maximum deflection for kinematic method when indentation velocity derivative \( \dot{a}_{BH} \) is used. Apparently, deflection is equal to 0.0001 \( \mu \)m, i.e. less than initial one by six orders of magnitude and less by four orders of magnitude compared to method 2 results. Trends shown in figure 4 indicate that the errors dependencies on product \( z^*k \) are power-mode, but exponents are different:

\[
\Delta_{\text{method1}} \approx \frac{8 \times 10^5}{(z-k)^2} ; \quad \Delta_{\text{method2}} \approx \frac{8 \times 10^6}{(z-k)^3} ; \quad \Delta_{\text{method3}} \approx \frac{8 \times 10^5}{(z-k)^5} \tag{4}
\]

i.e. increase of cuts number \( k \) improves accuracy of method 3 in the most material way; and in the least material way (though, significant) – method 1 accuracy.
Degree of accuracy improvement
Product of number of teeth and number of cuts

Figure 5. Comparative accuracy analysis of the two kinematic methods.

Figure 5 is more informative in terms of showing the degree of accuracy improvement for two kinematic points calculation methods on the $\Sigma_3$ (i.e. close to involute) compared to deflections of the wraparound jags (i.e. jagged curve tangent to involute). Two bottom curves – are minimum and maximum accuracy improvements of method 2. Two top curves – are minimum and maximum accuracy improvements of method 3, which is described above and includes use of derivative $\dot{a_{BH}}$.

4. Conclusion
Indention derivative was used in kinematic method of envelope points lumping given the wraparound points. Simple example demonstrated that in this case accuracy of kinematic calculation of coordinates under the same number of cuts increases by 3 to 6 orders of magnitude. It is assumed that in the general case use of the derivative $\dot{a_{BH}}$ improves accuracy not by 3 to 6 orders of magnitude but only by 2 to 3 orders of magnitude, since it was involute to be considered as envelope and in case of its formation second derivative is $\ddot{a_{BH}}$=0. In other engagements, especially in space ones, forth derivative may be not zero, that would lead to calculation accuracy reduction.

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