Design division mixed fractions materials using PMRI and lesson study

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Abstract One of the problem in modern mathematical learning process is the mathematics concept presented by the teacher to students such as machines. So, the concepts are used by students as finished product without knowing the origin and how to reconstruct them. The effort that can be done by the teacher to overcome this problem by designing learning wherever possible starting with presenting realistic problems to drive and support students understanding abstract idea in mathematics. In this article we will discuss the design mixed fraction division based on PMRI, which is a type of design research. Design research has three cycles, namely preliminary design, classroom experiment, and retrospective analysis. This Design is aimed for supporting students understanding the concept division mixed fraction then use it into problem solving. In this article only the preliminary study phase will be discussed, which includes literature study activities, curriculum analysis, and the design of the initial prototype based on non-formal trials conducted at the SMP Srijaya Negara Palembang. An HLT (Hypotetical Learning Trajectory) has been produced regarding division of mixed fraction using image representation. This material not only designed using PMRI but also Lesson Study as learning system. So the design contain three activities, which are two activities about sharing task and one activity about jumping task.

Keywords: Design Mixed Fraction Division, PMRI, Lesson Study, Design Research

1. Introduction

Four pillars of learning according to UNESCO, namely; learning to know (getting knowledge to prepare students to become human beings who compete in the future), learning to do (applying knowledge in the form of skills that can be used by students and participating in the global community), learning to be (providing analytical skills and social self to enable individuals to develop their psycho-social potential), and learning to live together (developing knowledge to enable individuals and communities to live in peace and harmony) [17].

In the curriculum 2013 fraction is one of the basic material in learning mathematics which is studied at the level of primary and junior secondary education. Fractions are also often used in everyday life. However, fractions are always a tough challenge for students, even middle grades. One way to teach fraction material is by representation. Representation plays a very important role in studying fractions as quoted by Cramer and Wyberg as follows: “Representation should be treated as essential elements in supporting student’s understanding of mathematical concepts and relationships; in communicating mathematical approaches, arguments, and understandings to one’s self and to the others.” [5].

It was explained that a concrete model is an important form of representation and is needed to support students understanding and performing fraction operations. Representation can help students...
clarify frequently confusing ideas in pure symbolic form [18]. When faced with fractional operations especially division, students can solve them using an algorithm but do not know where the algorithm came from and why the algorithm was used. So that representation (including images, context, student language, and symbols) is important for building / making ideas that are meaningful to students. Furthermore, research related to fraction is done by [5], in the study explained that the causes of student errors are weak understanding or lack of understanding of concepts in fractions, and in particular there is no understanding of the symbolic representation of fractions. Based on the above, fraction concept learning and fraction operation are expected to be able to use models, images, contexts, student languages, and symbols as representations.

Problems in modern mathematics learning are mathematical concepts presented by teachers to students such as machines, so that mathematical procedures are used by students as finished products [14]. Providing fraction operating material that is not balanced with a strong understanding of concepts can lead students to understand fractions as limited instrumental and procedural [2] and also memorizing without meaning or rote learning [18], which in general can ultimately impact the low students' abilities in fractions. Mechanistic learning has an impact on the meaninglessness of student learning because mathematics is presented separately from the context that students can understand at the beginning of learning. So that the mathematical concept will be quickly forgotten by students and students will be difficult to apply the concept. When children learn mathematics in an isolated fashion, divorced from experienced reality, it will be quickly forgotten and they will not be able to apply it [19].

The standard process of presenting mathematical learning from teacher to student must be in a unified whole, meaning that concepts, procedures, and intellectual processes in mathematics must be related to a mathematical content [7]. Unfortunately, the mathematics learning method used by the teacher is not always in accordance with the characteristics of the students and the characteristics of the learning material. The teacher gives more explanations on the board and the students passively only record the teacher's information [20].

Concrete objects and objects around the environment can be used as a context for learning mathematics in building mathematical linkages through social interaction. The use of concrete objects as a beginning of the modeling process by students. The model can be interpreted as a bridge from real problems to formal mathematics. One type of realistic model has a characteristic bottom-up approach where students develop their own models and then the model is used as the basis for developing formal mathematics [4]. The realistic type in the idea of an iceberg that floats in the middle of the sea. In the iceberg model there are four levels of activity, namely (1) mathematical environmental orientation, (2) material models, (3) building stone and (4) formal mathematics [3]. In teacher learning as a mediator facilitator, and evaluator, both process assessments and product evaluations through mathematics learning use the real world as a bridge for students to understand abstract mathematical ideas [10]. So far, learning only focuses on the target, learning oriented to the mastery of material competence, the fact is that mathematics learning in particular is only oriented towards mastering material targets has proven to be successful in short-term "remembering" competencies, but fails to equip children to solve problems in long-term life [11]. The approach that can be used in fraction learning is Indonesian Realistic Mathematics Education (PMRI).

PMRI is an adaptation of Realistic Mathematics Education (RME) where mathematics learning is a human activity and mathematics must be linked significantly to the context of students' daily lives as a source of development and as an application area through both horizontal and vertical mathematical processes [20]. There are five levels in learning fraction operations, namely the introduction of fractions according to the level of students, regulating strategies for delivering fractions, sorting rules in operating equations in fractions, students operating independently and performing their own results by following rules for fraction operations with accurate. A problem through the context at each step of the fraction learning [15]. This opinion is in accordance with the expectations of mathematics learning where at every opportunity, mathematics learning should begin with the introduction of problems that are appropriate to the situation (contextual problem). According [15] contexts that can be used in
fraction learning include the context of weight, length, time, price, and so on. In this study researchers designed learning in the form of contextual problems as a starting point for students to find mathematical ideas in learning fraction distribution operations. Furthermore, the design guidelines for activities in this learning rest on the Hypothetical Learning Trajectory (HLT) which contains activities to support the learning of fraction distribution material.

Lesson Study is a collaborative process from a group of teachers to jointly: (1) identify learning problems that are felt by the teacher (one or a group of teachers) (2) plan the steps of learning (as a solution identified problems), (3) implement learning carried out by one teacher selected (agreed upon), temporarily other teachers observe the learning process, (4) evaluate the learning process what has been done (5) improve results-based learning planning evaluation (6) carry out learning again, (7) reevaluate learning which has been implemented, and (8) share (disseminate) experiences and findings from the results of the evaluation to other teachers [5].

The activities were designed using a Lesson Study for Learning Community (LSLC) system. Lesson study (or jugyō kenkyū) is a process of improving teaching originating from Japanese basic education and applying it widespread professional development practices. Working in small groups, teachers collaborate with each other, meet to discuss learning goals, planning actual class lessons or called "research lessons", observing how their ideas work in direct learning with students, and then report the results so that other teachers can use them [16]. Lesson study is not a learning method or learning strategy, but in lesson study can be selected and applied various methods or learning strategies that are appropriate to the situation, conditions, or learning problems faced by teachers and students. In lesson study, the teacher must change the teaching-oriented classical learning process (Teacher Center Learning) into student-centered learning.

This research designs learning using research design, PMRI, and Lesson study as a learning system. According to [21] the merging of lesson study and design research can make students collaborate well, so that learning mathematics is expected to be meaningful and easy for students. Students with high ability could complete the jump task, while students with low achievement had problems in completing the jump task. However, through collaborative learning with the password "please teach me", students who have problems with mathematical concepts finally, can complete the jumping task with HOTS question categories [22]. So the researcher designed this study which used design research, PMRI, and lesson study.

2. Methodology
This research method uses design research, namely validation study. In its implementation design research is a repetitive process (a cyclical process of thought experiment and instruction experiment. The meaning of repetitive processes (cyclic) is the iterative process to get a learning trajectory through thought experiments (Thought experiments) to learning experiments. Cycle in validation study through 3 stages [1]. Furthermore the stages would be discussed in the table below:

| Validation study stage | Activity |
|------------------------|----------|
| Preparing and preliminary design | Study the literature before designing various learning activities during the study. Based on theoretical and literary studies, develop HLT (contains a series of activities that contain the conjecture of student thinking). HLT design by expert (Teacher and Researcher) |
| Design Experiment | Cycle 1 (pilot experiment) HLT design trials (involving a minimum of 6 sample students with low, moderate, and high ability outside the research subjects) to determine the students' initial abilities. The revised HLT design is based on the teacher's model suggestions |
and the results of interviewing sample student.

Cycle 2 (teaching experiment)
Applying HLT which has been revised by the model teacher in the pilot experiment stage to explore students’ strategies and thoughts in learning. The researcher is only an observer

Retrospective Analysis
Analyze the results of teaching experiments to plan activities or to develop designs in the next activity. In this stage researcher compare the HLT and Actual Learning Trajectory which happen in the learning prosess

The implementation of Lesson Study is emphasized in 3 stages, namely Plan (plan or design), Do (implement), and See (observe, and after that reflect the results of observations) [13]. Plan Phase, is the first phase, here a group of teacher identified problems are found in the classroom. The identification of problems with teaching material, schedules, students’ characterization, class conditions, teaching methods, teaching media, and evaluation instruments toward teaching process and result. In the Do phase, teacher implemented the lesson plan which has been designed in plan phase. The teacher and expert observed the process using the prepared observation sheet. To support it, the observer recorded the lesson, they observed students expresion along leraning prosess and finally the See phase, where the teacher implemented the was given time to state plan lesson teacher’s feeling during implementation both for themselves and their students. Next, time was given to observers, both experts and other teachers, to share the data they are collected on the student’s activities are followed by showing of the video. The teacher of presentation, then, asked to respond to the observers' comments. The important thing is to be developed as the base to make improvements for the next teaching [9].

Lesson study composed by two tasks that need to be used as training when conducting collaborative learning and high-level thinking processes, namely sharing task and jumping task activities. Sharing task is a task that students can still do. While jumping tasks are tasks where material is rather difficult. This type of assignment is given so that students can think more critically and be challenged so that students will experience a 'leap' of learning to encourage them to think harder and to get something from what is learned. In its application, this two-step task is presented at the core activities where the learning process is taking place. So the teacher actually acts as a facilitator when students enter the group and collaboratively conduct discussions, build dialogue and listen to each other to focus their thoughts on a problem or task assigned. Task sharing is given a few moments after a teacher reviews a topic of learning. During a jumping task, each student is given a task or exercise in the form of a question or other form where the level is much higher than the sharing task. Collaborative learning using practice during jumping tasks will benefit both students who are considered to have below average competence and those at a higher level of competence. Low-ability students will get a better leap of learning, namely the learning process that starts from 'development' to 'basic' rather than the opposite where students always get the learning process from the ground up to development [10].

3. Result and Discussion

3.1. Initial Design
The initial design of the Student Worksheet (LKPD) by the researcher takes the form of three main activities, namely activities 1 and 2 for task sharing, and activity 3 for jumping tasks. Activity 1 student was given a problem about the situation of pouring 4½ liters of syrup into smaller containers on liter measure, ½ liter, and 1 ½ liter. Next, the activities of 2 students were given a problem about dividing 20 ounces of bread mixture into breads measuring 1 ¼ ounces and 2 ½ ounces. In activity 1
and 2 students were asked to resolve the mixed fraction number distribution situation informally through questions that led students to get to 'find' fraction sharing algorithms.

**Figure 1.** Design Activity 1 as a sharing task

Activity 1

Mrs. Zainab is a household scale passion fruit syrup producer. Ahead of Lebaran day, Zainab produces 4 1/2 liters of syrup every day. Mrs. Zainab packs her syrup into three packages, namely 1/4 liter bottle packaging, 1/2 liter bottle packaging, and 11/2 liter bottle packaging. How many bottles of 1/4 liter syrup can Zainab make from 4 1/2 liters of syrup? Your answer is using pictures!

**Figure 2.** Design Activity 2 as a sharing task

Activity 2

Mrs. Tika is a bread maker. Today Mrs. Tika will make chocolate sandwiches and cheese sandwiches. To make the bread, Mrs. Tika prepared each 2kg of dough for chocolate sandwiches and cheese sandwiches. Then each dough is weighed 1 1/4 ounce for chocolate sandwich while each cheese sandwich requires 2 1/2 ounces of dough. After weighing then each mixture is formed into dots.

**Figure 2.** Design Activity 2 as a sharing task

After students are able to accomplish activities 1 and 2, then activity 3 is given as a jumping task. Jumping task here is to give assignments or problems that challenge students, the given questions are categorized as difficult. So it is expected that in this phase there will be an interaction between high and low ability students through the "please teach me" mechanism. Students who have less ability ask for help to be taught by students who have high abilities. In activity 3 a problem is presented about choosing a transportation service by considering time and cost. This question is designed not only to stimulate students' analytical skills but also the ability to argue because when summing up the final answer, students are asked to provide recommendations on which transportation services are more effective to choose from.
3.2. Validation

At this stage, the researcher validates the initial design of the LKPD with colleagues to see the practicality of the three activities designed by the researcher and LKPD as a whole. The results are carried out the following revisions:

Mrs. Zainab is a household scale passion fruit syrup producer. Ahead of Lebaran day, Zainab produced three types of syrup packaging, namely a 1/4 liter bottle, a 1/2 liter bottle, and a 11/2 liter bottle, each of which was produced 4 1/2 liters of syrup every day.

How many bottles of 1/4 liter syrup can Zainab make from 4 1/2 liters of syrup? Illustrate your answer using pictures! From your picture, how many bottles of 1/4-liter syrup can you make from 1 liter of syrup?

**Figure 4.** Activity 2 After Revised

Revision is in activity 2, which is to change the overall editorial question and change the number 13½ to 4½. According to colleagues, this revision needs to be done because editorial activity 2 in the
initial design is considered ambiguous for students. While changing numbers also needs to be done because activity 2 aims to lead students to find concepts, so researchers decide to use numbers that are easier. Teacher validation At this stage, the researcher validated the initial design of the LKPD with the Srijaya Negara Middle School teacher to see the practicality and validity of the three activities designed by researchers and LKPD as a whole. The results are carried out the following revisions.

**Fractions as Representations of Parts of a Whole**

Aci is a grade 7 junior high school student, today was given a packet of chocolate by his uncle who had just returned from Bali. Aci brought the chocolate to school. He wants to eat the chocolate with classmates.

1. If Aci eats the chocolate with Bella, Cici, Dewi, Erna, and Fifi, on the condition that everyone must get the exact same portion. Describe the piece of chocolate that everyone will have! And state each person’s part into a fractional clash!

2. If Aci ate the chocolate with 11 other friends, with the condition that everyone must get the exact same portion. Describe the piece of chocolate that everyone will have! And state each person’s part into a fractional clash!

3. If there are 36 students in Aci’s class, and Aci wants everyone to get 1 block of chocolate, how many packs of chocolate should Aci bring? Draw the illustrations!

**Figure 5.** Additional activity as apperception

According to the teacher, before entering main activities (sharing task and jumping task) one apperception activity needs to be added, the purpose of which is to recall students' knowledge of fraction representation in the form of drawings. The design of perception activity designed by researchers here also uses contextual problems, namely regarding the distribution of chocolate. In this case students are asked to represent various forms of fractions through pieces of chocolate.

The revision is in the initial design activity 2, namely exchanging activity 2 in the initial design with activity 1, the reason, the problem of activity 2 in the initial design is more arbitrary so it is more suitable to be placed in activity 1. Next also revise the editorial question, change the number $13\frac{1}{2}$ to $4\frac{1}{2}$, and provide an illustration of the bottle image on the question. According to the teacher, this revision needs to be done so that students better understand the purpose of the problem.
Mrs. Zainab is a household scale passion fruit syrup producer. Ahead of Lebaran day, Zainab produced three types of syrup packaging, namely a 1/4 liter bottle, a 1/2 liter bottle, and a 1 1/2 liter bottle, each of which was produced 4 1/2 liters of syrup every day.

How many bottles of 1/4 liter syrup can Zainab make from 4 1/2 liters of syrup? Illustrate your answer using pictures! From your picture, how many bottles of 1/4-liter syrup can you make from 1 liter of syrup?

**Figure 6.** Switching activity 1 to activity 2 and adding figure to represent bottle

### 3.3. Small Group

| Student | Activity 1 | Activity 2 | Activity 3 | % Correct |
|---------|------------|------------|------------|-----------|
| T1      | ✓          | ✓          | ✓          | ✓         | ✓         | 100%       |
| S1      | ✓          | ✓          | x          | x         | x         | 71.4%      |
| R1      | ✓          | ✓          | x          | x         | x         | 28.5%      |
| T2      | ✓          | ✓          | ✓          | ✓         | ✓         | 85.7%      |
| S2      | ✓          | ✓          | x          | x         | x         | 42.8%      |
| R2      | ✓          | ✓          | x          | x         | x         | 28.5%      |
| % Correct | 100%       | 100%       | 50%        | 66.7%     | 66.7%     | 33.3%      | 16%        |

There should be no more students who cannot answer the problems given in task sharing and jumping tasks, but in fact based on the table above there is still a gap between high, medium and low ability students. This might happen because the "please teach me" process has not been carried out maximally. So that the role of the teacher is expected to coordinate students who are "problematic" to want to ask for help, and students who are considered capable, volunteered to help his friends who have problems. Based on the table above, we can also draw the conclusion that activities 1 and 2 are sharing tasks, while activity 3 is a jumping task.

### 4. Conclusions

Based on the results of student answers analysis and interviews with student. Researchers concluded that, apperception activities should be given a separate meeting before main learning is carried out. The activities on sharing tasks aim to guide students to find mathematical concepts or ideas. Problems in jumping task can be stated as jumping if> 50% of students cannot finish it. Activities 1 and 2 are sharing tasks. Activity 3 is a jumping task. The learning design using PMRI and LSLC is suitable to be applied in Sriyaja Negara Junior High School.
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