Analysis of vibration resistance reliability of directional probe framework for MWD logging system

Qing Liu\textsuperscript{1,3} and Junjie Wu\textsuperscript{2}

\textsuperscript{1}Shengli College, China University of Petroleum, Dongying, Shandong 257000, China
\textsuperscript{2}Dongying Constellation Petroleum Science & Technology Co., Ltd, Dongying, Shandong 257000, China
\textsuperscript{3}E-mail: 000199@slcupc.edu.cn

Abstract. This paper discusses the failure mode of structure under complex working conditions of oil drilling. The method of selecting the main failure mode of directional probe framework under the condition of vibration is deduced. The calculation of the vibration resistance reliability for the framework of directional probe is simplified. The anti-vibration reliability index of directional probe framework is calculated by an example in this paper. The influence of machining error and additional quality on the vibration resistance reliability of directional probe framework are also analyzed. The results show that the main failure modes of the directional probe framework are different with different additional mass. Therefore, the research results have certain design guidance significance.

1. Introduction
The directional probe framework is an important part of the MWD (Measurement while Drilling) system. Furthermore, the framework is the carrier of the electronic device and the key component of the directional probe [1-3].

A complex system structure usually consists of a variety of structures. The statistical characteristics of system load can be transformed into the statistical characteristics of structural load by mathematical method. Then we can analyze the reliability of each structure. But the reliability of each structure can not be directly equal to the reliability of the whole system. In fact, the analysis and calculation of the reliability of whole system is very complicated. Most systems can be divided into statically determinate system and statically indeterminate system. Failure of each structure for statically determinate system will result in failure of the whole system. So the reliability of the system is less than that of each structure. In statically indeterminate systems, however, problems arise only when every structure in the system has problems, so the reliability of this system must be greater than that of each structure. The actual project is much more complicated than the theoretical research. And the failure forms and modes of the whole structure are various. It is difficult to determine the reliability of structure system and the reliability of individual components simply. This is especially true for the anti-vibration reliability of the directional probe framework considering the complex working conditions [4-5].
2. Reliability analysis based on ANSYS
Analysis of structural reliability using ANSYS can solve many problems. For example, the uncertainty of the parameters can be solved according to the uncertainty of the input data in the system. The determination of structural failure probability characteristics due to uncertainty of input data can be defined. The allowable range of structural behavior can be determined by the known allowable failure probability, maximum deformation, maximum stress, etc. The determination of the most influential characteristics on input parameters and failure probability. The calculation of the sensitivity of the output result relative to the input data. The correlation coefficient between input parameter and output result parameter is given etc. The probabilistic design system (PDS) of ANSYS provides ten distribution forms of random variables: Gaussian distribution, Trongauss distribution, Log-normal Type I Distribution, Log-normal Type II Distribution, Triangular Distribution, Uniform Distribution, Exponential Distribution, Beta Distribution, Gamma Distribution and Weibull distribution. Among them, the Log-normal Type I Distribution and Log-normal Type II Distribution are both log-normal distribution, but the input methods are different.

The basic process based on ANSYS structural reliability analysis can be divided into the following two steps. The first step is to successively parameterize presentation of object members, initial assignment, construction of finite element model, loading, solving and extracting the corresponding calculation results using the "* GET" command in turn, assign the value to the target object parameters; The second step is to enter the analysis module, specify the analysis files defined above, select the definition input variables (analysis object members) and the correlation coefficient between them, define the distribution type of each input variable follows, the distribution function and its parameters, specify the analysis tools and methods, execute the analysis loops, and save the analysis results in file form. The analysis flow is shown in Figure 1.

![Figure 1. Data flow of PDS in ANSYS.](image)

3. Failure mode of directional probe framework under vibration condition
Suppose the function is: \( Z = R - S \), \( R \) stands for structural resistance, \( S \) stands for load action effect. For loads, the load roughness index \( LRI \) (Load Roughness Index) is a dimensionless index to measure the relative relation between structural resistance and the discrete degree of load effect [4-6], which is defined as
In the formula, \( \sigma_S \) and \( \sigma_R \) stand for the standard deviation of structural resistance and load effect respectively.

The relationship between the load roughness index and the failure mode of structure can be discussed by introducing load roughness index. Considering the effect of underground vibration load, the vibration load is the control load in the evaluation for structure of directional probe framework. So we can ignore other non-vibration loads, it can be reduced to a single load when discussing the correlation of failure modes.

Let the two linear function functions of the probe framework be:

\[
Z_1 = \sum_{i=1}^{n} a_i R_i - S \quad \quad Z_2 = \sum_{i=1}^{n} b_i R_i - S
\]  

The correlation coefficient between the above two functional functions is

\[
\rho_{z_1 z_2} = \left( \sum_{i=1}^{n} a_i b_i \frac{\sigma_R^2}{\sigma_S} + 1 \right) \left( \sum_{i=1}^{n} a_i^2 \frac{\sigma_S^2}{\sigma_S} + 1 \right) \left( \sum_{i=1}^{n} b_i^2 \frac{\sigma_S^2}{\sigma_S} + 1 \right)^{1/2}
\]  

(3)

In the formula, \( \sigma_R \) and \( \sigma_S \) stands for the standard deviation of the resistance and the load effect respectively. From Formula (1), we get

\[
\frac{\sigma_R^2}{\sigma_S^2} = \frac{1}{LRI_i^2} - 1
\]  

(4)

Replace Formula (4) with Formula (3),

\[
\rho_{z_1 z_2} = \left[ \sum_{i=1}^{n} a_i b_i \frac{1}{LRI_i^2} - 1 \right] \left[ \sum_{i=1}^{n} a_i^2 \frac{1}{LRI_i^2} - 1 + 1 \right] \left[ \sum_{i=1}^{n} b_i^2 \frac{1}{LRI_i^2} - 1 + 1 \right]^{1/2}
\]  

(5)

In this way, we can determine the relationship between the correlation coefficient and the load roughness index between the failure modes of the directional probe skeleton.

Under the control load, the load roughness index is approximately 1. From Formula (3), the correlation coefficient is about 1. Any two failure modes of a structure are approximately completely correlated, i.e. linearly correlated. According to the theory of probability statistics, the sufficient and necessary condition for complete linear correlation between two random variables is that there is a linear relationship between random variables \( X, Y \) with probability 1, i.e.

\[
P(Y=ax+b)=1
\]  

(6)

In the formula, \( a, b \) are constants, and \( a>0 \) (positive correlation).

For two failure modes of the structure, \( Z_1(R,S)=0, Z_2(R,S)=0 \). According to the above conclusion, if there are constants \( a, b \), and \( a>0 \) satisfying the formula,

\[
P(Z_2=aZ_1+b)=1
\]  

(7)

There is a membership relationship between the failure domains of the two functions. Assume that \( Z>0 \) and \( Z\leq0 \) indicates that the structure is in a reliable state and failure state, respectively. According to the value of constant \( b \), there is a membership relationship between the failure domains of the two failure modes which can be divided into the following three cases.

i when \( b=0 \), there is

\[
P(Z_2\leq0|Z_1\leq0)=1 \quad \quad P(Z_1\leq0|Z_2\leq0)=1
\]  

\[
P(Z_2>0|Z_1>0)=1 \quad \quad P(Z_1\leq0|Z_2>0)=1
\]  

(8)

It can be seen from the above relations that the failure modes \( Z_1 \) and \( Z_2 \) are both invalid and reliable at the same time, and the two are equivalent.

ii when \( b<0 \), there is
\[
P(Z_1 \leq 0 | Z_2 \leq 0) = 1, \quad P(Z_1 \leq 0 | Z_2 > 0) \neq 1
\]
\[
P(Z_2 > 0 | Z_1 \leq 0) \neq 1, \quad P(Z_2 > 0 | Z_1 > 0) = 1
\]

The failure of failure mode \( Z_1 \) must lead to the failure of \( Z_2 \), otherwise it is not. Another case is that the reliability of failure mode \( Z_2 \) necessarily leads to the reliability of \( Z_1 \), otherwise it is not. Therefore, the failure domain of failure mode \( Z_1 \) belongs to the failure domain of failure mode \( Z_2 \).

\[
\text{iii then } b > 0, \quad P(Z_2 \leq 0 | Z_1 \leq 0) \neq 1, \quad P(Z_1 \leq 0 | Z_2 \leq 0) = 1
\]
\[
P(Z_2 > 0 | Z_1 > 0) = 1, \quad P(Z_1 > 0 | Z_2 > 0) \neq 1
\]

The failure of failure mode \( Z_2 \) must lead to the failure of \( Z_1 \), otherwise it is not. Another case is that the reliability of failure mode \( Z_1 \) necessarily leads to the reliability of \( Z_2 \), otherwise it is not. Therefore, the failure domain of failure mode \( Z_1 \) contains the failure domain of failure mode \( Z_2 \).

According to the formula of reliability index, we know that

\[
\beta_1 = \frac{\mu_{Z_1}}{\sigma_{Z_1}}, \quad \beta_2 = \frac{\mu_{Z_2}}{\sigma_{Z_2}}
\]

In the formula, \( \beta_1, \beta_2, \mu, \beta_1, \beta_2, \mu_{Z_1}, \mu_{Z_2}, \sigma_{Z_1}, \sigma_{Z_2} \) are the reliability index, mean and standard deviation of failure mode \( Z_1 \) and \( Z_2 \). From Formulas (4) - (6), we can get

\[
\mu_{Z_i} = a\mu_{Z_1} + b, \quad \sigma_{Z_i} = a\sigma_{Z_1} + b
\]

Replace Formula (11) with Formula (12)

\[
\beta_2 = (a\mu_{Z_1} + b)(a\sigma_{Z_1})^{-1} = \beta_1 + b(a\sigma_{Z_1})^{-1}
\]

Because \( a > 0, \sigma_{Z_1} > 0 \), from Formula (13): i when \( \beta_1 = \beta_2, b = 0 \); ii when \( \beta_1 > \beta_2, b < 0 \); iii when \( \beta_1 < \beta_2, b > 0 \). Combined with Formula (8) to Formula (10), it can be seen that under underground vibration load, failure mode with low failure probability must fail when failure mode with high failure probability (That means the failure mode with high probability of failure will fail first), otherwise it is not. Thus, it can be concluded that the reliability of the structural system is approximately determined by its weakest failure mode. So, the calculation can be greatly simplified.

4. Examples

The material used for directional probe framework is aviation grade superhard aluminum LC4, which belongs to typical 6 sensor framework structure. The outer diameter of the framework was 35.5 mm, the overall length was 602 mm, and the monomer mass was 0.56 kg. The finite element models are shown in Figures 2 to 5. In order to ensure the operation speed, some rounded corners and small diameter holes are simplified.

**Figure 2.** Model for whole framework.

**Figure 3.** Model for local of framework (1).
Finite element model of the directional probe framework structure was built by ANSYS software. All details of the directional probe framework are simulated by SOLID92 element in the model.

We can consider three limit states of underground vibration condition: The radial displacement of the directional probe framework does not exceed 1/1000 of which height. The skeleton stress does not exceed the yield stress. The shear force of the screw at the installation of the directional probe framework and the outer pipe reaches the limit value. Functional functions can be written as

\[
Z_1 = [\Delta] - \Delta \\
Z_2 = [\sigma] - \sigma \\
Z_3 = [S] - S
\]  \hspace{1cm} (14)

In the formula, the value of \([\Delta]\) takes 1/1000(H/1000=0.602) of the total height of the directional probe framework, the value of \([\sigma]\) takes the yield stress (420 MPa) of the directional probe framework, and the value of \([S]\) takes the limit shear force 14.8 KN.

The statistical characteristics of each random variable are shown in Table 1.

| Random Variables                  | Pattern            | Mean Value | Coefficient of Variation |
|-----------------------------------|--------------------|------------|--------------------------|
| **Downhole vibration load**       | Extreme I          | 5g(50m/s²) | 0.3                      |
| **Vibration load incidence angle**| Equidistribution   | 180°       | 0.58                     |
| **Additional mass coefficient of framework** | Equidistribution   | 0.1        | 0.01                     |
| **Elastic modulus of framework materials** | Normal              | 75GPa      | 0.05                     |
| **Yield strength of skeleton materials** | Lognormal           | 420MPa     | 0.15                     |

The PDS system of ANSYS does not provide the distribution of extreme Type I distribution. Custom sampling of underground vibration loads is required. After considering the factors of efficiency and reliability, the inverter method is used for sampling [7-8]. The inverse transformation method is a common method for generating random variables. The pseudo-random number \(R\) of \([0,1]\) is first generated, and we make it equal to the distribution function of random variables.

\[
F(x) = R \quad \text{or} \quad x = F^{-1}(R)
\]  \hspace{1cm} (15)

In the formula, \(x\) is the random variable of the distribution function \(F(x)\), and \(R\) is the uniformly distributed pseudo-random number of \([0,1]\). The underground vibration load follows an extreme Type I distribution.
The inverse function of $F(x)$ is

$$x = -\alpha \ln(-\ln R) + k$$

(17)

The MATLAB random number function unifrnd() can be used to generate (0,1) uniformly distributed random numbers (Taking into account that $F(x)$ inverse function is defined in a domain of 0 $<$ $R$ $<$ 1, the ends are rounded off.). Bring it into Formula (17) to get the random number of $x$. The standard deviation of the sample is 0.609 according to the calculation of stdev() function and the mean value of the sample is 5.010. The difference between the calculated results and the standard deviation and mean value of underground vibration load (0.6 and 5, respectively) is very small. A plot of probability distribution values agrees well with the probability density function of extreme Type I, which is shown in Figure 6. The sample can be considered credible.

![Figure 6. Underground vibration probability distribution density.](image)

We used Monte-Carlo simulation to perform 50000 calculations. The final analysis results are as follows: the probability of $Z_1$ $<$ 0 is 7.5075%, the probability of $Z_2$ $<$ 0 is 4.5890% and the probability of $Z_3$ $<$ 0 is 0%. The weakest failure mode of the directional probe frame is $Z_1 = |\Delta| - \Delta < 0$, which is the main failure mode. Based on generalized reliability indicators

$$\beta = -\Phi^{-1}(P_f)$$

(18)

In the formula, $\Phi^{-1}$ is the inverse function of the standard normal distribution function, and $P_f$ is the failure probability. $\beta_i$=1.4365 can be obtained from the Standard Normal Distribution Function Table. The difference of additional quality coefficient of framework is great. Calculated the corresponding reliability index of the directional probe frame under the condition that the underground vibration load is 3 g (low vibration), which is shown in Table 2.

| Coefficient for Additional Mass of Framework | Probability for $Z_1$ $<$ 0/% | Probability for $Z_2$ $<$ 0/% | Probability for $Z_3$ $<$ 0/% | Main failure Mode | Reliability Indicators |
|---------------------------------------------|-----------------------------|-----------------------------|-----------------------------|-------------------|------------------------|
| 0.15                                        | 0.1389                      | 0.1651                      | 0                           | $Z_2$             | 2.9381                 |
| 0.20                                        | 0.1940                      | 0.0935                      | 0                           | $Z_1$             | 2.8877                 |
| 0.25                                        | 0.1546                      | 0.2518                      | 0                           | $Z_2$             | 2.8049                 |
| 0.30                                        | 0.2608                      | 0.2349                      | 0                           | $Z_1$             | 2.7934                 |
5. Conclusions
We got these conclusions from the study:

(1) The results of Monte-Carlo simulation are accurate, when the sampling number is large enough. The vibration resistance reliability of the directional probe framework can be calculated according to the ANSYS reliability analysis flow.

(2) The selection method of the main failure modes of the skeleton anti-vibration can be deduced through discussing on the failure mode of framework structure under vibration load. It can be seen that under underground vibration load, failure mode with low failure probability must fail when failure mode with high failure probability fails first, otherwise it is not. Thus, it can be concluded that the reliability of the structural system is approximately determined by its weakest failure mode.

(3) An example is given to calculate the vibration reliability of the directional probe framework. In this paper, the influence of material parameters and additional mass coefficient on the vibration-resistant reliability of directional probe framework is analyzed. The results show that if the additional mass coefficient is different, the main failure modes of the framework are also different. The research results have some reference value for the design and calculation of the actual directional probe framework.

References
[1] Wang Lu, Wang Yu, Deng Yaqi, Aboelmagd Noureldin and Li Pingfei 2020 Drilling trajectory survey technology based on 3D RISS with a single fiber optic gyroscope[J]. Optik 203
[2] Li Ming, Gao Boyue, Zhang Qiang and Jiang Tianjie 2019 Implementation of fault analysis and solutions for directional probe [J]. Petroleum Pipes and Instruments 5(06) 92-94
[3] Wang Zhiwei 2018 Application of accelerometer and fluxgate in directional probe[J]. Western Prospecting Engineering 30(08) 26-28+30
[4] Rosyid D M 1992 Elemental reliability index-based system design for skeletal structures[J]. Structural Optimization 4(1) 1-16
[5] Wang Shan and He Shuiqing 1993 Analysis and design of structural reliability[M]. Beijing: National Defense Industry Press
[6] Wu Junjie 2009 Analysis on seismic reliability of aging petroleum platform[D]. China University of Petroleum (East China)
[7] Gao Huixuan 1995 Statistical calculations[M]. Beijing: Peking University Press
[8] Mi Xiaoqing 2005 Statistical calculation and analysis[M]. Beijing: Tsinghua University Press