SOLAR NEUTRINOS AND THE SUN

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ABSTRACT

We present updated standard solar models (SSMs) that incorporate the latest results for nuclear fusion rates, recently published. We show helioseismic results for high and low metallicity compositions and also for an alternative set of solar abundance, derived from 3D model atmospheres, which give intermediate results. For the high and low metallicity models, we show that current solar neutrino data can not differentiate between models and that a measurement of the CNO fluxes is necessary to achieve that goal. A few additional implications of a hypothetical measurement of CNO neutrinos, both in terms of solar and stellar physics, are discussed.

1. Introduction

We present and discuss the results of our most up-to-date solar models. Part of this work is devoted to the solar abundance problem, that is, the mismatch between results of solar models and helioseismic inferences on solar structure when most the recent and sophisticated (using a 3D solar model atmosphere) determination of the solar composition is used. Results for neutrino fluxes are also presented. Together with a new analysis of solar neutrino data that incorporates latest results by the Borexino experiment, these results show that solar neutrino data can be equally well reproduced by solar models with both the high and low metallicity compositions. Currently, solar neutrinos can not yield information towards the solution of the solar abundance problem. However, this situation could be modified if a direct determination of CNO neutrinos is achieved. If such a measurement were done today, it would lead to a direct measurement of the central C+N content of the Sun where uncertainties from solar models and neutrino properties represent only 10%. Possible implications of such a measurement are discussed both in terms of solar and stellar physics. Finally, we speculate with the possibility that the early evolution of the Sun and the interaction with its protoplanetary disk can be constrained by determining the central content of C+N. This might be of relevance in studies of the evolution of protoplanetary disks and planet formation theories.

2. Solar Models

2.1. Astrophysical Factors
A decade after the critical evaluation of the pp chain and CN cycle rates published in the Solar Fusion I paper\(^1\), a new revision, Solar Fusion II\(^2\), has established a new set of recommended values and uncertainties for the pp-chains and CNO-bicycle cross sections. Results in Solar Fusion II account for the large effort of the nuclear physics community, both experimental and theoretical, during the last 10 years. Astrophysical factors have been regularly updated in SSM\(^3\) during this period. We now adopt as our standard the recommended values in Solar Fusion II; the most relevant changes with respect to the previous values used in SSM calculations\(^4\) are given in Table 1. Changes in the central values of key reactions are modest: +2% for \(S_{11}\), -2% for \(S_{33}\), -5% for \(S_{17}\), +6% for \(S_{1,14}\). The uncertainties are larger, in some cases by up to a factor of 2, than previous values. This has implications for the theoretical uncertainties in the neutrino fluxes, although uncertainties in the solar composition continue to dominate in most cases.

| Reaction  | SFII (keV-b)       | Previous (keV-b)     |
|-----------|--------------------|----------------------|
| \(S_{11}\) | \(4.01 \times 10^{-22}(1 \pm 0.010)\) | \(3.94 \times 10^{-22}(1 \pm 0.004)\) |
| \(S_{33}\) | \(5.21 \times 10^{3}(1 \pm 0.052)\)   | \(5.4 \times 10^{3}(1 \pm 0.06)\)   |
| \(S_{34}\) | \(0.56(1 \pm 0.054)\)               | \(0.567(1 \pm 0.03)\)               |
| \(S_{17}\) | \(2.08 \times 10^{-2}(1 \pm 0.077)\)| \(2.14 \times 10^{-2}(1 \pm 0.038)\)|
| \(S_{1,14}\) | \(1.66(1 \pm 0.072)\)             | \(1.57(1 \pm 0.08)\)              |
| \(R(\text{pep})/R(pp)\) | ↑ 2.5%          | —                   |

2.2. Solar Composition(s)

Most of the results presented here are based on two different sets of solar abundances. One is that of Grevesse & Sauval (1998), hereafter GS98\(^5\); the other, that from Asplund et al. (2009), hereafter AGSS09\(^6\). For both sets we adopt the meteoritic scale for all refractory elements; silicon is the anchor point between the photospheric and meteoric scales. The adopted solar abundances are of fundamental importance because the surface metal-to-hydrogen abundances ratio is a constraint solar models have to fulfill. Many properties of the resulting model depend on the adopted solar metallicity, or composition. In particular, the metal-to-hydrogen mass fractions ratio in the solar surface is \((Z/X)_{\odot} = 0.0229\) according to GS98, but only 0.0178 according to AGSS09. The large change is mostly the result of a strong reduction (about 30-40%) in the CNO and Ne abundances.

The difference between GS98 and AGSS09 has a profound impact on solar models that has been widely discussed in the literature, particularly in relation to structural quantities in the models that can be compared with results from helioseismology. We briefly review these results in § 3.1. Several reasons lie behind the reduction
of the solar CNO abundances in but the dominating effects are: use of a 3D-hydrodynamics model atmosphere, better selection of spectral lines (identification of blends), detailed treatment of radiative transport in the line-formation modeling including non-local thermodynamic equilibrium for some elements.

Caffau and collaborators (hereafter CO\textsuperscript{5}BOLD) have also embarked in a similar task, the determination of solar abundances using 3D state-of-the-art model atmospheres. Interestingly, although the underlying structure of the model atmospheres by both Asplund’s and the CO\textsuperscript{5}BOLD groups are very similar, derived abundances for the CNO elements are not; results are summarized in Table 2 (error bars are included for CNO elements). Whereas differences arise from a number reasons, the selection of spectral lines used by each group for the abundance determinations seems to be the dominant one. The treatment of blends and line broadening, for example, introduces non-negligible differences between the results of different groups.

We note the CO\textsuperscript{5}BOLD results lay between the GS98 and AGSS09 values, in fact, they are consistent within errors with both of them. Unfortunately, CO\textsuperscript{5}BOLD abundances have not been obtained at the moment for all elements and have to be complemented with other sources; abundances for these elements are given in parenthesis.

It is not among the goals of the present work to make a critical analysis of the goodness of the different solar abundance determinations present in the literature, neither to discuss the detailed results of solar models for each set of abundances. We content ourselves by adopting GS98 and AGSS09 as two different standards which probably represent two extreme cases and on which to base our discussion. In addition, we briefly present helioseismic results for a solar model using the CO\textsuperscript{5}BOLD abundances.

| Element | GS98      | AGSS09    | CO\textsuperscript{5}BOLD |
|---------|-----------|-----------|---------------------------|
| C       | 8.52 ± 0.06 | 8.43±0.05 | 8.50±0.06                 |
| N       | 7.92 ± 0.06 | 7.83±0.05 | 7.86±0.12                 |
| O       | 8.83 ± 0.06 | 8.69±0.05 | 8.76±0.07                 |
| Ne      | 8.08       | 7.93      | (8.05)                    |
| Mg      | 7.58       | 7.53      | (7.54)                    |
| Si      | 7.56       | 7.51      | (7.52)                    |
| Ar      | 6.40       | 6.40      | (6.50)                    |
| Fe      | 7.50       | 7.45      | 7.52                      |
| Z/X     | 0.0229     | 0.0178    | (0.0209)                  |
3. Results

3.1. Helioseismology

The impact of solar abundances in the helioseismic properties of solar models has been widely discussed in the literature.\cite{9,10,11,12} Solar models that use the AGSS09 composition are not in agreement with helioseismic inferences of the solar interior structure. Discrepancies manifest themselves in a variety of ways: mismatch in the determination of the depth of the convective envelope, low solar surface abundance of helium, differences in the sound and density profiles, too low mean molecular weight in the solar core. We suggest the interested reader to refer to the vast literature on the subject, a flavor of which is given in the references above, for details. Problems are present for standard solar models; they are even more acute for non-standard models that attempt to include poorly understood dynamic effects.\cite{13} Most updated results for SSMs using our two reference compositions, GS98 and AGSS09, have been recently presented\cite{14} and show small changes with respect to previous models. In Fig. 1 we show the relative sound speed difference for SSMs with GS98 and AGSS09. For each composition, the dashed line shows results for previous models, i.e. models using the set of astrophysical factors given in the third column of Table 1, whereas the solid line corresponds to models computed using the Solar Fusion II recommended values. Changes are small but noticeable, particularly towards the center ($R < 0.2 R_\odot$), and within the range of sensitivity of current helioseismic data.\cite{15} The responsible for the changes is the slight increase in $S_{11}$ because it produces a decrease in the temperature
of the central regions of the Sun. Since for an ideal gas \( c^2 \propto T/\mu \) (here \( c \) is the sound speed, \( T \) the temperature, and \( \mu \) the mean molecular weight), and changes in \( \mu \) are negligible, then the model sound speed is reduced accordingly.

As can be expected from the abundances listed in Table 2, when the CO\(^5\)BOLD values are used, the resulting solar structure lays somewhere in between SSMs that use either GS98 or AGSS09. To illustrate this, the sound speed profile is given in Fig. 2. The improvement in the sound speed profile model with CO\(^5\)BOLD abundances with respect to the model with AGSS09 comes about because the location of the base of the convective envelope in the former is in better agreement with helioseismology. Other helioseismic constraints, e.g. surface helium abundance, show a similar behavior, with CO\(^5\)BOLD abundances giving results almost equally distant from GS98 and AGSS09. Values for these quantities are included in Fig. 2.

![Figure 2: Relative sound speed difference for SSMs using the different solar abundances listed in Table 2. Values for the depth of the convective envelope in units of the solar radius and the mass fraction of the surface helium abundance are also shown. For reference, the helioseismic values are \( R_{\text{CZ}}/R_\odot = 0.713 \pm 0.001 \) and \( Y_s = 0.2485 \pm 0.0035 \).](image)

3.2. Solar Neutrinos

The new SFII nuclear reaction rates do alter the predicted solar neutrino fluxes, particularly the fluxes associated with the ppIII chain and CN-cycle, mechanisms for solar hydrogen burning that are relatively unimportant energetically. Changes in the nuclear rates in SFII with respect to our previous set of preferred values have been described in § 2.1. Model predictions for the neutrino fluxes and associated uncertainties are presented in Table 3 for both reference solar compositions, AGSS09 and GS98. The third column of the table quantifies the changes with respect to results using older set of preferred nuclear rates \(^{[1]}\). The most significant changes are a 5% decrease in the predicted \(^8\)B flux primarily because of the increase in \( S_{11} \) and the increase in the \(^{13}\)N flux due to the larger \( S_{1,14} \) and central abundance of C. The
increase in C is a consequence of the lower SFII value for $^{15}\text{N}(p,\gamma)^{16}\text{O}$, a reaction that competes with the CN I cycle reaction $^{15}\text{N}(p,\alpha)^{12}\text{C}$ and allows mass to flow out of the CN I cycle into CN II.

Table 3 also includes the updated solar neutrino fluxes inferred from all available neutrino data [14]. The analysis includes the recent more precise $^7\text{Be}$ measurement [16], which is the main change with respect to previous analysis [17, 18, 19]. In Figure 3 we show for the relevant neutrino fluxes a comparison between the two SSMs and solar fluxes, normalized to the GS98 SSM values. For the $^{13}\text{N}$ and $^{15}\text{O}$ fluxes only upper limits can be established with current solar neutrino data. Except for the pp flux, experimental values lay in between SSM predictions for the two reference solar compositions. For comparison of the SSM predictions with the fluxes inferred from neutrino data, we use the $\chi^2$ function defined in [19], with updated errors and correlations. We find $\chi^2_{\text{GS98}} = 3.5$ and $\chi^2_{\text{AGSS09}} = 3.4$, leading in both cases to $P_{\text{agr}}^{\text{GS98,AGSS09}} = 90\%$. The new fusion cross sections from SFII and the new Borexino results lead to both models predicting solar neutrino fluxes in excellent agreement with inferred ones. From this analysis, we conclude that, currently, solar neutrinos can not differentiate between solar compositions. In both cases, excellent agreement with data is achieved.

| Flux | SFII-GS98     | SFII-AGSS09     | $\Delta$ | Solar       |
|------|---------------|-----------------|----------|-------------|
| pp   | 5.98(1 ± 0.006) | 6.03(1 ± 0.006) | +0.1\%  | 6.05(1^{+0.015}_{-0.011}) |
| pep  | 1.44(1 ± 0.011) | 1.47(1 ± 0.012) | +2\%    | 1.46(1^{+0.010}_{-0.014}) |
| hep  | 8.04(1 ± 0.30)  | 8.31(1 ± 0.30)  | +1.6\%  | 8.18(1^{+0.4}_{-0.5})    |
| $^7\text{Be}$ | 5.00(1 ± 0.07) | 4.56(1 ± 0.07) | -1.7\%  | 4.82(1^{+0.04}_{-0.05})  |
| $^8\text{B}$ | 5.58(1 ± 0.14) | 4.59(1 ± 0.14) | -5\%    | 5.00(1 ± 0.03)            |
| $^{13}\text{N}$ | 2.96(1 ± 0.14) | 2.17(1 ± 0.14) | +5\%    | $\leq 6.7$                |
| $^{15}\text{O}$ | 2.23(1 ± 0.15) | 1.56(1 ± 0.15) | +5-6\%  | $\leq 3.2$                |
| $^{17}\text{F}$ | 5.52(1 ± 0.17) | 3.40(1 ± 0.16) | +2\%    | $\leq 59$                 |
| $\chi^2/P_{\text{agr}}$ | 3.5 / 90\% | 3.4 / 90\% | —       | —                       |

### 4. CNO neutrinos: what can be learned?

The Sun is almost entirely powered by proton-proton captures. However, nuclear energy generated by the CNO bi-cycle dominates for more massive (1.2 $M_\odot$) or more evolved stars. Detection of solar CNO neutrinos is the only direct observational proof of CNO as a source of nuclear energy in stars. Current SSMs predict that the CNO bi-cycle contribute less than 1\% to the solar luminosity; CNO solar neutrinos can test this result.

If, on the other hand, we assume nuclear processes in stars are well understood, a measurement of CNO solar neutrinos can be used to reveal properties of the solar interior. In particular, $^{13}\text{N}$ and $^{15}\text{O}$ neutrinos have energies and fluxes which should be within the capabilities of current and forthcoming liquid scintillator detectors like
Borexino, KamLAND, and SNO+. Recently, a new analysis technique for subtracting the $^{210}$Bi background from the Borexino signal has been presented \cite{20} that could allow detection of $^{13}$N and $^{15}$O neutrinos in a Borexino-like detector with 1 yr of data. The prospects for precise measurements of CNO neutrinos are further improved with larger detectors like SNO+.

A direct goal of measuring CNO neutrinos is that they can be used to determine abundance of carbon plus nitrogen in the solar core. As discussed above, current solar neutrino data not only does not discriminate between high and low-metallicity solar compositions but can not even favor one of them. Let us assume an optimistic scenario in which both the $\Phi({}^{13}\text{N})$ and $\Phi({}^{15}\text{O})$ fluxes are measured to 10% precision. Moreover, let the central values perfectly align with results from one of the SSMS discussed above. If the GS98 SSMS values are used, i.e. $\Phi_{\text{exp}}({}^{13}\text{N}) = 2.96 \times 10^8 \text{cm}^{-2}\text{s}^{-1}$ and $\Phi_{\text{exp}}({}^{15}\text{O}) = 2.17 \times 10^8 \text{cm}^{-2}\text{s}^{-1}$, then we would get $\chi^2_{\text{GS98}} = 3.7(88\%)$ and $\chi^2_{\text{AGSS09}} = 13.9(8\%)$; i.e. measuring $\Phi({}^{13}\text{N})$ and $\Phi({}^{15}\text{O})$ at this level of precision would clearly favor solar models with a particular C+N content over the other. Clearly, these assumptions make up for a very favorable case, but give a flavor of what can potentially be achieved.

A more targeted approach can be used to determine the C+N abundance in the solar core directly. One possible way has been laid out where the $^8\text{B}$ solar neutrino flux is used as a thermometer of the solar core \cite{21}. Because the temperature dependence of the nuclear reactions that produce CNO and $^8\text{B}$ neutrinos is very similar, many sources of uncertainty in the solar models affect them in the same way and can be
almost completely cancelled out; these are the environmental factors. Using power-law expansions of solar neutrino fluxes\textsuperscript{22}, \(\Phi(^{13}\text{N} + ^{15}\text{O})\) can be expressed as a function of \(\Phi(\text{^{8}B})\) as\textsuperscript{21}:

\[
\frac{\Phi(^{13}\text{N} + ^{15}\text{O})}{\Phi_{\text{SSM}}(^{13}\text{N} + ^{15}\text{O})} = \frac{X(C + N)}{X_{\text{SSM}}(C + N)} \left[ \frac{\Phi(\text{^{8}B})}{\Phi_{\text{SSM}}(\text{^{8}B})} \right]^{0.828} \times [1 \pm 0.03(\text{exp}) \pm 0.026(\text{env}) \pm 0.035(\text{LMA}) \pm 0.10(\text{nucl})].
\]

Uncertainties in this equation are: 3\% from the experimental \(^{8}\text{B}\) measurements, 2.6\% from remainder of environmental uncertainties, 3.5\% from neutrino parameters, and 10\% from nuclear cross sections. These uncertainty sources are experimental and under control, and can be eventually improved (except for the negligible environmental component). The SSM only acts as a scaling factor (and determining the precise value of the power-law exponent, but this changes little for different solar models). A hypothetical measurement of \(\Phi(^{13}\text{N} + ^{15}\text{O})\) can directly yield a determination of the central \(\text{C+N}\) abundance. The uncertainty from all other sources besides the hypothetical measurement amounts, today, to \(\approx 10\%\) and are dominated by the nuclear physics part. For reference, the difference in the \(\text{C+N}\) abundance brought about by the change from the GS98 to the AGSS09 solar abundances is of the order 35\%.

There is not direct information about the composition of the solar (or any other star) core. Helioseismic information about the solar core is mostly sensitive to the mean molecular weight and temperature; information about metallicity is of indirect nature, and usually degenerate with other quantities, such as the radiative opacity of stellar matter. CNO neutrinos would give direct information on the abundance of metals, in particular \(\text{C+N}\), in the solar central regions. This would be in itself an outstanding achievement. Together with other constraints on the solar composition, this could potentially be useful to constrain, among other processes, the efficiency of settling of heavy elements in the Sun.

Recently, it has been suggested that there are systematic differences between the solar surface abundances and those of other stars, otherwise very similar to the Sun, depending on the presence and characteristics of the planetary system in each star\textsuperscript{23}. Tentatively, the cause for these differences has been ascribed to an interplay between refractory elements been preferentially locked in planetesimals and protoplanetary disk material been accreted to the Sun. If true, this idea would suggest the surface composition of the Sun does not reflect the original composition of the protosolar nebula but rather it is the outcome of a mixture between composition of the primordial nebula and of chemically processed material from the protoplanetary disk. The solar core, however, keeps memory of the primordial composition.

Because CNO neutrinos offer a unique way of determining solar core metal content (at least of \(\text{C}\) and \(\text{N}\)), the interesting possibility arises they can be used to constraint the evolution and interaction between the early Sun and its protoplanetary disk. A first step towards assessing the possibility the Sun has suffered such an accretion phase
has been recently discussed\textsuperscript{14}. In that work different accretion histories (varying accreted masses and compositions) have been considered and the resulting structure of solar models, both in helioseismic and solar neutrino aspects, analyzed. For reasons of space, we can not summarize those results here. However, in terms of our present discussion, possibly the most interesting result is that applying Eq. 1 (developed for SSMs) to the non-SSMs that include accretion, the central C+N abundance of the models can be recovered with very good precision. Using the $^8\text{B}$, $^{13}\text{N}$, and $^{15}\text{O}$ fluxes for each of the models with accretion, we can use Eq. 1 to estimate the central content of C+N in the model, $X_{\nu}(\text{CN})$. The relative difference between this estimation and the actual C+N central abundance of the model, $X_{\text{mod}}(\text{CN})$, is shown as a function of $X_{\text{mod}}(\text{CN})$ in Fig. 4. For illustration, results for the GS98 and AGSS09 SSMs are shown with black vertical lines. Neutrino fluxes alone allow the determination of the central C+N abundance with 6% accuracy for all models considered which, as can be seen, have quite large variations in the metallicity. This is better than the current intrinsic uncertainty in the relation, 10%, dominated by uncertainties in astrophysical factors ($S_{1,14}$ and $S_{17}$). Results, therefore, are encouraging: scaling relations between neutrino fluxes and solar composition derived from SSMs seem to be applicable in quite general cases, even for cases where the central metallicity changes by more than a factor of 2, above of what can be reasonably expected given helioseismic and current solar neutrino constraints\textsuperscript{14}.

![Figure 4: Neutrino fluxes: comparison between experimental results and SSMs. Vertical lines denote $1\sigma$ model uncertainties. Solar values are represented by solid horizontal lines; dotted lines are $1\sigma$ uncertainties. All fluxes normalized to GS98 SSM values.](image)

### 5. Final remarks

Latest changes in the input physics of SSMs have a small effect on helioseismic quantities and do not have an impact on the solar abundance problem. A comparison of the new analysis of solar neutrino data, including the latest results by Borexino,
against model predictions show that current neutrino data can be equally described by SSMs with high and low metallicity compositions. Both experimental $^8$B and $^7$Be fluxes lay right in between both model predictions for both GS98 and AGSS09 compositions. On the other hand, a precise measurement of the $^{13}$N and $^{15}$O fluxes allows to extract the central C+N abundance with a model uncertainty of about 10%. To the extent we have been able to test this result, it seems valid regardless of SSMs being an accurate representation of the solar structure. A measurement of the CNO fluxes, and as a corollary of the central C+N abundance, has a wide range of implications not only for solar but also for stellar physics; here we have briefly discussed a few of them: direct experimental determination of CNO-cycles as source of nuclear energy in stars, solar abundance problem, mixing processes in the Sun and, more tentatively, early phases of solar evolution and interaction with the protoplanetary disk.

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