Partonic structure of $\pi$ and $\rho$ mesons from data on hard exclusive production of two pions off nucleon

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Abstract

We fitted the $\pi\pi$ mass distribution in the range $0.5 \leq M_{\pi\pi} \leq 1.1$ GeV measured in hard exclusive positron-proton reactions at HERA by the form dictated by QCD at leading twist level. Extracted parameters are related to valence quark distribution in the pion, and to the pion and $\rho$ meson distribution amplitudes. We obtain, for the first time, a measurement of the second Gegenbauer coefficient of the $\rho$ meson distribution amplitude: $a_2^{(\rho)} = -0.10 \pm 0.20$ for a photon virtuality of $\langle Q^2 \rangle = 21.2$ GeV$^2$.

1. Introduction

Owing to QCD factorisation theorem for hard exclusive reactions \cite{1} the dependence of the amplitude of the reaction

$$\gamma^*_L(q) + T(p) \to \pi^+\pi^- + T'(p')$$

(1)

with longitudinally polarised virtual photon on the di-pion mass $M_{\pi\pi}$ factors out in an universal (independent of the target) factor. At leading order in $\alpha_s(Q^2)$ this factor has the form \cite{2}:

$$\mathcal{A}(M_{\pi\pi}) \propto \int_0^1 \frac{dz}{z} \Phi^I(z, \zeta, M_{\pi\pi}; Q^2).$$

(2)

Here $\Phi^I(z, \zeta, M_{\pi\pi}; Q^2)$ is the two-pion light cone distribution amplitude ($2\pi$DA) \cite{3}, which depends on $z$–longitudinal momentum carried by the quark, $\zeta$ characterising the distribution of longitudinal momentum between the two pions \cite{4} and the invariant mass of produced pions $M_{\pi\pi}$, the superscript $I$ standing for isospin of produced pions ($I = 0, 1$).

\textsuperscript{1}With $Q^2 \sim 2(p \cdot q) \gg \Lambda_{QCD}^2$ and $M_T^2, M_T^{f'}, (p' - p)^2, M_{\pi\pi}^2 \ll Q^2$.

\textsuperscript{2}For detailed definition of kinematical variables $z$ and $\zeta$ see refs. \cite{3, 4}.
The dependence on the virtuality of the incident photon $Q^2$ is governed by the ERBL evolution equation [3].

For the process (1) at small $x_B = Q^2/2(p \cdot q)$ the production of two pions in the isoscalar channel is strongly suppressed relative to the isovector channel [4], because the former is mediated by $C$-parity odd exchange. At asymptotically large $Q^2$ QCD predicts the following simple form for the isovector $2\pi$ DA [2, 6]:

$$\Phi_{I,\text{asy}}^{I=1}(z, \zeta, M_{\pi\pi}) = 6z(1-z)(2\zeta - 1)F_\pi(M_{\pi\pi}) ,$$

(3)

where $F_\pi(M_{\pi\pi})$ is the pion electro-magnetic (e.m.) form factor in time-like region, measured with high precision in low energy experiments [10, 11]. From eqs. (2,3) we conclude that at asymptotically large $Q^2$ QCD predicts unambiguously the shape of the di-pion mass distribution. Asymptotically the dependence of the amplitude on $M_{\pi\pi}$ has the form:

$$\mathcal{A}_{\text{asy}} \propto \beta(M_{\pi\pi}) F_\pi(M_{\pi\pi}) \cos \theta ,$$

(4)

where \( \beta(M_{\pi\pi}) = \sqrt{1 - \frac{4m_\pi^2}{M_{\pi\pi}^2}} \) is the velocity of pions in their centre of mass system (cms) and \( \theta \) is the angle between the directions of the positive pion and the momentum of produced $\pi^+\pi^-$ system in the $\pi^+\pi^-$ cms. This angle is related to \( \zeta \) in the following way:

$$\cos \theta = \frac{2\zeta - 1}{\beta(M_{\pi\pi})} .$$

(5)

The corresponding di-pion mass distribution has asymptotically the form

$$\frac{dN(M_{\pi\pi})}{dM_{\pi\pi}} \propto M_{\pi\pi} \beta(M_{\pi\pi})^3 |F_\pi(M_{\pi\pi})|^2 .$$

(6)

The asymptotic shape for any di-meson production (mesons $M_1, M_2$) was derived in [7, 8, 9], where it was related to the cross section of $e^+e^- \rightarrow M_1, M_2$ at low $\sqrt{s}$.

At non-asymptotic $Q^2$ values, the $2\pi$DA deviates from its asymptotic form (3). This deviation can be described by a few parameters which can be related to quark distributions (skewed and usual) in the pion and to distribution amplitudes of mesons ($\pi, \rho$, etc... ), for details see [2].

2. Deviations from the asymptotic form

The first non-trivial deviation from the asymptotic form of the $2\pi$DA occurs in $P$ and $F$ waves [2]. Generically their effect on $M_{\pi\pi}$ dependence of the hard amplitude can be written as:

$$\mathcal{A}(M_{\pi\pi}) \propto \beta(M_{\pi\pi}) e^{i\delta_1} |F_\pi(M_{\pi\pi})| \left(1 + D_1(M_{\pi\pi})\right) P_1(\cos \theta)$$

$$+ \ \beta(M_{\pi\pi})^3 e^{i\delta_3} D_2(M_{\pi\pi}) P_3(\cos \theta) ,$$

(7)

where $P_l(\cos \theta)$ are Legendre polynomials and $\delta_1(M_{\pi\pi})$ and $\delta_3(M_{\pi\pi})$ are the $P$-wave and the $F$-wave $\pi\pi$ scattering phase shifts, which are well known from low-energy experiments.
The functions $D_{1,2}(M_{\pi\pi})$ describe the deviation of the amplitude’s $M_{\pi\pi}$ dependence from the asymptotic form. These functions are real and can be parametrised as:

$$D_1(M_{\pi\pi}, Q^2) = A_1(Q^2) e^{b_1 M_{\pi\pi}^2} - \frac{6m_{\pi}^2}{M_{\pi\pi}^2} A_2(Q^2) e^{b_2 M_{\pi\pi}^2}$$

$$D_2(M_{\pi\pi}, Q^2) = A_2(Q^2) e^{b_3 M_{\pi\pi}^2}.$$

The dependence of $A_{1,2}(Q^2)$ on $Q^2$ is governed by the QCD evolution and in leading order is given by:

$$A_{1,2}(Q^2) = A_{1,2}(\mu_0) \left( \frac{\alpha_s(Q^2)}{\alpha_s(\mu_0)} \right)^{50/(99-6n_f)}.$$

(8)

With increasing $Q^2$, the parameters $A_{1,2}(Q^2)$ go logarithmically to zero and one reproduces the asymptotic formula (4). The parameters $A_{1,2}(Q^2)$ are directly related to partonic structure of $\pi$ and $\rho$ mesons, see section 4 and ref [2].

The parameter $b_3$ is $Q^2$-independent but is $M_{\pi\pi}$ dependent. The latter dependence is fixed by $\pi\pi$ scattering phase shifts (see for derivation [2]):

$$b_3(M_{\pi\pi}) = \bar{b}_3 + \text{Re} \frac{M_{\pi\pi}^2}{\pi} \int_{4m_{\pi}^2}^{\infty} ds \frac{\delta_3(s)}{s^2 (s - M_{\pi\pi}^2 - i0)}.$$

(9)

In above equations $\bar{b}_i$ are subtraction constants of corresponding dispersion relations for functions $D_{1,2}(M_{\pi\pi})$, see details in [2].

Using expression (7) we can derive the form of the $M_{\pi\pi}$ distribution:

$$\frac{dN(M_{\pi\pi})}{dM_{\pi\pi}} = N \left[ \beta(M_{\pi\pi})^3 M_{\pi\pi} |F_{\pi}(M_{\pi\pi})|^2 \left(1 + D_1(M_{\pi\pi})\right)^2 + \frac{3}{7} M_{\pi\pi}^5 \beta(M_{\pi\pi})^7 D_2(M_{\pi\pi})^2 + \text{higher waves } l \geq 5 \right],$$

(10)

where the higher partial waves can be safely neglected.

3. Angular distributions of produced pions

Another way to obtain sensitivity to the non-asymptotic parameters $A_{1,2}(Q^2)$ is to study the two pion angular distributions. From the expression of the amplitude (7) one can derive the $M_{\pi\pi}$ dependence of intensity densities $\langle Y^m_l(\theta, \phi) \rangle$ defined as:

$$\frac{d}{dM_{\pi\pi}} \langle Y^m_l \rangle = M_{\pi\pi} \beta(M_{\pi\pi}) \int_{-1}^{1} d(cos \theta) \int_{0}^{2\pi} d\varphi |Y^m_l(\theta, \phi)|^2 |A(M_{\pi\pi}, \theta, \phi)|^2,$$

(11)

where $Y^m_l(\theta, \varphi)$ are spherical harmonics, $\varphi$ is the azimuthal angle between the pion decay plane and plane formed by momentum of the virtual photon and total momentum of produced $\pi^+\pi^-$ system.
For the first nontrivial intensity density $\langle Y_2^0 \rangle$ we have:

\[
\frac{d}{dM_{\pi\pi}} \langle Y_2^0 \rangle \propto M_{\pi\pi} \left[ \beta(M_{\pi\pi})^3 |F_\pi(M_{\pi\pi})|^2 \left( 1 + D_1(M_{\pi\pi}) \right)^2 \right. \\
+ \frac{9}{7} \beta(M_{\pi\pi})^5 |F_\pi(M_{\pi\pi})| \left( 1 + D_1(M_{\pi\pi}) \right) D_2(M_{\pi\pi}) \cos \left[ \delta_1(M_{\pi\pi}) - \delta_3(M_{\pi\pi}) \right] \\
\left. + \frac{2}{7} \beta(M_{\pi\pi})^7 D_2(M_{\pi\pi})^2 \right].
\]

The combination $\langle Y_0^0 - \sqrt{5}/2 Y_2^0 \rangle$ is especially sensitive to the deviations from asymptotic form because it is exactly zero asymptotically:

\[
\frac{d}{dM_{\pi\pi}} \langle Y_0^0 - \sqrt{5}/2 Y_2^0 \rangle \propto M_{\pi\pi} \left[ \beta(M_{\pi\pi})^5 |F_\pi(M_{\pi\pi})| \left( 1 + D_1(M_{\pi\pi}) \right) \right. \\
\times \quad D_2(M_{\pi\pi}) \cos \left[ \delta_1(M_{\pi\pi}) - \delta_3(M_{\pi\pi}) \right] \\
\left. - \frac{1}{9} \beta(M_{\pi\pi})^7 D_2(M_{\pi\pi})^2 \right].
\]  

Let us note, however that the expressions for intensity densities obtained above are based on leading twist expression for the amplitude (7), in particular, the contribution of transversely polarised photon is neglected (it contributes at higher twist level). Since in the combination of intensity densities (12) the leading twist contribution tends to cancel, the effect of higher twists (for which the cancellation is not expected) can be numerically large.

The study of angular distribution allows to make separation of productions by longitudinally (leading twist) and transversely (higher twist) polarised photons. Indeed, we can form combinations of intensity densities which get contributions only from $\gamma_L^n$, so we minimise the contributions of higher twists. One of such combinations is, for example,

\[
\frac{d}{dM_{\pi\pi}} \langle Y_0^0 + \sqrt{5}/2 Y_2^0 \rangle \propto M_{\pi\pi} \left[ \beta(M_{\pi\pi})^3 |F_\pi(M_{\pi\pi})|^2 \left( 1 + D_1(M_{\pi\pi}) \right)^2 \right. \\
+ \frac{6}{7} \beta(M_{\pi\pi})^5 |F_\pi(M_{\pi\pi})| \left( 1 + D_1(M_{\pi\pi}) \right) \right. \\
\times \quad D_2(M_{\pi\pi}) \cos \left[ \delta_1(M_{\pi\pi}) - \delta_3(M_{\pi\pi}) \right] \\
\left. + \frac{1}{3} \beta(M_{\pi\pi})^7 D_2(M_{\pi\pi})^2 \right].
\]

The details of the derivation and the analysis of the data will be presented elsewhere.

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3 Under assumption of $s$--channel helicity conservation (SCHC), which holds with good accuracy experimentally.

4 Strictly speaking this combination gets also contribution from transverse polarisation of the photon, but it occurs only in $F$-wave and therefore is expected to be small.
The advantage of the formalism presented in this section is that it maximally uses the information on pion e.m. form factor $F_\pi(M_{\pi\pi})$ and pion phase shifts $\delta_l(M_{\pi\pi})$ at low $M_{\pi\pi}$ which are very well known from low energy experiments. For example, from the known phase shifts $\delta_1(M_{\pi\pi})$ and $\delta_3(M_{\pi\pi})$ we conclude that the term proportional to $\cos[\delta_1(M_{\pi\pi}) - \delta_3(M_{\pi\pi})]$ changes the sign around $M_{\pi\pi} = 0.8$ GeV, so to increase the sensitivity to this term it would be interesting to consider a “$M_{\pi\pi}$ asymmetry” for various observables:

$$\left( \int_{2m_\pi}^{0.8} dM_{\pi\pi} - \int_{0.8}^{M_{\pi\pi}^{\text{max}}} dM_{\pi\pi} \right) \frac{d}{dM_{\pi\pi}} \text{(an observable)}.$$(14)

4. Expected values for the parameters

By crossing relations the parameter $A_2$ is related to the third moment of the valence quark distribution in the pion [2]:

$$A_2(Q^2) = \frac{7}{6} M_3(Q^2) = \frac{7}{6} \int_0^1 dx \, x^2 \, (u_{\pi^+} - \bar{u}_{\pi^+}) .$$ (15)

If the parametrisation suggested in [12] is used for the quark distributions in the pion, we obtain the values of the parameter $A_2(Q^2)$ listed in Table 1.

| $Q^2$(GeV$^2$) | $A_2$ |
|----------------|-------|
| 2              | 0.110 |
| 4              | 0.099 |
| 10             | 0.089 |
| 15             | 0.085 |

Table 1: Values of the parameter $A_2$ as function of $Q^2$, obtained using eq. (15) and quark distribution in the pion suggested in [13].

The value of $A_1$ is constrained by the soft pion theorem [3]:

$$A_1(Q^2) = a_2^{(\pi)}(Q^2) - A_2(Q^2),$$

where $a_2^{(\pi)}$ is the second Gegenbauer coefficient of the pion distribution amplitude. Unfortunately this parameter is not very well measured. In ref. [13] the value of $a_2^{(\pi)} = 0.19 \pm 0.13$ at $Q^2 = 2.4$ GeV$^2$ is quoted.

Additionally the parameters $A_1(Q^2)$ and $\bar{b}_1$ can be related to the second Gegenbauer coefficient $a_2^{(\rho)}(Q^2)$ of the $\rho$ meson distribution amplitude as [3]:

$$a_2^{(\rho)}(Q^2) = A_1(Q^2)e^{\bar{b}_1 M_{\pi\pi}^2} .$$ (16)

Up to now there is no direct experimental information about $a_2^{(\rho)}(Q^2)$. 

5
The $M_{\pi\pi}$ dependence of the parameter $b_3$ is estimated to be weak in the range of $0.5 < M_{\pi\pi} < 1.1$ GeV and is then replaced by a constant $\bar{b}_3$. The parameters $\bar{b}_i$ were estimated in the instanton model of QCD vacuum [2] to be around the following values:

$$
\begin{align*}
\bar{b}_1 &= -0.75 \text{ GeV}^{-2} \\
\bar{b}_2 &= -0.75 \text{ GeV}^{-2} \\
\bar{b}_3 &= 0.75 \text{ GeV}^{-2}.
\end{align*}
$$

5. Results of fits to the HERA data

Recently data on di-pion mass distribution in hard exclusive reaction was measured at HERA by the reaction

$$
e + p \rightarrow e + p + \pi^+ + \pi^-.
$$

This section presents attempts to fit HERA data by the leading twist parametrisations described in the sections above.

In order to study the $Q^2$ evolution of the di-pion mass distribution three sets of data, the ZEUS photoproduction data ($Q^2 \approx 0$) [14], the ZEUS low $Q^2$ data ($0.25 < Q^2 < 0.85$ GeV$^2$) [15] and the H1 high $Q^2$ data ($2.5 < Q^2 < 60$ GeV$^2$) [16] are used. The mean $W$ value for these samples is around 70 GeV, where $W$ is the energy in the photon-proton cms ($W^2 \approx Q^2/x_{Bj} - Q^2$). For the three samples, the di-pion mass distribution was studied in the ranges $0.55 < M_{\pi\pi} < 1.2$ GeV, $0.55 < M_{\pi\pi} < 1.1$ GeV and $0.50 < M_{\pi\pi} < 1.1$ GeV, respectively. The main background to eq. (18) consists of events in which the proton diffractively dissociates into hadrons, and is estimated to be around 20 % for the ZEUS samples and around 10 % for the H1 sample. This background does not distort the di-pion mass distribution discussed in previous sections if the mass of recoiled baryonic system is much smaller than the hard scale $Q^2$. The contamination from the production of $\omega$ (decaying into $\pi^+\pi^-\pi^0$) and $\phi$ (decaying into $K^+K^-$ or $\pi^+\pi^-\pi^0$) mesons was estimated to be few percents for the ZEUS samples and 7 % for the H1 sample. This background is mainly situated at low mass $M_{\pi\pi} < 0.6$ GeV.

Fig. 1 presents the di-pion mass distribution (black points) for six $Q^2$ values: the first distribution corresponds to the ZEUS photoproduction sample, the two following ones to the ZEUS low $Q^2$ sample and the three last ones to the H1 sample. For the H1 data, the distributions are corrected bin per bin for the production of $\omega$ and $\phi$ mesons.

Firstly, we attempt to fit the HERA data with the simplest formula of the asymptotic form (6) where the di-pion mass distribution is directly proportional to the square of the pion e.m. form factor. This latter was recently precisely measured in low energy experiments [11] (see also [10]), and we use directly these experimental data for $|F_{\pi}(M_{\pi\pi})|$. The result of the fits is superposed to the data in Fig. 1 (black curves). The asymptotic

5 Note that the di-pion mass distributions measured at HERA are not corrected for the transverse photon production. Nevertheless for $Q^2$ greater than 2 GeV$^2$ the longitudinal cross section dominates the transverse cross section.

6 At leading twist the energy dependence factories out. The shape of the di-pion mass distribution is then energy independent in the present formalism.
form does not reproduce the distribution in photoproduction and at low $Q^2$ as expected, but describes well the data at high $Q^2$, the $\chi^2/ndf$ of the fits for the six $Q^2$ bins of Fig. 1 being respectively 1625/25, 169/10, 69/10, 30.5/23, 16.6/21 and 11.6/18. We see that di-pion mass distributions in soft (low $Q^2$) and hard (large $Q^2$) regimes are essentially different and are described by different physics. Therefore the parametrisations designed for soft processes [17, 18, 19] are not relevant for hard processes, and the values of the parameters extracted from large $Q^2$ data are not related directly to physical observables.

Secondly, we study the deviations from the asymptotic form of the di-pion mass distribution measured at HERA. As the formalism is valid in the hard regime $Q^2 \gg \Lambda_{QCD}^2$, only the H1 samples ($Q^2 > 2.5$ GeV$^2$) are considered. The parametrisation (11) is fitted to the data, with three free parameters: the normalisation $N$, and the parameters $A_1$ and $A_2$. For the parameters $\bar{b}_1, \bar{b}_2,$ and $\bar{b}_3$ we use the values given by the calculation in the instanton model of QCD vacuum, see eq. (17). The result of the fits is presented in Fig. 2 as the black curves, and the values obtained for the parameters are listed in Table 2. The three fits have a $\chi^2/ndf$ value smaller than one.

| $Q^2$ = 3.1 GeV$^2$ | $Q^2$ = 7.2 GeV$^2$ | $Q^2$ = 21.2 GeV$^2$ |
|---------------------|---------------------|---------------------|
| $N = 1.59 \pm 0.21$ | $N = 1.11 \pm 0.55$ | $N = 0.66 \pm 0.30$ |
| $A_1 = 1.10 \pm 0.18$ | $A_1 = 0.26 \pm 0.49$ | $A_1 = -0.16 \pm 0.34$ |
| $A_2 = -0.34 \pm 0.50$ | $A_2 = 0.22 \pm 0.45$ | $A_2 = 0.18 \pm 0.41$ |
| $\chi^2/ndf = 20.6/21$ | $\chi^2/ndf = 16.5/19$ | $\chi^2/ndf = 11.3/16$ |

Table 2: Values of the normalisation $N$, the parameters $A_1$ and $A_2$, and the $\chi^2/ndf$ obtained from the fit of eq. (11) to the H1 data [10], using the values given in eq. (17) for the parameters $\bar{b}_1, \bar{b}_2,$ and $\bar{b}_3$.

We observe an indication for a decrease of the parameter $A_1$ when $Q^2$ increases, what is expected from QCD evolution, see eq. (8). In general the values of parameters $A_1$ and $A_2$ are in agreement with theoretical expectations (see section 4). However the errors of the parameters $A_1$ and $A_2$ are large, due to limited statistic available. Nevertheless the sensitivity of the data to the parameter $A_1$ is encouraging. More precise data from the 1997-99 years data taking at HERA would bring considerably more accurate information on these parameters which are related directly to the valence quark distribution in the pion, and pion and $\rho$ meson distribution amplitudes.

6. Discussion of the results and conclusions

The analysis presented here is based on the leading order, twist-2 formalism. The power corrections (higher twist contributions) to the amplitude of the reaction (1) are systematically neglected. The size of higher twist corrections might be rather large [20, 21], so that

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The asymptotic form (11) is expected to work only in the hard regime $Q^2 \gg \Lambda_{QCD}^2$, also let us stress once more that the form of di-pion mass distribution of eq. (10) also makes sense only in the hard regime.
they constitute the main theoretical systematic uncertainty in our analysis. On general grounds the higher twist effects should be smaller for such observables as the shape of the di-pion mass distribution considered here, also the study of angular distribution of produced pions as discussed in section 3 can be used to minimise the contributions of higher twists. Another source of theoretical errors is that we fixed values of parameters \( \bar{b}_i \) by the model values (17). Hopefully more precise data will allow us to measure these parameters.

To be on more safe side in respect to higher twist corrections we use the highest values of \( \langle Q^2 \rangle = 21.2 \) GeV\(^2\) available in the data sample to evaluate observables related to partonic structure of the pion and \( \rho \) meson. The results for the third moment of valence quark distribution in the pion \( M_3 \) (see eq. (13)), the second Gegenbauer coefficients of the pion and \( \rho \) meson distribution amplitudes (\( a_2^{(\pi)} \) and \( a_2^{(\rho)} \) respectively) at \( \langle Q^2 \rangle = 21.2 \) GeV\(^2\) are presented in Table 3.

| Quantity | Our analysis | Other sources |
|----------|--------------|---------------|
| \( M_3 \) | 0.15 ± 0.35 | 0.07 \[12\], 0.085 ± 0.005 \[22\] |
| \( a_2^{(\pi)} \) | 0.02 ± 0.50 | 0.14 ± 0.09 \[13\] |
| \( a_2^{(\rho)} \) | -0.10 ± 0.20 | — |

Table 3: Values of the third moment of valence quark distribution in the pion \( M_3 \), and the second Gegenbauer coefficient of the pion and \( \rho \) meson distribution amplitudes (\( a_2^{(\pi)} \), \( a_2^{(\rho)} \)) at \( \langle Q^2 \rangle = 21.2 \) GeV\(^2\) obtained in our analysis, and in \[12, 13, 22\].

Although, due to the limited statistic, the error bars for physical observables are large we see that the obtained values are in agreement with other experiments. This agreement is especially interesting because previously the restrictions for \( a_2^{\pi} \) and \( M_3 \) were obtained from completely different measurements: the value of second Gegenbauer coefficient \( a_2^{\pi} \) was obtained in ref. \[13\] from analysis of data on \( \gamma \pi \) transition form factor at large \( Q^2 \), whereas the third moment \( M_3 \) is restricted by \( \pi N \) Drell-Yan data \[12, 22\]. We see that the formalism to extract the partonic structure of the pion and \( \rho \) mesons suggested here is complementary to already known methods.

Our analysis allowed us to obtain for the first time an experimental estimate of the second Gegenbauer coefficient of the \( \rho \) meson distribution amplitude \( a_2^{(\rho)} = -0.1 \pm 0.2 \) at \( \langle Q^2 \rangle = 21.2 \) GeV\(^2\). Unfortunately the precision of determination of \( a_2^{(\rho)} \) is still low to discriminate between different model predictions for this quantity (QCD sum rule: 0.12 ± 0.06 \[25\], 0.05 ± 0.01 \[24\] and instanton model: 

\[ -0.07 \] \[4, 27\]). With increasing of statistics the accuracy of determination of the parameters related to the partonic structure of the mesons can be considerably improved. Also the studies of angular distributions

\[ ^8 \] Lattice calculations of \( M_3 \) are in agreement with results of analysis of \[12, 22\]: 0.09 ± 0.03 \[24\] and 0.10 ± 0.02 \[26\] at \( \langle Q^2 \rangle = 21.2 \)
of produced pions as discussed in section 3 can considerably increase the sensitivity to parameters of partonic structure of pions and their resonances.

As a final remark we note that the formalism developed here can be easily generalised to the case of production of other mesons in hard exclusive reactions (e.g. $K\bar{K}$, $3\pi$, $4\pi$, $KK\pi\pi$, etc.). The study of meson mass and angular distributions of produced meson can provide us with rich information on partonic structure of these mesons and their resonances. Interesting prediction can be made [7, 8, 9] for the asymptotic shape of meson cluster mass distribution for the reaction:

$$\gamma_L^* + p \rightarrow (\text{hadrons}) + N', \quad (19)$$

where the mass of the hadron cluster $M_X^2 \ll Q^2$. In this case at asymptotically large $Q^2$ we have:

$$\frac{dN}{dM_X^2} \propto R(M_X), \quad \text{with} \quad (20)$$

$$R(\sqrt{s}) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}.$$

It would be interesting to check experimentally this asymptotic formula and try to detect deviations from it. Generically, this deviation is described by the vacuum correlator of two light-ray operators. We showed here that the asymptotic expression (20) works pretty well at low $M_X$ (for $Q^2 > 7$ GeV$^2$) where the $2\pi$ channel dominates over all other hadronic channels, it would be interesting to check how good this formula works at higher $M_X$. Also it would be interesting to see whether the formula like (20) is applied also for multimeson partial cross section.

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Figure 1: Dipion mass distribution for six $Q^2$ values. The black points correspond to measurements from the ZEUS [14, 15] and H1 [16] experiments. The curves show the result of the fit of the asymptotic form (6).
Figure 2: Di-pion mass distribution for three $Q^2$ values. The black points correspond to measurements from the H1 [10] experiment and the curves show the result of the fit of eq. (10).