On the origin of families of quarks and leptons—predictions for four families

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Abstract. The approach unifying all the internal degrees of freedom—proposed by one of us—offers a new way of understanding families of quarks and leptons: a part of the starting Lagrange density in $d (= 1 + 13)$, which includes two kinds of spin connection fields—the gauge fields of two types of Clifford algebra objects—transforms the right-handed quarks and leptons into left-handed ones manifesting in $d = 1 + 3$ the Yukawa couplings of the Standard Model. We study the influence of the way of breaking symmetries on the Yukawa couplings and estimate properties of the fourth family—the quark masses and the mixing matrix, investigating the possibility that the fourth family of quarks and leptons appears at low enough energies to be observable with the new generation of accelerators.
1. Introduction

The Standard Model of the electroweak and strong interactions (extended by assuming nonzero masses of the neutrinos) fits with around 25 parameters, and constrains all the existing experimental data. It leaves unanswered, however, many open questions, among which are questions about the origin of families, the Yukawa couplings of quarks and leptons and the corresponding Higgs mechanism. Understanding the mechanism for generating families, their masses and mixing matrices might be one of the most promising ways to physics beyond the Standard Model.

The approach unifying spins and charges [1]–[10] might—by offering a new way of describing families, that is by assuming two kinds of Clifford operators, the known Dirac one and another one, anticommuting with the Dirac one1—give a possible explanation about the origin of the Yukawa couplings.

It was demonstrated in [5], [7]–[10] that a left-handed $SO(1, 13)$ Weyl spinor multiplet includes, if the representation is analyzed in terms of the subgroups $SO(1, 3), SU(2), SU(3)$ and the sum of the two $U(1)$s, all the spinors of the Standard Model—that is the left-handed $SU(2)$ doublets and the right-handed $SU(2)$ singlets of quarks and leptons. Correspondingly does a simple starting Lagrange density for spinors, which carries in $d = (1 + 13)$-dimensional space nothing but two kinds of spins (no charges) and interacts with vielbeins and two kinds of spin connections, manifest in $d = (1 + 3)$-dimensional space—but after appropriate breaks of

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1 The second kind of Clifford algebra object, anticommuting with the Dirac one, was derived by Mankoč Borštnik [1, 2] through the Grassmann algebra and was later rewritten [6, 11, 12] in terms of the Dirac-like operators, which can be interpreted as the right multiplication of the Clifford algebra objects, if the Dirac ones are understood as the left multiplication. It is easy to understand what the left and the right multiplication mean, if one expresses the spinor representations as products of binomials of the Clifford algebra objects as proposed in [12].
the starting symmetry—all the gauge fields assumed by the Standard Model, as well as the mass matrices leading to the observed masses of quarks and leptons and the measured mixing matrices.

There are a lot of open questions to be answered before the approach unifying spins and charges can be accepted as a theory to describe what we observe (if it passes all the tests at all). We are studying them step-by-step. Let us write down some of these questions and try to seek some of the possible answers.

(i) Is the use of the second kind of the Clifford algebra objects II, IIA and IIB the right way to describe families? The Dirac operators successfully describe the spin degree of freedom in $d = (1 + 3)$. This second kind of the Clifford algebra objects exists as a mathematical tool determining the equivalent representations with respect to the Dirac operators. Since we do observe families, which manifest as equivalent representations, it seems meaningful to describe families with these new Clifford algebra operators.

There are several attempts in the literature to explain the origin of families. In [13], for example, the authors investigate the possibility that the unification of the charges (described by $SO(10)$) and the family quantum number (described by $SO(8)$) within the group $SO(18)$ might be the right way to understand the replication of the family of quarks and leptons at low energies. In [14], the authors assume the Standard Model group to the third power to reproduce families. In [15], the authors assume the $SU(4)_L \times SU(2)_L \times SU(4)_R \times SU(2)_R$ groups to end up with three families. In the approach unifying spins and charges spinors (due to two types of $\gamma^a$ operators) carry two indices, one index takes care of the ordinary spin, the other of the family. There are the ordinary $S^{ab}$, which transform one state of a spinor representation into another, while $\tilde{S}^{ab}$ transform the family index. The Standard Model assumptions manifest that these two degrees of freedom are coupled at least through the Yukawa couplings. In the approach, these two degrees of freedom are coupled from the very beginning in a simple Lagrange density for spinors.

(ii) Has the universe more than $(1 + 3)$ dimensions at all (besides the internal degrees of freedom, manifested by the spin and the charges)? We have made several studies of properties of space with more than $(1 + 3)$ dimensions [16]–[20]. These and other not yet published studies makes one of the authors of this paper (NSMB) believe that such a possibility is very probable. (The Kaluza–Klein-like theories and the string theories also postulate more than the observed $(1 + 3)$ dimensions.)

(iii) Can one at all come to almost massless (in comparison with the scales where breaks should occur) spinors and gauge fields which we observe after breaking symmetries? In [21], Witten says that this is almost impossible. We proposed in [18, 20] the boundary condition for spinors in $d = (1 + 5)$ compactified on a finite disk that ensures masslessness of spinors in $d = (1 + 3)$, allowing at the same time charges of only one sign. We hope that such a toy model might be extended to the case when $d = (1 + 13)$ breaks into $d = (1 + 7)$. After each break massless spinors of only one handedness survive. All the rest have a mass of the scale of breaking the symmetry.

(iv) How do symmetries break? We do not yet know how this happens and what is responsible for each of the breaks. We are at this point assuming the way of breaking, which leads to particular nonzero vacuum expectation values of fields, in agreement with what we observe, trying to learn from these studies more about possible breaks. The assumed way of breaking then leads to particular properties of mass matrices, which limits the number.
of free parameters and correspondingly enables predictions. In principle, from the simple starting action all the properties of the quarks and the leptons are calculable, which is an extremely difficult problem. We remind the reader that even much simpler problems are not yet solved, like how can one from the simple starting action for molecules and atoms, where the interaction comes from the electrodynamics, end up with a crystal?

(v) How do all the starting families manifest after the symmetries break? All the spinors of all the families gain the mass of the scale of breaking after the symmetries break, except the ones protected by the boundary conditions [18] or by the charge (like it is in the Standard Model, where the left-handed spinors are assumed to carry different weak charges from the right-handed partners and accordingly stay massless unless the weak charge breaks). In our case, eight families acquire masses on two scales of breaking two symmetries, as explained in section 3. After the first break, the last four families stay massless because of the charge-protection mechanism (section 3.2): the left-handed partners, like in the Standard Model, carry namely a charge different from the right-handed ones. The eight times eight mass matrices break into two times four matrices, with the first four families decoupled from the last four families. The last four families are what we have described in this paper. They acquire mass after the weak charge breaks. The last three families are candidates for the observed ones. The fourth family is yet to be discovered, if it does not decouple from the lower three. If it does decouple from the three lightest families, the fourth family is a candidate for forming the dark matter, and so also is the fifth family, which is in our way of breaking symmetries decoupled from the last four families.

(vi) Why do we not see the new gauge fields connected with the second kind of the spin? We do not yet know the answer to this question. In the calculations, we assume that they are too heavy to be observed. This question is under consideration.

(vii) Do the weak gauge fields obtain in the approach unifying spins and charges the measured masses? We do not present in this paper properties of the weak gauge fields after the symmetries break. It is not difficult to see from the starting Lagrange density, and taking into account also [18], which part of spin connection fields contribute after the assumed breaks to the mass of the weak fields. To evaluate how the nonzero gauge fields, appearing in the Yukawa couplings, determine the mass of the weak fields is not so simple. To understand this and the dynamics of these fields, which behave as scalar fields, a demanding study is needed. The project in hand is focussed on this issue. We would like to remind the reader that once the effective Lagrange density for spinors in $d = (1 + 3)$ is derived, the properties of the gauge fields are determined, which is what one can learn if one tries to understand the assumptions of the Standard Model [22].

The paper [10] analyzes, how terms, which lead to masses of quarks and leptons, appear in the approach unifying spins and charges as a part of the spin connection and vielbein fields. No Higgs is needed in this approach to ‘dress’ the right-handed spinors with the weak charge, since the terms of the starting Lagrangean, which include $\gamma^0\gamma^s$, with $s = 7, 8$, do the job of a Higgs field.

In this paper, we study properties of mass matrices (Yukawa couplings) following from the approach unifying spins and charges after assuming some (two) possible ways of breaking the starting symmetry. To calculate from the starting Lagrange with only one (or at the most two) parameter(s), all the properties of the observed quarks and leptons is, as we said above, a too ambitious project at this moment (even in the limit when gravity can be treated—as it can be in

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this particular case, since breaks are supposed to occur far below the Planck scale—as ordinary
gauge fields), because of all possible perturbative and nonperturbative effects. Instead, we study
the influence of particular breaks of symmetries on properties of mass matrices. We calculate
on a tree level and leave the fields determining mass matrices after the assumed breaks as free
parameters to be determined by the experimental data. In this way, we try to understand how
might the right way of breaking symmetries go, if our approach is the right way beyond the
Standard Model, and whether the fourth family of quarks and leptons might appear at energies
observable with new accelerators. We present masses and mixing matrices for the four families
of quarks. The assumed breaks of symmetries relate mass matrix elements quite strongly.

The results are in agreement with the experimental data [23].

Although we are still far from being able to calculate from the simple starting action in
\( d = 1 + 13 \), the properties of the families of quarks and leptons as manifested at measurable
energies directly—each break causes perturbative and nonperturbative effects, which are by
themselves hard problems (not yet solved even in hadron physics)—the approach manifests
several appreciable features:

a. In one Weyl representation in \( d = 1 + 13 \) all the quarks and the leptons of one family
appear, but only the left-handed quarks and leptons carry the weak charge, whereas the
right-handed ones are weak chargeless.

b. The starting Lagrange density offers the mechanism for generating families by including
two kinds of the Clifford algebra objects.

c. It is a part of the starting Lagrange density in \( d = 1 + 13 \), which transforms the right-handed
weak chargeless spinors into the left-handed weak charged spinors manifesting the Yukawa
couplings of the Standard Model.

2. Action for chargeless Weyl spinors in \( d = (1 + 13) \) and appearance of families of
quarks and leptons

This section repeats briefly the approach unifying spins and charges as presented in [10]. We
assume that only a left-handed Weyl spinor in \((1 + 13)\)-dimensional space exists. A spinor
carries only spin (no charge) and interacts accordingly with only the gauge gravitational fields—
with the spin connections and the vielbeins. We assume two kinds of the Clifford algebra
objects and allow accordingly two kinds of gauge fields [1]–[10]. One kind is the ordinary
gauge field (gauging the Poincaré symmetry in \( d = 1 + 13 \)). The corresponding spin connection
fields appear for spinors as gauge fields of \( S^{ab} \) (equation (2)) defined in terms of \( \gamma^a \), which are
the ordinary Dirac operators

\[
S^{ab} = \frac{1}{2} (\gamma^a \gamma^b - \gamma^b \gamma^a),
\]

\[
\{\gamma^a, \gamma^b\}, = 2\eta^{ab}.
\]

These gauge fields manifest at ‘physical energies’ as all the gauge fields of the Standard Model,
and they also contribute—by connecting the right-handed weak chargeless quarks or leptons to
the left-handed weak charged partners within one family of spinors—to the diagonal terms of
mass matrices.

The second kind of gauge fields in our approach is responsible for the appearance of
families of spinors and accordingly also for couplings among families, contributing to diagonal
matrix elements as well. This might explain, together with the first kind of the spin connection fields, the Yukawa couplings of the Standard Model of the electroweak and color interactions. The corresponding spin connection fields appear for spinors as gauge fields of $\tilde{S}^{ab}$

$$\tilde{S}^{ab} = \frac{1}{2} (\tilde{\gamma}^a \tilde{\gamma}^b - \gamma^b \tilde{\gamma}^a),$$

(2)

$$\{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+ = 2\eta^{ab}, \quad \{\tilde{\gamma}^a, \gamma^b\}_+ = 0, \quad \{S^{ab}, \tilde{S}^{cd}\}_- = 0$$

with $\tilde{\gamma}^a$ as the second kind of the Clifford algebra objects $[2, 11]$.

Following $[10]$, we write the action for a Weyl (massless) spinor in $d(=1+13)$-dimensional space as follows:

$$S = \int d^d x \mathcal{L},$$

$$\mathcal{L} = \frac{1}{2} (E \tilde{\gamma}^a p_{0a} \psi + \text{h.c.}) = \frac{1}{2} (E \tilde{\gamma}^a f^a_{\alpha} p_{0a} \psi) + \text{h.c.}$$

(3)

Here, $f^a_{\alpha}$ are vielbeins (inverted to the gauge field of the generators of translations $e^a_{\alpha} e^b_{\beta} = \delta^a_{\beta}$ and $e^a_{\alpha} f^b_{\beta} = \delta^b_{\alpha}$), with $E = \det(e^a_{\alpha})$, while $\omega_{aba}$ and $\tilde{\omega}_{aba}$ are the two kinds of the spin connection fields, the gauge fields of $S^{ab}$ and $\tilde{S}^{ab}$, respectively. (We kindly ask the reader to read about the properties of these two kinds of the Clifford algebra objects---$\gamma^a$ and $\tilde{\gamma}^a$ and of the corresponding $S^{ab}$ and $\tilde{S}^{ab}$---and about the technique used here in $[10]$ or $[11, 12]$.)

One Weyl spinor representation in $d = (1 + 13)$ with the spin as the only internal degree of freedom manifests, if analyzed in terms of the subgroups $SO(1, 3) \times U(1) \times SU(2) \times SU(3)$ in four-dimensional ‘physical’ space as the ordinary ($SO(1, 3)$) spinor, with all the known charges of one family of the left-handed weak charged and the right-handed weak chargeless quarks and leptons of the Standard Model. To manifest this, we make a choice of $\tau^{Ai} = \sum_{s,t} c^{Ai}_{st} S^{st}$, with $c^{Ai}_{st}$ chosen in such a way that $\tau^{Ai}$ fulfills the commutation relations of the $SU(3)$, $SU(2)$ and $U(1)$ groups: $\{\tau^{Ai}, \tau^{Bj}\}_- = i\delta^{AB} f^{Ai jk} \tau^{Ak}$, with the structure constants $f^{Ai jk}$ of the corresponding groups, where the index $A$ identifies the charge groups ($A = 1$ identifies $SU(2)$ and the weak charge, $A = 2$ denotes one of the two $U(1)$ groups—the one following from $SO(1, 7) = A = 3$ denotes the group $SU(3)$ and the color charge and $A = 4$ denotes the group $U(1)$ following from $SO(6)$) and index $i$ identifies the generators within one charge group. We have:

$$\tau^{11} := \frac{1}{2} (S^{38} + S^{67}), \quad \tau^{12} := \frac{1}{2} (S^{57} + S^{68}), \quad \tau^{13} := \frac{1}{2} (S^{56} - S^{78}),$$

$$\tau^{21} := \frac{1}{2} (S^{56} + S^{78}), \quad \tau^{31} := \frac{1}{2} (S^{912} - S^{1011}), \quad \tau^{32} := \frac{1}{2} (S^{911} + S^{1012}), \quad \tau^{33} := \frac{1}{2} (S^{910} - S^{1112}),$$

$$\tau^{34} := \frac{1}{2} (S^{912} - S^{1013}), \quad \tau^{35} := \frac{1}{2} (S^{913} + S^{1014}), \quad \tau^{36} := \frac{1}{2} (S^{1114} + S^{1213}), \quad \tau^{37} := \frac{1}{2} (S^{1113} + S^{1212}),$$

$$\tau^{38} := \frac{1}{2\sqrt{3}} (S^{910} + S^{1112} - 2S^{1214}), \quad \tau^{41} := -\frac{1}{2} (S^{910} + S^{1112} + S^{1314}), \quad \text{and} \quad Y = \tau^{41} + \tau^{21}$$

(4)

The reader can find this analysis in $[10]$. We proceed in the following way. We make a choice of the Cartan subalgebra set with $d/2 = 7$ elements in $d = 1 + 13$:

$$S^{03}, \quad S^{12}, \quad S^{56}, \quad S^{78}, \quad S^{910}, \quad S^{1112}, \quad S^{1314}.$$
Cartan subalgebra set:

In particular, the vector

We chose the starting vector to be an eigenvector of all the members of the Cartan set.

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8 u

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it represents a right-handed weak chargeless u-quark with spin up and with the color

respectively, all of which are eigenvectors of

projectors [10]

Then we express the basis for one Weyl in \(d = 1 + 3\) as products of nilpotents and projectors [10]

\[
ab_{(k)} := \frac{1}{2} \left( \gamma^a + \eta^{aa}_{ik} \gamma^b \right), \quad ab_{[k]} := \frac{1}{2} \left( 1 + i \gamma^a \gamma^b \right),
\]

(5)

respectively, all of which are eigenvectors of \(S^{ab}\)

\[
S^{ab}_{(k)} := \frac{k}{2} \frac{ab_{(k)}}{ab_{[k]}}, \quad S^{ab}_{[k]} := \frac{k}{2} \frac{ab_{[k]}}{ab_{(k)}}.
\]

(6)

We chose the starting vector to be an eigenvector of all the members of the Cartan set.

In particular, the vector \((+)(+)(+)[−][+] (−)\) has the following eigenvalues of the Cartan subalgebra set: \(\frac{i}{2}, \frac{i}{2}, \frac{i}{2}, \frac{i}{2}, -\frac{i}{2}, \frac{i}{2}, -\frac{i}{2}\), respectively. With respect to the charge groups, it represents a right-handed weak chargeless u-quark with spin up and with the color \((-1/2, 1/(2\sqrt{3})\) = \(τ^{33}, τ^{38}\). (How the ordinary group theoretical way of analyzing spinors is done can be found in many text books, also in [13, 32].)

Accordingly, we may write one octet of the left-handed and the right-handed quarks of both spins and of one color charge as presented in Table 1. All the members of the octet of the table 1 can be obtained from the first state by the application of \(S^{ab}; (a, b) = (0, 1, 2, 3, 4, 5, 6, 7, 8)\). The operators of handedness are defined

\[
\begin{array}{cccccccccc}
\hline
i & |^a ψ_i) & \Gamma^{(1,3)} & S^{12} & \Gamma^{(4)} & τ^{13} & τ^{21} & τ^{33} & τ^{38} & τ^{41} & Y & Y' \\
\hline
1 & \psi_1^c & 0 & 3 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 & 1 & 2/3 & 1/6 & 0 & -1 & 1/2 & -1 & 1/2 & -1/2 & 1/(2\sqrt{3}) & 1 & 6 & 2/3 & -1/3 \\
2 & \psi_2^c & 0 & 3 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 & -1 & 1/2 & 1/6 & 0 & -1 & 1/2 & -1 & 1/2 & -1/2 & 1/(2\sqrt{3}) & 1 & 6 & 2/3 & -1/3 \\
3 & \psi_3^c & 0 & 3 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 & -1 & -1/2 & 1/6 & 0 & -1 & 1/2 & -1 & 1/2 & -1/2 & 1/(2\sqrt{3}) & 1 & 6 & 2/3 & -1/3 \\
4 & \psi_4^c & 0 & 3 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 & -1 & -1/2 & 1/6 & 0 & -1 & 1/2 & -1 & 1/2 & -1/2 & 1/(2\sqrt{3}) & 1 & 6 & 2/3 & -1/3 \\
5 & \psi_5^c & 0 & 3 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 & -1 & -1/2 & 1/6 & 0 & -1 & 1/2 & -1 & 1/2 & -1/2 & 1/(2\sqrt{3}) & 1 & 6 & 2/3 & -1/3 \\
6 & \psi_6^c & 0 & 3 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 & -1 & -1/2 & 1/6 & 0 & -1 & 1/2 & -1 & 1/2 & -1/2 & 1/(2\sqrt{3}) & 1 & 6 & 2/3 & -1/3 \\
7 & \psi_7^c & 0 & 3 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 & -1 & -1/2 & 1/6 & 0 & -1 & 1/2 & -1 & 1/2 & -1/2 & 1/(2\sqrt{3}) & 1 & 6 & 2/3 & -1/3 \\
8 & \psi_8^c & 0 & 3 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 & -1 & -1/2 & 1/6 & 0 & -1 & 1/2 & -1 & 1/2 & -1/2 & 1/(2\sqrt{3}) & 1 & 6 & 2/3 & -1/3 \\
\hline
\end{array}
\]

Table 1. The 8-plet of quarks—the members of \(SO(1, 7)\) subgroup, belonging to one Weyl left-handed (\(\Gamma^{(1,1,3)} = -1 = \Gamma^{(1,7)} \times \Gamma^{(0)}\)) spinor representation of \(SO(1, 13)\). It contains the left-handed weak charged quarks and the right-handed weak chargeless quarks of a particular color (\(-1/2, 1/(2\sqrt{3})\)). Here, \(Γ^{(1,3)}\) defines the handedness in \((1 + 3)\) space, \(S^{12}\) defines the ordinary spin (which can also be read directly from the basic vector), \(τ^{13}\) defines the weak charge, \(τ^{21}\) defines the \(U(1)\) charge from \(SO(1, 7)\), \(τ^{33}\) and \(τ^{38}\) define the color charge and \(τ^{41}\) defines another \(U(1)\) charge, which together with the first one defines \(Y = τ^{41} + τ^{21}\) and \(Y' = τ^{41} − τ^{21}\). The vectors are eigenvectors of all the members of the Cartan subalgebra set (\(\{S^{03}, S^{12}, S^{56}, S^{78}, S^{9^{10}}, S^{11^{12}}, S^{13^{14}}\}\)). The reader can find the whole Weyl representation in [10].

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as follows: \( \Gamma^{(1, 13)} = i2^7 S^{03} S^{12} S^{56} \cdots S^{13 14} \), \( \Gamma^{(1, 3)} = -i2^2 S^{03} S^{12} \), \( \Gamma^{(1, 7)} = -i2^4 S^{03} S^{12} S^{56} S^{78} \), \( \Gamma^{(6)} = -2^3 S^{010} S^{1112} S^{1314} \) and \( \Gamma^{(4)} = 2^3 S^{56} S^{78} \).

Quarks of the other two color charges and the color chargeless leptons are distinguishable from this octet only in the part which determines the color charge and \( \tau^4 \) (\( \tau^4 = 1/6 \) for quarks and \( \tau^4 = -1/2 \) for leptons). (They can be obtained from the octet given in table 1 by the application of \( S_{ab} \); \( (a, b) = (9, 10, 11, 12, 13, 14) \) on these states. In particular, \( S^{913} \) transforms the right-handed \( \bar{u}_R^\tau \)-quark of the first column into the right-handed weak chargeless neutrino of the same spin \((++)(+) \mid (+)(++) \mid (+) [+] [+] \), while it has \( \tau^4 = -1/2 \), and accordingly \( Y = 0 \) and \( Y' = -1 \).) One notices that \( 2\tau^4 \) measures the baryon number of quarks, whereas \(-2\tau^4 \) measures the lepton number. Both are conserved quantities with respect to the group \( SO(1, 7) \).

We can formally rewrite the Lagrangean of equation (3), so that it manifests the usual Lagrange density for spinors in \( d = (1 + 3) \) of the Standard Model of the electroweak and color interactions before the Higgs field breaks the \( SU(2) \times U(1) \) symmetry, and that it manifests the Yukawa couplings as well

\[
\mathcal{L} = \bar{\psi} \gamma^m \left( p_m - \sum_{A, i} g^A \tau^4 A^4_{m i} \right) \psi + \sum_{s=7, 8} \bar{\psi} \gamma^s \psi_0 \psi + \text{the rest.} \tag{7}
\]

Here, \( p_{0s} = p_s - \frac{1}{2} \omega_{s t} \omega_{s t} - \frac{1}{2} \tilde{\omega}_{s t} \tilde{\omega}_{s t}; \) \( (s', t) \in (5, 6, \ldots) \), while \( A^4_{m i}, m = 0, 1, 2, 3 \), denote the gauge fields (expressible in terms of \( \omega_{s t m} \)) corresponding to the charges defined by the generators \( \tau^4 \). One easily sees from table 1 that the operator \( \gamma^0 \gamma^7 \) (or \( \gamma^0 \gamma^8 \) or any superposition of these two operators) transforms the right-handed weak chargeless \( \bar{u}_R^\tau \) quark of the first row into the left-handed weak charged \( \bar{u}_L^\tau \) quark of the same spin and the color charge presented in the seventh row—which is just what the Higgs field together with \( \gamma^0 \) does in the Standard Model. We assume that breaks of the starting symmetry \( SO(1, 13) \) (the Poincaré symmetry in \( d = 1 + 13 \) and the symmetry in the \( \tilde{S}_{ab} \) sector) lead first to \( SO(1, 7) \times SU(3) \times U(1) \), where all the spinors are massless, while further breaks lead to \( SO(1, 3) \times U(1) \times SU(3) \), manifesting the observed symmetries and the Yukawa couplings with the observed masses of quarks and leptons and the mixing matrices. If we find out how and at which scales the break of \( SO(1, 7) \) to \( SO(1, 3) \times SU(2) \times U(1) \) (possibly via \( SO(1, 3) \times SO(4) \)) occurs, the approach could accordingly offer the explanation of why we see spinors carrying beside the spin also the weak, the electromagnetic and the color charge, why each of the charges couple with a different coupling constant to the corresponding gauge fields, what are the ratios of these coupling constants, why until now have we been able to see only three families, all three of different masses and at which energy scale the next family occurs.

We are not yet able to answer all these questions. Assuming that in particular two ways of breaking symmetries could occur, we are, in this paper, trying to find out possible connections between breaks of symmetries and the symmetries of the corresponding Yukawa couplings, and to predict accordingly what are the properties of the fourth family in each of these two ways of breaking symmetries. The larger the symmetries of the Yukawa couplings after the assumed breaks of symmetries, the smaller the number of free parameters in the Yukawa couplings following from our approach and the more predictive our approach unifying spins and charges in this simple application of it.
The terms responsible for the Yukawa couplings in our approach can be rearranged to be written in terms of nilpotents \((\pm)\) as follows:

\[
y^\nu p_{\nu s} = (+) p_{0\nu s} + (-) p_{\nu s-},
\]

with \(s = 7, 8\) and \(p_{0\nu s} = p_{\nu s} \mp ip_{\nu s}\) and we can write

\[
\bar{S}^{ab} = \frac{1}{2} \{\bar{\varphi}(k) + (-k)\}(\bar{\varphi}(k))^{bc} - (k)(\bar{\varphi}(k))^{bc}
\]

for any \(c\). We can accordingly rewrite \(-\sum_{(a,b)} \frac{1}{2} \{\bar{\varphi}(k)\}^{ac} \bar{\omega}_{abc} = -\sum_{(a,b),k,l} \sum_{(ac),(bd),k,l} \{\bar{\varphi}(k)\}^{ac} \bar{\omega}_{abcd}\)

\(\bar{A}^{kl}_{\pm}(ac),(bd)\). The diagonal matrix elements are expressed as the operators \(\bar{A}^Y_{\pm}((ac),(bd))\), with the pairs \((a,b)\) in the first sum running over all the indices which do not characterize the Cartan subalgebra, with \(a, b = 0, \ldots, 8\), while the two pairs \((ac)\) and \((bd)\) in the second sum denote only the Cartan subalgebra pairs for \(SO(1, 7)\) we only have the pairs \((03),(12),(03),(56),(03),(78),(12),(56),(12),(78),(56),(78)\); \(k\) and \(l\) run over four possible values, so that \(k = \pm i\), if \((ac) = (03)\) and \(k = \pm 1\) in all other cases, while \(l = \pm 1\).

Having the spinor basis written in terms of projectors and nilpotents (table 1) and knowing the relations of equation (14), it turns out that it is convenient to rewrite the mass term \(\mathcal{L}_Y = \sum_{s=7,8} \bar{y}^\nu y^\nu p_{\nu s} \psi\) in equation (7) as follows:

\[
\mathcal{L}_Y = \bar{\psi} \gamma^0 \psi = \left\{ \begin{array}{c}
\mathcal{L}_Y = \bar{\psi} \gamma^0 \left\{ \sum_{y=\gamma^{21},\gamma^{41}} \bar{y}^\nu A^Y_{\pm} \gamma^\nu \right\} \\
+ \sum_{y=\gamma^{21},\gamma^{41}} \bar{y}^\nu \gamma^\nu A^Y_{\pm} \left\{ \sum_{\bar{\omega}(k)(l)(\bar{\varphi}(k))^{ac} \bar{\omega}_{abcd} = -\sum_{(ac),(bd),k,l} \sum_{(ac),(bd),k,l} \{\bar{\varphi}(k)\}^{ac} \bar{\omega}_{abcd}\}
\end{array} \right\}
\]

Taking into account that \((+)(+) = 0 = (-)(-)\), whereas \((+)(-) = (+)\) and \((-)(+) = (-)\), we recognize that equation (10) distinguishes between the u-quark (only the terms with \((-)\) give nonzero contributions) and the d-quarks (only the terms with \((+)\) give nonzero contributions) and accordingly also between the neutrino and the electron. We also see that the first two rows contribute to the diagonal elements of the mass matrices, while the second two contribute to their nondiagonal elements. Both diagonal and nondiagonal elements are expressible in terms of the gauge fields \(\omega_{abc}\) and \(\bar{\omega}_{abc}\). The diagonal matrix elements are expressed as the gauge fields of the operators \(\tau^{21}, \tau^{41}\) as well as the operators \(\tilde{\tau}^{13} := \frac{1}{2}(\tilde{\varphi} - \tilde{\varphi})\) and \(\tilde{\tau}^{21} := \frac{1}{2}(\tilde{\varphi} + \tilde{\varphi})\).

Taking into account that \(-\frac{1}{2} S^{\nu} \omega_{nu} = \tau^{21} A^{\nu}_{\pm} + \tau^{41} A^{\nu}_{\pm}, -\frac{1}{2} \tilde{S}^{\nu} \bar{\omega}_{nu} = \tilde{\tau}^{21} A^{\nu}_{\pm} + \tilde{\tau}^{41} A^{\nu}_{\pm}\) and \(-\frac{1}{2} \bar{S}^{mn} \bar{\omega}_{mn} = \tilde{\bar{\omega}}^{13} \tilde{\bar{\omega}}^{13} + \tilde{\bar{\omega}}^{21} \tilde{\bar{\omega}}^{21} + \tilde{\bar{\omega}}^{41} \tilde{\bar{\omega}}^{41}\), with the pairs \((\nu,n) = (0,3), (1,2); (s,t) = (5,6), (7,8), (9,10)\), belonging to the Cartan subalgebra and \(\Omega_{\pm} = \Omega_{\gamma} \pm i \Omega_{8}\), where \(\Omega_{7}, \Omega_{8}\) mean any of the above fields.

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\(\tilde{\omega}_{ab7}, \tilde{\omega}_{ab8}\), we find \(A_{1}^{13} = -(\omega_{56\pm} - \omega_{78\pm})\), \(A_{2}^{21} = -\frac{1}{2}(\omega_{56\pm} + \omega_{78\pm})\) and \(A_{4}^{41} = -\frac{1}{2}(\omega_{910\pm} + \omega_{1112\pm} + \omega_{1314\pm})\), and equivalently \(A_{1}^{13} = -(\omega_{56\pm} - \omega_{78\pm})\), \(A_{2}^{21} = -\frac{1}{2}(\omega_{56\pm} + \omega_{78\pm})\), \(A_{4}^{41} = -\frac{1}{2}(\omega_{910\pm} + \omega_{1112\pm} + \omega_{1314\pm})\). We point out that this is true only before any symmetries break. We repeat that \(\omega_{abc} = f_{c}^{a} \omega_{aba}\) and \(\tilde{\omega}_{abc} = f_{c}^{a} \tilde{\omega}_{aba}\).

We have for the nondiagonal mass matrix the elements

\[
\tilde{A}_{\pm}^{\mp}(ab, (cd)) = -\frac{i}{2} \left(\tilde{\omega}_{ac\pm} - \frac{i}{r} \tilde{\omega}_{bc\pm} - i\tilde{\omega}_{ad\pm} - \frac{1}{r} \tilde{\omega}_{bd\pm}\right),
\]

\[
\tilde{A}_{\pm}^{\mp}(ab, (cd)) = -\frac{i}{2} \left(\tilde{\omega}_{ac\pm} + \frac{i}{r} \tilde{\omega}_{bc\pm} + i\tilde{\omega}_{ad\pm} - \frac{1}{r} \tilde{\omega}_{bd\pm}\right),
\]

\[
\tilde{A}_{\pm}^{\mp}(ab, (cd)) = -\frac{i}{2} \left(\tilde{\omega}_{ac\pm} + \frac{i}{r} \tilde{\omega}_{bc\pm} - i\tilde{\omega}_{ad\pm} + \frac{1}{r} \tilde{\omega}_{bd\pm}\right),
\]

\[
\tilde{A}_{\pm}^{\mp}(ab, (cd)) = -\frac{i}{2} \left(\tilde{\omega}_{ac\pm} - \frac{i}{r} \tilde{\omega}_{bc\pm} + i\tilde{\omega}_{ad\pm} + \frac{1}{r} \tilde{\omega}_{bd\pm}\right),
\]

with \(r = i\), if \((ab) = (03)\) and \(r = 1\) otherwise. We simplify the index \(kl\) in the exponent of fields \(\tilde{A}_{\pm}^{kl}(ab, (cd))\) to \(\pm\), omitting \(i\).

The way of breaking either of the two symmetries—the Poincaré one and the symmetry determined by the generators \(\tilde{S}_{ab}\) in \(d = 1 + 13\)—strongly influences the Yukawa couplings of equation (10), relating the parameters \(\tilde{\omega}_{abc}\) and influencing the coupling constants.

In this paper, we assume two ways of breaking symmetries, and investigate under what conditions each of these two ways of breaking symmetries lead to the current measured properties of fermions.

### 2.1. Properties of Clifford algebra objects

Since \(S_{ab}^{\pm} = \frac{i}{2} \gamma_{a}^{b}, \) for \(a \neq b\) (for \(a = b, S_{ab}^{\pm} = 0\)), it is useful to know the following properties of \(\gamma_{a}^{b}\)s, if they are applied on nilpotents and projectors

\[
\gamma_{a}^{b}(k) = \eta_{a}^{b} [-k], \quad \gamma_{b}^{ab}(k) = -ik [-k],
\]

\[
\gamma_{a}^{b}[k] = (-k), \quad \gamma_{b}^{ab}[k] = -ik \eta_{a}^{b} (-k).
\]

Accordingly, for example, \(S_{ac}^{\pm}(k_1(k_2)) = -i \frac{1}{2} \eta_{a}^{d} \eta_{c}^{e} [-k_1][k_2] \). The operators, which are an even product of nilpotents

\[
\gamma^{\pm}_{(ab, cd), k_1, k_2} = (\pm k_1)(\pm k_2),
\]

appear to be the raising and lowering operators for a particular pair \((ab, cd)\) belonging to the Cartan subalgebra of the group \(SO(q, d - q)\), with \(q = 1\) in our case. There are always four possibilities for products of nilpotents with respect to the sign of \((k_1)\) and \((k_2)\), since \(k_1 = \pm i, l = 1, 2\) or \(k_2 = \pm 1, l = 1, 2\) (whether we have \(i\) or \(1\) depends on the character of the indices of the Cartan subalgebra: \(i\) for the pair \((03),\) and \(i\) otherwise). We can make use of \(R\) and \(L\) instead of \(k_1, k_2\) to distinguish between the two kinds of lowering and raising operators in...
appear (equivalently as when a pair \((\pm 1, 7)\), respectively, since they distinguish between right-handed weak chargeless states and left-handed weak charged states: When applied to states of inappropriate handedness \(\tau_{(ab, cd), k_1, k_2}^{\pm} 0312\) gives 0. For example, \(\tau_{(03,12), R}^{\pm} 0312\) is the raising \((+i)(+)\) and lowering \((-i)(-)\) operator, respectively, for a right-handed quark or lepton, while \(\tau_{(03,12), L}^{\pm} 0312\) is the corresponding left-handed raising and lowering operator, respectively, for left-handed quarks and leptons. Being applied on a weak chargeless \(u^c_R\) of a color \(c\) and of the spin \(1/2\), \(\tau_{(03,12), R} 0312\) transforms it into a weak chargeless \(u^c_R\) of the same color and handedness but of the spin \(-1/2\), whereas \(\tau_{(03,78), R} = (-i)(-)\) transforms a weak chargeless \(u^c_R\) of any color and of the spin \(1/2\) into the weak charged \(u^c_L\) of the same color and the same spin, but of the opposite handedness.

It is useful to have in mind [11, 12] the following properties of the nilpotents \((\tilde{k})\):

\[
\begin{align*}
(k)(k) &= 0, & (k)(-k) &= \eta^{a\alpha}[k], & [k][k] &= [k], & [k][k] &= 0, \\
(k)[k] &= 0, & [k][k] &= 0, & [k][k] &= 0,
\end{align*}
\]

which the reader can easily check by taking into account equation (12).

### 2.2. Families of spinors

Commuting with \(S^{ab}\), \(\tilde{S}^{ab}\), \(\tilde{S}^{ab}\) generate equivalent representations, which we recognize as families. To evaluate the application of \(\tilde{S}^{ab}\) on the starting family, presented in table 1, we take into account the Clifford algebra properties of \(\tilde{\gamma}^a\). We find

\[
\begin{align*}
\tilde{\gamma}^a (k) &= -i\eta^{a\alpha}[k], & \tilde{\gamma}^a(k) &= -k[k], \\
\tilde{\gamma}^a &[-k] = i \tilde{\gamma}^a[k], & \tilde{\gamma}^a[k] &= -k[k].
\end{align*}
\]

Accordingly, it follows that

\[
\begin{align*}
(k)(k) &= 0, & (-k)(k) &= -\eta^{a\alpha}[k], & (-k)[-k] &= i(-k), & (k)[-k] &= 0, \\
(k)[k] &= i(k), & (-k)[+k] &= 0, & (-k)[-k] &= 0, & (k)[-k] &= -i\eta^{a\alpha}[-k].
\end{align*}
\]

The operators, which are an even product of nilpotents in the \(\tilde{\gamma}^a\) sector

\[
\tilde{\tau}_{(ab, cd), k_1, k_2}^{\pm} = (\pm k_1)(\pm k_2),
\]

appear (equivalently as \(\tilde{\tau}_{(ab, cd), k_1, k_2}^{\pm} 0312\) in the \(S^{ac}\) sector) as the raising and lowering operators, when a pair \((ab), (cd)\) belongs to the Cartan subalgebra of the algebra \(\tilde{S}^{ac}\), transforming a member of one family into the same member of another family. For example: \(\tilde{\tau}_{(03,12), -1,-1} 0312\) transforms the right-handed \(u^c_R\) quark from table 1 into the right-handed \(u^c_R\) quark \(u^c_R = [+i][+] \times (+)(+) \times [-][+](-),\) which has all the properties with respect to the operators \(S^{ab}\), the same as \(u^c_R\) from table 1.
3. From eight to four families of quarks and leptons

Assuming that the break of the symmetry from $SO(1, 13)$ to $SO(1, 7) \times SU(3) \times U(1)$ makes all the families except the massless ones determined by $SO(1, 7)$ very heavy (of the order of $10^{15}$ GeV or heavier), we end up with eight families: $\tilde{S}_{ab}$, with $(a, b) \in \{0, 1, 2, 3, 5, 6, 7, 8\}$ (or equivalently the products of nilpotents $(k_1)(k_2)$, with $k_1, k_2$ equal to $\pm 1$ or $\pm i$, while $(ab), (cd)$ denote two of the four Cartan subalgebra pairs $\{(03), (12), (56), (78)\}$) generate $2^{8/2-1} = 8$ families. The first member of the $SO(1, 7)$ multiplet in table 1 (the right-handed weak chargeless u$^c$-quark with spin 1/2, for example, as well as the right-handed weak chargeless neutrino with spin 1/2—both differ only in the part which concerns the $SU(3)$ and the $U(1)$ charge ($U(1)$ from $SO(6)$), and which stay unchanged under the application of $\tilde{S}_{ab}$ with $(a, b) \in \{0, 1, 2, 3, 5, 6, 7, 8\}$) appears in the following 8 families:

\[
\begin{align*}
\text{I.} & \quad (03)_{12}^{56} \quad | \quad 78 \quad | \quad \cdots \\
\text{II.} & \quad (+i)^{[+]} \quad | \quad (+)^{[+]}) \quad | \quad \cdots \\
\text{III.} & \quad (+i)^{[+]} \quad | \quad [+]^{[+]} \quad | \quad \cdots \\
\text{IV.} & \quad (+i)^{[+]}) \quad | \quad (+)^{[+]}) \quad | \quad \cdots \\
\text{V.} & \quad (+i)^{[+]}) \quad | \quad [+]^{[+]} \quad | \quad \cdots \\
\text{VI.} & \quad (+i)^{[+]} \quad | \quad [+]^{[+]}) \quad | \quad \cdots \\
\text{VII.} & \quad (+i)^{[+]}) \quad | \quad (+)^{[+]}) \quad | \quad \cdots \\
\text{VIII.} & \quad (i)^{[+]}) \quad | \quad (+)^{[+]}) \quad | \quad \cdots \\
\end{align*}
\]

The remaining seven members of each of the above eight families can be obtained, as in table 1, by the application of the operators $\tilde{S}_{ab}$ on the above particular member (or with the help of the raising and lowering operators $\tau_{\pm (ab), (cd), k_1, k_2}$). One can easily conclude (by checking the quantum numbers represented in table 1) that each of the eight states of equation (18) indeed represents the right-handed weak chargeless quark (or the right-handed weak chargeless lepton, depending on what appears for $| \quad \cdots$ in equation (18)).

The way of further breaking the symmetry $SO(1, 7) \times U(1) \times SU(3)$ influences strongly properties of the mass matrix elements determined by equation (10). We assume two particular ways of breaking the symmetry $SO(1, 7) \times U(1)$ and study under which conditions the two ways of breaking symmetries can reproduce the known experimental data.

To come from the starting action of the proposed approach (with at the most two free parameters) to the effective action manifesting the Standard Model of the electroweak and color interaction—in this paper, we treat only the Yukawa part of the Standard Model action—and further to the observed families as well as to make predictions for the properties of a possible fourth family, we make the following assumptions:

(i) The break of symmetries of the group $SO(1, 13)$ (the Poincaré group in $d = 1 + 13$) into $SO(1, 7) \times SU(3) \times U(1)$ occurs in a way that in $d = 1 + 7$ massless spinors with the charge $SU(3) \times U(1)$ appear. (Our work on the compactification of a massless spinor
\(d = 1 + 5\) into \(d = 1 + 3\) and a finite disk gives us some hope that such an assumption might be justified \([18, 20, 31]\). The break of symmetries influences both the (Poincaré) symmetry described by \(S^{ab}\) and the symmetries described by \(\tilde{S}^{ab}\).

(ii) Further breaks lead to two (almost) decoupled groups of four massive families, well separated in masses.

(iii) We make estimates on a ‘tree level’.

(iv) We assume the mass matrices to be real and symmetric expecting that the complexity and the nonsymmetric properties of the mass matrices do not influence masses considerably and the real part of the mixing matrices of quarks and leptons. In this paper, we do not study CP (charge parity symmetry) breaking.

The following two ways of breaking symmetries leading to four ‘low-lying’ families of quarks and leptons are chosen:

(a) First, we assume that the break of symmetries from \(SO(1, 7) \times U(1) \times SU(3)\) to the observed symmetries in the ‘low-energy’ regime occurs so that all the nondiagonal elements in the Lagrange density (equation (10)) caused by the operators of the type \((\tilde{k}) (\tilde{l})\)

\[
\begin{align*}
&ab \\
&cd
\end{align*}
\]

or of the type \((\tilde{k}) (\tilde{l})\), with either \((ab)\) or \((cd)\) equal to \((56)\), are zero. In the ‘Poincaré’ sector this assumption guarantees the conservation of the electromagnetic charge \(Q = S^{56} + \epsilon^{41}\) by the mass term, since the operators \((\tilde{k}) (\tilde{l})\) transform the u-quark into the d-quark and vice versa. We extend this requirement also to the operators \((\tilde{k}) (\tilde{l})\). This means that all the fields of the type \(\tilde{A}_{\pm}^{kl}((ab), (cd))\), with either \(k\) or \(l\) equal to \(\pm\) and with either \((ab)\) or \((cd)\) equal to \((56)\), are put to zero. Then the eight families split into decoupled two times four families. One easily sees that the diagonal matrix elements can be chosen in such a way that one of the two sets of four families has much larger diagonal elements than the other (which guarantees correspondingly also much higher masses of the corresponding fermions). Accordingly, we are left with studying the properties of one of the four families, decoupled from the other four families. We present this study in section 3.1.

(b) In the second way of breaking symmetries from \(SO(1, 7) \times U(1) \times SU(3)\) to the observed ‘low-energy’ sector, we assume that no matrix elements of the type \(S^{ms} \omega_{msc}\) or \(\tilde{S}^{sm} \tilde{\omega}_{smc}\), with \(m = 0, 1, 2, 3\) and \(s = 5, 6, 7, 8\), are allowed. This means that all the matrix elements of the type \(\tilde{A}_{\pm}^{kl}((ab), (cd))\), with either \(k\) or \(l\) equal to \(\pm\) and with \((ab)\) equal to \((03)\) or \((12)\) and \((cd)\) equal to \((56)\) or \((78)\), are put to zero. This means that the symmetry \(SO(1, 7) \times U(1)\) breaks into \(SO(1, 3) \times SO(4) \times U(1)\) and further into \(SO(1, 3) \times U(1)\). Again, the mass matrix of eight families splits into two times decoupled four families. We recognize that in this way of breaking symmetries the diagonal matrix elements of the first four families are again much larger than the diagonal matrix elements of the last four families. We study the properties of the four families with the lower diagonal matrix elements in section 3.2.

To simplify the problem, we assume in both cases (a) and (b), that the mass matrices are real and symmetric. To determine free parameters of the mass matrices by fitting masses and mixing matrices of four families to the measured values for the three known families within the known accuracy, is by itself quite a demanding task. And we hope that after analyzing two
The assumption that there are no matrix elements of the type \( (ab), (cd) \), \( k, l = \pm \) and \( (ab), (cd) = (03), (12), (78) \) are expressible with the corresponding \( \theta_{abab} \) fields (equation (10)). They then accordingly determine the properties of the four families of u-quarks. The mass matrix is not yet required to be symmetric and real.

| \( \alpha \) | \( II_R \) | \( III_R \) | \( IV_R \) |
|---|---|---|---|
| \( I_L \) | \( A^I_u \) | \( \tilde{A}_u^-((03), (12)) \) | \( \tilde{A}_u^+(03), (78)) \) | \( -\tilde{A}_u^-((12), (78)) \) |
| \( II_L \) | \( \tilde{A}_u^-((03), (12)) \) | \( A^II_u \) | \( \tilde{A}_u^+(12), (78)) \) | \( -\tilde{A}_u^-(03), (78)) \) |
| \( III_L \) | \( \tilde{A}_u^-((03), (78)) \) | \( -\tilde{A}_u^-((12), (78)) \) | \( A^III_u \) | \( \tilde{A}_u^+(03), (12)) \) |
| \( IV_L \) | \( \tilde{A}_u^-((12), (78)) \) | \( -\tilde{A}_u^-((03), (78)) \) | \( \tilde{A}_u^-(03), (12)) \) | \( A^IV_u \) |

**Table 3.** The mass matrix of the four families of d-quarks obtained within the approach unifying spins and charges under the assumptions (I)–(iii) and (a) (in section 3). Comments are the same as in table 2.

| \( \beta \) | \( II_R \) | \( III_R \) | \( IV_R \) |
|---|---|---|---|
| \( I_L \) | \( A^I_d \) | \( \tilde{A}_d^+(03), (12)) \) | \( -\tilde{A}_d^-((03), (78)) \) | \( \tilde{A}_d^+(12), (78)) \) |
| \( II_L \) | \( \tilde{A}_d^-((03), (12)) \) | \( A^II_d \) | \( \tilde{A}_d^+(12), (78)) \) | \( -\tilde{A}_d^-(03), (78)) \) |
| \( III_L \) | \( -\tilde{A}_d^-((03), (78)) \) | \( \tilde{A}_d^-((12), (78)) \) | \( A^III_d \) | \( \tilde{A}_d^+(03), (12)) \) |
| \( IV_L \) | \( -\tilde{A}_d^-((12), (78)) \) | \( \tilde{A}_d^-((03), (78)) \) | \( -\tilde{A}_d^-((03), (12)) \) | \( A^IV_d \) |

possible breaks of symmetries, even such a simplified study can help us understand the origin of families and predict properties of the fourth family.

### 3.1. Four families of quarks in proposal no. 1

The assumption that there are no matrix elements of the type \( \tilde{A}_d^H((ab), (cd)) \), with \( k = \pm \) and \( l = \pm \) (in all four combinations) and with either \( (ab) \) or \( (cd) \) equal to \( (56) \) leads to the following four families (corresponding to the families I, II, IV, VIII in equation (18)):

\[
\begin{align*}
\text{I.} & \quad 03 & 12 & 56 & 78 \\
\text{II.} & \quad 03 & 12 & 56 & 78 \\
\text{III.} & \quad 03 & 12 & 56 & 78 \\
\text{IV.} & \quad 03 & 12 & 56 & 78
\end{align*}
\]

and to the corresponding mass matrices presented in tables 2 and 3. It is easy to see that the parameters can be chosen so that the last four families, decoupled from the first four, have much higher diagonal matrix elements and determine accordingly fermions of much higher masses.
If required that the mass matrices are real and symmetric, one ends up with the matrix elements for the u-quarks as follows: \( \tilde{A}_u^+((03), (12)) = \frac{1}{2}(\bar{\omega}_{127a} + \bar{\omega}_{018a}) = \tilde{A}_u^-((03), (12)), \)
\( \tilde{A}_u^+((03), (78)) = \frac{1}{2}(\bar{\omega}_{387a} + \bar{\omega}_{078a}) = \tilde{A}_u^-((03), (78)), \)
\( \tilde{A}_u^+((12), (78)) = -\frac{1}{2}(\bar{\omega}_{277a} + \bar{\omega}_{187a}) = -\tilde{A}_u^-((12), (78)), \)
\( \tilde{A}_u^-((12), (78)) = -\frac{1}{2}(\bar{\omega}_{277a} + \bar{\omega}_{187a}) = -\tilde{A}_u^-((12), (78)), \)
\( \tilde{A}_u^-((03), (78)) = \frac{1}{2}(\bar{\omega}_{387a} - \bar{\omega}_{078a}) = \tilde{A}_u^-((03), (78)) \) and \( \tilde{A}_u^-((03), (12)) = -\frac{1}{2}(\bar{\omega}_{327a} - \bar{\omega}_{018a}) = \tilde{A}_u^-((03), (12)). \) The diagonal terms are \( A^{\Pi}_u = \tilde{A}_u^+(\bar{\omega}_{127a} - \bar{\omega}_{038a}), \)
\( A^{\Pi}_u = \tilde{A}_u^+(\bar{\omega}_{787a} - \bar{\omega}_{038a}), \)
and \( A^{\Pi}_u = \tilde{A}_u^+(\bar{\omega}_{018a} - \bar{\omega}_{038a}).\) One obtains equivalent expressions also for the d-quarks: \( \tilde{A}_d^+((03), (12)) = \frac{1}{2}(\bar{\omega}_{018d} + \bar{\omega}_{127d}) = \tilde{A}_d^-((03), (12)), \)
\( \tilde{A}_d^+((03), (78)) = \frac{1}{2}(\bar{\omega}_{387d} + \bar{\omega}_{078d}) = \tilde{A}_d^-((03), (78)), \)
\( \tilde{A}_d^+((12), (78)) = -\frac{1}{2}(\bar{\omega}_{277d} + \bar{\omega}_{187d}) = -\tilde{A}_d^-((12), (78)), \)
\( \tilde{A}_d^-((12), (78)) = -\frac{1}{2}(\bar{\omega}_{277d} + \bar{\omega}_{187d}) = -\tilde{A}_d^-((12), (78)), \)
\( \tilde{A}_d^-((03), (78)) = \frac{1}{2}(\bar{\omega}_{387d} - \bar{\omega}_{078d}) = \tilde{A}_d^-((03), (78)) \) and \( \tilde{A}_d^-((03), (12)) = -\frac{1}{2}(\bar{\omega}_{327d} - \bar{\omega}_{018d}) = \tilde{A}_d^-((03), (12)). \) The diagonal terms are \( A^{\Pi}_d = \tilde{A}_d^+(\bar{\omega}_{018d} - \bar{\omega}_{038d}), \)
\( A^{\Pi}_d = \tilde{A}_d^+(\bar{\omega}_{787d} + \bar{\omega}_{038d}), \)
and \( A^{\Pi}_d = \tilde{A}_d^+(\bar{\omega}_{127d} + \bar{\omega}_{038d}).\) Different parameters for the members of the families are due to different expressions for the matrix elements, different diagonal terms, contributed by \( S^{ab} \omega_{ab\pm} \) and also due to perturbative and nonperturbative effects which appear through breaks of symmetries.

Let us assume that the mass matrices are real and symmetric (assumption (iv) in section 3) and in addition that the break of symmetries leads to two heavy and two light families and that the mass matrices are diagonalizable in a two-step process [24, 25], so that the first diagonalization transforms the mass matrices into block-diagonal matrices with two \( 2 \times 2 \) submatrices. We follow [25] (where the reader can find all the details). It is easy to prove that a \( 4 \times 4 \) matrix is diagonalizable in two steps only if it has a structure

\[
\begin{pmatrix}
A & B \\
B & C = A + kB
\end{pmatrix}
\]

Since \( A \) and \( C \) are assumed to be symmetric \( 2 \times 2 \) matrices, \( B \) must be as well. The parameter \( k \), which is an unknown parameter, has the property \( k = k_u = -k_d \), where the indices \( u \) and \( d \) denote the \( u \) and the \( d \) quark, respectively. The above assumption requires that \( \bar{\omega}_{277a} = 0, \bar{\omega}_{277d} = -\frac{k}{2} \bar{\omega}_{197a}, \bar{\omega}_{078a} = \frac{k}{2} \bar{\omega}_{387a}, \bar{\omega}_{078d} = -\frac{k}{2} \bar{\omega}_{078a} \) and \( \delta = u, d \).

Under these assumptions, the matrices diagonalizing the mass matrices are expressible with only three parameters, and the angles of rotations in the \( u \)-quark case are related to the angles of rotations in the \( d \)-quark case as follows:

\[
\tan^{a,b} \varphi_{u,d} = \left( \sqrt{1 + (a,b A_{\eta_{a,d}})^2} \right) + a,b A_{\eta_{a,d}},
\]

\[
a A_{\eta_{a}} = -b A_{\eta_{d}}, \quad b A_{\eta_{a}} = -a A_{\eta_{d}}
\]

with \( a \), which determines the last two times two matrices and \( b \) the first two times two matrices after the first step diagonalization. Then the angles of rotations in the \( u \) and the \( d \) quark case are related: (i) for the angle of the first rotation (which leads to two by diagonal matrices), we find \( \tan \varphi_u = \tan^{-1} \varphi_d \) with \( \varphi_u = \frac{\pi}{4} - \frac{\varphi}{2} \). (ii) For the angles of the second rotation in the sector \( a \) and \( b \), we correspondingly find for the \( u \)-quark \( a,b \varphi_u = \frac{\pi}{4} - \frac{a,b \varphi}{2} \) and for the \( d \)-quark \( a,b \varphi_d = \frac{\pi}{4} + \frac{a,b \varphi}{2} \).
It is now easy to express all the fields \( \tilde{\omega}_{abc} \) in terms of the masses and the parameters \( k \) and \( \alpha, \beta \eta_{a,d} \):

\[
\tilde{\omega}_{018u} = \frac{1}{2} \left[ \frac{m_{u2} - m_{u1}}{\sqrt{1 + (\alpha \eta_u)^2}} + \frac{m_{u4} - m_{u3}}{\sqrt{1 + (\beta \eta_u)^2}} \right],
\]

\[
\tilde{\omega}_{078u} = \frac{1}{2} \left[ \frac{\alpha \eta_u (m_{u2} - m_{u1})}{\sqrt{1 + (\alpha \eta_u)^2}} - \frac{\beta \eta_u (m_{u4} - m_{u3})}{\sqrt{1 + (\beta \eta_u)^2}} \right],
\]

\[
\tilde{\omega}_{127u} = \frac{1}{2} \left[ \frac{\alpha \eta_u (m_{u2} - m_{u1})}{\sqrt{1 + (\alpha \eta_u)^2}} + \frac{\beta \eta_u (m_{u4} - m_{u3})}{\sqrt{1 + (\beta \eta_u)^2}} \right],
\]

\[
\tilde{\omega}_{187u} = \frac{1}{2} \left[ \frac{m_{u2} - m_{u1}}{\sqrt{1 + (\alpha \eta_u)^2}} + \frac{m_{u4} - m_{u3}}{\sqrt{1 + (\beta \eta_u)^2}} \right],
\]

\[
\tilde{\omega}_{387u} = \frac{1}{2} \left[ \frac{(m_{u4} + m_{u3}) - (m_{u2} + m_{u1})}{\sqrt{1 + (\alpha \eta_u)^2}} \right],
\]

\[
a_u = \frac{1}{2} \left( m_{u1} + m_{u2} - \frac{\alpha \eta_u (m_{u2} - m_{u1})}{\sqrt{1 + (\alpha \eta_u)^2}} \right),
\]

with

\[
a_u = A_u - \frac{1}{2} \tilde{\omega}_{038u} + \frac{1}{2} \left( \frac{k}{2} - \sqrt{1 + \left( \frac{k}{2} \right)^2} \right) (\tilde{\omega}_{078u} + \tilde{\omega}_{187u}),
\]

and equivalently for the d-quarks, where \( \alpha, \beta \eta_u \) stays unchanged (equation (20)).

The experimental data offer the masses of six quarks and the corresponding mixing matrix for the three families (within the measured accuracy and the corresponding calculational errors). Due to our assumptions the mixing matrix is real and antisymmetric

\[
V_{ud} = \begin{pmatrix}
c(\varphi)c(\varphi') & -c(\varphi)s(\varphi') & -s(\varphi)c(\varphi') & s(\varphi)s(\varphi') \\
c(\varphi)s(\varphi') & c(\varphi)c(\varphi') & -s(\varphi)s(\varphi') & -s(\varphi)c(\varphi') \\
s(\varphi)c(\varphi') & -s(\varphi)s(\varphi') & c(\varphi)c(\varphi') & -c(\varphi)s(\varphi') \\
s(\varphi)s(\varphi') & s(\varphi)c(\varphi') & c(\varphi)s(\varphi') & c(\varphi)c(\varphi')
\end{pmatrix},
\]

where

\[
\varphi = \varphi_\alpha - \varphi_\beta, \quad \alpha \varphi = \alpha \varphi_\alpha - \alpha \varphi_\beta, \quad \alpha \varphi_\beta = -\frac{\alpha \varphi + \beta \varphi}{2}
\]

with the angles described by the three parameters \( k, \alpha \eta_u \text{ and } \beta \eta_u \).

We present numerical results in the next section. The assumptions which we made left us with the problem of fitting 12 parameters for both types of quarks to the experimental data. Since the parameter \( k \), which determines the first step of diagonalization of mass matrices,
turns out (experimentally) to be very small, the ratios of the fields \( \tilde{\omega}_{abc} \) for u-quarks and d-quarks (\( \tilde{\omega}_{abc} \)) are almost determined with the values \( a^b \eta_i \) (equation (20)), and we are left with 7 parameters, which should be fitted twice to three masses of quarks and (in our simplified case) three angles within the known accuracy.

3.2. Four families of quarks in proposal no. 2

The assumption made in the previous subsection (3.1) takes care—in the \( S_{ab} \) sector—that the mass term conserves the electromagnetic charge. The same assumption was also made in the \( \tilde{S}_{ab} \) sector.

In this subsection, we study the break of the symmetries from \( SO(1, 7) \times U(1) \times SU(3) \), down to \( SO(1, 3) \times U(1) \times SU(3) \), which occurs in the following steps. First, we assume that all the matrix elements \( \tilde{A}_{ab}^{(0)}((ab), (cd)) \), which have \((ab)\) equal either to \((03)\) or \((12)\) and \((cd)\) equal either to \((56)\) or \((78)\), are equal to zero, which means that the symmetry \( SO(1, 7) \times U(1) \) breaks into \( SO(1, 3) \times SU(4) \times U(1) \).

We then break \( SO(4) \times U(1) \) in the sector \( \tilde{S}_{ab} \tilde{\omega}_{abc} \), \( s = 7, 8 \), so that at some high scale one of \( SU(2) \) in \( SO(4) \times U(1) \) breaks together with \( U(1) \) into \( SU(2) \times U(1) \) and then—at a much lower scale, which is the weak scale—the break of the symmetry of \( SU(2) \times U(1) \) into \( U(1) \) appears.

The break of the symmetries from \( SO(1, 7) \times U(1) \) to \( SO(1, 3) \times SO(4) \times U(1) \) makes the eight families decouple into two times four families, arranged as follows:

\[
\begin{align*}
\text{I.} & & \text{II.} & & \text{III.} & & \text{IV.} & & \text{V.} & & \text{VI.} & & \text{VII.} & & \text{VIII.} \\
& [+i][+] \mid (+)[+] \mid \ldots & & (+)[+] \mid (+)[+] \mid \ldots & & (+)[+] \mid (+)[+] \mid \ldots & & (+)[+] \mid (+)[+] \mid \ldots & & (+)[+] \mid (+)[+] \mid \ldots & & (+)[+] \mid (+)[+] \mid \ldots & & (+)[+] \mid (+)[+] \mid \ldots
\end{align*}
\]

\[\text{(24)}\]

We shall see that the parameters of the second four families lead accordingly to much higher masses.

In equation (10), we have rearranged the terms \( \tilde{S}_{ab}^{ab} \tilde{\omega}_{abcz} \) for \( a, b = 0, 1, 2, 3, 5, 6, 7, 8 \) in terms of the raising and lowering operators, which are products of nilpotents \( (\pm k_1) (\pm k_2) \), with \((ab), (cd)\) belonging to the Cartan subalgebra. Introducing the notation for the particular lowering and raising operators as follows:

\[
\begin{align*}
\tilde{t}_{N_r}^+ = & - (3i)(+) \quad \tilde{t}_{N_r}^- = - (3i)(-), & \tilde{t}_{N_r}^+ = & (3i)(+), & \tilde{t}_{N_r}^- = & (-3i)(-), \\
\tilde{t}_1^+ = & (1)(-), & \tilde{t}_1^- = & (-)(+), & \tilde{t}_2^+ = & (1)(+), & \tilde{t}_2^- = & (-)(-), \\
\end{align*}
\]

\[\text{(25)}\]

and for the diagonal operators

\[
\begin{align*}
\tilde{N}_+ = & \frac{1}{2} (\tilde{S}_{12} + i \tilde{S}_{03}), & \tilde{N}_- = & \frac{1}{2} (\tilde{S}_{12} - i \tilde{S}_{03}), & \tilde{t}_{13} = & \frac{1}{2} (\tilde{S}_{56} - \tilde{S}_{78}), & \tilde{t}_{23} = & \frac{1}{2} (\tilde{S}_{56} + \tilde{S}_{78}),
\end{align*}
\]

\[\text{(26)}\]
we can write
\[
\frac{1}{2} \tilde{S}_{ab} \tilde{\omega}_{ab\pm} = \frac{\tilde{g}^m}{\sqrt{2}} \left( -\tilde{\tau}^{+}_{N_i} \tilde{A}_{\pm}^{\dagger N_i} - \tilde{\tau}^{-}_{N_i} \tilde{A}_{\pm}^{- N_i} + \tilde{\tau}^{+}_{N_i} \tilde{A}_{\pm}^{+ N_i} + \tilde{\tau}^{-}_{N_i} \tilde{A}_{\pm}^{- N_i} \right) \\
+ \frac{\tilde{g}^1}{\sqrt{2}} \left( -\tilde{\tau}^{1+}_{A_{\pm}} + \tilde{\tau}^{1-}_{A_{\pm}} \right) + \frac{\tilde{g}^2}{\sqrt{2}} \left( \tilde{\tau}^{2+}_{A_{\pm}} + \tilde{\tau}^{2-}_{A_{\pm}} \right) \\
+ \tilde{g}^m \left( \tilde{N}^3_+ \tilde{A}_{\pm}^{3 N_i} + \tilde{N}^3_- \tilde{A}_{\pm}^{-3 N_i} \right) + \tilde{g}^1 \left( \tilde{\tau}^{13}_{A_{\pm}} + \tilde{\tau}^{23}_{A_{\pm}} \right) + \tilde{g}^4 \tilde{A}^4_{\pm}. \quad (27)
\]

If the fields \( \tilde{A}^{kl}_{\pm}((ab)(cd)) \) in equation (11) and the fields in equation (27) are taken together with the coupling constants \( \tilde{g}^i, i = 1, 2, 4, m \) (taking care of the running in the \( \tilde{S}_{ab} \) sector) they are in one to one correspondence. For example, \(-\frac{\tilde{g}^m}{\sqrt{2}} \tilde{\tau}^{+}_{N_i} \tilde{A}_{\pm}^{1 N_i} = -\left( -\tilde{g}^s \right) \tilde{A}_{\pm}^{+ 1} \).

We assume that at the break of \( SO(4) \times U(1) \) into \( SU(2) \times U(1) \) appearing at some large-scale new fields \( \tilde{A}^1_{\pm} \) and \( \tilde{A}^2_{\pm} \) manifest (in a similar way in the Standard Model new fields occur when the weak charge breaks) that
\[
\tilde{A}^{23}_{\pm} = \tilde{A}^{1}_{\pm} \sin \tilde{\theta}_2 + \tilde{A}^{Y\dagger}_{\pm} \cos \tilde{\theta}_2, \quad \tilde{A}^{41}_{\pm} = \tilde{A}^{1}_{\pm} \cos \tilde{\theta}_2 - \tilde{A}^{Y\dagger}_{\pm} \sin \tilde{\theta}_2, \quad (28)
\]
and accordingly also new operators
\[
\tilde{Y} = \tilde{\tau}^{41} + \tilde{\tau}^{23}, \quad \tilde{Y}' = \tilde{\tau}^{23} - \tilde{\tau}^{41} \tan \tilde{\theta}_2. \quad (29)
\]

It then follows for the \( \tilde{S}_{ab} \tilde{\omega}_{ab\pm} \) sector of the mass matrix
\[
\frac{1}{2} \tilde{S}_{ab} \tilde{\omega}_{ab\pm} = \frac{\tilde{g}^m}{\sqrt{2}} \left( -\tilde{\tau}^{+}_{N_i} \tilde{A}_{\pm}^{\dagger N_i} - \tilde{\tau}^{-}_{N_i} \tilde{A}_{\pm}^{- N_i} + \tilde{\tau}^{+}_{N_i} \tilde{A}_{\pm}^{+ N_i} + \tilde{\tau}^{-}_{N_i} \tilde{A}_{\pm}^{- N_i} \right) + \frac{\tilde{g}^1}{\sqrt{2}} \left( -\tilde{\tau}^{1+}_{A_{\pm}} + \tilde{\tau}^{1-}_{A_{\pm}} \right) \\
+ \frac{\tilde{g}^2}{\sqrt{2}} \left( \tilde{\tau}^{2+}_{A_{\pm}} + \tilde{\tau}^{2-}_{A_{\pm}} \right) + \tilde{g}^m \left( \tilde{N}^3_+ \tilde{A}_{\pm}^{3 N_i} + \tilde{N}^3_- \tilde{A}_{\pm}^{-3 N_i} \right) + \tilde{g}^1 \left( \tilde{\tau}^{13}_{A_{\pm}} + \tilde{\tau}^{23}_{A_{\pm}} \right) + \tilde{g}^4 \tilde{A}^4_{\pm}. \quad (30)
\]

Here, \( \tilde{Y} \) = \( \tilde{g}^{-4} \cos \tilde{\theta}_2 \), \( \tilde{Y}' = \tilde{g}^{-2} \cos \tilde{\theta}_2 \) and \( \tan \tilde{\theta}_2 = \frac{\tilde{g}^4}{\tilde{g}^{-2}} \).

Let, at a weak scale, the matrix \( SU(2) \times U(1) \) break further into \( U(1) \) leading again to new fields
\[
\tilde{A}^{13}_{\pm} = \tilde{A}_{\pm} \sin \tilde{\theta}_1 + \tilde{Z}_{\pm} \cos \tilde{\theta}_1, \quad \tilde{A}^{13'}_{\pm} = \tilde{A}_{\pm} \cos \tilde{\theta}_1 - \tilde{Z}_{\pm} \sin \tilde{\theta}_1, \quad (31)
\]
and resulting in new operators
\[
\tilde{Q} = \tilde{\tau}^{13} + \tilde{Y} = \tilde{S}^{56} + \tilde{\tau}^{41}, \quad \tilde{Q}' = -\tilde{Y} \tan^2 \tilde{\theta}_1 + \tilde{\tau}^{13}, \quad (32)
\]
with \( \tilde{\tau} = \tilde{g}^{Y} \cos \tilde{\theta}_1, \tilde{g}' = \tilde{g}^{1} \cos \tilde{\theta}_1 \) and \( \tan \tilde{\theta}_1 = \frac{\tilde{g}^{-4}}{\tilde{g}^{-2}} \). If \( \tilde{\theta}_2 \) appears to be very small and \( \tilde{g}^{-2} \tilde{A}^{2\pm}_{\pm} \) and \( \tilde{g}^{Y} \tilde{A}^{Y}_{\pm} \tilde{Y}' \) very large, the last four families (decoupled from the first four families) appear to be very heavy in comparison with the first four families. The first four families of mass matrices (evaluated on a tree level) for the u-quarks (−) and the d-quarks (+) are presented in table 4.

In table 4, \( a_{\pm} \) stands for the contribution to the mass matrices from the \( \tilde{S}_{ab} \tilde{\omega}_{ab\pm} \) part (which distinguishes among the members of each particular family) and from the diagonal terms of the \( \tilde{S}_{ab} \tilde{\omega}_{ab\pm} \) part. The mass matrix in table 4 is in general complex. To be able to estimate the properties of the four families of quarks, we assume (as in subsection 3.1) that the mass matrices are real and symmetric. We then treat the elements as they appear in table 4 as free parameters and fit them to the experimental data. Accordingly, we rewrite the mass matrix in table 4 in the form presented in table 5.

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Table 4. The mass matrix for the lower four families of the u-quarks (with the sign $-$) and the d-quarks (with the sign $+$).

|     | I  | II  | III | IV |
|-----|----|-----|-----|----|
| I   | $a_{\pm}$ | $\frac{g^m}{\sqrt{2}} \tilde{A}_{\pm}^{-N}$ | $\frac{g^1}{\sqrt{2}} \tilde{A}_{\pm}^{1+}$ | 0  |
| II  | $\frac{g^m}{\sqrt{2}} \tilde{A}_{\pm}^{-N}$ | $a_{\pm} + \frac{1}{2} g^m (\tilde{A}_{\pm}^{2N} + \tilde{A}_{\pm}^{2N})$ | 0  | $-\frac{g^1}{\sqrt{2}} \tilde{A}_{\pm}^{1+}$ |
| III | $\frac{g^1}{\sqrt{2}} \tilde{A}_{\pm}^{1-}$ | 0 | $a_{\pm} + \tilde{e} \tilde{A}_{\pm} + g^m \tilde{Z}_{\pm}$ | $\frac{g^m}{\sqrt{2}} \tilde{A}_{\pm}^{1+}$ |
| IV  | 0  | $\frac{g^1}{\sqrt{2}} \tilde{A}_{\pm}^{1-}$ | $\frac{g^m}{\sqrt{2}} \tilde{A}_{\pm}^{-N}$ | $a_{\pm} + \tilde{e} \tilde{A}_{\pm} + g^m \tilde{Z}_{\pm} + \frac{1}{2} g^m (\tilde{A}_{\pm}^{3N} + \tilde{A}_{\pm}^{3N})$ |

Table 5. The mass matrix from table 4, taken in this case to be real and parameterized in a transparent way. The symbols $-$, $+$ denote the u-quarks and the d-quarks, respectively.

|     | I  | II  | III | IV |
|-----|----|-----|-----|----|
| I   | $a_{\pm}$ | $b_{\pm}$ | $-c_{\pm}$ | 0  |
| II  | $b_{\pm}$ | $a_{\pm} + d_{1\pm}$ | 0 | $-c_{\pm}$ |
| III | $c_{\pm}$ | 0 | $a_{\pm} + d_{2\pm}$ | $b_{\pm}$ |
| IV  | 0  | $c_{\pm}$ | $b_{\pm}$ | $a_{\pm} + d_{3\pm}$ |

The parameters $b_{\pm}$, $c_{\pm}$, $d_{i\pm}$, $i = 1, 2, 3$ are expressible in terms of the real and symmetric part of the matrix elements of table 4. We present the way of adjusting parameters to the experimental data for the three known families in the next section.

4. Numerical results

The two types of mass matrices in section 3 followed from the two assumed ways of breaking symmetries from $SO(1, 7) \times U(1) \times SU(3)$ down to the observable $SO(1, 3) \times U(1) \times SU(3)$ in the scalar (with respect to $SO(1, 3)$) part determining the Yukawa couplings. Since the problem of deriving the Yukawa couplings explicitly from the starting Lagrange density of the approach unifying spins and charges is very complex, in this paper, we make a rough estimation for each of the two proposed breaks of symmetries in order to see whether the approach could be the right way to go beyond the Standard Model of the electroweak and color interactions and what the approach teaches us about the families. We hope that the perturbative and nonperturbative effects manifest at least to some extent in the parameters of the mass matrices, which we leave to be adjusted so that the masses and the mixing matrix for the three known families of quarks agree (within the declared accuracy) with the experimental data. We also investigate the possibility of making predictions about the properties of the fourth family.
4.1. Experimental data for quarks

We take the experimental data for the known three families of quarks from [26, 27]. We use for masses the data

\[ m_u/\text{GeV} = (0.0015 - 0.005, 1.15 - 1.35, 174.3 - 178.1), \]

\[ m_d/\text{GeV} = (0.004 - 0.008, 0.08 - 0.13, 4.1 - 4.9). \]

Predicting four families of quarks and leptons at ‘physical’ energies, we require the unitarity condition for the mixing matrices for 4, rather than 3 measured families of quarks [26]

\[
\begin{pmatrix}
0.9730 - 0.9746 & 0.2157 - 0.2278 & 0.0032 - 0.0044 \\
0.220 - 0.241 & 0.968 - 0.975 & 0.039 - 0.044 \\
0.006 - 0.008 & 0.035 - 0.043 & 0.07 - 0.0993
\end{pmatrix},
\]

\[ |V_{td}/V_{ts}| = 0.208^{+0.008}_{-0.006} \text{ or } 0.16 \pm 0.04. \]  
(34)

We keep in mind that the ratio of the mixing matrix elements \( |V_{td}/V_{ts}| \) includes the assumption that there exist only three families.

4.2. Results for proposal no. 1

We see that within the experimental accuracy, the (real part of the) mixing matrix may be assumed to be approximately symmetric up to a sign and then accordingly parameterized with only three parameters. Equation (21) offers for the ways of breaking the symmetry \( SO(1, 7) \times U(1) \times SU(3) \) down to the observable \( SO(1, 3) \times U(1) \times SU(3) \) proposed in section (3.1) the relations among the proposed elements of the two mass matrices for quarks on one and the masses of quarks and the three angles determining the mixing matrix on the other side. We have 7 parameters to be fitted to the six measured masses and the measured elements of the mixing matrix within the experimental accuracy. We use the Monte-Carlo method to adjust the parameters to the experimental data presented in equations (33) and (34). We allow the two quark masses of the fourth family to lie in the range from 200 GeV to 1 TeV. The obtained results for \( k \) and the two \( a_{h\eta} \) are presented in table 6. In table 7, the fields \( \tilde{\omega}_{abc} \) are presented. One notices that the Monte-Carlo fit keeps the ratios of the \( \tilde{\omega}_{abc} \) very close to 0.5 (\( k \) is small, but not zero). In equation (35), we present the corresponding masses for the four families of quarks

\[ m_u/\text{GeV} = (0.0034, 1.15, 176.5, 285.2), \]

\[ m_d/\text{GeV} = (0.0046, 0.11, 4.4, 224.0), \]

and the mixing matrix for the quarks

\[
\begin{pmatrix}
0.974 & 0.223 & 0.004 & 0.042 \\
0.223 & 0.974 & 0.042 & 0.004 \\
0.004 & 0.042 & 0.921 & 0.387 \\
0.042 & 0.004 & 0.387 & 0.921
\end{pmatrix},
\]

\[ |V_{td}/V_{ts}| \]  
(36)

For the ratio \( |V_{td}/V_{ts}| \), we find in equation (36) the value around 0.1. The estimated mixing matrix for the four families of quarks predicts quite strong couplings between the fourth and the other three families, limiting some of the matrix elements of the three families as well.
Table 6. The Monte-Carlo fit to the experimental data [26, 27] for the parameters $k$, $a\eta$ and $b\eta$ determining the mixing matrices for the four families of quarks is presented.

|   | u   | d   |
|---|-----|-----|
| $k$ | −0.085 | 0.085 |
| $a\eta$ | −0.229 | 0.229 |
| $b\eta$ | 0.420 | −0.440 |

Table 7. Values for the parameters $\tilde{\omega}_{abc}$ in MeV for the u-quarks and the d-quarks (section 3.1) as obtained by the Monte-Carlo fit relating the parameters and the experimental data.

|   | u   | d   | u/d |
|---|-----|-----|-----|
| $|\tilde{\omega}_{018}|$ | 21 205 | 42 547 | 0.498 |
| $|\tilde{\omega}_{078}|$ | 49 536 | 101 042 | 0.490 |
| $|\tilde{\omega}_{127}|$ | 50 700 | 101 239 | 0.501 |
| $|\tilde{\omega}_{187}|$ | 20 930 | 42 485 | 0.493 |
| $|\tilde{\omega}_{387}|$ | 230 055 | 114 042 | 2.017 |
| $a$ | 94 174 | 6237 |

4.3. Results for proposal no. 2

In section 3.2, assumptions about the way of breaking the symmetries (from $SO(1, 7) \times U(1) \times SU(3)$ to the ‘physical’ ones $SO(1, 3) \times U(1) \times SU(3)$) leave us with two sets of four families of very different masses for the u- and the d-quarks. In table 5, the mass matrices for the lighter of the two groups of four families of quarks are presented in a parameterized way under the assumption that the mass matrices are real and symmetric.

There are six free parameters in each of the two mass matrices. The two off-diagonal elements together with three out of the four diagonal elements determine the orthogonal transformation, which diagonalizes the mass matrix (subtraction of a constant times the unit matrix does not change the orthogonal transformation). The four times four matrix is diagonalizable with the orthogonal transformation depending upon six angles (in general with $n(n−1)/2$). We use the Monte-Carlo method to fit the free parameters of each of the two mass matrices to the elements of the quark mixing matrix equations (34) and the quark masses equations (33) of the three known families. One notices that the matrix in table 5 splits into two times two matrices, if we make parameters $c_{±}$ equal to zero. Due to the experimental data, we expect that $c_{±}$ must be small. The quark mixing matrix is assumed to be real (but not also symmetric as it was in section 4.2). Since there are more free parameters than the experimental data to be fitted, we look for the best fit in dependence on the quark masses of the fourth family. Assuming for the fourth family quark masses the values $m_{u_4} = 285$ GeV and $m_{d_4} = 215$ GeV, the Monte-Carlo fit gives the following mass matrices (in MeV) ($(-b, -a) \cup (a, b)$ meaning...
that both intervals are taken into account) for the u-quarks:

\[
\begin{pmatrix}
(9, 22) & (-150, -83) & (-50, 50) & (-306, 304) \\
(-150, -83) & (1211, 1245) & (-306, 304) & (-50, 50) \\
(-50, 50) & (-306, 304) & (171600, 176400) & (-150, -83) \\
(-306, 304) & (-50, 50) & (-150, -83) & (200000, 285000)
\end{pmatrix}
\]

(37)

and for the d-quarks

\[
\begin{pmatrix}
(5, 11) & (8.2, 14.5) & (-50, 50) & (-198, -174) \\
(8.2, 14.5) & (83 - 115) & (-50, 50) & (8.2, 14.5) \\
(-50, 50) & (-198, -174) & (4260 - 4660) & (-14.5, -8.2) \\
(-198, -174) & (8.2, 14.5) & (-14.5, -8.2) & (200000, 215000)
\end{pmatrix}
\]

(38)

These mass matrices correspond to the following values for the quark masses (only the central values are written):

\[m_u/\text{GeV} = (0.005, 1.220, 171, 285),\]

\[m_d/\text{GeV} = (0.008, 0.100, 4.500, 215),\]

and to the following absolute values for the quark mixing matrix (only the central values are written):

\[
\begin{pmatrix}
0.974 & 0.226 & 0.00412 & 0.00218 \\
0.226 & 0.973 & 0.0421 & 0.000207 \\
0.0055 & 0.0419 & 0.999 & 0.00294 \\
0.00215 & 0.000414 & 0.00293 & 0.999
\end{pmatrix}
\]

(39)

with 80% confidence level. We get \(V_{ud}/|V_{ts}| = 0.128 - 0.149\).

For higher values of the two masses of the fourth family, the matrix elements of the mixing matrix \(V_{ij}\) and \(V_{ui}\), \(i = d, s, t,\) slowly decrease—decoupling very slowly the fourth family from the first three. For \(m_u = 500\,\text{GeV} = m_d\), for example, we obtain \(V_{d4} < 0.000\,\text{93}, V_s < 0.000\,\text{13}, 0.000\,\text{28} < V_b < 0.000\,\text{48}, V_u < 0.000\,\text{93}, V_c < 0.000\,\text{15}\) and \(0.000\,\text{28} < V_{4t} < 0.000\,\text{48}\).

5. Discussions and conclusions

We have studied in this paper whether the approach of one of us \([1]–[10]\) unifying spins and charges might answer those of the open questions of the Standard Model of the electroweak and color interactions which are connected with the appearance of families of fermions, of the Yukawa couplings and of the weak scale.

Starting from the Lagrange density for spinors in \(d(=1 + 13)\)-dimensional space with two kinds of fields (equation (3))—the gauge fields \(\omega_{abc}\) of the Lorentz group \((S^{ab} = \frac{1}{4}(\gamma^a\gamma^b - \gamma^b\gamma^a))\) and the gauge fields \(\bar{\omega}_{abc}\) of \(\bar{S}^{ab} = \frac{1}{4}(\bar{\gamma}^a\bar{\gamma}^b - \bar{\gamma}^b\bar{\gamma}^a)\), with \(\bar{\gamma}^a\) which anticommute with \(\gamma^a\), \(\{\gamma^a, \bar{\gamma}^b\} = 0\)—we end up, by assuming two possible breaks of symmetries (sections 3.1 and 3.2), at the ‘physical’ scale with two types of four families of quarks and leptons corresponding to the two chosen ways of breaking symmetries. In this paper, we study only
the properties of quarks. Parameterization of the quark masses depends strongly on the chosen way of breaking symmetries.

One Weyl spinor in \(d = 1 + 13\), if analyzed in terms of the Standard Model groups \(SO(1, 3) \times SU(2) \times U(1) \times SU(3)\), manifests the left-handed quarks and leptons carrying the weak charge and the right-handed quarks and leptons without the weak charge. It is a long way from the starting simple Lagrange density for spinors carrying only the spins and interacting with only the vielbeins and the two kinds of spin connections (equation (3)) to the observed properties of quarks and leptons. To treat breaking of the starting symmetries properly, taking into account all perturbative and nonperturbative effects, boundary conditions and other effects (by treating gauge gravitational fields in the same way as ordinary gauge fields, since the scale of breaking \(SO(1, 13)\) is supposed to be far from the Planck scale) is a huge project.

In this paper, we estimate the part \(\psi^+ \gamma^0 \gamma^s p_\mu \psi\) with \(s = 7, 8\), of the starting Lagrange density, which determines the Yukawa couplings (equations (7) and (10)) and does what in the Standard Model the Higgs field does. The two chosen ways of breaking the starting symmetries (sections 3.1 and 3.2) in the \(S^{ab}\) sector differ in the part breaking \(SO(1, 7) \times U(1)\) with eight massless families to \(SO(1, 3) \times U(1)\) where four families have low enough masses for all the four to be possibly observable at ‘physical energies’, while the remaining four families with much higher masses are decoupled from the last four families.

In one of the two ways of breaking symmetries, we assume that there are no matrix elements of the type \(\tilde{A}_{kl}^{ab}\), with \(k = \pm\) and \(l = \pm\) (in all four combinations) and with either \((ab)\) or \((cd)\) equal to \((56)\). (In the Poincaré sector, such a choice guarantees the conservation of the electromagnetic charge.) We also assume that the mass matrices are symmetric and real (hoping that this assumption does not influence considerably the real part of the mixing matrices and the masses) and diagonalizable in two steps.

In the second choice of breaking the starting symmetries, we instead assume that all the matrix elements \(\tilde{A}_{kl}^{ab}\) which have \((ab)\) equal either to \((03)\) or \((12)\) and \((cd)\) equal to \((56)\) or \((78)\) are equal to zero, which means that the symmetry \(SO(1, 7) \times U(1)\) breaks into \(SO(1, 3) \times SO(4) \times U(1)\). We then break \(SO(4) \times U(1)\) in the sector \(\tilde{S}^{ab}\), \(s = 7, 8\), so that at some high scale one of \(SU(2)\) in \(SO(4) \times U(1)\) breaks together with \(U(1)\) into \(SU(2) \times U(1)\) and then—at much lower scale—the break of the symmetry of \(SU(2) \times U(1)\) to \(U(1)\) appears. The last four families acquire their masses with the second break, that is the break of the weak charge. We again end up with four families decoupled from the much heavier four families with the quark mass matrices differing strongly from the mass matrices in the first choice. We assume again that the mass matrices are real.

We make the calculations on the tree level, obtaining mass matrices for quarks in both the chosen ways parameterized with the fields, whose strengths depend on the way and the scale of breaking symmetries. We let the perturbative and nonperturbative effects be (at least to some extent) included in the fields, for which we assume that they are free parameters to be determined by fitting the masses and the mixing matrix to the known experimental data within the known accuracy.

The symmetries of the mass matrices in the first chosen way of breaking the starting symmetries locate (after assuming real and symmetric mass matrices, diagonalizable in two steps) the masses of the four families to be in the region for which the analyses in [23, 28, 29] show that it is experimentally allowed. The second choice of breaking the symmetries (although each of the mass matrices have only two off-diagonal elements) does not determine the masses of the fourth quark family, leaving these masses as free parameters. Both choices predict rather
strong coupling among the observed three and the fourth family. The fourth family decouples in the second choice of breaking symmetries from the first three for quite high values for the fourth family quark masses. The calculated ratio $|V_{td}|/|V_{ts}|$ differs for both assumed ways of breaking the symmetries from the measured one (we obtain $|V_{td}|/|V_{ts}|$ equal to 0.128–0.149 in the second case), which is expected since the measured value is obtained with the inclusion of the calculations made under the assumption that there are only three families.

Both the chosen ways are very approximate. To state which of the two ways is more trustable, further (more demanding) calculations have to be made. However, it seems quite acceptable that breaks of symmetries go in both sectors—the Poincaré one and the one connected with $\tilde{S}^{ab}$—through two steps of breaking the symmetries $SO(1,7) \times U(1)$ (as suggested by the second way of breaking symmetries) resembling in both steps the Standard Model way of breaking the symmetry in the spinor sector, suggesting that the second way might be the right one, although one can not at all expect that the break of symmetries in both sectors manifests in the same way. This paper is to be understood as a first step to further calculations, which should at the end tell whether our way of describing charges and families is the right way beyond the Standard Model of the electroweak and color interactions.

What are the further steps in analyzing the possibility that the approach unifying spins and charges is the right way to answer the open questions of the Standard Model of the electroweak and color interactions? Since the ways of breaking symmetries chosen in this paper to come to the effective Lagrange density of the Standard Model determines, as written in the introductory section 1, also properties of the effective Lagrange density for the gauge fields and the scalar fields, we have studied within the same approach and assumed the same two ways of breaking the starting symmetries, the gauge field sector and the scalar fields sector: the vacuum expectation values of the scalar fields and the dynamics of both kinds of fields. Since the lepton sector is as important as the quark sector, we have studied the lepton sector as well, although it seems [25] that calculations beyond the tree level are for leptons, in particular for neutrinos, very important. Since the approach unifying spins and charges predicts more families, we shall investigate the possibility that dark matter, which contributes 6–7 times more to the mass of the observed part of our universe, is made out of the families which (almost) decouple from the observed three [30]. We also note that within the approach unifying spins and charges, the explanation of a possible new Higgs discussed in several papers in the literature are very natural here since the perturbation theory below the tree level of course has terms which might be interpreted as the new Higgs.

The approach unifying spins and charges makes two assumptions: (i) spinors carry only spin and no charges, but in $d > 1 + 3$ and (ii) there are two Clifford algebra objects used for describing the properties of spinors, the Dirac one determining the spin and the charges and the second one determining the families: the two degrees of freedom being strongly entangled through the Yukawa couplings.

Let us end this paper by admitting again all the difficulties which the approach unifying spins and charges confronts (most of the difficulties were already pointed out in the introduction and also in this conclusion section). Although the starting action for a spinor (and for the corresponding spin connections and vielbeins) is very simple and transparent, breaking down the starting symmetries of $SO(1,13)$ to the observed low-energy sector, is a very demanding problem which we must prove is at all solvable. We have solved problems step-by-step under some assumptions (and also through simplified toy models), but most of the problems are still confronting us. From these approximate steps to the consistent implementation of the approach.
of one of us (NSMB) is a long and very demanding way which even might not lead to what
we hope it does: to the right way beyond the Standard Model. In this paper, we break ‘by
hand’ all the symmetries when testing which one might lead to the observed phenomena. We
hope, however, that we shall prove that it does break spontaneously and why it does (and we
are intensively studying possible reasons for such breaks). But because of all the nonadiabatic
effects, the real calculations will hardly go without explicit breaking of symmetry (as we see
in hadron physics). What makes one of us (SNMB) trust the approach as the right way beyond
the Standard Model is that the two Clifford algebra objects do exist, the interplay of which
manifests, if breaking appropriately the starting symmetry, the properties of one family of
quarks and leptons, the Yukawa couplings among the families and correspondingly also the
gauge fields.

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