Transformer Ratio at Wakefield Excitation by Train of Electron Bunches with Linear Growth of Current in Dielectric Resonator Electron–Positron Collider

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Abstract. The efficiency of electron bunch acceleration by wakefield, excited in two-beam electron-positron dielectric resonator collider by train of electron bunches is determined by transformer ratio ($TR$). The train of homogeneous bunches, in which current grows linearly along a train, is considered. The length of homogeneous bunch equals to half of excited wavelength $0.5\lambda$. Interval between bunches is $p\lambda$, $p = 1, 2, \ldots$. The conditions have been formulated, when the wakefield pulses from all electron bunches are added. Bunch is injected in dielectric resonator, when the back wavefront of the wakefield pulse is on the bunch-injection boundary of the resonator/cavity. Bunch leaves the resonator, when the first wavefront of wakefield pulse is on the end of the resonator. The conditions for a large $TR$ have been formulated. The wakefield and $TR$ have been derived after $N$ bunch. The computer simulation has been performed taking into account wakefield pulse dispersion spreading and electron bunch transition radiations. The results of analytical investigation are compared with the results of computer simulation. It has been shown that the dependence $TR = 2N$ is performed. Advantage of this mode with large transformer ratio that some injected bunch with decreased current becomes witness. When this bunch is injected it gets in maximum accelerating field.

The infinite periodical mode of accelerator operation has been proposed. In this mode, witness-bunches and driver-bunches are injected according to periodic mode. In the infinite periodical mode, short shaped trains of driver-bunches are alternated with witnesses. Large transformer ratio and identical decelerating field for all driver-bunches are provided by this long periodic train.
1. Introduction
Transformer ratio (TR) can be approximately determined as the ratio $TR = E_{ac}/E_{dr}^{-1}$ [1]. There $E_{ac}$ is the accelerating wakefield for witness-bunch and $E_{dr}$ is the maximum decelerating wakefield for driver-bunches. Certainly that $E_{ac} \approx E_{dr}$, if $TR \approx 1$. Earlier $TR$ increasing in plasma [2-11], in waveguide (dielectric-loaded) [12-13], for collinear accelerating scheme of electron bunches [14], for laser pulse electron acceleration [15-18], for acceleration of electrons in metallic cavity [19] and for the collinear wakefield acceleration schemes in single and many mode structures [20] has been studied. Authors investigate the dielectric loaded cavity scheme of $TR$ increasing in this article. Dielectric loaded cavity scheme has an advantage for acceleration of electrons because of wakefield addition and accumulation [21-25] when train of electron bunches is used as driver.

Starting from certain bunch number the injected bunches do not interact at the waveguide entry with wakefield, excited by the first bunch. Because of this only limited number of bunches takes part in coherent addition of their wakefields in a waveguide. The difference of $v_{b} - v_{gr}$ determines this number of bunches. There $v_{b}$ is the velocity of the bunches. The wakefield pulse spreads with the group velocity $v_{gr}$. For real conditions this number so small and this limits wakefield amplitude. Using the cavity scheme has been proposed in [26] to get rid of such difficulty. The face planes “trap” the wakefield pulse that is excited by short bunches of train.

At time of the injection of next bunch the wakefield pulse should be situated at the cavity’s front (face plane) of the bunch injection. Wakefields from bunches should lead to a coherent wakefields’ addition of a large number of bunches.

Numerical simulation [27] has shown that in the cavity case wakefield is reflected both from the front of the bunch injection and from the cavity exit. The wakefield amplitude should increase with additional bunches.

Ways of transformer ratio increasing in the cavity is very important. This task is studied in this article in the dielectric loaded cavity. This scheme of transformer ratio increasing has been used in [3, 4] for excitation of wakefield in plasma.

We apply the charges linearly growing along the train according to the ratio $Q_{1}:Q_{2}:Q_{3}...$ as 1:3:5 ... [3-5, 24, 28, 29] similar to the plasma and waveguide schemes. There $Q_{i}$ is the i-th bunch charge.

The each bunch is uniform along its axis. The bunch length is used $L_{bn} = 0.5 \lambda$, $\lambda$ is the excited wavelength, determined by Cerenkov condition $k v_{b} = \Omega_{w}$, $k = 2\pi \lambda^{-1}$, $\Omega_{w}$ is the wave frequency.

The purposes of the article:
- to prove that the high $TR$ value in the cavity can be provided by ramped train of electron bunches under the conditions: $\Omega_{p}^{\perp} = (p + 0.5)(1 - v_{gr}^{-1})^{-1}$, $\Lambda = \pi v_{gr}^{-1} \Omega_{p}^{\perp}$, $p$ is the positive integer (see equations (1), (2)), $\Omega_{b}$ is the repetition frequency of bunches, $\Lambda$ is the cavity length;
- to suggest for the continuous operation of the accelerator the periodic regime, because of the bunch charge is not large infinitely, and to formulate conditions for providing the high transformer ratio in periodic regime;
- to prove low boundary effects on the transformer ratio growth when bunches number increases.

We study the cavity pumping with infinite quality factor $Q$, due to the slightly effects of the $Q$-factor on $TR$ value (in our case). For number $K$ of bunches, satisfying $K < Q \pi^{-1}$, one can neglect dumping. To get the same $TR = 2K$ in the case of finite $Q$-factor it is necessary, one can show, only slightly to change the charges ratio $Q_1 : Q_2 : Q_3 ...$

2. Transformer ratio at wakefield excitation in dielectric-loaded cavity by a train of electron bunches with linear growth of current along train

The cavity length $\Lambda$ meets the two conditions (1) (see below equation (1)). First front of every bunch of the train is injected into the cavity from the left at the time of the returning the back front of the excited wakefield pulse, formed by previous bunches, to the bunch-injection front of the cavity (see figure 1 (a)). Every bunch leaves the cavity at the time, when first front of the excited by previous bunches wakefield pulse comes to exit of the cavity (see figure 1 (b)). Rough shape of the wakefield pulse, formed by previous $K$ bunches and being formed by $(K+1)$-th bunch, at the time when $(K+1)$-th bunch reaches to the cavity center, we see in figure 2. Coherent addition of wakefields from all driver-bunches is realized at the two conditions (1) (see equation (1)).

$$
\tau = 2\pi \Omega_b^{-1} = 2\Lambda v_{gr}^{-1} = \pi(2p+1)\left(\Omega_a \left(1 - v_{gr}^{-1} v_b\right)\right)^{-1}
$$

(1)

**Figure 1.** A shape of the excited wakefield pulse, when $(K+1)$-th bunch gets into the cavity (a) and leaves the cavity (b). $(K+1)$-th bunch is shown in red rectangle. The wakefield amplitude after $(K+1)$-th bunch (shown in figure 1 (b)) is higher than the wakefield amplitude before $(K+1)$-th bunch (shown in figure 1 (a)). The reflecting planes for wakefield pulse are located at $z = 0$ and $z = \Lambda$.

**Figure 2.** A shape of the excited field pulse, when $(K+1)$-th bunch gets into the center of the cavity. Direction of travel of the electron bunch is shown in red arrow. Direction of travel of the wakefield pulse is shown in black arrows. One can see that decelerating wakefield is of small amplitude into area of $(K+1)$-th bunch location.
These two conditions are the large transformer ration condition and the condition of coherent addition of wakefields from all driver-bunches. There \( \tau = 2\Lambda v_g^{-1} \) is the excited wakefield pulse period of the longitudinal oscillations with group velocity \( v_g \) between bunch-injection front of the cavity (left in figure 1 (a)) and bunch-exit front of the cavity (right in figure 1 (a)), \( 2\pi\Omega_p^{-1} \) is the period between two consecutive injections of bunches, \( \Omega_p \) is the repetition frequency of bunches, \( p \) is the integer number (positive).

Unlike of the plasma scheme [3, 4] and waveguide case in our case of the cavity for TR increasing the ratio of the excited wave frequency \( \Omega_w \) and the repetition frequency of bunches \( \Omega_p \) needs to be changed to (see equation (1))

\[
\Omega_w \Omega_p^{-1} = (p + 0.5)(1 - v_g^{-1} \lambda)^{-1}
\]  

The expression (2) has been obtained from the requirement equation (1) that decelerating field \( E_z \) for all driver-bunches should be equal to decelerating field \( E_m \) for the first bunch. This is completed when the accelerating field comes to the back front of the returned wakefield pulse when the each bunch is injected. At addition of a larger excited (by \( N \)-th bunch) decelerating field to the smaller accelerating field (after \( (N - 1) \)-th bunch) of the pulse, the decelerating wakefield is obtained to be equal to decelerating field \( E_m \) for the first driver-bunch. Then a large TR and increase of accelerating field are provided.

For integer or half-integer \( \Lambda \lambda^{-1} \) the condition equation (1) can be reduced to

\[
\tau = 2\Lambda v_g^{-1} = 2\Lambda v_p^{-1} = \pi(2P + 1)\Omega_p^{-1},
\]

if the new value \( P = p + 2\Lambda \lambda^{-1} \) is used, which is the positive integer. From the second condition (3) (see equation (3)) one can derive \( P = 2\Lambda \lambda^{-1} v_g^{-1} - 0.5 \), taking into account \( \Omega_w = 2\pi v_g \lambda^{-1} \).

The distribution of the current \( I_{in} \) of such train of uniform in the longitudinal direction bunches equals to:

\[
I_{in}(z,t) = I_{in}(2K - 1), K \geq 1,
\]

\[
0 < v_b \left(t - \tau(K - 1)\right) - z < L_{in}, F = \left(\Lambda + L_{in}\right)v_p^{-1}, \tau(K - 1) < t < \tau(K - 1) + F
\]

(see in figure 3). \( I_{in} \) is the first bunch current, \( K \) is the number of injected bunches.

**Figure 3.** The bunches are uniform, along the axis, cylinders. The current of ramped train of these bunches are distributed according to linear dependence.

Let us apply the function [13]:

\[
Z_z = [RQ^{-1}](0.5\Omega_w)\cos(\Omega_w T)
\]

Considering one-mode approximation and using this function, we obtain the excited field in the cavity for the time interval \( \tau(K - 1) < t < \tau(K - 1) + F \) at field pulse excitation by the \( 1^{st} \) bunch

\[
E_z(z,t) = E_m\{[\theta(v_b T_b - z) - \theta(v_b T_b - L_{in} - z)]S +
\]

(6)
\[
[\theta(v_{T_k} - z)]2KS + [\theta(v_{T_k} + \Lambda - z)]2(K-1)S,
\]
\[S = \sin(\Omega_z T), I = 1 - v_{T_k}v_{y_k}^{-1}, T_k = t - \tau(K-1),
\]
\[\theta(y)\] is the function of Heaviside, \(E_m = [RQ^{-1}](0.5I_{bm}), T = t - zv_{y_k}^{-1}, [RQ^{-1}]\) is known figure of quality for considered wave of this cavity [13]. Equation (6) has been obtained by integrating a wakefield, created by a very short bunch, along the bunch with taking into account the current distribution. The first term describes decelerating field inside the \(K^{th}\) bunch, and the second term presents the field in the past of the \(K^{th}\) bunch, the third term presents field excited by previous \(K-1\) bunches. We can derive \(TR\) after the \(K^{th}\) bunch. We get large value of 2nd term of equation (6) and the largest value of first term of (6). After analyzing these values, we can find that \(TR\) is equal to 2\(K\) after the \(K^{th}\) bunch. Previous authors obtained this in [3, 5, 6] for plasma waveguide schemes. One can show that the \(K^{th}\) bunch decelerating field equals to the first bunch decelerating field.

For the validation of the method, one can tend the length of the bunch to zero and to obtain the wakefield, excited by a very short bunch.

In \(TR\) calculations and for comparison with analytical \(TR\) we use parameters which are close to those in [30]: the dielectric permeability \(\varepsilon = 1.725\), the external dielectric radius \(R_{m} = 250 \mu m\), \(\Lambda = 5.56 mm\). The bunch is simulated by 10 identical bunches, each of which is a short disk of charge \(Q_{b} = 2nC\). The transversal shape of bunch is Gaussian. Also we use the bunch energy \(W_{bm} = 1 GeV\), the radius of bunch \(R_{bm} = 150 \mu m\), the bunch length \(L_{bm} = 0.5\lambda\). We use the fully dielectric-loaded cavity in our investigations, without channel for bunches. In figure 4 one can see the z-distribution of the excited electric field \(E_{z}\), when the first, second and third bunch is on the distance \(z = 0.15 cm\) from the bunch-injection cavity front. Because \(TR\) depends on longitudinal coordinate, we use \(TR\) average value along cavity. For first bunch the average value along cavity \(TR\) is \(TR_{1} \approx 1.9\); for second bunch the average value of \(TR\) is \(TR_{2} \approx 3.35\), for 3rd bunch the average \(TR\) is \(TR_{3} \approx 5.03\). We consider that \(TR\) is smaller than value \(TR = 2K\), obtained from the results, without considering of transition radiation and dispersion spreading. But taking into account of the transition radiation and dispersion spreading does not change considerably of \(TR\) value. The results of computer simulation qualitatively agree with the results of analytical calculations.

![Figure 4](image_url)

**Figure 4.** Wakefield \(E_{z}\) (in statvolt/cm), excited by first (a), second (b) and third (c) bunches.

Wakefield \(E_{z}\) shown in \(z = 0.15 cm\) from the point of the injection of bunches.

The coordinate is in \(cm\).
For increase of power transmission from driver-bunches to the excited field and after that to witness-bunches we consider a chain of dielectric-loaded cylindrical cavities. We suggest decelerating field \( E_{dc} = 80 \text{MV/m} \) and \( TR = 12 \) at \( K = 6 \) and accelerating field \( E_{ac} \approx 1 \text{GV/m} \). Then at length 250 m of accelerator, driver-bunches of energy 20 GeV are entirely decelerated (or on each of 10 decelerating parts, each of dimension 25 m, driver-bunches of energy 2 GeV are entirely decelerated). Then the maximum energy of witness-bunches in this accelerator is equal to 250 GeV.

3. Infinite periodic train of short ramped trains of driver-bunches, alternated with witness-bunches

At continuous operation of the accelerator, behind every \( G-th \) driver-bunch a witness of comparatively high current is injected. Behind witness-bunch the value of the wakefield falls off from

\[
E_{ac} = KE_i \quad \text{to} \quad \mu E_{ac} = \left( K - G \right) E_i, \quad \mu = \left( K - G \right) K^{-1} < 1, \quad G = K \left( 1 - \mu \right).
\]

\( E_i \) is the field behind the first bunch. Then \( TR_e \) in periodic case is equal to

\[
TR_e \approx \left[ KE_i + \left( K - G \right) E_i \right] \left( 2E_{dc}^{-1} \right)^{-1} = \left( K - 0.5G \right) E_i E_{dc}^{-1}
\]

We apply that \( TR = 2K \) known at \( G = 0 \). In this case \( E_i E_{dc}^{-1} = 2 \) and the coupling of \( TR_e \) with \( \mu \) and with number of bunches \( K \) of the train, after that the periodic asymptotic field is set, is

\[
TR_e \approx K \left( 1 + \mu \right)
\]

\( E_{dc} \) is the largest decelerating field for the bunch-driver.

One can derive that the largest \( TR_e \) is equal to \( TR_e = 2K - 1 \).

Figure 5. The current of the infinite periodic train. After short ramped train of \( K \) bunches the infinite train is a periodic train. High-current witness there is in each period. After it the short-ramped train of the driver-bunches is followed.

Two witness-bunches shown by arrows.

4. Conclusions

The conditions for a large transformer ratio in dielectric loaded cavity/resonator have been formulated. The wakefield and transformer ratio have been derived after \( N \) bunch analytically and by computer simulation. As a result of study one can conclude that transformer ratio can become large in considered case when wakefield is excited in dielectric loaded cavity by ramped train of bunches. Each bunch is the longitudinally uniform cylinder of high-energy electrons. Current of bunches increases along train according to linear dependence.

We evaluated the specific parameters for the accelerator.
The computer simulation has been performed taking into account dispersion spreading and transition radiations. Taking into account of the transition radiation and dispersion spreading does not change considerably value of transformer ratio. The results of computer simulation qualitatively agree with the results of analytical calculations.

Advantage of this mode with large transformer ratio that some injected bunch with decreased current becomes witness. When this bunch is injected it gets in back wavefront of pulse and in maximum accelerating field.

The infinite periodical mode of accelerator operation has been proposed. In this case witness-bunches and driver-bunches are injected according to periodic mode. In the infinite periodical mode, short shaped trains of driver-bunches are alternated with witnesses. Large transformer ratio and identical decelerating field for all driver-bunches are provided by this long periodical train.

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