Color-octet heavy quarkonium productions
in $Z^0$ decays at LEP

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Abstract

We consider the energy and the polar angle distributions of $J/\psi$'s produced via the color-singlet and the color-octet ($^3S_1^{(8)}$) mechanisms in $Z^0 \to J/\psi + X$ at LEP. Since both distributions of the $J/\psi$ produced via color-octet mechanism are significantly different from those via color-singlet mechanism, these observables can be used as tests of color-octet production mechanism for heavy quarkonia. We also discuss $Z^0 \to \Upsilon + X$ and $W \to J/\psi (\text{or} \Upsilon) + X$ in brief.

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Heavy quarkonium productions in high energy processes have long been studied in perturbative QCD along with the color-singlet model. However, the recent measurements of $J/\psi$ and $\psi'$ productions at the Tevatron put serious questions on such theoretical approaches \cite{1}. In short, the data for $p\bar{p} \rightarrow \psi' + X$ is larger than the color-singlet model by a factor of $\sim 30$. In order to resolve this discrepancy, Braaten and Fleming suggested a color-octet fragmentation of an energetic gluon into $\psi'$ by emission of soft gluons \cite{2}. This approach requires one parameter, $\langle 0 | O_{\psi'}^{S_1^3}(3S_1^1) | 0 \rangle$, the matrix element of a color-octet, dimension-six, four-quark operator in the Nonrelativistic QCD, which is the effective field theory for QCD relevant to the heavy quarkonium physics \cite{3}. Braaten and Fleming showed that if one fixes this parameter to fit the measured cross section, then the observed spectrum in $p_T$ is reproduced. Then, Cho and Leibovich \cite{4} extended this approach in order to go beyond the fragmentation picture (which is applicable only to the limit $p_T^2 \gg m_{\psi'}^2$), including other color-octet intermediate states such as $^1S_0^8(1S_0^0)$ and $^3P_J^{o=1,2}(3P_J^0)$ in terms of two more NRQCD matrix elements, $\langle 0 | O_{\psi'}^{S_0^1}(1S_0^1) | 0 \rangle$ and $\langle 0 | O_{\psi'}^{P_0^3}(3P_0^3) | 0 \rangle$. This is especially relevant for the case of inclusive $\Upsilon(nS)$ productions at the Tevatron, since the $p_T$ in this case is not that large compared to $m_\Upsilon$ and thus the fragmentation picture is not valid any more. Overall descriptions for $\psi(')$ and $\Upsilon(nS)$ productions at the Tevatron seem reasonably good by now, although some color-octet matrix elements are shown to lead too large contributions to $J/\psi$ photoproductions \cite{3} and $B \rightarrow J/\psi + X$ \cite{6}.

In view of this, it is important to test the idea of color-octet mechanism in the heavy quarkonium productions in places other than at the Tevatron. One can think of the $S-$wave charmonium production in $B$ decays \cite{3,7}, the angular dependence of $J/\psi$ production cross section in the $e^+e^-$ annihilations \cite{8}, $Z^0 \rightarrow J/\psi + X$ \cite{9,10}, $J/\psi$ productions at the $\gamma p$ collision \cite{11,12}, the associated $J/\psi + \gamma$ hadroproduction \cite{13} and many others. All of these have formed an active research field recently \cite{12}.

In this letter, we reconsider the inclusive heavy quarkonium production in $Z^0$ decays, which is accessible at the current LEP experiments. This study is partly motivated by the recent report by OPAL collaboration \cite{13} that they have observed excess of events for
$Z^0 \to \Upsilon(nS) + X$ for $n = 1, 2, 3$, larger than theoretical expectation by a factor of $\sim 10$

$$\sum_{n=1}^{3} B(Z^0 \to \Upsilon(nS) + X)_{\text{exp}} \simeq (1.2^{+0.9}_{-0.6} \pm 0.2) \times 10^{-4},$$  \hspace{1cm} (1)

compared to the $b$–quark fragmentation contribution $\text{[14]}$,

$$\sum_{n=1}^{3} B(Z^0 \to \Upsilon(nS) + X)_{\text{frag}} \simeq 1.6 \times 10^{-5}. \hspace{1cm} (2)$$

Similar excess were also observed in the prompt $J/\psi$ and $\psi'$ production in $Z^0$ decays, although the experimental errors are quite large $\text{[15]}$:

$$B(Z^0 \to J/\psi + X) = (3.0 \pm 2.3) \times 10^{-4}, \hspace{1cm} (2.6 \times 10^{-5}) \hspace{1cm} \text{(3)}$$

$$B(Z^0 \to \psi' + X) = (2.2 \pm 1.5) \times 10^{-4}, \hspace{1cm} (1.1 \times 10^{-5}) \hspace{1cm} \text{(4)}$$

where the results for the heavy quark fragmentations $\text{[14]}$ are shown in the parentheses for comparison. The issues of the branching ratios and the energy distributions of $J/\psi$ in $Z^0 \to J/\psi + X$ have been addressed already in Refs. $\text{[9]}$ and $\text{[10]}$. In this letter, we suggest another observable, the polar angle distribution of $J/\psi$ (relative to the $e^+e^-$ beam directions at LEP) for the color-singlet and the color-octet contributions, in order to provide another independent check of the idea of color-octet mechanism in heavy quarkonium productions.

In order to study the angular distribution of $J/\psi$ produced in the $Z^0$ decays at LEP, it is convenient to define $S(E)$ and $\alpha(E)$ as

$$\frac{d^2\Gamma_{1,8}}{dE_{\psi}d\cos\theta_{\psi}} \equiv S_{1,8}(E_{\psi}) \left(1 + \alpha_{1,8}(E_{\psi}) \cos^2\theta_{\psi}\right), \hspace{1cm} \text{(5)}$$

following the case of $e^+e^- \to \gamma^* \to J/\psi + X$ at CLEO energy, $\sqrt{s} \approx 10.6 \text{ GeV} \text{[8,16]}$. The subscripts 1, 8 denote the color-singlet and the color-octet contributions, respectively. Since we are concerned with the polar angle ($\cos \theta_{\psi}$) distribution of $J/\psi$ in the rest frame of $Z^0$ as well as the energy distribution, we have to include the polarization of the $Z^0$ produced through the $e^+e^-$ annihilations at LEP (with $\sqrt{s} = M_Z$). This effect can be conveniently described by replacement of $\epsilon^{\mu}(Z)\epsilon^{*\nu}(Z)$ in the decay process by the density matrix $\rho^\mu_\nu$, 

$$\rho^\mu_\nu = \frac{1}{3} \left( -g^{\mu\nu} + \frac{Z^{\mu}Z^{\nu}}{M_Z^2} \right) - \frac{i}{2M_Z} \epsilon^{\mu\lambda\tau} Z_\lambda P_\tau - \frac{1}{2} Q^{\mu\nu}, \hspace{1cm} \text{(6)}$$
where $Z^\mu$ is the 4-momentum of $Z^0$.

The vector polarization $P^\mu$ and the tensor polarization $Q^{\mu\nu}$ in Eq. (3) can be obtained in the case of $e^+(k_2) e^-(k_1) \rightarrow Z^0$ process as following [17]:

$$P^\mu = \frac{\Delta^\mu}{M_Z} \frac{2g_V^e g_A^e}{(g_V^e)^2 + (g_A^e)^2}$$

$$Q^{\mu\nu} = \frac{1}{3} \left( -g^{\mu\nu} + \frac{Z^\mu Z^\nu}{M_Z^2} \right) + \frac{\Delta^\mu \Delta^\nu}{M_Z^2},$$

(7)

where $\Delta^\mu \equiv (k_1 - k_2)^\mu$.

After obtaining the invariant amplitudes for the color-singlet and the color-octet processes, we have performed the symbolic manipulations over the squared amplitude with the above polarization tensors and integrated over the phase space, using REDUCE and Mathematica. The resulting expressions for $S_{1,8}(E)$ and $\alpha_{1,8}(E)$ are rather lengthy, and will be shown separately elsewhere [18]. Our formalism with the density matrices for polarized particles can be applied to other processes where $Z^0$'s are produced via different mechanism from the $e^+e^- \rightarrow Z^0$. For such processes, the density matrix for $Z^0$ will be different from Eqs. (7). The explicit relation between the density matrices of $J/\psi$ and $Z^0$ in $Z^0 \rightarrow J/\psi + X$ for an arbitrary $Z^0$ polarization will be discussed there, too.

Let us first consider the conventional color-singlet contribution to $Z^0 \rightarrow J/\psi + X$ at LEP. The most dominant contribution at LEP energy scale is known [9,10] to come from $Z^0 \rightarrow (c\bar{c})(3S^1_1) + c + \bar{c}$, which can be further approximated into the (anti)c–quark fragmentation into $J/\psi$ as [14]. In this work, we employ the full amplitude for the color-singlet $J/\psi$ production without using the fragmentation approximation. The relevant Feynman diagrams are shown in Fig. 1. The corresponding amplitude can be obtained as usual in the covariant gauge. For simplicity, we do not show the amplitude for the color-singlet $J/\psi$ production in $Z^0 \rightarrow J/\psi + X$ explicitly. However, the corresponding expressions for $S_1(E)$ and $\alpha_1(E)$ will be given later [18].

Next, we consider $Z^0 \rightarrow q\bar{q} + g$ followed by $g \rightarrow 3S^1_1 \rightarrow J/\psi + (\text{soft hadrons})$ with $q = u, d, c, s, b$. The relevant Feynman diagrams are shown in Fig. 2. Other diagrams are higher orders in $v^2$, or suppressed by the short distance factor $m_Q^2/M_Z^2$, and thus irrelevant.
to $Z^0 \rightarrow q\bar{q} + J/\psi(\text{or } \psi', \Upsilon(nS))$. It is convenient to use the following variables:

$$s \equiv (Z - P)^2 = (q_1 + q_2)^2,$$

$$t \equiv (Z - q_1)^2 = (P + q_2)^2,$$

$$u \equiv (Z - q_2)^2 = (P + q_1)^2,$$

with $s + t + u = M_Z^2 + M_\psi^2 + 2m_q^2$, and $P$, $q_1$ and $q_2$ are the 4-momenta of $J/\psi$, $q$ and $\bar{q}$, respectively. The color-octet contribution of $J/\psi$ production via the process $Z_0 \rightarrow q\bar{q} + J/\psi$ factorizes into short and long distance pieces as

$$\mathcal{M}(Z_0 \rightarrow q\bar{q}J/\psi)_{\text{octet}} = \mathcal{M}(Z_0 \rightarrow q\bar{q}g^*(\rightarrow c\bar{c}[3 S_1^{(8)}])]_{\text{short distance}} \times \mathcal{M}(c\bar{c}[3 S_1^{(8)}] \rightarrow J/\psi)_{\text{long distance}}.$$

In terms of $s, t, u$ variables, the amplitude for the process $Z_0 \rightarrow q\bar{q}g^*(\rightarrow c\bar{c}[3 S_1^{(8)}])$ is given by

$$\mathcal{M}(Z_0 \rightarrow q\bar{q}g^*(\rightarrow c\bar{c}[3 S_1^{(8)}])]_{\text{octet}} = \frac{eg_\gamma^2}{M_{J/\psi} \sin \theta_W \cos \theta_W} (\mathcal{M}_{1\mu} + \mathcal{M}_{2\mu}) \epsilon_{\mu}^Z(Z) \epsilon^*_{\psi}(P),$$

where $g_V$ and $g_A$ for the up-type and the down-type quarks are given as

$$\langle g_V, g_A \rangle = \begin{cases} 
  \left( +\frac{1}{4}, -\frac{2}{3} \sin^2 \theta_W, -\frac{1}{4} \right), & \text{for } q = u, c, t, \\
  \left( -\frac{1}{4}, +\frac{1}{3} \sin^2 \theta_W, +\frac{1}{4} \right), & \text{for } q = d, s, b.
\end{cases}$$

The long distance contribution due to the color-octet fragmentation of a gluon into the color-singlet heavy quarkonium $J/\psi$, $\psi'$ or $\Upsilon(nS)$ is described in terms of a parameter $\mathcal{M}_8(nS)$, which depends on the heavy quarkonium state one considers. Cho and Leibovich have determined these parameters for $nS = \Upsilon(nS), J/\psi$ and $\psi'$ : for example,

$$\sum_{n=1}^{3} \mathcal{M}_8(\Upsilon(nS)) \equiv \sum_{n=1}^{3} |\mathcal{M}(b\bar{b}[3 S_1^{(8)}] \rightarrow \Upsilon(nS))|^2 = 6.4 \times 10^{-3} \text{ GeV}^2,$$

$$\mathcal{M}_8(J/\psi(1S)) \equiv |\mathcal{M}(c\bar{c}[3 S_1^{(8)}] \rightarrow J/\psi)|^2 = 0.68 \times 10^{-3} \text{ GeV}^2.$$
After one averages and sums the squared amplitude over the initial and the final spin states using Eqs. (6), and performs the phase space integration, one easily finds various distributions as well as the branching ratios.

We first show the $z(\equiv 2E_\psi/M_Z)$ dependence of $\alpha_{1,8}(E_\psi)$ for the color-singlet (in dashed curve) and the color-octet (in solid curve) case respectively in Fig. 3 (a). We note that the $\alpha_1(E)$ abruptly changes its sign from $\sim -0.8$ to $+1$ near the end point of $z \approx 1$. This originates from the smallness of $\delta = 2M_\psi/M_Z \approx 6.6 \times 10^{-2}$. In Fig. 3 (b), we show the $\alpha(E_Y)$ for the case of $Z^0 \rightarrow Y(1S) + X$. The behavior near $z \approx 1$ becomes less abrupt here, as a result of the not-too-small $\delta \approx 0.42$ in this case. We have checked that this abrupt behavior near $z \approx 1$ dies out as we increase $\delta$.

Our energy distributions shown in Fig. 4 (a) agree with those obtained by [8] and [10], which is a check of our calculations based on the polarized $Z^0$ decays. The color-octet mechanism (shown in the solid curve) produces softer $J/\psi$’s compared to the color-singlet mechanism (shown in the dashed curve). Most $J/\psi$’s are predicted to have energy around $\sim 5.5$ GeV, if the color-octet mechanism works. Similar is true of the $Y$ energy distribution shown in Fig. 4 (b).

The $\cos \theta_\psi$ distributions($\theta_\psi$ is the angle between the initial electron and the final $J/\psi$ direction) can be obtained by integrating (5) over the heavy quarkonium energy $E_\psi$. The color-singlet contribution predicts the polar angle distribution to be

$$\frac{d\Gamma_1}{d \cos \theta_\psi} = (5.4 \times 10^{-2}) \left(1 + 0.92 \cos^2 \theta\right) \text{ MeV.} \quad (17)$$

$$\frac{d\Gamma_1}{d \cos \theta_Y} = (9.7 \times 10^{-3}) \left(1 + 0.73 \cos^2 \theta\right) \text{ MeV.} \quad (18)$$

On the other hand, the color-octet mechanism predicts

$$\frac{d\Gamma_8}{d \cos \theta_\psi} = (25.8 \times 10^{-2}) \left(1 + 0.34 \cos^2 \theta\right) \text{ MeV.} \quad (19)$$

\footnote{For the case of the $J/\psi$ production at CLEO energy through $e^+e^- \rightarrow \gamma^* \rightarrow J/\psi + X$ [8] [10], the corresponding $\delta = M_\psi/E = 3.1$ GeV/5.29 GeV $\approx 0.59$ is not small.}
\[ \frac{d\Gamma_8}{d\cos\theta_\psi} = (36.7 \times 10^{-3}) (1 + 0.27\cos^2\theta) \text{ MeV}. \]  

Adding these two contributions together, we get

\[ \frac{d\Gamma_{1+8}}{d\cos\theta_\psi} = (31.2 \times 10^{-2}) (1 + 0.44\cos^2\theta) \text{ MeV}. \]  

\[ \frac{d\Gamma_{1+8}}{d\cos\theta_\Upsilon} = (46.4 \times 10^{-3}) (1 + 0.37\cos^2\theta) \text{ MeV}. \]  

Therefore, we find that the color-octet mechanism makes the angular distributions flatter compared to that by the color-singlet mechanism alone. The polar angle distributions for \( Z \to J/\psi + X \) and \( Z \to \Upsilon + X \) are shown in Figs. 5 (a) and (b). For each plot, the color-singlet, the color-octet and the sum of the two are shown in the dotted, the dashed and the solid curves, respectively. The difference in the \( \theta_\psi \) distributions between the singlet and the octet contributions are not so prominent compared with the energy (\( E_\psi \)) distributions shown in Figs. 4 (a) and (b). However, using the number of \( \sim 600 \) \( J/\psi \)'s from \( Z^0 \to J/\psi + X \) at LEP, it would be possible to distinguish (21) from (17) from the \( \cos\theta_\psi \) distribution \[19\] (at least for the \( J/\psi \) production in \( Z^0 \) decays). Thus, the measurement of \( \cos\theta_\psi \) distribution could constitute another independent test of the idea of color-octet mechanism as a possible solution to \( \psi' \) anomaly at the Tevatron. Since the measurement of the polarization of \( \psi' \) is not easy, our option may be more viable experimentally.

Finally, we can obtain the branching ratios for \( Z^0 \to J/\psi(\text{or } \Upsilon(1S)) + X \) through the color-singlet and the color-octet mechanisms by integrating (17) – (23) over the whole \( \theta_\psi \). The results are shown in Table 1. The color-octet contribution is comparable to that through the color-singlet contributions (or equivalently, the \( c(\text{or } b) \)—quark fragmentation into \( J/\psi(\text{or } \Upsilon(1S)) \)) considered by Braaten, Cheung and Yuan \[14\]. When summed over the light flavor quantum numbers (\( q = u, d, c, s, b \)), the former actually becomes larger than the latter by a factor of \( 4 - 10 \). For the \( \psi' \), \( \Upsilon(2S) \) and \( \Upsilon(3S) \) production rates, the color-octet contributions should be scaled down by a factor of 0.3, 0.6 and 0.14, respectively, coming from \( \mathcal{M}_8(2S)/\mathcal{M}_8(1S) \). Our results on the branching ratio agree with the results obtained in Refs. \[12\] and \[10\], which is another check of our calculations based on polarized \( Z^0 \) decays.
Our analysis can be easily extended to the $W$ decays, $W \rightarrow q\bar{q} + g$ followed by $g \rightarrow J/\psi, (\text{or } \psi', \Upsilon(nS))$. Using the unitarity of the CKM matrix element $V_{qq'}$, one can easily show that

$$B(W \rightarrow J/\psi + X)_{\text{octet}} \approx 2 \times 10^{-4}, \quad (23)$$

compared to the charm quark fragmentation in the singlet model \cite{14},

$$B(W \rightarrow J/\psi + X)_{\text{frag}} \approx 4 \times 10^{-5}. \quad (24)$$

Again, the color-octet mechanism dominates the heavy quark fragmentation in the color-singlet model in $W$ decays, and this may be observed at LEP200. For Upsilon production, we have

$$B(W \rightarrow \Upsilon(1S) + X)_{\text{octet}} \approx 3.7 \times 10^{-5}. \quad (25)$$

In conclusion, we have considered the color-octet mechanism in the heavy quarkonium production in $Z^0$ decays. Both $Z^0 \rightarrow \Upsilon + X$ and $J/\psi + X$ are enhanced due to the color-octet mechanism by a factor of $\sim 10$ or so. Also, the energy and the polar angle distributions of $J/\psi$'s produced via color-octet mechanism are drastically different from those (via charm-quark fragmentation) in the color-singlet model. These two distributions can provide key tests for the idea of the color-octet gluon fragmentation which was invented to solve the $\psi'$ anomaly at the Tevatron. This idea might be the simultaneous solution to the $\psi'$ anomaly at the Tevatron and the excess of events in the channel $Z^0 \rightarrow \Upsilon + X$ at LEP, if the observed energy and polar angle distributions follow predictions made here and by other works \cite{11} \cite{10}. Similar phenomena occur in the case of inclusive $W$ decays into $J/\psi, \psi', \Upsilon(nS)$, and the excess of $W \rightarrow \Upsilon + X$ would constitute another test of the color-octet mechanism.

A remark is in order before closing. Some recent works on the color-octet contributions to $J/\psi$ photoproduction \cite{11} \cite{12} and $B \rightarrow J/\psi + X$ \cite{13} \cite{14} show that some of the color-octet matrix elements (via the $^1S_0^{(8)}$ and $^3P_J^{(8)}$) determined from the $J/\psi$ productions at the Tevatron \cite{15} might have been overestimated by an order of magnitude. However, in the $J/\psi$ productions
in $Z^0$ decays considered in this work, the color-octet $^3S_1^{(8)}$ channel is most dominant, with other color-octet channels being essentially negligible. Therefore, our predictions would be relatively stable.

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FIGURE CAPTIONS

Fig.1  Feynman diagrams for the color-singlet mechanism for $Z^0 \rightarrow (c\bar{c})(^3S_1^{(1)}) + c\bar{c}$.

Fig.2  Feynman diagrams for the color-octet mechanism for $Z^0 \rightarrow q\bar{q} + J/\psi$ with $q = u, d, c, s, b$.

Fig.3  The $\alpha_{1,8}(z)$ of the (a) $J/\psi$ and (b) $\Upsilon$ as functions of $z \equiv 2E_H/M_Z$ ($H = \psi, \Upsilon$) for the color-singlet (in dashed curve) and the color-octet (in solid curve) case respectively.

Fig.4  The energy distributions of the (a) $J/\psi$ and (b) $\Upsilon$. The color-octet contribution and the color-singlet contributions are shown in the solid and the dashed curves, respectively.

Fig.5  The polar angle distributions of the (a) $J/\psi$ and (b) $\Upsilon$ (relative to the $e^+e^-$ beam direction at the $Z$-peak). The color-octet contribution, the color-singlet contributions and the sum of the two are shown in the dashed, the dotted and the solid curves, respectively.
TABLES

TABLE I. Branching ratios for $Z^0 \rightarrow q\bar{q} + J/\psi$ (or $\Upsilon(1S)$) for $q = u, d, c, s, b$ with $m_u = 5$ MeV, $m_d = 10$ MeV, $m_s = 150$ MeV, $m_c = 1.5$ GeV and $m_b = 4.9$ GeV, and for the long-range matrix elements $\sum_{n=1}^{3} M_1(\Upsilon(nS)) = 2.3 \times 10^{-1}$GeV$^2$, $\sum_{n=1}^{3} M_8(\Upsilon(nS)) = 6.4 \times 10^{-3}$GeV$^2$, $M_1(J/\psi(1S)) = 6.1 \times 10^{-2}$GeV$^2$ and $M_8(J/\psi(1S)) = 6.8 \times 10^{-4}$GeV$^2$.

| $q$     | $B(Z^0 \rightarrow q\bar{q} + J/\psi)$ | $B(Z^0 \rightarrow q\bar{q} + \Upsilon(1S))$ |
|---------|---------------------------------------|----------------------------------------------|
| $u$     | $4.1 \times 10^{-5}$                  | $5.6 \times 10^{-6}$                         |
| $d, s$  | $5.4 \times 10^{-5}$                  | $7.2 \times 10^{-6}$                         |
| $c$     | $3.9 \times 10^{-5}$                  | $5.6 \times 10^{-6}$                         |
| $b$     | $4.4 \times 10^{-5}$                  | $6.5 \times 10^{-6}$                         |
| octet sum | $23.2 \times 10^{-5}$             | $32.1 \times 10^{-6}$                        |
| singlet | $5.6 \times 10^{-5}$                  | $9.6 \times 10^{-6}$                         |
| (fragmentation) | $(6.5 \times 10^{-5})$ | $(14.2 \times 10^{-6})$                  |
| total   | $28.8 \times 10^{-5}$                 | $41.7 \times 10^{-6}$                        |
FIG. 1. Feynman diagrams for the color-singlet mechanism for $Z^0 \rightarrow (c\bar{c})(^3S_1^{(1)}) + c\bar{c}$.

FIG. 2. Feynman diagrams for the color-octet mechanism for $Z^0 \rightarrow q\bar{q} + J/\psi$ with $q = u, d, c, s, b$. 

FIG. 3.  The $\alpha_{1,8}(z)$ of the (a) $J/\psi$ and (b) $\Upsilon$ as functions of $z \equiv 2E_H/M_Z$ ($H = \psi, \Upsilon$) for the color-singlet (in dashed curve) and the color-octet (in solid curve) case respectively.

FIG. 4. The energy distributions of the (a) $J/\psi$ and (b) $\Upsilon$. The color-octet contribution and the color-singlet contributions are shown in the solid and the dashed curves, respectively.
FIG. 5. The polar angle distributions of the (a) $J/\psi$ and (b) $\Upsilon$ (relative to the $e^+e^-$ beam direction at the $Z$-peak). The color-octet contribution, the color-singlet contributions and the sum of the two are shown in the dashed, the dotted and the solid curves, respectively.