A Note on the Hitchin System in a Background $B$-field

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The space of solutions to the Hitchin equations on the dual torus with punctures determines the Higgs branch of certain impurity theories. An alternative description of this Higgs branch is provided, in terms of the proper deformation of Hitchin system with deformation parameter given by a $B$-field. For the dual torus minus the singular points we construct explicit solutions to the $B$-deformed Hitchin equations by reducing them to principal chiral model equations and then using deformation quantization methods.
**Introduction**

Bound states of $N$ coincident parallel D-branes are described by a SU($N$) supersymmetric gauge theory on the world-volume of the D-branes. The scalar fields of this theory are matrices in the adjoint representation of the gauge group SU($N$) and are interpreted as the matrix transverse coordinates of the D-branes [1]. This gauge theory possesses many features already found in gauge theories on noncommutative spaces [2].

Recently the study of matrix compactifications on tori has been intensively worked out in the context of noncommutative geometry. The consideration of a constant three form $C_{-ij}$ on the torus has shown to deform the underlying gauge theory of toroidal compactification [3,4]. This deformation turns the algebra of functions on the dual torus into the algebra with the Moyal $\ast$-product defined in terms of the background $B$-field as a symplectic form on the dual torus with its constant component as the deformation parameter $\zeta_B$. By considering Type IIA theory with $N$ D0-branes on $T^2$ in a background $B$-field one obtains the similar deformed gauge theory [5].

$(2,0)$ field theories in six dimensions can be interpreted, in the light of matrix theory, as a sigma model whose target space is the moduli space (with some singularities corresponding to small instantons) of Yang-Mills (YM) instantons on $\mathbb{R}^4$ [6]. A DLCQ description of these theories leads to the introduction of the Fayet-Iliopoulos (FI) terms in the gauge theory in order to resolve the singularities of the moduli space [7]. From the space-time point of view FI parameters are interpreted as the constant value of the background $B$-field. The resulting $(2,0)$ theory can be reinterpreted as a non-local field theory, with a finite non-locality scale $\zeta$, describing YM instantons on noncommutative $\mathbb{R}^4$ [8]. In [8] it was shown that the Higgs branch of $N$ D0-branes inside $k$ D4-branes with D0 and D4 branes connected by the expectation value of a $B$-field flux, parametrizes the instanton moduli on the noncommutative $\mathbb{R}^4$ and thus a noncommutative version of the ADHM construction of Yang-Mills instantons can be carried out. Generalizations to the equivariant ADHM construction on noncommutative ALE spaces and its comparison to Nakajima’s description of instantons on ALE spaces are given in [10].

The above system of D0 and D4-branes has also been recently discussed in the context of impurity theories [11]. There it was shown that the Higgs branch of impurity theories has a hyper-Kähler structure and is given by the moduli space of one and two-dimensional (compact) reductions of self-dual Yang-Mills equations. The Nahm and Hitchin equations with impurity terms given by the fundamental hypermultiplets are introduced by the longitudinal D4-branes. These results have been used to solve elliptic models of $\mathcal{N} = 2$ gauge
theories via compactification to three dimensions. The Coulomb branch of elliptic models is mapped to the Higgs branch of five-dimensional theories with three-dimensional impurities. Thus its solution is given by solving a Hitchin system on a Riemann surface with punctures.

Thus the Higgs branches of the mentioned theories do not receive quantum corrections and they will be deformed under the presence of the $B$-field. In the present note we argue that the Higgs branch of impurity theories, that is, the moduli space of Hitchin equations with impurities will be deformed under the presence of a background $B$-field. We will show that Moyal deformed Hitchin equations can be solved explicitly through the BFFLS formalism of deformation quantization.

This paper is dedicated to Professor Jerzy F. Plebański on the occasion of his 70th birthday. Although exact solutions of the Einstein equations have been his constant preoccupation in physics he has also made some of the leading contributions to self-dual gravity whose formalism was shown to be relevant to describe the geometry of $\mathcal{N} = 2$ strings. Noncommutative deformation of the Hitchin equations and their solutions are natural solutions of integrable systems present in non-perturbative gauge theories coming from string theory and also from matrix theory. I hope that the modest contribution to understanding these systems presented here is appropriate on this occasion.

**Impurity Vacua in a $B$-field**

In it was shown that the Higgs branch of the bound state of $N$ D0-branes and $k$ D4-branes wrapped on a circle is given by the Nahm equations with impurities determined by hypermultiplets coming from D0-D4 strings. The corresponding system for a compactification of the D4-branes on $T^2$ gives the Hitchin equations with impurities localized at certain points on the dual torus $\hat{T}^2$ with real coordinates $\tilde{\sigma}$ and $\tilde{\sigma}'$

$$F_{z\bar{z}} + [\Phi, \overline{\Phi}] = \frac{1}{2R_1 R_2} \sum_{p=1}^{k} (Q^p \otimes Q^{*p} - \tilde{Q}^{*p} \otimes \tilde{Q}^p)$$

$$\overline{D} \Phi = -\frac{1}{2R_1 R_2} \sum_{p=1}^{k} \delta^2(\tilde{z} - \tilde{z}_p)Q^p \otimes \tilde{Q}^p$$

where $F_{z\bar{z}} = \partial \overline{A} - \overline{\partial} A + [A, \overline{A}]$ and $\overline{D} = \overline{\partial} + [\overline{A}, \partial]$ with $\partial = \partial_z$, $\overline{\partial} = \partial_{\overline{z}}$, $A = A_z$ and $\overline{A} = A_{\overline{z}}$. Here $z = \tilde{\sigma} + i\tilde{\sigma}'$, $\overline{z} = \tilde{\sigma} - i\tilde{\sigma}'$, $\partial = \frac{1}{2}(\partial_{\tilde{\sigma}} - i\partial_{\tilde{\sigma}'})$ and $\overline{\partial} = \frac{1}{2}(\partial_{\tilde{\sigma}} + i\partial_{\tilde{\sigma}'})$, $\Phi$ is a complex
scalar field in the adjoint representation of SU(N) and \( R_1, R_2 \) are the radii of the compact directions along the D4-branes. \( A \) and \( \Phi \) fields are functions on \( \hat{T}^2 \) and they come from the adjoint hypermultiplets associated to the D0-D0 strings. \( Q \) and \( \tilde{Q} \) are \( k \) complex scalars belonging to \( k \) fundamental hypermultiplets coming from D0-D4 strings, and \( \tilde{z}_p = (z, \bar{z})_p \) are the positions of the \( p \)-th field \( Q \) in the dual torus \( \hat{T}^2 \). Scalar fields of \( Q \) never become fields on the dual torus because they come from longitudinal D4-branes. Eqs. (1) and (2) should be understood modulo gauge transformations and the corresponding moduli space of solutions have no quantum corrections and therefore the study of the Higgs branch lead to exact results using only the classical equations (1) and (2).

Turning on a \( B \)-field flux on the impurity theory induces the presence of FI terms \( \xi_i \) such that the \( \mathcal{D} \)-flatness conditions are modified to \( \mathcal{D} = \xi_i \). Thus moduli space deformed by \( B \)-flux is again interpreted as the description of YM instantons on noncommutative \( \mathbb{R}^2 \times \hat{T}^2 \) [13].

An alternative description of Higgs branch with a \( B \)-field can be done through the Moyal deformation of self-dual YM equations [8] following the lines of [3,5]. Thus away from the singularities at the points \( \tilde{z}_p \) and in the presence of a \( B \)-field we have the Moyal deformed Hitchin equations on the noncommutative dual torus \( \hat{T}_B^2 \) without punctures

\[
F_{\bar{z}z}(\tilde{z}) + \{\Phi(\tilde{z}), \Phi(\tilde{z})\}_B = 0
\]

(3)

\[
\bar{\nabla}\Phi(\tilde{z}) + \{\bar{A}(\tilde{z}), A(\tilde{z})\}_B = 0,
\]

\[
\bar{\nabla}\Phi(\tilde{z}) + \{\bar{A}(\tilde{z}), A(\tilde{z})\}_B = 0,
\]

(4)

Here the Moyal bracket is defined by \( \{f, g\}_B \equiv \frac{1}{\kappa_B} (f \ast g - g \ast f) \) for \( f, g \) functions on the torus. The \( \ast \)-product is given by \( f \ast g = f \exp(\frac{i}{\kappa_B} \xi_{ij} \partial_i \partial_j) g \) with \( \xi_B = B \), the deformation parameter. The FI parameters \( \xi \) are encoded in the \( B \)-deformation of Eqs. (1) and (2). As one takes \( B \to 0 \) one recovers straightforwardly system (1) and (2). This is equivalent to taking \( \xi \to 0 \). Eqs. (3) and (4) can be seen as dimensional reduction of self-dual Yang-Mills equations on noncommutative \( \mathbb{R}^2 \times \hat{T}^2 \) to noncommutative \( \hat{T}^2 \). From now on we will work with Eqs. (3) and (4) and in the next section we attempt to find solutions for them.
Looking for Solutions of the Hitchin Equations in a $B$-field

The Hitchin equations are two-dimensional reductions from self-dual Yang-Mills theory \cite{15}. These equations are defined on any Riemann surface of genus $g$ and certain marked points. The moduli space of Hitchin equations possesses an hyper-Kähler structure. The original application of these equations was to study the moduli space of stable vector bundles on an arbitrary Riemann surface \cite{15}. Hitchin equations were also used brilliantly also to gain more insight about this moduli space from the point of view of a Hamiltonian integrable system \cite{15,16}. Later Donaldson showed that Hitchin equations can be reduced to the study of twisted harmonic maps on a hyperbolic space of negative curvature, and in this context some solutions can be obtained \cite{17}. Our goal in this section is to obtain solutions for the Moyal $B$-deformed Hitchin equations (3) and (4). In order to do it we first show the equivalence of Hitchin equations and the principal chiral model at the classical level and then we use the BFFLS formalism to look for explicit solutions \cite{14}.

Following \cite{13,17} (see also \cite{18}) we define new connections $A = A - \Phi$ and $\mathcal{A} = A + \Phi$. It can be easily shown that $A$ and $\mathcal{A}$ are the components of a flat connection on $\hat{T}_B^2$ if the Hitchin equations (3) and (4) are fulfilled. That means $A$ and $\mathcal{A}$ satisfy

$$F_{\mathcal{A}}(\tilde{z}) = \bar{\partial}A(\tilde{z}) - \partial A(\tilde{z}) + \{A(\tilde{z}), \mathcal{A}(\tilde{z})\}_B = 0. \tag{5}$$

Here we have used the existence of an harmonic map $g : \hat{T}^2 \to G_*$, which satisfy $A(\tilde{z}) = g^{-1}(\tilde{z}) * \bar{\partial}g(\tilde{z})$ and $\mathcal{A}(\tilde{z}) = g^{-1}(\tilde{z}) * \partial g(\tilde{z})$. $G_*$ is an infinite-dimensional Lie group which is defined by $G_* = \{g = g(\tilde{z}) \in C^\infty(\hat{T}^2); g * g^{-1} = g^{-1} * g = 1\}$ and $g^{-1}$ is the inverse mapping in the group. Defining $H(\tilde{z}) \equiv g(\tilde{z}) * \Phi(\tilde{z}) * g^{-1}(\tilde{z})$, one can show that Hitchin’s equations (3),(4) are equivalent to the system

$$g^{-1}(\tilde{z}) * \left(\bar{\partial}H + \partial H - 2\{H, H\}_B\right) * g(\tilde{z}) = 0, \tag{6}$$

$$\bar{\partial}H(\tilde{z}) = \{\overline{H}(\tilde{z}), H(\tilde{z})\}_B, \quad \bar{\partial}\overline{H}(\tilde{z}) = \{H(\tilde{z}), \overline{H}(\tilde{z})\}_B. \tag{7}$$

Furthermore one can define $J = 2H$ and $\overline{J} = -2\overline{H}$ and the above system is equivalent to the principal chiral model (PCM)

$$\partial J(\tilde{z}) - \bar{\partial}J(\tilde{z}) + \{J(\tilde{z}), J(\tilde{z})\}_B = 0, \tag{8}$$

$$\partial \overline{J}(\tilde{z}) + \bar{\partial}\overline{J}(\tilde{z}) = 0 \tag{9}$$
with \( J(\tilde{z}) = h^{-1}(\tilde{z}) \ast \partial h(\tilde{z}) \) and \( \bar{J}(\tilde{z}) = h^{-1}(\tilde{z}) \ast \partial \bar{h}(\tilde{z}) \). These equations can be derived from the Lagrangian

\[
L_{PCM} = -\frac{\zeta^2}{2} \int d^2 \tilde{z} \text{Tr}_N \left( h^{-1}(\tilde{z}) \ast \partial h(\tilde{z}) \ast h^{-1}(\tilde{z}) \ast \partial \bar{h}(\tilde{z}) \right).
\]  

(10)

Equations (9) and (10) are precisely a suitable Moyal deformation of the PCM equations. These equations are classically equivalent to the equations \([19]\)

\[
i \partial \bar{\partial} \Theta(\tilde{z}) + \frac{1}{2} \{ \partial \Theta(\tilde{z}), \bar{\partial} \Theta(\tilde{z}) \}_B = 0
\]

(11)

where \( J = -\frac{1}{2} \partial \Theta \) and \( \bar{J} = \frac{1}{2} \bar{\partial} \Theta \) with Lagrangian

\[
L_H = \int d^2 \tilde{z} \text{Tr}_N \left( \frac{1}{2} (\partial \Theta \ast \partial \Theta + \bar{\partial} \Theta \ast \bar{\partial} \Theta) + \frac{2}{3} \Theta \ast \{ \partial \Theta, \bar{\partial} \Theta \}_B \right).
\]

(12)

From the quantum point of view the \( \Theta \) model and the PCM are inequivalent because both models are renormalized in a different way. As we mentioned before the Higgs branch is determined exactly by classical equations thus equivalence of Eqs. (8),(9) and (12),(13) we will use is justified.

After having shown equivalence between Hitchin equations (3)(4) and the PCM equations (9)(10) we now attempt to find solutions to the latter equations. Before that it is convenient going back to real coordinates \( \tilde{\sigma} \). Thus Eq. (12) is rewritten as

\[
\partial_{\tilde{\sigma}}^2 \Theta + \partial_{\tilde{\sigma}'}^2 \Theta + \{ \partial_{\tilde{\sigma}} \Theta, \partial_{\tilde{\sigma}'} \Theta \}_B = 0
\]

(13)

where \( \Theta = \Theta(\tilde{\sigma}, \tilde{\sigma}') \). This equation looks like the heavenly equation discussed in \([20]\). However in this case \( \Theta \) is a function on the dual torus and not on a four-dimensional self-dual space. Thus solutions of (14) do not determine any self-dual metric. But one can still attempt to solve Eq. (14) using the Weyl-Wigner-Moyal formalism. The correspondence between matrices \( \Theta \) and matrix-valued functions \( \Theta(\tilde{\sigma}) \) on the dual torus is given by \([21,22,3]\)

\[
\sigma_N^{-1} : \text{Mat}_N \rightarrow C^\infty(\hat{T}^2)
\]

\[
\sigma_N^{-1}(\Theta) = \Theta(\tilde{\sigma}) = \sum_{(m,n) \in \mathbb{Z}^2} \Theta_{m,n} \exp\left(i\left(\frac{m}{R_1} \tilde{\sigma} + \frac{n}{R_2} \tilde{\sigma}'\right)\right)
\]

(14)
where \( \hat{R}_1 = \frac{1}{2\pi R_1}, \hat{R}_2 = \frac{1}{2\pi R_2} \). \( \text{Mat}_N \) is the set of \( N \times N \) non-singular matrices representing the Lie algebra \( \text{su}(N) \) and \( C^\infty(\hat{T}^2) \) is the set of smooth functions on the dual torus \( \hat{T}^2 \).

On the other hand it is well known that the basis of the Lie algebra \( \text{su}(N) \) can be seen as a two-indices infinite algebra. The elements of this basis are denoted by \( \exp\left(i\left(\frac{m}{R_1} + \frac{n}{R_2}\right)\right) \) and they satisfy the two-indices infinite Lie algebra \[ [L_m, L_n] = \frac{N}{\pi} \sin\left(\frac{\pi N}{N} m \times n \mod Nq\right), \] (15)

where \( m = (m_1, m_2) \), \( n = (n_1, n_2) \) and \( m \times n := m_1n_2 - m_2n_1 \). The large \( N \) limit \((N \to \infty)\) of algebra (15) gives the area-preserving diffeomorphism algebra \( \text{sdiff}(\hat{T}^2) \).

The correspondence (14) can be seen as the composition of two mappings. The first one is a Lie algebra representation of \( \text{su}(N) \) (for finite \( N \)) into a Lie algebra \( \hat{G} \) of self-adjoint operators acting on the Hilbert space \( L^2(R) \), given by

\[ \Psi: \text{su}(N) \to \hat{G}, \quad \Theta \mapsto \Psi(\Theta) := \hat{\Theta}. \] (16)

The second mapping is a genuine Weyl correspondence \( W^{-1} \) which establishes a one to one correspondence between the algebra \( B \) of self-adjoint linear operators acting on \( L^2(R) \) and the space of real smooth functions \( C^\infty(\hat{T}^2) \) where \( \hat{T}^2 \) is seen as the classical phase-space. This correspondence \( W^{-1}: B \to C^\infty(\hat{T}^2) \) is given by

\[ \Theta(\tilde{\sigma}, \tilde{\sigma}'; \zeta_B) \equiv W^{-1}(\hat{\Theta}) := \int_{-\infty}^{\infty} <\tilde{\sigma} - \frac{\xi}{2} \hat{\Theta}|\tilde{\sigma} + \frac{\xi}{2} > \exp\left(\frac{i}{\zeta_B} \xi \tilde{\sigma}'\right) d\xi, \] (17)

for all \( \hat{\Theta} \in B \) and \( \Theta \in C^\infty(\hat{T}^2) \). Thus from the identification of \( B \) with \( \hat{G} \), it follows that the correspondence \( \sigma^{-1}_N \) is equal to the map composition \( \sigma^{-1}_N = W^{-1} \circ \Psi \) for finite \( N \) and it is actually a Lie algebra isomorphism.

In Ref. [20] it was shown how to solve Eq.(14) for the case of self-dual gravity. In what follows we apply the same method to find solutions for this equation. We set \( N = 2 \), obtaining \( 2 \times 2 \) matrices forming the Lie algebra \( \text{su}(2) \). Thus matrices \( \Theta \) can be expanded as

\[ \Theta = \sum_{a=1}^{3} \theta_a \tau_a \] (18)

where \( \tau_a \ (a = 1, 2, 3) \) constitutes a basis of \( \text{su}(2) \) and \( \theta_a \) are some constant numbers. Eq. (17) leads to
\[ \hat{\Theta} = \sum_{a=1}^{3} \theta_a \Psi(\tau_a) \]  

where \( \Psi(\tau_a) \) \((a = 1, 2, 3)\) are a basis for the Lie subalgebra \( \mathfrak{su}(2) \) of unitary operators and are given by:

\[
\Psi(\tau_1) = i\alpha \hat{\sigma}' + \frac{1}{2\zeta_B} (\hat{\sigma}^2 - 1) \hat{\sigma}, \quad \Psi(\tau_2) = -\alpha \hat{\sigma}' + \frac{i}{2\zeta_B} (\hat{\sigma}^2 + 1) \hat{\sigma} \quad \text{and} \\
\Psi(\tau_3) = -i\alpha \hat{1} - \frac{1}{\zeta_B} \hat{\tau}' \hat{\sigma}' .
\]

Inserting equations for \( \Psi(\tau_a) \) into Eq. (20) and then using (18) we obtain finally the solution

\[
\Theta(\hat{\sigma}, \hat{\sigma}') = \frac{i}{2} \theta_1 \hat{\sigma}(\hat{\sigma}'^2 - 1) - \frac{1}{2} \theta_2 \hat{\sigma}(\hat{\sigma}'^2 + 1) - i\theta_3 \hat{\sigma} \hat{\sigma}' + B \cdot (\alpha + \frac{1}{2})(-\theta_1 \hat{\sigma}' - i\theta_2 \hat{\sigma}' + \theta_3). \tag{20}
\]

When \( \zeta_B(= B) \rightarrow 0 \) we get the simple solution

\[
\Theta_0(\hat{\sigma}, \hat{\sigma}') = \frac{i}{2} \theta_1 \hat{\sigma}(\hat{\sigma}'^2 - 1) - \frac{1}{2} \theta_2 \hat{\sigma}(\hat{\sigma}'^2 + 1) - i\theta_3 \hat{\sigma} \hat{\sigma}' \tag{21}
\]

where \( \Theta_0 \) is the lower term of the series \( \Theta = \Theta_0 + \sum_{n=1}^{\infty} \zeta_B^n \Theta_n \).

Thus Hitchin’s equations can be solved by reducing them to the PCM equations and then using the WWM formalism. Solutions for the Hitchin equations in a \( B \)-field depend explicitly on the deformation parameter \( B \) as we have shown in Eq. (21). In this sense the moduli space of deformed Hitchin equations is deformed and therefore the Higgs branch will also be deformed. The same procedure can be carried over to the Higgs branch given by the solutions of Nahm’s equations away from impurities [11]. In that case the presence of the background \( B \)-field turns the Nahm equations into

\[
\frac{dT_i}{ds} + \{ T_0, T_i \}_B + \frac{1}{2} \varepsilon_{ijk} \{ T_j, T_k \}_B = 0. \tag{22}
\]

Solutions of these equations can be obtained straightforwardly following the lines of [24].

Finally Hitchin equations on the cylinder \( \mathbb{R} \times \mathbb{S}^1 \) can be obtained from matrix string theory compactified on \( \mathbb{S}^1 \) [25,26]. Hitchin equations results form the BPS condition in the supersymmetry transformations. Compactification on a further circle \( \mathbb{S}^1 \) one can find the equivalence with a supersymmetric Yang-Mills theory on a \( 2 + 1 \)-dimensional spacetime \( \mathbb{R} \times \hat{T}^2 \). If one can include a background \( B \)-field in this picture, and check the BPS condition one obtains \( B \)-deformed Hitchin equations on a dual torus \( \hat{T}^2_B \) of the type (3)(4). Solutions of these systems following the lines of the present paper would be an alternative than the Toda equation method discussed in [26].
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