The study of elastic properties of fractured porous rock based on digital rock

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Abstract. Knowledge of the elastic properties of fractured rock plays a significant role in accurate and rigorous reservoir description. In this paper, we carry out a systematic study about the effect of fracture width, fluid properties on elastic properties of reservoir rocks based on digital rock technology. Digital rock models with different fracture width are reconstructed via a superposition reconstruction procedure which impose a fracture on an isotropic host rock. A modified Finite Element Method (FEM) is adopted to investigate rock elastic properties. The results show that the presence of fracture leads to a strong anisotropy of rock. The five elastic constants of the resultant TI medium are derived as a function of the properties of the fracture width and fluid bulk modulus. The aspect ratio of P-wave velocity and S-wave velocity ($V_p/V_s$) is a function of fluid bulk modulus and can be used to identify fractured gas reservoir. The results allow one to have a better understanding about the correlations of fracture, fluid and elastic properties of reservoir rocks.

1. Introduction
Rock elastic parameters play a significant role in rock mechanics, geophysical exploration and sonic logging interpretation. The elastic properties of granular rocks have been well studied over the last few years. Fractured rocks are distinguished from the granular rocks because of their high heterogeneity and anisotropy. Conventional experiments are difficult to investigate the impact of fracture width, fluid properties on the elastic properties of fractured rock. Thus, analytical or numerical simulation techniques are indispensable methods in studying rock elastic properties. The Conventional rock physics models are based on either empirical relations from laboratory data or theoretical models based on simple idealized microstructures, such as rigorous bounds [1,2], effective medium theories [3], simple deterministic models [4,5], or empirical relationships [6]. These rock physics models have given important insights into understanding rock elastic properties. However, these models are almost always over-simplified and can not explain or describe the elastic properties of complex geometries. With the development of digital rock technology, some of the aforementioned shortcomings have been overcome, which have allowed to obtain precise geometric structures of fractured rock and directly calculate the rock’s petrophysical properties [7,8].
Garboczi and Day (1995)[9] proposed a method of using the finite element method to calculate the elastic parameters of composite materials based on digital image. Arn et al. (2002)[10] first use the finite element method to calculate the elastic parameters of Fontainebleau sandstone based on digital rock, and the numerical simulation results are in excellent agreement with available experimental data. The feasibility of the finite element method in the numerical calculation of elastic parameters of digital rock is proved. Makarynska et al. (2008)[11] calculated elastic properties of partially saturated rock via FEM. The numerical results are compared to the Gassmann theory combined with Wood’s formula (GW) for a mixture of pore fluids, which is exact for a monomineralic macroscopically homogeneous porous medium. Zhao et al. (2013)[12] proposed a method to generate fractured digital rock and analysis the effect of fracture on electrical properties of fractured rock. To our knowledge, there have not been many systematic studies about the effect of fracture width, fluid properties on elastic properties of fractured rock based on digital rock technology.

Single fracture is the basic element of the rock fracture network, so the elastic characteristics of the porous rock with a single fracture is the foundation of the study of the elastic characteristics of the fractured reservoir. In this paper, we first briefly recall the algorithm used to generate fractured digital rock. We then discuss the finite element method that we employ to carry out the elastic simulations. Finally, the relationship between elastic parameters and fracture width, fluid properties are studied based on digital rock physics.

2. Methods

2.1. Method to construct fractured digital rock

It is difficult to carry physical experiment on fracture rock and investigate the effect of micro-factors (such as fracture width) on elastic parameters quantitatively. Therefore, how to build a fractured digital rock which can reflect the characteristics of true fracture is the key to the elastic simulation of fractured rock. In this paper, the fractured digital rock is constructed via superimposing a fracture on host rock. The host rock can be generated by experimental methods or numerical methods.

At present, there are many fracture models which are used in the investigation of fractured rock properties, such as parallel plate model [13], penny model [14]. To some extent, these models reveal the pore space characteristics of fractured rock, but these models consider the fracture as a smooth surface of flat plate or disc without considering the roughness of the fracture surface itself. Rock physics experiments show that the surface of rock fracture has a high degree of spatial correlation, and has self-affine fractal statistics [15]. For fracture surface of rocks, a universal value of the roughness exponent is $H = 0.8$ [16,17,18]. In our research, we assume that there is no overlap of the two fracture surfaces, and the bottom surface can be expressed by the function $Z(x,y)$ in the Cartesian coordinate system. The self-affine property of the fracture surface shows that it exhibits scale invariance under scale transformation. That is, when $x \rightarrow \lambda_x x, \ y \rightarrow \lambda_y y, \ z \rightarrow \lambda_z z$, we assume that $\lambda_x = \lambda_y = \lambda$ and $\lambda_z = \lambda^H$, then

$$z(x,y) = \lambda^{-H} z(\lambda x, \lambda y)$$ \hspace{1cm} (1)

Where $\lambda$ is the scale factor, $H$ is roughness exponent or Hurst exponent.

The most common mathematical model for self-affine fractals is the fractional Brownian motion (fBm). In this paper, we adopt a modified successive random addition (MSRA) algorithm to generate fBm surface [12,19], which is simple, efficient in computation. We assume that there is no overlap of the two fracture surfaces, and the fracture width is $d$, then the up surface can be expressed as:

$$z_u(x,y) = z(x,y) + d$$ \hspace{1cm} (2)

Fig. 1 shows the procedure to generate fractured digital rock. The self-affine fracture surface generated via MSRA method and the roughness exponent is 0.8(Fig.1(a) and Fig.1(b)). Fig. 1(c) shows the host digital rock generated via CT scan. After the host rock and fracture are generated separately,
the digital rock with a single fracture constructed by superimposing the fracture on the host rock (Fig. 1(d)).

![Figure 1](image1.png)

**Figure 1.** The procedure to generate fractured digital rock. (a) fracture surface with $H=0.8$, (b) fracture without overhang, (c) host rock generated via CT scan, (d) fracture digital rock constructed via a superimpose method.

2.2. Simulation Method and theory

Garboczi (1998) [20] developed a finite element program designed to study the linear elastic properties. The method is effective to compute the linear elastic properties of homogeneous rocks whose microstructure information has been stored in a 2D or 3D image. Given bulk modulus and shear modulus of each component of the rock, The FEM uses a variational formulation of the linear elastic equations and finds the solution by minimizing the elastic energy using a fast conjugate-gradient method, then the final elastic displacement distribution at each pixel is determined, and the two independent parameters ($\lambda$ and $\mu$) of isotropic rock can be obtained.

For fractured rock, we can run six independent FEM simulations to solve for the deformation field of six different applied external orthogonal strains [21]. In this way, we can construct any particular applied macroscopic strain as a combination of the selected strain basis $\varepsilon_i^\alpha$. Where $\alpha$ enumerates the basis vectors, and $i$ represents the Voigt index in the basis vector. In Fig. 2, a schematic diagram of the six strain basis vectors is given. The basis vectors are listed as follows:

$\varepsilon^1 = (1,0,0,0,0,0)$, $\varepsilon^2 = (0,1,0,0,0,0)$, $\varepsilon^3 = (0,0,1,0,0,0)$, $\varepsilon^4 = (0,0,0,1,0,0)$, $\varepsilon^5 = (0,0,0,0,1,0)$, $\varepsilon^6 = (0,0,0,0,0,1)$

![Figure 2](image2.png)

**Figure 2.** Loading conditions. For the six external macroscopic basis strains, (a)–(c) is the uniaxial compression loading and (d)–(f) is the sheer strain.
By choosing one of the strain basis vectors, the six corresponding macroscopic stresses can be obtained. According to the linear relationship of the elastic equation, the six macroscopic stresses define a column of the stiffness matrix. This is sufficient for calculating all 36 elements (Voigt notation) of the elastic stiffness tensor. In our modeling, the fracture horizontally penetrate in an isotropic host rock, thus the fractured rock is transversely isotropic with a vertical symmetry axis (VTI). There are five independent elements in the effective elastic stiffness tensors and the Voigt stiffness matrix has the form [22]:

\[
\begin{bmatrix}
    c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\
    c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\
    c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\
    0 & 0 & 0 & c_{44} & 0 & 0 \\
    0 & 0 & 0 & 0 & c_{44} & 0 \\
    0 & 0 & 0 & 0 & 0 & c_{66}
\end{bmatrix}, \quad c_{66} = \frac{1}{2} (c_{11} - c_{12}) \quad (3)
\]

\(c_{11}\) and \(c_{33}\) correspond to P-wave are parallel and perpendicular to the fracture plane respectively, while \(c_{44}\) and \(c_{66}\) related to S-wave are perpendicular and parallel to the fracture plane respectively. We can run tow independent FEM simulations to get all the required parameters. The vertical sound speeds for P- and S-waves can be expressed as [23]:

\[V_p = \sqrt{\frac{C_{33}}{\rho}}, \quad (4)\]
\[V_s = \sqrt{\frac{C_{44}}{\rho}}, \quad (5)\]

Where \(\rho\) is the rock density.

3. Results and discussion

In this paper, fractured porous digital rocks are reconstructed via superimposing the fracture to rock matrix. Elastic properties of fracture porous rocks are studied based on digital rock technology. Table 1 shows the simulation parameters following [22]:

| Mineral | Bulk modulus | Shear modulus | Density |
|---------|--------------|---------------|---------|
| Calcite | 76.8Gpa      | 32.0Gpa       | 2.71g/cm³ |

In our research, we only investigate the effect of fracture and fluid on rock elastic properties, so it is necessary to ensure that the host rock is homogeneous. Thus we use the process-based method to generate the host rock.

3.1. The effect of fracture width on rock elastic properties

In order to study the effect of fracture width on rock elastic properties, we build a group of fractured digital rock with an aperture of 2 to 12 pixels with a step of 2 pixels. The host rock porosity is 10%, the rock size is 200 × 200 × 200 cubic pixels, and the resolution is 5μm / pixel, so the fracture width is 10μm, 20μm, 30μm, 40μm, 50μm, 60μm. The finite element method is used to estimate the elastic properties of the fractured rock.

Fig. 3(a) shows the relationship between elastic parameter and the fracture aperture when the rock is dry. From the figure we can see that the stiffness element \(C_{11}\) and \(C_{33}\) separate with each other, so as
$C_{44}$ and $C_{66}$. That indicates the presence of fracture makes the pore space structure more complex and leads to a strong anisotropy of rock. $C_{11}$ is parallel to the fracture plane, thus it can be supported by the rock framework, so it is of little change with increasing fracture aperture. While $C_{33}$ is perpendicular to fracture plane, in this direction the rock is “soft” caused by fracture, so it decreases with increasing fracture aperture. Fig. 3(b) shows the relationship between velocity and the fracture aperture when the rock is dry. The velocity of P-wave and S-wave ($V_p$ and $V_s$) decreases with the increase of fracture aperture. When the fracture aperture is large enough, $V_p$ and $V_s$ tend to 0.

![Figure 3](image-url)  
**Figure 3.** The relationship between elastic parameter and fracture aperture.

### 3.2. The effect of fluid on elastic properties of saturated rock with fracture

The existence of fracture will lead to the elastic anisotropy of reservoir rocks, the fluid filled in the fracture also has an impact on the elastic properties of fractured rock. One of the important issues related to elastic properties in fractured rock is the effect of pore fluid on anisotropy. In this section, we investigate the effect of fluid bulk modulus on stiffness elements, wave velocity and anisotropy parameter ($\epsilon$ and $\gamma$). The simulation result are shown in Fig. 4. Since $C_{11}$ is parallel to the fracture plane, it is of little change with increasing fluid elastic modulus, $C_{13}$ is perpendicular to the fracture plane, in this direction rock becomes more flexible and compressible, and it increases with increasing fluid modulus (Fig. 4(a)). This is because the pore fluid plays a supporting role in the pores and fracture. With the increase of the elastic modulus of the fluid, $C_{44}$ and $C_{66}$ nearly unchanged, which shows that the effect of fluid bulk modulus on shear elastic constant are relatively small. Fig. 4(b) shows the relationship between velocity and fluid bulk modulus. From the picture, $V_s$ nearly unchange with the increase of the fluid bulk modulus, which also indicate the S-wave is not sensitive to fluid properties. While $V_p$ increases with the increase of the fluid bulk modulus, and the growth rate decreases with the increase of fluid bulk modulus. Fig. 4(c) shows the relationship between $V_p/V_s$ and fluid bulk modulus, from the figure can be seen, the aspect ratio of $V_p/V_s$ increases with the increase of fluid modulus, but the growth rate gradually slowed down. When the spaces (pore and fracture) are fully saturated with water or oil, the shear wave velocity changed little. However, when the reservoir space containing gas (fluid modulus near 0), $V_p/V_s$ decreases sharply with the decrease of fluid modulus, which indicates we can identify fractured gas reservoir via $V_p/V_s$. From Fig. 4(d), we observe an overall decrease of the anisotropy parameters with the increase of fluid modulus.
4. Conclusion
In this paper, we performed systematic analysis of the effect of fracture width and fluid bulk modulus on elastic properties of fractured rock based on digital rock technology. The results show that the stiffness elements parallel to fracture plane change little with the increase of fracture width. The stiffness elements which are perpendicular to the fracture plane gradually decrease with the increase of fracture width. The velocity of P-wave and S-wave ($V_p$ and $V_s$) decreases with the increase of fracture aperture. When the fracture aperture is large enough, $V_p$ and $V_s$ tend to 0. The pore fluid plays a supporting role in the pores and fracture. The S-wave is not sensitive to fluid properties. The aspect ratio of P-wave velocity and S-wave velocity ($V_p/V_s$) is a function of fluid bulk modulus and can be used to identify fractured gas reservoir. The anisotropic parameters decrease with the increase of fluid modulus. The elastic properties of fractured reservoir rock still need to be further studied based on digital rock, with the development of digital rock technology, we’ll be able to analyze larger samples with higher resolutions and fracture density, aspect ratio, incident angle and azimuth will be studied based on digital rock in the future.

Acknowledgements
This work was supported in part by China Natural Science Foundation (Grant No. 41804125) and Natural Science Basic Research Plan in Shaanxi Province of China (Grant No. 2018JQ4043) and Scientific Research Program Funded by Shaanxi Provincial Education Department (Grant No. 18JK0619).

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