The Complexity of Controlling Candidate-Sequential Elections

Edith Hemaspaandra
Department of Computer Science
Rochester Institute of Technology
Rochester, NY 14623, USA

Lane A. Hemaspaandra
Department of Computer Science
University of Rochester
Rochester, NY 14627, USA

Jörg Rothe
Institut für Informatik
Heinrich-Heine-Universität Düsseldorf
40225 Düsseldorf, Germany

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Abstract
Candidate control of elections is the study of how adding or removing candidates can affect the outcome. However, the traditional study of the complexity of candidate control is in the model in which all candidates and votes are known up front. This paper develops a model for studying online control for elections where the structure is sequential with respect to the candidates, and in which the decision regarding adding and deleting must be irrevocably made at the moment the candidate is presented. We show that great complexity—PSPACE-completeness—can occur in this setting, but we also provide within this setting polynomial-time algorithms for the most important of election systems, plurality.

1 Introduction and Related Work

This paper introduces a framework for the study of online candidate control in sequential elections. After introducing this issue, we provide a real-world motivating example, formalize the problem, provide a number of results, and suggest directions for future work.

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We will carefully define candidate control, in particular in our sequential setting, in detail later. However, as a quick initial sense of what candidate control traditionally has meant, so that this introduction makes sense, candidate control refers to trying to ensure by adding or deleting candidates, that a given candidate wins (or does not win). Usually, one is limited in how many candidates one is allowed to add or delete. Computational social choice is particularly interested in the algorithmic and complexity issues here: How hard it is to decide if in a given setting one can by such an action achieve one’s goal?

Previous work on candidate control of elections has been in the model of full-information, simultaneous voting. This is a problem, since in quite a few real-world settings—from TV singing/dancing talent shows to university faculty-hiring processes—candidates are introduced, and appraised by the voters, in sequence. We provide a natural model for sequential candidate evaluation, a framework for evaluating the computational complexity of controlling the outcome within that framework, and results on the range such complexity can take on. We hope this paper will lead to further work examining temporally involved candidate control, and we conclude with some open directions.

Our model of the process’s goal, having the chair try to guarantee a goal under the most hostile of responses, is inspired by the area of online algorithms [BE98], and was used for online manipulation in [HHR14] and for online voter control [HHR12b]. The just-mentioned papers [HHR14, HHR12b] adopt a snapshot-in-time view of voter-sequential elections (other work about or related to voter-sequential elections includes [Slo93, DP01, Ten04, DE10, XC10, PP13]), unlike this paper, which takes a snapshot-in-time view with regard to candidates being the objects that are sequentially added. This view is also in part inspired by the work of Chevaleyre et al. [CLM+12], who study the possible winner problem when new candidates are added. Note, however, that their model and ours differ greatly. For example, while in their model addition of candidates is not a choice (they all are just added, at once, and the question is whether the previous votes can be extended to include this block of new candidates such that a given candidate wins), in our model the chair has a choice each time a new candidate shows up, and the preferences are gradually revealed just at that moment to include that new candidate. Another different but related work, done independently of and appearing after the preliminary version of this paper [HHR12a], is that of Oren and Lucier [OL14], whose model involves votes arriving one at a time and in-the-moment choices over bundles of goods. Electoral control has, in the standard (i.e., not online) setting, been studied intensively in many papers since the seminal work of Bartholdi, Tovey, and Trick [BTT92], e.g., although this is a far from complete list, [HHR07, ENR09, FHHR09, BEH+10, FHH11, FHH11, FHH11, FHH13, Men13, RS13, EHH14, FHH14, EFRS15, EHH15], see also the detailed survey [FR16].

2 Motivating Example

In an author’s school, faculty hiring happens basically as follows. On some Mondays, a candidate visits, gives a talk, and meets with faculty members. Then each of the department’s rank-and-file faculty members sends by email to the faculty and department chair her ranking of all the candidates so far, namely, by inserting the new candidate into the preference order she sent after the previous candidate. The chair typically follows up by phoning the candidate a day or two after the visit, so
that phoning occurs after the chair has seen the faculty rankings generated by the candidate’s visit. Moving now from reality to (slight?) fiction, let us imagine that the chair in that followup can easily choose to scare away a candidate (“Oh, did I remember to mention that if you come, your office will be a shared closet in our lovely basement, I’ll help you broaden yourself by teaching a wide range of introductory courses, and I see in you a real talent for extensive committee work which I’ll put to good use?”). But let us further assume that the chair cannot do this more often than a certain threshold, as otherwise the rank-and-file faculty will realize the chair is manipulating the process and will revolt. So, how should the chair use this power of candidate suppression to most effectively ensure that one of the candidates the chair likes will, at the end of the process, win the election (under the faculty preferences, among the candidates not scared away)?

This example nearly perfectly captures the topic and model of this appendix. We are moving what in the literature is called “candidate control” [BTT92] (in the example, of the sort known as “constructive control by deleting candidates”) from its existing setting of simultaneous elections into a setting where preferences are set/revealed sequentially and the chair, right after the preferences related to an introduced candidate are revealed, must use-or-forever-lose the ability to suppress that candidate.

We also are interested—again moved to a sequential setting—in constructive control by adding candidates, a natural analogue of the above, and in destructive versions of both adding/deleting candidates, which are the same issues except the chair’s goal is to ensure that none of a certain set of hated candidates is hired.

Bartholdi, Tovey, and Trick [BTT92] defined non-online versions of the constructive-deletion notion used above and a precursor of the constructive-addition notion used above. The non-online versions of the constructive-addition notion used above and both destructive notions used above are from Hemaspaandra, Hemaspaandra, and Rothe [HHR07], although destructivity had been introduced even earlier by Conitzer, Sandholm, and Lang [CSL07] for a different type of attack known as manipulation. (However, as mentioned earlier in the paper, we are consistently following the now more standard nonunique-winner model, rather than the unique-winner model.)

3 Formalizing the Problem

Let us discuss how to formalize this into a decision problem whose complexity can be studied. We’ll do so here in detail just for constructive control by deleting candidates, and then will describe, by altering that, how the other online candidate-control problems are captured. Let $E$ denote the underlying election system: a mapping from candidates and votes over the candidates (with preferences typically as strict, linear orderings) to a set of winners. The candidates left standing at the end (i.e., not deleted by the chair) will be fed into this election system along with the votes (with each vote’s preference order masked down to that set of still-standing candidates).

The input will capture a “moment of decision” for the chair. That is, the input will give the history of the process up to the given point, and then will ask whether there is some action of the chair that can ensure she will get a happy outcome. We must make it clear what we mean by this.

The input will be the set of candidates, the set of voters, the order in which the candidates will be presented, a flag denoting which the current candidate is, a bound $k$ on the maximum number
of candidates the chair can suppress, an ordering $\sigma$ of how the chair views all candidates (to put this in the context of our motivating example from Section 2 this is as if the department chair had the c.v.'s ahead of time and has evaluated them already), a specific candidate $d$ such that the chair’s goal is to ensure that there is an election winner from the set $\{c \mid c \geq_\sigma d\}$ (i.e., $d$ or some candidate the chair likes better than $d$ is a winner), and the history up to the current moment in time (which means for each candidate before the current one a bit saying whether the chair deleted that candidate, and a preference order for each voter over all the candidates up to and including the current one—we could also make this just over all as-yet nondeleted candidates, but let us make it over all candidates so far, though it doesn’t affect the eventual results; we prefer this because it allows the history of the voting situation to be part of the instance). And the question being asked in this decision problem is whether there is some decision the chair can make about the current candidate (to delete, or not to delete) such that, assuming that the chair at each future decision is free to act in light of the information revealed up to that point, the chair can ensure that the winner set will have nonempty intersection with the candidates she likes, $\{c \mid c \geq_\sigma d\}$, regardless of what else happens in the election (i.e., even if the revealed preferences are highly unfavorable to the chair’s wishes).

The decision problem (i.e., language) here is simply the set of all inputs where the answer to that question is yes. Let us call this problem online-$\mathcal{E}$-constructive-control-by-deleting-candidates (online-$\mathcal{E}$-CCDC, for short).

Although we used a somewhat informal wording above, there is a more formally satisfying phrasing that captures the same notion using alternating quantifiers: Does there exist a legal move by the chair about the current candidate, such that for all possible settings of the information revealed after this up to the chair’s next decision, there exists a legal next decision by the chair, such that . . . . . . such that the winner set contains either $d$ or some candidate the chair likes more than $d$.

Briefly, the “adding”-candidates analogue (of the above deleting-candidates case), denoted by online-$\mathcal{E}$-CCAC, is almost the same—except the input contains a “certainly in the election” set of candidates, and a (disjoint) set of “potential additional” candidates, and a presentation ordering over the union of those two sets, and the rest is analogous (so for potential-addition candidates before the current one the input tells whether the chair added them, etc.).

And these constructive-control deleting and adding cases each have a “destructive control” sibling, online-$\mathcal{E}$-DCDC and online-$\mathcal{E}$-DCAC, where the question is whether the chair can ensure that no one “$d$ or worse” is a winner (i.e., the chair can ensure that no member of $\{c \mid d \geq_\sigma c\}$ is a winner).

For destructive control by deleting candidates, there is a special issue as to whether the chair can simply start deleting some or all candidates who are “$d$ or worse,” thus perhaps ruthlessly obtaining her goal. Our default model—call it the “non-hand-tied chair” model—is that the chair may delete some, but never all, of the candidates who are “$d$ or worse.” An alternative model—call it the “hand-tied chair” model—is that the chair may never delete anyone who is “$d$ or worse.” The results we mention in this appendix for destructive control by deleting candidates hold equally well for both those models. In both these models, in any legal input instance, the “previous actions” by the chair cannot violate the model, e.g., in the hand-tied chair model, if the history shows that some candidate in $\{c \mid d \geq_\sigma c\}$ was deleted, then the input is rejected, as it is illegal. The history the
snapshot provides can legally contain dumb actions, but it cannot contain illegal ones.

As always, in the language of multiagent systems candidates are alternatives and voters are agents. So though about “elections,” this model is equally well about preference aggregation in multiagent systems in which the alternatives are sequentially revealed and evaluated by the agents, and another party is trying to control the outcome.

4 Complexity Results

Let us assume that our election system’s (E’s) winner-determination problem (i.e., “Is candidate c a winner under this election system, if the candidates and votes are C and V?”) is in polynomial time. Then it is easy to see from the quantifier approach mentioned above that all our above online candidate control problems are in PSPACE. The PSPACE upper bound remains valid even if we restrict E’s winner problem not to P but rather to PSPACE.

Clearly, not all election systems will require the full power of PSPACE for mounting control attacks. It is easy to construct artificial systems where all these control attacks have polynomial-time algorithms. But a more important question is whether the PSPACE upper bound is itself too enormous. Can such tremendous control complexity be realized, even for election systems whose winner problems must be in polynomial time?

The answer is yes. Although the construction is not simple, we have by setting up appropriate election systems and reductions from intractable problems, shown that for each of the problems defined above, there is an election system with a polynomial-time winner problem for which the online control problem of the given type is PSPACE-complete.

Briefly put, the construction enmeshes issues of formulas into election systems in a way that so tightly incorporates and interprets formulas, variables, and assignments, that one can—by using a careful reduction and some legal preprocessing transformations—ensure that the process of the online control attempt can succeed exactly if the input to a PSPACE-complete formula-problem that transformed into that problem is a positive instance.

Theorem 4.1 1. For each election system E with a polynomial-time winner problem, online-E-CCDC, online-E-CCAC, online-E-DCDC (in both the non-hand-tied and the hand-tied chair model), and online-E-DCAC are in PSPACE.

2. There exist election systems E and E′ with polynomial-time winner problems such that online-E-CCDC, online-E-CCAC, online-E′-DCDC (in both the non-hand-tied and the hand-tied chair model), and online-E′-DCAC are PSPACE-complete.

Proof. 1. The famous characterization of PSPACE as alternating polynomial time, due to Chandra, Kozen, and Stockmeyer [CKS81], establishes the upper bounds for these four problems: Each can be solved by an alternating Turing machine in polynomial time, and thus by a deterministic polynomial-space Turing machine.

1The first statement of Theorem 4.1 holds even for election systems whose winner problems are in PSPACE.
2. For proving the lower bounds, we will define $\leq^m_n$-reductions from the PSPACE-complete problem QBF to our online candidate-control problems. In fact, we will prove that these problems are PSPACE-complete even when limited to the case of there being one voter.

We start by providing the $\leq^m_n$-reduction from QBF to online-$\mathcal{E}$-CCAC for the election system $\mathcal{E}$ defined as follows. Interpret each candidate as a pair $(F, i)$, where $F$ is a boolean formula and $i$ a nonnegative integer. If there are any syntactic problems, or if any two candidates have distinct boolean formulas, then everyone loses. Otherwise, all candidates have the same boolean formula, call it $\hat{F}$. Let $\ell$ be the number of variables in $\hat{F}$ (e.g., $\hat{F} = (x_1 \lor \neg x_2) \iff (x_1 \land \neg x_3 \lor x_3)$ has three variables: $x_1$, $x_2$, and $x_3$). We assume that $\ell \geq 1$ (otherwise, $\hat{F}$ is syntactically illegal, so everyone loses). Now, if (a) $\ell$ is odd or (b) the candidate set does not contain $(\hat{F}, i)$ for every even $i$, $0 \leq i \leq \ell$, or (c) there are two or more voters, then everyone loses. Otherwise, lexicographically order the $\ell$ variables by their names. We will refer to the lexicographically $i$th among them as $v_i$ for the purpose of this proof. For each odd $i$, $1 \leq i \leq \ell$, set $v_i$ to true if and only if there is a candidate named $(\hat{F}, i)$. For each even $i$, $2 \leq i \leq \ell$, set $v_i$ to true if and only if the single voter prefers candidate $(\hat{F}, i)$ to candidate $(\hat{F}, 0)$. Now, if $\hat{F}$ is true under this assignment then everyone wins; else everyone loses. This ends the specification of election system $\mathcal{E}$. Note that the winner problem for $\mathcal{E}$ is in P.

We now $\leq^m_n$-reduce QBF to online-$\mathcal{E}$-CCAC. Let $G$ be a given QBF instance, i.e., we want to know whether $G$ belongs to QBF. Without loss of generality, let $G$ be of the form $(\exists w_1)(\forall w_2) \cdots (\exists w_{j-1})(\forall w_j)[\varphi(w_1, w_2, \ldots, w_j)]$, where $j \geq 1$ and $\varphi$ is a propositional formula. Now rewrite $G$ and $\varphi$ so that their actual variable names are lexicographically ordered in the order $w_1, w_2, \ldots, w_{j-1}$. Then remove all quantifiers. Call what that creates $\hat{F}$; it is basically $\varphi$ with variable names adjusted as above.

We will map this to the following instance of online-$\mathcal{E}$-CCAC:

- the initial set of already qualified candidates is $\{(\hat{F}, 0), (\hat{F}, 2), \ldots, (\hat{F}, 2j)\}$;
- the set of spoiler candidates that can potentially be added is $\{(\hat{F}, 1), (\hat{F}, 3), \ldots, (\hat{F}, 2j-1)\}$;
- the addition bound $k$ is $j$;
- the chair’s preference order $\sigma$ is $(\hat{F}, 2j) >_\sigma (\hat{F}, 2j-1) >_\sigma \cdots >_\sigma (\hat{F}, 0)$;
- the distinguished candidate $d$ is $(\hat{F}, 0)$;
- the presentation order of the candidates is $(\hat{F}, 0), (\hat{F}, 1), \ldots, (\hat{F}, 2j)$;
- the current candidate, $c$, for whom the chair has to make a decision now as to whether to add her is $(\hat{F}, 1)$; and
- the voter set is a single voter whose preference with regard to $(\hat{F}, 0)$ and $(\hat{F}, 1)$ is (actually irrelevant for the reduction but we are required by our model to provide it): $(\hat{F}, 0)$ is preferred to $(\hat{F}, 1)$.

\footnote{Our reductions will always map to formulas of the form $(\exists w_1)(\forall w_2) \cdots (\exists w_{j-1})(\forall w_j)[\cdots]$.}
This completes the description of the ≤ₚm-reduction from QBF to online-ε-CCAC, which clearly is computable in polynomial time. It remains to prove that it is correct. This, however, is easy to see: By definition of the election system ε, G ∈ QBF if and only if the chair can make some decision about the current candidate c (to add, or not to add) such that, assuming that the chair at each future decision is free to act in light of the information revealed up to that point, the chair can ensure that the winner set will have a nonempty intersection with the candidates she likes, \{(\hat{F},i) \mid (\hat{F},i) ≥_σ (\hat{F},0)\} (which happens to be all candidates) regardless of what else happens in the election.

PSPACE-completeness of online-ε-CCDC is proven very similarly. We use the same election system ε, and from a given QBF instance G (as above) we construct an online-ε-CCDC instance, where we start with the candidate set \{(\hat{F},i) \mid 0 ≤ i ≤ 2j\} and now set the deletion bound to k = j. Everything else in the reduction remains the same. Again, it follows that G ∈ QBF if and only if the chair can make some decision about the current candidate c (to delete, or not to delete) such that, assuming that the chair at each future decision is free to act in light of the information revealed up to that point, the chair can ensure that at least one candidate (all of which the chair likes) wins, regardless of what else happens in the election.

The destructive cases can be handled quite similarly. The only difference is that we define election system ε′ to be just like ε except that we change every occurrence of “everyone loses” to “everyone wins” and every occurrence of “everyone wins” to “everyone loses.” In our reductions from QBF to the destructive control problems, using the same σ as the chair’s preference order and the same distinguished candidate d = (\hat{F},0) will be fine. In particular, it will not conflict with either the non-hand-tied or the hand-tied chair model: Under each of those models, specifying this d and this σ means “we cannot delete candidate (\hat{F},0),” so deleting (\hat{F},0) would be illegal; therefore, to keep this candidate from winning, the only way is to ensure the “everyone loses” triggers due to (\exists w_1)(\forall w_2)⋯(\exists w_{j-1})(\forall w_j)[\hat{F}(w_1,w_2,\ldots,w_j)] holding. But this implies for each considered case of destructive online candidate control (online-ε-DCDC in both the non-hand-tied and the hand-tied chair model and online-ε-DCAC) that the given control instance is positive if and only if G ∈ QBF.

We now turn to online candidate control for some specific election system widely in use: plurality. The following result shows that both constructive and destructive online control by adding and deleting candidates is an easy problem for (candidate-sequential) plurality voting, in sharp contrast with the corresponding standard control problems, which are NP-complete for plurality (for the two constructive cases, this is due to the fact that the two relevant unique-winner-model results of [BTT92, HHR07] have been verified in [FHH14] to also hold in the nonunique-winner model; for the two destructive cases, this is due to the fact that we have verified that the two relevant unique-winner-model results of [HHR07] also hold in the nonunique-winner model).

**Theorem 4.2** online-plurality-CCDC, online-plurality-CCAC, online-plurality-DCDC (in both the non-hand-tied and the hand-tied chair model), and online-plurality-DCAC are in P.

**Proof.** Consider an input to the problem online-plurality-CCDC as defined above. So we are focused on some current “moment of decision” for the chair (recall what this means from page 3).
We describe a polynomial-time algorithm for the question: Does the chair have a current action that will ensure her of reaching her goal?

Let $d$ be the distinguished candidate and $\sigma$ be the chair’s preference. We refer to the candidates in $\{a \mid a \geq_\sigma d\}$ as good candidates and to the other ones as bad candidates. As required in this moment of decision described on page 3 each of the already revealed candidates have their flags set as to whether or not they have been deleted, and we assume that the votes are currently masked down to the still-standing candidates (i.e., to the already revealed, yet not deleted candidates up to this point in time).

Rather than analyze directly what to do in this moment, let us note that we have at most two choices in this moment. Either we leave $c$ in, or (only if the number of allowed deletions has not been already expended with the already done deletions) we remove $c$. Each of those cases leaves a relatively pure situation, in which $c$ is no longer special. And so what we will now do is show how to analyze such a pure situation, i.e., to say, in such a situation, whether the chair has a forced win. (Knowing how to do that in polynomial time implicitly resolves our problem. We just check the two cases and see if at least one is a forced win for the chair—except if the deletion bound was already expended then we check only the case where $c$ is kept in and we see if that is a forced-win setting.)

So, what does such a case look like? The setting is we now are given: the set of candidates; an ordering $\sigma$ over the candidates; a designated candidate $d$ (recall: our goal is to ensure that the winner set has nonempty intersection with the set $\{a \mid a \geq_\sigma d\}$); an order $\tau$ in which the candidates are being revealed, where the candidates in a nonempty prefix of $\tau$ (i.e., the ones who have had the preferences among them revealed so far) are flagged as to being either removed or kept in (and all candidates after that in $\tau$ are not yet flagged as being in or out); all votes but masked down to just the still-standing candidates (the already revealed but not deleted candidates); and a natural number $k \geq 0$, which will be how many deletions are left to use.\footnote{The prefix of $\tau$ is nonempty because “$c$” will already have been revealed, since we call this polynomial-time routine only, see the previous paragraph, when at least one candidate has been set as in-or-out.}

And we need to know whether we have a forced win even if the universe is perfectly hostile to us, i.e., we need to know whether no matter what votes are revealed as the process progresses we will win (i.e., the winner set will have a nonempty intersection with $\{a \mid a \geq_\sigma d\}$).

Let $b$ be the number of bad candidates currently unrevealed.

Our polynomial-time algorithm proceeds as follows. If there is no voter, every candidate that is left standing at the end will be a plurality winner (with score zero). So in this case our algorithm accepts if there is at least one good candidate revealed but undeleted, or not yet revealed. And otherwise our algorithm rejects. So henceforward, let us assume that the number of voters is at least one.

If all candidates are already revealed, just analyze whether a good candidate is among the winners, and if so, return success on this case. Otherwise, we go on as follows.

We now check whether any good candidates are currently winning. If no good candidate wins

\footnote{If from the prefix of $\tau$ and the flags we find more deletions have happened than the problem originally allowed, a case that won’t actually ever happen within the way we are calling this algorithm, we are then already in a no-win case since we, the chair, back in the given history already violated the deletion bound. So if this (impossibly) were to happen, then we would return the fact that forced-success along a legal path from our current state is impossible, since we’ve already violated the rules.}
at the current moment, then (i) if there is at least one unrevealed good candidate and the number $b$ of unrevealed bad candidates is at most $k$, then we have a forced win here; (ii) otherwise return that we don’t have a forced win here (for example, we certainly won’t win if the votes will be eventually revealed to show that the future—i.e., as yet unrevealed—bad candidates are a top segment of every voter’s vote).

Otherwise, at least one good candidate is currently winning (according to the votes masked down to the still-standing candidates up to the current point). We first check whether $k \geq b$, i.e., whether all bad candidates after $c$ can be deleted. If this is not the case, then much as above there is no hope for the chair to be sure to reach her goal, since for example even a single future bad candidate that cannot be deleted may be ranked on top of every vote in the worst case.

Let us finally consider (with all the things eliminated above not being on the table as possibilities that we have to deal with here) the case where at least one good candidate is currently winning and $k \geq b$. If there are no revealed, still-standing bad candidates, return that we have a forced win (as there will be no bad candidates at the end and at least one good one, so there will certainly be a good one among the winners), and so henceforward assume we have at least one revealed, still-standing bad candidate.

Let $B$ be the highest current plurality score among all revealed, still-standing bad candidates, according to the votes masked down to the revealed, still-standing candidates.

A best strategy for the universe, regarding seeking to defeat the chair’s goals, is as follows.

- All future bad candidates are in a top-segment of each vote. This forces the chair to expend $k - b$ deletions on removing those future bad candidates.

- The interesting twist here is what a hostile universe can do with future good candidates. Note that future good candidates can, if not deleted, steal first-place votes from existing good candidates that are winning or otherwise doing well.

The most effective attack a hostile universe can do, given that it is already going to force us to remove all future bad candidates, is to leave $B$ as the strongest score of any bad candidate (the universe has no tool left to increase beyond $B$ since it is—for a different compelling reason—making us spend our deletions to burn away all future bad candidates, and the existing bad candidates already are revealed and so can only move down in how many votes they are at the top of as more candidates are revealed in the votes; but moving them down would play against the universe’s interests here). So the universe’s best strategy is to also use new good candidates to try to decrease the scores of all revealed, still-standing good candidates to at most $B - 1$, while also ensuring that no added good candidates score more than $B - 1$. Of course, the chair would try to stop this, by deleting good candidates as needed (at most $k - b$ deletions of good candidates though, on top of the $b$ deletions of bad candidates that the universe can force to be needed bad-candidate deletions).

Let $g$ be the number of future good candidates. If $k - b \geq g$, we (the chair) have a forced win (recall that a good candidate is currently winning), as we can remove all of those candidates.

What about the case $k - b < g$? Note that the universe then can add $g - (k - b)$ good candidates (of our choice, not its choice, but it turns out that our choice won’t help us here against an optimally hostile universe). The most effective thing the universe can try to do is to use these new good candidates to try to draw all the revealed, still-standing good candidates down to at most $B - 1$ first-place votes, while also not letting any still-standing-at-the-end good candidate (including ones
added after the moment this problem is looking at) get more than \( B - 1 \) first-place votes.

Let \( S \) denote the set of all revealed, still-standing candidates \( i \) for which \( \text{score}(i) \geq B \). Note that the “surplus” of each member \( i \) of \( S \) is \( \text{score}(i) - (B - 1) \), and the universe is trying to in effect shed these surpluses onto future good candidates that don’t get removed by the chair, while ensuring that none of those candidates itself ends up with \( B \) or more first-place votes. The universe will have at least \( \|S\| + g - (k - b) \) candidates over which to do this spreading.

It is thus not hard to see that if

\[
\left\lceil \frac{\sum_{i \in S} \text{score}(i)}{\|S\| + g - (k - b)} \right\rceil \geq B,
\]

then we (the chair) have a forced win, since we can ensure that at least one current or added good candidate will have score at least \( B \). And if the above ceiling is less than \( B \), we do not have a forced win, as the universe can by adding good candidates ensure that all good candidates still-standing at the end have score at most \( B - 1 \).

This completes our polynomial-time algorithm (that is “called” up to two times—with \( c \) left in, and if \( k > 0 \) then also with \( c \) removed), and so gives us an overall polynomial-time algorithm for online-plurality-CCDC, as promised.

That online-plurality-CCAC is in P can be shown similarly. A difference is that now we have two types of candidates: qualified candidates who are certainly in the election and spoiler candidates who can possibly be added by the chair. Therefore, instead of speaking of candidates to be deleted (or not), our algorithm will now speak of spoiler candidates to be added (or not). In particular, when we are in the case that at least one good candidate is currently winning (according to the votes masked down to the still-standing candidates up to the current point), in order to check whether no bad candidate after \( c \) can enter the election (which would immediately destroy the chair’s hopes), we now need to check whether all bad candidates after \( c \) are spoiler candidates (which the chair simply doesn’t add).

Now, the hostile universe’s best strategy against the chair (leading to our worst-case scenario) can be described similarly to the above best strategy of the universe in the case of online-plurality-CCDC, except that we consider “good candidates after \( c \) that are qualified” instead of “good candidates after \( c \) that cannot be deleted.”

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6How does the universe do this, given that it doesn’t know which up to \( k - b \) good candidates we will delete? If it did know which, how to do this balancing is immediate. But what it will do is it will as the votes are revealed for each future good candidate make them consistent on the up-to-then revealed candidates with what it would do with those votes if that one were to be the next member of its set of at least \( g - (k - b) \) good candidates to add. If the chair removes the candidate, it will then try that same approach again with the next to-be-revealed good candidate. Not knowing which still-in-the-future-at-that-point candidates will be allowed in is not a problem, since the universe does not have to commit to their locations in the votes until the moment the candidate is revealed for possible deleting by the chair. So by the end, the universe has indeed added a set of at least \( g - (k - b) \) good candidates such that no still-standing good candidate has more than \( B - 1 \) first-place votes. Note of course that this process never requires the universe to have a member of \( S \) take a point “away” from another member of \( S \), which is good as that simply can’t be done. Rather, it is the incoming good votes that are used to drain away the points from the members of \( S \).

7Note that the chair can decide not to add any of the future spoiler candidates; only the number of spoiler candidates that can be added is limited. In fact, the chair will never add a bad spoiler candidate coming after \( c \). Note also that just a single future candidate that is both qualified and bad would kill off the chair’s chances to reach her goal, since this candidate will take the top position of each vote in the worst case.
The destructive cases can be handled analogously as well, with just a few changes. Consider online-plurality-DCDC with essentially the same notation used above for online-plurality-CCDC. In particular, $d$, $\sigma$, $c$, $k$, $b$, and $B$ have the same meaning, except that the distinguished candidate $d$ has now turned from a good into a bad candidate, since the chair’s goal now is to make sure that no one “d or worse” is a winner (i.e., we now refer to the candidates in $\{a \mid a >_\sigma d\}$ as good candidates and to the other ones as bad candidates). We can handle both the non-hand-tied chair model (where, recall, the chair may delete some, but never all, bad candidates) and the hand-tied chair model (where, recall, the chair may never delete any bad candidate). However, here we will simply mention just some key differences, starting with the former. For instance, when we are in the case that at least one good candidate is currently winning (according to the votes masked down to the still-standing candidates up to the current point) and we need to check whether all bad candidates after $c$ can be deleted in the non-hand-tied chair model, we have to check whether either some already revealed bad candidate up to now has been labeled as undeleted and $k \geq b$, or no already revealed bad candidate up to now has been labeled as undeleted and $b = 0$. If this is not the case, there is no hope for the chair to reach her goal, as even a single future bad candidate that cannot be deleted in the non-hand-tied chair model will be ranked on top of every vote in the worst case and thus wins. That is, the chair doesn’t have a forced win in this case. Similarly, the rest of the argumentation can be slightly adapted to show that online-plurality-DCDC in the non-hand-tied chair model is in P.

The proof that in the hand-tied chair model also online-plurality-DCDC is in P differs from the above proof only slightly. For instance, instead of checking whether all bad candidates showing up after $c$ can be deleted, we now check only whether $b = 0$, as we are not allowed to delete any bad candidate in the hand-tied chair model.

Finally, incorporating in the proof of online-plurality-CCAC $\in$ P the changes corresponding to those that turned the proof of online-plurality-CCDC $\in$ P into a proof of online-plurality-DCDC $\in$ P (though disregarding the issue of whether the chair is hand-tied or not), we see that online-plurality-DCAC is in P as well.

5 Conclusions and Open Directions

This paper’s contribution is a model and a number of results for the research direction of candidate-sequential elections—a direction that we feel is of interest, not as a replacement for the study of voter-sequential elections, but as a notion that captures different but also important settings. Our results show that online candidate-control can be extremely complex, but that for the most important real-world election system, plurality, candidate control can be quite simple—even of polynomial time-complexity.

It will be important to seek further results for the complexity, in this model, of natural systems. It would also be interesting to formalize and study online control by partition of candidates in sequential elections. Another interesting direction will be to also give the chair limited or total control over the candidate presentation order; in political science, for example, in many settings control of agenda-order can be powerful.
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