Current theories of particle physics lead to the unavoidable conclusion that there must have been several phase transitions in the early universe. Further, in the context of these theories, it is possible that cosmological phase transitions would have produced topological defects that may be roaming our heavens today. A finding of these fossils from the early universe would provide a direct confirmation of the thermal history of the cosmos, insight into astrophysical phenomena, and, vital information about particle physics. The elimination of unobserved topological defects provides important constraints on particle physics and may also suggest novel cosmology. I describe some of the research on cosmic topological defects and recent efforts to address cosmological issues in condensed matter systems.

I. INTRODUCTION

The theoretical foundations of cosmology were laid by Einstein in 1915 with the discovery of General Relativity. In this framework, it became possible to describe mathematically the evolution of the universe and to address questions about its beginning and its end. Subsequently, the world of cosmology opened up with Hubble’s observation that distant galaxies are receding, thus leading to the conclusion that the universe is expanding. These historic discoveries marked the beginning of modern observational cosmology and initiated detailed investigations of our universe. Today we can answer questions that earlier we could not even imagine asking.

The observed expansion of the universe means that the younger universe was smaller and hotter. Using our current knowledge of physics, this leads to a picture of the universe when it was only a few minutes old and at a temperature of $10^{10}$ K. Remarkably, this picture can be (and has been) tested, since the light elements were “cooked” at this time and we can compare calculations of the cosmological fraction of elements like Hydrogen, Helium, Deuterium, and Lithium with their observed abundances. The success of “Big Bang Nucleosynthesis” gives us confidence in our understanding of the universe from a few minutes after the big bang.

In accelerator experiments, we have studied matter up to energies corresponding to temperatures of about $10^{29}$ K. The theoretical description of matter at such temperatures is given by the electroweak model due to Glashow, Salam and Weinberg. The triumph of the model was in the prediction of the existence of the $W^\pm$ and $Z$ bosons which were later discovered at CERN. Hence we feel fairly confident that we understand the behaviour of matter up to $10^{15}$ K.

The standard model of cosmology that has been so successful in its big bang nucleosynthesis predictions, when extrapolated back to a time of $10^{-10}$ s, predicts that the universe was at a temperature of about $10^{15}$ K and so must have been the arena for electroweak physics. Our confirmation of the electroweak model provides us with some confidence in our understanding of the universe at an age of $10^{-10}$ s, though we do not yet have any means to directly probe the universe of that time. At even earlier times, when the universe must have been at a temperature of about $10^{29}$ K, particle physicists believe the universe was the stage for the physics of “Grand Unified Theories” (GUTs). Here, we do not yet have a standard model of particle physics, but there are several candidates. The exploration of the consequences of particle physics (and in particular, GUTs) for cosmology, and vice versa, has become a subject in its own right.

The electroweak model and GUTs are based on a scheme called “spontaneous symmetry breaking” which, in lay terms, is another name for phase transitions. If these descriptions of particle physics are correct, the unavoidable implication is that the early universe must have seen phase transitions much like the freezing of water and the magnetization of iron. Then, the consequences of phase transitions that we observe in the laboratory can be expected to apply to the universe as well. In particular, relics of the high temperature phase of condensed matter systems called “topological defects” are routinely observed in the laboratory and similar relics of the early high temperature universe could exist in the present universe. In other words, these are possible fossils from the early universe.

The hunt for cosmic topological defects depends crucially on their properties. The last two decades have seen extensive research on topological defects and their potential role in cosmology. Very recently, the lack of experimental input has been relieved by enterprising condensed

*In recent times, there has been discussion of whether the particles that we know (eg. electrons) are actually topological defects [1]. This kind of idea has a long history and the possibility that electrons are objects with structure dates back to Abraham [2] and Lorentz [3]. I will not discuss this very interesting aspect of topological defects in the present article.
matter physicists who have been performing experiments in the laboratory to answer questions of great interest to cosmologists. But before explaining the possible role of topological defects in the cosmos and the lab, I need to describe some basics of modern particle physics.

II. INSIDE THE ATOM

Today, we observe four seemingly different forces in Nature. First is the force that holds us on the Earth, namely, gravity. Second is the force that keeps the atom together which is electromagnetism. Then there is the “weak” nuclear force which causes radioactivity and the “strong” nuclear force which holds the proton together.

Historically, electricity and magnetism were believed to be two different forces that were treated in a unified manner only once Maxwell wrote his equations. In particular, this means that there is only one coupling constant that describes the strength of the electric and magnetic forces. Today we understand electromagnetism as the simplest kind of “gauge theory”. In fact, the known non-gravitational forces are ascribed to the exchange of spin 1 particles called gauge particles which for the electromagnetic force is none other than the photon. In mathematical language, the photon is a particle of a gauge field \( A_\mu(t, \vec{x}) \). Now, it is well-known that there is a symmetry of electromagnetism related to the gauge transformation:

\[
A_\mu \to A'_\mu = A_\mu + \frac{1}{e} \partial_\mu \Lambda(t, \vec{x})
\]

where, \( \Lambda \) is an arbitrary function and \( e \) is a coupling constant. This symmetry is described by rotations in a complex plane as can be seen if we couple the photon to a complex scalar field \( \Phi \). Then an interaction term that preserves the gauge symmetry is

\[
|D_\mu \Phi|^2 \equiv |(\partial_\mu - ieA_\mu)\Phi|^2
\]

provided we also perform the transformation

\[
\Phi \to \Phi' = e^{i\Lambda(t,\vec{x})}\Phi.
\]  

(1)

This transformation is a (space-time dependent) rotation in the complex \( \Phi \) plane, and hence electromagnetism is invariant under rotations described by one angle \( \Lambda \). Such rotations form a group called \( U(1) \) (the group of unitary \( 1 \times 1 \) matrices) where the subscript \( Q \) is used to denote that the charge associated with the symmetry is ordinary electric charge.

The \( U(1) \) symmetry of the model can be “broken” or “hidden” in the vacuum if \( \Phi \) takes on a non-vanishing, fixed value in the lowest energy state. This can happen if, for example, there is a potential term for \( \Phi \) such as

\[
V(\Phi) = \frac{\lambda}{4} (|\Phi|^2 - \eta^2)^2.
\]

Then the lowest energy state is obtained with \( |\Phi| = \eta \) which is non-zero, and we say that \( \Phi \) has a “Vacuum Expectation Value” (VEV). As the VEV is not invariant under phase rotations, the \( U(1) \) symmetry is said to be spontaneously broken. Furthermore, by calculating thermal effects it can be shown that at high temperature, \( \Phi = 0 \) is the lowest energy state, while at low temperature \( |\Phi| = \eta \) is preferred. So, if we have a thermal bath of \( \Phi \) and \( A_\mu \) quanta, at high temperatures the system will have a \( U(1) \) symmetry which will be broken upon cooling. This symmetry breaking is depicted as:

\[ U(1) \to 1. \]

The reader unfamiliar with group theory might feel lost among the strange symbols such as \( U(1) \) and others to follow. It is best to simply think of these as shorthand notations for writing down all the transformations of the fields in the model that leave the physics of the system unchanged. So \( U(1) \) is just a convenient way of saying that the transformations that leave Maxwell’s equations unchanged correspond to rotations in the complex plane. Another example, closer to everyday experience, is the set of continuous transformations that leave a sphere unchanged. This is the group of all rotations in three dimensions and is denoted by \( SO(3) \).

Using a generalization of the gauge symmetry idea and spontaneous symmetry breaking, electromagnetism and the weak force have now been unified in the Glashow-Salam-Weinberg electroweak model. This unification is, however, different from that of electricity and magnetism since the unified model still has two coupling constants. The unification stems from the fact that the electromagnetic and weak forces are now described within a common framework. The electroweak model is based on the gauge symmetry

\[
SU(2)_L \times U(1)_Y
\]

(2)

which means that the group elements are direct products of special (determinant equal to one), unitary, \( 2 \times 2 \) matrices, and, phase factors as in \( U(1) \). The \( L \) subscript means that the \( SU(2) \) acts on certain (left-handed) fermions and the \( Y \) subscript denotes that the associated charge is “hypercharge” and serves to distinguish the \( U(1) \) symmetry from that of electromagnetism written as \( U(1)_Q \). There are four gauge fields in the electroweak model: three \( (W^a_\mu, a = 1, 2, 3) \) transforming under the \( SU(2)_L \) and one \( (Y_\mu) \) under the \( U(1)_Y \).

At this stage, it is not evident where electromagnetism is contained in the electroweak theory since there is no sign of \( U(1)_Q \) in (2). Also, the theory with the symmetry group (2) contains four different kinds of massless, spin 1 particles whereas we only see one (the photon). What happened to three of the four bosons?

Let us now introduce a Higgs (scalar) field, \( \Phi \), which transforms under the group elements in (2) and is in the
doublet representation of $SU(2)_L$ i.e. it should be a two complex component vector. $\Phi$ is now assumed to get a “Vacuum Expectation Value” (VEV), that is, $\Phi = \Phi_0 \neq 0$. So now transformations that change the value of $\Phi$ are not allowed. This means that the symmetry group in (2) is no longer valid and one must find the subgroup that leaves the VEV unchanged. This subgroup turns out to be a $U(1)$ group and is none other than $U(1)_Q$. Therefore, after spontaneous symmetry breaking, there is only one gauge field $(A_\mu)$ that is massless just as we observe, and there are three gauge fields $(W^\pm_\mu, Z^0_\mu)$ that are massive. So the massless photon can mediate long range forces, while the massive gauge bosons can only mediate short range (weak) forces. In this way, starting from a very symmetric situation one derives the vastly disparate electromagnetic and weak forces.

In this article, I will mainly be interested in the aspect of spontaneous symmetry breaking which in the electroweak model can be depicted as:

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_Q.$$  

In fact, this is not quite correct since the $SU(2)_L$ and the $U(1)_Y$ factors contain two elements that are common. (This is the center of $SU(2)_L$ which contains the elements $\pm 1$.) So the correct symmetry breaking is:

$$[SU(2)_L \times U(1)_Y]/Z_2 \rightarrow U(1)_Q.$$  

(3)

The precise structure of symmetry groups can be very important in the determination of the cosmological consequences of the model. So far we have ignored the strong force. The theory describing this force is called “Quantum Chromo Dynamics” (QCD) and is based on an unbroken $SU(3)_c$ group where the index $c$ stands for the “colour” charge. So the standard model is based on a product of three groups, that is,

$$[SU(3)_c \times SU(2)_L \times U(1)_Y]/(Z_3 \times Z_2)$$

(5)

With every group there is an associated gauge coupling constant and so the model has three gauge coupling constants which are denoted by $g_3$, $g_2$ and $g_1$ for the strong, weak and hypercharge factors.

In field theory it is known that coupling constants “run”. This means that the values of the coupling constants that one measures depend on the energy at which the measurement is performed. The rate of the running is determined by the renormalization group equations which we will not discuss here. But the point is that the three different coupling constants of the standard model seem to converge to the same value at an energy scale of about $10^{16}$ GeV (see Fig. 1). This suggests that there is only one coupling constant at high energies and most likely only one symmetry group. In other words, the suggestion is that there is “Grand Unification” described in terms of a grand unified group.

Let us denote the grand unified group by $G$. Then, as in the electroweak model, $G$ must break down to the standard model group which must be a subgroup of $G$:

$$G \rightarrow [SU(3)_c \times SU(2)_L \times U(1)_Y]/(Z_3 \times Z_2).$$

(4)

Two of the simpler examples of $G$ often seen in the literature are $SU(5)$ and $SO(10)$.

**III. A CHANGE OF PHASE**

As we have seen, a central idea in modern theories of particle physics is that there is spontaneous symmetry breaking. However, the idea actually originated in condensed matter physics in the context of phase transitions. To understand the connection between spontaneous symmetry breaking and phase transitions, consider a very simple phase transition in which a gas (or liquid) freezes to form a solid. In the gaseous phase, the molecules are flying around in random motion and the (infinite) container of gas is symmetric under translations:

$$\vec{x} \rightarrow \vec{x} = \vec{x} + \vec{\delta}$$

where $\vec{\delta}$ can be any arbitrary vector. That is, the symmetry group is that of all translations. Now, when the gas solidifies, the molecules are arranged on some lattice and the residual symmetry transformations are restricted to

$$\vec{\delta} = \vec{a}$$

where $\vec{a}$ is a vector from any one lattice site to another. Hence, translational symmetry has been broken (reduced) by the change of phase.
Going back to particle physics, the very successful electroweak theory is based on spontaneous symmetry breaking, and hence we are faced with the prospect of phase transitions in particle physics. If, somehow, we were to heat up the particle physics vacuum, at some high temperature we would be likely to find a new phase. For the electroweak phase transition to occur, we expect to need a temperature of about $10^{15}$ K. (The sun’s interior is at a mere ten million degrees.) In particle accelerators, we can achieve the corresponding energies, but only over a very small region and for a very short duration. So particle accelerators are not currently useful for studying the electroweak phase transition. (They are, however, being used to study the QCD phase transition at a temperature of $10^{10}$ K.) The GUT phase transition needs an exorbitant $10^{29}$ K and it would be hard to even dream of a machine that could attain such energies. However, the early universe must have seen temperatures corresponding to the electroweak transition at the age of $10^{-10}$ s and the GUT phase transition at $10^{-35}$ s, making it the natural environment for the study of high energy particle physics. At the same time, particle processes in the early universe must have determined the state of the current universe and so we would like to understand the cosmology of phase transitions. (For a review of cosmological phase transitions, see the article by M. Gleiser.)

An obvious question at this stage is: how can we study something that happened so long ago? To answer this, I must explain what topological defects are.

**IV. TOPOLOGY AND FRUSTRATION**

Let us return to the solidification of a gas. During this phase transition, the molecules of the gas that are in random motion have to line up into a regular lattice. If the gas is cooled quickly, each small volume of molecules starts lining up but there is not enough time for the distant parts of the gas to decide which line to choose. So molecules in different parts of the gas line up in a lattice but the orientation of the lattice is chosen independently. If the orientations are chosen in a certain way it may become impossible for the entire gas to freeze into a regular lattice. This can happen for topological reasons and the solidification might be frustrated. The end result is a solid with defects in its lattice. Since these defects are due to topological conditions, they are known as “topological defects”. (For reviews of topological defects in particle physics and cosmology, see[14].)

To illustrate topological defects in the particle physics context, consider the $U(1)$ model described in Sec. [1]. Spontaneous symmetry breaking occurs in this model when $\Phi(t, x)$ acquires a VEV (that is, becomes non-zero) at some time. However, as described in a seminal paper by Tom Kibble[15], the acquired value of $\Phi$ at different spatial points will, in general, be different. In particular, on a circle $C$ in space, parametrized by an angle $\theta$, we could have:

$$\Phi|_C = \eta e^{i\theta}, \quad \theta \in [0, 2\pi].$$

There is a topological index associated with this VEV of $\Phi$. (Basically, it is the number of times $\Phi$ wraps around the circle in the complex plane as we go around $C$.) Next consider the disk bounded by the circle $C$ (see Fig. 2). With the value of $\Phi$ on $C$ given in (5), because of the topology, it is possible to show that necessarily $\Phi = 0$ somewhere on the disk. But $\Phi = 0$ is the value of $\Phi$ in the unbroken symmetry phase. Hence the completion of the phase transition is frustrated because of the topology in the model. Also, the spatial point where $\Phi$ vanishes is not in the vacuum (because the vacuum corresponds to $\Phi \neq 0$) and hence, there is energy at this point. This energy configuration is called a topological defect.

**FIG. 2.** The winding of the field $\Phi$ around the circle $C$ forces $\Phi$ to vanish at a point on any surface spanned by $C$. By considering different surfaces bounded by $C$, we see that there is a one-dimensional locus of points at which $\Phi = 0$. Since $\Phi \neq 0$ in the vacuum, there is energy in the neighbourhood of the curve on which $\Phi = 0$. This energy is locked-in because to remove it, the field would have to be rearranged over an infinite region of space. The energy distribution around the curve with $\Phi = 0$ is a “string”.

In the case of the $U(1)$ model, we could consider any surface bounded by the circle $C$, and since $\Phi \neq 0$ everywhere on $C$, there will always be a point on the surface with $\Phi = 0$. Therefore there will be a one-dimensional locus of points where the phase transition has been frustrated and has $\Phi = 0$. This one-dimensional topological defect is called a “string” and was first theoretically described by Abrikosov in the condensed matter context[16], and by Nielsen and Olesen in the particle physics context[17].

Very similarly, phase transitions can get frustrated by topology in more complicated models. This can result in two-dimensional topological defects called “domain walls”, strings with junctions in them, point-like defects called “monopoles”, and, many hybrids. A distinction is also made between defects that have associated magnetic fields and those that have none. The former are...
called “local” or “gauge” defects while the latter are called “global” defects. Domain walls are always global, but strings and monopoles can be global or local. Local monopoles were discovered independently by ’t Hooft [2] and by Polyakov [3]. They are also known as “magnetic monopoles” and behave just like isolated North or South poles of a bar magnet. In addition to these topological defects, there is another defect called a “texture” in which the field Φ is forced to vanish at one point in space-time.

How can we determine the topological defects in any given model? The secret lies in the symmetry breaking pattern which in turn determines the topology of the vacuum manifold. The point is that, if a certain field configuration yields the lowest energy state of the system, transformations of this configuration by the elements of the symmetry group will also give the lowest energy state. For example, if a spherically symmetric system has a certain lowest energy value, this value will not change if the system is rotated. More mathematically, if the group $G$ breaks to a subgroup $H$ (as, for example, in (3) or (4)), and the system is in the lowest energy state which we denote by $S$, transformations of $S$ by elements of $G$ will leave the energy unchanged. In addition, transformations of $S$ by elements of $H$ will leave $S$ itself (and not just the energy) unchanged. So the many distinct ground states of the system are given by all transformations of $G$ that are not related by elements in $H$. This space of distinct ground states is called the “vacuum manifold” and is therefore given by the space of all elements of $G$ in which elements related by transformations in $H$ have been identified. The space is denoted by $G/H$ and mathematicians call it a “coset space”.

The outcome of the above discussion is that the symmetry breaking leads to the determination of the vacuum manifold which is some surface in an abstract mathematical space. Now think of the vacuum manifold as a surface like the surface of a ball (two sphere), or, the surface of a doughnut (torus). These surfaces have different topological properties. For example, one can draw a closed path on a torus that cannot be continuously shrunken to a point while all closed paths on a two sphere can be. One can also cover the two sphere with another two sphere (like an orange peel covers the orange) that cannot then be shrunk to a point. It is these properties that are crucial for the existence of topological defects.

If the vacuum manifold (i.e. coset space) has incontractable one spheres (paths), the model will have string solutions. (With a little thinking, the $U(1)$ example above can help to understand this claim.) If the vacuum manifold has incontractable two and/or three spheres, the model contains monopoles and/or textures respectively. If the vacuum manifold is disconnected, we will get domain wall solutions. The topology of various coset spaces has now been determined and is given by what are called “homotopy groups” and denoted by $\pi_n(G/H)$.

Mathematicians have prepared tables that give the homotopy groups for different choices of $G$ and $H$.

The basic fact to remember is that the symmetry breaking pattern determines the topology of the vacuum manifold and hence the topological defects. So given $G$ and $H$ we can determine the topological defects present in the system.

An important feature of topological defects is that they cannot be removed by locally rearranging the fields. In the string case, for example, the circle $C$ could be chosen to be at infinity and the removal of the string through the disk would require rearrangement of the field on an infinite portion of the disk. Any dynamical procedure to do this would need infinite energy and hence the string is permanently locked in.

The energy of a defect depends on the temperature at which it forms. Just to give an idea, monopoles formed at the GUT phase transition would weigh $\sim 10^{-8}$ gms, strings would have a linear energy density of about $10^{22}$ gms/cm, and, domain walls would have a surface energy density of about $10^{22}$ gms/sq-cm.

Not all phase transitions lead to topological defects. A prime example of such a transition is the electroweak phase transition. (GUT phase transitions always lead to magnetic monopoles.) Yet it should be mentioned that there can still be field configurations in the absence of topology that closely resemble topological defects. Examples of such configurations include “semilocal strings” found by Ana Achúcarro and me [14] and “electroweak strings” first found by Nambu [15]. Unlike topological defects, however, these configurations are not permanently locked in and can decay.

The possibility of topological defects in particle physics raises the hope that some of these may have been produced in a cosmological phase transition and could be observed in the universe today by their influence on astronomical, astrophysical and cosmological processes.

V. COSMOLOGICAL OBSERVATIONS

Cosmological surveys now cover a large fraction of our observable universe. Astronomers have mapped the luminous structure in slices of the sky out to a distance of several hundred megaparsecs (see Fig. 3) [16][17]. These maps of the universe show that galaxies are distributed on walls surrounding empty bubbles (voids). This comes as somewhat of a surprise because one’s first guess would be that galaxies are spread randomly in space.

Recently, another vital observational tool for the study of the early universe has become available. This is the

1 However, if there is a defect and an anti-defect in the system, they can mutually annihilate.
structure of the temperature fluctuations in the “Cosmic Microwave Background Radiation” (CMBR). The CMBR is light that is directly coming to us from a time when the universe was about 100,000 years old and at a temperature of about 3000 K. This “recombination” epoch is significant because protons and electrons combine to form hydrogen atoms at about 3000 K. After recombination, the universe contains electrically neutral atoms and, since light does not scatter off neutral atoms, it can travel freely to us. Before recombination, however, the matter in the universe is electrically charged and light scatters strongly. During this period, light propagates as if it were in a fog and so light from the pre-recombination universe cannot reach us. The CMBR is the earliest light we could possibly see and it is very significant that we have actually seen this light (see Fig. 4) [18].

FIG. 3. The points in the wedges show the distribution of galaxies in a slice of the sky as observed by the Las Campanas Redshift Survey. The survey covers three strips of the sky in the Northern hemisphere and another three strips in the Southern hemisphere. The larger angular width of the Northern hemisphere strips is shown on top of the figure (10° to 16° of Right Ascension). The smaller angular widths of the strips is about a few degrees and the Declination of each of the strips is specified on the side of the figure. The radial distance to a galaxy is measured in terms of a velocity corresponding to the observed redshift of the galactic light. The CMBR is extremely uniform in all directions. The uniformity is only spoilt by tiny fluctuations of about 1 part in 10⁵. In other words, the temperature of the CMBR is \( T = 2.7 \) K, no matter in which direction you choose to look but there are fluctuations \( \delta T \) in this temperature:

\[
\left( \frac{\delta T}{T} \right)_{rms} \approx 10^{-5}.
\]

Further, due to the growth in the number of observational experiments, it is now becoming possible to say something about the map of \( \delta T \) over the sky. The observations determine the temperature fluctuation on the sky at different angular scales and so one has quantities related to the spherical harmonics of \( \delta T \). The usual procedure in a calculation of the anisotropy is to decompose the temperature fluctuations on the sky (coordinates \( \theta \) and \( \phi \)) in spherical harmonics:

\[
\frac{\delta T}{T}(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta, \phi)
\]

and then calculate

\[
c_l = \langle |a_{lm}|^2 \rangle,
\]

where the angular brackets denote an ensemble average. Then it is conventional to find

\[
< Q > = \left[ \frac{l(l+1)c_l}{2\pi} \right]^{1/2} T_{cmb}
\]

as a function of \( l \), where \( T_{cmb} \) is the CMBR temperature measured in \( \mu K \). The calculated \( < Q > \)'s are then compared with observations (see Fig. 5) [19].

FIG. 4. The temperature of the CMBR in degrees Kelvin as measured at various frequencies in GHz. (FIRAS was a satellite borne experiment to measure the spectrum of the CMBR.)
The fluctuations of the temperature of the CMBR can be produced in two ways. The first is if the matter that the photons last scattered off (the “last scattering surface”) was not quite uniform, and the second is if there are objects between the last scattering surface and the present that disturb the photons. In the first case, the CMBR provides a very definite determination of the state of the 100,000 year old universe and in the second case, it provides a probe of the universe between now and the last scattering surface. By considering the details of the fluctuations of the CMBR, we hope to be able to derive both the state of the early universe and the intervening influences.

FIG. 5. The observed distribution of \( < Q > \) - a quantity related to the anisotropy of the CMBR in the \( l^{th} \) multipole moment (see text) - together with error bars. The curve is the prediction of an inflationary model. (The \( < Q > \) in this plot is normalized with an extra factor of \( (5/12)^{1/2} \) as compared to the definition in the text for historical reasons.)

VI. FOSSILS FROM THE EARLY UNIVERSE

Based on our knowledge of particle physics, the gradual cooling of the universe must have been punctuated by sharp phase transitions. This is similar to the violent climatic changes on earth that would have affected the otherwise gradual evolution of life. Further, just as we seek fossils of the early forms of life, we can seek fossils from the early universe in the form of topological defects. In fact, topological defects are our main hope of directly studying the very early universe.

The current belief that the electromagnetic, weak and strong forces unified at about \( 10^{16} \) GeV implies a cosmological GUT phase transition at a temperature of \( 10^{29} \) K at the young age of \( 10^{-35} \) s. Then we are led to consider the formation of topological defects corresponding to this scale. These defects could be magnetic monopoles, strings or domain walls.

Magnetic monopoles formed at the GUT scale would dilute with the expansion of the universe while keeping their number fixed. This means that the energy density in monopoles goes down as \( a^{-3} \) where \( a \) is the scale factor of the universe. However, the dominant energy in the early universe is radiation. The energy density of radiation not only gets diluted by the expansion but the energy of each radiation quanta also gets redshifted. Therefore the energy density in radiation falls off as \( a^{-4} \). This means that the energy in monopoles becomes more important as the universe expands. Following this argument by a more careful and detailed analysis, Zel’dovich and Khlopov \( [20] \), and John Preskill \( [21] \) found that GUT monopoles would start dominating the universe very early and would overclose the universe (i.e. the energy density in monopoles would exceed the critical density and the universe would recollapse in a very short time). This is clearly not the case.

Around 1980, the monopole overabundance problem led to a tension between a central belief in particle physics - that of grand unification - and cosmology. For consistency, either grand unification had to succumb, or, cosmology needed revision. The breakthrough was achieved when Alan Guth realized \( [22] \) that an exponential inflationary period in cosmology, during which the energy density in monopoles is diluted to acceptable levels, would alleviate the tension. The following years have seen a number of other solutions to the monopole problem but inflationary cosmology has survived because it also offers solutions to a number of other cosmological puzzles. (See Andrei Linde’s book on inflationary cosmology \( [23] \) for an account of the field.)

Domain walls formed at the GUT scale, like magnetic monopoles, would be a cosmological disaster. If we had one domain wall of mass per unit area equal to \( \sigma \) in our visible universe (size \( t \)), the total energy contained in it would be of order \( \sigma t^2 \). And the energy in all the other matter would be of order \( \rho t^3 \sim t/G \) where \( \rho \sim 1/Gt^2 \) is the energy density in matter and \( G \) is Newton’s gravitational constant. So the ratio of domain wall energy to other forms of energy is \( Gt \). By inserting the value of \( \sigma \) (10^52 gms/sq-cm) and \( G \), we find that domain walls start dominating the universe very early and would lead to a universe unlike ours. This rules out the formation of GUT domain walls. (Though, if the GUT model leads to an inflationary universe, GUT domain walls would be acceptable.)

Cosmic strings formed at the GUT scale are more benign than magnetic monopoles and domain walls. To see this, however, is considerably more difficult. The problem has been studied over the years using intensive computer simulations by three groups: Andy Albrecht and Neil Turok, Dave Bennett and Francois Bouchet, and Bruce Allen and Paul Shellard (see \( [24] \) for reviews). Analytic tools to study the problem have been devised by Tom Kibble, Dave Bennett, Ed Copeland and oth-
ers (references may be found in [7,6]). Most recently, Mark Hindmarsh, inspired by ideas from condensed matter physics, has devised a technique to study the evolution of domain walls in an expanding universe [25]. This seems like a promising approach to study the evolution of cosmic strings too, though the technique has not yet been applied to this problem.

The results on cosmic string evolution, first established by Bennett and Bouchet, were that the network evolves in such a way that the long strings shed very small loops and the energy density in strings remains a fixed small fraction of the energy density in the universe. This leads to the possibility that cosmic strings may have been produced at the GUT scale and could be “out there” for us to discover. In addition, cosmic strings would have influenced the light and matter around them and so it may be possible to detect this influence by careful observations of the present universe. In particular, cosmic strings may have left their imprint on the CMBR and the large-scale structure. The flip side of the coin is that if the effect of GUT scale cosmic strings on the CMBR and on large-scale structure formation disagrees with observation, we would be able to say that they do not exist and thus gain some important information about particle physics at very high energies.

The first is by realizing that a cosmic string that is illuminated on the backside by a light source would act as a lens for the source since the string curves the intervening spacetime. So a string would cause multiple images of a background quasar or galaxy. The observation of such an event would not only tell us that there are cosmic strings in the universe but it would also tell us where the string is currently located. With Andrew de Laix and Lawrence Krauss, I have recently been investigating this scheme for a cosmic string hunt [26]. In Fig. 8 the location of a string with several background sources is shown. The string causes the light from the sources to bend and the field appears as shown in Fig. 7. In this hunt for strings, there is an uncertainty in the details of the lensing pattern since the shape of cosmic strings is not precisely known. However, the limiting factor is the small probability for looking in the right direction for observing a string lensing event. Ongoing and planned surveys, however, will be covering roughly a quarter of the sky and should find GUT cosmic strings if they are there.

FIG. 6. The filled circles show the location of several hypothetical unlensed sources in the presence of a foreground string segment that was generated by computer simulation. The sharp kinks in the string are partially due to the fact that what is shown is the projection of the string onto a plane and, on small scales, due to the simulation grid used to generate the string. The inner box (dotted line) is 25" × 25".

FIG. 7. The appearance of the field of sources in the 25" × 25" size box shown in the previous figure due to gravitational lensing by the string. The stringy appearance of the lensed sources seems evident. The challenge in real surveys would be to pick out the stringy nature of the signal in a field of other astronomical objects.

There are two known ways to hunt for cosmic strings. A second way to search for strings is to seek their imprint on the CMBR. Just as a string distorts the images of background sources, Kaiser and Stebbins [27] showed that moving strings would change the energy of photons that pass by. Since the CMBR is background illumination for cosmic strings, it should have temperature fluctuations induced by strings. Ongoing experiments are
determining the CMBR fluctuations very carefully and theorists have been calculating the influence of strings on the background. The theoretical predictions depend on various other factors (such as the energy density in the universe). At the moment, the simplest cosmological model with strings does not appear to be consistent with the observations [28,29]. In another 5 years, with more and better data and with further characterization of the string network, we should be able to say with greater certainty if the observed anisotropy in the CMBR can be due to GUT cosmic strings.

In addition, as first pointed out by Zeldovich [30] and by Alex Vilenkin [31], it is possible to consider the influence of cosmic strings on the formation of structure (galaxies etc.) in the universe. Over the years, our understanding of the influence of cosmic strings on the matter in the universe has evolved. At first it was believed that cosmic string loops would be centers around which galaxies would form. Later the potential importance of long (infinite) strings for structure formation was realized by Silk and Vilenkin [32], Stebbins et al. [33] and by me [34]. This realization gained force once it was found that the string loops were too small to be of much importance in the formation of large-scale structure [35]. The implications of cosmic strings for structure formation continue to be worked out with great vigour by researchers like Albrecht, Allen, Brandenberger, Shellard, Stebbins, and others.

It should be added that there are great cosmological, astrophysical and theoretical uncertainties in the research on formation of large-scale structure by strings (eg. see the paper by Rees [36]). However, if the calculated distribution of large-scale structure due to strings roughly agrees with the observed distribution (Fig. 3), this would provide hope for the existence of strings. A disagreement would provide evidence against string seeded structure formation and hence a constraint on GUT models in particle physics.

As first pointed out by Hogan and Rees [37], there is yet another constraining observation that cosmic strings must satisfy - this is the observed limit on a cosmological background of gravitational waves. Since cosmic strings generate gravitational radiation, their energy density has to be low enough such that their gravitational radiation remain within limits imposed by the timing of the millisecond pulsar [38]. (A gravitational wave background would introduce noise in the millisecond pulsar timing beyond that what is observed and accounted for.) An estimate of the gravitational radiation from strings depends sensitively on the structure of the string network. Based on the current understanding of the network, the gravitational wave constraints are evaded by GUT strings, though by a small margin.

In an effort spearheaded by Turok [39], Spergel [40] and Durrer [41], cosmologists have also examined the influence of texture and other global defects on the CMBR and large-scale structure. Once again, analysis of the simplest theoretical models indicates that GUT scale global defects by themselves cannot simultaneously explain large-scale structure formation and the anisotropy of the CMBR.

The interest in the GUT phase transition comes from the underlying unification philosophy, the apparent convergence of the known coupling constants (Fig. 1), and, the cosmological relevance of the GUT energy scale. (The GUT energy scale seems suitable for laying out the seeds of density inhomogeneities that will later grow to become galaxies.) However, our knowledge of particle physics is not yet complete enough that we can say that the electroweak and GUT phase transitions were the only unifying cosmological phase transitions. Indeed, there are several particle physics models in which phase transitions would have occurred between the electroweak and GUT epochs. Defects produced at these epochs may not have been responsible for galaxy formation but it would be invaluable to know if they exist in the universe. Since these defects would be lighter, it is unlikely that they will be seen due to their gravitational interactions. Instead, to hunt them, one has to rely on their particle physics interactions which can lead to electromagnetic radiation and cosmic rays, an effort actively pursued by Bhattacharjee and others [12,43,44].

VII. DOWN ON EARTH

The fact that cosmological phase transitions and condensed matter phase transitions are described by the same physical principles, allows us to consider performing “cosmological experiments” in the lab. These are experiments in condensed matter systems that are motivated by cosmology. This idea was first suggested by Zurek [45]. For example, condensed matter physicists have studied topological defects for a long time and have been interested in their microphysical properties and also in how the system of defects relaxes with time thus leading to the completion of the phase transition. However, until now, they were not interested in the number of defects, or in the size distribution of vortices (strings) that are formed during a phase transition. Both these quantities are of crucial interest to cosmologists since the number and distribution of defects determines their astronomical and astrophysical relevance. Hence an experiment that studies the distribution of vortices produced during a phase transition would be called a “cosmological experiment”.

‡ The exchange of ideas between cosmologists and condensed matter physicists was greatly facilitated by a six month long program and a NATO workshop on topological defects held
Over the last several years, a number of condensed matter experiments of a cosmological flavour have been performed. First was the experiment in nematic liquid crystals by Chuang et al. where the authors studied the relaxation of a network of strings [13]. This was later followed by efforts to study the formation of defects in liquid crystals by Srivastava and collaborators [18]. The attention then turned to phase transitions closer to expected particle physics phase transitions and experiments studying the formation of vortices in $^3$He were carried out by Peter McClintock and his group in Lancaster [49]. Most recently, there have been a number of ingenious experiments in $^3$He conducted in Grenoble, Helsinki and Lancaster [50,51] that have also studied the formation of strings. (These are described in the article by A. Gill [52].)

A leading personality in the effort to connect particle physics and cosmology with $^3$He is Grisha Volovik. The point he has tried to convey to the physics community is that there are strong similarities between the basic structure of particle physics and $^3$He [53]. So one can imagine simulating particle physics processes of cosmological interest in $^3$He provided one is careful to ask the right questions. For example, as has been done with astounding success, it is possible to simulate the cosmological formation of strings in $^3$He since the formation of topological defects is not peculiar to details of the cosmological environment. At the same time, it seems that it may not be possible to simulate the cosmological evolution of strings in condensed matter systems since that depends on the Hubble expansion and the absence of strong dissipative processes, both of which are cosmological conditions and hard to find in the lab setting. Here I will describe another process that has been studied in $^3$He [54] and which is of great interest to cosmologists - this is the generation of matter, also called “baryogenesis”.

In the absence of an external magnetic field, $^3$He is known to have two superfluid phases which are called the A and B phases. At high temperature, $^3$He is invariant under rotations of the Cooper pair spin (S), orbital angular momentum (L), and, also, phase rotations of the wave-function that lead to the conservation of particle number (N). So the (continuous) symmetry group of $^3$He is:

$$G = SO(3)_S \times SO(3)_L \times U(1)_N \, .$$

The spontaneous symmetry breaking pattern for the transition into the A phase is:

$$G \rightarrow SO(2)_{S_3} \times U(1)_Q \, ,$$

where

$$Q = L_3 - \frac{N}{2} \, .$$

Note that in this symmetry breaking, the $SO(3)_S$ group breaks to $SO(2)_{S_3}$, while the remaining symmetry breaking pattern appears to be exactly that of the electroweak model. There are, however, subtle differences in certain discrete symmetries in the two models that are important in determining the topology of the vacuum manifold and hence, the topological defects. (Nonetheless, a direct analog of the non-topological electroweak string is present in $^3$He [53].) Another difference is that $^3$He does not contain fundamental gauge fields other than the electromagnetic fields. In the electroweak model, however, such gauge fields exist and are important. It is useful to be aware of these subtle differences because it allows us to meaningfully compare $^3$He experiments with particle theory expectations.

The common elements in $^3$He and (hypothetical) particle theory is the presence of non-trivial topology. Therefore processes such as the formation of topological defects can be studied in $^3$He and the results translated to the particle physics world. In addition, $^3$He contains quasiparticles that correspond to the fundamental particles (leptons and quarks) in particle theory. These quasiparticles interact with the order parameter of $^3$He just as the fundamental fermions interact with the electroweak gauge fields. So $^3$He does contain “effective” gauge fields besides the ordinary electromagnetic fields. This similarity is very valuable since the behaviour of fermions in fixed background gauge fields can be simulated by the interaction of quasiparticles in the background of some order parameter configuration in $^3$He. Indeed this is precisely what is needed to simulate the violation of baryon number in $^3$He.

In vacuum the energy of a free fermion is given by:

$$E = \pm \sqrt{p^2 + m^2} \, ,$$

where $p$ is the momentum and $m$ is the mass of the fermion ($c$ has been set to 1). Therefore to create a fermion and antifermion pair from the vacuum requires at least an energy equal to $2m$. In the presence of certain scalar and gauge field configurations, however, the dispersion relation for fermions can display a “zero mode” (see Fig. 5). This can be seen by solving the Dirac equation in the non-trivial scalar and gauge field background. If the Dirac equation has a solution with zero energy eigenvalue then this solution is the zero mode. (Alternatively, the existence of the zero mode follows from certain mathematical “index theorems” which I will not describe here.) For example, there can be a zero mode in the background of a string lying along the $z$-axis. Effectively this says that there are fermions trapped on the string that behave as if they are massless as long as they stay on...
the string. But these trapped fermions can travel along the string and their dispersion relation in the one dimensional space of the string is:

$$E = \pm p_z.$$ 

FIG. 8. a) The energy versus the momentum of fermions along the $^3$He vortex (assumed to lie along the $z-$axis). The crucial feature in this spectrum is the presence of a zero mode (the $n = 0$ line) which crosses from the negative energy to the positive energy region. The application of an electric field lifts the level of the Dirac sea along the $n = 0$ mode and particles move from the vacuum ($E < 0$) to the physical world ($E > 0$). This is the anomalous production of fermions from the vacuum. b) The spectrum of $u$ and $d$ quarks on strings in the electroweak model. As the level of the Dirac sea rises (or falls), $u$ and $d$ quarks are produced (or destroyed) in such a way that the total electric charge, $q$, remains conserved but the total baryon number $B$ is violated. ($C$ is the charge under an operation called “particle conjugation”.)

The existence of zero modes leads to the anomalous creation of fermions (Fig. 8). An intuitive picture is that, if an electric field is applied along the string, the Dirac sea can rise as a whole and particles from the Dirac sea can get pushed into positive vacuum states along the zero mode. This can happen in string-like configurations in the electroweak model of particle physics and also along vortices in $^3$He. The anomalous creation of fermions in the electroweak model leads to the creation of matter (baryons) over antimatter (antibaryons) or vice versa, while the anomalous creation of quasiparticles in $^3$He leads to the violation of total quasiparticle momentum and is observed as an excess force on moving vortices.

In a cosmological scenario, such processes together with other suitable conditions such as thermal non-equilibrium and CP violation can lead to the creation of matter in the universe (baryogenesis).

When we apply an electric field $E$ along a string that carries magnetic field $B$, the rate of production of fermions of charge $q$ is:

$$\dot{n} = \frac{q^2}{4\pi^2} E \cdot B,$$

where $n$ is the number density of fermions. (The electric field itself can be induced via Faraday’s law if the string moves across an ambient magnetic field.)

The anomaly equation (7) is applicable to both the electroweak model and $^3$He. The possibility of anomalous generation of baryon number along strings was discussed by Witten [55]. In the electroweak case, a non-Abelian generalization of (7) leads to the possibility of anomalous baryon charge on electroweak string knots (see Fig. 9) as I showed in collaboration with George Field [56], and, Jaume Garriga [57]. In $^3$He, the anomaly equation leads to quasiparticle production. The measurable quantity, however, is the momentum, $P$, carried off by the anomalously created quasiparticles:

$$\partial_t P = \frac{1}{2\pi^2} \int d^3x (p_F \hat{l}) E \cdot B,$$

where, $p_F$ is the Fermi momentum and $\hat{l}$ is the orientation of the Cooper pair angular momentum.

In the Cooper pair plus quasiparticle system, momentum is obviously exactly conserved. In the absence of the anomaly, the momentum in the Cooper pairs and quasiparticles is separately conserved. Due to the anomaly, however, momentum is transferred from the $^3$He vacuum (Cooper pairs) to the quasiparticles and vice versa. This transfer of momentum leads to an extra force on moving vortices:

$$F = \partial_t P = \pi \hbar NC_0 \hat{z} \times (v_n - v_L),$$

where $N$ is the winding of the vortex, the coefficient $C_0$ is a temperature dependent coefficient, the vortex lies in the $\hat{z}$ direction, and $v_L - v_n$ is the vortex line velocity with respect to the normal fluid.
The Manchester group, led by Henry Hall and John Hook, used a clever experimental setup in which an array of vortices was created by rotating a sample of $^3$He. A diaphragm placed within the sample had two orthogonal modes of oscillation which could be driven electrically and also detected. Oscillations in one of the modes was used to create the relative velocity $v_L - v_n$. The extra force on the vortices given by eq. (8) produces forces perpendicular to the driven mode of oscillation and hence couples to the other oscillation mode of the diaphragm. The oscillations in this orthogonal mode can then be measured. This leads to the measurement of quantities related to the coefficient $C_0$ at different temperatures. The results confirm the anomalous production of quasiparticles on the vortex.

![FIG. 9. A knotted configuration of electroweak strings that has associated baryon number. Here “baryon number” is defined in terms of particles trapped on the string and this is somewhat different from the usual meaning which is defined in terms of particles in the vacuum.](image)

The observation of “momentogenesis” in $^3$He confirms “baryogenesis” in the electroweak model. The experiments, however, do not say anything about the cosmological process of baryogenesis since these depend on various other cosmological factors such as departures from thermal equilibrium and CP violation.

**VIII. OUTLOOK**

In the study of the early universe, the last several decades have seen a remarkable confluence of ideas originating in vastly different branches of physics. Who could have imagined the possibility of fossils from the early universe and that one day we would be “digging” for them? That the mysteries of the atom could be revealed by astronomical observations, while the secrets of the big bang may be locked in particle accelerators? It requires an even further stretch of imagination to contemplate simulating the early universe in a vial of helium. Yet this is the current state of early universe cosmology and we can be sure of many equally surprising developments in the years to come.

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Tanmay Vachaspati enjoys research in cosmology particularly for its richness in diverse problems. His investigations of the early universe are stimulated by observations in astronomy and cosmology, developments in theoretical particle physics, gravitational phenomena and condensed matter experiments. A hope that spurs his research activities is the possibility that one day we will directly probe the universe within the first second of its existence.