M(atrix) Theory on $T^9/Z_2$ Orbifold and Twisted Zero-Branes

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Abstract

M(atrix) theory compactified on an orbifold $T^9/Z_2$ is studied. Via zero-brane parton scattering we find that each of the $2^9 = 512$ orbifold fixed points carry $-1/32$ units of zero-brane charge. The anomalous flux is cancelled by introducing a twisted sector consisting of 32 zero-branes that are spacetime supersymmetry singlets. These twisted sector zero-branes are nothing but gravitational waves propagating along the M-theory direction. There is no D0-partons in the untwisted sector, a fact consistent with holographic principle. For low-energy excitations, the orbifold compactification is described by ten-dimensional supersymmetric Yang-Mills theory with gauge group $SO(32)$.

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1 Introduction

Via D0-partons, M(atrix) theory \cite{1} defines a non-perturbative light-front Hamiltonian dynamics of M-theory \cite{2}, which unifies all known perturbative superstring theories at strong coupling regime. Regularizing zero-momentum by compactifying the longitudinal M-direction on a circle of radius \( R \), the M(atrix) theory is defined by dynamics of \( N \) D0-partons:\footnote{Our spinor conventions are as follows. We take Majorana representation so that \((16 \times 16)\ \Gamma_i\)'s are real and symmetric, \(i\bar{\Theta}\Gamma_i \equiv \Theta^T \) and \( \Gamma^{(11)} \Theta = +\Theta \):}

\[
S_M = \mathrm{Tr}_N \int d\tau \left( \frac{1}{2R} (D_\tau X^I)^2 + \frac{R}{4} [X', X']^2 + \Theta^T D_\tau \Theta + iR\Theta^T \Gamma_I [X^I, \Theta] \right). \tag{1}
\]

where \( X^I \) and \( \Theta^\alpha \) denote 9 bosonic and 16 spinor coordinates of 0-brane partons \((I = 1, \cdots, 9 \) and \( \alpha = 1, \cdots, 16)\). The non-dynamical gauge field \( A_\tau \) that enters through covariant derivatives \( D_\tau X^I \equiv \partial_\tau X^I - i [A_\tau, X^I] \) and \( D_\tau \Theta^\alpha \equiv \partial_\tau \Theta^\alpha - i [A_\tau, \Theta^\alpha] \) projects the physical Hilbert space to a gauge singlet sector and ensures invariance under area-preserving diffeomorphism transformation. In the infinite-momentum light-front frame, out of thirty-two supersymmetries of M-theory, only sixteen are realized as dynamical supersymmetries. The other sixteen become kinematical supersymmetries. Thus, defining their supergenerators as \( i\epsilon \) and \( \xi \) respectively, the M(atrix) theory is invariant under the following supersymmetry transformations

\[
\begin{align*}
\delta X^I &= -2\epsilon^T \Gamma^I \Theta \\
\delta \Theta &= \frac{i}{2} \left( [\Gamma_I D_\tau X^I + \frac{1}{2} \Gamma_{IJ} [X^I, X^J]] \epsilon + \xi \right) \\
\delta A_\tau &= -2\epsilon^T \Theta.
\end{align*} \tag{3}
\]

The sixteen dynamical and sixteen kinematical supersymmetry charges are given by:

\[
\begin{align*}
Q_\alpha &= \sqrt{R} \mathrm{Tr} \left( \Gamma^I \Pi_I + \frac{i}{2} \Gamma_{IJ} [X^I, X^J] \right) \Theta_\beta, \\
S_\alpha &= \frac{2}{\sqrt{R}} \mathrm{Tr} \Theta_\alpha
\end{align*} \tag{4}
\]

respectively.

One outstanding issue in M(atrix) theory is proper description of M-theory compactification. Generically, compactification onto shrinking \( d \)-dimensional space is defined in M(atrix) theory by a \((d + 1)\)-dimensional quantum theory with an appropriate number of supersymmetries dictated by the holonomy of the space. For \( d \leq 3 \), the quantum theory is Yang-Mills gauge theory, which is (super)renormalizable. For \( d > 3 \), the gauge theory becomes non-renormalizable, hence, should be replaced by yet-to-be-found fixed-point theory. Nevertheless, at low-energy and at appropriate limits of moduli space, the gauge theory should provide an effective M(atrix) field theory description to the compactification. As such, though admittedly limited, M(atrix) gauge theory description of compactification in all dimensions should reveal many useful information of (compactified) quantum M-theory.

\[
\Gamma_i = \begin{pmatrix}
0 & \sigma_i^a \\
\sigma_i^a & 0
\end{pmatrix}, \quad i = 1, \cdots, 8; \quad \Gamma_9 = \begin{pmatrix}
-\delta_{ab} & 0 \\
0 & +\delta_{ab}
\end{pmatrix}. \tag{2}
\]

\[\]
Indeed, with such an effective M(atrix) theory approach, we have studied previously the $T^5/Z_2$ orbifold compactification and were able to many nontrivial M-theory dynamics inherent to this compactification including anomalous G-flux $[4, 5, 8]$ and spacetime spectrum $[8]$.

In this paper, we keep this attitude, and study effective gauge theory description of M(atrix) theory compactified on $T^9/Z_2$ orbifold, viz. compactification of all transverse dimensions. On $T^9/Z_2$ orbifold, $2^9 = 512$ fixed points are present and act as potential sources of anomalous charge. In section 2, we probe each orbifold fixed point via D0-parton scattering and find that it carries an anomalous gravity flux of $-1/32$ unit. We find that cancellation of total gravity flux then require introduction of a twisted sector consisting of 16 units of longitudinal gravitons and their $Z_2$ images, viz. 32 twisted D0-branes. The resulting effective M(atrix) theory is thus given by ten-dimensional supersymmetric Yang-Mills theory with gauge group $SO(32)$, the only possible anomaly-free gauge theory in ten-dimensions. In section 4, to understand the peculiarity of the M(atrix) theory on $T^9/Z_2$, we compare it with other previously studied orbifolds $S^1/Z_2$ $[3]$, $M^2$ (Möbius strip) $[6]$, $T^5/Z_2$ $[5]$ and $T^8/Z_2$ $[10]$, the fixed points of $T^9/Z_2$ may carry anomalous M-theory charges and in turn create uncancelled vacuum energy tadpole. In this section, using the prescription defined in Ref. $[3]$, we explore the potential anomalous M-theory charges via D0-parton scattering $[3]$.  

2 Anomalous Gravity Flux and Twisted Sector

Consider $T^9/Z_2$ orbifold on which the M(atrix) theory is compactified. On the orbifold there are $2^9 = 512$ fixed points. Near each orbifold fixed point, the D0-parton perceives the space locally as $R^9/Z_2$. This orbifold is described by $Z_2$ involution of M(atrix) theory defined on the covering space $R^9$, viz. the defining M(atrix) quantum mechanics Eq. (1). The Chan-Paton condition to the $Z_2$ orbit is then given by:

\[ X^I = -M \cdot X^{IT} \cdot M^{-1} \]
\[ \Theta_\alpha = \Gamma_\perp M \cdot \Theta^{T}_\alpha \cdot M^{-1} , \quad \Gamma_\perp = \Gamma^1 \cdots \Gamma^9. \]

Since $\Gamma_\perp = \Gamma^{(11)}$ in our convention and the spinor $\Theta$ is defined such that $\Gamma^{(11)}\Theta = +\Theta$, we find that the $Z_2$ involution projects U(2N) covering space gauge group to SO(2N) and remove the sixteen kinematical supersymmetries completely. The latter peculiarity is rooted to the fact we have compactified all transverse directions. In section 4, we will explain this peculiarity in yet another way in terms of M(atrix) theory R-symmetry and decomposition of kinematical and dynamical supersymmetries thereof.

As in the other M(atrix) orbifolds previously studied $S_1/Z_2$ $[3]$, $M_2$ (Möbius strip) $[3]$, $T^5/Z_2$ $[3]$ and $T^8/Z_2$ $[10]$, the fixed points of $T^9/Z_2$ may carry anomalous M-theory charges and in turn create uncancelled vacuum energy tadpole. In this section, using the prescription defined in Ref. $[3]$, we explore the potential anomalous M-theory charges via D0-parton scattering $[3]$.

2.1 Probing Gravity Flux via Parton Scattering

We have previously prescribed fixed-target D0-parton scattering as a probe of potentially anomalous M-theory charges localized at orbifold fixed points $[3]$. According to the prescrip-

\[ ^3 \text{After this part of work was completed, we have learned an independent work by Ganor et.al.} [10], \text{in which the same parton scattering result on} T^{3^5}/Z_2 \text{as ours was obtained.} \]
tion, the anomalous charge is probed by comparing the D0-parton scattering off the orbifold to
that in the flat space:

\[ V_{\text{orientifold}}(r, v) \equiv V_{\mathbb{Z}_2 \text{orbit}}(r, v) - V_{\text{flat \ space}}(r, v). \] (5)

As a local probe, we place a 0-brane parton. A 0-brane moving slowly near the fixed point
will experience the presence of fixed point or, equivalently, a mirror 0-brane parton approaching
toward the probing 0-brane parton. As such we expect that the effect of orbifold fixed point
amounts to a net force equivalent to two-body 0-brane scattering. Dynamics of a zero-brane
parton scattering off the fixed point is described by SO(2) M(atrix) theory quantum mechanics.
Consider \( A_9 = ivt \sigma_2/2 \), where \( v \) denotes the relative velocity between the probe zero-brane and
the image zero-brane. In the instantaneous limit, the two-body potential is calculated from the
forward scattering phase shift:

For the \( T_9/\mathbb{Z}_2 \) orbifold we consider presently, we continue utilizing the same prescription
to probe possible anomalous charges. Since the D0-parton scattering is a local process, we
consider momentarily region near any of the 2^9 orbifold fixed points.

With the choice of \( SO(2N) \) D0-parton scattering off the orbifold fixed point is described
by \( SO(2) \) gauge theory. Thus, there is no force coming from the scattering. A straightforward
quadratic expansion shows that there is no massive modes at all! This means that the probing
0-brane parton moves freely in the orbifold space! This means that there is no \( v^4/r^7 \) force one
should expect from the consideration of M(atrix) theory. We thus need to add a zero-brane to
take into account of the effects.

On the other hand, on the covering space, D0-D0 brane scattering is described by \( SU(2) \)
quantum mechanics. Hence, we find that the presence of \( \Omega_9 \) orientifold gives a net force:

\[ V_{\text{orientifold}} = 0 - \frac{v^4}{r^7} = -\frac{v^4}{r^7}. \] (6)

Clearly, each orientifold carries negative anomalous D0-brane (anti-D0-brane) charges. On
\( T_9/\mathbb{Z}_2 \), D0-brane charge conservation requires introduction of twisted sector, which will main-
tain conservation of D0-brane charge on \( T_9/\mathbb{Z}_2 \). Consider a twisted D0-brane of charge \( Q \)
located at the orientifold fixed point. By scattering off D0-brane parton, we find that the twisted
D0-brane exerts a force to the parton of

\[ V_{\text{twisted}} = 2 \cdot (2Q) \frac{(v/2)^4}{(r/2)^7} = 2^5 Q \frac{v^4}{r^7}. \] (7)

Here, the first factor of 2 is the difference of reduced mass between the D0-parton and its image
scattering versus D0-parton and twisted D0-brane scattering and the second factor of 2 is the
D0-brane charge as measured in the covering space. We conclude that we need \(-\frac{1}{32}\) units of
twisted D0-brane charge localized at the orientifold fixed points. Had we taken USp gauge
group, then there is no charge localized at the fixed point. This indicates that SO gauge group
is the correct choice to the M(atrix) theory.

We thus find that local cancellation of anomalous gravity flux requires introduction of a
twisted sector consisting of thirty-two D0-partons. One peculiarity of these twisted D0-partons
as the spectrum of the twisted sector is that they are supersymmetric in M(atrix) theory
side while the corresponding twisted sector in two-dimensional M-theory are supersymmetry
singlets \[7, 8\]. However, this is not a contradiction. It is well-known that purely bosonic or fermionic adjoint multiplets that are singlets under supersymmetry can exist in \((0+1)\)-dimensions. Using this fact, we can associate the twisted sector D0-parton states with twisted sector spinors of \((1+1)\)-dimensional M-theory localized at each fixed points. These spinors are chiral since they carry a positive BPS charge. Related aspects has been observed also in Ref. \[?\] with string and M-theory contexts.

What about total D0-parton flux conservation? Since the transverse directions are all compactified (i.e. finite-in-all-directions compactification), the total D0-parton flux has to vanish. The flux cannot leak to longitudinal direction since they are already boosted to the speed of light! These twisted D0-branes themselves. Furthermore, since D0-brane charge conservation is already saturated by the twisted sector, there is no room for the D0-partons in the untwisted sector!

2.2 Effective M(atrix) Gauge Theory on \(T^9/\mathbb{Z}_2\)

Let us now extend the analysis to \(T^9/\mathbb{Z}_2\) orbifold in the limit the orbifold shrinks to zero size. M(atrix) theory in this limit is conveniently described by first T-dualizing the covering space \(T^9\) and then project onto \(\mathbb{Z}_2\) orbits.

The T-duality of M(atrix) theory is defined in terms of T-duality transformation of D0-parton themselves. In the limit each sides of \(T^9/\mathbb{Z}_2\) orbifold shrinks to zero, infinitely many multiply winding open string become light. They are more conveniently describedonce all directions of covering space \(T^9\) is T-dualized so that winding configurations are mapped into momentum modes. Sum over all possible multiple winding quantum number is then mapped into sum over all possible momentum quantum number, hence, Fourier transformation of M(atrix) theory variables. The Fourier transformation results in ten-dimensional supersymmetric Yang-Mills theory on dual parameter space \(\tilde{T}^9\). At least at classical level, the Yang-Mills theory is consistent with \(U(2N)\) covering space gauge group.

The effective M(atrix) gauge theory for \(T^9/\mathbb{Z}_2\) orbifold is then obtained by modding out with order-2 involution \(\mathbb{Z}_2 = \otimes \cdot P\) of orientation reversal

\[
\Omega : \quad A^M(y, t) \rightarrow \Omega \cdot A^M(\Omega y, t) \cdot \Omega^{-1} = M \cdot A^M_{T}(y, t) \cdot M^{-1}
\]

\[
\Psi_{\alpha}(y, t) \rightarrow \Omega \cdot \Psi_{\alpha}(\Omega y, t) \cdot \Omega^{-1} = M \cdot \Psi_{\alpha}^{T}(y, t) \cdot M^{-1}.
\]  

(8)

and parity transformation

\[
P : \quad A^M(y, t) \rightarrow P \cdot A^M(P y, t) \cdot P^{-1} = - A^M(+y, t)
\]

\[
\Psi_{\alpha}(y, t) \rightarrow P \cdot \Psi_{\alpha}(P y, t) \cdot P^{-1} = \Gamma^{(11)} \Psi_{\alpha}(+y, t).
\]  

(9)

Noting that \(\Gamma^{(11)} \Psi = + \Psi\) in our convention, we find, at least classically, the effective M(atrix) gauge theory for \(T^9/\mathbb{Z}_2\) orbifold compactification is given by ten-dimensional supersymmetric gauge theory with gauge group \(SO(2N)\) or any sub-group thereof.

We have already identified the twisted sector consists of sixteen D0-branes and their images. On \(T^9/\mathbb{Z}_2\), they are localized at 512 orbifold fixed points. After T-duality on all directions, however, these twisted sector D0-branes are delocalized and fill in the bulk of dual parameter space \(\tilde{T}^9\). We thus conclude that the twisted sector is also described by ten-dimensional Yang-Mills theory! In fact, in ten-dimensional \(\mathcal{N} = 1\) rigid supersymmetry, there is no other possible
supermultiplets than Yang-Mills one. Thus it is not surprising that both untwisted and twisted sector are described by Yang-Mills gauge theory.

Combining both the untwisted and the twisted sectors, the effective M(atrix) gauge theory follows from Fourier transform of $\mathbb{Z}_2$ invariant configuration of $2(N + 16)$ D0-partons and the result is ten-dimensional supersymmetric Yang-Mills theory with gauge group $SO(2N + 32)$. Quantum consistency of the M(atrix) gauge theory requires that the gauge group is free from gauge anomalies. This singles out the gauge group to be $SO(32)$, hence, $N = 0$! This effective M(atrix) gauge theory with vacuous untwisted sector is precisely what agrees with the simple argument given earlier that on $T^9/\mathbb{Z}_2$ (in fact for any "finite-in-all-directions" compactification) there is no transverse space left over into which the D0-parton flux can extend over. The only D0-partons one can place on $T^9/\mathbb{Z}_2$ are the 32 ones which neutralize the anomalous gravity charges localized at the orbifold fixed points. Dynamics of these neutralizing D0-partons are described by ten-dimensional $SO(32)$ supersymmetric Yang-Mills gauge theory.

3 Spacetime Symmetry of M-theory versus R-Symmetry of M(atrix) Theory

In due course of studying effective M(atrix) gauge theory, we have encountered several peculiarities unique to $T^9/\mathbb{Z}_2$ compactification. In particular we have seen that the untwisted sector D0-partons are completely suppressed and that the twisted sector spectrum that are two-dimensional spacetime M-theory supersymmetry singlets are described in M(atrix) gauge theory by Yang-Mills supermultiplets of ten-dimensional parameter space. In order to have better understanding of these peculiarities, we now compare $S^1/\mathbb{Z}_2$, $T^5/\mathbb{Z}_2$ and $T^9/\mathbb{Z}_2$ orbifold compactifications in a unified manner. In particular, we pay attention to the way spacetime Lorentz symmetry in M-theory is realized in terms of R-symmetry in corresponding M(atrix) gauge theory.

The little group of spacetime Lorentz symmetry of M-theory is $SO(9)$. In M(atrix) theory description of M-theory, this symmetry is realized as R-symmetry of extended supersymmetry. Under $SO(9)_R$ of M(atrix) theory, nine transverse space fields $X^i$’s transform as vector representation, while fermionic superpartners transform as sixteen-dimensional spinor representation. Let us see how the one-to-one correspondence between spacetime symmetry and R-symmetry is realized upon compactification of M-theory (under which the M(atrix) theory becomes decompactified). In particular, we are interested in the realization of supercharges in both descriptions.

Once compactified on $S_1$, M-theory reduces to Type IIA superstring, whose supercharges are two Majorana-Weyl spinors of opposite $(9+1)$-dimensional chirality, hence, nonchiral. The corresponding M(atrix) theory is $(1+1)$-dimensional supersymmetric Yang-Mills theory with sixteen supercharges. It turns out that the M(atrix) theory is also nonchiral, consisting of eight $(1+1)$-dimensional Majorana-Weyl spinors for each chirality. The sixteen supercharges are thus realized as $(8,8)$. Upon compactification, the R-symmetry is now reduced to $SO(8)_R \subset SO(9)_R$. Under the R-symmetry, the eight right-moving supercharges transform as spinor representation while the eight left-moving supercharges transform as conjugate spinor representation. It is also obvious that half of the M-theory supercharges that become one right-handed $d=10$ Majorana-Weyl spinor become eight M(atrix) theory supercharges that are right-handed $d=2$
Majorana-Weyl spinors. We thus find that spacetime symmetry of M-theory is realized as
R-symmetry of M(atrix) theory, in which the spinor chirality in each sides are correlated via
spinor decomposition.

Heterotic M(atrix) theory is obtained by taking a quotient of $\mathbb{Z}_2$ involution. The involution
projects out supercharges in left-handed spinor representation in both M-theory and M(atrix)
theory sides. It should then be noted that the twisted sectors in each side have left-handed
chirality while supercharges are right-handed. In M-theory side, the twisted sector consists of
d=10, (1,0) gauge supermultiplet. In M(atrix) theory side, the twisted sector fields are d=2,
(8,0) supersymmetry singlet, left-handed spinors.

As a next nontrivial compactification, consider M-theory compactified on $T_5$. M-theory
on $T_5$ has six-dimensional (2,2) supersymmetry consisting of two Weyl supercharges of each
chirality. The original spacetime symmetry SO(10,1) is reduced to SO(5,1)×SO(5). the latter
is nothing but R-symmetry in d=6 M-theory. With two Weyl spinors in d=6, we have global
symmetry USp(4) = SO(5). As before, the little group of M-theory spacetime symmetry is
SO(4)⊂SO(5,1). The corresponding M(atrix) theory is d=6 super-Yang-Mills theory whose
supercharges are (1,1). In M(atrix) theory side, the little group is realized as R-symmetry,
viz., SO(4)⊂SO(9). The sixteen supercharges are realized as two d=6 Weyl spinors of both
chirality. Each Weyl spinors give rise to USp(2) = SU(2) global symmetry, which are one part
of SO(4) = SU(2)×SU(2). The M-theory supercharges , which were chiral, are now nontrivial
for only one of the two SU(2)’s in M(atrix) theory. Again, M-theory and M(atrix) theory chi-
ralities of supercharges are related each other via decomposition of spinors. Upon $\mathbb{Z}_2$ quotient,
we drop left-handed Weyl supercharge in M-theory and corresponding R-charged supercharge
in M(atrix) theory, which is again left-handed. The twisted sectors in each descriptions are
hypermultiplets and fermions all transform as right-handed.

Let us proceed further and consider M-theory compactification on $T_9$. In M-theory side,
we have D=2, (16,16) supersymmetry, viz, supercharges comprise of sixteen right-handed and
sixteen left-handed Majorana-Weyl spinors. The M-theory has SO(9) R-symmetry and each
sixteen supercharges transform as Majorana spinor representation of SO(9) R-symmetry. The
corresponding M(atrix) theory is d=10 super-Yang-Mills theory. The M(atrix) theory super-
charge consist of single, sixteen dimensional Majorana-Weyl spinor. An amusing point is that
the M-theory is nonchiral but the M(atrix) theory is chiral. This can be traced back to the
fact we have chosen D0-branes, not anti-D0-branes, as M(atrix) theory partons. Therefore, in
this case, the other supercharge of opposite chirality is hidden as kinematical supersymmetry
charge. Upon $\mathbb{Z}_2$ orientifolding, we drop left-handed supercharges in M-theory and kinematical
supercharge in M(atrix) theory.

4 Discussions

In this paper we have studied M(atrix) theory description of compactified M theory on $T^9/\mathbb{Z}_2$
orbifold. In the large volume limit, through zero-brane parton scattering off the orbifold fixed
points, we have found that the fixed point carries anomalous gravi-photon flux of one unit.
In order to cancel the anomalous flux a twisted sector consisting of thirty-two D0-partons is
introduced. Effectively, the 512 states of the D0-parton BPS multiplet that are split due to
spontaneously broken kinematical supersymmetry and cancels locally the anomalous gravity
flux at each of 512 orbifold fixed points.
Because all the nine transverse directions are compactified, we have seen that the $\mathbb{T}^9/\mathbb{Z}_2$ orbifold entails certain peculiarities not encountered in lower dimensional compactifications. In the infinite momentum frame, the D0-parton flux are extended entirely into transverse directions. Since all transverse directions are compactified, the D0-partons charge conservation requires that no D0-partons to be present. This thinning of parton degrees of freedom, however, stops at thirty-two for $\mathbb{T}^9\mathbb{Z}_2$, since the orbifold fixed points carry anomalous gravity flux and thirty-two D0-partons are needed to cancel them. We have shown that this fits nicely with the fact that effective M(atrix) gauge theory is the unique anomaly-free SO(32) supersymmetric Yang-Mills theory in ten dimensions.

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References

[1] T. Banks, W. Fischler, S. Shenker and L. Susskind, Phys. Rev. D (1997) [hep-th/9610043).
[2] E. Witten, Nucl. Phys. B443 (1995) 85.
[3] N. Kim and S.-J. Rey, M(atrix) Theory on an Orbifold and Twisted Membrane, to be published in Nucl. Phys. B, [hep-th/9701139]
[4] D. Kabat and S.-J. Rey, T-Duality and Wilson Lines in Heterotic M(atrix) Theory, to be published in Nucl. Phys. B, [hep-th/9705099]
[5] N. Kim and S.-J. Rey M(atrix) Theory on $\mathbb{T}^5/\mathbb{Z}_2$ and Five-Branes, hep-th/9705132
[6] N. Kim and S.-J. Rey Non-Orientable M(atrix) Theory, hep-th/9710192
[7] K. Dasgupta and S. Mukhi, Orbifolds of M-Theory, Nucl. Phys. B465 (1996) 399, hep-th/9512196
[8] E. Witten, Five-Branes and M-Theory on an Orbifold, Nucl. Phys. B463 (1996) 383, hep-th/9612219
[9] E. Witten, Flux Quantization in M-Theory and The Effective Action, hep-th/9609122
[10] O.J. Ganor, R. Gopakumar and S. Ramgoolam, Higher Loop Effects in M(atrix) Orbifolds, hep-th/9705188
[11] K. Dasgupta, D. Jatkar and S. Mukhi, Gravitational Couplings and $\mathbb{Z}_2$ Orientifolds, hep-th/9707224
S. Mukhi, Orientifolds: The Unique Personality of Each Spacetime Dimension, hep-th/9710004