Deterministic generation of Gaussian pure state in quasilocal dissipative system

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(Dated: May 5, 2014)

This paper shows that an arbitrary Gaussian pure state can be deterministically generated in a dissipative open system that has quasilocal interactions between the subsystems and couples to the surrounding environment in a local manner. A quasilocal interaction, which means that the interaction occurs among only a few subsystems, is a crucial requirement for practical engineering of a dissipative system. The key idea is that first an auxiliary system having a local interaction with the environment is prepared and then that auxiliary system is coupled to the underlying target system via a set of two-body Hamiltonians in such a way that a desired pure state is generated. Moreover, we show that even with a simple single-mode auxiliary system, the deterministic generation of an arbitrary approximate Gaussian cluster state is possible, by devising an appropriate switching scheme. We discuss in a specific example how much a dissipation-induced pure Gaussian state can be perturbed by decoherence and parameter error.

PACS numbers: 03.65.Yz, 42.50.Dv

I. INTRODUCTION

Preparing a desired pure state is a crucial task in quantum information technologies. However, in a realistic situation any quantum system unavoidably interacts with the surrounding environment and is often described by the following Markovian master equation:

\[
\frac{d}{dt} \rho(t) = -i[H, \rho(t)] + \sum_{i=1}^{m} \left( L_i \rho(t) L_i^\dagger - \frac{1}{2} L_i^\dagger L_i \rho(t) - \frac{1}{2} \rho(t) L_i^\dagger L_i \right),
\]

where \( H \) is the system Hamiltonian and \( L_i \) is the coupling operator representing the interaction between the system and the \( i \)th environment channel. Usually the state \( \rho(t) \) in Eq. (1) dissipatively evolves in time towards a mixed state and never recover its purity. However, it has been shown in several papers \[1\] that by engineering suitable pairs of the system operators \( H \) and \( L_i \), the state \( \rho(t) \) can be uniquely moved to a pure steady state. This means that a desired pure state can be deterministically generated without specific initialization of the system. We actually find some applications of this environment engineering approach to entangled state generation \[8\]–\[16\] and further some advanced quantum information processing such as quantum computation \[17\], memories \[18\], and distillation \[19\].

Here we mention the quasilocal interaction. An operator is called quasilocal if it acts only on a given site of the system and its neighborhood; particularly in this paper we call the interaction quasilocal if it occurs between only two nodes. In the case of a quantum oscillator network whose \( i \)th node is a single-mode oscillator of variable \( x_i = (q_i, p_i) \), an example of a quasilocal coupling is \( L = q_1 + q_2 \). However, to generate a desired pure state such as a highly entangled state via the above-mentioned dissipation-based method, the system is often required to have non-quasilocal (global) interaction with the environment. Let us consider a seven-node quantum oscillator network depicted in Fig. 1(a). This network has internal couplings among the nodes \( x_3-x_6 \) and \( x_4-x_5 \) through a Hamiltonian \( H \) in addition two global couplings of the nodes \( (x_1, x_2, x_3, x_4) \) and \( (x_5, x_6, x_7) \) to the environments, the operators of which are, for instance, \( L_1 = q_1 + q_2 + q_3 + q_4 \) and \( L_2 = p_5 + p_6 + p_7 \). From a practical viewpoint, clearly this kind of global interaction is hard to implement. Actually, there have been several proposals to engineer a dissipative system having a quasilocal interaction with the environment, which yet deterministically produces a useful pure state. For instance, it was shown in Ref \[10\] that a qubit two-
dimensional cluster state can be generated via a quasilocal dissipative process and Ref. [10] demonstrated the case of a uniformly distributed entangled state. Moreover, for general finite-dimensional systems, Ticozzi and Viola provided a condition to determine whether a given pure state can be stabilized under fixed locality constraints [20]. However, in the infinite-dimensional case, only a few specific results have been reported [6, 10, 12].

The result obtained in Ref. [8] is that for any pure Gaussian state we can always engineer an open Gaussian system whose unique steady state is identical to that pure state. However, as mentioned above, this open Gaussian system often has to have a global interaction with the environment, such as the system shown in Fig. 1(a). In this paper, nonetheless, we show that an arbitrary pure Gaussian state can be dissipatively generated in a certain extended system having only local interaction with the environment. The idea is described as follows. First, we couple the underlying target system, via a set of two-body Hamiltonians, to a certain auxiliary system having local interactions with the environment. That is, the extended open system composed of the target and the auxiliary systems contains only quasilocal interactions [see Fig. 1 (b)]. Then, based on the result of Ref. [6], we will show that a Gaussian pure state generated in the original system of interest, which can include global interactions with the environment, is also generated in the above-mentioned extended system. This configuration is actually a generalization of the result of Ref. [8], where dissipation-induced generation of a two-mode squeezed state in atomic ensembles was demonstrated.

In addition, we present a simple schematic that only utilizes a single-mode auxiliary system. More specifically, even in the case where the above-mentioned result requires us to have a multimode auxiliary system, it will be shown that only a single-mode auxiliary system can serve to achieve the same goal. The key idea is the use of a switching scheme of the auxiliary system; that is, in the case of the system depicted in Fig. 1, a single-mode auxiliary system first couples to the subsystems $x_1, \ldots, x_4$ and then switches to couple to the subsystems $x_5, x_6, x_7$. A particularly important result is that, with this switching scheme, an arbitrary approximate Gaussian cluster state [21, 22] can be generated dissipatively. This is a generalization of the switching scheme proposed by Li et al. in Ref. [12], which demonstrated dissipative generation of several types of four-mode cluster states in an actual physical setup.

Finally, we examine how much a specific dissipation-induced pure state is robust against some perturbation. The system is a pair of two atomic ensembles that dissipatively generates a two-mode squeezed state [8]; here damping decoherence and parameter uncertainty are further taken into account. Then we find the optimal system parameters that maximize the entanglement between the atomic ensembles. A point worth noting is that these optimal parameters are out of the range where the auxiliary cavity mode can be adiabatically eliminated; that is, this is an example where the extended system is really robust against some perturbation compared to only the target system obtained through the adiabatic elimination.

**Notation.** Let $X = (X_{ij})$ be a matrix whose entry $X_{ij}$ is an operator on a Hilbert space or a complex number. Then $X^\top = (X_{ji})$ denotes the transpose of $X$. Also, $X^\dagger$ means the Hermitian conjugate of $X$; i.e., $X^\dagger = (X_{ij}^\dagger)$ for an operator $X_{ij}$ and $X^\dagger = (X_{ij}^\dagger)$ for a complex number $X_{ij}$. We use $\Re(X)$ and $\Im(X)$ to denote the real and imaginary parts of $X$. For a vector of operators $x$, we define the anti-commutator of $x$ by
\[
\{x, x^\top\} = xx^\top + (xx^\top)^\top.
\]

**II. GAUSSIAN DISSIPATIVE SYSTEM**

In this section, we describe a general Gaussian dissipative system in terms of a quantum stochastic differential equation (QSDE) and present the condition for the steady state of this system to be pure [8]. The time evolution of a general open quantum system is generated by the unitary operator $U(t)$ subjected to the following QSDE (in Itô form) [24]:
\[
dU(t) = \left\{ \sum_{i=1}^{m} \left( L_i dA_i(t)^\dagger - L_i^\dagger dA_i(t) \right) - \left( \sum_{i=1}^{m} \frac{1}{2} L_i^\dagger L_i + iH \right) dt \right\} U(t), \tag{2}
\]
with $U(0) = I$ (we have not included the scattering term). Here $A_i(t)$ is the annihilation process on the $i$th environment vacuum field; this satisfies the quantum Itô rule, e.g., $dA_i(t)dA_j(t)^\dagger = \delta_{ij} dt$. Let $p$ be the initial state of the system and $\lvert 0 \rangle$ be the vacuum state of the whole environment field. The master equation (1) is obtained by differentiating $\rho(t) = U(t)(\rho \otimes |0\rangle \langle 0|)U(t)^\dagger$ and tracing out the field state.

Next, let $(q_i, p_i)$ be the canonical conjugate pair of the $i$th quantum oscillator, which satisfies the canonical commutation relation (CCR) $q_i p_j - p_j q_i = i\delta_{ij}$. Defining the vector of observables $x = [q_1, \ldots, q_n, p_1, \ldots, p_n]^\top$, we can write the CCR as
\[
xx^\top - (xx^\top)^\top = i\Sigma_n, \quad \Sigma_n = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix},
\]
where $I_n$ denotes the $n \times n$ identity matrix (we often drop the subscript $n$). A general linear open system can be characterized by the following Hamiltonian $\hat{H}$ and the coupling operators $L = [L_1, \ldots, L_m]^\top$:
\[
\hat{H} = \frac{1}{2} xx^\top G x, \quad L = C x,
\]
where $G = G^\top \in \mathbb{R}^{2n \times 2n}$ and $C \in \mathbb{C}^{m \times 2n}$. From Eq. (2), the QSDE of $x(t) = \ldots
\[ [U(t)\dagger q_1U(t), \ldots, U(t)\dagger q_nU(t), U(t)\dagger p_1U(t), \ldots, U(t)\dagger p_nU(t)]^\top \]
is given by

\[ dx(t) = Ax(t)dt + BdW(t), \]

where \( A = \Sigma_n(G + \bar{C}^\top \Sigma_m \bar{C}/2) \) and \( B = \Sigma_n \bar{C}^\top \) with \( \bar{C} = \sqrt{2} \text{Re}(ic)^\top, \text{Im}(ic)^\top \text{)}^\top \). Also, we have defined the field operators \( q_i = (A_i + A_i^\dagger)/\sqrt{2}, p_i = (A_i - A_i^\dagger)/\sqrt{2}, \) and \( W = [q_1, \ldots, q_m, p_1, \ldots, p_m]^\top \).

A Gaussian system can be fully characterized by only the mean vector \( \langle x \rangle = [(q_1), \ldots, (q_n), (p_1), \ldots, (p_n)]^\top \) and the covariance matrix \( V = \langle (\Delta x, \Delta x^\top) \rangle /2 \) with \( \Delta x = x - \langle x \rangle \). Note that a Gaussian state is pure if and only if \( 1/\sqrt{2^n} \det(V) = 1 \). The importance of the linear system \( \dot{x} = Ax + B \) is that, if the initial state of this system is Gaussian, then it preserves the Gaussianity of the state for all time. In particular, the covariance matrix \( V(t) \) obeys the following Lyapunov differential equation:

\[ \frac{d}{dt} V(t) = AV(t) + V(t)A^\top + \frac{1}{2} BB^\top. \]

Hence the covariance matrix of the steady state is given by the solution of the algebraic Lyapunov equation

\[ AV + VA^\top + \frac{1}{2} BB^\top = 0. \]

This equation has a unique solution if and only if \( A \) is a Hurwitz matrix; i.e., all eigenvalues of \( A \) are in the open left half complex plane.

Now we are interested in a linear Gaussian system whose steady state is uniquely pure. It has been shown in Ref. [8] that an arbitrary pure Gaussian state can be dissipatively generated, if the system matrices \( G \) and \( C \) can be freely chosen. In particular, the following result is useful in this paper:

Theorem 1 [8]. Suppose that Eq. (4) has a unique solution \( V \). Then, this is the covariance matrix of a pure Gaussian state if and only if the following matrix equations are satisfied:

\[ \left(V + \frac{i}{2} \Sigma_n\right) C^\top = 0, \quad \Sigma_n G V + V(\Sigma_n C)^\top = 0. \]

III. PURE GAUSSIAN STATE GENERATION VIA QUASILOCAL DISSIPATIVE ENVIRONMENT

To generate a certain desirable pure Gaussian state dissipatively, the system is often required to have a global interaction with the environment; that is, for instance, the coupling operator must be of the form \( L = q_1 + q_2 + q_3 + \cdots \), which corresponds to \( C = (1, 1, 1, \ldots) \).

In this section, we show that such a global interaction between the system and the environment can always be avoided by constructing an auxiliary system that quasilocally couples to the target system and locally couples to the environment.

\[ \text{A. The extended dissipative system} \]

In addition to the underlying target system consisting of \( n \) quantum oscillators, we consider an auxiliary system consisting of \( m \) oscillators. Let \( x = [q_1, \ldots, q_n, p_1, \ldots, p_n]^\top \) and \( \tilde{x} = [\tilde{q}_1, \ldots, \tilde{q}_m, \tilde{p}_1, \ldots, \tilde{p}_m]^\top \) be the vectors of canonical conjugate pairs of each system. We assume that the decoherence of the target system is negligible, i.e., the system operators are

\[ H = \frac{1}{2} x^\top G x, \quad L = 0, \]

where \( G = G^\top \in \mathbb{R}^{2n \times 2n} \). The auxiliary system has no self-Hamiltonian and dissipates through typical damping channels, hence the system operators are given by

\[ \tilde{H} = 0, \quad \tilde{L} = \sqrt{\kappa} \tilde{a}, \]

where \( \tilde{a} = [\tilde{a}_1, \ldots, \tilde{a}_m]^\top \) is a vector of annihilation operators, i.e., \( \tilde{a}_i = (\tilde{q}_i + i\tilde{p}_i)/\sqrt{2} \). The two systems couple via the following interaction Hamiltonian:

\[ H_{\text{int}} = i(\tilde{a}^\dagger C x - x^\top C^\dagger \tilde{a}), \]

with \( C \in \mathbb{C}^{m \times 2n} \). Note that \( H_{\text{int}} \) describes a quasilocal interaction between the two systems, because it can always be decomposed into a sum of two-body Hamiltonians as follows:

\[ H_{\text{int}} = \sum_{i,j} (x^{(i)^\top} K^{(ij)} \tilde{x}^{(j)} + \tilde{x}^{(i)^\top} K^{(ij)^\top} x^{(j)}), \]

where \( x^{(i)} = [q_i, p_i]^\top \) and \( \tilde{x}^{(i)} = [\tilde{q}_i, \tilde{p}_i]^\top \). We will see later that this quasilocal Hamiltonian \( H_{\text{int}} \) has the same effect as the coupling operator of the system originally of interest, i.e., \( L = Cx \), when choosing \( G \) and \( C \) so that they satisfy the condition of Theorem 1. Defining \( C = \sqrt{2} [\text{Re}(ic)^\top, \text{Im}(ic)^\top]^\top \), we can rewrite Eq. (7) as

\[ H_{\text{int}} = \frac{1}{2} (x^\top C^\dagger \tilde{x} + \tilde{x}^\top C^\top x). \]

Then, the overall system vector \( \xi(t) = [x(t)^\top, \tilde{x}(t)^\top]^\top \) obeys the following linear QSDE:

\[ d\xi(t) = A\xi(t)dt + BdW(t), \]

where

\[ A = \begin{bmatrix} \Sigma_n G & \Sigma_n C^\top \\ \Sigma_n C & -\kappa I_{2m}/2 \end{bmatrix}, \quad B = - \begin{bmatrix} 0 \\ \sqrt{\kappa} I_{2m} \end{bmatrix}. \]

The covariance matrix \( V(t) = \langle (\Delta \xi(t), \Delta \xi(t)^\top) \rangle /2 \) evolves in time through Eq. (8) and its steady solution, if it exists, is obtained by solving Eq. (11). To avoid confusion, we again present the same equation

\[ AV + VA^\top + \frac{1}{2} BB^\top = 0, \]
where $A$ and $B$ are now given in Eq. 9. Recall here that we were originally interested in the system with the Hamiltonian $H = x^TGx/2$ and the coupling operator $L = Cx$, which thus obeys the linear dynamics

$$dx(t) = A_1 x(t) dt + B_1 dW(t),$$

where $A_1 = \Sigma_n (G + \bar{C}^\top \Sigma_m \bar{C})/2$ and $B_1 = \Sigma_m \bar{C}^\top$. Note that the dissipation channel $L = Cx$ can be global. Its steady covariance matrix in particular is our concern,

$$A_1 V_1 + V_1 A_1^\top + \frac{1}{2} B_1 B_1^\top = 0. \quad (12)$$

The following theorem states that, if the system $\Sigma_n$ has a pure steady state, then the extended system $\Sigma_n \Sigma_m$ can dissipatively produce the same pure Gaussian state.

**Theorem 2.** Suppose that Eq. (12) has a unique solution $V_1$ that corresponds to a pure Gaussian state. Then Eq. (10) has a unique solution $V = \text{diag}(V_1, I_{2m}/2)$. 

**Proof.** First, we prove that Eq. (10) has a unique solution, which is equivalent to that $A$ is a Hurwitz matrix. For this purpose, let us set $A^\top \eta = \lambda \eta$; i.e., $\eta \in \mathbb{C}^{2(n+m)}$ is an eigenvector of $A^\top$ and $\lambda \in \mathbb{C}$ is the corresponding eigenvalue. Then, multiplying Eq. (10) by $\eta^\top$ from the left and by $\eta$ from the right, we have $\Re(\lambda) = -\eta^\top BB^\top \eta/4\eta^\top V \eta$. This further becomes $\Re(\lambda) = -\eta_1^\top \eta_2/4\eta^\top V \eta$, where we have defined $\eta = [\eta_1^\top, \eta_2^\top]^\top$ with $\eta_1 \in \mathbb{C}^{2n}$ and $\eta_2 \in \mathbb{C}^{2m}$. Now let us assume $\eta_2 = 0$; then, from $A^\top \eta = \lambda \eta$ we obtain

$$G \Sigma_n^\top \eta_1 = \lambda \eta_1, \quad \bar{C} \Sigma_n^\top \eta_1 = 0. \quad (13)$$

Moreover, noting the assumption that Eq. (12) has a unique solution

$$V_1 = \int_0^\infty e^{A_1 t} \left( \frac{1}{2} B_1 B_1^\top \right) e^{A_1^\top t} dt,$$

we find that $\eta_1$ satisfies

$$\eta_1^\top V_1 \eta_1 = \frac{1}{2} \int_0^\infty \| C \Sigma_n^\top e^{A_1^\top t} \eta_1 \|^2 dt.$$

However, this takes zero due to Eq. (13) and the Cayley-Hamilton theorem. This conclusion contradicts $V_1 > 0$, hence we have $\eta_2 \neq 0$; this further leads to $\Re(\lambda) < 0$, implying that $A$ is a Hurwitz matrix.

Next let us prove that $V = \text{diag}(V_1, I_{2m}/2)$ is the solution of Eq. (10). When representing $V$ in a block matrix form $V = [V_1, V_{12}; V_{12}^\top, V_2]$, these entries satisfy

$$\Sigma_n GV_1 + V_1 (\Sigma_n G)^\top + \Sigma_n \bar{C}^\top V_{12} + V_{12} (\Sigma_m \bar{C})^\top = 0,$$

$$V_1 (\Sigma_m \bar{C})^\top + \Sigma_m \bar{C}^\top V_2 + \Sigma_n GV_{12} - \kappa V_{12}/2 = 0,$$

$$\Sigma_m \bar{C} V_{12} + V_{12} (\Sigma_m \bar{C})^\top - \kappa V_2 + \kappa I_{2m}/2 = 0. \quad (14)$$

Now the assumption is that $V_1$ corresponds to a unique pure steady state, hence from Theorem 1 we have

$$V_1 (\Sigma_m \bar{C})^\top + \frac{1}{2} \Sigma_n \bar{C}^\top = 0, \quad \Sigma_n GV_1 + V_1 (\Sigma_n G)^\top = 0. \quad (15)$$

Note that the former condition comes from the fact that Theorem 1 yields $V_1 \Re(iC)^\top - \Sigma_n \Re(iC)^\top /2 = 0$ and $V_1 \Re(iC)^\top + \Sigma_n \Re(iC)^\top /2$. Then, we find that the set of matrices $V_{12} = 0$, $V_2 = I_{2m}/2$, and $V_1$ satisfying Eq. (15) is the solution to Eq. (14). Because Eq. (14) has a unique solution as shown in the former part of this proof, $V = \text{diag}(V_1, I_{2m}/2)$ is the unique solution.

This theorem states that, once we find a suitable pair of matrices $G$ and $C$ such that the system $\Sigma_n$ dissipatively and uniquely generates a desired pure Gaussian state, the same goal can be achieved by alternatively constructing the extended system $\Sigma_n \Sigma_m$. Note again that the system $\Sigma_m$ can couple to the environment globally, while the extended system $\Sigma_n \Sigma_m$ locally couples to the environment through the operator $\tilde{L}_1 = \sqrt{\kappa_a}$ and its internal modes quasilocally couple with each other through the Hamiltonian $H_{\text{int}}$. That is, the target system stabilizing a desired pure state can be realized as a subsystem of an extended system having local coupling to the environment and quasi-local internal couplings among the nodes.

**Remark 1.** As an auxiliary system, we usually take a system with very fast modes that can be adiabatically eliminated. In our case, even when the assumption of Theorem 2 does not hold, by taking $\kappa$ sufficiently large, the auxiliary system rapidly converges to the vacuum and the mode $\tilde{x}(t)$ can be adiabatically eliminated. Then, the target system is approximated by the system whose coupling operator is $\tilde{L}_1 = 2 \bar{C} x/\sqrt{\kappa}$. In this sense, Theorem 2 implies that we can treat the system as if the auxiliary modes were heavily damped as long as the pure steady state condition is satisfied.

### B. Generation of two-mode squeezed state

We consider here a two-mode Gaussian system studied in Ref. 8. Although for this system the requirement of quasilocality is already satisfied, this example clearly illustrates our idea.

The physical setup is depicted in Fig. 2; the target system is two atomic ensembles trapped in a ring-type cavity, while the auxiliary system corresponds to this optical cavity composed of two propagating modes. These two systems interact with each other via external pulse lasers with Rabi frequencies $\Omega_s$ and $\Omega_a$, ($i = 1, 2$). Here we assume that the number of atoms in each ensemble is sufficiently large; then the collective spin component of the atomic ensemble can be approximated by an annihilation operator and consequently the interaction Hamiltonian is
interaction Hamiltonian can be simply written as

\[ H_{\text{int}} = \left\{ a_1^\dagger (\sqrt{N_1} \beta_{u_1} a_1 + \sqrt{N_2} \beta_{s_2} a_2^\dagger) + \text{H.c.} \right\} + \left\{ a_2^\dagger (\sqrt{N_1} \beta_{s_1} a_1^\dagger + \sqrt{N_2} \beta_{u_2} a_2) + \text{H.c.} \right\}, \]

where \( a_i \) and \( \tilde{a}_i \) are the annihilation operators of the \( i \)th atomic ensemble and the \( i \)th cavity mode, respectively. Here \( N_i \) denotes the number of atoms of the \( i \)th ensemble, and also

\[ \beta_{u_i} = \frac{\Omega_{u_i} g^*_u}{2\Delta_u}, \quad \beta_{s_i} = \frac{\Omega_{s_i} g^*_s}{2\Delta_s}, \quad i = 1, 2, \]

where \( g_u \) and \( \Delta_u \) denote the coupling strength and the detuning, respectively. If we set the parameters as \( N_i = N, g_u = g_s = g \ (i = 1, 2) \), and \( \Delta_u = \Delta_s = \Delta \), then the interaction Hamiltonian can be simply written as

\[ H_{\text{int}} = \frac{\sqrt{N_2}}{2\Delta} \left[ \left\{ a_1^\dagger (\Omega_{u_1} a_1 + \Omega_{s_2} a_2^\dagger) + \text{H.c.} \right\} + \left\{ a_2^\dagger (\Omega_{s_1} a_1^\dagger + \Omega_{u_2} a_2) + \text{H.c.} \right\} \right]. \]

Further, let us set the Rabi frequencies as \( \Omega_{u_1} = \Omega_{u_2} = \Omega > 0 \) and \( \Omega_{s_1} = \Omega_{s_2} = r\Omega \), where \( r \in [0, 1) \) is a parameter. Then, the interaction Hamiltonian is of the form [1] with

\[ iC = \frac{\mu}{\sqrt{2}} \begin{bmatrix} 1 & r & i^{-r} \\ r & 1 & -i \\ i & -r & 1 \end{bmatrix}, \quad (16) \]

where \( \mu = \sqrt{Ng}\Omega/2\Delta \). For \(^{87}\text{Rb}\) atoms, the spontaneous emission of each atom is negligible and thus we can set \( L = 0 \), for typical values of the parameters, \( \mu, \kappa \approx 100 \) kHz and \( r = 0.8 \) [3]. Also, the number of atoms is large enough so that the self Hamiltonian of the atomic ensembles is assumed to be 0, i.e., \( H = 0 \) or equivalently \( G = 0 \).

The coupling operator of the cavity is given by \( \tilde{L} = \sqrt{\kappa} \tilde{a} \), where \( \kappa \) is the damping rate, and we assume that the detuning of the cavity is 0, i.e., \( \tilde{H} = 0 \).

Now, for the matrix \( C \) given by Eq. (16) and \( G = 0 \), the Lyapunov equation [12] has the following unique solution:

\[ V_1 = \frac{1}{2} \begin{bmatrix} \cosh(2\xi) & -\sinh(2\xi) & 0 \\ -\sinh(2\xi) & \cosh(2\xi) & 0 \\ 0 & 0 & \cosh(2\xi) \sinh(2\xi) \end{bmatrix}, \]

where \( \xi = \tan^{-1}(r) \). This is the covariance matrix of a pure two-mode squeezed state. Therefore, from Theorem 2, the Lyapunov equation [12] has a unique solution \( V = \text{diag}(V_1, I_2/2) \); that is, the pair of atomic ensembles acquires the two-mode squeezed state at steady state, while the auxiliary cavity mode becomes a trivial coherent state. Equivalently, the whole four-mode atom-cavity system generates the same atomic steady state as that generated in the two-atomic-ensemble system with the Hamiltonian \( H = 0 \) and the coupling operators

\[ L_1 = \mu(a_1 + ra_2), \quad L_2 = \mu(a_2 + ra_1). \]

Another physical realization of this purely dissipative system was proposed in Refs. [14, 15].

**IV. DISSIPATION-INDUCED PURE CLUSTER STATE WITH SINGLE ENVIRONMENT CHANNEL**

As shown in the preceding section, an arbitrary pure Gaussian state can be generated in the target system, by introducing a certain \( m \)-mode auxiliary system that locally couples to \( m \) environment channels. From an engineering viewpoint, it is clearly convenient if we can achieve the goal with the auxiliary system having a small number of modes \( m \). In particular, let us consider the simple case \( H = 0 \); i.e., the system Hamiltonian \( H \) is negligible compared to the interaction Hamiltonian, as in the case of Sec. III B. In this case, we need the condition \( m = n \), which is understood from Theorem 1 together with the fact that, for a covariance matrix \( V \) corresponding to a pure Gaussian state, \( V + i\Sigma_n/2 \) has an \( n \)-dimensional kernel [28]. However, this requirement is very demanding especially when \( n \) is large.

In this section we present a general solution to the above-posed problem. That is, we prove that even for a general \( n \)-mode system with \( H = 0 \), only a single-mode auxiliary system coupling to a single environment channel introduces a dissipative mechanism that drives the system state to an arbitrary approximate Gaussian cluster state [21, 22]. This result is significant in the sense that a most simple dissipative system deterministically generates a most useful quantum state from the quantum information viewpoint. The key idea is the use of a switching scheme of the interaction Hamiltonian between the target and the auxiliary systems, which is a
generalization of the idea proposed by Li et al. in Ref. [12].

A. The switching scheme

As introduced above, in this section we consider a most simple system; that is, the system’s Hamiltonian is negligible ($H = 0$) and the auxiliary system is single-mode ($m = 1$), in addition to the assumptions $L = \hat{H} = 0$ and $\hat{L} = \sqrt{\kappa}\hat{a}$. We represent the interaction Hamiltonian in the following form:

$$H_{\text{int}} = \mu \sum_{j=1}^{n} \left\{ \alpha_j^\dagger (\alpha_j a_j + \beta_j a_j^\dagger) + \text{H.c.} \right\}, \quad (17)$$

where $a_j$ ($j = 1, \ldots, n$) denotes the $j$th annihilation operator of the target system. $\alpha_j, \beta_j \in \mathbb{C}$ and $\mu \in \mathbb{R}$ are parameters.

First let us consider the case $n = 1$ and set the parameters as $\alpha_1 = 1$ and $\beta_1 = \mu \in [0, 1)$. Then, the corresponding $C$ matrix in Eq. (17) is given by $C = \mu [1 + \mu i(1 - r)] / \sqrt{2}$ and Eq. (12) has the following unique solution:

$$\begin{bmatrix} \xi \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \left[ e^{-2\xi} & 0 \\ 0 & e^{2\xi} \right] \end{bmatrix}, \quad (18)$$

where $\xi = \tan^{-1}(r)$. This is the covariance matrix of a pure squeezed state, hence from Theorem 2 we find that Eq. (12) has a unique solution $V = \text{diag}(V_1, I_2/2)$. As a result, when $n = 1$, the target system acquires a pure squeezed state while the auxiliary system becomes vacuum, through the interaction Hamiltonian in switching.

Next we consider the case of $n > 1$. In this case, as mentioned above, Eq. (12) does not have a unique solution, hence Theorem 2 cannot be directly applied. Now let us consider a unitary matrix $U \in \mathbb{C}^{n \times n}$ and the unitary coordinate transformation $a' = U a$ of the vector of system variables $a = [a_1^\dagger, \ldots, a_n^\dagger]$. In terms of the quadratures $q'_i = (a_i + a_i^\dagger)/\sqrt{2}$ and $p'_i = (a_i - a_i^\dagger)/\sqrt{2}i$, we find that $x' = [q'_1, \ldots, q'_n, p'_1, \ldots, p'_n]^\dagger$ is the symplectic and orthogonal transformation of $x$ as follows:

$$x' = S x, \quad S = \begin{bmatrix} \Re(U) & -\Im(U) \\ \Im(U) & \Re(U) \end{bmatrix}. \quad (19)$$

Note that, since $S$ is symplectic, $x'$ satisfies the CCR: $x' x'^\dagger - (x' x'^\dagger)^\dagger = i S \Sigma_n S^\dagger = i \Sigma_n$.

We explain here the idea of switching. At the $k$th switching stage, the parameters $(\alpha_j^{(k)}, \beta_j^{(k)})$ of the interaction Hamiltonian (17) are chosen as follows:

$$\alpha_j^{(k)} = U_{kj}, \quad \beta_j^{(k)} = r U_{kj}, \quad r \in [0, 1), \quad (20)$$

where $U_{kj}$ is the $(k, j)$ element of the unitary matrix introduced above. With this choice, the interaction Hamiltonian (12) is written as

$$H_{\text{int}}^{(k)} = \mu \left\{ \alpha_j^\dagger (a'_k + ra_k^\dagger) + \text{H.c.} \right\}. \quad (21)$$

This is no more than the Hamiltonian discussed in the case $n = 1$. Hence, in the $k$th switching stage the $k$th mode $(q'_k, p'_k)$ deterministically changes to a pure squeezed state with covariance matrix (18). Note that during the $k$th stage, the other system variables $(q'_\ell, p'_\ell), \ell \neq k$ do not change at all. Therefore, applying the interaction Hamiltonian (20) repeatedly by changing $k = 1, 2, \ldots, n$, the corresponding system variable $(q'_k, p'_k)$ gets squeezed in this order, as schematically shown in Fig. 3. In particular, if the initial state is the ground state with respect to $x'$ (and thus as well), all the off-diagonal elements of the covariance matrix remain 0 and then the system’s covariance matrix $V'_1 = (\{\Delta x', \Delta x'^\dagger\})/2$ becomes $V'_1 = \text{diag}(e^{-2\xi} I_2, e^{2\xi} I_n)/2$. As a result, the steady state of the target system of the mode $x = S^\dagger x'$ is a pure Gaussian state with covariance matrix

$$V_1 = \frac{1}{2} S^\dagger \begin{bmatrix} e^{-2\xi} I_2 & 0 \\ 0 & e^{2\xi} I_n \end{bmatrix} S. \quad (22)$$

This is a unitary transformed pure squeezed state; we will see later that this state can represent any approximate Gaussian cluster state by appropriately choosing the unitary matrix $U$ or equivalently the switching parameters $(\alpha_j^{(k)}, \beta_j^{(k)})$.

B. Generation of CV cluster state

A Gaussian cluster state is an entangled state of great importance, particularly in one-way quantum computation [22, 23]. Hence, this state should be a target that is dissipatively generated with the scheme presented in this paper, particularly with the switching scheme. Thus we show here how to chose the unitary matrix $U$ so that the covariance matrix (21) represents a given target Gaussian cluster state.

Here is the definition of a Gaussian cluster state [22]: Let $A = A^\dagger \in \mathbb{R}^{n \times n}$ be the adjacency matrix representing the graph structure of a cluster state of interest; i.e.,
the \((i,j)\) element of \(A\) represents the weight of the coupling between the \(i\)th and the \(j\)th nodes of the network. Then the approximate Gaussian cluster state is defined as a state satisfying
\[
\lim_{\alpha \to \infty} \text{cov}(p - Aq) = 0, \tag{23}
\]
where \(\alpha \in \mathbb{R}\) is a certain parameter contained in the state; it often corresponds to a squeezing parameter. Also we have defined \(\text{cov}(x) = \langle \{\Delta x, \Delta x\} \rangle / 2\). It is of course impossible in reality to take the limit \(\alpha \to \infty\), hence we call the state with finite \(\alpha\) the approximate Gaussian cluster state.

Now we describe a relation that connects a given adjacency matrix \(A\) and the unitary matrix \(U\) characterizing the covariance matrix \((22)\). As will be mentioned later in Remark 2, one such relation has already been obtained in Refs. \([23, 25]\) particularly for the aim of constructing a concrete optical process corresponding to \(U\), but we here provide an alternative method in rather an abstract way.

**Proposition 1.** Let \(A = A^T \in \mathbb{R}^{n \times n}\) be the adjacency matrix of a given Gaussian cluster state and define \(N = -(I_n + A)\). Then the polar decomposition \(N = RU\) yields a unitary matrix \(U\) and a real matrix \(R\). The Gaussian state with covariance matrix \((22)\) characterized by this unitary \(U\) then satisfies
\[
\text{cov}(p - Aq) = \frac{1}{2} (I_n + A^2) e^{-2\zeta}, \tag{24}
\]
hence it is an approximate Gaussian cluster state converging to the ideal one in the limit of \(\zeta \to \infty\).

**Proof.** First, \(R\) is real because \(R = \sqrt{NN^T} = I_n + A^2 \in \mathbb{R}^{n \times n}\). Next, for the Gaussian state with covariance matrix \((22)\) we have
\[
\text{cov}(p - Aq) = [-A, I_n] \frac{1}{2} S^T \begin{bmatrix} e^{-2\zeta} I_n & 0 \\ 0 & e^{2\zeta} I_n \end{bmatrix} S [-A] = \frac{1}{2} e^{-2\zeta} F_1 F_1^T + \frac{1}{2} e^{2\zeta} F_2 F_2^T,
\]
where \(F_1 = \Re(U)A + \Im(U)\) and \(F_2 = \Im(U)A - \Re(U)\). Noting that \(RU = N\) and \(R\) is real, we have \(\Re(U) = -R^{-1} A\) and \(\Im(U) = -R^{-1}\). Hence we have \(F_1 = -R^{-1}(I_n + A^2)\) and \(F_2 = 0\). Furthermore, noting that \(I_n = UU^T = R^{-1} N N^T R^{-1}\), we have \(RR^T = NN^T = I_n + A^2\), thus \(F_1 F_1^T = (I_n + A^2) R^{-1} R^{-1} (I_n + A^2) = I_n + A^2\). As a result, we have \(\text{cov}(p - Aq) = (I_n + A^2) e^{-2\zeta} / 2\).

The merit of this result is that the unitary matrix \(U\) is straightforwardly constructed from a given \(A\) compared to the result of Refs. \([23, 25]\), although in this case \(U\) does not have a clear correspondence to some optical realizations. However, Eq. \((21)\) clarifies how to physically implement the interaction Hamiltonian \((17)\).

**Remark 2.** The approximate Gaussian cluster state with covariance matrix \((22)\) can be deterministically generated on optical fields, by the following method \([23, 25]\):

We prepare \(n\) independent and identical squeezed light fields, and mix them via some passive optical devices such as a beam splitter and a phase shifter in a specific order determined from the adjacency matrix \(A\). The collection of these transformations is totally represented by a unitary matrix; if we denote that unitary matrix as \(U^\dagger\), the covariance matrix of the output fields is identical to Eq. \((22)\). That is, the unitary transform in the switching scheme corresponds to the scattering process on optical fields, and the dissipation-induced pure squeezed states correspond to the initially-prepared optical squeezed states. Hence, the presented switching scheme can be interpreted as a dissipative counterpart to the optical scheme proposed in Refs. \([23, 25]\). The biggest difference between these two schemes is that in the dissipative case we consider a state generated in matter whereas the optical state exists in a flying light field; the former can be later manipulated or stored, while the latter is suited for propagating quantum information. A specific relation between these two regimes was discussed in Ref. \([29]\).

**Remark 3.** In Proposition 1 the polar decomposition is used to find the appropriate unitary matrix \(U\), but it is clear from the proof that only a certain decomposition of the form \(N = RU\) with unitary \(U\) and real \(R\) really gives the same relation \((24)\). Here we see that, besides the polar decomposition, the Gram Schmidt procedure also serves as a convenient method to obtain such a decomposition. Let \(U\) be a unitary matrix whose row vectors are obtained from the Gram Schmidt procedure of the row vectors of \(N\). Then we immediately have the relation \(N = RU\) with a real lower triangular matrix \(R\), which is consistent with the RQ decomposition. Hence the Gram Schmidt procedure also yields the unitary matrix satisfying the condition in Proposition 1.

### C. Physical realization

We can implement the presented switching scheme for \(n\) atomic ensembles (target system) trapped in a single-mode optical cavity (auxiliary system) in a similar configuration studied in Sec. \(\[\]\). The physical setup in the case \(n = 4\) is depicted in Fig. \(\[\]\). Let \(a_j\) be the annihilation operator approximating the collective spin component of the \(j\)th atomic ensemble and \(\widetilde{a}_j\) the annihilation operator of the cavity mode. Then the interaction Hamiltonian is given by \([\ref{\ref{}}]\).

\[
H_{\text{int}} = \frac{\sqrt{N} q}{2\Delta} \sum_{j=1}^{n} \left[ \widetilde{a}_j^\dagger (\Omega_{u_j} e^{i\phi_{u_j}} a_j + \Omega_{s_j} e^{i\phi_{s_j}} a_j^\dagger) + \text{H.c.} \right],
\]
where \(\phi_{\bullet} \in [0, 2\pi)\) is the laser phase. The switching scheme shown in Sec. \([\ref{\ref{}}]\) suggests that, at the \(k\)th switching stage, we choose the parameters as
\[
\Omega_{u_j} e^{i\phi_{u_j}} = \Omega U_{kj}, \quad \Omega_{s_j} e^{i\phi_{s_j}} = r\Omega U_{kj}^*, \quad r \in [0, 1), \tag{25}
\]
where \(\Omega > 0\) is a parameter. The unitary matrix \((U_{kj})\) is determined from the target Gaussian cluster state. As
proven in Sec. IV A, the whole state of the atomic ensembles deterministically reaches this target, if they are all initially set to the ground states.

D. Example

Let us consider the problem of dissipatively generating a four-node square cluster state, depicted in Fig. 5, in the atomic ensemble system discussed in the preceding section. The connecting edges between the nodes are equally weighted and thus the adjacency matrix $A$ of this graph state is given by

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}. $$

We follow Proposition 1 to determine a unitary matrix $U$ characterizing the switching law (20). In particular, here we utilize the Gram-Schmidt procedure (see Remark 3). To make the calculation simple, we orthogonalize the following matrix:

$$N = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} N = \begin{bmatrix} i & -i & 0 & 0 \\ 0 & 0 & i & -i \\ i & 0 & 1 & 1 \\ 1 & 1 & i & 0 \end{bmatrix}. $$

Then, $U$ is given by

$$U = \begin{bmatrix} -i[1, -1, 0, 0]/\sqrt{2} \\ -i[0, 0, 1, -1]/\sqrt{2} \\ -i[i, i, 2, 2]/\sqrt{10} \\ [2, 2, i, i]/\sqrt{10} \end{bmatrix}. $$

Actually with this choice we have

$$\text{cov}(p - Aq) = \frac{1}{2} e^{-2\xi} \begin{bmatrix} 3 & 2 & 0 & 0 \\ 2 & 3 & 0 & 0 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 2 & 3 \end{bmatrix},$$

thus the state approximates well the target square cluster state when large $\xi$ is taken.

The switching law (25) with the unitary matrix (26) clarifies how to choose the laser parameters as shown below. At the first switching stage, they are determined from the first row vector of $U$ as

$$\Omega_{u_i} = \frac{\Omega_{s_i}}{r} = \frac{1}{\sqrt{2}} \Omega, \quad i = 1, 2,$$

$$\Omega_{u_i} = \Omega_{s_i} = 0, \quad i = 3, 4,$$

$$\phi_{u_1} = \frac{3}{2} \pi, \quad \phi_{u_2} = \frac{1}{2} \pi, \quad \phi_{u_3} = \frac{1}{2} \pi, \quad \phi_{u_4} = \frac{3}{2} \pi.$$

Through the interaction Hamiltonian with these parameters, the CCR pair $(q_i', p_i')$ gets squeezed in the long time limit. Then we switch the parameters and set:

$$\Omega_{u_i} = \Omega_{s_i} = 0, \quad i = 1, 2,$$

$$\Omega_{u_i} = \frac{\Omega_{s_i}}{r} = \frac{1}{\sqrt{2}} \Omega, \quad i = 3, 4,$$

$$\phi_{u_3} = \frac{3}{2} \pi, \quad \phi_{u_4} = \frac{1}{2} \pi, \quad \phi_{u_3} = \frac{1}{2} \pi, \quad \phi_{u_4} = \frac{3}{2} \pi.$$

Then the second node $(q_2', p_2')$ gets squeezed. By repeating a similar procedure, each CCR pair $(q_i', p_i')$, $i = 1, \ldots, 4$ becomes a squeezed state with covariance matrix (13). As a result, if each atomic ensemble is in the ground state at the initial time, the whole state changes to a pure Gaussian state with covariance matrix (15) according to the theory developed in Secs. IV A and IV B, now we know that any approximate Gaussian cluster state can be deterministically generated in atomic ensembles trapped in a single-mode cavity. Moreover, as demonstrated here, the appropriate switching law can be systematically constructed, once the target cluster state is specified.
V. PERTURBATION TO PURE STEADY STATE

In this section, we reconsider the two atomic ensembles discussed in Sec. III B, taking into account some specific perturbations added to the system. Actually, an atomic ensemble constructed in a cavity often loses coherence due to spontaneous emission. In addition to this kind of quantum effect, in practice any system contains some parameter uncertainties, which can bring a serious loss of coherence as well when aiming to dissipatively generate a pure state. In the case of an atomic ensemble, the number of atoms is usually never determined exactly. We take these two typical perturbations and evaluate how much the steady state is affected by these losses. In particular, we find the optimal squeezing level and the cavity damping rate that maximize the entanglement. Although these investigations do not straightforwardly provide new insight into the quasilocality discussed throughout this paper, the result will clarify a merit of enlarging the system rather than focusing only on the target system obtained by adiabatic elimination.

A. Decoherence effect

First we assume that the atomic ensembles are subject to a loss of atomic coherence, in which case the corresponding coupling operator is represented by \( L = \sqrt{\gamma}[a_1, a_2]^T \), with \( \gamma \) the decoherence rate. The coefficient matrix of the driving term of the system, given in Eq. (9), is then changed to

\[
A' = \left[ \begin{array}{cc} 0 & \Sigma_2 \bar{C}^T \\ \Sigma_2 \bar{C} & -\kappa I_4/2 \end{array} \right] + \left[ \begin{array}{cc} -\gamma I_4/2 & 0 \\ 0 & 0 \end{array} \right] = A + \Delta A,
\]

where \( \bar{C} \) is defined via Eq. (18). The eigenvalues of \( A \) are given by

\[
\lambda_{\pm} = -\frac{\kappa}{4} \pm \frac{1}{4} \sqrt{\kappa^2 - 16\mu^2(1-r^2)}.
\]

A notable point is that, when large squeezing is introduced, i.e., \( r \approx 1 \), the eigenvalue \( \lambda_+ \) approaches zero, even in the case \( \gamma \ll \mu, \kappa \). This loss of stability of the system implies that a desirable convergence of the state is prevented. Thus the perturbation \( \Delta A \) is not negligible, particularly when aiming to generate a large entangled state. Now the coefficient matrix of the diffusion term of the system is \( B' = \text{diag}(\sqrt{\gamma}I_4, \sqrt{\kappa}I_4) \) and the Lyapunov equation \( A'V + VA'^T + B'B'^T/2 = 0 \) has the following explicit solution:

\[
V_1 = \text{diag} \left( rc + 1/2, -c, -c, rc + 1/2 \right),
\]

where \( c = 4\kappa/(\kappa + \gamma)(\kappa + \gamma + 4(1-r^2)) \). Here, for simplicity, we have replaced \( \kappa/\mu \) and \( \gamma/\mu \) by \( \kappa \) and \( \gamma \), respectively. The entanglement of this two-mode Gaussian state can be quantified by the logarithmic negativity and is given by

\[
E_N = \max\{0, -\log(2\nu)\}, \quad \nu = \frac{\kappa\gamma^2 + 4\kappa(1-r)^2 + \gamma(\kappa^2 + 4(1-r^2))}{2(\kappa + \gamma)(\kappa \gamma + 4(1-r^2))}.
\]

Figure 6 shows \( E_N \) versus the squeezing level \( \xi = \tanh^{-1}(r) \) and the cavity damping rate \( \kappa \) for the cases (a) \( \gamma = 0 \), (b) \( \gamma = 0.01 \), and (c) \( (\gamma, \epsilon) = (0.01, 1.1) \).

![Logarithmic negativity](image)
a very small decoherence rate $\gamma = 0.01$, as depicted in Fig. [3] (b). In particular, it is apparent from the figure that large squeezing brings about a large loss of entanglement, which is yet consistent with the fact that a large squeezed state is usually very fragile to decoherence. It is also reasonable that now $\kappa$ affects $E_N$ in two ways: Large $\kappa$ induces rapid leaking of the photons through the cavity light field, while small $\kappa$ means that the cavity mode interacts with the atomic ensembles many times, implying many emissions of the photons into the cavity field. Consequently, there exists an optimal set of the parameters $\kappa$ and $r = \tanh(\xi)$ that maximizes $E_N$:

$$\kappa_* = 2\sqrt{1 - r_*^2},$$

$$r_* = \frac{1}{2(\gamma^2 + 1)} \left( 2 + d + \frac{\gamma^2(\gamma^2 - 3)}{d} \right),$$

(28)

where $d = [-\gamma^6 + 5\gamma^4 - 2\gamma^2 + (\gamma^2 + 1)\sqrt{-\gamma^4(\gamma^2 - 4)}]^{1/3}$. Particularly in the case $\gamma = 0.01$, these optimal values are given by $r_* \approx 0.97$ and $\kappa_* \approx 0.52$, and $E_N$ then takes the value of about 2.91.

Remark 4. It should be pointed out that the above optimal parameters are out of the range where the auxiliary cavity mode can be adiabatically eliminated; that is, as discussed in Remark 1 in Sec. III A, the auxiliary system can be adiabatically eliminated only when $\kappa$ is sufficiently large, but clearly in this case $E_N$ goes down to zero. In this sense, the system studied here provides an example where the extended system is really robust against decoherence compared to only the target system obtained through adiabatic elimination.

B. Parameter uncertainty

Since the condition imposing the system to have a pure steady state is described by a set of algebraic equations, it is easily violated by some parameter changes. In the case of the atomic system under consideration, the numbers of atoms of each ensemble must be exactly the same, i.e., $N_1 = N_2$, but it is fairly unrealistic. Hence let us examine here how much the difference between $N_1$ and $N_2$ affects the entanglement of the steady state. We particularly set $\epsilon = \sqrt{N_2/N_1} = \sqrt{1.1}$; as usual, the number of trapped atoms is of order $10^6$, and such relatively large uncertainty (10%) can actually happen. Figure [3](c) shows the logarithmic negativity $E_N$, where the decoherence due to the spontaneous emission discussed in the preceding section is additionally taken into account. As expected, further degradation of the entanglement is observed and in almost all ranges of the parameters $\xi$ and $\kappa$ the state is no longer entangled. Nevertheless, surprisingly, it is not a uniform degradation: actually, as in the previous case, by engineering the system with optimal parameters $(\kappa^*_x, r^*_x)$, we obtain a steady state that still has a relatively large entanglement of $E_N = 2.41$, which is only a 17% loss of entanglement compared to the ideal value of $E_N = 2.91$, where there is no uncertainty (i.e., $\epsilon = 1$). In other words, by constructing the system with these parameters $(\kappa^*_x, r^*_x)$, we can guarantee the entangled state with at least $E_N = 2.41$, against the uncertainty of the difference of the number of atoms up to 10%. This robustness property indicates the possible effectiveness of the dissipation-based method for state preparation even in a realistic situation, as long as the system parameters are appropriately determined.

VI. CONCLUSION

The main results of this paper are twofold: First, we have shown that an arbitrary Gaussian pure state can be deterministically generated via the local dissipative environment, by constructing an appropriate auxiliary system. Second, we have shown that, even when only a single-mode auxiliary system is available, a well-tuned switching scheme allows us to stabilize any approximate Gaussian cluster state in a dissipative way. The former is a generalization of the scheme proposed in Ref. [8] that yields a dissipation-induced two-mode squeezed state, while the latter is that of Ref. [12] where deterministic generation of several four-mode cluster states was demonstrated.

The essential mechanism for bringing quasilocality to the system considered in this paper is that, for a Gaussian system, any interaction Hamiltonian is always a sum of two-body (hence quasilocal) Hamiltonians. This implies that as long as this kind of interaction Hamiltonian is taken when constructing an auxiliary system, any non-Gaussian (target) system couples to the environment quasilocally. This would be an interesting approach to explore a general method of constructing a desired quasilocal dissipative environment for general non-Gaussian systems.

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