A Black Hole Conjecture and Rare Decays in Theories with Low Scale Gravity

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Abstract

In models with large extra dimensions, where the fundamental gravity scale can be in the electroweak range, gravitational effects in particle physics may be noticeable even at relatively low energies. In this paper, we perform simple estimates of the decays of elementary particles with a black hole intermediate state. Since black holes are believed to violate global symmetries, particle decays can violate lepton and baryon numbers. Whereas previous literature has claimed incompatibility between these rates (e.g. \textit{p}-decay) and existing experimental bounds, we find suppressed baryon and lepton-violating rates due to a new conjecture about the nature of the virtual black holes. We assume here that black holes lighter than the (effective) Planck mass must have zero electric and color charge and zero angular momentum – this statement is true in classical general relativity and we make the conjecture that it holds in quantum gravity as well. If true, the rates for proton-decay, neutron-antineutron oscillations, and lepton-violating rare decays are suppressed to below experimental bounds even for large extra dimensions with TeV-scale gravity. Neutron-antineutron oscillations and anomalous decays of muons, \textit{\tau}-leptons, and \textit{K} and \textit{B}-mesons open a promising possibility to observe TeV gravity effects with a minor increase of existing experimental accuracy.

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1 Introduction

In physics, we sometime run into the so-called hierarchy problems. This happens when we find two or several quantities which are different by many orders of magnitude whereas, a priori, we would have expected them to be more or less at the same scale.

A very well known hierarchy problem in high energy physics is related to the huge gap between the Planck mass $M_{Pl} \sim 10^{19}$ GeV, which sets the energy scale for gravitational interactions, and the electroweak scale $M_{EW} \sim 10^3$ GeV, which instead is joined with the particle physics world. In recent years, models with extra dimensions have been proposed [1]-[4], where the true fundamental gravity scale can be as low as a few TeV and where the Planck mass is a mere effective long-distance 4-dimensional parameter. For a recent review see [5]. In these scenarios, gravity becomes phenomenologically interesting for high energy physics and we may have realistic possibilities to observe and study quantum gravity phenomena at future colliders.

However, one of the constraints on theories with large extra dimensions is the rate of rare processes such as proton decay. Current experimental bounds depend on the decay mode in question [6], with several channels bounded by $\tau_p > 10^{33}$ years [7, 8]. Contributions to $p$-decay [9] include those from GUT-scale intermediate bosons that mediate baryon number violation; these contributions can be suppressed below experimental bounds by additional symmetries as discussed by [1, 2, 10, 11, 12, 13]. In addition, several authors have worried that proton decay and other rare B- and L-violating decays via virtual black holes (BHs) can be exceedingly rapid. According to common belief, the decay/evaporation of BHs does not conserve any global $U(1)$-quantum numbers and, in particular, baryonic, B, and leptonic, L, charges [14]. The usual description of this phenomenon is based on enumeration of all possible operators which do not conserve global charges, normalized to the Planck scale with dimensionless coefficients of order unity.

The suggestion that proton decay can be mediated by a virtual BH was put forward about 30 years ago by Zeldovich [15] and subsequently considered by [16, 17, 18, 19]. The probability of this process in the standard frameworks with $M_{Pl} \sim 10^{19}$ GeV is very low, but with smaller Planck mass in TeV range, the baryon number violating processes through formation of an intermediate virtual BH become much more efficient. Adams et al. [20] argued that experimental limits on the proton lifetime constrain the quantum gravity scale to be larger than $10^{16}$ GeV, implying that the size of the large extra dimensions is less than $l < 10^9/n \times 10^{-30}$ cm, where $n$ is the number of such dimensions. Arkani-Hamed and Schmaltz [21] propose reducing the proton decay rate by a scenario in which the Standard Model fields are confined to a thick wall in extra dimensions, with fermions “stuck” at different points in the wall; then couplings between them are suppressed due to the exponentially small overlaps of their wave functions.

Here, instead, we suggest that $p$-decay via virtual BHs may be more suppressed than previously thought. We propose a “conjecture” that, just as in classical gravity, sub-Planck-mass BHs in quantum gravity can only exist with zero local charge (electric or color) and zero angular momentum. In classical general relativity, it can be shown that a BH with mass smaller than the (effective) Planck mass must have zero electric charge and zero angular momentum (see Section 4); otherwise no horizon can exist and the naked singularity is exposed. Moreover, it can be shown that it is impossible to make the electric charge of a classical BH larger than its mass, $Q^2 > M^2$, in Planck units, by an influx of charged particles on the horizon (see e.g. [22, 23]). In this paper we make the (as yet unproven) conjecture that this result remains true in quantum gravity. Then the virtual BHs that
mediate interactions such as proton decay must have zero spin and be electrically neutral. As a consequence, a four-body collision (3 quarks and a lepton) is required for formation of the BH intermediate state, which leads to a suppressed proton decay.

We compute rare baryon and lepton violating decays as well as $(n - \bar{n})$-oscillations due to intermediate virtual BHs and find that these rates are in agreement with existing experimental bounds even for TeV scale fundamental gravity. In preparation is a second paper, in which we will investigate a mechanism for generating the baryon number of the universe using BH-mediated processes.

The content of the paper is as follows. In Sec. 2 we briefly review TeV-scale gravity models and in Sec. 3 the standard approach to gravitationally induced proton decay. In Sec. 4 we discuss our BH conjecture. In Sec. 5 we consider rare lepton-violating decays with a BH intermediate state, giving an estimate of their rates. In Sec. 6 we discuss baryon-violating processes, proton decay and $(n - \bar{n})$-oscillations, with a BH intermediate state. In Sec. 7 some anomalous decays of particles with heavy quarks, $t$ and $b$, are evaluated. The results are summarized in Sec. 8. The paper closes with an appendix where we review present lower bounds on the magnitude of the possible fundamental gravity scale $M_*$.

## 2 TeV-gravity models

In 1998 Antoniadis, Arkani-Hamed, Dimopoulos and Dvali proposed a “geometric” solution to the hierarchy problem of high energy physics, where the observed weakness of gravity (at long distances) would be related to the presence of large compact extra dimensions [1, 2]. Motivated by string theory, the observable universe would be a 4-dimensional brane embedded in a $(4+n)$-dimensional bulk, with the Standard Model particles confined to the brane, while gravity is allowed to propagate throughout the bulk. In such scenarios, the Planck mass $M_{Pl}$ becomes an effective long-distance 4-dimensional parameter and the relation with the fundamental gravity scale $M_*$ is given by

$$M_{Pl}^2 \sim M_*^{2+n} R^n,$$

(1)

where $R$ is the size of the extra dimensions. If these extra dimensions are “large”, i.e. $R \gg M_{Pl}^{-1} \sim 10^{-33}$ cm, then the fundamental gravity scale can be as low as a few TeV and therefore of the same order of magnitude as $M_{EW}$. If we assume $M_* \sim 1 \text{ TeV}$, we find:

$$R \sim 10^{(30/n)-17} \text{ cm}.$$

(2)

In this approach, however, the hierarchy problem is not really solved but shifted instead from the hierarchy in energies to a hierarchy in the size of the extra dimensions which are much larger than $1/\text{TeV} \sim 10^{-17}$ cm but much smaller than the 4-dimensional universe size.

The case $n = 1$ is excluded because from Eq. (2) we would obtain $R \sim 10^{13}$ cm and therefore strong deviations from Newtonian gravity at solar system distances would result. For $n \geq 2$, $R \lesssim 100 \mu$m and nowadays we have no experimental evidence against a modification of gravitational forces in such a regime [24]. Interesting variations of these models can lower the fundamental gravity scale with the use of non-compact extra dimensions [4].

If gravitational interactions become strong at the TeV scale, quantum gravity phenomena are in the accessible range of future experiments in high energy physics. In particular, there is a fascinating possibility that hadron colliders (such as LHC) will be BH factories.
(for a review, see e.g. Ref. [25], criticisms can be found in Refs. [26, 20]). From the classical point of view, we expect BH production in collision of two particles with center of mass energy \( \sqrt{s} \) if these particles approach each other so closely that they happen to be inside the event horizon of a BH with mass \( M_{BH} \approx \sqrt{s} \). Semiclassical arguments, valid for \( M_{BH} \gg M_* \), predict the BH production cross-section

\[
\sigma \approx \pi R_{BH}^2(M_{BH}),
\]

where \( R_{BH}(M_{BH}) \) is the horizon radius of a BH of mass \( M_{BH} \). In the case of an uncharged and non-rotating BH in \((4+n)\)-dimensions, the horizon radius is obtained from the \((4+n)\)-dimensional Schwarzschild metric [27]

\[
R_S = \frac{1}{\sqrt{\pi} M_*} \left( \frac{M_{BH}}{M_*} \right)^{\frac{1}{n+1}} \left[ \frac{8 \Gamma\left(\frac{n+3}{2}\right)}{n+2} \right]^{\frac{1}{n+1}},
\]

where we have ignored possible effects of the gravitational field of the brane and have assumed \( R_{BH} \) much smaller than the size of the extra dimensions, so that the boundary conditions which come from compactification can be neglected. On the contrary, if \( R_S \) is larger than some extra dimensions, the Schwarzschild radius is given by a lower dimensional BH solution. We have introduced here two quantities, \( R_{BH} \) for the horizon radius of a generic BH and \( R_S \), the same for a Schwarzschild BH. Since in what follows we will deal with Schwarzschild BHs only, these quantities are the same.

### 3 Spacetime foam and proton decay

In this section, we discuss the previous works [19, 20] in which BHs from the spacetime foam give rise to proton decay. We raise some objections and, in the next section, turn to our conjecture which implies that the decays of this section do not take place.

Since at the moment a reliable quantum theory of gravity is lacking, the standard approach is based on semiclassical calculations. For example, virtual BHs can be obtained considering the path integral for \( N \) non-interacting BHs which, in a 4-dimensional spacetime, is (the generalization to a \((4+n)\)-dimensional spacetime can be found in Ref. [20])

\[
Z \sim \int_0^\infty \frac{1}{N!} \sum_{N=0}^\infty \left( \frac{V}{L_{Pl}^3} \right)^N \exp\left( -\frac{4\pi N m^2}{M_{Pl}^2} \right),
\]

where \( V \) is a normalization volume and \( L_{Pl} \) is the Planck length. From Eq. (5) we can define a probability distribution of having \( N \) BHs with mass \( m \) and, computing the expectation value of their mass and of their number density, we find \( \langle m \rangle \sim M_{Pl} \) and a density of roughly one BH per Planck volume. This is the so called spacetime foam picture [30, 31] where the spacetime is filled with tiny virtual BHs which arise as quantum fluctuations out of the vacuum. These BHs exist (for very short time, of order of one Planck time, as a consequence of the uncertainty principle) in the same sense as electron-positron pairs exist in the vacuum of quantum electrodynamics.

In this context we consider the decay of the proton due to virtual BHs from the spacetime foam. We take the proton as an object of volume \( V_p \) equal to its Compton wavelength cubed,
that is \( V_p \sim m_p^{-3} \), with three valence point-like quarks inside. Probably a better estimate is \( V_p \sim \Lambda_{QCD}^{-3} \), where \( \Lambda_{QCD} = 100 - 300 \text{ MeV} \). Thus in the estimates presented below \( \Lambda_{QCD} \) may be substituted instead of \( m_p \). The probability of finding two quarks in the same Planck volume is simply given by the ratio of the Planck volume to the box volume \((m_p/M_{Pl})^3\). If we include the fact that a virtual BH should be formed at the same time when these two quarks are packed so closely, we obtain an additional \( m_p/M_{Pl} \) suppression factor and, by dimensional analysis, we arrive at the decay rate

\[
\Gamma \sim \frac{m_p^5}{M_{Pl}^4}.
\]

(6)

Since BH evaporation conserves energy, charge and angular momentum, this expression is an estimate of the rate of the following process:

\[
q + q \rightarrow \bar{q} + l,
\]

(7)

where \( q (\bar{q}) \) is a quark (anti-quark) and \( l \) a charged lepton.

The same result can be obtained by different arguments. The rate of the process (7) can be estimated as:

\[
\dot{n}/n = n\sigma_{BH} = \sigma_{BH}|\psi(0)|^2,
\]

(8)

where \( n \sim m_p^3 \) is the number density of quarks inside the proton and \( \sigma_{BH} \) is their interaction cross-section through formation of a virtual BH. Since the interaction arises from a dimension six operator, the amplitude has a factor \( 1/M_{Pl}^2 \) and the cross section can be estimated as

\[
\sigma_{BH} \sim m_p^2/M_{Pl}^4.
\]

(9)

Again, we obtain the result in Eq. (6).

Inserting the standard Planck mass \( M_{Pl} \sim 10^{19} \text{ GeV} \) into Eq. (9), we predict the proton lifetime of the order of \( 10^{45} \) years and there is no problem with the current experimental bound. On the other hand, in models with large extra dimensions the fundamental gravity scale is \( M_* \) and, replacing \( M_{Pl} \) with \( M_* \sim 1 \text{ TeV} \), leads to quite short-lived proton with the lifetime \( \tau_p \sim 10^{-12} \text{ s} \). Hence, in order to avoid contradiction with the present experimental constraints, Adams et al. \[20\] require \( M_* \gtrsim 10^{16} \text{ GeV} \).

However, some criticisms may be raised in this connection. In fact, even if in some cases vacuum fluctuations lead to observable phenomena, such as the Casimir force, they are doubtless not well understood when we consider quantum gravity effects related to formation of small virtual BHs. We may encounter problems, even with the standard Plank mass of \( 10^{19} \text{ GeV} \), if we apply the same considerations to leptons, which are supposed to be elementary ones (not consisting of some “smaller” parts). In this case much faster gravitationally induced decays can be expected, because we do not need to wait for two constituent particles to be within the same Planck volume. Hence, in the arguments leading to Eq. (6), only the \( m/M_{Pl} \) suppression factor remains. For example, the rate of \( \mu \rightarrow e\gamma \) could be as large as \( \Gamma \sim m_\mu^2/M_{Pl}^4 \). This estimate is based on time uncertainty relation and gives rise to the decay branching ratio about \( 10^{-4} \), which contradicts the experimental bound by 6 orders of magnitude.

However, these considerations are pretty inaccurate and cannot rigorously lead us to the conclusion that the spacetime foam picture with the standard Planck mass is incorrect. In
particular, the result presented above for the rate of the decay $\mu \rightarrow e\gamma$, which is inversely proportional to the first power of the Planck mass looks very suspicious, one should expect at least the second power, and possibly indicates a sickness of this kind of argument, because in any normal theory the probability of a process in the leading order must contain an even power of the coupling constant. This is true in the usual approach, according to which the decay $\mu \rightarrow e\gamma$ is described by the dimension five operator, $F_{\mu\nu}\langle \psi\sigma^{\mu\nu}\psi \rangle/M_{Pl}$ so that $\Gamma \sim m_\mu^3/M_{Pl}^2$. It does not contradict the experimental bound with the normal Planck mass, $M_{Pl} = 10^{19}\text{ GeV}$, but still it puts a very strong lower limit on the effective Planck mass, $M_* \gtrsim 10^{13}\text{ GeV}$.

In fact practically all “natural” estimates with the low scale gravity lead to too high rates of processes with baryonic and/or leptonic number violation. One possible conclusion is that the gravity scale must be high, close to $M_{Pl}$, or that there is some new exclusion principle which forbids an easy formation of virtual BH. In what follows we will explore the latter option, requiring zero charge and angular momentum of virtual BHs because non-zero charge and momentum are classically forbidden for very light ones with $m < M_*$. Simultaneously here enters another important assumption that virtual BHs have masses of the order of the masses of the initial state (see the next section).

4 A Classical Black Hole Conjecture

In classical general relativity, a BH with mass smaller than the (effective) Planck mass cannot be formed if it has a non-zero electric charge or if it rotates. Here we will make the conjecture that this result remains true in quantum gravity. Hence the virtual BHs that mediate interactions such as proton decay must be electrically neutral and spinless.

The classical condition that a BH less massive than the Planck mass must have vanishing angular momentum and electric charge can be seen as follows. We can examine the 4 dimensional Kerr-Neumann or Reissner-Nordstrom solutions to see that a horizon does not exist if the BHs have electric charge or angular momentum. The position of the horizon is given by the expression \[ R_{BH} = \frac{M_{BH}}{M_{Pl}} + \sqrt{\left( \frac{M_{BH}}{M_{Pl}} \right)^2 - Q^2 - J^2 \left( \frac{M_{BH}}{M_{Pl}} \right)^2}, \] \[ \text{(10)} \]

where $Q$ and $J$ are respectively electric charge and angular momentum of BH. It is clear that in 4D space-time the horizon cannot be formed if \[ \left( \frac{M_{BH}}{M_{Pl}} \right)^2 < \frac{Q^2}{2} + \sqrt{\frac{Q^4}{4} + J^2}. \] \[ \text{(11)} \]

In the absence of a horizon, there would be naked singularities which are not allowed. Indeed if condition (11) is fulfilled, the Kerr-Neumann metric allows to reach the physical singular point $r = 0$ from some large $r$ in finite time without crossing any metric singularity. Probably BHs with such low masses cannot be created. See e.g. problem 17.17 in book [23] where it is shown that it is impossible to charge an already existing Reissner-Nordstrom BH up to $Q > M_{BH}/M_{Pl}$ by any physical process. So a naked singularity cannot be created.

This result is true only in classical physics and quantum gravity effects may change the bound (11). Moreover, the time-energy uncertainty relation could allow a virtual BH to have mass/energy larger than the mass of the initial particle. Still in the absence of
anything better we assume that only diagrams with uncharged and non-rotating virtual
BHs can participate in light \(m < M_{Pl}\) particle decays.

As for the color charge, we expect a similar picture, though the situation is much less
clear. For classically large BHs any color charge should be screened at the microscopic
distances of the order of the inverse QCD scale, \(R_{QCD} = 1/\Lambda_{QCD}\), so there are no colored
hairs at large distances but they can be present at the distances smaller than \(R_{QCD}\). The
analysis of classical colored BH properties can be found in Ref. [32]. For microscopically
small BH, considered here, the screening radius \(R_{QCD}\) is usually large in comparison with
the radius of the BH. Thus for sub-Planck-mass BHs we can apply the same prescription
as is done above for electrically charged BH and require colorless states, even if an analytic
bound such as Eq. (11) does not exist. On the other hand, weak \(SU(2)\) charges can be
safely neglected, since the symmetry is broken via the Higgs mechanism and BHs do not
manifest charges if they are related to massive gauge bosons [33]. To be more precise, weak
hairs may exist but only for a short time, \(\tau_{W} \sim 1/M_{W}\), where \(M_{W}\) is the mass of weak
gauge boson, \(W\) or \(Z\). Since this time is much shorter than the life-time of the processes
considered below we neglect weak hairs of BHs and allow for arbitrary quantum numbers
with respect to weak \(SU(2)\).

A similar situation should hold in higher dimensions but instead of \(M_{Pl}\) an effective
gravitational scale \(M_{*}\) should be substituted. If the Compton wavelength of an elementary
particle, \(\lambda_{C} = 1/m_{i}\), is much smaller than the size of the extra dimensions, it is natural to
believe that gravity “inside” an elementary particle becomes multidimensional.

In addition to this conjecture of neutral and non-rotating BHs we impose some, maybe
even more questionable, conditions in calculations/estimates of the amplitude of reactions
with broken global quantum numbers due to virtual BH. In essence we suggest a set of
rules which do not respect some of the usual conditions existing in quantum field theory,
in particular crossing relations between amplitudes. For example, we allow a virtual BH to
decay into, say, a proton and an electron, but we do not allow a proton to form a BH plus
a positron, with the same amplitude. The picture that we have in mind is a kind of time
ordering: a BH could be formed in a collision of a neutral system of particles in the s-channel
whereas a BH cannot be in the t-channel of a reaction. We assume that BHs can be formed
out of positive energies of real particles only and not from virtual energies of particles in
closed loops. For example, BH cannot be formed by vacuum fluctuations, despite the fact
that, according to the standard picture, vacuum fluctuations might create a pair or more of
virtual particles both with positive and negative energies. The mass of the BH should be of
the order of the energy of incoming (or outgoing) particles. In an attempt to describe this
in terms of the usual language we come to a version of the old non-covariant perturbation
theory with all virtual particles having positive energies. It corresponds to the choice of
only one mass-shell pole in the Feynman Green’s functions. This rule allows only for BHs
with masses which are of the order of the energies of the initial (or final) particles, as we
postulated above. It may look very strange, to say the least, but virtual BHs are not well
defined objects and we do not know what happens with space-time at the relevant scales.
Taken literally these rules would lead to violation of some sacred principles of the standard
theory (locality, Lorentz invariance, and more). Let us remind the reader, however, that the
existing attempts in the literature to invoke virtual BHs are based on standard quantum
field theory in a situation where it is almost surely inapplicable.

So it is not excluded that many properties of the standard field theory are broken,
including even Lorentz invariance and locality. We cannot of course present any serious
arguments in favor of our construction but it predicts quite impressive phenomena with
clear signatures based on a very simple set of rules and if these effects are discovered, the approach may be taken more seriously. Our goal here is to formulate a reasonable(?) set of rules which may possibly describe processes with virtual BHs and are, at least, not self-contradictory. Based on these rules we will study the phenomenological consequences which are quite rich and may be accessible to experiments after a minor increase of accuracy.

5 Lepton number violating decays mediated by black holes

Gravitational interactions are known to become stronger at short distances or at high energies and in the standard theory they are expected to become strong at \( E \sim M_{PL} \sim 10^{19} \text{ GeV} \) or at the distances about \( 10^{-33} \text{ cm} \). In TeV-gravity models, we can expect non-negligible gravitational effects much earlier at the energy scales of contemporary particle physics, at relatively low energies, about electroweak scale. Let us remind the reader that rare decays have been often used to probe small distances and sometimes have given information about heavy particles prior to their discovery. They can play the same important role to investigate models with the fundamental gravity scale in the TeV range.

5.1 Muon decay \( \mu \to 3e \)

In this connection we will consider \( \mu^- \) decay into \( e^- e^- e^+ \), which violates muonic/electronic number conservation due to formation of a virtual BH in the intermediate state. The diagram describing this process is presented in Figure 1a.

First, the muon emits a virtual photon; then the photon produces an electron-positron pair. Next the muon and the positron form a Schwarzschild BH. Since the BH does not respect muon-number conservation, it can decay into an \( e^+ e^- \)-pair. This is not the Hawking radiation [34] because the latter is a semiclassical process, which can be realized for “large” BHs, but is essentially a quantum gravity phenomenon. We cannot reliably calculate its amplitude, since it is surely non-perturbative, but assume that it can be estimated on dimensional grounds assuming that numerical coefficients are of order unity. We also assume that the BH decays predominantly into Standard Model particles on our brane [35] (however, since bulk emission of gravitons becomes more relevant as the number of extra dimensions increases [36], even processes with missing energy may play an interesting role). BH decay conserves energy, angular momentum and gauge charges, but violates global symmetries. As we have already mentioned the process presented in Figure 1b violates the conservation of lepton family number.

Let us now make a rough estimate of the rate of this decay. The emission of the virtual photon and its subsequent transformation into an \( e^+ e^- \)-pair leads to the suppression of the probability by the factor of \( (\alpha/2\pi)^2 \).

By dimensional arguments, the amplitude of the decay \( \mu^- e^+ \) into \( e^+ e^- \) through a virtual BH is proportional to \( g_2^2/M_{BH}^2 \), where \( M_{BH} \) is the mass of BH and \( g_2 \) is the dimensionless coupling constant of BH to two fermions. This coupling constant must be proportional to the strength of gravitational coupling. Here we make the assumption that \( g_2 \sim R_S E \), where \( E \) is the total energy of all colliding particles which make the virtual BH in their center of mass system, i.e. \( E = M_{BH} \). Thus \( g_2/M_{BH} = R_S \). As is written above, \( R_S \) is the Schwarzschild gravitational radius. For \( R_S \) we use Eq. 1 with the last factor in square brackets taken to be \( \sqrt{\pi} \) in the case of multidimensional gravity, while \( R_S = M_{BH}/M_{Pl} \) for the standard gravity.
We have identified the energy of the colliding or outgoing particles with the BH mass, while $R_S$ describes the strength (or better to say, weakness) of gravitational interaction. We emphasize that this is an assumption about the nature of the (quantum) gravitational interaction with the BH. As for the mass of the BH we assume that it is of the order of the muon mass, $M_{BH} \approx m_\mu$, or more precisely, $M_{BH} = E_{e^+} + E_{e^-}$ in their rest frame. In principle one might worry that a virtual BH might have a much larger mass, e.g. larger than $M_*$, and in this case it could be both charged and rotating. For very heavy BHs in the intermediate state the amplitude should be suppressed by an inverse power of BH mass and hence it could be weaker than the amplitude with light neutral BH. On the other hand, if a heavy charged and/or rotating BH could be formed in low-body (e.g. two-body) collision of the constituents of the initial particle, while neutral BH demands more particles for its creation, the processes with heavy BHs would be less suppressed and could be dangerously efficient, despite an absence of the factor $(\alpha/2\pi)^2$. A much larger $M_*$, beyond TeV range would then be necessary in this case to avoid conflict with experiment.

Quantum field theory allows for any masses of virtual particles. However, it may be not true for virtual BHs which are surely nonperturbative and quite complicated quantum fluctuations of space-time. We suggest that such heavy intermediate black holes are not allowed. Our hypothesis is that virtual BH’s are in some sense real, i.e. their mass is of the order of the real characteristic energy of colliding particles which form BH in the process under scrutiny. This assumption contradicts the statement of quantization of BH masses with steps of the order of $M_*$ but in the absence of a serious theory it may be as good as any other. At least, it is very simple and in some sense natural: for formation of BH we need real available energy which is of the order of the decaying particle mass.

The amplitude of $\mu \rightarrow 3e$ decay corresponding to the diagram in Fig. 1a is equal to:

$$A(\mu \rightarrow 3e) = \frac{\alpha}{\pi} \ln \left( \frac{M_e^2}{m_\mu^2} \right) \left[ (\gamma_{23})^{2n+1} (\bar{\psi}_2 \psi_3)(\bar{\psi}_1 \psi_\mu) - (\gamma_{13})^{2n+1} (\bar{\psi}_1 \psi_3)(\bar{\psi}_2 \psi_\mu) \right],$$

where $\gamma_{ij} = M_{BH}^{(ij)}/M_*$ come from the factor $g_5^2/M_{BH}^2 = R_S^2$ (see Eq. (4)) and $\alpha = 1/137$; the log-factor comes from logarithmically divergent triangle part of the diagram with the ultraviolet cut-off taken at the gravity scale $\Lambda_{UV} \sim M_*$; $\psi_j$ are the Dirac spinor wave functions of the corresponding particles. The amplitude is antisymmetric with respect to interchange of two final state electrons 1 and 2. The mass of the virtual BH is taken to be equal to the energy of the $e^+e^-$ pair emitted by this BH. The upper indices $M_{BH}^{(ij)}$ indicate which particles are emitted by BH. In what follows we substitute for simplicity for $M_{BH}^{(ij)}$ its average value

$$\langle \left( M_{BH}^{(12)} \right)^2 \rangle = \langle (m_\mu^2 - 2m_\mu E_3) \rangle = \kappa m_\mu^2,$$

where $\kappa = 1/2 - 1/3$. Taking the necessary traces and integrating over three body phase space we find the decay width:

$$\Gamma(\mu \rightarrow 3e)_n = \frac{\alpha^2 m_\mu}{2^{11/3}} \left( \ln \left( \frac{M_e^2}{m_\mu^2} \right) \right)^2 \left( \frac{m_\mu}{M_*} \right)^{4(1+\frac{1}{n+1})} \kappa^{\frac{2}{n+1}},$$

Two factors $1/2$ come from averaging over the spin of the initial muon and from $1/2!$ due to two identical particles in the final state.
In the case of the ordinary (3+1)-dimensional gravity with $M_* = M_{Pl}$, the decay rate is roughly equal to

$$\Gamma(\mu \to 3e)_{4D} \sim 0.001 \left(\frac{\alpha}{2\pi}\right)^2 \frac{m_\mu^9}{M_{Pl}^8}$$

(15)

and is negligibly small. In the higher dimensional case with $n = 2$:

$$\Gamma(\mu \to 3e)_2 \approx 6 \cdot 10^{-31} \text{ GeV} \left(1 + 0.11 \ln \frac{M_*}{\text{TeV}}\right)^2 \left(\frac{\text{TeV}}{M_*}\right)^{4/3} (3\kappa)^{2/3}.$$ 

(16)

Since the total decay rate of the muon is

$$\Gamma_{tot} \approx 3 \cdot 10^{-19} \text{ GeV}$$

(17)

then, assuming $M_* \sim 1 \text{ TeV}$, we obtain the branching ratio about $2 \cdot 10^{-12}$ for $n = 2$ which is close to the present experimental constraint [8]:

$$BR(\mu^- \to e^- e^+ e^-) \bigg|_{Exp} < 1.0 \cdot 10^{-12}.$$ 

(18)

For larger $n$ the decay rate would be in stronger disagreement with the experimental bound. However even a minor increase in the value of $M_*$ would avoid the contradiction. It is intriguing that these estimates with $M_* \sim \text{ TeV}$ are quite close to the existing bounds on muon number violating decay $\mu \to 3e$.

5.2 Lepton number violation in $e^+ e^-$ collisions

The conservation of muon number can be also violated in the reaction

$$e^+ + e^- \to \mu + e$$

(19)

or similar reactions with any other leptons in the final state. A virtual BH can be formed here just from the initial $e^+ e^-$ pair and the cross-section of this reaction would be about

$$\sigma(e^+ e^- \to \mu e) \approx 7 \cdot 10^{-39} \text{ cm}^2 \left(\frac{M_{BH}}{100 \text{ GeV}}\right)^{2+\frac{4}{n+1}} \left(\frac{\text{TeV}}{M_*}\right)^{4+\frac{4}{n+1}}.$$ 

(20)

This may be observable in high energy $e^+ e^-$ collisions. We again note that the predicted cross-section [20] is surprisingly close to the current limits on lepton flavor violation search at the $Z^0$ resonance [3]. One should keep in mind, however, that the total angular momentum of the initial $e^+ e^-$ pair must vanish. This means that the annihilation should proceed from the $s$-state because the total spin of the initial particles is zero. This demands the same sign of helicity of $e^+$ and $e^-$ in their center of mass frame. On the other hand, the annihilation through $Z$-boson proceeds with total angular momentum equal one and demands opposite helicities of $e^+$ and $e^-$. 

5.3 Muon decay $\mu \to e\gamma$

One can obtain an estimate of the decay rate $\mu \to e\gamma$ along the same lines. In the simplest diagram (see Figure 1b) the probability is suppressed by an additional power of $\alpha$, but the ratio of two-body to three-body phase space compensates this extra suppression, so that the
probability of $\mu \rightarrow e\gamma$ decay through the considered mechanism would be approximately the same as $\mu \rightarrow 3e$.

We should keep in mind however, that there is an additional ambiguity related to the second loop in diagram of Fig. 1, which contains three virtual electrons. If we first integrate over the $(\mu e\gamma)$-loop, the second loop would also be logarithmic, because one electron propagator, common for both loops, would be reduced to a point. The log-factor related to the second loop should be taken with the loop counting factor, $1/(2\pi)^2$ and the result would be somewhat smaller than the naive estimate without logarithmic and loop counting factors. There is an important difference between the first and the second loops. The particles in the first loop interact with the BH at one point and the BH propagator does not enter into consideration. In the second loop the interaction with the BH is not pointlike and the statement about its ultraviolet behavior was made assuming the standard form of the BH propagator, $\sim (P_{BH}^2 - M_{BH}^2)^{-1}$. It is unknown whether or not this is true. Possibly covariant perturbation theory is not applicable to the case of virtual BH and naive estimates of divergences are not valid (see Sec. 4). This problem is also encountered in Sec. 6.3, where it is essential for the calculation of the time of $(n - \bar{n})$-oscillations.

5.4 $\tau$ decays

The widths of the analogous processes with nonconservation of the $\tau$-flavor number in $\tau$-lepton decays would be enhanced by the ratio $(m_{\tau}/m_{\mu})^{5+4/(n+1)}$, which is $6 \cdot 10^7$ for $n = 2$. Since the $\tau$ life-time is $10^7$ times shorter than that of the muon, the expected branching ratios of the decays $\tau \rightarrow 3l$ or $\tau \rightarrow l\gamma$ would be an order of magnitude larger than those for similar muon decays, i.e. maximally around $10^{-11}$. It surely does not contradict the existing bounds, which are of order of $10^{-6} - 10^{-7}$ [8]. It is unclear if such decays may be reachable in not too distant future.

If the virtual BH in the diagram in Fig. 1 emits a $q\bar{q}$-pair, instead of a lepton-antilepton pair, it would lead to decay of $\tau$-lepton into semileptonic channel, $\tau^- \rightarrow l^- +$ mesons. The branching ratio (BR) for this inclusive process is expected at the same level as the BR for $\tau \rightarrow 3e$ decays, or somewhat smaller, because a decay of BH into a pseudoscalar state (e.g. $\pi$-meson) may be suppressed and the dominant decay channel should be a scalar one.

There can also be interesting modes of $\tau$ decays with non-conservation of baryon and lepton numbers, as e.g. $\tau^- \rightarrow e^- e^+\bar{p}$, $e^- e^- p$ and analogous ones with neutrons and neutrinos. The diagrams describing such decays are essentially the same as Fig. 1h, with replacement of the initial $\mu$ with $\tau$ and with a BH which emits three quarks and one lepton instead of two leptons. The effective Lagrangian describing e.g. such decays has the form:

$$\frac{\alpha}{\pi} \ln \left(\frac{m_{\tau}^2}{m_{\tau}^2} \right) \frac{g_2 g_4}{M_{BH}^2} \bar{\psi}_l \psi_{\tau} \psi_{\bar{q}} \psi_l,$$  \hspace{1cm} (21)

where different $\psi$’s are spinor wave functions/operator of the corresponding fermions and $g_n$ is the coupling constant of BH to $n$ fermions. According to the arguments presented in the preceding sections, creation of BHs in multiparticle collisions should be suppressed due to the necessity for several particles to meet in the same small volume and thus, e.g. $g_4$ should contain the BH radius to the fourth power, while the rest of the necessary dimension is supplied by the BH mass:

$$g_4 = R_s^4 M_{BH}, \text{ and } g_2 = R_s M_{BH}. \hspace{1cm} (22)$$
The amplitude of the decay $\tau \rightarrow pl\bar{l}$ is determined by the matrix element of this operator between an initial $\tau$ and final $3l + p$ states. The matrix element of making a proton out of three quarks is:

$$\langle p|\psi_3^3|\text{vac}\rangle \sim \frac{m_q^3}{(2\pi)^4} \bar{u}_p,$$

where $m_q \approx 300$ MeV is the constituent quark mass, that is the characteristic energy scale of the process, and $u_p$ is the spinor wave function of the proton.

We do not distinguish in this section between the masses of light constituent quarks, $m_q \approx 300$ MeV and the characteristic scale of strong interaction, $\Lambda_{QCD} \sim 100$ MeV. However, in future sections of the paper, where we consider proton decay and neutron-antineutron oscillations, the difference may be important.

It is straightforward now to calculate the decay rate

$$\Gamma(\tau \rightarrow pl\bar{l}) \sim 10^{-3} \left(\alpha \frac{M^2}{m_\tau}\right)^2 \left(\ln \frac{m_q^2}{m_\tau^2}\right) \left(\frac{m_\tau}{M_*}\right)^6 \left(\frac{m_q}{M_*}\right)^{4+\frac{10}{n+1}} m_\tau,$$

where the first factor, $10^{-3}$, comes from three-body phase space. The life-time with respect to this decay would be extremely long: for $M_* \sim 1$ TeV and $n = 2$, we obtain $\tau \sim 10^{16}$ years. We have omitted here the loop counting factors but the effect is tiny even without them.

### 5.5 $K$-mesons decays

Similar considerations can be applied to rare decays of $K$-mesons. In particular, if two quarks constituting $K^0$-meson might form BH, this BH could decay into any neutral combination of two leptons, $e^+e^-, \mu^+\mu^-$ and $\mu^\pm e^\mp$. The amplitude of this decay can be estimated as:

$$A(K \rightarrow 2l) \approx g_{Kqq} \left(\frac{g_2}{M_{BH}}\right)^2 \frac{m_q^2}{(2\pi)^2} \bar{\psi}_1\psi_2,$$

where $g_{Kqq}$ is the Yukawa coupling constant of $K$-meson to $\bar{q}q$-pair and the third factor came from the integration of the loop of $\bar{q}q$ pair which combines into $K$-meson. The corresponding decay width is given by

$$\Gamma(K^0 \rightarrow l\bar{l}) = \frac{g_{Kqq}^2 m_K}{4(2\pi)^3} \left(\frac{m_q}{M_*}\right)^4 \left(\frac{m_K}{M_*}\right)^{\frac{4}{n+1}}.$$

We have assumed that $M_{BH} = m_K$ and $E = m_K$ since no energy is taken away to other particles. With $g_{Kqq} \sim 1$, $m_q = 300$ MeV, $M_* = 1$ TeV and $n = 2$, we obtain for the life-time of this decay

$$\tau(K^0 \rightarrow l\bar{l}) \approx 0.16 \text{ s}.$$

For $K_2^0$ it gives the branching ratio $BR(K^0 \rightarrow l\bar{l}) \sim 10^{-7}$, which is 4-5 orders of magnitude above the experimental bounds: for example, $BR(K^0 \rightarrow e^+e^-) < 10^{-11}$ and $BR(K^0 \rightarrow e^\pm \mu^{\mp}) < 5 \cdot 10^{-12}$. To avoid the contradiction we should either take $M_* > 3$ TeV or to see if there is a possible suppression mechanism for this decay in this scenario. In fact, there is one. It is natural to expect that BH should have the quantum numbers of the
vacuum, i.e., it should be a scalar object. Hence the $K$-meson, which is a pseudoscalar, cannot transform to BH directly, but should emit some other particle in such a way that the remaining combination of the quark-antiquark system would be scalar. The simplest way is to emit a $\pi^0$-meson, while the remainder would make a BH which would decay into $ll$. This mechanism means in particular that two body decays $K \to ll$ are suppressed and not dangerous.

The amplitude of the decay $K \to \pi ll$ is described by the diagram in Fig. 2 and is equal to

$$A(K \to \pi ll) = \frac{g_K^2 g_{\pi S} m_q^2}{(2\pi)^2 M_{BH}^2} \bar{\psi}_l \psi_{l'}$$

where $g_{K\pi S}$, the coupling constant of $K$ and $\pi$ to the scalar state of quark-antiquark pair, has dimension of inverse mass. The factor $m_q^2/(2\pi)^2$ comes from the quark loop. The mass of BH is $M_{BH}^2 = (p_{l'} + p_{l})^2 = m_K^2 + m_{\pi}^2 - 2m_K E_\pi$. The life-time with respect to this decay, after integration over 3-body phase space with $M_{BH}$ depending upon the pion energy, $E_\pi$, is equal to:

$$\tau(K \to \pi ll) = 0.85 \cdot 10^2 s \left(\frac{g_{K\pi S} m_\pi}{m_K}\right)^{-2} \left(\frac{M_\pi}{\text{TeV}}\right)\left(\frac{\text{TeV}}{m_K}\right)^{4+n} \cdot \left(\frac{300\text{ MeV}}{m_q}\right)^4 \left(\frac{6.4 \cdot 10^{-3}}{f_n}\right),$$

where $f_n$ is related to integration over phase space:

$$f_n = \int_\mu (1+\mu^2)^2 dx \sqrt{x^2 - \mu^2} \left(1 + \mu^2 - 2x\right)^{1+\frac{2}{n+1}}.$$

Here $\mu = m_\pi/m_K$. The factor $6.4 \cdot 10^{-3}/f_n$ is equal to 1 for $n = 2$, to 0.82 for $n = 3$, and to 0.58 for $n = 7$.

The experimental bounds on the branching ratios of $K_L^0 \to \pi^0 2l$ decays are about $(3 - 5) \cdot 10^{-10}$, while its life-time is $5.2 \cdot 10^{-8}$ sec. Thus for $n = 2$, $M_\pi = 1$ TeV, and $g_{K\pi S} m_\pi \sim 1$ we are on the verge of discovery of such decays. For larger $n$ a larger $M_\pi$ or smaller $g_{K\pi S}$ are needed.

Similar estimates can be presented for decays of charged $K$-mesons, e.g.,

$$K^+ \to \pi^+ \mu^+ e^-, \pi^+ 2\nu, \text{ etc.}$$

Experimental bounds on the branching ratios of these decays are about $3 \cdot 10^{-11}$. The absolute probability of decay (31) is approximately the same as that of $K^0 \to \pi^0 ll$. Since the total life-time of $K^+$ is 4 times shorter than that of $K_L^0$, the predicted branching ratios are 4 times smaller and, according to the discussion above, they may be close to the existing bounds.

This model has a few interesting features/signatures. The dominant anomalous decay mode is three body. The charge of the emitted pion is the same as the charge of the initial $K$. The probabilities of the decays with charged and neutral leptons in the final states are approximately the same. The rather large magnitude of the branching ratios of these anomalous decays of $K$-mesons make them very interesting/promising candidates in the search for non-conservation of global lepton quantum numbers.

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Figure 1: a) Muon decay $\mu \to 3e$ with BH intermediate state. b) Diagram for the process $\mu \to e\gamma$.

Figure 2: Kaon decay $K^0 \to \pi^0 e^-\mu^+$. $K$-meson probably cannot transform directly to a BH, because it is a pseudoscalar particle; instead, the emission of a $\pi^0$ leaves a scalar $q\bar{q}$ system.

Moreover, we note another appealing feature of all the decays with a BH intermediate state: if undiscovered weakly interactive light particles exist, such as for example possible sterile neutrinos or axions, they should be emitted by the BH with the same probability of the other light particles, if compatible with the BH quantum numbers. In fact, tiny BHs may provide a unique opportunity to discover such a kind of particles because, if the so-called “equivalence principle” holds, gravity couples to any form of energy with the same strength and does not distinguish one type of particle from another. Obviously, the emitted weakly interactive particles cannot be seen by the detector and the event appears as a process with violation of energy, momentum and, possibly, angular momentum.

6 Baryon number violating processes mediated by black holes

6.1 Proton decays

Proton decay is described by a more complicated diagram, as shown in Fig. 3. To create an electrically neutral, colorless, nonrotating BH, one of the three quarks in the proton must emit a lepton pair through $\gamma$, $W$, or $Z$ exchange; one of these leptons, together with the three quarks, may then form such a BH devoid of any quantum numbers.

Since the BH is now formed in a 4-body collision, the probability of BH creation would be suppressed by an additional small ratio squared of BH volume to the proton volume.
The ratio of volumes appears in the second power because we have two more particles in the initial state in comparison with BH creation in $\mu \rightarrow 3e$ decay. In the latter case the BH could be produced in a two-body collision. This leads to the suppression factor $(R_S \Lambda_{QCD})^6$, where $\Lambda_{QCD} \sim 100$ MeV is the inverse proton size (in what follows we skip the sub-QCD). In fact, the situation is more complicated and a one parameter description may be impossible because the proton size is roughly the inverse pion mass, while the characteristic quark energies in the proton, which determine the coupling to BH are of the order of the constituent quark mass, i.e. $m_q \approx 300$ MeV.

We will come to the same conclusion studying the transformation $3qe^- \rightarrow \bar{q}q$ through a virtual BH, as is considered in Sec. [9]. The amplitude of the reaction is proportional to $g_2 g_4/M_{BH}^2 = R_S^5$, where $g_2$ is the coupling constant of BH to $n$ fermions; $g_4$ is dimensionless, while $g_4$ has dimension of inverse mass cubed, see Eq. (22). (Notice that in the case of $\mu \rightarrow 3e$ decay the amplitude is proportional to $g_4^2$ and that’s why the decay probability should be much larger.)

The amplitude of the proton decay corresponding to Fig. 3 is equal to:

$$A(p \rightarrow l^+ \bar{q}q) = \frac{\alpha}{\pi} \ln \frac{M_p^4}{m_q^2} \frac{\Lambda^3 R_S^5}{(2\pi)^4} \psi_l \psi_p \bar{\psi}_q \psi_q. \quad (32)$$

It leads to the decay rate:

$$\Gamma(p \rightarrow l^+ \bar{q}q) = \frac{m_p \alpha^2}{212 \pi^{1/3}} \left( \ln \frac{M_p^2}{m_q^2} \right)^2 \left( \frac{\Lambda}{M_\psi} \right)^6 \left( \frac{m_p}{M_\psi} \right)^{4 + \frac{10}{n+1}} \int_0^{1/2} dx x^2 (1-2x)^{1 + \frac{2}{n+1}}. \quad (33)$$

Correspondingly the lifetime of the proton with respect to the inclusive decay $p \rightarrow \bar{q}ql^+$ is:

$$\tau_p \approx 10^{20} \text{ years} \left( \frac{M_\psi}{\text{TeV}} \right)^{10 + \frac{10}{n+1}} \left( \frac{\text{TeV}}{m_p} \right)^{10 + \frac{10}{n+1}} \left( \frac{100 \text{MeV}}{\Lambda} \right)^6 \ln^{-2} (M_\psi/\text{TeV}) f_p^{-1}(n), \quad (34)$$

where $f_p(n)$ is 1, 1.3 and 2.2 for $n = 2, 3$ and 7 respectively.

The best experimental lower bounds at the level $\tau_p > 10^{33}$ years [8] are established for the modes $p \rightarrow e^+\pi^0$ and $p \rightarrow \nu K^+$. For all other 2-body and some three-body modes the bounds are at the level of $10^{32}$ years. The disagreement of our result with experiment can be easily avoided if we take a slightly larger $M_\psi$, still even smaller than 3 TeV. On the other hand, we should keep in mind that our estimates are by no means rigorous; they are only true up to an order of magnitude (and possibly some omitted factors would allow $M_\psi$ to be as low as 1 TeV). For example, if we take into account that the proton inverse size, $\Lambda \sim 100$ MeV is three times smaller than the quark mass $m_q \sim 300$ MeV, the life-time might become larger by the factor $(m_q/\Lambda)^6 \sim 10^3$.

There is one more argument indicating that the decay life-time [34] may be underestimated. By assumption, the intermediate virtual BH is a scalar and since we believe that gravity does not break parity, such BH cannot go into a pseudoscalar particle. In other words, the pair $\bar{q}q$ cannot form a pseudoscalar meson or vector meson. Thus we come to the important conclusion that proton must have predominantly 3-body decay modes: lepton plus two mesons with the two mesons in scalar state. This immediately shifts our estimates for the probability of decay into $(l^+ + 2$ mesons) down by an order of magnitude because of smaller phase space of scalar state. On the other hand the decay into three leptons is not influenced by this argument and its life time should be given by Eq. [34]. Bearing in mind that the experimental lower bounds on the proton life-time with respect to three
body lepton channel, $l^+ l^+ l^- \ (l = e, \mu)$ are at the level $(8 - 5) \cdot 10^{32}$ years we see that proton decays are on the verge of experimental discovery if $M_\star$ is slightly larger than or about 2 TeV.

There are quite peculiar signatures specific to the model of proton decay considered here. First, as we have already mentioned, the decays should be mostly 3-body ones. Second, the final state particles must always contain a positron, $e^+$, or a positive muon, $\mu^+$. The branching ratio into three lepton channel is predicted to be larger than that into $e^+(\mu^+)$ and two mesons, because it is natural to expect that the probability of BH decay into a neutral combination of two leptons (or antileptons, or lepton and antilepton) is more or less the same as the probability of the decay into two quarks, while the probability of the subsequent quark transformation into two mesons is smaller than one, because other channels are open. The energy spectrum of emitted leptons (with the same charge as proton) is cut-off at higher energies due to the factor $(1 - 2E/m_p)^{1+10/(n+1)}$, see Eq. (33).

6.2 Neutron-antineutron oscillations

Another process where non-conservation of baryons is actively studied by experiments is neutron-antineutron transformation. While in many cases, as e.g. SU(5) GUT or electroweak theory, neutron-antineutron oscillations are impossible or completely negligible, because they demand change of baryonic number by 2, $\Delta B = 2$, the gravitational breaking of global symmetries does not respect any selection rule and the oscillation time may be reasonably small. In the framework of the approach presented here, the neutron-antineutron oscillations are described by the diagram of Figure 4. An estimate of this diagram is very uncertain and the result should be taken with great caution. There are two loops containing weak $W$ or $Z$ bosons. Both these loops are logarithmic and if the ultraviolet cutoff is given by the effective Planck scale, $M_\star$, their contribution is not suppressed as an inverse power of the weak boson mass. Logarithmically divergent part of such loop diagram gives the factor:

$$\frac{\alpha}{\pi} \ln \left( \frac{\Lambda_{UV}^2}{m_Z^2} \right)$$

(35)

where $\Lambda_{UV}$ is the ultraviolet cutoff, which is probably reasonable to take equal to the effective Planck mass, $\Lambda_{UV} = M_\star$. The amplitude of neutron-antineutron transformation contains this factor squared.
The other part of the diagram, containing the lepton loop, is linearly divergent after we perform integration in the loops containing weak bosons. The integral should be cut-off at the same energy scale as the neighboring triangle diagrams with weak bosons, i.e. at $M_\ast$. In other words, the linear divergent part as usually vanishes and the integral is proportional to the external momentum which, in this case, is of the order of $M_\ast$. (See discussion in Sec. 5, two paragraphs below Eq. (20)).

As a result of this rather frivolous estimate we obtain for the amplitude of the transition of three quarks into three antiquarks:

$$L_{\Delta B=2} = \left[ \frac{\alpha}{\pi} \ln \left( \frac{M_\ast^2}{m_Z^2} \right) \right]^2 M_\ast (R_3^4 E)^2 (\bar{\psi} C \psi)^3,$$

(36)

where the first factor comes from two triangle parts of the diagram, Eq. (35), the second factor $M_\ast$ is the ultraviolet cut-off of the linearly divergent loop with three lepton lines and one virtual BH and the next factor is the coupling constant of BH to 4 particles. The last term is the product of 6 quark operators and $C$ is the charge conjugation matrix.

Now it is straightforward to obtain the time of neutron-antineutron oscillations taking the matrix element $\langle n | L_{\Delta B=2} | \bar{n} \rangle$. Since the effective energy cutoff in $3q$-transition into neutron is the QCD scale, $\Lambda$, we obtain:

$$\tau_{n\bar{n}} = \left[ \frac{2\alpha}{\pi} \ln \left( \frac{M_\ast}{m_Z} \right) \right]^{-2} \left( \frac{M_\ast}{\Lambda} \right)^{7+\frac{8}{n+1}} \Lambda^{-1}$$

(37)

(note that we have omitted here the huge loop counting factor $(2\pi)^8$, because the result is weak anyhow). With $n = 2$ and $M_\ast \sim 1$ TeV and $\Lambda = 100$ MeV it corresponds to an oscillation time of about $3 \cdot 10^{19}$ s which is twelve-thirteen orders of magnitude below the existing experimental limit: direct searches for $n \rightarrow \bar{n}$ processes using reactor neutrons put the upper limit on the mean time of transition in vacuum [5]:

$$\tau_{n\bar{n}} > 8.6 \cdot 10^7 \text{ s},$$

(38)

while the limit found from nuclei stability is slightly stronger:

$$\tau_{n\bar{n}} > 1.3 \cdot 10^8 \text{ s}.$$  

(39)

If the theoretical prediction of Eq. (37) were true, the chances to observe $(n - \bar{n})$-oscillations in the reasonable future are negligible. However, one can obtain much more optimistic predictions if there exist supersymmetric partners of the usual particles, as considered in the following subsection.

### 6.3 Supersymmetric extension

Some of our estimates, such as for $(n - \bar{n})$-oscillations, would be different if there exist supersymmetric partners of the standard model particles. Since supersymmetry remains a hypothesis, not yet proven by experiment, and nothing is known about the masses of superpartners, except for lower limits on their values, more ambiguity is introduced into the calculations. Consequently we here consider anomalous processes with inclusion of superpartners as a separate subsection.
Figure 4: \((n - \bar{n})\)-oscillation mediated by a virtual BH. If we consider only Standard Model particles, the effect is negligible.

The spins of the SUSY particles may differ by 1/2 from their standard model partners, with all other quantum numbers being the same. In particular, there could be scalar quarks (s-quarks) or spin-1/2 partners of vector bosons mediating interactions. Existence of new types of elementary particles would modify both neutron-antineutron transformation and proton decay.

In this case, one of the quarks in the neutron can emit a neutralino, \(\chi^0\), and become a squark, \(\tilde{q}\). This \(\tilde{q}\), together with remaining quarks, can form a neutral and spinless BH. This BH in turn may decay into two antiquarks, \(2\bar{q}\), and anti-squark, \(\bar{\tilde{q}}\). The latter captures \(\chi^0\) and becomes the usual antiquark, \(\bar{q}\). This completes the transformation of three quarks into three antiquarks (see Fig. 5). The analogous process with emission of a gluino does not help, because after this emission the remaining two quarks and one squark become colored.

To find the amplitude corresponding to this diagram we need to calculate the contribution from the loop containing two s-quark propagators, one neutralino propagator, and, most problematic, a propagator of the virtual BH, about which we do not have much information. Possibly the result would be less ambiguous if we were to use the old non-covariant perturbation theory with particles on mass shell with positive energies. The last condition is important for definition of the vertex of interaction of BH with particles entering into it or emitted by it. Because of that, the total energy of incoming or outgoing particles is always of the order of the energy of the initial state and the mass of BH is of the same order. The necessity to use non-covariant perturbation theory in our description of interaction of particles with virtual BHs leads to breaking of Lorentz invariance. Another argument in favor of non-covariant perturbation theory is the impossibility to make Wick rotation with virtual BH – at least we do not know how to do that.

The effective Lagrangian corresponding to the diagram in Fig. 5 is the following:

\[
L^{\text{susy}}_{\Delta B=2} = \frac{\alpha g_3^2}{2\pi} \frac{m_\chi}{m_{\text{SUSY}} M_{\text{BH}}^2} (\bar{\psi} C \psi)^3, \tag{40}
\]

where \(g_3 = R_3^2 M_{\text{BH}}\) is the coupling constant of BH to 3 particles, two of which have spin 1/2 and one is scalar, \(\alpha \sim 0.01\) is the electroweak coupling constant at characteristic SUSY scale; \(m_\chi\) is the mass of neutralino, \(m_{\text{SUSY}}\) is the mass of other superpartners, and we assume that they are all of the same order of magnitude, \(m_\chi \sim m_{\text{SUSY}}\). A subtle point is the value of the BH mass. According to the arguments presented above we take it to be of the same order of magnitude as the energies of the external particles; again, in the absence of any fundamental theory describing behavior of virtual BH, we can consider this at best a plausible assumption.
Figure 5: \((n - \bar{n})\)-oscillation with supersymmetric particles. In this case the observation of the phenomenon may be accessible to future experiments.

Taking the matrix element of operator (40) between \(n\) and \(\bar{n}\) states we find:

\[
\langle \bar{n} | L_{\Delta B=2}^{\text{susy}} | n \rangle = \frac{\alpha}{2\pi} \left(\frac{\Lambda}{2\pi}\right)^6 \left(\frac{M_{BH}}{M_*}\right)^{4+n} \left(\frac{M_*}{M_{BH}}\right)^{6+n} \left(\frac{m_{\text{susy}}}{300 \text{GeV}}\right)^{4+n} \left(\frac{\text{GeV}}{1 \text{ TeV}}\right)^{4+n} \left(\frac{M_*}{\text{TeV}}\right)^{4+n} \right). \tag{41}
\]

Here \(\Lambda\) is the effective energy which enters in calculating of the matrix element 3 quark operators over the neutron state, it is usually taken about 100 MeV; and \((2\pi)^8\) is the loop counting factor – there are 4 loops with virtual quarks which go either into \(n\) or \(\bar{n}\) and each loop provides with \(1/(2\pi)^2\).

Taking all the factors together we find for the time of \((n - \bar{n})\)-oscillations:

\[
\tau_{\bar{n}n} \approx 3 \cdot 10^9 \text{ sec} \cdot 10^{12 \frac{12}{n+1} - 4} \left(\frac{100 \text{ MeV}}{\Lambda}\right)^6 \left(\frac{m_{\text{susy}}}{300 \text{GeV}}\right) \left(\frac{\text{GeV}}{M_{BH}}\right)^{4+n} \left(\frac{M_*}{\text{TeV}}\right)^{4+n} \left(\frac{M_*}{M_{BH}}\right)^{6+n} \left(\frac{M_{BH}}{\text{TeV}}\right)^{4+n} \left(\frac{m_{\text{susy}}}{300 \text{GeV}}\right)^{4+n} \left(\frac{\text{GeV}}{1 \text{ TeV}}\right)^{4+n} \tag{42}
\]

This result looks quite promising. If \(M_*\) is not too much larger than 1 TeV and the SUSY partners are not far from 300 GeV, the chances to observe neutron-antineutron transformations are very good. According to the model presented here their observation would indicate not low scale gravity but also low energy SUSY; but this is probably too far fetched.

We also note that the contribution of SUSY partners to proton decay is negligible.

7 Heavy quark decays

We discuss here decays of heavier quarks or mesons containing such quarks. Though the experimental accuracy in their decays are much lower than for proton or muon, the effects may be amplified because of larger masses and it is quite probable that the manifestation of low scale gravity will be first observed in such decays.

Let us start from the heaviest, the \(t\)-quark. Since it has a very large mass, \(m_t = 175 - 180\) GeV, close to the assumed gravity scale, one can expect that B-nonconserving decays of the \(t\)-quark may be noticeably enhanced. A B-nonconserving decay \(t \to 4q + l\) is described by a diagram of the type presented in Fig. 6. The same considerations as those presented in Sec. 5 lead to the decay rate:

\[
\Gamma \sim \varepsilon_5 \left(\frac{\alpha_{\text{QCD}}}{2\pi}\right)^2 \left(\frac{m_t}{M_*}\right)^{10+n} \left(\frac{M_{BH}}{\text{TeV}}\right)^{10+n} \left(\frac{m_{\text{susy}}}{300 \text{GeV}}\right)^{10+n} \left(\frac{\text{GeV}}{1 \text{ TeV}}\right)^{10+n} m_t, \tag{43}
\]
where $\epsilon_5 \sim 10^{-10}$ is the 5-body phase space normalized to the $t$-quark mass.

The total decay rate of the $t$-quark is known to be:

$$\Gamma_{tot} \approx \alpha m_t \approx 7 \cdot 10^{-3} m_t.$$  (44)

Using Eqs. (43) and (44) we obtain the branching ratio (always assuming $M_* \sim 1$ TeV):

$$BR_{\Delta B \neq 0} \sim 10^{-20} - 10^{-19}$$  (45)

for $n$ between 2 and 7. Probably the decay rate (43) is underestimated because of too large phase space suppression factor and the branching ratio is somewhat larger. In particular it would be larger for B-nonconserving decays of mesons or baryons containing $t$-quark because phase space suppression in this case would be much milder. On the other hand, the loop counting factor, omitted above, would play its destructive role.

At the present time there are no experimental restrictions on nonconservation of the baryonic number in decays of particles containing $t$-quark. However, a noticeable baryonic charge non-conservation in $t$-quark decays and some other anomalously enhanced decays of mesons containing $t$-quark may be accessible to future experiments.

More realistic possibility may be in the leptonic sector: for example, the BH in Fig. 6 can emit an $e^\pm \mu^\mp$-pair, leading to the violating the family lepton number decay $t \rightarrow u e \mu$. Its amplitude is

$$A(t \rightarrow u e \mu) = \frac{\alpha_{QCD}}{\pi} \ln \left( \frac{M_*}{m_t} \right)^2 R_S^2 \tilde{\psi}_u \psi_t \tilde{\psi}_\mu \psi_e.$$  (46)

The predicted decay rate is

$$\Gamma(t \rightarrow u e \mu) = \frac{\alpha^2_{QCD} m_t}{16\pi^5} \ln^2 \left( \frac{M_*}{m_t} \right)^2 \left( \frac{m_t}{M_*} \right)^{4+\frac{4}{\alpha + 1}} \int_0^{1/2} dx x^2 (1 - 2x)^{1+\frac{2}{\alpha + 1}}$$  (47)

and, for $n = 2$, we expect a BR at the level of $10^{-9}$.

On the other hand, $t$-quark may be not the best candidate for search of anomalous decays induced by virtual BH because $m_t > m_W$ and the total decay width of $t$-containing particles is quite large due to the open channel into $W$ and $b$-quark.

Probably a better place to search for low scale gravity effects could be decays of $b$-quark or, to be more precise, decays of mesons containing $b$-quark, in particular $B$-mesons. On one hand, $b$-quark is lighter than $t$-quark, its mass is about 4.5 GeV, and this makes the effects weaker. On the other hand, the total decay widths of particles containing $b$-quark are smaller than those with $t$-quark (because $b$ is not heavy enough to decay into $W$ boson and lighter quarks) and this would enhance branching ratio of anomalous decays induced by gravity.

As an example, let us consider a decay of $B^0$-meson consisting of a heavy $b$-quark and light $\bar{d}$-quark. As is known from QCD such a system of light and heavy quark has the size of the order of $\Lambda^{-1}$ and the characteristic energy of the light quark about $\Lambda$. Its decay into two light leptons can be considered in the same way as muon or $K$-meson decays in Sec. 5. One should expect that an uncharged and nonrotating virtual BH can be formed directly in collision of $b$ and $\bar{d}$ and it is not necessary to invoke any other virtual particles. If this is true (but remember that BH should probably be a scalar and not a pseudoscalar), then the amplitude of $B$-meson decay into two quarks or two leptons is given by the expression

$$A(B \rightarrow l l) = \frac{g_B^2}{M_B^2} \frac{4}{(2\pi)^4} \int \frac{d^4p (p+p_b+m_\mu m_\nu)}{(p^2-m_q^2)(p_1^2-m_Q^2)} \tilde{\psi}_l \psi_l,$$  (48)
where $g_{Bqq}$ is the coupling constant of $B$-meson to two quarks and $p_1 = p - p_B$.

The cut-off of this integral is determined by the strong interaction scale $\Lambda$ and could be described by a formfactor:

$$F_1 = F(p_B^2/m_B^2, \frac{p_B^2}{m_B^2}, \frac{p_q^2}{m_q^2})$$

which vanishes if the participating particles are too far from the mass shell. However, it is rather complicated to impose this condition in the standard form of the Feynman integral \[48\]. We will use to this end the on-mass-shell representation of the Green’s functions:

$$G(x, t) \sim \int \frac{d^3p}{(2\pi)^3} E \exp (-iEt + ipx),$$

where $E = \sqrt{p^2 + m^2}$. The one loop diagram is now described by the expression

$$\int \frac{d^3p F_2(p^2)}{E_q E_b (E_B - E_q - E_b)} \left( m_b^2 - p_b p_b + m_b m_q \right).$$

The last term in brackets comes from the fermionic trace and contains the product of four-momenta of the virtual fermions. The new formfactor $F_2$ is a function of three-momentum $p^2$ because all the particles are on-mass-shell but energy is not conserved in each vertex. This form-factor is determined by the interaction potential and is cut-off at $|p| \sim \Lambda$.

This one-loop integral \[51\] can be easily estimated giving the result $\sim \Lambda^2$. An important fact is that it does not contain the mass of heavy quark, $m_b$ in the denominator.

The decay width of $B$-meson into the channel $B \rightarrow ll$ or $B \rightarrow \bar{q}q$ can now be estimated as

$$\Gamma(B \rightarrow ll) \approx \frac{m_B g_{Bqq}^2}{2^{3/5}} \left( \frac{m_B}{M_*} \right)^4 \left( \frac{m_b}{m_B} \right) \frac{4}{\pi^5} \left( \frac{\Lambda}{m_B} \right)^4.$$  \[52\]

For $g_{Bqq} = 1$, $n = 2$, $\Lambda = 100$ MeV and $M_* = 1$ TeV we find the life-time $\tau_B \sim 3 \cdot 10^{-3}$ s. The total life-time of $B^0$ is $1.5 \cdot 10^{-12}$ s. Thus the branching ratio of anomalous decays with the chosen values of the parameters should be about $5 \cdot 10^{-10}$. This result is below the existing experimental bounds. The branching ratios of $B^0$ into $e^+e^-$, $\mu^+\mu^-$, $e\mu$ are all bounded by approximately $10^{-7}$. We repeat, however, that estimated branching ratio may be true if pseudoscalar BHs are allowed.

If the virtual BH must be scalar, and we consider the decay $B \rightarrow 2l$, then before collapsing into BH the system $bd$ should emit a light pseudoscalar (PS) meson, $P$, which is later to be absorbed by the final state leptons, but the probability of that is very low (it is a weak interaction process). Thus the 3-body decays should dominate in the same way as found for $K$-meson decays in Sec. 5.

The amplitude of the decay $B \rightarrow P + 2l$, where $P$ is a light PS-meson, is determined by the similar loop integral as above with the only change that $m_B^2 - 2E_pm_b$ is substituted instead of $m_B^2$. This leads to further deviation from the mass-shell pole and the integral is suppressed by approximately an order of magnitude in comparison with the previous case, i.e. $\sim 0.1\Lambda^2$. Correspondingly the decay width is equal to

$$\Gamma(B \rightarrow Pll) \approx \frac{10^{-2} m_B (g_{BPSTM_B})^2}{2^{3/5}} \left( \frac{\Lambda}{m_B} \right)^4 \left( \frac{m_B}{M_*} \right)^{4+\frac{4}{\pi^5}} \int_0^{1/2} dx x(1-2x)^{1+\frac{2}{\pi^5}}.$$  \[53\]
Figure 6: Top decay with BH intermediate state and violation of baryon and lepton numbers.

where $g_{BPS}$ is the coupling constant of transition of $B$-meson into PS-meson $P$ and a scalar state of two quarks. It has dimension of inverse mass and it may be natural to assume that $g_{BPS} \sim 1/\Lambda$ (this is a quite strong coupling).

For $g_{BPS} \sim 1/\Lambda$, $n = 2$, $\Lambda = 100$ MeV and $M_* = 1$ TeV the life-time with respect to this decay is 0.2 s. It leads to the branching ratio $BR(B \rightarrow Pll) \approx 10^{-11}$. This relatively large branching is related to a huge coupling $g_{BPS} \sim 1/\Lambda$.

An interesting process could be a decay of $B^0$ into $p+e^-$. The branching ratio of this decay should be suppressed with respect to the decay of $B^0$ into two leptons at least by the factor $(R S \Lambda)^{6} \lesssim 10^{-24}$, because now the BH emits four particles and therefore the amplitude of the process is proportional to $g_{2} g_{4}/M_{BH}^{2}$. It makes this decay impossible to observe in foreseeable future.

There can be some other $B$-meson decays as well, with violation of baryon or lepton numbers or not, where virtual BH could give noticeable contributions and an observation of possible anomalously large branching ratios might indicate on the effects of virtual BHs.

8 Conclusions

Observable effects of gravity in particle physics are an interesting and fascinating possibility allowed by models with large extra dimensions, where the fundamental gravity scale can be set in the TeV range. In such a framework, we have considered rare decays with BH intermediate states. Since BH decays violate global symmetries, we expect lepton and baryon numbers non-conserving processes. We elevate the classical requirement of zero charge and zero angular momentum for sub-Planck-mass black holes to a general conjecture even in quantum gravity. In that case, the predicted B or L non-conserving decay rates or the frequency of neutron-antineutron oscillations are interestingly close to the existing experimental bounds but not in conflict with them for $M_* > 1$ TeV. It is striking that neutron-antineutron oscillations and some anomalous decays of $K$ and $B$-mesons are very close to the existing experimental bounds and, thus, quite promising for observation of TeV gravity effects.

We have taken $M_*$ as low as possible, namely about 1 TeV. With higher $M_*$ all experimental bounds can be easily satisfied, but new physics becomes harder to test. Unfortunately we do not know how large can be $M_*$; hence it is impossible to reject our model by experiment. Even with $M_*$ slightly larger than 1 TeV all effects discussed here would be strongly suppressed.
The model presented here is very speculative and includes plenty of wishful thinking. On the other hand, it gives very interesting testable predictions with specific signatures absent in other models. We repeat that with the “natural” value of $M_* = 1$ TeV the predictions for the magnitude of new effects are quite close to the existing experimental accuracy.

The way in which virtual BHs are treated here implies some striking features, e.g., possible breaking of Lorentz invariance. It may mean, in particular, that the magnitude of effects in different coordinate frames may be significantly different – a reincarnation of the old concept of the ether.

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Appendix: Short review on current constraints on $M_*$

To begin with, we stress that, here and throughout the paper, as fundamental gravity scale we take the quantity $M_*$, which is related to the $(4+n)$-dimensional gravitational constant $G_*$ of the $(4+n)$-dimensional Einstein-Hilbert action by the simple relation

$$M_*^{n+2} = \frac{1}{G_*}.$$  \hfill (54)

In the trivial case $n = 0$, we have $M_*^2 = M^2_{Pl} = 1/G_N$. In the literature there are at least other two popular conventions. Some details on the different possibilities can be found, for example, in Ref. [29].

In addition to this, we now assume that the $(4+n)$-spacetime is given by $M_4 \times T^n$, where $M_4$ is the standard 4-dimensional spacetime we know and $T^n$ is an $n$-dimensional torus of radius $R$. In this special case, the volume of the extra dimensions is finite and equal to $(2\pi R)^n$ and the relation between the standard 4D Planck mass $M_{Pl} = 1.22 \cdot 10^{19}$ GeV and the fundamental gravity scale $M_*$ is

$$M_{Pl}^2 = (2\pi R)^n M_*^{n+2}.$$  \hfill (55)

With these two statements in mind, we now review present experimental lower bounds on the magnitude of $M_*$. From now on, we follow Ref. [8].

Since a TeV gravity scale allows the emission of gravitons at colliders [39], constraints on $M_*$ can be obtained looking for missing energy in processes such as $e^+e^- \to \gamma G$ (the probability that the graviton $G$ interacts with the detector is suppressed by the standard Planck mass $M_{Pl}$, and therefore negligible). The non-observation of such events at LEP leads to the 95% CL bounds [40]

$$M_* > 1.43, 0.76, 0.47, 0.33, 0.25 \text{ TeV}$$  \hfill (56)

for $n = 2 − 6$.

Much more stringent constraints for $n < 4$ can be obtained by astrophysical considerations. We note however that these bounds require the existence of gravitons lighter than about 100 MeV and can be evaded if gravitons acquire small extra contributions to their masses, as suggested in [41, 42, 43] (in the scenario of Ref. [43], even the $n = 1$ case cannot be safely excluded). In fact, the astrophysical environments used to constraint $M_*$ are
characterized by a typical energy per particle of 10 – 100 MeV and an effective graviton mass of 100 MeV or more prevents a copious production of them. These scenarios leave instead unchanged collider constraints, where the typical energy is roughly 100 GeV and a 100 MeV mass is unimportant.

Since gravitons are weakly interactive particles, a possible their production in a supernova event can compete with neutrino cooling. Neutrinos detection from SN1987A requires

\[ M_\ast \gtrsim 12.5, 1.0 \text{ TeV} \]  \hspace{1cm} (57)

for \( n = 2 \) and 3 respectively.

On the other hand, if we consider all gravitons produced by all the supernovae in the history of the universe, we can expect a diffuse gamma ray background due to the graviton decay into photons. The non-observation of such a diffuse background by EGRET satellite puts the bound

\[ M_\ast \gtrsim 34, 2.6 \text{ TeV} \]  \hspace{1cm} (58)

for the \( n = 2 \) and the \( n = 3 \) case.

Finally, noting that gravitons produced in supernovae events are not high relativistic particles, we can expect that many of them remain gravitationally bound to the neutron star remnant and that their subsequent decay reheats the surface of the star. The measured luminosity of some pulsars leads to the very stringent bounds

\[ M_\ast \gtrsim 670, 20 \text{ TeV} \]  \hspace{1cm} (59)

always for \( n = 2 \) and 3.

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