Coherent chirped pulse laser network in Michelson phase conjugating configuration.

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The mechanisms of nonlinear phase-locking of a large fiber amplifier array are analyzed. It is shown that Michelson phase conjugating configuration with double passage through array of fiber amplifiers have the definite advantages compared to one-way fiber array coupled in a Mach-Zehnder configuration. Regardless to amount of synchronized fiber amplifiers Michelson phase-conjugating interferometer is expected to do a perfect compensation of the phase-piston errors and collimation of backwardly amplified fiber beams on entrance/output beamsplitter. In both configurations the nonlinear transformation of the stretched pulse envelope due to gain saturation is capable to randomize the position of chirp inside envelope thus it may reduce the visibility of interference pattern at output beamsplitter. A certain advantages are inherent to the sech-form temporal envelope because of exponential precursor and self-similar propagation in gain medium. The Gaussian envelope is significantly compressed in a deep gain saturation regime and frequency chirp position inside pulse envelope is more deformed.

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I. INTRODUCTION

The chirped pulse laser amplifiers (CPA) aimed to generation of a strong optical fields and optical acceleration of electron and ion beams have attracted a special attention from the point view of compactness and versatility [1,2]. The possible applications range from medical tissue treatment to spallation and further to strong field fundamental physics [3]. Recent highly promising trend is a coherent summation of the stretched subnanosecond laser pulses produced by a thousands of fiber amplifiers each operating at millijoule level. This chirped spatially smooth pulse of a tens of Joules may be compressed into femtosecond pulse by a diffraction gratings [4]. The advantage of this approach is in massive parallelism which is possible due to commercial availability thus carefully tested 30–50 fs fiber master oscillators, millijoule level short length ($L_f \sim 1–10 \text{ m}$) fiber amplifiers and reliable fiber beam splitters developed in recent decades due to telecommunications needs.

The most impressive project [1] presumes the usage of $30fs$ fiber master oscillator emitting $10^{-6}J$ pulses with a tens of kilohertz repetition rate and fiber stretcher in order to produce $\sim 900ps$ frequency chirped pulse. The elongation of pulse in time domain is necessary to eliminate self-focusing and other nonlinear effects during amplification process. The Bespalov-Talanov filamentation instability [5] is suppressed because of smallness of the fiber’s core diameter $(D \sim 10–120\mu km)$ compared to the most dangerous filament transverse size in rare-earth doped laser amplifiers $\ell_\perp = (2\pi k/\sqrt{2n_2 I(z,t)})^{-1} \sim 200–400\mu km$ [6], where $kn_2 I(z,t)$ is Kerr nonlinearity of refractive index induced by optical intensity $I(z,t)$. Next stage involves gradual pulse amplification in a multiplexing set of standard fiber amplifiers. Each amplifier supports single spatial mode and preserves frequency chirp, required to compression inside the output pair of diffraction gratings. The output compressor also became a standard component since 1985 [4]. The current technology bottleneck is a method of the coherent summation [7] of the $N_f \sim 10^{3–4}$ single spatial mode laser beams into a single transversely smooth beam with preservation of the frequency chirp to ensure grating compression.

For this purpose a variety of the linear and nonlinear beam coupling techniques is being studied [8–12]. The usage of 50/50 beam splitters and Glan-Thompson like polarization cubes inside Mach-Zehnder interferometers [10] (fig.1a) requires the adjustment of the optical path difference with an accuracy of $\lambda/(20–50)$ and perfect overlapping of elementary Gaussian beams. Such an adjustment is a quite routine procedure for a low frequency thermal and tension noises [1] because their noise maximum locates below $10Hz$. Nevertheless despite the commercial availability of electro-optically controlled phase-shifters and liquid crystal light valve wavefront controllers the operation of a thousands beam-splitting units might overcomplicate a system and increase the system cost and operation expenses. The similar difficulty is a necessity of providing a high degree of the spatial overlapping of $10^{3–4}$ elementary Gaussian beams amplified inside fiber array in a sequence of $10^{3–4}$ demultiplexing (beam combining) beamsplitters. The more serious complications are due to nonlinear self-phase modulation (SPM or B-integral discrepancy) [12] and temporal envelope transformation because of gain saturation of laser amplifiers [14]. The nanosecond laser pulses with exponential envelope $\text{sech}(t – zn_0/c)$ move in a self-similar style with superluminal envelope speed while Gaussian pulses demonstrate self-steepening of pre-
FIG. 1: (Color online) A) Chirped pulse amplification and compression in Mach-Zehnder configuration. MO is master oscillator, CPA is chirped pulse amplifying array, BS\textsubscript{fiber} is entrance beamsplitter tree, which may be both free space and fiber array, M are ordinary retro-mirrors. BST is free-space output binary BS tree. A\textsuperscript{2}*T is target of volume \(\lambda^3\). B) Chirped pulse amplification and compression in Michelson configuration. PCM is degenerate four-wave mixing (DFWM) phase-conjugating mirror, \(\chi(t)/2\) is Pockels time-dependent chirp preserving decoupler, BST is free-space entrance binary BS tree dividing MO beam into \(N_f = 2^{N_{ex}}\) amplified beams. Backward phase-conjugated amplified emission is combined into single beam by this Michelson BST.

cursor and subluminal speed with a stronger deformation of envelope. We will show that this effect is essential for both preservation of the frequency chirp magnitude and its location within stretched pulse envelope \(f(t = z n_0/c)\).

In this work we theoretically analyze the coherent beam summation technique inside binary tree of beam splitters (binary BST) without \(\lambda/(20 - 50)\) accuracy adjustment\[8, 11\]. This is possible with the aid of phase-conjugating hence self-adjusting interferometry\[15, 19\]. The diffraction will be taken into account within framework of the split-step factorizable model\[20, 21\]. Propagation inside single Gaussian mode fibers and paraxial free space intervals is given by conventional exact solutions\[22, 23\]. The gain inside fibers and nonlinear transformation elements will be considered in a ray approximation\[21\]. The most attention will be paid to phase-locking of fiber array via reflection from phase-conjugating mirror (PCM)\[15, 24\] (fig.4,5). In this approach the incident signal \(E_f(z, t, \vec{r})\) records the information about gain medium in dynamic hologram written inside PCM\[23\]. The reflected phase-conjugated signal \(E_b(z, t, \vec{r})\) propagates as "backward in time" replica with appropriate recovery into the smooth single spatial mode beam inside a sequence of the Michelson beam splitters (fig.1b). The thin slice PCM with instantaneous response inside a third order \(\chi(3)\) Kerr dielectric medium\[27, 29\] will be considered as a solution compatible with preservation of the temporal profile of the chirped pulse. The basic difference between Mach-Zehnder and Michelson configurations is that binary BST is placed in opposite parts of amplifier array (fig.1a,b), although in both cases it is constructive interference inside a sequence of beam splitters that ensures the coherent lossless output. The Michelson amplitude division binary BST gives a substantial reduction of losses compared to wavefront division phase-conjugator\[20\].

The paper is organized as follows. In section II the model is formulated for temporal envelope having frequency chirp inside array of fibers each having variable gain \(G_{mn}\) and different length \(L_{mn}\) resulting in phase piston errors \(\Delta\phi_{mn}\). The section III describes the procedure of phase-conjugation with preservation of frequency chirp and \(\Delta\phi_{mn}\) compensation. Section IV is devoted to backward nonlinear amplification of the phase conjugated replica in fiber amplifiers array. The conditions of the chirp preservation are formulated qualitatively for Gaussian and \(sech\) temporal envelopes \(\text{fig.6,7}\). The asymptotic behavior of output interference pattern for large \(N_f\) is studied in section V. In concluding section VI the results are formulated.

II. MODEL FORMULATION

The features of a single passage Mach-Zehnder fiber amplifier network (fig.1a) and double-passage Michelson phase-conjugating amplifier network (fig.1b) will be analyzed for electric fields of pulses \(E_{G0}\) and \(E_{E0}\), which initially \((z = 0)\) have transform limited temporal envelopes of Gaussian and hyperbolic secant form:

\[
E_{G0,se}(z, t, \vec{r}) \equiv E^0 \cdot \exp \left(-\frac{\tau^2}{2 \tau^2_{Gs}}\right) \cdot f_{G0,se}(\tau)
\]

\[
\cdot \exp \left[i \theta_{G0,se}(\tau)\right] \cdot \exp \left(-i \omega t + ik z\right),
\]

\[
\theta_{G0,se}(\tau) \equiv k n_2 \cdot L_{str} \cdot |E_{G0,se}(z, t, r = 0)|^2,
\]

\[
f_{G0}(\tau) = \exp \left(-\frac{\tau^2}{2 \tau^2_{Gs}}\right), f_{se}(\tau) = sech \left[\tau / \tau_{se}\right],
\]

which undergoes frequency modulation due to Kerr nonlinearity \(\chi(3)\). Here \(E^0\) is electrical field amplitude at stretcher output \((z = L_{str})\), \((z, t, \vec{r})\) is coordinate system, collocated with propagation axis \(z\), \(\tau = t - z n_0 / c\), \(\tau_{Gs}, \tau_{se} \sim 30 - 50fs\) are the pulse durations for MO and \(\sim 900ps\) for stretched pulse in amplifiers, \(k_z \sim k = n_0 \omega / c\) is wavenumber, \(n_0\) is linear refractive index, \(v_g = \partial \omega / \partial k = c / n_0\), \(L_{str}\) is the length of the medium (stretcher) having \(\chi(3)\) susceptibility, \(r = |\vec{r}|\), \(D\) is radius of Gaussian fundamental transverse mode, \(\theta_{G0,se}(\tau)\) is proportional to the breakup integral \(B\):

\[
\theta_{G0,se}(\tau) \sim B(\vec{r} = 0, \tau) = \frac{2 \pi n_2}{\lambda} \int_0^{L_{str}} |E_{G0,se}(z, \tau, r = 0)|^2 dz, \quad \tau_{se} = 3 \chi(3) / 8 n_0.
\]

calculated as a nonlinear phase accumulated over distance \(L_{str}\) for a given moment \(\tau\) of envelope propagating along fiber axis \((r = 0)\), \(n_2 |E_{G0,se}|^2\) is Kerr refractive index, \(n_2 \sim 10^{-20} n^2 / W\) for typical glasses,
n_2 \sim 10^{-19} m^2/W for crystals alike Nd:YAG and even more for resonant media e.g. sodium vapor, $n_0$ is linear refractive index of stretcher. This oversimplified estimate of self-phase modulation is adequate for short Kerr medium where group velocity dispersion $\beta = \partial^2 k/\partial \omega^2$ is negligible in nonlinear Schrödinger equation (NLS) for slowly varying envelopes $E_{G,s}(z, \tau)$:

$$i \frac{\partial E_{G,s}(z, \tau)}{\partial z} = \frac{\beta}{2} \frac{\partial^2 E_{G,s}(z, \tau)}{\partial \tau^2} - \gamma |E_{G,s}(z, \tau)|^2 \times E_{G,s}(z, \tau), \quad A_{eff} = \frac{\int \int_{-\infty}^{\infty} |E_{G,s}(z, \tau)|^2 \partial^2 \tau^2}{\int \int_{-\infty}^{\infty} |E_{G,s}(z, \tau)|^4 \partial^2 \tau^2}, \quad (3)$$

where $\gamma = (n_2 \omega)/(CA_{eff})$, $A_{eff}$ is effective area of a fiber. Frequency modulation in each given moment $t$ is provided by $\delta \omega_{G,s}(\tau) = \partial \omega_{G,s}(\tau)/\partial \tau$. The phase factors $\phi_{G,s}(\tau)$ as a function of a local time $\tau = t - z/v_g$ are shown in fig.4a for sech envelope (eq.) and for Gaussian envelope (eq.) at fig.4b. In both cases the linear frequency chirp appears near the parabolic maximum of pulse envelope. The split-step approach to NLS means the separate integration of nonlinear phase-modulation and dispersion terms. Let us split NLS in two slices only. This removes from consideration the possible effects of temporal solitons formation [22] and their interaction during pulse stretching in fiber from femtosecond to nanosecond duration. Instead the smooth analytical formulas relevant to temporally elongated nanosecond pulse are used. The remained integration is due to group velocity dispersion (GVD) $\beta = \partial^2 k/\partial \omega^2$:

$$i \frac{\partial E_{G,s}(z, \tau)}{\partial z} = \frac{\beta}{2} \frac{\partial^2 E_{G,s}(z, \tau)}{\partial \tau^2}, \quad (4)$$

The solution via Fourier transform of initial value (Cauchy) problem from $z = 0$ to any $z > 0$ is given by:

$$E_{G,s}(z, \tau) = \int_{-\infty}^{\infty} E_{G,s}(z = 0, \omega) \exp \left[ i \frac{\beta}{2} \omega^2 z - i \omega \tau \right] d\omega, \quad (5)$$

where spectra of Gaussian and sech envelopes at output facet of master oscillator are given by:

$$E_{Gs}(z = 0, \omega) = \int_{-\infty}^{\infty} \exp \left[ -\tau^2 / 2 \tau_{Gs}^2 \right] \exp[i\omega \tau] d\tau = \frac{\tau_{Gs} \sqrt{\pi}}{2} \exp[-\omega^2 \tau_{Gs}^2 / 4],$$

$$E_{Gs}(z = 0, \omega) = \int_{-\infty}^{\infty} \frac{sech \left[ \tau / \tau_{Gs} \right]}{\tau_{Gs}} |\exp[i\omega \tau]| d\tau = \frac{\pi \tau_{Gs} \sqrt{\pi}}{2} sech \left[ \frac{\pi \omega \tau_{Gs}}{2} \right]. \quad (6)$$

The broadening of pulses due to GVD $\beta$ is given by:

$$E_{Gs}(z, \omega) = (2\pi \tau_{Gs})^{1/2} \exp \left[ -\omega^2 \tau_{Gs}^2 / 4 + i \omega^2 \beta z / 2 \right],$$

$$E_{Gs}(z, \omega) \sim sech \left[ \pi \omega (\tau_{Gs} - \beta \omega z) / 2 \right]. \quad (7)$$

Both cases demonstrate the self-similar stretching due to propagation along $z$. Propagation of chirped pulses is somewhat more complicated due to nonlinear phase $\theta(\tau)$:

$$E_{Gs}(L_{str}, \omega) = \int_{-\infty}^{\infty} \exp \left[ -\tau^2 / 2 \tau_{Gs}^2 \right] \exp[i\omega \tau + i \theta_{Gs}(\tau)] d\tau,$$

$$E_{Gs}(L_{str}, \omega) = \int_{-\infty}^{\infty} sech \left[ \tau / \tau_{Gs} \right] \exp[i\omega \tau + i \theta_{Gs}(\tau)] d\tau. \quad (8)$$

The explicit formulas for the chirped spectra of the $E_{G,s}(z, \omega)$ for $z > L_{str}$ for the short (quasiclassical) distance $z$ are obtained via stationary phase method [21, 31] because $\theta_{G,s}(\tau)$ has a slowly varying parabolic extremum near $\tau = 0$. The spectrum for Gaussian pulse is modified as:

$$E_{Gs}(z, \omega) \approx \tau_{Gs} \sqrt{2 \pi} \frac{2}{\sqrt{2} \pi \omega^2 \tau_{Gs}^2} \exp[-\omega^2 \tau_{Gs}^2 / 4] \times \exp \left[ i \frac{\beta}{2} \omega^2 z + i k_{n2} \cdot L_{str} \cdot |E_{Gs}(0, \tau = 0)|^2 + i \pi / 4 \right], \quad (9)$$

whereas for sech($\tau$) pulse $\omega -$ dependent spectrum is:

$$E_{Gs}(z, \omega) \approx \tau_{Gs} \sqrt{2 \pi} \frac{2}{\sqrt{2} \pi \omega^2 \tau_{Gs}^2} \times \exp \left[ i \frac{\beta}{2} \omega^2 z + i k_{n2} \cdot L_{str} \cdot |E_{Gs}(0, \tau = 0)|^2 + i \pi / 4 \right]. \quad (10)$$

The recovered temporal envelopes of nanosecond duration $E_{G,s}(z, \tau)$ have a linear frequency modulation (chirp) near pulse maximum in such a way that precursor is red-shifted with respect to the pulse tail [4]. The same frequency chirp is produced by a pairs of diffraction gratings, prisms with chromatic aberrations and grisms (prisms with grooved gratings).

Let us extend now the model for phase locking of multiple beams amplified in fibers or bulk crystals. Using temporal envelopes $E_{G,s}(z, \tau)$ the high frequency field $\mathcal{E}_{mn}$ in each fiber channel $(m, n)$ is:

$$\mathcal{E}_{mn}(z, t, \vec{r}) \equiv E_{mn} \cdot \exp \left[ -(\vec{r} - \vec{r}_{mn})^2 / 2D^2 \right].$$

$$E_{G,s}(z, \tau) \cdot \exp \left[ -i \omega t + ik_{n2} \cdot z + i \Delta \phi_{mn} \right]. \quad (11)$$

Consider the spatially periodic lattice with period $p$ composed of the polarization preserving fiber laser amplifiers whose output facets are located at points $\vec{r}_{mn}$, where $\vec{r}$ is transverse coordinate, $z$ is coordinate along pulse propagation direction, $t$ is time. The linearly polarized output field of this array $\mathcal{E}_{f}(z, t, \vec{r})$ is a superposition of the elementary partial waves each having transversely Gaussian profile with radius $D$ [23, 32]:

$$\mathcal{E}_{f}(z, t, \vec{r}) \equiv \exp[-i \omega t + ik_{n2} \cdot E_{G,s}(z, \tau) \sum_{m,n} E_{mn} \cdot \exp\left[-(\vec{r} - \vec{r}_{mn})^2 / 2D^2 + i \Delta \phi_{mn} \right], \quad (12)$$
where $E_{mn}$ is electric field amplitude, $E_{Ga,Se}(z,\tau) = f_{Ga,Se}(t - z/v_g) \cdot exp[i\theta_{Ga,Se}(t - z/v_g)]$ is temporal envelope function, $\Delta\phi_{mn}$ is a random phase piston shift labelled by indices $(m, n)$, induced by fiber’s length variation $L_{mn}$, heating or stress inside a given fiber $(m, n)$ amplifier [26].

III. PHASE-CONJUGATION OF LASER ARRAY EMISSION WITH CHIRPED PULSE TEMPORAL ENVELOPE

The accurate control of the phase piston errors $\Delta\phi_{mn}$ with accuracy about $\lambda/(20 - 50)$ is well studied for the small number ($N_f \approx 2 - 4$) of phase-locked fiber amplifiers and a reasonable energy efficiency (more than 95 percents) had been reported already [10]. In most cases the Mach-Zehnder interferometry was used because it ensures temporal envelope preservation for $\lambda/(20 - 50)$ path difference between synchronized beams. The question is in robustness of this technique for a large number $N_f \approx 2^{N_{ex}} = 4096 - 32768$ of a phase locked laser amplifiers. Here $N_{ex} \sim 12 - 15$ is a number of superpositions experienced by each elementary beam inside binary BST to achieve perfect smooth beam combination. The total number of required beamsplitters $N_{bs}$ grows linearly when $N_f$ is increased. Starting from $N_f = 2$ when one beamsplitter is needed $(N_{bs} = 1)$ it is clear that network which combine $N_f = 2^{N_{ex}}$ fiber lasers contains $N_{bs} = (N_f/2)/(1 - 0.5) = N_f - 1$ beamsplitters (sum of geometric progression). This network requires not only careful adjustments of the phase lags $\Delta\phi_{mn}$ before each beam splitter. The other urgent requirement is to ensure perfect spatial overlapping of transverse Gaussian profiles of the each beam at the each of $N_f - 1$ beamsplitters of the coherent beam summation network.

This task would require an intense usage of the microprocessors and substantial computing resources. The preliminary evaluation of computing and micropositioning resources might be performed in the following way. Indeed each of the $N_{bs}$ beamsplitters has six degrees of freedom. In addition position of the each elementary Gaussian beam directed to combining beamsplitter is controlled by two transverse coordinates, two angles and focal point location. Thus the upper bound on the total amount of dynamical variables to be controlled in Mach-Zehnder network (fig.1a) is $N_{var} = (N_f - 1) * 6 + 2 * (N_f - 1) - (2 + 2 + 1)$. The alternative method of phase locking of multiple beams amplified in fibers or bulk crystals is Michelson phase-conjugator (fig.1b) which looks experimentally attractive compared to Mach-Zehnder interferometric schemes (fig.1a) [3][4]. In addition to currently studied nonlinear beam coupling techniques with one pass propagation which use second order parametric processes with $\chi^{(2)}$ nonlinearities [12] or four-wave mixing $\chi^{(3)}$ processes we consider double-pass configuration [13] which is a well-proven tool for compensation of the phase-piston errors $\Delta\phi_{mn}$ in amplifying channels [30].

For the first sight stimulated Brillouin scattering (SBS) looks feasible for phase-conjugation of chirped laser pulses of nanosecond duration. The conceptual difficulty is in accuracy of reproduction of the temporal envelope $f(t - z/v_g)$ [33]. For the long chirped pulse $\tau_{Ga,Se} > \tau_{ph} \sim 10^{-9}$ sec the phase modulation (linear in time frequency chirp) will be distorted by a random phase jumps separated by interval $\sqrt{2\pi}G_{obs} \tau_{ph}$ caused

![FIG. 2: (Color online) Number of beams splitters BS required for coherent summation of $N_f = 8$ fiber amplifiers. This binary tree placed at output of Mach-Zehnder interferometer (fig.1a) is a pyramid with ratio of elements in adjacent layers 0.5. As it easily seen by summation of the clearly visible geometric progression, number of BS and mirrors M required is exactly $N_f - 1$.][figure2]

![FIG. 3: (Color online) Number of degrees of freedom controlled for ensured constructive interference in BS. Gaussian beams (which have hyperboloid isosurfaces) at entrance ports of BS are controlled by two transverse coordinates, two paraxial angles and one longitudinal parameters (distance towards Gaussian beam waist). Shifted Gaussians in output ports form interference pattern similar to the two plane wave intersection or "equal tilt fringes".][figure3]
by finite lifetime of acoustical phonons $\tau_{ph} = 1/\Gamma$ [24]. Here $G_{s,0} \cong 25 - 30$ is SBS increment (gain growth rate), $\Gamma = 2\pi(k_0 + k_s)^2/3\rho_0$ is sound damping rate (spontaneous Brillouin scattering linewidth) [28, 33]. $\eta$ is viscosity of Brillouin medium. On the other hand for the short nanosecond laser pulses having $\tau_{G_s,se} \sim 10^{-9}$ sec SBS reflected wave $E_b(z,t,\vec{r})$ is not able to follow the frequency modulation of the pump wave $E_f(z,t,\vec{r})$ because of inertia of acoustic wave $Q(z,t,\vec{r})$.

Indeed SBS equations of motion for the scalar slowly varying envelope optical fields, i.e. $E_f$ moving in the positive $z$-direction and $E_b$ moving oppositely are:

$$\frac{\partial E_f(z,t,\vec{r})}{\partial z} + \frac{n_0}{c} \frac{\partial E_f}{\partial t} + \frac{i}{2k_p} \nabla^2 E_f = \frac{i\gamma_{SBS}\omega_p}{4c n_0 \rho_0} Q E_b$$
\hspace{4cm} (13)

$$\frac{\partial E_b(z,t,\vec{r})}{\partial z} - \frac{n_0}{c} \frac{\partial E_b}{\partial t} - \frac{i}{2k_s} \nabla^2 E_b = -\frac{i\gamma_{SBS}\omega_s}{4c n_0 \rho_0} E_f Q^*,$$
\hspace{4cm} (14)

with dimensionless slowly varying acoustical perturbation complex amplitude $Q$ [28]:

$$v_{ac} \frac{\partial Q(z,t,\vec{r})}{\partial z} + \frac{\partial Q}{\partial t} + \frac{\Gamma Q}{2} = E_f E_b \frac{i\gamma_{SBS}(k_p + k_s)^2}{16\pi \omega_{ac}}$$
\hspace{4cm} (15)

where $\gamma_{SBS} = \rho (\partial \epsilon/\partial p)_s$ is electrostrictive coupling constant [35], $\rho$ is density of SBS medium, $n_0$ is refractive index, $v_{ac}$ is speed of sound.

As is shown experimentally and computationally in many works the phase of reflected Stocks pulse in the limit $\tau_{G_s,se} \gg \sqrt{2\pi G_{s,ph}\tau_{ph}}$ experiences a random phase jumps $\Delta \Phi_{pump}$ uniformly distributed in the interval $(-\pi, \pi)$ [24]. For the broadband SBS pump radiation when characteristic correlation time is much shorter than lifetime of acoustic phonons $\tau_{ph}$ the Stocks wave envelope is modulated by random phase jumps $\Delta \Phi_{pump}$ of pump wave $E_f(z,t)$ [22]. In such a case linear frequency chirp is not reproduced by SBS mirror. The short pulse limit $\tau_{G_s,se} \ll \sqrt{2\pi G_{s,ph}\tau_{ph}}$ means the fixed carrier frequency of acoustical phonons. Consequently the carrier frequency difference $\omega_{ac} = \omega_f - \omega_b$ between pump and Stocks photons do not feel the frequency chirp of incident wave $E_f(z,t)$. Hence phase-conjugated replica $E_b(z,t)$ having carrier frequency $\omega_b$ cannot be compressed after reflection from SBS PCM.

For this reason the usage of phase-conjugating mirror with instantaneous response e.g. mirror based on $\chi^{(3)}$ Kerr-like instantaneous nonlinearity [28, 29] looks attractive. The wavefront reversal via degenerate four-wave mixing (DFWM) [23] is described by nonlinear Schrödinger equation for the wide area $\chi^{(3)}$ slice:

$$\frac{\partial E_{PC}^{G_s,se}(z,t,\vec{r})}{\partial z} = \frac{i}{2k} \nabla^2 E_{PC}^{G_s,se} - \frac{i\gamma_{PC}}{2} |E_{PC}^{G_s,se}|^2 E_{PC}^{G_s,se},$$
\hspace{4cm} (16)

where $E_{PC}^{G_s,se}$ is a superposition of the four optical fields having Gaussian or sech temporal envelope:

$$E_{G_s,se}^{PC}(z,t,\vec{r}) = E_1 + E_2 + E_f + E_b, \quad (17)$$

where $E_1$ and $E_2 = E_1^\dagger$ are phase conjugated pump beams with chirped spectra, $E_f$ is fiber array output beam, corrugated by phase-piston errors $\Delta \phi_{mn}$ (Eq.12), $E_b \sim E_f^\dagger$ is phase-conjugated replica generated within DFWM PC mirror. Reflectivity of PC mirror ($R_{PCM}$) and PC fidelity ($K_{PCM}$) are given by conventional equation for the four-mixing phase-conjugation inside thin Kerr slice [28, 29] (fig.2):

$$\frac{\partial E_{G_s,se}^{PC}(z,t,\vec{r})}{\partial z} = \frac{i\gamma_{PC}}{2} E_1 E_2 f^2(\tau) E_{G_s,se}^{PC}(z,t,\vec{r}),$$
\hspace{4cm} (18)

where Kerr nonlinearity of DWFM PCM is $\gamma_{PC} = k 3\chi^{(3)}/8\pi$, $L_{PCM}$ is thickness of the Kerr slice. Temporal dependence of PCM reflectivity (fig.4) preserves the envelope form near pulse maximum and asymptotic dependencies at precursor and pulse tail. The undepleted pump reflection regime [27] with $R_{PCM} \cong 0.2$ is considered here to avoid development of Bespalov-Talanov instability [5, 6, 28]. Experimentally $E_1, E_2$ are smooth counter propagating Gaussian beams with phase matched wavefronts. The spatial structure of factorized incident $E_f$ and phase-conjugated beams $E_b$ is given by a following procedure. Let us decompose $E_f$ in Fourier plane-wave series with randomly tilted wavevectors $\vec{k}_{pq}$. For the sake of computational convenience consider the superposition of waves emitted by output fiber facets as plane waves. The period of fiber array $p \sim 100\mu m$ is taken comparable to fiber mode at output microlens $2D \sim 100\mu m$. The resulting interference pattern in Fresnel zone, i.e. at the distance $z = \sim D^2/\lambda$ from fiber array output plane is identical to field which passed through randomly corrugated phase-plate (fig.5). The 2D Fourier sum of plane waves with "global" wavevectors $\vec{K} = \vec{k} + \vec{k}_{pq}$ may be reduced to the 1D Fourier sum where each plane wave
with "local" wavevector $\vec{K} = \vec{K}_s + \vec{K}_M$ is emitted by a randomly tilted smooth area located at equivalent phase plate in near field [36]:

$$E_f(z,t,\vec{r}) \cong \exp[-i\omega t + ikz + i\theta_{Gs,Ss}(\tau)]$$

$$\cdot f_{Gs,Ss}(\tau) \sum_M a_M \cdot \exp[i\vec{K}_M \cdot \vec{r}],$$

where $\vec{K}_M$ is randomly tilted vector of partial speckle plane wave, $a_M$ is Fourier amplitude, $f_{Gs,Ss}(\tau)$ is temporal envelope. The phase-conjugated replica $E_b$ is:

$$E_b(z,t,\vec{r}) \cong \exp[-i\omega t - ikz + i\theta_{Gs,Ss}(\tau)] \cdot f_{Gs,Ss}(\tau)$$

$$\sum_M a'_M \cdot \exp[-i\vec{K}_M \cdot \vec{r}],$$

The interference pattern inside PC mirror is given by [37]:

$$I_{\text{speckle}}(z,t,\vec{r}) = |E_f(z,t,\vec{r}) + E_b(z,t,\vec{r})|^2.$$

The 3D distribution of intensity $I_{\text{speckle}}(z,t,\vec{r})$ is a random collection of pairs of helices with opposite handedness [36] (fig.5). This feature of PC mirror ensures the phase-matched propagation of the phase-conjugated replica $E_b(z,t,\vec{r})$ [34]. The remarkable feature of this phase-conjugating laser interferometer technique [15, 37] is a perfect compensation of the phase-piston errors $\Delta \phi_{mn}$ backward propagation through the amplifying array [30] (fig.5). As a result the backwardly amplified beams will be collected in beamsplitter tree (fig.1b) in a single smooth beam identical to master oscillator output. The $\pi$-shift between reconstructed chirped backward waves which is necessary for decoupling from master oscillator MO [15] may be produced by appropriate modulation inside $\lambda(t)/2$ Pockels cell (fig.1b).

The requirement of low PCM reflection $R_{PCM} \cong 0.2$ imposes an additional link between backward gain in fiber array and energy of pump beams $E_1, E_2$ in PCM. According to IZEST-ICAN project [1] the output energy between final BS and compressor (fig.1b) must be about 30 – 40 J. For the realistic nanosecond spatially smooth beams $E_1, E_2$ with energy $W_{PCM,pump} \sim 0.1 – 0.5 J$ the reflected chirped pulse energy cannot exceed $W_{PCM,back} \sim 0.02 – 0.1 J$. Consequently the backward gain in fiber network $G_{mn} = \sigma_{Yb} \int_{0}^{L_{mn}} N_0(z')dz'$ should be large enough to reach the level of 30 – 40 J. The reduction of gain $G_{mn}$ would require a more energetic pump beams $E_1, E_2$.

**IV. TEMPORAL ENVELOPE DEFORMATION DUE TO GAIN SATURATION.**

The nonlinear amplification of the light pulse in rare earth doped fiber [38, 39] modifies temporal envelopes. For incoherent amplification regime of short laser pulse [14] $T_1 \gg T_2$ in a presence of self-focusing the pulse propagation in each $(m,n)$ fiber with gain $G_{mn} = \sigma_{Yb} \int_{0}^{L_{mn}} N_0(z')dz'$ is described by nonlinear Shrodinger-Franz-Nodvik equation [3]:

$$\frac{\partial E_{f,b}(z,t,\vec{r})}{\partial z} \pm \frac{n_e}{c} \frac{\partial E_{f,b}}{\partial t} + \frac{i}{2k_p} \nabla^2 E_{f,b} =$$

$$\frac{\sigma_{Yb} N(z,t)}{2} E_{f,b} + i k n_d |E_f + E_b|^2 \cdot E_{f,b},$$

(22)

where $\sigma_{Yb} \sim 10^{-20} cm^2$ is stimulated cross section of $Yb^{3+}$ resonant transition, $T_1, T_2$ are longitudinal and transversal relaxation times. The ultimate optical flux $F_{cm}$ is limited by $\text{F}_{lim} \sim h c / 2 \sigma_{Yb}$. The dynamics of population inversion $N(z,t)$ follows to rate equation:

$$\frac{\partial N(z,t)}{\partial t} = -\sigma_{Yb} N(z,t) \cdot |E_f + E_b|^2 + \frac{N_0(z) - N(z,t)}{T_1},$$

(23)

Indeed duration $\tau_{Gs,Ss} \cong 10^{-9} sec$ of CPA pulse [6] defines the incoherent dynamics of amplification $T_2 << \tau_{Gs,Ss} << T_1$ [14]. For a geometry of laser network under consideration (fig.1a,b) the average length of fiber amplifier set $< L_{mn} >= L_{f} \sim 1 – 10$ meters exceeds spatial length of pulse $c \cdot \tau_{Gs,Ss} \leq 30$ cm by an order of magnitude.

Thus backward wave $E_b$ is amplified without interference with forward wave $E_f$:

$$\frac{\partial E_{f,b}(z,t,\vec{r})}{\partial z} \pm \frac{n_e}{c} \frac{\partial E_{f,b}(z,t)}{\partial t} =$$

$$\sigma_{Yb} N_0(z) E_{f,b}(z,t) \times$$

$$\exp[-2\sigma_{Yb} \int_{-\infty}^{t} |E_{f,b}|^2 d\tau] - i k n_2 |E_{f,b}(z,t)|^2 E_{f,b}(z,t),$$

(24)
where integration is over the whole pulse prehistory $-\infty < \tau < t$. As a result the envelope of backward pulse $E_b$ is transformed due to gain saturation in accordance with exact solution [6, 14]:

$$E_b(z,t) = \frac{E_b(L_f,t) \cdot \exp\left[-ikn_2 \int_{-\infty}^{z} |E_b(z',t)|^2 dz\right]}{\sqrt{1 - \exp[\int_{-\infty}^{z} |E_b(z',t)|^2 dz]}}$$

(25)

The different regimes of pulse transformation are possible in this incoherent case. The most interesting feature is the self-similar regime of propagation which happens for initial conditions $E_{f,b}$ when both precursor and tail have an exponential form. This initial condition corresponds to specially formed hyperbolic sech$(t - zn_0/c)/\tau_{\infty}$ pulse of nanosecond duration. In this regime the maximum of intensity shifts toward the pulse demonstrating seemingly superluminal propagation [14]. This regime is best suited for the chirp preservation. The exact temporal profile of phase-conjugated pulse for sech$^3(\tau)$ input pulse and $N_0(z) = N_0$ is given by:

$$E_{b_{sech}}(z = 0, \tau) = \frac{E_b(L_f)\text{sech}^3(\tau/\tau_{\infty}) \cdot \exp\left[-ikn_2 \int_{-\infty}^{L_f} |E_b(\tau)\text{sech}^3(\tau/\tau_{\infty})|^2 dz\right]}{\sqrt{1 - \exp[\int_{-\infty}^{L_f} |E_b(\tau)\text{sech}^3(\tau/\tau_{\infty})|^2 dz]}}$$

(26)

Despite the speed of envelope maximum reaches $V \sim (6 - 10) \cdot c$ this regime of amplification does not violate causality. When the envelope maximum reaches a certain moment at precursor, which corresponds to oscillator threshold, the rectangular front moving with the speed $c$ appears. Fig.(6 a,b) shows the deformation of $E_b(z = 0, \tau) \sim \text{sech}^3(\tau)$ temporal profile for small signal gain increments in the range $G = \sigma_{vb}N_0L_f \approx 2 - 9$. The deep saturation shifts the pulse maximum and chirp towards precursor for a several hundreds of picoseconds in a self-similar way (Fig.6c,d). The asymmetry induced by gain saturation is weak due exponential precursor. The chirp varies smoothly for tens of megahertz.

The other initial condition for $E_b(z = 0, \tau)$ corresponds to Gaussian $\exp(-\tau^2/\tau_{\infty}^2)$ nanosecond pulse. The exact temporal profile of phase-conjugated pulse for $\exp^3(-\tau^2/\tau_{\infty}^2)$ input pulse is:

$$E_{b_{gauss}}(z = 0, \tau) = \frac{E_b(L_f)\exp(-3\tau^2/\tau_{\infty}^2) \cdot \exp\left[-ikn_2 \int_{-\infty}^{L_f} |E_b(\tau)\exp(-3\tau^2/\tau_{\infty}^2)|^2 dz\right]}{\sqrt{1 - \exp[\int_{-\infty}^{L_f} |E_b(\tau)\exp(-3\tau^2/\tau_{\infty}^2)|^2 dz]}}$$

(27)

where $\text{erf}(6\tau/\tau_{\infty}) = \int_{-\infty}^{6\tau} \exp(-6\tau^2/\tau_{\infty}^2) d\tau$. The deformation of $E_{b_{gauss}}(z = 0, \tau) \sim \exp^3(-\tau^2/\tau_{\infty}^2)$ temporal profile for small signal gain increments in the range $G = \sigma_{vb}N_0L_f \approx 2 - 6$ is shown at Fig.7. The deep saturation also shifts the pulse maximum and chirp along precursor. But temporal shift is more than a nanosecond and asymmetry induced by gain saturation is much stronger. The pulse is shortened and chirp varies in a range of the hundreds of megahertz.

V. DISCUSSION. FIGURE OF MERIT FOR A PHASE-LOCKED OUTPUT.

For the time modulated carrier frequency $\omega$ the interference pattern of the each pair of the fiber amplified waves inside phase-locking Mach-Zehnder and Michelson interferometers is characterized by effectiveness of beam combination taking into account random phase-piston errors [17, 40]. Suppose that beams are perfectly overlapped at beamsplitter and phase piston error is al-
most compensated. Let us assume that beams $E_{1,2}(\omega)$ have identical spectral power $P_0(\omega)$ [31]. The remained phase-jitter $\Delta \Phi_{1,2}(\omega)$ is due to chirp deformation in amplification. The effectiveness is measured by figure of merit (FOM) which is defined as a visibility of interference pattern at beamsplitter BS for each given spectral harmonic [9,10]:

$$FOM_{12}(\omega) = \frac{P_{comb}(\omega) - P_{idle}(\omega)}{P_{comb}(\omega) + P_{idle}(\omega)}$$

$$= \frac{|E_1(\omega) + E_2(\omega) \cdot \exp(i\Delta \Phi_{1,2}(\omega))|^2}{|E_1(\omega)|^2 + |E_2(\omega)|^2},$$

(28)

where $P_{comb}(\omega)$ is a spectral component of optical flux measured in output port of beam splitter, $P_{idle}(\omega)$ is a spectral power in idle port. For the above assumption of equal spectral density:

$$FOM_{12}(\omega) = \frac{|E_1(\omega) + E_2(\omega) \cdot \exp(i\Delta \Phi_{1,2}(\omega))|^2}{|E_1(\omega)|^2 + |E_2(\omega)|^2}$$

$$\pm 2P_0(\omega) \cos[\Delta \Phi_{1,2}(\omega)] = \pm \cos[\Delta \Phi_{1,2}(\omega)].$$

(29)

The figure of merit for all spectral components is evaluated as integral over all frequencies:

$$FOM_{12} = C \cdot \int s(\omega) FOM_{12}(\omega) d\omega$$

$$= \pm C \cdot \int s(\omega) \cos[\Delta \Phi_{1,2}(\omega)] d\omega, \quad C = 1/ \int s(\omega) d\omega,$$

(30)

where $s(\omega)$ is normalised spectral intensity.

The FOM for the $N_f = 2^{N_e}$ beams coherently added to each other at all components of the binary BST (fig.1) may be evaluated as interference pattern of $2^{N_e}$ beams with fluctuating phases $\Delta \Phi_{mn}(\omega)$. When phase piston errors $\Delta \phi_{mn}$ are eliminated by precise adjustment in Mach-Zehnder scheme or phase-conjugation in Michelson scheme, the result of constructive interference at output beamsplitter B reads as:

$$E_b(z = BS, \omega) = \frac{1}{\sqrt{N_f}} \sum_{n} E_0(\omega) \exp[i\Phi_{mn}(\omega)] =$$

$$E_0(\omega) \sqrt{s(\omega)} N_f \sum_{n} \exp[i\Phi_{mn}(\omega)].$$

(31)

This gives the spectral intensity $I(\omega)$ at output as:

$$I(\omega) = E_b(\omega) E_b^*(\omega) = |E_0(\omega)|^2 s(\omega) N_f +$$

$$\sum_{n,m \neq n} \sum_{n} \cos[i\Phi_{mn}(\omega)],$$

(32)

where $\Phi_{mn}(\omega)$ is relative phase fluctuation between $m$ and $n$ fiber channel at frequency $\omega$. The figure of merit over all $\omega$ for $N_f$ channels is given by:

$$FOM = \frac{\int [I(\omega) - (N_f \cdot s(\omega) - I(\omega))] d\omega}{\int N_f \cdot s(\omega) d\omega} = \frac{C}{N_f} \times$$

$$\int s(\omega)[(2 - N_f) + \frac{2}{N_f} \sum_{n,m \neq n} \sum_{n} \cos[i\Phi_{mn}(\omega)]] d\omega,$$

(33)
where substitution of relative $FOM_{mn}$ between two channels gives:

$$FOM = \left[ \frac{2}{N_f} - 1 \right] + \frac{2}{N_f^2} \sum_{n,m \neq n}^{N_f} \sum_{n}^{N_f} \cdot FOM_{nm}. \quad (34)$$

Final evaluation may be simplified else under assumption $FOM_{nm} = FOM_{12}$ [40]:

$$FOM_{\text{large}} \equiv \left[ \frac{2}{N_f} - 1 \right] + 2\left[ 1 - \frac{1}{N_f} \right] \cdot FOM_{12}. \quad (35)$$

In the framework of above formulated model the asymptotic behavior (large $N_f$) of array network with small interchannel phase fluctuations $\Phi_{mn}(\omega)$ demonstrates the tendency to $FOM$ of a single pair of channels [40].

VI. CONCLUSIONS

In this work the theory of phase-locking configurations of the fiber amplifying network composed of $N_f$ lasers is presented. It is shown that a single pass binary $BST$ tree Mach-Zehnder combiner is sensitive to variations of $N_{var} = (N_f - 1) \ast 6 + 2 \ast (N_f - 1) \ast (2 + 2 + 1)$ degrees of freedom. The growth of $N_{var}$ is linear with respect to size of fiber array. On the other hand the double pass Michelson configuration with DFWM phase-conjugating mirror automatically adjusts backwardly reflected signal $E_b$ with fibers and $N_{BS}$ beamsplitters. The essential condition of proper PCM operation is that information about fiber network stored within PC mirror dynamical hologram [25] should be large enough to remember layout of the entrance binary $BST$ and distribution of the phase-pistons $\Delta \phi_{mn}$. This issue is tightly connected with the phase conjugation fidelity (pump / signal correlation $K_{PCM}$) whose high value ($\geq 0.9$) is experimentally compatible with low PC reflectivity $R_{PCM} \sim 0.2$.

Preliminary evaluation shows that proper energy efficiency of the Michelson configuration (fig.1b) requires weakly saturated gain of forward pulse $E_f(\tau)$ up to $\sim 100 \mu J$ with almost immediate (within $\sim 10 \text{ns}$) amplification of the backward pulse $E_b(\tau)$ from $\sim 20 \mu J$ up to $\sim 2 - 3 mJ$. These $\sim 0.8 J$ per pulse losses are small compared to heating losses in fiber CFA evaluated at the level of 10-20 percents [1]. As is shown already [41] the saturated gain for backward pulse could provide 2.2$mJ$ level output per each large mode area photonic crystal fiber amplifier. Thus smooth spatial mode $30 - 40 J$ chirped pulse output before compressor looks quite realistic.

In Michelson interferometric configuration the phase-piston errors of the each pair of beams $\Delta \phi_{mn}$ are compensated via phase-conjugating action of DFWM mirror [15]. On the contrary the Mach-Zehnder scheme is highly sensitive to variations of optical path’s and orientations of beamsplitters [42]. The optimization of the figure of merit requires the reasonable balance of gains $G_{mn}$ inside fiber amplifier’s network in order to avoid a randomization of the maxima of temporal envelopes and corresponding random displacement of frequency chirp $\delta \omega_{mn}(t)$ in a set of fiber amplifiers. The gain values in Michelson fiber network $G_{mn}$ are tightly linked with reflectivity of PC mirror and energy of DFWM PCM pump beams.

In both cases the distortions of temporal profile $E_{b_{Ga.s.s.}}(z = 0, \tau)$ seriously affect the frequency chirp. The sech envelope is less sensitive to nonlinear distortion in fiber amplifier with saturated gain because of exponential precursor. There is a definite range of fiber gain $G_{mn}$ where the pulse maximum moves towards the precursor with superluminal speed with sufficiently small displacement of the chirp towards precursor. The Gaussian temporal profile demonstrates much stronger deformation of the envelope due to saturation of the fiber gain $G_{mn}$ because Gaussian precursor decelerates pulse maximum and the envelope $E_{b_{Ga}}(z = 0, \tau)$ becomes steeper.

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