Rumor spreading model considering the influence of public opinion environment

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Abstract. Rumor spreading has a profound influence on daily lives. There are several factors influencing rumor spreading. In this paper, we assume that there are two kinds of individuals in the crowd: those who do not know the rumors and those who learn about the rumors. And in the latter category, there are two attitudes toward rumors: those who are convinced of rumors and those who hold an objective attitude. Those convinced and objective individuals form a public opinion environmental factor conducive to the spread of rumors. Based on the public opinion environmental influence whose intensity decays with time, a model with two attitudes toward rumors is established. With numerical simulations, we illustrated the conversion state of three categories because of public opinion environmental factors, analyzing the sensitivity of the model. The results showed that it is beneficial to control the rumor propagation when controlling the influence of those who are convinced of rumors and those who hold an objective attitude rather than constraining one of them individually. And it was commonsensical that increasing the attenuation intensity of the two attitude rumors was also an effective way conducive to the control.

1. Introduction

Rumor is a typical situation during social development, which plays an important role in humans’ daily life. It is often defined like this: some explanations in which the public are interested or which can arouse controversy, regardless of its true or false, are propagated in a rather short time [1]. After entering the information age, along with the rapid development of the media, the means of rumor communication has also undergone tremendous changes, and the rumors nowadays have become more influential. Because of the uncertainty inherent of rumors and the strong demand for extracting unknown information, this makes unsubstantiated false information spread everywhere. What’s worse, people may generate misleading cognition for events, which can cause panic and social unrest. Thus, it is necessary to design a model to study the mechanism and law of rumor spreading.

The research on the rumor spreading model originated in the 1960s. Because of many similarities between the spread of rumors in the interpersonal network and the spread of infectious diseases, the existing rumor propagation models are mostly based on infectious disease models. Daley and Kendal [2] conducted a comparative study of infectious diseases and rumors, summarizing the similarities and differences between rumors and infectious diseases. Bettencourt et al. [3] Considered the interaction...
between individuals and individuals and the environment to discuss interaction effects, to analyze some phenomena and problems in the real society, and study the dynamics of opinion communication. Such research aroused great interest among scholars. With the development of complex network models, Newman [4] believed that the DK model did not consider the topological characteristics of social networks and could only describe the propagation process on small-scale social networks. Furthermore, relevant scholars discussed the impact of communication mechanisms under different social network models. Complex network theory has developed rapidly in the past 10 years. The complex network model represented by the small world model [5] and the scale-free model [6] can describe the characteristics of many real networks. For example, Moreno Y et al. [7] established a rumor propagation model on a scale-free network, and compared the conclusions obtained by computer simulation and random analysis. Zhu et al. [8] established an SIR (susceptible-infected-removed) model to explore the spread of rumors on complex social networks. On the basis of SIR, the mean field theory and Monte Carlo method were used to study the propagation dynamics behavior on a uniform network, and a new SIR model was constructed by Wang et al [9]. Zhao studied the SIHR rumor propagation model with memory mechanism and forgetting mechanism on homogeneous network [10] and heterogeneous network [11], and then considered the time-varying forgetting rate when establishing the model [12]. Sun Rui et al. [13] considered the differences in the immunity of individuals in actual complex networks to rumors, and studied the rumor propagation model under non-uniform propagation rates.

As the main subject in the process of communication, the influence of the individual on the decision-making process of the rumor process cannot be neglected. Scholars proceed from the individual level and carry out further research on the process of rumor communication, such as the forgetting mechanism and rebuttal mechanism. Nekovee [14] first studied the existence of spontaneous forgetting mechanism in the process of rumor propagation, and revised the classic SIR rumor propagation model. Jaeger et al. [15] considered the factors of communicator credibility and found that rumors would be more likely to be adopted when credibility was high. Considering the influence of personal interest and refutation mechanism on the spread of rumors, Ran et al. [16] proposed the IWSR (ignorant-weakly spreader-strongly spreader-removal) rumor propagation model. Because individuals play different roles in the spread of rumors, Yang [17] designed a new state transfer function for each node based on the degree of different nodes in the network and proposed a new rumor-propagating ILSR model. Zhang Ju-Ping [18] considered the speed of forgotten rumors and establishes a SITR (susceptible-infective-true-removed) rumor propagation model. Askarizadeh et al. [19] proposed an evolutionary game model. The rumor control center sends anti-rumor messages to consider the rumor control mechanism, to study the factors affecting people's decision-making and the influence of people's decision-making on rumor propaganda and control.

In addition to individual factors, the influence of macroscopic factors cannot be ignored. Kawachi et al. [20] analyzed different ways of communication and the influence of different groups on the spread of rumors. Afassinou et al. [21] believed that people with education were more likely to prevent rumors spreading, so considering the rate of forgetting and the rate of education, the SEIR model was proposed. Wang et al. [22] considered negative and positive social reinforcement and proposed an improved CSR model for disseminating rumors in mobile social networks. As the research matured further, scholars' research on public opinion had obvious interdisciplinary characteristics. Many scholars from the fields of physics and sociology have studied the public opinion information. As a social collective behavior, rumors have similarities with the macroscopic behaviors of physical systems. And individual motivations and interactions are similar to microscopic movements in physical systems. At present, the main theory of applied physics research rumors is the phase change theory [23]. Castellano et al. [24] also used statistical physics to study and verify the theoretical results of social communication dynamics.

Generally speaking, public opinion environmental factors that the increasing number of individuals who are convinced of rumors can promote the rumor spreading. We design a rumor diffusion dynamics model for the influence of public opinion environmental factors on spread efficiency of different
attitudes to rumors. Then we verify the influence of parameter variation on rumor propagation by numerical simulation and the corresponding control strategy is proposed.

The organization of this paper is as follows. In section 2, we design a rumor spreading model which took public opinion environmental factors conducive to the spread into consideration. In section 3, we analyzed the stability of the model and numerical simulations are presented to demonstrate our results. In section 4, the conclusions are provided and we discuss some possible improvements for future study.

2. Rumor spreading model considering the public opinion environmental factor

In this section, the population considered by the model does not have a population moving in and out. According to whether different individuals get the rumors and the attitudes toward rumors after getting them, the crowd is divided into three parts: those who are convinced of rumors, $I_1$ (the convinced individual); those who hold an objective attitude toward rumors, $I_2$ (the objective individual); those who do not know rumors, $S$ (the unknown individual).

The rumor itself is accompanied by uncertainty, and due to the strong demand for unknown information, unfounded and unproven false information spreads everywhere, making the public feel serious about the incident itself. Now, the public opinion environment can play an important role in the rumor spreading process. For example, by using some theories of pseudoscience or spreading some information of predicted disaster which can cause the public scare, like wars or earthquakes, people are easy to trust the rumors, which makes rumors spread more quickly.

Here are two hypotheses about the way rumors spread in the crowd. First, only through human-to-human contact, rumors can be successfully spread, ignoring the possibility of successful communication through other media. Second, there is a certain genetic relationship in rumors spreading, which means that the process of spreading the rumor is along with the attitude of the original holder of the rumor.

After an unknown individual $S$ hears the rumor from other individuals, he/she may have two possible choices: to be transformed into $I_1$ (the convinced individual) or $I_2$ (the objective individual). Let the conversion rates be $a$ and $b$ respectively. The rumors have a certain timeliness. Over time, the attenuation rate from $I_1$ to $S$ is $u$, while the attenuation rate from $I_2$ to $S$ is $v$. There also exists a conversion rate between $I_1$ and $I_2$, because of the mutual contact of these two types of people with different attitudes to rumors, which is expressed by the conversion rate.

With the rumor spreading, the composition of the entire crowd would change, thus making the public opinion environment produce corresponding changes. This will affect the efficiency of the rumors spreading, which is considered in this paper. And it is universally believed that the increase of rumor will have an impact on the public opinion. Ordinary people are often more likely to believe a rumor when exposed in this environment. We define such promotion as a public opinion environmental factor $M$. The value is related to that of $I_1$ and $I_2$. However, public opinion environmental factors can cause a positive feedback to the rumor spreading with convinced attitude. Because $M$ is the average effect on the crowd, there is no unit. Considering the unity of the dimension, we divide $I_1, I_2$ by the total number of people $N$ to eliminate the influence of the unit. And we try to use the form of the index $e$ to eliminate the sensitivity of public opinion environmental factors $M$ caused by changes in $I_1$ and $I_2$.

The flow chart corresponding to the above assumptions and conditions is shown in Figure 1.

The meaning of each symbol in Figure 1 is shown in Table 1.

Figure 1 shows the change law of $I_1$, $I_2$, $S$ and the influence of $M$ on them.

For $I_1$, there are three variations in the number of people per unit time. First part of the variations is the addition of $aWXY / N$, when some unknown Individuals $S$ after contacting the convincing individuals $I_1$ become new convincing individuals $I_1$ with the influence of $M$. Second part is the addition of $cYZ / N$, when some objective individuals $I_2$ after contacting the convincing individuals $I_1$
become new convincing individuals $I_1$. Third part is the reduction of $uY$, because of the attenuation rate of $I_1$. So, the change of the number of the convincing individuals per unit time is:

$$\frac{dY(t)}{dt} = aWX/N - uY/N + cYZ/N.$$

Figure 1. The flow diagram of rumor spreading model.

Table 1. The notations of the model.

| Notation | Description |
|----------|-------------|
| $S$      | The unknown individuals |
| $I_1$    | The convinced individuals |
| $I_2$    | The objective individuals |
| $M$      | The public opinion environmental factor |
| $X(t)$   | The number of the unknown individuals at the time $t$ |
| $Y(t)$   | The number of the convinced individuals at the time $t$ |
| $Z(t)$   | The number of the objective individuals at the time $t$ |
| $W(t)$   | The size of the public opinion environmental factor at the time $t$ |
| $a$      | The conversion rate from $S$ to $I_1$ |
| $b$      | The conversion rate from $S$ to $I_2$ |
| $c$      | The conversion rate between $I_2$ and $I_1$ |
| $u$      | The attenuation rate from $I_1$ to $S$ |
| $v$      | The attenuation rate from $I_2$ to $S$ |
| $r_1$    | The degree of the influence of $I_1$ on the public opinion environmental factor |
| $r_2$    | The degree of the influence of $I_2$ on the public opinion environmental factor |
| $k$      | The natural attenuation rate of the public opinion environmental factor |
| $N$      | The total number of the crowd |

For $I_2$, there are also three variations in the number of people per unit time. First part of variations is the addition of $bXZ/N$, when some unknown Individuals $S$ after contacting the objective individuals $I_2$ become new objective individuals $I_2$. Second part is the reduction of $cYZ/N$, when some objective individuals $I_2$ after contacting the convincing individuals $I_1$ become new convincing individuals $I_1$. Third part is the reduction of $vZ$, because of the attenuation rate of $I_2$. So, the change of the number of the objective individuals per unit time is:

$$\frac{dZ(t)}{dt} = bXZ/N - vZ - cYZ/N.$$

Because the whole population $N$ is constant, we get $\frac{dX(t)}{dt} = -\frac{dY(t)}{dt} - \frac{dZ(t)}{dt}$. So, $\frac{dX(t)}{dt} = -aWX/N - bXZ/N + uY/N + vZ$.

Because $M$ is influenced by $I_1$ and $I_2$, we can get $\frac{dW(t)}{dt} = r_1e^{Y/N} + r_2e^{Z/N} - kW$.

So the rumor spreading model of different attitudes which involves public opinion environmental factors is established, as follows:
\[
\begin{align*}
\frac{dX(t)}{dt} &= -\frac{aWXY}{N} - \frac{bXZ}{N} + uY + vZ, \\
\frac{dY(t)}{dt} &= \frac{aWXY}{N} - uY + \frac{cYZ}{N}, \\
\frac{dZ(t)}{dt} &= \frac{bXZ}{N} - vZ - \frac{cYZ}{N}, \\
\frac{dW(t)}{dt} &= \frac{r_x e^N + r_e e^N - kW}{N}, \\
N &= X + Y + Z, \\
u &\leq v.
\end{align*}
\]

(1)

3. The results of numerical simulation

3.1. The result of model stability

In equation (1), let \( r_1 = 0.5 \), \( r_2 = 0.3 \), \( k = 0.2 \), \( u = 0.3 \), \( v = 0.4 \), \( c = 0.1 \), \( X(0) = 1000 \), \( Y(0) = 1 \), \( Z(0) = 10 \), \( W(0) = 1 \). And we choose \( a \) and \( b \) in different values, the result is shown in Figure 2.

![Figure 2](image)

**Figure 2.** The stability of \( X(t), Y(t), Z(t) \) with different \( a, b \) values (a) \( a = 0.04, b = 0.2 \) (b) \( a = 0.04, b = 0.7 \) (c) \( a = 0.1025, b = 0.8 \) (d) \( a = 0.5, b = 0.4 \)
It can be seen from Figure 2 that, in equation (1), \( X(t), Y(t) \) and \( Z(t) \) tend to be stable over time. When the conversion rate \( a \) or \( b \) changes, the stability value changes relatively. We define the stable values of \( X(t), Y(t), Z(t) \) as \( \overline{S}, \overline{T}_1, \overline{T}_2 \). Table 2 shows several specific states.

**Table 2.** The different states of \( \overline{S}, \overline{T}_1, \overline{T}_2 \) with different values of \( a, b \).

| \( a \)  | \( b \)  | State of stability | Figure       |
|------|------|----------------|--------------|
| 0.04 | 0.2  | \( \overline{S} > 0, \overline{T}_1 = 0, \overline{T}_2 = 0 \) | Figure 2(a) |
| 0.04 | 0.7  | \( \overline{S} > 0, \overline{T}_1 = 0, \overline{T}_2 > 0 \) | Figure 2(b) |
| 0.1025 | 0.8 | \( \overline{S} > 0, \overline{T}_1 > 0, \overline{T}_2 > 0 \) | Figure 2(c) |
| 0.5  | 0.4  | \( \overline{S} > 0, \overline{T}_1 > 0, \overline{T}_2 = 0 \) | Figure 2(d) |

As can be shown in Table 2, for different values, the model tends to be stable over time. But the structure of the crowd after stabilization is different. Let's explore the variation of \( a \) and \( b \) in each component group.

### 3.2. Results without considering the public opinion environmental factor

First, let \( r_1 = 0, r_2 = 0, k = 0 \), which means \( \overline{M} = W(t) = W(0) = 1 \). From the equation (1), it is known that the public opinion environmental factor is not effective for the rumor spreading process at this time. The other parameters are taken that \( u = 0.3, v = 0.4, c = 0.1 \), and the initial values are taken that \( X(0) = 1000, Y(0) = 1, Y(0) = 10, W(0) = 1 \). Let \( a, b \) change from 0 to 1 with step 0.01. The corresponding results of \( \overline{S}, \overline{T}_1, \overline{T}_2 \) are shown in Figure 3(a), and the coexistence states of \( \overline{S}, \overline{T}_1, \overline{T}_2 \) are shown in Figure 3(b).

**Figure 3.** (a) \( \overline{S}, \overline{T}_1, \overline{T}_2 \) as the function of \( a, b \) (b) the coexistence state of \( \overline{S}, \overline{T}_1, \overline{T}_2 \) with respect to \( a, b \), while \( \overline{S} > 0, \overline{T}_1 = 0, \overline{T}_2 = 0 \) in zone I, \( \overline{S} > 0, \overline{T}_1 = 0, \overline{T}_2 > 0 \) in zone II, \( \overline{S} > 0, \overline{T}_1 > 0, \overline{T}_2 > 0 \) in zone III, \( \overline{S} > 0, \overline{T}_1 > 0, \overline{T}_2 = 0 \) in zone IV.

Figure 3 describes when \( [a, b] \in [0, 0.3] \times [0, 0.4] \), \( \overline{S} > 0, \overline{T}_1 = 0, \overline{T}_2 = 0 \), which means only unknown individuals exist in the crowd. We can see that the rumor propagation fails, because the attenuation rate of those who have attitudes to the rumor originally including \( I_1 \) and \( I_2 \) is greater than the conversion rate of rumor propagation. It is enough to offset the propagation intensity of rumors so that the rumor cannot spread successfully.
When $[a, b] \in [0, 0.3] \times [0.4, 1]$, the conversion rate $a$ is smaller than the attenuation rate $u$ while the conversion rate $b$ is greater than the attenuation rate $v$. As a result, $I_2$ appears. And there are only two types of people ($S$ and $I_1$), which means the rumor of the objective attitude still exists in the system.

In the same way, when $[a, b] \in [0.3, 1] \times [0, 0.4]$, the system presents the coexistence state of $S$ and $I_1$, and there are only two types of people who do not know the rumors and are convinced in the rumors in the system, and firmly believe that the attitude rumors do not disapper in the system, which means the rumor of the convinced attitude are existing in the system.

When $[a, b] \in [0.3, 1] \times [0.4, 1]$, $a > u$ and $b > v$, so that two types of the rumors in the system can coexist. It is worth noting that there will be three components existing in the system although the coexistence conditions are rather severe, which can be seen from the small yellow area in the Figure 3(b). In conclusion, increasing the attenuation rate of rumors and reducing the conversion rate can make rumors difficult to spread. So, we might control rumors by increasing its decay rate.

3.3. Results with considering the public opinion environmental factor

Let $r_1 = 0.5, r_2 = 0.2, k = 0.2, u = 0.3, v = 0.4, c = 0.1$. and the initial values are taken that $X(0) = 1000, Y(0) = 1, Z(0) = 10, W(0) = 1$. Let $a, b$ from 0 to 1 with step 0.01. The changes of $S, I_1, I_2$ with respect to $a, b$ are shown in Figure 4(a), and the coexistence state of $S, I_1, I_2$ with respect to $a, b$ are shown in Figure 4(b).

![Figure 4](image)

(a) $S, I_1, I_2$ as the function of $a, b$ (b) the coexistence state of $S, I_1, I_2$ with respect to $a, b$, while $S > 0, I_1 > 0, I_2 > 0$ in zone I, $S > 0, I_1 = 0, I_2 > 0$ in zone II, $S > 0, I_1 > 0, I_2 = 0$ in zone III, $S > 0, I_1 > 0, I_2 = 0$ in zone IV.

It can be seen from Figure 4 that there are four coexistence states in the system under this set of parameters. But due to the effect of the public opinion environmental factor, people who are convinced of rumors can only exist in a case where the conversion rate $a$ is much smaller. Therefore, in the system, the possibility of the coexistence state of $S$ and $I_1$ is greatly increased, and the state where $S$, $I_1$ and $I_2$ coexist is less likely to occur.

Since the public opinion environmental factor is related to the parameters $r_1$, $r_2$ and $k$, the effects of these three parameters are considered below.

Let $a = 0.1, b = 0.6, k = 0.2, u = 0.3, v = 0.4, c = 0.1$, and the initial values are taken that $X(0) = 1000, Y(0) = 1, Z(0) = 10, W(0) = 1$. Let $r_1, r_2$ range from 0 to 1 with step 0.01. The changes of

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\( \bar{S}, \bar{T}_1, \bar{T}_2 \) are shown in Figure 5(a), and the coexistence state of \( \bar{S}, \bar{T}_1, \bar{T}_2 \) with respect to \( r_1, r_2 \) are shown in Figure 5(b).

![Figure 5](image)

Figure 5. (a) \( \bar{S}, \bar{T}_1, \bar{T}_2 \) as the function of \( r_1, r_2 \) (b) the coexistence state of \( \bar{S}, \bar{T}_1, \bar{T}_2 \) with respect to \( r_1, r_2 \), while \( \bar{S} > 0, \bar{T}_1 = 0, \bar{T}_2 > 0 \) in zone I, \( \bar{S} > 0, \bar{T}_1 > 0, \bar{T}_2 > 0 \) in zone II, \( \bar{S} > 0, \bar{T}_1 > 0, \bar{T}_2 = 0 \) in zone III.

It can be seen from Figure 5 that under this set of parameters, the system has three coexistence states. The effect of public opinion environmental factor always exists in the system and it can be known from equation (1) that the effects of the two types of rumors on the public opinion environmental factor determine the states of the system. When the value of \( r_1 + r_2 \) is relatively small, \( S \) and \( I_2 \) coexist in the system, that is, there is no individual who is convinced in the rumor in the system. But no matter how small \( r_1 \) is, increasing \( r_2 \) can make individuals who are convinced in the rumor appear; no matter how small \( r_2 \) is, increasing \( r_1 \) can make individuals who are convinced in the rumor appear. Therefore, to eliminate those who are convinced in rumors, the sum of the effects of \( I_1 \) and \( I_2 \) on the public opinion environmental factor needs to be less than a certain threshold. Meanwhile, increasing \( r_1 \) or \( r_2 \) will increase the impact of \( I_1 \) and \( I_2 \) on the public opinion environmental factor, which will re-emerge those who are convinced in the rumor.

The effect of \( k \) is discussed below.

Let \( a = 0.1, r_1 = 0.5, r_2 = 0.2, u = 0.3, v = 0.4, c = 0.1, S(0) = 1000, I_1(0) = 1, I_2(0) = 10, M(0) = 1 \). Take \( b = 0.3, b = 0.6 \) respectively. Let \( k \) range from 0 to 1 with step 0.01. The changes of \( \bar{S}, \bar{T}_1, \bar{T}_2 \) with respect to \( k \) are shown in Figure 6.

It can be seen from Figure 6 that with the increase of the natural attenuation rate of public opinion environmental factors \( k \), the number of individuals who do not know the rumors in the system increases while individuals who are convinced in rumors decrease. And when \( k \) rises, the number of each type of people in the system tends to be stable. In Figure 6(a), with the growth of \( k \), \( I_1 \) decreases to 0, \( I_2 \) does not appear, and there finally only people who do not know the rumors exist in the system. In Figure 6(b), as \( k \) increases, \( I_1 \) suddenly decreases to 0 after decreasing to a certain positive value, but the number of \( S \) does not change. At the same time, those decreasing \( I_1 \) individuals show the \( I_2 \) state, that is, \( I_1 \) suddenly disappears and \( I_2 \) suddenly appears with an unchangeable total quantity, which is an interesting phenomenon. In short, increasing the natural attenuation rate of public opinion environmental factors is conducive to reducing the number of people who are convinced in the rumor, but too much emphasis on increasing the natural attenuation rate is ineffective.
Figure 6. $\overline{S}, \overline{I}_1, \overline{I}_2$ as the function of $k$, (a) $a = 0.1, b = 0.3$ (b) $a = 0.1, b = 0.6$.

Figure 7. $\overline{S}, \overline{I}_1, \overline{I}_2$ as the function of $c$, (a) $a = 0.05, b = 0.2$ (b) $a = 0.05, b = 0.7$ (c) $a = 0.5, b = 0.2$ (d) $a = 0.1, b = 0.6$. 
The effect of $c$ is below.

Let $r_1 = 0.5$, $r_2 = 0.2$, $k = 0.2$, $u = 0.3$, $v = 0.4$, $S(0) = 1000$, $I_1(0) = 1$, $I_2(0) = 10$, $M(0) = 1$. And we choose $a$ and $b$ in different values. Let $c$ range from 0 to 1 with step 0.01. The changes of $S$, $I_1$, $I_2$ with respect to $k$ are shown in Figure 7.

It can be seen from Figure 7 that with the increase of the conversion rate $c$, the number of $S$, $I_1$, $I_2$ does not change. So we choose $c = 0.1$ as the initial parameter, which would not cause some errors to the simulation results.

The effect of $u$, $v$ is below.

Let $r_1 = 0.5$, $r_2 = 0.2$, $k = 0.2$, $c = 0.1$, $S(0) = 1000$, $I_1(0) = 1$, $I_2(0) = 10$, $M(0) = 1$. And we choose $u$ and $v$ in different values. Let $a$, $b$ change from 0 to 1 with step 0.01. The coexistence states of $S$, $I_1$, $I_2$ are shown in Figure 8.

**Figure 8.** The coexistence state of $S$, $I_1$, $I_2$ with respect to $a$, $b$ (a) $u = 0.2$, $v = 0.4$ (b) $u = 0.2$, $v = 0.8$ (c) $u = 0.4$, $v = 0.6$ (d) $u = 0.6$, $v = 0.8$.

It can be seen from Figure 8 that $u$, $v$ can change the size of each zone in different directions. The changes of $u$, $v$ would not lead to the changes of the coexistence states. In order to show the zone III more obvious, we set the initial values of parameters $u = 0.3$, $v = 0.4$. 
4. Conclusions

In this paper, a rumor spreading dynamics model was studied which considered the propagation with two different attitudes in the closed crowd, which was influenced by the public opinion environmental factor. The effects of the public opinion environmental factor and various parameter can be verified by numerical simulation.

Through the research of this paper, we have drawn the following conclusions:

1. The rumors of different attitudes have different propagation efficiency and attenuation rates in the process of rumor spreading. As time goes by, the final stable results are different. Therefore, when studying the issue of rumor propagation, it is necessary to consider the original individuals’ attitude toward the rumors.

2. It is widely believed that the success of rumors propagation can be related to the number of initial rumors. However, it is surprising to find, through numerical simulations, that the success of propagation is independent of the initial number of rumors, but related to the propagation efficiency of rumors and the attenuation rate of rumors. Therefore, we can control the spread of rumors by affecting the propagation efficiency and attenuation rate of rumors.

3. Public opinion environmental factors can significantly promote the spread of the convinced rumors, which makes people more inclined to believe in the rumors. Therefore, we can also control the public opinion environmental factors by affecting the influence of convincing and objective individuals on the public opinion environmental factor.

4. It is worth noting that the rumors of different attitudes have a competitive relationship, and it is very difficult to make them coexist. Therefore, this enlighten that it is possible for us to regulate the existence of various rumors of different attitudes in the crowd to achieve the expected state.

For future research, we will concentrate on three possible extensions.

First of all, the attitudes mentioned in this paper only have two attitudes: convinced and objective. However, in the real world, the degree of belief in rumors obeys a probability distribution. It is the same with the transmission efficiency. And then we can establish mathematical models with different degree of belief based on different individuals.

Second, the degree of influence of public opinion environmental factors is determined by not only the number of rumors in the population, but also the educational background of the population, the degree of development of the media, and the government's control. Therefore, we can modify and improve the model by collecting the data from the specific population background.

Finally, in this paper, the model ignores the attenuation of information between individuals due to distance as a result of adopting the average field method. We will try to contribute a more precise model to investigate rumor propagation through the network structure.

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