Origin of $10^{15} - 10^{16}$ G Magnetic Fields in the Central Engine of Gamma Ray Bursts

Rafael S. de Souza

IAG, Universidade de São Paulo, Rua do Matão 1226, Cidade Universitária, CEP 05508-900, São Paulo, SP, Brazil
E-mail: rafael@astro.iag.usp.br

Reuven Opher

IAG, Universidade de São Paulo, Rua do Matão 1226, Cidade Universitária, CEP 05508-900, São Paulo, SP, Brazil
E-mail: opher@astro.iag.usp.br
Abstract: Various authors have suggested that the gamma-ray burst (GRB) central engine is a rapidly rotating, strongly magnetized, ($\sim 10^{15} - 10^{16}$ G) compact object. The strong magnetic field can accelerate and collimate the relativistic flow and the rotation of the compact object can be the energy source of the GRB. The major problem in this scenario is the difficulty of finding an astrophysical mechanism for obtaining such intense fields. Whereas, in principle, a neutron star could maintain such strong fields, it is difficult to justify a scenario for their creation. If the compact object is a black hole, the problem is more difficult since, according to general relativity it has “no hair” (i.e., no magnetic field). Schuster, Blackett, Pauli, and others have suggested that a rotating neutral body can create a magnetic field by non-minimal gravitational-electromagnetic coupling (NMGEC). The Schuster-Blackett form of NMGEC was obtained from the Mikhail and Wanas’s tetrad theory of gravitation (MW). We call the general theory NMGEC-MW. We investigate here the possible origin of the intense magnetic fields $\sim 10^{15} - 10^{16}$ G in GRBs by NMGEC-MW. Whereas these fields are difficult to explain astrophysically, we find that they are easily explained by NMGEC-MW. It not only explains the origin of the $\sim 10^{15} - 10^{16}$G fields when the compact object is a neutron star, but also when it is a black hole.

Keywords: modified gravity, magnetic fields, gamma ray bursts theory.
1. Introduction

Gamma-ray bursts (GRBs) are short and intense pulses of soft $\gamma$ rays. The bursts last from a fraction of a second to several hundred seconds. Magnetic fields in GRBs are believed to play a crucial role in the internal engine as a possible way to power and collimate the relativistic outflow \cite{1, 2}. Energy considerations require extremely large magnetic fields on the order of $10^{15}$ G \cite{3, 4, 2}. The relativistic outflow is a Poynting flux (with negligible baryon content) \cite{3}.

Usov \cite{4} suggested that GRBs arise during the formation of rapidly rotating highly magnetized neutron stars. Very strong magnetic fields could form and the pulsar could produce a relativistic Poynting flux flow. Various authors have investigated this magnetar-GRB connection \cite{5, 6, 7, 8, 9, 10, 11, 12, 13, 14}.

Thompson \cite{15} reviewed the arguments for proto-magnetars as possible GRB engines. Long duration GRBs ($\sim$ several hundred seconds) are, in general, attributed to core-collapse supernova. A core-collapse supernova leaves behind a hot proto-neutron star that cools on the Kelvin-Helmholtz time scale $\tau_{KH}$ ($\sim 10 - 100$s) by radiating neutrinos. Assuming that neutrino absorption on free nucleons dominates heating, a characteristic...
thermal pressure is $\sim 3 \times 10^{28} \text{ergs cm}^{-3}$. Setting $B^2/8\pi$ equal to this pressure requires $B \sim 10^{15}$ G. Thus if $B$ is involved in collimating and accelerating the relativistic jet, a field $B \gtrsim 10^{15}$ G is necessary. Like beads on a wire, the magnetic field lines force the plasma around the proto-magnetar into co-rotation with the stellar surface out to the Alfven surface, where the magnetic energy density equals the kinetic energy density of the outflow. A neutron star with a millisecond spin period has a reservoir of rotational energy of $\sim 2 \times 10^{52} P^{-2} \text{ergs}$, where $P$ is the period of rotation of the neutron star in milliseconds. Thus, the rotational energy of the proto-magnetar can be tapped and could produce a GRB if its rotational period is $\sim$ milliseconds.

In summary, the reasons for considering millisecond proto-magnetars as the central engine of long-duration GRB are:

1. the reservoir of rotational energy is in the range required to power GRBs;

2. the characteristic time of the long-duration GRBs is on the order of the Kelvin-Helmholtz cooling time, $\tau_{KH}$;

3. there is an observational connection between core collapse supernovae and long duration GRBs; and

4. the magnetic field of a magnetar $\gtrsim 10^{15}$ G is strong enough to accelerate and collimate the outflow.

The major problem of the magnetar model is the origin of the $\sim 10^{15} - 10^{16}$ G fields. As noted by Spruit [16], inheritance of the magnetic field from a main sequence progenitor and dynamo action at some stage of the progenitor is not sufficient to explain the fields of magnetars. A runaway phase of exponential growth would be needed to achieve sufficient field amplification. The magnetorotational instability is a possibility, but the magnetic field would probably decay rapidly by magnetic instabilities.

In a core collapse supernova, a black hole can be formed instead of a neutron star. The justification of the presence of strong magnetic fields in that case is more difficult since, according to general relativity, black holes have no magnetic fields.
The Schuster-Blackett (S-B) conjecture \cite{17, 18} suggests a gravitational origin of the magnetic fields in rotating neutral bodies, i.e., a non-minimal gravitational-electromagnetic coupling (NMGEC). An early attempt to make a theory that encompasses the S-B conjecture was made by Pauli \cite{19}. Latter, attempts were made by Bennet et al. \cite{20}, Papapetrou \cite{21}, Luchack \cite{22}, and Barut \cite{23}. The majority of these studies were based on the five-dimensional Kaluza-Klein formalism. This formalism was used in order to describe a unified theory of gravitation and electromagnetism in such a way so that the S-B conjecture is obtained. Opher and Wchoski \cite{24} applied the S-B conjecture for the origin of magnetic fields in rotating galaxies and proposed that the $B \sim 10^{-6} - 10^{-5}$ G magnetic fields in spiral galaxies are directly obtained from NMGEC. Mikhail et al. \cite{25} showed that Mikhail and Wanas tetrad theory of gravitation (MW) \cite{26} predicts the S-B conjecture of NMGEC. We call this the NMGEC-MW theory.

We investigate here the possibility that NMGEC-MW is the origin of the intense magnetic fields $10^{15} - 10^{16}$ G, connected with GRBs produced by rotating neutron stars or black holes. In section 2, we discuss NMGEC and in section 3, NMGEC-MW. In section 4, we apply NMGEC-MW to GRBs. Our conclusions and discussion are presented in section 5.

2. NMGEC

The Schuster-Blackett conjecture relates the angular momentum $L$ to the magnetic dipole moment $m$:

$$m = \left[ \beta \frac{\sqrt{G_N}}{2c} \right] L, \quad (2.1)$$

where $\beta$ is approximately a constant, on the order of unity \cite{24}, $G_N$ the Newtonian constant of gravitation, and $c$ is the speed of light. The angular momentum $L$ is

$$L = I \Omega, \quad (2.2)$$

where $\Omega = 2\pi P^{-1}$ is the angular velocity, $P$ the rotational period, and $I$ is the moment of inertia. The dipole moment $m$ is related to the magnetic field $B$ by
\[
B = \frac{3(m \cdot r)r - m|r|^2}{|r|^5},
\]  

(2.3)

where \( r \) is the distance from \( m \) to the point at which \( B \) is measured.

The observations and experiments supporting the S-B conjecture include the early work of Blackett \[18\] and Wilson \[27\]. Observational evidence for the S-B conjecture was compiled by Sirag \[28\], who compared the predictions of Eq. (2.1) to the observed values of the ratio of the magnetic moment to the angular momentum for the Earth, Sun, the star 78 Vir, the Moon, Mercury, Venus, Jupiter, Saturn, and the neutron star Her X-1. The value of \( \beta \) for all of these objects was \( \sim 0.1 \) to within a factor of ten.

3. NMGEC-MW

A possibility for obtaining the Schuster-Blackett relation from first principles was proposed by Mikhail \textit{et al.} \[25\]. They used one of the solution of the generalized tetrad field theory of Mikhail and Wanas \[26\]. The solution used is of the type FIGI \[29\] which can represent a generalized non-zero electromagnetic and gravitational field outside a neutral rotating spherically symmetric body \[30\]. Two main reasons why MW used the tetrad space for the formalism of the generalized field theory are:

1. It is based on 16 independent field variables instead of the 10 variables \( g_{\mu\nu} \) of the ordinary Riemannian space; and

2. It has 11 independent tensors in space of the two symmetric tensors \( g_{\mu\nu} \) and \( R_{\mu\nu} \) of ordinary Riemannian space.

Mikhail \textit{et al.} \[25\] evaluated the field equations, assuming a symmetric solution for the Reissner-Nordström metric,

\[
ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -f dt^2 + \frac{dr^2}{f} + r^2(d\theta^2 + \sin^2\theta d\phi^2),
\]

\[
f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2},
\]

(3.1)
where the $x^\mu$ are spatial-temporal coordinates, $\mu, \nu = 0, 1, 2, 3$, and $M$ and $Q$ are the mass and charge of the object, respectively. Mikhail et al. evaluated the non-vanishing components of the electromagnetic field tensor $F^{\mu\nu}$. They studied effects of rotation. Taking the electric charge to be zero, they found an extra term in the electromagnetic tensor, corresponding to a magnetic field. Thus, magnetic fields are generated as a result of the rotation of the body, as in the S-B conjecture. The surface magnetic field for a rotating body of mass $M$ (grams), radius $R$ (cm), and angular velocity $\Omega$ ($sec^{-1}$) is

$$B_p = \frac{9}{4} \sqrt{\frac{2M}{R}} G_N \Omega G.$$  

(3.2)

Comparing Eq. (3.2) with Eq. (2.1), we have

$$B_p = \frac{4\beta G^{1/2}_N}{5Rc} M \Omega G$$  

(3.3)

and obtain

$$\beta = \frac{45c}{8G_N^{1/2} \phi^{1/2}}.$$  

(3.4)

where $\phi = 2M/R$. In units of solar mass ($M_\odot$) and solar radius ($R_\odot$), we have

$$\beta \approx 2730 \left[ \frac{R_\odot}{R_\odot} \frac{M_\odot}{M} \right]^{1/2}.$$  

(3.5)

For a black hole, $M/R$ is approximately a constant and $\beta \sim 5$. For a 2 solar mass neutron star, $\beta \sim 10$.

4. Central Engine of the Gamma-Ray Burst

We first assume that the central rotating compact object is a black hole. At the end of the section, NMGEC-MW is applied to a rotating proto-magnetar.

The maximum amount of energy which can be extracted from a black hole is the rotational energy, $E_{rot}$

$$E_{rot} = Mc^2 - M_{irr}c^2,$$  

(4.1)
where $M_{\text{irr}}$ is the irreducible mass of the black hole,

$$M_{\text{irr}} = \sqrt{\frac{S_{\text{BN}}}{4\pi k_B}} M_{\text{Planck}}.$$  \hfill (4.2)

$S_{\text{BN}} = A_{BH} k_B c^3/4G_N h$ is the entropy, $A_{BH}$ the surface area, and $M_{\text{Planck}} = \sqrt{\hbar / G_N}$ the Planck mass. The rotational energy of a black hole with angular momentum $J$ is a fraction of the black hole mass $M$,

$$E_{\text{rot}} = f(\alpha) M c^2,$$  \hfill (4.3)

$$f(\alpha) = 1 - \sqrt{\frac{1}{2} \left[ 1 + \sqrt{1 - \alpha^2} \right]},$$  \hfill (4.4)

where $\alpha = Jc/M^2 G_N$ is the rotation parameter. For a maximally rotating black hole ($\alpha = 1$), $f = 0.294$.

From Eq. (3.3), the magnetic field at the black hole horizon is

$$B_p = \frac{9}{4} M \sqrt{\frac{5G_N \alpha}{R^3 c}} \approx 8.13 \times 10^8 \left( \frac{M}{M_\odot} \right)^{1/2} \left( \frac{R}{R_\odot} \right)^{-3/2} G.$$  \hfill (4.5)

In order to evaluate Eq. (4.5), we use characteristic parameters for the black hole: $\alpha \sim 0.1 - 1$ and $M \sim 2.5 \ M_\odot$ \cite{2}. The horizon radius for a rotating Kerr black hole is

$$r_{\text{BN}} = \frac{r_{\text{Sh}}}{2} \left[ 1 + \sqrt{1 - \alpha^2} \right],$$  \hfill (4.6)

where $r_{\text{Sh}} = 2G_N M/c^2$ is the Schwarzschild radius.

The prediction for magnetic fields in GRBs, using Eq. (4.5), is $B_p \sim 10^{15} - 10^{16}$ G. We conclude that the NMGEC-MW theory predicts the required magnetic fields in rotating black holes. For a rapidly rotating proto-magnetar, we obtain from Eqs. (2.1)-(2.3) and (3.5)

$$B_p = 5.4 \times 10^{13} \beta P^{-1} G,$$  \hfill (4.7)

where $\beta \sim 10$ and $P$ is the period in seconds. For $P \ll 1$ s, we obtain $B_p \gtrsim 10^{15}$ G.
5. Conclusions and Discussion

Magnetic fields $\simeq 10^{15} - 10^{16}$ G have been suggested to be connected with central engines of GRBs. We considered the possibility that GRBs are powered by a rapidly rotating highly magnetized central compact object, a black hole or a neutron star. Since these magnetic fields are difficult to be obtained by astrophysical mechanisms, we considered the possibility that the fields are created by NMGEC-MW.

For GRBs a magnetic field $\sim 10^{15} - 10^{16}$ G is required to produce the Poynting flux needed to supply the energy observed within the required time. The fields predicted by NMGEC-MW in Eq. (4.3) for a black hole and Eq. (4.7) for a neutron star, are in agreement with models requiring central engines with $\gtrsim 10^{15}$ G fields. We conclude that NMGEC-MW is a possible mechanism for creating $\gtrsim 10^{15}$ G fields in the central engine of GRBs due to a rotating neutron star or black hole.

From the results of our paper, one might believe that stellar mass black holes lose the rotation generated in their formation very quickly due to magnetic dipole emission from the strong magnetic field generated by NMGEC-MW. We might then conclude that only the youngest stellar black holes observed have non-zero rotation. However, stellar mass black holes are only observed in accreting binaries. Accretion from the stellar companion makes the black hole visible in X-rays. The accretion is able to spin-up the black holes. Thus, old black holes, as well as young black holes, can have non-zero rotation.
Acknowledgments

R.S.S. thanks the Brazilian agency FAPESP for financial support (04/05961-0). R.O. thanks FAPESP (06/56213-9) and the Brazilian agency CNPq (300414/82-0) for partial support. We would like to thanks an unidentified referee for helpful comments.

References

[1] Piran T., *AIPC* **784** (2005) 164.

[2] Lee, H. K.; Wijers, R. A. M. J. and Brown, G. E., *Phys. Rept.* **325** (2000) 83.

[3] Piran, T., *Rev. Mod. Phys.* **76** (2005) 1143.

[4] Usov, V. V., *Nature* **357** (1992) 472.

[5] Thompson, C., *Mon. Not. R. Astron. Soc.* **270** (1994) 480.

[6] Katz, J. I., *Astrophys. J.* **490** (1997) 633.

[7] Kluźniak W., and Ruderman M., *Astrophys. J.* **508** (1998) 113.

[8] Wheeler, J. C., Yi, I., Höfflich, P., Wang, L., *Astrophys. J.* **537** (2000) 810.

[9] Thompson, T. A., Chang, P., Quataert, E., *Astrophys. J.* **611** (2004) 380.

[10] Metzger, B. D., Thompson, T. A., Quataert, E. *Astrophys. J.* **659** (2007) 561.

[11] Bucciantini, N., Thompson, T. A., Arons, J., Quataert, E., and Del Zanna, L., *Mon. Not. R. Astron. Soc.* **368** (2006) 171.

[12] Komissarov, S. S., and Barkov, M. V., *Mon. Not. R. Astron. Soc.* **382** (2007) 1029.

[13] Bucciantini, N., Quataert, E., Arons, J., Metzger, B. D., Thompson, T. A., *Mon. Not. R. Astron. Soc.* **380** (2007) 1541.

[14] Yu, Y. W., and Dai, Z. G., *Astron. Astrophys.* **470** (2007) 119.

[15] Thompson, T. A., *Rev. Mex. Astron. Astrophys.* **27** (2007) 80.

[16] Spruit, H. C., *AIPC* **983** (2008) 391.

[17] Schuster, A., *Proc. Phys. Soc. London.* **24** (1911) 121.

[18] Blackett, P. M. S., *Nature* **159** (1947) 658.
[19] Pauli, W., *Ann. Phys.* (Leipzig) **18** (1933) 305.

[20] Bennet *et al.*, *Proc. R. Soc. London A* **198** (1949) 39.

[21] Papapetrou, A., *Philos. Mag.* **41** (1950) 399.

[22] Luchak, G., *Can. J. Phys.* **29** (1952) 470.

[23] Barut, A. O. and Gornitz, T., *Found. Phys.* **15** (1985) 433.

[24] Opher, R. and Wichoski, U. F., *Phys. Rev. Lett.* **78** (1997) 787.

[25] Mikhail, F. I., Wanas, M. I. and Eid, A. M., *Ap&SS* **228** (1995) 221.

[26] Mikhail, F. I. and Wanas, M. I., *Proc. R. Soc. London. A* **356** (1977) 471.

[27] Wilson, H. A., *Proc. R. Soc. London A* **104** (1923) 451.

[28] Sirag, S. P., *Nature* **278** (1979) 535.

[29] Mikhail, F. I. and Wanas, M. I., *Int. J. Theor. Phys.* **20**(1981) 671.

[30] Wanas, M. I., *Int. J. Theor. Phys.* **24**(1985) 638.