On the canonical quantization of anomalous SU(N) chiral Yang-Mills models

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1 Introduction

The chiral gauge models are known to suffer from anomalies leading to the inconsistency of the quantum theory [1, 2, 3, 4, 5, 6, 7]. This inconsistency may be avoided by modification of the classical action consisting in adding to it the Wess-Zumino (WZ) action modeling the anomaly [4, 5, 6, 8].

If the symplectic form of the modified action is nondegenerate, we have a well-defined action in which anomalous contribution of old fields is canceled with the similar one of new WZ fields. As a result, one has a unitary gauge theory on the physical subspace. However, in many interesting cases the symplectic form is degenerate and one must deal with Dirac machinery for quantization of constrained systems. It is important to know whether this machinery preserves the gauge invariance of the theory or not. Due to the fact that WZ action is the first order one in its fields the symplectic form must be degenerate at least for all odd-dimensional groups such as $SU(2k)$, $dim SU(2k) = 4k^2 - 1$. The particular case of two dimensional $SU(2)$ and four dimensional $SU(3)/SO(3)$ models was considered in [14, 15]. In this paper we consider the general case of SU(N) group with degenerate symplectic form in four space-time dimensions.

The plan of paper is as follows. In the next section we briefly review the method proposed by Faddeev and Shatashvili for quantization of anomalous Yang-Mills theory. In the third section, we describe its generalization for degenerate WZ actions and show that this generalization leads to gauge invariant quantum theory.

2 Anomalous SU(N) Yang-Mills model

Consider the four dimensional Yang-Mills model described by the following classical action:

$$ S_{cl} = \int d^4x \left( -\frac{1}{4} (F_{\mu\nu}^a)^2 + i \bar{\psi} \hat{\nabla} (A) \psi \right), $$

(1)
where \( \hat{\nabla}(A) = \gamma^\mu (\partial_\mu + A_\mu) \) and \( \psi \equiv \frac{1}{2}(1 + \gamma_5)\psi \) is chiral fermion field in the fundamental representation of the gauge group generated by \( \lambda^a \) satisfying

\[
[\lambda^a, \lambda^b] = f^{abc} \lambda^c, \quad \frac{1}{2} \text{tr} \lambda^a \lambda^b = -\delta^{ab}.
\]  

(2)

Due to the gauge invariance the action (1) have a set of first class constraints \( G^a(x) \) with the following Poisson bracket algebra

\[
\{G^a(x), G^b(y)\} = f^{abc} G^c(x) \delta(x - y)
\]  

(3)

\[
\{H, G^a(x)\}|_{G=0} = 0
\]  

(4)

This enables one to impose a gauge fixing condition. Let us use the temporal gauge \( (A_0 = 0) \). In this case one has constraints \( G \) dropped out from the classical action which now become a nondegenerate one. This action can be quantized in a usual way. The loosed Lorentz invariance will be restored on the physical subspace of the Hilbert space consisting of vectors satisfying

\[
\hat{G}|_{ph} = 0
\]  

(5)

Writing (5) one tacitly suppose that the (quantum) algebra of operators \( G \) will be identical to (3) and (4) with substitution \( \{,\} \rightarrow i[,,] \). Unfortunately that is not the case for anomalous theories. Quantum corrections destroy the gauge invariance in such theories.

Indeed the fermionic determinant

\[
\det \hat{\nabla}(A) = \int e^{iS_{cl}(A, \bar{\psi}, \psi)} d\bar{\psi} d\psi
\]  

is not gauge invariant

\[
\frac{\det \hat{\nabla}(A^g)}{\det \hat{\nabla}(A)} = e^{i\alpha_1(A, g)},
\]  

(7)

and

\[
\alpha_1(A, g) = \int d^4x \left[ d^{-1} \kappa(g) - \frac{i}{48\pi^2} \epsilon^{\mu \nu \lambda \sigma} \text{tr} [(A_\mu \partial_\nu A_\lambda + \partial_\mu A_\nu A_\lambda + A_\mu A_\nu A_\lambda)g_\sigma - \frac{1}{2} A_\mu g_\nu A_\lambda g_\sigma - A_\mu g_\nu g_\lambda g_\sigma] \right]
\]  

(8)

we use the notations

\[
\int d^4x d^{-1} \kappa(g) \equiv -\frac{i}{240\pi^2} \int_{M_5} d^5x \epsilon^{pqrs t} \text{tr} (g_p g_q g_r g_s g_t)
\]  

(9)

\[
g_\mu = \partial_\mu g g^{-1}.
\]  

(10)

In eq. (9) the integration goes over a five-dimensional manifold whose boundary is the usual four-dimensional space.

The particular form of local (mod 2\pi) functional \( \alpha_1(A, g) \) depends essentially on the computation scheme (regularization) used for calculation of the determinant but it cannot be annihilated in any admissible scheme. In eq. (8) a special choice of computation scheme is used. This choice spoils the whole \( SU(N) \) gauge symmetry opposite to one of ref. [12] where a maximal subgroup isomorphic to \( SO(N) \) is preserved.
From eq. (7) one can easily see that $\alpha_1(A, g)$ satisfies $(mod\ 2\pi)$ identity (1-cocycle condition):

$$\alpha_1(A^h, g) - \alpha_1(A, hg) + \alpha_1(A, h) = 0, \quad h, g \in SU(N),$$

and difference given by a different computational scheme consists in adding to $\alpha_1$ the gauge variation of a local (finite) counterterm (trivial 1-cocycle) –

$$\alpha_0(A^g) - \alpha_0(A).$$

As a consequence of the gauge non-invariance of the determinant (3) the modification of the constraint algebra (3,4) occurs. In particular the commutator for quantum operators $G^a$ will acquire Schwinger term (infinitesimal 2-cocycle)

$$[G^a(x), G^b(y)] = i f^{abc} \delta(x - y) + a^{ab}(x, y)$$

which generally does not vanish on the constraint surface $G = 0$. This fact makes condition (3) inconsistent and in this case one is stressed to loose either Lorentz (and gauge) invariance or unitarity.

To repair this situation and have a gauge invariant quantum theory one can modify the quantization procedure as proposed by L.D.Faddeev and S.L.Shatashvili [9]. According to this one must consider a modified action

$$S_{mod} = S_{cl} + \alpha_1(A, g)$$

instead of the classical one and quantize it after imposing gauge condition ($A_0 = 0$). If the new action (14) is well defined i.e. it has nondegenerate symplectic form one has restored the gauge invariance of the quantum theory with constraints $G$ obeying the old algebra (3,4). Also one has a number of additional degrees of freedom carried by the gauge group valued fields $g$ and the path integral representation for the generating functional is given by

$$Z = \int dA d\phi \delta(A_0) [\det \omega(A^g)]^{1/2} e^{iS + \alpha_1(A, g)}.$$  

where $\det \omega(A^g)$ is the determinant of the symplectic form [3],

$$\omega(A^g) = -\frac{i}{48\pi^2} \epsilon_{ijk} \text{tr} \{ \delta g g^{-1} \delta g g^{-1} (A^g_i \partial_j A^g_k + \partial_i A^g_j A^g_k + A^g_i A^g_j A^g_k) - \delta g g^{-1} \partial_i A^g_j \delta g g^{-1} A^g_k \}.$$  

But as was mentioned in the Introduction for some $SU(N)$ groups this action can have degenerate symplectic form. This means that there are a number of additional primary constraints that can generate for example some secondary constraints and so on [14, 15]. In this case one should modify the Faddeev-Shatashvili method to include the whole tower of the constraints. In particular one must verify if the additional constraints do not destroy the gauge invariance of the quantum theory.

### 3 Degenerate case

To quantize the action (14) with degenerate symplectic form let us firstly introduce a parametrization of the gauge group element $g \in G \equiv SU(N)$ by fields $\phi^A, A = 1, \ldots, \text{dim } G$

$$g = g(\phi)$$

(17)
In terms of these fields the action of the theory in first order formalism looks as follows

\[ S = \int d^4x \left\{ E_i^a \dot{A}_i^a - \frac{i}{48\pi^2} \epsilon_{ijk} \text{tr} \dot{A}_i^a\{A_j, g_k\} + \Gamma^A \dot{\phi}^A \right. 
+ \frac{i}{48\pi^2} \epsilon_{ijk} \left( A_i \partial_j A_k + \partial_i A_j A_k + A_i A_j A_k + A_i g_j A_k - A_i g_j g_k \right) g_A \dot{\phi}^A 
+ \frac{i}{2} (E^2 + B^2) - i \bar{\psi} \dot{\gamma}_i \nabla_i \psi + A_0^a G^a \] 

(18)

where \( E \) and \( B \) are "electric" and "magnetic" components of the gauge field strength \( F_{\mu\nu}, \Gamma^A \dot{\phi}^A \)
is the term of eq.(1), \( g_A \equiv \frac{\delta g(\phi)}{\delta x^a} g^{-1}, \epsilon_{ijk} \) is three dimensional antisymmetric tensor \((i=1,2,3)\). From eq.(18) one can see that one has a set of constraints – Gauss law

\[ G^a = \nabla_i E_i + i \psi^+ \lambda^a \psi - \frac{i}{48\pi^2} \epsilon_{ijk} \text{tr} \lambda^a \left( 2\{\partial_l A_j, g_k\} - g_l g_j A_k - A_l g_j g_k + A_i A_j g_k - g_i A_j A_k - g_i A_j g_k - g_i g_j g_k \right) \] 

(19)

As we have already mentioned we impose the gauge condition \( A_0 = 0 \) this will exclude the constraints \( G^a \) from our analysis on the classical level.

Canonical momenta for the fields \( A_i^a \) are given by shifting of \( E \)

\[ E_i^a \rightarrow \Pi_i^a = E_i^a - \frac{i}{48\pi^2} \epsilon_{ijk} \text{tr} \lambda^a\{A_j, g_k\} \] 

(20)

Introducing also the canonical momenta for the fields \( \phi^A \) one gets a set of constrains

\[ \varphi_A = p_A - \Gamma A + \] 

\[ + \frac{i}{48\pi^2} \epsilon_{ijk} \left( A_i \partial_j A_k + \partial_i A_j A_k + A_i A_j A_k + A_i g_j A_k - A_i g_j g_k \right) g_A \] 

(21)

Also one has an equivalent set of constraints \( \varphi_a \) given by

\[ \varphi_a = g_A^a \varphi_A = 0 \] 

(22)

where \( g_A^a \) is the inverse to the matrix \( g_A^a = \frac{1}{2} \text{tr} g_A \lambda^a \). The matrix of Poisson brackets of the constraints (21) is just the symplectic form for fields \( \phi^A \)

\[ \omega_{AB} = \{ \varphi_A(x), \varphi_B(y) \} = \frac{i}{48\pi^2} \epsilon_{ijk} \left( \frac{1}{2} [g_A, g_B] (\tilde{A}_i \partial_j \tilde{A}_k + \partial_i \tilde{A}_j \tilde{A}_k + \tilde{A}_i \partial_j \tilde{A}_k) + \right) 
\] 

\[ + \frac{1}{2} (g_A \tilde{A}_i g_B \partial_j \tilde{A}_k - g_B \tilde{A}_i g_A \partial_j \tilde{A}_k) \delta(x - y) \] 

(23)

where \( \tilde{A}_i \equiv (A_i + g_i) \). But it will be more convenient to use the equivalent set of constraints (22) for which the matrix of Poisson brackets is

\[ \omega_{ab} = \{ \varphi_a(x), \varphi_b(y) \} = \frac{i}{48\pi^2} \epsilon_{ijk} \left( \frac{1}{2} [\lambda_a, \lambda_b] (\tilde{A}_i \partial_j \tilde{A}_k + \partial_i \tilde{A}_j \tilde{A}_k + \tilde{A}_i \partial_j \tilde{A}_k) + \right) 
\] 

\[ + \frac{1}{2} (\lambda_a \tilde{A}_i \lambda_b \partial_j \tilde{A}_k - \lambda_b \tilde{A}_i \lambda_a \partial_j \tilde{A}_k) \delta(x - y) \] 

(24)

To write the transformation law of (24) one can observe that \( g_a^A g^{-1} \) can be expanded in terms of \( \lambda_a \)

\[ \lambda_a \rightarrow g \lambda_a g^{-1} = \Lambda_a^b (g) \lambda_b, \quad \det \Lambda = 1 \] 

(25)
From eqs. (24,25) one can see that the matrix (24) is transformed under gauge group action as follows

$$\omega_{ab}(x) \rightarrow \Lambda^c_a(x) \omega_{cd}(x) \Lambda^d_b(x),$$

where $$\omega_{ab}(x,y) = \omega_{ab}(x) \delta(x-y)$$.

If the matrix $$\omega_{ab}$$ is a degenerate one it has a set of linearly independent null vectors $$z^b_R$$, $$R = 1, \ldots, K < N \equiv \text{dim}G$$

$$\omega_{ab} z^b_R(\tilde{A}) = 0,$$

where $$K$$ is corank of the matrix $$\omega_{ab}$$.

One can find these null vectors as follows. Consider the following antisymmetric (isotopic) tensor

$$z^{a_1 \ldots a_K}(x) = \epsilon^{a_1 \ldots a_K b_1 b_2 \ldots b_{N-K-1} b_{N-K}} \omega_{b_1 b_2}(x) \ldots \omega_{b_{N-K-1} b_{N-K}}(x)$$

(28)

This tensor has the following properties

$$z^{a_1 \ldots a_K}(x) \omega_{a_1 b} = 0$$

(29)

$$z^{a_2 \ldots a_K}(x) z^{b_2 \ldots a_K}(x) \sim P^a_b$$

(30)

where $$P^a_b$$ in the last equation stands for the projector on zero subspace of the matrix $$\omega$$. To prove (29) one can consider a maximal nonzero minor $$\omega_{\bar{a} \bar{b}}$$. All other lines and columns of the matrix $$\omega_{ab}$$ are linear combinations of elements $$\omega_{\bar{a} \bar{b}}$$. One can see that only they contribute for eq. (28). In fact $$z^{a_1 \ldots a_K}$$ is skew product of all $$\omega_{\bar{a} \bar{b}}$$. So contracting $$z$$ with $$\omega_{ab}$$ one will have antisymmetric combination which is equal to zero because there is a component $$\omega_{\bar{a} \bar{b}}$$ which meets itself at least twice in this antisymmetric product. Another way to prove eq. (29) is to use Darboux theorem. Eq. (30) can be obtained by expansion of product of two $$\epsilon$$ in antisymmetric combinations of delta symbols.

It is clear that for any set of null vectors $$\{z_R\}$$ the skew product of these vectors must be proportional to the tensor (28)

$$\epsilon^{P \ldots R} z^{a_1}_{P} \ldots z^{a_K}_{R} \sim z^{a_1 \ldots a_K}$$

(31)

where $$\epsilon^{P \ldots R}$$ is $$K$$ dimensional antisymmetric symbol with $$\epsilon^{1 \ldots K} = 1$$, and vice versa any set of linear independent vectors satisfying (31) is set of null vectors of $$\omega$$.

In particular if there is the only null vector it is given by

$$z^a(x) = \epsilon^{a b_1 b_2 \ldots b_{N-2} b_{N-1}} \omega_{b_1 b_2}(x) \ldots \omega_{b_{N-2} b_{N-1}}(x)$$

(32)

In this case the gauge group must be odd dimensional.

In case when there are more than one constraint one has the combinations of constraints

$$\psi^{a_2 \ldots a_K}(x) = z^{a_1 a_2 \ldots a_K}(x) \varphi_{a_1}(x)$$

(33)

which commute with all the constraints $$\varphi_a$$ but they are not all independent.

So one must either introduce a set of linear independent vectors satisfying (31) or equivalently select the subset of constraints from (33).

Now suppose that one has chosen such set of null vectors $$z^a_R$$ as solution of the equation

$$\epsilon^{P \ldots R} z^{a_1}_P \ldots z^{a_K}_R = z^{a_1 \ldots a_K}.$$
These vectors are invariant under gauge group action
\[ z_R^a \to (\Lambda^{-1})^a_b z_R^b \]  
(35)
So the set of independent commuting constraints consist of
\[ \varphi_Q = z_Q^a \varphi_a. \]  
(36)

Now following the Dirac procedure one should impose the conservation of these constraints getting in this way the secondary constraints,
\[ \bar{\varphi}_Q(x) \equiv \{ H, \varphi_Q(x) \} = z_Q^a(x)\{ H, \varphi_a(x) \} = 0, \]  
(37)
where equalities are hold up to the primary constraint combinations and
\[ \{ H, \varphi_a \} = -i / 48\pi^2 \epsilon_{ijk} \text{tr} \lambda^a \{ E_i, F_{jk} \} - E_i \tilde{A}_j \tilde{A}_k + \tilde{A}_i E_j \tilde{A}_k - \tilde{A}_i \tilde{A}_j E_k. \]  
(38)
One can see that the commutator (38) transforms covariantly under gauge group action
\[ \{ H, \varphi_a \} \to \Lambda^a_b \{ H, \varphi_b \}. \]  
(39)
Due to this the secondary constraints will be invariant under gauge group transformations. This shows that imposing of the secondary constraints will not destroy the gauge invariance if the determinant of matrix of all constraints Poisson brackets is also gauge invariant.

Now one can unify the secondary constraints with the primary ones to get the set of constraints \( \varphi_I, (I) = (a, Q) \) and consider the matrix of the Poisson brackets of this set of constraints. It has the following block structure
\[ ||\omega_{IJ}|| = \begin{pmatrix} \{ \varphi_a, \varphi_b \} & \{ \varphi_a, \bar{\varphi}_Q \} \\ -\{ \varphi_a, \bar{\varphi}_P \} & \{ \bar{\varphi}_Q, \bar{\varphi}_P \} \end{pmatrix} \]  
(40)
Let us prove firstly the gauge invariance of its determinant. In order to do this consider its gauge transformation law.

The transformation law of the block \( \{ \varphi_a, \varphi_b \} \) is given by (26). To find the transformation law of the block \( \{ \varphi_a, \bar{\varphi}_Q \} \) let us remind that the constraints have the following structure
\[ \bar{\varphi}_Q(x) \equiv [H, \varphi_Q(x)] = z_Q^a(x)[H, \varphi_a(x)] = 0, \]  
(42)
where
\[ \Phi_{ia}^b(x)\delta(x - y) = \{ E_i^a(x), \varphi_a(y) \} \]  
(43)
is gauge covariant.

So the Poisson bracket has the form
\[ \{ \varphi_a, \bar{\varphi}_Q \} = \{ \varphi_a, E_i^a \} \Phi_{iQ}^a + E_i^a g_a^A \{ p_A, \Phi_{iQ}^a \} \]  
(44)
The first term in (43) is equal to \( \Phi_{iQ}^a \Phi_{ib}^b \delta(x - y) \) and it is gauge covariant (in indice \( a \)). The remaining term can be rewritten as follows
\[ E_i^a g_a^A \{ p_A, \Phi_{iQ}^a(\tilde{A}) \} = E_k^a g_a^A \frac{\delta \Phi_{kQ}^b(\tilde{A})}{\delta \tilde{A}_i} \frac{\delta g_i}{\delta \tilde{A}}, \]  
(44)
where
\[ g_A^a \frac{\delta g_i}{\delta \phi^A} = \lambda_a \delta_j^i (x - y) + g_B^a \nabla_i^g g_B \delta(x - y) \] (45)
\[ \nabla_i^g g_B \equiv \partial_i g_B - [g_i, g_B] \] (46)

Since (45) transforms covariantly it follows that the last term in (43) is also covariant.

Unfortunately proving the gauge invariance of the last block \( \{ \tilde{\phi}_P, \tilde{\phi}_Q \} \) is not so easy. It probably holds only on the surface of all second class constraints but luckily the determinant of \( \omega \) does not depend on this block.

Indeed let us consider the following antisymmetric block matrix which can be obtained from our by splitting the set of constraints \( \phi_a \) in the subset of commuting constraints \( \phi_Q \) and the subset of second class ones \( \tilde{\phi}_a \)

\[
\begin{pmatrix}
\{ \phi_a, \phi_b \} & \{ \phi_a, \phi_S \} & \{ \phi_a, \tilde{\phi}_Q \} \\
\{ \phi_R, \phi_b \} & \{ \phi_R, \phi_S \} & \{ \phi_R, \tilde{\phi}_Q \} \\
\{ \tilde{\phi}_P, \phi_b \} & \{ \tilde{\phi}_P, \phi_S \} & \{ \tilde{\phi}_P, \tilde{\phi}_Q \}
\end{pmatrix} \equiv \begin{pmatrix} \tilde{\omega} & 0 & a \\ 0 & 0 & b \\ -a^T & -b^T & c \end{pmatrix}
\] (47)

with nondegenerate block \( \tilde{\omega} = -\tilde{\omega}^T \), \( c = -c^T \) and quadratic block \( b \). One can see that this matrix is degenerate if and only if the block \( b \) is degenerate. Using the properties of the determinant one has

\[ \det \begin{pmatrix} \tilde{\omega} & 0 & a \\ 0 & 0 & b \\ -a^T & -b^T & c \end{pmatrix} = \det \begin{pmatrix} \tilde{\omega} & 0 & 0 \\ 0 & 0 & b \\ 0 & -b^T & 0 \end{pmatrix} = \det \tilde{\omega} (\det b)^2 \] (48)

i.e. it does not depend on the block \( c \).

Since the relevant part of \( \omega \) transforms by multiplication on orthogonal matrix the gauge invariance of the determinant is proved.

Let us note that Poisson brackets of the primary constraints are ultralocal i.e. they contains only delta functions and not its derivatives. From the other hand the Poisson brackets containing secondary constraints are not ultralocal. They contains not only ”pure” delta functions but the derivatives of the delta function are also present. This feature could complicate the further analysis in case when \( \omega \) is degenerate. Indeed, to find now the null vectors of such matrix one have to solve a system of differential equations. And the absence of the manifest gauge invariance of \( \omega \) itself will still complicate the proving of the gauge invariance (if it exists) of the final theory.

Fortunately there are some indications that in a general position point the procedure of the reproduction of the constraints stop here.

We know that \( \omega \) is degenerate if and only if the block \( b_{PQ} = \{ \tilde{\phi}_P, \tilde{\phi}_Q \} \) is degenerate. Now consider the subspace of phase space such that \( E = 0 \). Note that this subspace belongs to the surface of the secondary constraints. So consider the matrix \( b \) on this subspace. There it has the following form (see eq.(13))

\[ b_{PQ} = \text{tr} \Phi_i P \Phi_i Q \] (49)

and if constraints \( \tilde{\phi}_Q = \text{tr} E_i \Phi_i Q \) are independent in the vicinity of some point of phase space which is natural then [13] must be nondegenerate in this point. Moreover this will hold nearly everywhere due to the fact that determinant is a polynomial in the fields.
Considering $\omega$ to be nondegenerate one can write the path integral representation for the generating functional

$$Z = \int \exp i(\Pi \dot{A} + p_A \phi^A - \frac{1}{2}(E^2 + B^2) + S_f(\bar{\psi}, \psi, A)) \delta(\varphi_I) \det ||\omega_{IJ}(x - y)|| d\Pi dA dp d\phi d\bar{\psi} d\psi$$

where $S_f$ is the fermionic part of the action. Integrating over $p$ and changing variables back $\Pi \rightarrow E$

$$Z = \int \exp i(S(\bar{\psi}, \psi, A) + \alpha_1(A, g)) \delta(\varphi_Q) \det ||\omega_{IJ}(x - y)|| dE dA d\phi d\bar{\psi} d\psi$$

(51)

Since gauge invariance is restored one can choose the physical subspace by imposing

$$G^a|ph >= 0$$

(52)

with physical observables having the form

$$F(\Phi, g) = F(\Phi^b, h^{-1}g) = F(\Phi^g)$$

(53)

where $\Phi = (A, E, \bar{\psi}, \psi)$.

The number of the Lagrangean degrees of freedom can be established in the following way. It is equal to the number of (Lagrangean) fields minus half number of the second class constraints. In the case when only the secondary constraints appear one has

$$N_{df} = N_A + (N_p + N_{\phi}) - \frac{1}{2}N_\phi + N_{\text{fermions}} = 2N + \frac{1}{2}(N - K) + N_{\text{fermions}}$$

(54)

where $N = \dim G$ and fermionic degrees of freedom are denoted as $N_{\text{fermions}}$.

**Discussions:** We have shown that despite of degeneracy of the symplectic form of Wess-Zumino action one can perform the canonical quantization of the anomalous $SU(N)$ Yang-Mills theory and obtain finally a gauge invariant quantum theory with new degrees of freedom and additional constraints caused by degeneracy.

One can alternatively consider the initial anomalous theory with the constraints $G_a$ satisfying algebra $[3]$ and canonically quantize it. In our approach it corresponds to the quantization in $g = 1$ gauge. Due to the coincidence

$$\omega_{ab}|_{g=1} = a_{ab}$$

(55)

one has for $a$ the same set of null vectors $z_Q(A)$. One can calculate the commutator $\{H, G^a\}$, for example, using Bjorken-Johnson-Low formula in a manner similar to calculation of the commutator $\{G, G\}$ in ref.[16]. Performing this one will have for it

$$\{H, G^a\} = -\frac{i}{48\pi^2} \epsilon_{ijk} \operatorname{tr} \lambda^a(\{E_i, F_{jk}\} - E_i A_j A_k + A_i E_j A_k - A_i A_j E_k) = \{H, \varphi_a\}|_{g=1}.$$ 

(56)

After this one can write the path integral representation for generating functional

$$Z = \int d\Phi e^{iS} \delta(G_a) \det a$$

(57)

Using Faddeev-Popov trick one can formally pass from this integral to one in the gauge $A_0 = 0$ obtaining the result identic to eq.(51). The last quantization procedure will always give the
gauge invariant theory because the shifting of the fields by gauge transformation is performed only after the quantization. But to write the Poisson brackets one uses formulas from quantum commutators which is rather formal procedure. Nevertheless in the case when our first sheme gives the gauge invariant theory last one must agree with it.

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References

[1] L.S.Adler, Phys.Rev. 177 (1969) 5 (11) 2426–2438.

[2] W.A.Bardeen, Phys. Rev. 184 (1969) 5 1848–1859.

[3] D.J.Gross, R.Jackiw, Phys. Rev. D 6 (1972) 2 477–493.

[4] R.Stora, Algebraic structure and topological origin of anomalies in: Progress in gauge field theory Ed. H.Lehman, New–York, Plenum Press 1984, p.87–102.

[5] B.Zumino, Algebraic origin of anomalies in: Relativity groups and topology II Ed. B.S.DeWitt and R.Stora, Amsterdam, North Holland 1984, p.1293.

[6] L.D.Faddeev, Phys. Lett. 145B (1984) 1,2 81–86.

[7] L.D.Faddeev, S.L.Shataashvili, Theor. Math. Phys. 60 (1985) 770 (Teor. Mat. Fiz. 60 (1984) 2 225).

[8] J.Wess, B.Zumino, Phys. Lett. 37B (1971) 1 95–97.

[9] L.D.Faddeev, S.L.Shataashvili, Phys. Lett. 167B (1986) 2 225–228.

[10] J.Mickelsson, Commun. Math. Phys. 97 (1985) 361.

[11] R.Jackiw, R.Rajaraman, Phys. Rev. Lett. 54 (1985) 12 1219–1221.

[12] S.A.Frolov, A.A.Slavnov and C.Sochichiu, Phys. Lett. 301B (1993) 59-66.

[13] S.L.Shataashvili, Theor. Math. Phys. 71 (1987) 40 (in Russian).

[14] S.A.Frolov, A.A.Slavnov and C.Sochichiu, Canonical quantization of the degenerate WZ action including chiral interaction with gauge field, hep-th/9411182, submitted to Mod.Phys.Lett.

[15] S.A.Frolov, A.A.Slavnov and C.Sochichiu, SO(N) invariant Wess-Zumino action and its quantization, hep-th/9412164, submitted to Teor.Mat.Fiz.

[16] A.Yu.Alekseev, Ya. Madaichik, L.D.Faddeev, S.L.Shataashvili, Theor. Math. Phys. 73 (1988) 1149 (Teor. Mat. Fiz. 73 (1987) 2 187–190)