Various companies choose to outsource the delivery of part of their services, so as not to deviate from its core business and improve the service level. This approach leads to a new type of organizations, so-called networked and virtual enterprises, where possibly a great number of companies work together without having direct contact but through a broker, as an intermediary, that streamlines the relationships between them. To enable high level efficiency, as well as some other functional requirements, the meta-organizations and brokering services are conceived as environments and services for networked and virtual enterprises operation and dynamic reconfigurations, representing a model of organizations-of-organizations, as an implementation of one of the Industry 4.0 models and ecosystem for networked and virtual enterprises dynamic reconfiguration. In this paper, the meta-organizations with embedded brokering services, modelled as call centres, are analyzed. Various simulations are presented, based on Erlang’s formulas for some of design and performance measures parameters evaluation, such as service level, average waiting time, agent occupancy and service traffic intensity.

**Keywords:** Brokering service, Call center, Erlang formula, Meta-Organizations, Virtual Enterprises, Industry 4.0

1. INTRODUCTION

In response to market demand, many companies choose a helpdesk service to support the customer after the product or service sale. So, they choose to disaggregate their activities in order to keep the focus on their core business, and delivering part of their services, to other companies. In this way, we predict a better quality of service they provide.

These companies providing employed service have complete knowledge of the product concerned, and work with the company producing the product in a sort of symbiosis, in which both take advantage of this service, pointing thus the concept of virtual enterprises (VE) to network organizations and collaborative work. Companies disaggregate activities, approaching true virtual organizations with knowledge centers that interact largely through mutual interest, rather than systems of authority [1].

Some companies resort to external services, for service to their customers. An example of how these external services could be organized is Call Centre.
tiality, by reducing the risk of non-compliance, and also by the integration of the companies, accounting for streamline, plan and control their use of outsourcing.

The capacity of a broker to act on behalf of a client to guide the selection of the most suitable product has been well known and the extension of this concept in the context of broker’s participation as an agent within an electronic market is a natural progression [4].

Thus, the broker selects service providers and bridges between two entities, in most cases, between the producer of a product and the customer. The Broker is the main agent of agility [5]. The broker has high importance in the organizational reconfiguration of enterprises.

Organizations have evolved substantially since the first fundamental theories of organizational design, adapting over time until the present. Companies increasingly rely on its partners to ensure their own success. [6].

To enable high level efficiency, as well as some other functional requirements, the meta-organizations and brokering services are conceived as environments and services for networked and virtual enterprises operation and dynamic reconfigurations, representing a model of organizations-of-organizations, as an implementation of one of the Industry 4.0 models and ecosystem for networked and virtual enterprises dynamic reconfiguration.

In this paper, the meta-organizations with embedded brokering services, modelled as call centres, are analysed. Various simulations are presented, based on Erlang’s formulas for some of the design and performance measures parameters evaluation, such as service level, average waiting time, agent occupancy and service traffic intensity.

It is shown that modelling the meta-organizations with embedded brokering services, as call centres, using the Erlang’s formulas, represents a significant contribution to meta-organizations and brokering services design parameters and performance evaluation, not presented in up to date literature.

2. META-ORGANIZATION AND ITS ROLE IN DYNAMIC RECONFIGURABILITY OF NETWORKED ANDVIRTUAL ORGANIZATIONS

According to [7] Meta-Organization (MO) is a high-level organization, meaning an organization of organizations.

Organizations-of-organizations could be also represented as a system-of-systems, due to homomorphic relationship between some dimensions of their architectures. System-of-systems could be considered a resilient and adaptable IT ecosystem, “a special type of system of systems in which multiple systems with various degrees of autonomy achieve common goals while adapting to the given environment...” [8].

System-of-systems, as an IT ecosystem, is one of the fundamental features, or components, of Industry 4.0 [9]. In that way, it could be said that Meta-Organizations, as organizations-of-organizations, represent an Industry 4.0 model implementation and ecosystem for virtual enterprises dynamic reconfiguration, meaning dynamic creation, operation, reconfiguration and dissolution, providing a sustainable environment.

In [10] the meta-organizations are also defined as organizations-of-organizations that have assumed the form of associations. Members cannot be forced by the association to become members of it, and even when they are members, they are free to leave the association whenever they want.

According to [6] “Meta-organizations comprise networks of firms or individuals not bound by authority based on employment relationships but characterized by a system-level goal”.

In order to ensure trust between a meta-organization and its members and to ensure trust to each other, there are mutual contracts [7]. The contracts between the members of the meta-organizations are indirect, meaning that each member make the contract with the meta-organization institution that define its relationship to other members of the meta-organization.

From the definitions previously referred to, one can state that a network of companies can have a meta-organization, but not necessarily. Within the meta-organization can exist one or more virtual enterprises. When the meta-organization is considered as enabler of virtual enterprises, meaning that virtual enterprises are not feasible without the meta-organization as environment for its generation, dynamic reconfiguration and operation, it could be said that meta-organizations are virtual enterprises generators.

The authors in [7] state “The collaborative component of the MO and its members has two levels: 1) Stable relation: relation established by membership in MO, and 2) Dynamic relation: relations dynamically established among members within the MO.

The stable relation is celebrated by a contract made between the MO and the member, in order to assure terms and conditions (rights and obligations) between these parties, including relations between MO and a member, and between a member and other members.

Rules, infractions, and penalties are regulated by the MO and bounded in the contract for conflicts resolution and decision-making problems. Also, this contract regulates the members’ activities, for example, two members developed a joint work and one member considers that another member abused, for any reason, and did not fulfil the MO rules, the self-considered injured member could report to the MO the incident, in order to the MO mediate and apply a resolution to this situation based in the contract.

The dynamic relation is established by negotiation between members and there are three dimensions in changes: 1) members; 2) relation of the members in network, and 3) Ubiquitous Virtual Enterprise (UVE) project. The dynamic relation starts with a hierarchical relation, where the member owner/leader of the UVE project - idea/service/product - (Client) defines the initial terms of the relation with other members (Resources). This relation could be:

a) Owned: a member owns the project in 100% (knowledge and capital) and subcontracts (outsourcing) other members to provide products and services;

b) Co-owned closed: a group of members owns the project, and project shares are divided in accordance with all group members;
c) Co-owned open: all members could contribute to the project, having a share of the project depending on the contribution;
d) Open: all members could contribute freely to the project, and this project does not have ownership, it is open to the community of members”.

3. META-ORGANIZATION AS A CALL-CENTRE

A call centre is an installation conceived to support the delivery of any interactive service by “telephone” calls; usually an office space with multiple workstations occupied by agents to make and/or receive calls [11]. A call centre aims to make the interface between the customer and the company.

The call centre is an example of queuing systems: calls arriving, waiting in a virtual queue and which are, then, answered by an agent. These services are often modeled as systems of rows, M/M/s model in standard terminology in theory of queues - the model of Erlang C. The model of Erlang C assumes that 
calls arrive with an average arrival rate known, with a number of agents defined statistically identical and with service time following an exponential distribution. This model also requires all customers to wait as long as necessary to receive the service without disconnect the call [12]. The Erlang model is well known in engineering, and it is used for many other applications, such as referred in [13,14].

A call centre is comprised of human resources, operators, and technological resources, such as computers and communications equipment. This set interacts with customers and receives a large number of simultaneous connections in the call centre.

The MO are modeled as a call centre under the following assumptions:

1. Similarity between meta-organization and call center: in the meta-organization, brokers play a role similar to the role of call center agents. According to [15] “Given the functions that are assigned to the Broker under the different models of Agil/Virtual Enterprises, makes it a necessary agent but assuming different functions depending on the model that fits. In fact the Broker is a flexible/dynamic agent, since so for each model Agil/Virtual Enterprise it will have to adapt to functions as requested by the initiator/home agent Agil/Virtual Enterprise.”

2. However, there is a major difference between call centre and meta-organization. This difference is related to relationship between agents and call center in the call centre, and between brokers and meta-organization in the meta-organization. In the call centre agents are call centre employees with a fixed contract and respond to the call centre management, while in the meta-organization brokers are independent and respond to meta-organization just under governance rules.

3. However, abstracting the relationship nature between the agents and call centres, and between brokers and MO, and assuming: 1) that brokers in MO work under the same regime as agents in call centre (full-time 8 hours per day, that they must attend the call, etc) and 2) that clients behave in accordance with Erlang C model, and the behavior and performance of brokers in the MO could be modeled as the behavior and performance agents in the call centre. Under the referred assumptions the performance of MO could be modeled as the performance of call centre.

4. By the MO performance we consider the probability that one client enterprise’s request for service will be satisfied.

5. The time and quality of the client enterprise’s requests satisfaction will determine the MO’s capability to serve as an enable of dynamic reconfiguration of virtual enterprise.

6. Therefore, the objective of one MO is to provide as fast as possible and with the maximum quality as possible to the client enterprise’s requests. One of the success factors for accomplishing this objective is number of brokers (agents in call centre) that act within the MO, in the function of requests arrival rate and time for processing one request, in accordance with Erlang C model.

4. CALL-CENTRES AND THE ERLANG MODELS FOR EVALUATING CALL-CENTRES’ SERVICE LEVEL

The model M/M/s is a simple model since it considers a single queue system with multiple service processors (service desks). This system is characterized by having a Poisson arrival rate and exponential response times. The arrival rate \( \lambda \) is independent of the system state, and follows a FIFO (First-In-First-Out) answering system.

The Poisson distribution is the law of the processes equivalent to the waiting time. In terms of the theory of probability and statistics, the Poisson distribution is a discrete probability distribution which describes the number of events of behavior that occurs in a fixed period of time if these events occur with a known average rate and independently of the time elapsed since the previous event.

The Poisson distribution is a probability distribution that applies to the occurrence of rare events, such as quality control, probability of defects, accidents, etc. [16].

The Poisson distribution is constructed with a single parameter, lambda (\( \lambda \)), equation (1), [16]. This parameter represents both the mean and the variance. This distribution has a strong approach to binomial distribution.

\[
f(x) = \frac{-\lambda x^k}{k!}
\]

Agner Krarup Erlang (1878 - 1929) was a Danish mathematician. Erlang began its work by applying probability theory to the problems of telephone traffic and in 1909 published his first work “The Theory of Probabilities and Telephone Conversations”. This work demonstrates that telephone calls randomly distributed follow the law of Poisson distribution [17].

Erlang developed Erlang B formula and Erlang C formula.

The Erlang B formula calculates the blocking probability of calls. The locking occurs when the
number of lines is insufficient to receive all the amount of calls. Incoming calls that cannot be answered are forwarded to a voice mail, forcing people to disconnect the call. The only way to receive the service would be to hang up and reconnect [18].

Thus, Erlang B calculates the probability of calls becoming blocked for a given traffic intensity and a given number of agents. The Erlang B formula is [19]:

\[
E_b = \frac{u^m}{m!} \sum_{k=0}^{\infty} \frac{u^k}{k!}
\]

where:
- \(u\) - traffic intensity
- \(m\) - Number of agents.

Unlike the situation in which it uses the Erlang B formula, Erlang C formula allows a customer remains on hold, without having to disconnect the call (3) [19]. The formula developed by A.K. Erlang known as Erlang formula C is as follows,

\[
E_c = \frac{m \cdot u^m}{m-u} m! + \sum_{k=0}^{m-1} \frac{u^k}{k!}
\]

According to [20], the Erlang C formula allows to calculate the probability that a customer will have to wait for a resource, but in reality it does not mean that the customer will wait. Therefore, it is desirable that Erlang C measure has the smallest possible value. Also according to [20], it is know that the agent occupancy (4) is given by

\[
\rho = \frac{u}{m}
\]

while the traffic intensity is calculated by

\[
u = \lambda \cdot Ts
\]

where:
- \(\lambda\) – average arrival rate
- Ts - Average call duration.

Thus, considering (4) and (5), the Erlang C formula takes the following form (6) [20]:

\[
E_c = \frac{m \cdot u^m}{m-u} m! + \frac{u^m}{m!} + \sum_{k=0}^{m-1} \frac{u^k}{k!}
\]

According to [20], the service level is calculated by formula (7)

\[
W(t) = 1 - Ec(m,u) \cdot e^{-\frac{(m-u)t}{Ts}}
\]

where:
- \(u\) - traffic intensity
- \(m\) - number of agent
- \(t\) - target answer time
- \(Ts\) – Average call duration.

Through the Erlang C formula, it is still allowed to calculate the Average Waiting Time, \(T_w\), proposed by [20]. This will allow to have an idea of how long the client can expect to be served.

\[
T_w = \frac{E(m,u) \cdot Ts}{m(1-\rho)}
\]

Following, we present the analysis of a meta-organization design parameters and performance for VE dynamic reconfiguration for the case of design changes in automotive industry.

5. ANALYSING A META-ORGANIZATION AND BROKERING SERVICE DESIGN PARAMETERS AND PERFORMANCE FOR VE DYNAMIC RECONFIGURATION FOR THE CASE OF DESIGN CHANGES IN AUTOMOTIVE INDUSTRY

5.1 The case of design changes in automotive industry

Given the case study for [21], it is estimated that there are 453.000 product changes annually within the company and between the company and the service providers. These product changes can be translated into service requirements (seen at the call center as a call), which represents an interval of 50 seconds per service request, if the company work 24 hours a day, or 17 seconds, if the company operates 8 hours a day 1. Thus, in case the company running 24 hours a day, the arrival rate (7) is given by:

\[
\lambda = \frac{1}{17} = 0,06
\]

and in case the company running 8 hours per day, the arrival rate (8) is given by:

\[
\lambda = \frac{1}{50} = 0,06
\]

So, there were held a number of simulations for this case study of manufacturing, with these fixed arrival rates (to 0.06 and 0.02).

1 Although these data are relatively old, for the purpose of this research, this is not too relevant, considering that 1) the nowadays data are not too different, and 2) the research objective is not an analysis of a particular industrial case but the design and performance analyses methodology for evaluation of the design and performance parameters.
5.2 Design parameters and performance measures analysed

The design parameters of the MO organization for ve
dynamic reconfiguration considered correspond to the
following independent variables of the models:
1 - Average arrival rate
2 - Average operation duration (service time duration)
3 - Number of agents
The performance measures for the MO organization for
dynamic reconfiguration correspond to the following
dependent variables of the models (7), (8), (4), (5)
respectively:
F1 - Service level
F2 - Average waiting time
F3 - Agent occupancy
F4 –Traffic intensity (of services)

5.3 Service level analyses

The first simulation conducted for this case study was to
calculate the service level according to the average dura-
tion of operation, Ts, and arrivals rates, $\lambda = 0.02$ and $\lambda = 0.06$. As it is not described in the case, what is the average
length of operation, there are used-Ts values varying
between 120 seconds and 3600 seconds. Also used the
number of agents, an average of 50 agents online.

Figure 1, presents the simulation results, and it can be
seen that as the average operation time increases the
service level decreases. For the arrival rate $\lambda = 0.02$, len-
gth of service greater than 1200 seconds with a number of
50 agents, the company can not maintain a service level
of 90%. These data indicate that to maintain the same
arrival rate and service level would automatically be
necessary to increase the number of agents.

A similar situation occurs for an arrival rate of $\lambda = 0.06$, except that in this situation the average operating
time drops to about 550 seconds, meaning that for this
value the company can not maintain service levels
above 90%. Here again, it would be necessary to
increase the number of agents to maintain the same
service rate and maintain a high level of service.

Figure 2. Service level according to the number of agents
and varying the arrival rate, Ts=600s

5.4 Average waiting time analyses

The third simulation conducted for this case study was
to calculate the average waiting time based on the
number of agents, $m$, and arrivals rates, $\lambda = 0.02$ and $\lambda = 0.06$. As in the original case description, the opera-
tion duration is not presented, the duration of operation
is fixed for the value of 600s, as in the precious
simulations.

For this situation, the simulations were performed
with the number of agents varying between 25 agents
and 75 agents. As the number of agents increases, the
average waiting time decreases. This trend is expected
given that the average duration of operation is fixed.

However, it is important to point out that for the
arrival rate $\lambda = 0.02$, the waiting time is always close to
0 seconds, regardless of the number of agents that has
been chosen. As for the arrival rate $\lambda = 0.06$, it is
possible to verify that there is a peak waiting time above
60 seconds, the time begins to decrease from 45 agents,
waiting time reaching values very close to 0 seconds.
This situation is shown in Figure 3.
5.5 Agent occupancy analyses

The fourth simulation conducted for this case study was to calculate the agent occupancy according to the average duration of operation, $T_s$, and the arrival rates $\lambda = 0.02$ and $\lambda = 0.06$. For this situation, the average number of agents is fixed on 50 agents.

As the average duration of operation increases, the agent occupancy rate also increases. It appears in Figure 4, which for an arrival rate of $\lambda = 0.06$, the situation begins to be unsustainable for 1000 seconds of the average duration of operation. In this situation, the ideal would be to increase the number of agents.

Already for a $\lambda = 0.02$ arrival rate, it appears that the agent occupancy rate starts to becomes stifling, for the 3000 seconds average time of operation.

The fifth simulation conducted for this case study was the calculation of agents occupancy rate based on the number of agents and arrival rates, $\lambda = 0.02$ and $\lambda = 0.06$. For this situation, remained the average operating time of 600 seconds. The number of agents continues to vary between 25 agents and 75 agents.

As the number of agents increases, the agent occupancy rate decreases. It can be seen by Figure 5, for the arrival rate $\lambda = 0.02$, agent occupancy rate never exceeds 50%, which also allows to conclude that in this situation, the waiting time will also be very low.

As for the arrival rate $\lambda = 0.06$, it appears that the agent occupancy is untenable to 35 agents. Only when the number of agents increases to 45 agents, the occupancy rate decreases and therefore also reduces the waiting time. For this situation, the ideal would be to have always 45 agents at least, to be able to sustain this arrival rate for the average operation time assumed.

The sixth simulation conducted for this case study was the calculation of agents occupancy rate according to the number of agents and average duration of operation time for the arrival rate $\lambda = 0.06$. The different fixed numbers of agents used were 25, 50 and 75 agents.

In this situation, it turns out that for maximum agents 75, the average operation should not be more than 1200 seconds, both for the sake of the agent and the customer. In this situation, the agent starts to feel saturation effects, as well as the customer too, the average waiting time begins to be high. The situation is shown in Figure 6.

The seventh simulation conducted for this case study was the calculation of agents occupancy rate according to the number of agents and average duration of operation time for the arrival rate $\lambda = 0.02$. The different fixed number of agents used were 25, 50 and 75 agents.

The situation is shown in Figure 7. In this situation, it turns out that for maximum agents 75, the average operation should not exceed 3200 seconds. To a maximum of 25 agents, the average operation should not exceed 1100 seconds, for both the agents and the customers. In this situation, the agent begins to feel the effects of saturation, and for the client the average waiting time begins to be high.

5.6 Traffic intensity analyses

The eighth simulation conducted for this case study was to calculate the traffic intensity for the assumed average duration of operating time for the arrival rates $\lambda = 0.02$ and $\lambda = 0.06$, for an average of 50 agents.
In this situation, it turns out that the traffic intensity increases as the operation time increases. The situation is shown in Figure 8. The tendency of the functions was expected, taking into account the definition of traffic intensity given by multiplying the average arrival rate by the operation time.

Figure 8 - Traffic intensity according to the average duration operation and varying the arrival rate

The ninth simulation performed, represented in the Figure 9, for this case study was to calculate the traffic intensity according to the number of agents and the arrival rates $\lambda = 0.02$ and $\lambda = 0.06$, for an assumed average time of 600 seconds of operation duration.

In this situation, it turns out that the traffic intensity is maintained regardless of the number of agents. The tendency of the functions was expected, taking into account that the traffic intensity definition is given by multiplying the average arrival rate by the operation time.

Figure 9 - Traffic intensity according to the number of agents and arrivals rate

6. CONCLUSION

The Erlang formula C is used to model call centres. However, using the Erlang C formula for modelling and analysing a MO is limited as it implies only specific organizational and operational model of the MO, which is perfectly viable, but reduced compared with the general model of MO.

Consequently, further research should address development of new models for designing, managing and analysing MO that can address its general model.

Naturally, the special interest for the future research is validation of all models in the real life scenarios, in industry and related services.

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