Asymmetric diffusion in the delta-kicked rotor with broken symmetries

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We report an experimental investigation of momentum diffusion in the \( \delta \)-function kicked rotor where time symmetry is broken by a two-period kicking cycle and spatial symmetry by an alternating linear potential. The momentum diffusion constant is thus modified by kick-to-kick correlations which show a momentum dependence. We exploit this, and a technique involving a moving optical potential, to create an asymmetry in the momentum diffusion that is due entirely to the chaotic dynamics.

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Classical systems with chaotic dynamics can exhibit very different behavior in the quantum limit. One such system that has been frequently studied is the delta-kicked rotor (DKR), modeled using laser-cooled atoms in a pulsed periodic optical potential. Purely quantum mechanical phenomena such as dynamical localization \(^1\) (the quantum suppression of classical momentum diffusion), quantum resonances \(^2\) (ballistic rather than diffusive energy growth) have been observed. Asymmetry in the DKR has been produced via accelerator modes \(^3\) where the maximum momentum is transferred in each pulse to coherently impart a large amount of momentum to a particle.

Recently DKR systems where the timing of the kicks is subject to change, either by timing noise \(^4\) or by using a two-period cycle of kicks with small \(^5\) or large \(^6\) deviation from period one have been studied. The two cases are interesting to compare in that timing noise was found to destroy classical correlations that give rise to fluctuations in the diffusion rate, whereas for the two-period cycle the classical correlations acquire a momentum dependence. For the large deviations from period 1 (the \(2\delta \) kicked rotor) new families of correlations were shown to exist and verified experimentally.

In this paper we report an experimental realisation of the kicked rotor with broken time and space symmetry that was described theoretically in \(^7\). We used cold atoms in a pulsed optical lattice to model the delta kicked rotor with a two-period kicking cycle to break time symmetry. We use an optical lattice that is moving with constant velocity in the laboratory frame in order to probe the momentum dependence of the momentum diffusion rate in this case. We then use an accelerating optical lattice to additionally break spatial symmetry. This proof-of-principle experiment is, we believe, the first demonstration of an asymmetric momentum diffusion that is the result of purely chaotic dynamics and does not depend on specific structures in phase space.

An optical lattice formed by two counter-propagating laser beams may be used to trap laser-cooled atoms in a one-dimensional periodic potential \(^8\), \(^9\), \(^10\) the Hamiltonian for which is

\[
H = \frac{p^2}{2M} + V_0 \cos(2k_Lx)
\]

where \(M\) is the mass of the atom, \(k_L = 2\pi/\lambda\) the laser wavevector and \(V_0\) the potential depth. If the optical lattice is applied as a series of short (\(\delta\)-function) pulses with period \(T\), then we may compare this with the Hamiltonian for the \(\delta\)-kicked rotor as written in the usual dimensionless form:

\[
\mathcal{H} = \frac{\rho^2}{2} + K \cos(\phi) \sum \delta(\tau - n)
\]

where \(K\) is the stochasticity parameter which describes the strength of the kick. Here \(\rho = 2Tk_{LP}/M\) is a scaled momentum, \(\phi = 2k_Lx\) a scaled position, \(\tau = t/T\) a scaled time and \(\mathcal{H} = 8\omega_RT^2\mathcal{H}/\hbar\) the scaled Hamiltonian. The commutation relation \([\phi, \rho] = i\delta\omega_RT\) gives the scaled unit of system action or effective Planck constant \(\hbar_{eff} = 8\omega_RT\) (\(\omega_R\) the recoil frequency) which may be controlled through the period of the pulses. The dynamics of the kicked rotor have been well studied, particularly through the use of cold atoms in pulsed optical lattices \(^8\). One important feature is the time-dependence of the momentum which grows diffusively up to an \(\hbar_{eff}\) dependent time, \(t^* \propto \hbar_{eff}^{-2}\), the quantum break time, before saturating. At times \(t < t^*\) to lowest order the diffusion constant, \(D \propto K^2\). Corrections to this arise from correlations between kicks which appear as Bessel functions \(^10\):

\[
D(K) = K^2 \left[ \frac{1}{2} - J_2(K) - J_1^2(K) + J_2^2(K) + \ldots \right]
\]

the effects of which have been observed as anomalous momentum diffusion for particular values of \(K\) \(^11\).

In \(^8\) it was shown that for the \(\delta\)-kicked rotor where time symmetry is broken by a two-period kicking cycle of periods \(T(1 + b): T(1 - b)\) where \(b < 1\), then for short times these correlations give rise to a momentum dependent diffusion constant \(D(K, \rho, b) \approx D_0 - C(2, \rho)\), where \(D_0 \approx K^2[1/2 - J_1(K)^2]\) and \(C(2, \rho) = K^2J_2(K)\cos(2\rho b)\)
arising from correlations between kick number $i$ and kick number $i + 2$. This correction has a finite lifetime, denoted the ratchet time, $t_{\text{rat}}$ in \[\text{[2]},\] which depends on the parameter $b$ as $t_{\text{rat}} \approx 1/Db^2$. This timescale is different from, and may be controlled independently of, the break time, which lead to the main conclusion of \[\text{[12]}\] that the clearest experimental signature of this phenomenon would require $t^*/t_{\text{rat}} \approx Db/h_{\text{eff}} \sim 1$.

By including a linear term of alternating sign in the kicking potential Jonckheere et al. also showed that the $C(2, \rho)$ term may be made locally asymmetric around $\rho=0$, as $C(2, \rho) \to K^2J_2(K)\cos(2\rho - A)$ where $A$ is the (scaled) gradient of the linear “rocking” term, even for parameters where, unlike \[\text{[13]}\] there are no significant stable structures remaining. It was suggested that this system may be used to observe an asymmetry in the momentum diffusion or to produce a chaotic momentum filtering. An analysis of the Floquet states of this perturbed period kicked rotor may be found in \[\text{[14]}\].

In our experiment we realize a model of the $\delta$-kicked rotor using laser-cooled cesium atoms in a far-off resonant pulsed optical lattice. The lattice is formed by two horizontal counter-propagating laser beams, $1/e$ radius (0.95±0.05 mm), with parallel linear polarizations (see figure \[\text{[I]}\] which produces a spatial variation of the AC Stark shift that is proportional to the local intensity, and hence sinusoidal.

The pulses are produced by rapidly switching the drive voltage to the acousto-optic modulators (AOMs) according to a pre-defined sequence. The time between the kicks may be altered in order to produce the two-period “chirped” kicking cycle described above. The experiment proceeds as follows. Cesium atoms are trapped and cooled in a standard six-beam magneto-optic trap (MOT) before further cooling in an optical molasses to an rms scaled momentum width of $\sigma_p \approx 4$. The molasses light is turned off using an AOM and the periodic “kicking” potential applied. The beams for the kicking potential are derived from a Ti:Sapphire laser with an output power of 1 W at 852 nm, detuned typically 2000

The momentum dependence of the diffusion constant may be probed by using a sample of cold atoms with a non-zero mean initial momentum, such as may be prepared by cooling in an optical molasses in the presence of a non-zero magnetic field \[\text{[15]}\]. A disadvantage of this technique is that the wings of the atomic distribution may easily extend beyond the field of view of the CCD camera for relatively low momentum. Instead we have used a moving optical lattice formed by laser beams with a controlled frequency difference to make the kicking potential, so that atoms which are stationary in the laboratory frame (remain in the center of the CCD picture) have a non-zero momentum in the rest frame of the optical potential. This is achieved by driving the AOMs at frequencies $f \pm \Delta f$ as shown in figure \[\text{[II]}\] such that the atomic momentum in the rest frame of lattice is $\rho_L = m\lambda^2\Delta f h_{\text{eff}}/4\pi\hbar$. Using this technique the mean momentum in the lattice frame, $\rho_L$, may be varied over a large range in order to sample several periods of the oscillation of the diffusion constant without the beams becoming significantly misaligned from the cloud of cold atoms.

We have investigated several values of the parameters $K$ and $h_{\text{eff}}$, but present here those from conditions similar to those in \[\text{[5]}\], that is $K = 3.3$ (10% error arising mainly from the measurement of the beam intensity) and $h_{\text{eff}} = 1$ for values of $b = 1/16$ and $1/32$. This value of $K$ was chosen as it corresponds to the first maximum of the Bessel function $J_3(K)$, and hence may be expected to produce the clearest experimental signature, i.e. the largest amplitude oscillations in momentum asymmetry. Although for the Standard Map phase space is not completely chaotic for $K = 3.3$, the introduction of the parameter $b$ ensures no stable structures remain.
For these experiments the period of the kicks is $T = 9.47 \mu s$ and pulses are square with duration typically $t_p = 296$ ns ($t_p/T = 1/32 \leq b$), which is sufficient for there to be no substantial effects on the diffusion constant due to the finite temporal width of the kicks in the region of $\rho_L = 0$ \cite{16} (for larger $\rho_L$ these effects become important and start to affect the data). An investigation of the effects on the asymmetry of momentum diffusion arising the finite width of the kicks was presented in \cite{17}.

We characterise the asymmetry of the momentum distribution after the kicks by the first moment of the distribution, $\langle p \rangle = \int \rho N(p)dp / \int N(p)dp$, and plot this as a function of $\rho_L$ as shown in figure \ref{fig:momentum_asymmetry}. We observe that the asymmetry for each experiment oscillates with a period $\pi/b$ in agreement with the theory of \cite{18}, and shown by the dashed lines which are $\propto \sin(32\pi \rho_L)$ (top panel of figure \ref{fig:momentum_asymmetry}) and $\propto \sin(16\pi \rho_L)$ (lower panel). The data appears to deviate from this line at higher values of $\rho_L \approx 32\pi$ which may be due either to the beams becoming mis-aligned from the cold atoms as the AOM beam deflection increases with $\Delta f$, or may be an effect of the finite width of the pulses. For $t_p = 296$ ns the momentum boundary occurs at $\rho_b = 65\pi$, and as shown in \cite{17} the maximum asymmetry due to the finite pulse width occurs at approximately $\rho_L = 3\pi$, and in the negative sense. We should also note a dc offset to the data, which we believe arises from an initial misalignment of the laser beams, or from a systematic error in locating the centre of mass. The signature of the kick-to-kick correlations, however, is the ac signal which is clearly shown in both sets of data.

To break spatial symmetry a linear ‘rocking’ term of alternating sign is included by accelerating the optical lattice \cite{19}. This is done by modulating the frequency of one of the laser beams in a linear manner using a second (phase-locked) arbitrary function generator by an amount $\pm \delta f$ in the time of the kick period $T$. In the accelerating frame an inertial term appears in the Hamiltonian

$$H = \frac{\rho^2}{2M} + V_0 \cos(2k_Lx') \pm \text{max}'$$

(4)

where the primes indicate variables in the accelerating frame. If, as before, this is now recast into dimensionless form we find that (dropping the primes for convenience)

$$\mathcal{H} = \frac{\rho^2}{2} + \left(K \cos(\phi) \pm A\phi\right) \sum \delta(\tau - n)$$

(5)

where the dimensionless potential gradient is related to the magnitude of the frequency modulation (acceleration of the lattice) by $A = 2\pi t_p \delta f$ for finite pulses of width $t_p$. Accelerating the potential thus provides a simple way of controlling the magnitude of $A$ and hence controlling the phase shift of the momentum-dependent diffusion constant in order to make it locally asymmetric around zero momentum.

For the accelerating lattice experiment the parameters were $K = 2.6$, $T = 9.47 \mu s$, (so $\hbar_{\text{eff}} = 1.$) $t_p = 296$ ns and $b = 1/16$. The number of kicks was 120. As the maximum frequency modulation amplitude allowed by the radio-frequency synthesizers was $\pm 1.25$ MHz this limits the range of $A$ achievable to $\pm 3\pi/4$. In order to observe one complete oscillation of the momentum diffusion constant, for some experiments an additional constant frequency offset was introduced between the laser beams such that in the rest frame of the lattice the mean atomic momentum was $\rho_L = 8\pi$. The asymmetry of the momentum distribution (calculated as above) was measured as a function of the amplitude of the frequency modulation of the laser beam, $\delta f$, for both $\rho_L = 0$ and $\rho_L = 8\pi$ and plotted as a function of $(2\rho_L b - A)/\pi$. Results are shown in figure \ref{fig:momentum_distribution} and can be seen to be sinusoidal (proportional to the local gradient of the diffusion constant) with a period of $2\pi$ as expected from the theory.

Examples of the momentum distributions obtained from this experiment are shown in figure \ref{fig:momentum_distribution}. All three

FIG. 2: Momentum asymmetry vs starting momentum in the lattice frame for $K = 3.3$, $\hbar_{\text{eff}} = 1$, $b = 1/32$ (filled squares) and $b = 1/16$ (open triangles). The dotted lines are sinusoidal with period $\pi/b$ and are intended as a guide only.
FIG. 3: Asymmetry (first moment of momentum distribution) vs $(2\rho b - A)/\pi$ for the accelerated lattice with chirped kicks experiment. Filled squares are data for $\rho_0 = 0$, open squares are $\rho_0 = 8\pi$. The asymmetry oscillates with a period of $2\pi$. The parameters for this experiment are $K = 2.6$, $b = 1/16$, $\bar{h}_{eff} = 1$ and 120 kicks.

FIG. 4: Example momentum distributions from the accelerating lattice with chirped kicks experiment. Following the example of Jonckheere et al. we plot the modulus of the first moment of the momentum distribution, i.e. $|\rho N(\rho)|$ for clarity. It can be seen that while the distribution for $A = 0$ (black line in figure 4) is almost symmetric $A = -\pi/2$ produces a large positive asymmetry and $A = +\pi/2$ a large negative asymmetry.

In conclusion we have shown that by breaking time and space symmetry in the delta-kicked rotor correlations between kicks give rise to a momentum-dependent diffusion rate. We have exploited this in order to demonstrate experimentally a system that exhibits an asymmetric momentum diffusion due only to chaotic dynamics, in contrast to previous work that relies on specific features or structures in phase space.

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[1] C. T. Burzucha, J. C. Robinson, F. L. Moore, B. Sundaram, Q. Niu, and M. G. Raizen, Phys. Rev. E 60, 3881 (1999).
[2] W. H. Oskay, D. A. Steck, V. Milner, B. G. Klappauf, and M. G. Raizen, Opt. Commun. 179, 137 (2000).
[3] M. K. Oberthaler, R. M. Godun, M. B. d’Arcy, G. S. Summy, and K. Burnett, Phys. Rev. Lett. 83, 4447 (1999).
[4] W. H. Oskay, D. A. Steck, and M. G. Raizen, Chaos, Solitons and Fractals 16, 409 (2003).
[5] T. Jonckheere, M. R. Isherwood, and T. S. Monteiro, Phys. Rev. Lett. 91, 253003 (2003).
[6] P. H. Jones, M. Stocklin, G. Hur, and T. S. Monteiro, 93, 223002 (2004).
[7] D. R. Meacher, Contemp. Phys. 39, 329 (1998).
[8] G. Grynberg and C. Mennerat-Robilliard, Phys. Rep. 355, 335 (2001).
[9] M. G. Raizen, Adv. At. Mol. Opt. Phys. 41, 43 (1999).
[10] A. B. Rechester and R. B. White, Phys. Rev. Lett. 44, 1586 (1980).
[11] B. G. Klappauf, W. H. Oskay, D. A. Steck, and M. G. Raizen, Phys. Rev. Lett. 81, 4044 (1998).
[12] T. S. Monteiro, P. A. Dando, N. A. C. Hutchings, and M. R. Isherwood, Phys. Rev. Lett. 89, 253003 (2002).
[13] T. Cheon, P. Exner, and P. Šeba (2002), arXiv:cond-mat/0203241.
[14] G. Hur, P. H. Jones, and T. S. Monteiro (2004), submitted to Phys. Rev. A, arXiv:physics/0407100.
[15] S.-Q. Shang, B. Sheehy, P. van der Straten, and H. Metcalf, Phys. Rev. lett. 65, 317 (1990).
[16] B. G. Klappauf, W. H. Oskay, D. A. Steck, and M. G. Raizen, Physica D 131, 78 (1999).
[17] P. H. Jones, M. Goonasekera, H. E. Saunders-Singer, and D. R. Meacher, Europhys. Lett. 67, 928 (2004).
[18] K. W. Madison, M. C. Fischer, and M. G. Raizen, Phys. Rev. A 60, R1767 (1999).