Grammar induction for mildly context sensitive languages using variational Bayesian inference

Technical Report*

Eva Portelance (Stanford University)
Chris Bruno (McGill University)
Daniel Harasim (École Polytechnique Fédérale de Lausanne)
Leon Bergen (University of California San Diego)
Timothy J. O’Donnell (McGill University)

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Abstract

The following technical report presents a formal approach to probabilistic minimalist grammar induction. We describe a formalization of a minimalist grammar. Based on this grammar, we define a generative model for minimalist derivations. We then present a generalized algorithm for the application of variational Bayesian inference to lexicalized mildly context sensitive language grammars which in this paper is applied to the previously defined minimalist grammar.

Keywords: grammar induction; probabilistic minimalist grammar; variational Bayesian inference; language modeling.

1 Introduction

It is well known that natural language exhibits complex syntactic relations such as long distance and cross-serial dependencies, as in examples (1) and (2).

(1) Long-distance displacement.

\[ \text{What did you see GAP.} \]

(2) Cross-serial dependencies. (Swiss German (Shieber, 1985))

\[ \text{...mer d’chind em Hans es huus lönd hälfe aastriiche} \]
\[ \text{...we the children Hans the house let help paint} \]
\[ \text{’...we let the children help Hans paint the house’} \]
\[ \text{...d’chind em Hans es huus lönd hälfe aastriiche} \]

*Email: portelan@stanford.edu; Corresponding author
Some of these dependencies cannot be expressed by simple context free grammars. The class of grammars which generate mildly context sensitive languages first described by Joshi (1985) are the best suited for describing natural language as they are capable of expressing long-distance and cross-serial dependencies in a meaningful way which captures the generalizations put forth by linguists. Many different formalisms have emerged within this class and have been shown to be equivalent, including tree adjoining grammars (Joshi, Kosaraju, & Yamada, 1969), combinatory categorial grammars (Steedman, 1987; Szabolcsi, 1992), and more expressive linear context free rewriting Systems (Weir, 1988), multiple context free grammars (Seki, Matsumura, Fujii, & Kasami, 1991) and minimalist grammars (E. Stabler, 1997; Harkema, 2001). Some of these grammars have been formally implemented as probabilistic language models.

Probabilistic language models allow us to build parsing systems which can capture the right semantic interpretations and to develop machines which interact with people more fluently. Such models are adaptable to new inputs and can help resolve both lexical (3) and syntactic (4) ambiguities which exist in natural language by presenting us with a scalable metric of "goodness" for parses, where the evaluation metric stems from a previously motivated and well-understood detailed theory. Furthermore, such models can be used to relate our grammar to quantitative empirical data (e.g., processing times, corpus frequencies, MEG/MRI data, etc.). From a theoretical point of view, probabilistic language models present an environment for testing the learnability of syntactic theories.

(3) **Lexical ambiguity**

a. The sweater was *worn* (by Mary). - Passive verb

b. The sweater was (very) *worn*. - Adjective

(4) **Syntactic ambiguity**

a. I watched [the movie *with Jim Carrey*]. - Jim Carrey acts in the movie.

b. I [watched [the movie] *[with Jim Carrey]*]. - Jim Carrey and I watched a movie together.

Probabilistic inference for grammar induction has been applied to combinatory categorial grammars (Y. Bisk & Hockenmaier, 2013, 2012a, 2012b; Y. Y. Bisk, 2015; Wang, 2016) and tree substitution grammar (Blunsom & Cohn, 2010), but none of these use a variational Bayesian approach. Given the complexity of grammar generative models, this approach may be seen as favorable, as it turns an inference problem into an optimization problem and avoids the issues which arise from computing the posterior directly. This method has been applied to context free grammars (Kurihara & Sato, 2004) and adaptor grammars (Cohen & Smith, 2010), but has yet to be applied to the wider class of grammars which derive mildly context sensitive languages including minimalist grammars for which this technical report presents the first steps towards defining a generative model and a variational inference updating protocol. In the first section, we define a minimalist grammar and then in the following section, present the generative model for this grammar. This is then followed by the third section where we present the current formalization of the mean-field variational Bayes inference algorithm applied to the grammar described in the previous section and a set of conditions for its application to other equivalent formalisms.
2 Minimalist grammars

Minimalist Grammars are a formalization of Chomsky (1995)’s *The minimalist program*, a language processing model which is the framework used for much of the current research in linguistic syntactic theory. The interest in studying these grammars is that they offer a direct line of comparison for resulting syntactic structure predictions within the linguistic minimalist literature. In the subsections which follow, we present a working definition of a minimalist grammar.

2.1 Formalization of a minimalist grammar

The following formalization is based on the work of Harkema (2001) and E. P. Stabler (2011)’s ‘directional minimalist grammars’. A minimalist grammar is composed of a lexicon \( \mathcal{L} \) and structure building operations, typically classified as *merge* and *move*.

Let \( G \) be a minimalist grammar, such that \( G = (\mathcal{L}, R) \).

- \( \mathcal{L} \) is a finite set of lexical items with are defined as \( \pi ::= f_1, f_2, \ldots, f_n \), where \( \pi \) is a phonological feature (or a word in our case) and \( f_1, f_2, \ldots, f_n \) are syntactic features which interact with the structure building operations and are categorized into four types as follows:
  1. category (e.g. v, d, p) - define the syntactic categories (verb, noun ...);
  2. selector (e.g. d= , =p) - select argument constituent to the left or right;
  3. licensor (e.g. +case, +wh) - select moving constituent;
  4. licensee (e.g. -case, -wh) - selected moving constituent.

- \( R \) are the structure building operations which allow lexical items to merge together into large units of meaning. They are defined further bellow.

Each lexical item is categorized by one category feature. If a lexical item has selector features, these are checked by merging with constituents of the corresponding category feature. A licensor feature is used to move a previously merged constituent with the corresponding licensee feature.

\( R \) contains two structure building operations, *merge* defined in (1) and *move* defined in (2). These operations apply to constituents, both simple (a single lexical item) and complex (a series of lexical items which have been merged). which re marked by a single ‘:’. The operations are defined as follows:

Let \( x \) be a category feature, \( \gamma, \delta \) be a list of syntactic features and \( \lambda_1, \ldots, \lambda_m, \epsilon_1, \ldots, \epsilon_n \) constituents.

1. **MERGE**

   - MERGE-L (merge a non-moving item to left)

\[
\begin{align*}
[s : x = \gamma, \ldots, \lambda_m] & \quad [t : x, \epsilon_1, \ldots, \epsilon_n] \\
[ts : \gamma, \ldots, \lambda_m, \epsilon_1, \ldots, \epsilon_n]
\end{align*}
\]
- **MERGE-R** (merge non-moving item to right)

\[
[s : \{x \gamma, \lambda_1, \ldots, \lambda_m\}] \quad [t : x, \iota_1, \ldots, \iota_n]
\]

\[st : \gamma, \lambda_1, \ldots, \lambda_m, \iota_1, \ldots, \iota_n\]

- **MERGE-m** (merge an eventually moving item)

\[
[s : \{=x/x\} \gamma, \lambda_1, \ldots, \lambda_m\} \quad [t : x\delta, \iota_1, \ldots, \iota_n]
\]

\[s : \gamma, \lambda_1, \ldots, \lambda_m, t : \delta, \iota_1, \ldots, \iota_n\]

2. MOVE

- **MOVE-1** (moving item to final landing position)

\[
[s : +y \gamma, t : -y, \lambda_1, \ldots, \lambda_m] \quad [ts : \gamma, \lambda_1, \ldots, \lambda_m]
\]

- **MOVE-2** (moving item which will move again)

\[
[s : +y \gamma, t : -y\delta, \lambda_1, \ldots, \lambda_m] \quad [s : \gamma, t : \delta, \lambda_1, \ldots, \lambda_m]
\]

Here, \(\lambda_1, \ldots, \lambda_m, \iota_1, \ldots, \iota_n\) represent constituents which have previously merged using MERGE-m into the derivation, but still carry licensee features - i.e. items which have not reached their final landing position in the syntactic structure.

The figure in (5) is an example of a derivation in this system using R and the following lexical items defined.

(5) Derivation of 'what did you see?'.

\[
\mathcal{L} = \{ \text{what} :: d -\text{wh}, \text{see} :: =d \ d= v, \text{you} :: d, \text{did} :: =v \ i, \emptyset :: =i +\text{wh} \ c \}
\]

\[
\text{MOVE}
\]

\[
\text{c}
\]

| \text{what did you see} |
|--------------------------|
| \text{MERGE} |
| +\text{wh} \ c, -\text{wh} |

\[
=\text{i} +\text{wh} \ c \quad \text{MERGE} \quad \text{i, -\text{wh}}
\]

\[\text{(did you see, what)}\]

\[=v \ i \quad \text{MERGE} \quad v, -\text{wh} \]

\[\text{(you see, what)}\]

\[d \quad \text{MERGE} \quad \text{you} \ d= v, -\text{wh} \]

\[\text{(see, what)}\]

\[=d \ d= v \ d -\text{wh} \quad \text{see} \quad \text{what}\]
1. MERGE-m: v selects d what;  
2. MERGE-L: v selects d you;  
3. MERGE-R: i selects v you see, what;  
4. MERGE-R: c selects i did you see, what;  
5. MOVE: -wh moves to satisfy +wh what did you see;  
6. All features are satisfied.

Thus, this minimalist grammar allows us to represent long distance displacements as this example shows. In the next section I develop a generative model based on this minimalist grammar.

### 3 The generative model for a minimalist grammar

Developing a generative model for minimalist grammars is not intuitively obvious for two reasons. The first reason is minimalist grammars have always been defined ‘bottom up’: they begin with a set of lexical items (this is called the enumeration in Chomsky (1995)’s original theory) and use the structure building rules to derive a tree rooted by a single category feature. However, when sampling, we need to adopt a ‘top-down’ approach to the derivation problem to make sure we only sample derivation trees which terminate - i.e. are rooted by a single category feature. The second problem we face is how to handle MOVE. This operation complicates things significantly because it introduces the concept of tuples of items. For this reason, for our initial implementation we have decided to define a generative model which excludes MOVE as a possibility. However, we hope to update this model to include the all the minimalist structure building operations soon. Essentially, what this means is that the following generative model is equivalent to a context free grammar.

Let $c$ be an arbitrary category feature. We define a Dirichlet prior over $\theta$ parameterized by a distribution over categories of lexical items $\alpha$. $\theta_c$ is the probability distribution over all possible feature sets which contain the category $c$ such that $\sum_{i=1}^{N} = 1$. Furthermore, $\theta_{l,c}$ is the probability of the lexical item $l$ of category $c$ and $\text{cat}(l)$ is a function which returns the category feature of $l$.

$$p(\theta) = \prod_{\forall c} \text{DIRICHLET}(\theta_c, \alpha_c)$$

where,

$$\alpha_c = [\alpha_{c,l}|\text{cat}(l) = c]$$

We are explicitly sampling a distribution over each category from a Dirichlet. Effectively, our probabilistic minimalist grammar is defined as $(G, \{\theta_c, \forall c \in L\})$.

We can generate a derivation tree using the following recursive structure. I define the function $\text{selects}(l)$ and the function $\text{cat}(l)$ which take a lexical item as input and return respectively the selector features of that item and the category feature of the item. Furthermore, we define a function $\text{lexitem}(c)$ which takes as input a category feature and returns a lexical item of that category. Then we can recursively sample structures for derived constituents using the following equation. We define $d_l$ to be a derivation headed by the lexical
item $l$. Let $x_1 \ldots x_k$ by selector features.

$$MG^{\text{cat}}(l_i) = \begin{cases} 
\theta_{l,c} \times \prod_{i=1}^{k} MG^x(d_{\text{LEXTEM}(x_i)}) \\
\text{selects}(l) = x_1 \ldots x_k \\
\text{selects}(l) = \emptyset
\end{cases}$$

(3)

Concretely, this returns the probability of the derivation $d_i$, which is the product of the probability of all the lexical items which compose it.

Finally, we define $S$, our corpus, where $S = \{s_1, s_2, \ldots, s_n\}$, and $D$, the set of possible derivations for $S$, where $D = \{d_1, d_2, \ldots, d_n\}$. We can now define the posterior we are interested in as follows:

$$p(D, \theta | S) = p(S | D, \theta) p(D | \theta) p(\theta)$$

(4)

Given that the denominator of this fraction is not a tractable problem, using variational Bayesian inference to approximate the posterior represents a feasible way of doing inference for this model.

4 Variational Bayesian inference for minimalist grammar induction

Mean field variational inference is based on the assumption that all variables in the approximated posterior are independent and therefore it can be factorized.

$$q(D, \theta | S) = q(D | S) q(\theta | S) = \prod_{i=1}^{n} q(d | s_i) \prod_{\forall c} q(\theta_c | S)$$

(5)

where $d$ is the set of possible derivations of $s_i$.

We define an incremental updating scheme for $q(D, \theta | S)$ as follows. First, we update $q(\theta | S)$.

$$q(\theta_c | S) = \text{DIRICHLET}(\theta_c, \hat{\alpha}_c)$$

(6)

where

$$\hat{\alpha}_c = [\hat{\alpha}_{c,l} | \text{cat}(l) = c]$$

(7)

then,

$$\hat{\alpha}_{c,l} = \alpha_{c,l} + \sum_{i=1}^{n} \sum_{d_i \in \omega(s_i)} q(d_i | s_i) \text{COUNT}(l, d_i)$$

(8)

$\alpha_{c,l}$ is a hyperparameter of the Dirichlet prior, $\omega(s_i)$ returns all the possible derivations for $s_i$ and $\text{COUNT}(l, d_i)$ returns the counts of $l$ in some derivation $d_i$ in $\omega(s_i)$, given the current estimate of the grammar. Finally, we proceed to update $q(\theta | S)$.

$$q(\theta | S) = \prod_{\forall c} q(\theta_c | S)$$

(9)
Second, we update $q(D|S)$. 

$$q(D|S) = \prod_{i=1}^{n} q(d_i|s_i)$$  \hspace{1cm} (10)$$

$$q(d_i|s_i) = \frac{\prod_{l} \tau(l)^{\text{count}(l,d_i)}}{\sum_{d_i \in \omega(s_i)} \prod_{l} \tau(l)^{\text{count}(l,d_i)}}$$  \hspace{1cm} (11)$$

where

$$\tau(l) = \exp[\psi(\hat{\alpha}_c,l) - \psi(\sum_l \hat{\alpha}_c,l)]$$  \hspace{1cm} (12)$$

and $\psi$ is the digamma function.

By updating $q(\theta|S)$ and $q(D|S)$ incrementally, and given that they are codependent, we can optimize $q(D, \theta|S)$.

5 Variational inference for mildly context sensitive language grammars

Given that variational Bayesian inference transforms an inference problem into an optimization one, unlike sampling-based methods of inference, the method can easily be translated to work for other models as long as they can satisfy certain assumptions. The previous example of variational inference applied to a minimalist grammar was built on three assumptions.

(6) Assumptions for generalized variational Bayesian inference for mildly context sensitive language grammars

1. The latent derivation trees are context-free;
2. The probabilities of the trees in (1) can be decomposed into the product of their components' probabilities;
3. There is a Dirichlet prior over lexical items/rules.

The first assumption supposes that each lexical item is well-formed and conditionally independent, however the linearization assumption usually given in CFGs is not required of mildly context sensitive language grammar derivation trees. The variational inference updating protocol presented can be generalized to any lexicalized mildly context sensitive language grammar formalism that satisfies these three assumptions.

6 Conclusion

In this paper, we presented the formalization of a minimalist grammar and a generative model for this grammar as well as a generalized variational Bayesian approach to mildly context sensitive language grammar induction. We demonstrated how this approach can be applied to a specific grammar instantiation within this equivalence class using the case of a minimalist grammar. In future work, we hope to implement a computable version of this algorithm which will be integrated into a parsing framework which is also currently being developed (Harasim, Bruno, Portelance, & O’Donnell, 2017). This new framework will then be used to test linguistic syntactic theories and provide a baseline for future development of unsupervised language learning models as well as more ‘human’-like natural language processing applications.
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