Δ33-medium mass modification and pion spectra

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Abstract. We study the π±-spectra obtained in 2, 4, 6 and 8 A GeV Au–Au collisions within the thermal model. We find that the main features of the data can be well described after we include the pions from the decay of the Δ-resonance with medium mass modification.

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1 Introduction

Heavy ion collision experiments, in the energy range of several GeV, permit the study of nuclear matter and hadron properties under extreme conditions of temperature and density. Here, we are interested in how the pion spectra are influenced by the in medium mass modification. Considering the most precise pion spectra available, obtained in the energy range 2–8 A GeV [1], we show that there is sensitivity to the medium modification of the mass splitting between Δ33 and N. The availability of both pion charge polarities allows us to address the pion isospin asymmetry, an important ingredient in our analysis.

A prominent feature of these π±-spectra is that they are not described by a single thermal source. Instead, the initial analysis by the experimental group used a two-component thermal model with two different source temperatures. The dynamical origin of such a model is at present open to discussion. On the other hand, the decay Δ33(1232) → N + π has long been recognized to be a significant mechanism of pion production in the reaction energy range considered here [2,3]. Moreover, the shape of the decay pion spectrum is different from the shape of a thermal spectrum [4].

Thus, in order to obtain the observed momentum spectra of pions, we consider the sum of the direct thermal pion component with the in-medium-decay of the thermal Δ33 component. Only decays near the kinetic freeze-out contribute in the second component: the early decay product pions are reequilibrated and are part of the pion thermal component. The number of particles at a given value of momentum (the spectrum) is established by considering statistical distributions which arise at temperature-dependent values of hadron masses. Below the kinetic (i.e., scattering) freeze-out temperature at about $T = 100 \pm 20$ MeV, in the free-streaming domain, the hadron mass values are restored to the free space value. Moreover, as hadrons begin to free-stream out of the interaction region, their mass returns to the free space value, the required energy is derived from the modification of the volume-vacuum energy. Since all particles actually observed experimentally had time to ‘recover’ from medium modifications to their properties, till now the experimental study of this medium effect has focused on the observation of the possible modification of the decay width in matter, especially of the φ-meson [5].

In order to describe the pion spectra we will show that the required magnitude of the mass medium modification is in agreement with the widely held believes about the medium dependent properties of strongly interacting particles in dense, hot matter [6,7]. Considering the quantum-hadrodynamic model [8], the cancellation of several contributions to the energy of a nucleon with magnitude of a few hundred MeV leads to the relatively small nuclear binding energy in normal nuclear matter. One of these large components is the scalar potential which modifies nucleon mass. At normal nuclear density it has a magnitude of several 100 MeV, and as the nuclear matter is squeezed, this potential increases such that the effective nucleon mass melts entirely (see for example Fig 2. in Ref. [8]). Already at relatively moderate temperatures the thermal melting is more important compared to the density effect. A way to express the hybrid temperature-density dependence of mass modification is by the substitution:

$$T \to T_{\text{eff}} \simeq \sqrt{T^2 + (\mu_b/2\pi)^2}.$$  (1)

where $\mu_b$ is baryochemical potential.

A brief description of the thermal model particle spectra is given in the following section [2]. An analysis of the
pion spectra based on the proposed model with medium mass modification is presented in section [3] followed by a summary and discussion of our work in section [4].

2 Particle spectra in the thermal model

An important question governing the validity of the study presented here is: In what way is the pion spectrum modified by the medium? When a medium-modified hadron emerges into medium free space, the hadron mass relaxes and the hadron picks up (or if appropriate releases) the energy from/to the vacuum, and its mass returns to the normal free space value. In such a process, within an isotropic medium, the translational momentum of a particle cannot change. However, along with the mass, the energy of the particle will also change. Since the momentum distribution of particles with \( p = E / v \) is not changed in the process of free space mass restoration:

1) the number of thermal particles with momentum \( p \) is governed by the thermal momentum distribution of the source, obtained with medium modified mass, for example in the Boltzmann (classical) limit:

\[
\frac{d^3 N}{d^3 p} \propto e^{-\sqrt{m(T)^2 + p^2} / T},
\]

and thus for particle spectra all masses are to be understood to be medium-modified;

2) one should be able to reconstruct a decaying resonance using the medium modified masses for both the resonance and its decay products, for example:

\[
m_{A}^{2}(T) = \left( E_{\pi} + E_{N} \right)^{2} - \left( p_{\pi} + p_{N} \right)^{2} = 2 \left( \sqrt{m_{\pi}^{2}(T) + p_{\pi}^{2}} \sqrt{m_{N}^{2}(T) + p_{N}^{2}} - p_{\pi} \cdot p_{N} \right) + m_{\pi}^{2}(T) + m_{N}^{2}(T). \tag{3}
\]

Here all momenta are as measured in the final state.

This discussion shows that in order to obtain the observed momentum spectra of produced particles, we have to use medium-modified masses in the usual thermal model expressions, and discuss the results as function of the measured momentum. Other than the medium effect, our analysis proceeds along previously established methods [9][10][11]. The thermal momentum distribution of primary pions is, by assumption, given by the standard thermal Bose expression, and the decay products, for example:

\[
\frac{dN}{dy} = 2\pi N_{\pi} \int \frac{\Upsilon_{\pi} m_{\pi} \cosh y}{e^{\beta m_{\pi} \cosh y} - \Upsilon_{\pi}} dp_{\pi}. \tag{7}
\]

We expect matter flow of the system in the longitudinal direction:

\[
\frac{dN(y)}{dy} = \int_{\eta_{\min}}^{\eta_{\max}} \frac{dN(\eta - y)}{dy}, \quad \beta_{L} = \tanh(\eta_{\max}), \tag{8}
\]

where \( \eta_{\max} = -\eta_{\min} \), from symmetry about the center of mass, and \( \beta_{L} \) is the maximum longitudinal velocity. Thus we have the distribution of thermal pions

\[
\frac{1}{2\pi} \frac{dN(y)}{dy} = N_{\pi} \int_{\theta_{\min}}^{\theta_{\max}} \frac{\Upsilon_{\pi} m_{\pi} \cosh(\eta - y)}{e^{\beta m_{\pi} \cosh(y - \eta)} - \Upsilon_{\pi}} d\eta. \tag{9}
\]

Then the rapidity distribution is

\[
\frac{dN(y)}{dy} = 2\pi N_{\pi} \int_{\eta_{\min}}^{\eta_{\max}} \frac{\Upsilon_{\pi} m_{\pi} \cosh(y - \eta)}{e^{\beta m_{\pi} \cosh(y - \eta)} - \Upsilon_{\pi}} dp_{\pi} d\eta. \tag{10}
\]

3 Analysis of the data

In order to model of the charged pion spectra [1]:

1) We describe the high \( p_{t} \) component of the pion \( m_{i} \) spectra in terms of a thermal Bose shape, which in fact in this case is essentially an exponential. In this way we obtain the effective inverse slope \( T \). This \( T \) comprises a combination of the intrinsic thermal temperature \( T_{th} \) and the shift due to the collective longitudinal and radial expansion flow. The longitudinal flow is required for an accurate description of the rapidity spectra.
2) We form the difference between the direct pion spectrum determined by the high \( m_t \) fit and the experimental spectrum. We then model this difference in terms of the pion decay spectrum \( \Delta_{33}(1232) \rightarrow N + \pi \). We form the pion spectrum from direct and decay components using a fitted strength of the not-scattered \( \Delta_{33} \)-decay component.

3) We find that the pion spectrum can be described precisely when we reduce all hadron masses using a uniform multiplicative factor. A further improvement is apparently possible when the radial flow is introduced, but we do not pursue such refinements here for reasons already discussed.

The thermal momentum distribution of primary pions is, by assumption, given by the thermal Bose distribution Eq. (4). In the center of the hot, dense collision zone, high pressure is produced. This pressure will cause a collective motion of the system which has been experimentally observed [14][5]. However, it is not possible for us to disentangle thermal and collective flow effects [4]. Thus, the \( m_\perp \)-slope we obtain comprises some Doppler-like shift due to source motion. Aside of the collective radial motion we expect that, in the longitudinal direction, the memory of the projectile-target motion will remain.

Such longitudinal flow \( \beta_L \) is considered in Ref. [1], employing a flat longitudinal rapidity profile bounded by \( \eta_{\text{max}} = -\eta_{\min} \), where \( \beta_L = \tanh(\eta_{\text{max}}) \). A somewhat simpler approach is to consider two fluids moving apart with a remainder of the original projectile and collective rapidity:

\[
\frac{d^2N(p_t, y)}{dp_t dp_y dy} = \frac{1}{2} \frac{d^2\tilde{N}(p_t, y - \eta_0)}{dp_t dp_y dy} + \frac{1}{2} \frac{d^2\tilde{N}(p_t, y + \eta_0)}{dp_t dp_y dy}
\]

We will fit \( \eta_0 \) to the experimental rapidity distributions. For the \( m_t \) spectrum inclusion of these two sources means that at \( y = 0 \) we use Eq. (4) substituting \( m_t \rightarrow m_t \cosh \eta_0 \).

The parameters of the thermal model are presented in the top section of table 1, where \( T_{\text{slope}} = T / \cosh \eta_0 \), \( T \) comprises the effect of transverse expansion dynamics. For large \( p_t \), the pion fugacity is irrelevant; the thermal spectrum has a Boltzmann shape and is dominated by direct thermal component, and the fugacity becomes another component in the yield normalization. In this limit value of the mass of the pion is also irrelevant, as it is negligible compared to the value of \( p_t \). Hence the domain of large \( p_t \) >> \( m_\pi \) fixes the value of \( T_{\text{slope}} \). There remains a major difference between experimental spectrum and a single component thermal spectrum as was noted in [1]. This difference is shown in Fig. 1. This difference appears similar at all reaction energies and is localized in to a momentum range \( p_t < 0.4 \) GeV. This suggests as the common and unaccounted mechanism of pion production the \( \Delta_{33} \)-resonance decay.

For small \( p_t \) there is considerable impact of the Bose nature of the pion on the spectra. Furthermore, the difference in the shape between the positive and negative pion spectra can be in part accommodated by differences in \( T_{\pi^\pm} \), and in part by the associated difference in the relative yields of the \( \Delta_{33} \)-resonances as indicated by the power of \( \lambda_{13} \) for each of the polarities, see last column in table 2. The branching into the different channels for each of the \( \Delta_{33} \)-resonances is relevant considering the isospin weights. Since we allowed for the isospin asymmetry in the fugacity, we could insist in normalizing the weights. Since we allowed for the isospin asymmetry in the fugacity, we could insist in normalizing the weights.

### Table 1. Top: statistical parameters used to describe \( \pi^\pm \) spectra; bottom: modified hadron masses as used at different beam energies.

| Beam Energy   | 2A GeV | 4A GeV | 6A GeV | 8A GeV |
|---------------|--------|--------|--------|--------|
| \( T_{\text{slope}} \) [MeV] | 121    | 128    | 133    | 135    |
| \( \eta_0 \)     | 0.15   | 0.25   | 0.3    | 0.3    |
| \( T \) [MeV]    | 122    | 132    | 139    | 141    |
| \( \lambda_{13} \) | 1.2    | 1.15   | 1.15   | 1.15   |
| \( \gamma \) [GeV] | 1.85   | 1.62   | 1.44   | 1.44   |
| \( N_{\pi^-} \) [GeV^{-3}] | 120   | 339    | 521    | 647    |
| \( N_{\pi^+} \) [GeV^{-3}] | 136   | 383    | 587    | 727    |
| \( N_{\Delta e^{+B}/T_{\text{ch}}} \) | 2.170  | 983    | 630    | 536    |
| \( m_t/m_\pi \) | 0.72   | 0.63   | 0.59   | 0.57   |
| \( m_\Delta \) [GeV] | 0.880  | 0.774  | 0.722  | 0.704  |
| \( m_N \) [GeV]   | 0.671  | 0.590  | 0.550  | 0.537  |
| \( m_\pi \) [GeV] | 0.100  | 0.088  | 0.082  | 0.080  |
| \( \chi^2 / \text{dof} \) | 1.16   | 1.20   | 1.04   | 1.19   |

### Table 2. Delta decays and fugacities.

| Decay Channel | \( \gamma \) |
|---------------|--------------|
| \( \Delta^{++} \rightarrow (\pi^+ + p) \) | \( \gamma_{13}^{1/2} \) |
| \( \Delta^{+} \rightarrow 1/3(\pi^+ + n), 2/3(\pi^0 + p) \) | \( \gamma_{13}^{1/2} \) |
| \( \Delta^0 \rightarrow 1/3(\pi^+ + p), 2/3(\pi^0 + n) \) | \( \gamma_{13}^{1/2} \) |
| \( \Delta^- \rightarrow (\pi^- + n) \) | \( \gamma_{13}^{1/2} \) |

To describe this difference spectrum quantitatively, we consider \( \Delta_{33} \) two body decay, and employ the well known results, see, e.g., Ref. [9][10].

\[
\frac{d\tilde{N}_{\text{de}}}{p_t dp_y dy} = \frac{m_R b}{4\pi p^*} \int_{y_R^-(\gamma)}^{y_R^+(\gamma)} \frac{dy_R}{\sqrt{m_R^2 \cosh^2(y - y_R) - p_t^2}} \times \int_{m_{\pi R}^-}^{m_{\pi R}^+} \frac{dm_{\pi R}}{(m_{\pi R} - m_{\pi R}^-)(m_{\pi R} - m_{\pi R}^+)} \frac{d\tilde{N}_R}{m_{\pi R} dE_R} dy_R,
\]

where \( b \) is the branching ratio, and \( p^* \) is the momentum of the decay particle in the \( \Delta_{33} \)-rest-frame. For the \( \Delta_{33} \)-resonance spectrum we assume at first a Boltzmann distribution, with \( T \) determined by the pion spectrum and the \( \Delta_{33} \)-fugacity shown in table 2.

\[
\frac{1}{2\pi m_{\pi R} dE_{\pi R} dy} = N_{\Delta e^{+B}/T_{\text{ch}}} \frac{T_{\Delta e^{+B}/T_{\text{ch}}}}{\exp(\beta m_{\pi R} \cosh y R)},
\]

\[\text{(13)}\]
In the resulting pion decay component we also introduce the longitudinal flow according to Eq. (11). In this way we allow effectively for the longitudinal motion of the $\Delta_{33}$-source. The total pion distribution is composed of the direct and the decay contributions.

When computed with the free space masses of $\pi, \Delta_{33}, N$ the resulting spectrum of decay pions is found to have the shape seen in Fig. 1 but is systematically shifted to a higher momentum than the experimental data. In order to reduce the energy scale to the level seen in the difference spectrum in Fig. 1 we scale down the three hadron masses $\pi, \Delta_{33}, N$ by the same factor, which is chosen in a qualitative manner. The resulting masses we use are stated in the bottom section of table 1; the modification is non-negligible. There is a tendency for the mass reduction effect to increase with collision energy. As is seen in Fig. 1 (solid lines), after this modification we describe the experimental data rather well.

We also have to allow for the presence of matter flow. In general the radial flow has the effect of flattening the $\Delta_{33}$ spectrum more than the $\pi$ spectrum, since the effective inverse slope $T$, which combines the intrinsic temperature $T_{th}$ with the blue-shift of flow, grows somewhat with particle mass and collective velocity $v$ [16]. We account for the difference that the flow effect has on $\pi$, and the much heavier $\Delta_{33}$ as follows: We keep all quantities except for the distribution $N_{\Delta}$ unchanged. We modify the $\Delta_{33}$-spectrum using a simplification of the model proposed in Ref. [17]. For the flow rapidity profile $\eta_k(r) = \eta_f(R/R)^\alpha$, we take $\alpha = 0$ and $\eta_f = 0.3$. We allow here a cylindrical fireball since this is not a significant element in our approach and the model is simple and transparent. The contributions from the $\Delta_{33}$ decay with radial flow are shown in Fig. 1 solely for 8 GeV beam energy, where this effect can be expected to be largest, as dashed lines. There is a mild improvement in the high $p_t$ part of the difference

![Fig. 1. Difference of experimental $\pi^\pm$ yields and the direct thermal (th) yields, for the mid-rapidity data bin [1]. The contribution from mass modified $\Delta_{33}$ decay is shown by solid line. Contribution from mass modified $\Delta_{33}$ decay with radial flow is shown by dashed line for the case of 8 GeV beam energy (top panels). Dot-dashed line shows the $\Delta_{33}$ decay contribution without mass modification.](attachment:image.png)
spectrum. These results suggest that the complete and detailed inclusion of the radial flow, which has the largest influence for highest reaction energy, will further improve the understanding of the difference spectra seen in Fig. 1.

The integral of the transverse momentum spectra leads to the rapidity distribution shown in Fig. 2. In performing this integral it is relevant for the rapidity dependent normalization to disentangle in the slope temperature the flow effect from the true temperature. Since we already have seen that matter flow is desired, it is not surprising that when we do this introducing for simplicity a longitudinal flow $\eta_0$, the rapidity spectrum is described very well. The resulting $\chi^2$/dof is given in the bottom of table 1. It should be remembered that this is not a best fit optimizing the 8 parameters (4 statistical, 3 normalization parameters, and one mass scale parameter which determines the three hadron masses) at each reaction energy, and that we have treated the flow in somewhat cavalier, but as we think, precise enough manner.

### 4 Discussion and Summary

We have obtained an accurate description of pion $\pi^\pm$ spectra, consistent across the four reaction energies considered here. Our objective we had here was to show that the pion spectra we consider are sensitive to the introduction of the mass change parameter. We believe that we have demonstrated this in that we can explain quite satisfactorily the spectra considered. The relevant model parameters were determined in part by using the common knowledge and in part optimizing the parameters to the data. We did not make an effort to minimize the value of $\chi^2$, since several important physics elements are beyond the scope of the current approach.

We generally proceeded as follows: we chose a reasonable value of the hadron mass reduction parameter, the longitudinal flow $\eta_0$, and the two fugacities. Once this choice is made, the temperature $T$ (common for both $\pi^+$ and $\pi^-$), the two pion normalization factors $N_{\pi^\pm}$ and the $\Delta_{33}$-yield normalization are fitted to the data. We then tried another reasonable set of the three initial param-
eters in an effort to further reduce the $\chi^2$ and did this several times.

We would like to make a few additional remarks about the physical meaning of the parameters, possible future directions, and consistency of our approach:

1) One could argue that the softening of the pion decay spectrum, i.e., the difference spectrum seen in Fig. 1 is not due to mass modification but is due to decay pion scattering in matter. It is however hard to understand why this process at all reaction energies would have the effect of producing the $\Delta_{33}$-decay spectrum shape, with reduced energy scale. We note that though the data description we present is relatively good, at low $p_t$ there is some potential to improve the model, e.g. by considering the effect of pion scattering.

2) In Table 1 $N_\Delta$ is common for different isospin states of $\Delta$. For 6, 8 GeV beam energy cases, $N_\Delta$ and $N_\pi$ are comparable. However, $N_\Delta > N_\pi$ for 2 GeV beam energy. This suggests that the direct thermal pion component is dominated by the indirect pion production via the $\Delta_{33}$-resonance. Indeed, as is seen in Eq. (13), the $\Delta_{33}$-yield comprises a factor, $e^{\mu_B/T_{ch}}$, where $\mu_B$ is the baryon chemical potential. The rapid growth in the normalization factor with decreasing reaction energy is due to the expected increase of $\mu_B/T_{ch}$.

3) Hadron model calculations \cite{5} find mass shifts which are small, and decay width increases which are large. We note that the effect obtained in the bag model effect differs. The decrease in the hadron masses is accompanied by a decrease in transition energy, and thus the decay phase space diminishes. This reduces the width. There is a further potential reduction due to the reduction of the intrinsic decay matrix element, since for a smaller value of the bag constant $B$ the quark bag grows in size, diluting the strength of the reaction matrix element. We recall that at present an increase of mass shift has not been observed experimentally, while a decrease in width is not inconsistent with the available data. Thus a priori, a bag model description seems to agree better with data.

4) The consistency of our model can be tested by introducing the actual $\Delta_{33}$-resonance spectra obtained from a reconstruction by the invariant mass method of the mass modified $\Delta_{33}$-resonance using modified masses of decay pions and nucleons. Success of this would indeed be a convincing step supporting the path of analysis proposed here. Also, it would allow us to include in our approach the spectra of nucleons.

5) The matter flow field arises from initial conditions and ensuing dynamics driven by equations of state. Thus there is in principle a relation between the magnitude of the flow and the freeze-out condition (temperature, matter density) for each heavy ion reaction energy, if initial conditions could be understood. Our results are consistent with what may be called, in absence of theoretical understanding, the general wisdom about the magnitude of matter flow, temperature and density expected in conditions considered here.

In summary, we have studied the precision spectra of $\pi^\pm$ obtained for kinetic beam energy per nucleon 2, 4, 6, 8 A GeV at AGS. We can interpret the $\pi^\pm$ spectra as originating in a thermal fireball of dense matter. The necessary ingredients are aside of direct $\pi^\pm$ production, secondary $\Delta_{33}$-resonance decay, longitudinal flow, and a significant mass modification of the involved hadrons. In fact, the mass modification is the new physics element in the analysis presented, allowing a significant improvement of the data description using a single thermal source.

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References

1. J. L. Kluy et al. [E-0895 Collaboration], Phys. Rev. C 68, 054905 (2003) [arXiv:nucl-ex/0306033].
2. B. Hong et al. [FOPI Collaboration], Phys. Lett. B 407, 115 (1997) [arXiv:nucl-ex/9706001].
3. J. Barrette et al. [E814 Collaboration], Phys. Lett. B 351, 93 (1995) [arXiv:nucl-ex/9412002].
4. G. E. Brown, J. Stachel and G. M. Welke, Phys. Lett. B 253, 19 (1991).
5. R. Rapp and J. Wambach, Adv. Nucl. Phys. 25, 1 (2000) [arXiv:hep-ph/9909229].
6. G. E. Brown, V. Koch and M. Rho, Nucl. Phys. A 535, 701 (1991).
7. G. E. Brown and M. Rho, Phys. Rep. 269, 333 (1996).
8. B. D. Serot and J. D. Walecka, Int. J. Mod. Phys. E 6, 515 (1997) [arXiv:nucl-th/9701058].
9. J. Sollfrank, P. Koch and U. W. Heinz, Phys. Lett. B 252, 256 (1990).
10. J. Sollfrank, P. Koch and U. W. Heinz, Z. Phys. C 52, 593 (1991).
11. J. Letessier, A. Touss and J. Rafelski, Phys. Lett. B 475, 213 (2000) [arXiv:nucl-th/9911043].
12. J. Letessier and J. Rafelski, "Hadrons and quark - gluon plasma," Cambridge Monogr. Part. Phys. Nucl. Phys. and Cosmol. 18 1-397 (2002).
13. G. Torrieri, W. Broniowski, W. Florkowski, J. Letessier and J. Rafelski, Comp. Phys. Com. 167, 229 (2005), see: www.physics.arizona.edu/~torrieri/SHARE/share.html [arXiv:nucl-th/0404083].
14. M. A. Lisa et al. [EOS Collaboration], Phys. Rev. Lett. 75, 2662 (1995) [arXiv:nucl-ex/9502001].
15. P. J. Siemens and J. O. Rasmussen, Phys. Rev. Lett. 42, 880 (1970).
16. I. G. Bearden et al. [NA44 Collaboration], Phys. Rev. Lett. 78, 2080 (1997).
17. H. Dobler, J. Sollfrank and U. W. Heinz, Phys. Lett. B 457, 353 (1999) [arXiv:nucl-th/9904018].