Is timing noise important in the gravitational wave detection of neutron stars?

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In this paper we ask whether the phenomenon of timing noise long known in electromagnetic pulsar astronomy is likely to be important in gravitational wave (GW) observations of spinning-down neutron stars. We find that timing noise is strong enough to be of importance only in the young pulsars, which must have larger triaxialities than theory predicts for their GW emission to be detectable. However, assuming that their GW emission is detectable, we list the pulsars for which timing noise is important, either because it is strong enough that its neglect by the observer would render the source undetectable, or else because it is a measurable feature of the GW signal. We also find that timing noise places a limit on the observation duration of a coherent blind GW search, and suggest that hierarchical search techniques might be able to cope with this problem. Demonstration of the presence or absence of timing noise in the GW channel would give a new probe of neutron star physics.

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I. INTRODUCTION

Spinning triaxial neutron stars may provide a source of detectable gravitational radiation for the new generation of gravitational wave interferometers [1]. The known pulsar population will be targeted in forthcoming GW searches, as their locations are known to high accuracy and their rotational properties have been studied in some detail, greatly aiding the search. The GW amplitudes of these sources are likely to be low, so that only by coherently accumulating signal over long time intervals (months to years) is there any hope of making a positive detection [2]. This accumulation can be achieved by the process of matched filtering, where the noisy detector output is multiplied by a template waveform which remains in phase with the GW signal to better than one radian over the entire observation span, and the resulting product integrated.

The spin frequencies of almost all pulsars are observed to gradually decrease, presumably because of the loss of energy caused by electromagnetic and (hopefully) GW emission [3]. Apart from occasional glitches, this frequency change is gradual, so that pulsar physicists can model the rotation phase using only a few terms of a Taylor series of the form

$$\Phi = 2\pi \int f_0 + \dot{f}t + \ddot{f}t^2/2 + ... \, dt.$$ (1)

However, accurate radio timing observations reveal a small irregularity in this spin-down known as timing noise, occurring on long timescales, comparable with the likely GW observation timescales [4]. This noisy behavior of the electromagnetic signal may well be present in the GW signal too, as almost all parts of a neutron star are believed to be coupled together strongly on a timescale of seconds or less [5].

If present in the GW signal the timing noise would result in a phase difference between the idealized Taylor expansion and the real noisy GW signal which grows in time (see Equation (1) below). It follows that there would exist a timescale $T_{\text{decoherence}}$ at which a simple Taylor series of the above form will drift out of phase with the template by one radian, leading to a complete loss of signal-to-noise and preventing detection. This could be prevented by the GW observer going to the trouble of using the electromagnetically detected phase to generate the GW template. A method for achieving this has been devised by Pitkin & Woan for the Crab pulsar [6].

However, the timing noise should not be regarded simply as a nuisance, complicating the detection process. As described in section II, the strength of the timing noise can be used to probe stellar structure, giving a unique insight into neutron star physics. The accuracy with which the phase of the GW signal can be extracted from the noisy data stream is proportional to one over the signal-to-noise ratio [7], and so the GW phase measurement error decreases as $T^{-1/2}$, where $T$ is the duration of the coherent observation. It follows that there exists a timescale $T_{\text{detectable}}$ at which the strength of the timing noise in the GW data stream can be measured.

With these remarks in mind, we will pose and answer the following three questions in this paper:

- For each known pulsar, on what timescale $T_{\text{decoherence}}$ would the timing noise, if present in its GW signal, cause a simple Taylor series search template to drift out of phase with the actual signal, leading to a complete loss of signal-to-noise?

- For each known pulsar, on what timescale $T_{\text{detectable}}$ would the strength of GW timing noise be measurable?

- Is timing noise likely to be important when performing blind searches, i.e. GW searches for neu-
tron stars not currently observed as pulsars?
In all cases we have chosen to restrict our attention to observation durations of three years or less. This is partly because the computational costs of long coherent observations are very large \(8\), and partly because the interferometers are likely to be upgraded significantly over such a timescale, in which case analysis of a smaller set of newer, higher quality data might be more profitable.

The structure of this paper is as follows. We begin by summarizing the physical significance of timing noise in section II. In section III we assemble formulae necessary to estimate \(T_{\text{decoherence}}\) and \(T_{\text{detectable}}\). In section IV we describe how we estimate the strength of timing noise in the known pulsar population. In section V we provide both upper bounds and somewhat more realistic estimates of the GW amplitudes of these pulsars. In section VI we estimate the decoherence timescales of the pulsars. In section VII we estimate possible timescales for detectability of timing noise in the GW signal. In section VIII we briefly discuss the importance of timing noise for blind searches. We summarize our results in section IX.

II. THE PHYSICAL SIGNIFICANCE OF TIMING NOISE

Theoretically, the origin of timing noise is not understood. Eight different models of timing noise were considered in \(8\), where the statistical properties of the timing residuals of each model were calculated and compared with electromagnetic observations. Some of the models performed better than others, but a definitive identification of the basic mechanism at work proved impossible.

Nevertheless, it is instructive to consider three rather general scenarios, without linking them to any particular model of timing noise. In the first the whole star, including the pulsation-producing magnetosphere, rotates as a rigid body, so that the relative phase of the GW and electromagnetic wave (EW) is constant in time. In the second, the timing residuals represent a purely magnetospheric phenomenon, where the location of pulsation production, at a height of several neutron star radii above the surface, wanders randomly in longitude, without being any corresponding variation in the rotational phase of the star. In the third scenario, the variation in phase is ascribed to a weak random angular momentum exchange between the part of the star tied to the EW emission and the part tied to the GW emission. Denoting the moment of inertias of these two parts by \(I_{\text{EW}}\) and \(I_{\text{GW}}\), and the departure of their angular velocities from a smooth spindown law by \(\Delta \Omega_{\text{EW}}\) and \(\Delta \Omega_{\text{GW}}\), conservation of angular momentum then demands

\[
I_{\text{EW}} \Delta \Omega_{\text{EW}} + I_{\text{GW}} \Delta \Omega_{\text{GW}} = 0. \tag{2}
\]

Integrating twice and rearranging gives a relationship between the GW and EW phase residuals

\[
\Delta \Phi_{\text{GW}} = -\frac{I_{\text{EW}}}{I_{\text{GW}}} \Delta \Phi_{\text{EW}}. \tag{3}
\]

(As we are considering triaxial neutron stars, the GW emission is at twice the rotation frequency \(10\), so that the quantity \(\Delta \Phi_{\text{GW}}\) in Equation \(8\) is strictly only one-half of the GW phase residual). In this scenario, the anti-correlation of the gravitational and electromagnetic phase residuals would then allow us to probe the relative moments of inertia of the two parts of the star, as well as telling us that they are loosely coupled on the timescale of the phase wandering. For instance, if these two parts were the crust and fluid core, we would expect \(I_{\text{EW}}/I_{\text{GW}} \sim 10^{-2} \tag{11}\).

Theoretically, all parts of the star (apart from any superfluid pinned to the crust) are expected to be coupled strongly on timescales of the order of seconds \(2\), orders of magnitude smaller than the timescales of years on which the phase residuals vary. It therefore seems most likely that the first of the above three scenarios is correct. However, given the potential importance of this phase information, the following strategy suggests itself: Wherever possible, the GW analysis of a known pulsar should use the observed pulsar phase residuals \(\Delta \Phi_{\text{EM}}\) to demodulate the GW detector output prior to using a smooth spindown template of the form of equation \(10\). A way in which such an analysis could be performed for the Crab pulsar has been discussed recently by Pitkin & Woan \(8\). The demodulation itself should correct for GW phase wandering of the form

\[
\Delta \Phi_{\text{GW}} = \alpha \Delta \Phi_{\text{EW}}, \tag{4}
\]

with a range of different \(\alpha\) values used. The three scenarios above correspond to the cases \(\alpha = 1, 0, -I_{\text{EW}}/I_{\text{GW}}\), respectively. Measurement of a value of \(\alpha\) other than unity would challenge the standard picture of neutron star structure.

Of course, the addition of an extra parameter to an already computationally expensive search should only be done if the extra parameter is likely to have a significant effect upon the signal—either by making what would otherwise be an undetectable signal detectable, or by significantly improving the signal-to-noise of an already detectable one. It is precisely these issues that we address in this paper, to assess whether or not GW observers need go to the trouble of allowing for timing noise in their analyzes.

III. METHOD OF CALCULATION

Timing noise can be characterized as a random walk in one or a combination of the rotation phase, frequency or spindown rate \(4\). These three idealized behaviors are known as phase noise, frequency noise and spindown noise. Pulsar physicists quantify the strength of timing
noise by $\langle \Delta \Phi_{\text{EW}}^2 \rangle^{1/2}$, the root mean square phase residual separating the actual signal and the best Taylor series approximant to it, typically containing terms up to and including the first frequency derivative. For the three idealized forms of random walk this rms phase residual grows as:

$$\langle \Delta \Phi_{\text{EW}}^2 \rangle^{1/2} = kT^{n/2},$$

where $n = 1, 3, 5$ for phase noise, frequency noise and slow-down noise, respectively. The corresponding GW timing noise in our model is then:

$$\langle \Delta \Phi_{\text{GW}}^2 \rangle^{1/2} = |\alpha|kT^{n/2}. \tag{5}$$

### A. Calculation of the decoherence timescale

Taking our criterion for total decoherence to be $\langle \Delta \Phi_{\text{GW}}^2 \rangle^{1/2} = 1$ radian, Equation (5) gives

$$T_{\text{decoherence}} = \left( \frac{1}{|\alpha|k} \right)^{2/n}. \tag{6}$$

### B. Calculation of the detection timescale

A constant frequency periodic GW source is described by 7 parameters: two angles giving its direction in the sky, its frequency, the phase at time $t = 0$, two angles specifying the orientation of its spin axis, and its amplitude. When searching for GWs from a known pulsar, the first four of these are known to high accuracy, so only the last three need be fit for.

When timing noise is present in the GW signal (at least) two possible strategies present themselves. Firstly, we could imagine taking the total data stream and breaking it up into a small number of equal duration blocks, sufficiently short that each block can be analyzed without worrying about the effects of timing noise. The timing noise would then manifest itself as a smooth variation in the phase at $t = 0$ from block to block. This method has the advantage that we need make no assumptions about the relative phase of the EW and GW timing noises, i.e. does not assume a correlation of the form of Equation (6), but the disadvantage that if the number of data blocks is large the signal-to-noise ratio for each will be small, leading to inaccurate phase measurements.

We will pursue a second, somewhat simpler strategy in this paper. We will assume that the EW and GW timing noises are correlated as in Equation (6). Then $\alpha$ must be added as an eighth search parameter. This method has the advantage that the entire signal-to-noise of the observation can be brought to bear on the analysis. Of course, if the EW and GW timing noises are not correlated we would not expect any value of $\alpha$ to significantly maximize the signal-to-noise ratio, indicating that we must use the first method described above.

We present below a simple estimate of how long an observation time is required to measure $\alpha$, i.e. to obtain a value significantly different from zero.

The error in measuring the phase of a periodic GW signal is $\langle \Phi_{\text{GW}} \rangle$ given by

$$\delta \Phi_{\text{GW}} = \frac{N}{\rho}, \tag{7}$$

where $N$ is a small number which depends upon the number of parameters to be fit for. The signal-to-noise grows as $T^{1/2}$; we can make this time dependence explicit by writing

$$\delta \Phi_{\text{GW}} \approx \frac{N}{\rho_{1-\text{yr}}^{1/2}}, \tag{8}$$

where $\rho_{1-\text{yr}}$ is the signal-to-noise that would be attained for a one year observation, and $T_{\text{yr}}$ is the total observation duration in years.

We will take as our criterion for the detectability of timing noise in the GW signal that the rms phase wandering is equal to the GW phase measurement error, i.e. $\langle \Delta \Phi_{\text{GW}}^2 \rangle^{1/2} = \delta \Phi_{\text{GW}}$. Combining Equations (6) and (8) gives the time $T_{\text{equal}}$ at which this occurs

$$T_{\text{equal, yr}} = \left( \frac{N}{|\alpha|k\rho_{1-\text{yr}}} \right)^{2/(n+1)} \tag{9}$$

Of course, it is also necessary that the GW signal be detectable. Let us set the minimum signal-to-noise ratio for detection to some value $\rho_{\text{min}}$. Define this signal-to-noise to be attained for an observation duration $T_{\text{GW}}$. Using the result that the signal-to-noise grows as $T^{1/2}$ we have:

$$T_{\text{GW}} = \left( \frac{\rho_{\text{min}}}{\rho_{1-\text{yr}}} \right)^2. \tag{10}$$

The timing noise is only detectable when its magnitude exceeds the GW phase measurement error (i.e. $T > T_{\text{equal}}$) and the GW signal is detectable (i.e. $T > T_{\text{GW}}$). It therefore follows that the presence of timing noise can be detected in the GW data stream after an observation time $T_{\text{detectable}}$ equal to the maximum of these two values:

$$T_{\text{detectable}} = \max(T_{\text{equal}}, T_{\text{GW}}). \tag{11}$$

We would like to apply the above formulae to the entire pulsar population, estimating $T_{\text{detectable}}$ and $T_{\text{decoherence}}$ for each known pulsar. In order to make use of the formulae, we need to obtain estimates of the timing noise, parameterized by $k$ and $n$, and also of the GW signal strength, parameterized by $\rho_{1-\text{yr}}$.

### IV. THE TIMING NOISE

Ideally we would use observationally derived timing noise parameters $k$ and $n$ for the entire pulsar population. Unfortunately, a literature search shows that only
a minority of pulsars have been timed in sufficient detail to allow for estimation of the timing noise behavior. However, on the basis of the available data, Dewey & Cordes [13] have obtained a fitting formula that allows estimation of the strength of the timing noise of a pulsar in terms of its period $P$ and period derivative $\dot{P}$. We will therefore pursue the following strategy: For pulsars whose timing noise strength has been measured, we will use the observational data in our analysis. For pulsars whose timing noise hasn’t been measured (or at least weren’t provided by our literature search), we’ll use the fitting formula of Dewey & Cordes.

A. Pulsars of measured timing noise

A number of authors have published rms phase residuals $\langle \Delta \Phi_{EW}^2 \rangle^{1/2}$ for pulsars, typically after having fit the timing data to a Taylor expansion including the frequency and its first time derivative, over an interval of several years. The parameter $k$ can then be derived using the published values of $\langle \Delta \Phi_{EW}^2 \rangle^{1/2}$, $T$ and $n$ in Equation (13):

$$ k = \frac{\langle \Delta \Phi_{EW}^2 \rangle^{1/2}}{T^{n/2}}. \quad (13) $$

Unfortunately, in only a small subset of these observations was it possible to identify the nature of the walk, i.e. fix the value of $n$. For the sake of definiteness, we will therefore present results for $n = 3$; we have repeated calculations for $n = 1$ and $n = 5$ and found results which are not very different, with the $n = 3$ results being intermediate between the two other sets.

The results of our literature search for timing noise data are summarized in Table I. For reasons of convenience, we only selected pulsars of spin frequencies greater than 5 Hz, corresponding to GW frequencies greater than 10 Hz, as stars spinning more slowly than this will certainly not be of GW interest. The references of table I then provided a total of 53 pulsars. Note that the majority of the pulsars selected come from the Parkes Multibeam Study [14,15,16], which was specifically designed to find young pulsars.

B. Pulsars whose timing noise hasn’t been measured

In producing their fitting formula for timing noise, Dewey & Cordes make use of the ‘activity parameter’, defined as the logarithm of the rms phase residual for the pulsar in question divided by that of the Crab:

$$ A = \log \left( \frac{\langle \Delta \Phi_{EW}^2 \rangle^{1/2}}{\langle \Delta \Phi_{EW, Crab}^2 \rangle^{1/2}} \right). \quad (14) $$

The fitting formula is

$$ A = -1.4 \log P + 0.8 \log \dot{P} - 3.31. \quad (15) $$

where $P$ is the pulsar’s spin period in seconds and $\dot{P}$ is the dimensionless period derivative divided by $10^{-15}$. The Crab is observed to display frequency-type timing noise of the form:

$$ \langle \Delta \Phi_{EW, Crab}^2 \rangle^{1/2} = 0.24 \text{ radians } T_{\text{yrs}}^{3/2} \quad (16) $$

(see [4]). It follows that the activity parameter only makes sense for (Crab-like) frequency-type noise, as $A$ would otherwise be a function of time. We will therefore assume that all pulsars of measured timing noise display frequency-type noise. The above three equations can then be combined to give an estimate of the rms timing noise in any pulsar of known period and period derivative. Using $\langle \Delta \Phi_{EW}^2 \rangle^{1/2} / \langle \Delta \Phi_{EW, Crab}^2 \rangle^{1/2} = k/k_{Crab}$ in Equation (14) then allows calculation of $k$:

$$ k = k_{Crab} 10^4. \quad (17) $$

Application of this formula to pulsars of known timing noise shows that there exists a scatter of about an order of magnitude in the measured activity parameter values about the predicted ones [4]. This uncertainty should be borne in mind when reading the results sections of this paper—individual pulsars may be more or less noisy than assumed.

We used the above prescription for the 1182 pulsars listed in the Australia Telescope National Facility database [21] whose periods, period derivatives and distances from Earth were known, excluding the 53 of measured timing noise described in section 1A. i.e. the fitting formula was used for 1182 – 53 = 1129 pulsars.

V. GRAVITATIONAL WAVE AMPLITUDES

We also need to estimate the GW amplitudes of the pulsar population, giving the result in the form of the signal-to-noise for a one year coherent integration. The GW amplitude of a triaxial star a distance $r$ from Earth is given by:

$$ h = \left( \frac{2}{15} \right)^{1/2} \frac{G}{c^4} \frac{8 \Omega^2}{r} \Delta I, \quad (18) $$
where the normalization comes from averaging over all possible spin orientations of the source and $\Delta I$ is the difference between the two principal moments of inertia in the plane orthogonal to the spin $\hat{2}$. This is more conveniently expressed as a dimensionless number

$$\epsilon = \frac{\Delta I}{I}, \quad (19)$$

where $I$ is the moment of inertia of the non-rotating star; we will refer to $\epsilon$ as the ellipticity. Of course, the wave amplitudes at Earth are unknown as $\epsilon$ is unknown. An upper bound is often placed on these quantities by assuming that all the the spindown kinetic energy lost by the star is converted into GW energy, at fixed moment of inertia. In general the GW luminosity is given by

$$\dot{E} = \frac{32G}{5c^5} \Omega^6 (\Delta I)^2, \quad (20)$$

while the kinetic energy is $I\Omega^2/2$. Combining these results gives an upper bound on $h$ in terms of the pulsar’s spin period, period derivative and distance from Earth:

$$h_{\text{spindown}} = \frac{2}{r} \left( \frac{GIP}{c^3P} \right)^{1/2} \quad (21)$$

with a corresponding ellipticity

$$\epsilon_{\text{spindown}} = \left( \frac{5c^5 \dot{P} P^3}{32(2\pi)^4GI} \right)^{1/2}. \quad (22)$$

The results of such a calculation for all pulsars of known period, period derivative and distance from Earth are shown in Figure 1.

The ellipticities required to produce this level of spindown are of order $10^{-8}$ for the millisecond pulsars. Detailed modeling suggests that such an ellipticity could well be produced by strains in the neutron star crust [23]. However, for the younger pulsars, which include all those which lie above the noisecurves of the first generation detectors, the required ellipticities are much larger, around $10^{-4}$. Such a value is almost certainly unphysically large, suggesting that GW play little role in the energy budget of young neutron stars. With this in mind, in Figure 2 we show another set of estimated wave amplitudes, repeating the above calculation, but imposing a cut-off in $\epsilon$ of $10^{-7}$, i.e. setting $\epsilon = \max(\epsilon_{\text{spindown}}, 10^{-7})$. The value $\epsilon = 10^{-7}$ was chosen as it is at the upper end of what is considered plausible on physical grounds [23]. The wave amplitudes calculated for the millisecond pulsars are mostly unchanged, but the amplitudes for the young pulsars have been reduced by several orders of magnitude (most have fallen off the bottom of the figure). In the following sections we will present results assuming ellipticities ranging from $\epsilon_{\text{spindown}}$ down to $10^{-7}$. (It is probable that in reality pulsar ellipticities are even smaller than $10^{-7}$, but, as we shall see, we need not consider smaller values for the purposes of examining timing noise).

![Figure 1: Upper bounds on GW amplitudes assuming 100% conversion of spindown energy into GWs. The noisecurves are for a one year observation, and were produced using the fitting formulae of [22]. The ten red circles indicate those pulsars which have $T_{\text{decoherence}} < 10$ years; see Section VI.](image1)

![Figure 2: Upper bounds on GW amplitudes imposing at cut-off of $10^{-7}$ in $\epsilon$.](image2)

VI. RESULTS: THE DECOHERENCE TIMESCALE

We will now insert the timing noise strengths calculated in section IV into Equation 7 to obtain estimates of the timescale on which timing noise would cause a simple Taylor series to completely decohere from the GW signal, assuming that the latter is perfectly locked in phase with the EW signal (i.e. $\alpha = 1$ in Equation 4). These estimates are completely independent of the GW strength—they depend only upon the strength of the timing noise.

We find that 10 pulsars have decoherence timescales...
TABLE II: Pulsars with $T_{\text{decoherence}} < 3$ years and $T_{\text{GW}} < 3$ years, assuming the Initial LIGO noisecurve, 100% conversion of spindown energy to GW energy, and $\alpha = 1$.

| Name            | $T_{\text{decoherence}} /\text{years}$ | $T_{\text{GW}} /\text{years}$ |
|-----------------|--------------------------------------|-------------------------------|
| J0205+6449      | 2.0e+00                              | 2.8e+00                       |
| J0537-6910      | 1.1e+00                              | 2.3e+00                       |
| B0531+21        | 2.6e+00                              | 1.3e-02                       |

TABLE III: Pulsars with $T_{\text{decoherence}} < 3$ years and $T_{\text{GW}} < 3$ years, assuming the Advanced LIGO noisecurve, 100% conversion of spindown energy to GW energy, and $\alpha = 1$.

| Name            | $T_{\text{decoherence}} /\text{years}$ | $T_{\text{GW}} /\text{years}$ |
|-----------------|--------------------------------------|-------------------------------|
| J0205+6449      | 2.0e+00                              | 1.4e-04                       |
| B0531+21        | 2.6e+00                              | 1.9e-06                       |
| J0537-6910      | 1.1e+00                              | 6.2e-03                       |
| B0540-69        | 9.5e-01                              | 4.1e-03                       |
| J1124-5916      | 1.9e+00                              | 3.7e-03                       |
| B1509-58        | 1.4e+00                              | 2.0e-03                       |
| J1617-5055      | 2.5e+00                              | 1.1e-03                       |
| J1930+1852      | 1.9e+00                              | 3.3e-03                       |
| J2229+6114      | 2.6e+00                              | 1.7e-03                       |

of less than three years. To gain some insight, we show their positions in the GW amplitude–frequency plot in Figure 1 as open circles, assuming 100% conversion of spindown energy into GWs. As expected, most of these are young pulsars—only one has a GW frequency of less than 10 Hz. None of them are millisecond pulsars; the much weaker timing noise of the latter class of star leads to their having decoherence timescales many orders of magnitude longer that those of the young pulsars.

However, these estimates are only of interest if the GW signal is itself detectable. To this end, we will present results first assuming 100% conversion of spindown energy into GWs, and then assuming various bounds upon the allowed ellipticity.

A. Decoherence timescale assuming 100% conversion of spindown energy into GWs; probably unphysical

As stated above, we find that 10 pulsars have decoherence timescales of less than three years. However, even when making the optimistic assumption that 100% of the spindown energy is being converted into GW energy, some of these pulsars have GW detection timescales of more than three years, and are therefore of no interest.

To allow for this, the names, $T_{\text{decoherence}}$ and $T_{\text{GW}}$ values for the pulsars which satisfy $T_{\text{decoherence}} < 3$ years and $T_{\text{GW}} < 3$ years are given in Table II assuming the Initial LIGO noisecurve, and the corresponding results for Advanced LIGO are given in Table III.

Tables II and III represent the maximal set of pulsars for which timing noise could prevent detection. In fact, detection will be prevented only if $T_{\text{decoherence}} < T_{\text{GW}}$. For the 100% conversion of spindown energy to GW energy assumed in Tables II and III, this is the case only for the Initial LIGO observation of pulsars J0205+6449 and J0537-6910.

Note that only one pulsar has a predicted decoherence timescale of less than one year; namely pulsar B0540-69, which has $T_{\text{decoherence}} = 0.95$ years. We therefore see that, regardless of the GW signal strength, timing noise can only ruin a Taylor-series based GW detection for timescales of order a year or longer.

Motivated by the discussion of section II, if we set $|\alpha| = 10^{-2}$ in Equation (7) then, for frequency type noise, the decoherence timescales will be greater than those of Table III by a factor of $100^{2/3} \approx 21.5$; in this case there are no known pulsars with $T_{\text{decoherence}} < 3$ years.

B. Decoherence timescale assuming various upper bounds on the triaxiality

Putting an upper bound on the ellipticity will have no effect on the decoherence timescales but will increase the GW detection timescales, so that for each pulsar in the Tables II and III for some assumed $\epsilon_{\text{max}}$, the condition $T_{\text{decoherence}} < T_{\text{GW}}$ would be satisfied and timing noise would prevent GW detection. For instance, putting $\epsilon_{\text{max}} = 6 \times 10^{-7}$ the Advanced LIGO observation of the Crab satisfies the condition. However, repeating the analysis for the more physically plausible case where $\epsilon_{\text{max}} = 10^{-7}$, we find that the GW detection timescales of all pulsars in the table exceed 3 years, even for the Advanced LIGO noisecurve.

To sum up, timing noise could render as many as 3 pulsars undetectable using simple Taylor series for Initial LIGO, and as many as 9 for Advanced LIGO, but GW emission from these pulsars is only detectable if they have ellipticities in excess of those expected theoretically.

VII. RESULTS: THE DETECTION TIMESCALE

We will now combine the timing noise estimates of section IV with the GW amplitudes of section V to calculate the $T_{\text{detectable}}$ values for all known pulsars. We will present results assuming both the Initial and the Advanced LIGO noisecurves. In the results that follow we have set the parameter $N = 3$ in Equation (9). Cutler et al. (2003) have shown that for LISA observations at least, $N < 1.5$ for 40% of the possible spin orientations of the source, so our setting $N$ to twice this value is surely a conservative estimate of the phase error. Also, we have set $\rho_{\text{min}} = 5$ in Equation (11); for the directed searches considered here this will give an acceptably low false alarm rate (Reference [8], Eqn. 1.4).

Again, we will divide our results into two parts. In the first part we will assume complete conversion of spindown
energy to GW energy. For all but the millisecond pulsars these wave amplitudes are probably unrealistically large, so that this calculation gives a safe lower bound on the detectability timescale. We will then consider more realistic scenarios, where the assumed triaxiality $\epsilon$ is limited to some maximum value.

### A. Detection timescale assuming 100% conversion of spindown energy into GWs; probably unphysical

We will begin by assuming the the GW and EW signals are perfectly in phase (i.e. $\alpha = 1$ in Equation 4). In the case of Initial LIGO, the (measured) timing noise and (estimated) GW amplitude of the Crab are both large, so that its timing noise detectability time is very short, just $0.54\text{ years} \approx 200\text{ days}$. However, the significantly lower estimated GW amplitudes of the other pulsars (see Figure 1) leads to only two having $T_{\text{detectable}}$ values of less than 3 years. The names of all three pulsars with $T_{\text{detectable}} < 3$ years are collected in Table IV.

In the case of Advanced LIGO a total of 46 pulsars have $T_{\text{detectable}} < 3$ years; see Table V. The detectability timescale for the Crab is very short, just 0.058 years.

A summary of these results is provided in the first line of Table VI, where the number of pulsars with timing noise detectability timescales of less than three years are shown, for both Initial LIGO and Advanced LIGO.

Setting $|\alpha| = 10^{-2}$ leaves no pulsars with detectable timing noise using Initial LIGO, and only 7 for Advanced LIGO. These are listed in Table VII.

### B. Detection timescale assuming various upper bounds on the triaxiality

The number of pulsars with timing noise detectability timescales less than three years, for both Initial LIGO and Advanced LIGO, are shown in Table V for $\epsilon_{\text{max}} = 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}, 10^{-7}$. For reasons of brevity we have not tabulated the names of the pulsars falling in these categories; in any case they are a subset of the pulsars of Tables IV and V. In the case where $\epsilon_{\text{max}}$ takes on the huge value of $= 10^{-2}$, the results are the same as in section VLA as the triaxialities required to give 100% GW spindown are less than this. As $\epsilon_{\text{max}}$ is increased, the number of pulsars with $T_{\text{detectable}} < 3$ years decreases, falling to zero at $\epsilon_{\text{max}} = 10^{-7}$. Clearly, timing noise will only be detectable in the GW signal if the ellipticities of neutron stars are $10^{-6}$ or larger, i.e. at least an order of magnitude greater than theoretical modeling currently suggests.

### Table IV: Pulsars with $T_{\text{detectable}} < 3$ years, assuming 100% conversion of spindown energy into GW energy, the Initial LIGO noisecurve, and $\alpha = 1$.  

| Name                  | $T_{\text{detectable}}$/yrs |
|-----------------------|-----------------------------|
| J0205+6449            | 2.8e+00                     |
| B0531+21              | 5.4e-01                     |
| J0537-6910            | 2.3e+00                     |

### Table V: Pulsars with $T_{\text{detectable}} < 3$ years, assuming 100% conversion of spindown energy into GW energy, the Advanced LIGO noisecurve, and $\alpha = 1$.  

| Name                  | $T_{\text{detectable}}$/yrs |
|-----------------------|-----------------------------|
| B0114+58              | 2.6e+00                     |
| J0205+6449            | 1.4e+00                     |
| B0531+21              | 5.8e-02                     |
| J0537-6910            | 2.3e-01                     |
| B0540-69              | 1.9e-01                     |
| J0633+1746            | 2.6e+00                     |
| B0833-45              | 2.4e-01                     |
| J0834-4159            | 2.8e+00                     |
| J1040-5428            | 1.8e+00                     |
| J1055-6444            | 7.3e-01                     |
| J1105-6107            | 9.9e-01                     |
| J1112-6103            | 1.6e+00                     |
| J1124-5916            | 3.1e-01                     |
| B1259-63              | 1.7e+00                     |
| B1338-62              | 1.6e+00                     |
| J1420-6048            | 4.3e-01                     |
| J1509-5850            | 1.5e+00                     |
| B1509-58              | 2.1e-01                     |
| J1524-5625            | 1.0e+00                     |

### Table VI: A summary of our results for the detection timescale. The number of pulsars with $T_{\text{detectable}} < 3$ years, for both Initial LIGO and Advanced LIGO with $\alpha = 1$. The first line assumes 100% conversion of spindown energy into GW energy, while the remaining lines impose a cut-off in ellipticity as indicated.  

| $\epsilon_{\text{max}}$ | Number of pulsars | Initial LIGO | Advanced LIGO |
|--------------------------|-------------------|--------------|---------------|
| No bound                 | 3                 | 46           |
| $10^{-2}$                 | 3                 | 46           |
| $10^{-3}$                 | 2                 | 38           |
| $10^{-4}$                 | 2                 | 18           |
| $10^{-5}$                 | 0                 | 6            |
| $10^{-6}$                 | 0                 | 1            |
| $10^{-7}$                 | 0                 | 0            |
TABLE VII: Pulsars with $T_{\text{detectable}} < 3$ years, assuming 100% conversion of spindown energy into GW energy, and the Advanced LIGO noise curve, and $|\alpha| = 10^{-2}$.

| Name         | $T_{\text{detectable}}$/years |
|--------------|-------------------------------|
| J0205+6449   | 1.4e+00                       |
| B0531+21     | 5.8e-01                       |
| J0537-6910   | 2.3e+00                       |
| B0540-69     | 1.9e+00                       |
| B0833-45     | 1.7e+00                       |
| B1509-58     | 2.1e+00                       |
| J1617-5055   | 2.8e+00                       |

VIII. BLIND SEARCHES

A ‘blind search’ is a search for gravitational waves from a neutron star not currently known as a pulsar. The presence of timing noise in the GW signal for such a star is more pernicious than in the case of a directed search. In the latter case the electromagnetic pulsar data can be used to demodulate the timing noise from the data stream. In the case of a blind search this option is not available, so that the timing noise will impose a maximum duration on any blind search beyond which the smooth Taylor series approximation to the phase error by a significant amount (i.e. by about one radian).

In practice, a GW astronomer would perform a blind search by dividing the sky up into small patches, such that the Doppler shifts induced by the Earth’s spin and orbital motion are negligible over the patch $\mathbb{S}$. Then, for each such patch, a search over a range of spindown parameters $P$ is made. From the parametrization of equation (16), for a given spin period $P$ the strength of the timing noise is then determined, so that the decoherence timescale can then be estimated from equation (7). The maximum search duration is then given by the parametrized formula:

$$\log T_{\text{max}} = 2.62 + 0.93 \log P - 0.53 \log \dot{P}_{-15},$$

(23)

where $T_{\text{max}}$ is measured in years, $P$ in seconds and $\dot{P}_{-15}$ is the dimensionless period derivative divided by $10^{-15}$. This formula could be used by GW observers to limit the duration of a single coherent search.

However, there exists another limit on the length of a coherent blind search—namely that imposed by the requirement of doing the search in real time with finite computational power. Brady et al. have investigated the limits placed on the length of a single non-hierarchical coherent search in some detail. They found that for a star with a GW frequency $\lesssim 200$ Hz and spindown timescale $f/\dot{f} \gtrsim 1000$ years, the search is limited to a duration of $\lesssim 18$ days. Inserting these spindown parameters into Equation (23) gives a maximum search duration of $2.3$ years, 47 times longer. For a GW frequency of $\lesssim 1$ kHz and spindown timescale $\gtrsim 40$ years, Brady et al find a maximum search duration of just 0.8 days; timing noise gives a duration of 9.5 days, 12 times longer. It therefore appears that the finiteness of computational resources will place a more stringent limit on the duration of a single non-hierarchical coherent blind search than that due to timing noise.

However, the high computational costs of such searches have motivated GW astronomers to devise more computationally efficient hierarchical search techniques. As these, the full data set of duration $T_{\text{obs}}$ is split up into $N$ shorter pieces of duration $T_{\text{short}} = T_{\text{obs}}/N$. Each short piece is then coherently analysed by matched filtering, and the separate results then combined incoherently. There are two main formulations that have been proposed to achieve this: the ‘stack-slide’ method and the Hough transform. These will certainly be more computationally efficient than the single coherent analysis described above. For instance, Schutz & Papa found that a data set of $T_{\text{obs}} = 10^7$ seconds could be searched for GW signals of signal-to-noise ratio $\approx 23$ by splitting the data into blocks of duration $T_{\text{short}} = 14$ hours, using a 20 Gflop computer, although this did not include a search over spin-down parameters.

If it should prove that these hierarchical techniques are efficient enough that $T_{\text{obs}}$ should exceed the timing noise decoherence timescale, then the following strategy could be used, of a type first suggested by Brady & Creighton in the context of variable accretion rate systems: $T_{\text{short}}$ could be chosen to be significantly smaller than $T_{\text{max}}$ of Equation (23), so that the short coherent searches are not significantly degraded by the timing noise. The incoherent stage must then be performed allowing for the many possible phase shifts that timing noise could introduce. We suspect that this last step will be computationally expensive as the timing noise can map out many different paths through phase space, but we won’t attempt to quantify this cost here.

IX. CONCLUSIONS

In this paper we have addressed three issues.

First we asked for which of the known pulsars might timing noise prevent a GW detection, if the experimenter assumes only smooth Taylor-series spindown. Our main conclusions were as follows:

- If the GW timing noise is at the same level as the EW timing noise (i.e. $\alpha = 1$ in Eqn. (4)), then, assuming 100% conversion of spindown energy into GW energy, as many as 3 pulsars may be rendered undetectable by Initial LIGO (Table III), and as many as 9 for Advanced LIGO (Table III).

- If the GW timing noise is weaker, at the level of $|\alpha| = 10^{-2}$, no pulsars will be rendered undetectable by timing noise.

- Dropping the assumption of 100% conversion of spindown energy into GW energy and instead placing an upper bound on the ellipticities leads to
fewer stars for which decoherence may be an issue. Setting \( \epsilon_{\text{max}} = 10^{-7} \) or smaller we find that timing noise will not prevent the detection of any pulsars.

Next we asked for which of the known pulsars might timing noise be strong enough to be observed in the GW data stream. Our main conclusions were as follows:

- If the GW timing noise is at the same level as the EW timing noise (i.e. \( \alpha = 1 \)), then, assuming 100\% conversion of spindown energy into GW energy, timing noise will be detectable in 3 pulsars by Initial LIGO (Table IV), and in 46 pulsars for Advanced LIGO (Table V).

- If the GW timing noise is weaker, at the level of \( |\alpha| = 10^{-2} \), timing noise will not be detectable in the GW signal of any pulsars using Initial LIGO, and in only 7 using Advanced LIGO (Table VII).

- Dropping the assumption of 100\% conversion of spindown energy into GW energy and instead placing an upper bound on the ellipticities leads to fewer stars for which timing noise is detectable (Table V). Setting \( \epsilon_{\text{max}} = 10^{-7} \) or smaller we find that timing noise is not detectable in the GW signal of any pulsars.

Finally we asked how timing noise might affect blind GW searches, i.e. searches for GW emitters not currently observed as pulsars. Our main conclusion was that:

- Timing noise places a limit on the length of a data set that can be coherently analyzed (Equation 28).

Proposed hierarchical search techniques could perform searches over longer durations by allowing for the possibility of (many different possible realisations of) timing noise when the short coherent analyses are incoherently combined.

Table III is probably of most interest to today’s GW data analysts—it lists the pulsars for which timing noise is most important for the first generation interferometers. For these three pulsars, GW astronomers may wish to use electromagnetic timing residuals to improve their ability to search for these stars. A method of doing so for one of these (the Crab) was described recently by Pitkin & Woan.

To sum up, timing noise may be an important feature of the GW signal of some tens of young pulsars, but these stars must have very large ellipticities in order for the GW emission to be strong enough to be detectable. If observed in the GW signal, timing noise would provide a new insight into neutron star dynamics.

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