Semileptonic B Decays into Excited Charmed Mesons ($D_1$, $D_2^*$) in HQEFT

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Abstract

Exclusive semileptonic B decays into excited charmed mesons ($D_1$, $D_2^*$) are studied up to the order of $1/m_Q$ in the framework of the heavy quark effective field theory (HQEFT), which contains the contributions of both particles and antiparticles. Two wave functions $\eta^0_b$ and $\eta^0_0$, which characterize the contributions from the kinematic operator at the order of $1/m_Q$, are calculated by using QCD sum rule approach in HQEFT. Zero recoil values of other two wave functions $\kappa'_1$ and $\kappa'_2$ are extracted from the excited charmed-meson masses. Possible effects from the spin-dependent transition wave functions which arise from the magnetic operators at the order of $1/m_Q$ are analyzed. It is shown that the experimental measurements for the branching ratios of $B \to D_1 \, l\nu$ and $B \to D_2^* \, l\nu$ can be understood in the framework of HQEFT.
I. INTRODUCTION

Recently, studies on the semileptonic $B$ decays into excited charmed mesons become interesting in both experimental and theoretical sides. Experimentally, to precisely measure the branching ratios of the semileptonic $B$ meson into the groundstate charmed mesons, it also needs to measure precisely the branching ratios of the semileptonic $B$ decays into excited charmed mesons which is the main background for the former decays. Theoretically, it provides additional modes for testing the validity of effective theories, in particular, how good of the spin-flavor symmetry.

The semileptonic $B$ decays into excited charmed mesons have been discussed in [1,2] based on the framework of the usual heavy quark effective theory (HQET). Where the dependence of decay rates on wave functions was presented in a general form, the authors mainly deal with Isgur-Wise type function and the $1/m_Q$ order wave functions $\tau_1$ and $\tau_2$ which arise from the effective currents. The wave functions $\eta_i^Q$ arising from the chromomagnetic term in Lagrangian have been neglected, and those arising from the kinematic term in Lagrangian have been absorbed into the Isgur-Wise type function and not been considered separately. The detailed field theoretic calculations on these decays were presented in [3,4], where the Isgur-Wise type function and $\tau_1$, $\tau_2$ have been evaluated by using the QCD sum rule approach.

The purpose of this paper is to study the transitions between ground and excited state heavy-light mesons within the framework of heavy quark effective field theory (HQEFT) [5] in which both quark and antiquark effective fields with keeping quark-antiquark coupled terms have been considered. In particular, we focus on the semileptonic $B$ decays into the $j^P_l = \frac{3}{2}^+$ charmed meson doublet ($D_1$, $D_2^*$), where $j_l$ and $P$ are the spin and parity of the light degrees of freedom in the charmed mesons. It has been shown that the HQEFT can provide a consistent description on both exclusive [6,7] and inclusive [8,9] decays of heavy hadrons. It has been seen in these references that the HQEFT has many advantages with respect to the usual HQET. In the new framework, the values of $|V_{cb}|$ extracted from both exclusive and inclusive decays show a good agreement; the bottom hadron life time ratios can be well understood; $1/m_Q$ order corrections at zero recoil in both exclusive and inclusive decays automatically absent without imposing the equation of motion $iv \cdot DQ_v = 0$ when the physical observables are presented in terms of heavy hadron masses; less wave functions are needed; and there are interesting relations among meson masses and transition wave functions at zero recoil. In the most recent paper [7], the decay constants and binding energy of ground state heavy-light mesons as well as transition wave functions among them were studied consistently by using QCD sum rule approach in the framework of HQEFT. It was noticed that the HQEFT appears much more reliable than the usual HQET in describing the decays and transitions for the ground state mesons. Particularly, the $1/m_Q$ corrections to the heavy-light meson decay constants were found to be much smaller than the heavy quark mass, so that the scaling law of the decay constants is only slightly broken. This observation is unlike the usual HQET, which may lead to either the complete break down of the $1/m_Q$ expansion [10] or a large $1/m_Q$ correction [11] in evaluating the decay constants. Two transition wave functions $\kappa_1(y)$ and $\kappa_2(y)$ at the $1/m_Q$ order have also been evaluated in [7]. Their zero recoil values agree with those extracted from the ground state meson masses.
Our present paper is organized as follows. In Sec. II, the weak transition matrix elements relevant to \( B \to (D_1, D_2^*) \) decays are studied up to the order of \( 1/m_Q \) and parameterized by independent universal wave functions. The form factors and decay rates are then given in terms of those wave functions. The relevant normalization of the excited mesons is also discussed. In Sec. III, using the appropriate interpolating currents for excited heavy mesons, we derive the sum rules for two of the important wave functions concerned at \( 1/m_Q \) order, \( \eta_0^b \) and \( \eta_b^0 \). In Sec. IV, we present our numerical results obtained from the sum rule approach. The \( B \to (D_1, D_2^*) \) decay rates and branching ratios are discussed in detail. Finally, we come to our brief summary in Sec. V.

II. ANALYTIC FORMULAE FOR \( B \to (D_1, D_2^*) \) IN HQEFT

The matrix elements relevant to the semileptonic decays \( B \to (D_1, D_2^*) \) are the ones of vector and axial vector currents \( (V^\mu = \bar{c}\gamma^\mu b \) and \( A^\mu = \bar{c}\gamma^\mu\gamma^5 b) \) between \( B \) and the excited doublet \( (D_1, D_2) \). Usually, these matrix elements are parameterized as

\[
\begin{align*}
\langle D_1(v', \epsilon)|V^\mu|B(v)\rangle &= \sqrt{m_{D_1} m_B} [f_{V_1} \epsilon^\mu + (f_{V_2} v^\mu + f_{V_3} v'^\mu)(\epsilon \cdot v)], \\
\langle D_1(v', \epsilon)|A^\mu|B(v)\rangle &= i\sqrt{m_{D_1} m_B} f_A \epsilon^{\alpha\beta\gamma} \epsilon^\alpha v^\beta v'^\gamma, \\
\langle D_2^*(v', \epsilon)|A^\mu|B(v)\rangle &= \sqrt{m_{D_2} m_B} [k_A \epsilon^{\mu\alpha} v_\alpha + (k_A v^\mu + k_A v'^\mu) \epsilon^{\alpha\beta} v^\alpha v^\beta], \\
\langle D_2^*(v', \epsilon)|V^\mu|B(v)\rangle &= i\sqrt{m_{D_2} m_B} k_V \epsilon^{\mu\alpha\beta} \epsilon_\alpha v^\sigma v^\beta v'^\gamma, 
\end{align*}
\]

(2.1)

where the form factors \( f_i \) and \( k_i \) are dimensionless functions of \( y = v \cdot v' \), and \( \epsilon^\mu (\epsilon^{\mu\alpha}) \) is the polarization vector (tensor) of \( D_1 \) (\( D_2^* \)).

In the framework of HQEFT (its main formulation is presented in Appendix A) we can in general introduce an effective heavy hadron state \( |H_v\rangle \) for the witness of exhibiting manifestly the spin-flavor symmetry [6–8]. It is related to the hadron state \( |H\rangle \) in the full theory via

\[
\frac{1}{\sqrt{m_{H'} m_H}} \langle H'\bar{Q}\gamma Q|H\rangle = \frac{1}{\sqrt{\tilde{\Lambda}_{H'} \tilde{\Lambda}_H}} \langle H'_v\bar{J}_{eff}\gamma \int \! d^4x L_{eff}|H_v\rangle.
\]

(2.2)

with \( |H_v\rangle \) being normalized as

\[
\langle H_v|\bar{Q}_v^+ \gamma^\mu Q_v^+|H_v\rangle = 2\tilde{\Lambda} v^\mu
\]

(2.3)

where

\[
\tilde{\Lambda} = \tilde{\Lambda}_H - O(1/m_Q) = \lim_{m_Q \to \infty} \tilde{\Lambda}_H
\]

(2.4)

is taken to be heavy flavor independent, and it mainly reflects the effects of the light degrees of freedom in the heavy hadron characterizes the off-mass shellness of the heavy quark within the heavy hadron. Once \( \tilde{\Lambda} \) is chosen to be the flavor independent binding energy of the hadron \( |H\rangle \), one yields

\[
\tilde{\Lambda}_H \equiv m_H - m_Q.
\]

(2.5)
which is the total binding energy of hadron.

The hadronic matrix elements can be expanded according to the order of $1/m_Q$. By including the $1/m_Q$ order corrections which can arise from both the current expansion and the insertion of the effective Lagrangian, one gets

$$
\begin{aligned}
\frac{\tilde{A}_M}{m_M} \langle 0 | q \Gamma Q | M \rangle & \rightarrow \langle 0 | q \Gamma Q_{\nu}^+ | M_\nu \rangle - \frac{1}{2m_Q} \langle 0 | q \Gamma \frac{1}{i\not{q}_\perp} (i\not{q}_\perp)^2 Q_{\nu}^+ | M_\nu \rangle + O(1/m_Q^2), \\
\frac{\tilde{A}_{M'} \tilde{A}_M}{m_{M'} m_M} \langle M' | q \Gamma Q | M \rangle & \rightarrow \langle M'_\nu | q \Gamma Q_{\nu}^+ | M_\nu \rangle - \frac{1}{2m_Q} \langle M'_\nu | q \Gamma \frac{1}{i\not{q}_\perp} (i\not{q}_\perp)^2 Q_{\nu}^+ | M_\nu \rangle \\
& \quad - \frac{1}{2m_Q'} \langle M'_\nu | q \Gamma \frac{1}{i\not{q}_\perp} (i\not{q}_\perp)^2 Q_{\nu}^+ | M_\nu \rangle + O(1/m_Q^2).
\end{aligned}
$$

(2.6)

The form factors $f_i$ and $k_i$ can be parameterized by a set of universal wave functions. It is simplest to do this by using the trace formulism \[ 13 \]. The spin wave functions for the $j^P_l = \frac{1}{2}^-$ ground state mesons $B$, $B^*$ and $j^P_l = \frac{3}{2}^+$ charmed mesons $D_1$, $D_2^*$ are

$$
\mathcal{M}_\nu = \sqrt{\lambda} P_+ \begin{cases} 
-\gamma^5, & \text{for pseudoscalar meson} \\
\frac{\gamma^\mu}{\epsilon^\alpha \gamma_{\nu}^\mu}, & \text{for vector meson}
\end{cases}
$$

(2.7)

$$
\mathcal{F}_\nu^\mu = \sqrt{\lambda} P_+ \begin{cases} 
-\frac{1}{2} \gamma^5 \epsilon^\nu [g_{\mu}^\nu - \frac{1}{3} \gamma^\nu (\gamma^\mu - \nu^\mu)], & \text{for } D_1 \\
\epsilon^\mu \gamma_{\nu}^\mu, & \text{for } D_2^*
\end{cases}
$$

(2.8)

The matrices $\mathcal{M}_\nu$ and $\mathcal{F}_\nu^\mu$ satisfy the properties $\not{\epsilon} \mathcal{M}_\nu = \mathcal{M}_\nu \not{\epsilon}$, $\not{\epsilon} \mathcal{F}_\nu^\mu = \mathcal{F}_\nu^\mu \not{\epsilon}$ and $\mathcal{F}_\nu^\mu \gamma_\mu = \mathcal{F}_\nu^\mu \epsilon^\mu = 0$.

With respect to eq.\[(2.3)\] and the normalization of the $\frac{3}{2}^+$ excited states, we parameterize the matrix elements in eq.\[(2.6)\] as follows

$$
\begin{aligned}
\langle H_{D^*} | Q_{\nu}^+ \gamma^\mu Q_{\nu}^+ | H_0 \rangle & = \xi'(y) Tr[\mathcal{F}_{\nu}^\sigma \gamma^\mu \mathcal{F}_{\nu \sigma}^\alpha] \\
\langle H_{D^*} | Q_{\nu}^+ \gamma^\mu P_{i\nu} \cdot D_{\perp} Q_{\nu}^+ | H_0 \rangle & = -\kappa'_1(y) \frac{1}{N} Tr[\mathcal{F}_{\nu}^\sigma \gamma^\mu \mathcal{F}_{\nu \sigma}^\alpha] \\
\langle H_{D^*} | Q_{\nu}^+ \gamma^\mu P_{i\nu} \cdot D_{\perp} \sigma_{\alpha \beta} F_{\alpha \beta} Q_{\nu}^+ | H_0 \rangle & = \frac{1}{N} Tr[A_{\alpha \beta} \mathcal{F}_{\nu \sigma}^\gamma \gamma^\mu P_{i\nu} \cdot D_{\perp} \sigma_{\alpha \beta} F_{\alpha \beta} Q_{\nu}^+]
\end{aligned}
$$

(2.9)

with

$$
A_{\alpha \beta}^\sigma (v, v') = a_1 (g^{\sigma \alpha} g^{\sigma \beta} - g^{\sigma \beta} g^{\sigma \alpha}) + ia_2 g^{\sigma \alpha} g^{\sigma \beta} + a_3 g^{\sigma \alpha} (v^{\alpha} \gamma^{\beta} - v^{\beta} \gamma^{\alpha}) + ia_4 (g^{\sigma \alpha} g^{\sigma \beta} - g^{\sigma \beta} g^{\sigma \alpha}) + a_5 (g^{\sigma \alpha} v^{\beta} - g^{\sigma \beta} v^{\alpha}) \gamma^{\sigma} + ia_6 (g^{\sigma \alpha} v^{\beta} - g^{\sigma \beta} v^{\alpha}) v^{\sigma} + a_7 (g^{\sigma \alpha} v^{\beta} - g^{\sigma \beta} v^{\alpha}) \gamma^{\sigma} + a_8 (g^{\sigma \alpha} v^{\beta} - g^{\sigma \beta} v^{\alpha}) \gamma^{\sigma} + a_9 (g^{\sigma \alpha} v^{\beta} - g^{\sigma \beta} v^{\alpha}) \gamma^{\sigma}.
$$

(2.10)

Finishing the trace calculations, we get from \[(2.2)\] and \[(2.3)\]

$$
\begin{aligned}
2m_{D_1} v^\mu & = \frac{m_{D_1}}{2 \Lambda_{D_1}} \{ -2 \Lambda' \xi' + \frac{2}{m_c} (\kappa'_1(1) + 5 \kappa'_2(1)) \} (\epsilon^* \cdot \epsilon) v^\mu, \\
2m_{D_2} v^\mu & = \frac{m_{D_2}}{2 \Lambda_{D_2}} \{ 2 \Lambda' \xi' - \frac{2}{m_c} (\kappa'_1(1) - 3 \kappa'_2(1)) \} \epsilon^{\sigma \alpha} \epsilon_{\sigma \alpha} v^\mu.
\end{aligned}
$$

(2.11)
where
\[ \kappa'_{2} = -\frac{1}{3}(a_1 - a_2 - a_4 - a_6 - a_8) \] (2.12)

Eq. (2.11) yields
\[ \tilde{\Lambda}_D = \bar{\Lambda}' - \frac{1}{m_c}(\kappa'_1(1) + 5\kappa'_2(1)), \]
\[ \tilde{\Lambda}_{D^*} = \bar{\Lambda}' - \frac{1}{m_c}(\kappa'_1(1) - 3\kappa'_2(1)). \] (2.13)

In deriving these formulae we have used the normalization of the leading order wave function \( \xi'(1) \):
\[ \xi'(1) = 1, \] (2.14)

which is a direct result of eqs. (2.4) and (2.11).

Eq. (2.13) is similar to those relations for ground state mesons \[6, 7\]:
\[ \tilde{\Lambda}_D = \bar{\Lambda} - \frac{1}{m_c}(\kappa_1(1) + 3\kappa_2(1)), \]
\[ \tilde{\Lambda}_{D^*} = \bar{\Lambda} - \frac{1}{m_c}(\kappa_1(1) - \kappa_2(1)). \] (2.15)

For \( B \to (D_1, D_2^*) \) decays, we parameterize the relevant matrix elements as follows
\[ \langle H_{v'} \vert Q^j_\nu \Gamma Q^j_\nu \vert H_v \rangle = \tau(y) Tr[v_{\nu} \bar{F}^a \nu \Gamma M_{\nu}], \]
\[ \langle H_{v'} \vert Q^j_\nu \Gamma \frac{P_+}{i\nu \cdot D} D^2 \frac{Q^j_\nu}{\nu} \vert H_v \rangle = -\eta_{\nu}^b(y) \frac{1}{\Lambda} Tr[v_{\nu} \bar{F}^a \nu \Gamma M_{\nu}], \]
\[ \langle H_{v'} \vert Q^j_\nu \Gamma \frac{P_+}{-i\nu \cdot D} \bar{Q}^j_\nu \vert H_v \rangle = -\eta_{\nu}^c(y) \frac{1}{\Lambda} Tr[v_{\nu} \bar{F}^a \nu \Gamma M_{\nu}], \]
\[ \langle H_{v'} \vert \bar{Q}^j_\nu \Gamma \frac{P_+}{i\nu \cdot D} \bar{Q}^j_\nu \sigma_{\alpha\beta} F^{\alpha\beta} \bar{Q}^j_\nu \vert H_v \rangle = -\frac{1}{\Lambda} Tr[R_{\sigma\alpha\beta}^{b}(v, v') \bar{F}^\nu \Gamma P_+ i\sigma_{\alpha\beta} M_{\nu}], \]
\[ \langle H_{v'} \vert \bar{Q}^j_\nu \frac{i}{2} \sigma_{\alpha\beta} F^{\alpha\beta} \frac{P_+}{-i\nu \cdot D} \bar{Q}^j_\nu \vert H_v \rangle = -\frac{1}{\Lambda} Tr[R_{\sigma\alpha\beta}^{c}(v, v') \bar{F}^\nu i\sigma_{\alpha\beta} P_+ \Gamma M_{\nu}], \] (2.16)

where \( R_{\sigma\alpha\beta}^{b} \) and \( R_{\sigma\alpha\beta}^{c} \) can generally be written as
\[ R_{\sigma\alpha\beta}^{b}(v, v') = \eta_{\nu}^b v_{\sigma\alpha\gamma\beta} + \eta_{\nu}^b v_{\sigma\alpha\gamma\beta} + \eta_{\nu}^b g_{\sigma\alpha} v_{\beta'}, \]
\[ R_{\sigma\alpha\beta}^{c}(v, v') = \eta_{\nu}^c v_{\sigma\alpha\gamma\beta} + \eta_{\nu}^c v_{\sigma\alpha\gamma\beta} + \eta_{\nu}^c g_{\sigma\alpha} v_{\beta'}. \] (2.17)

\( \tau, \eta_{\nu}^{b} \) and \( \eta_{\nu}^{c} \) are Lorentz scalar wave functions of \( \nu \).

Now finishing the trace calculations, and taking into account eqs. (2.2), (2.4), (2.6), (2.13) and (2.15), one finds that \( f_i \) and \( k_i \) are related to the transition wave functions as follows
\[
\sqrt{6}f_{V_1} = (1 - y^2) \left\{ \frac{\tau}{2m_b\Lambda} (\kappa_1(1) + 3\kappa_2(1)) + \frac{\tau}{2m_c\Lambda'} (\kappa'_1(1) + 5\kappa'_2(1)) \right\}
+ \frac{1}{m_b\Lambda} \eta^b + \frac{1}{m_c\Lambda'} [\eta^c_1 - \frac{3}{2} \eta^c_3],
\]
\[
\sqrt{6}f_{V_2} = -3[\bar{\tau} + \frac{\tau}{2m_b\Lambda} (\kappa_1(1) + 3\kappa_2(1)) + \frac{\tau}{2m_c\Lambda'} (\kappa'_1(1) + 5\kappa'_2(1))]
- \frac{3}{m_b\Lambda} \eta^b - \frac{5}{m_c\Lambda'} [-\eta^c_1 + \frac{1}{2} \eta^c_3],
\]
\[
\sqrt{6}f_{V_3} = (y - 2)[\bar{\tau} + \frac{\tau}{2m_b\Lambda} (\kappa_1(1) + 3\kappa_2(1)) + \frac{\tau}{2m_c\Lambda'} (\kappa'_1(1) + 5\kappa'_2(1))]
+ \frac{1}{m_b\Lambda} (y - 2) \eta^b + \frac{1}{m_c\Lambda'} [\eta^c_1 (6 + y) - 2\eta^c_2 (1 - y) - \eta^c_3 (1 - \frac{3}{2} y)],
\]
\[
\sqrt{6}f_A = -(1 + y) \left\{ \frac{\tau}{2m_b\Lambda} (\kappa_1(1) + 3\kappa_2(1)) + \frac{\tau}{2m_c\Lambda'} (\kappa'_1(1) - 3\kappa'_2(1)) \right\}
+ \frac{1}{m_b\Lambda} \eta^b + \frac{1}{m_c\Lambda'} [\eta^c_1 - \frac{1}{2} \eta^c_3],
\]
\[
k_{A_1} = -(1 + y) \left\{ \frac{\tau}{2m_b\Lambda} (\kappa_1(1) + 3\kappa_2(1)) + \frac{\tau}{2m_c\Lambda'} (\kappa'_1(1) - 3\kappa'_2(1)) \right\}
+ \frac{1}{m_b\Lambda} \eta^b + \frac{1}{m_c\Lambda'} [\eta^c_1 - \frac{1}{2} \eta^c_3],
\]
\[
k_{A_2} = \frac{1}{m_c\Lambda'} \eta^c_2.
\]
\[
k_{A_3} = \left[ \frac{\tau}{2m_b\Lambda} (\kappa_1(1) + 3\kappa_2(1)) + \frac{\tau}{2m_c\Lambda'} (\kappa'_1(1) - 3\kappa'_2(1)) \right]
+ \frac{1}{m_b\Lambda} \eta^b + \frac{1}{m_c\Lambda'} [\eta^c_1 - \eta^c_2 - \frac{1}{2} \eta^c_3],
\]
\[
k_{V} = \left[ -\frac{\tau}{2m_b\Lambda} (\kappa_1(1) + 3\kappa_2(1)) + \frac{\tau}{2m_c\Lambda'} (\kappa'_1(1) - 3\kappa'_2(1)) \right]
- \frac{1}{m_b\Lambda} \eta^b - \frac{1}{m_c\Lambda'} [\eta^c_1 - \frac{1}{2} \eta^c_3],
\]
(2.18)

where
\[
\bar{\tau} = \tau - \frac{\eta^b_0}{2m_b\Lambda} - \frac{\eta^c_0}{2m_c\Lambda'},
\]
\[
\eta^b = -3\eta^b_1 - (1 - y) \eta^b_2 - \frac{1}{2} \eta^b_3.
\]
(2.19)

III. QCD SUM RULE EVALUATION FOR \( \tau, \eta^b_0 \) AND \( \eta^c_0 \)

In order to calculate the wave functions \( \tau \) and \( \eta^b_0, \eta^c_0 \), one may study the analytic properties of the three-point correlation functions

\[
i^2 \int d^4 x d^4 z e^{i(p' \cdot x - p \cdot z)} \langle 0 | T \{ \bar{J}^{\nu}_{1,+,3/2}(x), (\bar{Q} \Gamma Q)(0), J^I_{0,-,1/2}(z) \} | 0 \rangle = \Xi^\tau(\omega, \omega', y) L^\nu_{V,A},
\]
(3.1)
\[
i^2 \int d^4 x d^4 z e^{i(p' \cdot x - p \cdot z)} \langle 0 | T \{ J^{\alpha\beta}_{2,+,3/2}(x), (\bar{Q} \Gamma Q)(0), J^I_{0,-,1/2}(z) \} | 0 \rangle = \Xi^\tau(\omega, \omega', y) L^{\alpha\beta}_{V,A},
\]
(3.2)
\[ i^2 \int d^4 x d^4 y e^{i(p' \cdot x - p \cdot y)} \langle 0 | T \left\{ J_{1,+,j/2}^\nu(x), (\bar{Q} \Gamma \frac{P_+}{iD} D_\perp^2 Q)(0), J_{0,,-1/2}^\dagger(z) \right\} | 0 \rangle \]
\[ = \Xi^b_{\nu}(\omega, \omega', y) L_{V,A}^{\mu \nu}, \quad (3.3) \]
\[ i^2 \int d^4 x d^4 y e^{i(p' \cdot x - p \cdot y)} \langle 0 | T \left\{ J_{2,+,j/2}^{\alpha \beta}(x), (\bar{Q} \Gamma \frac{P_+}{iD} D_\perp^2 Q)(0), J_{0,,-1/2}^\dagger(z) \right\} | 0 \rangle \]
\[ = \Xi^b_{\nu}(\omega, \omega', y) L_{V,A}^{\mu \alpha \beta}, \quad (3.4) \]
\[ i^2 \int d^4 x d^4 y e^{i(p' \cdot x - p \cdot y)} \langle 0 | T \left\{ J_{1,+,j/2}^\nu(x), (\bar{Q} \tilde{D}_\perp^2 \frac{P_+}{-i\omega'} \Gamma Q)(0), J_{0,,-1/2}^\dagger(z) \right\} | 0 \rangle \]
\[ = \Xi^b_{\nu}(\omega, \omega', y) L_{V,A}^{\mu \alpha \beta}, \quad (3.5) \]
\[ i^2 \int d^4 x d^4 y e^{i(p' \cdot x - p \cdot y)} \langle 0 | T \left\{ J_{2,+,j/2}^{\alpha \beta}(x), (\bar{Q} \tilde{D}_\perp^2 \frac{P_+}{-i\omega'} \Gamma Q)(0), J_{0,,-1/2}^\dagger(z) \right\} | 0 \rangle \]
\[ = \Xi^b_{\nu}(\omega, \omega', y) L_{V,A}^{\mu \alpha \beta}, \quad (3.6) \]

with \( \Gamma \) being \( \gamma^\mu \) and \( \gamma^\mu \gamma^5 \) for vector and axial vector heavy quark currents separately. \( L_{V,A}^{\mu \nu(\rho \sigma \beta)} \) are Lorentz structures associated with the vector and axial vector currents (see Appendix B). \( J_{1,+,j/2}^\nu \) are the proper interpolating currents for the heavy-light mesons. Here

\[ J_{0,,-1/2}^{1\alpha} = \sqrt{\frac{1}{2}} \bar{Q} \gamma^\alpha q \quad \text{for pseudoscalar meson}, \]
\[ J_{1,,-1/2}^{1\alpha} = \sqrt{\frac{1}{2}} \bar{Q} \gamma^\alpha q \quad \text{for vector meson}, \quad (3.7) \]

for \( \frac{1}{2}^- \) ground state doublet, and

\[ J_{1,+,3/2}^{1\alpha} = \sqrt{\frac{3}{4}} \bar{Q}_v^+ \gamma^5 \gamma^\alpha (-i)(D_\perp^\alpha - \frac{1}{3} \gamma_\perp \gamma^\alpha \not{D}_\perp) q \quad \text{for } D_1, \]
\[ J_{2,+,3/2}^{1\alpha \beta} = \sqrt{\frac{1}{2}} \bar{Q}_v^+ \frac{(-i)}{2} (\gamma_\perp \gamma^\alpha D_\perp^\beta + \gamma_\perp \gamma^\beta D_\perp^\alpha - \frac{2}{3} \gamma_\perp \gamma^\alpha \gamma^\beta \not{D}_\perp) q \quad \text{for } D_2. \quad (3.8) \]

for \( \frac{3}{2}^+ \) doublet.

Generally, the proper current \( J_{j,P,j} \) for the state with quantum numbers \( j, P, j \) have been investigated in [3,8]. These currents were proved to satisfy the following conditions

\[ \langle 0 | J_{j,P,j}^{\alpha_1 \cdots \alpha_j}(0) | j', P', j'_l \rangle = i f_{P,j} \delta_{j,j'} \delta_{P,P'} \delta_{j,j'} \eta_{\alpha_1 \cdots \alpha_j}, \]
\[ i \langle 0 | T(J_{j,P,j}^{\alpha_1 \cdots \alpha_j}(x)J_{j',P',j'_l}^{\beta_1 \cdots \beta_j}(0))|0 \rangle = \delta_{j,j'} \delta_{P,P'} \delta_{j,j'} (-1)^j S g_{\perp}^{\alpha_1 \beta_1} \cdots g_{\perp}^{\alpha_j \beta_j} \]
\[ \times \int dt \delta(x - vt) \Pi_{P,j_l}(x) \quad (3.9) \]
in the limit \( m_Q \to \infty \). \( \eta_{\alpha_1 \cdots \alpha_j} \) is the polarization tensor for the spin \( j \) state, \( g_{\perp}^{\alpha \beta} = g^{\alpha \beta} - v^\alpha u^\beta \) is the transverse metric tensor, and \( \gamma_\perp = \gamma^\alpha - v^\alpha (v \cdot \gamma) \). \( S \) denotes symmetrizing the indices and subtracting the trace terms separately in the sets \( (\alpha_1 \cdots \alpha_j) \) and \( (\beta_1 \cdots \beta_j) \), \( f_{P,j} \) and \( \Pi_{P,j} \) are a constant and a function of \( x \) respectively, they depend only on \( P \) and \( j_l \).
Eqs. (3.9) implies that the sum rules in HQET (or HQEFT) for decay amplitudes derived from correlators containing such currents receive contributions only from one of the two states with the same spin-parity \((j, P)\) in the \(m_Q \to \infty\) limit. And starting from the leading order, the \(1/m_Q\) corrections to the decay amplitudes can then be calculated unambiguously order by order.

It should be noticed that the HQEFT differs from the HQET only in the \(1/m_Q\) corrections. Therefore \(f_{P,j}\) are the same in the two frameworks. For the ground state \(\frac{1}{2}^-\) mesons, the sum rule for \(f_{-1/2}\) is also known \([10,14]\) in the HQET. It was also checked again in the HQEFT (where \(F\) is the sum rule for it reads \(\frac{2}{T} \langle \bar{\Lambda} \Gamma | Q | \Lambda \rangle = \sum_{\Lambda V,A} S_{\Lambda V,A}(\Lambda V,A)\).

Our result is \(\frac{2}{T} \langle \bar{\Lambda} \Gamma | Q | \Lambda \rangle = \frac{1}{T} \langle \bar{\Lambda} \Gamma | Q | \Lambda \rangle\) \([7]\). Our result is

\[
f_{-1/2}^2 e^{-2\Lambda/T} = \frac{3}{16\pi^2} \int_0^{\infty} d\omega e^{-\omega/T} \omega^2 = \frac{\langle q \bar{q} \rangle}{2(1 + 4\alpha_s/3\pi)} - \frac{\langle q \bar{q} \rangle}{8\pi T^2} (1 + 4\alpha_s/\pi) - \frac{\langle q \bar{q} \rangle}{48\pi T} = SR_{-1/2}. \tag{3.10}
\]

For the \(\frac{3}{2}^+\) doublet, the sum rule for \(f_{3/2}\) is found to be \([13]\)

\[
f_{3/2}^2 e^{-2\Lambda/T} = \frac{1}{64\pi^2} \int_0^{\infty} d\omega e^{-\omega/T} \omega^4 + i \langle q \bar{q} \rangle_{\Lambda V,A}^2 - \frac{\langle q \bar{q} \rangle}{32\pi T} \frac{y + 5}{(y + 1)^2} = SR_{+3/2}. \tag{3.11}
\]

The leading order wave function \(\tau\) for \(B \to (D_1, D_2)\) decays is evaluated in \([1]\) through studying the three point correlation functions \((3.1)\) and \((3.2)\). The Borel transformed sum rule for it reads

\[
f_{-1/2} f_{+3/2} \tau e^{-\Lambda/T} = \frac{1}{2\pi^2} \int_0^{\infty} d\omega e^{-\omega/T} \omega^3 + i \frac{\langle q \bar{q} \rangle_{\Lambda V,A}^2}{12T} - \frac{\langle q \bar{q} \rangle}{96\pi} \frac{y + 5}{(y + 1)^2} = SR_{+}. \tag{3.12}
\]

The total external momenta in \((3.1)-(3.4)\) are \(p = m_Q v + k\) and \(p' = m_Q v' + k'\) with \(k\) and \(k'\) being the residual momenta of the heavy quarks. \(\Xi_{\Lambda V,A}^0(\Lambda V, y)\) and \(\Xi_{\Lambda V,A}^0(\Lambda V, y)\) are analytic functions of \(\omega = 2v \cdot k + O(1/m_Q)\) and \(\omega' = 2v' \cdot k' + O(1/m_Q)\) with discontinuities for their positive values. Saturating the correlators in eqs. \((3.3)-(3.6)\) with physical intermediate states in HQEFT, phenomenologically we represent them as follows

\[
\Xi_{\Lambda V,A}^{\mu(\mu\beta)} = \sum_{M', M} \frac{\langle 0 | J_{\mu(\mu\beta)}^{\Lambda V,A} | M' \rangle \langle M' | \bar{Q}^{\Lambda V,A} | 0 \rangle | M | J_{0,-1/2}^\Lambda | 0 \rangle}{(2\Lambda - \omega - i\epsilon)(2\Lambda' - \omega' - i\epsilon)} + \int_D dv dv' \rho_{\text{phys}}(v, v') \frac{\langle 0 | J_{\mu(\mu\beta)}^{\Lambda V,A} | M' \rangle \langle M' | \bar{Q}^{\Lambda V,A} | 0 \rangle | M | J_{0,-1/2}^\Lambda | 0 \rangle}{(2\Lambda - \omega - i\epsilon)(2\Lambda' - \omega' - i\epsilon)} L_{\Lambda V,A}^{\mu(\mu\beta)} + \text{subtractions}, \tag{3.13}
\]

with the first term in each equation being a double-pole contribution, and the second representing the higher resonance contributions in the form of a double dispersion integral over physical intermediate states in the proper integration domain \(D\).
The matrix elements in (3.13) and (3.14) can then be transformed into series of the matrix elements in the HQEFT in powers of $1/m_Q$ through (2.6). With (3.9), the first terms in (3.13) and (3.14) become in the limit $m_Q \to \infty$

\begin{equation}
\Xi^{b}_{pole} L^{\mu\alpha(\mu\beta)}_{V,A} = \frac{f_{+3/2} f_{-1/2}}{(2\Lambda - \omega - i\epsilon)(2\Lambda' - \omega' - i\epsilon)} \frac{-\eta^{b}_{0} L^{\mu\alpha(\mu\beta)}}{\Lambda} (3.15)
\end{equation}

\begin{equation}
\Xi^{c}_{pole} L^{\mu\alpha(\mu\beta)}_{V,A} = \frac{f_{+3/2} f_{-1/2}}{(2\Lambda - \omega - i\epsilon)(2\Lambda' - \omega' - i\epsilon)} \frac{-\eta^{c}_{0} L^{\mu\alpha(\mu\beta)}}{\Lambda'} (3.16)
\end{equation}

On the other hand, the correlation functions may be written as

\begin{equation}
\Xi^{b(c)}_{theo} = \int_{-\infty}^{\infty} d\nu d\nu' \frac{\rho^{b(c)}_{pert}}{(\nu - \omega - i\epsilon)(\nu' - \omega' - i\epsilon)} + \Xi_{NP} + subtractions (3.17)
\end{equation}

with the first term being perturbative contributions and the second non-perturbative ones. These can be calculated order by order in the framework of HQEFT by using the perturbation theory and the operator product expansion (OPE) as well. The Lorentz structure $L^{\mu\alpha(\mu\beta)}_{V,A}$ is extracted out and not presented in (3.17).

Assuming the quark-hadron duality, the sum rules for $\eta^{b}_{0}$ and $\eta^{c}_{0}$ read

\begin{equation}
\Xi^{b(c)}_{pole} = \int_{0}^{\nu_1} d\nu \int_{0}^{\nu_2} d\nu' \frac{\rho^{b(c)}_{pert}}{(\nu - \omega - i\epsilon)(\nu' - \omega' - i\epsilon)} + \Xi_{NP} + subtractions, (3.18)
\end{equation}

The next step of sum rule method is to perform the Borel operator

\begin{equation}
\hat{B}_{T}^{(\omega)} \equiv T \lim_{n \to \infty, -\omega \to \infty} \frac{\omega^n}{\Gamma(n)} \left( -\frac{d}{d\omega} \right)^n \text{ with } T = \frac{-\omega}{n} \text{ fixed} (3.19)
\end{equation}

to both sides of sum rules. Since there are two momentum variables $\omega$ and $\omega'$ for the correlators (3.3)-(3.6), a double Borel transformation $\hat{B}_{T}^{(\omega') \hat{B}_{T}^{(\omega)}}$ should be performed to them. This has the effect to suppress higher resonance contributions on one hand, and to enhance the importance of low dimension condensates on the other, and thirdly, it also eliminates the subtraction terms.

In QCD sum rule analysis for B semileptonic decays into ground state charmed mesons, it was argued [14–16] that the hadronic and perturbative spectral densities can not be locally dual to each other. But if one integrates the spectral densities over the "off-diagonal" variable $\nu_- = \frac{\nu - \nu'}{2}$, keeping the "diagonal" variable $\nu_+ = \frac{\nu + \nu'}{2}$ fixed, the quark-hadron duality is restored in $\nu_+$ for the integrated spectral densities. This method was also used in [7] to calculate the transitions between ground state mesons in the HQEFT.

In the present case, the initial and final states belong to different doublets and are asymmetric. If one uses an asymmetric triangle in the perturbative integral, however, the resulting wave functions or their derivatives will unfortunately be divergent at $y = 1$ [3]. For these reasons, here we shall follow the method used in [7,3], namely taking $t' = t = 2T$ and integrating first the spectral density over $\nu_-$ in the region $-\nu_+ < \nu_- < \nu_+$, we then obtain from the quark-hadron duality the following form

\begin{equation}
\tilde{\Xi}_{pole} = 2 \int_{0}^{\nu_{+}^{(1)}} d\nu_+ e^{-\nu_+/T} \tilde{\rho}_{pert}(\nu_+) + \tilde{\Xi}_{NP}, (3.20)
\end{equation}
where \( \tilde{\Xi} \) denotes the result obtained by applying double Borel operators to \( \Xi \), and

\[
\tilde{\rho}_{\text{pert}}(\nu_+) = \int_{-\nu_+}^{\nu_+} d\nu_- \rho_{\text{pert}}(\nu_+, \nu_-). \tag{3.21}
\]

In the present calculations, we will neglect the light quark mass and higher radiative corrections. For non-perturbative terms, we consider only the contributions of the quark condensate, the gluon condensate and the mixed quark-gluon condensate. The relevant Feynman diagrams are presented in Fig.1. From the sum rules, we arrive at the following results

\[
f_{+3/2} f_{-1/2} \frac{\eta_0^b}{\Lambda} e^{-\frac{(\tilde{\Xi} + \tilde{\Xi}')}{T}} = \frac{1}{8\pi^2} \int_0^{\omega_+^{(1)}} d\omega_+ e^{-\frac{\omega_+}{T} \omega_+^4} - \frac{\alpha_s \langle FF \rangle}{96\pi} \frac{7 - y}{(1 + y)^3} T - \frac{2\alpha_s \langle \bar{q} q \rangle}{3\pi} \frac{T^2}{(1 + y)^2} = \mathcal{S} \mathcal{R}_b
\]

\[
f_{+3/2} f_{-1/2} \frac{\eta_0^b}{\Lambda} e^{-\frac{(\tilde{\Xi} + \tilde{\Xi}')}{T}} = \frac{3}{8\pi^2} \int_0^{\omega_+^{(1)}} d\omega_+ e^{-\frac{\omega_+}{T} \omega_+^4} + \frac{\alpha_s \langle FF \rangle}{96\pi} \frac{9y + 1}{(1 + y)^3} T - \frac{2\alpha_s \langle \bar{q} q \rangle}{3\pi} \frac{(3 + 2y)T^2}{(1 + y)^2} = \mathcal{S} \mathcal{R}_c \tag{3.22}
\]

In the numerical calculations, we take the following typical values \( \alpha_s = g_s^2/4\pi \) for the condensates

\[
\langle \bar{q} q \rangle \approx -0.23 \text{ GeV}^3;
\]

\[
i \langle \bar{q} \sigma_{\alpha\beta} F_{\alpha\beta} q \rangle \approx -m_0^2 \langle \bar{q} q \rangle \text{ with } m_0^2 = 0.8 \text{ GeV}^2;
\]

\[
\alpha_s \langle FF \rangle \equiv \alpha_s \langle F_{a\alpha} F_{a\beta} \rangle \approx 0.04 \text{ GeV}^4. \tag{3.23}
\]

From Eqs.\( (3.10)-(3.12), (3.22) \), one easily gets

\[
\tau = \frac{\mathcal{S} \mathcal{R}_\tau}{\sqrt{\mathcal{S} \mathcal{R}_{-1/2} \times \mathcal{S} \mathcal{R}_{+3/2}}},
\]

\[
\frac{\eta_0^b}{\Lambda} = \frac{\mathcal{S} \mathcal{R}_b}{\sqrt{\mathcal{S} \mathcal{R}_{-1/2} \times \mathcal{S} \mathcal{R}_{+3/2}}},
\]

\[
\frac{\eta_0^b}{\Lambda'} = \frac{\mathcal{S} \mathcal{R}_c}{\sqrt{\mathcal{S} \mathcal{R}_{-1/2} \times \mathcal{S} \mathcal{R}_{+3/2}}}. \tag{3.24}
\]

The QCD higher order radiative corrections have not been included in eqs.\( (3.10)-(3.12) \) and\( (3.22) \). But what we will use in our numerical analysis is eqs.\( (3.24) \), which are ratios of the three-point correlators to the two-point correlators. Though the QCD radiative corrections may be large, one may expect that they may not influence the ratios in eqs.\( (3.24) \) significantly due to the cancelation of numerators and denominators.

**IV. NUMERICAL ANALYSIS OF WAVE FUNCTIONS AND DECAY RATES**

Now we turn to the numerical analysis of the sum rules obtained in the previous section. Imposing usual criterion that both higher order power corrections and the contributions of the continuum should not be very large, we find the proper sum rule “windows”:
0.7GeV \langle T \langle 1.2GeV \rangle \rangle . f_{-1/2} and f_{+3/2} have been studied in \cite{10,3} and the corresponding thresholds were found to be in the ranges 1.6GeV/\omega_c(2.2GeV and 2.7GeV/\omega_c^{*}(3.2GeV). In Fig.2 we present \tau(1), \eta^{b\prime}_0(1) and \eta^{c\prime}_0(1) as functions of \omega_c^{*} and \omega_c^{*\prime}(1), where \omega_c = 1.9GeV and \omega_c^{*\prime} = 2.95GeV are used. We find that \tau(1) are stable around the threshold \omega_c^{*} \approx 2.35GeV, whereas \eta^{b\prime}_0(1) and \eta^{c\prime}_0(1) are stable around a smaller threshold value \omega_c^{*\prime}(1) \approx 1.85GeV. With this analysis, we obtain the following values for the wave functions

\[ \tau(1) = 0.8 \pm 0.1GeV \]

\[ -\frac{\eta^{b\prime}_0(1)}{\Lambda} = 0.35 \pm 0.04GeV \]

\[ -\frac{\eta^{c\prime}_0(1)}{\Lambda^\prime} = 1.15 \pm 0.15GeV \]

(4.1)

Where the errors mainly arise from the threshold. In Fig.3, the variations of \tau, \eta^{b\prime}_0 and \eta^{c\prime}_0 with respect to y are shown, where we have used \omega_c^{*\prime} = 1GeV.

We now come to consider the \( B \rightarrow (D_1, D_2) \) decay rates. The differential decay rates are given by

\[
\frac{d\Gamma(B \rightarrow D_l l\bar{\nu})}{dy} = \frac{G_F^2 |V_{ub}|^2 m_B^5}{48\pi^3} r^3_1 \sqrt{y^2 - 1} \{2(1 - 2yr_1 + r_1^2)[f_{V_1}^2 + (y^2 - 1)f_{A_1}^2] + [(y - r_1)f_{V_1} + (y^2 - 1)(f_{V_1} + r_1f_{V_2})]^2 \}
\]

\[
\frac{d\Gamma(B \rightarrow D^*_l l\bar{\nu})}{dy} = \frac{G_F^2 |V_{ub}|^2 m_B^5}{144\pi^3} r^3_2 (y^2 - 1)^{3/2} \{3(1 - 2yr_2 + r_2^2)[k_{A_1}^2 + (y^2 - 1)k_{V_1}^2] + 2[(y - r_2)k_{V_1} + (y^2 - 1)(k_{A_1} + r_2k_{A_2})]^2 \}
\]

(4.2)

with the kinematically allowed ranges 1\langle y \langle 1.32 for \( B \rightarrow D_l l\bar{\nu} \) and 1\langle y \langle 1.31 for \( B \rightarrow D^*_l l\bar{\nu} \). The form factors \( f_i, k_l \) are related to the wave functions as shown in (2.18). The zero recoil values of \( \kappa_1 \) and \( \kappa_2 \) in (2.18) have been evaluated by fitting from the ground state meson masses \cite{5} and also by the QCD sum rule method \cite{7}. Similarly, \( \kappa'_1(1) \) can also be extracted from fitting the meson masses given in (2.13), i.e.,

\[ \kappa'_2(1) = \frac{m_c}{8}(m_{D^*_2}^2 - m_{D_1}^2). \]

(4.3)

To extract \( \kappa'_1(1) \), we consider the spin average mass of each doublet:

\[ \bar{m}_H = \frac{n_-m_{H^-} + n_+m_{H^+}}{n_- + n_+} \]

(4.4)

with

\[ n_{\pm} = 2j_{\pm} + 1, \quad j_{\pm} = j_{\pm} \pm \frac{1}{2}. \]

(2.13), (2.15) and (4.4) yield

\[
\kappa'_1(1) - \kappa_1(1) = \frac{|(\bar{m}_B - \bar{m}_B) - (\bar{m}_D - \bar{m}_D)|}{m_B - m_c},
\]

\[
\bar{\Lambda}' - \bar{\Lambda} = \frac{m_c(\bar{m}_B - \bar{m}_B) - m_c(\bar{m}_D - \bar{m}_D)}{m_B - m_c}.
\]

(4.5)
Here \( \bar{m}_{B(D)} \) are the average masses of the \( j_i^P = \frac{1}{2}^- \) ground state mesons, whereas \( \bar{m}'_{B(D)} \) are the ones of the \( j_i^P = \frac{3}{2}^+ \) doublet mesons.

When taking the average meson masses to be \( \bar{m}_D = 1.971 \text{GeV} \), \( \bar{m}'_D = 2.445 \text{GeV} \), \( \bar{m}_B = 5.314 \text{GeV} \) and \( \bar{m}'_B = 5.73 \text{GeV} \) and the quark masses \( m_b = 4.8 \text{GeV} \), \( m_c = 1.35 \text{GeV} \), we arrive at the following values

\[
\begin{align*}
\kappa_1(1) &= -0.56 \text{GeV}^2 \\
\kappa_2(1) &= 0.05 \text{GeV}^2 \\
\kappa'_1(1) &= -0.83 \text{GeV}^2 \\
\kappa'_2(1) &= 0.00675 \text{GeV}^2
\end{align*}
\] (4.6)

\( \eta_i^Q \) characterize the matrix elements of the chromomagnetic operator. They are often neglected from the argument that the chromomagnetic operator must have small effects due to the small \( D_2 - D_1 \) mass splitting [4]. In this section, we first neglect them but discuss their possible sizable effects late on. When \( \eta_i^Q \) are neglected, the formulae of decay rates become very simple because each differential decay rate depends only on a composite wave function.

\[
\begin{align*}
\frac{d\Gamma(B \to D_1 l \bar{\nu})}{dy} &= \frac{G_F^2 |V_{cb}|^2 m_B^5}{72\pi^3} r_1^3(y+1)(y^2-1)^{3/2}[(y-1)(1+1)(y-1)^2 + y(1-2y_1 + r_1^2)]\tau_1^2(y), \\
\frac{d\Gamma(B \to D_2 l \bar{\nu})}{dy} &= \frac{G_F^2 |V_{cb}|^2 m_B^5}{72\pi^3} r_2^3(y+1)(y^2-1)^{3/2}[(y+1)(1-2y_2 + r_2^2) + 3y(1-2y_2 + r_2^2)]\tau_2^2(y)
\end{align*}
\] (4.7)

with

\[
\begin{align*}
\tilde{\tau}_1(y) &= \tau - \frac{\eta_0^b}{2m_b\Lambda} - \frac{\eta_0^c}{2m_c\Lambda'} + \frac{\tau}{2m_b\Lambda}(\kappa_1 + 3\kappa_2 + (\kappa'_1 + 5\kappa'_2)), \\
\tilde{\tau}_2(y) &= \tau - \frac{\eta_0^b}{2m_b\Lambda} - \frac{\eta_0^c}{2m_c\Lambda'} + \frac{\tau}{2m_c\Lambda'}(\kappa_1 + 3\kappa_2 + (\kappa'_1 - 3\kappa'_2)).
\end{align*}
\] (4.8)

It is seen that the \( 1/m_Q \) corrections from the kinematic term do not change the relative values for the two differential decay rates. As the spin-symmetry breaking term \( \kappa'_2 \) arising from the mass splitting is small, without including the \( 1/m_Q \) corrections from the chromomagnetic terms, the relative value of their total decay rates should have the similar behavior as the one at the leading order, i.e.,

\[
R = \frac{\Gamma^0(B \to D_1 l \bar{\nu})}{\Gamma^1(B \to D_1 l \bar{\nu})} \sim \frac{\Gamma^0(B \to D_2 l \bar{\nu})}{\Gamma^1(B \to D_2 l \bar{\nu})}
\] (4.9)

which can be explicitly seen from Table 1, where we have used the input values \( m_b = 4.8 \text{GeV} \), \( m_c = 1.35 \text{GeV} \), \( |V_{cb}| = 0.038 \), the life time \( \tau_B = 1.6 \text{ps} \) and the thresholds \( \omega_c = 1.9 \text{GeV} \), \( \omega_{c*} = 2.95 \text{GeV} \), \( \omega_{c*} = 2.35 \text{GeV} \), \( \omega_{c*} = 1.85 \text{GeV} \). Note that both decay rates of \( B \to D_1 l \bar{\nu} \) and \( B \to D_2 l \bar{\nu} \) receive large \( 1/m_Q \) contributions (but not as large as the \( 1/m_Q \) contributions received by the \( B \to D_1 l \bar{\nu} \) decay rate given in [17] within the framework of the relativistic quark model based on the quasipotential approach. This is different from the discussions based on the quasipotential approach with the same structure of heavy quark
mass corrections predicted in the usual HQET \[17\], where the decay rate of \( B \to D_2^* \bar{\nu} \) is only slightly increased by subleading \( 1/m_Q \) corrections.

In comparison with the experimental data reported by CLEO \[18\] and ALEPH \[19\] groups, our result for the branching ratio of the \( B \to D_1 \bar{\nu} \) decay with the inclusion of \( 1/m_Q \) corrections is in agreement with both measurements. While for the \( B \to D_2^* \bar{\nu} \) decay, the result at \( m_Q \to \infty \) limit is within the CLEO upper limit but not within the ALEPH one. When including \( 1/m_Q \) contributions, we find that the resulting decay rate for \( Br(B \to D_2^* \bar{\nu}) \) seems to go over the CLEO upper limit though it may still be consistent within the large errors due to the big uncertainties of the choices of the thresholds. On the other hand, in deriving the results in Table 1, we have made the assumption that \( \eta_i^Q = 0 \). In general, the contributions of \( \eta_i^Q \) may not be neglected. Their effects have shown in Fig.4, where we have used the above sum rule results for \( \tau, \eta_i^b \) and \( \eta_i^c \) and the thresholds \( \omega_c = 1.9\text{GeV}, \omega_c^* = 2.95\text{GeV}, \omega_c^+ = 2.35\text{GeV} \) and \( \omega_c^{(1)} = 1.85\text{GeV} \). We have also made the assumption that \( \eta_i^Q \) and \( \tilde{\tau} \) have the similar dependence on \( y, \tilde{\tau} \approx \tilde{\tau}(1)/(1 + \frac{y^2}{a^2}) \) with \( a^2 \approx 0.7 \). It is seen from Fig.4 that \( \eta_i^b, \eta_i^c \) and \( \eta_i^b \) influence the branching ratios in different ways. When \( \eta_i^b \) becomes larger, \( Br(B \to D_1 \bar{\nu}) \) decreases while \( Br(B \to D_2^* \bar{\nu}) \) increases, and \( \eta_i^c \) influences the two branching ratios in the same manner as \( \eta_i^b \) does, but both branching ratios are not sensitive to \( \eta_i^c \). \( \eta_i^c \) has opposite effects on the two decay modes, i.e., its increment enlarges \( Br(B \to D_1 \bar{\nu}) \) but suppresses \( Br(B \to D_2^* \bar{\nu}) \). For \( \eta_i^b \), it always appears in the form \( \tilde{\tau} + \frac{\eta_i^b}{m_\Lambda} \), so it affects \( Br(B \to D_1 \bar{\nu}) \) and \( Br(B \to D_2^* \bar{\nu}) \) in the same way, i.e., a negative value of \( \eta_i^b \) may suppress both branching ratios. Some reasonable results for the branching ratios at certain values of \( \eta_i^Q \) are listed in Table 2 and Table 3. From Table 2, Table 3 and Fig.4, we see that both branching ratios of \( B \to D_1(D_2^*) \bar{\nu} \) may be suppressed after considering the possible effects due to the contributions of chromomagnetic terms at \( 1/m_Q \) order. The resulting two branching ratios can easily be made to be consistent with the experimental measurements when \( \eta_i^b, \eta_i^b, \eta_i^c \) and \( \eta_i^c \) are in the proper ranges.

V. SUMMARY

The HQEFT has been reviewed and applied to study the semileptonic \( B \) decays into excited charmed mesons \((D_1, D_2^*)\) with \( j_i^P = \frac{3}{2}^+ \). The form factors of the matrix elements relevant to these two decays have been expressed in terms of wave functions within the framework of HQEFT. It has been shown that with the inclusion of quark-antiquark coupled sectors, the relevant matrix elements can be parameterized by the leading order Isgur-Wise function and additional twelve wave functions \( \eta_i^Q (Q = b, c; i = 0, 1, 2, 3) \) and \( \kappa_i^j (j = 1, 2) \) appearing at \( 1/m_Q \) order. Two wave functions \( \tau_i^c \) and \( \tau_i^c \) characterizing the \( 1/m_Q \) corrections of effective current in the usual HQET \[42\] are absent in HQEFT.

By adopting proper interpolating currents for the heavy-light mesons, the leading order wave function \( \tau \) and two important wave functions \( \eta_i^b \) and \( \eta_i^c \) at \( 1/m_Q \) order have been calculated by the QCD sum rule approach. Zero recoil values of \( \kappa_i^1 \) and \( \kappa_i^2 \), which are \( 1/m_Q \) order wave functions for the matrix elements between \( \frac{3}{2}^+ \) excited states, have also been extracted from fitting the excited meson masses. By using these calculated results and also considering possible effects from wave functions \( \eta_i^Q (i = 1, 2, 3) \) arising from the chromomagnetic operators at \( 1/m_Q \) order, \( B \to D_1(D_2^*) \bar{\nu} \) decay rates and branching ratios
have been evaluated. When neglecting $\eta_i^Q (i = 1, 2, 3)$, we have shown that both of these decays receive large $1/m_Q$ corrections from $\eta_0^Q$ and $\kappa_i^{(c)}(1)$ ($i = 1, 2$). For $B \to D_1 l \bar{\nu}$ decay this is similar to the results obtained from quasiquark potential approach based on the same structure as in the usual HQET, but for $B \to D_2^* l \bar{\nu}$ decay the situation is quite different from the case described by that approach, where $Br(B \to D_2^* l \bar{\nu})$ only increases slightly when the $1/m_Q$ order contributions from $\tau_1^{(c)}$ are included. It has been shown that when considering all possible contributions from wave functions at the $1/m_Q$ order in the HQEFT, but without breaking the $1/m_Q$ expansion, the resulting branching ratios of the semileptonic $B$ decays into $\frac{3}{2}$ excited charmed mesons can agree well with the experimental measurements for the proper ranges of $\eta_i^Q$.

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APPENDIX A: LOCALITY OF HQEFT

For completeness and clarification, we briefly review the formulation of heavy quark effective field Lagrangian that keeps both effective quark and antiquark fields [5]. In particular, we would like to emphasize, by adding this appendix, that the HQEFT is a local effective theory and contains no non-local operators.

Firstly, denote the heavy quark (antiquark) field as

$$Q = Q^+ + Q^-,$$

(A1)

with $Q^+$ and $Q^-$ corresponding to the two solutions of the Dirac equation.

Defining

$$\hat{Q}_v^\pm = \frac{1 \pm i\nu}{2} Q^\pm, \quad R_v^\pm = \frac{1 \mp i\nu}{2} Q^\pm$$

(A2)

with $\nu^\mu$ an arbitrary four-vector satisfying $\nu^2 = 1$. Furthermore $\hat{Q}_v^\pm$ are related to the desired effective fields defined as

$$Q_v = e^{i\nu m_Q v \cdot x} \hat{Q}_v, \quad \bar{Q}_v = \bar{Q}_v e^{-i\nu m_Q v \cdot x}.$$  

(A3)

From the equation of motion of quark field and antiquark field, the original fields $Q$ and $\bar{Q}$ can be expressed by the new fields as follows,

$$Q_v^\pm = [1 + (1 - \frac{\nu \cdot P}{2m_Q})^{-1}] \hat{Q}_v^\pm \equiv \hat{\omega} Q_v^\pm$$

$$= e^{\pm im_Q v \cdot x} [1 + (1 + \frac{i\nu \cdot P}{2m_Q})^{-1}] Q_v^\pm \equiv e^{\pm im_Q v \cdot x} \omega Q_v^\pm,$$

$$\bar{Q}_v^\pm = \bar{Q}_v^\pm [1 + (1 - \frac{-i\nu \cdot P}{2m_Q})^{-1}] \equiv \bar{Q}_v^\pm \bar{\omega}$$

$$= \bar{Q}_v^\pm [1 + (1 - \frac{-i\nu \cdot P}{2m_Q})^{-1}] e^{\pm im_Q v \cdot x} \equiv \bar{Q}_v^\pm \bar{\omega} e^{\pm im_Q v \cdot x}.$$  

(A4)
\[ \tilde{D}^\mu, \tilde{D}_\parallel \text{ and } \tilde{D}_\perp \text{ are defined as} \]
\[ \begin{align*}
\int \kappa \tilde{D}^\mu \varphi & \equiv - \int \kappa D^\mu \varphi, \\
\tilde{D}_\parallel & \equiv \varphi (v \cdot D), \\
\tilde{D}_\perp & \equiv \varphi - \varphi (v \cdot D). 
\end{align*} \quad (A5) \]

The QCD Lagrangian becomes
\[ \mathcal{L}_{QCD} = \mathcal{L}_{\text{light}} + \mathcal{L}_{\text{eff}}, \quad (A6) \]
where \( \mathcal{L}_{\text{light}} \) represents the part of Lagrangian containing no heavy quarks, and
\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_v^{++} + \mathcal{L}_v^{+-} + \mathcal{L}_v^{-+} + \mathcal{L}_v^{--} \quad (A7) \]
with
\[ \begin{aligned}
\mathcal{L}_v^{++} &= \tilde{Q}_v^+ [i \tilde{D}_\parallel - m_Q + \frac{1}{2m_Q} i \tilde{D}_\perp (1 - \frac{i \tilde{D}_\parallel + m_Q}{2m_Q})^{-1} i \tilde{D}_\perp] \tilde{Q}_v^+ \equiv \tilde{Q}_v^+ \hat{A} \tilde{Q}_v^+ \\
\mathcal{L}_v^{+-} &= \tilde{Q}_v^+ [i \tilde{D}_\parallel + \frac{1}{2m_Q} i \tilde{D}_\perp (1 - \frac{i \tilde{D}_\parallel + m_Q}{2m_Q})^{-1} i \tilde{D}_\perp] \tilde{Q}_v^+ \equiv \tilde{Q}_v^+ \hat{A} \tilde{Q}_v^+ \\
\mathcal{L}_v^{-+} &= \tilde{Q}_v^+ [-i \tilde{D}_\perp + \frac{1}{4m_Q^2} (-i \tilde{D}_\parallel) (1 - \frac{i \tilde{D}_\parallel + m_Q}{2m_Q})^{-1} i \tilde{D}_\perp] \\
&\times (1 - \frac{i \tilde{D}_\parallel + m_Q}{2m_Q})^{-1} i \tilde{D}_\perp) \tilde{Q}_v^+ \equiv \tilde{Q}_v^+ \hat{B} \tilde{Q}_v^+ \\
\mathcal{L}_v^{--} &= e^{2m_Qv \cdot x} \tilde{Q}_v^+ [-i \tilde{D}_\perp + \frac{1}{4m_Q^2} (-i \tilde{D}_\parallel) (1 - \frac{i \tilde{D}_\parallel + m_Q}{2m_Q})^{-1} i \tilde{D}_\perp] \\
&\times (1 - \frac{i \tilde{D}_\parallel + m_Q}{2m_Q})^{-1} i \tilde{D}_\perp) \tilde{Q}_v^+ \equiv e^{2m_Qv \cdot x} \tilde{Q}_v^+ \hat{B} \tilde{Q}_v^+ \quad (A8) 
\end{aligned} \]

It is seen that all parts of the effective Lagrangian are local.

When quark fields and antiquark fields decouple completely, it is reasonable to deal with only section \( \mathcal{L}_v^{++} \) or \( \mathcal{L}_v^{--} \) independently. This is just the case considered in the framework of the usual HQET. To consider the finite quark mass corrections one should also include the contributions from \( \mathcal{L}_v^{++} \) and \( \mathcal{L}_v^{--} \) as well.

Similarly, the heavy quark currents can also be decomposed into four parts in a similar way,
\[ J(x) = \tilde{Q}(x) \Gamma Q(x) = \tilde{Q}^+ \Gamma Q^+ + \tilde{Q}^+ \Gamma Q^- + \tilde{Q}^- \Gamma Q^+ + \tilde{Q}^- \Gamma Q^- \\
\rightarrow J_{\text{eff}}(x) = \tilde{Q}_v^+ \hat{\omega} \tilde{\omega} \tilde{Q}_v^+ + \tilde{Q}_v^+ \hat{\omega} \tilde{\omega} \tilde{Q}_v^- + \tilde{Q}_v^- \hat{\omega} \tilde{\omega} \tilde{Q}_v^+ + \tilde{Q}_v^- \hat{\omega} \tilde{\omega} \tilde{Q}_v^- \\
= J_{v}^{++} + J_{v}^{+-} + J_{v}^{-+} + J_{v}^{--}. \quad (A9) \]

The matrix element on the r.h.s. of (2.2) is actually evaluated via
\[ \langle H'_v | J_{\text{eff}} e^{i \int \mathcal{L}_{\text{eff}} | H_v} = \langle H'_v | (J_{v}^{++} + J_{v}^{+-} + J_{v}^{-+} + J_{v}^{--}) e^{i \int (\mathcal{L}_v^{++} + \mathcal{L}_v^{+-} + \mathcal{L}_v^{-+} + \mathcal{L}_v^{--})} | H_v \rangle. \quad (A10) \]
Here $J_{\text{eff}}$ and $\mathcal{L}_{\text{eff}}$ shall include all 4 parts instead of only the $'++'$ parts in (A7) and (A9) as has been done in the usual HQET. The first sector $'++'$ of $J_{\text{eff}}$ contributes

$$\langle H_{\nu}' | Q_{\nu}^{i+} \tilde{\omega} \Gamma \dot{\omega} \dot{Q}_{v}^{+} e^{i \int \mathcal{L}_{\text{eff}} | H_{v} \rangle}. \quad (A11)$$

If neglecting all except $'++'$ sectors in $J_{\text{eff}}$ and $\mathcal{L}_{\text{eff}}$, we get

$$\langle H_{\nu}' | Q_{\nu}^{i+} \tilde{\omega} \Gamma \dot{\omega} \dot{Q}_{v}^{+} e^{i \int \mathcal{L}_{\text{eff}} | H_{v} \rangle},$$

which is just what the usual HQET treated.

The contributions from the $'+-'$ sector of $J_{\text{eff}}$ is

$$\langle H_{\nu}' | \tilde{Q}_{\nu}^{i+} \tilde{\omega} \Gamma \dot{\omega} \dot{Q}_{v}^{+} e^{i \int \mathcal{L}_{\text{eff}} | H_{v} \rangle}. \quad (A12)$$

In the case that both the initial and final states contain only heavy quarks (no heavy antiquarks), matrix elements such as

$$\langle H_{\nu}' | \tilde{Q}_{\nu}^{i+} \tilde{\omega} \Gamma \dot{\omega} \dot{Q}_{v}^{+} | H_{v} \rangle$$

do not contribute, and (A12) contributes only at higher perturbation order, i.e., only when $\mathcal{L}_{\text{eff}}$ is inserted into the matrix elements. Therefore the leading order contribution from (A12) should be

$$\langle H_{\nu}' | i \int dxdy T\{ (\tilde{Q}_{\nu}^{i+} \tilde{\omega} \Gamma \dot{\omega} \dot{Q}_{v}^{+}) (x), (\tilde{Q}_{v}^{i-} \dot{B} \dot{Q}_{v}^{+}) (y) \} | H_{v} \rangle \quad (A13)$$

and then the effective antiquark fields $\tilde{Q}_{v}^{-}(x)$ and $\tilde{Q}_{v}^{-}(y)$ should be contracted. Due to (A8), this will yield the propagator

$$\frac{-iP_{-}}{\not{\bar{\psi}} v \cdot p + m_{Q} + O(1/m_{Q})} \quad (P_{\pm} \equiv \frac{1 \pm \not{v}}{2},$$

which is of $O(1/m_{Q})$.

Contributions from other sectors of the current $J_{\text{eff}}$ can be treated in the same way. It is easy to see that $\langle H_{\nu}' | J_{v}^{++} e^{i \int \mathcal{L}_{\text{eff}} | H_{v} \rangle$ gives contributions of $O(1)$, and $\langle H_{\nu}' | J_{v}^{+-} e^{i \int \mathcal{L}_{\text{eff}} | H_{v} \rangle$ is $O(1/m_{Q})$, whereas $\langle H_{\nu}' | J_{v}^{-+} e^{i \int \mathcal{L}_{\text{eff}} | H_{v} \rangle$ is $O(1/m_{Q}^{2})$ since $\mathcal{L}_{1/m_{Q}}$ should be inserted twice, and each contraction of $\tilde{Q}_{v}^{-}$ and $\ddot{Q}_{v}^{-}$ gives a $1/m_{Q}$ suppression.

To be more clear, contracting $\tilde{Q}_{v}^{-}$ and $\ddot{Q}_{v}^{-}$ in (A13) yields

$$\langle H_{\nu}' | i \int d^{4}x d^{4}y \tilde{Q}_{\nu}^{i+} (x) \tilde{\omega} \Gamma \dot{\omega} \int \frac{d^{4}p}{(2\pi)^{4}} \frac{-iP_{-} e^{-iP_{-}(x-u)}}{\not{\bar{\psi}} v \cdot p + m_{Q} + O(1/m_{Q})} \dot{B} \dot{Q}_{v}^{+} (y) e^{i \int \mathcal{L}_{\text{eff}} | H_{v} \rangle. \quad (A14)$$

which can be written in the following form with replacing the momentum $p$ by the derivative $\partial$ and performing the integral of the momentum

$$\langle H_{\nu}' | i \int d^{4}x d^{4}y \tilde{Q}_{\nu}^{i+} (x) \tilde{\omega} \Gamma \delta (x - y) \tilde{\omega} \frac{P_{-}}{i \not{\bar{\psi}} v \cdot \partial + m_{Q} + O(1/m_{Q})} \dot{B} \dot{Q}_{v}^{+} (y) e^{i \int \mathcal{L}_{\text{eff}} | H_{v} \rangle, \quad (A15)$$


Using the same trick, one can treat all contributions from \( \langle H'_{v'} | J_{v'}^+ e^i \int \mathcal{L}_{\text{eff}} | H_v \rangle \) and obtains
\[
\langle H'_{v'} | J_{v'}^+ e^i \int \mathcal{L}_{\text{eff}} | H_v \rangle = \langle H'_{v'} | Q'^+_v \Gamma \omega [-i \mathcal{D}_\parallel - \frac{1}{2m_Q} i \mathcal{D}_\perp (1 - \frac{i \mathcal{D}_\parallel}{2m_Q})^{-1} i \mathcal{D}_\perp]^{-1} BQ^+_v e^i \int \mathcal{L}_{\text{eff}} | H_v \rangle
\]
which means that, effectively, one can reexpress \( J_{v'}^+ \) to be the effective current in which only the effective quark fields \( \tilde{Q}'_v^+ \) and \( Q_v^+ \) are used. Namely,
\[
J_{v'}^+ = \langle Q'^+ \Gamma Q' \rangle = \tilde{Q}'_v^+ \Gamma \omega \tilde{Q}_v^-
\]
\[
\rightarrow \tilde{Q}'_v^+ \Gamma \omega (-A^{-1}) B \tilde{Q}_v^+ = e^{i(m_Q v' - m_Q v)v} \tilde{Q}'_v^+ \Gamma \omega (-A^{-1}) BQ^+_v
\]
In an analogous way, one can reexpress \( J_{v}^- \), \( J_{v}^+ \), \( \mathcal{L}_{v}^+ \), and \( \mathcal{L}_{v}^- \) into the corresponding effective currents and Lagrangians by only using the effective quark fields \( \tilde{Q}_v^+ \) and \( Q_v^+ \). Consequently, we have
\[
\mathcal{L} \rightarrow \mathcal{L}_{\text{eff}} \rightarrow \mathcal{L}_{\text{eff}}^{++} = \mathcal{L}_{v}^{++} + \tilde{\mathcal{L}}_{v}^{++},
\]
\[
\tilde{\mathcal{L}}_{v}^{++} \equiv \langle \mathcal{L}_{v}^{++} + \mathcal{L}_{v}^+ + \mathcal{L}_{v}^- \rangle = \frac{1}{2m_Q} Q_v^+[i \mathcal{D}_\parallel] + \frac{1}{2m_Q} \frac{i \mathcal{D}_\perp (1 - \frac{i \mathcal{D}_\parallel}{2m_Q})^{-1} i \mathcal{D}_\perp}{i \mathcal{D}_\parallel} \frac{1}{i \mathcal{D}_\parallel} i \mathcal{D}_\perp
\]
\[
\times [1 - \frac{i \mathcal{D}_\parallel}{2m_Q} + \frac{1}{2m_Q} \frac{1}{i \mathcal{D}_\parallel} i \mathcal{D}_\perp - \frac{1}{2m_Q} \frac{1}{i \mathcal{D}_\parallel} i \mathcal{D}_\perp]^{-1} i \mathcal{D}_\parallel \frac{1}{i \mathcal{D}_\parallel} i \mathcal{D}_\perp + \frac{1}{2m_Q} \frac{1}{i \mathcal{D}_\parallel} i \mathcal{D}_\perp (1 - \frac{i \mathcal{D}_\parallel}{2m_Q})^{-1} i \mathcal{D}_\perp] Q_v^+
\]
\[
= \frac{1}{2m_Q} \tilde{Q}_v^+ A \frac{1}{i \mathcal{D}_\parallel} i \mathcal{D}_\perp [1 - \frac{i \mathcal{D}_\parallel}{2m_Q} + \frac{1}{2m_Q} \frac{1}{i \mathcal{D}_\parallel} i \mathcal{D}_\perp - \frac{1}{2m_Q} \frac{1}{i \mathcal{D}_\parallel} i \mathcal{D}_\perp]^{-1} i \mathcal{D}_\parallel \frac{1}{i \mathcal{D}_\parallel} i \mathcal{D}_\perp A \tilde{Q}_v^+,
\]
which represents the additional contributions to the effective Lagrangian \( \mathcal{L}_{v}^{++} \) that has been widely adopted in the usual HQET. This additional part may be regarded as the effective potential part of heavy quark due to the exchanges of virtual antiquarks. It is seen that when one imposes the on-shell condition \( A \tilde{Q}_v^+ = 0 \), i.e. \( \mathcal{L}_{v}^{++} = 0 \), the effective potential part \( \tilde{\mathcal{L}}_{v}^{++} \) also vanishes, i.e. \( \tilde{\mathcal{L}}_{v}^{++} = 0 \). For off-shell case \( \langle i \mathcal{D}_\parallel \rangle \sim \langle i \mathcal{D}_\perp \rangle \sim \Lambda \), or
\[
\frac{\langle (i \mathcal{D}_\perp)^2 \rangle}{2m_Q} \ll \langle i \mathcal{D}_\parallel \rangle,
\]
the leading contribution of the effective potential part is
\[
\tilde{\mathcal{L}}_{v}^{++}|_{\text{LO}} = \tilde{Q}_v^+ \frac{(i \mathcal{D}_\perp)^2}{2m_Q} Q_v^+.
\]
Correspondingly, the heavy quark current turns out to be
\[
J = \tilde{Q} \Gamma Q \rightarrow J_{\text{eff}} \rightarrow J_{\text{eff}}^{++} \equiv J_{v}^{++} + \tilde{J}_{v}^{++}
\]
\[
= e^{i(m_Q v' - m_Q v)v} \{ \tilde{Q}_v^+ \Gamma Q_v^+ + \frac{1}{2m_Q} \tilde{Q}_v^+ \Gamma \frac{1}{i \mathcal{D}_\parallel} (i \mathcal{D}_\perp)^2 Q_v^+ + \frac{1}{2m_Q} \tilde{Q}_v^+ (-i \mathcal{D}_\parallel)^2 \frac{1}{-i \mathcal{D}_\parallel} \Gamma Q_v^+ + \frac{1}{4m_Q} \tilde{Q}_v^+ \Gamma \frac{1}{i \mathcal{D}_\parallel} i \mathcal{D}_\perp (i \mathcal{D}_\parallel) \}
\]
\[ x_i \hat{P}_Q Q_v^+ + \frac{1}{4m_Q^*} \tilde{Q}_v^+ (-i \hat{P}_{\perp})(-i \hat{P}_{\parallel})(-i \hat{P}_{\parallel}) \frac{1}{-i \hat{P}_{\parallel}} \Gamma Q_v^+ \\
+ \frac{1}{4m_Q^* m_Q} \tilde{Q}_v^+ (-i \hat{P}_{\parallel})^2 \frac{1}{-i \hat{P}_{\parallel}} \Gamma \frac{1}{i \hat{P}_{\parallel}} (i \hat{P}_{\parallel})^2 Q_v^+ + O(\frac{1}{m_Q^{(i)}}) \] 
\equiv J_{\text{eff}}^{(0)} + J_{\text{eff}}^{(1/m_Q)} \quad (A20)

with \( J_{\text{eff}}^{(0)} \) the leading term \( J_{\text{eff}}^{(0)} = e^{i(m_Q v - m_Q v')x} \tilde{Q}_v^+ \Gamma Q_v^+ \) and \( J_{\text{eff}}^{(1/m_Q)} \) the remaining terms in \( J_{\text{eff}} \).

**APPENDIX B: LORENTZ STRUCTURES**

Here we present the general Lorentz structures of \( L_{\nu\mu}^{\mu\alpha\beta} \) appeared in eqs. (3.1)-(3.6).

\[ L_{V}^{\mu\nu} = \frac{1}{\sqrt{6}} \left[(y^2 - 1)g^{\mu\nu} + 3v^\mu v^\nu + (1 - 2y)v^\mu v'^\nu - 3yv^\mu v'^\nu - (y - 2)v'^\mu v'^\nu \right], \quad (B1) \]

\[ L_{A}^{\mu\nu} = i \frac{1}{\sqrt{6}} (1 + y) \epsilon^{\mu\alpha\beta} v_\alpha v'_\beta, \quad (B2) \]

\[ L_{2V}^{\mu\alpha\beta} = -\frac{i}{2} \left( \epsilon^{\mu\alpha\sigma}(v_\beta - v'_\beta y) + \epsilon^{\mu\beta\sigma}(v_\alpha - v'_\alpha y) \right) v_\sigma v'_\rho, \quad (B3) \]

\[ L_{A}^{\mu\alpha\beta} = \frac{1}{3}(1 + y)g^{\alpha\beta}(v^\mu - v'^\mu) - \frac{1}{2}(1 + y)g^{\alpha\mu}(v^\beta - v'^\beta y) - \frac{1}{2}(1 + y)g^{\beta\mu}(v^\alpha - v'^\alpha y) \\
+ \frac{1}{2}(1 - y)v^\alpha v^\beta v'^\mu + \frac{1}{2}(1 - y)v^\alpha v'^\beta v'^\mu - \frac{1}{3}(1 + y)v^\alpha v'^\beta v^\mu \\
- \frac{2}{3}(1 + y)v^\alpha v'^\beta v'^\mu + (v'^\alpha v'^\beta + v^\alpha v^\beta)v'^\mu. \quad (B4) \]
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Fig.1. Feynman diagrams contributing to $\eta^b_0$ and $\eta^c_0$ at the order concerned. The box at the up of each diagram represents the $1/m_Q$ order heavy-heavy currents in eqs. (3.1)-(3.6).
Fig. 2a

Fig. 2b
Fig. 2. Wave functions $\tau, \eta^0_b$ and $\eta^0_c$ at the zero recoil point $y = v \cdot v' = 1$. 

Fig. 3. Graph showing the relationship between $\tau$ and $\eta$ for different energies $E$.
Fig. 3b

Fig. 3c

Fig. 3. Variations of the wave functions $\tau$, $\eta_0^b$ and $\eta_0^c$ with respect to $y$ at $T = 1\text{GeV}$. 
Fig.4c

Fig.4. Branching ratios of $B \to D_1(D^*_2)$ decays at different values of $\eta^Q_i$. a. $\eta^b(1) = -0.4\text{GeV}^2$; b. $\eta^b(1) = -0.6\text{GeV}^2$; c. $\eta^b(1) = -0.8\text{GeV}^2$. In these figures $\eta^c_2(1) = -0.6\text{GeV}^2$ is used, and we assumed that $\eta^Q_i$ and $\bar{\tau}$ have the same monopole.
Table 1. Decay rates $\Gamma$ (in units of $|V_{cb}/0.038|^2 \times 10^{-15}$ GeV) and branching ratios $\text{BR}$ (in %) for $B \to D_1(D_2^*)l\bar{\nu}$ decays in the infinitely heavy quark mass limit and taking account of first order $1/m_Q$ corrections. $R$ is a ratio of branching ratios taking account of $1/m_Q$ corrections to branching ratios in the infinitely heavy quark mass limit.

| $m_Q \to \infty$ | $B \to D_1l\bar{\nu}$ | $B \to D_2^*l\bar{\nu}$ |
|------------------|------------------------|------------------------|
| $\Gamma$         | $1.87 \pm 0.53$        | $3.24 \pm 0.94$        |
| $\text{Br}$      | $0.45 \pm 0.13$        | $0.79 \pm 0.23$        |
| with $\eta^Q_i$  | $2.42 \pm 0.65$        | $4.05 \pm 1.09$        |
| and $\kappa^{(1)}_i$ | $0.59 \pm 0.16$    | $0.99 \pm 0.26$        |
| $R$              | $1.30 \pm 0.54$        | $1.25 \pm 0.05$        |

Table 2. Branching ratios (in %) at some values of $\eta^Q_i$. $\eta^2_1(1) = -0.6\text{GeV}^2$ and the monopole assumption are used in deriving the branching ratios in this table.

| $\eta^b(1)(\text{GeV}^2)$ | $\eta^l(1)(\text{GeV}^2)$ | $\eta^2_1(1)(\text{GeV}^2)$ | $\text{Br}(B \to D_1l\bar{\nu})$ | $\text{Br}(B \to D_2^*l\bar{\nu})$ |
|---------------------------|---------------------------|---------------------------|-------------------------------|-------------------------------|
| $-0.4$                    | $-0.2$                    | $-0.1$                    | $0.70$                        | $0.52$                        |
|                           | $-0.2$                    | $0.1$                     | $0.80$                        | $0.41$                        |
|                           | $0$                       | $0.2$                     | $0.47$                        | $0.58$                        |
|                           | $0$                       | $0.4$                     | $0.58$                        | $0.46$                        |
|                           | $0.2$                     | $0.4$                     | $0.32$                        | $0.71$                        |
|                           | $0.2$                     | $0.6$                     | $0.42$                        | $0.58$                        |
| $-0.6$                    | $-0.2$                    | $-0.1$                    | $0.66$                        | $0.45$                        |
|                           | $-0.2$                    | $0.1$                     | $0.75$                        | $0.35$                        |
|                           | $0$                       | $0.2$                     | $0.43$                        | $0.51$                        |
|                           | $0$                       | $0.4$                     | $0.53$                        | $0.40$                        |
|                           | $0.2$                     | $0.4$                     | $0.29$                        | $0.63$                        |
|                           | $0.2$                     | $0.6$                     | $0.39$                        | $0.51$                        |
| $-0.8$                    | $-0.2$                    | $-0.1$                    | $0.61$                        | $0.40$                        |
|                           | $-0.2$                    | $0.1$                     | $0.71$                        | $0.30$                        |
|                           | $0$                       | $0.2$                     | $0.40$                        | $0.45$                        |
|                           | $0$                       | $0.4$                     | $0.49$                        | $0.35$                        |
|                           | $0.2$                     | $0.4$                     | $0.27$                        | $0.56$                        |
|                           | $0.2$                     | $0.6$                     | $0.35$                        | $0.45$                        |
Table 3. Branching ratios (in %) at some values of $\eta_i^Q$. $\eta_2^Q(1) = 0$GeV$^2$ and the monopole assumption are used in deriving the branching ratios in this table.

| $\eta_b^Q$(1)(GeV$^2$) | $\eta_1^Q$(1)(GeV$^2$) | $\eta_2^Q$(1)(GeV$^2$) | $\text{Br}(B \to D_1 l \bar{\nu})$ | $\text{Br}(B \to D^*_2 l \bar{\nu})$ |
|------------------------|------------------------|------------------------|-------------------------------|-------------------------------|
| -0.4                   | -0.2                   | -0.1                   | 0.63                          | 0.61                          |
|                        | -0.2                   | 0.1                    | 0.73                          | 0.49                          |
|                        | 0                      | 0.2                    | 0.42                          | 0.67                          |
|                        | 0                      | 0.4                    | 0.52                          | 0.55                          |
|                        | 0.2                    | 0.4                    | 0.29                          | 0.81                          |
|                        | 0.2                    | 0.6                    | 0.38                          | 0.67                          |
| -0.6                   | -0.2                   | -0.1                   | 0.58                          | 0.54                          |
|                        | -0.2                   | 0.1                    | 0.68                          | 0.43                          |
|                        | 0                      | 0.2                    | 0.38                          | 0.60                          |
|                        | 0                      | 0.4                    | 0.48                          | 0.48                          |
|                        | 0.2                    | 0.4                    | 0.26                          | 0.73                          |
|                        | 0.2                    | 0.6                    | 0.35                          | 0.60                          |
| -0.8                   | -0.2                   | -0.1                   | 0.54                          | 0.48                          |
|                        | -0.2                   | 0.1                    | 0.63                          | 0.37                          |
|                        | 0                      | 0.2                    | 0.35                          | 0.53                          |
|                        | 0                      | 0.4                    | 0.44                          | 0.42                          |
|                        | 0.2                    | 0.4                    | 0.24                          | 0.66                          |
|                        | 0.2                    | 0.6                    | 0.32                          | 0.53                          |