Radiative corrections to co-annihilation processes

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2. Analysis of stop co-annihilation
3. QCD corrections
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New physics models for dark matter

- Dark matter constitutes of Weakly Interacting Massive Particles, that are stable due to new symmetry (SUSY, UED, Little Higgs, ...)

- Within model, relic density can be computed

Boltzmann equation \( \frac{dn}{dt} = -3Hn - \langle \sigma_{\text{eff}}v \rangle (n^2 - n_{\text{eq}}^2) \)

Thermally averaged annihilation cross-section

\[
\langle \sigma_{\text{eff}}v \rangle = \frac{\int_{4m^2}^{\infty} ds \sqrt{s - 4m^2} \ W \ K_1(\sqrt{s}/T)}{16m^4 T K_2^2(m/T)}, \quad W = \int d\Omega |\mathcal{M}|^2
\]

Relic density today: \( \Omega h^2 \sim \text{const.} \times \left[ \int_{m/T_{\text{freezeout}}}^{\infty} dx \frac{\langle \sigma_{\text{eff}}v \rangle(x)}{x^2} g_*^{1/2} \right]^{-1} \)
New physics models for dark matter

- DM particles expected to be *weakly interacting*
  - Annihilation cross-section can be computed reliably
  - Radiative corrections small

- New symmetry predicts large spectrum of new particles

- Extra particles affect dark matter abundance
  - Need collider data for parameters

- If mass close DM particle mass, *co-annihilation* can occur
Co-annihilation

Mass of new particle $\tilde{X}$ close to WIMP $\tilde{\chi}_1^0$

- Freeze-out of $\tilde{X}$ and $\tilde{\chi}_1^0$ at roughly same temperature
- Annihilation in parallel (co-annihilation)
- Reduction of total dark matter density

Example:
MSSM involving co-annihilation with scalar top
Stop-neutralino co-annihilation

- **Lightest neutralino** $\tilde{\chi}_1^0$ is good dark matter candidate in supersymmetry (for R-parity conservation)
- For bino $\tilde{\chi}_1^0$, $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow X$ is typically very small
  → Too large relic density in many SUSY scenarios
- If mass of $\tilde{t}_1$ close to mass of $\tilde{\chi}_1^0$:
  → Stop-neutralino **co-annihilation** reduces dark matter density
- Light stop $\tilde{t}_1$ is mainly $\tilde{t}_R$
  Assume further that $\tilde{\chi}_1^0$ is mainly bino
Typical parameter regions

Green: Relic density consistent with WMAP

Co-annihilation for $\Delta m \lesssim 30 \ GeV$

Contribution from processes:

- $\tilde{\chi}_1^0 \tilde{\tau}_1$: 85%
- $\tilde{\chi}_1^0 \tilde{\chi}_1^0$: 5%
- $\tilde{\ell}_1 \bar{\tilde{\ell}}_1$: 10%

$m_{\tilde{\chi}_1^0} = 118 \ GeV$

$m_{\tilde{\ell}_1} = 138 \ GeV$

$\Omega h^2 = 0.112$
QCD corrections to $\tilde{\chi}_1^0 - \tilde{t}_1$ annihilation

- Contrary to $\tilde{\chi}_1^0 \tilde{\chi}_1^0$ annihilation, $\tilde{\chi}_1^0 \tilde{t}_1$ and $\tilde{t}_1 \tilde{t}_1$ annihilation receive (large) QCD corrections.

- Relevant sub-processes for $\tilde{\chi}_1^0 - \tilde{t}_1$ annihilation:
  $m_{\tilde{\chi}_1^0} = 118$ GeV, $m_{\tilde{t}_1} = 138$ GeV

1% $t\gamma$
2% $tZ$

49.5% $\tilde{\chi}_1^0 \tilde{t}_1 \to W^+ b$

47.5% $\tilde{\chi}_1^0 \tilde{t}_1 \to t g$

Diagrams:
- $\tilde{\chi}_1^0 \tilde{t}_1 \to W^+ b$
- $\tilde{\chi}_1^0 \tilde{t}_1 \to t g$
- $\tilde{\chi}_1^0 \tilde{t}_1 \to W^+ b$
- $\tilde{\chi}_1^0 \tilde{t}_1 \to t g$
Calculation of QCD corrections

One-loop ($\alpha_s$) SUSY-QCD corrections to $\tilde{\chi}_1^0 \tilde{t}_1 \rightarrow W^+ b$ and $\tilde{\chi}_1^0 \tilde{t}_1 \rightarrow t g$

Standard techniques for virtual and real contributions

Virtual corrections (examples):

Real corrections:

Contribution with on-shell intermediate top belongs to $\tilde{\chi}_1^0 \tilde{t}_1 \rightarrow t g$  
$\rightarrow$ To be subtracted
Results for $\tilde{\chi}_1^0 - \tilde{t}_1$ annihilation cross-section

- **Few % corrections**
- **Can be larger (up to $\sim 30\%$)** for $m_{Wb} \approx m_t$ due to interference effects
- **Large corrections (up to $\sim 50\%$)**
- **Scale dependence only slightly reduced**
Results for $\tilde{\chi}_1^0 - \tilde{t}_1$ annihilation cross-section

- Few % corrections
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$\tilde{\chi}_1^0 \tilde{t}_1 \rightarrow W^+ b$

$\tilde{\chi}_1^0 \tilde{t}_1 \rightarrow tg$

Preliminary

$\sqrt{s} = 500$ GeV

$\mu_0 = \sqrt{s}/2$
QCD corrections to $\tilde{t}_1$-$\tilde{t}_1$ annihilation

$\tilde{t}_1$-$\tilde{t}_1$ annihilation typically contributes only 10% to relic abundance

But: During freeze-out phase stops are slowly moving and receive large Coulomb corrections:

$$\frac{\Delta \sigma}{\sigma} \sim \frac{\alpha_s}{v} \sim \mathcal{O}(1)$$

Multiple gluon exchange gives terms $(\alpha_s/v)^n$

→ Need to be resummed

Similar to QED corrections for $\tilde{\chi}^0_1$-$\tilde{\chi}^{\pm}_1$ co-annihilation in focus point

Hisano, Mastumoto, Nagai, Saito, Senami '06
Coulomb correction in NRQCD

Resummed effect of Coulombic gluon corrections can be computed in non-relativistic QCD

\[
\left[ -\frac{\Delta}{m\tilde{t}_1} + V(r) \right] \psi(r) = E \psi(r)
\]

\[
V(r) = -C_F \frac{\alpha_s}{r}, \quad E \to E + i\Gamma
\]

Correction for cross-section \( \frac{\Delta \sigma}{\sigma} = |\psi(0)|^2 \)

At higher orders \( V \) receives logarithmic corrections

\[
V(q^2) = -C_F \frac{4\pi\alpha_s}{q^2} \left[ 1 + \frac{\alpha_s}{4\pi} \left( \frac{31}{9} C_A - \frac{20}{9} T_F n_f + \beta_0 \log(\mu^2/q^2) \right) + \ldots \right]
\]

Since \( q = m\tilde{t}_1 v \ll m\tilde{t}_1 \), they are typically large
Coulomb correction in NRQCD

Generally, logarithms $\log(v)$ should also be resummed

Advanced methods: pNRQCD, vNRQCD

Hoang et al. '00
Beneke, Signer, Smirnov '99
Hoang, Manohar, Stewart, Teubner '01,02
Hoang '04
Penin, Piñeda, Smirnov, Steinhauser '04
Piñeda, Signer '06

Here: Simple estimate using only NRQCD with NNLO QCD potential
Schröder '99

Leading contribution comes from S-wave
→ Corresponds to 1S-stopponium bound state

Include stopponium decay width $\Gamma$ in Schrödinger equation from

$$\langle \tilde{t}_1 \tilde{t}_1 \rangle_{1S} \rightarrow gg, W^+W^- \quad \Rightarrow \quad \Gamma \approx 5 \text{ MeV}$$

→ Very small correction

Intrinsic $\tilde{t}_1$ decay width is also very small for $m_{\tilde{t}_1} \sim \mathcal{O}(100 \text{ GeV})$
Results for $\tilde{t}_1 - \tilde{t}_1$ annihilation cross-section

$m_{\tilde{t}_1} = 122$ GeV

- Huge enhancement for
  
  $$\beta = \sqrt{1 - \frac{4 m_{\tilde{t}_1}^2}{s}} \lesssim 0.4$$
  
i.e. $\frac{\sqrt{s}}{2 m_{\tilde{t}_1}} \gtrsim 1.09$

- Question when bound state effects kick in

- In freeze-out phase soft effects are cut off by temperature
  
  $$T_{\text{freeze-out}} \sim \frac{1}{20} m$$

- Further improvements of theoretical prediction necessary...
Effect of radiative corrections on relic density

- Medium corrections through corrections to $\tilde{\chi}_1^0\tilde{t}_1$
- Corrections larger for small masses

- Large $\tilde{t}_1\tilde{t}_1$ corrections in co-annihilation region
- WMAP preferred region shifts in parameter space
Effect of radiative corrections on relic density

QCD corrections to $\tilde{\chi}_1^0 \tilde{t}_1$ only

QCD correction to $\tilde{\chi}_1^0 \tilde{t}_1$ and $\tilde{t}_1 \tilde{t}_1$

- Medium corrections through corrections to $\tilde{\chi}_1^0 \tilde{t}_1$
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**Connection to colliders**

Precise measurement of $m_{\tilde{t}_1}$ and $m_{\tilde{\chi}^0_1}$ would allow to predict $\Omega_{CDM}h^2$

**Benchmark scenario:**

$m_{\tilde{\chi}^0_1} = 107.2$ GeV  
$m_{\tilde{t}_1} = 122.5$ GeV  
$\cos \theta_{\tilde{t}} = 0.0105$

$\delta m_{\tilde{t}_1} = 1$ GeV  
$\delta m_{\tilde{\chi}^0_1} = 0.3$ GeV

Carena, Freitas '06

However, large Coulombic correction introduces theoretical error

**But:**

Coulombic effect can be tested in $e^+e^- \rightarrow \tilde{t}_1 \tilde{t}^*_1$
Conclusions

- Radiative corrections can have important impact on co-annihilation processes.

- For neutralino co-annihilation, QCD corrections can reduce the predicted relic density by up to 50%.

- Analysis uses only simple treatment of Coulombic QCD threshold corrections.
  - More improvements necessary.