An impact of non-Newtonian blood viscosity on hemodynamics in a patient-specific model of a cerebral aneurysm

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Abstract. Despite of the recent achievements in the research of blood rheology, the effect of non-Newtonian blood viscosity on hemodynamic parameters in cerebral aneurysms is not clarified. The purpose of this study is to investigate the role of different rheological models for prediction of hemodynamic parameters in the patient-specific model of a saccular aneurysm. Transient flow simulations were conducted using OpenFOAM CFD Toolbox. The simulations employed different viscosity models: Newtonian, Power Law, Bird-Carreau, Casson and Local viscosity model. The similar flow patterns were observed both for Newtonian and non-Newtonian fluids in the patient-specific model of the aneurysm. The average difference between Newtonian and non-Newtonian models was about 12%. However for some specific points in the region of recirculating flow the difference was significantly higher - from 20% to 63%. The non-Newtonian blood behaviour influenced the streamlines distribution in the aneurysm region, producing a larger recirculating zone in the center of the aneurysm. The results showed that the major differences between Newtonian and non-Newtonian fluids were found in regions of low velocities and recirculating zones.

1. Introduction
A cerebral aneurysm is a major cerebrovascular disorder which can lead to subarachnoid hemorrhage. There are many known factors contributing to the intracranial aneurysms growth and rupture like hemodynamic stress, arterial wall degeneration, inflammation processes etc., however pathogenesis involving the formation of a cerebral aneurysm remains unclear. A majority of researchers supports a hemodynamic theory of aneurysm genesis and consider a wall shear stress (WSS) as one of the main factors contributing to development of cerebral aneurysms [1-4]. WSS is a friction force which blood flow exerts on the vessel wall that commonly used to evaluate a hemodynamic stress. WSS depends on viscosity value and velocity gradient near the vessel wall, which should be precisely defined for correct evaluation of the stress. A viscosity of blood non-linearly depends on the velocity gradient; however most of the researchers assume blood as a fluid with constant viscosity (Newtonian fluid), which may lead to overestimation of WSS and velocity gradients in some cases [5-7].

The effect of Newtonian viscosity assumption on hemodynamic forces within cerebral aneurysms was evaluated in the study [8]. CFD analysis was performed both for Newtonian and non-Newtonian viscosity models. Newtonian fluid model resulted in a higher aneurysmal time-averaged WSS...
(TAWSS), specifically in the areas of slow flow. In another study Evju et al. [9] investigated the effect of 4 different viscosity models, 2 different inflow conditions and 2 different outflow conditions in 12 aneurysms of middle cerebral artery. They found a strong correlation between the different viscosity models and boundary conditions. No strong correlations were found between the different WSS metrics and the geometrical metrics. Also Morales et al. investigated the role of blood viscosity on intra-aneurysmal hemodynamics in coiled aneurysm models [10]. Numerical simulations were performed to compare the flow fields for Newtonian and non-Newtonian models of fluid. By comparing two fluid models, it was found that Newtonian fluid overestimated the intra-aneurysmal hemodynamics in coiled models. In the high-viscosity regions the viscosity was elevated more than twice and flow velocity was near zero. However for the untreated simulation the Newtonian model does not significantly affected the overall intra-aneurysmal hemodynamics in comparison with non-Newtonian model. Additionally Gambaruto et al. analyzed the sensitivity of the flow field with respect to geometry of cerebral artery and selected fluid model [11]. They found that the sensitivity to geometry variability was greater, but comparable, to the one of the rheological model.

Despite of the recent achievements in the research of blood rheology, further studies should be done to clarify the effect of a non-Newtonian blood behavior on hemodynamic parameters in the cerebral arteries. In this study we provide an insight of blood dynamics in the patient-specific aneurysm model, using different non-Newtonian models of blood. The influence of viscosity models on intra-aneurysmal hemodynamics is investigated by computational fluid dynamics. Three most widely used non-Newtonian viscosity models are employed to represent blood behavior. Moreover using an available data of blood viscosity, the Local viscosity model is proposed for high-precision simulation of viscosity changes at low shear rates. The numerical results are compared with the data obtained from simulations utilizing the Newtonian fluid assumption.

2. Materials and methods

2.1. Patient-specific model

A patient-specific model of a cerebral artery was obtained by CT-angiography from a 54 years old patient with a saccular aneurysm. The raw data was post-processed to construct a 3D STL model of the internal carotid artery including an aneurysm sac and inlet and outlet segments. The model was processed to exclude artificial defects. The final geometry of the cerebral artery with the aneurysm is presented in Figure 1. The aneurysm is characterized by the following dimensions. The diameter of the inlet segment $d_{inlet}$ was 3.2 mm and diameters of the outlet segments $d_{outlet}$ were 3 mm. The aneurysm height $a_{height}$ was 12 mm, the aneurysm width $a_{width}$ was 11 mm, the aneurysm depth $a_{depth}$ was 7 mm and the aneurysm neck $a_{neck}$ was 8.13 mm.

2.2. Boundary conditions for the aneurysm model

A realistic velocity curve was applied at the inlet segment of the artery (Figure 2). Average velocity over the cardiac cycle was $\bar{u} = 238$ mm/s. The applied velocity curve corresponds to flow rate of 7 l/h, which is physiologically relevant for the cerebral artery. For simulations with non-Newtonian fluid models a pulse frequency $f$ of 60 beats per minute (one beat per second) was used. A free outflow boundary condition was imposed at the outlet segments of the artery.
Figure 1. Patient-specific model of the cerebral aneurysm: a) aneurysm sac at different projections; b) near-wall cell layers.

Figure 2. Inlet velocity curve for the studied saccular aneurysm model.

In order to compare the simulation results for Newtonian and non-Newtonian fluid models the dimensionless parameters - Reynolds number $Re$ and Strouhal number $St$ should be the same [12, 13]. To calculate $Re$ for non-Newtonian fluid a representative viscosity value $\eta_{rep}$ should be found. This value can be obtained using the coefficients $k$ and $n$ of a Power Law model (see Section 2.3) [12, 14]:

$$\eta_{rep} = k \left( \frac{2\pi}{d_{inlet}} \right)^{n-1} \frac{\pi}{4} \left( \frac{3n+1}{4n} \right)^n = 3.5 \text{ mPa}\cdot\text{s}.$$  

Using the representative viscosity value the Reynolds number at the inlet segment of the artery could be calculated as

$$Re = \frac{\pi \cdot d_{inlet} \cdot \rho}{\eta_{rep}} = 228,$$

where blood density $\rho$ is 1050 kg/m$^3$.

The corresponding representative shear rate $\gamma_{rep}$ for the non-Newtonian fluid was calculated by formula:
\[ \gamma_{\text{rep}} = \frac{8 \cdot \bar{u}}{d_{\text{inlet}} \left( \frac{3n+1}{4n} \right)^{\frac{n}{n-1}}} = 467 \text{ s}^{-1}. \]

The Womersley number \( \alpha \) could be found using the equation [14]:

\[ \alpha = \frac{d_{\text{inlet}}}{2} \sqrt{\frac{2\pi f}{(n_{\text{rep}}/\rho)}} = 2.1. \]

The Strouhal number \( \text{St} \) was

\[ \text{St} = \frac{f \cdot d_{\text{inlet}}}{\bar{u}} = 0.01344. \]

For a correct comparison, the Womersley number \( \alpha \) as well as Reynolds \( \text{Re} \) and Strouhal \( \text{St} \) numbers for Newtonian and non-Newtonian fluids should be the same [12, 13]. Therefore the average inlet velocity \( \bar{u} \) and pulse frequency \( f \) were adjusted to obtain similar \( \text{Re} \) and \( \text{St} \) for Newtonian fluid model. The average velocity for Newtonian fluid model was \( \bar{u} = 0.254 \text{ m/s} \) and frequency \( f \) was 64 beats per minute.

2.3. Fluid viscosity

The two types of viscosity models were used to represent blood: Newtonian and non-Newtonian. A Newtonian model is the simplest assumption for modeling of blood rheology. This model considers blood as fluid with constant viscosity:

\[ \eta(\gamma) = \eta, \quad (1) \]

where \( \eta \) is dynamic viscosity; \( \gamma \) is shear rate. The viscosity \( \eta = 4.1 \text{ mPa}\cdot\text{s} \) was used to represent the Newtonian viscosity for numerical studies. This value corresponds to kinematic viscosity of \( \nu = 3.905 \times 10^{-6} \text{ m}^2/\text{s} \).

To represent the non-Newtonian blood behavior three most widely used models were employed: Power Law, Bird-Carreau and Casson.

Power Law model:

\[ \eta(\gamma) = k \gamma^{n-1}, \quad (2) \]

where \( k \) is consistency index; \( n \) is an index. The viscosity is bounded by minimum \( \eta_{\text{min}} \) and maximum \( \eta_{\text{max}} \) values respectively. The following parameters for the Power Law model were used in this study: \( k = 0.0117642; \ n = 0.8092; \ \eta_{\text{min}} = 3.5 \text{ mPa}\cdot\text{s} \) and \( \eta_{\text{max}} = 14 \text{ mPa}\cdot\text{s} \).

Bird-Carreau model:

\[ \eta(\gamma) = \eta_{\text{min}} + (\eta_{\text{max}} - \eta_{\text{min}}) \left\{ \left[ 1 + (k \gamma)^2 \right]^{(n-1)/2} \right\}. \quad (3) \]

The parameters of Bird-Carreau model were: \( k = 0.6046; \ n = 0.3742 \). Viscosities \( \eta_{\text{min}} \) and \( \eta_{\text{max}} \) were the same as for the Power Law model.

Casson model:

\[ \eta(\gamma) = \sqrt{\frac{\tau_y}{\gamma} + \sqrt{k}}. \quad (4) \]

where \( \tau_y \) is yield stress. The parameter values of \( \tau_y = 3.6; \ k = 4.1 \) were used to represent blood with the Casson model.

Additionally a Local viscosity model was proposed:

\[ \eta(\gamma) = a_i + b_i (\gamma - \gamma_i) + c_i (\gamma - \gamma_i)^2 + d_i (\gamma - \gamma_i)^3, \]

\[ \left[ \gamma_i, \gamma_i \right], \ i = \{1, 2, \ldots, S\}. \quad (5) \]
where \( a_i, b_i, c_i \) and \( d_i \) are coefficients of \( i \)-th spline; \( S \) is a number of splines. The Local viscosity model uses a set of cubic splines to precisely interpolate the measured blood viscosity. For every time moment the viscosity value is computed using the current velocity gradient at the point.

The presented parameter values for non-Newtonian viscosity models were chosen in such a way to minimize an error between viscosity curves for blood and its corresponding rheological model. The least squares method was employed for this purpose. The viscosity curves for selected fluid models (1)-(5) are presented in Figure 3.

![Figure 3. Viscosity over shear rate for the different fluid models.](image)

2.4. Numerical simulation

SnappyHexMesh tool was used to generate a hexahedral computational mesh with maximum edge length of 0.128 mm for numerical studies. The mesh was additionally refined by adding five cell layers (cell height is 0.0375 mm) near the vessel wall for precise computation of the near-wall velocity gradients (see Figure 1b). Mesh independence check was carried out to ensure that results were not affected by selected mesh size. The average \( y^+ \) value was 0.74. Two million cells were sufficient to capture the flow details in the regions of interest. The same computational mesh was used both for Newtonian and non-Newtonian cases. The flow simulations were based on the three-dimensional incompressible Navier-Stokes and continuity equations:

\[
\rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla P + \nabla \cdot \mathbf{\tau},
\]

where \( \mathbf{u} \) is blood velocity; \( P \) is blood pressure; \( \tau \) is a stress tensor.

\[
div \mathbf{u} = 0,
\]

\[
\mathbf{\tau} = \eta \left( \nabla \mathbf{u} + \nabla \mathbf{u}^T \right),
\]

where \( \Delta t = 1 \times 10^{-3} \) s. Open source CFD toolbox OpenFOAM (OpenFOAM Foundation Ltd, London, UK) was used to conduct numerical studies. Second order numerical schemes were used.
for spatial and temporal discretization. The PIMPLE algorithm was employed for solving the equations (6)-(8).

3. Results
The main hemodynamic parameters in the patient-specific aneurysm model were analyzed to estimate the impact of different rheological models of blood on hemodynamic parameters in the aneurysm sac. For this purpose the region of interest, including the aneurysm sac, was extracted from the dataset. Hemodynamic parameters were analyzed during the systolic peak, which corresponds to the maximal hemodynamic stress. A velocity distribution for the central cross-section of the aneurysm model is presented in Figure 4. The similar flow patterns were found both for Newtonian and non-Newtonian fluids. The high velocity zones were observed near the left and bottom sides of the aneurysm dome, whereas zones of low velocity were found in the center of the aneurysm and in the aneurysm bleb. The maximum velocity value was predicted by Newtonian model; however the relative difference between Newtonian and non-Newtonian models for the maximum velocity was about 5%. To analyze an impact of different fluid models on the velocity distribution a set of 36 key points was employed. Position of these points is presented in Figure 5. Table 1 summarizes the observed velocity values at the key points for used fluid models. According to the Table 1 the Newtonian model showed a higher velocity magnitude in the aneurysm region then other fluid models. The Local viscosity model was used as a reference for comparison with other fluid models, because it precisely represents the measured viscosity of blood. The average difference between Newtonian and Local viscosity model was about 12%. However in some specific points at the region of recirculating flow the difference between the models was significantly higher - from 20% to 63%.

A viscosity distribution for the central cross-section of the aneurysm dome is shown in Figure 6. The high viscosity region was located near the center of the aneurysm sac and in the aneurysm bleb. The maximum viscosity values were observed near the left-bottom region of the aneurysm cross-section. In these regions the viscosity value was 5.27 mPa•s, which is approximately 28.5% higher than for the Newtonian model. However the maximum viscosity value was not reached by any fluid model. The Power Law model showed an underestimation of high viscosity in comparison with the Local viscosity model as well as both Bird-Carreau and Casson models showed an overestimation on 5% and 21.38% respectively.

![Figure 4. Velocity distribution at the central cross-section.](image-url)
Figure 5. Position of the key points for Newtonian and Local viscosity fluid models.

Table 1. Velocity (m/s) at the key points for the different fluid models.

|     | Newtonian | Power Law | Bird-Carreau | Casson | Local viscosity | Δ, % |
|-----|-----------|-----------|---------------|--------|-----------------|------|
| P1  | 1.44      | 1.34      | 1.36          | 1.40   | 1.33            | 7.3  |
| P2  | 1.21      | 1.16      | 1.15          | 1.14   | 1.16            | 4.6  |
| P3  | 1.60      | 1.51      | 1.51          | 1.51   | 1.51            | 5.5  |
| P4  | 0.95      | 0.91      | 0.89          | 0.90   | 0.90            | 5.2  |
| P5  | 1.94      | 1.83      | 1.83          | 1.84   | 1.83            | 5.7  |
| P6  | 0.21      | 0.20      | 0.20          | 0.19   | 0.19            | 6.0  |
| P7  | 0.10      | 0.07      | 0.11          | 0.13   | 0.08            | 23.4 |
| P8  | 1.94      | 1.84      | 1.84          | 1.83   | 1.84            | 5.1  |
| P9  | 0.12      | 0.11      | 0.10          | 0.10   | 0.11            | 6.5  |
| P10 | 0.15      | 0.11      | 0.15          | 0.14   | 0.12            | 22.2 |
| P11 | 0.14      | 0.15      | 0.13          | 0.11   | 0.14            | 1.7  |
| P12 | 0.13      | 0.13      | 0.12          | 0.10   | 0.12            | 6.9  |
| P13 | 0.28      | 0.26      | 0.27          | 0.27   | 0.26            | 8.7  |
| P14 | 0.54      | 0.53      | 0.53          | 0.51   | 0.53            | 2.3  |
| P15 | 1.09      | 1.04      | 1.03          | 1.02   | 1.04            | 4.4  |
| P16 | 0.08      | 0.08      | 0.08          | 0.11   | 0.08            | 0.3  |
| P17 | 0.23      | 0.18      | 0.19          | 0.13   | 0.19            | 17.1 |
| P18 | 0.11      | 0.07      | 0.09          | 0.15   | 0.07            | 35.1 |
| P19 | 0.21      | 0.22      | 0.21          | 0.18   | 0.22            | 9.4  |
| P20 | 0.13      | 0.11      | 0.11          | 0.14   | 0.11            | 16.5 |
| P21 | 0.09      | 0.10      | 0.10          | 0.12   | 0.09            | 0.2  |
| P22 | 0.06      | 0.08      | 0.06          | 0.05   | 0.07            | 12.3 |
| P23 | 0.25      | 0.22      | 0.20          | 0.11   | 0.22            | 11.9 |
| P24 | 0.09      | 0.04      | 0.09          | 0.34   | 0.08            | 7.5  |
| P25 | 0.38      | 0.38      | 0.38          | 0.29   | 0.37            | 2.3  |
| P26 | 0.10      | 0.16      | 0.16          | 0.14   | 0.16            | 63.0 |
| P27 | 0.13      | 0.11      | 0.11          | 0.14   | 0.11            | 14.5 |
| P28 | 0.47      | 0.46      | 0.47          | 0.40   | 0.46            | 2.8  |
| P29 | 0.60      | 0.57      | 0.57          | 0.58   | 0.57            | 4.3  |
| P30 | 0.03      | 0.04      | 0.04          | 0.04   | 0.04            | 20.2 |
| P31 | 0.12      | 0.14      | 0.12          | 0.11   | 0.13            | 7.4  |
| P32 | 0.72      | 0.67      | 0.67          | 0.68   | 0.67            | 7.1  |
| P33 | 0.13      | 0.14      | 0.13          | 0.13   | 0.14            | 9.2  |
| P34 | 1.04      | 0.97      | 0.98          | 0.98   | 0.97            | 6.6  |
| P35 | 1.10      | 1.05      | 1.05          | 1.04   | 1.05            | 4.5  |
| P36 | 1.29      | 1.24      | 1.24          | 1.22   | 1.24            | 4.0  |
**Figure 6.** Viscosity distribution for the different fluid models.

**Figure 7.** Computed streamlines during the systolic peak.
Streamlines were computed for all fluid models to visualize a flow pattern in the aneurysm sac. The obtained streamlines are presented in Figure 7. The inlet flow separates on two parts: the first one flow along the parent artery; another one produces a swirl motion in the region of aneurysm. In the center of a zone of recirculation the lowest velocity values were detected. The non-Newtonian blood behavior influenced a streamlines distribution in the aneurysm region producing a larger recirculating zone in the center of the aneurysm.

Using calculated velocity gradients and viscosity values a WSS distribution was obtained for all fluid models. Computed WSS distribution is presented in Figure 8. The general pattern of WSS distribution over the aneurysm surface was similar for Newtonian and non-Newtonian models. For the most regions the WSS value was approximately 20 Pa. However for zones of low WSS, the Newtonian model overestimated the highest value of WSS on 16.5% in comparison with the Local viscosity model.

![Figure 8. Wall shear stress distribution during the systolic peak.](image)

The results showed that non-Newtonian properties had the significant influence on blood flow characteristics in the studied case, especially at zones of recirculation, and should be considered in the future investigations of cerebral hemodynamics.

4. Discussion
The purpose of the present study was to investigate the effect of non-Newtonian behavior of blood on hemodynamics in a realistic cerebral artery and to evaluate an influence of different viscosity models on the resulting hemodynamic parameters. In this work we presented detailed numerical simulations of hemodynamics in the patient-specific model of the saccular aneurysm using different rheological models: Newtonian, Power Law, Bird-Carreau, Casson and Local viscosity.

Hemodynamic factors such as velocity and pressure gradients, regions of high and low shear stresses play an important role in forming cerebral aneurysms. This is especially important at bifurcations, where the flow is disturbed and secondary flows are created. The Newtonian model of
fluid is a common assumption for simulation of blood flow in large arteries, however for small arteries, especially for cerebral arteries with aneurysm; this assumption may not be precise in all cases, since in regions with slow and recirculating flow blood exhibits a non-Newtonian behavior. Moreover it should be noted that the used non-Newtonian fluid models are only the simplifications of blood properties and not necessary depict the blood internal forces. In the present study to overcome the errors of approximation, which are the results of applying a certain non-Newtonian model, the Local viscosity model was used. The Local viscosity model is based on a set of cubic splines which precisely interpolate the measured viscosity data.

Blood flow dynamics was studied under physiologically relevant flow conditions which were adjusted to keep the similar dimensionless parameters between Newtonian and non-Newtonian fluid models. The vessel wall was assumed to be rigid. According to the study [15] the aneurysm wall has a lack of elastic and therefore exhibits a reduced compliance. Hence an assumption of rigid walls is reasonable. Aneurysm geometry has a significant influence on size and location of zones with slow recirculating flow.

In this study, the highest differences between Newtonian and non-Newtonian fluid models were observed especially at these zones, which is in agreement with the recent study by Carty et al. [16]. Furthermore, Gibaly et al. [17] found, that despite of the general agreement of flow patterns, at low velocity zones the non-Newtonian fluid model predicted lower shear stresses compared to Newtonian model, which could significantly affect hemodynamic parameters at these specific regions. In our case the average difference between Newtonian and Local viscosity models was about 12%; however, at some specific points inside zones of recirculating flow the difference was from 20% to 63%. It also states that non-Newtonian model should be used for flow studies in the cerebral aneurysms. Gambaruto et al. [18] employed a Newtonian model and two generalized non-Newtonian models – Carreau and Cross viscosity model for flow studies in cerebral aneurysm models and found variability of the results between the different viscosity models. In contrast, in our study we showed that even usage of patient-specific viscosity data with standard non-Newtonian models could produce significant differences, especially at the low shear rates zones, compared to the precise blood viscosity curve (Local viscosity model). Moreover, standard non-Newtonian viscosity models could not fit perfectly the viscosity values both at high and low shear rates simultaneously due to predefined form of the curve. Either low or high shear rates values could be fitted well. This is supported by the recent study by Suzuki et al. [19], demonstrating that independent of the aneurysm size, numerical simulations with Newtonian or non-Newtonian fluid model could produce values different from those of the patient-specific viscosity model. Additionally, the results of our study correlate with the recent study by Meng et al. which found that a usage of Newtonian assumption could overestimate the hemodynamic parameters, while underestimate the potential of thrombus formation [20].

The model example, which utilized the patient-specific model of a cerebral artery with aneurysm, demonstrated the importance of choosing a specific non-Newtonian fluid model for study of hemodynamics in a realistic aneurysm. The realistic geometry of the cerebral artery played an important role in the intra-aneurysmal distribution of hemodynamic parameters. The intra-aneurysmal flow showed a complex vortex structure. The increment of viscosity in the center of the vortex and in the aneurysm dome led to reduction of the flow due to the viscous forces. This hemodynamic conditions are very important for thrombus formation and clinically relevant for a decision making process.

Our study extends the recent studies that evaluate effect of non-Newtonian rheological properties of blood on distribution of hemodynamic parameters in regions of bifurcation and aneurysm, however it has some limitations. The obtained numerical results were not validated by high precision measurement technique such as LDA or PIV, e.g. [21]. The number of cases should be extended to derive the statistical implications from the results. Also the assumption of rigid walls was used, which could potentially affect the viscosity distribution. Future studies will address the listed limitations. Furthermore, more investigations should be done to evaluate the viscosity changes in patient-specific models of cerebral aneurysms after a treatment with flow-diverter stents where zones of stagnant flow
are created. The further studies include the investigation of viscosity changes in patient-specific models of cerebral aneurysms treated by the flow-diverter.

5. Conclusion
This work presents a numerical study of hemodynamics in a patient-specific model of the cerebral aneurysm using different rheological models of blood. A Newtonian model overestimates the maximum velocity magnitude compared to non-Newtonian models. The study showed that the major differences between Newtonian and non-Newtonian fluids are observed in regions of low velocities and recirculating zones. It was found that for the patient-specific model of the aneurysm the difference in computed velocity magnitude varied from 20% to 63% at some points. It confirms that non-Newtonian model of blood should be used for studies of hemodynamics in the cerebral arteries. Also it is demonstrated that the Local viscosity model allows eliminating approximation errors and it could be used for the cases where the measured data of blood viscosity is available and high precision of estimation of hemodynamic parameters is necessary.

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