A Millimeter-Wave Half-Waveplate Based on Field Transformation

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Abstract—In this paper, we proposed a half-wave plate, which works at millimeter-wave band, based on a field transformation(FT) method with wide incident angle and broad working bandwidth. The proposed waveplate is composed of periodically arranged two dielectric layers with sub-wavelength heights. The effective electromagnetic(EM) parameters of the proposed waveplate can be tuned by manipulating the heights of the two dielectric layers. As a result, a broadband EM half-waveplate is achieved in millimeter-wave region, which obtains 90% conversion efficiency across the frequency band from 24 to 37 GHz. The performance of the waveplate is verified through full-wave simulation. Such half-waveplate can be easily realized through 3D printing technology.

Keywords—Half-waveplate; Field transformation; Broadband;

I. INTRODUCTION
Manipulating polarization has great potentialities in various scientific and engineering realms. The half-waveplate can achieve polarization conversions between transverse electric(TE) and transverse magnetic(TM), left-handed circular polarization (LCP) and right-handed circular polarization(RCP). Over the last decades, various physical mechanisms, such as those used in circular dichroism of chiral media[1], Brewster effects[2] and birefringence of anisotropic materials[3] from both natural and artificial structures have been explored. Different devices have been proposed to convert polarizations through these studies. Nevertheless, most of these designs still more or less have limitations in conversion efficiency, bandwidth, complexity for fabrication and dimension requirements.

In this paper, we show a remarkable degree of fundamental advances by applying the FT technique in the design of waveplates to achieve polarization conversion. Such FT method is applied connecting with effective medium theory. The waveplate is optimized for broadband operation with more than 90% conversion efficiency across the frequency band from 24 to 37 GHz and an incident angle range up to 60°. Another advantage of our design is can be easily fabricated based on 3D printing technology.

II. DESIGN AND SIMULATION
We are inspired by the FT method which proposed by Jensen Li[4]. Let us confine to 2D case of in-plane wave propagations on the x-y plane with both material properties and fields invariant in the z direction. The fields are denoted with a subscript “(0)” are in the virtual space. The applied FT method is defined by

$$
\begin{align*}
E_0 \rightarrow \cos \phi \begin{bmatrix} 
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi 
\end{bmatrix} E_0 \\
H_0 \rightarrow \begin{bmatrix} 
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi 
\end{bmatrix} H_0 
\end{align*}
$$

(1)

There are additional off-diagonal terms which is induced by \( \phi \) in permittivity and permeability tensors. When \( \phi \) is not equal to 0, the permittivity and permeability tensors can then be written as

$$
\begin{align*}
\epsilon &= \begin{bmatrix} n & 0 & A_y \\
0 & n & -A_x \\
A_y & -A_x & n
\end{bmatrix}, \\
\mu &= \begin{bmatrix} n & 0 & -A_y \\
0 & n & A_x \\
-A_y & A_x & n
\end{bmatrix}
\end{align*}
$$

(2)

where

$$
A_x = \left( \frac{1}{k_0} \right) \frac{\partial \phi}{\partial x}, \quad A_y = \left( \frac{1}{k_0} \right) \frac{\partial \phi}{\partial y}
$$

(3)

and \( k_0 \) is the wavenumber of the vacuum. If we assume \( \phi = \pi/2 \), we can get an equation for this FT method:

$$
\begin{align*}
E_z &= iH_0 \\
H_z &= E_0
\end{align*}
$$

(4)

the FT medium now acts as a polarization converter from TE to TM or LCP to RCP with \(-\pi/2\) phase shift, operating as a half-waveplate, as shown in Fig 1. We have a medium from \( y = 0 \) to \( y = h \), and \( \pi/2 \) correspond to waveplate as shown in Fig 1. The medium parameters become \( A_x = 0, -A_y = \phi/(kh) \). It is worth noting that the waveplate design obtained here is independent of incidence angle.

![Fig. 1. Schematic diagram of FT transmitted waveplate](image-url)
The main job for realizing this half-waveplate is to obtain a material with the off-diagonal terms in permittivity and permeability as described in Eq(2). The requirement of permeability could be eliminated by a mathematical approximation. Such a modification can be achieved based on equation as shown below[5]:

\[
\begin{pmatrix}
1 & 0 & \frac{2A_x}{n^2} \\
0 & 1 & \frac{2A_y}{n^2} \\
\frac{2A_z}{n^2} & \frac{2A_z}{n^2} & 1
\end{pmatrix}, \quad \mu = 1
\]

(5)

Here, we propose to realize off-diagonal terms in permittivity by utilized effective medium theory[6]. As Fig 2 shown, the red material’s permittivity \(\varepsilon_1\) is 10, its height \(d_1\) is 0.68mm, the blue material’s permittivity \(\varepsilon_2\) is 1, its height \(d_2\) is 5mm, and \(\eta = d_2/d_1\). We arranged the two materials intersecting along one direction periodically to get a waveplate whose permittivity is \(\varepsilon'\).

![Schematic diagram of the realization of the FT waveplate.](image_url)

The permittivity \(\varepsilon\) of the medium after rotation can be adjusted by controlling \(\varepsilon'\) which is determined by \(d_1, d_2\) and \(\eta\) in Eq(6), Eq(7) and Eq(8).

\[
\varepsilon' = \begin{pmatrix}
\varepsilon_x' & 0 & 0 \\
0 & \varepsilon_y' & 0 \\
0 & 0 & \varepsilon_z'
\end{pmatrix}
\]

(6)

\[
\varepsilon_x' = \frac{\varepsilon_x + \eta \varepsilon_z}{1 + \eta}, \quad \varepsilon_y' = \frac{1}{1 + \eta} \left( \frac{1}{\varepsilon_x} + \frac{1}{\varepsilon_z} \right)
\]

(7)

\[
\varepsilon = \begin{pmatrix}
\frac{(\varepsilon_x + \varepsilon_z)}{2} & 0 & -\frac{(\varepsilon_x - \varepsilon_z)}{2} \\
0 & \varepsilon_y' & 0 \\
-\frac{(\varepsilon_x - \varepsilon_z)}{2} & 0 & \frac{(\varepsilon_x + \varepsilon_z)}{2}
\end{pmatrix}
\]

(8)

The thickness of whole waveplate is 12.5mm, and simulation results of different operational modes through CST from 20GHz to 40GHz are shown in Fig. 3, where \(\theta\) is the incident angle. Due to the approximation procedure induces large diagonal terms simultaneously with off-diagonal terms in the permittivity tensors, the center frequency has slightly shifted to a higher frequency.

![Fig. 3. Full wave simulation results](image_url)

As we can see, for TE to TM and LCP to RCP conversion, nearly 90% 3dB bandwidth is realized at 24-37 GHz with incident angles up to 50°. For the incident angle of 60°, a 3 dB bandwidth over 26-33 GHz is still achieved. When the incident angle gradually increases to 60°, the propagation distance of EM waves in the waveplate is getting longer, which induces a lower center frequency.

III. CONCLUSION

In summary, a half-waveplate based on FT method is designed. It achieves polarization conversion efficiency over 90% across frequency band from 24 to 37 GHz. The polarization conversion performance has been verified through full-wave simulations. We will utilize 3D printer to carry out the experimental fabrication and measurement of the proposed waveplate.

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