The non-linear probability distribution function in models with local primordial non-Gaussianity

Tsz Yan Lam* and Ravi K. Sheth*

Center for Particle Cosmology, University of Pennsylvania, 209 S. 33rd Street, Philadelphia, PA 19104, USA

ABSTRACT

We use the spherical evolution approximation to investigate non-linear evolution from the non-Gaussian initial conditions characteristic of the local \( f_{nl} \) model. We provide an analytic formula for the non-linearly evolved probability distribution function (PDF) of the dark matter which shows that the underdense tail of the non-linear PDF in the \( f_{nl} \) model should differ significantly from that for Gaussian initial conditions. Measurements of the underdense tail in numerical simulations may be affected by discreteness effects, and we use a Poisson counting model to describe this effect. Once this has been accounted, our model is in good quantitative agreement with the simulations. In principle, our calculation is an important first step in programs which seek to reconstruct the shape of the initial PDF from observations of large-scale structures in the Ly\( \alpha \) forest and the galaxy distribution at later times.

Key words: methods: analytical – dark matter – large scale structure of Universe.

1 INTRODUCTION

The most common inflation model (the single scalar field, slow-roll inflation) predicts the primordial perturbations to be approximately described by the Gaussian statistics. Detections of non-Gaussianity can discriminate between different models (e.g. Maldacena 2003; Babich, Creminelli & Zaldarriaga 2004; Buchbinder, Khoury & Ovrut 2008; Khoury & Piazza 2008; Silvestri & Trodden 2008, and references therein), so the study of various cosmological probes of primordial non-Gaussianity has attracted much recent attention.

For example, the cosmic microwave background (CMB) has been used to constrain non-Gaussianity (Creminelli et al. 2006; Hikage et al. 2008; Yadav & Wandelt 2008; Komatsu et al. 2009). In addition, various aspects of large-scale structure have also been examined as probes of non-Gaussianity. These include halo abundances and bias (Lucchin, Matarrese & Vittorio 1988; Koyama, Soda & Taruya 1999; Matarrese, Verde & Matarrese 2008; Afshordi & Tolley 2008; Carbone, Verde & Matarrese 2008; Desjacques, Seljak & Iliev 2008; Dalal et al. 2008; Lo Verde et al. 2008; Slosar et al. 2008); higher order effects on the galaxy power spectrum (Izumi & Soda 2007; McDonald 2008; Slosar et al. 2008; Sefusatti & Komatsu 2007); and the galaxy bispectrum (Scoccimarro, Sefusatti & Zaldarriaga 2004; Sefusatti & Komatsu 2007). Slosar (2009) has discussed how to optimize the constraint on \( f_{nl} \) by applying weightings on subsamples from a single tracer. Primordial black holes would also be used to constrain non-Gaussianity (Bullock & Primack 1997; Khlopov 2008).

Recently, Grossi et al. (2008) used \( N \)-body simulations to study the effect of primordial non-Gaussianity on the non-linear probability distribution function (PDF) of the dark matter field. They found that departures from the Gaussian prediction are strongest in underdense regions and on small scales. The main goal of the present study is to provide some analytic understanding of their results. We do this by studying the non-linear evolution of the dark matter PDF. When the initial conditions are Gaussian, then the non-linear evolution can be modelled by the spherical and ellipsoidal collapse models (Lam & Sheth 2008a,b). In these models, the evolution of the PDF depends on two ingredients: a model for the dynamical evolution from an initial state to a final one, and the appropriate average over the initial states. In particular, the spherical and ellipsoidal collapse models approximate the non-linear dynamics as \( 1 \rightarrow 1 \) or \( 3 \rightarrow 1 \) mappings from the initial to the final state that do not depend on the Gaussianity of the initial conditions. Primordial non-Gaussianity enters only because it determines the initial set of states. As a result, essentially all of the machinery developed for the Gaussian case can be simply carried over to the present study.

We recap the definitions of the local non-Gaussian \( f_{nl} \) model in Section 2.1. The effect of smoothing the initial distribution is described in Section 2.2. Section 2.3 describes the non-linear PDF associated with the spherical evolution model and compares the results with the Gaussian
case. Section 2.4 compares our predictions with measurements in simulations. We summarize our results and discuss some applications in Section 3.

2 THE PDF OF THE DENSITY IN THE LOCAL NON-GAUSSIAN MODEL

In the $f_{\text{cd}}$ models of current interest, the initial density fluctuation field is only mildly perturbed from the Gaussian. Therefore, following Lo Verde et al. (2008) and Desjacques et al. (2008), we use the Edgeworth expansion as a convenient way to summarize our results. Because we are interested in mass scales which are substantially larger than that of a single collapsed halo – the regimes studied by Lo Verde et al. (2008) and Desjacques et al. (2008) – we are certainly in the regime where the Edgeworth expansion is a useful approximation.

2.1 The unsmoothed initial distribution

The primordial perturbation potential $\Phi$ of the local non-Gaussian field is

$$\Phi = \phi + f_{\text{cd}}(\phi^2 - \langle \phi^2 \rangle),$$

(1)

where $\phi$ is a Gaussian potential field and $f_{\text{cd}}$ is a scalar. Note that our definition of $f_{\text{cd}}$ is of opposite sign to that defined by, for example, the Wilkinson Microwave Anisotropy Probe (WMAP) team (Komatsu et al. 2009). Equation (1) includes the first two terms of what is potentially an infinite series in $f_{\text{cd}}$. However, in what follows, we will assume that these higher order terms can all be ignored. For this simplified model, we will study the change in the PDF of non-linear overdensity compared to the Gaussian initial conditions. We will use $P_{\delta}(k)$ to represent the power spectrum of $\delta$; in what follows, we will set $P_{\delta}(k) = A k^{n_s-3}$, where $n_s \approx 1$, and $A$ is a normalization constant that is fixed by requiring that the rms fluctuations on $8 h^{-1}$ Mpc in the associated non-Gaussian initial density field (which we will define shortly) have value $\sigma_s$.

We define $D$ as the real, symmetric $3 \times 3$ tensor whose components are proportional to the second order derivatives of the potential $\Phi$: $D_{ij} \equiv \delta_{ij} \phi + f_{\text{cd}}(\phi_i \phi_j + \phi_j \phi_i)$,

(2)

where $\phi_i = \partial_i \phi$ and $\phi_j = \partial_j \phi$. We will sometimes refer to $D$ as the shear or deformation tensor associated with the potential $\Phi$.

Correlations between the $\Phi_i$ will be very useful in what follows. These depend on the correlations between $\phi$ and its derivatives at the same spatial location. However, because $\phi$ is Gaussian, they can be computed easily. Specifically,

$$\langle \phi \phi \rangle = \sigma_0^2,$$

$$\langle \phi_i \phi_j \rangle = \frac{\sigma_i^2 \delta_{ij}}{3},$$

$$\langle \phi_i \phi_j \phi_k \rangle = 0,$$

$$\langle \phi_i \phi_j \phi_k \phi_l \rangle = \frac{\sigma_i^2 \sigma_j^2 \delta_{ij} \delta_{kl}}{15},$$

(3)

where $\delta_{ij}$ is the Kronecker delta function, and we have defined $\delta_{ijkl} \equiv \delta_{ij} \delta_{kl} + \delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}$.

$$\sigma_i^2 \equiv \int \frac{dk}{2\pi^2} P_{\delta}(k) M^2(k, z) W^2(k R) (-k^2)^i,$$

(4)

where

$$M(k, z) = \frac{3c^2 D(z)}{5c^2 H_0^2} T(k)$$

(7)

and $T(k)$ is the cold dark matter (CDM) transfer function. (Strictly speaking, we are currently interested in the limit in which $W = 1$. We have defined the more general expression so that it can be used in the following subsections.) Since we will be looking at the PDF at the present time, we will suppress the $z$ dependence in the following discussion. Thus,

$$\langle \Phi_i \Phi_{kl} \rangle = \frac{\sigma_{NG}^2}{15} \delta_{ijkl},$$

(8)

and

$$\langle \Phi_i \Phi_{kl} \Phi_{mn} \rangle = 2 f_{\text{cd}}^2 \frac{\gamma_{NG}}{135} \left[ \delta_{ij} \delta_{klmn} + \delta_{il} \delta_{kmn} + \delta_{im} \delta_{jkl} \right] + O(f_{\text{cd}}^3),$$

(9)

where

$$\frac{\sigma_{NG}^2}{15} = \sigma_i^2 + 4 f_{\text{cd}}^2 \left[ \frac{\sigma_i^2}{3} + \frac{\sigma_i^2 \sigma_j^2}{15} \right]$$

(10)

and

$$\gamma_{NG} = -2 \frac{\sigma_i^2}{3}. $$

(Note if equation (1) is an approximation to a model in which terms of higher order in $f_{\text{cd}}$ also contribute, then our $f_{\text{cd}}^2$ contribution to $\sigma^2_{NG}$ should be modified; see, for example, Hikage et al. 2006.)

2.2 The smoothed overdensity field in linear theory

In the spherical evolution model, the quantity of interest is the linear overdensity $\delta_l$ smoothed on some scale $V_l$. Most of the complication in $f_{\text{cd}}$ models arises from the fact that the effect of smoothing is non-trivial. This non-triviality is a consequence of the fact that a smoothed Gaussian field is itself Gaussian, but this self-similarity does not hold for generic random fields.
We now calculate the distribution function of the initial overdensity $\delta_i$ smoothed on scale $R_i$ in the $f_{\text{NL}}$ model. Our goal is to approximate this initial PDF using the Edgeworth expansion:

$$p(\delta_i| R_i) \, d\delta_i = \frac{e^{-v(R_i)^2/2}}{\sqrt{2\pi}} \left\{ 1 + \frac{\sigma_{\text{NG}}(R_i) S_1(R_i)}{6} H_3(v(R_i)) + \ldots \right\} \, dv(R_i),$$

(11)

where $v(R_i) = \delta_i/\sigma_{\text{NG}}(R_i)$ and $H_3(v) = v(3v^2 - 3)$, and we neglect terms higher than $S_3(R_i)$. This is slightly inconsistent because we will be keeping the $O(d^3)$ in the evaluation of the variance (see below). However, this truncation is justified for $f_{\text{NL}} \sim 100$, since the next order term in the Edgeworth expansion, proportional to $S_3(R_i)$, has a sub-per-cent level contribution compared to the first order correction in the variance [the second term in equation (12), which is a few per cent effect on the variance]. Thus, to proceed, we must specify how to calculate $\sigma_{\text{NG}}$ and $\sigma_{\text{NG}} S_1$.

The Fourier transform of the initial overdensity is related to the Fourier transform of $\Phi$ by $\delta_i \equiv -k^2 M(k) \Phi$. So spherical symmetry implies that the power spectrum and bispectrum of $\delta_i$ are

$$P_{\delta_i}(k) = (-k^2)^2 M^2(k) P_{\Phi}(k) = k^2 M^2(k) \left[ P_{\Phi}(k) + \frac{2 f_{\text{NL}}^3}{(2\pi)^3} \int dq P_{\Phi}(q) P_{\Phi}(|k - q|) \right],$$

(12)

$$B_{\delta_i}(k_1, k_2, k_{12}) = (-k_1^2)(-k_2^2)(-k_{12}^2) M(k_1) M(k_2) M(k_{12}) B_{\Phi}(k_1, k_2, k_{12}),$$

(13)

$$B_{\delta_i}(k_1, k_2, k_{12}) \equiv 2 f_{\text{NL}} \left[ P_{\Phi}(k_1) P_{\Phi}(k_2) + \text{cyclic} \right] + O(f_{\text{NL}}^3).$$

(14)

From equations (8) and (9), we have

$$\sigma_{\text{NG}}^2 \equiv \langle \delta_i^2 \rangle = \frac{1}{(2\pi)^2} \int \frac{dk}{k} 4\pi k^2 M^2(k) P_{\Phi}(k) W^2(k R_i),$$

(15)

$$\sigma_{\text{NG}} S_1 \equiv \langle \delta_i \rangle \sigma_{\text{NG}} = \frac{f_{\text{NL}}^2}{(2\pi)^3} + O(f_{\text{NL}}^3)$$

where

$$2f_{\text{NL}}^2 \gamma_{\text{NG}}^3 (R_i) = -\frac{2}{(2\pi)^2} \int \frac{dk_1}{k_1} k_1^2 W(k_1 R_i) \int \frac{dk_2}{k_2} k_2^2 W(k_2 R_i) \int d\mu_{12} W(k_{12} R_i) B_{\Phi}(k_1, k_2, k_{12}),$$

(17)

with $\mu_{12} \equiv \cos \theta_{12}$, and $\theta_{12}$ is the angle between $k_1$ and $k_2$, $k_{12}^2 \equiv k_1^2 + k_2^2 + 2k_1 k_2 \mu_{12}$.

To second order in $f_{\text{NL}}$, the variance $\sigma_{\text{NG}}^2(R_i)$ is the sum of two terms; one is the same as for the Gaussian, and the second, which is proportional to $f_{\text{NL}}^2$, involves a convolution of the power spectrum $P_{\Phi}$ with itself. This term has two infrared singularities (at $q = 0$ and $k = q$, respectively) for $n_s < 4$. Fortunately, these can be removed by rewriting the power spectrum as

$$P_{\Phi}(k) = P_{\Phi}(k) + \frac{2 f_{\text{NL}}^3}{(2\pi)^3} \int dq \left[ P_{\Phi}(q) P_{\Phi}(|k - q|) - P_{\Phi}(k) P_{\Phi}(q) - P_{\Phi}(k) P_{\Phi}(|k - q|) \right],$$

(18)

where $P_{\Phi}(k)$ has a new normalization (McDonald 2008). For $|f_{\text{NL}}| \lesssim 100$, this renormalization is just a few per cent effect, but the procedure is essential for removing the singularities.

Equation (17) indicates that $\langle \delta_i^3 \rangle$ scales linearly with $f_{\text{NL}}$. On scales larger than about $100h^{-1}\text{Mpc}$, the integral which defines $\gamma_{\text{NG}}^3$ can be approximated analytically (Scoccimarro et al. 2004), but on smaller scales, the integral must be evaluated numerically. For models of interest, $\sigma S_1$ is only weakly scale dependent: for example, it is $\approx -0.02(f_{\text{NL}}/100)$ on $\sim 100h^{-1}\text{Mpc}$, and is less than a factor of 2 larger on scales $\sim 1h^{-1}\text{Mpc}$ (e.g. fig. 1 in Scoccimarro et al. 2004).

This means that $|\sigma_{\text{NG}} S_1| \ll 1$ on the scales of interest in this paper, justifying our use of the Edgeworth expansion. In addition, note that if $\sigma_{\text{NG}}(R_i) S_1(R_i)$ were independent of $R_i$, then the Edgeworth expansion would be a function of $v$ only. In this case, the initial (non-Gaussian) PDF would be scale independent, in the sense that the PDF would have the same functional form for all smoothing scales, just as it does for the Gaussian. Over a sufficiently narrow range of scales, this is a reasonable approximation. Note that this scale-dependence of the initial (non-Gaussian) PDF occurs even though $f_{\text{NL}}$ itself is (by hypothesis) a constant. See Lo Verde et al. (2008) for a discussion of the additional complications associated with scale-dependence in $f_{\text{NL}}$.

2.3 The PDF of the smoothed, evolved, non-linear overdensity

The previous section showed how to calculate the distribution of the initial (linear theory) overdensity $\delta_i$. Non-linear evolution changes these distributions; this section shows how to estimate the non-linear PDF of the overdensity. As noted in the introduction, this can be done by following the same steps as in Lam & Sheth (2008b), but with the non-Gaussian initial distributions. For brevity, we show results for the spherical evolution model only.

In essence, this approach assumes that the non-linear density $\rho = 1 + \delta = M/\bar{\rho} V$ in a region of volume $V$ containing mass $M$ is related to its linear overdensity $\delta_i$ by a $1 \to 1$ mapping. As a result, the cumulative distribution of the evolved PDF at fixed $V$ is simply related to that of the initial PDF smoothed on scale $V_1 \propto M$. In particular,

$$\rho^2 p(\rho|V) = p_{\text{NG}}[\delta_i(\rho)|V_i(\rho)] v \frac{\ln v}{\ln \rho},$$

(19)
where \( p_{NL}(\delta_1|V) \) is the initial PDF of \( \delta_1 \) at a given smoothing scale \( V_1 = \rho \, V = (1 + \delta) V \) (see, e.g. section 2.1 of Lam & Sheth 2008b, for the algebra which leads to this expression). Note that for spherical volumes, \( (1 + \delta) = (R_1/R)^3 \): the non-linear overdensity is simply related to the ratio of the initial and evolved sizes. Equation (19) does not guarantee that the PDF is correctly normalized. To ensure that it is, we set \( \rho' \equiv N \rho \) and \( \rho^2 \, p'(\rho') \equiv \rho^2 \, p(\rho) \) and determine the value of \( N \) that is required so that \( \int dp' \rho' \, p'(\rho') \) and \( \int dp' \rho' \, p'(\rho') \) both equal unity (see discussion in Lam & Sheth 2008b). All that remains is to specify the mapping between \( \rho \) and \( \delta_1 \). In the spherical model, this is given by

\[
\rho' \equiv N \rho = \frac{M}{\rho V} = \left( \frac{R_1}{R} \right)^3 \approx \left( 1 - \frac{\delta}{\delta_c} \right)^{-\delta_c},
\]

where the second to last expression shows that the non-linear density is simply related to the ratio of the initial and evolved (comoving) sizes of the spherical patches, and the final expression, with \( \delta_c \approx 5/3 \), provides an excellent approximation to the exact spherical evolution model (Bernardeau 1994; Sheth 1998). In the exact model, \( \delta_c \) depends slightly on cosmology; the value \( \delta_c = 1.66 \) is appropriate for the ΛCDM cosmology that is the current standard.

Equations (20) and (11), when inserted into equation (19) imply that

\[
\rho^2 \, p(\rho|V) = \frac{1}{\sqrt{2\pi\sigma^2(\rho)} \exp \left\{ -\frac{\delta_1^2(\rho)}{2\sigma^2(\rho)} \right\} \left\{ 1 + \frac{\sigma(\rho)S_1(\rho)}{6} H_5 \left( \frac{\delta_1(\rho)}{\sigma(\rho)} \right) \right\} \left[ 1 - \frac{\delta(\rho)}{\delta_c} + \frac{\gamma_\rho}{6} \delta_1(\rho) \right],
\]

where \( p_{\Sigma} \) denotes the smoothed non-linearly evolved PDF associated with Gaussian initial conditions, \( \sigma(\rho) = \sigma_{NL}(R_1) \), and \( \gamma_\rho \equiv -3 \ln(\sigma^2/\ln M) \) is not to be confused with the skewness parameter we defined earlier. Finally, we set \( \rho' \equiv N \rho \) and \( \rho^2 \, p'(\rho') \equiv \rho^2 \, p(\rho) \) to ensure that \( \int dp' \rho' \, p'(\rho') \) and \( \int dp' \rho' \, p'(\rho') \) both equal unity (as discussed above).

The non-Gaussian modification contributes the final term of the right-hand side of equation (21). Since \( \delta_1 = 0 \) when \( \rho = 1 \), there is little or no correction to the Gaussian case at \( \rho \approx 1 \). However, there is an effect at the low- and high-density tails. To see what it is, note that \( \sigma \, S_3 \) is only a weak function of \( R \), so it is a weak function of \( \rho \), and hence the main \( \rho \) dependence is due to \( H_5 \). To see what this dependence is, suppose that \( \gamma_\rho = 6/5 \) (this is close to its actual value on the scales we consider when comparing with simulations in the next section). Given \( \delta_1 = \delta(1 - \rho^{-1/3}) \) and \( \sigma(\rho) = \sigma_0 \rho^{-\gamma_\rho/3} \), then \( \delta_1/\sigma \approx (5/3)(1 - \rho^{-1/3})(\sigma_0 \rho^{-1/3}) \approx (5/3 \sigma_0)(\rho^{-1/3} - \rho^{-2/3}) \), where \( \sigma_0 \) is the variance of the initial field when \( \rho = 1 \). When \( \rho \gg 1 \), then \( \delta_1/\sigma \approx (5/3 \sigma_0 \rho^{-1/3}) \rho^{-1/3} \), so \( H_5 \propto \rho^{-6/5} \). As a result, for \( f_{\lambda 0} > 0 \), the high-density tail of the PDF is increasingly suppressed compared to the Gaussian case as \( \rho \) increases. At low densities, \( \rho \ll 1 \), then \( \delta_1/\sigma \approx -(5/3 \sigma_0 \rho^{-2/3}) \), so \( H_5 \propto \rho^{-6/5} \): the low-density tail is enhanced for \( f_{\lambda 0} > 0 \). It is interesting to contrast this with the behaviour in the tails of the PDF of the linearly extrapolated field (equation 11). In this case, the correction to the tails scales as \( (1 + V^2 \sigma S_3/6) \), so it is approximately symmetric for \( V \gg 1 \) and \( V \ll 1 \). In contrast, for the non-linear field, the change is stronger for \( \rho \ll 1 \) than for \( \rho \gg 1 \), so we expect the underdense tail to be a good probe of \( f_{\lambda 0} \). This asymmetry is induced by how the initial smoothing scale is determined; including the (weak) scale-dependence of \( \sigma \), \( S_3 \) will further enhance the change in the underdense region. For sufficiently large \( \delta_1/\sigma \), the non-Gaussian piece can be negative, signaling that our truncation of the Edgeworth expansion was inappropriate. Fortunately, we are generally only interested in scales that are not significantly affected by this truncation problem.

### 2.4 Comparison with simulations

We compare the predictions of our approach with measurements of the non-linear PDF in numerical simulations from Desjacques et al. (2008). These followed the evolution of 1024^3 particles in a periodic cube of sides 1600 h^{-1} Mpc. The background cosmology is spatially flat, dominated by a cosmological constant, having \( (\Omega_m, \Omega_b, n_s, h, \sigma_8) = (0.279, 0.0462, 0.96, 0.7, 0.81) \), so the particle mass was \( 3 \times 10^{11} \, h^{-1} \, M_{\odot} \). The simulations sample the space of \( \delta_1/\sigma \) by discrete particles. This produces discreteness effects which are largely irrelevant, except in the least dense tails of the PDF. (e.g., the average number of particles in spherical cells with radius 4h^{-1} Mpc is \( \approx 70 \). So discreteness effects are severe at \( \rho \ll 1/7 \).) We account for this using the Poisson model:

\[
p(N|V) = \int dM \, p(M|V) \, p(N|M),
\]

where \( p(N|M, V) = (M/m_p)^N \exp(-M/m_p)/N! \), with \( m_p \) equal to the particle mass (Sheth 1996). We then set \( \rho = N/\bar{\rho} \) when plotting the results.

The symbols in the top panel of Fig. 1 show the measured PDF for counts in spheres of radius 8h^{-1} Mpc for \( f_{\lambda 0} = 100 \). The dashed curve which is closest to these symbols shows equation (22) for \( f_{\lambda 0} = 100 \). The dashed, solid and dotted curves show the predictions of equation (21) for \( f_{\lambda 0} = -100, 0 \) and 100. The bottom panel shows the ratio of the counts in the \( f_{\lambda 0} \) models to those when \( f_{\lambda 0} = 0 \). The outer set of curves show equation (21) and the inner set show the effect of accounting for discreteness effects using equation (22). This shows that the PDF for positive \( f_{\lambda 0} \) is slightly skewed towards underdense regions, the reverse is true for negative \( f_{\lambda 0} \), and our model provides an excellent description of these trends.

Fig. 2 shows a similar analysis of the PDF on smaller scales, for which the discreteness effects are more pronounced: at small \( \rho \), the symbols in the top panel lie well above the curves associated with equation (21). However, our Poisson model for the discreteness effect appears to be rather good: the prediction associated with equation (22) provides a good description of the measurements. The bottom panel
Figure 1. Non-linear overdensity PDF using the spherical collapse model in cells of radius $8h^{-1}\text{Mpc}$. Long-dashed (red), solid (black) and dotted (red) curves in the upper panel show $\ln[\rho(\rho)]$ as a function of $\ln\rho$ for $f_{nl} = -100, 0$ and 100. Symbols show the PDF measured in the $f_{nl} = 100$ simulation, and long-dashed–short-dashed (cyan and $f_{nl} = -100$) and dot–dashed (cyan, $f_{nl} = 100$, and only in lower panel) curves show the associated Poisson-sampled prediction (equation 22). The lower panels show the log of the ratio between the $f_{nl} \neq 0$ predictions and that for Gaussian initial conditions, for which $f_{nl} = 0$. Filled and empty symbols are similar ratios of the measured PDFs for $f_{nl} = -100$ and 100, respectively.

Figure 2. Same as previous figure, but for cells of radius $4h^{-1}\text{Mpc}$.

shows that discreteness effects tend to wash out the differences between the different $f_{nl}$ runs, but that our Poisson model does an excellent job of accounting for this effect (the predictions at small $\rho$ are almost indistinguishable from the measurements).

3 DISCUSSION

We used the spherical evolution model to study the non-linear evolved PDF of the dark matter density field in the local primordial non-Gaussian $f_{nl}$ model. (The spherical model is able to provide a good description of the non-linear PDF when $f_{nl} = 0$.) For currently acceptable values of $f_{nl}$, our approach shows that the signatures of primordial non-Gaussianity are small, but are most evident in underdense regions (equation 21 and related discussion). The PDF measured in simulations can be affected by discreteness effects, especially in small underdense cells. The effect of this can be approximated by using a Poisson counting model (equation 22). Once this is done, our model is in very good agreement with measurements in numerical simulations (Figs 1 and 2), so we hope our equation (21) will be useful in studies which require knowledge of the non-linear evolved PDF.
Recently, following the same logic for why cluster abundances should be good probes of primordial non-Gaussianity, Kamionkowski, Verde & Jimenez (2009) have suggested that void abundances should also be good probes. Our results provide further motivation for studying underdense regions. We are in the process of developing a more complete model of voids and void shapes (following Sheth & van de Weygaert 2004). In addition, our analysis provides an important first step for using the distribution of flux decrements in the Ly$\alpha$ forest (e.g. Desjacques, Nusser & Sheth 2007; Kim et al. 2007) to constrain non-Gaussianity (following Gaztañaga & Croft 1999, for the case of Gaussian initial conditions). In particular, our work complements the recent simulation analysis of Viel et al. (2009), who found that the regions of the forest that are associated with underdense regions are particularly sensitive to $f_{nl}$.

Finally, note that the parameter which describes the non-Gaussianity in the smoothed initial and final fields, $\sigma_3$, is only weakly scale-dependent (at least in the local form we studied here). Therefore, our analysis indicates that the non-Gaussian distribution of the resulting non-linear PDF (equation 21) can be written in terms of the scaled variable $\nu = \delta_l / \sigma$. So one could formulate a reconstruction of the initial $f_{nl}$ field analogously to how this is done for the Gaussian case (Lam & Sheth 2008b). We have not pursued this further.

ACKNOWLEDGMENTS

We thank V. Desjacques for help with measuring the PDFs in his simulations, R. Scoccimarro and E. Komatsu for many helpful discussions, and the referee for a helpful report. Thanks also to S. Cole, K. Dolag, M. Grossi, W. Hu, Y. P. Jing, Z. Ma, T. Nishimichi and R. Scoccimarro for discussions about resolution effects on the setting up of initial conditions in simulations.

REFERENCES

Afshordi N., Tolley A. J., 2008, Phys. Rev. D, 78, 123507
Babich D., Creminelli P., Zaldarriaga M., 2004, J. Cosmol. Astro-Part. Phys., 8, 9
Bernardeau F., 1994, A&A, 291, 697
Buchbinder E. I., Khoury J., Ovrut B. A., 2008, Phys. Rev. Lett., 100, 171302
Bullock J. S., Primack J. R., 1997, Phys. Rev. D, 55, 7423
Carbone C., Verde L., Matarrese S., 2008, ApJ, 684, L1
Creminelli P., Nicolis A., Senatore L., Tegmark M., Zaldarriaga M., 2006, J. Cosmol. Astro-Part. Phys., 5, 4
Dalal N., Doré O., Huterer D., Shirokov A., 2008, Phys. Rev. D, 77, 123514
Desjacques V., Nusser A., Sheth R. K., 2007, MNRAS, 374, 206
Desjacques V., Seljak U., Ilic I. T., 2008, preprint (arXiv:0811.2748)
Gaztañaga E., Croft R. A. C., 1999, MNRAS, 309, 885
Grossi M., Branchini E., Dolag K., Matarrese S., Moscardini L., 2008, MNRAS, 390, 438
Hikage C., Komatsu E., Matsubara T., 2006, ApJ, 653, 11
Hikage C., Matsubara T., Coles P., Liguori M., Hansen F. K., Matarrese S., 2008, MNRAS, 389, 1439
Izumi K., Soda J., 2007, Phys. Rev. D, 76, 083517
Kamionkowski M., Verde L., Jimenez R., 2009, J. Cosmol. Astro-Part. Phys., 1, 10
Kilopov M. Y., 2008, preprint (arXiv:0801.0116)
Khouri J., Piazza F., 2008, preprint (arXiv:0811.3633)
Kim T.-S., Bolton J. S., Viel M., Hahnelt M. G., Carswell R. F., 2007, MNRAS, 382, 1657
Komatsu E. et al., 2009, ApJS, 180, 330
Koyama K., Soda J., Taruya A., 1999, MNRAS, 310, 1111
Lam T. Y., Sheth R. K., 2008a, MNRAS, 389, 1249
Lam T. Y., Sheth R. K., 2008b, MNRAS, 386, 407
Lo Verde M., Muehner R., Shandera S., Verde L., 2008, J. Cosmol. Astro-Part. Phys., 4, 14
Lucchin F., Matarrese S., Vittorio N., 1988, ApJ, 330, L21
McDonald P., 2008, Phys. Rev. D, 78, 123519
McEwen J. D., Hobson M. P., Lasenby A. N., Mortlock D. J., 2008, MNRAS, 388, 659
Maldaeva J., 2003, J. High Energy Phys., 5, 13
Matarrese S., Verde L., 2008, ApJ, 677, L77
Matarrese S., Verde L., Jimenez R., 2000, ApJ, 541, 10
Robinson J., Baker J. E., 2000, MNRAS, 311, 781
Scoccimarro R., Sefusatti E., Zaldarriaga M., 2004, Phys. Rev. D, 69, 103513
Sefusatti E., Komatsu E., 2007, Phys. Rev. D, 76, 083004
Sheth R. K., 1996, MNRAS, 279, 1310
Sheth R. K., 1998, MNRAS, 300, 1057
Sheth R. K., de Weygaert R., 2004, MNRAS, 350, 517
Silvestri A., Trodden M., 2008, preprint (arXiv:0811.2176)
Slosar A., 2009, JCAP, 3, 4
Slosar A., Hirata C., Seljak U., Ho S., Padmanabhan N., 2008, J. Cosmol. Astro-Part. Phys., 8, 31
Taruya A., Koyama K., Matsubara T., 2008, Phys. Rev. D, 78, 123534
Viel M., Branchini E., Dolag K., Grossi M., Matarrese S., Moscardini L., 2009, MNRAS, 393, 774
Yadav A. P. S., Wandelt B. D., 2008, Phys. Rev. Lett., 100, 181301

This paper has been typeset from a TeX/UTF8 file prepared by the author.

© 2009 The Authors. Journal compilation © 2009 RAS, MNRAS 395, 1743–1748