Resonances and Electroweak Observables at the ILC

W. Kilian, J. Reuter
Deutsches Elektronen-Synchrotron DESY, D–22603 Hamburg, Germany

Precise measurements of the interactions of electroweak vector bosons at the ILC yield information about the physics of electroweak symmetry breaking. In order to combine this with the possible observation of new resonances or the effects of new strong interactions at the LHC, we need to relate resonance parameters with low-energy observables. We derive these relations for a generic setup and draw conclusions about the new-physics reach of the ILC in the electroweak sector.

1. INTRODUCTION

The previous generation of precision experiments, in particular data from LEP and SLC, have established electroweak interactions as a spontaneously broken gauge theory. However, the underlying physics that triggers the formation of a scalar (Higgs) condensate and thus breaks the symmetry is still unknown. The possible scenarios range from purely weakly interacting models, such as the minimal Standard Model with a light Higgs boson and its supersymmetric generalizations, to strongly-interacting settings that could indicate the opening-up of further gauge sectors or extra dimensions.

The electroweak gauge bosons $W$ and $Z$ acquire longitudinal polarization components via their interaction with the Higgs condensate. Thus, a comprehensive study of weak-boson interactions amounts to a nontrivial measurement of parameters that are related to the unknown symmetry-breaking sector. It may happen that this new physics involves resonances in the elastic scattering of vector bosons and, in analogy with the form factors of QCD, in the form factors of vector boson production. The possible resonances include scalar, vector or tensor states. In strongly-interacting models, we expect new physics to emerge around the scale of unitarity saturation, $1.2 \text{ TeV}$ [1]. Alternatively, the scattering amplitudes and form factors might be featureless while saturating the unitarity limit at high energies.

At the LHC, the energy effectively available for $WW$ scattering processes may allow for direct observation of these effects, at least in some of the possible channels [2]. The ILC energy is limited to about $1 \text{ TeV}$, therefore direct observation [3] is less likely. However, the virtual exchange of a massive resonance has an indirect effect on the measurable interactions of vector bosons, and therefore is encoded in the values of anomalous couplings in the low-energy effective theory. Given the limited amount of data that will be available at the LHC for these processes, the complementary measurement of low-energy parameters at the ILC [4] is essential for obtaining a complete picture.

The effective theory is given by the chiral Lagrangian formalism for the (Higgsless) Standard Model [5, 6]. The terms in the Lagrangian are expanded in terms of powers of $1/4\pi v$, where $v = 246 \text{ GeV}$ is the electroweak scale. The fields are defined in such a way that an $SU(2)_L$ invariance is manifest. A corresponding $SU(2)_R$ invariance, that after spontaneous symmetry breaking would result in a $SU(2)_{L+R}$ custodial symmetry, is also included, but explicitly broken down to the hypercharge $U(1)$ subgroup in a well-defined way.

The Standard Model with a Higgs boson is included in this framework as a special case, where a scalar resonance is coupled to the electroweak sector with a specific coupling strength.

In this talk, we present results for the low-energy effects of resonances that can be coupled to vector bosons. Given a measurement of the low-energy parameters (anomalous couplings), this allows one to relate LHC (high-energy) and ILC (low-energy) observables in the electroweak sector in a straightforward way. In [7, 8], this program is carried out explicitly, using a new determination of the ILC sensitivity to the anomalous couplings in vector boson interactions.
2. THE CHIRAL LAGRANGIAN FRAMEWORK

The Chiral Lagrangian is a version of the generic low-energy Lagrangian of electroweak physics that is particularly suited for studying the symmetry properties of the possible interactions. The electroweak field strengths are written in a manifestly $SU(2)_L \times SU(2)_R$ covariant way:

$$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + ig[W_\mu, W_\nu], \quad B_{\mu\nu} = \Sigma (\partial_\mu B_\nu - \partial_\nu B_\mu) \frac{v^3}{2} \Sigma^\dagger, \quad (1)$$

where $W=W^k \tau^k$. The symmetry breaking field $\Sigma$ contains the longitudinal degrees of freedom in terms of the Goldstone triplet $w \equiv w^k \tau^k$,

$$\Sigma = \exp \left( -\frac{i}{v} w \right), \quad (2)$$

where the exponential parameterization is a convenient one out of infinitely many equivalent choices. Furthermore, there is a vector field $V$ derived from the covariant derivative $D\Sigma$ and a $SU(2)_R$-breaking spurion field $T$,

$$V = \Sigma (D\Sigma)^\dagger = -(D\Sigma)\Sigma^\dagger, \quad T = \Sigma^3 \Sigma^\dagger. \quad (3)$$

With these definitions, the basic Lagrangian reads [5, 6, 8]

$$\mathcal{L}_0 = -\frac{1}{2} \text{tr} \{ W_{\mu\nu} W^{\mu\nu} \} - \frac{1}{2} \text{tr} \{ B_{\mu\nu} B^{\mu\nu} \} - \frac{v^2}{4} \text{tr} \{ V_\mu V^\mu \} + \beta_1 \mathcal{L}_0' + \sum_i \alpha_i \mathcal{L}_i. \quad (4)$$

The list of lowest-order anomalous couplings is given by the operators

$$\mathcal{L}_0' = \frac{v^2}{4} \text{tr} \{ TV_\mu \} \text{tr} \{ TV^\mu \}, \quad (5a)$$

$$\mathcal{L}_1 = gg' \text{tr} \{ B_{\mu\nu} W^{\mu\nu} \}, \quad (5b)$$

$$\mathcal{L}_2 = ig' \text{tr} \{ B_{\mu\nu} [V^\mu, V^\nu] \}, \quad (5c)$$

$$\mathcal{L}_3 = ig \text{tr} \{ W_{\mu\nu} [V^\mu, V^\nu] \}, \quad (5d)$$

$$\mathcal{L}_4 = (\text{tr} \{ V_\mu V^\nu \})^2, \quad (5e)$$

$$\mathcal{L}_5 = (\text{tr} \{ V_\mu V^\mu \})^2, \quad (5f)$$

$$\mathcal{L}_6 = \text{tr} \{ V_\mu V^\nu \} \text{tr} \{ TV^\mu \} \text{tr} \{ TV^\nu \}, \quad (5g)$$

$$\mathcal{L}_7 = \text{tr} \{ V_\mu V^\nu \} (\text{tr} \{ TV_\mu \})^2, \quad (5h)$$

$$\mathcal{L}_8 = \frac{1}{2} g^2 (\text{tr} \{ TW_{\mu\nu} \})^2, \quad (5i)$$

$$\mathcal{L}_9 = \frac{1}{2} ig \text{tr} \{ TW_{\mu\nu} \} \text{tr} \{ T[V^\mu, V^\nu] \}, \quad (5j)$$

$$\mathcal{L}_{10} = \frac{1}{4} (\text{tr} \{ TV^\mu \})^2 (\text{tr} \{ TV_\nu \})^2. \quad (5k)$$

In this list, there are three operators ($\mathcal{L}_0'$, $\mathcal{L}_1$, $\mathcal{L}_8$) that affect gauge-boson propagators directly (oblique corrections). They are related to the $S, T, U$ parameters [9]. Three additional operators ($\mathcal{L}_2, \mathcal{L}_3, \mathcal{L}_9$) contribute to anomalous triple gauge couplings [10]. The remaining five operators ($\mathcal{L}_4$-$\mathcal{L}_7$ and $\mathcal{L}_{10}$) induce anomalous quartic couplings only.

For the purpose of this talk, we have restricted ourselves to the purely bosonic part of the effective Lagrangian. This is appropriate if the new physics is not directly coupled to the fermions of the Standard Model. The extension to fermionic interactions is straightforward; we have to add all kinds of four-fermion operators together with a list of operators that couple bilinear fermion currents to bosonic currents and thus affect the effectively measured $W$ and $Z$ couplings to fermions. Furthermore, operators that violate C and/or CP can be added as well; we do not include these in the present work.
3. RESONANCES

If we do not consider direct couplings to SM fermions, the properties of new resonances are encoded in their couplings to the bosonic degrees of freedom. These terms have to be added to the Lagrangian (4). The couplings are organized in powers of $1/M$, where $M$ is the resonance mass. If $M \gg v$ (or if, in the light-Higgs case, we want to have a renormalizable model), we can truncate the series after the leading term(s), so only a finite number of parameters is involved.

A further organizing principle is the number of spurions $T$ that have to be introduced. Without these, i.e., in the custodial symmetric (or isospin-conserving) case, only certain combinations are allowed: a resonance coupled to $WW/ZZ/WZ$ pairs must have the $(I, J)$ [isospin,spin] assignments $(0, 0)$, $(2, 0)$, $(0, 2)$, $(2, 2)$, or $(1, 1)$. However, the presence of the isospin-breaking coupling of the $B$ field in the leading-order Lagrangian precludes exact isospin conservation. Allowing for breaking terms, we have to consider all isospin/spin assignments of resonances up to 2. In this talk, we omit the spin-2 and isospin-2 states for brevity; for a full account, see [8].

For each resonance type, we write down the extra terms in the Lagrangian. A direct consequence is the result for the two-boson partial decay widths, which we compute in the limit $M \gg M_W, M_Z$ where the Goldstone-boson equivalence theorem can be used.

In the ILC context, we are interested in the low-energy effects. Therefore, we also compute the result from integrating out the resonance at tree level, retaining terms up to order $(v/M)^4$. This result is expressed in values for the anomalous couplings $\beta_1$ and $\alpha_i$. Note that other sources of anomalous couplings, namely loop contributions within the Standard Model and beyond, are not given here.

3.1. Scalar Resonances

Scalar resonances are of particular interest since the most prominent representative of this class, a $I = 0$ scalar boson, serves as a Higgs boson if its couplings take particular values. In extended models with Higgs bosons, there are also scalar resonances with higher isospin. For instance, in the MSSM the $(H^+, H^0, H^-)$ triplet can be viewed as an $I = 1$ triplet. As another example, some Little Higgs models contain a complex triplet $((\phi^+, \phi^0, \phi^0)$, which under isospin decomposes into a real $I = 2$ quintet and a singlet.

3.1.1. Scalar Singlet: $\sigma$

This state has two independent linear couplings, $g_{\sigma}$ and $h_{\sigma}$. The latter violates isospin. Neglecting self-couplings etc. that do not contribute to the order we are interested in, the Lagrangian is

$$\mathcal{L}_\sigma = -\frac{1}{2} [\sigma (M_\sigma^2 + \varphi^2) \sigma + 2\sigma j], \quad \text{where} \quad j = -\frac{g_{\sigma} v}{2} \text{tr} \{V_\mu V^\mu\} - \frac{h_{\sigma} v}{2} (\text{tr} \{TV_\mu\})^2.$$  (6)

The Higgs boson corresponds to the special values $g_{\sigma} = 1$ and $h_{\sigma} = 0$. Integrating out $\sigma$, we get zero shifts for all anomalous couplings, except for

$$\alpha_5 = g_{\sigma}^2 \left( \frac{v^2}{8M_\sigma^2} \right), \quad \alpha_7 = 2g_{\sigma}h_{\sigma} \left( \frac{v^2}{8M_\sigma^2} \right), \quad \alpha_{10} = 2h_{\sigma}^2 \left( \frac{v^2}{8M_\sigma^2} \right).$$  (7)

In the high-mass limit, the $\sigma$ width is given by

$$\Gamma_\sigma = \frac{g_{\sigma}^2 + \frac{1}{2}(g_{\sigma} + 2h_{\sigma})^2}{16\pi} \left( \frac{M_\sigma^3}{v^2} \right).$$  (8)

This includes $\sigma \to W^+W^-$ and $\sigma \to ZZ$. 0311
3.1.2. Scalar Triplet: $\pi$

Coupling a scalar triplet $\pi$ to vector boson pairs requires isospin violation. The Lagrangian is

$$\mathcal{L}_\pi = -\frac{1}{4} \text{tr} \left\{ \pi (M^2 + D^2) \pi + 2\pi j \right\} \quad \text{with} \quad j = h_\pi v \nu \frac{1}{2} \text{tr} \{ T V^\mu \} + h'_\pi v \nu \frac{1}{2} T \text{tr} \{ V^\mu V^\nu \} + \frac{k_\pi v}{2} T \left( \text{tr} \{ T V^\mu \} \right)^2. \quad (9)$$

Evaluating the effective Lagrangian, the nonvanishing parameters are

$$\alpha_5 = 2h'^2_\pi \left( \frac{v^2}{16M^2_\pi} \right), \quad \alpha_6 = h^2_\pi \left( \frac{v^2}{16M^2_\pi} \right), \quad \alpha_7 = 2h_\pi (h_\pi + 2k_\pi) \left( \frac{v^2}{16M^2_\pi} \right), \quad \alpha_9 = 4k_\pi (h_\pi + k_\pi) \left( \frac{v^2}{16M^2_\pi} \right). \quad (10)$$

The partial widths for the decay into vector boson pairs are different for charged and neutral pions:

$$\Gamma_{\pi^\pm} = \frac{1}{16\pi} \left( \frac{M^2_\pi}{v^2} \right), \quad \Gamma_{\pi^0} = \frac{h'^2_\pi + \frac{1}{8} (h_\pi + h'_\pi + 2k_\pi)^2}{16\pi} \left( \frac{M^3_\pi}{v^2} \right). \quad (11a)$$

If there is approximate isospin conservation we expect the total widths to be dominated by fermion pairs and by three-boson decays, analogous to the pions of QCD.

3.2. Vector Resonances

In contrast to scalar resonances, vector resonances contribute to operators involving field strengths and thus contribute to two- and three-gauge boson couplings. For that reason, and because in the isospin-conserving case the leading contribution vanishes, we expand up to order $1/M^4$.

3.2.1. Vector singlet: $\omega$

The Lagrangian is

$$\mathcal{L}_\omega = \frac{1}{2} \left[ \omega_\mu ((M^2 + \partial^2)g^\mu\nu - \partial^\nu \partial^\mu) \omega_\nu + 2\omega_\mu j^\mu \right] \quad \text{with} \quad j_\mu = i \frac{h_\omega v^2}{2} \text{tr} \{ T V_\mu \} + \frac{2h_\omega}{M^2_\omega} \partial_\nu \text{tr} \{ T W_\nu W^\nu_\rho \}. \quad (12)$$

Expanding up to second order and expressing the result in the canonical operator basis, we obtain the nonvanishing coefficients

$$\beta_1 = h^2_\omega \frac{v^2}{2M^2_\omega}, \quad \alpha_1 = \alpha_2 = \alpha_4 = \alpha_7 = h^2_\omega \left( \frac{v^2}{2M^2_\omega} \right)^2, \quad \alpha_5 = \alpha_6 = \alpha_8 = \alpha_9 = -h^2_\omega \left( \frac{v^2}{2M^2_\omega} \right)^2. \quad (13a)$$

The $\omega$ boson can decay into $W^+ W^-$ but not into $ZZ$, and the pair decay width is

$$\Gamma_\omega = \frac{h^2_\omega + \frac{4h'^2_\omega}{M^2_\omega} v^2}{48\pi} M_\omega. \quad (14)$$

3.2.2. Vector triplet: $\rho$

The Lagrangian for a vector triplet $\rho$ contains anomalous magnetic moments and takes the form

$$\mathcal{L}_\rho = \frac{1}{4} \text{tr} \left\{ \rho_\mu \left( M^2 g^\mu\nu + D^2 g^\mu\nu - D^\nu D^\mu + 2i\mu_\rho g^\mu W^\mu_\rho + 2\mu'_\rho g^\mu B^\mu_\rho \right) \rho_\nu + 2\rho_\mu j_\mu \right\}, \quad (15)$$

where

$$j_\mu = ig_\rho v^2 V_\mu + ig'_\rho v^2 T \text{tr} \{ T V_\mu \} + \frac{4\mu^2}{M^2_\rho} D_\nu (W_\rho^\nu W_\rho^\mu) + \frac{4\mu'_\rho}{M^2_\rho} D_\nu (B_\rho^\nu W_\rho^\mu) + \frac{4\rho^\mu}{M^2_\rho} D_\nu (T \text{tr} \{ T W_\rho^\mu W_\rho^\nu \}). \quad (16)$$
Including a mass splitting $\Delta M_{\rho}$, we get the nonvanishing parameters

\begin{align}
\beta_1 &= 4h_{\rho}(g_{\rho} + h_{\rho}) \frac{v^2}{2M_{\rho}^2} - (g_{\rho} + 2h_{\rho})^2 \frac{v^2 \Delta M_{\rho}^2}{2M_{\rho}^4}, \\
\alpha_1 &= (g_{\rho} + 2h_{\rho})^2 \left( \frac{v^2}{2M_{\rho}^2} \right)^2, \\
\alpha_2 &= (-g_{\rho}^2 + 4h_{\rho}^2 + g_{\rho} h_{\rho})^2 \left( \frac{v^2}{2M_{\rho}^2} \right)^2, \\
\alpha_3 &= g_{\rho}(g_{\rho} + 2h_{\rho}) (1 + \mu_{\rho}) \left( \frac{v^2}{2M_{\rho}^2} \right)^2, \\
\alpha_4 &= (g_{\rho} - 2h_{\rho})^2 \left( \frac{v^2}{2M_{\rho}^2} \right)^2, \\
\alpha_5 &= -(g_{\rho} - 2h_{\rho})^2 \left( \frac{v^2}{2M_{\rho}^2} \right)^2, \\
\alpha_6 &= 8g_{\rho} h_{\rho} \left( \frac{v^2}{2M_{\rho}^2} \right)^2, \\
\alpha_7 &= -8g_{\rho} h_{\rho} \left( \frac{v^2}{2M_{\rho}^2} \right)^2, \\
\alpha_8 &= -4h_{\rho}(g_{\rho} + h_{\rho}) \left( \frac{v^2}{2M_{\rho}^2} \right)^2, \\
\alpha_9 &= -2 \left( h_{\rho}(g_{\rho} + 2h_{\rho}) + g_{\rho} h_{\rho} h_{\rho} \right) \left( \frac{v^2}{2M_{\rho}^2} \right)^2. 
\end{align}

These parameters modify the triple gauge couplings $g_1^Z$, $\kappa_\gamma$ and $\kappa_Z$, along with oblique corrections and quartic gauge couplings. At the same order of the expansion, there are also contributions to $\lambda_{\gamma}, \lambda_Z$ that correspond to dimension-6 operators not listed here (see [8]). As a particular result, we note that the relation $\lambda_{\gamma} = \lambda_Z$ is violated by these terms.

A charged $\rho$ resonance can decay into $W^\pm Z$ and $W^\pm \gamma$. For the neutral $\rho$, the Landau-Yang theorem forbids $ZZ$ and $\gamma\gamma$ final states. The partial widths are

\begin{align}
\Gamma_{\rho^\pm \rightarrow W^\pm Z} &= \frac{(g_{\rho} + 2h_{\rho})^2 + 2(c_{\ell_{\rho}}^4 + \frac{1}{2} s_{\ell_{\rho}}^4)^2}{48\pi} M_{\rho}, \\
\Gamma_{\rho^\pm \rightarrow W^\pm \gamma} &= \frac{2(s_{\ell_{\rho}}^2 - \frac{1}{4}s_{\ell_{\rho}}^4)^2}{48\pi} M_{\rho}, \\
\Gamma_{\rho^0} &= \frac{(g_{\rho} - 2h_{\rho})^2 + 2(\ell_{\rho} + \frac{1}{2} \ell_{\rho}^4)^2}{48\pi} M_{\rho}.
\end{align}

4. CONCLUSIONS

The low-energy effect of a resonance is proportional to its couplings squared. The same holds for the partial width for the decay into vector boson pairs. The total width is bounded from below by the partial widths, while an upper limit is given by the resonance mass: if the would-be width becomes larger than that, the resonance parameterization loses its meaning. Combining these two observations, we see that the maximally allowed coupling value, and thus the strongest possible low-energy effect, is present if the partial width is equal to the mass, and there are no other decay channels.

We conclude that for each given channel and coupling parameter, we obtain an inequality of the form

\[ |\Delta \alpha_i| \lesssim 4\pi k_i \left( \frac{v^2}{M^2} \right)^2, \]

where $k_i$ is some numerical factor. In the scalar and tensor cases, the width is proportional to $M^3$, while the leading effect on anomalous couplings is proportional to $1/M^2$. In the vector case, the width scales with the mass, but all anomalous couplings except for $\beta_1$ get contributions only at order $1/M^4$. The net effect is of order $1/M^4$ for (almost) all channels and couplings. This clearly limits the physics reach of low-energy measurements.

The significant exception is the $\beta_1$ parameter, which is equivalent to the $\rho$ parameter. If isospin is violated, this parameter receives contributions of the order $1/M^2$, cf. (13a, 17a). We thus confirm the expectation that this parameter is most sensitive to new-physics effects. However, we also see that there are ways to suppress this contribution: (i) If vector-boson interactions conserve isospin ($h_{\rho} = 0$, $\Delta M_{\rho} = 0$), the parameter is zero. (ii) For the triplet case, there is also a pseudo-symmetric limit $h_{\rho} = -g_{\rho}, \Delta M_{\rho} = 0$ where $\beta_1$ vanishes. (iii) There may be some cancellation with the mass splitting, or with extra terms not accounted for by tree-level resonance exchange. In particular, we have to keep in mind that fermionic couplings and loop effects could also be relevant.
It is interesting to note that, at order $1/M^4$, several different couplings of vector resonances are present and affect low-energy observables. In particular, there are contributions of anomalous magnetic moments. These cannot be measured by scanning resonance curves at LHC or at a higher-energy hadron or lepton collider. Thus, the precision measurement capabilities of the ILC add independent information, regardless of the state of LHC high-energy measurements.

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