A high-precision segmented Paul trap with minimized micromotion for an optical multiple-ion clock

Karsten Pyka, Norbert Herschbach, Jonas Keller, and Tanja E. Mehlstäubler
Physikalisch-Technische Bundesanstalt, Bundesallee 100, D-38116 Braunschweig, Germany

June 22nd, 2012

Abstract
We present a new setup to sympathetically cool $^{115}$In$^+$ ions with $^{172}$Yb$^+$ for optical clock spectroscopy. A first prototype ion trap made of glass-reinforced thermoset laminates was built, based on a design that minimizes axial micromotion and offers full control of the ion dynamics in all three dimensions. We detail the trap manufacturing process and the characterization of micromotion in this trap. A careful calibration of the photon-correlation spectroscopy technique demonstrates a resolution of 1.1 nm in motional amplitude of our measurements. With this method, we demonstrate a sensitivity to systematic clock shifts due to excess micromotion of $|\langle \Delta \nu / \nu \rangle_{\text{mm}}| = 8.5 \times 10^{-20}$. Owing to our on-board filter electronics on the ion trap chips no rf phase shifts could be resolved at this level. We measured rf fields over a range of 400 µm along the ion trap axis and demonstrate a region of 70 µm where an optical frequency standard with a fractional inaccuracy of less than $\leq 1 \times 10^{-18}$ due to micromotion can be operated.

1 Introduction

In the past years, the improvement of optical clocks based on single-ion spectroscopy has lead to frequency comparisons with a relative inaccuracy as low as $|\Delta \nu / \nu| < 1 \times 10^{-17}$ [1]. Most promising candidates have shown to be two-species ion systems, in which the clock ion is sympathetically cooled and the detection is realized through quantum logic spectroscopy [2]. To reduce systematic shifts related to excess micromotion for such a two-ion system, linear Paul traps are used, which ideally have zero axial micromotion due to their geometry.

While single-ion clocks show an excellent potential to reach lowest frequency inaccuracy, they are limited in their short-term stability by the intrinsically low signal-to-noise ratio (SNR), as only one ion contributes to the clock signal. One approach to improve the short-term stability of the clock measurement, thus reducing lengthy averaging times, is to increase the spectroscopy pulse time on long-lived atomic states [1, 3]. This naturally limits the number of available clock ion candidates and places severe requirements on the clock laser stability.

Proposals have been made to develop an optical frequency standard based on multiple ions with increased SNR [4, 5]. In our approach, we sympathetically cool indium ions with ytterbium ions. Here, the direct spectroscopy of indium as the clock ion avoids quantum logic and facilitates scaling-up the clock read-out of multiple ions [4].

However, with a larger number of clock ions the residual axial micromotion of a linear ion trap and its associated systematic frequency shifts become an issue [6]. In the best optical clock comparison [1] a shift of the clock ion by a distance of only 3 µm along the trap axis leads to a relative frequency shift due
to axial micromotion of \((\Delta \nu/\nu)_{\text{mm}} \approx -2.7 \times 10^{-17}\).

In recent studies, various techniques in different trap geometries have been used to investigate micromotion and motional amplitudes of a few nanometers have been demonstrated for a single ion \([7, 8, 9, 10, 11, 12]\). However, for an optical clock with multiple ions, it is imperative to engineer trap structures capable of storing linear chains of ions with the lowest possible axial rf fields over an extended range together with a high-level control of the ion dynamics.

In quantum information experiments with trapped ions, where fast ion transport is required and relevant time scales are in the millisecond regime or below, miniaturized scalable trap structures have been investigated \((13, 14, 15, 16, 17, 18, 19, 20, 21)\). In \([4]\) we proposed a design for a scalable linear ion trap for an optical clock, in which up to ten ions can be stored per trapping segment within the Lamb-Dicke regime. Since observed excess heating rates scale with the ion-electrode distance \(d\) as \(\frac{d\text{Heat}}{dt} \propto d^{-4}\) \([22, 23]\), the trap electrode dimensions were chosen to be of millimeter size, to be able to reach long interrogation times that are not limited by the heating of the ion. A strong effort is put on optical access, an electrode scheme avoiding rf phase shifts on the electrodes, and precise dimensions of the trap segments to minimize axial rf fields to a level required for clock spectroscopy.

In this paper we present our experimental setup to test high-precision ion traps and demonstrate successful operation of a first prototype. The atomic system used to characterize the ion trap is \(^{172}\text{Yb}^+\). The laser system for this ion, the vacuum system and the detection scheme are described in section 2. In section 3, the design and the fabrication process of the prototype trap are explained in detail. In the last section, we focus on high-resolution micromotion measurements and compare experimentally determined axial rf fields to FEM calculations of this trap. We demonstrate a sensitivity of photon-correlation spectroscopy to micromotion induced frequency shifts, like 2nd order Doppler shift and Stark shift, of \(|(\Delta \nu/\nu)_{\text{mm}}| < 10^{-19}\). With this method, we demonstrate, to our knowledge for the first time, an ion trap that is capable of trapping multiple ions along the trap axis for high-precision spectroscopy with a micromotion induced relative frequency inaccuracy of about \(1 \times 10^{-18}\).

## 2 Experimental setup

### 2.1 Laser systems for the trapping of \(^{172}\text{Yb}^+\)

The advantage of the even isotope \(^{172}\text{Yb}^+\), used to characterize the ion trap, is the absence of a hyperfine structure and thus a low number of electronic states that have to be addressed. Four different diode lasers are used for photoionization, cooling and detection, and repumping. An overview of the relevant energy levels including their linewidth and transition wavelength is given in Fig. 1.

![Figure 1: Left: Photoionization of \(^{172}\text{Yb}\) via two-step laser excitation. Right: Partial level-scheme of \(^{172}\text{Yb}^+\) with atomic transitions used for cooling and repumping.](image)

For photoionization, a frequency-doubled external cavity diode laser (ECDL) system is used to resonantly excite neutral \(^{172}\text{Yb}\) to its \(^1\text{P}_1\) state. A single pass through a Brewster-cut periodically-poled potassium titanyl phosphate (PPKTP) crystal is sufficient to generate a power of \(P_{399\text{nm}} \approx 80 \mu\text{W}\), out of which 7 \(\mu\text{W}\) are sent to the atoms. The beam is guided through the trap vertically, whereas the atomic beam of neutral Yb passes horizontally through the trap. Thereby, Doppler-shifts and broadening due to the high temperature of the atoms are avoided. In fact, a scan of the laser shows a full-width half maximum of only 35 MHz at a saturation...
parameter of \( s = 2\Omega^2/\Gamma^2 \lesssim 1 \), where \( \Omega \) is the Rabi-frequency and \( \Gamma \) the decay rate of the excited state.

An optically amplified and frequency-doubled diode laser system (Toptica TA SHG pro) delivers an output power of \( P_{369\text{nm}} \approx 55 \text{mW} \) at a wavelength of 369.5 nm. This light is used to cool and detect the ions by driving the \( ^2S_{1/2} \leftrightarrow ^2P_{1/2} \) transition of \( ^{172}\text{Yb}^+ \). It is also used for the second excitation step of the photoionization process. The laser is divided into three beams, all sent to the trap via polarization-maintaining optical fibers. All beams are collimated and imaged into the trap with a waist of \( w_{369\text{nm}} \approx 80 \mu\text{m} \), where the cooling beam in the vertical direction is overlapped with the photoionization beam, see Fig. 3. The power used for Doppler cooling and detection of a single ion is \( \approx 3 \mu\text{W} \), which gives a saturation parameter of \( s_{369\text{nm}} \approx 0.6 \) for each beam. In order to perform spectroscopy on Coulomb crystals in more than one segment, the beams can be expanded along the trap axis.

Due to the decay channel from the cooling transition to the long-lived \( ^2D_{3/2} \) state, constant cooling of the ion is only possible by depleting the population of this dark state. Therefore, another laser beam with a power of 20 mW is sent to the ions, driving the \( ^2D_{3/2} \leftrightarrow ^3[3/2]_{1/2} \) transition at a wavelength of 935.2 nm. The beam is guided in an optical fiber and imaged with a \( f = 500 \text{mm} \) lens along the trap axis (Z, see Fig. 3), having a minimum waist of \( w_{935\text{nm}} \approx 125 \mu\text{m} \) at the trap center.

Collisions with the background gas in the vacuum chamber can populate the metastable \( ^2F_{7/2} \) state by a combination of non-radiative and radiative decays [24]. This state has an estimated lifetime of several years [3, 25], so the ion is lost for the experimental sequence and the decoupling from the cooling cycle leads to heating and eventual loss from the trap. On longer time scales of minutes quenching of the metastable state population via collisions with heavier background molecules such as \( \text{H}_2\text{O} \) is observed. For a deterministic clean-out of the \( ^2F_{7/2} \) state, a second repump laser at 638.6 nm is used in the experiment. About 5.5 mW of this light is overlapped with the 935.2 nm laser beam, again having a waist of \( w_{639\text{nm}} \approx 125 \mu\text{m} \) at the trap center.

A wavelength meter based on a Fizeau interferometer\(^1\) with an 8-channel multi-mode fiber switch is used to stabilize all lasers. The specified absolute frequency accuracy is 60 MHz in a range of 350 nm to 1100 nm for a single-mode fiber connection, and 200 MHz for the multi-channel switch operation. However, the stability of measured frequencies is 1 MHz from shot to shot. This is determined by measuring the frequency of an ultra-stable laser with a sub-Hz linewidth and a few 100 mHz/s drift, see Fig. 2. Together with a temperature drift of \( \sigma_T \approx 0.2 \text{K/h} \) a drift of the wavemeter readout of less than 3 MHz per hour is observed. All lasers are measured in the red and near-infrared range.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.png}
\caption{Temporal stability of the wavemeter readout. Shown is the Allan deviation of a typical frequency measurement of our stable reference laser at 822 nm with sub-Hz linewidth and drift of \(< 1 \text{ Hz/s}\) using the WS-7 wavemeter.}
\end{figure}

\subsection*{2.2 Experimental apparatus and detection scheme}

The vacuum system is designed for testing and characterization of different trap geometries and provides versatile optical and electrical access. The cylindrical chamber is based on a DN250 flange with adapter flanges on the side and bottom for windows and electrical feedthroughs.

Six windows are mounted pairwise in the horizontal plane and allow laser beams to pass along the...

\(^1\)High Finesse, WS-7
Figure 3: Schematic top view of the experimental setup. The trap is placed in front of the re-entrant vacuum viewport. Three laser beams ($H_1$, $H_2$, $V$) at 369.5 nm are used for the measurement of micromotion in all spatial dimensions. Laser beams along $Z$ provide light at 638.5 nm and 935.2 nm for repumping. The photoionization laser at 398.9 nm is overlapped with the beam in $y$ direction.
trap axis (Z) as well as under an angle of $\theta \approx \pm 25^\circ$ to the z-axis (H1 and H2). One vertical laser beam (V) passes perpendicularly to the horizontal plane through the ion trap. With this setup, we are able to measure excess micromotion of the ion in all three dimensions and characterize the trap. A detailed scheme is shown in Fig. 3.

For detection, a re-entrant viewport is mounted horizontally and perpendicularly to the trap axis with a distance of 23 mm between the vacuum window and the trap center. A self-built lens system with standard spherical 1 inch lenses and a working distance of 37 mm provides a light collection efficiency of 2%. The lens design is a retrofocus lens with a numerical aperture $NA = 0.27$. In order to assure wavefront errors smaller than $\lambda/4$ the lens positions have been optimized with a commercial ray-tracing software. With a magnification $V = 25$ and a CCD-chip pixel size of 16 $\mu$m, we obtain a spatial resolution of 0.6 $\mu$m/px on our camera.

The detection scheme is twofold: it is possible to either detect with the electron multiplying CCD (EM-CCD) camera\(^2\) or with a photomultiplier\(^3\) (PMT). Also, it is possible to detect ions with both devices simultaneously using a 50/50 beam splitter. The latter configuration was used during the measurements presented in this paper.

The vacuum in the apparatus is maintained by an ion getter pump, having a nominal pumping speed of 20 mbar l/s for air and a titanium sublimation pump. The residual pressure after pre-baking, re-opening and inserting the ion trap, followed by a modest bake-out at 60°C is $1 \times 10^{-8}$ Pa.

The atomic ovens are made of a vertical tantalum tube with spot-welded copper wires at the ends to provide the heating current. The atomic beam is collimated with a copper shield with two slits of $\approx 1$ mm in the horizontal direction, assuring atomic flux through the loading segment, while the spectroscopy segments do not get contaminated. An isotope-enriched sample of ytterbium is used, as the natural abundance of the isotopes $^{172}$Yb and $^{173}$Yb are 22% and 16%, respectively. Their photoionization resonances are separated by only 55 MHz and show considerable overlap. A frequency scan of the photoionization laser is shown in Fig. 4.

For our segmented linear Paul trap various dc voltages necessary for axial confinement of the ions and for micromotion compensation are generated by a 13 bit analog output PCI-card\(^4\). A low-pass filter and a distribution box combine the axial confinement voltages and radial compensation voltages that are applied on a common trap electrode. A detailed scheme is shown in Fig. 5. For the inner electrodes, the compensation voltages provided by the PCI-card are divided down to improve the resolution of the field, which is applied to shift the ions in the radial direction ($U_{tc}$). The range and resolution of all three fields are summed up in Table 1.

Furthermore, the high-voltage rf field for the radial confinement, that is necessary in order to achieve high secular frequencies for $^{172}$Yb$^+$ is provided by a helical resonator, designed according to [27]. With a free-standing coil, made of a 5 mm thick copper wire, an unloaded quality factor of $Q_{unloaded} \approx 1050$ has been realized. Connected to the trap, the quality factor is $Q_{loaded} \approx 640$ and the resonance frequency for driving the trap is $\Omega_{rf} = 2\pi \times 25.67$ MHz. The amplitude of the rf voltage is measured by a calibrated

\(^2\)Andor: DU-897, 512 px $\times$ 512 px, quantum efficiency QE = 85 % for 200 nm – 370 nm

\(^3\)Hamamatsu: R7207-01, Bialkali window, QE $\geq$ 20 % for 160 nm – 650 nm

\(^4\)National Instruments: NI 6723 with 32 analog outputs
thin wire pick-up, which is placed inside the helical resonator tank.

With this system, the ion trap can be driven with an rf voltage amplitude of 1500 V using an rf power < 2 W.

3 The ion trap

3.1 Specifications

The trap introduced in this paper is based on a design published in [4]. The main goal is to obtain a scalable structure of an array of linear Paul traps, where chains of about 10 ions can be stored in the Lamb-Dicke regime in each trapping segment, with axial rf fields at negligible level. This means micromotion induced systematic fractional frequency shifts need to be below $1 \times 10^{-18}$. For this, FEM calculations had been carried out to estimate the effects of alignment uncertainties and machining tolerances on the rf potential. In order to test the trap design and the new experimental setup, a simplified prototype trap has been built from easily machinable thermoset wafers, featuring one loading segment and two spectroscopy segments. The realized electrode structure with five electronically isolated segments and the geometric dimensions are shown in Fig. 5.

The rf electrodes $U_{rf}$ only carry the high rf voltage for the radial confinement of the ions, whereas the rf ground electrodes $GND_{rf}$ provide dc voltages for the axial confinement of the ions as well as the micromotion compensation in all three dimensions. The inner electrodes opposite to the rf electrodes are used for axial confinement as well as micromotion compensation, while the outer electrodes are used for micromotion compensation only. Table 1 shows the dc voltages that can be applied, together with their resolution and the corresponding dc electric field calculated at the position of the ion.

For the axial confinement, $U_t$ is applied to the electrodes neighbouring the used trap segment. A typical value for most measurements presented in this paper is $\omega_{ax,yb} \approx 2\pi \times 116$ kHz at $U_{t,n-1} = U_{t,n+1} = 4$ V and $U_{t,n} = 0$ V. As those electrodes are controlled independently, they can also be used for micromotion.

![Figure 5: Trap geometry and electronic configuration. All rf ground electrode segments are dc isolated from each other. With individual voltages $U_{t,n}$ axial confinement is realized. A differential voltage $U_{tc,n}$ provides compensation fields in radially diagonal direction in each segment $n$. A differential voltage $U_{ec,n}$ on the outer compensation electrodes provides an independent second field vector to move the ions to any position in the xy-plane.](image)

| axis | $\Delta U_{dc}$ | $U_{dc,\text{range}}$ | $\Delta E_{dc}$ | $E_{dc,\text{range}}$ |
|------|----------------|----------------------|----------------|----------------------|
| $U_{tc}$ | x | 0.6 mV | ±2.5 V | 0.32 V/m | 1325 V/m |
| | y | 0.40 V/m | 1650 V/m |
| $U_{ec}$ | x | 2.9 mV | ±12 V | 0.45 V/m | 1860 V/m |
| | y | 0.05 V/m | 204 V/m |
| $U_t$ | z | 2.9 mV | ±12 V | 0.30 V/m | 612 V/m |
compensation as well as shifting the ion along the trap axis to measure residual rf electric fields. Here, a field resolution of $\Delta E_{z, \text{min}} = 0.30 \text{ V/m}$ corresponds to a spatial shift of $0.3 \mu m$ at $\omega_{x, \text{Yb}} = 2\pi \times 116 \text{ kHz}$.

For micromotion compensation in radial direction, $U_{\text{ec}}$ and $U_{\text{tc}}$ are used. $U_{\text{ec}}$ generates a field, which has its strongest vector component (90%) along the x-axis and is solely used to shift the ion along this direction. $U_{\text{tc}}$ generates a field with strong components in both x and y direction, which makes it necessary to compensate with $U_{\text{ec}}$ in x direction when shifting the ion along the y-axis. The equally spaced quadrupole electrode geometry leads to an efficient rf trapping potential in radial direction with a calculated loss factor of $L = 1.29$ [28].

For clock operation, radial secular frequencies of more than 1 MHz can be reached with indium ions at an rf trap voltage of 1500 V. For the trap characterization with $^{172}\text{Yb}^+$, a lower rf voltage amplitude of $U_{rf} \approx 810 \text{ V}$ is used, resulting in radial secular frequencies of $\omega_{\text{rad, Yb}} \approx 2\pi \times 484 \text{ kHz}$. At this rf amplitude the trap depth is $2.2 \text{ eV}$ in x direction and $3.4 \text{ eV}$ in y direction. The principal axes of the trap are rotated with respect to the x-axis by about $36^\circ$ and $126^\circ$.

### 3.2 Trap fabrication

The prototype ion trap is made of Rogers 4350BTM, a glass-reinforced, ceramic-filled hydrocarbon thermost with low rf losses ($\tan \delta < 0.0037$) [5]. This makes it possible to generate a high trapping rf voltage with low input power. The electrodes are made of laser structured $200 \mu m$ thick wafers, with a $35 \mu m$ conductive copper layer and a $10 \mu m$ gold thick-film. A thin nickel layer is used as an adhesion promoter.

Four boards are stacked on top of each other and mounted on a carrier board as shown in Fig. 6. Two identical boards (1), one of which is rotated by $180^\circ$ around the trap axis relative to the other, form the quadrupole trap. They are separated by four $1 \text{ mm}$ thick spacers placed in the corners. An additional board (2) is attached on top of each of the trap boards, with spacers of $0.25 \text{ mm}$ thickness. The stack is glued on a $1.5 \text{ mm}$ thick carrier board (3), which provides gold pins as dc voltage connectors to the multipin feedthrough and a mount for the rf connection. The rf lead is a $0.1 \text{ mm}$ thick, oxygen-free copper foil cut into $5 \text{ mm}$ wide strips of equal length to avoid phase shifts on the rf electrodes.

To prevent coupling of rf power into the dc voltage sources as well as coupling of high-frequency noise onto the trap electrodes, low-pass filters are mounted directly on the trap boards close to the dc electrodes. Non-magnetic and UHV-proof SMD components [6] are soldered with a lead free UHV compatible solder [7]. The copper foil strips used for the rf connection are soldered onto the boards as well.

After fitting all boards with the electrical components, they were aligned under the cross-hairs of a microscope [8] and fixed one after the other using a glue with low outgassing and low shrinkage [9]. This glue has the advantage of being both heat and UV curable. After stabilizing the boards with a broadband UV lamp, the complete stack was heated in an oven for 30 min at $130^\circ C$ to ensure that all glue was cured. The dc leads on the individual electrode boards were then connected to the carrier board with a ball bonder using $30 \mu m$ thick gold wires. Finally, the gold pins were soldered to the carrier board for the dc voltage connection. The link to the vacuum feedthrough is made with Kapton coated wires with crimped connectors. Putting the whole trap stack under the microscope again revealed a translational misalignment of about $20 \mu m$ along the trap axis and less than $0.05 \text{ mrad}$ rotational misalignment between the trap boards.

---

5 see http://www.rogerscorp.com, not available anymore, but current 4360 shows same specifications

6 Resistors: $R = 300 \Omega$, Barry Industries - partnumber: RP0402BA-3003 JN-91. Capacitors: $C = 4.7 \text{nF}$, Novacap - partnumber: 0402 C472 J500 PH-HB. Low-pass cutoff frequency: $\nu_{\text{cutoff}} = 113 \text{ Hz}$

7 Kester: 80Sn19Ag1Cu

8 Leitz UWM 3410 Gen2, see http://outgassing.nasa.gov/ for vacuum compatibility
4 Characterization of the prototype trap

4.1 Trap operation

As a first test of the ovens and the ion trap, a single lens was used to observe the whole trapping region. The fluorescence of neutral Yb atoms was observed in the trace of the photoionization beam passing through the loading segment, segment 3 in Fig. 7. By moving the laser beam through all trap segments and detecting atoms only in segment 3, we were able to verify the alignment of the atomic beam aperture. Ions were loaded and shuttled into all three trap segments. Using the detection lens with magnification of $V = 25$, we are able to resolve single ions in a Coulomb crystal in one trap segment, see Fig. 7. Deterministic loading of single ions was demonstrated.

The radial secular frequencies were measured for a single $^{172}$Yb$^+$ ion by amplitude modulation of the rf electric field. For an rf voltage amplitude of $U_{rf} \approx 810$ V and a dc voltage $U_t \approx 0.05$ V, two radial secular frequencies of $\omega_{rad,1} = 2\pi \times (490 \pm 0.5) \text{ kHz}$ and $\omega_{rad,2} = 2\pi \times (472 \pm 0.5) \text{ kHz}$ were measured.
Here, the rf voltage is deduced from the calibrated pick-up voltage $U_{\text{mon}}$ of the helical resonator with $K = U_{\text{rf}} / U_{\text{mon}} = 5400 \pm 340$. The observed trap frequencies are, within a few percent, in good agreement with the calculated rf potential.

### 4.2 Photon-correlation spectroscopy

A single $^{172}\text{Yb}^+$ ion is loaded into the trap to measure residual micromotion in our trap by photon-correlation spectroscopy [6]. This technique measures the rf field induced motion of the ion via its 1st order Doppler-shift on a broad atomic line with natural linewidth $\Gamma_{\text{nat}}$. In order to resolve the weak modulation of the ion fluorescence at the trap drive frequency, its scattering rate is correlated with the phase of the rf trap voltage. A time-to-amplitude-converter (TAC) generates pulses with a height dependent on the time $T$ between a START-signal, triggered by the detection of a photon by the PMT, and a STOP-signal generated by the rf voltage of the trap. A multi-channel-analyzer yields a histogram of these pulses sorted in height and thus, in time difference $T$. Thereby, the modulation amplitude of the ion’s fluorescence at the trap frequency $\Omega_{\text{rf}}$ is observed. A detailed description of the method can be found in [6].

As an example, Fig. 8 shows the histograms of two measurements of the radial micromotion component along the y-axis of a single ion as measured by the laser beam along V. For each measurement, the data acquisition time is 30 s. In the measurement shown in Fig. 8, where micromotion is detected along the vertical laser beam, a weak laser beam along H1 is present to cool the ion in the axial direction. The average photon count rate at a detuning of $\omega_{\text{laser}} - \omega_{\text{ion}} = -\Gamma/2$ is about $10^{800}$ s$^{-1}$, including a background of about $1800$ s$^{-1}$.

Using the laser beams in H1, H2 and V, respectively, the micromotion can be measured in all dimensions. The velocity amplitude of rf induced ion motion $v_{\text{mm}}$ is evaluated using the linearization of the line profile, which is a valid approximation for $kv_{\text{mm}} \ll \Gamma$. In this case, the signal contribution $S_{\text{det,i}}$
of each beam $i$ can be written as

$$S_{\text{det},i} = \frac{S_{\text{max},i}}{2} + S_{\text{mod},i} \sin(\Omega_{\text{rf}} t + \varphi_i), \quad (1)$$

with $i=H1, H2, V$, where $S_{\text{max},i}/2$ is the ion’s fluorescence at $\omega_{\text{laser}} - \omega_{\text{ion}} = -\Gamma/2$, which corresponds to half of the maximum fluorescence for a (saturation-broadened) Lorentzian probability distribution. $S_{\text{mod},i}$ is the fluorescence modulation amplitude and $\varphi_i$ is the phase of the signal relative to the rf voltage. To improve the fit of the data, the frequency of the rf trigger signal is $\Omega_{\text{rf}}/2$, so that the data set contains two rf periods, i.e. $T = 0..4\pi/\Omega_{\text{rf}}$. From the individual measurements along laser beams H1, H2 and V, the velocity components along the trap axes are extracted using:

$$k \nu \frac{v_{\text{mm},y}}{\Gamma_{\text{nat}}} = \frac{S_{\text{mod},V} \sqrt{1 + \nu}}{S_{\text{max},V} f_c}, \quad (2)$$

$$k \nu \frac{v_{\text{mm},x}}{\Gamma_{\text{nat}}} = C_x \sqrt{A^2 + B^2 + 2AB \cos(\varphi_{H2} - \varphi_{H1})} \sin(\Omega_{\text{rf}} t + \varphi_i), \quad (3)$$

$$k \nu \frac{v_{\text{mm},z}}{\Gamma_{\text{nat}}} = C_x \sqrt{A^2 + B^2 - 2AB \cos(\varphi_{H2} - \varphi_{H1})}, \quad (4)$$

with $C_x, x = \frac{1}{2\cos \theta_{x,x} f_c}$ and $A, B = \frac{S_{\text{mod},H1,H2} \sqrt{1 + s_{H1,H2}}}{S_{\text{max},H1,H2}}$.

Here, $k = 2\pi/369.5 \text{nm}$ and $\theta_{x,y} = 65^\circ$ and $\theta_z = 25^\circ$ are the projection angles of the laser beams H1 and H2 to the trap axes x and z, respectively. Saturation broadening of the atomic line is taken into account by the individual saturation parameters $s_i$ of each laser beam.

In addition, the measured signal is corrected by a factor $f_c$, due to the finite lifetime of the excited state of the ion. Only when the lifetime of the excited state is much shorter than the modulation period of the signal ($\tau \ll T_{\text{rf}}$), the full signal modulation $m_0 = S_{\text{mod}}(0)/S_{\text{mod}}$ is observed. In general, the fluorescence of an ion, when excited by a periodic signal, follows the differential equation $S(t) = \tau^{-1} S(t) + S_{\text{drive}}(t)$, with a periodic term $S_{\text{drive}}(t)$ [29]. The general solution to this equation is:

$$S(t) = e^{-t/\tau} \left( c_1 + \int_{-\infty}^t S_{\text{drive}}(t') e^{t'/\tau} dt' \right), \quad (5)$$

which consists of a fast exponential decay and the damped response to the modulation. For the fluorescence of a laser-cooled ion with micromotion, this modulation is $S_{\text{drive}}(t) = S(0) + S_{\text{mod}}(0) \sin(\Omega_{\text{rf}} t)$, with $S_{\text{mod}} \propto |\vec{k} \cdot \vec{v}_{\text{mm}}|$. For times larger than the natural decay time $\tau$, (5) gives:

$$S(t) = \tau S(0) + \frac{S(0)}{\tau S_{\text{mod}}} \frac{\tau^{-1} \sin(\Omega_{\text{rf}} t) - \Omega_{\text{rf}} \cos(\Omega_{\text{rf}} t)}{\tau^2 - \Omega_{\text{rf}}^2}, \quad (6)$$

$$m_0 = \frac{S(0)}{\tau S_{\text{mod}}} \left( \tau^{-2} + \Omega_{\text{rf}}^2 \right)^{-1/2} \sin(\Omega_{\text{rf}} t + \varphi_i).$$

From this the modulation $m_{\text{det}}$ of the detected fluorescence is derived to be

$$m_{\text{det}} = \frac{S(0)}{S_{\text{mod}}} \frac{\tau^{-2} + \Omega_{\text{rf}}^2}{\tau S_0} = m_0 \cdot f_c, \quad (7)$$

giving a reduction in contrast compared to the modulation $m_0$ of

$$f_c = \frac{1}{\sqrt{1 + (\Omega_{\text{rf}} / \tau)^2}}. \quad (8)$$

With a lifetime $\tau = 8 \text{ns}$, this correction term amounts to $f_c = 0.61$.

The finite saturation parameter $s_i \approx 0.6$ of each laser beam leads to a reduction of the measured micromotion velocity amplitude of about 26%. It is evaluated before the measurement by scanning the cooling laser over the resonance and fitting a Lorentzian profile to the scan. The fit gives the effective linewidth $\Gamma$ and hence the saturation parameter for each laser beam with a systematic relative uncertainty of about 8%, which is the major contribution to the systematic uncertainty of the data. Compared to that, the uncertainty in the angles of the laser beams to the trap axes is negligible and not taken into account.

Great care is taken to correctly subtract the background signal due to laser straylight from the fitted offset $S_{\text{max},i}/2$. The detection laser itself contributes
about 5–10% to the background. In the case of measuring micromotion along \( V \), laser beam \( H1 \) is present during the measurement to compensate axial heating of the ion. For this purpose the power in beam \( H1 \) is reduced by a factor of 14 and no contribution to the modulation amplitude is detectable. Only a constant offset is subtracted from \( S_{\text{max},V}/2 \).

The resolution achieved in the measurements is limited in general by intensity and frequency fluctuations of the spectroscopy laser and has been evaluated as a scatter of the data from repetitive measurements in all laser beams. The highest resolution is obtained for the micromotion \( y \) component measured along laser beam \( V \) with a scatter of \( \sigma_{\text{rf},y} = 21 \text{ V/m} \). For the measurements along the trap axis this uncertainty is 22 V/m. The lowest resolution is obtained in \( x \) direction with \( \sigma_{\text{rf},x} = 49 \text{ V/m} \). The fit to the data contributes an additional statistical uncertainty of 8 V/m. All systematic and statistical error contributions are added quadratically.

As shown in Fig. 8, an electric field of \( \Delta E_x = 0.9 \text{ V/m} \), corresponding to \( \Delta U_{\text{ec}} = 5.8 \text{ mV} \), has to be applied to produce a detectable micromotion along the \( y \)-axis. This corresponds to an rf field amplitude of \( E_{\text{rf},y} = (51 \pm 23) \text{ V/m} \), or an excess micromotion amplitude of 1.1 nm. With \( (\Delta \nu/\nu)_{\text{mm}} = -v_{\text{mm}}^2/2c^2 \), this gives a relative frequency shift due to time dilation of \(-8.5 \times 10^{-20}\). Here, the resolved rf field is limited only by the resolution of the dc voltage control on the electrodes that moves the ion radially. It is worthwhile mentioning that the rf field induced Stark shift is about an order of magnitude smaller than the shift due to time dilation, as shown in [4], and therefore negligible in this work.

4.3 3D micromotion measurement in the trap

To characterize and test the photon-correlation method quantitatively, a single ion was first shifted in both radial directions of the trap, where the strong rf quadrupole potential dominates and its field gradients can be determined accurately from our measurements of the secular frequencies. In Fig. 9a the micromotion measurement is shown for an ion shifted along the \( x \) direction and in Fig. 9b for an ion shifted along the \( x \) direction. The graphs show the measured rf electric field component perpendicular to the direction of the ion shift as a function of the ion position in the trap. The position of the ion is calculated according to (16) in [6] using the measured radial secular frequencies and the calculated dc electric fields applied by changing \( U_{\text{tc}} \) and \( U_{\text{ec}} \). For comparison, the rf electric field obtained by the FEM calculations is plotted together with the measured data as a function of the ion position.

The measurement of the \( y \) component of the field is in good agreement with the calculation, whereas the \( x \) component shows a deviation of about 25%. This can be explained by the fact, that the Rogers4350BTM boards are elastic. Pictures taken from the assembled trap stack show that the outer compensation boards slightly bend away from the rf trap boards in the center and therefore have a larger distance to the trap center in \( x \) direction. This leads to a decreased static electric field at a given compensation voltage \( U_{\text{ec}} \). A FEM calculation, in which the distance between the compensation and trap boards is varied, shows a decrease of the dc field in \( x \) direction of about 30% for an increase of the electrode distance of 0.25 mm, which is reasonable looking at the taken pictures.

On the other hand, along the \( y \) direction the ion is shifted by applying \( U_{\text{tc}} \) on the rf ground electrodes on the quadrupole trap boards. As these electrodes are more rigidly machined from one continuous wafer and measured secular frequencies have confirmed the expected rf quadrupole geometry, no relevant deviations in the applied dc fields, created by \( U_{\text{tc}} \), are expected here. This gives a strong argument for the good agreement between the measurement and the calculations in the case of the vertical ion shift, where \( E_{\text{rf},x} \) is measured.

A defined jump of 180° of the fitted phase of the photon-correlation signal is observed, when the ion is moved radially across the rf node, see Fig. 9c. The data point at \( x = 0 \) corresponds to non-resolvable micromotion, Fig. 8a. Thus, within the resolution of \( \sigma_{\text{rf},y} = 23 \text{ V/m} \) of our measurement no micromotion amplitude due to rf phase shifts between the electrodes could be detected, which would lead to a smooth transition of the phase of the cross-correlation signal [6].
Figure 10: Residual axial rf electric field along trap axis derived from measured micromotion amplitude (diamonds) compared to axial rf field estimated from FEM calculations (solid line). The dashed line indicates the rf field, for which the relative frequency shift induced by micromotion is smaller than $10^{-18}$ for an indium ion optical clock. The zero axial position corresponds to the ion position for $U_{ax,1} = U_{ax,3} = 4$ V, where the measurement was initiated.

To measure residual on-axis rf fields within the trapping segment, a single ion was shifted in the direction of the trap axis and the micromotion was measured in all dimensions. The laser beams have been adjusted while shifting the ion, in order to prevent systematic uncertainties due to a change in the intensity at the ion position.

The axial rf field is shown in Fig. 10 and compared to FEM calculations, that include the contribution to the rf electric field due to the finite length of the trap as well as the slits between the electrodes. Alignment and machining uncertainties and their effect on the rf field have been investigated in [4]. According to that, variations in the width of the slits between the electrodes due to machining tolerances would affect the measured field gradient and can be an explanation for the slightly larger rf fields.

For the radial components of the rf field we experimentally could verify a value of 115 V/m or below in the range of $40 < z < 170$ µm in all dimensions. In a two day measurement, a sudden change of excess micromotion was observed for the data set taken the next day after reloading ions and not compensating micromotion anew, shown in Fig. 11. This is attributed to charge build-up during the loading process, as the measurements were conducted in the center segment, which is adjacent to the loading segment and not fully protected by the atomic beam aperture.

For optical clock operation with In$^+$ ions, the amplitudes of axial rf fields need to be $|E_{rf}| \leq 115$ V/m to guarantee a small enough fractional frequency shift due to time dilation of $|((\Delta\nu/\nu)_{mm}| = 4E_{rf}e^2/(m^2\Omega_{rf}^2) \leq 1 \times 10^{-18}$. In both radial directions micromotion can be compensated to a level of $\sigma_{rf,y} = 23$ V/m and $\sigma_{rf,x} = 50$ V/m, only limited by
Figure 9: a) and b) RF field as function of ion displacement in radial directions x and y. To check the consistency of the measured micromotion amplitudes, expected rf field amplitudes have been calculated from the measured radial secular frequencies (solid lines). c) Phase $\phi_y$ of the cross-correlation signal in y direction. For $x = 0$, corresponding to the data in Fig. 8a, no fit was possible for $-1 \leq \phi_y \leq 2$.

Figure 11: Residual radial rf field along trap axis measured together with the axial field component as cross check, the x component is shown in a), the y component is shown in b). The measurements are done over two days, with the same trap parameters and compensation voltages. The offset of the data between $z \approx 0...75 \mu m$ compared to the rest is due to a slow drift in the compensation over night.

5 Conclusion

We performed high-precision micromotion measurements in a novel segmented linear Paul trap which is designed for optical clock spectroscopy on linear...
Coulomb crystals. Using a single $^{172}$Yb$^+$ ion as a high-resolution electric field probe we demonstrated a sensitivity of photon-correlation spectroscopy to electric dc fields as low as $\Delta E_{\text{dc}} \leq 0.9 \text{ V/m}$. This allowed us to resolve micromotion to a level where micromotion induced systematic frequency shifts lead to a fractional inaccuracy below $10^{-19}$ for an optical frequency standard.

With this, we were able to show that already the prototype trap, made of printed circuit boards, features a region of 130 $\mu$m along the trap axis in which the relative frequency shift due to micromotion is $|\langle \Delta \nu / \nu \rangle_{\text{mm}}| \leq 1 \times 10^{-18}$ at a trap voltage of $U_{\text{rf}} = 810 \text{ V}$.

![Figure 12: Coulomb crystal with fluorescing $^{172}$Yb$^+$ ions and sympathetically cooled $^{115}$In$^+$ ions. The indium ions (indicated with circles) are confirmed by measuring the mass-dependent secular modes of the crystal via parametric excitation.](image)

In our experiment, we have implemented sympathetic cooling of $^{115}$In$^+$ ions via laser cooling of $^{172}$Yb$^+$ ions, see Fig. 12. For such a multi-ion clock based on the $^{115}$In$^+$ clock transition, a region of 70 $\mu$m can be used for clock spectroscopy, when the ion trap is operated at $U_{\text{rf}} = 1500 \text{ V}$ and $\omega_{\text{In,rad}} = 2\pi \times 1.5 \text{ MHz}$, $\omega_{\text{In,ax}} = 2\pi \times 225 \text{ kHz}$. This allows trapping of a chain of about 12 $^{115}$In$^+$ ions for optical clock operation with $|\langle \Delta \nu / \nu \rangle_{\text{mm}}| \leq 10^{-18}$.

Based on the design presented and tested in this paper, an ion trap of laser-cut AlN wafers will be machined. With this material, a better mechanical stiffness and precision can be obtained and exclusively non-magnetic materials will be used. We expect an improvement in the performance in terms of axial micromotion as well as heat conductivity for precise ion trap temperature evaluation. In addition, a larger number of trap segments will allow us to increase the number of ions used for spectroscopy.

We thank Christian Tamm, Max Harlander and Yves Colombe for fruitful discussions on ion trap technology and Kihwan Kim for providing SMD parts for preliminary tests. Ekkehard Peik and Kristijan Kuhlmann are thanked for a careful reading of the manuscript. This work was supported by the cluster of excellence QUEST.

### References

[1] C. W. Chou et al., Phys. Rev. Lett. 104, (2010) 070802.
[2] P. O. Schmidt et al., Science 309, (2005) 749–752.
[3] M. Roberts et al., Phys. Rev. Lett. 78, (1997) 1876–1879.
[4] N. Herschbach et al., Appl. Phys. B 107, (2012) 891–906.
[5] C. Champenois Phys. Rev. A 81, (2010) 043410.
[6] D. J. Berkeland et al. J. Appl. Phys. 83, (1998) 5025–5033.
[7] S. Narayanan et al., J. Appl. Phys. 110, (2011) 114909.
[8] D. T. C. Allcock et al., Appl. Phys. B 107, (2012) 913–919.
[9] Y. Ibaraki et al., Appl. Phys. B 105, (2011) 219–223.
[10] G. Wilpers et al., Nature Nanotechnology 7, (2012) 572–576.
[11] B. L. Chuah et al., preprint arXiv:1211.0101v1.
[12] S. C. Doret et al., New J. Phys. 14, (2012) 073012.
[13] M. J. Madsen et al., Appl. Phys. B 78, (2004) 639–651.
[14] J. P. Home and A. M. Steane Quantum Inform. Comput. 6, (2006) 5.
[15] D. Stick et al., Nat. Phys. 2 (2006) 36–39.
[16] S. Schulz et al., Fortschr. Phys. 54, (2006) 648–665.

[17] W. K. Hensinger et al., Appl. Phys. Lett. 88, (2006) 034101.

[18] D.R. Leibrandt et al. Quantum Inform. Comput. 9, (2009) 11.

[19] U. Tanaka et al., J. Phys. B At. Mol. Opt. Phys. 42, (2009) 154006.

[20] C. E. Pearson et al., Phys. Rev. A 73, (2006) 032307.

[21] D. T. C. Allcock et al., Appl. Phys. B 107, (2012) 913–919.

[22] Q. A. Turchette Q A et al., Phys. Rev. A 61, (2000) 063418.

[23] N. Daniilidis et al., New J. Phys. 13, (2011) 013032.

[24] M. M. Schauer et al., Phys. Rev. A 79, (2009) 062705.

[25] B. C. Fawcett and M. Wilson, Atom. Data and Nucl. Data Tables 47, (1991) 241.

[26] D. Das et al., Phys. Rev. A 72, (2005) 032506.

[27] W. W. Macalpine and R. O. Schildknecht, Proc. of the IRE (1959) 2099.

[28] C. A. Schrama et al., Opt. Comm. 101, (1993) 32–36.

[29] E. Peik, PhD Thesis (Max-Planck-Institute for Quantum Optics, Garching, 1993).