Design and Research on Modification Method of Finite Element Dynamic Model of Concrete Beam Based on Convolutional Neural Network

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Abstract. Because of the uncertainty of the measurement results, the dynamic inverse problem equation is called the stochastic model correction equation. In order to make this equation or method can be used smoothly in actual engineering, it is aimed at low modal orders and small samples in actual engineering. Condition, this paper proposes a concrete beam finite element dynamic model correction method based on convolutional neural network technology. This method trains the convolutional neural network algorithm by expanding the data set, so that the accuracy of the finite element dynamic model correction method is obtained. Improve, and the method is feasible in actual engineering.

1. Introduction
Dynamic model modification is an important model modification method. The traditional dynamic-based model correction method mainly constructs the dynamic inverse problem equation based on the eigenvalue equation according to the natural frequency, mode shape, modal curvature, flexibility matrix, etc. of the structure. That is, the structural parameters or their correction coefficients are solved by constructing the model correction equation.

When measuring uncertainty, this equation is the stochastic model correction equation. The conventional method is to repeatedly solve the deterministic model correction equation through Monte Carlo simulation of a large sample. This process involves repeated inversions of large matrices, and at the same time, it will encounter the problem of ill-conditioned equations, which makes it difficult to get close to the actual project. In order to avoid multiple calculations of numerical simulation during model modification, it is a common practice to train agent models to simplify the prediction results. The accuracy of the surrogate model prediction results affects the reliability of the dynamic model correction results. Therefore, for the finite element dynamic model correction method, an effective way to improve its reliability is to obtain a high nonlinear fitting ability and high precision under the condition of a small sample. The agency model.

2. Research status and research route design

2.1. Research status of finite element dynamic model correction method
Accurate material parameters determine the rationality and accuracy of the finite element analysis results. The finite element dynamic model correction method based on measured data provides a method of obtaining material parameters close to the actual conditions of the project. At present, many scholars have done a lot of research on this point. Comprehensively, there are currently two finite element dynamic model correction frameworks. The first is the indirect analysis method, which expresses the finite element dynamic model as an optimization problem, and the optimal solution is obtained through multiple calculations of the finite element; the second is the direct analysis method, which establishes the mapping from the effect quantity to the material parameters of the finite element model Relationship, directly predict the parameter value of the finite element model material through the effect quantity.

In the indirect analysis method, it is necessary to simulate the entire material system many times, but its larger number of finite element model units, a larger amount of feature value extraction, and a longer calculation time. In order to reduce the time required for model modification, a nonlinear mathematical model with a certain error with the finite element model is usually used as a substitute for the finite element model, that is, the proxy model. Out of the requirement for the number of times of feature value extraction, and at the same time restricting the number of training samples composed of finite element model cannot be too large, scholars have conducted research on different agent models and optimization algorithms. For example, the particle swarm optimization algorithm and the ant colony optimization algorithm are combined with the artificial neural network agent model.

2.2. Research route of finite element dynamic model correction method
Aiming at the problem that the direct analysis method under the condition of low modal order and small samples has lower analysis accuracy than the indirect analysis method in the actual engineering, a model correction direct analysis method based on convolutional neural network is proposed; for convolutional neural network The network has high requirements for the number of training samples, and multi-output correlation vector machines are used to expand the number of training samples.

Combined with the concrete beam numerical model, by comparing the direct analysis and indirect analysis of the output correlation vector machine algorithm by the order of the modal parameters and the number of samples, discuss and determine the feasibility of the number of samples and the optimization accuracy of the modal order and determine the expansion training The method of sample size, and by comparing the accuracy of the correction results of the multi-output correlation vector machine direct, indirect analysis method and the multi-output correlation vector machine convolutional neural network direct analysis method, the superiority of the method is determined. The flow chart of the research route is shown in Figure 1.

![Fig 1. Flow chart of the research route.](image-url)
3. Research on the method of finite element dynamic model modification

3.1. Direct and indirect analysis theory and mathematical model of finite element dynamic model modification

3.1.1. Indirect analysis method. The modification of structural dynamic model is generally expressed as a problem of seeking optimal solution. The optimized objective function reflects the deviation of the recognition mode and the modal parameters calculated by the finite element method. In general, the optimization problem can be expressed as

\[ \min g(d) = \sum_{i=1}^{n_1} y_i \left( \frac{f_i^m - f_i^c(d)}{f_i^m} \right)^2 + \beta \sum_{i=1}^{n_2} \{1 - \text{MAC}[\Phi_i^m, \Phi_i^c(d)]\} \]

subject to \( d \in D_x \) (1)

Where \( d \) is the material parameter to be corrected, which should be within a certain range of \( D_x \); \( f_i^m \) and \( \Phi_i^m \) are the corresponding structural natural frequencies and vibration modes obtained through actual measurements; given material parameters \( d \), \( f_i^c(d) \) and \( \Phi_i^c(d) \) are the natural frequency and mode shape obtained by finite element calculation; \( y_i \) and \( \beta \) are the weight coefficients; MAC is the modal confidence criterion index.

3.1.2. Direct analysis method. The direct analysis method is to directly establish the relationship from the modal parameters to the material parameters. Its mathematical model is the inverse function of the mathematical model of profile analysis.

\[ P = f^{-1}(F) = g(F) \] (3)

In formula (3), \( f^{-1}(F) \) is the inverse function of \( f(F) \), and \( g(F) \) represents the nonlinear mapping relationship from modal parameters to material parameters. Similar to the indirect analysis method, different material parameters and corresponding modal parameter data obtained through finite element in advance, these data constitute the training sample of the proxy model. When training the surrogate model, the modal parameters are used as input and the material parameters are used as output. The surrogate model directly establishes a nonlinear mapping from the modal parameters to the material parameters, thus avoiding the indirect analysis method to solve the optimization problem. Then, the trained proxy model is used to directly predict the corresponding material parameters based on the obtained concrete beam parameters.

\[ P^* = g(F^*) \] (4)

3.2. Principle and implementation steps of concrete beam finite element dynamic model modification method based on convolutional neural network

In order to avoid multiple calculations in numerical simulations, proxy models are used instead of finite element calculations, so the number of training samples used by proxy models is limited. The algorithm based on convolutional neural network is an algorithm for regular learning of a large number of samples, and there are certain requirements for the number of training samples. The multi-output correlation vector machine indirect analysis method has higher accuracy, but the multi-output correlation vector machine direct analysis method is less accurate. In the training model stage, the high-precision feature of the multi-output correlation vector machine indirect analysis method is used to expand the data set, which can improve the direct analysis the accuracy of the law.
3.2.1. The principle of multi-output correlation vector machine algorithm. Traditional machine learning methods often lead to overfitting in order to minimize the error between the actual target value \( t \) and the predicted value. In order to avoid this situation, the multi-output correlation vector machine model assumes that the weights are Gaussian distribution, and the nonlinear relationship between input and output is modeled as:

\[
t = W\Theta(x) + \epsilon \tag{5}
\]

Where \( \epsilon \) the Gaussian noise is vector of zero mean and diagonal covariance matrix, \( S = \text{diag}(\sigma_1^{-2}, \sigma_2^{-2}, \ldots, \sigma_l^{-2}) \) and \( \Theta(x) = [1, K(x, x_1), \ldots, K(x, x_l)]^T \) is a basis function matrix. Research shows that the prediction accuracy of the Gaussian kernel function RVM is better, so the Gaussian kernel function \( k(x_i, x_j) = \exp(-|x_i - x_j|/\theta^2) \) is used as the kernel function of the multi-output correlation vector machine.

Suppose the diagonal matrix \( A = \text{diag}(a_1^{-2}, a_2^{-2}, \ldots, a_l^{-2}) \) Take each parameter as a hyperparameter to determine the relevant basis function Correlation, and independently control the variance of each weight, \( W \) is used as the weight of the \( s \)-th component of the output vector \( t \). In order to obtain the correct estimation of the model parameters, the likelihood of the data is marginalized:

\[
p\left\{\{t\}_{i=1}^m \mid A, S\right\} = \int p\left\{\{t\}_{i=1}^m \mid W, S\right\}p(W \mid A)dW = \prod_{s=1}^{m} \int N(\tau_s \mid \Gamma_{s}W_s, \sigma_s^2)N(W_s \mid 0, A)dW
\]

\[
= \prod_{s=1}^{m} |H_s|^{-\frac{1}{2}} \exp\left(-\frac{1}{2} \tau_s^T H_s^{-1} \tau_s \right) \tag{6}
\]

Where \( H_s = \sigma_s^2 I + \Gamma A^{-1}, \Gamma^T = [\Theta(x_1), \Theta(x_2), \ldots, \Theta(x_l)] \) is the design matrix, and the vector \( \tau_s \) contains the output vector All measurement samples of the \( s \)-th component oft.

The negative logarithm of the equation is written as a function of hyperparameters in the following form:

\[
L(\alpha) = -\sum_{s=1}^{m} \left\{ \log |H_s| - \tau_s^T H_s^{-1} \tau_s \right\} \tag{7}
\]

Then use the optimal hyperparameters and noise parameters to obtain the optimal weight matrix. The optimized parameters can be expressed as:

\[
A_{\text{opt}} = \text{diag}(a_1^{\text{opt}}, \ldots, a_l^{\text{opt}}); \Sigma_{s}^{\text{opt}} = [(\sigma_s^{\text{opt}})^{-2} \Gamma^T \Gamma + A_{\text{opt}}]^{-1}
\]

\[
\mu_{s}^{\text{opt}} = (\sigma_s^{\text{opt}})^{-2} \sum_{s}^{\text{opt}} \tau_s^T \tau_s; W_{\text{opt}} = [\mu_1^{\text{opt}}, \ldots, \mu_m^{\text{opt}}]^T \tag{8}
\]

For the new input vector \( x^* \), the output is:

\[
t^* = W_{\text{opt}}\Theta(x^*) \tag{9}
\]

3.2.2. Implementation steps of convolutional neural network algorithm. Convolutional neural network is a feedforward neural network, usually composed of input layer, convolutional layer, pooling layer and fully connected.

For the input layer, the convolutional neural network requires the input matrix format to be a square matrix. If missing, it can be filled with a predetermined square matrix. The general methods are: repeat the existing data, use 0 to occupy the matrix or map to a square matrix according to the function. For the low modal order of the direct analysis method, a square matrix is constructed by using the matrix multiplication method on the input vector:
The convolutional layer is the core of the convolutional neural network. The convolution operation is similar to filtering. Its weight can be adjusted by iteration. It is mapped to a fixed-size output window after a fixed-size input window and a convolution kernel are dot-produced. The window size is smaller than the size of the input matrix, so the input matrix is traversed by shifting the window to realize the feature extraction of the input matrix. Considering that the modal order is generally small, the SAME convolution kernel whose output matrix dimension is not reduced after convolution is adopted. The specific formula is as follows:

\[
\text{Input}_{n \times n} = x_{n \times 1} \cdot x_{n \times 1}^T
\]  

(10)

\[
z(u, v) = \sum_{i=\infty}^{0} \sum_{j=-\infty}^{0} \text{Input}_{i,j} \cdot k_{u-i,v-j}
\]  

(11)

Where \( k \) is the weight value in the convolution kernel, and \( \text{Input}_{i,j} \) represents the value of the input layer in the input window.

The main purpose of the pooling layer is to reduce the dimensions of the next layer while retaining the characteristics of the matrix, and at the same time prevent overfitting; while the fully connected layer plays a role of classification in the convolutional neural network, and the end of the model is designed with two Layer fully connected layer, one of which is to reduce the dimension of the input two-dimensional matrix and output it as a one-dimensional vector, and the second layer is to conform the output of the neural network in the model to the preset output format.

4. Conclusion

This paper discusses the influence of the number of samples and modal order on the direct analysis and indirect analysis of the multi-output correlation vector machine algorithm in the dynamic model modification through the concrete beam finite element model. In order to solve the problem of large errors in the direct analysis method under the order of states, a direct analysis method is proposed that expands the data set with a multi-output correlation vector machine and combines the convolutional neural network to realize the correction of the finite element dynamic model. This method can further improve the prediction accuracy of this method by improving the prediction accuracy of the expanded data set and carrying out research on the construction of the convolutional neural network framework.

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