A STOCHASTIC PRODUCTION PLANNING PROBLEM

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Abstract. Stochastic production planning problems were studied in several works; the model with
one production good was discussed in [3]. The extension to several economic goods is not a trivial
issue as one can see from the recent works [8], [9] and [13]. The following qualitative aspects of the
problem are analyzed in [9]: the existence of a solution and its characterization through dynamic
programming/Hamilton Jacobi Bellman (HJB) equation, as well as the verification (i.e., the solution
of the HJB equation yields the optimal production of the goods). In this paper, we stylize
the model of [8] and [9] in order to provide some quantitative answers to the problem. This is possible
especially because we manage to solve the HJB equation in closed form. We point to a fixed point
characterization of the optimal production rates. Among other results, we find that the optimal
production rates adjusted for demand are the same across all the goods and they also turn to be
independent of some model parameters. Moreover we show that production rates (adjusted for
demand) are increasing in the aggregate number of goods produced, and they are also uniformly
bounded. Numerical experiments show some patterns of the output.

Key Words and Phrases: Stochastic production problem, stochastic control, fixed point.

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