In this paper, a bearingless switched reluctance motor (BSRM) with sharing suspension windings, which simplifies the winding structure and reduces the complexity of the control circuit, is analysed. A torque analytical model is derived using the Maxwell stress tensor method, which provides better results, compared with the linear model in the magnetic saturated conditions. A finite element analysis (FEA) model of the prototype is built and used for comparison with the torque analytical model. The results show that the torque analytical model provides accurate results in the entire magnetic saturation region, which verify the feasibility of this model. In addition, the self-decoupling characteristics of this type of motor are illustrated using analytical model analysis, FEA and experimental analysis. The results indicate that BSRMs with sharing suspension windings can be flexibly controlled without using complex self-decoupling algorithms of conventional BSRMs. The presented work provides a theoretical basis for the control strategy.

1 INTRODUCTION

Switched reluctance motors (SRMs) are acknowledged for simple rotor structure, low cost and independent phase control features. They are applied in various industries including aircraft, mining and vehicles. The concept model of a bearingless switched reluctance motor (BSRM) was proposed in [1], which avoids situations including bearing wear and lubrication failure during high-speed operation of normal SRMs. A set of suspension windings was added to a normal SRM, which changes the air gap magnetic field distribution and induces a stable radial force [1, 2]. BSRMs have the merits of both bearingless motor and SRMs, including simple structure, robustness and strong anti-interference [3–5].

A BSRM is a highly coupled non-linear system; thus, the establishment of its analytical model is required for the exploration of the strategies used for effective control of the system. Several approaches have been proposed for the analytical modelling purposes. In [6], an equivalent magnetic circuit model was established, and the ellipse path was used to obtain the torque analytical model of BSRMs with sharing suspension windings. In [7], a decoupling strategy was presented based on the torque analytical model derived by the Maxwell stress tensor method. In [8], a torque analytical model with single-phase and dual-phase conduction modes was proposed using the Maxwell stress tensor method.

Due to the coupling relationship between the torque control and the radial force control, a specific decoupling algorithm or an improved motor structure must be introduced to achieve decoupling. Several control schemes have been proposed for this purpose. For conventional 12/8 BSRMs, a decoupling control scheme based on neural network inverse system and improved torque analytical models was carried out to control the radial force and torque separately as suggested in [9]. A 12/4 dual windings BSRM in [10] realised the self-decoupling because of the special inductance profile caused by the wide rotor pole structure. In [11], the electromagnetics and decoupling characteristics of a BSRM with a segmented rotor structure were analysed. In [12], a BSRM with a biased permanent magnet used two rotors to control the radial force and torque, respectively. In [13], a simple structure of stator windings of a BSRM used a 12/14 pole hybrid stator structure, which retained the characteristics of self-decoupling while it increased the power density and reduced the iron core loss. BSRMs with
sharing suspension windings have self-decoupling characteristics, which can be realised without changing the structure of the stators and rotors.

This paper is in the area of BSRMs with sharing suspension windings. In view of the complicated coupling between radial force control and torque control of conventional BSRMs, a simple linear equation is used to describe the magnetic saturation characteristics. The self-decoupling characteristics of this motor are verified, which shows the advantages of the sharing suspension winding structure.

This paper is organised as follows. First, the structural features of BSRM with sharing suspension windings are explained in Section 2. The torque analytical model derived using the Maxwell stress tensor method and the self-decoupling characteristics are verified in Section 3. In Section 4, the self-decoupling characteristics are proved by using finite element analysis (FEA) and experimental verification. Finally, the conclusion of this paper is provided in Section 5.

2 | THE STRUCTURE CHARACTERISTICS OF THE BSRM WITH SHARING SUSPENSION WINDINGS

A schematic of sharing suspension windings BSRM and power converters is shown in Figure 1 [14]. $N_a$, $N_b$, and $N_c$ are turns of the main windings; $N_x$ and $N_y$ are turns of the suspension windings. The main windings are the same as that of traditional BSRMs. The adjacent three stator teeth of A, B and C phases share a suspension winding. The suspension winding changes the magnetic field distribution of air gaps and generates the radial force. When the rotor eccentricity occurs, the position of the rotor is rectified by adjusting the suspension winding current $i_x$ and $i_y$ to accord with the reference position. Using feedback signals from controllers, the rotor can be stably suspended by adjusting the magnitude and direction of suspension winding current in case of eccentricity.

The BSRMs with sharing suspension windings mainly have two advantages: First, only two sets of suspension windings are used to control radial displacement regardless of the number of motor phases, which effectively simplify and lower the failure rate of the control circuit. Second, the BSRMs with sharing suspension windings realise the self-decoupling and reduce the difficulty of the control strategy.

3 | TORQUE ANALYTICAL MODEL

3.1 | Derivation of torque analytical model

The Maxwell stress tensor method is a cost-efficient means to obtain the radial force of the motor, which uses an arbitrary curved surface surrounding the rotor teeth to calculate the stress tensor and obtain the torque $T$. The resultant force and moment in the magnetic mass of a given volume $V$ are equivalent to the resultant force of all tensions on the surface $\delta$ surrounding the volume $V$. The normal force $F_n$ and tangential force $F_t$ are as follows [15]:

$$F_n = \frac{1}{2\mu_0} \int_\delta (B_n^2 - B_t^2) dA,$$  (1)

$$F_t = \frac{1}{\mu_0} \int_\delta B_n B_t dA.$$  (2)
where $\mu_0$ is the vacuum permeability, $B_n$ is the normal magnetic density, $B_t$ is the tangential magnetic density, and $A$ is the area of surface integration.

In order to simplify Equations (1) and (2), the area component can be converted into a line integral. The selected integral path should be perpendicular or parallel to the direction of the magnetic field so that the $F_t$ component is 0. As shown in Figure 2, the radial and tangential force on the cross-section of the rotor are denoted by $F_{nr}$ and $F_{nt}$, respectively, $g_0$ is the width of air gap, and $\theta$ is the rotor angle.

When the magnetic field is perpendicular to the integral path, from Equation (1), the normal force is

$$ F_{nr} = \frac{1}{2\mu_0} \int S B_n^2 dA, $$

when the magnetic field is parallel to the integral path, from Equation (1), the tangential force is

$$ F_{nt} = -\frac{1}{2\mu_0} \int S B_t^2 dA. $$

The Ampere's law is used to find the main magnetic field intensity $H_m$ and the fringing magnetic field intensity $H_f$ at the air gap. An elliptical path is used to obtain the fringing magnetic field intensity [2] as suggested in Figure 3. When the stator and rotor poles are aligned, the rotor angle $\theta$ is 0. $r$ is the rotor radius, $g_0 + a_1 r |\theta|$ is the length of the semi-major axis, and $a_1$ is defined as

$$ a_1 = \frac{r |\theta|}{g_0 + r |\theta|}. $$

The average length of the ellipse path $l_f$ can be expressed as

$$ l_f = \frac{\pi}{4} (g_0 + a_1 r |\theta|). $$

Referring to Figure 3, the tangential force $F_{nt}$ and torque $T$ can be obtained by integrating the air gap flux density along paths 1 to 2 and 3 to 4:

$$ F_{nt} = \frac{b}{2\mu_0} \left( \int_3^4 B_m^2 dl - \int_1^2 B_f^2 dl \right), $$

$$ T = F_{nt} r = \frac{br}{2\mu_0} \left( \int_3^4 B_m^2 dl - \int_1^2 B_f^2 dl \right), $$

where $b$ is the length of the laminations of the rotor.

In unsaturated conditions, the magnetic field intensity in the ferromagnetic material is subtle and ignored. As shown in Figure 1, taking the air gap 4 as an example, when the main windings of $B$ phase are energised and current is applied to the $Y$-direction suspension windings

$$ H_m g_0 = N_{mab} + N_{m1}, $$

$$ H_f l_f = N_{fab} + N_{f1}. $$

Positive torque will be generated at 1, 2, 3, and 4, and negative torque will be generated at 6 and 8, respectively.

$$ T = (F_{f4} + F_{f3} + F_{f2} + F_{f1} - F_{f6} - F_{f8}) r $$

$$ = \frac{br \mu_0}{2} \left[ \left( H_{m4}^2 + H_{m3}^2 + H_{m2}^2 + H_{m1}^2 \right) g_0 - \left( H_{f4}^2 + H_{f3}^2 + H_{f2}^2 + H_{f1}^2 \right) l_f \right] $$

$$ = -\frac{br \mu_0}{2} \left[ \left( H_{m6}^2 + H_{m8}^2 \right) g_0 - \left( H_{f6}^2 + H_{f8}^2 \right) l_f \right].$$
Assuming a constant $i_f$, an analytical model neglecting magnetic saturation can be derived from Equations (9) to (11).

$$T = 2\mu_0 br (N_{l_f,mb})^2 \left( \frac{1}{g_r} - \frac{1}{l_r} \right).$$  
(12)

In saturated conditions, the magnetic field intensity in the ferromagnetic material $H_f$ is considered as shown in Figure 3. The relationship between the magnetic field intensity and the magnetomotive force is

$$H_{ma(k)} + H_{ag(d-g)} = N_{i_{wb}} + N_{j_f},$$  
(13)

$$H_{j_f} + H_{ag(d-l_f)} = N_{i_{wb}} + N_{j_f},$$  
(14)

where $i_{wb}$ is the main winding current of $B$ phase, $d$ is the equivalent length of ferromagnetic material flux path including the length of rotor pole $d_r$, the length of stator pole $d_s$, and compensation coefficient $\delta$ [16],

$$d = d_r + d_s + \delta,$$  
(15)

and from the magnetic flux continuity law:

$$H_{ma} = \mu_r H_f,$$  
(16)

where $\mu_r$ is the relative magnetic permeability of the steel sheet ferromagnetic material.

A linear equation is used to express the relationship between $1/\mu_0 \mu_r$ and $H_f$ in saturated conditions of the iron core:

$$\frac{1}{\mu_0 \mu_r} = k_1 H_f + k_2,$$  
(17)

where $k_1$ and $k_2$ are fitting coefficients. This is cost-efficient, compared with the polynomial fitting method, and improves the calculation speed of the processor.

From Equations (13) to (17), the main magnetic field intensity $H_{ma}$ is

$$H_{ma} = \frac{(M + U_k \mu_0) + \sqrt{M^2 + U_k^2(\mu_0)^2} + 2(M - 2g)U_k \mu_0}{2g k_1 \mu_0},$$  
(18)

where $U_k$ is the magnetomotive force, $M$ is $g_r k_1 \mu_0 (d - g)$, and $F$ is $l_r + k_2 \mu_0 (d - l_r)$. The fringing magnetic field intensity $H_{j_f}$ at the air gap is

$$H_{j_f} = \frac{(F + U_k \mu_0) + \sqrt{F^2 + U_k^2(\mu_0)^2} + 2(F - 2l_r)U_k \mu_0}{2l_r k_1 \mu_0}.$$  
(19)

The intensity of the magnetic field at other air gaps can be obtained using the same approach.

Due to the non-linear features of ferromagnetic material, the influence of magnetic saturation on the torque of the motor is taken into account and $i_f$ of each air gap is assumed to be equal; using Equations (11), (18) and (19) the torque is given by

$$T = 2\mu_0 br \left[ M^2 + 2N_{l_f,mb}k_1 \mu_0 (2M - g) + (k_1 \mu_0)^2 N_{l_f,mb}^2 \right]$$

$$- \frac{br}{l_r k_1 \mu_0} \left[ F^2 + 2N_{l_f,mb}k_1 \mu_0 (2F - l_r) + (k_1 \mu_0)^2 N_{l_f,mb}^2 \right].$$  
(20)

Equations (12) and (20) are the torque analytical models suitable for unsaturated and saturated conditions, respectively.

### 3.2 Analysis of self-decoupling characteristics

From Equations (17) and (20), the proposed analytical models have a low correlation with the variables $i_r$ and $i_f$. Referring to Figure 1, when the main windings of $B$ phase and $Y$-direction suspension windings are energised, the negative torque generated at pole 6 and 8 neutralises the change of positive torque at pole 1, 2, 3 and 4 determined by the suspension winding current. During operation, some stator poles produce positive torque, while other stator poles produce negative torque under the excitation of suspension windings. In other words, the torque is almost unaffected by the suspension winding current.

### 3.3 Verification of torque analytical model

A finite element (FE) model is built for the prototype motor using the parameters provided in Table 1. The magnetic density observation point is set at the stator pole to explore the magnetic circuit saturation characteristics of the motor.

The saturation characteristics of the motor are analysed using the Finite Element Method (FEM). The linear increasing trend and actual increasing trend of the relationship between the magnetic flux density and the magnetomotive force are given in Figure 4, which are used to determine the areas of saturation.

When the total magnetomotive force is approximately 348 ampere-turns, the corresponding air gap magnetic field intensity is $1.392 \times 10^6 \text{A/m}$. Meanwhile, the magnetic circuit begins to saturate and ceases a linear increasing trend. Equations (20) and (17) are used before and after the saturation of the magnetic circuit, respectively. In order to verify the accuracy of the analytical model before the saturation of the magnetic circuit, the comparison results of FEA and calculation results of Equation (20) are given. When the centre of the stator and rotor teeth are aligned, the $Y$-direction suspension windings and main windings of $B$ phase are energised to analyse the effects of the main winding current, suspension winding current and the position angle of the rotor. Figure 5 shows the comparison results between the analytical model and the FE model in the case of a constant main winding current. It shows that the torque
TABLE 1 Parameters of bearingless switched reluctance motor with sharing suspension windings

| Parameters                      | Values  |
|--------------------------------|---------|
| Stator/rotor pole number       | 12/8    |
| Main winding turn number       | 72      |
| Suspension winding turn number | 60      |
| Stator pole arc width (mm)     | 12      |
| Rotor pole arc width (mm)      | 14      |
| Stator outer diameter (mm)     | 155     |
| Stator inner diameter (mm)     | 90      |
| Stator yoke thickness (mm)     | 10      |
| Rotor outer diameter (mm)      | 89.5    |
| Rotor inner diameter (mm)      | 61      |
| Rotor yoke thickness (mm)      | 12      |
| Air gap length (mm)            | 0.25    |
| Rated power (kW)               | 1       |
| Rated current (A)              | 5       |
| Rated speed (rpm)              | 1500    |
| Rotor laminations length (mm)  | 132     |
| Stator and rotor core materials| DW540-50|

Figure 4 shows the relationship between flux density and the magnetomotive force.

The relationship between flux density and the magnetomotive force:

![Image](image1)

Analytical model calculation results coincide with FEA results under different excitation conditions of suspension windings.

During the actual operation, inevitable local saturation of the BSRMs leads to the non-linear increment of torque; thus, the simplified linear model is not suitable for describing the torque characteristics when the total magnetomotive force is more than 348 ampere-turns; Equation (17) is used to describe the torque characteristics when the magnetic circuit is saturated.

![Image](image2)

During the actual operation, inevitable local saturation of the BSRMs leads to the non-linear increment of torque; thus, the simplified linear model is not suitable for describing the torque characteristics when the total magnetomotive force is more than 348 ampere-turns; Equation (17) is used to describe the torque characteristics when the magnetic circuit is saturated.

Figure 5 Torque at different suspension winding current (\(i_{sb} = 1.833 \, A\))

![Image](image3)

The results of the proposed analytical model fit with the FEA results of the saturated and unsaturated conditions with different main winding currents applied to the motor.

The results of the proposed analytical model fit with the FEA results of the saturated and unsaturated conditions, which show that approximate processing methods for obtaining Equations (17) and (20) are reasonable. From Figure 6, when the magnetic circuit is saturated, the torque analytical model results match FEA results, which illustrates that the model is feasible. However, from Figures 5 and 6, in case the overlap area of the stator and rotor is large, that is, when \(\theta\) is from 0 to \(2^\circ\), the error between the analytical model and FEA is significant. Herein, the magnetic field intensity at the root of the stator teeth is significantly greater than the stator teeth, which may further lead to

![Image](image4)

Figure 6 Torque at different main winding current (\(i_y = 1.5 \, A\))

![Image](image5)
local saturation. The proposed model is based on a uniform distribution of the magnetic circuit, which leads to an increase in the calculated value of the magnetic field strength at the air gap. As consequences, the torque obtained from the proposed model is larger than the FEA results.

4 | VERIFICATIONS OF SELF-DECOUPLING CHARACTERISTICS

4.1 | FEA result analysis

FEM is used to analyse the self-decoupling characteristics. The BSRM with sharing suspension windings and the conventional BSRM are modelled using parameters provided in Table 1. The FEA results are provided in Figures 7(a) and (b), respectively.

The FEA results of BSRMs applied with stable main winding current under the suspension winding current from 0 to 4 A are provided in Figure 7. It can be clearly seen in Figure 7 that the torque of conventional BSRM is easily affected by the change of suspension winding current, which is unfavourable for stable torque control. The torque of BSRM with sharing suspension windings is hardly affected by suspension winding current even if it is adjusted into a large range. This design realises the self-decoupling of radial force control and torque control without using the decoupling algorithm, and thus avoids introducing a complex coupling relationship.

4.2 | Experimental verification

The experiment platform of prototype BSRM with sharing suspension windings is provided in Figure 8, which includes AC power supply, dSPACE controller, power conversion circuit, current sampling circuit and the prototype motor. The parameters of the prototype motor are given in Table 1, and the DC link voltage of the power converter is 260 V.

In order to verify the self-decoupling characteristics of the motor, the influence of the suspension winding current and the main winding current on the torque is considered. Under no-load conditions, the rotor mechanical motion equation is

\[
J \frac{d\omega}{dt} = \sum T - F \omega, \tag{21}
\]

where \( J \) is the moment of inertia, \( \omega \) is the angular velocity, \( F \) is the damping coefficient. The difference between the sum of torque and resistance is proportional to angular acceleration, and if there is no sudden change in torque, it will not cause a change in angular velocity. As the rotation speed \( n \) is proportional to the angular velocity \( \omega \), it can be used to infer the torque variation.

The influence of the main winding current \( i_{mb} \) and the suspension winding current \( i_y \) on rotation speed is obtained using experiments as shown in Figure 9.
The results in Figure 9(a) show that although there is a sudden change of the Y-direction suspension winding current $i_y$ (0.518 A/div), the speed is fairly constant. The experiment shows that a change of the main winding current (0.11 A/div) brings significant changes to the speed of the motor (20 rpm/div). These suggest the speed is hardly affected by the Y-direction suspension winding current but mainly determined by the main winding current. Figure 9(b) shows that the fluctuation of the suspension winding current will not change the speed under stable operation. This confirms the torque is hardly affected by the suspension winding current. The X-direction suspension windings are also analysed using this approach and obtained close results. It can be seen that the torque control and radial force control are decoupled in this BSRM with sharing suspension windings.

5 | CONCLUSION

BSRMs generally work in the state of magnetic saturated conditions, which increase the difficulty of establishing the torque analytical model for the stable control. In this paper, the characteristics of magnetic saturation are described by a simple linear equation, which effectively simplified the torque analytical model. The establishment of the torque analytical model considering magnetic saturation only used the structure parameters of the motor and the characteristics of the ferromagnetic material without the support of the FEA results. The comparison between the analytical model and the FE model shows that the modelling effect is good except in cases of small rotor angles. This issue can be solved using the design of control strategies, which is a future work of this paper. This analytical model can be applied to control systems for real-time torque control without using mass storage, which is cost-friendly, compared with the commonly used 2-D look-up table method. The self-decoupling characteristics of BSRM with sharing suspension windings are analysed using the proposed model, which are also illustrated by FEA and experimental results. These indicate the torque control can be decoupled from the radial force control without using a comprehensive decoupling scheme. This model can be used to design a control scheme for other BSRMs with sharing suspension windings.

REFERENCES

1. Takemoto, M., et al.: A design and characteristics of switched reluctance type bearingless motors. In: Proceeding of the 4th International Symposium on Magnetic Suspension Technology, Gifu City, Japan, pp. 49–63 (1998)
2. Takemoto, M., et al.: Improved analysis of a bearingless switched reluctance motor. IEEE Trans. Ind. Appl. 37(1), 26–34 (2001)
3. Chiba, A., Rahman, M.A., Fukao, T: Radial force in a bearingless reluctance motor. IEEE Trans. Magn. 27(2), 786–790 (1991)
4. Yuan, Y., Sun, Y., Huang, Y.: Radial force dynamic current compensation method of single winding bearingless flywheel motor. IET Power Electron. 8(7), 1224–1229 (2015)
5. Chen, L., Hofmann, W.: Modelling and control of one bearingless 8–6 switched reluctance motor with single layer of winding structure. In: Proceedings of the 2011 14th European Conference on Power Electronics and Applications, Birmingham, UK, pp. 1–9 (2011)
6. Wang, X., Ge, B., Wang, J.: Mathematical modeling of a novel bearingless switched reluctance motor. COMPEL—Int. J. Comput. Math. Electr. Electron. Eng. 33(1/2), 376–397 (2014)
7. Sun, Y., et al.: A new decoupling control strategy for bearingless switched reluctance motors based on improved mathematical model. In: 2011 International Conference on Electrical Machines and Systems, Beijing, China, pp. 1–5 (2011)
8. Ahmed, F., Kalita, K., Nemade, H.B.: Torque and controllable radial force production in a single winding bearingless switched reluctance motor with a speed-controlled drive operation. Int. Trans. Electr. Energ. Syst. 30(5), e12312 (2020)
9. Li, H., et al.: Decoupling control of temperature and pressure based on expected dynamic method. In: 2020 Chinese Control and Decision Conference (CCDC), Hefei, China, pp. 2409–2413 (2020)
10. Zhou, J., et al.: Decoupling mechanism of torque and levitation-force control for 12/4 dual-winding bearingless switched reluctance motor. In: 2016 19th International Conference on Electrical Machines and Systems (ICEMS), Chiba, Japan, pp. 1–6 (2016)
11. Huang, Y., et al.: Design and analysis of a novel bearingless segmented switched reluctance motor. IEEE Access 7, 94342–94349 (2019)
12. Wang, H., et al.: A novel bearingless switched reluctance motor with a biased permanent magnet. IEEE Trans. Ind. Electron. 61(12), 6947–6955 (2014)
13. Xu, Z., et al.: Design and analysis of novel 12/14 hybrid pole type bearingless switched reluctance motor with short flux path. J. Electr. Eng. Technol. 7(5), 705–715 (2012)
14. Wang, X., et al.: A novel bearingless switched reluctance motor. COMPEL—Int. J. Comput. Math. Electr. Electron. Eng. 31(6), 1681–1695 (2011)

15. Garrigan, N.R., et al.: Radial force characteristics of a switched reluctance machine. In: Conference Record of the 1999 IEEE Industry Applications Conference. Thirty-Forth IAS Annual Meeting (Cat. No.99CH36370), Phoenix, AZ, USA, vol. 4, pp. 2250–2258 (1999)

16. Hao, Y., et al.: Torque analytical model of switched reluctance motor considering magnetic saturation. IET Electr. Power Appl. 14(7), 1148–1153 (2020)

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