Synergy between the LHC and the ILC \textsuperscript{a}

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ABSTRACT

In this talk I explain in brief the motivations behind undertaking a study of the LHC-ILC interplay. I will give information about the activities of the LHC-LC study group as well as the study group document. I will illustrate the scope of the document by taking a few examples from the document which has appeared on the archives\textsuperscript{1} since this talk was presented.

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Synergy between the LHC and the ILC

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Introduction

The field of high energy physics is poised at a very interesting juncture at present, with the Large Hadron Collider (LHC) ready to start colliding protons on protons at CERN in the year 2007 and the two detectors CMS and ATLAS getting geared to start studies at the LHC. The particle physics community pins its hopes on the LHC to shed light on the crucial question of the origin of mass of all the fundamental particles. At the same time, there is now a world wide consensus that the next big facility in High Energy Physics should be an International $e^+e^-$ collider which has to be a Linear Collider; the International Linear Collider: the ILC. A look at the history of high energy colliders brings up many examples of the complementary roles played by the hadronic and leptonic colliders in furthering the frontiers of our knowledge through an interplay and feedback between the two types of colliders. There are many examples in the past where a new particle has been discovered at one machine, and its properties have been studied in detail with measurements at the other. Similarly, experimental results obtained at one machine have often given rise to predictions that have led to new searches at the other machine, resulting in ground-breaking discoveries. The current state of play in the field of High Energy Physics, the long time that has to necessarily elapse between the conceptual design of an accelerator and the actual commissioning of experiments and the very high stakes in physics studies at the next generation colliders; both on the economic and physics front, make it imperative that we as a community assess the desired energy, the luminosity and the timing of this planned ILC vis-a-vis the Physics Goals that we all are hopeful to reach at the LHC.

For a critical assessment of the above issues, a very close interaction between the experimental communities involved in the LHC and the ILC studies is absolutely essential. A LHC/LC study group was formed in 2002 in the
ECFA/DESY framework with the aim of achieving this. Given the worldwide nature of the LC study groups\textsuperscript{2} and the International nature of the LHC itself, such a study group also took a worldwide character very soon. At present the LHC/LC study group contains about 116 members which includes theorists, members of the CMS and ATLAS collaborations, members of all the LC study groups\textsuperscript{3} as well as contact persons from the Tevatron. The study group had series of meetings over a period of two years, including a very serious and vigorous activity in the TeV Collider Workshop at Les Houches in 2003. More information on the activities of the Working Group can be obtained from the webpage: www.ippp.dur.ac.uk/\textasciitilde georg/lhclc. The International Linear Collider Steering Committee (ILCSC), also unanimously supported the idea of such a study group.

Since the Physics case for the LHC and the ILC, each with its own virtues, has been clearly established, this working group basically wanted to look in detail how the two can complement each other. The aim was to study how information obtained at both the machines can be used most optimally to get more conclusive and effective answers to the very fundamental questions of the structure of the space and time that the HEP community is asking at present. The aim was not to compare which of the two colliders can do better, but rather how measurements at the ILC might give pointers to new benchmarks for measurements to be performed at the LHC. While the information from the ILC may not affect the triggering it may certainly affect the luminosity/detector upgrades as well provide a yet sharper focus to the LHC studies by eliminating some of the possibilities of extensions beyond the SM and/or narrowing down the allowed parameter space in the context of a specific model. Indeed, the upgrades at the Tevatron have benefited from information obtained from the precision measurements at the LEP.

The study group therefore aimed at identifying issues where the cross-talk between the two can increase the utility of both. For the sake of definiteness it was assumed that the LHC will run for about 20 years and that the ILC will come into operation after the LHC has been running for a few years. This will be possible if the somewhat aggressive time table for the ILC, envisaged by the ILCSC, can be adhered to. The possibilities of the cross-talk were analysed assuming a generic situation that the Tevatron and the LHC will see new physics, but the nature of new physics will not be entirely clear.

At the time of the conference the LHC/ILC study group document was close to its final form and is now available as a hep-ph preprint\textsuperscript{1}, submitted for publication. The document contains work done by about 116 authors, discussed over seven meetings, contains about 470 pages. A large number of examples of complementarity and cross-talk between the two colliders have
been identified and studied. The studies show that indeed there are many scenarios where the LHC experiments can benefit from knowledge obtained from the ILC and vice versa. While no examples were found where the triggering at the LHC could be affected by the input from studies at the ILC, points for further studies were identified which may reveal such examples.

**LHC/ILC interplay**

We begin the discussion of the interplay by reminding ourselves of the different virtues of the two machines. The strongest point about the LHC, which is a $pp$ collider with $\sqrt{s} = 14$ TeV is of course that it is already under construction and has a large mass reach for direct discoveries of new physics. Even though the initial state kinematics of the constituent collisions in the hard interaction is not known, conservation of the transverse momentum $P_t$ allows to make good kinematical measurements. The composite and the strongly interacting nature of the colliding protons implies that at the LHC one will always have underlying events and the QCD backgrounds need to be known accurately.

The ILC, which envisages $e^+e^-$ collisions with a cm energy $\sqrt{s} = 0.5–1$ TeV, certainly will have a lesser reach in energy but has the strong point of doing high precision measurements due to the cleaner environment and precise knowledge of the kinematics and polarisation of the initial state. Backgrounds are of course much less severe, the options of $\gamma\gamma$, $\gamma p$ collisions open up new avenues to study the physics of the EW symmetry breaking and the physics beyond the SM. The high precision of measurements possible at the ILC can make it sensitive to the *indirect effects* of the same particles which the LHC expects to be able to produce *directly*. Thus information from a lower energy ILC can feed back into studies at the LHC. This is indeed the simplest form of synergy. Indeed we have seen the interplay between the top quark mass estimation from the precision EW measurements and direct measurements from the Tevatron. We also see the impact of $m_t$ measurement from the Tevatron on the limits for the SM Higgs boson. Precision measurements from the ILC can thus tell sometimes LHC where to focus the effort. Precision measurements at the LHC, though not impossible, are difficult at the LHC and hence will be possible only after a few years. These can thus benefit (and help us realise the LHC potential completely) from a feedback from the ILC. The capabilities of the ILC are of course clearly not restricted only to precision measurements, but also include making discoveries which at times will be difficult or impossible at the LHC. Qualitative statements made above are obvious and have to be supported by quantitative studies, which are indeed present in the document.

Specifically three different kinds of scenarios for the cross-talks have been
discussed in the document. First is the simple 'linear' addition of the utility of both the machines, where the ILC data can help clear up the underlying structure of the new physics of which the Tevatron and LHC will offer us a glimpse. This scenario does not require, for example, an overlap in the operational period of the two accelerators. Second scenario involves a higher level of synergy, where a combined analysis/interpretation of the LHC and the ILC data can make the total bigger than the sum and help, in particular, reduce the model dependencies in the analysis. This is not unlike the effect of a reanalysis of the older JADE data on the determination of $\alpha_s$. While this may not require a strict overlap of the two machines, given the time it takes to develop the data analysis tools and the huge amount of the LHC data that would need to be archived, least amount of time difference, including a negative one, in the operational lives of the two will help matters. If there is indeed time overlap, data from the ILC could influence the second phase of the LHC by providing input to the upgrade options for the LHC machines and detectors. Again, not wholly unlike the synergy between the LEP and Tevatron, where the upgrade of Tevatron detectors has been affected by the data from LEP. The benefits from both the machines will be maximal in the last case. I will take different examples from the document in the subjects of EW symmetry breaking; establishing and understanding the Higgs mechanism, Supersymmetry (SUSY) including the Supersymmetric Higgses and last but not the least the issues in EW symmetry breaking which are alternates to the Higgs mechanism such as dynamical symmetry breaking or alternatives to SUSY such as Little Higgs etc. It goes without saying that what is presented here is a very small and incomplete sample of the results in the document.

 EW Symmetry breaking: the Higgs mechanism

All of us are quite sure that if the SM Higgs exists, the LHC will be able to observe the SM Higgs and afford measurements of its various properties such as the width, relative couplings etc., to an accuracy of about 10–15% by the end of the high luminosity run. The ILC will of course be capable of profiling the SM Higgs with a great degree of accuracy even in the low energy, moderate luminosity option, except for the $t\bar{t}H$ coupling and a full reconstruction of the Higgs potential. One of the questions addressed in the document in this context was to see if and how this situation can be improved with a LHC/ILC cross-talk.

Top Yukawa coupling measurements

A good measurement of the $Ht\bar{t}$ coupling, $g_{t\bar{t}H}$, is quite essential to be able to confirm the Higgs mechanism as the origin of fermion masses. It will be
While couplings of the Higgs to all the other fermions can be measured to a high precision at a low energy (< 500 GeV) ILC, a precision measurement of $g_{t\bar{t}H}$ will require $\sqrt{s} \simeq 800 – 1000$ GeV. The LHC measurement is model dependent. Combining the ILC precision measurements of the branching ratios of the Higgs into different channels, along with LHC measurements of the $\sigma(pp \rightarrow H + X) \times \text{B.R.}$ for various final states, one can determine the $t\bar{t}H$ coupling in a model independent way. The dashed line in Fig.1 shows the relative error of measurement of $g_{t\bar{t}H}$ that can be achieved with the LHC and a low energy, moderate luminosity ILC. The lower solid curve shows that for an ILC to achieve a similar accuracy on its own will require a much higher luminosity and of course a higher energy which may be possible in the second stage of an ILC. So this is an example where the cross-talk increases utility of both, the LHC and a low energy ILC.

Higgs self coupling measurements

The complementarity of the two machines for these two measurements is truly remarkable. For low Higgs masses $m_H < 140$ GeV, a 500 GeV ILC offers the best chance with a luminosity of 1 ab$^{-1}$ with LHC offering no possibilities at all. On the other hand for a heavier Higgs the situation is reversed. However, even then the reconstruction of the scalar potential will require a luminosity upgrade of the LHC and precision information on the $g_{HWW}, \Gamma_H$ and $g_{t\bar{t}H}$ from...
a low energy ILC will be an important input. The present study \(^6\) explores the possibilities of the LHC/ILC synergy here. This is an example where the LHC/ILC studies have identified further work that needs be done to address issues of systematic uncertainties etc. so as to confirm the conclusions arrived here.

**Strong EW symmetry breaking**

If a higgs boson with a mass less than the upper bounds set by the unitarity arguments is not seen at the LHC, it would imply that the EW symmetry breaking dynamics has to be tested in \(W/Z\) scattering processes. Such studies are also interesting in the context of the new ideas such as the Little Higgs models. Separate studies exist for the LHC and the ILC which estimate how well the \(WW\) scattering probes this dynamics. Due to the different theoretical and experimental approximations it is not possible to combine the results of these analyses numerically. One can make the following qualitative statement. The LHC and the ILC are sensitive to different and/or complementary channels. The LHC clearly has the sensitivity to higher mass resonances, but the ILC has the ability, for example, to separate the \(WW\) final state from the \(ZZ\) final states. In general, both at the LHC and the ILC, the studies find large correlations among different parameters of the model of the strong EW symmetry breaking. Combined analysis of the LHC and the ILC data, would clearly reduce these correlations. A full and efficient use of the LHC data would require detailed information on multi-fermion final states such as the angular distributions etc. from the ILC. Combination with data from a sub TeV ILC will be absolutely essential in disentangling the states that the LHC will be capable of producing. Combined LHC/ILC studies are now in progress \(^7\).

**Supersymmetry**

Supersymmetry is arguably the ‘standard’ ‘BSM: Beyond the Standard Model’ physics. If TeV scale Supersymmetry exists, signal for some Supersymmetric particle is sure to show up at the LHC. The LHC studies have already shifted gear from exploring the ‘discovery’ potential to exploring the SUSY ‘spectroscopy’ \(^4\). The latter, which consists of determining the properties of SUSY particles such as their masses and couplings, has been performed mainly in a model dependent manner. Given the plethora of SUSY models which correspond to different SUSY breaking mechanisms and essentially reflects our ignorance of the SUSY breaking mechanism \(^8\), model independent analyses will be welcome in the context.
Determination of sparticle masses, SUSY parameters: LHC/ILC synergy

At the LHC, the SUSY signal will be caused by the sparticle production in pairs and the decay of these involving long decay chains ending, for the R-parity conserving case, in the lightest Supersymmetric particle, the LSP $\tilde{\chi}^0_1$. Since the LSP is 'lost' the sparticle mass determination at the LHC will be basically done by using the 'edges' from the decay chains, e.g. $\tilde{\chi}^0_2 \rightarrow \tilde{\chi}^0_1 l^+ l^-$. This procedure gives rise to an obvious problem of rather strong correlation between the determined sparticle mass and that of the LSP. Thus the accuracy of the sparticle mass determination will be strongly affected by the precision with which the LSP mass is known. The analyses clearly show that the error in the gluino mass $\Delta m_g \sim \Delta m_{\tilde{\chi}^0_1}$, the exact relation depending upon the experimental analysis such as the jet scale uncertainty. The issue of how the mass measurements at the LHC could be improved by information available from the ILC, was studied using the results of the ATLAS analysis of the completely simulated events for the point SPS1a. This point has a light sparticle spectrum and both the LHC and the ILC have reach for a number of lighter sparticles.

The RMS values of the mass distribution in the case of the LHC alone, and combined with measurements from the ILC. All numbers in GeV.

|          | LHC      | LHC+LC   |
|----------|----------|----------|
| $\Delta m_{\tilde{\chi}^0_1}$ | 4.8      | 0.05 (LC input) |
| $\Delta m_{\tilde{\chi}^0_2}$ | 4.7      | 0.08     |
| $\Delta m_{\tilde{\chi}^0_3}$ | 5.1      | 2.23     |
| $\Delta m_{\tilde{t}_R}$   | 4.8      | 0.05 (LC input) |
| $\Delta m_{\tilde{t}_L}$   | 5.0      | 0.2 (LC input)  |
| $\Delta m_{\tilde{z}}$     | 5-8      | 0.3 (LC input)  |
| $\Delta m_{\tilde{d}}$     | 8.7      | 4.9      |
| $\Delta m_{\tilde{q}_R}$   | 7-12     | 5-11     |
| $\Delta m_{\tilde{b}_1}$   | 7.5      | 5.7      |
| $\Delta m_{\tilde{b}_2}$   | 7.9      | 6.2      |
| $\Delta m_{\tilde{g}}$     | 8.0      | 6.5      |

Figure 2: Mass correlation plots. Dots: LHC alone. Vertical bands: Fixing $m_{\tilde{\chi}^0_1}$ to within $\pm 2\sigma$ with LC input ($\sigma = 0.2\%$).

The plot in Fig.2 shows clearly the above-mentioned correlations. An accurate determination of the $m_{\tilde{\chi}^0_1}$, with the precision indicated by the two vertical lines in Fig. 2 at the ILC, will certainly reduce the error in the determination of, for example, $m_{\tilde{b}_1}$. The numbers in Table show how an accurate input from
the ILC on masses of the sparticles that are accessible to the ILC, will be able to substantially improve the accuracy of the mass determination at the LHC for those whose masses are beyond the reach of the ILC. The jet measurement seems to be the limiting factor for the accuracies possible with a combined analysis of the LHC and the ILC data. This is an example where the study has isolated a feature of LHC analysis which could be improved upon, so as to increase the overall precision of sparticle mass determination at the LHC. The sparticle masses so determined can then be used to determine the pattern of SUSY breaking.\[11\]

The wide error bands in the left panel of Fig. 3 around $M_U$ are based on present data, and the the expected errors on the spectrum of supersymmetric particles from LHC measurements within mSUGRA. The narrow bands demonstrate the improvement expected by future GigaZ analyses and the measurement of the complete spectrum at “LHC+ILC”. Thus LHC/ILC synergy can indeed help sharpen our knowledge of the High Scale physics beyond the SM. As a matter of fact an analysis\[12\] for the SPS1a point shows that no convergence in the global fits to the MSSM parameters is possible without including the ILC/LHC results.

One very interesting demonstration of the feedback from ILC into LHC studies and vice versa was seen in a study of heavier neutralino/chargino sector and the SUSY parameter determination\[13\]. They look at a point where the $\tilde{\chi}_4^0$ can be produced only at the LHC. The heavier states are notoriously difficult to study at the LHC. The measurements of the $\tilde{\chi}_1^\pm, \tilde{\chi}_2^0, i = 1, 2$ at the ILC can

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**Figure 3:** Left panel shows area around the unification point $M_U$ defined by the meeting point of $\alpha_1$ with $\alpha_2$. Right hand panel shows the experimental accuracies for the branching ratios $\text{BR}(h \to b\bar{b})$ and $\text{BR}(h \to WW^*)$ at the LC, indicated by a vertical and horizontal band, respectively in comparison with the theoretical prediction in the MSSM.
determine the SUSY parameters in a model independent way and the $m_{\tilde{\chi}^0_4}$ is then predicted. The ILC can thus tell the LHC where to look. Armed with this knowledge, the LHC analysis to search for $\tilde{\chi}^0_4$ can be tuned better. The study in the context of ATLAS detector shows that the error in $m_{\tilde{\chi}^0_4}$ determination at the LHC can go down from 5 GeV to 2.5 GeV, for a $m_{\tilde{\chi}^0_4} = 378.3$ GeV if such information is available from the ILC to the LHC analysis. Further the value of $m_{\tilde{\chi}^0_4}$ determined at the LHC can then be fed back into the ILC analysis thus increasing the accuracy of the SUSY parameter determination there. This is need the case of information from both the colliders feeding back into the study at the other. In another study a strategy for using LHC/ILC together for a determination of the mixing parameters for the third generation of squarks at the LHC has been outlined.

**SUSY-Higgs at the LHC/ILC**

In turn the last mentioned information is very significant for a precision prediction of the mass of the 'light' higgs $h$, in SUSY and its branching ratios. A study shows how the combined knowledge can improve the significance of testing the consistency of the MSSM from accurate measurements of the B.R. of $h$. This is illustrated in the right panel of Fig. 3. The medium shaded (light blue) region indicates the range of predictions in the MSSM being compatible with the assumed experimental information from the LHC and the ILC.

This is a case where the accurate $m_t$ determination and the precision measurements of the B.R. of the $h$ from the ILC along with the information available from the strongly interacting sparticle sector (out of the reach of ILC) together can give information on the SUSY parameters such as the trilinear coupling $A_t$. This in turn can give pointers to the SUSY phenomenology at the LHC. This again is an excellent example of the synergy.

In case of non-universal gaugino masses, SUSY can also make the lightest higgs $h$ 'invisible' due to decays into neutralinos. In this case the $\tilde{\chi}^0_1$ necessarily has a substantial higgsino component, affecting the sparticle search at the LHC. Detection of such a $h$ at the LHC is difficult, if not impossible. As shown in the left panel of Fig. 4 cosmology constraints disfavour a large region in the parameter space where this can happen. Even then, substantial portions of the parameter space where this may happen are still allowed. Needless to say that an ILC can detect a $h$ with 'invisible' decay products quite easily. If the 'invisibility' is due to SUSY, one expects enhanced production of $h$ in $\tilde{\chi}^0_1$ decays, as shown in the right panel of Fig. 4. Thus this is a case where ILC input will play a useful role in pinning down the SUSY scenario.
Contact interactions, new gauge theories:

Due to the developments in the Little Higgs Models there has been new impetus to look at theories with an extended gauge sector. Equally important are the new ideas of the Extra Dimensional models which predict Kaluza Klein (KK) excitation of the gauge bosons. A result of the investigation of the possibility of discriminating between these different models which predict extra gauge bosons and the role of the LHC/ILC synergy in this is present in the report\textsuperscript{17}. Again, it demonstrates ample scope for the LHC/ILC synergy. For example, LHC can see new resonances, at a mass that is not accessible to the ILC whereas the ILC using the LHC pointers can measure couplings through a simple study of multi-fermion final states. One can then use the precision measurements at a GigaZ to distinguish between these different models.

The left panel of the Fig.5 indicate the ability of an ILC to distinguish between the different models by a measurement of the couplings of a new $Z'$ ($m_{Z'} < 4.5 \text{ TeV}$) for which the LHC has a reach, whereas the right panel indicates the ability to do so through the EW precision measurements of $\sin^2 \theta_W, m_W$. The much narrower inner circle indicates the reach of a GigaZ.
Figure 5: Left panel indicates The 95% C.L. contours on leptonic $Z'$ couplings, assuming the mass of the $Z'$ is measured at LHC. SM prediction, LEP/SLD and GigaZ expected precision compared with $Z'$ models. The ellipse in the lower part of the Fig. corresponds to the current experimental accuracy where as the inner one indicate reach at the GigaZ. The left, upper ellipse indicates the SM prediction.

Conclusions

Only a very small sample of the LHC/ILC study group document was presented here. The document contains many more examples, in 1) EW physics, 2) QCD and Top Physics, 3) Studies of the Higgs Potential 4) CP studies in the Higgs sector, 5) Extra dimensional models, 7) Radion-Higgs separation, 8) Little Higgs studies, 9) NonMinimal Supersymmetric Standard Model (NMSSM) etc., where the LHC/ILC synergy has been demonstrated in a quantitative fashion. New points for further studies have been also been identified. In some cases, fully simulated LHC and ILC events have been used for analysis. Many are still studies by phenomenologists which need experimental simulations. This document is but just a beginning having scratched only the surface. Certainly the more and new studies need to be performed. However, the LHC/ILC study document has indeed shown that it is hard to believe that after the ILC turns on, no new questions will be asked of the LHC. All this points towards the need of having some overlap in the running life of the two machines the LHC and the ILC.

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References

1. LHC/LC Study Group et al., arXiv:hep-ph/0410364.
2. See for example, http://www.interactions.org/linearcollider,
   http://hepwww.physics.yale.edu/lc/.
3. J. A. Aguilar-Saavedra et al. [ECFA/DESY LC Physics Working Group 
   Collaboration], arXiv:hep-ph/0106315; 
   T. Abe et al. [American Linear Collider Working Group Collaboration], 
   in Proc. of Snowmass 2001, ed. N. Graf, arXiv:hep-ex/0106055; 
   K. Abe et al. [ACFA Linear Collider Working Group Coll.], arXiv:hep- 
   ph/0109166; see: lcdnb.kek.jp/RMdraft/.
4. J. G. Branson, D. Denegri, I. Hinchliffe, F. Gianotti, F. E. Paige and 
   P. Sphicas [ATLAS and CMS Collaborations], Eur. Phys. J. directC 4 
   (2002) N1; 
   ATLAS Coll., Technical Design Report, CERN/LHCC/99-15 (1999); 
   CMS Coll., Technical Proposal, CERN/LHCC/94-38 (1994).
5. K. Desch, M. Schumacher, S. Dawson, A. Juste, L. Reina and D. 
   Wackeroth, in Ref. 1.
6. U. Bauer, T. Plehn and D. Rainwater, in Ref. 1.
7. T. Barklow, S. Boogert, G. Cerminara, W. Kilian, A. Krokhotine, 
   K. Mönig, A.F. Osorio, in Ref. 1.
8. See for example, Theory and Phenomenology of Sparticles, M. Drees, R. 
   M. Godbole and P. Roy, World Scientific, January 2005.
9. M. Chiorboli, A. De Roeck, B.K. Gjelsten, K. Kawagoe, E. Lytken, 
   D. Miller, P. Osland, G. Polesello and A. Tricomi, in Ref. 1.
10. For example, see, H.-U. Martyn and G. Weiglein, in Ref. 1.
11. B. Allanach, G.A. Blair, S. Kraml, H.-U. Martyn, G. Polesello, W. Porod, 
    P.M. Zerwas, in Ref. 1.
12. P. Bechtle, K. Desch and P. Wiene mann, in Ref. 1
13. K. Desch, J. Kalinowski, G. Moortgat-Pick, M.M. Nojiri and G. Polesello, 
    in Ref. 1.
14. J. Hisano, K. Kawagoe and M.M. Nojiri, in Ref. 1.
15. K. Desch, E. Gross, S. Heinemeyer, G. Weiglein and L. Živković, in Ref. 
    1.
16. F. Boudjema, G. Bélanger, R.M. Godbole, in Ref. 1.
17. D. Bourilkov, M. Dittmar, A. Djouadi, S. Godfrey, A. Nicollerat, 
    F. Richard, S. Riemann, T. Rizzo, in Ref. 1.