Probing a Four Flavour vis-a-vis Three Flavour Neutrino Mixing for UHE Neutrino Signals at a 1 Km$^2$ Detector

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Abstract

We consider a four flavour scenario for the neutrinos where an extra sterile neutrino is introduced with the three families of active neutrinos and study the deviation from three flavour scenario in the ultra high energy (UHE) regime. We calculate the possible muon and shower yields at a 1 Km$^2$ detector such as ICECUBE for these neutrinos from distant UHE sources namely Gamma Ray Bursts (GRBs) etc. Similar estimations for muon and shower yields are also obtained for three flavour case. Comparing the two results we find considerable differences of the yields for these two cases. This can be useful for probing the existence of a fourth sterile component using UHE neutrino flux.
1 Introduction

This is now established in different oscillations and other experiments that neutrinos occur in three active flavours. But the existence of a fourth sterile neutrino has been proposed and pursued since long as also in recent times. The neutrino oscillation data from experiments like liquid scintillator neutrino detector or LSND [1,2,3] could not be satisfactorily explained by three neutrino oscillation framework. There are observed excess in LSND data that is consistent with $\bar{\nu}_\mu - \bar{\nu}_e$ oscillation with $0.2 \leq \Delta m^2 \leq 10 \text{ eV}^2$. But this mass square difference is not consistent with $\Delta m^2_{21}$ or $\Delta m^2_{32}$ obtained from solar or atmospheric neutrino experiments. This is also substantiated from the analysis of excess observed by miniBoone experiment for both $\bar{\nu}_\mu - \bar{\nu}_e$ and $\nu_\mu - \nu_e$ oscillations [4,5]. These results suggest the existence of an additional fourth neutrino with mass square splitting $\Delta m^2_{41} >> \Delta m^2_{32}$. This fourth neutrino, if exists will not have other Standard Model couplings as indicated by the LEP experiment of $Z$ boson decay width. Hence this additional neutrino if exists, is referred to as sterile neutrino. In addition there are reactor neutrino anomalies reported by experiments where lower rates are found for $\bar{\nu}_e$ from nuclear reactors at a distance which is too short for any effective neutrino oscillation among standard neutrinos [6,7,8]. Lower rate has also been observed at 3$\sigma$ for $\nu_e$’s from $^{51}$Cr and $^{37}$Ar sources in solar neutrino experiments with gallium [9,10,11,12,13].

Several current experiments are analyzing their data including a fourth sterile neutrino and give bounds on different oscillation parameters. The MINOS experiment [14] measures $\nu_\mu$ oscillations using charged current (CC) and neutral current (NC) interactions in a long baseline experiment with a far and near detector that has a long baseline separation of 734 km. The MINOS and its upgraded MINOS+ experiment, from the analysis of their data have recently put constraints on sterile neutrino oscillation parameters ($\sin^2 \theta_{24} - \Delta^2_{41}$) [15,16]. NOvA experiment on the other hand is another long baseline neutrino experiment that look for $\nu_\mu - \nu_s$ oscillation (with $\nu_\mu$ beam from NuMI at Fermilab) through NC interaction in a long baseline experiment with a baseline distance from near and far detector of 810 km. NOvA experiment search for the oscillation in disappearance channel of active neutrino flux in the near and far detector.

With new data from reactor and other short and long baseline neutrino experiments such as MINOS [14]-[25], Daya Bay [25]-[32], Bugey [33] etc. and their analyses considering the active-sterile neutrino oscillation give new bounds on active-sterile mixing angles and $\Delta m^2$. There are other future long baseline experiments such as DUNE (Deep Underground neutrino experiment) [34,35,36,37], T2HK [38,39,40] etc. that may throw more light on neutrino oscillation physics and the active-sterile neutrino oscillation search will be enriched. For example, for DUNE which is a long baseline experiment with the baseline length of about 1300 km between Fermilab, the neutrino source and the detector at Sanford
Underground Research Facility or SURF at South Dakota, the neutral current data would be useful in case active neutrinos oscillate to sterile neutrinos [41].

In this work, we adopt four (3+1) neutrino scheme where we have three active neutrinos and one sterile neutrino and a four flavour oscillation scenario instead of the usual three active neutrino case. We also separately consider the three active neutrino scenario and the three flavour oscillations. Our purpose is to explore the possibility of an experimental signature that would or would not indicate the existence of a sterile neutrino. To this end we consider ultra high energy (UHE) neutrinos from distant extragalactic sources and their detection possibilities in a large terrestrial neutrino telescope such as ICECUBE [43]. High energy events such as Gamma Ray Bursts or GRBs can produce such neutrinos through their particle acceleration mechanism. GRBs are thought to occur by the bouncing off of infalling accreted matter on a failed star that has possibly turned into a black hole. In the process, a powerful shock wave progresses outwards with energies as high as \( \sim 10^{53} \text{ ergs} \) or more in the form of a “fireball”. The protons inside such a fireball, being accelerated thus, interacts with \( \gamma \) by the process of cosmic beam dump while the pions are produced which in turn decays to ultra high energy neutrinos.

The UHE neutrinos therefore will ideally be produced from the decay of pions by GRB process in a ratio \( \nu_e : \nu_\mu : \nu_\tau = 1 : 2 : 0 \). These neutrinos will suffer flavour oscillations or suppressions while traversing to a terrestrial detector. Because of the astronomical distances that the GRBs are from the earth, the oscillatory part \( (\sin^2(\Delta m^2[L/AE])) \) of the oscillation probability equation averages out (\( L \) and \( E \) are the baseline length and energy of the neutrinos respectively while \( \Delta m^2 \) denotes the mass square difference of any two neutrino species). Thus one is left with, in the oscillation probability equations, just three oscillation parameters namely the three mixing angles \( \theta_{12}, \theta_{23} \) and \( \theta_{13} \) in case of three active neutrino scenario while for the (3+1) four neutrino scheme considered here, there are three additional mixing angles namely \( \theta_{14}, \theta_{24} \) and \( \theta_{34} \) that account for the mixing of the three active neutrinos with the fourth sterile neutrino. We adopt in this work the experimental best fit values for the three active neutrino mixing angles namely \( \theta_{12}, \theta_{23} \) and \( \theta_{13} \) obtained from the analyses of data from solar neutrinos, atmospheric neutrinos, reactor and accelerator neutrinos etc. But the active-sterile mixing angles are not known with certainty. However, as discussed earlier in this section, bounds or limits on these unknown mixing angles are obtained from the data analyses of various other reactor or accelerator based neutrino experiments. With new long baseline experiments coming up along with more and more data available from the existing experiments, these bounds are expected to be more stringent.

As mentioned earlier we consider here the UHE neutrinos from GRBs and in this work we estimate the possible detection yield at a kilometer square detector such as ICECUBE [43] for the four neutrino (3+1) oscillation scheme considered in this work. Similar estimations
is also made with the usual three active neutrino scheme and their oscillations. We consider in this work two kinds of signals namely the muon track signal and the shower/cascade shower that may be produced by the charged current (CC) and neutral current (NC) interactions of GRB neutrinos during its passage through the earth rock as also in a ICECUBE like detector. The muons are obtained when the UHE $\nu_\mu$ from GRB reaches earth and interacts with the earth’s rock while moving through the earth towards the detector. The CC interactions of $\nu_\mu$ and $\nu_\tau$ yield $\mu$ and $\tau$ respectively ($\nu_\alpha + N \rightarrow \alpha + X$, where $\alpha \equiv \mu$ or $\tau$). The $\mu$s are detected by the track mvents in an ice detector through its Cerenkov light. The $\tau$ can be detected by “double bang” events or “lollipop” events. The first bang of “double bang” event is produced at the site of first CC interaction $\nu_\tau + N \rightarrow \tau + X$ when a $\tau$ track followed by a cascade would be generated and the second bang of hadronic or electromagnetic shower occurs when $\nu_\tau$ is regenerated from the decay of $\tau$ in the fiducial volume of the detector. A lollipop event is one when the first bang could not be detected but the $\tau$ track can be detected or reconstructed along with the second bang. In the case of an inverse lollipop event, the first bang and the neutrino track could be obtained while the second bang evades detection. In this work we do not consider these events related to $\nu_\tau$ CC interaction as these detections are not very efficient and could be significant only in an energy window of $\sim 2$ PeV – 10 PeV. However, in this work, we include in our analysis the muon track signal that can be obtained from $\nu_\tau$ from the process $\nu_\tau \rightarrow \tau \rightarrow \bar{\nu}_\mu \mu \nu_\tau$. The CC interactions of $\nu_e$ produce electromagnetic showers. Shower events are also considered from the neutral current (NC) interaction of neutrinos of all active flavours. The computations for these events are performed for both (3+1) scheme and three active flavour scheme. We then compare our results for these two scenarios.

We also calculate the effective Majorana $m_{ee}$ for the present (3+1) neutrino (three active and one sterile) framework and obtain its variation with the mass of the lightest neutrino. We then compare our results with the known bounds from the neutrino double beta decay experiments. We find that for lower mass of the lightest neutrino, the inverted hierarchy of neutrino masses in (3+1) scenario may barely satisfies these limits.

The paper is organised as follows. In Section 2 we give a brief discussion of the formalism for UHE neutrino fluxes from diffused GRBs as well as that from a single GRB. These flux of neutrino experiences flavour oscillations as it propagates from the GRB sources and reaches the earth. Neutrino fluxes at the earth from those high energy sources (GRBs) are calculated for both the cases with three active neutrinos and their oscillations and three active and one sterile neutrinos ((3+1) scheme) where a four neutrino oscillation scenario is considered. Section 2 is divided into four subsections. Subsection 2.1 furnishes the calculation of both (3+1) flavour and 3 flavour neutrino oscillation probabilities while subsection 2.2 deals with the UHE neutrino fluxes for four and three flavour cases from GRBs, on reaching the Earth. The analytical expressions for the total number of neutrino
induced muons and shower events from diffused GRB sources at 1 Km$^2$ ICECUBE detector are addressed in Subsection 2.3 while the same from a single GRB is discussed in Subsection 2.4. The calculational results are discussed in Section 3 for diffused GRB neutrino fluxes as also for neutrino fluxes from each of the different single GRBs at given red shifts. The neutrinoless double beta decay in (3+1) flavour scenario is given in Section 4. Finally in Section 5 the paper is summarised with concluding remarks.

2 Formalism

2.1 Four and Three Neutrino Oscillations

In general the probability for a neutrino $|\nu_\alpha\rangle$ of flavour $\alpha$ to oscillate to a neutrino $|\nu_\beta\rangle$ of flavour $\beta$ is given by [15] (considering no CP violation in neutrino sector)

$$P_{\nu_\alpha \rightarrow \nu_\beta} = \delta_{\alpha\beta} - 4 \sum_{j>i} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \sin^2 \left( \frac{\pi L}{\lambda_{ij}} \right).$$  \hspace{1cm} (1)

In the above, $i, j$ denote the mass indices, $L$ is the baseline distance and $U_{\alpha i}$ etc. are the elements of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix [16] such that

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle,$$  \hspace{1cm} (2)

$|\nu_i\rangle$ being the $i^{th}$ mass eigenstate. The oscillation length $\lambda_{ij}$ is given by

$$\lambda_{ij} = 2.47 \text{Km} \left( \frac{E}{\text{GeV}} \right) \left( \frac{\text{eV}^2}{\Delta m^2_{ij}} \right),$$  \hspace{1cm} (3)

with $E$ being the neutrino energy and $\Delta m^2_{ij}$ is the mass square difference of $i^{th}$ and $j^{th}$ neutrino mass eigenstates. The baseline $L$ of UHE neutrinos are generally of astronomical distance. With $\Delta m^2_{ij} L/E \gg 1$ for UHE neutrinos from distant GRB or AGN, the oscillatory part in the probability equation is averaged to half. Thus,

$$\left\langle \sin^2 \left( \frac{\pi L}{\lambda_{ij}} \right) \right\rangle = \frac{1}{2}. $$  \hspace{1cm} (4)

The probability equation (Eq. (1)) is then reduced to

$$P_{\nu_\alpha \rightarrow \nu_\beta} = \delta_{\alpha\beta} - 2 \sum_{j>i} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j}$$

$$= \delta_{\alpha\beta} - \sum_i U_{\alpha i} U_{\beta i} \left[ \sum_{j \neq i} U_{\alpha j} U_{\beta j} \right]$$

$$= \sum_j \left| U_{\alpha j} \right|^2 \left| U_{\beta j} \right|^2,$$  \hspace{1cm} (5)
where we use the unitarity condition

$$\sum_i U_{\alpha i} U_{\beta i} = \delta_{\alpha\beta} \ .$$

(6)

For four flavour scenario, where a fourth sterile neutrino $\nu_s$ is considered along with the usual three flavours $\nu_e, \nu_\mu$ and $\nu_\tau$, the neutrino flavour eigenstates and mass eigenstates are related through

$$
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau \\
\nu_s
\end{pmatrix}
= 
\begin{pmatrix}
\tilde{U}_{e1} & \tilde{U}_{e2} & \tilde{U}_{e3} & \tilde{U}_{e4} \\
\tilde{U}_{\mu1} & \tilde{U}_{\mu2} & \tilde{U}_{\mu3} & \tilde{U}_{\mu4} \\
\tilde{U}_{\tau1} & \tilde{U}_{\tau2} & \tilde{U}_{\tau3} & \tilde{U}_{\tau4} \\
\tilde{U}_{s1} & \tilde{U}_{s2} & \tilde{U}_{s3} & \tilde{U}_{s4}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3 \\
\nu_4
\end{pmatrix} \,,
$$

(7)

where $\tilde{U}_{\alpha i}$ etc. ($i$ being the mass index ($i = 1, 2, 3, 4$) and $\alpha$ being the flavour index ($\alpha = e, \mu, \tau, s$)) are the elements of the PMNS mixing matrix for the 4-flavour case, which can be generated by the successive rotations ($R$) (in terms of four mixing angles $\theta_{14}, \theta_{24}, \theta_{34}, \theta_{13}, \theta_{12}, \theta_{23}$) \[7\] as

$$
\tilde{U} = R_{34}(\theta_{34}) R_{24}(\theta_{24}) R_{34}(\theta_{14}) R_{23}(\theta_{23}) R_{13}(\theta_{13}) R_{12}(\theta_{12}) \, ,
$$

(8)

where we consider no CP violation \[4\] in neutrino sector and hence the CP phases are absent. Considering the present 4-flavour scenario to be the minimal extension of 3-flavour case by a sterile neutrino, the matrix $\tilde{U}$ can be written as

$$
\tilde{U}_{(4 \times 4)}
= 
\begin{pmatrix}
c_{14} & 0 & 0 & s_{14} \\
-s_{14}s_{24} & c_{24} & 0 & c_{14}s_{24} \\
-c_{24}s_{14}s_{34} & -s_{24}s_{34} & c_{34} & c_{14}c_{24}s_{34} \\
-c_{24}s_{14}c_{34} & -s_{24}c_{34} & -s_{34} & c_{14}c_{24}c_{34}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3 \\
\nu_4
\end{pmatrix} \times
\begin{pmatrix}
\nu_{u1} & \nu_{u2} & \nu_{u3} & 0 \\
\nu_{\mu1} & \nu_{\mu2} & \nu_{\mu3} & 0 \\
\nu_{\tau1} & \nu_{\tau2} & \nu_{\tau3} & 0 \\
0 & 0 & 0 & 1
\end{pmatrix} \, ,
$$

(9)

$$
\begin{pmatrix}
c_{14} \nu_{u1} & c_{14} \nu_{u2} & c_{14} \nu_{u3} & s_{14} \\
-s_{14}s_{24} \nu_{u1} + c_{24} \nu_{\mu1} & -s_{14}s_{24} \nu_{u2} + c_{24} \nu_{\mu2} & -s_{14}s_{24} \nu_{u3} + c_{24} \nu_{\mu3} & c_{14}s_{24} \\
-c_{24}s_{14}s_{34} \nu_{u1} & -c_{24}s_{14}s_{34} \nu_{u2} & -c_{24}s_{14}s_{34} \nu_{u3} & c_{14}c_{24}s_{34} \\
-s_{24}s_{34} \nu_{\mu1} + c_{34} \nu_{\tau1} & -s_{24}s_{34} \nu_{\mu2} + c_{34} \nu_{\tau2} & -s_{24}s_{34} \nu_{\mu3} + c_{34} \nu_{\tau3} & +c_{34} \nu_{\tau3}
\end{pmatrix} \, ,
$$

(10)

\[4\] Although the evidence of CP violation in lepton sector is yet to be established, an analysis of T2K data sets a best fit value of $\delta = -\pi/2$ but with only 2$\sigma$ C.L. Hence we neglected the CP violation in our work.
where $U_{ei}$ are the matrix elements of 3-flavour neutrino mixing matrix

$$U_{(3 \times 3)} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix}. \tag{11}$$

The matrix $U_{(3 \times 3)}$ can be expressed as the successive rotations

$$U = R_{23}R_{13}R_{12}, \tag{12}$$

where

$$R_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad R_{13} = \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix}, \quad R_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}. \tag{13}$$

Therefore

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}s_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}. \tag{14}$$

Following Eq. \[5\] the oscillation probability $P_{(4 \times 4)}$ for 4-flavour case can now be represented as \[48\]

$$P = \begin{pmatrix} P_{ee} & P_{e\mu} & P_{e\tau} & P_{e\nu} \\ P_{\mu e} & P_{\mu\mu} & P_{\mu\tau} & P_{\mu\nu} \\ P_{\tau e} & P_{\tau\mu} & P_{\tau\tau} & P_{\tau\nu} \\ P_{\nu e} & P_{\nu\mu} & P_{\nu\tau} & P_{\nu\nu} \end{pmatrix} \equiv XX^T, \tag{15}$$

with

$$X = \begin{pmatrix} \vert \tilde{U}_{e1} \vert^2 & \vert \tilde{U}_{e2} \vert^2 & \vert \tilde{U}_{e3} \vert^2 & \vert \tilde{U}_{e4} \vert^2 \\ \vert \tilde{U}_{\mu1} \vert^2 & \vert \tilde{U}_{\mu2} \vert^2 & \vert \tilde{U}_{\mu3} \vert^2 & \vert \tilde{U}_{\mu4} \vert^2 \\ \vert \tilde{U}_{\tau1} \vert^2 & \vert \tilde{U}_{\tau2} \vert^2 & \vert \tilde{U}_{\tau3} \vert^2 & \vert \tilde{U}_{\tau4} \vert^2 \\ \vert \tilde{U}_{s1} \vert^2 & \vert \tilde{U}_{s2} \vert^2 & \vert \tilde{U}_{s3} \vert^2 & \vert \tilde{U}_{s4} \vert^2 \end{pmatrix}. \tag{16}$$

Similarly for 3-flavour scenario the probability $P_{(3 \times 3)}$ takes the form

$$P = AA^T \tag{17}$$

where

$$A = \begin{pmatrix} \vert U_{e1} \vert^2 & \vert U_{e2} \vert^2 & \vert U_{e3} \vert^2 \\ \vert U_{\mu1} \vert^2 & \vert U_{\mu2} \vert^2 & \vert U_{\mu3} \vert^2 \\ \vert U_{\tau1} \vert^2 & \vert U_{\tau2} \vert^2 & \vert U_{\tau3} \vert^2 \end{pmatrix}. \tag{18}$$
2.2 UHE Neutrino Fluxes from GRBs

From the GRBs the neutrino (antineutrino) flavours are expected to produce in the ratio

\[ \nu_e : \nu_\mu : \nu_\tau = 1 : 2 : 0. \]

The isotropic flux \([42, 49]\) for \(\nu_\mu\) and \(\bar{\nu}_\mu\) estimated by summing over all the sources is given as (Gandhi et al. \([50]\))

\[ F(E_\nu) = \frac{dN_{\nu_\mu + \bar{\nu}_\mu}}{dE_\nu} = N \left( \frac{E_\nu}{1\text{GeV}} \right)^{-n} \text{cm}^{-2}\text{s}^{-1}\text{sr}^{-1}\text{GeV}^{-1}. \]  

(19)

In the above,

\[ N = 4.0 \times 10^{-13} \quad n = 1 \text{ for } E_\nu < 10^5 \text{ GeV}, \]
\[ N = 4.0 \times 10^{-8} \quad n = 2 \text{ for } E_\nu > 10^5 \text{ GeV}. \]

Therefore the fluxes of the corresponding flavours (same for both neutrinos and antineutrinos since no CP violation is considered in the neutrino sector) can be expressed as

\[ \frac{dN_{\nu_\mu}}{dE_\nu} = \phi_{\nu_\mu} = \frac{dN_{\bar{\nu}_\mu}}{dE_\nu} = \phi_{\bar{\nu}_\mu} = 0.5F(E_\nu), \]
\[ \frac{dN_{\nu_e}}{dE_\nu} = \phi_{\nu_e} = \frac{dN_{\bar{\nu}_e}}{dE_\nu} = \phi_{\bar{\nu}_e} = 0.25F(E_\nu). \]  

(20)

These neutrinos suffer flavour oscillations as they reach the terrestrial detector due to the astronomical baseline length. Thus in the process the \(\nu_\mu\) can oscillate to \(\nu_\tau\) and/or to other flavours on reaching the earth. The flux of neutrino flavours for four and three flavour cases, on reaching the earth will respectively be

\[ F^4_{\nu_e} = P^4_{\nu_e \rightarrow \nu_e} \phi_{\nu_e} + P^4_{\nu_e \rightarrow \nu_\mu} \phi_{\nu_\mu}, \]
\[ F^4_{\nu_\mu} = P^4_{\nu_\mu \rightarrow \nu_e} \phi_{\nu_e} + P^4_{\nu_\mu \rightarrow \nu_\mu} \phi_{\nu_\mu}, \]
\[ F^4_{\nu_\tau} = P^4_{\nu_\tau \rightarrow \nu_e} \phi_{\nu_e} + P^4_{\nu_\tau \rightarrow \nu_\mu} \phi_{\nu_\mu}, \]
\[ F^4_{\nu_s} = P^4_{\nu_s \rightarrow \nu_e} \phi_{\nu_e} + P^4_{\nu_s \rightarrow \nu_\mu} \phi_{\nu_\mu}, \]  

(21)

and

\[ F^3_{\nu_e} = P^3_{\nu_e \rightarrow \nu_e} \phi_{\nu_e} + P^3_{\nu_e \rightarrow \nu_\mu} \phi_{\nu_\mu}, \]
\[ F^3_{\nu_\mu} = P^3_{\nu_\mu \rightarrow \nu_e} \phi_{\nu_e} + P^3_{\nu_\mu \rightarrow \nu_\mu} \phi_{\nu_\mu}, \]
\[ F^3_{\nu_\tau} = P^3_{\nu_\tau \rightarrow \nu_e} \phi_{\nu_e} + P^3_{\nu_\tau \rightarrow \nu_\mu} \phi_{\nu_\mu}. \]  

(22)

In the above \(F^4_{\nu_\alpha}(F^3_{\nu_\alpha})\) is the flux for the species \(\nu_\alpha\), \(\alpha\) being the flavour index and \(P^4_{\nu_\alpha}(P^3_{\nu_\alpha})\) is the corresponding oscillation probability for 4(3) flavour scenario.
Cosmic neutrino flux (Eq. (21)) in the far distance can be expressed as a product of 
\( P_{(4 \times 4)} (= X X^T) \) and the intrinsic flux \( \phi_{\nu_e} (\alpha = e, \nu, \tau, s) \) in the matrix form

\[
\begin{pmatrix}
F_{\nu_e}^4 \\
F_{\nu_\mu}^4 \\
F_{\nu_\tau}^4 \\
F_{\nu_s}^4
\end{pmatrix}
= X X^T \times \begin{pmatrix}
\phi_{\nu_e} \\
\phi_{\nu_\mu} \\
\phi_{\nu_\tau} \\
\phi_{\nu_s}
\end{pmatrix}.
\tag{23}
\]

Assuming the standard ratio of intrinsic neutrino flux i.e.

\[
\phi_{\nu_e} : \phi_{\nu_\mu} : \phi_{\nu_\tau} : \phi_{\nu_s} = 1 : 2 : 0 : 0.
\]

Now by using the above assumption and Eq. (16), Eq. (23) can be rewritten as

\[
\begin{pmatrix}
F_{\nu_e}^4 \\
F_{\nu_\mu}^4 \\
F_{\nu_\tau}^4 \\
F_{\nu_s}^4
\end{pmatrix}
= \begin{pmatrix}
|\bar{U}_{e1}|^2 & |\bar{U}_{e2}|^2 & |\bar{U}_{e3}|^2 & |\bar{U}_{e4}|^2 \\
|\bar{U}_{\mu1}|^2 & |\bar{U}_{\mu2}|^2 & |\bar{U}_{\mu3}|^2 & |\bar{U}_{\mu4}|^2 \\
|\bar{U}_{\tau1}|^2 & |\bar{U}_{\tau2}|^2 & |\bar{U}_{\tau3}|^2 & |\bar{U}_{\tau4}|^2 \\
|\bar{U}_{s1}|^2 & |\bar{U}_{s2}|^2 & |\bar{U}_{s3}|^2 & |\bar{U}_{s4}|^2
\end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ 2 \\ 0 \end{pmatrix} \phi_{\nu_e}.
\tag{24}
\]

From Eq. (21) it then follows that

\[
F_{\nu_e}^4 = \left| U_{e1} \right|^2 (1 + |U_{\mu1}|^2 - |U_{\tau1}|^2 - |U_{s1}|^2) + \left| U_{e2} \right|^2 (1 + |U_{\mu2}|^2 - |U_{\tau2}|^2 - |U_{s2}|^2) \\
+ \left| U_{e3} \right|^2 (1 + |U_{\mu3}|^2 - |U_{\tau3}|^2 - |U_{s3}|^2) + \left| U_{e4} \right|^2 (1 + |U_{\mu4}|^2 - |U_{\tau4}|^2 - |U_{s4}|^2)) \phi_{\nu_e},
\]

\[
F_{\nu_\mu}^4 = \left| U_{\mu1} \right|^2 (1 + |U_{\mu1}|^2 - |U_{\tau1}|^2 - |U_{s1}|^2) + \left| U_{\mu2} \right|^2 (1 + |U_{\mu2}|^2 - |U_{\tau2}|^2 - |U_{s2}|^2) \\
+ \left| U_{\mu3} \right|^2 (1 + |U_{\mu3}|^2 - |U_{\tau3}|^2 - |U_{s3}|^2) + \left| U_{\mu4} \right|^2 (1 + |U_{\mu4}|^2 - |U_{\tau4}|^2 - |U_{s4}|^2) \phi_{\nu_e},
\]

\[
F_{\nu_\tau}^4 = \left| U_{\tau1} \right|^2 (1 + |U_{\mu1}|^2 - |U_{\tau1}|^2 - |U_{s1}|^2) + \left| U_{\tau2} \right|^2 (1 + |U_{\mu2}|^2 - |U_{\tau2}|^2 - |U_{s2}|^2) \\
+ \left| U_{\tau3} \right|^2 (1 + |U_{\mu3}|^2 - |U_{\tau3}|^2 - |U_{s3}|^2) + \left| U_{\tau4} \right|^2 (1 + |U_{\mu4}|^2 - |U_{\tau4}|^2 - |U_{s4}|^2) \phi_{\nu_e},
\]

\[
F_{\nu_s}^4 = \left| U_{s1} \right|^2 (1 + |U_{\mu1}|^2 - |U_{\tau1}|^2 - |U_{s1}|^2) + \left| U_{s2} \right|^2 (1 + |U_{\mu2}|^2 - |U_{\tau2}|^2 - |U_{s2}|^2) \\
+ \left| U_{s3} \right|^2 (1 + |U_{\mu3}|^2 - |U_{\tau3}|^2 - |U_{s3}|^2) + \left| U_{s4} \right|^2 (1 + |U_{\mu4}|^2 - |U_{\tau4}|^2 - |U_{s4}|^2) \phi_{\nu_e}.
\tag{25}
\]

Similarly for 3-flavour scenario we can write Eq. (22) by using Eq. (17 - 18) as

\[
\begin{pmatrix}
F_{\nu_e}^3 \\
F_{\nu_\mu}^3 \\
F_{\nu_\tau}^3 \\
F_{\nu_s}^3
\end{pmatrix}
= \begin{pmatrix}
|U_{e1}|^2 & |U_{e2}|^2 & |U_{e3}|^2 \\
|U_{\mu1}|^2 & |U_{\mu2}|^2 & |U_{\mu3}|^2 \\
|U_{\tau1}|^2 & |U_{\tau2}|^2 & |U_{\tau3}|^2 \\
|U_{s1}|^2 & |U_{s2}|^2 & |U_{s3}|^2
\end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ 2 \\ 0 \end{pmatrix} \phi_{\nu_e}.
\tag{26}
\]
Finally Eq. (26) can be written as

\[
F_{\nu_e}^3 = |U_{\nu_e 1}|^2 (1 + |U_{\mu 1}|^2 - |U_{\tau 1}|^2) + |U_{\nu_e 2}|^2 (1 + |U_{\mu 2}|^2 - |U_{\tau 2}|^2)
\]
\[
+ |U_{\nu_e 3}|^2 (1 + |U_{\mu 3}|^2 - |U_{\tau 3}|^2) \phi_{\nu_e} ,
\]

\[
F_{\nu_{\mu}}^3 = |U_{\mu 1}|^2 (1 + |U_{\mu 1}|^2 - |U_{\tau 1}|^2) + |U_{\mu 2}|^2 (1 + |U_{\mu 2}|^2 - |U_{\tau 2}|^2)
\]
\[
+ |U_{\mu 3}|^2 (1 + |U_{\mu 3}|^2 - |U_{\tau 3}|^2) \phi_{\nu_{\mu}} ,
\]

\[
F_{\nu_{\tau}}^3 = |U_{\tau 1}|^2 (1 + |U_{\mu 1}|^2 - |U_{\tau 1}|^2) + |U_{\tau 2}|^2 (1 + |U_{\mu 2}|^2 - |U_{\tau 2}|^2)
\]
\[
+ |U_{\tau 3}|^2 (1 + |U_{\mu 3}|^2 - |U_{\tau 3}|^2) \phi_{\nu_{\tau}} .
\]  

(27)

### 2.3 Detection of UHE Neutrinos from Diffused GRB Sources

The most promising way of detection is by looking for upward-going muons produced by \(\nu_{\mu}\) CC interactions. Such upward-going muons cannot be misidentified from muons produced in the atmosphere. The detection of \(\nu_{\mu}\)'s from GRBs can be observed from the tracks of the secondary muons.

The total number of secondary muons can be observed in a detector of unit area is (following \[51\], \[52\], \[53\])

\[
S = \int_{E_{thr}}^{E_{\nu max}} dE_\nu dN_\nu d\sigma_{shadow}(\cos \theta_z)P_{\mu}(E_\nu, E_{\mu min}).
\]  

(28)

The phenomenon of earth shielding can be described by the shadow factor \(P_{\text{shadow}}(E_\nu)\), which is defined to be an effective solid angle divided by \(2\pi\) for upward-going muons. This is a function of the energy-dependent neutrino-nucleon interaction length \(L_{\text{int}}(E_\nu)\) in the earth and the column depth \(z(\theta_z)\) for the incident neutrino zenith angle \(\theta_z\). For the case of isotropic fluxes, the attenuation can be represented by this shadow factor, which is given by

\[
P_{\text{shadow}}(E_\nu) = \frac{1}{2\pi} \int_{-1}^{0} d\cos \theta_z \int d\phi \exp[-z(\theta_z)/L_{\text{int}}(E_\nu)],
\]  

(29)

where interaction length \(L_{\text{int}}(E_\nu)\) is given by

\[
L_{\text{int}} = \frac{1}{\sigma_{\text{int}}(E_\nu)N_A}.
\]  

(30)

In the above expression, \(N_A (= 6.023 \times 10^{23}\text{mol}^{-1} = 6.023 \times 10^{23}\text{cm}^{-1})\) is the Avogadro number and \(\sigma_{\text{int}}(= \sigma_{\text{NC}} + \sigma_{\text{CC}})\) is the total (charged-current plus neutral-current) cross-section. The column depth \(z(\theta_z)\) can be expressed as

\[
z(\theta_z) = \int \rho(r(\theta_z, l))dl.
\]  

(31)
In Eq. (31), \( \rho(r(\theta, \phi), l) \) represents the density of the Earth. To a good approximation, the Earth may be considered as a spherically symmetric ball consisting of a dense inner and outer core and a lower mantle of medium density. In our work we consider a convenient representation of the matter density profile of the Earth, which is given by the Preliminary Earth Model [54]. The neutrino path length entering into the earth is \( l \).

The probability \( P_\mu(E_\nu, E_{\mu(\text{min})}) \) for a muon arriving in the detector with an energy threshold of \( E_{\mu(\text{min})} \) is given by

\[
P_\mu(E_\nu, E_{\mu(\text{min})}) = N_A \sigma^{cc}(E_\nu) \langle R(E_{\mu}; E_{\mu(\text{min})}) \rangle ,
\]

where \( \langle R(E_{\mu}; E_{\mu(\text{min})}) \rangle \) is the average range of a muon in rock.

The energy loss rate of muons with energy \( E_\mu \) due to ionization and catastrophic losses like bremsstrahlung, pair production and hadro production is expressed as [52]

\[
\left\langle \frac{dE_\mu}{dX} \right\rangle = -\alpha - \frac{E_\mu}{\xi}.
\]

The constants \( \alpha \) and \( \xi \) in Eq. (33) describe the energy losses and the catastrophic losses respectively in the rock. These two constants are computed as

\[
\alpha = 2.033 + 0.077 \ln[E_\mu(GeV)] \times 10^3 \text{ GeV cm}^2 \text{ gm}^{-1}, \quad \frac{1}{\xi} = 2.033 + 0.077 \ln[E_\mu(GeV)] \times 10^{-6} \text{ GeV cm}^2 \text{ gm}^{-1},
\]

for \( E_\mu \leq 10^6 \text{ GeV} \) [55] and otherwise [56]

\[
\alpha = 2.033 \times 10^{-3} \text{ GeV cm}^2 \text{ gm}^{-1}, \quad \frac{1}{\xi} = 3.9 \times 10^{-6} \text{ GeV cm}^2 \text{ gm}^{-1}.
\]

The average range for a muon of initial energy \( E_{\mu(\text{initial})} \) and final energy \( E_{\mu(\text{min})} \) is given by

\[
R(E_{\mu(\text{initial})}, E_{\mu(\text{min})}) = \int_{E_{\mu(\text{min})}}^{E_{\mu(\text{initial})}} \frac{dE_\mu}{\langle dE_\mu/dX \rangle} \approx \frac{1}{\xi} \ln \left( \frac{\alpha + \xi E_{\mu(\text{initial})}}{\alpha + \xi E_{\mu(\text{min})}} \right).
\]

As mentioned earlier, we also consider the muon events from the decay of \( \tau (\tau \leftarrow (\nu_\tau + N \rightarrow \tau + X)) \) which is produced via the CC interaction of \( \nu_\tau \) at earth.

The muon events from charge current interactions can be computed by replacing \( \frac{dN_\mu}{dE_\nu} \) in Eq. (28) by \( F^4_{\nu_\mu} \) from Eq. (25) and \( F^3_{\nu_\mu} \) from Eq. (27) for the cases of 4-flavour scenario and 3-flavour scenario respectively. As mentioned earlier, we also consider the muon events from the decay of \( \tau (\tau \leftarrow (\nu_\tau + N \rightarrow \tau + X)) \) which is produced via the CC interaction of \( \nu_\tau \) at earth.

The only possibility of considering this process is that this \( \tau \) decays after a very short path length back to \( \nu_\tau \) plus leptons and the process occurs with the probability of 0.18 [44, 57]. Using Eq. (28 - 36) the number of such muon events can be computed.
We consider the shower events from CC interaction of $\nu + e$ and from the NC interactions of all three active flavours. For the shower case we have considered the whole detector volume $V$ and neglected any specific track events. For the shower case the event rate is given by

$$S_{sh} = V \int_{E_{thr}}^{E_{\nu\max}} \frac{dN_{\nu}}{dE_{\nu}} P_{\text{shadow}}(E_{\nu}) \int dy \frac{1}{\sigma^i} \frac{d\sigma^i}{dy} P_{\text{int}}(E_{\nu}, y). \quad (37)$$

In the above expression, $\sigma^i = \sigma^{\text{CC}}$ for the electromagnetic shower and $\sigma^i = \sigma^{\text{NC}}$ when $\nu_e$ NC interactions are considered. The probability that a shower produced by the neutrino interactions is given by

$$P_{\text{int}} = \rho N_A \sigma^i L, \quad (38)$$

where $\rho$ is the matter density and $L$ is the length of the detector. According to the case of shower events $\frac{dN_{\nu}}{dE_{\nu}}$ in Eq. 37 is replaced by $F_{\nu e}^4$, $F_{\nu \mu}^4$, $F_{\nu \tau}^4$ from Eq. 25 and $F_{\nu e}^3$, $F_{\nu \mu}^3$, $F_{\nu \tau}^3$ from Eq. 27 for the cases of 4-flavour scenario and 3-flavour scenario respectively.

### 2.4 Detection of Neutrinos from a Single GRB

In this subsection we consider muon events from the neutrinos for the case of a single GRB. We follow a similar approach as in section 2.3 (diffuse GRB case) for the purpose. Besides the expression for flux for a single GRB being different from that of the case for diffuse GRBs, the zenith angle $\theta_z$ (used in Eq. 29) is now fixed for a particular GRB. Thus the expression for $P_{\text{shadow}}$ is now modified as

$$P_{\text{shadow}} = \exp\left[-z(\theta_z)/l_{\text{int}}(E_{\nu})\right]. \quad (39)$$

The earth density should also be accordingly computed for a fixed $\theta_z$.

For the case of isotropic emission from the source, the secondary neutrino flux $\frac{dN_{\nu\theta}}{dE_{\nu\theta}}$ (the total number of secondary neutrinos emitted from a single GRB at redshift $z'$ per unit observed neutrino energy $E_{\nu\theta}$ that are incident on the earth) is given by

$$\frac{dN_{\nu\theta}}{dE_{\nu\theta}} = \frac{dN_{\nu}}{dE_{\nu}} \frac{1}{4\pi r^2(z')} (1 + z'), \quad (40)$$

where the comoving radial coordinate distance ($r(z')$) of the source is expressed as

$$r(z') = \frac{c}{H_0} \int_0^{z'} \frac{dz''}{\sqrt{\Omega_\Lambda + \Omega_m (1 + z'')}}. \quad (41)$$

In a spatially flat Universe $\Omega_\Lambda + \Omega_m = 1$, where $\Omega_\Lambda$ is energy component to the critical energy density of the Universe and $\Omega_m$ is the contribution of the matter density to the
energy density of the Universe in units of the critical energy density. The speed of light is denoted as \( c \) and \( H_0 \) is the Hubble constant. The values of the constants adopted in our calculation are \( \Omega_\Lambda = 0.684, \Omega_m = 0.316 \) and \( H_0 = 67.8 \text{ Km sec}^{-1} \text{ Mpc}^{-1} \) \( [58] \).

The neutrino spectrum \( \frac{dN_\nu}{dE_\nu} \) in Eq. (40) is expressed as

\[
\frac{dN_\nu}{dE_\nu} = N \times \min \left( 1, \frac{E_\nu}{E_{\nu}^{br}} \right) \frac{1}{E_\nu^2}.
\] (42)

In the above, \( N \) is normalization constant and \( E_{\nu}^{br} \) is the neutrino spectrum break energy. The latter \( (E_{\nu}^{br}) \) is a function of the Lorentz factor of the GRB \( (\Gamma) \), photon spectral break energy \( (E_{\gamma,\text{MeV}}^{br}) \) and is given by the expression,

\[
E_{\nu}^{br} = 10^6 \frac{\Gamma_{2.5}^2}{E_{\gamma,\text{MeV}}^{br}} \text{GeV},
\] (43)

where, \( \Gamma_{2.5} = \Gamma/10^{2.5} \). The normalization constant \( N \) can be written as

\[
N = \frac{E_{\text{GRB}}}{1 + \ln(E_{\nu,\text{max}}/E_{\nu,\text{min}}^{br})}.
\] (44)

In the above \( E_{\nu,\text{max}}, E_{\nu,\text{min}} \) respectively represent lower and upper cutoff energy of the neutrino spectrum. At the time of neutrino emission from a single GRB the total amount of energy released is \( E_{\text{GRB}} \), which is 10% of the total fireball proton energy.

With the neutrino flux from a single GRB computed using Eq. (42 - 44), the same methodology as in the diffuse case is now followed to obtain the muon and shower yield at square kilometer detector such as ICECUBE.

3 Calculations and Results

In this section the calculations and results for the neutrino induced muons and the shower events as estimated for a Km\(^2\) detector are described. The UHE neutrinos considered here are a) from diffused neutrino flux and b) from a single GRB.

3.1 Diffused neutrino flux

The possible secondary muon and shower yields at a 1 Km\(^2\) detector such as ICECUBE for the cases of (3+1) flavour as well as 3 flavour UHE neutrinos from distant GRB sources are calculated by using Eqs. (19 - 27) and Eqs. (28 - 38). We can also calculate the same for the cases of both 4 flavour and 3 flavour UHE neutrinos from single GRB sources by solving Eqs. (19 - 27) and Eqs. (39 - 44). The density profile of the earth following the Preliminary Earth Reference Model from \( [54] \) and \( \nu N \) interaction cross-sections including charged-current, neutral-current and their sum from \( [50] \) have been used to calculate the
secondary fluxes. For all the calculations in this work the detector threshold energy $E_{th}$ is taken to be $E_{th} = 1$ TeV. In the present calculations we assume $E_{\text{max}} = 10^{11}$ GeV.

For the purpose of our analysis, we have considered a ratio $R$ between the muon and the shower events, which is defined as

$$R = \frac{T_\mu}{T_{\text{sh}}} \quad \text{(45)}$$

where

$$T_\mu = S(\nu_\mu) + S(\nu_\tau)$$
$$T_{\text{sh}} = S_{\text{sh}}(\nu_e \text{ CC interaction}) + S_{\text{sh}}(\nu_e \text{ NC interaction}) + S_{\text{sh}}(\nu_\mu \text{ NC interaction}) + S_{\text{sh}}(\nu_\tau \text{ NC interaction})$$

and the quantities $S$ and $S_{\text{sh}}$ are defined in Eq. (28) and Eq. (37) respectively. In 4 flavour and 3 flavour scenario the above mentioned ratio $R$ is denoted as $R_4$ and $R_3$ respectively.

The motivation of our work is to show how the neutrino induced muon and the shower fluxes from distant UHE sources namely diffused GRB are affected in case a sterile neutrino exists in addition to the three active neutrinos. For this purpose we have made a comparison of the ratio $R$ between the (3+1) scenario and 3 active neutrino scenario. The calculations are made for three different sets of value of the sterile mixing angles namely $\theta_{14}, \theta_{24}$ and $\theta_{34}$ while the mixing angles for 3 neutrino mixing are adopted as the current best fit values for them. Needless to mention that the other oscillation parameter $\Delta m^2$ plays no role for this case as the oscillation part is averaged out due to astronomical baseline length. The limits on four flavour mixing angles ($\theta_{14}, \theta_{24}, \theta_{34}$) are chosen following the 4-flavour analysis of different experimental groups such as MINOS, Daya Bay, Bugey, NOvA [15, 25, 33, 59, 60, 61, 62, 63, 64]. The upper limits on $\theta_{24}$ and $\theta_{34}$ obtained from NOvA [60] are $\theta_{24} \leq 20.8^0$ and $\theta_{34} \leq 31.2^0$ assuming $\Delta m^2_{41} = 0.5$ eV$^2$. However according to MINOS analysis [15] $\theta_{24} \leq 7.3^0$ and $\theta_{34} \leq 26.6^0$ for the same value of $\Delta m^2_{41}$. ICECUBE-DeepCore [65] results considering $\Delta m^2_{41} = 1$ eV$^2$ suggests $\theta_{24} \leq 19.4^0$ and $\theta_{34} \leq 22.8^0$. Therefore, in the present work we vary both $\theta_{24}$ and $\theta_{34}$ within the limit $2^0 \leq \theta_{24} \leq 20^0$ and $2^0 \leq \theta_{34} \leq 20^0$. We also consider limits on $\theta_{14}$ such that $\theta_{14} \leq 4^0$, consistent with the results from the combined analysis by MINOS, Daya Bay and Bugey-3 [25] (in the range $0.2$ eV$^2 \leq \Delta m^2_{41} \leq 2$ eV$^2$). Using these limits on $\theta_{14}, \theta_{24}, \theta_{34}$ we compute the ratio $R_4$ and $R_3$ for diffuse flux. In Table 1, we furnish the computed values of $R_4$ for two representative sets of values for $\theta_{14}, \theta_{24}$ and $\theta_{34}$. The computed value for $R_3$, the muon to shower ratio for the three flavour case, is also furnished for comparison. From Table 1 it is obvious that the muon yield to shower ratio increases by considerable proportion from the ratio for three flavour case (for the particular
Table 1: Comparison of the muon to shower ratio for a diffused GRB neutrino flux for the 4 flavour (3+1) case compared with the same for 3 flavour case for two sets of active sterile neutrino mixing angle. See text for details.

| $\theta_{14}$ | $\theta_{24}$ | $\theta_{34}$ | $R_4$ (in 4f) | $R_3$ (in 3f) |
|---------------|---------------|---------------|---------------|---------------|
| 3°            | 5°            | 20°           | 9.48          | 1.80          |
| 4°            | 6°            | 15°           | 9.68          | 1.80          |

choices furnished in Table 1, this increase by more than five times) if a fourth sterile neutrino is assumed to be present in nature in addition to the three usual active neutrinos.

Figure 1: Variation of $R_4$ with $\theta_{24}$ and $\theta_{34}$ for (a) $\theta_{14} = 1°$ and (b) $\theta_{14} = 4°$. See text for details.

We have also explored how the ratio $R_4$ varies with different values of active-sterile mixing angles. In Fig. 1 we show the variations of $R_4$ with $\theta_{24}$ and $\theta_{34}$ for two fixed values of $\theta_{14}$ namely $\theta_{14} = 1°$ (Fig. 1a) and $\theta_{14} = 4°$ (Fig. 1b). From Fig. 1 it may be noted that the maximum value of the ratio $R_4$, i.e., $R_{4}^{\text{max}}$ is $\sim 6$ times higher than $R_3$.

3.2 Single GRB

We have made similar exercise for the neutrinos from a single GRB instead of diffused neutrino flux from several GRBs. A particular GRB occurs at a fixed zenith angle and at a definite redshift with respect to an observer at Earth. We have used two sets of active-sterile mixing angles for our calculations as given in Table 1. The active neutrino mixing angles are fixed at their current experimental values. With these sets of parameters we estimate the neutrino induced muons in a Km$^2$ detector for the UHE neutrinos from a GRB at different
Figure 2: Variation of the neutrino induced muons from single GRBs with different redshifts at a fixed zenith angle $\theta_z = 10^\circ$. “set 1” and “set 2” correspond to the two sets of values for active-sterile mixing angles given in Table 1.

Figure 3: Variation of the neutrino induced muons from the GRB with different GRB energies at a fixed zenith angle ($\theta_z = 10^\circ$). “set 1” and “set 2” are as in Fig. 2.

redshifts. The results are obtained using Eqs. (39 - 44) and Eqs. (14 - 37). The values of the parameter such as the Lorentz factor $\Gamma$, photon spectral break energy $E_{BR, \gamma, MeV}$ etc. required to calculate the neutrino flux from a single GRB are chosen as $\Gamma = 50.12$ and $E_{BR, \gamma, MeV} = 0.794$. These values are adopted from Table 1 of ref [53]. The results are shown in Fig. 2. In Fig. 3 we show the neutrino induced muons with different GRB energies. From both Fig. 2 and Fig. 3 it can be observed that the case of four flavour mixing cannot be distinguished from three flavour mixing as there is no significant deviation as observed in the case of diffused flux discussed earlier in Sect. 3.4.
4 Neutrinoless double beta decay in 3+1 scenario

Figure 4: The variation of the effective Majorana neutrino mass with the lightest neutrino mass for normal hierarchy and inverted hierarchy in 4 flavour (3 active + 1 sterile) scenario. The pair of red lines and the pair of green lines indicate the limits obtained from different experiments (see text). For lower $m_0$ only inverted hierarchy satisfies experimental limits.

In earlier section we have presented how a four flavour scenario with three active and one sterile neutrino can affect the neutrino flux for diffused and single GRB sources when compared with conventional three flavour approach. However, these studies do not provide any information about mass of the sterile neutrino or more precisely $\Delta m^2_{41}$ ($\Delta m^2_{43}$) for normal (inverted) hierarchy of neutrino mass. This is obvious as study of GRB fluxes involve large distance and mass square oscillation is therefore averaged out. However, sterile neutrino in the present 3 + 1 framework can affect the phenomena of neutrinoless double beta decay. The effective Majorana mass for observable neutrinoless double beta decay in 3 + 1 scenario is given as

$$m_{ee} = \sum_{i=1-4} |U_{ei}|^2 m_i,$$  \hspace{1cm} (47)

where we have neglected the Majorana phases. The above Eq. (47) can be rewritten in terms of mixing angles

$$m_{ee} = |c_{14} c_{12} c_{13}|^2 m_1 + |c_{14} s_{12} c_{13}|^2 m_2 + |c_{14} s_{13}|^2 m_3 + |s_{14}|^2 m_4.$$ \hspace{1cm} (48)

We consider that the sterile neutrino with mass $m_4$ is heavier than light active neutrinos. Therefore, the effective Majorana mass in case of normal ordering of active neutrinos is given as

$$m_{ee} = |c_{14} c_{12} c_{13}|^2 m_1 + |c_{14} s_{12} c_{13}|^2 \sqrt{m_1^2 + \Delta m^2_{21}} + |c_{14} s_{13}|^2 \sqrt{m_1^2 + \Delta m^2_{31}}$$

$$+ |s_{14}|^2 \sqrt{m_1^2 + \Delta m^2_{41}}.$$ \hspace{1cm} (49)
Similarly for the case of inverted hierarchy of active neutrinos, the expression in Eq. (48) can be rewritten as

\[
m_{ee} = |c_{14}c_{12}c_{13}|^2 \sqrt{m_3^2 + \Delta m_{23}^2 - \Delta m_{21}^2} + |c_{14}s_{12}c_{13}|^2 \sqrt{m_3^2 + \Delta m_{23}^2} + |c_{14}s_{13}|^2 m_3
\]

\[
+ |s_{14}|^2 \sqrt{m_3^2 + \Delta m_{43}^2}.
\]

Hence, for normal (inverted) hierarchy, \(m_1\) (\(m_3\)) is the lightest neutrino mass which we will denote as \(m_0\) for simplicity. From Eqs. (49-50), it can be easily observed that the effective Majorana mass \(m_{ee}\) depends on new physics involving sterile neutrino mixing angle \(\theta_{14}\) and mass square difference \(\Delta m_{41}^2\) (or equivalently \(\Delta m_{43}^2\)). In the present work we investigate the effects of these parameters on effective Majorana mass for neutrinoless double beta decay. Since, \(m_3\) is the lightest neutrino in case of inverted hierarchy, \(\Delta m_{43}^2 = m_4^2 - m_0^2\) is equivalent to \(\Delta m_{41}^2 = m_4^2 - m_0^2\) appearing in the expression of Eq. (49) for normal hierarchy.

In Fig. 4 we plot the variation of effective Majorana mass with lightest neutrino mass \(m_0\) varied within the range \(10^{-3} \text{ eV} \leq m_0 \leq 1 \text{ eV}\) for both normal and inverted hierarchy of neutrino mass using best fit values of active neutrino mixing angles \(\theta_{12}\) and \(\theta_{13}\). The shaded region shown in gray (black) in Fig. 4 corresponds to the normal (inverted) hierarchy of active neutrinos. We consider a conservative limit on mixing angle in between \(0^{\circ} \leq \theta_{14} \leq 4^{\circ}\) and the range of \(\Delta m_{41}^2\) from 0.2 eV\(^2\) to 2 eV\(^2\) consistent with the exclusion limits on \(\theta_{14}\) obtained from combined results of MINOS, Daya Bay and Bugey-3 experiments [25] and references therein) for normal hierarchy. We have assumed the same range of \(\theta_{14}\) and \(\Delta m_{43}^2\) for the case of inverted hierarchy of neutrino mixing. From Fig. 4, it can be easily observed that for inverted hierarchy (IH), the specified range of \(m_0\), \(\theta_{14}\) and \(\Delta m_{43}^2\) effective neutrino mass \(m_{ee}\) is almost constant for smaller values of \(m_0\) (0.001 to 0.01 eV). For higher values of \(m_0\), \(m_{ee}\) tends to increase proportionally with \(m_0\). Similar trend is observed for normal hierarchy (NH) of neutrino mass when \(m_0 \geq 0.1 \text{ eV}\) is considered. However, for smaller values of \(m_0\) (\(\leq 0.1 \text{ eV}\)) the effective neutrino mass \(m_{ee}\) in case of normal hierarchy tends to decrease. The observed upper limit on effective Majorana neutrino mass obtained from the combined of KamLAND-Zen [66] and EXO-200 [67] is 0.2-0.4 eV corresponds to the region within the pair of red lines shown in Fig. 4. Therefore, in the above specified range NH and IH are indistinguishable. Stringent limit on \(m_{ee}\) is further obtained from KamLAND-Zen [68] (region within the horizontal green lines in Fig. 4) with \(m_{ee} \sim 0.06 - 0.16 \text{ eV}\) probing the near inverted hierarchy regime. From Fig. 4 it can be easily observed that lightest neutrino mass \(m_0\) must be larger than 0.1 eV for higher values of \(m_{ee}\). However, for inverted hierarchy, lightest neutrino mass \(m_0\) can be smaller (\(\sim 0.02 \text{ eV}\)) when the limits on \(m_{ee}\) from KamLAND-Zen [68] is taken into account. It is to be noted that in the present discussion we have neglected the Majorana phases. However, one should consider all the Majorana phases. Extensive study of effective neutrino mass including all the Majorana phases has been presented in a recent work [69] using \(\sin^2 \theta_{14} = 0.019\) for \(\Delta m_{41}^2 = 1.7 \text{ eV}^2\).
For further details see [69] and references therein.

5 Summary and Conclusions

We investigate the deviations of ultra high energy (UHE) neutrino signatures obtained from GRB events in a Km$^2$ detector (such as ICECUBE) for a 3+1 neutrino framework from usual three active neutrino. We consider a four flavour scenario with three light active neutrinos and one sterile neutrino. The ratio of muon events to the shower events are calculated for both the three flavour and four flavour cases which are denoted as $R_3$ and $R_4$. Using the present limits on active sterile mixing obtained from different neutrino experiments along with the active neutrino mixing results, we found that the maximum value of the ratio of muon events with respect to shower events $R_4^{\text{max}}$ can be six to eight times larger for 3+1 mechanism when compared with normal three active neutrino formalism $R_3$. Therefore, the present analysis shows that any excess of such events detected in a Km$^2$ detector over that predicted for three neutrino mixing case can clearly indicate the presence of active sterile neutrino mixing. Thus UHE neutrino from distant GRB can be a probe to ascertain the existence of a sterile neutrino. In addition, we have also investigated neutrino induced muon events from a single GRB in the present framework of 3+1 neutrino and compared the results with the three flavour scenario. For a single GRB, with the observed bounds on active sterile neutrino mixing, there is no significant deviation from three active neutrino results. Therefore, for a single GRB, it is difficult to discriminate between usual three neutrino and four flavour (3 active + 1 sterile) formalism. We further investigate the bounds on light neutrino mass in the present four neutrino scheme obtained from neutrinoless double beta decay search results. We found that for normal hierarchy, using the present bounds on active sterile mixing and the bounds from neutrinoless double beta decay, we estimate the order of light neutrino mass in our work. We found that for inverted hierarchy, lightest neutrino mass can be as small as $\sim 0.02$ eV when bounds from KamLAND is considered.

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