Three-Slit Ghost Interference and Nonlocal Duality

Mohd Asad Siddiqui
Centre for Theoretical Physics, Jamia Millia Islamia, New Delhi, India.

A three-slit ghost interference experiment with entangled photons emerging from a spontaneous parametric down-conversion (SPDC) is theoretically analyzed using wave-packet dynamics. A nonlocal duality relation is derived which connects the path distinguishability of one photon to the interference visibility of the other.

I. INTRODUCTION

Quantum entanglement and nonlocality are two aspects of correlations which are intimately related to each other[1]. Such fundamental aspects of quantum theory are extensively studied[2] and today also its an emerging field of research. The correlated properties of entangled two-photon states have attracted attentions, due to their extensive applications in quantum optics and quantum information[3, 4]. As a result, Strekalov et.al demonstrated a ghost interference experiment[5], which show a nonlocal behaviour with spontaneous parametric down-conversion (SPDC) source $S$, a common method of producing entangled photons, conventionally called signal and idler beam[6–8], are then split by a polarized beam splitter into two beams, detected in coincidence by two distant pointlike photon detectors $D_1$ and $D_2$.

A double-slit is in the path of photon 1, and the detector $D_1$ is kept behind (see FIG. 1(a)), no first order interference is observed for photon 1, surprisingly, as one would normally expect Young’s double-slit interference. Also when the photon 2 is detected by $D_2$, in coincidence with a fixed detector $D_1$, the double-slit interference pattern is observed (see FIG. 1(b)), even though there is no double-slit in the path of photon 2. Many interesting outcomes are due to the spatial correlations which is with twin photons, produced in parametric down-conversion[9].

The two slit experiment has also been studied extensively in context of wave-particle duality and Bohr’s principle of complementarity. The fact that the wave and particle nature cannot be observed at the same time, appears to be so fundamental that Bohr elevated it to the level of a new principle, known as, the principle of complementarity[10]. Bohr stressed that the wave nature of particle, characterized by interference, and the particle nature, characterized by which way (i.e., which path) information, are mutually exclusive. The principle then quantitatively analysed, to see, what extent the two natures could be simultaneously observed[11, 12]. The Englert-Greenberger-Yasin (EGY) duality relation[11, 12] is local, in the sense that when we talk of which-path distinguishability, we talk of the which-path knowledge of the same particle giving interference pattern.

Recently, a nonlocal duality relation[13] was derived for two slit experiment, which relate the which-path information of one particle to the fringe visibility of the other.

Of late, a new interest has been developed towards the three-slit interference experiments[14–16], whilst for different reasons.

In this paper, we propose and theoretically analyze a three-slit ghost interference experiment performed with entangled photons emerging from a SPDC. Also a nonlocal duality relation is derived which connects the path distinguishability of one photon to the interference visibility of the other.

II. THREE SLIT GHOST INTERFERENCE

In our proposed experiment, the two slits are replaced by three slits, in the earlier setup (see FIG. 2). The entangled photons 1 and 2 from the SPDC source $S$ show an interference pattern, similar to the pattern observed from three slits experiment. Even though photon 2 never passes through the region between the source $S$ and three slits, we see an interference pattern for photon 2, as if a beam of photon 2 with a source located at the position of detector $D_1$, get split by three-slits.
III. WAVE-PACKET ANALYSIS

In order to theoretically analyze the ghost interference\[17\], a generalized EPR state\[18\] is used, which unlike the EPR state\[19\], is well behaved and fully normalized.

$$\Psi(z_1, z_2) = C_1 \int_{-\infty}^{\infty} dp \, e^{-p^2/4\hbar^2}\sigma^2 e^{-ipz_1/\hbar} e^{-i(z_1+z_2)/4\hbar^2}$$

where $C_1$ is a normalization constant, and $\sigma, \Omega$ are certain parameters whose physical significance will become clear in the following. In the limit $\sigma, \Omega \to \infty$ the state reduces to the EPR state.

The pair of photons are assumed to travel in opposite directions along the x-axis, and the entanglement is in the z-direction. We will ignore the dynamics along the x-axis as it does not affect the entanglement. We assume that during the evolution for time $t$, the photon travels a distance equal to $ct$. Integration performed over $p$ in Eq. (1) gives:

$$\Psi(z_1, z_2) = \sqrt{\frac{\sigma}{\pi \omega}} e^{-(z_1-z_2)^2/\sigma^2} e^{-(z_1+z_2)/4\Omega^2}. \quad (2)$$

The uncertainty in positions and the wave-vector of two photons, along the z-axis, is given by

$$\Delta z_1 = \Delta z_2 = \sqrt{\Omega^2 + 1/4\sigma^2},$$

$$\Delta k_{1z} = \Delta k_{2z} = \frac{1}{2} \sqrt{\sigma^2 + \frac{1}{4\Omega^2}}. \quad (3)$$

The above equation gives the position and momentum spread of the photons in the z-direction.

If the state at time $t = 0$ is $\psi(z, 0)$, then the wave-packet of photon, after time $t$ will evolve as

$$\psi(z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(ik_z z - i\omega(k_z) t)\tilde{\psi}(k_z, 0)dk_z, \quad (4)$$

where $\tilde{\psi}(k_z, 0)$ is the Fourier transform of $\psi(z, 0)$ with respect to $z$. The photon approximately travels in the x-direction, but can slightly deviate in the z-direction, so it can pass through slits which are located at different $z$-positions, and therefore its true wave-vector will be given by,

$$\omega(k_z) = c\sqrt{k_x^2 + k_z^2}. \quad (5)$$

Since the photon travel along x-axis, hence for $k_x \gg k_z$, one can write $k_z \approx k_0$, where $k_0 = 2\pi/\lambda$. The dispersion along z-axis can be approximated by

$$\omega(k_z) \approx ck_0 + ck_z^2/2k_0. \quad (6)$$

Using this, Eq. (4) becomes

$$\psi(z, t) = \frac{e^{-i\omega(k_0) t}}{2\pi} \int_{-\infty}^{\infty} \exp(ik_z z - i\omega(k_z)/2k_0)\tilde{\psi}(k_z, 0)dk_z. \quad (7)$$

In case of entangled photons, after time $t_0$, photon 1 reaches the triple slit ($ct_0 = L_2$), and photon 2 travels a distance $L_2$ towards detector $D_2$. Therefore, the state of the entangled photons after time $t_0$ is given by:

$$\psi(z_1, z_2, t_0) = \frac{e^{-2i\omega(k_0) t_0}}{4\pi^2} \int_{-\infty}^{\infty} dk_1 \exp(ik_1z_1 - i\omega(k_1)/2k_0) \times \int_{-\infty}^{\infty} dk_2 \exp(ik_2z_2 - i\omega(k_2)/2k_0)\tilde{\psi}(k_1, k_2, 0), \quad (8)$$

where $\tilde{\psi}(k_1, k_2, 0)$ is the Fourier transform of $\psi(z, 0)$ with respect to $z_1, z_2$.

To investigate the effect on the entangled state, one can use two different approaches. The first, most obvious is to model a potential for three slits, and calculate its evolution in that potential. We will follow the second, a comparatively easier approach, here we capture the essence of the effect of triple slit on the state, without going into tedious calculations. When the state interacts with a single-slit, we assume, that a Gaussian wave-packet emerges from that slit, centered at its location, whose width is related to the width of the slit.

Consider the three slits A, B and C, and the wave packet passes through be, say, $|\phi_A(z_1)\rangle$, $|\phi_B(z_1)\rangle$ and $|\phi_C(z_1)\rangle$, respectively. Some part of the state of particle 1 will be blocked, represented by $\chi(z_1)$. All these states are orthogonal, and the actual state of particle 1 can be expanded in this basis.

$$|\Psi(z_1, z_2, t_0)\rangle = |\phi_A\rangle\langle\phi_A|\Psi\rangle + |\phi_B\rangle\langle\phi_B|\Psi\rangle + |\phi_C\rangle\langle\phi_C|\Psi\rangle + |\chi\rangle\langle\chi|\Psi\rangle. \quad (9)$$

The terms $|\phi_A\rangle\Psi\rangle$, $|\phi_B\rangle\Psi\rangle$, $|\phi_C\rangle\Psi\rangle$, $|\chi\rangle\Psi\rangle$ are states of particle 2 and can be written explicitly as follows.

$$\psi_A(z_1) = \langle\phi_A(z_1)|\Psi(z_1, z_2, t_0)\rangle,$$

$$\psi_B(z_1) = \langle\phi_B(z_1)|\Psi(z_1, z_2, t_0)\rangle,$$

$$\psi_C(z_1) = \langle\phi_C(z_1)|\Psi(z_1, z_2, t_0)\rangle,$$

$$\psi_{\chi}(z_1) = \langle\chi(z_1)|\Psi(z_1, z_2, t_0)\rangle. \quad (10)$$

The entangled state after particle 1 passes through triple-slit will be given by:

$$|\Psi(z_1, z_2)\rangle = |\phi_A\rangle|\psi_A\rangle + |\phi_B\rangle|\psi_B\rangle + |\phi_C\rangle|\psi_C\rangle + |\chi\rangle|\Psi_{\chi}\rangle, \quad (11)$$
where $|\phi_A\rangle$, $|\phi_B\rangle$ and $|\phi_C\rangle$ are states of particle 1, and $|\psi_A\rangle$, $|\psi_B\rangle$ and $|\psi_C\rangle$ are states of particle 2. The first three terms represent the amplitude of particle 1 passing through these slits, and the last term represents the amplitude of being blocked or reflected. The linearity of the Schrödinger equation assures that the first three terms and the last term evolve independently. Since the experiment considers only those photon 1 which passes through the triple slit, we can throw away the last term. This will not change anything except the renormalization of the state.

We also assume that $|\phi_A\rangle$, $|\phi_B\rangle$ and $|\phi_C\rangle$, are Gaussian wave-packets:

$$
\phi_A(z_1) = \frac{1}{(\pi/2)^{1/4}\sqrt{\epsilon}} e^{-(z_1-z_0)^2/\epsilon^2},
\phi_B(z_1) = \frac{1}{(\pi/2)^{1/4}\sqrt{\epsilon}} e^{-(z_1+z_0)^2/\epsilon^2},
\phi_C(z_1) = \frac{1}{(\pi/2)^{1/4}\sqrt{\epsilon}} e^{-(z_1-z_0)^2/\epsilon^2},
$$

(12)

where $+z_0, 0, -z_0$ are $z$-position’s of slit A, B and C, respectively, and $\epsilon$ be their widths. Using (10) and (11), wavefunctions $|\psi_A\rangle$, $|\psi_B\rangle$, and $|\psi_C\rangle$ can be calculated, which, after normalization, have the following form

$$
\psi_A(z_2) = C_2 \ e^{-\frac{(z_2-z_0')^2}{\epsilon^2}},
\psi_B(z_2) = C_2 \ e^{-\frac{z_2^2}{\epsilon^2}},
\psi_C(z_2) = C_2 \ e^{-\frac{(z_2+z_0')^2}{\epsilon^2}},
$$

(13)

where

$$
\Gamma = \frac{\epsilon^2 + \frac{\lambda^2}{\sigma^2} + \frac{\lambda^2}{4\pi^2\sigma^2} + \frac{2i\hbar t_0}{m} + \frac{2i\hbar t_0}{m}}{1 + \frac{\lambda^2}{\sigma^2} + \frac{2i\hbar t_0}{m} + \frac{1}{4\pi^2\sigma^2} + \frac{2i\hbar t_0}{m}},
$$

$$
z_0' = \frac{4i\hbar t_0 + z_0}{4i\hbar t_0 + z_0},
$$

and

$$
C_2 = (2/\pi)^{1/4}(\sqrt{\Gamma_R} + \frac{\Gamma_i}{\sqrt{\Gamma_R}})^{-1/2}.
$$

(14)

Here $\Gamma_R$, $\Gamma_i$ are the real and imaginary parts of $\Gamma$, respectively.

Thus, the state which emerges from the triple slit, has the following form

$$
\Psi(z_1, z_2) = c \left( e^{-\frac{(z_1-z_0)^2}{\epsilon^2}} e^{-\frac{(z_1-z_0')^2}{\epsilon^2}} + e^{-\frac{z_2^2}{\epsilon^2}} e^{-\frac{z_2^2}{\epsilon^2}} e^{-\frac{(z_1+z_0)^2}{\epsilon^2}} + e^{-\frac{(z_1-z_0)^2}{\epsilon^2}} e^{-\frac{(z_1+z_0')^2}{\epsilon^2}} \right),
$$

(15)

where $c = (1/\sqrt{\pi})(\sqrt{\Gamma_R} + \frac{\Gamma_i}{\sqrt{\Gamma_R}})^{-1/2}$. The above expression is obtained by dropping the phase factor of Eq.(11), as it is not important for our final analysis. Eq.(15) represents three wave-packets of photon 1, of width $\epsilon$, and localized at $-z_0$, 0 and $+z_0$, entangled with three wave-packets of photon 2, of width $\sqrt{\frac{\epsilon^2}{\epsilon^2 + 1}}$, localized at $-z_0$, 0 and $+z_0$.

At this early stage one can notice the amplitude of photon 1 through slits A, B and C, which are correlated to spatially separated wave-packets of photon 2. Thus, in principle when one detect the photon 2, he can know which slit, A, B or C, the photon 1 passed through. By Bohr’s principle of complementarity, if one knows which slit photon 1 passed through, no interference pattern will be seen. This is the fundamental reason for non-observance of interference pattern showed by photon 1 in the ghost interference experiment.

Before reaching detector $D_2$, the particle 2 further evolves for time $t$, thus transforms the state (14) to

$$
\Psi(z_1, z_2, t) = C_t \left( \exp \left[ \frac{-(z_1-z_0)^2}{\epsilon^2 + iL_1\lambda/\pi} \right] \exp \left[ \frac{-(z_2-z_0')^2}{\Gamma + iL_1\lambda/\pi} \right] \right.
\left. + \exp \left[ \frac{-z_2^2}{\epsilon^2 + iL_1\lambda/\pi} \right] \exp \left[ \frac{-z_2^2}{\Gamma + iL_1\lambda/\pi} \right] \right),
$$

(15)

where

$$
C_t = \frac{1}{\sqrt{\pi}\sqrt{\epsilon + iL_1\lambda/\pi}\sqrt{\Gamma_R} + (\Gamma_i + iL_1\lambda/\pi)/\sqrt{\Gamma_R}}.
$$

When the correlation because of entanglement between the photons are good, one can make further approximations: $\Omega \gg \epsilon$, $\Omega \gg 1/\sigma$ and $\Omega \gg 1$. In this limit,

$$
\Gamma \approx \gamma^2 + 2i\hbar t_0/\mu, \quad z_0' \approx z_0,
$$

(16)

where $\gamma^2 = \epsilon^2 + 1/\sigma^2$.

The wave-function (15) represents the combined state of two photons when they reach the detector $D_1$ and $D_2$. Now if $D_1$ and $D_2$ are located at $z_1$ and $z_2$ respectively, the probability density of their coincident count is given
by

\[ P(z_1, z_2) = \vert \Psi (z_1, z_2, t) \vert^2 \]

\[ = \vert C_t \vert^2 \left( \exp \left[ \frac{-2(z_1 - z_0)^2}{\epsilon^2 + \left( \frac{L_1}{\pi e} \right)^2} - \frac{2(z_2 - z_0)^2}{\gamma^2 + \left( \frac{AD}{\pi \gamma} \right)^2} \right] \right. \]

\[ + \exp \left[ \frac{-2z_1^2}{\epsilon^2 + \left( \frac{L_1}{\pi e} \right)^2} - \frac{2z_2^2}{\gamma^2 + \left( \frac{AD}{\pi \gamma} \right)^2} \right] \]

\[ + \exp \left[ \frac{2(z_1 + z_0)^2}{\epsilon^2 + \left( \frac{L_1}{\pi e} \right)^2} - \frac{2(z_2 + z_0)^2}{\gamma^2 + \left( \frac{AD}{\pi \gamma} \right)^2} \right] \]

\[ + \exp \left[ \frac{-2z_1^2 + z_0^2 - 2z_1z_0 - 2z_2^2 + z_0^2 - 2z_2z_0}{\epsilon^2 + \left( \frac{L_1}{\pi e} \right)^2} \right. \]

\[ \times 2 \cos \left[ (z_1^2 - 2z_1z_0)\xi_1 + (z_0^2 - 2z_2z_0)\xi_2 \right] \]

\[ + \exp \left[ \frac{-2z_1^2 + z_0^2 + 2z_1z_0 - 2z_2^2 + z_0^2 + 2z_2z_0}{\epsilon^2 + \left( \frac{L_1}{\pi e} \right)^2} \right] \]

\[ \times 2 \cos \left[ (z_1^2 + 2z_1z_0)\xi_1 + (z_0^2 + 2z_2z_0)\xi_2 \right] \],

where

\[ \xi_1 = \frac{\lambda L_1/\pi}{\epsilon^4 + \left( \frac{L_1}{\pi e} \right)^2}, \quad \xi_2 = \frac{\lambda D/\pi}{\gamma^4 + \left( \frac{AD}{\pi \gamma} \right)^2}, \]

\[ D = L_1 + 2L_2, \quad \text{and} \]

\[ C_t = \frac{1}{\sqrt{\pi} \sqrt{\epsilon + \left( \frac{\lambda L_1}{\pi e} \right)^2 \sqrt{\gamma^4 + \left( \frac{AD}{\pi \gamma} \right)^2}}}. \]

\[ \vert \Psi (0, z_2, t) \vert^2 = \vert C_t \vert^2 \left[ \exp \left( \frac{-2z_0^2}{\epsilon^2 + \left( \frac{L_1}{\pi e} \right)^2} \right) - \frac{2(z_2^2 + z_0^2)}{\gamma^2 + \left( \frac{AD}{\pi \gamma} \right)^2} \right] \]

\[ \times 2 \cosh \left[ \frac{4z_2z_0}{\gamma_D^2} \right] \left( 1 + \frac{\cos \left[ 4z_2z_0/AD \right]}{\cosh \left[ 4z_2z_0/\gamma_D \right]} \right) \]

\[ + \exp \left( \frac{-2z_0^2}{\gamma_D^2} \right) + 2 \exp \left( \frac{-2z_2^2}{\epsilon^2 + \left( \frac{L_1}{\pi e} \right)^2} - \frac{2(z_2^2 + z_0^2)}{\gamma^2 + \left( \frac{AD}{\pi \gamma} \right)^2} \right) \]

\[ \times \left( \exp \left( \frac{2z_2z_0}{\gamma_D^2} \right) \cos \left[ \frac{2z_2z_0/\gamma_D + \beta}{\beta} \right] \right) \]

\[ + \exp \left( \frac{-2z_2z_0}{\gamma_D^2} \right) \cos \left[ \frac{2z_2z_0/\gamma_D + \beta}{\beta} \right] \right) \right) \]

where

\[ \gamma_D^2 = \gamma^2 + (\lambda D/\pi \gamma)^2, \quad \text{and} \]

\[ \beta = \frac{z_0^2 \pi}{\lambda} \left( \frac{1}{L_1 + \frac{D}{L}} \right). \]

Neglecting \( \beta \), we get

\[ \vert \Psi (0, z_2, t) \vert^2 = \vert C_t \vert^2 \left( \exp \left( \frac{-2z_0^2}{\epsilon^2 + \left( \frac{L_1}{\pi e} \right)^2} \right) - \frac{2(z_2^2 + z_0^2)}{\gamma^2 + \left( \frac{AD}{\pi \gamma} \right)^2} \right) \]

\[ \times 2 \cosh \left[ \frac{4z_2z_0}{\gamma_D^2} \right] \left( 1 + \frac{\cos \left[ 4z_2z_0/AD \right]}{\cosh \left[ 4z_2z_0/\gamma_D \right]} \right) \]

\[ + \exp \left( \frac{-2z_0^2}{\gamma_D^2} \right) + 2 \exp \left( \frac{-2z_2^2}{\epsilon^2 + \left( \frac{L_1}{\pi e} \right)^2} - \frac{2(z_2^2 + z_0^2)}{\gamma^2 + \left( \frac{AD}{\pi \gamma} \right)^2} \right) \]

\[ \times \left( \exp \left( \frac{2z_2z_0}{\gamma_D^2} \right) \cos \left[ \frac{2z_2z_0/\gamma_D + \beta}{\beta} \right] \right) \]

\[ + \exp \left( \frac{-2z_2z_0}{\gamma_D^2} \right) \cos \left[ \frac{2z_2z_0/\gamma_D + \beta}{\beta} \right] \right) \]

where

\[ \kappa_1 = \frac{4\pi z_0}{\lambda D}, \quad \kappa_2 = \frac{2\pi z_0}{\gamma_D}. \]

For \( \gamma^2 \ll \lambda D/\pi \), (19) represents an interference pattern for photon 2 with fringe widths \( (w_2 = \frac{2\pi}{\kappa_1}) \), due to slit A and C, A and B, and B and C, are respectively given by,

\[ (w_2)_{AC} \approx \frac{\lambda D}{2z_0}, \]

\[ (w_2)_{AB} = (w_2)_{BC} \approx \frac{\lambda D}{2z_0}. \]

(20)

IV. RESULTS

A. Ghost interference

We analyze three slit ghost interference experiment, the entangled photons with same wave-length \( \lambda \), and the detector \( D_1 \) is fixed at \( z_1 = 0 \). In that case, (17) reduces

This is the ghost interference, the distance \( D \) in the formula is the distance from the three-slits, right through the source to the detector \( D_2 \), (see FIG. 2).
B. Nonlocal wave-particle duality

To find the duality relation, we place the which-way detector behind these slits, (see FIG. 3), this makes the which-path information available to the experimenter. The which-way detector should have three states, which correlate with the particle, when it passes through each slit. Let the states be \( |d_1\rangle, |d_2\rangle, |d_3\rangle \), corresponds to the particle through slit 1, 2 and 3, respectively. Without the loss of generality, we assume that the states \( |d_1\rangle, |d_2\rangle, |d_3\rangle \) are normalized, but not necessarily mutually orthogonal. If \( |d_1\rangle, |d_2\rangle, |d_3\rangle \) are mutually orthogonal, one can find the Hermitian operator, which will give different eigenvalues, this corresponds to a particle travelling through each slit. Thus we get the information about the path of the particle, without ambiguity. If \( |d_1\rangle, |d_2\rangle, |d_3\rangle \) are not mutually orthogonal, the path of the particle will not be fully distinguishable.

The which-way distinguishability \(^{10}\) for particle 1 is given by

\[ D_1 ≡ 1 - \frac{1}{3} (|\langle d_1 | d_2 \rangle|^2 + |\langle d_2 | d_3 \rangle|^2 + |\langle d_1 | d_3 \rangle|^2), \] (21)

the value lies in the range \( 0 \leq D_1 \leq 1 \).

Let us see the effect of which-path detector on the ghost interference given by particle 2. We assume that the two particles move in opposite directions along the x-axis, and the entanglement is in the z-direction. We get the following states.

\[
Ψ(z_1, z_2, t) = C_t \left( |d_1\rangle \exp \left[ \frac{-(z_1 - z_0)^2}{\epsilon^2 + \frac{i(L_1\lambda}{\pi} \right] \exp \left[ \frac{-z_0^2}{\Gamma + \frac{iL_1\lambda}{\pi} \right] \\
+ |d_2\rangle \exp \left[ \frac{-z_1^2}{\epsilon^2 + \frac{iL_1\lambda}{\pi} \right] \exp \left[ \frac{-z_2^2}{\Gamma + \frac{iL_1\lambda}{\pi} \right] \\
+ |d_3\rangle \exp \left[ \frac{-(z_1 + z_0)^2}{\epsilon^2 + \frac{iL_1\lambda}{\pi} \right] \exp \left[ \frac{-(z_2 + z_0)^2}{\Gamma + \frac{iL_1\lambda}{\pi} \right] \right)
\]

where

\[ C_t = \frac{1}{\sqrt{\pi} \sqrt{\epsilon + iL_1\lambda/\epsilon\pi} \sqrt{\Gamma_r + (\Gamma_i + iL_1\lambda/\pi)/\sqrt{\Gamma_r}} \]

The probability density at \( z_1 = 0 \), given by \( |Ψ(0, z_2, t)|^2 \), has the following form

\[
|Ψ(0, z_2, t)|^2 = |C_t|^2 \left[ \exp \left[ \frac{-2z_2^2}{\gamma_D^2} \right] \right] (1 + 2 \cosh \left[ \frac{4z_2 z_0}{\gamma_D^2} \right] ) \\
x \exp \left[ \frac{-2z_0^2}{\epsilon^2 + \left( \frac{L_1\lambda}{\pi} \right)^2} \right] + 2 |\langle d_1 | d_2 \rangle| \\
x \exp \left[ \frac{-z_2^2}{\epsilon^2 + \left( \frac{L_1\lambda}{\pi} \right)^2} - \frac{2z_2^2 + z_0^2 - 2z_2 z_0}{\gamma_D^2} \right] \\
cos \left[ 2z_2 z_0 \xi_2 - z_2^2 (\xi_1 + \xi_2) \right] \\
+ 2 |\langle d_1 | d_3 \rangle| \exp \left( -\frac{2z_2^2}{\epsilon^2 + \left( \frac{L_1\lambda}{\pi} \right)^2} - \frac{2(z_2^2 + z_0^2)}{\gamma_D^2} \right) \\
x \cos \left[ 4z_2 z_0 \xi_2 + 2 |\langle d_2 | d_3 \rangle| \right] \\
x \cos \left[ 2z_2 z_0 \xi_2 + z_0^2 (\xi_1 + \xi_2) \right] \right] \\)

(23)

Visibility of the interference fringes is conventionally defined as

\[ V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \] (24)

where \( I_{max} \) and \( I_{min} \) represent the maximum and minimum intensity in neighbouring fringes, respectively. Maxima and minima of (23) will occur at points where the value of each cosine is 1 and -1/2 , respectively, provided we ignore \( z_0^2 (\xi_1 + \xi_2) \) term. The visibility of particle 2 can then be written down as

\[ V_2 = \frac{3 \left( |\langle d_1 | d_2 \rangle| e^{\frac{2z_2 z_0}{\gamma_D^2}} + |\langle d_1 | d_3 \rangle| e^{-\frac{2z_2 z_0}{\gamma_D^2}} \right) \alpha + |\langle d_1 | d_2 \rangle| e^{\frac{2z_2 z_0}{\gamma_D^2}} + |\langle d_1 | d_3 \rangle| e^{-\frac{2z_2 z_0}{\gamma_D^2}} \right)}{\alpha + |\langle d_1 | d_2 \rangle| e^{\frac{2z_2 z_0}{\gamma_D^2}} + |\langle d_1 | d_3 \rangle| e^{-\frac{2z_2 z_0}{\gamma_D^2}} + |\langle d_2 | d_3 \rangle| e^{\frac{2z_2 z_0}{\gamma_D^2}}} \]

(25)

where

\[ \zeta = \left( \frac{1}{\epsilon^2 + \left( \frac{L_1\lambda}{\pi} \right)^2} + \frac{1}{\gamma_D^2} \right) \] and

\[ \alpha = 2 \left( \exp \left[ \frac{z_0^2}{\gamma_D^2} \right] + 2 \exp \left[ -z_0^2 \zeta \cdot \cosh \left[ \frac{2z_2 z_0}{\gamma_D^2} \right] \right] \right) \]

The maximum visibility one can theoretically get when \( z_0 < \frac{L_1\lambda}{\pi} \), and \( z_0 < \frac{L_1\lambda}{\pi} \). The actual fringe visibility will be less than or equal to that, and can be written as

\[ V_2 \leq \frac{3 \left( |\langle d_1 | d_2 \rangle| + |\langle d_1 | d_3 \rangle| + |\langle d_2 | d_3 \rangle| \right)}{2 \left( 1 + |\langle d_1 | d_2 \rangle| + |\langle d_1 | d_3 \rangle| + |\langle d_2 | d_3 \rangle| \right)}. \] (26)

Using (21) the above equation gives

\[ V_2 + \frac{2D_1}{3 - D_1} \leq 1. \] (27)

The duality relation derived here is very similar to the duality relation derived earlier for a three-slit interference
The big difference is that, in three-slit experiment we talk of the path distinguishability and the fringe visibility for the same particle. In three-slit ghost interference, we show that the relation is between different particles, i.e. the path distinguishability of particle 1 is related with the fringe visibility of particle 2.

V. CONCLUSION

In conclusion, we have analyzed the complementarity between which-way information and interference fringe visibility for ghost interference, for SPDC entangled particles passing through three slits. We also derive a three-slit nonlocal duality relation which connects the path distinguishability of one photon to the interference visibility of the other which means erasing the which-path information of photon 1 recovers the interference pattern of photon 2 and vice-versa.

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