Close approach maneuvers around an oblate planet

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Abstract. There are many applications of the close approach maneuvers in astronautics, and several missions used this technique in the last decades. In the present work, those close approach maneuvers are revisited, but now considering that the spacecraft passes around an oblate planet. This fact changes the distribution of mass of the planet, increasing the mass in the region of the equator, so increasing the gravitational forces in the equatorial plane. Since the present study is limited to planar trajectories, there is an increase in the variation of energy given by the maneuver. The planet Jupiter is used as the body for the close approach, but the value of $J_2$ is varied in a large range to simulate situations of other celestial bodies that have larger oblateness, but the same mass ratio. This is particularly true in recent discovered exoplanets, and this first study can help the study of the dynamics around those bodies.

1. Introduction

Several papers in the literature describe the close approach maneuvers, like the ones shown in references [1] to [20]. Using the model given by the "patched conics" to explain the close approach maneuvers, it is assumed that the mission can be divided in three stages, all of them studied under Keplerian motion. It is also considered that the system consists of three bodies: $M_1$, a massive body in the center of the Cartesian system; $M_2$, a smaller body in a Keplerian orbit around $M_1$; and $M_3$, a spacecraft that is traveling in an orbit around $M_1$ when it passes close to $M_2$. This passage changes the orbit of $M_3$, which leads to a variation of eccentricity, semi-major axis, velocity, energy and angular momentum of $M_3$. This phenomenon is also called a swing-by maneuver. The advantages of this maneuver are the savings of time and fuel for a particular mission. According to these assumptions, the orbits of $M_1$ and $M_2$ remain unchanged. For the motion of the spacecraft, the model of two bodies is used in the initial phase, for the spacecraft-central body system. In the second stage, it is assumed that the spacecraft passes through the secondary body in an open trajectory. The third step is again assumed to be a system formed by two bodies, spacecraft-central body. The goal of this maneuver is to modify the Keplerian trajectories spacecraft-central body from the first to the third stage. Figure 1 illustrates this sequence. In this figure, A and B are the initial and final points of the trajectory during the close approach, $\delta$ is the angle of curvature of the spacecraft, $\Psi$ is the angle between the periapsis line and the line connecting $M_1$ and $M_2$, $V_2$ is the velocity of $M_2$ with respect to $M_1$, $V_{\infty}$ and $V_{\infty}^*$ are the velocities of $M_3$ with respect to $M_2$, when approaching and leaving $M_2$, respectively, and $r_p$ is the periapsis distance of the close approach.
2. Equations of Motion
To get more accurate results, it is important to improve the mathematical model used to describe this problem. The study is limited to planar motion, because this region is where the effects of the oblateness of the planet is maximized. The planar circular restricted three-body problem is an excellent mathematical model to use in this type of study, because it allows both primaries to act all the time in the motion of the spacecraft. Based on this model, it is possible to get the spacecraft equations of motion, as follows [21].

This problem is studied using the canonical system of units. The unit of mass will be the total mass of the system, where the mass of Sun ($M_1$) is added to the mass of the planet ($M_2$) and the spacecraft ($M_3$) is assumed to have negligible mass. In this system, the adimensional mass of $M_2$ is given by the mass ratio:

$$\mu = \frac{M_2}{M_1 + M_2} \quad (1)$$

The mass of $M_1$ is given by:

$$(1 - \mu) \quad (2)$$

Using a fixed reference system, where the origin is located at the centroid of the two massive bodies, it is possible to develop the equations of motion. The x-axis is the line connecting $M_1$ and $M_2$ and the vertical axis is the line perpendicular to the x-axis. In this system, $M_1$ and $M_2$ have positions that are given by:

$$\begin{align*}
\bar{x}_1 &= -\mu r \cos \nu \\
\bar{y}_1 &= -\mu r \sin \nu \\
\bar{x}_2 &= (1 - \mu) r \cos \nu \\
\bar{y}_2 &= (1 - \mu) r \sin \nu
\end{align*} \quad (3)$$

Where $r$ is the distance between the two primaries, given by:

$$r = \frac{1 - e^2}{1 + e \cos \nu} \quad (4)$$

and $\nu$ is the true anomaly of $M_2$. In this system, the equations of motion of the spacecraft are given by:

$$\begin{align*}
\dot{x} &= -\frac{(1 - \mu) (\bar{x} - \bar{x}_1)}{r_1^3} - \frac{\mu (\bar{x} - \bar{x}_2)}{r_2^3} \\
\dot{y} &= -\frac{(1 - \mu) (\bar{y} - \bar{y}_1)}{r_1^3} - \frac{\mu (\bar{y} - \bar{y}_2)}{r_2^3}
\end{align*} \quad (5)$$
Where \( r_1 \) and \( r_2 \) are the distances from the spacecraft to \( M_1 \) and \( M_2 \), respectively, given by:

\[
\begin{align*}
\frac{\dot{y}}{r_1^3} &= \frac{(y-y_1)}{r_1^3} - \frac{\mu (y-y_2)}{r_2^3} \quad \text{(6)} \\
\end{align*}
\]

Taking into account the disturbance due to the oblateness \( (J_2) \) of the secondary body \( M_2 \), there is a new term in the potential energy, given by:

\[
U_{J_2} = -\mu R^2 \left( -\frac{1}{2} \right) \quad \text{(9)}
\]

That can be expressed in terms of \( x-y \) components by equations 11 and 12:

\[
\begin{align*}
\ddot{x} &= -\frac{3}{2} \mu R^2 J_2 \frac{(x-x_2)}{r_2^5} \quad \text{(11)} \\
\ddot{y} &= -\frac{3}{2} \mu R^2 J_2 \frac{(y-y_2)}{r_2^5} \quad \text{(12)}
\end{align*}
\]

The main problem will be to simulate and classify trajectories passing close to Jupiter (\( M_2 \)). For this task we assume the presence of three bodies: Sun (\( M_1 \)), Jupiter (\( M_2 \)) and the spacecraft (\( M_3 \)). Thus, the problem is to study the motion of the spacecraft near the encounter with the planet Jupiter. The energy (E) and the angular momentum (C) are calculated before and after the close encounter to detect the change of the orbit due to this maneuver. The simulations are made by numerical integration of the equations (5) and (6), adding the effect of the oblateness shown in equations (10), (11) and (12). When the spacecraft reaches the points A and B, far away from Jupiter, the energy and angular momentum before and after the passage, respectively, are calculated assuming a two-body dynamics Sun-spacecraft. The orbits are classified in four categories [13]: direct elliptical (negative energy and positive angular momentum), retrograde elliptical (negative energy and angular momentum), direct hyperbolic (positive energy and angular momentum), and retrograde hyperbolic (positive energy and negative angular momentum).

3. Results

The results consist of graphs showing the variation of the orbit of the spacecraft due to the encounter with Jupiter considering the oblateness of this planet. The horizontal axis represents the angle of approach and the vertical axis represents the velocity of the spacecraft at the periapsis. They are related to the orbits in accordance with the rules shown in table 1:

| Before:                        | After:                        | Direct Elliptical | Retrograde Elliptical | Direct Hyperbolic | Retrograde Hyperbolic |
|--------------------------------|-------------------------------|-------------------|-----------------------|-------------------|-----------------------|
| Direct Elliptical              | A                             | E                 | I                     | M                 |
| Retrograde Elliptical          | B                             | F                 | J                     | N                 |
| Direct Hyperbolic              | C                             | G                 | K                     | O                 |
| Retrograde Hyperbolic          | D                             | H                 | L                     | P                 |

The letter Z is black and indicates that the spacecraft did not leave Jupiter, so it remains orbiting around the planet. The three independent variables that describe the maneuver are: 1) \( r_p \), the distance from the periapsis, kept constant at 1.3 radius of Jupiter; 2) \( V_p \), the velocity at the periapsis; 3) \( \Psi \), the
approach angle. The graphs were obtained for the spacecraft trajectories considering several values for the oblateness of Jupiter: 0.0, 0.0147 (the real value), 0.0735 (five times the real value), 0.1470 (ten times the real value). The angle $\Psi$ goes from 180º to 360º, because there is a symmetry in the system and the range from 0º to 180º is a mirror image of the range shown here, just replacing escapes by captures. The velocities where chosen in the range from 4.1 to 4.7 canonical units, because this is the regions where captures and escapes occur. The regions below and above these values are dominated by orbits that stay around Jupiter or has negligible effects, respectively. The graphs below show the results. Figures 2 to 5 show that the general forms of the figures are not changed by the oblateness of the planet, but some particular regions are strongly modified by the $J_2$ term. Note, for example, the bottom right corner of the figures 2 to 5. Other examples are the middle and the bottom left corner of these figures.

To emphasize those modifications new graphs were made, showing the bottom right corner in more details. It is noted that trajectories of type K (that represents orbits that are direct hyperbola before and after the close approach) are transformed in I (that represents orbits that are direct ellipse before the close approach and direct hyperbola after that). The physical reason for this fact is the extra force generated by the largest concentration of mass near the equator of the planet. Figures 6 to 9 show a detailed view for $\Psi$ between 330º and 360º and $V_p$ between 4.1 and 4.2 canonical units, the region where the effects are stronger. These figures show clearly how the $J_2$ has a great influence in the modification of the trajectories in that region.

Figure 10 shows the changes in the trajectories of the spacecraft using two different values for $J_2$: 0 and 0.147. It is shown a change from the type of trajectory K (green) to I (red). In this example, the velocity at the periapsis is 4.12 canonical units and the angle of approach $\Psi$ is 330º.

**Figure 2.** $J_2 = 0$.  
**Figure 3.** $J_2 = J_2$ of Jupiter.
Figure 4. $J_2 = 5$ times $J_2$ of Jupiter.

Figure 5. $J_2 = 10$ times $J_2$ of Jupiter.

Figure 6. $J_2 = 0$

Figure 7. $J_2 = J_2$ of Jupiter
4. Conclusions
It was showed that the inclusion of the oblateness of the planet causes a variation in the trajectories of the spacecraft. Several values for $J_2$ were used and it was observed a modification of the trajectories, in particular in the border lines between different families of orbit, where there are strong changes in the types of orbits caused by the oblateness of the planet. The physical reason for this variation is that the oblateness changes the mass distribution of the body, reducing the average distances from each mass.
element to the spacecraft, which is equivalent to an increase in the mass of the planet, thereby increasing the effect of the passage of the vehicle.

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