Robust Non-Coherent Beamforming for FDD Downlink Massive MIMO

François Rottenberg*, Ming-Chun Lee*, Thomas Choi*, Jianzhong Zhang† and Andreas F. Molisch*,
*University of Southern California, Los Angeles, CA, USA,
†Samsung Research America, Richardson, TX, USA

Abstract—Designing beamforming techniques for the downlink (DL) of frequency division duplex (FDD) massive MIMO is known to be a challenging problem due to the difficulty of obtaining channel state information (CSI). Indeed, since the uplink-downlink bands are disjoint, the system cannot rely on channel reciprocity to estimate the channel from uplink (UL) pilots as in time division duplexing (TDD) system. Still, in this paper, we propose original designs for robust beamformers that do not require any feedback from the users and only rely on the transmission of UL pilots. The price to pay is that the beamformer is non-coherent in the sense that it does not leverage full knowledge of the phase of each multipath component. A large variety of novel designs are proposed under different criterion and partial phase knowledge.

Keywords—Massive MIMO, FDD, Robust Beamforming, Non-Coherent.

I. INTRODUCTION

The large benefits promised by massive MIMO technology critically rely on the knowledge of CSI at the base station (BS). In FDD, the BS cannot rely on channel reciprocity from UL pilots to obtain this CSI since the UL and DL bands are disjoint. This makes the acquisition of DL CSI much more challenging. Many works have been proposed in the literature to address this problem. Most of them rely on a limited amount of feedback from the users based on DL pilots, e.g., [1]–[3]. However, this has the disadvantage of generating an overhead and requires the channel to be static between transmission of the DL pilots, feedback, and use of the CSI in beamforming. An interesting alternative is channel extrapolation from the UL to the DL band as it completely removes the overhead. Such extrapolation can be realized by using high-resolution techniques [4, 5]. The theoretical performance of such methods was studied by us in [6] and we showed that the MSE of the extrapolated channel scales with the ratio of the extrapolation range and the uplink bandwidth.

However, high-resolution techniques suffer from a large implementation complexity. Moreover, channel extrapolation becomes increasingly difficult for large FDD separation between UL-DL bands, as the phase of each path becomes inaccurate. In this paper, we investigate the design and the performance of robust non-coherent beamformer at the BS, which is particularly justified when the phase of each path is completely or partially unknown. Though suboptimal (except in particular cases), non-coherent beamformers can be designed relying only on UL pilots (through the spatial covariance of the channel) and thus remove the need of a DL feedback, while leveraging the knowledge of only partial phase knowledge. This problem was actually already studied in [7, 8].

The main novelty of this paper firstly resides in a more general formulation including partial phase knowledge through their covariance matrix and secondly in analyzing a large variety of robustness criteria. Indeed, various designs are considered, first in the single-user (SU) case, including maximizing the stationary signal-to-noise ratio (SNR) or the worst-case performance. In the multiple-user (MU) case, both zero forcing (ZF) and signal-to-noise-and-leakage ratio (SNLR) are investigated.

Notations: Vectors and matrices are denoted by bold lowercase and uppercase letters, respectively. Superscripts *, † and H stand for conjugate, transpose and Hermitian transpose operators. The symbols |·|, ∥·∥2, tr(·), E(·), S(·) and H(·) denote the complex modulus, the Frobenius norm, imaginary unit, trace, expectation, imaginary and real parts, respectively. Vector v_{max}(X) is the eigenvector of matrix X associated to its largest eigenvalue, denoted by \lambda_{max}(X).

II. CHANNEL MODEL

We consider a massive MIMO system composed of a single BS equipped with a total of N antennas and K single-antenna users. The BS and the users communicate using the cyclic prefix-orthogonal frequency division multiplexing (CP-OFDM) modulation. We denote by h_k(f, t) ∈ \mathbb{C}^{N \times 1} the channel vector of user k, evaluated at subcarrier frequency f and at time t. As depicted in Fig. 1, we assume that h_k(f, t) is composed of a total of L_k multipath components and the l-th path is fully characterized by its delay \tau_{k,l}, its Doppler shift ν_{k,l}, its complex gain a_{k,l} and its azimuth and elevation angles \phi_{k,l} and \theta_{k,l} at the BS,

\begin{align}
&h_k(f, t) = \sum_{l=1}^{L_k} a_{k,l} e^{-j2\pi f \tau_{k,l} + j2\pi f \nu_{k,l}},
&\quad (1)
\end{align}

where \mathbf{a}(\phi, \theta, f) ∈ \mathbb{C}^{N \times 1} is the array steering vector evaluated in direction (\phi, \theta) and at frequency f. We refer the interested reader to [9] for more information on the underlying assumptions of this model. In the following, we rewrite (1) as

\begin{align}
&h_k(f, t) = \sum_{l=1}^{L_k} a_{k,l}(f, t) e^{j\nu_{k,l}(f, t)},
&\quad (2)
\end{align}

Fig. 1: Massive MIMO multipath propagation channel.
where $a_{k,l}(f,t)$ is the spatial signatures of path $l$ and $\epsilon_{k,l}(f,t)$ is the so-called instantaneous phase uncertainty of path $l$. We refer to the previous model as non-coherent channel model. This is motivated by the fact that the phases $\epsilon_{k,l}(f)$ of each path are highly sensitive variables. In the UL band, if pilots are regularly sent by user $k$, the phases can be accurately estimated. However, if the BS uses a different band for the DL, as in FDD operation, the estimation/extrapolation of the phase $\epsilon_{k,l}(f)$ is much more intricate [9]. Note that, in the special case $\epsilon_{k,l}(f,t) = 0$, no uncertainty remains and the model becomes coherent again.

On the other hand, the spatial signatures $a_{k,l}(f,t)$ appearing in [2] have a more deterministic nature, as they depend on the power of each path and its incoming direction $(\phi_{k,l}, \theta_{k,l})$. Moreover, the vectors $a_{k,l}(f,t)$ can be estimated from UL pilot estimates and extrapolated in the DL band, e.g., for FDD operations. This is why we will assume in the remainder of this paper that we have perfect knowledge of the spatial signatures of each user.

In the following, we consider the transmission of an OFDM symbol at a specific subcarrier frequency and a multicarrier symbol. For clarity and without loss of generality, we drop the frequency-time dependence in (4)

$$ h_k = \sum_{l=1}^{L_k} a_{k,l} e^{j\epsilon_{k,l}} = A_k v_k, $$

where $A_k = (a_{k,1}, ..., a_{k,L_k}) \in \mathbb{C}^{N \times L_k}$ and $v_k = (e^{j\epsilon_{k,1}}, ..., e^{j\epsilon_{k,L_k}})^T \in \mathbb{C}^{L_k \times 1}$. In DL, the BS uses the beamforming vector $g_k \in \mathbb{C}^{N \times 1}$ to transmit the symbol $s_k$ to user $k$. The symbols $s_k$ have variance $p_k$ and are uncorrelated. The beamforming vectors are normalized so that $\|g_k\|^2 = 1$. The received sample at user $k$ can then be written as

$$ r_k = g_k^H h_k s_k + \sum_{k' \neq k} g_k^H h_k s_{k'} + w_k, $$

where $w_k$ is zero mean additive complex circularly symmetric white Gaussian noise of variance $\sigma^2$.

### III. Robust Beamforming

The challenge that we address in this section is the design of the beamforming vector $g_k$ for fixed power $p_k$ given the uncertainty on $h_k$ through the phases $\epsilon_{k,l}$, that we consider as random variables. In the following, we assume that the users are capable of estimating their equivalent channel $g_k^H h_k$ during data reception and thus can perform a phase rotation on $r_k$ (coherent detection), which is implicitly captured by the considered objective functions. We first focus on the single-user case before addressing the multi-user case.

#### A. Single-User Case

In this case, no inter-user interference (IUI) is present and we drop the subscript $k$ in every user-dependent notation for clarity. Equation (3) simplifies to

$$ r = g^H h s + w, $$

where $h = A v$ and $v = (e^{j\epsilon_1}, ..., e^{j\epsilon_L})^T$. The single-user problem simplifies to maximizing the beamforming power $|g^H h|^2$, which provides robustness against additive noise.

1) Stationary Beamforming Power Criterion: Since the instantaneous value of $h$ is not known we propose first to maximize the so called stationary beamforming power obtained by averaging $|g^H h|^2$ over the statistics of the instantaneous phase uncertainties $\epsilon_i$. Given the phase correlation matrix $R = \mathbb{E}(vv^H)$, we find $\mathbb{E}(|h|^2) = ARA^H$ and the optimization can be formulated as

$$ g_{\text{Station.}} = \arg \max_g \mathbb{E}|g^H h|^2 \text{ s.t. } \|g\|^2 \leq 1 $$

$$ = \arg \max_g g^H ARA^H g \text{ s.t. } \|g\|^2 \leq 1. $$

The solution can be easily found as the dominant eigenvector of matrix $ARA^H$

$$ g_{\text{Station.}} = \mathbf{v}_{\max}(ARA^H), $$

which physically implies that the BS should form a beam in the dominant spatial direction, resulting in a stationary beamforming power equal to the largest eigenvalue of $ARA^H$ that we denote by $\lambda_{\max}(ARA^H)$. This performance has to be compared to the one of the optimal coherent beamforming vector $g = h/\|h\|$, obtained as a special case of above derivation for an all-one $R$ matrix and achieving a stationary beamforming power $\mathbb{E}(|h|^2) = tr(ARA^H)$. Denoting by $\tilde{r}$ the rank of $ARA^H$, we can easily find the following upper and lower bounds for $\lambda_{\max}(ARA^H)$

$$ \frac{tr(ARA^H)}{\tilde{r}} \leq \lambda_{\max}(ARA^H) \leq tr(ARA^H). $$

In practice, knowledge of $R$ might be difficult to obtain. The most pessimistic case is then to consider i.i.d. uniformly distributed in $[0, 2\pi]$ phases, giving $R = I_L$. Depending on the conditioning of $AA^H$, the two following extreme cases can be distinguished: i) the upper bound of (7) is reached by the non-coherent beamformer in the case of a rank-one channel, i.e., $\tilde{r} = 1$, which is likely to happen if the user has one strongly dominant spatial direction as user locations 1 and 2 in Fig. 2. In that case, the coherent and non-coherent beamforming gains are equivalent in the stationary/"average" sense. ii) the lower bound of (7) arises if the channel has $\tilde{r}$ orthogonal equipowered spatial directions. For instance, if the three paths arriving at user location 3 in Fig. 2 are spatially well separated and of equivalent power, the non-coherent beamformer will perform about three times ($\approx 4.8$ dB) worse than the coherent one. In the most pessimistic case, the channel has $\tilde{r} = N$ equipowered orthogonal spatial modes and the stationary performance corresponds to the one of the uniform beamformer $g = 1/\sqrt{N}$ implying that the knowledge of spatial signatures does not provide any advantage over an agnostic BS.

2) Worst-Case Beamforming Power Criterion: Only the stationary beamforming power is maximized in (5) while no guarantees are provided on the instantaneous one. In this sense, the user might be subject to fades across time and frequency. To avoid this, we could instead design the beamformer vector in order to maximize the worst-case performance, which we formalize as

$$ g_{\text{Worst}} = \arg \max_g \min_{\epsilon_1, ..., \epsilon_L} |g^H h|^2 \text{ s.t. } \|g\|^2 \leq 1, $$

$$ = \arg \max_g \min_{\epsilon_1, ..., \epsilon_L} |\sum_{l=1}^{L} g^H a_l e^{j\epsilon_l}|^2 \text{ s.t. } \|g\|^2 \leq 1. $$
worst-case can make a few first observations. Depending on the channel three spatially well separated paths with similar power. Fig. 2: Different potential user locations. Location 1 is pure stationary paths coming from the same direction, having the same power be unavoidable. For instance, consider the case of a zero coherent beamforming.

We can also see that, if at least one path is orthogonal to but different phases \( \epsilon \), both criteria. Furthermore, investigating further the problem which is non convex but can be minimized using numerical methods \([10]\). Since has the form of a difference of convex (DC) functions in \( \epsilon \), one can check that, for a fixed \( l \), the beamforming vector arises if the two paths have opposite phases, i.e. \( \epsilon_1 = -\epsilon_2 \mod 2\pi \) and is independent of the beamforming vector \( g \). If such a case occurs in practice, the BS can either (i) still use a non-coherent beamformer and rely on coding across time and frequency to recover from potential fades; (ii) transmit one pilot in the direction \( a_1 \), obtain some feedback information from the user and perform coherent beamforming.

We can also see that, if at least one path is orthogonal to all others (as user location 3 in Fig. 2), it is possible to avoid a zero worst-case by using a beamforming vector being the matched filter of the orthogonal path spatial signature. These observations illustrate that a specific propagation scenario might be optimistic regarding the stationary beamforming power while performing very bad in the worst-case sense and vice-versa, as illustrated by user locations 2 and 3 in Fig. 2. Note also that user location 1 performs very well regarding both criteria. Furthermore, investigating further the problem (5) and assuming that \( \epsilon_1 \in [0, 2\pi], \forall l \), it is possible to show that it is equivalent to solving

\[
\mathbf{g}_{\text{Worst}} = \arg \max_l |g^H_{l}a_l| - \sum_{l=1, l \neq l'}^{L} |g^H_{l}a_l| \text{ s.t. } \|g\|^2 \leq 1,
\]

which intuitively implies that the worst-case arises if the amplitude related to the strongest path is affected by destructive interference from all other paths. If the value of the objective function is zero, it implies that a zero worst-case is unavoidable. One can check that, for a fixed \( l' \), the objective has the form of a difference of convex (DC) functions in \( g \), which is non convex but can be minimized using numerical methods \([10]\). Since \( L \) is finite and known, one can solve the problem for each \( l' \in \{1, ..., L\} \) and find then optimal \( l' \). This is summarized in the Algorithm 1.

### Algorithm 1: Max worst-case beamforming power

For each \( l' \in \{1, ..., L\} \), iteratively solve

\[
\min_g -|g^H_{l'}a_{l'}| + \sum_{l=1, l \neq l'}^{L} |g^H_{l}a_l| \text{ s.t. } \|g\|^2 \leq 1
\]

(0) Initialize \( g^{(0)} \) and \( n = 0 \).

1. Linearize \(-|g^H_{l'}a_{l'}|\) around \( g = g^{(n)} \) by using subgradient and update \( g^{(n+1)} \) by solving the convexified problem.

2. \( n \to n + 1 \), go back to step (1) until convergence.

**Solution:** Keep optimal \( g \) over \( l' \).

#### 1) Zero Forcing Design: Looking at (3), a straightforward choice to completely remove the IUI is to ensure that \( g^H_k h_{k'} = 0, \forall (k, k') \) s.t. \( k \neq k' \), implying that the signal intended to each user does not interfere with other users. Since \( h_{k'} = A_{k'} v_{k'} \), this can be satisfied if \( g^H_k \) lies in the left null space of the interference matrix \( A_k \triangleq (A_1, ..., A_{k-1}, A_{k+1}, ..., A_K) \) of dimension \( N \times L_k \) with \( L_k = \Delta \sum_{k'=1,k' \neq k}^{K} L_{k'} \). Let us denote the rank of \( A_k \) by \( r_k \). We define its singular value decomposition (SVD) as

\[
A_k = (\mathbf{U}_{k,1} \quad \mathbf{U}_{k,2}) \left( \begin{array}{ccc} \Sigma_k & 0_{N-r_k \times L_k-r_k} \\ 0_{r_k \times L_k-r_k} & 0_{r_k \times r_k} \end{array} \right) \mathbf{V}_k^H,
\]

where \( \mathbf{U}_{k,1} \in \mathbb{C}^{N \times r_k} \), \( \mathbf{U}_{k,2} \in \mathbb{C}^{N \times N-r_k} \), \( \Sigma_k \in \mathbb{R}^{r_k \times r_k} \) and \( \mathbf{V}_k \in \mathbb{C}^{L_k \times r_k} \) and \( \mathbf{V}_k \in \mathbb{C}^{L_k \times L_k} \). The ZF condition implies that \( g_k \) should adopt the following generic form

\[
g_k = P_k \tilde{g}_k, \quad P_k = U_{k,1}^{H} U_{k,2}^{H} \in \mathbb{C}^{N \times N}.
\]

We refer to the projection matrix \( P_k \) as a pre-beamforming matrix (similarly as was done in [11] for separating groups of users). Note that a projection matrix satisfies \( P_k = P_k^H \) and \( P_k^H P_k = P_k \). If the ZF criterion is fulfilled, the received signal at user \( k \), free from IUI, is given by

\[
r_k = \tilde{g}_k^H P_k h_k s_k + w_k = \tilde{g}_k^H \tilde{h}_k s_k + w_k,
\]

where \( \tilde{h}_k \triangleq P_k h_k = P_k A_k v_k \) is the equivalent channel after pre-beamforming. Note that (9) is freed from IUI and has a similar shape as (4) in the single-user case with an equivalent channel \( h_k \) instead of \( \tilde{h}_k \). This implies that the techniques developed in the single-user case can be straightforwardly applied in the multi-user case, i.e., the beamforming vector is then obtained by: (i) replacing the expression of matrix \( A_k \) and vector \( a_{k,l} \) in the single-user case by matrix \( P_k A_k \) and vector \( P_k a_{k,l} \) respectively, (ii) finding the optimal \( \tilde{g}_k \), and (iii) applying the pre-beamforming matrix \( P_k \).

For instance, the beamforming vector that would maximize the stationary beamforming power is given by

\[
g_{k,\text{Station}}^{ZF} = P_k v_{\text{Max}}(P_k A_k R_k A_k^H P_k) = v_{\text{Max}}(P_k A_k R_k A_k^H),
\]

where \( R_k \triangleq \mathbb{E}(v_k v_k^H) \). One can wonder about the impact of the pre-beamforming matrix \( P_k \). As depicted in Fig. 3 depending on channel conditions, three cases can be distinguished: (i) the spatial signatures of user \( k = 1 \) are orthogonal to the ones of all other users, i.e., \( A_1^H A_k = 0 \) implying that \( P_k A_k = A_k \) and hence, no performance loss is induced; (ii) the spatial signatures of user \( k = 2 \) are not orthogonal to the ones of all other users but cannot be written as an exact linear combination of them, inducing some performance loss,
and (iii) the spatial signatures of user 3 can be expressed as a linear combination of the ones of user 2 implying that \( P_3 A_3 = 0 \) and hence, no degrees of freedom are left after the pre-beamforming operation.

The discussion of the previous paragraph induces that a clever multiplexing of the user in the time-frequency-spatial plane should be performed similarly as in the coherent case. If the channels of two users experience closely related spatial structure, it might be preferable to "orthogonalize" them using different time-frequency resources rather than in the spatial domain. Another alternative is the transmission of a few pilots across different time-frequency resources rather than in the spatial plane should be performed similarly as in the coherent case.

At low SNR, the beamforming vector converges to the single-user solution previously derived in (9)

\[
\lim_{\rho_k \to +\infty} g_{k, \text{Station.}}^{\text{RZF}} = v_{\max}(A_k R_k A_k^H),
\]

which makes sense as the system is noise limited and IUI is negligible. We now analyze the high SNR regime. To simplify the analysis, we consider the i.i.d. phase case \( \mathbf{R}_k = I_{L_k}, \forall k \). Using the SVD of \( A_k \) and the Woodbury matrix inversion lemma, we can rewrite the inverse as

\[
(A_k \hat{A}_k^H + \rho_k I_N)^{-1} = (\hat{U}_{k,1} \Sigma_k^2 \hat{U}_{k,1}^H + \rho_k I_N)^{-1}
= \rho_k^{-1} \left(I_N - \hat{U}_{k,1} \left(\rho_k \Sigma_k^{-2} + I_{r_k}\right)^{-1} \hat{U}_{k,1}^H\right).
\]

Using the fact that \( P_k = I_N - \hat{U}_{k,1} \hat{U}_{k,1}^H \) and taking the limit, we find

\[
\lim_{\rho_k \to +\infty} g_{k, \text{Station.}}^{\text{RZF}} = v_{\max}\left(P_k A_k A_k^H\right).
\]

This result corresponds to the ZF solution obtained in (10) for \( \mathbf{R}_k = I_{L_k} \), which again makes sense as the system is limited by IUI.

IV. Simulation Results

This section aims at numerically validating the proposed designs. The channel frequency response is generated according to (1). We consider a single OFDM symbol so that the time dependence can be discarded. The bandwidth is set to 10 MHz. The path parameters (gains, delays and angles) are generated by the QuaDRiGa toolbox [13] according to the 3GPP TR 36.873 v12.5.0 specifications [14]. Both line-of-sight (LOS) and non-line-of-sight (NLOS) types of the model are used. Moreover, we generate the path parameters related to 100 users locations randomly distributed in a radius of 200 meters around the BS; the BS height is 20m above ground. The BS is equipped with a \( N = 128 \) isotropical elements (16 Horiz. × 8 Vert.) rectangular array. We assume that the spatial signatures \( a_{k,l} \) are perfectly known at the BS. We consider the pessimistic case \( \mathbf{R}_k = I_{L_k} \).

For computing the worst-case beamforming (BF), we implement Algorithm 1 using the CVX matlab toolbox [15]. For the sake of comparison, we also plot the performance of coherent beamforming, i.e., when the phases associated to each spatial signature is perfectly known. In the single-user case, we also include the performance of a uniform beamer \( \mathbf{g} = 1/\sqrt{N} \), which represents the case of no channel knowledge.

Single-User Case: We first consider the NLOS case. For one random location, Fig. 4 (a) plots the performance of the proposed non-coherent beamformers. On the one hand, the stationary beamformer performs well on average but is subject to potential fades. On the other hand, the worst-case beamformer has a relatively stable performance across frequency. One can check that there is an approximately 10 dB loss in beamforming power with respect to the coherent beamformer. These observations are further confirmed in Fig. 4 (b) which shows the cumulative density function (CDF) of the beamforming power estimated over all user locations and frequencies. The LOS case is considered in Fig. 4 (c). In that case, the penalty of non-coherent beamformers is much
closed-form expressions and easy to compute in terms of non-coherent beamformers. Most of these designs are in respect of the NLOS and LOS performances are shown in Fig. 5 and 6 respectively.

**Multi-User Case:** We now consider the multi-user case of each user is set to 10 dB, i.e., $\rho_k^{-1} = 10$ dB. No power loading and scheduling are implemented even though we select at random 100 combinations of 5 users. The sum of path gains of each user $K = 5$ users, NLOS.

K = 5 users, NLOS

Fig. 5: Multi-user case: CDF over 100 user selections in NLOS.

K = 5 users, LOS

Fig. 6: Multi-user case: CDF over 100 user selections in LOS.

reduced. Indeed, since there is one strongly dominant path, it is sufficient to form a beam in that direction and knowledge of the instantaneous phases of other paths is less important.

Multi-User Case: We now consider the multi-user case with $K = 5$ users. From the 100 generated user locations, we select at random 100 combinations of 5 users. The sum of path gains of each user $\alpha_{k,l}$ is normalized to one and the SNR of each user is set to 10 dB, i.e., $\rho_k^{-1} = 10$ dB. No power loading and scheduling are implemented even though we expect that they are crucial for non-coherent beamforming. The NLOS and LOS performances are shown in Fig. 5 and 6 respectively.

V. CONCLUSION

In this paper, we have investigated the design of robust non-coherent beamformers. Most of these designs are in closed-form expressions and easy to compute in terms of complexity and thanks to the fact that they do not rely on any user feedback, directly applicable in FDD massive MIMO systems.

REFERENCES

[1] A. Adhikary, J. Nam, J. Ahn, and G. Caire, “Joint Spatial Division and Multiplexing - The Large-Scale Array Regime,” IEEE Transactions on Information Theory, vol. 59, no. 10, pp. 6441–6463, Oct 2013.
[2] X. Rao and V. K. N. Lau, “Distributed Compressive CSIT Estimation and Feedback for FDD Multi-User Massive MIMO Systems,” IEEE Trans. Signal Process., vol. 62, no. 12, pp. 3261–3271, June 2014.
[3] M. Barzegar Khalilisarai, S. Haghighatshoar, X. Yi, and G. Caire, “FDD Massive MIMO via UL/DL Channel Covariance Extrapolation and Active Channel Sparsification,” IEEE Trans. Wireless Commun., vol. 18, no. 1, pp. 121–135, Jan 2019.
[4] M. Pan, A. F. Molisch, P. Orišek, and A. Okazaki, “Super-Resolution Blind Channel Modeling,” in 2011 IEEE International Conference on Communications (ICC), Kyoto, Japan, June 2011, pp. 1–5.
[5] W. Yang, L. Chen, and Y. E. Liu, “Super-Resolution for Achieving Frequency Division Duplex (FDD) Channel Reciprocity,” in 2018 IEEE 19th Internat. Workshop on Signal Proc. Adv. in Wireless Communications, Kalamata, Greece, June 2018, pp. 1–5.
[6] F. Rottenberg, R. Wang, J. Zhang, and A. F. Molisch, “Channel Extrapolation in FDD Massive MIMO: Theoretical Analysis and Numerical Validation,” in 2019 IEEE Global Communications Conference (GLOBECOM), Waikoloa, USA, Dec. 2019.
[7] P. Zetterberg and B. Ottersten, “The spectrum efficiency of a base station antenna array system for spatially selective transmission,” IEEE Trans. Veh. Technol., vol. 44, no. 3, pp. 651–660, Aug 1995.
[8] K. Hugl, J. Laurila, and E. Bonek, “Downlink beamforming for frequency division duplex systems,” in Seamless Interconnection for Universal Services. Global Telecommunications Conference. GLOBECOM’99., vol. 4, Dec 1999, pp. 2097–2101 vol.4.
[9] F. Rottenberg, T. Choi, P. Luo, J. Zhang, and A. F. Molisch, “Performance Analysis of Channel Extrapolation in FDD Massive MIMO Systems,” arXiv preprint arXiv:1904.00798, 2019.
[10] A. L. Yuille and A. Rangarajan, “The Concave-Convex Procedure,” Neural Computation, vol. 15, no. 4, pp. 915–936, 2003.
[11] A. Adhikary, J. Nam, J. Ahn, and G. Caire, “Joint Spatial Division and Multiplexing - The Large-Scale Array Regime,” IEEE Transactions on Information Theory, vol. 59, no. 10, pp. 6441–6463, Oct 2013.
[12] E. Björnson, M. Bengtsson, and B. Ottersten, “Optimal Multiuser Transmit Beamforming: A Difficult Problem with a Simple Solution Structure [Lecture Notes],” IEEE Signal Processing Magazine, vol. 31, no. 4, pp. 142–148, July 2014.
[13] S. Jaekel, L. Raschkowski, K. Börner, and L. Thiele, “QuadDRiGa: A 3-D Multi-Cell Channel Model With Time Evolution for Enabling Virtual Field Trials,” IEEE Trans. Antennas Propag., vol. 62, no. 6, pp. 3242–3256, June 2014.
[14] “3GPP TR 36.873 v12.5.0,” Tech. Rep., 2017.
[15] M. Grant and S. Boyd, “CVX: Matlab software for disciplined convex programming, version 2.1,” Mar. 2014.