Aspects of the confinement mechanism in Landau gauge QCD

Kai Schwenzer
Institut für Physik, Karl-Franzens Universität Graz, Universitätsplatz 5, 8010 Graz, Austria

I analyze the IR fixed point structure of Landau gauge QCD. Precisely the fixed point with a strong kinematic singularity of the quark-gluon vertex that proved crucial for the recently proposed confinement mechanism in the quenched approximation is absent in dynamical QCD. Therefore, the IR singularities do not induce asymptotic quark confinement but the long-range interaction is screened by unquenching loops at scales of the order of the quark mass. This provides the prerequisite for a microscopic description of deconfinement and string breaking. The fixed points determine the qualitative form of the heavy quark potential and may be relevant for hot and dense matter.

Quark confinement is a manifestly non-perturbative phenomenon which is inherently scale dependent and features several crucial aspects that a possible mechanism has to explain. In order to give such an explicit dynamical confinement mechanism in terms of the underlying colored degrees of freedom this entails to actually specify them and correspondingly to fix a gauge. In the absence of dynamical quarks the most direct signature for confinement is an area law of large Wilson loops [1] that can be directly related to a linear rising potential between static color sources. In momentum space Green’s functions this long-range physics is encoded in the infrared (IR) regime. The IR divergences of Green’s functions are expected to offer an explanation for confinement since the early work of Weinberg [2]. In Landau gauge Mandelstam [3] provided such a mechanism in a simplified approximation to the gluon Dyson-Schwinger equation (DSE) leading to a strongly divergent gluon propagator that scales $\sim 1/p^4$ in the IR limit. Yet, it has been shown recently that an IR enhanced gluon propagator is incompatible with the DSEs for the gluonic vertices [4]. Instead there is a consistent IR scaling fixed point of Yang-Mills theory (YMT) where the ghost sector of the theory dominates in the IR limit whereas the gluon dynamics is suppressed [5]. This scaling solution provides a mechanism for gluon confinement within the scenarios of Kugo-Ojima [7] and Gribov-Zwanziger [8, 9]. It extends to arbitrary Green’s functions [10] and kinematics [11], represents the unique IR scaling fixed point [11] and is also obtained with other functional methods [11]. However, it posed a puzzle how suppressed gluons can provide the strong interaction between color sources as seen in lattice QCD [1].

Recently it was shown in the quenched approximation that this fixed point of Landau gauge YMT indeed induces a strong IR quark interaction that leads to a linear, confining potential between quarks [1]. This confinement mechanism relies on a self-consistently enhanced kinematic divergence of the quark-gluon vertex in the soft-gluon limit that the external gluon momentum vanishes. The question is then how unquenching effects can circumvent this static confinement and thereby account for string breaking and hadronization seen in collider experiments. Such a mechanism must be related to scales of the order of the masses of created hadrons. In this work I outline a comprehensive analysis of the IR fixed point structure of Landau gauge QCD in the unquenched case. It confirms the screening of gluonic interactions via quark loops in the on-shell region that could “break the gluonic string” and drive the hadronization process. The non-perturbative dynamics of QCD is described by the system of DSEs which is generated algorithmically [13]. The DSEs form an infinitely coupled tower of equations that in general requires approximations. Yet, it can be shown [14] that the equations for all $n$-point functions with $n > 4$ are linear in the sense that the corresponding $n$-point function appears at most once in each loop graph. As far as the IR singularities are concerned there is thereby no self-consistent enhancement mechanism in these equations. They also include no tree level term and are hence effectively determined by lower order Green’s functions. Therefore, any IR enhancement has to be generated by the primitively divergent Green’s functions and the truncation can be restricted to the latter. Moreover, it has been found previously [11] that all contributions including a 4-gluon vertex are strongly IR suppressed so that this vertex can be neglected as well. However, the DSE system features a peculiarity compared to other functional approaches like the functional renormalization group (FRG) [11] or $n$-particle irreducible (nPI) actions [15] in that it involves a bare vertex in every DSE so that IR strength can be missing in the lowest order diagrams when they involve enhanced vertices. Since ghosts and quarks do not couple at the tree level this requires to include the ghost-quark vertex to consider all IR leading terms. The considered truncation is then given by the DSEs in the gauge and matter sector in figs. [1] and [2].

For the analysis of the IR regime a flexible power counting scheme [11, 13, 10] has been developed that yields the IR fixed point structure without an actual solution of the non-linear integral equations. Here I only sketch the main steps of the analysis which is similar to the one previously performed in the gauge sector [4] and extends the one in the quenched approximation [12]. The details will be presented in a forthcoming article [13]. The infrared analysis relies on the observation that YMT is scale invariant at the classical level. Quantum fluctuations in-
roduce a dynamical scale $\Lambda_{\text{QCD}}$ and in QCD the matter sector introduces in addition explicit mass scales, but it is experimentally established that strong interaction has no scales far below the MeV scale. Correspondingly, it is a reasonable assumption that in the IR regime $p \ll \Lambda_{\text{QCD}}$ the scale dependence of all Green’s functions is given by a power law scaling in terms of the the dimensionless ratio $p = p/\Lambda_{\text{QCD}}$, parametrized for the propagators by

$$D_{\mu \nu}^{ab} (p) \sim (p^2)^{\delta_{gl} - 1}, \quad G^{ab} (p) \sim (p^2)^{\delta_{gh} - 1}, \quad S (p) \sim (p^2)^{\delta_q}$$

where $\delta_{gl}$, $\delta_{gh}$ and $\delta_q$ denote the anomalous IR exponents, distinguished by lower indices. The vertices depend on several external momenta and can feature divergences in various kinematic configurations that are described by different IR exponents $\delta_{gl}$, $\delta_{gh}$, distinguished by upper indices. In the uniform limit that all external momenta vanish these exponents are $\delta_{gg}$ for the ghost-gluon vertex, $\delta_{3g}$ for the 3-gluon vertex, $\delta_{ug}$ for the quark-gluon vertex and $\delta_{qgh}$ for the quark-ghost-gluon vertex. In addition there are exponents $\delta_{gggh}$, $\delta_{ggh}$, $\delta_{ghh}$ when two external momenta vanish, $\delta_{ggh}$, $\delta_{ghh}$, $\delta_{gg}$, $\delta_{ggh}$, $\delta_{gggh}$, $\delta_{ghh}$, $\delta_{qgh}$, $\delta_{ghh}$ when a single external momentum vanishes and finally $\delta_{ghh}$ when only the difference of the two external quark respectively ghost legs is soft. There are different tensor structures these are the exponents of the IR leading part(s). The loop integrals in the DSEs are dominated by the singularities of the integrand and receive contributions from loop momenta in the vicinity of all external momentum and mass scales $\Lambda_{\text{QCD}}$, i.e. in QCD also by scales of the order of the constituent quark masses. In the IR limit there is

![Figure 1: The gauge part of the considered coupled DSE system. Ellipses denote graphs involving neglected 4- and 5-point functions and closed quark loops are absent in quenched QCD.](image1)

![Figure 2: The matter part of the considered DSE system.](image2)

a clear scale separation and one can divide the contributions from soft and hard loop momenta. By determining the IR scaling of these distinct contributions from the different loop graphs via a power counting analysis the DSE system translates to a coupled set of algebraic equations for the anomalous exponents. In each DSE the IR leading term on the right hand side, corresponding to the smallest IR exponent, determines the IR behavior of the Green’s function. When hard momenta are present there are various possible cancellations in the IR analysis that affect the canonical scaling of Green’s functions. These arise from gluon transversality, the symmetry of Green’s functions $[4]$ and from the Dirac structure $[12]$. There is an additional suppression of the contribution from hard loop momenta $k$ where the quark propagator can be expanded in the soft external momentum $p$

$$Z((k+p)^2) = \frac{Z(k^2)}{ik + M(k^2)} \left(1 - \frac{2p \cdot k}{k^2 + M^2(k^2)} + \cdots \right)$$

which shows that if there is no constituent quark mass $m_q = M(0) = 0$ or another hard scale in the integral the leading term is scale-free and is removed in the renormalization procedure so that the propagator is actually suppressed in $p$. The gauge propagators feature an analogous suppression by $p^2 [4]$. In the power counting analysis this is considered by symbols that take values $\mu_q = \frac{1}{2}$ and $\mu_{gl} = 1$ when the propagators are massless, i.e. $\delta_q = - \frac{1}{2}$ and $\delta_{gl} = 0$, and which vanish otherwise. Analogous symbols $m_q^2$ and $\mu_q^2$ are introduced at the tree level.

The power counting analysis for the DSE system figs. $[1]$ and $[2]$ results in an extensive system of coupled algebraic equations for the above 18 IR exponents $[14]$. Fortunately, there are important constraints on the IR exponents, given in $[4]$, and analogous ones in the quark sector that simplify the system considerably. Previously such constraints had been obtained from the assumption that a skeleton expansion of the vertices should not explicitly diverge. Yet, the same constraints are obtained independently from the complementary set of FRG equations without additional assumptions $[11]$ $[16]$. Since the equations for the quark-ghost vertex are linear they can be solved by successive replacements of the possible solutions and the connection between the exponents in different kinematic limits provides further complementary constraints $[14]$. Applying the constraints yields that the fermionic 3-point vertices are not divergent when only a fermion momentum vanishes $\delta_{gg} = \delta_{gg} = 0$ and leaves the simplified system for the IR exponents of the propagators

$$1 - \delta_{gh} = \min \left(1; \delta_{gg} + \delta_{gh} + \delta_{gl} + 1\right)$$

$$1 - \delta_q = \min \left(1; \delta_{gg} + \delta_{gh} + \delta_{gl} + 1; \mu_q\right)$$

$$1 - \delta_{gl} = \min \left(1; \delta_{gg} + \delta_{gh} + \delta_{gl} + 1; \mu_q; \delta_{gg} + \mu_{gl}\right)$$

$$\left[\delta_{gg} + 2\delta_{gh} + 2\delta_{gg} + \mu_q\right]$$
and of the remaining primitively divergent vertices

\[
\delta_{gg}^{u} + \frac{1}{2} = \min\left(\frac{1}{2}; \cdots; 2\delta_{gg}^{u} + \delta_{gh} + 2\delta_{gl} + \frac{1}{2}; 2\delta_{q} + 2\mu_{gl} + \mu_{q} + \frac{1}{2}; 2\delta_{gg}^{u} + 2\delta_{gl} + \frac{1}{2}; 2\delta_{gg}^{u} + 2\delta_{gg}^{u} + \delta_{gh} + 2\delta_{gl} + \frac{1}{2}; \right)
\]

\[
\delta_{gg}^{gl} = \min\left(0; \delta_{gh} + 2\delta_{gg}^{u} + \delta_{gl} + \frac{1}{2}; 2\delta_{gg}^{u} + 2\delta_{gg}^{u} + 2\delta_{gl} + 3\delta_{gh} + 2\delta_{gl} + 1\right)
\]

\[
\delta_{gg}^{u} + \frac{1}{2} = \min\left(\frac{1}{2}; 2\delta_{gg}^{u} + 3\delta_{gh} + \frac{1}{2}; 2\delta_{gg}^{u} + 1; 2\delta_{gg}^{u} + 1 + \frac{1}{2} + \mu_{gl}; 2\delta_{gg}^{u} + \delta_{gh} + \frac{1}{2} + \mu_{gl}
\]

\[
\delta_{gg}^{gl} = \min\left(0; \delta_{gh} + 1; 2\delta_{gg}^{u} + 2\delta_{gl} + \frac{1}{2}; 2\delta_{gg}^{u} + 2\delta_{gg}^{u} + \delta_{gh} + \delta_{gl} + \frac{1}{2}; \right)
\]

\[
\delta_{gg}^{u} = \min\left(0; \cdots; \delta_{gg}^{u} + 2\delta_{gg}^{u} + 3\delta_{gh} + \delta_{gh} + \delta_{gl} + \frac{1}{2}; \right)
\]

\[
\delta_{gg}^{gl} = \min\left(0; \cdots; \delta_{gg}^{u} + 2\delta_{gg}^{u} + 3\delta_{gh} + 2\delta_{gl} + \frac{1}{2}; \right)
\]

where the contributions from different graphs in figs. [1] and [2] are separated by semicolons and contributions from different regions of the same loop integral by commas. It is instructive to recall the solution of the quenched system [12] first, where the terms in brackets arising from closed quark loops are absent. There the quark-gluon vertex features different solutions with a trivial respectively a strongly enhanced IR scaling. In the DSEs the latter arises from the IR region of the ghost loop diagram in fig. [2] corresponding to the first non-trivial term in eqs. [5] and [6]. Inserting the dressed quark-ghost vertex, results in a non-Abelian graph with a ghost-triangle, representing the IR leading contribution [4] to the 3-gluon vertex in eq. [6]. Since such a fully dressed non-Abelian graph arises directly both in the FRG [11] and \( \alpha \)PI actions [13], the dynamics seems to be represented more efficiently in these approaches. More importantly, for the strong soft-gluon singularity the unquenching contributions in the last term in brackets in eqs. [3] and [6] are incompatible with the gauge sector, e.g. \( 1 - \delta_{gl} \leq \delta_{gg}^{gl} \n \). The starting point for the full solution of the unquenched system is eq. [1] for the ghost propagator. As it stands the bare term is leading and in addition to the trivial, perturbative solution that is also realized in the ultraviolet (UV) regime one obtains a decoupling solution with a bare ghost and an IR finite gluon propagator [14]. Due to the suppressed gluonic interaction there are no IR enhanced Green’s functions. The decoupling solution seems to be found in current lattice studies [19]. Yet the study of the IR regime of a gauge theory on the lattice is a notoriously hard problem and there are several issues like the present implementation of the gauge condition and the related contamination with Gribov copies that demand caution [18]. In particular, since there is another solution that explains important qualitative aspects of strong interaction physics better: For an IR enhanced propagator the renormalization procedure can also be performed in a way that the tree level term in the ghost equation is cancelled identically. This yields the scaling solution where \( \delta_{gl} = -2\delta_{gh} \equiv 2\kappa \) [5], depending on a parameter \( \kappa \geq 0 \) since the gluon equation [3] is then identically fulfilled by the ghost-loop exponent [4]. The correspondingly subleading contribution from unquenching loops with hard momenta in eq. [6] yields the important constraint \( \delta_{gg}^{gl} \geq 1 - 2\kappa - \mu_{q} \). The scaling solution depends on the three qualitatively different cases of an infinite, a finite or a vanishing constituent quark mass in eq. [2]. The first case is similar to the quenched limit, but here quarks are entirely static and merely act as sources. Using the available constraints the solution of the residual system is possible without further assumptions and the resulting IR fixed points of QCD are given in table I. Strikingly for all QCD fixed points the gauge sector is not altered by the quark dynamics compared to the YMT result [4]. The main result of this article is, however, that due to the above constraint the solution with a strong soft-gluon singularity of the quark-gluon vertex is ruled out in dynamical QCD and only the solution with a trivial soft-scaling remains. This happens since hard virtual quark loops with momenta around the quark mass screen the long-range gluonic interaction.

Now let us discuss the impact of the IR divergences on quark confinement which should in the case of mesons be described by the 4-quark vertex DSE given in the first line of fig. [3]. The quarks in hadrons have momenta of order of hundreds of MeV and correspondingly the relevant kinematics for confinement is given by hard external quark momenta. Though the exchanged gluon momentum becomes soft when they are far spatially separated. Using the IR exponents in the static limit given in table I the first diagrams in fig. [3] do not induce strong long-range interactions since IR strength is missing due to the bare quark-gluon vertex [12]. However, this is not the case for the last graph with the higher order 4-quark-gluon vertex. Its DSE in the second line of fig. [3] includes graphs where all four quark-gluon vertices attached to the exchanged soft gluons are dressed. The kinematic regime where the small loop in the first line is dominated by momenta of the order of the quark mass and the one in the second line is soft yields a far stronger divergence. In the heavy mass limit the hard loop shrinks to a point and the
leading corrections reduce to graphs similar to the first two in the 4-quark DSE but with all four quark-gluon vertices dressed - again in complete agreement with the FRG \cite{11} or nPI methods \cite{13}. With the strong soft-gluon singularity of the dressed quark-gluon vertices this yields an IR interaction that scales $\sim 1/p^4$ corresponding to a linear, confining potential in coordinate space \cite{12}. In contrast, in the unquenched case the quark-gluon vertex is not enhanced corresponding to a 4-quark interaction $\sim p^8\kappa$ which yields a strongly suppressed long-range behavior and cannot be interpreted by a non-relativistic potential. Instead, the long-range interaction, found in the static respectively quenched case, is screened by virtual quark loops at scales of the order of the quark mass and dynamical QCD is indeed not asymptotically confining. Yet, in the heavy mass limit $m_q \gg \Lambda_{QCD}$ and for momenta $p\ll m_q$ where the quenched limit is valid approximation one can expect a linear potential over distances for which $V\sim \sigma x \ll 2m_q$. In the short distance regime $p\sim 1/x \gg \Lambda_{QCD}$ the perturbative UV fixed point is realized which yields an interaction $\sim 1/p^2$ corresponding to a Coulomb potential whereas the potential picture breaks down for distances $x \lesssim 1/m_q$. Correspondingly the qualitative form of the heavy quark potential is entirely determined by the fixed points of QCD as shown schematically in fig. 3. The arising screening scale is precisely of the order of the masses of corresponding mesonic states and thereby it is not hard to imagine that “the string breaks” and real particles are produced when the system is sufficiently excited. This is described by higher Green’s functions, like the 6-quark vertex, and it remains to be shown that only color singlets are produced. However, due to the intermediate rise in fig. 3 it is clear that produced quarks are confined into bound states when their excitation energy is small enough. The presence of light quarks surely goes beyond the present IR analysis and requires computations at finite hadronic scales. Finally the above fixed points could be relevant for hot and dense matter. In the limit of asymptotic temperature $T$ and/or chemical potential $\mu$ the quark masses become irrelevant in comparison and the chiral cases in table I which do not induce quark confinement, may be relevant for some scale region. Yet, in addition to the perturbative fixed point, describing complete deconfinement, the chiral scaling fixed points feature a confined gauge sector. The stronger one exhibits a Coulombic quark interaction as in the perturbative case and may be relevant for the strongly coupled plasma observed in heavy ion collisions. In contrast, the weaker one corresponds to a quark sector that completely decouples from a strongly coupled gauge sector in the IR limit. Interestingly, such a behavior has been independently found in an analysis of the low energy dynamics of ungapped modes at high $\mu$ \cite{33}.

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Table I: The anomalous power laws of the leading Green’s functions for the different fixed points of QCD distinguished by the constituent quark mass $m_q$. These power laws are only valid up to logarithmic corrections. The full verteces include also the canonical scaling dimension $-1$ for the gauge propagators and $\frac{1}{2}$ for the gauge 3-point vertices in the uniform limit. The value of $\kappa$ is fixed by an explicit IR solution and the best known value is $\kappa \approx 0.59$ \cite{6}. There are additional fixed points in the chiral case where the quark propagator is IR divergent \cite{14}, yet physically they are excluded by the current quark masses.

| Decoupling | $\delta_{pl}$ | $\delta_q$ | $\delta_m$ | $\delta_{pl}^u$ | $\delta_{pl}^d$ | $\delta_{pl}^u$ | $\delta_{pl}^d$ | $\delta_{pl}^{\mu}$ | $\delta_{pl}^{\mu}$ | $\delta_{pl}^{\mu}$ |
|------------|--------------|----------|-----------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| static ($m_q \rightarrow \infty$) / quenched | $-\kappa$ | $2\kappa$ | $\kappa$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ |
| massive ($m_q > 0, m_q^0 \geq 0$) | $-\kappa$ | $2\kappa$ | $\kappa$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ |
| chiral ($m_q = m_q^0 = 0$) | $-\kappa$ | $2\kappa$ | $\kappa$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ |
| perturbative ($\forall m_q$, both IR & UV) | $0$ | $1$ | $-\frac{1}{2} \vee 0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ |

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