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A Medium Model Leading to Analogy of Major Physics Laws

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The attempt of this paper is to suggest a new theoretical medium to support the efforts of building particle models as mechanical field structures. The medium model is based on two simple fundamental assumptions. All the properties of the medium, as counterparts in this medium of the well-known physics laws, including energy and momentum conservations, Lorentz transformation and special relativity, electromagnetic interaction, etc., are derived from these two assumptions. The governing equations are established based on the derived properties. Some steady solutions as well as the feasibility of interpreting these solutions as particles are also briefly discussed.

Keywords: Ether field; medium model; Lorentz transformation; relativity; electromagnetic field.

1. Introduction

The effort to find the fundamentals of particles and interactions has a long history and has never ended. In parallel to the quantum mechanics — Standard Model — string theory approach, there are some “non-mainstream” attempts to model elementary particles as certain kind of field structures. About 150 years ago, William Thomson (Lord Kelvin) made the “vortex atom” conjecture, and spent decades trying to approve it. Albert Einstein devoted most of his later years trying to find a uniform theory to explain particles and interactions. Many other attempts were made in more recent times. A big number of them are related to topological solitons and with knot and link structures (e.g., the Kamchatnov–Hopf soliton,4,5 the Skyrme–Faddeev model,4,5 and the glueball model6). Arminjon proposed to interpret gravity as pressure force in an ideal fluid.7 Dmitriyev also made a series of conjectures to interpret particles as structures in a mechanical medium.8,9
The purpose of this paper is to develop a simple but rich medium model, which potentially can serve as a new medium for the structures to interpret particles and interactions. We shall borrow a convenient name and call this medium ether field. However, we shall neither employ any preset property of the historical concept of ether as medium of light nor shall we preassume any known physical laws and apply them in a preset manner. The field model is based on two basic assumptions. From them we will derive a series of properties of the medium, which interestingly are almost exact analogs of the basic physical laws, including momentum and energy conservations (thus the motion laws and equations), Lorentz transformation (thus special relativity), electromagnetic (EM) field and a “gravity-like” interaction. If soliton-like solutions of the field equations exist (which in general is yet to be proved but some examples are given in this paper) and can be interpreted as particles, these properties of the medium potentially will provide a set of new explanations for the corresponding physical laws.

2. Ether, as a New Medium Model

Consider a scalar, non-negative real function of space and time, denoted by $\Omega(r, t)$. Borrowing a convenient name, we call $\Omega$ the ether field, and call the “amount” of ether field over a region of space, $Q = \int_V \Omega \, dv$ ($V \subseteq R^3$), ether. In other words, $\Omega$ is considered as density of ether over space.

We will assume that ether is conserved. This is the first of the two fundamental assumptions for ether field. Intuitively, this means a block of ether can move from one place to another, and can contract or expand, but the total amount (as integral of field strength over the volume) remains the same.

Strictly speaking, description of conservation relies on a reference system to enable the measurement of space (size, volume), time and speed. So this assumption actually implies the existence of such a reference system, and implies that speed (velocity) can be defined over each point of the field.

Under this assumption (hereafter in this paper we will call it “ether conservation assumption” or ECA), the following continuity equation holds:

$$\frac{\partial}{\partial t} \Omega + \nabla \cdot (\Omega \mathbf{u}) = 0.$$  \hspace{1cm} (2.1)

For a block of stationary field, we further define energy as $E = \int_V \Omega^2(r) \, dv$. Obviously, here we take square of ether field strength as “energy density”. Note that this is just a formalistic definition, and “energy” is another borrowed name. It does not bring any preassumed property, e.g., conservation. Indeed, conservation of energy will be derived from the basic assumptions.

Also note that this definition is only for stationary field. Energy of dynamic field will be defined in a later section.

Now let us consider a rectangular block of stationary and uniform field $\Omega_0$, with $L$ as length and $s$ as area of the left-hand side $S$ (Fig. 1). Here $S$ is considered the
interface between the ether inside the block and the ether outside. When $S$ moves, we assume that no ether will pass through it. According to ECA, amount of ether on each side of $S$ will remain unchanged.

In the case that $S$ moves towards right a small distance $\delta$ and other sides of the block remain in the same positions, assuming this procedure is quasi-static, i.e., field remains stationary and uniform all the time, after the movement, field strength inside the block becomes

$$\Omega' = \Omega_0 \frac{L}{L - \delta},$$

and energy of the block becomes

$$E' = \Omega'^2 s(L - \delta).$$

As $\delta \ll L$, the change of energy is

$$\Delta E = E' - E_0 = \Omega_0^2 sL \frac{\delta}{L - \delta} \approx \Omega_0^2 s\delta.$$

While no ether being exchanged between both sides of $S$, the quasi-static movement of $S$ compresses (if $\delta > 0$) or expands (if $\delta < 0$) the field in the block, thus causing energy to increase or decrease in it. The amount of increased/decreased energy is proportional to square of the field strength (i.e., energy density), area of $S$ and distance of movement in the normal direction, but is independent of volume of the field block (as long as $L \gg \delta$ and the procedure is quasi-static).

The same calculation applies to a general case of an area element (i.e., an area of interface) moving quasi-statically in a stationary field $\Omega$. As ether is compressed on one side and expanded on the other side, energy increases on one side and decreases on the other, by the same amount. This is equivalent to an energy exchange between the two sides. Assuming the normal distance that the area element moves is $\delta$, the amount of exchanged energy is

$$\Delta E \approx \Omega^2 s\delta = P_0 s\delta \quad (\text{here } P_0 \triangleq \Omega^2). \quad (2.2)$$

Obviously, if we make analogy between ether field and ideal gases, we can define $P_0$ as pressure of stationary field. Then $P_0 s$ will be the "press force", and $P_0 s\delta$ will be the "work" done against the press force. Equation (2.2) means this work is equal to the energy exchanged during the procedure. In other words, if we define pressure, press
force and work in the above way, “mechanical energy” of stationary ether field is conserved in the quasi-static procedures.

Note that for ether field, “energy” by definition means solely the “mechanical energy”, whose volume density is equal to the square of field strength. Unlike gases, there is no temperature, inner energy or other form of energy considered here. The conservation of “mechanical energy” described above thus means conservation of energy for stationary ether field.

As a side note, the above definitions of pressure, press and work are following the definition of energy (as square of field strength integrated over space). In principle, we can also define energy in a more general way as $\tilde{E} = \int_v \Omega^n dv \ (n > 1)$, and the definition of pressure would change accordingly to $\tilde{P} = (n - 1)\Omega^n$. By applying ECA, energy conservation of stationary field can be derived in a similar way.

In the rest of this paper, unless specially mentioned, we will only consider the case of $n = 2$.

3. Sound Propagation Assumption

According to ECA and the definition of energy, when a block of ether contracts or expands, amount of ether remains the same, but energy varies when volume changes. For the same amount of ether, the smaller the volume, the greater is the energy.

When field is not uniform, the regions where field is stronger have higher “energy density” (and higher “pressure”). A natural supposition is that ether will diffuse from high-pressure regions to low-pressure regions.

The diffusion will cause perturbations in the field, similar to density perturbations (sounds) propagating in gases. In the latter case, speed of sound is a property of the gas. It varies for different gases and in different temperatures. In ether field, however, there is no temperature or other factors to impact the speed. Therefore, we will assume that sounds (perturbations) propagate in a constant speed, denoted as $c$, in ether field. This is the second basic assumption. We will call it “sound propagation assumption”, or SPA, hereafter in this paper.

In order to further analyze this “constant speed” and its impacts, we will need to consider the details of perturbation waves in the field. Similar to waves in gases, the “wavefront” could be in various of shapes. To simplify, we will take a small sample area, $W$, from the wavefront, and approximately consider it as a flat area (i.e., consider the wave as plane wave in the small region around $W$). We will also take a volume element on each side of $W$, and approximately consider that field strength and velocity are uniform within each of the volumes. (Thus, there will be a small step in field strength and velocity at $W$, as shown in Fig. 2.)

Note that $W$ here as a sample area of wavefront is different from the sample area of ether interface in Sec. 2 (i.e., side $S$ in Fig. 1). Both $W$ and $S$ are imaginal area elements in the field. But as mentioned in Sec. 2, when $S$ moves, we assume that no ether will pass through it, while $W$ here as a wavefront moves in sound speed, passing through ether on its way.
For convenience in further discussions, we will call a surface a Lagrangian surface or Lagrangian interface if no ether passes through it, and call a surface/interface otherwise a Eulerian surface or Eulerian interface.

In other words, Lagrangian surfaces/interfaces are “material surfaces/interfaces”, and are decided by state of the field. Side S in Sec. 2 is an example. Eulerian interfaces or surfaces, on the other hand, are just geometry frames used to examine the field. Wavefront area $W$ can be considered as a special case of Eulerian interface (which according to SPA always moves in sound speed).

In the rest of this section and later ones (Secs. 4–9), we will employ this “sample area method” to analyze the effects of SPA, and derive the energy and momentum conservation laws. Sections 4–8 will concentrate on the cases that ether velocity is in the normal direction of the sample area, while Sec. 9 will discuss the cases with velocity in tangential direction (i.e., the three-dimensional cases).

First, let us consider a simple case. Assume that the field is stationary before being perturbed, and a perturbation as a step $\varepsilon$ in field strength is propagating from left to right (Fig. 3). According to SPA, the step $\varepsilon$ (or the “wavefront” area $W$) moves in a fixed speed $c$, regardless of strength of $\varepsilon$.

As $W$ moves towards right at speed $c$, the net amount of ether that moves with it in unit time would be $J = \varepsilon sc$ (here $s$ is the area of $W$). We will call $J$ the “ether flow”, and call $\frac{J}{s} = \varepsilon c$ the “flow density” hereafter in this paper.

As assumed, speed of ether on each side of $W$ is uniform. To meet ECA and support the movement of wavefront $W$, the field on the left-hand side of $W$ needs to

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Fig. 2. A sample area of wavefront and volume elements around it.

Fig. 3. A perturbation propagating in a stationary field.
be moving in a certain speed, \( v \), to provide flow \( J \). This requires \((\Omega + \varepsilon)s v = \varepsilon sc\), or \( v = \frac{sc}{R + \varepsilon} \).

In a further case (Fig. 4), if field \( \Omega \) is moving in an initial speed \( v_0 \) before being perturbed (assume \( v_0 < c \)), in other words, the perturbation is propagating in a moving field, the assumption of SPA here is that wavefront \( W \) still travels at the speed of \( c \) (relative to the observer, rather than to the field).

In this case, fields on both sides of \( W \) are moving in certain speed, and the “net ether flow” to support \( W \) moving in sound speed is provided by the difference of flow between the two sides. If field speed on the left-hand side is \( v \), ECA requires

\[
(\Omega + \varepsilon)v s - \Omega v_0 s = \varepsilon sc \quad \text{or} \quad \frac{v}{c}(\Omega + \varepsilon) - \frac{v_0}{c}\Omega = \varepsilon.
\]

Obviously, this is different from the case of sound in gases. In gas case, the sound speed is relative to the gas. If the gas is moving, the speed of sound in the gas relative to the observer is the (Galilean or Lorentz) transformation of speed of the gas and the sound speed in stationary gas. In the suggested ether field here, however, SPA assumes that sound speed in ether field is constant relative to the observer, even when sound is traveling in a moving field.

To a certain degree, this is rather the “ether version” of the light speed constancy principle in relativity. Note that in the ether field model here, light and photon are supposed to be “high-level” phenomenon, and are not directly related to perturbations/sounds in the field yet (a rough conjecture about light and photon will be discussed at the end of this paper). As a result of that, \( c \) here is just the speed of sound in ether field, and is (temporarily) not directly related to light. But if we put \( c \) in a similar position as the speed of light, sound propagation assumption here is obviously similar to light speed constancy principle in special relativity.

In ideal gases, sound speed can be derived from mechanical properties (i.e., elasticity and inertia). Here in ether field, we set constancy of sound speed as an assumption. Thus, this assumption implies certain mechanical properties of the field. This essential assumption (i.e., SPA), together with the ECA, will lead to energy and momentum conservations and other mechanical properties of ether field, which are to be discussed in the following sections.
4. Acceleration of Ether

With ECA and SPA, let us reconsider the stationary field block in Sec. 2 (Fig. 1) in a dynamic way (instead of the quasi-static way). Assume that the left-hand side, $S$, moves towards right with a steady speed $\theta$ within a short period of time $\tau$ (Fig. 5, $\theta < c$). Similarly as in Sec. 2, $S$ is considered a Lagrangian interface between ether inside and the outside of the block, thus, no ether will pass through it. The movement of $S$ will push the ether next to it towards right. This ether, adding to the field in the block, will cause non-uniformity and form a perturbation, which will then propagate to the right, in sound speed.

If there was no sound propagation, the ether pushed to the right would have accumulated unlimitedly. Sound propagation spreads out the ether, limits the accumulation and sets the field strength to a level in which the ether flow of sound propagation and the flow pushed to the right by movement of $S$ counterbalance.

The amplitude of perturbation, $\varepsilon$, can be calculated from this balanced flow. As side $S$ moves in the speed of $\theta$, the amount of ether it pushes towards right per unit time is $\Omega\theta s$. This would be equal to the amount of ether spread out with the perturbation per unit time. That is

$$\Omega\theta s = \varepsilon(c - \theta)s.$$  

Thus

$$\varepsilon = \frac{\theta}{c - \theta} \Omega \quad \text{or} \quad \Omega + \varepsilon = \frac{c}{c - \theta} \Omega.$$  

(4.1)

$\varepsilon$ can also be calculated from the equivalence of ether on the initial and end states of the block. At $t = 0$, the block lays between point $A$ and point $C$ (Fig. 5); field strength is $\Omega$ and the block is stationary. At $t = \tau$, the block is now between point $B$ and point $C$, with field strength of $\Omega + \varepsilon$. Equivalence of ether on these two states gives the same result as (4.1).

Strictly speaking, $\varepsilon$ here is the average amplitude of the perturbation. But according to (4.1), $\varepsilon$ is decided by $\theta$ (and $\Omega$). While $\theta$ keeps steady, $\varepsilon$ is stable. If we set $\tau$ as a very short time period, $\varepsilon$ would be the instantaneous amplitude.

![Fig. 5. Perturbation caused by side of a field block moving in a fixed speed.](image-url)
Between side S and the wavefront (which reaches point C at \( t = \tau \)), ether should be moving in a certain speed, \( v \), to provide the flow. This requires

\[
(\Omega + \varepsilon)sv = \varepsilon sc \quad \text{or} \quad v = \frac{\varepsilon sc}{(\Omega + \varepsilon)s} = \theta. \quad (4.2)
\]

In other words, at \( t = \tau \), ether in the block has already been accelerated to the same speed as side S.

Same analysis applies to the case that side S moves outwards of the block. In that case, \( \theta \) has a negative value, and Eqs. (4.1) and (4.2) still stand. \( \varepsilon \) in this case is also negative (i.e., a negative perturbation).

Similar analysis also applies to a block with an initial speed. Assume that the block is moving in an initial speed \( v_0 \) in the \( x \)-direction. At \( t = 0 \), side S starts moving in speed \( \theta \), and keeps this steady speed until \( t = \tau \) (Fig. 6). In this case, the extra flow caused by movement of S is \( \Omega(\theta - v_0)s \). It can be positive or negative depending on the sign of \( \theta - v_0 \). At \( t = \tau \), the perturbation reaches point C. At the same time, also reaching point C is the ether that was at point \( C_0 \) at \( t = 0 \) (distance from \( C_0 \) to \( C \) is \( \tau v_0 \)). If we take the block between A and \( C_0 \) at \( t = 0 \) as the initial block, it becomes the block between B and C at \( t = \tau \). Applying ECA we have

\[
(\Omega + \varepsilon)(\tau c - \tau \theta)s = \Omega(\tau c - \tau v_0)s
\]

or

\[
\varepsilon = \frac{\theta - v_0}{c - \theta} \Omega \quad \text{(4.3)}
\]

or

\[
\Omega + \varepsilon = \frac{c - v_0}{c - \theta} \Omega, \quad \text{(4.4)}
\]

and the speed of ether in the block at \( t = \tau \) is also the same as side S, i.e.,

\[
v = \frac{\varepsilon sc + \Omega sv_0}{(\Omega + \varepsilon)s} = \theta. \quad \text{(4.5)}
\]
We can apply (4.3) on either the upstream side or the downstream side. If ether in
the block is moving away from the considered side, \( v_0 \) is positive; otherwise \( v_0 \) is negative.
We can also apply (4.3) for the cases of \( S \) moving inward (\( \theta > 0 \)) and outward
(\( \theta < 0 \)) of the block.

In a special case, if field outside the block on a side is dedicatedly “arranged” to
have proper strength and speed, it is possible that the side is pushed or pulled to stop
at \( t = 0 \) and remains stationary until \( t = \tau \). In this case, we have \( \theta = 0 \) and (4.3)–
(4.5) still hold.

In summary, for a block of ether with an initial speed of \( v_0 \), \(-c < v_0 < c\), if a side
perpendicular to \( v_0 \) moves in a steady speed in a short period of time, the ether
within the inference zone of that side (zone within distance sound travels in the
time period) will be accelerated/decelerated to the same speed of the side. These
accelerations/decelerations are consequences of sound propagations, ruled by ECA
and SPA.

5. Energy and Momentum of Dynamic Fields

With the analysis of ether acceleration/deceleration, let us further consider a block of
ether with uniform strength \( \Omega \) and speed \( v \) in its length direction. Same as before,
denote length of the block as \( L \) and area of the left- or right-hand side as \( s \).

Assume that at \( t = 0 \), both sides on the left and right are pushed/pulled to stop.
According to (4.3), there will be negative and positive perturbations on the upstream
and downstream sides, respectively. The strengths of the perturbations are \( \varepsilon = \pm \frac{v}{c} \Omega \). At \( t = 0 \), these two perturbations start propagating towards each other,
at the speed of sound [Fig. 7(b)]. At \( t = \frac{L}{2c} \), they meet at the mid-point of the block
[Fig. 7(c)]. At this point of time, all ether in the block is stationary. The definition of
stationary field energy in Sec. 2 thus applies, and we can calculate the total energy of
the block as

\[
E = (\Omega - \varepsilon)^2 \frac{L}{2} s + (\Omega + \varepsilon)^2 \frac{L}{2} s = \left( 1 + \frac{v^2}{c^2} \right) \Omega^2 L s.
\]

![Fig. 7](https://example.com/fig7.png)

(a) \( t = 0 \). (b) \( 0 < t < L/(2c) \). (c) \( t = L/(2c) \).

A block with initial speed is “blocked” to stop on both sides.
As assumed, both sides do not move after \( t = 0 \). Thus, there is no “work” done by the sides on the block. If we define energy of a dynamic field block with speed \( v \), field strength \( \Omega \) and volume \( V \) as

\[
E = \left( 1 + \frac{v^2}{c^2} \right) \Omega^2 V, \quad (5.1)
\]

energy of the block keeps the same at start state \( [t = 0, \text{shown in Fig. 7(a)}] \) and end state \( [t = L/(2c), \text{shown in Fig. 7(c)}] \).

Furthermore, at any time between \( t = 0 \) and \( t = L/(2c) \), if we apply (5.1) to the moving portion of the block [i.e., field between \( C_1 \) and \( C_2 \) in Fig. 7(b)] and add it to the energy of stationary portions (i.e., between \( A \) and \( C_1 \) and between \( C_2 \) and \( B \)), it can be found that the total energy stays the same at any time.

In other words, if we define energy of dynamic field as (5.1), energy is conserved during this procedure.

There is another conserved quantity in this procedure. Consider the interfaces on both left- and right-hand sides. As they are both stationary after \( t = 0 \), the definition of pressure for stationary field in Sec. 2 applies. On the left-hand side the pressure is \((\Omega - \varepsilon)^2\) while on the right-hand side it is \((\Omega + \varepsilon)^2\). The time both perturbations reach the mid-point of the block is \( \tau = \frac{L}{2c} \). At this time, the whole block is stationary. If we calculate the product of press and time (i.e., the “impulse”) on each side, we can find the difference of it between the left- and right-hand sides as

\[
\Delta I = (\Omega + \varepsilon)^2 s \frac{L}{2c} - (\Omega - \varepsilon)^2 s \frac{L}{2c} = \frac{2\Omega^2}{c^2} v L s.
\]

If we define \( \frac{2\Omega^2}{c^2} \) as “mass density”, and define \( \frac{2\Omega^2}{c^2} v \) as “momentum density”, momentum of the block at \( t = 0 \) would be \( \frac{2\Omega^2}{c^2} v L s \) and momentum at \( t = \tau \) is 0 (as the whole block is stationary at \( t = \tau \)). Change in momentum in this case is equal to difference of impulse provided by the left- and right-hand sides during the procedure, i.e., momentum is conserved.

The same conclusion could be obtained at any time between \( t = 0 \) and \( t = \tau \), by comparing momentum change in the block to net impulse provided by both sides.

6. Local Energy and Momentum Conservations

With the definitions of energy and momentum for dynamic field, we can now revisit the case of a field block accelerated by movement of its side.

For the first example discussed in Sec. 4 (Fig. 5), on the right-hand side, as field is stationary before the perturbation arrives, pressure there is always \( \Omega^2 \). On the left-hand side, field is not stationary. We can denote the pressure on the left-hand side by \( P \). At the start state, the block lays between point \( A \) and point \( C \); field strength is \( \Omega \); speed is 0; and volume is \( \tau cs \). At the end state, the block is between \( B \) and \( C \); field is
\[ \Omega + \varepsilon; \text{ speed is } \theta; \text{ and volume is } (\tau c - \tau \theta)s. \] At \( t = \tau \), the left-hand side \( S \) (with pressure \( P \)) has done a work of \( P\tau \theta s \) to the block. The right-hand side did not move, so there is no work done there. In order to have energy change equal to sum of works done to the block, we need

\[ \left( 1 + \frac{\theta^2}{c^2} \right) (\Omega + \varepsilon)^2 (\tau c - \tau \theta)s - \Omega^2 \tau c s = P\tau \theta s. \]

Applying (4.1), we have

\[ P = \left( 1 - \frac{\theta^2}{c^2} \right) (\Omega + \varepsilon)^2. \]

In other words, if pressure at a point where field strength is \( \Omega \) and speed is \( v \) is defined as

\[ P = \left( 1 - \frac{v^2}{c^2} \right) \Omega^2, \quad (6.2) \]

pressure on the left-hand side satisfies (6.1) and energy is conserved during the acceleration procedure in this case.

On the other hand, at the end state, mass density is \( \frac{2(\Omega + \varepsilon)^2}{c^2} \) and momentum density is \( \frac{2(\Omega + \varepsilon)^2}{c^2} \theta \). With (6.1), we can straightforwardly confirm that change in momentum is equal to net impulse provided to the block by both sides during the acceleration, i.e., momentum is conserved.

It is also straightforward to extend these conclusions to the acceleration/deceleration of a block with non-zero initial speed (shown in Fig. 6).

Recalling (5.1), the total energy of a block consists of two portions: The static portion \( \Omega^2 V \) and the kinetic portion \( \Omega^2 \frac{v^2}{c^2} V \). These are the only two forms of energy in the field. The static energy can also be considered as a potential energy. In an imagined scenario that the ether in the block evenly permeates throughout the whole space (and there is no other ether in the space), the static energy would be equal to the work done to push the ether back to the block (quasi-statically). On the other hand, as per (6.1), pressure is equal to the difference between static energy density and kinetic energy density.

In summary, for uniform field blocks, in one-dimensional motions (in which acceleration or deceleration is in the same direction as initial speed), if energy, mass, momentum and pressure, work, impulse are defined in the above way, two conservation laws hold: Change of energy is equal to the total net work done on the block; change of momentum is equal to the net impulse given by the sides.

The restriction of uniformity here is not substantively necessary. If the field is not uniform, we can split the block into small blocks, approximately considering each block to be uniform, and set time period to a very short one. The above analysis still applies.
Also note that the conservations are only inside field blocks thus far. Strictly speaking, they are just local level of energy and momentum conservations. To extend them to the whole field, i.e., to global level, it is necessary to confirm that the energy/momentum a block gains/loses is exactly the same in amount to those the adjacent blocks lose/gain. This will be discussed in the next section.

Obviously, these conservation properties depend on definitions of energy, momentum and other quantities. The conservations in this sense can be considered as mathematical inferences of those definitions. At the same time, they are also consequences of ECA and SPA. To a certain degree, they can also be considered as natural properties of the theoretical medium in which ECA and SPA stand. Definitions of energy, momentum, etc. are just the proper tools to expose these properties.

Speed of sound, as speed of distribution of non-uniformities, plays an important role in the procedures. Sound speed decides the amplitudes of perturbations and easiness of acceleration. This explains why sound speed, mostly combined with ether speed (as the ratio of ether speed to sound speed, or the “Mach number”, $v/c$), appears in the definitions of mass, momentum and energy.

7. Interface and Interaction

Unlike in the gas cases, in ether field, we shall not consider anything other than ether. In other words, we consider ether as the only being in space. There is no other “object” to impact motion of ether. Changes in motion status of ether are solely caused by ether-to-ether interactions.

Furthermore, because ether is the only being, interaction between ether is local. It only happens between parts of ether that are “touching” each other. There is neither remote interaction nor other kinds of “force”, as there is no other being to act as medium for remote interaction or other kinds of force.

For an interface in the field, ether on both sides are “pushing” each other. Pressure [defined as (6.2)] describes the strength of the “pushes”. According to SPA, if the “pushes” on both sides are not balancing, i.e., pressure around the interface is not uniform, the ether nearby will react to “correct” the non-uniformity. The reaction/correction appears as a perturbation, or sound, and the quickness of nearby ether to be involved in the reaction/correction is prescribed by SPA with the fixed speed of sound.

So, generally speaking, it is local non-uniformity that causes motion status to change. To “correct” the non-uniformity, the involved ether near an unbalanced point in the field (i.e., a point where gradient of pressure is non-zero) will change motion status and redistribute. This redistribution can then impact other ether, causing new non-uniformities and further changes. As series of changes go on, the effect spreads as a perturbation, at sound speed. This is the way ether interacts, and the way motion status changes.
As assumed, all these changes follow ECA and SPA. They can be considered as results of these two assumptions. Generally speaking, ECA rules how motions impact ether distributions (accumulations and decumulations), and SPA rules how ether distributions (non-uniformities) impact motions. These two assumptions together form the fundamental of the “ether field theory” this paper is attempting to establish.

As defined in Sec. 3, ether does not pass through a Lagrangian interface. For a Lagrangian interface to be steady (stationary or moving in an unchanged speed), ECA requires normal speed of ether at the interface to be continued, and SPA requires pressures to be balanced. If at least one of these is “unbalanced”, ether around the interface will be pushed/pulled and change motion status.

Consider two respectively uniform blocks. At $t = 0$, the field strength and velocity are $\Omega_1$, $v_1$ and $\Omega_2$, $v_2$, respectively. $v_1$ and $v_2$ are parallel, and both blocks share a side, $S$, perpendicular to $v_1/v_2$ (Fig. 8). Let us check the motion status of this shared side, as a Lagrangian interface.

In a general case, $v_1$ and $v_2$ can be different and pressures on both sides of $S$ may not be the same. In this case, $S$ would move towards the low-pressure side, to balance the differences. Assume that the speed at which $S$ moves is $\theta$, stable at least within a short period of time.

According to (4.4) and (4.5), fields on both sides will be pulled/pushed to the same speed as $S$ (i.e., $\theta$), and field strengths on both sides will, respectively, become

$$
\Omega_1' = \frac{c + v_1}{c + \theta} \Omega_1 \quad \text{and} \quad \Omega_2' = \frac{c - v_2}{c - \theta} \Omega_2. \tag{7.1}
$$

Pressure is a function of field strength and speed. In this one-dimensional case, as motions are only in the normal direction of the interface, equal pressures with equal normal speeds mean equal field strengths, i.e., $\Omega_1' = \Omega_2'$ (unless $v_1 = v_2 = c$ pressures are both zero).

![Fig. 8. Movement of an interface in the field.](image)
Denoting the common field strength as \( \Omega' = \Omega'_1 = \Omega'_2 \), with (7.1), we have

\[
\begin{align*}
\Omega' &= \frac{1}{2} \left[ \left( 1 + \frac{v_1}{c} \right) \Omega_1 + \left( 1 - \frac{v_2}{c} \right) \Omega_2 \right], \\
\theta &= \frac{\left( 1 + \frac{v_1}{c} \right) \Omega_1 - \left( 1 - \frac{v_2}{c} \right) \Omega_2}{\left( 1 + \frac{v_1}{c} \right) \Omega_1 + \left( 1 - \frac{v_2}{c} \right) \Omega_2}, \\
\varepsilon_1 &= \Omega' - \Omega_1 = \frac{1}{2} \left( -\Omega_1 \left( 1 - \frac{v_1}{c} \right) + \Omega_2 \left( 1 - \frac{v_2}{c} \right) \right), \\
\varepsilon_2 &= \Omega' - \Omega_2 = \frac{1}{2} \left( \Omega_1 \left( 1 + \frac{v_1}{c} \right) - \Omega_2 \left( 1 + \frac{v_2}{c} \right) \right), \\
\Omega'^2 &= \left( 1 - \frac{\theta^2}{c^2} \right) \Omega'^2 = \left( \Omega_1 \left( 1 + \frac{v_1}{c} \right) \right) \cdot \left( \Omega_2 \left( 1 - \frac{v_2}{c} \right) \right) \cdot \left( \Omega_1 \left( 1 - \frac{v_1}{c} \right) + \Omega_2 \left( 1 + \frac{v_2}{c} \right) \right) \cdot \left( \Omega_2 \left( 1 + \frac{v_2}{c} \right) \right).
\end{align*}
\]  

\( \Omega' \) here is the balanced pressure of the field around \( S \) after it moves; \( \varepsilon_1 \) and \( \varepsilon_2 \) are the amplitudes of the two perturbations that started from \( S \) towards both directions.

As pressure balances on the interface \( S \) after \( t = 0 \), works done by both the blocks are the same in amount, positive on one side and negative on the other. Energy gained on one side of the interface is equal to energy lost on the other side. With conservation inside each block, total energy is conserved during this procedure of interaction between blocks. This extends energy conservation from local level (i.e., inside the block) to global level (i.e., inter-block and thus the whole field). Same is for momentum conservation.

8. Ether Flow and Ether Motion

Applying (7.2)–(7.6), we can review a few special cases, which will be helpful for further discussions.

**Case I.** If at \( t = 0 \), two adjacent blocks are both stationary but with different field strengths, i.e., \( v_1 = v_2 = 0 \), \( \Omega_1 \neq \Omega_2 \) (Fig. 9), with (7.2)–(7.5) we have

\[
\Omega' = \frac{1}{2} \left( \Omega_1 + \Omega_2 \right), \quad \theta = \frac{\Omega_1 - \Omega_2}{\Omega_1 + \Omega_2} \frac{1}{c}, \quad \varepsilon_1 = -\frac{1}{2} \left( \Omega_1 - \Omega_2 \right), \quad \varepsilon_2 = \frac{1}{2} \left( \Omega_1 - \Omega_2 \right).
\]  

(a) Two stationary blocks at \( t = 0 \).  
(b) Field is averaged out by \( \varepsilon_1 \) and \( \varepsilon_2 \).

Fig. 9. Movement of interface with discontinuity in stationary field.
Note that the difference in original field strength is split into two perturbations, \( \varepsilon_1 \) and \( \varepsilon_2 \), one positive and one negative, each has an amplitude equal to half of the difference. Fields on both sides are averaged out when these perturbations spread out from the interface.

The ether flow densities corresponding to \( \varepsilon_1 \) and \( \varepsilon_2 \) are \( J_1 = \varepsilon_1 c \) and \( J_2 = \varepsilon_2 c \), respectively, or in vector form, \( \mathbf{J}_1 = \varepsilon_1 \mathbf{e}_x \) and \( \mathbf{J}_2 = \varepsilon_2 \mathbf{e}_x \). Note that when \( \Omega_1 > \Omega_2 \) we have \( \varepsilon_1 < 0 \). In this case \( \mathbf{J}_1 \) is a negative perturbation, propagating to the left (i.e., \(-\mathbf{e}_x\)-direction).

**Case II.** If at \( t = 0, v_2 = 0 \) and \( \frac{\Omega_1}{c} = \Omega_2 \), we have

\[
\Omega' = \Omega_1, \quad \theta = v_1, \quad \varepsilon_1 = 0, \quad \varepsilon_2 = \frac{\Omega_1 v_1}{c}.
\]  

(8.2)

In this case, the end state is exactly the same as the original field on the left. This is the case of a perturbation \( \varepsilon = \frac{\Omega}{c} \) propagating in a stationary field \( \Omega \), which was discussed in Sec. 3 (Fig. 3).

For a field with speed \( v \), if we formalistically divide the field strength \( \Omega \) into two portions: static portion, \( \omega_s \), and dynamic portion, \( \omega_d \), as

\[
\omega_s = \Omega \left(1 - \frac{v}{c}\right), \quad \omega_d = \frac{\Omega v}{c},
\]  

(8.3)

ether flow in the field can be calculated as \( J_d = \omega_d c = \Omega v \). In other words, it can be imagined as if the dynamic portion \( \omega_d \) is moving in sound speed, and the static portion \( \omega_s \) keeps stationary. For convenience purpose, we will call \( J_d \) the "sound flow", and call this analysis method "sound splitting" of ether field. It is a kinematically equivalent description of the moving field.

In the above case, the static portion on the left, \( \omega_{s1} \), is equal to \( \Omega_2 \) on the right. Kinematically speaking, we can consider that the sound flow passes through the interface without causing any change in the static portion.

In the cases that \( \omega_{s1} \neq \Omega_2 \), we can treat \( \omega_{s1} \) and \( \Omega_2 \) as two stationary blocks as in case I, and add the two perturbations \( \varepsilon_1 \) and \( \varepsilon_2 \) with the original sound flow \( \omega_d \). This gives the same results as (7.2)–(7.6).

**Case III.** Applying sound splitting to the general case in Sec. 7, we can split the fields in the two blocks into four parts namely

\[
\omega_{s1} = \Omega_1 \left(1 - \frac{v_1}{c}\right), \quad \omega_{s2} = \Omega_2 \left(1 - \frac{v_2}{c}\right), \quad \omega_{d1} = \Omega_1 \frac{v_1}{c}, \quad \omega_{d2} = \Omega_2 \frac{v_2}{c}.
\]  

(8.4)

Then treat \( \omega_{s1} \) and \( \omega_{s2} \) as two stationary blocks as in case I. If \( \omega_{s1} \neq \omega_{s2} \), the difference will split into two perturbations,

\[
\varepsilon_1, \varepsilon_2 = \pm \frac{1}{2} (\omega_{s1} - \omega_{s2}),
\]

which bring two flows in \( \pm \mathbf{e}_x \)-directions:

\[
\mathbf{J}_{\varepsilon_1} = \varepsilon_1 \mathbf{e}_x, \quad \mathbf{J}_{\varepsilon_2} = \varepsilon_2 \mathbf{e}_x.
\]
With the sound flows in the original blocks

\[ J_{d1} = \omega_{d1} c = \Omega_1 v_1, \quad J_{d2} = \omega_{d2} c = \Omega_2 v_2, \]

we have totally four flows. They can be combined in a way described below to calculate the field strengths and speeds in different areas.

If two flows are in the same direction, they will both keep moving in sound speed, and the amplitudes add up in the overlapping area. In the example shown in Fig. 10(a), from the left to point \( A \), the dynamic field is \( \omega_d = \varepsilon_1 + \varepsilon_2 \); between \( A \) and \( B \) dynamic field is \( \omega_d = \varepsilon_1 \); from point \( B \) to the right, there is no dynamic field. In all the areas, static field remains unchanged from the original static field \( \omega_{d0} \). The actual field strength and speed can be calculated according to (8.3).

If two flows meet from opposite directions, we can also consider as if each of them keeps moving forward in sound speed, but in the overlapping area, part of the flows will stop each other and become static field. The difference of the two flows will form a new sound flow. In the example shown in Fig. 10(b), two flows \( J_1 = \varepsilon_1 c \) and \( J_2 = \varepsilon_2 c \) from the left and right meet each other. Assuming \( \varepsilon_2 \geq \varepsilon_1 \), in the area from the left to point \( A \), dynamic field is \( \omega_d = \varepsilon_1 \) and static field is \( \omega_s = \omega_{s0} \); between \( A \) and \( B \), \( \omega_d = \varepsilon_2 - \varepsilon_1, \omega_s = 2\varepsilon_1 + \omega_{s0} \); from \( B \) to the right, \( \omega_d = \varepsilon_2, \omega_s = \omega_{s0} \).

This is an equivalent (and intuitive) way to calculate field status changes near an interface. The outcomes are the same as results using (7.2)–(7.6).

**Case IV.** For the general case in Sec. 7, we can have another way to calculate the kinematically equivalent results.

Rewrite (7.2) and (7.3) as

\[
\begin{align*}
\Omega'c &= \frac{1}{2}[(c + v_1)\Omega_1] + \frac{1}{2}[(c - v_2)\Omega_2], \\
\theta/c &= \frac{1}{2}[(c + v_1)\Omega_1] - \frac{1}{2}[(c - v_2)\Omega_2].
\end{align*}
\]

(8.5)

(8.6)

It seems that the impacts of the two blocks to the interface are \( \frac{1}{2}[(c + v_1)\Omega_1] \) and \( \frac{1}{2}[(c - v_2)\Omega_2] \), respectively. In other words, for a field block \( \Omega \) with velocity \( v = ve_v \),

**Fig. 10.** Sound flows and combination.
the impacts in the “upstream” direction (−e_v) and “downstream” direction (e_v) can be measured by \( \frac{1}{2} \Omega(c - v) \) and \( \frac{1}{2} \Omega(c + v) \), respectively. These two properties are flow-like. If we define two flows, named up-flow and down-flow, respectively, as

\[
J_+ = \frac{1}{2} \Omega(c + v), \quad J_- = \frac{1}{2} \Omega(c - v),
\]

and assume they both move in sound speed, we have

\[
J_+ + J_- = \Omega c, \quad J_+ - J_- = \Omega v.
\]

Then for the interface in Sec. 7, we have four flows (\( J_{1+}, J_{1-} \) and \( J_{2+}, J_{2-} \), shown in Fig. 11), all in sound speed, interacting in the way described as in case III. The field strength and speed in the settled area thus are

\[
\begin{align*}
\Omega' &= \frac{1}{c} (J_{1+} + J_{2-}), \\
\theta &= \frac{J_{1+} - J_{2-}}{J_{1+} + J_{2-}} c,
\end{align*}
\]

which are the same as in (7.2) and (7.3).

In certain sense, the two flows, \( J_+ \) and \( J_- \), describe the capability of a field block to expand in the downstream and upstream directions. In an extreme case that the block is placed in real empty space (i.e., \( \Omega = 0 \) outside of the block), the block will expand in these two directions with flows of \( J_+ \) and \( J_- \), respectively.

For convenient purpose, we will call this way of analysis “flow splitting”. Note that all the splitting analysis methods described in this section (i.e., sound splitting and flow splitting) are just kinematically equivalent ways to describe and calculate ether motions. They give the same results as the original formulas (7.2)–(7.6), which are derived from the basic assumptions, ECA and SPA.

From the splitting analysis, we can “extract” two special properties of a flow in the field. First, the “wavefront” of these split flows is always moving in sound speed. Second, density of a flow, as field strength multiplied by speed, keeps the same along the way, until it meets another flow in opposite direction and cancels each other (partly or totally). As a result of that, in the sense of kinematical equivalence, we can consider that either all the ether in the field is moving in certain speed to support the flow or only a portion of the ether (the dynamic portion) is moving in sound speed to provide the flow, as long as the product of speed and field strength remains the same.
In the case that field strength changes along the way, the flow density will not change, but the actual speed of the field to support the flow will change accordingly.

Basically, these properties are still from ECA and SPA. They will be extended to three-dimensional cases in further discussion.

In previous discussions, we often examined a block of ether by "pushing" it on its sides, or "stopping" it on the left or right. With flow splitting, we can now describe these procedures in a more intuitive way. For a block $\Omega$ with speed $v$, assuming $v$ is from left to right, to push its left-hand side to certain speed, what we need is to have the field outside of the interface at $t = 0$ with the same static field as in the block, and a dynamic field to provide the corresponding flow for the required speed. To "stop" the block from the left, we only need to have a stationary field outside on the left at $t = 0$ with the same strength as the static portion in the block, i.e., $\Omega(1 - \frac{v}{c})$. On the right-hand side, a stationary field with strength of $\Omega(1 + \frac{v}{c})$ serves the same purpose.

As the "pushing" and "pulling" are done by flows reaching the interface at a time, movement of the interface starts immediately, and as long as the flows are steady, speed of the interface keeps steady.

In previous sections, we also considered a Lagrangian surface as no ether passes through it. But in the sense of kinematical equivalence, it is the same as a surface with multiple flows passing through it and the "net flow" is zero. Flow analysis and Lagrangian/Eulerian interfaces are just for kinematical analysis. In the sense of field dynamics, in the actual cases each point in the field only has one velocity, which (by multiplying the field strength at that point) provides the net ether flow density.

9. Three-Dimensional Motions

The property that once generated, density of a sound flow will not change until it meets another sound flow in opposite direction, will be extended to three-dimensional cases, to explore energy and momentum conservations in field with three-dimensional motions and interactions.

SPA also needs to be extended to three-dimensional cases. For one-dimensional motions, we assumed that perturbations (sounds) travel at a constant speed relative to the observer, regardless of motion status of the field in which perturbations are traveling. We will further assume that this applies to the cases that velocity of the field is not in parallel to the propagating direction of the perturbation. In other words, sound speed in ether field is not impacted by either the longitudinal or the lateral speed of the field.

Consider a uniform block of field with strength of $\Omega$ and velocity of $v = v_x e_x + v_y e_y + v_z e_z$. Recall the energy and pressure definitions, (5.1) and (6.2). Total energy and pressure should not change when the coordinate system changes directions. So they are still $E = (1 + \frac{\nu^2}{c^2})\Omega^2 V$ and $P = (1 - \frac{\nu^2}{c^2})\Omega^2$.

For convenience, we can combine $v_y$ and $v_z$ to a lateral speed $v_\perp$ as $v_\perp^2 = v_y^2 + v_z^2$, and rewrite the velocity as $v = v_x e_x + v_\perp e_\perp$. Accordingly, the sound flow can be split as $J = \Omega v_x e_x + \Omega v_\perp e_\perp = J_x + J_\perp$. 
Consider all the sides of the block as Lagrangian surfaces and assume that at \( t = 0 \), the block is pushed/pulled to stop in the \( x \)-direction from both of its sides in \( x \). Similar to the discussion in Sec. 5, two perturbations, \( \varepsilon = \pm \frac{v_x}{c} \), will start from both sides in \( x \) and move towards the center of the block. In the lateral direction \( \mathbf{e}_\perp \), the field will keep moving with the unchanged flow density \( J_\perp \). To maintain this flow density, when field strength changes, speed in \( \mathbf{e}_\perp \) will change accordingly.

At \( t = \frac{L}{(2c)} \), the two perturbations meet at the center of the block in \( x \)-direction. The field strengths of the left half (\( \Omega_1' \)) and the right half (\( \Omega_2' \)) are now

\[
\Omega_1' = \Omega - \varepsilon = \Omega \left( 1 - \frac{v_x}{c} \right), \quad \Omega_2' = \Omega + \varepsilon = \Omega \left( 1 + \frac{v_x}{c} \right).
\]

At this point of time, speed in \( x \)-direction is zero for the whole block. Speeds in \( \mathbf{e}_\perp \)-direction, \( v_{\perp 11} \) (in the left half) and \( v_{\perp 12} \) (in the right half), can be calculated from the unchanged flow density \( J_\perp = \Omega v_\perp \) and new field strengths \( \Omega_1' \) and \( \Omega_2' \). That gives

\[
v_{\perp 11}' = \frac{v_\perp}{1 - \frac{v_x}{c}} \quad \text{and} \quad v_{\perp 12}' = \frac{v_\perp}{1 + \frac{v_x}{c}}.
\]

Assuming \( v_x > 0 \), we obviously have \( \Omega_1' < \Omega < \Omega_2' \) and \( v_{\perp 11}' > v_\perp > v_{\perp 12}' \) (and \( \Omega_1' v_{\perp 11}' = \Omega v_\perp = \Omega_2' v_{\perp 12}' \)). In other words, in the left half, when the perturbation passes by, speed in \( x \)-direction becomes zero; field strength decreases from \( \Omega \) to \( \Omega_1' \); and the lateral speed increases from \( v_\perp \) to \( v_{\perp 11}' \). At different locations in \( x \), lateral speed increases at different times. The left-most part speeds up first when the perturbation starts from there, then other parts along the \( x \)-direction speed up in sequence. As a result of that, if the initial block is in cuboid shape, the left half will become a parallelepiped [parallelogram in the top–down view, as shown in Fig. 13 (c)]. The volume however keeps the same as in the initial block. Similar procedure happens in the right half of the block. There the lateral speed decreases from \( v_\perp \) to \( v_{\perp 12}' \) when the perturbation sweeps from right towards the block center, and the shape changes to a parallelepiped as well [Fig. 13(c)].

After \( t = 0 \), sides in \( x \)-direction stay steady, thus no work is done to the block in this direction. In \( \mathbf{e}_\perp \)-direction, the \(+\mathbf{e}_\perp\) side and \(-\mathbf{e}_\perp\) side (i.e., the upstream side and the downstream side) have the same pressure and speed all the time. Net work done to the block by these sides is also zero.

![Fig. 12. A field block with initial lateral speed being blocked to stop.](image-url)
At $t = L/(2c)$, applying (5.1), we can calculate the energies in the left half and the right half and add them together. That gives

$$E = \frac{V}{2} \left( 1 + \frac{v_{x}^2}{c^2} \right) \Omega_1^2 + \frac{V}{2} \left( 1 + \frac{v_{x}^2}{c^2} \right) \Omega_2^2 = \left( 1 + \frac{\nu_x^2 + \nu_{x1}^2}{c^2} \right) \Omega^2 V = \left( 1 + \frac{\nu^2}{c^2} \right) \Omega^2 V,$$

which is the same as energy of the initial block.

It is also straightforward to confirm that energy keeps the same at any moment when $0 < t < L/(2c)$.

For momentum, in the $e_x$-direction, after $t = 0$, the pressures on both sides are, respectively,

$$P_{x-} = \left( 1 - \frac{v_{x1}^2}{c^2} \right) \Omega_1^2 \quad \text{and} \quad P_{x+} = \left( 1 - \frac{v_{x2}^2}{c^2} \right) \Omega_2^2.$$

It can be calculated that change of momentum is equal to difference of impulses in the $e_x$-direction.

In the $e_{\perp}$-direction, as pressures on both sides are equal all the time, net impulse to the block is zero. Momentum can be calculated from $\Omega_1$, $\Omega_2$ and $v_{x1}$, $v_{x2}$. It remains the same as in the initial block.

Applying similar analysis to the case that a block with speed in the $e_{\perp}$-direction is accelerated/decelerated in the $e_x$-direction, it can be confirmed that energy conservation stands, and momentum is conserved in each direction of $e_x$ and $e_{\perp}$.

Same as in one-dimensional cases, for a Lagrangian interface to be steady, normal speed and pressure need to be the same on both sides. In three-dimensional cases, if the speed in tangential direction is not the same on different sides of an interface, it is possible to have different field strengths in different sides while the pressure is still balancing.

As pressure is the same on both sides, when a Lagrangian interface moves, work done on one side is equal to work done on the other. This means energy can transfer
through interfaces without gain/loss in the three-dimensional cases, and the total energy is conserved. Same is the case for momentum.

10. Ether Field Equations

Apparently, ether is quite similar to ideal gas: Compressible, following momentum and energy conservations. On the other hand, for ether field, pressure and energy density are known functions of field strength and velocity. This greatly simplifies the “constitutive relation”. Another special property of ether compared to gases is that mass does not conserve. Ether is conserved, instead.

Splitting the field into small-volume elements, with the definition of pressure, and applying the ether, momentum and energy conservations to the elements, we can derive three basic equations in a way similar to the derivation of Navier–Stokes equations in fluid mechanics.

The three equations, named ether equation, momentum equation and energy equation, respectively, hereafter in this paper, are

\[
\begin{align*}
\frac{\partial}{\partial t} \Omega + \nabla \cdot (\Omega \mathbf{u}) &= 0, \\
\frac{\partial}{\partial t} \left( \frac{2\Omega^2}{c^2} \mathbf{u} \right) + \nabla \cdot \left( \frac{2\Omega^2}{c^2} \mathbf{u}\mathbf{u} \right) &= -\nabla \left( \left( 1 - \frac{u^2}{c^2} \right) \Omega^2 \right), \\
\frac{\partial}{\partial t} \left( 1 + \frac{u^2}{c^2} \right) \Omega^2 + \nabla \cdot \left( \left( 1 + \frac{u^2}{c^2} \right) \Omega^2 \mathbf{u} \right) &= -\nabla \cdot \left( \left( 1 - \frac{u^2}{c^2} \right) \Omega^2 \mathbf{u} \right). 
\end{align*}
\]

(10.1) (10.2) (10.3)

Actually, the ether equation was also given in Sec. 2 as (2.1), as a direct conclusion of ECA. Momentum equation and the energy equation are based on the definitions of momentum and energy in Sec. 5 and the related conservation properties derived in previous sections.

As an alternative format, energy equation (10.3) can be simplified to

\[
\frac{\partial}{\partial t} \left( 1 + \frac{u^2}{c^2} \right) \Omega^2 + 2\nabla \cdot (\Omega^2 \mathbf{u}) = 0. 
\]

(10.4)

As the only unknowns here are field strength and velocity, these three equations, as a system, are overdetermined. However, they are consistent, as shown below.

Expanding (10.2) as

\[
\Omega^2 \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \frac{\partial \Omega^2}{\partial t} + \Omega^2 \mathbf{u} \cdot \nabla \mathbf{u} + \Omega^2 \mathbf{u} \nabla \cdot \mathbf{u} + \mathbf{u} \cdot (\mathbf{u} \cdot \nabla \Omega^2) = -\frac{1}{2} \nabla ((c^2 - u^2)\Omega^2),
\]

dot-multiplying both sides by velocity, and applying the identity

\[
\mathbf{u} \cdot (\mathbf{u} \cdot \nabla \mathbf{u}) = \frac{1}{2} \mathbf{u} \cdot \nabla u^2,
\]

we have

\[
\Omega^2 \frac{\partial u^2}{\partial t} + 2u^2 \frac{\partial \Omega^2}{\partial t} + \Omega^2 \mathbf{u} \cdot \nabla u^2 + 2\Omega^2 u^2 \nabla \cdot \mathbf{u} + 2u^2 (\mathbf{u} \cdot \nabla \Omega^2) \\
= -\mathbf{u} \cdot \nabla ((c^2 - u^2)\Omega^2). 
\]

(10.5)
Multiplying (10.1) by \(2c^2\Omega\), we get
\[
c^2 \frac{\partial \Omega^2}{\partial t} + 2c^2\Omega \nabla \cdot \mathbf{u} + c^2 \mathbf{u} \cdot \nabla \Omega^2 = 0. \tag{10.6}
\]
Adding (10.6) to (10.5) gives
\[
\begin{align*}
\frac{\partial}{\partial t} ((c^2 + u^2)\Omega^2) + u^2 \frac{\partial \Omega^2}{\partial t} + \Omega^2 \mathbf{u} \cdot \nabla u^2 + 2\Omega^2 u^2 \mathbf{v} \cdot \mathbf{u} + 2u^2 \mathbf{u} \cdot \nabla \Omega^2 \\
+ 2c^2 \Omega^2 \nabla \cdot \mathbf{u} + c^2 \mathbf{u} \cdot \nabla \Omega^2 \\
= -\mathbf{u} \cdot \nabla ((c^2 - u^2)\Omega^2)
\end{align*}
\]
or
\[
\begin{align*}
\left[ u^2 \frac{\partial \Omega^2}{\partial t} + u^2 (\mathbf{u} \cdot \nabla \Omega^2) + 2\Omega^2 u^2 \nabla \cdot \mathbf{u} \right] \\
+ \frac{\partial}{\partial t} ((c^2 + u^2)\Omega^2) + u^2 (\mathbf{u} \cdot \nabla \Omega^2) + \Omega^2 \mathbf{u} \cdot \nabla u^2 + 2c^2 \Omega^2 \nabla \cdot \mathbf{u} + c^2 \mathbf{u} \cdot \nabla \Omega^2 \\
= -\mathbf{u} \cdot \nabla ((c^2 - u^2)\Omega^2).
\end{align*}
\]
Applying (10.1), the first term can be simplified as
\[
\begin{align*}
\left[ u^2 \frac{\partial \Omega^2}{\partial t} + u^2 (\mathbf{u} \cdot \nabla \Omega^2) + 2\Omega^2 u^2 \nabla \cdot \mathbf{u} \right] \\
= 2\Omega u^2 \frac{\partial \mathbf{\Omega}}{\partial t} + 2\Omega u^2 (\mathbf{u} \cdot \nabla \mathbf{\Omega}) + 2\Omega u^2 \nabla \cdot \mathbf{u} \\
= 2\Omega u^2 \left[ \frac{\partial \mathbf{\Omega}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{\Omega} + \Omega \nabla \cdot \mathbf{u} \right] = 0.
\end{align*}
\]
Thus we have
\[
\begin{align*}
\frac{\partial}{\partial t} ((c^2 + u^2)\Omega^2) + u^2 (\mathbf{u} \cdot \nabla \Omega^2) + \Omega^2 \mathbf{u} \cdot \nabla u^2 + 2c^2 \Omega^2 \nabla \cdot \mathbf{u} + c^2 \mathbf{u} \cdot \nabla \Omega^2 \\
= -\mathbf{u} \cdot \nabla ((c^2 - u^2)\Omega^2)
\end{align*}
\]
or
\[
\begin{align*}
\frac{\partial}{\partial t} ((c^2 + u^2)\Omega^2) + \Omega^2 \mathbf{u} \cdot \nabla (c^2 + u^2) + (c^2 + u^2) \nabla \cdot (\Omega^2 \mathbf{u}) \\
= -(c^2 - u^2)\Omega^2 \nabla \cdot \mathbf{u} - \mathbf{u} \cdot \nabla ((c^2 - u^2)\Omega^2)
\end{align*}
\]
or
\[
\frac{\partial}{\partial t} ((c^2 + u^2)\Omega^2) + \nabla \cdot ((c^2 + u^2)\Omega^2 \mathbf{u}) = -\nabla \cdot ((c^2 - u^2)\Omega^2 \mathbf{u}),
\]
which is just the energy equation (10.3).

In other words, energy equation can be derived from ether equation and momentum equation. In this sense, energy conservation can be considered as a consequence of ether and momentum conservations.

Recall that energy and momentum conservations were derived from the same sound propagation procedure. They co-exist in the same procedure, and are both results of SPA and ECA. It is not a surprise that they are not independent (and are consistent).
As a result of that, we can take away the energy equation from the system (and use it as a derived equation if need). This leaves the ether equation and the momentum equation to form a closed equation system:

\[
\begin{align*}
\frac{\partial}{\partial t} \Omega + \nabla \cdot (\Omega \mathbf{u}) &= 0, \\
\frac{\partial}{\partial t} (\Omega^2 \mathbf{u}) + \nabla \cdot (\Omega^2 \mathbf{u} \mathbf{u}) &= -\frac{c^2}{2} \nabla \left( 1 - \frac{\mathbf{u}^2}{c^2} \right) \Omega^2.
\end{align*}
\]

This is the fundamental equation system for ether field. In the following sections, we will concentrate on the properties of this system and the possible solutions.

11. Background Field

According to SPA, ether will diffuse from high-pressure regions to the low-pressure regions. Unless moving in sound speed (thus pressure is zero), field in an area cannot stay stable if surrounded by empty space (i.e., space with zero field strength). For any structure (formed by stationary field or field moving slower than sound speed) to stably exist, there must be a non-zero field in the background, to provide positive pressure as the “boundary conditions”. Field structures may then exist in low-pressure regions inside this background field (BGF), balanced by certain kind of motions.

We will assume the existence of such a background field. It suffuses throughout the whole space.

This actually sketches a picture of the universe, from the ether field point of view. Ether, as the only being in the universe, permeates throughout the space. Non-uniformities cause motions. Motions can cause field redistributions (and non-uniformities), i.e., motion and non-uniformity are reciprocal causations. Some motions appear as perturbations (sounds), propagating through the field in a fixed speed. Some motions (e.g., central motions) may exist within a region in the field. This kind of motion structures, if steady and with certain stable properties, form particles.

All the field structures (including the background field) follow the same basic equations. This is primarily the simple model of universe this paper is attempting to build.

Here, “field structures” refer to those regions where field strength and velocity are forming certain stable patterns. We will also call development and interaction of field structures as “procedures” or “events”. However, there is no clear distinction between field structures and the background field. Ether field, existing in the whole space, acts as a whole following the same equations in all the places, all the time. Background field is not necessarily uniform or steady. Instead, there can be all kinds of distributions and motions. We can extract stable patterns as “field structures” or particles for particular interest. But in principle, background field, all the structures, interactions and events could be described by solving the field equations under certain initial/boundary conditions.
On the other hand, the equations and the description of the field are based on certain measurement systems. In this picture of universe, ether is what it is, acts as how it acts. The measurement systems are artificial (defined by human). Measurement results depend on many factors (e.g., scope of the measurement, motion status of the observer), and every observer relies on the measurement results to describe and understand the field. So different observers can have different descriptions for the same field and events. Moreover, as ether is the only being in the space, there are no other objects to refer to when doing measurements. What can be employed to measure the field is the field itself. Strictly speaking, clocks and rulers are just certain field structures, which follow the same equations as do other parts of the field that clocks and rulers are used to measure.

As a result of that, measurements have limitations. Certain properties are the same for all observers (i.e., are absolute). Others can be different when measured by different observers (i.e., are relative). The relative properties may also follow certain rules of transformation between different observers’ views.

There are also properties that are fundamentally not detectable (not measurable). One example is the (absolute) field strength. The basic field equations are homogeneous in $\Omega$. If $(\Omega, u_x, u_y, u_z)$ is a solution to the equations, $(k\Omega, u_x, u_y, u_z)$ is also a solution (where $k$ is a positive constant and field strengths in the boundary/initial conditions are also multiplied by $k$). This means that an observer inside certain boundary can only measure relative field strength inside the boundary. Absolute strength is not measurable, as absolute strength of the field does not have any impact on the physics laws that the observed procedures follow.

We will call this “strength relativity” of ether field.

12. Motion Relativity and Lorentz Transformation

As basic assumptions/postulates, ECA and SPA are universal. They are assumed to stand regardless of strength, uniformity and motion status of the field, regardless of motion status of the observer. From different observers’ views, space (distance between events, size of field structures, etc.), time (duration between events), field strength, velocity, etc. can be different even for the same field and events. However, these properties always follow ECA and SPA.

As ECA and SPA always stand, so do the ether field equations, which are derived from ECA and SPA. This is regardless of motion status of observers. In other words, motion status of an observer (reference system) in ether field does not impact the physics laws that the observed ether procedures satisfy.

This is another relative property of the field. It is a natural inference of the basic/universal assumptions ECA and SPA. We will call it “motion relativity” hereafter in this paper.

In an obvious sense, this is an “ether version” of the (special and general) principles of relativity that Albert Einstein introduced about a century ago to build special and general theories of relativity. However, it is slightly different in the
approach towards this principle here. In Einstein’s theories, principle of relativity is a postulate independent of light speed constancy. Here in ether field theory, we set sound (light) speed constancy as a postulate (SPA), which together with another postulate (ECA) leads to momentum and energy conservations and the basic field equations. As ether is considered the only being in the space, the field equation system determines all physics procedures, or in other words, contains all the physics laws. Wherever SPA and ECA stand, the equations apply and physics laws stand the same.

In this sense, principle of relativity is “included” in motion relativity. As consequences, we would expect ether field to be “relativistic”, or in other words, ether field theory to be consistent with theory of relativity. This is the main topic for the rest of this section (special relativity) and next section (possible analogy of gravity in ether field).

Let us consider two different observers (reference systems), $O$ and $O'$, in the same region of the field. $O$ can be in arbitrary motion status but $O'$ relative to $O$ moves in a steady velocity, $v(|v| < c)$. Obviously, each of $O$ and $O'$ will have its own measurement/observation of the background field and the field structures/events. But for both of them, SPA and ECA stand. Perturbations (sounds) travel in the same fixed speed relative to each of them, and observations of the field from $O$ and $O'$, respectively, satisfy the same basic equations.

From light speed constancy and principle of relativity, Einstein derived Lorentz transformation, which describes the relations between time and space measured from different (inertial) reference systems. If we let sound speed take the position of light speed, and consider that all physics laws are from ECA and SPA (thus are the same for different reference systems), we have counterparts of the two assumptions Einstein used, and can do similar derivation that leads to an ether version of Lorentz transformation (in which light speed is replaced by sound speed).

Naturally, we would then expect this transformation to be consistent with the field equation system, or in other words, expect the field equations to be Lorentz-invariant. If this stands, ether field structures and motions, as solutions of the field equations, would then be “relativistic”.

The rest of this section is devoted to the proof of Lorentz invariance for the three basic equations [i.e., (10.1)–(10.3)].

For convenience, let us consider the standard configuration in which origin points of reference systems $O(x, y, z, t)$ and $O'(x', y', z', t')$ are coincident at $t = 0$, $x$ and $x'$ align, $y$ and $y'$, $z$ and $z'$ are parallel, respectively, and the velocity of $O'$ relative to $O$ (denoted as $v$) is in the $x$-direction (and $0 < |v| < c$). Field strength and speed measured from the two reference systems are, respectively, denoted as $(\Omega, u_x, u_y, u_z)$ and $(\Omega', u'_x, u'_y, u'_z)$. With Lorentz factor $\alpha = \sqrt{1 - \frac{v^2}{c^2}}$, the Lorentz transformations read

$$x' = \frac{1}{\alpha} (x - vt), \quad y' = y, \quad z' = z, \quad t' = \frac{1}{\alpha} \left( t - \frac{vx}{c^2} \right)$$
or
\[ x = \frac{1}{\alpha} (x' + vt'), \quad y = y', \quad z = z', \quad t = \frac{1}{\alpha} \left( t' - \frac{vx'}{c^2} \right) \]

and
\[
\frac{\partial}{\partial x} = \frac{1}{\alpha} \frac{\partial}{\partial x'} - \frac{1}{\alpha} \frac{v}{c^2} \frac{\partial}{\partial t'}, \quad \frac{\partial}{\partial y} = \frac{\partial}{\partial y'}, \quad \frac{\partial}{\partial z} = \frac{\partial}{\partial z'}, \quad \frac{\partial}{\partial t} = \frac{1}{\alpha} \frac{\partial}{\partial t'} - \frac{v}{\alpha} \frac{\partial}{\partial x'}, \quad (12.1)
\]
hence
\[
\begin{align*}
\nu_x' &= \frac{u_x - v}{1 - \frac{u_x v}{c^2}}, \\
u_y' &= \frac{\alpha u_y}{1 - \frac{u_x v}{c^2}}, \\
u_z' &= \frac{\alpha u_z}{1 - \frac{u_x v}{c^2}}, \\
\Omega' &= \frac{\Omega}{\alpha} \left( 1 - \frac{u_x v}{c^2} \right)
\end{align*}
\]
or
\[
\begin{align*}
u_x &= \frac{u_x' + v}{1 + \frac{u_x v}{c^2}}, \\
u_y &= \frac{\alpha u_y}{1 + \frac{u_x v}{c^2}}, \\
u_z &= \frac{\alpha u_z}{1 + \frac{u_x v}{c^2}}, \\
\Omega' &= \frac{\Omega'}{\alpha} \left( 1 + \frac{u_x v}{c^2} \right).
\end{align*}
\]

Note that the transformation of field strength can be derived from ECA and the transformation of dimensions (volume).

From the above, it is straightforward to confirm that pressure is Lorentz-invariant, i.e.,
\[ \Omega^2 (c^2 - u^2) = \Omega^2 (c^2 - u'^2). \]
Actually, \( \Omega^2 (c^2 - u_x^2) \), \( \Omega u_y \), \( \Omega u_z \) and \( \Omega^2 u_y u_z \) are also Lorentz-invariant. These (and a few other identities) will be used in the following derivations.

### 12.1. Lorentz invariance of ether equation

Substituting the transformations (12.1) to ether equation (10.1), we have
\[
\frac{1}{\alpha} \frac{\partial}{\partial t'} \Omega - \frac{v}{\alpha} \frac{\partial}{\partial x'} \Omega + \frac{1}{\alpha} \frac{\partial}{\partial x'} (\Omega u_x) - \frac{1}{\alpha} \frac{v}{c^2} \frac{\partial}{\partial t'} (\Omega u_x) + \frac{\partial}{\partial y'} (\Omega u_y) + \frac{\partial}{\partial z'} (\Omega u_z) = 0
\]
or
\[
\frac{\partial}{\partial t'} \left( \frac{\Omega}{\alpha} \left( 1 - \frac{vu_x}{c^2} \right) \right) + \frac{\partial}{\partial x'} \left( \frac{\Omega}{\alpha} (u_x - v) \right) + \frac{\partial}{\partial y'} (\Omega u_y) + \frac{\partial}{\partial z'} (\Omega u_z) = 0.
\]
Applying (12.2) and the invariance of \( \Omega u_y \) and \( \Omega u_z \), it becomes
\[
\frac{\partial}{\partial t'} \Omega' + \frac{\partial}{\partial x'} (\Omega' u_x') + \frac{\partial}{\partial y'} (\Omega' u_y') + \frac{\partial}{\partial z'} (\Omega' u_z') = 0. \quad (12.3)
\]
This is the ether equation in system $O'(x', y', z', t')$. In other words, the ether equation is Lorentz-invariant.

### 12.2. Lorentz invariance of momentum equation in lateral directions

With Lorentz invariance of pressure, momentum equations in $y$- and $z$-directions (i.e., the lateral momentum equations) can be transformed in a similar way as for ether equation. Taking $y$-direction as an example, the momentum equation in $y$,

$$
\frac{\partial}{\partial t}(\Omega^2 u_y) + \frac{\partial}{\partial x}(\Omega^2 u_x u_y) + \frac{\partial}{\partial y}(\Omega^2 u_y u_y) + \frac{\partial}{\partial z}(\Omega^2 u_y u_z) = -\frac{1}{2} \frac{\partial}{\partial y}(\Omega^2(c^2 - u^2)),
$$

can be transformed to

$$
\frac{1}{\alpha} \frac{\partial}{\partial t'}(\Omega^2 u_y) - \frac{v}{\alpha} \frac{\partial}{\partial x'}(\Omega^2 u_y) + \frac{1}{\alpha} \frac{\partial}{\partial x'}(\Omega^2 u_y u_x) - \frac{1}{\alpha} \frac{v}{c^2} \frac{\partial}{\partial y'}(\Omega^2 u_y u_x) + \frac{\partial}{\partial y'}(\Omega^2 u_y u_y) + \frac{\partial}{\partial z'}(\Omega^2 u_y u_z)
$$

$$
= -\frac{1}{2} \frac{\partial}{\partial y'}(\Omega^2(c^2 - u'^2)).
$$

With (12.2) and the invariance of $\Omega u_y$, $\Omega u_z$ and pressure, it is also

$$
\frac{\partial}{\partial t'}\left(\frac{\Omega^2 u_y}{\alpha}(1 - \frac{v u_x}{c^2})\right) + \frac{\partial}{\partial x'}\left(\frac{\Omega^2 u_y}{\alpha}(u_x - v)\right) + \frac{\partial}{\partial y'}(\Omega^2 u_y u_y') + \frac{\partial}{\partial z'}(\Omega^2 u_y u_z')
$$

$$
= -\frac{1}{2} \frac{\partial}{\partial y'}(\Omega^2(c^2 - u'^2))
$$

or

$$
\frac{\partial}{\partial t'}(\Omega^2 u_y') + \frac{\partial}{\partial x'}(\Omega^2 u_x' u_y') + \frac{\partial}{\partial y'}(\Omega^2 u_y' u_y') + \frac{\partial}{\partial z'}(\Omega^2 u_y' u_z')
$$

$$
= -\frac{1}{2} \frac{\partial}{\partial y'}(\Omega^2(c^2 - u'^2)),
$$

(12.4)

which is exactly the momentum equation in $y'$-direction in $O'(x', y', z', t')$.

### 12.3. Transformation of energy equation

As mentioned in Sec. 10, energy equation (10.3) can be derived from ether equation (10.1) and momentum equation (10.2). Theoretically speaking, once (10.1) and (10.2) are Lorentz-invariant, (10.3) is invariant as well.

However, as shown below, transformation of the energy equation (10.3) in reference system $O$ leads to components of both the energy equation and the longitudinal momentum equation in system $O'$. Similarly (discussed in Sec. 12.4), transformation of the longitudinal momentum equation in $O$ also leads to both the energy equation and longitudinal momentum equation components in $O'$. In other words, neither the energy equation nor the longitudinal momentum equation transforms to solely the counter equation in $O'$. As a result,
neither of them can be proved Lorentz invariant separately (at least by the approach taken in this paper). Nevertheless, the Lorentz invariance shows up when the two transformed equations are considered together (discussed in Sec. 12.4).

Take the simplified format of energy equation, (10.4),

$$\frac{\partial}{\partial t} \left( \left( 1 + \frac{u^2}{c^2} \right) \Omega^2 \right) + 2 \nabla \cdot \left( \Omega^2 \mathbf{u} \right) = 0,$$

which expands to

$$\frac{\partial}{\partial t} \left( c^2 \Omega^2 + \Omega^2 u_x^2 + \Omega^2 u_y^2 + \Omega^2 u_z^2 \right) + 2c^2 \frac{\partial}{\partial x} (\Omega^2 u_x)
+ 2c^2 \frac{\partial}{\partial y} (\Omega^2 u_y) + 2c^2 \frac{\partial}{\partial z} (\Omega^2 u_z) = 0.$$

Applying the transformations, it becomes

$$\frac{1}{\alpha} \frac{\partial}{\partial t'} \left( c^2 \Omega^2 + \Omega^2 u_x^2 + \Omega^2 u_y^2 + \Omega^2 u_z^2 \right) - \frac{\alpha}{\partial x'} \left( c^2 \Omega^2 + \Omega^2 u_x^2 + \Omega^2 u_y^2 + \Omega^2 u_z^2 \right)
+ 2c^2 \frac{\partial}{\partial x'} (\Omega^2 u_x) - 2 \frac{\alpha}{\partial y'} (\Omega^2 u_x) + 2c^2 \frac{\partial}{\partial y'} (\Omega^2 u_y) + 2c^2 \frac{\partial}{\partial z'} (\Omega^2 u_z) = 0$$

or

$$\frac{1}{\alpha} \frac{\partial}{\partial t'} \left( c^2 \Omega^2 + \Omega^2 u_x^2 + \Omega^2 u_y^2 + \Omega^2 u_z^2 - 2 \Omega^2 u_x v \right)
- \frac{\alpha}{\partial x'} \left( c^2 \Omega^2 + v \Omega^2 u_x^2 + v \Omega^2 u_y^2 + v \Omega^2 u_z^2 - 2c^2 \Omega^2 u_x \right)
+ 2c^2 \frac{\partial}{\partial y'} (\Omega^2 u_y) + 2c^2 \frac{\partial}{\partial z'} (\Omega^2 u_z) = 0. \quad (12.5)$$

The first term can be further transformed as

$$\frac{1}{\alpha} \frac{\partial}{\partial t'} \left( c^2 \Omega^2 + \Omega^2 u_x^2 + \Omega^2 u_y^2 + \Omega^2 u_z^2 - 2 \Omega^2 u_x v \right)
= \frac{1}{\alpha} \frac{\partial}{\partial t'} \left[ c^2 \Omega^2 + \Omega^2 (u_x^2 - 2u_x v + v^2) - v^2 \Omega^2 + \Omega^2 u_y^2 + \Omega^2 u_z^2 \right]
= \frac{1}{\alpha} \frac{\partial}{\partial t'} \left[ \Omega^2 (c^2 - v^2) + \Omega^2 (u_x - v)^2 + \Omega^2 u_y^2 + \Omega^2 u_z^2 \right]
= \frac{1}{\alpha} \frac{\partial}{\partial t'} \left[ c^2 \alpha^2 \Omega^2 + \alpha^2 \Omega^2 u_x^2 + \Omega^2 u_y^2 + \Omega^2 u_z^2 \right]
= \frac{1}{\alpha} \frac{\partial}{\partial t'} \left[ c^2 \Omega^2 \left( 1 + \frac{u_x^2}{c^2} \right)^2 + \Omega^2 u_x^2 - \frac{v^2}{c^2} \Omega^2 u_x^2 + \Omega^2 u_y^2 + \Omega^2 u_z^2 \right]
= \frac{1}{\alpha} \frac{\partial}{\partial t'} \left( c^2 \Omega^2 + 2 \Omega^2 u_x v + \Omega^2 u_x^2 + \Omega^2 u_y^2 + \Omega^2 u_z^2 \right)
= \frac{1}{\alpha} \frac{\partial}{\partial t'} \left( c^2 \Omega^2 + \Omega^2 u_x^2 + \Omega^2 u_y^2 + \Omega^2 u_z^2 \right) + \frac{1}{\alpha} \frac{\partial}{\partial t'} (2 \Omega^2 u_x v).$$
Applying the identity $\alpha^2 = (1 + \frac{u_x v}{c^2})(1 - \frac{u_x v}{c^2})$, the second term of (12.5) can be transformed as

$$
-\frac{1}{\alpha} \frac{\partial}{\partial x'} (c^2 \Omega^2 + \Omega^2 u_x^2 + \Omega^2 u_y^2 + \Omega^2 u_z^2 - 2c^2 \Omega^2 u_x)
$$

$$
= -\frac{1}{\alpha} \frac{\partial}{\partial x'} \left[ \Omega^2 (c^2 v - c^2 u_x + c^2 u_x^2) + \Omega^2 u_y^2 + \Omega^2 u_z^2 \right]
$$

$$
= \frac{1}{\alpha} \frac{\partial}{\partial x'} \left[ \frac{c^2 \Omega^2}{1 - \frac{u_x v}{c^2}} \left( \frac{u_x - v}{c^2} \right) \left( \frac{u_x - v}{c^2} + u_x \right) - \Omega^2 u_y^2 - \Omega^2 u_z^2 \right]
$$

$$
= \frac{1}{\alpha} \frac{\partial}{\partial x'} \left[ \frac{c^2 \Omega^2}{1 - \frac{u_x v}{c^2}} \left( \frac{u_x + u_x' + v}{1 + \frac{u_x v}{c^2}} \right) - \Omega^2 u_y^2 - \Omega^2 u_z^2 \right]
$$

$$
= \frac{1}{\alpha} \frac{\partial}{\partial x'} \left[ 2c^2 \Omega^2 u_x' + \Omega^2 u_y^2 v + c^2 \Omega^2 v - \Omega^2 v^2 - \Omega^2 u_z^2 \right].
$$

The last two terms of (12.5) can be transformed as

$$
2c^2 \frac{\partial}{\partial y'} (\Omega^2 u_y) + 2c^2 \frac{\partial}{\partial z'} (\Omega^2 u_z)
$$

$$
= 2c^2 \frac{\partial}{\partial y'} \left( \Omega^2 u_y' \frac{1 + \frac{u_x v}{c^2}}{\alpha} \right) + 2c^2 \frac{\partial}{\partial z'} \left( \Omega^2 u_z' \frac{1 + \frac{u_x v}{c^2}}{\alpha} \right)
$$

$$
= 2c^2 \frac{1}{\alpha} \frac{\partial}{\partial y'} (\Omega^2 u_y') + 2 \frac{\partial}{\partial y'} (\Omega^2 u_y' \frac{u_x v}{\alpha})
$$

$$
+ 2c^2 \frac{1}{\alpha} \frac{\partial}{\partial z'} (\Omega^2 u_z') + 2 \frac{\partial}{\partial z'} (\Omega^2 u_z' \frac{u_x v}{\alpha})
$$

$$
= 2c^2 \frac{1}{\alpha} \frac{\partial}{\partial y'} (\Omega^2 u_y') + 2c^2 \frac{1}{\alpha} \frac{\partial}{\partial z'} (\Omega^2 u_z')
$$

$$
+ 2 \frac{\partial}{\partial y'} (\Omega^2 u_y' u_y') + 2 \frac{\partial}{\partial z'} (\Omega^2 u_z' u_z').
$$

Substituting these transformed terms back to (12.5), we have

$$
\frac{1}{\alpha} \frac{\partial}{\partial t'} (c^2 \Omega^2 + \Omega^2 u_x^2 + \Omega^2 u_y^2 + \Omega^2 u_z^2) + \frac{1}{\alpha} \frac{\partial}{\partial t'} (2\Omega^2 u_x v)
$$

$$
+ \frac{1}{\alpha} \frac{\partial}{\partial x'} (2c^2 \Omega^2 u_x' + \Omega^2 u_x^2 v + c^2 \Omega^2 v - \Omega^2 u_y^2 - \Omega^2 u_z^2)
$$

$$
+ 2c^2 \frac{1}{\alpha} \frac{\partial}{\partial y'} (\Omega^2 u_y') + 2c^2 \frac{1}{\alpha} \frac{\partial}{\partial z'} (\Omega^2 u_z') + 2 \frac{\partial}{\partial y'} (\Omega^2 u_y' u_y')
$$

$$
+ 2 \frac{\partial}{\partial z'} (\Omega^2 u_z' u_z') = 0.
$$
which is
\[
\frac{1}{\alpha} \frac{\partial}{\partial t'} (c^2\Omega^2 + \Omega^2 u_x'^2 + \Omega^2 u_y'^2 + \Omega^2 u_z'^2) + 2c^2 \frac{1}{\alpha} \frac{\partial}{\partial x'} (\Omega^2 u_x') \\
+ 2c^2 \frac{1}{\alpha} \frac{\partial}{\partial y'} (\Omega^2 u_y') + 2c^2 \frac{1}{\alpha} \frac{\partial}{\partial z'} (\Omega^2 u_z') + \frac{2v}{\alpha} \frac{\partial}{\partial t'} (\Omega^2 u_x') \\
+ \frac{2v}{\alpha} \frac{\partial}{\partial x'} (\Omega^2 u_x'^2) + \frac{2v}{\alpha} \frac{\partial}{\partial y'} (\Omega^2 u_x'u_y') + \frac{2v}{\alpha} \frac{\partial}{\partial z'} (\Omega^2 u_x'u_z') \\
+ \frac{v}{\alpha} \frac{\partial}{\partial x'} (c^2\Omega^2 - \Omega^2 u_x'^2 - \Omega^2 u_y'^2 - \Omega^2 u_z'^2) = 0
\]
or
\[
\frac{c^2}{\alpha} \left[ \frac{\partial}{\partial t'} \left( \left( 1 + \frac{u'^2}{c^2} \right) \Omega^2 \right) + 2\nabla' \cdot (\Omega^2 u') \right] \\
+ \frac{2v}{\alpha} \left[ \frac{\partial}{\partial t'} (\Omega^2 u_x') + \frac{\partial}{\partial x'} (\Omega^2 u_x'^2) + \frac{\partial}{\partial y'} (\Omega^2 u_x'u_y') \right] \\
+ \frac{\partial}{\partial z'} (\Omega^2 u_x'u_z') + \frac{1}{2} \frac{\partial}{\partial x'} (\Omega^2 (c^2 - u'^2)) \right] = 0. \quad (12.6)
\]

Obviously, the first term contains all the components of the energy equation in the new reference system \( O' \), and the second term has all components of the longitudinal momentum equation in \( O' \). In other words, (12.6) is a combination of the energy equation and the longitudinal momentum equation in \( O' \).

For convenience, we can denote the left-hand side of the energy equation (10.4) by \( \Gamma_e \), and denote the left-hand side of the longitudinal momentum equation (with the force term moved to the left-hand side) by \( \Gamma_{mx} \). That is
\[
\Gamma_e \triangleq \frac{\partial}{\partial t} \left( \left( 1 + \frac{u'^2}{c^2} \right) \Omega^2 \right) + 2\nabla' \cdot (\Omega^2 u), \\
\Gamma_{mx} \triangleq \frac{\partial}{\partial t} (\Omega^2 u_x) + \frac{\partial}{\partial x} (\Omega^2 u_x'^2) + \frac{\partial}{\partial y} (\Omega^2 u_x'u_y) \\
+ \frac{\partial}{\partial z} (\Omega^2 u_x'u_z) + \frac{1}{2} \frac{\partial}{\partial x} (\Omega^2 (c^2 - u'^2)).
\]

With that, the energy equation (10.4) and the longitudinal momentum equation in reference system \( O \) can be, respectively, written as \( \Gamma_e = 0 \) and \( \Gamma_{mx} = 0 \); their counterequations in system \( O' \), which are yet to be proved, are \( \Gamma'_e = 0 \) and \( \Gamma'_{mx} = 0 \); and the transformed energy equation (12.6) can be written as
\[
\frac{c^2}{\alpha} \Gamma'_e + \frac{2v}{\alpha} \Gamma'_{mx} = 0. \quad (12.7)
\]

Solely from (12.7), we cannot conclude \( \Gamma'_e = 0 \) and \( \Gamma'_{mx} = 0 \). In order to derive that conclusion, we will need further relation between \( \Gamma'_e \) and \( \Gamma'_{mx} \). This can be obtained by transforming the longitudinal momentum equation too to system \( O' \), which leads to another combination of \( \Gamma'_e \) and \( \Gamma'_{mx} \).
12.4. Lorentz invariance of energy equation and longitudinal momentum equation

In the $x$-direction, the momentum equation is

$$
\frac{\partial}{\partial t}(\Omega^2 u_x) + \frac{\partial}{\partial x}(\Omega^2 u_x^2) + \frac{\partial}{\partial y}(\Omega^2 u_x u_y) + \frac{\partial}{\partial z}(\Omega^2 u_x u_z) = -\frac{1}{2} \frac{\partial}{\partial x}(\Omega^2(c^2-u^2)).
$$

(12.8)

The left-hand side can be transformed to

\[
\frac{1}{\alpha} \frac{\partial}{\partial t'}(\Omega^2 u_x) - \frac{v}{\alpha} \frac{\partial}{\partial x'}(\Omega^2 u_x) + \frac{1}{\alpha} \frac{\partial}{\partial x'}(\Omega^2 u_x^2) - \frac{1}{\alpha} \frac{v}{c^2} \frac{\partial}{\partial t'}(\Omega^2 u_x^2) \\
+ \frac{\partial}{\partial y'}(\Omega^2 u_x u_y) + \frac{\partial}{\partial z'}(\Omega^2 u_x u_z) \\
= \frac{1}{\alpha} \frac{\partial}{\partial t'} \left( \Omega^2 u_x \left( 1 - \frac{u_x v}{c^2} \right) \right) + \frac{1}{\alpha} \frac{\partial}{\partial x'}(\Omega^2 u_x (u_x - v)) + \frac{\partial}{\partial y'}(\Omega^2 u_x u_y) + \frac{\partial}{\partial z'}(\Omega^2 u_x u_z) \\
= \frac{1}{\alpha} \frac{\partial}{\partial t'} \left( \Omega^2 \left( 1 - \frac{u_x v}{c^2} \right) u_x - \frac{\alpha^2}{c^2} \right) \\
+ \frac{1}{\alpha} \frac{\partial}{\partial x'} \left( \Omega^2 \left( 1 - \frac{u_x v}{c^2} \right) u_x - \frac{\alpha^2}{c^2} \right) \\
+ \frac{\partial}{\partial y'} (\Omega u_x (\Omega u_y)) + \frac{\partial}{\partial z'} (\Omega u_x (\Omega u_z)) \\
= \frac{1}{\alpha} \frac{\partial}{\partial t'} \left( \Omega^2 u_x (1 + \frac{u_x v}{c^2}) \right) + \frac{1}{\alpha} \frac{\partial}{\partial x'}(\Omega^2 u_x u_x (1 + \frac{u_x v}{c^2})) \\
+ \frac{\partial}{\partial y'} (\Omega u_x + v)(\Omega u_y') + \frac{\partial}{\partial z'} (\Omega u_x + v)(\Omega u_y') \\
= \frac{1}{\alpha} \frac{\partial}{\partial t'}(\Omega^2 (u_x + v)) + \frac{1}{\alpha} \frac{\partial}{\partial x'}(\Omega^2 u_x (u_x + v)) + \frac{1}{\alpha} \frac{\partial}{\partial y'}(\Omega^2 (u_x + v) u_y') \\
+ \frac{1}{\alpha} \frac{\partial}{\partial z'}(\Omega^2 (u_x + v) u_z') \\
= \frac{1}{\alpha} \left[ \frac{\partial}{\partial t'}(\Omega^2 u_x') + \frac{\partial}{\partial x'}(\Omega^2 u_x') + \frac{\partial}{\partial y'}(\Omega^2 u_x u_y') + \frac{\partial}{\partial z'}(\Omega^2 u_x u_z') \right] \\
+ \frac{\nu}{\alpha} \left[ \frac{\partial}{\partial t'}(\Omega^2 u_y') + \frac{\partial}{\partial x'}(\Omega^2 u_y') + \frac{\partial}{\partial y'}(\Omega^2 u_y') + \frac{\partial}{\partial z'}(\Omega^2 u_z') \right],
\]

and the right-hand side is

\[
-\frac{1}{2} \frac{\partial}{\partial x}(\Omega^2(c^2-u^2)) = -\frac{1}{2} \frac{\partial}{\partial x}(\Omega^2(c^2-u^2)) \\
= -\frac{1}{2\alpha} \frac{\partial}{\partial x'}(\Omega^2(c^2-u^2)) + \frac{\nu}{2\alpha} \frac{\partial}{\partial t'}(\Omega^2(c^2-u^2)) \\
= -\frac{1}{2\alpha} \frac{\partial}{\partial x'} \left( \Omega^2(c^2-u^2) \right) + \frac{\nu}{2\alpha} \frac{\partial}{\partial t'} \left( \Omega^2 \left( 1 - \frac{u_x^2}{c^2} \right) \right),
\]

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Consider (12.10) and (12.11) together. As (12.8) thus becomes
\[
\frac{1}{\alpha} \left[ \frac{\partial}{\partial t'} \left( \Omega^2 u_x \right) + \frac{\partial}{\partial x'} \left( \Omega^2 u_x' \right) + \frac{\partial}{\partial y'} \left( \Omega^2 u_y \right) + \frac{\partial}{\partial z'} \left( \Omega^2 u_z \right) \right] \\
+ \frac{v}{\alpha} \left[ \frac{\partial}{\partial t'} \left( \Omega^2 u_x \right) + \frac{\partial}{\partial x'} \left( \Omega^2 u_x' \right) + \frac{\partial}{\partial y'} \left( \Omega^2 u_y \right) + \frac{\partial}{\partial z'} \left( \Omega^2 u_z \right) \right] \\
= -\frac{1}{\alpha} \frac{\partial}{\partial x'} \left( \frac{1}{2} \Omega^2 (c^2 - u^2) \right) + \frac{v}{\alpha} \frac{\partial}{\partial t'} \left( \frac{1}{2} \Omega^2 \left(1 - \frac{u^2}{c^2} \right) \right),
\]
which is
\[
\frac{1}{\alpha} \left[ \frac{\partial}{\partial t'} \left( \Omega^2 u_x' \right) + \frac{\partial}{\partial x'} \left( \Omega^2 u_x \right) + \frac{\partial}{\partial y'} \left( \Omega^2 u_y \right) + \frac{\partial}{\partial z'} \left( \Omega^2 u_z \right) \right] \\
+ \frac{v}{\alpha} \left[ \frac{\partial}{\partial t'} \left( \frac{1}{2} \Omega^2 \left(1 + \frac{u^2}{c^2} \right) \right) + \frac{\partial}{\partial x'} \left( \Omega^2 u_x \right) + \frac{\partial}{\partial y'} \left( \Omega^2 u_y \right) + \frac{\partial}{\partial z'} \left( \Omega^2 u_z \right) \right] = 0,
\]
or
\[
\frac{1}{\alpha} \left[ \frac{\partial}{\partial t'} \left( \Omega^2 u_x' \right) + \frac{\partial}{\partial x'} \left( \Omega^2 u_x \right) + \frac{\partial}{\partial y'} \left( \Omega^2 u_y \right) \right] \\
+ \frac{\partial}{\partial z'} \left( \Omega^2 u_z' \right) + \frac{1}{2} \frac{\partial}{\partial x'} \left( \Omega^2 (c^2 - u^2) \right) \\
+ \frac{v}{2\alpha} \left[ \frac{\partial}{\partial t'} \left( \left(1 + \frac{u^2}{c^2} \right) \Omega^2 \right) + 2 \nabla' \cdot (\Omega^2 \mathbf{u}') \right] = 0. \tag{12.9}
\]
Similar to (12.7) in Sec. 12.3, (12.9) here is also a combination of the longitudinal momentum equation and the energy equation in system $O'$. Applying the denotations in Sec. 12.3, (12.9) can be written as (12.10) below. Putting it together with (12.7) we have
\[
\begin{cases}
\frac{v}{2\alpha} \Gamma_e + \frac{1}{\alpha} \Gamma_{mx} = 0, \\
c^2 \frac{\Gamma_e}{\alpha} + \frac{2v}{\alpha} \Gamma_{mx} = 0.
\end{cases} \tag{12.10}
\]
(12.11)

Consider (12.10) and (12.11) together. As $0 < |v| < c$, $\alpha \neq 0$. The two combinations on the left-hand side of both equations are linearly independent. They both vanish if and only if $\Gamma_e = 0$ and $\Gamma_{mx} = 0$. In other words, as (12.10) and (12.11) both stand, we can conclude $\Gamma_e = 0$ and $\Gamma_{mx} = 0$, which are just the two counterequations we need:
\[
\frac{\partial}{\partial t'} \left( \left(1 + \frac{u^2}{c^2} \right) \Omega^2 \right) + 2 \nabla' \cdot (\Omega^2 \mathbf{u}') = 0,
\]
\[
\frac{\partial}{\partial t'} \left( \Omega^2 u_x' \right) + \frac{\partial}{\partial x'} \left( \Omega^2 u_x \right) + \frac{\partial}{\partial y'} \left( \Omega^2 u_y \right) + \frac{\partial}{\partial z'} \left( \Omega^2 u_z' \right) = -\frac{1}{2} \frac{\partial}{\partial x'} \left( \Omega^2 (c^2 - u^2) \right).\]
This closes the proof of Lorentz invariance for all equations (under the standard configuration).

Existence of ether does not necessarily lead to absolute space/time. In the ether model proposed in this paper, based on ECA and SPA, the field equations by their nature are Lorentz-invariant. This means time and space are relative (and relativistic) in this proposed ether field. It is a requirement/conclusion from the field equations which are derived from the two basic assumptions, ECA and SPA. In this sense, we can consider that the ether field equations actually imply Lorentz transformations, or in other words, the proposed ether field theory, based on ECA and SPA, organically implies special relativity.

13. Non-uniform Motions and Gravity-like Force

Even with the background field throughout the space, the concept of inertial reference system is only for multiple observers that each is moving in steady velocity relative to each of the others. For a single reference system in the field, it is not possible (and not necessary) to know whether it is an inertial system or an accelerating one. For example, from a certain reference system $A$, field in a region may be flat (i.e., with uniform strength and uniform/steady velocity). But from another system $B$ (that is not in steady velocity relative to $A$), field in this region can be non-flat. Even in this special case, the observation from $A$ and that from $B$ are equivalent, as we do not know whether the field is “indeed” flat or not without employing a reference system. System $A$ is not special to be considered as an inertial one (even though it is “following the field” in certain sense). Neither is system $B$.

In general, the observations from system $A$ and from system $B$ could have complicated relation. However, according to motion relativity, they all follow ECA and SPA, thus follow the same equations. By further analyzing the equations, we may find some clues to understand the relation.

In particular, pressure is a function of field strength and speed, while force density is gradient of pressure. Obviously, force could be different for different observers. On the other hand, momentum equation requires that total force is equal to change rate of momentum. The difference of forces from different reference systems should be reflected in the momentum equation.

Expanding both sides of momentum equation (10.2) and applying the identity of

$$ \mathbf{V} \cdot \nabla \mathbf{V} = \frac{1}{2} \nabla V^2 - \mathbf{V} \times (\nabla \times \mathbf{V}) $$

we have

$$ \frac{2 \Omega}{c^2} \frac{\partial}{\partial t} (\Omega \mathbf{u}) + \frac{2 \Omega \mathbf{u}}{c^2} \frac{\partial \Omega}{\partial t} + \frac{2}{c^2} (\nabla \cdot (\Omega \mathbf{u})) (\Omega \mathbf{u}) - \frac{2}{c^2} (\Omega \mathbf{u}) \times (\nabla \times (\Omega \mathbf{u})) + \nabla \frac{\Omega^2 u^2}{c^2} = \nabla \frac{\Omega^2 u^2}{c^2} - \nabla \Omega^2 $$

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The second term vanishes according to the ether equation. Thus we have

\[
\frac{2\Omega}{c^2} \frac{\partial}{\partial t} (\Omega u) + \frac{2\Omega}{c^2} \left[ \frac{\partial \Omega}{\partial t} + \nabla \cdot (\Omega u) \right] - \frac{2}{c^2} (\Omega u) \\
\times (\nabla \times (\Omega u)) + \nabla \frac{\Omega^2 u^2}{c^2} = \nabla \frac{\Omega^2 u^2}{c^2} - \nabla \Omega^2.
\]

The second term vanishes according to the ether equation. Thus we have

\[
\frac{2\Omega}{c^2} \frac{\partial}{\partial t} (\Omega u) - \frac{2}{c^2} (\Omega u) \times (\nabla \times (\Omega u)) + \nabla \frac{\Omega^2 u^2}{c^2} = \nabla \frac{\Omega^2 u^2}{c^2} - \nabla \Omega^2 \tag{13.1}
\]

or

\[
\frac{2\Omega}{c^2} \frac{\partial}{\partial t} (\Omega u) - \frac{2}{c^2} (\Omega u) \times (\nabla \times (\Omega u)) = -\nabla \Omega^2. \tag{13.2}
\]

There is a common term, \( \nabla \frac{\Omega^2 u^2}{c^2} \) (denoted as \( \mathbf{G} \) hereafter for convenience), on both sides of (13.1). But on different sides, this term is from different sources.

On the right-hand side of (13.1), the two terms represent two parts of pressure gradient, or two sources of force. They can be considered as forces due to non-uniformity of kinetic energy density and non-uniformity of static energy density, respectively. These two terms together represent all the forces apply to ether in the field.

On the left-hand side of (13.1), \( \mathbf{G} \) is part of the momentum change rate. If we consider \( \mathbf{A} = \Omega \mathbf{u} \) as ether flow density in the field, \( \mathbf{G} = \nabla \frac{\Omega^2 u^2}{c^2} = \nabla \frac{\nabla \cdot \Omega u}{c^2} \) is the gradient of ether flow density (square). It can be considered as momentum change caused by non-uniform motion, or momentum change rate (over time) due to momentum flow change over space. The other two terms on the left-hand side of (13.1) can be considered as local momentum change rate over time and momentum change rate due to rotation, respectively. These three terms together form the total momentum change rate, which is equal to total force according to the momentum equation.

As \( \mathbf{G} \) appears on both sides of the equation, it cancels itself. In other words, the momentum change caused by non-uniform motion is always balanced by the force due to non-uniform kinetic energy. The momentum equation thus becomes (13.2). However, when we calculate the total force, this term is always a part of it. Same is for total acceleration [total momentum change rate, as on the left-hand side of (13.1)].

\( \frac{\Omega^2 u^2}{c^2} \) is non-negative and is proportional to the square of speed. At the points where field speed is zero, \( \frac{\Omega^2 u^2}{c^2} \) reaches the minimum and \( \mathbf{G} \) vanishes. Equivalently speaking, at any point in the field, from a local body-following reference system, ether speed is zero and this gradient term vanishes.

In other words, as a force term, \( \mathbf{G} \) can be locally vanished by choosing a proper reference system. This is similar to the feature of gravity (which is balanced out by inertia force in a free-falling reference system). In this sense, this force term \( \mathbf{G} \) is similar to gravity to a certain degree. We will call \( \mathbf{G} \) a “gravity-like” force, or “ether-Gravity” (or “e-Gravity” in short), hereafter in this paper. The prefix “e” indicates
that it is in ether field level, rather than in particle level as our normal understanding of gravity.

In general relativity, gravity is considered a property of space (i.e., curvature of spacetime). As a possible analog here, we consider this e-Gravity as a property of ether field over space. For different reference systems, field strength and speed vary, so does this term. The variation is not due to cancellation by inertial force. It is due to differences in field properties measured from different reference systems.

Generally speaking, by transforming the momentum equation to (13.1), we can, respectively, split the momentum change rate on the left-hand side and force on the right-hand side into several items. As the e-Gravity term shows up on both sides, it cancels itself from the equation. The details of this term thus can be temporarily “ignored” by this cancellation. If we find the solutions to Eq. (13.2), we can then calculate the gravity term from field strength and velocity, and add it back to both sides of the equation to find the total force and total momentum change rate.

Another important property of gravity is being proportional to gravitational mass. To have a more complete analogy to gravity, it is necessary to have a counterpart of gravitational mass in ether field and link force term $G$ to it. This is yet to be explored, especially after particle solutions of the field equations are found.

14. Electromagnetic Interaction in Ether Field

Rewriting (13.2) and putting it together with the ether equation, we have

$$\frac{\partial}{\partial t} \Omega + \nabla \cdot (\Omega \mathbf{u}) = 0, \quad (14.1)$$

$$\frac{\partial}{\partial t} (\Omega \mathbf{u}) - \mathbf{u} \times (\nabla \times (\Omega \mathbf{u})) = -\nabla c^2 \Omega. \quad (14.2)$$

Substituting the variables by $\varphi = c^2 \Omega$, $\mathbf{A} = \Omega \mathbf{u}$, $\mathbf{B} = \nabla \times \mathbf{A}$ and $\mathbf{E} = \mathbf{B} \times \mathbf{u}$, the above equations become

$$\begin{cases}
\frac{1}{c^2} \frac{\partial}{\partial t} \varphi + \nabla \cdot \mathbf{A} = 0, \\
\frac{\partial}{\partial t} \mathbf{A} + \mathbf{E} = -\nabla \varphi.
\end{cases} \quad (14.3)$$

$$\frac{\partial}{\partial t} \mathbf{B} = -\nabla \times \mathbf{E}. \quad (14.6)$$
Taking divergence of (14.4) and curl of $B = \nabla \times A$, applying Lorentz gauge condition, we can further obtain the wave equations for the two potentials:

$$\frac{1}{c^2} \frac{\partial^2 \varphi}{\partial t^2} - \nabla^2 \varphi = \nabla \cdot E,$$

$$\frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} - \nabla^2 A = -\frac{1}{c^2} \frac{\partial E}{\partial t} + \nabla \times B.$$ (14.8)

Except for that electric charge and current are not yet involved, all these equations [(14.3)–(14.8)] are exactly the same in appearance here as in Maxwell’s electromagnetic theory (with Lorentz gauge condition).

These surprising similarities naturally lead to a conjecture that electromagnetic field may be explainable/interpretable by ether field, or may actually be just the above alternative forms of ether field.

Note that the electromagnetic field in our current understanding is based on electric charge (and current, moment) of particles, and particles are expected to be ether field structures. In this sense, electromagnetic field in our current understanding is at “particle level”, while the ether field properties used in the above analogy are at “ether field level”. Further details will need to be explored for the analogy here which is trying to link properties in different levels. Especially, we will need a counterpart for electric charge in the (yet to be obtained) particle solutions to make the analogy complete.

Historically, there have been similar attempts to analogize electromagnetic field theory to fluid dynamics. It is Maxwell who in the years he was constructing the electromagnetic theory first suggested that vector potential behaves like a moving medium.\(^\text{12}\) Examples of more recent attempts can be found in some recent publications.\(^\text{13,14}\) All these analogies seem revealing some similar similarity between electromagnetic field and fluids. Most of these attempts are based on the fluid model described by Navier–Stokes equations. The suggested analogy in this paper, however, is based on ether field. As a new medium model, ether has some unique features compared to Navier–Stokes fluids. This brings some detailed differences for the above analogy.

Obviously, the analogy here, if it stands, will bring some totally new understanding of the electromagnetic field, as well as a series of new properties and consequences, which are yet to be further explored/examined.

For example, the scalar potential $\varphi = c^2 \Omega$ and vector potential $A = \Omega \mathbf{u}$ are now directly related to each other. The latter is just the flow density of the former. Similar is for electric field $E = B \times \mathbf{u}$ and magnetic field $B$. In this case $E$ is relying on $B$. Wherever $E$ exists, $B$ has to be non-zero. Also, $E$ and $B$ will be orthogonal to each other. All these need to be further examined in detail, especially when particle solutions of the ether field are available and electric charge has a counterpart in the solutions.

Another feature brought by this analogy is that Lorentz gauge condition is now mandatory, as it is just an alternative form of ether conservation.
At the same time, momentum equation (13.2) can be rewritten in “force format” as

$$\frac{2\Omega}{c^2} \frac{\partial}{\partial t} (\Omega u) - \frac{2\Omega}{c^2} u \times (\nabla \times (\Omega u)) = -\frac{2\Omega}{c^2} \nabla c^2 \Omega.$$ 

Denoting $\rho = \frac{2\Omega}{c^2}$ and doing the substitution with the electromagnetic terms, the above equation becomes

$$\rho \left( \frac{\partial}{\partial t} A + E \right) = -\rho \nabla \varphi. \quad (14.9)$$

Recall that (13.2) is just a simplified format of the momentum equation. The right-hand side of (13.2) or (14.9) is a force term. With the analogies above, we can now consider this term as the force related to the analogized electromagnetic field. It is the force density that applies to ether in the field. Very roughly speaking, it can be considered as an “ether-level” force similar to Lorentz force in particle level. We will borrow one more name and call it “e-Lorentz force” hereafter.

The right-hand side of the momentum equation (13.1) contains two terms of force. The first term was considered as a gravity-like force while now the second term is considered as the field-level Lorentz force. From ether field point of view, these are the only two kinds of force in the field. For complicated field structures, these two force terms can be complicated. But if described by field equations, all forces can always be split into two portions, namely e-Gravity and e-Lorentz force, as the two terms on the right-hand side of (13.1).

In the current electromagnetic field and gravity theories, electric charge is the source of electric field. It is considered a property of particles and is carried by the particles, thus is concentrated at a point (for point-particles) or in a small volume (for non-point-particles). Similar is for gravitational mass. On the other hand, electric field and gravity field around are considered to be created by the charge and mass, proportional to the amount of charge/mass and anti-proportional to the square of distance.

From ether field point of view, however, particles are conjectured as non-local field structures. A particle is not concentrated at a point or in a small volume. The field is the particle itself, rather than the “force field” it generates. Wherever the force exists, the field there is part of the particle structure.

In the short range, the particle solutions may have complicated structures and the two force terms (i.e., e-Gravity and e-Lorentz) can be complicated, especially when two or more particles are close to each other. In our current understanding, there are four kinds of force in particle level (i.e., EM interaction, strong interaction, weak interaction and gravity). These could be the equivalent effects of the two force terms and their combinations. In the long range, the overall effects of e-Gravity and e-Lorentz force on a whole particle might converge to the inverse-square gravity and Lorentz force of our normal understanding.

If this is the case, the particle properties like charge and gravitational mass would be the total effect of the related field properties, and could be calculated in certain way from the field-level properties.
Generally speaking, when we calculate total effects by integrating certain properties over a volume in space, in certain cases (e.g., in inverse-square field), the integral could be a fixed value if certain point or region is included, and vanishes otherwise. If this is the case for the particle solutions, it may explain the concentration of electric charge and gravitational mass in our current understanding. The concentration of charge and mass on point-particles could actually be a mathematical effect rather than a real physical distribution, from ether theory point of view.

On the other hand, as the field properties like strength, velocity and vorticity are being linked to electromagnetic properties by the analogy, so could the gravity-like force term. We can rewrite this term as \( G = \nabla \left( \frac{\Omega \kappa^2}{c^2} \right) = \nabla \frac{A^2}{c^2} \), and \( A \) here is just the vector potential according to the analogy. In other words, the e-Gravity could be calculated from vector potential (as an “electromagnetic property” in ether field), and is linked to the e-Lorentz force \( -\rho \nabla \psi \). This is a significant consequence that (if analogized to the case of normal gravity and Lorentz force) does not seem to be seen in known observations of gravity and electromagnetic field (nor, though, does it seem to be clearly contradicting the known observations/experiments). This would be a strong requirement that needs to be met for the analogies to gravity and electromagnetic force to stand. We can use it as a hard standard in the future to examine any further detailed analogy brought by particle solutions and counterparts of electric charge and gravitational mass.

15. Steady Solutions

Obviously, seeking for solutions of field equations to analogize particles is an essential goal of this proposed ether field theory. To a certain degree, this is to reactivate Lord Kelvin’s “vortex atom” conjecture in a new way and to pursue the ambition Lord Kelvin and many others have had since then.

In this and the following sections, we will discuss about the overall view of the equations and the possible “particle solutions”, the relation/similarity to the fluid dynamics open problem and a special solution as example for further discussion. In an approximate way, we will also discuss about the possible interpretation of light and photons, as well as particles and anti-particles from the ether field point of view.

As the exact general solutions are still yet to be obtained, most of the discussions in the rest of this paper are just as conjectures or preliminary thoughts, which hopefully might provide some clues for further exploration beyond this paper.

As discussed in Sec. 11, there must be a BGF that suffuses throughout the whole space to provide a positive pressure as boundary conditions. Field structures are then expected to exist inside the background field, as solutions of the field equations with the boundary conditions.

According to the equations and the properties discussed in previous sections, there can be all kinds of motions in the field. Among them can be some central motions that form vortex-like structures in the field. In those structures,
pressure near the structure center should be in general lower than pressure in the surrounding field, so that the gradient of pressure can provide the necessary centripetal force to support the central motions. This kind of structures, if stable, would be the field structures we are looking for to analogize particles (namely the “particle solutions”).

In general, particles are moving relative to each other (and relative to BGF). So a particle (as a field structure) in general would not be steady. To simplify the discussion however, we will first concentrate on the case of a steady flow in a stationary, flat and infinitely large BGF. This will allow us to explore the possible single-particle solutions without inference from other particles.

In steady case, momentum equation (13.2) and the ether equation can be rewritten with the notations of \( A = \frac{\nabla \times A}{C_1} \) and \( \psi = \frac{\mu}{C_1^2} \) as

\[
\begin{align*}
\{ A \times (\nabla \times A) &= \nabla \psi, \\
\nabla \cdot A &= 0.
\}
\]

(15.1) (15.2)

This is exactly the same as the steady Euler equations for incompressible fluids.\(^{15,16}\)

Note that in appearance ether is “compressible”, as field density is not a constant. However, the corresponding steady Euler equation is the one for incompressible fluids. The flow density vector, \( A \), which is corresponding to velocity in incompressible fluids, is divergenceless.

As per (15.1), \( A \) is perpendicular to \( \nabla \psi \), hence \( A \cdot \nabla \psi = 0 \). This further leads to \( \mathbf{u} \cdot \nabla \Omega = 0 \). In other words, ether velocity \( \mathbf{u} \) is perpendicular to gradient of ether strength in steady flow. Applying this to (15.2), we can further obtain \( \nabla \cdot \mathbf{u} = 0 \). That is, the velocity field in steady ether flow is also divergenceless.

With (15.1) and (15.2), every solution of incompressible Euler equations can be converted to a steady solution in ether field. In this way, the particle solutions we are seeking for are linked to the possible solutions of steady Euler equations in fluid mechanics.

Despite the great interest and efforts, general solution of the steady Euler equations is still an open problem in mechanics.\(^{15}\) It is part of the more general open problem of solving the Navier–Stokes equations, which is one of the Millennium Prize Problems by Clay Mathematics Institute.\(^{16}\) The research on this problem is ongoing actively, which hopefully will soon lead to the general solutions that will also benefit the study of ether field.

There are, however, some special solutions presented so far. We can use them as examples to preliminarily discuss about the possible interpretation of particles.

Assuming that vector field \( A \) is divergenceless [thus satisfying (15.2)], and is everywhere collinear to its curl, the left-hand side of (15.1) will vanish. If the field strength is uniform, the right-hand side of (15.1) also vanishes and (15.1) is satisfied. \( A \) and \( \psi \) in this case form a solution of the equation system.
This is the Beltrami flow in fluids mechanics. The left-hand side of (15.1) (which is the Lamb vector of \(A\)) is zero everywhere. In this case, \(A\) and its curl are linked by a scalar function of space, denoted as \(\lambda\). That is

\[ A = \lambda \nabla \times A. \]

As \(A\) is divergenceless, we have

\[ \nabla \cdot A = \nabla \cdot (\lambda \nabla \times A) = 0 \quad \text{or} \quad \nabla \lambda \cdot (\nabla \times A) = 0. \]

Except for \(\nabla \times A = 0\) (which is a trivial case), there are two cases for \(\lambda\) and \(A\) to meet this condition: Either \(\lambda\) is a constant thus \(\nabla \lambda = 0\) (in this case, \(A\) is called a linear Beltrami flow) or \(\lambda\) is not a constant (i.e., \(A\) is a non-linear Beltrami flow) but \(\nabla \lambda\) is everywhere perpendicular to \(A\).

A good example for linear Beltrami flow is the engine vector of the curl operator in sphere coordination system\(^{17}\):

\[
\begin{align*}
A_r &= K_n n (n + 1) r^{-3/2} J_{n+1/2} (r) P_n (\cos \theta), \\
A_\theta &= K_n r^{-1/2} [J_{n-1/2} (r) - n J_{n+1/2} (r)] P_n ^1 (\cos \theta), \\
A_\phi &= -K_n r^{-1/2} J_{n+1/2} (r) P_n ^1 (\cos \theta).
\end{align*}
\]

Here \(K_n\) is a constant; \(n\) is an integer (as index of the solution series); \(J_{n+1/2} (r)\) is the Bessel function of the first kind; \(P_{n-1} ^1 (\cos \theta)\) is the Legendre function and \(P_{n} ^1 (\cos \theta)\) is the associated Legendre function.

This solution can be derived in a straightforward way by assuming the field is axisymmetric and the curl of the vector potential \(A\) is equal to \(A\) itself in spherical coordinate system \((r, \theta, \phi)\). That is

\[
\mathbf{\omega} = \nabla \times \mathbf{A} = \frac{1}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} (\sin \theta A_\phi) \right] \mathbf{e}_r - \frac{1}{r} \left[ \frac{\partial}{\partial r} (r A_\phi) \right] \mathbf{e}_\theta + \frac{1}{r} \left[ \frac{\partial}{\partial r} (r A_\theta) - \frac{\partial A_r}{\partial \theta} \right] \mathbf{e}_\phi = \mathbf{A}.
\]

Also assume that \(A_\phi\) is a function of \(r\) and \(\theta\) with separable variables. In that case we can let \(A_\phi = \omega_\phi = f (r, \theta) = R(r) \cdot \Theta (\cos \theta)\) and obtain a Legendre equation and a spherical Bessel equation for which solutions are

\[
\Theta (\cos \theta) = C_1 P_n ^1 (\cos \theta) \quad \text{and} \quad R (r) = C_2 \sqrt{\frac{\pi}{2 r}} r^{-1/2} J_{n+1/2} (r).
\]

These give \(f (r, \theta)\) and the three velocity components as in (15.3).

When \(n = 0\), it is a stationary field (i.e., a trivial solution).

When \(n = 1\), if we set the constant \(K_1 = 3 \sqrt{2 \pi} c \Omega\) (so that the maximum speed in the field is equal to sound speed; field strength \(\Omega\) is a constant here as assumed), velocity is

\[
\begin{align*}
u_r &= \frac{3c}{r^3} (\sin r - r \cos r) \cos \theta, \\
u_\theta &= -\frac{3c}{2r^3} (r^2 \sin r - \sin r + r \cos r) \sin \theta, \\
u_\phi &= \frac{3c}{2r^2} (\sin r - r \cos r) \sin \theta.
\end{align*}
\]
The three velocity components and pressure (along the radius, in the $\theta = \pi/2$ plane) are shown in Fig. 14.

At the radii where $\sin r = r \cos r = 0$, velocity components $u_r = 0$, $u_\theta = 0$ (and $u_\varphi$ reaches local extremums). The field is separated by these nodal spheres into spherical layers, with one vortex-ring-like structure existing in each layer. Flow and pressure distribution are shown in Fig. 15.
For higher orders, i.e., $n \geq 2$, the whole field is also split into homocentric spherical layers, with $n$ vortex rings in each layer. The flow and pressure for the cases of $n = 2$ and $n = 3$ are shown in Figs. 16 and 17, respectively.

There are other Beltrami flows presented. Some of them are physically less meaningful as solutions of the steady ether equations (e.g., periodic in space; with

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Fig. 16. Velocity and pressure of the second-order spherical solution.

Fig. 17. Velocity and pressure of the third-order spherical solution.
non-zero velocity in infinity, etc.). Related to Beltrami flows, the Kamchatnov–Hopf soliton\textsuperscript{3} has brought a lot of attention due to its nice features. But it is a solution of the magnetohydrodynamic equations (rather than the Euler equation). The Hopf fibration, if treated as a velocity field, is a (non-linear) Beltrami flow. But as the velocity has divergence, it is not a solution of the steady ether equations (which require divergenceless velocity).

In a more general case, if we require non-zero value on both sides of (15.1), the flow becomes the generalized Beltrami flow in fluid mechanics. In this case, the field strength is not uniform, but velocity is still divergenceless and is everywhere perpendicular to gradient of the field strength.

There are some exact solutions for this case; see, e.g., Refs. 18 and 19. Among them are a family of solutions related to Hill’s spherical vortex.\textsuperscript{20} For them, vorticity only exists inside a spherical or ellipsoidal region (the vortex core), and vanishes elsewhere. If these solutions are interpreted as particles, certain properties will only exist inside the core and vanish outside.

16. Particles and Properties

To link field structures to real particles, the properties of particles, like mass (gravitational and inertial), electric charge, momentum, etc. need to be linked to and explained by properties of the field. However, this will strongly rely on the (yet to be obtained) exact solutions. Before enough exact solutions are available for quantitative study, we can only try to qualitatively preview the possible aspects of the solutions, as well as the relations to the corresponding field properties.

As mentioned previously, so far the properties like mass, energy, force density, etc. are defined based on the field, i.e., are at “field level”. If steady solutions exist and are interpreted as particles, the overall properties of the particle (i.e., the “particle-level properties”) would need to be redefined as certain integral of the field-level properties.

In particle level, most of the properties are determined according to interactions (either the particle-to-particle interactions or the interactions between particles and the “force field” they are in). From ether field point of view, force only exists between adjacent ether, and the “force field” is part of the particle structures. Thus, a particle involved with long-range interactions cannot be a point or in a small region. Those particles as field structures should normally be infinitely large. When interacting with other particles, the field (and thus the particle itself) will change. In other words, interactions will unavoidably impact the particles themselves.

As discussed, there are only two kinds of force at the field level: e-Gravity and e-Lorenz force. These two forces are thus expected to be related to the particle-level kinetic properties (mass, momentum, energy, etc.) and the electromagnetic properties (charge, magnetic moment, etc.). However, the particle properties might not be just simply the integral of the field-level properties over space.
In principle, to understand the exact interaction between two particles, it would be necessary to find a solution of the field equations with two particle-like structures co-existing in the same boundary conditions. This could be complicated and would normally be a dynamic solution rather than a steady one. Exact interaction between multiple particles would be even more complicated, and likely will need to be studied with approximate and/or statistical methods.

As a preliminary attempt, however, we can take the spherical vortex solution discussed in last section to examine some properties of the e-Gravity and e-Lorentz force terms.

For this solution, as field strength \( u^2 \) is a constant, e-Lorentz force vanishes everywhere. If electric charge is relying on e-Lorentz force only and this solution is interpreted as a particle, it must be an electroneutral one. This applies to other solutions with linear Beltrami flow as well.

On the other hand, vector potential \( \mathbf{A} = \Omega \mathbf{u} \) and its curl are non-zero almost everywhere (except for the sphere center), thus a non-zero magnetic field \( \mathbf{B} = \nabla \times \mathbf{A} \) exists. But as \( \mathbf{B} \) and \( \mathbf{u} \) are everywhere parallel, electric field \( \mathbf{E} = \mathbf{B} \times \mathbf{u} \) is always zero.

For a solution with generalized Beltrami flow (which has non-zero Lamb vector), electric field will not be everywhere zero. For instance, Hill’s spherical vortex, if considered as a particle, will have non-zero electric and magnetic fields inside the core sphere. However, the exact meaning of these related to the electrical charge and magnetic moment of the whole particle is not very clear yet.

With solution (15.4), we have

\[
\mathbf{G} = \nabla \frac{\Omega^2 u^2}{c^2} = \frac{\Omega^2}{c^2} \nabla u^2 = \frac{\Omega^2}{c^2} (T_1'(r) \cos^2 \theta + T_2'(r)) \mathbf{e}_r - \frac{\Omega^2}{c^2 r} T_1(r) \sin(2\theta) \mathbf{e}_\theta.
\]

Obviously, with \( \theta \) existing in the above equations, e-Gravity in this case is not only in the radial direction. It also exists in the latitude direction \( \mathbf{e}_\theta \) and varies with \( \theta \). The force component in radial direction is dependent on \( \theta \) as well.

When \( r \) is small (i.e., close to “particle center”), e-Gravity changes greatly with \( r \). In the far field, except for certain polar angles (i.e., at both poles and the equator), components of the e-Gravity term in both radial and latitude directions are approaching \( r^{-3} \).

Note that this is within a single field structure. It is the “gravity density” that applies to the field in the solution. This in appearance is quite different than the gravity in normal understanding between the particles, which is only in radial
direction, exactly spherically symmetric and inverse-square. More details of these discrepancies will need to be further explored.

To interpret particles as ether field structures, there is another important feature yet to be explored. In real physical world, each particle (in static state) has special and fixed values of mass (energy), charge, etc. For an ether structure to be linked to these fixed properties, the structure needs to have fixed dimension and a fixed field strength distribution, as well as a fixed velocity field.

As the basic equations are homogeneous to field strength, the field strength in the particle can be proportional to the field strength in boundary condition (i.e., proportional to the strength of BGF). A possible case is that particles under the same boundary conditions have fixed ratio on mass and certain other properties, but when boundary conditions (BGF) change, the absolute quantity of the properties will change proportionally.

Sound speed provides an upper limit for velocity field, but whether this limit is reached in each stable solution remains a question. Similar situation is for absolute dimensions of field structures.

Overall, there must be a certain mechanism (e.g., maximization or minimization of certain action) to determine the stability of field structures and decide the fixed strength, fixed velocity and fixed dimensions of them, hence explain the fixed (and quantized) properties of particles. This is a missing part of the whole conjecture that is yet to be found.

### 17. Particle, Anti-particle and Photon

According to (13.2) or (14.9) and (15.1), in steady solutions, the e-Lorentz force is equal to the Lamb vector of vector potential, i.e., \( \nabla \psi = A \times (\nabla \times A) \). The angle between \( A \) and its curl \( \nabla \times A \) thus plays an important role.

In the Beltrami flow case, \( A \) and its curl are collinear and Lamb vector vanishes. In principle, this can happen with two non-trivial cases (i.e., \( \nabla \times A \neq 0 \)): curl is parallel or anti-parallel to the vector potential.

For example, in an axisymmetric case, if the vector field \( A = (A_r, A_\theta, A_\phi) \) is divergenceless and \( A \) is everywhere parallel to \( \nabla \times A \), the vector field \( A^* = (A_r, A_\theta, -A_\phi) \) would also be divergenceless and is anti-parallel to its curl \( \nabla \times A^* \).

Both \( A \) and \( A^* \) have vanished Lamb vector and satisfy the steady equations (15.1) and (15.2), thus both are solutions.

In this case, \( A \) and \( A^* \) are chiral symmetrical to each other. As \( A \) and \( A^* \) have the same magnitude for each vector component, if both are considered as particle solutions, the two particles will have some similar or related properties. For example, as the Lamb vector vanishes, e-Lorentz force is zero in either case. Thus the two particles would be both electroneutral. They can also have the same gravitational mass in particle level. However, the angular momentums along \( z \)-axis would be opposite to each other due to different chiralities.
To a certain degree, this seems matching the case of an electroneutral particle–anti-particle pair (e.g., neutron and anti-neutron).

For the generalized Beltrami flow case, $A$ and its curl are not everywhere collinear. Lamb vector thus is not everywhere zero. In the axisymmetric case, if the vector field $A = (A_r, A_\theta, A_\phi)$ and its curl form a directional angle of $\theta$, by changing the direction, chirality or magnitude of components, etc., there might be three other cases related to $A$ and have the directional angles from vector potential to its curl as $-\theta$, $\pi - \theta$ and $\pi + \theta$, respectively. A possible conjecture is that these four cases (including $A$) might form two charged particle–anti-particle pairs (like electron–anti-electron, proton–anti-proton).

On the other hand, mathematically, there are plenty of vortex-filament-like solutions for the steady equations. In the cylindrical coordinate system, if the steady flow is axisymmetric and is uniform along the $z$-axis, i.e., $u_r = 0$, $u_z = u_z(r)$, $u_\theta = u_\theta(r)$, $\Omega = \Omega(r)$, $\frac{\partial}{\partial r} = 0$, $\frac{\partial}{\partial \theta} = 0$, $\frac{\partial}{\partial z} = 0$, the momentum equation in $r$-direction becomes

$$\frac{\partial (c^2 \Omega^2 - \Omega^2 u_\theta^2 - \Omega^2 u_z^2)}{\partial r} = \frac{2 \Omega^2 u_\theta^2}{r}. \quad (17.1)$$

If we further have $u_z = 0$ (i.e., a non-swirl flow), the equation is

$$\frac{\partial (c^2 \Omega^2 - \Omega^2 u_\theta^2)}{\partial r} = \frac{2 \Omega^2 u_\theta^2}{r}. \quad (17.2)$$

Let $p(r)$ be a differentiable function of $r$ $(r \geq 0)$, positive and monotonically increasing [i.e., $p > 0$ and $p' = \frac{dp(r)}{dr} > 0$], then $\Omega = \sqrt{p + \frac{rp'}{2}}$, $u_\theta = c \sqrt{\frac{rp'}{2p+rp'}}$ would be a solution of (17.2) and $p(r)$ in this case is the pressure of the non-swirl flow.

For the swirl flow case ($u_z \neq 0$), we have the freedom to pick proper functions as pressure $p(r)$ and swirl speed $u_z$ to construct solutions for Eq. (17.1). As a simple example, if we let $u_z = u_\theta = \frac{1}{\sqrt{2}} \tilde{u}$, (17.1) becomes

$$\frac{\partial (c^2 \Omega^2 - \Omega^2 \tilde{u}^2)}{\partial r} = \frac{\Omega^2 \tilde{u}^2}{r},$$

and $\Omega = \sqrt{p + rp'}$, $\tilde{u} = c \sqrt{\frac{rp'}{p+rp'}}$ is a solution [with the same assumption on $p(r)$ as above].

The vortex filaments in these solutions (swirl or non-swirl) are straight and infinitely long. They may or may not exist in the field with the exact aspects as in these solutions. However, for a vortex-sphere-like or vortex-ring-like particle solution, in the far-field range along the symmetric axis, the flow can be approximated as a vortex filament (Fig. 18). By considering the properties of vortex filaments, we might be able to approximate the possible behavior in the far field of the particle solutions along the symmetric axis.
In fluid mechanics, vortex filaments are being carefully studied. For the vortex filaments that are solutions of incompressible Euler equations, Hasimoto applied the Local Induction Approximation (LIA) and described motion of filaments by the nonlinear Schrödinger equation (NLSE). Soliton solutions can then be obtained for the NLSE.

If similar approximation can be applied to the filament-like flow in the far-field/near-axis regions of particle solutions, and also leads to NLSE, the soliton waves along the filament would be a good analogy for photons. (Also see similar analogy/analysis made by Dmitriyev.)

Qualitatively speaking, when a particle is “accelerated” (i.e., the near-center portion of the particle solution entirely changes motion status), a perturbation wave would be generated along the filament. This is similar to the case of a photon emitting from an accelerated charged particle.

Furthermore, if photons are soliton wave packets in the filaments, a natural explanation could be offered on why light is transverse and with linear/circular polarization. Many packets moving together could form the electromagnetic wave (light) of our normal understanding. But in finer details these wave packets are separated. This might bring an explanation for the concept of photon.

The NLSE solitons are moving along the filament and each soliton is in a fixed speed (related to the shape of the soliton, so it is not a common speed for all solitons). In a particle solution, the symmetric axis is a straight line (unless the particle is strongly impacted by other particles and loses symmetricity). This might explain why photons propagate along straight lines.

As shape and speed are unchanged, the extra energy a soliton wave packet brings to the filament would be fixed and related to features of the packet (e.g., the wavelength). This may explain the quantum feature of light, and might even lead to a quantitative explanation and exact calculation for the Planck’s constant.

Fig. 18. Far-field, near-axis flow of a swirl vortex ring that can be approximated as a vortex filament.
With different values of the parameters, the soliton wave could be in different shapes. In certain cases, the filament might break and reconnect itself and form new vortex rings, as new particles. This might imply an explanation of the electron-pair effect.

Again, these are just rough conjectures so far. Further explorations are needed and will be relying on the yet to be obtained exact solutions of the fundamental equation system.

References

1. W. Thomson (Lord Kelvin), On vortex atoms, *Proc. R. Soc. Edinb.* VI (1867) 94–105.
2. W. Thomson (Lord Kelvin), On vortex motion, *Trans. R. Soc. Edinb.* 25 (1868) 217–260.
3. A. M. Kamchatnov, Topological soliton in magnetohydrodynamics, *Sov. Phys.-JETP* 55 (1) (1982) 69–73.
4. L. Faddeev, Quantisation of solitons, Report No. IAS Print-75-QS70 Institute for Advanced Study, Princeton, NJ (1975).
5. L. Faddeev and A. J. Niemi, Stable knot-like structures in classical field theory, *Nature* 387 (1997) 58–61.
6. R. Buniy and T. Kephart, A model of glueballs, *Phys. Lett. B* 576 (2003) 127–134.
7. M. Arminjon, Scalar theory of gravity as pressure force, *Rev. Roum. Sci. Tech. Ser. Méc. Appl.* 42(1–2) (1997) 27–57.
8. V. P. Dmitriyev, Mechanical analogies for the Lorentz gauge, particles and antiparticles, *Apeiron* 7 (2000) 173–183.
9. V. P. Dmitriyev, Mechanics of Schrödinger mechanics, arXiv:physics/0401004 [physics.ed-ph].
10. A. Einstein, Zur elektrodynamik bewegter korper, *Ann. Phys.* 322(10) (1905) 891–921.
11. A. Einstein, Die grundlage der allgemeinen relativitätstheorie, *Ann. Phys.* 354(7) (1916) 769–822.
12. D. M. Siegel, *Innovation in Maxwell’s Electromagnetic Theory* (Cambridge University Press, 1991).
13. J. J. Thomson, On the analogy between the electromagnetic field and a fluid containing a large number of vortex filaments, *Lond. Edinb. Dublin Philos. Mag. J. Sci.* 12(80) (1931) 1057–1063.
14. H. Marmanis, Analogy between the Navier-Stokes equations and Maxwell’s equations: Applications to turbulence, *Phys. Fluids* 10 (1998) 1428.
15. J. D. Gibbon, The three-dimensional Euler equations: Where do we stand?, *Physica D* 237(14–17) (2008) 1894–1904.
16. C. L. Fefferman, Existence and smoothness of the Navier-Stokes equation, in *The Millennium Prize Problems* (Clay Mathematics Institute, Cambridge, 2006), pp. 57–67.
17. C. C. Shi, Y. N. Huang and Y. S. Chen, On the Beltrami flows, *Acta Mech. Sin.* 8(4) (1992) 289–294.
18. C. Y. Wang, Exact solutions of the steady-state Navier-Stokes equations, *Annu. Rev. Fluid Mech.* 23(1) (1991) 159–177.
19. H. K. Moffatt, Generalised vortex rings with and without swirl, *Fluid Dyn. Res.* 3 (1988) 22–30.
20. M. J. M. Hill, On a spherical voltex, *Philos. Trans. R. Soc. Lond. A* 185 (1894) 213–245.
21. H. Hasimoto, A soliton on a vortex filament, *J. Fluid Mech.* 51(3) (1972) 477–485.