Prioritizing the Performance Evaluation Indicators of the Academic E-learning System using Fuzzy Approach

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Abstract

The present study is an attempt to prioritize the assessment indexes of Virtual education system performance through a hybrid method of Balanced Scorecard (BSC), Analytic Hierarchy Process (AHP) and TOPSIS. This research presents a multi-criteria decision-making approach for evaluation of virtual academic education system. This is a descriptive-analytic study, as well as an applied study in terms objective and results. This study was conducted in two stages: qualitative and quantitative. In the qualitative and quantitative stage, the samples were selected through purpose sampling and through census respectively in this research the experts' views in relation to the compliance of performance evaluation aspects with the BSC perspectives were used. The data measuring instrument in this study was prepared in the following two sections: the first part of the questionnaire dealt with the personal information and the second part was dedicated to identification of performance Status and importance of the selected indexes. Exploratory factor analysis and varimax rotation were used to assess the construct validity of the questionnaire. Cronbach’s alpha was used to determine the reliability of the test and its value was estimated about 0.81. Two other questionnaires were also prepared. The first questionnaire was used to determine the weight different performance evaluation aspects based on the concept of AHP technique and the second questionnaire was used to prioritize the key indexes for assessment of virtual education system's performance based on the concept of TOPSIS technique. The results of this study can be applicably used to plan and improve the performance of virtual education systems, furthermore, these results can be effective in development of BSC in IT management and e-learning systems and can present a suitable technique (composed of the two techniques) to summarize the results obtained from the evaluation of different BSC perspectives.

Keywords: AHP, BSC, E-learning, Performance, TOPSIS

1. Introduction

Performance evaluation and performance management, more generally, is a process by which we can obtain useful information on how to do effective things to reinforce positive behavior and eliminate inappropriate and unnecessary behavior. Performance evaluation is an aspect of system performance management which has been implemented mainly through the use of financial indexes¹. In recent decades, issues such as organizational learning, knowledge creation and innovation capacity have been taken into account as the determinants of competitive advantage and this concentration has stemmed from the advent of globalization, increased competition and unprecedented technological progress, especially in the field of information and communication technology that's why organizations seek comprehensive performance measuring indices and prioritize them in pressure². Kaplan and Norton established a new method performance measurement named the Balanced

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Scorecard (BSC) that evaluates the performance of the organization from four perspectives: financial, customer, internal processes, growth and learning\(^4\). The study also seeks to use the BSC approach to evaluate the e-learning services in virtual learning systems. Given that in the performance evaluation system using the BSC technique, the experts' views are used qualitatively and in terms of words, therefore measurement of collected opinions from the population through certain and fuzzy techniques can be criticized due to denial of uncertainties and subjective judgments\(^5\). By utilizing the fuzzy concepts in evaluations, verbal expressions can be used in form of natural language dialog statements to evaluate the performance assessment indexes and by linking those terms with the appropriate membership functions, more convenient and more accurate analyses can be applied on the values of indicators\(^6\). Therefore, in this paper fuzzy AHP and fuzzy TOPSIS techniques are used to weigh and prioritize the performance assessment indexes.

### 2. Review of Literature

Simons believes that control and performance measurement systems, information procedures and official affairs are key pivots which are used by managers to maintain or improve organizational activity patterns. Based on this definition, any performance evaluation system has four main goals:

- Any management and performance measurement system aims to transfer data.
- Control and performance measurement systems, indicate procedures and formal affairs.
- Control and performance measurement systems should be designed to be used by managers.
- Managers use control and performance measurement systems to maintain or improve organizational activity patterns\(^7\).

According to the above definition, the present study seeks to design a model and identify indexes that would cover the four abovementioned goals and analyze a range of indexes in the context of the BSC perspectives. Although the BSC approach causes the performance evaluation to be carried out in a multidimensional manner rather than with mere focus on financial criteria, issues such as huge volumes of information, and dogmatic judgments complicate the performance evaluation process\(^8\). The multi-criteria decision-making techniques are suitable to overcome the complexity of performance evaluation through the BSC technique. Many studies have shown that the Balanced Scorecard is a suitable technique for determining the performance evaluation indexes\(^9\).

Universities and non-profit organizations have used the Balanced Scorecard model as a factor for giving effectiveness to their activities\(^1\). In an article entitled multidimensional organizational Assessment, Bentes et al.\(^8\) have used a hybrid approach of BSC and AHP to evaluate three functional units of an organization according to the BSC perspectives. Yuksel et al.\(^9\) determined the performance level of a business based on the integration of Balanced Scorecard model with fuzzy analytic network process and their perspectives and strategies. Moayeri et al.\(^6\) used the Fuzzy Analytic Hierarchy Process (FAHP) and fuzzy TOPSIS method to propose a method for selection of math teachers based on the performance criteria and sub-criteria. In addition, considerable research has been conducted in the performance assessment of e-learning system. Seraji\(^10\) investigated a conceptual model is based on management decisions, technological and pedagogical. Rezvani and Dargahi\(^11\) classified features of e-learning system based on Kano model and offered main indicators of Performance assessment of e-learning system. Jami and Hosseinzadeh\(^12\) proposed hierarchical model (AHP) to prioritize indicators of performance assessment of e-learning system. Zanjirchi and Moradi\(^13\) proposed audit model to assess quality of e-learning system in the framework of total quality management with fuzzy approach. Salmeron\(^14\) proposed to build an augmented fuzzy cognitive map-based for modeling critical success factors in e-learning system. Mahmood and Hafeez\(^15\) assessed the performance of an e-learning software system to ensure its teaching and learning quality, contextual relevance and longer operational life to achieve economies of scale. McGill et al.\(^16\) examined conditions associated with continuation of e-learning initiatives in universities. But Identification of a wide and effective range of performance evaluation indexes in virtual education systems within the framework of performance evaluation dimensions and with the balanced scorecard approach is the point which has been taken into close considerations in this study as one of the essential needs of virtual education centers at universities.

### 3. Methodology

It is a descriptive-survey research and a type of applied study. Sample society consists of directors and professors...
of e-learning centers of Iran universities. This research is both qualitative and quantitative. In the qualitative part, the samples were chosen with systematic sampling that they were 15 people and in quantitative part, they were also 25 people. Moreover, in order to choose sample, the random cluster sampling has been used. The research questions posed in this research are as follows:

- What are the performance evaluation indicators of e-learning services in Iran universities?
- How is the allocation of mentioned indicators in balanced scorecard perspectives?
- What are prioritizing the performance evaluation indicators of e-learning services in Iran universities?

4. Identification of Evaluation Indicators of E-learning Services in Iran Universities

In order to identify the required criteria for performance evaluation of e-learning services, the balanced scorecard method has been used. Since balanced scorecard creates balance among financial and nonfinancial criteria, internal and external beneficiaries, short-term and long-term goals has a better performance in comparison to other methods of performance evaluation. For this purpose, the initial indicators of performance evaluation of e-learning services including 53 indicators were identified by analysis of research literature.

Then, by collecting the e-learning professors and experts’ views and applying the component analysis and varimax rotation, the final indicators of performance evaluation of e-learning services have been identified and finally 34 indicators have been confirmed.

5. Allocation of E-learning Services Indicators in BSC Perspectives

In this step by asking e-learning professors and experts’ views, it has been tried that compiled dimensions in balanced scorecard perspectives to be allocated according to Figure 1. Experts have stated their views corresponding to dimensions conformity with fourfold perspectives.

![Balanced Scorecard Perspectives](image)

**Figure 1.** Dimensions of performance evaluation of e-learning in Iran universities in balanced scorecard perspectives.
of balanced scorecard in the form of sentences such as absolutely proper, proper, nearly proper, improper and absolutely improper and after analysis of the dimensions conformity of e-learning services rate, learning process, access, teaching and supporting, educational content, hardware substructure, software substructure, speed, professor’s trait, interactions of roles, university management and structure and image with related perspectives have been estimated respectively: 0/821, 0/846, 0/917, 0/812, 0/780, 0/851, 0/936, 0/823, 0/847, 0/823, 0/957, 0/837.

6. Materials of Data Collection and their Validity and Reliability

The required data was collected in the form of a questionnaire. The validity of the research questionnaire has been confirmed by 20 persons of directors and e-learning professors. In order to measure its validity, the exploratory component analysis and varimax rotation have been used. The Cronbach’s alpha method has also been used for calculating the reliability of materials and the amount of Cronbach’s alpha for each of the twelve dimensions of e-learning services has been calculated more than 7/0.

By applying the exploratory component analysis method and carrying out the KMO and Bartlett’s tests, the indicators with component load less than 0/3 were eliminated from the questionnaire. In this regard, Kline believes that “the indicators which their component load is less than 0.3 or their statistics is smaller than absolute value 2 are weak indicators and will be eliminated from measurement model.”22,23. The results of factor loads were mentioned in Tables 1, 2. Finally, by considering the estimated component loads, 34 performance evaluation indicators in the shape of twelve dimensions were confirmed (Table 3).

| Table 1. Results of KMO and Bartlett’s tests |
|---------------------------------------------|
| Sig. | Square Root | KMO | Dimensions | Row |
|------|--------------|-----|------------|-----|
| 0/001 | 474/171 | 0/728 | E-learning services rate | C1 |
| 0/001 | 537/125 | 0/721 | Teaching-learning process | C2 |
| 0/001 | 492/123 | 0/716 | Access | C3 |
| 0/001 | 646/317 | 0/747 | Educational content | C4 |
| 0/001 | 672/120 | 0/801 | Hardware substructure, communications and security | C5 |
| 0/001 | 514/670 | 0/745 | Software substructure | C6 |
| 0/001 | 156/678 | 0/728 | Speed | C7 |
| 001/0 | 563/113 | 0/761 | Knowledge management | C8 |
| 001/0 | 527/451 | 0/792 | Professor’s trait | C9 |
| 001/0 | 461/056 | 0/703 | Interactions of roles | C10 |
| 001/0 | 539/217 | 0/745 | Management and structure | C11 |
| 001/0 | 551/165 | 0/737 | University image | C12 |

7. Weighting the Performance Evaluation Criteria of e-learning System

In 1996, Chang24 presented a very simple method for expanding the hierarchical analysis process to fuzzy space. This method expanded on the basis of mathematical average of experts’ views and hourly normalized method and by use of the fuzzy triangular numbers was

| Table 2. Results of component loads estimation after varimax rotation |
|---------------------------------------------|
| 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | Row |
|-----|----|----|---|---|---|---|---|---|---|---|---|-----|
| .   | .   | .   | . | . | . | . | . | . | . | . | . | 0/737 | 1 |
| .   | .   | .   | . | . | . | . | . | . | . | . | . | 0/685 | 2 |
| .   | .   | .   | . | . | . | . | . | . | . | . | . | 0/638 | 3 |
| .   | .   | .   | . | . | . | . | . | . | . | . | . | 0/710 | 4 |
| .   | .   | .   | . | . | . | . | . | . | . | . | . | 0/649 | 33 |
| .   | .   | .   | . | . | . | . | . | . | . | . | . | 0/613 | 34 |
Table 3. Final table of performance evaluation indicators of e-learning system in the shape of criteria

| Evaluation Indicators                                                                 | Dimensions/Criterion                                                                 |
|--------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------|
| Acceptability of fixed and variable tuition fees                                     | e-learning services rate                                                             |
| Cost price reduction of knowledge resources bank and e-content production             |                                                                                     |
| Allocation of content and knowledge resources bank according to scientific levels     | Teaching-learning process                                                             |
| Acceptability of educational supervision and guidance in students’ learning process    |                                                                                     |
| Improvement of learning patterns                                                     |                                                                                     |
| Easy access to virtual education system at any time and place                         | Access                                                                               |
| Access to all knowledge, content and interactive resources                           |                                                                                     |
| Simple and favorable use of virtual education system                                 |                                                                                     |
| Access to education of system user and content and knowledge banks                   |                                                                                     |
| The amount of updated and interactive SCO application in educational content and learning processes | Educational content                                                                 |
| The conformity rate of produced content with educational standards and virtual education |                                                                                     |
| The application amount of student evaluation SCO in learning process                  |                                                                                     |
| Existence of rich SCO bank, educational storages and educational content              |                                                                                     |
| Existence of proper servers fit to educational system needs and number of students    | Hardware infrastructure, communications and security                                  |
| Proper internet and internet bandwidth for covering the educational system            |                                                                                     |
| Access to required equipment and computer site (for execution of some teaching-learning processes, test center and production process of content) |                                                                                     |
| Existence of information and network security equipment                               |                                                                                     |
| Access to virtual education system compatible with virtual education standards        | Software infrastructure                                                             |
| Access to content production materials and production modules and learning process archive |                                                                                     |
| Existence of web conference tool for visual interactions                              |                                                                                     |
| Existence of special educational networks and associations                           |                                                                                     |
| Conformity of input connections, framework and all programs of educational system with software production standards |                                                                                     |
| feasiability of working with virtual education system with the least possible bandwidth (without disturbance of performance) | Speed                                                                              |
| Presentation of technical-supporting, executive and educational services promptly     |                                                                                     |
| The amount of new concepts creation as the worthwhile knowledge                      | Knowledge management                                                                 |
| The sharing amount of created knowledge and referring to the above-mentioned knowledge |                                                                                     |
| Ability of academic guidance of students in learning processes and interactive space and constant supervision | Professor's Characteristic                                                           |
| Ability of e-content production, educational objects and proper evaluation of educational programs |                                                                                     |
| Encompassing the communicative pattern among roles of the system (professor/expert/student) | Interactions of roles                                                              |
| Ability of the virtual educational system in presentation of performance reports of roles and related interactions |                                                                                     |
| Structure conformity of virtual education system of university/ university with virtual education system | Management and structure                                                             |
| High ability and motivation of the educational directors and experts of system in interaction with students |                                                                                     |
| Proper fame of the university                                                        | University image                                                                     |
| Reliance-creation for university brand                                               |                                                                                     |
welcomed warmly by researchers. The performance steps of this method are as follows:

**Step 1: Hierarchical tree drawing:** In this step to draw the structure of decision hierarchy by use of goal, criterion and alternative levels.

**Step 2: Matrix formation of couple comparisons:** By use of the decision-maker view, to form the comparisons matrix by applying the triangular fuzzy numbers $A_{ij}$ according to the views of some decision-makers.

$$
\tilde{A} = \begin{bmatrix}
(1, 1, 1) & \tilde{a}_{12} & \tilde{a}_{13} & \cdots & \tilde{a}_{1n} \\
\tilde{a}_{21} & (1, 1, 1) & \tilde{a}_{23} & \cdots & \tilde{a}_{2n} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & (1, 1, 1) & \tilde{a}_{nn}
\end{bmatrix}
$$

Fuzzy Judgment Matrix

That in this matrix $p_{ij}$ is the number of decision-makers about the expression priority of $i$ over $j$.

**Step 3: Mathematical average of views:** Calculate the mathematical average of decision-makers' views in the form of following matrix:

$$
\tilde{a}_{ij} = \frac{\sum_{k=1}^{p_{ij}} a_{ijk}}{p_{ij}} \quad i, j = 1, 2, \ldots, n
$$

**Step 4: Calculation of total of sentence elements:** Calculate the total of elements of sentences:

$$
\tilde{S}_i = \sum_{j=1}^{n} \tilde{a}_{ij} \quad i = 1, 2, \ldots, n
$$

**Step 5: Normalizing:** Normalize the total of sentences based on the following method:

$$
\tilde{M}_i = \tilde{S}_i \otimes \left[ \sum_{i=1}^{n} \tilde{S}_i \right]^{-1} \quad i = 1, 2, \ldots, n
$$

If we demonstrate $\tilde{S}_i$ in the form of $(l_i, m_i, u_i)$, the above relation will be calculated as the following:

$$
\tilde{M}_i = \left( \frac{l_i}{\sum_{i=1}^{n} m_i}, \frac{m_i}{\sum_{i=1}^{n} m_i}, \frac{u_i}{\sum_{i=1}^{n} m_i} \right)
$$

**Step 6: Identification of probability degree of being larger:** We calculate the probability degree of being larger of each $\mu_i$ over other $\mu$ is and name it $d'(A_i)$.

The probability degree of being larger of the fuzzy triangular number $\alpha_2 = (l_2, m_2, u_2)$ over the fuzzy triangular number $\alpha_1 = (l_1, m_1, u_1)$ equals to:

![Figure 2. Priority of two triangular fuzzy numbers.](image)
\[ V(M_2 > M_1) = \text{Sub}_{2 \times 2} \{ \min(\mu_{M_1}(x), \mu_{M_2}(y)) \} \] (5)

We can also state this relation as the following:

\[ V(M_2 > M_1) = \text{hgt}(M_2 \cap M_1) = \mu_{M_2}(d) \]

\[ \begin{align*}
V(M_2 > M_1) &= \begin{cases}
1 & m_2 \geq m_1 \\
0 & l_2 \geq l_1 \\
(\frac{m_2 - \mu_2}{(m_2 - \mu_2) - (m_1 - l_2)} & - \frac{l_2 - \mu_2}{m_1 - l_1})
\end{cases}
\end{align*} \] (6)

“d” is the coordinate of highest point in the joint area and collision of two membership functions (Figure 2).

For comparison of \( M_1 \) and \( M_2 \), calculation of both amounts of \( M_1 \geq M_2 \) and \( V(M_1 \geq M_2) \) is essential. The probability degree of being larger of the convex fuzzy number \( M \) over \( K \) the other convex fuzzy number \( M_i; i = 1, 2 \ldots k \) is divided as the following:

\[ d'(M) = V(M \geq M_1, M_2, \ldots, M_k) = V[(M \geq M_1), (M \geq M_2), \ldots, (M \geq M_k)] = \min V(M \geq M_i) \quad i = 1, 2 \ldots k \] (7)

**Step 7: Normalization:** By normalizing the weights vector, the normalized weights are obtained:

\[ W = \left[ \frac{d'(A_1)}{\sum_{i=1}^{n} d'(A_i)}, \frac{d'(A_2)}{\sum_{i=1}^{n} d'(A_i)}, \ldots, \frac{d'(A_n)}{\sum_{i=1}^{n} d'(A_i)} \right]^T \] (8)

Above weights are definite (non-fuzzy) weights. By repetition of this process, the weights of all matrices will be obtained. By doing these calculations, the following results are obtained respectively.

**Step 8: Weights compound:** By compounding the weights of alternative and criteria, the final weights are obtained.

\[ \bar{U}_i = \sum_{j=1}^{n} \tilde{w}_j \tilde{r}_{ij} \quad \forall i \] (9)

### 8. Identifying the Definite Weight of BSC Perspectives and Dimensions of Each of these Perspectives by Use of Hierarchical Model

**Step 1:** The decision hierarchical tree of this project is Figure 3:

**Step 2, 3 and 4:** For doing the couple comparisons, the following verbal sentences were used, are defined in Table 4:

| Fuzzy Number | Verbal Sentences               | Row |
|--------------|-------------------------------|-----|
| (1,1,1)      | Absolutely equal preference   | 1   |
| (0.5,1,1.5)  | Approximately equal preference| 2   |
| (1,1.5,2)    | Low preference                | 3   |
| (1.5,2,2.5)  | High preference               | 4   |
| (2,2.5,3)    | Very high preference          | 5   |
| (2.5,3,3.5)  | Absolutely high preference    | 6   |

Table 5 shows the mathematical average of experts’ views. In the last columns of these tables, the total of sentences elements has been shown.

Based on results of the above table, prioritizing the second level criteria over weighting the performance evaluation criteria of e-learning system is as following (Table 6 and 7):

- Customer perspective.
- Internal processes perspective.
- Financial perspective.
- Growth and innovation perspective.
Table 5. The average of couple comparisons over weighting the performance evaluation criteria of e-learning system

| Goal    | P1                     | P2                     | P3                     | P4                     | Total                  | Normalized             |
|---------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
| P1      | (1,1,1)                | (0.667,1,2)            | (0.5,0.667,1)          | (1.5,2,2.5)            | (3.667,4.667,6.5)      | (0.154,0.258,0.465)    |
| P2      | (0.5,1,1.5)            | (1,1,1)                | (0.5,1,1.5)            | (2,2.5,3)              | (4.5,5.7)              | (0.168,0.304,0.501)    |
| P3      | (1,1,5,2)              | (0.667,1,2)            | (1,1,1)                | (1.5,2,2.5)            | (4.167,5.5,7.5)        | (0.175,0.304,0.537)    |
| P4      | (0.4,0.5,0.667)        | (0.333,0.4,0.5)        | (0.4,0.5,0.667)        | (1,1,1)                | (2.133,2.4,2.834)      | (0.089,0.133,0.203)    |
| Total   |                        |                        |                        |                        | (13.967,18.067,23.834) |                        |

Table 6. The final weights matrix of criteria over weighting the performance evaluation criteria of e-learning system

| Elements                                          | Final weights of criteria |
|---------------------------------------------------|---------------------------|
| Financial perspective (P1)                        | 0.287                     |
| Customer perspective (P2)                         | 0.333                     |
| Internal processes perspective (P3)               | 0.333                     |
| Growth and innovation perspective (P4)            | 0.047                     |

Table 7. The final weights matrix of alternatives over weighting the performance evaluation criteria of e-learning system

| Prioritizing based on the definite weight | Final weights of alternatives | Element |
|------------------------------------------|-------------------------------|---------|
| 6                                        | 0.083                         | C1      |
| 3                                        | 0.102                         | C2      |
| 11                                       | 0.072                         | C3      |
| 1                                        | 0.105                         | C4      |
| 7                                        | 0.077                         | C5      |
| 4                                        | 0.098                         | C6      |
| 8                                        | 0.076                         | C7      |
| 12                                       | 0.047                         | C8      |
| 2                                        | 0.103                         | C9      |
| 9                                        | 0.076                         | C10     |
| 5                                        | 0.088                         | C11     |
| 10                                       | 0.073                         | C12     |

9. Prioritizing the Performance Evaluation Indicators of Virtual Education

In this section by use of the obtained definite weight from the past step, we will prioritize the alternatives. Fuzzy Spectrum and Analogous Verbal Sentence is defined in Table 8.

Table 8. Fuzzy spectrum and analogous verbal sentence

| Fuzzy number | Verbal sentence | Row |
|--------------|----------------|-----|
| (0,0.05,0.15)| Absolutely low | 1   |
| (0.1,0.2,0.3)| Low            | 2   |
| (0.2,0.35,0.5)| Relatively low | 3   |
| (0.3,0.5,0.7)| Average        | 4   |
| (0.5,0.65,0.8)| Relatively high | 5   |
| (0.7,0.8,0.9)| High           | 6   |
| (0.85,0.95,1)| Absolutely high | 7   |

The obtained results of alternatives evaluation based on the criteria according to fuzzy numbers and sentences of above table have been shown in Table 9. The inserted numbers in this table is the fuzzy average of experts’ views. The weight of each criterion has also been obtained based on the experts’ polling.

In continuation we will analyze the results of fuzzy TOPSIS technique steps for prioritizing the alternatives.

Step 1: Formation of decision-making matrix of alternatives evaluation: This matrix has been shown in Table 2.

Step 2: Normalization of decision-making matrix: In this step we should alter the fuzzy decision-making matrix of alternatives evaluation to a fuzzy normalized matrix \( \tilde{R} \) for obtaining the matrix, one of the following relations should be used:

Relation 1: \( \tilde{R} = \left[ \tilde{r}_{ij} \right]_{mn} \)

\( i = 1, 2, \ldots, m \)

\( j = 1, 2, \ldots, n \)

\( m \): Number of alternatives

\( n \): Number of criteria

If the fuzzy numbers are in the form of \((a,b,c)\), \( \tilde{R} \) this is a normalized matrix to be obtained as following:

- If criterion is positive:

Relation 2: \( \tilde{r}_{ij} = \left( \frac{a_{ij}}{c_{ij}}, \frac{b_{ij}}{c_{ij}}, \frac{c_{ij}}{c_{ij}} \right) \)
In this relation $c^*_j$ is the maximum amount of $c$ in criterion $j$ among all of the alternatives. The relation number (3) states this subject:

**Relation 3:** $c^*_j = \max_i c_{ij}$

- If criterion is negative:

**Relation 4:** $\bar{r}_{ij} = \left(\frac{a^*_i}{c^*_j}, \frac{a^*_i}{b^*_j}, \frac{a^*_i}{a^*_j}\right)$

In this relation $a^*_j$ is the minimum amount of ‘a’ in criterion $j$ among all of the alternatives. Relation number (5) states this subject:

**Relation 5:** $a^*_j = \min_i a_{ij}$

The results of normalization have been shown in Table 10.

**Step 3:** Formation of fuzzy weighted normalized matrix ($\tilde{V}$)

**Relation 6:** $\tilde{V} = [\tilde{v}_{ij}]_{mn}$, $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$

**Relation 7:** $\tilde{v}_{ij} = \bar{r}_{ij} \otimes \tilde{w}_j$

In this relation $\bar{r}_{ij}$ is the obtained normalized matrix from step 2 and $\tilde{w}_j$ is the fuzzy weighted matrix of criterion $j$.

Table 11 shows the fuzzy weighted normalized matrix:

**Step 4:** Identification of fuzzy positive ideal solution ($FPIS, A^+$) and fuzzy negative ideal solution ($FPIS, A^-$) for criteria.

**Relation 8:** $A^+ = (v^*_1, v^*_2, \cdots, v^*_n)$

**Relation 9:** $A^- = (v^-_1, v^-_2, \cdots, v^-_n)$

In this research, the fuzzy positive ideal solutions and fuzzy negative ideal solutions introduced by Chen will be used for all criteria. These solutions are as follows:

**Relation 10:** $v^*_i = (1, 1, 1)$

**Relation 11:** $v^-_i = (0, 0, 0)$

**Step 5:** Calculation of total distance between each alternative and fuzzy positive ideal and fuzzy negative ideal solutions:

If $A$ and $B$ are two fuzzy numbers expressed as follows, then distance between these two fuzzy numbers is obtained through relation (12):

$$D(A, B) = \sqrt{\frac{1}{3} \left[ (a_2 - a_1)^2 + (b_2 - b_1)^2 + (c_2 - c_1)^2 \right]}$$

Table 9. Fuzzy scores of alternatives evaluation (decision-making matrix)

|     | C1          | C2          | C3          | …… | C10         | C11         | C12         |
|-----|-------------|-------------|-------------|-----|-------------|-------------|-------------|
| A1  | (0.79,0.89,0.96) | (0.34,0.53,0.72) | (0.38,0.56,0.74) | …… | (0.46,0.62,0.78) | (0.54,0.68,0.82) | (0.66,0.77,0.88) |
| A2  | (0.76,0.86,0.94) | (0.38,0.53,0.68) | (0.24,0.41,0.58) | …… | (0.38,0.53,0.68) | (0.54,0.68,0.82) | (0.5,0.65,0.8)  |
| …… | ……         | ……         | ……         | …… | ……         | ……         | ……         |
| A34 | (0.49,0.65,0.8)  | (0.38,0.56,0.74) | (0.18,0.32,0.47) | …… | (0.34,0.53,0.72) | (0.71,0.83,0.92) | ……         |

Table 10. Fuzzy normalized matrix

|     | C1          | C2          | C3          | …… | C10         | C11         | C12         |
|-----|-------------|-------------|-------------|-----|-------------|-------------|-------------|
| A1  | (0.823,0.927,1) | (0.34,0.53,0.72) | (0.38,0.56,0.74) | …… | (0.489,0.66,0.83) | (0.54,0.68,0.82) | (0.66,0.77,0.88) |
| A2  | (0.792,0.896,0.979) | (0.38,0.53,0.68) | (0.24,0.41,0.58) | …… | (0.404,0.564,0.723) | (0.54,0.68,0.82) | (0.5,0.65,0.8)  |
| …… | ……         | ……         | ……         | …… | ……         | ……         | ……         |
| A34 | (0.51,0.677,0.833) | (0.38,0.56,0.74) | (0.18,0.32,0.47) | …… | (0.362,0.564,0.766) | (0.46,0.62,0.78) | (0.71,0.83,0.92) |

Table 11. Fuzzy weighted normalized matrix

|     | C1          | C2          | C3          | …… | C10         | C11         | C12         |
|-----|-------------|-------------|-------------|-----|-------------|-------------|-------------|
| A1  | (0.068,0.077,0.083) | (0.035,0.054,0.073) | (0.027,0.04,0.053) | …… | (0.037,0.05,0.063) | (0.048,0.06,0.072) | (0.048,0.056,0.064) |
| A2  | (0.066,0.074,0.081) | (0.039,0.054,0.069) | (0.017,0.03,0.042) | …… | (0.031,0.043,0.055) | (0.048,0.06,0.072) | (0.037,0.047,0.058) |
| …… | ……         | ……         | ……         | …… | ……         | ……         | ……         |
| A34 | (0.042,0.056,0.069) | (0.039,0.057,0.075) | (0.013,0.023,0.034) | …… | (0.027,0.043,0.058) | (0.04,0.055,0.069) | (0.052,0.061,0.067) |
Considering the above explanations about calculating the distance between two fuzzy numbers, we will obtain the distance between each of these alternatives and fuzzy positive ideal and fuzzy negative ideal solutions.

**Table 12. Ranking of alternatives**

| Rank | CC  | Distance to negative ideal | Distance to positive ideal | Alternatives | Row |
|------|-----|----------------------------|---------------------------|--------------|-----|
| 1    | 0.058 | 0.698                      | 11.314                    | A1           | 1   |
| 6    | 0.052 | 0.623                      | 11.393                    | A2           | 2   |
| 7    | 0.051 | 0.617                      | 11.396                    | A3           | 3   |
| 20   | 0.048 | 0.576                      | 11.438                    | A4           | 4   |
| 15   | 0.049 | 0.59                       | 11.425                    | A5           | 5   |
| 31   | 0.042 | 0.508                      | 11.507                    | A6           | 6   |
| 33   | 0.041 | 0.487                      | 11.53                     | A7           | 7   |
| 32   | 0.041 | 0.493                      | 11.527                    | A8           | 8   |
| 34   | 0.04  | 0.475                      | 11.54                     | A9           | 9   |
| 5    | 0.052 | 0.629                      | 11.385                    | A10          | 10  |
| 25   | 0.047 | 0.559                      | 11.456                    | A11          | 11  |
| 24   | 0.047 | 0.562                      | 11.456                    | A12          | 12  |
| 12   | 0.049 | 0.594                      | 11.419                    | A13          | 13  |
| 11   | 0.05  | 0.597                      | 11.419                    | A14          | 14  |
| 28   | 0.045 | 0.542                      | 11.474                    | A15          | 15  |
| 30   | 0.045 | 0.536                      | 11.481                    | A16          | 16  |
| 26   | 0.046 | 0.551                      | 11.465                    | A17          | 17  |
| 3    | 0.057 | 0.679                      | 11.333                    | A18          | 18  |
| 16   | 0.049 | 0.59                       | 11.43                     | A19          | 19  |
| 21   | 0.048 | 0.573                      | 11.444                    | A20          | 20  |
| 23   | 0.047 | 0.566                      | 11.452                    | A21          | 21  |
| 22   | 0.047 | 0.567                      | 11.449                    | A22          | 22  |
| 4    | 0.053 | 0.637                      | 11.378                    | A23          | 23  |
| 27   | 0.046 | 0.551                      | 11.466                    | A24          | 24  |
| 14   | 0.049 | 0.591                      | 11.427                    | A25          | 25  |
| 29   | 0.045 | 0.537                      | 11.482                    | A26          | 26  |
| 18   | 0.048 | 0.582                      | 11.434                    | A27          | 27  |
| 10   | 0.05  | 0.599                      | 11.417                    | A28          | 28  |
| 19   | 0.048 | 0.577                      | 11.443                    | A29          | 29  |
| 17   | 0.049 | 0.584                      | 11.431                    | A30          | 30  |
| 2    | 0.058 | 0.693                      | 11.319                    | A31          | 31  |
| 8    | 0.051 | 0.615                      | 11.403                    | A32          | 32  |
| 13   | 0.049 | 0.592                      | 11.425                    | A33          | 33  |
| 9    | 0.05  | 0.606                      | 11.411                    | A34          | 34  |

**Relation 13:** \( d_i = \sum_{j=1}^{n} d(\tilde{v}_j - \tilde{v}_i) \)

**Relation 14:** \( d_i^- = \sum_{j=1}^{n} d(\tilde{v}_j - \tilde{v}_i) \)

**Step 6:** Calculating the relative closeness of alternative \( i \) to the ideal solution. This relative closeness is defined as following:

**Relation 15:** \( CC_i = \frac{d_i^-}{d_i^- + d_i^+} \quad i = 1, 2, \ldots, m \)

**Step 7:** Ranking of alternatives: based on the descending order, the existing alternatives of the problem can be ranked. Each alternative which has a greater \( CC \) is better. The results have been presented in Table 12.

**10. Conclusion**

This study is an attempt to evaluate the performance of e-learning service in virtual education centers at universities of Iran. In the present study, the weights of BSC include customer perspective with a final weight of 0.3338, internal processes perspective with a final weight of 0.3330, the financial perspective with the final weight of 0.287 and innovation and growth perspective with the final weight of 0.047. That shows the customer perspective is of great importance in evaluating the performance of virtual education systems.

In the customer perspective, indexes such as “knowledge and content resources allocation commensurate with the scientific level” and “simple and user-friendly interface of virtual education system” are of great importance as influential indexes for increasing confidence and satisfaction of students.

In the internal processes perspective “indexes such as availability of a virtual education system compatible with the standards of virtual education, possibility of working with the virtual education with the least bandwidth (without any performance disorderliness), the applications of interactive and updated SCO’s (sharable content object) in the educational content and learning trends” have been estimated as effective indexes for increasing student and teacher satisfaction.

In the financial perspective “acceptability of fixed and variable tuitions” is a high priority. And in the growth and learning perspective, the adaptation of the university structure (virtual education center at the university) with the virtual education system is an index of great importance in the development of virtual education and also
one of the main challenges for universities in Iran. The confidence-building index towards the university brand as an effective index for attraction and academic achievements of students, and modification of the society’s cultural approach towards virtual education is an important index which is of high priority in this study.

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