Constraints on supersymmetric models from catalytic primordial nucleosynthesis of beryllium

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Abstract. The catalysis of nuclear reactions by negatively charged relics leads to increased outputs of primordial \textsuperscript{6}Li and \textsuperscript{9}Be. In combination with observational constraints on the primordial fractions of \textsuperscript{6}Li and \textsuperscript{9}Be, this imposes strong restrictions on the primordial abundance and the lifetime of charged relics. We analyze the constraints from the catalysis of \textsuperscript{9}Be on supersymmetric models in which the gravitino is the lightest supersymmetric particle and a charged slepton—such as the lighter stau—the next-to-lightest supersymmetric particle (NLSP). Barring the special cases in which the primordial fraction of the slepton NLSP is significantly depleted, we find that the \textsuperscript{9}Be data require a slepton NLSP lifetime of less than $6 \times 10^{3}$ s. We also address the issue of the catalytic destruction of \textsuperscript{6}Li and \textsuperscript{9}Be by late-forming bound states of protons with negatively charged relics, finding that it does not lead to any significant modification of the limit on the slepton lifetime.

Keywords: dark matter, big bang nucleosynthesis, cosmology of theories beyond the SM, physics of the early universe

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1. Introduction

Physics of the early Universe keeps proving to be an invaluable tool for probing particle physics models and, in particular, models of new physics beyond the standard model. One of the most important viability checks of these models results from the epoch of big bang nucleosynthesis (BBN), i.e. from the early Universe at temperatures of $T \lesssim 1$ MeV. The combination of standard model physics, general relativity and the experimental determination of the baryon-to-photon ratio from the anisotropies of the cosmic microwave background [1] forms the framework for the standard BBN (SBBN) which makes firm predictions for the primordial abundances of light elements such as deuterium, helium and lithium. The comparison of SBBN predictions with observationally determined primordial fractions of these elements provides an important consistency check of standard cosmology and serves as a remarkably powerful discriminator among models of new physics [2]. The three most generic ways in which new physics can affect the outcome of BBN are the change in the timing of the reactions caused, for example, by new significant contributions to the Hubble expansion rate [2], the non-thermal processes from late annihilation and decays of heavy particles [3]–[11], and the thermal catalysis of nuclear reactions caused by electromagnetically or strongly interacting relics [12].

Catalyzed big bang nucleosynthesis (CBBN) was already discussed almost twenty years ago in [13,14]. However, only in the last two years after the appearance of [12] has a lot of work been done [15]–[33] in order to refine various aspects of catalysis and to understand the implications of CBBN in the framework of specific models. The significant interest in CBBN is also fueled by its direct connection with collider physics. Indeed, the relics causing catalysis could be directly produced at the Large Hadron Collider (LHC) because of their electromagnetic and/or strong interactions. One of the most interesting, and perhaps one of the most natural, frameworks is, in this context, the supersymmetric (SUSY) extension of the standard model. There a spectrum with the gravitino $\tilde{G}$ as the lightest supersymmetric particle (LSP) and a charged scalar lepton $\tilde{l}_1$—such as the lighter stau $\tilde{\tau}_1$—as the next-to-lightest (NLSP) is a commonplace occurrence even if one
adopts restrictive assumptions on the soft SUSY breaking sector [34]–[36], [17, 19], [26]–[28]. While the gravitino LSP is a promising candidate for dark matter [37]–[44], the charged slepton NLSP can be long-lived and thus leads to CBBN [12], [17]–[20], [21]–[23], [25]–[28], [32]. If such a scenario is realized in nature, each standard model superpartner produced at the LHC will cascade down to the long-lived $\tilde{l}_1$ NLSP. As the lightest standard model superpartner, the $\tilde{l}_1$ NLSP will then appear as a quasi-stable muon-like particle that can escape the collider detector before decaying into the gravitino; cf [45, 40, 41, 46, 44] and references therein. Thus, one would find signatures that are very different from the excess in missing energy expected in the alternative neutralino LSP scenarios.

The most dramatic catalytic enhancement is seen in the $^6$Li and $^9$Be production triggered by the formation of bound states of $^4$He with a (generic) negatively charged relic that we call $X^-$. The catalytic path to $^6$Li and $^9$Be is shown by the following sequence of transformations [12, 31]:

$$X^- \rightarrow (^4\text{He}X^-) \rightarrow ^6\text{Li},$$  

$$X^- \rightarrow (^4\text{He}X^-) \rightarrow (^8\text{Be}X^-) \rightarrow ^9\text{Be}. \tag{1.1} \tag{1.2}$$

Although the ($^4\text{He}X^-$) system has a binding energy of about 350 keV, its formation is delayed until $T = 8$ keV by an overwhelmingly large number of energetic photons that photodissociate ($^4\text{He}X^-$). Thus, the ($^4\text{He}X^-$) bound state serves essentially as a ‘bottleneck’ for $^6$Li production, whereas the path to $^9$Be goes through the ‘double bottleneck’ of ($^4\text{He}X^-$) and ($^8\text{Be}X^-$). The key for the nuclear catalysis is an enormous enhancement of the reaction rates in the photonless recoil reactions mediated by $X^-$ [12, 31]:

$$(^4\text{He}X^-) + D \rightarrow ^6\text{Li} + X^-,$$  

$$(^8\text{Be}X^-) + n \rightarrow ^9\text{Be} + X^- . \tag{1.3} \tag{1.4}$$

Indeed, since the rates of these catalyzed reactions exceed the SBBN rates for the production of $^6$Li and $^9$Be by many orders of magnitude, one finds a strong sensitivity of the efficiency of the primordial $^6$Li and $^9$Be production to the abundance of $X^-$ at relevant times. For the case of $X^-$ being a thermal relic, this abundance is governed by the $X^-$ annihilation rate and by the $X^-$ lifetime $\tau_{X^-}$. In turn, observationally inferred limits on the primordial abundances of both $^6$Li and $^9$Be will impose significant constraints on the lifetime of $X^-$, its mass and its interactions. The limits on the lifetime and the abundance are particularly interesting in view of the possible catalytic solution to the so-called $^7$Li problem [12, 21], which is a persistent discrepancy between the predicted primordial amount of $^7$Li and a factor of 2–3 lower observations of $^7$Li in the atmospheres of metal-poor stars.

For the gravitino LSP scenarios in which $X^-$ is identified with a negatively charged slepton NLSP, $X^- = \tilde{l}_1$, the constraints imposed by primordial $^6$Li catalysis have already been analyzed in detail in a number of publications [12], [17]–[22], [26]–[28], [32, 33]. Assuming a standard cosmological history that leads to a typical thermal $\tilde{l}_1$ relic abundance, the bound from $^6$Li catalysis translates into an upper limit on the $\tilde{l}_1$ lifetime of $\tau_{\tilde{l}_1} = \tau_{X^-} \lesssim 5 \times 10^3$ s [12, 20, 22, 26]. In collider-accessible regions of the parameter space, this limit is found to be considerably more restrictive than the BBN.
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The constraints associated with electromagnetic/hadronic energy release from $\tilde{\tau}_1$ decays [17]–[19], [22, 28, 32]. The $\tau_{\tilde{\tau}_1}$ limit implies a gravitino mass $m_{\tilde{G}}$ well below 10% of the snepton NLSP mass $m_{\tilde{\tau}_1}$ for $m_{\tilde{\tau}_1} \lesssim O(1 \text{ TeV})$ [18]. This seems to exclude a kinematical determination of $m_{\tilde{G}}$ at the next generation of particle accelerators, which might have been feasible for $0.1 m_{\tilde{\tau}_1} \lesssim m_{\tilde{G}} < m_{\tilde{\tau}_1}$ [48, 49]. Consequently, the $\tau_{\tilde{\tau}_1}$ limit puts a big question mark over the idea of a collider test of supergravity via the microscopic determination of the Planck scale [47], as well as the $m_{\tilde{G}}$-dependent collider test of thermal leptogenesis [42]. Moreover, a post-inflationary reheating temperature above $T_R \simeq 10^7 \text{GeV}$, and thereby even the viability of thermal leptogenesis with hierarchical right-handed heavy Majorana neutrinos, seems to be disfavored by the $^6$Li constraint within the constrained minimal supersymmetric standard model (CMSSM) for a standard cosmological history [19, 26, 28]. In the CMSSM, the splitting between $m_{\tilde{\tau}_1}$ and $m_{\tilde{\tau}_1}$ required to evade the $\tau_{\tilde{\tau}_1}$ limit translates also into a lower limit on the gaugino mass parameter, which is assumed to take on a universal value $m_{1/2}$ at the scale of grand unification. Indeed, for the natural gravitino LSP mass range in gravity-mediated SUSY breaking scenarios, the cosmologically favored region can be associated with a mass range of the colored superparticles (e.g. a gluino mass of $m_{\tilde{g}} \gtrsim 2.5 \text{ TeV}$) for which it will be very difficult to probe SUSY at the LHC [26, 28].

In this paper we analyze the constraints imposed on SUSY models by the catalysis of a primordial $^9$Be abundance. While—based on the results of [31]—we do not expect the constraints from $^9$Be to be considerably tighter than those from $^6$Li, this analysis is warranted for a number of reasons:

(1) Observations of $^6$Li are extremely difficult because its lines are not resolved spectroscopically with respect to the lines of $^7$Li. The claim of a $^6$Li plateau’ with metallicity [50] at $\sim 10\%$ of the $^7$Li abundance is being challenged in a recent paper [51] and a new re-analysis of $^6$Li data is warranted as some of the observations may turn out to provide only upper limits. In fact, even the value of an upper limit on the primordial $^6$Li abundance is subject to discussions. Many papers adopt upper limits on primordial $^6$Li/H $\equiv n_{^6\text{Li}}/n_{\text{H}}$ within a range from $10^{-11}$ to $10^{-10}$. Unlike $^6$Li, $^9$Be is firmly detected in a significant number of stars at low metallicity, and its observational status is not in doubt. For the latest data on the $^9$Be abundance in metal-poor stars, see, e.g., [52]–[54].

(2) $^6$Li is more fragile than $^7$Li and would burn more efficiently at lower temperatures. Therefore, if there is a (yet unconfirmed) stellar mechanism (see, e.g., [55]) that resolves the lithium problem, i.e. that depletes $^7$Li by a factor of two or three, $^6$Li would have been depleted by an even larger factor. Such a stellar mechanism, however, would affect $^9$Be less than either $^7$Li or $^6$Li since both $^7$Li and $^6$Li are more fragile than $^9$Be, which thereby provides a more robust bound on new physics.

(3) The nuclear physics rates that enter in the calculation of $^9$Be catalysis are dominated by resonances. Given the wealth of experimental information on the $^9$Be resonances [56], this may eventually allow for very reliable calculations of the catalytic rates.

While stating strong bounds on primordial abundances/lifetimes of $X^-$, a lingering question remains: is there an ‘island’ of allowed lifetimes around $\tau_{X^-} \sim 10^6 \text{ s}$? One point emphasized in [13, 15, 25] is that bound states of $X^-$ with protons, $(pX^-)$, may have a
Table 1. Properties of \((pX^-)\) and \((^4\text{He}X^-)\) bound states. For \((^4\text{He}X^-)\), the binding energy \(E_b\) includes a finite charge radius correction \([12]\). Bohr radii \(a_B\) are quoted for idealized Bohr-type bound states. The given recombination temperatures \(T_r\) are understood as the temperatures at which the corresponding photodissociation rate becomes equal to the Hubble rate.

| Bound state | \(E_b\) (keV) | \(a_B\) (fm) | \(T_r\) (keV) |
|-------------|----------------|---------------|---------------|
| \((^4\text{He}X^-)\) | -347 | 3.6 | 8 |
| \((pX^-)\) | -25 | 29 | 0.6 |

significant impact on the primordial abundances of elements such as \(^6\text{Li}, ^7\text{Li}, ^7\text{Be}\) (and \(^9\text{Be}\)). Another point is related to \(X^-\) decays with very energetic decay products and the associated possibility of an environment in which \(^6\text{Li}\) may be destroyed efficiently. The second point has already been addressed \([17, 22, 32]\), and it has been found that late energy injection cannot suppress \(^6\text{Li}\) down to an acceptable level if it is significantly overproduced at 8 keV \([17, 22, 32]\). Despite its importance, the issue of possible \((pX^-)\) catalysis remained largely unresolved. While the importance of charge exchange reactions of \((pX^-)\) with \(^4\text{He}\) that may reduce the abundance of \((pX^-)\) was already mentioned in the early paper \([13]\), subsequent publications either ignored this issue \([15]\) or generally underestimated the impact of charge exchange reactions \([25]\). Recognizing its importance for the whole CBBN paradigm, we revisit the catalysis by \((pX^-)\) bound states. Indeed, in this paper, we put this issue to rest by reaching the conclusion that less than 10% of \(^6\text{Li}\) and \(^9\text{Be}\) synthesized at 8 keV could possibly be affected by \((pX^-)\) catalysis, whereas typically suppression factors in excess of 100 are needed in order to evade the corresponding limits. This clarifies that the islands in the parameter region with large abundances/large lifetimes, which were suggested to remain viable in \([25]\), cannot exist.

This paper is organized as follows. In the next section we estimate the charge exchange reaction rates relevant for \((pX^-)\)-mediated catalysis and consider their consequences for lithium and beryllium. In section 3 we analyze the bounds on the lifetime/abundance of \(X^-\) imposed by observations of \(^9\text{Be}\) in stellar atmospheres at low metallicities. The resulting constraints on SUSY models with the gravitino LSP and a charged slepton NLSP are given in section 4. We reach our conclusions in section 5.

2. \((pX^-)\) catalysis and charge exchange reactions

We begin the discussion of the possibility of \((pX^-)\)-mediated catalysis by reminding the reader of the basic properties of the \((pX^-)\) and \((^4\text{He}X^-)\) bound states. Table 1 lists the corresponding binding energies \(E_b\), Bohr radii \(a_B\) and recombination temperatures \(T_r\), where \(T_r\) is understood as the temperature below which the rate of photodissociation of a bound state becomes smaller than the Hubble expansion rate. As the table suggests, the recombination of \((pX^-)\) bound states becomes efficient only after the temperature drops below 1 keV, which corresponds to an age of the Universe of the order of a few weeks or so, and we will assume in this section that the \(X^-\) lifetime \(\tau_{X^-}\) is large enough to allow for this recombination to happen. Clearly, the presence of a large number density of negatively charged particles \(n_{X^-}\) during the recombination with helium leads to an overproduction
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of $^6\text{Li}$ and $^9\text{Be}$ [12,31] due to the nuclear catalysis at $8$ keV. To be specific, we choose $Y_{X^-} \equiv n_{X^-}/n_B = 10^{-2}$ (with $n_B$ denoting the baryon number density), $\tau_{X^-} \to \infty$, and using our previous results [12,31,26] determine the abundances of lithium and beryllium at $T = 1$ keV from CBBN production:

$$\frac{^6\text{Li}}{\text{H}}|_{T=1 \ \text{keV}} \approx 8 \times 10^{-8}; \quad \frac{^9\text{Be}}{\text{H}}|_{T=1 \ \text{keV}} \approx 3 \times 10^{-10}, \quad (2.1)$$

which is about three orders of magnitude above the observational bounds; cf section 3 below. Some of the synthesized $^6\text{Li}$ and $^9\text{Be}$ will be in bound states with $X^-$. Below $T = 1$ keV, the concentration of $(pX^-)$ is controlled by a rapidly diminishing photodissociation rate and by depletion through the charge exchange reaction [13]:

$$(pX^-) + ^4\text{He} \to (^4\text{He}X^-) + p. \quad (2.2)$$

This reaction may have a very large rate as its cross section is determined by the actual size of the $(pX^-)$ bound state which is of the order of $a_B \approx 30$ fm (cf table 1). In fact, most of the recombined states $(pX^-)$ are immediately intercepted by the reaction (2.2) so that the resulting $(pX^-)$ abundance would remain quite small at all temperatures.

From studies of charge exchange reactions of muons on hydrogen, it is known that the muon is captured into highly excited states that have large orbital momenta and large principal quantum numbers. The radii of these excited orbits of muonic hydrogen are comparable to the Bohr radius of ordinary hydrogen. In the case of (2.2), the capture would mainly proceed to the $n = 3$ and 4 levels of the $(^4\text{He}X^-)$ bound state. To estimate the cross section for the reaction (2.2), we employ a semiclassical approximation in which the motion of helium is described by a classical trajectory while the proton is treated quantum mechanically. The large values of $n$ and $l$ of the resulting $(^4\text{He}X^-)$ bound states give some justification to this treatment.

Calling $R$ the separation between $^4\text{He}$ and $X^-$ (or, more generally, the separation between $X^-$ and the incoming nucleus of charge $Z$), we now investigate the $R$ value at which the proton loses its ability to bind to $X^-$. The one-dimensional slice of the proton potential energy in the field of $X^-$ and $^4\text{He}$:

$$V(r) = -\frac{\alpha}{r} + \frac{\alpha Z}{|r - R|}, \quad (2.3)$$

is plotted in figure 1. The limit of $R \to \infty$ corresponds to an unperturbed binding of the proton to $X^-$ with a binding energy of $E_b = -25$ keV (cf table 1). For $Z > 1$ and finite $R$, the curve has a maximum at positive values of $r$, referred to as $V_{\text{max}}$. As the $^4\text{He}$ nucleus comes closer, $R$ decreases. For $R$ values below some critical distance $R_{c1}$, the binding energy of the proton becomes positive so that the tunneling of the proton to $r \to +\infty$ starts to become viable. For even smaller values of $R$, one can find another ‘critical’ distance $R_{c2}$ at which the probability for the tunneling of the proton becomes comparable to 1 due to the fly-by of the $^4\text{He}$ nucleus. In principle, this is not an easy quantum mechanical problem and we use further simplifications to overcome that. In order to estimate $R_{c1}$, we employ the variational calculation of the proton energy in the potential (2.3) by using the trial wavefunction for the ground state:

$$\psi(\mu, \nu) = \exp[-(\mu - \nu)R/(2a)] \times (1 + \nu R/b)^2, \quad (2.4)$$
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Figure 1. Potential energy of the proton in the field of $X^-$ at $r = 0$ and an incoming nucleus at $r = -R_{c2}$. The potential energy is plotted along the line connecting $X^-$ with $^4$He (solid line) or $^6$Li (dashed line), respectively. As the distance between the incoming nucleus and $X^-$ decreases, the potential well becomes more narrow, and the proton ground state energy level is pushed upward. The critical deconfinement distance $R_{c2}^{^4\text{He},^6\text{Li}}$ is defined as the distance at which the energy of the bound state found variationally using (2.4) becomes larger than the height of the barrier $V_{\text{max}}$ to the right of $X^-$. 

where $\mu$ and $\nu$ are elliptic coordinates and $a$ and $b$ the minimization parameters. The coordinates are defined as $\mu = (r_1 + r_2)/(2R)$ and $\nu = (r_1 - r_2)/(2R)$, where $r_1$ and $r_2$ are the proton–nucleus and proton–$X^-$ distances, respectively. We calculate the energy of the ground state $E_{\text{var}}^{\text{pX}}$ as a function of the distance between $X^-$ and the incoming nucleus $R$. We determine $R_{c1}$ from $E_{\text{var}}^{\text{pX}}(R_{c1}) = 0$ and estimate $R_{c2}$ from $E_{\text{var}}^{\text{pX}}(R_{c2}) = V_{\text{max}}$, which describes the situation when even a metastable bound state simply cannot exist. The cross section for the charge exchange reaction is then approximated by the geometric one with the impact parameter $\rho = R_{c2}$, $\sigma = \pi R_{c2}^2$, which essentially assumes a deconfinement probability of 1 for $R \leq R_{c2}$. This approximation would fail for very fast incoming nuclei. For the case considered in this paper, however, the fly-by time is much longer than the period with which the proton orbits within ($pX^-$). The results of our estimates are presented in table 2. As can be seen from this table, a $^4$He–$X^-$ distance of $\sim 95$ fm is sufficient to release the proton from the bound state. Consequently, our estimate points to a very large cross section of almost 300 bn for the charge exchange reaction (2.2). For $Z = 1$ we have a cross section $\sim 2\pi a_B^2$, which compares well with the results for the charge exchange cross section in the case of muon–hydrogen scattering [57].
Table 2. Deconfining distances and charge exchange reaction cross sections on the \((pX^-)\) target for incoming nuclei with different charges \(Z\). The \(R_{c1}\) values are given in units of the \((pX^-)\)-Bohr radius \(a_{B}^{(pX^-)}\) and in units of fm.

| \(Z\) | \(R_{c1}\) at \(E = 0\) | \(R_{c2}\) at \(E = V_{\text{max}}\) | \(\sigma = \pi R_{c2}^2\) in \(\text{bn}\) |
|------|-----------------|-----------------|------------------|
| 1    | 1.4 (40 fm)     | 1.4 (40 fm)     | 51 bn            |
| 2    | 3.7 (107 fm)    | 3.3 (95 fm)     | 280 bn           |
| 3    | 5.8 (167 fm)    | 4.7 (135 fm)    | 580 bn           |
| 4    | 7.9 (230 fm)    | 5.7 (160 fm)    | 850 bn           |

Using this cross section, we incorporate the charge exchange reactions in the network of Boltzmann equations and calculate the residual concentration of \((pX^-)\) in a wide temperature range. In the limit of infinite lifetimes, \(\tau_{X^-} \to \infty\), we find that the abundance of \((pX^-)\) reaches its peak at around \(T = 0.7\) keV. Its maximum abundance at these temperatures can be well approximated as

\[
n_{(pX^-)}^{\text{max}} / n_p \simeq 4 \times 10^{-7} \left( \frac{Y_{X^-}}{10^{-2}} \right),
\]

where we made the safe assumption\(^4\) of \(Y_{X^-} \lesssim Y_{\text{4He}}\), which ensures the linear scaling in (2.5).

Reference [25] makes the somewhat surprising suggestion that even a tiny fraction of surviving \((pX^-)\) bound states may cause a significant reduction in the \(6\)Li abundance, given the significant uncertainty in the nuclear rates. It is easy to see, however, that two types of processes are possible for the colliding \(6\)Li–\((pX^-)\) system: the charge exchange reaction and the nuclear reaction:

\[
\begin{align*}
\text{A:} & \quad (pX^-) + 6\text{Li} \to 6\text{LiX^-} + p, \\
\text{B:} & \quad (pX^-) + 6\text{Li} \to X^- + ^4\text{He} + ^3\text{He}.
\end{align*}
\]

At first, we completely ignore process A and concentrate on process B which destroys \(6\)Li. To illustrate our point, we will assign the maximal possible rate to process B, which is given by the unitarity bound in the \(s\)-channel:

\[
\langle \sigma v \rangle_{\text{B}}^{\text{max}} = \frac{\pi}{m_{\text{6Li}}^2} \langle v^{-1} \rangle = \frac{\sqrt{2\pi}}{m_{\text{6Li}} \sqrt{m_{\text{6Li}}} T_g^{1/2}} = 1.4 \times 10^8 \frac{1}{T_g^{1/2}},
\]

where the last expression is calculated in units of \(N_A^{-1} \text{cm}^3 \text{s}^{-1} \text{mol}^{-1}\) and \(T_g = T / 10^9 \text{K}\). Comparing the destruction rate to the Hubble rate at a fiducial temperature of \(T_g = 0.008\), we find that the former is much smaller:

\[
\frac{\langle \sigma v \rangle_{\text{B}}^{\text{max}} n_{(pX^-)}^{\text{max}}}{H} \bigg|_{T_g = 0.008} \simeq 0.03 \ll 1
\]

\(^4\) Focusing on a minimal MSSM particle content, \(Y_{X^-}\) is determined by the standard chemical decoupling of \(X^-\) from the primordial plasma for which \(Y_{X^-} \lesssim Y_{\text{4He}}\) holds unless \(m_{X^-} \gtrsim \mathcal{O}(4 \text{ TeV})\); see section 4 for a detailed discussion of \(Y_{X^-}\).
for $Y_{X^-} = 0.01$. This tells us that at most 3% of the $^6$Li synthesized at 8 keV could potentially be affected by $(pX^-)$ states via process B, and therefore the whole issue of nuclear uncertainties is irrelevant given the strength of the charge exchange reactions.

Even though we find that process B makes no impact on $^6$Li, it is still interesting to examine whether (2.8) corresponds to a realistic rate. As can be seen from figure 1, for reaction B to occur, the proton has to tunnel to the left through a distance of at least 100 fm inside the Coulomb barrier of $^6$Li in order to trigger its decay to two helium nuclei. Notice that the field of $X^-$ does not facilitate this tunneling in any way. Effectively, this is the same distance for tunneling that a free $\sim 50$ keV energy proton would have to overcome in a $p + ^6$Li $\rightarrow ^4$He $+ ^3$He reaction. Therefore, one expects an exponential suppression of the corresponding probability by $\exp(-\sqrt{E_G/50}$ keV) $\ll 1$, where $E_G$ is the Gamow energy. At the same time, the deconfining rate is 100% as long as the impact parameter is less than or equal to $R_{cd}$, leading to the inevitable conclusion that the rate of the charge exchange reaction A greatly exceeds that of the nuclear process B:

$$\frac{\langle \sigma v \rangle_B}{\langle \sigma v \rangle_A} \ll 1.$$  \hspace{1cm} (2.10)

Returning to the calculation of [25] which gives a very large estimate for process B, we believe that this estimate is probably an artifact of assuming a ‘frozen’ profile for the proton wavefunction. In contrast, in the correct approach, the proton wavefunction is easily polarized and deconfined by the incoming heavier nucleus. Further doubts in the validity of the estimates in [25] are cast by the answer for the cross section at extremely small energies. For example, it follows from [25] that $\sigma_B$ has almost an atomic size cross section, whereas this is a nuclear reaction between objects of nuclear size. The key difference between our treatment of $(pX^-)$-induced catalysis and that of [25] is the significant underestimation of the strength of the charge exchange reactions in the latter work, which in turn leads to overestimates in expressions analogous to our equations (2.5) and (2.9).

To conclude this section, we consider an interesting way of having an impact of $(pX^-)$ on $^6$Li. Indeed, a successive chain of charge exchange reactions can lead to molecular states that are finally destroyed in nuclear reactions with protons:

$$(pX^-) + ^6$Li $\rightarrow (^6$Li$X^-) + p,$  \hspace{1cm} (2.11)

$$(pX^-) + (^6$Li$X^-) $ $\rightarrow (^6$Li$X_2^-) + p,$  \hspace{1cm} (2.12)

$$(pX^-) + (^6$Li$X_2^-) $ $\rightarrow (^6$Li$X_3^-) + p,$  \hspace{1cm} (2.13)

$$(^6$Li$X_3^-) + p $ $\rightarrow ^7$Be $+ 3X^- $ or $^4$He $+ ^3$He $+ 3X^-.$  \hspace{1cm} (2.14)

In the last step of this chain, $(^6$Li$X_3^-)$ ‘ammonium’ has a chance for a nuclear interaction with protons or helium unsuppressed by a residual Coulomb barrier since $(^6$Li$X_3^-)$ is a very compact object. A similar chain exists for $^7$Be and $^9$Be where the sequence of charge exchange reactions can proceed until Be–$X^-$ ‘methane,’ $(^6$Be$X^-)$. It is important to note that the efficiency of this chain reaction depends very sensitively on the concentration of the $(pX^-)$ bound states and on the mass of the $X^-$ particle. The latter enters through the average relative velocity of two heavy objects, e.g. $(pX^-)$ and $(^6$Li$X^-)$, which in turn scales as $m_{X^-}^{-1/2}$. Therefore, in the limit of an infinitely heavy $X^-$ and with our
Figure 2. Evolution of primordial abundances as a function of time (or temperature $T_9$) from the input $Y_{X^-} = 0.01$, $m_{X^-} = 100$ GeV and $\tau_{X^-} \rightarrow \infty$. The solid (red) line of the ($pX^-$) abundance reaches its maximum of $\sim 4 \times 10^{-7}$ at $T_9 \simeq 0.008$. Around the same temperatures, the yield of unbounded $X^-$, $Y_{X^-}^{\text{free}}$, starts to decline more rapidly since it is removed by the recombination with $p$ followed by the charge exchange reaction on $^4\text{He}$. Using the charge exchange rates estimated in this section, we run the set of Boltzmann equations to determine the residual concentrations of ($pX^-$) and of the molecular bound states of $^6\text{Li}$ with $X^-$. The results are plotted in figure 2. As one can see, an initial concentration of $X^-$ per nucleon of $Y_{X^-} = 10^{-2}$ results in a ($pX^-$) abundance that never exceeds the maximum (2.5), leading to a progressively diminishing number of molecular states.

There is another plausible mechanism for the destruction of ($^6\text{Li}X^-$) that is related to the recoil of $^6\text{Li}$ freed in the decay of $X^-$. Typical kinetic energies of $^6\text{Li}$ after the decay are comparable to that on the orbit $\sim 700$ keV. In the center-of-mass frame with plasma protons, this corresponds to energies of $\sim 100$ keV. At such a center-of-mass energy, some of the $^6\text{Li}$ nuclei released in the decays of the bound states will be destroyed. However, given that only a small fraction of $^6\text{Li}$ is locked in bound states with $X^-$ and that the destruction rate is smaller than the thermalization rate, it is safe to conclude that this mechanism cannot also lead to large overall depletion factors for $^6\text{Li}$. The same argument applies to $^9\text{Be}$. 

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To conclude this section, neither lithium nor beryllium synthesized in CBBN processes at 8 keV would be affected in any significant way by the subsequent generation of \((pX^-)\) bound states. Thus, the part of the parameter space with a typical freeze-out \(X^-\) abundance and a long \(X^-\) lifetime is confidently ruled out, which is shown in more detail in the following sections.

3. \(^9\text{Be}\) constraints on the \(X^-\) lifetime and abundance

In order to constrain the \((\tau_{X^-}, Y_{X^-})\) parameter space from the catalytic path (1.2) to \(^9\text{Be}\) [31], we need to set an upper limit on its primordial abundance from existing observations. It is generally accepted that the galactic evolution of the abundances of Be, along with Li and B, are dominated by cosmic-ray nucleosynthesis. While Be is burned rapidly in stellar centers, it is produced in cosmic rays by the spallation reactions of fast protons and \(\alpha\) particles hitting ambient CNO nuclei [58]. As a consequence, the abundances of Be and O are linked, leading to a secondary scaling, \(\text{Be} \propto \text{O}^2\) [59]. On the other hand, inverse spallation reactions of CNO nuclei, both produced and accelerated in supernovae, will give a Be yield that is essentially independent of the metallicity of the interstellar medium. Such primary processes, leading to \(\text{Be} \propto \text{O}\), are expected to play a major role during the early galactic epochs [60].

The produced Be is subsequently supplemented in the outer layers of stars. Thus, old stars which are far from the galactic center (and thereby less affected by the galactic chemical evolution) bear the potential to encode any pre-galactic origin of Be. Indeed, Be has been observed in a number of Population II halo stars at very low metallicities \([\text{Fe/H}] \lesssim -2.5; [A] = \log_{10} A + 12\). Particularly noteworthy is the detection in the star G 64–12 at \([\text{Fe/H}] \simeq -3.3\) [52]. The star’s high Be value of \(\log_{10}(\text{Be/H}) \simeq -13.05\) might suggest a possible flattening in the Be trend during the early evolutionary phases of our galaxy [52]. Whether this really points to a primordial plateau or whether this indicates a Be dispersion at lowest metallicities [54] is not clear at present.

Figure 3(a) shows the original Be detection in the star G 64–12\(^5\) (filled dot) along with a subset of data points taken from figure 3a of [52]. The data of figure 3(b) are taken from figure 6b of the recent work [61] which also uses \([\text{O/H}]\) as a metallicity tracer. The latter paper discusses the implications of a new temperature scale on the abundances of Li, Be and B. In principle, different assumed physical parameters which characterize the stellar atmosphere may result in large systematic shifts of the inferred abundances. In this regard, it is important to note that Be is not overly sensitive to the assumed surface temperature of the halo dwarfs [61]. In the following we shall thus take a pragmatic approach. In both figures 3(a) and (b), we obtain the least-squares weighted mean (dashed lines) for a representative sample of stars at lowest metallicities. From the variance of the fit, we can extract a nominal 3\(\sigma\) upper limit (solid lines) on primordial \(^9\text{Be}\). From figure 3(b), we find

\[
\log_{10}\text{Be/H}|_{\text{high}} = -12.68 \Rightarrow ^9\text{Be/H} \leq 2.1 \times 10^{-13}.
\]  

(3.1)

Conversely, figure 3(a) yields \(^9\text{Be/H} \lesssim 10^{-13}\) while fitting only the last two data points with \([\text{O/H}] < -1.3\) in figure 3(b) would give \(^9\text{Be/H} \lesssim 1.3 \times 10^{-13}\). In our context,
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Figure 3. Observations of Be in Pop II halo stars. In the left panel (a), the data is taken from figure 3a of [52] and is plotted as a function of [Fe/H]. The right panel (b) shows the data from figure 6b of [61] where [O/H] provides the metallicity indicator. The filled dots depict the data points associated with the star G 64–12. The solid lines give the inferred nominal upper limits on $^9$Be from the weighted mean (dashed lines) of a sample of stars at lowest metallicity. Also shown in (b) is a fit of a primary scaling of Be; see main text.

those values are less conservative so that we use (3.1) in the following. In figure 3(b) we have additionally fitted for a primordial component, $^9$Be/H$_p$, in combination with a primary scaling, $^9$Be/H = $\kappa$ O/H. It seems, however, that a purely primary mechanism with $\kappa \simeq 2.9$ fits the data best since $^9$Be/H$_p$ comes out negligibly small\footnote{For a proper comparison between different assumed surface temperature scales and corresponding fits of primary versus secondary scaling, see [61].}. Finally, we are aware that neither of the fitted mean values in figure 3 is very good in terms of $\chi^2$. However, a firm conjecture of a Be plateau is not the purpose of this work, and indeed (3.1) does provide a sufficiently conservative limit to work with.

We can now confront the constraint (3.1) with the CBBN yield of $^9$Be obtained by solving the associated Boltzmann equations. The central input parameter for the catalytic production of $^9$Be and $^6$Li is the abundance of $X^-$ at the time of its recombination with $^4$He. Above 10 keV, we can track the resulting ($^4$He$X^-$) abundance by using the Saha equation since photodissociation proceeds rapidly. Only at $T \simeq 8$ keV, ($^4$He$X^-$) starts to build up efficiently, and we couple it into the full set of Boltzmann equations. We use a $^4$He–$X^-$ recombination cross section that is based on the work of [12]. It takes into account the finite size of the nucleus and includes $\alpha$ captures into 1S as well as 2S states. For the cross section of catalyzed $^6$Li production (1.3), we employ the result of a nuclear three-body calculation [20]. The path to $^9$Be proceeds via ($^6$Be$X^-$) bound states which are formed by the radiative fusion $^4$He + ($^4$He$X^-$) → ($^8$Be$X^-$) + $\gamma$. From there, $^9$Be is subsequently produced by resonant neutron capture ($^8$Be$X^-$) + n → $^9$Be + $X^-$ [31];
see the appendix for the associated cross sections. In our code we can neglect the formation of \( ^8\text{Be}X^- \) that proceeds via molecular bound states \( ^4\text{He}X^2_2 \) \cite{31}. This process becomes important only for a combination of large \( Y_X^- \) and large \( \tau_{X^-} \), i.e. a parameter region which is already excluded by \(^6\text{Li} \) overproduction. Also note that, at the time when \( ^8\text{Be}X^- \) form, their photodissociation is not important because of the high binding energy \( E_b(^8\text{Be}X^-) \simeq 1430 \) keV \cite{12}. Finally, for \( T_B < 0.2 \), the SBBN \( n \) abundance can already be tracked well by including the processes \( D + D \rightarrow n + ^3\text{He}, \) \( T + D \rightarrow n + ^3\text{He}, \) and \( ^3\text{He} + n \rightarrow p + T \) in the reaction network \cite{62}. Those cross sections, as well as the one from \( p \)-induced \( ^6\text{Li} \) destruction, can be found, for example, in \cite{63}. It is important to note that we assume the SBBN central value for the deuterium abundance. The early decays of \( X^- \) may result in an injection of nucleons into the system. This typically drives the deuterium abundance upward, resulting in an enhanced number of neutrons at later times and therefore in an increased output of \(^9\text{Be} \), with the general scaling \( ^9\text{Be} \sim \text{const} \times (D/D_{\text{SBBN}})^2 \). We choose to disregard this effect, noting its model-dependent character. We are allowed to do so since its inclusion can only make the \(^9\text{Be} \) derived bound on the \( X^- \) abundance stronger.

We should mention at this point that anyone attempting a precision calculation of \(^9\text{Be} \) within the CBBN framework would have to include an additional channel related to the early production of beryllium, as pointed out in \cite{21}. This channel consists of \( X^- \) capture on \(^7\text{Be} \) with a subsequent \( p \)-induced reaction producing \( ^8\text{BX}^- \), which then beta decays to the \( ^8\text{BeX}^- \) bound state:

\[
X^- \rightarrow (^7\text{Be}X^-) \rightarrow (^8\text{BX}^-) \rightarrow (^8\text{BeX}^-) \rightarrow ^9\text{Be}. \tag{3.2}
\]

The efficiency of this chain is directly proportional to a rather small \(^7\text{Be} \) abundance. Therefore, the final output of \(^9\text{Be} \) via (3.2) is never very large, but could reach the level of \( \sim \text{O}(10^{-13}) \) for large abundances of \( X^- \). For the purpose of setting limits on particle physics models, we are allowed to ignore this early chain (3.2), noting that it is generally subdominant and also model-dependent. In particular, the chain (3.2) depends on the properties of \( X^- \) \cite{21}, as well as on the non-thermal processes that can affect the \(^7\text{Be} \) abundance\(^7\).

Figure 4 shows the evolution of catalyzed \(^6\text{Li} \) and \(^9\text{Be} \) production from the solution of the corresponding set of Boltzmann equations below \( T = 10 \) keV. We parameterize \( Y_{X^-} \) by the \( X^- \) abundance prior to decay by introducing \( Y_{X^-}^{\text{dec}} \), where the superscript ‘\( \text{dec} \)’ stands for decoupling, and by the \( X^- \) lifetime \( \tau_{X^-} \), so that the (total) \( X^- \) abundance at any moment during BBN is given by \( Y_{X^-}(t) = Y_{X^-}^{\text{dec}} \times \exp(-t/\tau_{X^-}) \). In particular, to obtain the curves in figure 4, the values \( Y_{X^-}^{\text{dec}} = 5 \times 10^{-4} \) and \( \tau_{X^-} = 5 \times 10^3 \) s are used. When the ‘bottleneck’ abundances of \( ^4\text{He}X^- \) and \( ^8\text{BeX}^- \) form, the catalytic paths (1.1) and (1.2) to \(^6\text{Li} \) and \(^9\text{Be} \) open up, resulting in the asymptotic values \( ^9\text{Be}/H \simeq 10^{-13} \) and \( ^6\text{Li}/H \simeq 3 \times 10^{-11} \). The dashed line shows the neutron abundance and the dotted line the free \( X^- \) abundance, which is dominated by its exponential decay. We remark in passing that residual recombinations of \(^4\text{He} \) with \( X^- \) lead to the crossing of the \( ^4\text{He}X^- \) and \( X_{\text{free}} \) lines at late time.

\(^7\) In addition, as pointed out in a recent reference \cite{75}, the beta decay of \(^8\text{B} \) occurs predominantly to the excited states of \(^8\text{Be} \), which will likely result in a break-up of \((^8\text{Be}X^-) \) and further reduction of the efficiency of the (3.2) chain.
Figure 4. Evolution of catalyzed $^6$Li and $^9$Be production shown together with the formation of the ‘bottleneck’ abundances of ($^4$He$X^-$) and ($^8$Be$X^-$) for $Y_{X^-}^{\text{dec}} = 5 \times 10^{-4}$ and $\tau_{X^-} = 5 \times 10^3$ s. The dashed line gives the neutron abundance while the dotted line shows the abundance of free $X^-$. In figure 5 we obtain exclusion boundaries from catalyzed $^9$Be and $^6$Li production in the ($\tau_{X^-}, Y_{X^-}^{\text{dec}}$) parameter space. For the convenience of the reader, the $X^-$ number density $n_{X^-}^{\text{dec}}$ normalized to the entropy density $s$ is given on the $y$ axis on the right-hand side. Above the solid line, $^9$Be is in excess with respect to (3.1) and thus excluded. The shown band reflects the uncertainties in the observational determination of $^6$Li. On the lower border, $^6$Li/H = $10^{-11}$ is fulfilled while $^6$Li/H = $10^{-10}$ holds on the upper border of the band. The cross indicates the exemplary parameter point considered in figure 4. At large lifetimes, the linear scaling of $^6$Li with $Y_{X^-}$ can easily be seen from the boundaries of the band. Note that we find $^9$Be/$^6$Li in the interval between $10^{-3}$ and $10^{-2}$, whenever CBBN is efficient, which confirms the observation already made in [31].

4. Implications for supersymmetric models

Let us now address the implications of the results derived above to SUSY extensions of the standard model in which the gravitino $\tilde{G}$ is the LSP and a charged slepton $\tilde{l}_1$ the NLSP [37, 34, 39, 40, 35, 36, 41], [17]–[19], [26]–[28], [33]. As the spin-3/2 superpartner of the graviton, the gravitino is an extremely weakly interacting particle with supergravity couplings [64, 65] that are suppressed by the (reduced) Planck scale $M_P = 2.4 \times 10^{18}$ GeV. Thereby, the negatively charged $\tilde{l}_1$ can be the long-lived $X^-$ with a lifetime
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Figure 5. Contour plot of CBBN abundance yields of $^6$Li and $^9$Be in the $(\tau_{X^-}, Y_{X^-}^{\text{dec}})$ plane. The solid line shows the limit (3.1). The region above this line is excluded by $^9$Be overproduction. The lower (upper) boundary of the band corresponds to $^6$Li/H $= 10^{-11} (10^{-10})$. The $y$ axis on the right-hand side indicates the $X^-$ number density $n_{\text{dec}}^{X^-}$ normalized to the entropy density $s$. The cross shows the parameter point considered in figure 4.

$\tau_X = \tau_{l_1}$ governed by the decay $\tilde{l}_1 \rightarrow \tilde{G} l^8$:

$$\tau_{l_1} \simeq \Gamma^{-1}(\tilde{l}_1 \rightarrow \tilde{G} l) = \frac{48\pi m_{\tilde{G}}^2 M_P^2}{m_{l_1}^5} \left(1 - \frac{m_{l_1}^2}{m_{\tilde{G}}^2}\right)^{-4}.$$  \hfill (4.1)

Indeed, $\tau_{l_1} \gtrsim 10^4 \text{s}$ occurs in a large region of natural values of the gravitino mass $m_{\tilde{G}}$ and the slepton mass $m_{l_1}$, as illustrated by the $\tau_{l_1}$ contours (dotted lines) in figure 6.

For a standard cosmological history with a post-inflationary reheating temperature $T_R$ above the decoupling temperature of the $\tilde{l}_1$ NLSP, $T_f \lesssim m_{\tilde{l}_1}/20$ \cite{37}, the $\tilde{l}_1$ NLSP freezes out of the primordial plasma as a cold thermal relic so that its yield after decoupling $Y_{\text{dec}}^{\tilde{l}_1}$ is governed by its mass and its annihilation rate. Thereby, $Y_{\text{dec}}^{\tilde{l}_1}$ becomes sensitive to the mass spectrum and the couplings of the SUSY model. In this work, we work with a representative yield that is quite typical for an electrically charged massive thermal relic \cite{37,39,19} \footnote{We assume $R$-parity conservation. For the case of broken $R$-parity, see, e.g., \cite{67}.}: \footnote{For a recent thorough study of the decoupling yield of a charged relic, see \cite{68}.}

$$Y_{\text{dec}}^{\tilde{l}_1} \equiv \frac{n_{\tilde{l}_1}}{n_B} = 2 Y_{\text{dec}}^{\tilde{l}_1} = 0.8 \times 10^{-3} \left(\frac{m_{\tilde{l}_1}}{100 \text{ GeV}}\right),$$ \hfill (4.2)
Figure 6. Cosmological constraints on the masses of the gravitino LSP and a charged slepton NLSP for a yield $Y_{\tilde{l}_1}^{\text{dec}}$ given by (4.2). The gray band indicates $\Omega_{\tilde{G}}^{\text{NTP}} \in \Omega_{\tilde{G}}^{\text{DM}}$. Above this band, $\Omega_{\tilde{G}} > 0.126$. On the thin solid line labeled with $f$ values only $f \Omega_{\tilde{B}}$ is provided by $\Omega_{\tilde{G}}^{\text{NTP}}$. The dotted lines show contours of $\tau_{\tilde{l}_1}$. Due to CBBN, the region below the solid and the long-dashed–dotted (red) lines is disfavored by observationally inferred abundances of $^9\text{Be}$ and $^6\text{Li}$, respectively. The effect of electromagnetic and hadronic energy injection on primordial D disfavors the regions inside the short-dashed–dotted (blue) curves and to the right or inside of the short-dashed (blue) curves, respectively. The region below the dashed (green) line is disfavored by the effect of electromagnetic energy injection on $^3\text{He}/D$. While the constraints from hadronic energy injection are obtained for a purely ‘right-handed’ $\tilde{l}_1 \simeq \tilde{l}_R$ NLSP, the ones from electromagnetic energy injection are valid for the $\tilde{\chi}_1$ NLSP case with a visible electromagnetic energy of $E_{\text{vis}} = \epsilon_{\text{em}} = 0.3 E_\tau$ released in $\tilde{\chi}_1 \rightarrow \tilde{G}_\tau$.  

where $n_{\tilde{l}_1}$ denotes the total $\tilde{l}_1$ number density assuming an equal number density of positively and negatively charged $\tilde{l}_1$. Note that the yield (4.2) is in good agreement with the curve in figure 1 of [37] that has been derived for the case of a purely ‘right-handed’ $\tilde{\chi}_1 \simeq \tilde{\tau}_R$ NLSP with a mass that is significantly below the masses of the lighter selectron and the lighter smuon, $m_{\tilde{\tau}_1} \ll m_{\tilde{e}_1, \tilde{\mu}_1}$, and with a bino-like lightest neutralino, $\tilde{\chi}_1^0 \simeq \tilde{B}$, that has a mass of $m_{\tilde{B}} = 1.1 m_{\tilde{\tau}_1}$. In the case of an approximate slepton mass degeneracy, $m_{\tilde{\tau}_1} \lesssim m_{\tilde{e}_1, \tilde{\mu}_1} \lesssim 1.1 m_{\tilde{\tau}_1}$, the $\tilde{\tau}_1$ NLSP yield (4.2) can become twice as large due to slepton coannihilation processes [37, 19]. Approaching the $\tilde{\chi}_1^0$ $\tilde{\tau}_1$ coannihilation region, $m_{\tilde{\chi}_1^0} \approx m_{\tilde{\tau}_1}$, even larger enhancement factors occur; see, e.g., figure 3 in [19]. On the other
hand, a sizable left–right mixing of the stau NLSP is associated with an increase of its MSSM couplings and thus with a reduction of $Y_{\tilde{l}_1}$. Moreover, an exceptional reduction of $Y_{\tilde{l}_1}$ can also occur in a non-standard thermal history with late-time entropy production after the decoupling of the $\tilde{l}_1$ NLSP and before BBN \cite{69,19,20} or in low $T_R$ scenarios \cite{23}. Noting that both cases require substantial modifications to the MSSM field content at or below the weak scale, we disregard such possibilities and focus in the remainder of this work on the more generic $Y_{\tilde{l}_1}$ values described by (4.2).

Confronting (4.2) with our limits shown in figure 5, we obtain the CBBN constraints shown in figures 6–8. In each figure, it is the region to the right of the long-dashed–dotted (red) lines and the one to the right of the solid line in between those lines that is disfavored by $^6\text{Li}/H |_p \leq 10^{-11}$ and $10^{-10}$ and by the $^9\text{Be}$ limit (3.1), respectively. While the emphasis is on the limits from CBBN of $^9\text{Be}$ (and $^6\text{Li}$), the following additional cosmological constraints are shown for comparison:

- Since each $\tilde{l}_1$ NLSP decays into one $\tilde{G}$ LSP, these decays lead to a non-thermally produced (NTP) gravitino density \cite{37,40}:

$$\Omega_{NTP}^{\tilde{G}} h^2 = m_{\tilde{G}} Y_{\tilde{l}_1}^{\text{dec}} n_B(T_0) h^2 / \rho_c,$$

where $\rho_c /[n_B(T_0) h^2] = 42.1 \text{ GeV}$ \cite{66}. This contributes to the relic gravitino density $\Omega_{\tilde{G}}$ which should not exceed the observationally inferred dark matter density $\Omega_{\text{DM}}$ and thus imposes an additional constraint on the model. Accordingly, we show shaded regions in figures 6–8, in which the $\Omega_{NTP}^{\tilde{G}} h^2$ values obtained with (4.2) agree with the nominal 3σ range of $\Omega_{\text{dm}} h^2$ inferred with a restrictive six-parameter ‘vanilla’ model from the three-year dataset of the Wilkinson Microwave Anisotropy Probe (WMAP) satellite \cite{[1]}:

$$\Omega_{\text{DM}}^{3\sigma} h^2 = 0.105^{+0.021}_{-0.030},$$

with $h = 0.73^{+0.04}_{-0.03}$ denoting the Hubble constant in units of 100 km Mpc$^{-1}$ s$^{-1}$. In each figure, the parameter space above the shaded region is disfavored by the dark matter constraint $\Omega_{NTP}^{\tilde{G}} \leq \Omega_{\text{DM}}$. With any additional contribution to $\Omega_{\text{DM}}$—such as an axion density or a thermally produced gravitino density $\Omega_{\tilde{G}}^{\text{TP}}$—this constraint can become even more restrictive. In figure 6 this is indicated by the thin solid contours labeled with $f = 0.1, 0.01, 10^{-3}$ and $10^{-4}$, on which (4.3) obtained with (4.2) satisfies $f \Omega_{NTP}^{\tilde{G}} = 0.126$, respectively.

- In a $\tilde{l}_1$ NLSP decay, standard model particles are emitted in addition to the gravitino which can affect the abundances of the primordial light elements. While the associated hadronic/electromagnetic energy release seems to affect the CBBN constraints from $^6\text{Li}$ and $^9\text{Be}$ only mildly—as discussed in the introduction—it could alter in a substantial way the primordial fractions of D/H and $^3\text{He}/^4\text{He}$ \cite{40,35,41,17,22}. The effect of hadronic energy injection on primordial D disfavors the regions inside the short-dashed–dotted (blue) curves shown in figures 6–8\textsuperscript{10}. These curves are obtained

\textsuperscript{10} Additional constraints on hadronic energy release are imposed by the primordial abundances of $^4\text{He}$, $^3\text{He}/^4\text{He}$, $^7\text{Li}$ and $^6\text{Li}/^7\text{Li}$ \cite{5,6,8}–\cite{10,17}. However, in the region allowed by the $^9\text{Be}$ and $^6\text{Li}$ constraints from bound-state effects, i.e. $\tau_{\tilde{l}_1} \lesssim 10^4 \text{ s}$, the considered D constraint on hadronic energy release is the dominant one, as can be seen, for example, in figures 38–41 of \cite{9} and in figures 6–8 of \cite{10}.
from the upper limits on $Y_{\mathrm{dec}}^{\tilde{l}_1}$ that are given in figure 11 of [41] as derived from a computation of the four-body decay of a purely ‘right-handed’ $\tilde{\ell}_1 \simeq \tilde{\ell}_R$ NLSP into the gravitino, the tau and a quark–antiquark pair [41]. Note that these upper limits are based on the severe and conservative upper bounds on the released hadronic energy (95% CL) obtained in [9] for observationally inferred values of the primordial D abundance (see references cited in [9]):

$$\left( \frac{D}{H} \right)_{\text{mean}} = (2.78^{+0.44}_{-0.38}) \times 10^{-5} \Rightarrow \text{severe constraint,} \quad (4.5)$$

$$\left( \frac{D}{H} \right)_{\text{high}} = (3.98^{+0.59}_{-0.67}) \times 10^{-5} \Rightarrow \text{conservative constraint.} \quad (4.6)$$

Without trying to give extra credence to a rather high value of D/H in (4.6), following [9], we simply take it as a limiting value for D/H.

The regions disfavored by electromagnetic energy injection are shown in figure 6 only. Here it is the region to the right or inside of the short-dashed (blue) curves and the region to the right of the long-dashed (green) line that are disfavored by the primordial abundances of D and $^3$He/D, respectively. These curves are obtained for the stau NLSP case $\tilde{l}_1 = \tilde{\tau}_1$, i.e. for a ‘visible’ electromagnetic energy of $E_{\mathrm{vis}} = \epsilon_{\mathrm{em}} = 0.3 E_\tau$ of the tau energy $E_\tau = (m_\tau^2 - m_\tilde{G}^2 + m_\tilde{l}_1^2)/2m_\tilde{l}_1$ released in $\tilde{\tau}_1 \rightarrow \tilde{G}\tau$\textsuperscript{11}, where the $D_{\mathrm{em}}^{\text{sev}}$ and $^3$He/D constraints result from the $Y_{\mathrm{NLSP}}$ limits given in figure 42 of [9] and the $D_{\mathrm{em}}^{\text{cons}}$ constraint from the $Y_{\mathrm{NLSP}}$ limit given in figure 6 of [7]; see also figure 9 (lower panel) in [41]. As noted before, the elevated content of D leads to the enhancement of CBBN-produced $^6$Li and $^9$Be. For example, if non-thermal processes boost the deuterium abundance to the level of (4.6), it would lead to an enhancement of the $^6$Li output by a factor of $\sim 2$, while the corresponding enhancement factor in the case of $^9$Be is about 4.

Simple comparison shows that the $^9$Be constraint (together with the one from $^6$Li) provides the most restrictive upper limit on $m_\tilde{G}$ for a given $m_{\tilde{l}_1}$ in the collider-accessible region below 1 TeV. Indeed, in the most conservative case, $f = 1$, the dark matter constraint $\Omega_N^{\text{NTP}} \leq \Omega_{\text{DM}}$ disfavors ($m_{\tilde{G}}, m_{\tilde{l}_1}$) combinations associated with $m_\tilde{G} \gtrsim 200$ GeV and $m_{\tilde{l}_1} \gtrsim 800$ GeV, which is a mass range that will be difficult to probe at the LHC or at the International Linear Collider (ILC). Moreover, the ‘electromagnetic’ $D_{\mathrm{em}}^{\text{cons}}$ and $^3$He/D constraints are always less restrictive than the CBBN constraints from $^9$Be and $^6$Li. Only the ‘hadronic’ constraint $D_{\mathrm{had}}$ can potentially compete with the CBBN constraints. For instance, this occurs for ($m_\tilde{G}, m_{\tilde{l}_1}$) combinations associated with $m_\tilde{G} \gtrsim 60$ GeV and a heavy $\tilde{l}_1$ NLSP with $m_{\tilde{l}_1} \gtrsim 900$ GeV. Accordingly, the presented CBBN constraints from $^9$Be and $^6$Li are the most relevant ones in the $m_{\tilde{l}_1}$ range that will be accessible at the next generation of particle accelerators.

For $m_{\tilde{l}_1}$ below 1 TeV, the new $^9$Be constraint can also be considered as the most robust BBN constraint. Indeed, the difference between the contours labeled with $D_{\mathrm{em}}^{\text{cons}}$ and $D_{\mathrm{em}}^{\text{sev}}$ demonstrates that the D constraints are associated with a significant uncertainty

\textsuperscript{11} For the selectron NLSP, $\tilde{l}_1 = \tilde{e}_1$, with $E_{\mathrm{vis}} = \epsilon_{\mathrm{em}} = (m_\tau^2 - m_\tilde{\tau}_1^2 + m_\tilde{e}_1^2)/2m_\tilde{e}_1$, given by the ‘full’ electron energy released in $\tilde{e}_1 \rightarrow \tilde{G}e_\tau$, the bounds become more severe as shown explicitly in figure 12 of [41]. However, a comparison with figure 5 shown above shows that the constraints from electromagnetic energy injection will still be significantly less restrictive than the CBBN constraints from $^9$Be and $^6$Li.

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related to the assumed upper limit on the primordial D/H fraction; cf (4.5) and (4.6). The uncertainty associated with assumed upper limits on the primordial $^6\text{Li}$/H fraction, which can differ by (even more than) an order of magnitude, is indicated by the difference between the $^6\text{Li}$ contours obtained for $^6\text{Li}/\text{H}|_p \leq 10^{-11}$ and $10^{-10}$. Indeed, since the observational status of $^9\text{Be}$ is in better shape than the one of $^6\text{Li}$ and since $^9\text{Be}$ is less fragile than $^6\text{Li}$ and $^7\text{Li}$, and thus less affected by stellar mechanisms, the $^9\text{Be}$ constraint (which is represented by a single line) can be considered to be more robust than the $^6\text{Li}$ constraint.

Here we would like to emphasize that the $^9\text{Be}$ and $^6\text{Li}$ constraints are the ones that are the least sensitive to the precise value of $Y_{\tilde{l}}^{\text{dec}}$ in the region $Y_{\tilde{l}}^{\text{dec}} \gtrsim 10^{-4}$. This results from the fact that the limits are very steep in that region, as can be seen in figure 5. Indeed, a yield that is twice as large as (4.2) will affect the position of the $^9\text{Be}$ and $^6\text{Li}$ constraints only very mildly. In contrast, such an enhanced yield—as encountered, for example, in the case of slepton coannihilations—leads to significant changes of the dark matter constraint and the BBN constraints associated with hadronic/electromagnetic energy injection, as can be seen explicitly in figure 17 of [18]. It should also be noted that an elevated slepton yield can lead to an additional non-thermal output of $^6\text{Li}$ for $\tau_{\tilde{l}} \gtrsim \text{ few } \times 10^8$ s. This is because energetic spallation debris of destroyed $^4\text{He}$ nuclei from slepton decays can hit ambient $^4\text{He}$ and thereby fuse $^6\text{Li}$ [8]–[10]. This mechanism depends sensitively on the hadronic branching ratio $B_h$ of the four-body slepton decay into the gravitino, the associated lepton and a quark–antiquark pair for which typically $B_h \ll 3 \times 10^{-3}$ for $m_{\tilde{l}} \lesssim 2$ TeV (see figure 5 of [41]). Indeed, as discussed in [29], for those branching ratios, the effect of CBBN on $^6\text{Li}$ is the dominant one in the region which is not already excluded by the D constraint. Thus, for $m_{\tilde{l}} \lesssim 1.5$ TeV, our obtained limits on $^6\text{Li}$ overproduction are only marginally affected by the hadronic energy release of $\tilde{l}$ decays. However, for larger slepton masses, i.e. for scenarios of large $Y_{\tilde{l}}^{\text{dec}}$ in conjunction with $B_h > 10^{-3}$, the hadronic production of $^6\text{Li}$ becomes efficient so that only a simultaneous treatment of both effects can decide on the accurate $^6\text{Li}$ BBN output. Note that this can make our presented limits on $^6\text{Li}$ only stronger. Thus, we are on the conservative side when neglecting such additional contributions.

Considering the CBBN constraints in figures 6–8, one finds that the constraints from $^9\text{Be}$ and $^6\text{Li}$ are in close proximity to each other. This coincidence is, of course, related to the fact that the ratio of maximally allowed values for $^9\text{Be}$ and $^6\text{Li}$ is in rough agreement with the efficiency of producing $^9\text{Be}$ and $^6\text{Li}$ per each long-lived negatively charged slepton. Given the close proximity of these limits, it is tempting to speculate about a possible CBBN origin of primordial abundances of $^6\text{Li}$ and $^9\text{Be}$ at lowest metallicities [31]. This possibility applies to the gravitino dark matter scenarios that are located at or only slightly to the left of these constraints. It will be interesting to see whether a solution of the $^7\text{Li}$ problem can also be found in these scenarios. Indeed, several ways to solve the $^7\text{Li}$ problem have been proposed that could be relevant in this region [8, 36, 12, 17, 21, 25, 30]. However, a definitive answer will require an elaborate treatment of BBN in which all relevant effects from bound-state formation and electromagnetic/hadronic energy release are included simultaneously.

Having discussed the generic features of the $^9\text{Be}$ constraint and its comparison with other BBN constraints, we now would like to address its implications. Our task is
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Figure 7. Contours of \( m_\tilde{G} \) (dotted lines) as a function of \( \tau_\tilde{l}_1 \) and \( m_\tilde{l}_1 \). Assuming a slepton yield \( Y_{\tilde{l}_{1}\text{dec}} \) given by (4.2), constraints from CBBN of \(^{9}\text{Be}\) and \(^{6}\text{Li}\) are obtained as shown by the solid line and the long-dashed–dotted (red) lines, respectively. For a purely ‘right-handed’ \( \tilde{l}_1 \approx \tilde{l}_R \) NLSP with a yield (4.2), BBN constraints from effects of hadronic energy injection on D are obtained as indicated by the short-dashed–dotted (blue) lines. In the shaded region, \( \Omega_{NTP}^\text{T} h^2 \) agrees with (4.4).

facilitated by the \(^{9}\text{Be}\) constraint being very close to the one from \(^{6}\text{Li}\), which makes those implications similar to the ones of the \(^{6}\text{Li}\) constraint [12], [17]–[20], [22], [26]–[28], [33]. In this respect, section 2 becomes important in which we show that the possibility of allowed islands in the parameter region with large \( Y_{\tilde{l}_1}/\text{large } \tau_\tilde{l}_1 \)—which was advocated to remain viable in [25]—does not exist. Our present work does thereby reassure the conclusions drawn from the \(^{6}\text{Li}\) constraint in a decisive way:

(1) The gravitino mass \( m_\tilde{G} \) is constrained to values well below 10% of the slepton NLSP mass \( m_\tilde{l}_1 \) for \( m_\tilde{l}_1 \lesssim \mathcal{O}(1 \text{ TeV}) \). This can be read off conveniently from the CBBN constraints shown in figure 6 and also from figure 7 in which \( m_\tilde{G} \) contours (dotted lines) are given as a function of \( \tau_\tilde{l}_1 \) and \( m_\tilde{l}_1 \). In particular, \( 0.1 m_\tilde{l}_1 \lesssim m_\tilde{G} < m_\tilde{l}_1 \) and thereby the kinematical determination of \( m_\tilde{G} \) proposed in [47] remains cosmologically disfavored at least at the next generation of particle accelerators.

(2) The CBBN constraints disappear for a gravitino mass of \( m_\tilde{G} \lesssim 200 \text{ MeV} \) provided \( m_\tilde{l}_1 \gtrsim 80 \text{ GeV} \) as supported by the non-observation of long-lived charged sleptons at the Large Electron Positron Collider (LEP) [66]. This can be seen in figure 6.
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Figure 8. Contours of $T_{R}^{\max}$ imposed by $\Omega_{TP}^{G} h^2 + \Omega_{TP}^{NTP} h^2 \leq \Omega_{DM} h^2 \leq 0.126$ for $c = 1$ (dashed lines) and $c = 7$ (dotted lines) as a function of $\tau_{\tilde{l}_1}$ and $m_{\tilde{l}_1}$. The other cosmological constraints are identical to the ones shown in figure 7.

Accordingly, for gauge-mediated SUSY breaking leading to small values of $m_{\tilde{G}}$, the CBBN constraint can be irrelevant. However, for $m_{\tilde{G}} \gtrsim 10$ GeV, as obtained in gravity-mediated SUSY breaking, the CBBN constraints impose a lower limit of $m_{\tilde{l}_1} > 400$ GeV, as can be seen in figures 6 and 7. With CMSSM relations between the masses of the superparticles, this translates into a lower limit on the gluino mass $m_{\tilde{g}} > 2.5$ TeV [26]–[28]. Thereby, the $^9$Be constraint also points within the CMSSM to a cosmologically favored mass range that will be difficult to probe at the LHC.

(3) The $^9$Be constraint imposes an upper limit on the lifetime $\tau_{\tilde{l}_1}$ that ranges between $6 \times 10^{3}$ s at $m_{\tilde{l}_1} = 100$ GeV and $3 \times 10^{3}$ s at $m_{\tilde{l}_1} = 1.5$ TeV for a charged slepton NLSP with $Y_{\tilde{l}_1}^{dec}$ described by (4.2), as can be seen in figure 7. Indeed, when the value of $Y_{\tilde{l}_1}^{dec}$ increases by a factor of 15, the $\tau_{\tilde{l}_1}$ limit decreases by only a factor of two. This mild $m_{\tilde{l}_1}$ dependence of the $\tau_{\tilde{l}_1}$ limit reflects the fact that the $^9$Be constraint is quite insensitive to the precise value of $Y_{\tilde{l}_1}^{dec}$. While the $m_{\tilde{G}}$ contours and the D constraints shown in figure 7 are specific to the gravitino LSP scenario with unbroken $R$-parity, the CBBN limits on $\tau_{\tilde{l}_1}$ shown in figure 7 apply to any scenario with a long-lived charged slepton described by the yield (4.2) and can thus be relevant for axino LSP scenarios [70, 71, 44] and scenarios with $R$-parity violation [67] as well.

(4) Gravitino dark matter can originate not only from NLSP decays but also from thermal scattering of particles in the hot primordial plasma. Thereby, the relic gravitino
density $\Omega_G$ receives an additional contribution $\Omega^{\text{TP}}_G$ that depends basically linearly on the reheating temperature $T_R$ after inflation [38], [41]–[43]. In turn, the dark matter constraint, $\Omega^{\text{TP}}_G + \Omega^{\text{NTP}}_G \leq \Omega_{\text{DM}}$, can be translated into a conservative upper limit [33]:

$$T_R \leq \frac{2.37 \times 10^9}{c^2} \text{ GeV} \left( \frac{\Omega_{\text{DM}} h^2 \Omega^{\text{NTP}} G}{0.1} \right) \left( \frac{\tau_{l_1}}{10^4 \text{ s}} \right)^{1/2} \left( \frac{m_{l_1}}{100 \text{ GeV}} \right)^{1/2} \equiv T^{\text{max}}_R, \quad (4.7)$$

which depends on the ratio of the gluino mass $m_{\tilde{g}}$ and the $\tilde{l}_1$ NLSP mass, $c \equiv m_{\tilde{g}}/m_{l_1} > 1$, at the weak scale. In figure 8, contours of $T^{\text{max}}_R$ obtained with $\Omega_{\text{DM}} h^2 \leq 0.126$ and $\Omega^{\text{NTP}}_G$ given by the yield (4.2) are shown for the limiting case $c = 1$ (dashed lines) and for the case $c = 7$ (dotted lines), which is typical for universal soft SUSY breaking parameters at the scale of grand unification. Thus, also with the $^9\text{Be}$ constraint and with $\Omega^{\text{NTP}}_G$ included, $c = 1 \ (c = 7)$ is found to be associated with a $T^{\text{max}}_R$ value of $3 \times 10^9$ GeV ($10^8$ GeV) in the cosmologically favored region. Indeed, the $T_R$ constraints for $c = 7$ are consistent with earlier findings within the CMSSM and other constrained scenarios [19, 26, 72, 27, 28]. Since the $^9\text{Be}$ constraint imposes $\tau_{l_1} \lesssim 6 \times 10^3 \text{ s}$, as discussed above, associated upper limits on the mass ratio $c$ for a given lower limit on the reheating temperature—such as $T_R > 10^9$ GeV required by successful thermal leptogenesis with hierarchical heavy right-handed Majorana neutrinos—can be inferred from figure 3 of [33].

Let us conclude this section by considering prospects at future colliders. If the gravitino LSP scenario with a not too heavy charged slepton NLSP is realized in nature, the production and analysis of the (quasi-) stable charged sleptons will be a realistic option. Thereby, collider measurements of $m_{\tilde{g}}$ will become available [45, 46]. Moreover, with an experimental reconstruction of at least some part of the SUSY model, one will be able to calculate $Y^{\text{dec}}_{\tilde{l}_1}$ reliably for a standard thermal history with $T_R$ above the decoupling temperature $T_I$ of the slepton NLSP. Confronting the obtained $Y^{\text{dec}}_{\tilde{l}_1}$ with the CBBN constraints from $^9\text{Be}$ and $^6\text{Li}$ shown in figure 5 can then provide an upper limit on the lifetime $\tau_{l_1}$ of the long-lived slepton. Assuming the gravitino LSP scenario, this $\tau_{l_1}$ constraint together with the measured $m_{\tilde{l}_1}$ will imply an upper limit on the gravitino mass $m_{\tilde{G}}$, as can be seen in figures 6 and 7. In addition, if $\tau_{l_1}$ can be measured, for example, by analyzing $\tilde{l}_1$ decays in a collider detector [45, 48] or in some additional stopper material [73, 74, 71, 49], one should be able to compare the experimentally determined combination $(\tau_{l_1}, m_{l_1})$ with the CBBN constraints shown in figure 7 (without any assumption on the gravitino LSP scenario). While a finding of $(\tau_{l_1}, m_{l_1})$ in the region disfavored by CBBN could point to a non-standard cosmological history with late-time entropy production [69, 19, 20] or to a low reheating temperature [23], it would be most remarkable to find $(\tau_{l_1}, m_{l_1})$ in close vicinity of the CBBN constraints. Notwithstanding a rather large number of ‘ifs’ compounded in the previous sentences, the collider measurements could provide in this way an independent test for a hypothesis of the CBBN origin of $^6\text{Li}$ and $^9\text{Be}$ at lowest metallicities. Complementary to that it will be exciting to see new analyses of $^9\text{Be}$ and $^6\text{Li}$ data from future astrophysical observations.
5. Conclusions

From observations of beryllium in Population II halo stars at very low metallicities, we have extracted a nominal upper limit on primordial beryllium of $^{9}$Be/H $\leq 2.1 \times 10^{-13}$. This limit allows one to set interesting constraints on models in which the primordial $A = 8$ divide is bridged by catalytic effects. Considering the primordial catalysis of $^{9}$Be via bound-state effects of a negatively charged massive relic $X^{-}$ [31], we have derived $\tau_{X^{-}}$-dependent upper limits on the $X^{-}$ yield prior to decay, $Y_{X^{-}}^{\text{dec}}$. For a typical relic abundance $Y_{X^{-}}^{\text{dec}} \gtrsim 3 \times 10^{-4}$ (10$^{-4}$), we find that this $^{9}$Be limit translates into an upper limit on the $X^{-}$ lifetime of $\tau_{X^{-}} \lesssim 6 \times 10^{4}$ s (10$^{4}$ s), which is quite comparable with the $\tau_{X^{-}}$-limit inferred from the primordial catalysis of $^{6}$Li. Moreover, in the region where CBBN is efficient, we confirm that the ratio of the synthesized elements of $^{9}$Be/$^{6}$Li lies in the range $10^{-3}$–$10^{-2}$ [31], which provides perhaps the most model-independent prediction in the whole CBBN paradigm.

We have clarified that the presence of $(pX^{-})$ bound states cannot relax the $Y_{X^{-}}^{\text{dec}}$ limits at long lifetimes $\tau_{X^{-}}$ in any substantial way. Indeed, we have shown explicitly that late-time effects of $(pX^{-})$ bound states can affect the lithium and beryllium abundances synthesized at $T \simeq 8$ keV by not more than 10%. Any substantial formation of $(pX^{-})$ at $T \simeq 0.7$ keV is immediately intercepted by the very efficient charge exchange reaction of $(pX^{-})$ with $^{4}$He. This comes as no surprise given the large size of the $(pX^{-})$ system $\sim$30 fm and the fact that the proton deconfinement probability approaches unity already for a $^{4}$He–$X^{-}$ distance of $\sim$95 fm. In particular, we find that the fractional density of protons in bound states does not exceed the level of $\sim 10^{-8}$ for $Y_{X^{-}} \lesssim Y_{4\text{He}}$. Correspondingly, even with a $(pX^{-})$-induced $^{6}$Li-destruction cross section as large as the unitarity limit, at most a few percent of the synthesized $^{6}$Li could be destroyed. By the same argument, the $^{9}$Be yield also remains unaffected by late-time catalysis. Thus, we find that the possibility of allowed islands in the parameter region with typical $Y_{X^{-}}^{\text{dec}}$ and large $\tau_{X^{-}}$—which was advocated in [25]—does not exist.

Applying the $\tau_{X^{-}}$-dependent upper limits on $Y_{X^{-}}^{\text{dec}}$ derived from the primordial catalysis of $^{9}$Be, we have analyzed the new $^{9}$Be constraint in SUSY models in which the gravitino is the LSP and a long-lived charged slepton the NLSP, $\tilde{l}_{1} = X^{-}$. For typical values of the slepton NLSP yield after decoupling, the $^{9}$Be constraint obtained in this paper is found in close vicinity to the constraint from the primordial catalysis of $^{6}$Li. Accordingly, the implications of the $^{9}$Be constraint for SUSY models do not differ much from the case of $^{6}$Li. The important advantage of $^{9}$Be-derived constraints is due to the fact that $^{9}$Be is firmly detected in a significant number of stars at low metallicities, while the status of $^{6}$Li observations remains somewhat questionable. Another great virtue of $^{9}$Be-derived constraints is due to the expectations that an $^{9}$Be abundance would remain less affected compared to the one of $^{6}$Li by any stellar mechanism that might have caused the depletion in $^{7}$Li. Therefore, one would not expect any serious depletion factors between the (hypothetical) primordial fraction of $^{9}$Be and the actual observationally determined abundances of $^{9}$Be.

One could question the calculational status of the CBBN chain (1.2) leading to $^{9}$Be. The rates for the first two reactions, resulting in $(^{4}\text{He}X^{-})$ and $(^{8}\text{Be}X^{-})$, are determined by electromagnetic interactions and thus are not associated with large nuclear uncertainties. The final step on the way to $^{9}$Be is the neutron capture by the $(^{8}\text{Be}X^{-})$ bound state.
It is dominated by a resonant transition and the associated rate is believed to have a factor of a few uncertainty [31]. However, given the wealth of existing experimental data on the $^9\text{Be}$ resonances, and rapid progress in nuclear calculations of few-nucleon systems, one could hope that reasonably precise calculations of the catalytic rates may become available.

The main shortcoming of our analysis is that only the catalytic effects below 10 keV are taken into account. In a generic framework of a hypothetical $X^-$ particle, this is fully justified. However, in the specific SUSY model with gravitino LSP/slepton NLSP, one could go one step further by combining the catalytic effects with the energy injection effects. Fortunately for us, the question of $^9\text{Be}$ synthesis is somewhat decoupled from the question of energy injection. The connection is mainly due to the modification of the deuterium abundance: for example, hadronic energy input at $T \sim 30$ keV would lead to a larger deuterium abundance, which in turn would enhance the neutron abundance, resulting in a larger abundance of $^9\text{Be}$. Thus, the energy injection would lead to abundances of $^9\text{Be}$ that are somewhat larger than the ones that we determined by using the standard input for the deuterium abundance. Since the main idea of our paper is to derive a conservative limit on SUSY models from $^9\text{Be}$, we can stay on the conservative side and disregard this modification, which would make the $^9\text{Be}$ limit only stronger.

In comparison to other cosmological constraints on gravitino LSP scenarios with a long-lived charged slepton NLSP, we find that the $^9\text{Be}$ constraint (together with the one from $^6\text{Li}$) is the most relevant one in the collider-accessible region of slepton masses below 1 TeV. Indeed, if a SUSY scenario with a long-lived $\tilde{l}_1$ is realized in nature, one might be able to determine the combination $(\tau_{\tilde{l}_1}, m_{\tilde{l}_1})$ at collider experiments. Assuming a standard cosmological history, one might also ‘invert’ the collider data and infer $Y_{^6\text{Li}}^{\text{dec}}$. Independently of the assumption of the gravitino LSP, these quantities can then be confronted with constraints on the $(\tau_{\tilde{l}_1}, Y_{^6\text{Li}}^{\text{dec}})$ parameter space imposed by the primordial catalysis of $^6\text{Li}$ and $^9\text{Be}$. Thereby, the CBBN constraints can be considered as predictions that could be tested in upcoming high-energy experiments. It will be most remarkable if collider measurements point to a $(\tau_{\tilde{l}_1}, m_{\tilde{l}_1})$ combination in the vicinity of the CBBN constraints. Indeed, this could provide an experimental hint for the primordial catalysis being the origin of existing abundances of both $^9\text{Be}$ and $^6\text{Li}$ at lowest metallicities. However tenuous the BBN–LHC connection may seem at the moment, we expect a lot more clarity to be brought to this issue in the coming years.

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Appendix. CBBN reaction rates below 10 keV

In the following, we collect the key reaction rates, \(N_A\langle \sigma v \rangle\), used in the numerical solutions of the Boltzmann equations. They are given in units of cm\(^3\) s\(^{-1}\) mol\(^{-1}\) and \(T_9 = T/10^9\) K.

- **Recombination and photodissociation of \(X^-\):**
  \(\text{^4He} + X^- \rightarrow (\text{^4He}X^-) + \gamma : \quad 7900\ T_9^{-1/2},\)
  \((\text{^4He}X^-) + \gamma_{bg} \rightarrow \text{^4He} + X^- : \quad 1.85 \times 10^{10}\ T_9^{-2}\ \exp(-4.03/T_9),\)
  \(p + X^- \rightarrow (pX^-) + \gamma : \quad 3980\ T_9^{-1/2},\)
  \((pX^-) + \gamma_{bg} \rightarrow p + X^- : \quad 1.18 \times 10^9\ T_9^{-2}\ \exp(-0.29/T_9).\)

- **Charge exchange reactions:**
  \((pX^-) + \text{^4He} \rightarrow (\text{^4He}X^-) + p : \quad 3.9 \times 10^{10}\ T_9^{-1/2},\)
  \((pX^-) + \text{^6Li} \rightarrow (\text{^6Li}X^-) + p : \quad 6.45 \times 10^{10}\ T_9^{1/2},\)
  \((pX^-) + (\text{^6Li}X^-) \rightarrow (\text{^6Li}X^-_2) + p : \quad 3.37 \times 10^9\ T_9^{1/2}\ (1\ \text{TeV}/m_{X^-})^{1/2},\)
  \((pX^-) + (\text{^6Li}X^-_2) \rightarrow (\text{^6Li}X^-_3) + p : \quad 5.25 \times 10^8\ T_9^{1/2}\ (1\ \text{TeV}/m_{X^-})^{1/2}.\)

- **\text{^6Li} and \text{^9Be} catalysis (from [20] and [31], respectively):**
  \((\text{^4He}X^-) + D \rightarrow \text{^6Li} + X^- : \quad 2.37 \times 10^8\ (1 - 0.34\ T_9)\ T_9^{-2/3}\ \times\ \exp\left(-5.33\ T_9^{-1/3}\right),\)
  \(^{\text{^4He}} + (\text{^4He}X^-) \rightarrow (\text{^9Be}X^-) + \gamma : \quad 10^8\ T_9^{-3/2}\ \exp\left(-0.95\ T_9^{-1/3}\right)\ \times\ \exp\left(-1.02/T_9\right)\ +\ 0.66\ \exp\left(-1.32/T_9\right),\)
  \(^{\text{^9Be}X^-} + n \rightarrow \text{^9Be} + X^- : \quad 2 \times 10^9.\)

*Note added.* After the submission of this paper, a dedicated nuclear physics study of some CBBN reactions has appeared [75]. It supports the conclusion of this paper about the large rate for the charge exchange reactions that remove \((pX^-)\). At the same time, this work finds non-negligible shifts, O(100 keV), of the resonant energy levels employed in the \(^{\text{^9Be}}\) production chain. This may affect the overall efficiency of \(^{\text{^9Be}}\) production, and further investigations of the nuclear rates are needed.

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