An Improved Rotor Position Estimation Method for PMSM Based on Pulsating Square-Wave-Type Voltage Injection

Cheng Lin¹, ², Jiaxu Sun¹
¹National Engineering Laboratory for Electric Vehicles, Beijing Institute of Technology, Beijing, 100081, China
²Collaborative Innovation Center of Electric Vehicles in Beijing, Beijing Institute of Technology, Beijing, 100081, China
ethan_sjx@126.com

Abstract. This paper presents a pulsating high-frequency square-wave-type voltage signal injection based sensorless rotor position estimation method for permanent magnet synchronous motors (PMSM). In order to increase the injection frequency so as to enhance the control system's bandwidth, square-wave-type voltage is injected instead of sinusoidal voltage. To avoid disturbances, the sampling frequency is set as equal to the PWM switching frequency so that the injection frequency can be half of the switching frequency. Due to the characteristics of the square-wave-type voltage, the signal processing is further simplified without the use of filters, which can eliminate the time delay and increase the system's computational efficiency and feasibility. As a result, the rotor position estimation performance is improved and the sensorless control system's dynamics is enhanced. Results of simulation verify the effectiveness of the proposed method and its applicability to PMSM sensorless control systems.

1. Introduction

In recent years, "electrification" has become one of the main development trends in the auto industry. Motors are important energy conversion units for both electric vehicles and hybrid vehicles. Permanent magnet synchronous motors (PMSM), due to their various advantages, have gained more applications in the field of electric vehicles [1]. Classical vector control methods are generally applied, which require the rotor position and rotating speed as feedback information. Thus position sensors are used, but they can cause problems such as increasing the cost and size, reducing system's reliability and so on [2, 3]. To eliminate position sensors and related cable connections, sensorless rotor position estimation methods develop, which can be classified into three categories: methods based on back electromotive force [4-7], methods based on high-frequency signal injection [7-11] and methods based on artificial intelligence [12].

Methods based on high-frequency signal injection are commonly used for rotor position estimation of PMSM operating in low speed (including standstill) state. Signals injected mainly include rotating high-frequency voltage and pulsating high-frequency voltage, and the latter has a relatively wider application range [13, 14]. Many efforts have been made within this area. Wang et al. combined the pulsating high-frequency voltage signal injection method and a sliding mode observer based method to realize PMSM's sensorless wide-speed-range operation [8]. Liu et al. analyzed the influences of several parameters of the PMSM control system on the estimation accuracy of pulsating high-frequency voltage injection method, and summarized the influence rules [9]. Yoon et al. proposed a
high-frequency square-wave-type voltage injection method so as to increase the frequency of the injected signal, as a result of which the rotor position estimation performance was improved and the system's bandwidth was increased [10]. Park et al. proposed a simple sensorless algorithm based on high-frequency voltage injection method, which could omit the use of filters to eliminate signal delay and simplify signal processing [11]. However, to ensure rotor position estimation accuracy and PMSM's stable operation, the injected signal's frequency cannot be too high [14]. Also the filterless signal processing can still be further simplified.

In this paper, an improved sensorless rotor position estimation method for PMSM based on pulsating high-frequency square-wave-type voltage signal injection is proposed. Firstly, the principle of the proposed estimation method is explained. Secondly, the simplified signal processing method is presented. Lastly, the performance of the proposed method is verified by simulation with its results displayed and analyzed.

2. Proposed sensorless rotor position estimation method

In this section, the induced stator current under the pulsating high-frequency square-wave-type voltage excitation is deduced to explain the principle of the proposed method. Moreover, the simplified signal processing method that omits the use of filters is presented.

2.1. Induced current under the pulsating high-frequency square-wave-type voltage excitation

For interior permanent magnet synchronous motors (IPMSM), the voltage model in the rotor reference $d-q$ frame can be expressed as

$$
\begin{align*}
    \frac{du_d}{dt} &= R_s i_d + L_d \frac{di_d}{dt} - \omega_L i_q \\
    \frac{du_q}{dt} &= R_s i_q + L_q \frac{di_q}{dt} + \omega_L (L_d i_d + \psi_f)
\end{align*}
$$

(1)

where, $u_d$ and $u_q$ are the stator voltage components in the $d$- and $q$-axis respectively, $i_d$ and $i_q$ are the stator current components in the $d$- and $q$-axis respectively, $R_s$ is the stator resistance, $L_d$ and $L_q$ are the stator inductances in the $d$- and $q$-axis respectively, $\omega_L$ is the rotor electrical speed, and $\psi_f$ is the permanent magnet flux linkage.

When a high-frequency voltage signal is injected, its frequency can be much higher than the fundamental frequency, so the relationship between the induced high-frequency current and the injected voltage in the real rotor reference $d-q$ frame can be described as

$$
\begin{align*}
    \frac{du_{in}}{dt} &= L_d \frac{di_{in}}{dt} \\
    \frac{du_{in}}{dt} &= L_q \frac{di_{in}}{dt}
\end{align*}
$$

(2)

where, the subscript "in" represents injected quantities.

In order to accurately estimate the rotor position, the relationship between the real rotor reference $d-q$ frame and the estimated rotor reference $\hat{d}-\hat{q}$ frame is built as shown in figure 1. Define the rotor position estimation error $\hat{\theta}$ as

$$
\hat{\theta} = \theta - \hat{\theta}
$$

(3)

where, $\theta$ is the real rotor position, and $\hat{\theta}$ is the estimated rotor position.
Figure 1. The relationship between the real rotor reference frame and the estimated rotor reference frame.

Using the coordinate transformation, equation (2) can be transformed into the estimated rotor reference $\hat{d} - \hat{q}$ frame as

$$
\begin{bmatrix}
\frac{d}{dt} \hat{i}_d^{\text{in}} \\
\frac{d}{dt} \hat{i}_q^{\text{in}}
\end{bmatrix}
= \begin{bmatrix}
\cos \hat{\theta}_e & -\sin \hat{\theta}_e \\
\sin \hat{\theta}_e & \cos \hat{\theta}_e
\end{bmatrix}
\begin{bmatrix}
\frac{1}{L_d} & 0 \\
0 & \frac{1}{L_q}
\end{bmatrix}
\begin{bmatrix}
\cos \hat{\theta}_e & \sin \hat{\theta}_e \\
-\sin \hat{\theta}_e & \cos \hat{\theta}_e
\end{bmatrix}
\begin{bmatrix}
\hat{u}_{d\text{in}} \\
\hat{u}_{q\text{in}}
\end{bmatrix}
$$

(4)

where, the superscript "\(^\ast\)" represents quantities in the estimated rotor reference $\hat{d} - \hat{q}$ frame.

The proposed method only injects the square-wave-type voltage signal into the $\hat{d}$-axis, which can be described as

$$
\begin{cases}
\hat{u}_{d\text{in}} = V_{\text{in}} (-1)^k \\
\hat{u}_{q\text{in}} = 0
\end{cases}
$$

(5)

where, $V_{\text{in}}$ is the amplitude of the injected voltage signal ($V_{\text{in}} > 0$), $k$ is the system's discrete sampling instant. Hence the induced high-frequency current can be deduced as

$$
\begin{bmatrix}
\frac{d}{dt} \hat{i}_d^{\text{in}} \\
\frac{d}{dt} \hat{i}_q^{\text{in}}
\end{bmatrix}
= V_{\text{in}} (-1)^k 
\begin{bmatrix}
\cos^2 \hat{\theta}_e + \sin^2 \hat{\theta}_e \\
\frac{L_d}{L_d + L_q} \\
\frac{L_q}{(L_q - L_d) \sin 2 \hat{\theta}_e} \\
\frac{2L_dL_q}{2L_dL_q}
\end{bmatrix}
$$

(6)

The discrete form of equation (6) can be written as

$$
\begin{bmatrix}
\Delta \hat{i}_d^{\text{in}} \\
\Delta \hat{i}_q^{\text{in}}
\end{bmatrix}
= V_{\text{in}} (-1)^k \Delta T 
\begin{bmatrix}
\cos^2 \hat{\theta}_e + \sin^2 \hat{\theta}_e \\
\frac{L_d}{L_d + L_q} \\
\frac{L_q}{(L_q - L_d) \sin 2 \hat{\theta}_e} \\
\frac{2L_dL_q}{2L_dL_q}
\end{bmatrix}
$$

(7)

where, "\(\Delta\)" represents the difference between the present value and the previous value of sampling instants, $\Delta T$ is the signal sampling period.

Define $f(\hat{\theta}_e)$ as

$$
f(\hat{\theta}_e) = \text{sign}(\hat{u}_{d\text{in}}) \Delta \hat{i}_q^{\text{in}} = V_{\text{in}} \Delta T \frac{(L_q - L_d) \sin 2 \hat{\theta}_e}{2L_dL_q} = \frac{V_{\text{in}} \Delta T (L_q - L_d)}{L_dL_q} \hat{\theta}_e
$$

(8)
Apparently, when \( f(\hat{\theta}_e) \) is adjusted to zero, the value of the rotor position estimation error \( \hat{\theta}_e \) will become zero as well, meaning that the estimated rotor position has converged to the real rotor position. Using a position observer with \( f(\hat{\theta}_e) \) as input, the rotor position information can be obtained.

2.2. Simplified signal processing method

Due to the voltage signal injected, the measured stator current consists of a fundamental component \( \hat{i}_{fq} \) and a high-frequency component \( \hat{i}_{qin} \) (taking the current in \( \hat{q} \) -axis as an example) as

\[
\hat{i}_q = \hat{i}_{fq} + \hat{i}_{qin}
\]  

Usually filters are used to realize the separation of current signals, but they can cause the signal time delay and reduce the estimation accuracy. Owing to the characteristics of the injected square-wave-type voltage, the current signal processing can be realized through arithmetic calculation as follows without the use of any filters.

Previous studies measured the stator currents twice every pulse-width modulation (PWM) period and set the injection frequency equal to the switching frequency. However, when the injection frequency is above half of the switching frequency, disturbances will appear [14]. In this paper, the currents sampling frequency is set the same as the switching frequency, meaning that the stator currents are measured once every PWM period, and the injection frequency can be controlled as half of the switching frequency. Figure 2 shows the timing sequence of several signals.

![Figure 2. Timing sequence of several signals.](image)

Taking the current in \( \hat{q} \) -axis as an example, it can be written as

\[
\begin{align*}
\hat{i}_q[k-1] &= \hat{i}_{fq}[k-1] + \hat{i}_{qin}[k-1] \\
\hat{i}_q[k] &= \hat{i}_{fq}[k] + \hat{i}_{qin}[k]
\end{align*}
\]  

(10)

Between two sampling instants, the fundamental current components can be approximately considered as unchanged, which can be expressed as
As shown in figure 2, there is a relationship that can be described as

\[
\begin{align*}
\hat{i}_{qf}^{qf \text{ in}}[k-1] &= \hat{i}_{qf}^{qf \text{ in}}[k] \\
\hat{i}_{qf}^{qf \text{ in}}[k] &= \hat{i}_{qf}^{qf \text{ in}}[k-1] - \frac{\Delta \hat{i}_{qf}^{qf \text{ in}}[k]}{2} \\
\hat{i}_{qf}^{qf \text{ in}}[k] &= \hat{i}_{qf}^{qf \text{ in}}[k] + \frac{\Delta \hat{i}_{qf}^{qf \text{ in}}[k]}{2}
\end{align*}
\]

From equation (9) to equation (11), \(\hat{i}_{qf}^{qf \text{ in}}\) and \(\Delta \hat{i}_{qf}^{qf \text{ in}}\) at sampling instant \(k\) can be deduced as

\[
\begin{align*}
\hat{i}_{qf}^{qf \text{ in}}[k] &= \frac{1}{4} (\hat{i}_{qf}^{qf \text{ in}}[k-1] + \hat{i}_{qf}^{qf \text{ in}}[k]) \\
\Delta \hat{i}_{qf}^{qf \text{ in}}[k] &= \hat{i}_{qf}^{qf \text{ in}}[k] - \hat{i}_{qf}^{qf \text{ in}}[k-1]
\end{align*}
\]

Here division operation is omitted when calculating \(\Delta \hat{i}_{qf}^{qf \text{ in}}[k]\). Hence the separation of the current signals is realized through simple algebraic calculation, which can eliminate the signal time delay and increase the system's computational efficiency.

Figure 3 presents the block diagram of the improved rotor position estimation method.

3. Results of simulation and analyses

In this section, the performance of the proposed method is tested based on MATLAB/Simulink. The proposed method can be applied to any type of PMSM with saliency in its rotor impedance. Here, the simulation test was conducted based on the nominal parameters of an IPMSM as shown in table 1.

Table 1. IPMSM nominal parameters.

| Quantity          | Symbol | Value | Unit |
|-------------------|--------|-------|------|
| Stator resistance | \(R_s\) | 0.33  | Ω    |
| Stator inductances| \(L_d\) | 5.2   | mH   |
|                   | \(L_q\) | 17.4  | mH   |
| Flux linkage      | \(\psi_f\) | 0.646 | Wb   |
The PWM switching frequency was set as 10 kHz, the sampling frequency as 10 kHz and the injected voltage frequency as 5 kHz. The amplitude of the square-wave-type voltage was 15 V and the PI gains needed were tuned properly.

Figure 4 shows the rotor position estimation performance on a constant speed operation at 200 r/min. The real rotor position $\theta_e$, the estimated rotor position $\hat{\theta}_e$ and the rotor position estimation error $\tilde{\theta}_e$ are presented. As shown in figure 4, the estimated rotor position has virtually no noise and the rotor position estimation error is negligible. It can be concluded that the proposed method is able to estimate the rotor position with no time delay.

![Figure 4. Simulation results of rotor position estimation.](image-url)
Figure 5 shows the speed estimation performance on a constant speed operation at 200 r/min. The real speed $n$, the estimated speed $\hat{n}$ and the speed estimation error $\dot{n}$ are displayed. As shown in figure 5, the speed estimation error is slight, verifying that the proposed method can be applied to PMSM sensorless control systems.

4. Conclusion
This paper proposed an improved rotor position estimation method for PMSM. The frequency of the injected square-wave-type voltage signal is controlled as half of the PWM switching frequency so that the system's bandwidth is enhanced and disturbances are avoided. The signal processing is simplified to not only omit the time delay but increase the computational efficiency. As a result, the rotor position estimation performance is improved and the sensorless control system can be more easily implemented. Results of simulation have verified the effectiveness of the proposed method and its applicability to PMSM sensorless control systems.

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