On the transmission of light through a single rectangular hole

F. J. García-Vidal,1 Esteban Moreno,1 J. A. Porto,1 and L. Martín-Moreno2

1Departamento de Física Teórica de la Materia Condensada, Universidad Autónoma de Madrid, E-28049 Madrid, Spain
2Departamento de Física de la Materia Condensada, ICMA-CSI C, Universidad de Zaragoza, E-50009 Zaragoza, Spain

In this Letter we show that a single rectangular hole exhibits transmission resonances that appear near the cutoff wavelength of the hole waveguide. For light polarized with the electric field pointing along the short axis, it is shown that the normalized-to-area transmittance at resonance is proportional to the ratio between the long and short sides, and to the dielectric constant inside the hole. Importantly, this resonant transmission process is accompanied by a huge enhancement of the electric field at both entrance and exit interfaces of the hole. These findings open the possibility of using rectangular holes for spectroscopic purposes or for exploring non-linear effects.

PACS numbers: 42.25.Bs, 42.79.Ag, 78.66.Bz, 41.20.Jb

Since the pioneering work of Ebbesen et al.1 reporting extraordinary optical transmission (EOT) through two-dimensional (2D) hole arrays perforated in optically thick silver films, the study of the transmission properties of subwavelength apertures (holes or slits) has become a very active area of research in electromagnetism. In the last few years, EOT has been found in other frequency regimes, as THz2,3 and microwave4. Nowadays there is a wide consensus that EOT in hole arrays is linked to the excitation of the surface electromagnetic (EM) modes5,6 that decorate the surfaces of structured metals.

Very recently, different experiments have focused on the influence of hole shape on the transmission properties of both 2D hole arrays7,8,9 and single subwavelength holes10 in the optical regime. All these studies show strong polarization dependencies in rectangular holes and also that rectangular holes exhibit higher transmittance than square or circular holes with the same area. Interestingly, it was also found that single rectangular holes can support transmission resonances, even in the sub-wavelength regime.

In this Letter we present the, up to our knowledge, first theoretical study on the dependence on hole shape of the transmittance through a single hole. We find that, as a difference with circular holes11, single rectangular holes can exhibit strong transmission resonances in all frequency ranges. One of these resonances appears close to cutoff, with a peak transmittance controlled simply by the ratio between the long and short sides of the rectangle. Additionally, we show that the presence of a material filling the hole greatly boosts the transmittance. Associated to these transmission resonances, there is a very strong increase of the electric field at the hole.

Figure 1 shows schematically the system under study. A rectangular hole of sides $a_x$ and $a_y$ perforated on a metallic film of thickness $h$. The structure is illuminated by a $p$-polarized plane wave with the magnetic field pointing along the $y$-direction.

![Diagram of a single rectangular hole of sides $a_x$ and $a_y$ perforated on a metal film of thickness $h$. The structure is illuminated by a $p$-polarized plane wave with the magnetic field pointing along the $y$-direction.](image)

Figure 1: Diagram of a single rectangular hole of sides $a_x$ and $a_y$ perforated on a metal film of thickness $h$. The structure is illuminated by a $p$-polarized plane wave with the magnetic field pointing along the $y$-direction.
tailed account of this method that was developed to treat an arbitrary number of indentations can be found in [14]. In this method, the EM fields in both reflection (I) and transmission (III) vacuum regions are expressed in terms of EM eigenmodes $|\vec{k}\sigma>$, characterized by the in-plane component of the wavevector $\vec{k}$, and the polarization $\sigma$. Inside the hole, the EM field is expanded in terms of all EM waveguide eigenmodes. After matching appropriately the EM fields at the two interfaces of the system ($z = 0$ and $z = h$), the formalism provides the full EM field in all space in terms of the projection onto waveguide eigenmodes of the electric field at both hole entrance and exit interfaces. In all calculations presented in this letter (normal incidence illumination and $a_x, a_y < \lambda$), we have checked that considering just the first TE eigenmode ($|TE>$) is enough to obtain very accurate results for the transmittance so, for simplicity, we present our formalism just for this case. Then, in terms of the modal amplitudes $E$ and $E'$, the electric field bivector $\vec{E} = (E_x, E_y)^T$ ($T$ standing for transmission) at the hole entrance and exit can be written as $|\vec{E}(z = 0) = E|TE>$ and $|\vec{E}(z = h) = -E'|TE>$, respectively. Here we have used Dirac’s notation, with $<\vec{r}|TE> = (1, 0)^T \sin(\pi(y/a_y + 1/2))/\sqrt{N}, N = a_x/a_y/2$ being a normalization factor. The equations that $E$ and $E'$ must satisfy are:

\begin{align}
(G - \Sigma) E - G_V E' &= I_0, \\
-G_V E + (G - \Sigma) E' &= 0.
\end{align}

where $I_0 = 2i <k_0p|TE>$ is the external illumination term, i.e., the overlap integral between the incident plane wave $|k_0p>$ and mode $|TE>$. For the case of normal incidence, and normalizing the incident EM field such that the incoming energy flux over the hole area is unity, a simple calculation gives $I_0 = 4\sqrt{2}/\pi$. $\Sigma$ and $G_V$ are two magnitudes that only depend on the characteristics of TE mode inside the hole: $\Sigma = Y_{TE}/\tan(qy)$ and $G_V = Y_{TE}/\sin(qy)$; $q_y = \sqrt{k_x^2 - (\pi/a_y)^2}$ is the propagation constant of the fundamental TE mode, $Y_{TE} = q_y/k_o$ its admittance and $k_o$ is $2\pi/\lambda$.

The self-illumination of the hole, via vacuum modes, is controlled by $G = i/(2\pi)^2 \sum_{\sigma} \int d\vec{k}Y_{\vec{k}\sigma}|<TE|\vec{k}\sigma>|^2$ (with $Y_{\vec{k}\sigma}$ being the admittance of the EM vacuum mode $|\vec{k}\sigma>$ [15]). For the case of rectangular holes,

\begin{align}
G &= \frac{a_xa_y}{8\pi^2k_o} \int_{-\infty}^{+\infty} dk_x \int_{-\infty}^{+\infty} dk_y \frac{k_x^2 - k_y^2}{k_o^2 - k_x^2 k_y^2} \sin^2\left(\frac{k_x a_x}{2}\right) \\
&\quad \times \left[\sin(k_y a_y + \pi) + \sin(k_y a_y - \pi)\right]^2
\end{align}

where $k^2 = k_x^2 + k_y^2$. Notice that, in our formulation, Re($G$) comes from the coupling to evanescent modes in vacuum and Im($G$) from the radiative modes.

After obtaining $E$ and $E'$ from (1), the normalized-to-area transmittance ($T$) is calculated as:

\begin{equation}
T = \frac{G_V}{Y_{k_{0p}}} \text{Im}[E^*E']
\end{equation}

Figures 2 and 3 illustrate the dependence of $T$ with the two geometrical parameters involved (ratio $a_y/a_x$ and thickness $h$). In both cases we consider normal incidence. As previously said, all results are scalable, and we have chosen as unit length the cutoff wavelength of the TE waveguide inside the hole, $\lambda_c = 2a_y$.

![Figure 2](image_url)

**FIG. 2:** Normalized-to-area transmittance ($T$) versus wavelength (in units of the cutoff wavelength, $\lambda_c$), for a normal incident plane wave impinging at a rectangular hole, for different ratios $a_y/a_x$. Metal thickness is $h = a_y/3$. For comparison, in the inset we plot $T$ versus wavelength for a single square (black line) and circular (red line) holes.

Figure 2 renders $T$ for the case $h = a_y/3$, for different values of $a_y/a_x$. As clearly seen in this figure, a transmission peak develops at approximately $\lambda_c$, with increasing maximum transmittance and decreasing linewidth as $a_y/a_x$ increases. In the case of square or circular holes there is also a resonance close to cutoff, but a very faint one (see inset of Fig.2). In these last two cases, bellow cutoff the normalized-to-area maximum transmittance is of order of $1$, i.e., approximately the amount of light that is directly impinging at the hole. In all cases, above cutoff $T$ decreases strongly with $\lambda$, both due to the fact that the fields inside the hole are evanescent and that, in the extreme subwavelength regime, an incident wave couples very poorly to the hole [15].

The dependence of $T(\lambda)$ with metal thickness is shown in Fig. 3, for a rectangular hole with $a_y/a_x = 10$. Apart from the previously discussed transmission peak located at $\lambda \approx \lambda_c$, which spectral position essentially remains independent of $h$, a series of transmission resonances emerge as the depth of the hole is increased. As we will show later on, these are Fabry-Perot resonances in
rectangular holes, similar to the ones recently found in sub-wavelength 1D slits [17, 18, 19].

The resonant characteristics of the transmittance through rectangular holes, and their dependence on geometrical parameters can be worked out analytically from set of equations (1). For the case we are analyzing (a symmetric structure with respect to the plane \( z = h/2 \)), maximum transmission appears when the field intensities at the entrance and exit sides of the aperture are equal, i.e. \(|E| = |E'|\). From (1), this occurs when \(|G - \Sigma| = |G_V|\), condition that after some straightforward algebra implies

\[
2 \text{Re}(G) = \frac{|G|^2 - Y_{TE}^2}{Y_{TE}} \tan(q_z h) \tag{4}
\]

There are several wavelengths at which this transcendental equation is satisfied, providing the exact spectral location of the transmission peaks (\(\lambda_{res}\)) found in Figs. 2 and 3. Ignoring the shift in the spectral dependence due to the EM coupling to vacuum modes (this is, setting \( G \to 0 \), which is the appropriate limit for \( a_x, a_y \ll \lambda \)), Eq. (4) transforms into \( Y_{TE} \tan(q_z h) = 0 \), which is the usual Fabry-Perot condition for the existence of a standing wave inside the hole. Note that this last equation allows the solution \( q_z = 0 \), so a transmission peak located at around the cutoff wavelength is expected, irrespective of the geometrical parameters \( a_y/a_x \) (see Fig.2) and \( h \) (see Fig.3). In general, Eq. (4) predicts the shifts in the Fabry-Perot resonant wavelengths due to the coupling to both radiative and evanescent vacuum modes.

Using the resonance condition [Eq.(4)], from Eqs.(1) and (3) and after some straightforward algebra, we obtain that the normal-incidence \( T \) at resonance, \( T_{res} \) is given by:

\[
T_{res} = \frac{|I_0|^2}{4 \text{Im}(G)} \tag{5}
\]

A very good simple approximation to \( T_{res} \) can be obtained realizing that, in the extreme subwavelength limit (\( a_x, a_y \ll \lambda \)), Eq. (2) gives \( \text{Im}(G) \approx 32 a_x a_y/(3\pi\lambda^2) \). We have checked that this expression holds even for \( a_y \approx \lambda/2 \) therefore, for \( \lambda > 2a_y \) we find

\[
T_{res} \approx \frac{3 \lambda^2}{4\pi a_x a_y} \tag{6}
\]

Recalling that \( T \) is the normalized-to-area transmissivity, this expression implies that the total amount of light emerging from a rectangular hole is, at least for the resonance appearing close to cutoff, independent of the length of the short side! Although derived for rectangular holes, Eq. (6) seems to be more general as the same expression was found for circular holes [20], with the term \( a_x a_y \) replaced by the area of the circular hole. The important point in rectangular holes is that, for the polarization chosen, the transmittance peak appearing at cutoff depends only on the long side (\( \lambda_{res} = 2a_y \)), resulting in a transmittance \( T_{res} \approx (3/\pi) a_y/a_x \) close to cutoff. This is the main result of this Letter, as it predicts a huge transmission enhancement in a single rectangular hole with large aspect ratio.

FIG. 4: \( T \) for a normal incident plane wave versus wavelength for a rectangular hole with \( a_y/a_x = 10 \) and different values of \( \epsilon \) inside the hole. Metal thickness is \( h = a_y/3 \). Dashed and dotted lines show the behavior of Eqs.(5) and (6), respectively. Orange curve shows \( T \) versus \( \lambda \) in the limit \( h \to 0 \) for a rectangle with aspect ratio \( a_y/a_x = 10 \).

Additionally, even for a fixed aspect ratio \( a_y/a_x \), Eq. (6) gives us a clue for further enhancing the transmission: filling the hole with a material with dielectric constant...
\( \epsilon > 1 \), as this increases the cutoff wavelength. In fact, in the definition of quantities appearing in Eqs. (1) and (3), the only place in which \( \epsilon \) enters is in the propagation constant associated to mode \( |TE| > \) which now reads:

\[
q_z = \sqrt{\epsilon k_0^2 - (\pi/a_y)^2}.
\]

Therefore, the spectral position of resonances depend on \( q_z \) (and therefore on \( \epsilon \)) but the transmittance at resonance is still given by Eq. (5). As a result, filling the hole with a dielectric would redshift the Fabry-Perot transmission peak appearing close to cutoff to \( \approx 2\sqrt{\epsilon a_y} \) (for thick enough metal films, see below) and, more importantly, increase its transmittance. This is illustrated in Fig. 4, which renders the transmission spectra for rectangular holes of aspect ratio \( a_y/a_x = 10 \), in a metallic film of thickness \( h = a_y/3 \), for several values of \( \epsilon \). Note that this way of increasing the transmission through the hole by filling it with material with \( \epsilon > 1 \) can be also operative for the case of circular [21] or square holes. Remarkably, in rectangular holes this mechanism acts almost independently of the enhancement due to the aspect ratio, so \( T_{res} \) is proportional to both \( a_y/a_x \) and \( \epsilon \).

It is worth analyzing the limit \( h \to 0 \) of the transmissivity of single rectangular holes, the analogue of Bethe’s study for circular holes [10]. By taking this limit in the set of equations (1) and Eq.(3), we can obtain an expression for \( T \) valid for all wavelengths:

\[
T = \frac{|I_0|^2}{4|G|^2} \Im(G),
\]

which, as expected, is independent of \( \epsilon \). The behavior of this magnitude as a function of wavelength is rendered in Fig. 4 (orange curve) for the case \( a_y = 10a_x \). As metal thickness is decreased, the resonance moves from a location close to \( 2\sqrt{\epsilon a_y} \) to a much shorter wavelength, given by the resonant condition \( \text{Re}(G) = 0 \) (notice that Eq.(7) at resonance gives the same expression as Eq.(5) for this case). Interestingly, in the extreme subwavelength limit, as \( \text{Re}(G) \propto \lambda \) and \( \Im(G) \propto 1/\lambda^2 \), \( T \) decreases with \( \lambda \) as \( 1/\lambda^4 \), as in the case of circular holes [10].

It is also interesting to analyze the enhancement of the EM-fields associated to this resonant phenomenon. Naively, one would expect that the intensity of the E-field at the entrance and exit sides of the hole (\( |E|^2 \) and \( |E'|^2 \)) should be proportional to the transmittance. However, the direct evaluation of \( |E|^2 = |E'|^2 \) at the resonant condition given by Eq. (4) yields:

\[
|E|^2_\text{res} = |E'|^2_\text{res} = \frac{|I_0|^2}{4|\Im(G)|^2},
\]

leading to an enhancement of the intensity of the E-field (with respect to the incident one) that scales with \( \lambda_{res}^4/\epsilon(a_ya_x)^2 \), square of the enhancement in the transmittance (see Eq.(6)). This implies that in the process of resonant transmission, light is highly concentrated on the entrance and exit sides of the hole but only a small fraction of this light is finally transmitted.

These effects (huge enhancements of both transmission and field amplitude) should be readily observable in the microwave or THz frequency ranges. In these regimes, holes with very large aspect ratio can be manufactured and, furthermore, dielectric materials presenting large positive dielectric functions are also available.

We would like to end with some comments about the transferability of our results to the optical regime. Although our results agree with the experimental finding that transmittance resonances increase and become better defined with increased aspect ratio, they have only a semi-quantitative value in this regime, especially for short sides not much larger than twice the skin depth (i.e. \( \approx 50nm \)). Additionally, the influence of localized surface plasmon modes on the transmittance is not capture by our model. The role played by these modes on the transmittance is a point that deserves further theoretical investigation.

Financial support by the Spanish MCyT under contracts MAT2002-01534 and MAT2002-00139, and the EC under Projects No. No. FP6-2002-IST-1-507879 and FP6-NMP4-CT-2003-505699 is gratefully acknowledged.

[1] T.W. Ebbesen et al., Nature (London) 391, 667 (1998).
[2] J. Gomez-Rivas et al., Phys. Rev. B 68, 201306 (2003).
[3] D. Qu, D. Grischowsky, and W. Zhang, Opt. Lett. 29, 896 (2004).
[4] M. Beruete et al., Opt. Lett. 29, 2500 (2004).
[5] L. Martín-Moreno et al., Phys. Rev. Lett. 86, 1114 (2001).
[6] W.L. Barnes et al., Phys. Rev. Lett. 92, 107401 (2004).
[7] R. Gordon et al., Phys. Rev. Lett. 92, 037401 (2004).
[8] K.J. Klein Koerkamp et al., Phys. Rev. Lett. 92, 183901 (2004).
[9] H. Cao, and A. Nahata, Opt. Express 12, 3664 (2004).
[10] A. Degiron, H.J. Lezec, N. Yamamoto, and T.W. Ebbesen, Opt. Commun. 239, 61 (2004).
[11] A. Roberts, J. Opt. Soc. Am. A 4, 1970 (1987).
[12] J.R. Suckling et al., Phys. Rev. Lett. 92, 147401 (2004).
[13] L. Martín-Moreno and F.J. García-Vidal, Opt. Express 12, 3619 (2004).
[14] J. Bravo-Abad, F.J. García-Vidal, and L. Martín-Moreno, Phys. Rev. Lett. 93, 227401 (2004).
[15] Y_{ks} = k_s/k_0 and Y_{kp} = k_p/k_0 (for s- and p- polarization, respectively) where |k|^2 + k_{s}^2 = k_{p}^2.
[16] H. Bethe, Phys. Rev. 66, 163 (1944).
[17] J.A. Porto, F.J. García-Vidal, and J.B. Pendry, Phys. Rev. Lett. 83, 2845 (1999).
[18] F. Yang and J.R. Sambles, Phys. Rev. Lett. 89, 63901 (2002).
[19] J. Bravo-Abad, L. Martín-Moreno, and F.J. García-Vidal, Phys. Rev. E 69, 026601 (2004).
[20] Y. Leviatan, R.F. Harrington, and J.R. Mautz, IEEE Trans. Antenas Propag., AP-30, 1153 (1982).
[21] F.J. García de Abajo, Opt. Express 10, 1475 (2002).