Experimental realization of non-Abelian gauge potentials and topological Chern state in circuit system

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Abstract

Gauge fields, both Abelian and non-Abelian type, play an important role in modern physics. It prompts extensive studies of exotic physics on a variety of platforms. In this work, we present building blocks, consist of capacitors and inductors, for implementing non-Abelian gauge fields in circuit system. Based on these building blocks, we experimentally synthesize the Rashba-Dresselhaus spin-orbit interaction. Using operational amplifier, to break the time reversal symmetry, we further provide a scheme for designing the topological Chern circuit system. By measuring the chiral edge state of the Chern circuit, we experimentally confirm its topological nature. Our scheme offers a new route to study physics related to non-Abelian gauge field using circuit systems.

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Gauge fields, including the Abelian type that originates from classical electrodynamics, and the non-Abelian type introduced by Yang and Mills [1], are cornerstones in modern physics. They also play pivotal roles in topological states [2, 3]. Recently, gauge fields have been widely studied on various platforms [4–6], including ultracold atoms [6–14], and photonic [5, 15–23] systems, because these artificial systems can offer clean experimental platforms as well as highly controllable freedoms, enabling the realization of exotic physics that are not easily realized in high energy systems or solids. For a non-Abelian gauge field, it requires degenerate energies and noncommutative, matrix-form gauge potential. So far, except for a few works that have implemented non-Abelian gauge field in cold atoms [14, 24, 25] and photonic [22, 23, 26–30], it still remains a great challenge to easily implement it.

A number of recent articles using circuit systems to study various topological states have appeared, including the design of topological insulators [31–39], Weyl states [33, 40–42], non-Hermitian systems [43, 44], 4D TIs [45–48], topological Anderson insulator [49], and higher-order topological states [50–53]. Circuit systems have the unique advantage, that the properties of the circuit are independent of the shape of the system and only related to the topological geometry of the interconnections of the circuit devices. Since the wires between devices can be connected using bridging approach, and not constrained by distance and spatial dimension, circuit can be used to design connecting matrix with arbitrary nearest neighbors or in arbitrary dimensions. On the other hand, by engineering the number of nodes within the unit cell, the internal degrees of freedom of the system can be controlled. These properties make circuit systems highly suitable for studying the non-Abelian gauge field related physics. In this work, we first give the building blocks for the implementation of non-Abelian gauge field in circuits, and then present two circuit systems that are related to non-Abelian physics. The first example is the implementation of two dimensional Rashba-Dresselhaus spin-orbit interaction. The Rashba-Dresselhaus type splitting are obtained by measuring the resonance frequency of the circuit. In the second example, we propose for the first time a concrete design of a Chern circuit using non-Abelian gauge potential, and experimentally reveal its topological nature by measuring the edge states.

We start with the Yang-Mills Hamiltonian with non-Abelian gauge field

\[ H_{YM} = \frac{1}{2m} [(p_x + A_x)^2 + (p_y + A_y)^2], \]  

(1)
where \( m \) is the effective mass of carrier and the gauge filed component \( A_{x(y)} \) is a Hermitian matrix. We assume that the vector potential \( A_{x(y)} \) takes the form of \( A_x = (\alpha' + \beta')\sigma_x \) and \( A_y = (\alpha' - \beta')\sigma_y \), which do not commute with each other. Substituting \( A_{x(y)} \) into Hamiltonian 1 we get the standard Rashba-Dresselhaus spin-orbit interaction (SOI) \cite{54, 55}

\[
H = \frac{p_x^2}{2m} + \alpha(p_x\sigma_x + p_y\sigma_y) + \beta(p_x\sigma_x - p_y\sigma_y) + \text{const.},
\]  

(2)

where \( \sigma_x(y) \) are Pauli matrices acting on the pseudo spins, \( p_x = p_{x1} + p_{x2}, \alpha = \alpha'/2m \) and \( \beta = \beta'/2m \) are the Rashba and Dresselhaus SOI constants, respectively. In many literatures, Rashba SOI denotes \( k_x\sigma_y - k_y\sigma_x \), while Dresselhaus SOI denotes \( k_x\sigma_y + k_y\sigma_x \). They are equivalent to the notations used in this paper by a pseudo-spin rotation \( \sigma_x \rightarrow -\sigma_y \) and \( \sigma_y \rightarrow \sigma_x \).

Before presenting the scheme for implementing the non-Abelian gauge field in the form of the Rashba-Dresselhaus SOI and the topological Chern state in the circuit lattice, we first give the circuit structure that implements the non-commutative hopping as shown in fig.1. Cell \( m \) and \( n \) are composed of inductors which are connected to form a ring. Cell \( C \) consists of capacitors which connect the nodes in cell \( m \) and \( n \) as shown in fig.1a. A similar structure is used to realize the QSHE in refs \cite{31, 32}. According to Kirchhoff’s current law, the admittance equation of the circuit in the AC steady state case is given as

\[
\begin{pmatrix}
i_m \\
i_n
\end{pmatrix} = \begin{pmatrix}
\frac{1}{j\omega}L & j\omega C_{mn} \\
j\omega C_{mn}^T & \frac{1}{j\omega}L
\end{pmatrix}\begin{pmatrix}
v_m \\
v_n
\end{pmatrix},
\]  

(3)

where \( \omega \) is the frequency, \( j = \sqrt{-1} \), \( m \) and \( n \) are indices for inductor cells, \( i_{m(n)} = (i_{m(n)1}, i_{m(n)2}, ..., i_{m(n)N}) \), \( v_{m(n)} = (v_{m(n)1}, v_{m(n)2}, ..., v_{m(n)N}) \), \( i_{m(n)\tau} \) is the total current sunk into node \( m(n)\tau \), \( v_{m(n)\tau} \) is the voltage at node \( m(n)\tau \), \( \tau = 1, 2, ..., N \), and \( N \) is the number of nodes in the inductor cell. \( L \) is the admittance matrix for inductors in cell \( m \) and \( n \), which reads \( L = (-2I_N + P + P^{-1}) \). \( I_N \) is the \( N \)-by-\( N \) unit matrix. \( P \) is the permutation matrix for permuting 1 to 2, 2 to 3, ..., \( N-1 \) to \( N \), and \( N \) to 1. The matrix \( C_{mn} \) describes the connection configuration of the capacitors that connects nodes in \( m \) and \( n \) cells.

In the absence of external source, the conservation of charge condition requires that the sum of currents sunk into each node equals 0. Equation 3 can be rewritten in the form of a tight-binding Hamiltonian for the eigenvalue problems

\[
\begin{pmatrix}
0 & UC_{mn}U^T \\
UC_{mn}^T U^T & 0
\end{pmatrix}\begin{pmatrix}
\tilde{v}_m \\
\tilde{v}_n
\end{pmatrix} = \frac{\Lambda}{\omega^2} \otimes I_2 \begin{pmatrix}
\tilde{v}_m \\
\tilde{v}_n
\end{pmatrix},
\]  

(4)
Figure 1. The building blocks for constructing non-Abelian gauge potential. (a) Schematic diagram of the onsite cell $m$, $n$ and the connecting cell $C$. In cell $m$ and $n$, inductors are connected in a loop structure. Cell $C$ consists of capacitors that connect the nodes in cell $m$ and $n$. (b to e) The connection configurations in cell $C$ that can achieve the $\pm \sigma_{0,1,2,3}$ formula hopping matrix.

where matrix $U$ is used to diagonalize the matrix $P_1$ in $L$, $\Lambda = U L U^\dagger$ is a diagonal matrix, $\tilde{v}_{m,n} = U v_{m,n}$, and the eigenvalue $\omega$ is the resonant frequency of the circuit system. Below, we discuss the case in which the inductor cell contains three nodes. The case with more nodes is discussed in the appendix. An interesting thing is that by designing the $C_{mn}$ matrices, the matrix $U C_{mn} U^\dagger$ can take the form of a Pauli matrices. In figure 1(b to e), we give the configurations of the cell $C$ that enables $\sigma_{0,1,2,3}$ types of tunneling matrix. In this configuration, the voltage on the nodes can be viewed as a quasi-particle undergoing non-Abelian tunneling, including flip of the pseudo spins and the change of phase, along neighbour links. This modules are the key for our implementation of the SOI and topological Chern state in circuit.

Now, we present the circuit for realizing the Rashba-Dresselhaus SOI. As shown in figure
2, the nearest neighbor hopping in the $x(y)$ direction employs the $\sigma_x(\sigma_y)$ hopping module, and the next nearest neighbor in the $x$ and $y$ direction employs the $\sigma_0$ hopping modules. An advantage of the circuit system is evidenced here, namely the ability to achieve farther nearest-neighbor hopping through wire connections, which is not easily achieved in rigid materials. For the two-dimensional circuit network in figure 2(a, b), using the method similar to that used to write eq. 3 and eq. 4, we can obtain Hamiltonian of the form

$$H_2 = (2\cos 2k_1 + 2\cos 2k_2 - 13)I_2$$

$$+ 2 \begin{pmatrix} 0 & \cos k_1 - i\sqrt{3}\cos k_2 \\ \dagger & 0 \end{pmatrix},$$

(5)

We're interested in the 2-by-2 block Hamiltonian 5. The frequency dispersion of Hamiltonian 5 are shown in Fig. 2(c). The dispersion near point $(\pm \pi/2, \pm \pi/2)$ has the form of Rashba-Dresselhaus SOI type splitting. Expanding Hamiltonian 5 near point $(\pi/2, \pi/2)$, keeping to the first order of k, we obtain the Rashba-Dresselhaus SOI Hamiltonian $H(k) = -17\sigma_0 + \alpha(k_1\sigma_1 + k_2\sigma_2) + \beta(k_1\sigma_1 - k_2\sigma_2)$, where $\alpha = -\sqrt{3} - 1$ and $\beta = \sqrt{3} - 1$ are Rashba and Dresselhaus SOI constant, respectively. The nature frequencies of $h_1$ can be tuned away from the frequencies of Hamiltonian 5. Here, we added grounded capacitors to the nodes of the inductors cells, which does not affect the form of the hopping matrices, but leaves $h_1$ without resonant frequency solutions. Figure 2(d) shows the comparison between the experimentally measured frequency dispersion (black curves) and the theoretical result (red curves). Since the devices are not ideal, the capacitors with a $\pm 5\%$ tolerance and the inductor with a $\pm 5\%$ tolerance and 43 $m\Omega$ internal resistance are used to fabricate the circuit board, the experimentally measured dispersion shows certain broadening and with a small shift in frequency compare to the theoretical results.

Up to now, we synthesize the Rashba-Dresselhaus SOI in the circuit lattice. This scheme can be easily extended to 1D or 3D lattice for other types of SOI. Since the SOI type non-Abelian gauge filed does not break time reversal symmetry, a further design is required to implement the topological Chern state that breaking time reversal symmetry is necessary. Based on the Haldane model, reference [36, 37] theoretically show the schemes for implementing the topological Chern state using the negative impedance converter and Hall
resistor. Despite these theoretical proposals, the synthesis and observation of Chern circuit remain experimentally elusive.

In the following we present the design of the Chern circuit based on the non-Abelian gauge field and experimentally confirm the topological nature of Chern circuit.

The design is shown in fig. 3. The devices in the red dashed box (3(a)) constitute an integrator. The integrator and capacitor are connected in series and then connected in parallel with the inductor. On each node, they are grounded through resistor $R$.

The effective Hamiltonian for this circuit lattice has a similar form to Hamiltonian 5. Now $h_1 = 9i/(2CR \omega) + 8 \cos k_1 + 12 \cos k_2 - 23$ and the 2-by-2 block Hamiltonian is given by

$$\left(\sum_{i=0}^{3} d_i(k) \sigma_i(\tilde{v}_2, \tilde{v}_3)^T = -\frac{3}{CL\omega^2}(\tilde{v}_2, \tilde{v}_3)^T, \right.\tag{6}$$

where $d_0(k) = -23$, $d_1(k) = 2\cos k_1$, $d_2(k) = 2\sqrt{3}\cos k_2$, $d_3(k) = -3\sqrt{3}/(2CR\omega) + 2\sqrt{3}\sin k_1 + 2\sqrt{3}\sin k_2$. Here, $d_3$ is a function of $\omega$. The eigenvalues $\omega$ and non-zero wave functions $(\tilde{v}_2, \tilde{v}_3)^T$ can be obtained by solving $\text{Det}[\Sigma d_i \sigma_i + 3/CL\omega^2\sigma_0] = 0$. $h_1$ is non-Hermitian, and its frequencies are complex that can be set to decay exponentially with time by choosing proper device parameters, with frequency values far from the frequency of Hamiltonian 6. In order to determine the topological nature of Hamiltonian 6, we calculate the Chern number of the system by the means of the Wilson loop. Using the device parameters listed in figure 3, the Wilson loop is calculated as shown in Figure 3(d), indicating a Chern number of 1.

Non-zero Chern numbers are associated with the chiral edge states at the boundary of the system, which is usually measured experimentally to determine the topological properties of the system. To measure the edge states, we fabricate a circuit lattice with $30 \times 3$ unit cells, which takes periodic boundary condition in the $x$ direction and open boundary condition in the $y$ direction. Exciting the circuit at node 1 in the $(1, 1)$ cell with frequency sweep form $160$ kHz to $260$ kHz, measuring the amplitude and the phase of the voltages at all nodes as a function of frequency, we obtained the chiral edge state as shown in figure 3(e). The red curves in figure 3(e) are the edge states obtained by calculating the dispersion of the strip using the model 6. The two bands near frequency $200$ kHz have opposite chiralities and are located on the upper and lower boundaries of the strip, respectively. Since the edge states are very localized in the $y$ direction, we can only detect the edge states on the $y = 1$ boundary with the excitation source at $(1, 1)$ cell, while the edge state on the $y = 3$ edge are
not detected. These results are consistent with the property of the topological edge state. The issues of the curves becoming broadened can be improved by choosing devices with less tolerance and using inductors with lower internal resistance.

As evidenced by the examples, our scheme suggests a circuit approach to design new physics related to non-Abelian gauge fields. The versatile physical effects of the non-Abelian gauge field, such as the non-Abelian Aharonov–Bohm effect\cite{56}, the Aharonov–Casher effect\cite{57}, the novel motion of the quasi-particle, etc., can be studied in the circuit system. The on and off states of circuit branches are easy to control with modern techniques, which makes it possible to change the circuit structure to obtain a controlled gauge field, topological state, or the internal degrees of freedom in the unit cell. The topological nature of the circuit can lead to new applications for circuit systems, e.g. chips working with topological properties are insensitive to fabrication errors, local damage, etc. Our solution is easily generalized to design non-Abelian gauge fields and topological states in arbitrary dimensions, including fractional dimensions.

Based on electro-mechanical-acoustic analogy, our scheme can be generalized to mechanical and acoustic systems for the implementation of non-Abelian physics in these systems. In combination with superconducting devices, our scheme has the potential to investigate physics related to superconductivity.

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The circuit was implemented on FR4 printed circuit board. Components consist of inductors $L$ (1.8 $\mu$H with a $\pm$5% tolerance and 43 $m\Omega$ series resistance), capacitors $C$ (56 $nF$ with a $\pm$5% tolerance), operational amplifier AD8057ARTZ-REEL7, and resistors $R_1$ (15 $\Omega$ with a $\pm$1% tolerance), $R_2$ (30 $\Omega$ with a $\pm$1% tolerance), $R_3$ (100 $M\Omega$ with a $\pm$1% tolerance). The voltage, including the amplitude and phase, at each node is probed by Rohde&Schwarz vector network analyzer ZNL6 5kHz-6GHz.

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Figure 2. The Rashba-Dresselhaus SOI circuit system. (a) Schematic and circuit diagram of the 2D Rashba-Dresselhaus SOI circuit. The green (yellow) block is the nearest neighbor hopping module $+\sigma_{1(2)}$ in the $x(y)$ direction. The pink block is the next-nearest neighbor hopping module $+\sigma_0$ in $x$ and $y$ directions. Their configuration details are given in fig.1. The cyan blocks are the inductor cells, where each node has a grounded capacitor with the same capacitance to the capacitors in the $\sigma_{0,1}$ modules. (b) PCB layout of the Rashba-Dresselhaus SOI circuit. The red dashed box correspond to a unit cell. The green, orange and purple dashed box correspond to the $\sigma_1$, $\sigma_2$, and $\sigma_0$ module, respectively. (c) Frequency dispersion of the circuit. The Rashba-Dresselhaus type splitting is present at $(\pm\pi/2, \pm\pi/2)$ point. (d) Comparison of the experimentally measured frequency dispersions (black) with the theoretical results (red curves). The $k$-line passes through the point $(\pi/2, \pi/2)$, along the $k_1$ direction.
Figure 3. The topological Chern circuit system. (a) Schematic and circuit diagram of the 2D Chern circuit. The cyan blocks $L//\int$ are the onsite modules, which detailed in (b). The green (yellow) block is the nearest neighbor hopping module in the $x(y)$ direction. The gray block is the nearest neighbor hopping module in $x$ and $y$ directions. Their configuration details are given in figure 1. (b) Schematic of a cyan cell of fig (a). The integrator is marked by a red dotted box. The red resistor is use to eliminate the initial voltage and increase the stability of the integrator. The integrator is series connected with a red capacitor (the capacitance is three times the capacitance of a black capacitor in the integrator) and then connected in parallel with the inductor $L$. Each node has a green ground resistor, which has twice the resistance compared to the resistors in the integrator. (c) PCB layout of one unit cell of the topological Chern circuit. The devices in the blue dashed box correspond to $L//\int$ module given in fig. (b). The green (yellow) dashed box correspond to the $\sigma_1 + i\sqrt{3}\sigma_3$ ($\sigma_2 + i\sqrt{3}\sigma_3$) module in fig. (a). (d) The calculated Wilson loop of the circuit system which indicate the topological Chern number is equal to 1. (e) Comparison of the experimentally measured frequency dispersions (black) with the theoretical results (red curves).