VR CCD Photometry of Variable Stars in the Globular Cluster NGC 4147

Sneh Lata1, A. K. Pandey1, J. C. Pandey1, R. K. S. Yadav1, Shashi B. Pandey1, Aashish Gupta2, Tarun Bangia1, Hum Chand1, Mukesh K. Jaiswar1, Yogesh C. Joshi1, Mohit Joshi1, Brijesh Kumar1, T. S. Kumar1, Biman J. Medhi1,3, Kuntal Misra1, Nandish Nanjappa1, Jaysreekar Pant1, Purushottam3, B. Krishna Reddy1, Sanjit Sahu1, Saurabh Sharma1, Wahab Uddin1, and Shobhit Yadav1

1 Aryabhatta Research Institute of Observational Sciences, Manora Peak, Nainital 263002, Uttarakhand, India; sneh@aries.res.in
2 PDPM Indian Institute of Information Technology Design & Manufacturing Jabalpur Dumna Airport Road, Dumna-482005 Jabalpur, Madhya Pradesh, India
3 Department of Physics, Gauhati University, Jalukbari, Guwahati 781014, India

Received 2019 March 28; revised 2019 April 29; accepted 2019 May 15; published 2019 July 8

Abstract

We present results of a search for variable stars in a region of the globular cluster NGC 4147 based on photometric observations with a 4K × 4K CCD imager mounted at the axial port of the recently installed 3.6 m Devasthal optical telescope at Aryabhatta Research Institute of Observational Sciences, Nainital, India. We performed time photometry of NGC 4147 in the V and R bands, and identified 42 periodic variables in the region of NGC 4147, 28 of which have been detected for the first time. Seventeen variable stars are located within the half-light radius ≤ 0.48, of which 10 stars are newly identified variables. Two of the 10 variables are located within the core radius ≤ 0.07. Based on their location in the V/(V – R) color–magnitude diagram and variability characteristics, seven, eight, five, and one newly identified probable member variables are classified as RRc, EA/E, EW, and SX Phe, respectively. The metallicity of NGC 4147 estimated from the light curves of RRab stars is consistent with that obtained from the observed V/(V – R) color–magnitude diagram.

Key words: globular clusters: general – stars: variables: RR Lyrae

Supporting material: data behind figures

1. Introduction

The study of variable stars in globular clusters has been carried out by several groups using different methods and techniques (e.g., Clement et al. 2001; Pritzl et al. 2002; Layden et al. 2003; Clementini et al. 2005; Corwin et al. 2006; Baker et al. 2007 and Kopacki et al. 2008). Globular clusters normally consist of low-mass Population II stars and provide an ideal environment for the search of RR Lyrae stars. The early investigations of globular clusters show that more than 90% of the known variables were of RR Lyrae type. With the advent of new technology, other types of variables in the globular clusters have also been discovered. Even after new discoveries of variables, RR Lyrae stars still dominate the variability populations in globular clusters. They now constitute less than 70% of known variables (see Clement 2017).

NGC 4147 is a relatively small globular cluster of radius 0.48, which is located 21 kpc from the Galactic center and 19 kpc from the Sun and has low metallicity [Fe/H] = −1.83 (Harris 1996; 2010 edition). The 0.48 radius given in the Harris catalog (Harris 1996; 2010 edition) refers to the half-light radius of the cluster. The core and tidal radii of the cluster are 0.09 and 6.1, respectively (Harris 1996; 2010 edition). The reddening toward the cluster NGC 4147 is found to be E(B – V) = 0.02 mag (Harris 1996; 2010 edition). Previous studies by Castellani & Quarta (1987), Arellano et al. (2004), and Stetson et al. (2005) have shown that, although NGC 4147 is a relatively metal-poor cluster with a predominantly blue horizontal branch (HB) in its color–magnitude diagram (CMD), its RR Lyrae stars have short periods, characteristic of more metal-rich Oosterhoff type I clusters. Meanwhile, the large proportion of RRc variables is typical of an Oosterhoff II cluster.

Up to now, the only known variables in NGC 4147 are RR Lyrae. With a much better plate scale and the large-aperture telescope, it should be possible to identify more RR Lyrae and other variable stars in the crowded central region of the globular cluster NGC 4147. With this aim, observations of the globular cluster NGC 4147 were taken using the 4k × 4k CCD camera, the first light imaging instrument mounted at the axial port of the recently installed 3.6 m Devasthal optical telescope (DOT) at the Aryabhatta Research Institute of Observational Sciences (ARIES), Devasthal campus, India (Pandey et al. 2018). A time series photometry was carried out on six nights from 2017 March 23 to 2017 April 9 in order to search for the photometric variables in the cluster NGC 4147. With these new observations, we present the precision of photometry and compare the present results with previously published work. This paper is organized as follows. In the next section, we present observations and data reduction. In subsequent sections, we present variable identification and discuss their association with the cluster on the basis of the V/(V – R) CMD, proper motions, and geometric probabilities. We also discuss the period determination of known and newly detected variable stars. Finally, we present the study on RR Lyrae variables as well as other variables detected in the region of the globular cluster NGC 4147.

2. Observations and Data Reduction

The CCD observations were carried using the 3.6 m DOT (f/9 Alt-azimuth mounting) at ARIES, Devasthal (Nainital), India. The telescope is equipped with a 4k × 4k CCD imager, which is mounted on the main port of the telescope. The CCD imager has a pixel size of 15 μm and a plate scale of 0.095 arcsec per pixel, covering an approximately 6.23 × 6.23 arcmin² field.
of view over the sky. The observations were taken in 4 × 4 binning mode with 1000 kHz readout. We made observations in the \( V \) and \( R \) bands on six nights between 2017 March 23 and April 9. In total, we collected 339 and 302 frames in the \( V \) and \( R \) bands, respectively. The exposure times vary from 30 to 50 s. The FWHM of the observations varies between \( \sim 0^\prime 7 \) and \( 1^\prime 0 \).

The sky brightness was found to be high due to the bright Moon on two nights. Bias and twilight flats were also taken along with the target field. The log of observations is given in Table 1.

The preprocessing of the images was performed using IRAF. The instrumental magnitudes of the stars were obtained using the DAOPHOT package (Stetson 1987). Both aperture and PSF photometry were performed because PSF photometry gives better results for the crowded region. For the PSF photometry, we have selected bright isolated stars across the field to construct a characteristic point-spread function (PSF) for the images. The PSF photometry of all the sources was obtained using the ALLSTAR task. Figure 1 shows the observed field of the globular cluster NGC 4147. The data of the \( V \) and \( R \) bands were merged by matching the coordinates using a radial-matching tolerance of 1'. We detected 1057 stars in both the \( V \) and \( R \) bands. The DAOMATCH (Stetson 1992) routine of DAOPHOT was used to find the translation, rotation, and scaling solutions between different photometry files, whereas DAOMASTER (Stetson 1992) was used to match the point sources. DAOMASTER was also used to remove the effects of frame-to-frame flux variation due to airmass and exposure time. This task makes the mean flux level of each frame equal to the reference frame by an additive constant. The first target image on 2017 March 25 was taken as the reference frame.

The intensity-weighted mean instrumental magnitudes in the \( V \) and \( R \) bands given by DAOMASTER (Stetson 1992) were transformed into the standard ones using the photometric data of standard stars marked in the field of NGC 4147 by Arellano et al. (2004). We also used data of several stars taken as standard by Stetson et al. (2005) to calibrate the present instrumental magnitudes. The following transformation equations were obtained:

\[
V = v + (-0.198 \pm 0.021) \times (V - R) + 4.800 \pm 0.022 \\
R = (1.036 \pm 0.032) \times (v - r) - 0.0874 \pm 0.019,
\]

where \( v \) and \( r \) are the instrumental magnitudes in the \( V \) and \( R \) bands, respectively.

The \( X \) and \( Y \) coordinates of the reference image were transformed into right ascension and declination using the CCMAP and CCTRAN tasks available in IRAF. The average photometric error of the data was estimated using observations of each star. The average photometric error of the stars along with the identified variables as a function of standard magnitude in the \( V \) and \( R \) bands is shown in Figure 2. We plotted only those stars with photometric error \( \leq 0.02 \) mag up to \( V \sim 18 \) mag, whereas all the identified variables, irrespective of their photometric errors, are plotted in Figure 2. The photometric errors in the \( (V - R) \) colors of the stars have been calculated as

\[
\sigma_{V-R} = \sqrt{(\sigma_V^2 + \sigma_R^2)}.
\]

2.1. Comparison with Previous Photometry

We have compared the present photometric results with the previous photometry given by Stetson et al. (2005) and Wang et al. (2000). Data were downloaded from the Vizier catalog for both previous photometries. The catalog by Stetson et al. (2005) lists \( BV \) data for 91 stars, while Wang et al. (2000) presented photometry for 115 stars. Cross-identification yields 79 and 94 common stars with Stetson et al. (2005) and Wang et al. (2000), respectively. Figure 3 shows the difference \( \Delta \) (in the sense of present minus previous photometry) in \( V \) and \( (V - R) \) between our photometry and previous photometries. The filled circles represent the difference between the present \( V \) magnitudes and those by Stetson et al. (2005) while unfilled circles show the difference between present photometry and the photometry by Wang et al. (2000). Figure 3 indicates that present \( V \) magnitudes match well with those given by Stetson et al. (2005), whereas comparison of present \( V \) magnitudes with those obtained by Wang et al. (2000) indicates fair agreement up to \( V \sim 16.0 \) mag. The present \( V \) magnitudes become fainter than those of Wang et al. (2000) after \( V \sim 16.0 \) mag. The \( (V - R) \) colors obtained by Wang et al. (2000) are redder and become even redder with the increase of \( V \) magnitudes.

2.2. Variables Identification

The corrected instrumental magnitudes of the variables provided by DAOMASTER were converted into standard ones by obtaining transformation equations using standard stars. We obtained transformation equations as follows:

\[
V = (1.000 \pm 0.002) \times v + 4.845 \pm 0.010 \\
R = (0.998 \pm 0.007) \times r + 4.797 \pm 0.005.
\]

The color term was not used in the above equations as it is found to be insignificant (see also Arellano et al. 2004). We identified variable stars by inspecting their light curves visually. The light curves of all the stars were displayed using a code written in IDL, each of them was visually inspected, and those that showed periodic/regular brightness variation were selected. We have detected a total of 42 variables in the region of NGC 4147. Among these 42 variables, 28 variables have been detected for the first time, and these variables are termed newly identified variables. Stars V20 and V31 could not be detected in the \( R \) band while we could not detect V12 in the \( V \) band. The optical data of variable stars along with their identification numbers are listed in Table 2. Column 1 of Table 2 gives the identification number of a variable. We adopted the same nomenclature for the known variable stars that was used by earlier studies (see Stetson et al. 2005). The

| S. No. | Date of Observations | Object | \( V \) (\( N \times \text{Exp.} \)) | \( R \) (\( N \times \text{Exp.} \)) |
|--------|---------------------|--------|-------------------------------|-------------------------------|
| 1      | 2017 Mar 23         | NGC 4147 | ...                          | 5 × 300 s                     |
| 2      | 2017 Mar 24         | NGC 4147 | 45 × 40 s                    | 45 × 40 s                     |
| 3      | 2017 Mar 25         | NGC 4147 | 39 × 50 s                    | 39 × 50 s                     |
| 4      | 2017 Mar 28         | NGC 4147 | 80 × 50 s                    | 38 × 50 s                     |
| 5      | 2017 Apr 8          | NGC 4147 | 58 × 30 s                    | 58 × 30 s                     |
| 6      | 2017 Apr 9          | NGC 4147 | 117 × 30 s                   | 117 × 30 s                    |

Note. \( N \) and Exp. represent the number of frames obtained and exposure time, respectively.
identification numbers for the new variable stars are assigned in continuation of past work. The positions of the identified variables are shown by the circles in Figure 1. This clearly shows that most of the variables identified in the present study are found to be located toward the cluster center. To understand the spatial distribution of the identified variables in a better way, the radial distance \( r \) of each variable is determined by estimating the cluster center \( (\text{R.A.} = 12^h:10^m:6.31 \text{ and decl.} = +18^d:32^m:33^s42) \) and given in Table 2. Two newly identified variables, V29 and V31, are located within the core radius \( \leq 0\prime09 \). Seventeen variable stars including V29 and 31 are found to be distributed within the half-light radius \( \leq 0\prime48 \), of which 10 stars, namely V24, V27, V28, V29, V30, 31, V32, V33, V35, and V36, are newly identified variables. Data of all the detected variable stars in the present work is available. The format of the data is as follows. The first, second, and third columns represent the JD, magnitude, and error, respectively.

3. Identification of Probable Members of NGC 4147

In order to understand the nature of variable candidates, it is necessary to find out their association with the globular cluster NGC 4147. We have used the \( V/(V - R) \) CMD and proper motion to identify probable members of the cluster as the sample of identified variables is contaminated by the field population. The \( V/(V - R) \) CMD of the cluster NGC 4147...
discussed in the next section clearly reveals contamination due to field star population.

3.1. V/(V − R) CMD

Figure 4 shows the $V/(V − R)$ CMD for all stars in the region of NGC 4147 that were common in both $V$ and $R$ bands. In this figure, larger filled circles represent variables identified in the present work. The continuous curve shows the theoretical model by Girardi et al. (2002), while the dotted line represents the zero-age HB (ZAHB) locus, which was calculated using the Princeton–Goddard–PUC (PGPUC) stellar evolutionary code based on Valcarce et al. (2012, 2013). The PGPUC code is available for chemical compositions ranging from $Z = 1.60 \times 10^{-4}$ to $1.57 \times 10^{-2}$ (−2.25 $\lesssim$ [Fe/H] $\lesssim$ −0.25). Helium abundances from $Y = 0.23$ to 0.37, and an alpha-element enhancement of $[\alpha/\text{Fe}] = 0.3$. During calculation of the ZAHB locus, the values of the mass, $Z$, $Y$, and $[\alpha/\text{Fe}]$ were taken to be 0.8 $M_\odot$, 0.001, 0.23 and 0.3, respectively. In Figure 4, we did not plot those stars that have large photometric errors $>0.02$ mag up to $V \sim 18$ mag. However, we considered and plotted those variable stars that have photometric errors $>0.02$ mag. In this CMD, we could not plot three stars, namely V12, V20, and V31, as their $(V − R)$ colors were not available. The $V/(V − R)$ CMD clearly shows a well-defined main-sequence, giant, supergiant, and HB. From the $V/(V − R)$ CMD, we also noticed that there is no gap present in the HB. The HB morphology of the cluster was discussed in detail by Stetson et al. (2005), and they found that NGC 4147 has a predominantly blue HB, which is different from Oosterhoff type I globular clusters.

An attempt has been made to determine the age and distance of the cluster NGC 4147 by comparing present observations with the theoretical models of Girardi et al. (2002). The model of Girardi et al. (2002) for $Z$ (relative metal abundance) $=0.001$
and Y (relative helium abundance) = 0.23 gives the best fit to the observed distribution of stars in the CMD of NGC 4147. The reddening toward the cluster region was estimated along with the age and distance modulus by comparing present observations with theoretical models. The distance modulus \((V - M_V)\) and the age of the cluster are thus obtained to be 16.40 mag and 12.58 Gyr, respectively. The derived age of NGC 4147 is consistent with the age of other globular clusters. Using the distance modulus, the distance to the cluster was calculated to be 17.49 kpc. The value of the reddening \(E(V - R)\) toward the cluster region is estimated as 0.04 mag which further yielded \(E(B - V) = E(V - R)/0.65\) reddening value as 0.06 mag. The metallicity \([\text{Fe}/\text{H}]\) of stars can be generally calculated using the following relation \([\text{Fe}/\text{H}] = \log(Z/X) - \log(Z/X)_\odot\), where \(Z/X\) is the metal to hydrogen ratio (Smith 1995; Harris 1996: 2010 Edition). The above conversion is used if the elemental distribution is assumed to follow solar abundances. The metals in stars are often grouped into two categories: \(\alpha\) elements and Fe elements. The old metal-poor stars are typically \(\alpha\) enhanced in the halo and bulge of our Galaxy \(([\text{Fe}/\text{H}] < -0.6\) and \([\alpha/\text{Fe}] \sim 0.3 - 0.4\); e.g., Cáceres & Catelan 2008 and references therein). Therefore, to determine the relative iron abundance with respect to the solar iron abundance for any \([\alpha/\text{Fe}]\) value, we used the relation given by Valcarce et al. (2012, 2013). Thus, the value of metallicity \([\text{Fe}/\text{H}]\) with \([\alpha/\text{Fe}] = 0.3\) comes out to be \(-1.47\). If we consider the value of \([\alpha/\text{Fe}] = 0.4\), the metallicity \([\text{Fe}/\text{H}]\) comes out to be \(-1.57\).

All of the present variable candidates except V44 are found to be located on the HB, red giant branch, and blue stragglers region, and these could be probable members of the cluster. Due to the lack of R-band data, the membership of the newly identified variables V20 and V31 cannot be determined.

### 3.2. Membership Probability Based on Proper Motions and Geometric Distribution

The proper motion data by Wang et al. (2000) were also used to determine the membership of the identified variables. Wang et al. (2000) estimated the proper motion of stars having magnitudes \(B \lesssim 17.0\) mag using three-epoch data. Per their criterion, they can regard a star having membership probability \(\leq 0.67\) as a nonmember. Wang et al. (2000) found that there are quite a few stars with large membership probabilities located above the HB. From their location in the \(V/(V - R)\) CMD of NGC 4147, they suggested the existence of a second HB, and it might be due to their binary nature. For the larger dispersion in the member star sequence, Wang et al. (2000) suggested that there could be crowding problems near the cluster center that reduced the accuracy of the photometric data.

The present data of variables were cross-matched with the Wang et al. (2000) data within the 1″ matching radius, and 23 stars were found in common between these two data sets. The known variables V2, V3, V4, V6, V8, V10, V11, V12, V14, V16, and V17 detected in the present work have membership probabilities \(\geq 0.70\), hence these could be members of the cluster. Wang et al. (2000) considered V13 as a nonmember of the cluster as its membership probability has been found to be 0.70. Proper motion data of the known variable star V1 are not known. Only 10 newly identified variables have proper motion data, of which four stars, V24, V31, V37, and V42, have membership probabilities \(\geq 0.70\). For stars V20, V23, V28, V32, V33, and V47, membership probability was found to be either zero or \(\leq 0.70\). The proper motions of cross-matched stars along with their membership probability are listed in Table 3.

We also used proper motion values of variable stars taken from the Gaia astrometric mission (Gaia Collaboration et al. 2018). The Gaia astrometric mission was launched in 2013 to measure positions, parallaxes, proper motions, and photometry to obtain physical parameters for millions of stars. There are 36 variable stars that have proper motion values. To calculate the membership probabilities using the proper motion data of these 36 stars, we tried a parametric approach that is commonly used as described in Vasilevskis et al. (1958). This approach assumes a normal bivariate distribution for proper motion values for both field and cluster stars. The distribution of proper motion values for field stars around our cluster followed no standard form and therefore, we used the nonparametric approach as described in Cabrera-Cano & Alfonso (1990) through its implementation Clusterix (http://clusterix.cerit-sc.cz/). It iteratively improves empirically determined cluster and field star distributions, without any assumption about their shape and also uses positional data as supplementary information. The proper motion cutoff was set to 10 mas yr\(^{-1}\) and the radius of the cluster to 6.5′. Table 3 lists proper motion data and membership probability of 36 variable stars. Table 3 shows that all these stars excluding V29 have membership probability more than 80%.

Rozyczka et al. (2017) discussed membership probabilities of the variable stars based on their proper motions, spatial distribution, and CMD location. They allow for the fact that stars with proper motion membership probabilities less than 70% might still be cluster members. In the case of M22, they proposed that if the proper motion probability of the star is less than 70%, but if it has a geometric probability of >99% with...
The determined geometric membership probabilities of variable stars are listed in Table 3.

To assign the membership status of the stars when no proper motions are available, Rozyczka et al. (2017) suggested another membership criteria based on geometric probabilities \(>70\%\), CMD location, and distance from the cluster center. Using these criteria, stars V21, V22, V25, V26, V27, V29, V30, V33, V35, V36, V38, V39, V40, V41, V43, and V46 could be probable members of the cluster. Two stars, V26 and V44, located outside the half-light radius and have geometric probabilities \(<70\%\) can be considered as nonmembers. The membership status of the stars is marked as “1,” whereas nonmembers are flagged as “0.”

The condition that if the star’s CMD location is appropriate for its variability type, and its distance from the cluster center is less than \(3/3\) (which is the half-light radius for M22), the star could be a member of the cluster. This could be the situation for V28, V32, V33, and V47 in NGC 4147. These stars are located in the crowded central region of the cluster within the half-light radius \(\lesssim 0.48\), where it might have been difficult to derive accurate proper motions and distances. On the other hand, V20 and V23 both have membership probabilities less than 70\% and are located much farther from the cluster center. They are probably field stars. The geometric membership probabilities of stars were calculated using the relation \(P_{geom} = 1 - \pi r^2 / S\) given by Rozyczka et al. (2017), where \(r\) is the radial distance from the cluster center in arcseconds and \(S\) is the size of the field of view in arcsec\(^2\). The value of \(S\) is taken as 139726.644 arcsec\(^2\).
4. Period Determination

We have used the Lomb–Scargle (LS) periodogram (Lomb 1976; Scargle 1982) to determine the most likely periods of the variable stars. This periodogram gives better periods even if the data are taken at irregular intervals. Periods derived from the LS periodogram were further confirmed using the NASA exoplanet archive periodogram service. Light curves of variable stars were folded using these derived periods. The phased light curves were visually inspected, and we opted for the period that gives the best folded light curve. Figures 5 and 6 display the phased light curves of known and newly identified variable stars, respectively. The $R$-band light curves of variables were phased with the period obtained from their $V$-band light curves. For better representation of the phased light curves for the 28 newly discovered variable stars, we folded their light curves with the phase bin of 0.01.

We could not determine the period of star V1 on the basis of present data, due to our short observational baseline. We have combined present data of star V1 with those of Arellano et al. (2004) to determine its period. A period of 0.50039 days for V1 was determined using the LS periodogram. There was a

---

5 http://exoplanetarchive.ipac.caltech.edu/cgi-bin/Pgram/nph-pgram
Figure 6. The phased light curves of the newly identified variable stars in the $V$ and $R$ bands. The data used to create this figure are available.
systematic offset of 0.1 mag between the present and Arellano et al. (2004) data. We brought the present magnitudes to the magnitudes of Arellano et al. (2004) by applying the offset. The same procedure has been adopted in the case of variables V6, V7, and V12 to determine their periods. The phased light curves of V1, V6, V7, and V12 in both V and R bands are plotted in Figure 7. The periods and amplitudes in the V and R bands of 42 variable stars are given in Table 2.

5. RR Lyrae Variables

5.1. Known

The present study consists of 14 known variables that have already been discovered and studied. The studies of the cluster NGC 4147 by Davis (1917) and Baade (1930) discovered one and three variable stars, respectively. The photometric study by Sandage & Walker (1955) reported 10 variable stars in the cluster region. Newburn (1957) studied variable stars and discovered three variable candidates. A study on variable stars in NGC 4147 was presented by Mannino (1957). Clement (1997) provided a list of 17 RR Lyrae stars in the region of NGC 4147. Arellano et al. (2004) reported results of V- and R-band CCD photometry of the known RR Lyrae stars in NGC 4147 and derived significantly improved periods and discovered one new variable, V18. The periodicities of most variables were revised, and new ephemerides were calculated by Arellano et al. (2004). They detected the Blazhko effect in two stars, namely V2 and V6, and found three, V5, V9, and V15, which had been previously reported as variables, to be nonvariables. The variable V18 they discovered has a period of
Figure 7. The phased light curves of V1, V6, V7, and V12 in the V and R bands. The filled squares show present data while filled circles represent data of Arellano et al. (2004). In the case of V12 isolated data points are displayed by asterisk symbol. The data used to create this figure are available.
∼0.49205 days and amplitude of ∼0.15 mag. They also estimated the physical parameters of RR Lyrae variables. Stetson et al. (2005) presented a detailed study on the photometry and astrometry of the cluster NGC 4147. They reanalyzed five exposures of the cluster obtained with WFPC2 on the Hubble Space Telescope and presented calibrated CMDs and color–color diagrams. They also found morphological properties that were generally consistent with previous published works. The star V18 discovered by Arellano et al. (2004) was found to be nonvariable by Stetson et al. (2005). They also detected one new variable V19 and characterized it as an RR Lyrae star.

The previously known RR Lyrae variables in the present work have periods in the range from ∼0.26 to ∼0.60 days and the amplitude of their brightness variation in the V-band ranges from ∼0.34 to ∼1.17 mag. Their amplitudes, periods, and location in the $V/(V - R)$ CMD are in agreement with those of RR Lyrae-type stars. The known RR Lyrae variables have already been classified as RRab and RRc type on the basis of their variability characteristics in earlier works. The RRab variables V1, V2, V6, and V7 vary with larger $V$ amplitudes in the range of ∼1.02 to ∼1.17 mag. The periods of these RRab stars vary from ∼0.49 to ∼0.61 days. All known RRc variables, namely V3, V4, V8, V10, V11, V14, V16, and V17, have amplitudes in the range of ∼0.34 to ∼0.71 mag, while their brightness varies with the periods between ∼0.26 to ∼0.40 days. The periods of the cataloged variables V3, V4, V6, V7 V8, V10, V11, V12, V13, and V17 are revised and found to be in agreement with previously determined periods. There are two stars, V2 and V14, whose periods derived using the present data are different from the periods reported in previous studies. We could not detect brightness variation in stars V5, V9, V15, and V18 using present data. This confirms the conclusion reached by Stetson et al. (2005) and the fact that these stars are no longer considered to be variables. Star V19 could not be detected in the present work because it was outside of the present field of view.

Now, we would like to discuss those known variables whose periods, amplitudes, and light-curve shapes have been found to be slightly different from those of previous studies.

In the case of star V2, the derived period of 0.335070 days from the present data is not consistent with that of the earlier derived period (Arellano et al. 2004). Further, the light curve of V2 could not be phased well with this derived period. The present data was also not phased well with the period of ∼0.49 days reported by Newburn (1957), Mannino (1957), Arellano et al. (2004), and Stetson et al. (2005). The LS power spectrum of V2 is shown in Figure 8(a), where the prominent peak

Figure 8. Power spectrum of star V2. Plot (a) represents the power spectrum obtained using the LS algorithm for the present $V$-band data, while plot (b) shows the power spectrum obtained from the CLEANed algorithm. The short dashed line shows the power spectrum obtained using the $V$-band data of Arellano et al. (2004). The long dashed and continuous curves represent present data in the $V$ and $R$ bands, respectively. Plot (c) shows the power spectrum of V2 obtained using the LS algorithm in combination with previous data (Arellano et al. 2004), whereas plot (d) displays the power spectrum obtained from the CLEANed algorithm.

The Astronomical Journal, 158:51 (18pp), 2019 July Lata et al.
The power spectrum obtained using the present data and that corresponds to a period of 0.335070 days; however, a period of 0.493055397 days. The combined data could not be phased to periods 0.331181093 days and 0.49 days is also present in the data. We compared between the cleaned power spectra clearly show the period of 0.335070 days in the present study compared with Arellano et al. (2004). The phased light curves of V2 in the upper panel shows the CLEANed power spectrum of the data of Arellano et al. (2004). Because the LS power spectrum is noisy. The CLEAN algorithm (Roberts et al. 1987) was therefore used to determine the period. Figure 8(b) shows the CLEANed power spectrum of the present data in the V and R bands and the V-band data of Arellano et al. (2004). CLEANed power spectra were obtained by using the loop gain of 0.1 and iteration of 100. The CLEANed power spectra clearly show the period of 0.335070 days from the present data and ~0.49 days from the Arellano et al. (2004) data. We have also applied the same approach as applied for V1 to determine the period of star V2. The LS and CLEANed power spectra of the combined data are shown in Figures 8(c) and (d), respectively. Both power spectra show prominent peaks at periods 0.331181093 days and 0.493055397 days. The combined data could not be phased well with a period of 0.493055397 days as can be seen from its light curve displayed in Figure 9. This indicates that both periods may be present in the data. Star V2 is a known RRab-type variable. The period of ~0.33 days in the present study seems to be too short for an RRab-type variable, and this might be due to the short observational baseline. The ~0.33 day period is an alias because of the uncertainty in the number of cycles elapsed in one day. A close inspection of the light curves plotted for V2 in Figure 5 reveals an overlap between the descending branch and ascending branch at minimum light. This is very clear in the R-band light curve and to a lesser extent in the V-band light curve. It is an indication that the ~0.33 day period is too short. A complicating factor in the case of V2 is that it is a Blazhko variable. This has been well illustrated by Arellano et al. (2004) with their extensive data coverage. The data in this present work were obtained on only six nights, and it appears that observations were obtained at different Blazhko phases. We assume that this is the reason for the disconnect near maximum light in the light curves plotted in Figure 9. In view of this, it is not surprising that period searches with the current data do not favor the ~0.49 day period. Also, it is not surprising that the combined data of Arellano et al. (2004) and the current study might favor the ~0.33 day period because there are more observations in the current study compared with Arellano et al. (2004). For further study, the period of V2 is adopted as ~0.493 days.

The V amplitudes of V6 and V7 are derived to be ~1.02 and ~1.09 mag, respectively. The present light curves for V7 show significant scattering in both V and R bands, indicating the presence of the Blazhko effect (see Figure 7). However, Arellano et al. (2004) did not find any sign of the presence of the Blazhko effect in V7.

A large fraction of data for stars V3 and V11 did not phase well with their derived periods (see Table 2). This suggests the presence of the Blazhko effect in these stars.

Star V12 was classified as an RRab-type variable. We could detect V12 only in the R band. In the bottom panel of Figure 7, we show the folded light curve of V12, where the filled squares represent the present data while the filled circles show data from Arellano et al. (2004). A few data points from Arellano et al. (2004), which are isolated from the light curve of V12, are shown by the asterisk symbol in Figure 7. Excluding these data points, the amplitude of this star was found to be ~1.32 mag in the R band, which is smaller than that derived by Arellano et al. (2004). A similar conclusion was also drawn by Stetson et al. (2005) about the light curve and amplitude of V12.

The period and amplitude of the star V13 derived from the present data are consistent with those of previous studies. Arellano et al. (2004) suggested it to be a double-mode RRd star or a Blazhko-type RRc. Its light curve shows a significant scatter at the maximum, whereas its present light curve in the V band shows negligible scatter. However, they also mentioned that their observations are not sufficient to confirm whether it is an RRd star or a Blazhko-type RRc-type star. Stetson et al. (2005) reported that their data did not phase with the period given by Arellano et al. (2004). From the present observations, we confirm that V13 is an RRc-type variable.

The period prior to the star V14 was found to be ~0.26 days. Previous studies have reported its period to be ~0.52 days (Newburn 1957), ~0.25 days (Arellano et al. 2004), and 0.35 days (Stetson et al. 2005). For this star, we found a slightly different period from that of Arellano et al. (2004) and Stetson et al. (2005). The previous studies also indicated that V14 is pulsating with different periods.

5.2. Newly Identified

Stars V24, V29, V32, V33, V35, V36, V42, and V47 are new variables located on the HB of NGC 4147. All of them can reasonably be assumed to be RR Lyrae stars but another consideration is their amplitudes in the V and R bands.
Amplitudes of pulsating variables are found as a function of color. As illustrated in Table 10 of Stetson et al. (2005), the amplitude of variability is always largest in $B$ and smallest in the $I$ band for RR Lyrae stars. Considering the above fact, along with the periods and shape of the light curves, of the eight new HB stars, we classify stars V29, V32, V33, V35, and V42 as RR Lyrae stars. The amplitudes of their brightness variation in the $V$ band are in the range of $\sim 0.17$ to $\sim 0.62$ mag, whereas their periods of variability are found to be in the range of $\sim 0.28$ to $\sim 0.30$ days. The $R$ amplitudes of these variables except V31 vary from $\sim 0.12$ to $\sim 0.55$ mag. Furthermore, their periods and amplitudes suggest that they could be probable RRc-type variables.

In the $V/(V - R)$ CMD, star V28 is located at the beginning of the HB i.e., very close to the red giant branch. It has been considered as a probable member of the cluster because it is located at a radial distance of $\sim 0.16$ from the cluster center. Apart from its location in the CMD, the period, amplitude, and shape of the light curve of V28 closely resemble those of RRc-type variables. Hence, it can be classified as an RRc-type variable.

Star V31 is found to be lying almost in the center of the cluster at a radial distance of 0.047. With a $V$ magnitude of 16.547 mag, star V31 is comparable to other HB stars. Its period and $V$ amplitude are $\sim 0.282$ days and 0.476 mag, respectively. The shape of the light curve, period, and amplitude indicate that V31 might belong to the RRc-type population.

To confirm the classification of newly identified RRc variables, we plotted them along with the known RR Lyrae variables except V12 in the Bailey diagram (amplitude versus period diagram), which is shown in Figure 10. The filled and unfilled circles represent known RRab and RRc pulsators, respectively, while filled squares show the newly identified RRc-type variables. As shown in Figure 10, the known RRab and RRc variables are found to be distributed in two different regions, similar to previous studies in other globular clusters.

Figure 10. The amplitude vs. period diagram (Bailey diagram) for RRab and RRc variables. Unfilled and filled circles show known RRc and RRab variables, respectively. The newly identified RRc-type variables are shown by filled squares.

All the new RRc variables as shown in Figure 10 are located firmly among known RRc variables, and this further suggests that their variability type is similar to RRc-type variables. Here, we note that star V42 has a slightly lower amplitude than the other RRc variables. Hence, the present study identifies seven variables as RR Lyrae stars and classifies them as RRc type.

5.3. Fourier Decomposition and Physical Parameters of RR Lyrae Variables

The Fourier decomposition technique consists of fitting the observed brightness variation of pulsating stars with the Fourier series

$$f(\phi) = A_0 + \sum_{k=1}^{4} A_k \cos(2\pi k\phi + \psi_k),$$

where $\phi = (t - t_0)/P$ is the pulsational phase and $t_0$ is the time of maximum brightness. The best-fit Fourier series for one RRc star as a sample is shown in Figure 11, and the best-fit Fourier parameters are given in Table 4. The structure of the light curve can be quantified in terms of a combination of low-order Fourier coefficients, in particular the amplitude ratio, $R_{k1} = A_k/A_1$, and the phase difference $\psi_k = \psi_{k1} - k\psi_1$, where $k = 1, 2, 3, 4$. In the case of RRab variables, the Fourier decomposition was done using a sine series. A careful inspection of light curves for V3 and V11 reveals that these stars show the Blazhko effect. Therefore, we have divided V3 and V11 data into two parallel light curves with different natures as shown in Figure 12, and these variables are designated as V3(1), V3(2) and V11(1), V11(2). The relation between the pulsation of RR Lyrae variables and their physical parameters can be obtained from hydrodynamic pulsation models with defined physical parameters, which are used to generate theoretical light curves. The physical parameters, $[\text{Fe/H}], T_{\text{eff}},$ and $M_V$ for RRab stars were calculated using the equations given by Jurcsik & Kovacs (1996), Kovacs & Jurcsik (1996), and Jurcsik (1998). These physical parameters derived for RRab variables are given in Tables 5. For RRc stars, using their phase difference parameter $\psi_{31}$, the metallicity $[\text{Fe/H}]$ is calculated from the relation $[\text{Fe/H}] = 52.466P^2 - 30.075P + 0.131\psi_{31}^2 + 0.982\psi_{31} - 4.198\psi_{31}P + 2.424$ given by Morgan et al. (2007), where $P$ represents the period of the variable star. The calculated values of $[\text{Fe/H}]$ for RRc-type stars are listed in Table 5.
| ID | Band | A0  | A1  | R21    | R31    | R41    | ε21 | ε31 | ε41 |
|----|------|-----|-----|--------|--------|--------|-----|-----|-----|
| 1  |      |     |     | 0.351 ± 0.003 | 0.412 ± 0.010 | 0.262 ± 0.009 | 0.142 ± 0.008 | 5.606 ± 0.024 | 4.839 ± 0.035 | 3.175 ± 0.059 |
| 2  |      |     |     | 0.261 ± 0.010 | 0.368 ± 0.044 | 0.252 ± 0.038 | 0.135 ± 0.037 | 4.672 ± 0.158 | 4.551 ± 0.194 | 4.562 ± 0.302 |
| 3  |      |     |     | 0.353 ± 0.004 | 0.408 ± 0.012 | 0.352 ± 0.009 | 0.238 ± 0.008 | 5.722 ± 0.024 | 5.249 ± 0.031 | 3.171 ± 0.040 |
| 4  |      |     |     | 0.611 ± 0.161 | 0.605 ± 0.123 | 0.280 ± 0.058 | 4.375 ± 0.170 | 4.269 ± 0.217 | 3.357 ± 0.355 |
| 5  |      |     |     | 0.285 ± 0.042 | 0.289 ± 0.024 | 0.395 ± 0.025 | 0.206 ± 0.023 | 5.213 ± 0.083 | 4.238 ± 0.067 | 3.694 ± 0.121 |

**Table 4**

The Fourier light-curve fitting parameters of individual RRab, RRc, and other variables.
5.4. Oosterhoff Classification and Distance Determination

NGC 4147 was classified by Castellani & Quarta (1987) as an Oosterhoff type I despite having a low metallicity (−1.83; Harris catalog). Oosterhoff (1939) divided globular clusters into two groups: Oosterhoff type I and Oosterhoff type II. The Oosterhoff type I globular clusters are found to be more metal-rich ([Fe/H] > −1.7) than Oosterhoff type II globular clusters ([Fe/H] < −1.7; Smith 1995). Recently, Villanova et al. (2016) presented an extensive spectroscopic study on the globular cluster NGC 4147 and found its metallicity [Fe/H] = −1.84, which is comparable to the typical metallicity of halo globular clusters. Stetson et al. (2005) estimated the value of [Fe/H] as −1.55. The present mean value of [Fe/H] for RRab variables estimated using hydrodynamic pulsation models is −1.658. For RR stars, the present mean value of [Fe/H] comes out to be −1.746, which is on the scale of Zinn & West (1984). The iron abundance estimated for RRab stars is on the Jurcsik & Kovacs (1996) scale, while the metallicity of NGC 4147 adopted by Harris (1996; 2010 edition) is on the Zinn & West (1984) scale. Therefore, to convert the present mean value of [Fe/H] for RRab stars into the Zinn & West (1984) scale, we used the relation ([Fe/H] = 1.431[Fe/H]ZW + 0.88) given by Jurcsik (1995). On the scale of Zinn & West (1984), the mean metallicity of RRab stars is, thus, translated as −1.774. The present average metallicity of RRab and RRc variables is −1.760, which is slightly smaller than that (−1.57) obtained from the cluster CMD by fitting theoretical models and found to be closer to the value provided in the Harris catalog compared with that obtained by Stetson et al. (2005). The present mean [Fe/H] value obtained from RRab and RRc variables seems to be consistent with that of Oosterhoff type II globular clusters.

The Oosterhoff classification of globular clusters is also discussed based on the mean periods of RRc and RRab variables, and the ratio of the number of RRc to RRab variables. For globular clusters, van Agt & Oosterhoff (1959) determined the mean periods of the RRc and RRab variables to be 0.319 and 0.549 days, respectively. Clement et al. (2001) found the mean RRab and RRc periods for globular clusters to be 0.559 days and 0.326 days, respectively, for Oosterhoff type I, and 0.659 days and 0.368 days, respectively, in the case of Oosterhoff type II. Based on the mean periods of the RRc and RRab variables, and the ratio of the number of RRc to RRab variables, Stetson et al. (2005) already gave a full discussion of the Oosterhoff classification of NGC 4147. They found that the mean periods of the RRc and RRab variables were characteristic of an Oosterhoff type I cluster, but the ratio of the number of RRc to RRab variables was characteristic of type II. Because the present work identified seven new RRc-type variables, we revised the value for the mean period of RRc-type stars only and the ratio of the number of RRc to RRab variables. The calculated mean period of RRc is found to be ~0.317 days. The present mean period of RRc stars is in good agreement with that given by van Agt & Oosterhoff (1959). The number ratio of the RRc including newly identified RRc variables to the total number of RR Lyrae-type variables is calculated as NC/(NC + NAb) = 0.76, which is larger than that (0.67) obtained by Stetson et al. (2005). This confirms the blue HB morphology as concluded by Stetson et al. (2005).

The mean value of the absolute magnitude, M_V, of RRab stars is determined to be 0.792 mag. From this mean M_V, the distance modulus (V − M_V) of the cluster NGC 4147 is calculated and found to be about 16.25 mag, which gives the distance to the cluster as 17.30 kpc. The mean apparent V magnitude of RRab variables is taken to be 17.049 mag, which was calculated from the V magnitudes given in Table 2. The above derived distance of the cluster NGC 4147 agrees with that (17.49 kpc) obtained from the V/(V − R) CMD.

6. Other Variables

There are several studies that show that globular clusters also contain variables other than RR Lyrae stars. These could be SX Phe, eclipsing binaries, semiregular (SR) and other types (McNamara 1995; Kaluzny 1996; Mazur et al. 2003; Arellano et al. 2011; Kopacki & Pigulski 2012; Kopacki 2015; Martinazzì et al. 2015). Recently, Kaluzny et al. (2016) and Rozyczka et al. (2017), in the case of globular clusters NGC 2301 and M22, presented numerous light curves for eclipsing binaries and SX Phe variables, and classified them based on their light curves, periods, and amplitudes. The present work contains 28 new variables, seven of which have been classified as RRc variables (see Section 5.2). Out of the 21 remaining variables, 17 variables, namely V21, V22, V24, V25, V27, V30, 34, V36, V37, V38, V39, V40, V41, V43, V45, V46 and V47 could be eclipsing binaries, out of which stars V24, V27, V30, and V36 are found to be located within the half-light radius of the cluster. These stars, according to the variability types listed in the Moscow General Catalog of Variable Stars (GCVS), are classified as Eclipsing binary systems (E), Algol (Beta Persei)-type eclipsing systems (EA), and W Ursae Majoris-type eclipsing variables (EW). Based on the light curves, stars V24, V36, V37, V39, and V46 are classified as EA type, while stars V22, V25, and V30 are classified as E type, and variables V21, V27, V40, V41, and V47 as EW-type binaries.

The shape of the light curves for V34 and V43 look like RR Lyrae variables. Their variability characteristics do not match with their location on the V/V − R CMD. Therefore, these two could be probable RR Lyrae stars that might belong to the field population, and these are called suspected field stars. In Section 3.2, V34 and V43 were considered to be probable members of the cluster on the basis of their geometric probabilities, where we did not consider variability type. We could not classify star V45 on the basis of its light curve, period, and amplitude.

The probable member variable, V38, is found to be located 0°846 away from the cluster center, and in the V/(V − R) CMD it lies in the location of blue stragglers. The location in the V/(V − R) CMD, period, and amplitude of V38 suggest that it could be a probable candidate of SX Phe-type variables.

Stars V20, V23, V26, and V44 are identified as field stars based on their geometric probabilities. Among these four stars, V20 and V44 seem to be EA-type variable based on their light curves.

We could not detect any SR-type variable in the present cluster, and this might be due to the short span of the observations. In the CMD of globular clusters, SR-type
variables are located near the top of the red giant branch, brighter than the HB. According to the GCVS, they pulsate with a period from tens of days and longer. A good discussion of the SR variables in M13 was recently presented by Osborn et al. (2017) in which they have shown CMD positions, periods, and classifications of SR variables.

6.1. Classification Based on Fourier Parameters

The automated classification of variable stars is necessary because of large data sets. Recently, Graham et al. (2017) discussed the challenges in automated classification of variable stars. A useful tool for automated classification is the Fourier decomposition of light curves. Nowadays, the Fourier decomposition technique is used to distinguish between eclipsing binaries and RR Lyrae variables. An illustration of this technique is given in Masci et al. (2014), about the automated classification of periodic variable stars. Their paper deals with RR Lyrae-, EA-, and EW-type variables, and they use the Fourier phase difference, $\phi_21$ and $\phi_31$, as well as the amplitudes $A2$ and $A4$.

In view of the above, to see the distribution and verify the present classification of variables, we plotted all of the present identified variables in various combination of their Fourier parameters in Figure 13. The Fourier parameters of all present eclipsing binaries, field stars, and the unclassified star V45 including one SX Phe were estimated in a manner similar to that discussed in Section 5.3 and listed in Table 4. The A1/A4 diagram of Figure 13 shows that RR Lyraes and eclipsing binaries are populated in different regions. In Figure 13, the A1/A4 diagram shows more clear separation of RR Lyraes and eclipsing binaries in comparison to the A2/A4 and A3/A4 diagrams. The A2/A4 diagram segregates different types of variable stars more than the A3/A4 diagram but not to the extent of the A1/A4 diagram. In the present study, it seems that EW-type variables are located with the EA type in the A1/A4, A2/A4, and A3/A4 diagrams, but in Figure 3 of Masci et al. (2014), within the given range of 0.05 < $A2$ < 0.02 and 0.05 < $A4$ < 0.01, EW-type variables are found to be distributed among EA-type variables. The distribution of present EA-, EW-, and RRC-type variables seems to be consistent with that given by Masci et al. (2014). It seems that the present classification of eclipsing binaries based on their light curves is reasonable. The $\psi_{31}/\psi_{41}$ diagram shows that new RRC member variables are located among previously known RRC-type variables, while from this diagram it is very difficult to discuss the nature of the present eclipsing binaries.

7. Summary

The VR CCD photometry and detection of 42 periodic variables in the region of NGC 4147 are presented using data taken from the 3.6 m DOT, ARIES, India. For the majority of the known variables, we present better light curves. Twenty-eight new variables were detected in the present work, and most of them belong to the HB and red giant branch. Ten new variable stars are distributed within the half-light radius $\leq 0.48$ of the cluster, two of which are located within the core radius $\leq 0.09$. The membership of variable candidates was established on the basis of the $V/(V - R)$ CMD, proper motions, and
geometric probabilities, and a majority of them are found to be members of the cluster. Seven newly identified probable members of the cluster could be RRc type, while we have classified eight new members as AE/E, five as EW-type eclipsing binaries, and one as an SX Phe-type variable. The iron abundance estimated from RRab- and RRc-type variables is indicative of NGC 4147 being an Oosterhoff type II cluster. The distance modulus of the cluster derived from the light curves of RRab variables using the Fourier decomposition technique is $\sim$16.25 mag, which agrees with that (16.40 mag) obtained from the cluster $V/(V-R)$ CMD by fitting theoretical models.

The authors are thankful to the anonymous referee for critical suggestions/comments. We are thankful to the governing council chairmen, Prof. K. Kasturirangan, Prof. S. K. Joshi, Prof. G. Swarup, and all of the PMB members for their guidance in successfully installing the 3.6 m DOT. The authors are highly grateful to the former director of ARIES, Prof. Ram Sagar for his significant contribution towards 3.6 m DOT. We thank our colleagues, Dr. Maheswar Gopinathan, Dr. Amitesh Omar, Dr. Alok C. Gupta, and Dr. Santosh Joshi, for their contribution to the 3.6 m DOT project. The authors acknowledge all of the technical, administrative, and supporting staff of ARIES for their wholehearted support in the realization of the 3.6 m DOT project.

Figure 13. Relative Fourier parameters for all variables identified in the present work. The different symbols used are filled circles (known RRab), unfilled circles (known RRc), crosses (new RRc type including two suspected field stars V34 and V43), unfilled triangles (EA/E including SX Phe and two field stars V20 and V44), square (EW), and filled triangle (V23, V26 and V45). The crowded region at the left corner of the amplitude ratio plots is zoomed into in the inset of these plots.

ORCID iDs
Sneh Lata @ https://orcid.org/0000-0001-9367-1580
Yogesh C. Joshi @ https://orcid.org/0000-0001-8657-1573
Saurabh Sharma @ https://orcid.org/0000-0001-5731-3057

References
Arellano, F. A., Arévalo, M. J., Lázaro, C., et al. 2004, RMxAA, 40, 209
Arellano, F. A., Figuera, J. R., Giridhar, S., et al. 2011, MNRAS, 416, 2265
Baade, W. 1930, AN, 239, 353
Baker, J. M., Layden, A. C., Welch, D. L., & Webb, T. M. A. 2007, AJ, 133, 139
Cabrera-Cano, J., & Alfaro, E. J. 1990, A&A, 235, 94
Cáceres, C., & Catelan, M. 2008, ApJS, 179, 242
Castellani, V., & Quarta, M. L. 1987, A&A, 71, 1
Clement, C. 1997, A Third Catalogue of Variable Stars in Globular Clusters, http://www.astro.utoronto.ca/~clement/cat/clusters.html
Clement, C. 2017, EPJWC, 152, 01021
Clement, C. M., Muzzin, A., Duffen, Q., et al. 2001, AJ, 122, 2587
Clementini, G., Gratton Raffaele, G., Bragaglia, A., et al. 2005, ApJ, 630, 145
Corwin, T. M., Sumerel, A. N., Pritzl, B. J., et al. 2006, AJ, 132, 1014
Davis, H. 1917, PASP, 29, 260
Gaia Collaboration et al. 2018, A&A, 616, 13
Girardi, L., Bertelli, G., Bressan, A., et al. 2002, A&A, 391, 195
Graham, M., Drake, A., Djorgovski, S. G., Mahabal, A., & Donalek, C. 2017, EPJWC, 152, 03001
Harris, W. E. 1996, AJ, 112, 1487
Jurcsik, J. 1995, AcA, 45, 653
Jurcsik, J. 1998, A&A, 333, 571
Jurcsik, J., & Kovacs, G. 1996, A&A, 312, 111
