Current-induced nonreciprocity and refraction-suppressed propagation in a multilayered graphene-dielectric crystal

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Abstract
Nonreciprocal photonic devices play a significant role in regulating the propagation of electromagnetic waves. Here we theoretically investigate the nonreciprocal properties of transverse magnetic modes in a multilayered graphene-dielectric crystal under an applied DC bias. We find that drifting electrons driven by the external DC electric field can give rise to extremely asymmetric dispersion diagrams. Furthermore, when the drifting electrons travel antiparallel to the normal component of the incident wave vector, negative refraction can be strongly suppressed, causing the energy of light to flow along the direction of the electric current. Our theoretical findings can be used to design nonreciprocal optoelectronic devices and enable light to propagate without refraction.

Keywords: nonreciprocity, graphene, drifting electrons, surface plasmon polaritons, negative refraction, transfer-matrix methods, multilayered graphene-dielectric crystals

1. Introduction
Optical nonreciprocity can be achieved by breaking time-reversal symmetry (Lorentz reciprocity) in photonic systems [1, 2]. Nonreciprocal photonic devices based on time-reversal symmetry breaking play a significant role in manipulating the flow of light. In general, such devices allow light to propagate in one direction, but suppress the flow of light in the opposite direction [3, 4]. There are various ways to break the time-reversal symmetry, including optical nonlinearities [5–8], spatiotemporal modulations [9–15], and magneto-optical effects [16–31]. Very recently, the issue of truly unidirectional surface plasmon polaritons (SPPs), based on magneto-optical effects, has attracted much attention. According to [25], if nonlocality and dissipation were taken into consideration, strictly unidirectional SPPs cannot exist in the nonreciprocal plasmonic systems. However, in a follow-up theoretical study, the authors found that a class of truly one-way propagating SPPs, located in the upper bulk-mode bandgap characterized by the so-called gap Chern number, can robustly exist at the interface between an opaque material and a magnetized plasma, even though nonlocal effects and realistic dissipation were included [28–30]. This shows that nonlocality may play an important role in studying the topic of nonreciprocal responses, but the inclusion of nonlocal effects does not hinder the emergence of truly unidirectional SPPs in some cases.

Recently, nonreciprocal responses in plasmonic platforms have been realized by applying an external DC voltage [32–42]. Such a novel approach can lead to a Doppler frequency shift of SPPs [35–37], wherein forward-propagating (backward-propagating) waves traveling parallel (antiparallel) to the direction of drifting electrons obtain a blueshift (redshift) and the corresponding propagating length is enhanced (suppressed) [36, 37]. Thus, the dispersion diagram of the SPPs will become extremely asymmetric, resulting in nonreciprocal responses. However, the effect of drifting electrons on a multilayered graphene-dielectric crystal has, to the best of our knowledge, not been explored.
In this work, we investigate the nonreciprocal properties of transverse magnetic (TM) modes in a multilayered graphene-dielectric crystal subjected to an external electric field. Using the transfer matrix method, we find that the band structure of TM modes and the corresponding hyperbolic isofrequency contours (IFC) become asymmetric as drift velocity increases. Moreover, we show that drifting electrons driven by the electric field can suppress the refraction angles of the wave vector and group velocity. Such properties may offer a novel route to control light propagation.

2. Theoretical model

Consider a periodic multilayer structure consisting of graphene sheets and dielectric materials as shown in figure 1. Since the transverse electric modes maintain time-reversal symmetry in the presence of an external bias and the corresponding dispersion lacks hyperbolic-like characteristic [43–45], here we mainly focus on the TM-polarized modes. By using the transfer matrix method and applying the Bloch theorem, the dispersion relation for the multilayered graphene-dielectric crystal can be expressed as [46, 47]

\[
\cos(K_Bd) = \cos(k_d) - \frac{1}{2} \frac{\omega_d^{\text{drift}}}{\omega_0\varepsilon_d} k_z \sin(k_d),
\]

where \(K_B\) is the Bloch wave vector, \(d\) is the period, \(\varepsilon_0\) is the vacuum permittivity, \(\varepsilon_d\) is the relative permittivity, \(k_z = \sqrt{\varepsilon_d(\omega/e)^2 - k_x^2}\) (assuming \(k_y = 0\)) is the \(z\) component of the wave vector, and \(\sigma^{\text{drift}}\) is the nonlocal conductivity of graphene with drifting electrons. In the absence of a DC bias voltage, the general nonlocal conductivity of graphene can be obtained by employing the random-phase approximation [48–52]. Then we have

\[
\sigma_g(\omega, k_z) = i\omega \frac{e^2}{k_z^2} \Pi(\omega, k_z),
\]

with

\[
\Pi(\omega, k_z) = \frac{k_z^4}{4\pi^2 \sqrt{\omega^2 - k_z^4}} \times \left[ G \left( \frac{2k_Fv_F + \omega}{k_zv_F} \right) - G \left( \frac{2k_Fv_F - \omega}{k_zv_F} \right) \right] - \frac{2\epsilon}{\pi\nu_F\hbar}.
\]

Applying a DC bias voltage to the multilayered graphene-dielectric crystal can induce drifting electrons in each graphene sheet (see the green arrow in figure 1). In this scenario, the corresponding nonlocal conductivity can be modified as [39–41]

\[
\sigma^{\text{drift}}_g(\omega, k_z) = \frac{\omega}{\omega - k_zv_0} \sigma_g(\omega - k_zv_0, k_z),
\]

where \(v_0\) is the velocity of drifting electrons traveling along the \(x\) direction. Such a current-carry nonlocal conductivity model can be obtained either by using a quantum mechanical method based on the self-consistent field approach [40], or by solving the semiclassical Boltzmann transport equation [41]. A detailed discussion about the variation of the current-carrying nonlocal graphene conductivity with either \(k_z\) or \(\omega\) can be found in [39]. It should be noted that the Doppler-shifted factor \(\omega - k_zv_0\) caused by the drifting electrons makes the nonlocal conductivity of graphene become nonreciprocal, i.e. \(\sigma^{\text{drift}}_g(\omega, k_z) \neq \sigma^{\text{drift}}_g(\omega, -k_z)\). This implies that time-reversal symmetry breaking in graphene-based plasmonic systems can be achieved by applying an electrical DC bias [37, 39].

3. Numerical results and discussions

The dispersion relation of TM-polarized propagating waves is given by solving equation (1). Here we consider \(\varepsilon_d = 4\), \(E_F = 0.1\) eV, \(\gamma = -500\) fs, \(d = 100\) nm and \(v_F = 10^6\) m s\(^{-1}\). These parameters are used throughout the rest of the paper. As shown in figure 2, the orange shaded regions correspond to the plasmonic (polaritonic) modes arising from the coupling of SPPs between adjacent graphene layers, while green color regions depict the photonic modes conformed to the light line [54–58]. Note that the allowed band of the plasmonic mode will shrink progressively to the dispersion of a SPP for a single graphene layer (represented by blue curves in figure 2), as the period of the structure increases [59]. Since the dispersion curve for the plasmonic mode is characterized by hyperbolas in the wave vector space, and most importantly, such a hyperbolic dispersion characteristic is responsible for negative refraction [54, 60], in what follows we focus only on the plasmonic mode. In figure 2(a), in the absence of DC electrical
current, \( v_0 = 0 \), the dispersion diagrams associated with the forward plasmonic band \( k_x > 0 \) and backward plasmonic mode \( k_x < 0 \) are symmetric about the \( k_x = 0 \) axis. In other words, the time-reversal symmetry is preserved. However, as illustrated in figure 2(b), increasing the electron drift velocity to \( 0.65v_F \) gives rise to an extremely asymmetric band structure, namely, \( \omega(k_x) \neq \omega(-k_x) \). Hence, a strong nonreciprocal response of TM-polarized modes also emerges in a multilayered graphene-dielectric nanostructure under an applied DC bias.

In particular, the red arrows showed in figure 2 indicate the band-crossing (degeneracy) point, where the photonic band and plasmonic band touch each other \([56–58]\). Expanding the dispersion equation equation \((1)\) in powers of \( k_zd \), and then substituting \( K_B = 0 \) into the resulting expansion, yielding \([57]\)

\[
k_z^2 = \frac{12}{d^2} \frac{d}{d + 4d} \left( \frac{i \sigma_{\text{drift}}}{2 \omega_0 \varepsilon_d} \right).
\]

Next, plugging \( k_z = 0, k_x = -\sqrt{\varepsilon_d \omega/c} \) and \( v_0 = 0.65v_F \) into equation \((6)\), we readily obtain the operation frequency of the band-crossing point for the backward plasmonic mode \( f \approx 8.97 \text{ THz} \) (\( f = \omega / 2\pi \)).

To further examine the feature of nonreciprocal responses, we plot the IFC of the multilayered graphene-dielectric crystal with different electron drift velocity values in figure 3. For \( v_0 = 0 \) (see the green curves), the hyperbolic contour of the plasmonic mode remains symmetric. Remarkably, the presence of the drifting electrons makes the hyperbolic contour strongly asymmetrical with respect to the \( k_z = 0 \) axis.

**Figure 2.** Dispersion diagrams of TM-polarized modes for different drift velocity values. The red arrows in the insets indicate the band-crossing points. Colored regions correspond to the allowed bands, wherein the photonic modes are shown by green color and the plasmonic modes are represented by orange color. The blue line depicts the dispersion curve of a single graphene sheet with the same dielectric permittivity \( \varepsilon_d = 4 \). Other parameters are \( E_F = 0.1 \text{ eV}, d = 100 \text{ nm}, v_F = 10^6 \text{ m s}^{-1} \) and \( \gamma^{-1} = 500 \text{ fs} \).
As shown in figure 3(a), at the operation frequency $f = 8.5$ THz, the backward plasmonic mode ($k_x < 0$) undergoes the so-called topological transition with the increase of the drift velocity, and finally forms a closed ellipse [24]. Interestingly, as the operation frequency increases to 8.97 THz, such a closed ellipse will shrink to a point (i.e. the band-crossing point). By contrast, as clearly seen in figure 3(b), for the operation frequency corresponding to the band-crossing point $f = 8.97$ THz, increasing the drift velocity gradually causes the dispersion curve of the backward plasmonic mode to become almost flat. Such an interesting characteristic and the resultant effect on negative refraction will be discussed in what follows. Meanwhile, for the Bloch wave vector $K_B = 0$ and drift velocity $v_0 = 0$, the magnitudes of the wave vectors for the forward plasmonic mode and backward plasmonic mode are equivalent. Nevertheless, as the drift velocity increases, in stark contrast to the forward plasmonic mode, the magnitude of the backward wave vector has a considerable increase. This means that the band gap, which is the magnitude of the difference between the wave vector of two plasmonic modes (forward mode and backward mode), becomes asymmetric about the $k_x = 0$ axis. Such an asymmetric band gap is useful for examining the one-way total transmission [16].

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We now consider the effect of drifting electrons on the refraction angle. As schematically shown in the inset of figures 4(a) and (b), Let us suppose a TM-polarized wave with the wave vector $k_i$ oblique incident from a dielectric material with permittivity $\varepsilon_i$ to the multilayered graphene-dielectric crystal that carries a DC electrical current. The group velocity (direction of energy flow) for the TM-polarized mode (see the red arrows in the figure 4) can be written as [27, 31]

$$V_g = \frac{\partial \omega}{\partial k_x} \hat{x} + \frac{\partial \omega}{\partial K_B} \hat{z}.$$  

As seen in figure 4(b), when the electrons are traveling toward the $-x$ direction, the corresponding IFC is nearly flat, which means that incident waves with different wave vectors can be converted to TM-polarized waves with almost the same wave vectors. Such a nearly flat IFC, also called as canalization, can be used to suppress diffraction [60, 61]. According to the boundary continuity condition of the electromagnetic wave propagation, the tangential component of the incident wave vector $k_i = \omega/c$ should be continuous at the interface between an isotropic material and a graphene-based photonic crystal [62, 63], namely, $K_B = k_i \sqrt{\varepsilon_i} \sin \theta$. The refraction angles of the wave vector and group velocity can be defined as [62]

$$\theta_k = \arctan \left( \frac{K_B}{k_x} \right),$$  

$$\theta_g = \arctan \left( \frac{\partial \omega}{\partial K_B} \frac{\partial \omega}{\partial k_x} \right).$$  

respectively. Figure 4(c) shows the refraction angles of the wave vector and group velocity for different drift velocity values. When the drifting electrons move along the +x direction, that is, the direction of the drifting electrons is parallel to the normal component of the incident wave vector, the drift velocity has an insignificant impact on the refraction angle. By contrast, for the electrons moving in the −x direction, the refraction angle of the wave vector gradually decreases with the increase of the drift velocity. Notably, the refraction angle of the group velocity goes down from 80° to nearly 0°, a drop of 80°. It should be noted that the refraction angle of the wave vector as well as group velocity drop to nearly 0°, that is, drifting electrons driven by an external electrical field can drag the wave vector and group velocity to the same direction, which makes the electromagnetic wave energy flow along the direction of the DC current. Such properties not only can be used to design tunable nonreciprocal devices but also offer a novel route to control the light propagation.

4. Conclusion

In summary, we have theoretically discussed the dispersion characteristics of a multilayered graphene-dielectric crystal that carries a DC current. Our numerical results indicate that drifting electrons driven by an external electric field can lead to asymmetric hyperbolic dispersion diagrams. Furthermore, when the drifting electrons travel antiparallel to the normal component of the incident wave vector, the negative refraction will be suppressed strongly. This means that the drifting electrons with a large enough drift velocity value can drag the wave vector and group velocity to the same direction, which makes the electromagnetic wave energy flow along the direction of the DC current. Such properties not only can be used to design tunable nonreciprocal devices but also offer a novel route to control the light propagation.

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Figure 4. (a), (b) Isofrequency contours of the graphene-dielectric periodic nanostructure with drifting electrons traveling in different directions at a given drift velocity \( v_0 = 0.65 v_f \). Each inset shows a TM-polarized wave incident from an isotropic material with permittivity \( \varepsilon = 4 \), where the green arrow represents the direction of the drifting electrons. The red arrow indicates the direction of the group velocity, while the dashed line represents the continuity of the tangential component of \( k_i \) at the interface. (c) Refraction angles of the wave vector and group velocity vector as functions of incident angles at a given drift velocity \( v_0 \). (d) Refraction angles for the wave vector and group velocity vector as functions of incident angles at a given drift velocity \( v_0 = 0.65 v_f \). Each inset shows a TM-polarized wave incident from an isotropic material with permittivity \( \varepsilon = 4 \), where the green arrow represents the direction of the drifting electrons. The red arrow indicates the direction of the group velocity, while the dashed line represents the continuity of the tangential component of \( k_i \) at the interface. (c) Refraction angles of the wave vector and group velocity vector as functions of incident angles at a given drift velocity \( v_0 \). (d) Refraction angles for the wave vector and group velocity vector as functions of incident angles at a given drift velocity \( v_0 = 0.65 v_f \). Each inset shows a TM-polarized wave incident from an isotropic material with permittivity \( \varepsilon = 4 \), where the green arrow represents the direction of the drifting electrons. The red arrow indicates the direction of the group velocity, while the dashed line represents the continuity of the tangential component of \( k_i \) at the interface. (c) Refraction angles of the wave vector and group velocity vector as functions of incident angles at a given drift velocity \( v_0 \).
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