Stationary models for fast and slow logarithmic spiral patterns in disc galaxies

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ABSTRACT

A recent wavelet analysis on multiwavelength image data of the nearby spiral galaxy NGC 6946 revealed a multi-arm spiral structure that persists well into the outer differentially rotating disc region. The extended spiral arms in polarized radio-continuum emission and in red light appear interlaced with each other, while the spiral arms in emissions of total radio continuum, of Hα from H II regions, and of neutral hydrogen all trace the red-light spiral arms, although to a somewhat lesser extent. The key issue now becomes how to sustain extended slow magnetohydrodynamic (MHD) density wave features in a thin magnetized disc with a flat rotation curve.

We describe here a theoretical model to examine stationary non-axisymmetric logarithmic spiral configurations constructed from a background equilibrium of a magnetized singular isothermal disc (MSID) with a flat rotation curve and with a non-force-free azimuthal magnetic field. It is found analytically that two types of stationary spiral MSID configurations may exist, physically corresponding to the two possibilities of fast and slow spiral MHD density waves. Such stationary MHD density waves are possible only at proper MSID rotation speeds.

For the fast MSID configuration, logarithmic spiral enhancements of magnetic field and gas density are either in phase in the tight-winding regime or shifted with a spatial phase difference $\pi/2$ for open spiral structures. For the slow MSID configuration, logarithmic spiral enhancements of magnetic field and gas density are either out of phase in the tight-winding regime or shifted with a spatial phase difference $\pi/2$ for open spiral structures and persist in a flat rotation curve. For NGC 6946, several pertinent aspects of the slow MSID scenario with stationary logarithmic spiral arms are discussed. The two exact solutions can be also utilized to test relevant numerical MHD codes.

Key words: accretion, accretion discs – MHD – waves – ISM: general – galaxies: individual: NGC 6946 – galaxies: spiral.

1 INTRODUCTION

Magnetic field image mapping from polarized non-thermal radio-continuum emissions of the nearby spiral galaxy NGC 6946 started more than two decades ago (e.g., Klein et al. 1982; Harnett, Beck & Buczilowski 1989; Ehle & Beck 1993; Beck et al. 1996). In terms of the large-scale spiral structure of magnetic field, what distinguishes NGC 6946 from other nearby spiral galaxies such as M51, M31, NGC 2997, etc. was the key observation of Beck & Hoernes (1996) that its spiral arms in optical bands and in polarized radio-continuum emissions were interlaced (rather than overlapped) with each other (fig. 2 of Beck & Hoernes 1996) in the central disc portion of almost rigid rotation, although features of similar spiral arm displacements were also suspected earlier in the late-type spiral galaxies IC 342 (e.g., Krause, Hummel & Beck 1989) and M83 (e.g., Sukumar & Allen 1989).

From the perspective of large-scale galactic magnetohydrodynamics (MHD) and in the context of M51 and NGC 6946, Fan & Lou (1996) proposed the physical concepts of fast and slow spiral MHD density waves that could manifest in a rotating, magnetized, self-gravitating thin gas disc (Fan & Lou 1997, 1999; Lou & Fan 1997, 1998a,b; Rogava, Poedts & Heirman 1999; Hau & Chou...
In the tight-winding or WKBJ regime, spiral enhancements of magnetic field and surface mass density are grossly in phase for fast MHD density waves (FMDWs), while these two spiral enhancements are significantly phase shifted for slow MHD density waves (SMDWs). The underlying physical rationale for multi-wavelength emissions of galactic spiral structures is that, except for a time delay of \(-10^7\) yr in star formation, the high-density gas regions would eventually give rise to optical arms as a result of incessant cloud and star formation activities which further lead to enhancements of random small-scale magnetic field with stronger total radio-continuum emissions. More specifically, if the relativistic cosmic-ray electron gas is largely confined to the galactic disc with a more or less smooth distribution, then polarized radio-continuum emission arms and optical arms would appear interlaced with each other for SMDWs in NGC 6946 (Fan & Lou 1996). For galactic FMDWs, in contrast, the total and polarized radio-continuum emissions are always in competition to a certain extent because the spiral enhancements of gas density and magnetic field are grossly in phase. It is the time delay in cloud and star formation that partially helps retain polarized radio-continuum signals from being completely overwhelmed by total radio-continuum emissions that are usually much stronger. Examples of galactic FMDWs include M51, NGC 2997, M31 etc. (Lou & Fan 1998a, 2000; Lou, Han & Fan 1999; Lou et al. 2002) and perhaps, the circumnuclear spiral structures of NGC 1097 and NGC 6951 on kiloparsec scales (Barth et al. 1995; Lou et al. 2001b).

In terms of kinetic dynamo models, there were suggestions that spatial variations of the dynamo number (e.g., Moss 1998; Shukurov 1998) or an azimuthal variation of only the correlation time of interstellar turbulence (e.g., Rohde, Beck & Elstner 1999) may produce interlaced optical and magnetic spiral structures. We believe that the dynamo theory is powerful in understanding the origin of galactic magnetic fields when seed fields are sufficiently weak. With the growth of a mean magnetic field, stellar density waves coupled with MHD density waves in the interstellar medium gradually form large-scale spiral structures that are observable in various wavebands. We have described on several occasions earlier (e.g., Lou & Fan 2000) that the dynamo process and the MHD density wave process are important in understanding different aspects of galactic magnetic fields. For example, the dynamo process is crucial in generating random small-scale magnetic fields along optical spiral arms. In terms of the physical principle, both processes are contained in the full MHD equations. In terms of the mathematical treatment, it is the intrinsic non-linearity and the multiple scales of the problem that has prevented so far a self-consistent solution of the two MHD processes. We discuss here the MHD density wave aspect of the spiral structure problem that is not immediately connected with the details of the turbulent dynamo mechanism for generating galactic magnetic fields.

The discovery (Ferguson et al. 1998) of recent star formation activities in the extreme outer regions of disc galaxies (NGC 6946 included) with narrow faint large-scale spiral arms supports the physical notion that the magnetized neutral hydrogen H I disc is an integral component in the overall MHD density wave scenario (Lou et al. 1999, 2002). The association of broad weak H I arms and faint stellar arms with Hα arms of H II regions as well as their correlations with polarized radio-continuum emission arms offer a comprehensive view for the manifestation of MHD density waves in a spiral galaxy. A recent wavelet analysis on multi-wavelength image data of NGC 6946 by Frick et al. (2000) indicated that interlaced spiral arms in red light and polarized radio continuum extend well into the disc region of a largely flat rotation curve (e.g., Tacconi & Young 1989; Sofue 1996). For earlier model prescriptions (Lou & Fan 1998a), SMDWs may manifest extensively in an almost rigidly rotating disc portion but are restricted to a narrow ring-like zone with a flat rotation curve. So, the major challenge one has to confront with is to figure out a way of sustaining an extended manifestation of SMDW patterns in a flat rotation curve, because the original galactic SMDW model was developed for the nearly rigid rotating portion of a disc with a background azimuthal field that is force free (Fan & Lou 1996; Lou & Fan 1998a).

In this Letter, we advance a stationary logarithmic spiral model for magnetized singular isothermal disc (MSID) configurations. There exist stationary fast and slow MSID configurations with logarithmic spiral arms, corresponding to FMDWs and SMDWs respectively. While the slow MSID configuration model is idealized, we note none the less that some salient features might be pertinent to NGC 6946. In particular, we point out a sensible way of resolving the key issue of an extended manifestation of SMDW features in a flat rotation curve. For more specifics of NGC 6946, the reader is referred to observations by Beck & Hoernes (1996), Ferguson et al. (1998), and Frick et al. (2000) as well as references contained therein.

### 2 STATIONARY MSIDS WITH LOGARITHMIC SPIRAL PATTERNS

Our model is constructed using the cylindrical coordinates \((r, \theta, \phi)\). For an axisymmetric MSID background, the angular rotation speed is \(\Omega = aD/r\) where \(a\) is the isothermal sound speed and \(D\) is a dimensionless rotation parameter. On large spatial and temporal scales, the frozen-in condition of the magnetic field in an interstellar medium is valid and the ideal MHD serves as a useful first approximation. If magnetic field lines were ‘connected’ along spiral arms, then the disc differential rotation would have tightly wound them up during the lifetime of a spiral galaxy.\(^1\) This ‘magnetic field winding dilemma’ is similar to the ‘winding dilemma’ of optical spiral arms which, in fact, stimulated the early development of galactic density wave theory (Lin & Shu 1964; Goldreich & Lynden-Bell 1965; Shu 1970; Binney & Tremaine 1987; Bertin & Lin 1996). The background magnetic field \(B_0\) with a non-force-free scaling \(r^{-1/2}\), is thus taken to be azimuthal to avoid the ‘magnetic field winding dilemma’ (Lynden-Bell 1966; Roberts & Yuan 1970). The Alfvén wave speed \(C_\Lambda\) in an MSID is defined by \(C_\Lambda^2 = (\int \hat B^2 dz)/(4\pi \Sigma_0)\) where \(\Sigma_0 \propto r^{-1}\) is the surface mass density. The epicyclic frequency \(\kappa\) is defined by \(\kappa^2 = (2\Omega r) (r^2 \Omega dr)/dr = 2\Omega^2\). To attribute a fraction \((1 - F)\) of the total gravitational potential \(\phi_t\) in the background equilibrium to an axisymmetric dark matter halo that is unresonant to coplanar MHD perturbations in an MSID, one may expediently write \(\phi_t = F\phi_t^*(1 - F)\phi_t^*\) where \(\phi_t^* = d\phi_t/4\pi\) and \(F\phi_t^*\) is due to the MSID with \(0 < F < 1\) (e.g. Syer & Tremaine 1996; Shu et al. 2000). The background rotational equilibrium of an MSID is thus characterized by

\[
-a^2D^2 \frac{r}{r} = a^2 \frac{r}{r} - \frac{d\phi_t}{dr} \frac{C_\Lambda^2}{2r^2},
\]

\(^1\) Alternatively, should a strong turbulent magnetic diffusion indeed operate, then a mean radial magnetic field may survive in the dynamo scenario without such a winding problem. One might even start an MHD perturbation analysis in a non-azimuthal background magnetic field.
where \( F \Phi _0 = 2 \pi G \Sigma _0 \) from the Poisson equation and \( G \) is the gravitational constant. In the disc plane at \( z = 0 \), the gravity and the net Lorentz force are radially inward (an outward magnetic pressure force but a stronger inward magnetic tension force), while the gas pressure and centrifugal forces are radially outward. For a partial MSID with \( 0 < P < 1 \), the surface mass density is simply given by

\[
\Sigma _0 = \frac{F[a^2(1 + D^2) - C_A^2/2]}{2 \pi G r}.
\] (2)

For non-axisymmetric coplanar MHD perturbations that are sinusoidal in azimuthal angle \( \theta \) and stationary in an inertial frame of reference, besides a common periodic factor of \( \exp (-i m \theta) \), we denote the radial variations \( S(r) \), \( U(r) \), \( J(r) \), \( V(r) \), \( R(r) \), and \( Z(r) \) for perturbations in surface mass density, radial velocity, specific vertical angular momentum, gravitational potential, radial magnetic field, and azimuthal magnetic field, respectively. The index \( m \) represents the number of spiral arms.

From the MHD equations (e.g., Lou & Fan 1998a), we derive the following two-dimensional equations for stationary perturbations in an inertial frame of reference, namely

\[
m \Omega S + \frac{1}{r} \frac{d}{dr} (r \Sigma _0 i U) + \frac{m \Sigma _0}{r^2} J = 0
\] (3)

for the mass conservation,

\[
m \Omega i U + \frac{2m \Omega U}{r} = - \frac{d \Phi }{dr} - \frac{C_A^2 S}{2 \Sigma _0 r} + \frac{C_A^2 m i U}{m \Omega r}
\]

\[- \frac{C_A^2}{r} \left( \frac{d}{dr} \left( \frac{1}{2r} \right) \right) \left( \frac{i U}{m \Omega r^{1/2}} \right) \]

(4)

for the radial momentum equation with \( \Phi = a^2 S \Sigma _0 + V \),

\[
m \Omega J + \frac{r \kappa ^2}{2 \Omega} i U = - m \Phi + \frac{C_A^2 i U}{2 m \Omega r}
\] (5)

for the azimuthal momentum equation,

\[
V(r) = - G \int _0 ^{\infty } S(\zeta ) \cos (m \chi ) \frac{d \zeta }{\zeta ^2 + r^2 - 2 \xi r \cos \chi }^{1/2}
\] (6)

for the Poisson integral relation,

\[
Z = - \frac{1}{m} \frac{d (r i R)}{dr}
\] (7)

for the divergence-free condition of the magnetic field, and

\[
iR = \frac{R \Phi i U}{m \Omega r}
\] (8)

for the radial component of the magnetic induction equation. Equations (3)–(8) contain both aligned and unaligned or spiral solutions (Kalnajs 1973; Shu et al. 2000). We here focus on the latter. Note that distortions to the background magnetic field are accompanied by radial and azimuthal flow velocities.

For the unaligned or spiral case of coplanar stationary MHD perturbations in an MSID, straightforward combinations of equations (3)–(5) lead to the two equations below,

\[
\left\{ m^2 \Omega ^2 r^2 - \kappa ^2 r^2 + C_A^2 = C_A \left[ m^2 - \left( \frac{C_A^2}{2 \Omega ^2 r^2} - \frac{3}{2} \right) \right] \times \left( \frac{C_A^2}{2 \Omega ^2 r^2} - 2 \right) \right\} \frac{d i U}{dr} = m \Omega r \left( \frac{d \Phi }{dr} + 2 \Phi - \frac{C_A^2 S}{2 \Sigma _0} \right)
\]

(9)

\[
+ C_A^2 \left( \frac{d}{dr} \left( \frac{C_A^2}{2 \Omega ^2 r^2} - \frac{5}{2} \right) \right) \left( \frac{m \Omega r S}{\Sigma _0} - \frac{m \Phi }{m \Omega r} \right) = 0.
\] (10)

For a logarithmic spiral of a constant pitch angle \( \delta \), one has the known potential-density pair \( S = sr^{-3/2 + i \alpha} \) and \( V = q r^{-1/2 + i \alpha} \) where the parameter \( \alpha \) characterizes the radial variation of perturbations and the two constant coefficients \( s \) and \( v \) are related by \( v = - 2 \pi G N_m (\alpha) \) with \( N_m (\alpha) \) being the Kalnajs function (Kalnajs 1971). As we shall solve equations (3)–(10) exactly without invoking the tight-winding or WKBJ approximation, \( \delta \) is the positive \( \delta \) allowed values of \( \alpha \) that \( N_m (\alpha) \) is positive, and the relation between \( \alpha \) and \( \delta \) can be either small (the WKBJ regime) or large (open structures). Unless one starts with a non-axisymmetric background, there is no angular variation in \( \delta \). Consistent with the logarithmic spiral forms of \( S \) and \( V \), one writes \( i U = i u r^{-1/2 - i \alpha} \) where \( u \) is another constant coefficient. Substitutions of these expressions into equations (9) and (10) lead to an exact cancellation of the imaginary part and the solution criterion or dispersion relation for stationary logarithmic spirals in an MSID becomes

\[
m^2 \Omega ^2 - \left\{ 2 \Omega ^2 + \frac{C_A^2}{r^2} a^2 \right\} - \frac{2 \pi G N_m (\alpha) \Sigma _0}{r} \]

(11)

\[
\times \left( m^2 + a^2 + \frac{1}{4} \right) \left( \frac{C_A^2}{r^2} \right) = m^2 \Omega ^2
\]

\[
+ \left\{ \frac{a^2}{r^2} \right\} - \frac{2 \pi G N_m (\alpha) \Sigma _0}{r} \]

\[
\times \left( m^2 + a^2 + \frac{1}{4} \right) \left( \frac{C_A^2}{r^2} \right) = 0,
\] (11)

reminiscent of the dispersion relation for spiral FMDWs and SMDWs (Fan & Lou 1996; Lou & Fan 1998a; Lou et al. 2001a). These logarithmic spiral MHD density waves propagate in both radial and azimuthal directions relative to the MSID and appear stationary in an inertial frame of reference only for proper values of \( D^2 \) or \( a^2 D^2 \). Criterion (11) is quadratic in \( D^2 \), and gives rise to two allowed values of \( a^2 D^2 \) corresponding to stationary FMDWs and SMDWs, respectively, as viewed in an inertial frame of reference. For \( C_A^2 = 0 \), criterion (11) readily reduces to equation (37) of Shu et al. (2000). In the limit of \( \alpha \rightarrow 0 \), we also recover the so-called ‘breathing mode’ regime (Lemos, Kalnajs & Lynden-Bell 1991). In the quadratic equation (11) for \( a^2 D^2 \) with \( |m| \approx 2 \), the coefficient of \( a^2 D^2 \) is positive, the coefficient of \( a^2 D^2 \) is negative, and the remaining coefficient is positive. One can show (Lou, in preparation) that the determinant \( \Delta \) of equation (11) is positive for \( |m| \approx 2 \) so that there are two positive roots of \( a^2 D^2 \). We refer to the larger and the smaller as the plus- and minus-sign solutions of \( a^2 D^2 \) that correspond to fast and slow MSID configurations, respectively.

By equations (7), (8) and (10) with the logarithmic spiral forms of \( S \), \( V \), and \( i U \) and the relation between \( S \) and \( V \), we further
derive
\[
Z = -\frac{B_0 \delta}{\Omega \Sigma_0} \left[ \frac{a^2}{\Sigma_0} - 2\pi G N_* (\alpha) - \Omega^2 r \right] 
\]
\times \left[ \alpha^2 + i\alpha \left( \frac{C_A}{2\Omega^2 r^2} - \frac{3}{2} \right) \right] \left[ \alpha^2 + \left( \frac{C_A}{2\Omega^2 r^2} - \frac{3}{2} \right)^2 \right]^{-1}
\tag{12}
\]
to examine the spatial phase relationship between perturbations of surface mass density \(\Sigma\) and azimuthal magnetic field \(Z\). In the limit of \(\alpha \to 0\), one has \(Z \to 0\), but \(iu\) given by equation (10) remains non-zero. For a small \(\alpha \neq 0\) such that \(\alpha^2\) may be ignored relative to the \(i\alpha\) term, \(Z\) and \(S\) are out of phase by \(\sim \pm \pi/2\). In the tight-winding or WKBJ regime of \(\alpha \to \infty\), one obtains
\[
Z = -\frac{B_0 \delta}{\Sigma_0 \Omega r^2} \left[ a^2 - 2\pi G N_* (\alpha) \Sigma_0 r - \Omega^2 r^2 \right].
\tag{13}
\]
By relation (11) for stationary logarithmic spirals in an MSID and relation (12) for \(Z\) and \(S\), we obtain
\[
Z \propto \pm S \left[ a^2 + i\alpha \left( \frac{C_A}{2\Omega^2 r^2} - \frac{3}{2} \right) \right]
\tag{14}
\]
for the plus- and minus-sign solutions of \(a^2 D^2\), respectively. For a sufficiently large \(\alpha\) in the WKBJ regime, one may ignore the imaginary part of equation (14), and \(Z\) is roughly in- and out-of-phase with \(S\) for the plus- and minus-sign solutions, respectively. This outcome is consistent with the earlier results of FMDWs and SMDWs in the tight-winding regime (Fan & Lou 1996; Lou & Fan 1998a). For a small \(\alpha\), we may drop the \(\alpha^2\) term relative to the \(i\alpha\) term, and \(Z\) either precedes or lags behind \(S\) by a phase difference of \(\sim \pi/2\) for both stationary fast and slow logarithmic spiral MSID configurations. We also note that the radial velocity is \(U(r) = m \Omega r Z / (a B_0)\) and the azimuthal velocity is
\[
J(r) = \left( \frac{C_A}{2\Omega^2 r^2} - 1 \right) \frac{\Omega}{m} \left( \frac{a^2}{\Omega \Sigma_0} - \frac{2\pi G N_* (\alpha)}{\Omega} \right) S.
\tag{15}
\]
In principle, these interrelations among gas density, magnetic fields, and velocity components offer potential observational tests (e.g., Visser 1980a,b). Furthermore, the exact analytical solutions derived above can be utilized to test relevant numerical MHD codes and provide a valuable basis for numerical exploration in non-linear stationary as well as time-dependent domains.

3 APPLICATION TO THE SPIRAL GALAXY NGC 6946

A realistic spiral galaxy contains a massive dark matter halo, a stellar disc, and a magnetized gas disc. The halo controls the rotation curve and helps preventing rapid bar-type instabilities (e.g., Ostriker & Peebles 1973). On large scales, density waves in the stellar disc and MHD density waves in the magnetized gas disc are dynamically coupled through the mutual gravity. A comprehensive treatment of such a problem involves heavy mathematics (Lou, in preparation). It suffices to state here the basic conclusions relevant to observational diagnostics. As already known (Lou & Fan 1997, 1998b, 2000; Lou et al. 2001a), spiral enhancements of stellar and surface mass densities are largely locked in phase with each other in a composite system of fluid stellar and gas discs coupled through the mutual gravity. This provides an important benchmark for examining spatial phase relationships among spiral structures as revealed at various electromagnetic wavelengths. In terms of the observational diagnostics, formation of young bright massive stars outlines high-density gas arms in blue light; enhancements of surface density of older stars in the stellar disc give rise to broad smooth spiral arms in red light as well as near-infrared bands; in addition to producing stronger Hα emissions from HII regions, star formation activities in high-density gas arms also lead to enhancements of small-scale random and entangled magnetic fields which, when submerged in a gas of relativistic cosmic-ray electrons, lead to strong total radio-continuum emissions; meanwhile, spiral arms of enhanced large-scale regular magnetic field with less disturbances on smaller scales manifest prominently in polarized radio-continuum emissions with higher degrees of polarization; while star formation activities diminish gradually at larger radii along spiral arms, moderate enhancements along neutral hydrogen H1 arms may still be distinguishable and correlate with magnetic arms and red-light arms (Lou et al. 1999, 2002). In all these aspects, observations of NGC 6946 (Kamphuis & Sancisi 1993; Beck & Hoernes 1996; Ferguson et al. 1998; Frick et al. 2000) agree qualitatively with our theoretical results regarding structural interrelations of logarithmic spiral arms derived from multi-wavelength data. The main results of our analysis are that for stationary logarithmic spirals in an MSID in the tight-winding approximation, enhancements of magnetic field and surface mass density are out of phase for the slow MSID logarithmic spiral structure and that this spiral structure persists indeed in a disc with a flat rotation curve.

We note that in our MSID model, \(\Sigma_0\) scales as \(\sim r^{-1}\) while \(|S|\) scales as \(\sim r^{-3/2}\) with the \(|S|/\Sigma_0\) ratio scaling of \(\sim r^{-1/2}\). For a composite MSID system (Lou, in preparation), this ratio scaling in a thin fluid stellar disc remains essentially the same. The background \(|B_0|\) scales as \(\sim r^{-1/2}\) while both \(|Z|\) and \(|S|\) scale as \(\sim r^{-1}\) with the \(|(Z)^2 + |S|^2|^{1/2}/B_0\) ratio scaling of \(\sim r^{-1/2}\). In comparison with fig. 7 of Frick et al. (2000) and given considerable uncertainties, there are no clear trends for such radial scalings; this discrepancy remains an open issue to be resolved. To supplement potential tests of further observations, we note that both \(|U|\) and \(|J|/r|S|\) for the radial and azimuthal velocity deviations scale as \(\sim r^{-1/2}\).

A limitation of our linear construction of fast and slow MSID configurations is that only the perturbation magnetic field remains tangential to the spiral arm \((|R/Z| = m \alpha = \tan \delta)\) by equation (7) (Lou & Fan 1998a). By equation (8), the pitch angle of the total magnetic field orientation is small given typical galactic parameters. One way out of this is to have large magnetic field deviations (significantly distorted magnetic ovals) into the non-linear regime with \(Z > B_0\) so that the total magnetic field pitch angle will remain less than but close to \(\delta\).

In the context of an isopedically magnetized SID with poloidal magnetic field threading across the disc, stationarity conditions analogous to criterion (11) were interpreted as onsets of barred-spiral instabilities by Shu et al. (2000) through extensive numerical explorations. In the same spirit, criterion (11) should lead to onsets of either fast or slow spiral MSID configurations for the plus- and minus-sign solutions of \(a^2 D^2\), respectively. Should our proposed scenario contain an element of truth, the important question as to why NGC 6946 happens to be around such an onset condition of slow MSID with logarithmic spiral arms remains open. Perhaps, the mass of the dark-matter halo might play a decisive role as it dominantly determines the disc rotation speed and curve profile.
4 DISCUSSION

Given that an H I disc is usually larger than a normal optical spiral pattern, the fainter optical spiral arms from H II regions in the outer part of a disc galaxy as revealed by deep Hα exposures (Ferguson et al. 1998) provide an important observational diagnostics for probing the large-scale MHD density wave scenario. One can normally detect large-scale H I spiral arm structures in disc galaxies, although some are fuzzy and less regular. The weaker Hα counterparts of these H I arms reinforce the notion of MHD density waves, because for a magnetized H I gas disc submerged with relativistic cosmic-ray electrons, non-thermal radio-continuum emissions would offer complementary magnetic field information. This prospect requires more high-quality multi-wavelength observations of nearby spiral galaxies.

In a real disc galaxy such as NGC 6946, non-linear processes are likely involved. Our perturbation approach for stationary MSID with logarithmic spirals should be viewed as only indicative of possible features of fast and slow MSID spiral structures. Non-linear spiral disturbances can give rise to collections of much distorted and misaligned ovals for stellar orbits (Kalnajs 1973), gas streamlines (Shu et al. 2000) and coplanar magnetic field lines (Lou & Fan 1998a) such that a linear analysis might not be adequate for large deviations in flow and magnetic field with comparable radial and azimuthal components. While our MHD density wave scenario avoids the ‘winding dilemma’, such non-linear configurations of either fast or slow MSID may give rise to logarithmic MHD spiral shocks and pose considerable challenges that would require a combination of analytical and numerical techniques (e.g., Galli et al. 2001). Finally, we note that similar physical considerations might be relevant to multi-wavelength observations of M83 (NGC 5236; Beck and Neininger, private communications) and IC342 (Krause et al. 1989).

We have so far focused on possible unaligned or spiral MSID configurations. As for the aligned case of stationary MSID configurations constructed from equations (3)–(8), one also obtains two possible values \(a_{2D}^1\) corresponding to strictly azimuthal propagations, relative to the disc, of MHD density waves that appear stationary in an inertial frame of reference. Note that the aligned case of Shu et al. (2000) essentially corresponds to a rescaled (in terms of their two parameters \(\eta\) and \(\Theta\)) purely azimuthal propagation of density waves in an isopedically magnetized SID. For \(m = 2\), the aligned case could be relevant to bar configurations in a magnetized disc of differential rotation. For one of the two aligned MSID configurations, enhancements of surface mass density and azimuthal magnetic field are out of phase, while for the other these two enhancements are in phase. These configurations may represent intriguing possibilities that might bear observational consequences on large-scale barred structures involving magnetic fields. We also note, in reference to discussions about equations (12)–(15), that for open (i.e. large \(\alpha\)) unaligned or spiral fast and slow MSID configurations, enhancements of surface mass density and azimuthal magnetic field are phase shifted by a phase difference of \(\pm \pi/2\); these open unaligned structures, when superposed with a central bulge, may bear resemblances to barred spiral structures.

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