Robot manipulator end-effector orientation setting methods

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Abstract. The paper presents the approach expanding the possible ways of the manipulated object orientation setting, and consequently the robot manipulator end-effector required orientation when maintaining the process equipment through the use of the rotation matrices $3 \times 3$ and one of the Euler angles systems. The algorithm of identifying the complete set of the permitted combinations of angles between the axes of the base coordinate system and the manipulated object one clearly specifying the robot manipulator end-effector desired orientation in the Cartesian coordinates selected space is proposed. Fifty four ways of the orientation defining based on the rotation matrices $3 \times 3$ and six ways on the basis of the Euler angles were obtained. The specified angles combinations make it possible to simplify the integration of the coordinate systems related to the manipulated object and robot manipulator end-effector.

Key-words: manipulated object, robot manipulator end-effector, orientation, rotation matrix $3 \times 3$, Euler angles

1. Introduction
The industrial robots are one of the main automation facilities when developing the robotic process systems as part of the flexible manufacturing systems. In performing the basic process operations or maintaining the process equipment by the robot, the manipulator end-effector must be oriented in accordance with the required angular location of the manipulated object. The coordinate system related to the manipulated object or used when describing the process operations should be appropriately correlated to the manipulator end-effector coordinate system. This is particularly important when maintaining the metal cutting equipment with computer numerical control that uses a sufficiently diverse location of the coordinate systems related to the tool, processed blank and their movements direction [1].

In robotics for specifying the manipulator end-effector desired orientation in the Cartesian coordinates space, the off-diagonal elements (direction cosines) of the rotation matrix $3 \times 3$ or the Euler angles system [2] applied in astronomy and including the precession, nutation and self-rotation angles are most frequently used.

When the off-diagonal elements are used, three angles serve as the preset ones: two angles fixing the axis $z_C$ position of the coordinate system $O_zx_0y_0$ determining the manipulated object required angular position relative to the axes $x_0$ and $y_0$ of some basic or inertial coordinate system $O_0z_0x_0y_0$ and the angle between the axes $x_0$ and $y_C$ of the mentioned coordinate systems (figure 1).

In the robotic systems design practice depending on the process tasks setting, the other angles combinations can be known in advance and consequently are the most convenient ones for setting. It should be added that the coordinate axes simple re-designation cannot always be used, since they are often rigidly connected to the robot links (including the gripper) or to the elements certain axes of the processing equipment maintained by the robot. For example, when using the widely known special Denavit-Hartenberg coordinate systems [3, 4], the axis $x_C$ of the coordinate system associated with the grasping should be directed perpendicular to the axis $z$ of the coordinate system located on the manipulator previous link, which in turn is directed along the rotation axis or parallel to the last kinematic pair guide depending on its type. The preliminary recalculation of the manipulated object
orientation angles known in advance by the processing for obtaining the corresponding angular values of the mentioned conventional systems might not have been accepted due to the trigonometric functions ambiguous nature and can result in the additional uncertainties or manipulator gripper orientation errors

2. Problem statement
In this regard, in order to expand the developers capabilities when defining the required angular position (orientation) of the manipulated object and consequently the robot gripper, the paper objective is to identify the complete set of permitted combinations of angles between the axes of the assigned base coordinate system \( O_0x_0y_0z_0 \) and system \( O_Cx_Cy_Cz_C \) clearly specifying the manipulated object orientation in the Cartesian coordinates selected space.

![Figure 1](image)

**Figure 1.** The manipulated object (manipulator end-effector) orientation defining in the inertial space by the angles \( x_0^\wedge z_C, y_0^\wedge z_C \) and \( x_0^\wedge y_C \).

3. Theory
For applying the rotation matrix 3*3 for manipulator end-effector position and orientation further calculations, three of nine angles should be known. Moreover, the remaining six angles can be defined by means of the six corresponding equations system [5]. In this case, the theoretically possible number of angles combinations to be specified is calculated as the number of combinations of nine in threes:

\[
C_9^3 = \frac{9!}{3!(9-3)!} = 84
\]

However, not all the combinations have the conceptual meaning in terms of the orientation and can be used for the orientation defining.

As mentioned above, one of the most common methods of the manipulated object orientation defining which is used in robotics is assigning its coordinate axes angular position by the rotation matrix 3*3 off-diagonal elements. It should be noted that such an approach is sufficiently illustrative and convenient from the engineering point of view and can be generalized to other possible angles combinations to be assigned (should be known).

The given approach can be generalized to all the appropriate cases of the manipulated object orientation defining by using the following algorithm:

- orientation of one of three coordinate axes of the manipulated object relative to two axes of three ones of the base coordinate system (fixation of two orienting degrees of freedom of the manipulated object);
orientation of one of the two remaining coordinate axes of the manipulated object relative to one of three axes of the base system (fixation of the remaining third orienting degree of freedom of the manipulated object).

As is well known, the rotation matrix $M_{OC}$ 3*3 has the following form:

$$
M_{OC} = \begin{bmatrix}
C(x_0^x x_c) & C(x_0^y y_c) & C(x_0^z z_c) \\
C(y_0^x x_c) & C(y_0^y y_c) & C(y_0^z z_c) \\
C(z_0^x x_c) & C(z_0^y y_c) & C(z_0^z z_c)
\end{bmatrix}
$$

where the symbol $C$ is the trigonometric function "Cosine"; $x_0^x x_c$, ..., $z_0^z z_c$ are the angles between the axes $x_0$, $y_0$, $z_0$ of the base coordinate system and the axes $x_c$, $y_c$, $z_c$ of the coordinates associated with the manipulated object (manipulator end-effector).

The possible variants of defining the gripper orientation are revealed in accordance with the algorithm accepted above.

The position of the gripper axis $x_c$ can be defined relative to any two axes of the coordinate system $O_0x_0y_0z_0$, that is by two angles combination of the matrix $M_{OC}$ first column. Consequently, there are three variants of specifying the axis $x_c$ position, which can be represented by the following rotation matrix 3*3 (see below the first columns of formulas (1), (2) and (3), in which the angles cosines to be specified are highlighted in bold and underlined):

$$
M_{OC} = \begin{bmatrix}
C(x_0^x x_c) & C(x_0^y y_c) & C(x_0^z z_c) \\
C(y_0^x x_c) & C(y_0^y y_c) & C(y_0^z z_c) \\
C(z_0^x x_c) & C(z_0^y y_c) & C(z_0^z z_c)
\end{bmatrix}
$$

$$
M_{OC} = \begin{bmatrix}
C(x_0^x x_c) & C(x_0^y y_c) & C(x_0^z z_c) \\
C(y_0^x x_c) & C(y_0^y y_c) & C(y_0^z z_c) \\
C(z_0^x x_c) & C(z_0^y y_c) & C(z_0^z z_c)
\end{bmatrix}
$$

$$
M_{OC} = \begin{bmatrix}
C(x_0^x x_c) & C(x_0^y y_c) & C(x_0^z z_c) \\
C(y_0^x x_c) & C(y_0^y y_c) & C(y_0^z z_c) \\
C(z_0^x x_c) & C(z_0^y y_c) & C(z_0^z z_c)
\end{bmatrix}
$$

As noted above, such defining the angles deprives the system of two orienting degrees of freedom. For the manipulated object complete identifying in the orientation, now it is enough to set the position of the axes $y_c$ or $z_c$ relative to any of the three axes of the base coordinate system by using one of the six angles which direction cosines values are located in the second and third columns of the matrix $M_{OC}$ (for example, in matrix (1) the angle $x_0^y y_c$ is accepted as the mentioned above angle, in formula (1) it is highlighted in bold and underlined). Figure 2 shows the manipulated object orientation defining version corresponding to the angles highlighted in bold in matrix (1).
Figure 2. The manipulator end-effector orientation defining by the angles $x_0^x_C$, $y_0^y_C$ and $x_0^y_C$.

Therefore, when using the axis $x_C$ as the initially assigned one, there are 18 options of defining the manipulated object orientation: nine options when using the second column of matrixes (1)-(3); nine options when using the third column of matrixes (1)-(3). Providing the similar arguments concerning the axes $y_C$ and $z_C$, 36 more options of the manipulated object orientation defining can be obtained:

- for the initial positioning of the axis $y_C$, two angles combination of the matrix $M_{0C}$ second column should be used (in this case the axes $x_C$ or $z_C$ are oriented by the angles of the first and third columns of this matrix):

$$
M_{0C} = \begin{bmatrix}
C(x_0^x_C) & C(x_0^y_C) & C(x_0^z_C) \\
C(y_0^x_C) & C(y_0^y_C) & C(y_0^z_C) \\
C(z_0^x_C) & C(z_0^y_C) & C(z_0^z_C)
\end{bmatrix}
$$

$$
M_{0C} = \begin{bmatrix}
C(x_0^x_C) & C(x_0^y_C) & C(x_0^z_C) \\
C(y_0^x_C) & C(y_0^y_C) & C(y_0^z_C) \\
C(z_0^x_C) & C(z_0^y_C) & C(z_0^z_C)
\end{bmatrix}
$$

$$
M_{0C} = \begin{bmatrix}
C(x_0^x_C) & C(x_0^y_C) & C(x_0^z_C) \\
C(y_0^x_C) & C(y_0^y_C) & C(y_0^z_C) \\
C(z_0^x_C) & C(z_0^y_C) & C(z_0^z_C)
\end{bmatrix}
$$

- for the initial positioning of the axis $z_C$, two angles combination of the matrix $M_{0C}$ third column should be used (the axes $y_C$ or $x_C$ are oriented by the angles of the first and second columns of this matrix):

$$
M_{0C} = \begin{bmatrix}
C(x_0^x_C) & C(x_0^y_C) & C(x_0^z_C) \\
C(y_0^x_C) & C(y_0^y_C) & C(y_0^z_C) \\
C(z_0^x_C) & C(z_0^y_C) & C(z_0^z_C)
\end{bmatrix}
$$

$$
M_{0C} = \begin{bmatrix}
C(x_0^x_C) & C(x_0^y_C) & C(x_0^z_C) \\
C(y_0^x_C) & C(y_0^y_C) & C(y_0^z_C) \\
C(z_0^x_C) & C(z_0^y_C) & C(z_0^z_C)
\end{bmatrix}
$$

$$
M_{0C} = \begin{bmatrix}
C(x_0^x_C) & C(x_0^y_C) & C(x_0^z_C) \\
C(y_0^x_C) & C(y_0^y_C) & C(y_0^z_C) \\
C(z_0^x_C) & C(z_0^y_C) & C(z_0^z_C)
\end{bmatrix}
$$
Thus, under the accepted approach, 54 variants of 84 possible combinations are the appropriate variants of the manipulated object orientation defining. Another common method of the manipulator end-effector orientation defining is using the Euler angles [1] (figure 3). One shall generalize the Euler method to other possible options of the manipulator end-effector orientation defining.

The manipulated object orientation defining by Euler method is known to provide the manipulated object sequential rotation by the following three angles (figure 3):

• rotation by the precession angle $x_0^x\mathbf{C}$ about the axis $z_0$;
• rotation by the nutation angle $z_0^z\mathbf{C}$ about the rotated axis $x_C$;
• rotation by the self-rotation angle $x_C^xCC$ around the rotated axis $z_C$.

The similar transformations can be performed by using the other coordinate axes. In addition to the axis $z_0$, the rotation by the precession angle can be performed about the axes $x_0$ and $y_0$, and the nutation and rotation angles are performed round the other two axes in sequence as it is the case in the original version. Hence, in addition to the main way of the manipulated object orientation defining, five more possible ways can be obtained.

All six possible ways of the manipulated object orientation defining which are obtained on the basis of the Euler method should be specified.

Version 1 (the main one): the given version is described above and is shown in figure 1.

Version 2:
• rotation by the precession angle $x_0^x\mathbf{C}$ about the axis $z_0$;
• rotation by the nutation angle $z_0^z\mathbf{C}$ about the rotated axis $y_C$;
• rotation by the self-rotation angle $y_C^yCC$ about the rotated axis $z_C$.

Version 3:
• rotation by the precession angle $z_0^z\mathbf{C}$ about the axis $y_0$;
• rotation by the nutation angle $y_0^y\mathbf{C}$ about the rotated axis $z_C$;
• rotation by the self-rotation angle $z_C^zCC$ about the rotated axis $y_C$.

Version 4:
• rotation by the precession angle $z_0^z\mathbf{C}$ about the axis $y_0$;
• rotation by the nutation angle $y_0^y\mathbf{C}$ about the rotated axis $x_C$;
• rotation by the self-rotation angle $x_C^xCC$ about the rotated axis $y_C$.

Version 5:
• rotation by the precession angle $y_0^y\mathbf{C}$ about the axis $x_0$.

The manipulated object orientation defining by Euler method is known to provide the manipulated object sequential rotation by the following three angles (figure 3):

\[
M_{OC} = \begin{bmatrix}
C(x_0^x\mathbf{C}) & C(x_0^y\mathbf{C}) & C(x_0^z\mathbf{C}) \\
C(y_0^x\mathbf{C}) & C(y_0^y\mathbf{C}) & C(y_0^z\mathbf{C}) \\
C(z_0^x\mathbf{C}) & C(z_0^y\mathbf{C}) & C(z_0^z\mathbf{C})
\end{bmatrix}
\]
rotation by the nutation angle $x_0^x_C$ about the rotated axis $y_C$;
rotation by the self-rotation angle $y_C^y_{CC}$ about the rotated axis $x_C$.

Version 6:
rotation by the precession angle $y_0^y_C$ about the axis $x_0$;
rotation by the nutation angle $x_0^x_C$ about the rotated axis $z_C$;
rotation by the self-rotation angle $z_C^z_{CC}$ about the rotated axis $x_C$.

The example of the proposed approach application is the coordinate systems presented in figures 1 and 2, and demonstrating the possibility of specifying the manipulated object orientation at any random, location of the coordinate system $O_C z_C x_C y_C$ relative to the object, which is most convenient from the technological point of view.

4. Results
The algorithm enabling to identify all the possible methods combinations of defining the required in accordance with the process orientation of the manipulated object and consequently of the robot manipulator end-effector when implementing the corresponding translations, namely 54 translations based on the rotation matrix 3*3 direction cosines application and six translations based on the Euler method, is proposed.

5. Results discussion
The direction cosines appropriate combinations and the complete set of the angles combinations similar to the Euler angles were chosen of all the theoretically possible combinations by using the proposed approach to analyzing the rotation matrix direction cosines combinations and possible sequences of coordinates rotation similar to the Euler angles.

6. Conclusions
The proposed methods of defining the manipulated object orientation make it possible to expand the industrial robots application scope when maintaining the processing equipment and to simplify the integration of the robots coordinate systems with the equipment coordinate systems which is especially important in planning the robot manipulator end-effector movements in the operating area of the numerically controlled machine tools in automating the loading and unloading.

7. References
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