Computerized Generation of Asymmetric Straight Bevel Gear

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Abstract. To carry out any type of analysis and study the gear drives, the primary steps include the graphical representation of the gear drive under consideration. In this study, the mathematical formulations of the tooth surface, tooth root surface, and tooth root transition surface of symmetric straight bevel gear were formulated based on the planning principle. The proposed modification has been done on the mathematical formulation of the symmetric tooth profile to consider the effect of asymmetric tooth profiles (which are characterized by different pressure angles on the loaded and unloaded sides). As a result of this study, a computer program based on the above mathematical formulation has been built to represent the final shape of symmetric and asymmetric teeth profiles of straight bevel gear for different design parameters.

Keywords. Tooth surface, Tooth Root Surface, Tooth Root transition surface, Gear Planning Principle, Symmetric Tooth Profile, Asymmetric Tooth Profile.

1. Introduction
The bevel gear blank has a conical shape, so the tooth is a taper in thickness and height. The straight bevel gear is regarded as a simple form of bevel gears used for transfer power between two intersecting shafts. One of the essential bevel gear parameters is the intersecting shaft angle; this angle is 90° (right angle) in many applications. Some applications involve the shaft angle is being more than 90° while in other applications, it is less than 90° [1]. M. Q. Abdullah and Ismael [2] introduced a new form of generation and meshing simulation for involute helical gear. The proposed generation is based on two rack-cutters' imaginary movement, one used to generate gear, and the other used to generate the pinion. M.Q. Abdullah and Ismael [3] generate and simulate the hypoid bevel gear by using the face hobbing process; this generation process is done by the mathematical formulation of the cutter blade movement. Then, the final form of the tooth surface of the hypoid bevel gear is drawn with the aid of a computer program. M. Q. Abdullah and Jwege[4] introduced the mathematical formulation of the actual form of symmetric tooth profile of spur gear by using shaping process. This actual form has been developed to consider the effect of asymmetric tooth profile of such gear. The above mathematical formulation obtained for symmetric and asymmetric tooth profile can be introduced in graphical form based on a computer program has been built for different design parameter. M. Q. Abdullah, Hatem, and Mansoor [5] introduced a mathematical formulation of a new version of Novikov helical gear combining crowned involute profile with double circular arc profile. The final shape of this type of gear can be got by using SOLIDWORKS program based on the derived mathematical formulation. M. Q. Abdullah and Mansoor [6] formulate the general mathematical equation with different parameters used to represent the final shape of noncircular gear.
This equation may be used to give any centrode shape depending on the selected parameters. The hobbing process can also be used to generate a tooth with a double circular arc profile and distributed over the pitch curve of noncircular gear. Then depending on the derived mathematical formulation, the final shape of noncircular gear derive may be achieved using the SOLIDWORKS program. Most of the above research papers mentioned the generation process for standard helical gear and standard hypoid gear, also the generation of asymmetric tooth profile of spur gear. In this work, the mathematical generation process for symmetric and asymmetric tooth profiles of straight bevel gear has been achieved.

2. Coordination of straight bevel gear during planning process

In the process of cutting teeth of straight bevel gear, three coordinates system must be set up according to the cutting process, as shown in Figure 1, [7]:

1. \( O_1(x_1, y_1, z_1) \): represent reference coordinates system fixed at the midpoint of tooth thickness (big side of the tooth of straight bevel gear). Axis \( (x_1) \) is set in the horizontal direction of increasing thickness of the tooth. Axis \( (z_1) \) is set in the direction of increasing cone angle (go away from the tooth). Axis \( (y_1) \) is set by applying the right-hand rule.
2. \( O_2(x_2, y_2, z_2) \): represent cutter coordinate system tightly fixed on the cutter; at the beginning of the cutting process, this coordinate coincides with the reference coordinate system.
3. \( O_3(x_3, y_3, z_3) \): represent the coordinates of gear, and it always rotates with gear blank in the cutting process. The origin of these coordinates is created as a result of the intersection of the axis \( y_1 \) and axis \( l \).

![Figure 1. Coordinates of straight bevel gear during the cutting process.](image-url)
The cutter used to generate these teeth have two movements; one is the rectilinear translation along the apex \((u_{o1})\) of conical blank, and the second is rotation about the axis of the tool \((\omega_2)\). During the machining process, as shown in Figure 1, the conical blank rotates about axis I as \(\omega_3\). After a certain period of cutting process time, the distance between \(O_1\) and \(O_2\) is \(l\), and the angular position of the blank coordinate system relative to the reference coordinate is \(\phi\). So, the above distance \((l, \phi)\) can be represented in vector form:

\[
l = u_{o1}t \\
\phi = \omega_3t
\]  

If one takes any position of the cutting tool on gear conical blank, for example, point N shown in ‘figure 1’, so the radial vector of this point on cutting coordinate \((\theta_2)\) is \(r^{(2)}\) while the radial vector of blank gear coordinate \((O_3)\) is \(r^{(3)}\). \(\omega_3\) and linear velocity \(u_{o1}\) represent the dominant parameters in the process of generation tooth profile [7].

3. Meshing equation of straight bevel gear

In the process of generating the teeth profile of straight bevel gear, the relative velocity \(u_{12}\) is perpendicular to the unit tangent vector \(n_1\) at the point of contact between the blank tooth surface and the cutting surface. Thus, the relation between these variables can be written as [7]:

\[
n_1 \cdot u_{12} = 0
\]

The above two variables (relative velocity \(u_{12}\) and unit tangent vector \(n_1\)) can be obtained when the cutting tool at the initial position (in the initial position, the reference coordinate \(O_1\) system coincide with the cutter coordination system \(O_2\)). Let take the contact point of the cutter at point N with coordinate \(x_1, y_1, z_1\), so that the relation between cutting face velocity of the cutter \(u_1\) and the tooth surface velocity \(u_2\) can be described as:

\[
u_1 = u_{o1}i_1 - u_{o1}k_1
\]

\[
u_{o2} = \omega_3(L - l)\tan\delta
\]

\[
u_2 = \omega^{(3)} + r^{(3)}
\]

\[
\omega^{(3)} = \omega_3 k
\]

\[
r^{(3)} = x_1i_1 + (y_1 + (L - l)\tan\delta)j_1 + z_1k_1
\]

So, the relative velocity between the cutting surface and tooth surface can be found by subtracting Equation (4) and Equation (6) to get:

\[
u_{12} = y_1\omega_3i_2 - x_1\omega_3j_1 - u_{o1}k_1
\]

Equation (9) represents the relative velocity in the reference coordinate \(O_1\) at the initial position. Then, to find the relative velocity at the cutter coordinate system \(O_2\) at any meshing position during the machining process, transformation must be used to convert the radial position vector from \(O_2\) to \(O_1\) as described in the following equation:

\[
r^{(1)} = M_{02}r^{(2)} + r_{02}
\]

Where; \(r^{(2)} = [x_2y_2z_2]^T\)

Substituting Equation (10) in Equation (9) gives the relative velocity \(u_{12}\) in \(O_2\) as shown in the following equation.

\[
u_{12} = y_2\omega_3i_2 - (x_2 - \phi(L - l)\tan\delta)\omega_3j_2 - u_{o2}k_2
\]

4. Generation of straight bevel gear with symmetric tooth profile

The cutter used in the process of generation teeth profiles of straight bevel gear consist of two edges; as shown in Figure 2, each edge cuts one tooth surface of the gear [7].
Figure 2. Cutter profile.

The cutting edge surface consist of three sections:

1. Section AD is used to generate the tooth surface of the gear.
2. Section DE is used to generate the tooth root transition surface of the gear.
3. Section EF is used to generate the tooth root surface.

The length of the segment CA and segment CD is obtained from the following equation:

\[
CA = \mu_a = h_a / \cos \beta \\
CD = \mu_f = (h_f - r + r \sin \beta) / \cos \beta
\]  

The parameter describes in Equation (12) and (13) vary along the length of the tooth, as shown in Figure 2, according to these equations:

\[
h_a = h_{a1} - l_1 \tan \theta_a
\]
\[
h_f = h_{f1} - l_1 \tan \theta_f
\]

Also, the thickness of the tooth is varying along the length from the big end to the small end according to the following equation:

\[
s = s_z (1 - \frac{h_f}{h_a})
\]

So, the cutting surface equation of the part AD of the cutter can be described in the following equation:
where ε vary from 0 to b (tooth width) and μ vary from μf to μu. Also, the cutting surface equation of the part DE of the cutter can be described in the following equation:

\[ r^{(2)} = \left( \frac{d}{2} - r \cos \xi \right) i_2 + \left( -h_f + r \cos \xi \right) j_2 + uk_2 \]  

(18)

Where, \( d = s + 2( h_f - r ) \tan \beta + \left( \frac{2r}{\cos \beta} \right) \)

Finally, the cutting surface equation of the part EF can be described in the following equation:

\[ r^{(2)} = wi_2 - h_fj_2 + uk_2 \]  

(19)

Where, \( w \) vary from \( d/2 \) to \( (d + h_f)/2 \). As shown in Figure 2, the cutter is symmetric about the neutral axis, so the cutting surface equation of the other side can be derived by the symmetric transformation. The tooth surface equation of straight bevel gear is the coordinate transformation combined with the meshing equation, as shown in the following equation:

\[ r^{(3)} = M_{32} r^{(2)} + r^{(3)}_{12} \]  

(20a)

\[ r^{(3)}_{12} = M_{31} r_{12} + r^{(3)}_{1} \]  

(20b)

\[ n_1 \ast u_{12} = 0 \]  

(20c)

The symmetric tooth surface equation of straight bevel gear can be obtained by substituting the cutting surface equations, and meshing equation into Equation (20) gives the following equation which is used to generate teeth profile of the straight bevel gear:

\[ x_3 = x_2 \cos \phi - y_2 \sin \phi + (L - l_1) \phi \tan \delta \cos \phi - (L - l_1) \tan \delta \sin \phi \]  

(21)

\[ y_3 = x_2 \sin \phi - y_2 \cos \phi + (L - l_1) \phi \tan \delta \cos \phi - (L - l_1) \tan \delta \sin \phi \]  

(22)

\[ z_3 = z_2 + l_1 \]  

(23)

5. Generation of Straight Bevel Gear with Asymmetric Tooth Profile

In this case, there are different pressure angle on each side of the tooth profile of the straight bevel gear, one for the loaded side \( \beta_L \) and other for unloaded side \( \beta_U \). So the process using in the generation of symmetric tooth profiles must be adapted to take into account the effect of asymmetry property. Therefore, the process used to generate the asymmetric tooth profile must be done into two-part, the first part for the loaded side and the other part for the unloaded side.

5.1. Generation of loaded side

In this part (right part), the cutting surface equation of the part AD from the cutter can be represented in the following equation:

\[ r_L = \left( \frac{s}{2} - \mu_1 \sin \beta_L \right) i_2 + (\mu_1 \cos \beta_L) j_2 + uk_2 \]  

(24)

Also, the cutting surface equation of the part DE from the cutter can be represented by the following equation [5]:

\[ r_L = \left( \frac{d}{2} - r \cos \xi \right) i_2 + \left( -h_f + r \cos \xi \right) j_2 + uk_2 \]  

(25)

Where, \( d = s + 2( h_f - r ) \tan \beta_L + \left( \frac{2r}{\cos \beta_L} \right) \)

Finally, the cutting surface equation of part EF from the cutter can be represented by the following equation:

\[ r_L = wi_2 - h_fj_2 + uk_2 \]  

(26)
The tooth surface equation of the loaded side of the straight bevel gear can be obtained by substituting the cutting surface Equations (24), (25), (26) and meshing equation into Equation (20), which gives the following equation that is used to generate loaded side teeth profile of the straight bevel gear:

\[
\begin{align*}
 x_{3L} &= x_{2L}\cos\phi - y_{2L}\sin\phi + (L - l_1)\phi \tan\delta \cos\phi - (L - l_1)\tan\delta \sin\phi \\
 y_{3L} &= x_{2L}\sin\phi - y_{2L}\cos\phi + (L - l_1)\phi \tan\delta \cos\phi - (L - l_1)\tan\delta \sin\phi \\
 z_{3L} &= z_{2L} + l_1
\end{align*}
\] (27) (28) (29)

5.2. Generation of Unloaded Side

In this part (half part), the cutting surface equation of the part GH from the cutter can be represented in the following equation:

\[
r_U = -\left(\frac{s}{s} - \mu_1\sin\beta_U\right)i_2 + (\mu_1\cos\beta_U)j_2 + uk_2
\] (30)

Also, the cutting surface equation of the part HI from the cutter can be represented by the following equation:

\[
r_U = -\left(\frac{d}{2} - r\cos\xi\right)i_2 + (-h_f + r\cos\xi)j_2 + uk_2
\] (31)

Where; \( d = s + 2(h_f - r)\tan\beta_L + \left(\frac{2r}{\cos\beta_U}\right) \)

Finally, the cutting surface equation of part EF of the cutter can be represented by the following equation:

\[
r_U = -wi_2 - h_fj_2 + uk_2
\] (32)

The tooth surface equation of the unloaded side of straight bevel gear can be obtained by substituting the cutting surface Equations (30), (31), (32) and meshing equation into Equation (20), which gives the following equation that is used to generate unloaded side teeth profile of straight bevel gear:

\[
\begin{align*}
 x_{3U} &= x_{2U}\cos\phi - y_{2U}\sin\phi + (L - l_1)\phi \tan\delta \cos\phi - (L - l_1)\tan\delta \sin\phi \\
 y_{3U} &= x_{2U}\sin\phi - y_{2U}\cos\phi + (L - l_1)\phi \tan\delta \cos\phi - (L - l_1)\tan\delta \sin\phi \\
 z_{3U} &= z_{2U} + l_1
\end{align*}
\] (33) (34) (35)

6. Results

The two generated parts of the loaded and unloaded sides of the asymmetric tooth profile of straight bevel gear are joined together to form the final mathematical simulation used to represent the final shape of the asymmetric straight bevel gear. The final mathematical simulation obtained for symmetric and asymmetric tooth profile is represented graphically by using MATLAB program version 16 and SOLIDWORKS program version 14 for different gear geometry parameters such as module, reference pitch angle, number of teeth, thickness, addendum, addendum angle, dedendum angle, root fillet radius, and pressure angle. Table (1) shows the geometrical parameters used to represent the final form of full straight bevel gear. The first step in the generation process is to draw the profile of straight bevel gear by using the parameter described in table (1), and then by using the SOLIDWORKS program, the profile was revolved from 0° to 360° degree to create the final conical shape, as shown in Figure 3 and 4. The second step in the generation process is programming the mathematical formulation of symmetric and asymmetric teeth profile of straight bevel gear by using MATLAB program, then the points obtained for the tooth profile is imported to the SOLIDWORKS program and fit on the conical model, as shown in Figure 5. The third step in the generation process involved a lofted cut of the points fit on the conical blank, as shown in Figure 6. A circular pattern is then made to complete the final shape of symmetric and asymmetric straight bevel gear, as shown in Figure 7. Figures 8 and 9 represent the final form of symmetric and asymmetric straight bevel gear, while Figure 10 and 11 represent the final form of assembly straight bevel gear for symmetric and asymmetric teeth profile.
Figure 3. 2D drawing of gear based on table 1.

Figure 4. 3D conical shape model.

Figure 5. Points of asymmetric tooth profile fit on the conical blank.
Figure 6. Tooth generation process on the conical blank.

Figure 7. Full teeth generation process on the conical blank.

Figure 8. 3D model of asymmetric straight bevel gear with 14.5 pressure angle on the loaded side and 35 pressure angle on the unloaded side.
Figure 9. 3D model of symmetric straight bevel gear with 14.5 pressure angle.

Figure 10. 3D assembly model of asymmetric straight bevel gear.

Figure 11. 3D assembly model of symmetric straight bevel gear.
Table 1. Gear geometry parameter.

| Gear terms (unit)                  | Gear (1)          | Gear (2)          |
|-----------------------------------|-------------------|-------------------|
| Pitch diameter (mm)               | $d_1 = 140 \text{ mm}$ | $d_2 = 140 \text{ mm}$ |
| Number of teeth                   | $Z_1 = 20$        | $Z_2 = 20$        |
| Module                            | $m = 7$           | $m = 7$           |
| Pressure angle (degree)           | $\beta = 14.5^\circ$ | $\beta = 14.5^\circ$ |
| Face width (mm)                   | $b = 33 \text{ mm}$ | $b = 33 \text{ mm}$ |
| Addendum (mm)                     | $h_{a1} = 7 \text{ mm}$ | $h_{a2} = 7 \text{ mm}$ |
| Dedendum (mm)                     | $h_{f1} = 8.75 \text{ mm}$ | $h_{f2} = 8.75 \text{ mm}$ |
| Reference pitch angle (degree)    | $\delta = 45^°$   | $\delta = 45^°$   |
| Cone distance (mm)                | $R = 98.994 \text{ mm}$ | $R = 98.994 \text{ mm}$ |
| Addendum angle                    | $\theta_{a1} = 4.04^\circ$ | $\theta_{a2} = 4.04^\circ$ |
| Dedendum angle                    | $\theta_{f1} = 5.05^\circ$ | $\theta_{f2} = 5.05^\circ$ |
| Tip angle (degree)                | $\delta_{a1} = 49.04$ | $\delta_{a2} = 49.04$ |
| Root angle (degree)               | $\delta_{f1} = 39.95$ | $\delta_{f2} = 39.95$ |
| Tip outside diameter (mm)         | $d_{a1} = 149.899 \text{ mm}$ | $d_{a2} = 149.899 \text{ mm}$ |
| Thickness of the tooth (mm)       | $s_1 = 10.9956 \text{ mm}$ | $2 = 10.9956 \text{ mm}$ |

7. Conclusion
Throughout the present study, the following conclusions can be drawn:

1. Based on the gear planning principle, a mathematical formulation of the asymmetric teeth profiles of straight bevel gear has been achieved by modifying the mathematical formulation of the symmetric teeth profiles of such gear for different gear design parameters.
2. The 3D final shape of the asymmetric and symmetric teeth profiles of straight bevel gear can be obtained using MATLAB and SOLIDWORKS programs based on the derived mathematical formulation.
3. The 3D assembly model of a pair of asymmetric and symmetric teeth profiles of straight bevel gear was established using the SOLIDWORKS program.

Nomenclatures

- $n_1$: Unit tangent vector.
- $u_{12}$: Relative velocity between cutting surface and blank tooth surface.
- $M_{a2}$: Transformation matrix coordinate from $O_2$ to $O_1$.
- $r_{02}$: Column matrix of the center of coordinates $O_2$ in $O_1$.
- $r$: Corner radius of the cutter.
- $\beta$: Pressure angle.
- $h_a$: Addendum.
- $h_f$: Dedendum.
- $h_{at}$: Big end addendum.
- $h_{f1}$: Big end dedendum.
- $\theta_a$: Addendum angle.
- $\theta_f$: Dedendum angle.
- $s$: Tooth thickness.
- $s_1$: Big end tooth thickness.
- $M_{32}$: The coordinate transformation matrices of $O_2$ to $O_3$.
- $M_{31}$: The coordinate transformation of $O_1$ to $O_3$.
- $r_{12}^{(3)}$: The column matrices of origin $O_2$ in $O_3$.
- $r_{1}^{(3)}$: The column matrices of origin $O_1$ in $O_3$. 
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