Numerical simulation of far field acoustics of an airfoil using vortex method and 2-D FW-H equation

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Abstract. Airfoil self noise is the main noise source of large wind turbine blades and aircraft wings. Unsteady flow field of NACA0012 airfoil has been simulated numerically in this paper using vortex method, and validation has also been done. Far field acoustics of circular cylinder at Reynolds number equals 150, 200 and 2000 have been simulated and validated numerically in this paper, as unsteady load of target object being acoustic source. The result of far field acoustics of circular cylinder shows that the maximum sound pressure points are in the line perpendicular to the coming flow, which is consistent with other papers. And comparison with paper about NACA0012 airfoil at Re=200 shows that the directivity is in a good agreement. Work of this paper paves the way to further investigation of acoustics prediction of wind turbine blades.

1. Introduction

Confirmation of experiment shows that the main sound sources of large wind turbine blades focus on the blades tip [¹] (not pure tip). Noise from blade airfoil is the main sound sources of large wind turbines. Main prediction way of airfoil noise of large wind turbines is semi-empirical expression after NACA0012 airfoil experiments [²]. However, accuracy of the expression would be affected by the accuracy of boundary layer parameters and shape of airfoil. As the development of computer technology, numerical prediction has been an efficient method for wind turbine blades noise. Numerical methods such as usual business software, LES [³] and CAA [⁴], take the unsteady flow field near the airfoil as being the sound source, and combine with the far field radiation model, then obtain the far-field acoustics. However, consumption of source and time is too large to apply these methods in a wide range. Vortex method is a maturely fast and efficient method to calculate unsteady flow field, combined with far field acoustic model, noise prediction model can be set up.

As a result, static NACA0012 airfoil will be simulated using vortex method in this paper, and aerodynamic results validated with comparison. As unsteady load of target object being acoustic source, far field acoustics of NACA0012 airfoil at Re=200 have been simulated here. This method provides an efficient tool for noise prediction of wind turbine blades, and also laid a solid foundation for further research on sound of wind turbine blades under the influence of airfoil motion and unsteady coming flow.
2. Governing equation and numerical method

As the Reynolds number of large wind turbines is usually at $10^6$ and more, and viscosity is very small, here we adopt two-dimensional uncompressible vortex dynamic equation without viscosity as the governing equation:

$$ \frac{d\vec{\omega}}{dt} = 0 $$ (1)

In the former formula, $\vec{\omega}$ is vorticity. Former formula means the total vorticity is a constant. Motion of vortex particles in the field is affected by potential flow, vortexes in the boundary layer and flow. Vortexes’ velocity is obtained by 2D Biot-Savert formula:

$$ \frac{d\vec{x}}{dt} = \vec{u} + \vec{u}_\infty $$ (2)

In the former formula, $\vec{x}$ is location of vortex particle, $\vec{u}$ is velocity affected by other vortexes, $\vec{u}_\infty$ is potential velocity. Dirichlet non-slipping boundary condition is adopted on the boundary:

$$ \vec{v}_i \cdot \vec{s} = 0 $$ (3)

In the former formula, $\vec{v}_i$ is surface velocity, $\vec{s}$ is the surface tangential unit vector.

The process of calculation is as follows:

1. Assume that there is a thin vortex layer on the object surface, the strength of the vortex is $\gamma_m$ and its tangential velocity satisfies the non-slipping boundary condition. As the velocity of surface vortex is constituted by potential flow and induced velocity by other vortexes. After the non-slipping boundary condition, vortex strength can be obtained. This process is also called Martensen analysis [8].

$$ -\frac{1}{2} \gamma_m + \frac{1}{4\pi} \int \frac{\vec{n}_m \times \left( \left( \gamma_n \times \vec{r}_{mn} \right) \times \vec{n}_m \right) ds}{r_{mn}^3} + \vec{n}_m \times \left( \vec{u}_\infty \times \vec{n}_m \right) = 0 $$ (4)

Former formula shows the sum of tangential velocity on surface point $m$ is 0, and the velocities are: self-induced velocity (the first item), induced velocity by other vortexes (the second item) and potential velocity (the last item). In the formula, $\vec{n}_m$ is the normal unit vector at point $m$, $\vec{r}_{mn}$ is displacement vector of point $m$ to point $n$.

2. Determine the vortex shedding location. Two shedding vortexes way are usually adopted in the program. And for the blunt body, the shedding locations are near the separation points, but near the trailing edge for streamline body. The vortex strength is $\Delta \Gamma = 1/2 \gamma^2 \Delta t$.

3. Convection and location update of newborn vortex particles. The velocity contains two parts, the first part is induced velocity by Z free vortex particles, and the second part is induced velocity by vortex on the boundary layer.

$$ \tilde{\vec{u}}_m = \sum_{n=1}^{Z} \Delta \Gamma_n \vec{U}_{mn} + \sum_{n=1}^{N} \gamma_n \Delta s_n \vec{U}_{mn} $$ (5)

In the former formula, $\vec{u}_m$ is velocity of number $m$ vortex particle, $\Delta s_n$ is vortex length on the surface, and $\vec{U}_{mn}$ is induced factor. Attention should be paid that number of free vortex particles in the flow field is different at every time moment and update should be done to Z.

At last, repeat the first step. Then, complete the calculation.

Pressure at every point on the surface at every time moment can be got by Bernoulli’s equation:

$$ p_0 + \frac{1}{2} \rho v^2 = \frac{1}{2} \rho \vec{u}^2 $$ (6)

In the former formula, $p_0$ is the atmospheric pressure. After we obtain the pressure at every point, we can get the coefficient of pressure, lift and drag.

As unsteady load of target object being acoustic source and with Fourier transform, far field sound pressure of airfoil can be got by two-dimensional FW-H equation [6] in frequency domain.
\[\hat{p}(x, \omega) = \left(\frac{ke^{2ikr}}{8\pi r}\right)^{1/2} \sum_{n=1}^{N} \hat{x} \cdot n \hat{p}_n(x_n, \omega_n) \exp(-i\omega x_n) ds \]  

(7)

Former formula means the sound pressure at far point \( x \) on the \( \omega \) frequency. \( \omega_D = \omega D \) is Doppler frequency, \( D = \omega_D/\omega \) is Doppler factor, \( D = 1 - \hat{n} \cdot \hat{x} / c \), \( k \) is sound wave number \( k = \omega / c \), \( c \) is sound speed. \( r \) means distance between far point and airfoil centre. \( \hat{p}_n \) is unsteady load at surface point \( x \) on the \( \omega_D \) frequency. \( \hat{x} \) is unit vector of far field point. \( n \) is the normal unit vector of airfoil surface point.

3. Numerical model and validation

Calculation of sound field of NACA0012 airfoil is divided into two parts. First is near-field unsteady flow field, and second is radiated sound field.

This section shows the validation of vortex method for near-field unsteady flow field, and two-dimensional FW-H equation for far field sound pressure. Firstly, numerical simulation is taken out for NACA0012 airfoil with coming flow \( u_\infty = 1.0 \) and attack angle \( \alpha = 20^\circ \). Secondly, numerical simulation of far field sound pressure is taken out for circular cylinder at \( Re = 150, 200 \) and \( 2000 \).

3.1. Numerical validation of vortex method for near-field unsteady flow

Figure 1 shows the lift and drag coefficients \( C_l \) and \( C_d \) of NACA0012 airfoil at \( Re = 200 \). After a period of time, the coefficients come to a steady condition with small amplitudes.

![Figure 1. Lift and drag coefficients of NACA0012 airfoil at Re=200](image)

The alternate vortexes form and shed at the upper and lower trailing edges. From figure 2 we know that the time period \( T \) is a little less than 3 second. Statistics of the result shows the time period is 2.7875s and frequency \( f \) is 0.3584/s compared with paper \( f = 0.35/s \). [7]

![Figure 2. Streamlines of flow field of NACA0012 airfoil (Re=200)](image)

3.2. Validation of two-dimensional FW-H equation for far sound field
This section gives several examples to validate the accuracy of formula (7). Far acoustic field of circular cylinder at Re=150, 200 and 2000, NACA0012 airfoil at Re=200 are shown firstly. Secondly, the acoustic field of NACA0012 airfoil is given out.

Circular cylinder with diameter D=1.0, coming flow $u_\infty=1.0$ and 68 and Re=150 is coming first. The Mach No. is 0.003 and 0.2. Figure 3 shows $C_l$ comparison \[8\]. Though the coming flow is different, the result is almost same. That’s to say, the Mach No. has little influence on aerodynamic parameters \[8\]. The St No. is 0.177 and 0.165 for coming flow $u_\infty=1.0$ and 68.0 compared with 0.183 of paper \[8\].

Because of the frequency formula, the maximum sound pressure is got when the sound frequency is equal to vortex shedding frequency. Figure 4 shows sound pressure comparison at $r=75D$. Though different coming flow speed, sound pressure are agreed very well with the paper \[8\].

Figure 3. $C_l$ comparison for different coming flow (Re=150).

Result of Re=200 is coming next. Figure 5 shows sound spread in the flow field. Because we has known that Mach No. has little influence on the result, here we give the coming flow $u_\infty=1.0$. Based on the correct simulation of circular cylinder at Re=200 (amplitude of lift coefficient is $\pm 0.6$, but $\pm 0.68$ for paper \[7\]), figure 6(a) shows far field sound pressure figure at $r=15D$.

From figure 6(a) we see that the maximum sound pressure is much smaller than that of paper, and Doppler Effect is obvious in reference paper. That’s because Mach No. of reference paper is $Ma=0.2$. Two main reasons to explain the difference of maximum sound pressure are: 1, the amplitude of lift coefficient is much bigger than that here, while its value is the key to maximum sound pressure. 2, Self-programming LES method is used in reference paper, so the sound pressure is caused not only by dipole sound source from unsteady surface load, but also quadrupole sound source from turbulent stress. Although most paper believes that sound pressure from quadrupole is much smaller than that from dipole, especially when Re is very small, the flow field is not totally laminar and two dimensional for circular cylinder at Re=200. Figure 6(b) shows the directivity diagram comparison when the amplitude of lift coefficient is instead $\pm 0.70$, the good agreement can be see, which proves reason 1.

Figure 5. Sound spread of circular cylinder (Re=200).

Figure 6. Comparison of directivity diagram for $r=15D$ (Re=200).
Result of circular cylinder of Re=2000 is showed below. The prediction is taken out under the correct calculation of circular cylinder at Re=150 and 200. The maximum sound pressure is $6 \times 10^{-4}$ which is much bigger than that of Re=150 and 200.

Figure 7. Directivity diagram for r=75D (Re=2000).

For unsteady flow field with periodic vortex shedding, radiated noise is tonal and the frequency is that of vortex shedding. That’s to say, the radiated sound is caused by vortex shedding, which is consistent to many other papers. Correct near-field unsteady flow field calculated using discrete vortex method and correct far-field acoustics using two-dimensional FW-H equation show that the program adopted here is reliable.

4. Result analysis

This section shows the result of NACA0012 airfoil with Re=200 and $\alpha=20^\circ$ based on the result of section 3.1. Figure 8 shows directivity diagram at the distance of r=10c, where c is the airfoil chord. Once the author [7] said the order of sound pressure maybe wrong, but the trend of the directivity is correct. So here we focus on the directivity only, and nice agreement between the results here with reference can be seen. While here Mach number is 0.003, which is much smaller than reference Ma=0.2, so the Doppler Effect is not obvious. From section 3.2 we know that the Mach number has little influence on aerodynamic parameters and sound pressure, as a result, here we confirm the same. Figure 9 shows the sound spread in the range about 7 times the wavelength.

Figure 8. Comparison of directivity diagram of NACA0012 (left is from paper, and right is result here).

Figure 9. Sound spread figure of NACA0012.

5. Conclusions

Numerical simulation of sound pressure of NACA0012 airfoil at far-field is taken out in this paper using vortex method and two-dimensional FW-H equation.

That calculation has two parts, the first part is unsteady flow field using vortex method, and the second part is far-field sound pressure using two-dimension FW-H equation with the unsteady surface load as the sound source. Correctness of the methods is validated firstly. As for unsteady flow field of NACA0012 airfoil, the frequency of vortex shedding is in good agreement with reference. As for sound pressure of circular cylinder at different Reynolds numbers, the directivity diagram and sound spread figure are very similar to paper results, and also show that the tonal noise is mainly from vortex shedding.
For sound pressure in far-field of NACA0012 airfoil, the directivity of sound pressure in the far field is in a good agreement with reference.

While wind turbine blades runs at Reynolds number $10^6$ and more, results here pave the way for the further research.

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