The dynamical behavior of homogeneous scalar-field spacetimes with general self-interaction potentials

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Abstract The dynamics of homogeneous Robertson–Walker cosmological models with a self-interacting scalar field source is examined here in full generality, requiring only the scalar field potential to be bounded from below and divergent when the field diverges. In this way we are able to give a unified treatment of all the already studied cases—such as positive potentials which exhibit asymptotically polynomial or exponential behaviors—together with its extension to a much wider set of physically sensible potentials. Since the set includes potentials with negative inferior bound, we are able to give, in particular, the analysis of the asymptotically anti De Sitter states for such cosmologies.

Keywords Cosmological models · Homogeneous scalar fields

1 Introduction

Scalar fields have attracted a great deal of attention in cosmology, since the discovery that such a field can act as an “effective” cosmological constant in driving inflation [18]. From this first break-through, which aroused from the simplest conceivable model, i.e. that of a non self-interacting field, the research on scalar field spacetimes has been
constantly growing, and quite more general scenarios have been considered, such as scalar fields coupled with perfect fluids, non-minimal couplings, and alternate theories of gravity [1–8].

A scalar field spacetime can be viewed, like other matter models coupled with gravity such as perfect fluids, as a solution of the Einstein field equations which depends on the choice of the equation of state for the matter. In the case of the scalar field, the role of the equation of state is played by the self-interaction potential $V$, which is equal to zero in the “standard” inflationary solution. For several reasons, however, it is difficult to believe that this function vanishes. For instance, dimensional reduction of fundamental theories to four dimensions typically gives rise to self-interacting scalar fields with exponential potentials, coupled to four-dimensional gravity. Therefore exponential potentials have been widely considered in the literature (see [27] and references therein) also with the aim of uncovering possible large-scale observable effects [11,25,26,28]. Of course, however, at the present status of our knowledge the specific functional form of $V$ is quite unsure and, as a consequence, it would be optimal to classify the dynamical behavior of the spacetimes in dependence of all the possible choices of the potential function. Relevant attempts have been made also in this more general direction. Actually, the application of dynamical systems techniques has proven very useful and, due mainly to the works [10,19–24], we know the dynamical behavior of scalar field spacetimes for a wide class of non-negative potentials; for such potentials therefore, as will be explained below, the contribution of the present paper relies in a simplification and completion of known results.

Non negativity of the potential means that the energy density of the scalar field has a positive lower bound. However, in many issues and especially when string theory comes into play, it becomes relevant to inspect spacetimes in which the potential still has a lower bound but this bound is negative. From the physical point of view, it should be noted that, although care must be given to the fact that local positivity of energy density might be violated in these spacetimes, under certain somewhat general conditions potentials of this kind generate solutions with positive total energy [17]. In particular, of course, a constant negative potential generates an “equilibrium” state which is just the Anti de Sitter solution (AdS); therefore, a potential with a negative minimum generates a class of spacetimes which have an AdS “equilibrium” (see e.g. [5,15,16]). In the present paper we thus study the dynamical behavior of FRW scalar field cosmological models, imposing only very general conditions on the scalar field potential; our hypotheses essentially reduce to ask the potential to be bounded from below and divergent when the field diverges. Within this quite general framework, we give a unified treatment of all the relevant known cases—such as asymptotically polynomial and exponential behaviors—as well as its completion to a much wider set of possible, physically sensible potentials, also in presence of negative (i.e. AdS) minima. It is worth mentioning that our treatment can be applied to the “reverse” problem as well, i.e. to homogeneous scalar field collapse. This issue is treated in companion papers [12–14].

The paper is organized as follows. First, we formulate the field equations as a first order, regular dynamical system for the scalar field, its time derivative, and the scale factor. Then, we study in full generality the behavior of the trajectories of this system in dependence of the properties of the potential function. Finally, the issue of recollapse