Numerical Analysis of Inner Flow Loss Evolutions in a Low-Aspect Ratio Transonic Compressor Rotor at Windmilling Conditions

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Abstract. This paper conducts a detailed numerical investigation to study the evolutions of flow topology and aerodynamic losses in a low-aspect ratio compressor rotor operating from freewindmill condition (near stall point) to highly loaded windmill condition (near choke point). Firstly, a simplified numerical calculation method based on energy transfer between rotor blade and working fluid is employed to capture the windmill points at low rotational speeds. Then, the influences of windmill on tip leakage loss are analyzed. At windmilling conditions, the tip leakage flow travels across the clearance from suction to pressure side and causes blockages near the pressure surface in the casing end-wall region, which is quite different from the flow phenomenon at freewindmill conditions. Finally, the influences of windmill on flow separations are investigated. In tip region, with the rotor operating from freewindmill to highly loaded windmill condition, the flow separation switches from suction surface separation to pressure surface separation, and both the intensity and size are increased. Unlike near the tip, the hub separations always appear on the suction surface and a radial growth is observed near the trailing edge at the highly loaded windmilling condition, which is not presented at both the near stall and windmill points.

1. Introduction
For an aircraft, when the combustor is unlit and the shaft rotating speed is suddenly decreased, the aero-engine will provide no power to the compressor. In this situation, the rotational components in a compressor will freewheel under the influences of the incoming moving air and this is the windmilling condition. At this condition, the work input to the compressor is far less than the energy transmitted from the incoming air to the compressor, and some stages are operating in a turbine mode. At this moment, the engine drag becomes negligible and will directly affect the glide angle and flight stability when an aircraft is attempted to land. Moreover, as the compressor freewheels, the evaluation of the mass flow, pressure and temperature in the combustor is very essential because it has a significant influence on the successful relight of the engine. Therefore, it is very important to have investigations on the aerodynamics of compressors at the windmilling operating point.

To improve the relight capability of the engine, it is required to provide accurate predictions for the key performance parameters at windmilling conditions through prediction models or physical experiments. Many investigations have been carried out on the development of prediction models, such as windmill performance modeling methods [1-3], turbofan sub-idle performance model [4, 5], and real-
time aero-thermodynamic model [6]. In the meanwhile, Mishra et al. [7] also carried out experimental tests for a twin-spool low bypass turbofan engine to get the windmill performance characteristics and the data of the full engine for light-off at different altitudes with different flight Mach numbers were also obtained based on an altitude test facility. Even though these prediction methods and experimental results can provide the overall performance of an engine under the windmilling condition, they can hardly give the inner aerodynamics of the engine and the studies on the aerodynamics of compressors at this working condition become very essential.

In recent years, much attention has been focused on the investigations of aerodynamics of the fan stage in high bypass ratio turbofans at windmilling conditions. Previous research work by Prasad and Lord [8] based on a high-bypass ratio fan stage under engine-out windmilling condition showed that high aerodynamic losses occurred in both the rotor and stator vane due to highly off-design working condition (highly negative incidences). In the meanwhile, it was also found that a large amount of total pressure loss was presented in the stator passage because the rotor outflows left the blade at approximately the metal angle [9, 10]. Ortolan et al. [11] conducted an experimental and numerical investigation on the influences of downstream stator on the evolutions of rotor outflow topology in low-speed fans at highly loaded windmilling conditions. They found that the stator modifications could hardly affect the rotor outflow topologies, but presented a significant influence on the local aerodynamic losses in non-rotating rows and the operating mass flow in the rotor blade passage. Moreover, Gunn and Hall [12] made a loss and deviation analysis in both a low-speed rig fan and a transonic fan, and the attention was focused on the comparison results between the design point and the windmilling condition. The results in the rig fan showed that the flow separations occurred on the pressure surface in the stator along most of the blade span, which led to local flow blockages and changed the rotor work profile. In the meanwhile, the transonic rotor deviation at the windmilling condition was different from that at the design point and showed an increment at the hub region. Dufour et al. [13] studied the flow through the fan stage of a high bypass ratio turbofan under windmilling condition and reported three-dimensional (3-D) separation topology pattern in the stator blade passage. Moreover, the nonlinear harmonic method was also used to make unsteady analyses of the fluid over the stage section near the tip and reported that the modal content of the rotor/stator interactions downstream the stator was quite specific.

The above analyses for the aerodynamics at windmill were made based on the classical high-aspect ratio fan blades, but different physics and flow structures would be expected to appear for the compressors with low-aspect ratio blades [13]. Goto et al. [14] conducted an experimental and numerical study on the unsteady flow fields in an axial-flow compressor to reveal the flow structures in the stator at windmill. They found that severe flow separations were presented on the pressure surface of the stator blade due to extremely negative incidence angle and a large vortex was generated in the separation region, which produced serious blockages and total pressure losses. Gill et al. [15] described a complete four quadrant based on the compressor operating conditions for a three stage low-speed axial compressor. In different quadrants, the compressor was forced to work at various combinations of rotating direction, flow direction and pressure rise, which also included a windmilling operating condition.

From the above investigations in open literature, it can be seen that the reports concerning the aerodynamics of the compressors with low-aspect ratio blades at windmilling conditions are still quite limited, and the research works involving the inner flow-field details and the loss evolutions in low-aspect ratio compressors are still required. In view of this point, the paper has conducted a numerical investigation on the evolutions of flow topologies and aerodynamic losses in a low-aspect ratio transonic compressor rotor when its operation covers the near stall point (compressor mode), windmill point and highly loaded windmilling condition (turbine mode) at one rotating speed line. Firstly, a simplified numerical calculation method has been developed to justify the windmill points for the low-aspect ratio rotor. Then, the overall profiles of some key aerodynamic parameters and the associated aerodynamic losses have been given. Finally, the attention has been focused on the analyses and discussion on the evolutions of tip leakage loss and separation losses at both the tip and hub regions during the operation mode transformation. The motivation of this work is to gain more insight into the physics of the flow structure evolutions in a low-aspect ratio compressor rotor at windmill.
2. Research Object and Computational Techniques

2.1. Research Object
A high-performance low-aspect ratio transonic compressor rotor, NASA Rotor 67 designed by NASA Lewis Research Center, has been chosen as the research compressor because it is a widely used and well documented test case [16]. This compressor rotor is the first-stage rotor of a two-stage fan. The rotor solidity varies from 3.11 at the hub to 1.29 at the tip and the inlet and outlet hub-to-tip ratios are 0.375 and 0.478, respectively. The other major design parameters are summarized in Table 1.

| Table 1. Design parameters of the NASA Rotor 67 |
|-----------------------------------------------|
| Blade parameters | Value |
| Blade number | 22 |
| Inlet relative tip Ma | 1.38 |
| Blade aspect ratio | 1.56 |
| Tip solidity | 1.27 |
| Tip clearance (mm) | 1 |
| Rotating speed (rpm) | 16043 |
| Tip speed (m/s) | 429 |
| Mass flow (kg/s) | 33.25 |
| Total pressure ratio | 1.63 |

2.2. Numerical Methods
The 3-D steady Reynolds-averaged Navier-Stokes equations in the rotating frame are solved by using a commercial flow solver NUMECA FINE-Turbo EURANUS through a finite volume method. Time integration uses a four-stage Runge-Kutta scheme with implicit residual smoothing, while the spatial discretization employs a second-order central-differenced scheme. The one-equation Spalart-Allmaras model is utilized for turbulence closure, which is based on the previous studies concerning turbomachines [17-19]. Some acceleration techniques, such as multi-grid method and local time stepping, are also adopted in the simulation processes.

A single-blade passage computational model is applied in this work, with periodic boundary conditions along the pitchwise direction. Figure 1 gives a description of the computational model. The inlet of the computational domain is set at 2.5 times the axial chord length (at blade mid-span) upstream the leading edge, while the outlet is set at 2.5 times the axial chord length downstream the trailing edge. In the meanwhile, total pressure and total temperature are imposed at the inlet and the inflow is set as the axial direction. At the outlet of the computational domain, the averaged static pressure is specified based on the radial equilibrium. A perfect gas is selected as the working fluid, and the non-slip and adiabatic conditions are imposed on all the solid boundaries in the computational domain.

![Figure 1. Schematic graph of the computational model](image-url)
The computational domain is discretized with a multi-block O4H-type structured mesh. The blade surface is surrounded by an O-type mesh, while the remaining regions are filled with H-type mesh. The tip gap region is meshed with a butterfly topology, composed of 17 grid points in spanwise locations to model the local clearance flows. The computational mesh for the compressor rotor is depicted in Figure 2. In all the simulations, the y+ values are kept less than 5 near the solid walls.

A grid independence study is conducted by taking into account three different grid points (Grid A, Grid B and Grid C) to avoid the influences of grid size on inner flow-field solutions. All these numerical simulations are based on the same boundary conditions both at the inlet and outlet of the computational domain. The Grid A is the coarse mesh which has approximately 0.33 million grid points. The Grid B represents the medium mesh with a total number of 0.87 million grid points, while the Grid C is the fine mesh containing about 1.25 million grid points. Figure 3 gives the results of the grid independence study. On the one hand, the distributions of the rotor efficiency along the radial direction are compared. As shown in Figure 3(a), the computed adiabatic efficiency for Grid B corresponds well with that for Grid C along the full blade span, and both of the two numerical results show notable deviations with the results using grid A, especially at the spanwise locations from 35% to 65% span and from 75% to 90% span. On the other hand, the radial distributions of the total pressure ratio for different meshes are also compared. As depicted in Figure 3(b), the numerical simulation results with grid B and Grid C have shown similar variation trends along the full spanwise locations, both of which show notable differences from that for Grid A from the hub to 90% blade span. On the whole, to balance the computational accuracy and time cost, the Grid B is utilized to perform the numerical simulations in this study.
2.3. CFD Validations
To evaluate the CFD method and tool, the numerical results are compared with the available experimental data. The detailed experimental results are measured by using the laser anemometer, which are provided by Strazisar et al. [16]. The comparisons are mainly based on the compressor performance maps and the inner flow-field details.

Figure 4 gives the comparisons of the performance maps between the CFD results and the experimental data at the 100% rotational speed. For the adiabatic efficiency, the CFD results agree well with the experimental results over the whole operating range. As for the total pressure ratio, the CFD results are slightly lower than the experimental data. It can be found that the stable operating range predicted by the CFD is lower than that of the experiment. Overall, the CFD results match well with the experimental data.

Figure 3. Results of the grid independence study

Figure 4. Comparisons of the compressor performance maps between the CFD and the experimental results
The total temperature and total pressure distributions along the radial direction at the rotor outlet are compared between the CFD results and the experimental data. The comparison results are shown in Figure 5. For the total temperature distribution in Figure 5(a), it can be found that the CFD results agree very well with the experiment along the full blade span. As for the total pressure distribution in Figure 5(b), a good agreement between the CFD results and the experimental data is also observed, except that there are small deviations near the tip.

![Figure 5. Comparisons of the radial distributions of the total temperature and total pressure between the CFD results and the experimental data](image)

As for the numerical validations by other researchers, it is also observed that the performance maps and inner flow-fields are well predicted by NUMECA FINE-Turbo and have a good agreement with the available experimental data [20, 21]. Based on the above validation works, it is indicated that the CFD tool and method used in this study can provide reliable and accurate flow-field details and performance map predictions for a transonic axial-flow compressor.

3. Simplified Numerical Calculation Method for Windmill Conditions

At windmilling conditions, the engine would provide no power and the compressor rotor is in a freewheeling mode. In this situation, there is a critical windmilling point where the compressor produces no work to the incoming fluid and the incoming fluid also provides no work to the compressor. This paper has developed a simplified numerical calculation method to justify the critical windmill point.

Figure 6 gives a schematic graph of a rotor blade in a Cartesian coordinate system. The Z coordinate is along the axial direction. Take an elemental volume $\delta A$ on the blade surface for an example, the force imposed on the elemental volume by the working fluid, which is equal to that on the fluid by the elemental volume, can be expressed as:

$$\delta \vec{F} = P(\vec{n}_A \cdot \delta A)$$  \hspace{1cm} (1)

Where $P$ is the static pressure imposed on the elemental volume and $n_A$ is the normal vector for the elemental volume $\delta A$. 

![Figure 6. Schematic graph of a rotor blade in a Cartesian coordinate system](image)
Figure 6. Schematic graph of the force on the elemental volume in the rotor blade

Since the elemental volume is rotating around the Z axis, the torque to the fluid imposed by the elemental volume can be expressed as:

$$\delta \vec{M} = \vec{r} \times \delta \vec{F}$$  \hspace{1cm} (2)

$$\delta \vec{M}_Z = \delta \vec{M} \cdot \vec{n}_A = (\vec{r} \times \delta \vec{F}) \cdot \vec{n}_A$$  \hspace{1cm} (3)

In this situation, the axial component of the torque to the fluid by the whole blade can be obtained by integrating the $\delta M$ over the blade surface and is expressed as:

$$M_Z = \sum \delta M_Z = \sum [(\vec{r} \times \delta \vec{F}) \cdot \vec{n}_A]$$  \hspace{1cm} (4)

The total work input to the fluid is as follows:

$$W = M_Z \cdot \omega$$  \hspace{1cm} (5)

Where $\omega$ represents rotating speed of the rotor blade. From the equation (5), it can be seen that when the rotor blade does work to the fluid, the total work input $W$ is positive and the axial component of the torque $M_Z$ is also positive. When the fluid in turn does work to the rotor blade, the total work input $W$ is negative and the axial component of the torque $M_Z$ is also negative. When it comes to the situation that $M_Z$ is equal to zero, it is supposed to be the windmill point where there is no work exchange between the rotor blade and the working fluid.

Figure 7 gives the maps of axial torque versus mass flow rate for the compressor rotor, meanwhile the red line shows the boundary line where the axial torque of the blade is zero. It can be seen that when the compressor operates at low rotational speeds, there are operating points in which the torque is equal to zero. It means that the windmill points exist on these rotational speed lines. Moreover, with the decrease of the rotational speed, the windmill point moves towards the stability limit of the compressor, and the range of blades operating in turbine mode also increases and the same time. When the rotational speed drops to 10% of the design speed, the whole characteristic line is below the dividing line, which means the blades will work completely in the turbine mode. However, when the compressor operates at high rotational speeds, it can be observed that the rotor blade operates without windmilling conditions.
Figure 7. Maps of axial torque versus mass flow rate for the compressor rotor

According to equation (5), the windmill points are selected based on the fact that the axial component of the total torque of the blade is equal to zero. Figure 8 presents the variations of the operating mass flow rates at the windmill points with different rotational speeds. It is found that as expected, the windmill point moves towards higher operating mass flow with the increment of the rotational speed. Moreover, this variation has shown an approximately linear trend.

Figure 8. Variations of the operating mass flows at windmill points with different rotational speeds

For this low-aspect ratio compressor rotor, as the windmill points can be determined, the loss evolutions in the blade passage will then be analyzed from the freewindmill condition (compressor mode) to the highly loaded windmilling condition (turbine mode) at one rotating speed line. In this work, the 30% rotational speed line has been selected as shown in Figure 9. The near stall point (freewindmill condition), the windmill point (freewheeling mode) and the near choke point (highly loaded windmilling condition) are marked on this operating line to give descriptions of the flow topology and loss evolutions in the rotor blade passage.
4. Results and Discussion

In this section, the radial profiles of some key aerodynamic performance parameters are compared at different operating points to present the physical characteristics of the windmill condition. Then, the evolutions of the flow topology and aerodynamic losses including the tip leakage loss, tip and hub separation losses are fully analyzed at the near stall, windmill and near choke points. The main purpose is to give a full description of the flow topology and structures in the low-aspect ratio compressor rotor at windmilling conditions.

4.1. Comparisons of Performance Parameters

For a compressor rotor, when the blade is operating at the near stall condition, windmill point and near choke condition, the blade airfoil sections at different spanwise locations will operate in different mode, especially at the windmilling conditions. Therefore, the radial profiles of some key performance parameters are compared for the different operating conditions to justify the physical characteristics of the rotor at the windmilling condition.

Figure 10 compares the incidence angle distributions along the radial direction for the near stall condition, windmill point and near choke condition. It can be observed that the rotor blade operates at negative incidence angles along the full blade span at the windmill point. For the highly loaded windmilling condition, the rotor blade suffers from highly negative incidence angle in comparison with that at the windmill point. As for the near stall condition, the rotor blade operates at positive incidence angles along the full blade span, suffering from highly positive incidences at the hub region. The results of the incidence distributions suggest that the rotor blade works in different operation modes for the near stall, windmill and near choke conditions. The work exchange between the rotor blade and the working fluid will be further analyzed to identify the specific operation mode of the compressor.

To measure the work input to the working fluid by the rotating blade, the blade load coefficient can be used to describe the radial work distributions and defined as:

$$
\psi = \frac{(h_{out} - h_{in})}{U^2}
$$

where $h$ represents the stagnation enthalpy and $U$ denotes the rotor rotational velocity. In this study, the rotational velocity at the rotor tip has been utilized.
The radial work distributions for different operating conditions are compared in Figure 11. It is found that at the windmill point, the rotor blade does work to the fluid from the hub to about 45% blade span, while the rotor blade accepts the work from the fluid in the outward spanwise locations. Overall, there is no net work exchange between the rotor blade and the working fluid. At the near choke condition (also the highly loaded windmill condition), there is no work input to the fluid any more along the full blade span, which means that the rotor blade freewheels. As for the near stall condition, it can be seen that approximately all the blade airfoil sections operate in a compressor mode in the radial locations and it is indicated that the rotor blade can normally input work to the fluid. The results show that there are two operating modes (compressor mode and turbine mode) at low rotational speeds for the compressor rotor and the rotor will switch the operation between these two modes at different mass flow conditions on the rotational speed line.

The overall blade load and total pressure ratio for different operating points are summarized in Table 2. At the windmill point, the rotor blade load is very close to zero showing almost no net work input,
while the total pressure ratio at this condition is very close to 1. When the rotor operates as compressor mode at the near stall condition, the total pressure ratio is very low due to the low rotational speed. However, at the highly loaded windmilling condition, the blade load becomes highly negative and the total pressure ratio is far less than 1, indicating the rotor operating in a turbine mode.

**Table 2.** Comparisons of load coefficient and total pressure ratio of the compressor rotor at different operating conditions

| Parameters   | Near stall | Windmill | Near choke |
|--------------|------------|----------|------------|
| Load coefficient | 0.55       | 0.01     | -1.66      |
| Total pressure ratio | 1.04       | 0.99     | 0.81       |

Furthermore, to estimate the overall aerodynamic loss of the compressor at these three operating points, the entropy loss coefficient is introduced to measure the overall aerodynamic loss and defined as follows:

\[
\xi = \frac{T \Delta s}{0.5V^2}
\]

Where \( V \) is the velocity evaluated at the rotor inlet in the relative frame, \( s \) represents the entropy and \( T \) is the static temperature. The advantage of the entropy loss coefficient is that it is effective for both the rotor and stator blades.

The entropy losses of the investigated rotor are compared for different operating conditions and the results are depicted in Figure 12. In comparison with the aerodynamic loss at the near stall point, the rotor suffers from higher loss at the windmill point. However, when the compressor operates at the highly loaded windmilling condition, the suffered aerodynamic loss will rapidly increase, which is much higher than those at both the near stall point and the windmill point. In the following sections, the attention will be focused on the evolutions of the major aerodynamic losses in the blade passage for the near stall, windmill and near choke points.

**Figure 12.** Comparison of the entropy loss of the rotor at different operating conditions

4.2. Effect of Windmill on Tip Leakage Loss

For the compressor rotor, the tip leakage loss is a major source of aerodynamic losses and can significantly influence the compressor performance. At normal operating conditions, the leakage flow will travel across the tip clearance under the pressure difference between the pressure surface and suction surface and then shear with the mainstream at the outlet of the clearance, leading to leakage vortex near
the casing. The leakage vortex develops along the streamwise direction and results in flow blockages and losses in the blade passage. In this section, the evolutions of the tip leakage loss from the near stall point to the highly loaded windmilling condition will be discussed.

In general, the leakage vortex will cause a low momentum fluid region near the casing due to the mixing between the leakage and main flows. In this situation, the intensity and size of the low momentum region could be utilized to evaluate the interaction loss of the leakage flow and the mainstream. To give a clear view of the tip leakage flow trajectory and the interaction between the leakage and main flows, the entropy distributions on the blade-to-blade surfaces in the tip region are compared at different operating conditions. As shown in Figure 13(a), since the rotor blade is operating in a compressor mode, the leakage flow travels across the tip clearance from the pressure surface to the suction surface and interacts with the main flow, leading to leakage vortex and the associated blockages in the blade passage near the pressure surface of the adjacent blade. In comparison, at the windmill point as depicted in Figure 13(b), it can be observed that the leakage flow crosses the tip clearance from the blade suction surface to the pressure surface and results in a low momentum fluid region near the pressure surface, which is quite different from the leakage flow phenomenon at compressor mode. Moreover, a large area of flow blockage occurs on the blade pressure surface along the streamwise direction, leading to high aerodynamic losses. For the highly loaded windmilling condition as described in Figure 13(c), a similar flow phenomenon as that at the windmill point is also observed. However, it is noted that the intensity and size of the interaction between the leakage and main flows are much higher at the highly loaded windmilling condition than those at the windmill point. As a result, the size of the flow blockage in the blade-to-blade direction has also been notably increased.

![Entropy distributions](image)

**Figure 13.** Comparison of entropy distribution on the blade-to-blade surface near the tip (98% blade span) at different operating conditions

To further present the developments of flow blockages and local aerodynamic loss along the streamwise direction, the entropy distributions are plotted on the cross planes in the tip region as shown in Figure 14. At the near stall condition in Figure 14(a), like the compressor operating at high rotational speed, the interaction between the tip leakage flow and the mainstream has increased the casing end-wall boundary layers, especially near the pressure side of the adjacent blade. When the compressor operates at the windmill point in Figure 14(b), the leakage flow coming out of the clearance from the pressure side has caused notable leakage vortex near the pressure surface in the casing end-wall region. The resultant flow blockage develops along the streamlines and its size increases in both the radial and circumferential directions, leading to a notable blockage area near the tip at the rotor outlet. Further, at the highly loaded windmilling condition in Figure 14(c), it can be seen that both the intensity and size
of the flow blockages are much higher than those at the windmill point, resulting in much higher local aerodynamic losses. Based on the above results, it is indicated that the leakage flow in the tip region will be deteriorated at windmilling conditions (from the windmill point to the highly loaded windmilling conditions) in comparison with that at the near stall condition in freewindmill mode. Moreover, the casing end-wall flow blockages and the associated aerodynamic losses are also notably aggravated at the highly loaded windmilling point.

Figure 14. Comparison of the entropy distributions on the cross planes in the blade passage at different operating conditions

4.3. Effect of Windmill on Tip Flow Separations

For the compressor rotor operating under off-design conditions at low rotational speeds, flow separation loss is also a major source of losses in the blade passage [22, 23]. In this section, the influences of windmill on the flow separations in the rotor tip region are discussed.

Since the blade load has a significant influence on the separation onset in the blade passage, the pressure coefficient is introduced to measure the load distribution on the blade surface and defined as:

$$C_p = \left( \frac{P_{wall} - P_{in}}{P_{in}^* - P_{in}} \right)$$  \hspace{1cm} (8)

Where $P_{wall}$ is the static pressure on the blade surface. $P_{in}^*$ and $P_{in}$ represent the total pressure and static pressure at the inlet of the computational domain.

Figure 15 compares the pressure coefficients on the blade surface for different operating conditions in the tip region. At the near stall condition, as in a compressor mode, the static pressure on the pressure surface is higher than that on the suction surface along the full chordwise direction. In this situation, the rotor blade can still input work to the working fluid. As the mass flow increases, the pressure on the pressure surface becomes lower than that on the suction surface from the blade leading edge and there is an intersection on the pressure coefficient line between the pressure side and the suction side at about 60% chordwise location for the windmill point. The results indicate that the rotor blade accepts work from the working fluid from the leading edge to approximately 60% chordwise location, while the rotor blade still inputs work to the working fluid from the 60% chordwise location to the trailing edge. At the highly loaded windmilling condition, the intersection on the pressure coefficient line between the pressure side and the suction side has been pushed backward to about 75% chordwise location. It can be also found that the pressure difference between the suction side and the pressure side has been enlarged compared with that at the windmill point. The results suggest that the areas of the chordwise locations accepting work from the working fluid also increase and the rotor blade has lost the ability to input work to the working fluid at the highly windmilling condition.
Figure 15. Comparison of the pressure coefficients on the blade surface for different operating conditions in the tip region (90% blade span)

Further, the relative Mach number distributions on the blade-to-blade surfaces near the tip are compared at different operating conditions to give descriptions of flow separations on the blade airfoil sectional surfaces. The comparison results are shown in Figure 16. At the near stall condition in Figure 16(a), the flow separation occurs near the leading edge and covers almost the full axial chord length locations as the compressor operates in a compressor mode. When the compressor operates at the windmill point in Figure 16(b), the flow separation has switched from the suction surface separation to the pressure surface separation originated from the leading edge since the rotor blade is working under negative incidence condition as depicted in Figure 10. With the mass flow further increasing to the highly loaded windmilling condition, it is found that the flow separation still exist on the pressure surface, but the intensity of the separation at the leading edge has increased compared with that at the windmill point.

Figure 16. Comparison of the relative Mach number distributions on the blade-to-blade surfaces for different operating conditions in the tip region (90% blade span)

With regard to the local aerodynamic losses, Figure 17 compares the entropy distributions on the blade-to-blade surface at different operating conditions. It can be observed that the aerodynamic loss is
presented on the suction surface at the near stall condition, while at the windmill point the aerodynamic loss is mainly shown on the pressure surface. Moreover, when the rotor blade operates in the highly loaded windmilling condition, the flow separation still occur on the blade pressure surface but has caused much higher aerodynamic loss than that at both the near stall point and the windmill point.

Figure 17. Comparison of the entropy distributions on the blade-to-blade surfaces for different operating conditions in the tip region (90% blade span)

4.4. Effect of Windmill on Hub Flow Separations
Hub corner separations might also occur in a well designed compressor even at the design point and will always cause large amount of local aerodynamic losses and flow blockages. In this section, the major attention has been focused on the influences of windmill on the hub corner separations.

The pressure coefficient distributions on the blade surfaces for different operating conditions are compared near the hub as shown in Figure 18. At the near stall point, the static pressure on the pressure surface is higher than that on the suction surface along the full chordwise direction and the rotor operates in a compressor mode, which presents a similar flow phenomenon as that in the blade tip region. In this situation, the rotor blade inputs work to the working fluid along the full axial chord length locations. For the windmill point, it can be also noted that the pressure on the pressure surface becomes lower than that on the suction surface from the blade leading edge, but the intersection point on the pressure coefficient line between the pressure side and the suction side has moved forward to about the 15% chordwise location at the hub region in comparison with that at the tip region. The results indicate that even though the rotor blade accepts work from the working fluid from the leading edge to approximately 15% chordwise location, on the whole, the overall work input to the working fluid by the rotor blade is positive, which is different from the result near the tip. It is also indicated that the blade airfoil sections in the hub region have shown better operating conditions than those near the tip at the windmill point, which corresponds well to the results shown in Figure 11. At the same time, a similar variation trend of the pressure coefficient distributions has been observed at the highly loaded windmilling condition. The intersection on the pressure coefficient line between the pressure side and the suction side has been moved forward to about 30% chordwise location compared with that near the blade tip. It means that the rotor blade airfoil near the hub has operated closer to the compressor mode than that near the tip.
Figure 18. Comparison of the pressure coefficient distributions on the blade surfaces for different operating conditions in the hub region (10% blade span)

Figure 19 describes a comparison of the entropy distributions on the blade-to-blade surfaces at different operating conditions near the hub. At the near stall condition, no notable flow separations have been observed on the suction surface at the hub region. With the operating mass flow increasing, the separation region starts to appear on the suction surface near the trailing edge at both the windmill point and the highly loaded windmilling condition. Moreover, it is noted that the rotor blade suffers from much higher separation loss at the trailing edge at the highly loaded windmilling condition than that at the windmill point. However, unlike in the blade tip region, no notable flow separations have been observed on the pressure surface near the hub from the freewindmill to the highly windmilling condition.

Figure 19. Comparison of the entropy distributions on the blade-to-blade surface for different operating conditions in the hub region (10% blade span)
The developments of the hub corner separations along the streamwise direction at different operating conditions have been given in Figure 20. As the operating condition varies from the near stall point to the highly loaded windmill point, the hub corner separations always exist on the suction surface and the intensity and size of the separation regions have been increased along the streamwise direction. In the meanwhile, it can be noted that there is a radial development of the hub corner separation near the trailing edge at the highly loaded windmilling condition, which is not shown at both the windmill point and the near stall point. The results show that the flow separations have only caused high aerodynamic losses on the blade suction surface near the trailing edge at the hub region, rather than on the pressure surface as in the tip region.

![Figure 20. Comparison of the entropy distributions on the cross planes in the blade passage for different operating conditions near the hub (10% blade span)](image)

5. Summary and Conclusions
This paper has conducted a numerical investigation on the evolutions of flow topology and aerodynamic losses in a low-aspect ratio transonic compressor from the normal operating condition to the highly loaded windmilling condition including windmill point at one rotating speed line. The motivation is to gain more insight into the physics of the flow structure evolutions in a low-aspect ratio compressor rotor at windmilling conditions. The main conclusions can be summarized as follows:
(1) For the investigated low-aspect ratio compressor, the rotor blade operates without windmilling conditions at high rotational speed, while the windmill point occurs at low rotational speed. Moreover, the results show that the operating mass flow rate at the windmill point has presented a nearly linear variation trend with the increase of the rotational speed. On one rotational speed line, the rotor will operate in a compressor mode at the near stall point. As the mass flow increases, there will be net work exchange between the rotor and the fluid at the windmill point, with work input to the fluid at the hub region and work input to the blade from the fluid at the tip region. At highly loaded windmilling condition, the rotor blade will totally operate in a turbine mode along the full span. In the meanwhile, the loss of the rotor blade will rapidly increase compared with that at both the near stall and windmill points.

(2) At the near stall condition, as the rotor operates in a compressor mode, the tip leakage flow travels across the clearance from the pressure side to the suction side and then lead to leakage vortex moving to the pressure surface of the adjacent blade. Unlike the near stall condition, the tip leakage flow at the windmill point travels across the clearance from the suction side to the pressure side and results in low momentum fluid region near the pressure surface, thus leading to flow blockages in the casing end-wall region. When the rotor blade operates at the highly loaded windmilling condition, a similar flow phenomenon as that at the windmill point is also found and both the intensity and size of the flow blockages have been notably increased.

(3) In the blade tip region, the flow separation is onset near the leading edge and develops on the suction surface along the full chordwise direction at the near stall condition. With the mass flow decreasing, the suction surface separation has switched to the pressure surface separation from the leading edge at the windmill point. As the mass flow further decreases to the highly loaded windmilling condition, the intensity and size of the flow separation has increased compared with those at the windmill point. On the whole, the pressure surface separation has led to much higher aerodynamic losses than the suction surface separation. Moreover, the separation loss increases rapidly with the rotor blade operating from compressor mode to turbine mode, especially at the highly loaded windmilling condition.

(4) In the blade hub region, the flow separations always appear on the suction surface rather than on the pressure surface in the tip region. At the near stall condition, no notable flow separations have been observed on the suction surface. As the operating condition varies from the near stall point to the highly loaded windmill point, both the intensity and size of the separation regions have been increased along the streamwise direction. In the meanwhile, it is noted that there is a radial development of the hub corner separation near the trailing edge at the highly loaded windmilling condition, which is not shown at both the windmill point and the near stall point.

**Nomenclature:**
- $C_p$ = pressure coefficient
- $h$ = stagnation enthalpy
- $M$ = torque
- $n_A$ = normal vector
- $P^*$ = total pressure
- $P$ = static pressure
- $s$ = entropy
- $T^*$ = total temperature
- $T$ = static temperature
- $U$ = rotor rotational velocity
- $W$ = work
- $y^+$ = non-dimensional wall distance

**Greek Symbols:**
- $\gamma$ = specific heat ratio
- $\pi$ = total pressure ratio $= P_{out}^*/P_{in}^*$
- $\zeta$ = entropy loss coefficient
- $\eta$ = adiabatic efficiency $= [(P_{out}^*/P_{in}^*)^{(\gamma-1)/\gamma-1}] / (T_{out}^*/T_{in}^*)^{\gamma-1}$
ψ = load coefficient
ω = rotor rotating speed
δA = the elemental volume

Subscripts:
in = inlet
out = outlet
PS = pressure surface
SS = suction surface
wall = blade surface
z = axial direction

Abbreviations:
CFD = computational fluid dynamics
LE = leading edge
TE = trailing edge

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