$\mathcal{O}(\alpha_s)$ Spin-Dependent Weak Structure Functions *

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La Plata Preprint 11-94 (6 December 1994)

Abstract

The complete next to leading logarithmic ($\mathcal{O}(\alpha_s)$) corrections to the spin dependent weak deep inelastic structure functions $g_1$, $g_3$ and $g_4$, are calculated using dimensional regularization within the HVBM method. Analysing the quark and gluon initiated contributions to these corrections for different values for the quark masses, a consistent factorization prescription for spin dependent quark distributions, which safely removes soft contributions, is defined. It is shown that within this scheme, quark initiated corrections are comparable in magnitude to those of gluonic origin, even though their contributions to the moments are small.

*Partially supported by CONICET-Argentina.
**Introduction**

The very recent release of new and more precise data on polarized deep inelastic scattering experiments [1] have considerably increased the interest in many aspects of the internal spin structure of nucleons and their related experiments [2]. Among them, a growing number of parametrizations for polarized parton distributions, designed to reproduce the experimental data, and cross sections for spin dependent processes, thought as further constraints for the distributions and cross check for the data, are available at present [3–5].

However, the increasing precision of the experiments and the alternative of having large contributions from gluon initiated processes, require both parton distributions and cross sections being obtained at least at order $\alpha_S$ and with a consistent prescription for the regularization and subtraction of soft contributions. It has been shown [3], in the context of the photon-gluon fusion process, that a careful analysis of the regularization procedures for collinear singularities is essential for isolating and subtracting soft contributions. This analysis not only sheds light on the controversy about this point, but emphasizes the importance of working in a consistent factorization scheme. The same kind of analysis, but for weak boson-gluon fusion processes, is clearly more involved due to the presence of different mass scales in the same process and must be carefully carried out.

In recent years, several works have been presented in connection with spin dependent weak structure functions [7–9]. Among them, those addressing to $O(\alpha_S)$ corrections, only deal with gluon initiated corrections, and, in most cases, without worrying about the proper subtraction of soft contributions. In the present paper we calculate the complete next to leading logarithmic -quark and gluon initiated- corrections to the spin dependent weak deep inelastic structure functions, using dimensional regularization [10] and treating $\gamma_5$ and $\epsilon_{\mu\nu\rho\sigma}$ according to the original proposal by t’Hooft and Veltman and systematized by Breitenlohner and Maison [11]. This method, hereafter referenced as HVBM, has all the advantages of dimensional regularization, can be implemented at higher orders of perturbation theory and is free from inconsistencies. In addition, it has been shown that soft contributions to different
processes involving massless quarks can clearly be identified and substracted in this scheme \[12\]. The presence of non zero masses, eventually large compared to the factorization scale \(\mu_{\text{fact}}\), changes this situation requiring a different factorization prescription. This problem is also addressed.

Finally, we discuss the phenomenological consequences of the alluded corrections in the structure functions, discriminating contributions from different origins and using parton distributions, defined with the above mentioned factorization prescription, which fit the available electromagnetic data.

**Structure functions and parton distributions**

In this section we begin establishing the definition of the weak structure functions that will be used throughout this paper and also our notation for parton distributions beyond leading order.

The spin dependent component of the hadronic tensor, in the case of a longitudinally polarized target, can be written as

\[
W^{\mu\nu} = -ie^{\mu\nu\rho\sigma} q_{\rho} g_{\sigma} + \left( -g^{\mu\nu} + \frac{q^{\mu} q^{\nu}}{q^2} \right) g_3 \\
+ \frac{1}{P \cdot q} \left( P^\mu - \frac{P \cdot q q^\mu}{q^2} \right) \left( P^\nu - \frac{P \cdot q q^\nu}{q^2} \right) g_4
\]

where \(q\) and \(P\) are the four-momentum of the exchanged virtual boson and the nucleon target respectively. With this definition, the structure functions have, at lowest order, the following form in terms of parton distributions

\[
g^B_j(x) = \sum_i C^{B}_{ij}(x) \Delta q_i(x) \tag{2}
\]

where \(x\) is the usual Bjorken variable i.e.

\[
x = \frac{-q^2}{2P \cdot q} \tag{3}
\]
\( \Delta q_i \) are the spin dependent quark distributions, and the coefficients \( C_{ij}^B \) are given in tables 1 and 2. The index \( i \) runs over the quark flavours and \( B \) indicates the boson exchanged in the process under consideration.

The next to leading logarithmic corrections arise, at order \( \alpha_s \), from the diagrams in figures 1, 2 and 3. The evaluation of the diagram in figure 1a corresponds to the lowest order expressions for the structure functions (equation 2). Diagrams 1b and 1c give no contribution for massless quarks in the Landau gauge. The interference between 1a and 1d, and also the diagrams of figure 2, produce both infrared and collinear contributions. While the infrared contributions cancel each other, the divergencies of collinear origin remain and have to be factorized in the definition of parton distributions. The diagrams of figure 3 also have collinear divergencies that have to be removed in the same way. In order to deal with the occurring divergencies we use dimensional regularization. In the diagrams considered above, \( \gamma_5 \) matrices are present not only due to the helicity projectors, but also because of the weak interaction vertex. As we have mentioned before, we have chosen to deal with this object following the HVBM proposal. Matrix elements can be straightforwardly calculated with the program TRACER [13], which masters most of the intricacies of the method.

By means of the usual projector technique, we obtain the \( \alpha_s \) contributions to the structure functions from the phase space integrated matrix elements (see appendix A for calculational details)

\[
g_j^B(x) = \sum_i \int_x^1 \frac{dz}{z} C_{ij}^B(z) \left\{ \left[ \delta(1 - z) + \frac{\alpha_s}{2\pi} \Delta P_{qq}(z)(\ln \frac{Q^2}{\mu^2} - \frac{1}{\epsilon}) + \frac{\alpha_s}{2\pi} \Delta f_j^q(z) \right] \Delta q_i^0(\frac{x}{z}) \\
+ \left[ \frac{\alpha_s}{2\pi} \Delta P_{qg}(z)(\ln \frac{Q^2}{\mu^2} - \frac{1}{\epsilon}) + \frac{\alpha_s}{2\pi} \Delta f_j^g(z) \right] \Delta g_0^0(\frac{x}{z}) \right\} \tag{4}
\]

The sum in this equation runs over the flavours of those quarks and antiquarks that participate in the vertex. \( \Delta P_{qq} \) and \( \Delta P_{qg} \) are the usual Altarelli-Parisi evolution kernels [14]. \( \mu \) is the scale introduced by dimensional regularization and

\[
\frac{1}{\epsilon} = \frac{1}{\bar{\epsilon}} + \ln 4\pi - \gamma_E \tag{5}
\]

where \( \epsilon \) is defined through \( d = 4 - 2\epsilon \), being \( d \) the space-time dimension. \( \Delta q_i^0 \) and \( \Delta g^0 \) are
the bare densities of quarks and gluons respectively. The term proportional to \( \delta(1-z) \) is the lowest order contribution arising from diagram 1a, while \( \Delta f_q \) and \( \Delta f_g \) are the quark and gluon initiated finite non logarithmic corrections.

Defining the scale dependent next to leading logarithmic quark distributions as

\[
\Delta q_i(x, Q^2) = \Delta q_i^0(x) + \frac{\alpha_s}{2\pi} \int x^1 \frac{dz}{z} \left[ \Delta P_{qq}(z)(\ln \frac{Q^2}{\mu^2} - \frac{1}{\epsilon}) + \Delta f_{q}^0(z) \right] \Delta q_i^0\left(\frac{x}{z}\right)
\]

\[
+ \frac{\alpha_s}{2\pi} \int x^1 \frac{dz}{z} \left[ \Delta P_{qg}(z)(\ln \frac{Q^2}{\mu^2} - \frac{1}{\epsilon}) + \Delta f_{g}^0(z) \right] \Delta q_i^0\left(\frac{x}{z}\right)
\]

\[ (6) \]

the structure functions are then given by

\[
g_j^B(x) = \sum_i C_{ij}^B(x) \Delta q_i(x, Q^2) + \frac{\alpha_s}{2\pi} \sum_i \int x^1 \frac{dz}{z} C_{ij}^B(z) \left[ \Delta f_j^q(z) - \Delta f_i^q(z) \right] \Delta q_i\left(\frac{x}{z}, Q^2\right)
\]

\[
+ \frac{\alpha_s}{2\pi} \sum_i \int x^1 \frac{dz}{z} C_{ij}^B(z) \left[ \Delta f_j^g(z) - \Delta f_i^g(z) \right] \Delta g\left(\frac{x}{z}, Q^2\right)
\]

\[ (7) \]

This definition factorizes the poles accounting for collinear singularities and the logarithmic terms associated with the Altarelli-Parisi evolution in the quark distributions. The terms \( \Delta f_{q}^0 \) and \( \Delta f_{g}^0 \), are designed to absorb eventual soft contributions from the non logarithmic corrections. Notice that in the usual \( \overline{\text{MS}} \) scheme this substraction terms are not included.

Modern parton distributions are usually defined in what is called a variable (scale dependent) flavour number scheme \([10]\). In schemes of this kind, quarks whose masses are smaller than the typical energy scale \( \mu_{\text{phys}} \) are considered massless in the evolution. For those quarks with masses larger than the scale, there are no associated parton distributions. In this paper we adopt this kind of scheme, however we keep in mind the absolute mass hierarchy for factorization purposes.

\( \mathcal{O}(\alpha_s) \) corrections

In this section we calculate the soft and hard contributions corresponding to the different diagrams. We begin with those related to the box diagrams of figure 3. In reference \([11]\) this
kind of diagram, but for an electromagnetic interaction, was evaluated using dimensional regularization within the HVBM method obtaining

\[ g_1^{\gamma}_{(\text{parton})} = e_i^2 \frac{\alpha_s}{2\pi} \Delta P_{qg}(\ln \frac{Q^2}{\mu^2} - \frac{1}{\epsilon}) + e_i^2 \frac{\alpha_s}{2\pi} \frac{1}{2} \left[2(2z - 1)(\ln \frac{1 - z}{z} - 1) + 2(1 - z)\right] \] (8)

The subscript \( \text{parton} \) means that in order to obtain the structure function, the expression must be convoluted with the appropriate bare parton distribution. Doing this and comparing with the electromagnetic version of equation (4), \( \Delta f_g \) can be identified with

\[ \Delta f_g (z) = \frac{1}{2} \left[(2z - 1)(\ln \frac{1 - z}{z} - 1) + 2(1 - z)\right] \] (9)

It has been shown \[6\] that the last term in equation (9) has a soft origin in the case of massless quarks, and therefore must be factorized. This means that, for massless quarks, \( \tilde{\Delta} f_{qg} \) must be fixed accordingly

\[ \tilde{\Delta} f_{qg}^{m<\mu_{fact}} = 1 - z \] (10)

However if we are dealing with quarks whose masses are smaller than the physical scale but are larger than the scale that defines soft and hard phenomena, they can be considered as massless for the evolution (active flavour) but the last term in equation (9) corresponds to a hard contribution and should not be factorized, i.e.

\[ \tilde{\Delta} f_{qg}^{m>\mu_{fact}} = 0 \] (11)

Of course, this implies a non vanishing gluonic contribution to the first moment of \( g_1^{em} \) only for light flavours \( m < \mu_{fact} \) independently of which are active \( m < \mu_{phys} \).

The preceeding discussion defines the gluonic part \( \tilde{\Delta} f_{qg} \) of our factorization prescription. \( \tilde{\Delta} f_{qg} \) will be fixed after we evaluate the quark initiated processes. In the following we calculate the analogue of equation (9) for the weak structure functions and verify that our prescription factorizes soft contributions.

For \( g_1 \), the finite non logarithmic contribution \( \Delta f_g \) is identical for both the electromagnetic and weak structure functions. The effect of the weak vertex is completely absorbed in
the coefficient $C_{11}^B$. If the exchanged boson does not produce change in the quark flavour, the factorization prescription works in complete analogy to the electromagnetic case. However, if there is change of flavour and one of the quarks has a mass larger than the factorization scale, whereas the other does not, then there is a soft contribution arising from the lighter but not from the heavier. The factorization prescription accounts for this as it should.

For $g_3$ and $g_4$, $\Delta f_g$ vanishes. Due to equation (11), the contribution associated with the heavier flavours via $\Delta_f^g$ also vanishes. When the exchanged boson is a $Z^0$, the contribution from the lighter quarks and antiquarks cancel between each other leading to a vanishing gluonic contribution to the structure functions. However, for flavour changing bosons there is no such cancellation. For example, if we deal with a $W^+$ and in a 4-flavour scheme, only $d, \bar{u}, s, c$ participate, and using equation (7) we have

$$g_3^{W+}(x, Q^2) \mid_{\text{gluonic}} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} \sum_{i = d, \bar{u}, s, c} C_{i3}^{W+} \left[ - \Delta f_i^g(z) \right] \Delta g \left( \frac{x}{z}, Q^2 \right)$$

This result agrees, if the limit $m^2/Q^2 \to 0$ is taken, with that of reference [8], where the gluonic contribution to the structure functions was evaluated using quark masses and a transverse momentum cut-off as regulator.

The quark initiated non logarithmic correction $\Delta f_q$ comes from the processes in figures 1 and 2. The amplitudes associated to diagrams 2a and 2b are straightforwardly evaluated and integrated in the corresponding phase space. Calculated in the Landau gauge and using the HVBM method, the vertex correction amounts to replace

$$\gamma_{\mu}(1-a\gamma_5) \leftrightarrow \gamma_{\mu}(1-a\gamma_5) \left\{ 1 + \frac{\alpha_s}{4\pi} \frac{4\pi \mu^2}{3} \left( \frac{4\pi \mu^2}{Q^2} \right)^\epsilon \frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} \left[ \frac{-2}{\epsilon^2} - \frac{2}{\epsilon} - 8 \right] \right\}$$

As in this gauge the quark self energy vanishes at order $\alpha_s$ for massless quarks, there is no quark wave function renormalization, all the effect of the interference between diagrams 1a and 1d is the replacement of the delta function in equation (4) by the delta times the factor between brackets in equation (13). Adding this contribution to those of diagrams 2a and 2b, and isolating the logarithmic and pole terms, we identify the $\Delta f_q$ term.
For $g_1^B$ the non logarithmic correction results to be

$$
\Delta f_1^q(z) = \frac{4}{3} \left[ (1 + z^2) \left( \frac{\ln(1-z)}{1-z} \right)_+ - \frac{3}{2} \left( \frac{1}{1-z} \right)_+ - \frac{1 + z^2}{1-z} \ln z \right. \\
\left. + 2 + z - \left( \frac{9}{2} + \frac{1}{3} \pi^2 \right) \delta(1-z) \right] - \frac{16}{3} (1-z)
$$

(14)

The last term can be traced back to have a collinear origin for massless quarks, so this defines the correspondent tilde-term

$$
\widetilde{\Delta f}^q_{m<\mu_{fact}} = -\frac{16}{3} (1-z)
$$

(15)

Analogously, we fix for heavy quarks

$$
\widetilde{\Delta f}^q_{m>\mu_{fact}} = 0
$$

(16)

Equations (15) and (16) complete the definition of the factorization prescription we adopt. This prescription agrees with the one used in reference [12] for massless quarks but is clearly different for massive quarks.

Finally, for $g_3$ and $g_4$ the corrections are given by

$$
\Delta f_3^q(z) = \Delta f_1^q(z) + \frac{4}{3} (1 + z)
$$

(17)

and

$$
\Delta f_3^q(z) = \Delta f_1^q(z) + \frac{4}{3} (1 - z)
$$

(18)

The next to leading order quark initiated corrections to the structure functions $g_3$ and $g_4$ ($\Delta f_{3,4}^q - \widetilde{\Delta f}^q$) are identical to those obtained for the unpolarized structure functions $F_1$ and $F_2$, due to the same tensorial structure at partonic level [17].

In a scheme where the regularization is performed keeping explicitly the masses of the quarks and introducing a cut off in transverse momentum, the hard contributions are obtained taking into account in each process the mass hierarchy. However, with the factorization prescription we propose, the non logarithmic terms $\Delta f^q$ and $\Delta f^g$ are not dependent on the kind of boson exchanged or the relation between the quark masses and the factorization
scale. This kind of dependence is absorbed in the process independent $\tilde{\Delta f}$ terms showing clearly the universal character of factorization.

**Numerical Results**

In this section we analyse the relevance of the correction terms we have just calculated. All the analysis on $O(\alpha_s)$ corrections to the spin dependent weak structure functions available in the literature deal only with the gluon initiated corrections. Even more, most studies on spin dependent electromagnetic structure functions and parton distributions disregard the quark initiated corrections either not including them in equation (7) ($\Delta f_q = 0$) or approximating them by their effect in the moment of the structure function ($\Delta f_q = -\frac{\alpha_s}{\pi} \delta(1 - z)$).

As the contribution to the moment is of order $\frac{\alpha_s}{\pi}$, it is usually assumed that these corrections are small, however as the $\Delta f_q$ are non trivial functions, nothing guarantees that the corrections are not comparable to the others for some $x$ interval. Of course, one can always define a factorization prescription in such a way that some correction terms are absorbed in the parton distributions (at least for certain structure functions) provided the prescription is implemented consistently in other processes. This, however changes radically the interpretation of parton distributions. It is important then, to have an estimate of the relative weight of the corrections within the scheme employed and particularly in one that factorizes soft contributions in each process.

We begin comparing numerically the quark and gluon initiated corrections to the structure functions (second and third terms in equation (7), respectively), between each other and with the tree level part (first term of the same equation). In order to make the estimates, we need a set of spin dependent parton distributions defined at order $\alpha_s$ and within the factorization scheme proposed. We build the set taking for the bare densities the functional forms suggested by the spin dilution model [3] and fix the free parameters of it in such a way that the available data on spin dependent electromagnetic structure functions is reproduced.
This procedure reduces considerably the number of parameters to be fixed and includes in
the analysis other theoretical and experimental ingredients which are detailed in reference
[3].

As the next to leading order evolution kernels for the polarized case haven’t been calcu-
lated yet, in order to evaluate data at different values of $Q^2$ one can either use leading order
evolution kernels or consider observables with a moderate dependence as the asymmetries at
an average $Q^2$ value of 10 GeV$^2$. It has been shown [18] that the effect of the $Q^2$ evolution
in the asymmetries is small when compared to the ambiguities associated with the fitting
procedure and the experimental errors. Figures (4) and (5) show the agreement between the
electromagnetic data and the asymmetries calculated with the set.

In Figures (6), (7) and (8) we show the contributions to different structure functions
$x g_1^\gamma(x), x g_1^{W^+}(x)$ and $x g_3^{W^+}(x)$, discriminating their different origins (naive, quark initi-
ated correction, gluon initiated correction). The figures clearly show that even though the
moment of the quark initiated corrections are smaller than those of the gluons, which have
it main contribution in the very small $x$ region, the quark initiated corrections are greater
$(x g_1^\gamma(x), x g_3^{W^+}(x))$ or comparable $(x g_1^{W^+}(x))$ to the gluonic ones in most of the $x$ interval.
Compared to the naive contribution, the quark initiated correction can be as large as 20% of the former, while the $\alpha_s^2$ approximation ammount to 7%. Notice also, that there is a non
vanishing gluon contribution to $x g_3^{W^+}(x)$. In this structure function the gluon contributions
associated with quarks $\bar{u}$ and $d$ in the box diagram, cancel each other while there is no such
cancellation between $s$ and $\bar{c}$ due to the different factorization properties they have (equation
11).

**Conclusions**

We have computed the complete $\mathcal{O}(\alpha_s)$ corrections to the spin dependent deep inelas-
tic scattering weak structure functions introducing a factorization prescription that safely
removes soft contributions taking into account the problem of mixing different mass scales
in a same process. Using a set of spin dependent parton distributions that reproduces the 
electromagnetic data, we have shown that the quark initiated corrections are larger or com-
parable to those gluon initiated even though their moments are not.

Acknowledgements

We thank L.N.Epele, H.Fanchiotti and C.A.García Canal for helpful discussions and carefully reading the manuscript.

Appendix A

This appendix contains some calculational details related to the definitions of structure 
functions at partonic level.

In addition to the hadronic tensor (eq.1), one can also define a partonic one which has a 
similar structure, but is written in terms of partonic structure functions.

\begin{equation}
w^{\mu\nu} \equiv -ie^{\mu\nu\rho\sigma} \frac{q_\rho p_\sigma}{p \cdot q} g_1^{\text{partonic}} + (-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}) g_3^{\text{partonic}} \\
+ \frac{1}{p \cdot q} (p^\mu - \frac{p \cdot q}{q^2} q^\mu) (p^\nu - \frac{p \cdot q}{q^2} q^\nu) g_4^{\text{partonic}}
\end{equation}

The phase-space integrated matrix elements of either the quark or the gluon initiated di-
agrams of figs.1, 2 and 3 give the quark and gluon components of the partonic tensor, 
respectively. The partonic structure functions must be convoluted with the appropriatd 
 bare parton distributions in order to obtain the usual hadronic ones.

In order to isolate each partonic structure function it is customary to define the following proyectors

\begin{align}
P_{1}^{\mu\nu} & = i\epsilon^{\mu\nu\rho\sigma} \frac{q_\rho p_\sigma}{2p \cdot q} \\
P_{3}^{\mu\nu} & = \frac{1}{2(1-\epsilon)} \left[ -g^{\mu\nu} + \frac{4z^2}{Q^2} p^\mu p^\nu \right] \\
P_{4}^{\mu\nu} & = \frac{z}{(1-\epsilon)} \left[ -g^{\mu\nu} + \frac{4z^2(3-2\epsilon)}{Q^2} p^\mu p^\nu \right]
\end{align}
where

\[ z = \frac{Q^2}{2p \cdot q}. \] (21)

This implies

\[ g_{i \text{ partonic}} = \frac{1}{4\pi} \int d\Gamma \Delta |M|^2_{\mu \nu} P_{\mu \nu} \] (22)

where the spin dependent amplitude \( \Delta |M|^2_{\mu \nu} = \frac{1}{2} \left[ |M_+|^2_{\mu \nu} - |M_-|^2_{\mu \nu} \right] \) is defined in terms of those for partons whose polarization is parallel(+) or antiparallel(−) to that of the target and is normalized so that the tree-level diagram fig.1a gives a \( \delta(1 - z) \). The n-dimensional phase-space \( d\Gamma \) within the HVBM method is given by

\[ \int d\Gamma = \frac{1}{8\pi \Gamma(1-\epsilon)} \int_0^1 dy \int_0^{sy(1-y)} d\hat{k}^2 \hat{k}^{-2(1+\epsilon)} \] (23)

for two outgoing particles. \( \hat{k}_\mu \) is the n-4 dimensional component of the momentum of one of the outgoing partons.

**Appendix B**

The bare parton distributions inspired in the spin dilution model and used in this paper can be effectively parametrized as

\[ x \Delta q(x) = A_q x^{B_q} (1 - x)^{C_q} (1 + D_q x + E_q x^2) \] (24)

where the parameters are given in Table (3). Notice that these parameters are not the free parameters of the modified spin dilution model to be adjusted!. The former include information about the unpolarized parton distributions.
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Figure Captions

Figure 1  a) Lowest order graph for DIS; b),c) and d) virtual gluon correction graphs to a).

Figure 2 Real gluon emission corrections to 1a).

Figure 3 Gluon contribution to DIS at order $\alpha_s$

Figure 4 The spin-dependent proton asymmetry given by the fit compared to SMC, E-143 and earlier EMC data.

Figure 5 The spin-dependent neutron asymmetry given by the fit compared to E-142 data and the combined proton and deuteron SMC data.

Figure 6 The the naive contribution and the quark and gluon initiated corrections to the spin-dependent structure function $x g_1^\gamma(x)$

Figure 7 The same as figure 6 for $x g_1^{W^+}(x)$.

Figure 8 The same as figure 6 for $x g_3^{W^+}(x)$.

Table Captions

Table 1 Values for the coefficients $C_{ij}^B$ defined in equation (2).

Table 2 Axial and vector couplings for the $Z^0$ exchange.

Table 3 Parameters of the spin dependent parton distributions.
\[ B = \gamma \quad B = W^+ \quad B = Z^0 \]

\begin{align*}
  j = 1 & \quad \epsilon_i^2/2 & 1 & (C_{V_i}^2 + C_{A_i}^2)/2 \\
  j = 3 & \begin{cases} 
    i = q & 0 & -1 & -C_{V_i}C_{A_i} \\
    i = \bar{q} & 0 & 1 & C_{V_i}C_{A_i} 
  \end{cases} \\
  j = 4 & \begin{cases} 
    i = q & 0 & -2z & -2z C_{V_i}C_{A_i} \\
    i = \bar{q} & 0 & 2z & 2z C_{V_i}C_{A_i} 
  \end{cases}
\end{align*}

**Table 1:** Values for the coefficients \( C_{ij}^B \) defined in equation (2).

\[ C_{V_i} \frac{1}{2} - \frac{4}{3} \sin^2 \theta_W \quad \frac{1}{2} + \frac{2}{3} \sin^2 \theta_W \]

**Table 2:** Axial and vector couplings for the \( Z^0 \) exchange.

|   | \( A \) | \( B \) | \( C \) | \( D \) | \( E \) |
|---|---|---|---|---|---|
| \( u_u \) | 1.052 | 0.704 | 4.085 | 2.467 | 17.34 |
| \( d_d \) | -1.066 | 0.7685 | 2.179 | -1.645 | 0.4527 |
| \( \pi \) | 1.956 | 1.504 | 4.676 | -3.391 | 2.981 |
| \( \bar{d} \) | 1.035 | 1.734 | 2.905 | -3.051 | 2.324 |
| \( \bar{s} \) | 0.0932 | 1.594 | 8.694 | -2.149 | 32.709 |
| \( \tau \) | -2.171 | 57.27 | 173.6 | -20.36 | 2.279 |
| \( g \) | 3.828 | 0.5243 | 2.287 | -2.747 | 1.953 |

**Table 3:** Parameters of the spin dependent parton distributions.
This figure "fig1-1.png" is available in "png" format from:

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This figure "fig1-2.png" is available in "png" format from:

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