Mathematical modeling for egg production and egg weight curves in a synthetic white leghorn

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ABSTRACT

This study was conducted to evaluate 8 mathematical models, namely, Logistic (LM), Morgan Mercer Flodin (MMF), Polynomial Fit (PF), Rational Function (RF), Sinusoidal Fit (SF), Quadratic fit (QF), Gompertz function (GF), and Modification Compartmental Model (MCM) fitted to weekly egg production and egg weight of synthetic White Leghorn (SWL) population 21 to 40 wk of age of 5 generations (2015-16 to 2019-20). The relevant data for the present investigation were collected from SWL population, maintained in the department of Animal Genetics and Breeding, LUVAS, Hisar (India). The efficiency or reliability of the models were obtained by various criteria of goodness of fit such as coefficients of determination ($R^2$), Root Mean Square Error (RMSE), Akaike's Information Criterion (AIC), Bayesian Information Criterion (BIC), graphical analysis, and Chi-square test. The results indicated that RF, MCM, SF, and PF were best models for fitting weekly egg production curve due to higher values of $R^2$ and low values of RMSE, AIC, and BIC as compare to remaining models. In case of weekly egg weight, the best values of goodness of fit criteria were showed by MMF model followed by MCM and LM model. The results indicated that these models could be conveniently used for fitting for weekly egg production and egg weight in synthetic white leghorn, respectively.

Key words: egg production curves, mathematical models, synthetic White Leghorn

INTRODUCTION

Poultry is one of the fastest growing segments of agricultural sector in India with an annual growth of more than 14%, contributing million tones or 3.6% of the global egg production with annual growth rate of egg production as 6 to 7% (5th International Poultry and Livestock Expo, 2017). The total poultry of the country has been increased by 16.81% over previous census and became 851.81 million during year 2019. Over 45.78% increase in backyard poultry and total backyard poultry is 317.07 Million in 2019 (20th livestock census). Egg production is an important part of the commercial poultry industry. Appropriate mathematical functions precisely represent the entire production phase of the chicken and provide suitable means for biological comparisons and interpretations. Egg production curves help in predicting egg production, determining the optimum culling age of breeders and help making economical decisions (Brown et al., 1976; Aggrey, 2002; Savegnago et al., 2012). Several models have been proposed by different workers for the analysis of egg production curves in poultry (Anang and Indrijani, 2006). Egg production is a result of many genes through biochemical, anatomical, and physiological processes. Egg production in poultry is a complex quantitative trait and shows considerable individual variation over the laying period. Statistical models used to describe poultry egg production curves support to many management decisions to be taken to increase egg production (Wolc et al., 2011). Prediction of total egg production as early as possible during the laying cycle using part records facilitates a poultry breeder to select breeding birds early thereby helping in reducing the cost of egg production/day-old chicks (Ganesan et al., 2011). Use of mathematical models to accurately fit egg production curves is necessary to make economic projections for laying hens (Adams and Bell, 1980; Kiran, 1998). It is also of great importance in practical poultry breeding for making predictions about egg production on an annual or any other chosen period basis, to facilitate early selection of the breeder birds (Bindya et al., 2010, Abraham and Murthy, 2017;
Ahmadu et al., 2017). Most of the nonlinear models presented in the literature to fit egg production have curve parameters with a biological interpretation, which makes it possible to summarize in 3 or 4 parameters what the egg production pattern is like. Use of nonlinear models to fit the egg production of different populations makes it possible to compare egg production between them. In this manner, egg production can be compared between selected and nonselected lines of hens to study changes to egg production curves caused by selection processes. The behavioral trends in production like the time of peak and decline as well as the persistency of lay for the timely management of housing dates, egg marketing and labor needs of the enterprise can also be studied from these egg production curves.

The present investigation was intended to evaluate various mathematical models to explain average weekly egg production and egg weight in a synthetic White Leghorn (SWL) population and to compare their relative accuracy. The results would be helpful for setting optimizing feeding strategies in order to improve egg performance of synthetic White Leghorn.

MATERIALS AND METHODS

Location and Research Data

The research work was undertaken at Department of Animal Genetics and Breeding, Lala Lajpat Rai University of Veterinary and Animal Sciences, Hisar located 29.15 latitude and 75.72 longitudes in western Haryana (India). It is situated at an elevation of 216-meters above sea level. The study area falls under subtropical climatic conditions of India, where temperature goes to 50°C during hot periods (June-Aug) and up to 0°C during cold periods (Dec-Feb). The sheds were open-sided with proper ventilation and protection through iron nets. Data pertaining to the present study was collected from records weekly hen housed egg production from 21 to 40 wk of age for social sciences (SPSS 20.0 version). The mortality rates were approximately 2% and thus, the records of those pullets survived up to the end of the investigation were only considered for the egg production curve analysis.

General Management

The chicks of synthetic White Leghorn were brooded and reared hatch wise using standard managemental practices. The chicks were vaccinated against Marek’s disease, Ranikhet, Gumboro, and Fowl pox at appropriate ages. Cockerels were separated from the pullets at 8 wk of age. At 20 wk of age, the body weights were recorded and pullets were housed in layer houses hatch-wise. The pullets shifted in individual cages of each pullet was maintained to record the age at first egg, egg production up to 40 wk of age.

Statistical Analysis

Varous mathematical models were examined for fitness to explain the average weekly egg production of combined data of 5 generations (G1- 546, G2- 464, G3- 541, G4- 619, G5- 616) of 2,786 pullets the SWL population. For the fitting of egg weight curve under mathematical models in SWL, weekly average egg weights were used. The data was collected by weighing 100 random eggs of a week (5th generation-2019–2020 only). The following eight mathematical models were fitted independently to average weekly data of egg production and egg weight as follows:

1) Model-I: Logistic Model \( Y = (A) / (1 + B e^{-C x}) \)
2) Model-II: MMF Model \( Y = (AB + C x^D) / (B + x^D) \)
3) Model-III: Polynomial Fit of nth degree \( Y = B_0 + B_1 x + B_2 x^2 + B_3 x^3 + B_4 x^4 \)
4) Model-IV: Rational Function \( Y = (A + B x) / (1 + C x + D x^2) \)
5) Model-V: Sinusoidal Fit \( Y = A + B \cos(C x + D) \)
6) Model-VI: Quadratic Fit \( Y = B_0 + B_1 x + B_2 x^2 \)
7) Gompertz function \( Y = A \exp[-B \exp(-C x)] \)
8) Modification Compartmental Model (Yang et al., 1989) \( Y = A \exp(-B x)/1 + \exp(-C(-x-D)) \)

Where, \( Y \), Average weekly egg production in a particular period of recording; \( x \), The week (of production) in which egg production was recorded; \( A, B, C, D \), Model parameters as defined in a particular model; \( A \), is an asymptote (a scale of parameter); \( B \), sets, the displacement along the x-axis (the rate of decrease in laying ability); \( C \), sets, the week (y scaling) the reciprocal indicator of the variation in sexual maturity; \( D \), the mean age of sexual maturity; \( B_0, B_1, B_2, B_3, B_4 \) are coefficients under quadratic and polynomial function, \( x \), is times (week), and \( e \), is mathematical constant. The model parameters, \( A, B, C, D \) and \( E \) were derived using the least squares curve fit method suggested by Speigel and Stephens (2000). The adequacy of the models was obtained by the coefficients of determination (\( R^2 \)), Root Mean Square Error (RMSE), Akaike’s Information Criterion (AIC) and Bayesian Information Criterion (BIC). The statistical analysis was performed using Statistical package for social sciences (SPSS 20.0 version).

RESULTS

The observed and the expected average weekly egg production and egg weight on fitting the mathematical...
models, the $R^2$, RMSE, AIC and BIC of the fitted models and their model parameters for the selected mathematical models for white leghorn strain are presented in Tables 1 and 2, respectively.

**Egg Production and Egg Weight Curves in the Pullets**

Pullets of the white leghorn strain showed increasing trend in the observed egg production at the initial weeks of production from 1 to 12 wk during the weeks approaching peak production and diminished thereafter. The egg production in the pullet showed faster increase during the initial 2 wk of production followed by a gradually increase during 3rd, 4th, and 5th wk of egg production. Subsequently, sudden increases were observed on 6 wk of egg production and then increased gradually until peak productions of 6.12 were attained. The production thereafter gradually diminished, average weekly egg production by the end of 20 wk reached to 4.87. Pullets of the white leghorn showed increasing trend in the observed egg weights at the initial weeks of production from 1 to 5 wk but later their egg weight acquired a linear trend during the weeks approaching peak production.

**Mathematical Models Fitted to Egg Production Curves in White Leghorn**

The $R^2$ values of the fitted models for the white leghorn, ranged from 0.889 to 0.978 for weekly egg production and 0.926 to 0.997 for weekly egg weight presented on Table 2. All the models provided acceptable fit to the data. However, for the weekly egg production model selection criteria, the $R^2$ values for the RF, MCM, SF, PF, QF, LM, MMF, and GF were 0.978, 0.997, 0.974, 0.973, 0.953, 0.899, 0.895, and 0.889, respectively. RF, MCM, SF, and PF, models were similar and had the highest coefficient of determination ($R^2$) values of ($R^2 > 0.97$), while all other models recorded the least fitting to egg production curve ($R^2 < 0.97$). Consequently, RF, MCM, SF, and PF models showed lowest values of RMSE, AIC, and BIC values than remaining model.

In case of weekly egg weight, the highest coefficient of determination was estimated by MMF models ($R^2 = 0.997$), with lowest RMSE, AIC, and BIC values each for the fitting the weight data. Additionally, MCM, LM, PF, GF, and RF also showed better adequacy ($R^2 > 0.97$). Least prediction value was estimated by SF model with $R^2$ value of 0.925.

The Rational Function had the highest model adequacy and fulfills all criteria for selecting best optimum model, closely followed by MCM model for weekly egg production. Similarly, for weekly egg weight, MMF was the best model, closely followed by MCM model. The results indicated that RF, MCM and MMF model gave best fit compared to all other models in white leghorn.

**Comparison of the Observed and Estimated Egg Production Using the Fitted Models**

Models which had high $R^2$ values and low standard errors, AIC and BIC values were in greater agreement with the observed values. Parameter estimate of “a” for weekly egg number of RF and GF model gave asymptote value of (2.145; 5.625) and egg weight of MMF and SF model showed that (37.26; -23.24), respectively. The growth rate “c” for RF and GF model were (0.08; 0.25) for egg number and MMF and SF model for weekly egg weight were (55.20; 0.003), respectively. Graphical representation of actual weekly egg number/egg weight and fitted models are shown in Figures 1 and 2. A judgments of the values of coefficient

| Table 1. Comparison of observed and estimated average egg number up to 40 wk of age in pullets. |
|---------------------------------------------------------------|
| **Egg production (Number)** | **Observed** | **RF** | **MCM** | **SF** | **PF** | **QF** | **LM** | **MMF** | **GF** |
| T (WK) | 1.24 | 2.39 | 2.35 | 2.36 | 2.35 | 1.96 | 2.08 | 2.5 | 3.45 |
| 1 | 2.58 | 2.65 | 2.64 | 2.62 | 2.6 | 2.5 | 2.52 | 2.58 | 3.56 |
| 2 | 3.02 | 2.94 | 2.94 | 2.92 | 2.91 | 2.99 | 2.97 | 2.81 | 3.67 |
| 3 | 3.47 | 3.25 | 3.26 | 3.26 | 3.26 | 3.44 | 3.42 | 3.2 | 3.78 |
| 4 | 3.64 | 3.59 | 3.61 | 3.62 | 3.63 | 3.85 | 3.83 | 3.68 | 3.89 |
| 5 | 4.13 | 3.94 | 3.96 | 3.98 | 3.98 | 4 | 4.21 | 4.19 | 4.14 |
| 6 | 4.29 | 4.32 | 4.32 | 4.35 | 4.36 | 4.54 | 4.5 | 4.52 | 4.13 |
| 7 | 4.35 | 4.66 | 4.66 | 4.69 | 4.71 | 4.82 | 4.76 | 4.81 | 4.26 |
| 8 | 4.88 | 4.99 | 4.99 | 5.01 | 5.02 | 5.06 | 4.96 | 5.02 | 4.39 |
| 9 | 5.06 | 5.29 | 5.28 | 5.29 | 5.29 | 5.26 | 5.12 | 5.17 | 4.52 |
| 10 | 5.49 | 5.54 | 5.53 | 5.5 | 5.52 | 5.42 | 5.24 | 5.27 | 4.66 |
| 11 | 6.12 | 5.73 | 5.71 | 5.68 | 5.67 | 5.54 | 5.33 | 5.35 | 4.9 |
| 12 | 6.90 | 5.84 | 5.82 | 5.79 | 5.77 | 5.61 | 5.4 | 5.41 | 4.95 |
| 13 | 5.88 | 5.87 | 5.86 | 5.83 | 5.81 | 5.64 | 5.45 | 5.45 | 5.1 |
| 14 | 5.69 | 5.81 | 5.82 | 5.8 | 5.79 | 5.63 | 5.49 | 5.48 | 5.26 |
| 15 | 5.57 | 5.69 | 5.57 | 5.7 | 5.58 | 5.5 | 5.5 | 5.42 |
| 16 | 5.36 | 5.51 | 5.53 | 5.54 | 5.54 | 5.54 | 5.54 | 5.52 | 5.58 |
| 17 | 5.34 | 5.28 | 5.32 | 5.33 | 5.35 | 5.54 | 5.54 | 5.54 | 5.75 |
| 18 | 4.99 | 5.03 | 5.03 | 5.05 | 5.06 | 5.18 | 5.56 | 5.55 | 5.93 |
| 19 | 4.87 | 4.75 | 4.74 | 4.74 | 4.74 | 4.96 | 5.56 | 5.56 | 6.11 |
| Chi-square | 0.12 | 0.12 | 0.13 | 0.14 | 0.25 | 0.49 | 0.53 | 2.35 |

Abbreviations: GF, Gompertz function; LM, logistic model; MCM, modification compartmental model; MMF, Morgan Mercer Flodin, Polynomial Fit, RF, rational function; QF, quadratic fit; SF, sinusoidal fit; WK, week. Chi-square values were non-significant.
of determination ($R^2$) and standard errors together with the status of significance of Chi-square values for different generations using the eight mathematical models revealed RF to be the best fit, very closely followed by MCM for weekly egg production and MMF Model to be the best fit, very closely followed by MCM for weekly egg weight. The other models, SF Model, PF of $n^{th}$ degree, QF, LM, and GF also gave good fits in a more or less similar order. From the overall results, it was understood that the ranking of the models based on their efficiency random bred white leghorn strain were similar for the first 3 best fitting ones, viz., RF, MCM, and SF Model. Hence, these 3 models were identified to be the best and due to their simple form they are recommended for predictive purposes in white leghorn strain.

**DISCUSSION**

Egg production curve describes the relation between number of eggs and time of the laying period. The most important use of egg production models in poultry is to estimate the economic and genetic worth by predicting the total egg production from partial records. One of the main concerns for the poultry breeder is how to best define egg production as a trait for selection. The rate of egg production changes over time, and can be represented in terms of a "production curve". It was found that when the generations of the genetic group were pooled, the estimated weekly egg production for almost all the models fitted was slightly deviant from the observed values for the period from 21 wk to 26th wk of age. From 25th wk onward, there was close proximity between the observed and expected values till the attainment of 52 wk of age. The curves of the SWL population as presented by the models indicated that goodness of fit criteria was generally high for most of the models ($R^2 > 0.89$) for weekly egg number and egg weight. This result is within the range reported by Murthy (1998), Darmani Kuhi et al. (2003), Forni et al. (2008), and

**Table 2.** Estimates of model parameters, coefficient of determination, root mean square.

| Models  | A   | B    | C    | D    | E    | R2   | ADJ R2   | RSS   | RMSE  | AIC  | BIC  |
|---------|-----|------|------|------|------|------|----------|-------|-------|------|------|
| Egg production (Number) |     |      |      |      |      |      |          |       |       |      |      |
| RF      | 2.145 | 0.049 | -0.08 | 0.003 |      | 0.978 | 0.972    | 0.571 | 0.195 | -22.887 | -25.683 |
| MCM     | 44.2 | 0.103 | -0.23 | 13    |      | 0.977 | 0.970    | 0.587 | 0.197 | -22.647 | -25.443 |
| SF      | 3.864 | -1.96 | 0.187 | 0.509 |      | 0.974 | 0.967    | 0.653 | 0.208 | -21.722 | -24.518 |
| PF      | 2.169 | 0.133 | 0.049 | -0.004 | 0.001 | 0.973 | 0.963    | 0.685 | 0.221 | -19.306 | -22.801 |
| QF      | 1.386 | 0.597 | -0.02 |      |      | 0.953 | 0.944    | 1.2   | 0.273 | -18.437 | -20.533 |
| LM      | 5.584 | 2.329 | -0.33 |      |      | 0.899 | 0.880    | 2.569 | 0.400 | -11.825 | -13.922 |
| MMF     | 5.611 | 0.003 | 2.486 | -3.284 |      | 0.895 | 0.867    | 2.678 | 0.422 | -9.464  | -12.260 |
| GF      | 5.625 | 1.437 | 0.258 |      |      | 0.889 | 0.868    | 2.815 | 0.419 | -11.031 | -13.127 |
| Egg weight |     |      |      |      |      |      |          |       |       |      |      |
| MMF     | 37.257 | 79.596 | 55.204 | 3.322 |      | 0.997 | 0.996    | 2.05  | 0.369 | -11.785 | -14.581 |
| MCM     | 56.458 | 0.001 | -0.366 | -0.576 |      | 0.986 | 0.982    | 8.447 | 0.750 | 0.513  | -2.282 |
| LM      | 55.316 | 0.793 | -0.391 |      |      | 0.985 | 0.982    | 8.833 | 0.743 | -1.988 | -3.195 |
| PF      | 30.329 | 6.268 | -0.596 | 0.025 | 0.001 | 0.984 | 0.978    | 9.222 | 0.811 | 3.275  | -0.218 |
| QF      | 55.419 | 0.614 | 0.345 |      |      | 0.982 | 0.978    | 10.626 | 0.814 | 0.506  | -1.500 |
| RF      | 29.872 | 11.584 | 0.144 | 0.002 |      | 0.979 | 0.973    | 12.589 | 0.916 | 3.979  | 1.183 |
| SF      | 36.306 | 2.827 | -0.099 |      |      | 0.926 | 0.912    | 43.087 | 1.641 | 12.666 | 10.569 |
|        | -23.24 | 23.34 | 0.003 | 0.042 |      | 0.925 | 0.906    | 43.096 | 1.695 | 14.668 | 11.872 |

Abbreviations: A, weekly egg number/egg weight; AIC (Model Efficiency), Akaike Information Criterion; B, Inflection Point; C, Asymptote 1; D, Asymptote 2; For PF model, A-E are partial regression coefficients (equivalent to $B_0$ to $B_4$); RMSE, root mean square error; R2, coefficient of determination.

Figure 1. Fitting of eight mathematical models to egg production curve in a synthetic white leghorn.
Narinc et al. (2010) in chicken. The high $R^2$ values in the present study indicate that the models adequately described the observed data on egg production traits. The effect on model parameter showed that the constant “$a$” (asymptote value) for weekly egg number and egg weight for the present study was close agreement of previous report (Oni, 1997). As far as the goodness of fits is concerned, the best value (the lowest) of the AIC gives the closest representation of data (Marc, 2007). However, the suitability of the model for egg production curves depends not only on its general goodness of fits but also on the ability of describing the asymptotic phase of the curve (Macciotta et al., 2011).

The results indicated that RF, MCM, SF, and PF models gave best fit to egg production curve compared to all other models in white leghorn. This result was in conformity with the findings of various research workers who adopted different models in layer type chicken viz., Gavora et al. (1982), Cason and Britton (1988), Lal et al. (2003), Narinc et al. (2010), and Savegnago et al. (2012). Similar to present results, Abraham and Murthy (2017) studied 6 mathematical models for egg number, namely, Logistic, MMF, Polynomial Fit, Rational Function, Sinusoidal Fit, and Quadratic fit and they reported Rational Function ($R^2$: 94.08–97.22%) and Polynomial Fit ($R^2$: 93.26–96.67%) as the best fitting models. On the other hand, Thomas et al. (1994) reported QF as best model for egg production curve in layers.

For weekly egg weight, all models showed high level of adequacy with maximum due to MMF models ($R^2 = 0.997$), with lowest RMSE, AIC, and BIC values each for the fitting the weight data. Additionally, MCM, LM, PF, GF, and RF also showed better adequacy ($R^2 > 0.97$). Ahmadu et al. (2017) studied four models (Richard, Gompertz, exponential, and logistic) and reported Richard Model outperformed the other models in modeling body weight and egg weight in layers. However, there are very scarce reports available on fitting egg weight in layers.

The deviations of the expected values (derived using the fitted models) from the observed values were tested by Chi-square for testing the goodness of fit considering the reference period from 21 to 40 wk of age. Irrespective of the generations, the computed Chi-square values were non-significant which indicated a good fitness of all the models for the weekly egg production and egg weight during the entire period from 21 to 40 wk of age. The standard errors estimated for the models remained lower wherever the coefficient of determination was higher. The criteria of $R^2$ values and standard errors, together with the status of nonsignificance of Chi-square values were used to assess the applicability and fitness of the models.

Appropriate mathematical functions precisely represent the entire production phase of the chicken and provide suitable means for biological comparisons and interpretations. Egg production curves help in predicting egg production, determining the optimum culling age of breeders and help making economical decisions. The purpose of modeling the production curve in poultry eggs is to achieve a more detailed analysis of the egg production cycle and describe the curve phases and duration. The curve also facilitates the production prediction, the long-term projection of eggs yield, and economic planning of production and decision-making, among others (Gavora et al., 1982; Yang et al., 1989; Groen et al., 1998).

**CONCLUSIONS**

The mathematical models used in this study showed high level of accuracy for fitting the weekly egg production and egg weight of synthetic White Leghorn. On the basis of goodness of fit criteria, best models viz., RF, MCM, SF, and PF for weekly egg production and MMF, MCM, and LM for weekly egg weight were identified. Additionally, these models could be conveniently adopted due to their simple nature in terms of...
parameters. The results indicated that these models could be conveniently used for fitting for weekly egg production and egg weight in synthetic white leghorn, respectively.

ACKNOWLEDGMENTS

We thank to worthy Vice-Chancellor, LUVAS, Hisar for providing the necessary facility required to conduct this study.

Author contributions: MAS and CSP designed the study, collected data and wrote the manuscript. CSP and YCB analyzed the data. ASY revised the paper. All authors approved the final version of the paper.

DISCLOSURES

The authors declare no conflict of interest.

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