Tobological feedback for superconducting states

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Abstract. We discuss feedback effects that stabilize the superconducting states by the induced topological term in the effective Lagrangian. The chiral feedback effect due to the Chern-Simons-like term for quasi-two-dimensional system with time-reversal symmetry breaking (TRSB) was studied in [1, 2]. We consider the extension of the chiral feedback to three-dimensional TRSB system and investigate similar feedback effects for quasi-two-dimensional or three-dimensional time-reversal symmetric systems.

1. Introduction
The p-wave superfluid \textsuperscript{3}He has two distinct phases in zero fields. The pairing order parameter of A-phase has point nodes and, in the weak coupling approximation, B-phase with a fully gapped order parameter gains more condensation energy than A-phase. The reason for the overcoming stability of A-phase is the spin fluctuation feedback [3, 4]. More recently, it was found that the chiral p-wave pairing, which is a two-dimensional and superconducting analog of \textsuperscript{3}He-A phase, would have another type of feedback effect [1, 2]. The chiral pairing leads to a spontaneous induction of a Chern-Simons(CS)-like term in the low energy effective action for the electromagnetic gauge field [5, 6, 7, 8]. Due to the charge-flux attachment by the CS-like term, each electron in a chiral Cooper pair feels the Lorentz force that strengthens the attractive interaction. The topological nature of pairing thus causes this chiral feedback effect and supports the pairing itself. We then present our attempt to find topological feedback mechanisms for the other types of superconducting states. We investigate the three-dimensional extension of the chiral feedback effect for TRSB Weyl superconductors with point nodes. The helical p-wave state, which is the time-reversal-symmetric (TRS) extension of the chiral p-wave state, would have a feedback effect caused by the induced BF term (TRS extension of the CS term) leading to the spin-dependent Lorentz force. The three-dimensional extension is also possible for the TRS pairing state.

2. Weyl (A-phase type) feedback
We introduce a feedback mechanism on 3D Weyl superconducting states (Weyl SC), which point nodes and break time-reversal symmetry. We consider the simplest example, i.e. \textsuperscript{3}He-A-type superconducting state of

\begin{equation}
\tilde{d}(\vec{k}) = |\Delta_0|(\hat{k}_x \pm i\hat{k}_y)e_z, \quad \tilde{\Delta}(\vec{k}) = i\sigma_\nu\sigma_\mu d_\nu d_\mu \hat{k}_\nu, \quad \hat{k}_i = \frac{k_i}{k_F},
\end{equation}
which has point nodes at $\vec{k} = (0, 0, \pm k_F)$. We suppose the parabolic dispersion $\epsilon(\vec{k}) = \frac{\vec{k}^2 - k_F^2}{2m_e}$, for simplicity. The CS-like term

$$\frac{\sigma_{xy}}{2} \epsilon_{0ijk} (A_0 \partial_i A_j + A_i \partial_j A_0), \quad (i, j = x, y)$$

(2)

also appears in the low-energy effective action of this pairing state [5, 6, 7, 8]. We will see below that the term gives a positive feedback.

As with the chiral feedback, a charge-density–charge-current interaction induced by the CS-like term Eq. (2) contributes to the pairing interaction additionally. The density–current correlation function connected with this interaction just below $T_c$ is

$$\pi_{0j}(\vec{q}) = k_B T_c e^2 \sum_m \int \frac{d^3 \vec{k}}{(2\pi)^3}$$

$$\times \text{Tr} \left[ G(i\omega_m, \vec{k} + \vec{q}) \Delta(\vec{k} + \vec{q}) G(-i\omega_m, -\vec{k} - \vec{q}) \frac{2k_j + q_j}{2m_e} G(-i\omega_m, -\vec{k}) \Delta^1(\vec{k}) G(i\omega_m, \vec{k}) \right],$$

(3)

where $G(i\omega_m, \vec{k}) = \frac{1}{i\omega_m - \epsilon(\vec{k})}$, $\omega_m = (2m + 1)\pi k_B T_c$, and the time dependence is ignored.

Due to the symmetry, we can express

$$\pi_{0j}(\vec{q}) = i\epsilon_{ijk} q_k f_k(\vec{q}) + \text{(symmetric in } l \leftrightarrow j), \quad f_k(\vec{q}) = -\frac{i}{2!} \epsilon_{ijk} \frac{\partial \pi_{0j}(\vec{q})}{\partial q_i},$$

(4)

where $\epsilon_{ijk}$ is totally antisymmetric tensor. Using similar approximations used in [1, 2], we have an approximate form of $f_k(\vec{q})$ as

$$f_k(\vec{q}) \simeq -\frac{\epsilon^2 N(0) (\epsilon_{ijk} d^*_{\nu j} d_{\nu j})}{24m_e (\pi k_B T_c)^2} \frac{1}{1 + \gamma \xi^2 q^2},$$

(5)

where $N(0)$ is the density of state at Fermi level, $\xi = \frac{\nu_F}{2\pi k_B T_c}$ is a coherence length, and $\gamma$ is an $O(1)$ numerical factor.

Substituting Eq. (5) into the correlation function Eq. (4), we have

$$\pi_{0\beta}(\vec{q}) = \frac{\epsilon^2 N(0) (\epsilon_{ijk} d^*_{\nu j} d_{\nu j})}{24m_e (\pi k_B T_c)^2} \frac{\epsilon_{ijk} q_i}{\sqrt{1 + \gamma \xi^2 q^2}}.$$  

(6)

To estimate this contribution, we calculate a correction to the 4th order term in the GL free energy expressed as

$$\Delta F_{\text{Weyl}} = k_B T_c \int \frac{d^3 q}{(2\pi)^3} D_{00}(\vec{q}) \pi_{0\alpha}(\vec{q}) D_{\alpha\beta}(\vec{q}) \pi_{\beta\delta}(\vec{q}),$$

(7)

where, $D_{00}(\vec{q}) = \frac{1}{q^2 + l_{TF}^2}$, and $D_{\alpha\beta}(\vec{q}) = -\frac{\delta_{\alpha\beta}}{q^2}$. $D_{00}(\vec{q})$ and $D_{\alpha\beta}(\vec{q})$ are gauge field propagators in Coulomb gauge with the Thomas-Fermi screening length $l_{TF}$. Substituting Eq. (6), we have

$$\Delta F_{\text{Weyl}} \approx \alpha^2 k_F l_{TF} T_c N(0) (\epsilon_{ijk} d^*_{\nu j} d_{\nu j})^2 \frac{108\pi^2 N(0) (\epsilon_{ijk} d^*_{\nu j} d_{\nu j})^2}{108\pi^2 N(0) (\pi k_B T_c)^2} < 0,$$

(8)

where, $\epsilon_{ijk} d^*_{\nu j} d_{\nu j} = 2i\chi |\Delta_0|^2$ and $\chi = \pm 1$ denotes chirality ($z$-component of the relative angular momentum). The correction is negative and the effect favors the $^3$He-A-type state. Comparing with the quasi-2D result, $1/d$ is replaced with $k_F$ but qualitative nature is not changed.
3. Extension to Time reversal symmetric system

For time reversal symmetric (TRS) systems the BF-term $\epsilon_{ij}(a_0 \partial_i A_j + A_i \partial_j a_0)$, that is a time reversal symmetric extension of the CS term, is induced in the low-energy effective action. In the BF-term, the electrostatic gauge field $A_0$ is replaced with a “spin” gauge field $a_0$ which shows the spin fluctuation, and spin-density–charge-current interaction is induced.

3.1. Feedback effect for helical $p$-wave (quasi 2D)

In this section, we discuss the feedback effect for $p$-wave superconducting states in quasi-2D system with time reversal symmetry, i.e., helical $p$-wave states. They are categorized into two types,

$$|\text{parallel}\rangle = \frac{1}{2}(|s_\chi = +1, \chi = +1\rangle + |s_\chi = -1, \chi = -1\rangle),$$

$$|\text{anti-parallel}\rangle = \frac{1}{2}(|s_\chi = +1, \chi = -1\rangle + |s_\chi = -1, \chi = +1\rangle),$$

where $|\text{parallel}\rangle (|\text{anti-parallel}\rangle)$ is the superposition of two chiral $p$-wave states each of which has $\chi$ parallel (anti-parallel) to $s_\chi$.

The $d$-vectors of them are

$$\vec{d}(\vec{k}) = \frac{1}{2}(d_{\uparrow\uparrow}^{p(\rho)}(\vec{k}) + d_{\downarrow\downarrow}^{p(\rho)}(\vec{k})), \quad (11)$$

where

$$d_{\uparrow\uparrow}^{p(\rho)}(\vec{k}) = \Delta_0 \left(\vec{k}_x - i\vec{k}_y\right)(\vec{e}_x + i\vec{e}_y), \quad d_{\downarrow\downarrow}^{p(\rho)}(\vec{k}) = \Delta_0 \left(\vec{k}_x + i\vec{k}_y\right)(\vec{e}_x - i\vec{e}_y).$$

We analyze a feedback effect for these states. The correlation function through spin-density–charge current interaction $H_{\text{int}} = \sum_{\vec{q}} g_{ij}^{\mu} \rho^{\mu}(-\vec{q}) J_{\vec{q}}^{i}(-\vec{q})$ has the form

$$\pi_{ij}^{\mu}(\vec{q}) = -k_B T C e \sum_m \int \frac{d^d k}{(2\pi)^d} \int \frac{d^3 k}{(2\pi)^3} \times \text{Tr} \left[ \sigma^{\mu} G(i\omega_m, \vec{k} + \vec{q}) \Delta(\vec{k} + \vec{q}) G(-i\omega_m, -\vec{k} - \vec{q}) \frac{2k_j + q_j}{2m_e} G(-i\omega_m, -\vec{k}) \Delta^\dagger(\vec{k}) G(i\omega_m, \vec{k}) \right], \quad (12)$$

where the energy band is parabolic, $\epsilon(\vec{k}) = \frac{k_x^2 + k_y^2 - k_z^2}{2m_e}$ and the spin-orbit interaction is ignored for simplicity. We assume a layered structure with spacing $d$ in $z$-direction just like the system assumed in the chiral feedback.

Following the way used in the previous section, we obtain a spin-density–charge-current correlation function of the form

$$\pi_{ij}^{\mu}(\vec{q}) = i\epsilon_{ij} q_l f^{\mu}(\vec{q}) + (\text{symmetric in } l \leftrightarrow j), \quad f^{\mu}(\vec{q}) \sim e \frac{N(0)}{(\pi k_B T)^2} \frac{\epsilon_{ij} \epsilon_{\mu\nu} q^\nu q^\rho d^{d}_{\rho}}{4m_e \sqrt{1 + \gamma^2 q_\perp^2}}, \quad (13)$$

where $q_\perp^2 = q_x^2 + q_y^2$, and the circular symmetry is used for the approximation in the last line.

The spin-density–charge-current interaction contributes additionally to the pairing interaction. To estimate this contribution, we calculate a correction to the 4th order term in the GL free energy expressed as

$$\Delta F_h = k_B T C \int \frac{d^d q}{(2\pi)^d} g_0^{\mu\nu}(\vec{q}) \pi_{ij}^{\mu}(\vec{q}) D_{ij}(\vec{q}) \pi_{ij}^{\nu}(\vec{q}), \quad (14)$$
where the propagators for (spin) gauge fields are

\[ D_{ij}(q) = -\frac{\delta_{ij}}{q^2}, \quad g_{0\mu}^{\alpha\beta}(q) = \frac{3k_F^2I_0}{q^2} \left( 1 - \frac{q_F^2}{12k_F} q^2 + \xi^2 \right) \delta^\mu^\nu, \]  

(15)

where \( q^2 = q_x^2 + q_y^2 + q_z^2 \), and a magnetic correlation length \( \xi_s = \frac{1}{12k_F^2} \frac{I_0N(0)}{1 + I_0N(0)} \) shows the Stoner enhancement. Using similar approximations in Ref. [2], we have

\[ \Delta F_h \approx -\frac{3\pi \alpha I_0 k_F^2 \xi_s T_c}{32\gamma} \frac{N(0)}{d F} \left( \epsilon_{ij} \epsilon_{\mu\nu} d_{\mu}^* d_{\nu} \right)^2, \]  

(16)

where \( (\epsilon_{ij} \epsilon_{\mu\nu} d_{\mu}^* d_{\nu})^2 = |\Delta_0|^4 \chi^2 (1 + s^2) \) is positive definite for both parallel and anti-parallel states. Hence, the helical feedback effect supports both parallel and anti-parallel states equally.

3.2. \(^3\)He-B phase type feedback

We study a feedback mechanism for a \(^3\)He-B phase type superconductor, i.e., an extension of the helical feedback to 3-dimensional system in this section. The order parameter has the form

\[ \Delta(k) = \Delta_0 (i\sigma_y \vec{\sigma}) : d(k), \quad \vec{d} = \frac{1}{2} \left[ (\vec{e}_z \hat{k}_x + \vec{e}_x \hat{k}_y) + (\vec{e}_y \hat{k}_y + \vec{e}_z \hat{k}_z) + (\vec{e}_x \hat{k}_z + \vec{e}_z \hat{k}_z) \right], \]  

(17)

of the superposition of helical \( p \)-waves on each plane.

The spin-density–charge-current correlation function is almost same with the helical one except for the intermediate fermions’ spatial degrees of freedom and written as,

\[ \pi_{ij}^{\alpha\beta}(q) = i\epsilon_{ijk} q_{jl} f_{\alpha}^{\mu}(q), \quad f_{\alpha}^{j}(q) \approx \frac{eN(0)}{48m_e\beta} \frac{\pi}{(\pi k_B T_c)^3} \frac{1}{\sqrt{1 + \gamma \xi^2 q^2}}, \]  

(18)

We obtained Eq. (18) by similar approximations used in Eq. (5).

By using the correlation function Eq. (18), we calculate a feedback correction to the 4th order term in the GL free energy as

\[ \Delta F_{B-phase} \approx -\frac{\alpha I_0 k_F^2 \xi_s T_c N(0) (\epsilon_{ij} \epsilon_{\mu\nu} d_{\mu}^* d_{\nu})^2}{576\gamma \pi^2 T_F} \frac{(\pi k_B T_c)^2}{(\pi k_B T_c)^2}, \]  

(19)

which is negative definite. In 3D TRS system, the feedback also arises and supports the states.

4. Summary

We study about extensions of the chiral feedback for 3D TRSB system, and quasi-2D and 3D TRS systems. It was found that those effects stabilize the states. Because of topological nature of interaction terms, all of these effects are also universal. In addition, the Stoner enhancement could strengthen the feedback significantly in TRS cases.

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References

[1] J. Goryo and M. Sigrist, J. Phys. C, 12 L599 (2000).
[2] J. Goryo and M. Sigrist, Europhys. Lett. 57, 578 (2002).
[3] W.F. Brinkman and P. W. Anderson, Phys. Rev. A, 10 2386 (1974)
[4] Y. Kuroda, Prog. Theor. Phys., 53 349 (1975).
[5] G. E. Volovik, Sov. Phys. JETP 67 1804 (1988).
[6] J. Goryo and K. Ishikawa, Phys. Lett. A 260 294 (1999).
[7] N. Read and D. Green, Phys. Rev. B 61 10267 (2000).
[8] A. Furusaki, M. Matsumoto, and M. Sigrist, Phys. Rev. B 64 054514 (2001).