Solving Fractional Coupled EW and Coupled MEW Equations Using Bernstein Collocation Method

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Abstract. This paper deals with approximation solution for coupled of space-time-fractional of both the equal width wave equation (FCEWE) and the modified equal width wave equation (FCMEWE) using Bernstein polynomials with collocation method and employing the Caputo definition for fractional derivatives. The method reduces the coupled system to a system of algebraic equations which is simple in handling and gives the best results.

Keywords. Bernstein polynomials, Collocation method, Caputo fractional derivative.

1. Introduction

Continuous progress altogether areas of life encouraged researchers not only to review ordinary, partial, and integral differential equations, but also to strive within field of studying and finding numerical, approximate, and exact solutions for fractional differential equations that appeared in the seventeenth century by discussing the derivative of order (1/2) through a mathematician (Leibnitz). Then other scholars continued to figure within the development and study of those derivatives to the present day[1]. The numerical and approximate solutions of those derivatives are studied in various fields like mathematics, physics, and engineering because most of those derivatives might not have exact solutions[2]. Scientists have provided a variety of definitions during this field, each definition has special features and properties, for instance (Riemann), (Liouvill), (Grunwald) and (Letnikov). During this paper, we use the Caputo definition.

Polynomials play a prominent role within the analysis and approximation has been utilized in various fields that require mathematical solutions as useful mathematical tools that distinguished with precision, and simply differentiation and integral. Bernstein's polynomial is one among the foremost common polynomials in solving differential equations of all types by numerical methods, and a lot of research supports our words like solving ordinary and partial differential equations using adomian decomposition method with modified Bernstein Polynomials[3], Bernstein polynomials in solving space-time fractional diffusion equation[4], solving time-fractional order telegraph equation by Bernstein polynomials[5], applications to fractional differential equations[6], and others. The collocation method used with Bernstein's polynomials, it is a way for the numerical solution of equations (differential, partial and integral) and therefore, the summary of its idea is that choosing a specific area for possible solutions (polynomials up to a certain degree), note that each method has its own solution mechanism. The Bernstein collocation method reduces the problem to a system of non-linear algebraic equations. Many research followed this method and that we will mention a number of them: see [7], [8] and [9]
This research sought to seek out an approximate solution to the equal width wave fractional 
differential equation (FCEWE) and the modified equal width wave equation (FCMEWE) utilized in the 
fields of physics and electrical engineering. Many studies tried to find numerical, approximate and exact 
solutions to those equations like Sine-Gordon expansion[10], the tanh-sech method[11], the simple 
hyperbolic tangent Ansatz method [12] and sine-cosine method[13], the bright soliton solutions and 
singular solutions are constructed for space-time fractional EW and modified EW equations[14], the 
Riccati–Bernoulli sub-ODE method[15], modified Kudryashov method [16], the generalized (G'/G)-
expansion method[17] for the space-time fractional nonlinear partial differential

\[ F(u, D_{\alpha}^r u, D_{\beta}^s u, D_{\gamma}^t u, D_{\delta}^v u, D_{\epsilon}^w u, \ldots) = 0 \]
\[ G(v, D_{\alpha}^r v, D_{\beta}^s v, D_{\gamma}^t v, D_{\delta}^v v, D_{\epsilon}^w v, \ldots) = 0, \quad 0 < r_1, r_2 \leq 1 \]

Where \( u(x, t) \) and \( v(x, t) \) are solutions for the system of nonlinear fractional partial differential 
equations, where \( F \) and \( G \) are polynomial in \( u \) and \( v \) respectively.

This paper is arranged as follows: Section 2, the Basic definitions and properties are presented. The 
proposed strategy is exhibit step by step in Section 3, and in Section 4, all these steps are applied to find 
the approximate solutions and supported by illustrative graphs, then the research was concluded with 
conclusions.

2. Basic definitions and properties used
This part deals with some basic definitions and properties of the methods used to find an 
approximate solution, and we will mention them in succession:

Caputo fractional derivative
The definition of Caputo fractional derivative is:

\[ D_{\alpha}^n f(x) = \frac{1}{\Gamma(n-\alpha)} \int_0^x \frac{f^{(n)}(\tau)}{(x-\tau)^{\alpha}} d\tau, \quad 0 \leq n - 1 < \alpha < n \]

\( D_{\alpha} \) be used to represent Caputo fractional differential operator, especially when \( f(x) = x^n, \)

\[ D_{\alpha}^n x^n = \begin{cases} 
0 & \text{if } n \notin N \text{ and } n < |\alpha| \\
\left(\frac{\Gamma(n+1)}{\Gamma(n+1-\alpha)}\right) x^{n-\alpha} & \text{if } N_0 = \{0,1,2,\ldots\}, \quad N = \{1,2,\ldots\}. \text{ Upper limit function } |\alpha| \text{ is the smallest positive integral when } |\alpha| > a; \text{ lower limit function } |\alpha| \text{ is the biggest positive integral where } |\alpha| < a. \quad [18] 
\end{cases} \]

Bernstein polynomials
In numerical analysis, the Bernstein polynomial by Sergei Natanovich Bernstein is a polynomial 
resulting from a linear combination of the basis of Bernstein's polynomials, and the polynomials are 
vanishing at the endpoint of a & b. It is frequently used to find approximate solutions based on 
solutions supported by the Stone-Weierstrass approximation theory.

Definition 1: The Bernstein polynomials of degree n associated with function \( y(x) \) defined on the 
interval \([a, b]\) is

\[ B_n(y; x) = \frac{1}{(b-a)^n} \sum_{i=0}^{n} \binom{n}{i} (x-a)^i (b-x)^{n-i} y(x_i) \]

Where \( x_i = a + i \frac{b-a}{n}, \quad i=0,1,2,\ldots,n \)

In this search, we used Bernstein basis polynomials defined as follows:

Definition 2: The Bernstein basis polynomials of degree n defined on the interval \([a, b]\) are given as:

\[ B_i^n = \binom{n}{i} (x-a)^i (b-x)^{n-i}, \quad i=0,1,\ldots,n \]

where the binomial coefficient is \( \binom{n}{i} = \frac{n!}{i!(n-i)!} \)

For convenience, we usually set \( B_{i,n} = 0 \) if \( i < 0 \) or \( i > n \). [19]

Some useful bases and properties for Bernstein polynomials, see[2],[20]and[21]
The collocation method will be utilized to approximate solutions. In spite of the fact that the general procedure itself is not very precisely defined, it essentially includes forming an approximate solution as a linear combination of a convenient set of functions, the coefficients of which are dictated by requiring the combination to fulfill the differential equation at specific points (collocation at these points) which is:

\[
x_i = \frac{2k_i-1}{2n} \text{ and } t_i = \frac{2k_i-1}{2n} \quad i = 1, 2, ..., n
\]

Convergence analysis

For studying the convergence in Bernstein’s polynomials, let:

\[
\tag{6}
\sum u_{x_j}^p (x) = u(x)
\]

where \( u(x) \) is integrable function in interval \([0,x]\) which can be approximate by Bernstein polynomials and \( U = [u_0, u_1, ..., u_n]^T \) is coefficient which needs to obtain .

Then, if \( u \in C^{n+1}[0,x] \), and \( U^T \eta(x) \) is the best approximation of \( u \) out of \( W = \text{span}\{\eta_0, \eta_1, ..., \eta_n\} \), then the biggest error is produced as follows:

\[
\tag{7}
\|u - U^T \eta\|_2 \leq \frac{\sqrt{\frac{2n+3}{2(n+1)}}}{\sqrt{2n+3}}
\]

Where \( M = \max_{x \in [0,1]} |u^{(n+1)}(x)| \), \( S = \{x_0, x_n\} \}

For the proof and more theorems about the Convergence analysis and the maximum error, see [18] and [20]

3. Procedure steps

The steps of the proposed strategy to find the approximate solution of Eq. (1) are listed below:

Step 1 : Defining a Bernstein’s polynomial \( B_{i,n}(x) \) in the interval \([0,1]\) and we get the following formula :

\[
\tag{8}
B_{i,n}(x) = \binom{n}{i} x^i (1-x)^{n-i}, \quad i=0,1,2,...,n
\]

Step 2 : We use the simpler formula, which often falls within the properties of the Bernstein polynomial, which is:

\[
\tag{9}
B_{i,n}(x) = \sum_{k=0}^{n-i} (-1)^k \binom{n}{k} \binom{n-i}{k} x^{i+k}
\]

This is to facilitate the process of finding the fractional partial derivative of the equation (I)

Step 3 : The Caputo definition is applied to the formula (9) and finds the fractional partial derivative

\[
\tag{10}
D^\alpha B_{i,n}(x) = \sum_{k=0}^{n-i} (-1)^k \binom{n}{k} \binom{n-i}{k} D^\alpha (x^{i+k})
\]

Assuming that \( D^\alpha B_{i,n}(x) = B_{i,n}(x) \), we get :

\[
\tag{11}
B_{i,n}(x) = \sum_{k=0}^{n-i} (-1)^k \binom{n}{k} \binom{n-i}{k} \frac{\Gamma(i+k+1)}{\Gamma(i+k+1-\alpha)} x^{i+k-\alpha}
\]

Step 4 : Using the collocation method we impose a solution for each of \( u(x,t) \) and \( v(x,t) \)

If \( u(x,t) \in L^2([0,1] \times [0,1]) \) and \( v(x,t) \in L^2([0,1] \times [0,1]) \), then consider \( (n+1) \times (n+1) \) terms

\[
\tag{12}
u(x,t) = \sum_{i=0}^{n} \sum_{j=0}^{n} u_{ij} B_{i,n}(x) B_{j,n}(t)
\]

\[
v(x,t) = \sum_{i=0}^{n} \sum_{j=0}^{n} v_{ij} B_{i,n}(x) B_{j,n}(t)
\]

where \( u_{ij} \) and \( v_{ij} \) \( (i=0,1,...,n; \quad j=0,1,...,n) \) coefficients must be found.

Step 5 : We find the fractional partial derivatives for each term of the equation (12) with the help of a program specially designed to perform this task using Matlab(R2019a) software.

These are the entirety of the steps required to solve the system of equations mentioned in (1). We now present the solution in detail when the value of \( n \) is fixed.

4. Application

In order to find the approximate solution of space-time fractional of both the equal width wave equation(FCEWE) and the modified equal width wave equation (FCMEWE) the application was divided into two sections,
4.1. Solution of space – time fractional CEWE
Consider of the form:

\[ D_t^\alpha u(x,t) + e D_x^\alpha u^2(x,t) - \mu D_{xxt}^{\alpha/2} u(x,t) + e D_x^\alpha v^2(x,t) = 0 \]
\[ D_t^\alpha v(x,t) + e D_x^\alpha v^2(x,t) - \mu D_{xxt}^{\alpha/2} v(x,t) = 0 \]  

(13)

To solve the system in (13), the proposed method was implemented assuming that \( e = \mu = 1 \) for various values of the fractional derivative \( \alpha \) at \( x_i = t_i, i \in (0,1) \).

Case (1): when \( n=2 \)

\[ D_t^\alpha u(x,t) = \sum_{i=0}^{2} \sum_{j=0}^{2} u_{ij} B_{i,j}(x) D_t^\alpha (B_{j}^a(t)) \]
\[ = u_{00} B_{0,0}^a(x) B_{0,0}^a(t) + u_{01} B_{0,1}^a(x) B_{0,1}^a(t) + u_{02} B_{0,2}^a(x) B_{0,2}^a(t) + u_{10} B_{1,0}^a(x) B_{1,0}^a(t) + u_{11} B_{1,1}^a(x) B_{1,1}^a(t) + u_{12} B_{1,2}^a(x) B_{1,2}^a(t) + u_{20} B_{2,0}^a(x) B_{2,0}^a(t) + u_{21} B_{2,1}^a(x) B_{2,1}^a(t) + u_{22} B_{2,2}^a(x) B_{2,2}^a(t) \]

Where

\[ B_{0,0}^a(x) B_{0,0}^a(t) = (1-2x+x^2) * \frac{r(1)}{r(1-\alpha)} t^{-\alpha} - 2 \frac{r(2)}{r(2-\alpha)} t^{-1-\alpha} + \frac{r(3)}{r(3-\alpha)} t^{2-\alpha} \]
\[ B_{0,1}^a(x) B_{0,1}^a(t) = (1-2x+x^2) * \frac{r(2)}{r(2-\alpha)} t^{-\alpha} - 2 \frac{r(3)}{r(3-\alpha)} t^{2-\alpha} \]
\[ B_{0,2}^a(x) B_{0,2}^a(t) = (1-2x+x^2) * \frac{r(3)}{r(3-\alpha)} t^{2-\alpha} \]
\[ B_{1,0}^a(x) B_{1,0}^a(t) = 2x - 2x^2) * \frac{r(1)}{r(1-\alpha)} t^{-\alpha} - 2 \frac{r(2)}{r(2-\alpha)} t^{-1-\alpha} + \frac{r(3)}{r(3-\alpha)} t^{2-\alpha} \]
\[ B_{1,1}^a(x) B_{1,1}^a(t) = 2x - 2x^2) * \frac{r(2)}{r(2-\alpha)} t^{-\alpha} - 2 \frac{r(3)}{r(3-\alpha)} t^{2-\alpha} \]
\[ B_{1,2}^a(x) B_{1,2}^a(t) = 2x - 2x^2) * \frac{r(3)}{r(3-\alpha)} t^{2-\alpha} \]
\[ B_{2,0}^a(x) B_{2,0}^a(t) = \frac{r(1)}{r(1-\alpha)} x^{2-2a} - 2 \frac{r(2)}{r(2-\alpha)} x^{1-2a} + \frac{r(3)}{r(3-\alpha)} x^{2-2a} * \frac{r(1)}{r(1-\alpha)} t^{-\alpha} - 2 \frac{r(2)}{r(2-\alpha)} t^{-1-\alpha} + \frac{r(3)}{r(3-\alpha)} t^{2-\alpha} \]
\[ B_{2,1}^a(x) B_{2,1}^a(t) = \frac{r(2)}{r(2-\alpha)} x^{1-2a} - 2 \frac{r(3)}{r(3-\alpha)} x^{2-2a} * \frac{r(1)}{r(1-\alpha)} t^{-\alpha} - 2 \frac{r(2)}{r(2-\alpha)} t^{-1-\alpha} + \frac{r(3)}{r(3-\alpha)} t^{2-\alpha} \]
\[ B_{2,2}^a(x) B_{2,2}^a(t) = \frac{r(3)}{r(3-\alpha)} x^{2-2a} * \frac{r(1)}{r(1-\alpha)} t^{-\alpha} - 2 \frac{r(2)}{r(2-\alpha)} t^{-1-\alpha} + \frac{r(3)}{r(3-\alpha)} t^{2-\alpha} \]

The same steps are returned to calculate \( D_t^\alpha v(x,t) \) and \( D_{xxt}^{\alpha/2} v(x,t) \).
As for $D_α u^2(x, t)$ and $D_α v^2(x, t)$ squaring is performed, finding results, and then taking the fractional partial derivative of the variable x, using Caputo method. Matlab (R2019a) software was used to find these solutions, yields:

$$(34) \quad D_α (B_{0.2}(x))^2 = -4 \left( \frac{r(2)}{r(2-y)} \right) x^{1-y} + 6 \left( \frac{r(3)}{r(3-y)} \right) x^{2-y} - 4 \left( \frac{r(4)}{r(4-y)} \right) x^{3-y} + \frac{r(5)}{r(5-y)} x^{4-y}$$

$$(35) \quad D_α (B_{1.2}(x))^2 = 4 \left( \frac{r(3)}{r(3-y)} \right) x^{2-y} - 8 \left( \frac{r(4)}{r(4-y)} \right) x^{3-y} + 4 \left( \frac{r(5)}{r(5-y)} \right) x^{4-y}$$

$$(36) \quad D_α (B_{2.2}(x))^2 = 2 \left( \frac{r(2)}{r(2-y)} \right) x^{1-y} - 6 \left( \frac{r(3)}{r(3-y)} \right) x^{2-y} + 6 \left( \frac{r(4)}{r(4-y)} \right) x^{3-y} - 2 \left( \frac{r(5)}{r(5-y)} \right) x^{4-y}$$

$$(37) \quad D_α (B_{0.2}(x)B_{1.2}(x)) = 2 \left( \frac{r(2)}{r(2-y)} \right) x^{1-y} - 6 \left( \frac{r(3)}{r(3-y)} \right) x^{2-y} + 6 \left( \frac{r(4)}{r(4-y)} \right) x^{3-y} - 2 \left( \frac{r(5)}{r(5-y)} \right) x^{4-y}$$

$$(38) \quad D_α (B_{0.2}(x)B_{2.2}(x)) = \left( \frac{r(3)}{r(3-y)} \right) x^{2-y} - 2 \left( \frac{r(4)}{r(4-y)} \right) x^{3-y} + \left( \frac{r(5)}{r(5-y)} \right) x^{4-y}$$

$$(39) \quad D_α (B_{1.2}(x)B_{2.2}(x)) = 2 \left( \frac{r(2)}{r(2-y)} \right) x^{1-y} - 2 \left( \frac{r(3)}{r(3-y)} \right) x^{2-y} + 2 \left( \frac{r(4)}{r(4-y)} \right) x^{3-y} - \left( \frac{r(5)}{r(5-y)} \right) x^{4-y}$$

After employing the collocation method with Bernstein polynomials and applying Caputo’s definition of the fractional derivative, the results are expressed in Fig. (1), which shows solutions with different fractional derivatives ($α$) and different values of (x) and (t).

**Figure 1.** The approximation solutions of FCEWE using different values of $α$, $n=2$ and $e=μ=1$.

Case (2): when $n=3$

In order to find solutions when $n=3$, third-order polynomials are constructed, derivatives are found and all the steps are repeated in Case (1). For $D_α u^2(x, t)$ where $n=3$ the results as follows:

$$(40) \quad D_α (B_{0.3}(x))^2 = \left( \frac{r(7)}{r(7-y)} \right) x^{6-y} - 6 \left( \frac{r(6)}{r(6-y)} \right) x^{5-y} + 15 \left( \frac{r(5)}{r(5-y)} \right) x^{4-y} - 20 \left( \frac{r(4)}{r(4-y)} \right) x^{3-y} + 15 \left( \frac{r(3)}{r(3-y)} \right) x^{2-y} - 6 \left( \frac{r(2)}{r(2-y)} \right) x^{1-y}$$

$$(41) \quad D_α (B_{0.3}(x)B_{1.3}(x)) = 3 \left( \frac{r(7)}{r(7-y)} \right) x^{6-y} - 15 \left( \frac{r(6)}{r(6-y)} \right) x^{5-y} + 30 \left( \frac{r(5)}{r(5-y)} \right) x^{4-y} - 30 \left( \frac{r(4)}{r(4-y)} \right) x^{3-y} + 15 \left( \frac{r(3)}{r(3-y)} \right) x^{2-y} - 3 \left( \frac{r(2)}{r(2-y)} \right) x^{1-y}$$

$$(42) \quad D_α (B_{0.3}(x)B_{2.3}(x)) = -3 \left( \frac{r(7)}{r(7-y)} \right) x^{6-y} + 12 \left( \frac{r(6)}{r(6-y)} \right) x^{5-y} - 18 \left( \frac{r(5)}{r(5-y)} \right) x^{4-y} + 12 \left( \frac{r(4)}{r(4-y)} \right) x^{3-y} - 3 \left( \frac{r(3)}{r(3-y)} \right) x^{2-y}$$

$$(43) \quad D_α (B_{0.3}(x)B_{3.3}(x)) = \left( \frac{r(7)}{r(7-y)} \right) x^{6-y} - 3 \left( \frac{r(6)}{r(6-y)} \right) x^{5-y} - 3 \left( \frac{r(5)}{r(5-y)} \right) x^{4-y} - \left( \frac{r(4)}{r(4-y)} \right) x^{3-y}$$
\begin{align}
(44) & \quad D_x^\alpha (B_{1,3}(x))^2 = 9 \left( \frac{r(7)}{r(7-y)} \right) x^{6-y} - 36 \left( \frac{r(6)}{r(6-y)} \right) x^{5-y} + 54 \left( \frac{r(5)}{r(5-y)} \right) x^{4-y} - \\
& \quad 36 \left( \frac{r(4)}{r(4-y)} \right) x^{3-y} + 9 \left( \frac{r(3)}{r(3-y)} \right) x^{2-y} \\
(45) & \quad D_x^\alpha (B_{1,3}(x)B_{2,3}(x)) = -9 \left( \frac{r(7)}{r(7-y)} \right) x^{6-y} + 27 \left( \frac{r(6)}{r(6-y)} \right) x^{5-y} - 27 \left( \frac{r(5)}{r(5-y)} \right) x^{4-y} + \\
& \quad 9 \left( \frac{r(4)}{r(4-y)} \right) x^{3-y} \\
(46) & \quad D_x^\alpha (B_{1,3}(x)B_{3,3}(x)) = 3 \left( \frac{r(7)}{r(7-y)} \right) x^{6-y} - 6 \left( \frac{r(6)}{r(6-y)} \right) x^{5-y} + 9 \left( \frac{r(5)}{r(5-y)} \right) x^{4-y} \\
(47) & \quad D_x^\alpha (B_{2,3}(x))^2 = 9 \left( \frac{r(7)}{r(7-y)} \right) x^{6-y} - 18 \left( \frac{r(6)}{r(6-y)} \right) x^{5-y} + 9 \left( \frac{r(5)}{r(5-y)} \right) x^{4-y} \\
(48) & \quad D_x^\alpha (B_{2,3}(x)B_{3,3}(x)) = -3 \left( \frac{r(7)}{r(7-y)} \right) x^{6-y} + 3 \left( \frac{r(6)}{r(6-y)} \right) x^{5-y} \\
(49) & \quad D_x^\alpha (B_{3,3}(x))^2 = \left( \frac{r(7)}{r(7-y)} \right) x^{6-y}
\end{align}

for different fractional derivatives(\(\alpha\)) and different values of \(x\) and \(t\) we obtain the results shown in Figure (2).

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{The approximation solutions of FCEWE using different values of \(\alpha, n=3\) and \(e=\mu=1\).}
\end{figure}

4.2. Solution for fractional CMEW equations

Consider space – time fractional CMEW for finding functions \(u(x,t)\) and \(v(x,t)\) in the form :
\begin{align}
(50) & \quad D_x^\alpha u(x,t) + eD_x^\alpha u(x,t) - \mu D_x^\alpha u(x,t) + eD_x^\alpha v^3(x,t) = 0 \\
& \quad D_x^\alpha v(x,t) + eD_x^\alpha v^2(x,t) - \mu D_x^\alpha v(x,t) = 0
\end{align}

The procedure for solving (FCMEW) is not different from the process of solving (FCEWE) as main steps, the only difference is to find the terms \(D_x^\alpha u^3(x,t)\) & \(D_x^\alpha v^3(x,t)\). The same previous steps in finding the values of \(u\) and \(v\) are followed, the results have been presented by the graph for \(n=2\) and \(e=\mu=1\). we will suffice to provide some values as follows and the Fig. (3) shown the results.

\begin{align}
(51) & \quad D_x^\alpha (B_{2,2}(x))^3 = \left( \frac{r(7)}{r(7-y)} \right) x^{6-y} - 6 \left( \frac{r(6)}{r(6-y)} \right) x^{5-y} + 15 \left( \frac{r(5)}{r(5-y)} \right) x^{4-y} - \\
& \quad 20 \left( \frac{r(4)}{r(4-y)} \right) x^{3-y} + 15 \left( \frac{r(3)}{r(3-y)} \right) x^{2-y} - 6 \left( \frac{r(2)}{r(2-y)} \right) x^{1-y} + \left( \frac{r(1)}{r(1-y)} \right) x^0-y \\
(52) & \quad D_x^\alpha (B_{2,2}(x))^3 = -8 \left( \frac{r(7)}{r(7-y)} \right) x^{6-y} - 24 \left( \frac{r(6)}{r(6-y)} \right) x^{5-y} - 24 \left( \frac{r(5)}{r(5-y)} \right) x^{4-y} + \\
& \quad 8 \left( \frac{r(4)}{r(4-y)} \right) x^{3-y} \\
(53) & \quad D_x^\alpha (B_{2,2}(x))^3 = \left( \frac{r(7)}{r(7-y)} \right) x^{6-y}
\end{align}
Figure 3. the approximation solutions of FCMEWE using different values of $\alpha$, $n=2$ and $e=\mu=1$.

5. Conclusion
This study, the approximation solution of Fractional Coupled EWE and Fractional Coupled MEWE have been found by using collocation method based on Bernstein basis polynomials to a discrete variable $(x,t)$ becomes $(x_i, t_i)$, the Figures showed the application is effective and have useful advantages, no gaps appeared within the curves compared to some studies that used trigonometric functions to solve the same system, the closer the value of the fractional partial derivative to $\alpha \approx 1$ the wave appears more smooth, uniform and equaled in width.

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References
[1] A. Chatterjee, U. Basu, and B. N. Mandal, “Numerical algorithm based on Bernstein polynomials for solving nonlinear fractional diffusion-wave equation,” Int. J. of Advances Applied Mathematics Andm., vol. 5, no. 2, pp. 9–15, 2017.
[2] M. M. Khader and R. T. Alqahtani, “Approximate solution for system of fractional non-linear dynamical marriage model using Bernstein polynomials,” J. Nonlinear Sci. Appl., vol. 10, no. 03, pp. 865–873, 2017, doi: 10.22436/jnsa.010.03.02.
[3] A. F. Qasim and E. S. Al-Rawi, “Adomian Decomposition Method with Modified Bernstein Polynomials for Solving Ordinary and Partial Differential Equations,” J. Appl. Math., vol. 2018, 2018, doi: 10.1155/2018/1803107.
[4] A. Baseri, E. Babolian, and S. Abbasbandy, “Normalized Bernstein polynomials in solving space-time fractional diffusion equation,” Adv. Differ. Equations, vol. 2017, no. 1, pp. 1–25, 2017, doi: 10.1186/s13662-017-1401-1.
[5] M. Asgari, R. Ezzati, and T. Allahviranloo, “Numerical solution of time-fractional order telegraph equation by bernstein polynomials operational matrices,” Math. Probl. Eng., vol. 2016, no. 2, 2016, doi: 10.1155/2016/1683849.
[6] H. Khalil, “Brenstien polynomials and applications to fractional differential equations,” Comput.
Methods Differ. Equations, vol. 3, no. 1, pp. 14–35, 2015.

[7] S. N. Shihab and M. N. M. Ali, “Collocation Orthonormal Bernstein Polynomials Method for Solving Integral Equations,” Eng. & Tech. Journal, Vol. 33, Part (B), No. 8, 2015.

[8] S. Yalçınbaş and H. Görler, “Bernstein collocation method for solving the first order Nonlinear differential equations with the mixed Non-Linear conditions,” Math. Comput. Appl., vol. 20, no. 3, pp. 160–173, 2015, doi: 10.1080/17426596.2015.1012021.

[9] A. Akyüz-Daşcıoğlu and N. Isler Acar, “Bernstein collocation method for solving linear differential equations,” Gazi Univ. J. Sci., vol. 26, no. 4, pp. 527–534, 2013.

[10] A. Korkmaz, O. E. Hepson, K. Hosseini, H. Rezazadeh, and M. Eslami, “On The Exact Solutions to Conformable Time Fractional Equations in EW Family Using Sine-Gordon Equation Approach,” no. December, pp. 1–13, 2017, doi: 10.20944/preprints201712.0188.v1.

[11] H. Ma, X. Meng, H. Wu, and A. Deng, “Exact solutions of the space-time fractional equal width equation,” Therm. Sci., vol. 23, no. 4, pp. 2307–2313, 2019, doi: 10.2298/TSCI19042313M.

[12] O. E. Hepson, “Hyperbolic Tangent ansatz method to space time fractional modified KdV, modified EW and Benney–Luke Equations.” 2018, [Online]. Available: www.preprints.org.

[13] K. R. Raslan, T. S. EL-Danaf, and K. K. Ali, “New exact solution of coupled general equal width wave equation using sine-cosine function method,” J. Egypt. Math. Soc., vol. 25, no. 3, pp. 350–354, 2017, doi: 10.1016/j.joems.2017.03.004.

[14] A. Korkmaz, “Exact solutions of space-time fractional EW and modified EW equations,” Chaos, Solitons and Fractals, vol. 96, pp. 122–138, 2017, doi: 10.1016/j.chaos.2017.01.015.

[15] S. Z. Hassan and M. A. E. Abdelrahman, “Solitary wave solutions for some nonlinear time-fractional partial differential equations,” Pramana - J. Phys., vol. 91, no. 5, 2018, doi: 10.1007/s12043-018-1636-8.

[16] K. R. Raslan, T. S. EL-Danaf, and K. K. Ali, “Exact Solution of Space-Time Fractional Coupled EW and Coupled MEW Equations Using Modified Kudryashov Method,” Commun. Theor. Phys., vol. 68, no. 1, pp. 49–56, 2017, doi: 10.1088/0253-6102/68/1/49.

[17] D. Lu and S. Ye, “Optical Solitary Wave Solutions of the Space-Time Fractional Modified Equal-Width Equation and their Applications,” Int. J. Math. Res., vol. 8, no. 1, pp. 1–20, 2019, doi: 10.18488/journal.24.2019.81.1.20.

[18] W. Li, L. Bai, Y. Chen, S. Dos Santos, and B. Li, “Solution of linear fractional partial differential equations based on the operator matrix of fractional bernstein polynomials and error correction,” Int. J. Innov. Comput. Inf. Control, vol. 14, no. 1, pp. 211–226, 2018, doi: 10.24507/ijicic.14.01.211.

[19] A. S. Bataineh, “Bernstein polynomials method and it’s error analysis for solving nonlinear problems in the calculus of variations: Convergence analysis via residual function,” Filomat, vol. 32, no. 4, pp. 1379–1393, 2018, doi: 10.2298/FIL1804379B.

[20] H. Song, M. Yi, J. Huang, and Y. Pan, “Bernstein polynomials method for a class of generalized variable order fractional differential equations,” IAENG Int. J. Appl. Math., vol. 46, no. 4, pp. 437–444, 2016.

[21] A. Saadatmandi, “Bernstein operational matrix of fractional derivatives and its applications,” Appl. Math. Model., vol. 38, no. 4, pp. 1365–1372, 2014, doi: 10.1016/j.apm.2013.08.007.