1. Introduction

Understanding hadronic interactions at high energies represents a very important ingredient for modeling high energy cosmic ray air showers. In this paper, we present a new approach to simulate hadronic interactions, including hadron-hadron, hadron-nucleus, and nucleus-nucleus scattering, in the energy range roughly between $10^{11}$ and $10^{17}$ eV. Such collisions are very complex, being composed of many components, and therefore some strategy is needed to construct a reliable model. The central point of our approach is the hypothesis, that the behavior of high energy interactions is universal (universality hypothesis). So, for example, the hadronization of partons in nuclear interactions follows the same rules as the one in electron-positron annihilation; the radiation of off-shell partons in hadronic collisions is based on the same principles as the one in deep inelastic scattering. We construct a model for hadronic interactions in a modular fashion. The individual modules, based on the universality hypothesis, are identified as building blocks for more elementary interactions (like $e^+e^-$, lepton-proton), and can therefore be studied in a much simpler context. With these building blocks under control, we can provide a quite reliable model for nucleus-nucleus, hadron-nucleus and hadron-hadron scattering, providing in particular very useful tests for the complicated numerical procedures using Monte Carlo techniques.

1.2 The Universality Hypothesis

Generalizing proton-proton interactions, the structure of nucleus-nucleus scattering should be as follows: there are elementary inelastic interactions between individual nucleons, realized by partonic “half-ladders”, where the same nucleon may participate in several of these elementary interactions. Also elastic scatterings are possible, represented by parton ladders. Although such diagrams can be calculated in the framework of perturbative QCD, there are quite a few problems: important cut-offs have to be chosen, one has to choose the appropriate evolution variables, one may question the validity of the “leading logarithmic approximation”, the coupling of the parton ladder to the nucleon is not known, the hadronization procedure is not calculable from first principles and so on. So there are still many unknowns, and a more detailed study is needed.

Our starting point is the universality-hypothesis, saying that the behavior of high-energy interactions is universal. In this case all the details of nuclear interactions can be determined by studying simple systems in connection with using a modular structure for modeling nuclear scattering. One might think of proton-proton scattering representing a simple system, but
this is already quite complicated considering the fact that we have in general already several elementary interactions. It would be desirable to study just one elementary interaction, which we refer to as “semihard Pomeron”, which will be done in the next section.

1.3 The semihard Pomeron

In order to investigate the semihard Pomeron, we turn to an even simpler system, namely lepton-nucleon scattering. A photon is exchanged between the lepton and a quark of the proton, where this quark represents the last one in a “cascade” of partons emitted from the nucleon. The squared diagram represents a parton ladder. In the leading logarithmic approximation (LLA), the virtualities of the partons are ordered such that the largest one is close to the photon [1, 2]. If we compare with proton-proton scattering, we have ordering from both sides with the largest virtuality in the middle, so in some sense the hadronic part of the lepton-proton diagram represents half of the elementary proton-proton diagram, and should therefore be studied first. In fact such statements are to some extent commonly accepted, but not carried through rigorously in the sense that also for example the hadronization of these two processes is related.

But first we investigate the so-called structure function $F_2$, related to the lepton-proton cross section via [3]

$$\frac{d\sigma}{dx \, dQ^2} = L(x, Q^2) \, F_2(x, Q^2) \quad (1)$$

with a calculable factor $L$. The variable $Q^2$ represents the absolute value of the photon virtuality, and $x$ is the Bjorken-$x$ variable. $F_2$ represents the hadronic part of the diagram, and is, using eq. [3], measurable. In lowest order and considering only leading logarithms of $Q^2$, we find

$$F_2(x, Q^2) = \sum_j e_j^2 \, x \, f^j(x, Q^2) \quad (2)$$

with

$$f^j(x, Q^2) = f^j(x, Q_0^2) + \sum_{ij} \int_x^1 \frac{d\xi}{\xi} \int_{Q_0^2}^{Q^2} \frac{dQ'^2}{Q'^2} \, f^i(\xi, Q'^2) \, \frac{\alpha_s}{2\pi} \, P_i^j \left( \frac{x}{\xi} \right). \quad (3)$$

Iterating this equation obviously represents a parton ladder with ordered virtualities, coupled in a nonperturbative way to the nucleon. To account for the perturbative part, we introduce a so-called QCD evolution function $E_{QCD}^{ij}(Q_0^2, Q_1^2, x)$, representing the evolution of a parton cascade from scale $Q_0^2$ to $Q_1^2$. This function is calculated in an iterative way based on eq. [3].

Next we have to determine the $x$-distribution of the first parton of the ladder. We consider a Pomeron contribution

$$\varphi^i_P(x) = C_P \otimes E^i_{soft \, P} \quad (4)$$

and a Reggeon contribution

$$\varphi^i_R(x) = C_R \otimes E^i_{soft \, R}, \quad (5)$$

with $E^i_{soft \, P}$ and $E^i_{soft \, R}$ representing a soft Pomeron and a Reggeon respectively, and $C$ is the Pomeron/Reggeon-nucleon coupling [4]. The sum of these two contributions is the total initial distribution $\varphi^i(x)$. The distribution at scale $Q^2$ is

$$f^i = \sum_i \varphi^i \otimes E^{ij}_{QCD} \quad (6)$$
The structure function is then calculated as

$$F_2(x, Q^2) = \sum_j e_j^2 x f^j(x, Q^2).$$  \hspace{1cm} (7)$$

For $Q = Q_0$, the $\Pi P$-contribution is a function which peaks at very small values of $x$ and then decreases monotonically towards zero for $x = 1$, the $\Pi R$-contribution on the other hand has a maximum at large values of $x$ and goes towards zero for small values of $x$. The precise form of $f$ depends crucially on the exponent for the Pomeron-nucleon coupling, and we find a good agreement for $\beta_{\Pi} = \frac{1}{2}$.

We are now in a position to write down the expression $G_{\text{semi}}$ for a cut semihard Pomeron, representing an elementary inelastic interaction in $pp$ scattering. We can divide the corresponding diagram into three parts. We have the process involving the highest parton virtuality in the middle, and the upper and lower part representing each an ordered parton ladder coupled to the nucleon. According to the universality hypothesis, the two latter parts are known from studying deep inelastic scattering, representing each the hadronic part of the DIS diagram. So we get, for given impact parameter $b$ and given energy squared $s$,

$$G_{\text{semi}} = \sum_{ij} \int d\xi^+ d\xi^- dQ^2 \frac{d\sigma_{\text{Born}}^{ij}}{dQ^2}(\xi^+ \xi^- s, Q^2).$$  \hspace{1cm} (8)$$

In addition to the semihard Pomeron, one has to consider the expression representing the soft Pomeron \[.] The latter one, $G_{\text{soft}}$, is the Fourier transform of a Regge pole amplitude $A \sim s^{\alpha(t)}$. So an elementary inelastic interaction in an energy range of say $10 - 10^4$ GeV is therefore written as

$$G_{\text{tot}} = G_{\text{semi}} + G_{\text{soft}}.$$  \hspace{1cm} (9)$$

Up to this point, we are able to calculate cross sections, namely $F_2$, which is essentially the photon-proton cross section, in case of lepton-proton scattering, and the jet cross section, which can be obtained by integrating $G_{\text{semi}}$ over impact parameter, in case of proton-proton scattering.

1.4 Hadron Production

As discussed in the last chapter, there exist observables (cross sections) which can be calculated without detailed knowledge about hadron production, but our main goal is to calculate particle production. Using the Monte Carlo technique, this amounts to generating first parton configurations, and then, in a second step, hadron configurations.

Let us start again with the case of lepton-nucleon scattering. We generate a parton configuration, based on the expression for $F_2$ as discussed in the previous chapter. The next step consists of generating with certain probabilities hadron configurations, starting from a given parton configuration. We cannot calculate those probabilities within QCD, so we simply provide a recipe, the so-called string model. The first step consists of mapping a partonic configuration into a string configuration. For this purpose, we use the colour representation of the parton configuration: a quark is represented by a colour line, a gluon by a colour-anticolour pair. One then follows the colour flow starting from a quark via gluons, as intermediate steps, till one finds an antiquark. The corresponding sequences

$$q - g_1 - g_2 \ldots - g_n - \bar{q}$$  \hspace{1cm} (10)$$
are identified with kinky strings, where the gluons represent the kinks. Such a string decays into hadron configurations with the corresponding probabilities given in the framework of the theory of classical relativistic strings.

The above discussion of how to generate parton and hadron configurations is not yet complete: the emitted partons are in general off-shell and can therefore radiate further partons. This so called timelike radiation is taken into account using standard techniques. The mapping of parton to hadron configurations still works the same way as discussed above.

In case of proton-proton interactions one generates parton configurations based on the expression for $G_{\text{semi}}$ as discussed in the last chapter. Hadron configurations are generated according to the same principles as for lepton scattering. Actually, our treatment of generating parton configurations for an elementary pp interaction is absolutely compatible with deep inelastic scattering, it is based on the same building blocks, in particular on the evolution functions.

Our procedure can now be used to treat the case of many semihard Pomerons, for $p-p$ as well as nuclear scattering. A detailed discussion will be given in a future publication.

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