Odd-even staggering and shell effects in nuclear charge radii

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(Dated: August 3, 2021)

Abstract

A unified theoretical model reproducing charge radii of known atomic nuclei plays an essential role to make extrapolations in the regions of unknown nuclear size. Recently developed new ansatz which phenomenally takes into account the neutron-proton short-range correlations (np-SRCs) can describe the discontinuity properties and odd-even staggering (OES) effect of charge radii along isotopic chains remarkably well. In this work, we further review the modified rms charge radii formula in the framework of relativistic mean field (RMF) theory. The charge radii are calculated along various isotopic chains that include the nuclei featuring the $N = 50$ and $82$ magic shells. Our results suggest that RMF with and without considering correction term give almost similar trend of nuclear size for some isotopic chains with open proton shell, especially the shrink phenomena of charge radii at strong neutron closed shells and the OES behaviors. This suggests that the np-SRCs has almost no influence for some nuclei due to the strong coupling between different levels around Fermi surface. The weakening OES behavior of nuclear charge radii is observed generally at completely filled neutron shells and this may be proposed as a signature of magic indicator.

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I. INTRODUCTION

Nuclear charge radii characterize the charge density extension in space and determine the Coulomb interaction in nuclei provide access to nuclear structure information [1]. Plenty of methods are employed to perform measurements of nuclear charge radii, such as muonic atom x rays ($\mu^-$) [2], high-resolution laser spectroscopy techniques [3]–[8], high energy elastic electron scattering ($e^-$) [9]–[11] and isotope shifts (ISs) [12], [13], etc. So far, more available charge radii data are provided in nuclear chart [14]. As one of the important input quantities in astrophysics, credible data of finite nuclei charge radii play an important role in theoretical study [15]. Moreover, reliable predictions can also serve as useful guides for experimental detection of charge radii of nuclei far away from the $\beta$-stability line.

In general, nuclei charge radii often display the emergence of simple patterns and regular behaviours or global properties that the variation of charge radii along isotopic chains represent discontinuous features [4], [10], [15]. These remarkably abrupt changes in charge radii are observed naturally across the neutron-closure shells, namely the kinks at $N = 20, 28, 50, 82, 126$ [14], [16]–[27]. The strong shell structure results in the parabolic-like shapes of charge radii with respect to the variation of neutron number. In addition, the odd-even staggering (OES) effect that the nuclear charge radii of odd-neutron isotopes are smaller than the averages of their even-neutron neighbours, are generally observed throughout the nuclear chart [14].

With the accumulation of experimental data, many empirical relations and microscopic models have been proposed to investigate the variations of nuclear charge radii. The general nuclear size is ruled by $A^{1/3}$ law through introducing the shell and isospin effects [28], [29]. Similarly, $Z^{1/3}$-dependence for nuclear charge radius is also directly proposed in Ref. [30]. The sophisticated Garvey-Kelson (GK) relation has been transformed to describing the nuclear charge radius [31], [32], but its extrapolating ability is limited [33], [34]. For heavy or superheavy neutron-rich elements, it is noteworthy that their bulk properties are barely obtained due to short half-lives [35]. Thereby, $\alpha$-decay chain provides a possible method to extract the properties of parent nuclei. Recent works attempt to deduce the charge radii based on the $\alpha$-decay properties [36], [37], even cluster and proton emission data [38]. The microscopic nuclear structure models based on the mean-field approach such as Hartree-Fock-Bogoliubov (HFB) model [39], [40] and relativistic mean field (RMF) theory [41], [42] can
reveal the inner nuclear interactions self-consistently. As encountered in ab initio calculations with chiral effective field theory (EFT) interactions [13], these models can not reproduce the fine structure of nuclear charge radii well. In recent years, Bayesian neural networks as an alternative approach was devoted to describe the charge radii in nuclear chart [13-15], but the underlying physical mechanism is unclear.

Various mechanisms have been proposed to elucidate the evolution of nuclear charge radii, such as the core polarization by valence neutrons [16, 17], precise knowledge of radial moments [18], adjacent nucleons relations [19], quadrupole deformation [20], etc. The sophisticated Fayans EDF model, in which a novel density-gradient term is introduced into the pairing interaction, can reproduce the staggering effects of charge radii [21, 22]. This model demonstrates that surface pairing components play an essential role. By contrast, the phenomenally modified charge radii formula which associates to the neutron-proton (np) pairs correlations is proposed under the relativistic mean field (RMF) theory within NL3 parameterization set [23]. This new ansatz can remarkably describe the discontinuity properties and OES effects of charge radii along isotopic chains [24]. As argued in Ref. [25], the short-range correlations (SRCs) originating from np pairing contribute to the root-mean-square (rms) charge radius of finite nuclei. The np correlations play an essential role to characterize the OES of nuclear charge radii [25] and the magnitude of the neutron skin in asymmetric nuclei [26].

As mentioned in Ref. [24], the effect of np-SRCs has an influence on the computed rms charge radius. In some cases, however, this effect has no contributions to charge radius. In Ref. [27], it points out that pairing does not play a crucial role in the origin of the kink at magic number and OES behaviors. This means the experimental data can be reproduced predominately at the mean-field level. To understand these uncertainties, we further check the recently developed approach by studying charge radii in rich experimental data chains that include \( N = 50 \) or 82 magic shell. We focus on the variations of nuclear charge radii and the OES behaviors along isotopic chains and aim to make a further complement for our new ansatz.

This paper is organized as follows. In Sec. II, we briefly describe the theoretical model. In Sec. III, we present the results and discussions. A short summary and outlook are provided in Sec. IV.
II. THEORETICAL MODEL

The relativistic mean field (RMF) theories have made remarkable successes in describing various nuclear physics phenomena \cite{57-67}. For the version of nonlinear self-consistent Lagrangian density, nucleons are described as Dirac particles which interact via the exchange of $\sigma$, $\omega$ and $\rho$ mesons. The electromagnetic field is served as photon. The effective Lagrangian density

$$
\mathcal{L} = \bar{\psi} [i \gamma^\mu \partial_\mu - M - g_\sigma \sigma - \gamma^\mu (g_\omega \omega_\mu + g_\rho \vec{\tau} \cdot \vec{\rho}_\mu + e A_\mu)] \psi \\
+ \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{2} g_2 \sigma^2 - \frac{1}{4} g_3 \sigma^4 \\
- \frac{1}{4} \Omega^{\mu\nu} \Omega_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{4} c_3 (\omega^\mu \omega_\mu)^2 \\
- \frac{1}{4} \vec{R}_\mu \cdot \vec{R}_\mu + \frac{1}{2} m_\rho^2 \rho^\mu \cdot \vec{\rho}_\mu + \frac{1}{4} d_3 (\rho^\mu \rho_\mu)^2 \\
- \frac{1}{4} F^{\mu\nu} F_{\mu\nu},
$$

(1)

where $M$ is the mass of nucleon and $m_\sigma$, $m_\omega$, and $m_\rho$, are the masses of the $\sigma$, $\omega$ and $\rho$ mesons, respectively. Here $g_\sigma$, $g_\omega$, $g_\rho$ and $e^2/4\pi$ are the coupling constants for $\sigma$, $\omega$, $\rho$ mesons and photon, respectively. The NL3 parameter set is employed \cite{68}. In order to obtain the ground state properties, the Hamiltonian of the system becomes $H' = H - \lambda \langle Q \rangle$, the second term on the right hand represents the linear constraint part \cite{69}. In general, the mean-square charge radius of a nucleus assumes the form (in units of fm$^2$) \cite{11,70}

$$
R_{ch}^2 = \frac{\int r^2 \rho_p(r) d^3r}{\int \rho_p(r) d^3r} + 0.64 \text{ fm}^2,
$$

(2)

where the first term corresponds to the point-like density distribution and then the second term accounts for the finite size of proton \cite{70}. However, the shell closure and OES effect of charge radii can not be reproduced well \cite{52}. Since the modified expression has been proposed in the following way:

$$
R_{ch}^2 = \frac{\int r^2 \rho_p(r) d^3r}{\int \rho_p(r) d^3r} + 0.64 \text{ fm}^2 + \frac{a_0}{\sqrt{A}} \Delta D \text{ fm}^2.
$$

(3)

The last term on the right hand is the modified term which associates to the Cooper pair condensation \cite{74}. The quantity $A$ is the mass number and $a_0$ is a normalization constant by fitting the reliable experimental data of charge radii. The quantity $\Delta D = |D_n - D_p|$ represents the difference of Cooper pair condensation between neutron and proton. It is calculated self-consistently by solving the state-dependent BCS equations with a $\delta$ force interaction \cite{72,73}. More details and discussions are shown in Ref. \cite{52}.
III. RESULTS AND DISCUSSIONS

A. Cooper pair components

In this work, we focus on the behavior of charge radii along isotopic chains with respect to the variation of neutron number. The pairing strength is generally determined by fitting to the odd-even mass staggering \[^{14}\]. In order to reflect the universality of our results, the pairing strength is \( V_0 = 322.8 \text{ MeV fm}^3 \[^{52}\].

![Graph showing Cooper pair components of proton (solid circle) and neutron (open square) along Kr isotopic chain.](image)

FIG. 1. Cooper pair components of proton (solid circle) and neutron (open square) are shown along Kr isotopic chain.

In Eq. (3), the modified term is associated to the Cooper pair condensation function with the following form:

\[
D_{n,p} = \sum_{k>0} u_k v_k.
\]

This quantity can represent a measurement of the number of Cooper pairs in the BCS wave function \[^{71}\]. The difference of \( D_n \) and \( D_p \) is employed to equivalently measure the np-SRCs components in the rms charge radius. As shown in our previous work \[^{52}\], the fine structure of finite nuclei charge radius is determined by the modified term. In Ref. \[^{54}\] the authors demonstrate that the np-SRCs has no contribution to charge radii for open shell nuclei. This can be easily understood in our new ansatz that \( D_n \) and \( D_p \) might be comparable for open shell nuclei due to the strong coupling between different levels around the Fermi surface.
From this point of view, the quantities $D_{n,p}$ as a function of neutron number are plotted along Kr ($Z = 36$) isotopic chain in Fig. 1. One can find the values of quantity $D_n$ and $D_p$ are indeed close for neutron and proton. Consequently, the modified part has slightly influence on the rms charge radii. In order to clarify this phenomenon, more isotopic chains are investigated.

B. Variations of charge radii along isotopic chains

![Diagram](image)

FIG. 2. Charge radii of krypton (a), strontium (b), tellurium (c) and xenon (d) isotopes are obtained by the RMF(BCS) (dashed line) and the modified rms charge radii expression RMF(BCS)* methods (open diamond), respectively. The experimental data are taken from Ref. [14] (filled circle).

In Fig. 2, charge radii of krypton (a), strontium (b), tellurium (c) and xenon (d) isotopes are obtained by the RMF(BCS) method and the modified rms charge radii formula RMF(BCS)*. For krypton (a) and strontium (b) isotopes, the charge radii of these two isotopic chains shrink systematically with increasing neutron number until $N = 50$. The rapid increasing trends are shown across $N = 50$, but the slope of changes of strontium isotopic
chain is larger than krypton’s. The similar rapid increasing of charge radii across \( N = 50 \) is studied earlier in RMF within NL-SH parameterization set \[14\]. These nuclei with \( N = 50 \) closed shells are more difficult to be excited than their neighbors, which is evidenced by their relatively stable properties \[10, 16, 77\]. This suggests that the emergency of simple and regular patterns of rapidly increasing of charge radii across \( N = 50 \) are common features observed in self-bound many-nucleon systems \[14\].

For \( ^{75}\text{Kr} \), there exists a large deviation between experimental data and the calculated result. The distinctive aspect that the shape deformation has an influence on the charge radius should be considered carefully \[28, 50\]. The experimental data indicates the possible quadrupole deformation parameter \( \beta_{20} \approx 0.27 \) for nucleus \( ^{75}\text{Kr} \) \[78\]. However, in our calculation, \( \beta_{20} \) is around 0.48 for this nucleus. Along Sr isotopic chain, the charge radii beyond the neutron number \( N = 60 \) appear to increase with surprisingly leaping slope. As argued in Ref. \[79\], some particular isotopes, as in the region around \( Z = 40, N = 60 \) and \( Z = 62, N = 90 \), are considered to present a rapid onset of deformation. Actually, for \( ^{98-100}\text{Sr} \), the calculated quadrupole deformation parameters \( \beta_{20} \) are almost 0.45. For \( ^{81-88}\text{Sr} \) isotopes, the rms charge radii formula with correction term can reproduce experimental data well. But the rms charge radii of some proton-rich nuclei are slightly overestimated by the modified formula.

Remarkably, the emergency of simple and regular patterns of rapidly increasing of charge radii across \( N = 82 \) are common features observed in various different isotopic chains \[14, 16\]. As shown in Fig. 3 (c) and (d), this feature is also observed for tellurium and xenon isotopic chains. However, these two methods almost give similar results with the increasing neutron number. In order to further elaborate the discontinuity aspects of nuclear charge radii across \( N = 82 \) neutron shell, in Fig. 3 the calculated results for barium (a), cerium (b), neodymium (c) and samarium (d) isotopic chains are also shown. We can find both of two methods almost give similar trend with the increase of neutron number, especially the shrinking phenomena approaches \( N = 82 \) regions. This discontinuity aspect is presented evidently owing to the relatively stable properties of closed-shell nuclei \[23, 24\]. This has been attributed to the rather small isospin dependence of spin-orbit term in RMF model \[80\].

The parabolic-like shape of nuclear charge radii are observed between two strong closure shells \[14, 16\]. This distinct feature is evidently observed between \( N = 20 \) and \( N = 28 \) along calcium isotopic chain \[14, 13\]. In addition, this peculiar phenomenon can also be found
FIG. 3. Charge radii of barium (a), cerium (b), neodymium (c) and samarium (d) isotopes are obtained by the RMF(BCS) method (dashed line) and the modified rms charge radii expression RMF(BCS)* (open diamond). The experimental data are taken from Ref. [14] (filled circle).

dramatically in latest study of cadmium [23] and tin [24] isotopic chains. In remarkable contrast to calcium isotopes, the amplitude of parabolic-like shape of nuclear charge radii has been reduced. As shown in Ref. [52], the convex behavior of nuclear charge radii between two strong closure shells can be well reproduced by the modified formula. Therefore, we may infer that the parabolic-like shape of nuclear charge radii can also be observed between $N = 82$ and $N = 126$ closed shells, but with smaller amplitude. Thus more reliable experimental data are urgently needed.

C. OES behaviors in nuclear charge radii

As mentioned above, the OES effects of nuclear charge radii are generally observed throughout the nuclear chart [14]. The possible mechanisms have been proposed, such as blocking of ground state quadrupole vibrations by the odd neutron [81] and core polarization by valence neutrons [16, 47]. Meanwhile, phenomenological four-particle correlations
FIG. 4. Odd-even staggering effects of charge radii along krypton (a), strontium (b), tellurium (c) and xenon (d) isotopic chains are shown. The experimental data are taken from Ref. 14.

or $\alpha$-particle clustering are also supposed to produce the OES of nuclear charge radii [55]. Another theoretical approach which includes three- or four-body part in an effective residual interaction is also introduced to reproduce normal OES of nuclear charge radius well [82, 83]. In addition, the special deformation effects also lead to the large staggering, especially in very neutron deficient mercury and gold isotopes [84, 85]. In Ref. [86], it pointed out that the size of the neutron pairing energy had an influence on the large OES of the mercury nuclear charge radii near the $N = 104$ mid-shell region, and the shape coexistence is also observed.

In order to emphasize these phenomena, the three-point formula has been employed to extract the charge radii staggering effects along isotopic chains as follows [51]:

$$
\Delta r = \frac{1}{2} [R(N - 1, Z) - 2R(N, Z) + R(N + 1, Z)],
$$

where $R(N, Z)$ is rms charge radius. In Fig. 4, odd-even staggering effects of charge radii along krypton (a), strontium (b), tellurium (c) and xenon (d) isotopic chains are shown with and without the modified term. As shown in this figure, we can find that both of these two
methods can reproduce the OES staggering effect. But for $^{85,86,90,91}$Sr and $^{128,129,132,133}$Xe, the OES behaviors with RMF(BCS) approach can not follow the trend of experimental data with respect to RMF(BCS)$^*$ method. Meanwhile, the modified expression slightly overestimates the OES charge radii staggering, especially for tellurium isotopes. As demonstrated in Ref. [52], one can properly describe the OES of charge radii by adjusting the parameter $a_0$ in Eq. (4). In order to keep the global description, we will not further perform a fine turning in this work.

![Graph](image)

FIG. 5. Odd-even staggering effects of charge radii along barium (a), cerium (b), neodymium (c) and samarium (d) isotopic chains are shown. The experimental data are taken from Ref. [14].

In Fig. 5, the OES effects of charge radii along barium (a), cerium (b), neodymium (c) and samarium (d) isotopic chains are also shown. As encountered in tellurium isotopes, the OES behaviors are also slightly overestimated in barium and neodymium isotopic chains. In cerium isotopic chain, the experimental results are not shown due to the absence of odd-mass nuclei data [14]. However, both of these two approaches give almost similar trend with the increase of neutron number except $^{142−146}$Ce isotopes.

The values of three-point OES formula emphasizes the flattening of the isotopic depen-
vidence of the charge radii along isotopic chains [21]. In Fig. 4 (a) and (b), OES of charge radii along krypton \((Z = 36)\) and strontium \((Z = 38)\) isotopic chain can not strictly follow the general oscillation trend at \(N = 50\). We definitely label this weakening behavior as “abnormal staggering effects”. Actually, these phenomena are also observed at neutron magic number \(N = 28\) (Ca), 50 (Sr, Y, Zr), 126 (Pb), etc [14]. As shown in Figs. 1 and 4, similar case is encountered at \(N = 82\) closed-shell. This maybe provide a signature to identify the evidences of shell closure effects along isotopic chains in nuclear chart. Therefore, more available experimental data are needed to verify these arguments.

IV. SUMMARY AND OUTLOOK

The emergency of rapidly increasing of charge radii are commonly observed features across \(N = 50\) and 82 neutron-rich closed shells throughout nuclear chart [14]. The latest studies further demonstrate these discontinuity phenomena along cadmium and tin isotopic chains [23, 24]. In this work, the modified formula is employed to study charge radii along rich-data even-proton isotopic chains, such as krypton \((Z = 36)\), strontium \((Z = 38)\), tellurium \((Z = 52)\), xenon \((Z = 54)\), barium \((Z = 56)\), cerium \((Z = 58)\), neodymium \((Z = 60)\) and samarium \((Z = 62)\) elements. Our results can reproduce the shrink phenomena and abrupt increasing of charge radii around \(N = 50\) and 82 neutron magic number. The similar scenario is encountered at \(N = 28\) and 126 [52]. From calculated results, we can find both of these methods present similar results or modified formula shows slight improvement. This means that the np pairs correction has almost no influence on charge radii for these nuclei with open proton shells and this is consistent with Ref. [54].

Our results can reproduce the OES behaviors of charge radii, but this trend is overestimated at neutron magic number. Based on \(\Delta r\)’s definition, this seemingly corresponds to the inverse OES of charge radii, namely anomalous OES behavior. As shown in Ref. [52], the weakening of OES behaviors of charge radii are evidently found at \(N = 28\) and 126 closed-shells. Actually, this debilitating tendency can be observed naturally at neutron magic number [14]. We propose this commonly observed signature as an terrestrial probe to capture the magicity properties throughout nuclear chart. We should mention that many possible mechanisms are proposed to explain the fine structure of nuclei size [26, 16, 15, 54], especially those about the unpaired nucleons near magicity numbers. It is still an open ques-
tion to include the short-range correlations self-consistently in density functional theory.

The atomic nucleus is formed by two different kind of Fermions (protons and neutrons) that interact mainly by the electromagnetic and strong forces. It is pointed out that new data in neutron-rich nuclei all exhibit an intriguingly simple increase in charge radii across closed-shells \[1\]. Moreover, the electromagnetic properties of isotopes around magic numbers of protons and neutrons have been found to exhibit astonishingly simple trends. As demonstrated in Ref. \[54\], the np-SRCs which originate from short-range neutron-proton tensor interaction will cause protons to move far away from the center of nucleus. For open shell nuclei, the quantity of Cooper pairs components come from protons and neutrons are roughly comparable due to the strong coupling between different levels around Fermi surface. That is why both of these two approaches give almost similar results. Therefore, this makes a complement for our new ansatz as a unified theoretical model to describe the nuclear size quantitatively throughout nuclear chart.

V. ACKNOWLEDGEMENTS

This work is supported by the Reform and Development Project of Beijing Academy of Science and Technology under Grant No. 13001-2110. This work is also supported in part by the National Natural Science Foundation of China under Grants No. 11705118, No. 11975096, No. 11635003, No. 11025524, No. 11161130520, the National Basic Research Program of China under Grant No. 2010CB832903.

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