Does matter differ from vacuum?

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Abstract
A structured collection of thought provoking conclusions about space and time is given. Using only the Compton wavelength $\lambda = \hbar/mc$ and the Schwarzschild radius $r_s = 2Gm/c^2$, it is argued that neither the continuity of space-time nor the concepts of space-point, instant, or point particle have experimental backing at high energies. It is then deduced that Lorentz, gauge, and discrete symmetries are not precisely fulfilled in nature. In the same way, using a new and simple Gedankenexperiment, it is found that at Planck energies, vacuum is fundamentally indistinguishable from radiation and from matter. Some consequences for supersymmetry, duality, and unification are presented.

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The limits of textbook physics

Matter and space-time are the fundamental entities of our description of nature. Why do particles have the masses they have? The quantum theory of the electromagnetic, weak and strong nuclear forces cannot answer the question. Why does space-time have three plus one dimensions? The generally accepted theory of the structure of space-time, general relativity, cannot explain this simple observation (and in fact does not even allow to ask this question). One concludes that the description of nature by these theories is still incomplete.

The search for a more complete, unified description of nature is also driven by another, even more compelling motivation: quantum mechanics and general relativity contradict each other; they describe the same phenomena in incompatible ways. This well-known fact surfaces in many different ways.\[1\]

There is a simple, but not well known way to state the origin of the contradiction between general relativity and quantum mechanics. Both theories describe motion using objects, made of particles, and space-time. Let us see how these two concepts are defined.

A particle – and in general any object – is defined as a conserved, localisable entity which can move. (The etymology of the term ‘object’ is connected to this fact.) In other words, a particle is a small entity with conserved mass, charge etc., which can vary its position with time.

At the same time, in every physics text time is defined with the help of moving objects, usually called ‘clocks’, or with the help of moving particles, such as those emitted by light sources. Similarly, also the length unit is defined with objects, be it a oldfashioned ruler, or nowadays with help of the motion of light, which is a collection of moving particles as well.

The rest of physics has sharpened the definitions of particles and space-time. In quantum mechanics one assumes space-time given (it is included as a symmetry of the Hamiltonian), and one studies in all detail the properties and the motion of particles from it, both for matter and radiation. In general relativity and especially in cosmology, the opposite path is taken: one assumes that the properties of matter and radiation are given, e.g. via their equations of state, and one describes in detail the space-time that follows from them, in particular its curvature. But one fact remains unchanged throughout all these advances in standard textbook physics: the two concepts of particles and of space-time are defined with the help of each other. To avoid the contradiction between quantum mechanics and general relativity and to try to eliminate their joint incompleteness requires eliminating this circular definition. As argued in the following, this necessitates a radical change in our description of nature, and in particular about the continuity of space-time.

For a long time, the contradictions in the two descriptions of nature were avoided by keeping them separate: one often hears the statement that quantum mechanics is valid at small dimensions, and the other, general relativity, is valid at large dimensions. But this artificial separation is not justified and obviously prevents the solution of the problem. The situation resembles the well-known drawing by M.C. Escher, where two hands, each holding a pencil, seem to draw each other. If one takes one hand as a symbol for space-time, the other as a symbol for particles, and the act of drawing as a
symbol for the act of defining, one has a description of standard, present day physics. The apparent contradiction is solved when one recognizes that both concepts (both hands) result from a hidden third concept (a third hand) from which the other two originate. In the picture, this third entity is the hand of the painter.

In the case of space-time and matter, the search for this underlying concept is presently making renewed progress. The required conceptual changes are so dramatic that they should be of interest to anybody who has an interest in physics. Some of the issues are presented here. The most effective way to study these changes is to focus in detail on that domain where the contradiction between the two standard theories becomes most dramatic, and where both theories are necessary at the same time. That domain is given by the following well-known argument.

2 Planck scales

Both general relativity and quantum mechanics are successful theories for the description of nature. Each of them provides a criterion to determine when Galilean physics is not applicable any more. (In the following, we use the terms ‘vacuum’ and ‘empty space-time’ interchangeably.)

General relativity shows that it is necessary to take into account the curvature of space-time whenever one approaches an object of mass \( m \) to distances of the order of the Schwarzschild radius \( r_S \), given by

\[
    r_S = \frac{2Gm}{c^2}.
\]  

(1)

Approaching the Schwarzschild radius of an object, the difference between general relativity and the classical \( 1/r^2 \) description of gravity becomes larger and larger. For example, the barely measurable gravitational deflection of light by the sun is due to an approach to \( 2.4 \times 10^5 \) times the Schwarzschild radius of the sun. In general however, one is forced to stay away from objects an even larger multiple of the Schwarzschild radius, except in the vicinity of neutron stars, as shown in table 1; for this reason, general relativity is not necessary in everyday life. (An object smaller than its own Schwarzschild radius is called a black hole. Following general relativity, no signals from the inside of the Schwarzschild radius can reach the outside world; hence the name ‘black hole’.)

Similarly, quantum mechanics shows that Galilean physics must be abandoned and quantum effects must be taken into account whenever one approaches an object at distances of the order of the Compton wavelength \( \lambda_C \), which is given by

\[
    \lambda_C = \frac{\hbar}{mc}.
\]  

(2)

Of course, this length only plays a role if the object itself is smaller than its own Compton wavelength. At these dimensions one observes relativistic quantum effects, such as particle-antiparticle creation and annihilation. Table 1 shows that the ratio \( d/\lambda_C \) is near or smaller than 1 only in the microscopic world, so that such effects are not observed in everyday life; therefore one does not need quantum field theory to describe common observations.
Taking these two results together, the situations which require the combined concepts of quantum field theory and of general relativity are those in which both conditions are satisfied simultaneously. The necessary approach distance is calculated by setting \( r_S = 2\lambda_C \) (the factor 2 is introduced for simplicity). One finds that this is the case when lengths or times are of the order of

\[
\begin{align*}
  l_{Pl} &= \sqrt{\frac{\hbar G}{c^3}} = 1.6 \cdot 10^{-35} \text{ m}, \text{ the Planck length}, \\
  t_{Pl} &= \sqrt{\frac{\hbar G}{c^5}} = 5.4 \cdot 10^{-44} \text{ s}, \text{ the Planck time}. 
\end{align*}
\]

(3)

Whenever one approaches objects at these dimensions, general relativity and quantum mechanics both play a role; at these scales effects of quantum gravity appear. The values of the Planck dimensions being extremely small, this level of sophistication is not necessary in everyday life, neither in astronomy nor in present day microphysics.

However, this sophistication is necessary to understand why the universe is the way it is. The questions mentioned at the beginning – why do we live in three dimensions, why is the proton 1834 times heavier than the electron – need for their answer a precise and complete description of nature. The contradictions between quantum mechanics and general relativity make the search for these answers impossible. On the other hand, the unified theory, describing quantum gravity, is not yet finished; but a few glimpses on its implications can already taken at the present stage.

Note that the Planck scales are also the only domain of nature where quantum mechanics and general relativity come together; therefore they provide the only possible starting point for the following discussion. Planck [13] was interested in the Planck units mainly as natural units of measurement, and that is the way he called them. However, their importance in nature is much more pervasive, as will be seen now.

### 3 Farewell to instants of time

The difficulties arising at Planck dimensions appear when one investigates the properties of clocks and meter bars. Is it possible to construct a clock which is able to measure time intervals shorter than the Planck time? Surprisingly, the answer is no [14, 15], even though in the relation \( \Delta E \Delta t \geq \hbar \) it seems that by making \( \Delta E \) arbitrary large, one can make \( \Delta t \) arbitrary small.

Any clock is a device with some moving parts; such parts can be mechanical wheels, matter particles in motion, changing electrodynamic fields – e.g. flying photons –, decaying radioactive particles, etc. For each moving component of a clock, e.g. the two hands of the dial, the uncertainty principle applies. As has been discussed in many occasions [16, 17] and most clearly by Raymer [18], the uncertainty relation for two non-commuting variables describes two different, but related situations: it makes a statement about standard deviations of separate measurements on many identical systems, and it describes the measurement precision for a joint measurement on a single system. Throughout this article, only the second viewpoint is used.

Now, in any clock, one needs to know both the time and the energy of each hand, since otherwise it would not be a classical system, i.e. it would not be a recording device. One therefore needs the joint knowledge of non-commuting variables for each moving component of the clock; we are interested in the component with the largest time uncertainty, i.e. imprecision of time measurement \( \Delta t \). It is evident that the
smallest time interval $\delta t$ which can be measured by a clock is always larger than the
time imprecision $\Delta t$ due to the uncertainty relation for its moving components. Thus
one has

$$\delta t \geq \Delta t \geq \frac{\hbar}{\Delta E} \quad (4)$$

where $\Delta E$ is the energy uncertainty of the moving component. This energy uncertainty $\Delta E$ is surely smaller than the total energy $E = mc^2$ of the component itself. (Physically, this condition means that one is sure that there is only one clock; the case $\Delta E > E$ would mean that it is impossible to distinguish between a single clock, or a
clock plus a pair of clock-anticlock, or a component plus two such pairs, etc.) Furthermore, any clock provides information; therefore, signals have to be able to leave it. To
make this possible, the clock may not be a black hole; its mass $m$ must therefore be
smaller than the Schwarzschild mass for its size, i.e. $m \leq c^2 l/G$, where $l$ is the size of
the clock (here and in the following we neglect factors of order unity). Finally, the size $l$ of the clock must be smaller than $c \delta t$ itself, to allow a sensible measurement of the
time interval $\delta t$: one has $l \leq c \delta t$. (It is amusing to study how a clock larger than $c \delta t$
stops being efficient, due to the loss of rigidity of its components.) Putting all these
conditions together one after the other, one gets

$$\delta t \geq \frac{\hbar G}{c^5 \delta t},$$
or

$$\delta t \geq \sqrt{\frac{\hbar G}{c^5}} = t_{Pl} \quad (5)$$

In summary, from three simple properties of every clock, namely that one is sure to
have only one of it, that one can read its dial, and that it gives sensible readouts, one
gets the general conclusion that clocks cannot measure time intervals shorter than the
Planck time.

Note that this argument is independent of the nature of the clock mechanism.
Whether the clock is powered by gravitational, electrical, plain mechanical or even
nuclear means, the relations still apply. Note also that gravitation is essential in this
argument. A well-known study on the limitations of clocks due to their mass and their
measuring time has been published by Salecker and Wigner [19] and summarized in
pedagogical form by Zimmerman [20]; the present argument differs in that it includes
both quantum mechanics as well as gravity, and therefore yields a different, lower, and
much more fundamental limit.

The same result can also be found in other ways. [21] For example, any clock small
enough to measure small time intervals necessarily has a certain energy spread, as
described by quantum mechanics. Following general relativity, any energy density
induces a deformation of space-time, and signals from that region arrive with a certain
delay. The energy uncertainty of the source leads to a uncertainty in the delay. The
expression from general relativity [11] for the deformation of the time part of the line
element due to a mass $m$ is $\delta t = mG/lc^3$. If the mass varies by $\Delta E/c^2$, the resulting
uncertainty $\Delta t$ in the delay, the external accuracy of the clock, is

$$\Delta t = \frac{\Delta E G}{l c^3}. \quad (6)$$
Now the energy uncertainty of the clock is bounded by the uncertainty relation for time and energy, given the internal time accuracy of the clock. This internal accuracy must be smaller or equal than the external one. Putting this all together, one again finds the relation $\delta t \leq t_{Pl}$. One is forced to conclude that in nature there is a minimum time interval. In other words, at Planck scales the term “instant of time” has no theoretical nor experimental backing. It therefore makes no sense to use it.

4 Farewell to points in space

In a similar way one can deduce that it is not possible to make a meter bar or any other length measuring device that can measure lengths shorter than the Planck length, as one can find already from $l_{Pl} = c t_{Pl}$. The straightforward way to measure the distance between two points is to put an object at rest at each position. In other words, joint measurements of position and momentum are necessary for every length measurement. Now the minimal length $\delta l$ that can be measured is surely larger than the position uncertainty of the two objects. From the uncertainty principle it is known that each object’s position cannot be determined with a precision $\Delta l$ smaller than that given by $\Delta l \Delta p = \hbar$, where $\Delta p$ is the momentum uncertainty. Requiring to have only one object at each end means $\Delta p < mc$, which gives

$$\delta l \geq \Delta l \geq \frac{\hbar}{mc}. \quad (7)$$

Furthermore, the measurement cannot be performed if signals cannot leave the object; they may not be black holes. Their masses must therefore be so small that their Schwarzschild radius $r_S = \frac{2Gm}{c^2}$ is smaller than the distance $\delta l$ separating them. Dropping again the factor of 2, one gets

$$\delta l \geq \sqrt{\frac{\hbar G}{c^3}} = l_{Pl}. \quad (8)$$

Another way to deduce this limit reverses the role of general relativity and quantum theory. To measure the distance between two objects, one has to localize the first object with respect to the other within a certain interval $\Delta x$. This object thus possesses a momentum uncertainty $\Delta p \geq \hbar/\Delta x$ and therefore possesses an energy uncertainty $\Delta E = c(\Delta x)^2 + (\Delta p)^2)^{1/2} \geq \hbar c/\Delta x$. But general relativity shows that a small volume filled with energy changes the curvature, and thus the metric of the surrounding space.\cite{2, 11} For the resulting distance change $\Delta l$, compared to empty space, one finds\cite{22, 24, 25, 26, 27} the expression $\Delta l \approx G\Delta E/c^4$. In short, if one localizes a first particle in space with a precision $\Delta x$, the distance to a second particle is known only with precision $\Delta l$. The minimum length $\delta l$ that can be measured is obviously larger than each of the quantities; inserting the expression for $\Delta E$, one finds again that the minimum measurable length $\delta l$ is given by the Planck length.

As a remark, the Planck length being the shortest possible length, it follows that there can be no observations of quantum mechanical effects for situations in which the corresponding de Broglie or Compton wavelength would be even smaller. This is one of the reasons why in everyday, macroscopic situations, e.g. in car-car collisions, one
never observes quantum interference effects, in opposition to the case of proton-proton collisions.

In summary, from two simple properties common to all length measuring devices, namely that they can be counted and that they can be read out, one arrives at the conclusion that lengths smaller than the Planck length cannot be found in measurements. Whatever the method used, whether lengths are measured with a meter bar or by measuring time of flight of particles between the end points: one cannot overcome this fundamental limit. It follows that in its usual sense as entity without size, the concept of “point in space” has no experimental backing. In the same way, the term “event”, being a combination of “point in space” and “instant of time”, also loses its meaningfulness for the description of nature.

These results are often summarized in the so-called generalized uncertainty principle

\[ \Delta p \Delta x \geq \frac{\hbar}{2} + f \frac{G}{c^3} (\Delta p)^2 \],

(9)

where \( f \) is a numerical factor around unity. A similar expression holds for the time-energy uncertainty relation. The first term on the right hand side is the usual quantum mechanical one. The second term, negligible at everyday life energies, plays a role only near Planck energies. It is due to the changes in space-time induced by gravity at these high energies. One notes that the generalized principle automatically implies that \( \Delta x \) can never be smaller than \( f^{1/2} l_{Pl} \).

The generalized uncertainty principle is derived in exactly the same way in which Heisenberg derived the original uncertainty principle \( \Delta p \Delta x \geq \hbar/2 \) for an object: by using the deflection of light by the object under a microscope. A careful re-derivation of the process, not disregarding gravity, yields equation (9). [22]

For this reason all approaches which try to unify quantum mechanics and gravity must yield this relation; indeed it appears in canonical quantum gravity [28], in superstring theory [29], and in the quantum group approach [30], sometimes with an additional term proportional to \( (\Delta x)^2 \) on the right hand side of equation (9). [31]

Quantum mechanics starts when one realizes that the classical concept of action makes no sense below the value of \( \hbar \); similarly, unified theories such as quantum gravity start when one realizes that the classical concepts of time and length make no sense near Planck values. However, the usual description of space-time does contain such small values; it claims the existence of intervals smaller than the smallest measurable one. Therefore the continuum description of space-time has to be abolished in favor of a more appropriate one.

This is clearly expressed in a new uncertainty relation appearing at Planck scales. [36] Inserting \( c \Delta p \geq \Delta E \geq \hbar/\Delta t \) into equation (1) one gets

\[ \Delta x \Delta t \geq \hbar G/c^4 = t_{Pl} l_{Pl} \],

(10)

which of course has no counterpart in standard quantum mechanics. A final way to convince one-self that points have no meaning is that a point is an entity with vanishing volume; however, the minimum volume possible in nature is the Planck volume \( V_{Pl} = l_{Pl}^3 \).

Space-time points are idealizations of events. But this idealization is incorrect. Using the concept of “point” is equivalent to the use of the concept of “ether” a century ago:
one cannot measure it, and it is useful to describe observations only until one has found
the way to describe them without it.

5 Farewell to the space-time manifold

But the consequences of the Planck limits for time and space measurements can be
taken much further. To put the previous results in a different way, points in space and
time have size, namely the Planck size. It is commonplace to say that given any two
points in space or two instants of time, there is always a third in between. Physicists
sloppily call this property continuity, mathematicians call it denseness. However, at
Planck dimensions, this property cannot hold since intervals smaller than the Planck
time can never be found: thus points and instants are not dense, and between two points
there is not always a third. But this means that space and time are not continuous.
Of course, at large scales they are – approximately – continuous, in the same way that
a piece of rubber or a liquid seems continuous at everyday dimensions, but is not at
small scales. This means that to avoid Zeno’s paradoxes resulting from the infinite
divisibility of space and time one does not need any more the system of differential
calculus; one can now directly dismiss the paradoxes because of wrong premises on the
nature of space and time.

But let us go on. Special relativity, quantum mechanics and general relativity all
rely on the idea that time can be defined for all points of a given reference frame.
However, two clocks at a distance \( l \) cannot be synchronized with arbitrary precision.
Since the distance between two clocks cannot be measured with an error smaller than
the Planck length \( l_{\text{Pl}} \), and transmission of signals is necessary for synchronization, it is
not possible to synchronize two clocks with a better precision than the time \( l_{\text{Pl}}/c = t_{\text{Pl}} \),
the Planck time. Due to this impossibility to synchronize clocks precisely, the idea of
a single time coordinate for a whole reference frame is only approximate, and cannot
be maintained in a precise description of nature.

Moreover, since the difference between events cannot be measured with a precision
better than a Planck time, for two events distant in time by this order of magnitude,
it is not possible to say which one precedes the other with hundred percent certainty.
This is an important result. If events cannot be ordered at Planck scales, the concept
of time, which is introduced in physics to describe sequences, cannot be defined at all.
In other words, after dropping the idea of a common time coordinate for a complete
frame of reference, one is forced to drop the idea of time at a single “point” as well.
For example, the concept of ‘proper time’ loses its sense at Planck scales.

In the case of space, it is straightforward to use the same arguments to show that
length measurements do not allow us to speak of continuous space, but only about
approximately continuous space. Due to the lack of measurement precision at Planck
scales, the concept of spatial order, of translation invariance and isotropy of the vac-
uum, and of global coordinate systems lack experimental backing at those dimensions.

But there is more to come. The very existence of a minimum length contradicts
special relativity, where it is shown that whenever one changes to a moving coordinate
system a given length undergoes a Lorentz contraction. A minimum length cannot
exist in special relativity; therefore, at Planck dimensions, space-time is neither Lorentz
invariant, nor diffeomorphism invariant, nor dilatation invariant. All the symmetries
at the basis of special and general relativity are thus only approximately valid at Planck scales.

Due to the imprecision of measurement, most familiar concepts used to describe spatial relations become useless. For example, the concept of metric also loses its usefulness at Planck scales. Since distances cannot be measured with precision, the metric cannot be determined. One deduces that it is impossible to say precisely whether space is flat or curved. In other words, the impossibility to measure lengths exactly is equivalent to fluctuations of the curvature. ([22, 32]

In addition, even the number of space dimensions makes no sense at Planck scales. Let us remind ourselves how one determines this number experimentally. One possible way is to determine how many points one can choose in space such that all their distances are equal. If one can find at most \( n \) such points, the space has \( n - 1 \) dimensions. One recognizes directly that without reliable length measurements there is no way to determine reliably the number of dimensions of a space at Planck scales with this method.

Another simple way to check for three dimensions is to make a knot in a shoe string and glue the ends together; if it stays knotted under all possible deformations, the space has three dimensions. If it can be unknotted, space has more than three dimensions, since in such spaces knots do not exist. Obviously, at Planck dimensions the measurement errors do not allow to say whether a string is knotted or not, because at crossings one cannot say which strand lies above the other.

There are many other methods to determine the dimensionality of space. ([33] All these methods use the fact that the concept of dimensionality is based on a precise definition of the concept of neighborhood. But at Planck scales, as just mentioned, length measurements do not allow us to say with certainty whether a given point is inside or outside a given volume. In short, whatever method one uses, the lack of reliable length measurements means that at Planck scales, the dimensionality of physical space is not defined. It should therefore not come as a surprise that when approaching those scales, one could get a scale-dependent answer, different from three.

The reason for the troubles with space-time become most evident when one remembers the well-known definition by Euclid: ([34] “A point is that which has no part.” As Euclid clearly understood, a physical point, and here the stress is on physical, cannot be defined without some measurement method. A physical point is an idealization of position, and as such includes measurement right from the start. In mathematics however, Euclid’s definition is rejected, because mathematical points do not need metrics for their definition. Mathematical points are elements of a set, usually called a space. In mathematics, a measurable or a metric space is a set of points equipped in addition with a measure or a metric. Mathematical points do not need a metric for their definition; they are basic entities. In contrast to the mathematical situation, the case of physical space-time, the concepts of measure and of metric are more fundamental than that of a point. The difficulty of distinguishing physical and mathematical space arises from the failure to distinguish a mathematical metric from a physical length measurement.

Perhaps the most beautiful way to make this point clear is the Banach-Tarski theorem or paradox, which shows the limits of the concept of volume. ([35] This theorem states that a sphere made of mathematical points can be cut into six pieces in such
a way that two sets of three pieces can be put together and form two spheres, each of the same volume as the original one. However, the necessary cuts are “infinitely” curved and thin. For physical matter such as gold, unfortunately – or fortunately – the existence of a minimum length, namely the atomic distance, makes it impossible to perform such a cut. For vacuum, the puzzle reappears: for example, the energy of its zero-point fluctuations is given by a density times the volume; following the Banach-Tarski theorem, since the concept of volume is ill defined, the total energy is so as well. However, this problem is solved by the Planck length, which provides a fundamental length scale also for the vacuum.

In summary, physical space-time cannot be a set of mathematical points. But the surprises are not finished. At Planck dimensions, since both temporal order and spatial order break down, there is no way to say if the distance between two near enough space-time regions is space-like or time-like. One cannot distinguish the two cases. At Planck scales, time and space cannot be distinguished from each other. In summary, space-time at Planck scales is not continuous, not ordered, not endowed with a metric, not four-dimensional, not made of points. If we compare this with the definition of the term manifold, (a manifold is what locally looks like an euclidean space) not one of its defining properties is fulfilled. We arrive at the conclusion that the concept of a space-time manifold has no backing at Planck scales. Even though both general relativity and quantum mechanics use continuous space-time, the combination of both theories does not. This is one reason why the idea is so slow to disappear.

6 Farewell to observables and measurements

To complete this state of affairs, if space and time are not continuous, all quantities defined as derivatives versus space or time are not defined precisely. Velocity, acceleration, momentum, energy, etc., are only defined in the classical approximation of continuous space and time. Concepts such as ‘derivative’, ‘divergence-free’, ‘source free’, etc., lose their meaning at Planck scales. Even the important tool of the evolution equation, based on derivatives, such as the Schrödinger or the Dirac equation, cannot be used any more.

In fact, all physical observables are defined using at least length and time measurements, as is evident from any list of physical units. Any such table shows that all physical units are products of powers of length, time (and mass) units. (Even though in the SI system electrical quantities have a separate base quantity, the ampere, the argument still holds; the ampere is itself defined by measuring a force, which is measured using the three base units length, time, and mass.) Now, since time and length are not continuous, observables themselves are not continuously varying. This means that at Planck scales, observables (or their components in a basis) are not to be described by real numbers with – potentially – infinite precision. Similarly, if time and space are not continuous, the usual expression for an observable quantity $A$, namely $A(x,t)$, does not make sense: one has to find a more appropriate description. Physical fields cannot be described by continuous functions at Planck scales.

In quantum mechanics this means that it makes no sense to define multiplication of observables by real numbers. Among others, observables do not form a linear algebra. (One recognizes directly that due to measurement errors, one cannot prove that ob-
servables do form such an algebra.) This means that observables are not described by operators at Planck scales. Moreover, the most important observables are the gauge potentials. Since they do not form an algebra, gauge symmetry is not valid at Planck scales. Even innocuous looking expressions such as \( [x_i, x_j] = 0 \) for \( x_i \neq x_j \), which are at the basis of quantum field theory, become meaningless at Planck scales. Even worse, also the superposition principle cannot be backed up by experiment at those scales.

Similarly, permutation symmetry is based on the premise that one can distinguish two points by their coordinates, and then exchange particles at those two locations. As just seen, this is not possible if the distance between the two particles is small; one concludes that permutation symmetry has no experimental backing at Planck scales. Even discrete symmetries, like charge conjugation, space inversion, and time reversal cannot be correct in that domain, because there is no way to verify them exactly by measurement. CPT symmetry is not valid at Planck scales.

All these results are consistent: if there are no symmetries at Planck scales, there also are no observables, since physical observables are representations of symmetry groups. In fact, the limits on time and length measurements imply that the concept of measurement has no significance at Planck scales.

7 Can space-time be a lattice? Can it be dual?

Discretizations of space-time have been studied already fifty years ago. The idea that space-time is described as a lattice has also been studied in detail, for example by Finkelstein and by ’t Hooft. It is generally agreed that in order to get an isotropic and homogeneous situation for large, everyday scales, the lattice cannot be periodic, but must be random. Moreover any fixed lattice violates the result that there are no lengths smaller than the Planck length: due to the Lorentz contraction, any moving observer would find lattice distances smaller than the Planck value.

If space-time is not a set of points or events, it must be a set of something else. Three hints already appear at this stage. The first necessary step to improve the description of motion starts with the recognition that to abandon “points” means to abandon the local description of physics. Both quantum mechanics and general relativity assume that the phrase ‘observable at a point’ had a precise meaning. Due to the impossibility of describing space as a manifold, this expression is no longer useful. The unification of general relativity and quantum physics forces a non-local description of nature at Planck scales.

The existence of a minimal length implies that there is no way to physically distinguish locations that are spaced by even smaller distances. One is tempted to conclude that therefore any pair of locations cannot be distinguished, even if they are one meter apart, since on any path joining two points, any two nearby locations cannot be distinguished. One notices that this situation is similar to the question on the size of a cloud or that of an atom. Measuring water density or electron density, one finds non-vanishing values at any distance from the center; however, an effective size can still be defined, because it is very improbable to see effects of a cloud’s or of an atom’s presence at distances much larger than this effective size. Similarly, one guesses that space-time points at macroscopic distances can be distinguished because the probability that they will be confused drops rapidly with increasing distance. In short, one is thus led to
a probabilistic description of space-time. It becomes a macroscopic observable, the statistical, or thermodynamic limit of some microscopic entities.

One notes that a fluctuating structure for space-time would also avoid the problems of fixed structures with Lorentz invariance. This property is of course compatible with a statistical description. In summary, the experimental observations of Lorentz invariance, isotropy, and homogeneity, together with that of a minimum distance, point towards a fluctuating description of space-time. In the meantime, research efforts in quantum gravity, superstring theory and in quantum groups have confirmed independently from each other that a probabilistic description of space-time, together with a non-local description of observables at Planck dimensions, indeed resolves the contradictions between general relativity and quantum theory. To clarify the issue, one has to turn to the concept of particle.

Before that, a few remarks on one of the most important topics in theoretical physics at present: duality. String theory is build around a new supposed symmetry space-time duality (and its generalizations), which states that in nature, physical systems of size $R$ are indistinguishable from those of size $l^2_{\text{Pl}}/R$. (In natural units this symmetry is often written $R \leftrightarrow 1/R$.) Since this relation implies that there is a symmetry between systems larger and smaller that the Planck length, it is in contrast with the result above that lengths smaller than the Planck length do not make sense. Physical space-time therefore is not dual; but since present string theory includes the assumption that intervals of any size can be used to describe nature, it needs the duality symmetry to filter out the thus introduced unphysical situations. Duality does this by explaining that any system which would have smaller dimensions than the Planck length is in fact a system larger than this length.

8 Farewell to particles

Apart from space and time, in every example of motion, there is some object involved. One of the important discoveries of the natural sciences was that all objects are made of small constituents, called elementary particles. Quantum theory shows that all composite, non-elementary objects have a simple property: they have a finite, non-vanishing size. This property allows us to determine whether a particle is elementary or not. If it behaves like a point particle, it is elementary. At present, only the leptons (electron, muon, tau and the neutrinos), the quarks, and the radiation quanta of the electromagnetic, the weak and the strong nuclear interaction (the photon, the W and Z bosons, the gluons) have been found to be elementary. A few more elementary particles are predicted by various refinements of the standard model. Protons, atoms, molecules, cheese, people, galaxies, etc., are all composite (see table 1). Elementary particles are characterized by their vanishing size, their spin, and their mass.

The size of an object, e.g. the one given in table 1, is defined as the length at which one observes differences from point-like behavior. This is the way in which, using alpha particle scattering, the radius of the atomic nucleus was determined for the first time in Rutherford’s experiment.

Speaking simply, the size $d$ of an object is determined by measuring the interference pattern in the scattering of a beam of probe particles. This is the way one determines sizes of objects when one looks at them, using scattered photons. In order to observe
such interference effects, the wavelength $\lambda$ of the probe must be smaller than the object size $d$ to be determined. The de Broglie wavelength of the probe is given by the mass and the relative velocity $v$ between the probe and the unknown object, i.e. one needs $d > \lambda = \hbar/(mv) \geq \hbar/(mc)$. On the other hand, in order to make a scattering experiment possible, the object must not be a black hole, since then it would simply swallow the infalling particle. This means that its mass $m$ must be smaller than that of a black hole of its size, i.e., from equation (1), that $m < dc^2/G$; together with the previous condition one gets
\[ d > \sqrt{\frac{\hbar G}{c^3}} = l_{Pl}. \] (11)

In other words, there is no way to observe that an object is smaller than the Planck length. There is thus no way in principle to deduce from observations that a particle is point-like. In fact, it makes no sense to use the term “point particle” at all. Of course the existence of a minimal length both for empty space and for objects, are related. If the term “point of space” is meaningless, then the term “point particle” is so as well. Note that as in the case of time, the lower limit on length results from the combination of quantum mechanics and general relativity. (Note also that the minimal size of a particle has nothing to do with the impossibility, quantum theory, to localize a particle to within better than its Compton wavelength.)

The size $d$ of any elementary particle, which following the conventional quantum field description is zero, is surely smaller than its own Compton wavelength $\hbar/(mc)$. Moreover, we have seen above that a particle’s size is always larger than the Planck length: $d > l_{Pl}$. Eliminating the size $d$ one gets a condition for the mass $m$ of any elementary particle, namely
\[ m < \frac{\hbar}{c l_{Pl}} = \frac{\hbar c}{G} = m_{Pl} = 2.2 \cdot 10^{-8} \text{ kg} = 1.2 \cdot 10^{19} \text{ GeV}/c^2 \] (12)

(This limit, the so-called Planck mass, corresponds roughly to the mass of a ten days old human embryo, or, equivalently that of a small flea.) In short, the mass of any elementary particle must be smaller than the Planck mass. This fact is already mentioned as “well-known” by Sakharov [42] who explains that these hypothetical particles are sometimes called ’maximons’. And indeed, the known elementary particles all have masses well below the Planck mass. (Actually, the question why their masses are so incredibly much smaller than the Planck mass is one of the main questions of high energy physics. But this is another story.)

There are many other ways to arrive at this mass limit. For example, in order to measure mass by scattering – and that is the only way for very small objects – the Compton wavelength of the scatterer must be larger than the Schwarzschild radius; otherwise the probe would be swallowed. Inserting the definition of the two quantities and neglecting the factor 2, one gets again the limit $m < m_{Pl}$. (In fact it is a general property of descriptions of nature that a minimum space-time interval leads to an upper limit for elementary particle masses. [43]) The importance of the Planck mass will become clear shortly.

Another property connected with the size of a particle is its electric dipole moment. The stand model of elementary particles gives as upper limit for the dipole moment of...
the electron $d_e$ a value of

$$|d_e| < 10^{-39} \text{ m e}$$

where $e$ is the charge of the electron. This value is ten thousand times smaller than $l_{Pl}$; using the fact that the Planck length is the smallest possible length, this implies either that charge can be distributed in space, or that estimate is wrong, or that the standard model is wrong, or several of these. Only future will tell.

Let us return to some other strange consequences for particles. In quantum field theory, the difference between a virtual and a real particle is that a real particle is on shell, i.e. it obeys $E^2 = m^2c^4 + p^2c^2$, whereas a virtual particle is off shell, i.e. $E^2 \neq m^2c^4 + p^2c^2$. Due to the fundamental limits of measurement precision, at Planck scales one cannot determine whether a particle is real or virtual.

But that is not all. Since antimatter can be described as matter moving backwards in time, and since the difference between backwards and forwards cannot be determined at Planck scales, matter and antimatter cannot be distinguished at Planck scales.

Particles are also characterized by their spin. Spin describes two properties of a particle: its behavior under rotations (and if the particle is charged, the behavior in magnetic fields) and its behavior under particle exchange. The wave function of spin 1 particles remain invariant under rotation of $2\pi$, whereas that of spin 1/2 particles changes sign. Similarly, the combined wave function of two spin 1 particles does not change sign under exchange of particles, whereas for two spin 1/2 particles it does.

One sees directly that both transformations are impossible to study at Planck scales. Given the limit on position measurements, the position of the axis of a rotation cannot be well defined, and rotations become impossible to distinguish from translations. Similarly, position imprecision makes the determination of precise separate positions for exchange experiments impossible. In short, spin cannot be defined at Planck scales, and fermions cannot distinguished from bosons, or, differently phrased, matter cannot be distinguished from radiation at Planck scales. One can thus easily imagine that supersymmetry, a unifying symmetry between bosons and fermions, somehow must become important at those dimensions. Let us now move to the main property of elementary particles.

### 9 Farewell to mass

The Planck mass divided by the Planck volume, i.e. the Planck density, is given by

$$\rho_{Pl} = \frac{\rho}{G^2\hbar} = 5.2 \cdot 10^{96} \text{ kg/m}^3$$

and is a useful concept in the following. If one wants to measure the (gravitational) mass $M$ enclosed in a sphere of size $R$ and thus (roughly) of volume $R^3$, one way to do this is to put a test particle in orbit around it at a distance $R$. The universal law of gravity then gives for the mass $M$ the expression $M = Rv^2/G$, where $v$ is the speed of the orbiting test particle. From $v < c$, one thus deduces that $M < c^2R/G$; using the fact that the minimum value for $R$ is the Planck distance, one gets (neglecting again factors of order unity) a limit for the mass density, namely

$$\rho < \rho_{Pl}.$$
In other words, the Planck density is the maximum possible value for mass density. In particular, in a volume of Planck dimensions, one cannot have a larger mass than the Planck mass.

Interesting things happen when one starts to determine the error $\Delta M$ of the mass measurement in a Planck volume. Let us return to the mass measurement by an orbiting probe. From the relation $GM = rv^2$ one follows $G\Delta M = v^2 \Delta r + 2vr \Delta v > 2vr \Delta v = 2GM \Delta v / v$. For the error $\Delta v$ in the velocity measurement one has the uncertainty relation $\Delta v \geq \hbar / (m \Delta r) + \hbar / (MR) \geq \hbar / (MR)$. Inserting this in the previous inequality, and forgetting again the factor of 2, one gets that the mass measurement error $\Delta M$ of a mass $M$ enclosed in a volume of size $R$ follows

$$\Delta M \geq \frac{\hbar}{cR}.$$  \hspace{1cm} (16)

Note that for everyday situations, this error is extremely small, and other errors, such as the technical limits of the balance, are much larger.

As a check, let us take another situation, and use relativistic expressions, in order to show that the result still holds. Imagine having a mass $M$ in a box of size $R$ and weighing the box. (It is supposed that either the box is massless, or that its mass is subtracted by the scale.) The mass error is given by $\Delta M = \Delta E / c^2$, where $\Delta E$ is due to the uncertainty in kinetic energy of the mass inside the box. Using the expression $E^2 = m^2 c^4 + p^2 c^2$ one gets that $\Delta M \geq \Delta p \geq \hbar / c$, which again reduces to equation (16).

For a box of Planck dimensions, the mass measurement error is given by the Planck mass. But from above we know that the mass that can be put inside such a box is itself not larger than the Planck mass. In other words, the mass measurement error is larger (or at best equal) to the mass contained in a box of Planck dimension. In other words, if one builds a balance with two boxes of Planck size, one empty and the other full, as shown in the figure, nature cannot decide which way the balance should hang! Note that even a repeated or a continuous measurement would not resolve the situation: the balance would then randomly change inclination.

The same surprising answer is found if instead of measuring the gravitational mass one measures the inertial mass. The inertial mass for a small object is determined by touching it, i.e., physically speaking, by performing a scattering experiment. To determine the inertial mass inside a region of size $R$, a probe must have a wavelength smaller that $R$, and thus a corresponding high energy. A high energy means that the probe also attracts the particle through gravity. (One thus finds the intermediate result that at Planck scales, inertial and gravitational mass cannot be distinguished. Also the balance experiment shown in the figure makes this point: at Planck scales, the two effects of mass are always inextricably linked.) In any scattering experiment, e.g. in a Compton-type experiment, the mass measurement is performed by measuring the wavelength of the probe before and after the scattering experiment. We know from above that there always is a minimal wavelength error given by the Planck length $\l_{PL}$. In order to determine the mass in a Planck volume, the probe has to have a wavelength of the Planck length. In other words, the mass error is as large as the Planck mass itself: $\Delta M \geq M_{Pl}$. This limit is thus a direct consequence of the limit on length and space measurements.

This result has an astonishing consequence. In these examples, the measurement error is independent of the mass of the scatterer, i.e. independent of whether one
starts with a situation in which there is a particle in the original volume, or if there is none. One thus finds that in a volume of Planck size, it is impossible to say if there is one particle or none when weighing it or probing it with a beam! In short, vacuum, i.e. empty space-time can not be distinguished from matter at Planck scales. Another, often used way to express this is to say that when a particle of Planck energy travels through space it will be scattered by the fluctuations of space-time itself, making it thus impossible to say whether it was scattered by empty space-time or by matter. These surprising results stem from the fact that whatever definition of mass one uses, it is always measured via length and time measurements. (This is even so for normal weight scales: mass is measured by the displacement of some part of the machine.) And the error in these measurements makes it impossible to distinguish vacuum from matter.

To put it another way, if one measures the mass of a piece of vacuum of size $R$, the result is always at least $\frac{\hbar}{cR}$; there is no possible way to find a perfect vacuum in an experiment. On the other hand, if one measures the mass of a particle, one finds that the result is size dependent; at Planck dimensions it approaches the Planck mass for every type of elementary particle. This result is valid both for massive and massless particles.

Using another image, when two particles are approached to lengths of the order of the Planck length, the error in the length measurements makes it impossible to say whether there is something or nothing between the two objects. In short, matter and vacuum get mixed-up at Planck dimensions. This is an important result: since both mass and empty space-time can be mixed-up, one has confirmed that they are made of the same “fabric”, confirming the idea presented at the beginning. This idea is now commonplace in all attempts to find a unified description of nature.

This approach is corroborated by the attempts of quantum mechanics in highly curved space-time, where a clear distinction between the vacuum and particles is not possible; the well-known Unruh radiation [45], namely that any observer either accelerated in vacuum or in a gravitational field in vacuum still detects particles hitting him, is one of the examples which shows that for curved space-time the idea of vacuum as a particle-free space does not work. Since at Planck scales it is impossible to say whether space is flat or not, it follows that it is also impossible to say whether it contains particles or not.

In summary: the usual concepts of matter and of radiatiation are not applicable at Planck dimensions. Usually, it is assumed that matter and radiation are made of interacting elementary particles. The concept of an elementary particle is that of an entity which is countable, point-like, real and not virtual, with a definite mass, a definite spin, distinct from its antiparticle, and most of all, distinct from vacuum, which is assumed to have zero mass. All these properties are found to be incorrect at Planck scales. One is forced to conclude that at Planck dimensions, it does not make sense to use the concepts of ‘mass’, ‘vacuum’, ‘elementary particle’, ‘radiation’, and ‘matter’.

One then can ask at which dimension particles and vacuum can be distinguished. This is obviously possible at everyday dimensions, e.g. of the order of 1 m, and is experimentally possible up to $10^{-18}$ m. Present ideas in particle physics suggest that particles are defined at least until the scale of unification of the electromagnetic, weak
and strong interaction, i.e. until about $10^{-31}$ m. Somewhere between there and the Planck length of $10^{-35}$ m the distinction between particles and space-time becomes impossible, perhaps in some gradual way.

10 Unification

In this rapid walk, one has thus destroyed all the experimental pillars of quantum theory: the superposition principle, space-time symmetry, gauge symmetry, and the discrete symmetries. One also has destroyed the foundations of general relativity, namely the existence of a space-time manifold, of the field concept, the particle concept, and of the concept of mass. It was even shown that matter and space-time cannot be distinguished. It seems that one has destroyed every concept used for the description of motion, and thus made the description of nature impossible. One naturally asks whether one can save the situation.

To answer this question, one needs to see what one has gained from this sequence of destructive arguments. First, since it was found that matter is not distinguishable for vacuum, and since this is correct for all types of particles, be they matter or radiation, one has a clear argument to show that the quest for unification in the description of elementary particles is correct and necessary.

Moreover, since the concepts ‘mass’, ‘time’, and ‘space’ cannot be distinguished from each other, we also know that a new, single entity is necessary to define both particles and space-time. To find out more about this new entity, three approaches are being pursued in present research. The first, quantum gravity, especially the one using the Ashtekar’s new variables and the loop representation [7], starts by generalizing space-time symmetry. The second, string theory [8], starts by generalizing gauge symmetries and interactions, and the third, the algebraic quantum group approach, looks for generalized permutation symmetries [9].

The basic result of all these descriptions is that what is usually called matter and vacuum are two different aspects of one same “soup” of constituents. In this view, mass and spin are not intrinsic properties, but are properties emerging from certain configurations of the fundamental soup. The fundamental constituents themselves have no mass, no spin, and no charge. The three mentioned approaches to a unified theory have one thing in common: the fundamental constituents are extended. (In this way the non-locality required above is realized.) In a subsequent article we will give a collection of simple arguments for this result.

The present speculations describe the vacuum as a fluctuating tangle of extended entities and describe particles as knots or possibly even more complex structures. Such models explain why space has three dimensions, give hope to calculate the masses of particles, to determine their spin, and naturally confirm that vacuum has no energy at large scales, but high energy at small dimensions. At everyday energies and length scales, the tangle is not visible, the knots become point-like, and one recovers the smooth, empty vacuum and the massive, spinning, charged and pointlike particles. In this way, these speculative models avoid the circular definition of particles and space-time mentioned at the beginning. However, the details of this programme are still hidden in the impenetrable, but fascinating future of physics.
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[1] Gravity is curved space-time. Extensive research has shown that quantum field theory, the description of electrodynamics and of the nuclear forces, fails for situations with strongly curved space-times. In these cases the concept of particle is not uniquely defined; quantum field theory cannot be extended to include general relativity, i.e. gravity consistently. In short, quantum theory works only by assuming that gravity does not exist; and indeed, the gravitational constant does not appear in any consistent quantum field theory. (Even if one refuses to describe gravity by general relativity, e.g. by describing it by the naive idea that gravity is a force, the inclusion into quantum theory is not possible, due to the so-called non-renormalizability of gravitation.)

On the other hand, general relativity neglects the commutation rules between physical quantities discovered by experiments on a microscopic scale. General relativity assumes that position and momentum of material objects can be given the meaning of classical physics. General relativity assumes that quantum mechanics does not exist; it ignores Planck’s constant $\hbar$. Most dramatically, the contradiction is shown by the failure of general relativity to describe the pair creation of spin 1/2 particles, a typical quantum process. Wheeler [2, 3] and others [4, 5] have shown that in such a case, the topology of space necessarily has to change; in general relativity however, the topology of space is fixed. In short, the description of nature of general relativity contradicts that of quantum mechanics, and in order to describe matter correctly, our concept of space-time has to be changed.

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Note that if physical space is not a manifold, the various methods could give different answers for the dimensionality. For example, for linear spaces without norm, the number of dimensions cannot be defined at all.

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Table

| Object          | size: diameter d (m) | mass m (kg)      | Schwarzschild radius $r_S$ | ratio $d/r_S$ | Compton wave length $\lambda_C$ | ratio $d/\lambda_C$ |
|-----------------|----------------------|-----------------|-----------------------------|--------------|---------------------------------|-------------------|
| galaxy          | $\approx 1 \text{ Zm}$ | $\approx 5 \cdot 10^{40}$ | $70 \text{ Tm}$ | $\approx 10^7$ | $\approx 10^{-83}$ m | $\approx 10^{104}$ |
| neutron star    | $10 \text{ km}$ | $2.8 \cdot 10^{30}$ | $4.2 \text{ km}$ | $2.4$ | $1.3 \cdot 10^{-73}$ m | $8.0 \cdot 10^{76}$ |
| sun             | $1.4 \text{ Gm}$ | $2.0 \cdot 10^{30}$ | $3.0 \text{ km}$ | $4.8 \cdot 10^5$ | $1.0 \cdot 10^{-73}$ m | $8.0 \cdot 10^{81}$ |
| earth           | $13 \text{ Mm}$ | $6.0 \cdot 10^{24}$ | $8.9 \text{ mm}$ | $1.4 \cdot 10^9$ | $5.8 \cdot 10^{-68}$ m | $2.2 \cdot 10^{74}$ |
| human           | $1.8 \text{ m}$ | $75$ | $0.11 \text{ ym}$ | $1.6 \cdot 10^{25}$ | $4.7 \cdot 10^{-45}$ m | $3.8 \cdot 10^{44}$ |
| molecule        | $10 \text{ nm}$ | $0.57 \text{ zg}$ | $8.5 \cdot 10^{-52}$ m | $1.2 \cdot 10^{43}$ | $6.2 \cdot 10^{-19}$ m | $1.6 \cdot 10^{10}$ |
| atom ($^{12}$C) | $0.6 \text{ nm}$ | $20 \text{ yg}$ | $3.0 \cdot 10^{-53}$ m | $2.0 \cdot 10^{43}$ | $1.8 \cdot 10^{-17}$ m | $3.2 \cdot 10^{7}$ |
| proton p        | $2 \text{ fm}$ | $1.7 \text{ yg}$ | $2.5 \cdot 10^{-54}$ m | $8.0 \cdot 10^{38}$ | $2.0 \cdot 10^{-16}$ m | $9.6$ |
| pion $\pi$      | $2 \text{ fm}$ | $0.24 \text{ yg}$ | $3.6 \cdot 10^{-55}$ m | $5.6 \cdot 10^{39}$ | $1.5 \cdot 10^{-15}$ m | $1.4$ |
| up-quark u      | $< 0.1 \text{ fm}$ | $0.6 \text{ yg}$ | $9.0 \cdot 10^{-55}$ m | $< 1.1 \cdot 10^{38}$ | $5.5 \cdot 10^{-16}$ m | $< 0.18$ |
| electron e      | $< 4 \text{ am}$ | $9.1 \cdot 10^{-31}$ | $1.4 \cdot 10^{-57}$ m | $3.0 \cdot 10^{39}$ | $3.8 \cdot 10^{-13}$ m | $< 1.0 \cdot 10^{-5}$ |
| neutrino $\nu_e$ | $< 4 \text{ am}$ | $< 3.0 \cdot 10^{-35}$ | $< 4.5 \cdot 10^{-62}$ m | n.a. | $> 1.1 \cdot 10^{-8}$ m | $< 3.4 \cdot 10^{-10}$ |

Table 1: The size, Schwarzschild radius, and Compton wavelength of some objects appearing in nature. A short reminder of the new SI prefixes: f: $10^{-15}$, a: $10^{-18}$, z: $10^{-21}$, y: $10^{-24}$, P: $10^{15}$, E: $10^{18}$, Z: $10^{21}$, Y: $10^{24}$.

Figure caption

Figure 1: A *Gedankenexperiment* showing that at Planck scales, matter and vacuum cannot be distinguished.
