Neutron Star Structure and the Neutron Radius of $^{208}$Pb

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We study relationships between the neutron-rich skin of a heavy nucleus and the properties of neutron-star crusts. Relativistic effective field theories with a thicker neutron skin in $^{208}$Pb have a larger electron fraction and a lower liquid-to-solid transition density for neutron-rich matter. These properties are determined by the density dependence of the symmetry energy which we vary by adding nonlinear couplings between isoscalar and isovector mesons. An accurate measurement of the neutron radius in $^{208}$Pb—via parity violating electron scattering—may have important implications for the structure of the crust of neutron stars.

It is an extrapolation of 18 orders of magnitude from the neutron radius of a heavy nucleus—such as $^{208}$Pb with $R_n \approx 5.5$ fm—to the approximately 10 km radius of a neutron star. Yet both radii depend on our incomplete knowledge of the equation of state of neutron-rich matter. Therefore, an accurate measurement of the neutron radius in $^{208}$Pb may have important implications for some neutron star properties.

Heavy nuclei are expected to have a neutron-rich skin. This important feature of nuclear structure arises because of the large neutron excess and because the Coulomb barrier reduces the proton density at the surface. The thickness of the neutron skin depends on the pressure of neutron-rich matter: the greater the pressure, the thicker the skin as neutrons are pushed out against surface tension. The same pressure supports a neutron star against gravity. Thus models with thicker neutron skins often produce neutron stars with larger radii.

Neutron stars are expected to have a solid crust of nonuniform neutron-rich matter above a liquid mantle. The phase transition from solid to liquid depends on the properties of neutron-rich matter. Indeed, a high pressure implies a rapid rise of the energy with density making it energetically unfavorable to separate uniform matter into regions of high and low densities. Thus a high pressure typically implies a low transition density from a solid crust to a liquid mantle. This suggests an inverse relationship: the thicker the neutron-rich skin of a heavy nucleus, the thinner the solid crust of a neutron star.

In this letter we study possible “data-to-data” relations between the neutron-rich skin of a heavy nucleus and the crust of a neutron star. These relations may impact neutron star observables. Indeed, properties of the crust are important for models of glitches in the rotational period of pulsars, for the shape and gravitational radiation of non-spherical rotating stars, and for neutron-star cooling. Note that the skin of a heavy nucleus and the crust of a neutron star are composed of the same material: neutron-rich matter at similar densities.

The Parity Radius Experiment (PREX) at the Jefferson Laboratory aims to measure the neutron radius in $^{208}$Pb via parity violating electron scattering. Parity violation is sensitive to the neutron density because the $Z^0$ boson couples primarily to neutrons. The result of this purely electroweak experiment could be both accurate and model independent. In contrast, all previous measurements of bulk neutron densities used hadronic probes that suffer from controversial uncertainties in the reaction mechanism (see for example Ref. [8]). PREX should provide a unique observational constraint on the thickness of the neutron skin in a heavy nucleus. In this letter we explore some of the implications of this measurement on the structure of neutron stars.

Microscopic calculations of the energy of neutron matter constrain both the neutron skin in $^{208}$Pb and the crust of a neutron star; see for example Ref. [9]. However, these calculations of infinite neutron matter are not directly tested by observable properties of finite nuclei such as their charge densities or binding energies. Moreover, nonrelativistic calculations of symmetric nuclear matter have not succeeded in predicting the saturation density. It thus becomes necessary to fit some properties of a three-body force in order to reproduce nuclear saturation. Indeed, the properties of $A=8$ pure neutron drops calculated in Ref. [10] may depend on the three-nucleon force used. Thus we feel that it is important to distinguish direct finite-nucleus measurements—such as PREX—from theoretical neutron-matter “observables” based solely on calculations. Indeed, PREX may provide an important test of these calculations.

We start with a relativistic effective field theory that provides a simple description of finite nuclei and a Lorentz covariant extrapolation for the equation of state of dense neutron-rich matter. The theory has an isoscalar-scalar $\phi$ (sigma) meson field and three vector fields: an isoscalar $V$ (omega), an isovector $b$ (rho), and the photon $A$. We now supplement the Lagrangian with new nonlinear sigma-rho and omega-rho couplings.
These couplings allow us to change the density dependence of the symmetry energy which changes both the thickness of the neutron skin in $^{208}$Pb and the neutron-star crust.

The interacting Lagrangian density is given by [12],

$$\mathcal{L}_{\text{int}} = \bar{\psi} \left[ g_\nu \phi - \left( g_\nu V_\mu + \frac{g_\rho}{2} \tau \cdot b_\mu + \frac{e}{2} (1 + \tau_3) A_\mu \right) \gamma^\mu \right] \psi$$

$$- \frac{\kappa}{3!} (g_\rho \phi)^3 - \frac{\lambda}{4!} (g_\rho \phi)^4 + \frac{\xi}{4!} \sigma_\nu (V_\mu V^\mu) + \frac{\xi}{4!} g_\rho^3 (b_\mu \cdot b^\mu)^2$$

$$+ g_\rho^2 (b_\mu \cdot b^\mu \left[ A_4 g_\rho^2 \phi^2 + A_v g_\rho^2 V_\mu V^\mu \right]. \quad (1)$$

We consider a number of different parameter sets. First, we note that the nonlinear rho coupling $\xi$ will modify the density dependence of the rho mean field and this could change the neutron-skin thickness. However, unless $\xi$ is made very large, this term was found to have a small effect [12]. Thus for simplicity, we set $\xi = 0$ in all our parameter sets. Second, note that we could have added a cubic sigma-rho interaction of the form:

$$\mathcal{L}_3 = M \Lambda_3 (g_\rho \phi) \left( g_\rho^2 b_\mu \cdot b^\mu \right).$$

A nonzero $\Lambda_3$ does change the thickness of the neutron skin in $^{208}$Pb—but at the expense of a change in the proton density. Therefore, we set $\Lambda_3 = 0$ and focus exclusively on $\Lambda_4$ and $\Lambda_v$.

We start with the original NL3 parameter set of Lalazissis, König, and Ring [13]. (Note that a small adjustment of the $NN\rho$ coupling constant was needed to fit the symmetry energy of nuclear matter at a Fermi momentum of $k_F = 1.15$ fm$^{-1}$; see text below). The NL3 set has $\zeta = \Lambda_4 = \Lambda_v = 0$ and provides a good fit to the ground-state properties of many nuclei. In this model symmetric nuclear matter saturates at $k_F = 1.30$ fm$^{-1}$ with a binding energy per nucleon of $E/A = -16.25$ MeV and an incompressibility of $K = 271$ MeV. All other parameter sets considered here have been fixed to the same saturation properties.

We now add the new nonlinear couplings $\Lambda_4$ and/or $\Lambda_v$ between the isoscalar and the isovector mesons. At the same time we adjust the strength of the $NN\rho$ coupling constant ($g_\rho$) from its NL3 value to maintain the symmetry energy of nuclear matter unchanged (see text below). Note that neither $\Lambda_4$ nor $\Lambda_v$ affect the properties of symmetric nuclear matter since $b_\mu \equiv 0$. Hence, the saturation properties remain unchanged. Our goal is to change the neutron density and the neutron-skin thickness in $^{208}$Pb while making very small changes to the proton density which is well constrained by the measured charge density [4]. The two new couplings ($\Lambda_4$ and $\Lambda_v$) change the skin thickness in $^{208}$Pb by similar amounts. Yet they have different high-density limits [3]. For $\Lambda_v = 0$ the symmetry energy is proportional to the baryon density $\rho$ in the limit of very high density, while it only grows like $\rho^{1/3}$ for nonzero $\Lambda_v$. This can produce noticeable differences in neutron star radii, as we show below.

Let us start with $\Lambda_4 = 0$. For a given omega-rho coupling $\Lambda_v$, we readjust only the $NN\rho$ coupling constant $g_\rho$ in order to keep an average symmetry energy fixed. The symmetry energy at saturation density is not well constrained by the binding energy of nuclei. However, some average of the symmetry energy at full density and the surface energy is constrained by binding energies. As a simple approximation we evaluate the symmetry energy not at full density $k_F \approx 1.30$ fm$^{-1}$ but rather at an average density corresponding to $k_F = 1.15$ fm$^{-1}$. Thus, all our parameter sets have a symmetry energy of 23.50 MeV at $k_F = 1.15$ fm$^{-1}$. Note that the original NL3 parameter set predicts a symmetry energy of 37.4 MeV at full saturation density and close to 23.50 MeV at $k_F = 1.15$ fm$^{-1}$ [13]. This simple procedure produces a nearly constant binding energy per nucleon for $^{208}$Pb as $\Lambda_v$ is changed, as can be seen in Table 1. Moreover, Table 1 shows that increasing $\Lambda_v$ reduces the neutron-skin thickness significantly—while maintaining the proton radius nearly constant. In the following we plot our results for a range of $\Lambda_v$ values for which the proton radius is within 0.01 fm of its $\Lambda_v = 0$ value.

To study the solid crust of a neutron star we make a simple random-phase-approximation (RPA) calculation of the transition density below which uniform neutron-rich matter becomes unstable against small amplitude density fluctuations. This provides a lower bound to the true transition density [13]. We start with the longitudinal dielectric function $\epsilon_L$, as defined in Eq. (68) of Ref. [4], evaluated at an energy transfer $q_0 = 0$ and at an arbitrary momentum transfer $q$. That is,

$$\epsilon_L(q_0 = 0, q) = \det (1 - D_L \Pi_L), \quad (2)$$

Here $\Pi_L$ is a longitudinal polarization matrix describing particle-hole excitations of a uniform system of protons, neutrons, and electrons in beta equilibrium, as given in Eq. (56) of Ref. [4]. The matrix $D_L$, describing meson and photon propagation, follows from Eq. (57) of Ref. [4]—but includes additional terms to account for the nonlinear nature of the meson self-couplings [17]. We estimate the transition density $\rho_c$ by computing the largest density at which $\epsilon_L(0, q) < 0$ for any given $q$.\[ TABLE I. Results for the NL3 Parameter Set. The binding energy per particle in $^{208}$Pb is B.E., $R_p$ is the proton and $R_n - R_p$ is the difference between neutron and proton radii in Pb. (Note that we do not include center of mass corrections.) Finally, $\rho_c$ is our estimate for the transition density of neutron-rich matter from a nonuniform to uniform phase.\]
FIG. 1. Estimate of the transition density from nonuniform to uniform neutron-rich matter versus neutron-proton radius in $^{208}$Pb. The curves are for the four parameter sets described in the text.

In Fig. 1 we display the transition density for various parameter sets (see Table II) as a function of the predicted difference in the root-mean-square neutron and proton radii $R_n - R_p$ in $^{208}$Pb. The curves are parameterized by different values of $\Lambda_v$, as shown in Table II. The NL3 parameter set saturates nuclear matter with a relatively small value of the nucleon effective mass: $M^* = M - g_\phi \phi = 0.59 M$. The parameter set Z271 saturates nuclear matter as NL3 but with $M^* = 0.70 M$. This set also has $\zeta = 0$. The two remaining curves in the figure are for parameter sets having $\zeta = 0.06$ and both saturate nuclear matter with $M^* = 0.80 M$. (Set Z271v has a nonzero $\Lambda_v$ while set Z2714 uses a nonzero $\Lambda_4$). Note that the scalar mass $m_s$ for parameter sets S271, Z271v, and Z2714 is adjusted to reproduce the proton radius in $^{208}$Pb as computed with NL3. Figure 1 displays a clear inverse correlation between the transition density and the neutron-skin thickness $R_n - R_p$. The transition density expressed in fm$^{-3}$ is about,

$$\rho_c \approx 0.16 - 0.39 \left( R_n - R_p \right), \tag{3}$$

with the skin thickness expressed in fm. Moreover, this correlation seems to be insensitive to $M^*$ or to using $\Lambda_4$ or $\Lambda_v$ to change $R_n - R_p$. These results suggest that a measurement of the neutron radius in $^{208}$Pb will provide considerable information on the transition density.

TABLE II. Model parameters used in the calculations. The parameter $\kappa$ and the scalar ($m_s$) and vector ($m_v$) masses are given in MeV. The nucleon and rho masses are kept fixed at $M = 939$ and $m_\rho = 763$ MeV, respectively.

| Model | $g_s^2$ | $g_v^2$ | $\kappa$ | $\lambda$ | $\zeta$ | $m_s$ | $m_v$ |
|-------|---------|---------|----------|----------|--------|-------|-------|
| NL3   | 104.387 | 163.558 | 3.860    | -0.01591 | 0      | 508.194| 782.5 |
| S271  | 85.992  | 116.766 | 6.68     | -0.01578 | 0      | 505    | 783   |
| Z271v | 53.785  | 70.069  | 6.17     | 0.15634  | 0.06   | 465    | 783   |

Note that Fig. 1 only shows our results. Yet all other calculations that we are aware of also give consistent results. For example, the nonrelativistic microscopic equation of state of Friedman and Pandharipande has a transition density of $\rho_c = 0.096$ fm$^{-3}$ according to Lorenz et al. [10]. For this equation of state Brown finds $R_n - R_p = 0.16 \pm 0.02$ fm [11]. These numbers are in excellent agreement with Eq. (3).

Brown also finds a linear relation between $R_n - R_p$ and the derivative of the energy of neutron matter versus density $dE/d\rho$ evaluated at $\rho = 0.1$ fm$^{-3}$ [13]. He considers a large variety of nonrelativistic Skyrme interactions. Our results for $dE/d\rho$ versus $R_n - R_p$ are completely consistent. We expect these common $dE/d\rho$ values to give similar $\rho_c$ values consistent with Eq. (3) for these Skyrme interactions. This is because $dE/d\rho$ is related to the pressure while $\rho_c$ depends on the density dependence of the pressure.

Finally, for the relativistic interaction TM1 of Sugahara and Toki [8], we calculate from Eq. (2) $\rho_c \approx 0.059$ fm$^{-3}$ and $R_n - R_p = 0.27$ fm. Again, the numbers are again in good agreement with Eq. (3).

In Fig. 2 we show the electron fraction per baryon $Y_e$ versus density for uniform neutron-rich matter in beta equilibrium. We include results only for the S271 parameter set as all other sets yield similar results. The different curves are for different values of $\Lambda_v$ which predict the indicated $R_n - R_p$ values. The curves start near the transition densities displayed in Fig. 1. The electron fraction $Y_e$ is determined by the symmetry energy while $R_n - R_p$ is sensitive to the density dependence of the symmetry energy. Therefore a measurement of $R_n - R_p$ constrains the growth of $Y_e$ with density. If $R_n - R_p$ is greater than about 0.24 fm, $Y_e$ becomes large enough to allow the direct URCA process [19] to cool down a 1.4 solar mass neutron star.

FIG. 2. Electron fraction $Y_e$ versus baryon density for uniform neutron-rich matter in beta equilibrium using.
the S271 parameter set (other sets yield similar results). The curves are for different values of $\Lambda_v$ that predict the indicated values of $R_n - R_p$ for $^{208}$Pb, in fm.

We now consider the radius $R$ of a 1.4 solar mass neutron star. For a given parameter set, $R$ increases with $R_n - R_p$. However, as one changes the parameter set to increase $M^*$ or $\zeta$ the equation of state becomes softer at high density. As a result $R$ decreases for fixed $R_n - R_p$. Also, the high density equation of state is softer with $\Lambda_v$ than with $\Lambda_4$ so $Z_{271v}$ gives slightly smaller stars than parameter set $Z_{2714}$. We conclude that $R$ is not uniquely constrained by a measurement of the neutron-skin thickness because $R_n - R_p$ only depends on the equation of state at normal and lower densities while $R$ is also sensitive to the equation of state at higher densities. Yet one may be able to combine separate measurements of $R_n - R_p$ and $R$ to obtain considerable information about the equation of state at low and high densities. For example, if $R_n - R_p$ is relatively large while $R$ is small this could suggest a phase transition. A large $R_n - R_p$ implies that the low density equation of state is stiff while a small $R$ suggests that the high density equation of state is soft. The transition from stiff to soft could be accompanied by a phase transition.

![Graph: Radius of a 1.4 solar mass neutron star versus neutron-minus-proton radius in $^{208}$Pb for the four parameter sets described in the text.](image)

**FIG. 3.** Radius of a 1.4 solar mass neutron star versus neutron-minus-proton radius in $^{208}$Pb for the four parameter sets described in the text.

In conclusion: 1) It is possible to fit nuclear observables—such as charge densities, binding energies, and single particle spectra—with effective field theories that predict a range of neutron-skin thicknesses. This can be done by adding nonlinear couplings between isoscalar and isovector meson fields or, in general, by adding interactions that modify the density dependence of the symmetry energy. We conclude that the neutron-skin thickness is not tightly constrained by these observables. Yet a measurement of the skin thickness will constrain the density dependence of the symmetry energy.

2) The density dependence of the symmetry energy is adjustable in our relativistic effective field models while still reproducing nuclear-matter properties and other ground-state observables. Indeed, our models can provide a Lorentz-covariant extrapolation for the high density equation of state with a symmetry energy that rises slower with density relative to earlier relativistic mean-field models.

3) The electron fraction $Y_e$ of neutron-rich matter in beta equilibrium is correlated with the neutron-skin thickness in $^{208}$Pb. The thicker the neutron skin the faster $Y_e$ rises with density. In our models a neutron-skin thickness of the order of 0.24 fm or larger suggests that $Y_e$ will become large enough to allow a direct URCA process to cool down a $1.4M_\odot$ neutron star.

4) We have found an inverse correlation between the neutron-skin thickness and the density of a phase transition from nonuniform to uniform neutron-rich matter. In our models the transition density obeys the approximate relation: $\rho_c \approx 0.16 - 0.39 (R_n - R_p)$, with $R_n - R_p$ in fm and $\rho_c$ in fm$^{-3}$. This suggests that a neutron skin measurement in $^{208}$Pb can provide important information on the thickness and other properties of the crust of a neutron star.

5) Microscopic calculations of the energy of neutron matter constrain the density dependence of the symmetry energy and hence the neutron-skin thickness in $^{208}$Pb. Therefore a neutron skin measurement may provide an important observational check on such calculations that is not provided by other observables. Moreover, a neutron-skin measurement may constrain three-body forces in neutron-rich matter.

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