The effect of external dynamic loads on the lifetime of rolling element bearings: accurate measurement of the bearing behaviour

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Abstract. Accurate prediction of the lifetime of rolling element bearings is a crucial step towards a reliable design of many rotating machines. Recent research emphasizes an important influence of external dynamic loads on the lifetime of bearings. However, most lifetime calculations of bearings are based on the classical ISO 281 standard, neglecting this influence. For bearings subjected to highly varying loads, this leads to inaccurate estimations of the lifetime, and therefore excessive safety factors during the design and unexpected failures during operation. This paper presents a novel test rig, developed to analyse the behaviour of rolling element bearings subjected to highly varying loads. Since bearings are very precise machine components, their motion can only be measured in an accurately controlled environment. Otherwise, noise from other components and external influences such as temperature variations will dominate the measurements. The test rig is optimised to perform accurate measurements of the bearing behaviour. Also, the test bearing is fitted in a modular structure, which guarantees precise mounting and allows testing different types and sizes of bearings. Finally, a fully controlled multi-axial static and dynamic load is imposed on the bearing, while its behaviour is monitored with capacitive proximity probes.

1. Introduction
Accurate prediction of the lifetime of rolling element bearings is a crucial step towards a reliable design of many rotating machines. Recent research emphasizes an important influence of external dynamic loads on the lifetime of bearings. However, most lifetime calculations of bearings are based on the classical ISO 281 standard, neglecting this influence. Advanced calculation schemes add an empirical factor to the static load, based on the severity of the vibrations. For bearings subjected to highly varying loads, this leads to inaccurate estimations of the lifetime, and therefore excessive safety factors during the design and unexpected failures during operation. During normal operation of a bearing, the surfaces of the rolling elements and the raceways are separated by a lubricant film. Due to dynamic excitations, this film can break up. If metal-to-metal contact between the rolling elements and the raceways occurs, fretting wear is inevitable. Recent research indicates that the failure rate of wind turbines is about 3 times higher than the failure rate of conventional generators [1]. This is partly due to the unique operational
conditions of the bearings, resulting from widely varying wind loads and high vibration levels [2].

A study conducted at the KU Leuven investigates the behaviour of rolling element bearings subjected to highly varying loads. In this way, it aims to define a clear relation between external dynamic excitations and their influence on the lifetime of a bearing. The behaviour of the bearing is analysed experimentally using a novel test rig. In literature, many different designs of bearing test rigs have been proposed. Most of these set-ups are able to apply a static load on a bearing, using for example hydraulics or springs. Some excite the bearing in its radial direction [3, 4], some in its axial direction [5] and some in both directions [6]. Bearing manufacturers even offer these types of test rigs, allowing the customers to test the quality of the grease they are using. Other types of test rigs load the bearing using an unbalance weight on a shaft [7], or through a set of gears using a gearbox [8, 9]. The latter rigs can be used to validate techniques developed to detect bearing damage in signals dominated by gear meshing noise. Although many research projects conducted in the past use bearing test rigs, none of the rigs are able to apply a fully controlled multi-axial static and dynamic load on the bearing. Therefore, a new type of test rig was developed in the framework of the study.

The main concept of the test rig is fairly simple. An electric motor drives a shaft through a flexible coupling. The shaft is supported by two bearings, forming a rigid spindle. At the end of the shaft, a third bearing is mounted. This is the test bearing. The load is directly applied on the stationary outer ring of the test bearing. Figure 1 shows the concept and an overview of the test rig. The proposed bearing test rig innovates in two ways. Firstly, the load applied on the test bearing is multi-axial, with a component in the radial and axial direction, and both static and dynamic in each direction. Thereby, it is possible to load the bearing as if it were built into a real machine, for example a gearbox. Secondly, different types of bearings, such as deep groove ball bearings and tapered roller bearings, and different sizes of bearings can be tested. The design allows easy adjustment of the test rig to mount bearings with different inner diameter, outer diameter and width, without compromising on performance. Validation of theories on a wide range of bearings is therefore possible. Both aspects are discussed further on in the text.

![Figure 1. Concept (a) and overview (b) of the test rig.](image)

This paper describes the optimisation of the test rig design to perform valuable bearing measurements. Bearings are very precise machine components. Their behaviour can only be measured in an accurately controlled environment. Much effort is needed to avoid disturbances due to noise from other components and due to external influences such as temperature variations. As explained by Marsh [10], measuring ball bearings means minimizing the contribution of external effects. In the framework of the study, precisely controlling the bearing load is required as well. First, the paper describes the design of the main spindle, aiming to reduce the influence of both external vibrations and temperature variations on the bearing measurements. Then, section 3 presents the modular structure in which the test bearing
Section 4 and 5 describe how both the load and the speed of the bearing are controlled. Section 6 introduces the use of capacitive sensors to precisely monitor the bearing behaviour. Finally, section 7 details about the requirements on the dynamics of each of the test rig components during the design.

2. Precise spindle motion

An ideal spindle allows motion in a single degree of freedom: pure rotation. Any movement in the remaining five degrees of freedom is undesired and may be classified as either error or a response to an external influence. The error can result from the spindle’s design and manufacturing, while the external influence can be thermal gradients or external vibrations. This section discusses different techniques applied to limit both errors and external influences as much as possible.

To support the main shaft of the test rig, two tapered roller bearings mounted in opposition to each other are used. Tapered roller bearings offer high load capacity and stiffness because of the line contact between the rollers and races. The trade-off in performance is that these bearings typically have relatively large error motions, because any geometrical errors in the rollers or races print through to the rotation by the same internal stiffness that provides the favorable load capacity [10]. In comparison, air bearing spindles and hydrostatic spindles run with higher accuracy. However, a substantial increase in expense should be expected if a high load carrying capacity of these spindles is required. Ref. [11] shows a nice implementation of a bearing test rig using wave journal bearings to support the shaft.

The tapered roller bearings are mounted in a cylindrical housing, as shown in Figure 2. Both bearing seats are milled during one operation, ensuring a precise alignment between the bearings afterwards. During mounting, one ring is displaced on its seat in such a way that the bearing arrangement reaches a desired preload. The preload is set with a shim, which accurately determines the distance between the inner rings of both bearings. Mounting the bearings with the right amount of preload has several advantages:

- The clearance in the bearings is removed. This ensures smooth running with higher running accuracy [12].
- The stiffness of the support system increases [13]. Due to the preload force, all rolling elements are now loaded, forming a stiffer structure.
- The lifetime of the bearings increases. The preload serves to provide a minimum load on the bearing and prevent bearing damage as a result of sliding movements of the rolling elements [14].

When increasing the preload, the clearance in each bearing is reduced. At a certain point, there is no more clearance. Then, all rolling elements are loaded. However, further increasing the preload would only reduce the bearing lifetime, as the load on the bearing increases. Therefore, special attention is paid to the determination of the optimal preload.

Mounting the spindle bearings in O arrangement (back-to-back) gives the designer the opportunity to reduce the effect of temperature variations. A well-considered choice of the distance between the bearings leads to a preload which is independent of the operating temperature, as explained in [12]. Recalling Figure 2, the roller cone apex R is the point where the bearing centre line intersects the projection of the inclined outer ring raceway. When the temperature of the bearings increases during operation, the shaft expands more than the housing in both axial and radial direction. In an O arrangement, the radial preload increases while the axial preload decreases. If the apices R of both bearings coincide at one point, the resulting preload is unaffected. The adjusted preload is then independent of the temperature. In the presented spindle, an optimal distance between the two bearings of 270 mm was found. This large distance is advantageous to reduce the radial forces on the bearings after applying
a radial load on the test bearing. Also, a greater distance reduces the error angle of the shaft due to imperfect alignment of both bearings. To reduce the influence of external vibrations, the spindle is clamped in a heavy and rigid frame. Section 7 discusses this in detail.

3. Both accurate and modular mounting

The test rig makes it possible to test bearings of different types, such as deep groove ball bearings, angular contact ball bearings and tapered roller bearings, and bearings of different sizes, with an inner bore diameter varying from 10 to 19 mm, an outer diameter varying from 20 to 52 mm and a width varying from 5 to 15 mm. In this way, commonly used bearings, for example from electric motors and gearboxes, can be tested. Furthermore, each different test bearing is mounted according to the specifications provided by the bearing manufacturer. Using inadequate or incorrect bearing seats generally leads to a strong reduction of the bearing lifetime. Conclusions related to the lifetime of the test bearing, made during the study, might then be faulty.

In most cases, modularity and accuracy are opposing factors during a design. As the modularity increases, more parts are used, the stiffness of the structure decreases, and so on. Aligning all components becomes more difficult. Nevertheless, this bearing test rig allows mounting different types and sizes of bearings, while precise alignment with the spindle is guaranteed. The current section explains how this is achieved.

The test bearing is mounted in a modular structure consisting of an auxiliary shaft and an intermediate sleeve. A cross-section is given in Figure 3. An auxiliary shaft is clamped in the spindle of the test rig using a mechanism called collet chuck. After inserting this auxiliary shaft, the locknut is tightened. In milling machines, the same principle is used to clamp the tool into the spindle. Since bending of the spindle in response to radial cutting forces is inadmissible, the collet chuck is designed to form a stiff connection between the spindle and the tool. Both shafts are perfectly aligned, due to the tapered shape of the mating parts. The test bearing is then mounted on the auxiliary shaft, which is adapted to its bore diameter. On the other hand, the housing of the test bearing is adapted to the test bearing using an intermediate sleeve. The bore diameter of the sleeve is adjusted to the outer diameter of the test bearing. A different sleeve is installed when a different bearing is tested.

The bearing manufacturer states a set of guidelines to determine the appropriate fits of the shaft and housing seat, depending on the selected bearing. Also, the required dimensional tolerances, tolerances for the cylindrical form of the seats and tolerances for perpendicularity of the abutments are provided. All of these conditions can be taken into account for each bearing, as the auxiliary shaft and intermediate sleeve are tailored to a specific test bearing.
4. Controlling the load

The test rig imposes a load on the bearing, controlled separately in the radial and axial direction. No coupling between the radial and axial force is allowed. Furthermore, the load has a static and dynamic component in both directions. No coupling between the static and dynamic force is allowed. In this way, it is possible to simulate different real-life situations, where i.e. gear meshing forces are acting on the bearing. This section explains how the required decoupling is ensured and how the magnitude and the action point of the load are controlled. Finally, it shows why the load is unaffected by temperature variations.

The static and dynamic components of the load are provided by a different type of actuator. Air springs are used to apply a static force up to 10 kN in each direction. The air springs consist of two parallel plastic disks, connected through a rubber sleeve. Pressurized air in this sleeve provides the static force. The force is controlled by pressure regulators in the air circuit.

The dynamic actuators provide a broadband dynamic force from 25 to 500 Hz. Electrodynamic shakers are used to generate this force. Figure 4 gives an overview of the actuator configuration. In the right part, the actuators are replaced by force vectors, providing a clear view on the design of the bearing housing. The static load is generated by four air springs, transferring their force to the bearing using an arm on the housing. Two air springs control the axial force \( F_{a, st} \) and two air springs control the radial force \( F_{r, st} \). The dynamic load is directly introduced on the bearing housing through the stingers of the shakers: one stinger for the axial direction \( F_{a,d} \) and one stinger for the radial direction \( F_{r,d} \).

Using air springs and shakers in this configuration, the different components of the force are decoupled. Firstly, the lateral stiffness of the springs is negligible to their longitudinal stiffness, with an upper limit of only 1% of the longitudinal stiffness. Therefore, the radial and axial static force are decoupled and controlled separately. Secondly, the dynamic load of the shakers is transferred to the bearing using stingers. These stingers are stiff in their longitudinal direction, but flexible in their lateral direction. Therefore, the radial and axial dynamic load are decoupled. Finally, the resonance frequency of the air springs is very low. Depending on the air pressure, the resonance frequency varies between 1.9 and 2.1 Hz. Above \( 2.1\sqrt{2} \) Hz, the air springs act as a vibration isolator. Therefore, the shaker excitation is not transferred to the frame through the air springs, meaning that the static and dynamic forces are decoupled.
The static and dynamic component of the load on the test bearing is well-known, as it is measured directly at the point where it is introduced. During a preceding calibration test, the relation between the air pressure and the delivered load of an air spring is determined for different operating points. Here, each air spring is mounted in a structure and connected to a load cell. In this way, the load cell measures the load applied by the tested spring. The resulting relation is used to set the air pressure based on the requested bearing load. During the calibration, it was seen that each air spring shows a different relation between the load and the pressure. However, as every spring is controlled with a separate regulator, a different relation can be used for each air spring. The dynamic load is measured using an ICP load cell connecting the stinger to the structure, as often done when using shakers.

The applied load should act in the centre of the bearing. Otherwise, an uncontrolled and unwanted tilting moment is introduced on the bearing. Therefore, the static actuators are positioned in a way that the resulting force vector in each direction acts through the centre of the bearing. For the dynamic actuators, this means that the excited structure should be balanced. It is important that the centre of gravity (COG) of this structure coincides with the centre of the bearing, as the dynamic force acts in the COG of the excited part. The excited structure, consisting mainly of the housing and sleeve of the test bearing and the upper part of the air springs, is balanced during the design of the housing using computer aided design (CAD) software.

The force delivered by the air springs only depends on the air pressure in the circuit. As this air pressure is controlled by electro-pneumatic regulators, the static force is independent of disturbance factors like thermal expansion of the shaft due to heating of the spindle bearings.

5. Controlling the speed
A 9.1 kW synchronous servo motor is connected to the main shaft using a backlash-free jaw-type coupling. In this way, the bearing can operate as built into a real machine, including run-up and run-down simulations. The rotational speed of the bearing, ranging from 0 to 3000 rpm, is precisely controlled in a feedback loop implemented in the controller of the motor. The influence of load variations is reduced as much as possible. The elastic spider in the coupling isolates the test bearing from torsional vibrations generated by the motor. It also dynamically decouples the main shaft and motor in the radial and axial direction, meaning that the rotor of the motor does not influence the dynamics of the shaft. This has an important influence on the first bending resonance of the shaft, which should be as high as possible, as discussed in the last section.
6. Precise bearing measurements

The test bearing is instrumented with sensors commonly used for condition monitoring of bearings. Four accelerometers are mounted on the housing, as close as possible to the bearing itself. One thermocouple is mounted in the seat of the bearing and measures the outer ring temperature. Finally, three proximity probes measure the bearing motion. More precisely, they measure the relative displacement between the inner and outer ring of the bearing. They are mounted on the housing of the test bearing, and pointed at a disk shaped part of the auxiliary shaft, as shown in Figure 5. One probe measures the radial bearing motion, while two other probes measure both the axial and tilt motion.

![Figure 5. Positioning of the capacitive probes.](image)

Due to for example raceway surface imperfections and variations on the geometry of the rolling elements, every rolling element bearing introduces a certain error motion. This error motion consists of a synchronous and asynchronous part. The synchronous part is the movement which repeats itself after each revolution. A polar plot of this error displays lobes that can occur once, twice or at some multiple of times per revolution. On the other hand, the asynchronous part does not repeat on successive rotations. Rolling element bearings exhibit a significant asynchronous error motion because of the planetary-type kinematics of the inner race, outer race and rotating balls that lead to non-integer relationships between shaft speed and the key rotational frequencies. Therefore, consecutive rotations will not yield identical error motion results even in the absence of measurement errors [10].

For different static and dynamic load cases, the error motion of the test bearing is analysed using the capacitive probes. The probes measure the relative movement of a target surface from their sensing area, with an accuracy up to 8 nm. In the presented test rig, they are fixed to a vibrating structure and pointed to a rotating shaft. This complicates the measurements significantly. In order to obtain accurate data of the bearing motion, several measures were taken. The next part of this section focuses on the measures related to the probe mounting, the last part discusses the measures related to the rotation of the target surface.
Capacitive sensors measure the movement of a target based on a change in capacitance between the sensor and the target. This capacitance depends on the dielectric constant of the medium between both conductors, which is assumed to be air. Contamination of the air gap changes the dielectric constant, and introduces an error [15]. During replacement of the test bearing, the bearing grease can possibly reach the target surface of the sensors and distort the measurements afterwards. However in this configuration, each of the sensors is pointed at a surface which is not contaminated by grease during a replacement. Also, the axial sensors are positioned far outside the centre of the bearing, increasing the lever arm of the tilt measurement and therefore improving the accuracy. Finally, as the housing of the test bearing is dynamically excited, tight clamping of the sensors is required. Each of the probes is mounted in a custom-made structure, consisting of a sleeve, a nut and an o-ring, as shown in Figure 5. The o-ring clamps the sensor, without damaging its thin casing.

During rotation of the shaft around its centre axis, radial and axial mechanical runout of the target surface distort the signal. Radial runout is measured due to deviations of the disk side surface from a true circle, concentric with the rotation axis, and these include low-frequency components such as eccentricity and out-of-roundness [16]. Axial runout is introduced due to deviations of the face side surface of the disk from a flat surface, perpendicular to the axis of rotation.

To minimize the mechanical runout, a precise surface finish of the target surface is crucial. Therefore, both the bearing seat and target surface of the probes are grinded during the same operation. A runout of less than 0.8 µm is achieved in both directions, during rotation of the shaft around the bearing seat. If a higher accuracy is required, several costlier surface finish techniques are available. For example, the shaft can be coated with a 50 µm layer of nickel, and turned with a diamond tool afterwards. A mirror finished surface with a runout of less than 0.1 µm can then be obtained.

The measurement can also be highly improved using error separation techniques. The two most common error separation techniques are the Donaldson reversal technique, extended by Estler for face error motion reversal, and the multiprobe separation technique. In spindle metrology, these two techniques are widely used to separate the runout of a clamped rotating artifact in a spindle and the spindle error motion. In this way, it is possible to precisely characterize the runout of the artifact or the the spindle error motion. Both techniques are thoroughly discussed in Ref. [10]. In the presented test rig, the test bearing can be seen as a spindle, and the separation techniques can be used to remove the effect of runout during a measurement. However, the Donaldson reversal technique is only applicable if the spindle error is purely synchronous. As the test bearing generates a high asynchronous error, the technique cannot be used. The multiprobe separation technique allows an asynchronous error of the spindle, but requires using three different sensors at specific locations for each direction to be measured. In the current design of the test rig, it is not possible to apply this technique.

Clearly, error separation techniques are not suited to be used here. However, error compensation is possible. As the shaft of the test bearing is just a small auxiliary shaft, it can be clamped in a very precise spindle supported by air bearings. This spindle has an error motion of less than 10 nm, which can be neglected in comparison to the runout of the shaft. It is used to characterize the runout of the shaft. During post-processing of the test rig signals, the measured runout is subtracted from the signals of the capacitive probes. An optical trigger and a reference mark on the shaft allow synchronisation. By applying this compensation, the error due to mechanical runout decreases strongly.

7. Dynamic behaviour of the test rig
When analysing a bearing using the test rig, the measurements should not be influenced by the dynamics of the rig. Recalling that the test bearing is excited in a range from 25 to 500
Hz, the structure should be resonance free up to $500\sqrt{2}$ Hz. Each of the components of the test rig is developed using finite element method (FEM) software to check this condition. The calculations are simplified by dividing the rig in two parts which are analysed separately. The analysis of both parts is discussed in this section. First, the part connected to the outer ring of the test bearing, consisting of the housing and sleeve of the test bearing, is reviewed. Then, the part connected to the inner ring of the test bearing, consisting of the spindle and the frame, is discussed.

The housing of the test bearing is a solid structure transferring the forces of four different air springs to the bearing. On the one hand, the structure should be very strong to handle the static load up to 10 kN in each direction of the bearing. On the other hand, the housing should be as light as possible to minimize the inertial forces and maximize the resonance frequencies. In the final design, the first mode of the assembly housing and sleeve occurs at a resonance frequency of 695 Hz. The mode is presented in Figure 6. For this FEM analysis, the weight of the air springs is modelled as a point mass, and the stiffness increase provided by the test bearing is neglected.

The frame of the test rig is optimised to keep the resonances of the rig outside the range of excitation. It is a closed and rigid structure, mounted on four bushings. The bushings dynamically decouple the frame from the environment. The six rigid body modes of the test rig moving on its bushings are all located between 5 and 16.4 Hz. Therefore, clear excitation is possible starting at a frequency of 25 Hz. The frame itself consists of an upper and a lower part, as shown in Figure 7. The upper part connects the different components of the test rig, while the lower part forms a stiff structure to improve the torsional behaviour of the test rig. The FEM analysis of the test rig shows a first torsional and bending mode of the frame at respectively 663 and 731 Hz. Figure 8 visualises the first torsional mode. The first bending mode of the spindle shaft occurs at 805 Hz. It can consequently be concluded that the entire structure is resonance free in the full excitation range of the shakers. For this analysis, the housing of the test bearing is modelled as a simple point mass, while the test bearing is modelled as a spring. Also, the bushings are modelled as three-dimensional springs.

![Figure 6. First mode of the assembly housing and sleeve.](image)

The frame of the test rig supports the motor, the spindle and the static actuators. More importantly, it guarantees precise alignment between the motor and the spindle. Due to the rigid connection between both spindle and motor, and the strict tolerances requested during manufacturing of the frame, the alignment is unaffected by the operational conditions of the rig. Also, the frame allows moving the main spindle, in order to replace the test bearing. Locking and unlocking the spindle in the frame is done using two bolts. These bolts provide a clamping force between the frame around the spindle, as shown in Figure 7. This technique is used in milling machines as well, since the initial alignment with other components such as the motor is preserved after each locking action.

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8. Conclusion
An innovative and versatile bearing test rig has been developed. The test rig allows easy adjustment to mount different types and sizes of rolling element bearings. The bearings can be preloaded up to 10 kN, and excited up to 500 Hz. In this way, a wide range of bearings can be tested in real-life conditions. The design is optimised to reduce mechanical noise originating from both the main spindle and the motor. Also, the influence of temperature variations and external vibrations is kept to a minimum. Each of the components is free of resonances in the entire range of excitation. As a result, this test rig is an interesting tool to investigate the behaviour of bearings subjected to highly varying loads. It will be used to analyse the effect of dynamic forces on the lifetime of bearings.

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