Losing Your Marbles in Wavefunction Collapse Theories

Rob Clifton and Bradley Monton

ABSTRACT

Peter Lewis ([1997]) has recently argued that the wavefunction collapse theory of GRW (Ghirardi, Rimini, and Weber [1986]) can only solve the problem of wavefunction tails at the expense of predicting that arithmetic does not apply to ordinary macroscopic objects. More specifically, Lewis argues that the GRW theory must violate the enumeration principle: that ‘if marble 1 is in the box and marble 2 is in the box and so on through marble n, then all n marbles are in the box’ ([1997], p. 321). Ghirardi and Bassi ([1999]) have replied that it is meaningless to say that the enumeration principle is violated because the wavefunction Lewis uses to exhibit the violation cannot persist, according to the GRW theory, for more than a split second ([1999], p. 709). On the contrary, we argue that Lewis’s argument survives Ghirardi and Bassi’s criticism unscathed. We then go on to show that, while the enumeration principle can fail in the GRW theory, the theory itself guarantees that the principle can never be empirically falsified, leaving the applicability of arithmetical reasoning to both micro- and macroscopic objects intact.

1 Wavefunction Collapse Theories and the Tails Problem

2 Lewis’s Counting Anomaly

3 Can the Counting Anomaly Be Avoided?

4 Is the Counting Anomaly Ever Manifest?

5 Is Suppressing the Manifestation of Anomalies Enough?

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1 Wavefunction Collapse Theories and the Tails Problem

The standard Schrödinger dynamics for a quantum system prescribes that its state vector \( |\psi(t)\rangle \) always evolves in time deterministically, and linearly (i.e., that \( |\psi(t)\rangle \)'s evolution is the sum of the separate evolutions of its components in any basis). The standard ‘eigenstate-eigenvalue link’ semantics for quantum states dictates that an observable \( O \) of a quantum system in state \( |\psi(t)\rangle \) possesses a determinate value at time \( t \) if and only if \( O|\psi(t)\rangle = o|\psi(t)\rangle \) for some eigenvalue \( o \) of \( O \) (i.e., if and only if the probability of finding \( o \) in a measurement of \( O \) is 1 at time \( t \)). Unfortunately, the standard dynamics and semantics for quantum states together give rise to the measurement problem; they force the conclusion that a cat can be neither alive nor dead, and, worse, that a competent observer who looks upon such a cat will neither believe that the cat is alive nor believe it to be dead. The standard way out of the measurement problem is to keep the standard semantics and temporarily suspend the standard dynamics by invoking the collapse postulate. According to this postulate, the state vector \( |\psi(t)\rangle \), representing a composite interacting ‘measured’ and ‘measuring’ system, stochastically collapses, at some time \( t' \) during their interaction, into one of \( |\psi(t')\rangle \)'s components in the interaction basis. The trouble is that this is not a way out unless one can specify the physical conditions necessary and sufficient for a measurement interaction to occur; for surely ‘measurement’ is too ambiguous a concept to be taken as primitive in a fundamental physical theory.

Collapse theories are designed to cure this defect in the collapse postulate. They specify the precise physical conditions under which collapses are more or less likely to occur, without treating ‘measurement' interactions as different from other interactions contemplated by quantum theory. Our main focus shall be on wavefunction collapse theories. These are theories in which the representation of a state vector \( |\psi(t)\rangle \) as a function \( \psi(t, r_1, \ldots, r_N) \) on the
configuration space of the system is taken to be fundamental, and a collapse increases the concentration of the amplitude of $\psi(t, r_1, \ldots, r_N)$ in some region of configuration space.

Collapse, so construed, faces two obstacles. First, the smaller the region in which a wavefunction’s amplitude is concentrated by a collapse, the higher the collapsed state’s dispersion will be in momentum space (by the uncertainty relation), and hence the more energy the system can possess after collapse. So wavefunction collapses had better not make a macroscopic system’s wavefunction too narrow, otherwise the system could spontaneously heat up in an observable way. Secondly, in order to be empirically adequate, wavefunction collapse theories need to predict that collapses of a microscopic system’s wavefunction rarely occur, because the standard Schrödinger evolution of a microscopic system is overwhelmingly confirmed through interference experiments. But it is a well-known feature of the Schrödinger equation that it prevents the wavefunction of a closed system of particles from ever developing a support confined to a bounded region of their configuration space (except at isolated instants of time)\(^2\). Now if a system’s wavefunction cannot be made arbitrarily narrow by a collapse, and if it can never be completely concentrated in a bounded region—i.e., if a system’s wavefunction must virtually always possess ‘tails’ going off to infinity—then the standard semantics will block the attribution of a determinate location to each particle in the system, as well as to the position of the system as a whole. For a macroscopic system, it would then appear that the wavefunction collapse theorist has little hope of finally putting the measurement problem to rest. How one

\(^2\)And this is not simply an artifact of nonrelativistic quantum mechanics. Restricting to the positive energy solutions of the Dirac equation, they all have infinite support (Thaller [1992]). In fact, this is a consequence of the following much more general result. If the Hamiltonian generating the time evolution of a free relativistic particle has a spectrum bounded from below, and the particle is localized to a bounded region at $t = 0$, then there is a nonzero probability of finding the particle \textit{arbitrarily far away at any} later time (Hegerfeldt [1995]).
should deal with this problem, known as the wavefunction *tails problem*, is the subject of the present note.

In fact there is a standard solution to the tails problem, which Albert and Loewer ([1996]) have recently argued for at length. The solution is to weaken the eigenstate-eigenvalue link as regards the position of a particle by taking a particle to be located in some *region* of space just in case its wavefunction is *almost* an eigenstate of being located in that region. Restricting attention to localizing the particle in a region sidesteps the problem that its wavefunction can never be infinitely narrow. And taking the high probability of finding a particle in a region to be sufficient for asserting that it actually is in that region sidesteps the problem of infinite tails. Specifically, Albert and Loewer propose a weakened eigenstate-eigenvalue link for position that they call PosR:

‘Particle *x* is in region *R*’ if and only if the proportion of the total squared amplitude of *x*’s wave function which is associated with points in *R* is greater than or equal to 1 − *p*. (Albert and Loewer [1996], p. 87)

Albert and Loewer require that *p* lie somewhere in the interval (0, 0.5) (for *p* ≥ 0.5 would allow one to say that a particle lies in disjoint regions); otherwise, they argue that *p* may be taken to have any of the small continuum of values that can underwrite the way we actually use the word ‘located’ ([1996], p. 90). The obvious generalization of PosR to a multi-particle system would be (where × denotes Cartesian product):

‘Particle *x* lies in region *R*\(_x\) and *y* lies in *R*\(_y\) and *z* lies in *R*\(_z\) and \ldots’ if and only if the proportion of the total squared amplitude of \(\psi(t, \mathbf{r}_1, \ldots, \mathbf{r}_N)\) that is associated with points in \(R_x \times R_y \times R_z \times \cdots\) is greater than or equal to 1 − *p*.

We shall call this generalization of PosR, which Albert and Loewer do not explicitly endorse, the *fuzzy link*.
At first glance it would seem that the fuzzy link, with a suitable value for $p$ selected, promises to yield an unproblematic interpretation of collapse theories, and allow them to fulfill their goal of representing everyday macroscopic objects, like cats, as possessing reasonably well-defined locations. However, Peter Lewis ([1997]) has recently argued that the ‘spontaneous localization’ wavefunction collapse theory of GRW (Ghirardi, Rimini, and Weber [1986]), in virtue of its need to rely on the fuzzy link to solve the tails problem, has the unacceptable consequence that arithmetic does not apply to ordinary macroscopic objects. We believe that what Lewis succeeds in showing is that the GRW theory, interpreted in terms of the fuzzy link, sometimes entails a failure of conjunction introduction; that is, it entails that there can be certain physical situations where a proposition $A_1$ is true, $A_2$ is true, $\ldots$, $A_n$ is true, yet the conjunction $A_1 \land A_2 \land \ldots \land A_n$ (or $(\forall i)A_i$) is false. However, we shall show that the GRW theory itself guarantees that conjunction introduction can never be experimentally falsified, leaving the applicability of arithmetic to macroscopic objects intact.

In the next section, 2, we briefly review the essentials of the GRW theory needed for Lewis’ argument, and then spell out the argument itself. In section 3, we show why the response to Lewis’ argument recently given by Ghirardi and Bassi ([1999]) is unsuccessful, and consider other possible responses. Section 4 contains our demonstration that the failure of arithmetic for macroscopic objects that Lewis alleges (what we prefer to call a failure of conjunction introduction) can never become manifest in a world governed by GRW wavefunction collapses. In our final section, 5, we briefly discuss the larger issue: What epistemic stance should be taken towards interpretations of quantum theory that are forced to posit ‘anomalies’ which are never made manifest?
2 Lewis’s Counting Anomaly

We begin by recalling the ingredients of the GRW theory that are important to assessing Lewis’ ([1997]) argument and Ghirardi and Bassi’s ([1999]) reply. According to the GRW theory, the quantum state of an $N$-particle system evolves in accordance with Schrödinger’s equation except when a ‘hit’ occurs on one of the particles in the system. When a ‘hit’ occurs on the $i$th particle, the total wave function $\psi(t, r)$ for the system (abbreviating $(r_1, \ldots, r_N)$ by $r$) instantaneously collapses to:

$$\psi'(t, r) = \frac{j(x - r_i)\psi(t, r)}{R_i(x)}.$$  \hspace{1cm} (1)

The jump factor $j$ effecting the hit is taken to be a normalized Gaussian of relatively narrow width $10^{-5}$ cm, and a hit on the $i$th particle is posited to occur with probability $10^{-15}$ per second, for any $i$. ($R_i(x)$ is simply a renormalization factor, equal to $\int |j\psi|^2 d^3r_i$.) The hit center $x$, specified in $j$’s first argument, is randomly chosen with probability distribution $|R_i(x)|^2 d^3x$. This ensures that the localization of the probability distribution for the $i$th particle to the region surrounding a point $x$ in space occurs with the probability given by the standard quantum-mechanical Born rule. For the quantum state of a microscopic system—a system with much less than $10^{15}$ particles—the particles in the system will almost never be hit, so their total wavefunction will almost never collapse but, rather, will just evolve in accordance with Schrödinger evolution. On the other hand, the GRW theory ensures that the total wavefunction of a macroscopic system—a system with more than $10^{15}$ particles—will collapse very rapidly.

There are some important features of GRW collapse that need to be kept in mind for what follows. Suppose an ordinary macroscopic object, like a marble, is in a superposition of different states, where each state corresponds to the marble being well-localized to a region of space, but the regions, which we denote by $L$ and $R_i$, are widely separated:

$$\psi(t, r) = c_L \psi_L(t, r) + c_R \psi_R(t, r).$$  \hspace{1cm} (2)
If one of the particles in the marble is hit (which is virtually inevitable, given
the number of particles in the marble), \( \psi(t, r) \) will most likely collapse to a
state which is similar to the state of one of the two terms appearing in (3).
This occurs because the position of each particle in the marble is correlated
to the position of every other particle, which ensures that when one particle
is hit, it is as if the same jump factor were hitting every particle. Thus, if
just one particle \( i \) is hit with a jump factor \( j(x - r_i) \), and if \( x \in L \) (which
will occur with probability \( \approx |c_L|^2 \)), then the result will be:

\[
\psi'(t, r) = c_L j(x - r_i)\psi_L(t, r)/R_i(x) + c_R j(x - r_i)\psi_R(t, r)/R_i(x).
\]  

(3)

Since the region in which \( \psi_R \) is large is, by hypothesis, a region where \( j \) is
vanishingly small, the norm of the second term is now very small relative to
the first. Thus a single hit has precipitated an effective collapse of the total
wavefunction \( \psi(t, r) \) onto its first term. The collapse is only effective since
the second term of (3) is never zero, and will form part of the residual ‘tails’
of the wavefunction \( \psi'(t, r) \) in configuration space. By contrast, suppose we
have a pair of non-interacting marbles in an unentangled product state:

\[
\psi(t, r, r') = \psi_1(t, r)\psi_2(t, r').
\]  

(4)

In this case, any number of hits on the particles in one of the marbles will
leave the other marble’s wavefunction unchanged. For example, a hit on the
ith particle in marble 1 will just produce:

\[
\psi'(t, r, r') = \frac{j(x - r_i)\psi_1(t, r)}{R_i(x)}\psi_2(t, r')
\]  

(5)

(where \( R_i(x) = \int |j\psi_1|^2 d^{3N}r \)). It follows that in the absence of entanglement
between the marbles, they will each be subject to independent GRW collapse

\(^3\)Our use of the phrase ‘effective collapse’ in this context should be sharply distin-
guished from the way that phrase is sometimes employed in the context of no-collapse
interpretations to refer to the fact that—due to ‘environmental decoherence’—the state of
a macrosystem can be treated for all practical purposes as if it were no longer involved in
a superposition of macroscopically distinguishable states.
processes that will preserve the product character of their total wavefunction for as long as they fail to interact.

We turn now to Lewis’s ([1997]) argument. He first considers a marble that can be either in or out of a box. The eigenstate of the marble being in the box is $|\text{in}\rangle$, while the eigenstate of the marble being outside the box is $|\text{out}\rangle$. Suppose the marble starts out in the state

$$\frac{1}{\sqrt{2}}(|\text{in}\rangle + |\text{out}\rangle)$$

(which would correspond to a wavefunction $\psi(t, r)$ with both $\int_{\text{in}} |\psi|^2 d^3N r$ and $\int_{\text{out}} |\psi|^2 d^3N r$ equal to $1/2$). Lewis observes that GRW’s collapse dynamics will almost always leave the system in either a state like

$$a|\text{in}\rangle + b|\text{out}\rangle,$$

or a state like

$$a|\text{out}\rangle + b|\text{in}\rangle,$$

where $1 > |a|^2 \gg |b|^2 > 0$. Applying the eigenstate-eigenvalue link to either collapsed state, the marble is neither in nor out of the box. But according to the fuzzy link for $|b|^2 \leq p$, the marble is either determinately in or determinately out of the box. Moreover, if we suppose the post-collapse state of the marble is in fact $|7\rangle$, so that according to the fuzzy link the marble is in the box, then when one measures the location of such a marble, one could obtain the result that the marble is out of the box. The probability of this happening, $|b|^2$, is extremely low, but according to the GRW theory it could happen. Still, there is nothing contradictory in this state of affairs. One simply has to accept that very rarely a measurement of a marble’s location will cause it to jump to a location disjoint from the location it had, according to the fuzzy link, prior to the measurement. Indeed, even in the absence of measurement, the marble could well make such a jump spontaneously. Lewis’s counting anomaly can be seen as an attempt to magnify this unlikely anomaly to the point of absurdity.
Thus, Lewis next considers a system of $n$ non-interacting marbles, each of which is in a state like (7):

$$|\psi\rangle_{\text{all}} = (a|\text{in}\rangle_1 + b|\text{out}\rangle_1) \otimes (a|\text{in}\rangle_2 + b|\text{out}\rangle_2) \otimes \cdots \otimes (a|\text{in}\rangle_n + b|\text{out}\rangle_n).$$

(9)

Lewis takes each $|\text{in}\rangle_i$ state to refer to localization within a single box which is the same for all the marbles. He also needs to assume that the marbles are noninteracting, which can be ensured by making the dimensions of the box sufficiently large. When one measures the location of each marble in turn, the probability that all $n$ marbles will be found to be in the box is

$$|\langle \psi | \text{all} | \langle \text{in} |_1 \otimes | \text{in} |_2 \otimes \cdots \otimes | \text{in} |_n \rangle|^2 = |a^n|^2 = |a|^{2n}.$$\((10)\)

Since $|a|^2 < 1$ (notwithstanding that $|a|^2$ could be quite close to 1), $|a|^{2n} \ll 1$ for sufficiently large $n$, which makes it highly unlikely that all the marbles will be found in the box. Lewis concludes from this that the state $|\psi\rangle_{\text{all}}$ ‘cannot be one in which all $n$ marbles are in the box, since there is almost no chance that if one looks one will find them there’ ([1997], p. 318). In other words, by applying the fuzzy link for $|a|^{2n} \leq p$ to $|\psi\rangle_{\text{all}}$, one obtains the result that not all the marbles are in the box. And this seems to contradict the results one obtains when one applies the fuzzy link on a marble-by-marble basis, where one gets the results that marble 1 is in the box, marble 2 is in the box, and so on through marble $n$.

4Alternatively, one could assume that each state $|\text{in}\rangle_i$ refers to a separate box, and that the $n$ boxes, one for each marble, are pairwise widely separated in space.

5Note the similarity between this result and the ‘lottery paradox’. For any particular ticket holder (in a lottery with a sufficiently large number of tickets) one is inclined to infer, from the ticket holder’s high probability of losing, that they will in fact lose (cf. ‘this particular marble is in the box’). Yet, if the same inference is made for each ticket holder in turn, we would apparently arrive at the absurd conclusion that we are certain that no one will win the lottery (cf. ‘all the marbles are in the box’). But note well: while we would be wholly within our rights to draw back from the inference from high probability to certainty that generates the lottery paradox, exercising the same freedom against Lewis’s counting paradox would mean rejecting the fuzzy link and, consequently, send the wavefunction collapse theorist right back to the tails problem.
Actually, as Lewis points out ([1997], p. 321), the contradiction only holds if one assumes the *enumeration principle*: ‘if marble 1 is in the box and marble 2 is in the box and so on through marble \(n\), then all \(n\) marbles are in the box’. This principle is but a special case of conjunction introduction: if \(A_1\) is true, \(A_2\) is true, and so on through \(A_n\), then \(A_1 \land A_2 \land \ldots \land A_n\) (that is, \((\forall i)A_i\)) is true. Thus the fuzzy link, the GRW theory, and conjunction introduction jointly entail a contradiction. The moral Lewis draws from this contradiction is that no-collapse interpretations of quantum theory not subject to the tails problem, such as Bohm’s ([1952]) theory, are to be preferred over wavefunction collapse theories like GRW’s. We believe this conclusion is far too quick, so our next task will be to examine some other possible routes around the contradiction.

### 3 Can the Counting Anomaly be Avoided?

Recently Ghirardi and Bassi ([1999]) have claimed that Lewis’s contradiction between the fuzzy link and and the enumeration principle (or conjunction introduction) in fact *fails to arise* in the GRW theory. They consider the system in state \(\psi_{\text{all}}\), and point out that it can be rewritten as the following superposition of \(2^n\) macroscopic states:

\[
\psi_{\text{all}} = a^n |\text{in}_1\rangle \otimes |\text{in}_2\rangle \cdots |\text{in}_n\rangle + a^{n-1}b |\text{out}_1\rangle \otimes |\text{in}_2\rangle \cdots |\text{in}_n\rangle \\
+ a^{n-1}b |\text{in}_1\rangle \otimes |\text{out}_2\rangle \cdots |\text{in}_n\rangle + \cdots + a^{n-2}b^2 |\text{out}_1\rangle \otimes |\text{out}_2\rangle \cdots |\text{out}_n\rangle \\
+ \cdots
\]

They then write:

The marbles are macroscopic objects, and, as such, they contain a number of particles of the order of Avogadro’s number. But it is the most fundamental physical characteristic of the GRW theory that it forbids the persistence of superpositions of states of this
kind. In particular for the case under consideration the precise
GRW dynamics will lead in about one millionth of a second to the
suppression of the superposition and the “spontaneous reduction”
of the state [i.e. $|\psi\rangle_{\text{all}}$] to one of its terms (with the probability
attached to it by its specific coefficient). (Ghirardi and Bassi
[1999], p. 708)

Ghirardi and Bassi go on to argue ([1999], Section 4) that even if we assume
$n$ to be so large that all the mass of the universe is used to constitute the
$n$ marbles, $|\psi\rangle_{\text{all}}$ is still overwhelmingly likely to GRW collapse to the first
term in (11) given how close $|a|^2$ will in fact be to 1 for an object the size of a
marble undergoing GRW collapse. But their main conclusion, with reference
to $|\psi\rangle_{\text{all}}$, is that “it is meaningless to make any statement about the location
of the marbles in such states simply because they cannot persist for “more
than a split second” ’ ([1999], p. 709).

Let us suppose that Ghirardi and Bassi are correct that GRW collapse
will rapidly produce a reduction of the state $|\psi\rangle_{\text{all}}$ to one of the terms in
(11) (setting aside, for the moment, to which term $|\psi\rangle_{\text{all}}$ is most likely to
collapse). Of course, if such a reduction were to occur, it would have to
happen in such a way that the squared modulus of the coefficient of one of
the terms in (11) became large, and the other coefficients became small; for
a perfect collapse to one of (11)’s terms would leave the individual wavefunc-
tions of the marbles without tails. Also recall from Section 2 that, since the
noninteracting marbles begin in the product state $|\psi\rangle_{\text{all}}$ (which is a product
state, notwithstanding the fact that $|\psi\rangle_{\text{all}}$ may be rewritten as in (11)), the
marbles’ final effectively collapsed state must again be a product state. So
no matter what GRW collapses occur in state (11), the final state must have
the form:

$$|\psi'\rangle_{\text{all}} = (a_1|\text{in}\rangle_1 + b_1|\text{out}\rangle_1) \otimes (a_2|\text{in}\rangle_2 + b_2|\text{out}\rangle_2) \otimes \cdots \otimes (a_n|\text{in}\rangle_n + b_n|\text{out}\rangle_n)$$

(12)
where all the \( a_i \)'s and \( b_i \)'s are nonvanishing. Ghirardi and Bassi’s claim, on behalf of \(|\psi'\rangle_{\text{all}}\), must then be that when we re-expand \(|\psi'\rangle_{\text{all}}\), as \(|\psi\rangle_{\text{all}}\) was expanded in (11) above, the absolute square of one of the terms’ coefficients (there will again be \( 2^n \) terms) will now be \( \geq 1 - p \). However, by hypothesis we started out in a state \(|\psi\rangle_{\text{all}}\) wherein all the squares of the coefficients in (11) were bounded above by \(|a|^2\), which itself was supposed to be much less than 1. So the only way for the marbles to end up as Ghirardi and Bassi claim is that a large number of the marbles in \(|\psi\rangle_{\text{all}}\), in a very short time, have either their \( a \) or \( b \) coefficients driven by GRW collapse processes closer (in absolute square) to 1 than \( a \)'s value was. That is, the only way to make sense of Ghirardi and Bassi’s claim (consistent with Lewis’s standing assumptions that the marbles form an isolated system and do not interact, assumptions that entail that the marbles will be subject to independent GRW collapses that preserve the product character of their total state) is to suppose that in the final state \(|\psi'\rangle_{\text{all}}\) various coefficients, say \( a_1, b_2, a_3, \) etc. will have their absolute squares much closer to 1 than \( a \)'s was, yielding a value \(|a_1|^2|b_2|^2|a_3|^2 \cdots \geq 1 - p\) that restores consistency with the enumeration principle.

But herein lies the rub. For the purposes of his argument, Lewis supposed that the initial product state of the marbles was already one in which \(|a|^2\) had been driven by GRW collapse as close to 1 as it can be (a perfectly legitimate assumption, since if there were no upper limit on \(|a|^2\), there would be no tails problem to begin with). He was then free to choose a sufficiently large value of \( n \) with which to run his argument. It follows that the collapse scenario Ghirardi and Bassi envisage, in which some of the \(|a|^2\)'s get still closer to 1, has already been taken into account by Lewis’s argument, and cannot supply a basis from which to launch a criticism of that argument. Here is another way to see the point. Suppose we grant that \(|\psi\rangle_{\text{all}}\) very quickly evolves to a state of form \(|\psi'\rangle_{\text{all}}\). Then Lewis would still be free to exploit this latter state as the starting point for his argument. He could, first,
drop from consideration those marbles where $|a_i|^2 < 1 - p$. For the marbles that remain, they will be in a product state $|\psi''\rangle_{\text{all}}$ again of form (12) where, now, all the $a_i$'s have absolute squares within $p$ of 1 (and thus each individual marble will be in the box, according to the fuzzy link). Lewis could, then, simply consider sufficiently many marbles in a product state formed from sufficiently many tensor products of $|\psi''\rangle_{\text{all}}$ with itself in order to guarantee, yet again, that the probability that they are all in the box is less than or equal to $p$!

This leads us to Ghirardi and Bassi’s other criticism of Lewis’s argument. In effect, they challenge Lewis’s application of the multi-particle fuzzy link to $|\psi\rangle_{\text{all}}$ (or, if you prefer, to tensor products of $|\psi''\rangle_{\text{all}}$ with itself) by questioning whether it would actually be possible to produce enough marbles so that the probability that they are all in the box in a state like $|\psi\rangle_{\text{all}}$ is less than or equal to some small number $p$. They calculate that if each marble possesses a mass of about 1 gram, which puts precise limits on the value of $a$, and if we allow ourselves the mass of the entire universe ($\approx 10^{53}$ grams) with which to constitute the marbles, which sets a limit on the value of $n$, then the overwhelmingly most likely configuration of the marbles in state $|\psi\rangle_{\text{all}}$ will, in actual fact, still be the one where they are all in the box. In other words, under the given assumptions about marbles and our universe, it turns out that $|a|^2 \geq 1 - p$ for any reasonably small value for $p$. We see no reason to doubt this. However, it is at best a contingent fact about our world, since no matter how close to 1 GRW collapses will make $|a|^2$ for a macroscopic object like a marble, one could always imagine a sufficiently massive universe where $n$ is large enough that $|a|^{2n}$ is close to 0 and the contradiction between the fuzzy link, conjunction introduction, and the GRW theory remains. We believe a stronger response to Lewis’s counting anomaly would be one in which the force of the anomaly is muted for reasons internal to the GRW theory itself. Moreover, although Lewis clearly wants the anomaly to obtain for macroscopic objects, there would still be something puzzling about its ob-
taining for microscopic objects like particles. Yet if we replace each marble with a particle, which will be hit only very rarely; there is no reason internal to the GRW theory why $|a|^2$ should have to be so close to 1 that $|a|^{2n}$ cannot be close to 0 for a large number $n$ of particles in our universe.

Where do these shortcomings in Ghirardi and Bassi’s reasoning leave their claim that $|\psi\rangle_{\text{all}}$ will be ‘transformed immediately into a perfectly reasonable (from the point of view of the enumeration principle) state’ ([1999], p. 709)? Let us see. We can either suppose that the marbles are microscopic particles, or that the universe is sufficiently massive; we shall persist in telling the story using marbles in the state $|\psi\rangle_{\text{all}}$ (and the product of $|\psi''\rangle_{\text{all}}$ with itself enough times would do equally well). Since the marbles are non-interacting by hypothesis, when one of the particles in a marble is hit, the states of the other marbles will not be affected. Obviously if no hits produce marbles jumping out of the box, then the failure of the enumeration principle will persist. So suppose, instead, that a collapse occurs in such a way that marble 1 jumps out of the box. This is a very rare occurrence, but with enough marbles, one of them is bound to make the jump, and we lose no generality in assuming it is marble 1. The state of the system will then be

$$|\psi\rangle_{\text{out}_1} = (c|\text{in}_1\rangle + d|\text{out}_1\rangle) \otimes (a|\text{in}_2\rangle + b|\text{out}_2\rangle) \otimes \cdots \otimes (a|\text{in}_n\rangle + b|\text{out}_n\rangle),$$

where $|d|^2 \approx |a|^2$ (supposing, once more, that the value $|a|^2$ is about as close to 1 as GRW collapse can achieve). Now let $A_1$ denote the proposition that ‘marble 1 is in the box’, $A_2$ that ‘marble 2 is in the box’, and so on. According to the fuzzy link applied to each marble in state $|\psi\rangle_{\text{out}_1}$, it is true that marble 1 is not in the box, that marble 2 is in the box, 3 is in the box, and so on through marble $n$. By conjunction introduction, then, $-A_1 \land A_2 \land \cdots \land A_n$ is true. Yet with the fuzzy link applied to all $n$ marbles, this conjunction is false, since

$$|\langle \psi|_{\text{out}_1} \otimes |\text{out}_1\rangle \otimes |\text{in}_2\rangle \otimes \cdots \otimes |\text{in}_n\rangle|^2 = |da^{n-1}|^2 = |d|^2|a|^{2(n-1)},$$

$$|$14$
and, by hypothesis, $|d|^2|a|^{2(n-1)} ≈ |a|^{2n} ≤ p$ for sufficiently large $n$. Thus, the contradiction still obtains and the initial failure of conjunction introduction will simply be propagated via GRW evolution into the failure of another instance of conjunction introduction. This remains true no matter how many marbles make jumps or how rapidly GRW collapses occur. For at the moment after any number of jumps have occurred, there will always remain some conjunction relative to which conjunction introduction fails. The conclusion, we think, is inevitable: GRW collapse evolution in fact cannot suppress the failure of conjunction introduction in a sufficiently large isolated system of non-interacting marbles that evolve from a product state.

There are at least two other possible strategies one might adopt to avoid Lewis’s counting anomaly in the GRW theory. Both strategies involve modifying the fuzzy link.

One route around Lewis’s counting anomaly might be to employ different values for the fuzzy link’s $p$: one value, $p$, when the fuzzy link is applied to an individual marble’s probability distribution, and another value, $p_{\text{all}}$, when the link is applied to their total distribution as determined by $|\psi\rangle_{\text{all}}$. For any range of values in the interval $(0, 0.5)$ that one believes it is necessary to insist upon in order to underwrite our uses of the term ‘located’, one could always make sure both $p$ and $p_{\text{all}}$ are chosen from within one’s preferred range in such a way that $(1-p)^n ≥ 1-p_{\text{all}}$. If, then, each of the marbles has at least $1-p$ of its probability concentrated within the box, i.e. if $|a|^2 ≥ 1-p$, then each will get counted as actually in the box, and the conjunction of all those assertions will have the minimum necessary probability of $1-p_{\text{all}}$ for it to be counted as true as well! On the other hand, if $|a|^2 < 1-p$, so that no single marble is in the box, or if $|a|^{2n} < 1-p_{\text{all}}$, so that they are not all in the box, then the issue of a failure of conjunction introduction does not arise. Unfortunately, this clever strategy simply resurrects the tails problem. To guard against failures of conjunction introduction, one would always need to choose $p$ so that $(1-p)^n ≥ 1-p_{\text{all}} > 0.5$. But this means that as $n$ grows
large, $p$ must be chosen closer and closer to zero no matter what value is assumed for $p_{\text{all}}$. Moreover, as we have seen, there will be always some value, $|a|^2$, close to 1 such that GRW collapses cannot localize the marbles to the box with a higher probability than $|a|^2$ (and this would also be true were the marbles replaced by particles). Therefore, we could consider a sufficiently large value of $n$ such that one is forced by the inequality $(1 - p)^n > 0.5$ to choose a $p$ satisfying $|a|^2 < 1 - p$. In that case, though no failure of conjunction introduction ensues, the fuzzy link will dictate that no marble will be in the box. So no matter how well GRW collapses can concentrate the marbles’ wavefunctions in the box, we would be forced to conclude that no marble can be in the box, returning us right back to the tails problem.

Second, one might argue that the correct way to resolve the tails problem is to endorse just PosR—the fuzzy link applied only on an individual particle basis. One could then take facts about the joint positions of systems of particles, like marbles and collections of marbles, to supervene directly on facts about the positions of individual particles, rather than on their total wavefunction. Albert and Loewer themselves only formulate the fuzzy link for individual particles, and their willingness to suppose that ‘the value of the cat’s aliveness is determined by the positions of the particles that make it up’ ([1996], p. 86) suggests that they might prefer this strategy. Certainly this strategy would allow one to assert that all the marbles are in the box, notwithstanding their total state $|\psi\rangle_{\text{all}}$. However, since $|\psi\rangle_{\text{all}}$ is almost an eigenstate of ‘all the marbles are in the box’ being false, this strategy would require that the wavefunction collapse theorist not simply weaken the eigenstate-eigenvalue link between truth and probability 1, but sever this link entirely. And if one is willing to entertain the thought that events in a quantum world can happen without being mandated or made overwhelmingly likely by the wavefunction, then it is no longer clear why one should need to solve the measurement problem by collapsing wavefunctions! Another reason not to restrict to PosR alone is that it seems arbitrary to apply
a semantic rule for quantum states to a single-particle system, but not to a multi-particle system. Indeed, to the extent that one supposes there to be a plausible intuitive connection between an event’s having high probability according to a theory, and the event actually occurring, one is hard-pressed to resist the intuition in the multi-particle case. Finally, it is not clear how one could even make the distinction between PosR and the (full) fuzzy link in more sophisticated theories, like the ‘continuous spontaneous localization’ theory of Ghirardi, Grassi, and Pearle ([1990]), where talk of particles is replaced by talk of systems in near eigenstates of local mass density.

Even if one does not regard the above considerations as decisive against modifying the fuzzy link, neither Lewis nor Ghirardi and Bassi question this link, and we shall argue in the next section that the price of sometimes abandoning conjunction introduction is not near as high as Lewis portrays. But lest one think that conjunction introduction is an analytic truth, it is worth pointing out that abandoning analogous principles in the context of the interpretation of quantum theory is not unprecedented. In the quantum logic that Kochen and Specker ([1967]) advocate, conjunctions of quantum-mechanically incompatible propositions are not syntactically well-formed, and in Bell’s ([1986]) quantum logic, conjunction introduction is an invalid inference rule. Moreover, as Clifton ([1996], p. 386) points out, various modal interpretations deny property composition, a principle which roughly says that if system $S_1$ has some property and system $S_2$ has some property, then system $S_1 + S_2$ has the corresponding joint property. Property composition can be seen as a version of conjunction introduction, and note that the ‘conjuncts’, in this case, are compatible.

But, it will be argued: so much the worse for quantum logics and modal interpretations! However, as Albert and Loewer ([1996], p. 87) point out, even before multi-particle systems are considered, the GRW theory, interpreted via the PosR rule, violates the principle of property intersection: that if particle $x$ lies in region $\Delta$, and $x$ lies in $\Delta'$, then $x$ must lie in $\Delta \cap \Delta'$. The
violation of this principle is an easy consequence of PosR’s identification of ‘x lying in Δ’ with ‘x having a probability of at least 1-p of being found in Δ’.

And it is at least arguable that once property intersection fails, so must conjunction introduction—at least if we want ‘x lying in Δ’ to mean ‘no part of x lies outside of Δ’.

For if no part of x lies outside of Δ, and no part lies outside of Δ’, then, in particular, no part of x lies inside Δ but outside Δ’, nor does any part of x lie inside Δ’ but outside Δ. By conjunction introduction, then, it follows that no part of x lies outside of Δ ∩ Δ’, i.e., that x lies in Δ ∩ Δ’, and we have derived property intersection. (Clifton ([1996], pp. 381-2) gives this same argument to highlight the import of property intersection’s violation in Healey’s modal interpretation.)

We are not suggesting by these remarks that anomalies, such as the failure of conjunction introduction, should simply be swept under the carpet. Indeed, Albert and Loewer take a major part of their task to be to provide reasons for thinking that with a sufficiently small value chosen for p in PosR, violations of property intersection ‘aren’t going to be worth bothering about’ ([1996], p. 89). Our final task will be to make the same kind of point in relation to Lewis’ multiple-particle failure of conjunction introduction (though, as we have seen above, playing with the value of p will be of no help).

4 Is the Counting Anomaly Ever Manifest?

Recall that Lewis presents his argument as demonstrating a failure of the enumeration principle. He gives the following argument for why we should not give up this principle:

If we want to maintain that the enumeration principle breaks down at some point, then, we must maintain that the process of counting marbles breaks down—that counting cannot be applied to sufficiently large systems of marbles. Since counting is the foundation of arithmetic, this is tantamount to saying that
Lewis then argues, and we agree, that the cost of holding that arithmetic does not apply to large systems of marbles is too high. But we do not agree that Lewis has established the conditional that we italicized above. The trouble is that Lewis fails to operationalize the process of counting marbles by explicitly modelling the process itself in terms of collapsing GRW wavefunctions. Indeed, by calling the failure at issue a failure of the enumeration principle, with all the operational connotations of the term ‘enumeration’, Lewis fails to keep the failure of conjunction introduction distinct from its empirical falsification. Moreover, taking the objects at issue to be macroscopic marbles conveys the impression that the relevant failure of ‘enumeration’ is somehow already manifest in state $|\psi\rangle_{\text{all}}$, but that is only the case if we allow the enumerator the powers of Maxwell’s demon. Lewis shows signs of being aware of this concern, when he writes (just before the remarks quoted above):

> To deny the enumeration principle, we must deny that if we put marble 1 in the box, and we put marble 2 in the box, and so on through marble $n$, then there are $n$ marbles in the box. This process of putting marbles in a box is essentially one of counting the marbles [italics ours].

However, if marbles are ‘put into’ the box one-by-one, presumably interactions between marble placing devices and marbles will have to take place. But then it is no longer clear how one could thereby prepare the state $|\psi\rangle_{\text{all}}$, which presupposes that the marbles are not entangled in any way with each other or their environment. In any case, rather than seeking to make Lewis’s ‘putting’ metaphor physically concrete, we shall simply grant that $|\psi\rangle_{\text{all}}$ has been prepared by some means and, instead, ask whether the marbles’ (or particles’) instantiation of a failure of conjunction introduction can ever become manifest to a physical observer who takes it upon herself to tally them
up. Our answer is ‘No’, and it will be clear that our considerations also go through for a system in state $|\psi\rangle_{\text{out}1}$ and in any other of the states to which $|\psi\rangle_{\text{all}}$ (or relevantly similar product states of micro- or macroscopic objects) can evolve.

The most straightforward way to manifest a failure of conjunction introduction for a system in state $|\psi\rangle_{\text{all}}$ would be to empirically establish the truth of each of $A_1, A_2, \ldots, A_n$, together with an independent empirical test of the truth of $\neg(A_1 \land A_2 \land \cdots \land A_n)$.

An ideal measurement of whether marble 1 is in the box would correlate orthogonal states of a macroscopic measuring apparatus to the $|\text{in}\rangle$ and $|\text{out}\rangle$ states of the marble. Since this must be done for all $n$ marbles, $n$ apparatuses must be used. The marbles/apparatuses system will evolve from the state

$$[(a|\text{in}\rangle_1 + b|\text{out}\rangle_1) \otimes \cdots \otimes (a|\text{in}\rangle_n + b|\text{out}\rangle_n)] \otimes |\text{ready}\rangle_{M_1} \otimes \cdots \otimes |\text{ready}\rangle_{M_n} \quad (15)$$

to the state

$$(a|\text{in}\rangle_1|\text{in}^\prime\rangle_{M_1} + b|\text{out}\rangle_1|\text{out}^\prime\rangle_{M_1}) \otimes \cdots \otimes (a|\text{in}\rangle_n|\text{in}^\prime\rangle_{M_n} + b|\text{out}\rangle_n|\text{out}^\prime\rangle_{M_n}). \quad (16)$$

Since $|b|^2 \leq p$, the fuzzy link dictates that each apparatus records that its marble is in the box. This procedure does not yet qualify as a counting procedure, since we have not yet modelled an apparatus which records how many marbles are in the box. One might think that the information about each individual marble in the marble apparatuses, which could be further correlated to different memory stores in an observer’s brain, could simply be combined ‘in thought’ to get direct information about how many marbles are in the box. However, if we are not simply going to beg the question against verifying the enumeration principle, acquiring information about the marble count must itself be modelled in the GRW theory by a further interaction with the marbles/apparatuses system or within the observer’s brain.

Let us turn, then, to a generic counting procedure that will establish how many marbles are in the box. No doubt there are many ways to implement
counting physically, but the general scheme will need to involve a measure-
ment on the marbles/apparatus system that is the equivalent of asking the
question: ‘How many marbles are in the box?’ Let \( O \) be an observable with
\( n + 1 \) eigenvalues \( \alpha_i \), where the \( \alpha_i \)-eigenspace of the operator associated with
\( O \) is the subspace spanned by all the terms in the superposition (16) which
have as coefficient \( a^i b^{n-i} \). Thus, a system in an \( \alpha_i \)-eigenstate of \( O \) is one
where exactly \( i \) of the \( n \) marbles are in the box. Consider a measurement
of the observable \( O \) on a system in state (16). After the measurement, the
system is in the state

\[
|\psi\rangle_{\text{count}} = a^n |\phi\rangle_{\text{out}_0} |O = n\rangle_M + a^{n-1} b |\phi\rangle_{\text{out}_1} |O = n - 1\rangle_M + \cdots + b^n |\phi\rangle_{\text{out}_n} |O = 0\rangle_M \tag{17}
\]

where

\[
|\phi\rangle_{\text{out}_0} = |\text{in}\rangle_1 |\text{in}'\rangle_{M_1} |\text{in}\rangle_2 |\text{in}'\rangle_{M_2} \cdots |\text{in}\rangle_n |\text{in}'\rangle_{M_n}
\]

\[
|\phi\rangle_{\text{out}_1} = |\text{out}\rangle_1 |\text{out}'\rangle_{M_1} |\text{in}\rangle_1 |\text{in}'\rangle_{M_2} \cdots |\text{in}\rangle_n |\text{in}'\rangle_{M_n}
\]

\[
+ |\text{in}\rangle_1 |\text{in}'\rangle_{M_1} |\text{out}\rangle_1 |\text{out}'\rangle_{M_2} \cdots |\text{in}\rangle_n |\text{in}'\rangle_{M_n}
\]

\[
\vdots
\]

\[
+ |\text{in}\rangle_1 |\text{in}'\rangle_{M_1} |\text{in}\rangle_1 |\text{in}'\rangle_{M_2} \cdots |\text{out}\rangle_n |\text{out}'\rangle_{M_n}
\]

\[
|\phi\rangle_{\text{out}_n} = |\text{out}\rangle_1 |\text{out}'\rangle_{M_1} |\text{out}\rangle_2 |\text{out}'\rangle_{M_2} \cdots |\text{out}\rangle_n |\text{out}'\rangle_{M_n}.
\]

Since \( |a|^{2n} \leq p \), by the fuzzy link it is not the case that all \( n \) marbles are
in the box. However, each individual marble is still in the box, since for all
\( i \), \( |\langle \psi \rangle_{\text{count}} |\text{in}\rangle_i|^2 \geq 1 - p \). Thus, we have a violation of the enumeration
principle and hence conjunction introduction.

But this does not mean that a failure of the rules of counting has now
become manifest! The state \( |\psi\rangle_{\text{count}} \) is highly unstable given the GRW
dynamics, since we see from (17) that it is an entangled superposition of
states of macroscopic systems, where the various terms of (17) markedly
differ as to the location of the pointer on \( M \)’s dial that registers the value of \( O \). Thus, the GRW dynamics dictates that it is very likely that the total system will effectively collapse onto one of the terms in (17), and that it will do so very quickly, given how many particles in \( M \), the \( M_i \) apparatuses, and the marbles have the potential to be hit\(^6\). If the effective collapse is onto the state \( |\phi\rangle_{\text{out}} |O = n\rangle_M \), then clearly no failure of conjunction introduction becomes manifest, since the results of the various individual apparatuses in that state are in agreement with \( M \). What if the system effectively collapses onto some other term of (17), such as \( |\phi\rangle_{\text{out}1} |O = n - 1\rangle_M \)? In this case, since \( |\phi\rangle_{\text{out}i} \) is itself an entangled state, since its terms (pairwise) differ as to location of at least one of the marbles, and since the \( M_i \) apparatuses and marbles are macroscopic (or, if we are enumerating particles instead of marbles: since their number is extremely large), there will a further quick, effective collapse to one of \( |\phi\rangle_{\text{out}i} \)’s terms. Suppose, for example, that the total state (effectively) ends up as

\[
(|\text{out}_1\rangle |\text{out}_2\rangle_M |\text{in}_2\rangle_{M_2} \cdots |\text{in}_n\rangle_{M_n} |\text{in}_n\rangle_M) |O = n - 1\rangle_M. \tag{18}
\]

Then once again the results of the individual apparatuses are in agreement with \( M \)’s registration of the ‘in’ count as \( n - 1 \), and no failure of conjunction introduction has become manifest.

The same conclusion holds no matter what ‘in’ count \( M \) ends up registering. And if we think, again, of our observer, she might well come to believe that not all the marbles are in the box by looking at the pointer of the \( M \) apparatus and not finding the result ‘\( O = n \)’. Whereas before looking she

---

\(^6\)What is important here is that the total number of particles involved in (17)’s entangled state is sufficiently macroscopic. Thus, the macroscopic apparatuses \( M_i \) and \( M \) could be replaced by single particles, whose positions act as the measurement pointers, and GRW dynamics would still guarantee effective collapse to one of the terms in (17). (And if, further, the collection of marbles is replaced by a collection of particles sufficiently large to produce Lewis’s counting anomaly, their number will also more than likely suffice to produce the same sort of effective collapse of (17).)
might have held the belief, for each individual marble, that it is in the box, she could now, if the system quickly evolved to $[LS]$, believe that marble 1 is out of the box, each of the others are in the box, and that there are $n-1$ marbles in total in the box. She had no empirical justification for forming any belief about the ‘in’ count prior to totalling up the marbles. But upon totalling them up, she will have to initiate a process that amounts to the same thing as setting up the measurement interaction with $M$ and looking at its final pointer reading. It will then be a consequence of her instantiating this process that its outcome in fact agrees with her most current beliefs about the individual marbles!

This conclusion is inescapable even if we allow the observer to ‘form her own opinion’ about the marble count without bothering to look at the pointer’s location on $M$’s dial, so long as we model the observer’s opinion forming process within the GRW theory. Of course, we do not presume to know the contingent details of brain physiology that would be needed for a complete GRW model. However, it suffices to assume that whatever brain interactions instantiate ‘forming opinions about marble counts’, that process is veridical in the following sense. If, prior to making a judgement about the marble count, the observer were in the state

$$|\psi_{\text{out}_0}\rangle|\text{count?’}\rangle = |\text{in}\rangle_1|\text{in’}\rangle_{M_1}|\text{in}\rangle_2|\text{in’}\rangle_{M_2} \ldots |\text{in}\rangle_n|\text{in’}\rangle_{M_n}|\text{count?’}\rangle$$ (19)

—with the different $M_i$ states correlated to separate memory stores in her brain, and $|\text{count?’}\rangle$ denoting the initial state of that part of her brain that stores arithmetical judgements—then, afterwards, she should be in the state

$$|\psi_{\text{out}_0}\rangle|\text{The count is n}\rangle;$$ (20)

and, mutatis mutandis, for

$$|\psi_{\text{out}_1}\rangle|\text{The count is n - 1}\rangle, \ldots, |\psi_{\text{out}_n}\rangle|\text{The count is 0}\rangle.$$ (21)

Thus, if we make the minimal assumption that our observer would be competent to form opinions about marble counts when each individual marble
is *completely* localized (i.e., localized according to the standard eigenstate-eigenvalue link) either inside or outside of the box, it follows that her brain will instantiate exactly the same interaction with the marbles that we have supposed is instantiated by $M$, and, hence, that exactly the same conclusions that we drew above apply. To put it the other way around: the only way to arrange things so that our observer *could* falsify the enumeration principle would be to suppose that she was never a competent enumerator to begin with.

We have shown that, if one first measures the marbles individually, and then enumerates the collection as a whole, no failure of conjunction introduction can become manifest; the process of counting marbles cannot break down. Alternatively, one can consider what happens if one first measures the system as a whole, and then the marbles individually. After $O$ is measured, the marbles/$M$-apparatus system ends up in a state like that of $|\psi\rangle_{\text{count}}$, except without the states of the apparatuses $M_1$ through $M_n$. As before, since this is now an entangled state, the system will effectively collapse onto a state such as

$$|\text{out}\rangle_1|\text{in}\rangle_2\cdots|\text{in}\rangle_n|O = n - 1\rangle_M. \quad (22)$$

When the locations of each marble are now measured individually, the system will in all likelihood end up effectively in state (18), and thus no failure of conjunction introduction becomes manifest. Since the collapse to (18) is only effective, it is also possible, albeit highly unlikely, for the system to further evolve from being effectively (18) to being effectively some other state, like

$$ (|\text{in}\rangle_1|\text{in}'\rangle_{M_1}\cdots|\text{in}\rangle_n|\text{in}'\rangle_{M_n})|O = n\rangle_M \quad (23)$$

To further underscore the point that this conclusion is robust under differing assumptions about brain physiology, note that our considerations remain valid even when we suppose that our observer is like Albert’s ([1992], Figure 7.15) science-fictional character ‘John-2’—capable of registering information about the outside world in the state of a single brain particle. For, as observed in the previous footnote, it is only the sum total of all the particles in $M$, the $M_i$’s, and the marbles (or particles) being counted that needs to be macroscopic.
so that marble 1 jumps back in the box. But, even in such an unlikely scenario, the various apparatuses will still be in agreement at the end of the day; no failure of conjunction introduction is manifest. And, again, one could run through the same kind of treatment within the brain of an observer.

By considering cases like this, the general strategy of our argument becomes apparent. To manifest a failure of conjunction introduction, one has to get an (animate or inanimate) apparatus which measures the system as a whole appropriately correlated with the system, and one has to get (animate or inanimate) apparatuses which measure the locations of each marble appropriately correlated with each marble. Once all that is done, the requisite entanglement between the marbles (or particles) will be established and the dynamics of the GRW theory will guarantee that the system will either be in, or almost instantaneously evolve to, a state where the various apparatuses are in agreement and no failure of arithmetic is ever manifest. The strength of this response to Lewis’s counting anomaly is that it applies no matter how large we suppose the universe to be, and it applies just as well to counting particles as it does to counting marbles.

5 Is Suppressing the Manifestation of Anomalies Enough?

We have seen that the GRW theory, together with the fuzzy link, entails that conjunction introduction can fail for multi-particle systems. We also noted that, even for a single particle, there is the anomaly that property intersection can fail. Moreover, quantum systems can instantaneously jump between disjoint regions of space, though for a macrosystem this will virtually never happen.

To this list, we must also add that full blown action-at-a-distance can be instantiated at the microlevel. Consider two non-interacting particles (not marbles), L and R, each of which can either be in or out of a box, but their
boxes are widely separated in space, on the left and right. Suppose that at some time $t$ their joint state happens to be:

$$a|\text{in}\rangle_L|\text{in}\rangle_R + b|\text{out}\rangle_L|\text{out}\rangle_R,$$

(24)

with $|b|^2 \leq p$. Consider a sufficiently small time interval $T$ around $t$ over which the free Schrödinger evolution of the particles does not invalidate the inequality $|b|^2 \leq p$. We can also suppose that during $T$ the state (24) does not GRW collapse, because the probability for hits is negligibly low with only two particles in the system. Then, applying the fuzzy link to (24), both particles are determinately in their boxes throughout the interval $T$.

However, suppose that during $T$ the left-hand particle were subjected to a measurement of whether it is in or out of its box, producing the state:

$$a|\text{in}\rangle_L|\text{in}\rangle_M|\text{in}\rangle_R + b|\text{out}\rangle_L|\text{out}\rangle_M|\text{out}\rangle_R.$$  

(25)

Since $M_L$ is macroscopic, the probability of GRW collapse during $T$ is now extremely high. Of course, it is most likely that an effective collapse to the first term of (25) would occur. But it is certainly not impossible that the effective collapse would be to the second term, in which case the particle on the right, according to the fuzzy link, would have to switch from being determinately in its box to being determinately out. Notice that such a switch would have to have been brought about through action-at-a-distance, since in the absence of a measurement interaction on the distant left-hand particle (the ‘action’), the right-hand particle (‘at-a-distance’) would have remained in its box during $T$. (We are, of course, well aware that even when $|b|^2 \not\leq p$ there could be a jump in the state of the right-hand particle; but that jump would not be from one determinate state of affairs to another as interpreted via the fuzzy link.)

Is the fact that the GRW theory contains within it ‘mechanisms’ that suppress the manifestation of fuzzy link anomalies sufficient reason to continue to take the theory seriously? A sceptic might incline towards the view
set forth in Reichenbach’s ([1948]) ‘The Principle of Anomaly in Quantum Mechanics’ that one should always impose on any interpretation of quantum theory the requirement that there be no action-at-a-distance behind quantum phenomena. More generally, Reichenbach appears to argue against attributing an object any kind of behaviour that is radically different from the way the object manifests itself to us:

Speaking of unobserved objects is meaningful only if such objects are related to observed ones. If we say that a tree exists while we do not look at it, or while nobody looks at it, we interpolate an unobserved object between observables; and we select the interpolated object in such a way that it allows us to carry through the principle of causality. For instance, we observe that a tree casts a shadow; when we see a tree shadow without looking at the tree, we say that the tree is still in its place and thus satisfy the principle of causality. More precisely speaking, we select an interpolation which makes the causal laws of unobserved objects identical with those of observed ones. This qualification is necessary because otherwise we could interpolate different objects and construct for them peculiar causal laws; for example, we could assume that the unobserved tree splits into two trees, which however cast only one shadow. It is the postulate of identical causality for observed and unobserved objects which makes statements about unobserved objects definite... The postulate itself is neither true nor false, but a rule which we use to simplify our language. ([1948], p. 341)

One can ignore the conventionalist overtones of this passage and still agree that occurrences behind the phenomena should be described, as far as is possible, in a way that is continuous with the manifest world.

On the other hand, in the course of a discussion of the confusion between the instrumental and objective interpretation of wavefunctions, Reichenbach
opines: ‘The confusion of interpretations is one of the weak spots of the customary discussion of quantum-mechanical issues; it has blinded the eyes of some physicists to the extent that they do not see the causal anomalies *unavoidable for every interpretation* [italics ours]’ ([1948], p. 345). Indeed, from what we have said (at the end of Section 3) about no-collapse interpretations, it would not be hard to make a case that *no* interpretation of quantum theory can be entirely anomaly-free. The more important issue, it seems to us, is the *status* of an alleged anomaly, i.e., how *it* should be interpreted.

The GRW theory, considered as a theory about the evolution of wavefunctions, is perfectly consistent with classical logic and arithmetic. It is only once we relate wavefunctions to our ordinary language via the fuzzy link that all the anomalies we have discussed can crop up. This suggests that one should sharply distinguish the fundamental ontology of the GRW theory, viz. wavefunctions evolving and collapsing in configuration space, from the implications the fuzzy link has for how we are licensed to *talk* about a world governed by the GRW theory. Fuzzy link semantics, on this view, does not add anything of ontological import to the GRW theory, but simply provides a way of mapping our ‘particle’ language onto a theory whose fundamental language concerns wavefunctions. The fuzzy link, for some particular value of \( p \), would then have something of the status of a postulate that (to echo Reichenbach above) ‘is neither true nor false, but a rule which we use to simplify our language’.

Certainly the argument for this construal of the fuzzy link (apparently endorsed by Albert and Loewer ([1996], p. 91)) needs to be more fully developed. But, supposing it can be, we do not see any reason, in the case of wavefunction collapse theories, not to answer ‘Yes’ to the question in the title of this section.

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Departments of Philosophy and History and Philosophy of Science, University of Pittsburgh, Pittsburgh, PA 15260 (e-mail: rclifton+@pitt.edu).

Department of Philosophy, 1879 Hall, Princeton University, Princeton, NJ 08544-1006 (e-mail: bjmonton@princeton.edu).

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