An Experimental Study of Longitudinal Incisional Grooves in a Mixed Bedrock–Alluvial Channel

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Abstract
Natural bedrock rivers have various bedforms created by erosion. Flow-parallel incisional grooves formed longitudinally in bedrock are one common example of such bedforms. Although several studies have been conducted regarding these grooves, their formation processes are not well understood. In this study, we conducted a flume experiment to investigate the relationship between the flow structure and longitudinal grooves. The experimental results strongly suggest that longitudinal grooves are formed by moving sediment concentrated in multiple longitudinal pathways by turbulence-driven secondary flows. The sediment preferentially abrades the bedrock along these flow-parallel pathways resulting in longitudinal grooves in the bedrock. Measurements of the flow velocity distribution show that the positions of secondary flow cells producing the initial formation of the grooves are altered by the formation of those grooves. Because displaced secondary flows tend to make the sediment collide with the sidewalls of the longitudinal grooves, the grooves grow wider over time and some grooves partially combine with other adjacent grooves. The initial maximum number of longitudinal grooves $N_{max}$ strongly depends on the river width-depth ratio $B/D$, which defines the number of secondary flow cells, and can be expressed as $N_{max} = 0.5B/D$. However, because some grooves coalesce with other grooves due to the effects of the displacement of secondary flows, the average number of grooves showed a relationship that can be expressed as $N = 0.41B/D$. Based on this relationship, we inversely estimated the flow discharge of the Abashiri River using the number of longitudinal grooves observed in the river. The result was consistent with the observed annual maximum flow discharge of the river. This suggests that the number of longitudinal grooves can be used as an indicator for estimation of the formative flow discharge in bedrock rivers.

1. Introduction
Abrasion due to saltating gravels is a dominant erosion factor in bedrock rivers (e.g., Chatanantavet & Parker, 2009; Inoue et al., 2017; Johnson & Whipple, 2010; Lamb et al., 2015; Sklar & Dietrich, 2004). The increase of sediment supply has two conflicting effects regarding bedrock erosion (Gilbert, 1877). One is the tool effect, which refers to increased bedrock erosion due to collision with transported gravel, and the other is the cover effect, which refers to decreased bedrock erosion due to the protection of the bedrock by deposited gravel (Sklar & Dietrich, 1998, 2001). Therefore, identifying the spatial distribution of transported gravel and deposited gravel is important to understand the erosional landforms in bedrock rivers (Nelson et al., 2014).

Wohl (1993) reported that multiple parallel longitudinal grooves are formed in the Piccaninny Creek (Australia). Figure 1 shows the longitudinal grooves carved into the almost homogeneous sandstone in the Abashiri River (Japan). Because such longitudinal grooves are also formed in experimental channels made of homogeneous cohesive beds imitating bedrock (Wohl & Ikeda, 1997), their formation is due to the spatial distribution of bed load, rather than spatial variation of rock strength. The experimental results of Wohl and Ikeda (1997) indicated that bed forms changed from multiple longitudinal grooves to a single, broad groove with increasing bed slope. Finnegan et al. (2007) performed experiments to study formation and evolution of a groove with a small width-depth ratio, and Nelson and Seminara (2011) numerically modeled the same experiments. They reported that a groove was formed due to bed load impact wear and that the groove width increased with increasing ratio of sediment supply to transport capacity. Wohl (1993) and Inoue et al. (2016) suggest that multiple longitudinal vortices (i.e., turbulence-driven secondary flows) are a factor for the spatial disparity in the distribution of bed load, but no evidence is shown. The experimental results of Blankaert et al. (2010) showed that the number of longitudinal vortices is scaled with the water depth on the fixed flat bed.
Inoue et al. (2016) proposed a plane two-dimensional riverbed deformation model considering bedrock erosion and showed that multiple longitudinal grooves are formed when sediment supply is relatively small. However, because the model proposed by Inoue et al. (2016) does not consider three-dimensional vortices, the longitudinal grooves are formed due to the uneven distribution of bed load caused by random perturbations artificially introduced at the upstream end of the simulation domain. Therefore, a two-dimensional depth-averaged model cannot analyze how the factors defining the vortex structure (such as the water depth, bed slope, and sediment supply to capacity ratio) influence the number of longitudinal grooves.

Multiple longitudinal vortices are generated across the entire transverse direction by nonisotropic turbulence, and the magnitude of the secondary flow is weak (a few percent) compared with that of the main flow (Blanckaert et al., 2010). Longitudinal sand ridges are formed by turbulence-driven secondary flows in alluvial rivers (e.g., Colombini, 1993; Nezu, 2005; Nezu et al., 1985; Suzuki et al., 2014). These sand ridges are sedimentary landforms created by the accumulation of sand due to cross-stream components of shear stress induced by secondary flows. On the other hand, longitudinal grooves in bedrock are erosional landforms created by collision between the sediment and the bedrock. The differences in the shapes of these two types of landforms are likely to have an impact on the secondary flow structure. However, the interaction between the development of longitudinal incisional grooves and turbulence-driven secondary flows has not been clarified through experimental or computational investigation.

In this study, we examined the formation factors of longitudinal grooves in bedrock rivers through a flume experiment in which detailed flow velocity measurements were conducted, while also examining the relationship between the number of longitudinal grooves, water depth, bed slope, and gravel supply. Sections 2 and 3, respectively, explain the method and results of the flume experiment. In section 4, we discuss how longitudinal grooves and secondary flows interact with each other and whether it is possible to inversely estimate the formative flow discharge of a natural bedrock river based on the number of longitudinal grooves.

2. Experimental Method

2.1. Experimental Conditions

The experiment was conducted using a straight laboratory flume with a length of 25 m and a width of 50 cm (Figure 2). Water was supplied by a pump and gravel was manually supplied from the upstream end to ensure a uniform lateral distribution. Table 1 shows the experimental conditions. Runs 1–4 are a 2 × 2 matrix of water depth ($D = 3$ or $7$ cm) and bed slope ($S = 0.01$ or $0.02$). We changed the water depth because the number of longitudinal vortices is scaled with the flow depth (Blanckaert et al., 2010). We selected 0.01 and 0.02 gradients because Wohl and Ikeda (1997) reported that the multiple grooves were formed with a 0.01 gradient, and a single groove was formed with a 0.02 gradient. The influence of sediment supply suggested by Inoue et al. (2016) was examined by comparing Runs 4 and 5.

The flume had a mortar bed simulating bedrock, which had a thickness of 10 cm. The mixing ratio was 1:20:3.3 by weight (White Portland cement:0.2 mm silica sand:water). The curing time is two days. The mortar's strength was roughly spatially constant, with the average uniaxial compressive strength of 1.11 MPa. The strength was measured by needle penetration test developed by MARUTO Co., in Japan (e.g., Dipova, 2018). Our mortar is harder than the mortar of Mishra et al. (2018). The eroded mortar was transported as suspended load. The mortar bed was demolished after the completion of each case and a new flume bed was rebuilt. It was confirmed in preliminary experiments that the mortar bed would not be eroded due to flow shear stress under the experimental conditions of this study. Therefore, the erosion of the mortar bed (hereinafter referred to as “bedrock bed”) observed in this experiment was associated with abrasion of the bed by sediment collisions.
A fixed bed was used for the first 1 m from the upstream end of the flume to avoid rapid erosion at the introduction point of the sediment supplied to the flume. A fixed bed was also used for the downstream end (1 m), where the erosion tends to progress more quickly as the sediment moves faster due to the hydraulic control at the tailgate of the flume (i.e., essentially a free overflow). Haviv et al. (2006) suggested that the length of flow acceleration zone upstream of a waterfall is 2–4 times the flow depth (maximum length is roughly 30 cm in our experiments).

2.2. Measurement Method

Before Run 1, the two-dimensional cross-sectional profile of lateral and vertical flow velocity 10.5 m from the upstream end was measured using a two-dimensional electromagnetic current meter (VMT2-200-04PL, KENEK). There were 336 measurement points in total with 1-cm vertical intervals from 1 to 7 cm from the bed (7 points) and 1-cm transverse intervals from the 1- to the 48-cm point from the right bank (48 points). Measurement was conducted for 1 min at each point with a time step of 0.01 s, and the temporal average value was calculated. Since cross-sectional measurement using the electromagnetic current meter takes time (roughly 11 hr), it is impossible to perform during the experiments (4 hr). Therefore, no sediment was supplied during these velocity measurements so that no bedrock erosion would occur. Other conditions (i.e., flow discharge, bed slope, and flow depth) were the same as Run 1. After velocity measurement using a two-dimensional electromagnetic current meter, we rebuilt the mortar bed to unify the strength (curing day) of the mortar bed.

During Run 1 (i.e., after the sediments were supplied), an ultrasonic velocity profiler (UVP-DUO, Met-Flow) was installed on the left bank side at 10.5 m from the upstream end to measure the cross-sectional profile of lateral flow velocity. Although UVP can measure flow velocity instantaneously, the method can measure

### Table 1

**Experimental Conditions**

| Experiment case | Initial water depth D (cm) | Bed slope S | Sediment supply rate $q_{bs}$ (ℓ/min) | Initial width-depth ratio B/D | Initial sediment supply to capacity ratio $q_{bs}/q_{bc}$ | Flow discharge $Q$ (ℓ/s) | Froude number $Fr$ | Duration of water supply $T$ (hr) |
|-----------------|---------------------------|-----------|-----------------------------|------------------|-----------------------------|----------------------|-----------------|-----------------|
| Run 1           | 7.1                       | 0.01      | 1.56                        | 7.1              | 0.45                        | 46.0                 | 1.6             | 4               |
| Run 2           | 3.0                       | 0.01      | 1.56                        | 16.7             | 1.48                        | 14.5                 | 1.8             | 4               |
| Run 3           | 7.2                       | 0.02      | 1.56                        | 6.8              | 0.14                        | 65.0                 | 2.1             | 4               |
| Run 4           | 3.3                       | 0.02      | 1.56                        | 15.1             | 0.45                        | 20.5                 | 2.2             | 4               |
| Run 5           | 3.2                       | 0.02      | 3.12                        | 15.6             | 0.92                        | 20.5                 | 2.3             | 4               |

*Note. The major independent variables are water depth, bed slope, and sediment supply. The others are the related variables that were not systematically varied.*
only a one-dimensional profile. The transducer of the UVP-DUO was inserted into a cylinder installed at a height of 1 cm from the flume bed, with the tip of the transducer adhered to the acrylic flume wall using gel. The measurement time step was 0.1 s. Each measurement lasted for 2 min. To understand the change in the flow velocity profile before and after the erosion of the bedrock, measurements were performed twice: once at the beginning of sediment supply and once 3 hr later. The measurement range described in the manual of UVP-DUO is 0 to 50 cm. However, in our experience, the range that can be measured properly is about 5 to 45 cm.

During all runs (i.e., after the sediment supply was started), the water depth, the water surface, and bed elevations were measured at 2-m intervals over a 14-m-long section starting from the 2-m point and ending at the 16-m point, at the center of the flume, once every hour. Photos of the flume were taken from above with a digital camera once every hour to capture the areal fraction of alluvial cover on the bedrock.

In all runs, the water supply and the sediment supply were stopped at 240 min (4 hr). The sediments were moving slightly until the water flow completely stopped. The bed elevation was measured using a laser level gauge 3 times in total: once before water supply was started and twice after water flow completely stopped (before and after the removal of the deposited gravels). Measurements using the laser level gauge were conducted in a 12-m-long section starting at the 2-m point from the upstream end and ending at the 14-m point. The measurement intervals were 0.15 m in the longitudinal direction and 5 mm in the transverse direction. The bedrock erosion depth was calculated from the difference between the bed elevation before the removal of the deposited gravels and the initial bed elevation. The alluvial thickness was calculated from the difference between the bed elevations before and after the removal of the deposited gravels.

The sediment transport capacity in this experiment was calculated by combining the following bed load formula proposed by Wong et al. (2008) with the equation of the critical Shields number for bedrock given by Inoue et al. (2014), which is the power approximation to the model based on a force balance and the logarithmic velocity distribution.

\[ q_{bc} = 2.66(\tau_c - \tau_{c*})^{1.5} \sqrt{R_0 g d^3} \]  
\[ \tau_c = 0.027(k_s/d)^{0.75} \]

where \( \tau_c \) is the Shields number (\( DS/R_0g \)), \( D \) is the average observed water depth, \( S \) is the bed slope, \( R_0 \) is the specific gravity of the gravel in water (1.65), \( g \) is the gravitational acceleration (9.81 m/s\(^2\)), \( d \) is the diameter of the supplied grains (5 mm), \( \tau_{c*} \) is the critical Shields number, and \( k_s \) is the hydraulic roughness height. \( k_s \) was inversely calculated using the logarithmic velocity profile equation as expressed in terms of the Keulegan equation.

\[ U = \frac{6.25 + 5.75 \log_{10} \left( \frac{D}{k_s} \right)}{u_*} \]

where \( U \) is the depth-averaged velocity (\( =Q/BD \)), \( u_* \) is the friction velocity (\( =\sqrt{gDS} \)), \( Q \) is the flow discharge, and \( B \) is the flume width.

3. Experimental Results

3.1. Formation Process of Longitudinal Grooves

In Run 1, multiple longitudinal incisional grooves were formed on the bedrock as shown in Figure 2. Figures 3a–3e show the vertical photographs, erosion depth of the bedrock, and thickness of the gravel deposited on the bedrock (alluvial thickness) in Run 1. The vertical photographs taken 30 and 60 min after the beginning of the experiment (Figures 3a and 3b) show that the gravels were concentrated into four longitudinal pathways. As erosion progressed more quickly on areas where more gravels were transported relative to other areas, multiple longitudinal grooves were formed on the bedrock (at the 120-min mark; Figure 3c). As more bed load was concentrated into these longitudinal grooves, the gravel started to accumulate on the bottom of the longitudinal grooves. This reduced the vertical erosion of the longitudinal grooves, while the width of the longitudinal grooves widened due to lateral erosion of the inside walls of the longitudinal
grooves (at the 180- and 240-min marks; Figures 3d and 3e). This process was also described for a single groove by Finnegan et al. (2007) and modeled by Nelson and Seminara (2011). The erosion depth of the longitudinal grooves after 4 hr from the beginning of the experiment was 2 to 6 cm (Figure 3f), and the thickness of the gravel deposited in the longitudinal grooves was 1.5 to 3 cm (Figure 3g).

The cross-sectional profile of flow velocity measured with the electromagnetic current meter shows that there were six vortices, each of which was rotating in the opposite direction from the adjacent vortex, near the lateral center of the flume (Figure 4). In addition, two small vortices were observed near both sidewalls of the flume. Since the structure of these vortices strongly resembles the structure of turbulent-driven secondary-flow vortices measured by Blanckaert et al. (2010), the vortices observed in this study also appear to be induced by turbulence-driven secondary flows.

Figure 5a shows the cross-sectional profile of lateral flow velocity at z = 1 cm measured with the electromagnetic current meter, and Figure 5b shows the same information measured with the UVP. Although some conditions (measuring instruments and sediment supply) are different, both figures show similar distributions, as they show observation results over the initial flat bed. The blue dashed, vertical lines in Figure 5 indicate the points where the lateral flow velocity shifts from positive to negative in Figure 5b. A change from positive to negative means that the lateral velocity converges. Comparing this shift against the riverbed landforms after the experiment shown in Figures 5d and 5e, it indicates that longitudinal erosional grooves have been formed at the points indicated by the blue lines because the bed load particles tend to be concentrated at these points.

Figure 5c shows the lateral flow velocity after 3 hr and indicates that the points where the flows converge (the red dashed, vertical lines) have moved to the ridges of the longitudinal grooves, except for the central ridge at x = 10.5 that was lost due to lateral erosion. This suggests that the positions of the turbulence-driven secondary flow cells have moved because of the topographic change associated with the development of the longitudinal grooves.
3.2. Effects of Water Depth, Bed Slope, and Sediment Supply on the Groove Spacing

In Run 2 \((D= 3 \text{ cm and } S= 0.01)\), in which the water depth was smaller than Run 1 \((D\approx 7 \text{ cm and } S= 0.01)\), bed forms similar to antidunes and alluvial landforms similar to alternate bars were formed (Figures 6a–6e). No longitudinal grooves were observed on the bedrock, and a diagonal shallow groove was formed (Figure 6f). Because the nondimensional sediment supply, \(q_{bs}/q_{bc}\), of the initial flatbed in Run 2 was 1.48 (Table 1), which is higher than the threshold for transition from a bedrock bed to a gravel bed \((q_{bs}/q_{bc}= 1)\), the bedrock bed gradually transformed into a complete gravel bed from upstream to downstream (Figures 6e and 6g).

In Run 3 \((D\approx 7 \text{ cm and } S= 0.02)\) and Run 4 \((D\approx 3 \text{ cm, } S= 0.02)\), where \(q_{bs}/q_{bc}\) was less than 1 because the bed slope is steeper than Runs 1 and 2, multiple longitudinal grooves formed in both cases (Figure 7). When water depth was deeper (Run 3), three to four deep grooves were formed, whereas when water depth was shallower (Run 4), five to nine shallow grooves were formed. The cross-sectional profiles of bedrock erosion depth at \(x = 6.0 \text{ m and } 12.0 \text{ m}\) are shown in Figures 7g and 7h, respectively, for reference.

Comparing Run 1 (Figure 3) and Run 3 (Figure 7), where the water depths were equal (7 cm) but the bed slopes were different \((0.01 \text{ in Run 1 and } 0.02 \text{ in Run 3})\), shows that three to four longitudinal grooves were formed in both cases. However, the width and depth of the longitudinal grooves were different: the longitudinal grooves in Run 3 (Figures 7c and 7d) were narrower and deeper than those observed in Run 1 (Figures 5d and 5e). This difference is because in Run 3, where the bed slope was steeper, the alluvial cover of the bottom of the longitudinal grooves did not occur to the degree seen in Run 1 (the areal fractions of alluvial cover at 240 min are 0.38 in Run 1 and 0.20 in Run 3) and, therefore, vertical erosion progressed more aggressively.

In Run 5, for which the conditions were the same as Run 4 except for the sediment supply \((q_{bs}/q_{bc}= 0.45 \text{ in Run 4, and } 0.92 \text{ in Run 5})\), longitudinal grooves were observed until 60 min, but then the areal fraction of alluvial cover gradually increased, eventually making the flume bed into a complete gravel bed (Figure 8). Before coverage with the gravel, five to eight longitudinal grooves were observed, which is approximately the same number as Run 4, where the sediment supply was lower. The bed transitioned to a gravel bed after 60 min because \(q_{bs}/q_{bc}\) became larger than 1 due to the increase in roughness associated with gravel deposition and bedrock erosion (Figure 9).

Based on the above, when \(q_{bs}/q_{bc} < 1\), the experimental observations confirmed that the number of longitudinal grooves depends on the water depth and increases when the water depth is smaller (Figure 9, Runs 3 and 4). Moreover, the results also showed that the bed slope did not significantly affect the number of longitudinal grooves (Figure 9, Runs 1 and 3). Although the effect of sediment supply on the number of grooves is...
Figure 5. Cross-sectional profile of lateral flow velocity and riverbed shape in Run 1. (a) Cross-sectional profile of lateral flow velocity at a height of 1 cm from the initial bed measured with the electromagnetic velocity meter. No sediment was supplied and thus no erosion has occurred on the bedrock. The flow toward the right bank is positive. (b) Cross-sectional profile of lateral flow velocity at a height of 1 cm from the initial bed measured with UVP. This was immediately after starting sediment supply, so almost no erosion has occurred. (c) Cross-sectional profile of lateral flow velocity measured with the UVP after 180 min. Longitudinal grooves were formed in the bedrock. (d and e) Relative riverbed height from the initial bed level after 240 min at x = 3.6 m and 10.5 m, respectively. The solid line indicates the bedrock surface, the dotted lines indicate the sediment surface, the blue dashed vertical lines indicate the positions where flows converged at 0 min, and the red dashed vertical lines indicate the positions where flows converged after 180 min.
small when $q_{bs}/q_{bc} < 1$ (Figure 9, Runs 4 and 5), a diagonal groove is formed due to the formation of alluvial alternate bars when $q_{bs}/q_{bc} > 1$ (Figure 9, Run 2).

3.3. Relationship Among Sediment Supply to Capacity Ratio, Areal Fraction of Alluvial Cover, and Erosion Rate

Figures 10a–10d show chronological change in the water depth, the bed slope, the hydraulic roughness height, and the sediment supply to capacity ratio for the experiments of this study, respectively. According to Figure 10d, the sediment supply to capacity ratio in Runs 1, 3, and 4 slightly increased, but the increases were small. On the other hand, in Runs 2 and 5, where the bedrock was mostly covered with gravel, the sediment supply to capacity ratio increased after 60 min from the beginning of the experiment because the sediment-transport capacity dropped as the riverbed roughness increased due to groove formation and gravel deposition. Increase in roughness due to groove formation and gravel deposition, and subsequent decrease in transport capacity, has been observed previously in experiments by Finnegan et al. (2007). In addition, the sediment supply to capacity ratio asymptotically approached 1 in Runs 2 and 5 because the bed gradient gradually increased as the gravel accumulated from upstream, which translated into an increase in transport capacity.

The areal fraction of alluvial cover increased over time (Figure 10e). In Runs 2 and 5, the areal fraction of alluvial cover approached 1 as the sediment supply to capacity ratio exceeded 1. In other runs, the gravel accumulated at the bottom of the longitudinal grooves and the bedrock was partially covered by gravels. There was a positive correlation between time-averaged sediment supply to capacity ratio and time-averaged areal fraction of alluvial cover (Figure 10f). Although this trend is roughly consistent with the model proposed by Sklar and Dietrich (2004) ($p_c = q_{bs}/q_{bc}$), the experimental cover fraction is smaller than their model because the sediments passed through on the initial smooth bedrock surface with little accumulation. In order to accurately represent this phenomenon, it is necessary to combine a model that takes into account the
interaction between bedrock roughness and alluvial cover (e.g., Inoue et al., 2014; Johnson, 2014) and a model that takes into account temporal and spatial changes in the alluvial cover (Inoue et al., 2016; Nelson & Seminara, 2011; Zhang et al., 2015). This will be an exciting challenge for the future.

According to the concept of the cover effect proposed by Sklar and Dietrich (2004), the erosion rate $E$ decreases linearly with the increase of the areal fraction of alluvial cover $p_c$ and the supply-capacity ratio $q_{bs}/q_{bc}$, when the sediment supply is the same.

$$E = q_{bs}(1-p_c) = q_{bs}(1-q_{bs}/q_{bc}) \quad (4)$$

However, in this study, the relationship between the erosion rate and the areal fraction of alluvial cover in Runs 1 to 4, where sediment supply was equal, was not linear, but rather complicated (Figures 10g and 10h). One of the factors behind this was the lateral erosion of the longitudinal grooves. Mishra et al. (2018) showed experimentally that the sidewalls do not get covered by the gravel deposited on the riverbed and thus are not subject to the cover effect. When the lateral erosion of the longitudinal grooves is dominant, the erosion of the whole flume may increase even when the areal fraction of alluvial cover of the rock bed is high. This hypothesis is supported by the evidence that the erosion rate of Run 1, in which shallow and wide
longitudinal grooves were formed, was greater than that of Run 3, in which deep and narrow grooves were formed.

4. Discussion
4.1. Interrelationship Between the Development of Longitudinal Grooves and Turbulent-Induced Secondary Flows
Nezu et al. (1985); Nezu (2005), Colombini (1993), and Suzuki et al. (2014) show that turbulence-driven secondary flows are formed in flat straight flumes. The cell size of a turbulence-driven secondary flow is

Figure 8. Experimental results of Run 5 (water depth $\approx$ 3 cm, bed slope = 0.02, and sediment supply rate = 3.12 ℓ/min and the supply to capacity ratio = 0.92). (a) A photograph of alluvial cover after 30 min. (b) A photograph of alluvial cover after 60 min. (c) A photograph of alluvial cover after 120 min. (d) A photograph of alluvial cover after 180 min. (e) A photograph of alluvial cover after 240 min. (f) Contour map of bedrock erosion depth. (g) Alluvial thickness on the bedrock. In the vertical photographs, the dark parts indicate the sediment and the bright parts indicate the bedrock. After multiple longitudinal grooves were formed, the bedrock was covered with gravel.

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Figure 9. The number of grooves observed in our experiments. The error bars show the maximum and minimum values. Comparison between Runs 3 and 4 shows the effect of water depth. Comparison between Runs 1 and 3 shows the effect of bed slope. Comparison among Runs 2, 4, and 5 shows the effect of sediment supply-capacity ratio.
equivalent to the water depth (e.g., Nezu, 2005). Therefore, the number of secondary flow vortices will be approximately \(B/D\). Since one longitudinal erosional groove is formed between two vortices (Figure 4), the number of longitudinal grooves is expressed by the following equation.

\[
N = 0.5B/D
\]  

Figure 10. (a–e) Chronological change in the water depth, the bed slope, the hydraulic roughness height, the sediment supply to capacity ratio, and the areal fraction of alluvial cover, respectively. (f) Relationship between the time-averaged sediment supply to capacity ratio and the time-averaged areal fraction of alluvial cover. (g) Relationship between the erosion rate and the areal fraction of alluvial cover. (h) Relationship between the time-averaged sediment supply to capacity ratio and the erosion rate. In Figures 10g and 10h for investigating the cover effect, we have compared Runs 1–4 where sediment supply (i.e., tools effect) is equal. The erosion rate of Run 1 is larger than that of Run 3 despite the larger cover effect (i.e., \(p_c \) or \(q_{bs}/q_{bc} \)).
In addition to the experimental results of this study, the experimental results of Wohl and Ikeda (1997), Sumner et al. (2016), and Finnegan et al. (2007) (Table 2) are plotted in Figure 11. These results showed a strong positive correlation between the number of longitudinal grooves and the width-depth ratio. The gray solid line in the figure is a linear regression (through zero) of the relationship between the maximum number of longitudinal grooves and the river width-depth ratio, which can be expressed as $N = 0.49B/D$. This equation indicates that the maximum number of longitudinal grooves is defined by the number of turbulence-driven secondary flows.

The black solid line and the gray dashed line are regression (through zero) of the relationships between the river width-depth ratio and the average number of longitudinal grooves, and between the width-depth ratio and the minimum number of longitudinal grooves, which can be expressed as $N = 0.41B/D$ and $N = 0.34B/D$, respectively. The reason that the average number of longitudinal grooves was less than $0.5B/D$ is because some longitudinal grooves coalesced with others during the development process.

There are two major patterns for the combination of longitudinal grooves. The first pattern is that the sediment transport paths curve and combine with adjacent paths. The second pattern is that the grooves are combined due to lateral erosion. The first pattern was clearly observed at $x = 10$ m in Run 3 and at $x = 12.5$ m in Run 4. The longitudinal grooves in Run 4 were also weakly meandering around $x = 8.5$ m, although they did not combine with each other. The second pattern, which is the combination of grooves due to lateral erosion, was notable between $x = 8$ and $x = 12$ in Run 1.

One of the factors that caused the weak meandering of sediment transport paths and the lateral erosion of longitudinal grooves is the displacement of lateral flow velocity shown in Figure 5. In alluvial rivers, sand ridges are formed where near-bed lateral velocity converges, which has the effect of stabilizing the positions of longitudinal vortices (Colombini, 1993; Nezu, 2005; Figure 12a). On the other hand, in bedrock rivers, the bedrock is eroded, and grooves are formed where transported sediment gathers. Because of this pattern, the positions of the longitudinal vortices are not stabilized and are instead displaced (Figure 5). We interpret this pattern as suggesting that the displacement of the points where lateral flow velocity is concentrated promoted the lateral erosion of the longitudinal grooves (Figure 12b). The displacement of longitudinal vortices is an interesting phenomenon unique to bedrock rivers, which was observed for the first time in this experiment.

Although a single broad channel with undulating walls was formed with 0.02 gradient in the experiment of Wohl and Ikeda (1997), multiple longitudinal grooves were formed with 0.02 gradient in our experiments. Because the Froude number of their experiment was close to 1, the flow tends to oscillate between subcritical and supercritical areas along the streamwise extent of the channel, as suggested by descriptions in their paper (Wohl & Ikeda, 1997). The subcritical and supercritical transitions generally give large streamwise variations in flow with fast, converging

### Table 2

| Experiment | Wohl and Ikeda (1997) | Sumner et al. (2016), Case 3 | Finnegan et al. (2007) |
|------------|----------------------|-----------------------------|------------------------|
| Number of longitudinal grooves | 3–6 | 15–17 | 1 |
| River width $B$ (cm) | 20 | 90 | 30 (top 38 cm, bottom 22 cm) |
| Riverbed gradient $S$ | 0.01 | 0.005 | 0.016–0.026 |
| Water depth $D$ (cm) | 2 | 2.5 (maximum 2.8, minimum 2.3) | 10 |
| Particle diameter $d$ (mm) | 1 | 0.77 | 6 |
| Flow discharge $Q$ ($\ell$/s) | 2 | 10 | 40 |
| Sediment supply $Q_{bs}$ ($\ell$/min) | 0.13$^a$ | 0.16 | 0.006–0.0034$^b$ |

$^a$Converted assuming that 220 g/min translates to a density of 2.65 g/cm$^3$ with a porosity of 0.35. $^b$Converted assuming that 11–59 g/min translates to a density of 2.65 g/cm$^3$ with a porosity of 0.35.
flow areas and slower diverging flow areas. All of the experiments we carried out had a Froude number significantly greater than 1, and we did not see the strong streamwise variability described by Wohl and Ikeda (1997). We think that turbulence-driven secondary flows will be ineffective in situations where subcritical-supercritical transitions occur; for these cases, we would expect the formation of a single groove with undulating walls and less streamwise continuity than was observed in our (high Froude

Figure 12. Conceptual diagram of the formation process of longitudinal grooves. (a) In alluvial rivers, sand ridges are formed where flow is concentrated, which stabilizes the positions of the secondary flow cells (Nezu, 2005). (b) In bedrock rivers, areas where flows concentrate are eroded. As longitudinal grooves develop, the positions of the secondary flow cells move or the rotation directions of the secondary flow cells are reversed.
Table 3
Observation Data of the Abashiri River

| Name of river | Abashiri River |
|---------------|---------------|
| River width δ (m) | 48 |
| Riverbed gradient S | 0.0029 |
| Number of longitudinal grooves N | 8–16 |
| Roughness height kγ (m) | 0.22a |
| Flow discharge Q (m³/s) | 199b |

aValue inversely calculated using equation (3), using the flow discharge (648 m³/s) and water depth (3.39 m) observed by the Hokkaido Development Bureau during the flood in 2001. bAverage annual maximum flow discharge from 1955 to 2016 observed by Hokkaido Development Bureau.

Multiple longitudinal grooves are formed in an almost straight channel around 42 km from the river mouth. The bed is composed of sandstone, and gravel is deposited at the bottom of multiple longitudinal grooves. The water depth of the Abashiri River is estimated to be 1.47–2.04 m based on the relationship between the gray solid line and the gray dashed line in Figure 11 and the river width and the number of longitudinal grooves observed in the river (Table 3). Next, by substituting the estimated water depth, observed bed slope, and river width (Table 3) into equation (3), the flow discharge is calculated to be 153–271 m³/s (average value from black solid line is 185 m³/s). These values are roughly the same as the annual maximum flow discharge observed in the Abashiri River (199 m³/s).

Although this is just one sample, the above result suggests that the flow discharge of bedrock rivers can be estimated based on the number of longitudinal grooves. Moreover, this result implies that the decreasing of the number of longitudinal grooves due to coalescing of adjacent grooves does not continue indefinitely in natural rivers. The displacement of the secondary flow cells is forcibly caused by the development of longitudinal grooves. If the riverbed becomes flatter due to the combination of longitudinal grooves, the secondary flow cells return to their initial positions. Therefore, it is likely that the number of grooves does not decrease below a certain value. Exploration of the longer-term fluctuation of the number of longitudinal grooves will be another exciting challenge for the future.

The bedrock erosion resistance will not have a great influence on the number of longitudinal grooves initially formed because it does not change the number of secondary flow cells. Instead, the bedrock erosion resistance can affect the period of the groove number fluctuation described above. Flow fluctuations can complicate the number and shape of the grooves by changing the number of secondary flow cells. However, the range of flow discharge that contributes to the groove formation may not be large because a small discharge without gravel movement will not affect groove formation.

4.2. Inverse Estimation of Flow Discharge Using the Number of Longitudinal Grooves

Here we examine if we can inversely estimate the flow discharge of a natural river based on the number of longitudinal grooves. The Abashiri River (Figure 1) is in Hokkaido, Japan (43°44′50″N, 144°02′57″E). The catchment area is 1,380 km², and the average annual rainfall is 800 mm. The difference in the topographical profiles of ridges and grooves has a strong implication on the positions of turbulence-driven secondary flows. While sand ridges in alluvial rivers tend to stabilize the positions of the secondary flow cells (Nezu, 2005), longitudinal grooves in bedrock tend to displace the secondary flow cells. The formation mechanism of these grooves has both a similarity and a difference relative to the formation mechanism of sand ridges in alluvial rivers. The similarity is that the number and spacing of longitudinal grooves strongly depend on the river width-depth ratio. The difference is that while sand ridges are formed due to deposition along sediment transport paths, longitudinal grooves are formed as a result of abrasion by the sediment along the sediment transport paths. For this reason, the cross-sectional topographic profiles of sand ridges and longitudinal grooves are symmetrical.

The difference in the topographical profiles of ridges and grooves has a strong implication on the positions of turbulence-driven secondary flows. While sand ridges in alluvial rivers tend to stabilize the positions of the secondary flow cells, the number of grooves in bedrock will displace the secondary flow cells. As a result, the positions of flow concentration change over time in the longitudinal incisional grooves,
which can promote lateral erosion and combining of the grooves. The displacement of secondary flow cells accompanying this topographic change is a phenomenon unique to bedrock rivers, as discovered for the first time in this study.

Longitudinal grooves are formed when the sediment supply to capacity ratio is less than 1. The riverbed slope and sediment supply affect the depth and width of longitudinal grooves but have no effect on the number of grooves. The number of longitudinal grooves \( N \) is controlled by the river width-depth ratio \( B/D \), which defines the number of secondary flow cells. This relationship, including the effect of the combining of the grooves, is expressed as \( N = 0.41B/D \). Using this relationship, we inversely estimated the flow discharge of the Abashiri River from the observed number of longitudinal grooves. The estimated flow discharge was consistent with the observed annual maximum flow discharge.

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