Proposal for a New Reconstruction Technique for SUSY Processes at the LHC

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Abstract
When several sparticle masses are known, the kinematics of SUSY decay processes observed at the LHC can be solved if the cascade decays contain sufficient steps. We demonstrate four examples of this full reconstruction technique applied to channels involving leptons, namely a) gluino mass determination, b) sbottom mass determination, c) LSP momentum reconstruction, and d) heavy higgs mass determination.

1. INTRODUCTION
The potential of the LHC for SUSY parameter determination has been studied in great detail for the past seven years \cite{1,2,3,4,5,6}. One of the most promising methods involves the selection of events from a single decay chain near the kinematic endpoint. Information on the masses involved in the cascade decay can be extracted from the endpoint measurements. It has been established that one can achieve a few percent accuracy for sparticle mass reconstruction using this technique with sufficient statistics.

In this paper we propose a new method for reconstructing SUSY events which does not rely only on events near the endpoint. Instead one kinematically solves for the neutralino momenta and masses of heavier sparticles using measured jet and lepton momenta and a few mass inputs.

To illustrate the idea we take the following cascade decay chain
\begin{equation}
\tilde{g} \rightarrow \tilde{b}b \rightarrow \tilde{\chi}_2^0 bb \rightarrow \tilde{\chi}_1^0 b\ell\ell. \tag{1}
\end{equation}
This decay chain is approximately free from SM background with appropriate cuts. The five SUSY particles which are involved in the cascade decay have five mass shell conditions;

\begin{align*}
m_{\tilde{g}}^2 &= p_{\tilde{\chi}_1^0}^2, \\
m_{\tilde{b}}^2 &= (p_{\tilde{\chi}_1^0} + p_{\ell_1})^2, \\
m_{\tilde{\chi}_1^0}^2 &= (p_{\tilde{\chi}_1^0} + p_{\ell_1} + p_{\ell_2})^2, \\
m_{\tilde{\chi}_2^0}^2 &= (p_{\tilde{\chi}_1^0} + p_{\ell_1} + p_{\ell_2} + p_b)^2, \\
m_{\tilde{\chi}_1^0}^2 &= (p_{\tilde{\chi}_1^0} + p_{\ell_1} + p_{\ell_2} + p_{b_1} + p_{b_2})^2. \tag{2}
\end{align*}

Of these five masses, \(m_{\tilde{g}}, m_{\tilde{b}}\) and \(m_{\tilde{\chi}_1^0}\) can be measured at the LHC using first generation squark cascade decays with an accuracy of \(\sim 10\%\) (the mass difference is measured more precisely). Moreover, with input from a future high energy Linear Collider these masses might be determined with an accuracy \(\sim O(1\%)\). We therefore assume for the present work that the masses of the two lighter neutralinos and of the right handed slepton are known, and we ignore the corresponding errors.

For a \(bb\ell\ell\) event, the equations contain six unknowns \((m_{\tilde{g}}, m_{\tilde{b}}\) and \(p_{\tilde{\chi}_1^0}\)) which satisfy five equations. For two \(bb\ell\ell\) events, we have ten equations while we only have ten unknowns (two neutralino four momenta, \(m_{\tilde{g}}\) and \(m_{\tilde{\chi}_1^0}\)). Mathematically, one can obtain the sbottom and gluino masses and all neutralino momenta if there are more than two \(bb\ell\ell\) events.
Table 1: Some sparticle masses in GeV at SPS1a.

| $m_{\tilde{g}}$ | $m_{\tilde{b}_1(2)}$ | $m_{\tilde{\chi}^0_2}$ | $m_{\tilde{\ell}_R}$ | $m_{\tilde{\chi}^0_1}$ |
|----------------|---------------------|---------------------|---------------------|---------------------|
| 595.2          | 491.9(524.6)        | 176.8               | 136.2               | 96.0                |

We call this technique the “mass relation method” as one uses the fact that sparticle masses are common for events which go through the same cascade decay chain. Note events need not be near the endpoint of the decay distribution to be relevant to the mass determination. In the next section we demonstrate the practical application of this method to measurement of the masses of the gluino and sbottom.

As a byproduct of the technique, once the mass of the squark and of all the sparticles involved in the decay are known, the momentum of the lighter neutralino can be fully reconstructed, and this further constrains the event.

In SUSY events sparticles are always pair produced and there are two lightest neutralinos in the event. If squark decays via $\tilde{q} \to \tilde{\chi}^0_1 q \to \tilde{\chi}^0_1 \tilde{\ell} \to \tilde{\chi}^0_1 l$ can be identified on one side of the event then the neutralino momentum can be reconstructed as described above. The transverse momentum of the lightest neutralino in the other cascade decay can then be obtained using the following equation

$$p_T^{(\tilde{\chi}^0_1(2))} = p_T^{(\text{miss})} + p_T^{(\tilde{\chi}^0_1(1))},$$

provided that there are no hard neutrinos involved in the decay. This transverse momentum can be used to constrain the cascade decay of the other sparticle.

For the case where the other squark decays via $\tilde{q} \to \tilde{\chi}^+_1 q \to \tilde{\chi}^0_1 q W$ followed by $W \to q' q''$, the chargino mass can be determined by using Eq. (3) and the following relations,

$$p_{\tilde{q}} = p_{\tilde{\chi}^0_1(2)} + p_j + p_W,$$

$$p_{\tilde{q}}^2 = m_{\tilde{q}}^2,$$

where $p_j$ is the momentum of the selected high $p_T$ jet which comes from the squark decay and $p_W$ is the momentum of the two jet system consistent with the $W$ interpretation. The neutralino momentum resolution is important for the chargino mass reconstruction and we discuss this in section 3. The reconstruction will be discussed in a separate contribution.

The full reconstruction technique can be extended for higgs mass reconstruction. In section 4, we discuss the heavy higgs mass determination from the process $H \to \tilde{\chi}^0_2 \tilde{\chi}^0_2$ followed by $\tilde{\chi}^0_2 \to \tilde{\ell} \ell \to ll\tilde{\chi}^0_1$. This process is also useful for discovery of heavy higgs bosons. The four lepton momenta and missing momentum can be used to reconstruct the higgs mass assuming that the $p_T$ of the higgs boson is very small.

2. GLUINO CASCADE DECAY

We first discuss the results of a simulation study of the process where a gluino cascade decays into a sbottom at model point SPS1a. The relevant sparticle masses for this study are listed in Table 1. The events were generated using the HERWIG 6.4 generator and passed through ATLFAST, a parametrised simulation of the ATLAS detector.

We study only events which contain the cascade decay shown in Eq. (1). We then apply the following preselections to reduce backgrounds:

- $p_T^{\text{miss}} > 100$ GeV
- $M_{\text{eff}} > 600$ GeV
- at least 3 jets with $p_T > 150$ GeV, $p_T > 100$ GeV and $p_T > 50$ GeV.
- exactly two jets with $p_T > 50$ GeV tagged as $b$-jets
- exactly two OS-SF leptons with $p_{T1} > 20$ GeV, $p_{T2} > 10$ GeV, and invariant mass $40 \text{GeV} < m_{ll} < 78$ GeV.

The solution of Eq. (2) can be written in the following form:

$$m_{\tilde{g}}^2 = F_0 + F_1 m_{\tilde{b}}^2 \pm F_2 D,$$

where

$$D^2 \equiv D_0 + D_1 m_{\tilde{b}}^2 + D_2 m_{\tilde{b}}^4.$$  (5)

Here $F_i$ and $D_i$ depend upon $p_{\ell_i}$ and $p_{b_i}$ and the neutralino and slepton masses. In the event, there are two $b$ jets and we assume that the $b$ jet with larger $p_T$ originates from the $\tilde{b}$ decay. The two leptons must come from $\tilde{\chi}^0_2$ and $\ell$ decay. There are maximally four sets of gluino and sbottom mass solutions together with two lepton assignments for each decay, because we cannot determine from which decay the lepton originates. To reduce combinatorics we take the event pair which satisfies the following conditions:

- Only one lepton assignment has a solution to the Eq. (5)
- For a pair of events there are only two solutions and there is a difference of more than 100 GeV between the two gluino mass solutions.

![Fig. 1: $m_{\tilde{g}}$ obtained by using Eq. (5) for two $bb\ell\ell$ events.](image)

In Fig 1, we plot the minimum $m_{\tilde{g}}$ solution which satisfies the conditions given above. The peak position is consistent with the gluino mass, and the error on the peak position obtained by a Gaussian fit is around 1.7 GeV for 100 fb$^{-1}$. For the events used in the reconstruction, each event is used on average five times. Note that the $\sigma$ of the Gaussian fit is large ($\sim 56.7$ GeV) and is determined by the resolution on the momentum measurement of the four $b$-jets. It is worth stressing that the results presented here were produced by using a parametrised simulation of the response of the ATLAS detector to jets, based on the results of a detailed simulation. Results which crucially depend on the detailed features of the detector response, such as the possibility of discriminating the two sbottom squarks (see below) need to be validated by an explicit detailed simulation of the detector performed on the physics channel of interest. We only attempt here to evaluate the impact of the new technique on sparticle reconstruction.

Once the gluino mass has been determined one can reconstruct the sbottom mass by fixing the gluino mass to the measured value. Here one needs to solve only Eq.(5), which involves only two $b$-jets in the fit, and therefore errors due to the jet resolution are expected to be less than those for the gluino mass reconstruction.

For each event, there are two sbottom mass solutions $m_{\tilde{b}}(\text{sol1})$ and $m_{\tilde{b}}(\text{sol2})$, each sensitive to the gluino mass input. The difference between the gluino and sbottom mass solutions is however stable against variation in the assumed gluino mass. The mass itself may have a large error in the absolute scale, but the mass differences are obtained rather precisely, as is the case in the endpoint method.
In Fig. 2 (left), we plot the solutions for all possible lepton combinations in the $m_{\tilde{g}} - m_{\tilde{b}}$ (sol1) $m_{\tilde{g}} - m_{\tilde{b}}$ (sol2) plane. Here we use the $b$-parton momentum obtained from generator information. One of the solutions tends to be consistent with the input sbottom mass. Moreover the two decay modes $\tilde{g} \rightarrow \tilde{b}_1 b$ and $\tilde{b}_2 b$ are clearly separated.

We can compare the results from the previous analysis with those from the endpoint analysis$^{[12]}$, where one uses approximate the formula

$$p_{\tilde{\chi}^0_2} = \left(1 - \frac{m_{\tilde{\chi}^0_1}}{m_{\ell\ell}}\right) p_{\ell\ell}.$$

This formula is correct only at the endpoint of the three body decay $\tilde{\chi}^0_2 \rightarrow \chi^0_1 \ell\ell$, but is nevertheless approximately correct near the edge of $\tilde{\chi}^0_2 \rightarrow \ell\ell \rightarrow \ell\ell \tilde{\chi}^0_1$ for SPS1a. The sbottom mass obtained by using Eq.(6) is shown in Fig. 2(right). For this case, the $\tilde{b}_2$ peak at 70.6 GeV is not separated from the $\tilde{b}_1$ peak at 103 GeV.

**Fig. 2:** The distribution of $m_{\tilde{g}} - m_{\tilde{b}}$ calculated using the parton level $b$ momentum by solving Eq.(2) (left) and using the approximate relation Eq.(6) (right).

**Fig. 3:** As for Fig. 2 but with the $b$ jet momentum used instead of the $b$ parton momentum.
The $\tilde{b}_1$ mass, or the weighted average of the sbottom masses, is easily obtained. The $b$ jet resolution is not sufficient however to clearly separate the $\tilde{b}_1$ and $\tilde{b}_2$. This can be seen in Fig. 3 where the plots show the distributions corresponding to Fig.2(left) and (right) but now with the $b$ parton momenta replaced by $b$ jet momenta. For the endpoint analysis (Fig.3 right), a correct evaluation of the sbottom masses would require a fit taking into account the shape of the response of ATLAS to b-jets. In order to approximately evaluate the achievable statistical precision, a naive double gaussian fit was performed on the distribution shown in Fig.3 right, which corresponds to $\int dtL = 300$ fb$^{-1}$. The resulting statistical uncertainties are $\pm 1$ GeV ($\pm 2.5$ GeV) for the $m_{\tilde{g}} - m_{\tilde{b}_1}$ ($m_{\tilde{g}} - m_{\tilde{b}_2}$) peak positions respectively. Additional systematic uncertainties, not yet evaluated, as well a 1% error due to the uncertainty on the jet energy scale should also be considered. These numbers are obtained assuming the presence of two gaussian peaks in the data.

For the mass relation method the number of events available for the study is larger by a factor of 2 because events away from the endpoints can be used. We also use the exact formula for the mass relation method. Although the analysis is more complicated due to the multiple solutions, we believe it to be a worthwhile technique for use when attempting to reconstruct the $\tilde{b}_1$ and $\tilde{b}_2$ masses.

3. NEUTRALINO MOMENTUM RECONSTRUCTION

In this section, we discuss the reconstruction of the momentum of the lightest neutralino. As we have discussed already, the mass shell condition can be solved for long decay cascades, such as $\tilde{q} \rightarrow \tilde{\chi}^0_2 q \rightarrow \tilde{\ell} q \ell \rightarrow \chi^0_1 q \ell \ell$. For this process we have two neutralino momentum solutions for each lepton assignment. One may wonder if the solutions for the neutralino momentum might be smeared significantly, because of the worse jet energy resolution as compared to leptons, and the jet $p_T$ is generally much larger than the neutralino momentum for the cascade decay. In Fig. 4(left) we show the distribution of $p_T$(reco)/$p_T$(truth) for the point studied in [7]. Here we choose the correct lepton combination using generator information, and take the solution which minimizes $|p_T$(reco)/$p_T$(truth)$-1|$. Except for the case where we took the wrong jet as input the reconstructed $p_T$ is within 20% of the true neutralino momentum. The result for the gluino cascade decay into sbottom Eq.(1) is similar.

In Fig. 4(right) we show a similar reconstruction for the gluino cascade decay, but unlike Fig.4(left), we use both lepton combinations. We fix the gluino mass to the input value$^1$ and take events where one of the four sbottom mass solutions is consistent with the input sbottom mass such that $|m_{\tilde{b}_1} - m_{\tilde{b}_2}$(best)$| < 10$ GeV. We then take the solution where the sbottom mass is closest to the input $m_{\tilde{b}_1}$. There are still two $p_T$(reco) solutions, and we choose the one which minimize $\min(p_T$(reco)/$p_T$(truth), $p_T$(truth)/$p_T$(reco))$-1$. The neutralino momentum resolution is worse than that obtained using the correct lepton assignments only. Nevertheless a significant fraction of events are reconstructed with $0.8 < |p$(reco)/$p$(truth)$| < 1.2$.

4. HIGGS MASS RECONSTRUCTION

A promising decay for the observation of heavy and pseudo-scalar higgs bosons in the difficult region with intermediate $\tan \beta$ is the decay into two neutralinos. When both neutralinos decay through the chain

$$\tilde{\chi}^0_2 \rightarrow \tilde{\ell} R \ell \rightarrow \ell \ell \tilde{\chi}^0_1$$

the resulting signature consists of events with four isolated leptons (paired in opposite-sign same-flavour pairs) and no jet activity. The main SM backgrounds to this signature are $t\bar{t}$ production, where both the $b$-jets and the $W$s decay into leptons and $Zbb$ production. The key element for the rejection of these backgrounds is the fact that the leptons from $b$ decays are not isolated. A detailed study of the performance of lepton isolation in the detector is needed to assess the visibility of the signal. Additionally there is an important SUSY background, including irreducible backgrounds from direct slepton and

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$^1$Here we adopt an event selection which makes use of the true (input) gluino and sbottom mass values, although in practice fitted values would be used.
The calculated $\tilde{\chi}_1^0$ transverse momentum divided by the true transverse momentum. The decay $\tilde{q}_L \rightarrow \tilde{\chi}_2^0 q \rightarrow \tilde{\ell}_L \tilde{q} \ell \rightarrow \tilde{\chi}_1^0 q \ell \ell$ is studied for the model point used for the chargino study: $m_0 = 100$ GeV, $m_{1/2} = 300$ GeV, $A_0 = -300$ GeV, $\tan \beta = 6$, and $\mu > 0$. Only the correct lepton choice is used.

To demonstrate the power of the method, we apply it to Point SPS1a, for which the mass of the $A$ and of the $H$ is $\sim 394$ GeV. The BR into $\tilde{\chi}_2^0 \tilde{\chi}_1^0$ is 6% (1%) for the $A(H)$. We perform the study on 1000 events for $A \rightarrow \chi_2^0 \chi_2^0 \rightarrow \ell \ell \ell \ell$ corresponding approximately to the expected statistics for 300 fb$^{-1}$. We simply require 2 isolated leptons with $p_T > 20$ GeV and 2 further isolated leptons with $p_T > 10$ GeV, all within $|\eta| < 2.5$. The efficiency of these cuts is $\sim 60\%$.

We have not performed any background simulations because at this stage we only wish to explore the viability of the full reconstruction technique. The main problem for the reconstruction is the correct assignment of the leptons to the appropriate decay chain. The first selection is based on requiring a unique identification of the lepton pairs coming from the decays of the two $\tilde{\chi}_2^0$s. We therefore require that either of the following two criteria is satisfied:

- the flavour configuration of the leptons is $e^+ e^+ \mu^+ \mu^-$
- the lepton configuration is either $e^+ e^- e^+ e^-$ or $\mu^+ \mu^- \mu^+ \mu^-$, but for one of the two possible pairings the invariant mass of one of the pairs is larger than 78 GeV, i.e. above the lepton-lepton edge for the $\tilde{\chi}_2^0$ decay.

The total efficiency after these cuts is $\sim 30\%$. At this point, on each of the two legs there is still an ambiguity due to the fact that each lepton can be either the product of the first or of the second step in the decay chain. This gives 4 possible combinations. Furthermore, the full reconstruction results in a quartic equation which can have zero, two or four solutions. We show in Fig. 5 the distribution of the calculated $A$ mass for all of the retained combinations as a full line. The dashed line shows the combinations with the wrong lepton assignment. A clear and narrow peak emerges over the combinatorial background. The width is approximately 6 GeV, determined by the resolution of the measurement of the momentum of the leptons.

5. CONCLUSIONS

In this contribution, we have described a novel technique for reconstructing the mass and momenta of SUSY particles. This technique does not rely on any approximate formulae nor on endpoint measure-
ments. All events contribute to the sparticle mass determination and decay kinematics reconstruction, even if they are away from the endpoint of the distribution. The method may be particularly useful when the SUSY mass scale is large. In that case the statistics can be so low that the endpoint cannot be seen clearly while the SUSY sample itself is very clean.

The method applies most effectively when we know some of the sparticles’ masses exactly, because the number of unknown parameters in e.g. Eq. (2) is reduced. In the particular case where some of the sparticle masses are measured at a LC the sparticle cascades may be solved completely and study of the decay distributions and higher mass determination becomes possible at LHC.

When all the sparticle masses are known the neutralino momentum can be reconstructed if four sparticles are involved in the cascade decay. The sparticles would be pair produced, and if we can identify both of the cascade decay chains in the events then we only need six sparticles in the cascade decay to solve both of the neutralino momenta on account of the missing momentum constraint. The reconstruction of sparticle momenta provides us with an interesting possibility for studying the decay distribution at the LHC.

On the other hand, our method is not valid when some of the particles in the cascade produce hard neutrinos. This is unfortunately the case when the chargino decays into (s)leptons, when a $\tilde{\tau}$ is involved in the decay, or when a $W$ is produced and decays leptonically. If such SUSY decay processes dominate then this method may not be useful.

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