An asymptotic decrease of \( (m_p/m_e) \) with cosmological time, from a decreasing, small effective vacuum expectation value moving from a maximum in the early universe

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Abstract
The empirical, possible small variation downward by about \( 10^{-5} \), of the ratio of the proton mass to the electron mass, over a characteristic time interval estimated here to be about a billion years, is related to the decrease with time of a small, effective vacuum expectation value for a Goldstone-like pseudoscalar field which is present in the early universe. The same vacuum expectation value controls the magnitude of a very small, residual vacuum energy density today, which has decreased only a little in this model. The present time variation is estimated to be near to a definite limit, that is asymptotically approached as \( t \to \infty \).

Recently, new data [1] has given an indication that the ratio of the proton to electron mass \( \mu = (m_p/m_e) \), has decreased over a cosmological time interval. If interpreted in terms of an effective decrease in the proton mass, the data suggest a decrease by about \( (10 \text{ eV}) \times \mu \sim 18 \text{ keV} \), over a period of about the past twelve billion years. Natural questions which arise are then the following.

(1) Can one obtain the necessity for, and the direction of change, a decrease, independently of the possibility that coupling parameters such as \( \alpha_{em} \), depend upon time? Data on the latter possibility [2, 3], have stimulated the search for time variation of physical “constants” [4, 1]. Recent data [5, 6] have not yet confirmed a variation [2, 3]. Further data are forthcoming.

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(2) Can one obtain an estimate of the small absolute scale of the mass decrease, a few keV, by relating it to other small energy-scale effects, like a present, residual vacuum energy density (effective cosmological constant) of about $3 \times 10^{-47}$ GeV$^4$, and possibly to a neutrino mass of less than 0.1 eV?

(3) Is it possible to show that the time variation is approaching an asymptotic limit, as $t \to \infty$?

(4) Is it possible to estimate the cosmological time scale $\tau$, for significant mass decrease (on the above small energy scale), in terms of another cosmological parameter associated with the early universe? Possibly a significant decrease has also occurred in the first two billion years of the universe, as well as in the subsequent twelve billion years (present universe age $\sim 14$ Gyr).

The purpose of this note is to remark that there might be affirmative answers to all of these questions, and to make simple, but quantitative estimates of the relevant numbers. The basis for the results is the standard assumption that the main contributions to particle mass arise from the non-zero vacuum expectation values of spin-zero fields. It is usually assumed that such a vacuum expectation value arises at the stable minimum of some effective potential-energy density. Here, the first assumption leading to the results below, is that a small, particular vacuum expectation value of a pseudoscalar, Goldstone-like field $b$ (which spontaneously breaks CP invariance in a cosmological context) arises at a metastable (i.e. over a cosmological time interval $\tau$) maximum of an effective potential-energy density [7].

We note that the possibility that the standard, scalar-field inflationary dynamics in the very early universe originates at a maximum of an effective potential for a (classical) scalar (inflaton) field $\phi$, has already been considered in detail [8, 9, 10]. It has been shown that radiative corrections to a $\lambda \phi^4$ potential-energy density can set up an effective energy density with a maximum at or just above the Planck mass $M_{\text{Pl}} \cong 1.2 \times 10^{19}$ GeV, and a minimum just below $M_{\text{Pl}}$ [9]. Inflation occurs with the inflaton field at the maximum and during its movement to the minimum. The positive second derivative of the effective potential with respect to the field variable, at the minimum, corresponds to the large squared mass of inflaton quanta (estimated [7, 8] to be $m_\phi^2 \sim (5 \times 10^{11}$ GeV$)^2$). If metastable (i.e. essentially decoupled from big-bang radiation) these quanta can constitute dark matter today [7, 8, 11]. The pseudoscalar field $b$ can be connected with the scalar $\phi$, in a hypothetical, idealized model of a cosmological, spontaneously-broken chiral symmetry [7, 8]. However, the pseudoscalar field $b$ is a separate hypothesis from the scalar field $\phi$ whose vacuum energy density generates the hypothetical inflation near $t = 0$, and the $b$-field dynamics over cosmological time intervals is distinct. The hypothetical, small vacuum expectation value $F_b$ of the $b$ field, contributes a residual vacuum energy density of magnitude $|\rho_\Lambda| = |\lambda F_b^4| \sim 2.7 \times 10^{-47}$ GeV$^4$ for $F_b \sim 5.5$ eV [7], using the same, empirical [12, 13] value of the self-coupling parameter as for the $\phi$ field, $\lambda \sim 3 \times 10^{-14}$. An attempt is made to
obtain a separate estimate of $F_b$ by coupling $b$ to $\nu_\tau$ (with $g_{\nu_\tau}$) [7]. This gives rise to a (presumably largest) neutrino mass $m_{\nu_\tau} = \sqrt{\tilde{m}_{\nu_\tau}^2 + (g_{\nu_\tau}F_b)^2} \sim \sqrt{(g_{\nu_\tau}F_b)^2} \sim 0.055$ eV, for $F_b \sim 5.5$ eV and $g_{\nu_\tau} \sim 10^{-2}$ [7], and “bare” neutrino mass $\tilde{m}_{\nu_\tau} \sim 0$. This provides a quantitative representation (including the significant role of $\lambda$) of the often-remarked similarity between the empirical energy scales relevant for neutrino mass and for an effective cosmological constant. The above negative sign of the vacuum energy density can be changed to positive, by considering an (explicitly symmetry-breaking) effective energy density for the $b$ field to be $(1/2(\mu_b^2 b^2/4\lambda) + \lambda b^4)$ with $\mu_b^2 > 0$, instead of $(1/2(\mu_b^2 b^2/4\lambda) + \lambda b^4)$ with $\mu_b^2 < 0$. There is then a maximum [7] at $F_b$, with $F_b^2 = \mu_b^2/4\lambda$. The second derivative, the effective squared mass is $(\mu_b^2 - 12\lambda F_b^2) = -8\lambda F_b^2 < 0$. Thus, the second assumption here, which allows for the following results, is that quanta with negative squared mass (i.e. superluminal tachyons [14, 15]) are not present. This effectively prohibits strong long-range forces due to exchange of $b$ quanta. Thus, we assume that the main effect of a hypothetical coupling to quarks of a classical $b$ field is to give a mass contribution to primordial quarks. A coupling $g$ of the $b$ field to primordial ordinary quarks gives a quark mass contribution of $gF_b$; subsequently for three confined valence quarks, a nucleon mass contribution of $3gF_b$. We have assumed that primordial quarks have zero bare mass, and we do not consider possible thermal effects in this paper. We assume that it is at a later time $\sim 10^{-12}$ s, that electroweak symmetry-breaking generates the standard-model MeV mass contribution for light quarks, and we assume that this contribution then simply adds to the mass contribution estimated here, $gF_b$. This is the assumption that the electroweak mass term arises from the Higgs vacuum expectation value times a tiny coupling to the quark field which has acquired a mass term $gF_b$. Due to quark-antiquark pairs in the nucleon, the factor of 3 for the nucleon mass contribution, can probably be considered to be a minimal enhancement factor.

The above (standard) simple example of an effective potential-energy density, is used here only to illustrate a maximum, at $F_b$, and a point of zero second derivative, at $F_b/\sqrt{3}$. A classical field can remain at the maximum, i.e. with $db/dt = 0$. If we consider a very small perturbation which causes a variation with time of the effective $F_b$, then the direction of change must be assumed to be downward, so as to keep the related vacuum energy density bounded as $t \to \infty$. For the above example, the equation of motion implies that $b(t) = F_b(t)$ approaches zero as $t$ increases, where $F_b(t)$ is here used to denote an effective vacuum expectation value in different cosmological epochs. We must assume that this does not happen. Thus, the essential physical assumption is that $F_b(t)$ moves from a largest value at $t \sim 0$, toward zero, over cosmological time intervals.
parameterized by $\bar{t}$, but comes only a part of the way as $t \to \infty$, such that the second derivative then approaches zero through negative values. Although here, we have not obtained this from some hypothetical, effective potential, the idea is that such a behavior can be a physical possibility, which warrants being noted. Here we illustrate this possibility. We consider numerically a simple example for such a possible time dependence for the field $b(t) = F_b(t)$.

$$F_b^2(t) = \epsilon F_b^2(0) + \frac{(1 - \epsilon)F_b^2(0)}{1 + (t/\bar{t})^2}$$

with the parameter $\bar{t} \sim 2 \times 10^9$ yr. The limiting value is $\sqrt{\epsilon} F_b(0)$ as $t \to \infty$; $0 < \epsilon < 1$. So, we have

at $t \gtrsim 10^{-36}$ s \quad $3gF_b(0) \sim g(28.6 \text{ eV})$

at $t \sim \bar{t}$ \quad $3g\sqrt{(1 + \epsilon)/2F_b(0)} \sim g(23.4 \text{ eV})$ \quad (2)

at $t \gtrsim 14 \times 10^9$ yr \quad $\sim 3g\sqrt{\epsilon} F_b(0) \sim g(16.5 \text{ eV})$

Only for orientation, we have given the above numerical values using $\sqrt{\epsilon} \sim 1/\sqrt{3}$. From $t \gtrsim 10^{-36}$ s to 2 Gyr, there is a decrease of $-g(5 \text{ eV})$ in $m_p$. From $\sim 2$ Gyr to $\gtrsim 14$ Gyr, there is a decrease of $-g(7 \text{ eV})$. The asymptotic limit is approached in the present "old" universe. There are positive answers to the first three questions in the introduction. The direction of mass change is downward in the model. This direction is independent of the resolution of the issue of possible variation of coupling parameters with time. There is a limiting decrease as $t \to \infty$; this is related to the absence of a long-range force. Even with a sizable effective "magnification" factor $F^2$, $g \sim 10^3$, one obtains a small scale of mass change, $\sim \text{keV}$. Conceptually, this is related to a very small, residual vacuum energy density in the present epoch, and possibly to a very small neutrino mass. The electron mass can change, but the leptonic $b$ coupling may be like that estimated for neutrinos, $g_l \sim 10^{-2}$. Thus, the hypothetical downward change in $(m_p/m_e)$ is probably controlled by the downward change in $m_p$. The parameter $\bar{t}$ is possibly related to other dynamical quantities in the early universe. It might be connected with the ratio of vacuum expectation values, which ratio is numerically closely given in terms of the very small parameter $\lambda$ [11], that scales the primordial, vacuum energy densities: $\lambda^2 \sim F_b(0)/\sqrt{3} \phi_c \sim 5.5 \text{ eV}/10^{18} \text{ GeV} \sim 5.5 \times 10^{-27}$, for an inflaton mass $m_\phi \sim 2\sqrt{2}\sqrt{\lambda} \phi_c \sim 7.7 \times 10^{11}$ GeV. (Here $\phi_c \sim 10^{18}$ GeV, at an assumed minimum of zero for the inflaton effective potential.) Using the magnitude of the square root of the second derivative of the effective potential, $|m_b|$, as an

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$F^2$With reference to possible "magnification" of the effective $g$, it might be useful to note that the confinement of quarks at $\sim 10^{-6}$ s, does involve electroweak mass $\sim \text{MeV}$, being substantially increased to constituent quark mass. The QCD energy-scale parameter is $\Lambda \sim 220 \text{ MeV}$. 

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estimate for a minimal fluctuation $|\delta b|$ of a (classical) $b$ field at a potential maximum, and representing $|\delta b|$ as a geometric mean of the inverse of “initial” and “final” times, suggests $|\delta b| \sim \sqrt{\frac{1}{10^{-36} \text{ s}} \times \frac{1}{T}} > 2\sqrt{2}\sqrt{2 F_b(0)} = |m_b|$. Then, $T < (10^{-36} \text{ s}) \times (1/\lambda^4) \sim 3 \times 10^{16} \text{ s}$, where $10^{-36} \text{ s} \sim 1/m_\phi \sim \lambda^2/|m_b|$, is the time near the end of inflation generated by the $\phi$ field [11] $^F_3$. With this value for $T$, the expansion scale factor given approximately as $a(T) \sim (T/10^{-36} \text{ s})^{1/2} \sim (1/\lambda^2)^{1/2}$.

To recapitulate, the idea involves assumptions and numerical estimates. The unusual assumption is that the $b$ field can move from a small value at a maximum of its effective potential toward the value zero, over cosmological time intervals, but comes only a part of the way as $t \rightarrow \infty$. The second essential assumption is that when the second derivative of the effective potential with respect to the field is negative, quanta of the $b$ field with negative squared mass are not present to induce strong long-range forces. In the numerical estimates, an essential number is the empirically very small self-coupling parameter $\lambda$ for the inflaton ($\phi$) field; this parameter is common to the $b$ field self coupling in the model.

We conclude with the remark that two general ideas seem to receive support from the possible small decrease of $(m_p/m_e)$ with cosmological time [1]. One idea is that there is a small energy scale $F_b$, associated with the early universe [7, 11], in addition to the usual very high energy scales, i.e. inflaton mass and radiation temperature. Effects of the $b$ field and of the $\phi$ field are related, when depending upon the single parameter $\lambda$ [11]. The above smallness of $\lambda$ from the ratio of vacuum expectation values, can give the relatively slow evolution of the field and energy density on a short time scale for $\phi$, and possibly on a long time scale for $b$. An essential idea in this paper is a possible relationship of the very small parameter $\lambda$ to a cosmological time parameter $T$. Clearly, if $\lambda$ were to approach zero, then the residual vacuum energy density in the $b$ field would approach zero, and the interval $T$ would approach infinity. The second idea is that the small energy scale is capable of relating a very small effective cosmological constant today $^F_5$, and a small decrease of mass over a cosmological time interval.

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$^F_3$As counted from the time of $\phi$ leaving its value $\gtrsim M_{Pl}$ at the effective potential maximum.

There is no time-reversal symmetry for the vacuum in this model because of the presence of the $b$ field (odd under time reversal), and of a discontinuity in its behavior at $t \sim 0$.

$^F_4$It is interesting to note that the necessary minimal value of the expansion scale factor for an initial inflation over $\Delta t$ is a similar number. That is $a_{\text{infl}}(\Delta t) = e^{H_{\text{in}} \Delta t} \sim 1/\lambda^2 \sim 2 \times 10^{26}$, for $\Delta t \sim 1/H_{\text{in}} \times \ln(1/\lambda^2)$, where $H_{\text{in}}$ is the Hubble parameter as fixed by the initial vacuum energy density of the inflaton field, which is proportional to $\lambda$.

$^F_5$Perhaps this small energy scale limits the contributions of zero-point energies to a homogeneous energy density (possibly time dependent).
Appendix

The idea in this paper lends itself to a speculative conception of a progression of universes, without end and possibly with a most remote beginning, if any. An indefinitely long time interval in which the $b$ field has decreased a little, and the matter (and entropy) density has become vanishingly small, is joined to a negative time coordinate ($(\sqrt{\epsilon}F_b) \rightarrow -(\sqrt{\epsilon}F_b)$) at which the $\phi$ field is again established at a maximum ($|\phi| \rightarrow \sqrt{1/\lambda^2} |\sqrt{\epsilon}F_b|$, an “initial” condition explicitly related \cite{7} to a changing $b$ field near $t = 0$), causing another “initial”, brief inflation and a “big bang”. This is followed by another long time of matter dilution with a diminished vacuum energy density from a $b$ field $\sqrt{\epsilon}F_b$. Our motivation is that with such a concept of progression, some universes must acquire conditions of energy densities capable of supporting structure as we know it (with the $b$ field related to primordial CP violation, near $t = 0$), even if such conditions were not there in the remote past.

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