Ramsey interferometry as a witness of acceleration radiation

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We adapt a typical Ramsey interferometer by inserting a linear accelerator capable of accelerating an atom inside a single-mode cavity. We demonstrate that this simple scheme allows us to estimate the effects of acceleration radiation via interferometric visibility. By using a Rydberg-like atom, our results suggest that, for the transition regime of the order of GHz and interaction time of 1 ns, acceleration radiation effects can be observable for accelerations as low as $10^{17}$ m/s$^2$.

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I. INTRODUCTION

The Fulling-Davies-Unruh effect [1] is the premise to demonstrate that a linearly accelerated detector with acceleration $a$ in Minkowski space on a hyperbolic (constant acceleration) trajectory responds to the vacuum just as a detector at rest bathed by a Planck thermal distribution of quanta of a field at Unruh temperature $T_U = \hbar a/(2k_Bc^2)$ [2] where $\hbar$ is the Planck’s constant, $k_B$ the Boltzmann’s constant and $c$ the light speed in vacuum. On the other hand, it was shown [3, 4] that virtual processes in which atoms jump to an excited state while emitting a photon are an alternative way to view Unruh acceleration radiation. In a few words, by breaking and interrupting the virtual processes one can render the virtual photons real. Moreover in [5], it was shown that the radiation emitted by atoms falling into a black hole resembles, but is different from Hawking radiation, shedding light on the Einstein principle of equivalence between acceleration and gravity. The fundamental distinctions between Unruh effect and uniformly accelerated atoms inside cavities are discussed in [6].

The process of atomic excitation by means of interaction with a quantum field in vacuum can receive contributions from both vacuum fluctuations and radiation reaction [7]. This, in turn, can be extended to the study of atoms under uniform acceleration. For instance, the excitation rate and radiative energy shift of a two-level atom in uniform acceleration coupled to massless scalar field [8] and electromagnetic field [9] in an inertial frame were calculated in Minkowski vacuum and also in spacetimes with boundaries [10]. It was shown, for instance, that acceleration-induced perturbations lead to spontaneous excitation even in vacuum. In [11], it was studied the rate of change of atomic energy for an atom in uniform acceleration coupled to quantum electromagnetic field at a thermal state with temperature $T$ from a co-accelerated observer viewpoint. They have shown that the result is the same of a local inertial observer assuming the Unruh temperature in the co-accelerated frame.

Because the Unruh temperature is smaller than 1 K even for acceleration as high as $10^{22}$ m/s$^2$, its direct observation is not accessible with current technology. For this reason, there has been growing interest in practical scheme or experimental ideas to estimate the Unruh temperature [12–16]. Motivated by this consideration, in this contribution we consider a typical single particle Ramsey interferometer [17]. We modify the interferometer by inserting a linear accelerator (LA) (Cavity plus short-pulse laser) capable of accelerating an atom through a single-mode cavity. We show that this simple scheme allows us to estimate the effects of acceleration radiation via fringe visibility in an interferometric setup. In our theoretical analysis, we consider a Rydberg-like atom initially prepared in the ground state. We find that for the transition regime of the order of GHz and interaction time of 1 ns, the visibility encodes information about the acceleration radiation which is manifested experimentally for acceleration as low as $10^{17}$ m/s$^2$. Since ultra-high accelerations of Rydberg-like atoms have been obtained recently in [18, 19], the experimental implementation of the present setup is attainable by current technology.

The outline of the paper is as follows. In Sec. 2, we start by presenting a theoretical analysis of our setup and formulating an effective model for the system which is a two-level atom interacting simultaneously with a classical field and a quantum field. In Sec. 3, we evaluate finite-time corrections to the transition probability. In Sec. 4, we show how our setup works and calculate the interferometric visibility. The analysis and discussion of the results are presented in Sec. 5. Finally, our concluding remarks are addressed in Sec. 6.

II. THE SETUP

Consider a conventional Ramsey interferometer [17] for accelerated two-level atoms. Suppose that such two-level atoms are produced by the source in the ground $|g\rangle$ or excited $|e\rangle$ states and enter in the first Ramsey zone (cavity $R_1$) which has a field resonant or quasi-resonant with the transition $|g\rangle \leftrightarrow |e\rangle$ yielding a $\pi/2$ pulse on the atoms [17, 20]. After the atoms pass through the cavity $R_1$, they enter into cavity $C$ where they are resonantly coupled with a single mode electromagnetic field. Inside cavity $C$, the atoms are linearly accelerated by a short-pulse laser. After that, the atoms enter in the second Ramsey zone (cavity $R_2$) where their internal states are re-
combined and the interference pattern measured in the detector $D_g$. This setup is sketched in Fig. 1.

![Image](54x303 to 315x402)

**Figure 1.** The proposed scheme for single-particle interference acting as a witness of acceleration radiation. A key requirement in the practical realization of the acceleration mechanism is the ability to produce a short-pulse laser with intensity and frequency separately controlled. In addition, the short-pulse laser must be of low enough intensity so that it does not ionize or strongly perturb the internal states of the atom.

The Hamiltonian of the system includes two interaction terms. One of them is the quantum interaction of the atom and the quantum field inside the cavity $C$ and the other one is the semi-classical interaction of the atom and the classical field produced by the laser. The total Hamiltonian is described by the usual Jaynes-Cummings Hamiltonian \[ \hat{H} = \hat{H}_a + \hat{H}_f + \hat{H}_Q + \hat{H}_\text{int}, \] (1)

where $\hat{H}_a = \hbar \omega \hat{\sigma}_z$ is the Hamiltonian for a two-level atom with upper and lower levels $|\epsilon\rangle$ and $|g\rangle$, respectively, with $\hat{\sigma}_z = \frac{1}{2}(|g\rangle\langle g| - |\epsilon\rangle\langle \epsilon|)$. $\hat{H}_f = \sum_k \hbar \nu_k \hat{a}_k^\dagger \hat{a}_k$ is the Hamiltonian of the field where $\hat{a}_k$ and $\hat{a}_k^\dagger$ are photon creation and annihilation operators, and $\hat{H}_Q$ stands for the coupling of the atom with the quantum field inside the cavity $C$ which in the atomic frame of reference reads

\[ \hat{H}_\text{int} = \hbar \sum_k \lambda_k(t) [\hat{a}_k^\dagger \hat{\sigma}_- e^{i\nu_k t} - i\hat{k}z - i\omega \tau + \text{h.c.}]. \] (2)

In equation above, $\tau$ is the atom proper time ($t$ denotes the time in the inertial laboratory frame) and $\lambda_k(t)$ is the effective coupling constant between the atom and the field, which depends on the dipole moment and the field amplitude. Notice that the Hamiltonian (2) describes the atom-field interaction in the dipole and rotating-wave approximations. We also take $\lambda_k(t) = \lambda_k W(t)$, where we choose $W(t) = \exp \left( -\frac{|t|^2}{T^2} \right)$ as a gradual window function in order to describe the interaction between atom and field for a finite time interval during the flight in the linear accelerator. $T$ is a characteristic time such that $W(t \ll T) \approx 1$ and $W(t \gg T) \approx 0$. On the other hand, $\hat{H}_\text{int}$ stands for the coupling between the atom and the classical laser beam which in the dipole approximation is given by

\[ \hat{H}_\text{int} = \hbar \kappa \left( e^{-i\omega_L t} \hat{\sigma}_+ + e^{i\omega_L t} \hat{\sigma}_- \right), \] (3)

where $\omega_L$ is the laser beam frequency and $\kappa$ the coupling constant. $\hat{\sigma}_+ = |\epsilon\rangle\langle g|$ and $\hat{\sigma}_- = |g\rangle\langle \epsilon|$ are the atomic raising and lowering operators, respectively.

The total Hamiltonian of the system in the reference frame rotating at the frequency $\omega_L$ of the classical field, has the form

\[ \hat{\mathcal{H}}^{\text{RF}} = \hat{\mathcal{H}}_0 - \hbar \sum_k \delta_k \hat{a}_k^\dagger \hat{a}_k + \hbar \sum_k \lambda_k(t) [\hat{a}_k^\dagger e^{i\nu_k t} - i\hat{k}z - i\omega \tau + \text{h.c.}], \]

where $\delta_k = \omega_L - \nu_k$, and $\hat{\mathcal{H}}_0 = -\hbar \Delta \hat{\sigma}_z + \hbar \kappa (\hat{\sigma}_+ + \hat{\sigma}_-) + \text{dressing} \Delta = \omega_L - \omega$. The presence of the laser change the atom states $|\epsilon\rangle$ and $|g\rangle$ to the dressed states $|\pm\rangle$ and $|\mp\rangle$, respectively, see Fig. 2.

Let the coupling to the laser be much stronger than the coupling to the cavity modes, i.e., $|\kappa| \gg \lambda_k$. Thus, we can express $\hat{\mathcal{H}}^{\text{RF}}$ in the basis of eigenstates $|\pm\rangle$ of $\hat{\mathcal{H}}_0$ with

\[ \hat{\mathcal{H}}_0 |\pm\rangle = -\frac{\hbar (\Delta + \Omega)}{2} |\pm\rangle \]

where $\Omega = \sqrt{\Delta^2 + 4\kappa^2}$ is called the effective Rabi frequency. In the limit where the detuning $\Delta$ is large as compared to the $|\kappa|$, the effective Rabi frequency is given by $\Omega \approx \Delta + \delta \omega$ where $\delta \omega = \frac{4\kappa^2}{\Delta}$ corresponds to the shift in the transition frequency due to the interaction of the atom with the laser.

![Image](365x303 to 515x402)

**Figure 2.** Schematic representation of the dipole transition $|g\rangle \rightarrow |\epsilon\rangle$ at frequency $\omega$ coupled to the laser at frequency $\omega_L$ and the semiclassical dressed states.

The semiclassical dressed states are given by

\[ |+\rangle = \sin \theta |g\rangle + \cos \theta |\epsilon\rangle, \]
\[ |-\rangle = \cos \theta |g\rangle - \sin \theta |\epsilon\rangle, \]

where $\cos \theta = \sqrt{\frac{\frac{\Delta}{2} + \frac{\delta \omega}{2}}{\sqrt{\Delta^2 + 4\kappa^2}}}$ and $\sin \theta = \sqrt{\frac{\frac{\Delta}{2} - \frac{\delta \omega}{2}}{\sqrt{\Delta^2 + 4\kappa^2}}}$. Let’s introduce the raising and lowering operators for the dressed states basis $\hat{\pi}_+ = |+\rangle\langle -|$ and $\hat{\pi}_- = |\mp\rangle\langle \mp\rangle$. By using the semiclassical dressed state basis and operator notation we can rewrite $\hat{\mathcal{H}}^{\text{RF}}$ in the interaction picture as

\[ \hat{H}^{\text{I}} = \hbar \sum_k \lambda_k(t) [\sin \theta \cos \theta \hat{a}_k(t) e^{-i\delta_k t} + \hat{a}_k^\dagger(t) e^{i\delta_k t}] \hat{\pi}_z + \cos^2 \theta [\hat{a}_k^\dagger(t) \hat{\pi}_+ e^{-i(\Omega - \delta_k) t} + \hat{a}_k(t) \hat{\pi}_- e^{i(\Omega - \delta_k) t}] - \sin^2 \theta [\hat{a}_k^\dagger(t) \hat{\pi}_+ e^{i(\Omega - \delta_k) t} + \hat{a}_k(t) \hat{\pi}_- e^{-i(\Omega - \delta_k) t}] \]
where \( \hat{a}_k(\tau) = \hat{a}_k e^{-i\nu_k c t(\tau) + ikz(\tau) + i\omega \tau} \) and \( \hat{a}_k^\dagger(\tau) = \hat{a}_k^\dagger e^{i\nu_k c t(\tau) - ikz(\tau) - i\omega \tau} \). For \( \delta_k = \Omega \) and if \( |\lambda_k| \ll \Omega \) (strong driven regime), we can realize a rotating-wave approximation and choose from \( \hat{H}^\dagger \) the resonant terms

\[
\hat{H}_{\text{eff}} = \hbar \sum_k \lambda_k(\tau) \cos^2 \theta [\hat{a}_k(\tau) \hat{\pi}_+ + \hat{a}_k^\dagger(\tau) \hat{\pi}_-]
\]  

(4)

In the limit of large detuning \( \Delta \gg |\kappa| \) the atom is uniformly accelerated due a dipole force produced by the laser pulse \( \Psi \). For the uniformly accelerated atom traveling in the \( z \)-direction, the relation between the laboratory Minkowskian inertial frame with the accelerated frame is given by

\[
t(\tau) = \frac{c}{a} \sinh(\frac{a \tau}{c}), \quad z(\tau) = \frac{c^2}{a} \cosh(\frac{a \tau}{c}),
\]

where \( a \) is the acceleration. Thus the uniformly accelerated atom describes hyperbolic trajectories in the right-hand half Minkowski spacetime known as the right Rindler wedge defined by \( z > |\ell| \) (see, e.g., Ref. [25]). Moreover, for copropagating atom and field, \( k_z = k = \nu_k e/c \) and \( \lambda_k \) scales as \( \lambda_k = \lambda_k e^{-\frac{a \tau}{c}} \) for a uniformly accelerated atom due to the eletric field transformation properties to the atomic frame [2, 3]. The exponent in [2] becomes

\[
i\nu_k c t(\tau) - ikz(\tau) = \frac{i\nu_k c}{a} \sinh(\frac{a \tau}{c}) - \frac{i k c^2}{a} \cosh(\frac{a \tau}{c})
\]

\[
= -\frac{i\nu_k c}{a} e^{-\frac{a \tau}{c}}.
\]

Therefore, the effective interaction Hamiltonian (4) can be rewritten as

\[
\hat{H}_{\text{eff}} = \hbar \sum_k \lambda_k(\tau) \cos^2 \theta [\hat{a}_k \hat{\pi}_- e^{-i\omega \tau} e^{-\frac{a \tau}{c}} - i\omega \tau - \frac{a \tau}{c} + \text{h.c.}],
\]

(5)

For simplicity, we assume the single mode approximation for the coupling between the atom and the quantum field in the cavity \( C \) as in conventional derivations of the Jaynes-Cummings model [28]. Thus we may ignore the coupling between the atom and all field modes except with the mode \( k \) with frequency equal to \( \nu_k \). It is worth mention that this approximation produces accurate results if the evolution times are long (for details see, e.g., Ref. [29]).

### III. LEADING ORDER CORRECTION TO THE TRANSITION AMPLITUDE

In the interaction picture, the leading order unitary transformation induced by the Hamiltonian (4) in the weak coupling regime reads

\[
\hat{U} \approx 1 - iI(a) \cos^2 \theta \hat{a}_k^\dagger \hat{\pi}_- - iI^*(a) \cos^2 \theta \hat{a}_k \hat{\pi}_+,
\]

(6)

where the transition amplitude is

\[
I(a) = \lambda_k \int_{-\infty}^\infty d\tau \exp \left[ -\frac{i\nu_k c}{a} e^{-\frac{a \tau}{c}} - i\omega \tau - \frac{a \tau}{c} - \frac{|\tau|}{T} \right].
\]

By making the substitution \( x = \frac{\nu_k c}{a} e^{-\frac{a \tau}{c}} \), \( I(a) \) reduces to

\[
I(a) = \frac{\lambda_k}{\nu_k} \left( \frac{a}{\nu_k c} \right)^{\frac{a \tau}{c}} \int_{-\infty}^\infty dx \ x^\frac{a \tau}{c} e^{-ix} + \frac{\lambda_k}{\nu_k} \left( \frac{a}{\nu_k c} \right)^{\frac{a \tau}{c}} \int_0^{\nu_k c} dx \ x^{\frac{a \tau}{c} + \frac{a \tau}{c}} e^{-ix}.
\]

(7)

Recalling the definition of incomplete gamma functions [30] enables us to write \( I(a) \) as

\[
I(a) = -\frac{i\lambda_k}{\nu_k} \left( \frac{a}{\nu_k c} \right)^{\frac{a \tau}{c}} e^{-\frac{a \tau}{c}} \left[ \left( \frac{a}{\nu_k c} \right)^{\frac{a \tau}{c} + \frac{a \tau}{c}} - \frac{i\pi c}{a} \right] e^{\frac{a \tau}{c}} \Gamma(1 + \mu_- + \frac{\nu_k c}{a})
\]

\[+ \left( \frac{a}{\nu_k c} \right)^{\frac{a \tau}{c}} e^{-\frac{a \tau}{c}} \gamma(1 + \mu_+, \frac{\nu_k c}{a}).
\]

(8)

where \( \mu_{\pm} = \pm \frac{\nu_k c}{a} \). The expression above can be studied using the well known properties and asymptotic limits of incomplete gamma functions. In particular, let us consider the limit of high acceleration in which one obtains

\[
\Gamma(1 + \mu_-, \frac{\nu_k c}{a} \to 0) = \Gamma(1 + \mu_-),
\]

\[
\gamma(1 + \mu_+, \frac{\nu_k c}{a} \to 0) = 0.
\]

Now, substituting these results in (8), yields the transition probability:

\[
|I(a)|^2 \text{large } a = \frac{\lambda_k^2}{\nu_k^2} \left( \frac{a}{\nu_k c} \right)^{\frac{a \tau}{c}} e^{-\frac{a \tau}{c}} \Gamma(1 + \mu_-) \Gamma(1 + \mu_+).
\]

Furthermore we can expand in terms of \( \frac{1}{a T} \) in order to evaluate finite-time corrections to the transition probability. Thus, for \( \frac{1}{a T} \ll 1 \) and by using the relation \( \Gamma(1 - \frac{\nu_k c}{a}) \Gamma(1 + \frac{\nu_k c}{a}) = \frac{\nu_k c}{a} \sinh^{-1}(\frac{\nu_k c}{a}) \), the transition probability \( |I(a)|^2 \) can be written as

\[
|I(a)|^2 \approx \frac{2\pi \lambda_k^2 \nu_k}{a \nu_k} \left( \frac{1}{\nu_k c} \right)^{-\frac{a \tau}{c}} e^{-\frac{a \tau}{c} - 1} \left[ 1 - \frac{c}{a T} \left( 2 \ln \left( \frac{a}{\nu_k c} \right) \right. \right.
\]

\[+ \psi(\Omega_+) + \psi(\Omega_-) \left. \right] + \mathcal{O}(a T)^{-2}, \]

(9)

where \( \psi(z) \) is the digamma function and \( \Omega_{\pm} = 1 \pm \frac{\nu_k c}{a} \). It is clear that in the \( \frac{1}{a T} \to 0 \) limit, we recover the thermal spectrum as in the Unruh effect in free space. In other words, the radiation emitted by the accelerated atom in the cavity \( C \) corresponds to thermal radiation with temperature equal to the Unruh temperature \( T_U \).
IV. RAMSEY INTERFEROMETRY

In order to explain how the apparatus depicted in Fig. 1 works, let us assume that initially the system (atom plus quantum field in cavity C) is prepared in the state $|\psi_1\rangle = |0_{M_d}\rangle \otimes |g\rangle$, where $|0_{M_d}\rangle$ is the Minkowski vacuum state of the field defined in the inertial laboratory frame. The interaction between the atom and the cavity $R_1$ is chosen to produce a $\pi/2$ rotation in the Bloch sphere. This results in a transformation of the $|g\rangle$ state to the superposition state:

$$|\psi_1\rangle \rightarrow \frac{1}{\sqrt{2}} |0_{M_d}\rangle \otimes (|g\rangle + e^{i\phi}|e\rangle).$$

The probability amplitude of finding the atom in $|g\rangle$ or $|e\rangle$ between the cavities $R_1$ and $R_2$ accumulates a quantum phase. Thus, a phase shift $\Phi$ is introduced to the atomic state if the atom is in $|e\rangle$:

$$|\psi_2\rangle \rightarrow \frac{1}{\sqrt{2}} |0_{M_d}\rangle \otimes (|g\rangle + e^{i\phi}|e\rangle).$$

Now, let us consider that the two-level atoms cross a microwave cavity (cavity C) where they are linearly accelerated in the $z$-direction by a short-pulse laser, as shown in Fig. 1. From (9), we find that the interaction between the uniformly accelerated atom and the field mode $k$ produces the following transformations:

$$|0_{M_d}\rangle \otimes |\pm\rangle \rightarrow |0_{M_d}\rangle \otimes (-|\pm\rangle),$$

$$|0_{M_d}\rangle \otimes |\mp\rangle \rightarrow |0_{M_d}\rangle \otimes (|-\rangle - iI(a) \cos^2 \theta |1_{M_d}\rangle \otimes |\pm\rangle).$$

(10)

Such transformations show that when the uniformly accelerated atom is in the ground-state and the LA in the vacuum state, the coupling with the field mode in the LA does not affect the atom-field state. On the other hand, when the atom is in the excited state, a photon can be emitted inside the LA in the vacuum state which is called acceleration radiation. This mimics the superposition of an inertial and accelerated trajectory in a interferometer as discussed in [13]. Since we are interested in observing only the effect of the acceleration radiation in the interference pattern, we consider that the transition probability from the excited to the ground state or from the ground to the excited state is negligible when the non-accelerated atom interacts with the cavity C in the vacuum state, i.e., when the short-pulse laser is off. This is consistent with an atom-field interaction in the weak coupling regime and for very short interaction times [51]. By writing (9) in the $\{|+\rangle, |-\rangle\}$ basis and using Eq. (11), after the atom passes through the cavity C and propagates to cavity $R_2$, the state of the system reads

$$|\psi_3\rangle \rightarrow \frac{1}{\sqrt{2}} [ |0_{M_d}\rangle \otimes (|g\rangle + e^{i\phi}|e\rangle)$$

$$- iI(a)(e^{i\phi} \cos \theta + \sin \theta \cos^2 \theta |1_{M_d}\rangle \otimes |g\rangle$$

$$+ iI(a)(e^{i\phi} \cos \theta + \sin \theta \cos^2 \theta |1_{M_d}\rangle \otimes |e\rangle)].$$

Just as for the cavity $R_1$, the interaction time between the atom and the cavity $R_2$ is chosen so that it corresponds to a $\pi/2$-pulse interaction. Finally, after the atom passes through the cavity $R_2$, the final state of the system becomes

$$|\psi_4\rangle \rightarrow e^{i\frac{\Phi}{2}} \left\{ |0\rangle \otimes \left[ \cos \left( \frac{\Phi}{2} \right) |g\rangle + i \sin \left( \frac{\Phi}{2} \right) |e\rangle \right]$$

$$- iI(a)F_- |1\rangle \otimes |g\rangle - iI(a)F_+ |1\rangle \otimes |e\rangle \right\},$$

(12)

where

$$F_{\pm}(\theta, \Phi) = \frac{1}{2} \cos^2 \theta (e^{i\frac{\Phi}{2}} \cos^2 \Phi + e^{-i\frac{\Phi}{2}} \sin \Phi \cos \theta \pm \sin \theta).$$

The reduced density matrix $\hat{\rho}_A = \text{Tr}_M(|\psi_4\rangle\langle \psi_4|)$ of the atom is obtained using (12) and tracing out the field degrees of freedom. After some algebraic manipulations one obtains

$$\hat{\rho}_A = \frac{1}{N} \left\{ \begin{array}{l}
I(a)^2 |F_-|^2 + \cos^2 \left( \frac{\Phi}{2} \right) |g\rangle \langle g|
+ \left[ I(a)^2 F_+ F_-^* + i \sin \left( \frac{\Phi}{2} \right) \cos \left( \frac{\Phi}{2} \right) \right] |e\rangle \langle e|
+ \left[ I(a)^2 F_+ F_- - i \sin \left( \frac{\Phi}{2} \right) \cos \left( \frac{\Phi}{2} \right) \right] |g\rangle \langle e|
+ I(a)^2 |F_+|^2 + \sin^2 \left( \frac{\Phi}{2} \right) |e\rangle \langle e|
\end{array} \right\},$$

where the normalization factor $N$ is given by

$$N = 1 + |I(a)|^2 |F_+|^2 + |I(a)|^2 |F_-|^2,$$}

and $|I(a)|^2$ is given by [9].

From $\hat{\rho}_A$, we find that the probability to detect each atom in the $|g\rangle$ state by the detector $D_g$ is given by

$$P_g = \frac{1}{N} \left\{ \cos^2 \left( \frac{\Phi}{2} \right) + \frac{|I(a)|^2}{4} \cos^4 \theta \left[ 1 + \sin 2\theta (\cos \Phi - 1) \right]$$

$$+ \cos \Phi \sin^2 2\theta \right\},$$

$$= \frac{1}{N} \left\{ \cos^2 \left( \frac{\Phi}{2} \right) + \frac{|I(a)|^2}{4} \left( \frac{\delta \omega}{2(\Delta + \delta \omega)} \right)^2 \right\}$$

$$\times \left[ 1 + \frac{2\kappa}{(\Delta + \delta \omega)} (\cos \Phi - 1) + \frac{4\kappa^2}{(\Delta + \delta \omega)^2} \cos \Phi \right],$$

(13)

which is the Ramsey interference for the accelerated two-level atoms. The fringe visibility for the interference pattern is defined by $V = \frac{P_{\text{max}} - P_{\text{min}}}{P_{\text{max}} + P_{\text{min}}}$, where the max/min values are calculated with respect to phase $\Phi$. The probability $P_g$ has the maximum (minimum) value when $\Phi = 0$.
we plot the visibility difference as a function of acceleration by means of interference properties. It is known that de-excitation (excitation) of the atom by photon emission (absorption) is a source of loss of coherence. Here, we can observe from Eq. (14) that, because of the acceleration radiation emitted by the accelerated two-level atoms, loss of coherence effects are apparent showing a reduction in the fringe visibility, i.e., $V < 1$ (partially coherent). In the limit $a \to 0$ and $\kappa \to 0$ (short-pulse laser off), we get the visibility equal to 1 (completely coherent). We have to emphasize that this source of loss of coherence is produced by the dipole radiation effects by measuring the reduction in the fringe visibility of acceleration for different values of $\kappa$. In addition, we consider timescale of evolution much smaller than the lifetime of Rydberg states. To verify the loss of coherence effects induced by acceleration radiation, we compare the fringe visibility when the atom is inertial (short-pulse laser off) with the visibility when the atom is accelerated (short-pulse laser on). We define the visibility difference between the two situations as \( \delta V = V(0) - V(a) \).

In Fig. 3 we plot the visibility difference $\delta V$ as a function of the acceleration for two different interaction times $T = 1 \text{ ns}$ red (solid) line and $T \to \infty$ black (dashed) line. We consider the microwave regime of cavity field, i.e., $\omega$ and $\nu_k$ of the order of GHz, and for the coupling constants $\lambda_k = 50 \text{ MHz}$ and $\kappa = 200 \text{ MHz}$, where $\lambda_k \ll \omega$ (weak coupling regime) to guarantee the validity of the approximations used in Eq. (6).

We can observe that the difference between the fringe visibility of the accelerated and no accelerated interference pattern, monotonically increases when the acceleration grows. The loss in the fringe visibility can be understood as a result of the change in the internal state induced by acceleration radiation emitted due to the dipole approximation. In particular, the Fig. 3 shows that the visibility difference approaches to the value $\delta V \approx 10^{-5}$ for acceleration of the order of $5 \times 10^{17} \text{ m} \cdot \text{s}^{-2}$. Notice that, for an acceleration of $\approx 5 \times 10^{17} \text{ m} \cdot \text{s}^{-2}$ and an interaction time of the order of 1 ns, the cavity length is approximately $\approx 25 \text{ cm}$. The black dashed line in Fig. 3 shows the visibility difference as a function of acceleration for a long interaction time ($T \to \infty$). This asymptotic regime can be considered as an ideal curve. However, it is noteworthy that in the high acceleration regime the increase in interaction time corresponds to an increase in cavity size and most relativistic the atom becomes. In Fig. 4 we plot the visibility difference as a function of acceleration for different values of $\kappa$ and consequently of $\delta \omega$ in order to observe the influence of the shift in the transition frequency on the visibility difference. We can observe that the visibility difference increases when $\kappa$ and $\delta \omega$ increase. These results show that although the coupling with the classical laser pulse does not produce entanglement and decoherence the atom states get dressed producing a shift in the transition frequency which contributes to the loss of coherence.

Therefore, the proposed experimental setup which is based in a type of Ramsey interferometry with accelerated two-level atoms can be used to access acceleration radiation effects by measuring the reduction in the fringe
visibility. In addition, we would like to emphasize that our setup is idealized, since we have considered cavities with perfect mirrors. Investigation of the imperfection effects will be left for a future work.

VI. CONCLUSIONS

In Ref. [18], it was proposed the detection of the Unruh effect by measuring the difference in the Berry’s phase of accelerated and inertial detectors. It was found that the Unruh effect could be detected for accelerations as low as $10^{17}$ m/s$^2$. Although that proposal could be tested in any experimental setup capable to measure the Berry’s phase, a specific implementation was left for a future work. Here, we present a specific implementation to detect the analogous of this effect by measuring the fringe visibility of the Ramsey interferometry for accelerated two level atoms instead of Berry’s phase. We show that acceleration radiation emitted in a cavity in the vacuum state produces loss of coherence which can be observed by measuring the reduction of the fringe visibility. It is also shown that our setup can in principle detect a difference of $10^{-5}$ in the fringe visibility of Rydberg atoms with transition frequency of the order of GHz and accelerations as low as $5 \times 10^{17}$ m/s$^2$.

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