Formulations and algorithms for problems on rock mass and support deformation during mining

VM Seryakov
Chinakal Institute of Mining, Siberian Branch, Russian Academy of Sciences, Novosibirsk, Russia
E-mail: vser@misd.nsc.ru

Abstract. The analysis of problem formulations to calculate stress-strain state of mine support and surrounding rocks mass in rock mechanics shows that such formulations incompletely describe the mechanical features of joint deformation in the rock mass–support system. The present paper proposes an algorithm to take into account the actual conditions of rock mass and support interaction and the algorithm implementation method to ensure efficient calculation of stresses in rocks and support.

Two approaches to problems on stress state of mine support are currently widely applied. In the first approach, loads on support outback are determined based conditions of support installation and deformation, using various hypotheses of rock mass and support interaction. Then, the problem of stress state of mine support under the action of the determined loads is solved. In the second approach, the problem on joint deformation of mine and support interface is formulated. In this case, there are two stages of deformation. The first stage is free deformation of a tunnel boundary. The second stage is engagement of the tunnel boundary and support and their joint deformation [1–4].

So, in the first stage of deformation, free displacements of a tunnel boundary are found. These displacements are governed by external forces and connected with the partial release of initial stresses existing before the tunnel drivage. Thus, this partial stress release is to be determined at the first stage of the problem solution. The rest forces corresponding to the complete liberation of the initial stresses from the tunnel boundary affects both enclosing rock mass and support, and governs the stress state of the support and adjacent rock mass. The second stage is the problem on deformation of a nonuniform medium composed of rock mass and support having different physical properties. And here we have that determination method of the forces exerted on this nonuniform medium as well as their mechanisms remains yet uncertain.

Applied problem solving involves usually simpler formulations [1–4]. First, the problem on the adjacent rock deformation is solved as the problem on unsupported tunnel. Then, support subjected to normal compressive stresses applied to its outback surface is considered. By solving these two problems given the additional condition on the equality of the tunnel boundary and support outback displacements, it is assumed that formulated problem has been solved. The degree of approximation due to separate consideration of deformation of enclosing rocks and support is difficult to estimate.

The values of tunnel boundary displacement before support installation depend on the process variables of driving and supporting [5]. These values can be obtained from the analysis of in situ data of rock deformation in case of different designs of support systems. In [5], it is suggested to estimate tunnel boundary displacement before engagement with support by numerical solution of three-
dimensional elasticity problems on stress state of rocks surrounding a tunnel. With the known value of lag between the support and face, it is possible to evaluate free displacements of the tunnel boundary, $u_0$, $v_0$, which are the $\alpha X$- and $\alpha Y$-components of the displacement vector of the tunnel boundary before the support installation (in plane problem). Some formulations, which take into account that support is installed some time after partial relaxation of enclosing rocks from initial stresses, determine the support outback displacements $u_p$, $v_p$ using the relations: $u_p = u - u_0$; $v_p = v - v_0$, where $u$, $v$—projections of the displacement vector of unsupported tunnel [6, 7]. With such formulation, displacements of tunnel boundary with regard to its joint deformation with support have the same values as in the case of a tunnel without support. In other words, support has not influence on overall displacement of tunnel boundary. To estimate the effect exerted by the degree of such approximation on calculated data of stress state of support and adjacent rock mass, the problem formulation should more exactly condition their interaction.

Below we discuss a problem formulation assuming the deformation of rocks and support is described by an elastic body model. In this case, the first stage problem on deformation of adjacent rock mass can be solved by subjecting the external boundaries of the computational domain to the action of stresses conforming with the occurrence depth of the tunnel and by setting condition of the tunnel boundary to be free from external load. With the $X$-axis oriented horizontally and the $Y$-axis vertically, at the tunnel occurrence depth $H$ we have:

$$\sigma_x = -\gamma H; \quad \sigma_y = -\lambda \gamma H; \quad \tau_{xy} = 0. \quad (1)$$

Here, $\sigma_x$, $\sigma_y$, $\tau_{xy}$—normal and shear stress components in the plane $\alpha X\gamma$; $\gamma$—bulk weigh of overlying strata; $\lambda$—lateral earth pressure coefficient. Before the moment when support is installed, the tunnel boundary displaces by some a priori known values determined from the solution of a boundary value problem involving gradual increment in the stresses $\sigma_y$, $\sigma_z$ at the boundary of the computation domain. The first stage is assumed completed when the a priori known and calculated values of the unsupported tunnel displacement reach satisfactory agreement. In this manner, after the first stage of solution, we know stress state of adjacent rock mass and external forces that displace the tunnel boundary by the determined value.

By determining the dormant portion of external forces as the difference of the initial force and the forces applied at the first stage of deformation, we determine the second stage stresses at the computational domain boundary. At this stage, the external loading is applied to deform a nonuniform medium composed of rock mass and support subjected to the condition of their rigid engagement. This problems solution yields stress state of the support and additional stresses and strains corresponding to the second stage deformation. The external forces on the adjacent rock mass—support system at the second deformation stage can be given by $\sigma_y = -\alpha \gamma H; \quad \sigma_z = -\alpha \lambda \gamma H$ [1, 2]. The coefficient $\alpha$ is less than unit. We have said earlier that it is difficult to implement this algorithm in the problem on deformation of enclosing rock mass and tunnel support and, therefore, simplifications are involved. These simplifications are connected with various assumptions on the value of the coefficient $\alpha$.

Let us apply the proposed algorithm to estimate the degree of approximation in a traditional problem formulation on deformation of enclosing rock mass and support in a single tunnel of the assumption that the support is installed ‘instantaneously’. In case of the plane strain deformation, the computational domain is a half-plane with tunnel the boundary of which is in contact with a material attributed with mechanical properties of support. The inner surface of the supported tunnel is free from the external load, and the external boundary of the computational domain is subjected to forces conforming with the level of initial stresses in the center of the tunnel when being driven, i.e. Eq. (1).

This problems solution is a sump of solutions of two problems [8]. The first problem is on initial stresses in a nonuniform medium of enclosing rock mass and support. The external boundary of this nonuniform medium is set to experience external forces corresponding to the conditions (1). The second problem is the determination of additional stresses induced by the tunnel drivage. In this
problem, the external boundary of the computational domain is free from forces, and the internal boundary of the supported tunnel is subjected to the normal and shear forces which vanish when summed with the forces obtained from the solution of the first problem.

The traditional formulation of problems in case of ‘instantaneous’ installation of support in a tunnel shows that the tunnel is driven in the initial stress state different from the stress state generated by the rock mass weight and tectonics. This disagrees with the actual conditions of tunnel drivage and installation of support system in it. Support is installed in rock mass which is in the initial stress state governed by the overlying rock weight and the related tectonic external forces. After ‘instantaneous’ installation of support, its inner boundary is free from the external forces and normal and shear stresses.

The finite element calculation [9] of stress state of rock mass–support system in the traditional formulation of ‘instantaneous’ installation of the support is illustrated in Figure 1. The calculations involved the input data on the geometry of the tunnel, mechanical properties of rocks and support and the initial stress field. For rocks: Young’s modulus \( E = 50000 \) MPa; \( v = 0.25 \). For support: \( E = 100000 \) MPa; \( v = 0.2 \). The bulk weight of rocks and support is 0.03 MN/m\(^3\). The initial stress state of rock mass is conditioned by the rock mass weight and, according to Dinnik’s hypothesis:

\[
\sigma_y^0 = -\rho H; \quad \sigma_x^0 = -\rho H (1-v); \quad \tau_{xy}^0 = 0.
\]

The tunnel has the shape of a semi-circle with a radius of 7 m, the tunnel bottom occurs at the depth of 750 m. The support has a thickness of 0.5 m.

The concentration zones of the compressive stresses \( \sigma_2 \) in the support are observed closer to its junction with the tunnel floor. The compressive stresses reach maximum values at the inner boundary of the support. In the support in the tunnel arch, the stresses \( \sigma_2 \) are lower than in the concentration zones. The stresses \( \sigma_1 \) are not higher than 5 MPa almost everywhere in the support. In the support in the tunnel arch, zones of tensile stresses under 1 MPa are observed.

![Figure 1. Distribution of principal stresses (a) \( \sigma_1 \) and (b) \( \sigma_2 \) (MPa) in adjacent rock mass and support in case of ‘instantaneous’ installation.](image)

Now, let us apply the proposed algorithm and the related sequence of calculation of stress state in rock mass–support system in case of ‘instantaneous’ installation of support. First, we determine initial stress state of rock mass. ‘Instantaneous’ support leaves the initial stresses unaltered in the adjacent rock mass. The additional stresses corresponding to the formation of free space for a tunnel with the installed support are found by solving a boundary value problem with the normal and shear stresses preset at the rock mass and support interface, and these stresses go to zero when summed up with the corresponding components of the initial stress state.

This problem is solved using the method to calculate stress state of adjacent rocks and support with regard to the value of free displacement of the tunnel boundary up to the contact with the support [10]. The calculation of the tunnel drivage process by the method of initial stresses by means of turning to zero all stresses at finite elements inside the mined-out area divides the problem into two sub-problems. The first sub-problem determines mechanical state of rock mass during free displacement of rock mass toward the tunnel; the problem is solved when these displacement reach limit value. In case of ‘instantaneous’ support, this is the problem on the initial stress state of rock...
mass. The second sub-problem describes the process of joint deformation of adjacent rock mass and support.

The calculated results are shown in Figure 2. There is the qualitative agreement between these results and the traditional problem formulation. The highest quantitative discrepancy is observed in the calculation of the second principal stress. In the traditional formulation, support is subjected to higher compression. In concentration zones of compressive stresses, this discrepancy reaches 50%.

![Figure 2](image)

**Figure 2.** Distribution of principal stresses (a) $\sigma_1$ and (b) $\sigma_2$ (MPa) in adjacent rock mass and support in the proposed approach to calculating stress state with ‘instantaneous’ support.

**Conclusion**

The author has shown that the traditional formulation of the problem on stress state of rock mass–support system in case when support is ‘instantaneously’ installed in a tunnel corresponds to the conditions of tunnel drivage in stress field different from the initial stress field generated by the tectonics of weight of overlying strata.

The method to calculate stress state of rock mass–support system with regard to the value of free displacement of the tunnel boundary down to its contact with the support in case of its ‘instantaneous’ installation corresponds to the conditions of support installation in rock mass subjected to the initial stress state governed by overlying rock weight and the related tectonics. The quantitative difference in the calculated stresses in the concentration zones within the proposed approach as against the traditional problem formulation reaches 50%.

**References**

[1] Bulychev NS 1989 *Mechanics of Underground Structures in Examples and Problems* Moscow: Nedra (in Russian)

[2] Bulychev NS 1994 Mechanics of Underground Structures Moscow: Nedra (in Russian)

[3] Baklashov IV, Kartozia BA 1984 Mechanics of Underground Structures and Support Units Moscow: Nedra (in Russian)

[4] Kartozia BA, Fedunets BI, Shuplik MN et al 2003 *Mine and Underground Construction: University Textbook* Moscow: Gornaya Kniga Vol 2 (in Russian)

[5] Nasonov ID, Fedyukin VA, Shuplik MN 1992 *Technology of Underground Construction* Moscow: Nedra (in Russian)

[6] Protosenya AG, Dolgy EI, Ogorodnikov YuN et al 2003 *Mine and Underground Construction in Problems and Examples* Saint-Petersburg: Plekhanov Gorny Inst. (in Russian)

[7] Protosenya AG, Ogorodnikov YuN, Demenkov PA et al 2011 *Mechanics of Underground Structures* Saint-Petersburg: SPBGU-MANE (in Russian)

[8] Turchaninov IA, Iofis MA, Kasparyan EV 1989 Rock Mechanics Leningrad: Nedra (in Russian)

[9] Zienkiewicz O 1972 *The Finite Element Method in Engineering Science* McGraw Hill

[10] Seryakov VM 2015 Calculating stresses in support and sidewall rocks in stagewise face drivage in long excavations *J. Min. Sci.* Vol 51 No 4 pp 673–678