Visibility and stability of superstripes in a spin-orbit-coupled Bose-Einstein condensate

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Abstract. We consider a spin-1/2 Bose-Einstein condensate with equal Rashba and Dresselhaus spin-orbit coupling. After reviewing some relevant features of the quantum phases of the system, we present a short study on how their properties are changed by the presence of non-zero magnetic detunings and spin-asymmetric interactions. At small values of the Raman coupling and of the magnetic field the so-called stripe phase occurs, which displays both superfluidity and periodic density modulations, in analogy with supersolids. We finally review a recent proposal (Phys. Rev. A 90, 041604) to improve the visibility of the fringes, based on the space separation of the two spin components into a 2D bi-layer configuration and on the application of a π/2 Bragg pulse, and we show that this new configuration also yields a sizable increase of the stability of the stripe phase against magnetic fluctuations.

1 Introduction

The recent experimental realization of artificial gauge fields on neutral atomic Bose-Einstein condensates (BEC) [1,2,3,4] represents one of the most important achievements in the physics of ultracold gases. In the last few years, the nontrivial properties of such systems have attracted a broad interest, resulting in a wide number of papers devoted to this subject (see, for example, the reviews [5,6,7] and references therein).

A very relevant class of quantum gases coupled to synthetic gauge fields is represented by spin-orbit-coupled configurations. The first experimental implementation of spin-orbit coupling on a neutral atomic gas was performed by the NIST team in [4], where they managed to realize a BEC with equal Rashba [8] and Dresselhaus [9] spin-orbit couplings. The phase diagram of this system exhibits novel quantum phases, which include a stripe phase and a spin-polarized plane-wave phase [10,11]. The stripe phase is characterized by periodic modulations of the density profile resulting from the spontaneous breaking of translational symmetry, similar to what happens in supersolids [12]. Although experiments have already been made in the relevant range of parameters, a direct evidence of such modulations is still lacking, mainly due to the smallness of the amplitude and of the wavelength of the fringes.

In the first part of this paper we review the ground-state properties of a spin-orbit-coupled BEC in uniform matter, and we study how such properties are affected by the introduction of a non-zero magnetic detuning and of spin-asymmetric interaction

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strengths. We then discuss a combined procedure to make the stripes visible and stable [13], thus allowing for a direct experimental detection.

2 The model and the quantum phases

2.1 Single-particle Hamiltonian

We consider a spin-1/2 BEC with the kind of spin-orbit-coupling first realized by the NIST team in [4]. When written in a locally spin-rotated frame [14], the single-particle Hamiltonian describing the system is given by (we set $\hbar = m = 1$)

$$h_{\text{sp}} = \frac{1}{2}[(p_x - k_0 \sigma_z)^2 + p_{\perp}^2] + \frac{\Omega}{2} \sigma_x + \frac{\delta}{2} \sigma_z.$$  \hfill (1)

It accounts for the application of two counterpropagating and polarized laser fields, with wave vector difference $k_0$ chosen along the $x$ direction, in the presence of a nonlinear Zeeman field. The two lasers provide Raman transitions between the two spin states, with Raman coupling strength given by $\Omega$. The effective magnetic field $\delta$ is given by the sum of the true external magnetic field and of the frequency detuning between the two lasers (see, for example, [14]). The spin matrices entering the single-particle Hamiltonian (1) are the usual 2 $\times$ 2 Pauli matrices. It is worth pointing out that the operator $p$ entering (1) is the canonical momentum $-i\nabla$, with the physical velocity being given by $v_\pm = p \mp k_0 \hat{e}_x$ for the spin-up and spin-down particles. In terms of $p$ the eigenvalues of (1) are given by

$$\varepsilon_\pm(p) = \frac{p_x^2 + p_{\perp}^2}{2} + E_r \pm \sqrt{\left(k_0 p_x - \frac{\delta}{2}\right)^2 + \frac{\Omega^2}{4}}.$$  \hfill (2)

where $E_r = k_0^2/2$ is the recoil energy. The double-branch structure exhibited by the dispersion (2) reflects the spinor nature of the system. A peculiar feature of the dispersion (2), when $\delta = 0$, is that it displays, as a function of $p_x$, two degenerate minima at $\pm k_0 \sqrt{1 - (\Omega/4E_r)^2}$, both capable to host Bose-Einstein condensation. Notice that the wave vectors corresponding to such minima differ from $\pm k_0$ if $\Omega \neq 0$, and vanish for $\Omega = 4E_r$. For larger values of $\Omega$ the gas is in the single-minimum phase, where all the atoms occupy the $p = 0$ single-particle state.

2.2 Many-body ground state

The peculiar features of the single-particle dispersion (2) are at the origin of new interesting phases in the many-body ground state of the BEC. For a gas of $N$ particles enclosed in a volume $V$, in the presence of two-body interactions, the mean-field interaction Hamiltonian takes the form

$$H_{\text{int}} = \int d^3r \left[ \frac{g_{\uparrow\uparrow}}{2} n_\uparrow(n_\uparrow)^2 + \frac{g_{\downarrow\downarrow}}{2} n_\downarrow(n_\downarrow)^2 + g_{\uparrow\downarrow} n_\uparrow n_\downarrow \right],$$  \hfill (3)

where $g_{\sigma\sigma'} = 4\pi a_{\sigma\sigma'}$ ($\sigma, \sigma' = \uparrow, \downarrow$) are the coupling constants in the different spin channels, fixed by the corresponding scattering lengths $a_{\sigma\sigma'}$, while $n_{\uparrow,\downarrow}$ are the densities of the two spin components. The quantum phases predicted by mean-field theory depend on the value of the relevant parameters $k_0$, $\Omega$, $\delta$ and the interaction parameters $G_1 = n (g_{\uparrow\uparrow} + g_{\downarrow\downarrow} + 2g_{\uparrow\downarrow})/8$, $G_2 = n (g_{\uparrow\uparrow} + g_{\downarrow\downarrow} - 2g_{\uparrow\downarrow})/8$, $G_3 = n (g_{\uparrow\uparrow} - g_{\downarrow\downarrow})/4$
Stripe phase I
Single–minimum
phase III
Plane–wave
phase II
\( \bar{n}/\bar{n}(c) \)
\( k_1/k_0 \)

Fig. 1. Phase diagram of a spin-orbit-coupled BEC. The color represents the value of \( k_1/k_0 \). The white solid lines identify the phase transitions. The quantity \( \bar{n}(c) = E_r/(g\gamma) \) is the density at the tricritical point where the three phases meet. The diagram corresponds to a configuration with \( \gamma = (g - g_{\uparrow \downarrow})/(g + g_{\uparrow \downarrow}) = 0.0012 \) consistent with the value of [4].

[11], with \( \bar{n} = N/V \) the average density. In uniform matter, the ground-state wave function can be determined through a variational procedure based on the ansatz [11,15,16]

\[
\Psi(r) = \sqrt{\bar{n}} \left[ C_+ \left( \cos \theta_+ - \sin \theta_+ \right) e^{ik_+ x} + C_- \left( -\sin \theta_- + \cos \theta_- \right) e^{-ik_- x} \right],
\]  

where \( C_+ \) and \( C_- \) are coefficients satisfying the normalization constraint \( |C_+|^2 + |C_-|^2 = 1 \), and \( k_\pm \) represent the momenta at which Bose-Einstein condensation takes place. For a given value of \( k_0, \Omega, \delta \) and the two-body interaction strengths, the values of the variational parameters \( C_\pm, k_\pm \) and \( \theta_\pm \) can be calculated through a procedure of energy minimization, including both the single-particle [11] and the interaction [3] terms in the Hamiltonian. In particular, energy minimization with respect to \( k_\pm \) yields the general relationship \( 2\theta_\pm = \arccos (k_\pm/k_0) \) fixed by the single-particle Hamiltonian [11]. Once all the variational parameters have been determined, one can calculate key physical quantities like, for example, the momentum distribution, accounted for by the parameters \( k_\pm \), the densities \( n_\uparrow \) and \( n_\downarrow \) of the two spin components, the total density \( n = \Psi^\dagger \Psi = n_\uparrow + n_\downarrow \), the spin densities \( s_k = \Psi^\dagger \sigma_k \Psi \) with \( k = x, y, z \) and the corresponding spin polarizations \( \langle \sigma_k \rangle = N^{-1} \int d^3r \, s_k \).

The full variational calculation has been performed in [11], where the case of spin-symmetric coupling constants \( g_{\uparrow \uparrow} = g_{\downarrow \downarrow} \equiv g \) and zero detuning \( \delta \) was mainly considered. In this situation one finds \( k_+ = k_- = k_1 \) and \( \theta_+ = \theta_- = \theta \) for symmetry reasons. The ground state was found to be compatible with three distinct quantum phases; the corresponding phase diagram is shown in Fig. 1.

(I) Stripe phase. For small values of the Raman coupling \( \Omega \) and \( g > g_{\uparrow \downarrow} \), the ground state is a coherent superposition of the two plane-wave states \( e^{\pm ik_1 x} \) with equal weights (\( |C_+| = |C_-| = 1/\sqrt{2} \)), yielding a vanishing longitudinal spin polarization \( \langle \sigma_z \rangle \). The most striking feature of this phase is the appearance of density modulations in the form of stripes according to the law

\[
n(r) = \bar{n} \left[ 1 + \frac{\Omega}{2(2E_r + G_1)} \cos (2k_1 x + \phi) \right],
\]

where \( \phi \) is a phase parameter.
with the periodicity of the fringes $\pi/k_1$ fixed by the wave vector $k_1 = k_0 \sqrt{1 - [\Omega/(2(2E_r + G_1))]^2}$. These modulations appear as the result of a mechanism of spontaneous breaking of translational invariance, with the actual position of the fringes being given by the value of the phase $\phi$. The contrast in $n(r)$ is given by

$$\frac{n_{\text{max}} - n_{\text{min}}}{n_{\text{max}} + n_{\text{min}}} = \frac{\Omega}{2(2E_r + G_1)}$$

and vanishes as $\Omega \to 0$ as a consequence of the orthogonality of the two spin states entering Eq. (4) (in this limit $\theta \to 0$ and $k_1 \to k_0$). It is worth mentioning that the ansatz, Eq. (4), for the stripe phase provides only a first approximation, which ignores higher-order harmonics caused by the nonlinear interaction terms in the Hamiltonian.

(II) Plane-wave phase. For larger values of the Raman coupling, the system enters the so-called plane-wave phase (also called the spin-polarized or de-mixed phase), where Bose-Einstein condensation takes place in a single plane-wave state with momentum $p = k_1 \hat{e}_x$ ($C_- = 0$), lying on the $x$ direction (in the following we choose $k_1 > 0$). In this phase, the density is uniform and the spin polarization is given by $\langle \sigma_z \rangle = k_1/k_0$ with $k_1 = k_0 \sqrt{1 - [\Omega/(4(E_r - G_2))]^2}$. An energetically equivalent configuration is obtained by considering the BEC in the single-particle state with $p = -k_1 \hat{e}_x$ ($C_+ = 0$). The choice between the two configurations is determined by a mechanism of spontaneous symmetry breaking, typical of ferromagnetic configurations.

(III) Single-minimum phase. At even larger values of $\Omega$, the system enters the single-minimum phase (also called zero-momentum phase), where the condensate has zero momentum ($k_1 = 0$), the density is uniform, and the average spin polarization $\langle \sigma_z \rangle$ identically vanishes, while $\langle \sigma_x \rangle = -1$. The critical values of the Rabi frequencies $\Omega$ characterizing the various phase transitions can be identified by imposing that the chemical potential $\mu(\bar{n})$ and the pressure $P = \bar{n}\mu(\bar{n}) - \int \mu(\bar{n}) d\bar{n}$ be equal in the two phases at equilibrium. The transition between the stripe and the plane-wave phases has a first-order nature. In the low density (or weak coupling) limit, i.e. $G_1, G_2 \ll E_r$, the critical value of the Raman coupling $\Omega^{(I-II)}$ characterizing such transition is given by the density-independent expression [10,11]

$$\Omega^{(I-II)} = 4E_r \sqrt{\frac{2\gamma}{1 + 2\gamma}}$$

with $\gamma = G_2/G_1$. The transition between the plane-wave and the single-minimum phases has instead a second-order nature and takes place at the higher value $\Omega^{(II-III)} = 4(E_r - G_2)$, provided that the condition $\bar{n} < \bar{n}^{(c)}$ is satisfied. For higher densities one has instead a first-order transition directly between the stripe and the single-minimum phases. We also remark that, if $g < g_{\uparrow\downarrow}$, the stripe phase is energetically unfavorable, and only the plane-wave and the single-minimum phases are available.

2.3 Effects of non-zero detuning and spin-asymmetric interactions

The results discussed in the previous paragraph can be easily generalized to account for the presence of a non-vanishing magnetic detuning $\delta$ and of spin-asymmetric interactions $g_{\uparrow\uparrow} \neq g_{\downarrow\downarrow}$. In particular, the effects of an asymmetry in the intraspecies coupling constants can be compensated by choosing a magnetic detuning $\delta = -2G_{3\uparrow\downarrow}$, which ensures that the ground-state properties remain the same as in the symmetric
Fig. 2. Detuning versus Rabi coupling phase diagram in the experimental conditions of [4]. The parameters are $E_r = 2\pi \times 1.77$ kHz, density in the center of the trap $n_0 = 1.9 \times 10^{14}$ cm$^{-3}$ and the scattering lengths given in the main text.

In the most general case of arbitrary $\delta$ and $G_3$, the ground-state wave function can be still worked out by resorting to the ansatz (4), where now the two wave vectors $k_+$ and $k_-$ can have different values. The resulting phase diagram in the $\Omega-\delta$ plane is shown in Fig. 2. It is characterized by the occurrence of a stripe phase, where both momentum components of (4) are present (although they can have different weights $|C_+|$ and $|C_-|$), and by several plane-wave states, having different values of the momentum and hence of the magnetization [16].

The stripe phase occurs only in configurations where $g_{\uparrow\uparrow}g_{\downarrow\downarrow} > g_{\uparrow\downarrow}^2$, corresponding to the condition of miscibility of the two spin components in the absence of spin-orbit and Raman coupling. One can notice that, in systems with almost equal coupling constants, it occupies a very small region in the $\Omega-\delta$ plane. For example, in the case of the states $|\uparrow\rangle = |F = 1, m_F = 0\rangle$ and $|\downarrow\rangle = |F = 1, m_F = -1\rangle$ of $^{87}$Rb, where $a_{\uparrow\uparrow} = 101.41 a_B$ and $a_{\downarrow\downarrow} = a_{\uparrow\downarrow} = 100.94 a_B$, from Eq. (7) one finds the value $\Omega^{(1\rightarrow 0)} = 0.19 E_r$ for the critical Rabi coupling, while the critical magnetic detuning needed to bring the system from the stripe to the spin-polarized phase turns out to be of the order of $10^{-3} E_r$. The latter is actually proportional to the difference $\Delta\mu$ between the chemical potentials in the two phases. An analytic estimate can be obtained in the $\Omega \rightarrow 0$ limit, where one finds that the stripe phase is favored for values of the magnetic detuning $\delta$ such that $|\delta + 2G_3| \leq 4G_2$. At finite values of $\Omega$ the range of values of $\delta$ compatible with the stripe phase is further reduced. As a consequence, a tiny magnetic field (arising, for instance, from external fluctuations) can easily bring the system into the spin-polarized phases. The stability of the stripe phase can be strongly enhanced if one increases significantly the value of $G_2$, as we will discuss in Sect. 3.

The phase diagram of Fig. 2 also contains three different kinds of plane-wave phase. Those on the left region of the diagram, denoted by PW1 and PW2, correspond to the regime where the single-particle Hamiltonian has two local minima; PW1 and PW2 are favored for magnetic detunings $\delta$ larger and smaller than $-2G_3$, respectively. The state PW3 appears instead in the region where the single-particle Hamiltonian has one local minimum only. The red dashed lines represent the transition between the double-minimum and the single-minimum regimes; in drawing them we have taken the presence of spin-dependent interactions into account, yielding some corrections with respect to the single-particle results [16].
3 Experimental perspectives for the stripe phase

The stripe phase is doubtlessly the most intriguing phase appearing in the phase diagram of Sect. 2. It has been the object of several recent theoretical investigations \[10,19,20,21,22,23,24,25,26,27,28\]. The stripe phase is characterized by the spontaneous breaking of two continuous symmetries. The breaking of gauge symmetry yields superfluidity, while the breaking of translational invariance is responsible for the occurrence of a crystalline structure. The simultaneous presence of these two broken symmetries is typical of supersolids \[12,29,30,31\]. It has been shown, among other things, to be at the origin of the appearance of two gapless excitations as well as of a band structure in the excitation spectrum \[23\].

In the experiments of \[4\] and \[17\] a phase transition has been detected close to the theoretical prediction \(\Omega (I - II) = 0.19 E_r\) (see Eq. (7)) for the critical Raman coupling below which the occurrence of the stripe phase is expected. However, as we already mentioned in the introduction, there is still no direct experimental evidence of the periodic modulations in the density profile characterizing the stripe phase. The main reason is that, in the conditions of current experiments with spin-orbit-coupled \(^{87}\)Rb BECs \[18,32\], the contrast and the wavelength of the fringes are too small to be revealed. Another issue is represented by the fragility of the stripe phase against fluctuations of external magnetic fields, which has already been discussed in Par. 2.3. In \[13\] the authors proposed a procedure to make the experimental detection of the fringes a realistic perspective, improving their contrast and their wavelength, and increasing the stability of the stripe phase against magnetic fluctuations.

In order to achieve a larger value of the contrast \(6\), one needs to enlarge the range of values of \(\Omega\) compatible with the existence of the stripe phase. As can be seen from Eq. (7), an efficient way to increase the critical Raman coupling \(\Omega (I - II)\) is to reduce the value of the interspecies coupling constant \(g_{\uparrow \downarrow}\). A possibility is to look for hyperfine states characterized by a small (or tunable) interspecies scattering length. Here we discuss a different strategy, based on the idea of reducing the effective interspecies coupling by means of suitable trapping conditions. In particular, one can trap the atomic gas in a 2D configuration, with tight confinement of the spin-up and spin-down components around two different positions, displaced by a distance \(d\) along the \(z\) direction. This configuration can be realized with a trapping potential of the form

\[
V_{\text{ext}}(z) = \frac{\omega_z^2}{2} \left( z - \frac{d}{2} \sigma_z \right)^2
\]

produced either through magnetic gradient techniques or via spin-dependent optical potentials. Assuming a Gaussian profile \(\psi_{\pm} = (1/\sqrt{\pi a_z^2}) e^{-(z \mp d/2)^2/2a_z^2}\) for the \(z\) dependence of the spin-up and spin-down wave functions, with \(a_z = 1/\sqrt{\omega_z}\) the oscillator length along \(z\), the integration over \(z\) of the energy functional (3) gives rise to effective 2D coupling constants \(g_{\alpha \beta}\) given by

\[
\tilde{g}_{\uparrow \uparrow \downarrow \downarrow} = \frac{1}{\sqrt{2\pi a_z}} g_{\uparrow \uparrow \downarrow \downarrow}, \quad \tilde{g}_{\uparrow \downarrow} = \frac{1}{\sqrt{2\pi a_z}} g_{\uparrow \downarrow} e^{-d^2/2a_z^2}.
\]

In an analogous way one finds that also the effective Raman coupling, to be used in 2D, is lowered with respect to the physical coupling \(\Omega\) according to the law \(\tilde{\Omega} = e^{-d^2/4a_z^2} \Omega\), reflecting the reduction of the overlap between the two wave functions. Hence, the new configuration produced by a tight axial trapping potential with a spin-dependent displacement can be described formulating the Hamiltonian in 2D,

\[1\] In the present section we consider realistic spin-asymmetric interaction strengths \(g_{\uparrow \uparrow} \neq g_{\downarrow \downarrow}\), and we compensate the asymmetry by choosing \(\delta = -2G_3\).
Fig. 3. Integrated density profile $\int dydz n$ in the striped phase. (a) displays the situation without separation of the traps for the two spin components. (b) corresponds instead to traps separated along $z$ by a distance $d = a_z$, which increases the contrast of the fringes.

with the effective Raman coupling given by $\tilde{\Omega}$, and the interaction term obtained from the functional (3) with the replacement of the 3D densities with their 2D counterparts $\int dz n$ and of the coupling constants with the renormalized values (9). The main advantage with respect to the original 3D problem is that now, due to the relative separation of the atomic clouds of the two spin components, the effective $g_{\uparrow\downarrow}$ is reduced with respect to the two intraspecies coupling constants (see Eq. (9)). As a consequence, the ratio $\gamma$ appearing in Eq. (7) is now given by the expression

$$\gamma = \frac{G_2}{G_1} = \frac{g_{\uparrow\uparrow} + g_{\downarrow\downarrow} - 2g_{\uparrow\downarrow}e^{-d^2/2a_z^2}}{g_{\uparrow\uparrow} + g_{\downarrow\downarrow} + 2g_{\uparrow\downarrow}e^{-d^2/2a_z^2}}, \quad (10)$$

and is larger with respect to the 3D case, yielding an increased value of the critical effective Raman coupling, and thus of the largest reachable contrast of the fringes in the stripe phase. For example, choosing the value $d = a_z$ and the $^{87}$Rb hyperfine states mentioned in Par. 2.3 one finds the value $\gamma = 0.25$ for the ratio (10), to be compared with the value $\gamma = 0.0012$ for the $d = 0$ case.

Quantitative predictions for the novel configuration discussed above can be obtained by solving numerically the 3D Gross-Pitaevskii equation. In Fig. 3 we show the results for a gas of $N = 4 \times 10^4$ $^{87}$Rb atoms confined by a harmonic potential.

Another important change is that, due to the increase of the value of $\gamma$, the critical density $n^{(c)}$ can be significantly lowered with respect to the value in the $d = 0$ case, becoming of more realistic achievement in future experiments.
Fig. 4. Detuning versus effective Rabi coupling phase diagram in the conditions of Fig. 3b.

with frequencies \((\omega_x, \omega_y, \omega_z) = 2\pi \times (25, 100, 2500)\) Hz, the scattering lengths \(a_{\sigma\sigma'}\) and the recoil energy \(E_r\) equal to those reported in Par. 2.3 and consistent with Ref. [4]. Fig. 3a corresponds to \(d = 0\), while Fig. 3b corresponds to \(d = a_z = 0.22\) μm. In both Fig. 3a and Fig. 3b we have chosen values of the Raman coupling equal to one half the critical value to enter the plane-wave phase, in order to ensure more stable conditions for the stripe phase. In Fig. 3a this corresponds to \(\Omega = (1/2)\Omega^{(1-II)}(\gamma) = 0.095 E_r\) with \(\gamma = 0.0012\), while in Fig. 3b to \(\Omega = (1/2)e^{d^2/4a_z^2}\Omega^{(1-II)}(\gamma) = 1.47 E_r\) with \(\gamma = 0.25\). The plotted density corresponds to the 1D density as a function of the most relevant \(x\) variable, obtained by integrating the full 3D density along the \(y\) and \(z\) direction. The figure clearly shows that in the conditions of almost equal coupling constants (Fig. 3a) the density modulations are very small, while their effect is strongly amplified in Fig. 3b where the interspecies coupling is reduced with respect to the intraspecies values by a factor \(\sim 0.61\).

The suggested procedure has also the positive effect of making the stripe phase more robust against fluctuations of external magnetic fields. Indeed, the reduction of the interspecies coupling and the increase of the local 3D density, due to the tight axial confinement, yield a significant increase of the energy difference between the stripe and the plane-wave phases. For example, in the case considered in the above 3D Gross-Pitaevskii simulation with \(d = a_z\) (Fig. 3b), a magnetic detuning of the order of \(0.3 E_r\) is needed to bring the system into the spin-polarized phase (see the diagram in Fig. 4), while in the absence of displacement (Fig. 3a) the critical value is much smaller \((\sim 0.001 E_r\), see Par. 2.3 and Fig. 2).

Let us finally address the problem of the small spatial separation of the fringes, given by \(\pi/k_1\), which turns out to be of the order of a fraction of a micron in standard conditions. One possibility to increase the wavelength of the stripes is to lower the value of \(k_0\) by using lasers with a smaller relative incident angle. In the following we discuss a more drastic procedure which consists of producing, after the realization of the stripe phase, a \(\pi/2\) Bragg pulse with a short time duration (smaller than the time \(1/E_r\) fixed by the recoil energy), followed by the sudden release of the trap. This pulse can transfer to the condensate a momentum \(k_B\) or \(-k_B\) along the \(x\) direction, where \(k_B\) is chosen equal to \(2k_1 - \epsilon\) with \(\epsilon\) small compared to \(k_1\). The \(\pi/2\) pulse has the effect of splitting the condensate into various pieces, with different momenta. The situation is schematically shown in Fig. 5 for the spin-down component, where the initial condensate wave function, which in the stripe phase is a linear combination with canonical momenta \(\pm k_1\), corresponding to momenta \(k_0 - k_1\) and \(k_0 + k_1\) in the laboratory frame, after the Bragg pulse will be decomposed into six pieces. Two of them, those labeled in the lower part of the figure with momentum \(\sim 0\), will be
Fig. 5. Schematical description of the splitting of the spin-down component of the stripe wave function into different momentum components caused by a $\pi/2$ Bragg pulse transferring momentum $2k_1 - \epsilon$.

Fig. 6. Integrated density profiles $\int dz \, n$ (top) and $\int dy \, dz \, n$ (bottom) in the stripe phase, in the same conditions as Fig. 3b, after the application of a $\pi/2$ Bragg pulse transferring momentum $\pm 1.8 \hbar k_1$.

practically at rest after the pulse and are able to interfere with fringes of wavelength $2\pi/\epsilon$, which can easily become large and visible \textit{in situ}. It is worth noticing that these two latter pieces originate from the two different momentum components of the order parameter (4) in the stripe phase and involve $1/3$ of the total number of atoms. The corresponding interference effect would be consequently absent in the plane-wave
phase, where only one momentum component characterizes the order parameter. The other pieces produced by the Bragg pulse carry much higher momenta and will fly away rapidly after the release of the trap and of the laser fields. In Fig. 6 we show a typical behavior of the density profile obtained by modifying the condensate wave function in momentum space according to the prescription discussed above.

4 Conclusions

In this paper we have reviewed the ground-state properties of a spin-orbit-coupled BEC, focusing on the effects of the presence of a finite magnetic detuning and of spin-asymmetric coupling constants. The phase diagram includes a stripe phase, which is characterized by the presence of periodic modulations in the density profile. In the last part of the paper we have discussed a combined procedure to increase the visibility of such modulations and the stability of the stripe phase against magnetic fluctuations, thus favoring the exploration of this intriguing configuration in realistic experimental conditions.

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