The manipulated left-handedness in a rare-earth-ion-doped optical fiber by the incoherent pumping field

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The left-handedness was demonstrated in an $E_{r}^{3+}$-doped Zr$_{4}$F$_{6}$-B$_{6}$F$_{12}$-La$_{4}$F$_{3}$-AlF$_{3}$-N$_{6}$F (ZBLAFN) optical fiber modeled by a three-level quantum system. Under the electric and magnetic components of the probe field driving the transitions of $^{4}I_{15/2}$-$^{4}I_{13/2}$ and $^{4}I_{9/2}$-$^{4}I_{13/2}$ in the $E_{r}^{3+}$-doped optical fiber respectively, an increasing left-handedness was achieved by the incremental incoherent pumping field. However, the left-handedness damped when the incoherent pumping field drove the transition heavily. Our scheme may provide a solid candidate other than the coherent atomic vapour for left-handedness, and may extend the application of the rare-earth-ion-doped optical fiber in metamaterials via the external incoherent pumping field.

Keywords: Rare-earth-ion-doped optical fiber; three-level quantum system; left-handedness; incoherent pumping field

I. INTRODUCTION

Optical fibers doped with rare-earth ions, such as $E_{r}^{3+}$, $N_{d}^{3+}$, $T_{m}^{3+}$, $S_{M}^{3+}$, $H_{o}^{3+}$, $Y_{b}^{3+}$, and $P_{r}^{3+}$, have attracted significant scientific and industrial interests due to their applications in optical fiber amplifiers and fiber lasers[1–3]. Especially, the $E_{r}^{3+}$-doped optical fiber plays a much more important role in optical fiber communication[4–11]. And the emission transition $^{4}I_{15/2}$-$^{4}I_{13/2}$ in $E_{r}^{3+}$-doped fiber amplifier is the key element of modern telecommunication systems[4, 5]. In wavelength division multiplexing[6], the $^{4}I_{13/2}$-$^{4}I_{15/2}$ emission and the $^{4}I_{15/2}$-$^{4}I_{13/2}$ absorption transitions of $E_{r}^{3+}$ fulfill flat emission spectra and wavelength divergence. The transitions of $^{2}H_{11/2}$-$^{4}I_{15/2}$ and $^{4}F_{9/2}$-$^{4}I_{15/2}$ under infrared radiation excitation can convert fluorescence properties via $E_{r}^{3+}$-doped nanoparticles of GdPO$_{4}$[12].

On account of the flexible design and adjustable parameters comparing to their atomic counterparts, the $E_{r}^{3+}$-doped optical fiber systems play a more important role in practical application for quantum optics. And some nonlinear quantum optical phenomena, such as gain leveling[13], optical bistability and multi-stability[14], absorption-amplification response[15], enhanced index of refraction with vanishing absorption[16] were achieved in $E_{r}^{3+}$-doped optical fibers recently.

In this work, under the electric and magnetic components of the probe field coupling the transitions of $^{4}I_{15/2}$-$^{4}I_{13/2}$ and $^{4}I_{9/2}$-$^{4}I_{13/2}$, respectively, we theoretically investigate the feasibility of left-handedness via the incoherence pumping field in the $E_{r}^{3+}$-doped ZBLAFN optical fiber. For the simplify of experimental realization, we simulate the $E_{r}^{3+}$-doped optical fiber with an ordinary three-level quantum system, and the switching from increasing to decreasing left-handedness can be implemented simply by adjusting the pumping rate of the incoherent pumping field. In our scheme, a new left-handed material may be explored instead of artificial composite metamaterials[17], photonic crystal structures[18], transmission line simulation[19] and chiral media [20], which may extend the mediums for left-handedness and the application domain for $E_{r}^{3+}$-doped optical fiber in metamaterials

II. MODEL AND EQUATION

The $E_{r}^{3+}$-doped ZBLAFN optical fiber can be modeled by three-level quantum system shown in Fig. 1. The levels $|^{4}I_{15/2}⟩$, $|^{4}I_{13/2}⟩$ and $|^{4}I_{9/2}⟩$ in the $E_{r}^{3+}$ will behaved the $|1⟩$, $|2⟩$ and $|3⟩$ states, respectively. In such a modeled 3-level system, the parities of levels $|2⟩$ and $|3⟩$ are set to be identical, which is opposite to $|1⟩$. An incoherent pump field pumps atoms in level $|1⟩$ into upper

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level $|3\rangle$ with its pumping rate being $2\Gamma$. The possible optical transition $|1\rangle \leftrightarrow |2\rangle$ is mediated by a weak probe laser field with central frequency $\omega_c$ and Rabi frequency $\Omega_c = \vec{E}_0 d_{21}/\hbar$. Because of the parity selection rules, the two levels $|1\rangle$ and $|2\rangle$ with electric dipole element $d_{21} = \langle 2| \vec{d}|1\rangle \neq 0$ are coupled by the electric component of the weak probe field, where $\vec{d}$ is the electric dipole operator. The magnetic component of the probe field with frequency $\omega_b$ and Larmor frequency of $\Omega_b = \vec{B}_\mu \rho_{32}/2\hbar$ is applied to the magnetic-dipole transition $|2\rangle \leftrightarrow |3\rangle$, where $\mu_{32}$ is the corresponding magnetic-dipole matrix element. Interestingly, the transitions $|1\rangle \leftrightarrow |3\rangle$, $|1\rangle \leftrightarrow |2\rangle$ and $|3\rangle \leftrightarrow |2\rangle$ coupled by the incoherent pump field, the electric and magnetic components of the probe laser field respectively form a closed-loop configuration.

Then the semi-classical interaction Hamiltonian $H_{int}$ of this ionic system in an $E_{r}^{3+}$-doped ZBLAFN optical fiber is given in the interaction picture under the dipole and the rotating wave approximation as follows,

$$H_{int} = \Delta_e (|2\rangle\langle 2| + |3\rangle\langle 3| + \Delta_b |3\rangle\langle 3| + (\Omega_b |3\rangle\langle 2| + \Omega_e |2\rangle\langle 1| + H.c.)$$

(1)

where H.c. means Hermitian conjugation, $\Delta_e = \omega_e - \omega_{21}$ and $\Delta_b = \omega_b - \omega_{32}$ are the detunings of the electric and magnetic components of the probe laser field to the transitions $\omega_{21}$ and $\omega_{32}$, respectively. Then the equation of the time-evolution, i.e., the density matrix equations for the system can be described as $\frac{d\rho}{dt} = -i\frac{1}{\hbar}[H, \rho] + \Lambda \rho$ in Eq.(2), where $\Lambda \rho$ represents the irreversible decay part of the $E_{r}^{3+}$ ion system.

$$i\rho_{22} = -\Omega_b \rho_{32} + \Omega_b \rho_{23} - \Omega_e \rho_{12} + \Omega_e \rho_{21} - i\gamma_{21} \rho_{22} + i\gamma_{32} \rho_{33},$$

$$i\rho_{33} = -\Omega_b \rho_{23} + \Omega_b \rho_{32} - i(\gamma_{31} + \gamma_{32}) \rho_{33},$$

(2)

$$i\rho_{12} = -\Omega_c (\rho_{22} - \rho_{11}) + \Omega_b \rho_{13} - (\Delta_b + i\gamma_{21}/2 + i\Gamma) \rho_{12},$$

$$i\rho_{13} = \Omega_b \rho_{12} - \Omega_c \rho_{23} - [(\Delta_e + \Delta_b) + i\gamma_{31}/2 + i\Gamma] \rho_{13},$$

$$i\rho_{23} = -\Omega_b (\rho_{33} - \rho_{22}) - \Omega_e \rho_{13} - (\Delta_b + i\gamma_{21} + \gamma_{31} + \gamma_{32}) \rho_{23},$$

$\rho_{13}$,

where $\rho_{ij} = \rho^\dagger_{ij}$ (i,j=1,2,3) and the density matrix elements were constrained by the conditions: $\rho_{11} + \rho_{22} + \rho_{33} = 1$. Here, $\gamma_{ij}$ designates the decay rates from $|i\rangle$ to $|j\rangle$.

In the classical electromagnetic theory, the electric polarizability is a rank 2 tensor and defined by its Fourier transform $\vec{P}(\omega_e) = \epsilon_0 \alpha_e(\omega_e) \vec{E}(\omega_e)$, which can be calculated by the trace computation of the definition $\vec{P}(\omega_e) = \text{Tr}(\hat{\rho} \vec{d}) = \rho_{12} d_{21} + \text{c.c.}$. Here, in the $E_{r}^{3+}$-doped optical fiber we consider the electric polarizability of the incident field $\vec{E}_e$ carrying out at the frequency $\omega_e$. Therefore, we adopt the explicit $\omega_e$ dependence $\alpha_e(\omega_e) = \alpha_e$, and $\vec{E}_e$ was set to parallel to the atomic dipole $\vec{d}_{21}$ so $\alpha_e$ as to be a scalar. Then its expression can be represented as follows:

$$\alpha_e = \frac{\vec{d}_{21} \rho_{12}}{\epsilon_0 \vec{E}_e} = \frac{|d_{21}|^2 \rho_{12}}{\epsilon_0 \hbar \Omega_e},$$

(3)

The classical magnetic polarizations in the $E_{r}^{3+}$-doped optical fiber can be achieved in the same way, i.e., $\vec{P}_b(\omega_b) = \mu_0 \alpha_b \vec{B}(\omega_b)$, which can be obtained by the mean value of the atomic dipole moment operator via $\vec{P}_b = \text{Tr}(\hat{\rho} \vec{m}) = \rho_{12} \mu_{23} + \text{c.c.}$. For the simplification, we choose magnetic dipole is perpendicular to the induced electric dipole in accordance with the classical Maxwell’s electromagnetic wave-vector relation. Then the magnetization $\alpha_m$ is scalar, and its expression is as follows:

$$\alpha_m = \frac{\mu_0 \mu_{32} \rho_{23}}{B} = \frac{\mu_0 |\mu_{32}|^2 \rho_{23}}{\hbar \Omega_B}.$$  

(4)

The relative permittivity and relative permeability of the $E_{r}^{3+}$-doped optical fiber can be given according to the Clausius-Mossotti relations considering the local effect in dense medium[21, 22] as follows:

$$\epsilon_r = 1 + \frac{2}{3} N \alpha_e, \mu_r = 1 + \frac{2}{3} N \gamma_m.$$  

(5)
In the above, the expressions for the electric permittivity and magnetic permeability of this $E_r^{3+}$-doped optical fiber were obtained. In the section that follows, we will discuss its left-handedness via the permittivity, permeability and refractive index.

III. RESULTS AND DISCUSSION

With the steady solutions of Eq.(2) for $\rho_{12}$ and $\rho_{23}$, we can show the numerical results for the electric permittivity $\varepsilon_r$, magnetic permeability $\mu_r$ and refractive index $n$ in the $E_r^{3+}$-doped optical fiber. Before the calculation, some typical parameters should be selected. In the following numerical calculations, all the parameters will be scaled by $\gamma_{21}=90.6$ s$^{-1}$ [23], and we choose the decay rates as $\gamma_{31}=1.19\gamma_{21}$ and $\gamma_{32}=0.31\gamma_{21}$ from Ref.[23]. And we choose the average density for the $E_r^{3+}$ ions as $N=1.04 \times 10^{20} m^{-3}$, the electric transition dipole moment from $|2\rangle \leftrightarrow |1\rangle$ is chosen as $d_{21}=2.335 \times 1.602 \times 10^{-19} C \cdot m$ [24] and the typical magnetic transition dipole moment is chosen as $\mu_{32}=7.0 \times 10^{-23} C m^2 s^{-1}$[25, 26]. The Rabi frequency of the electric component of the probe laser field is set as $\Omega_{e}=0.5\gamma_{21}$ with $\Delta_b=0.25\gamma_{21}$.

According to the refraction definition of the left-handed material $n = -\sqrt{\varepsilon_r\mu_r}$ [27], we plot the permittivity $\varepsilon_r$, permeability $\mu_r$, and refractive index $n$ versus $\Delta_e/\gamma_{21}$ with different incoherent pumping frequencies $\Gamma$ in Fig.2, 3 and 4. The coincide intervals for negative $Re[\varepsilon_r]$ and $Re[\mu_r]$ will demonstrate the left-handedness in the $E_r^{3+}$-doped optical fiber. In Fig.2, we notice the value and interval for negative $Re[\varepsilon_r]$ are increasing when the incoherent pumping frequencies are varied by $\Gamma=1.2\gamma_{21}, \Gamma=1.3\gamma_{21}, \Gamma=1.4\gamma_{21}, \Gamma=1.5\gamma_{21}$. Which demonstrates the incoherent pump field coupling level $|1\rangle$ into upper level $|3\rangle$ can incur the creasing negative $Re[\varepsilon_r]$. On the same parametric condition $\mu_r$ was plot in Fig.3. It’s noted that $Re[\mu_r]$ maintains negative value in the interval of $[-15\gamma_{21}, 0]$ when the incoherent pumping field modulates its frequency, which shows the $E_r^{3+}$-doped optical fiber has negative $Re[\mu_r]$ in the same intervals as negative $Re[\varepsilon_r]$, and demonstrates the left-handedness in the $E_r^{3+}$-doped optical fiber.

![FIG. 2.](image)

FIG. 2. Real (solid lines) and imaginary (dashed lines) parts of permittivity $\varepsilon_r$ as a function of the rescaled detuning parameter $\Delta_e/\gamma_{21}$ for $\Gamma=1.2\gamma_{21}, \Gamma=1.3\gamma_{21}, \Gamma=1.4\gamma_{21}, \Gamma=1.5\gamma_{21}$.

The plots for $Re[n]$ in Fig.4 also demonstrate this. Fig.4 shows that $Re[n]$ maintain negative value in the interval of $[-15\gamma_{21}, 0]$ and its values are gradually enlarging when the incoherent pumping field is modulated in the same way as Fig.2.

The reason may come from the growing incoherent pumping field, which drives the transition $|3\rangle \leftrightarrow |1\rangle$ and brings out the changing populations between $|3\rangle$ and $|1\rangle$ in the $E_r^{3+}$-doped optical fiber. The variation of population between $|3\rangle$ and $|1\rangle$ results in the field-induced interference effect on the electric and magnetic polarization, which leads to the increasing left-handedness eventually.

Above all, the incoherent pumping field plays an important role in implementing left-handedness and attracts us mostly. What’s the result for the strongly incoherent pumping on the transition $|3\rangle \leftrightarrow |1\rangle$? In the following, the incoherent pumping rate was set as $\Gamma=10\gamma_{21}, \Gamma=15\gamma_{21}, \Gamma=20\gamma_{21}, \Gamma=25\gamma_{21}$ in Fig.5, 6 and 7, which are more stronger than those utilized before. As shown in Fig.5, 6, we note that the intervals for simultaneous negative $Re[\varepsilon_r]$ and $Re[\mu_r]$ are $[0, -15]$, which means a wide adjustable parameters for implementing left-handedness.
when the incoherent pumping field drives $|3⟩ \leftrightarrow |1⟩$ heavily. However, the increasing pumping rate $\Gamma$ leads to the damping values for $\text{Re}[\varepsilon_r]$ and $\text{Re}[n]$ in Fig.5 and Fig.7, even so the slightly rising $\text{Re}[\varepsilon_r]$ in Fig.6. As demonstrates the decreasing left-handedness when the incoherent pumping field drives $|3⟩ \leftrightarrow |1⟩$ heavily. It means the strong incoherent pumping field plays a destructive role in implementing left-handedness in the $E_r^{3+}$-dopped optical fiber.

From Fig.4 and Fig.7, we conclude that the different intensities of the incoherent pumping field result in the increasing or damping left-handedness in the $E_r^{3+}$-dopped optical fiber. The reason may qualitatively explain as the $E_r^{3+}$ ion’s closed-loop configuration. When the incoherent pumping field drives transition $|1⟩ \leftrightarrow |3⟩$ heavily, it influences the interferences between $|3⟩ \leftrightarrow |2⟩$ and $|1⟩ \leftrightarrow |2⟩$ simultaneously. The diminished quantum coherence and interference decrease the left-handedness and lead to the shrinking of $\text{Re}[n]$ in the $E_r^{3+}$-dopped optical fiber.

In our scheme, we used a three-level atomic system to model the emission transitions $^{4}I_{15/2} -^{4}I_{13/2}$ and $^{4}I_{13/2} -^{4}I_{15/2}$ in an $E_r^{3+}$-dopped ZBLAFN optical fiber. In order to achieve an asymptotic simulation result, the decay rates were set from some relevant research results[23]. Undoubtedly, these parameters utilized the simulating process promote the probability in the coming experiment.

From the above analysis, the left-handedness was demonstrated in an $E_r^{3+}$-dopped ZBLAFN optical fiber modeled by a three-level quantum system. In the literature, there was a quantum optical method, i.e., photonic resonant materials[26] to realize left-handed media.
which a controllable manipulation of the left-handedness by using the external fields (e.g., just like the incoherent pumping field in our scheme) interacting with coherent atomic vapour. However, in the current work, we achieved the left-handedness in the solid candidate which is the prominent feature other than the coherent atomic vapour. We would like to point out one can fruit the candidate of $E_r^{3+}$-dopped ZBLAFN optical fiber for left-handedness in our scheme. In the relevant experiment, the solid candidate may be liable to left-handedness. Meanwhile, the candidate for left-handedness of $E_r^{3+}$-dopped ZBLAFN optical fiber paved a solid candidate for left-handedness and extended the application of the rare-earth-ion-doped optical fiber in metamaterials via the external incoherent pumping field, which may attracted some interesting in the future.

IV. CONCLUSION

In conclusion, our scheme achieved left-handedness in an $E_r^{3+}$-dopped ZBLAFN optical fiber via the incoherent pumping field driving the transition of $^4I_{9/2} \rightarrow ^4I_{13/2}$. Through simulating $E_r^{3+}$ ion system with a three-level quantum system, a gradually growing left-handedness was achieved by the faint growth of the incoherent pumping field. However, the damping left-handedness emerges when the incoherent pumping field drives the transition strongly. Our scheme may propose a new avenue for implementing left-handedness and may extend the applied domain for rare-earth-ion-doped optical fiber in metamaterials.

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