Determining Cosmological Parameters from the Microwave Background

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Abstract

Recently funded satellites will map the cosmic microwave background radiation with unprecedented sensitivities and angular resolutions. Assuming only primordial adiabatic scalar and tensor perturbations, we evaluate how accurately experiments of this type will measure the basic cosmological parameters $\Omega$ (the total density of the Universe), $\Omega_b$ (the baryon density), $h$ (the Hubble constant), and $\Lambda$ (the cosmological constant). The proposed experiments are capable of measuring these parameters at the few-percent level. We briefly discuss the generality of these estimates and complications arising in actual data analysis.

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The slight temperature fluctuations in the cosmic microwave background (CMB), first
detected by the COBE satellite in 1992 [1], contain a wealth of information about the early
Universe [2,3]. By measuring the power spectrum of these anisotropies, we can hope to
extract information about the gross features of the Universe: the Hubble constant
\( H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1} \), which gives the expansion rate; the total energy density \( \Omega \)
in units of the critical density \( \rho_c = 3H_0^2/8\pi G \), which determines the geometry of the Universe; the
cosmological constant \( \Lambda \), the energy density of empty space; and the baryon density \( \Omega_b \) in
terms of the critical density. While information on each of these parameters may be obtained
from astronomical measurements, all are notoriously difficult to determine; current estimates
are not very precise and are dominated by systematic errors. The microwave background
promises a completely independent method of determining all of these parameters. This
article largely summarizes the conclusions of previously published work [4,5] to which we
refer the reader for details and more extensive references.

Here we estimate the precision with which these cosmological parameters will be deter-
mined by a high-resolution, high-sensitivity map of the microwave sky such as that produced
by the MAP satellite of NASA [6] (slated for launch in 2000) or ESA’s COBRAS/SAMBA
mission [7] (planned for 2004). The microwave sky is a statistical realization of an underlying
cosmological theory which predicts the temperature power spectrum:

\[
C(\theta) \equiv \left\langle \frac{\Delta T(\hat{\mathbf{q}}_1)}{T_0} \frac{\Delta T(\hat{\mathbf{q}}_2)}{T_0} \right\rangle 
\equiv \sum_{l=2}^{\infty} \frac{2l+1}{4\pi} C_l P_l(\cos \theta),
\]

where \( \Delta T(\hat{\mathbf{q}})/T_0 \) is the fractional temperature fluctuation in the direction \( \hat{\mathbf{q}} \), \( \hat{\mathbf{q}}_1 \cdot \hat{\mathbf{q}}_2 = \cos \theta \),
\( P_l \) are Legendre polynomials, and the brackets are an ensemble average over all observers;
the mean CMB temperature is \( T_0 = 2.726 \pm 0.010 \text{ K} \) [8]. A particular CMB measurement will
give an estimate for the values of the multipole moments \( C_l \). A simple model for a full-sky
mapping experiment which treats noise in each pixel as gaussian and neglects correlations
between pixels gives an estimated standard error in measuring each \( C_l \) as [9]

\[
\sigma_l = \left( \frac{2}{2l+1} \right)^{1/2} \left[ C_l + w^{-1} \exp(l^2\sigma_b^2) \right],
\]

where \( \sigma_b = 7.42 \times 10^{-3}(\theta_{\text{fwhm}}/1^\circ) \) for a gaussian beam, and the inverse weight per solid angle
\( w^{-1} \equiv (\sigma_{\text{pix}}\theta_{\text{fwhm}}/T_0)^2 \) is a pixel-size-independent measure of experimental noise. At small
\( l \) (large angles), the error estimate Eq. 2 is dominated by the first “cosmic variance” term,
while at large \( l \) (small angles) the noise increases exponentially due to the beam width.

A given cosmological theory will predict the \( C_l \) values. In the following analysis, we
consider the broad class of theories in which the primordial perturbations were adiabatic with
roughly a power-law spectrum. This includes all models based on inflation and encompasses
many currently popular models such as cold dark matter, mixed dark matter, open models,
and \( \Lambda \)-models. Outside of this class are isocurvature models, including defect models like
cosmic strings and textures; some comments on distinguishing between these two classes
of models are included below. The adiabatic models are described by the following set of parameters: \( \Omega, h, \Lambda \), and \( \Omega_b h^2 \), described above; the amplitudes and power-law indices of
the initial scalar and tensor perturbation spectra, $Q$, $r \equiv Q_T/Q_S$, $n_S$, and $n_T$, along with another parameter $\alpha \equiv dn_S/d\ln k$ which describes the deviation of the scalar spectrum from a perfect power law; the effective number of light-neutrino species at decoupling, $N_\nu$; and the total optical depth through the epoch of reionization, $\tau$. Given a set of values $s$ for these eleven cosmological parameters, we calculate the moments $C_l(s)$ using a semi-analytic technique [10,5]. The power spectrum can also be calculated by numerically evolving the relevant Boltzmann equations; an efficient and public code for this purpose has been provided in Ref. [11].

Now it is straightforward to determine the precision to which these parameters may be measured given the measurement error estimate Eq. 3. If the Universe is described by the underlying parameter set $s_0$, the probability distribution for observing a CMB power spectrum best fit by the parameter set $s$ is

$$P(s) \propto \exp \left[ -\frac{1}{2} (s - s_0) \cdot [\alpha] \cdot (s - s_0) \right]$$

(3)

where the curvature matrix $[\alpha]$ is given approximately by

$$\alpha_{ij} = \sum_l \frac{1}{\sigma_l^2} \left[ \frac{\partial C_l}{\partial s_i} \frac{\partial C_l}{\partial s_j} \right]_{s=s_0}.$$  

(4)

In statistical terminology, the matrix $[\alpha]$ is known as the Fisher information matrix [12]. The inverse of this matrix, the covariance matrix $[C] = [\alpha]^{-1}$, gives estimates for the uncertainties in measuring the parameters: when all parameters are fit simultaneously, the variance in $s_i$ is $C_{ii}$. If some of the parameters are fixed by other means, the variances on the rest are given by inverting the appropriate submatrix of $[\alpha]$.

In Fig. 1, we display the standard errors for the parameters $\Omega$, $\Lambda$, $h$, and $\Omega_b h^2$ given an underlying “standard CDM” model defined by the parameters $\Omega = 1$, $h = 0.5$, $\Omega_b h^2 = 0.01$, $\Lambda = 0$, 3 light neutrinos, no reionization, no tensor perturbations, and a flat initial power spectrum of scalar perturbations normalized to the COBE quadrupole, $Q = 18 \mu K$ [13]. Displayed as a function of beam size are the standard (“1-$\sigma$”) errors obtainable from a full-sky mapping experiment with two different noise levels $w^{-1} = 4.2 \times 10^{-15}$ and $1.3 \times 10^{-17}$. The first weight corresponds to the 90 GHz channel of MAP while the second corresponds to the 143 GHz channel of COBRAS/SAMBA. The large disparity in sensitivity is due to the difference between HEMT and bolometer technology: bolometers attain substantially better sensitivity but require active cooling to mK temperatures. In these frequency channels, MAP will have a nominal angular resolution of 0.29° and COBRAS/SAMBA a resolution of 0.17°. Note these error estimates assume no information about any of the parameters, i.e. an 11-parameter fit to the model. Our analysis assumes no systematic errors (e.g. in foreground removal, beam profile measurement, calculation of $C_l$’s) which lead to systematic misestimates of the cosmological parameters.

The CMB thus in principle offers the possibility of measuring the basic cosmological parameters with far better precision than traditional astronomical techniques can offer. A natural question is the generality of these results. We have assumed a particular cosmological model and a highly idealized experiment; what happens when these assumptions are relaxed?

We have calculated the expected variances for several different underlying cosmological models with similar results, unless the universe is substantially open or has undergone
FIG. 1. Standard errors from a full-sky mapping experiment as a function of beam width for noise levels $w^{-1} = 4.2 \times 10^{-15}$ (upper curve) and $1.3 \times 10^{-17}$ (lower curve). The underlying model is “standard CDM.”

significant reionization. In the first case, features in the power spectrum are shifted to smaller angular scales, weakening the parameter determination for a given beam size. However, this displacement of power spectrum features is a robust signature of an open universe which is difficult for any other cosmological model to mimic [14,15], so the geometry of the universe will still be determined to high precision. In the second case, if the total optical depth back to the last scattering epoch is of order unity, features in the power spectrum will be greatly reduced in amplitude, hindering parameter determination. Current degree-scale anisotropy detections from a variety of ground and balloon experiments make this possibility unlikely [16,17].

Structure formation may not have resulted from initial adiabatic perturbations outside of the horizon, as with the inflation-type models we have considered, but rather from isocurvature fluctuations in some type of defect model (i.e. cosmic strings or textures). We do not yet possess highly accurate calculations of the CMB power spectrum in such models,
but recent work suggests that the power spectrum will generically possess a substantially different structure than in the adiabatic case [15,18]. If the Universe is actually described by cosmic strings or textures, a microwave background map should provide an unambiguous signature, although the extent to which cosmological parameters could also be determined remains an open question.

The ultimate barrier to extracting the information in the CMB is foreground sources. At microwave frequencies, galactic synchrotron, free-free, and dust emission are can be of comparable amplitude to the CMB temperature fluctuations [19,20]; in the region of the galactic plane, the foregrounds swamp the anisotropy signal. Additionally, at angular scales below 10’ radio point sources may be a serious problem. The galactic plane will be cut from any CMB map as COBE did; statistical techniques for analyzing the resulting partial-sky map are well-known [21]. For the rest of the sky, the foreground signals can be removed because they possess frequency dependences much different from blackbody. MAP will measure in five frequency bands ranging from 22 to 90 GHz, while COBRAS/SAMBA will measure in nine channels from 30 to 900 GHz; these frequency spreads should be sufficient for foreground separation at a high level of accuracy.

Finally, all of our calculations of theoretical CMB power spectra are performed in linear perturbation theory. At scales below a half degree, various non-linear physical effects begin to contribute at a non-negligible level: gravitational lensing of the microwave background by large-scale structure [22]; the Sunyaev-Zeldovich effect from hot clusters [23]; and the Rees-Sciama effect from non-linear cluster evolution [24]. While these effects generally only give corrections to the $C_l$’s of a few percent, the errors induced by neglecting them are systematic. Any analysis of a high-resolution map, particularly at angular scales below 10’, should include all of these effects for accurate parameter determination. The estimates here are performed by truncating the sum in Eq. (4) at $l = 1000$. The COBRAS/SAMBA experiment can in principle make use of information at much smaller scales than this: the lower curves in the figure become flat below quarter-degree resolution because the measurement becomes dominated by cosmic variance out to $l = 1000$. If theoretical models are understood well enough, if foregrounds are not a serious problem, and if the beam is determined well enough to allow probing scales substantially smaller than the beam size, the error estimates presented here for COBRAS/SAMBA at the smallest angular scales could be surpassed.

In conclusion, upcoming experiments to map the cosmic microwave background at high sensitivity and angular resolution promise very exciting results. If the Universe is described by an inflation-type model with a near power-law spectrum of initial adiabatic perturbations, such a map will provide us with precise determination of the basic cosmological parameters $\Omega$, $\Lambda$, $h$, and $\Omega_b$. If on the other hand the Universe is described by a defect model or some other unanticipated possibility, these experiments will likely indicate this unambiguously. Either way, the next decade should bring a great increase in our knowledge of the fundamental properties of the Universe.
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