Small $x$ processes: Heavy Quark production at high energy

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Abstract

A brief summary is given of some recent perturbative QCD results on the evaluation of cross sections and on the structure of final states in processes with incoming hadrons at small $x$. A new Monte Carlo simulation which includes these theoretical features is described together with its applications to the study of heavy quark production in $ep$ collisions at very high energy.

Heavy flavour production is a process with a hard scale given by the quark mass $M$. At very high energies this scale is much smaller than the collision c.m. energy $\sqrt{s}$. This implies that together with the collinear logarithms (powers of $\ln M^2/\Lambda^2$) we must resum also powers of $\ln x$ with $x \approx M^2/s \ll 1$. Much progress has been made over the past few years in the theoretical understanding of small-$x$ processes. In particular the key achievements in the region $x \to 0$ are: (i) a better understanding of the “Lipatov” anomalous dimension [1, 2]; (ii) the resummation [3] to all loop order the leading contributions in the coefficient function; (iii) the extension to this region of the coherent branching process; (iv) the possibility to resum all these new contributions by Monte Carlo methods [4, 5, 6]. In this talks I will briefly summarize these theoretical results and describe a recent application of the Monte Carlo simulation to heavy flavour lepton production at Hera and higher energies.

1) Structure function.

The structure function is given in term of the space-like anomalous dimension $\gamma_N^S(\alpha_S)$ (the limit $x \to 0$ corresponds to $N \to 1$, where $N$ is the energy moment index). The leading contributions in $\gamma_N^S(\alpha_S)$ are given by an expansion in powers of $\alpha_S/(N-1)$ known since long time [1] and recently studied in the framework of hard processes [2]. The first terms of the “Lipatov” anomalous dimension are

$$\gamma_N^S(\alpha_S) = \frac{\bar{\alpha}_S}{N-1} + 2\zeta_3\left(\frac{\bar{\alpha}_S}{N-1}\right)^4 + 2\zeta_5\left(\frac{\bar{\alpha}_S}{N-1}\right)^6 + 12\zeta_3^2\left(\frac{\bar{\alpha}_S}{N-1}\right)^7 + \cdots$$

(1)

where $\bar{\alpha}_S = C_A \alpha_S/\pi$ and $\zeta_i$ is the Riemann zeta function. There are no leading terms of order $\alpha_S^2$, $\alpha_S^3$, and $\alpha_S^5$. Although each term is singular only at $N = 1$, this expansion

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develops a square root singularity at \( N = 1 + (4 \ln 2)\bar{\alpha}_S \). The presence of this singularity at \( N > 1 \) implies that the behaviour of the structure function for \( x \to 0 \) is more singular than that given by any finite number of loops. For fixed \( \alpha_S \) and small \( x \) the behaviour of the one loop structure function is

\[
x F^{(1)}(x, Q) \sim \exp \sqrt{a \ln(1/x)} \, , \quad a = 4\bar{\alpha}_S \ln(\frac{Q^2}{Q_s^2}) \ .
\]

By summing the all loop result in Eq. (1) one finds instead the following behaviour

\[
x F^{(\text{all})}(x, Q) \sim x^{-p} \, , \quad p = (4 \ln 2)\bar{\alpha}_S \ ,
\]

which is much more singular for \( x \to 0 \).

In spite of this quite different behaviour, it turns out that at this highly inclusive level the all-loop and the conventional one-loop formula give similar results [4]. This is partially due to the fact that the first correction to the one-loop expression of the anomalous dimension is to order \( \alpha^4 \). Thus the steeper behaviour of the structure function is seen only for very low \( x \). Moreover it has been pointed [5] that the steeper behaviour for \( \alpha_S \) running is even more asymptotic than for fixed \( \alpha_S \). This is due to the presence of the cutoff in the exchanged transverse momenta \( k_{ti} > Q_0 \). Although this condition is asymptotically negligible, it has some effect in reducing the evolution of the branching in the first steps. As a result the distribution at small \( x \) is somewhat reduced. For a detailed discussion see [5] and the contribution by Ryskin to this conference.

2) Cross section.

To compute the heavy flavour production cross section one convolutes the structure function with the coefficient function. Recently a way has been found to sum all leading terms \( \alpha^n/(N - 1)^n \) in the coefficient function as well. The key development [3] in this study has been a generalized factorization theorem in which one takes into account the off-shellness of the hard scattered parton. One introduces the hard elementary off-shell cross section \( \hat{\sigma}(k_t, Q) \) for a photon and a gluon to produce a heavy quark-antiquark pair, where \( q^2 = -Q^2 \) and \( k^2 = -k^2_t \) are the photon and gluon squared masses respectively, and the generalized proton structure function \( F(x, \mu, k_t) \) giving the probability (per unit of \( \ln x \)) of finding a gluon at longitudinal momentum fraction \( x \) and transverse momentum \( k_t \) in a hard process at the scale \( \mu \). Integrating this distribution over \( k_t < \mu \) one obtains the gluon structure function \( F(x, \mu^2) \). By studying \( \hat{\sigma}(k_t, Q) \) at Hera energy and \( M = 5\text{GeV} \) one has [6] that when \( W^2 \gg M^2 \gg Q^2 \gg \Lambda^2 \), \( W \) being the hadronic c.m. energy, the natural cutoff is around \( 4M^2 \), while for deep inelastic production with \( W^2 \gg Q^2 \gg M^2 \) the cutoff becomes \( Q^2 \). Therefore the hard scale is typically assumed to be \( \mu^2 = 4M^2 + Q^2 \). However at small \( Q^2 \) or at \( W \simeq M \), the dynamical suppression in \( k_t \) is at a significantly smaller scale.

In the conventional calculation, the heavy flavour cross section is obtained by convoluting the on-shell elementary cross section \( \hat{\sigma}(0, 0) \) and the gluon structure function \( F(x, \mu^2) \). This procedure has two main effects: (i) for \( k^2_t < \mu^2 \) the elementary cross section is overestimated by its on-shell value \( \hat{\sigma}(0, 0) \); (ii) the ‘tail’ of the cross section at \( k^2_t > \mu^2 \) is ignored.

Asymptotically, the second effect dominates and the cross section is expected [3] to be larger than the conventional on-shell Born approximation. At subasymptotic energies
the first effect is important and one overestimates the cross section. The Monte Carlo simulation based on the coherent branching algorithm [6] predicts that the b-quark lepton-production cross section at Hera energy is lower than the one obtained by a conventional one-loop calculation [7].

3) **Coherent branching for** \( x \to 0 \) **and** \( x \to 1 \).

It has been recently shown [2] that in the small \( x \) region one can resum the leading contributions of gluon emission by a branching algorithm which has the following two characteristics.

(i) **Phase space for the branching.** Destructive interference among soft gluons depletes the emission phase space and one finds that both for large and small \( x \) the emission takes place in the angular ordered region. Denoting by \( \theta_i, q_{ti} \) and \( z_i \) the angle, the transverse momentum with respect to the incoming hadron and the exchanged energy fraction of the \( i \) emitted gluon we have, for small \( z_i \),

\[
\{ \theta_{i+1} > \theta_i \} \sim \{ q_{ti+1} > z_i q_{ti} \}.
\]

(4)

For small \( z_i \) this phase space is larger than the conventional one corresponding to transverse momentum ordering and leading to the one-loop anomalous dimension.

(ii) **Non-Sudakov form factor.** In the region \( x \to 0 \) some of the gluons \( q_i \) have \( z_i \to 0 \). The corresponding virtual corrections contain \( \ln z_i \)-singular contributions, which factorize and exponentiate to give the following non-Sudakov form factor

\[
\Delta_{ns}(z_i, q_{ti}, k_{ti}) = \exp \left[ -\frac{C_A}{\pi} \alpha_S(k_{ti}) \ln \left( \frac{1}{z_i} \right) \ln \left( \frac{k_{ti}^2}{z_i q_{ti}^2} \right) \right],
\]

(5)

where \( k_{ti} \) is exchanged transverse momentum resulting after the emission of gluon \( q_i \), i.e. is the total transverse momentum of the system formed by all partons emitted within a cone of aperture \( \theta_i \). This form factor becomes negligible for finite \( x \) but for small \( x \) is important and has to be considered together with the usual Sudakov form factor.

The non-Sudakov form factor has the effect of screening the \( 1/z_i \) singularity of the gluon splitting function

\[
\Delta_{ns}(z_i, q_{ti}, k_{ti}) / z_i \to 0 \quad z_i \to 0.
\]

(6)

The \( k_{ti} \)-dependence in \( \Delta_{ns} \) makes the branching non local, i.e. dependent on the development of part of the emission process. Because of this non local \( k_{ti} \)-dependence, the new branching does not leads, for \( x \to 0 \), to the Altarelli-Parisi equation for the structure function. One obtains instead the Lipatov equation for the structure function which gives the “Lipatov” anomalous dimension.

Angular ordering is also the phase space constraints obtained from coherence of soft radiation in the large \( x \) region [5]. Therefore it is possible to construct a unified coherent branching algorithm valid both for \( x \to 0 \) and \( x \to 1 \), which for \( x \to 0 \) takes into account the mentioned results to all-loops, and for \( x \) finite takes into account all the leading contributions and the next-to-leading corrections important in the large \( x \) region. The important difference between the time-like and the space-like branching is the presence in the latter of the non-Sudakov form factor.
4) Unified equation for the structure function.

From the coherent branching algorithm valid both for small and large $x$, one obtains a unified equation for the generalized structure function $F(x, Q, k_t)$ giving the probability (per unit of $\ln x$) of finding a gluon at longitudinal momentum fraction $x$ and transverse momentum $k_t$ in a hard process at the scale $Q$. This hard scale gives the maximum available angle for the branching. Taking into account only the contributions which are singular for $x \to 0$ and 1, for the energy moment distribution one obtains

$$Q^2 \frac{\partial F_N(Q, k_t)}{\partial Q^2} = \int dz \frac{\alpha_S^N C_A}{\pi} z^{N-1} \left[ \left( \frac{1}{1-z} \right)_+ + \frac{1}{z} \Delta_{ns}(z, \frac{Q}{z}, k_t) \right] F_N\left(\frac{Q}{z}, k_t - \frac{1-z}{z} Q\right). \quad (7)$$

In the Kernel the emission of a gluon with energy fraction $z$ and transverse momentum $q_t = (1-z)/z Q$ is factorized. In the integrand we have the distribution before the emission of this gluon with the total transverse momentum $k_t - q_t$ and at the scale $Q/z$ which is given by angular ordering. The integration over the azimuthal direction of $q_t$ is understood. The term $()_+$ is the usual $1/(1-z)$ singularity for the soft gluon emission. Its regularization corresponds to the usual Sudakov form factor. For $N > 1$ we have $z = O(1)$ thus both the rescaling of $Q$ in $F_N$ and the non-Sudakov form factor can be neglected and the equation becomes the usual light cone expansion evolution equation. The regular $z(1-z) - 2$ term in the gluon splitting function is not included in this approximation. For $N \to 1$ the rescaling of $Q$ and the screening of the $1/z$ singularity by the non-Sudakov form factor are important. The equation has not any more the structure of an evolution equation and becomes equivalent to the Lipatov equation and gives the anomalous dimension in Eq. (1).

5) Monte Carlo simulation.

From the QCD coherent branching algorithm valid in all regions of $x$, we have constructed a Monte Carlo simulation program which has been applied to the study of heavy flavour leptoproduction at Hera and higher energies. We have compared the results of this program, which for small $x$ reproduces the mentioned “all-loop” contributions, with the one of the conventional “one-loop” branching, which for small $x$ reproduces only the first contribution of the anomalous dimension in (1). For large $x$ the two branchings are equivalent. In the following I give a short summary of the main results of Ref. [6].

In the study of heavy flavour leptoproduction, the most important differences between the improved all-loop branching and the conventional one-loop branching are seen in the final state gluon distributions. These differences arise from the additional phase space available for primary gluon emission in the all-loop evolution: the region of disordered transverse momenta is forbidden in the one-loop evolution, whilst in the new treatment it is allowed, although suppressed at very small momentum fractions by the non-Sudakov form factor. Therefore the number of emitted gluons is enhanced, especially at small $x$ and large angles, i.e. in the low-rapidity region. At present, these differences are small compared with uncertainties due to our lack of knowledge of the input gluon distribution. This underlines the importance of determining the gluon structure function experimentally down to the lowest possible values of $x$.

One of the most important feature of the new formulation is the suppression of large
energy and rapidity gaps, and large neighboring pair masses, in the distributions of primary emitted gluons. When the full colour structure of final-state branching is taken into account, this will have the effect of suppressing the production of high-mass colour-singlet combinations of partons. This in turn will produce a more local preconfinement of colour \cite{10,11} and permit a more direct connection between the perturbative parton shower and the observed hadron distributions.

The inclusive distributions generated by one-loop and all-loop evolution are rather similar. This is mostly due to a cancellation of leading higher-order corrections in inclusive observables, reflected in the absence of leading singularities of the “Lipatov” anomalous dimension in second and third order and to the presence, for running $\alpha_S$, of the cutoff in the exchanged transverse momenta.

The effects of the leading singularities of the coefficient function, represented by the off-shell photon-gluon subprocess cross section, are visible in the transverse momentum distribution of the interacting gluon. However, the asymptotically dominant effect of the high-$k_t$ tail is masked, even at very high energy, by the lower value of the off-shell cross section at $k_t\lesssim 2M$. Although the dynamical simplifications prevent firm predictions of the resulting cross sections, these results are confirmed qualitatively by recent analytical studies of sub-asymptotic effects \cite{12}.

The program \cite{6} used to generate these Monte Carlo results is available from the authors. However, it should be emphasized that the program is not an event generator: it produces weighted events only and the weight distribution is broad. It also makes numerous kinematical and dynamical simplifications. Therefore it not suitable for detailed quantitative phenomenology, but is intended rather as a theoretical tool to permit a first look at exclusive small-$x$ phenomena.

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