No evidence for the blue-tilted power spectrum of relic gravitational waves

Qing-Guo Huang and Sai Wang

State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Science, Beijing 100190, China
E-mail: huangqg@itp.ac.cn, wangsai@itp.ac.cn

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Abstract. In this paper, we constrain the tilt of the power spectrum of relic gravitational waves by combining the data from BICEP2/Keck Array and Planck (BKP) and the Laser Interferometer Gravitational-Waves Observatory (LIGO). Supposing the linearly uniform priors for both the tensor-to-scalar ratio $r$ and the tensor tilt $n_t$, we find $n_t = 0.66^{+1.83}_{-1.44}$ at the 68% confidence level from the data of BKP B-modes. By further adding the LIGO upper limit on the intensity of stochastic gravitational-wave background, the constraint becomes $n_t = -0.76^{+1.37}_{-0.52}$ at the 68% confidence level by assuming that the tensor amplitude has the similar order of the upper bounds from current CMB experiments. We find that there is no evidence for a blue-tilted power spectrum of relic gravitational waves and either sign of the index of tensor power spectrum is compatible with the current data.

Keywords: inflation, gravitational waves and CMBR polarization

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The primordial gravitational waves can be generated during inflation [1–5]. The simplest inflation models show the power spectrum of primordial gravitational waves to be adiabatic, Gaussian and nearly scale-invariant. The primordial gravitational waves contribute to both the total intensity and polarizations for the anisotropy of the cosmic microwave background (CMB) [6–12]. Especially, the effects of primordial gravitational waves on the B-mode polarization might be detectable at the range $\ell \lesssim 150$. Recently, Background Imaging of Cosmic Extragalactic Polarization (BICEP2) [13] reported an excess of B-mode power over the base lensed-ΛCDM expectation at the range $30 < \ell < 150$ with a significance of more than 5σ. However, there were debates on whether the B-mode signals detected by BICEP2 are resulted from the primordial gravitational waves or from the interstellar dust polarization [14–17]. Actually, the power of dust polarization has the same magnitude as the BICEP2 B-mode signals [18]. Most recently, a joint analysis of the B-mode data from BICEP2/Keck Array and Planck (BKP) [19] yielded

$$r = 0.048^{+0.035}_{-0.032}$$

at 68% confidence level, and $r_{0.05} < 0.12$ at the 95% confidence level (C.L.) which is compatible with the Planck temperature-only limit [20]. The significance is too low to claim a detection of primordial gravitational waves.

The tilt of power spectrum of relic gravitational waves is used to measure the feature of the tensor power spectrum. In general, the tensor tilt $n_t$ is defined by

$$P_t(k) = A_t \left( \frac{k}{k_p} \right)^{n_t},$$

where $P_t(k)$ denotes the amplitude of the primordial tensor power spectrum at scale $k$, and $k_p$ is the pivot scale which is fixed to be $k_p = 0.01\text{Mpc}^{-1}$ in this note. In the canonical single-field slow-roll inflation models, the tensor tilt is related to the tensor-to-scalar ratio by the consistency relation between $r$ and $n_t$, namely, $r = -8n_t$ [21]. In this scenario, because of the upper limit $r_{0.05} < 0.12$, the spectrum of relic gravitational waves is nearly scale-invariant. In the inflation model, the tensor tilt is generally predicted as $n_t = -2\epsilon$ [21, 22]. The inflation requires $\ddot{a}/a = H^2(1 - \epsilon)$ where $\epsilon = H/H^2$, and thus $-2 < n_t < 0$. However, there are also some alternatives to the inflation model, which predict different tensor tilts. For example, the ekpyrotic model [23] predicts a blue-tilted tensor power spectrum, i.e. $n_t = 2$. Here we take the tensor tilt $n_t$ as a fully free parameter in our analysis. Even though the current datasets are not good enough to check the consistency relation of canonical single-field slow-roll inflation, we hope that the data can be used to test the ekpyrotic scenario.

In this paper, we shall make a joint analysis of the CMB B-mode polarization and Laser Interferometer Gravitational wave Observatory (LIGO) [24] data to constrain the tensor tilt $n_t$. The CMB B-mode polarization data coming from a joint analysis of the BKP data [19] can be used to constrain the tilt of the power spectrum of relic gravitational waves at large angular scales. In this paper, we are just interested in the BB bandpowers $1 - 5$ which are taken between BICEP2/Keck Array and the 217 and 353 GHz bands of Planck. As a complement, the data of LIGO place an upper limit on the intensity of stochastic gravitational-wave background $\Omega_{GW}$ at the specific frequency band around 100Hz. Here we do not prefer to use the CMB TT and TE datasets because $C^{TT}_\ell$ and $C^{TE}_\ell$ are dominated by the scalar perturbations, and then the analysis will strongly depend on the shape of the power spectrum.
of scalar perturbations which is model-dependent. In this sense, our results given by fitting $C^{BB}_\ell$ only from BKP and the combination with LIGO are model-independent.

The data of LIGO refers to the upper limit on the intensity of stochastic gravitational-wave background

$$\Omega_{GW} < 5.6 \times 10^{-6}$$

at the frequency band around 100Hz at the 95% C.L. \[24\]. In general, the intensity of stochastic gravitational-wave background at the wavenumber $k$ is determined by \[25, 27\]

$$\Omega_{GW}(k) = \frac{P_t(k)}{12H_0^2} \left( \frac{\dot{T}(\eta_0, k)}{\dot{T}(\eta_0, k)} \right)^2,$$

where $\eta_0$ denotes the conformal time today, $H_0$ is the Hubble parameter today, $\dot{T}(\eta, k)$ is the transfer function of tensor perturbations, and the overdot denotes a cosmic time derivative $d/dt$. The wavenumber $k$ of relic gravitational waves is related to the frequency $f$ by $f = k/2\pi$. The tensor transfer function $T(\eta, k)$ describes the evolution of relic gravitational waves in the universe. It has an analytical approximation, namely, \[25–28\]

$$\dot{T}(\eta_0, k) = -\frac{3j_2(k\eta_0)\Omega_m}{\eta_0} \sqrt{1 + 1.36 \left( \frac{k}{k_{eq}} \right)^2 + 2.50 \left( \frac{k}{k_{eq}} \right)^2},$$

where the conformal time today $\eta_0 = 1.41 \times 10^4$ Mpc, and $k_{eq} = 0.073\Omega_m h^2$ Mpc$^{-1}$ denotes the wavenumber relating to the Hubble horizon at the time of matter-radiation equality. Here $\Omega_m$ and $h$ denote the matter density parameter and the reduced Hubble parameter today, respectively. Since LIGO is sensitive to the relic gravitational waves of the wavenumber $k \gg k_{eq}$, the intensity of stochastic gravitational-wave background today is given by \[27\]

$$\Omega_{GW}(k) \simeq \frac{15}{16} \frac{\Omega_m^2 A_s r}{H_0^2 \eta_0^3 k_{eq}^2} \left( \frac{k}{k_p} \right)^{n_t}$$

where $A_s$ is the amplitude of scalar power spectrum. The data of LIGO are a complement to the CMB data, since both observations refer to two quite different cosmological scales. Recently a multi-wavelength constraint on the tensor tilt was done in \[29\] where the contribution to the CMB B-mode from polarized thermal dust emission was not taken account.\[1\]

We add a prior into the public Markov Chain Monte Carlo sampler (CosmoMC) \[30\] to account for the LIGO upper limit on the intensity of stochastic gravitational-wave background. We consider two combined datasets: one is the BKP B-mode data; the other refers to a combination of the BKP data and the LIGO upper limit (BKP+LIGO). We just consider the BKP B-mode data with the bandpowers $1-5$ ($20 < \ell < 200$). Actually, the extra four bandpowers of BKP would lead little changes on our final results. The cosmological model considered here is the base lensed-$\Lambda$CDM model+tensor cosmology. The parameters of the base lensed-$\Lambda$CDM model are fixed as the “BKP cosmology parameters” in the CosmoMC, namely, $(\Omega_b h^2, \Omega_s h^2, 100\theta_{MC}, \tau, \ln(10^{10} A_s), n_s) = (0.0220323, 0.1203761, 1.0411, 0.0924518, 3.16, 0.9619123)$. Thus, the parameter space just includes the tensor-to-scalar ratio ($r$) and the tensor tilt ($n_t$).

\[1\]Our paper was submitted to arXiv on 9 Feb., 2015. The authors in \[29\] revised their paper and also “added BICEP2/KECK Planck cross analysis” in the third version on 16 Feb., 2015, and their result is consistent with ours. Here we stress that the results in \[29\] depend on the assumption of the shape of power spectrum of scalar perturbations because they adopted the CMB TT and TE power spectra data as well.
Because of the absence of the knowledge of the tensor amplitude, the constraint on the tensor tilt is expected to be prior-dependent. In this paper, we suppose the linearly uniform priors for both the tensor-to-scalar ratio and the tensor tilt, i.e. \( r \in [0, 1] \) and \( n_t \in [-4, 6] \). Our results are summarized in table 1. The marginalized contour plot and the likelihood distributions of the parameters \( r \) and \( n_t \) are illustrated in figure 1. If the BKP dataset is just considered, the constraints on the tensor-to-scalar ratio \( r \) and the tensor spectral index \( n_t \) are \( r < 0.099 \) (95% C.L.) and \( n_t = 0.66^{+1.83}_{-1.44} \) (68% C.L.), respectively. It is different from that in [31] where a blue-tilted tensor power spectrum is preferred at around 3\( \sigma \) level without taking into account the contribution to the B-mode from the polarized dust. If the BKP+LIGO dataset is considered, the constraints become \( r < 0.106 \) (95% C.L.) and \( n_t = -0.76^{+1.37}_{-0.52} \) (68% C.L.), respectively. Our results indicate that a scale-invariant power spectrum of relic gravitational waves is compatible with the data, and the canonical single-field slow-roll inflation is within 1\( \sigma \). One should note that the result is sensitive to the prior on \( r \), since there is only an upper bound on the tensor amplitude (i.e., \( r \lesssim 0.1 \)). As both constraints from BKP and LIGO are consistent with \( r = 0 \), the true tensor amplitude can be negligibly small. Our result could be changed if the order of the actual tensor amplitude is far below the current upper bound. The spectral property can be properly constrained once the tensor amplitude is determined.

In this paper, we make a constraint on the tilt of power spectrum of relic gravitational waves by combining the data of BKP B-mode and LIGO where the linearly uniform priors for both the tensor-to-scalar ratio and the tensor tilt are supposed. The bounds on the tensor tilt are given by \( n_t = 0.66^{+1.83}_{-1.44} \) at 68% C.L. for the BKP dataset, and \( n_t = -0.76^{+1.37}_{-0.52} \) at 68% C.L. for the BKP+LIGO dataset. We find that there is no evidence for the blue-tilted tensor power spectrum, and either sign of the tilt of tensor spectrum is expected to be prior-dependent. In this paper, we suppose the linearly uniform priors for both the tensor-to-scalar ratio and the tensor tilt, i.e. \( r \in [0, 1] \) and \( n_t \in [-4, 6] \). The spectral property can be properly constrained once the tensor amplitude is determined.

Table 1. The 68% and 95% limits for the parameters \( r \) and \( n_t \) from the BKP only and BKP+LIGO datasets. Here the pivot scale is \( k_p = 0.01 \text{Mpc}^{-1} \).

| parameter | BKP 68% C.L. | BKP 95% C.L. | BKP+LIGO 68% C.L. | BKP+LIGO 95% C.L. |
|-----------|---------------|---------------|-------------------|-------------------|
| \( r \)   | \( < 0.055 \) | \( < 0.099 \) | \( < 0.059 \)     | \( < 0.106 \)     |
| \( n_t \) | \( 0.66^{+1.83}_{-1.44} \) | \( 0.66^{+2.92}_{-3.32} \) | \( -0.76^{+1.37}_{-0.52} \) | \( -0.76^{+1.63}_{-2.21} \) |

In figure 1, the dominant contribution to the tensor power spectrum is suppressed on the very small scales corresponding to the LIGO and then the combination with LIGO does not affect the constraint on the negative \( n_t \). See the 2 and 3-\( \sigma \) contour plot in figure 1 in the region of \( n_t < 0 \). By contrast, the lower bound on \( n_t \) is still quite loose. The temperature auto-correlations are sensitive to the negative value of the tilt of tensor power spectrum [31, 32]. Thus, the constraint on the negative part of the tensor tilt will be significantly improved once the the CMB TT spectrum is taken into account. Even though we obtain certain constraints on the tensor tilt just from CMB B-mode and LIGO datasets, it is worthwhile to further combining other datasets, such as the Planck TT spectrum, to make a more stringent constraint on the tilt of tensor power spectrum in the near future.
Figure 1. The marginalized contour plot and likelihood distributions of the parameters $r$ and $n_t$ from the BKP only (red) and BKP+LIGO (blue) datasets. Here the pivot scale is $k_p = 0.01\text{Mpc}^{-1}$.

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