Lane reduction in driven 2d-colloidal systems through microchannels

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(Dated: March 23, 2022)

The transport behavior of a system of gravitationally driven colloidal particles is investigated. The particle interactions are determined by the superparamagnetic behavior of the particles. They can thus be arranged in a crystalline order by application of an external magnetic field. Therefore the motion of the particles through a narrow channel occurs in well-defined lanes. The arrangement of the particles is perturbed by diffusion and the motion induced by gravity. Due to these combined influences a density gradient forms along the direction of the particles. A reconfiguration of the crystal is observed leading to a reduction of the number of lanes.

In the course of the lane reduction transition a local melting of the quasi-crystalline phase to a disordered phase and a subsequent crystallization along the motion of the particles is observed. This transition is characterized experimentally and using Brownian dynamics (BD) simulations.

Pedestrians on a walkway or in a pedestrian zone generally move in a well-ordered fashion once the number of people per area exceeds a certain threshold. The reason for this behavior is that people moving in one direction try to avoid crossing paths with people moving in the opposite direction. This can be most easily achieved, if so-called lanes are formed in the motion of the individuals. This phenomenon has been observed theoretically in a large number of systems or even in the motion of animals. Most reports of lane formation have been based on simulations or theoretical calculations. Only recently lane formation could be demonstrated experimentally in a three dimensional system of oppositely charged colloids driven by an external electric field.

In this work we report on studies of the transport behavior of colloids in a quasi two-dimensional (2d) setup. The colloids are superparamagnetic, therefore the interaction energy can be continuously tuned by the application of an external magnetic field. The particles are driven through a narrow constriction (channel). In addition to the analogies mentioned earlier, such a system resembles the classical case of systems like a quantum point contact in mesoscopic electronics. These contacts show transport in electronic channels due to quantization effects. A classical version of a similar scenario can be built on a liquid helium surface, which is loaded with charges. In this system the formation of channels has been reported as well. The main advantage of the use of superparamagnetic colloids instead of electrons is given by the size of the colloids, which can be easily monitored in a standard video microscopy setup. All relevant parameters can be gathered from the configuration data.

We compare the experimental results gained from video microscopy with BD simulations of particle flow through constrictions under very similar conditions. The results of both cases show qualitative and quantitative agreement. The main focus of this discussion will be on the lane reduction transition, giving possible scenarios for the origin of this transition and describing the behavior of the particles in the transition region and its close vicinity.

Two particle reservoirs and a connecting channel are defined on a lower substrate using UV-lithography. SEM pictures of the channel setup and of the channel entrance together with some dried particles inside and outside of the channel can be seen in Fig. The channel (60 μm wide and 2 mm long) is filled with a suspension of superparamagnetic particles, which are commercially available (Dynal, particle diameters σ = 4.55 μm, suspended in water). Gravity confines the colloidal particles to the surface of the channel due to the density mismatch between the particles and the liquid. So the system is quasi two-dimensional, as long as the magnetic interactions do not lead to out-of-plane motion of the particles. Thus for an applied uniform magnetic field perpendicular to the monolayer the particle interaction is purely repulsive, and its strength at distance ri,j is given by

\[ V_{ij}(r_{ij}) = \left( \frac{\mu_0}{4\pi} \right) \frac{M^2}{r_{ij}^3} \]  

with the magnetic dipole moments \( M = \chi_{\text{eff}} B \) of the particles, which are proportional to the external magnetic field B. If no boundary conditions are imposed on the system, 2d phase transitions from a liquid phase to the hexatic phase and finally a crystalline phase can be observed as a function of the particle interactions, with boundary conditions the behavior becomes more complex. In transport through narrow channels the system orders in lanes at intermediate strength of the interactions. The importance of the pair-interaction can be characterized by the dimensionless interaction strength 

\[ \Gamma = \frac{\mu_0 M^2 \rho^{3/2}}{4\pi k_B T}, \] 

where \( \rho \) denotes the number density of the particles, \( k_B \) the Boltzmann constant, and T the temperature.

Even at moderate tilts of the experimental setup the particles cannot escape the channel and gravitationally induced transport between the two reservoirs has to take place inside the channel. At this moderate tilt the velocity of non-interacting particles depends linearly on the tilt. A small barrier surrounding the channel prevents additional particles from falling into the channel. Thus the number of particles is conserved in the system reservoirs-channel. The video...
microscopy setup allows for moderate tilting without losing optical alignment. Before starting the experiments the system is set up completely horizontal. We use configurations where the particles are either all confined in one reservoir or equally distributed along the channel and in both reservoirs. The susceptibility of the colloidal particles is $\chi_{\text{eff}} = 3.07 \times 10^{-11} \text{ Am}^2/\text{T}$. Thus an external magnetic field $B = 0.24 \text{ mT}$ corresponds to $\Gamma \approx 2.5$.

The Brownian dynamics (BD) simulations are based on an overdamped Langevin equation. This approach neglects hydrodynamic interactions as well as the short-time momentum relaxation of the particles. Both approximations are fully justified in the current experimental context. Typical momentum relaxation times are on the order of 100 μs and therefore much shorter than the repetition rate of the video microscopy setup (10 s). The colloidal trajectories $\mathbf{r}_i(t) = (x_i(t), y_i(t))$ ($i = 1, \ldots, N$) are approximated by the stochastic position Langevin equations with the friction constant $\xi$:

$$\frac{dx_i(t)}{dt} = -\nabla_{\mathbf{r}_i} \sum_{j \neq i} V_{ij}(r_{ij}) + F_{i}^{\text{ext}} + \xi_i(t). \quad (3)$$

The right hand side includes the sum of all forces acting on each particle, namely the particle interaction, the constant driving force $F_{i}^{\text{ext}} = mg \sin(\alpha) \hat{x}$ for the inclination $\alpha$ and the random forces $\xi_i(t)$. The latter describe the collisions of the solvent molecules with the $i$th colloidal particle and in the simulation are given by random numbers with zero mean, $\langle \xi_i(t) \rangle = 0$, and variance $\langle \xi_\alpha(t) \xi_\beta(0) \rangle = 2k_B T \xi(t) \delta_{ij} \delta_{\alpha \beta}$. The subscripts $\alpha$ and $\beta$ denote the Cartesian components. These position Langevin equations are integrated forward in time in a Brownian dynamics simulation using a finite time $\Delta t$ and the technique of Ermak [16,17]. Particles are confined to the channel by ideal elastic hard walls in $y$-direction. The channel end is realized as an open boundary. To keep the overall number density in the channel fixed, every time a particle leaves the end of the channel a new particle is inserted at a random position (avoiding particle overlaps) within the first 10% of the channel, acting as a reservoir.

Starting from a random particle distribution within the channel, we first calculate an equilibrium configuration ($F_{i}^{\text{ext}} = 0$) of a closed channel with ideal hard walls. Afterwards we apply the external driving force and allow the system to reorganize for $10^6$ time steps, before we evaluate the configurations. The time step $\Delta t = 7.5 \cdot 10^{-5} \tau_B$ is used, with $\tau_B = \xi \sigma^2 / k_B T$ being the time necessary for a particle in equilibrium to diffuse its own diameter $\sigma$. We choose $\xi = 3 \pi \eta \sigma$, with $\eta$ denoting the shear viscosity of the water. The simulations are done with 2000 – 4500 particles, for a channel geometry of $L_x = 800 \sigma$ and $L_y = (9 - 12) \sigma$, and $\Gamma$-values of $\Gamma \approx 70 - 950$.

A typical snapshot from the experiment of the particles moving along the channel is shown in Fig. 2(a). Similar snapshots we get from simulations with co-moving (Fig. 2(b)) and fixed boundary particles (Fig. 2(c)), i.e., the velocity is kept to zero for the particles at the channel wall. In most regions of the channel the particles are placed in a quasi-crystalline order. This behavior is due to the strength of the particle interactions caused by the external magnetic field (high $\Gamma$-values), which leads to quasi-crystalline behavior in unbounded systems as well. The formation of this order naturally gives rise to the formation of lanes in the motion of the particles along the channel. A similar layering phenomenon has been observed in channels under equilibrium conditions [7]. Additionally to this lane formation we observe, both in experiment and in simulation, a decrease of the number of lanes in the direction of motion. In between both regions therefore a region exists in which the particles cannot be well-ordered. This region is called the lane-reduction zone. In Fig. 2 these regions have been marked.

The reduction of the number of lanes originates from a density gradient along the channel. The local particle density inside the channel is shown in Fig. 3(a) and (b) together with the particle separations in $x$- and $y$-directions. In the experiment (Fig. 3(a)) the density decreases monotonically along the direction of the motion of the particles by about 20%.

FIG. 1: SEM-pictures of the channel: Shown are an overview of the channel and a zoom to the region of the channel entrance. Some dried particles inside and outside of the channel can be seen as well.

FIG. 2: (a) Video microscopy snapshot of colloidal particles moving along the lithographically defined channel. (b) Simulation snapshots for a channel (692 × 60 μm, $\Gamma \approx 2.5$) with ideal hard walls (573.3 × 45 μm, $\Gamma = 115$), (c) the same as in (b) with the particles at the walls (marked green) kept fixed (573.3 × 45 μm, $\Gamma = 902$). The blue rectangles mark the lane transition region.
average density in the channel shows fluctuations on the order of 10% as a function of time. The total increase in density, however, is less than 3% during the total time of the experiment. We therefore argue that the density gradient is formed in a quasi-static situation. This argument is confirmed by results of BD simulations (Fig. 3(b)), where the corresponding change in the local density can be observed at the position 20% simultaneously. Due to this behavior no non-monotonic tal changes back to a situation, where axis. At the point of the lane-reduction transition the crys-

This can be qualitatively confirmed by the following rough estimation: Starting from an ideal triangular configuration with a given number of lanes \( n_l \) in a channel of fixed width, we calculated the potential energy per particle for different particle densities by just scaling the channel length. Plots of these energies per particle for different values of \( n_l \) as function of the particle density are shown in Fig. 3(c). They show clear intersection points, indicating that for a stretched configuration with \( n_l \) lanes in \( x \)-direction it can become energetically more favorable to switch to a compressed configuration with \((n_l - 1)\) lanes. Also equilibrium BD simulations \((F_{\text{ext}} = 0)\) of closed channels with non-parallel walls result in a density gradient with decreasing channel width and show lane transitions. A snapshot is shown in Fig. 3(c).

The region of lane-reduction can be well localized by an appropriate local order parameter. We therefore divide the channel of width \( L_y \) into several bins in \( x \)-direction each containing \( n_{\text{bin}} \) particles and evaluate for different number of lanes \((n_l)\) the lane order parameter

\[
\Psi_{\text{lane},n_l} = \frac{1}{n_{\text{bin}}} \sum_{j=1}^{n_{\text{bin}}} e^{-\frac{2\pi (n_l - 1) x_j}{L_y} y_j},
\]

which is unity for \( n_l \) particles distributed equidistantly across the channel width starting at \( y = 0 \). As can be seen in Fig. 3(d), the lane order parameter exhibits a clear disconti-
The defect causes a local phase transition from crystalline to disordered behavior. Since the position of the lane reduction zone is mainly determined by the density gradient, its location remains stable with time on average. A more detailed analysis reveals, however, that the transition zone oscillates back and forth around this average position. At the transition the driven particles in the bulk lanes have to change the lane, causing the transition to move a little bit in direction of the flow. A particle changing into the edge lane can neutralize the defect of the transition locally. This causes a reconfiguration of the crystal, which in turn gives rise to repositioning of the lane reduction zone back to a region of higher density, i.e., back to the closest density mismatch between bulk and edge particles.

To summarize we have shown the formations of lanes and the occurrence of a lane-reduction transition in a 2d system of superparamagnetic colloids both experimentally and by Brownian dynamics simulations. The lane formation is induced by the repulsive particle interactions and the confining potential of the channel. Due to those two factors, a crystal forms in the channel. This crystal is stretched along the direction of the motion of the particles as a response to the boundary conditions at both ends of the channel and the particle interactions. Due to this stretching a lane-reduction transition occurs in the channel, at which the crystal locally disorders. After the transition the crystal is compressed in the direction of motion.

We gratefully acknowledge the support of the SFB 513, the SFB TR6 and the NIC, HLRS, and SSC.