Optimal Capacity Comparison of Decode-And-Forward Cooperative System w/wo Channel State Information

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ABSTRACT

In this paper, the optimal channel capacity of cooperative system with or without channel state information (CSI) has been analyzed by simultaneously taking both relay location and power allocation into consideration. The proposed model is a dual-hop relay system with signal from a source to a destination through the relay using the decode-and-forward (DF) strategy. The numerical results show that the optimal channel capacity of system with CSI is significantly better than the one without CSI.

INTRODUCTION

The relay technology is of vital importance for wireless communications. A relay can extend the transmission distance and reduce unnecessary power consumption. In the fifth generation (5G) wireless communication system, the high data transmission rate will be limited by transmission distance because of the characteristic of the extremely high frequency (EHF). Therefore, many studies focus on the mmWave relay systems [1-2]. Furthermore, as a promising solution, relay-assisted mobile-to-mobile (M2M) and device-to-device (D2D) cooperative communications are attracting more and more attention. In [3-4], the outage probability performance of relaying mobile D2D and M2M cooperative networks with transmit antenna selection were investigated. The power allocation problem was formulated for performance optimization. In [5], authors studied the optimal capacity in fading channels by taking the relay location into consideration. The effect of power allocation on the channel capacity of DF relay systems in orthogonal frequency division multiplexing (OFDM) transmission has been investigated in [6]. Obviously, most of the above literature only explored the optimal performance of cooperative communications under one adjustable parameter. It usually gives a local optimal solution when both relay location and power allocation are considered. In this paper, to obtain the global optimal solution, the optimal channel capacity of DF relay systems will be further studied by simultaneously taking both relay location and power allocation into account.

Channel estimation techniques are also very popular topic of discussion. In general, using channel estimation to obtain exact CSI can result in a better channel capacity but less bandwidth efficiency and more latency. To investigate the trade-off between
channel capacity and bandwidth efficiency as well as latency, the optimal channel capacity under unknown CSI and known CSI will be derived respectively.

**SYSTEM MODEL**

![System Model Diagram]

Figure 1. System model.

The system model of this paper, as shown in Figure 1, is a dual-hop relay system with the signal from a source (S) to a destination (D) through the relay (R) using DF strategy. According to [5], the capacity of S-R and R-D can be represented mathematically as:

\[
C_{SR} = \log_2 \left( 1 + \frac{|a_{SR}|^2 P_S}{N_0 d_{SR}^{n_{SR}}} \right), \quad (1)
\]

\[
C_{RD} = \log_2 \left( 1 + \frac{|a_{RD}|^2 P_R}{N_0 d_{RD}^{n_{RD}}} \right), \quad (2)
\]

where \( P_S \) and \( P_R \) are the transmission powers of the source and the relay, respectively, \( N_0 \) is the variance of additive white Gaussian noise (AWGN), \( d_{SR} \) and \( d_{RD} \) are the distances from the source to the relay and the relay to the destination, respectively, \( n_{SR} \) and \( n_{RD} \) are the path loss exponents, and \( a_{SR} \) and \( a_{RD} \) are the channel gains of the aforesaid two paths. Assumed those channel gains are independent complex Gaussian random variable with zero mean and \( 2\sigma^2 \) variance. According to [7], \(|a_{SR}|\) and \(|a_{RD}|\) are Rayleigh distributions, while \(|a_{SR}|^2\) and \(|a_{RD}|^2\) are exponential distributions. The cumulative distribution function (CDF) of \(|a_{SR}|^2\) and \(|a_{RD}|^2\) can be derived as:

\[
F_{|a|^2}(z) = \begin{cases} 
1 - e^{-\lambda z}, & z \geq 0 \\
0, & z < 0 
\end{cases}
\]

where \( \lambda = \frac{1}{2\sigma^2} \). Without the direct path between the source and destination, the capacity of the source to the destination with DF strategy can be shown as [7]:

\[
C_{DF} = E \left\{ \frac{1}{2} \min(C_{SR}, C_{RD}) \right\}. \quad (4)
\]

**ANALYSIS OF OPTIMAL CAPACITY UNDER UNKNOWN CSI**

In order to maximize the system capacity, the optimization problem can be shown as follows:
\[
\text{Max}_{0 \leq d_{SR} \leq 1, 0 \leq P_S \leq P_T} \text{Max}_{0 \leq d_{RD} \leq 1} \frac{1}{2} E\{\min(C_{SR}, C_{RD})\}, \tag{5}
\]

with
\[
P_S + P_R = P_T, \tag{6}
\]
\[
d_{SR} + d_{RD} = d_{SD} = 1, \tag{7}
\]

where \(P_T\) is the total transmit power. The distance from the source to the destination, \(d_{SD}\), is normalized to be 1. Let \(\Gamma\) denote the random variable of \(\frac{1}{2} \min(C_{SR}, C_{RD})\). The CDF, \(F_{\Gamma}(\gamma)\), of \(\Gamma\) can be expressed as:

\[
F_{\Gamma}(\gamma) = \text{prob}(\Gamma \leq \gamma) = \text{prob}\left(\frac{1}{2} \min(C_{SR}, C_{RD}) \leq \gamma\right)
= 1 - \left(1 - F_{C_{SR}}(2\gamma)\right)\left(1 - F_{C_{RD}}(2\gamma)\right), \tag{8}
\]

where \(F_{C_{SR}}\) and \(F_{C_{RD}}\) are the CDF of \(C_{SR}\) and \(C_{RD}\), respectively. Both of them can be denoted as:

\[
F_{C_{SR}}(2\gamma) = 1 - \exp\left\{-\lambda \frac{N_0 d_{SR}^n}{P_S} (2^{2\gamma} - 1)\right\}, \tag{9}
\]
\[
F_{C_{RD}}(2\gamma) = 1 - \exp\left\{-\lambda \frac{N_0 d_{RD}^n}{P_R} (2^{2\gamma} - 1)\right\}. \tag{10}
\]

Replace (9) and (10) into (8) then,

\[
F_{\Gamma}(\gamma) = 1 - \exp\left\{-\lambda \left(\frac{N_0 d_{SR}^n}{P_S} + \frac{N_0 d_{RD}^n}{P_R}\right) (2^{2\gamma} - 1)\right\}. \tag{11}
\]

Let \(k = \lambda \left(\frac{N_0 d_{SR}^n}{P_S} + \frac{N_0 d_{RD}^n}{P_R}\right)\) and differentiate both sides of (11), the probability density function (PDF) of \(\Gamma\) can be obtained as follows:

\[
f_{\Gamma}(\gamma) = (k \cdot \ln 4) \cdot 2^{2\gamma} \exp\{-k(2^{2\gamma} - 1)\}, \tag{12}
\]

and the channel capacity can be derived as:

\[
E[\Gamma] = \frac{1}{2} \int_0^\infty \log_2 \left(1 + \frac{u}{k}\right) e^{-u} du, \tag{13}
\]

where \(u = k(2^{2\gamma} - 1)\). Since the integral value in (13) is monotone decreasing with parameter \(k\), the channel capacity increases as \(k\) decreases. Therefore, the optimization problem of (5) can be changed into
\[
\begin{align*}
\min_{0 \leq d_{SR} \leq 1 \atop 0 \leq P_S \leq P_T} k &= \min_{0 \leq d_{SR} \leq 1 \atop 0 \leq P_S \leq P_T} \lambda \left( \frac{N_0 d_{SR}^{n_{SR}}}{P_S} + \frac{N_0 d_{RD}^{n_{RD}}}{P_R} \right) = \min_{0 \leq d_{SR} \leq 1 \atop 0 \leq P_S \leq P_T} \lambda N_0 \left( \frac{d_{SR}^{n_{SR}}}{P_S} + \frac{(1 - d_{SR})^{n_{RD}}}{P_T - P_S} \right) \quad (14)
\end{align*}
\]

By taking \( \frac{\partial k}{\partial d_{SR}} = 0 \) and \( \frac{\partial k}{\partial P_S} = 0 \), and after some manipulation, we obtain

\[
\begin{align*}
n_{SR} &\cdot \frac{d_{SR}^{n_{SR} - 1}}{(1 - d_{SR})^{n_{RD} - 1}} = \frac{P_S}{P_T - P_S}, \quad (15) \\
\frac{d_{SR}^{n_{SR} - 1}}{(1 - d_{SR})^{n_{RD} - 1}} &= \left( \frac{P_S}{P_T - P_S} \right)^2. \quad (16)
\end{align*}
\]

By combining (15) and (16), the optimal capacity occurs at the following condition

\[
\begin{align*}
\frac{d_{SR}^{(n_{SR} - 1)/2}}{(1 - d_{SR})^{(n_{RD} - 1)/2}} &= \frac{n_{RD}}{n_{SR}}. \quad (17)
\end{align*}
\]

Furthermore, if \( n_{SR} = n_{RD} = n \), we obtain \( d_{SR} = 0.5 \) and \( P_S = 0.5 P_T \). In other words, under \( n_{SR} = n_{RD} = n \), the optimal capacity occurs when the relay is located at the midpoint of \( d_{SD} \) and both source and relay have equal transmission power.

**ANALYSIS OF OPTIMAL CAPACITY UNDER UNKNOWN CSI**

Assume that the system has the CSI. It means both \( |a_{SR}|^2 \) and \( |a_{RD}|^2 \) as well as \( C_{SR} \) and \( C_{RD} \) are deterministic. Therefore, the optimization problem can be simplified as:

\[
\begin{align*}
\max_{0 \leq d_{SR} \leq 1 \atop 0 \leq P_S \leq P_T} C_{DF} &= \max_{0 \leq d_{SR} \leq 1 \atop 0 \leq P_S \leq P_T} \frac{1}{2} \{\min(C_{SR}, C_{RD})\}. \quad (18)
\end{align*}
\]

under the same constraints of (6) and (7). Since \( C_{SR} \) and \( C_{RD} \) are monotonically increasing and monotonically decreasing respectively, with variable \( P_S \), the optimal capacity occurs when \( C_{SR} = C_{RD} \), that is,

\[
\frac{|a_{SR}|^2 P_S}{d_{SR}^{n_{SR}}} = \frac{|a_{RD}|^2 P_R}{d_{RD}^{n_{RD}}}. \quad (19)
\]

With (6) and (19), the \( P_S \) can be obtained as:

\[
P_S = \left[ \frac{|a_{RD}|^2 d_{SR}^{n_{SR}}}{|a_{SR}|^2 d_{RD}^{n_{RD}} + |a_{RD}|^2 d_{SR}^{n_{SR}}} \right] P_T, \quad (20)
\]

and (18) can be reduced to

\[
\begin{align*}
\max_{0 \leq d_{SR} \leq 1} C_{DF} &= \max_{0 \leq d_{SR} \leq 1} \frac{1}{2} \log_2 \left( 1 + \frac{|a_{SR}|^2 |a_{RD}|^2 P_T}{N_0 (|a_{SR}|^2 d_{RD}^{n_{RD}} + |a_{RD}|^2 d_{SR}^{n_{SR}})} \right). \quad (21)
\end{align*}
\]
In (21), the maximum capacity occur when the denominator is minimal. Let
\[ h(d_{SR}) = |a_{SR}|^2 d_{RD}^{n_{RD}} + |a_{RD}|^2 d_{SR}^{n_{SR}} \] and \( \frac{dh}{d(d_{SR})} = 0 \), we obtain
\[ \frac{d_{SR}^{n_{SR} - 1}}{(1 - d_{SR})^{n_{RD} + 1}} = \frac{|a_{SR}|^{2n_{RD}}}{|a_{RD}|^{2n_{SR}}} \] (22)

If \( n_{SR} = n_{RD} = n \), we further obtain
\[ d_{SR} = \frac{\left(\frac{|a_{SR}|^2}{|a_{RD}|^2}\right)^\frac{1}{n-1}}{\frac{1}{n-1}} \] (23)

**NUMERICAL RESULTS**

The parameters used in our numerical analysis are listed in TABLE I:

| Parameters                                      | values  |
|-------------------------------------------------|---------|
| Path loss exponent of source to relay           | \( n_{SR} = 3 \) |
| Path loss exponent of relay to destination      | \( n_{RD} = 3.5 \) |
| Total power                                     | \( P_T = 10 \) |
| The variance of AWGN                            | \( N_0 = 1 \) |

Figure 2. Numerical result in unknown CSI.

Figure 2 shows the channel capacity versus the transmission power \( P_S \) of source node and the relay location \( d_{SR} \) under unknown CSI, while Figure 3 shows the relative performance under known CSI. In Figure 2, \( \lambda = 0.5 \). It means \( E(|a_{SR}|^2) = E(|a_{RD}|^2) = \lambda^{-1} = 2 \) for unknown CSI. To figure out the impact of CSI on channel capacity, we use \( |a_{SR}|^2 = |a_{RD}|^2 = 2 \) for known CSI (i.e. for Figure 3). From these two figures, we observe that the maximal channel capacity for unknown CSI is \( C_{DF,MAX} = 2.4119 \) which occur when \( P_S = 5.3 \) and \( d_{SR} = 0.492 \), while the maximal
channel capacity for known CSI is $C_{DF_{MAX}} = 3.2875$ occurring at $P_S = 5.12$ and $d_{SR} = 0.477$. Obviously, the latter is significantly better than the former. In other words, with accurate channel estimation, the channel capacity can be enhanced. By the way, the optimal results can also be calculated using bisection search method based on (17), (16) and (22), (20) for both unknown CSI and known CSI, respectively.

![Figure 3. Numerical result in known CSI.](image)

CONCLUSIONS

In this paper, the optimal channel capacity of DF cooperative system with or without CSI has been derived. Numerical results exhibit the CSI can be used to enhance the channel capacity. However, it suffers less bandwidth efficiency and more latency due to the needs of channel estimation and CSI feedback. The proposed model only took into account the path from the source to the destination through the relay. Whether the direct transmission path from source to destination can give the diversity gain on channel capacity is worthy of further study.

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