Order Statistics - Fuzzy Approach in Processing of Multichannel Images and Video Sequences

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1. Introduction

There exist different applications of the image processing, such as medical imaging, high definition television, virtual reality, remote sensing, ultrasound and radar imaging, etc. In these applications, it is necessary to restore an image (or frames of video sequence) and decrease a noise influence exploiting the filtering algorithms that form a part of a general image processing system. The images are corrupted by noise, in sensors employed or maybe, during signal transmission. Also, several kinds of noises are produced by natural phenomena (atmospheric, scattering, interference, etc.). Usually, real noises are described by different models, there exist impulsive, additive and multiplicative (speckle) ones. So, the image pre-processing efficient scheme should be one of important part in any vision application permitting to suppress a noise, saving the image performances, such as, edge and fine features preservation, and also the chromaticity properties for the multichannel (multispectral) images. This demands to have several efficient filtering schemes, which depend on noise type and priory information, in a pre-processing stage of image or video sequence processing system. The main objective of present chapter is to exhibit several justified approaches in restoration of the images and video sequences, which are usually affected by noise of different nature, which can be efficiently used in different applications of the multichannel (multispectral) images and sequences. Here, multispectral image is defined in such a way, where each a pixel is represented by a number of channels that carry out information about its spectral content. Multispectral images span the domain of images from conventional three-channel colour images to hyperspectral imagery with hundred of bands/channels used in remote sensing applications, medicine, spectrometry, etc. In literature, there exist a lot of algorithms that process two dimensional (2D) images (Franke et al., 2000); (Russo & Ramponi, 1996); (Schulte et al., 2006, 2007a, 2007b, 2007c); (Shaomin & Lucke, 1994); (Nie & Barner, 2006); (Morillas et al., 2005, 2006, 2008a, 2008b, 2009); (Camarena et al., 2008, 2010); (Ma et al., 2007); (Amer & Schröder, 1996); (Xu, 2009). We compare the proposed 2D fuzzy framework with recently presented 2D-INR filter based on fuzzy logic (Schulte et al., 2007b), where a noise is detected preserving the fine features and edges in an image. Also, other promising classes of 2D processing algorithms are employed as comparative ones: 2D-AMNF, 2D-AMNF2 (Adaptive Multichannel Filters)(Plataniotis & Androutsos et al., 1997); (Plataniotis & Venetsanopoulos, 2000); 2D-
AMNIF (Adaptive Multichannel Filter using Influence Functions) (Ponomaryov et al., 2005); (Plataniotis & Venetsanopoulos, 2000); 2D-GVDF (Generalized Vector Directional Filter) (Trahanias & Venetsanopoulos, 1996); 2D-CWVDF (Centered Weighted Vector Directional Filters) (Lukac et al., 2004); and finally, 2D-VMF_FAS (Vector Median Filter Fast Adaptive Similarity) (Smolka et al., 2003). These techniques have demonstrated the better results among a lot of others known in literature. The principal drawback of all 2D processing algorithms is that they use only one frame of a video sequence and principally cannot use temporal correlation that exists between neighbouring frames to distinguish and decrease noise or motion in an image. This does not permit to suppress a noise efficiently, as well as to preserve the fine image features and restore the image chromaticity properties. Temporal information should be taken into consideration in the processing of the neighbouring frames together but straight averaging in temporal area the corresponding pixels to smooth a noise may introduce “ghosting” artifacts in the presence of camera and object motion. Such artifacts can be removed by motion compensation where a number of algorithms have been proposed with different computational complexity (Balster et al., 2006); (Jovanov et al., 2009); (Kravchenko et al., 2009); (Kravchenko et al., 2009); (Mélange et al., 2008); (Mélange et al., 2008); (Zlokolica et al., 2005). Thus, a desirable video noise filter should distinguish noisy and motional pixels as well as collect enough similar pixels adaptively from temporal to spatial directions.

In this chapter, the fuzzy set theory and fuzzy logic that offer us a powerful tool for representing and processing human knowledge and intuition, incorporating them into the design are employed, which cannot be done using classical mathematical modelling. The fuzzy metric is considered more effective in comparison with classical measures, moreover, due to the non-stationarity of images and serious problems in distinguishing between noise, motions, and fine features and edges, fuzzy modelling is considered quite appropriate in video sequence filtering. Here, classical binary decisions are replaced by a gradual transition, which is more appropriate when dealing with complex systems. Unfortunately, a methodology, which gathers the advantages of each one of powerful techniques (order vector statistics and fuzzy set theory) usually employed in processing of images or video sequences, providing the better suppression noise capability, fine features preservation, as well as chromaticity characteristics, is not developed sufficiently. In present chapter, promising scheme is designed unifying the directional gradients and pixel angular divergence measure together with the robust vector order statistics processing techniques described previously (Ponomaryov, 2007); (Ponomaryov et al., 2010). The employing the designed fuzzy rules with fuzzy measure of a motion in a form of the membership degree in a 3D sliding-window gives the opportunity to preserve well the fine image features and restore the chromaticity properties. General operations of novel approach consist of the selection made in fuzzy means for any spectral band of an image: if there exist the edges and fine features, or noise, or may be some movement in the central pixel into sliding processing window. So, the framework does it possible to distinguish these characteristics inherent in multispectral images (or frames) using fuzzy rules designed in this chapter. They are applied to fuzzy-directional values to resolve the hypothesis: if a central pixel component is a corrupted one or not. In case of a corrupted pixel happened, some procedures in substitution of a central component with one of its neighbours are realized according to justified in fuzzy matter selection.
We also realize the adaptation of several 2D algorithms in filtering of 3D video data: 3D-MF, 3D-VGVDF (Trahanias & Venetsanopoulos, 1996), 3D-VVMF and 3D-VVDKNNVMF (Ponomaryov, 2007). Additionally, we have implemented the 3D-VKNNF, 3D-VATM (Zlokolica et al., 2006), and 3D-VAVDATM filters (Ponomaryov, 2007). Other fuzzy Logic techniques 3D algorithms (Saeidi et al., 2006), and (Zlokolica et al., 2006) are analyzed during modelling and in the simulation experiments. The first framework that is used to smooth Gaussian noise is the designed FDARTF_G (Fuzzy Directional Adaptive Recursive Temporal Filter for Gaussian Noise) that preserves the fine features, edges and chromaticity properties, and the second one, 3D-FCF (Fuzzy Temporal Spatial Colour Filter) operates in similar way as FDARTF_G only with some modifications for impulsive noise decreasing. To justify the effectiveness of introduced 3D techniques, the comparison with the better filtering frameworks that exist in video sequence processing were used (Zlokolica et al., 2006); (Ponomaryov et al., 2009); (Schulte et al., 2006b); (Schulte et al., 2006a); (Mélange et al., 2008). Reference filters: “Fuzzy Motion Recursive Spatio-Temporal Filter” (FMRSTF) (Zlokolica et al., 2006); an adaptation of FMRSTF employing only angles instead of gradients, named as “Fuzzy Vectorial Motion Recursive Spatio-Temporal Filter” (FVMRSTF); “Video Generalized Vectorial Directional Processing” (VGVDF) (Trahanias et al., 1996), “Video Median M-type K-Nearest Neighbour” (VVDKNNVMF) described in (Ponomaryov, 2007) were used as comparative in suppression of Gaussian noise, and algorithms 3D-MF, 3D-VGVDF, 3D-VVMF, 3D-VVDKNNVMF, 3D-VKNNF, 3D-VATM, 3D-VAVDATM filters were used as comparative ones to evaluate 3D-FCF rendering during the simulations and modelling experiments. Numerical simulations have shown the better performance of original framework that outperforms existed methods in suppression of a noise of different nature increasing performances of a colour image and/or video data. The objective criteria used in modelling and simulation experiments of the different filtering algorithms are the Peak Signal-to-Noise Ratio (PSNR), Mean Absolute Error (MAE) and Normalized Colour Difference (NCD), (Plataniotis & Venetsanopoulos, 2000); (Ponomaryov, 2007). Additionally, the subjective visual criterion in form of error of reconstructed multichannel image is used.

Several designed promising algorithms as well as better existed ones were implemented on the DSP platform realizing analysis of the sequences or images in a real time environment (Mullanix et al., 2003); (Gallegos-Funes et al., 2009); (Kravchenko et al., 2009).

The current chapter is organized as follows: Sec. 2 presents the model of noise usually employed in image processing applications and defines the objective criteria: PSNR, MAE and NCD. Sec. 3 exposes some promising recent schemes for simultaneous processing of different kinds of 2D-3D images and video sequences corrupted by noises (Gaussian and impulsive). Sec. 4 explains the original 2D-3D procedures to suppress additive and impulsive noises using two neighbouring frames for the motion, fine detail and edges, and noise detection in multichannel images and video sequences. Here, the numerous experimental results of modelling and simulations in form of the objective and subjective measures are presented, justifying the effectiveness of several proposed and existing approaches, and also the implementation of the better promising algorithms on the DSP platform realizing analysis of the sequences or images in a real time environment is discussed. A brief conclusion is drawn in Sec. 5.
2. Noise and Performance Criteria

Real-world still images and video sequences are affected by random fluctuations in intensity, colour, texture, object boundary, or shape, and also by blurring blocking, and colour distortions. There are a lot of complex reasons for these fluctuations and distortions, often due to factors, such as non-uniform lighting, random fluctuations in object surface orientation and texture, sensor limitations, etc. The processing of such images or frames in video sequences can be treated as a problem of statistical inference, which requires the definition of a statistical model corresponding to the image and noise pixels employing the random field models. Combined with various frameworks for statistical inference, such as maximum likelihood (ML) and Bayesian estimation, random field models are used in image restoration, enhancement, classification, segmentation, compression and synthesis. The general model of image-noise representation consists of the random field definition that represents the multidimensional signal and the random process, together with the joint density that models the corruption mechanism (Bovik, 2000).

Images are relatively broadband signals where the visual information may be at mid-to-high spatial frequencies, and significant image details: edges, lines, and textures typically contain higher frequencies. The classical but no efficient approach in noise suppression influence is the linear filtering algorithms where for a given filter type, different quality of smoothing can be received by adjusting the bandwidth of a linear filter.

2.1 Additive Noise

Optimal methods of linear filtering theory is useful when the corruption could be represented as a Gaussian process and the criterion of accuracy is the mean square error (MSE), but this assumption is not correct in most applications, for example in digital systems. Gaussian noise is a part of almost any signal where an additive Gaussian noise generally assumes zero-mean Gaussian distribution and is usually introduced during video acquisition. The additive model is most appropriate when the noise in this model is independent of an image. There are many applications of the additive model: thermal noise, photographic noise, and quantization noise, etc.

2.2 Impulsive Noise in Image

It is assumed that the noise process is impulsive noise if as a result many of the signal values do not change at all or change slightly and some signal values change dramatically, in other words, the change is clearly visible (Astola & Kuosmanen, 1997). In practice, the same number of bits is used to represent the noisy and the noise-free signal, usually 8 bits or 256 levels 0, 1,..., 255. The realistic impulsive noise is modelled as bit errors in the signal values during transmitting the images or video sequences over noisy digital links. It is easy to calculate for a binary symmetric channel with a given crossover probability that the contribution to the MSE from the most significant bit is approximately 3 times that of all the other bits. Impulses are also referred to as outliers.

Several types of impulsive models usually can be used. Some of them need the detail a priori information about the degradation process in each a channel for multichannel (or colour) multidimensional image. In our opinion, the complex models that need several parameters, which should be determined a priori or during the processing stage, have low tolerance, and so, such a model can produce confusion during the interpretation of filtering
results (Ponomaryov et al., 2005); (Ponomaryov, 2007); (Kravchenko et al., 2009). Below, we use the simple and in the same time the most severe model of impulsive noise from point of view of image degradation. This model needs only prior information about the probability \( p \) of random spikes appearance, which are independent in each a channel. Additionally, the amplitude of impulsive noise is modelled as uniformly distributed random value within the interval of given values \((0-255)\) for each a channel in the case of colour images.

### 2.3 Mathematical Solutions Applied in Image-Noise Models

We use the simplest model for additive Gaussian noise degradation

\[
x_z(i, j) = x_0(i, j) + n(i, j),
\]

where \( x_0(i, j) \) is original image (or sequence frame), \( x_z(i, j) \) is degraded image, and \( n(i, j) \) is Gaussian additive noise. Also, such a model for noise influence in the case of impulse noise degradation is employed (Ponomaryov, 2007); (Kravchenko et al., 2009):

\[
x(i, j) = n_l(x_0(i, j)),\quad n_l(x_0(i, j))=\begin{cases} \text{random values with probability } P & x_0(i, j) \text{ another case} \\
\end{cases}
\]

where \( x_0(i, j) \) is original image (or sequence frame), \( x(i, j) \) is degraded image, and \( n_l(x_0(i, j)) \) the above presented function.

In the case of multiplicative noise degradation, the model (2) can be represented in the form (Kravchenko et al., 2009):

\[
x_{\text{speckle}}(i, j) = n_m(x_0(i, j) \cdot x_0(i, j)),
\]

where \( n_m(i, j) \) denote multiplicative (speckle) noise.

The eqs. (1-3) represent the basic models in degradations by noise. For multichannel images it is necessary to apply eq. (2) for each a channel.

In the case of multidimensional image representation, the model (2)-(3) is changed, and for 3D discrete image can be rewritten as follows:

\[
x_{\text{speckle}}(i, j, k) = n_l(x_0(i, j, k) \cdot x_0(i, j, k)),
\]

where \( n_l(x_0(i, j, k)) \) is the functional \( n_l(x_0(i, j, k))=\begin{cases} \text{noise } n_i \text{ with probability } P & x_0(i, j, k), \text{ otherwise} 
\end{cases} \), and \( x_{\text{speckle}}(i, j, k) \) is a noisy observation (i.e., the recorded image) of the 3-D function \( x_0(i, j, k) \).
Let present several promising approaches that are used in video sequence filtering.

### 2.4 Objective and Subjective Criteria

To model and evaluate different filters and compare their performances, several criteria are used, such as: the peak signal-to-noise ratio (PSNR) for the evaluation of noise suppression; the mean absolute error (MAE) for quantification of edges and fine feature preservation and the normalized colour difference (NCD) (Plataniotis & Venetsanopoulos, 2000):

\[
PSNR = 10 \cdot \log \left( \frac{(255)^2}{MSE} \right) \text{ dB},
\]

\[
MAE = \frac{1}{M \cdot N} \sum_{i=1}^{M} \sum_{j=1}^{N} \| x(i, j) - x_0(i, j) \|_2,
\]

where \( MSE = \frac{1}{M \cdot N} \sum_{i=1}^{M} \sum_{j=1}^{N} \| x(i, j) - x_0(i, j) \|_2^2 \) is the mean square error, \( M, N \) are the image dimensions, \( x(i, j) \) is the 3D vector value of the pixel \((i, j)\) in the filtered colour image, \( x_0(i, j) \) is the corresponding 3D vector value of the pixel in the original uncorrupted image, and \( \| \cdot \|_{L_1}, \| \cdot \|_{L_2} \) are the \( L_1 \)- and \( L_2 \)-vector norms, respectively;

\[
NCD = \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} \| \Delta E_{Luv}(i, j) \|_{L_2}}{\sum_{i=1}^{M} \sum_{j=1}^{N} \| E_{Luv}^*(i, j) \|_{L_2}}.
\]

Here, \( \| \Delta E_{Luv}(i, j) \|_{L_2} = \left[ \left( \Delta L^*(i, j) \right)^2 + \left( \Delta u^*(i, j) \right)^2 + \left( \Delta v^*(i, j) \right)^2 \right]^{1/2} \) is the norm of colour (or multichannel) error; \( \Delta L^*, \Delta u^*, \) and \( \Delta v^* \) are the difference in the \( L^*, u^*, \) and \( v^* \) components, respectively, between the two colour vectors that present the filtered image and uncorrupted original one for each a pixel \((i,j)\) of an image; and \( \| E_{Luv}^*(i, j) \|_{L_2} = \left[ \left( L^* \right)^2 + \left( u^* \right)^2 + \left( v^* \right)^2 \right]^{1/2} \) is the \( L_2 \) norm or magnitude of the uncorrupted original image pixel vector in the \( L^* u^* v^* \) space. It has been proved that the NCD objective measure expresses well the colour distortion (Plataniotis & Venetsanopoulos, 2000).

### 3. Some Efficient Frameworks in Video Sequences Processing

Let present several promising approaches that are used in video sequence filtering.
3.1 Motion-Compensated 3-D LLMMSE Filter
In this approach (Yin et. al., 2007), an image-noise model is supposed to be a sum of an image and the signal-independent, additive, spatio–temporal invariant white noise. Uniform temporal filtering area is adaptively grown according to the motion estimation status of the adjacent candidate frames. So, the frames with higher temporal correlation are motion-compensated to the current one. The pixel aggregation algorithm is used to include the homogeneous adjacent pixels and exclude the outlier (noisy) pixels. An adaptive weighted local mean and variance improve the filtering performance. When a pixel within the filtering support deviates from the current pixel beyond a defined threshold in terms of intensity, its weight is decreased to deemphasize its contribution to the local mean and variance estimation.

The spatio–temporal LLMMSE estimate of the pixel at the spatial position \(i, j\) of the \(k\)-th frame is given by adaptive Wiener filtering algorithm

\[
\hat{x}(i, j, k) = \bar{x}(i, j, k) + \frac{\hat{\sigma}_s^2}{\hat{\sigma}_s^2 + \hat{\sigma}_n^2} [x(i, j, k) - \bar{x}(i, j, k)]
\]

where \(\bar{x}(i, j, k)\) is the mean estimate of the current pixel in the local spatio–temporal area, which is a cuboid window centered about the current pixel. In the same area, the variance estimate of \(x(i, j, k)\) can be computed, and also local estimate of dispersion can be found \(\hat{\sigma}_s^2 = \max[\hat{\sigma}_d^2 - \hat{\sigma}_n^2]\). The robust block-matching motion estimator is employed here, where all candidate motion vectors are checked to select the right motion vector within an adaptively uniform area with enough spatial gradients. With the motion field obtained, the adjacent frames are compensated with respect to the current frame selecting the data used in filtering stage. The 8 per 8 blocks are used for motion estimation, finally presenting the results in the form of the dark blocks that mark the temporal stationary data in the current data, which form the temporal filtering samples; on the other hand, the white blocks represent the regions containing temporal non stationarity on the data. In general, the more adjacent frames a filter are used, the higher denoising capability it can achieve. However, the more temporal blurring can be due to increasing imperfection of motion compensation. In general, the candidate frames having higher temporal correlations with respect to the current frame are selected to grow the temporal data to be filtered.

3.2 Inter-frame Model of Wavelet Coefficients
In this approach (Yin et. al., 2007), an image-noise model is supposed to be a sum of image (Mahbubur Rahman et. al., 2007). In order to take into account the correlation between the wavelet (WL) coefficients of any two neighbouring frames, a joint statistical model in form bivariate Gaussian distribution for the video wavelet coefficients can be used. The joint density function takes into account one of the essential variabilities of the video WL coefficients of the neighbouring frames (the motion). So, the video WL coefficients are zero-mean conditionally independent bivariate Gaussian random variables with slow-varying variance and covariance. This model is a base for developing a bivariate maximum a posteriori (MAP) estimator for spatial filtering of a noisy video.
Let define \( f_j(k) \) as the WL coefficients for a given sub-band of the \( j \)-th frame, where, for simplicity \( k \) is used to represent two-dimensional spatial indexes. The WL coefficients of the previous neighbouring frames are denoted as \( f_j(k) \). Because the correlation coefficient represents the linear relationship between the two random processes, so, for a given sub-band video WL coefficients, the amount of motion that exists between any two frames can be indirectly measured by correlation coefficient. So, it is preferable to use the sub-band dependent correlation parameter \( r \) as an index of the motion. The higher the value of \( r \) is, the lower the amount of motion between the sub-bands of the two neighbouring frames will be, and vice versa.

To define the joint density function WL coefficients for the current frame and any of the previous frames, the motion index is used. The bivariate Gaussian density function with a strong dependency between two random processes is elliptic, so, the coefficients of any two frames with very little motion can be modelled using this density function. For a relatively large motion, this coefficient can be assumed to be zero.

The joint PDF (Probability Density Function) of WL coefficients for the current frame and any of the previous frames is written as:

\[
w(x_i, x_{j-1}) = \begin{cases} 
\frac{1}{2\pi\sigma_i\sigma_j\sqrt{1-r^2}} \exp \left[ -\frac{1}{2\sqrt{1-r^2}} \left( \frac{x_i^2}{\sigma_i^2} + \frac{x_{j-1}^2}{\sigma_{j-1}^2} + 2r \frac{x_i x_{j-1}}{\sigma_i \sigma_{j-1}} \right) \right], & \text{if } 0 < r < 1 \\
\frac{1}{2\pi\sigma_i\sigma_j} \exp \left[ -\frac{1}{2} \left( \frac{x_i^2}{\sigma_i^2} + \frac{x_{j-1}^2}{\sigma_{j-1}^2} \right) \right], & \text{if } -1 < r < 0 
\end{cases}
\]  

(9)

If the pixels of the video frames are corrupted by additive white Gaussian noise and the variance is unknown, it may be estimated by applying the median absolute deviation method in the highest sub-band of noisy WL coefficient. Noisy WL is presented as a sum of coefficients of a frame and noise.

First, let develop a bivariate MAP estimator to estimate the image WL coefficients of the current frame denoted as \( x_i(k) \), applying the correlation information of the \( j \)-th previous frame into account. The variances and covariances are estimated from the bivariate maximum likelihood (ML) estimator. In the second step, the estimated coefficients \( x_i(k) \) are passed through a recursive temporal averaging filter for additional noise reduction. At the last step, the denoised coefficients, denoted as \( \hat{x}_i(k) \), are inverse transformed to obtain the denoised video frame. The bivariate MAP estimator for WL coefficients is defined in the current frame from the noisy versions of the current frame and the \( i \)-th previous frame and can be written using eq. (8) and Gaussian PDF for noisy observation:

\[
\hat{x}_i(k) \mid x_{j-1} = \begin{cases} 
\frac{\sigma_i^2(k)\bar{\sigma}_j^2(k)}{\sigma_j^2(k)\bar{\sigma}_j^2(k) + \sigma_n^2} \left[ x_{j-1} + \bar{\rho}(k) \frac{\bar{\sigma}_j^2(k)}{\bar{\sigma}_j^2(k) + \sigma_n^2} - \sigma_j^2(k) \right], & \text{if } 0 < r < 1 \\
\frac{\sigma_i^2(k)}{\sigma_j^2(k) + \sigma_n^2} x_{j-1}, & \text{if } -1 < r < 0 
\end{cases}
\]  

(10)

where

\[
\bar{\sigma}_j^2(k) = \left[ 1 - \bar{r}^2(k) \right] \sigma_j^2(k) + \sigma_n^2, \quad \text{for } i = j - 1, j - 2, \ldots, j - l
\]
Therefore, the statistical model for the near frame video WL coefficients is considered as locally independent and identically distributed (i.i.d.) bivariate Gaussian distribution with conditional mean, variance, and covariance that are calculated locally for each index \( k \).

### 3.3 Wavelet-Domain and Motion-Compensated Video Denoising

This video denoising approach (Jovanov et al., 2009) exploits the idea of the motion estimation resources from the video coding module for video denoising. A novel motion field filtering step refines the accuracy of the motion estimates to a degree that is required for video denoising. Additionally, a novel robust temporal filtering against errors in the estimated motion field is proposed. Here, it is assumed that the video sequences are contaminated with the additive white Gaussian noise, with zero mean and known variance. The denoising approach is based on spatio-temporal filtering that combines WL spatial filtering, which is preceded by pixel-domain temporal filtering.

The basic idea in temporal algorithm is to compare the MAD between the corresponding blocks with the average MAD, and decide if motion is present or not. The proposed motion filtering method is particularly effective in suppressing spurious background motion vectors. The threshold \( THR \) for motion detection in the \( k \)-th frame in this filtering step is used to decide whether motion exists in each block. If the MAD<\( THR \), both motion vector components are set to zero. Otherwise, the motion vector keeps its original value.

The idea of Motion Compensated Temporal Filter is to control switching between weaker and stronger temporal smoothing based on a motion detection variable. At positions where no motion was detected, a standard recursive temporal filter is applied. At moving positions the filtering is realized, but this time along the estimated motion trajectory, using different filter coefficients. This covers the situation when the estimated motion is not perfect permitting a different degree of temporal smoothing for moving and for non-moving areas.

The proposed motion compensated filter is written as

\[
d_{i,j}^k = (1 - m_{i,j}^k)(\alpha(1 + \varepsilon_{i,j}^k)d_{i,j}^k + (1 - \alpha)(1 - \varepsilon_{i,j}^k)d_{i-p,j-q}^{k-1}) + m_{i,j}^k(1 + \varepsilon_{i,j}^k)d_{i,j}^k + (1 - \beta)(1 - \varepsilon_{i,j}^k)d_{i-p,j-q}^{k-1}
\]

where \( \alpha \) and \( \beta \) are the fixed parameters in recursive filter in static and moving areas. The values \( 1 + \varepsilon_{i,j}^k \) and \( 1 - \varepsilon_{i,j}^k \) are data driven factors for these parameters. The factor \( 1 + \varepsilon_{i,j}^k \) increases the influence of current frame pixel value \( d_{i,j}^k \) in the case when prediction error \( \varepsilon \) is large. The influence of \( d_{i-p,j-q}^{k-1} \) on the filtering result is in this case simultaneously suppressed through \( 1 - \varepsilon_{i,j}^k \) (which is close to zero). Otherwise, when the prediction error \( \varepsilon \) is small, the factor \( 1 - \varepsilon_{i,j}^k \) is close to 1, enforcing smoothing along the estimated motion trajectory.

At the second stage of framework, the temporal filter is combined with a wavelet domain spatial filter using a fuzzy-logic version of the spatially adaptive Probability Shrink that is applied to each wavelet coefficient a shrinkage factor, which is a function of two measurements: the coefficient magnitude and a local spatial activity indicator that indicates the fine feature changes.
3.4 Video denoising by fuzzy motion and detail adaptive averaging-FMDAF

A fuzzy-rule-based algorithm for the denoising of video sequences (Melange et al., 2008) corrupted with additive Gaussian noise constitutes a fuzzy-logic-based improvement of a recent detail and motion adaptive multiple class averaging filter (MCA) (Zlokolica et al., 2003). Last framework to avoid the spatio-temporal blur, only takes into account neighbouring pixels from the current frame in case of detected motion. So, to preserve the details, the filtering should be less strong when large spatial activity is detected in a current window. The filtering window used in the framework is a $3 \times 3 \times 2$ sliding window, consisting of pixel windows in the current and previous frames. The output of the proposed filter for the central pixel in the window is determined as a weighted adaptive average of the pixel values in the $3 \times 3 \times 2$ window:

$$\hat{x}_i(r) = \frac{\sum_{r'} \sum_{t'=t-1} Q(r',t',r,t) x_{r,t}(r')}{\sum_{r'} \sum_{t'=t-1} Q(r',t',r,t)} .$$

The absolute greyscale difference (gradient) between the two spatial-temporal pixel positions is computed as $\Delta(r',t',r,t) = |x_{r,t}(r') - x_{r,t}(r)|$; the function indicating the local detail amount is presented by $\delta(r,t) = \left\{ \sum_{r'} \left[ x_{r,t}(r') - \overline{x}_{r,t}(r) \right]^2 \right\}^{1/2}$; and the motion indicator $m(r,t) = |\overline{x}_{r,t} - \overline{x}_{r,t-1}|$ is measured as the absolute difference between the average grey values in the windows for the current and previous frames. In the MCA filter, the pixels are classified into four discrete index classes, depending on the $\Delta(r',t',r,t)$ value. When details are detected in a region, higher weights are assigned to pixels that are similar to the pixel being filtered (pixels from the lower index classes, which have smallest $\Delta(r',t',r,t)$ values), preserving these details. In homogeneous regions, the difference in weight compared to pixels from the higher index classes will be smaller, and strong filtering is to be performed. Exponential model for averaging function $Q(r',t',r,t)$, which depends on the amount of detail, motion, and class index inversely proportional, has been used.

Fuzzy motion and detail adaptive video filter, FMDAF employs the idea of MCA framework and the values $\Delta(r',t',r,t)$, $\delta(r,t)$ and $m(r,t)$. The model of exponential functions is changed by fuzzy logic framework with linguistic variables, introducing one fuzzy set Large Difference for the values $\Delta(r',t',r,t)$ . If a difference $\Delta(r',t',r,t)$ has a membership degree one in the fuzzy set Large Difference, then this means that this difference is large for sure. A membership degree equal to zero exposes the certainty that the difference is not large.

A linguistic variable “Large” that has been proposed for the difference $\Delta(r',t',r,t)$, is also introduced for the motion value $m(r,t)$ and for the detail value $\delta(r,t)$ defining the fuzzy sets Large Motion and Large Detail. A linguistic variable “Reliable” to indicate whether a given neighbourhood pixel is reliable to be used in the filtering of the central window pixel, and is represented by the fuzzy set “Reliable Neighbourhood Pixel”. Finally, the weight $Q(r',t',r,t)$ for the pixel at position $(r',t')$ is now defined as the degree, to which it is reliable to be used
in the filtering of the central window pixel, i.e., its membership degree in the fuzzy set “Reliable Neighbourhood Pixel”. The presented fuzzy rule 1 or 2 are depended on whether current \( t' = t \) or previous \( t' = t - 1 \) frames positions.

**Fuzzy rule 1.** Assigning the membership degree in the fuzzy set “Reliable Neighbourhood Pixel” of the pixel at spatial position \( r' \) in the current frame \( (t' = t) \) of the window with central pixel position \( (r, t) \):

IF [the detail value \( \delta(r,t) \) is large AND the difference \( \Delta(r',t',r,t) \) is not large] OR [the detail value \( \delta(r,t) \) is not large] THEN the pixel at position \( (r',t') \) is a Reliable Neighbourhood Pixel for the filtering of the central window pixel.

**Fuzzy rule 2.** Assigning the membership degree in the fuzzy set “Reliable Neighbourhood Pixel” of the pixel at spatial position \( r' \) in the previous frame \( (t' = t - 1) \) of the window with central pixel position \( (r, t) \):

IF [the detail value \( \delta(r,t) \) is large AND the difference \( \Delta(r',t',r,t) \) is not large] OR [the detail value \( \delta(r,t) \) is not large] AND the motion value \( m(r,t) \) is not large THEN the pixel at position \( (r',t') \) is a Reliable Neighbourhood Pixel for the filtering of the central window pixel.

Finally the described framework FMDAF adapts better to motion than the RMCA method as results reported in paper (Melange et al., 2008) indicates.

### 4. Fuzzy-Angular Deviation Frameworks in Denoising of Video Sequences

#### 4.1. Additive Noise Suppression

##### 4.1.1. 2D Spatial Noise Filtering

The filtering procedure includes the Histogram Calculation, Noise Estimation, and Spatial Algorithm Operations. A mean value \( \bar{x}_\beta \) (\( \beta = \{\text{Red, Green, Blue}\} \) in a colour image) is found in a sliding 3x3 processing window; later, the angle between two vectors deviation is computed agree to (Ponomaryov et al., 2007), mean value \( \bar{X} = \{\bar{x}_R, \bar{x}_G, \bar{x}_B\} \), and central pixel \( Y = \{x_{cR}, x_{cG}, x_{cB}\} \) is calculated. Finally, the probabilities: \( p_j \), the mean value \( \mu \), the variance \( \sigma^2_\beta \), and standard deviation (SD) \( \sigma_\beta = \sqrt{\sigma^2_\beta} \) should be calculated. Two processing windows: large 5x5, and into it, small 3x3 one, are employed in this scheme.

Let denote as \( \theta_i = A(x_i, x_c) \) the angle deviation \( x_i \) in respect to \( x_c \) where \( i = 0,1,...,8, i \neq c, c = \text{central pixel} \). The Spatial Algorithm is employed realizing the following **IF-THEN** rule for filtering the first frame only:

**IF** (\( \theta_1 \text{ AND } \theta_3 \text{ AND } \theta_4 \text{ AND } \theta_6 \geq \tau_1 \)) **OR** (\( \theta_0 \text{ AND } \theta_2 \text{ AND } \theta_5 \text{ AND } \theta_7 \geq \tau_1 \)) **THEN** Mean Weighted Filtering **ELSE** Spatial Filtering Algorithm. The “AND” operation is defined as “Logical AND”, the “OR” operation is “Logical OR”. The **Mean Weighted Filtering Algorithm** is realized using angle deviations as weight criteria (Ponomaryov et al., 2007).

If the spatial algorithm is selected, the processing is realized in each a colour plane using locally adapted SD \( \sigma_\beta \) around of mean value \( \bar{x}_{R5x5} \) found in sliding 5x5 processing window, adjusting it as follows: If \( \sigma_\beta < \sigma'_\beta \) then \( \sigma_\beta = \sigma'_\beta \) otherwise \( \sigma'_\beta = \sigma_\beta \).
Let introduce for a central pixel $x_i = x(i, j)$ of a current sample the following neighbours in eight cardinal directions: $N, E, S, W, NW, NE, SE, SW$ (Schulte et al., 2007b), and also similarity measures for each a given plane ($\beta = \{\text{Red, Green, Blue}\}$):

$$\nabla_{(i,j)\beta}(i,j) = \left| \nabla_{\beta}(i+k,j+l) - \nabla_{\beta}(i,j) \right|, k, l \in \{-1, 0, 1\}.$$  \hfill (13)

These gradients are called “main gradient values”, and the point $(i, j)$ is “the centre of the gradient values”. Two “derived gradient values” are proposed, permitting to avoid blur in presence of the edges (Schulte et al., 2007b). Finally, these three gradient values are connected into one value called “fuzzy vectorial-gradient value” under IF-THEN rule: IF $\nabla_{\gamma\beta} < T_{\beta}, T_{\beta} = 2 \cdot \sigma_{\beta}, \text{THEN}$ it is calculated the angle deviation in each $\gamma$ direction from eight mentioned for main and derived vectorial values involved.

Let define the membership function to obtain “Fuzzy Main and Derived Vectorial-Gradient Values“:

$$\mu_{\text{BIG}} = \begin{cases} \max\{x, y\}, & \text{if } \nabla_{\gamma\beta} < T_{\beta} \\ 0, & \text{otherwise} \end{cases},$$

where $x = \alpha_{(M, D1, D2)\beta}, y = 1 - \left[ \frac{\nabla_{\gamma(M, D1, D2)\beta}}{T_{\beta}} \right], \alpha_{\beta} = 2 / [1 + \exp(\theta_{\beta})], M = \text{Main value}, D1 = \text{Derived1}, D2 = \text{Derived2},$  \hfill (14)

and $\theta_{\beta}$ is the angle deviation between vector pixels $[255, 255, x_{\gamma\beta}]$ and $[255, 255, x'_{\gamma\beta}]$ for each a colour channel. Finally, the process to obtain “Fuzzy Vectorial-Gradient Values” is defined as the Fuzzy Rule 1_2D_G:

**Fuzzy Rule 1_2D_G**: Fuzzy Vectorial-Gradient value is defined as $\nabla_{\gamma\beta} \alpha_{\gamma\beta}$, in such a way

IF ($\nabla_{\gamma\beta}^M \text{ is BIG AND } \nabla_{\gamma\beta}^{D1} \text{ is BIG}$) OR ($\nabla_{\gamma\beta}^M \text{ is BIG AND } \nabla_{\gamma\beta}^{D2} \text{ is BIG}$) THEN $\nabla_{\gamma\beta} \alpha_{\gamma\beta}$ is true.

Final step in filtering a noise is realized employing a Weighted Mean procedure with found weights:

$$y_{\beta\text{out}} = \sum_{\gamma} \omega_{\gamma} x_{\gamma\beta} / \sum_{\gamma} \omega_{\gamma}, y_{\beta\text{out}} = \sum_{\gamma} \omega_{\gamma} x_{\gamma\beta} / \sum_{\gamma} \omega_{\gamma}, \omega_{\gamma} = \nabla_{\gamma\beta} \alpha_{\gamma\beta}.$$  \hfill (15)

### 4.1.2. 3D Spatio-Temporal Noise Filtering

The “Temporal Algorithm” is designed realizing the motion detection in past and present frames of a video sequence for better preservation of the image characteristics.

The angle deviations and gradient values related to a central pixel in the present frame respect to its neighbours from past frame are found according to the first expression in the following equation:
\[
\left( \theta_i^1 = D(x_i^{t-1}, x_i^t), \nabla_i^1 = |x_i^{t-1} - x_i^t| \right), \left( \theta_i^2 = D(x_i^{t-1}, x_i^t), \nabla_i^2 = |x_i^{t-1} - x_i^t| \right), \\
\left( \theta_i^3 = D(x_i', x_i'), \nabla_i^3 = |x_i' - x_i'| \right)
\]

(16)

where \( i = 1, 2, \ldots, 8 \), \( x_{i,t} \) is a central pixel channel in the present frame, and \( t-1 \) and \( t \) mark the past and present frames, respectively. The angle and gradient values in both frames are calculated according to second equation in (16). Finally, the same parameters for the present frame are only employed, eliminating operations in past frame as in the third expression in eq. (16).

The Gaussian membership functions in the fuzzy sets SMALL and BIG for gradients and angular deviations are defined as:

\[
\mu_{\text{SMALL}}(\theta) = \begin{cases} 
1, \text{ if } \theta < \theta_1 \\
\exp[-(\theta - \theta_1)^2 / 2\sigma^2], \text{ otherwise}
\end{cases}, \quad \mu_{\text{SMALL}}(\nabla) = \begin{cases} 
1, \text{ if } \nabla < \nabla_1 \\
\exp[-(\nabla - \nabla_1)^2 / 2\sigma^2], \text{ otherwise}
\end{cases}
\]

(17a)

\[
\mu_{\text{BIG}}(\theta) = \begin{cases} 
1, \text{ if } \theta < \theta_2 \\
\exp[-(\theta - \theta_2)^2 / 2\sigma^2], \text{ otherwise}
\end{cases}, \quad \mu_{\text{BIG}}(\nabla) = \begin{cases} 
1, \text{ if } \nabla < \nabla_2 \\
\exp[-(\nabla - \nabla_2)^2 / 2\sigma^2], \text{ otherwise}
\end{cases}
\]

(17b)

where \( \theta_1 = 0.2, \theta_2 = 0.9, \nabla_1 = 60, \nabla_2 = 140 \), and \( \sigma^2 = 0.1 \) for \( \theta \) and \( \sigma^2 = 1000 \) for \( \nabla \), and the numerical values of parameters are chosen according to the optimum values of the PSNR and MAE criteria.

The designed fuzzy rules (see Fig.1) are used to detect the movement presence and/or noise analyzing pixel by pixel, and to form a sample of pixels with similar structures for the subsequent filtration. The fuzzy rules were designed to detect changes in magnitude and angle deviations between central and neighbouring pixels in \( t \) and \( t-1 \) frames. Procedure for fuzzy rules is as follows:

**Fuzzy Rule 2_3D_G:** Definition of the Fuzzy Vectorial-Gradient value \( SBB_{\beta_i} \): IF \( \theta_1^i \) is SMALL AND \( \theta_2^i \) is BIG AND \( \theta_3^i \) is BIG AND \( \nabla_1 \) is SMALL AND \( \nabla_2 \) is BIG AND \( \nabla_3 \) is BIG THEN \( SBB \) is true (Fig. 1 b)).

**Fuzzy Rule 3_3D_G:** Definition of the fuzzy Vectorial-Gradient value \( SSS_{\beta_i} \): IF \( \theta_1^i \) is SMALL AND \( \theta_2^i \) is SMALL AND \( \theta_3^i \) is SMALL AND \( \nabla_1 \) is SMALL AND \( \nabla_2 \) is SMALL AND \( \nabla_3 \) is SMALL THEN \( SSS \) is true (Fig. 1 c)).

**Fuzzy Rule 4_3D_G:** Definition of the fuzzy Vectorial-Gradient value \( BBB_{\beta_i} \): IF \( \theta_1^i \) is BIG AND \( \theta_2^i \) is BIG AND \( \theta_3^i \) is BIG AND \( \nabla_1 \) is BIG AND \( \nabla_2 \) is BIG AND \( \nabla_3 \) is BIG THEN \( BBB \) is true (Fig. 1 d)).
For the reconstruction of edges and fine details in the image, we use the following processing procedure: a) calculate the SD ($\sigma''$) in the double 3×3×2 window of the neighboring images, and b) compare the current SD with previous using the following rule: IF $\left(\sigma''_{\text{RED}} \geq 0.4 \times \sigma'_{\text{RED}} \right)$ AND $\left(\sigma''_{\text{GREEN}} \geq 0.4 \times \sigma'_{\text{GREEN}} \right)$ AND $\left(\sigma''_{\text{BLUE}} \geq 0.4 \times \sigma'_{\text{BLUE}} \right)$, THEN fuzzy rules 2, 3, 4, and 5; OTHERWISE, weighted mean filter. The latter filter is applied using 17 pixels from the 3×3×2 window. Using this procedure, it is possible to select the areas containing fine details and contours and subsequently filter the pixels from this area according to the fuzzy logic algorithms. The SD values are updated using the following sensitivity parameter $\alpha$: $\sigma' = \alpha(\sigma_{\text{total}} / 5) + (1 - \alpha)\sigma'_i$, $\sigma_{\text{total}} = (\sigma''_{\text{RED}} + \sigma''_{\text{GREEN}} + \sigma''_{\text{BLUE}}) / 3$.

This parameter is chosen as follows: $\alpha = 0.125$ for the weighted mean filter and the fuzzy rule SSS, $\alpha = 0.875$ for SSB and BBS, and $\alpha = 0.875$ in the case of BBS if the motion-noise confidence value is $(\text{motion\_noise}) = 1$; $\alpha = 0.125$ if $(\text{motion\_noise}) = 0$, and $\alpha = 0.5$ in other cases or if the fuzzy rule is not applied.

If number of pixels with fuzzy value SBB, or SSS, or BBS, or BBB is the biggest one against those that present other IF-THEN conditions, it should be employed the next filtering algorithm only for such the pixels that satisfy the established IF condition:

$$y_{\text{out}} = \frac{\text{# pixels}}{\text{# pixels}} \cdot SBB_{\text{pi}} \left/ \sum_{i=1}^{\text{# pixels}} SBB_{\text{pi}} \right.$$ or

$$y_{\text{out}} = \frac{\text{# pixels}}{\text{# pixels}} \cdot 0.5(x'_{\text{pi}} + x''_{\text{pi}}) \cdot SSS_{\text{pi}} \left/ \sum_{i=1}^{\text{# pixels}} SBB_{\text{pi}} \right.$$ or

$$y_{\text{out}} = \frac{\text{# pixels}}{\text{# pixels}} \cdot (1 - BBS_{\text{pi}}) \left/ \sum_{i=1}^{\text{# pixels}} (1 - BBS_{\text{pi}}) \right.$$ or

or, if the number of pixels with $BBB_{\text{pi}}$ value is the biggest one. Here the filtering results $y_{\text{out}} = (1 - \alpha) x'_{\text{pi}} + \alpha x''_{\text{pi}}$. In eq. (18), #pixels are the number of pixels that satisfy to mentioned IF-THEN condition; $x'_{\text{pi}}$, $x''_{\text{pi}}$ represent each a pixel in the past and present.
frames that satisfy the mentioned IF-THEN condition; \( y_{\text{final}} \) is the output in spatial temporal filtering. If there is no majority in pixels for any Fuzzy Rule, only the mean of central pixels from present and past frames are used.

During numerous simulations, different video colour sequences Miss America (MA), Flowers (F) and Chair (C) in RGB colour space (24 bits) and QCIF format (176x144 pixels in a frame) are used to qualify effectiveness of the proposed approach in suppression of a noise and compare it with known techniques. Mentioned video sequences present different texture characteristics; permitting a better understanding of the robustness of the proposed and existed filtering schemes. Video sequences were contaminated by Gaussian noise of different intensity from 0.0 to 0.05 in their SDs. The filtered frames were evaluated according to PSNR, MAE, NCD objective criteria, and also in subjective manner.

The proposed Fuzzy Directional Adaptive Recursive Temporal Filtering for Gaussian noise named as FDARTF.G was compared with another similar one, the FMRSTF, and with the FVMRSF (Fuzzy Vectorial Motion Recursive Spatial-Temporal Filtering Using Angles) that is the modification of FMRSTF, which combines the gradients and angles in processing.

Other two reference filters were: VGVDF.G, adapted to process three frames, and the VVDKNNVMF filter presenting good efficiency in comparison with other filtering procedures. The data presented in Table 1 show that the proposed algorithm effectively suppresses the low-intensity additive noise and is the best according to the majority of filtration criteria for the video sequences Flowers and Miss America. Fig. 2 presents filtering results for sequence Miss America through 100 frames, where the better noise suppression in form of PSNR measure can be observed for novel filtering scheme.

| Criteria | Flowers Frame 20, Gaussian noise SD = 0.005 | Miss America Frame 20, Gaussian noise SD = 0.005 |
|----------|---------------------------------------------|--------------------------------------------------|
|          | FMRSTF F | FVMRS TF | FDARTF.G | VVDKN VNF | VGVDF | FMRSTF F | FVMRS TF | FDARTF.G | VVDKN VNF | VGVDF |
| PSNR     | 26,192   | 26,01    | 27,31    | 25,36    | 25,46  | 29,93    | 29,91    | 32,51    | 29,80    | 30,66  |
| MAE      | 9,638    | 9,83     | 8,50     | 8,78     | 8,96   | 5,82     | 5,83     | 4,46     | 6,18     | 5,55   |
| NCD      | 0,016    | 0,017    | 0,015    | 0,015    | 0,017  | 0,02     | 0,02     | 0,016    | 0,021    | 0,02   |
|          | SD = 0.01 |          |          |          |        |          |          |          |          |        |
| PSNR     | 24,36    | 24,34    | 25,72    | 24,63    | 24,72  | 27,68    | 27,68    | 30,06    | 27,61    | 28,66  |
| MAE      | 11,93    | 11,97    | 10,44    | 9,92     | 10,15  | 7,48     | 7,5      | 6,07     | 8,14     | 7,21   |
| NCD      | 0,0206   | 0,0208   | 0,0187   | 0,0193   |        | 0,026    | 0,026    | 0,021    | 0,028    | 0,026  |

Table 1. Simulation results for proposed framework and comparative filters.

4.2. Impulsive Noise Suppression

4.2.1. 2D Noise Filtering

Similar as in additive noise suppression idea is realized in framework used in impulsive noise suppression. It is based on the fuzzy-set theory and directional characteristics of an image, angular deviations of the image pixels in neighbouring multichannel video frames when the final filtered image frames are formed. At the first stage the spatial filtration of the initial frame of a sequence is performed. The following time stage realizes the combined processing of current neighbouring frames of the sequence. This processing uses the fuzzy set theory, which makes it possible to improve noise suppression. At the final stage, the
spatial filtration mechanism in each current frame is employed again. Let consider gradients and angular deviations of pixels in order to estimate the similarity between pixels within sliding processing window in verification if the central pixel is distorted by noise or free of noise. For each of directions \( \gamma = \{N, E, S, W, NW, NE, SE, SW\} \) with respect to the central pixel \( x^\beta_c \), we introduce the gradient \( \nabla^\beta_{(k,l)} x(i, j) = |x^\beta_c(i, j) - x^\beta(i+k, j+l)| \) where \((i, j) = (0,0)\) within the processing window, with the index \( \beta \) determining the image components (red (R), green (G), and blue (B)), \((k, l) \in \{-1, 0, 1\}\). We also introduce the basic gradient and four related gradients calculated with respect to the former one, the index values being \((k, l) \in \{-2, -1, 0, 1, 2\}\) for each direction \( \gamma \) (see Fig. 3). Fig. 3 shows pixels in processing procedure for SE direction for the basic and four related components.

![PSNR Miss America](image)

Fig. 2. PSNR criterion values for proposed and reference filters for 100 frames of colour video sequence Miss America contaminated by Gaussian noise with SD=0.015.

![Basic and related directions](image)

Fig. 3. Basic and related directions for gradients and angle variance values.

Let determine the angular deviation \( \theta^\beta_\gamma \) between the multichannel vectors \( x_1 = (x^R_1, x^G_1, x^B_1) \) and \( x_2 = (x^R_2, x^G_2, x^B_2) \) for each a colour component along the direction \( \gamma \) (for the SE direction, in this case) agree to procedure given in (Ponomaryov et al., 2007). Also, let define the basic gradients \( \nabla^\beta_{(1,1)} x(i, j) = \nabla^\beta_{SE(i,j)}, \theta^\beta_{\gamma=SE(i,j)} \), the related gradients, and the angular deviations:
\[ F_{(0,2)}^\beta x(i-1, j+1) = F_{SE_{(\gamma)}}^\beta, \quad F_{(2,0)}^\beta x(i+1, j-1) = F_{SE_{(\gamma)}}^\beta, \quad F_{(0,0)}^\beta x(i-1, j+1) = F_{SE_{(\gamma)}}^\beta, \]

\[ F_{(2,0)}^\beta x(i+1, j-1) = F_{SE_{(\gamma)}}^\beta, \]

where the operator \( F \) determines gradient \( \nabla \) or angular deviation \( \theta \). Analogously, we find the gradients and the angular deviations for the basic value and four related values of other directions \( \gamma \).

We now introduce two fuzzy sets SMALL \( (S) \) and BIG \( (B) \). Then, we use the Gaussian membership functions for both gradients and angular deviations in these sets:

\[
\mu(F_{(\gamma)}^\beta \text{ SMALL}) = \begin{cases} 
1, & F_{(\gamma)}^\beta < \text{med}_F \\
\exp\left\{ -\left[ (F_{(\gamma)}^\beta - \text{med}_F)^2 / 2\sigma^2 \right] \right\}, & \text{other case} 
\end{cases} \tag{20a}
\]

\[
\mu(F_{(\gamma)}^\beta \text{ BIG}) = \begin{cases} 
1, & F_{(\gamma)}^\beta > \text{med}_F \\
\exp\left\{ -\left[ (F_{(\gamma)}^\beta - \text{med}_F)^2 / 2\sigma^2 \right] \right\}, & \text{other case} 
\end{cases} \tag{20b}
\]

Here, \( \sigma_1=1000 \), \( \text{med}_1=60 \) and \( \text{med}_2=10 \) for the fuzzy gradients \( (F_{(\gamma)}^\beta = \nabla_{(\gamma)}^\beta) \) in the BIG and SMALL fuzzy sets, \( \sigma_2=0.8 \), \( \text{med}_3=0.615 \) and \( \text{med}_4=0.1 \) for the fuzzy angular deviations \( (F_{(\gamma)}^\beta = \theta_{(\gamma)}^\beta) \) in the BIG and SMALL fuzzy sets. The novel fuzzy rules developed are based on both gradients and angular deviations. They are applied to determine whether the central pixel is a noise, or a no-noise pixel, or a local movement.

**The 2D fuzzy rule 1_2D** determines the value of the fuzzy gradient-angular measure \( \nabla_{(\gamma)}^\beta \theta_{(\gamma)}^\beta \):

IF \( (\nabla_{(\gamma)}^\beta \text{ is } B \otimes \nabla_{(\gamma)}^\beta \text{ is } S \otimes \nabla_{(\gamma)}^\beta \text{ is } B \otimes \nabla_{(\gamma)}^\beta \text{ is } S \otimes \theta_{(\gamma)}^\beta \text{ is } B \otimes \theta_{(\gamma)}^\beta \text{ is } B \text{ THEN } \nabla_{(\gamma)}^\beta \theta_{(\gamma)}^\beta \text{ is BIG} \), where \( A \otimes B = A \text{ AND } B \), \( A \otimes B = \min \left( A, B \right) \).

Combining eight fuzzy gradient-angular measures for each of the directions, we introduce the noise factor \( r^\beta \).

The 2D fuzzy rule 2_2D: IF \( \nabla_{(\gamma)}^\beta \theta_{(\gamma)}^\beta \text{ is } B \oplus \nabla_{(\gamma)}^\beta \theta_{(\gamma)}^\beta \text{ is } B \oplus \nabla_{(\gamma)}^\beta \theta_{(\gamma)}^\beta \text{ is } B \oplus \nabla_{(\gamma)}^\beta \theta_{(\gamma)}^\beta \text{ is } B \oplus \nabla_{(\gamma)}^\beta \theta_{(\gamma)}^\beta \text{ is } B \oplus \nabla_{(\gamma)}^\beta \theta_{(\gamma)}^\beta \text{ is } B \oplus \nabla_{(\gamma)}^\beta \theta_{(\gamma)}^\beta \text{ is } B \text{ THEN } r^\beta \text{ is BIG} \), where \( A \oplus B = \max \left( A, B \right) \).

Depending on whether a pixel is the noisy or is noise-free, we use the following filtration algorithm:
IF \( r^\beta \geq 0.3 \), fuzzy logic algorithm, otherwise \( y_{\text{output}}^\beta = x_C^\beta \) \hspace{1cm} (21)

Fuzzy pixel weights for the algorithm are given in the form \( \rho \left( \nabla_x^\beta \theta_x^\beta \right) = 1 - \nabla_x^\beta \theta_x^\beta \), which determines the value of the membership function for the fuzzy set NO BIG (noise free). At the same time, the weights for the central pixel are chosen as \( \xi_c^\beta = M \sqrt{1 - r^\beta} \). The spatial filtration algorithm based on the fuzzy logic includes the following operations:

1) Calculation of fuzzy weights on the basis of the ordering of pixels in the 3×3 window: \( x_\gamma^\beta = \{ x_{SW}^\beta, \ldots, x_{(i,j)}^\beta, \ldots, x_{NE}^\beta \} \), where the ordering statistics \( x_\gamma^\beta(1) \leq x_\gamma^\beta(2) \leq \cdots \leq x_\gamma^\beta(9) \) are determined from the inequality \( \rho \left( \nabla_x^\beta \theta_x^\beta \right)^{(1)} \leq \rho \left( \nabla_x^\beta \theta_x^\beta \right)^{(2)} \leq \cdots \leq \rho \left( \nabla_x^\beta \theta_x^\beta \right)^{(9)} \).

2) Determination of the quantities \( \text{sum}^\beta = \rho \left( \nabla_x^\beta \theta_x^\beta \right) \) for \( j = 9, 8, \ldots, 1 \), by decreasing \( j \) from 9 until \( \text{sum}^\beta \geq \rho_0 \), \( \rho_0 = \left( \sum_{j=1}^{9} \rho \left( \nabla_x^\beta \theta_x^\beta \right) + M \sqrt{1 - r^\beta} \right) / 2 \), \( M = 3 \). In this case, the pixel ordering number \( j \) satisfying this condition determines the \( j \)-th pixel chosen as a result of the filtration \( x_\gamma^\beta(j) = y_{\text{output}}^\beta \).

3) If \( j \leq 2 \), then the fuzzy weights are calculated with the allowance for the threshold \( \rho_t = \left( \rho_0 - \rho \left( \nabla_x^\beta \theta_x^\beta \right)^{(1)} - \rho \left( \nabla_x^\beta \theta_x^\beta \right)^{(2)} \right) / 2 \) for the parameter \( \text{sum}^\beta = \rho \left( \nabla_x^\beta \theta_x^\beta(j) \right) \), with \( j = 9, 8, \ldots, 1 \) decreasing until \( \text{sum}^\beta \geq \rho_t \). The ordering number \( j \) satisfying this condition determines the \( j \)-th pixel chosen as a result of the filtration \( x_\gamma^\beta(j) = y_{\text{output}}^\beta \).

### 4.2.2. 3D Impulsive Noise Filtering

The three-dimensional (3D) algorithm (3D-FCF) realized at the second time stage in processing video sequences is determined by filtration of neighbouring frames. This makes it possible to estimate the degree of movement and the noise level in the central pixel as a result of the application of the 5×5×2 sliding window that contains two neighbouring frames. We calculate difference values for both the gradients and the angular deviations between the \( (t - I) \)-th and the \( t \)-th frames.

Using the algorithm developed in the Section 4.2.1., we can derive the 3D methodology; detailed development of this algorithm is described in (Ponomaryov et al., 2009).

Using the pixels in both frames of the sequence we can compute the motion estimation and noise level present in the central pixel. In this way it is possible to filtering the noisy pixel or not filtering it because of no noise and no movement present in the central sample. Membership values are computed in same way; defining fuzzy sets SMALL (S) and BIG (B). This means that we deal again with the no-movement situation or no-noise situation in the
pixel sample that is subjected to processing. Gaussian functions are also used for the membership function. Again, they determine the fuzzy gradient-angular difference values. The fuzzy rules developed by this filter, are based on the difference values of both gradients and angular deviations. These rules are applied with the goal to determine whether the central pixel is a noise or it is no-noise. Otherwise, we deal with local movement.

There are four fuzzy rules designed to determine if the central pixel is in movement, is noisy or lacks both. The first 3D fuzzy rule designed determines the value of the first fuzzy gradient-angular difference; it characterizes the confidence level for a movement-noise event as applied to the central pixel when the values of fuzzy gradients and angular differences along direction $\gamma$ are analyzed. The second 3D fuzzy rule characterizes the confidence level with respect to the no movement-no noise event as applied to the central pixel along direction $\gamma$. In this case, the regions are classified as homogeneous ones, edges, and fine feature regions. The third 3D fuzzy rule allows us to estimate the existence and the level of movement or noise in the central pixel on the basis of fuzzy gradient-angular values for all directions. Finally the fourth 3D fuzzy rule determines the time stage of the video sequence filtration, in which the $j$-th pixel should be chosen as the final result. This is true if the pixel satisfies the conditions that provide the reconstruction of fine details and contours when fuzzy ordering statistics are used. In this case, the pixel nearest to the central one among all neighbouring pixels in the $t$-th and $(t-1)$-th frames of the video sequence is chosen.

The temporal stage of the filtration consists in selecting two pixels agree to 3D fuzzy rules designed. These two pixels are averaged to provide the filtering temporal result.

The characteristics of the 3D-FCF filter proposed and algorithms well known in the literature were studied with the use of standard criteria. We compared the PSNR expressed in decibels, the MAE, and the NCD. The video sequences Miss America (MA) and Flowers (F) in the QCIF format (176×144 pixels) were employed. The video sequences were distorted by impulsive noise of a different intensity and processed by various filters. The distortions in each image channel were independent of each other. Table 2 shows the test results for different standard filters. The table content confirms that the 3D-FCF algorithm developed by us is the best for estimates made by the MAE criterion averaged over 100 frames of the $F$ sequence within a wide noise intensity range. Thereby the problem of the efficient reconstruction of the edges and fine image features is successfully solved. At the same time, the values of the PSNR criterion show the superiority of the new algorithm compared to the others for intermediate intensity noise. In accordance with both the PSNR and MAE criteria, the new framework is the best in the case of the MA video sequence filtration for noise of low and intermediate intensities less than 20%. Table 3 shows the values of the NCD criterion for MA and F video sequences, which characterizes the chromatic properties of the filters. Here, the new 3D-FCF algorithm again demonstrates the best quality within a wide range of noise intensity. The efficient filtration of the F sequence that contains a texture varying from frame to frame, as well as noticeable variations of colours, confirms the robustness of the method proposed. Subjective perception by human viewer can be observed in Fig. 4 showing better performance of the designed 3D framework in comparison with known methods in MA frame, where novel algorithm preserves better the edges, fine features, and chromaticity properties against other filters.

Real-Time analysis was realized on the DSP (TMS320DM642, Texas Instruments) and is based on Reference Framework defined as RF5 (Mullanix & Magdic et al., 2003, Gallegos-Funes, et al., 2009). Table 4 presents the processing times in some 2D and 3D algorithms,
which have been implemented on DSP, demonstrating reliability of the proposed approach against better algorithms found in literature.

### Table 3. Averaged values of criteria MAE for video sequences F and MA.

| Filter |
|-------|
| 3D-FCF | 3D-MF | 3D-VVMF | 3D-VVDKNV MF | 3D-VGVDF | 3D-VAVDATM | 3D-VKNNF |
| ---- |
| MA | PSN R | MA | PSN R | MA | PSN R | MA | PSN R | MA | PSN R | MA | PSN R |
| 5 |
| 2.13 | 29.52 | 6.65 | 26.83 | 6.64 | 26.78 | 7.45 | 25.77 | 7.44 | 25.56 | 5.45 | 27.25 |
| 15 |
| 3.38 | 27.76 | 7.19 | 26.22 | 7.14 | 26.20 | 8.20 | 25.11 | 7.72 | 25.29 | 6.28 | 26.57 |
| 30 |
| 6.08 | 25.04 | 8.59 | 24.77 | 8.50 | 24.69 | 10.03 | 23.17 | 9.74 | 23.24 | 8.13 | 24.99 |

Table 2. Averaged values of criteria MAE and PSNR for video sequences F and MA.

### Table 4. Time processing for 20 frames of video sequence “Miss America” on DSP.

| Filter |
|-------|
| 3D-FCF | 3D-MF | 3D-VVMF | 3D-VVDKNV MF | 3D-VGVDF | 3D-VAVDATM | 3D-VKNNF |
| ---- |
| MA | PSN R | MA | PSN R | MA | PSN R | MA | PSN R | MA | PSN R |
| 5 |
| 0.37 | 39.59 | 2.51 | 35.12 | 2.54 | 34.86 | 3.11 | 33.48 | 2.91 | 33.76 |
| 15 |
| 1.18 | 34.33 | 2.70 | 34.37 | 2.71 | 34.18 | 3.43 | 32.38 | 2.85 | 33.71 | 1.71 |
| 30 |
| 3.58 | 28.26 | 3.35 | 31.95 | 3.31 | 31.82 | 4.39 | 28.48 | 3.28 | 31.61 |

Table 3. Averaged values of NCD for video sequence F and MA.

### Fig. 4. a) Zoomed image region of 10th Miss America frame contaminated by impulsive noise of 15% intensity, b) Designed 3D-FCF, c) 3D-MF, d) 3D-VVMF, e) 3D-VGVDF, f) 3D-VAVDATM; g) 3D-VATM; h) 3D-VKNNF.

### Table 5. Time processing for 20 frames of video sequence “Miss America” on DSP.

| Filters |
|---------|
| Processing time in seconds |
| Maximum | Average | Total |
| 3D-FCF | 7.533 | 7.440 | 148.806 |
| 3D-VVMF | 0.075 | 0.075 | 1.496 |
| 3D-VGVDF | 28.52 | 25.6 | 512.02 |
| 3D-VAVDATM | 25.551 | 24.867 | 497.356 |
5. Conclusions

Several promising frameworks in suppression of noise of different nature in video sequences are presented in this chapter. It has been designed novel approach that employs the 3D fuzzy-vector order statistics frameworks based on the fuzzy-set theory and the directional angular information available as a result of processing multichannel still images and neighbouring frames in the video sequences contaminated by additive or impulsive noise. The designed fuzzy rules characterize the presence of motion and noise in processing area of the pixels in two neighbouring frames. Novel approach has appeared to demonstrate the essential improvement of the processing quality compared to all known filters. The method developed was successful in the suppression of a noise, as well as in the reconstruction of edges and fine details of the images. The excellent performance of the new filtering scheme has been tested during numerous simulations in terms commonly used objective criteria PSNR, MAE, NCD, and MRCE, as well as the subjective visual perception presented in form of the visual analysis by human visual system of filtered video sequences. The approach also turned out to be extremely efficient in the reproduction of chromatic characteristics of frame in video sequences. Real-Time analysis of several promising 2D and 3D algorithms was realized on the DSP presenting available processing performance.

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