BROKEN SYMMETRY AND JOSEPHSON-LIKE TUNNELING IN QUANTUM HALL BILAYERS

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I review recent novel experimental and theoretical advances in the physics of quantum Hall effect bilayers. Of particular interest is a broken symmetry state which optimizes correlations by putting the electrons into a coherent superposition of the two different layers.

The various quantum Hall effects are among the most remarkable many-body phenomena discovered in the second half of the twentieth century. The fractional effect has yielded fractional charge, spin and statistics, as well as unprecedented order parameters. There are beautiful connections with a variety of different topological and conformal field theories of interest in nuclear and high energy physics.

The quantum Hall effect (QHE) takes place in a two-dimensional electron gas formed in a quantum well in a semiconductor host material and subjected to a very high magnetic field. In essence it is a result of a commensuration between the number of electrons, \( N \), and the number of flux quanta, \( N \Phi \), in the applied magnetic field. The electrons condense into distinct and highly non-trivial ground states ('vacua') formed at each rational fractional value of the filling factor \( \nu \equiv N/N\Phi \).

The essential feature of (most) of these exotic states is the existence of an excitation gap. The electron fluid is incompressible and flows rigidly past obstacles (impurities in the sample) with no dissipation. A weak external electric field will cause the fluid to move, but the excitation gap prevents the fluid from absorbing any energy from the electric field. Hence the current flow must be exactly at right angles to the field and the conductivity tensor takes the remarkable universal form

\[
\sigma^{xx} = \sigma^{yy} = 0; \quad \sigma^{xy} = -\sigma^{yx} = \nu e^2/h. \tag{1}
\]

Ironically, this ideal behavior occurs because of imperfections and disorder in the samples which localize topological defects (vortices) whose motion would otherwise dissipate energy. In a two-dimensional superconductor, such vortices undergo a confinement phase transition at the Kosterlitz-Thouless temperature and dissipation ceases. In most cases in the QHE, an analog of the Anderson-Higgs mechanism causes the vortices to be deconfined so that dissipation is strictly zero only at zero temperature. In practice, values of \( \sigma^{xx}/\sigma^{xy} \) as small as \( 10^{-13} \) are not difficult to obtain at dilution refrigerator temperatures.

Recent technological progress in molecular beam epitaxy techniques has led to the ability to produce pairs of closely spaced two-dimensional electron gases. Strong correlations between the electrons in different layers lead a great deal of completely new physics involving spontaneous interlayer phase coherence.
As we will discuss below, this is the first example of a QHE system with a finite-temperature phase transition. This transition is in fact a Kosterlitz-Thouless transition into a broken symmetry state which is closely analogous to that of a 2D superfluid. Recent remarkable tunneling experiments have observed something closely akin to the Josephson effect in superconducting tunnel junctions and have measured the dispersion of the superfluid Goldstone mode.

We begin with the simplest example of the integer QHE in a single layer system of spinless electrons at $\nu = 1$. The strong magnetic field quantizes the kinetic energy into discrete Landau levels separated in energy by the cyclotron energy $\hbar \omega_c \sim 100\text{K}$. Each level has a macroscopic degeneracy equal to $N_\Phi$. This degeneracy in the kinetic energy means that interactions are enormously important and have non-perturbative effects at fractional filling factors. However for $\nu = 1$, every state of the lowest Landau level (LLL) is occupied and, since there is a large kinetic energy gap to the next Landau level, interactions are (relatively) unimportant. It is this gap which makes the system incompressible. Since the lowest LLL is completely full, the state is a simple Slater determinant. In the Landau gauge this can be written in second quantized form $|\Psi\rangle = \prod_k c_k^\dagger |0\rangle$ where $k$ labels the set of single-particle states. In first-quantized form the state is most easily expressed in the symmetric gauge $\Psi(z_1, z_2, \ldots, z_N) = N \prod_{i<j} (z_i - z_j)e^{-\frac{1}{4} \sum_m |z_m|^2}$ where $z_j \equiv (x_j + iy_j)/\ell$ is a dimensionless complex number representing the 2D position vector of the $j$th particle in units of the magnetic length $\ell$. The vandermonde polynomial factor in this Laughlin state is totally antisymmetric and is equivalent to a single Slater determinant filling all the orbitals in the LLL.

So far we have been ignoring the spin of the electrons. Various solid state effects make the Zeeman splitting much smaller than the LL splitting and so the degeneracy of each LL is effectively doubled when we include spin. Thus interactions turn out to be much more important than we have been naively assuming. At $\nu = 1$ the Coulomb interaction makes the spins spontaneously align into a very simple maximally ferromagnetic state

$$\Psi(z_1, z_2, \ldots, z_N) = \prod_{i<j} (z_i - z_j)e^{-\frac{1}{4} \sum_m |z_m|^2} |\uparrow\uparrow\uparrow\uparrow\ldots\uparrow\rangle. \quad (2)$$

In this state the spin part of the wave function is fully symmetric and so the spatial part must be the same fully antisymmetric wave function considered above. This wave function vanishes whenever any two electrons approach each other and thus optimizes the Coulomb exchange energy. Because (unlike an ordinary ferromagnetic metal) there is no kinetic energy cost to aligning the spins, the polarization is 100%. In the subsequent discussion we will assume that the weak Zeeman splitting combined with strong Coulomb exchange has frozen out the spin degree of freedom. This simplifying assumption is not necessarily valid in real systems at low magnetic fields however.

We turn now to the case of a QHE bilayer at total filling factor $\nu = 1$, that is, filling 1/2 in each layer. In nuclear physics the strong interaction between nucleons is largely independent of whether they are neutrons or protons. In that case it proves useful to define an isospin variable in which the up and down states of this new spin-1/2 degree of freedom represent the proton and neutron. Similarly here, if
the layer spacing $d$ is small compared to the electron spacing (which is a few times
the magnetic length $\ell$), then the Coulomb interaction between the particles is nearly
independent of which layer they are in. If we define an isospin or pseudospin which
labels the layer index, then the interactions are nearly rotationally invariant in the
pseudospin space. For $d = 0$ there is an exact SU(2) invariance and the ground
state at $\nu = 1$ is identical to the ferromagnetic case discussed above for real spins

$$|\Psi\rangle = \prod_k c_{k\uparrow}^{\dagger}|0\rangle. \quad (3)$$

The only difference is that the up arrow now represents an electron being in the
upper layer. The system is a pseudospin ferromagnet for precisely the same reason
that it is a real spin ferromagnet—this state optimizes the Coulomb exchange energy
by making the spatial part of the wave function fully antisymmetric. This state has
total pseudospin $S = \frac{N}{2}$. The particular realization above has $S^z = \frac{N}{2}$, but there
are a total of $2S + 1$ degenerate states with all possible pseudospin orientations.

Consider now what happens for small but finite layer separation $d$. In this limit
the Coulomb interaction between electrons in the same layer is slightly stronger
than for electrons in different layers. Thus the interactions are no longer pseudospin
rotationally invariant. For small $d$ the main effect of this is not to change the wave
functions of the eigenstates described above but simply to lift their degeneracy.
This is accurately represented by an ‘easy plane’ anisotropy term $H_a$ in the energy
of the pseudospin ferromagnet

$$H_a = \frac{e^2}{2C} (S^z)^2. \quad (4)$$

Because $S^z = (N_\uparrow - N_\downarrow)/2$ represents the charge imbalance between the two layers,
this term is simply the charging energy of the capacitor $C$ whose plates are the two
electron gases. This weak anisotropy prefers for the pseudospin magnetization to
lie in the xy plane so that $\langle S^z \rangle = 0$. A single spin lying in the xy plane at an angle
$\varphi$ with respect to the $x$ axis is a linear combination of the two up and down basis
states:

$$|\rightarrow\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle + e^{i\varphi}|\downarrow\rangle). \quad (5)$$

Hence the fully ferromagnetic many-body state with the same orientation is given
by

$$|\Psi\rangle = \prod_k \frac{1}{\sqrt{2}} \left( c_{k\uparrow}^{\dagger} + e^{i\varphi} c_{k\downarrow}^{\dagger} \right) |0\rangle \quad (6)$$
or equivalently in first quantization

$$\Psi(z_1, z_2, \ldots, z_N) = \prod_{i<j}^{N} (z_i - z_j) e^{-\frac{1}{4} \sum_{m} |z_m|^2} \rightarrow \rightarrow \rightarrow \ldots \rightarrow. \quad (7)$$

This is a very strange state. Even though there may be no possibility of tunneling
between the two layers, quantum mechanics allows the existence of states in which
we are uncertain which layer each electron is in. This state has this property—it
exhibits spontaneous interlayer phase coherence. Each electron is in a coherent superposition of the upper and lower layers, characterized by the phase angle $\phi$.

This state represents a broken gauge symmetry much like that in a superconductor. A superconductor spontaneously breaks the gauge symmetry associated with total charge. The bilayer QHE system is incompressible and has definite total charge. However, it has fluctuations in the charge difference between the two layers due to the uncertainty over which layer each electron is in. Hence it breaks the gauge symmetry associated with conservation of the charge difference between the two layers. To understand this, consider the gauge transformation induced by the unitary operator $U_-(\theta) = e^{\frac{i}{2}(N_+ - N_-)}$. The effect of this transformation on the field operators is

$$U_+ \Psi^\dagger \Psi = e^{-i \frac{\theta}{2}} \Psi^\dagger \Psi; \quad U_- \Psi^\dagger \Psi = e^{+i \frac{\theta}{2}} \Psi^\dagger \Psi. \quad (8)$$

The Hamiltonian is invariant under this $U(1)$ transformation

$$U_+ \theta U_- = H \quad (9)$$

since

$$[H, (N_+ - N_-)] = 0, \quad (10)$$

in the absence of tunneling between the layers. Examination of Eq. (9) however shows that the phase coherent state is characterized by the non-trivial order parameter

$$\psi(\vec{r}) \equiv \langle \Psi^\dagger (\vec{r}) \Psi (\vec{r}) \rangle = \frac{n_0}{2} e^{i \phi} \quad (11)$$

where $\Psi^\dagger (\vec{r})$ creates a particle in layer $\sigma$ at position $\vec{r}$ and $n_0 = 1/(2\pi \ell^2)$ is the total density. This order parameter is not gauge invariant

$$\psi(\vec{r}) \rightarrow \langle U_+ \theta \Psi^\dagger (\vec{r}) \Psi (\vec{r}) U_- \theta \rangle = e^{i \theta} \psi(\vec{r}). \quad (12)$$

Thus the state has less symmetry than the Hamiltonian, and it spontaneously breaks the $U(1)$ symmetry associated with conservation of $N_+ - N_-$. In a superconductor, the pair field order parameter $\chi(\vec{r}) \equiv \langle \Psi^\dagger (\vec{r}) \Psi (\vec{r}) \rangle$ transforms non-trivially under the gauge transformation associated with conservation of total charge $U_+ \theta = e^{\frac{i}{2}(N_+ + N_-)}$. The bilayer order parameter is however invariant under this transformation. The order parameter is charge neutral—it corresponds to pairing of particles and holes rather than particles and particles. Because of the charge neutrality, the pairs can condense despite the presence of the strong magnetic field. In contrast, the order parameter of a superconductor would be filled with vortices by the magnetic field which thus discourages condensation.

Note that we do not have the situation of a particle in one particular layer bound to a hole in the other layer. Rather we have one particle in each spatial orbital but we are uncertain which layer it is in. That is, the particle is in one layer and the hole in the other, but we do not know which is which. We see from the first quantized form of the state in Eq. (7) that there is a zero of the wave function whenever any two particles approach each other. Whether they are in the same or different layers does not matter. Thus if the particle is in the upper layer, there is
guaranteed to be a correlation hole directly underneath it in the other layer and vice versa. This explains the good correlation energy for this state.

An ideal experimental probe of the novel properties of the interlayer phase coherent state is quantum tunneling of electrons through the barrier separating the two layers via the weak perturbation

$$H_T = -\frac{\Delta_{\text{SAS}}}{2} \int d^2 r \, \Psi_\uparrow(\vec{r}) \Psi_\downarrow(\vec{r}) + \text{h.c.},$$

(13)

where $\Delta_{\text{SAS}}$ is the symmetric-antisymmetric single particle tunnel splitting. Because this perturbation changes the charge difference between the layers, it does not commute with rest of the Hamiltonian and one would naively expect that it would produce an energy shift in the ground state only in second order perturbation theory. However we see from Eq. (11) that through the magic of spontaneously broken symmetry, the tunneling term actually has a finite expectation value in the ground state

$$\langle H_T \rangle = -n_0 \frac{\Delta_{\text{SAS}}}{2} \int d^2 r \cos \varphi(\vec{r}).$$

(14)

Thus the response to tunneling appears in first order, because of the uncertainty over which layer each electron is in. The finite expectation value of the order parameter tells us that we can transfer an electron from one layer to the other and still be in the exact same quantum state! If the system is in the same quantum state, the energy change is zero. Therefore the process conserves energy only if the bias voltage across the tunnel junction is zero. Thus, much as in a superconducting Josephson junction, we expect an enormous zero bias anomaly in the tunnel current. This prediction, first made by Wen and Zee on the basis of the broken symmetry ground state proposed by Fertig was recently dramatically confirmed in a remarkable set of experiments by Spielman et al. The data shown in Fig. (1) represent the differential conductance (and in the lower panel the conductance) as a function of bias voltage in a sample with extremely weak tunneling ($\Delta_{\text{SAS}} \sim 85 \mu K$).

For large layer separation relative to electron spacing, the two layers are uncorrelated and it costs a lot of Coulomb energy to suddenly inject an electron into a layer by tunneling. The strong magnetic field destroys the fermi liquid state by making the kinetic energy degenerate and preventing the other electrons from moving away from the newly arrived electron. Only if the voltage is large enough to overcome the Coulomb gap will current be able to flow. This can be seen in the right panel of Fig. (1). Note that if the voltage is too large the current again decreases because the excess energy from the bias supply can not be absorbed by converting it to kinetic energy of the electrons.

For small $d/\ell$ the system undergoes a quantum phase transition into the interlayer phase coherent state and tunneling is dramatically enhanced near zero voltage. Unlike the true Josephson effect the dissipation is not infinitesimal on the supercurrent branch. Various proposals involving a finite phase coherence time have been made to explain the finite height and width of the differential conductance peak but this is a question which is still poorly understood and is a subject of current study. In order to understand the finite dissipation, it is necessary...
to understand the excitations above the phase coherent ground state. We now turn to this question.

Because of the U(1) symmetry, the energy of the coherent state can not depend on the global phase angle $\varphi$. Physically this just results from the fact that, in the absence of tunneling it is impossible for the electrons to know what the global phase angle is. However symmetry does not prevent the energy from depending on spatial gradients of $\varphi$. When the fluctuations out of the easy-plane are small we can parametrize the local pseudospin orientation vector $(\cos \varphi, \sin \varphi, m_z)$ in terms of the phase angle $\varphi$ and the conjugate ‘charge’ $m_z$. The leading terms in the gradient expansion for the energy yield

$$H = \int d^2 r \left( \frac{1}{2} \rho_s |\nabla \varphi|^2 + \frac{(\hbar n_0/2)^2}{2\Gamma} m_z^2 \right),$$

where $\Gamma$ is related to the capacitance per unit area. The pseudospin stiffness $\rho_s$ physically arises from the loss of optimal Coulomb exchange in the presence of a phase gradient. Because momentum in a magnetic field is related to position in real space, a phase gradient of the order parameter corresponds to a spatial displacement of the correlation hole so that it is no longer directly on top of the
particle. This increases the Coulomb energy. Microscopic Hartree-Fock calculations find \( \rho_s \sim 0.2 - 0.5K \) for typical sample parameters.

Because the phase is conjugate to the conserved ’charge’ \( m_z \), the action takes the form

\[
S = \int d^2r \int dt \left[ \frac{n_0}{2} \dot{\phi} m_z - H \right].
\]  

(16)

The first term can either be viewed as the Berry phase appropriate to a spin model or simply as the statement that the momentum density conjugate to \( \phi \) is \( p_\phi = \frac{\hbar}{2} m_z \). Integrating out the massive \( m_z \) fluctuations yields

\[
S = \frac{1}{2} \int d^2r \int dt \left[ \Gamma \dot{\phi}^2 - \rho_s |\nabla \phi|^2 \right].
\]  

(17)

This is the action of a superfluid with Goldstone mode velocity \( u = \sqrt{\frac{\rho_s}{\Gamma}} \).

Because the ‘charge’ \( m_z \) conjugate to \( \phi \) is the difference in the two layer charge densities, the supercurrent \( \vec{J}_- = \rho_s \nabla \phi \) associated with the Goldstone mode is antisymmetric in the layer index; that is, it corresponds to electronic currents flowing in opposite directions in the two layers. At finite temperatures the statistical mechanics of this novel superfluid is that of the 2D XY model and so it undergoes a true phase transition when vortices in the order parameter field unbind at the Kosterlitz-Thouless temperature (on the scale of \( \rho_s \)). While there does not yet exist direct evidence for this transition, we note that the tunnel peak at zero voltage begins to turn on at temperatures which are consistent with Hartree-Fock estimates of \( \rho_s \sim 100 - 500mK \).

We are now in a position to analyze the excitations produced by tunneling. From Eq. (14) we see that the tunneling operators that correspond to the two possible directions of tunneling must be \( T_{\pm} = -\lambda \int d^2r e^{i \varphi(\vec{r})} e^{\pm iqBx} \) where \( \lambda = n_0 \Delta \rho_s \) and the last term allows for the possibility that there is a tilted magnetic field which puts flux between the two layers. This can be derived by choosing the gauge \( \vec{A}_|| = xB_|| \hat{z} \) to describe the in-plane field and setting \( Q_B = \frac{eB_||}{\hbar} \).

In the true Josephson effect the current is first order in the tunneling amplitude. We will assume that there is sufficient decoherence (to be discussed below) that this does not occur here. The linear-response current can then be computed perturbatively in the tunneling using Fermi’s Golden Rule. For a sample of size \( L^2 \) the net tunnel current is

\[
I(V) = \frac{2\pi e \lambda^2 L^2}{\hbar} [S(Q_B,eV) - S(-Q_B,-eV)],
\]  

(18)

where, \( S(q, \hbar \omega) \) is the spectral density for the fluctuations of the order parameter at wavevector \( q \) and frequency \( \omega \), is proportional to the Fourier transform of the pseudospin correlation function \( \langle e^{i \varphi(\vec{r},t)} e^{-i \varphi(0,0)} \rangle \). Decoherence effects are included phenomenologically by adding spatial and temporal damping factors to this Fourier transform.

The current will have exhibit a peak (or in the case of the differential conductance, a derivative feature) which will occur at the voltage corresponding to the Goldstone mode energy and will disperse outwards with increasing wavevector (B field tilt). As Fogler and Wilczek have noted, this experiment is closely analogous
to the corresponding experiment in Josephson junctions of Eck et al. There a feature appears in the IV characteristic when the wave vector and energy matching conditions for the Swihart mode are achieved.

The differential conductance as a function of voltage at different values of $B_\parallel$ obtained by Spielman et al. is shown in the left panel of Fig. (2). In two dimensions at finite temperature there is no true broken symmetry, but order parameter fluctuations still have long-range power law correlations. This is the origin of the enormous peak in the differential conductance observed for $Q_B = 0$. At the lowest temperatures the half-width of this peak is only $\delta \sim 3 \mu\text{V}$. Within the perturbative model for the tunneling line shape this implies a phase coherence time $\tau_\phi = \hbar/\delta \sim 2 \times 10^{-10}\text{s}$. Using the collective mode velocity to obtain a measure of the coherence length yields $\xi \sim 2\mu\text{m}$. This is on the same scale as both the quantum to classical crossover length $\xi_T \sim \hbar u/k_B T$ and the Josephson penetration length $\lambda_J = \sqrt{4\pi e^2 \rho_s/\Delta_{\text{SAS}}}$. The latter might hint that the assumption that we can work perturbatively in the tunneling amplitude may be beginning to break down at the lowest temperatures. For finite voltage and finite tunneling the non-perturbative treatment is quite difficult because the system is not in equilibrium. Fogler and Wilczek have attacked this question in a 1D model by solving the classical equations of motion for the order parameter.

The left panel of Fig. (2) shows that application of the parallel magnetic field fairly quickly kills the central peak and a small side feature appears which disperses outward with increasing $B_\parallel$. Spielman et al. identify the inflection point as the center of this derivative feature and plot the resulting dispersion curve as shown in the right panel of Fig. (2). The dispersion is indeed linear and agrees to within about a factor of two of the predicted mode velocity of $\sim 10^4\text{m/s}$. It is perhaps not surprising that the measured mode velocity is somewhat lower since quantum fluctuations neglected in the Hartree-Fock approximation will lower the spin stiffness.

In addition to tunneling, another good probe of the phase coherent state would be interlayer drag. In a drag experiment one uses a sample with negligible tunneling, drives a current through the upper (say) layer and then measures the voltage drop in the lower layer (under zero current conditions for that layer). The ratio of drag induced voltage to the applied current is called the transresistance. For the case of ordinary fermi liquids, the electric field in the lower layer is directed oppositely to that in the drive layer in order to counteract the drag force due to momentum transferred from the drive layer. Because the rate of collisions between the fermi liquid particles vanishes as $T^2$ at low temperatures, the drag resistance is small and vanishes at zero temperature.

The simplest way to analyze drag in the present case is to define symmetric and antisymmetric (in layer index) currents $\vec{J}_\pm = \vec{J}_+ - \vec{J}_-$. Transport in the symmetric channel is that of an ordinary, nearly dissipationless, $\nu = 1$ Hall plateau. However transport in the antisymmetric channel is that of a superfluid. The antisymmetric electric field must therefore vanish. This means that the electric field in the lower layer must exactly match that in the drive layer. Hence the drag is very large (and does not vanish at $T = 0$) and the transresistance tensor $\rho^{12}$ is equal to the quantized Hall resistance tensor $\rho_{xx}^{12} \approx 0$; $\rho_{xy}^{12} = \frac{e^2}{h}$.

At the present time, there are still a variety of open issues. We do not have a
Figure 2. Left panel: Differential conductance vs. voltage for a variety of values of the parallel B magnetic $B_\parallel$. Inset: Magnified view showing the Goldstone feature dispersing outward in voltage with increasing $B_\parallel$. Black dots indicate inflection points which are used to determine the mode dispersion shown in the right panel. The velocity agrees to within about a factor of two of the value $\sqrt{\rho_s / \Gamma} \sim 10^4 \text{m/s}$ predicted from Hartree-Fock estimates of the spin stiffness and the compressibility parameters $\rho_s$ and $\Gamma$. After Spielman et al. Ref. 21

We do not understand why the central peak is not destroyed more rapidly with the addition of $B_\parallel$. The peak is still visible even when the Goldstone feature has moved out far enough to be distinct from it. Finally, we do not have a good understanding of the nature of the quantum phase transition or transitions that occur as the layer spacing is increased. Various scenarios have been suggested theoretically and there is some numerical evidence hinting that there is a single weakly first order transition.

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