MULTIVARIATE SIGNAL DENOISING BASED ON GENERIC MULTIVARIATE DETRENDED FLUCTUATION ANALYSIS

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ABSTRACT

We propose a novel multivariate signal denoising method that performs long-range correlation analysis of multiple modes in input data by considering inherent inter-channel dependencies of the data. That is achieved through a novel and generic multivariate extension of detrended fluctuation analysis (DFA) method - another contribution of this paper. Specifically, our proposed denoising method first obtains data driven multiscale signal representation using multivariate variational mode decomposition (MVMD) method. Then, the proposed generic multivariate DFA is used to reject the predominantly noisy modes based on their randomness scores. Finally, the denoised signal is reconstructed by summing the remaining modes albeit after the removal of the noise traces using the principal component analysis (PCA).

Index Terms—Multivariate signals, Detrended fluctuation Analysis, Multivariate variational mode decomposition.

1. INTRODUCTION

Multi-sensor systems have found widespread use in many applications including medical diagnosis, health monitoring, weather forecasting etc. Within these systems, a network of synchronized sensors is used to record signals originating from physical system(s) resulting in interdependent multichannel observations. Those observations, denoted by \( \mathbf{x}_i \in \mathbb{R}^m \), are modelled as a combination of the desired signal \( \mathbf{s}_i \in \mathbb{R}^m \) and the unwanted noise \( \psi_i \in \mathbb{R}^m \), as follows

\[
\mathbf{x}_i = \mathbf{s}_i + \psi_i, \quad \forall \ i = 1, \ldots, N. \tag{1}
\]

Estimation of true multivariate signal \( \mathbf{s}_i \) from raw signal recordings \( \mathbf{x}_i \) is a problem of considerable interest. To solve this problem, most of the existing algorithms are direct multichannel extensions of the popular multiscale approaches that have worked extremely well on univariate (single-channel) data. For instance, the sparsity of discrete wavelet transform (DWT) is exploited to reject noise via a multichannel expansion of the universal threshold [1]. Similarly, multiscale denoising approaches for multivariate data that are based on synchrosqueezed wavelet transform [2], multivariate empirical mode decomposition (MEMD) [3, 4] and translation invariant DWT aided by Mahalanobis distance measure [5], are extensions of [6, 7, 8, 9] respectively. Moreover, variational mode decomposition (VMD) algorithm [10] and its multivariate extension [11] have been employed for denoising [12, 13, 14]. In [12], detrended fluctuation analysis (DFA) [15] has been used to identify and reject the signal modes with predominant noise by estimating their long-range correlations.

In its original form, DFA only caters for single-channel time series data. While its multichannel extension exists [16], it processes each data channel in isolation thereby ignoring inter-channel correlations within multivariate data. To that end, we first develop a novel and generic multichannel extension of DFA, termed GMDFA in the sequel, that fully incorporates inter-channel correlations within data using Mahalanobis distance. Then, using that extension, we present a novel multichannel multiscale denoising method that first uses MVMD to decompose a multivariate data into multiple frequency modes; and then identifies (and rejects) the noisy modes using GMDFA. The efficacy of the proposed approach is demonstrated on a variety of real multichannel signals.

2. DETRENDED FLUCTUATION ANALYSIS

The detrended fluctuation analysis (DFA) is widely used to estimate the extent of long-range correlations in a nonstationary time series. The main advantage of using DFA is that it circumvents the artefacts of nonstationarity (e.g., local trend, noise etc.) which cause spurious scores in the otherwise used Hurst exponent method [15]. Specifically, DFA estimates a power law scaling exponent by observing natural variability of signal fluctuations around its local trend at different time scales. As a result, intrinsic fluctuations of a time series are extracted by detrending the slowly oscillating background that causes spurious scores [15] as described below:

Given a time series \( x_i, \forall i = 1, \ldots, N \); its normalized cumulative sum is obtained as follows: \( y_i = \frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x}) \), where \( \bar{x} \) denotes the signal mean. The resulting profile \( y_i \) is then divided into \( N_s = N/s \) segments of equal length \( s \) from both ends. Next, least squares polynomial fitting approach is employed on the resulting segments to estimate the local trend, denoted by \( \hat{y}_i \). Finally, a root mean squared (RMS) function \( F(s) \) of detrended fluctuations \( y_i - \hat{y}_i \) is obtained as

\[
F(s) = \left( \frac{1}{2N_s} \sum_{i=1}^{2N_s} \left( \frac{1}{s} \sum_{i=1}^{s} (y_i - \hat{y}_{i})^2 \right) \right)^{1/2}. \tag{2}
\]
Note from (2) that $F(s)$ is the root mean of local (segment) variances that is expected to increase with increase in the time scale $s$. This increase in $F(s)$, when described using the power law relation of the time scale $s$, i.e., a higher value of $\alpha$ indicates long-range correlations if $\alpha > 0.5$; while the cases of $\alpha = 0.5$ and $\alpha < 0.5$ suggest no-correlations and short-range correlations, respectively. Furthermore, $\alpha$ informs about the degree of smoothness of a time series, i.e., a higher value $\alpha$ indicates the presence of slow fluctuations while a lower $\alpha$ hints at rapid fluctuations [18].

The resulting insight gained through DFA renders it suitable in many signal processing related applications involving signal analysis [19] and denoising [12].

A multichannel DFA is presented in [16] using a straightforward multichannel generalization of (2) which is given by

$$F_m'(s) = \sqrt{\frac{1}{2N_m} \sum_{i=1}^{2N_m} \left( \frac{1}{N} \sum_{i=1}^{N} \sum_{n=1}^{m} (y_{in} - \bar{y}_{in})^2 \right)},$$

where $y_{in}$ and $\bar{y}_{in}$ respectively denote the profile and polynomial fit for the $n$th channel. Observe from (3) that the Euclidean norm of each $m$-variate error observation is used to formulate a multichannel fluctuation function $F_m'(s)$ in [16] which completely disregards the cross-channel correlations in the data and leads to spurious long range correlation scores.

3. PROPOSED METHODOLOGY

This section outlines our proposed multiscale multivariate signal denoising method. For this purpose, we first describe the proposed generic multichannel extension of DFA that underpins our denoising framework.

3.1. A Generic Multichannel Extension of DFA

To address the aforementioned weakness in the existing DFA in [14], we propose a novel multichannel extension of the DFA method that considers cross-correlations via Mahalanobis distance (MD) measure and may be seen as a generalization of [16]. The steps involved in the proposed Generic Multichannel DFA, termed GMMDFA, are given below:

Given a multivariate time series $x_i, \forall i = 1, \ldots, N$, where $x_i = [x_{i1}, \ldots, x_{im}]^T \in \mathbb{R}^m$ represents an $m$-variate observation at time index $i$, the cumulative sum $y_i$ is computed via

$$y_i = \frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x}),$$

where $\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$ denotes the multichannel mean.

Next, the signal $y_i = [y_{i1}, \ldots, y_{im}]$ for all $i = 1, \ldots, N$, is divided in $2N_s$ spatial segments by cutting it into $N_s = N/s$ segments of equal lengths starting from both ends of the series. Then, the local trend $\tilde{y}_i = [\tilde{y}_{i1}, \ldots, \tilde{y}_{im}]$ is estimated based on the quadratic polynomial fit of each channel

$$\tilde{y}_{in} = a_n \cdot i^2 + b_n \cdot i + c_n, \quad i = 1, \ldots, s,$$

where $a_n, b_n, c_n$ denote the coefficients required for the least square fit $\tilde{y}_{in} \sim y_{in}$. Here, quadratic polynomial is used to estimate the slowly varying background trend. We next provide the mathematical definition of Mahalanobis norm which forms the basis of our proposed method.

**Definition 1 (Mahalanobis norm)** Let $\Sigma$ denote a symmetric and positive definite covariance matrix of vector observations $z_i, \forall i = 1, \ldots, N$, we define the Mahalanobis norm $\|z_i\|_\Sigma = \sqrt{z_i^T \Sigma^{-1} z_i}$ that satisfies the following properties of a norm on that vector space $\mathbb{Z}$, i.e.,

1. $\|z\|_\Sigma > 0 \forall z \neq 0$;
2. $\|a z\|_\Sigma = 0$ iff $z = 0$;
3. $\|az\|_\Sigma = |a| \cdot \|z\|_\Sigma$ for a scalar $a$;
4. $\|z_1 + z_2\|_\Sigma \leq \|z_1\|_\Sigma + \|z_2\|_\Sigma$;

where the vectors $z, z_1$ and $z_2$ belong to the space $\mathbb{Z}$.

**Remark 1** Mahalanobis norm $\|z_i\|_\Sigma$ is a generalized multivariate norm because (a) it considers cross channel dependencies which are completely ignored within the $L_2$ norm; and (b) it performs variance normalization to remove variance bias across the channels (as depicted in Fig 1).

That can be observed from the following two cases of uncorrelated multivariate data where $\|z_i\|_\Sigma$ reduces to a form of $\|z_i\|_2$. Firstly, when $\Sigma = I_{m \times m}$ that denotes an identity matrix, $\|z_i\|_\Sigma$ is given by

$$\|z_i\|_{\Sigma = I_{m \times m}} = \sqrt{z_i^T I_{m \times m} z_i} = \sqrt{z_i^T z_i} = \|z_i\|_2.$$  

Secondly, when $\Sigma = \begin{bmatrix} \sigma_1^2 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & \sigma_m^2 \end{bmatrix}$ is a diagonal matrix where the vector $\sigma = [\sigma_1, \sigma_2, \ldots, \sigma_m]^T$ contains channel variances, $\|z_i\|_\Sigma$ is given by

$$\|z_i\|_{\Sigma = \sigma^T I_{m \times m}} = \sqrt{\frac{z_{i1}^2}{\sigma_1^2} + \ldots + \frac{z_{im}^2}{\sigma_m^2}} = \|z_i\|_2,$$

where $z_i = [z_{i1}, \ldots, z_{im}]^T$ and $\bar{z}_i = [\bar{z}_{i1}, \ldots, \bar{z}_{im}]^T$. 

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**Fig. 1:** Relocation of a set of points in 2D space (black circles) to another set (blue circles) through Mahalanobis distance norm. The black line shows the standard euclidean norm of a single point in 2D; the red line shows the corresponding Mahalanobis norm of the same point, which is more accurate since it considers cross-correlations and is robust to variance bias across the channels of multivariate data.
Finally, in the case of correlated multivariate data, Mahalanobis norm essentially computes the $L_2$ norm by uncorrelating the variance normalized vector observations as depicted in Fig. 1. For a special case of bivariate data, $\|z_i\|_2^2$ can be rewritten as

$$\|z_i\|_2^2 = \frac{1}{1 - \rho^2} \sqrt{\|z_i\|_2^2 - \frac{2\rho z_i z_{i+1}}{\sigma_1 \sigma_2}}.$$  

where $\rho$ denotes correlation coefficient and $\Sigma = \begin{pmatrix} \sigma_1^2 & \rho \sigma_1 \sigma_2 \\ \rho \sigma_1 \sigma_2 & \sigma_2^2 \end{pmatrix}.$

Based on remark 1, we utilize the generic (Mahalanobis) norm $\|y_i - \tilde{y}_i\|_2^2$ to formulate a purely multivariate fluctuation function $F_m^\Sigma(s)$ within MDFA, that is,

$$F_m^\Sigma(s) = \sqrt{\frac{1}{2\pi N} \sum_{\nu=1}^{2N} \sum_{i=1}^{(\nu+1)} (y_i - \tilde{y}_i)^T \Sigma^{-1} (y_i - \tilde{y}_i),}$$

where covariance matrix $\Sigma$ characterizes the interchannel dependencies within the detrended (or fluctuations) $y_i - \tilde{y}_i$. It is clear that (3) becomes a special case of (9) for identity covariance matrix, i.e., uncorrelated input multichannel data. For more interesting cases involving multichannel data that exhibit cross-channel correlations, (9) provides more informative fluctuation scores.

In order to perform multichannel scaling analysis in (9), $F_m^\Sigma(s)$ is computed for varying time scales where generally the range $s = 4, \ldots, 16$ is used [17]. Finally, a scaling exponent $\alpha$ is computed using power law representation of $F_m^\Sigma(s)$

$$F_m^{\Sigma}(s) = s^\alpha.$$  

(10)

In practice, $\alpha$ is calculated based on the slope of the plot between $\ln F_m^{\Sigma}(s)$ and $\ln s$ because $\log s_n F_m^{\Sigma}(s) = \frac{\ln F_m^{\Sigma}(s)}{\ln s}$, where $\ln$ denotes the natural logarithm operator.

### 3.2. Multivariate Denoising Using MVMD and GMDFA

Here, we present a multivariate signal denoising method that applies the proposed GMDFA on the data-driven modes of noisy signal obtained from MVMD, as discussed below:

3.2.1. Multiscale decomposition using MVMD

Multivariate VMD [11] is a generic multichannel extension of the VMD algorithm that decomposes a multivariate signal $z_i \in \mathbb{R}^m$ into $K$ number of predefined multivariate modulated oscillations $u_{k,i} \in \mathbb{R}^m$ which are based on a common frequency component across all channels.

### Fig. 2: Plot of scaling exponents $\alpha_k$, computed using proposed GMDFA, for MVMD modes of noisy Wind signal at 10 dB.

| Iterations | SDs (MDFA) | SDs (GMDFA) |
|------------|------------|-------------|
| $0.4$      | $0.45$     | $0.5$       |
| $0.46$     | $0.5$      | $0.5$       |
| $0.5$      | $0.5$      | $0.5$       |
| $0.52$     | $0.5$      | $0.5$       |
| $0.54$     | $0.5$      | $0.5$       |

| Scaling Exponent | $\text{MVMD}$ | $\text{GMDFA}$ |
|------------------|---------------|----------------|
| $0.4$            | $0.45$       | $0.5$       |
| $0.5$            | $0.5$       | $0.5$       |
| $0.55$           | $0.5$       | $0.5$       |

Within our proposed denoising approach, firstly MVMD is used to decompose a noisy multivariate signal $z_i$ into an ensemble of $K$ multichannel BLIMFs $u_{k,i}$ which comprise of modulated multivariate oscillations of a common frequency component. Among those, initial BLIMFs contain low frequency (or smooth) oscillations whereas the latter BLIMFs mostly comprise of high frequency fluctuations. This representation can be mathematically written as

$$z_i = \sum_{k=1}^{K} u_{k,i},$$

(11)

where $\{u_{k,i}\}_{k=1}^{K}$ denotes the set of initial BLIMFs containing majorly of (true) signal and $\{u_{k,i}\}_{k=K+1}^{K}$ denotes the BLIMFs with predominant noise. Next, MDFA is used to detect the predominant noise modes, i.e., $K_1$.

3.2.2. Rejection of predominantly noisy BLIMFs using MDFA

The proposed GMDFA is used to identify and discard predominantly noisy BLIMFs based on (a) their higher frequency content and (b) absence of long-range auto-correlations. In this regard, the comparative analysis of the scaling exponents $\alpha_k$, computed for each BLIMF $u_{k,i}$ using (12), is performed. Understandably, $\alpha_k$ should decrease for every higher order BLIMF of the MVMD owing to the presence of increasingly high frequency fluctuations and decreasing long-range correlations; that is evident from Fig. 2 that plots $\alpha_k$ for MVMD modes of a noisy trivariate wind signal.

Let $\beta_k$ denote the slope of the line connecting the exponents $\alpha_k$ and $\alpha_{k+1}$ for two consecutive modes, i.e.,

$$\beta_k = |\alpha_{k+1} - \alpha_k|.$$  

(13)

Then, $\beta_k$ quantifies the amount of change in the frequency of the fluctuations (or decrease in long-range correlations) when moving one mode to the other. That means, highest slope suggests maximum increase in frequency or maximum decrease in long-range correlations, i.e., largest increase in noise content. Consequently, the first mode after the highest slope, i.e., $u_{K_1+1}$, marks the beginning of predominantly noisy modes where $K_1$ may be computed as follows

$$K_1 = \arg \max_k \{\beta_1, \ldots, \beta_K\}.$$  

(14)

Subsequently, the modes $\{u_{k,i}\}_{k=K_1+1}^{K}$ are rejected as noise.
Table 1: Input versus output SNR values of various comparative multivariate signal denoising methods on real signals.

| Avg. In. SNR | Avg. Out. SNR | Avg. Out. SNR | Avg. Out. SNR | Avg. Out. SNR | Avg. Out. SNR |
|--------------|---------------|---------------|---------------|---------------|---------------|
|              | 10 dB         | 20 dB         | 30 dB         | 40 dB         | 50 dB         |
| Test Signal  | Bi. Scalar Signal | Tri. Wind Signal | Qd. Synthetic Signal | Bi. Scalar Signal | Tri. Wind Signal | Qd. Synthetic Signal |
| MWD          | bal.          | bal.          | bal.          | bal.          | bal.          | bal.          |
|              | 8.26          | 15.47         | 15.47         | 15.47         | 15.47         | 15.47         |
|              | 8.35          | 15.47         | 15.47         | 15.47         | 15.47         | 15.47         |
| MWD          | unbal.        | unbal.        | unbal.        | unbal.        | unbal.        | unbal.        |
|              | 8.40          | 15.47         | 15.47         | 15.47         | 15.47         | 15.47         |
| MMD          | bal.          | bal.          | bal.          | bal.          | bal.          | bal.          |
|              | 10.57         | 16.50         | 16.50         | 16.50         | 16.50         | 16.50         |
| MMD          | unbal.        | unbal.        | unbal.        | unbal.        | unbal.        | unbal.        |
|              | 10.57         | 16.50         | 16.50         | 16.50         | 16.50         | 16.50         |

Table 1 reports average output SNRs for $J = 20$ realizations from the comparative methods for all the input datasets (described above) at input SNR = $-2$, $2$, $6$ and $10$ dB. At each input noise level, we consider balanced noise (i.e., same input SNRs for all channels) and unbalanced noise cases (i.e., different input SNRs across different channels). To accentuate the best performing method, highest output SNRs are highlighted in bold for each input SNR. Observe that in most cases, the proposed MDD method yields highest output SNRs demonstrating the effectiveness of our method. Occasionally, at higher output SNRs, MMD outperforms our MDD method while MWD - generally regarded as a benchmark in multichannel signal denoising - remains competitive as well.

Finally, we inspect the visual quality of the reconstructed signal by displaying the denoised Sofar signals in Fig. 4 along with the noisy version at input SNR = $10$ dB. For meaningful qualitative analysis, we plotted original signal (shown using dotted line) in the background of the denoised signals (shown using solid line) in each case. Evidently, proposed MDD method yields best estimate of the original signal since it can estimate subtle details along with the slow variations, see Fig. 4 (d). On the contrary, MMD and MWD not only miss important signal details but also yield artifacts.

5. CONCLUSION

We have proposed a novel multivariate signal denoising method that is based on multiscale data representation and statistical signal properties. A novel and generic multichannel extension of detrended fluctuation analysis (DFA) underpins our denoising method which has been shown to outperform existing approaches owing to the full utilization of inter-channel correlations within input data through utilization of Mahalanobis distance measure.
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