Fixed rings in quotients of completed group rings

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Abstract

Let $k$ be $\mathbb{F}_p$ or $\mathbb{Z}_p$, let $G$ be a compact $p$-adic analytic group, and form its completed group algebra $kG$. Take a closed subgroup $\Gamma$ of $G$. We analyse the structure of the fixed ring of $kG/I$ under the conjugation action of $\Gamma$, for certain ideals $I$ induced from the $G$-centraliser of $\Gamma$, and we explain the consequences this has for the theory of the prime ideals of $kG$.

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Introduction

Our main objects of study are the completed group rings $kG$ of compact $p$-adic analytic groups $G$ over certain profinite rings $k$, otherwise known in special cases as Iwasawa algebras.

There is an ongoing project to understand the (prime) ideals in such rings: various more precise questions were posed in the survey paper [3] Questions G–O, and despite significant work on this front in the last decade, most of these questions still remain partly or entirely open. The most substantial results in this direction have been those of the paper [2], which fully answers [3] Question L, and answers [3] Questions N and O in the special case when the group $G$ is additionally assumed to be nilpotent.

Corollary B. The following corollary is now easy to deduce. In all of the below results, we may take $k$ to be either $\mathbb{F}_p$ or $\mathbb{Z}_p$.

**Theorem A.** Let $G$ be a $p$-valuable group, $\Gamma$ an arbitrary closed subgroup which we consider acting on $G$ by conjugation, and $H$ the centraliser in $G$ of $\Gamma$. If $I$ is a faithful $G$-prime ideal of $kH$, then the set of $\Gamma$-invariant elements of $kG/IkG$ is precisely $kH/I$.

This result, proved below as Theorems 4.9 and 4.11, is an analogue of Roseblade’s [8, Lemma 10], which was used to answer similar questions about group algebras of polycyclic groups. While Theorem A is not quite as general as [8, Lemma 10], in practice it is general enough to be used in the same contexts.

The main result of our paper is as follows. We state and prove a more precise version in the body of the paper.

**Corollary C.** If $kG$ is polycentral, then all faithful prime ideals are controlled by $Z$.

These results are proved only for $p$-valuable groups. Nonetheless, we remark that they have immediate consequences for more general compact $p$-adic analytic groups. For instance, in the case when $F$ is a finite field and $G$ is an arbitrary compact $p$-adic analytic group, recent work of the author [10] explicitly describes quotients of $FG$ by its minimal prime ideals in terms of rings of the form $EU$, where $E$ ranges over the finite extensions of the field $F$, and $U$ can be taken to range over the large uniform subquotients of $G$. These descriptions have been concretely applied in extending known results from uniform groups to larger classes of groups: see, for instance, [12] and [11] for two examples.

1 Preliminaries

Throughout this paper, $p$ will denote a fixed prime number, and $\mathbb{Z}_p$ will be the additive group (or ring) of $p$-adic integers.

Let $R$ be a ring equipped with a topology $T$. We say that $T$ is a ring topology on $R$ if the addition and multiplication maps $R \times R \to R$ are continuous with respect to $T$. In this case, we say $R$ is a topological ring (with respect to $T$).

Our standard reference for topological rings is [9].

**Lemma 1.1.** Let $R$ be a compact, totally disconnected (topological) ring and $I$ a closed ideal. Then $R/I$ is a compact, totally disconnected ring.
Proof. Clearly $R/I$ is a ring, and is compact as a quotient space. By [9, Theorem 5.4], the topology on $R/I$ is indeed a ring topology, so that $R/I$ is a compact ring. Now it follows from [9, Theorem 5.17(3)] that $R/I$ is totally disconnected.

Lemma 1.2. Profinite rings are compact and totally disconnected.

Proof. By definition, a profinite ring $R$ is an inverse limit of finite rings, and is hence a closed subspace of a product of finite rings. But finite rings are discrete, and hence compact and totally disconnected; and these properties are preserved under arbitrary products and closed subspaces.

Definition 1.3. A compact $p$-adic analytic (or compact $p$-adic Lie) group may be most simply defined as a closed subgroup of $GL_n(\mathbb{Z}_p)$ for some positive integer $n$. For the basic properties of these topological groups, we refer the reader to [4, §8] or [5, Ch. III, Definition 3.2.2]. In particular, it is known that every compact $p$-adic analytic group $G$ contains an open (hence finite-index) normal subgroup $H$ which is $p$-valuable, in the sense of [5, Ch. III, Definition 3.1.6]. These $p$-valuable groups are in some sense well-understood, as are the related class of uniform groups in the sense of [4, Definition 4.1].

Below, let $\mathcal{O}$ denote a fixed complete discrete valuation ring whose residue field is a finite field of characteristic $p$, and let $k$ denote a finite field of characteristic $p$.

Definition 1.4. Given a compact $p$-adic analytic group $G$, we may form its completed group ring over $\mathcal{O}$:

$$\mathcal{O}G := \lim_{\leftarrow U,n} (\mathcal{O}/\pi^n\mathcal{O})[G/U],$$

where

- $U$ ranges over the system of open normal subgroups of $G$,
- $n$ ranges over $\mathbb{N}$,
- $\pi$ is a uniformiser of $\mathcal{O}$,
- $R[G/U]$ is the usual group algebra of the (finite) group $G/U$ over the commutative ring $R$.

Compare [4, §7.1].

We may also form its completed group ring over $k$:

$$kG := \lim_{\leftarrow U} k[G/U],$$

with notation as above.

The topological rings $\mathbb{Z}_pG$ and $\mathbb{F}_pG$ are sometimes called the (respectively integral and modular) Iwasawa algebras of $G$.

Remark 1.5. $\mathcal{O}G$ is profinite, as each $(\mathcal{O}/\pi^n\mathcal{O})[G/U]$ is a finite ring. $kG$ is profinite, as each $k[G/U]$ is a finite ring.

2 Topological rings

Throughout this section, let $R$ denote either a complete discrete valuation ring whose residue field is a finite field of characteristic $p$, or a finite field of characteristic $p$.

Lemma 2.1. Let $G$ be a compact $p$-adic analytic group. Then $RG$ is compact and totally disconnected.

Proof. This follows from Lemma 1.2 and Remark 1.5.

Corollary 2.2. Let $G$ be a compact $p$-adic analytic group. Suppose that $\mathcal{J}$ is a fundamental system of neighbourhoods of zero for $RG$ consisting of open ideals, and let $I$ be an arbitrary closed ideal of $RG$. Then the canonical ring homomorphism

$$RG/I \rightarrow \lim_{\leftarrow J \in \mathcal{J}} (RG/(J + I))$$

is an isomorphism of topological rings.
Proof. By Lemmas [11] and [21] we see that $RG/I$ is a compact, totally disconnected ring. As $J$ is a fundamental system of neighbourhoods of zero in $RG$, by [9] Theorem 5.5, we have that $\{A_J : J \in \mathcal{F}\}$ is a fundamental system of neighbourhoods of zero in $kG/I$, where we set $A_J := (J + I)/I$. Each $A_J$ is easily seen to be an open ideal in $RG$, as it is the image of $J$ under the projection map $RG \to RG/I$, which is an open map (see e.g. [9] Theorem 5.2).

Now [9] Theorem 5.23 implies that $RG/I$ is topologically isomorphic to the inverse limit of the inverse system of rings $\{(RG/I)/A_J : J \in \mathcal{F}\}$. But $(RG/I)/A_J$ is naturally isomorphic to $RG/(J + I)$ by standard isomorphism theorems. (That this topological isomorphism is canonical can be read off from [9] Corollary 5.22 and the proof of [9] Theorem 5.23.)

3 Centralised group elements

Lemma 3.1. Let $G$ be a group, $\Gamma$ a subgroup, and $H$ a normal subgroup of $G$ centralising $\Gamma$. Suppose, for some fixed $g \in G$, we have that $x^{-1}gx \in gH$ for all $x \in \Gamma$; that is, the function $\theta : \Gamma \to G$ defined by $\theta(x) = x^{-1}gx^{-1}$ has image in $H$. Then $\theta$ is in fact a group homomorphism with image contained in $Z(H)$.

Remark 3.2. This is mentioned (without proof) in the context of linear groups by Roseblade in [8]; the proof is easy, but we have chosen to supply it.

Proof. To see that $\theta$ is a group homomorphism, note that

$$\theta(x)\theta(y) = x^{-1}gyx^{-1}y^{-1}g^{-1},$$

and that the two boxed factors commute, as the first is an element of $H$ and the second is an element of $\Gamma$. This expression then simplifies to $\theta(x)\theta(y) = y^{-1}x^{-1}gxg^{-1} = \theta(xy)$ as required. To see that the image is central in $H$, we compute, for arbitrary $h \in H$,

$$h^{-1}\theta(x)h = h^{-1}x^{-1}gxg^{-1}h,$$

noting again that the boxed expressions commute for the same reason as above. Now, as $H$ is normal in $G$, there exists some $h' \in H$ such that $h^{-1}y = gh'g^{-1}$ and hence that $g^{-1}h = h'g^{-1}$. Making these substitutions, we can see that

$$h^{-1}\theta(x)h = x^{-1}gh^{-1}xh'g.$$

Finally, as before, $h^{-1}xh' = x$, and so this equation reduces to the statement that $h^{-1}\theta(x)h = \theta(x)$.

For the remainder of this section, let $G$ be a $p$-valuable group, $H$ a closed normal subgroup, and $\Gamma$ a closed subgroup centralising $H$ and acting on $G$ by conjugation. Let also $R$ be as in the previous section. Fix also a faithful $G$-prime ideal $I \triangleleft RH$. (Recall that an ideal $I$ in $RH$ is faithful if $\ker(H \to RH^\times \to (RH/I)^\times) = \{1\}$; $I$ is a $G$-ideal if it is invariant under conjugation by $G$; and $I$ is $G$-prime if it is a $G$-ideal with the property that, whenever $A$ and $B$ are $G$-ideals of $RH$ with $AB \subseteq I$, we must have either $A \subseteq I$ or $B \subseteq I$.)

Proposition 3.3. Suppose that, for some nonzero $a \in RH/I$ and some $g \in G$, we have that $y = ag \in RG/IRG$ is centralised by $\Gamma$. Then $g$ is also centralised by $\Gamma$.

Proof. Viewing $G$ as a subgroup of $(RG/IRG)^\times$, we see that we are in the situation of Lemma 3.1. Let $\theta$ be as in this lemma: we will prove that $\theta(x) = 1$ for all $x \in \Gamma$.

Write $I$ as the intersection of a $G$-orbit of prime ideals $I_1, \ldots, I_s$, which is possible due to [8] Theorem 3.18. As $a$ is assumed nonzero inside $RH/I$, we can take $a = \tilde{a} + I$ for some $\tilde{a} \in RH \setminus I$; and $\tilde{a} \not\in I$ implies that $\tilde{a} \not\in J_i$ for at least one $1 \leq i \leq s$. Choose such a $J_i$, and denote it simply by $J$.

Take $x \in \Gamma$ with $\theta(x) = z \in Z(H)$. This means that

$$ag = x^{-1}(ag)x = zag,$$

so that $(z - 1)\tilde{a} \in I \not\subseteq J$. Now, as $z$ is central in $H$, this implies that $(z - 1) \cdot RH \cdot \tilde{a} \in J$; and since $J$ is prime, we must now have $z - 1 \in J$, i.e. $z \in J^1$. But $J^1 = 1$, hence $\bigcap_{j \in G}(J^1)^g = J_1 \cap \cdots \cap J_s = 1$, and as $H$ is orbitally sound (by [2] Proposition 5.9), we have $J^1 = 1$. Hence $z = 1$.

Proposition 3.4. Suppose now that $y = a_1g_1 + \cdots + a_ng_n$ is centralised by $\Gamma$, where each $a_i \in RH/I$, and the $g_i$ are elements of $G$ which are pairwise distinct modulo $H$. If $G/H$ is $p$-valuable, then $a_ig_i$ is centralised by $\Gamma$ for each $1 \leq i \leq n$. 

\[\square\]
Proof. We will prove that, if $gH \in G/H$ has finite $\Gamma$-orbit, then it has trivial $\Gamma$-orbit. This will clearly suffice. Let $S$ be the stabiliser in $\Gamma$ of $gH$. Then $S$ has finite index in $\Gamma$, say index $p^f$. Hence, for all $x \in \Gamma$, we have
\[ g^{-1}x^{p^f}gH = x^{p^f}H, \]
an equality inside $G/H$. But, as $G/H$ is $p$-valuable, we may take $p^f$-th roots to show that $x$ fixes $gH$, i.e. $S = \Gamma$. \qed

4 Finite orbit sums

4.1 Generalities

Setup. Let $G$ be a $p$-valuable group, $\Gamma$ an arbitrary closed subgroup which we consider acting on $G$ by conjugation, and $H$ the centraliser of $G$ in $\Gamma$, which is automatically closed, isolated and normal.

Fix $F$ a finite field of characteristic $p$, and $O$ a CDVR with residue field $F$. Writing $FH$ and $OH$ to denote the completed group rings of $H$ (over $F$ and $O$ respectively), take arbitrary $G$-prime ideals $I \triangleleft FH$ and $J \triangleleft OH$.

For convenience, we will also use the following notation. Given any closed subgroup $K$ of $G$, set $\Omega_K := FK$ for the completed group ring of $K$ over $F$, and similarly $\Lambda_K := OK$. If $H \leq K$, we will also write $\Omega_K' := \Omega_K/\Omega_K$ and $\Lambda_K' = \Lambda_K/J\Lambda_K$.

Definition 4.1. Fix a transversal $T$ to $H$ in $G$. We define left modules as follows:
\[ R = \bigoplus_{t \in T}(\Omega_H)t, \quad S = \bigoplus_{t \in T}(\Lambda_H)t, \]
where $R$ is a left $\Omega_H'$-submodule of $\Omega_G'$, and $S$ is a left $\Lambda_H'$-submodule of $\Lambda_G'$. Note that, in fact, $R$ is a subring of $\Omega_G$, and $S$ is a subring of $\Lambda_G$.

Proposition 4.2. If $I$ is faithful, then $R^\Gamma = \Omega_H'$. If $J$ is faithful, then $S^\Gamma = \Lambda_H'$.

Proof. Both of the inclusions “$\subseteq$” are clear.

For the converse, suppose that the element $y = a_1t_1 + \cdots + a_nt_n \in R$ is centralised by $\Gamma$, where each $a_i \in \Omega_H'$ is nonzero, and the $t_i$ are distinct elements of $T$. Then, by Proposition 3.3 the elements $a_it_i \in R$ are centralised by $\Gamma$ for each $1 \leq i \leq n$; and hence, by Proposition 3.3 the element $t_i \in T$ is centralised by $\Gamma$ for each $1 \leq i \leq n$. But, by definition of $H$ as the centraliser of $\Gamma$, we see that $t \in T$ is centralised by $\Gamma$ if and only if $t \in H$. So we must have $n = 1$, $a_1 \in \Omega_H'$ and $t_1 \in H$, so that $y \in \Omega_H'$. The argument for $S$ is essentially identical. \qed

Choose a descending sequence
\[ G = G_1 > G_2 > G_3 > \ldots \]
of open normal (hence $\Gamma$-invariant) subgroups of $G$ such that $\bigcap_{n \geq 1} G_n = \{1\}$. For each $i \geq 1$, and each closed subgroup $K$ of $G$, write $K_i = G_i \cap K$.

For each $i \geq 1$, let $\varepsilon_{K,i}$ be the augmentation ideal $\ker(\Omega_K \to F[K/K_i])$. We will denote $\Omega_{K,i} = \Omega_K/\varepsilon_{K,i}$, and if $H \leq K$, we will also denote $\Omega_{K,i}' = \Omega_K/\varepsilon_{K,i} + I\Omega_K$. (When the group $K$ is understood to be equal to $G$, we will drop the subscript $G$.)

Note that $G$ naturally acts on $\Omega'$ and each $\Omega_i'$ by conjugation (via the surjection $G \to G/I'$), and that there are natural surjections $\pi_i : \Omega_i' \to \Omega_i$ and $\varphi_i : \Omega' \to \Omega_i'$ respecting the action of $\Gamma$.

Definition 4.3. Suppose the group $\Gamma$ acts (on the right) on the ring $A$, and $\mathcal{C}$ is a finite $\Gamma$-orbit of some element $a \in A$. We will write $\mathcal{C}$ to denote the sum of the orbit $\mathcal{C}$: that is, denoting by $\Gamma_a$ the stabiliser of $a$ in $\Gamma$, we set
\[ \mathcal{C} = \sum_{\gamma \in \Gamma_a \setminus \Gamma} a^{\gamma} \in A, \]
where $\gamma$ runs over a set of (right) coset representatives of $\Gamma_a$ in $\Gamma$.

For each $n \geq 1$, let $T_n = \varphi_n(T)$, a (finite) transversal to $H/H_n$ in $G/G_n$. By definition, the $T_n$ are compatible with the projection maps (in the sense that $\pi_{i+1}(T_{i+1}) = T_i$). Hence $T$ may be realised as the inverse limit of the $T_n$ (in the category of sets), and this construction is compatible with the realisation of $G$ as the inverse limit of the $G/G_n$ (in the category of groups).

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Lemma 4.4. As left $\Omega_{H,n}$-submodules of $\Omega'_{G,n}$, we have $\Omega'_{G,n} = \bigoplus_{t \in T_n} (\Omega'_{H,n})^t$.

Proof. Note first that $H/H_n \cong HG_n / G_n \leq G/G_n$, so that $\Omega_{G,n} = \Omega_{H,n} \ast A$, where $A$ is the finite group $G/HG_n$. Now, writing $\theta : \Omega_G \to \Omega_{G,n}$, we have $\theta(I\Omega_G) = \theta(I)\Omega_{G,n} = \theta(I) \cdot \Omega_{H,n} \ast A$, which, by [7] Lemma 1.4(ii) and its proof, is equal to $\theta(I) \ast A$. Hence, again by [7] Lemma 1.4(ii), we have

$$\Omega'_{G,n} = \Omega_{G,n} / \theta(I\Omega_G) \cong \Omega_{H,n} / \theta(A/\theta) \ast A \cong \Omega'_{H,n} \ast A.$$

Taking $T_n$ as a basis for the crossed product inside $\Omega'_{G,n}$ yields the desired result.

Lemma 4.5. As left $\Omega'_{H,n}$-submodules of $(\Omega'_{G,n})^\Gamma$, we have $(\Omega'_{G,n})^\Gamma = \sum_{t \in T_n} (\Omega'_{H,n})^\Gamma$.

Proof. The inclusion “$\supseteq$” is obvious, once we show that the expression $\Gamma$ makes sense as an element of $\Omega'_{G,n}$; and indeed, as each $t \in T_n$ has finitely many $\Gamma$-conjugates inside the (finite) group $G/G_n$, it has finitely many $\Gamma$-conjugates when considered as an element of the ring $\Omega'_{G,n}$.

For the reverse inclusion: let $x = a_1t_1 + \cdots + a_r t_r \in (\Omega'_{G,n})^\Gamma$, where $a_i \in \Omega'_{H,n}$ and $t_1, \ldots, t_r$ are distinct elements of $T_n$; this decomposition is possible due to Lemma 4.4. Then it is easy to see that $\Gamma$ permutes the set $\{a_1 t_1, \ldots, a_r t_r\}$, and so it falls into disjoint $\Gamma$-orbits $\Gamma a_1, \ldots, \Gamma a_r$, so that $x = \Gamma a_1 + \cdots + \Gamma a_r$. It suffices to show that each $\Gamma a_i$ can be written as $\Gamma a i$ for some $a_i \in \Omega'_{H,n}$ and $t \in T_n$. To this end, suppose (renumbering without loss of generality) that $\Gamma a_i = \{a_1 t_1, \ldots, a_m t_m\}$; but then $a_1 t_1 + \cdots + a_m t_m = a_1^\Gamma t_1 = a_1^\Gamma t_1$.

Lemma 4.6. The maps $\Omega'_{n+1} \to \Omega'_n$ form an inverse system with inverse limit $\Omega'$.

Proof. This follows from Corollary 2.2.

Remark 4.7. Lemmas 4.4, 4.5 may be proved also for $\Lambda'$ with minimal changes.

4.2 The case of a finite coefficient field

We focus first on the case of $\Omega'_{G}$. Retain all of the notation of the previous subsection.

Proposition 4.8. $(\Omega')^\Gamma = \mathbb{R}^\Gamma$.

Proof. The inclusion “$\supseteq$” is clear; the proof of “$\subseteq$” is simply a modification of [1] Proposition 2.1, which we outline below.

Take some nonzero element $\alpha \in (\Omega')^\Gamma$. We will view $\Omega'$ as the inverse limit of the $\pi_n$ by Lemma 4.6 and hence as a subring of the product $\prod \times \Omega'_2 \times \Omega'_1$. Then we may write $\alpha$ as $(\ldots, \alpha_3, \alpha_2, \alpha_1)$ with each $\alpha_i \in (\Omega_i')^\Gamma$. There is some minimal positive integer $r$ such that $\alpha_r \in \Omega'_r$ is nonzero. Now, by Lemma 4.4 we may write

$$\alpha_r = \sum_{t \in T_r} a_r^\Gamma t^n$$

for some choices of $a_r^\Gamma \in \Omega'_{H,r'}$.

Choose some term $a_0^\Gamma t_0$ with $a_0 \neq 0$. Exactly as in [1] Proposition 2.1, for each $i \geq 0$, we inductively choose $a_{i+1} \in \Omega'_{H_{r+r'+1}}$ and $t_{i+1} \in T_{r+r'+1}$ satisfying the following three properties:

(i) $\pi_{i+1}(a_{i+1}) = a_i$,
(ii) $\pi_{i+1}(t_{i+1}) = t_i$,
(iii) $|\text{orb}(t_{i+1})| = |\text{orb}(t_i)|$.

We refer the reader to [1] Proposition 2.1 for the details of this lifting procedure.

Property (ii) tells us that the $t_n$ converge to some element $t$ in the inverse limit $T$; property (i) tells us that the $a_n$ converge to some element $a_t$ in the inverse limit $\Omega'_{H}$. Moreover, by property (iii), $|\text{orb}(t)| = |\text{orb}(t_0)|$, and so in particular $|\text{orb}(t)|$ is finite. Hence $a_t^\Gamma \in \mathbb{R}^\Gamma$, and by construction, $\varphi_r(a_t^\Gamma) = a_0^\Gamma t_0$.

Repeating this process for each nonzero term in $\alpha_r$, we eventually find some finite sum $\beta = \sum_{t} a_t^\Gamma \in \mathbb{R}^\Gamma$ such that $\varphi_r(\beta) = \alpha_r$. Hence, inductively, we see that $\alpha$ can be approximated arbitrarily well by elements in $\mathbb{R}^\Gamma$.

We can now join this with Proposition 4.2.

Theorem 4.9. If $I$ is faithful, then $(\Omega'_{G})^\Gamma = \Omega'_{H}$. 


Proof. Let $R$ be as in Proposition 4.8, so that $(\Omega_G')^F$ is the closure in $\Omega_G'$ of $R^F$; and by Proposition 4.2, $R^F = \Omega_H^F$, which is already closed in $\Omega_G'$.

4.3 The case of a coefficient CDVR

We now move to $\Lambda_G'$.

Proposition 4.10. $(\Lambda')^F = \overline{S^F}$.

Proof. The inclusion “⊇” is clear; the proof of “⊆” is now an easy modification of Proposition 2.2, which we omit.

Theorem 4.11. If $J$ is faithful, then $(\Lambda_G')^F = \Lambda_H^F$.

Proof. As in Theorem 4.8.

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