Toward a standard model 2, via Kaluza ansatz 2

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Abstract

New results and perspectives precipitate from the (modified as) Kaluza ansatz 2 (KA2), whereby, instead of appending $n$ Planck-scale ($L_o$) compact SL dimensions to ordinary 4D spacetime, one augments $n$ such dimensions by 3 large ones. By KA2, the fundamental rôle of gravity in the dynamics of vacuum geometry is being conceded to the remaining fundamental interactions. The ground state in KA2 is of the form $\mathcal{M}^{n+1} = \mathcal{C}^{n+1} \times \mathbb{R}^3$, where the static (averaged-out over scales $L \gg L_o$) $\mathcal{C}^{n+1}$ carries effective torsion as relic of the deeper vacuum dynamics at Planck scale. For the simplest non-trivial implementation of KA2, the Bianchi IX subclass of $SU(2)$-invariant $B_{IX}^4$ provides the $\mathcal{C}^5 = B_{M}^4 \times S^1$, with the $S^1$ coming from ‘augmentability’, a complement to compactification. The classical action involves (i) the gravitational and EW sectors in elegant hierarchy, (ii) the higgsless emergence and full calculability of the EW gauge bosons masses and (iii) gravity as a necessarily effective field, hence non-quantizable. A conjectured $\mathcal{C}^{n+1}$ with $n \leq 7$ (to adjoin the strong interaction) toward a standard model 2, might also offer novel perspectives for supergravity.

Keywords: Kaluza-Klein theories, Taub string, hierarchy, torsion, higgsless EW masses, compactification, augmentability, standard model, supergravity, quantization of gravity.
1 Introduction

The main task at LHC may be impeded by the Higgs sector of the standard model, but the latter will require even deeper reform, if the former folds at LHC, because of its other fundamental problems, notably on hierarchy and the quantization of gravity. Curiously related, forty years before the collective formulation of the Higgs mechanism, the Kaluza ansatz was likewise received as a ‘clever artifact’ (for the enlargement of the then young theory of general relativity), to be likewise elevated subsequently to a fundamental notion, but its geometric elegance has remained unquestionably unique all along. The fundamental interactions can be segregated by physical aspects (dimension-less vs -full couplings and quantization) but not by a priori geometrical ones in a unified higher-dimensional context. Nevertheless, one may resort to the approach towards full geometrization via the standard Kaluza ansatz [1], for a complementary KA2 approach, in the sense that, instead of appending $n$ Planck-scale ($L_o$) compact SL dimensions to ordinary general-relativistic 4D spacetime, one can augment $n$ such dimensions of a $C^{n+1}$ proper vacuum [2] by 3 large ones.

By KA2, the central rôle of gravity in the dynamics of vacuum geometry is being conceded to the remaining fundamental interactions. The ground state in KA2 is of the form $\bar{M}^{n+4} = \bar{C}^{n+1} \times \mathbb{R}^3$, where the static (averaged-out over scales $L \gg L_o$) $\bar{C}^{n+1}$ carries effective torsion [2], [3] as relic of the deeper vacuum dynamics at Planck scale. The requirement of augmentability (a complement to that of spontaneous compactification) will curtail the already-small class of $C^{n+1}$. The latter can be a homogeneous space [4], [5] with vigorous or even chaotic dynamics, like Misner’s elusive mixmaster $B_4^M$ [6] in the Bianchi IX subclass of left-$SU(2)$ invariant $B_4^{IX}$, which also includes the Taub string [2], a pp wave $B_4^T$. In the static $\bar{B}_4^M$, $\bar{B}_4^T$, the effective torsion $\bar{T}^A$ (parallelizing in the second case) is explicitly calculable via the effective loss of the Ricci flatness in $B_4^M$, $B_4^T$ [2]. The simplest possible non-trivial ground state for K-A2 involves a $\bar{M}^8$ with a $C^5 = B_M^4 \times S^1$, where $\bar{B}_M^4$ is chosen for its round rather than squashed $S^3$; the $S^1$ factor is actually imposed by augmentability, as we will see, whereby the topology of the SL sections and the transitive $SU(2)$ invariance on them must be enlarged to at-least $S^3 \times S^1$ and $SU(2) \times U(1)$, respectively. For a comparative view, we will also cite the standard $M^8_o$ ground state (with Minkowski’s $M^4_o$) as depicted in the schemes

$$M^8_o(e^A, \Gamma^A_B) := M^4_o \times (S^3 \times S^1) \xrightarrow{\delta e} M^8(e^A, \Gamma^A_B), \quad (1.1)$$

$$\bar{M}^8(\bar{e}^A, \bar{\Gamma}^A_B) := (\bar{B}_M^4 \times S^1) \times \mathbb{R}^3 \xrightarrow{\delta \bar{e}, \delta \bar{\gamma}} \bar{M}^8(\bar{e}^A, \bar{\gamma}^A_B), \quad (1.2)$$

for the standard vs the KA2 approach. The difference between $M^8_o$ and $\bar{M}^8$ may at first appear to be rather trivial, because they both have the same topology and metric (or $e^A$ Cartan frames), hence also identical $\bar{\Gamma}^A_B$ Christoffel connections, so their only difference is
the presence of the effective torsion $T^A$ in the $\tilde{\gamma}^A_B$ connection of $\mathcal{M}^8$. Nevertheless, their difference in perspective and results will turn out to be fundamental. In either case we arrive at a ‘low-energy’ configuration, the $M^8(e^A, \Gamma^A_B), M^8(e^A, \gamma^A_B)$, respectively.

In the standard case, the process of starting with the $M^8_0(e^A, \Gamma^A_B)$ ground state to arrive by (1.1) at $M^8(e^A, \Gamma^A_B)$ is geometrically viewable as a tilt from excitation of the frames from their value $\bar{e}^A$ in $M^8_0$ to $e^A = \bar{e}^A + \delta \bar{e}^A$ in $M^8(e^A, \Gamma^A_B)$. The physical content of this excitation is, of course, the $SU(2) \times U(1)$ gauge-field potentials $A^I$, which, in the case of (1.2) for the KA2 approach, must also excite the torsion $T^A$ in $M^8$. By the holonomy theorems and the Cartan structure equations for any $(\mathcal{R}^A_B, T^A)$ set [3], the torsion $T^A$ (field-content and scale) is completely independent from the Riemannian part $R^A_B$ of the curvature $\mathcal{R}^A_B$. Thus, excitations under the KA2 approach to reach $M^8(e^A, \gamma^A_B)$ in (1.2) must be of the ‘metric and connection’ Palatini type, namely independent excitations of frames and of torsion, so they will necessarily involve (beyond $\kappa_0, L_0$) two new independent scales, the $\kappa$ and $L_1$, respectively. Classically they can be of virtually any amplitude, limited only by the strength $\kappa_0^{-2}$ of the Taub-string, which is of Planck scale, and likewise for $\mathcal{B}^4_M$. However, as with the otherwise stable Minkowski vacuum in the standard approach, the addition ‘by-hand’ of any mass in $\mathcal{B}^4_1$ or $\mathcal{B}^4_M$ would cause a mathematical singularity [5]. As long as this cannot be averted by the overlying torsion, mass terms in the respective actions can be generated only effectively by the geometry, or the vacuum stability will be lost.

Notes on notation: The indices $A, B, M \ldots$ run as $M = (\mu; m) = (0, 5, 6, 7; 1, 2, 3, 4)$ with $(1, 2, 3, 4)$ in the compact dimensions. In all our Cartan frames and duals $(e^M, E_N)$ we employ orthonormal $g_{AB} = \text{diag}(-1, +1, \ldots, +1)$ and $I = (i, 4) = (1, 2, 3, 4)$ indices for the $SU(2) \times U(1)$ left-invariant 1-forms with $d\ell^i = -\frac{1}{2} \epsilon_{ijk} \ell^j \ell^k$, $d\ell^4 = 0$. Due to isometries on $S^3 \times S^1$ there exist four transitive Killing vectors $\Xi_I$ and their components $\Xi_I^\mu$ remain invariant under both types of the Kaluza ansatz. Commutation relations between the $L_\mu$, $\Xi_\nu$ and Lie derivatives $\mathcal{L}_{\Xi_I}$ (by use of the duality relation $\ell^m(L_n) = \delta^m_n$, etc.) can be summarized as [1]

$$[L_I, L_m] = \delta^k_m \epsilon_{jk} L_i, \quad [\Xi_I, \Xi_m] = \delta^k_m \epsilon_{jk} \Xi_i, \quad \mathcal{L}_{\Xi_I} L_m := [\Xi_I, L_m] = 0, \quad \mathcal{L}_{\Xi_I} \ell^m = 0. \quad (1.3)$$

The general connection $\gamma^M_N$ and the Christoffel $\Gamma^M_N = \Gamma^M_{NP} e^P$ in the covariant derivatives $\mathcal{D}, \mathcal{D}$, respectively, are antisymmetric in $M, N$ just like the contorsion tensor-valued 1-form $K^M_N$ in $\gamma^M_N = \Gamma^M_N + K^M_N$ with $D e^M := d e^M + \Gamma^M_N e^N \equiv 0, \quad D e^4 = d e^M - \Gamma^M_N E_N \equiv 0$. The general curvature $\mathcal{R}^A_B$ includes its Riemannian part $R^A_B := d\gamma^A_B + \Gamma^A_P \wedge \gamma^P_B$, with the Weyl and Ricci tensors $W^A_{BMP}$ and $R_{MN} = R^P_{MPN}$. Cartan’s first and second structure equations involve the general curvature $\mathcal{R}^M_N$ and the torsion $T^M$ 2-forms as [3]

$$\mathcal{R}^A_B := d\gamma^A_B + \gamma^A_P \wedge \gamma^P_B = R^A_B + DK^A_B + K^A_P \wedge K^P_B = \frac{1}{2} \mathcal{R}^A_{BPN} e^N \wedge e^P, \quad (1.4)$$

$$T^M := D e^M = de^M + \gamma^M_N \wedge e^N = K^M_N \wedge e^N = \frac{1}{2} T^M_{NP} e^N \wedge e^P. \quad (1.5)$$
2 Tilting the frames in $\tilde{\mathcal{M}}^8(\bar{e}^A, \bar{\gamma}_B^A)$ towards $\mathcal{M}^8(e^A, \gamma_B^A)$

To implement K-A2, we must fix the frames etc for $\tilde{\mathcal{M}}^8(\bar{e}^A, \bar{\gamma}_B^A)$ in (1.2), then proceed with the tilt $e^A = \bar{e}^A + \delta \bar{e}^A$ in terms of $\mathcal{A}^I$ and (in the next section) with the excitation $\delta \tilde{\mathcal{A}}^{\mu}$. From $(\bar{e}^M; \bar{E}_N) = (\bar{e}^\mu = \delta^\mu_\mu dx^\mu, \bar{e}^m = L_0 \ell^m, \bar{E}_\nu = \delta_\nu^\mu \partial_\mu, \bar{E}_n = L_0^{-1} L_n)$, with trivial vierbeins $\delta^\mu_\mu$ for holonomic $\bar{e}^\mu$, we find $\bar{\gamma}_B^A = \Gamma_B^A + \bar{K}_B^A$ and the non-vanishing $\bar{\gamma}_j^i = 2\bar{\Gamma}_j^i = 2\bar{K}_j^i = \epsilon^i_{jk} \epsilon^k$; the Ricci and scalar contractions from the Riemannian part $(\bar{\mathcal{R}}^i_{jkl})$ of the full curvature $(\bar{\mathcal{R}}^i_{jkl})$ are $R_{ij} = 1/2 L_0^{-2} n_{ij}, R = 3/2 L_0^{-2}$, identical to those of $M^8(e^A, \Gamma_B^A)$ in (1.1). For the Riemann-Cartan geometry in $\tilde{\mathcal{M}}^8(\bar{e}^A, \bar{\gamma}_B^A)$ of (1.2), the parallelizing torsion gives a $\bar{K}_j^i = 1/2 \epsilon^i_{jk} \epsilon^k$ in $\bar{B}_M^4$, hence $\bar{R}^i_{jkl} = 0$. The vanishing of the Hilbert-Einstein-Cartan Lagrangian $\mathcal{L}_{\text{HEC}} = \mathcal{R}$ for $\tilde{\mathcal{M}}^8$ offers harmless simplicity until (4.1). The orthonormality relations between the Killing vectors $\Xi_I$ can be expressed in terms of a continuous angle parameter $\theta \in (0, \pi/2)$, the slicing angle $\theta$ (to be distinguished from Weinberg’s mixing angle$^1$ $\theta_W$), as

$$\Xi_I^m \Xi_J^\nu \eta_{mn} = \left( L_0 \sin \theta \right)^2 \eta_{ij} \delta^i_1 \delta^j_1 + \left( L_0 \cos \theta \right)^2 \eta_{44} \delta^1_1 \delta^4_1. \quad (2.1)$$

The scale $L_0$ of the components is imposed by the frames $\bar{e}^A$ and the lengths $L_0 / \sin \theta$, $L_0 / \cos \theta$ are proportional to the radii of $S^3$ and $S^1$ in any particular slicing of the $S^3 \times S^1$ torus, as fixed by $\theta$. These $\Xi_I$ provide a basis for tangent vectors on $S^3 \times S^1$, just like the $L_m$ do. However, while the $L_m$ are ab initio left-invariant, by $\mathcal{L}_{\Xi_I} L_m = 0$ etc., the $\Xi_I$ cannot possibly form a left-invariant basis, due to the $\mathcal{L}_{\Xi_I} \Xi_J \not= 0$ relations from (1.3). Therefore, under ordinary circumstances, the $\Xi_I$ would be an odd and cumbersome (albeit fully legitimate) basis to employ in left-invariant environments, such as those involving the round or even the squashed $S^3$. This observation will be useful to us later on.

The excitation of the frames in $e^A = \bar{e}^A + \delta \bar{e}^A$ (etc., via $\ell^m (L_n) = \delta^m_n$) in KA2 is linear in the gauge potentials $\mathcal{A}^I$ and identical to that of the standard case as

$$\bar{e}^A \rightarrow e^A := \bar{e}^A + g [\Xi \cdot \mathcal{A}]^m \delta^A_m \iff \bar{E}_B \rightarrow E_B = \bar{E}_B - g [\Xi \cdot \mathcal{A}]_\nu \delta^\nu_B, \quad (2.2)$$

where $g$ is a scaleless coupling parameter$^2$ and the potentials enter through the components of the diagonal tensor $[\Xi \cdot \mathcal{A}]$ of mixed (1,1) rank (to be discussed shortly) defined as

$$[\Xi \cdot \mathcal{A}]_\nu^m := \Xi^m_i A^i \sin \theta + \Xi^m_i A^i \cos \theta, \quad [\Xi \cdot \mathcal{A}]_\nu^i = \Xi_i A^i \sin \theta + \Xi_i A^i \cos \theta. \quad (2.3)$$

The transformations in (2.2) can be viewed as trivially reversible with the simple transfer of the terms involving $g [\Xi \cdot \mathcal{A}]$ on one or the other side of those relations, so as to formally

$^1$Under gauge symmetry breaking, $\theta$ could be identified with whatever particular value the $\theta_W$ has.

$^2$The basic scales, like the $L_0$, are carried by the frames. All other quantities must have either derivable or inherently independent scale (like the $L_1$, to be identified with the EW), or be scaleless as with, e.g., all entries in (1.3). In the scaleless coupling $g = \sqrt{2} \kappa / L_0$, the denominator de-scales $\Xi_I^m$ (circumstantially scaled by $L_0$ in (2.1)) and $\kappa$, identifiable as the $\sqrt{8\pi G_N}$ gravitational coupling, provides the missing scale.
also define $\bar{e}^A$ in terms of $e^A$ etc., with the same components of $g[\Xi \cdot A]$. This reveals an underlying tilt invariance, whereby tensorial components like $\Xi^m, A^\nu_m, [\Xi \cdot A]^m, [\Xi \cdot A]^m$ etc remain the same in either of $(\bar{e}^M, \bar{E}_N), (e^M, E_N)$. This is due to the ‘diagonality’ of $e^\mu = \bar{e}^\mu$ $E_n = \bar{E}_n$ and survives the generalized excitation of $\bar{e}^\mu$ to $e^\mu = e^\mu_\mu dx^\mu$, etc., to be introduced later-on by (4.1). This tilt invariance can simplify calculations considerably; its members also include volume elements like $\bar{\varepsilon} = \varepsilon$ and derivations from $\bar{E}_n = E_n$, but not ordinary partial derivatives from $E_\mu \neq E_\mu$. For the latter kind, by excitation of $\partial_\mu$ under the tilt of the frames in (2.2), a rigorous gravito-EW ‘minimal-coupling’ rule can be uncovered as

$$\bar{E}_\mu \rightarrow E_\mu = \bar{E}_\mu - g[\Xi \cdot A]_\mu \implies \partial_\mu \rightarrow \partial_\mu - g(\xi, A_\mu \sin \theta + \xi_4 A^4_\mu \cos \theta) \ . \quad (2.4)$$

To proceed with the calculation of the $L_{HEC}$ Lagrangian, we first note that we have not yet arrived at $\mathcal{M}^8(e^A, \gamma^A_B)$ of (1.2), because we have not yet excited the connection. Accordingly, we will employ an asterisk $\ast$ on our not-yet-excited intermediate $\gamma^M_N$, which, of course, has changed anyway from the $\bar{\gamma}^M_N$ value, due to the $\delta \bar{e}^A$ excitation, so its Christofel part is identical to the one involved in the standard Kaluza ansatz. The calculation towards the intermediate $L_{HEC}^* = \mathcal{R}^*$ (modulo surface terms) involves the basic preliminary result

$$de^m = g[\Xi \cdot F]^m - \frac{1}{2L_0} \delta^m_i \epsilon^i_{jk} e^j \wedge e^k, \quad [\Xi \cdot F]^m := \Xi^m F = \sin \theta + \Xi^m_4 \cos \theta F^4, \quad (2.5)$$

with the gauge-field strength $F$ as defined below. Its kinetic term emerges in $L_{HEC}^*$ as in the standard treatment, while the rest, formally included in $[GR + GEW \text{ terms}]$ sector, relates to gravity and torsion. In view of the $\mathcal{R} = 0$ result in $\mathcal{M}^8$ there will be no effective cosmological constant from reduction to 4 spacetime dimensions. These aspects of the

$$L_{HEC}^* = [GR + GEW \text{ terms}] - \frac{1}{2} \kappa^2 F^2, \quad F^I := dA^I + \frac{1}{2} g\sin \theta \delta^I_i \epsilon_{jk} A^j \wedge A^k, \quad (2.6)$$

Lagrangian will not be influenced by the mentioned generalization in (4.1), and our remarks following (2.1) do apply, of course, to the gauge-invariant environment established by (2.6). The slicing angle $\theta$ therein is fully redundant (with a trivial re-definition of $g\sin \theta$) and we could have dispensed with it and the Killing vectors altogether. In fact, it would have been easier to arrive at (2.6) by simply employing, instead of $(e^M, E_N)$, a left-invariant frame. For that, we could have simply used $L_o \delta^m_i A^I$ instead of $[\Xi \cdot A]^m$ in (2.2), and then proceed as usual to verify the claim. We conclude that, as long as the gauge symmetry is respected, any particular slicing of the torus is as good as any other, so the slicing angle $\theta$ must drop out in a symmetric environment, and it does. Indeed, the $\theta$-dependence of the $L_0 / \sin \theta$ and $L_0 / \cos \theta$ radii in the orthonormality relations between the Killing vectors in (2.1) works in conjunction with the standard choice in (2.3) for the dependence of $[\Xi \cdot A]$ on $\theta$, so the slicing angle is precisely canceled out. However, this seemingly ‘useless’ involvement of $\theta$ will prove crucial and irreplaceable for the implementation of the upcoming gauge-symmetry breaking by the KA2 approach, as we’ll see in the next section.
3 Excitation of the torsion to arrive at $\mathcal{M}^8(e^A, \gamma^A_B)$

For completion of the KA2 approach in (1.2), our last step involves the excitation of the torsion to $T^M = \bar{T}^M + \delta T^M$ and the induced $K^M_N = \bar{K}^M_N + \delta \bar{K}^M_N$. Both excitations are linear in $\mathcal{A}^I$ (as with (2.2) for $\delta \bar{e}^M$) and linearly related among themselves (by (1.5) etc) as

$$\delta \bar{K}_{MNP} = -\frac{1}{2}(\delta \bar{T}_{MNP} + \delta \bar{T}_{NPM} - \delta \bar{T}_{PMN}).$$  \hspace{1cm} (3.1)

However, before we proceed with the calculation of this (proportional to $\mathcal{A}^I$ and scaled by the mentioned $L_{1}$) $\delta \bar{T}^M$, we must make sure that this excitation of the connection is indeed independent, namely that the field-content of $\mathcal{A}^I$ has not been already exhausted towards the $\delta \bar{e}^M$ tilt of the frames in (2.2). As we’ll see in the last section, unspent degrees of freedom in $\mathcal{A}^I$ do survive in this case and they are precisely enough to accommodate the transverse degrees of freedom of the EW gauge bosons. We can easily find that any general connection $\gamma^M_N$, enlarged to $\gamma^M_N + K^M_N$, induces the $K^{MPN} K_{NPM} - K^{MPN}_{M} K^{N}_{PN}$ contribution (modulo surface terms) to an accordingly enlarged $\mathcal{L}_{\text{HEC}}$ Lagrangian. Thus, when the intermediate connection $\gamma^*^M_N$, is excited to $\gamma^M_N = \gamma^*^M_N + \delta \bar{K}^M_N$, the intermediate $\mathcal{L}^*_\text{HEC}$ in (2.6) will be accordingly elevated to final form with quadratic-in-$\delta \bar{K}$ terms, as

$$\mathcal{L}_{\text{HEC}} = [\text{GR} + \text{GEW terms}] - \frac{1}{2} \kappa^2 \mathcal{F}^2 + \delta \bar{K}^{APB} \delta \bar{K}_{BPA} - \delta \bar{K}^{MPN}_{M} \delta \bar{K}^{N}_{PN}. \hspace{1cm} (3.2)$$

Due to the implicit presence of quadratic-in-$\mathcal{A}^I$ terms (in the two last ones on the r.h.s.), we have already lost the $SU(2) \times U(1)$ left invariance which had previously covered the entire $\mathcal{L}^*_\text{HE}$ in (2.6), so gauge-symmetry breaking has already occurred in the $\mathcal{L}_{\text{HEC}}$ of (3.2), as a result of the excitation $\delta \bar{T}^M$ of the connection.

To replace $\sim$ with precise equality in $\delta T^M = \frac{1}{2} \delta \bar{T}^M_{NP} e^N \wedge e^P \sim L_{1}^{-1} \mathcal{A}^I$, we note that the missing tensorial factor on the r.h.s. must: depend only on the Killing vectors $\Xi_j$, have exactly one free group-index $I$ (to saturate the free-one on $\mathcal{A}^I$) and balance the rest of the free indices in that relation. To accomplish that, we must exploit the already-installed breaking of gauge invariance in (3.2), in the sense that there exists an unknown but specific angle $\theta_W$, by which the $S^3 \times S^1$ torus can be viewed as already sliced. By fixing (in-retrospect agreement with standard convention) the $I = 3$ direction, we can introduce a fixed mixing as $\Xi^b_{(W)} \sim (\Xi^b_2 \sin \theta_W + \Xi^b_4 \cos \theta_W)$ without loss of generality. This $\Xi^b_{(W)}$ times a $\Xi_I^p$ for the required free index $I$ (and antisymmetry for torsion) accommodates fully and precisely all the requirements on our tensorial factor as $\Xi^b_{(W)} \Xi^p_I$, so the final result is unique as

$$\delta T^p = \frac{g}{L_{1}} \eta^{pa} \eta_{bm} \eta_{pn} \Xi^b_{(W)} \Xi^p_I \mathcal{A}^I_{a} e^m \wedge e^n, \quad \Xi^b_{(W)} := \frac{1}{\sqrt{2L_{0}}} (\Xi^b_2 \sin \theta_W + \Xi^b_4 \cos \theta_W). \hspace{1cm} (3.3)$$

Thus, by (3.1), the only non-vanishing independent components of $\delta T^M$ and $\delta \bar{K}^M_N$ are

$$\frac{1}{2} \delta T^{npb} = -\delta \bar{K}^{npb} = \delta \bar{K}^{bnp} = \frac{g}{L_{0} L_{1}} \eta^{\mu \nu} \Xi^b_{(W)} \Xi^p_I \mathcal{A}^I_{\nu}. \hspace{1cm} (3.4)$$
To find explicitly the mass term already present in (3.2), we may re-write the latter as
\[
\mathcal{L}_{\text{HEC}} = [\text{GR} + \text{GEW terms}] - \frac{\kappa^2}{2} \mathcal{F}^2 - \kappa^2 M_{IJ} A_I^I A_J^I \eta^{\mu \nu}, \tag{3.5}
\]
wherefrom, by the tracelessness of contorsion from (3.4), we can read-out the identification
\[
\kappa^2 M_{IJ} A_I^I A_J^I \eta^{\mu \nu} = -\delta K^{APB} \delta K_{BPA}. \tag{3.6}
\]
The straightforward substitution of (3.4) in (3.6) quantifies the mass matrix etc., as
\[
M_{IJ} = (L : L_1)^{-2} \left[ (\Xi (W) )^2 \Xi_I^P \Xi_I^P - (\Xi (W) \cdot \Xi) I (\Xi (W) \cdot \Xi) J \right], \tag{3.7}
\]
where, having used the orthonormality relations from (2.1), we must now set \( \theta = \theta_W \). With \((\Xi (W) )^2 = 1\), as normalized in (3.3), we may express the mass-term in (3.7) as
\[
M_{IJ} = (L_1 \sin \theta_W)^{-2} \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & \frac{1}{2} & -\frac{1}{2} \tan \theta_W \\
0 & 0 & -\frac{1}{2} \tan \theta_W & \frac{1}{2} \tan^2 \theta_W
\end{pmatrix}. \tag{3.10}
\]

Either
\[
\Delta_I^J = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -\cos \theta_W & \sin \theta_W \\
0 & 0 & +\sin \theta_W & \cos \theta_W
\end{pmatrix}
\]
or
\[
\begin{pmatrix}
1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\
-i/\sqrt{2} & i/\sqrt{2} & 0 & 0 \\
0 & 0 & -\cos \theta_W & \sin \theta_W \\
0 & 0 & +\sin \theta_W & \cos \theta_W
\end{pmatrix}
\]
diagonalizes \( M_{IJ} \) to its eigenvalues as
\[
M_I^J = (L_1 \sin \theta_W)^{-2} \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & \frac{1}{2} \cos \theta_W & -0 \\
0 & 0 & 0 & 0
\end{pmatrix} = \begin{pmatrix}
m_W^2 & 0 & 0 & 0 \\
0 & m_Z^2 & 0 & 0 \\
0 & 0 & \frac{1}{2} m_Z^2 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}, \tag{3.12}
\]
so the gauge-boson masses are post-dicted as \( m_W = (L_1 \sin \theta_W)^{-1} \), \( m_Z = (L_1 \sin \theta_W \cos \theta_W)^{-1} \), and the \( \rho := m_W^2 / (m_Z \cos \theta_W)^2 \) parameter as \( \rho = 1 \). For the physical gauge bosons, actually read off (3.11) as \( W^\pm = (A^I \mp i A^I) / \sqrt{2} \), \( Z = -\cos \theta_W A^3 + \sin \theta_W A^4 \), \( B = \sin \theta_W A^3 + \cos \theta_W A^4 \), the \( \mathcal{L}_{\text{HEC}} \) of (3.5) takes on its standard expression, with the mass term therein as
\[
\kappa^2 M_{IJ} A_I^I A_J^I \delta^{\alpha \beta} = \kappa^2 \left( m_W^2 W^+ W^- + \frac{1}{2} m_Z^2 Z^2 \right). \tag{3.13}
\]
4 Discussion

In spite of its simplicity, the KA2 approach by (1.2) vs the standard (1.1) offers surprisingly far-reaching results and relevance to fundamental issues, hitherto unrelated. We will briefly expand toward some of them, after some clarifications and pending completions. As anticipated (in view of the $\mathcal{R}=0$ result in $\mathcal{M}^8$), in order to have a general curvature scalar $\mathcal{R}$ present in the [GR + GEW terms] general relativistic and gravito-EW sector in (3.5), we must allow for a generalized excitation of $\bar{e}^\mu$ to $e^\mu$ in (2.2), now re-defined in terms of the vierbeins $e^\mu_\rho$ and inverse $E^\rho_\nu$ (namely with $e^\mu_\rho E^\rho_\nu = \delta^\mu_\nu$) as

$$e^A = e^\mu_\rho dx^\rho \delta^A_\mu + (\bar{e}^m + g[\Xi \cdot A]^m) \delta^A_m \iff E_B = (E^\nu_\nu \partial_\nu - g[\Xi \cdot A]_\nu) \delta^A_\nu + \bar{E}_n \delta^A_n. \quad (4.1)$$

When we set out for the simplest non-trivial implementation of the KA2 approach, we initially anticipated to utilize one of the $\mathcal{M}^7 = \mathcal{B}^4_\mathrm{M} \times \mathbb{R}^3$ or $\mathcal{M}^7 = \mathcal{B}^3_\mathrm{M} \times \mathbb{R}^3$ ground states, instead of the finally employed $\mathcal{M}^8 = \mathcal{B}^4_\mathrm{M} \times S^1 \times \mathbb{R}^3$ in (1.2). The $S^1$ factor therein is not merely a ‘spectator’, but it is rather imposed by *augmentability* under KA2, as we’ll see. By general considerations [1], the 7-dimensional $\mathcal{M}^7$ proper vacuum would have involved a total of $7(7-3)/2 = 14$ independent states in its $\mathcal{L}_{\text{HEC}}$; the piecemeal count of the 2 graviton states in $\mathcal{M}^7$ plus the $SU(2)$ scalar and massless gauge boson states as $2 + 3(1) + 1 + 2 \cdot 3$ would have again given us precisely 14. This means that there would be no field-content left in the corresponding $\mathcal{A}^7$ for an independent variation of the torsion. Thus, for our $\mathcal{M}^7$ cases, we would end-up with a gauge-invariant $\mathcal{L}_{\text{HEC}}$ and no mass-term, hence with a KA2 approach redundant to the standard one. A non-redundant KA2 is achieved with the minimally augmented $C^5 = \mathcal{B}^4_\mathrm{M} \times S^1$ for the $\mathcal{M}^8 = \mathcal{B}^4_\mathrm{M} \times S^1 \times \mathbb{R}^3$ ground state in (1.2). Indeed, in this case, the general count of $8(8-3)/2 = 20$ states for $\mathcal{M}^8$ is not quite matched by the piecemeal count of states, *again* for massless (now $SU(2) \times U(1)$) gauge bosons, because $2 + 3(3+1)/2 + 3 + 1 = 17$. The surviving 3 degrees of freedom in $\mathcal{A}^7$ have been precisely enough to generate (by independent excitation of the torsion) the 3 transverse states of the mass term (3.13) in the Lagrangian (3.5). Thus, the notion of *augmentability* by KA2 emerges as complementary to the requirement for spontaneous compactification. As a result, $C^5 = \mathcal{B}^4_\mathrm{M} \times S^1$ is augmentable under KA2, but the $\mathcal{C}^4 = \mathcal{B}^1_{\mathrm{IX}}$ is not.

We are now better equipped to sum-up our results as follows.

(i) The gravitational and electroweak sectors have emerged in elegant *hierarchy* in a $\mathcal{L}_{\text{HEC}}$ of the form (3.5), in terms of the (identifiable as gravitational) coupling $\kappa$ and the EW scale $L_1$. The gravitational interaction is an effective one at scales $L >> L_0$ and its frames $e^\mu$, as defined by (4.1), are *in principle* calculable via Einstein’s equations. The latter will follow from any Lagrangian of the type (3.5), after the fashion of ‘electrovac equations’ and a ‘minimal-coupling’ rule, now elevated to gravito-EW vacuum equations and (2.4), respectively.
(ii) The higgsless emergence of the EW gauge-boson masses is fully calculable by (3.7-3.13), although the numerical value of $\theta_W$ would be calculable only with the employment of the $B^4_T$ of the Taub string (instead of the $B^4_M$) in (1.2), via the set of the radii of its squashed $S^3$, the $(1/\sqrt{3}, 1/\sqrt{3}, \sqrt{\pi/2} - 1)$ in units of $L_\circ$ [2]. The mass term in (3.5) has been produced by the geometry via excitation of the effective torsion, actually the only geometric element which could protect against mathematical singularities, if masses were to be added by-hand.

(iii) By KA2, if mathematical singularities (but not physical ones) were to be excluded from physical spacetime, the fundamental rôle of gravity in the dynamics of vacuum geometry is being conceded to the remaining fundamental interactions. Gravity does retain all its geometric aspects, but the dimensionful coupling $\kappa$ is now its only relation to Planck scale. At or close to that scale, where everything is actually part of a true proper vacuum, the meaning of a gravitational coupling is empty anyway. At the intermediate regime, where all other interactions are quantized (say, very widely around $L_1$), gravity would again be in a $L >> L_\circ$ environment, so it would remain classical there, as in ordinary 4D classical regime. It would then follow that gravity can only stand as an effective interaction or classical field in 4D, and as such it would have to be excluded from quantization.

Thus, by our findings via the KA2 approach, we may conjecture an augmentable $C^n$ to adjoin the strong interaction towards a standard model 2, which already includes gravity. In view of the reasonable $n \leq 7$ requirement, this would also offer the option of an analogous re-orientation in supergravity [1].

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