Nuclear Structure Aspects of the Neutrinoless $\beta\beta$-Decays

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In this article, we analyze some nuclear structure aspects of the $0\nu$ double beta decay nuclear matrix elements (NME). We give results for the decays of $^{48}\text{Ca}$, $^{76}\text{Ge}$, $^{82}\text{Se}$, $^{124}\text{Sn}$, $^{128}\text{Te}$, $^{130}\text{Te}$, and $^{136}\text{Xe}$, using improved effective interactions and valence spaces. We examine the dependence of the NME’s on the effective interaction and the valence space, and analyze the effects of the short range correlations and the finite size of the nucleon. Finally we study the influence of the deformation on the values of the NME’s.

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I. INTRODUCTION

The double beta decay is the rarest nuclear weak process. It takes place between two even-even isobars, when the decay to the intermediate nucleus is energetically forbidden or hindered by the large spin difference between the parent ground state and the available states in the intermediate nuclei. It comes in three forms: The two-neutrino decay $\beta\beta_{2\nu}$, is just a second order process mediated by the weak interaction. It conserves the lepton number and has been already observed in a few nuclei. The second mode, the neutrinoless decay $\beta\beta_{0\nu}$, needs an extension of the standard model of the electroweak interactions as it violates lepton number. A third mode, $\beta\beta_{0\nu,\chi}$ is also possible in some extensions of the standard model and proceeds via emission of a light neutral boson, a Majoron $\chi$. The last two modes, not yet experimentally observed, require massive neutrinos –an issue already settled by the recent measures by Super-Kamiokande [1], SNO [2] and KamLAND [3]. Interestingly, the double beta decay without emission of neutrinos would be the only way to sign the Majorana character of the neutrino and to distinguish between the different scenarios for the neutrino mass differences. In what follows we shall concentrate mostly in the $\beta\beta_{0\nu}$ mode. The uprising of interest in the observation of the $\beta\beta_{0\nu}$ decay was somewhat obscured by the analysis of the status of the calculations of the nuclear matrix elements, that enter in the lifetime of the decay together with the effective neutrino mass, made in ref. [4]. The authors concluded on a very pessimistic note, saying that the spread of the available values for the nuclear matrix elements was such that there was no hope to translate the experimental signal into useful input for the theories beyond the standard model. A critical assessment of the results of the many calculations available in the framework of the quasi particle random phase approximation (QRPA) has been made recently [5], with a much more optimistic conclusion. In this article we want to continue improving upon the reliability of the calculated nuclear matrix elements (NME) in the framework of large scale applications of the Interacting Shell Model (ISM). In addition, we put forward a process of benchmarking the different approaches, and study the stability of the NME’s under reasonable modifications of the nuclear structure inputs of the calculations.

II. DOUBLE BETA DECAYS

As we have already mentioned, some nuclei, otherwise nearly stable, decay emitting two electrons and two neutrinos ($2\nu$ $\beta\beta$) by a second order process mediated by the weak interaction, that has been experimentally measured in several favorable cases. The decay probability contains a phase space factor and the square of a nuclear matrix element

$$[T_{1/2}^{2\nu}]^{-1} = G_{2\nu}|M_{GT}^{2\nu}|^2$$

If the neutrinos are massive Majorana particles, the double beta decay can take place without emission of neutrinos ($0\nu$ $\beta\beta$). In this case the transition is mediated by terms that go beyond the standard model. The decay probability contains a phase space factor $G_{0\nu}$, the effective electron neutrino mass (a linear combination of the mass eigenvalues whose coefficients are elements of the mixing matrix) and the nuclear matrix element $M_{GT}^{(0\nu)}$.

$$[T_{1/2}^{(0\nu)}]^{-1} = G_{0\nu}\left(M_{GT}^{(0\nu)} - \left(\frac{g_V}{g_A}\right)^2 M_F^{(0\nu)}\right)^2 \left(\frac{m_\nu}{m_e}\right)^2$$

The $2\nu$ matrix element can be written as:

$$M_{GT}^{(2\nu)} = \sum_i \frac{\langle GD(J) | \hat{\sigma}^- | F \rangle (1^+ \hat{\sigma}^- | F \rangle)}{\Delta E_i}$$

The nuclear structure information entering in this matrix element consists on the wave function of the ground state of the father nucleus (F) and the wave function of the state of the grand daughter, GD(J), nucleus (J=0, J=1, or J=2) to which the decay proceeds. In addition, the wave functions and excitation energies of all
the $1^+$ states in the odd-odd daughter nucleus are in principle necessary. The spin-isospin operators in the nuclear medium are quenched by a factor $q \approx 1/g_A$, and this quenching factor is also required to reproduce the experimental data of the $2\nu$ double beta decays.

To compute exactly the $0\nu$ matrix element, we would also need the wave function of the ground state of the father nucleus and of the ground state grand daughter, but in addition we would need all the wave functions and excitation energies of all the $J^+$ states in the odd-odd daughter nucleus. Most conveniently, the matrix elements can be approximately obtained in the closure approximation, that is good to better than 90% due to the high momentum of the virtual neutrino in the nucleus ($\approx 100$ MeV) Hence, the matrix element can be written as:

$$M_{\nu}^{(0\nu)} = (\mathcal{G}D|h(\vec{r}_1 - \vec{r}_2)|)(\hat{\sigma}_1 \cdot \hat{\sigma}_2)(t_1^\dagger t_2)|F\rangle \quad (4)$$

where $h(\vec{r}_1 - \vec{r}_2)$ is the neutrino potential ($\approx 1/r$). In this approximation no knowledge of the intermediate nucleus is directly implied.

The transition operators are usually obtained from the Hamiltonian of Doi et al. However, according to recent claims, additional terms originating in the coupling to the virtual pions, should give non-negligible contributions. The finite size of the nucleon and the short range correlations need to be taken into account in the calculation of the two body matrix elements of the $0\nu$ two-body transition operators as well.

Perhaps the most relevant issue for our mastering on the neutrinoless double beta decays is to get a better insight in the physical content of the two body transition operator. Only with this knowledge could one decide whether the nuclear wave functions that enter into the calculation of the NME’s contain the degrees of freedom that correspond to the correlations to which the operator couples dominantly.

The two body decay operators can be written generically as:

$$M_{\nu}^{(0\nu)} = \sum_J \sum_{i,j,k,l} M_J^{i,j,k,l} \left((a_i^\dagger a_j^\dagger)^J (a_k a_l)^J\right)^0 \quad (5)$$

In words, what the operator does is to annihilate two neutrons in the parent nucleus and to create two protons. The NME is the overlap of the resulting object with the grand daughter ground state.

The contributions to the $0\nu$ matrix element as a function of the $J$ of the of the decaying pair have a very telling structure as can be seen in Figure 1. The dominant contribution corresponds to the $J=0^+$ pairs, while all the other pairs have much smaller contributions, but all of them of sign opposite to the leading term. If the initial and final wave functions had seniority zero, only the leading term would contribute and the matrix element would be maximal. This is a first indication, relating the pairing content of the nuclear wave functions and the neutrinoless double beta decay operator.

### III. INTERACTING SHELL MODEL (ISM) AND QRPA CALCULATIONS

In the quest for better wave functions to describe the double beta decay processes, two main avenues have been explored. The interacting shell model in larger and larger valence spaces, with ever improving effective interactions, and the quasiparticle RPA. Ideally, both methods should be able to produce good spectroscopy for parent, daughter and grand-daughter, even better if it extends to a full mass region, correct total Gamow Teller strengths and strength functions, $2\nu$ matrix elements, etc. In brief, the goal is to have a description as close to perfect as possible of the dynamics of the nuclei involved in the transition. Until rather recently, ISM calculations that could encompass most of the relevant degrees of freedom of the nuclei of interest, were only available for the lighter emitters, and, as a consequence, most systematic studies were performed in the QRPA framework. As we will discuss in the next section, all the potential double beta emitters are now within the reach of the ISM except $^{150}\text{Nd}$, that, being deformed, is also out of the reach of the spherical QRPA calculations.

In the ISM approach, the valence spaces contain a number of orbits that is "small" compared to the QRPA, however, all the possible ways of distributing the valence particles among the valence orbits are taken into account. In the QRPA calculations, the number of active orbits is larger, but only 1p-1h and 2p-2h excitations from the normal filling are considered (and not all of them).

The effective interactions used in the ISM calculations are usually G-matrices whose monopole behavior is fitted to the spectroscopic properties of a large region of nuclei,
in general those comprised between to magic closures for
the neutrons and for the protons. In some cases the inter-
actions are plainly fitted to a set of experimental masses
and excitation energies. In the QRPA description, the
starting point is provided by realistic or schematic in-
}  

TABLE I: Update of the ISM $0\nu$ results  

|       | $\langle m_\nu \rangle$ for $T_{1/2} = 10^{25}$ y. | $M_{1G}(0\nu)$ | 1-$\chi_F$ |
|-------|-----------------------------------------------|----------------|-----------|
| $^{48}$Ca | 0.85                                           | 0.67            | 1.14      |
| $^{76}$Ge | 0.90                                           | 2.35            | 1.10      |
| $^{82}$Se | 0.42                                           | 2.26            | 1.10      |
| $^{124}$Sn | 0.45                                          | 2.11            | 1.13      |
| $^{128}$Te | 1.92                                         | 2.36            | 1.13      |
| $^{130}$Te | 0.35                                        | 2.13            | 1.13      |
| $^{136}$Xe | 0.41                                        | 1.77            | 1.13      |

All these spaces are accessible to large scale ISM
descriptions. The Strasbourg-Madrid [10] Shell-Model
 codes can deal with problems involving bases of $O(10^{10})$
Slater determinants, using relatively modest computa-
tional resources.

V. UPDATE OF THE ISM $0\nu$ RESULTS

In the valence spaces $r_3$-$g_{9/2}$ ($^{76}$Ge, $^{82}$Se) and $r_4$-$h_{11/2}$
($^{124}$Sn, $^{128-130}$Te, $^{136}$Xe) we have obtained high quality
effective interactions by carrying out multi-parametrical
fits [11] whose starting point is given by realistic G-
matrices [12]. With these interactions the beta decay
properties of a large set of nuclei are well reproduced.
The $2\nu$ double beta decay half-lives are found in rea-
sonably good agreement with the experimental results
as well. In the valence space proposed for $^{90}$Zr, $^{100}$Mo,
$^{110}$Pd and $^{116}$Cd, our results are still preliminary and
subject to further improvement both on the interaction
side and on the removal of the yet necessary truncations.

Our results are obtained in the closure approxima-
tion, with the short range and finite size corrections mod-
elled as described in [13]; using $r_0=1.2$ fm to make the
matrix element dimensionless; with $g_A=1.25$; and with-
out higher order contributions to the nuclear current. A
preliminary estimation of the higher order contributions
gives a reduction of the ISM NME’s in the range of 10%.
Our present best values are collected in Table I. $\chi_F$ is
defined as:

$$\chi_F = \frac{\left(\frac{g_V}{g_A}\right)^2}{M_{1G}(0\nu)}$$  (6)

Except in the case of doubly magic $^{48}$Ca, whose NME
is severely quenched, all the other values cluster around
a value $M_{1G}(0\nu)=2.5$. The limits of the effective neutrino
mass for a half life limit of $10^{25}$ y, that incorporate the
phase space factors, show a mild preference for some of
the potential emitters.

In Table II we compare the ISM results with the most
recent QRPA calculations including the higher order cor-
rections discussed before. The range of values of the
NME’s shown in the table is that given by the authors,
and derives from the different choices of $g_{pp}$ and $g_A$ used in the calculations, as well as from the use or not of a renormalized version of the QRPA. In addition, in order to make the comparison more transparent, we have selected the results that treat the short range correlations by means of a Jastrow factor. Overall, the two sets of QRPA calculations are now compatible. The ISM predictions of the NME’s are systematically smaller than the QRPA central values, except in the case of $^{136}$Xe. This nucleus is semi-magic, and the present experimental limit on its $2\nu$ decay half life is surprisingly large, perhaps indicating that some subtle cancelation mechanism is at work. For the others, a plausible explanation of the discrepancy, relating it to the implicit seniority truncations present in the spherical QRPA calculations, will be presented elsewhere.

VI. EXPLORING THE DEPENDENCES OF THE ISM RESULTS

A. Effective interaction

The ISM results depend only weakly on the effective interactions provided they are compatible with the spectroscopy of the region. For instance, in the $pf$ shell we have three interactions that work properly, KB3 [16], FPD6 [17] and GXPF1 [18]. Their predictions for the $2\nu$ and the neutrinoless modes are quite close to each other (see table III). Similarly, in the $r_3g$ and $r_4h$ spaces, the variations among the predictions of spectroscopically tested interactions are small (10-20%).

B. Finite size and short range corrections

There has been a certain debate among QRPA practitioners about the amount of the reduction of the NME’s due to the short range correlations [19]. The debate has become milder after the publication of ref. [14]. In the ISM description, once the finite size of the nucleon is taken into account by means of the standard dipole form factor [20], the effect of the short range correlations, that we model by a Jastrow ansatz following ref. [21], is about one half of what it would be without it. In fact, the ISM corrections for finite size and short range effects with the Jastrow prescription are quite close in percentage to the QRPA values. Short range and finite size corrections proceed mainly through the reduction of the $J^\pi=0^+$ pair contribution. Preliminary ISM calculations using a softer prescription for the short range correlations do not change this picture at all. In fact, the relative values of the ISM NME’s with respect to the QRPA ones seem to be independent of the choice of the (common) prescription used for the the short range corrections.

C. The influence of deformation

Changing adequately the effective interaction used in the calculations, we can increase or decrease the deformation of parent, grand-daughter, or both, at will. In this manner, we can gauge the effect of these variations on the decays. We have artificially changed the deformation of $^{48}$Ti and $^{48}$Cr adding an extra $\lambda Q \cdot Q$ term to the effective interaction. The results are presented in Fig. 2. Positive values of $\lambda$ increase the deformation while negative values reduce it. For zero values of both $\lambda$’s, $^{48}$Cr is already well deformed, while $^{48}$Ti is transitional. The circle to the upper left corresponds to the spherical-spherical situation, the square to the upper right to equally deformed Titanium and Chromium, and the diamond to the bottom left to a spherical Titanium and a very deformed Chromium. Our conclusion is that a large mismatch of deformation can reduce the $\beta\beta$ matrix elements by factors as large as 2-3. This exercise indicates that the effect of defor-
nation is very important and cannot be overlooked. We have reached similar conclusions for heavier emitters and a systematic exploration in realistic cases is under way.

VII. ISM CALCULATIONS IN QRPA-LIKE VALENCE SPACES

The ISM valence space for the $^{76}$Ge and $^{82}$Se decays has traditionally encompassed the orbits $1p_{\frac{3}{2}}$, $0f_{\frac{5}{2}}$, $1p_{\frac{1}{2}}$, and $0g_{\frac{9}{2}}$. In the QRPA, two major oscillator shells are taken into account; $0f_{\frac{5}{2}}$, $1p_{\frac{3}{2}}$, $0f_{\frac{7}{2}}$, $1p_{\frac{1}{2}}$, $0g_{\frac{9}{2}}$, $1d_{\frac{5}{2}}$, $0g_{\frac{7}{2}}$, $2s_{\frac{1}{2}}$, and $1d_{\frac{3}{2}}$.

As a first step toward a more complete benchmarking, we have evaluated the influence of the $2p$-$2h$ jumps from the $1f_{\frac{7}{2}}$ orbit – $^{56}$Ni core excitations – in our results for the $^{82}$Se decay. Similar calculations for the $^{76}$Ge decay are under way. The calculation in the full $rg$ space plus $2p$-$2h$ proton excitations from the $0f_{\frac{5}{2}}$ orbit gives a $20\%$ increase of $M_{\nu\nu}$, but probably we overestimate the amount of core excitations. Our $0f_{\frac{5}{2}}$ proton occupancies, 7.71 and 7.69 in $^{82}$Se and $^{82}$Kr are smaller than the BCS occupancies of Rodin et al. [5], 7.84 and 7.84. Therefore the above $20\%$ must be taken as a very conservative upper bound. The $2\nu$ matrix element remains nearly constant, even if the total Gamow-Teller strengths, $(GT^+)$ and $(GT^-)$, increase from 0.15 to 0.34 and from 20.5 to 26.9. The larger $GT^+$ strength is somehow compensated by a larger cancellation among the contributions of the different intermediate states and by a shift of the centroid of the strength toward higher energy.

We have also computed the $^{136}$Xe decay in the $r$-$h$ space including $2p$-$2h$ excitations from the $0g_{\frac{9}{2}}$ proton orbit and the matrix element increases less than $10\%$. In another set of calculations, we have included $2p$-$2h$ neutron excitations toward the $0h_{\frac{1}{2}}$ and $1f_{\frac{5}{2}}$. The occupancies that we obtain are relatively large ($0.25$ neutrons in each orbit) and the effect is to increase the matrix element by $15\%$. It is interesting to note that the increase with the two orbits simultaneously active is equivalent to that obtained including one or another orbit separately. Therefore there is no pile-up of the contributions of the small components of the wave function. As a preliminary conclusion, the ISM results seem to be robust against the inclusion of small components of the wave function.

VIII. CONCLUSIONS

Large scale shell model calculations with high quality effective interactions are available or will be in the immediate future for all but one of the neutrinoless double beta emitters. The theoretical spread of the values of the nuclear matrix elements entering in the lifetime calculations is greatly reduced if the ingredients of each calculation are examined critically and only those fulfilling a set of quality criteria are retained. A concerted effort of benchmarking between ISM and QRPA practitioners would be of utmost importance to increase the reliability and precision of the nuclear structure input for the double beta decay processes.

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