Morbidity-Mortality Table Construction for Eleven Chronical Diseases (ECD) Using Constant Force Assumption

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Abstract. This paper describes the construction of morbidity-mortality table if the dataset is only available at ages interval. The table used to measure the interaction between morbidity and mortality on the eleven chronic diseases (ECD). The probabilistic framework is based on the multiple-states Markov model. The transition intensities are assumed constant in each age interval. This paper finds high probability someone stays in state ECD and higher probability someone dies because of ECD than other causes.

1. Introduction
Chronical diseases are the leading causes of death and disability worldwide. Disease rates from these conditions are accelerating globally, advancing across every region, and pervading all socioeconomic classes. Moreover, chronic diseases cause a substantial financial burden and can push individuals and households into poverty. The World Health Report [1] By 2020, chronic diseases contribution is expected to rise to 73% of all deaths and 60% of the global burden of disease. Moreover, 79% of the deaths attributed to these diseases occur in developing countries. Only 20% of chronic disease deaths occur in high-income countries, while 80% occur in low and middle-income countries, most of the world’s population lives. Chronic diseases also affect both women and men almost equally.

The chronic diseases discussed in this paper are Mental and behavioral disorders, diseases of the nervous system, heart diseases, hypertension, stroke and cerebrovascular disease, aneurysm, chronic obstructive pulmonary disease, asthma, HIV, diabetes mellitus, acute kidney failure, and chronic kidney disease. Then, this selected chronic diseases named eleven chronic diseases (ECD).

When chronic diseases characterize the mortality profile, the analysis of mortality no longer suffices for a correct evaluation of the population’s health. As a consequence, in a demographic stage, characterized by an increase in life expectancy, a standard life table is no longer sufficient and it is necessary to analyze the morbidity profile of the population. Analysis of the available data needs a suitable morbidity-mortality model and their interaction.

This paper focused on the morbidity-mortality table constructions as a means to estimates the transition probabilities between states. The probabilistic framework is based on multiple-states Markov model. A person’s health condition considered as a state. Then the transition
intensities assume constant on each ages interval. The dataset used in this paper is hospital morbidity and mortality data in ages interval. Then, the numerical application is performed.

2. Multiple States Model
To represent the incidence of ECD, a multiple states model is used. Someone’s health condition considered as a status. Let $S(t); t = 0, 1, 2, ..., T$ be a Markovian process describing the development of a single policy in discrete time, where the random variable $S(t)$ represents the state of the process at time $t$ and $[0, T]$ be a fixed finite time horizon. The ECD incidences is modeled by a multiple states model with a finite state space $S = \{1 = \text{Non-ECD}, 2 = \text{ECD}, 3 = \text{dead}\}$ set of transitions according to Figure 1.

![Figure 1. Model for The Incidence of ECD](image)

3. Constant Force Assumption
In actuarial science and demography, $\mu_x$ is called the force of mortality. It represents the instantaneous rate of mortality at a certain age measured on an annualized basis. $\mu$ can be assumed exponentially distributed. Exponential distribution has a lack of memory property. Its shown in the force of mortality which is constant, i.e.

$$\mu_x = \mu_{x+t} = \mu$$

This exponential assumption also named linear assumption or Balducci assumption.

If $l_{x+t}$ exponentially distributed with $l_{x+t} = a.b^t$, then, for $t = 0$ we get $a = l_x$ and for $t = 1$ we get $b = \frac{l_{x+1}}{l_x}$. Exponential interpolation model formulated as follows

$$i_l_{x+t} = i_{l_x} \left(\frac{i_{l_{x+1}}}{i_{l_x}}\right)^t$$

$$= i_{l_x} (i_{p_x})^t$$

This exponential assumption also called constant force assumption.

4. Morbidity-Mortality Decrement Table
4.1. Flow Equation, Orientation Equation, Integration Equation
Morbidity-mortality table as a model to measure the interaction between morbidity and mortality of chronical diseases. The following notation is used.

- $l_x$: The number alive at exact ages $x$
- $d_{x+t}$: The number of decrements between exact ages $x$ and $x + t$
- $lL_x$: The number of person-years lived by the life table population between ages $x$ and $x + t$
\( M_x \): Observed central rate of decrement between exact ages \( x \) dan \( x + t \)

\( m_x \): The life table Central rate of decrement between exact ages \( x \) dan \( x + t \)

General multiple states model can be specified algebraically by three sets of equations, called flow, orientation, and integration equations.

(i) Flow Equation

Related the number alive in table 1 at age \( x + t \) to the number alive in table 1 at ages \( x \), and increment/decrement from table between ages \( x \) and \( x + t \).

\[
1_l x^{x+t} = 1_l x + \sum_{i=1, i \neq 1}^k t^d_{ix} - \sum_{i=1, i \neq 1}^k t^d_{ix} - \frac{1}{t} d^3_{ix}
\]

where

\( 1_l x \): Number alive ini table 1 at ages \( x \)

\( t^d_{ix} \): Increment from table \( i \) to table 1 between ages \( x \) and \( x + t \)

\( t^d_{ix} \): Decrement from table 1 to table \( i \) between ages \( x \) and \( x + t \)

(ii) Orientation Equation

Relate the observed central rate of decrement and life table central rate of decrement. Assume that each observed has stationer distribution.

\[
i^d_M x = i^d m x = \frac{i^d L x}{i L x}
\]

For all possible \( i \) and \( j \)

(iii) Integration Equation

Relate the function \( l_x \) and \( L_x \). If \( i l_x \) is assumed to be a exponential function of age over the interval \( x \) to \( x + t \).

\[
i l x + s = i l x \left( \frac{i l x + 1}{i l x} \right)^s
\]

Then the integration equation is as follows

\[
i^j L x = \int_0^t i l x + s ds
\]

\[
= \int_0^t i l x \left( \frac{i l x + 1}{i l x} \right)^s ds
\]

\[
= i l x \left( \frac{i l x + 1}{i l x} \right)^s \left[ \frac{1}{ln \left( \frac{i l x + 1}{i l x} \right)} \right] 0
\]

\[
= \frac{i l x + t - i l x}{ln (i l x + t) - ln (i l x)}
\]

\[
= \frac{t (i l x + t - i l x)}{ln (i l x + t) - ln (i l x)}
\]

Figure 1 shows three states for model ECD. Therefore the three set corresponding equation is as follows
(i) Flow Equation

\[ \begin{align*}
1l_{x+t} &= 1l_x - \frac{1}{t}d_x^2 - \frac{1}{t}d_x^3 \\
2l_{x+t} &= 2l_x + \frac{1}{t}d_x^2 - \frac{2}{t}d_x^3 \\
3l_{x+t} &= 3l_x + \frac{1}{t}d_x^3 + \frac{2}{t}d_x^3
\end{align*} \]

(ii) Orientation Equation

\[ \begin{align*}
1M_x^2 &= \frac{1}{t}d_x^2 \\
1M_x^3 &= \frac{1}{t}d_x^3 \\
2M_x^3 &= \frac{2}{t}d_x^3
\end{align*} \]

(iii) Integration Equation

\[ \begin{align*}
1L_x &= \frac{t(1l_{x+t} - 1l_x)}{\ln(1l_{x+t}) - \ln(1l_x)} \\
2L_x &= \frac{t(2l_{x+t} - 2l_x)}{\ln(2l_{x+t}) - \ln(2l_x)}
\end{align*} \]

4.2. Transition Probabilities
Transition probabilities define as follows

\[ t^jP_x^i = Pr[Y(x+s) = j|Y(x) = i] \quad (1) \]

\[ t^jP_x^i = \frac{t^jP_x^i}{1l_x} \quad (2) \]

where

- \( t^jP_x^i \): Expected number of person in closed group population in \( i \) at ages \( x \) who will also in state \( j \) at ages \( x+t \)
- \( 1l_x \): Expected number of person in state \( i \) at ages \( x \)
- \( i^hP_x^j \): expected decrement from state \( h \) to state \( j \) between ages \( x \) and \( x+t \) in closed group population in state \( i \) at ages \( x \)
- \( i^hP_x^j \): Total decrement from state \( h \) to state \( j \) after ages \( x+t \) at the same group population.

Therefore,

\[ \begin{align*}
i^hP_x^j &= i^hP_x^j - t^hP_x^j \\
i^hP_x^j &= - \sum_{h=1, h\neq j}^{k+1} i^hP_x^j + \sum_{h=1, h\neq j}^{k+1} i^hP_x^j \end{align*} \]
The flow equation for ECD model:

\[
\begin{align*}
\frac{d^2}{dt^2} x & = \frac{1}{t} \left( \frac{d}{dx} x \right) - \frac{1}{t} \left( \frac{d}{dx} x \right) + \frac{1}{t} \left( \frac{d}{dx} x \right) \\
\frac{d^3}{dt^3} x & = -\frac{1}{t} \left( \frac{d}{dx} x \right) + \frac{1}{t} \left( \frac{d}{dx} x \right) + \frac{1}{t} \left( \frac{d}{dx} x \right) \\
\frac{d}{dt} x & = \frac{1}{t} \left( \frac{d}{dx} x \right) - \frac{1}{t} \left( \frac{d}{dx} x \right) + \frac{1}{t} \left( \frac{d}{dx} x \right)
\end{align*}
\]

The orientation equation for ECD model:

\[
\frac{d^2}{dt^2} x = \frac{1}{t} \left( \frac{d}{dx} x \right) - \frac{1}{t} \left( \frac{d}{dx} x \right) + \frac{1}{t} \left( \frac{d}{dx} x \right)
\]

The integration equation will be:

\[
\frac{d}{dt} x = \frac{1}{t} \left( \frac{d}{dx} x \right) - \frac{1}{t} \left( \frac{d}{dx} x \right) + \frac{1}{t} \left( \frac{d}{dx} x \right)
\]

where \(i, j = 1, 2 ; h = 1, 2, 3 ; j \neq h\)

The integration equation will be:

\[
\frac{d}{dt} x = \frac{1}{t} \left( \frac{d}{dx} x \right) - \frac{1}{t} \left( \frac{d}{dx} x \right) + \frac{1}{t} \left( \frac{d}{dx} x \right)
\]

where \(i, j = a, b\) with initial condition \( \frac{d}{dt} x = \frac{d}{dt} x \) dan \( \frac{d}{dt} x = 0 \) where \( i \neq j \).

Transition probabilities matrix for ECD are as follow.

\[
\begin{bmatrix}
\frac{1}{t} p_{11} & \frac{1}{t} p_{12} & \frac{1}{t} p_{13} \\
0 & \frac{1}{t} p_{22} & \frac{1}{t} p_{23} \\
0 & 0 & 1
\end{bmatrix}
\]

Then the estimation of each transition probability is obtained as follows:
(i) Probability someone in state non-ECD at age \( x \) stay in state non-ECD at age \( x + t \)

\[
\begin{align*}
\frac{1}{t} M^2_x &= \frac{1}{t} d^2_x \\
\frac{1}{t} M^2_x &= \frac{1}{t} \frac{d^2_x}{L_x} \\
\ln(\frac{1}{t} p^1_x) &= -t \frac{1}{t} M^2_x \left( \frac{1}{d^2_x} \right) \\
\ln(\frac{1}{t} p^1_x) &= -t \left( \frac{1}{t} M^2_x + \frac{1}{t} M^3_x \right) \\
\frac{1}{t} p^1_x &= \exp(-t \left( \frac{1}{t} M^2_x + \frac{1}{t} M^3_x \right))
\end{align*}
\]

(ii) Probability someone in state non-ECD at age \( x \) becomes ECD at age \( x + t \)

\[
\begin{align*}
\frac{1}{t} P^2_x &= \frac{1}{t} \frac{d^2_x}{L_x} \\
\frac{1}{t} P^2_x &= \frac{1}{t} \frac{d^2_x - d^3_x}{L_x} \\
\frac{1}{t} P^2_x &= \frac{M^2_x}{L_x} - \frac{M^3_x}{L_x} \\
\frac{1}{t} P^2_x &= \frac{1}{t} M^2_x + \frac{1}{t} M^3_x \left( 1 - \exp\left( -t \left( \frac{1}{t} M^2_x + \frac{1}{t} M^3_x \right) \right) \right)
\end{align*}
\]

(iii) Probability someone in states non-ECD at age \( x \) die at age \( x + t \)

\[
\begin{align*}
\frac{1}{t} P^3_x &= \frac{1}{t} M^3_x \\
\frac{1}{t} P^3_x &= \frac{1}{t} M^3_x \left( 1 - \exp\left( -t \left( \frac{1}{t} M^2_x + \frac{1}{t} M^3_x \right) \right) \right)
\end{align*}
\]

(iv) Probability someone in state ECD at agr \( x \) stay in state ECD at age \( x + t \)

\[
\frac{2}{t} P^2_x = \exp\left( -t \frac{2}{t} M^3_x \right)
\]

(v) Probability someone in state ECD at age \( x \) die at age \( x + t \)

\[
\frac{2}{t} P^3_x = 1 - \exp\left( -t \frac{2}{t} M^3_x \right)
\]

4.3. Transition Intensity

For all \( i \neq j \), \( t P^{ij}_x \) are differentiable function \( t \). Define the transition intensity as follows:

\[
\mu_x^{ij} = \lim_{h \to 0} \frac{h P^{ij}_x}{h}
\]

Then we get the transition intensity as follows

\[
\begin{align*}
\mu_{x}^{12} &= \frac{1}{t} M^2_x \\
\mu_{x}^{13} &= \frac{1}{t} M^3_x \\
\mu_{x}^{23} &= \frac{2}{t} M^3_x
\end{align*}
\]
5. Numerical Application

5.1. Dataset Description

To investigate the differences in transition probabilities of ECD on women and man, the model is tested on both the genders. Specifically, our analysis focuses on Indonesian males and females using data from the year 2018. The data used in this paper is hospital morbidity-mortality data from RST.Tk III Wijayakusuma Purwokerto, Banyumas, Indonesia. The dataset is structured as follows:

- The number of death among ECD
- The number of death because of other causes
- The number of patient with ECD and non-ECD

The number of healthy people can be calculated as the difference between the Banyumas population and the number of the patient given by the hospital dataset.

5.2. Morbidity-Mortality Table

Ages interval in this paper follows the ages interval in the RST. Wijayakusuma Purwokerto morbidity dataset. The morbidity-mortality table constructed from the dataset is as follows:

Table 1. Morbidity-Mortality Life Table for Male ECD

| Entry Ages of Interval | Non-ECD | ECD | Death |
|------------------------|---------|-----|-------|
|                        | $l_x$   | $d_x$ | $q_x$ |
| 0                      | 69,917  | 124  | 1     |
| 5                      | 69,792  | 182  | 0     |
| 15                     | 69,609  | 269  | 4     |
| 25                     | 69,336  | 3,983| 19    |
| 45                     | 65,334  | 8,496| 50    |
| 65                     | 56,788  | 4,252| 45    |

Table 2. Morbidity-Mortality Life Table for Female ECD

| Entry Ages of Interval | Non-ECD | ECD | Death |
|------------------------|---------|-----|-------|
|                        | $l_x$   | $d_x$ | $q_x$ |
| 0                      | 65,673  | 114  | 1     |
| 5                      | 65,558  | 207  | 1     |
| 15                     | 65,350  | 623  | 2     |
| 25                     | 64,725  | 3,983| 7     |
| 45                     | 60,735  | 8,101| 38    |
| 65                     | 52,596  | 4,398| 35    |

The value of $l_x$, $d_x$, $q_x$ and $p_x$ can be obtained from dataset, and the value of $l_{x+t}$, $l_x$ and $q_x$ can be obtained using flow equation. From the Table 1 and 2, there are seen that the number of person alive in status 1 will decrease and the number of person alive in status 2 and 3 will increase.
Table 3. Central Rate of Decrement for Male

| Entry Ages of Interval | Non-ECD | ECD |
|------------------------|---------|-----|
|                        | $\frac{1}{2}M_x^2$ | $\frac{1}{2}M_x^3$ | $\frac{3}{2}M_x^3$ |
| 0                      | 0.0003550 | 0.00000029 | 0.00000000 |
| 5                      | 0.0002611 | 0.00000140 | 0.0063208 |
| 15                     | 0.0003872 | 0.00000558 | 0.0014186 |
| 25                     | 0.0029585 | 0.00001411 | 0.002616 |
| 45                     | 0.0069684 | 0.00004100 | 0.003921 |
| 65                     | 0.0022246 | 0.00002350 | 0.001508 |

Table 4. Central Rate of Decrement for Female

| Entry Ages of Interval | Non-ECD | ECD |
|------------------------|---------|-----|
|                        | $\frac{1}{2}M_x^2$ | $\frac{1}{2}M_x^3$ | $\frac{3}{2}M_x^3$ |
| 0                      | 0.0003475 | 0.00000030 | 0.00000000 |
| 5                      | 0.0003163 | 0.00000150 | 0.0056003 |
| 15                     | 0.0009579 | 0.00000314 | 0.003474 |
| 25                     | 0.0031758 | 0.00000315 | 0.002080 |
| 45                     | 0.0071604 | 0.00000336 | 0.001926 |
| 65                     | 0.0024958 | 0.00001999 | 0.000968 |

Based on equation 4, 5, and 6, the transition intensities equal to the central rate of decrement. Table 3 and 4 show that the transition intensities of a person moving from non-ECD state to ECD state increase with increasing ages and decrease at the age of 65 years old, in both genders. The same condition occurs in the intensity of the transition from non-ECD state to death state. On the other hand, the highest transition intensities from state ECD to state ECD and death occurs at age five-years-old for both genders.

Table 5. Transition Probabilities for Male

| Entry Ages of Interval | Non-ECD | ECD |
|------------------------|---------|-----|
|                        | $\frac{1}{2}P_x^2$ | $\frac{1}{2}P_x^3$ | $\frac{3}{2}P_x^3$ |
| 0                      | 0.99821217 | 0.00177353 | 0.00000000 |
| 5                      | 0.99737792 | 0.00260775 | 0.0063208 |
| 15                     | 0.99607809 | 0.00386444 | 0.0014186 |
| 25                     | 0.94228107 | 0.05744490 | 0.002616 |
| 45                     | 0.86919521 | 0.13003949 | 0.003921 |
| 65                     | 0.92433261 | 0.07487497 | 0.001508 |
Table 6. Transition Probabilities for Female

| Entry Ages of Interval | Non-ECD | ECD         |
|------------------------|---------|-------------|
|                        | $iP_x^1$ | $iP_x^2$ | $iP_x^3$ | $2P_x^2$ | $2P_x^3$ |
| 0                      | 0.9982489 | 0.0017359 | 0.0000152 | 1.0000000 | 0.0000000 |
| 5                      | 0.9968272 | 0.0031575 | 0.0000153 | 0.9455362 | 0.0544638 |
| 15                     | 0.9904361 | 0.0095333 | 0.0000306 | 0.9965322 | 0.0034678 |
| 25                     | 0.9383546 | 0.0615373 | 0.0001081 | 0.9958483 | 0.0041517 |
| 45                     | 0.8659916 | 0.1338327 | 0.0006257 | 0.9961544 | 0.0038456 |
| 65                     | 0.9157160 | 0.0836185 | 0.0006654 | 0.9966175 | 0.0033825 |

Table 5 and 6 shows the transition probability someone stays in state non-ECD decrease with increasing ages but increase at age 65 years old, in both genders. The same condition occurs in the probability of the transition from state non-ECD to state ECD and state non-ECD to state death. On the other hand, the highest transition probabilities from state ECD to state death occurs at age five-years-old for both genders.

6. Conclusions

This paper has described the construction of a morbidity-mortality table considering the aspects of the multiple-states life table theory with the Markov chain, assuming the number of people living in a state at age $x$ to $x+t$ is exponentially distributed. Then, this paper finds that for both genders, the transitions probability someone stays in state non-ECD decrease with increasing ages and increase at age 65 years-old, transitions probability someone move from state non-ECD to state ECD increase with increasing ages and decrease at age 65 years-old, and probability someone die because of non-ECD increase with the increasing ages. On the other hand, the probability of the transition someone stays in state ECD and move from state ECD to death does not have the same characteristic in both genders. However, the highest probability someone died because of ECD is identical for both gender, i.e., at age five-years-old. Furthermore, in both gender, from ages five-years-old probability someone dies because of ECD is higher than other causes.

References

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