A note on modeling a void generation and contraction in complex plasma

Abstract
An empty region (void) has been observed in many experiments on complex (dusty) plasma in particular which were taken place at the International Space Station. Here the known model of Avinash, Bhattacharjee and Hu of formation of void in complex plasma is considered to provide simulations of voids evolution from initially unstable flow mode. The model has been transformed into the divergent form. The MUSCL scheme of the second approximation order in space and in time has been applied for the hydrodynamic part of the model and a MINMOD limiter has been used for obtaining monotonic solution. As a result the dynamics of void’s growth and its partial contraction to the center have been obtained at the final stage of the void evolution.

Keywords: dusty plasma, complex plasma void, numerical methods, MUSCL scheme, plasma simulation

Nomenclature

\[ \begin{align*}
n_d, n_e & = \text{densities of dust and electrons components} \\
v_d, v_i & = \text{velocities of dust and ions components} \\
E & = \text{electric field currency} \\
F_d & = \text{ion-drag force} \\
D_\alpha & = \text{diffusion coefficient} \\
t, x, y & = \text{time and spatial variables} \\
a, b & = \text{fit parameters of ion-drag force approximation} \\
\mu & = \text{coefficient of ions mobility} \\
\alpha_0 & = \text{friction coefficient} \\
\tau_i, \tau_e & = \text{coefficients of normalized temperature for dust and ion species} \\
n_i & = \text{amount of the grid points in the x-direction} \\
\end{align*} \]

Introduction
Recently a field connected with the research of complex plasma is widely investigated.\(^1\)\(^-\)\(^3\) One of the first complex plasma patterns containing a domain which is free of dust particles (void) was observed in the course of the experiments on the board of the International Space Station.\(^4\) Afterwards similar patterns were found at the laboratory conditions.\(^4\)\(^-\)\(^6\) To describe such a pattern formation the model of Avinash, Bhattacharjee and Hu was introduced.\(^7\)\(^-\)\(^9\) In this model the evolution of the dynamics of a single symmetric void from an equilibrium state is described by electro-hydrodynamic equations which take into account the effect of an ion attraction force as a nonlinear function of the speed of ions. The algorithm for modeling the appearance of complex plasma void by means of the model has been presented for the case of cylindrical geometry of the electrical field.\(^6\) Also, generation of a single symmetric void and concentric symmetrical voids in unmoving flows of low-temperature complex plasma has been obtained.\(^5\)\(^,\)\(^10\) A study of generation of both voids in moving flows\(^11\) and unmoving media was presented.\(^12\) Recently the Lax’s scheme with re-calculation\(^13\) and the complex conservative difference scheme\(^14\) have been used for the hydrodynamic part of the model. At the same time, the MUSCL scheme is a widely using tool in hydrodynamic\(^14\) and plasma simulations.\(^11\) This paper is devoted to the MUSCL scheme application to a void evolution with respect to the contraction process. The simulations are based on the algorithm with using a predictor-corrector procedure for obtaining the second approximation order in space and in time and a MINMOD limiter is used for obtaining monotonic solution. Presented results can be used in research of dusty plasma dynamics under the conditions of microgravity which may associate with the conditions in space.

Model

Modeling is based on the model Avinash et al.\(^8\) Governing equations considered in dimensionless form with respect to the normalizing parameters\(^1\)\(^-\)\(^3\) have been applied:

\[ \begin{align*}
\frac{\partial v_d}{\partial t} + v_d \frac{\partial v_d}{\partial x} &= F_d - E - \alpha_0 v_d - \tau_e \frac{\partial n_d}{\partial x}, \\
\frac{\partial n_d}{\partial t} &= \frac{\partial (n_d v_d)}{\partial x} + D_\alpha \frac{\partial^2 n_d}{\partial x^2}, \\
\frac{d n_e}{d x} &= -\frac{n_E}{\tau_i}, \\
\frac{d E}{d x} &= 1 - n_e - n_d, \\
F_d &= \frac{a E}{b + \left| v_i \right|^\mu}, \quad v_i = \mu E.
\end{align*} \]

Here concentrations of dust and electrons components are \(n_d, n_e\);
velocities of dust and ions components are \( v_d \) \( v_i \); electric field currency is \( E \); ion-drag force is \( F_d \); diffusion coefficient is \( D \). In the system (1)-(5); (1) is the dust momentum and (2) is the dust continuity equations, (3) is the balance equation of electrons neglecting the electron inertia, the Poisson law which completes the nonlinear system of equations, and the expressions for ion–drag force presented in (4, 5) constitute the governing equations. Also, one can consider equations (1) and (2) as the hydrodynamic part of the model\(^9\)-\(^12\) and the equations (3)-(5) are the electrostatic part of it.

**Methods**

The hydrodynamic part of the model can be expressed in the divergent form

\[
\frac{\partial \mathbf{u}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = \mathbf{f},
\]

where \( \mathbf{u} = \left( \begin{array}{c} n_d \\ v_d \end{array} \right) \), \( \mathbf{F} = \left( \begin{array}{c} n_d v_d \\ 0.5n_d^2 + \tau_d \ln n_d \end{array} \right) \), \( \mathbf{f} = \left( \begin{array}{c} D \frac{\partial^2 n_d}{\partial x^2} \\ F_d - E - \alpha n_d \end{array} \right) \).

Earlier the centered scheme was applied to the hydrodynamic part of the model as well as the Lax’s scheme with re-calculation.\(^10\) The eq. (6) was approximated with the second order using central scheme and the method of Runge-Kutta of the fourth order was used to calculate (3), (4).\(^10\) Also, in the numerical simulations the next restrictions were imposed: if \( n_d \geq 1 \) then \( n_d \) was applied by 1 and \( v_d \) was applied by \( v_d^0 \). Here we apply the MUSCL scheme to (6). The algorithm is free of any restrictions on \( n_d \) and \( v_d \). A predictor-corrector procedure is used for obtaining the second approximation order in space and in time. To obtain monotonic shape of the numerical solution a MINMOD limiter based on the values \( n_d \) is utilized. Parameters of the simulations are collected in Table 1.

| Parameter | Value |
|-----------|-------|
| \( D \)   | 0     |
| \( \tau_d \) | 0.125 |
| \( \tau_i \) | 0.001 |
| \( a \)   | 7.5   |
| \( b \)   | 1.6   |
| \( \tau_0 \) | 2     |
| \( \mu \) | 1.5   |
| \( n_e^0 \) | 0.999 |
| \( E_0 \) | \( 10^4 \) |

**Results**

The evolution of void generation in time in the initially unmovin flow \( (v_d^0=0) \) is presented in Figure 1 and Figure 2. The dynamics of dusty particles density \( n_d \) is shown in Figures 1A-1D and the dynamics of dusty particles velocity \( v_d \) is shown in Figures 2A-2D. Here two-dimensional plots for \( n_d, v_d \) obtained by the rotation technique are presented. The mechanism of generation of voids is based on the superposition of the electrical field and the ion attraction force action. Void develops in time from the uniformly distributed density and velocity values with constant parameters. The initial circular structure is shown in Figure 1A, the void becomes saturated at \( t=300 \) and the contraction process occurs up to \( t=1000 \).
Conclusion

Generation of a pattern with empty region (void) has been modelled numerically in complex plasma. The modeling is based on the Avinash, Bhattacharjee and Hu model of a void formation. The model has been reduced to the divergent form and the MUSCL scheme of the second approximation order has been used for the hydrodynamic part of the model. Dynamics of the void parameters such as density and velocity of dust particles has been obtained. It has been shown that after the initial stage of the circular void structure formation and before the steady state establishing the intermediate stage occurs of a void subsequent expansion and partial contraction to the center.

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Conflict of interest

The author declares that there is no conflict of interest.

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