Dynamics of interacting phantom scalar field dark energy in
Loop Quantum Cosmology

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Abstract
We study the dynamics of a phantom scalar field dark energy interacting with dark matter in loop quantum cosmology (LQC). Two kinds of coupling of the form $\alpha \rho_m \dot{\phi}$ (case I) and $3\beta H (\rho_\phi + \rho_m)$ (case II) between the phantom energy and dark matter are examined with the potential for the phantom field taken to be exponential. For both kinds of interactions, we find that the future singularity appearing in the standard FRW cosmology can be avoided by loop quantum gravity effects. In case II, if the phantom field is initially rolling down the potential, the loop quantum effect has no influence on the cosmic late time evolution and the universe will accelerate forever with a constant energy ratio between the dark energy and dark matter.

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I. INTRODUCTION

Recently, present accelerating expansion of our universe has been confirmed by many observations, such as the cosmic microwave background (CMB) anisotropy, type Ia supernovae, and large scale galaxy surveys [1, 2]. In order to explain this observed phenomena, dark energy is assumed to exist in the universe within the framework of general relativity. Dark energy is an exotic energy component with negative pressure and accounts for about 72% of present total cosmic energy. In addition, observations also show that there is another dark component in the universe, i.e., dark matter, accounting for about 25% of total cosmic energy today. The simplest candidate of dark energy is the cosmological constant. It is, however, plagued with the so-called coincidence problem and the cosmological constant problem [3]. Thus some dynamical scalar fields, such as quintessence [4], phantom [5], quintom [6] and hessence [7], are proposed as possible candidates of dark energy. It is worthy to note, however, that for these scalar field models the coincidence problem still remains. Although the two dark components are usually studied under the assumption that there is no interaction between them, one can not exclude such a possibility. In fact, researches show that a presumed interaction may help alleviate the coincidence problem [8]. Therefore interacting dark energy models have attracted a great deal of interest.

Most of the present studies on dark energy are carried out in the framework of classical Einstein gravity. However, it is commonly believed that quantum gravity effects would play a role in the evolution of the universe. Therefore, it is desirable to examine the properties of dark energy in a theory of quantum gravity. One of such theories which we are interested in the present paper is Loop Quantum Gravity (LQG) (see e.g. [9, 10, 11, 12] for reviews), which is a nonperturbative background independent theory. At the quantum level, the classical spacetime continuum is replaced by a discrete quantum geometry and the operators corresponding to geometrical quantities have discrete eigenvalues. LQG has been applied in cosmological context as seen in various literature where it is known as Loop Quantum Cosmology (LQC) [13, 14, 15, 16]. The effects of loop quantum gravity modify the standard Friedmann equation by adding into it a correction term $-\rho^2/\rho_c$ which
essentially encodes the discrete quantum geometric nature of spacetime \[16, 17, 18\]. Let us note that although extra terms such as those derived in Ref. \[19\] are in principle possible, the effect of such terms is negligible for all practical purposes \[20\]. When this correction term becomes dominant, the universe begins to bounce and then oscillates forever. Therefore both the future singularity and the singularity at semi-classical regime can be avoided \[16, 18, 21, 22, 23\]. Recently, the dynamics of phantom, quintom and hessence in loop quantum cosmology have been studied \[23, 24, 25\] and behaviors different from that in the standard FRW cosmology, such the avoidance of future singularities, are found. At this point, it should also be noted here that the LQC modification, although very interesting, is only derived for pure isotropy and is not very clear how it fits within broader models.

In this paper, we will discuss the dynamics of a phantom scalar field dark energy, coupled to dark matter in loop quantum cosmology, to see in what way the LQG effects would affect the cosmological evolution of the system. Two kinds of interactions of the form, \(\alpha \rho_m \dot{\phi} \[26\]\) and \(3\beta H (\rho_{\phi} + \rho_m) \[8\]\), will be studied. It is worth pointing out that the first coupling could arise in string theory or after a conformal transformation of Brans-Dicke theory, while the second is motivated by analogy with dissipation of cosmological fluids and has been proposed for a possible dynamical solution to the coincidence problem. Let us note here that the phantom field, although a viable candidate of dark energy, also has some strange properties, such as being unstable to vacuum decay and the violation of the dominant energy condition.

II. LOOP QUANTUM COSMOLOGY

By incorporating the effects of loop quantum gravity which essentially encode the discrete quantum geometric nature of spacetime, the effective modified Friedmann equation
in a flat universe is given by  \[ H^2 = \frac{\rho}{3} \left( 1 - \frac{\rho}{\rho_c} \right), \] (1)

where \( H \equiv \dot{a}/a \) is the Hubble parameter, \( \rho \) is the total energy density, and a dot denotes the derivative with respect to cosmic time \( t \). Here we set \( 8\pi G \equiv 1 \); and the critical loop quantum density is

\[ \rho_c \equiv \frac{\sqrt{3}}{16\pi^2 \gamma^3 G^2 \hbar}, \] (2)

where \( \gamma \) is the dimensionless Barbero-Immirzi parameter. Let us note here that it has been suggested that \( \gamma \simeq 0.2375 \) by the black hole thermodynamics in LQG \([21, 22]\). Differentiating Eq. (1) and using the conservation equation of cosmic total energy

\[ \dot{\rho} + 3H (\rho + p) = 0, \] (3)

one obtains the effective modified Raychaudhuri equation

\[ \dot{H} = -\frac{1}{2} (\rho + p) \left( 1 - 2 \frac{\rho}{\rho_c} \right), \] (4)

where \( p \) is the total pressure. Actually, as shown in \([29]\), the effective modified Raychaudhuri equation can be also derived directly by using the Hamilton’s equations in LQC, without assuming the energy conservation.

**III. DYNAMICS OF THE INTERACTING PHANTOM SCALAR FIELD DARK ENERGY IN LQC**

Let us suppose that there are only the phantom scalar field dark energy and dark matter in a spatially-flat universe. The Lagrangian for the phantom scalar field fluid is

\[ \mathcal{L} = (1/2)\partial^\mu \phi \partial_\mu \phi - V(\phi) , \] (5)

where \( V(\phi) \) is the potential of the phantom field. Therefore, the energy density and pressure for the phantom field can be expressed as

\[ \rho_\phi = -\frac{1}{2} \dot{\phi}^2 + V(\phi) , \] (6)

\[^1\text{It is interesting to note that this kind of modified Friedmann equation also appears in cosmological braneworld models with a single timelike extra dimension \([28]\).}\]
and
\[ p_\phi = -\frac{1}{2} \dot{\phi}^2 - V(\phi). \]  

We assume that there is an interaction \( \Gamma \) between the phantom dark energy and dark matter. A positive \( \Gamma \) corresponds to energy transferring from phantom to dark matter and vice versa for a negative one. Therefore the energy densities for the phantom scalar field and dark matter satisfy the following equations
\[ \dot{\rho}_\phi + 3H(1 + w_\phi)\rho_\phi = -\Gamma, \]  
\[ \dot{\rho}_m + 3H\rho_m = +\Gamma, \]  
and the Raychaudhuri equation. The evolutionary equation for the phantom field and the modified Friedmann equation can be expressed as
\[ \dot{H} = -\frac{1}{2} \left( \rho_m - \dot{\phi}^2 \right) \left( 1 - \frac{2\rho}{\rho_c} \right), \]  
\[ \ddot{\phi} = -3H\dot{\phi} + V' + \frac{\Gamma}{\phi}, \]  
\[ H^2 = \frac{1}{3} \left( \rho_m - \frac{1}{2} \dot{\phi}^2 + V \right) \left( 1 - \frac{\rho}{\rho_c} \right), \]  
where \( V' \equiv dV/d\phi \). In this paper we only consider the case of exponential potential \( V(\phi) = V_0 \exp(-\lambda \phi) \) with a positive constant \( \lambda \).

In order to study the dynamics of the above system, we introduce the following dimensionless variables
\[ x \equiv \frac{\dot{\phi}}{\sqrt{6}H}, \quad y \equiv \frac{\sqrt{V}}{\sqrt{2}H}, \quad z \equiv \frac{\rho}{\rho_c}, \quad \frac{d}{dN} \equiv \frac{1}{H} \frac{d}{dt}, \]  
where \( N \equiv \ln a \) is the e-folding number and is used as an independent variable instead of cosmological time. Using the above definitions, the effective modified Friedmann equation, namely Eq. (12), can be rewritten as
\[ \left( \frac{\rho_m}{3H^2} - x^2 + y^2 \right)(1 - z) - 1 = 0, \]  
and Eq. (10) becomes
\[ \frac{\dot{H}}{H^2} = -\left[ \frac{3}{2} \left( \frac{1}{1 - z} + x^2 - y^2 \right) - 3x^2 \right](1 - 2z). \]  

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In addition, we will be interested in three scalar quantities, which are, respectively, the phantom fractional density parameter $\Omega_\phi$, the effective phantom equation of state $w_\phi$ and the effective equation of state for total cosmic energy $w_{\text{eff}}$ given by

$$
\Omega_\phi = -x^2 + y^2, \quad w_\phi = \frac{x^2 + y^2}{x^2 - y^2}, \quad w_{\text{eff}} = (1 - z)\left[-x^2 - y^2\right].
$$

Using the Eqs. (11, 14, 15), we obtain the autonomous equations:

$$
x' = -3x - \sqrt{\frac{3}{2}} \lambda y^2 + \frac{\Gamma}{\sqrt{6}H^2}\phi - x\left[3x^2 - \frac{3}{2}\left(\frac{1}{1 - z} + x^2 - y^2\right)\right](1 - 2z),
$$

$$
y' = -\sqrt{\frac{3}{2}} \lambda xy - y\left[3x^2 - \frac{3}{2}\left(\frac{1}{1 - z} + x^2 - y^2\right)\right](1 - 2z),
$$

$$
z' = -3z - 3z(1 - z)(-x^2 - y^2).
$$

Let $f \equiv x', g \equiv y', h \equiv z'$. Then using the following condition

$$
(f, g, h)\big|_{(x_c, y_c, z_c)} = 0,
$$
we can obtain the critical points $(x_c, y_c, z_c)$ of the autonomous system.

Next, we will study, by examining the stability of the these critical points using the standard linearization and stability analysis, the dynamics of two different interaction cases, i.e., $\Gamma_1 = \alpha \rho_m \dot{\phi}$ and $\Gamma_2 = 3\beta H(\rho_m + \rho_\phi)$ between the dark energy and dark matter.

**A. Case I: $\Gamma = \alpha \rho_m \dot{\phi}$**

This kind of interaction could arise from string theory or scalar-tensor theory [26]. The dynamics of this interacting phantom model was studied in the standard FRW cosmological framework in Ref. [30] and it was found that energy transfer either from the phantom field to the dark matter or vice versa yields similar cosmological consequences, and the energy density of the phantom field increases with the cosmic expansion, leading to unwanted future singularity. In this section, we will study the cosmological evolution of this interacting model in the framework of loop quantum cosmology. The autonomous
Eq. (17) can be rewritten as follows

\[
x' = -3x - \sqrt{\frac{3}{2}} \lambda y^2 - \frac{\sqrt{6}}{2} \alpha \left( \frac{1}{1 - z} + x^2 - y^2 \right) - x \left[ 3x^2 - 3 \left( \frac{1}{1 - z} + x^2 - y^2 \right) \right] (1 - 2z), \tag{19a}
\]

\[
y' = -\sqrt{\frac{3}{2}} \lambda xy - y \left[ 3x^2 - 3 \left( \frac{1}{1 - z} + x^2 - y^2 \right) \right] (1 - 2z), \tag{19b}
\]

\[
z' = -3z - 3z(1 - z)(-x^2 - y^2). \tag{19c}
\]

Then we obtain five critical points:

- **Point (A)**: \( \left( \frac{\sqrt{6} \alpha}{3}, \ 0, \ 0 \right) \),
- **Point (B)**: \( \left( -\frac{\lambda}{\sqrt{6}}, \ \sqrt{1 + \frac{\lambda^2}{6}}, \ 0 \right) \),
- **Point (C)**: \( \left( -\frac{\lambda}{\sqrt{6}}, \ -\sqrt{1 + \frac{\lambda^2}{6}}, \ 0 \right) \), \tag{20}
- **Point (D)**: \( \left( \frac{\sqrt{6}}{2 \alpha}, \ 0, \ 1 - \frac{2\alpha^2}{3} \right) \),
- **Point (E)**: \( \left( \frac{\sqrt{6}}{2 \alpha + \lambda}, \ \frac{\sqrt{2\alpha^2 - 3 + 2\alpha \lambda}}{\sqrt{2(\alpha + \lambda)}}, \ 0 \right) \).
The eigenvalues, $\mu$, of the coefficient matrix of the linearized equations for these critical points $A, B, C, D$ and $E$ can be expressed respectively as

- **Point (A):**
  \[ \mu_1 = -\left(\alpha^2 + \frac{3}{2}\right), \quad \mu_2 = 2\alpha^2 - 3, \quad \mu_3 = -\frac{2\alpha^2 - 3 + 2\alpha\lambda}{2}. \]

- **Point (B):**
  \[ \mu_1 = \lambda^2, \quad \mu_2 = -3 - \frac{1}{2}\lambda^2, \quad \mu_3 = -3 - \lambda(\alpha + \lambda), \]

- **Point (C):**
  \[ \mu_1 = \lambda^2, \quad \mu_2 = -3 - \frac{1}{2}\lambda^2, \quad \mu_3 = -3 - \lambda(\alpha + \lambda), \]

- **Point (D):**
  \[ \mu_1 = -\frac{3}{2}\left(1 + \sqrt{1 + \frac{6 - 4\alpha^2}{\alpha^2}}\right), \]
  \[ \mu_2 = -\frac{3}{2}\left(1 - \sqrt{1 + \frac{6 - 4\alpha^2}{\alpha^2}}\right), \]
  \[ \mu_3 = -\frac{3\lambda}{2\alpha}, \]

- **Point (E):**
  \[ \mu_1 = -\frac{3\lambda}{\alpha + \lambda}, \]
  \[ \mu_2 = -\frac{3}{4}\frac{2\alpha + \lambda}{\alpha + \lambda} \left(1 + \sqrt{1 + \frac{8[3 + \lambda(\lambda + \alpha)][2\alpha^2 + 2\alpha\lambda - 3]}{3[2\alpha + \lambda]^2}}\right), \]
  \[ \mu_3 = -\frac{3}{4}\frac{2\alpha + \lambda}{\alpha + \lambda} \left(1 - \sqrt{1 + \frac{8[3 + \lambda(\lambda + \alpha)][2\alpha^2 + 2\alpha\lambda - 3]}{3[2\alpha + \lambda]^2}}\right). \]

In what follows we will analyze the stability of these critical points:

- **For point A:**
  This fixed point is physically meaningless since $\Omega_\phi = -\frac{2\alpha^2}{3} < 0$.

- **For point B, C:**
  The existence of both points are only dependent on $\lambda$. This can be understood as a result of the fact that the $\alpha$-dependent term in Eq. (17a) vanishes when $\Omega_\phi \to 1$ and $z \to 0$. Since the sign of $\mu_1$ is always opposite to the sign of $\mu_2$, these two points are saddle points. However, these $\alpha$-independent critical points are found to be always stable and correspond to the future singularity in the standard FRW cosmology [30]. Therefore
the future singularity appearing in the standard cosmology can be avoided by the loop quantum gravity effect.

- For point D:
  This fixed point is physically meaningless since \( \Omega_\phi = -\frac{3}{\alpha^2} < 0 \).

- For point E:
  The same as in the standard cosmology, this point is also unstable in LQC.

From the above analysis, we conclude that there is not any stable node for this kind of interacting dark energy model in LQC, and the future singularity appearing in the standard cosmology can be avoided. Our result of analytical discussions agrees with that of the numerical calculations obtained in Ref. [31].

**B. Case II: \( \Gamma = 3\beta H(\rho_\phi + \rho_m) \)**

This type of interaction is motivated by analogy with dissipation of cosmological fluids and has been proposed for a possible dynamical solution to the coincidence problem [8]. The dynamics of this interacting phantom scalar field model in the standard FRW cosmology has been studied in Ref. [30] and it was found that there are two kinds of late time attractors. If the phantom field initially rolls down the potential the universe will accelerate forever and the total cosmic energy density decreases with the cosmic expansion, while if the phantom field initially climbs up its potential the universe will end with a big rip. Here we discuss the dynamics of this interacting model in LQC. For convenience, we introduce another variable \( \xi \equiv \sqrt{\rho_m/3H} \). The autonomous equations can be rewritten as:

\[
x' = -3x - \sqrt{\frac{3}{2}} \lambda y^2 - \frac{3\beta}{2x(1-z)} - x \left[ 3x^2 - \frac{3}{2} \left( \frac{1}{1-z} + x^2 - y^2 \right) \right] (1 - 2z), \quad (21a)
\]

\[
y' = -\sqrt{\frac{3}{2}} \lambda xy - y \left[ 3x^2 - \frac{3}{2} \left( \frac{1}{1-z} + x^2 - y^2 \right) \right] (1 - 2z), \quad (21b)
\]

\[
z' = -3z - 3z(1-z)(-x^2 - y^2), \quad (21c)
\]

\[
\xi' = -3\xi \left[ \frac{1}{2} + \left( x^2 - \frac{1}{2} \xi^2 \right) (1 - 2z) - \frac{\beta}{2\xi^2(1-z)} \right]. \quad (21d)
\]
From Eq. (14) it is easy to see that there are only three independent equations in the above system. This system has four critical points:

Point A:

\[ x_A^2 = \frac{1}{2}(\sqrt{1 + 4\beta} - 1), \quad y_A = 0, \]
\[ \xi_A^2 = \frac{1}{2}(\sqrt{1 + 4\beta} + 1), \quad z_A = 0. \]

Obviously this solution is physically meaningless since \( \Omega_\phi < 0 \). The solutions for the other three critical points are tedious and we do not present the details here. However, these solutions satisfy the following set of equations:

\[ \beta = f(x), \tag{23a} \]
\[ y^2 = -x^2 - \frac{\sqrt{6}\lambda x}{3} + 1, \tag{23b} \]
\[ z = 0, \tag{23c} \]
\[ \xi^2 = 2x^2 + \frac{\sqrt{6}\lambda x}{3}, \tag{23d} \]

where we have defined a cubic function

\[ f(x) \equiv x \left( 2x + \frac{\sqrt{6}\lambda}{3} \right) \left( 1 - \frac{\sqrt{6}\lambda x}{3} \right). \tag{24} \]

There is a critical point B with \( x_B < 0 \), if

\[ 0 < \beta \leq f \left( \frac{-\lambda - \sqrt{\lambda^2 + 12}}{2\sqrt{6}} \right), \tag{25} \]

and this point B corresponds to an initially climbing-up phantom field [30]. There are two other critical points (C and D) with \( x_{C,D} > 0 \). One is physically meaningless and we label it by D. Point C exists for

\[ 0 < \beta \leq \min \left[ f \left( \frac{-\lambda + \sqrt{\lambda^2 + 6}}{\sqrt{6}} \right), f \left( \frac{-\lambda + \sqrt{\lambda^2 + 12}}{2\sqrt{6}} \right) \right], \tag{26} \]

and it corresponds to an initially rolling-down phantom field.

Employing the standard techniques in the linearization and stability analysis, we can obtain three independent evolution equations of the linear perturbations. Using the eigenvalues of the coefficient matrix for points B and C, found according to Eq. (23), we find
that the critical point $B$ is a saddle point, while point $C$ is a stable critical point, and it is, therefore, a late time attractor. This is different from what was obtained in the standard FRW cosmology where both points $B$ and $C$ are late time attractors [30]. In the standard FRW cosmology, if the field initially rolls down the potential, the universe will enter a final state without Big Rip described point $C$, while for an initially climbing-up phantom field, the universe will enter a final state described by point $B$ and end with a Big Rip. Therefore, the future singularity can be avoided by the loop quantum effect in LQC for the case in which the field initially climbs up the potential. As a result, in any case, there is no Big Rip in LQC. In Fig. (1) we show the stability regions for parameter space $(\lambda, \beta)$. One can see that the effect of loop quantum gravity breaks the stability of the initially climbing-up field but leave that of the initially rolling-down field intact. This shows that if the universe does not evolve to a big rip in the standard cosmology its evolution seems to be uninfluenced by the loop quantum effect.

Now, we will show results of numerical analysis we have carried out on this interacting phantom scalar field dark energy model in LQC. In Figs (2, 3) we show the evolution curves of $H$ and $\rho$ for an initially climbing-up phantom field with different values of the coupling constant between the dark energy and dark matter. From these two figures we can see that $H$ climbs up to reach its maximum value when the total cosmic energy density $\rho$ reaches the value $\frac{\rho_c}{2} = 0.75$, and $H$ goes down to zero when $\rho$ reaches the maximum $\rho = \rho_c$. After that, the universe contracts and then bounces. As time goes on, the universe will undergo oscillations with increasing frequency which may eventually blow up. Consequently, this seems to give rise to a new singularity. This kind of behavior of possible infinite frequency of oscillation also appears in the case of the interaction of the form $\Gamma = \alpha \dot{\phi} \rho_m$, as found in Ref. [31]. In addition, the larger the value of $\beta$, the later our universe enters the oscillating regime.

In Figs. (4, 5, 6, 7), we give the numerical results for an initially rolling-down phantom field with the requirement given in Eq. (26). Fig. (4) shows the evolutionary properties of the universe with different initial conditions. Apparently the trajectories converge to the same final state determined only by parameter $\lambda$. Figs. (5, 6) show the evolutionary curves of $H$ and the effective equation of state for total cosmic energy $w_{\text{eff}}$. We find that
\( H \) does not oscillate and \( w_{\text{eff}} \) approaches finally to a constant which is less than \(-\frac{1}{3}\) but larger than \(-1\). Thus the universe will keep accelerating forever while its energy density decreases with the cosmic expansion. Fig. [7] shows that in the final state the energy ratio of the phantom and dark matter reaches a constant. Therefore the coincidence problem can be alleviated.

IV. CONCLUSION

In conclusion, we have studied in loop quantum cosmology the dynamical system of a phantom field coupled to dark matter through an interaction of the form \( \alpha \dot{\phi} \rho_m \) (case I) or \( 3\beta H(\rho_\phi + \rho_m) \) (case II). The exponential potential for the phantom is used. For case I, there is a late time attractor solution in the standard FRW cosmology which corresponds to a big rip of the universe; whereas in LQC this solution transforms to be unstable, thus the big rip singularity which appears in the standard cosmology can be avoided by the loop quantum effect.

For case II, it was found, in the standard FRW cosmology, that if the phantom field is initially rolling down the potential, the dynamical system has a late time attractor and the universe will accelerate forever, while if the field initially climbs up the potential, there is also a late time attractor but the universe will end with a big rip. By studying the dynamics of this interacting phantom model in LQC, we find that the universe with an initially climbing-up phantom field will oscillate forever; therefore the future singularity can be avoided by loop quantum effect; while for an initially rolling-down phantom field the universe will have the same late time evolution as that in the standard cosmology, i.e., the universe will accelerate forever with a constant ratio between the energies of the phantom and dark matter and the total energy density of the universe will decrease with the cosmic expansion. Therefore, in case II, the loop quantum effect only intervenes when the universe will evolve to a future singularity, and it sits idle when otherwise.
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FIG. 1: The stability regions of \((\lambda, \beta)\) parameter space for case II. In LQC, in the region I the rolling-down critical point (Point C) is a stable late time attractor; in region II there are no stable critical points. However, in the standard FRW cosmology, in the region I, both the climbing-up scaling solution and the rolling-down scaling solution are the stable late time attractors, while in the region II, the climbing-up solution is the stable late-time attractor. III represents the region of the solutions without physical meaning.

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FIG. 2: The evolution of $H$ for a climbing-up phantom field with different coupling constants $(\beta = 0.2, 0.8)$ and $\lambda = 1, V_0 = 1, \rho_c = 1.5, \dot{\phi}_0 = -0.4$ in case II.

FIG. 3: The evolution of cosmic total energy density for a climbing-up phantom field with different coupling constants $(\beta = 0.2, 0.8)$ and $\lambda = 1, V_0 = 1, \rho_c = 1.5, \dot{\phi}_0 = -0.4$ in case II.
FIG. 4: The convergence of different initial conditions to the attractor solution in the $(x, y, z)$ phase space for a rolling down phantom field in case II. The red point denotes the critical point $C$.

FIG. 5: The evolution of $H$ with time for a rolling-down phantom field with $\beta = 0.2, \lambda = 1, V_0 = 1, \rho_c = 1.5$ and $\dot{\phi}_0 = 0.4$. 
FIG. 6: The evolution of the effective equation of state for the total cosmic energy in a rolling-
down phantom model with $\beta = 0.2, \lambda = 1, V_0 = 1, \rho_c = 1.5$ and $\dot{\phi}_0 = 0.4$.

FIG. 7: The evolutionary curves of the fractional densities of a phantom field (dashed line) and
dark matter (solid line) for a rolling-down phantom model with $\beta = 0.2, \lambda = 1, V_0 = 1, \rho_c = 1.5$
and $\dot{\phi}_0 = 0.4$. 