Abstract—Energy Storage Systems (ESS) are expected to play a critical role in future energy grids. ESS technologies are primarily employed for reducing the stress on grid and the use of hydrocarbons for electricity generation. However, in order for ESS option to become economically viable, proper sizing is highly desired to recover the high capital cost. In this paper we propose a system architecture that enables us to optimally size the ESS system according to the number of users. We model the demand of each customer by a two-state Markovian fluid and the aggregate demand of all users are multiplexed at the ESS. The proposed model also draws a constant power from the grid and it is used to accommodate the customer demand and charge the storage unit, if required. Then, given the population of customers and their stochastic demands, and the power drawn from the grid we provide an analytical solution for ESS sizing using the underflow probability as the main performance metric, which is defined as the percentage of time that the system resources fall short of demand. The results indicate that significant savings in terms of ESS size can be achieved. Such insights very important in designing the system planning phases of future energy grid infrastructures.

I. INTRODUCTION

Over the last few years there has been a pressing need to reduce the use of hydrocarbons for electric power generation. Such generation typically occurs during peak hours, that is only around 10-12% of the time, due to employing fast start, high cost, and usually environmentally unfriendly generation options. This current state of affairs has adverse effects on both utilities and consumers. For utilities, the increase in system peak to average load ratio reduces the capacity utilization for low cost generators. Furthermore, due to peak demand the distribution and transmission network may get more congested, thus requiring capacity upgrades. From consumers’ standpoint, on the other hand, it raises the unit electricity prices and may lead to service interruptions. Moreover, it has been anticipated that the power grid operations will become more stressed and less secure with the penetration of intermittent renewable resources and electric vehicles (both pure and plug-in hybrid) in the next decade.

As the energy storage technology evolves, distributed energy storage is becoming a viable option to address the aforementioned issues. ESSs located at distributed level can be used for applications such as peak shaving and load leveling to reduce the stress on the grid. Hence, the use of ESS will lead to savings in system fuel consumption and emissions due to time of use. It will further reduce the congestion, especially on the distribution grid, enable renewable energy penetration, and increase the overall utilization of power generation portfolio. For customers the corresponding savings will be reflected in electricity tariffs. Utilities have already started deploying shared-based community energy storage on a broad scale. Other potential applications include aiding the demand in large scale electric vehicle charging stations and reducing the peak consumption in smart residential and business complexes composed of large number of users. This is depicted in Figure 1.

Considering the cost (both capital and operational) of candidate ESS technologies (depicted in Table I) and the importance of the applications, the optimal sizing of the storage unit is a critical step for maximizing the previously stated benefits. Over-provisioning ESS size entails costly and underutilized assets, whereas under-provisioning reduces its operating lifetime (e.g., frequently exceeding allowable depth of charge level degrades its health). Existing sizing guidelines (according to peak demand) provide loose upper bounds and hence increase the system cost. The sizing approach should consider the stochastic behavior of customer demand, especially the peak fluctuation patterns. To that end, the contributions of this paper can be summarized as follow.

- We show that the individual demand of single class customers can be modeled as two-state Markovian fluids and analyze the system using stochastic theory of fluid dynamics.
- We propose an architecture to size energy storage systems serving large number of single class customers. Aggregated demand is first met by a deterministic grid power, and the fluctuations are accommodated by ESS. We show that our model ensures grid reliability at all times.
- We provide general design guidelines for optimal allocation of ESS and power grid resources. We provide explicit

| Technology                  | Power Subsystem Cost $/kW | Energy Storage Subsystem Cost $/kWh | Round-trip Efficiency (%) |
|-----------------------------|---------------------------|------------------------------------|---------------------------|
| Advanced Lead-acid Battery  | 400                       | 330                                | 80                        |
| Zinc/bromine Batteries      | 400                       | 400                                | 70                        |
| Lithium-ion Batteries       | 400                       | 600                                | 85                        |
| Flywheels                   | 600                       | 1600                               | 95                        |
| Supercapacitors             | 500                       | 10000                              | 95                        |
solutions for ESS underflow probability that is used as the main metric for dimensioning the grid and ESS resources for varying number of customers. This approach enables system planners to solve the sizing problem in a more efficient manner for varying customer population.

The remainder of this paper is organized as follows: in Section II we present the related literature. In Section III we provide the mathematical model for the sizing problem. Finally, in Section IV we provide various numerical examples for different design considerations.

II. RELATED WORK

Recently, there has been an increasing body of literature on the role of ESS in utility applications, the evaluation of candidate ESS technologies, and methodologies for proper ESS sizing for in utility applications [1], [4], [6]–[11]. We start by reviewing some of the most important utility applications that require storage units. Overall, these applications are differentiated through their required output power and duration of support. ESS technologies for end-user peak shaving applications are used to regulate the variability in voltage levels (e.g., voltage sags and flickers) and typically require high-power short-duration ESS support. On the other hand, in load leveling applications, ESS can be charged during the times of low energy demand, and the storage energy can be used during peak hours (typically hours and requires higher capacity). Furthermore, ESS owners can sell stored energy at higher rates during peak hours (energy arbitrage) in order to make profit. In this paper we mainly assume that ESS is used for load leveling applications to keep the peak demand fluctuations as small as possible. Finally, ESSs are required to foster the penetration of renewable generation as they are used to smoothen the intermittency of such resources.

The rationale behind the optimal sizing approaches is to make the ESS economically viable so that utilities and consumers can enjoy the aforementioned savings. The work in [9] presents dynamic programming based sizing approach for industrial peak saving applications. Studies in [2] and [12] present a ESS sizing methodology for small scale charging stations and evaluate the candidate technologies using tools from queuing theory, where they coupled the performance metric “probability of not serving customers” with the ESS size to solve the provisioning problem. Similar metric is also used in [13] and [14] where they map the dynamics of the ESS with a buffer, that is used to hold data in communication systems, and solve the sizing problem according to “underflow probability” for renewable energy applications. From power engineering point of view, the sizing problem is usually solved via simulation techniques [15] and the overview of sizing methodologies can be found in [16]. However, to the best of our knowledge, this is the first study that couples the ESS dimensioning problem with the number of customers using the storage unit.

The methodology proposed in this paper is rooted in Asynchronous Transfer Model (ATM) networks which deals with a similar architecture, in which users fill up a buffer with voice packets and a constant rate server processes requests and empties the buffer. The goal is to find the probability that the buffer level exceeds some threshold. This was first proposed by Anick [17] and the solution of the desired probability distribution function was improved in [18] and [19]. Furthermore, our assumption on modeling the individual user demand with an “On/Off” fluid model has been made in [3], [20] and [21] which solve the power resource provisioning problem is solved in applicants without ESS.

III. PROBLEM FORMULATION

A. System Model

We consider a smart grid application in which the demands of $i = \{1, 2, ..., N\}$ users are accommodated by an ESS size of $B$ and constant grid power amount of $C$. It has been successfully shown in [3], [20], [21] and [22] that the pattern of a single power consumer is well-represented by a two state “On/Off” process. The service provider (in this case the operator of the ESS and the grid power) receives the demand request of user $i$ according to a Poisson process of rate $\lambda$. During the “On” period user $i$’s demand is uniform and represented by $R_p$. Considering the variety in customer demands, the durations of the customers demands are assumed to be exponentially distributed with parameter $\mu$. Note that the probability of being in the “On” state is $p_{On}=\frac{\lambda}{\lambda+\mu}$ and the mean demand of the user $i$ is $\frac{1}{\lambda+\mu}R_p$. The conceptual overview of the system model is represented in Figure 2.

Notice that the benefits of this architecture are multi-facet. First during the low demand periods, the excess power can...
be stored in the ESS and during peak hours it can be used to meet customer demands. As a result, peak demand and the costs of end users are minimized. Also, drawing constant power from the grid isolates the grid from stochastic changes in the customer side and hence prevents grid components from unpredictable demand fluctuations. Furthermore, system operator can draw a long term contract and enjoy lower prices. It is assumed that the power rating of the ESS is large enough to serve the demand when all users are at “On” state.

Considering \( N \) such multiplexed user demand requests, the composite model of each independent two state models can be represented by an \((N+1)\)-state continuous-time Markov chain (depicted in Figure 3). Note that for each state \( i \in \{0, 1, \ldots, N\} \), stationary probability distribution \( \pi = [\pi_0, \pi_1, \ldots, \pi_N] \) can easily be computed solving for \( \pi M = 0 \), where \( M \) is the infinitesimal generator matrix and can be constructed by:

\[
M = \begin{pmatrix}
-N\lambda & N\lambda & 0 & \cdots & 0 \\
\mu & -(\mu + (N-1)\lambda) & (N-1)\lambda & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & \lambda \\
0 & 0 & 0 & \cdots & -N\mu
\end{pmatrix}
\]  \hspace{1cm} (1)

The blocking probabilities will be used to dimension the storage devices for a target underflow probability and different number of customers under the stochastic regime given above.

As a baseline approach, the network operator solves the resource provisioning problem by serving the aggregate demand with the grid power \( C \), and capacity planning will be performed according to the peak rate demand given by \( C=NR_p \). However, by exploiting the statistical aspects of the customer demand (e.g., in an “On/Off” model, each source does not always draw power since \( 0<p_{on}<1 \)) and by employing an ESS, system operator can reduce the stress on the grid. To that end, the main thrust of this paper is to compute the required size of energy storage size \( B \), for given the grid capacity \( C \), the superposition of \( i\in\{1,\ldots,N\} \) aforementioned flows are multiplexed in the ESS, and the probability that the available resources fall short of demand is upper bounded by some target \( \epsilon \). Note that, based on this approach system operator is guaranteed to serve the customer demand majority of the time.

\begin{align*}
\text{overflow probability is } & P\{\bar{x} > B\}. \\
\text{Let } \bar{L}(t) \text{ represent the packets generated by user } i, \text{ hence the aggregate demand of all users fill the buffer at a rate of } \sum_i \bar{L}(t) - C. \text{ Further, denote the workload in the buffer of the equivalent system at time } t \text{ by } \bar{S}(t), \text{ then the rate of change in the work trajectory, defined as the realization of a particular workload, is:}
\end{align*}

\[
\frac{d\bar{S}(t)}{dt} = \begin{cases}
0, & \text{if } \bar{S}(t) = B & \text{and } \sum_i \bar{L}(t) > C, \\
0, & \text{if } \bar{S}(t) = 0 & \text{and } \sum_i \bar{L}(t) < C, \\
\sum_i \bar{L}(t) - C, & \text{otherwise}
\end{cases} \hspace{1cm} (2)

In a similar manner, let us define \( L_i(t) \) and \( S(t) \) as the charge requests of single smart grid user and the workload at the storage unit, respectively. Hence, for the rate of change in the charge level of the ESS is:

\[
\frac{dS(t)}{dt} = \begin{cases}
0, & \text{if } S(t) = B & \text{and } \sum_i L_i(t) < C, \\
0, & \text{if } S(t) = 0 & \text{and } \sum_i L_i(t) > C, \\
C - \sum_i L_i(t), & \text{otherwise}
\end{cases} \hspace{1cm} (3)

Without loss of generality it is clear that there is a one-to-one mapping between the queuing model and the ESS model and the workload trajectories of the two system such that \( S(t)+\bar{S}(t)=B, \forall t \) (detailed analysis is presented in [20]). Hence computing the overflow probability in the queuing model gives us the underflow probability in the ESS model. To this end, let us define continuous random variable \( x \) as the ESS depletion level. Next, we will provide guidelines to compute the ESS provisioning problem that is defined by finding \( B \) such that \( P\{x \leq 0\} \leq \epsilon \) or equivalently for the queue model \( P\{\bar{x} \geq B\} \leq \epsilon \).

\[ C. \text{ System Analysis} \]

Define \( F_i(t, x) \) for \( i\in\{1,2,\ldots,N\} \) and \( x \geq 0 \), as the cumulative probability distribution of the ESS charge level i.e., the probability that ESS level is less than or equal to
Then solution of the cumulative distribution function can be obtained by writing them in their complete form:

\[ F_i(t + \Delta t, x) = (N - (i - 1)) \lambda \Delta t F_{i-1}(t, x) + (i + 1) \mu \Delta t F_{i+1}(t, x) \]

+ \((1 - ((N - i)\lambda + i\mu)) \Delta t) F_i(t, x) + (i - C) \mu \frac{\partial F_i}{\partial x}(t, x)\),

where \(o(\Delta t^2)\) represents that other terms approach to zero more rapidly than \(\Delta t\) as \(\Delta t \to 0\). Now by taking the limit \(\lim_{\Delta t \to 0} F_i(t + \Delta t, x)\), (4) simplifies to:

\[ \frac{\partial F_i(x, t)}{\partial t} = (N - (i - 1)) \lambda F_{i-1}(x) + (i + 1) \mu F_{i+1}(x) \]

\[ - ((N - i)\lambda + i\mu) F_i(x) - (i - C) \mu \frac{\partial F_i}{\partial x}(x). \]

It is further assumed that steady state conditions holds, that is \(\frac{\partial F_i(x, t)}{\partial t} = 0\) and hence \(\frac{\partial F_i(x, t)}{\partial x} \to F_i(x)\). Using the steady state (5) becomes:

\[ (i - C) \lambda \frac{\partial F_i}{\partial x} = (N - (i - 1)) \lambda F_{i-1}(x) \]

\[ - ((N - i)\lambda + i\mu) F_i(x) - (i - C) \mu \frac{\partial F_i}{\partial x}(x). \]

Note that without loss of generality \(F_{i-1}(x) = F_{N+1} = 0\). Then solution of the cumulative distribution function can be obtained by writing them in their complete form:

\[ -C\mu \frac{dF_0(x)}{dx} = -N\lambda F_0(x) + \mu F_1(x) \]

\[ (1 - C)\mu \frac{dF_1(x)}{dx} = N\lambda F_0(x) - ((N - 1)\lambda + \mu) + 2\mu F_2(x) \]

\[ (2 - C)\mu \frac{dF_2(x)}{dx} = (N - 1)\lambda F_1(x) - ((N - 2)\lambda + 2\mu) + 3\mu F_3(x) \]

\[ \vdots \]

\[ (N - C)\mu \frac{dF_N(x)}{dx} = \lambda F_{N-1}(x) - N\mu F_N(x). \]

Now, define \((N+1)\)-element row vector \(F(x) \triangleq \left[F_0(x), F_1(x), ..., F_N(x)\right]\), then the set of equalities given in (6) can be written in the following matrix form:

\[ \frac{dF(x)}{dx} = D F(x)Q, \]

where \(D\) is a diagonal matrix defined as \(D \triangleq \text{diag} [-C\mu, (1 - C)\mu, ..., (N - C)\mu]\) and \(M\) is given in (1).

The solution of the first order differential equation given in (8) can be expressed by a sum of exponentials. The general solution requires computing \((N + 1)\) eigenvalues of the matrix \(MD^{-1}\) and the general solution is expressed as [17]:

\[ F(x) = \sum_{i=0}^{N} \alpha_i F_i e^{\alpha_i x} \]

where the vector \(\alpha_i\) represents the undetermined coefficients and \(z_i\) is the \(i\)-th eigenvalue corresponding to the eigenvector \(\Phi_i\) that solves \(z_i \Phi_i D = \Phi_i M\). The solution of the differential equation in (9) is provided by Morrison [18] for normalized set of parameters that will be explained next.

1) ESS Underflow Probability: In this subsection we provide the solution of the cumulative distribution function \(F(x)\) for underflow probability with respect to number of users \(N\), ESS size \(B\) and grid power \(C\). In order to simplify the notation we normalize the system parameters. Without loss of generality, time is measured in units of a single average “On” time \((1/\mu)\). Also unit customer demand is measured according to peak rate (that is \(R_p\)). With these simplifications, mean customer demand becomes \(\lambda/(\mu + \lambda)\). Define \(\kappa\) as the ESS per user \((B/N)\) and \(\varsigma\) as the grid power allocated per source \((C/N)\). In order for the system to be stable, \(\varsigma\) should be between the mean and the peak rate demand (to handle the mean stochastic demand and the fluctuations),

\[ \frac{\lambda}{1 + \lambda} < \varsigma < 1. \]

Furthermore, for stability purposes the system capacity \(C\) should be greater than the mean customer demand. Hence, we denote the variable \(v\) as the power above the mean demand allocated per user as:

\[ v = \varsigma - \frac{\lambda}{1 + \lambda}. \]

Then the asymptotic expression for the buffer underflow probability \(F_N(B)\) for a given number of customers, ESS size, and the power allocated per user can be expressed as [18]:

\[ F_N(B) = \frac{1}{2} \sqrt{\frac{u}{\pi \varsigma (\varsigma + \lambda (1 - \varsigma))}} \left[ e^{-N\varsigma} e^{-2\sqrt{\varsigma + \lambda (1 - \varsigma)} N B} \right] \times e^{-\varsigma B}, \]
We use the aforementioned normalized values (unit time is measured in peak demand - \( R_p \)) and unit demand is measured in peak demand - \( R_p \). We start by exploring the relations between the number of users, ESS size (in \( R_p \mu \) units) and the corresponding underflow probability for a given system capacity \( C \). Charge request rate per single user \( \lambda \) is set to 2 (two charge requests of size \( R_p \) arrives in unit time), and the mean capacity above the mean demand per user is set to \( \nu = 0.035 \). Then, using (11) one can easily compute the total system capacity as \( C = 0.3683N \) units. In Figure 5 ESS sizing is evaluated for user population \( N \) from 400 to 800. This result can be used in various ways. First, given user population \( N \), system operator can choose the ESS size according to a certain underflow probability. For instance, for a large scale EV charging facility (e.g., located in shopping mall, airports [3]) with \( N = 400 \) charging slots in order to accommodate 99% of the customer demand the ESS size should be selected as \( B = 9 \times 10 (= R_p \times 0.5 (= \mu^{-1}) = 45 \text{kWh} \). One important thing to notice is that as the user population increases the required ESS size reduces due to the increase in multiplexing gains. Another important observation is that instead of sizing the ESS to meet the entire customer demand, just by rejecting a few percentage of customers great savings in the storage size, hence in terms of total system cost, can be achieved.

Another design consideration from the system operator point of view would be the following. Suppose that the system operator employs an already acquired ESS size of \( B = 5 \), then she is interested in the amount of power to draw from the grid so that she can guarantee to meet certain level of demand (e.g., 99% etc.). To that end, the underflow probability for a range of system capacity per user \( \varsigma \) and user population is evaluated in Figure 6. Obviously as the capacity per user increases the underflow probability goes to zero. Similar to previous evaluation, as the number of users increase, due to multiplexing gains the same percentage of customers can be accommodated with less amount of resources. Next the relation between the
ESS capacity and the grid power is investigated for a fixed number of users $N=500$. Each curve depicted in Figure 7 represents a contour of underflow probability and the Buffer-Grid Power ($B$-$C$) combinations to reach the same underflow probabilities. Obviously in order to serve more customers (less $\epsilon$) more grid and ESS capacity are required. Moreover this result can be useful from financial analysis standpoint. For a specific project system (number of customers $N$, underflow probability) designer can analyze the unit cost of ESS and grid resources. Then the optimal combination of the ESS-Grid power can be obtained at the intersection of the cost curve and the contours given here. Three different cases for the cost are illustrated in Figure 7. As a future work, we are aiming to develop cost models for energy storage and power grid to optimally compute grid and ESS resources.

The primary motivation for the employment of the ESS is to reduce the stress on the grid and improve the utilization of power system components (e.g., power generation etc.). Thus, our final evaluation is on the percentage of reduction on the power grid for a fixed ESS size ($B=5$) and for different underflow probabilities. This time arrival rate is set to $\lambda=4$ and the comparison for varying arrival rates are done according to peak demand allocation. It can be seen in Figure 8 that multiplexing resources lead to great reduction on the power grid.

V. CONCLUSION

In this paper we provided a methodology to size energy storage systems for peak hour utility applications like load leveling, peak shaving, and energy arbitrage. The input of the system was the constant grid power and the number of customers and we computed the ESS size using the underflow probability. We further showed how different design parameters can be used in system planning phase.

We are aiming to expand this work in the following ways. In this paper we assumed single class loads multiplexed at energy storage. However recent measurement based studies show that home appliance loads can be classified into multiple classes of On-Off sources. Hence, we will consider multiple classes of loads scenario and examine if it is cost effective to employ one ESS for the aggregated demand (resource pooling) or allocate each different ESS to each class. Inspired by the results in Figure 7 final research direction includes developing an long term economic model to optimally allocate grid and ESS resources.

ACKNOWLEDGMENT

This publication was made possible by NPRP grant # 6-149-2-058 from the Qatar National Research Fund (a member of Qatar Foundation). The statements made herein are solely the responsibility of the authors.

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