THE UNIVERSE WAS REIONIZED TWICE

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ABSTRACT

We show that the universe was reionized twice, first at \( z \approx 15-16 \) and again at \( z \approx 6 \). Such an outcome appears inevitable when normalizing to two well-determined observational measurements, namely, the epoch of the final cosmological reionization at \( z \approx 6 \) and the density fluctuations at \( z \approx 6 \), which in turn are tightly constrained by \( \mathrm{Ly}_{\alpha} \) forest observations at \( z \approx 3 \). These two observations most importantly fix the product of star formation efficiency and the ionizing photon escape fraction from galaxies at high redshift. The only major assumption made is that the initial mass function of metal-free, Population III stars is top-heavy. To the extent that the relative star formation efficiencies in gaseous minihalos with \( \mathrm{H}_2 \) cooling and large halos with atomic cooling at high redshift are unknown, the primary source for the first reionization is still uncertain. If star formation efficiency in minihalos is at least 10\% of that in large halos, then Population III stars in the minihalos may be largely responsible for the first reionization; otherwise, the first reionization will be attributable largely to Population III stars in large halos. In the former case, \( \mathrm{H}_2 \) cooling in minihalos is necessarily efficient. We show that gas in minihalos can be cooled efficiently by \( \mathrm{H}_2 \) molecules and that star formation can continue to take place largely unimpeded throughout the first reionization period, as long as gas is able to accumulate in them. This comes about thanks to two new mechanisms for generating a high X-ray background during the Population III era put forth here, namely, X-ray emission from the cooling energy of Population III supernova blast waves and that from miniquasars powered by Population III black holes. Consequently, \( \mathrm{H}_2 \) formation in the cores of minihalos is significantly induced to be able to counteract the destruction by Lyman-Werner photons produced by the same Population III stars. In addition, an important process for producing a large number of \( \mathrm{H}_2 \) molecules in relic \( \mathrm{H} \, \mathrm{II} \) regions of high-redshift galaxies, first pointed out by Ricotti, Gnedin, & Shull in 2001, is quantified here for Population III galaxies. It is shown that \( \mathrm{H}_2 \) molecules produced by this process may overwhelm the dissociating effects of the Lyman-Werner photons produced by stars in the same Population III galaxies. As a result, the Lyman-Werner background may not build up in the first place during the Population III era. The long cosmological reionization and reheating history is complex. From \( z \approx 30 \), Population III stars gradually heat up and ionize the intergalactic medium, completing the first reionization at \( z \approx 15-16 \), followed by a brief period of \( \Delta z \approx 1 \), during which the intergalactic medium stays completely ionized because of sustained ionizing photon emission from concomitant Population III galaxies. The transition from Population III stars to Population II stars at \( z \approx 13 \) suddenly reduces, by a factor of \( \approx 10 \), the ionizing photon emission rate, causing hydrogen to rapidly recombine, marking the second cosmological recombination. From \( z \approx 13 \) to 6, Compton cooling by the cosmic microwave background and photoheating by the stars self-regulate the Jeans mass and the star formation rate, giving rise to a mean temperature of the intergalactic medium maintained nearly at a constant of \( \approx 10^4 \) K. Meanwhile, recombination and photoionization balance one another such that the intergalactic medium stays largely ionized during this stage, with \( n_{\mathrm{H}^+}/n_{\mathrm{H}} \approx 0.6 \). Most of the star formation in this period occurs in large halos with dominant atomic line cooling. We discuss a wide range of implications and possible tests for this new reionization picture. In particular, the Thomson scattering optical depth is increased to 0.10 ± 0.03, compared to 0.027 for the case of only one rapid reionization at \( z = 6 \). Upcoming Wilkinson Microwave Anisotropy Probe observations of the polarization of the cosmic microwave background should be able to distinguish between these two scenarios. In addition, properties of minihalos at high redshift (\( z \geq 6 \)) will be very different from previous expectations; in particular, they will be largely deprived of gas, perhaps alleviating the cosmological overcooling problem.

Subject headings: cosmic microwave background — cosmology: theory — intergalactic medium — supernovae: general

1. INTRODUCTION

The conventional view is that the universe becomes reionized at some point in the redshift range \( z = 6-10 \), when the UV emission rate in galaxies with virial temperatures greater than \( \approx 10^4 \) K (where hydrogen atoms are efficient coolants) exceeds the overall recombination rate of the intergalactic medium (IGM) (see, e.g., Barkana & Loeb 2001; Madau 2002). Pre-WMAP (Wilkinson Microwave Anisotropy Probe) cosmic microwave background (CMB) experiments place an upper limit of \( \tau_{\mathrm{e}} \approx 0.3 \) on the Thomson scattering optical depth due to the IGM, giving an upper bound for the reionization epoch at \( z \leq 22 \) (see Venkatesan, Tumlinson, & Shull 2003 for references and a summary of past and ongoing CMB experiment results on \( \tau_{\mathrm{e}} \)). At the time of this writing the redshift range (for the final reionization epoch) has been narrowed to a point at \( z \approx 6 \), as suggested by recent observations of high-redshift
quasars from the Sloan Digital Sky Survey (see, e.g., Becker et al. 2001; Cen & McDonald 2002; Barkana 2002; Fan et al. 2002; Litz et al. 2002).

However, the thermodynamic state of the IGM at $z > 6$ is quite uncertain but has profound implications on many observations, including future CMB experiment results, such as those from WMAP and Planck. In addition, a new picture for the initial mass function (IMF) of Population III stars is emerging from a number of recent theoretical studies of the collapse of primordial gas clouds at high redshifts (Abel, Bryan, & Norman 2002; Bromm, Coppi, & Larson 2002; Nakamura & Umemura 2002), which should have significant bearings on the IGM at high redshift. Here Population III stars are defined to be metal-free stars with a top-heavy IMF, and Population III galaxies are defined to be galaxies where Population III stars form.

In this paper we present a new scenario. In the context of the standard cold dark matter (CDM) cosmological model we show that the universe was first reionized at $z = 15–16$ by Population III stars and again reionized at $z = 6$. Following the first reionization, the transition from Population III stars to Population II stars occurs. At this point photoionization becomes insufficient to counterbalance the rapid recombination process, and the IGM recombines to become opaque to Ly$\alpha$ and ionizing photons, again! As time progresses, with increasing density fluctuations and the nonlinear mass-scale, the star formation rate gradually picks up. At $z \sim 6$ the global star formation rate exceeds the global recombination rate, and the universe is completely reionized for the second time and stays ionized henceforth. From the first cosmological reionization through the second cosmological reionization the mean temperature of the IGM is maintained at $\sim 10^4$ K, balanced between Compton cooling and photoheating, and hydrogen is more than half-ionized, balanced between recombination and photoionization.

We show that this new reionization picture is inevitable, as long as the Population III IMF is top-heavy, thanks to a new, powerful constraint placed on the product of star formation efficiency and the ionizing photon escape fraction from galaxies at high redshift, which otherwise would be unknown. This constraint comes from two solid pieces of observation: (1) the universe is required to be reionized at $z \sim 6$ and (2) the density fluctuation in the universe at $z \sim 6$ is well determined by the same small-scale power traced by the observed Ly$\alpha$ forest at $z \sim 3$. Two other assumptions are that there is a universal transition from Population III stars to Population II stars at some redshift determined by the level of the metal enrichment of the IGM and that metals from Population III star formation are blown out from the dwarf Population III galaxies into the IGM.

While it is likely that the ionizing sources for the second cosmological reionization are stars in large galaxies with efficient atomic line cooling (see Fig. 13), the ionizing sources for the first cosmological reionization are uncertain to the extent that we do not know what the relative star formation efficiencies in large halos and minihalos are. We define “minihalos” as those whose virial temperature is less than $\sim 8 \times 10^5$ K, where only H$_2$ cooling is possible in the absence of metals, and we define large halos as those with virial temperatures above $\sim 8 \times 10^5$ K, capable of cooling via atomic lines. We show that, if star formation efficiency in minihalos with H$_2$ cooling is not more than a factor of 10 less efficient than in large halos with atomic cooling, then Population III stars in minihalos may be largely responsible for the first reionization. Conversely, the first reionization may be attributable largely to Population III stars in large halos.

In order to enable efficient H$_2$ cooling in minihalos, it is necessary to maintain an adequate level of H$_2$ molecule fraction within their cores. As is well known, H$_2$ molecules are fragile and easily destroyed by photons in the Lyman-Werner (LW) bands ($11.18–13.6$ eV; Field, Sommervill, & Dresser 1966; Stecher & Williams 1967), to which the universe is largely transparent. Primeval H$_2$ molecules are thought to be completely destroyed well before enough ionizing photons are produced to reionize the universe (Gnedin & Ostriker 1997; Haiman, Rees, & Loeb 1997; Tegmark et al. 1997). Therefore, an adequate production rate of H$_2$ molecules is required to counteract the destruction rate to keep the H$_2$ fraction at a useful level conducive to further fragmentation and star formation. A sufficiently high X-ray background at high redshift could serve as a requisite catalyst for forming H$_2$ molecules by deeply penetrating into and generating a sufficient number of free electrons in the cores of minihalos (Haiman et al. 1996; Haiman, Abel, & Rees 2000; Ricotti, Gnedin, & Shull 2001, hereafter RGS; Glover & Bland 2003, hereafter GB) or the IGM (Oh 2001; Venkatesan, Giroux, & Shull 2001).

We put forth two new mechanisms for generating a high X-ray background during the Population III era. We point out that at $z = 13–20$ Population III supernova remnants and miniquasars powered by Population III black holes are efficient X-ray emitters. The much higher density of the interstellar medium at high redshift results in a more rapid and earlier cooling phase of the supernova blast waves, emitting a large fraction of the cooling energy in X-ray energy ($\sim 1$ keV). (In contrast, supernova blast waves in the local galaxies are known not to cool efficiently until their temperatures have dropped well below the X-ray regime; see, e.g., Cox 1972; Chevalier 1974, 1982.) Miniquasars powered by Population III black holes of mass $\sim 10–100 M_\odot$ are expected to emit a large fraction of the radiation in X-rays at $\geq 1$ keV. The combined X-ray emission from these two sources is sufficient to enable efficient H$_2$ formation and cooling in the cores of minihalo galaxies in the redshift period in question.

In addition, an important process for producing a large number of H$_2$ molecules in relic H ii regions of Population III galaxies is quantified. It is shown that H$_2$ molecules produced by this process may overwhelm the LW photons produced by stars in the same Population III galaxies. As a result, the LW background may not have built up in the first place during the Population III era. In combination, we suggest that star formation in gas-rich minihalos can continue to take place largely unimpeded throughout the first reionization period.

The outline of this paper is as follows. Two new sources of X-ray radiation, namely, the Population III supernova cooling radiation and X-ray emission from miniquasars, are quantified in § 2. In § 3 we quantify the H$_2$ formation in relic H ii regions of Population III galaxies. In § 4 we compute in detail the evolution of the IGM from $z \geq 20$ to 6. We discuss possible implications and significant consequences and tests of this new scenario in § 5 and conclude in § 6. Throughout, a spatially flat CDM cosmological model with $\Omega_M = 0.25$, $\Omega_b = 0.04$, $\Lambda = 0.75$, $H_0 = 72$ km s$^{-1}$ Mpc$^{-1}$, $\sigma_8 = 0.8$, and a power spectrum index $n = 1$ is adopted (see, e.g., Bahcall et al. 2003).
2. X-RAY EMISSION FROM POPULATION III SUPERNOVAE AND MINIQUASARS AND MOLECULAR HYDROGEN FORMATION IN MINIHALOS

The important role that H$_2$ molecules play in the collapse of primordial gas clouds to form metal-free stars has a long history of investigation (Hirasewa 1969; Hutchins 1976; Silk 1977; Hartquist & Cameron 1977; Shchekinov & Edelman 1978; Yoshii & Sabano 1979; Tohline 1980; Carlberg 1981; Lepp & Shull 1984; Yoshii & Saio 1986; Lahav 1986; Stahler 1986; Shapiro & Kang 1987; Uehara et al. 1996; Nakamura & Umemura 1999, 2001, 2002; Larson 1995, 2000; Abel et al. 1998; Abel, Bryan, & Norman 2000, 2002; Bromm, Coppi, & Larson 1999, 2002; Fuller & Couchman 2000; Machacek, Bryan, & Abel 2001). The emerging picture, from a number of recent theoretical studies (Abel et al. 2002; Bromm et al. 2002; Nakamura & Umemura 2002), for the mass of metal-free stars in the collapsing primordial gas clouds (induced by H$_2$ cooling) at redshift $z \sim 10-30$ is that these primordial gas clouds fragment to form very massive stars with mass $M \geq 100 M_{\odot}$. This outcome appears to be determined by the Jeans mass of the collapsing cloud, involving a complicated interplay between cooling and fragmentation. For galaxies at the redshift in question ($z \sim 13-20$), the initial (i.e., interstellar) gas density is less than the threshold value of $10^5$ cm$^{-3}$ identified by Nakamura & Umemura (2001, 2002), and fragmentation is to occur at low density with a fragment mass of $\sim 100 M_{\odot}$. We adopt this new theory for Population III stars, called very massive stars (VMSs). However, we note that the main conclusions drawn in this paper will remain unchanged as long as the IMF of Population III stars is substantially top-heavy (see, e.g., Umeda & Nomoto 2003).

Many authors have examined the possibility of Population III stars in minihalos with a normal IMF exerting significant feedback on subsequent structure formation (Couchman & Rees 1986), reionizing and reheating the universe (Couchman 1985; Fukugita & Kawasaki 1994; Haiman & Loeb 1997), or partially reionizing the universe (see, e.g., Gnedin & Ostriker 1997; Gnedin 2000a; RGS). The primary apparent difficulty for Population III stars in minihalos to reionize the universe has been identified with the destruction of H$_2$ molecules by photons in the LW bands resulting from the same Population III stars, long before the completion of the reionization process (Gnedin & Ostriker 1997; Haiman, Rees, & Loeb 1996; Tegmark et al. 1997); the H$_2$ photodissociation time becomes shorter than the Hubble time when the ionizing radiation intensity at the Lyman limit reaches $\sim 10^{-24}$ ergs cm$^{-2}$ s$^{-1}$ Hz$^{-1}$ sr$^{-1}$ at the redshift of interest, about 3 orders of magnitude below what is required to ionize the universe.

We point out two new important sources for X-ray emission in this section, capable of creating a high X-ray background at high redshift to enable the formation of enough H$_2$ molecules to induce continuous cooling and star formation in minihalos during the Population III era.

2.1. X-Ray Emission from Population III Supernova Remnants

We use simple analytic means to estimate the X-ray emission from Population III supernova remnants, assuming that a spherical supernova blast wave propagates into a uniform-density interstellar medium. A spherically expanding supernova remnant at sufficiently late times follows the simple, adiabatic, self-similar Sedov-Taylor solution (Sedov 1959; Taylor 1950; Shklovsky 1968; Cox 1972; Ostriker & Cowie 1981). Specifically, the shock radius ($R_s$), shock velocity ($V_s$), and postshock temperature ($T_s$) (for $\gamma = 5/3$) obey

$$R_s = 21.7 t_4^{2/5} (E_{52}/n_0^{1/5}) \text{ pc},$$

$$V_s = 83.9 t_4^{-3/5} (E_{52}/n_0^{1/5}) \text{ km s}^{-1},$$

$$T_s = 8.45 \times 10^6 t_4^{-6/5} (E_{52}/n_0^{2/5}) \text{ K},$$

where $E_{52}$ is the explosion energy in units of $10^{52}$ ergs, $t_4$ is the time elapsed since the onset of explosion in units of $10^4$ yr, and $n_0$ is the density of the interstellar medium in units of cm$^{-3}$.

The Sedov-Taylor phase ends with a rapid cooling phase, resulting in the formation of a thin dense shell (see, e.g., Ostriker & McKee 1988). This radiative cooling phase of the shocked gas sets in abruptly approximately when the cooling time is equal to the time elapsed, i.e., when

$$t = \frac{3n_e k T_s}{\Lambda(T_s) n_e^2},$$

where 100% hydrogen is assumed in computing the internal energy and the electron density $n_e$ for simplicity (but not for the cooling function $\Lambda$; see below); $k$ is Boltzmann’s constant; $n_e = 4n$ is the postshock electron density; and $\Lambda(T_s)$ is the volume cooling function for a primordial plasma with 76% hydrogen and 24% helium by mass, taken from Sutherland & Dopita (1993). Since the blast waves of the first generation of supernovae propagate into a primordial gas free of metals, metal cooling is nonexistent. More importantly, complex processes due to dust need not be considered; in particular, X-ray emission would not suffer significantly from dust absorption (Ostriker & Silk 1973; Burke & Silk 1974; Draine & Salpeter 1979; Shull 1980; Wheeler, Mazurek, & Sivaraniakrishnan 1980; Tielens et al. 1987; Draine & McKee 1993). Combining equations (3) and (4) and solving for $T_s$ as a function of $n$ gives the results shown in Figure 1, for two cases with $(E_{52}, Z/Z_{\odot}) = (5.0, 0.0)$ and $(0.1, 1.0)$, where the former may be appropriate for supernovae/hypervovae resulting from very massive Population III stars (Woosley & Weaver 1982; Ober, El Eid, & Fricke 1983; Bond, Arnett, & Carr 1984; Nakamura et al. 2001; Heger & Woosley 2002) and the latter for present-day normal supernovae.

A critical point to note is that the density of the general interstellar medium should scale with $(1 + z)^3$, being significantly higher at high redshift than that of the local interstellar medium. This assumption should hold in the context of cosmological hierarchical structure formation for the following reasons. First, the mean gas density scales with $(1 + z)^3$. Second, halos at low and high redshift in cosmological simulations show similar properties when density and length are measured in their respective comoving units (see, e.g., Navarro, Frenk, & White 1997; Del Popolo 2001). Third, the spin parameters (i.e., angular momentum distribution) of both high- and low-redshift halos have very similar distributions peaking at a nearly identical value, $\lambda \sim 0.05$ (Peebles 1969; White 1984; Barnes & Efstatiou 1987; Ueda et al. 1994; Steinmetz & Bartelmann 1995; Cole & Lacey 1996; Bullock et al. 2001; R. Cen et al. 2003, in preparation). Thus, cooling gas in galaxies at low and high redshift should collapse by a similar factor before the
structure becomes dynamically stable (e.g., rotation support sets in), resulting in interstellar densities scaling as \((1 + z)^3\). Direct simulations (Abel et al. 2000, 2002; Bromm et al. 2002) suggest a gas density of \(10^3-10^4\) cm\(^{-3}\) by the end of the initial free fall for minihalos at \(z \approx 20\), verifying this simple analysis. The same argument was given in Kaiser (1991) on a somewhat larger scale for clusters of galaxies. Peebles (1997), using the same scaling of density with respect to redshift, suggested that an early epoch of galaxy formation may be favored. Observationally, the higher density in bulges of spiral galaxies or elliptical galaxies is consistent with their earlier formation epochs. In Figure 1 we have used \(n(z) = n_0(1 + z)^3\), with \(n_0 = 1\) cm\(^{-3}\) being the density of the local interstellar medium, to translate the bottom \(x\)-axis \((n)\) to the top \(x\)-axis \((1 + z)\).

In Figure 1 we see that at \(z = 15.5\) (or \(n = 4492\) cm\(^{-3}\)), the blast wave enters the rapid cooling phase at a postshock temperature slightly above \(1.0 \times 10^7\) K. Most of the energy is radiated away during the brief cooling phase (Falle 1975, 1981) with photon energy \(h\nu \sim kT\). (Subsequent evolution of the cooling shell will be subject to various instabilities; see, e.g., Chevalier & Imamura 1982; Vishniac 1983; Bertschinger 1986; Cioffi, McKee, & Bertschinger 1988. Following that, the evolution enters a snowplow phase driven by the pressure of a still hot interior gas; McKee & Ostriker 1977; Ostriker & McKee 1988.) For a primordial gas cooling at \(1.0 \times 10^7\) K, it is found that \((40\%, 31\%, 18\%)\) of the instantaneously radiated energy is at photon energies above \((0.8, 1.0, 1.5)\) keV. Clearly, a significant amount of the total energy of the supernova blast wave will be turned into X-ray photons. As we see below, \(z = 15-16\) is identified as the redshift of the first cosmological reionization by Population III stars. At higher redshift, the emitted photons from supernova remnant shell cooling would be still harder.

As a consistency check we find that, for \(n = 4492\) cm\(^{-3}\), at the onset of the rapid cooling phase, the elapsed time is \(t_{\text{rad}} = 857\) yr, the shell radius is \(r_{\text{rad}} = 2.09\) pc, and the swept-up interstellar medium mass is \(M_{\text{rad}} = 4169\) \(M_\odot\). The fact that \(M_{\text{rad}}\) is much larger than the mass of the supernova ejecta \((M_e \sim 100\) \(M_\odot\)) guarantees the Sedov-Taylor solution for the regime in question (until cooling sets in). The fact that the shell radius \(r_{\text{rad}}\) is much smaller than the size of galaxies \((\gtrsim 10\) pc\) and that \(M_{\text{rad}}\) is much smaller than the total baryonic mass in the galaxies \((\gtrsim 10^4\) \(M_\odot\)) in question indicates that the blast wave at the cooling time is still sweeping through the high-density interstellar medium, as was assumed. As another consistency check, the time required for electrons and ions to reach temperature equilibrium, is \(t_{\text{eq}} = 1.4 \times 10^5 E_{52}^{-1/4} n^{-3/7} = 162\) yr for the case considered, which is much shorter than \(t_{\text{rad}}\), indicating that electrons can radiate away the shock-heated thermal energy and that the Sedov-Taylor phase is valid. In contrast, in the starburst model for active galactic nuclei (AGNs; Terlevich et al. 1992), the proposed supernova remnants interact with a much higher density medium \((n \sim 10^3\) cm\(^{-3}\)) such that the Sedov-Taylor phase is never reached because of extremely rapid cooling before thermalization.

In brief, a large fraction of supernova explosion energy in Population III galaxies at \(z \sim 13-20\), possibly as large as \(f_X \sim 0.30\), is shown to be converted into X-ray photons with energy greater than 1 keV. The X-ray background produced by this process is shown below to be able to play a positive feedback role in the formation of and cooling by \(H_2\) molecules in minihalos.

Let us now proceed to identify the characteristics of the composite spectrum of the background radiation field produced by both emissions from the VMSs and the thermal emission from VMS supernova blast waves. A relevant parameter for our purpose is the ratio of the energy in the LW bands \((h\nu = 11.18-13.6\) eV\) to the energy in photons with \(h\nu > 1\) keV, \(\Psi_{\text{III}} \equiv E_{\text{LW}}/E_{\text{1keV}}\). Photons in the LW bands \((11.18-13.6\) eV\) are primarily responsible for photoionizing \(H_2\) molecules and are not heavily absorbed by intervening intergalactic atomic hydrogen (except for the sawtooth modulation by the atomic Lyman line series; Haiman, Abel, & Rees 2000, hereafter HAR). Hard X-ray photons at \(h\nu > 1\) keV, on the other hand, are also largely unabsorbed by intervening atomic hydrogen and helium and are capable of penetrating deeply into the cores of minihalos to produce free electrons through both direct photoionization and secondary photoelectron ionization. The abundance of \(H_2\) molecules in the cores of minihalos is primarily a result of the competition between the two. In contrast, photons with energy in the range 13.6 eV–1 keV are heavily absorbed by atomic hydrogen and helium prior to complete reionization of the universe and thus have little effect on the formation of \(H_2\) molecules in the cores of minihalos (but see RGS for a positive feedback due to ionizing photons on the surface of the expanding \(H\) ii regions and in relic \(H\) ii regions). We may write \(\Psi_{\text{III}}\) approximately as

\[
\Psi_{\text{III}} \approx \frac{0.007 M_{\text{VMS}} c^2 f_X}{\xi_{\text{IMF}} E_{\text{ex}} f_X},
\]  

where the coefficient 0.007 indicates thermonuclear energy.
efficiency; \( f_{\text{LW}} \) is the fraction of energy in the LW bands emitted by VMSs; \( c \) is the speed of light; \( M_{\text{VMS}} \) is the characteristic mass of VMSs (\( \sim 200 M_\odot \); see § 3); \( E_{\text{x}} \) is the explosion energy of a typical VMS; and \( \xi_{\text{IMF}} \) is inserted to take into account the effect that some VMSs (e.g., nonrotating stars with \( M \geq 260 M_\odot \); Rakavy, Shaviv, \& Zinnam 1967; Bond et al. 1984; Glatzel et al. 2001; Heger \& Woosley 2002) do not produce supernovae (but collapse wholly to black holes). The supernova explosion energy, \( E_{\text{ex}}, \) released by VMSs in the mass range \( 140-260 M_\odot \) is approximately \( 10^{52}-10^{53} \text{ ergs} \) (Woosley \& Weaver 1982; Ober et al. 1983; Bond et al. 1984; Nakamura et al. 2001; Heger \& Woosley 2002). The VMS has approximately a blackbody radiation spectrum with an effective temperature of \( \sim 10^{7.2} \text{ K} \) (Tumlinson \& Shull 2000; Bromm, Kudritzki, \& Loeb 2001), for which it is found that \( f_{\text{LW}} = 1.5 \times 10^{-2}, \) yielding

\[
\Psi_{\text{III}} = 2.5\xi_{\text{IMF}} f_{\text{X}}^{-1} \left( \frac{E_{\text{ex}}}{5 \times 10^{52} \text{ ergs}} \right)^{-1} \frac{M}{200 M_\odot}. \tag{6}
\]

Is it small enough to enhance \( H_2 \) formation in minihalos? Let us compare it to the results of a systematic study of the effect of X-ray photons on the cooling of minihalo gas by HAR. HAR investigated the effect using a \( J_\nu \propto \nu^{-1} \) background radiation spectrum, with an upper cutoff at 10 keV. Using \( \Psi \) to parameterize their results, the findings of HAR are that, when

\[
\Psi_{\text{HAR}} \leq \frac{\ln(13.6/11.18)}{\ln(10/1)} = 5.1 \tag{7}
\]

(for \( \xi_{\text{X}} = 0.16 \)), \( H_2 \) formation is enhanced and cooling by \( H_2 \) is sufficient to allow for the gas in the cores of minihalos with virial temperature \( T_v \geq 1000 \text{ K} \) to collapse to form stars. We note that in evaluating \( \Psi_{\text{HAR}} \) above, we have taken into account a factor of \( \sim 6 \) underestimation of \( \Psi_{\text{HAR}} \) in the original work of HAR due to a factor of 6 underestimate of the LW photon absorption cross section by \( H_2 \) in the original work of HAR due to a factor of 6 underestimation of the LW photon absorption cross section by \( H_2 \) molecules (Z. Haiman 2002, private communication; RGS; GB). The correction factor of 6 is a lower limit in the optically thin case for \( H_2 \) molecules for the cores of minihalos. In the optically thick limit, a substantially larger correction factor needs to be applied (i.e., \( \Psi_{\text{HAR}} \) would be substantially larger than 5.1), although the exact effect would require a recalculation of the results in HAR. It thus appears that the X-ray emission from Population III supernova remnants alone may be able to produce enough X-ray photons relative to the number of photons in the LW bands such that the production rate of \( H_2 \) molecules dominates the destruction rate in most of the minihalos. It is still possible that, in the case that \( \xi_{\text{IMF}} \) and/or \( f_{\text{X}} \) are significantly lower than their adopted fiducial values, the effect due to X-ray emission from Population III supernova remnants would possibly be diminished; detailed studies would be required to more carefully quantify this.

### 2.2. X-Ray Emission from Miniquasars Powered by Population III Black Holes

Another direct consequence of Population III VMSs is an inevitable production of a significant number of black holes. Population III VMSs more massive than \( 260 M_\odot \) would eventually implode, carrying the entire mass to form black holes (Rakavy et al. 1967; Bond et al. 1984; Glatzel et al. 1985; Woosley 1986) without producing explosions. Population III VMSs less massive than \( 140 M_\odot \), after about 3 million years of luminous life, would explode as supernovae or hypernovae, leaving behind black holes of mass \( \sim 10-50 M_\odot \) (Heger \& Woosley 2002). These Population III black holes could accrete gas from the surroundings to shine as miniquasars. The likelihood of gas accreting onto these black holes is at least as high as in their lower redshift counterparts since the gas density is higher and halos tend to have somewhat lower spins at high redshift. In the case of the very massive Population III black holes (\( M \geq 260 M_\odot \)) formed without explosion, gas may be ready to accrete immediately since the surrounding gas has not been blown away. The dynamics of the black holes in these environments in the context of cosmological structure formation is a complex subject and can only be treated in separate works. Here we make the assumption that these black holes would accrete gas and grow.

The characteristic gas temperature of a disk powered by accretion onto a black hole is (Rees 1984)

\[
T_e = 1.3 \times 10^7 M_\odot^{1/4} \text{ K}, \tag{8}
\]

where \( M_\odot \) is the black hole mass in units of \( 10^2 M_\odot \). For \( M_{\text{BH}} = (200, 20) M_\odot \), we have \( T_e = (1.3 \times 10^7, 2.4 \times 10^7) \text{ K} \). While the spectral energy distribution (SED) of quasars powered by supermassive black holes is known to contain a significant fraction in X-rays (e.g., Elvis et al. 1994), the miniquasars at high redshift powered by much smaller black holes will be conspicuous in X-rays, probably emitting predominantly in the X-ray band from both thermal and nonthermal emission. A somewhat more quantitative argument may be made as follows. The SED of observed quasars powered by supermassive black holes of mass \( \sim 10^8 M_\odot \) contains a substantial amount of energy in the X-rays, but the largest concentration of energy appears to peak at \( \sim 12-13 \text{ eV} \) (the UV bump; Elvis et al. 1994), barring the unknown gap between UV and X-ray. Under the reasonable assumption that the SED peak frequency scales with the characteristic temperature \( T_e \), then the peak frequency for Population III black hole–powered miniquasars would be shifted (by a factor of \( \sim 100 \) compared to quasars) to \( \nu \sim 1 \text{ keV} \).

Let us now compute the X-ray background produced by miniquasars powered by Population III black holes. We put this quantitatively in the context of the ratio of energy in LW photons to that of X-ray photons. First, an order-of-magnitude estimate: if a miniquasar has accreted a fraction \( \eta_Q \) of the initial mass of the black hole, the radiated energy will be

\[
E_Q = \eta_Q \alpha \frac{c^2 M_{\text{BH}}}{10^{-3}} \tag{9}
\]

where \( \alpha \) is the radiative efficiency. Then the ratio \( \Psi \) may be written as

\[
\Psi_Q = \frac{0.1 \Omega_{\text{f}} c^2 f_{\text{LW}} \alpha \eta_Q}{0.1 \Omega_{\text{f}} c^2 f_{\text{BH}} f_{\text{X}} f_{\text{esq}}} = 10^{-3} \eta_Q^{\frac{1}{5}} \alpha_{\text{f}}^{\frac{1}{5}} f_{\text{BH}}^{-1} f_{\text{X}}^{-1} f_{\text{esq}}, \tag{10}
\]
by miniquasars with photon energy greater than 1 keV; and $f_{eq}$ is the X-ray escape fraction from miniquasars. We have used $f_{1\text{W}} = 1.5 \times 10^{-2}$. The value of $\alpha_{0.1}$ is thought to be close to unity (Rees 1984), and the value of $f_X$ is expected to be of the order of unity. The fraction of mass in Population III stars ultimately collapsing to black holes $f_{BH}$ is also of the order of unity if Population III stars are either very massive, $\sim 100 M_\odot$, or $\gg 1 M_\odot$. The accretion fraction $\eta$ would depend on the gas accretion rate and the formation time of the black hole; parameterization by equation (10) would be fairly accurate if black holes formed a long time ago. But a more accurate way to compute the X-ray energy from miniquasars is to integrate over time over all miniquasars by parameterizing each radiating at $\xi_{\text{Edd}}$ times the Eddington luminosity. The results are shown in Figure 2, where the solid and dotted curves show $\Psi_Q$ with radiative efficiency of $\alpha = (0.1, 0.03)$, respectively. The global Population III star formation rate is computed using a Press-Schechter (1974) formula considering all halos with virial temperature greater than 1000 K. The reason that the dotted curve with lower radiative efficiency ($\alpha = 0.03$) lies below (i.e., emits less X-ray photons) the solid curve is that the black hole mass in this case grows at a faster rate, being inversely proportional to the radiative efficiency.

Another useful way to estimate the growth of mass in black holes is to follow the local observed black hole mass–bulge mass relation, $M_{BH} \sim 0.006 M_{\text{bulge}}$ (Magorrian et al. 1998). If the same ratio of the mass in black holes to bulge mass, which at high redshift may be equated to the total stellar mass, holds at high redshift, then we can readily compute the energy liberated by gas accretion onto Population III black holes as

$$\Psi_Q = \frac{0.007 \epsilon_f f_{III} f_{1\text{W}}}{0.006 \times 1.0 \times 10^{23} L_{\text{Edd}} f_{III} f_{BH} f_X f_{eq}} = 0.88 \alpha_{0.1} \xi_{\text{Edd}}^{-1} \frac{f_{eq}}{0.2} f_{BH}^{-1} f_X^{-1} .$$

(11)

The horizontal dashed line in Figure 2 indicates $\Psi_Q$ computed this way, with radiative efficiency of $\alpha = 0.1$ and X-ray escape fraction $f_{eq} = 0.2$.

From Figure 2 it is evident that $\Psi_Q$ is significantly smaller than $\Psi_{\text{HAR}} = 5.1$ (eq. [7]) at $z \leq 2$, required to induce sufficient H$_2$ cooling in minihalos (HAR). The higher value of $\Psi_Q$ at higher redshift ($z \geq 20$) is due to the fact that the age of the universe becomes much shorter than the Eddington time of $\sim 4 \times 10^8$ yr and the black holes have not had enough time to accrete a substantial amount of gas; the age of the universe is $1.9 \times 10^9$ yr at $z = 20$ (for $H = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_M = 0.25$, and $\Lambda = 0.75$). We stress that, if for miniquasars $\alpha$ has a value very close to unity, then the X-ray emission from miniquasars would not be significant until $z \leq 9$.

In summary, two distinctive mechanisms, namely, supernova remnants and miniquasars, both appear to be able to generate enough X-ray emission at $z \sim 13$–20 to provide positive feedback on star formation in subsequent (other) minihalos by making a sufficient number of X-ray photons relative to the number of destructive photons produced in the LW bands. The combination of the two should ensure that enough X-ray radiation is produced as a result of Population III star formation. In addition, there may be other, significant X-ray emission mechanisms, such as inverse Compton emission (see, e.g., Hogan & Layzer 1979; Oh 2001) or massive X-ray binaries (see, e.g., Bookbinder et al. 1980; Helfand & Moran 2001). Moreover, additional positive feedback mechanisms, such as that proposed by Ferrara (1998) due to enhanced H$_2$ formation in the supernova cooling shells, that put forth by RGS from the enhanced H$_2$ formation at the surfaces of Strömgren spheres of individual Population III galaxies, H$_2$ formation in the IGM exposed to an X-ray background (Oh 2001; Venkatesan et al. 2001), and that quantified in § 3 due to H$_2$ formation in relic H II regions produced by Population III galaxies, will further help promote H$_2$ formation. Taking all the processes together, it appears likely that the chief obstacle to continuous H$_2$ formation and cooling in minihalos is removed.

Since the assumption that X-ray emission produced by these two processes related to Population III star formation can produce positive feedback to subsequent star formation is important, it is warranted to have independent checks. We compare our results to those of GB. We compare to their X-ray emission model due to inverse Comptonization, which has luminosity density $L_X = 7.7 \times 10^{23} (\nu_0 / \nu) f_e \text{ ergs s}^{-1} \text{ Hz}^{-1} (M_\odot \text{ yr}^{-1})^{-1}$, where $\nu_0 = 1 \text{ keV}$ and $f_e$ is the fraction of supernova energy transferred to relativistic electrons. Integrating $L_X$ over a range 1–10 keV, we obtain $E_X = 4.28 \times 10^{42} f_e \text{ ergs s}^{-1} (M_\odot \text{ yr}^{-1})^{-1}$. The luminosity at the LW bands in GB is $L_{1\text{W}} = 1.1 \times 10^{28} \text{ ergs s}^{-1} (M_\odot \text{ yr}^{-1})^{-1}$, which after integration over LW bands gives $E_{1\text{W}} = 6.4 \times 10^{42} \text{ ergs s}^{-1} (M_\odot \text{ yr}^{-1})^{-1}$. Taking the ratio of the above two luminosities yields $\Psi_{GB} = E_{1\text{W}} / E_X = 150$ for...
the most optimistic model that they use with $f_c = 0.1$. For this model, GB find that the critical virial temperature of halos where cooling is sufficient is lowered from $8 \times 10^3$ to about $2 - 3 \times 10^3$ K at $z \sim 17$ (see Fig. 6 in GB), a significant effect. While even this model is already sufficient to enable molecular cooling in halos with $T_v > 2000$ K to reionize the universe at $z \sim 17$ (see Fig. 5), it is expected that the much smaller $\Psi_{\text{III}}$ computed above for Population III stars in our model would further drive the critical virial temperature to much lower values and enable star formation in almost all minihalos, consistent with the conclusions reached by HAR.

3. FORMATION OF H$_2$ MOLECULES IN RELIC H II REGIONS PRODUCED BY POPULATION III GALAXIES

While the positive feedback due to a high X-ray background produced by Population III supernovae and mini-quasars powered by Population III black holes seems sufficient to sustain continuous gas cooling in gaseous minihalos, we quantify another important source for production of H$_2$, which was first pointed out by RGS for relic H II regions of high-redshift galaxies with metal-free stars of Salpeter IMF (Tumlinson & Shull 2000).

Each Population III galaxy creates an H II region (Strömgren sphere) around it. Since the lifetime of a Population III galaxy of $3 \times 10^6$ yr is comparable to and somewhat shorter than the hydrogen recombination time, $t_{\text{rec}} = 1.3 \times 10^7$ yr at $z \sim 20$, we may assume for simplicity, without causing substantial errors, that the size of the Strömgren sphere is just determined by the number of ionizing photons emitted, $N_{\text{ion}}$. We follow the evolution of such a relic H II region after the death of a Population III galaxy, in the absence of any external radiation field. We solve a set of rate equations and energy equation for relevant species in an H II region free of metals. The relevant reaction coefficients are taken from Abel et al. (1997).

Figure 3 shows the fraction of H$_2$ molecules as a function of redshift for four H II regions formed at $z = 17, 18, 19, 20$, evolved up to redshift $z = 15$, which is identified as the first reionization epoch by Population III stars (§ 4). We assume an average clumping factor of 15 for this calculation for gas surrounding and inside Population III galaxies; the adopted clumping factor of 15 is only a rough guess, based on the argument that the clumping factor for gas surrounding halos (including gas inside the halos for the H II regions in question) should be significantly larger than the mean IGM clumping, which is a few at the redshift range, $z = 15-20$, in question (see Fig. 12). The typical size of the H II regions around Population III galaxies at the redshift in question is $\sim 10-100 \, h^{-1} \, kpc$ comoving.

We see that the H$_2$ fraction in H II regions shows a sharp rise at an early time and flattens out at later times. The reason for this asymptotic convergence may be understood by examining Figure 4, where the temperature evolution of the gas in the region is shown. The H$_2$ formation rate decreases with decreasing temperature (Abel et al. 1997), causing the total H$_2$ abundance to flatten out below 1000 K. On the high-temperature end, since the H$_2$ formation rate only becomes significant after the temperature has dropped below about 7000 K, we see a slight delay ($\Delta z \sim 1.5$) in the ascent of the H$_2$ abundance in Figure 3.
at higher redshifts, this ratio would still be higher.) Then we obtain $R_{\text{relic}} = 0.13$. Since about 10 LW photons are needed to photodissociate one H$_2$ molecule (RGS; GB), the break-even point for the ratio of the number of H$_2$ molecules produced over the number of LW photons produced is about 0.1; i.e., no net molecules would be produced. It thus appears that H$_2$ molecules produced in the relic H II regions produced by Population III stars may be sufficient to offset the production of LW photons by the same stars. Consequently, the LW radiation background may not be able to build up during the Population III era. However, a more careful calculation, taking into account the density fluctuations among others, is needed to precisely quantify this positive feedback effect. But we defer such a calculation to a future work. Nevertheless, it seems possible that this positive feedback in relic H II regions may be more effective than that proposed by Ferrara (1998), due to enhanced H$_2$ formation in the supernova cooling shells, and that put forth by RGS from the enhanced H$_2$ formation at the surfaces of Strömgren spheres of individual Population III galaxies.

In summary, it appears that LW radiation background may have not been able to build up to destroy H$_2$ molecules in subsequent minihalos. In the event that LW radiation background was able to build up, its intensity would have been substantially reduced. The fact that the X-ray background produced by Population III supernovae and mini-quasars powered by Population III black holes has already been capable of countering an undiminished LW radiation background (i.e., by ignoring all the H$_2$ molecules produced by these positive feedback processes) leads us to conclude that subsequent star formation in minihalos will not be hindered by previous star formation in the Population III era.

### 4. THE PROCESS OF COSMOLOGICAL REIONIZATION

#### 4.1. Computational Method

We examine the evolutionary history of the IGM from a very early redshift ($z \gg 20$) until the universe is completely reionized at $z \sim 6$, as suggested by recent observations. Rather than taking a brute force approach, which will be deferred to a later work, we use a new, fast, Monte Carlo-like approach to economically sample parameter space. This method, we hope, should be capable of capturing the essential physical processes involved.

At any time the gas in the universe consists of $N$ distinct regions in the two-dimensional phase space specified by $(T_i, y_i)$, where $T_i$ and $y_i$ are the temperature and neutral hydrogen fraction, respectively, and $N$ is a varying number. Each phase-space region $i$ contains $f_i$ mass fraction of the total gas in the universe, and its mean gas density is assumed to be equal to the global mean; the sum of all $f_i$ is unity. For each region $i$, we solve a combined set of equations simultaneously to follow the coupled evolution of gas and star formation:

$$\frac{df_i}{dt} = \frac{dN_{\text{ion}}}{dt} \frac{1}{\bar{y}} f_i ,$$

$$\frac{dT_i}{dt} = -\frac{\Sigma_i (T_i, y_i, C_i, z)}{3k} ,$$

$$\frac{dy_i}{dt} = \alpha C_i \bar{n}(1 - y_i)^2 - \beta C_i \bar{n} y_i (1 - y_i) ,$$

where $\bar{n}$ is the global mean hydrogen number density; $\bar{y}$ is the global average of the neutral hydrogen fraction; $\Sigma_i$ is the net cooling rate per unit mass (including Compton cooling, atomic line cooling, recombination cooling, photoheating due to ionization, and adiabatic cooling due to cosmic expansion), which is a function of $T_i, y_i, \text{clumping factor } C_i$ (see below), and redshift $z$; $k$ is Boltzmann’s constant; $\alpha$ is the case B hydrogen recombination coefficient; $\beta$ is the hydrogen collisional ionization coefficient; all relevant coefficients are taken from Cen (1992); $dN_{\text{ion}}/dt$ is the number of ionizing photons emitted per unit volume per unit time, equal to

$$\frac{dN_{\text{ion}}}{dt} = c^* \epsilon_{\text{UV}} c^2 \bar{n} \frac{d\psi}{13.6 \text{ eV} dt} ,$$

where $c^*$ is the star formation efficiency; $\epsilon_{\text{UV}}$ is the fraction of energy (in units of rest mass of stars formed, which is approximately equal to $1 \times 10^{-4}$ for Population II stars with low metallicity and Salpeter IMF and $1.8 \times 10^{-3}$ for massive Population III stars) turned into hydrogen-ionizing photons; $\bar{n}$ is the mean global gas mass density; $c$ is the speed of light; $f_{\text{esc}}$ is the ionizing photon escape fraction from galaxies; $13.6 \text{ eV}$ is used in conjunction with the appropriate definition for $\epsilon_{\text{UV}}$ such that the number of hydrogen-ionizing photons is properly computed, although it is understood that the average ionizing photon energy is somewhat greater than $13.6 \text{ eV}$; and $d\psi/dt$ is the global rate of fraction of gas formed into stars, equal to

$$\frac{d\psi}{dt} = \sum_i f_i \frac{d\psi_i}{dt} ,$$

where $d\psi_i/dt$ is the rate of the fraction of gas formed into stars in region $i$. At each time step, we use the Press-Schechter (1974) formula,

$$\psi_i (> M, z) = \text{erfc} \left[ \frac{\delta_c}{\sqrt{2 \sigma(M, z)}} \right] ,$$

where $\psi_i$ is the complimentary error function and $\sigma(M, z)$ is the rms density fluctuation smoothed over a region of mass $M$ at $z$ and $\delta_c$ is a constant equal to 1.69, indicating the amplitude of the density fluctuations on a top-hat sphere that collapses at $z$ (Gunn & Gott 1972). The threshold halo mass $M$ is basically the Jeans mass, given the IGM temperature of region $i$; i.e., only those halos whose mass is greater than the Jeans mass at that time will accrete gas and contribute to the star formation at that time step. In practice, however, the threshold mass $M$ depends on the history of the gas involved, as well as on nontrivial effects due to dynamically dominant dark matter and is thus best determined by detailed simulations. We use the empirically determined “filter mass” formula from full hydrodynamic simulations by Gnedin (2000b):

$$M_f = 1.0 \times 10^8 \kappa \left( \frac{\Omega_M}{0.25} \right)^{-1/2} \left( \frac{T_i}{10^4 \text{ K}} \right)^{3/2} \left( \frac{1+z}{10} \right)^{-1.5} ,$$

where at the relevant redshift range, $z \sim 6$–$20$, the empirically determined constant $\kappa$ is found to be 0.5–1.6. The results obtained in §§3.2 and 4.3 do not depend on $\kappa$ sensitively in the indicated range of $\kappa$. No. 1, 2003
As has been validated in §§ 2 and 3 at the Population III era, star formation occurs in all halos where gas is able to collect, and is not limited to large halos where atomic hydrogen cooling becomes efficient. It is, however, unclear whether H$_2$ cooling would still be efficient in the Population II era. We show that even in the absence of the X-ray background, a high residual ionization fraction can be maintained after the first reionization, thanks to the long hydrogen recombination time. Let us make a simple estimate by computing the ratio of the recombination time to the dynamical time of a region of overdensity $\delta$ with ionization fraction $x$. We find

$$\frac{t_{\text{rec}}}{t_{\text{dyn}}} = 0.8 \left(\frac{1 + z}{16}\right)^{-3/2} \left(\frac{\delta}{10^5}\right)^{-1/2} \left(\frac{x}{10^{-2}}\right)^{-1}$$

(19)

for $T = 5000$ K. It is thus clear that an ionization fraction of the order of $10^{-3}$ to $10^{-2}$ will be maintained for $\delta$ as high as $10^5$ (the core density is about $\delta = 10^6-10^7$), as long as the gas is ionized at least to that level at a higher redshift (of course, the first reionization raises the gas ionization to a much higher level than this). Such a level of ionization is adequate to enable the formation of a sufficient amount of H$_2$ molecules to supply ample cooling at $T \lesssim 10^4$ K (HAR; Tegmark et al. 1997), and star formation should continue to take place in any gaseous halo. Looking forward to the results, it turns out that the condition for H$_2$ cooling has a rather small effect for the second cosmological reionization, since the amount of intergalactic gas able to accrete onto minihalos is very small.

The halo mass within the virial radius $M_v$ is related to the virial temperature at $z$ by the standard formula

$$M_v = 1.2 \times 10^8 \left(\frac{\Omega_M}{0.25}\right)^{1/2} \left(\frac{1 + z}{10}\right)^{-3/2} \left(\frac{T_v}{10^4 \text{ K}}\right)^{3/2} \text{M}_\odot,$$

(20)

where $T_v$ is the virial temperature in kelvins and $\Omega_M$ is the matter density at zero redshift in units of the closure density. Figure 5 relates the virial temperature at each redshift to the halo mass for the five indicated virial temperatures. Figure 6 shows the fraction of gas in halos as a function of redshift, with virial temperatures greater than the five indicated values, $T_v = (200, 1000, 2000, 4000, 8000)$ K.

The clumping factor in each region affects local atomic processes and is defined to be $C_i = \psi_i C_{\text{halo}} + (1 - \psi_i)$, where $C_{\text{halo}}$ is the effective clumping factor of virialized gas per unit mass and the remaining gas is assumed to have a clumping factor equal to unity for simplicity. We adjust $C_{\text{halo}}$ such that the overall $\langle C \rangle$ averaged over all regions matches detailed simulations at some appropriate redshift. Note that the adjustment on $C_{\text{halo}}$ is necessary in order to take into account the complicated radiation shielding effect affecting halos.

Having defined the relevant formulae, let us go back to examine the main evolution equations more closely. Equation (12) is the rate equation for mass in region $i$ that is photoionized, i.e., the rate of the amount of gas swept by the ionization front. The sum of newly ionized gas over all the phase-space regions at each time step creates a separate (H II) region in the phase space with a mean temperature equal to

$$T_{N+1} = \sum_{i=1}^{N} f_i (T_i + y_i T_{\text{ion}}),$$

(21)

where the first term inside the parentheses on the right-hand side is the temperature of region $i$ and the second term accounts for photoheating, with $T_{\text{ion}}$ set to $2 \times 10^4$ K (varying $T_{\text{ion}}$ from $1 \times 10^4$ to $3 \times 10^4$ K does not significantly change the results). We set the initial neutral fraction of the newly created H II region to $y_{N+1} = 10^{-4}$, although results obtained do not depend on the exact choice for this value, as long as it is a small number. The initial mass fraction of this new H II region is equal to

$$f_{N+1} = \frac{dN_{\text{ion}}}{dt \Delta t},$$

(22)

where $\Delta t$ is the time step. We formulate that all phase-space
regions coexist in real space when averaged over a sufficiently large volume and photoionization ionizes all phase-space regions proportionally, as indicated by equation (12). Equations (13) and (14) are the energy equation and rate equation, respectively, for neutral hydrogen in each phase-space region.

The attentive reader may have noticed the central idea behind this simple approach. That is, this approach attempts to mimic the stochastic percolation process during cosmological reionization, where individual H II regions are created and their subsequent evolution followed. New H II regions created may originate from a combination of regions, some of which have previously been ionized but subsequently recombined, some of which were never ionized, and some of which may be recently created H II regions but now get percolated.

We do not include a photoionization term on the right-hand side of equation (14); rather, equivalently, we remove the fraction of gas in that region that is being ionized and add it to a new phase-space region according to equations (12) and (22). This is equivalent to saying that gas is ionized when the ionization front passes through and is not ionized uniformly.

One important difference between this method and another commonly used method that computes the filling factor of H II regions (see, e.g., Shapiro & Giroux 1987; Haiman & Loeb 1997; Madau, Haehnelt, & Rees 1999; Miralda-Escude, Haehnelt, & Rees 2000) is that we follow the evolution of all regions including H I, H II, and partially ionized regions. This is important for our purpose, chiefly because at high redshift, H II regions experience rapid recombination and cooling. Needless to say, this method is still highly simplified. More sophisticated calculations would require simulations that follow detailed three-dimensional radiative transfer (see, e.g., Norman, Paschos, & Abel 1998; Abel, Norman, & Madau 1999; Razoumov & Scott 1999; Kessel-Deynet & Burkert 2000; Gnedin & Abel 2001; Cen 2002) but will be carried out in our future work.

We start a simulation at a very early redshift, say, z = 100, when the gas may be safely assumed to be cold and neutral. For simplicity, we set the initial gas temperature to be equal to the microwave temperature, although the gas may be cooler than that, but the results would not change noticeably. Thus, the initial condition is: $T_1 = 2.73(1 + z_i)$ K (where $z_i$ is the starting redshift), $y_1 = 1$, $C_1 = 1$, $f_1 = 1$, and the initial number of entries in the phase space $(T_i, y_i)$ is $N = 1$. $N$ increases linearly with the number of time steps taken. We can afford to make the time steps sufficiently small to be able to follow the ionization and energy equations accurately in this simple approach.

To self-consistently compute the IMF, we follow the metal enrichment history of the IGM. Most importantly, there is a critical transition at some point in redshift in the metallicity of the IGM, when a certain amount of Population III stars have formed. Subsequent to that transition, sufficient metal cooling would cool collapsing gas inside halos to temperatures lower than those achievable by H$_2$ cooling, resulting in the “normal” Population II stars with a “normal” Salpeter-like IMF. According to Oh et al. (2001), this transition occurs when a fraction, $3 \times 10^{-4}$ to $1.2 \times 10^{-4}$, of the total gas is formed into VMSs with masses in the range 140–260 M$_\odot$. Without fine tuning, it is likely that VMSs with masses outside this range should exist.

Bearing that in mind, we assume conservatively for our calculations a threshold fraction of $1 \times 10^{-4}$. We note that between the formation of stars and metal enrichment of the IGM there should be a delay in time. To accommodate this time lag, we assume that 0.2 H (where H is the Hubble time at that time) after a fraction $1 \times 10^{-4}$ of total gas has formed into VMSs, the transition from Population III VMSs to normal stars with a Salpeter-like IMF takes place.

The emission spectrum of metal-free VMSs with mass greater than $100 \ M_\odot$ is relatively simple and has, to a very good approximation, a blackbody spectrum with an effective temperature of $T_{\text{eff}}$ K, radiating at the Eddington luminosity $L_{\text{Edd}} = 1.3 \times 10^{38} (M/M_\odot) \ ergs \ s^{-1}$ for about $3 \times 10^6$ yr, which translates to a hydrogen-ionizing photon production rate of $1.6 \times 10^{48}$ photons s$^{-1}$ M$^{-1}$ for VMSs (El Eid, Fricker, & Ober 1983; Bond et al. 1984; Bromm et al. 2001; Schaerer 2002). The hydrogen-ionizing photon rate for normal Population II stars with a Salpeter IMF is assumed to be $8.9 \times 10^{46}$ photons s$^{-1}$ M$^{-1}$, corresponding to a UV emission efficiency of $1 \times 10^{-4}$ (in units of the rest mass energy of the total amount of stars formed) with a low metallicity ($Z \leq 10^{-2}$ $Z_\odot$) and a low mass cutoff of $0.1 \ M_\odot$ adopted from the Bruzual-Charlot stellar synthesis library (Bruzual A 2003). With the high effective temperature, VMSs are efficient emitters of ionizing photons with approximately 10 times more hydrogen-ionizing photons per unit stellar mass than stars with a Salpeter IMF.

We use a very conservative star formation efficiency for minihalos with H$_2$ cooling, $\epsilon_{\text{H}_2} = 0.002$ (Abel et al. 1997, 2000; Bromm et al. 2001, 2002), the fraction of gas formed into stars out of an amount of gas virialized in minihalos. Star formation efficiency in halos with efficient atomic cooling is assumed to be $\epsilon_{\text{H}_1} = 0.1$.

### 4.2. Normalizing the Ionizing Photon Escape Fraction

The most important factors determining the ionization process are $c^*$, $f_{\text{es}}$, and $C$. The first two can be combined (see eq. [15]). Thus, effectively, there are two primary factors, $c^* f_{\text{es}}$ and $C$. For the sake of explanation and taking a very conservative stance, we use a ratio $c^* H_1/c^* H_2 = 50$, as indicated in the previous subsection, to maximize the radiation emission for the second reionization when Population II stars are responsible and minimize the emission from Population III stars, which are responsible for the first reionization. Consequently, we are left with the freedom to adjust $f_{\text{es}}$ and $C$.

Both theoretically and observationally, we have essentially no direct knowledge about $f_{\text{es}}$ at the high redshift in question. One may argue that smaller galaxies at high redshift would result in a higher escape fraction. On the other hand, one may also argue that higher density at high redshift may yield a lower escape fraction (see, e.g., Ricotti & Shull 2000). Perhaps more important, the escape fraction would critically depend on the distribution of stars inside galaxies. It thus seems most productive to seek empirical constraints on $f_{\text{es}}$. Our approach is to require that the universe is reionized at $z = 6$, as observations suggest (see, e.g., Becker et al. 2001; Cen & McDonald 2002; Barkana 2002; Fan et al. 2002; Litz et al. 2002). With such a normalization point and the fact that the clumping factor $C$ is well constrained by Lyα forest observations, we are able to tightly constrain the range of $f_{\text{es}}$. Since the same range of waves in the density fluctuation power spectrum traced by the Lyα
forest is mostly responsible for the clumping of the IGM at $z = 6$, this direct observational constraint is very powerful.

Figure 7 shows the required ionizing photon escape fraction from galaxies as a function of the clumping factor of the IGM at $z = 6$. The cross-shaded area indicates the range of the clumping factor constrained by Ly$\alpha$ forest observations (Croft et al. 2002). Based on the latest high signal-to-noise ratio Ly$\alpha$ forest observations, Croft et al. (2002) give a constraint on $\sigma_8 = 0.78 \pm 0.22$ (2$\sigma$) for a flat CDM model with $\Omega_M = 0.25$ and $\Lambda = 1 - \Omega_M$. Using slightly different values of $\Omega_M$ does not significantly change the range of $\sigma_8$. We use simulations to empirically obtain the clumping factor of H II regions at $z = 6$. The high-resolution simulations of Gnedin & Ostriker (1997) are the best simulations for this purpose. They give a clumping factor for H II regions of $C_{\text{HII}} = 35$ at $z = 6$ for a low-$\sigma_8$ CDM model with $\sigma_8 = 0.67$, $\Omega_M = 0.35$, and $h = 0.70$ (see Fig. 2 of Gnedin & Ostriker 1997). Since the CDM model at the high redshift behaves like an Einstein–de Sitter universe, we can scale their results empirically on the scale. Second, nonlinear evolution tends to steepen the power spectrum index at small scales to a universal value approximately equal to roughly $-1.1$, thus washing out the initial differences. Third, by redshift $z \sim 6$, in any reasonable model based on CDM, the fraction of mass residing in halos of interest becomes about 10% or greater, indicating that low-$\sigma$ peaks have taken the charge. However, the reionization process at higher redshift, where high-$\sigma$ peaks are responsible for most of the ionizing radiation emission, depends sensitively on $\sigma$. Reducing $\sigma$ would delay the overall reionization process and vice versa. For an excellent discussion, see Venkatesan (2002) and Venkatesan et al. (2003). We plan to return to this topic in detail in a future paper.

Figure 8 shows the epoch of the first reionization as a function of the normalized escape fraction. The cross-shaded region is constrained by the normalization that the universe is reionized at $z = 6$ (see Fig. 7). We see that the required constraint by the Ly$\alpha$ forest observations limits the first reionization epoch to $z = 15-16$.

We note that the required ionizing photon escape fraction of $0.15$–$0.23$ from galaxies at $z \sim 6$ is significantly higher than those of local starburst galaxies. It should be stressed that the clumping factor predicted by Gnedin-Ostriker simulations, although at high resolution, is nevertheless still underestimated. Thus, the actual required ionizing photon escape fraction from galaxies may be somewhat higher than indicated. For local starburst galaxies, Hurwitz, Jelinsky, & Dixon (1997) give $f_{\text{es}} \leq 0.032, 0.052,$ and $0.11$ (2$\sigma$) for Mrk 496, Mrk 1267, and IRAS 08339+6517, respectively ($\leq 0.57$ in the case of Mrk 66). Deharveng et al. (2001) give an escape fraction of $f_{\text{es}} < 0.062$ for Mrk 54. Heckman et al. (2001) give $f_{\text{es}} \leq 0.06$ for local starburst galaxies. Theoretical models (Dove & Shull 1994; Dove, Shull, & Ferrara 2000) give an estimate of $f_{\text{es}} = 0.02$–$0.10$. Thus, we conclude that, in the context of a Λ CDM model, galaxies at $z \sim 6$...
appear to demand a higher ionizing photon escape fraction than local starburst galaxies. (Note that we have already assumed a reasonable, but perhaps somewhat high, star formation efficiency of $C_{\text{H}} = 0.1$ for galaxies at $z \sim 6$.) A seemingly consistent explanation for this trend could be that smaller or denser galaxies allow more ionizing photons to escape. On extrapolation, it seems plausible that a higher ionizing photon escape fraction may be expected for galaxies at still higher redshift, those responsible for the first cosmological reionization. For simplicity and being conservative, we have assumed that the ionizing photon escape fraction from galaxies at $z \geq 6$ is equal to that as determined in Figure 7.

4.3. Detailed Evolution of Intergalactic Medium

Let us now examine the evolution of the IGM in the context of a standard CDM model. We use the following model parameters: $c^h = 0.002$, $c^a = 0.1$, and $\langle C \rangle = 25$ at $z = 6$ and $\delta_{\text{a}} = 0.15$. This model lies at the lower (left-hand) bound of the cross-shaded region in Figure 7. Any model to the right of this model would require a higher ionizing photon escape fraction and thus would yield a higher first reionization redshift. A spatially flat CDM cosmological model with $\Omega_M = 0.25$, $\Omega_b = 0.04$, $\Lambda = 0.75$, $H_0 = 72$ km s$^{-1}$ Mpc$^{-1}$, and $\sigma_8 = 0.8$ is used. The results do not sensitively depend on small variations in the cosmological model parameters. We remind the reader that in our formalism described in § 4.1 we have the freedom to adjust $C_{\text{halo}}$ to match the required $\langle C \rangle$ at $z = 6$; we find that $C_{\text{halo}} = 702$ with the adopted cosmological model provides a match to the required $\langle C \rangle = 25$ at $z = 6$.

Figure 9 shows the global hydrogen neutral fraction and the complimentary ionized fraction as a function of redshift. The first reionization at $z = 15.5$, as well as the sustained ionized state until $z = 13.2$, is made possible by Population III VMSs. The redshift $z = 13.2$ marks the transition from Population III stars to Population II stars, which occurs $0.2t_H$ after a fraction $10^{-4}$ of the total gas has formed into Population III stars (see § 4.1), causing the emission of hydrogen-ionizing photons to plunge. The suddenly reduced hydrogen-ionizing photon emission is no longer able to counter the rapid hydrogen recombination process, resulting in the second cosmological recombination at $z = 13.2$. Since a very small neutral fraction suffices to blank out all Ly$\alpha$ emission, the universe essentially becomes opaque to Ly$\alpha$ photons from $z = 13.2$ to 6.

However, hydrogen is significantly ionized with $n_{\text{H}/n_{\text{H}} > 0.6$ throughout the long second reionization process from $z = 13.2$ to 6. To facilitate a better understanding, it is useful to show some important timescales involved. We show in Figure 10 the ratio of hydrogen recombination time over the Hubble time and the ratio of Compton cooling time over the Hubble time. It is noted that at the redshift and density of interest, Compton cooling dominates over other cooling terms, although adiabatic cooling starts to become important approaching the end of the second reionization period. We immediately see that the hydrogen recombination time is significantly longer than the Compton cooling time, both of which are shorter than the Hubble time at $z \geq 8$. Therefore, the IGM at early times heated by the photoionization would subsequently cool down more rapidly than recombining, resulting in an overcooled but significantly ionized IGM, in the absence of subsequent photoheating.

More results are shown in Figures 11, 12, 13, and 14 for the evolution of the mean temperature, the clumping factor, the stellar fraction, and the number of ionizing photons, respectively, as a function of redshift. In Figure 11 we see a sustained ascent in the mean temperature of the IGM from $z \sim 30$ up to the redshift of the first reionization, $z = 15.5$. Subsequently, the mean temperature of the IGM is roughly maintained at $10^4$ K because of the works of two counter-balancing effects: substantial cooling reduces the Jeans mass of the IGM and would increase the star formation rate, which would then provide increased heating by photoionization. Thus, the mean temperature of the IGM is

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**Fig. 9.**—Global mean of the hydrogen neutral (solid line) and complimentary ionized (dashed line) fraction as a function of redshift.

**Fig. 10.**—Ratio of the recombination time to the Hubble time (solid curve) and ratio of the Compton cooling time to the Hubble time (dashed curve) a function of redshift.
self-regulated because of the competition between the Compton cooling and photoheating. For the CDM cosmological model, it happens that the star formation activities are at a level such that the two balancing terms are comparable in magnitude, resulting in a fairly mild evolution of the IGM mean temperature, seen in Figure 11.

Fig. 11.—Evolution of the mean IGM temperature as a function of redshift during the second cosmological reionization process. The solid vertical tick indicates the first reionization epoch. The dashed vertical tick indicates the transition epoch from Population III stars to Population II stars. The dotted vertical tick indicates the second reionization epoch.

Fig. 12.—Evolution of the mean clumping factor as a function of redshift during the second cosmological reionization process, consistent with direct numerical simulation of Gnedin & Ostriker (1997). The solid vertical tick indicates the first reionization epoch. The dashed vertical tick indicates the transition epoch from Population III stars to Population II stars. The dotted vertical tick indicates the second reionization epoch.

Fig. 13.—The y-axis shows the evolution of the fraction of gas formed into stars as a function of redshift. The solid vertical tick indicates the first reionization epoch. The dashed vertical tick indicates the transition epoch from Population III stars to Population II stars. The dotted vertical tick indicates the second reionization epoch.

Fig. 14.—Cumulative number of ionizing photons per hydrogen atom produced as a function of redshift. The solid vertical tick indicates the first reionization epoch. The dashed vertical tick indicates the transition epoch from Population III stars to Population II stars. The dotted vertical tick indicates the second reionization epoch.

From Figure 12 it is evident that in the period preceding the first reionization from $z = 18$ to $15.5$ a large fraction of gas is unable to accrete onto minihalos because of the heated temperature of the IGM, and this results in a large decrease in the clumping factor of the IGM, followed by a mild increase in the clumping factor in the brief ionized state from $z = 15.5$ to $13.2$. From $z = 13.2$ to $6$ the clumping factor of the IGM increases steadily, parallel to the increase in the nonlinear mass scale with time, which is then followed by a very brief, moderate drop immediately after $z = 6$. 
when there is an increase in the temperature of the IGM from $T \sim 10^4$ to $2 \times 10^4$ K (see Fig. 11) due to the completion of the second reionization.

Some interesting features may be found in Figures 13 and 14. We see in Figure 13 that, although star formation in minihalos (dashed curve) dominates that in large halos (solid curve) at $z \geq 17$, the latter gradually takes over and becomes dominant at $z \leq 17$. Therefore, while Population III stars in minihalos may be a very important source of heavy elements to enrich the IGM, the first cosmological reionization is largely due to Population III stars in large halos. As a result, at least for this particular case, the first reionization would have occurred even without the validation presented in §§2 and 3 that H$_2$ cooling is sufficient in minihalos during the first reionization process. About $6 \times 10^{-3}$ of the total gas has formed into Population III VMSs by the time the universe was first reionized. We note that we have adopted a very conservative ratio of $n_{\text{H}_2}^*/n_{\text{H}_2}^* = 0.1/0.002 = 50$ for this example. If, for example, the ratio were 10, then Population III stars in minihalos would largely be responsible for first reionization. By the time the universe was reionized for the second time, about 1% of the total gas has formed into stars.

From Figure 14 we see that at the epoch of the first reionization about two ionizing photons per baryon have been produced by Population III stars. About $10^{-4}$ fraction of gas formed into Population III stars at a slightly later epoch and the delayed transition from Population III stars to Population II stars occur at $z = 13.2$, clearly seen in Figures 9, 11, and 12. More than 10 ionizing photons per baryon have been produced by the time the universe is reionized for the second time at $z = 6$.

In summary, the cosmological reionization process has several interesting characteristics. First, the universe was generically reionized twice, once at $z = 15-16$ by Population III VMSs and again at $z \sim 6$ by normal Population II stars. Most of the Population III VMSs responsible for the first cosmological reionization would come from large halos if star formation efficiency in minihalos is less than a few percent of that of large halos; in this case, our argument that H$_2$ cooling is efficient in minihalos during the first cosmological reionization, while still true, may not be necessary. Otherwise, most of the Population III VMSs responsible for the first cosmological reionization would come from minihalos, where efficient H$_2$ cooling is required, and the first reionization redshift would be higher than $z = 15-16$. Second, the IGM is maintained at a rather “warm” temperature, $T \sim 10^4$ K, from the first reionization through the second reionization; i.e., the universe is significantly heated up even in this “dark age” preceding the second, complete reionization of the universe at $z \sim 6$. Third, the IGM is kept at a significantly ionized state throughout the second reionization period. Finally, the overwhelming fraction of star formation activity responsible for producing the ionizing photons in the period from the first reionization through the second reionization is in large halos.

5. DISCUSSION

The possible indication of a more top-heavy initial stellar mass function in early galaxies or earlier stages of galaxies than the present-day IMF (Salpeter 1955) was suggested five decades ago (Schwarzschild & Spitzer 1953). Cosmological consequences of the first-generation, massive stars at high redshift have been discussed in a variety of contexts (see, e.g., Layzer & Hively 1973; Carr 1977; Rees 1978; Rowan-Robinson, Negroponte, & Silk 1979; Puget & Heyvaerts 1980; Tarbet & Rowan-Robinson 1982; Carr, Bond, & Arnett 1984; Haiman & Loeb 1997; Barkana & Loeb 2001).

The reionization picture presented here would have a wide range of profound implications for many aspects of structure formation. There are many questions that need to be addressed. It is beyond the scope of this paper to explore any of these issues in significant detail, and we will study these and other relevant issues in subsequent investigations. But here we give some simple estimates or analyses for some selected issues.

5.1. Initial Metal Enrichment of the Intergalactic Medium

Direct observational evidence for massive Population III stars has just been emerging recently. While Luck & Bond (1985) and others have previously indicated the need for VMSs ($M > 100 \ M_\odot$) to explain the overabundant $\alpha$-elements, Qian, Wasserburg, and collaborators (Wasserburg & Qian 2000; Qian & Wasserburg 2000, 2002; Oh et al. 2001) have recently stressed the unique signature of Population III VMSs and suggested that Population III stars could promptly produce the observed abundance patterns of metal-poor stars (McWilliam et al. 1995; Ryan, Norris, & Beers 1996; Rossi, Beers, & Sneden 1999; Norris, Ryan, & Beers 1997, 2001; Norris et al. 2002; Hill et al. 2002; Depagne et al. 2002).

Given the shallow potential wells of Population III galaxies, supernova explosions would likely blow away the ejecta along with a large fraction of the diffuse interstellar gas (Mac Low & Ferrara 1999; Mori, Ferrara, & Madau 2002), at least for minihalos. From Figures 9 and 11–14 we see that the transition from Population III stars to Population II stars occurs at a later time than the first reionization; i.e., Population III stars ionized the universe for the first time. Oh et al. (2001) have suggested this possibility based on the observed transition of the abundance pattern at [Fe/H] = $-3$ in metal-poor stars summarized by Qian and Wasserburg (Wasserburg & Qian 2000; Qian & Wasserburg 2001, 2002), who found that at lower values VMSs dominate the enrichment, whereas at higher values a sharp rise in the abundances of the heavy $r$-process elements such as Ba and Eu in galactic halo stars with [Fe/H] $\geq -3$ signifies the occurrence of Type II supernovae of normal stars with masses of $10-60 \ M_\odot$.

One may expect to see the Population III star abundance patterns in low-density regions of the universe, such as Ly$_\alpha$ clouds or voids, where subsequent additional enrichment may be minute. Recent work on Ly$_\alpha$ clouds suggests that metal enrichment by Population III stars is consistent with Ly$_\alpha$ cloud observations and that the necessary enrichment occurs prior to $z \sim 4.6$ (the highest epoch for the available Ly$_\alpha$ cloud data analyzed; Qian, Sargent, & Wasserburg 2002).

One obvious advantage for Population III supernovae to enrich the IGM with the first metals at $z \gg 4$ is that it is much easier to relatively uniformly disperse the metals across the IGM, for two reasons. First, the disturbances to the density and velocity fields are smaller at high redshift, because each Population III galaxy needs to fill only a very small IGM volume. Second, any significant large-scale motions would decay away rapidly, in the absence of dynamical support. As a result, the excellent agreement found between simulations and observations with respect to the Ly$_\alpha$ forest at
z = 2–4 (Cen et al. 1994b; Zhang, Anninos, & Norman 1995; Hernquist et al. 1996; Miralda-Escudé et al. 1996; Bond & Wadsley 1997; Theuns et al. 1998) would not be altered, although recent simulations also indicate that low-redshift galactic winds would not spoil the main properties of the Lyα forest produced by previous simulations (Theuns et al. 2002).

5.2. Intergalactic Magnetic Field

Another possible consequence of supernova explosion and expulsion of the gas into the IGM is the magnetic feedback to the IGM. Rees (1994) first pointed out the importance of resident supernova remnants in galaxies being a substantial large-scale seed field for galactic dynamos. Let us give a simple estimate here for the contribution of the Population III supernova remnants to the intergalactic magnetic field. Following Rees (1994), we should use local observations as a guide. The Crab Nebula (a plerion) has a magnetic field of strength $B \sim 100$ $\mu$G currently occupying a sphere of radius $r \sim 1$ pc, totaling a flux of $\psi_{\text{Crab}} \sim 3 \times 10^{33}$ G cm$^2$. Assuming flux conservation and a Population III magnetic bubble filling factor of unity gives $B_{\text{IGM}} = [(f_{\text{III}}/\rho_0(1+z_0)^{3/2})(M_{\text{III}}/M_{\text{Crab}})^{5/3}]^{1/2}B_{\text{Crab}}$, where $M_{\text{III}} \sim 100 M_\odot$ is the mass of a typical Population III star; $M_{\text{Crab}} \sim 5 M_\odot$ is the zero-age main-sequence progenitor star for the Crab; $f_{\text{III}} \sim 2.5 \times 10^{-4}$ is the fraction of baryons formed into Population III stars (see Fig. 14) by redshift $z = 13.2$ when the transition from Population III to Population II occurs; $\rho_0$ is the mean baryonic density at zero redshift; $z_0$ is the redshift of ejection of the magnetic field into the IGM; and we have assumed that the magnetic flux is approximately proportional to the stellar mass. Inserting all the numbers gives $B_{\text{IGM}}(z = 13.2) \sim 1 \times 10^{-9}$ G at $z = 13.2$ and, subsequently, $B_{\text{IGM}}(z) \sim 1 \times 10^{-9}[(1+z)/14.2]^{5/3}$ G. This magnetic field, having a significantly larger amplitude than that produced by gravitational shocks in the collapse of large-scale structure (Kulsrud et al. 1997), could serve as a seed field for subsequent galaxy formation. The mean separation between Population III galaxies is of the order of 100 comoving kpc. Thus, if Population III supernovae are responsible for enriching the IGM relatively uniformly to a metallicity of about 1/1000 of the solar value, it is also likely that the magnetic field lines originating from supernovae remnants would be stretched to fill up the intergalactic space, resulting in an initial magnetic field possibly coherent on scales as large as ~100 kpc.

In addition, miniquasars powered by Population III black holes may produce mini-radio jets, as would have been implied by the observational fact that radio jets are observed in accretion disks on a wide range of scales in a wide variety of astrophysical systems. The magnetic field from miniquasars may be as important as or more important than that from the Population III stars, but a reliable estimate is difficult.

5.3. Population III Black Holes

Without fine-tuning the IMF of Population III stars, it seems likely that ~200 $M_\odot$ black holes from Population III stars more massive than 260 $M_\odot$ and smaller black holes ($M_{\text{BH}} \sim 10$–50 $M_\odot$) from Population III stars less massive than 140 $M_\odot$ would form. The possible bimodality of the distribution of black hole mass is interesting, but the consequences are too complicated to be easily outlined. Those Population III black holes will be building blocks for subsequent structures, and the question is, where will they go? In general, since there is no strong correlation between the small-scale structures that form Population III galaxies and later, much larger structures, one would expect to find those black holes in all environments on large scales, including globular clusters, galactic disks, galaxy halos, and intergalactic space. It is possible that the dynamic formation of halos and galaxies in hierarchical structure formation may be significantly altered in the presence of these massive black holes, especially in the cores of the relevant structures, such as globular clusters, galaxies, etc. The number density of Population III black holes of mass $\sim 100 M_\odot$ can be estimated by assuming that a fraction, $f_{\text{III BH}}$, of the Population III stellar mass ends in black holes of the indicated mass, which gives $6.3 \times 10^{-4} f_{\text{III BH}}(\Omega_0 h^2/0.023) \times (f_{\text{III}}/10^{-4})$ per comoving Mpc$^3$; for $f_{\text{III}}$ is the fraction of gas formed into Population III stars. Thus, roughly $10^3–10^4$ per comoving Mpc$^3$ Population III black holes of mass $\sim 100 M_\odot$ would form, comparable to the number density of globular clusters, and the mass density is comparable to present mass density in supermassive black holes in the centers of galaxies (see, e.g., Merritt & Ferrarese 2001). The interesting agreement between this result and that of Madau & Rees (2001) with regard to the mass density of Population III black holes is traceable to the fact that the 3 $\sigma$ density peaks used in the model of Madau & Rees (2001) happen to yield about the same collapsed fraction as ours.

From the inferred number density of Population III black holes, it is quite possible that many globular clusters could be seeded by those black holes. There is clearly no shortage of Population III black holes to provide seeds for later supermassive black holes seen in centers of local galaxies (see, e.g., Tremaine et al. 2002), AGNs, and quasars (see, e.g., Rees 1984, 1990).

How could these black holes be observed? Miralda-Escudé & Gould (2000) pointed out the possible existence of stellar black hole clusters at the Galactic center. Their reasoning may be equally applied here. Since the Population III star mass fraction is of the order of 10$^{-4}$ of the total baryonic mass, as our calculation indicates, there should be $\sim 10^3$ black holes with a mass of either 100 or 10 $M_\odot$ (noting that the mass fraction in black holes will be a factor about 10 lower for less massive black holes). The reader is encouraged to refer to Miralda-Escudé & Gould (2000) and also Sigurdsson & Rees (1997) for a detailed discussion of possible manifestations of such a black cluster, bearing in mind that the black holes here are more massive and may enhance some of the effects discussed.

In the hierarchical structure formation process, these Population III black holes are expected to grow (there are a large number of e-folding times available) and merge, along with larger cosmic structures. The merger of Population III black holes with supramassive black holes at the centers of galaxies may be detectable by the gravitational wave experiment, the Laser Interferometer Space Antenna (LISA) (see, e.g., Menou, Haiman, & Narayanan 2001; Hughes 2002).

5.4. Detecting Population III Galaxies and Population III Hypernovae

We examine the observability of Population III galaxies and their associated hypernovae with respect to SIRTF and the James Webb Space Telescope (JWST).
The physical sizes of the galaxies in the redshift range $z = 13-15$ are unknown, but it seems unlikely that they will be larger than their lower redshift counterparts. Steidel et al. (1996) find that the great majority of $z > 3$ objects have a half-light radius of $\sim 2$ kpc, which, if placed at $z_S = 17$, would subtend $\sim 0.05$ (for the adopted model), more than a factor of $\sim 10$ below the angular resolution of SIRTF. In other words, these high-redshift galaxies are point sources to SIRTF. The flux density (the power per unit antenna area and per unit frequency interval) of a point source (without cosmological surface brightness dimming) at frequency $\nu$ is $S = P(\nu(1+z_S)/(1+z_S))d_{em}$ (Weinberg 1972, p. 453), where $z_S$ is the source redshift and $\nu$ is the observed photon frequency. $P$ is the intrinsic power, the power emitted per unit solid angle and per unit frequency interval, related to the luminosity $L$ of the source by $P = L/4\pi\Delta\nu$, where $\Delta\nu$ is the rest-frame bandwidth at which the source has a luminosity $L$; $d_{em}$ is the comoving distance to the source. Using a rest-frame band of $0.09-0.27 \mu$m (corresponding to the observer’s band, $1.3-3.8 \mu$m) and $z_S = 13.2$, we obtain

$$S = 1.45 \frac{L}{10^9 L_\odot} \text{nJy}.$$  \hspace{1cm} (23)

The point-source detection sensitivity at the 1 $\sigma$ level (for a relatively long integration time, $>500$ s) of the Infrared Array Camera (IRAC) on SIRTF at $3.5-4.5 \mu$m is $S_{1\sigma} \approx [(100/\mu) + 0.65] \mu$Jy, where $\mu$ is the integration time in seconds. The second term inside the brackets is the confusion noise limit due to faint unresolved sources, modeled by Franceschini et al. (1991).

The total luminosity of a Population III galaxy ranges from $L = 1 \times 10^5 (c_H^2/0.002)(M_\odot/10^9 M_\odot) L_\odot$ for minihalos to $1 \times 10^{10} (c_H^2/0.1)(M_\odot/2 \times 10^9 M_\odot) L_\odot$ for large halos; only a small fraction of the total luminosity is in the indicated band. It is clear that no Population III galaxies hosted by minihalos will be directly detectable by SIRTF, even without the confusion noise limit. Perhaps only the high-mass end of Population III galaxies with mass $>10^9 M_\odot$ (which would give $>0.7 \mu$Jy) may be detectable by SIRTF, in the presence of confusion sources. The sensitivity of the JWST is close to 1 nJy (Stockman & Mather 2000). Therefore, Population III galaxies with large halos will be detectable by JWST up to a redshift of $z \sim 15$, whereas Population III galaxies with minihalos may still be beyond reach of even JWST. Our calculations indicate that a large fraction of Population III galaxies will be detectable by JWST; a more detailed treatment will be reserved for a future paper.

The total number of Population III hypernovae is

$$N_{HN} = 2.5 \times 10^{-5} \rho_0 (4\pi/3) R_H^3 / M_{III} = 1.3 \times 10^{16}$$

assuming that 25% of the Population III stellar mass ends in black holes), where $M_{III} = 100 M_\odot$ for Population III star mass is used, $R_H = 6000 h^{-1}$ Mpc is the comoving Hubble radius, and $\rho_0$ is the comoving mean baryonic density. We can estimate the Population III hypernova surface number density at any given time to be

$$\Sigma_{HN} = \frac{1.3 \times 10^{16} \Delta_{HN}(1+z)}{t_H} \frac{1}{4\pi \times 12 \times 10^7 \text{arcmin}^2} \approx 2.3 \text{arcmin}^{-2},$$  \hspace{1cm} (24)

where $\Delta_{HN}$ is the time duration of each hypernova event in the hypernova rest frame and $t_H$ is the age of universe at $z = 13.2$; the first parenthesized term in the above equation takes into account the finite duration of each supernova event, and an intrinsic duration for the hypernovae $\Delta_{HN} = 1$ yr is used; the last term, 0.60, takes into account that the observable redshift interval $z \geq 13.2$ is 60% of the total volume of the universe then. This hypernova surface density is comparable to the surface density of supernovae derived by Miralda-Escudé & Rees (1998), based on the observed metallicity in the Ly$\alpha$ clouds.

Assuming the optical luminosity of a hypernova to be $1 \times 10^{50} L_\odot$ (about 10 times that of the local normal supernovae), the flux density of Population III hypernovae at $z = 13.2$ will be at a level $\sim 0.014 \mu$Jy. Even without background confusion (if the variable source has a variability timescale shorter than the indicated integration time), in order to detect such a flux density at the 1 $\sigma$ level, an integration time of 6 days on SIRTF IRAC will be required, which seems impractical. Even if this long integration is achievable, the fact that the variability timescale of a hypernova at the considered redshift may be quite long indicates that it will be extremely difficult to directly detect high-redshift hypernovae, unless their flux is significantly amplified.

In the Appendix we derive the probability of strong gravitational lensing by massive clusters of galaxies and find that a random source behind a massive galaxy cluster will have a probability

$$P_{\text{clust}}(\mu) = \frac{6.7 \times 10^{-2}}{\mu^2} \left(\frac{\Omega_{\text{FOV}}}{25 \text{arcmin}^2}\right)^{-1}$$

being magnified by $\geq \mu$, where $\mu$ is the lensing magnification and $\Omega_{\text{FOV}}$ is the field of view of a telescope. The number of Population III hypernovae, magnified by $\geq \mu$, per field of view centered on a massive cluster is then

$$N_{HN} = P_{\text{clust}}(\mu) \Omega_{\text{FOV}} \Sigma_{HN},$$

where $\Sigma_{HN}$ is the surface density of hypernovae indicated by equation (24). In order for the IRAC camera of SIRTF to detect at a $3 \sigma$ statistical level a point source, which would have a flux density of $S$ in the absence of gravitational lensing magnification, one requires that

$$S = \mu \sqrt{100/\mu},$$

where the point-source confusion limit is removed and $\mu$ is the integration time in seconds. We plot the number of hypernovae, $N_{HN,\text{det}}$, per field of view detected at a $3 \sigma$ confidence level against integration time $\tau$ in Figure 15, assuming $L_{\text{HN}} = 1 \times 10^{10} L_\odot$ giving an un lensed flux density of $S = 14.5$ nJy at the observed wavelength of several $\mu$m. To put the matter in perspective, 1000 target fields each with a 10 ks integration time would detect $\sim 300$ multiply lensed Population III hypernovae at a $3 \sigma$ confidence level.

We note that since only highly magnified Population III hypernovae will be observable, the observed candidates should have distinct features: the multiple images will significantly help the identification process. If the cluster lens is known, its lens potential derived independently elsewhere would further constrain the image configurations. Once such images are found, periodic monitoring up to a few years would eventually verify the transient nature of the Population III hypernovae. Since the total number of detectable hypernovae is just proportional to the total integration time, one could choose an optimal strategy such that the observation is most sensitive to the expected splittings of the images: a shorter exposure will require a higher
magnification from a more central region of the lens, where splittings may decrease because of a smaller velocity dispersion at the center of the cluster.

At a flux density of a level of $\sim 10$ nJy, Population III hypernovae will be detectable by JWST (Stockman & Mather 2000) without gravitational lensing magnification. The possible observable duration of 4–5 yr for the initial bright phase of Population III hypernovae, corresponding to an 80–90 day intrinsic duration (Woosley & Weaver 1986), will be a signature.

5.5. Effects on the Cosmic Microwave Background

The earlier reionization would significantly increase the electron Thomson scattering optical depth. This is so because the IGM ionization fraction remains appreciable during the period between the first reionization ($z = 15.5$) and the second reionization ($z = 6.1$). Figure 16 shows the cumulative Thomson scattering optical depth as a function of redshift. We see that the Thomson scattering optical depth increases from $\tau_e(z < 6.1) = 0.027$ to $\tau_e(z < 20) = 0.097$. Evidently, the first reionization boosts the optical depth of the second reionization by a factor of $\sim 3$, making the total Thomson optical depth much more significant. Given the narrow allowed range of the first reionization epoch between $z = 15$ and 16 (Fig. 8), our prediction of $\tau_e \sim 0.10 \pm 0.03$ [where $\Delta \tau_e = 0.03$ is approximately estimated considering uncertainties on $C(z)$, $f_{\text{es}}(z)e^*(z)$, and the IMF] is fairly robust, where most of uncertainties come from uncertainties in the IMF and $f_{\text{es}}(z)e^*(z)$. The $1 \sigma$ lower bound results if no significant number of Population III stars were ever formed, while the $1 \sigma$ upper bound is obtained for the case in which $f_{\text{es}}(z)e^*(z)$ is assumed to increase with redshift (still assuming that star formation efficiency in minihalos is much less efficient than in large halos). However, in the event that Population III and Population II star formation efficiency in minihalos is much more efficient than assumed, then $\tau_e$ could be significantly higher, possibly reaching 0.18.

Including possible double reionization of He II (Wyithe & Loeb 2003; Venkatesan et al. 2003), $\tau_e$ could increase by up to 16%. The implications for the CMB (for an excellent recent review, see Hu & Dodelson 2002), in particular, for polarization of the CMB (see, e.g., Seljak 1997; Zaldarriaga 1997, Kamionkowski, Kosowsky, & Stebbins 1997), are significant.

We follow the recent analysis by Kaplinghat et al. (2003) on the detectability of polarization of the CMB. Kaplinghat et al. (2003) show that the WMAP satellite will be able to measure $\tau_e$ to an accuracy of 0.02–0.03 (1 $\sigma$). Thus, it appears that WMAP will be able to distinguish the two reionization scenarios at about a 2–3 $\sigma$ confidence level: the standard reionization scenario in which the universe is fully ionized at $z < 6$ and fully neutral at $z > 6$ (giving $\tau_e = 0.027$) versus the reionization scenario presented here in which the universe is fully ionized at $z < 6$ and partially neutral at $z = 6-20$ (giving $\tau_e = 0.097$). According to Kaplinghat et al. (2003), the Planck surveyor will probably be able to probe the detailed reionization history in addition to discriminating between the standard reionization model and the current reionization model at a very high confidence level.

5.6. Nondetection of Population III Stars Locally

There is a long history of searching for Population III stars (Bond 1970, 1981; Hills 1982; Bessell & Norris 1984, 1987; Carr 1987; Cayrel 1996; McWilliam et al. 1995; Ryan et al. 1996; Rossi et al. 1999; Beers 2000; Norris et al. 1997, 2001, 2002; Hill et al. 2002; Depagne et al. 2002) or primeval galaxies (Partridge 1974; Davis & Wilkinson 1974). If Population III stars are as massive as suggested, perhaps it is not surprising that no single Population III star should have been found, while over 100 metal-poor stars with $-4 < [\text{Fe/H}] < -3$ have been found (Cayrel 1996), since all Population III stars would end as either supernovae or black holes. The proposed model, unfortunately, would
mark the end of the search for Population III stars in the local universe.

5.7. Detectability of Galaxies Beyond $z = 6$

It is interesting to note, from Figure 9, that there is a substantial redshift range beyond the end redshift of the second reionization, $z = 6$, where the IGM is already substantially ionized. For example, $n_{H_1}/n_H \sim 0.1-0.2$ at $z \sim 6.0-7.5$. This feature of an extended redshift interval of high ionization appears to be generic. This result would have interesting consequences on the observability of galaxies beyond the second reionization epoch. Taking the result at face value, for a galaxy at $z = 6.5$, the Strömgren sphere produced by the galaxy will be a factor of about 2 larger than that in the case in which the galaxy is embedded in a completely neutral IGM. This would help reconcile the seemingly conflicting observational claims: on one hand, the universe appears to be reionized at $z \sim 6$ when the ionizing radiation background is seen to rise sharply (see, e.g., Becker et al. 2001; Cen & McDonald 2002; Fan et al. 2002); on the other hand, Ly$\alpha$ galaxies at a redshift as high as $z \gtrsim 6.5$ (Hu et al. 2002; Kodaira et al. 2003) have been detected (see Haiman 2002 for an alternative explanation). The optical depth due to the damping wing of neutral hydrogen outside the Strömgren is in general reduced by a factor of $(n_{H_1}/n_H)^{4/3}$ (Miralda-Escudé & Rees 1998; Cen & Haiman 2000; Madau & Rees 2000), which is a factor of 0.08 for $n_{H_1}/n_H = 0.15$. One prediction is that the number of observable Ly$\alpha$ galaxies would quickly thin out beyond $z \sim 7$, because of the combined effect of a rapid increase of the neutral fraction beyond $z \sim 7$ (see Fig. 9) and a decrease in the number of large Ly$\alpha$ galaxies. More detailed treatment of this important subject will be done in a separate paper. An observational campaign by several groups (Hu et al. 2002; Rhoads et al. 2000; Ajiki et al. 2002) to detect high-redshift Ly$\alpha$ may be able to probe the detailed reionization structure near the end of the second cosmological reionization.

5.8. Hydrogen 21 cm Line due to Minihalos Prior to the Second Cosmological Reionization: A Test

The possibility of probing the high-$z$ universe with hydrogen 21 cm line absorption or emission has been suggested and investigated in various contexts by many authors (Hogan & Rees 1978; Scott & Rees 1990; Subramanian & Padmanabhan 1993; Kumar, Padmanabhan, & Subramanian 1995; Bagla, Nath, & Padmanabhan 1997; Madau, Meikien, & Rees 1997; Shaver et al. 1999; Tozzi et al. 2000; Carilli, Gnedin, & Owen 2002; Iliev et al. 2002; Furlanetto & Loeb 2002). The mean, neutral medium expanding with the Hubble flow at redshift $z$ would produce an optical depth for the hydrogen 21 cm resonant absorption that is (Shklovsky 1960)

$$\tau(z) = \frac{n(z) \Omega_b h^2}{H(z) g_1 (g_2 + g_1) A_{21}^2} \frac{c^3}{8 \pi^2 k T_{	ext{sp}}} \frac{h \nu}{k T_{	ext{sp}}} \times \left( \frac{1 + z}{8} \right)^{3/2},$$

where $H(z)$ is the Hubble constant at redshift $z$, $n(z)$ is the mean atomic hydrogen number density at $z$, $g_1 = 1$ and $g_2 = 3$ are the statistical weights of the lower and upper hyperfine levels, respectively; $\nu$ is the frequency of the hydrogen 21 cm line; $T_{\text{sp}}$ is the spin temperature of the atomic hydrogen; $\nu$ is the frequency of the 21 cm line; $c$ is the speed of light; $k$ is Boltzmann’s constant; $A_{21}$ is the spontaneous decay rate of the hyperfine transition of atomic hydrogen; $\Omega_M$ and $\Omega_b$ are total matter and baryonic matter density, respectively, at $z = 0$ in units of closure density; and $h = H_0/100$ km s$^{-1}$ Mpc$^{-1}$, where $H_0$ is the Hubble constant at $z = 0$. Clearly, the uniform medium would only cause a very modest absorption at $z \sim 6-15$. Minihalos would cast significantly larger optical depth. For 21 cm absorption by atomic hydrogen in virialized minihalos, we may rewrite equation (28) approximately as

$$\tau_h(z) = \frac{178 n(z) r_v}{\sigma_v} \frac{g_2}{g_1 (g_2 + g_1)} A_{21}^2 \frac{c^3}{8 \pi^2 k T_{	ext{sp}}} \frac{h \nu}{k T_{	ext{sp}}},$$

where $r_v$ and $\sigma_v$ are the virial radius and velocity dispersion within the virial radius, respectively, and the factor 178 is the relative overdensity of matter within the virial radius. Using relations $\sigma_v^2 = GM_c/2r_v$ and $M_c = 178(4\pi/3)\rho(z) r_v^3$ [where $\rho(z)$ is the mean total density at $z$] and the definition of Hubble constant, we can transform equation (29) to

$$\tau_h(z) = 2 \sqrt{178 \pi(z)}$$

$$= 0.023 \left( \frac{T_{	ext{sp}}}{200 \text{ K}} \right)^{-1} \frac{\Omega_b h^2}{0.02} \frac{\Omega_M h^2}{0.15} \frac{1}{-1/2} \left( \frac{1 + z}{8} \right)^{3/2}.$$

We see that gaseous minihalos with $T_{\text{sp}} \sim 10^2-10^3$, if they exist, could produce significant optical depths at 21 cm at a level of 0.01–0.1 (Furlanetto & Loeb 2002; see also Carilli et al. 2002 for a numerical treatment). Note that the spin temperature $T_{\text{sp}}$ would be bracketed by the CMB temperature and the kinetic temperature (approximately the virial temperature) of a minihalo.

Furlanetto & Loeb (2002) have pointed out that the 21 cm forest produced by minihalos prior to cosmological reionization will be detectable by the next generation of low-frequency radio telescopes, such as the Low Frequency Array and the Square Kilometer Array. However, this will only be possible if the minihalos could retain their gas, as they would in the conventional reionization scenario. In the reionization scenario presented here, as indicated in Figure 14, no significant amount of gas will be able to accumulate in minihalos throughout the second reionization period. Consequently, in our scenario, 21 cm forest lines, which would otherwise be produced by minihalos without the first reionization, have been largely wiped out. It is noted that 21 cm absorption measurements would require bright background radio sources to be present at very high redshift. Similarly, all hydrogen 21 cm emission line forests due to minihalos, as many as $\sim 100$ lines per unit redshift at $z \sim 9$, as predicted by Iliev et al. (2002) in the standard reionization scenario, will be absent in the present reionization scenario. In addition, the 21 cm emission spatial and spectral signature at $z \geq 6$ (Madau et al. 1997; Tozzi et al. 2000) will also be significantly altered in the sense that signals due to minihalos will be removed.
Hydrogen 21 cm observations, in either absorption or emission, will be a definitive test to distinguish between the two reionization scenarios.

5.9. Metal Absorption Lines at \( z > 6 \): A Probe of the Second Reionization Process and Initial Metal Enrichment History

Oh (2002) pointed out that some metal absorption lines may be observable at high redshift. He identified \( \text{O} \, \text{i} (\lambda 1302) \) and \( \text{Si} \, \text{ii} (\lambda 1260) \) lines, along the hydrogen Ly\( \alpha \) line, as the best candidates for tracing neutral hydrogen because of the proximity of their ionizational potentials to the hydrogen ionization potential. Since both of these metal species trace closely the neutral hydrogen density, the \( \text{O} \, \text{i} \) and \( \text{Si} \, \text{ii} \) absorption forests may provide a way to probe the reionization history. Two possible utilizations are noted here. First, both \( \text{O} \, \text{i} \) and \( \text{Si} \, \text{ii} \) absorption forests are expected to thin out at \( z \leq 7 \), which could provide a direct probe of the end of the second reionization process. We note that, for the IGM, which is primarily metal-enriched by Population III stars, the abundances of \( \alpha \)-elements such as oxygen and silicon relative to the solar value (Heger & Woosley 2002) would greatly increase the optical depth of the oxygen \( \text{O} \, \text{i} \) and \( \text{Si} \, \text{ii} \) absorption forests, at the apparent low metallicity (defined by \( \text{Fe} / \text{H} \sim 10^{-3} \)). This would substantially increase the number of observable \( \text{O} \, \text{i} \) and \( \text{Si} \, \text{ii} \) lines predicted by Oh (2002). Second, the relative optical depths of \( \text{O} \, \text{i} \) and \( \text{Si} \, \text{ii} \) absorption lines may shed light on the relative abundance of \( \text{O} \) to \( \text{Si} \), giving useful information on synthesis in Population III stars (Qian & Wasserburg 2000, 2002). In addition, gradually increasing the contribution from Population II supernovae (Type II) to the IGM metals with time may also show up in the redshift evolution of the relative optical depths of \( \text{O} \, \text{i} \) and \( \text{Si} \, \text{ii} \) absorption lines, possibly providing information on the overall metal-enrichment history of the universe at those early times.

5.10. Comparisons with Some Other Works

The author was made aware of an earlier paper by Wyithe & Loeb (2003), who investigated the reionization histories due to stars and quasars. They assume that minihalos do not form stars at the redshift of interest but systematically explore the two-dimensional parameter space spanned by the ionizing photon escape fraction and the transition redshift from a VMS IMF and a normal IMF. They pointed out that, for some restricted region in the parameter space, \( \text{H} \) and/or \( \text{He} \, \text{ii} \) may be reionized twice. The primary differences between their study and the current study are threefold. First, we show that \( \text{H} \, \text{ii} \) cooling, and thus star formation, in minihalos takes place unimpeded, as long as gas is able to accrete onto them, whereas they do not consider star formation in minihalos. Second, we significantly constrain the product of the ionizing photon escape fraction and star formation efficiency directly using the Ly\( \alpha \) forest and the final reionization epoch observations. As a consequence, we show that it seems inevitable that the universe will be reionized first by Population III stars in either minihalos or large halos and second by Population II stars. Finally, we use a new computational method to follow self-consistently the evolution of the entire IGM, including \( \text{H} \, \text{ii} \) regions, \( \text{H} \, \text{i} \) regions, and partially ionized regions. The two studies are, however, complementary in their different approaches.

Venkatesan et al. (2003) also examined the consequences of Population III metal-free stars on the reionization of the universe. They indicated the possibility of reionizing \( \text{He} \, \text{ii} \) twice with an incomplete first attempt of reionizing \( \text{He} \, \text{ii} \).

Mackey, Bromm, & Hernquist (2003) presented a three-phase reionization picture. With a top-heavy VMS IMF, they pointed out that the first transition from the molecular cooling phase to the atomic cooling phase occurs at \( z \sim 30 \), when their demand of 10 LW photons per baryon is met. The proposed first transition in Mackey et al. (2003) is dictated by the lack of \( \text{H} \, \text{ii} \) cooling in minihalos. We pointed out here positive feedback, and as a result, there is no such transition. Mackey et al. (2003) find that at \( z = 15–20 \) the IGM has been enriched to a level of \( 10^{-3.5} \, Z_\odot \) and that the second transition to a normal IMF occurs. Their assessment of the transition epoch from Population III to Population II is consistent with, but at a slightly higher redshift than, our calculation. Their universe, however, is not reionized until \( z \sim 6 \). It is noted in passing that Mackey et al. (2003) pointed out a novel Population II.5 stellar population associated with cooling shells of Population III supernovae. In summary, the primary differences between this study and that by of Mackey et al. (2003) are two-fold. First, we indicated that the positive feedback mechanisms on \( \text{H} \, \text{ii} \) formation allow star formation to occur in minihalos throughout the first reionization period up to \( z \sim 13.2 \). Second, we significantly constrain the product of the ionizing photon escape fraction and the star formation efficiency by the direct Ly\( \alpha \) forest and final reionization epoch observations.

5.11. Uncertainties

Perhaps the most uncertain in the chain of derivation is the IMF of the Population III stars. However, most of the main conclusions, including positive feedback of Population III star formation and two times of cosmological reionization, are likely to hold for any sufficiently top-heavy IMF. While the simulations with subsolar mass resolution (Abel et al. 2002; Bromm et al. 2002) have clearly shown the formation of Population III VMSs, it is yet unclear what the angular momentum transport mechanisms are. It is possible, in principle, that hydrodynamic processes, included in the quoted simulations, could transport the unwanted angular momentum outward, as the simulators have advocated. If local star formation is a guide, one could imagine that very massive Population III stars may form in binaries or multiples, in which case angular momentum removal could be easily achieved. It is also intriguing to note that, if we extrapolate the observed ratio of a supermassive black hole to bulge mass (see, e.g., Tremaine et al. 2002), then one would expect to find a black hole of mass \( \sim 10^7–10^8 \, M_\odot \) at the center of each minihalo of mass \( 10^5–10^6 \, M_\odot \). Thus, perhaps one should not be too surprised that nature has managed to form a compact object (star) of mass \( \sim 100 \, M_\odot \) at the center of a minihalo, if the quoted observations are any empirical guide; it may be argued, angular momentum-wise, that it is a less stringent task to collapse a higher density gas cloud at higher redshift to form a star than a lower density gas cloud at lower redshift to form a black hole, as observed.

We have ignored the possible contribution from quasars to the second reionization process. If the emission rate of ionizing photons from quasars is proportional to the star formation rate, as indicated by low-redshift observations (Boyle & Terlevich 1998; Cavaliere & Vittorini 1998), then, to zeroth order, all that is needed is to renormalize \( f_{\text{es}} \) such
that the second reionization is completed at \( z \sim 6 \), and the results would remain largely unchanged.

6. CONCLUSIONS

The conclusions of this paper consist of two sets:

1. We put forth two new mechanisms for generating a high X-ray background during the Population III era, namely, X-ray emission from the cooling energy of Population III supernova blast waves and from miniquasars powered by Population III black holes. We show, consequently, that \( \text{H}_2 \) formation in the cores of minihalos is significantly induced, more than enough to compensate for destruction by LW photons produced by the same Population III stars. In addition, another, perhaps dominant, process for producing a large number of \( \text{H}_2 \) molecules in relic \( \text{H} \, \text{II} \) regions created by Population III galaxies, first pointed out by RGS, is quantified here. We show that \( \text{H}_2 \) molecules produced by this process may overwhelm the LW photons produced by the stars in the same Population III galaxies. As a result, the LW background may not be able to build up to affect \( \text{H}_2 \) molecules in minihalos. In combination, we suggest that cooling, and hence star formation, in minihalos can continue to take place largely unimpeded throughout the first reionization period, as long as gas is able to accumulate in them.

2. We show that the IGM is likely to have been reionized twice, first at \( z = 15-16 \) and again at \( z = 6 \). Under a very conservative assumption for the star formation efficiency in minihalos with \( \text{H}_2 \) cooling, the first reionization may be attributable largely to Population III stars in large halos; in this case, processes promoting \( \text{H}_2 \) formation in the cores of minihalos, such as those mentioned above in set 1, do not really matter. In contrast, if star formation efficiency in minihalos is not more than a factor of 10 less efficient than in large halos, then Population III stars in minihalos may be largely responsible for the first reionization. In either case, it seems likely that Population III stars in minihalos make a large contribution to the first phase of the metal enrichment of the IGM approximately up to a level where the transition from Population III stars to Population II stars occurs.

This apparent inevitability of two reionizations is reached based on a joint constraint by two observational facts: (1) the universe is required to be reionized at \( z \sim 6 \) and (2) the density fluctuation in the universe at \( z \sim 6 \) is well determined by the same small-scale power traced by the \( \text{Ly} \alpha \) forest observed and well measured at \( z \sim 3 \). As a result, the product of star formation efficiency and ionizing photon escape fraction from galaxies at high redshift is well constrained, dictating the fate of the cosmological reionization process.

The prolonged reionization and reheating history of the IGM is more complicated than usually thought. Because both cooling and recombination timescales are much shorter than the Hubble time at the redshift in question, we devise a new, improved computational method to follow the evolution of the IGM in all phases, including \( \text{H} \, \text{II} \) regions, \( \text{H} \, \text{I} \) regions, and partially ionized regions. The overall reionization process may be separated into four joint stages:

1. From \( z \sim 30 \) to \( 15-16 \), Population III stars gradually heat up and ionize the IGM, and the first reionization occurs at the end of this stage, when the mean temperature of the IGM reaches \( \sim 10^4 \text{K} \).

2. The first stage is followed by a brief period with a redshift interval of the order of \( \Delta z \sim 1 \), where the IGM stays completely ionized due to sustained ionizing photon emission by forming Population III stars. During this period, the temperature of the IGM is maintained at \( \sim 10^4 \text{K} \).

3. The transition from Population III stars to Population II stars sets in at the beginning of this stage (\( z \sim 13 \)). The abruptly reduced (by a factor of \( \sim 10 \)) ionizing photon emission rate causes hydrogen to rapidly recombine, and the universe once again becomes opaque to \( \text{Ly} \alpha \) and Lyman continuum photons, marking the second cosmological recombination. From this time until \( z = 6 \), Compton cooling by the CMB and photoheating by the stars self-regulate the Jeans mass and the star formation rate. The mean temperature of the IGM is maintained at \( \sim 10^4 \text{K} \) from \( z \sim 13 \) to \( 6 \). Meanwhile, recombination and photoionization balance one another such that the IGM stays largely ionized during this stage, with \( n_{\text{HI}}/n_{\text{HII}} \geq 0.6 \). Most of the star formation in this period occurs in large halos with dominant atomic line cooling.

4. At \( z = 6 \), the global star formation rate again surpasses the global recombination rate, resulting in the second reionization of the universe. The second reionization is predominantly due to stars formed in halos where atomic line cooling is efficient.

There is a wide range of interesting implications from this new reionization picture presented here. Highlights of a few follow:

1. The magnetic field originating from massive stars could pollute the IGM with a large-scale coherent field (\( 1 \sim 100 \text{ kpc comoving} \)) of the order of \( \sim 10^{-9} \text{ G} \) at \( z \sim 13-15 \).

2. The number density of Population III massive black holes (10–300 \( M_\odot \)) is comparable to that of globular clusters and should seed later structure formation.

3. Direct detection of Population III hypernovae/supernovae is very difficult, but a systematic search for gravitational lensing–magnified Population III hypernovae/supernovae targeted at massive clusters of galaxies may turn out to be fruitful; SIRTF may be able to detect them. SIRTF may also be able to detect some of the large Population III galaxies. JWST should be able to detect both large Population III galaxies and Population III hypernovae.

4. The Thomson scattering optical depth is increased to 0.10 ± 0.03 (compared to 0.027 for the case of only one rapid reionization at \( z = 6 \)), which will have significant implications on the polarization observations of the CMB. Upcoming WMAP results should be able to distinguish between these two scenarios.

5. Under the present scenario it is not a surprise that no Population III stars have been found in the local universe.

6. The IGM, while opaque to \( \text{Ly} \alpha \) photons, is highly ionized (\( n_{\text{HI}}/n_{\text{HII}} \sim 0.1-0.2 \)) up to redshift \( z \sim 7 \). This may reconcile the observation, indicating the second reionization at \( z \sim 6 \) with the detection of \( \text{Ly} \alpha \) galaxies at \( z \geq 6.5 \).

7. In contrast to the conventional reionization scenario, 21 cm lines prior to \( z = 6 \), which would otherwise be produced by minihalos in the absence of the first cosmological
reionization, are predicted not to exist. This would provide a definitive test of the scenario proposed here. For that matter, any predictions related to minihalos at these redshifts would be altered.

8. Finally, minihalos will be largely deprived of gas at a higher redshift (\( z \geq 6 \)) than previously thought. This could alleviate the cosmological overcooling problem (see, e.g., White & Rees 1978).

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**APPENDIX**

**MAGNIFICATION OF POPULATION III SOURCES BY GRAVITATIONAL LENSING**

We determine the fraction of area in the background source plane at redshift \( z_S \) that is magnified by a factor of \( \mu \) by foreground objects, primarily clusters of galaxies. The magnification regime in which we are interested is \( \mu \geq 10 \). We first derive a general relationship between the magnification (\( \mu \)) and source plane coordinate (\( x \)), \( \mu \propto 1/x \), for any singular density profile. Then we use the singular isothermal sphere, thanks to its ease of analytic treatment, that has the same \( \mu \propto 1/x \) relation, to model clusters of galaxies to compute the gravitational lensing cross section at the high-\( \mu \) end. The computed lensing cross section is cross-checked by observations to ensure consistency.

We express the magnification \( \mu \) as a function of \( x \), the source plane proper coordinate (Fig. 17). It is well known that \( \mu \propto 1/x \) in the small-\( x \), high-\( \mu \), limit, for both a point-mass lens (Vietri & Ostriker 1983) and a singular isothermal sphere (SIS) lens (Turner, Ostriker, & Gott 1984). For simplicity, we consider only axisymmetric lenses. We generalize this property of \( \mu \) in the small-\( x \), high-\( \mu \) limit to any lens system as long as the lensing bending angle \( \theta \), as a function of proper impact parameter \( b \), does not increase as fast as \( b \). It is noted that \( \theta \propto b \) for a uniform mass sheet, \( \theta = \) constant for an SIS lens, and \( \theta \propto 1/b \) for a point mass. From Figure 17 we can readily write down the lens equation as

\[
\alpha + \beta = \frac{b}{D_L}, \quad \beta = \frac{x}{D_S}, \quad \alpha D_S = \theta D_{LS}.
\]  

(A1)

The above equation relates the angular position of the image, \( \alpha + \beta \), to the position of the source, \( \beta \) (both with respect to some reference direction), through the bending angle \( \theta \). \( D_S, D_L, \) and \( D_{LS} \) are the angular diameter distance between the observer and the source, between the observer and the lens, and between the lens and the source, respectively; \( b \) is the proper impact parameter relative to the lens center. Expressing the bending angle \( \theta \) as \( \theta = \theta_0(b/b_0)^m \) (\( m < 1 \)), we can rewrite the above lens equation as

\[
\theta_0(b/b_0)^m = b D_S/(D_L D_{LS}) - x/D_{LS}.
\]  

(A2)

In the limit \( x \to 0 \), the solution for the impact parameter is \( b = b_0 [b_0 D_S/(\theta_0 D_L D_{LS})]^{1/(m-1)} \) (Einstein radius) and \( dx/db = (1 - m) D_S/D_L \). It is noted that the requirement \( m < 1 \) is to guarantee a solution, i.e., the existence of an Einstein
radius. Then it is straightforward to show that, at the small-x limit, the magnification is

$$\mu(x) = \frac{b}{x} \frac{db}{dx} \left( \frac{D_S}{D_L} \right)^2 = \frac{K}{x},$$  \hspace{1cm} (A3)

with the constant $K$, independent of $x$, being

$$K = \left( \frac{b_0 D_S}{b_0 D_L D_{LS}} \right)^{1/(m-1)} \frac{D_S}{D_L} \frac{b_0}{1-m}.$$

(A4)

We note that, for any (centrally) singular density profile, $m$ is less than unity. It has been shown by Syer & White (1996) and Subramanian, Cen, & Ostriker (2000) that the density profile in the inner regions of dark matter halos, which are formed through hierarchical gravitational clustering/merging in the conventional Gaussian structure formation models, is $\rho(r) \propto r^{-(3+\gamma)/5}$, where $\gamma$ is the linear power spectrum index at the relevant scales. For the scales of interest, we have $n \sim -2$ to $-4$ for CDM models, giving $\rho(r) \propto r^{-1.5}$ to $r^{-1.0}$. These inner slopes of halos were borne out in $N$-body simulations (Navarro et al. 1997; Moore et al. 1999). Thus, we should expect $\mu \propto 1/x$ at small $x$ for halos formed in hierarchical structure formation models, including those of galaxy cluster size. Moreover, cooling and subsequent condensation of baryons in the centers of halos may further steepen the density profiles in the inner regions.

Having deduced the universal $\mu \propto 1/x$ relation at the small-$x$ end, we now carry out further calculations by adopting the SIS model, because of its analytical simplicity and because of its astrophysical relevance, as shown in many statistical studies (Gunn & Gott 1972; Tyson 1983; Turner et al. 1984; Hinshaw & Krauss 1987; Narayan & White 1988; Wu 1989; Fukugita & Turner 1991; Mao 1991). This is further justified if $\mu$ is normalized at a somewhat lower value, where some data from current observations exist. We note that, for individual multiply imaged observed quasars, it is clear that one needs to include ellipticities of the lenses for realistic modeling. For example, one cannot produce quadrupoles with an axisymmetric lens (Narayan & Grossman 1989; Blandford et al. 1989). However, for our purpose of calculating the magnification cross section at moderate-to-high range, $\mu \sim 10–100$, the assumption of axisymmetry of lenses should be adequate. For an SIS lens, the bending angle $\theta_0$, conveniently independent of impact parameter, is (Turner et al. 1984)

$$\theta_0 = 4\pi \left( \frac{\sigma_\parallel}{c} \right)^2,$$

(A5)

where $c$ is the speed of light and $\sigma_\parallel$ is the line-of-sight velocity dispersion of the SIS lens. With equation (A5) we can solve equation (A2) with the following solution at the $x \rightarrow 0$ limit: $b = b_0 D_L D_{LS}/D_S$ and $db/dx = D_L/D_S$. Inserting this solution into equation (A3) yields the small-$x$ limit of the $x$-$\mu$ relation for an SIS lens:

$$x(\mu) = \frac{4\pi}{\mu} \left( \frac{\sigma_\parallel}{c} \right)^2 D_{LS}.$$

(A6)

An SIS lens would subtend a solid angle in the source plane $\Delta \Omega$ within which the luminosity of a source is magnified by $\geq \mu$: $\Delta \Omega = \pi x^2 (\mu)/D_S^3$. Thus, the total probability of a random source at $z_S$ being magnified by $\geq \mu$ can be obtained by adding up all the foreground lenses. This is done by integrating $\Delta \Omega/4\pi$ over lenses of all masses and over all redshifts up to the source redshift $z_S$, resulting in a double integral:

$$P_{random} = \int_0^\infty \int_0^{z_S} \frac{\pi x^2}{4\pi D_S^3} n(M_A, z) 4\pi r^2 \frac{d^3r}{dz} dM,$$

(A7)

where $n(M_A, z)$ is the mass function of halos at redshift $z$. To make the subsequent calculation more analytically tractable, we assume

$$n(M_A, z) = (1+z)^{-w} g(M_A),$$

(A8)

where

$$g(M_A) = (4 \times 10^{-5}/M_\odot)(M_A/M_\odot)^{-1} \left[ 1 + (M_A/M_\odot)^{-1} \right] \exp(-M_A/M_\odot) h^3 \text{ Mpc}^{-3}$$

(A9)

is the differential cluster mass function at $z = 0$. taken from Bahcall & Cen (1993). $M_A$ is the observed cluster mass within the Abell radius ($r_A = 1.5 h^{-1} \text{ Mpc}$), and $M_\odot = 1.8 \times 10^{14} h^{-1} M_\odot$. The term $(1+z)^{-w}$ is intended to approximately describe the evolution of the overall cluster mass function with redshift. The cluster mass, $M_A$, defined within the Abell radius, $r_A$, may be related to $\sigma_\parallel$ for the SIS model by

$$\sigma_\parallel^2 = \frac{GM_A}{2r_A},$$

(A10)

where $G$ is the gravitational constant. Combining equations (A6) and (A10) indicates that $x^2 \propto M_A^2$. It is instructive to examine the behavior of the following term, $x^2 g(M_A)$, which contains all the dependence of the integrand of the integral in equation (A7) on $M_A$. Since $x^2 g(M_A)$ is constant at the low-$M_A$ end and $x^2 g(M_A) \propto (M_A/M_\odot) \exp(-M_A/M_\odot)$ at the high-$M_A$ end, the integral over $M_A$ in equation (A7) is convergent on both ends and dominated by massive clusters near the exponential downturn at about $M \sim 1 \times 10^{15} M_\odot$. Consequently, for our purpose, we need only to be accurate on the
high-mass end near the exponential downturn for $g(M_A)$ and its redshift distribution characterized by parameter $w$. Substituting equations (A6), (A8), (A9), and (A10) into equation (A7) (and making use of the simple relations in an $\Omega_0 = 1$ universe: $D_L = \frac{R_H}{H}\left[1 - 1/(1 + z_L)^{1/2}\right]$, $D_S = \frac{R_H}{H}\left[1 - 1/(1 + z_S)^{1/2}\right]$, $D_{LS} = \frac{R_H}{H}\left[1/(1 + z_L)^{1/2} - 1/(1 + z_S)^{1/2}\right]$, and $dr(z)/dz = \frac{1}{2}R_H(1 + z)^{-3/2}$, where $R_H = 2c/H/6000$ h Mpc is the comoving Hubble radius) yields

$$P_{\text{random}} = \frac{2B\pi G^2 R_H^3}{c^4\mu^2}(1 - \frac{1}{\sqrt{1 + z_S}})^{-2}\int_0^\infty M_A^2 g(M_A) dM$$
$$\times \int_0^{z_S} \left(1 - \frac{1}{\sqrt{1 + z}}\right)^2 \left(\frac{1}{\sqrt{1 + z} - \frac{1}{\sqrt{1 + z_S}}}\right)^2 (1 + z)^{-3/2 - w} dz$$
$$= 1.0 \times 10^{-5} B^3 g^2 R_H^2 M_2^2 \left(1 - \frac{1}{\sqrt{1 + z_S}}\right)^{-2}\int_0^\infty t(1 + t^{-1}) \exp(-t) dt$$
$$\times \int_0^{z_S} \left(1 - \frac{1}{\sqrt{1 + z}}\right)^2 \left(\frac{1}{\sqrt{1 + z} - \frac{1}{\sqrt{1 + z_S}}}\right)^2 (1 + z)^{-3/2 - w} dz.$$  \hspace{1cm} (A11)

All constants in equation (A11) are in cgs units, except $R_H$, which is in Mpc. Note that we have also inserted a constant $B$ in equation (A11), which serves to absorb the uncertainties due to other factors that either cannot be accurately treated here or are unknown, including background cosmology, deviations of density profiles from singular isothermal spheres, gravitational lensing due to other astronomical objects, and uncertainties in the observed cluster mass function. The normalization of $B$ will be set by comparing to observations at $\mu > 2$. Now expanding all the constants and integrating equation (A11) with respect to $t$ ($\equiv M_A/M_\odot$), we obtain

$$P_{\text{random}} = \frac{0.036 B}{\mu^2} \left(1 - \frac{1}{\sqrt{1 + z_S}}\right)^{-2} I(z_S, w) ,$$  \hspace{1cm} (A12)

where $I(z_S, w) \equiv \int_0^{z_S} \left[1 - 1/(1 + z)^{1/2}\right] \left[1/(1 + z)^{1/2} - 1/(1 + z_S)^{1/2}\right] (1 + z)^{-3/2 - w} dz$. The integral $I(z_S, w)$ can be solved analytically (but the resultant expression is quite lengthy), and here we just give the final numbers for specific $z_S$ and $w$. For $z_S = 17.0$, $P_{\text{random}} = (1.1 \times 10^{-3}, 6.8 \times 10^{-4}, 4.2 \times 10^{-4}, 2.1 \times 10^{-4}) B/\mu^2$, for $w = (0.0, 0.5, 1.0, 2.0)$, respectively. It shows that $P_{\text{random}}$ only weakly depends on $w$, because the integral is dominated by the moderate-redshift range $z \sim 0.5-1.0$.

It is justified to only consider the large splitting events since the lensing cross section is dominated by massive clusters that give rise to large splittings ($>1^\circ$). For the sake of concreteness, we adopt $w = 0.5$, which is consistent with the relatively mild evolution of cluster density up to a redshift of about unity (see, e.g., Bahcall, Fan, & Cen 1997). As we show below, the final results ($P_{\text{cl}}$) turn out to be extremely weakly dependent on $w$. Furthermore, we take $B = 3.4$ to normalize $P_{\text{random}}$:

$$P_{\text{random}}(\mu) = \frac{2.0 \times 10^{-3}}{\mu^2} ,$$  \hspace{1cm} (A13)

which gives $P_{\text{random}}(\mu = 2) = 4.0 \times 10^{-4}$. Since we are integrating the observed clusters to obtain the lensing probability and the same clusters are also responsible for observed gravitational lensing events, we can make a consistency check. For the Hewitt & Burbidge (1989) quasar catalog of 4250 quasars, there are two confirmed multiple-image lens systems with splitting greater than $1^\circ$, which corresponds to a multiple lensing probability of $4.7 \times 10^{-4} (\cong 2/4250)$, i.e., $P = 4.7 \times 10^{-4}$ for $\mu > 2$ (in the singular isothermal sphere case). Thus, our adopted normalization is consistent with the observed value of $P = 4.7 \times 10^{-4}$ for $\mu > 2$. Considering possible selection biases (Turner 1980; Cen et al. 1994a; Kochanek 1995) and the countervailing effect that the observed quasar sample is at a significantly lower redshift than $z = 17$ considered here, we argue that the adopted normalization is reasonable. From this we see that a random source at $z_S = 17$ has a rather small probability of $\leq 10^{-3}$ of being very strongly lensed.

We now examine the case in which we do not sample the sky randomly but rather observe a set of selected regions centered on massive clusters. The cross section for magnification $\geq \mu$ in the source plane is $\pi x^2(\mu)$, so the solid angle within which the sources will be magnified by $\geq \mu$ is $\pi x^2(\mu)/D_3^2$. Integrating $\pi x^2(\mu)/D_3^2$ over a preselected set of clusters with mass $M_A \geq M_{\text{lim}}$ in the redshift range $z = z_1$ to $z_2$ and dividing the integral by the number of clusters preselected gives the mean solid angle for source magnification $\geq \mu$:

$$\langle \Omega(\geq \mu) \rangle = \frac{\int_{M_{\text{lim}}}^{\infty} B' g(M_A) dM \int_{z_1}^{z_2} \pi x^2/D_3^2 (1 + z)^{-w} 4\pi r^2 (dr/dz) dz}{N_{\text{cl}}(z_1, z_2, M_{\text{lim}})} ,$$  \hspace{1cm} (A14)

where $N_{\text{cl}}(z_1, z_2, M_{\text{lim}}) \equiv \int_{M_{\text{lim}}}^{\infty} g(M_A) dM \int_{z_1}^{z_2} (1 + z)^{-w} 4\pi r^2 (dr/dz) dz$ is the total number of clusters selected (a normalization factor). Dividing the mean solid angle for source magnification $\geq \mu$ by the field of view yields the mean probability of a source in such selected fields being magnified by $\geq \mu$:

$$P_{\text{cl}}(\mu) = \frac{\langle \Omega(\geq \mu) \rangle}{\Omega_{\text{FOV}}} ,$$  \hspace{1cm} (A15)

where $\Omega_{\text{FOV}}$ is a telescope's field of view. As an example, let us take $M_{\text{lim}} = 1 \times 10^{15} M_\odot$, $z_1 = 0.0$, and $z_2 = 0.4$, which yields $N_{\text{cl}}(0.0, 0.4, 1 \times 10^{15}) = (3020, 2669, 2362, 1852)$, for $w = (0.0, 0.5, 1.0, 2.0)$, respectively, in an $\Omega_0 = 1, H = 50$ km s$^{-1}$
Mpc⁻¹ universe. Similar to equation (A11), we can integrate equation (A14) for $z_S = 6.0$, $z_I = 0.0$, $z_Z = 0.4$, and $M_{\text{min}} = 1.0 \times 10^{15} M_\odot$, and the result is $P_{\text{clust}}(\mu) = (6.7 \times 10^{-2}, 6.7 \times 10^{-2}, 6.8 \times 10^{-2}, 6.9 \times 10^{-2}/\mu^2(\Omega_{\text{FOV}}/25 \text{arcmin}^2)^{-1}$. Although an adjustment parameter $B$ is included in equation (A14), which is different from $B$ in equation (A11), to reflect the fact that the uncertainties involved are somewhat different (e.g., since the probability is normalized per cluster, the uncertainty in the amplitude of the observed cluster mass function cancels out), for clarity and simplicity, we adopt $B = B = 3.3$, which is justified because the difference in various uncertain factors between $B$ and $B$ is relatively small given the relative weak dependence of both $P_{\text{random}}$ and $P_{\text{clust}}$ on $w$. The result indicates that $P_{\text{clust}}(\mu)$ is nearly independent of $w$, unlike $P_{\text{random}}$, which depends on $w$, giving

$$P_{\text{clust}}(\mu) = \frac{6.7 \times 10^{-2}}{\mu^2} \left( \frac{\Omega_{\text{FOV}}}{25 \text{arcmin}^2} \right)^{-1}.$$  

(A16)

Comparing $P_{\text{clust}}$ with $P_{\text{random}}$ for the case with $w = 0.5$, for example, we see that one gains a factor of 34 in the cluster-centered survey compared to a random survey, for a field of view of $\Omega_{\text{FOV}} = 25 \text{arcmin}^2$. The relative gain decreases with increasing size of the field of view, and the two probabilities become equal at $\Omega_{\text{FOV}} = 838 \text{arcmin}^2$, above which the calculation of $P_{\text{clust}}$ is no longer valid because of multiple clusters in a single field. Note that although the mean probability of a source within the field of view is inversely proportional to $\Omega_{\text{FOV}}$, the total number of magnified sources per field of view is $P_{\text{clust}}\Omega_{\text{FOV}}\Sigma_{\text{sec}}$ (where $\Sigma_{\text{sec}}$ is the surface density of sources, either galaxies or supernovae in this study), which is independent of $\Omega_{\text{FOV}}$. The physical reason for having a relative increase in the probability of magnified sources is that the clusters of galaxies are rare targets, and hence, a random field of view has a small probability of intersecting a cluster. We note that $P_{\text{random}}$ is still valid even in the case of multiple clusters in a single field. However, the calculation of $P_{\text{random}}$ breaks down when multiple clusters are precisely aligned along the line of sight. But such cases are likely to be negligibly few. We further note that the spatial clustering of galaxy clusters would further boost the gain of $P_{\text{clust}}$ over $P_{\text{random}}$, but we do not consider this effect here.

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