Euler [6] derived in 1936 a more general expression for the quantum nonlinearities in the Lagrangian of QED, and a complete calculation in the low-frequency limit by Euler and Kockel [5] in 1935. Subsequently, Heisenberg and Euler [6] derived in 1936 a more general expression for the quantum nonlinearities in the Lagrangian of quantum electrodynamics (QED), and a complete calculation of light-by-light scattering in QED was published until very recently. In 2013, a measurement of light-by-light scattering has remained elusive.

Magnetic field, Born and Infeld [11] proposed in 1934 a unitarian conceptually distinct nonlinear modification of the Lagrangian of QED:

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \rightarrow$$

$$\mathcal{L}_{\text{BI}} = \beta^2 \left( 1 - \sqrt{1 + \frac{1}{2\beta^2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{16\beta^4} (F_{\mu\nu} \tilde{F}^{\mu\nu})^2} \right).$$

where $\beta$ is an a priori unknown parameter with the dimension of [mass]$^2$ that we write as $\beta \equiv M^2$, and $\tilde{F}_{\mu\nu}$ is the dual of the field strength tensor $F_{\mu\nu}$. Interest in Born-Infeld theory was revived in 1985 when Fradkin and Tseytlin [12] discovered that it appears when an Abelian vector field in four dimensions is coupled to an open string, as occurs in models inspired by $M$ theory in which particles are localized on lower-dimensional “branes” separated by a distance $\approx 1/\sqrt{\beta} = 1/M$ in some extra dimension. (Remarkably, the maximum field strength is related to the fact that the brane velocity is limited by the velocity of light [13], confirming the insight of Born and Infeld [11].) Depending on the specific brane scenario considered, $M$ might have any value between a few hundred GeV and the Planck scale $\sim 10^{19}$ GeV. For the purposes of this Letter, we consider only the relevant terms of fourth order in the gauge field strengths in Eq. (1).

To date, there has been no strong lower limit on the Born-Infeld scale $\beta$ or, equivalently, the brane mass scale $M$ and the brane separation $1/M$. A constraint corresponding
to $M \gtrsim 100$ MeV was derived in Ref. [14] from electronic and muonic atom spectra, though the derivation was questioned in Ref. [15]. Measurements of photon splitting in atomic fields [16] were considered in Ref. [17], where it was concluded that they provided no limit on the Born-Infeld scale and it was suggested that measurements of the surface magnetic field of neutron stars [18] might be sensitive to $M = \sqrt{\beta} \sim 1.4 \times 10^{-3}$ GeV. More recently, measurements of nonlinearities in light by the PVLAS Collaboration [19] were somewhat more sensitive to the individual nonlinear terms in Eq. (1), but they were not sensitive to the particular combination appearing in the Born-Infeld theory, as discussed in Ref. [20], where more references can be found.

Here, we show that the agreement of the recent ATLAS measurement of light-by-light scattering with the standard QED prediction provides the first limit on $M$ in the multi-GeV range, excluding a significant range extending to

$$M \gtrsim 100 \text{ GeV},$$

entering the range of interest to brane theories. This limit is obtained under quite conservative assumptions, and plausible stronger assumptions would strengthen our lower bound to $M \gtrsim 200$ GeV.

One may also consider a Born-Infeld extension of the standard model in which the hypercharge $U(1)_Y$ gauge symmetry is realized nonlinearly, in which case the limit (2) is relaxed to

$$M_Y = \cos \theta_W M \gtrsim 90 \text{ GeV},$$

where we have used $B^\nu_Y = \cos \theta_W A^\nu_{EM} - \sin \theta_W Z^\nu$ and $\sin^2 \theta_W = 0.23$, with $\theta_W$ being the weak mixing angle. As a corollary of this lower limit on the $U(1)_Y$ brane scale, we recall that Arunasalam and Kobakhidze recently pointed out [21] that the standard model modified by a Born-Infeld $U(1)_Y$ theory has a finite-energy electroweak monopole [22,23] solution $M_Y$, whose mass they estimated as $M_M \approx 4$ TeV $+ 72.8 M_Y$. Such a monopole is less constrained by Higgs measurements than electroweak monopoles in other extensions of the standard model [24], and hence of interest for potential detection by the ATLAS [25], CMS, and MoEDAL experiments at the LHC [26]. However, our lower limit $M_Y \gtrsim 90$ GeV (2) corresponds to a 95% C.L. lower limit on the mass of this monopole $M_M \gtrsim 11$ TeV, excluding its production at the LHC.

Following the suggestion of Ref. [8], we consider ultraperipheral heavy-ion collisions in which the nuclei scatter quasistatically via photon exchange: Pb + Pb $\rightarrow$ Pb$^{(*)} + X$, as depicted in Fig. 1, effectively acting via the equivalent photon approximation (EPA) [27] as a photon-photon collider. (The authors of Ref. [8] also considered the observability of light-by-light scattering in $pp$ collisions at the LHC but concluded that they were less promising than Pb-Pb collisions.) The EPA allows the electromagnetic field surrounding a highly relativistic charged particle to be treated as equivalent to a flux of on-shell photons. Since the photon flux is proportional to $Z^2$ for each nucleus, the coherent enhancement in the exclusive $\gamma \gamma$ cross section scales as $Z^4$, where $Z = 82$ for the lead (Pb) ions used at the LHC. This is why heavy-ion collisions have an advantage over proton-proton or proton-nucleus collisions for probing physics in electromagnetic processes [8]. Photon fusion in ultraperipheral heavy-ion collisions has been suggested as a way of detecting the Higgs boson [28,29], and, more recently, the possibility of constraining new physics beyond the standard model (BSM) in this process was studied in Refs. [30,31].

As was already mentioned, the possibility of directly observing light-by-light scattering at the LHC was proposed in Ref. [8], and this long-standing prediction of QED was finally measured earlier this year with 4.4$\sigma$ significance by the ATLAS Collaboration [9] at a level in good agreement with calculations in Refs. [8,10]. The compatibility with the standard model constrains any possible contributions from BSM physics. Born-Infeld theory is particularly interesting in this regard, as constraints from low-energy optical and atomic experiments have yet to reach the sensitivity of interest for measuring light-by-light scattering [19,20].

The leading-order cross section for unpolarized light-by-light scattering in Born-Infeld theory in the $\gamma \gamma$ center-of-mass frame is given by [17,32]

$$\sigma_{BI}(\gamma \gamma \rightarrow \gamma \gamma) = \frac{1}{2} \int d\Omega \frac{d\sigma_{BI}}{d\Omega} = \frac{7 m_{\gamma \gamma}^6}{1280 \pi \beta^4},$$

where $m_{\gamma \gamma}$ is the diphoton invariant mass and the differential cross section is

$$\frac{d\sigma_{BI}}{d\Omega} = \frac{1}{4096 \pi^2 \beta^4} \left(3 + \cos \theta\right)^2.$$
normalizations), as solid blue and dashed red lines, respectively. We see that the Born-Infeld distribution is less forward peaked than that for QED. For the latter, we used the leading-order amplitudes for the quark and lepton box loops in the ultrarelativistic limit from Ref. [33], omitting the percent-level effects of higher-order QCD and QED corrections, as well the \( W^\pm \) contribution that is negligible for typical diphoton center-of-mass masses at the LHC.

The total exclusive diphoton cross section from \( \text{Pb} + \text{Pb} \) collisions is obtained by convoluting the \( \gamma\gamma \to \gamma\gamma \) cross section with a luminosity function \( dL/d\tau \) [34],

\[
\sigma_{\text{excl}} = \int_{\tau_0}^{1} d\tau \frac{dL}{d\tau} \sigma_{\gamma\gamma \to \gamma\gamma}(\tau).
\]

We have introduced here a dimensionless measure of the diphoton invariant mass, \( \tau \equiv m_{\gamma\gamma}^2/s_{\text{NN}} \), where \( \sqrt{s_{\text{NN}}} = 5.02 \text{ TeV} \) is the center-of-mass energy per nucleon pair in the ATLAS measurement. The luminosity function, derived, for example, in Ref. [34], can be written as an integral over the number distribution of photons carrying a fraction \( x \) of the total Pb momentum:

\[
\frac{dL}{d\tau} = \int_\tau^1 dx_1 dx_2 f(x_1) f(x_2) \delta(\tau - x_1 x_2),
\]

where the distribution function \( f(x) \) depends on a nuclear form factor. We follow Ref. [34] in adopting the form factor proposed in Ref. [29], while noting that variations in the choice leads to \( \sim 20\% \) uncertainties in the final cross sections [8]. A contribution with a nonfactorizable distribution function should also be subtracted to account for the exclusion of nuclear overlaps, but this is not a significant effect for the relevant kinematic range, causing a difference within the 20\% uncertainty [31] from the photon luminosity evaluated numerically using the STARLIGHT code [35]. For \( \sqrt{s_{\text{NN}}} = 5.5 \text{ TeV} \) and \( m_{\gamma\gamma} > 5 \text{ GeV} \), we obtain a QED cross section of \( \sigma_{\text{excl}}^{\text{QED}} = 385 \pm 77 \text{ nb} \), in good agreement with Ref. [8]. The ATLAS measurement is performed at \( \sqrt{s_{\text{NN}}} = 5.02 \text{ TeV} \) and for \( m_{\gamma\gamma} > 6 \text{ GeV} \), for which we find \( \sigma_{\text{excl}}^{\text{QED}} = 220 \pm 44 \text{ nb} \).

This total \( \gamma\gamma \to \gamma\gamma \) cross section is reduced by the fiducial cuts of the ATLAS analysis, which restrict the phase space to a photon pseudorapidity region \( |\eta| < 2.4 \) and require photon transverse energies \( E_T > 3 \text{ GeV} \) and the diphoton system to have an invariant mass \( m_{\gamma\gamma} > 6 \text{ GeV} \), with a transverse momentum \( p_T < 2 \text{ GeV} \) and an acoplanarity \( A_{\phi} = 1 - \Delta\phi/\pi < 0.01 \). We simulate the event selection using Monte Carlo sampling, implementing the cuts with a 15\% Gaussian smearing in the photon transverse energy resolution at low energies and 0.7\% at higher energies [9,36] above 100 GeV. Since the differential cross section does not depend on \( \phi \), we implement the acoplanarity cut as a fixed 85\% efficiency, following the ATLAS analysis [9]. The total reduction in yield for the QED case is a factor \( e \sim 0.30 \), which results in a fiducial cross section \( \sigma_{\text{fid}}^{\text{QED}} = 53 \pm 11 \text{ nb} \) for \( \sqrt{s_{\text{NN}}} = 5.02 \text{ TeV} \), in good agreement with the two predictions of 45 and 49 nb quoted by ATLAS [9].

Following this validation for the QED case, we repeat the procedure for the Born-Infeld cross section. Since the Born-Infeld \( \gamma\gamma \to \gamma\gamma \) cross section grows with energy, the dominant contribution to the cross section comes from the \( \tau \lesssim 0.2 \) part of the integral, compared with \( \tau \lesssim 10^{-4} \) for the QED case. We show in Fig. 3 the distributions of the \( \sigma(\gamma\gamma \to \gamma\gamma) \) cross section multiplied by the photon flux luminosity factor—normalized by the total exclusive cross section—as functions of the invariant diphoton mass distribution, for the QED case in the upper panel and in Born-Infeld theory with \( M = \sqrt{\beta} = 200 \text{ GeV} \) in the lower panel.

We see that the invariant-mass distribution in the Born-Infeld case extends to \( m_{\gamma\gamma} > M \), where the validity of the tree-level Born-Infeld Lagrangian may be questioned because the Taylor expansion of the square root in the nonpolynomial Born-Infeld Lagrangian (1) could break down. With this in mind, we use two approaches to place plausible limits on \( M = \sqrt{\beta} \). In the first and most conservative method, we consider \( \gamma\gamma \) scattering only for \( m_{\gamma\gamma} \lesssim M \), while in the second approach we integrate the \( \gamma\gamma \) cross section (4) up to the diphoton invariant mass where the unitarity limit \( \sigma_{\text{BI}} \sim 1/m_{\gamma\gamma}^2 \) is saturated, beyond which we assume that the cross section saturates the unitarity limit and falls as \( \sim 1/m_{\gamma\gamma}^2 \).

We find fiducial efficiencies for the cutoff and unitarization approaches to be \( e \sim 0.39 \) and 0.14, respectively. While the \( E_T \) and \( \eta \) cuts have much less effect than for QED, as expected from the difference in the angular distributions visible in Fig. 2, the larger invariant masses appearing in the Born-Infeld case are much more affected by the \( p_{T\gamma\gamma}^\text{min} \) requirement.

Our calculations of the corresponding \( U(1)_{EM} \) Born-Infeld fiducial cross sections are plotted in the upper

![Graph](image-url)
Panel of Fig. 4 as a function of $M = \sqrt{\beta}$: the green curve is for the more conservative cutoff approach, and the blue curve assumes that unitarity is saturated. These calculations are confronted with the ATLAS measurement of $\sigma_{\text{fid}} = 70^{+24}_{-17} \text{ (stat)} \times 10^{+17}_{-10} \text{ (syst)} \text{ nb}$, assuming that these errors are Gaussian and adding them in quadrature with a theory uncertainty of $10 \text{ nb}$. We perform a $\chi^2$ fit to obtain the 95% C.L. upper limit on a Born-Infeld signal additional to the 49 nb standard model prediction. (We neglect possible interference effects that are expected to be small due to the different invariant-mass and angular distributions involved.) This corresponds to the excluded range shaded in pink above $\sigma_{\text{fid}}$, which translates to the limit $M = \sqrt{\beta} \gtrsim 100(190) \text{ GeV}$ in the cutoff (unitarized) approach, as indicated by the green (blue) vertical dashed line in Fig. 4.

These limits could be strengthened further by considering the $m_{\gamma\gamma}$ distribution shown in Fig. 3(b) of Ref. [9], where we see that all of the observed events had $m_{\gamma\gamma} < 25 \text{ GeV}$, in line with expectations in QED, whereas, in the Born-Infeld theory, most events would have $m_{\gamma\gamma} > 25 \text{ GeV}$. Calculating a ratio of the total exclusive cross section of QED for $m_{\gamma\gamma} > 6 \text{ GeV}$ and $> 25 \text{ GeV}$ as $\sigma_{\text{excl}}^{m_{\gamma\gamma} > 6 \text{ GeV}} / \sigma_{\text{excl}}^{m_{\gamma\gamma} > 25 \text{ GeV}} \sim 0.02$, we estimate a 95% C.L. upper limit of $\sim 2 \text{ nb}$ for $m_{\gamma\gamma} > 25 \text{ GeV}$. The corresponding exclusion plot is shown in the lower panel of Fig. 4, where we see a stronger limit $M = \sqrt{\beta} \gtrsim 210(330) \text{ GeV}$ in the cutoff (unitarized) approach, with the same color coding used previously.

Our lower limit on the QED Born-Infeld scale $M = \sqrt{\beta} \gtrsim 100 \text{ GeV}$ is at least 3 orders of magnitude stronger than the lower limits on $M = \sqrt{\beta}$ obtained from previous measurements of nonlinearities in light [14–17,19,20]. Because of the kinematic cuts made in the ATLAS analysis, our limit does not apply to a range of values of $M \lesssim 10 \text{ GeV}$ for which the nonlinearities in Eq. (1) should be taken into account. However, our limit is the first to approach the range of potential interest for string or $M$ theory constructions since models with (stacks of) branes...
separated by distances $1/M : M = O(1)$ TeV have been proposed in that context [37]. Our analysis could clearly be refined with more sophisticated detector simulations and the uncertainties reduced. However, in view of the strong power-law dependence of the Born-Infeld cross section on $M = \sqrt{\beta}$ visible in Eq. (4), the scope for significant improvement in our constraint is limited unless experiments can probe substantially larger $m_{\gamma\gamma}$ ranges. In this regard, it would be interesting to explore the sensitivities of high-energy $e^+ e^-$ machines considered as $\gamma \gamma$ colliders.

As mentioned in the introduction, Arunasalam and Kobakhidze recently pointed out [21] that the standard model modified by a Born-Infeld theory of the hypercharge $U(1)_Y$ contains a finite-energy monopole solution with mass $M_M = E_0 + E_1$, where $E_0$ is the contribution associated with the Born-Infeld $U(1)_Y$ hypercharge, and $E_1$ is associated with the remainder of the Lagrangian. Arunasalam and Kobakhidze estimated [21] that $E_0 \approx 72.8 M_Y$, where $M_Y = \cos \theta_W M$, and Cho et al. had previously estimated [23] that $E_1 \approx 4 \text{ TeV}$. (Both of these estimates are at the classical level, and quantum corrections have yet to be explored.) Combining these calculations and using our lower limit $M \gtrsim 100 \text{ GeV}$ (2), we obtain a lower limit $M_M \gtrsim 11 \text{ TeV}$ on the $U(1)_Y$ Born-Infeld monopole mass. (For completeness, we recall that it was argued in Ref. [21] that nucleosynthesis constraints on the abundance of relic monopoles require $M_M \lesssim 23 \text{ TeV}$. Unfortunately, this is beyond the reach of MoEDAL [26] or any other experiment at the LHC [25], but it could lie within reach of a similar experiment at any future 100-TeV $pp$ collider [38], or of a cosmic ray experiment.

In this Letter we have restricted our attention to possible Born-Infeld modifications of $U(1)$ gauge factors and their constraints in light-by-light scattering only. We plan to examine in the future the experimental constraints from measurements at the LHC on possible Born-Infeld extensions of the SU(3)$_C$ and SU(2)$_L$ gauge symmetries of the standard model.

The work of J. E. and N. E. M. was supported partly by STFC Grant No. ST/L000326/1. The work of T. Y. was supported by a Junior Research Fellowship from Gonville and Caius College, Cambridge, England. We thank Vasiliki Mitsou for drawing our attention to Ref. [21], and her and Albert De Roeck, Igor Ostrovskiy, and Jim Pinfold of the MoEDAL Collaboration for their interest and the relevant discussions. T. Y. is grateful for the hospitality of King’s College London, where part of this work was completed.

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