Application of magnetic field for improvement of energy spread of an electron beam

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Abstract. Electron diffraction is not solely a powerful method to study fundamental physics but has also been applied to quantum sensors. A low-coherence electron source results in a drop of fringe contrast. One approach to enhance a visibility of the diffraction pattern is to improve a longitudinal coherence of an electron beam. When electrons pass through a magnetic field, they experience a force which bends them to circular paths. Electrons with different energies will travel along the different paths. By placing a slit behind the magnetic field, the width of the electron energy distribution will become narrower and hence an improvement of the beam coherence. However, this method reduces the intensity of the electron beam. The simulation was performed to optimize the slit width for electron diffraction experiments with respect to the energy spread of the beam and electron flux.

1. Introduction

After the experiment on electron interference was first performed in 1927 by Davisson and Gemer \cite{1}, the matter waves have become a subject of interest. The advance in technology has made a leap of progress in the research based on the interferometer for different particles, for instance, electrons \cite{2}, ions \cite{3}, neutrons \cite{4} and atoms \cite{5}. Additionally, electron interferometry has been applied in many applications such as demonstration of Talbot-Lau effect \cite{6, 7}, double-slit and electron biprism interference \cite{7}. The famous of a near-field electron diffraction is Talbot-Lau effect. The electron Talbot-Lau effect has been used in a precision measurement of the earth magnetic field \cite{8}. In addition, an electron interference pattern is sensitive to vibrating or disturbing noise, therefore a compact electron matter wave interferometer for sensor technology has been created based on biprism interferometer \cite{9}. A visibility of the electron diffraction pattern plays a major role in improving the interferometer’s sensitivity. The experiments with the fullerene molecule C\textsubscript{60} \cite{10} indicated that the broader the velocity distribution of an electron beam leads to the absence of higher order interference fringes. The visibility of an electron interference pattern can be improved by selecting only electrons with similar energy ranges. This leads to a low energy spread of an electron beam and hence a higher sensitivity. In this paper, we present an approach to select electrons according to their energies by using magnetic field including the simulation-based design of the system.

2. System design

The system presented in this work was designed based on materials and equipment available at our laboratory. Here, a tungsten filament was used as an electron source. The filament, which was heated
by applying an electric current, emitted electrons. The freed electrons were then accelerated through a potential difference of $V$ toward an anode. According to Lorentz force law, an electron moving with a speed $v$ through a magnetic field $\vec{B}$ experiences a force

$$\vec{F} = -e(\vec{v} \times \vec{B}),$$

(1)

where $e$ is the magnitude of the elementary charge. This force causes the curvature movement of the electron. The radius of curvature of the electron path can be calculated by

$$R = \frac{mv}{eB},$$

(2)

where $m$ is the electron mass. Electrons with different velocities, therefore travel the different paths.

![Figure 1. System design consists of a Helmholtz coil (red), a mu-metal cylinder (gray), an adjustable slit and a detector.](image)

As mentioned before, our system was designed for electron diffraction experiments. The experiments have to be carried out inside the vacuum chamber because the mean free path of the electrons in the air is very short. Therefore, our system will be placed inside a DN100CF 6-way cross vacuum chamber which has been equipped with optical components used for other experiments. Due to space constraints, a compact system was required. According to the availability of materials and equipment at our laboratory, a Helmholtz coil was used to create a magnetic filed. The Helmholtz coil was covered by a mu-metal cylinder that acted as a Faraday shield. The electrons felt the Lorentz force only while they were moving inside the cylinder. The cylinder had two openings, one on the front and the other one on the side as shown in figure 1. The electrons that passed through the first opening were bent by the magnetic field of the Helmholtz coil. Only the electrons with the certain energies could pass through the second opening and reach the detector located behind the second opening. Additionally, an adjustable slit was place between the detector and the second opening in order to make our system more flexible. The energy range of the electrons reaching a detector could be selected by changing the width of the slit.

3. Magnetic field of Helmholtz coil

A constant current $I$ along a circular loop of wire creates a magnetic field $B$. For a loop centered at the origin, the magnetic field at point $P$ (see figure 2) can be calculated by using Biot-Savart’s law

$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{s} \times \vec{r}}{r^3},$$

(3)

where $\mu_0$ is the permeability of free space. For off-axis magnetic field, an evaluation of the integral directly from Biot-Savart’s law is very complicated. Alternatively, the magnetic field can be calculated from the vector potential [11].
\[ \vec{A} = \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{l}}{r} = \frac{\mu_0 I a}{\pi \sqrt{a^2 r^2 + 2 a r \sin \theta}} \left[ \frac{(2 - k^2) K(k^2) - 2 E(k^2)}{k^2} \right], \]  

where \( K(k^2) \) and \( E(k^2) \) are complete elliptic integrals of the first and second kind, respectively. The argument of the elliptic integral is

\[ k^2 = \frac{4 a \sin \theta}{a^2 + r^2 + 2 a \sin \theta}. \]  

Here, the cross section of the wire is not taken into account.

Once the vector potential is obtained, the magnetic field can be derived from

\[ \vec{B} = \nabla \times \vec{A}. \]  

The magnetic field components in Cartesian coordinates can be written as \[12\]

\[ B_x = \frac{C x}{2 \alpha^2 \beta \rho} \left[ (a^2 + r^2)E(k^2) - \alpha^2 K(k^2) \right] \]  

\[ B_y = \frac{C y}{2 \alpha^2 \beta \rho} \left[ (a^2 + r^2)E(k^2) - \alpha^2 K(k^2) \right] \]  

\[ B_z = \frac{C}{2 \alpha^2 \beta} \left[ (a^2 - r^2)E(k^2) + \alpha^2 K(k^2) \right], \]

where \( \rho^2 = x^2 + y^2 \), \( r^2 = x^2 + y^2 + z^2 \), \( \alpha^2 = a^2 + r^2 - 2a \rho \), \( \beta^2 = a^2 + r^2 + 2a \rho \), \( k^2 = 1 - \alpha^2 / \beta^2 \), \( \gamma = x^2 - y^2 \) and \( C = \mu_0 I / \pi \).

For our setup shown in figure 1, electrons would enter the cylinder at a point halfway between the two loops, i.e., on the x-y plane with \( z = 0 \). The filed components along x and y axes are zero, whereas the z axis component is

\[ B_z = \frac{N C}{\alpha^2 \beta} \left[ (a^2 - r^2)E(k^2) + \alpha^2 K(k^2) \right]. \]

Since the field components in x and y directions at the point where electrons enter the cylinder are zero, the Lorentz force acting on the electrons with initial velocity perpendicular to z-axis will have only x and y components. Therefore, the paths of the electrons moving on the x-y plane can be describe by 2D vectors.
4. Results and discussions
In our system, a Helmholtz coil of radius 2.5 cm was used. The maximum current applied to the coil was determined by the current threshold of the wire. Since the allowed current was quite low, i.e., $I_{\text{max}} \sim 1$ A, a stronger magnetic field could be obtained by increasing the number of turn $N$ or by reducing the distance between the loops $d$. Here we used $d = 5$ mm with $N=50$ turns. The magnetic field created by the Helmholtz coil was shielded by a 1-mm-thick mu-metal cylinder.

The electrons emitted from the tungsten filament were accelerated through a potential difference of 400 V. If the potential energy was totally converted to the kinetic energy of the electrons, they would travel with velocity of $\sim 1.2 \times 10^7$ m/s. These electrons moved parallel to x-y plane along the y-axis. When they entered the mu-metal cylinder at the point (0,-25,0) mm, they started to feel the Lorentz force and moved along curved paths. Figure 3 shows the paths of the electrons traveling through the magnetic field generated by the Helmholtz coil with different applied currents from 0.5 A to 1.0 A. The electrons that can reach the detector had to fulfill 2 conditions. First, they had to move along the curved paths that led them to the second opening, otherwise they would hit the cylinder wall. Second, when the electrons exited the cylinder, their velocity components in the x and z directions had to be small or $\sim 0$. Otherwise, the high-divergence beam would expand rapidly over a distance. When it passed through the slit placed behind the cylinder, its intensity dropped significantly. Therefore, the low-divergence beam was preferable. The selection of the applied current have a great impact on the quality of the output beam. The careful selection of the applied current regarding the electron velocity is required. The optimum current for the given parameters was 0.744 A. With this current, the electron with velocity of $1.2 \times 10^7$ m/s moved parallel to the x-axis when it exited the mu-metal cylinder. Additionally, they passed through the center of the second opening.

Figure 3. Paths of the electrons with velocity of $1.2 \times 10^7$ m/s traveling through the magnetic field generated by Helmholtz coil with different applied currents.

Figure 4. Paths of the electrons with different energies traveling through the magnetic field generated by Helmholtz coil with applied current of 0.744 A. The second opening of the mu-metal cylinder is indicated by a red line.

Figure 4 shows paths of the electrons with different velocities. Here, the current of 0.744 A was applied to the coil. It can be clearly seen that the electrons with velocity between $1.05 \times 10^7$ m/s and $1.40 \times 10^7$ m/s can also pass through the second opening of the mu-metal cylinder. Since thermionic emission from a tungsten cathode had energy spread of 1-3 eV [13], it was assumed that the electron beam from our source has Gaussian distribution with the full width at half maximum (FWHM) of $1 \times 10^6$ m/s. When the slit was taken into account, the distribution of the electron velocity became narrower (see figure 5). The FWHM and the intensity of the output beam for different slit widths are shown in table
1. On the one hand, the slit with small width leads to a lower energy spread of the beam. On the other hands, it strongly reduces the intensity of the beam.

Table 1. The full width at half maximum of the electron velocity distribution and the intensity of the output beam for different slit widths. The intensity is present as a percentage of the input beam intensity

| Slit width [mm] | FWHM [m/s] | Intensity [%] |
|-----------------|------------|---------------|
| 0.50            | $2.8 \times 10^5$ | 37.06         |
| 0.75            | $4.2 \times 10^5$ | 54.15         |
| 1.00            | $5.4 \times 10^5$ | 67.70         |
| 1.50            | $7.4 \times 10^5$ | 85.83         |
| 2.00            | $8.8 \times 10^5$ | 95.16         |
| input beam      | $1.0 \times 10^6$ | 100.00        |

Figure 5. The velocity distribution of the input beam (blue line) and the output beam for slit width of 2.00 mm (red line).

Figure 6. (a) The full width at half maximum of the electron velocity distribution and (b) the intensity of the output beam for a slit with width of 0.50 mm, 0.75 mm, 1.00 mm, 1.50 mm, and 2.00 mm.
In case of broader beams, the beam intensity dropped strongly when it passed through the cylinder as shown in figure 6. Since the number of the electrons which did not fulfill the conditions discussed above were high, the significant part of the electrons were filtered out by the cylinder. The width of the electron velocity distribution significantly decreased. When the slit was applied, only the electrons with the velocities close to the desire value could reach the detector. The velocity distribution became narrower. Despite the different in velocity distribution of the input beams, the distribution of the output beams were similar.

5. Conclusion
It has been shown that our approach helps to reduce the energy spread of the beam. However, this method drastically reduces the beam intensity, especially the input beam with a broader distribution. Our system will be applied in the electron interference experiment which requires an electron beam with the FWHM of the velocity distribution less than $5.5 \times 10^5$ m/s. Therefore, the optimal width of the slit is 1.0 mm. Since the dark count of our microchannel plate detector is only $3 \text{s}^{-1} \cdot \text{cm}^{-2}$, the output intensity of 67.70% of the initial beam will be sufficient for the experiment. Nevertheless, the intensity of the input beam can be finely adjusted during the experiment by regulating the current through the filament.

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