Variation of the velocity and the frequency of a periodic signal along the world line of the emitter

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Abstract
The variation of the velocity of a periodic signal and its frequency along the world line of a standard emitter (at rest with an observer) are considered in a space with affine connections and metrics. It is shown that the frequency of the emitted periodic signal is depending on the kinematic characteristics of the motion of the emitter in space-time related to its shear and expansion velocities. The same conclusions are valid for a standard clock moving with an observer.

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Short title: Variation of velocity and frequency of a standard periodic signal

1 Introduction
1. Modern problems of relativistic astrophysics as well as of relativistic physics (dark matter, dark energy, evolution of the universe, measurement of velocities of moving objects etc.) are related to the propagation of signals in space or in space-time. The basis of experimental data received as results of observations of the Doppler effect or of the Hubble effect gives rise to considerations about the theoretical status of effects related to detection of signals from emitters moving relatively to observers carrying detectors in their laboratories. Nevertheless, in the last decades, there is no essential evolution of the theoretical models related to new descriptions of the Doppler and Hubble effects corresponding to the recent development of new mathematical models for the space-time. In the astronomy and astrophysics standard theoretical schemes for measuring velocities are used related to classical mechanics and / or special and general relativity [1], [2].

2. The incoming periodic signals sent by an emitter moving relatively to an observer (detector) are compared with periodic signals of an emitter lying at rest with the observer. On this basis, the change of frequency and velocity of the incoming periodic signals leads to conclusions about velocities and accelerations of objects moving with respect to the observer.
1.1 Standard periodic emitter

1. A **standard emitter** is an emitter moving together with an observer (detector) in space-time and lying at rest in the proper frame of reference of the observer (detector). The proper frame of reference of the standard emitter could be identified with the proper frame of reference of the observer (detector).

2. In all considerations of effects related to the propagation of periodic signals, it is assumed that an emitter is moving together with the observer, i.e. an emitter is at rest in the proper frame of reference of the observer (detector) and is used as a standard emitter with respect to which the incoming signals from other emitters are compared with respect to their velocity and frequency. On this basis, the variations of the frequency of periodic signals of an emitter with relative velocity and accelerations with respect to an observer (detector) could be established and the relative velocity and acceleration could be measured. It is generally assumed that a standard emitter has constant frequency on its way in space-time because of the fact that it is at rest with respect to the observer and no changes based on Doppler effect could occur. This assumption is based on the standard expression of the Doppler effect in classical mechanics and special relativity, where no acceleration of the proper frame of the observer is taken into account. But if the observer (detector) is moving with acceleration on (nongeodesic in $V_n$-spaces or non-auto parallel) world line in space-time the things could change and we could have results different from the standard assumption for the constant frequency of an emitter moving in space-time together with an observer. The question arises how the proper motion of a standard emitter could influence the velocity and the frequency of its periodic signals.

In this paper the variation of the absolute value of the velocity and the frequency of signals sent by a standard emitter are considered in the proper frame of reference of the emitter. In Section 2 the variation of the velocity of a periodic signal along the world line of a standard emitter is considered. In Section 3 the variation of the frequency of a periodic signal along the world line of a standard emitter is determined. In Section 4 the variation of the velocity and the frequency of periodic signals of a standard oscillator (clock) are found. Some concluding remarks comprise the Section 5. It is shown that the general belief that the frequency and the absolute value of the velocity of periodic signals sent by a standard emitter does not change on the world line of the emitter needs to be revised.

1.2 Methods for determining space distances and relative velocities

1. In the classical (non-quantum) field theories different models of space-time have been used for description of the physical phenomena and their evolution. The 3-dimensional Euclidean space $E_3$ is the physical space used as the space basis of classical mechanics. The 4-dimensional (flat) Minkowskian space $\mathcal{M}_4$ is used as the model of space-time in special relativity. The (pseudo) Riemannian spaces $V_4$ without torsion are considered as models of space-time in general relativity. In theoretical gravitational physics (pseudo) Riemannian spaces without torsion as well as (pseudo) Riemannian spaces $U_4$ with torsion are proposed as space-time grounds for new gravitational theories. To the most sophisticated models of space-time belong the spaces with one affine connection.
and metrics \((L_n, g)\)-spaces] and the spaces with affine connections and metrics \([\mathcal{T}_n, g]\)-spaces] [3, 4].

2. All considerations related to the relative motions of objects with respect to each other are made on the basis of the notions of relative velocity and relative acceleration. The relative velocity is usually considered as radial (centrifugal, centripetal) velocity and tangential (Coriolis) velocity.

There are two different approaches for measuring radial velocities of cosmic objects:

- a covariant approach related to proper times (spectroscopic method) and
- a co-ordinate approach related to time and distances in a co-ordinate system (identified as the proper frame of reference of the Sun) (astrometric method) [1].

The covariant method is usually specialized for a defined co-ordinate system. This leads to difficulties related to the different definitions of spectroscopic and kinematic quantities which are not directly connected to the real measurement of the radial or tangential velocities of astronomical objects. The comparison between the co-ordinate quantities and the spectroscopic data leads to introduction of notions such as "barycentric radial-velocity measure" and "astrometric radial velocity", where the notion of real radial velocity is avoided. The reason for the introduction of the above notions is the lack of covariant method for describing, on the one side, the relative velocities and accelerations and, on the other side, the lack of relations between these kinematic characteristics and their corresponding Doppler shifts and Hubble shifts. Recently, it has been shown that the introduced in \((L_n, g)\)-spaces kinematic characteristics related to relative velocities and relative accelerations could be in simple way expressed in terms of radial (centrifugal, centripetal) and tangential (Coriolis') velocities and accelerations [5]. These velocities and accelerations could be considered as the velocities and accelerations of an emitter from point of view of an observer.

1.3 Periodic signals. Definitions and properties

The notion of periodic signal could be defined from physical and from mathematical point of view.

From physical point of view a periodic signal in a \((\mathcal{T}_n, g)\)-space, considered as a model of space-time, is characterized by:

- A periodic process, characterized by its direction and frequency, transferred by an emitter and received by an observer (detector).

- A periodic process with finite velocity of propagation from point of view of the observer, characterized by its absolute value of the velocity of propagation.

From mathematical point of view a periodic signal in a \((\mathcal{T}_n, g)\)-space, considered as a model of space-time, is characterized by:

- Isotropic (null) contravariant vector field \(\tilde{k} : g(\tilde{k}, \tilde{k}) = 0\), \(\tilde{k} \in T(M)\), \(\dim M = n\), \(\text{sgn } g = n - 2\) or \(\text{sgn } g = -n + 2\), determining the direction of the propagation of a periodic signal in space-time. \(M\) is the differentiable
manifold with dimension \( n \), provided with affine connections and metrics, \( T(M) \) is the tangent space over \( M : T(M) = \cup_{x \in M} T_x(M) \).

- Non-isotropic contravariant vector field \( u : g(u, u) = e = \pm l_u^2 \neq 0 \). The vector field \( u \in T(M) \) is interpreted as the velocity vector field of an observer (detector).

- \( l_u^2 = \pm g(u, u) > 0 \), interpreted as the finite velocity of a periodic signal, determining the absolute value of the velocity of a periodic signal with respect to the proper frame of reference of an observer. The sign before \( g(u, u) \) is depending on the signature of the metric of the space-time.

- Scalar product of \( \tilde{k} \) and \( u : g(\tilde{k}, u) = \omega > 0 \), interpreted as the frequency of a periodic signal, determining the frequency of a periodic signal with respect to the proper frame of reference of an observer (detector).

A frame of reference is determined by the set of three geometric objects [6]:

- A non-null (time-like if \( \dim M = 4 \)) contravariant vector field \( u \in T(M) \).
- A tangent sub space \( T_x^\perp u(M) \) orthogonal to \( u \) at every point \( x \in M \), where \( u \) is defined.
- (Contravariant) affine connection \( \nabla = \Gamma \). It determines the type of transport along the trajectory to which \( u \) is a tangent vector field. \( \Gamma \) is related to the covariant differential operator \( \nabla_u \) along \( u \).

Then the definition of a frame of reference reads

The set \( \text{FR} \sim [u, T^\perp u(M), \nabla = \Gamma, \nabla_u] \) is called frame of reference in a differentiable manifold \( M \) considered as a model of the space or of the space-time.

2 Variation of the velocity of a periodic signal along the world line of a standard emitter

Let us now consider the motion of a standard emitter moving with an observer. First of all, let us recall the definitions for a periodic signal and its theoretical description in \( (L_n, g) \)-spaces. Since all other spaces, considered as mathematical models of a space or of a space-time, are included in these types of spaces, all results would be valid for \( (L_n, g) \)-spaces, \( U_n \), \( V_n \)-spaces (spaces with affine connection and metrics, (pseudo) Riemannian spaces with torsion, (pseudo) Riemannian spaces without torsion) etc.

Let \( \tau \) be the proper time (parameter) of the world line (trajectory) \( x^i(\tau), i = 1, \ldots, n, \dim M = n, n = 4 \) in a space-time of a standard emitter [i.e. of an emitter lying at rest with an observer (detector)]. The tangent vector field along the world line of the standard emitter could be written in the form

\[
\frac{du}{d\tau} = \frac{dx^i}{d\tau} \cdot \partial_i = u^i \cdot \partial_i , \quad u^i = \frac{dx^i}{d\tau} .
\]

(1)

Its absolute length \( l_u \) defined by the expression [3]

\[
\pm l_u^2 = g(u, u) = g_{ij} \cdot u^i \cdot u^j , \quad g_{ij} = f^k \cdot f^l \cdot g_{kl} ,
\]

(2)
where
\[ g = g_{ij} \cdot dx^i \cdot dx^j , \quad dx^i \cdot dx^j = \frac{1}{2} \cdot (dx^i \otimes dx^j + dx^j \otimes dx^i) , \quad (3) \]
\[ f^k_i = f^k_i(x^i) \in C^\infty(M) . \]
could be interpreted as the absolute value of the velocity of the periodic signal, measured in the proper frame of reference of the observer (detector) now identified with the proper frame of reference of the standard emitter.

The space direction of the propagation of a periodic signal is given by the contravariant vector field
\[ k_\perp = g^{ij} \cdot h_i \cdot \tilde{k}_j \cdot \partial_i , \quad (4) \]
where
\[ g(\tilde{k}, \tilde{k}) = 0 , \quad \mathcal{G} = g^{ij} \cdot \partial_i \cdot \partial_j , \]
\[ \partial_i \cdot \partial_j = \frac{1}{2} \cdot (\partial_i \otimes \partial_j + \partial_j \otimes \partial_i) , \]
\[ h_u = g - \frac{1}{2} \cdot g(u, u) \cdot g(u) \otimes g(u) , \]
\[ g(u, u) = \pm l_u^2 = e \neq 0 . \]

The contravariant isotropic (null) vector field \( \tilde{k} \) determines the direction of the periodic signal in the space-time.

The frequency \( \omega \) of the periodic standard signal (the periodic signal sent by a standard emitter) is determined by the relations [7]:
\[ \omega = g(u, \tilde{k}) = l_u \cdot g(\tilde{n}_\perp, k_\perp) , \quad (5) \]
where
\[ k_\perp = \mp l_\perp \cdot \tilde{n}_\perp = \mp \frac{\omega}{l_u} \cdot \tilde{n}_\perp , \quad g(\tilde{n}_\perp, \tilde{n}_\perp) = \mp 1 . \quad (6) \]

Once more, a standard emitter is an emitter staying at rest with respect to an observer (detector). This means that the world lines of emitter and observer are identical in the frame of reference of the observer (emitter). In other words, the emitter and the frequency meter are at rest to each other and are moving together in space-time.

The notion of a clock is closely related to the notion of a periodic signal. A clock is a physical device consisting of an oscillator running at some angular frequency \( \omega \) and a counter that counts the cycles. The period of the oscillator, \( T = 2\pi/\omega \), is calibrated in some standard oscillator. The counter simply counts the cycles of the oscillator. Since some epoch, or the event at which the count started, we say that a quantity of time equal to \( NT \) has elapsed, if \( N \) cycles have been counted” [8].

Let us now consider the variation of the absolute value \( l_u \) of the velocity of a periodic signal of a standard emitter. For this purpose we consider the change of \( l_u^2 \) along the vector field \( u \). Since \( l_u^2 = \pm g(u, u) \), we have the relations
\[ \nabla_u(l_u^2) = u(l_u^2) = 2 \cdot l_u \cdot u(l_u) = 2 \cdot l_u \cdot \frac{dl_u}{d\tau} =\]
\[ = \pm \nabla_u[g(u, u)] = \pm [(\nabla_u g)(u, u) + 2 \cdot g(u, u)] , \quad (7) \]
\[ a = \nabla_u u . \]
where \( a = \nabla_u u \) is the acceleration of the standard emitter along its world line, i.e. \( a \) is the deviation of the world line from the corresponding auto-parallel world line \( \nabla_u u = 0 \). Therefore,

\[
l_u \cdot \frac{dl_u}{d\tau} = \pm [g(u, a) + \frac{1}{2} \cdot (\nabla_u g)(u, u)] .
\]  

(8)

**Special case:** \( U_{n-}, U_{n-}, \nabla_{n-}, \) and \( V_{n-} \)-spaces. \( \nabla_u g = 0 \) for \( \forall u \in T(M) \).

\[
l_u \cdot \frac{dl_u}{d\tau} = \pm g(u, a) = \pm g(u, a_{||}) ,
\]

\[
a_{||} = \frac{1}{e} \cdot g(u, a) \cdot u , \quad e = g(u, u) .
\]

**Special case:** General relativity in \( V_{n-} \)-spaces. In general relativity the absolute value of a light signal is normalized to \( l_u = c = \text{const.} \), or \( l_u = 1 \). Then \( g(u, a) = g(u, a_{||}) = 0 \). The acceleration \( a = a_{||} = \nabla_u g_u(a) \) is orthogonal to the vector field \( u \). It is lying in the sub space orthogonal to \( u \). There are two reasons for the normalization of \( u \): one is from mathematical point of view, and the other is from physical point of view. From mathematical point of view, every non-null (non-isotropic) contravariant vector field \( u \) could be normalized by the use of the absolute value \( l_u \) of its length in the form

\[
\frac{u}{l} = \pm \frac{c_0}{l_u} \cdot u , \quad g(u, u) = \frac{c_0^2}{l_u^2} \cdot g(u, u) = \pm c_0^2 ,
\]

\( c_0 = \text{const.} \neq 0 \).

From physical point of view, it is assumed that a light signal is propagating with constant absolute value \( l_u = \text{const.} \) from point of view of the frame of reference of an observer. This point of view leads to its mathematical realization by means of the normalization of the vector field \( u \).

**Special case:** Auto-parallel motion of a standard emitter. This type of motion is described by the equations

\[
a = f \cdot u , \quad f \in C^r(M) , \quad r \geq 2 ,
\]

\[
a = 0 .
\]

In the case \( a = f \cdot u \), it follows that

\[
\frac{1}{l_u} \cdot \frac{dl_u}{d\tau} = f \pm \frac{1}{2 \cdot l_u^2} \cdot (\nabla_u g)(u, u) .
\]

In the case \( a = 0 \), it follows that

\[
\frac{1}{l_u} \cdot \frac{dl_u}{d\tau} = \pm \frac{1}{2 \cdot l_u^2} \cdot (\nabla_u g)(u, u) .
\]

**Special case:** Geodesic motion of a standard emitter in general relativity in \( V_{n-} \)-spaces. Since \( \nabla_u g = 0 \), the auto-parallel (geodesic) motion described by \( a = f \cdot u \) or \( a = 0 \) leads to the relations

\[
\frac{1}{l_u} \cdot \frac{dl_u}{d\tau} = f ,
\]

\[
\frac{1}{l_u} \cdot \frac{dl_u}{d\tau} = 0 .
\]
If the term $f \cdot u$ is interpreted as a type of friction then in a $V_n$-space we have the relation

$$l_u = l_{u0} \cdot \exp(f \cdot d\tau), \quad l_{u0} = \text{const.}$$

If $f > 0$, $l_u$ will increase with the time. If $f < 0$, $l_u$ will decrease with the time. Therefore, if we use a geodesic equation in its non-canonical form ($a = 0$) then the absolute value of the velocity of a standard periodic signal would change during a time period, where the proper time of the world line of the standard emitter does not appear as the affine parameter of the geodesic trajectory.

In the case $a = 0$, it follows that

$$\frac{1}{l_u} \cdot \frac{dl_u}{d\tau} = 0, \quad l_u \neq 0,$$

$$\frac{dl_u}{d\tau} = 0, \quad l_u = l_{u0} = \text{const.} \neq 0.$$

Therefore, the absolute value $l_u$ of a standard periodic signal is a constant quantity along the geodesic world line of the standard emitter. This means that if a standard emitter is moving on a geodesic trajectory in a (pseudo) Riemannian space without torsion then the absolute value $l_u$ of its periodic signals will be a constant quantity along this geodesic world line, i.e. $l_u = l_{u0} = \text{const.} \neq 0$.

Special case: Weyl’s spaces with torsion ($Y_n$-spaces, Weyl-Cartan spaces). For this type of spaces the condition

$$\nabla_u g = \frac{1}{n} \cdot Q_u \cdot g, \quad \text{dim} \ M = n,$$

is fulfilled. Then

$$\frac{1}{l_u} \cdot \frac{dl_u}{d\tau} = \frac{1}{2 \cdot n} \cdot Q_u \pm \frac{1}{l^2_u} \cdot g(u, a).$$

Special case: Auto-parallel motion of a standard emitter in Weyl’s spaces with torsion.

$$\nabla_u g = \frac{1}{n} \cdot Q_u \cdot g, \quad a = 0,$$

$$l_u = \left(\frac{l^2_u}{u} + \frac{1}{n} \cdot \int Q_u \cdot d\tau\right)^{1/2}, \quad l^2_{u0} = \text{const.} > 0.$$

Therefore, if a standard emitter is moving in a Weyl’s space with torsion on an auto-parallel world line then the absolute value of the velocity of its periodic signals would change with the time under the existence of the quantity $Q_u$ as a characteristic of this type of spaces. If, further, $Q_u = -n \cdot d\varphi/d\tau$, $\varphi(x^i(\tau)) = \varphi(\tau)$ is a dilaton field in a $Y_n$-space, then

$$l_u = \left[l^2_u - \varphi(\tau)^{1/2}\right], \quad l^2_{u0} = \text{const.} > 0.$$

The dilaton field $\varphi$ influences the absolute value of the velocity of a periodic signal emitted by a standard emitter. This fact could be used as a check-up of the existence of a dilaton field in a Weyl-Cartan space considered as a model of space-time.
In the further considerations the question could arise under which conditions a standard emitter could send periodic signals with constant absolute value of its velocity along its world line, measured in the proper frame of reference of the emitter.

If we consider the conditions under which the absolute value $l_u$ of the velocity of a periodic signal appears as a conserved quantity in a $(\mathcal{T}_n, g)$-space, we can prove the following propositions:

**Proposition 1.** The necessary and sufficient condition for the absolute value $l_u$ of the velocity of a periodic signal to be a conserved quantity along the world line of a standard emitter with tangent vector field $u$ is the condition

$$g(u, a) = -\frac{1}{2} \cdot (\nabla_u g)(u, u),$$

equivalent to the condition

$$(\mathcal{L}_u g)(u, u) = 0.$$  \hspace{1cm} (9)

The proof is trivial. One should only take into account the relations

$$l_u \frac{dl_u}{d\tau} = \pm [g(u, a) + \frac{1}{2} \cdot (\nabla_u g)(u, u)], \quad l_u \neq 0,$$

$$\mathcal{L}_u [g(u, u)] = u[g(u, u)] = \nabla_u [g(u, u)] = (\mathcal{L}_u g)(u, u).$$

**Special case:** $V_n$-spaces as models of space-time in general relativity theory. Since $\nabla_u g = 0$, we have as necessary and sufficient conditions the conditions

$$g(u, a) = 0 \triangleq (\mathcal{L}_u g)(u, u) = 0.$$  \hspace{1cm} (10)

**Proposition 2.** A sufficient condition for the absolute value $l_u \neq 0$ of the velocity of a periodic signal of an emitter to be a constant quantity ($l_u = \text{const.} \neq 0$) along the world line of the emitter with tangent vector field $u$ is the condition

$$\mathcal{L}_u g = 0,$$  \hspace{1cm} (11)

i.e. if the vector field $u$ is tangent vector field to the world line of a standard emitter then the absolute value $l_u$ of the velocity of its periodic signals is a constant quantity ($l_u = \text{const.} \neq 0$) along the world line of the emitter when $u$ is a Killing vector field.

The proof is trivial.

**Special case:** $V_n$-spaces as models of space-time in general relativity theory.

(a) A sufficient condition for $l_u = \text{const.} \neq 0$ is the condition for the world line of the standard emitter to be a geodesic trajectory in a given $V_n$-space, i.e. $\nabla_u u = a = 0$.

(b) A sufficient condition for $l_u = \text{const.} \neq 0$ is the vector field $u$ as tangent vector field to the world line of the standard emitter to be a Killing vector field, i.e. $\mathcal{L}_u g = 0$.

### 3 Variation of the frequency of a periodic signal along the world line of an emitter

If an emitter is used as a standard emitter by an observer on his world line for comparison of the incoming periodic signals from another emitters then the
variation of the frequency of the standard emitter is of great importance for the correct determination of the frequency of the incoming signals.

From the explicit form of the frequency \( \omega \)
\[
\omega = l_u \cdot g(\bar{n}_\perp, k_\perp) ,
\] (12)
where
\[
k_\perp = \pm \frac{\omega}{l_u} \cdot \bar{n}_\perp , \quad g(\bar{n}_\perp, u) = 0 ,
\] (13)
it follows that
\[
\nabla u \omega = u \omega = \frac{d\omega}{d\tau} = \nabla_u [l_u \cdot g(\bar{n}_\perp, k_\perp)] =
= ul_u \cdot g(\bar{n}_\perp, k_\perp) +
+ l_u \cdot [(\nabla_u g)(\bar{n}_\perp, k_\perp) + g(\nabla_u \bar{n}_\perp, k_\perp) + g(\bar{n}_\perp, \nabla_u k_\perp)] .
\] (14)

Remark. If we use the relations
\[
l_u = \lambda \cdot \nu = \lambda \cdot \frac{\omega}{2 \cdot \pi} ,
\] (15)
\[
\omega = 2 \cdot \pi \cdot \frac{l_u}{\lambda} = l_u \cdot g(\bar{n}_\perp, k_\perp) ,
\] (16)
we can find the expression for the corresponding to \( \omega \) length \( \lambda \) of the periodic signal in the form
\[
\lambda = \frac{2 \cdot \pi}{g(\bar{n}_\perp, k_\perp)} ,
\] (17)
and therefore,
\[
g(\bar{n}_\perp, k_\perp) = 2 \cdot \frac{\pi}{\lambda} .
\] (18)

The projection of \( k_\perp \) on its unit vector \( \bar{n}_\perp \) has exact relation to the length \( \lambda \) of the periodic signal with frequency \( \omega \).

After straightforward computations we obtain the following relation
\[
h_u(\mathcal{L}_u \bar{n}_\perp, \bar{n}_\perp) + d(\bar{n}_\perp, \bar{n}_\perp) + \frac{1}{2} \cdot (\nabla_u g)(\bar{n}_\perp, \bar{n}_\perp) = 0 ,
\] (19)
where \( d \) is the deformation velocity tensor
\[
d = \sigma + \omega + \frac{1}{n - 1} \cdot \theta \cdot h_u .
\] (20)

The trace free covariant symmetric tensor \( \sigma \) is the shear velocity tensor. The antisymmetric covariant tensor \( \omega \) is the rotation velocity tensor. The invariant \( \theta \) is the expansion velocity invariant [9, 3].

The vector \( k_\perp \) is directed in the line of sight, and the vector field \( u \) is orthogonal to it. The vector field \( u \) is a tangent vector field to the world line of the standard emitter. If we wish to consider the world line of the emitter and the line of sight as co-ordinates lines along which we consider the propagation of the periodic signals (the absolute value of their velocity and direction) then we should admit the validity of the relations (the second condition is fulfilled by the construction of \( k_\perp \))
\[
\mathcal{L}_u k_\perp = 0 , \quad g(u, k_\perp) = 0 .
\] (21)
Remark. The above relations are the necessary and sufficient conditions for the existence of co-ordinates curves to which $u$ and $k_\perp$ appear as tangent vectors at every point of the corresponding curve \[10\].

By the use of the expressions

$$
\mathcal{L}_u k_\perp = \mathcal{L}_u (\mp \frac{\omega}{l_u} \cdot \vec{n}_\perp) = \\
= \mp \left[ \frac{d}{dt} \left( \frac{\omega}{l_u} \right) \right] \cdot \vec{n}_\perp \mp \frac{\omega}{l_u} \cdot \mathcal{L}_u \vec{n}_\perp = 0 , \\
\mathcal{L}_u \vec{n}_\perp = -\left[ \frac{d}{dt} \left( \log \frac{\omega}{l_u} \right) \right] \cdot \vec{n}_\perp , \quad u = \frac{d}{dt} , \\
h_u(\mathcal{L}_u \vec{n}_\perp, \vec{n}_\perp) + d(\vec{n}_\perp, \vec{n}_\perp) + \frac{1}{2} \cdot (\nabla_u g)(\vec{n}_\perp, \vec{n}_\perp) = 0 ,
$$

after substitution of the explicit form for $\mathcal{L}_u \vec{n}_\perp$ in the last above expression, the relation follows

$$
-\left[ \frac{d}{dt} \left( \log \frac{\omega}{l_u} \right) \right] \cdot h_u(\vec{n}_\perp, \vec{n}_\perp) + d(\vec{n}_\perp, \vec{n}_\perp) + \frac{1}{2} \cdot (\nabla_u g)(\vec{n}_\perp, \vec{n}_\perp) = 0 . \quad (22)
$$

3.1 Explicit form of the equation for $\omega$

Since

$$
h_u(\vec{n}_\perp, \vec{n}_\perp) = g(\vec{n}_\perp, \vec{n}_\perp) = \mp 1 \quad , \quad (23)
$$

we have

$$
\frac{d}{dt} \left( \log \frac{\omega}{l_u} \right) = \mp \left[ d(\vec{n}_\perp, \vec{n}_\perp) + \frac{1}{2} \cdot (\nabla_u g)(\vec{n}_\perp, \vec{n}_\perp) \right] .
$$

On the other side,

$$
d(\vec{n}_\perp, \vec{n}_\perp) = (\sigma + \omega + \frac{1}{n-1} \cdot \theta) h_u(\vec{n}_\perp, \vec{n}_\perp) = \\
= \mp \left[ \frac{1}{n-1} \cdot \theta \mp \sigma(\vec{n}_\perp, \vec{n}_\perp) \right] ,
$$

$$
\frac{1}{n-1} \cdot \theta \mp \sigma(\vec{n}_\perp, \vec{n}_\perp) = H , \quad d(\vec{n}_\perp, \vec{n}_\perp) = \mp H \quad . \quad (25)
$$

The function $H = H(\tau)$ is the s.c. Hubble function \[5\], \[7\]. Therefore, we can write now the expression

$$
\frac{d}{dt} \left( \log \frac{\omega}{l_u} \right) = H \mp \frac{1}{2} \cdot (\nabla_u g)(\vec{n}_\perp, \vec{n}_\perp) . \quad (26)
$$

If we introduce the abbreviation

$$
\nabla(\vec{n}_\perp, \vec{n}_\perp) = \mp \left[ d + \frac{1}{2} \cdot (\nabla_u g)(\vec{n}_\perp, \vec{n}_\perp) \right] = \\
= H \mp \frac{1}{2} \cdot (\nabla_u g)(\vec{n}_\perp, \vec{n}_\perp) , \quad (27)
$$

$$
\nabla = \mp \left[ d + \frac{1}{2} \cdot (\nabla_u g) \right] , \quad (29)
$$
we obtain the equation for $\omega$
\[
\frac{d}{d\tau} (\log \frac{\omega}{l_u}) = \nabla(\vec{n}_l, \vec{n}_\perp) .
\] (30)

Its solution follows in the form
\[
\frac{\omega}{l_u} = \frac{\omega_0}{l_{u0}} \cdot \exp\left( \int \left[ H \mp \frac{1}{2} \cdot (\nabla u g)(\vec{n}_l, \vec{n}_\perp) \right] d\tau \right) ,
\]
\[
\omega_0 = \text{const.}, \quad l_{u0} = \text{const.}
\]

Since
\[
\omega = 2 \cdot \pi \cdot \frac{l_u}{\lambda} , \quad \frac{\omega}{l_u} = \frac{2 \cdot \pi}{\lambda} , \quad l_u = \frac{\omega \cdot \lambda}{2 \cdot \pi} ,
\] (31)
\[
\frac{\omega_0}{l_{u0}} = \frac{2 \cdot \pi}{\lambda_0} , \quad \lambda_0 = \text{const.}
\]

we have
\[
\lambda = \lambda_0 \cdot \exp\left\{ - \int \left[ H \mp \frac{1}{2} \cdot (\nabla u g)(\vec{n}_l, \vec{n}_\perp) \right] d\tau \right\} .
\] (33)

Therefore, the length $\lambda$ of the standard periodic signal will decrease in the proper frame of the standard emitter (observer) if
\[
\nabla(\vec{n}_l, \vec{n}_\perp) = H \mp \frac{1}{2} \cdot (\nabla u g)(\vec{n}_l, \vec{n}_\perp) > 0 , \quad \lambda < \lambda_0 ,
\] (34)

and $\lambda$ will increase when
\[
\nabla(\vec{n}_l, \vec{n}_\perp) = H \mp \frac{1}{2} \cdot (\nabla u g)(\vec{n}_l, \vec{n}_\perp) < 0 , \quad \lambda > \lambda_0 .
\] (35)

If
\[
\nabla(\vec{n}_l, \vec{n}_\perp) = H \mp \frac{1}{2} \cdot (\nabla u g)(\vec{n}_l, \vec{n}_\perp) = 0 ,
\] (36)

then
\[
\lambda = \lambda_0 = \text{const.}
\]

3.2 Relation between the variation of the frequency and the variation of the absolute value of a standard periodic signal

From the equation for $l_u$
\[
\frac{1}{2} \cdot \frac{d^2 l_u}{d\tau^2} = \pm [g(u, a) + \frac{1}{2} \cdot (\nabla u g)(u, u)]
\] (37)

we can find the expression for $l_u$
\[
\frac{d^2 l_u}{d\tau^2} = \pm 2 \cdot [g(u, a) + \frac{1}{2} \cdot (\nabla u g)(u, u)] = \pm 2 \cdot \tilde{V} ,
\]
\[
\tilde{V} = g(u, a) + \frac{1}{2} \cdot (\nabla u g)(u, u) .
\]
Then
\[ l_u = (l_{u0}^2 + 2 \cdot \int \tilde{V} \cdot d\tau)^{1/2} \quad , \quad l_{u0}^2 = \text{const.} > 0 \quad . \quad (38) \]

If we now substitute the expression for \( l_u \) in the expression for \( \omega \) we can find
the general relation for the variation of the frequency \( \omega \) under the variation of
the absolute value \( l_u \) of a standard periodic signal with the proper time of the
emitter
\[
\omega = \omega_0 \cdot (1 \pm \frac{2}{l_{u0}^2} \cdot \int [g(u, a) + \frac{1}{2} \cdot (\nabla_u g)(u, u)] \cdot d\tau)^{1/2} .
\]
\[ \cdot \exp\left(\int [H \mp \frac{1}{2} \cdot (\nabla_u g)(\tilde{n}_\perp, \tilde{n}_\perp)] \cdot d\tau\right) . \quad (39) \]

As a final result, for the variations of the absolute value \( l_u \), the frequency \( \omega \), and the length \( \lambda \) of a standard periodic signal with the proper time of a
standard emitter we obtain the relations
\[
l_u = (l_{u0}^2 + 2 \cdot \int [g(u, a) + \frac{1}{2} \cdot (\nabla_u g)(u, u)] \cdot d\tau)^{1/2} \quad , \quad (A) \]
\[ l_{u0}^2 = \text{const.} > 0 \quad , \]
\[
\omega = \omega_0 \cdot (1 \pm \frac{2}{l_{u0}^2} \cdot \int [g(u, a) + \frac{1}{2} \cdot (\nabla_u g)(u, u)] \cdot d\tau)^{1/2} .
\]
\[ \cdot \exp\left(\int [H \mp \frac{1}{2} \cdot (\nabla_u g)(\tilde{n}_\perp, \tilde{n}_\perp)] \cdot d\tau\right) , \quad (B) \]
\[
\lambda = \lambda_0 \cdot \exp\{-\int [H \mp \frac{1}{2} \cdot (\nabla_u g)(\tilde{n}_\perp, \tilde{n}_\perp)] \cdot d\tau\} , \quad (C) \]
\[ \lambda_0 = \text{const.} \]

Special case: \( U_n^- \), \( V_n^- \), \( U_n^- \), and \( V_n^- \)-spaces. For these types of spaces \( \nabla_u g = 0 \), \( \nabla = \mp d \), \( \nabla(\tilde{n}_\perp, \tilde{n}_\perp) = H \), \( \tilde{V} = g(u, a) \).
\[
l_u = (l_{u0}^2 + 2 \cdot \int g(u, a) \cdot d\tau)^{1/2} \quad , \quad l_{u0}^2 = \text{const.} > 0 \quad , \]
\[
\omega = \omega_0 \cdot (1 \pm \frac{2}{l_{u0}^2} \cdot \int g(u, a) \cdot d\tau)^{1/2} \cdot \exp(\int H \cdot d\tau) .
\]
\[ \lambda = \lambda_0 \cdot \exp(-\int H \cdot d\tau) , \quad \lambda_0 = \text{const.} \]

Special case: \( U_n^0 \), \( V_n^0 \), \( U_n^0 \), and \( V_n^0 \)-spaces. \( \nabla_u g = 0 \), \( \nabla = \mp d \), \( \nabla(\tilde{n}_\perp, \tilde{n}_\perp) = H = H_0 = \text{const.} \), \( \tilde{V} = g(u, a) \).
\[
\mp d(\tilde{n}_\perp, \tilde{n}_\perp) = H_0 = \text{const.} . \quad , \]
\[
\mp \left[\frac{1}{n-1} \cdot \theta \mp \sigma(\tilde{n}_\perp, \tilde{n}_\perp)\right] = H_0 \quad ,
\]
\[ \sigma(\tilde{n}_\perp, \tilde{n}_\perp) = \pm \frac{1}{n-1} \cdot \theta + H_0 \ , \]
\[ l_u = (l_{u0}^2 \pm 2 \cdot \int g(u, a) \cdot d\tau)^{1/2} \ , \ l_{u0}^2 = \text{const.} > 0 \ , \]
\[ \omega = \omega_0 \cdot (1 \pm \frac{2}{l_{u0}^2} \cdot \int g(u, a) \cdot d\tau)^{1/2} \cdot \exp(H_0 \cdot \tau) \ , \]
\[ \omega_0 = \text{const.} , \]
\[ \lambda = \lambda_0 \cdot \exp(-H_0 \cdot \tau) \ , \ \lambda_0 = \text{const.} \]
Special case: \( U_n^--, V_n^-, U_n^-, \) and \( V_n^- \)-spaces. \( \nabla_u g = 0, \nabla = \mp d, \nabla(\tilde{n}_\perp, \tilde{n}_\perp) = H = H_0 = \text{const.}, \nabla = g(u, a) = 0. \)
\[ l_u = l_{u0} \ , \ l_{u0} = \text{const.} > 0 \ , \]
\[ \omega = \omega_0 \cdot \exp(H_0 \cdot \tau) , \ H_0 = \text{const.} , \]
\[ \lambda = \lambda_0 \cdot \exp(-H_0 \cdot \tau) , \ \lambda_0 = \text{const.} \]

Therefore, if \( g(u, a) = 0 \) and \( H = H_0 \) the absolute value \( l_{u0} \) of the velocity of a periodic signal of a standard emitter is a constant quantity along the world line of the emitter. The frequency \( \omega \) as well as the length \( \lambda \) of the signal are depending exponentially on the time parameter \( \tau \). In general relativity in \( V_n^- \)-spaces \((n = 4)\), a standard periodic light signal of a standard emitter will have a constant value \( l_{u0} \) of its velocity if the acceleration \( a \) is orthogonal to the vector field \( u \) or if the world line of the standard emitter is a geodesic trajectory \((a = 0)\) in space-time.

Special case: Shear free and expansion free \((\overline{L}_n, g)\)-spaces. \( \sigma = 0, \theta = 0 \). For shear free and expansion free \((\overline{L}_n, g)\)-spaces \( H = 0 \) and we have the relations
\[ l_u = (l_{u0}^2 \pm 2 \cdot \int g(u, a) + \frac{1}{2} \cdot (\nabla_u g)(u, u) \cdot d\tau)^{1/2} \ , \]
\[ l_{u0}^2 = \text{const.} > 0 \ , \]
\[ \omega = \omega_0 \cdot (1 \pm \frac{2}{l_{u0}^2} \cdot \int g(u, a) + \frac{1}{2} \cdot (\nabla_u g)(u, u) \cdot d\tau)^{1/2} \cdot \exp(\mp \frac{1}{2} \cdot \int (\nabla_u g)(\tilde{n}_\perp, \tilde{n}_\perp) \cdot d\tau) \ , \]
\[ \lambda = \lambda_0 \cdot \exp(\pm \frac{1}{2} \cdot \int (\nabla_u g)(\tilde{n}_\perp, \tilde{n}_\perp) \cdot d\tau) \ . \]

4 Variation of the velocity and the frequency of periodic signals of a standard oscillator (clock)

Let us now consider the change of the frequency of periodic signals of a standard oscillator used as a standard clock in the frame of reference of an observer. The period of the oscillator, \( T = 2 \cdot \pi / \omega \), is calibrated in some standard oscillator. 13
If the frequency $\omega$ is changing along the world line of the oscillator then the period $T$ is also changing in the corresponding form

$$T = \frac{2 \cdot \pi}{\omega} = T_0 \cdot (1 \pm \frac{2}{\omega l_u^2} \cdot \int [g(u, a) + \frac{1}{2} \cdot (\nabla_u g)(u, u)] \cdot d\tau)^{-1/2},$$

$$\cdot \exp(- \int [H \mp \frac{1}{2} \cdot (\nabla_u g)(\vec{n}_\perp, \vec{n}_\perp)] \cdot d\tau) \quad ,$$

$$T_0 = \frac{2 \cdot \pi}{\omega_0} = \text{const.} \quad (40)$$

Therefore, a standard oscillator (clock) would not change its frequency and period if and only if the following conditions are fulfilled

$$g(u, a) + \frac{1}{2} \cdot (\nabla_u g)(u, u) = 0 \quad , \quad (41)$$

$$H \mp \frac{1}{2} \cdot (\nabla_u g)(\vec{n}_\perp, \vec{n}_\perp) = 0 \quad . \quad (42)$$

These conditions are in generally not fulfilled even in the Einstein theory of gravitation, where the above relations should also be valid in their special forms

$$g(u, a) = 0 \quad , \quad (43)$$

$$H = 0 \quad . \quad (44)$$

In all cosmological models in general relativity with Hubble function $H = H(\tau)$ different from zero and $g(u, a) = 0$ a standard clock will move in space-time with a period $T$ obeying the condition

$$T = \frac{2 \cdot \pi}{\omega} = T_0 \cdot \exp(- \int H \cdot d\tau) \quad . \quad (45)$$

Furthermore, the condition $g(u, a) = 0$ is considered as a corollary of the assumption of the constant value $l_u = c = \text{const.}$ or $l_u = 1$ of the speed of light. There is no unique physical argument for the last assumption if a theory of gravitation has to describe the behavior of physical systems including the speed of propagation of their interactions on the basis of its own structures.

## 5 Conclusions

In the present paper the variation of the absolute value and the frequency of a periodic signal sent by a standard emitter is considered. The obtained results contradict with the general belief that a standard emitter does not change the frequency of its periodic signals on its world line considered also as the world line of an observer (detector) moving together with the standard emitter. The periodic signals of a standard emitter could have constant period and frequency only under very specific conditions. They should be fulfilled if an exact comparison with incoming signals is required. In all other cases there is a difficult task for an observer to make conclusions about the real variations of the frequency of an incoming periodic signal in his proper frame of reference without knowing the exact kinematic characteristics of the tangent vector field of his world line. If the kinematic characteristics related to the notion of relative velocity of an
observer (shear, rotation, and expansion velocities) are given (or found by the use of a theoretical scheme) then the variation of the frequency of a periodic signal, incoming to the observer, could be easily determined. The possible solutions of this problem are worth to be investigated in an other paper. The same conclusions are valid if we try to compare the periods of different clocks on the basis of a standard clock (oscillator, emitter) moving with an observer.

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