Discrete-time Sliding Mode Control for MR Vehicle Suspension System

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Abstract. This paper presents control performance of a full-vehicle suspension system featuring magnetorheological (MR) dampers via a discrete-time sliding mode control algorithm (DSMC). A cylindrical MR damper is designed by incorporating Bingham model of the MR fluid and the field-dependent damping characteristics of the MR damper are evaluated. A full-vehicle suspension model installed with independent four MR dampers is constructed and the governing equations which include vertical, pitch and roll motion are derived. A discrete-time control model is established with considering system uncertainties and a discrete-time sliding mode controller which has inherent robustness to model uncertainty and external disturbance is formulated. Vibration control performances under bump excitation are evaluated and presented.

1. Introduction
In general, vehicle suspension system provides improved passenger comfort as well as improved handling with the effective isolation from road disturbance. Recently, research works on vibration control of the semi-active suspension system using electrorheological (ER) and magnetorheological (MR) fluids have been significantly increased. The semi-active suspension system can offer a desirable performance generally enhanced in the active mode without requiring large power consumption and expensive hardware compared to passive and active suspension. Petek [1] proposed a electrically controlled semi-active shock absorber using ER fluid and demonstrated the control performance. Carlson et al. [2] proposed a commercially available MR damper which is applicable to on-and-off-highway vehicle suspension system. Most of analytical and experimental researches for the ER and MR suspension system have limited their scope to a quarter car model. Moreover, research work focusing on the reliability of the suspension control system, which is easily subjected to parameter uncertainties and external disturbance in practice, is considerably rare. Choi et al. [3] evaluated control performance of ER full-vehicle suspension system by incorporating continuous-time sliding mode controller (CSMC) into hardware-in-the-loop simulation. In general, the CSMC has been designed based on the continuous-time control model. Thus its implementation by a digital computer requires certain sampling process. However, practical implementation of the CSMC is sometimes very difficult, because the sampling may bring not only degradation of control performance but also instability of the control system with an unnecessary large gain. This leads to the discrete-time sliding
Consequently, the main contribution of this study is to evaluate control performance of the discrete-time sliding mode controller (DSMC) for full-vehicle MR suspension system. A cylindrical MR damper is designed based on Bingham characteristics of the MR fluid and the field-dependent damping force is evaluated. A mechanical model of the full-vehicle MR suspension is constructed and equations of motion of the vehicle system are derived by considering vertical, pitch and roll motion. After establishing the discrete-time control model with model uncertainty in the state-space form, a DSMC is designed and implemented to the system. Vibration control performance is evaluated under bump excitation.

2. Modelling of full-vehicle suspension system

The schematic configuration of the proposed MR damper is shown in figure 1. The MR damper consists of cylinder, piston, gas chamber and floating piston which can compensate the volume of rod caused by the movement of piston. It is assumed that the MR fluid is incompressible and that pressure in one chamber is uniformly distributed. Thus, the damping force of the MR damper can be expressed as follows [4];

$$F_{MR} = k_x p_x + c_x \dot{p}_x + (A_p - A_i) (2L/h) \alpha H \text{sgn}(\dot{p}_x), H = NI / 2g$$  \hspace{1cm} (1)

where $p_x$ and $\dot{p}_x$ are the piston displacement and velocity, respectively, $\text{sgn}(\cdot)$ is signum function, $k_x$ is the effective stiffness due to gas pressure, $c_x$ is the effective damping due to the fluid viscosity, $A_p$ and $A_i$ represent the piston head and piston rod areas, respectively. $L$ is the length of magnetic pole, $h$ is gap between the magnetic poles, $H$ is applied magnetic field, $N$ is number of coil turns, $g$ is the gap between magnetic poles and $I$ is input current. Here, $\alpha$ and $\beta$ are intrinsic value of the MR fluid to be experimentally determined. In this study, for the MR fluid, the commercial product (MRF132-LD) from Lord Corporation is used and the experimental constants $\alpha$ and $\beta$ are identified as 0.083 and 1.25, respectively. Figure 2 presents damping force with respect to the piston velocity at various magnetic fields. It is clearly observed that the damping force is increased as the magnetic field increases. As a specific case, the damping force of 203.1 N at piston velocity of 0.0628 m/s is increased up to 1165.5 N by applying the input current 2.0 A.

Figure 3 presents a mechanical model of the full-vehicle MR suspension system equipped with four MR dampers. The vehicle body itself is assumed to be rigid and has degrees of freedom in the vertical pitch and roll direction. The equations of motion for the full-vehicle model are derived as follows [3];

$$M_{\dot{z}_x} = -f_{11} - f_{12} - f_{13} - f_{14} + F_{MR1} + F_{MR2} + F_{MR3} + F_{MR4}$$

$$J_{\dot{\phi}_1} = a f_{12} + a f_{13} - b f_{14} - a F_{MR1} - a F_{MR2} + b F_{MR3} + b F_{MR4}$$

$$J_{\dot{\phi}_2} = -c f_{12} + d f_{13} - c F_{MR1} + d F_{MR2} - c F_{MR3} + d F_{MR4}$$

$$m_{z_{a1}} \ddot{z}_{a1} = f_{11} - f_{12} - F_{MR1}, m_{z_{a2}} \ddot{z}_{a2} = f_{12} - f_{13} - F_{MR2},$$

$$m_{z_{a3}} \ddot{z}_{a3} = f_{13} - f_{14} - F_{MR3}, m_{z_{a4}} \ddot{z}_{a4} = f_{14} - f_{14} - F_{MR4}$$  \hspace{1cm} (2)

Figure 3 presents a mechanical model of the full-vehicle MR suspension system equipped with four MR dampers. The vehicle body itself is assumed to be rigid and has degrees of freedom in the vertical pitch and roll direction. The equations of motion for the full-vehicle model are derived as follows [3];
Here $f_m = k_s (z_m - z_u) + c_s (\dot{z}_m - \dot{z}_u)$ and $f_u = k_s (z_u - z_m)$ for $i = 1, 2, 3, 4$. $M$ is the sprung mass and $m_i$, $i = 1, 2, 3, 4$ is the unsprung mass. $J_\theta$ is the pitch mass moment of inertia and $J_\phi$ is the roll mass moment of inertia. $k_s$ ($i = 1, 2, 3, 4$) is the total stiffness coefficient of the suspension spring and $c_s$ ($i = 1, 2, 3, 4$) is the stiffness coefficient of the suspension and it is equal to $c_s$. $k_s$ ($i = 1, 2, 3, 4$) is the stiffness coefficient of tire. Here, $z_m$, $z_u$, and $z_i$ ($i = 1, 2, 3, 4$) are the vertical displacement of sprung mass, unsprung mass, and excitation, respectively. $g_z$, $\theta$ and $\phi$ are the vertical displacement, pitch angular displacement and roll angular displacement. In equation (2), $a$, $b$, $c$ and $d$ are the distance between the front damper and center of gravity (C.G.) of the sprung mass, the distance between the rear and C.G., the distance between the left and C.G. and the distance between the right and C.G., respectively. By considering system uncertainties such as number of riding person and payload, the state-space control model can be obtained as follows:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + Dw(t) = (A_0 + \Delta A(t))x(t) + (B_0 + \Delta B(t))u(t) + Dw(t)$$

(3)

where $x$, $u$ and $w$ are state vector, control input vector and disturbance vector, respectively. $A_0$, $\Delta A$, $B_0$, $\Delta B$ and $D$ are the nominal part of the system matrix, uncertain part of the system matrix, nominal part of the control input matrix, uncertain part of the control input matrix and the disturbance matrix, respectively. It is assumed that all elements of the system and input matrices are piecewise smooth functions. By using the zero-order-hold (ZOH) method, the discrete-time control model can be expressed as follows:

$$\dot{x}(k+1) = (\Phi_0 + \Delta \Phi)x(k) + (\Gamma_0 + \Delta \Gamma)u(k) + Dw(k)$$

(4)

where $\Phi_0$ and $\Gamma_0$ are the nominal system and nominal input matrix. $\Delta \Phi$ and $\Delta \Gamma$ are corresponding uncertain parts.

### 3. Controller design and performance evaluation

For the system in equation (4), a sliding surface in the state space is defined as follows:

$$s(k) = Cx(t)$$

(5)

where $C$ is the surface gradient vector which should be determined so that the sliding surface itself is stable. For the discrete-time system, the controller should be chosen to satisfy the sliding condition and convergence condition as follows:

$$\left[ s(k+1) - s(k) \text{sgn}(s(k)) \right] < 0, \left[ s(k+1) + s(k) \text{sgn}(s(k)) \right] > 0$$

(6)

For the attenuation of vibration chattering in a settled phase and also the enhancement of robustness to the system, equivalent separation method is adopted in this work [5]. At the outside of the sliding region, the equivalent controller and discontinuous controller deliver the state trajectory to the sliding region. At the inside of the sliding region, the $\beta$-equivalent controller only delivers the state to the
origin point through the sliding surface. Equivalent controller, $\beta$-controller and discontinuous controller are determined as follows:

$$u_{eq} = - (C \Phi_0)^{-1} (C \Phi_0 - C)x(t)$$

$$u_{eq,d} = - (C \Gamma_0)^{-1} (C \Phi_0 - \beta C)x(t), \quad 0 \leq \beta < 1$$

$$u_d = - h(k) \text{sgn}(C \Phi_0 s(k)) \sum_{i=1}^{n} |x_i(k)|$$

where $h(k) = 0$ for the inside of the sliding region and $h_i(k) < h(k) < h_i(k)$ for the outside of the sliding region.

Control performance of the proposed full-vehicle MR suspension system with DSMC is evaluated. Time response of the MR suspension system for the bump excitation with constant velocity of 3.08km/h, which can make first resonant frequency of vehicle suspension system, is presented in figure 4. It is clearly observed that the vertical displacement of sprung mass, pitch angular displacement and tire deflection are reduced by applying the control input determined by the DSMC. This directly implies that ride comport as well as steering stability of a vehicle can be improved by employing the proposed DSMC associated with the semi-active MR suspension system.

4. Conclusion
A DSMC was formulated for vibration control of a full-vehicle MR suspension system. A cylindrical MR damper was designed based on Bingham characteristics of MR fluid and the field-dependent damping force was investigated. After constructing of mechanical model of the full-vehicle MR suspension system, equations of motion of the vehicle system were derived. By considering the system uncertainties, a discrete-time control model of the vehicle system was constructed in the state space. A DSMC was then designed and implemented to the control system. Vibration control performance of the DSMC was evaluated under bump excitation. It has been clearly demonstrated that vibration of the vehicle can be effectively suppressed by applying the proposed control methodology. It is finally remarked that the road test of the proposed control system for MR suspension system will be carried out in the near future.

Acknowledgments
This work was supported by Acceleration Research (Center for ER Fluid Technology and Application) of MOST/KOSEF. This financial support is gratefully acknowledged.

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