NEUTRINOS, OSCILLATIONS, AND BIG BANG NUCLEOSYNTHESIS

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Abstract
The role of neutrinos in big bang nucleosynthesis is reviewed. Neutrino oscillations in the early universe both in resonance and non-resonance case are briefly discussed. BBN and supernova limits on heavy sterile neutrinos with the mass in 10-100 MeV range are presented and compared with direct experimental bounds.
1 Basics of BBN

Big Bang Nucleosynthesis is known as one of the three solid pillars of the Standard Cosmological Model (SCM); the other two are the Cosmic Microwave Background Radiation (CMBR) and General Relativity (GR). BBN describes creation of light elements in the early hot universe in a very (or rather, depending upon the attitude) good agreement with astronomical observations. The theory predicts mass fraction of $^4\text{He}$ (with respect to total baryonic matter) about 25%, the relative number densities of deuterium, $^2\text{H}$, and $^3\text{He}$ at the level a few $\times 10^{-5}$ each, and a few $\times 10^{-10}$ of $^7\text{Li}$. The predictions span 9 orders of magnitude and well fit the data.

The characteristic temperature and time scales for the processes of light element formations are respectively $T = (\sim \text{MeV}) - 70 \text{ keV}$ and $t = 0.1 - 10^3 \text{ sec}$. The relation between the time and temperature (the cooling rate) at this stage is given by

$$ (t/\text{sec})(T/\text{MeV})^2 = 0.74 \left(\frac{10.75}{g_*}\right)^{1/2} $$

(1)

Where $g_*$ describes the particle content of the primeval plasma. In the SCM it is equal to 10.75 with the following contributions: 2 from photons, 7/2 from electron-positron pairs, and $3 \cdot 7/4$ from three neutrino flavors. Any other form of energy present in the plasma is parametrized by the contribution into $g_*$ as additional neutrino flavors, $\delta g_* = (7/4)(N_\nu - 3)$, though these hypothetical forms of energy could be either in the form of massive particles, or vacuum-like energy, or anything else. Because the cosmological cooling rate depends upon $g_*$, BBN is sensitive to any form of energy from MeV down to tens of keV range.

There are of course some baryons in the plasma, building material for light nuclei, but they do not have a noticeable contribution into the total energy density because they are quite rare. Their number density, $n_B$, is characterized by the parameter

$$ \eta_{10} = \frac{n_B}{n_\gamma} = \text{a few} \times 10^{-10} $$

(2)

As we see in what follows, the output of light elements produced at BBN depends upon $\eta_{10}$. It is the only unknown parameter of the standard theory. In fact, the data on light element abundances are used to determine $\eta_{10}$. Till the last year it was the only reasonably accurate way. Direct astronomical observations of the fraction of baryonic matter in the universe give about 10% of the necessary amount of baryons. A dominant part of baryons is not visible. This year the measurements
of the relative heights of the first and second peaks in the angular fluctuations of CMBR permitted to determine $\eta_{10}$ independently \[1\):

$$\eta_{10}^{CMBR} = 5.7 \pm 1.0$$

(3)

This value is rather close to $\eta_{10}$ determined from BBN (see below).

Production of light elements crucially depends upon the available number of neutrons. The latter is determined by the competition between the rate of the reactions of neutron-proton transformation

$$n + \nu_e \leftrightarrow p + e^-,$$  
$$n + e^+ \leftrightarrow p + \bar{\nu}$$

(4)

and the cosmological expansion rate,

$$H = 5.44\sqrt{\frac{g_*}{10.75}} \frac{T^2}{m_{Pl}}$$

(5)

where $m_{Pl} = 1.22 \cdot 10^{19}$ GeV is the Planck mass. The rate of the reactions is $\Gamma_{np} \sim G_F^2 T^5$ and at $T > 0.7$ MeV reactions are faster than expansion and neutron-to-proton ratio follows equilibrium value, $(n/p)_{eq} = \exp(-\Delta m/T)$, where $G_F = 1.166 \times 10^{-5}$ GeV$^{-2}$ is the Fermi coupling constant and $\Delta m = 1.293$ MeV is the neutron-proton mass difference. When the temperature drops below $\sim 0.7$ MeV the reaction become slow in comparison with expansion and the ratio $n/p$ would remain constant (frozen) if not the neutron decay with the life-time $\tau_n = 886.7$ sec. As a result of the decay the neutron-to-proton ratio slowly decreases as $\exp(-t/\tau_n)$. It goes on till the temperature drops down to the temperature of nucleosynthesis, $T_{BBN} = 60 - 70$ keV, when formation of light elements abruptly and quickly begins. Practically all neutrons end their lives in $^4He$ since the latter has the largest binding energy. Mass fraction of $^4He$ rather weakly, logarithmically, depends upon the number density of baryons, $\eta_{10}$. This quantity determines the temperature of BBN and respectively the number of neutrons that survived to this moment. One can see that $T_{BBN}$ is a very mild function of $\eta_{10}$. On the other hand, helium-3 and especially deuterium are very sensitive to $\eta_{10}$ because these elements disappear in two-body baryon collisions whose probability is proportional to the number density of baryons. By this reason the amount of primordial deuterium serves as “baryometer”, measuring $\eta_{10}$. The calculated abundances of light elements as functions of $\eta_{10}$ are presented in figs. \[1, 2, 3\], taken from the review \[2\].

According to the recent measurements the observational data on primordial mass fraction of $^4He$ is concentrated near two centers $Y_p = 0.234 - 0.238$ \[4, 5, 6\] and $Y_p = 0.244$ \[7\]. The first value corresponds to $\eta_{10} \approx 2$, while the other is close
to 4-5. The analysis of ref. 3) was based on the data of the work 4) but with an account of ionization corrections.

Recent measurements of the number fraction of deuterium, \(D/H\), in high red-shift Lyman-alpha clouds which presumably are not contaminated by stellar processes and have its primordial quantity not diminished by burning is stars, give the results between 4 and 1.5 in units of \(10^{-5}\) 5, 6, 7, 8, 9, 10, 11, 12, 13). The corresponding values of \(\eta_{10}\) are between 4.5 and 8.5. There are also several measurements of very high deuterium values, 15-20 in the same units 4), for the references to earlier papers see e.g. 14). However it was argued recently in ref. 15) that the system at red-shift \(z = 0.7\), where the high deuterium value has been observed, has a complex velocity field and cannot be used for identification of deuterium because the deuterium line corresponds to velocity -81 km/sec and can be easily mimicked by the turbulent velocity field. It is concluded in the paper that high deuterium is practically excluded.

Figure 1: The predicted \(^4\)He abundance (solid curve) and the 2\(\sigma\) theoretical uncertainty 3). The horizontal lines show the range indicated by the observational data.
2 Role of neutrinos in BBN

There are several physical effects that make neutrinos important for the nucleosynthesis. First, as we have already mentioned above, neutrinos contribute to the total cosmological energy density and through it to the cooling rate, see eq. (1). The freezing temperature of $n/p$-transformation depends on $g_*$ as $T_{fr} \sim g_*^{1/6}$ and the larger is $g_*$, the larger becomes $n/p$-ratio and more $^4He$ is produced. Another effect is a decrease of the time interval which is necessary to reach the nucleosynthesis temperature $T_{BBN}$ (the latter does not depend on $g_*$). Both effects work in the same direction and result in an increase of the mass fraction of $^4He$. An addition of one extra neutrino species leads to the increase of $Y_p$ by approximately 5%. For deuterium the effect is of the same sign and stronger, see figs. 4, 5.

The energy distributions of neutrinos are assumed to have the Fermi-Dirac equilibrium form:

$$f_{\nu_j} = [1 + \exp (E/T - \xi_j)]$$

where $E$ is the neutrino energy, and $\xi_j$ is dimensionless chemical potential, $\xi_j = \mu_j/T$, and $j$ denotes neutrino flavour, $j = e, \mu, \tau$. In the standard model is assumed
Figure 3: The predicted $^7$Li abundance (solid curve) and the $2\sigma$ theoretical uncertainty $[3]$. The horizontal lines show the range indicated by the observational data.

that lepton asymmetry of neutrinos is negligibly small or in other words $\xi_j = 0$. The impact of chemical potentials on light elements abundances is two-fold. First, degenerate neutrinos, i.e. those with a non-zero $\xi$, have a larger energy density, corresponding to:

$$g_* = 10.75 \left[ 1 + 0.3488 \sum_j \left( 2 \left( \frac{\xi_j}{\pi} \right)^2 + \left( \frac{\xi_j}{\pi} \right)^4 \right) \right]$$

Thus a non-zero $\xi$ results in an increase of $^4$He and deuterium.

The second effect is operative only for electronic neutrinos because the latter are directly involved in the reactions $[4]$. If $\nu_e$ are degenerate, then the equilibrium $n/p$ ratio is shifted by the factor $\exp(-\xi_e)$ and the effect may have either sign depending on the sign of $\xi_e$. The most accurate recent bounds $[17]$ on the values of the chemical potentials based on a combined analysis of the angular spectrum of CMBR and BBN with the Deuterium fraction $D/H = (3.0 \pm 0.4) \cdot 10^{-5}$, are

$$-0.01 < \xi_e < 0.2, \quad |\xi_{\mu,\tau}| < 2.6$$

The third possible phenomenon is a non-equilibrium distortion of neutrino spectrum. By the reasons specified above it has an especially strong effect on
BBN for the case of electronic neutrinos. Normally neutrinos are well in thermal equilibrium which is not distorted by the cosmological expansion even if neutrino interactions is switched off at low temperatures, $T < (1-2)$ MeV. The latter is true only for massless (or very light) particles. The spectrum may be distorted if there are some new massive particles (or some neutrinos are heavy) which can decay or annihilate into light neutrinos after their decoupling. The effect could be large but even in the standard model with the usual massless neutrinos a significant spectrum distortion exists [18, 19]. Initially at large temperatures neutrinos were in strong thermal contact with electron-positron plasma, but when the temperature dropped down the contact became weaker and neutrino and electromagnetic components of the plasma became almost independent. Later $e^+e^-$ annihilation at $T \leq m_e$ heats up the electromagnetic component of the plasma and the temperatures of neutrinos and electrons become different. Residual annihilation of hotter electrons, $e^+e^- \rightarrow \bar{\nu}\nu$, distorts neutrino spectrum. The effect was accurately calculated by the numerical solution of the integro-differential kinetic equations. The most precise calculations performed by two different groups and by different methods [20, 21] show a very good agreement. According to their results an excess of the energy densities for $\nu_e$
Figure 5: Deuterium-to-hydrogen by number as a function of the number of massless neutrino species. Notations are the same as in fig. (4).

and $\nu_\mu$ and $\nu_\tau$ are respectively

$$\delta \rho_{\nu_e}/\rho_{\nu_e} = 0.94\% \quad \text{and} \quad \delta \rho_{\nu_\mu}/\rho_{\nu_\mu} = 0.40\%$$

However the impact of this effect on $^4\text{He}$ is extremely small, at the level of $10^{-4}$. On the other hand, such correction may be in principle observed in high precision measurements of CMB anisotropies by future PLANCK satellite mission \cite{22,23}, if the canonical model can be tested with the accuracy of about 1% or better. A change in neutrino energy density with respect to the standard case would result in a shift of equilibrium epoch between matter and radiation, which is imprinted on the form of the angular spectrum of fluctuations of CMBR. The total energy density of relativistic matter in the standard model is given by

$$\Omega_{\text{ref}} = \Omega_\gamma \left[ 1 + 0.68 \frac{N_\nu}{3} \left( \frac{1.401 T_\nu}{T_\gamma} \right)^4 \right]$$

where $\Omega_\gamma$ is the relative energy density of cosmic electromagnetic background radiation (CMBR) and $T_\gamma$ is the photon temperature. The corrections discussed in this section and electromagnetic corrections of ref. \cite{23} could be interpreted as a change of $N_\nu$ from 3 to 3.04.
3 Effects of neutrino oscillations on BBN

In the case of oscillations between active neutrinos, i.e. $\nu_e$, $\nu_\mu$, and $\nu_\tau$, there would be no impact on BBN, if neutrinos were initially in thermal equilibrium with vanishing chemical potentials. Kinetic equilibrium is established automatically at $T > 2 \text{ MeV}$ due to a large weak interaction rate in comparison with the expansion rate. Vanishing of neutrino chemical potentials is the standard assumption but it is not necessarily true. If neutrinos are degenerate, oscillations between different flavors could have an impact on the production of light elements. The result depends not only on the parameters of the oscillations but also on the initial values of the lepton asymmetries in different sectors. More interesting physical effects originate in the case of oscillations between active ($\nu_a$) and sterile ($\nu_s$) neutrinos. Initially $\nu_s$ should be absent in the primeval plasma. They can be produced only through the mixing with active ones but the mixing in matter is strongly suppressed at high temperatures.

All three effects discussed in the previous section through which neutrinos could influence BBN, are operative in the case of oscillations between active and sterile neutrinos: 1) new relativistic species (namely $\nu_s$) are created, enlarging $N_\nu$; 2) in the case of resonance oscillations a large lepton asymmetry may be generated in the sector of active neutrinos; 3) spectrum of active neutrinos, especially of $\nu_e$, can be distorted by oscillations.

If there exist one or several types of sterile neutrinos, i.e. neutrinos which do not any interaction with other particles and manifest themselves only through non-diagonal mixing in the mass matrix, the eigenstates of the free Hamiltonian can be parametrized in the following way:

$$\nu_1 = \nu_a \cos \theta_{vac} + \nu_s \sin \theta_{vac}, \quad \nu_2 = -\nu_a \sin \theta_{vac} + \nu_s \cos \theta_{vac}$$

(11)

where $\theta_{vac}$ is the mixing angle in vacuum. The effects of medium are described by the refraction index (or, what is practically the same, by effective potential). The latter can be calculated from the Dirac equation averaged over thermal bath of cosmic plasma. The equation, roughly speaking, has the form:

$$i \partial \psi_a = \frac{G_F}{\sqrt{2}} \left( \bar{\psi}_a O_\alpha \psi_b \right) O_\alpha \psi_a$$

(12)

where $O_\alpha = \gamma_\alpha (1 + \gamma_5)$. The factor in the r.h.s. in front of $\psi_a$, averaged over plasma, gives the effective potential, $(G_F/\sqrt{2}) \langle \bar{\psi} O_\alpha \psi \rangle = \delta_{aa} V_{eff}$. The space component of this operator vanishes due to isotropy of the plasma, while the time component is
non-zero if the plasma is charge asymmetric. For the normally assumed magnitude of any cosmological charge asymmetry at the level $10^{-9} - 10^{-10}$ this term is subdominant and the main contribution to refraction index comes from the so-called non-local term.

The operators $\psi_a$ in the l.h.s. and r.h.s. of eq. (12) may be at different space-time points because the weak interaction is not a local one but is mediated by an exchange of intermediate $W$ or $Z$ bosons. So the interaction is not the product of currents taken in the same space-time point. The contribution of non-locality is inversely proportional to $m_{W,Z}^2$ but still of the first order in the coupling constant. Taking all contributions together we obtain:

$$V_{\text{eff}} = \pm C_1 \eta G_F T^3 + C_2^a G_F^2 T^4 E^\alpha \alpha^{-1}$$

where $E$ is the neutrino energy, $\alpha = 1/137$, $C_1 = 0.95$, $C_2^e = 0.61$, and $C_2^\mu,\tau = 0.61$. The numerical values of the coefficients $C_j$ are calculated for thermally equilibrium bath and for temperatures below 100 MeV when muons were not present in the plasma. The coefficient $\eta$ is the charge asymmetry of the cosmological plasma, including all particle species interacting with $\nu_a$:

$$\eta^{(e)} = 2\eta_{\nu_e} + \eta_{\bar{\nu}_e} + \eta_e - \eta_n/2 \quad \text{(for } \nu_e \text{)},$$

$$\eta^{(\mu)} = 2\eta_{\nu_{\mu}} + \eta_{\bar{\nu}_e} + \eta_{\bar{\nu}_{\mu}} - \eta_n/2 \quad \text{(for } \nu_{\mu} \text{)},$$

and $\eta^{(\tau)}$ for $\nu_\tau$ is obtained from eq. (13) by the interchange $\mu \leftrightarrow \tau$. The individual charge asymmetries, $\eta_X$, are defined as the ratio of the difference between particle-antiparticle number densities to the number density of photons: $\eta_X = (n_X - n_{\bar{X}})/n_\gamma$. The charge asymmetry terms come with negative sign for neutrinos and with positive sign for antineutrinos.

At small $T$ the charge asymmetry term may be larger than the non-local one but at smaller temperatures the vacuum term, $\delta m^2 \cos 2\theta_{\text{vac}} / 2E$ dominates for $\delta m^2 > 10^{-7}$ eV$^2$. So for the “normal” value of the cosmological charge asymmetry the first term in eq. (13) is practically always subdominant.

An important difference between neutrino oscillations in stellar environment and in cosmology is that in the former the loss of coherence is not important, neutrinos can be described by wave function and first order effects (in terms of the coupling constant) given by the effective potential are sufficient for description of all essential physics. Situation in cosmology is more complicated. Neutrino production and annihilation, as well as elastic scattering at non-zero angle could be quite strong at high temperatures, so the coherence is quickly destroyed and the density matrix
formalism should be used \[23, 26, 27, 28\). The kinetic equation for density matrix in the cosmological background has the form

\[
\dot{\rho} = \left( \frac{\partial}{\partial t} - H p \frac{\partial}{\partial p} \right) \rho = i [\mathcal{H}_m + V_{\text{eff}}, \rho] + \int d\tau (\bar{\nu}, l, \bar{l}) \left( f_l f_l A^+ - \frac{1}{2} \{ \rho, A^+ \} \right) + \\
\int d\tau (l, \nu', l') \left( f_{\nu'} B^+ B - \frac{1}{2} f_{\nu} \{ \rho, B^+ \} \right)
\]

(16)

where \( \rho \) is the density matrix of the oscillating neutrinos, \( f_l \) is the distribution function of other leptons in the plasma, \( d\tau \) is the phase space element of all particles participating in the reactions except for the neutrinos in question. It has essentially the same form as the usual collision term in kinetic equation for the number density, when oscillations are absent:

\[
I_{\text{coll}}^l = -\left( \frac{2\pi}{E_i} \right)^4 \sum_{Z,Y} \int d\nu_Z d\nu_Y \delta^4(p_i + p_Y - p_Z) \left| A(i + Y \rightarrow Z) \right|^2 \\
f_l \prod_Y f \prod_Z (1 \pm f) - \left| A(Z \rightarrow i + Y) \right|^2 \prod_Z f \prod_{i+Y} (1 \pm f)
\]

(17)

Here \( Y \) and \( Z \) are arbitrary, generally multi-particle states, \( \prod_Y f \) is the product of phase space densities of particles forming the state \( Y \), and

\[
d\nu_Y = \prod_Y d\mathbf{p} \equiv \prod_Y \frac{d^3\mathbf{p}}{(2\pi)^3 2E}
\]

(18)

The signs ’+’ or ’−’ in \( \prod (1 \pm f) \) are chosen for bosons and fermions respectively.

The first commutator term in the r.h.s. is first order in the interaction. It is the usual contribution from refraction index which does not break coherence. The last two terms are second order in the interaction and are related respectively to annihilation, \( \nu\bar{\nu} \leftrightarrow \bar{l}l \), and elastic scattering, \( \nu l \leftrightarrow \nu' l' \). The quantum statistics factors, \( (1 - f) \), and \( (I - \rho) \) are neglected here. They can be easily reconstructed, see ref. \[27\). In the interaction basis and in the case of active-sterile mixing the matrices \( A \) and \( B \) have only one non-zero entry in the upper left corner equal to the amplitude of annihilation or elastic scattering respectively; the upper ’+’ means Hermitian conjugate.

The exact form of the coherence breaking terms presented above is quite complicated. In many cases an accurate description can be obtained with a much simpler anzats:

\[
\dot{\rho} = \ldots - \{ \Gamma, (\rho - \rho_{\text{eq}}) \}
\]

(19)

where the multi-dots denote contributions from the neutrino mixing in medium described by the commutator term in eq. \([10]\), \( \rho_{\text{eq}} \) is the equilibrium value of the density
matrix, i.e. the unit matrix multiplied by the equilibrium distribution function \((\mathfrak{I})\), and the matrix \(\Gamma\) that describes the interaction with the medium, is diagonal in the flavor basis; it is expressed through the reaction rates. If we take for the latter the total scattering rate, including both elastic scattering and annihilation, we obtain in the Boltzmann approximation \(^{29}\):

\[
\Gamma_0 = 2\Gamma_1 = g_a \frac{180\zeta(3)}{7\pi^4} G_F^2 T^4 p .
\]  

(20)

In general the coefficient \(g_a(p)\) is a momentum-dependent function, but in the approximation of neglecting \([1 - f]\) factors in the collision integral it becomes a constant \(^{30}\) equal respectively to \(g_{\nu_e} \simeq 4\) and \(g_{\nu_\mu,\tau} \simeq 2.9\) \(^{31}\). In ref. \(^{32}\) more accurate values are presented that are found from the thermal averaging of the complete electro-weak rates (with factors \([1 - f]\) included), which were calculated numerically from using the Standard Model code of ref. \(^{20}\). This gives \(g_{\nu_e} \simeq 3.56\) and \(g_{\nu_\mu,\tau} \simeq 2.5\).

Let us first consider non-resonance oscillations. The effective mixing angle in the medium is given by

\[
\sin 2\theta_m = \frac{a \sin 2\theta_{vac}}{a \cos 2\theta_{vac} - V_{eff}}
\]

(21)

where \(a = \delta m^2 / 2E\). In the pioneering papers \(^{33},^{34}\) and in many subsequent ones the production rate of sterile neutrinos was estimated as:

\[
\Gamma_s = \langle \Gamma_a \sin^2 2\theta_m \sin^2 \delta m^2 t/2E \rangle
\]

(22)

where \(\Gamma_a\) is the interaction rate of active neutrinos and after averaging over time one can substitute \(\langle \sin^2(\delta m^2 t/2E) \rangle = 1/2\). Demanding that the energy density of \(\nu_s\) produced by oscillations does not exceed the energy corresponding to \(\Delta N_{\nu}\) neutrino species, one can obtain the following limit

\[
\delta m^2 \sin^4 \theta_{vac} < 6 \cdot 10^{-3} \left( \frac{T_a}{3 \text{ MeV}} \right)^6 \Delta N_{\nu}^2 \text{ eV}^2 ,
\]

(23)

where \(T_a\) is the decoupling temperature of active neutrinos, \(\nu_a\). In ref. \(^{33}\) the latter was taken 3 MeV for \(\nu_e\) and 5 MeV for \(\nu_\mu,\tau\). It corresponds to account only of the annihilation rate. In ref. \(^{34}\) the decoupling temperatures were \(T_e = 1.8\) eV and \(T_{\mu,\tau} = 2.7\) eV, corresponding to account of the total reaction rate. To resolve the problem one has to turn to the kinetic equations for the density matrix with the exact form of the coherence breaking terms \(^{14}\). Introducing real and
imaginary parts for the non-diagonal elements of the density matrix according to
\( \rho_{as} = \rho_{sa}^* = R + iI \), one finds:
\[
\dot{\rho}_{aa}(p_1) = -FI - \int A^2_{el} [\rho_{aa}(p_1)f_1(p_2) - \rho_{aa}(p_3)f_1(p_4)] - \\
\int A^2_{ann} [\rho_{aa}(p_1)\bar{\rho}_{aa}(p_2) - f_1(p_3)f_1(p_4)],
\]
\( \dot{\rho}_{ss}(p_1) = FI \), \hspace{1cm} (24)
\[
\dot{R}(p_1) = WI - \frac{1}{2} R \left[ \int d\tau(l_2, \nu_3, l_4)A^2_{el}f_1(p_2) + \
\int d\tau(\tilde{\nu}_2, l_3, \tilde{l}_4)A^2_{ann}\rho_{aa}(p_2) \right],
\]
\( \dot{I}(p_1) = -WR - \frac{F}{2}(\rho_{ss} - \rho_{aa}) - \frac{1}{2} I \left[ \int d\tau(l_2, \nu_3, l_4)A^2_{el}f_1(p_2) + \
\int d\tau(\tilde{\nu}_2, l_3, \tilde{l}_4)A^2_{ann}\rho_{aa}(p_2) \right],
\]
\( \dot{\bar{R}}(p_1) = WI - \frac{1}{2} R \left[ \int d\tau(l_2, \nu_3, l_4)A^2_{el}f_1(p_2) + \
\int d\tau(\tilde{\nu}_2, l_3, \tilde{l}_4)A^2_{ann}\rho_{aa}(p_2) \right],
\)
\( \dot{\bar{I}}(p_1) = -WR - \frac{F}{2}(\rho_{ss} - \rho_{aa}) - \frac{1}{2} I \left[ \int d\tau(l_2, \nu_3, l_4)A^2_{el}f_1(p_2) + \
\int d\tau(\tilde{\nu}_2, l_3, \tilde{l}_4)A^2_{ann}\rho_{aa}(p_2) \right],
\)
\( \dot{\bar{R}}(p_1) = WI - \frac{1}{2} R \left[ \int d\tau(l_2, \nu_3, l_4)A^2_{el}f_1(p_2) + \
\int d\tau(\tilde{\nu}_2, l_3, \tilde{l}_4)A^2_{ann}\rho_{aa}(p_2) \right],
\)
\( \dot{\bar{I}}(p_1) = -WR - \frac{F}{2}(\rho_{ss} - \rho_{aa}) - \frac{1}{2} I \left[ \int d\tau(l_2, \nu_3, l_4)A^2_{el}f_1(p_2) + \
\int d\tau(\tilde{\nu}_2, l_3, \tilde{l}_4)A^2_{ann}\rho_{aa}(p_2) \right],
\)

The integration is taken over the phase space according to:
\[
d\tau = \frac{1}{2E_1} \int \frac{d^3p_2}{(2\pi)^32E_2^2} \frac{d^3p_2}{(2\pi)^32E_2^2} \frac{d^4p_2}{(2\pi)^44\delta^4(p_1 + p_2 - p_3 - p_4)} \]
\( \omega_{osc} = \left( F^2 + W^2 \right)^{1/2} \)

The amplitude of elastic scattering and annihilation with proper symmetrization factors can be taken from tables of ref. [20].

We will formally solve equations (26,27) to express the real and imaginary parts \( R \) and \( I \) of the non-diagonal components through the diagonal ones. The relevant equations can be written as
\[
\dot{R} = WI - \frac{1}{2} R (\rho_{aa} - \rho_{ss}), \hspace{1cm} (29)
\]
\[
\dot{I} = -WR + \frac{F}{2}(\rho_{ss} - \rho_{aa}) - \frac{1}{2} I (\rho_{aa} - \rho_{ss}). \hspace{1cm} (30)
\]

In the limit when the oscillation frequency
\[
\omega_{osc} = \left( F^2 + W^2 \right)^{1/2}
\]
is much larger than the expansion rate, the solution is given by stationary point approximation, i.e. by the condition of vanishing the r.h.s.:
\[
R = \frac{FW}{2(W^2 + \Gamma_0^2/4)} (\rho_{aa} - \rho_{ss}) \hspace{1cm} (31)
\]
\[
I = \frac{F\Gamma_0}{4(W^2 + \Gamma_0^2/4)} (\rho_{aa} - \rho_{ss}) \hspace{1cm} (32)
\]

In the non-resonant case, when \( W \neq 0 \), usually the condition \( W^2 \gg \Gamma^2/4 \) is fulfilled and
\[
R \approx (\sin 2\theta_m/2) (\rho_{aa} - \rho_{ss}) \hspace{1cm} (33)
\]
\[
I \approx (\sin 2\theta_m \Gamma_0/4W) (\rho_{aa} - \rho_{ss}) \hspace{1cm} (34)
\]
where $\theta_m$ is the mixing angle in matter and in the limit of small mixing

$$\tan 2\theta_m \approx \sin 2\theta_m \approx \frac{F}{W} = \frac{\sin 2\theta}{\cos 2\theta + (2E_{\text{eff}}/\delta m^2)}$$

(36)

Now we can substitute the expression for $I$ into eqs. (24,25) and obtain a closed system of equations for the two unknown diagonal elements of density matrix which is easy to integrate numerically. In the case that the number density of sterile neutrinos is small and the active neutrinos are close to equilibrium, as often the case, we obtain the following equation that describes the production of sterile neutrinos by the oscillations

$$\dot{\rho}_{ss} \approx \left(\sin^2 2\theta_m \Gamma_0/4\right) f_{eq}$$

(37)

An important conclusion of this derivation is that this production rate is by factor 2 smaller than the approximate estimates used in practically all earlier papers. An explanation of this extra factor $1/2$ is that the time derivative of $\rho_{ss}$ is proportional to imaginary part of the non-diagonal component of the density matrix and the latter is proportional to $\Gamma_1 = \Gamma_0/2$.

The equations for the diagonal elements of density matrix can be solved analytically in the limit of small number density of sterile neutrinos, $\rho_{ss} \ll \rho_{aa}$ and for the case of kinetic equilibrium of active neutrinos, so that in Boltzmann approximation their distribution takes the form:

$$\rho_{aa} = C(x) \exp(-y)$$

(38)

where dimensionless variables $x = 1 \text{ MeV} a(t)$ and $y = pa(t)$ are introduced, with $a(t)$ being the cosmological scale factor, normalized at high $T$ as $a = 1/T$. Here $C(x)$ can be understood as an effective chemical potential, $C(x) = \exp[\xi(x)]$, same for $\nu$ and $\bar{\nu}$. A justification for this approximation is a much larger rate of elastic scattering (that maintains the form (38) of $\rho_{aa}$) with respect to annihilation rate that forces $\xi$ down to zero (or $\xi = -\bar{\xi}$ in the case of non-zero lepton asymmetry).

This approach is similar to the standard calculations of cosmological freezing of species. Now we can integrate both both sides of eq. (24) over $d^3 y$ so that the contribution of elastic scattering disappears and the following ordinary differential equation describing evolution of $C(x)$ is obtained:

$$\frac{dC}{dx} = -\frac{k_l}{x^4} \left[ C^2 - 1 + \frac{10(1 + g_L^2 + g_R^2)}{24(1 + 2g_L^2 + 2g_R^2)} \int dy \ y^3 e^{-y} \sin^2 2\theta_m \right]$$

(39)
The first term in the r.h.s. of this equation came from annihilation and the second one from the oscillations; the contribution of elastic scattering disappeared after integration over $d^3y_1$.

We have assumed above that $\sin 2\theta_m \ll 1$ and thus the term $\sim (\sin 2\theta_m)^2dC/dx$ has been neglected. It is a good approximation even for not very weak mixing. The constants $k_l$ are given by

$$k_l = \frac{8G_F^2(1 + 2g_L^2 + 2g_R^2)}{\pi^3 H x^2}$$  \hspace{1cm} (40)

so that $k_e = 0.17$ and $k_{\mu,\tau} = 0.098$. The charge asymmetry term in neutrino refraction index was also neglected. It is reasonable in the non-resonant case if the asymmetry has a normal value around $10^{-9} - 10^{-10}$.

Eq. (39) can be solved analytically if $|\delta| = |1 - C| \ll 1$ and after some algebra we obtain the following result for the increase of the effective number of neutrino species induced by mixing of active neutrinos with sterile ones:

$$\Delta N_\nu = \frac{1}{9\pi^2} \frac{\sin^2 2\theta_{\nu e}^\text{vac} G_F^2(1 + g_L^2 + g_R^2)}{\sqrt{\beta_l} H x^2} \frac{g_\nu(T_{\nu s}^\text{prod})}{10.75}$$  \hspace{1cm} (41)

where

$$\beta_e = \frac{2.34 \cdot 10^{-8}}{\delta m^2 \cos 2\theta} \quad \text{and} \quad \beta_{\mu,\tau} = \frac{0.65 \cdot 10^{-8}}{\delta m^2 \cos 2\theta}$$  \hspace{1cm} (42)

Substituting numerical values of the parameters we obtain:

$$(\delta m^2_{\nu e,\nu e}/\text{eV}^2) \sin^4 2\theta_{\nu e}^\text{vac} = 3.16 \cdot 10^{-5}(g_\nu(T_{\nu s}^\text{prod})/10.75)^3(\Delta N_\nu)^2$$  \hspace{1cm} (43)

$$(\delta m^2_{\nu e,\nu e}/\text{eV}^2) \sin^4 2\theta_{\nu e}^\text{vac} = 1.74 \cdot 10^{-5}(g_\nu(T_{\nu s}^\text{prod})/10.75)^3(\Delta N_\nu)^2$$  \hspace{1cm} (44)

Here another factor $g_\nu$ came from the Hubble parameter.

These results can be compared with other calculations. They are approximately 2 orders of magnitude stronger than those presented in refs. [33], where too high freezing temperature for weak interaction rates was assumed and the limit was obtained: $\delta m^2 \sin^4 2\theta < 6 \cdot 10^{-6}\Delta N_\nu^2$. In ref. [34] the limit was $\delta m^2 \sin^4 2\theta < 3.6 \cdot 10^{-4}\Delta N_\nu^2$. (All these are given for mixing with $\nu_e$.) On the other hand, the limits obtained in ref. [34] are approximately 6 time stronger than those found above (43,44). They are: $\delta m^2 \sin^4 2\theta < 5 \cdot 10^{-6}\Delta N_\nu^2$ for mixing with $\nu_e$ and $\delta m^2 \sin^4 2\theta < 3 \cdot 10^{-6}\Delta N_\nu^2$ for mixing with $\nu_{\mu,\tau}$. The difference by factor 6 between these results and eq. (43,44) can be understood in part by the factor 2 difference in the interaction rate, according to eq. (37), which gives factor 4 difference in the limits. The remaining difference by roughly factor 1.5 could possibly be prescribed
to different ways of solution of kinetic equations or to the fact that the increase in
the number density of sterile neutrinos is accompanied by the equal decrease in the
number density of active neutrinos if the production of the latter by inverse anni-
hilation is not efficient. This phenomenon is missed in kinetic equations with the
simplified form of the coherence breaking term (19), which are mostly used in the
literature, while equations (24-27) automatically take that into account. However,
for sufficiently large mass difference, $\delta m^2$, the effective temperature of $\nu_s$ production is:

$$T_{\nu_s{\text{prod}}} = (12 - 15) (3/y)^{1/3} (\delta m^2/{\text{eV}}^2)^{1/6} \text{ MeV}$$ (45)

Here the first number is for mixing of $\nu_s$ with $\nu_e$ and the second one is for mixing
with $\nu_\mu$ or $\nu_\tau$. It is larger than the temperature of the annihilation freezing, so
the active neutrino states are quickly re-populated and the mentioned above affect
could be significant only for a small mass difference. Much weaker bounds obtained
in ref. [35] resulted from an error in the coherence breaking terms in kinetic equations
for the non-diagonal matrix elements of the density matrix.

The result (43) includes only a rise of the total cosmological energy density
due to production of sterile neutrinos. In the case of mixing with $\nu_e$ the distortion
of spectrum of the latter is also important for BBN. This issue is addressed in
refs. [36, 37, 38].

If the mass difference between $\nu_s$ and $\nu_\alpha$ is negative then the MSW-resonance
transition might take place in cosmological background. The resonance may also
exist with an arbitrary sign of $\delta m^2$ if the initial value of the asymmetry is suffi-
ciently large but we will not consider this case. A very interesting phenomenon
was discovered in ref. [39]. It was found that initially very small lepton asymme-
try ($\sim 10^{-9}$) can rise almost to unity because of oscillations. The effect is based
on the dependence of the refraction index on the cosmological charge asymmetry
(the first term in eq. (13)). Since this term comes with different signs to neutrinos
and antineutrinos, the resonance conditions for the same value of momenta could
be reached e.g. earlier for $\nu$ than for $\bar{\nu}$. This would give rise to a more efficient
transformation of neutrinos in comparison with antineutrinos and the asymmetry
could start rising. This creates a positive feed-back effect and exponential rise of
the asymmetry at initial stage. In fact, initially the asymmetry very strongly drops
down and only after some rather short period the explosive rise begins. The first
results were obtained by numerical solution of kinetic equations for the density ma-
trix, with some reasonable approximations made. In subsequent works more precise
numerical methods have been developed. The present-day state of art and the list
Figure 6: Evolution of $L^{(e)} = 2L_{\nu_e} + \eta$ for $\nu_e \to \nu_s$ oscillations with $\sin^2 2\theta_0 = 10^{-8}$ and, from left to right, $\delta m^2/eV^2 = -0.25, -0.5, -1.0, -2.0, -4.0$ obtained from numerically solution of the quantum kinetic equations. The initial $L_{\nu_e} = 0$ is taken and $\eta = 5 \times 10^{-10}$ is assumed. The low temperature evolution is weakly dependent on these values.

of relevant references can be found e.g. in the recent papers [40], [41], [42]. Numerical solution presents a serious challenge because of quickly oscillating functions under the sign of the integral over neutrino momentum which enters the charge asymmetry term. On the other hand, the presence of fast and slow variables permits to separate them and to find an approximate analytical solution [43]. Numerical and analytical results are in a very good agreement in the coinciding range of parameters. The evolution of $\nu_e$-asymmetry according to calculations of ref. [44] for $\sin^2 2\theta = 10^{-8}$ and several values of mass difference is presented in fig. 6.

Immediately after the large rise of lepton asymmetry due to oscillations between active and sterile neutrinos was discovered, it was argued [45] that the resulting asymmetry is large but chaotic, i.e. its sign is essentially unpredictable. The sign of the asymmetry is very sensitive to the oscillation parameters and the input of numerical calculations. As a result of this feature the sign of lepton asymmetry might be different in different causally non-connected domains [46]. This could have interesting implications and, in particular, would lead to inhomogeneous nucleosynthesis. The existence of chaoticity was confirmed in several subsequent
papers but again in the frameworks of simplified thermally averaged equations. On the other hand, the analysis of possible chaoticity performed in ref. on the basis of numerical solution of kinetic equations with a full momentum dependence shows a different picture. Most of parameter space is not chaotic, while in the region where chaoticity is observed numerical calculations are not reliable. Analytical calculations of ref. also do not show any chaoticity. At the present time the problem remains unresolved.

A very interesting phenomenon of spatial fluctuations of lepton asymmetry was found in ref. . Neutrino oscillations in the presence of initially small baryonic inhomogeneities could give rise to domains with different signs of lepton asymmetry. This effect is different from chaotic amplification of asymmetry discussed above. As we have already mentioned the initial asymmetry first drops down to an exponentially small value and after that started to rise, also exponentially, with a larger integrated exponent. Since the value of the asymmetry in the minimum could be extremely small, it is sensitive to small perturbations and the final sign of asymmetry could be determined by them. This goes not in a trivial way as e.g. spatial fluctuations of the sign in the minimum but somewhat more tricky and related to diffusion term in kinetic equations. As is argued in the original paper (see also ref. ), small spatial fluctuations of the cosmological baryon asymmetry, though they do not create change of sign in the minimum, would induce formation of domain with super-horizon sizes (at the moment of their creation) with large lepton asymmetry of different signs.

Let us consider the equation, modeling evolution of the lepton asymmetry in the presence of small spatial inhomogeneities, used in ref. . Following notations of this paper let us denote the asymmetry of active neutrinos of flavor as . The combination that enters the refraction index of is , where consists of the contributions of other neutrino species, baryons, and electrons. Due to electric charge neutrality the last two are not independent. The preexisting fluctuations in asymmetry of neutrinos would be erased due to a large neutrino mean free path in cosmic plasma after neutrino decoupling. So the background asymmetry can be written as , where the first term is homogeneous and the last one could be inhomogeneous and related to the fluctuations in the baryon number. Now we will neglect the universe expansion (it is essential for quantitative estimates but not for a qualitative result) and will solve the evolution equations in somewhat different way, permitting more accurate evaluation of the effects of diffusion. The
evolution of the neutrino asymmetry is described by the equation:

$$\dot{L}_{\nu\alpha}(\vec{x}, t) = a(t) \left[ 2 L_{\nu\alpha}(\vec{x}, t) + \bar{L} + \delta B(\vec{x}) \right] + D(t) \nabla^2 L_{\nu\alpha}(\vec{x}, t)$$

(46)

where $D(t)$ is the diffusion coefficient, and the function $a(t)$ is initially negative and generate an exponential decrease of the asymmetry, but at some critical time $t_c$ it changes sign and this creates a huge rise of the asymmetry. It is essential that, while $a(t)$ is negative, the asymmetry drops down to a very small value. In a more accurate formulation $a$ would also depend on the asymmetry itself, but in what follows we are interested in rather small values of the asymmetry, where the non-linear effects are not important.

The solution to this equation can be found by the Fourier transform and we obtain:

$$L_{\nu\alpha}(x, t) = \bar{L} \int_{t_{in}}^{t} dt' a(t') e^{2 \int_{t'}^{t} dt'' a(t'')} +$$

$$+ \int_{t_{in}}^{t} dt' a(t') e^{2 \int_{t'}^{t} dt'' a(t'')} \int d^3 k \bar{L}_{\nu\alpha}(\vec{k}, t_{in}) e^{-k^2 \int_{t_{in}}^{t} dt'' D(t'')} +$$

$$+ \int_{t_{in}}^{t} dt' a(t') e^{2 \int_{t'}^{t} dt'' a(t'')} \int d^3 k e^{i\vec{k}\vec{x}} \delta \hat{B}(\vec{k}, t_{in}) e^{-k^2 \int_{t_{in}}^{t} dt'' D(t'')}$$

(47)

Here “hut” indicates the Fourier transform of the corresponding function. The first integral in this expression can be explicitly taken because the integration measure $dt'a(t')$ is exactly the differential of the exponential. The integration of this term gives:

$$\frac{1}{2} \bar{L} \left[ \exp \left( 2 \int_{t_{in}}^{t} dt a(t_{2}) \right) - 1 \right]$$

(48)

So we obtained a rising term (after some initial decrease) plus a constant initial value of $\bar{L}$.

Moreover, the second term can be further integrated because the initial value $\bar{L}(\vec{k}, t_{in})$ is supposed to be homogeneous and so its Fourier transform is just delta-function, $\delta^3(k)$. The integral gives

$$L_{\nu\alpha}^{(in)} \exp \left( 2 \int_{t_{in}}^{t} dt a(t_{2}) \right)$$

(49)

where $L_{\nu\alpha}^{(in)}$ means the initial value, i.e. taken at $t = t_{in}$. So if we forget about the constant term $\bar{L}/2$, we would have $(L_{\nu\alpha}^{(in)} + \bar{L}/2)$ multiplied by the rising exponent. One would get exactly this expression if one solves the equation for $\dot{L}_{\nu\alpha}$ in homogeneous case.
Let us now consider the last term which, according to the arguments of ref. [49], could change the sign of the rising asymmetry, i.e. this last term could become the dominant one. To evaluate the integral let us substitute:

$$\delta \hat{B}(\vec{k}, t_{in}) = \int d^3 x_1 e^{i\vec{k}\cdot\vec{x}_1} \delta B(\vec{x}_1)$$

(50)

where \(\delta B(\vec{x}_1)\) is the initial value of the inhomogeneous term. Now we can make integration over \(d^3k\). We have the integral of the type

$$\int d^3k \exp[-S^2k^2 + i\vec{k}(\vec{x} - \vec{x}_1)]$$

(51)

the scalar product of vectors \(\vec{k}\) and \(\vec{r} = \vec{x} - \vec{x}_1\) is equal to \(\vec{k}(\vec{x} - \vec{x}_1) = kr \cos \theta\), and

$$S^2(t) = \int_{t_{in}}^{t} dt_2 D(t_2)$$

(52)

Integration over angles in \(d^3k = 2\pi k^2 dk d\cos \theta\) is trivial, it gives \(\sin kr/kr\). The remaining integration can be done as follows:

$$\int d\vec{k} k \sin kr \exp[-S^2k^2] = (d/dr) \int dk \cos kr \exp[-S^2k^2]$$

(53)

and the last integral can be taken if we expand the range of integration from minus to plus infinity. Introducing new variable \(\vec{x}_1 = \vec{x} - S(t_1)\vec{\rho}\) we finally obtain

$$\int dt_1 a(t_1) e^{2\int_{t_1}^{t} dt_2 a(t_2)} \int d^3 \delta B(\vec{x} - S(t_1)\vec{\rho}) e^{-\vec{\rho}^2}$$

(54)

This is the contribution the lepton asymmetry \(L_{\nu_a}\) generated by the (small) baryonic inhomogeneities. Its asymptotic rise at large \(t\) is similar to the rise of other terms but its exponential decrease at intermediate stage could be considerably milder and, as a result, this term could become dominant with the sign determined by the sign of the fluctuations in the baryon asymmetry. We can check this on a simple example assuming that the function \(a(t)\) has the form \(a(t) = a_1(t - t_c)\) and that fluctuations of the asymmetry are described by one harmonic mode: \(\delta B(\vec{x}) = \epsilon_B \cos \vec{k}_0 \vec{x}\). This form of \(\delta B\) could be inserted either into eq. (54) or into initial eq. (48) and we find for the oscillating part of the asymmetry (up to a constant coefficient):

$$\delta L(\vec{x}) = \epsilon_B \cos \vec{k}_0 \vec{x} e^{a_1(t - t_c)^2 - S(t)k_0^2} \left[ \int_{t_c - t_{in}}^{t_c} dt_1 e^{-a_1 t_1^2 + S(t_1)k_0^2} + \int_0^{t_c - t_{in}} dt_1 e^{-a_1 t_1^2} \left( e^{S(t_1)k_0^2} - e^{-S(t_1)k_0^2} \right) \right]$$

(55)

Both terms rise as \(\exp[a_1(t - t_c)^2]\), i.e. in the same way as the other homogeneous terms (we assume that \(S(t)\) is finite at large \(t\) and not too large). The first term is
exponentially suppressed as \(\exp[-a_1(t_c - t_{in})]\) also at the same level as the homogeneous terms. The second term, which vanishes in homogeneous case (\(k_0 = 0\) or \(S = 0\)) is not exponentially suppressed. In the limit of large \(a_1\) the integral can be evaluated as \(\sim S(0) k_0^2/a_1^3\). It is small but not exponentially small. Thus, it is easy to imagine the situation when the last term dominates and resonance enhancement of lepton asymmetry in the background of small fluctuations of baryon asymmetry could create domains with large and different lepton asymmetry. The effect is very interesting and deserves more consideration.

4 Heavy sterile neutrinos (with 10-100 MeV mass)

A hypothesis that such neutrinos may exist originated from the observation of the KARMEN anomaly in the time distribution of the charged and neutral current events induced by neutrinos from \(\pi^+\) and \(\mu^+\) decays at rest\(^{51}\). A suggested explanation of this anomaly was the production of a new neutral particle in pion decay

\[
\pi^+ \rightarrow \mu^+ + x^0 , \tag{56}
\]

with the mass 33.9 MeV, barely permitted by the phase space, so that this particle moves with non-relativistic velocity. Its subsequent neutrino-producing decays could be the source of the delayed neutrinos observed in the experiment. Among possible candidates on the role of \(x^0\)-particle was, in particular, a 33.9-MeV sterile neutrino\(^{52}\). According to ref. \(^{53}\), cosmology and astrophysics practically exclude the interpretation of the KARMEN anomaly by a 33.9 MeV neutrino mixed with \(\nu_\tau\). It agrees with a later statement by the KARMEN collaboration made at Neutrino 2000\(^{54}\) that the anomaly was not observed in upgraded detector KARMEN 2, but the question still remains which area in the mass-mixing-plane for heavy sterile neutrinos can be excluded. This issue was addressed recently by NOMAD collaboration in direct experiment\(^{55}\) and in ref. \(^{56}\) by considerations of big bang nucleosynthesis and the the duration of the supernova (SN) 1987A neutrino burst.

The mixing of heavy sterile neutrinos with the active ones couples the former to \(Z^0\) boson and induce the decay

\[
\nu_2 \rightarrow \nu_1 + \ell + \bar{\ell}, \tag{57}
\]

where \(\ell\) is any lepton with the mass smaller than the mass \(m_2\) of the heavy neutrino. If \(m_2 < 2m_\mu\) the decay into \(\bar{\mu}\mu\) and \(\bar{\tau}\tau\) is kinematically forbidden. If \(\nu_s\) is mixed
either with $\nu_\mu$ or $\nu_\tau$, the life-time is expressed through the mixing angle as:

$$
\tau_{\nu_s} \equiv \Gamma_{\nu_2}^{-1} = \frac{1.0 \text{ sec}}{(M_s/10 \text{ MeV})^3 \sin^2 2\theta}.
$$

(58)

For the mixing with $\nu_e$ the numerator is 0.7 sec; the difference is due to the charged-current interactions.

There are several effects operating in different directions, by which a heavy unstable sterile neutrino could influence big-bang nucleosynthesis. First, their contribution to the total energy density would speed up the expansion and enlarge the frozen neutron-to-proton ratio. Less direct but stronger influence could be exerted through the decay products, $\nu_e$, $\nu_\mu$, and $\nu_\tau$, and $e^\pm$ and through the change of the temperature evolution, $T_\nu/T_\gamma$. The impact of $\nu_\mu$ and $\nu_\tau$ on BBN is rather straightforward: their energy density increases with respect to the standard case and this also results in an increase of $r_n$. This effect can be described by the increased number of effective neutrino species $N_\nu$ during BBN. The increase of the energy density of $\nu_e$, due to decay of $\nu_s$ into $\nu_e$, has an opposite effect on $r_n$. Though a larger energy density results in faster cooling, the increased number of $\nu_e$ would preserve thermal equilibrium between neutrons and protons for a longer time and correspondingly the frozen $n/p$-ratio would become smaller. The second effect is stronger, so the net result is a smaller $n/p$-ratio. There is, however, another effect of a distortion of the equilibrium energy spectrum of $\nu_e$ due to $e^\pm$ produced from the decays of $\nu_s$. If the spectrum is distorted at the high-energy tail, as is the case, then creation of protons in the reaction $n + \nu_e \rightarrow p + e^-$ would be less efficient than neutron creation in the reaction $\bar{\nu}_e + p \rightarrow n + e^+$. We found that this effect is quite significant. Last but no least, the decays of $\nu_s$ into the $e^+e^-$-channel will inject more energy into the electromagnetic part of the primeval plasma and this will diminish the relative contribution of the energy density of light neutrinos and diminish $r_n$.

In refs. 53, 56) the distribution functions of neutrinos were calculated from kinetic equations in Boltzmann approximation and in a large part of parameter space they significantly deviate from equilibrium. The distributions of electrons and positrons were assumed to be very close to equilibrium because of their very fast thermalization due to interaction with the photon bath. However, the evolution of the photon temperature, due to decay and annihilation of the massive $\nu_s$ was different from the standard one, $T_\gamma \sim 1/a(t)$, by an extra factor $(1 + \Delta) > 1$ and this was explicitly calculated from the energy balance condition. At sufficiently high temperatures, $T > T_W \sim 2 \text{ MeV}$, light neutrinos and electrons/positrons were in strong contact, so that the neutrino distributions were also very close to the
equilibrium ones. If $\nu_s$ disappeared sufficiently early, while thermal equilibrium between $e^\pm$ and neutrinos remained, then $\nu_s$ would not have any observable effect on primordial abundances, because only the contribution of neutrino energy density relative to the energy density of $e^\pm$ and $\gamma$ is essential for nucleosynthesis. Hence a very short-lived $\nu_s$ has a negligible impact on primordial abundances, while with an increasing lifetime the effect becomes stronger. Indeed at $T < T_W$ the exchange of energy between neutrinos and electrons becomes very weak and the energy injected into the neutrino component is not immediately redistributed between all the particles. The branching ratio of the decay of $\nu_s$ into $e^+e^-$ is approximately $1/9$, so that the neutrino component is heated much more than the electromagnetic one. As we mentioned above, this leads to a faster cooling and to a larger $n/p$-ratio.

If the equilibrium number density of sterile neutrinos is reached, it would be maintained until $T_f \approx 4\sin^2\theta - 2/3$ MeV. This result does not depend on the heavy neutrino mass because they annihilate with massless active ones, $\nu_2 + \bar{\nu}_a \rightarrow all$. The heavy neutrinos would be relativistic at decoupling and their number density would not be Boltzmann suppressed if, say, $T_f > M_s/2$. This gives

$$\sin^2 2\theta (\delta m^2/\text{MeV}^2)^{3/2} < 500.$$  \hfill (59)

If this condition is not fulfilled the impact of $\nu_s$ on BBN would be strongly diminished. On the other hand, for a sufficiently large mass and non-negligible mixing, the $\nu_2$ lifetime given by Eq. (58) would be quite short, so that they would all decay prior to the BBN epoch. (To be more exact, their number density would not be frozen, but follow the equilibrium form $\propto e^{-m_2/T_f}$.)

Another possible effect that could diminish the impact of heavy neutrinos on BBN is entropy dilution. If $\nu_2$ were decoupled while being relativistic, their number density would not be suppressed relative to light active neutrinos. However, if the decoupling temperature is higher than, say, 50 MeV pions and muons were still abundant in the cosmic plasma and their subsequent annihilation would diminish the relative number density of heavy neutrinos. If the decoupling temperature is below the QCD phase transition the dilution factor is at most $17.25/10.75 = 1.6$. Above the QCD phase transition the number of degrees of freedom in the cosmic plasma is much larger and the dilution factor is approximately 5.5. However, these effects are essential for very weak mixing, for example the decoupling temperature would exceed 200 MeV if $\sin^2 2\theta < 8 \times 10^{-6}$. For such a small mixing the lifetime of the heavy $\nu_2$ would exceed the nucleosynthesis time and they would be dangerous for BBN even if their number density is 5 times diluted.

Sterile neutrinos would never be abundant in the universe if $\Gamma_s/H < 1$. In
fact we can impose a stronger condition demanding that the energy density of heavy neutrinos should be smaller than the energy density of one light neutrino species at BBN ($T \sim 1 \text{ MeV}$). Taking into account a possible entropy dilution by factor 5 we obtain the bound:

$$\left(\frac{\delta m^2}{\text{MeV}^2}\right) \sin^2 2\theta < 2.3 \times 10^{-7}.$$  \hspace{1cm} (60)

Parameters satisfying this conditions cannot be excluded by BBN.

If $\nu_s$ (or to be more precise $\nu_2$) mass is larger than 135 MeV, the dominant decay mode becomes $\nu_2 \rightarrow \pi^0 + \nu_a$. The life-time with respect to this decay can be found from the calculations \cite{57,58} of the decay rate $\pi^0 \rightarrow \nu \bar{\nu}$ and is equal to:

$$\tau = \left[ \frac{G_F^2 m_s (m_s^2 - m_\pi^2)}{16\pi} f_\pi^2 \sin^2 \theta \right]^{-1} = 5.8 \cdot 10^{-9} \text{ sec} \left[ \sin^2 \theta \frac{m_s (m_s^2 - m_\pi^2)}{m_\pi^4} \right]^{-1}$$ \hspace{1cm} (61)

where $m_s \equiv m_2$ is the mass of the sterile neutrino (we always assume that the mixing angle is small so that $\nu_2 \approx \nu_s$), $m_\pi = 135 \text{ MeV}$ is the $\pi^0$-mass and $f_\pi = 131 \text{ MeV}$ is the coupling constant for the decay $\pi^+ \rightarrow \mu + \nu_\mu$. The approximate estimates of ref. \cite{56} permit one to conclude that for the life-time of $\nu_2$ smaller than 0.1 sec, and corresponding cosmological temperature higher than 3 MeV, the decay products would quickly thermalize and their impact on BBN would be small. For a life-time larger than 0.1 sec, and $T < 3 \text{ MeV}$, one may assume that thermalization of neutrinos is negligible and approximately evaluate their impact on BBN. If $\nu_s$ is mixed with $\nu_\mu$ or $\nu_\tau$ then electronic neutrinos are not produced in the decay $\nu_s \rightarrow \pi^0 \nu_a$ and only the contribution of the decay products into the total energy density is essential. As we have already mentioned, non-equilibrium $\nu_e$ produced by the decay would directly change the frozen $n/p$-ratio. This case is more complicated and demands a more refined treatment.

The $\pi^0$ produced in the decay $\nu_s \rightarrow \nu_a + \pi^0$ immediately decays into two photons and they heat up the electromagnetic part of the plasma, while neutrinos by assumption are decoupled from it. We estimate the fraction of energy delivered into the electromagnetic and neutrino components of the cosmic plasma in the instant decay approximation. Let $r_s = n_s/n_0$ be the ratio of the number densities of the heavy neutrinos with respect to the equilibrium light ones, $n_0 = 0.09T_3^3$. The total energy of photons and $e^+e^-$-pairs including the photons produced by the decay is

$$\rho_{em} = \frac{11}{2} \frac{\pi^2}{30} T^4 + r_s n_0 \frac{m_s}{2} \left(1 + \frac{m_\pi^2}{m_s^2}\right),$$ \hspace{1cm} (62)

while the energy density of neutrinos is

$$\rho_\nu = \frac{21}{4} \frac{\pi^2}{30} T^4 + r_s n_0 \frac{m_s}{2} \left(1 - \frac{m_\pi^2}{m_s^2}\right).$$ \hspace{1cm} (63)
The effective number of neutrino species at BBN can be defined as

\[ N_{\nu}^{(\text{eff})} = \frac{22}{7} \frac{\rho_{\nu}}{\rho_{\text{em}}} \]  

(64)

Because of the stronger heating of the electromagnetic component of the plasma by the decay products, the relative role of neutrinos diminishes and \( N_{\nu}^{(\text{eff})} \) becomes considerably smaller than 3. If \( \nu_s \) are decoupled while relativistic their fractional number at the moment of decoupling would be \( r_s = 4 \) (two spin states and antiparticles are included). Possible entropy dilution could diminish it to slightly below 1. Assuming that the decoupling temperature of weak integrations is \( T_W = 3 \text{ MeV} \) we find that \( N_{\nu}^{(\text{eff})} = 0.6 \) for \( m_s = 150 \text{ MeV} \) and \( N_{\nu}^{(\text{eff})} = 1.3 \) for \( m_s = 200 \text{ MeV} \), if the frozen number density of \( \nu_s \) is not diluted by the later entropy release and \( r_s \) remains equal to 4. If it was diluted down to 1, then the numbers would respectively change to \( N_{\nu}^{(\text{eff})} = 1.15 \) for \( m_s = 150 \text{ MeV} \) and \( N_{\nu}^{(\text{eff})} = 1.7 \) for \( m_s = 200 \text{ MeV} \), instead of the standard \( N_{\nu}^{(\text{eff})} = 3 \). Thus a very heavy \( \nu_s \) would result in under-production of \(^4\text{He}\). There could, however, be some other effects acting in the opposite direction.

Since \( \nu_e \) decouples from electrons/positrons at smaller temperature than \( \nu_\mu \) and \( \nu_\tau \), the former may have enough time to thermalize. In this case the temperatures of \( \nu_e \) and photons would be the same (before \( e^+ e^- \)-annihilation) and the results obtained above would be directly applicable. However, if thermalization between \( \nu_e \) and \( e^\pm \) was not efficient, then the temperature of electronic neutrinos at BBN would be smaller than in the standard model. The deficit of \( \nu_e \) would produce an opposite effect, namely enlarging the production of primordial \(^4\text{He}\), because it results in an increase of the \( n/p \)-freezing temperature. This effect significantly dominates the decrease of \( N_{\nu}^{(\text{eff})} \) discussed above. Moreover even in the case of the decay \( \nu_2 \to \pi^0 + \nu_\mu,\tau \), when \( \nu_e \) are not directly created through the decay, the spectrum of the latter may be distorted at the high energy tail by the interactions with non-equilibrium \( \nu_\tau \) and \( \nu_\mu \) produced by the decay. This would result in a further increase of \(^4\text{He}\)-production. In the case of direct production of non-equilibrium \( \nu_e \) through the decay \( \nu_2 \to \pi^0 + \nu_e \) their impact on \( n/p \) ratio would be even much stronger.

To summarize, there are several different effects from the decay of \( \nu_s \) into \( \pi^0 \) and \( \nu \) on BBN. Depending upon the decay life-time and the channel these effects may operate in opposite directions. If the life-time of \( \nu_2 \) is larger than 0.1 sec but smaller than 0.2 sec, so that \( e^\pm \) and \( \nu_e \) establish equilibrium the production of \(^4\text{He}\) is considerably diminished so that this life-time interval would be mostly forbidden. For life-times larger than 0.2 sec the dominant effect is a decrease of the energy density of \( \nu_e \) and this results in a strong increase of the mass fraction of \(^4\text{He}\). Thus large life-times should also be forbidden. Of course there is a small
part of the parameter space where both effects cancel each other and this interval of mass/mixing is allowed. It is, however, difficult to establish its precise position with the approximate arguments used in ref. 56).

Thus, in the case of $\nu_s \leftrightarrow \nu_{\mu,\tau}$ mixing for $m_s > 140$ MeV we can exclude the life-times of $\nu_s$ roughly larger than 0.1 sec, except for a small region near 0.2 sec where two opposite effects cancel and the BBN results remain undisturbed despite the presence of sterile neutrinos. Translating these results into mixing angle according to eq. (61), we conclude that mixing angles $\sin^2 \theta < 5.8 \cdot 10^{-8} m_\pi / m_s / ((m_s / m_\pi)^2 - 1)$ are excluded by BBN. Combining this result with eq. (60) we obtain the exclusion region for $m_s > 140$ MeV:

$$5.1 \cdot 10^{-8} \frac{\text{MeV}^2}{m_s^2} < \sin^2 \theta < 5.8 \cdot 10^{-8} \frac{m_\pi}{m_s (m_s / m_\pi)^2 - 1}.$$  \hspace{1cm} \text{(65)}$$

In the case of $\nu_s \leftrightarrow \nu_e$ mixing for $m_s > 140$ MeV the limits are possibly stronger, but it is more difficult to obtain reliable estimates because of a strong influence of non-equilibrium $\nu_e$, produced by the decay, on neutron-proton reactions.

The constraints on the mass/mixing of $\nu_s$ from neutrino observation of SN 1987A are analyzed in some detail in ref. 53) and based on the upper limit of the energy loss into a new invisible channel because the latter would shorten the neutrino burst from this supernova below the observed duration.

The results are summarized in fig. 7. The region between the two horizontal lines running up to 100 MeV are excluded by the duration of the neutrino burst from SN 1987A. BBN excludes the area below the two upper dashed lines if the heavy neutrinos were abundant in the early universe. These two upper dashed lines both correspond to the conservative limit of one extra light neutrino species permitted by the primordial $^4$He-abundance. The higher of the two is for mixing with $\nu_{\mu,\tau}$ and the slightly lower curve is for mixing with $\nu_e$. In the region below the lowest dashed curve the heavy neutrinos are not efficiently produced in the early universe and their impact on BBN is weak. For comparison we have also presented here the region excluded by NOMAD Collaboration 55) for the case of $\nu_s \leftrightarrow \nu_\tau$ mixing. A more accurate consideration would probably permit to expand the excluded region both in the horizontal and vertical directions.

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Figure 7: Summary of the exclusion regions in the ($\sin^2 \theta$-$M_s$)-plane.

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