Seebeck effect in a thermal QCD medium in the presence of strong magnetic field

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Abstract

The strongly interacting partonic medium created post ultrarelativistic heavy ion collision experiments exhibits a significant temperature-gradient between the central and peripheral regions of the collisions, which in turn, is capable of inducing an electric field in the medium; a phenomenon known as Seebeck effect. The effect is quantified by the magnitude of the induced electric field per unit temperature-gradient - the Seebeck coefficient ($S$). We study the coefficient, $S$ with the help of the relativistic Boltzmann transport equation in relaxation-time approximation, as a function of temperature ($T$) and chemical potential ($\mu$), wherein we find that with current quark masses, the magnitude of $S$ for individual quark flavours as well as that for the partonic medium decreases with $T$ and increases with $\mu$, with the electric charge of the flavour deciding the sign of $S$. The emergence of a strong magnetic field ($B$) in the non-central collisions at heavy-ion collider experiments motivates us to study the effect of $B$ on the Seebeck effect. The strong $B$ affects $S$ in multifold ways, via : a) modification of phase-space due to the dimensional reduction, b) dispersion relation in lowest Landau level (occupation probability), and c) relaxation-time. We find that a strong $B$ not only decreases the magnitudes of $S$’s of individual species, it also flips their signs. This leads to a faster reduction of the magnitude of $S$ of the medium than its counterpart at $B = 0$. We then explore how the interactions among partons described in perturbative thermal QCD in the quasiparticle framework affect the Seebeck effect. The interactions indeed affect the coefficient drastically. For example, even in strong $B$, there is no more a flip of the sign of $S$ for individual species and the magnitudes of $S$ of individual species as well as that of the medium get enhanced in comparison with the current quark mass description at either $B = 0$ or $B \neq 0$.

Keywords: Seebeck effect, QCD, quasiparticle description, QCD, strong magnetic field, Boltzmann Transport equation

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I Introduction

The formation of a Quark-Gluon Plasma (QGP) under extreme temperatures and/or chemical potentials and its subsequent confinement into interacting hadrons has been an intense area of research for more than three decades. Ultra relativistic Heavy-Ion Collisions (UHRICs) at the CERN Super Proton Synchrotron (SPS), Brookhaven National Laboratory Relativistic Heavy Ion Collider (RHIC), and Large Hadron Collider (LHC) accelerators, reach center of mass energies that are much larger than the critical energy density required for a transition from hadrons to QGP as predicted by lattice QCD calculations[1]. Experimental data for hadrons of low to medium transverse momenta from the RHIC appeared to quantitatively agree with theoretical results obtained from the macroscopic description of QGP using ideal fluid dynamics, indicating that the viscosity of the matter created in the early stages post heavy ion collisions is small, thus establishing the nature of flow of QGP as that of a perfect fluid[2–5]. Since then, understanding the strongly interacting medium formed under extreme temperatures via studying its transport coefficients has been a relevant and a challenging task. In an amazing theoretical discovery, Kovtun, Son and Starinets conjectured that all substances have the value of the ratio $\eta/s = 1/4\pi$ (in units with $\hbar = k_B = c = 1$) as the lower limit[6]. The smallness of this ratio indeed helped to explain the flow data[7]. There are theoretical results that indicate that the ratio of bulk viscosity to entropy density $\zeta/s$ may attain a maximum value in the vicinity of phase transition, in agreement with lattice QCD simulations[8–11].

The effect of thermal conductivity on the medium has also been studied, specifically in relation to the determination of the critical point in the QCD phase diagram[12]. Several methods have been employed in the evaluation and study of these transport coefficients, which include perturbative QCD, different effective models, etc[13–15].

Two ultrarelativistic highly charged ions colliding with a finite impact parameter can give rise to large magnetic fields[16]. These fields can be as large as $eB \sim 10^{-1}m^2_\pi$ ($\approx 10^{17}$ Gauss) for SPS energies, $eB \sim m^2_\pi$ for RHIC energies and $eB \sim 15m^2_\pi$ for LHC energies[17]. The created magnetic field was believed to be strong for a very short span of time ($\sim 0.2$ fm for RHIC energies) whereafter it decays very fast[18, 19]. However, it was later pointed out[20, 21] that owing to a finite electrical conductivity, $\sigma_{el}$ of the plasma, the magnetic field does not decay very rapidly and hence contributes non trivially towards the evolution of the medium[22]. As such, the effect of magnetic field on the transport coefficients also needs to be investigated[22].

In this work, the thermoelectric behaviour of the QGP medium is analyzed via the relevant transport coefficient, $\text{viz.}$, the Seebeck coefficient. The first of such effects was discovered by T.J. Seebeck in 1821 wherein he showed that an electromotive force was generated on heating the junction between two dissimilar metals[23]. This phenomenon
of conversion of a temperature-gradient in a conducting medium into an electric current is termed as the Seebeck effect and depends on the bulk properties of the medium involved. When a temperature-gradient is established in a conducting medium, the more energetic charge carriers diffuse from the region of higher temperature to the region of lower temperature, resulting in the creation of an electric field. The diffusion stops when the created electric field becomes strong enough to impede the further flow of charges. The Seebeck coefficient is defined as the electric field (magnitude) generated in a conducting medium per unit temperature-gradient when the electric current is set to zero[24, 25], i.e. \( \vec{E} = S \vec{\nabla}T \) (\( S \) is the Seebeck coefficient). Conventionally, the Seebeck coefficient is taken to be positive if the thermoelectric current flows from the hotter end to the colder end. Thus, the Seebeck coefficient is positive for positive charge carriers and negative for negative charge carriers, its magnitude being very low for metals (only a few micro volts per degree Kelvin temperature-gradient) whereas much higher for semiconductors (typically a few hundred micro volts per degree Kelvin temperature-gradient)[26]. It is to be noted that while for condensed matter systems, a temperature gradient is sufficient to give rise to an induced current, this is not the case with a system such as the electron-positron plasma or the QGP. This is because in a condensed matter system, the ions are stationary and the majority charge carriers are responsible for conduction of electric current. However, in a medium consisting of mobile charged particles and antiparticles, a temperature gradient will cause them to diffuse in the same direction, giving rise to equal and opposite currents, which cancel. In such media, therefore, in addition to a temperature-gradient, a finite chemical potential is also required for a net induced current to exist. Thermoelectric properties have been an extensive area of investigation in the field of condensed matter physics over the past three decades. Some of the notable works include the study of the Seebeck effect in superconductors[27–31], Seebeck effect in the graphene superconductor junction[32], electric and thermoelectric transport properties of correlated quantum dots coupled to superconducting electrode[33], transport coefficients of high temperature cuprates[34], thermoelectric properties of a ferromagnet-superconductor hybrid junction[35], Seebeck coefficient in low dimensional correlated organic metals[36], etc.

The deconfined hot QCD medium created post heavy ion collisions can possess a significant temperature-gradient between the central and peripheral regions of the collisions. Majority of collisions in such experiments being non-central, a strong magnetic field perpendicular to the reaction plane is also expected to be created and could be sustained by a finite electrical conductivity of the medium. Study of thermoelectric properties in the context of heavy ion collisions is still uncharted territory, apart from a single paper[37], wherein the authors calculated the Seebeck coefficient of a baryon rich hot hadronic gas with zero meson chemical potential, modelled by the Hadron Resonance Gas (HRG) model.
at chemical freeze-out\cite{38, 39} with a resonance mass cut-off of 2.25 GeV. However, in the present work, we wish to do the aforesaid investigation on a color deconfined medium of quarks and gluons. In addition, we also explore the effects of a strong magnetic field and quasiparticle description of the medium constituents, on the Seebeck effect, where the quasiparticle/effective masses of the partons (\textit{mainly} quarks) are evaluated from perturbative thermal QCD up to one-loop. The noteworthy differences from a hadronic medium are two-fold: i) The degrees of freedom are more fundamental, \textit{i.e.} the elementary quarks and gluons instead of mesons and baryons. ii) The system is relativistic, \textit{i.e.} $m_f \ll T$ ($m_f$ refers to the mass of $f$\textsuperscript{th} flavour).

The paper is organised as follows: In Section II, we discuss the thermoelectric effect in a thermal QCD medium by transport theory, using the Relativistic Boltzmann Transport Equation (RBTE) in the relaxation-time approximation. In subsections, II. A and II. B, we quantify the effect by the Seebeck coefficients with the current quark masses of the partons in the absence and presence of a strong magnetic field, respectively. In Section III, we calculate the same, taking into account the interactions in the medium by perturbative thermal QCD in a strong magnetic field, which in turn generates (quasiparticle) masses for the partons. In Section IV, the conclusions are drawn and the results are summarized.

II  Seebeck effect in hot partonic medium with current quark masses

In this section, we construct the framework for studying the thermoelectric effect in general for a hot partonic medium and then estimate the Seebeck coefficient for the individual species as well as for the composite medium in subsection II.A. Subsequently in II.B, we introduce a strong external magnetic field to the aforesaid study and study the effects on the Seebeck coefficient.

II.A  Seebeck coefficient in the absence of strong magnetic field

In this subsection, we will begin with the relativistic Boltzmann Transport equation (RBTE) to estimate the infinitesimal disturbance due to the externally applied infinitesimal electric field in a hot partonic medium. The rate of change of the distribution function of the $i$\textsuperscript{th} flavour possessing quark chemical potential $\mu_i$, immersed in a thermal medium of quarks and gluons at temperature $T$ is given by RBTE

$$p^\mu \frac{\partial f_i(x, p)}{\partial x^\mu} + q_i F^{\rho\sigma} p_\sigma \frac{\partial f_i(x, p)}{\partial p^\rho} = C[f_i(x, p)],$$  \hspace{1cm} (1)
where \( C[f_i(x, p)] \) is the collision term and \( F^{\rho\sigma} \) is the electromagnetic field strength tensor. Since the treatment of the collision term using quantum scattering theory is a very cumbersome problem, we resort to the relaxation-time approximation, which is valid when the deviation of the distribution function from equilibrium \( (\delta f_i) \) is much smaller than the equilibrium distribution function itself, \( i.e. \delta f_i \ll f_i^{\text{iso}} \). In this approximation, the collision term is given by

\[
C[f_i(x, p)] \simeq -\frac{p_\nu u_\nu}{\tau_i} f_i \delta f_i, \tag{2}
\]

where \( u_\nu = (1, 0, 0, 0) \), is the 4-velocity of the medium in the local rest frame and \( \tau_i \) is the relaxation time of the \( i \)-th flavour in hot partonic medium \([41]\)

\[
\tau_i(T) = \frac{1}{5.1 T \alpha_s^2 \log(1/\alpha_s(T, \mu_i)) [1 + 0.12(2N_f + 1)]}. \tag{3}
\]

Here, \( \alpha_s \) is the strong coupling, which runs with the temperature and chemical potential as \([42]\)

\[
\alpha_s(T, \mu_i) = \frac{g^2(T, \mu_i)}{4\pi} = \frac{6\pi}{(33 - 2N_f) \ln \left( \frac{2\pi^2 T^2 + \mu_i^2 / \Lambda_{\text{QCD}}^2}{\Lambda_{\text{QCD}}^2} \right)}. \tag{4}
\]

where \( \mu_i \) is the quark chemical potential (of the \( i \)-th flavour), \( N_f \) is the number of flavours in the medium and \( \Lambda_{\text{QCD}} \sim 0.200 \text{ GeV} \).

Thus, once the collision integral (eq.2) is known, one can in principle obtain the infinitesimal disturbance, \( \delta f_i \) from RBTE (eq.1). For the thermoelectric effect, we are interested only in the quark distribution for which the equilibrium distribution for the \( i \)-th flavour, \( f_i \), is given by

\[
f_i = \frac{1}{\exp \left( \frac{w_i - \mu_i}{T} \right) + 1}. \tag{5}
\]

Suppose an infinitesimal electric field, \( \vec{E} = F^{i0} \text{ or } F^{0i} \) disturbs the above equilibrium distribution in phase-space infinitesimally and the infinitesimal disturbance from equilibrium \( (\delta f_i) \) also satisfies the RBTE. Thus, the disturbance is obtained by considering the \( \rho = i \) and \( \sigma = 0 \) components in RBTE (eq.1).

\[
\delta f_i = \frac{\vec{p}_i \tau_i}{\omega_i} f_i (1 - f_i) \left( -\frac{1}{T^2} \right) \nabla_{\vec{p}} T(\vec{r}) + 2q_i f_i (1 - f_i) \left( \vec{E}(\vec{r}) \cdot \vec{p}_i \right) \frac{\tau_i}{\omega_i} T, \tag{6}
\]

which, in turn, produces the induced four-current through the relation

\[
j_\mu = \sum_i q_i g_i \int \frac{d^3 p}{(2\pi)^3 \omega_i} p_\mu \left[ \delta f_i^q(x, p) - \delta f_i^q(x, p) \right], \tag{7}
\]
where \(q_i\) and \(g_i\) are the charge and degeneracy factors of the \(i^{\text{th}}\) quark flavour, respectively.

We thus obtain the spatial-part of the induced four-current, i.e. the induced current density

\[
\vec{j}_i = \frac{g_i q_i T_i}{2\pi^2} \left[ \int_0^\infty \frac{p_i^4}{\omega_i^2(p)} \left\{ f_i(1 - f_i)(\omega_i(p) - \mu_i) + \bar{f}_i(1 - \bar{f}_i)(\omega_i(p) + \mu_i) \right\} \left( -\frac{1}{T^2} \right) \nabla_T T(\vec{r}) \right.
\]

\[
+ 2q_i \int_0^\infty \frac{p_i^4}{\omega_i^2(p)} \left\{ f_i(1 - f_i) + \bar{f}_i(1 - \bar{f}_i) \right\} \frac{\vec{E}_i}{T(\vec{r})} \bigg] .
\]  
\((8)\)

The above current density is set equal to zero to yield the electric field due to the temperature-gradient \([44]\). Thus, the relation between the temperature-gradient in the coordinate space and the induced electric field is obtained:

\[
\vec{E}_i = \frac{1}{2Tq_i} \frac{I_2}{I_1} \nabla_T T(\vec{r}),
\]  
\((9)\)

For a single species, the degeneracy factor and the relaxation time cancel out from the numerator and denominator. Thus, the induced electric field can be recast in the form

\[
\vec{E}_i = \frac{1}{2Tq} \frac{I_2}{I_1} \nabla_T T(\vec{r}),
\]  
\((10)\)

where the integrals \(I_1\) and \(I_2\) are defined by

\[
I_1 \equiv \int_0^\infty \frac{p^4}{\omega^2(p)} \left\{ f(1 - f) + \bar{f}(1 - \bar{f}) \right\},
\]  
\((11)\)

\[
I_2 \equiv \int_0^\infty \frac{p^4}{\omega^2(p)} \left\{ f(1 - f)(\omega(p) - \mu) - \bar{f}(1 - \bar{f})(\omega(p) + \mu) \right\},
\]  
\((12)\)

where the chemical potential \((\mu_i)\) for all flavours are taken the same, i.e. \(\mu_i = \mu\).

Therefore, the coefficient of the temperature-gradient in the above relation (eq.\(10\)) gives the Seebeck coefficient \((S)\) for a single species

\[
S = \frac{1}{2Tq} \frac{I_2}{I_1}.
\]  
\((13)\)

We compute the coefficient, \(S\) as a function of temperature at fixed chemical potentials, \(\mu_q = 30\) MeV, 40 MeV and 50 MeV to observe the Seebeck effect in a hot and dense medium.
Figure 1: Variation of Seebeck coefficient of $u$ (left) and $d$ (right) quarks with temperature for different fixed values of chemical potentials.

We observe that the Seebeck coefficients (magnitudes) for $u$ and $d$ quark (Figs. 1a and 1b) decrease with the temperature for a fixed chemical potential, which is due to the fact that the net number density, $(n - \bar{n})$ (which is proportional to the net charge) decreases with the temperature for a fixed $\mu$. However, the coefficient is found to increase with chemical potential at a given temperature. This is because a larger $\mu$ is indicative of a larger surplus of particles over anti-particles, which, in the case of $u$ quark implies a larger abundance of positive over negative charges, leading to a larger thermoelectric current and hence a larger $S$. However, for the $d$ quark, a larger $\mu$ would mean a larger abundance of negative charges (particles) over positive charges (anti-particles), leading to a more negative value of $S$. Here, the sign of the Seebeck coefficient is solely determined by the sign of the electric charge the particle carries, because the other factors in the coefficient- the integrals $I_1$ and $I_2$ (eq.(13)), for both the quarks, are positive as can be seen from Fig(3) and Fig(4). The current quark masses of $u$ and $d$ quarks being very close to each other leads to almost identical values of the $I_1$ and $I_2$ integrals for both the quarks. As such, the magnitude of the electric charge of $u$ quark being twice that of $d$ quark is directly reflected in the magnitude of Seebeck coefficient of $d$ quark being almost twice that of the $u$ quark.
The Seebeck coefficient for $S$ quark shows the same characteristics as that of $u$ and $d$ quarks. The $I_1$ and $I_2$ integrals are positive for the $s$ quark as well. However, its mass is almost 100 times more than that of $u$ or $d$. This leads to a larger range of $I_1$ and $I_2$ values for the $s$ quark as compared to the $d$ quark. However, the ratio $I_1/I_2$ yields values that are not too different from that of $d$ quark (Fig (3) & Fig (4)). The electric charge for the $s$ quark is the same as $d$ quark, so they exhibit close agreement in the values of respective individual Seebeck coefficients.

Figure 3: Variation of $I_1$ integrals for $u$ (left), $d$ (middle) and $s$ (right) quarks with temperature for different fixed values of chemical potential.
After having calculated the Seebeck coefficient for a thermal medium consisting of a single species, we move on to the more realistic case of a multi-component system, which in our case corresponds to multiple flavours of quarks in the QGP. However, gluons being electrically neutral, do not contribute to the thermoelectric current, therefore, the total electric current in the medium is the vector sum of currents due to individual species:

\[
\vec{J} = \vec{J}\,(1) + \vec{J}\,(2) + \vec{J}\,(3) + \cdots
\]

\[
= \left( \frac{q_1^2 g_1 \tau_1}{T \pi^2} (I_1) + \frac{q_2^2 g_2 \tau_1}{T \pi^2} (I_2) + \cdots \right) \vec{E} - \left( \frac{q_1^2 g_1 \tau_1}{2T \pi^2} (I_1) + \frac{q_2^2 g_2 \tau_2}{2T \pi^2} (I_2) + \cdots \right) \nabla T(\vec{r}).
\]

(14)

Setting the total current, \( \vec{J} = 0 \) as earlier, we get the induced electric field,

\[
\vec{E} = \frac{1}{2T} \sum_i \frac{q_i g_i \tau_i}{q_i^2 g_i \tau_i} (I_1) \nabla T(\vec{r}),
\]

(15)

which yields the Seebeck coefficient for the multi-component medium:

\[
S = \frac{1}{2T} \sum_i q_i g_i \tau_i (I_2)/q_i^2 g_i \tau_i (I_1).
\]

(16)

All quarks have the same degeneracy factor and their relaxation times (seen from eq.(3)) are also identical for each flavour. Hence, the total Seebeck coefficient for the multi-component systems can be rewritten as

\[
S = \frac{\sum_i S_i q_i^2 (I_1)}{\sum_i q_i^2 (I_1)},
\]

(17)

which could be viewed as a weighted average of the Seebeck coefficients of individual species \( (S_i) \) present in the medium. In our calculation, we have considered only three flavours of quarks, viz: \( u, d \) and \( s \), thus, the explicit expression comes out to be:

\[
S = \frac{4S_u(I_1)u + S_d(I_1)d + S_s(I_1)s}{4(I_1)u + (I_1)d + (I_1)s},
\]

(18)
where $S_u, S_d, S_s$ denote the individual Seebeck coefficients for the $u, d$ and $s$ quarks respectively. Likewise, the $I_i$ integrals for different flavours are denoted by the respective flavour indices.

![Variation of total Seebeck coefficient with temperature for different fixed values of chemical potentials.](image)

Figure 5: Variation of total Seebeck coefficient with temperature for different fixed values of chemical potentials.

As can be seen from Fig(5), the Seebeck coefficient of the medium is positive and decreases with the temperature. Like earlier for single species, it increases with the chemical potential. Although $S_d$ and $S_s$ are both negative, the relative magnitudes of $S_u$, $S_d$ and $S_s$ are such that eq.(18) renders the Seebeck coefficient of the medium positive with a small magnitude.

The magnitude of the Seebeck coefficient is the magnitude of electric field produced in the medium for a unit temperature-gradient. Qualitatively, it is a measure of how efficiently a medium can convert a temperature-gradient into electricity. The sign of the Seebeck coefficient expresses the direction of the induced field with respect to the direction of temperature gradient, which is conventionally taken to point towards the direction of increasing temperature. A positive value of the Seebeck coefficient means that the induced field is in the direction of the temperature-gradient. In the convention mentioned above, this will happen when the majority charge carriers are positively charged. As expected, individual Seebeck coefficient is positive for a positively charged species ($u$ quark) and negative for a negatively charged species ($d$ and $s$ quarks).
II.B Seebeck coefficient in the presence of a strong magnetic field

In the presence of magnetic field, we decompose the quark momentum into components longitudinal ($p_L$) and transverse ($p_T$) to the direction of the magnetic field. Quantum mechanically the energy levels of the $i^{th}$ quark flavour get discretized into Landau levels, so the dispersion relation becomes

$$\omega_{(i,n)}(p_L) = \sqrt{p_L^2 + m_i^2 + 2n|q_i B|},$$  \tag{19}$$

where $n = 0, 1, 2, \ldots$ are quantum numbers specifying the Landau levels. It is well known that in the strong magnetic field (SMF) limit (characterised by $|q_f B| \gg T^2$, where $B$ is the magnetic field and $q_f$ is the electric charge of the $f^{th}$ flavour), quarks are rarely excited thermally to higher Landau levels owing to the large energy gap between the levels, which is of the order of $\sqrt{|eB|}$\cite{45}. Therefore, they are constrained to be populated exclusively in the lowest Landau level ($n=0$), implying that the quark momentum in the presence of a strong magnetic field is purely longitudinal\cite{46-48}. Taking the magnetic field to be in the 3-direction, we identify $p_L$ with $p_3$, so the above dispersion relation is simplified into a relation for a one-dimensional free particle:

$$\omega_i(p_3) = \sqrt{p_3^2 + m_i^2}.$$  \tag{20}$$

Thus, the equilibrium quark distribution function in SMF limit becomes:

$$f_{i,B} = \frac{1}{e^{\beta(\omega_i - \mu_i)} + 1}.$$  \tag{21}$$

Owing to the quark momentum being purely longitudinal in the presence of a strong magnetic field, the electromagnetic current generated in response to the electric field ($J_3$) is also purely longitudinal.

$$J_3 = \sum_i q_i g_i \int \frac{d^3 p}{(2\pi)^3} \omega_i p_3 \left[ \delta f_i^q(\tilde{x}, \tilde{p}) - \delta f_i^{\bar{q}}(\tilde{x}, \tilde{p}) \right],$$  \tag{22}$$

where, $\tilde{x} = (x_0, 0, 0, x_3)$ and $\tilde{p} = (p_0, 0, 0, p_3)$. In addition, as an artifact of strong magnetic field, the density of states in two spatial directions perpendicular to the direction of magnetic field becomes $|q_i B|$\cite{49, 50}, i.e.

$$\int \frac{d^3 p}{(2\pi)^3} \rightarrow \frac{q_i B}{2\pi} \int \frac{dp_3}{2\pi}.$$  \tag{23}$$

The infinitesimal change in the distribution function in the strong magnetic field is thus obtained from the RBTE in the relaxation-time approximation,

$$p^0 \frac{\partial f_{i,B}}{\partial t} + p^3 \frac{\partial f_{i,B}}{\partial x_3} + q_i F_{03} p_3 \frac{\partial f_{i,B}}{\partial p^0} + q_i F^{30} p_0 \frac{\partial f_{i,B}}{\partial p_3} = -\frac{p^0}{\tau_B} \delta f_{i,B},$$  \tag{24}$$
where \( \tau_i \) denotes the relaxation-time for quarks in the presence of strong magnetic field, which, in the Lowest Landau Level (LLL) approximation is given by[51]:

\[
\tau_i(T, B) = \frac{w_i}{\alpha_s(eB)C_2m_i^2(e^{\beta \omega_i} + 1)} \left[ \int dp^3 \frac{1}{w_i(e^{\beta \omega_i} + 1)} \right],
\]

which has been evaluated for massless quarks. However, it has been shown in Ref.[52] that the effect of finite quark mass in the evaluation of scattering cross sections is very small, and hence, the relaxation-time is largely unaffected. \( C_2 = \frac{4}{3} \) is the Casimir factor and \( \alpha_s(eB) \) is the running coupling constant, which runs primarily with the magnetic field, since in the SMF regime \((eB >> T^2)\), the strong magnetic field is the largest energy scale. Its form is given in [53, 54]

\[
g^2(eB) = \frac{4\pi}{\alpha_s^0(\mu_0)^{-1}} + \frac{11N_c}{12\pi} \ln \left( \frac{\Lambda_{QCD}^2 + M_B^2}{\mu_0^2} \right) + \frac{1}{3\pi} \sum_f |q_f| \beta_f,
\]

where \( \alpha_s^0(\mu_0) \) is given by

\[
\alpha_s^0(\mu_0) = \frac{12\pi}{11N_c \left( \mu_0^2 + M_B^2 \right)^{\frac{1}{2}}},
\]

and \( M_B \) is interpreted as the ground-state mass of two gluons connected by the fundamental string \((\sim 1 \text{ GeV})\) with the string tension, \( \Omega = 0.18 \text{ GeV}^2 \) and \( \Lambda_{V} \) and \( \mu_0 \) are taken as 0.385 GeV and 1.1 GeV, respectively.

Now, the infinitesimal change for quark and anti-quark distribution functions can be obtained from the RBTE (eq.24) in SMF regime as

\[
\delta f_{i,B} = -\frac{\tau_B^i}{p^0} \frac{p_z f_{i,B}(1 - \bar{f}_{i,B})}{T} \left[ \frac{\omega_i - \mu_i}{T} (\vec{\nabla}T)_z - 2q_i E_z \right],
\]

\[
\delta \bar{f}_{i,B} = -\frac{\tau_B^i}{p^0} \frac{p_z \bar{f}_{i,B}(1 - \bar{f}_{i,B})}{T} \left[ \frac{\omega_i + \mu_i}{T} (\vec{\nabla}T)_z + 2q_i E_z \right],
\]

which gives the induced current density from (22) for a single species,

\[
J_z = (\vec{\nabla}T)_z \left[ \frac{gg|qB|}{T(2\pi)^2} \frac{d\rho_z}{\omega^2} \beta_z^2(\tau^B) \left\{ \bar{f}(1 - \bar{f}) \frac{\omega + \mu}{T} - f(1 - f) \frac{\omega - \mu}{T} \right\} \right] - 2qE_z \left[ \frac{gg|qB|}{T(2\pi)^2} \frac{d\rho_z}{\omega^2} \beta_z^2(\tau^B) \left\{ \bar{f}(1 - \bar{f}) + f(1 - f) \right\} \right].
\]

Defining the following integrals

\[
H_1 = \int \frac{d\rho_z}{w_i^2} \tau_B^i \beta_z^2 \left\{ -\bar{f}(1 - \bar{f})(\omega + \mu) + f(1 - f)(\omega - \mu) \right\},
\]

\[
H_2 = \int \frac{d\rho_z}{w_i^2} \tau_B^i \beta_z^2 \left\{ \bar{f}(1 - \bar{f}) + f(1 - f) \right\},
\]

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the current density (3rd-component) from eq.(29) can be recast in the form
\[ J_z = (\nabla T)_z \frac{qg|qB|}{T^2(2\pi)^2} H_1 - E_z \frac{qg|qB|}{T(2\pi)^2} 2qH_2 \] (32)

As earlier, the induced electric field due to the temperature-gradient is obtained by setting \( J_z = 0 \),
\[ E_z = \frac{1}{2TqH_2} (\nabla T)_z. \] (33)
The proportionally constant gives the Seebeck coefficient:
\[ S = \frac{1}{2Tq H_2} \] (34)

Figure 6: Variation of Seebeck coefficient of \( u \) (left) and \( d \) (right) quarks with temperature for a fixed chemical potential and magnetic field.

As can be seen from Fig.(6), the variation of the individual Seebeck coefficients (magnitudes) of the \( u \) and \( d \) quarks with temperature and chemical potential shows the same trend as in the earlier case. However, the sign of the Seebeck coefficient in this case is negative for \( u \) quark and positive for \( d \) quark. This is opposite to what was encountered earlier. This is because the \( H_1 \) integrals for both \( u \) and \( d \) quarks turn out to be negative in this case (Fig.(8)) and the \( H_2 \) integrals positive (Fig.(9)). Considered along with the dependence on the particle charge, this explains the sign of the Seebeck coefficient for \( u \) and \( d \) quarks. The Seebeck coefficients for the \( u \) and \( d \) quarks in this case turn out to be about 1 order of magnitude smaller, compared to their \( B = 0 \) counterparts.
Figure 7: Variation of \( s \) quark Seebeck coefficient with temperature for a fixed magnetic field and different fixed values of chemical potential.

The sign of the \( s \) quark Seebeck coefficient is again opposite to that of the \( B = 0 \) case owing to the \( H_1 \) integral for \( s \) quark being negative. The magnitude of the Seebeck coefficient rises with temperature upto about \( T = 270 \) MeV and starts decreasing therefrom. Although the ratio \( H_1/H_2 \) is an increasing function of \( T \) for the entire temperature range, it does not increase fast enough after \( T = 270 \) MeV to compensate for the rising temperature (in the denominator of \( S \); eq.(34)).

It should be noted that contrary to the case of pure thermal medium, the relaxation time here is momentum dependent. As such, it cannot be taken out of the momentum integrations \( (H_1, H_2) \) and hence does not cancel out in \( S \).

Figure 8: Variation of \( H_1 \) integrals for \( u \) (left), \( d \) (middle) and \( s \) (right) quarks with temperature.
Now we generalize our formalism to a medium consisting of multiple species, therefore, the total current is given by the sum of currents due to individual species:

\[
J_z = J_z^1 + J_z^2 + J_z^3 + \cdots
\]

\[
= (\vec{\nabla} T)_z \left\{ \frac{q_1 g_1 |q_1 B|}{T^2 (2\pi)^2} (H_1)_1 + \frac{q_2 g_2 |q_2 B|}{T^2 (2\pi)^2} (H_1)_2 + \frac{q_3 g_3 |q_3 B|}{T^2 (2\pi)^2} (H_1)_3 + \cdots \right\}
- E_z \left\{ \frac{q_1 g_1 |q_1 B|}{T (2\pi)^2} 2q_1 (H_2)_1 + \frac{q_2 g_2 |q_2 B|}{T (2\pi)^2} 2q_2 (H_2)_2 + \frac{q_3 g_3 |q_3 B|}{T (2\pi)^2} 2q_3 (H_2)_3 + \cdots \right\}
\]

(35)

Again, the Seebeck coefficient of the medium in a strong magnetic field is obtained by setting \( J_z = 0 \),

\[
S = \frac{1}{2T} \sum_i q_i |q_i B| (H_1)_i, \tag{36}
\]

which could be further expressed in terms of the weighted average of individual Seebeck coefficients.

\[
S = \frac{\sum_i S_i |q_i|^3 (H_2)_i}{\sum_i |q_i|^2 (H_2)_i}. \tag{37}
\]

Thus, unlike the Seebeck coefficient in the absence of magnetic field (eq.13), both the individual as well as total Seebeck coefficient of the medium depend on the relaxation-time in the presence of a strong magnetic field.

Figure 9: Variation of \( H_2 \) integrals for \( u \) (left), \( d \) (middle) and \( s \) (right) quarks with temperature.
Figure 10: Variation of Seebeck coefficient of the medium with temperature for a fixed chemical potential in the absence and presence of a magnetic field.

We can now visualize the sole effect of strong magnetic field on the (total) Seebeck coefficient from the comparison of $B \neq 0$ and $B = 0$ results in Fig. (10). Unlike the $B = 0$ (red line) case, the total Seebeck coefficient in strong $B$ (black line) becomes negative, indicating that the induced electric field is opposite to the direction of temperature-gradient. However, the magnitude of total Seebeck coefficient decreases with the temperature and increases with the chemical potential, much like, the $B = 0$ case. However, the strong magnetic field enhances the magnitude of $S$ by one order of magnitude, compared to the $B = 0$ case.

III Seebeck effect of hot partonic medium in a quasiparticle model

Quasiparticle description of quarks and gluons in a thermal QCD medium in general, introduces a thermal mass, apart from their current masses in QCD Lagrangian. These masses are generated due to the interaction of a given parton with other partons in the medium, therefore, quasiparticle description in turn describes the collective properties of the medium. However, in the presence of strong magnetic field in the thermal QCD medium, different flavors acquire masses differently due to their different electric charges. Different versions of quasiparticle description exist in the literature based on different effective theories, such as Nambu-Jona-Lasinio (NJL) model and its extension PNJL model [55–57], Gribov-Zwanziger quantization [58, 59], thermodynamically consistent quasiparticle model [60], etc. However, our description relies on perturbative thermal QCD, where the medium generated masses for quarks and gluons are obtained from the
poles of dressed propagators calculated by the respective self-energies at finite temperature and/or strong magnetic field.

### III.A Seebeck coefficient in the absence of magnetic field

In the quasiparticle description of quarks and gluons in a thermal medium with 3 flavours, all flavours (with current/vacuum masses, \( m_i << T \)) acquire the same thermal mass \([61,62]\)

\[
m^2_T = \frac{g'^2(T)T^2}{6},
\]

which is, however, modified in the presence of a finite chemical potential \([63]\)

\[
m^2_{T,\mu} = \frac{g'^2(T)T^2}{6} \left( 1 + \frac{\mu^2}{\pi^2T^2} \right),
\]

where \( g' \) is the running coupling constant already mentioned in eq.(4).

We take the quasiparticle mass (squared) of \( i \)th flavor in a pure thermal medium to be \([60]\):

\[
m^{' 2}_{iT} = m^2_i + \sqrt{2} m_i m_T + m^2_T,
\]

where \( m_i \) is the current quark mass of the \( i \)th flavour. So the dispersion relation for the \( i \)th flavour takes the form

\[
\omega^2_i(p) = \vec{p}^2_i + m^2_i + \sqrt{2} m_i m_T + m^2_T.
\]

Using this expression of \( \omega_i(p) \) in the quark distribution functions as well as in the integrals \( I_1 \) and \( I_2 \) (eq.12), we proceed in a similar fashion and evaluate the individual Seebeck coefficients for the \( u, d \) and \( s \) quarks in quasiparticle description from eq.(13).
As can be seen from Figs. 11a and 11b, the Seebeck coefficients of $u$ and $d$ quarks show a trend similar to their current quark mass counterparts in the absence of magnetic field (Figs. 1a & 1b) and their magnitudes decrease with temperature and increase with chemical potential. The $I_1$ and $I_2$ integrals for both quarks are also found to be positive and as such, the sign of the coefficient is again determined by the electric charge of the particle. The change due to the quasiparticle description adopted here, is a slight increase in the magnitudes of the Seebeck coefficients for both quarks.

Similar to the current quark mass case (in Fig. 2), the coefficient for $s$ quark decreases in
magnitude with increasing temperature (Fig. 12). Owing to its negative electric charge and the positive value of $I_1, I_2$ integrals, the sign of the coefficient is negative. Thus, the overall behaviour of individual Seebeck coefficients in the quasiparticle description is similar to that of the current mass description with slightly enhanced magnitudes. Numerically the average percentage increase for the $u, d$ and $s$ quarks are around 19.52%, 19.70% and 24.38%, respectively.

Similarly, the total Seebeck coefficient of the medium in quasiparticle description

![Graph showing variation of Seebeck coefficient with temperature for different chemical potentials.](image)

Figure 13: Variation of Seebeck coefficient of the medium with temperature for different fixed values of chemical potential.

(in Fig. 13) is found to have a small positive value, which decreases with the temperature and increases with the chemical potential as earlier, but with a significantly elevated magnitude in comparison with the current quark mass case (in Fig 5).

### III.B Seebeck coefficient in the presence of strong magnetic field

In the presence of magnetic field, only quarks are affected while the gluons are not directly influenced. As a result, only the quark-loop of the gluon self-energy will be affected and the gluon-loops remains altered. Furthermore, only quarks contribute to the thermoelectric effect, and hence, we proceed to calculate the thermal quark mass in the presence of a strong magnetic field, which can be obtained from the pole ($p_0 = 0, p \to 0$ limit) of the full propagator.

As we know, the full quark propagator can be obtained self-consistently from the
Schwinger-Dyson equation (assuming massless flavours, which is assumed to be true at least for light flavours),

\[ S^{-1}(p_\parallel) = \gamma^\mu p_{\parallel \mu} - \Sigma(p_\parallel), \quad (42) \]

where \( \Sigma(p_\parallel) \) is the quark self-energy at finite temperature in the presence of strong magnetic field. We can evaluate it up to one-loop

\[ \Sigma(p_\parallel) = \frac{4}{3} g^2 i \int \frac{d^4k}{(2\pi)^4} \left[ \gamma_\mu S(k) \gamma_\nu D^{\mu\nu}(p-k) \right], \quad (43) \]

where \( \Sigma(p_\parallel) \) denotes the Casimir factor and \( g \) represents the running coupling already mentioned in eq.(26). \( D^{\mu\nu}(p-k) \) is the gluon propagator, which is not affected by the magnetic field, so its form is given by

\[ D^{\mu\nu}(p-k) = \frac{ig^{\mu\nu}}{(p-k)^2}. \quad (44) \]

However, the quark propagator, \( S(K) \) in the strong magnetic field limit, is affected and is obtained by the Schwinger proper-time method at the lowest Landau level \((n=0)\) in momentum space,

\[ S(k) = ie^{-\frac{k^2}{m_i^2}} \left( \gamma^0 k_0 - \gamma^3 k_z + m_i \right) \frac{k^2 - m_i^2}{k_\parallel - m_i^2} (1 - \gamma^0 \gamma^3 \gamma^5), \quad (45) \]

where the 4-vectors are defined as \( k_\perp \equiv (0, k_x, k_y, 0) \), \( k_\parallel \equiv (k_0, 0, 0, k_z) \).

Next we obtain the form of quark and gluon propagators at finite temperature in the imaginary-time formalism and subsequently replace the energy integral \((\int \frac{dp_0}{2\pi})\) by Matsubara frequency sum. However, in a strong magnetic field along \( z \)-direction, the transverse component of the momentum becomes vanishingly small \((k_\perp \approx 0)\), so the exponential factor in eq.(45) becomes unity and the integration over the transverse component of the momentum becomes \(|q_f B|\). Thus, the quark self-energy in eq.(43) at finite temperature in the SMF limit will be of the form

\[ \Sigma(p_\parallel) = \frac{2g^2 |q_f B|}{3\pi^2} \sum_n \int dk_z \left[ \frac{(1 + \gamma^0 \gamma^3 \gamma^5) (\gamma^0 k_0 - \gamma^3 k_z) - 2m_i}{k_0^2 - \omega_k^2} \right] \frac{1}{[(p_0 - k_0)^2 - \omega_{pk}^2]}. \quad (46) \]

where \( \omega_k^2 = k_z^2 + m_i^2, \omega_{pk}^2 = (p_z - k_z)^2 \) and \( L^1 \) and \( L^2 \) are the two frequency sums, which are given by

\[ L^1 = T \sum_n \frac{1}{k_0^2 - \omega_k^2} \frac{1}{[(p_0 - k_0)^2 - \omega_{pk}^2]}, \quad (47) \]

\[ L^2 = T \sum_n \frac{1}{k_0^2 - \omega_k^2} \frac{1}{[(p_0 - k_0)^2 - \omega_{pk}^2]} \quad (48) \]
We first do the frequency sums \[64\] and then integrate the momentum \(k_z\) to obtain the simplified form of quark self-energy eq.\((46)\) \[66\] as
\[
\Sigma(p_{\parallel}) = \frac{g^2 |q_i B|}{3 \pi^2} \left[ \frac{\pi T}{2 m_i} - \ln(2) \right] \left[ \frac{\gamma^0 p_0}{p_{\parallel}^2} + \frac{\gamma^3 p_z}{p_{\parallel}^2} + \frac{\gamma^0 \gamma^5 p_z}{p_{\parallel}^2} + \frac{\gamma^3 \gamma^5 p_0}{p_{\parallel}^2} \right]. \tag{49}
\]

To solve the Schwinger-Dyson equation eq.\((42)\), one needs to first express the self-energy at finite temperature in magnetic field in a covariant form \[67, 68\],
\[
\Sigma(p_{\parallel}) = A(p_0, p) \gamma^\mu u_\mu + B(p_0, p) \gamma^\mu b_\mu + C(p_0, p) \gamma^5 \gamma^\mu u_\mu + D(p_0, p) \gamma^5 \gamma^\mu b_\mu , \tag{50}
\]
where \(u^\mu (1,0,0,0)\) and \(b^\mu (0,0,0,-1)\) denote the preferred directions of heat bath and magnetic field, respectively. The form factors, \(A, B, C\) and \(D\) are computed in strong \(B\) with LLL approximation as
\[
A = \frac{g^2 |q_i B|}{3 \pi^2} \left[ \frac{\pi T}{2 m_i} - \ln(2) \right] \frac{p_0}{p_{\parallel}^2}, \tag{51}
\]
\[
B = \frac{g^2 |q_i B|}{3 \pi^2} \left[ \frac{\pi T}{2 m_i} - \ln(2) \right] \frac{p_z}{p_{\parallel}^2}, \tag{52}
\]
\[
C = - \frac{g^2 |q_i B|}{3 \pi^2} \left[ \frac{\pi T}{2 m_i} - \ln(2) \right] \frac{p_z}{p_{\parallel}^2}, \tag{53}
\]
\[
D = - \frac{g^2 |q_i B|}{3 \pi^2} \left[ \frac{\pi T}{2 m_i} - \ln(2) \right] \frac{p_0}{p_{\parallel}^2} . \tag{54}
\]

Then the self-energy \((50)\) can be expressed in terms of chiral projection operators \((P_R\) and \(P_L\)) as
\[
\Sigma(p_{\parallel}) = P_R [(A - B) \gamma^\mu u_\mu + (B - A) \gamma^\mu b_\mu] P_L + P_L [(A + B) \gamma^\mu u_\mu + (B + A) \gamma^\mu b_\mu] P_R . \tag{55}
\]

Hence, the Schwinger-Dyson equation is able to express the inverse of the full propagator in terms of \(P_L\) and \(P_R\),
\[
S^{-1}(p_{\parallel}) = P_R \gamma^\mu X_\mu P_L + P_L \gamma^\mu Y_\mu P_R , \tag{56}
\]
where
\[
\gamma^\mu X_\mu = \gamma^\mu p_{\parallel\mu} - (A - B) \gamma^\mu u_\mu - (B - A) \gamma^\mu b_\mu , \tag{57}
\]
\[
\gamma^\mu Y_\mu = \gamma^\mu p_{\parallel\mu} - (A + B) \gamma^\mu u_\mu - (B + A) \gamma^\mu b_\mu . \tag{58}
\]

Thus, the effective propagator is finally obtained by inverting eq. \((56)\)
\[
S(p_{\parallel}) = \frac{1}{2} \left[ P_R \gamma^\mu Y_\mu P_L + P_L \gamma^\mu X_\mu P_R \right] , \tag{59}
\]
where

\[
\frac{X^2}{2} = X_1^2 = \frac{1}{2} [p_0 - (A - B)]^2 - \frac{1}{2} [p_z + (B - A)]^2 ,
\]

(60)

\[
\frac{Y^2}{2} = Y_1^2 = \frac{1}{2} [p_0 - (A + B)]^2 - \frac{1}{2} [p_z + (B + A)]^2 .
\]

(61)

Thus, the thermal mass (squared) for \(i\)th flavor at finite temperature and strong magnetic field is finally obtained by taking the \(p_0 = 0, p_z \rightarrow 0\) limit in either \(X_1^2\) or \(Y_1^2\) (because both of them are equal),

\[
m_{iT,B}^2 = \frac{g^2 q_i B}{3\pi^2} \left[ \frac{\pi T}{2m_i} - \ln(2) \right],
\]

(62)

which depends both on temperature and magnetic field. The quark distribution functions with medium generated masses in the absence and presence of magnetic field therefore manifest the interactions present in the respective medium in terms of modified occupation probabilities in the phase space and thus affect the Seebeck coefficients. The quasiparticle (or effective) mass of \(i\)th quark flavor is generalized in finite temperature and strong magnetic field into

\[
m_{iT,B}^2 = m_i^2 + \sqrt{2} m_i m_{iT,B} + m_{iT,B}^2 .
\]

(63)

Now, using the above quasiparticle mass in the distribution function and proceeding identically, we compute the individual Seebeck coefficients from eq.(34) as a function of temperature, which is shown in Fig. 14.

![Figure 14: Variation of Seebeck coefficient of u (left) and d (right) quarks with temperature for a fixed chemical potential and magnetic field.](image)

We find that the magnitudes of both \(u\) and \(d\) quark Seebeck coefficients decrease with the temperature and increase with the chemical potential. In quasiparticle description,
the $H_1$ integrals for both $u$ and $d$ quarks becomes positive, unlike the case with current quark masses in strong $B$ (Fig. 6), so the sign of the individual Seebeck coefficient is decided only by the electric charge of quarks, similar to the $B = 0$ case. The magnitudes of the Seebeck coefficients are found to increase by two-order of magnitude over their current quark mass case counterparts (seen in Fig. 6), which could thus be attributed to the quasiparticle description.

![Figure 15: Variation of $s$ quark Seebeck coefficient with temperature for a fixed magnetic field and different fixed values of chemical potential.](image)

The sign of the Seebeck coefficient for $s$-quark in quasiparticle description now becomes negative, opposite to the case of current quark mass description (Fig. 7). Again, this is because the quasiparticle description flips the signs of $H_1$ integral for the $s$ quark from negative (in Fig. 8) to positive, so the deciding factor for the sign of the coefficient is the sign of the electric charge of $s$ quark (which is negative). Furthermore, the variation of the magnitude of Seebeck coefficient with temperature in quasiparticle description is quite different compared to the current quarks mass case (Fig. 7) and is rather similar to that of $d$ quark (Fig. 14b) but with smaller magnitude.

Comparison between the $B = 0$ and $B \neq 0$ results within the quasiparticle description reveals summarily that the percentage increase is more pronounced at lower temperatures. The average percentage increase over the entire temperature range is 467.61\% and 212.63\% for $u$ and $d$ quarks, respectively. However, the percentage increase for $s$ quark is -36.81\%, suggesting that the $s$ quark Seebeck coefficient in the presence of a strong $B$ is smaller in magnitude than its pure thermal ($B = 0$) counterpart (Fig. 12). Hence, in the presence of strong $B$, the Seebeck effect depends strongly on the interactions among the constituents in the medium, encoded by the appropriate quasiparticle description.
Figure 16: Variation of $I_1$ integrals for $u$ (left), $d$ (middle) and $s$ (right) quarks with temperature for different fixed values of chemical potential.

Figure 17: Variation of $I_2$ integrals for $u$ (left), $d$ (middle) and $s$ (right) quarks with temperature for different fixed values of chemical potential.

Once the individual Seebeck coefficients of $u$, $d$ and $s$ quarks have been evaluated in quasiparticle description, we compute the weighted average of the above individual coefficients to obtain the (total) Seebeck coefficient of the medium as a function of temperature from eq.(37). This is shown in Fig. 18. To see the effects of magnetic field in quasiparticle description, we have also displayed the same in the absence of magnetic field (shown by black solid line in Fig. 13) in the same figure for better visual effects.
Figure 18: Variation of Seebeck coefficient of the medium with temperature for a fixed chemical potential in the absence and presence of a magnetic field.

The succinct observation is that the strong magnetic field makes the Seebeck coefficient in quasiparticle description negative and larger by one-order of magnitude than its immediate counterpart in the absence of magnetic field. The negative sign physically implies that the induced electric field acts opposite to the direction of temperature-gradient. Moreover, the magnitude of the coefficient decreases faster with the temperature at large chemical potential ($\mu=50$ MeV).

IV Summary and conclusions

In this paper, we have investigated the thermoelectric phenomenon of Seebeck effect in a hot QCD medium in two descriptions: i) when the quarks are treated in QCD with their current masses and ii) when the quarks are treated in quasiparticle model. The emergence of a strong magnetic field in the non-central events of the ultra-relativistic heavy ion collisions provides a further impetus to carry out the aforesaid investigations both in the absence and presence of a strong magnetic field, in order to isolate the effects of strong magnetic fields and interactions present among partons. For this purpose, the Seebeck coefficients are calculated individually for the $u$, $d$ and $s$ quarks, which, in turn, give the Seebeck coefficient of the medium via a weighted average of the individual coefficients. Thus, effectively, four different scenarios have been analysed:

1. Current mass description with $B = 0$.
2. Current mass description with $B \neq 0$. 
3. Quasiparticle description with $B = 0$.

4. Quasiparticle description with $B \neq 0$.

Comparison between the cases 1 and 3 is able to decipher the effect of the interactions among the partons through the quasiparticle description on the Seebeck effect in the absence of strong magnetic field, where the magnitudes of Seebeck coefficient of individual species as well as that of the medium get(s) slightly enhanced with respect to the current mass description. The sign of the individual Seebeck coefficients is positive for positively charged particles ($u$ quark) and negative for negatively charged particles ($d$ and $s$ quarks) except for the case 2, where, owing to the $H_1$ integral becoming negative (for each quark), the situation is reversed. Comparison between cases 2 and 4 brings out the sole effect of the quasiparticle description in the presence of a strong magnetic field, where it is seen that the magnitude of the coefficients are amplified. The sign of the Seebeck coefficient of the medium, is however same in both the cases. Lastly, the comparison between cases 3 and 4 brings forth the sole effect of strong constant magnetic field on the Seebeck effect in the quasiparticle description. The variation of individual and total Seebeck coefficients with temperature and chemical potential are found to show similar trends in both the cases but with enhanced magnitudes in the latter case.

The trend of overall decrease (increase) of the magnitude of Seebeck coefficient with the increase in temperature (chemical potential) is seen for all cases. However, in the quasiparticle description, the magnitude of the coefficient gets enhanced in the presence of strong magnetic field, so the inclusion of interactions among partons plays a crucial role in thermoelectric phenomenon in thermal QCD.

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