What is Beta?

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Abstract. Measurements of the cosmological density parameter ($\Omega$) using techniques that exploit the gravity-induced motions of galaxies yield, in linear perturbation theory, the degenerate parameter combination $\beta = \Omega^{0.6}/b$, where the linear bias parameter $b$ is the ratio of the fluctuation amplitudes of the galaxy and mass distributions. However, the relation between the mass and the galaxy density fields depends on the complex physics of galaxy formation, and it can in general be non-linear, non-local, and stochastic. There is a growing consensus that the one-parameter model for bias is oversimplified. This leaves us with the following question: What is the quantity that is actually being measured by the different techniques? In order to address this question, we present estimates of $\beta$ from galaxy distributions constructed by applying a variety of biased galaxy formation models to cosmological N-body simulations. We compare the values of $\beta$ estimated using two different techniques: the anisotropy of the redshift-space power spectrum and an idealized version of the POTENT method. In most cases, we find that the bias factor $b = \Omega^{0.6}/\beta$ derived from redshift-space anisotropy or POTENT is similar to the large-scale value of $b_\sigma(R)$ defined by the ratio of rms fluctuation amplitudes of the galaxy and mass distributions. However, non-linearity of bias and residual effects of non-linear gravitational evolution both influence $\beta$ estimates at the 10-20% level.

1. Analysis

We estimate $\beta$ from a variety of simulated galaxy distributions, created by applying simple local and deterministic biasing algorithms to the mass distributions obtained from cosmological N-body simulations.

The cosmological models that we use are: (1) $\Omega = 1.0$, CDM model with a tilted power spectrum designed to simultaneously satisfy both COBE and cluster normalization constraints. (2) $\Omega = 0.4$, CDM, cluster normalized model. (3) $\Omega = 0.2$, CDM, cluster normalized model. Cluster normalization requires that $\sigma_{8,m} \Omega^{0.6} = 0.55$ (White, Efstathiou & Frenk 1993).

The biasing algorithms that we use are: (1) Semi-analytic: An empirical bias prescription derived by Narayanan et al. (1999, in prep.) which characterizes the relation between the galaxy and mass density fields in the semi-analytic galaxy formation models of Benson et al. (1999). (2) Sqrt-Exp. (Square-root Exponential): A biasing prescription in which $(1 + \delta_g) \propto \sqrt{(1 + \delta_m)} e^{\alpha(1+\delta_m)}$. (3)
Power-law: A biasing prescription in which $(1 + \delta_g) \propto (1 + \delta_m)^\alpha$. Here, $(1 + \delta_g)$ and $(1 + \delta_m)$ are the galaxy and mass over-densities, respectively.

All the bias models are designed to yield galaxy distributions with an rms fluctuation, in top-hat spheres of radius $12h^{-1}\text{Mpc}$, of $\sigma_{12} \approx 0.7$. The values of $\Omega^{0.6} / b_\sigma(12)$ for the galaxy distributions are $0.6$, $0.58$, and $0.56$ for $\Omega = 1.0$, $0.4$, and $0.2$, respectively. All the results have been averaged over four independent realizations of the cosmological models. The simulation volumes are periodic cubes, with a box size of $400h^{-1}\text{Mpc}$.

The first method that we use to estimate $\beta$ involves the anisotropy of the redshift-space power spectrum. Kaiser (1987) and Cole, Fisher & Weinberg (1994) showed that in linear theory, and under the plane-parallel approximation, the multipole moments of the redshift-space power spectrum may be used to estimate $\beta$. Here we employ two different $\beta$ estimators: $P^S(k)/P^R(k)$, the ratio of the angle-averaged redshift-space power spectrum (monopole) to the real-space power spectrum, and $P_2(k)/P_0(k)$, the ratio of the quadrupole and monopole moments of the redshift-space power spectrum. These ratios are, in principle, measurable, and they are related to $\beta$ as follows: $P^S(k) / P^R(k) = 1 + \frac{3}{5}\beta + \frac{1}{2}\beta^2$, $P_2(k) / P_0(k) = (\frac{4}{3}\beta + \frac{2}{5}\beta^2) / (1 + \frac{3}{2}\beta + \frac{1}{5}\beta^2)$. These ratios yield an estimate of $\beta$ at each wavenumber $k$, and the scale dependence of this estimate is caused mainly by non-linearity in the peculiar velocity field. It is possible to obtain a global estimate of $\beta$ by modeling this non-linearity. We use two such models in our analysis: (1) Exponential velocity distribution model (Cole, Fisher & Weinberg 1995), where we assume that the galaxies have uncorrelated small scale peculiar velocities which are drawn from an exponential distribution. We use this model in our analysis of $P^S(k)/P^R(k)$ (the results of which are shown in Fig. 1). (2) Empirical model (Hatton & Cole 1999), where the scale dependence of $P_2(k)/P_0(k)$ is found empirically by examining a large number of N-body simulations spanning a broad range of parameter space.

The second method that we use to estimate $\beta$ involves a direct comparison between the mass and galaxy density fields. The POTENT method (Bertschinger & Dekel 1989) reconstructs the full three dimensional peculiar velocity field $\mathbf{v}$ from the observed radial peculiar velocity field, allowing a measurement of $\beta$ directly from the linear theory relation $\nabla \cdot \mathbf{v} = -\beta H \delta_g$. We estimate $\beta$ using an idealized POTENT method, in which we have perfectly reconstructed the three dimensional peculiar velocity field. We obtain the value of $\beta$ by fitting a line of the form $y = ax + c$ to the $-\nabla \cdot \mathbf{v}$ vs. $\delta_g$ relation. We perform this fit in the region: $-0.5 < \delta_g < 0.5$. By repeating this procedure for different smoothing of the velocity and density fields, we can measure $\beta$ as a function of scale (see Fig. 1).

2. Conclusions

Figure 1 compares $\beta$ estimates from the two methods described previously to the function $\beta_\sigma(R) = \Omega^{0.6} / b_\sigma$. Here, $b_\sigma(R)$ is the bias function defined as $b_\sigma(R) = \sigma_g(R) / \sigma_m(R)$, where $\sigma_g(R)$ and $\sigma_m(R)$ are the rms fluctuations of the galaxy and mass density fields, smoothed with a Gaussian filter of radius $R$, of the galaxy and mass distributions, respectively. In our full analysis, we also compare our $\beta$
estimates to the function \( \beta_P(k) = \Omega^{0.6}/b_P(k) \). Here \( b_P(k) \) is the bias function defined as \( b_P(k) = \sqrt{\frac{P_g(k)}{P_m(k)}} \), where \( P_g(k) \) and \( P_m(k) \) are the power spectra of the galaxy and mass distributions, respectively. In the case of a simple, linear bias model, \( \delta_g = b \delta_m \), all these definitions of \( b \) are equivalent, i.e., \( b_\sigma = b_P = b \).

In most cases, we find that the bias factor \( b = \Omega^{0.6}/\beta \) derived from redshift-space anisotropy or POTENT is similar to the large-scale value of \( b_P(k) \) or \( b_\sigma(R) \) defined by rms fluctuation amplitudes. Discrepancies at the 10-20% level may reflect non-linear dynamics as much as non-linearity of bias, since they also occur for unbiased models. In greater detail: (1) Measurements of \( \beta \) derived by fitting the Cole et al. (1995) exponential velocity distribution model to \( P_S(k)/P_R(k) \) underestimate the large-scale values of \( \beta_P \) by about 10-20% for all cosmological models and biasing prescriptions. (2) Measurements of \( \beta \) derived by fitting the Hatton & Cole (1999) empirical model to \( P_2(k)/P_0(k) \) accurately reproduce the large-scale values of \( \beta_P \) for all cosmological models and biasing prescriptions. (3) Measurements of \( \beta(R) \) derived from a POTENT-like comparison of \(-\nabla \cdot \mathbf{v}\), the negative divergence of the peculiar velocity field, to \( \delta_g \), the galaxy density field, underestimate the large-scale values of \( \beta_\sigma \) by about 10-20% for all cosmological models and biasing prescriptions, with the exception of the Sqrt-Exp. biasing prescription. The Sqrt-Exp. model gives erratic results because the non-linearity of the bias model makes the estimated \( \beta \) sensitive to the range of \( \delta_g \) used in the linear fit. (4) Estimates of \( \beta \) from the anisotropy of the redshift-space power spectrum approximately agree with those from the POTENT method on large scales, except in the case of the Sqrt-Exp. model.

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Figure 1. Comparison of $\beta$ estimates from different methods, as a function of scale. Each panel shows $\beta(R)$ for a particular cosmology and biasing prescription. The dashed line represents the function $\beta_\sigma(R) = \Omega_0^{0.6}/b_\sigma(R)$. Here, $b_\sigma(R)$ is the bias function defined as $b_\sigma(R) = \sigma_g(R)/\sigma_m(R)$, where $\sigma_g(R)$ and $\sigma_m(R)$ are the rms fluctuations of the galaxy and mass density fields, smoothed with a Gaussian filter of radius $R$. The solid and dotted lines show the estimates of $\beta$, along with their $1\sigma$ uncertainties, derived from fitting the exponential velocity distribution model (Cole et al. 1995) to $P^S(k)/P^R(k)$. The points represent POTENT-like $\beta$ estimates, derived by fitting a line to the measured relation between $-(\nabla \cdot \mathbf{v})$ and $\delta_g$, when the velocity and density fields are smoothed with Gaussian filters of different radii, $R$. Each point represents the weighted average over four independent simulations, and its $1\sigma$ error is computed as the error in the weighted mean. With the exception of the galaxy distributions created using the Sqrt-Exp. biasing prescription, the large-scale values of $\beta$ derived from the $-(\nabla \cdot \mathbf{v}) - \delta_g$ relation are similar to the large-scale values of $\beta_\sigma$, underestimating them by about 10-20%. Furthermore, these estimates are in approximate agreement with $\beta$ estimates derived from the anisotropy of the redshift-space power spectrum.