Mathematical analysis of the model of a low-melting metal coating on the surface of the guide

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Abstract. The work presents the asymptotic solution of the system of differential equations in the form of series in powers of the parameter due to the melt of the surface of the guide covered with a low-melting metal alloy and the rate of dissipation of mechanical energy, taking into account the dependence of the viscosity of the lubricant on pressure. Subsequently, a self-similar solution was found, as a result of which the fields of velocities, pressure and the main operating characteristics, load capacity and friction force were determined when the working gap was not completely filled with lubricant.

1. Introduction
The increase in loads on friction units, which affect their durability, efficiency and reliability, depends mainly on the design and quality of the bearings [1-9]. Therefore, when designing new machines and mechanisms, it becomes necessary to take into account additional factors. In the existing theoretical calculation methods [10-12] for plain bearings, one of the drawbacks in most cases is the fact that the incomplete (pre-emergency state) filling of the working gap with the lubricant is not taken into account at all, or is partially taken into account.

Thus, the development of a mathematical calculation model with an incomplete filling of the working gap with a lubricant when the surface of the guide is coated with a low-melting metal alloy is one of the promising areas of theoretical research in modern tribology. The latter determines the novelty and relevance of the results obtained.

2. Materials and Methods
A mathematical model is proposed that describes the motion of an incompressible true-viscous lubricant for a "thin layer" taking into account the dependence of viscosity and many additional factors in case of incomplete (pre-emergency state) filling of the working gap with lubricant from pressure. Also, a comparative analysis of the newly obtained results and the existing ones was carried out, which confirmed the approximation of the new model for solving practical problems.

A wedge-shaped sliding bearing is considered. It is assumed that the surface between the slider and the guide is not completely filled with lubricant, the slider is stationary, and the guide is covered with a low-melting metal alloy and moves around the axis Oy at speed $u^\ast$ (Fig. 1).

The following dependence gives the dependence of the viscosity of the lubricant on pressure:

$$\mu' = \mu_0 e^{q'p'},$$  \hspace{1cm} (1)
where $\mu_0$ is the characteristic viscosity, $\mu'$ is the coefficient of dynamic viscosity of the lubricant, $p'$ is the hydrodynamic pressure in the lubricating layer, $\tilde{\alpha}$ is a constant.

In the Cartesian coordinate system (Fig. 1), the equation of the linear slider contour and the guide molten surface are written as:

$$y' = h_0 + x' \tan \alpha, \quad y' = -\Phi(x), \quad (2)$$

where $h_0$ is the thickness of the lubricating film in the initial section, $\Phi_0$ is the thickness of the molten layer in the initial section, $\Phi(x)$ is the function characterizing the thickness of the melt of the guide surface, $\alpha$ is the angle of inclination of the linear contour of the slider to the axis $Ox'$. The initial basic equations are the dimensionless equation of motion of a viscous incompressible fluid in the "thin layer" approximation, the continuity equation and the equation that determines, taking into account the expression for the rate of dissipation of mechanical energy, the molten contour of the guide. In the Cartesian coordinate system, the above system of equations with boundary conditions will be written in the form:

$$\frac{\partial v}{\partial y} = 0, \quad \frac{\partial^2 v}{\partial y^2} = -\frac{dp}{dx}, \quad \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = 0, \quad \frac{d\Phi(x)}{dx} = K \int_{-\Phi(x)}^{\Phi(x)} \left( \frac{\partial v}{\partial y} \right)^2 dy, \quad (3)$$

$$v = 0, \quad u = 0 \quad \text{at} \quad y = h(x) = 1 + \eta x, \quad v = -1, \quad u = 0 \quad \text{at} \quad y = -\Phi(x),$$

$$\Phi = -\frac{h^*}{l} \quad \text{at} \quad x = 0, \quad p(x_1) = p(x_2) = 0, \quad \Phi(0) = 0. \quad (4)$$

where $u, v$ are the components of the velocity vector of the lubricating medium, $\eta = \frac{ln \alpha}{h_0}, \quad K = \frac{2 \mu^*}{h_0 L'}$ are the parameters due to the melt and the rate of dissipation of mechanical energy, $L'$ is the specific heat of fusion per unit volume, $x_1, x_2$ are the angular coordinates of the beginning and end of the free surface of the lubricant, respectively.

Dimensionless quantities are related to dimensional quantities by the following relationships:

$$x' = lx, \quad y' = h_0 y, \quad v_x = u' v, \quad v_y = u' \frac{h_0}{l} u, \quad p' = p^* \rho, \quad p^* = \frac{\mu^* L'}{h_0^2}, \quad \mu' = \mu^* \mu, \quad \alpha = p^* \tilde{\alpha}. \quad (5)$$

Let us introduce a notation, let $z = e^{-\omega p}$. Differentiation of both sides of the equality leads to the following form of equation (3):

$$\frac{\partial^2 v}{\partial y^2} = -\frac{1}{\alpha} \frac{dz}{dx}, \quad \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = 0, \quad z \frac{d\Phi(x)}{dx} = K \int_0^{\Phi(x)} \left( \frac{\partial v}{\partial y} \right)^2 dy, \quad (6)$$

with the boundary conditions:

$$v = 0, \quad u = 0 \quad \text{at} \quad y = h(x) = 1 + \eta x, \quad v = -1, \quad u = 0 \quad \text{at} \quad y = 0,$$

$$z(x_1) = z(x_2) = 1. \quad (7)$$
Write down the boundary conditions \( u \) and \( v \) for on the contour \( y = -\Phi(x) \):

\[
\begin{align*}
v(0 - H(x)) &= v(0) - \left( \frac{\partial v}{\partial y} \right)_{y=0} \cdot H(x) - \left( \frac{\partial^2 v}{\partial y^2} \right)_{y=0} \cdot H^2(x) + \ldots = -1 \\
u(0 - H(x)) &= u(0) - \left( \frac{\partial u}{\partial y} \right)_{y=0} \cdot H(x) - \left( \frac{\partial^2 u}{\partial y^2} \right)_{y=0} \cdot H^2(x) + \ldots = 0
\end{align*}
\]

We seek the asymptotic solution of the system of differential equations (3) - (4) and (8) in the form of series in powers of the parameter \( K \):

\[
v(x, y) = v_0(x, y) + K v_1(x, y) + K^2 v_2(x, y) + \ldots
\]

\[
u(x, y) = u_0(x, y) + K u_1(x, y) + K^2 u_2(x, y) + \ldots
\]

\[
\Phi(x) = -\Phi_0(x) - K \Phi_1(x) - K^2 \Phi_2(x) - \ldots
\]

\[
z(x) = z_0(x) + K z_1(x) + K^2 z_2(x) + \ldots
\]

Substituting (9) into the system of differential equations (3) - (4), we obtain the equations:

– for zero approximation (not taking into account melting):

\[
\begin{align*}
\frac{\partial^2 v_0}{\partial y^2} &= -\frac{1}{\alpha} \frac{dz_0}{dx}, \quad \frac{\partial v_0}{\partial x} + \frac{\partial u_0}{\partial y} = 0, \quad \frac{d\Phi_0(x)}{dx} = 0,
\end{align*}
\]

with the boundary conditions:

\[
v_0 = 0, \quad u_0 = 0 \quad \text{at} \quad y = 1 + \eta x, \quad v_0 = -1, \quad u_0 = 0 \quad \text{at} \quad y = 0,
\]

\[
z_0(x_1) = z_0(x_2) = 1, \quad \Phi_0 = \frac{h^*}{l}.
\]

– for the first approximation (taking into account the melting of the low-melting coating):

\[
\begin{align*}
\frac{\partial^2 v_1}{\partial y^2} &= -\frac{1}{\alpha} \frac{dz_1}{dx}, \quad \frac{\partial v_1}{\partial x} + \frac{\partial u_1}{\partial y} = 0, \quad -z_0 \frac{d\Phi_1(x)}{dx} = \int_0^{h(x)} \left( \frac{\partial v_0}{\partial y} \right)^2 dy,
\end{align*}
\]

with the boundary conditions:

\[
v_1 = \left( \frac{\partial v_0}{\partial y} \right)_{y=0} \cdot \Phi_1(x), \quad u_1 = \left( \frac{\partial u_0}{\partial y} \right)_{y=0} \cdot \Phi_1(x),
\]

\[
v_1 = 0, \quad u_1 = 0 \quad \text{at} \quad h(x) = 1 + \eta x, \quad z_1(x_1) = z_2(x_2) = 0.
\]

The solution to the problem for the zero approximation (not taking into account melting) will be sought in the form:

\[
v_0 = \frac{\partial \psi_0}{\partial y} + V_0(x, y), \quad u_0 = -\frac{\partial \psi_0}{\partial y} + U_0(x, y)
\]

\[
\psi_0(x, y) = \bar{\psi}_0(\xi); \quad \xi = \frac{y}{h(x)}.
\]
Substitution of (14) into the system of differential equations (10) - (11) allows obtaining the following system of differential equations:

\[
\begin{aligned}
\ddot{\psi}'' &= \tilde{C}_2; & \quad \ddot{v}'' &= \tilde{C}_1; & \quad \tilde{u}' - \xi \ddot{v} = 0; & \quad \frac{dz_0}{dx} = -\alpha \left( \frac{\tilde{C}_1}{h^2(x)} + \frac{\tilde{C}_2}{h^3(x)} \right);
\end{aligned}
\]

with the boundary conditions:

\[
\begin{aligned}
\ddot{\psi}_0(0) &= 0, & \ddot{\psi}_0'(0) &= 0, & \ddot{u}_0(0) &= 0, & \ddot{v}_0(0) &= 0; & \\
\ddot{u}_0(0) &= 0, & \ddot{v}_0(0) &= -1; & \int_0^1 \ddot{v}_0(\xi) \, d\xi &= 0.
\end{aligned}
\]

By direct integration, we get

\[
\begin{aligned}
\tilde{\psi}_0(\xi) &= \frac{\tilde{C}_1}{2} (\xi^2 - \xi), & \tilde{v}_0(\xi) &= \tilde{C}_1 \frac{\xi}{2} + \left( 1 - \frac{\tilde{C}_1}{2} \right) \xi - 1, & \tilde{C}_1 &= -6.
\end{aligned}
\]

From the condition \( z_0(x_1) = z_0(x_2) = 1 \) up to terms of the second-order of smallness \( O(\eta^2) \) for \( \tilde{C}_2 \), we obtain the expression:

\[
\tilde{C}_2 = 6 \left( 1 + \frac{\eta}{2} (x_2 - x_1) \right).
\]

Now we can write:

\[
z_0 = 3\eta \alpha \left( (x - x_1)^2 - (x - x_2)(x_2 - x_1) \right) + 1.
\]

The definition \( \Phi_1(x) \), taking into account equation (17), is reduced to the following equation:

\[
\frac{d\Phi_1(x)}{dx} = \frac{h(x)}{z_0} \left[ \frac{\tilde{\psi}_0''(\xi)}{h^2(x)} + \frac{\tilde{v}_0''(\xi)}{h(x)} \right] \, d\xi.
\]

Integration of equation (20) leads to:

\[
\Phi_1(x) = \left[ \int_0^1 \frac{\Delta dx}{h^2(x)} + \int_0^1 \frac{\Delta dx}{h^3(x)} + \int_0^1 \frac{\Delta dx}{h(x)} \right] \cdot 0.9998.
\]

\[
\Delta_1 = \int_0^1 \left( \Phi_0''(\xi) \right)^2 \, d\xi = \frac{\tilde{C}_2}{12}; \quad \Delta_2 = \int_0^1 2\Phi_0''(\xi) \tilde{v}_0''(\xi) \, d\xi = \tilde{C}_2; \quad \Delta_3 = \int_0^1 (\tilde{v}_0''(\xi))^2 \, d\xi = 4.
\]

The solution of equations (21) – (22) up to terms of the second-order of smallness leads to:

\[
\Phi_1(x) = 0.9998 \left[ 13x - \frac{25}{2} \eta x^2 + 9\eta (x_2 - x_1) \right].
\]

Taking into account (23), boundary conditions (13) have the following form:

\[
v_1 = 0, \quad u_1 = 0 \quad \text{at} \quad y = 1 + \eta x, \quad u_1 = \left( \frac{\partial u_0}{\partial y} \right)_{y=0} = 0,
\]

\[
v_1(x) = \left[ 1 + 5\eta x - 3\eta (x_1 + x_2) \right] \cdot \Phi_1(x), \quad z_1(x_1) = z_2(x_2) = 0.
\]
The solution of the equation of motion of a viscous incompressible fluid for the first approximation (12) taking into account the boundary conditions (24) using the Leibniz formula on the parameters is as follows:

\[
\frac{\partial}{\partial x} \int_{x_0}^{x_1} v_1(x)dy = 0, \quad \text{for } z_1 \text{ we get:}
\]

\[
z_1 = -\alpha \left[ 117\eta \left(x_1^3 - x^3\right) + \frac{117}{2} \eta (x_1 + x_2) \left(x_2^3 - x^3\right) \right] \cdot 0.9998,
\]

(25)

then

\[
z = z_0 + Kz_1 = 2.99\alpha \eta \left[ (x-x_1) - (x-x_1) (x_2-x_1) - K \left( 39 \left(x_2^3 - x^3\right) + \frac{39}{2} (x_1 + x_2) \left(x_2^3 - x^3\right) \right) \right] + 1
\]

(26)

Applying the expansion of the function \( e^{-\alpha p} \) up to terms inclusive \( O(\alpha^2) \) and solving the resulting equation up to \( O(\alpha^4) \), for the hydrodynamic pressure we obtain:

\[
p = 2.99\eta \left[ (x-x_1) - (x-x_1) (x_2-x_1) - K \left( 39 \left(x_2^3 - x^3\right) + \frac{39}{2} (x_1 + x_2) \left(x_2^3 - x^3\right) \right) \right].
\]

(27)

Determine the bearing capacity and friction force:

\[
W = p l \int_{x_1}^{x_2} dx = \frac{\mu_1 l l^2}{h_0} \left[ 2.99\alpha \eta \left[ \frac{x_1^2}{3} - \frac{x_1^3}{3} - x_1 \left(x_2^3 - x_1^3\right) + x_1 \left(x_2 - x_1\right) \right] - \left(\frac{x_2^2}{2} - \frac{x_1^2}{2}\right) (x_2 - x_1) + \right.

\left. + x_1 \left(x_2 - x_1\right)^2 - K \left[ 39 \left(x_2^4 - x_1^4\right) - x_1^4 \left(x_2^3 - x_1^3\right) + \frac{39}{2} \left(x_2^3 (x_2 - x_1) - (x_1 + x_2) \left(x_2^3 - x_1^3\right) \right) \right] \right],
\]

(28)

\[
L = \mu_0 \int_{x_1}^{x_2} \frac{\partial v_0}{\partial y} \bigg|_{y=0} + K \frac{\partial v_1}{\partial y} \bigg|_{y=0} dx = \frac{1 - \alpha p + \frac{\alpha^2 p^2}{2}}{1,0018} \left[ (x_2 - x_1) + \frac{5}{2} \eta (x_2^3 - x_1^3) - 3\eta (x_2^2 - x_1^2) + \right.

\left. + K \left( x_2 - x_1 \right) + 2\eta (x_2^3 - x_1^3) - \frac{117}{2} \left(x_2^3 - x_1^3\right) + \frac{117}{4} \eta (x_1 + x_2) \left(x_2^3 - x_1^3\right) \right].
\]

In an experimental study, a wedge-shaped sliding bearing with a low-melting metal coating made of Wood’s alloy was considered (Table 1). Based on the results of the experiments, the value of the friction coefficient was determined. The value makes it possible to judge the presence of a hydrodynamic friction regime both when the bearing is operating with a lubricant with Newtonian properties, and when the fusible coating of the guide surface is melted from Wood’s alloy. The temperature regime and the transition of the hydrodynamic friction regime to boundary friction were also determined. The analysis of experimental studies shows that the melt of a low-melting coating made of Wood’s alloy affects the friction coefficient 2.5 – 4 times more intensively than the rheological properties of the applied liquid lubricants. A set of experimental studies, which confirmed the reliability of the developed theoretical models and the data of their numerical analysis, in the considered range of design and operational parameters of wedge-shaped sliding bearings with low-
melting metal coatings from Wood's alloy, as a result of satisfactory convergence of theoretical and experimental results.

Table 1. Comparison of theoretical and experimental research

| №  | Theoretical research                      | Experimental research                      |
|----|------------------------------------------|-------------------------------------------|
|    | Thrust bearing without fusible coating   | Wood alloy low-melting metal-coated thrust bearing |
|    |                           | Wedge-shaped slide bearing with wood alloy low-melting metal coating |
| 1  | 0,0038                                   | 0,0019                                   | 0,0028       |
| 2  | 0,0042                                   | 0,0021                                   | 0,0031       |
| 3  | 0,0045                                   | 0,0022                                   | 0,0034       |
| 4  | 0,0047                                   | 0,0023                                   | 0,0035       |
| 5  | 0,0051                                   | 0,0024                                   | 0,0037       |

3. Results
Theoretical studies have shown that the bearing capacity with a low-melting metal coating of the guide surface, taking into account the dependence of the general rheological properties of the lubricant used and the melt of a low-melting metal coating with valid viscosity properties on pressure, increases ≈ by 5 – 8% with an increase in the parameter $\alpha$, which characterizes the dependence of viscosity on pressure, and the length $(x_1 - x_2)$ of the loaded region, and the coefficient of friction, in this case, decreases ≈ by 8 – 9%.

Based on the calculated models obtained in the theoretical part, an experimental study was carried out, because of which the area of promising operation of the developed tribosystem was determined.

As a result of experimental studies, tribological characteristics have been determined, which make it possible to judge the presence of the duration of the hydrodynamic friction regime and the reliability of the developed theoretical calculation models and the data of their numerical analysis.

4. Conclusions
New multivariable expressions have been developed for the main operating characteristics (bearing capacity and friction force) of a wedge-shaped sliding bearing, taking into account the rheological properties of a true-viscous lubricant with an incomplete filling of the working gap, and also taking into account the melt of the guide surface covered with a low-melting metal alloy.

An assessment of the influence of the parameters of variable factors caused by the melt of the surface of the guide covered with a low-melting metal alloy and the dependence of the viscosity of the lubricant on pressure when the working gap is not completely filled is given.

The obtained refined design models of wedge-shaped sliding bearings allow adjusting the ratio of its bearing capacity and the coefficient of friction as a result of varying the coating from a low-melting metal coating on the surface of the guide.

Satisfactory convergence of theoretical and experimental studies was established in support of the theoretical conclusions.
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