The hard scale in the exclusive $\rho$ meson production in diffractive DIS

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Abstract

We re-examine the issue of the pQCD factorization scale in the exclusive $\rho$ production in diffractive DIS from the $k_t$-factorization point of view. We find that this scale differs significantly from, and it possesses much flatter $Q^2$ behavior than widely used value $(Q^2 + m_{\rho}^2)/4$. With these results in mind, we discuss the $Q^2$ shape of the $\rho$ meson production cross section. We introduce rescaled cross sections, which might provide further insight into the dynamics of $\rho$ production. We also comment on the recent ZEUS observation of energy-independent ratio $\sigma(\gamma^* p \to \rho p)/\sigma_{tot}(\gamma^* p)$.

1 Introduction

The exclusive production of vector mesons in diffractive DIS

$$\gamma^{(*)} p \to V p$$

turned out to be an ideal testing ground \[1\] \[2\] \[8\] of many predictions of the famous color-dipole approach, \[1\] \[5\] \[6\]. Within this formalism, the basic quantity is the cross section of the color dipole interaction with the target proton $\sigma_{dip}(x, r)$, which can be approximately related to the conventional gluon density $G(x_g, \overline{Q}^2)$ inside the proton, \[4\]. The hard scale $Q^2$, at which the gluon density should be taken, is set both by the virtuality and the mass of the vector meson, and in the case of heavy quarkonium is approximately equal to $\frac{1}{4}(Q^2 + m_V^2)$. The latter result can be also obtained in the direct DGLAP-inspired calculations of the relevant Feynman diagrams, see, for example, the calculations of the exclusive production of $J/\psi$ mesons in diffractive DIS \[7\].

Although the identification of the pQCD factorization scale with $\frac{1}{4}(Q^2 + m_V^2)$ is valid only in the case of non-relativistic vector mesons, there is a hope that the same relation will hold for the light vector mesons and for large virtualities as well. This hope leads to the prediction of a remarkable universality: the cross section of light and heavy mesons will exhibit the same $Q^2$ behavior, if plotted against $Q^2 + m_V^2$ rather than $Q^2$ alone.

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On the experimental side, the first data showed that cross sections of \( \rho \), \( \omega \), \( \phi \) and \( J/\psi \) mesons, taken at equal values of \( Q^2 + m_V^2 \) and corrected by corresponding \( SU(4) \) factors, were indeed very close to each other, \([8]\). It is interesting to note that a similar dependences on \( Q^2 + m_V^2 \) were observed not only in the magnitudes of the cross sections of different vector meson production, but also in the values of the energy growth exponent, which is related to the Pomeron intercept, as well as in the slopes of the forward diffraction cone.

However, with the advent of new, more accurate data of \( \rho \) and \( J/\psi \) production, it became clear that this scaling was only approximate. Recently, ZEUS concluded \([9]\) that the \( J/\psi \) production cross section is typically 40\% higher that the cross sections of the light vector meson production. This difference is especially obvious on the plots of \( \sigma_L \) and \( \sigma_T \) separately, \([8]\).

A natural question arises as to what extent the universality of the \( Q^2 = (Q^2 + m_V^2)/4 \) as the relevant hard scale is valid. Since there is almost no room for doubt in the case of \( J/\psi \) production, the question can be formulated as what is the relevant pQCD factorization scale of the high \( Q^2 \) production of \( \rho \) mesons \(^1\).

The answer to this question was, in fact, given already in \([3]\). Within the color dipole formalism, the pQCD factorization scales for the longitudinally and transverse produced \( \rho \) mesons were shown to be \( \bar{Q}^2(\rho_L) \approx 0.15 \cdot (Q^2 + m_V^2) \) and \( \bar{Q}^2(\rho_T) \approx (0.07 \div 0.1) \cdot (Q^2 + m_V^2) \). This is notably smaller than \( (Q^2 + m_V^2)/4 \), and suggests that even the highest \( Q^2 \) experimental points in \( \rho \) production correspond to, at most, semiperturbative values of \( \bar{Q}^2(\rho) \).

In fact, the exact value of the pQCD factorization scale can be affected by the shape of the unintegrated gluon distribution. Unfortunately, when \([3]\) appeared, no numerically reliable parametrizations of dipole cross section or of the unintegrated gluon structure function were available, and one was bound to a semiquantitative guess. The situation changed two years ago, when numerically accurate, simple, and ready-to-use parametrizations of the unintegrated gluon structure function \( F(x_g, \vec{\kappa}) \) were obtained from the analysis of proton structure function \( F_{2p} \), \([10]\). These parametrizations were devised for \( x_{Bj} < 0.01 \) and for the entire domain of relevant \( Q^2 \) values. They were put in the basis of the \( k_t \)-factorization calculations of both light and heavy vector meson production cross sections and yielded rather good description of the available data, \([11, 12, 13]\). These fits now allow for a quantitative reanalysis of the hard scale in the \( \rho \) meson production. This is performed in the present paper.

Numerically accurate understanding of the hard scale in the \( \rho \) production is demanded in many applications. For example, a need for the hard scale arises when one attempts to understand the \( Q^2 \) behavior of the \( \rho \)-meson electroproduction. The early data could be successfully parametrized by a simple law

\[
\sigma(\gamma^* p \rightarrow \rho p) \propto \frac{1}{(Q^2 + m_{\rho}^2)^n}
\]

with \( n = 2.32 \pm 0.10 \) (ZEUS, \([14]\)) or \( n = 2.24 \pm 0.09 \) (H1, \([15]\)). However, with the advent of high precision data in a much broader \( Q^2 \) region, it became clear that such powerlike fits have very limited applicability domain. A natural question has been raised

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\(^1\)To be definite, we will deal with \( \rho \) mesons, but the general conclusions are obviously valid for all light vector mesons.
as what would be the most insightful and physically motivated fit to the $Q^2$ behavior of
\[ \sigma(\gamma^* p \rightarrow \rho p). \]

In virtually any description of the exclusive $\rho$ meson production, one has to deal with
the gluon content of the proton. Clearly, in order to experimentally study the underlying
mechanism of the vector meson production, one might want to get rid of the “trivial”
source of the $Q^2$ behavior that arises due to the gluon density. A simple way to do
this would be to divide the cross sections by the conventional gluon density squared and
study the $Q^2$ behavior of the result. However, this procedure requires knowledge of the
hard scale, at which the gluon density should be calculated.

Another issue that demands the understanding of the hard scale in $\rho$ production is
related to the energy behavior of the ratio
\[ r_\rho = \frac{\sigma(\gamma^* p \rightarrow \rho p)}{\sigma_{\text{tot}}(\gamma^* p)}, \]

The pQCD improved Regge model predicts that these cross sections rise with energy as
\[ \sigma(\gamma^* p \rightarrow \rho p) \propto (W^2)^{2\Delta_{\text{IP}}} \; ; \; \sigma_{\text{tot}}(\gamma^* p) \propto (W^2)^{\Delta_{\text{IP}}}, \]

where $\Delta_{\text{IP}} \equiv \alpha_{\text{IP}} - 1$ is the Pomeron intercept, so that the ratio is predicted to rise as
\[ r_\rho \propto (W^2)^{\Delta_{\text{IP}}}. \]

In last years, a large amount of data became available on the $\rho$ meson photo- and
electroproduction in a broad range of the total photon-proton energy $W$ and the photon
virtuality $Q^2$. This enabled the ZEUS collaboration to study the energy behavior
of $r_\rho$ at several values of $Q^2$. The results, [9], posed an apparent challenge: throughout
the whole $Q^2$ range, from $Q^2 = 0$ up to $Q^2 = 27 \text{ GeV}^2$, the ratio $r_\rho$ was found to be
energy independent. The conclusion of [9] was that neither Regge model, nor pQCD
approach can explain this constancy.

However, the Pomeron intercept is known to significantly depend on the hard scale
involved in the interaction, see experimental data [14] [15] and results of the
phenomenological analysis of [10]. Thus, when testing prediction (3), one must make
sure that both intercepts in (2) are taken at the same hard scale.

In [9] these scales were identified with $Q^2$ for the total virtual photoabsorption cross
section and with $Q^2_\rho = Q^2 \equiv (Q^2 + m_{\rho}^2)/4$ for the $\rho$ meson production cross section, so
that the experimental results were given for the quantity:
\[ r_\rho = \frac{\sigma(\gamma^* p \rightarrow \rho p)(W^2, Q^2_\rho)}{\sigma_{\text{tot}}(\gamma^* p)(W^2, Q^2)}. \]

Note that here the $\rho$ meson production cross section and the total virtual photoabsorption
cross section are taken at different virtualities.

As we argue in this paper, the true scale of the $\rho$ production can noticeably deviate
from $(Q^2 + m_{\rho}^2)/4$, especially at very small and very large $Q^2$. Thus, it appears that the
mismatch of the scales can be at least one of the sources of the discrepancy observed.

The structure of the paper is the following. In Section 2 we briefly review the results of
the $k_t$-factorization approach to the exclusive production of vector mesons in diffractive
DIS. For future guidance, we also show how these results simplify in the case of heavy
vector mesons. In Section 3 we study which gluons are relevant for the interaction, and
settle the factorization scales $\overline{Q}_L^2$ and $\overline{Q}_T^2$ for longitudinal and transverse amplitudes for any $Q^2$. Section 4 serves as an application of the results obtained to the study of the $Q^2$ behavior of the $\rho$ meson electroproduction cross section. In this Section we also propose new quantities, the rescaled cross sections, which might provide additional insight into the mechanism of the interaction. Finally, Section 5 is devoted to the discussion of the results and to the conclusions.

2 Exclusive production of vector mesons

Within the familiar color dipole formalism [4, 5], the production of a vector meson is viewed as a three-step process. First, at distances of the order of the coherence length $\ell_c \sim 1/(m_N x_{Bj})$ (where $x_{Bj}$ is Bjorken $x$ and $m_N$ is the nucleon mass) upstream the target, the incident photon splits into a $q\bar{q}$ pair, a color dipole. Then, interaction between the color dipole and the target takes place. This $q\bar{q}$ pair receives longitudinal momentum transfer, so that its invariant mass squared turns positive and close to the mass of a corresponding vector meson. Then, at large formation distances, this $q\bar{q}$ pair projects onto hadronic final states.

At high energies, the typical longitudinally size of the interaction region, of the order of radius of the proton, is much smaller than both the coherence length and the formation length. This allows one to treat the color dipole frozen during the interaction, and the basic quantity that appears in the approach is the diagonal color dipole cross section $\sigma_{\text{dip}}(x,r)$, which is related to the gluon density of the target proton. The analysis of [3] yielded the result that the pQCD factorization scales for the longitudinally and transverse produced $\rho$ mesons were $\overline{Q}_L^2(\rho_L) \approx 0.15 \cdot (Q^2 + m_V^2)$ and $\overline{Q}_T^2(\rho_T) \approx (0.07 \div 0.1) \cdot (Q^2 + m_V^2)$.

In this paper we reexamine this issue within the $k_t$-factorization approach, which is closely related to the color dipole formalism. As said above, our analysis is based on the parametrizations of the unintegrated gluon structure function that were given in [10].

2.1 The basic amplitude

Figure 1: The QCD-inspired diagrams for $\gamma^* p \to V p$ process with two gluon $t$–channel. Only Diagr.(a) contributes to the imaginary part of amplitudes.

Within the $k_t$-factorization approach, the imaginary part of the amplitude of the vector meson production is given by discontinuity of the diagram shown in Fig. 1a. The upper blob should be understood as sum of four diagrams, shown in Fig. 2. We denote the quark and gluon loop transverse momenta and the momentum transfer by $\vec{k}$, $\vec{\kappa}$, and $\vec{\Delta}$, respectively. The fraction of the photon lightcone momentum carried by quark is

\[ \kappa + \frac{1}{2}\Delta \]

\[ \kappa - \frac{1}{2}\Delta \]
denoted by $z$, while the fractions of the proton light cone momentum carried by the two gluons are $x_1$ and $x_2$. With this notation, the imaginary part of the amplitude can be written in a compact form:

$$ImA = s \frac{c v \sqrt{4 \pi \alpha_{em}}}{4 \pi^2} \int \frac{d^2 \vec{k}}{k^4} \alpha_s(q^2) F(x_1, x_2, \vec{r}_1, \vec{r}_2) \int \frac{dz d^2 \vec{k}}{z(1 - z)} \psi^*(z, \vec{k}) \cdot I(\lambda_V, \lambda_\gamma).$$ (4)

where $\vec{r}_{1,2} = \vec{k} \pm \frac{1}{2} \Delta$. The helicity-dependent integrands $I(\lambda_V, \lambda_\gamma)$ have form

$$I^S(L, L) = 4QMz^2(1 - z)^2 \left[ 1 + \frac{(1 - 2z)^2}{4z(1 - z)} \frac{2m}{M + 2m} \right] \Phi_2;$$

$$I^S(T, T) = (\varepsilon \vec{V}^*)[m^2 \Phi_2 + (\bar{k} \vec{\Phi}_1)] + (1 - 2z)^2(\bar{k} \vec{V}^*)(\bar{\varepsilon} \vec{\Phi}_1) \frac{M}{M + 2m} - (\bar{\varepsilon} \vec{k})(\bar{V}^* \Phi_1) + \frac{2m}{M + 2m}(\bar{\varepsilon} \vec{k})(\bar{V}^*) \Phi_2;$$

$$I^S(L, T) = 2Mz(1 - z)(1 - 2z)(\bar{\varepsilon} \vec{\Phi}_1) \left[ 1 + \frac{(1 - 2z)^2}{4z(1 - z)} \frac{2m}{M + 2m} \right] - \frac{Mm}{M + 2m}(1 - 2z)(\bar{\varepsilon} \vec{k}) \Phi_2;$$

$$I^S(T, L) = -2Qz(1 - z)(1 - 2z)(\bar{V}^* \vec{k}) \frac{M}{M + 2m} \Phi_2.$$ (5)

where

$$\Phi_2 = -\frac{1}{(\bar{\varepsilon} + \vec{\tau})^2 + \varepsilon^2} - \frac{1}{(\varepsilon - \vec{\tau})^2 + \varepsilon^2} + \frac{1}{(\bar{\varepsilon} + \vec{\Delta}/2)^2 + \varepsilon^2} + \frac{1}{(\varepsilon - \vec{\Delta}/2)^2 + \varepsilon^2};$$

$$\vec{\Phi}_1 = -\frac{\bar{\varepsilon} + \vec{\tau}}{(\bar{\varepsilon} + \vec{\tau})^2 + \varepsilon^2} - \frac{\varepsilon - \vec{\tau}}{(\varepsilon - \vec{\tau})^2 + \varepsilon^2} + \frac{\bar{\varepsilon} + \vec{\Delta}/2}{(\bar{\varepsilon} + \vec{\Delta}/2)^2 + \varepsilon^2} + \frac{\varepsilon - \vec{\Delta}/2}{(\varepsilon - \vec{\Delta}/2)^2 + \varepsilon^2},$$

with $\vec{\tau} \equiv \vec{k} - (1 - 2z)\Delta/2$ and $\varepsilon^2 = z(1 - z)Q^2 + m_0^2$. Finally, the strong coupling constant is taken at $q^2 = \max[\varepsilon^2 + \vec{k}^2, \vec{n}^2]$.

In the absence of $\Delta - \vec{r}$ correlations, and for a very asymmetric gluon pair, the off-forward gluon structure function $F(x_1, x_2, \vec{r}_1, \vec{r}_2)$ that enters (4) can be approximately related to the forward gluon density $F(g, \vec{r})$ via

$$F(x_1, x_2 \ll x_1, \vec{r}_1, \vec{r}_2) \approx F(g, \vec{r}) \cdot \exp \left( -\frac{1}{2} b_{g\gamma} \Delta^2 \right).$$
Here \( x_g \approx 0.41 x_1 \); the coefficient 0.41 is just a convenient representation of the off-forward to forward gluon structure function relation found in [15]. Numerical parametrizations of the forward unintegrated gluon density \( F(x_g, \vec{r}) \) for any practical values of \( x_g \) and \( \vec{r}^2 \) can be found in [10]. The slope \( b_{3F} \) can be experimentally accessed in the high-mass elastic diffraction.

The vector meson wave function \( \psi_V(z, \vec{k}) \) describes the projection of the quark-antiquark pair onto the physical vector meson. It is normalized so that the forward value of the vector meson form factor is unity, and the free parameters are chosen to reproduce the experimentally observed value of the vector meson electronic decay width \( \Gamma(V \to e^+e^-) \). In what concerns the shape of the radial wave function, we followed a pragmatic strategy. Namely, we took a simple Ansatz for the wave function, namely, the oscillator type wave function and performed all calculations with it. In order to control the level of uncertainty, introduced by the particular choice of the wave function, we redid calculations with another wave function Ansatz, namely, the Coulomb wave function, and compared the results. Since these two wave functions represent the two extremes (very compact and very broad wave functions that still lead to the same value of the electronic decay width), this difference gives a reliable estimate of the uncertainty. It is given typically by a factor of 1.5 for magnitudes of the cross sections, while in the observables that involve ratios of the cross sections (including slopes and intercepts) this uncertainty is reduced. Details can be found in [13, 11].

Note also that when deriving (5), we treated the vector mesons as 1S wave states and used the pure 5-wave spinorial structure \( S^\mu \) instead of \( \gamma^\mu \), [19].

### 2.2 The heavy meson limit and the scaling property

It is instructive to look at the production amplitude (4) in the heavy quarkonium limit. In this case, the vector mesons is essentially non-relativistic, \( z \to 1/2, \vec{k}^2 \ll \vec{m}_V^2 \), and

\[
\epsilon^2 = z(1-z)Q^2 + m_\gamma^2 \approx \frac{1}{4}(Q^2 + m_V^2) \equiv \overline{Q}^2.
\]

Even in the photoproduction limit, the mass of the meson sets the hard energy scale of the interaction, and one should expect the DGLAP-inspired leading \( \log \overline{Q}^2 \) approximation to be a viable approach to the problem. The DGLAP approximation of the heavy vector meson production implies the following ordering

\[
\vec{k}^2 \ll \vec{r}^2 \ll \overline{Q}^2,
\]

which leads to simplifications in the production amplitude. Consider the forward scattering. In this case only helicity conserving amplitudes survive, and one obtains

\[
I(L, L) = \vec{r}^2 \frac{8Qm_V}{(Q^2 + m_V^2)^2}; \quad I(T, T) = \vec{r}^2 \frac{8m_V^2}{(Q^2 + m_V^2)^2}.
\]

We reiterate that this simplification works while \( \vec{r}^2 \) is sufficiently small, namely, \( \vec{r}^2 \ll \overline{Q}^2 \). In the spirit of the DGLAP approach, the integrands \( I(\lambda_V; \lambda_s) \) do not depend on \( \vec{r}^2 \), aside from the overall \( \vec{r}^2 \) factor, whose origin is the color neutrality of the proton. Then, integration of \( \vec{r}^2 \) immediately leads to the conventional gluon density:

\[
\int_0^\infty \frac{d\vec{r}^2}{\kappa^2} \alpha_s(\vec{r}^2) F(x_g, \vec{r}^2) I(T, T) \to \frac{8m_V^2}{(Q^2 + m_V^2)^2} \int_0^{\overline{Q}^2} \frac{d\vec{r}^2}{\kappa^2} \alpha_s(\vec{r}^2) F(x_g, \vec{r}^2) \approx \frac{8m_V^2}{(Q^2 + m_V^2)^2} \cdot \alpha_s(\overline{Q}^2) \cdot G(x_g, \overline{Q}^2).
\]
The integration of the wave function can be related to the electronic decay widths of the vector meson. The result for the forward differential cross section reads, \[7\]:

\[
\frac{d\sigma}{d|t|} \bigg|_{t=0} = \frac{\pi^3}{12\alpha_{em}} m_V \Gamma(V \to e^+e^-) \left[ \frac{\alpha_s(Q^2) \cdot G(x_g, Q^2)}{Q^6} \right]^2.
\]

Note that in this limit the difference between the pure S-wave vector meson and the vector meson with \(\gamma^\mu\) coupling vanishes.

Eq.\(6\) identifies the hard scale that defined the gluon content of the proton in such a reaction with the quantity \(Q^2 = (Q^2 + m^2_\rho)/4\). Although this derivation is valid only for very heavy vector mesons, it is often hoped that for the light mesons the hard scale will have the same form. We show below that this is not true.

This result, although an approximation, suggests the scaling property of the vector mesons production: the \(Q^2\) behavior of all vector mesons cross sections should be proportional to each other if taken at equal \(Q^2 + m^2_V\). Note that \(6\) does not suggest that the absolute values of the cross sections should be equal, even when corrected by the relevant flavor content factors \(c^2_V\), simply due to the fact that this approach does not predict universality of \(m_V \cdot \Gamma(V \to e^+e^-)\).

3 The scale in exclusive \(\rho\) production

In this section we perform an analysis of the amplitude \(4\), taken for simplicity at \(\vec{\Delta} = 0\), and find what values of the transverse gluon momenta are essential in the interaction. These values will define the hard scale of the conventional gluon density. Again, this analysis is essentially similar to what was done in \(3\).

3.1 Weight functions: mapping the glue

We start with a qualitative analysis of the production of longitudinal \(\rho\) mesons at high \(Q^2\). Let us for a moment neglect the scaling violations and the other sources of marginal \(\vec{k}^2\) behavior, and rewrite the amplitude \(4\) in the following simplified form:

\[\text{Im} A_{L \to L} \propto \int \frac{d\vec{k}^2}{\vec{k}^2} F(x_g, \vec{k}) \cdot W_L(\vec{k}^2) ; \quad W_L \approx \frac{1}{\vec{k}^2} \int_0^1 dz z(1-z) \int d^2 \vec{k} \psi(z, \vec{k}) \Phi_2\]

with

\[\Phi_2 = -\frac{1}{(\vec{k} + \vec{\kappa})^2 + \varepsilon^2} - \frac{1}{(\vec{k} - \vec{\kappa})^2 + \varepsilon^2} + \frac{2}{\vec{k}^2 + \varepsilon^2} .\]

At large \(Q^2\), the most of the \(\vec{\kappa}^2\) integration comes from the logarithmic region with \(\vec{\kappa}^2 \ll \vec{\kappa}^2 \ll \varepsilon^2\). This leads to simplifications:

\[W_L(\vec{\kappa}^2) \approx \int_0^1 dz z(1-z) \int d^2 \vec{k} \psi(z, \vec{k}) \cdot \frac{2}{\varepsilon^2(\varepsilon^2 + \vec{\kappa}^2)} .\]

If we denote by \(\langle \varepsilon^2 \rangle\) the typical values of \(\varepsilon^2 = z(1-z)Q^2 + m^2_\rho\) that dominate the integral \(7\), then the weight factor \(W_L(\vec{\kappa}^2)\) stays almost constant at \(\vec{\kappa}^2 \ll \langle \varepsilon^2 \rangle\), and quickly decreases for \(\vec{\kappa}^2 > \langle \varepsilon^2 \rangle\). It should have a form of “smoothed step function”, and effectively cuts off from above the \(\kappa^2\) region essential for the interaction. If we further
approximate it by the exact step function \( \theta(\kappa^2 - \bar{Q}_L^2) \) with properly defined cut-off scale \( \bar{Q}_L^2 \). Then the integration (7) can be done exactly:

\[
\int_0^\infty \frac{d\kappa^2}{\kappa^2} F(x_g, \kappa) W_L(\kappa^2) \Rightarrow W_L(0) \cdot \int_0^\infty \frac{d\kappa^2}{\kappa^2} F(x_g, \kappa) = W_L(0) \cdot G(x_g, \bar{Q}_L^2). \tag{8}
\]

This transition is precisely what is implied in the DGLAP approach, recall derivation of (6). Thus, a procedure that would yield the values of the relevant hard scale, at which the conventional gluon density should be taken, is the following: take the weight function \( W_L(\kappa^2) \) and find the value \( \bar{Q}_L^2 \) that would lead to the most justified replacement (8).

Obviously, the scale \( \bar{Q}_L^2 \) should be close to \((\varepsilon^2 + \bar{r}_2)\). Due to the broad wave function of the \( \rho \), the typical values of \( z(1-z) \) will be less than 1/4. As a result, the scale of the gluon density in (8) should be noticeably softer than \((Q^2 + m_\rho^2)/4\).

In the case of the production of the transversely polarized \( \rho \) mesons, the contribution of the very asymmetric \( q\bar{q} \) pairs is enhanced. This leads to typical values of \( z \) still smaller, and the departure of the corresponding factorization scale \( \bar{Q}^2_T \) from \((Q^2 + m_\rho^2)/4\) becomes even more prominent.

### 3.2 Numerical results

The above expectations are corroborated by the numerical analysis. The functions \( W_L(\kappa^2) \) and \( W_T(\kappa^2) \) are defined now via

\[
\frac{1}{s} Im A_{L\rightarrow L} \equiv \int \frac{d\kappa^2}{\kappa^2} F(x_g, \kappa) \cdot W_L(\kappa^2); \quad \frac{1}{s} Im A_{T\rightarrow T} \equiv \int \frac{d\kappa^2}{\kappa^2} F(x_g, \kappa) \cdot W_T(\kappa^2)
\]

for all values of \( Q^2 \). Their behavior for \( Q^2 = 100 \text{ GeV}^2 \) is illustrated in Fig. (3) where we show the ratios \( W_L(\kappa^2)/W_L(0) \) and \( W_T(\kappa^2)/W_T(0) \) as functions of \( \kappa^2 \). One sees that these weight functions start decreasing at \( \kappa^2 \ll Q^2 \) and reach 1/2 at \( \kappa^2 \approx 15 \text{ GeV}^2 \) and \( \kappa^2 \approx 10 \text{ GeV}^2 \), respectively. This shows that the above mentioned effect of the broad wave function leads to significant softening of the relevant scale.

The factorization scales \( \bar{Q}_L^2 \) and \( \bar{Q}_T^2 \) can be defined, in principle, in several ways. As a trial definition, one can look at the \( \kappa^2 \) points, where \( W_i(\kappa^2) \) reach \( \frac{1}{s} W_i(0) \). Defined so, the longitudinal and transverse scales were found to be approximately equal to \( \frac{1}{s}(Q^2 + 2.0 \text{ GeV}^2) \) and \( \frac{1}{s}(Q^2 + 2.6 \text{ GeV}^2) \), respectively. The numbers \( \frac{1}{s} \) and \( \frac{1}{s} \) are very close to 0.15 and 0.07 - 0.1 obtained in [9].

The gluon density, however, has itself significant \( \kappa^2 \) dependence, [10]. Namely, in the region \( \kappa^2 \sim 1 \div 10 \text{ GeV}^2 \) and very small \( x_g \) \( (x_g \lesssim 10^{-3}) \) (which corresponds, at fixed \( W = 75 \text{ GeV} \), to values of \( Q^2 \lesssim 10 \text{ GeV}^2 \), \( F(x_g, \kappa) \) is a strongly rising function of \( \kappa^2 \).

At larger \( Q^2 \), the effective \( x_g \) grows, and the unintegrated gluon density becomes flat. Finally, at large enough \( Q^2 \) (for \( W = 75 \text{ GeV} \), this corresponds to \( Q^2 \gtrsim 100 \text{ GeV}^2 \)), the unintegrated gluon density decreases with \( \kappa^2 \) growth in the region \( \kappa^2 \sim 1 \div 10 \text{ GeV}^2 \). Therefore, the span of effectively contributive \( \kappa^2 \) will extend to higher values of \( \kappa^2 \) (at small \( Q^2 \)) or reduce to smaller values of \( \kappa^2 \) (at high \( Q^2 \)).

In order to take this into account, it is more useful to define the hard scales via the following implicit relations:

\[
\frac{1}{W_i(0)} \int_0^\infty \frac{d\kappa^2}{\kappa^2} F(x_g, \kappa) W_i(\kappa^2) \equiv G(x_g, \bar{Q}^2_i) \quad i = L, T \tag{9}
\]
Figure 3: The normalized weight functions $W_L(\kappa^2)/W_L(0)$ and $W_T(\kappa^2)/W_T(0)$ calculated at $Q^2 = 100$ GeV$^2$. The $\kappa^2$ values where $W_L$ and $W_T$ reach 1/2 are noticeably softer than $(Q^2 + m_\rho^2)/4$.

Figure 4: The $Q^2 \rightarrow Q_L^2$ and $Q^2 \rightarrow Q_T^2$ mapping in the $\rho$ production. The heavy meson analysis expectation $(Q^2 + m_\rho^2)/4$ is also shown.

Fig. 4 shows the values of $Q_L^2$ (solid line) and $Q_T^2$ (dashed line), defined according to $\mathbf{11}$, as functions of $Q^2$. These values start from 0.63 GeV$^2$ and 0.4 GeV$^2$, respectively, in the photoproduction limit, and slowly grow with $Q^2$ rise. At $Q^2 = 27$ GeV$^2$ (the highest $Q^2$ data point from H1 data on $\rho$ production), these values are only around
4.5 GeV$^2$ and 3 GeV$^2$, respectively. This confirms the conclusion of [3] that even at largest $Q^2$ where data are available, we still deal with semiperturbative situation. It is interesting to note that a better fit to these curves is given by a non-linear, rather than linear approximation:

$$\mathcal{Q}_L^2 \approx 1.5 \cdot \mathcal{Q}_T^2 \approx 0.45 \cdot (Q^2 + 1.5)^{0.7},$$

where all quantities are expressed in GeV$^2$.

The same Figure shows also, by dotted line, the expectation $(Q^2 + m^2_\rho)/4$ inspired by the heavy meson analysis. This expectation starts from 0.15 GeV$^2$, which is noticeably smaller than $\mathcal{Q}_L^2(0)$ and $\mathcal{Q}_T^2(0)$, and rises with $Q^2$ significantly faster than $\mathcal{Q}_L^2$ and $\mathcal{Q}_T^2$.

3.3 The $Q^2$ dependence of the weight functions

In the heavy meson approximation [4], the scale $\mathcal{Q}^2 = (Q^2 + m^2_\rho)/4$ defines not only the hard scale of the gluon density, but also the absolute value of the cross section. It is interesting to check whether such a universality holds for the $\rho$ production. This can be done by studying the $Q^2$ dependence of the $W_L(0)$ and $W_T(0)$. Motivated by (6), we introduce the “effective twists” $\varepsilon^2_L$ and $\varepsilon^2_T$ via the following relations:

$$W_L(0) = 0.153 \cdot Q \cdot \frac{\alpha_s(\varepsilon^2_L)^2}{(\varepsilon^2_L)^2}; \quad W_T(0) = 0.153 \cdot m_\rho \cdot \frac{\alpha_s(\varepsilon^2_T)^2}{(\varepsilon^2_T)^2}.$$ (11)

Here, factor 0.153 GeV = $\sqrt{\Gamma(\rho \to e^+e^-)/4\pi m_\rho}$ is the numerical value of all the residual factors present in the definition of $W_i$. This definition of the “effective twists” $\varepsilon^2_L$ and $\varepsilon^2_T$ is such that in the heavy meson limit one would recover the familiar $\mathcal{Q}^2 = 0.25 \cdot (Q^2 + m^2_\rho)$. These “effective twists” were found to be approximately equal to

$$\varepsilon^2_L \approx 0.23 \cdot (Q^2 + 1.1); \quad \varepsilon^2_T \approx 0.16 \cdot (Q^2 + 1.3).$$ (12)

Again, all quantities are expressed here in GeV$^2$. It is interesting to note that here, as well as in [10], the virtuality $Q^2$ sums with $M^2 \sim 1 \div 1.5$ GeV$^2$ instead of widely assumed $m_\rho = 0.6$ GeV$^2$. This is not surprising, since the mass of the $\rho$ meson does not have much relevance to the interaction of the $q\bar{q}$ dipole with the proton.

4 The $Q^2$ behavior of the $\rho$ production cross section

Let us turn to an issue that is closely related to the above discussion of the hard scales in the $\rho$ production, namely, to the $Q^2$ behavior of the exclusive $\rho$ meson production cross section.

As mentioned in the Introduction, the early data on $\rho$ mesons were successfully parametrized (in the moderate and high-$Q^2$ region) by a simple law

$$\sigma(\gamma^*p \to \rho p) \propto \frac{1}{(Q^2 + m^2_\rho)^n}$$

with $n = 2.32 \pm 0.10$ (ZEUS, [14]) or $n = 2.24 \pm 0.09$ (H1, [15]). However, subsequent data made it clear that such power fits have very limited applicability. One thus can ask
what would be the best way to parametrize this behavior. Note that this is not merely a question of how to fit the data, but rather a question of what are the main sources of $Q^2$ behavior of the cross section.

Let us take the heavy meson limit result (6) as a starting point. It states that the cross section should quickly fall with $Q^2$ due to powers of $Q^2 + m^2$, the fall being just slightly tamed by the rise of the gluon density. In order to check whether or not these are the only sources of the $Q^2$ behavior of the $\rho$ meson cross section, let us consider a rescaled cross section

$$\Sigma(Q^2) = \sigma(\gamma^*p \rightarrow \rho p) \cdot \frac{(Q^2)^3 \cdot b(Q^2)}{G(x_g, Q^2) \cdot \alpha_s(Q^2)^2}.$$  \hspace{1cm} (13)

Here, as before, $Q^2 \equiv (Q^2 + m^2)/4$. Note again that the gluon density is taken here at constant energy ($W = 75$ GeV), so that $x_g = 0.41(Q^2 + m^2)/W^2$ also depends on $Q^2$. From the heavy meson limit expression (6) one might hope that $\Sigma(Q^2)$ is roughly independent of $Q^2$, since all the sources of $Q^2$ dependence are removed.

![Rescaled Cross Sections](image)

**Figure 5:** The rescaled cross section $\Sigma(Q^2)$ defined according to (13).

Fig. 5 shows the $k_t$-factorization predictions for $\Sigma(Q^2)$ as function of $Q^2$. One observes that this rescaled cross section has quite a flat shape, from very low to very high $Q^2$, regardless of the exact wave function used.
In principle, all the observables used in (13) are accessible in experiment. The only delicate issue would be the choice of the gluon density, especially at low $Q^2$, since the DGLAP fits to conventional gluon density are available only for $Q^2 \gtrsim 1$ GeV$^2$ and can differ significantly from the $k_t$-factorization results, see [10]. It would be still interesting to see, whether the experimental data possess such a scaling.

In the light of the results of the previous section, it is somewhat surprising that (13), inspired by the heavy meson limit, is nevertheless rather flat. If, however, we take a look at $\sigma_L$ and $\sigma_T$ separately, and define

$$
\Sigma_L(Q^2) = \frac{\sigma_L}{Q^2} \cdot \frac{Q^8 \cdot b(Q^2)}{\left[ G\left(x_g, Q^2\right) \cdot \alpha_s(Q^2) \right]^2}; \quad \Sigma_T(Q^2) = \frac{\sigma_T}{m^2_p} \cdot \frac{Q^8 \cdot b(Q^2)}{\left[ G\left(x_g, Q^2\right) \cdot \alpha_s(Q^2) \right]^2}, \quad (14)
$$

we will see that this remarkable $Q^2$-independence holds for $\Sigma_L(Q^2)$, but not for $\Sigma_T(Q^2)$, Fig. 6, left panel. This difference between $\Sigma_L(Q^2)$ and $\Sigma_T(Q^2)$ can be regarded as a direct proof that the hard scales relevant for longitudinal and transverse cross sections are different.

If we take into account the results of the previous section and introduce

$$
\tilde{\Sigma}_L(Q^2) = \frac{\sigma_L}{Q^2} \cdot \frac{(\varepsilon_L^2)^4 \cdot b(Q^2)}{\left[ G\left(x_g, Q^2\right) \cdot \alpha_s(\varepsilon_L^2) \right]^2}; \quad \tilde{\Sigma}_T(Q^2) = \frac{\sigma_T}{m^2_p} \cdot \frac{(\varepsilon_T^2)^4 \cdot b(Q^2)}{\left[ G\left(x_g, Q^2\right) \cdot \alpha_s(\varepsilon_T^2) \right]^2}, \quad (15)
$$

then these rescaled quantities have flat $Q^2$ shape both for longitudinal and transverse cross sections, Fig. 6, right panel. Moreover, $\tilde{\Sigma}_L(Q^2)$ and $\tilde{\Sigma}_T(Q^2)$ are close to each other, which should be expected by the construction of the “effective twists” and factorization

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**Figure 6:** (left panel) The rescaled cross sections $\Sigma_L(Q^2)$ and $\Sigma_T(Q^2)$ defined according to (14). The $Q^2$ behavior of $\Sigma_L(Q^2)$ and $\Sigma_T(Q^2)$ are very different. (right panel) The rescaled cross sections $\tilde{\Sigma}_L(Q^2)$ and $\tilde{\Sigma}_T(Q^2)$ defined according to (15). The two quantities are close to each other and both have flat $Q^2$ behavior.
scales. These curves are not exact constants due to non-zero average value of $\bar{\Delta}^2$ and due to the presence of helicity-flip amplitudes.

Again, it would be very interesting to see a similar analysis of the experimental data in terms of $\tilde{\Sigma}(Q^2)$. In particular, it would be interesting to see whether such an analysis can help resolve the long-standing $\sigma_T$-problem of the $k_t$-factorization predictions [12, 13].

5 Discussion and conclusions

The most of the analysis of the previous sections was done with the oscillator wave function. The same analysis with other wave function Ansätze will yield slightly different numbers in (10) and (12). Nevertheless, the general picture will remain the same. Namely, several observations are stable against the variations of the exact shape of the wave function:

- at smaller $Q^2$ ($Q^2 \lesssim 3 \div 5 \text{ GeV}^2$) the DGLAP factorization scale is larger than $(Q^2 + m_\rho^2)/4$;
- at large enough $Q^2$ ($Q^2 \gtrsim 10 \text{ GeV}^2$ for the transverse amplitude and $Q^2 \gtrsim 20 \text{ GeV}^2$ for the longitudinal amplitude), the factorization scale is significantly smaller than $(Q^2 + m_\rho^2)/4$. This should be taken as a word of caution against an unwarranted application of the DGLAP approach to the problem of $\rho$ meson production even at high $Q^2$;
- the overall $Q^2$ dependence of the pQCD factorization scale is significantly flatter than $(Q^2 + m_\rho^2)/4$. This is mostly due to the specific way the $\bar{\kappa}^2$-behavior of the unintegrated gluon density changes, as the $Q^2$ increases (at fixed $W$).
- the presence of $m_\rho$ in the often used scale $(Q^2 + m_\rho^2)/4$ is misleading, since the $\rho$ meson mass has little relevance to the color dipole interaction with the target proton. Instead, $Q^2$ appears in combinations of the form of $Q^2 + M^2$ with $M^2 \approx 1 \div 1.5 \text{ GeV}^2$.

The numerically accurate understanding of the relevant hard scales in the $\rho$ production allowed us to study in detail the $Q^2$ dependence of the $\rho$ production cross section. With the help of the rescaled cross sections (13)–(15), Figs. 5 and 6, we showed once more that the production of transverse and longitudinal vector mesons is governed by distinct hard scales. When we took into account the difference of the scales just found, we observed a very close similarity between the rescaled cross sections $\tilde{\Sigma}_L(Q^2)$ and $\tilde{\Sigma}_T(Q^2)$. It would be very interesting to see the results of a similar analysis of the experimental data. This analysis might help resolve the long-standing problem of too small $\sigma_T$ at high $Q^2$.

Let us also comment on a contribution to the recent puzzle of energy independence of $r_\rho = \sigma(\gamma^* p \rightarrow \rho p)/\sigma_{\text{tot}}(\gamma^* p)$ ratio. As was discussed in this paper, at higher $Q^2$, the true factorization scale is smaller than $(Q^2 + m_\rho^2)/4$. Inverting the argument, one can state that $\sigma_{\text{tot}}(Q^2_{\text{tot}} = (Q^2 + m_\rho^2)/4)$ is expected to be harder than $\sigma_\rho(Q^2)$. This should enhance the expected value of the Pomeron intercept in $\sigma_{\text{tot}}$ and reduce the prediction of the energy behavior of $r_\rho$. Unfortunately, our numerical analysis showed that this effect is marginal and does not lead to resolution of the problem.

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