RADIATION PRESSURE-SUPPORTED ACCRETION DISKS: VERTICAL STRUCTURE, ENERGY ADVECTION, AND CONVECTIVE STABILITY

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ABSTRACT

By taking into account the local energy balance per unit volume between the viscous heating and the advective cooling plus the radiative cooling, we investigate the vertical structure of radiation pressure-supported accretion disks in spherical coordinates. Our solutions show that the photosphere of the disk is close to the polar axis and therefore the disk seems to be extremely thick. However, the density profile implies that most of the accreted matter exists in a moderate range around the equatorial plane. We show that the well-known polytropic relation between the pressure and the density is unsuitable for describing the vertical structure of radiation pressure-supported disks. More importantly, we find that the energy advection is significant even for slightly sub-Eddingon accretion disks. We argue that the non-negligible advection may help us understand why the standard thin disk model is likely to be inaccurate above \( \sim 0.3 \) Eddington luminosity, which was found by some works on black hole spin measurement. Furthermore, the solutions satisfy the Solberg-Høiland conditions, which indicate the disk to be convectively stable. In addition, we discuss the possible link between our disk model and ultraluminous X-ray sources.

Key words: accretion, accretion disks – black hole physics – convection – hydrodynamics – instabilities

1. INTRODUCTION

The standard thin accretion disk model (Shakura & Sunyaev 1973) has been widely applied to X-ray binaries and active galactic nuclei. Due to the basic assumption of energy balance between viscous heating and radiative cooling, such a model was known to be invalid for the super-Eddington accretion case, where the advective cooling is probably significant. Instead, the slim disk model (Abramowicz et al. 1988) was introduced to describe super-Eddington accretion disks. However, there exists some conflict between theory and observation. The theory predicts that the advection is negligible for \( L \lesssim L_{\text{Edd}} \) (e.g., Watarai et al. 2000; Sadowski 2011), where \( L_{\text{Edd}} \) is the Eddington luminosity, which indicates that the standard thin disk model should be valid up to \( L_{\text{Edd}} \). On the contrary, some works on the black hole spin measurement showed that the standard disk model is likely to be inaccurate for \( L \gtrsim 0.3L_{\text{Edd}} \) (e.g., McClintock et al. 2006). Moreover, even the recent general model for optically thick disks (e.g., Sadowski et al. 2011), which unifies the standard thin disk and the slim disk, could not help us obtain a self-consistent spin parameter for \( L \gtrsim 0.3L_{\text{Edd}} \) (e.g., Straub et al. 2011). In our opinion, the above conflict may be resolved if the vertical structure is well incorporated.

Most previous works on accretion disks focused on the radial structure in cylindrical coordinates \((r, \phi, z)\). For the vertical structure, however, a simple well-known relationship, \( H = c_s/\Omega_K \) or \( H\Omega_K/c_s = \text{constant} \), was widely adopted, where \( H \) is the half-height of the disk, \( c_s \) is the sound speed, and \( \Omega_K \) is the Keplerian angular velocity. Such a relationship comes from vertical hydrostatic equilibrium with two additional assumptions. One is the approximation of gravitational potential: \( \psi(R, z) \simeq \psi(R, 0) + \Omega_K^2 z^2/2 \), and the other is a one-zone approximation or a polytropic relation \( p_{\text{tot}} = K \rho^{\gamma+1}/N \) in the vertical direction (e.g., Hoshi 1977), where \( p_{\text{tot}} \) is the total (gas plus radiation) pressure and \( \rho \) is the density. Obviously, the above assumptions work well for geometrically thin disks, but may be inaccurate for a mass accretion rate \( \dot{M} \) approaching the Eddington one, \( M_{\text{Edd}} \), for which the disk is probably not thin. Consequently, the relationship \( H\Omega_K/c_s = \text{constant} \) may be invalid for \( M \gtrsim M_{\text{Edd}} \).

Without the potential approximation, our two previous works investigated the geometrical thickness of accretion disks and the validity of the relationship \( H\Omega_K/c_s = \text{constant} \). Gu & Lu (2007) adopted the explicit gravitational potential in cylindrical coordinates and found that the above relationship is inaccurate for \( M \gtrsim M_{\text{Edd}} \), and therefore the disk can be geometrically thick. Gu et al. (2009) used spherical coordinates to avoid the approximation of the gravitational potential, and found that an advection-dominated accretion disk is likely to be quite thick. In these two works, however, the polytropic relation is still adopted in the vertical direction, which takes the place of the energy balance per unit volume between viscous heating and advective cooling plus radiative cooling. The validity of such a polytropic relation, however, remains questionable, in particular for large \( M \) due to dominant radiation pressure.

The purpose of this paper is to revisit the vertical structure of radiation pressure-supported disks by taking into account the local energy balance and to study the variation of energy advection with mass accretion rates. The paper is organized as follows. Equations and boundary conditions are derived in Section 2. A global view of the solutions in the \( m-r \) diagram is presented in Section 3. For a typical radius \( r = 10r_g \), the vertical structure and the energy advection are investigated in Section 4. The two-dimensional solutions and the convective stability are studied in Section 5. The summary and discussion are in Section 6.

2. EQUATIONS AND BOUNDARY CONDITIONS

2.1. Equations

We consider a steady state axisymmetric accretion disk in spherical coordinates \((r, \theta, \phi)\) and use the Newtonian potential, \( \psi = -GM/r \), where \( M \) is the black hole mass. Following
Narayan & Yi (1995), we assume $v_0 = 0$ for simplicity, which means a hydrostatic equilibrium in the $\theta$ direction. Simulations (e.g., Ohnaga et al. 2005, Figure 3), however, revealed that $v_0$ will be significant for extremely high accretion rates such as $M = 1000 L_{\text{Edd}} / c^2$. As shown in the following sections, our solutions mainly correspond to $M$ around $M_{\text{Edd}}$. For such accretion rates, the validity of $v_0 = 0$ remains a question.

The basic equations of continuity and momentum take the forms (e.g., Kato et al. 2008):

$$
\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho v_r) = 0 ,
$$

(1)

$$
v_r \frac{\partial v_r}{\partial r} - \frac{v_r^2}{r} = - \frac{GM}{r^2} - \frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{\kappa_{\text{es}}}{c} F_r,
$$

(2)

$$
- \frac{v_r^2}{r} \cot \theta = - \frac{1}{\rho} \frac{\partial p}{\partial \theta} + \frac{\kappa_{\text{es}}}{c} F_\theta,
$$

(3)

$$
v_r \frac{\partial}{\partial r} (r v_\phi) = \frac{1}{\rho r^3} \frac{\partial}{\partial r} (r^3 \tau_{r\phi}),
$$

(4)

where $v_r$ and $v_\phi$ are respectively the radial and azimuthal velocity, $F_r$ and $F_\theta$ are respectively the radial and vertical radiation flux, $p$ is the gas pressure, $\kappa_{\text{es}}$ is the opacity of electron scattering, and $\tau_{r\phi}$ is the $r \phi$ component of the viscous stress tensor, $\tau_{r\phi} = \nu r \rho \partial (v_\phi / r) / \partial r$. Following the spirit of $\alpha$ stress prescription, we assume the kinematic viscosity coefficient $v = \alpha c_s^2 r / V_k$, where $c_s$ is the sound speed defined below (Equation (7)), and $V_k = (GM/r)^{1/2}$ is the Keplerian velocity.

We stress that, even though the $\alpha$ stress prescription has been widely adopted for theoretical studies, simulations of magnetorotational turbulence have shown that the stress does not scale well locally with the pressure. For instance, the simulations on thin disks by a shearing box showed that the time- and box-averaged results are likely to support that the stress is proportional to the thermal (gas plus radiation) pressure (e.g., Hirose et al. 2009a, Figure 3). However, Figure 16 of Hirose et al. (2009b) shows that the maximal thermal pressure is located on the equatorial plane, whereas Figure 11 shows that the maximal stress is obviously not at the same place. These two figures reveal that the stress is not proportional to the pressure locally.

In the present study, for simplicity, we will keep the local $\alpha$ stress prescription for numerical calculation, which is a weak point of this work.

The energy equation including gas and radiation is written as (e.g., Ohnaga et al. 2005)

$$
\nabla \cdot [(e + E)u] = - p \nabla \cdot v - \nabla v \cdot P - \nabla \cdot F + \Phi_{\text{es}},
$$

(5)

where $e$ and $E$ are the internal energy density of the gas and the radiation, respectively. $P = f E$ is the radiation pressure tensor, $\Phi_{\text{es}}$ is the viscous dissipative function, and the radiation flux $F$ is expressed as

$$
F = - \frac{\lambda c}{\rho \kappa_{\text{es}}} \nabla E.
$$

(6)

In this work, we focus on the region inside the photosphere, so we can take the well-known Eddington approximation, i.e., $\lambda = 1/3$ and the Eddington tensor $f = I/3$.

Since we only study the radiation-pressure-supported disks, the gas pressure $p$ and the gas internal energy density $e$ will be dropped in our calculation. In order to avoid directly solving the partial differential equations, some assumptions on the radial derivatives ($\partial / \partial r$) are required. Following the spirit of self-similar assumptions (e.g., Begelman & Meier 1982; Narayan & Yi 1995), we adopt the following radial derivatives for $c_s$ and $E$:

$$
\begin{align*}
\frac{\partial \ln c_s}{\partial \ln r} &= - \frac{3}{2} ; \\
\frac{\partial \ln E}{\partial \ln r} &= - \frac{5}{2},
\end{align*}
$$

where the sound speed $c_s$ is defined as

$$
c_s^2 \equiv \frac{E}{3 \rho}.
$$

(7)

Based on the above two radial derivatives, the following four derivatives can be inferred from Equations (1)–(7):

$$
\begin{align*}
\frac{\partial \ln |v_r|}{\partial \ln r} &= - \frac{1}{2} ; \\
\frac{\partial \ln v_\phi}{\partial \ln r} &= - \frac{1}{2} ; \\
\frac{\partial \ln \rho}{\partial \ln r} &= - \frac{3}{2} ; \\
\frac{\partial \ln F_r}{\partial \ln r} &= -2.
\end{align*}
$$

With all the above derivatives, we can remove the “$\partial / \partial r$” terms in Equations (2) and (4)–(6), and the following equations are then obtained from Equations (2)–(6):

$$
\begin{align*}
\frac{1}{2} v_r^2 + \frac{5}{2} c_s^2 + v_\phi^2 - v_K^2 &= 0, \\
v_\phi^2 \cot \theta &= - \frac{r \kappa_{\text{es}}}{c} F_\theta, \\
v_r &= \frac{3 \alpha c_s^2}{2 V_k}, \\
- \frac{1}{2} v_r (3 \rho v_\phi^2 - E) &= \frac{1}{\sin \theta} \frac{d}{d \theta} (\sin \theta F_\theta), \\
dE \frac{d \theta}{d \theta} &= - \frac{3 \rho \kappa_{\text{es}}}{c} F_\theta, \\
F_r &= - \frac{5 c E}{6 \rho \kappa_{\text{es}}^2}.
\end{align*}
$$

(8)

The seven equations, Equations (7)–(13), enable us to solve for the seven variables: $v_r, v_\phi, c_s, \rho, E, F_r$, and $F_\theta$. There are two differential equations in this system. In addition, the position of the surface is unknown. Thus, there are in total three boundary conditions required to determine a unique solution.

### 2.2. Boundary Conditions

An obvious boundary condition on the equatorial plane is $F_r = 0$. However, this condition is not applicable for numerical calculation since it is automatically matched as indicated by Equation (9). Combining Equations (9) and (11), we can derive the following equation:

$$
\cot \theta \frac{d}{d \theta} (v_\phi^2) = v_\phi^2 + \frac{r v_r \kappa_{\text{es}}}{2c} (3 \rho v_\phi - E).
$$

(14)

An alternative boundary condition on the equatorial plane is then obtained from the above equation (the left-hand side is zero and thus the right-hand side should also be zero):

$$
v_\phi^2 + \frac{r v_r \kappa_{\text{es}}}{2c} (3 \rho v_\phi - E) = 0 \quad (\theta = \frac{\pi}{2}).
$$

(15)
The parameter space is divided into three regions by two parallel solid lines. The middle region, denoted as “radiation pressure,” corresponds to the radiation pressure-supported disk, which is our main interest in this work. An example solution for \( m = 0.6 \) at \( r = 10r_g \) (filled circle) is shown in Figure 2. The solutions for various \( m \) at a typical radius \( r = 10r_g \) (vertical dashed line) are focused on in Section 4. The two-dimensional solutions for \( m = 0.6 \) (horizontal dot-dashed line) are studied in Section 5.

The second boundary condition is a definition of the surface. We define the photosphere as the position above which the optical depth is around unity. The condition can be written as

\[
\tau_{\text{eq}} = r\kappa_{\text{eq}}\rho^2 \frac{d\rho}{d\theta} = 1 \quad (\theta = \theta_0),
\]

where \( \theta_0 \) \((0 < \theta_0 < \pi/2)\) is the polar angle of the photosphere. The third condition is related to the mass accretion rate:

\[
\dot{M} = -2\pi r^2 \rho_{\text{eq}} \sin \theta \, d\theta.
\]

3. SOLUTIONS IN \( \dot{m}-r \) DIAGRAM

In our calculation we set \( \dot{M} = 10 \dot{M}_\odot, \kappa_{\text{es}} = 0.34 \text{ cm}^2 \text{ g}^{-1} \), and \( \alpha = 0.02 \), where the value of \( \alpha \) is taken from recent simulations (Hirose et al. 2009a). The Eddington accretion rate is expressed as \( \dot{M}_{\text{Edd}} = 4\pi GM/\eta c\kappa_{\text{es}} \), where \( \eta \) is the radiative efficiency of the flow. We choose \( \eta = 1/16 \) since it is comparable to the Schwarzschild black hole efficiency of 0.057. The dimensionless accretion rate is defined as \( \dot{m} \equiv \dot{M}/\dot{M}_{\text{Edd}} \).

With the equations and boundary conditions in Section 2, we can numerically derive the \( \theta \)-direction distribution of the physical quantities for a given \( \dot{m} \) at a certain radius \( r \). The radiation pressure-supported disk solutions in the \( \dot{m}-r \) diagram are shown in Figure 1, where \( r_g \equiv 2GM/c^2 \) is the Schwarzschild radius. The parameter space is divided into three regions by two parallel solid lines, roughly with \( m \propto r \). The region above the upper solid line is denoted as “Outflow,” where we cannot find solutions. No solution exists, probably due to the assumption of \( v_\theta = 0 \) in advance. In our view, a real flow located in this region may have \( v_\theta \neq 0 \) and the inflow accretion rate may decrease inward. The physical understanding could be that, for high accretion rates and particularly for the inner radii, the viscous dissipation may be sufficiently large such that the radiation pressure is too strong to be balanced by the gravitational force. Thus, outflows may be driven by the radiation pressure and the inflow \( \dot{m} \) drops inward. On the other hand, simulations of supercritical accretion flows (e.g., Ohsuga et al. 2005, Figure 6) showed that the inflow accretion rate roughly follows the \( \dot{m} \propto r \) relationship for \( \dot{m} = 1000L_{\text{Edd}}/c^2 \) at \( r_{\text{out}} = 500r_g \) (corresponding to \( \dot{m} = 62.5 \) due to the definition of \( M_{\text{Edd}} \) with \( \eta = 1/16 \)). The slope of the upper solid line in Figure 1, which may be regarded as the maximal accretion rate due to our calculation, agrees well with the slope in the above simulations.

The region under the lower solid line is denoted as “Gas pressure,” where no solution is found either. In our understanding, it is probably because the gas pressure cannot be ignored in this region, which may be in conflict with the radiation pressure-supported assumption. We point out that the lower solid line in this diagram is higher than the well-known line that separates the inner and middle regions of standard thin disks (Shakura & Sunyaev 1973). The reason is that the gas and radiation pressure are comparable for the latter, whereas the radiation pressure-supported disk may require the accretion rate to be higher such that the radiation pressure sufficiently dominates over the gas pressure, and therefore the effect of gas pressure on the vertical structure can be completely ignored.

The region between the two solid lines, denoted as “Radiation pressure,” which means that the radiation pressure completely dominates, corresponds to the solutions of main interest in this work. In Section 4, we will focus on the vertical structure and the energy advection at a typical radius, \( r = 10r_g \), as indicated by the vertical dashed line in Figure 1. In Section 5, we will study the two-dimensional solutions for a typical accretion rate \( \dot{m} = 0.6 \) in the range \( 6r_g < r < 12r_g \) and \( 0 < \theta < \pi/2 \), as indicated by the horizontal dot-dashed line. In addition, we point out that for inner radii such as \( 3 \sim r_g \), the two solid lines in Figure 1 may deviate from a real black hole accretion system due to the Newtonian potential used in this work.

4. SOLUTIONS AT A TYPICAL RADIUS \( r = 10r_g \)

4.1. Vertical Structure

In this section, we will focus on the solutions at the typical radius \( r = 10r_g \). Figure 2 shows the vertical structure of the disk with \( \dot{m} = 0.6 \). In Figure 2(a), the dot-dashed, dotted, solid, and dashed lines show the vertical distribution of the dimensionless density \( (\rho/\rho_0) \), radial velocity \( (v_r/v_K) \), azimuthal velocity \( (v_\phi/v_K) \), and sound speed \( (c_s/v_K) \), respectively, where \( \rho_0 \) is the density on the equatorial plane. It is seen that \( \rho \) significantly decreases, whereas \( c_s \) and \( |v_r| \) increases, from the equatorial plane to the surface. In Figure 2(b), the solid line shows the variation of \( \tau_{\text{es}} \) defined in Equation (15), where the photosphere \( (\tau_{\text{es}} = 1) \) is located at \( \theta_0 = 4^\circ \), quite close to the polar axis. The disk seems to be extremely thick according to the position of the photosphere. However, the profile of \( \rho \) implies that most of the accreted matter exists in a moderate range around the equatorial plane, such as \( \pi/4 < \theta < 3\pi/4 \), which is clearer in Figures 4 and 5 (discussed below). The dashed line shows the variation of \( |F_0|/cE \). There exists \( |F_0|/cE \lesssim 1/3 \) for the whole solution, which indicates that the Eddington approximation is valid and the solution is therefore self-consistent.

Using a more general viscosity, Begelman & Meier (1982) studied a geometrically thick, radiation pressure-supported model for supercritical accretion disks. They showed that there exists a narrow empty funnel along the rotation axis with half-opening angle \( \lesssim 4.6^\circ \). As seen in our Figure 2(a), the density

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**Figure 1.** Solutions in the \( \dot{m}-r \) diagram. The parameter space is divided into three regions by two parallel solid lines. The middle region, denoted as “radiation pressure,” corresponds to the radiation pressure-supported disk, which is our main interest in this work. An example solution for \( m = 0.6 \) at \( r = 10r_g \) (filled circle) is shown in Figure 2. The solutions for various \( m \) at a typical radius \( r = 10r_g \) (vertical dashed line) are focused on in Section 4. The two-dimensional solutions for \( m = 0.6 \) (horizontal dot-dashed line) are studied in Section 5.

\[
\tau_{\text{es}} = r\kappa_{\text{es}}\rho^2 \frac{d\rho}{d\theta} = 1 \quad (\theta = \theta_0),
\]

where \( \theta_0 \) \((0 < \theta_0 < \pi/2)\) is the polar angle of the photosphere. The third condition is related to the mass accretion rate:

\[
\dot{M} = -2\pi r^2 \rho_{\text{eq}} \sin \theta \, d\theta.
\]
drops sharply close to the photosphere, and thus a nearly empty funnel also seems to exist in our model. The difference is that the disk surface in Begelman & Meier (1982) is the location where some physical quantities such as \( m = \varphi < \pi \) and \( 0 = \pi/2 - \theta < \pi \). The disk surface in our model is defined as the location where Equation (15) is matched, and no divergence appears in our solutions.

For a real disk with \( m = 0.6 \), the photosphere may exist between \( \theta = 45^\circ \) and \( \pi/3 \), the present result \( (\approx 4^\circ) \). Our argument is as follows. There are two possible reasons that may cause the present photosphere to be quite close to the polar axis. One is that we have ignored the radiation force from one side (e.g., \( \theta = \theta_0 \) and \( 0 < \varphi < \pi \)) to the other (e.g., \( \theta = \theta_0 \) and \( \pi < \varphi < 2\pi \)). The other reason is that we consider only the \( r \varphi \) component of the stress tensor, which may cause inaccurate results for small \( \theta \), such as strong shearing of the angular velocity \( \Omega \) in the vertical direction, where \( \Omega = v_\varphi / (r \sin \theta) \). Nevertheless, the \( r \varphi \) component assumption may work well for moderate \( \theta: \pi/4 < \theta < 3\pi/4 \). The fact that the surface condition could not be matched in the range \( \pi/4 < \theta < 3\pi/4 \) indicates that the half-opening angle of the disk \( (\pi/2 - \theta_0) \) is likely to be larger than \( \pi/4 \).

The profile of \( c_s \) in Figure 2(a) is quite different from that in the previous works with a vertical polytropic assumption (e.g., Hoshi 1977; Gu et al. 2009). Under the polytropic relation \( p_{\text{tot}} = K \rho^{1+1/N} \) (normally \( 1.5 < N < 3 \)), and for the radiation pressure-dominated case, \( p_{\text{tot}} \) can be replaced by \( E/3 \), \( c_s \) will decrease continuously from the equatorial plane to the surface. The reason for the opposite behavior of \( c_s \), as implied in Figure 2(a), is that \( m \) drops faster than \( E \) from the equatorial plane to the surface. Figure 3 shows the variation of the quantity \( d \ln E / d \ln \rho \) with \( \theta \) for \( m = 0.5 \) (dashed line), \( m = 0.6 \) (solid line), and \( m = 1 \) (dotted line). If the polytropic relation works well, \( d \ln E / d \ln \rho \) should be a constant of \( 1 + 1/N \).

It is clearly shown in Figure 3 that, however, \( d \ln E / d \ln \rho \) varies significantly with \( \theta \) rather than being a constant. More importantly, \( d \ln E / d \ln \rho < 1 \) indicates that \( N \) is negative and thus unacceptable. We therefore argue that the polytropic relation should be unsuitable for describing the vertical structure of radiation pressure-supported disks. Moreover, since the energy advection is relevant to \( c_s \) (e.g., \( Q_{\text{adv}} = M c_s^3/2 \pi R^3 \) in Abramowicz et al. 1995, where \( Q_{\text{adv}} \) is the advective cooling rate per unit area), we may expect essentially different results on the strength of advection.

4.2. Energy Advection

Figure 4 shows the variation of the vertically averaged advection rate \( m \), where the mass accretion rate \( m \) is defined as \( f_{\text{adv}} = Q_{\text{adv}} / Q_{\text{vis}} \). The quantities \( Q_{\text{adv}} \) and \( Q_{\text{vis}} \) are expressed as follows:

\[
Q_{\text{adv}} = r \int_{\theta_0}^{\pi - \theta_0} q_{\text{adv}} \sin \theta \, d\theta, \quad (17)
\]

\[
Q_{\text{vis}} = r \int_{\theta_0}^{\pi - \theta_0} q_{\text{vis}} \sin \theta \, d\theta, \quad (18)
\]

where \( q_{\text{adv}} = -v_r E/2r \) and \( q_{\text{vis}} = -3 \rho v_r v_\varphi^2 / 2r \) are respectively the advective cooling rate and the viscous heating rate per unit volume, as implied by the left-hand side of Equation (11).

The solid line in Figure 4 corresponds to the total accretion rate integrating from \( \theta_0 \) to \( \pi - \theta_0 \), as shown by Equation (16), whereas the dashed line corresponds to the specific accretion rate integrating from \( \theta = \pi/4 \) to \( \theta = 3\pi/4 \), i.e.,

\[
M_{\pi/4} = -2 \pi r \int_{\pi/4}^{3\pi/4} \rho v_r \sin \theta \, d\theta. \quad (19)
\]
The reason why we calculate for $M_{\pi/4}$ is that the $r\phi$ stress assumption may work well for $\pi/4 < \theta < 3\pi/4$. As shown by the horizontal range of the solid and dashed lines, most of the accreted matter exists in this specific range, e.g., $\dot{m}_{\pi/4} = 0.52$ corresponding to $\dot{m} = 0.6$. The figure also shows that $f_{adv}$ rapidly increases with increasing $\dot{m}$ in the range $0.5 \lesssim \dot{m} \lesssim 1.1$. More importantly, the value of $f_{adv}$ ($0.2 \lesssim f_{adv} \lesssim 0.8$) indicates that the energy advection is significant even for sub-Eddington accretion disks.

Such a result is quite different from the previous one, where advection was found to be significant only for the super-Eddington accretion case. Watarai et al. (2000) introduced an elegant formula to describe the $M-L$ relationship based on their numerical solutions under the well-known Paczyński–Wiita potential (Paczyński & Wiita 1980). Their Equations (15)-(19) imply that, for the position $r = 10r_g$, advection is negligible for $M \lesssim 67L_{\text{Edd}}/c^2$ ($\sim 4M_{\text{Edd}}$). For the whole disk, advection is negligible for $M \lesssim 20L_{\text{Edd}}/c^2$ ($1.25M_{\text{Edd}}$). Such a critical $M$ for the whole disk was confirmed by some recent global solutions under general relativity. Figure 4.11 of Sadowski (2011) shows that advection is negligible for $L \lesssim L_{\text{Edd}}$ for any spin parameter $a_*$. For $a_* = 0$, i.e., the Schwarzschild black hole, the critical $M$ is just around $M_{\text{Edd}}$. In our opinion, the different results on advection between the above two works and ours are related to the different approaches in describing the vertical structure.

A significant difference is that Watarai et al. (2000) and Sadowski et al. (2011) chose cylindrical coordinates whereas we adopt spherical coordinates. Of course, the final results should not depend on the coordinates used. However, as pointed out by Abramowicz et al. (1997), there are some interesting differences between the equations written in cylindrical and spherical coordinates. There is no centrifugal force in the $z$ direction in cylindrical coordinates, whereas there is no gravitational force in the $\theta$ direction in spherical coordinates. Abramowicz et al. (1997) claimed that it is exactly this property that makes the spherical coordinates much better adapted for describing the flow near the black hole horizon. Here, we argue that spherical coordinates are more suitable for describing geometrically thick disks as follows. In cylindrical coordinates, in the $z$ direction, whether with a polytropic relation between the pressure and the density (Watarai et al. 2000), or with local energy balance (Sadowski et al. 2011), an approximation for the gravitational force, i.e., $\partial \psi / \partial z = \Omega^2_z z$, was adopted to describe the vertical structure. Such an approximation will probably be invalid for $z/r \gtrsim 1$. In particular, for $z \to \infty$, the approximate force goes to infinity whereas the real force ought to vanish. Thus, cylindrical coordinates seem unsuitable for studying geometrically thick disks. In other words, a geometrically thick disk solution in cylindrical coordinates may not be self-consistent. Sadowski et al. (2011) limited their solutions to $M < 2M_{\text{Edd}}$ probably due to this reason. As shown by their Figure 10, the maximal value of $H/r$ for $M = 2M_{\text{Edd}}$ is $\sim 0.4$. For higher $M$, the value of $H/r$ will be even larger thus the solution based on the approximate force may be inaccurate. On the contrary, in spherical coordinates, there is no need to use an approximation for the gravitational force. The centrifugal force in the $\theta$ direction, which takes the place of the $z$-direction gravitational force in cylindrical coordinates, is derived in this work by solving the vertical differential equations. Thus, our approach to the vertical structure seems to be more reasonable.

We agree that the solutions in Sadowski et al. (2011) are likely to be self-consistent since their $H/r$ is significantly less than unity, in particular for the solutions with $M \lesssim M_{\text{Edd}}$. Then what are the reasons for the quantitative difference in the advection for $M \lesssim M_{\text{Edd}}$ between their solutions and ours? In our understanding, there exist three possible reasons as follows. First, as mentioned in Section 3 of Sadowski et al. (2011) for their numerical methods, the vertical structure is derived using a given advection factor $f_{adv}$ in advance. The value of $f_{adv}$ is probably obtained by solving the radial structure on the equatorial plane. Moreover, their $f_{adv}$ is assumed to be uniform in the $z$ direction. On the contrary, we obtain a varying $f_{adv}$ by solving the vertical equations. As shown by our Figure 8, $f_{adv}$ increases significantly with $z$. We can therefore expect that the vertically averaged $f_{adv}$ at a cylindrical radius will also be significantly larger than that at $z = 0$, which may explain why our $f_{adv}$ is larger than that in Sadowski et al. (2011). Second, Sadowski et al. (2011) assumed a uniform $v_R$ and $v_\phi$ in the $z$ direction and used the Keplerian strain to calculate the viscous dissipation. In our method, however, we include varying $v_z$ and $v_\theta$ in the vertical direction, and the viscous dissipation is calculated based on $v_z$ instead of $v_R$. Third, Sadowski et al. (2011) assumed $v_r = 0$ whereas we have $v_\theta = 0$. As stressed by Abramowicz et al. (1997), since the stationary accretion flows resemble quasi-spherical flows ($\theta_\phi \approx \text{constant}$) much more than quasi-horizontal flows ($H \approx \text{constant}$), $v_\theta = 0$ may be a more reasonable approximation than $v_r = 0$. Moreover, the above three reasons may also be responsible for the different results in the convective stability, as will be discussed in Section 5.3.

4.3 Spin Problem for $L \gtrsim 0.3L_{\text{Edd}}$

As mentioned in Section 1, some works on black hole spin measurement showed that the standard thin disk model is likely to be inaccurate for $L \gtrsim 0.3L_{\text{Edd}}$ (e.g., McClintock et al. 2006; Straub et al. 2011). One explanation is that the inner disk edge is still located at the innermost stable circular orbit (ISCO), but its emission is shaded by the outer disk. Thus, the inner disk radius obtained from spectral fitting is not true. However, Weng & Zhang (2011) showed that the disks in black hole and neutron star X-ray binaries trace the same evolutionary pattern for $L \gtrsim 0.3L_{\text{Edd}}$. In addition, for the neutron star system XTE J1701-462, the boundary emission area remains nearly constant despite the varying luminosity of the disk (Lin et al. 2009, Figure 17), which indicates that the neutron star’s surface is not shaded. Weng & Zhang (2011) therefore argued that the inner disk of
the black hole system should not be shaded either due to the similar phenomenon. They suggested that the inner disk radius moves outward because of the increasing radiation pressure.

In our opinion, from energy advection, it is easy to understand why the standard disk model seems to be inaccurate above the black hole system should not be shaded either due to the Variation of averaged height. We define an averaged dimensionless height as being ignored, may be inaccurate. We can therefore expect that, even for the Paczyński-Wiita heating rate at a smaller radius such as $r = 10 r_g$. On the other hand, for the same $r$ in Figure 4. On the other hand, for the same $m$, since the viscous heating rate at a smaller radius such as $r = 5 r_g$, then the real $f_{\text{adv}}$ at $10 r_g$ may be smaller than the values showed in Figure 4. We would point out that, compared with the Paczyński–Wiita potential, the Newtonian potential in the present work may magnify the viscous heating rate at small radii such as $10 r_g$, and thus the real $f_{\text{adv}}$ at $10 r_g$ may be larger than that at $r = 10 r_g$, so will the advection factor be. We can therefore expect that, even for the Paczyński–Wiita potential, the advection at the position close to the ISCO should be non-negligible for $m \sim 0.3$. Consequently, the standard thin disk model, based on the energy balance between the viscous heating and the radiative cooling with the advective cooling being ignored, may be inaccurate.

### 4.4. Vertical Height

Figure 2 shows that $\rho$ decreases significantly with decreasing $\theta$, and Figure 4 implies that most of the accreted matter exists in the range $\pi/4 < \theta < 3\pi/4$. In order to have a clearer view, we define an averaged dimensionless height as $\Delta \theta \equiv \Sigma/2\rho \rho_0$, where the surface density $\Sigma$ takes the form:

$$\Sigma = r \int_{\theta_0}^{\pi - \theta_0} \rho \sin \theta \, d\theta.$$  

Figure 5 shows the variation of $f_{\text{adv}}$ with $\Delta \theta$ (solid line). Even though the photosphere is close to the polar axis, the averaged height $\Delta \theta$ is geometrically slim with $0.3 \lesssim \Delta \theta \lesssim 0.6$. Furthermore, the figure shows that $f_{\text{adv}}$ increases with increasing $\Delta \theta$ or $m$, which agrees with the classic picture. For quantitative comparison, we plot the function $f_{\text{adv}} = 1.5 \tan^2(\Delta \theta)$ (dashed line) in Figure 5 due to the relationship $f_{\text{adv}} \gtrsim (H/R)^2$ introduced by Abramowicz et al. (1995), which is equivalent to $f_{\text{adv}} \gtrsim \tan^2(\Delta \theta)$ here. It is seen that $f_{\text{adv}}$ is not well proportional to $\tan^2(\Delta \theta)$. In the range $0.6 < m < 1$ or $0.3 < f_{\text{adv}} < 0.8$, however, we may regard the formula $f_{\text{adv}} = 1.5 \tan^2(\Delta \theta)$ as a rough approximation.

### 5. Two-Dimensional Solutions and Convective Stability

#### 5.1. Two-dimensional Solutions

In Section 4, we focus on the solutions at a typical radius $r = 10 r_g$. In this section, we will study the disk solutions for various radii. Since the vertical solutions are based on the assumptions of partial derivatives in the radial direction (presented in Section 2.1), it is necessary to derive vertical solutions for various radii to check whether these assumptions are self-consistent. Following the example solution in Figure 2, we study the two-dimensional solutions for $m = 0.6$ in the range $6 r_g \leq r \leq 12 r_g$ and $0 < \theta \leq \pi/2$. Figure 6 shows the radial variations of $c_s$ and $E$ (solid lines) for five polar angles, i.e., $\theta = 90^\circ$, $75^\circ$, $60^\circ$, $45^\circ$, and $30^\circ$. For comparison, the radial profile of $v_r$, which is proportional to $r^{-1/2}$, is shown in Figure 6(a), and an example slope of $\propto r^{-5/2}$ is shown in Figure 6(b). The figure shows that $c_s$ and $E$ behave roughly as $r^{-1/2}$ and $r^{-5/2}$, respectively, which agrees with the original assumptions of radial derivatives. As mentioned in Section 2.1, once the radial derivatives of $c_s$ and $E$ are given, the other ones can be inferred from Equations (1)–(7). Thus, we can expect that the radial derivatives of $v_r$, $v_\phi$, $\rho$, and $F_r$ in the two-dimensional solutions should also be in agreement with the assumptions. Our solutions are therefore likely to be self-consistent.

In our calculation, the location of the photosphere does not vary much with the radius, i.e., $\theta_0 \lesssim 5^\circ$ for various radii. As discussed in Section 4.1, the real position of the photosphere is likely to be located in the range $5^\circ < \theta_0 < 45^\circ$. We will make some comparisons with simulations for the photosphere in Section 6. As shown in Figure 6, the derivatives of $c_s$ and $E$ deviate a little for $r \rightarrow 12 r_g$ and $\theta = 90^\circ$. Such a divergence may be well understood from Figure 1, which shows that the solution for $m = 0.6$ and $r \rightarrow 12 r_g$ is quite close to the lower solid line, which indicates that the gas pressure may not be negligible. In particular for the equatorial plane, the mass density has the maximal value there, thus the gas pressure may be most significant at this position.

#### 5.2. Solberg–Høiland Conditions

In this section, we will study the convective stability of the radiation pressure-supported disks. The well-known Solberg–Høiland conditions in cylindrical coordinates $(R, \phi, z)$ take the forms (e.g., Tassoul 2000):

$$\frac{1}{R^2} \frac{\partial l^2}{\partial R} - \frac{1}{C_p \rho} \nabla P \cdot \nabla S > 0,$$

$$- \frac{\partial P}{\partial z} \left( \frac{\partial l^2}{\partial R} \frac{\partial S}{\partial z} - \frac{\partial l^2}{\partial z} \frac{\partial S}{\partial R} \right) > 0,$$

where $l$ is the specific angular momentum per unit mass, $P$ is the total pressure, $C_p$ is the specific heat at constant pressure, and $S$ is the entropy expressed as

$$dS \propto d \ln \left( \frac{P}{\rho^\gamma} \right),$$

where $\gamma$ is the adiabatic index.
The $R$ and $z$ components of the well-known Brunt–Väisälä frequency are written as

\[ N_R^2 = -\frac{1}{\gamma \rho} \frac{\partial P}{\partial R} \frac{\partial}{\partial R} \ln \left( \frac{P}{\rho^\gamma} \right), \]

\[ N_z^2 = -\frac{1}{\gamma \rho} \frac{\partial P}{\partial z} \frac{\partial}{\partial z} \ln \left( \frac{P}{\rho^\gamma} \right), \]

and the epicyclic frequency takes the form:

\[ \kappa^2 = \frac{1}{R^2} \frac{\partial^2}{\partial R^2} \ln \left( \frac{P}{\rho^\gamma} \right). \]

Thus, the first Solberg–Høiland condition, Equation (21), can be simplified as

\[ N_{\text{eff}}^2 = N_R^2 + N_z^2 + \kappa^2 > 0, \quad (24) \]

where $N_{\text{eff}}$ is defined as the effective frequency. For accretion disks, there usually exists $\partial P/\partial z < 0$ (as shown in Figure 8), so the second Solberg–Høiland condition, Equation (22), reduces to

\[ \Delta_{\text{IS}} = \frac{\partial^2}{\partial R^2} \ln \left( \frac{P}{\rho^\gamma} \right) - \frac{\partial^2}{\partial z^2} \ln \left( \frac{P}{\rho^\gamma} \right) > 0. \quad (25) \]

In numerical calculations, we adopt $P = E/3$ and $\gamma = 4/3$ according to the radiation pressure-supported assumption.

5.3. Convective Stability

Based on the two-dimensional solutions for $m = 0.6$ in Section 5.1, we can obtain the variations of the physical quantities in cylindrical coordinates and therefore investigate convective stability using Equations (24) and (25). We take the cylindrical radius $R = 10r_g$ as a typical position in which to study the convective stability.

Figure 7 shows the $z$-direction variations of $\kappa^2$, $N_R^2$, $N_z^2$, and $N_{\text{eff}}^2$, and $\Delta_{\text{IS}}$, where the first four quantities are normalized by $\Omega_K^2$, and the last one is normalized by $v_K^2$. The positive values for both $N_{\text{eff}}^2$ and $\Delta_{\text{IS}}$ indicate that the disk should be convectively stable.

\[ \frac{\partial S}{\partial z} \propto \frac{\partial}{\partial z} \ln \left( \frac{E}{\rho^{4/3}} \right) > 0, \quad (26) \]

which is known as the Schwarzschild criterion for a constant angular velocity at a cylindrical surface ($\partial \Omega/\partial z = 0$), corresponding to the so-called barytropic flows where the pressure depends only on the density. As the dashed line in Figure 8 shows, the angular momentum $l$ (or equivalently the angular

**Figure 6.** Radial variations of $c_s$ and $E$ (solid lines) for the polar angle $\theta = 90^\circ$, $75^\circ$, $60^\circ$, $45^\circ$, and $30^\circ$ for $m = 0.6$. For comparison, the Keplerian velocity $v_K$ ($\propto r^{-1/2}$) and an example slope of $\propto r^{-3/2}$ are plotted by the dashed lines in (a) and (b), respectively.

**Figure 7.** $z$-direction variations of $\kappa^2$ (dot-dashed line), $N_R^2$ (dashed line), $N_z^2$ (dotted line), $N_{\text{eff}}^2$ (thick solid line), and $\Delta_{\text{IS}}$ (thin solid line) for $m = 0.6$ at a cylindrical radius $R = 10r_g$. The quantities $\kappa^2$, $N_R^2$, $N_z^2$, and $N_{\text{eff}}^2$ are normalized by $\Omega_K^2$, and $\Delta_{\text{IS}}$ is normalized by $v_K^2$. For the equatorial plane, the result of $N_{\text{eff}}^2 > 0$ can be inferred from Equation (15) of Narayan & Yi (1994), which reveals that the disk will always be convectively stable for $\gamma = 4/3$ at $z = 0$.

To further understand the convectively stable results for $z > 0$, we plot Figure 8 to show the $z$-direction variations of $\rho$, $E$, $l$, $E/\rho^{4/3}$, and the advection factor $f_{\text{adv}}$. It is seen that $\rho$ drops faster than $E$ with increasing $z$, so the quantity $E/\rho^{4/3}$ increases with $z$. From Equation (23) we immediately have

\[ \frac{\partial S}{\partial z} \propto \frac{\partial}{\partial z} \ln \left( \frac{E}{\rho^{4/3}} \right) > 0, \]
velocity $\Omega$) does not vary significantly with $z$. Thus, the convectively stable results are easy to understand from $\partial S/\partial z > 0$.

In a similar study (vertical structure based on the local energy balance) of the general model for optically thick disks, however, the disk was found to be convectively unstable (e.g., Sadowski et al. 2009, 2011). As interpreted using the three possible reasons in Section 4.2, the significant difference in the results between their works and ours is probably related to the different approaches in describing the vertical structure. In addition, here we mention two more details which may help us understand the difference in convective stability. First, the profiles of $\rho$ and $E$ in Figure 8 reveal that the absolute value of radial velocity $|v_r|$ increases with $z$ ($|v_r| \propto E/\rho$ inferred from Equations (7) and (10)). Compared with the uniform disk in Sadowski et al. (2011), our increasing $|v_r|$ with $z$ may result in a faster drop of $\rho$ in the $z$ direction (steeper slope of $\rho$) if we simply assume the mass supply to be comparable. Second, compared with Sadowski et al. (2011), the advection in our solutions is significantly stronger, which means that for the same $\dot{m}$ and thus a comparable viscous heating rate, the vertical radiation flux $F_z$ will be less in our results. Thus, $E$ may decrease slower in the $z$ direction (flatter slope of $E$) due to less $F_z$ and lower $\rho$ (inferred from Equation (6)). As indicated by Equation (26), the flatter slope of $E$ and the steeper slope of $\rho$ will both make a contribution to $\partial S/\partial z > 0$, and the disk is therefore likely to be convectively stable.

We would stress that our solutions are limited by the radiation pressure-supported case. Thus, the present results cannot directly show the convective stability of disks for either the gas pressure-supported case or in the case with comparable gas and radiation pressure. Actually, we have performed some additional calculations for thin disks in cylindrical coordinates to check the convective stability, following the method of Sadowski et al. (2011) but without considering advection. We found that the disk is convectively stable for the gas pressure-supported case, whereas the disk is convectively unstable in the case where radiation pressure is significant. Thus, we would agree with Sadowski et al. (2011) on the convectively unstable disks for moderate accretion rates such as $0.01 \sim 0.1 \dot{M}_{\text{Edd}}$, corresponding to significant radiation pressure and non-negligible gas pressure. Moreover, for $\dot{m} \lesssim 0.1$, the disk will be geometrically thin, and thus there is no difference between the assumptions $v_\theta = 0$ and $v_z = 0$, and $f_{\text{adv}}$ is probably negligible. As a consequence, the solutions of Sadowski et al. (2011) ought to be accurate.

Furthermore, as revealed by the profiles of $E$ and $\rho$ in Figure 8, $d \ln E/d \ln \rho$ is less than unity in the $z$ direction. Following the argument in Section 4.1, the polytropic relation also seems unsuitable in the $z$ direction. In addition, as shown in Figures 7 and 8, our example solution for $\dot{m} = 0.6$ at $R = 10R_g$ is terminated at $z/R = 0.66$. The reason is that the solutions in spherical coordinates is limited by $r = 12R_g$ (as shown by the horizontal dot-dashed line in Figure 1), which corresponds to $z/R = 0.66$ at $R = 10R_g$ in cylindrical coordinates.

5.4. Analysis of Convective Stability for $z \ll R$

For the region close to the equatorial plane, we can analyze the convective stability using the Taylor expansion method. Obviously, we have $\partial S/\partial z = 0$ at $z = 0$ from symmetric conditions. Thus, for $z \ll R$, the value of $\partial S/\partial z$ can be estimated using the second-order derivative at $z = 0$:

$$\frac{\partial S}{\partial z} \approx \frac{\partial^2 S}{\partial z^2} \quad (z \ll R).$$

(27)

Based on Equations (7)–(12), we can eliminate $v_r$, $E$, and $F_\theta$ and therefore obtain a set of three equations for the three quantities $v_\phi$, $c_s$, and $\rho$. Using the Taylor expansion method, together with the boundary condition of Equation (14), we derive the following three relationships for the second-order derivatives of $v_\phi$, $c_s$, and $\rho$ (the ram-pressure term in Equation (8) is ignored):

$$5c_s \frac{\partial^2 c_s}{\partial \theta^2} + v_\phi \frac{\partial^2 v_\phi}{\partial \theta^2} = 0,$$

$$\frac{1}{\rho} \frac{\partial^2 \rho}{\partial \theta^2} + 2 \frac{\partial^2 c_s}{\partial c_s^2} = -\frac{v_\phi^2}{c_s^2},$$

$$\frac{3}{\rho} \frac{\partial^2 v_\phi}{\partial \theta^2} = \frac{1}{2} \frac{\partial^2 \rho}{\partial \theta^2} + \frac{1}{2} \frac{\partial^2 c_s}{\partial c_s^2} + \frac{1}{v_\phi^2 - c_s^2} \left( v_\phi \frac{\partial^2 v_\phi}{\partial \theta^2} - c_s \frac{\partial^2 c_s}{\partial \theta^2} \right),$$

where $\tilde{\theta}$ is defined as $\tilde{\theta} = \pi/2 - \theta$, which is a small value in the analysis.

In our solutions, we have $c_s^2 \ll v_\phi^2$ at $z = 0$, e.g., $c_s^2/v_\phi^2 = 0.16$ for $\dot{m} = 0.6$ at $R = 10R_g$. For the simple case with $c_s^2/v_\phi^2 \ll 1$, the above three relationships give

$$\frac{\partial^2 \ln v_\phi}{\partial \theta^2} \approx -\frac{5}{16} \frac{v_\phi^2}{c_s^2} ; \quad \frac{\partial^2 \ln c_s}{\partial \theta^2} \approx \frac{1}{8} \frac{v_\phi^4}{c_s^4} ; \quad \frac{\partial^2 \ln \rho}{\partial \theta^2} \approx -\frac{1}{4} \frac{v_\phi^4}{c_s^4},$$

The second-order derivative of entropy at $z = 0$ can therefore be derived using the following coordinate transformation:

$$\frac{\partial^2 S}{\partial z^2} \approx \frac{1}{R} \frac{\partial}{\partial r} \left( \frac{c_s^2}{\rho^{1/3}} \right) \left( \frac{c_s^2}{\rho^{1/3}} \right) + \frac{1}{R} \frac{\partial}{\partial r} \left( \frac{c_s^2}{\rho^{1/3}} \right) \approx \frac{1}{R^2} \left( \frac{1}{3} \frac{v_\phi^4}{c_s^4} - \frac{1}{2} \right) > 0.$$

(28)

Thus, Equations (27) and (28) indicate $\partial S/\partial z > 0$ for the region close to the equatorial plane. The disk in this region is therefore likely to be convectively stable.
6. SUMMARY AND DISCUSSION

In this paper, we have studied the vertical structure, energy advection, and convective stability of radiation pressure-supported disks in spherical coordinates. In the θ direction, we replaced the pressure–density polytropic relation with the local energy balance per unit volume between the viscous heating and the radiative cooling, and obtained the distribution of physical quantities such as ρ, v_r, v_θ, c_s, E, and F_p. The photosphere was found close to the polar axis and therefore the disk seems to be extremely thick. However, most of the accreted matter exists in a moderate range around the equatorial plane such as π/4 < θ < 3π/4. We showed that the polytropic relation is unsuitable for describing the vertical structure of radiation pressure-supported disks. More importantly, we found that energy advection is significant even for slightly sub-Eddington accretion disks, which is quite different from the previous result that shows advection is of importance only for super-Eddington accretion disks. We argued that the non-negligible advection may help us understand why the standard thin disk model is likely to be inaccurate for some sources up to luminosities of a few 10^{10} erg s^{-1}. The disk will provide thermal radiation, which is normally not dominant because of the moderate m_{max}. On the other hand, the outflows may give a contribution to the non-thermal radiation through the bulk motion Comptonization (Titarchuk & Zanni 1998) or through the jet of the radiation-pressure driven and magnetically collimated outflow (Ohsuga & Mineshige 2011).

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