THE TOP QUARK AND THE HIGGS BOSON MASS
FROM LEP SLC AND CDF DATA

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Abstract

The impact of the new experimental data from LEP, SLC and
CDF on the top quark mass $m_{top}$ and the Higgs boson mass $m_{Higgs}$
is investigated. The determinations of $m_{top}$ and of an upper bound
on $m_{Higgs}$ are given, taking into account the experimental error on
the QED coupling constant $\alpha_{em}$ and on the $b$ quark mass $m_{b}$.
The relevance of higher order theoretical uncertainties is pointed out.

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Up to now the four LEP experiments at CERN collected roughly $8 \times 10^6$
$Z^0$ bosons, of which $3 \times 10^6$ have been produced in the 1993 resonance scan-
ning. This led to a substantial improvement in the measurement of the $Z^0$
parameters such as $M_Z$, $\Gamma_Z$, $\Gamma_b$, the asymmetries and so on. Meanwhile
other relevant experimental results have been achieved. First, the experiment
SLD at SLAC measured the value of the left-right asymmetry on a sample of
$5 \times 10^4 Z^0$'s, but with longitudinally polarized electrons ($P \simeq 0.62$), reaching
an accuracy competitive with LEP determination. Second, the experiment
CDF at FERMILAB improved the measurement of $M_W$, leading to
a better determination of the ratio $M_W/M_Z$. Last, but not least, very
recently CDF collaboration claimed for evidence of top quark production in

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\( \bar{p}p \) collisions at \( \sqrt{s} = 1.8 \) TeV, quoting a value for the top quark mass of \( m_{\text{top}} = 174 \pm 17 \) GeV \[4\].

At this point, it can be relevant to study the impact of the new experimental data on the determination of the fundamental parameters of the Minimal Standard Model, which is the goal of the present short note. Moreover, in the light of the presently achieved experimental accuracy, two more items should be taken into account. Firstly a particular care has to be devoted to the effect of the experimental error on the electromagnetic coupling constant \( \alpha_{\text{em}} \), coming from the parameterization of the light quark contribution to the vacuum polarization \[5\], and on the \( b \) quark mass \( m_b \). Secondly the theoretical uncertainty due to higher order effects in the Standard Model predictions has to be taken into account and properly quantified. It goes almost without saying that everything we are going to show is largely based on data presented at the winter conferences; in particular the averaging of \( R_b \) among the four LEP experiments is complicated and very preliminary so that this and other numbers, such as the correlation matrix, could very well change \[6\].

In order to attain the above stated goal, the electroweak library of the code TOPAZ0 \[7\] has been used. Very recent developments in the field of electroweak and QCD radiative corrections, such as \( O(G_{F}^{\mu}G_{F}^{\mu}) \) in \( \Delta \rho \), QCD corrections including \( b \) quark mass effects with running \( b \) quark mass and full \( O(\alpha_{\mu}^{\alpha}\alpha_{\mu}) \) effects \[8\], have been taken into account \[9\].

The indirect determination of the top quark mass \( m_{\text{top}} \) and the Higgs boson mass \( m_{H} \) have been studied in some detail. The data used are the experimental measurements of the \( Z^0 \) parameters, namely \( M_Z, \Gamma_Z, R, R_b, \sigma_{\text{had}}, g_V/g_A \) or the deconvoluted asymmetries, plus the best determination of the ratio \( M_W/M_Z \) (UA2 + CDF, weighted average). When the ratio \( g_V/g_A \) has been used in place of the asymmetries, the inclusion of the SLD measurement has been performed by taking the weighted average of the LEP and SLD experimental data (see Table 1). The experimental error on \( \alpha_{\text{em}}(M_Z), 1/\alpha_{\text{em}}(\text{light}) = 128.87 \pm 0.12 \), and on the \( b \) quark mass \( m_b, m_b = 4.7 \pm 0.2 \) GeV, and the experimental value of the top quark mass \( m_{\text{top}} \) as given by the direct determination of CDF (\( m_{\text{top}} = 174 \pm 17 \) GeV) have been included by proper penalty functions. Moreover we have used the presently available elements of the correlation matrix \[10\].

Let us begin with the top quark mass determination. The situation is
well summarized in Figs. 1-4, where $\chi^2$ versus $m_{\text{top}}$ is shown. For a given $m_{\text{top}}$ the corresponding $\chi^2$ has been obtained by minimizing the $\chi^2$ function with respect to $M_Z$ and $\alpha_s$ for $m_{\text{Higgs}}$ kept fixed at $m_{\text{Higgs}} = 300$ GeV ($M_Z$ constrained at $91.190 \pm 0.004$ GeV, no constraint on $\alpha_s$). Fig. 1 shows the $\chi^2$ in the following situations ($M_W/M_Z$ is always included): LEP data only (dash-dotted line), LEP + SLC data (solid line), LEP data + CDF constraint (dotted line) and LEP + SLC data + CDF constraint (dashed line). The ratio $g_V/g_A$ is used as summarizing the asymmetry data. The uncertainty due to the error on $\alpha_{\text{em}}(M_Z)$ and $m_b$ is propagated in the theoretical part of the $\chi^2$.

Fig. 2 shows the same content as Fig. 1, but with $\alpha_{\text{em}}(M_Z)$ and $m_b$ kept fixed at their central value, $1/128.87$ and $4.7$ GeV respectively. In Fig. 3 the effect of propagating the error on $\alpha_{\text{em}}(M_Z)$ and $m_b$ is pointed out by comparing a fit in which the error is taken into account (dashed line) with a fit in which it is neglected (solid line). Fig. 4, at last, is the same as Fig. 1 but with the individual asymmetries used in place of the combined value of $g_V/g_A$. The best determination of $m_{\text{top}}$ can be considered the one in which the whole set of experimental information is used, namely the one in which LEP + SLC data + CDF constraint (+ $M_W/M_Z$) are used, together with the propagation of the errors on $\alpha_{\text{em}}(M_Z)$ and $m_b$. In this case the best fit gives

$$m_{\text{top}} = 174.0^{+9.3}_{-9.6}^{+12.0}_{-12.5}^{+0.2}_{-3.4} \text{ GeV},$$

where, according to a commonly accepted procedure, the central value refers to $m_{\text{Higgs}} = 300$ GeV, the first error is statistical, the second one is obtained by allowing $m_{\text{Higgs}}$ to vary from 60 to 1000 GeV and the third one is due to higher order theoretical uncertainties. At best fit one obtains $\alpha_s = 0.124$ and $M_Z = 91.190$ GeV. The last uncertainty is connected to the unknown electroweak higher order terms, the truncation or not in perturbation theory, the electroweak and QCD scales and the factorization or not of QCD radiation. Actually the central value for $m_{\text{top}}$ deserves some additional explanation. It has been derived by choosing some of the options on the treatment of higher order EW terms such that we get the best agreement between TOPAZ0 and the other existing codes (BHM [1], LEPTOP [2] and ZFITTER [3]). If we use the same data set (LEP + SLC data + CDF constraint) and perform a three parameter fit ($M_Z, m_{\text{top}}, \alpha_s$) at $m_{\text{Higgs}}$ fixed, then the minimum of the $\chi^2$
corresponds to \( m_{Higgs} = 64 \) GeV (more about this later, see table 2). There is of course some degree of arbitrariness in fixing \( m_{Higgs} \) to 300 GeV and one could ask what happens if we derive results for \( m_{top} \) at the best value for \( m_{Higgs} \) kept fixed. Therefore we have performed a two parameter fit with respect to \( M_Z \) and \( \alpha_s \) for the Higgs mass fixed at 64 GeV. We obtain

\[
m_{top} = 161.9^{+0.4}_{-0.7} \text{ GeV,} \tag{2}
\]
corresponding to \( \alpha_s = 0.122 \). This result is confirmed by a three parameter fit on the same data set, namely a fit to \( M_Z \), \( \alpha_s \) and \( m_{Higgs} \) (without any constraint on \( m_{Higgs} \)), giving

\[
m_{top} = 161.9^{+13.9}_{-11.4} \text{ GeV,} \tag{3}
\]
corresponding to \( \alpha_s = 0.123 \) and \( m_{Higgs} = 64 \) GeV. Finally, by performing the same type of fit with the penalty function on \( m_{\mu} \), only slightly different results are obtained, namely

\[
m_{top} = 162.4^{+13.4}_{-9.6} \text{ GeV,} \tag{4}
\]
with \( \alpha_s = 0.122 \), \( m_{Higgs} = 68.5 \) GeV. All these values are found to be in good agreement with the results very recently obtained in [15].

Before making any comment it is worth noting that a slightly different situation appears if we neglect the SLD data. Actually a canonical fit at \( m_{Higgs} = 300 \) GeV gives

\[
m_{top} = 168.1^{+9.6+11.5}_{-9.9-11.8} \text{ GeV,} \tag{5}
\]
where the first error is statistical and the second one is due to a variation of \( m_{Higgs} \) from 60 to 1000 GeV, whereas a fit in which \( m_{Higgs} \) is left free provides

\[
m_{top} = 164.0^{+14.7}_{-13.7} \text{ GeV,} \tag{6}
\]
with at best fit \( m_{Higgs} = 187 \) GeV and \( \alpha_s = 0.124 \). The difference on the central values for \( m_{top} \) is smaller than the corresponding one appearing when the SLD data is included, reflecting the fact that the SLD asymmetry is about 3\( \sigma \) away from the corresponding LEP measurement. As a consequence of this the value of \( m_{Higgs} \) is driven towards the direct search boundaries and the central value for \( m_{top} \) depends strongly on the type of fit performed. On the
contrary we do not find large ($\approx 10$ GeV) deviations on $m_{\text{top}}$ from different fits if the SLD data is excluded. At last excluding CDF constraint, i.e for the data set LEP + SLC ($+M_W/M_Z$), the best fit gives $m_{\text{top}} = 174.0^{+11.0+17.0+0.3}_{-11.7-18.5-4.9}$ GeV, in good agreement with the result quoted in [16]. Moreover for the LEP data alone ($+M_W/M_Z$) we obtain $m_{\text{top}} = 165^{+12+17}_{-13-19}$ GeV in agreement with [17].

For the sake of comparison, it is worth quoting the value of $\alpha_s$ as obtained from a fit to $R$, which gives

$$\alpha_s = 0.1258 \pm 0.0060^{+0.0029+0.0007}_{-0.0031-0.0014};$$

(7)

where the first error is the experimental one, the second one comes from $m_{\text{top}} = 174 \pm 17$ GeV and $m_{\text{Higgs}} = 60 - 1000$ GeV and the last one is again due to theoretical uncertainty. This value has been obtained along the same lines of the one presented in [18]. If on the other hand we perform a fit to $M_Z, m_{\text{top}}, m_{\text{Higgs}}$ to the LEP + SLC data + CDF constraint for $\alpha_s$ fixed and derive the $\chi^2(\alpha_s)$ distribution, then we get $\alpha_s = 0.1218 \pm 0.0047$. The same fit excluding SLC gives instead $\alpha_s = 0.1242^{+0.0053}_{-0.0050}$.

At this point some comments are in order. The SLC measurement of $A_{LR}$ increases the fitted $m_{\text{top}}$ value of about 6-9 GeV with respect to the value given by LEP data only. Moreover when the asymmetries are individually entered in the fit instead of fitting the combined value of $g_V/g_A$, the inclusion of the SLC measurement leads to a clear rise of the $\chi^2$. This confirms that the SLC value is about 3 $\sigma$ away from the combined LEP value of $g_V/g_A$. Including the CDF constraint increases the fitted value of $m_{\text{top}}$ of about 3 GeV if SLC is not included, whereas it gives no effect on the central value of $m_{\text{top}}$ if SLC is included in the fit. In any case CDF constraint reduces the statistical error on $m_{\text{top}}$ of about 2 GeV and the error on $m_{\text{top}}$ due to the uncertainty on $m_{\text{Higgs}}$ of about 5 GeV. The uncertainty on the central value of $m_{\text{top}}$ generated by the error on $\alpha_{em}(M_Z)$ and $m_b$ can be quantified in about 2 GeV and finally the one due to the theoretical ambiguity on higher orders can be estimated to be around 4-5 GeV. It is also worth noting that the only $Z^0$ parameter which at present is non-standard is $R_b$, whose experimental value is larger than expected of about two standard deviations, if indeed the top quark is around 174 GeV. Excluding $R_b$ from the fit leads to an increasing of $m_{\text{top}}$ of 4-6 GeV.

As far as $m_{\text{Higgs}}$ determination is concerned, the $\chi^2$ as a function of $m_{\text{Higgs}}$ has been obtained by means of a three parameter fit with respect to $M_Z, m_{\text{top}}$
and $\alpha_s$ at $m_{\text{Higgs}}$ fixed. In principle one could expect some influence of the direct observation of the top quark on the theoretical predictions for $m_{\text{Higgs}}$. In order to point out such an effect the direct determination of $m_{\text{top}}$ by CDF at $m_{\text{top}} = 174 \pm 17 \text{ GeV}$ has been taken into account by including a proper penalty function. The situation is well described by the results shown in Table 2 (7 observables means fitting $g_V/g_A$, 11 observables means fitting the asymmetries). For the most complete set of data (LEP + SLC + CDF), the curves at 95% C.L. in the $m_{\text{top}}$–$m_{\text{Higgs}}$ plane are also shown in Fig. 5 for three different values of $\alpha_s$ and including the Higgs mass penalty function.

Predictions and corresponding errors from a fit to LEP+SLD+CDF data (average $g_V/g_A$) are given in Table 3, where $\sin^2 \vartheta(b)$ includes the universal $Z \to bb$ vertex corrections. The effect of the SLD measurement is to bring the $m_{\text{Higgs}}$ upper limit well below 1 TeV almost independently of the CDF constraint. The reason is that SLD wants $m_{\text{top}}$ large and $m_{\text{Higgs}}$ and $\alpha_s$ small in order to readjust as much as possible the LR asymmetry. The constraint on $m_{\text{Higgs}}$ is more a symptom of the clash between SLD and LEP than a reliable hint of $m_{\text{Higgs}}$ small. The information carried by the CDF constraint requires a careful examination. Actually it has been verified that without the CDF constraint the $\chi^2$ shape as a function of $m_{\text{Higgs}}$ is unstable with respect to normal fluctuations of the experimental data in the large $m_{\text{Higgs}}$ tail, in agreement with [14], whereas the inclusion of the CDF constraint renders the tail more stable under small perturbations of the data. In the case of $m_{\text{Higgs}}$ determination the theoretical uncertainty on EW higher orders plays a very relevant role. The situation is described in Fig. 6, where the $\chi^2$ as a function of $m_{\text{Higgs}}$ is plotted for the most complete set of data LEP + SLD + CDF (7 observables). Actually the $\chi^2$ is not a single curve but instead the whole band inside the two solid lines, describing the theoretical uncertainty on the Standard Model pseudo-observables. Inside this band we have reported the $\chi^2$ distribution as derived from TOPAZ0 in its default settings and also the one obtained from TOPAZ0 adapted for comparisons with other existing codes. This theoretical uncertainty leads to a corresponding uncertainty on the upper limit of some 200 GeV.

In conclusion, the last LEP, SLD and CDF data bring to an indirect determination of $m_{\text{top}}$ at $m_{\text{top}} = 174^{+9.3+12.0+0.2}_{-9.6-12.5-3.4} \text{ GeV}$ and allow to discuss an upper limit on $m_{\text{Higgs}}$ with some improvement with respect to the past.

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| Observable       | Value                          |
|------------------|-------------------------------|
| $M_Z$            | $91.1899 \pm 0.0044$ GeV      |
| $\Gamma_Z$      | $2497.1 \pm 3.8$ MeV          |
| $R$              | $20.789 \pm 0.04$             |
| $R_b$            | $0.2210 \pm 0.0019$           |
| $\sigma_{had}$  | $51.51 \pm 0.12$ nb          |
| $A^l_{FB}$       | $0.0170 \pm 0.0016$          |
| $A^\tau_{pol}$   | $0.150 \pm 0.010$            |
| $A^e$            | $0.120 \pm 0.012$            |
| $A^b_{FB}$       | $0.0970 \pm 0.0045$          |
| $A^c_{FB}$       | $0.072 \pm 0.011$            |
| $A_{LR}$         | $0.1668 \pm 0.0079$          |
| $g_V/g_A$(LEP)   | $0.0711 \pm 0.0020$          |
| $g_V/g_A$(LEP+SLD)| $0.0737 \pm 0.0018$          |
| $M_W/M_Z$        | $0.8814 \pm 0.0021$          |

Table 1: Experimental values
| Set of data                  | $\chi^2_{\text{min}}$ | $m_{Higgs}$ (GeV) | 95% C.L. |
|-----------------------------|-----------------------|-------------------|----------|
| LEP + SLD + CDF             | 7.5/7                 | 64                | 580      |
| LEP + CDF                  | 7.8/7                 | 187               | 1354     |
| LEP                        | 6.9/7                 | 76                | 986      |
| LEP + SLD                  | 6.6/7                 | 39                | 400      |
| LEP + SLD + CDF            | 17.1/11               | 53                | 511      |
| LEP + CDF                  | 9.7/11                | 165               | 1237     |

Table 2: Predictions for $m_{Higgs}$
| Observable | stat.         | Higgs       | theor.       |
|------------|---------------|-------------|--------------|
| $M_W$      | 80.321 GeV    | ±0.052      | ±0.019       |
| $\sin^2 \theta(l)$ | 0.2319 ±0.0003 | ±0.0002 ±0.0004 | +0.002 ±0.004 |
| $\sin^2 \theta(b)$ | 0.2331 ±0.0002 | ±0.0005 ±0.0002 | +0.0 ±0.0002 |

Table 3: Our predictions for $M_W$ and $\sin^2 \theta(l, b)$ for a fit to LEP+SLD+CDF data.
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