Reinforcement Learning with Parameterized Actions

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Abstract
We introduce a model-free algorithm for learning in Markov decision processes with parameterized actions—discrete actions with continuous parameters. At each step the agent must select both which action to use and which parameters to use with that action. We introduce the Q-PAMDP algorithm for learning in these domains, show that it converges to a local optimum, and compare it to direct policy search in the goal-scoring and Platform domains.

1 Introduction
Reinforcement learning agents typically have either a discrete or a continuous action space (Sutton and Barto 1998). With a discrete action space, the agent decides which distinct action to perform from a finite action set. With a continuous action space, actions are expressed as a single real-valued vector. If we use a continuous action space, we lose the ability to consider differences in kind: all actions must be expressible as a single vector. If we use only discrete actions, we lose the ability to finely tune action selection based on the current state.

A parameterized action is a discrete action parameterized by a real-valued vector. Modeling actions this way introduces structure into the action space by treating different kinds of continuous actions as distinct. At each step an agent must choose both which action to use and what parameters to use to execute it. For example, consider a soccer playing robot which can kick, pass, or run. We can associate a continuous parameter vector to each of these actions: we can kick the ball to a given target position with a given force, pass to a specific position, and run with a given velocity. Each of these actions is parameterized in its own way. Parameterized action Markov decision processes (PAMDPs) model situations where we have distinct actions that require parameters to adjust the action to different situations, or where there are multiple mutually incompatible continuous actions.

We focus on how to learn an action-selection policy given pre-defined parameterized actions. We introduce the Q-PAMDP algorithm, which alternates learning action-selection and parameter-selection policies and compare it to a direct policy search method. We show that with appropriate update rules Q-PAMDP converges to a local optimum. These methods are compared empirically in the goal and Platform domains. We found that Q-PAMDP out-performed direct policy search and fixed parameter SARSA.

2 Background
A Markov decision process (MDP) is a tuple \( \langle S, A, P, R, \gamma \rangle \), where \( S \) is a set of states, \( A \) is a set of actions, \( P(s, a, s') \) is the probability of transitioning to state \( s' \) from state \( s \) after taking action \( a \), \( R(s, a, r) \) is the probability of receiving reward \( r \) for taking action \( a \) in state \( s \), and \( \gamma \) is a discount factor (Sutton and Barto 1998). We wish to find a policy, \( \pi(a|s) \), which selects an action for each state so as to maximize the expected sum of discounted rewards (the return).

The value function \( V^\pi(s) \) is defined as the expected discounted return achieved by policy \( \pi \) starting at state \( s \)

\[
V^\pi(s) = \mathbb{E}_\pi \left[ \sum_{t=0}^{\infty} \gamma^t r_t \right].
\]

Similarly, the action-value function is given by

\[
Q^\pi(s, a) = \mathbb{E}_\pi \left[ r_0 + \gamma V^\pi(s') \right],
\]

as the expected return obtained by taking action \( a \) in state \( s \), and then following policy \( \pi \) thereafter. While using the value function in control requires a model, we would prefer to do so without needing such a model. We can approach this problem by learning \( Q \), which allows us to directly select the action which maximizes \( Q^\pi(s, a) \). We can learn \( Q \) for an optimal policy using a method such as Q-learning (Watkins and Dayan 1992). In domains with a continuous state space, we can represent \( Q(s, a) \) using parametric function approximation with a set of parameters \( \omega \) and learn this with algorithms such as gradient descent SARSA(\( \lambda \)) (Sutton and Barto 1998).

For problems with a continuous action space (\( A \subseteq \mathbb{R}^m \)), selecting the optimal action with respect to \( Q(s, a) \) is non-trivial, as it requires finding a global maximum for a function in a continuous space. We can avoid this problem using a policy search algorithm, where a class of policies parameterized by a set of parameters \( \theta \) is given, which transforms the problem into one of direct optimization over \( \theta \) for an objective function \( J(\theta) \). Several policy search approaches...
Parameterized Tasks

A parameterized task is a problem defined by a task parameter vector $\tau$ given at the beginning of each episode. These parameters are fixed throughout an episode, and the goal is to learn a task dependent policy. Kober et al. (2012) developed algorithms to adjust motor primitives to different task parameters. They apply this to learn table-tennis and darts with different starting positions and targets. Da Silva et al. (2012) introduced the idea of a parameterized skill as a task dependent parameterized policy. They sample a set of tasks, learn their associated parameters, and determine a mapping from task to policy parameters. Deisenroth et al. (2013) applied a model-based method to learn a task dependent parameterized policy. This is used to learn task dependent policies for ball-hitting task, and for solving a block manipulation problem. Parameterized tasks can be used as parameterized actions. For example, if we learn a parameterized task for kicking a ball to position $\tau$, this could be used as a parameterized action kick-to($\tau$).

3 Parameterized Action MDPs

We consider MDPs where the state space is continuous ($S \subseteq \mathbb{R}^n$) and the actions are parameterized: there is a finite set of discrete actions $A_d = \{a_1, a_2, \ldots, a_k\}$, and each $a \in A_d$ has a set of continuous parameters $X_a \subseteq \mathbb{R}^{m_a}$. An action is a tuple $(a, x)$ where $a$ is a discrete action and $x$ are the parameters for that action. The action space is then given by

$$A = \bigcup_{a \in A_d} \{(a, x) \mid x \in X_a\},$$

which is the union of each discrete action with all possible parameters for that action. We refer to such MDPs as parameterized action MDPs (PAMDPs). Figure 1 depicts the different action spaces.

We apply a two-tiered approach for action selection: first selecting the parameterized action, then selecting the parameters for that action. The discrete-action policy is denoted $\pi^d(a|s)$. To select the parameters for the action, we define the action-parameter policy for each action $a$ as $\pi^a(x|s)$. The policy is then given by

$$\pi(a, x|s) = \pi^d(a|s)\pi^a(x|s).$$

In other words, to select a complete action $(a, x)$, we sample a discrete action $a$ from $\pi^d(a|s)$ and then sample a parameter $x$ from $\pi^a(x|s)$. The action policy is defined by the parameters $\omega$ and is denoted by $\pi^a_\omega(x|s)$. The action-parameter policy for action $a$ is determined by a set of parameters $\theta_a$, and is denoted $\pi^a_{\theta_a}(x|s)$. The set of these parameters is given by $\theta = [\theta_{a_1}, \ldots, \theta_{a_k}]$.

The first approach we consider is direct policy search. We use a direct policy search method to optimize the objective function

$$J(\theta, \omega) = \mathbb{E}_{s_0 \sim D}[V^\pi_{\theta,\omega}(s_0)],$$

with respect to $(\theta, \omega)$, where $s_0$ is a state sampled according to the state distribution $D$. $J$ is the expected return for a given policy starting at an initial state.

Our second approach is to alternate updating the parameter-policy and learning an action-value function for the discrete actions. For any PAMDP $M = (S, A, P, R, \gamma)$ with a fixed parameter-policy $\pi^a_\omega$, there exists a corresponding discrete action MDP $M_\theta = (S, A_d, P_\theta, R_\theta, \gamma)$, where

$$P_\theta(s'|s, a) = \int_{x \in X_a} \pi^a_{\theta_a}(x|s)P(s'|s, a, x)dx,$$

$$R_\theta(r|s, a) = \int_{x \in X_a} \pi^a_{\theta_a}(x|s)R(r|s, a, x)dx.$$
Algorithm 1 Q-PAMDP($k$)

**Input:**
Initial parameters $\theta_0, \omega_0$
Parameter update method $\text{P-UPDATE}$
Q-learning algorithm $\text{Q-LEARN}$

**Algorithm:**

$\omega \leftarrow \text{Q-LEARN}^{(\infty)}(M_\theta, \omega_0)$

repeat
\[
\theta \leftarrow \text{P-UPDATE}^{(k)}(J_\omega, \theta)
\]
\[
\omega \leftarrow \text{Q-LEARN}^{(\infty)}(M_\theta, \omega)
\]
until $\theta$ converges

Algorithm 1 describes a method for alternating updating $\theta$ and $\omega$. The algorithm uses two input methods: $\text{P-UPDATE}$ and $\text{Q-LEARN}$ and a positive integer parameter $k$, which determines the number of updates to $\theta$ for each iteration. $\text{P-UPDATE}(f, \theta)$ should be a policy search method that optimizes $\theta$ with respect to objective function $f$. Q-LEARN can be any algorithm for Q-learning with function approximation. We consider two main cases of the Q-PAMDP algorithm: Q-PAMDP(1) and Q-PAMDP($\infty$).

Q-PAMDP(1) performs a single update of $\theta$ and then relearns $\omega$ to convergence. If at each step we only update $\theta$ once, and then update $\omega$ until convergence, we can optimize $\theta$ with respect to $H$. In the next section we show that if we can find a local optimum $\theta$ with respect to $H$, then we have found a local optimum with respect to $J$.

4 Theoretical Results

We now show that Q-PAMDP(1) converges to a local or global optimum with mild assumptions. We assume that iterating $\text{P-UPDATE}$ converges to some $\theta^*$ with respect to a given objective function $f$. As the $\text{P-UPDATE}$ step is a design choice, it can be selected with the appropriate convergence property. Q-PAMDP(1) is equivalent to the sequence

$\omega_{t+1} = W(\theta_t)$

$\theta_{t+1} = \text{P-UPDATE}(J_{\omega_{t+1}}, \theta_t),$

if $\text{Q-LEARN}$ converges to $W(\theta)$ for each given $\theta$.

**Theorem 4.1 (Convergence to a Local Optimum).** For any $\theta_0$, if the sequence

$\theta_{t+1} = \text{P-UPDATE}(H, \theta_t), \quad (1)$

converges to a local optimum with respect to $H$, then Q-PAMDP(1) converges to a local optimum with respect to $J$.

**Proof.** By definition of the sequence above $\omega_t = W(\theta_t)$, so it follows that

$J_{\omega_t} = J(\theta, W(\theta)) = H(\theta)$.

In other words, the objective function $J$ equals $H$ if $\omega = W(\theta)$. Therefore, we can replace $J$ with $H$ in our update for $\theta$, to obtain the update rule

$\theta_{t+1} = \text{P-UPDATE}(H, \theta_t).$

Therefore by equation [1] the sequence $\theta_t$ converges to a local optimum $\theta^*$ with respect to $H$. Let $\omega^* = W(\theta^*)$. As $\theta^*$ is a local optimum with respect to $H$, by definition there exists $\epsilon > 0$, s.t.

\[ ||\theta^* - \theta||_2 < \epsilon \implies H(\theta) \leq H(\theta^*). \]

Therefore for any $\omega$,

\[ \|\left(\theta^*\right) - \left(\theta \omega\right)\|_2 < \epsilon \implies \|\theta^* - \theta\|_2 < \epsilon \]

\[ \implies H(\theta) \leq H(\theta^*) \]

\[ \implies J(\theta, \omega) \leq J(\theta^*, \omega^*). \]

Therefore $(\theta^*, \omega^*)$ is a local optimum with respect to $J$. $\square$

In summary, if we can locally optimize $\theta$, and $\omega = W(\theta)$ at each step, then we will find a local optimum for $J(\theta, \omega)$. The conditions for the previous theorem can be met by assuming that $\text{P-UPDATE}$ is a local optimization method such as a gradient based policy search. A similar argument shows that if the sequence $\theta_t$ converges to a global optimum with respect to $H$, then Q-PAMDP(1) converges to a global optimum $(\theta^*, \omega^*)$.

One problem is that at each step we must re-learn $W(\theta)$ for the updated value of $\theta$. We now show that if updates to $\theta$ are bounded and $W(\theta)$ is a continuous function, then the required updates to $\omega$ will also be bounded. Intuitively, we are supposing that a small update to $\theta$ results in a small change in the weights specifying which discrete action to choose. The assumption that $W(\theta)$ is continuous is strong, and may not be satisfied by all PAMDPs. It is not necessary for the operation of Q-PAMDP(1), but when it is satisfied we do not need to completely re-learn $\omega$ after each update to $\theta$. We show that by selecting an appropriate $\alpha$ we can shrink the differences in $\omega$ as desired.

**Theorem 4.2 (Bounded Updates to $\omega$).** If $W$ is continuous with respect to $\theta$, and updates to $\theta$ are of the form

$\theta_{t+1} = \theta_t + \alpha_t \text{P-UPDATE}(\theta_t, \omega_t),$

with the norm of each $\text{P-UPDATE}$ bounded by

$0 < ||\text{P-UPDATE}(\theta_t, \omega_t)||_2 < \delta,$

for some $\delta > 0$, then for any difference in $\omega \epsilon > 0$, there is an initial update rate $\alpha_0 > 0$ such that

$\alpha_t < \alpha_0 \implies ||\omega_{t+1} - \omega_t||_2 < \epsilon.$

**Proof.** Let $\epsilon > 0$ and

$\alpha_0 = \frac{\delta}{\|\text{P-UPDATE}(\theta_t, \omega_t)\|_2}.$

As $\alpha_t < \alpha_0$, it follows that

$\delta > \alpha_t \|\text{P-UPDATE}(\theta_t, \omega_t)||_2$

$= ||\alpha_t \text{P-UPDATE}(\theta_t, \omega_t)||_2$

$= ||\theta_{t+1} - \theta_t||_2.$

So we have

$||\theta_{t+1} - \theta_t||_2 < \delta.$

As $W$ is continuous, this means that

$||W(\theta_{t+1}) - W(\theta_t)||_2 = ||\omega_{t+1} - \omega_t||_2 < \epsilon.$

$\square$
In other words, if our update to \( \theta \) is bounded and \( W \) is continuous, we can always adjust the learning rate \( \alpha \) so that the difference between \( \omega_t \) and \( \omega_{t+1} \) is bounded.

With Q-PAMDP(1) we want P-UPDATE to optimize \( H(\theta) \). One logical choice would be to use a gradient update. The next theorem shows that gradient of \( H \) is equal to the gradient of \( J \) if \( \omega = W(\theta) \). This is useful as we can apply existing gradient-based policy search methods to compute the gradient of \( J \) with respect to \( \theta \). The proof follows from the fact that we are at a global optimum of \( J \) with respect to \( \omega \), and so the gradient \( \nabla_\omega J \) is zero. This theorem requires that \( W \) is differentiable (and therefore also continuous).

**Theorem 4.3** (Gradient of \( H(\theta) \)). If \( J(\theta, \omega) \) is differentiable with respect to \( \theta \) and \( \omega \) and \( W(\theta) \) is differentiable with respect to \( \theta \), then the gradient of \( H \) is given by \( \nabla_\theta H(\theta) = \nabla_\theta J(\theta, \omega^*) \), where \( \omega^* = W(\theta) \).

**Proof.** If \( \theta \in \mathbb{R}^n \) and \( \omega \in \mathbb{R}^m \), then we can compute the gradient of \( H \) by the chain rule:

\[
\frac{\partial H(\theta)}{\partial \theta_i} = \frac{\partial J(\theta, W(\theta))}{\partial \theta_i} = \frac{\partial J(\theta, \omega^*)}{\partial \theta_i} + \sum_{j=1}^n \frac{\partial J(\theta, \omega^*)}{\partial \theta_j} \frac{\partial \theta_j}{\partial \theta_i} + \sum_{k=1}^m \frac{\partial J(\theta, \omega^*)}{\partial \omega_k^*} \frac{\partial \omega_k^*}{\partial \theta_i},
\]

where \( \omega^* = W(\theta) \). Note that as by definitions of \( W \),

\[
\omega^* = W(\theta) = \arg \max \ J(\theta, \omega),
\]

we have that the gradient of \( J \) with respect to \( \omega \) is zero \( \partial J(\theta, \omega^*)/\partial \omega_k^* = 0 \), as \( \omega \) is a global maximum with respect to \( J \) for fixed \( \theta \). Therefore, we have that

\[
\nabla_\theta H(\theta) = \nabla_\theta J(\theta, \omega^*).
\]

To summarize, if \( W(\theta) \) is continuous and P-UPDATE converges to a global or local optimum, then Q-PAMDP(1) will converge to a global or local optimum, respectively, and the Q-LEARN step will be bounded if the update rate of the P-UPDATE step is bounded. As such, if P-UPDATE is a policy gradient update step then Q-PAMDP by Theorem 4.1 will converge to a local optimum and by Theorem 4.4 the Q-LEARN step will require a fixed number of updates. This policy gradient step can use the gradient of \( J \) with respect to \( \theta \).

With Q-PAMDP(∞) each step performs a full optimization on \( \theta \) and then a full optimization of \( \omega \). The \( \theta \) step would optimize \( J(\theta, \omega) \), not \( H(\theta) \), as we do update \( \omega \) while we update \( \theta \). Q-PAMDP(∞) has the disadvantage of requiring global convergence properties for the P-UPDATE method.

**Theorem 4.4** (Local Convergence of Q-PAMDP(∞)). If at each step \( \theta \) of Q-PAMDP(∞) for some bounded set \( \Theta \):

\[
\theta_{t+1} = \arg \max_{\theta \in \Theta} J(\theta, \omega_t),
\]

\[
\omega_{t+1} = W(\theta_{t+1}),
\]

then Q-PAMDP(∞) converges to a local optimum.

**Proof.** By definition of \( W \), \( \omega_{t+1} = \arg \max_{\omega} J(\theta_{t+1}, \omega) \). Therefore this algorithm takes the form of direct alternating optimization. As such, it converges to a local optimum (Bezdek and Hathaway 2002).

Q-PAMDP(∞) has weaker convergence properties than Q-PAMDP(1), as it requires a globally convergent P-UPDATE. However, it has the potential to bypass nearby local optima (Bezdek and Hathaway 2002).

## 5 Experiments

We first consider a simplified robot soccer problem (Kitano et al. 1997) where a single striker attempts to score a goal against a keeper. Each episode starts with the player at a random position along the bottom bound of the field. The player starts with the ball in possession, and the keeper is positioned between the ball and the goal. The game takes place in a 2D environment where the player and the keeper have a position, velocity and orientation and the ball has a position and velocity, resulting in 14 continuous state variables.

An episode ends when the keeper possesses the ball, the player scores a goal, or the ball leaves the field. The reward for an action is 0 for non-terminal state, 50 for a terminal goal state, and \(-d\) for a terminal non-goal state, where \(d\) is the distance of the ball to the goal. The player has two parameterized actions: kick-to \((x, y)\), which kicks the ball towards position \((x, y)\); and shoot-goal \((h)\), which shoots the ball towards a position \(h\) along the goal line. Noise is added to each action. If the player is not in possession of the ball, it moves towards it. The keeper has a fixed policy: it moves towards the ball, and if the player shoots at the goal, the keeper moves to intercept the ball.

To score a goal, the player must shoot around the keeper. This means that at some positions it must shoot left past the keeper, and at others to the right past the keeper. However at no point should it shoot at the keeper, so an optimal policy is discontinuous. We split the action into two parameterized actions: shoot-goal-left, shoot-goal-right. This allows us to use a simple action selection policy instead of complex continuous action policy. This policy would be difficult to represent in a purely continuous action space, but is simpler in a parameterized action setting.

We represent the action-value function for the discrete action \(\pi\) using linear function approximation with Fourier basis features (Konidaris, Osentoski, and Thomas 2011). As we have 14 state variables, we must be selective in which basis functions to use. We only use basis functions with two non-zero elements and exclude all velocity state variables. We use the soft-max discrete action policy (Sutton and Barto 1998). We represent the action-parameter policy \(\pi_\theta\) as a normal distribution around a weighted sum of features \(\pi_\theta(x|s) = N(\theta^T \psi_\theta(s), \Sigma)\), where \(\theta\) is a matrix of weights, and \(\psi_\theta(s)\) gives the features for state \(s\), and \(\Sigma\) is a fixed covariance matrix. We use specialized features for each action. For the shoot-goal actions we are using a simple linear basis \((1, g)\), where \(g\) is the projection of the keeper onto the goal line. For kick-to we use linear features \((1, bx, by, bx^2, by^2, (bx - kx)/||b - k||_2, (by - b)\) for \(b \in \mathbb{R}^2\) and \(k \in \mathbb{R}^2\) representing the positions of the kick point and the keeper, respectively.
where \((b_x, b_y)\) is the position of the ball and \((k_x, k_y)\) is the position of the keeper.

For the direct policy search approach, we use the episodic natural actor critic (eNAC) algorithm \(\text{Peters and Schaal 2008}\), computing the gradient of \(J(\omega, \theta)\) with respect to \((\omega, \theta)\). For the Q-PAMDP approach we use the gradient-descent Sarsa(\(\lambda\)) algorithm for Q-learning, and the eNAC algorithm for policy search. At each step we perform one eNAC update based on 50 episodes and then refit \(Q_\omega\) using 50 gradient descent Sarsa(\(\lambda\)) episodes.

Return is directly correlated with goal scoring probability, so their graphs are close to identical. As it is easier to interpret, we plot goal scoring probability in figure 2. We can see that direct eNAC is outperformed by Q-PAMDP(1) and Q-PAMDP(\(\infty\)). This is likely due to the difficulty of optimizing the action selection parameters directly, rather than with Q-learning.

For both methods, the goal probability is greatly increased: while the initial policy rarely scores a goal, both Q-PAMDP(1) and Q-PAMDP(\(\infty\)) increase the probability of a goal to roughly 35%. Direct eNAC converged to a local maxima of 15%. Finally, we include the performance of SARSA(\(\lambda\)) where the action parameters are fixed at the initial \(\theta_0\). This achieves roughly 20% scoring probability. Both Q-PAMDP(1) and Q-PAMDP(\(\infty\)) strongly outperform fixed parameter SARSA, but eNAC does not. Figure 3 depicts a single episode using a converged Q-PAMDP(1) policy— the player draws the keeper out and strikes when the goal is open.

Next we consider the Platform domain, where the agent starts on a platform and must reach a goal while avoiding enemies. If the agent reaches the goal platform, touches an enemy, or falls into a gap between platforms, the episode ends. The state space consists of four variables \((x, \dot{x}, e_x, \dot{e}_x)\), representing the agent position, agent speed, enemy position, and enemy speed respectively. For learning \(Q_\omega\), as in the previous domain, we use linear function approximation with the Fourier basis. We apply a softmax discrete action policy based on \(Q_\omega\), and a Gaussian parameter policy based on scaled parameter features \(\psi_\omega(s)\).

Figure 5 shows the performance of eNAC, Q-PAMDP(1), Q-PAMDP(\(\infty\)), and SARSA with fixed parameters. Both Q-PAMDP(1) and Q-PAMDP(\(\infty\)) outperformed the fixed parameter SARSA method, reaching on average 50% and 65% of the total distance respectively. We suggest that Q-PAMDP(\(\infty\)) outperforms Q-PAMDP(1) due to the nature of the Platform domain. Q-PAMDP(1) is best suited to domains with smooth changes in the action-value function with respect to changes in the parameter-policy. With the Platform domain, our initial policy is unable to make the first jump without modification. When the policy can reach the second platform, we need to drastically change the action-value function to account for this platform. Therefore, Q-
Figure 5: Average percentage distance covered, averaged over 20 runs for Q-PAMDP(1), Q-PAMDP(∞), and eNAC in the Platform domain. Intervals show standard error.

Figure 6: A successful episode of the Platform domain. The agent hops over the enemies, leaps over the gaps, and reaches the last platform.

PAMDP(1) may be poorly suited to this domain as the small change in parameters that occurs between failing to making the jump and actually making it results in a large change in the action-value function. This is better than the fixed SARSA baseline of 40%, and much better than direct optimization using eNAC which reached 10%. Figure 6 shows a successfully completed episode of the Platform domain.

6 Related Work

Hauskrecht et al. (2004) introduced an algorithm for solving factored MDPs with a hybrid discrete-continuous action space. However, their formalism has an action space with a mixed set of discrete and continuous components, whereas our domain has distinct actions with a different number of continuous components for each action. Furthermore, they assume the domain has a compact factored representation, and only consider planning.

Rachelson (2009) encountered parameterized actions in the form of an action to wait for a given period of time in his research on time dependent, continuous time MDPs (TMDPs). He developed XMDPs, which are TMDPs with a parameterized action space (Rachelson 2009). He developed a Bellman operator for this domain, and in a later paper mentions that the TiMDPpoly algorithm can work with parameterized actions, although this specifically refers to the parameterized wait action (Rachelson, Fabiani, and Garcia 2009). This research also takes a planning perspective, and only considers a time dependent domain. Additionally, the size of the parameter space for the parameterized actions is the same for all actions.

Hoey et al. (2013) considered mixed discrete-continuous actions in their work on Bayesian affect control theory. To approach this problem they use a form of POMCP, a Monte Carlo sampling algorithm, using domain specific adjustments to compute the continuous action components (Silver and Veness 2010). They note that the discrete and continuous components of the action space reflect different control aspects: the discrete control provides the “what”, while the continuous control describes the “how” (Hoey, Schroder, and Althothali 2013).

In their research on symbolic dynamic programming (SDP) algorithms, Zamani et al. (2012) considered domains with a set of discrete parameterized actions. Each of these actions has a different parameter space. Symbolic dynamic programming is a form of planning for relational or first-order MDPs, where the MDP has a set of logical relationships defining its dynamics and reward function. Their algorithms represent the value function as an extended algebraic decision diagram (XADD), and is limited to MDPs with predefined logical relations.

A hierarchical MDP is an MDP where each action has subtasks. A subtask is itself an MDP with its own states and actions which may have their own subtasks. Hierarchical MDPs are well-suited for representing parameterized actions as we could consider selecting the parameters for a discrete action as a subtask. MAXQ is a method for value function decomposition of hierarchical MDPs (Dietterich 2000). One possibility is to use MAXQ for learning the action-values in a parameterized action problem.

7 Conclusion

The PAMDP formalism models reinforcement learning domains with parameterized actions. Parameterized actions give us the adaptability of continuous domains and to use distinct kinds of actions. They also allow for simple representation of discontinuous policies without complex parameterizations. We have presented three approaches for model-free learning in PAMDPs: direct optimization and two variants of the Q-PAMDP algorithm. We have shown that Q-PAMDP(1), with an appropriate P-UPDATE method, converges to a local or global optimum. Q-PAMDP(∞) with a global optimization step converges to a local optimum. Q-PAMDP(∞) with a global optimization step converges to a local optimum.

We have examined the performance of these approaches in the goal scoring domain and the Platformer domain. The robot soccer goal domain models the situation where a striker must out-maneuver a keeper to score a goal. Of these, Q-PAMDP(1) and Q-PAMDP(∞) outperformed eNAC and fixed parameter SARSA. Q-PAMDP(1) and Q-PAMDP(∞) performed similarly well in terms of goal scoring, learning policies that score goals roughly 35% of the time. In the Platform domain we found that both Q-PAMDP(1) and Q-PAMDP(∞) outperformed eNAC and fixed SARSA.
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