Natural Slow-Roll Inflation\textsuperscript{1}

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Abstract
It is shown that the non-perturbative dynamics of a phase change to the non-trivial phase of $\lambda \phi^4$-theory in the early universe can give rise to slow-rollover inflation without recourse to unnaturally small couplings.

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1. **Introduction**

The inflation paradigm solves a number of cosmological problems, including the horizon problem, the flatness-age problem and the problem of the origin of density inhomogeneities. Its essential feature is a super-luminal expansion of the universe at very early times. Models of inflation generally rely on the dynamics of some scalar field (the inflaton), which can also be thought of as an order parameter for the possible phases of a scalar field. At early times it is assumed that the inflaton is displaced from the absolute minimum of its potential, and the potential energy density thus produced is assumed to dominate. Such a vacuum energy density, inserted into the (standard Einstein) gravitational action, creates a period of exponential expansion of the universe with time until the scalar field attains its absolute minimum. One of the perhaps more natural mechanisms for localising the field away from its absolute minimum is based on the temperature-dependent effective potential (of a form similar to that in figure 1) of the scalar field, where at high temperature the minimum of the potential is at the origin but at temperatures below a critical temperature the absolute minimum takes some other value (any remaining local minimum at the origin being labeled the false vacuum). The first inflation models assumed a strong first order phase transition from the false to true vacuum and the time before this tunneling process occurred gives the duration of the exponential expansion. However, if this tunneling time is too large, the phase transition, turning false vacuum into true vacuum, will never be completed. In fact, a detailed calculation shows that the inflation model with a strong first order phase transition is very unlikely. Subsequent to this, many models were designed to overcome this problem whilst preserving the essential ingredients of inflation. One of the more successful approaches relies on a mechanism originally proposed by Albrecht and Steinhardt, and Linde where the effective potential of the order parameter has an extremely flat region near the false vacuum minimum implying that the evolution of the inflaton to the true vacuum is via a process of “slow rolling.” Such models successfully implement inflation but require unnaturally small couplings ($\sim 10^{-12}$) to satisfy the bounds on density fluctuations imposed by the cosmic microwave background radiation (CMBR). The mechanism arising from natural particle physics models which could produce such small couplings, or more generally, an extremely flat part of the potential, is not yet fully understood, and will be the subject of this letter.

Recently, some insight into the vacuum structure of massless $\varphi^4$-theory has been gained. The theory has two phases, a trivial phase and a non-trivial phase with non-vanishing condensate. The effective potential of the scalar field $\varphi$ was calculated in a renormalisation group invariant approach and it was found that in the trivial phase it coincides with the one-loop renormalisation group improved effective potential. However, in the non-perturbative phase, a dynamical mass for the scalar field is generated, and the vacuum energy density agrees with
the predictions from the scale anomaly. The effective potential is convex \([8]\) in both phases, implying \(\langle \varphi \rangle = 0\) and this means that \(\langle \varphi \rangle\) is not an appropriate order parameter for describing the vacuum structure of the theory. We will see below that an appropriate and convenient order parameter is \(\sqrt{\langle \varphi^2 \rangle}\). The effective potential of the scalar condensate \(\langle \varphi^2 \rangle\) was calculated in \([9]\) where it was found that at large temperature the system is in the perturbative phase (see figure 1) and that at a critical temperature the energy density of the non-perturbative phase becomes lower than that of the trivial phase, and a phase transition to the non-trivial vacuum occurs.

In this letter we present the effective action for the order parameter \(\sqrt{\langle \varphi^2 \rangle}\) at finite temperature \(T\) in a derivative expansion. We will find that the kinetic energy term acquires a temperature-dependent dynamical prefactor, which becomes large if \(\sqrt{\langle \varphi^2 \rangle} < T\). This prefactor implies that tunneling from the false vacuum state \((\sqrt{\langle \varphi^2 \rangle} = 0)\) to the true ground state is suppressed. The system can be quantitatively described in terms of an auxiliary field (with standard kinetic term) moving in a modified potential and this modified potential is of the form used in slow-rolling-inflation models. This can explain the slow roll phase transition of the inflaton to the true vacuum in a natural way without relying on an unnaturally small coupling.

2. Effective action of the scalar condensate in derivative expansion

Starting from the Euclidean generating function for \(\lambda \varphi^4\) theory

\[
Z[j] = \int \mathcal{D}\varphi \exp\left\{-\int d^4x \left( \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + \frac{m^2}{2} \varphi^2 + \frac{\lambda}{4!} \varphi^4 - j(\vec{x}) \varphi^2(\vec{x}) \right) \right\}
\]

and then linearising the \(\varphi^4\)-interaction with an auxiliary field \(M\) it is possible to calculate the effective potential (see figure 1) for the scalar condensate at finite and zero temperature (see refs. \([8, 9, 10]\) for details). Here we generalise the calculation to weakly space dependent condensates, and evaluate the effective action in a gradient expansion. To lowest order in the modified loop expansion \([8, 9]\) we have

\[
-\ln Z[j] = \int_0^{1/T} d\tau \int d^3x \left( -\frac{3}{2\lambda} [M - m^2 + 2j(\vec{x})^2] \right) + \frac{1}{2} \text{Tr} \ln(-\partial^2 + M(\vec{x})) ,
\]

where \(T\) is the temperature in units of Boltzmann’s constant and \(M\) now takes its mean field value. At finite temperature, the trace extends over all configurations in space-time which satisfy periodic boundary conditions in the Euclidean time direction. In Schwinger’s proper time regularisation the loop contribution is

\[
L = \frac{1}{2} \text{Tr} \ln(-\partial^2 + M) = -\frac{1}{2} \text{Tr}_3 \sum_n \int_{1/\Lambda^2} \frac{ds}{s} \exp\left\{-s(-\nabla^2 + M(\vec{x}) + (2\pi T)^2 n^2) \right\} ,
\]

where \(\Lambda^2\) is a cut-off parameter chosen to be large compared to both \(m^2\) and \(\lambda\) for the effective potential to be well approximated by the modified potential.

3
where $\Lambda$ is the proper time cutoff and the trace $\text{Tr}_3$ in (3) extends only over the space degrees of freedom. Using Poisson’s formula we obtain the alternative form

$$L = -\frac{1}{4\sqrt{\pi T}} \int \frac{ds}{s^{3/2}} \sum_{\nu} \exp(-\frac{1}{4sT^2} \nu^2) \text{Tr}_3 \exp[-s(-\nabla^2 + M(\vec{x}))],$$

(4)

which is more convenient for discussing the ultra-violet behaviour of the loop [9, 12]. The divergences arising from the loop in (2) can be absorbed into the bare parameters $\lambda, m, j$. Here we generalise the renormalisation procedure of [9] to space-time dependent sources $j(x)$, i.e.,

$$\frac{6}{\lambda} + \frac{1}{16\pi^2} \left( \ln \frac{\Lambda^2}{\mu^2} - \gamma + 1 \right) = \frac{6}{\lambda_R},$$

(5)

$$\frac{6}{\lambda} j(x) - \frac{3m^2}{\lambda} - \frac{1}{32\pi^2} \Lambda^2 = \frac{6}{\lambda_R} j_R(x) - \frac{3m^2}{\lambda_R},$$

(6)

$$\int d^4x \ j^2(x) - m^2 \int d^4x \ j(x) = 0,$$

(7)

where $\lambda_R, m_R, j_R$ are the renormalised quantities. In the following we only consider the massless case $m_R = 0$. In this case, equations (5) and (7) imply that

$$- \ln Z[j_R] = \int_0^{1/T} d\tau \int d^3x \left( -\frac{3}{2\lambda} M^2 - \frac{6}{\lambda_R} j_RM - \frac{1}{32\pi^2} \Lambda^2 M \right)$$

$$+ \frac{1}{2} \text{Tr} \ln(-\partial^2 + M(\vec{x})).$$

(8)

This generating functional is finite. The quadratic and logarithmic divergencies of the trace-term in (8) are cancelled by the terms $-\frac{1}{32\pi^2} \Lambda^2 M$, $-\frac{3}{2\lambda} M^2$ respectively [9]. The effective action can now be obtained by a Legendre transformation with respect to the renormalised source $j_R$, i.e.,

$$\Gamma[\varphi^2_c] := - \ln Z[j_R] + \int d^4x \ \varphi^2_c(x) j_R(x), \quad \varphi^2_c(x) := \frac{\delta \ln Z[j_R]}{\delta j_R(x)},$$

(9)

where the classical field $\varphi_c$ is defined by

$$\varphi^2_c(\vec{x}) = \frac{6}{\lambda_R} M(\vec{x}).$$

(10)

Due to field renormalisation, $\lambda_R \varphi^2_c$ is renormalisation group invariant [9], and we therefore refer to $M$ as the physical scalar condensate. The effective action for $M(\vec{x})$ is

$$\Gamma[M] = \int_0^{1/T} d\tau \int d^3x \left( -\frac{3}{2\lambda} M^2 - \frac{1}{32\pi^2} \Lambda^2 M(\vec{x}) \right) + \frac{1}{2} \text{Tr} \ln(-\partial^2 + M(\vec{x})).$$

(11)
We wish to evaluate this effective action for weakly space-dependent fields \( M(\vec{x}) \) in a gradient expansion. This can be more systematically accomplished by using the modified heat-kernel expansion proposed in ref. 11. Since we are only interested in the first two terms in the expansion however, we can follow a more direct, though perhaps less elegant, route to the same result. We expand \( M \) around an arbitrary point \( \vec{x}_0 \), i.e.,

\[
M(\vec{x}) = M(\vec{x}_0) + \eta(\vec{x}) =: M_0 + \eta(\vec{x}) ,
\]

and we may assume that only derivatives of \( \eta \) up to second order contribute to the loop. Using

\[
e^{-K_0+\Lambda} = e^{-K_0} + \int_0^1 dx \ e^{-xK_0} A e^{-(1-x)K_0} + \int_0^1 dx \int_0^{1-x} dy \ e^{-xK_0} A e^{-(1-x-y)K_0} A e^{-yK_0} + O(A^3) ,
\]

the loop (13), up to second order in the field \( \eta \), is

\[
L = L_0 + L_1 + L_2 + O(\eta^3),
\]

\[
L_2 = -\frac{1}{4\sqrt{\pi T}} \int \frac{ds}{s^{3/2}} \sum_\nu \exp\left(-\frac{1}{4sT^2} \nu^2 \right) \int_0^1 dx \ Tr_3 \left[ e^{-xK_0} \eta e^{-(1-x)K_0} \eta \right] s^2 ,
\]

where \( L_0 \) and \( L_1 \) denote the terms independent and linear in \( \eta \) and

\[
K_0 = s (-\nabla^2 + M_0) .
\]

\( L_2 \) is most easily evaluated in momentum space where the trace-term in \( L_2 \) becomes

\[
\int \frac{d^3p}{(2\pi)^3} \tilde{\eta}(\vec{p}) \{ \int \frac{d^3k}{(2\pi)^3} e^{-xs((\vec{p}+\vec{k})^2+M_0)} e^{-(1-x)s(\vec{k}^2+M_0)} \} \tilde{\eta}(\vec{-p}) ,
\]

with \( \tilde{\eta} \) denoting the Fourier transform of the field \( \eta(\vec{x}) \). If we confine ourselves to weakly space dependent fields \( \eta \), we may expand (14) up to second order in the external momentum \( \vec{p} \). The kinetic term of the \( \eta \)-field is

\[
L_{kin} = \frac{1}{192\pi^2 T} \int ds \sum_\nu \exp\left(-\frac{1}{4sT^2} \nu^2 \right) e^{-sM_0} \int \frac{d^3p}{(2\pi)^3} \tilde{\eta}(\vec{p}) \vec{p}^2 \tilde{\eta}(\vec{-p}) .
\]

Gathering together all terms which contribute to the effective action (12), and identifying \( M_0 \) and \( \eta(\vec{x}) \) with \( M(\vec{x}) \) is tedious but straightforward. The final result is

\[
\Gamma[M(\vec{x})] = \frac{1}{32\pi^2} \int d^3x \left\{ (1 + uf_0(u)) \frac{1}{12M} \nabla M \nabla M + U(M) \right\}
\]

\[
U(M) = \frac{1}{2} M^2 (\ln \frac{M}{\mu^2} - \frac{1}{2}) - \frac{3}{2\lambda_R} M^2 - M^2 \frac{1}{u^2} f_3(u) ,
\]
where $u = M/4T^2$ and the functions $f_\epsilon(x)$ are defined by

$$f_\epsilon(x) = \sum_{\nu \neq 0} \int_0^\infty \frac{ds}{s^\nu} e^{-sx} e^{-s^2/4}.$$  \hfill (20)

The functions $f_\epsilon$ decay exponentially for large $x$, i.e.,

$$f_\epsilon(x) \approx 2x^{-\frac{\nu+1}{2}} K_{\nu-1}(2\sqrt{x}) \approx \sqrt{\pi} x^{\frac{\nu+1}{2}} e^{-2\sqrt{x}},$$  \hfill (21)

where the $K_\nu(x)$ are the modified Bessel functions of the second kind. For $x \to 0$, the function $f_3$ approaches a finite value whereas $f_0(x)$ diverges, i.e.,

$$\lim_{x \to 0} f_3(x) = 2\zeta(4), \quad f_0(x) \sim \frac{\pi}{2} \frac{1}{x^{3/2}} \text{ as } x \to 0.$$  \hfill (22)

The renormalisation point dependence of the effective potential (19) can be removed in favour of the zero-temperature value of the scalar condensate $M_c$, defined by the minimum of (19), which gives

$$U(M) = \frac{1}{2} M^2 (\ln \frac{M}{M_c} - \frac{1}{2}) - M^2 \frac{1}{u^2} f_3(u).$$  \hfill (23)

The effective potential $U(M)$ and the function $1 + uf_0(u)$ are shown in figure 2 for $T = 0.25\sqrt{M_c}$ and it can be seen that at this temperature, the false minimum of $U(M)$ at $M = 0$ has higher energy density than the true ground state. The crucial additional observation is that in this false-vacuum region ($u \to 0$), the function $1 + uf_0(u)$ becomes large. A large prefactor in front of the kinetic term suppresses space dependent fluctuations implying that tunneling from the false to the true ground state is suppressed.

One might argue, on the other hand, that if the prefactor of the kinetic term is large, the derivative expansion itself is invalid. However, we are interested in configurations given by the classical equation of motion, and so we need not calculate the effective action for arbitrary field configurations. In fact, we will see that the large prefactor enforces classical configurations with small gradients and that higher derivative terms are suppressed even further. In particular, the kinetic energy for classical solutions is always small compared to the effective potential, justifying the expansion for our purposes.

3. The Slow-Rollover phase transition

In this section we investigate the type of phase transition the inflaton undergoes, if it tunnels from the false ground state to the true vacuum. To proceed further, it is convenient to introduce the order parameter

$$\varphi(\vec{x}) := \sqrt{M(\vec{x})}.$$  \hfill (24)
In terms of this order parameter the effective action \((18)\) is \((u = \varphi^2/4T^2)\)

\[
\Gamma[\varphi] = \frac{1}{24\pi^2} \frac{1}{T} \int d^3x \left\{ (1 + uf_0(u)) \frac{1}{2} \nabla \varphi \nabla \varphi + V(\varphi) \right\}
\]

\[
V(\varphi) = \frac{3}{4} U(M(\varphi)) = \frac{3}{8} \varphi^4 (\ln \frac{\varphi^2}{M_c} - \frac{1}{2}) - \varphi^4 \frac{1}{u^2} f_3(u).
\]

As can be seen in \((22)\), the prefactor of the kinetic term diverges when (classically) the inflaton is in the false ground state at some finite temperature \((\varphi/T \rightarrow 0)\). This would imply that no tunneling could occur and the inflaton would never leave the false vacuum. Note, however, that if an infrared cutoff \(m_0\) is used in the definition of the function \(f_0(x)\) in \((20)\)

\[
f_0(x) \rightarrow \sum_{\nu \neq 0} \int_0^\infty ds e^{-sm_0} e^{-sx} e^{-\frac{x^2}{4}},
\]

the prefactor of the kinetic term becomes finite. In the very early universe, of course, there is a natural infrared cutoff given by the horizon scale (i.e., \(H^{-1}\) where \(H = \dot{R}/R\) is the Hubble parameter with \(R\) the scale size of the universe). This cutoff should be used in any action, though for non-divergent quantities it makes a negligible difference at and below the GUT scale. We can very crudely estimate the size of this prefactor (at \(\varphi = 0\)) for a cut-off \(m_0 \simeq H^2 \simeq \pi M_c^2/2m_{Pl}^2\) where \(m_{Pl}\) is the Planck mass, if the inflationary transition occurs at the GUT scale \((T_c \sim M_c^{1/2} \sim 10^{15} \text{ GeV})\).

We have

\[
\frac{m_0}{T_c^2} \sim \frac{8\pi M_c}{m_{Pl}^2} \sim 10^{-8}
\]

Using the asymptotic form for \(f_0\) given by \((22)\), the prefactor of the kinetic term in the false vacuum state is then

\[
1 + \frac{m_0}{T_c^2} f_0\left(\frac{m_0}{T_c^2}\right) \approx \frac{\pi}{2} \frac{T_c}{\sqrt{m_0}} \sim 10^4
\]

We note here in passing, that the calculation in the previous section is for time-independent fields under the assumption of thermodynamic equilibrium, and that neither of these assumptions can be exactly satisfied in the early universe (in fact, for the Robertson-Walker metric it is not even possible to define equilibrium phase space distributions \([17]\)) though of course, we know they are very good approximations for many epochs in the early universe. We expect however, that the effect of the finite horizon size would be one of the dominant violations of these assumptions at and above the GUT scale (assuming no other phase transitions are occurring at that time). A more quantitative estimate of the magnitude of the prefactor \((23)\) would require a more refined calculation.
Perhaps a more appropriate way of proceeding from this point would be to calculate the bubble nucleation rate due to tunneling through the temperature dependent barrier in (26) (assuming as usual, that the processes of thermalisation and tunneling occur on two widely different time scales), but suppressed by the kinetic-energy prefactor. There are indications [18], however, that the transition may be only very weakly first order, or even second order, implying that such a calculation would not appropriate at this level of approximation and is thus beyond the scope of this letter.

The previous section does tell us, though, that in general not only the effective potential, but also the prefactor of the kinetic term is temperature dependent, and might be large for false vacuum configurations. This fact is well known in the description of collective phenomena in many-body systems [19]. In order to further investigate the implications of the temperature dependent prefactor of the kinetic term, we simulate its effect by using the following effective action

\[
\Gamma[\varphi] = \int d^4x \left\{ \frac{1}{2} F\left(\frac{\varphi}{T}\right) \partial_\mu \varphi \partial_\mu \varphi + V(\varphi) \right\},
\]

where

\[
F(x) = 1 + ce^{-x},
\]

\[
V(\varphi) = \frac{3}{8} \varphi^4 \left( \ln \frac{\varphi^2}{M_c} - \frac{1}{2} \right)
\]

(30)

where \(c\) is a (large) constant which, for now, we set equal to \(10^4\) from (29) and we see that the prefactor \(F(x)\) exponentially approaches 1 for large \(x\) but becomes large for small \(x\). We note at this point, that (31) describes a phase transition and Green’s functions in a heat bath and that once inflation starts, and the particle interaction rate becomes much less than \(H\), this is is no longer a good description [13]. We therefore assume that at some \(T_s < T_c\) the temperature in (30) becomes essentially fixed and we take (31) thereafter as a kinetic theory description [14], though of course the effective temperature of particle phase space distributions cools in the usual way (i.e., \(T \sim R^{-1}\) for massless particles and \(T \sim R^{-2}\) for massive ones).

To study the transition of the inflaton from the false ground state to the true ground state we must look for stationary points of the effective action (30). This is most easily achieved by mapping the order parameter \(\varphi\) onto an auxiliary field \(\psi\) defined by

\[
\psi := \int_0^{\varphi} \sqrt{F\left(\frac{\varphi'}{T}\right)} d\varphi'.
\]

(33)

Note that this mapping is invertible since \(\sqrt{F}\) is always positive. In terms of this
auxiliary field $\psi$ the action is
\[ \Gamma[\psi] = \int d^4x \left\{ \frac{1}{2} \partial_\mu \psi \partial_\mu \psi + V_{\text{mod}}(\psi) \right\}, \tag{34} \]
where the modified effective potential is
\[ V_{\text{mod}}(\psi) := V(\varphi(\psi)). \tag{35} \]
This is precisely what we require, the prefactor of the kinetic term in (30) has been eliminated and so (34) therefore describes the time-evolution of the field $\psi$ moving in a modified potential $V_{\text{mod}}$ in the standard way, i.e., varying the Einstein action for spatially homogeneous fields gives
\[ \ddot{\psi} + 3H \dot{\psi} + V'_{\text{mod}} = 0 \tag{36} \]
where again $H$ is the Hubble parameter. We stress that the time-evolution of the $\psi$-field is directly transferred to the time-evolution of the order parameter $\varphi$ since the mapping (33) is time independent. Figure 3 shows $V(\varphi)$ and $V_{\text{mod}}(\psi)$ for the choice (31) and (32). $V_{\text{mod}}(\psi)$ is very flat in the region of the false vacuum, enabling $\ddot{\psi}$ to be ignored and, as further described in the following section, a slow-rollover phase transition to occur.

4. Inflation
We now examine more closely the details and consequences of the phase transition in the above model as a mechanism for inflation.
For inflation we would like the scale of symmetry breaking to be of the order of the GUT scale and so we imagine $M_c$ in the above to be of order $(10^{15}\text{GeV})^2$. Far above the critical temperature, the absolute minimum of the potential is at the origin, and the field will be localised at $\varphi = 0$ in the standard way (see figure 1). There is no problem with this mechanism in the model we describe, in contrast to the usual slow-roll scenario where the coupling is so tiny that an initial thermal state is unlikely to occur \[3, 13, 16\]. The large prefactor of the kinetic term, which the $\varphi$-field incurs while it is near $\varphi = 0$, also confers the following two advantages during this epoch. First, it suppresses spatial gradients, making a smooth patch of homogeneous $\varphi$-field more probable. By the same token, it suppresses fluctuations of the $\varphi$-field within this patch, so that as the critical temperature is approached and then passed it is unlikely that thermal fluctuations will be strong enough to push the field to the new absolute minimum. The field will thus supercool in its false vacuum state.
Eventually, when the temperature gets low enough, the field will commence its slow roll to the absolute minimum (or alternatively, it will tunnel through the remaining potential barrier and then start to slow-roll). As we saw in the previous section, the
slow-rolling is again a consequence of the large kinetic energy prefactor. We now examine this era in a little more detail.

For the phenomenological function $F$ we have chosen in eqn. (31) it is in fact possible to perform the integral (33) analytically. For $c \gg 1$ and $\varphi/T \ll 1$, corresponding to large prefactor and hence slow rolling, (33) becomes

$$\psi \simeq 2\sqrt{c}T \left(1 - e^{-\varphi/2T}\right) \simeq \sqrt{c} \varphi. \quad (37)$$

Substituting this into (35) (and setting $T = 0$ to remove the $T$-dependent barrier as we are only concerned with slow-rolling) gives

$$V_{mod}(\psi) \simeq \frac{3\psi^4}{8c^2} \left( \ln \left( \frac{\psi^2}{cM_c} \right) - \frac{1}{2} \right) \quad (38)$$

during slow-rolling only. Of course (37) means that (38) has the same Coleman-Weinberg form as (32), however the extra factor $c^{-2}$ makes the effective “coupling” for the $\psi$-field extremely small because we expect $c$ to be very large. It also makes analysis during slow-rolling particularly simple, slow-rolling on the Coleman-Weinberg potential having been studied many times before [3, 13].

In order to justify the derivative expansion for our purposes, we estimate the magnitude of the kinetic term during the slow roll period. Using the equation of motion for the auxiliary field $\psi$ (36) (ignoring $\ddot{\psi}$ during slow rolling) and (37) we have

$$c \dot{\varphi}^2 \approx \left( \frac{V'_{mod}}{3H} \right)^2 = O\left( \frac{1}{c^4} \right), \quad (39)$$

which is thus small compared to the intrinsic scale (e.g., the zero-temperature vacuum-energy density) of the effective potential.

Ensuring that the slow-rolling does not conflict with bounds on density fluctuations $(\delta \rho/\rho)$ provided by the CMBR and assuming such fluctuations are also responsible for large scale structure $(\delta \rho/\rho \simeq 10^{-5})$ allows one to test the value for the parameter $c$ and also, as usual, implies inflation lasts long enough to solve the horizon, flatness and monopole problems. We first of all note that the energy density and pressure of the $\varphi$-field have the following form

$$\rho_{\varphi} = \frac{1}{2} F(\varphi/T) \dot{\varphi}^2 + V(\varphi) \quad (40)$$

$$p_{\varphi} = \frac{1}{2} F(\varphi/T) \dot{\varphi}^2 - V(\varphi) \quad (41)$$

plus gradient terms proportional to $(\nabla \varphi)^2/R^2$ which we ignore as usual. Now, even though the function $F$ is large for $\varphi$ near zero, during slow rolling, the potential term
will still dominate from (39). This implies that the amplitude of density fluctuations generated by inflation is given by
\[
\frac{\delta \rho}{\rho} \approx \frac{\delta \varphi V'(\varphi)}{c\dot{\varphi}^2} \approx \frac{\delta \psi V'_\text{mod}(\psi)}{\dot{\psi}^2}
\] (42)
where we have used (33) and (37). The last expression is just the usual form for fluctuations generated by a weakly coupled field so that we expect \(\delta \psi \approx H/2\pi\) and from the slow-rolling condition \(3H\dot{\psi} \approx -V'(\psi)\) (i.e., (36) with negligible \(\dot{\psi}\)), and where during inflation \(H^2 \approx \text{const} \approx \pi M_c^2/2m^2_{pl}\), from the \(T = 0\) \(\varphi\)-potential. So we get, again as expected,
\[
\frac{\delta \rho}{\rho} \approx \frac{3H^3}{V'_\text{mod}}
\] (43)
Pursuing the standard analysis further we find that we can use the approximation \(V'_\text{mod}(\psi) \approx \dot{\psi}^3/c^3\) during the slow roll and the value of \(\psi\), \(N\) e-folds before the end of inflation is \(\psi(N) \approx \left(8/3c^2\right)^{1/2} N^{3/2}\). Using these in (43) and solving for \(c\) yields
\[
c \approx \left(\frac{8N^3}{3}\right)^{1/2} \left(\frac{\delta \rho}{\rho}\right)^{-1}.\] (44)
So, for generating cosmological structure, which requires \(N \approx 50\) and \(\delta \rho/\rho \approx 10^{-5}\) from COBE, we would need a value \(c \approx 10^7\). This is somewhat larger, but nevertheless encouragingly similar to the crude estimate of (29). Furthermore, if we were to use, in (29), the horizon scale and temperature \(T_s\), when inflation commences, rather than the values at the critical temperature (where the minima in the effective potential are degenerate), we would expect a larger value for \(c\). We hope to verify this in a more detailed calculation.

We make a remark here on slow rolling in our model. It might appear at first from (38), that we have simply exchanged an unrealistically small coupling constant for an unrealistically large multiplicative constant. In the standard slow-rolling scenario, though, the tiny coupling really is unexplained, excepting perhaps supergravity models [5] which, however, suffer from their own problems [20, 21] (but incidentally, also offer a precedent for considering non-minimal kinetic terms [21]). In contrast, however, in the model we consider here, the large value of \(c\) simply occurs from the dynamics of the theory — in (29) its value is a consequence of the infrared cutoff given by the finite horizon size.

After the slow rolling inflation transition, comes reheating, as the \(\varphi\)-field drops into and oscillates about its new minimum. From figure 2, it can be seen that at this true minimum the kinetic-term prefactor has returned to its conventional value of \(\frac{1}{2}\), i.e., equations (37) and (38) are no longer good approximations. The \(\varphi\)-field thus becomes a field with a canonical kinetic energy, oscillating about its absolute minimum — perturbations in \(\varphi\) are no longer suppressed and most importantly,
the fact that the coupling of $\varphi$ can have a reasonable value means that the model does not of necessity have a low reheat temperature, since other particles which may couple to $\varphi$ can also have reasonable couplings.

In summary, we have described here a model which can undergo a slow-rolling phase transition as a natural consequence of the dynamics of the theory. The slow-rolling behaviour is due to a field dependent prefactor of the kinetic term in the effective action, and its large value in the false vacuum and during slow rolling is a natural consequence of the non-perturbative dynamics of the theory. In addition to natural slow-rolling, the model does not have some of the usual problems associated with the conventional slow-rolling scenario. In particular, there is no problem with thermal equilibrium as the explanation for localising the field in the false vacuum, and there is no necessity for a low reheat temperature.

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Figure captions

**Figure 1:** The temperature dependent effective potential $V(\varphi^2,T)$ for different values of the temperature $T$. At high temperature $T \gg T_c$ the minimum of potential is at $\varphi = 0$. At zero temperature, or the continuum (i.e., no periodicity in imaginary time), the absolute minimum (corresponding to the non-trivial phase of $\lambda \varphi^4$ theory) of the potential occurs at $\lambda \varphi^2/6 = M_c$, the magnitude of the scalar condensate.

**Figure 2:** The effective potential $U$ and the coefficient of the kinetic term $1 + u f_0(u)$ as functions of $M/M_c$ ($M \equiv \varphi^2$) at a temperature $T = 0.25 M_c^{1/2}$.

**Figure 3:** The potential of the order parameter $V(\varphi)$ and the modified potential $V_{\text{mod}}(\psi)$ for $c = 20$ in units of $M_c^2$. Fields $\varphi, \psi$ in units of $M_c^{1/2}$. 
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