Field-induced interaction of a pseudoscalar particle with photon in a magnetized $e^-e^+$ plasma

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Abstract

The effective interaction of a pseudoscalar particle with photon in plasma with the presence of a constant uniform magnetic field is investigated. It is shown that under some physical conditions the effective coupling between pseudoscalar particle and photon does not depend on medium parameters and particles momentum. The probability of the familon decay into photon pair in a strongly magnetized degenerate ultrarelativistic plasma is calculated.

1 Introduction.

For a long time there has been a stable interest to investigations of quantum processes in an external active medium, which can be presented by plasma as well as a magnetic field. Actually, both components of the active medium could exist in astrophysical objects. By this means, the interest to studies of elementary particle physics under extreme conditions, dense plasma, strong magnetic field and/or high temperature, is caused, in particular, by possible astrophysical applications. Of special interest are the processes with participation of a light weakly interacting particle. It should be noted that in previous studies of quantum processes in a dense stellar matter the main attention was given to the neutrino processes. This is due to the fact that neutrino plays determining role in astrophysical cataclysms like a supernova explosion and a coalescence of neutron stars.

However, the investigations of the other weakly interacting particles physics (axion, familon etc.) could be of interest for astrophysical applications also [1]. In view of weakly interaction with matter such particles could give a perceptible effect on the dynamics of a cooling star. In particular, the emission processes of these particles could give an additional contribution into the star energy losses [2–5]. The other interesting effect from astrophysics viewpoint is the possible asymmetry of emission of the weakly interacting particles from a supernova caused by the presence of an external magnetic field. This asymmetry could lead to the reactive force and, as a result, to the initial impetus of a pulsar (kick-velocity) [5].

In this paper we study the interaction of a pseudoscalar particle with photon in the electron-positron plasma with the presence of an external magnetic field. As a pseudoscalar particle we will consider the familon, the Nambu-Goldstoun boson arising as a result of the spontaneous

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breakdown of a global symmetry between fermion generations [6, 7]. Notice, that interaction between familon and photon becomes possible in an external magnetic field only because of the fact that familon does not have both anomalous $\Phi G \tilde{G}$ and $\Phi F \tilde{F}$ coupling in vacuum ($G$ and $F$ are gluonic and electromagnetic fields, respectively).

The field-induced effective familon-photon interaction is described by the loop Feynman diagram shown in Fig.1. and can be presented as

$$L_{\phi \gamma} = g_{\phi \gamma} \tilde{F}^{\alpha \beta} (\partial_\beta A_\alpha) \Phi.$$  \hspace{1cm} (1)

Here $A_\mu$ is the four-potential of the quantized electromagnetic field, $\Phi$ is the familon field, $\tilde{F}^{\alpha \beta} = \frac{1}{2} \varepsilon^{\alpha \beta \sigma \tau} F_{\sigma \tau}$ is the dual tensor of the external magnetic field $F_{\sigma \tau}$, $g_{\phi \gamma}$ is the effective familon-photon coupling in a magnetized plasma.

The Lagrangian (1) leads to the amplitude of the familon $\rightarrow$ photon transition in the following form

$$M_{\phi \rightarrow \gamma} = ig_{\phi \gamma} (\varepsilon^* \tilde{F} q),$$  \hspace{1cm} (2)

where $q_\mu = (\omega, \vec{q})$ is the photon 4-momentum, $\varepsilon_\mu$ is the photon polarization 4-vector.

In a magnetized plasma, along with the loop-channel of the familon $\rightarrow$ photon transition the additional channel caused by the presence of plasma, becomes possible, namely, the Compton-like familon-photon ”forward scattering” on plasma electrons and positrons (Fig.2). Notice, that the contribution of this process to the effective interaction between pseudoscalar particle and photon was not taken into account previously [4]. However, as it will be shown below, under some physical conditions the plasma contribution into the effective coupling $g_{\phi \gamma}$ could appear to be much more than the field one.

So, in a magnetized plasma the amplitude of the transition $\phi \rightarrow \gamma$ can be written in general case as the sum of the field and plasma contributions

$$M_{\phi \rightarrow \gamma} = M^F + M^{Pl}.$$  \hspace{1cm} (3)

Further we will calculate the plasma and field contributions into the effective familon-photon coupling in plasma with the presence of a magnetic field.

Figure 1: The field-induced interaction of a pseudoscalar particle and photon. Double lines indicate, that the influence of an external magnetic field is taken into account in the propagator of virtual fermions.

Figure 2: The diagrams describing the Compton-like familon $\rightarrow$ photon ”forward scattering” on plasma electrons.
2 Field-induced part of the effective coupling between pseudoscalar particle and photon in a magnetized plasma.

The field-induced part of the effective coupling $g_{\phi\gamma}$, can be derived from the diagram shown on Fig.1, where the sum over all virtual fermions is considered. The main contribution into this sum comes from an electron as the particle with the maximal specific charge $e/m_e$, which is the most sensitive to the external magnetic field influence.

The Lagrangian describing the familon-electron interaction is

$$L = \frac{c_e}{v_\phi} (\bar{\Psi}_e \gamma^\mu \gamma_5 \Psi_e) \partial_\mu \Phi,$$

where $c_e$ is a model dependent parameter of the order of unit, $v_\phi$ is the horizontal symmetry breaking scale, $\Psi_e$ is the electron field.

In the second order of the perturbation theory, the $S$-matrix element corresponding to the diagram shown in Fig.1 can be written as

$$S^F = \frac{-iec_e}{2v_\phi V \sqrt{\omega \omega'}} q_\mu \varepsilon_\nu \int d^4x d^4y Tr\{S(y,x) \gamma^\mu \gamma_5 S(x,y) \gamma^\nu\} e^{i(q'y)} e^{-i(qx)},$$

where $q^\alpha = (\omega, \vec{k})$ and $q'^\alpha = (\omega', \vec{k}')$ are the 4-momenta of the familon and photon respectively, $S(x,y)$ is the propagator of virtual electron in the loop, $e > 0$ is the elementary charge.

The electron propagator in the constant uniform magnetic field can be presented in the form [8]

$$S(x,y) = e^{i\Phi(x,y)} S(z),$$

here $z_\mu = x_\mu - y_\mu$.

The translational and gauge non-invariant part of the propagator is separated in the phase $\Phi(x,y)$ which can be defined in terms of an integral along an arbitrary contour

$$\Phi(x,y) = e \int_y^x d\xi^\mu \left[ A_\mu(\xi) + \frac{1}{2} F_{\mu\nu}(\xi - y)^\nu \right].$$

Taking into account that in the case of two-vertex fermion loop, the sum of the phases arising from two fermion propagators is zero

$$\Phi(x,y) + \Phi(y,x) = 0,$$

one can define the amplitude of the process by the standard manner [9]

$$S = \frac{i(2\pi)^4 \delta^4(q - q')}{2\omega V} M,$$

where the field-induced part of the amplitude can be presented in the form

$$M^F = \frac{-ec_e}{v_\phi} q_\mu \varepsilon_\nu^* \int d^4z e^{-i(qz)} Tr\{S(-z) \gamma^\mu \gamma_5 S(z) \gamma^\nu\}. $$

The one-loop two-point amplitude in an external magnetic field was intensively investigated previously [10–12]. The expression for the amplitude of the familon-photon conversion could be extracted, for example, from the book [12], where in the section 4 the field-induced one-loop amplitude of the transition $j \to f \bar{f} \to j'$ for various combinations of scalar, pseudoscalar, vector and pseudovector interactions of general currents $j$ and $j'$ with fermions are presented. In
particular, from the formula (4.22) of [12] corresponding to the axial-vector and vector vertices we obtain

\[ M^F = \frac{2i\alpha_{\mu}}{\pi \nu_\phi} (e^* \tilde{F} q) \int_0^1 du \int_0^\infty d\tau e^{-\Omega(u,\tau)}, \] (9)

\[ \Omega(u, \tau) = \tau \left( 1 - \frac{q^2}{4m^2} \right) - \frac{q^2}{2m^2} \left( \frac{ch(\eta u \tau) - ch(\eta \tau)}{\eta sh(\eta \tau)} \right), \]

where \( \varphi_{\alpha \beta} \) is the dimensionless external field tensor, \( \varphi_{\alpha \beta} = F_{\alpha \beta}/B, \) \( \tilde{\varphi}_{\alpha \beta} = \frac{1}{2} \varepsilon_{\alpha \beta \rho \sigma} \varphi^{\rho \sigma} \) is the dual tensor, \( \beta = eB, \) \( \eta = \beta/m_e^2 = B/B_e (B_e = m_e^2/e \simeq 4.41 \times 10^{13} \text{ G}) \)

\( q^2 = \left( q\tilde{\varphi} q \right) = q_1^2 + q_2^2, \) \( q_2^2 = q_2^2 - q_1^2 \) (it is assumed that a magnetic field is directed along the third axis, \( \vec{B} = (0, 0, B) \)), \( m_e \) is the electron mass, \( \alpha = e^2/4\pi \).

Comparing the expression (9) with the amplitude (2) we find the field contribution into the effective coupling of the familon-photon interaction in the form

\[ g_{\phi \gamma}^F = \frac{2e c \alpha}{\pi \nu_\phi} \int_0^1 du \int_0^\infty d\tau e^{-\Omega(u,\tau)}. \] (10)

It should be emphasized that, strictly speaking, the field-induced amplitude of the processes featuring pseudoscalar particles and therefore the effective coupling (10) are a non-uniqueness physical quantity because of the Adler triangle anomaly. Since the familon-photon interaction is free from the Adler anomaly, the effective familon-photon coupling must disappear in the local limit when \( q^2 = 0 \). Therefore obtaining the physically correct result it is necessary to perform the procedure of the substraction of the Adler anomaly which is reduced to the substraction from the field-induced coupling (10) the one in the limit \( m_e \to \infty \)

\[ g_{\phi \gamma}^F = \frac{2e c \alpha}{\pi \nu_\phi} \left( \int_0^1 du \int_0^\infty d\tau e^{-\Omega(u,\tau)} - 1 \right). \] (11)

The expression (11) describes the field induced part of the effective coupling.

3 Plasma part of the effective coupling between pseudoscalar particle and photon in a magnetized plasma.

The plasma contribution into the effective familon-photon coupling is caused by the Compton-like familon-photon ”forward scattering” on plasma electrons and positrons (Fig.2). With the Lagrangian (11) we find the S-matrix element corresponding to the diagrams of Fig.2 in the form

\[ S_{c-}^{PL} = \frac{ie c_e}{2 \nu_\phi \sqrt{\omega \omega'}} q_\mu \varepsilon_\nu^* \sum_{n=0}^\infty \sum_s \int d^4x d^4y dn e^{-i qx} e^{i q'y} \times \]

\[ \times \operatorname{Tr}(\bar{\psi}_e(p, x) \gamma^\mu \gamma_5 S(x, y) \gamma^\nu \psi_e(p, y) + \bar{\psi}_e(p, y) \gamma^\nu S(y, x) \gamma^\mu \gamma_5 \psi_e(p, x)), \]

where \( q^\alpha = (\omega, \vec{k}) \) and \( q'^\alpha = (\omega', \vec{k}') \) are the 4-momenta of the familon and photon respectively, \( p^\mu = (E_\alpha, \vec{p}) \) is the electron 4-momentum, \( E_\alpha = \sqrt{p_\alpha^2 + 2n \beta + m^2} \) is the energy, \( n \) is the Landau level number, \( \psi_e \) is the solution of the Dirac equation in magnetic field, \( S(x, y) \) is

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1 We use natural units in which \( c = \hbar = 1 \).
the electron propagator, \( dn_{e^-} \) is the phase space element of plasma electrons. In the external magnetic field, the number of plasma electrons in the gauge \( A^\mu = (0, 0, Bx, 0) \) is defined as

\[
\frac{dp_2 dp_3}{(2\pi)^2} L_2 L_3 f(E_n, \mu).
\]

Here \( L_1, L_2 \) are the auxiliary parameters which determine the normalization volume \( V = L_1 L_2 L_3 \), \( f(E_n, \mu) \) is the electron distribution function, which has the following form in the plasma rest frame

\[
f(E_n, \mu) = \frac{1}{e^{(E_n-\mu)/T} + 1},
\]

where \( \mu \) and \( T \) are the plasma chemical potential and temperature respectively.

The S-matrix element of the familon-photon "forward scattering" on plasma positrons can be obtained from (12) by substitution \( p \to -p \) in the solution of the Dirac equation and \( \mu \to -\mu \) in the distribution function.

The electron wave function in the above-mentioned gauge can be written as [13]

\[
\psi_e(p, x) = \frac{u_e(p, \xi) e^{-i(E_n x_0 - p_2 x_2 - p_3 x_3)}}{\sqrt{2E_n (E_n + m_e)L_2L_3}},
\]

where the bispinor amplitude, corresponding to the two projections of the electron spin on the field direction, \( s = \pm 1 \), are given by

\[
u_{s=1}(p, \xi) = \begin{pmatrix} 0 \\ (E_n + m_e)V_n(\xi) \\ -i\sqrt{2\beta n}V_{n-1}(\xi) \\ -p_3V_n(\xi) \end{pmatrix},
\]

\[
u_{s=1}(p, \xi) = \begin{pmatrix} (E_n + m_e)V_{n+1}(\xi) \\ 0 \\ p_3V_{n-1}(\xi) \\ i\sqrt{2\beta n}V_n(\xi) \end{pmatrix}.
\]

Here \( V_n(\xi) = \frac{\beta^{1/4}}{\sqrt{2\beta n!}} e^{-\xi^2/2} H_n(\xi) \), \( H_n(\xi) \) is the Hermite polynomials, \( \xi \) is the dimensionless variable, \( \xi = \sqrt{\beta}(x_1 + p_2/\beta) \).

The translational and gauge non-invariant phase of the electron propagator [6] in the considered gauge is reduced to the simple form

\[
\Phi(x, y) = -\frac{\beta}{2}(x_1 + y_1)(x_2 - y_2).
\]

The translational and gauge invariant part of the propagator \( S(z) \) has several representations. For our purposes it is convenient to take it in the form

\[
S(z) = -\frac{i}{4\pi} \int_0^\infty d\tau \frac{d^2p_\|}{(2\pi)^2} \left\{ [(p'\gamma)_\| + m_e] \Pi_- (1 + t\hbar \tau) + [(p'\gamma)_\| + m_e] \Pi_+ (1 - t\hbar \tau) - \frac{i\beta(z\gamma)_\|^2}{2t\hbar \tau} (1 - t\hbar^2 \tau) \right\}
\times \exp \left( -\frac{\beta z^2}{4t\hbar \tau} - \frac{\tau(m_z^2 - p'\|^2)}{\beta} - i(p'\gamma)_\| \right),
\]

(14)

where \( z_\mu = x_\mu - y_\mu, p'_\mu \) is the four-momentum of the virtual electron, \( d^2p'_\| = dp_0 dp'_3, \Pi_\pm = \frac{1}{2}(1 \pm i\gamma_1 \gamma_2), (p'\gamma)_\| = (p'_\gamma \rho^\mu_\| - p'_\gamma \gamma_3)_{\rho^\mu_\|}, (z\gamma)_\| = (z\gamma \rho^\mu_\| - z\gamma \gamma_3)_{\rho^\mu_\|} = z_1 \gamma_1 + z_2 \gamma_2. \)

After performing the partial integration of the Eq. (12) over 4-coordinates \( x \) and \( y \), impulses of virtual electron and the second component of plasma electron’s momentum \( p_2 \), the \( S \)-matrix
element of the familon-photon transition with coherent scattering on all plasma electrons and positrons can be expressed as

$$S_{Pl} = \frac{16\alpha m^2 \pi^3}{\omega V v_\phi} \left(\varepsilon^* \bar{F} q\right) \delta^4(q - q') \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \frac{dp_3}{E_n} \left\{ f(E_n, \mu) + f(E_n, -\mu) \right\} \times$$

$$\times \int_0^{\infty} ds \left( e^{i s (q_0^2 + 2(pq)_\parallel)} + e^{i s (q_0^2 - 2(pq)_\parallel)} \right) e^{-i q_\perp^2 \sin(2\beta s)/2\beta} \lambda_n \left( \frac{q_\perp^2}{\beta} \sin^2(\beta s) \right),$$

where \((pq)_\parallel = E\omega - p_3 q_3\), \(L_n(x)\) is the Laguerre polynomials, normalized by the condition

$$\int_0^{\infty} e^{-x} L_n^2(x) \, dx = 1.$$  

As one can see from Eq.(15), the four-dimensional delta function, corresponding to the energy and momentum conservation law is realized in the \(S\)-matrix element, as a consequence of the fact that both initial and final states are the neutral particles. Using the standard definition of an invariant amplitude we can write the plasma contribution into the \(g_{\phi\gamma}\) in the form:

$$g_{Pl}^{\phi\gamma} = \frac{-2\alpha m^2}{\pi v_\phi^2} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \frac{dp_3}{E_n} \left\{ f(E_n, \mu) + f(E_n, -\mu) \right\} \times$$

$$\times \int_0^{\infty} ds \left( e^{i s (q_0^2 + 2(pq)_\parallel)} + e^{i s (q_0^2 - 2(pq)_\parallel)} \right) e^{-i q_\perp^2 \sin(2\beta s)/2\beta} \lambda_n \left( \frac{q_\perp^2}{\beta} \sin^2(\beta s) \right).$$

Notice that the result is valid for electron-positron plasma in the presence of a constant magnetic field of an arbitrary strength.

4 The effective coupling between pseudoscalar particle and photon in the strongly magnetized plasma.

In this section we investigate the familon-photon interaction under the physical conditions when from both components of an active medium, a magnetic field and plasma, the magnetic field dominates, so the magnetic field strength appears to be the largest physical parameter

$$\beta \gg \mu^2, T^2, m_e^2.$$  

Such conditions could be realized, for example, in a supernova explosion or in a coalescence of neutron stars where an outside region of the neutrinosphere with strong magnetic field up to the \(10^{14} - 10^{16}\) G and rather rarified plasma can exist.

The field contribution into effective coupling in the strong magnetic field limit \((\eta = B/B_e \gg 1)\) can be reduced to the form

$$g_{\phi\gamma}^F = \frac{2c\alpha}{\pi v_\phi} H(z), \quad H(z) = \int_0^1 \frac{du}{1 - z(1 - u^2)} - 1,$$

where \(z = q_\parallel^2/4m_e^2\).
As for the plasma contribution to the effective coupling $g_{\phi\gamma}$, under the conditions when plasma electrons and positrons occupy the ground Landau level only ($n = 0$), we have from Eq. (16)

$$g_{\phi\gamma}^{pl} \simeq \frac{-4c_e\alpha m_e^2}{\pi v_\phi} q_\parallel^2 \int_{-\infty}^{+\infty} \frac{dp_3}{E} \frac{f(E, \mu) + f(E, -\mu)}{4(pq)_{\parallel}^2 - q_\parallel^4},$$

(19)

where $E = \sqrt{p_3^2 + m^2}$ is the electron (positron) energy on the ground Landau level.

With Eqs. (18) and (19), the effective coupling $g_{\phi\gamma}$ in the strongly magnetized plasma is

$$g_{\phi\gamma} = \frac{-2c_e\alpha}{\pi v_\phi} F(q_\parallel),$$

(20)

$$F(q_\parallel) = 2m_e^2 q_\parallel^2 \int_{-\infty}^{+\infty} \frac{dp_3}{E} \frac{f(E, \mu) + f(E, -\mu)}{4(pq)_{\parallel}^2 - q_\parallel^4} - H(z).$$

(21)

It is interesting to note, that the expression (20) is valid also for the familon (photon) propagating along the field direction ($q_\perp = 0$) independently on the value of an external magnetic field.

Really, considering $q_\perp = 0$ in the field contribution into the effective coupling (11) we immediately reproduce the result (18) obtained for the ground Landau level. As for the plasma contribution described by the expression (16), taking into account that

$$\lambda_n(0) = \begin{cases} 1, & n = 0, \\ 0, & n \neq 0, \end{cases}$$

(22)

one can see that the effective coupling $g_{\phi\gamma}$ is caused by the ground Landau level only.

So, the result (20), obtained for strongly magnetized plasma when plasma electrons and positrons occupy the ground Landau level only has a more wide area of application. It may be used even in the magnetic field of arbitrary strength for the familon (photon) propagating along the magnetic field.

The expression (20) is significantly simplified in some limiting cases:

- the case of relatively high familon (photon) energy ($\beta \gg q_\parallel^2 \gg m_e^2$). In this limit the plasma contribution is suppressed by the electron mass $m_e$, which is the smallest parameter of the problem and the effective coupling $g_{\phi\gamma}$ is caused mainly by the field contribution

$$g_{\phi\gamma} \simeq g_{\phi\gamma}^F \simeq \frac{-2c_e\alpha}{\pi v_\phi}.$$  

(23)

- the case of relatively small familon (photon) energy ($\omega^2 \ll m_e^2$) when familon propagates transversely on the magnetic field direction ($q_3 = 0$). In view of the asymptotic behavior of the function $H(z)$ at small argument, the major contribution into the effective coupling $g_{\phi\gamma}$ comes from plasma

$$g_{\phi\gamma} \simeq g_{\phi\gamma}^{pl} = \frac{2c_e\alpha}{\pi v_\phi} \int_{m_e}^{\infty} \frac{dE}{E} \frac{2} {p_3} \frac{dE}{dE} (f(E, \mu) + f(E, -\mu)).$$

(24)

- the case of a strongly magnetized degenerate ultrarelativistic plasma when familon energy

$$\omega < 2\mu$$

$$g_{\phi\gamma} \simeq \frac{-2c_e\alpha}{\pi v_\phi}.$$  

(25)

As one can see from Eqs. (22), (23) and (24), under some physical conditions the effective coupling $g_{\phi\gamma}$ does not dependent on the particle four-momentum.
The conversion of the familon into two photons.

The familon, being a stable particle in the vacuum, in the presence of a magnetized plasma can decay both into the electron-positron pair in tree-level [14] and into the photon pair in the loop level. The decay \( \phi \rightarrow \gamma + \gamma \) becomes possible owing to the effective coupling \( g_{\phi \gamma} \) of the familon-photon interaction in a magnetized plasma. Really, the Lagrangian of the familon-two photon interaction can be uniquely restored from (1) in the special case when the effective coupling \( g_{\phi \gamma} \) is a constant independent on particle momentum

\[
L_{\phi \gamma \gamma} = \frac{g_{\phi \gamma}}{4} \left( \tilde{F}^{\mu \nu} F_{\nu \mu} \right) \Phi, \tag{25}
\]

where \( F^{\alpha \beta} \) and \( \tilde{F}^{\alpha \beta} = \frac{1}{2} \varepsilon^{\alpha \beta \sigma \tau} F_{\sigma \tau} \) are the tensor and dual tensor of the quantized electromagnetic field.

In this section we investigate the conversion of a familon into two photons in the strongly magnetized degenerate ultrarelativistic plasma

\[
\beta \gg \mu^2 \gg T^2, m_e^2. \tag{26}
\]

assuming that the familon’s energy satisfies the condition

\[
2\mu > \omega \gg T. \tag{27}
\]

Notice, that under the condition (27) the process of the familon tree-level decay into electron-positron pair is suppressed by the statistical factors in strongly degenerate plasma.

In the strongly magnetized plasma as in the strong magnetic field there are two photons modes

\[
\varepsilon^{(1)}_\alpha = \frac{(q\varphi)_\alpha}{\sqrt{q_1^2}}, \quad \varepsilon^{(2)}_\alpha = \frac{(q\tilde{\varphi})_\alpha}{\sqrt{q_\parallel^2}}. \tag{28}
\]

So, in a general case the following channels of the familon conversion into the photon pair are possible: \( \phi \rightarrow \gamma^{(1)} + \gamma^{(1)} \), \( \phi \rightarrow \gamma^{(1)} + \gamma^{(2)} \), \( \phi \rightarrow \gamma^{(2)} + \gamma^{(2)} \).

To obtain the decay probability one has to analyze the photon dispersion in a magnetized plasma. By virtue of the fact that photon of the first mode does not interact with electrons and positrons occupying the lowest Landau level, the dispersion law for the first mode is the same as in a pure magnetic field [15]

\[
q^2 = -\frac{\alpha}{3\pi} q_1^2.
\]

As for the second photon mode, it turn out that its dispersion law is defined by the same function \( F(q_\parallel) \) as the familon-photon effective coupling in the strongly magnetized plasma [20]

\[
q^2 = \frac{2\alpha eB}{\pi} F(q_\parallel). \tag{29}
\]

The function \( F(q_\parallel) \) defined in Eq. (20) tends to unity, \( F(q_\parallel) \sim 1 \) in the case of strongly magnetized degenerate ultrarelativistic plasma, so

\[
q^2 = \frac{2\alpha eB}{\pi}.
\]

Since the massless familon can decay only into photons with \( q^2 < 0 \) under conditions considered, see Eqs. (26) and (27) the familon conversion into two photons of the first mode, \( \phi \rightarrow \gamma^{(1)} + \gamma^{(1)} \), is the only possible channel of the familon decay.
The amplitude of this decay can be immediately obtained from the Lagrangian and has the form

\[
M = g_{\phi\gamma} \frac{(q_1 \varphi q_2) (q_1 \bar{\varphi} q_2)}{\sqrt{q_{1\perp} q_{2\perp}}},
\]

(30)

where \( g_{\phi\gamma} \) is described by the Eq.(24), \( q^\mu = (\omega, \vec{q}) \), \( q_1^\mu = (\omega_1, \vec{q}_1) \), \( q_2^\mu = (\omega_2, \vec{q}_2) \) are the familon and the final photons 4-momenta.

The probability of the process \( \phi \rightarrow \gamma^{(1)} + \gamma^{(1)} \) under the conditions (27) is defined as

\[
\omega W = \frac{|M|^2}{64 \pi^2} \frac{d^3q_1}{\omega_1} \frac{d^3q_2}{\omega_2} \delta^4(q - q_1 - q_2).
\]

(31)

After integration over the phase space of the photons we find

\[
W \simeq \frac{g_{\phi\gamma}^2 \alpha^2 \omega^3 \sin^4 \theta}{1152 \pi^3},
\]

(32)

where \( \theta \) is the angle between the magnetic field direction and the vector \( \vec{q} \).

Notice that the result (32) can be applied to any pseudoscalar particle with the coupling of the type (4). However, in the general case, it is necessary to take into account the pseudoscalar-photon coupling in vacuum due to the Adler’s anomaly.

6 Conclusion.

We have studied the interaction between pseudoscalar particle and photon in an electron-positron plasma with the presence of a constant magnetic field. As an example of a pseudoscalar particle, we have considered the familon, associated with the spontaneous breakdown of a horizontal family symmetry. The most general expression for the plasma and field contributions into the effective coupling of familon-photon interaction in a magnetized plasma was obtained. It was found that under some physical conditions the effective coupling is a constant independent on particle 4-momentum. In particular, the strong magnetic field limit was considered when plasma electrons and positrons occupy the ground landau level only. It was shown that the result for the effective familon-photon coupling obtained for the strongly magnetized plasma is valid in the case of arbitrary magnetic field strength for the pseudoscalar particle propagating along a magnetic field as well.

As an example of possible application of the result obtained we have calculated the probability of the familon decay into two photons under physical conditions of strongly magnetized degenerate ultrarelativistic plasma.

We believe that the result presented in this paper would be useful for the investigations of interaction between pseudoscalar particle and photon in an active medium.

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