Analytical models for warehouse configuration

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The performance of a warehouse is impacted by how it is configured, yet there is no optimization model in the literature to answer the question of how to best configure the warehouse in terms of warehouse shape and the configuration of the dock doors. Moreover, the building blocks for such a model (put-away, replenishment, and order picking models that can be combined in an optimization model) are either not available (in the case of replenishment) or built on a set of inconsistent assumptions (in the case of put-away and order picking). Therefore, this article lays the foundation for more sophisticated warehouse configuration optimization models by developing the first analytical model for replenishment operation performance and extending put-away and order picking performance models. These new models are used to address a question motivated by industry: the optimal configuration of a case-picking warehouse in terms of the shape of the facility and whether the facility is configured with dock doors on one or both sides. An example is presented to demonstrate the use of the proposed models in answering such a question, quantifying the benefit of using an integrated approach to warehouse configuration.

Keywords: Warehouse shape, warehouse design, put-away, order picking, replenishment, random storage, class-based storage

1. Warehouse shape and door configuration

Warehouse design is a complex process that involves both structural and operational decisions that ultimately affect the overall performance of the warehouse. To further complicate the process, many of the design decisions are interrelated, leading to several design alternatives. Two such design decisions are warehouse shape and dock door configuration. We refer to the joint problem of these two design decisions as warehouse configuration.

The warehouse that we consider fulfills orders for cases and product is stored in pallet racks. The order picking locations for fast-moving items reside on the bottom level of a pallet rack and the upper levels are designated as reserve storage. Warehouses where an order picker travels along aisles to pick items represent the majority of warehouses (de Koster et al., 2006), and the most common forward area for picking cases is the ground floor of the pallet rack (Bartholdi and Hackman, 2011). Accordingly, we believe that our research is applicable to a broad range of warehouses.

The warehouse shape factor is defined as the width-to-depth ratio of the storage-rack area, which also impacts the overall shape of the building itself (Francis, 1967). Figure 1 illustrates two traditional warehouse layouts with parallel aisles where travel is rectilinear. The storage areas in the two layouts accommodate the same number of storage locations, but the shapes of the storage areas are very different. The shape of a warehouse directly impacts the number and length of the picking aisles. With rectilinear travel, it is clear from Figs. 1(a) and 1(b) that the shape of the warehouse affects the travel distance for put-away, order picking, and replenishment. Consequently, warehouse shape is an important design consideration.

Francis (1967) determined that for a centrally located Pickup and Deposit (P&D) point (or dock door) and single stops, as in a unit-load put-away operation, the optimal warehouse shape is such that the warehouse is twice as wide as it is deep. In fact, the 2:1 shape ratio has been accepted as “optimal” (Francis, 1967; Bassan et al., 1980). However, we will show that removing the assumption of a single P&D point results in a different optimal shape. For a unit-load warehouse that utilizes all dock doors and performs only put-away and retrieval operations, this finding is significant. In addition to put-away, one must also consider the order picking and replenishment operations in determining the optimal warehouse shape because these three operations represent the major travel components in the warehouse that we consider. The replenishment operation is necessary when there is a forward area for picking, as only fast-moving products are stored here in order to...
reduce the size of the area and, as a result, reduce travel. Replenishment occurs when a location in the forward area nears depletion.

Other factors affecting travel distance are the storage and routing policies. In order to reduce travel, fast-moving items are often stored in locations that are convenient to P&D points. Class-based storage groups items into classes based on their level of activity, where the fastest-moving items are located closest to the P&D points in order to reduce travel and storage within each class is random. Class-based storage is preferred to full-turnover-based storage, as full-turnover-based storage requires a repositioning of items when demand frequencies change (de Koster et al., 2006; Tompkins et al., 2010). Thus, considering class-based storage in the design phase helps to produce designs that are not overly sensitive to assumptions about demand patterns. Figure 2 displays three layouts for three classes of storage where the darker shades of color represent the fastest-moving items. When dock doors are located on only one side of the facility, the fast-moving items are located near the ends of the aisles closest to the dock doors as in Figs. 2(a) and 2(b). The diagonal layout in Fig. 2(a) places the fastest-moving items closest to a single-central (assumed) P&D point at the front of the warehouse, and the slower items are placed in locations that are farthest from the P&D point. The identical-aisle layout in Fig. 2(b) is a special case of the diagonal layout in Fig. 2(a), where the boundary for each storage class does not vary from one aisle to the next but is identical across all aisles. This configuration aims to reduce the distance traveled along the aisles. If dock doors are located on opposite sides of the facility, the fast-moving items are generally located in the centermost aisles (near a central pickup point) as illustrated in Fig. 2(c). This configuration seeks to reduce the number of aisles traveled, as well as the cross-aisle travel, and is often referred to as the within-aisle layout.

The travel distance for each class-based layout depends on the amount of storage and the activity level for each class of items, as well as the warehouse shape. Furthermore, the class-based storage layouts entail two different strategies for reducing travel (i.e., reducing cross-aisle travel versus within-aisle travel). Thus, the optimal warehouse shape is not obvious, and it would appear that practitioners would benefit by understanding how the warehouse configuration affects its performance.

To reduce the complexity of the models that we develop and to form the basis for validating our methodology, we restrict our consideration to layouts without center cross aisles. Cross aisles may reduce travel by allowing more opportunities for order pickers to change aisles, but inserting cross aisles requires additional warehouse space. Also, in cases where the pick density is high, additional cross aisles can result in longer picking tours. See Roodbergen et al. (2008) for a model to estimate travel for cross aisles with a random storage layout and Berglund and Batta (2012) for the optimal placement of cross aisles in a warehouse with class-based storage.

We were able to confirm that warehouse configuration is of interest to industry through two projects in the Center for Excellence in Logistics and Distribution (CELDi). A CELDi member organization was interested in determining how warehouse configuration affects the overall performance of their case-picking warehouse. Their only insight into this question was the above “optimal” 2:1 warehouse shape result. In addition, the company had an informal policy that more dock doors are preferred to fewer dock doors in designing a facility. Thus, most of their current facilities are configured with dock doors along both sides of the facility.
the facility and as close to a 2:1 ratio as is permissible given the site plan. Generalization of the work for this member organization was funded by the other members of CELDi.

Thus, the objective of our research is to help such organizations. We do so by first developing analytical models to estimate the expected travel for the put-away, order picking, and replenishment operations for random storage, as well as the class-based storage layouts in Figs. 2(b) and 2(c). Then, we use the models to investigate the optimal warehouse shape based on each operation. We believe that analytical models for overall warehouse design are preferred over simulation in evaluating design performance, as simulation is less conducive to generalization (Gu et al., 2010).

A primary result of this investigation is that the optimal shape varies considerably by the operation considered. Finally, we illustrate how to determine the optimal shape of a case-picking warehouse that considers the combined travel for put-away, order picking, and replenishment operations over a period of time. Even though we focus on the optimal warehouse shape as a design consideration, our models can be used collectively to assess overall warehouse design performance with regard to forward area size and layout, pallet rack height, as well as the shape of the pallet rack area.

2. Literature review

Francis (1967) was the first to consider the optimal shape of the warehouse and concluded that a 2:1 shape ratio is optimal for a single P&D point and a single stop. Bassan et al. (1980) considered multiple dock doors and concluded that the distance to a single point is minimized from the most centrally located door and that all doors should be as near as possible to the center of the warehouse. Although this is correct, they (incorrectly) concluded that a 2:1 warehouse shape is optimal for the case of a unit-load warehouse with multiple doors, as well as for a single P&D point.

Several routing policies have been suggested in the literature for order picking. The simplest strategy is the traversal policy, where the order picker enters every aisle that contains at least one pick location, traverses the entire aisle, and exits at the opposite end of the aisle (Hall, 1993). With the return policy, the order picker enters an aisle, travels to the farthest pick, and returns to the same end of the aisle that was entered (de Koster et al., 2007). With the midpoint strategy the order picker travels only as far as the midpoint of the aisle before returning; picks past the midpoint are obtained from the back cross aisle (Hall, 1993). With the largest gap policy, the order picker enters an aisle as far
as the largest gap between two adjacent pick locations or between the end of the aisle and the closest pick, thus avoiding the largest gap that does not contain picks (Hall, 1993). Finally, the composite strategy combines the traversal and return policies; an aisle is not traversed if returning results in less travel for a given aisle (de Koster et al., 2007).

Hall (1993) developed models to approximate the expected travel distances in an order picking warehouse for the traversal, return, midpoint, and largest-gap routing strategies based on a fixed area with a random storage policy and a centrally located P&D point. He found that for random storage, elongated (wider) warehouses are favorable as the number of picks increases. This finding is intuitive because if there are as many picks as there are aisles, then it is likely that all aisles would be traversed. In this situation, a more elongated warehouse would increase the number of aisles while also making the aisles shorter. By elongating the warehouse, the within-aisle travel is then reduced.

Petersen (1997) used simulation to compare the performance of the traversal, return, midpoint, largest-gap, composite (a hybrid of the return and traversal policies), and optimal routes in a random storage warehouse with 1000 storage locations by generating pick lists of five, 15, 25, 35, and 45 picks. He concluded that narrow, deeper warehouses are more effective at minimizing order picking travel for all of the strategies except the return policy. However, we attribute these results to the fact that he performed his analysis for 1000 storage locations and only considered shape ratios of 3:1, 2:1, 1:1, and 1:2. Furthermore, with only 1000 storage locations, it is conceivable (depending on the dimensions of the storage locations) that a pick list with more than 20 lines would result in more than one stop per aisle for the shape ratios considered. With more than one stop per aisle, narrow warehouses would indeed reduce travel, requiring fewer (but longer) aisles to be entered. He also concluded that the largest gap strategy is preferred for a smaller number of pick lines.

In a simulation-based evaluation of storage layouts and routing policies presented in Petersen (1999) the within-aisle layout was favorable to the diagonal layout, regardless of the pick list size in a warehouse with 10 aisles. Petersen's work (which assumed a warehouse with 10 aisles and 1000 items) did not consider warehouses of varying shapes.

An activity-based strategy that assigns items to storage locations based on a ratio of the required space to the order frequency is the Cube-per-Order-Index (COI), as first introduced by Heskett (1963). Caron et al. (1998) developed analytical models for order picking travel with a COI-based storage strategy where the warehouse is divided into two sections separated by a cross aisle. The first model estimated the expected travel for a return routing policy using a modified version of the identical-aisle layout, and the second model estimated travel for the traversal routing policy using a modified version of the within-aisle layout. For the return strategy model, the authors acknowledge that the within-aisle travel is overestimated because it is based on the average number of picks per aisle.

Hwang et al. (2004) also developed analytical models to determine order picking travel based on a COI storage policy. Their models included the return policy for the identical-aisle layout, the traversal policy for the within-aisle layout, and the midpoint policy for a perimeter layout (where the fastest-moving items are placed at both ends of an aisle, and the slowest-moving items are located in the innermost storage locations in the aisle). The return policy performed well for a small number of picks, and the traversal policy performed the best for a large number of picks ($N = 64$ to $80$). In general, however, the midpoint policy outperformed both the traversal and return policies in terms of minimizing order picking travel. Hwang et al. (2004) also found that a highly skewed ABC curve can significantly reduce travel regardless of the routing policy and that the best shape for a warehouse is a 2:1 width-to-depth ratio. However, only five shape ratios were considered, ranging from 0.45 to 1.75.

Class-based storage, on the other hand, groups items into classes based on their activity level, where the storage within a class is assumed to be random. Le-Duc and de Koster (2005) developed a model to estimate order picking travel in a class-based storage warehouse with the diagonal layout, where the percent of storage for each class can vary across aisles. The model is based on the return strategy and utilizes expected values to determine the number of picks in an aisle, as well as the number of picks in each class. The authors acknowledge that using expected values results in an overestimation of within-aisle travel. Le-Duc and de Koster (2005) were the first to consider the storage zone optimization problem for the diagonal aisle layout. They developed a heuristic procedure to determine the optimal boundaries for each storage zone in an aisle and found that the identical-aisle layout is a robust layout for minimizing travel, regardless of the number of stops per tour.

Chew and Tang (1999) modeled the order picking travel for the traversal policy based on the occupancy problem for any item-to-location assignment. Their model assumes a corner P&D location, and because it uses a single probability for visiting a given aisle, the model does not apply to the diagonal or identical-aisle layout.

Despite extensive research on the forward-reserve problem (Gu et al., 2007), no prior research exists that considers the expected travel for replenishments. Thomas and Meller (2012) recently presented a limited replenishment model. Pohl et al. (2011) modeled the expected dual-command travel distance in a unit-load warehouse with random storage. The travel-between portion of their model estimates the expected distance between two random points in a warehouse. However, in a replenishment operation there are two travel legs: the first is travel between the last replenishment location and the storage location for the current replenishment operation (and is similar to the travel-between
portion of dual-command); the second leg is travel between the storage location and the picking location for that product. As we discuss later, this second leg typically does not occur between two random points due to put-away strategies.

In summary, the research that has considered warehouse shape has been limited to the put-away operation or the order picking operation in isolation and only in the context of a random storage warehouse. Furthermore, most of the previous research has considered the travel to and from a single P&D point rather than multiple dock doors. In taking all of this research together, the best shape of a warehouse is not obvious and appears to be dependent upon which operations are considered (Thomas and Meller, 2012).

3. Optimal warehouse shape

In our travel time models, we assume a rectangular warehouse with aisles that are orthogonal to the side(s) of the facility containing dock doors, where the doors are located at points along the entire width of the warehouse. We consider both a one-sided and two-sided configuration of dock doors. Our models assume that aisles are continuously located, that storage locations are continuous, and that the side-to-side movement within an aisle is negligible. We assume a uniform usage of dock doors for put-away operations, rather than a single, centrally located dock door.

For order picking, the pick list is obtained from a centrally located pickup point, and the completed order is deposited at a uniformly distributed dock door before returning to the pickup point. We consider a traversal routing policy for random storage. Even though the largest-gap strategy outperforms the traversal strategy, we acknowledge that it is less commonly used in practice due to its complexity. The forward area for picking is the bottom level of storage within the picking aisles, with the reserve storage locations in the upper levels of storage. Thus, replenishment to the forward area occurs within the storage aisles. We model the expected distance for replenishment such that the worker enters or exits an aisle from the end that results in the least travel.

For the one-sided class-based storage layout, the shipping and receiving doors are located on only one side of the facility, and we utilize the identical-aisle layout in Fig. 2(b). With the fast-moving items concentrated at the ends of the aisles, it is likely that most of the picks in an aisle will not require travel past the center of the aisle (for a fairly skewed ABC curve), and the return policy minimizes the within-aisle travel for this layout. Le-Duc and de Koster (2005) investigated the optimal storage boundaries for a class-based storage warehouse using the diagonal layout, and they found that the identical-aisle layout is optimal for a large number of picks using the return policy (where the picker enters and exits from the same end of the aisle, traveling as far as the farthest pick). Furthermore, Le-Duc and de Koster found that the identical-aisle layout provided very good results for the return policy, regardless of the pick list size. Because this layout places the fast-moving items along the entire width of the facility that contains doors, it may also result in less congestion than the diagonal layout.

For the two-sided layout, the dock doors are located along both sides of the facility. Because there is a centrally located pickup point for obtaining the pick list and an effort to concentrate the pick locations of the fast-moving items, we propose the layout in Fig. 2(c) with the traversal routing strategy. This layout will significantly reduce both the across-aisle travel and the number of aisles that require traversal, as most of the picks will occur in the centermost aisles. Additionally, because all of the items in an aisle are the same class, this layout will allow a flow-through of product, making the storage locations convenient to both the shipping and receiving doors (Bartholdi and Hackman, 2011).

We now present models that extend current research in three areas:

1. We present travel time models for put-away and order picking that consider more than one P&D point (multiple dock doors) for both random storage, as well as the one-sided and two-sided class-based storage layouts.
2. We present new travel time models for replenishment operations for both random and class-based storage layouts. These models may provide useful insight to future research on the forward-reserve problem because forward-reserve models assume that the cost of replenishment can be specified a priori.
3. We consider warehouse shape for layouts with random and class-based storage and include an example that illustrates how the three warehouse operations (put-away, order picking, and replenishment) can be weighted to determine the best overall warehouse shape.

The remainder of this article is organized as follows. In Section 4 we present our models for the put-away operation for both random and class-based storage layouts. Models to determine the expected distance for order picking for random and class-based storage are presented in Section 5. In Section 6 we introduce models for the replenishment operation for both random and class-based storage, and in Section 7 we provide an example that illustrates how to determine the optimal warehouse shape when considering the put-away, order picking, and replenishment operations. Finally, in Section 8 we summarize our findings regarding warehouse shape for both random and class-based storage.
4. Put-away travel

In our models we denote the width of the warehouse as \( W \), the depth of the warehouse (parallel to the picking aisles) as \( L \) as shown in Fig. 3, and the number of levels of pallet rack as \( H \). Accordingly, the two horizontal travel components \( x \) and \( y \) denote the across-aisle travel and within-aisle travel, respectively. Vertical travel, denoted by \( z \), is required to access pallet locations above the bottom level of the pallet rack. We refer to the expected horizontal distance required for a put-away operation as \( E[D_{x,y}] \).

In our modeling we assume the height of the warehouse is fixed (see Parikh and Meller (2010) for a model to determine this parameter). In Section 4.4 we include an expression for the expected vertical travel to put away a pallet and then explain how to calculate the total travel time that includes both the horizontal and vertical travel components.

4.1. Uniform (one-sided or two-sided) doors with random storage

Using well-known results (Bozer, 1985) for the expected values of the maximum and minimum of two continuous uniform [0,1] random variables (and treating aisle locations as continuous random variables), the expected one-way horizontal travel for a put-away operation with random storage can be expressed as

\[
E[D_{x,y}] = W/3 + L/2 + g + a,
\]

where \( g \) is the depth of the staging area and \( a \) is the width of the end cross-aisle in Fig. 3.

**Theorem 1.** The optimal warehouse shape for a put-away operation with uniform door usage and random storage is \( 3/2 \).

**Proof.** We use the relationship for area \( (A = WL) \) and warehouse shape \( (r = W/L) \) to express the warehouse length and width in terms of the area and shape factor, \( r \):

\[
L = \frac{A}{r} \quad \text{and} \quad W = \sqrt{Ar}.
\]

Thus, the expression for expected travel can be written as

\[
E[D_{x,y}] = \frac{\sqrt{A}}{3} + \frac{\sqrt{A/r}}{2} + g + a.
\]

Taking the derivative with respect to \( r \) and setting it equal to zero, we have

\[
\frac{1}{2} \left( \frac{\sqrt{A}}{3} \right) r^{-\frac{3}{2}} - \frac{1}{2} \left( \frac{\sqrt{A}}{2} \right) r^{-\frac{3}{2}} = 0.
\]

Solving for \( r \) yields the optimal width-to-depth shape ratio:

\[
r^* = 3/2.
\]

This completes the proof.

With random storage, the optimal \( r \) value is the same regardless of a one-sided or two-sided configuration of dock doors. A similar exercise with one, centrally located door yields an optimal ratio of 2.0 (Francis, 1967). Thus, we can clearly see that warehouse design is sensitive to the door usage assumption; with the use of all doors the optimal shape is 1.5, but with a single, centrally located dock door the optimal shape is 2.0.

4.2. Uniform (one-sided) doors with class-based storage

The expected distance required for a put-away operation with class-based storage is different from the random storage model in that now we must consider the percent of storage for each class, as well as the frequency of put-aways for each class. For the one-sided layout, the objective is to determine how far into an aisle the operator must travel for the put-away operation. The expected one-way travel for a one-sided configuration with class-based storage is

\[
E[D_{x,y}] = W/3 + pL + g + a,
\]

where \( p \) is the fraction of the aisle that is traveled for a given turnover, or activity profile. Here we use a class-based ABC curve. In Fig. 4(a) the distance for each class within the aisle is shown as \( PSA_L, PBS_L, \) and \( PSC_L \), where \( PS \) is the percent of storage and the subscript is the storage class (such that \( PSA + PBS + PSC = 1 \)). Of interest is the optimal warehouse shape for the put-away operation in such a warehouse (the following result can be shown by convex analysis; a detailed proof can be found in the online Appendix A.1).
Result 1: The optimal warehouse shape for a put-away operation with uniform doors and a one-sided storage layout is $3p$.

The value of $p$ can be determined using the following:

$$p = PA_A(0.5PS_A) + PA_B(PS_A + 0.5PS_B) + PA_C(PS_A + PS_B + 0.5PS_C),$$

(2)

where $PA$ is the percent of activity and the subscript is the storage class. For a particular ABC curve where the percent of storage is 20/30/50 for class-A, class-B, and class-C items and the percent of activity is 80/15/5, the value of $p$ is

$$p = 0.17.$$ 

Therefore, the optimal value for $r$ for an 80/15/5 ABC curve is

$$r = 3(0.17) = 0.51.$$ 

Note that for an ABC curve where the percent of activity is 1/3/1/3/1/3 and the percent of storage is 1/3/1/3/1/3 for class-A, class-B, and class-C items, the value for $p$ is

$$p = 0.5 \left( \frac{1}{3} \right) \left( \frac{1}{3} \right) + \frac{1}{3} \left( \frac{1}{3} \right) + 0.5 \left( \frac{1}{3} \right) \left( \frac{1}{3} \right) + \frac{1}{3} \left( \frac{1}{3} \right) + \left( \frac{1}{3} \right) \left( \frac{1}{3} \right) + 0.5 \left( \frac{1}{3} \right) \left( \frac{1}{3} \right) = \frac{1}{2},$$

and therefore the optimal $r$ for uniform storage is $3/2$, which is the equivalent result from Theorem 1.

Thus far, we have used the ABC curve to determine the value $p$. In subsequent sections where we consider order picking and replenishment, we will use the parameter $S$ from Bender (1981) to refer to the skewness of the ABC curve:

$$Y = \frac{(1 + S)X}{S + X},$$

where $X$ is the percent of storage, $Y$ is the percent of activity, and $S$ is the skewness factor. For a set of points $(X_i, Y_i)$, where the values for $X_i$ represent the percent of storage and $Y_i$ represents the percent of activity for each item $i$, the value of $S$ can be determined with the least-squares method using the following expression:

$$\sum Y_i - (1 - S) \sum \frac{X_i}{Y_i} = 0.$$ 

We note here that $S$ is positive and approaches infinity for the case where all items approach an equal activity level (uniform distribution). Figure 4(b) displays the optimal values for $r$ for a range of ABC curve parameters, denoted by $S$. The table in online Appendix B lists values for $S$ and $p$ for a range of ABC curves.

4.3. Uniform (two-sided) doors with class-based storage

For a put-away operation with the two-sided, class-based layout, the across-aisle distance must be determined based on the frequency and layout of each class, whereas the distance into an aisle is equivalent to that of random storage (i.e., half the length of the aisle). The layout of class-A, class-B, and class-C items for a two-sided configuration is displayed in Fig. 5(a), and the distances to the center of each storage class can be defined as

$$D_1 = 0.25(PS_C)W,$$

(3)

$$D_2 = 0.5(PS_C)W + 0.25(PS_B)W,$$

(4)

$$D_3 = W/2 + 0.5(PS_A)W + 0.25(PS_B)W,$$

(5)

$$D_4 = W - 0.25(PS_C)W.$$  

(6)
The percent of activity for each storage class can be used to determine the across-aisle travel from a dock door (within a storage class) to an aisle in any storage class. Given the symmetry of the layout, the expected distance traveled from a door within $C_1$ is the same as the expected distance from a door within $C_2$; the expected distance from a door within $B_1$ is the same as the expected distance from a door within $B_2$. If the put-away aisle resides within the same class section as the door, then the expected travel is one-third of the width of the section.

The expected $x$-distance from a door within $C_1$ (or $C_2$) to some aisle is

$$E[ D_{x}^{C} ] = PA_C \left[ \left( \frac{1}{2} \right) \left( \frac{1}{3} \right) \left( \frac{1}{2} \right) PS_C W + \left( \frac{1}{2} \right) (D_4 - D_1) \right] + \left( \frac{1}{2} \right) (D_2 - D_1) + \left( \frac{1}{2} \right) (D_3 - D_1) + PA_A \left[ \frac{W}{2} - D_1 \right].$$

The expected $x$-distance from a door within $B_1$ (or $B_2$) is

$$E[ D_{x}^{B} ] = PA_C \left[ \left( \frac{1}{2} \right) (D_2 - D_1) + \left( \frac{1}{2} \right) (D_4 - D_2) \right] + PA_B \left[ \left( \frac{1}{2} \right) \left( \frac{1}{3} \right) \left( \frac{1}{2} \right) PS_B W + \left( \frac{1}{2} \right) (D_3 - D_2) \right] + PA_A \left[ \frac{W}{2} - D_2 \right].$$

The expected $x$-distance from a door within $A$ is

$$E[ D_{x}^{A} ] = PA_A \left[ \frac{1}{3} PS_A W \right] + PA_B \left[ \frac{W}{2} - D_2 \right] + PA_C \left[ \frac{W}{2} - D_1 \right].$$

Multiplying the three previous equations by the corresponding percent of storage for the dock door (probability of using the dock door) and then summing these equations, we have the total expected $x$-distance. The expected one-way horizontal travel can be expressed as

$$E[D_{x,y}] = E[D_x] + L/2 + g + a.$$ 

In the following result (proved via convex analysis; see online Appendix A.2), we use the parameter $q$ to denote the coefficient of $W$ (embedded in $E[ D_{x} ]$) as defined by Equations (7) to (9). The derivation of this parameter is included in Appendix C.

**Result 2:** The optimal warehouse shape for a put-away operation with uniform doors and a two-sided storage layout is $1/2q$.

If the percent of storage for class-A, class-B, and class-C items is 20/30/50 and the percent of activity is 80/15/5, the optimal ratio is $r = 1.89$.

Figure 5(b) displays the optimal values for $r$ for a range of ABC curves. As the value of $S$ decreases (a more skewed ABC curve), the optimal value of $r$ approaches 2.0; as the value of $S$ increases (toward a uniform distribution), the optimal value of $r$ is 1.5.
4.4. Summary

We have considered the optimal shape for the put-away operation for a random storage warehouse, as well as class-based storage layouts for two dock door configurations. The optimal shape for a random storage layout with uniform dock door usage is 1.5. For the one-sided, class-based storage layout, the optimal shape decreases below 1.5 as the skewness of the ABC curve increases. The optimal shape for the two-sided, class-based storage layout is between 1.5 and 2.0, where it approaches 2.0 as the skewness of the ABC curve increases.

Thus far we have only considered the horizontal travel for the put-away operation. The expected one-way vertical distance to put away a pallet in one of the \((H - 1)\) levels of storage rack (above level one) can be expressed as

\[
E[D_x] = P_h \left( 1 + \frac{(H - 1)}{2} \right) = P_h \left( \frac{H}{2} \right),
\]

where \(P_h\) is the height of a pallet opening. The total two-way travel time for a put-away operation, then, can be expressed as

\[
E[T] = 2 \left( \frac{E[D_{x,y}]}{v_{x,y}} + \frac{E[D_z]}{v_z} \right),
\]

where \(v_{x,y}\) and \(v_z\) are the horizontal and vertical speeds of the lift truck, respectively.

5. Order picking travel

Several strategies have been developed for routing order pickers in a warehouse. Hall (1993) presented analytical models for three strategies: traversal, midpoint, and largest gap. Because the traversal strategy is the most common policy used in practice, we consider this strategy for random storage and the two-sided, class-based storage layout. For the one-sided, class-based storage layout, we consider a return policy to take advantage of the reduction in within-aisle travel. We do not consider vertical travel in the pick tour because our problem definition states that all case picks come from the bottom level of storage.

5.1. Random storage policy

With a random storage policy, all pick locations are equally likely to be visited on a pick tour. The models presented by Hall (1993) assume that storage is random and travel occurs from a centrally located P&D point. Because we assume that the order will be deposited at a uniformly distributed dock door before returning to the central pickup point, we modify Hall’s equation to include the extra distance required for the dropoff. Hall’s work shows that for uniformly distributed pick locations, increasing the number of picks per route favors elongated warehouses (i.e., larger values of \(r\)) for a fixed storage area.

5.1.1. Traversal strategy with a centrally located pickup point and uniformly distributed dropoff point

For values of \(N\) greater than or equal to 5, Hall (1993) models the expected length of an order picking tour from a centrally located P&D point as

\[
E[D_{x,y}] = 2W \left[ \frac{(N - 1)}{(N + 1)} \right] + ML \left[ 1 - \left( \frac{(M - 1)}{M} \right)^N \right] + 0.5L,
\]

where \(N\) is the number of picks and \(M\) is the number of aisles in the warehouse. The first term in the expression is the expected \(x\)-distance (within-aisle travel) times the number of aisles and the probability that an aisle contains at least one pick. The final term accounts for the expected travel required to return to the front of the warehouse.

If the order is dropped off at a uniformly distributed dock door, the probability that the door is located within the \(x\)-distance traveled to fulfill the order (i.e., the range of aisles containing the items picked) is \((N - 1)/(N + 1)\). Figure 6 shows the distance from a dock door outside of the pick range to the closest pick.

The probability that a door lies outside of the pick range is

\[
\Pr(\text{drop-off door is outside the pick range}) = 1 - \left( \frac{N - 1}{N + 1} \right) = \frac{2}{N + 1}.
\]

If the dock door for the drop-off does lie outside the pick range, the expected one-way distance from the farthest pick position to the dock door is

\[
D_{\text{drop-off}} = 0.5 \left( \frac{1}{N + 1} \right) W.
\]

Multiplying the probability that the dock door lies outside of the pick range by the expected distance to the door outside the pick range, we have the expected one-way distance to a dock door outside the pick range:

\[
E[D_{\text{drop-off}}] = \frac{W}{(N + 1)^2}.
\]

The expected tour length (that includes the additional travel to a door located outside the pick range) is

\[
E[D_{x,y}] = 2W \left[ \frac{(N - 1)}{(N + 1)} + \frac{1}{(N + 1)^2} \right] + ML \left[ 1 - \left( \frac{(M - 1)}{M} \right)^N \right] + 0.5L. \tag{11}
\]

This distance assumes that the order picker would first travel in the direction that is opposite the dock door where
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the order will be deposited (so as to complete the tour by heading toward the order’s dock door). Figure 7 displays the optimal values of \( r \), where the number of picks range from 5 to 40 (for areas of 100,000 ft\(^2\) to 300,000 ft\(^2\), where the center-to-center aisle width is 20 feet).

From Fig. 7, as the number of picks increases, so does the optimal value for \( r \). This is consistent with Hall’s recommendation that an elongated warehouse is advantageous when there are several picks per tour (Hall, 1993). Furthermore, note that these values of \( r \) are much greater than 1.5 (the optimal shape for put-away operations in a random storage warehouse).

5.2. Class-based storage

For a class-based storage policy, fast-moving items are located in such a way to take advantage of shorter distances to the P&D points. Thus, the layouts for a one-sided and two-sided configuration employ two different routing strategies in order to achieve a minimum expected tour distance.

5.2.1. Return policy for an one-sided layout and a centrally located pickup and uniform deposit point

The return strategy involves entering and exiting from the same end of the aisle to retrieve items, traveling only as far as the farthest pick in the aisle. For a highly skewed class-based ABC curve, a return policy would require travel through only a small percent of the total aisle length. As suggested by Le-Duc and de Koster (2005), only the return policy is relevant for this layout. The across-aisle (\( x \)-travel) required for the return policy is the same as the first term in Equation (11)

\[
E[D_x] = 2W \left( \frac{N-1}{N+1} + \frac{1}{(N+1)^2} \right).
\]

For the within-aisle travel (\( y \)-distance), however, instead of traversing the entire aisle, a picker would only travel as far as the farthest pick in the aisle. To determine this distance, we consider the possibility of \( n \) picks in an aisle, ranging from 1 to \( N \), and calculate the associated probabilities. The \( n \) picks may be any combination of classes, so we enumerate over every possibility (i.e., two class-A picks and \((n-2)\) class-B picks, etc.), multiplying the probability and distance for each. We sum the distances for each class combination of \( n \) picks and multiply by the probability of \( n \) picks. Finally, we sum over all possible picks in an aisle to get the expected distance into an aisle:

\[
E[D_y] = 2M \sum_{n=1}^{N} \binom{N}{n} \left( \frac{1}{M} \right)^n \left( \frac{M-1}{M} \right)^{N-n} \times \left[ (PA)_A^n \left( \frac{n}{n+1} \right) PS_AL + (PA)_B^n \times PS_A + \left( \frac{n}{n+1} \right) PS_B \right] L \]

\[
+ (PA)_C^n \left[ PS_A + PS_B + \left( \frac{n}{n+1} \right) PS_C \right] L \]

\[
+ \sum_{i=1}^{n-1} \binom{n}{i} (PA)_A^{n-i} (PA)_B^i \times PS_A + \left( \frac{i}{i+1} \right) PS_B \right] L.
\]
curves with $S = 100000$ ft$^2$ and $300000$ ft$^2$. Warehouses with larger picking areas favor slightly more elongated warehouses compared to those with smaller picking areas, as with the random storage layout. The optimal shape becomes more elongated as the number of picks increases and as the skewness of the storage layout. The optimal shape decreases (toward a more uniform distribution).

\[ \text{Expected within-aisle distance} = \sum_{i=1}^{12} d_i + \left( 1 - \frac{\sum_{i=1}^{12} d_i}{2W} \right) \left[ 0.5 \left( W - 0.5 \sum_{i=1}^{12} d_i \right) \right]. \]

The expected within-aisle distance is simply the length of an aisle times the expected number of aisles traveled. The expected number of aisles is less than that of random storage because most of the picks are concentrated in the class-A aisles (that account for only a small percent of the total number of aisles for a fairly skewed ABC curve). First, we consider the probability for every possible class-combination of picks. Then, for each possibility, we calculate and total the expected number of class-A, class-B, and class-C picks. For a tour with $N$ pick locations, we can use the multinomial distribution to determine the probability of each scenario. For each pick, there are three possible types of picks (class-A, class-B, or class-C). The probability of $n_A$ class-A picks, $n_B$ class-B picks, and $n_C$ class-C picks is

\[ \Pr(n_A, n_B, n_C) = \frac{N!}{n_A!n_B!n_C!} (PA_A)^{n_A}(PA_B)^{n_B}(PA_C)^{n_C}. \]

Figure 9 illustrates the 12 combinations of picks that result in different $x$-distances, and the respective distances ($d_1$ to $d_{12}$) are included in online Appendix D. The total expected across-aisle distance is then the sum of each of the $d_i$ distances plus the probability that the dock door for the drop-off of the order is outside the picking range times half of the distance outside the picking range:

\[ \text{Expected across-aisle distance} = \sum_{i=1}^{12} d_i + \left( 1 - \frac{\sum_{i=1}^{12} d_i}{2W} \right) \left[ 0.5 \left( W - 0.5 \sum_{i=1}^{12} d_i \right) \right]. \]
approximation for the within-aisle travel is then
\[
E[D_y] = LM \sum_{i=0}^{N} \sum_{j=0}^{N-i} (PA_A)^i (PA_B)^j (PA_C)^{N-i-j} \times \left[ \frac{N!}{i!j!(N-i-j)!} \right] \times \left[ PS_A \left[ 1 - \left( 1 - \frac{1}{MPS_A} \right)^i \right] + PS_B \left[ 1 - \left( 1 - \frac{1}{MPS_B} \right)^j \right] + PS_C \left[ 1 - \left( 1 - \frac{1}{MPS_C} \right)^{N-i-j} \right] \right] + 0.5L.
\]

The total expected tour length is then
\[
E[D_{x,y}] = E[D_x] + E[D_y].
\]

Figures 10(a) and 10(b) illustrate the optimal shape for ABC curves with $S$ parameter values from 0.03 (very skewed) to 0.7 (hardly skewed) for warehouses with picking areas of 100 000 ft$^2$ and 300 000 ft$^2$. Again, warehouses with smaller picking areas favor slightly less elongated warehouses than those with larger picking areas. For the more skewed ABC curves ($S = 0.03, 0.07,$ and $0.10$), the optimal shape reaches a peak when the number of class-A picks is close to the number of class-A aisles. (For the 100 000 ft$^2$ picking area, this peak occurs for a smaller number of picks, as compared with the 300 000 ft$^2$ picking area, because there are fewer class-A aisles.) Then, as the number of picks increases, the shape decreases to achieve fewer (longer) aisles, as there are multiple class-A picks per aisle. However, when the number of class-B picks becomes a factor, the optimal shape increases such that the class-B aisles become shorter. This increase is very gradual because even though the number of picks is increasing, very few of these picks are class-B items for fairly skewed ABC curves. In addition, picking areas that are larger result in higher shape ratios, and the peak occurs at much higher values of $N$ because there are more aisles for each class of items. For the hardly skewed curve, the optimal shape increases steadily as the number of picks increases because there are significantly more class-B and class-C items.
The optimal shapes for the two-sided traversal strategy are much greater than the optimal shapes for the two-sided put-away operation (1.5 to 2.0). Furthermore, as the number of stops per tour increases, the optimal warehouse shape for the two-sided layout is significantly less as compared with traversal with random storage. With random storage, increasing the number of pick lines would result in more aisles traversed, but with the two-sided layout, likely the additional lines include primarily class-A aisles (with multiple pick lines).

5.3. Summary
The optimal warehouse shape for order picking with a random storage layout and a one-sided, class-based layout increases as the number of stops per tour increases. Both the one-sided and two-sided class-based layouts favor less-elongated warehouses as compared with random storage for a large number of picks, especially for more skewed ABC curves. This is intuitive because both class-based storage layouts aim to reduce the within-aisle travel component. For a small number of stops per tour (e.g., \( N < 10 \)), the one-sided layout favors less-elongated warehouses than the two-sided layout. As the number of stops per tour increases (with more class-B picks), there is less of an impact on the optimal warehouse shape for the two-sided layout. Furthermore, warehouse shape is a more significant factor for a random storage warehouse than one with class-based storage.

6. Replenishment travel
The replenishment operation begins at the picking location that was last replenished and involves travel to the reserve storage location for the next item to be replenished, followed by travel to the next replenishment location. We define \( \alpha \) as the probability that the replenishment location for an item resides within the same aisle as the reserve storage location for the item. If put-aways are truly random, \( \alpha = 1/M \). However, we contend that even in a random storage warehouse, some effort is made to place the reserve storage location for an item in the same aisle as the picking location of the item. Therefore, in general, \( \alpha \geq 1/M \).

For simplicity, we use locations 1, 2, and 3 to denote the location of the last replenishment, the reserve storage location for the next replenishment, and the location for the next replenishment, respectively. In our models, we assume that the worker exits toward the end of the aisle that minimizes travel. The distance models that we use are based on the probabilities shown in Fig. 11. As illustrated in Fig. 11, the replenishment distance depends on the aisle locations of the last replenishment, reserve storage for the next replenishment, and the next replenishment. Because location 3 is visited after location 2, the total expected distance for the case where locations 1 and 3 reside in the same aisle is the same as if three aisles were visited. This probability is included in the last scenario in Fig. 11. In the following sections, we present models to estimate the distance required for a replenishment for both random and class-based storage layouts.

6.1. Replenishment travel for random storage
For random storage, the replenishment locations and reserve storage locations are uniformly distributed within the aisle. In modeling the within-aisle travel for random storage, we calculate the expected distances such that if two (or all three) of the points are located in the same aisle, they can be in any order. The expected distance for each replenishment scenario is as follows:

Case 1 (all three locations are in the same aisle): Locations 1 and 2 are in the same aisle with probability \( 1/M \), and location 3 is in the same aisle as location 2 with probability \( \alpha \). Thus, all travel is in the same aisle as shown in Fig. 12, with three uniformly distributed
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Fig. 11. Probabilities for replenishment travel.

points (labeled a, b, and c, where any point can represent any of the three locations).

From Fig. 12, if location 1 is in position a or c, the expected horizontal distance is

\[ 0.5 \left( \frac{1}{2} L \right) + 0.5 \left( \frac{3}{4} L \right). \]

If location 1 is in position b, the expected distance is

\[ 0.5 \left( \frac{3}{4} L \right) + 0.5 \left( \frac{3}{4} L \right). \]

With an equal probability of location 1 being in any of the three positions, the total expected distance is

\[ \frac{2}{3} L. \]

Multiplying the probability of three locations in the same aisle by the expected distance, we have

\[ \frac{\alpha}{M} \left[ \frac{2}{3} L \right]. \]

Case 2 (locations 1 and 2 are in the same aisle): Locations 1 and 2 are in the same aisle with probability 1/\(M\), and location 3 is in a different aisle than location 2 with probability 1 – \(\alpha\). Thus, two uniformly distributed points are located in the same aisle, and the third location is in a different aisle. Figure 13 illustrates the two possibilities for within-aisle travel. In each case, the worker exits toward the end of the aisle that minimizes travel, such that backtracking does not occur between locations 1 and 2. Thus, the total within-aisle distance for the case of three locations in two aisles is

\[ \frac{2}{3} L + \frac{1}{2} L = \frac{7}{6} L. \] (13)

The expected across-aisle distance between the two aisles of interest is (1/3)\(W\). Multiplying the probability by the expected travel, we have

\[ \frac{1 - \alpha}{M} \left[ \frac{7}{6} L + \frac{1}{3} W \right]. \]

Case 3 (locations 1 and 2 are in different aisles, but locations 2 and 3 are in the same aisle): Locations 1 and 2 are in different aisles with probability (\(M - 1\))/\(M\), and locations 2 and 3 are in the same aisle with probability \(\alpha\). Again, the across-aisle travel is between two aisles as depicted in Fig. 14. The total within-aisle travel can be determined from Equation (13).

Fig. 12. Three locations in the same aisle.

Fig. 13. Possible routes with locations 1 and 2 in same aisle.
Thus, the expected across-aisle distance and within-aisle distance is the same as for Case 2. Multiplying the probability by the expected distance yields the following:

$$\frac{\alpha(M - 1)}{M} \left[ \frac{7}{6} L + \frac{1}{3} W \right].$$

Case 4 (three locations are in three different aisles): Locations 1 and 2 are in different aisles with probability \((M - 1)/M\), and locations 2 and 3 are also in different aisles with probability \((1 - \alpha)\). Thus, travel involves entering/exiting three aisles. Note that we include the case where locations 1 and 3 are in the same aisle here. The within-aisle travel to a uniform point in three different aisles is \(4L/2 = 2L\), and the across-aisle travel for three aisles is \(2W/3\):

$$\frac{(1 - \alpha)(M - 1)}{M} \left[ 2L + \frac{2}{3} W \right].$$

Taking into consideration all possible scenarios, we have the total expected horizontal distance for a replenishment operation in a random storage warehouse:

$$E[D_{x,y}] = \frac{\alpha}{M} \left[ \frac{2}{3} L \right] + \frac{\alpha(M - 1)}{M} \left[ \frac{7}{6} L + \frac{1}{3} W \right] + \frac{(1 - \alpha)}{M} \left[ \frac{7}{6} L + \frac{1}{3} W \right] + \frac{(1 - \alpha)(M - 1)}{M} \times \left[ 2L + \frac{2}{3} W \right].$$

As stated previously, if put-aways are completely random, then \(\alpha\) can be expressed as \(1/M\). Accordingly, the expected horizontal distance can be expressed as

$$E[D_{x,y}] = \frac{1}{M^2} \left[ \frac{2}{3} L \right] + \frac{M - 1}{M^2} \left[ \frac{7}{6} L + \frac{1}{3} W \right] + \frac{M - 1}{M^2} \left[ \frac{7}{6} L + \frac{1}{3} W \right] + \frac{(M - 1)^2}{M^2} \left[ 2L + \frac{2}{3} W \right],$$

and combining like terms, we have

$$E[D_{x,y}] = \frac{1}{M^2} \left[ \frac{2}{3} L \right] + \frac{2M - 2}{M^2} \left[ \frac{7}{6} L + \frac{1}{3} W \right] + \frac{(M - 1)^2}{M^2} \left[ 2L + \frac{2}{3} W \right].$$

Figure 15 depicts the optimal shape for different values of \(\alpha\) for picking areas of 100 000 ft\(^2\) and 300 000 ft\(^2\). The lower bound for \(\alpha\) is \(1/M\), and the optimal warehouse shape for this case is 2.81 and 2.88 for the 100 000 ft\(^2\) and 300 000 ft\(^2\) areas, respectively. The shape of the warehouse becomes more elongated for increasing values of \(\alpha\) with maximum optimal shapes of 3.40 and 3.44 for the picking areas considered. The optimal shape for the replenishment operation is greater than the optimal shape for the put-away operation but less than the optimal shape for order picking in a random storage warehouse.

### 6.2. Replenishment travel for one-sided layout with class-based storage

With the one-sided layout, we again use \(\alpha\) to represent the probability that the reserve storage location (location 2) is in the same aisle as the next replenishment location (location 3) and, as before, the reserve storage location is still randomly located within an aisle. However, the replenishment locations are not uniformly distributed, and we assume that a worker exits toward the end of the aisle that minimizes travel. Thus, now we must consider the storage class of the item being replenished (location 3) and the storage class of the previous replenishment (location 1).

For each of the four cases presented above for random storage, we now also include all possible combinations of storage classes for locations 1 and 3 (e.g., location 1 was a class-A item and location 3 is a class-A item or, location 1 was a class-B item and location 3 is a class-B item, etc.). After the distances for each of these scenarios have been determined, we then multiply each distance by its probability of occurrence and sum over all scenarios to
determine the total expected distance for a replenishment operation. For example, if location 1 is a class-A item and location 3 is a class-B item, the probability of occurrence is $P_{A_A} \times P_{A_B}$; this probability is then multiplied by the expected distance in traveling from a class-A replenishment location to a class-B replenishment location. The distance equations are included in online Appendix E.

Figure 16 illustrates the optimal $r$ values for a one-sided warehouse with picking areas of 100,000 ft$^2$ and 300,000 ft$^2$ for an 80/20 ABC curve. For ABC curves with different levels of skewness, there is no appreciable difference in optimal shape. The optimal shape increases slightly as $\alpha$ increases, but the shape is less elongated than for replenishments in a random storage warehouse. This is due to the reduced within-aisle travel for the replenishment operation with the one-sided, class-based storage layout. The optimal shape of the warehouse has a lower bound ($\alpha = 1/M$) of approximately 1.9 and an upper bound ($\alpha = 1$) of 3.0 for the picking areas considered. Also, the optimal shape for replenishment in the one-sided warehouse is greater than the optimal shape for the put-away operation but less than the optimal shape for order picking for an 80/20 ABC curve.

6.3. Replenishment travel for two-sided layout with class-based storage

The expected distance of the replenishment operation in a two-sided layout is also dependent on the storage class of the previous and next replenishment. However, for the two-sided layout, the storage class is no longer defined within the aisle; instead, each aisle contains a given storage class, and we assume that the reserve storage locations within the aisle are uniformly distributed. Travel across aisles, on the other hand, depends on the number of aisles in each storage class and on the class of the previous and last replenishment. Consequently, we present the distance equations according to the storage classes of locations 1 and 3, instead of ordering by the four cases defined previously. We do so because not all cases apply for a given pair of storage classes for locations 1 and 3. For example, if location 1 resides within class-A storage and location 3 resides within class-B storage, then it is not possible for all locations to reside in the same aisle.

Because the aisles are not identical in terms of the storage class (as was the case with the one-sided layout), the across-aisle travel will result in different distances, depending on the storage class of the aisle that contains the reserve storage location. For example, consider the case where location 1 is a class-A item and location 3 is a class-A item. For the case where the three locations are in three different aisles, the three aisles could all be class-A aisles, or the aisle with the reserve location could be a class-B or class-C aisle. Hence, the expected distances for these two scenarios (for the case where all locations are in different aisles) are different. The equations for the expected distance for the replenishment operation with the two-sided layout are included in online Appendix F.

Because the aisles are not identical, in terms of the activity profile, the optimal warehouse shape is now more dependent on the skewness of the ABC curve. Figure 17 displays the optimal shape for a fairly skewed ABC curve (80/20) and a hardly skewed ABC curve for picking areas of 100,000 ft$^2$ and 300,000 ft$^2$. The optimal shapes for the fairly skewed ABC curve range from approximately 3.5 to 8.6, and the optimal shapes for the hardly skewed ABC curve range from 3.0 to 4.3 for the picking areas considered. Thus, as the skewness of the ABC curve increases and as $\alpha$ increases, more elongated warehouses are preferred. Furthermore, the two-sided layout results in optimal warehouse shapes that are significantly higher compared to the one-sided layout. This is intuitive because the within-aisle travel for the two-sided layout has more of an impact on total travel than the across-aisle travel, especially for highly skewed ABC curves (resulting in significantly less across-aisle travel for the fast-moving items that span across a small number of aisles).
Table 1. Daily travel distance by operation

| Operation                  | Optimal shape, r | Distance per operation (ft) | Number of operations | Total travel distance (ft) |
|----------------------------|------------------|-----------------------------|----------------------|---------------------------|
| Put-away (1)               | 0.75             | 729.3                       | 700                  | 510 510.0                 |
| Pallet pick (1)            | 0.75             | 729.3                       | 50                   | 36 465.0                  |
| Case picking tour (12)     | 5.20             | 4698.3                      | 738                  | 3467 345.4               |
| Replenishment (19)         | 2.70             | 725.9                       | 650                  | 471 835.0                 |
| Total                      |                  | 4486 155.4                  |                      |                           |

6.4. Summary

The optimal warehouse shape for the replenishment operation results in warehouses that are generally more elongated compared with the put-away operation. For random storage warehouses with picking areas between 100 000 ft² and 300 000 ft², the optimal warehouse shape ranges from approximately 2.9 to 3.4 for increasing values of α, and the optimal shapes for the one-sided warehouse range from 1.9 to 3.0. The one-sided layout is very insensitive to the skewness of the ABC curve, as compared with the two-sided layout. As the skewness of the ABC curve increases, the optimal warehouse shape for the two-sided warehouse becomes significantly more elongated. In addition, the one-sided layout results in warehouse shapes that are less-elongated than the two-sided and random storage layouts. In the next section we demonstrate how to determine the optimal warehouse shape that considers put-away, order picking, and replenishment travel.

7. Warehouse shape example

Thus far we have presented the optimal warehouse shape for individual warehouse operations. However, the optimal warehouse shape should reflect all travel operations, so we will present an example to demonstrate how to determine the optimal warehouse shape that takes into account the horizontal travel for put-away, case-based order picking, and replenishment to the forward area.

Consider a warehouse with dock doors on one side only and with a storage area of 300 000 ft². The staging area (including the end cross aisles) is 50 feet. Items are stored according to the one-sided, class-based layout. The percent of activity and percent of storage are 65/20/15 and 20/30/50, respectively, for class-A, class-B, and class-C items. Pallet put-aways are such that 80% of the time, reserve storage locations are in the same aisle as their forward picking location. In addition, batches for order picking average 22 lines (or stops) per batch with approximately 1.2 picks per line. Incoming pallets, on average, consist of 30 cases.

An average day consists of 700 pallet put-aways, 50 pallet picks, and 19 500 case picks. With an average of 1.2 case picks per line (and 22 lines), the number of batches is approximately 738 (19 500/(22 × 1.2)). This results in 650 replenishments per day (assuming 19 500 picks and 30 cases per pallet).

Using Equation (2), the value for p is 0.25, and the optimal warehouse shape for the put-away operation is 0.75. This results in a storage area with a width of 474 feet and depth of 632 feet. Using Equation (1), the one-way distance for a put-away operation is 364.65 feet; thus, two-way travel requires 729.3 feet. The optimal shape for a pallet pick requires the same travel as for a pallet put-away, and again the optimal shape is 0.75.

The optimal warehouse shape for order picking with 22 lines per batch (using the return policy) results in an optimal warehouse shape of 5.2, with a width of 1249.0 feet and depth of 240.2 feet. Using Equation (12), the travel per batch is 4698.3 feet. For replenishment, the optimal warehouse shape for the one-sided layout with an α value of 0.8 is 2.7, and the corresponding distance is 725.9 feet using a storage area width of 900.0 feet and depth of 333.3 feet.

Table 1 displays the optimal shape for each operation and the total travel distance per day. Thus, the total distance for all operations represents a Lower Bound (LB) for the total minimum travel distance.

From Table 1, the optimal warehouse shape varies from 0.75 to 5.20 for the put-away, order picking, and replenishment operations. The optimal warehouse shape is not a linear function for all operations, and the optimal shape that considers all operations is not clear. To determine a composite warehouse shape, we plot the total travel distance of all operations for shapes ranging from 0.5 to 5.0 as shown in Fig. 18.

![Fig. 18. Total distance traveled for shapes ranging from 0.5 to 5.0.](image-url)
The minimum distance of 4689 006 feet occurs at a shape of 3.8, a difference of 202 851 feet more than the LB. From Fig. 18 it is clear that choosing an optimal warehouse shape that considers all operations can result in significant labor savings. For example, consider a company that uses a warehouse shape ratio of 2.0 as a rule-of-thumb to design its warehouses. This would result in a warehouse that is 3.9% above the optimal daily travel, resulting in $55 000 of additional labor per year (using a horizontal equipment speed of 250 fpm, $18 per hour labor rate, and 250 operational days per year).

Next we illustrate how well the optimally shaped warehouse obtained in the previous example (\( r = 3.8 \)) performs when various parameters change. Figure 19 depicts the optimal warehouse shape for the current ABC curve (where the top 20% of the items account for 65% of the pick lines, or 65/20), as well as curves where the top 20% of the items correspond to ±10% of the demand (i.e., 75/20 and 55/20). From Fig. 19, we see that as the skewness of the curve increases, the optimal shape decreases. Table 2 lists the total travel for the optimal shape for the new data (\( r^* \)) as well as the optimal shape with the original data (\( r = 3.8 \)) and then the percent difference. Thus, for this example, even though the optimal shape is sensitive to the demand curve used, the total travel distance is not (for both scenarios, the error is less than 1%). Thus, if the demand profile changed by ±10%, the solution of 3.8 would provide near-optimal travel distances for the previous example.

The probability of same-aisle replenishment (\( \alpha \)) has only a slight impact on the total travel for the previous example (\( \alpha = 0.8 \)), and the optimal warehouse shape does not change for \( \alpha \) values of 0.7 and 0.9 as shown in Fig. 20. Accordingly, the percent difference in total travel for both scenarios is zero as shown in Table 2.

We next consider the sensitivity of warehouse shape to the number of pick lines (equal to 22 lines per batch and 1.2 picks per line). Again we see that the optimal shape increases as the number of pick lines increases as indicated in Fig. 21. If the number of lines per batch decreased to 18 (and the number of picks per line increased to 1.4), the optimal shape would decrease to 3.1. Conversely, if the lines per batch increased to 26 (and the number of picks per line decreased to 1.0), the optimal shape would increase to 4.1. Nonetheless, the previously obtained shape of 3.8 results in travel that is less than 1% from the optimal for these two scenarios.

The example that we considered utilized the one-sided, class-based layout for a particular set of warehouse characteristics. A similar analysis could be performed for the two-sided, class-based storage layout.

**Table 2. Performance of the optimally shaped warehouse (\( r = 3.8 \)) as parameters change**

| Parameter                  | New value | New \( r^* \) | \( r^* \) | \( r = 3.8 \) | Percent difference |
|----------------------------|-----------|---------------|------------|---------------|-------------------|
| ABC skewness               | 55/20     | 4.4           | 5066 271   | 5073 700      | 0.15              |
|                            | 75/20     | 3.0           | 4269 165   | 4295 257      | 0.61              |
| Same-aisle replenishment, \( \alpha \) | 0.7      | 3.8           | 4718 185   | 4718 185      | 0.00              |
|                            | 0.9       | 3.8           | 4659 908   | 4659 908      | 0.00              |
| Lines, lines/batch         | 18, 1.4   | 3.1           | 4452 769   | 4465 437      | 0.28              |
|                            | 26, 1.0   | 4.1           | 5099 917   | 5103 707      | 0.07              |
8. Conclusions

A warehouse’s configuration affects the travel distances for put-away, order picking, and replenishment; thus, it is an important design consideration. We have presented new expressions for put-away, order picking, and replenishment operations for random storage and two class-based storage layouts. Our models include a uniform distribution of dock doors instead of a single P&D location that is commonly assumed in the warehousing literature. We presented structural results on the optimal shape of a warehouse under specific assumptions and graphical illustrations that could lead to useful rules of thumb for industry going forward. In terms of the overall optimization problem, we presented numerical results for the put-away, order picking, and replenishment distances in an example warehouse with a one-sided, class-based storage layout. The numerical results from our example illustrate that the optimal warehouse configuration is not consistent for all operations, and consideration must be given to the number of operations and distances associated with each operation. The total distance for all operations and dock-door configurations can be evaluated over the range of optimal shapes for individual operations to determine the optimal warehouse configuration that minimizes the total travel distance. Though we confined our results to an analysis of warehouse shape, the models presented can be used to evaluate a broad range of warehouses including varying design parameters such as the size of the forward area for random and class-based layouts, as well as the effect of activity distributions that change over time.

We believe that our research, which covers warehouses that fulfill orders at the case level by picking cases from pallet rack, provides a foundation for a more sophisticated examination of this problem. That is, additional aspects of this type of warehouse could be modeled and combined with our models to enlarge the solution space (e.g., travel time models for layouts with additional cross aisles). Also, the types of warehouses considered could be extended in the same vein. In particular, item-level picking would require a new set of order picking and replenishment performance models to be incorporated into the overall optimization framework. In addition, our optimization framework is based on enumerating over a range of warehouse shapes and dock door configurations. A more sophisticated treatment of this non-linear optimization problem may provide additional structural results of use to industry.

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Supplemental material

Supplemental data for this article can be accessed on the publisher’s website.

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