Recent Results in Neutrino Physics

José W. F. Valle

Instituto de Física Corpuscular - C.S.I.C., Departament de Física Teòrica, Universitat de València
46100 Burjassot, València, SPAIN

ABSTRACT

Present limits on neutrino masses are reviewed, along with the positive cosmological and astrophysical hints from dark matter, solar and atmospheric neutrino observations. If all these hints are due to neutrino physics, either neutrinos are closely degenerate, with a mass of about 2 eV, leading to neutrinoless $\beta\beta$ decay rate observable in the next round of experiments, or else a light sterile neutrino exists in nature. In either case the simplest seesaw scheme would be ruled out. However, one may consistently implement the quasi-degenerate neutrino scenario in extended seesaw models, e.g., based on SO(10). The light sterile neutrino possibility can be implemented in schemes with radiative mass generation, leading to the possibility of enhanced lepton flavour violating processes as well as neutrino oscillations observable at accelerators. Finally, I discuss an direct, but striking, possible manifestation of neutrino masses in the symmetry breaking sector of the electroweak theory: the invisibly decaying higgs boson. I describe how LEP data can be used to provide model independent limits on the higgs particle and also discuss the prospects for probing the associated physics at higher energies.

1 Preliminaries

No solid theoretical principle prevents neutrinos from having mass. Moreover, from the point of view of theory, it is rather mysterious that neutrinos seem to be so special when compared with the other fundamental fermions. Many attractive extensions of the standard model require neutrinos to be massive. This is the case, for example, in SO(10) or left-right symmetric theories, where the presence of right-handed neutrinos is required in order to realize the extra symmetry. On the other hand, there is, in these theories, a natural mechanism to understand the relative smallness of neutrino masses. In this case lepton number is part of the gauge symmetry and its feeble violation is related to the observed smallness of neutrino masses and to the V-A nature of the weak interactions.

This is by no means the only way to neutrino masses. Indeed, it has been realized in the early days that lepton number may be a spontaneously broken global symmetry. Since then there have been many other attractive suggestions of how to realize this idea in realistic scenarios. In this case, quite naturally, the observed smallness of neutrino masses does not require any large mass scale. The extra particles required to generate the neutrino masses have masses at scales accessible to present experiments. Such a low scale for lepton number breaking could have important implications not only in astrophysics and cosmology (e.g., electroweak baryogenesis) but also in particle physics as we will discuss below.

Whichever way one adopts, present theory is not capable, from general principles, of predicting the scale of neutrino masses any better than it can fix the masses of the other quarks and charged leptons, say the muon. One should at this point turn to experiment.

There are several limits on neutrino masses that follow from observation. The laboratory bounds may be summarized as:

$$m_{\nu_e} \lesssim 10 \text{ eV}, \quad m_{\nu_\mu} \lesssim 270 \text{ keV}, \quad m_{\nu_\tau} \lesssim 31 \text{ MeV} \quad (1)$$

1E-mail VALLE at vm.ci.uv.es or 16444::VALLE.

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Figure 1: $\beta\beta_0$ decay and Majorana neutrinos.

These limits follow purely from kinematics and have therefore the great advantage that they are the most model-independent of the neutrino mass limits. The experimental status of the limits on the $\nu_e$ mass have been extensively discussed here [6]. Note that the limit on the $\nu_\tau$ mass may be substantially improved at a tau factory [7]. In addition, there are limits on neutrino masses that follow from the nonobservation of neutrino oscillations. I address you to ref. [8] for a detailed discussion and compilation. As opposed to the limits in eq. (1) neutrino oscillation limits are correlated ones, involving neutrino mass differences versus mixing. Thus they rely on the additional assumption, although quite natural in gauge theories, that massive neutrinos do mix.

Apart from the above limits, there is an important one derived from the non-observation of the $\beta\beta_0$ nuclear decay process i.e. the process by which nucleus $(A, Z - 2)$ decays to $(A, Z) + 2 e^-$. This lepton number violating process would arise via neutrino exchange and, although highly favoured by phase space over the usual $2\nu$ mode, it proceeds only if the virtual neutrino is a Majorana particle. The decay amplitude is proportional to

$$\langle m \rangle = \sum_\alpha K_{c\alpha}^2 m_\alpha$$

(2)

where $\alpha$ runs over the light neutrinos. The non-observation of $\beta\beta_0$ in $^{76}\text{Ge}$ and other nuclei leads to the limit [9]

$$\langle m \rangle \lesssim 1 - 2 \text{ eV}$$

(3)

depending on nuclear matrix elements [10]. Even better sensitivity is expected from the upcoming enriched germanium experiments [11]. Although rather stringent, the limit in eq. (3) is rather model-dependent, and does not apply when total lepton number is an unbroken symmetry, as is the case for Dirac neutrinos. Even if all neutrinos are Majorana particles, $\langle m \rangle$ may differ substantially from the true neutrino masses $m_\alpha$ relevant for kinematical studies, since in eq. (2) the contributions of different neutrino types may interfere destructively, similarly to what happens in the simplest Dirac neutrino case, where the lepton number symmetry enforces that $\langle m \rangle$ automatically vanishes [11].

The $\beta\beta_0$ decay process may also be engendered through the exchange of scalar bosons, raising the question of which relationship the $\beta\beta_0$ decay process bears with the neutrino mass. A simple but essentially rigorous proof shows that, in a gauge theory, whatever the origin of $\beta\beta_0$ is, it requires neutrinos to be Majorana particles, as illustrated in Fig. (1). Indeed, any generic "black box" mechanism inducing neutrinoless double beta decay can be closed, by W exchange, so as to produce a diagram generating a nonzero Majorana neutrino mass, so the relevant neutrino will, at some level, be a Majorana particle [12].

Gauge theories may lead to new varieties of neutrinoless double beta decay involving the emission of light superweakly interacting spin zero particles [13]. One of these, called majoron, is the goldstone boson associated to the spontaneous violation of a global lepton number symmetry [4].

$$(A, Z - 2) \rightarrow (A, Z) + 2 e^- + J.$$
The emission of such light scalars would only be detected through their effect on the $\beta$ spectrum.

The simplest model with sizeable majoron emission in $\beta\beta$ decays involving an isotriplet majoron \[14\] leads to a new invisible decay mode for the neutral gauge boson with the emission of light scalars,

$$Z \rightarrow \rho + J,$$

now ruled out by LEP measurements of the invisible $Z$ width \[17\].

However it has been recently shown that a sizeable majoron-neutrino coupling leading to observable emission rates in neutrinoless double beta decay can be reconciled with the LEP results in models where the majoron is an isosinglet and lepton number is broken at a very low scale \[16\]. An alternative possibility was discussed in \[17\]. Recently there have been negative searches for the majoron emitting neutrinoless double beta decay by the Irvine and Heidelberg-Moscow groups which lead to a limit on the majoron-neutrino coupling of about $10^{-4}$ \[18\].

In addition to laboratory limits, there is a cosmological bound that follows from avoiding the overabundance of relic neutrinos \[19\]

$$\sum_i m_{\nu_i} \lesssim 50 \text{ eV}$$

This limit is also model-dependent, as it only holds if neutrinos are stable on cosmological time scales. There are many models where neutrinos decay into a lighter neutrino plus a majoron \[19\],

$$\nu_\tau \rightarrow \nu_\mu + J.$$ 

Lifetime estimates in seesaw type majoron models have been discussed in ref. \[20\]. Here I borrow the estimate of the model of ref. \[21\], given by curve C in Fig. \[2\]. The solid line gives the lifetime required in order to suppress the relic $\nu_\tau$ contribution. The dashed line ensures that the universe has become matter-dominated by a redshift of 1000 at the latest so that fluctuations have grown by the same factor by today \[22\]. Comparing curve C with the solid and dashed lines one sees that the theoretical lifetimes can be shorter than required. Moreover, since these decays are invisible, they are consistent with all astrophysical observations. Recently Steigman and collaborators have argued that many values of the $\nu_\tau$ mass can be excluded by cosmological big-bang nucleosynthesis, even when it decays \[23\]. This, however, still leaves open a wide region of theoretically interesting $\nu_\tau$ lifetime-mass values. It follows than that any effort to improve present neutrino mass limits is worthwhile. These include searches for distortions in the energy distribution of the electrons and muons coming from decays such as $\pi, K \rightarrow e\nu, \pi, K \rightarrow \mu\nu$, as well as kinks in nuclear $\beta$ decays \[24\].

## 2 Positive Hints for Neutrino Mass

In addition to the limits described in the previous section, observation also provides us with some positive hints for neutrino masses. These follow from cosmological, astrophysical and laboratory observations which I now discuss.

Recent observations of cosmic background temperature anisotropies on large scales by the COBE satellite, when combined with smaller scale observations (cluster-cluster correlations) indicate the need for the existence of a hot dark matter component, contributing about 30% to the total mass density, i.e. $\Omega_{HDM} \sim 0.3$ \[25, 26\]. For this the most attractive particle candidate is a massive neutrino, such as a $\nu_\tau$ of a few eV mass. This suggests the possibility of having observable $\nu_e$ to $\nu_\tau$ or $\nu_\mu$ to $\nu_\tau$ oscillations in the laboratory. The next generation of experiments CHORUS and NOMAD at CERN, and the P803 experiment proposed at Fermilab will probe this possibility \[27\].

Second, the solar neutrino data collected up to now by the two high-energy experiments Homestake and Kamiokande, as well as by the low-energy data on pp neutrinos from the GALLEX and SAGE experiments still pose a

\[4\] However, this lifetime limit is less reliable than the one derived from the critical density, as there is not yet an established theory for the formation of structure in the universe.
Figure 2: Estimated $\nu_\tau$ lifetime versus observational limits.

Persisting puzzle [28, 29]. Comparing the full data of GALLEX including their most recent ones, with the Kamiokande data, one can obtain the allowed one sigma region for $^7$Be and $^8$Be fluxes as the intersection of the region to the left of line labelled 91 with the region labelled KAMIOKA. The lines are normalized with respect to the reference solar model of Bahcall and collaborators. Including the Homestake data of course only aggravates the discrepancy [30], as can be seen from the fig xx. Thus the solar neutrino problem seems really a problem. The simplest astrophysical solutions are highly disfavored if all data are taken simultaneously, leading to the need of new physics in the neutrino sector [31]. The most attractive way to account for the data is to assume the existence of neutrino conversions involving very small neutrino masses $\sim 10^{-3}$ eV [32]. The region of parameters allowed by present experiments is illustrated in Fig. (4) [33] (for similar analyses, see ref. [34]). Note that the fits favour the non-adiabatic over the large mixing solution, due mostly to the larger reduction of the $^7$Be flux found in the former.

Finally, there are hints for neutrino masses from studies involving atmospheric neutrinos. Although the predicted absolute fluxes of neutrinos produced by cosmic-ray interactions in the atmosphere are uncertain at the 20 % level, their ratios are expected to be accurate to within 5 % [37]. An apparent decrease in the expected flux of atmospheric $\nu_\mu$’s relative to $\nu_e$’s arising from the decays of $\pi$’s and $K$’s produced in the atmosphere, and from the secondary muon decays has been observed in three underground experiments, Kamiokande, IMB and possibly Soudan2 [35]. This atmospheric neutrino deficit can be ascribed to neutrino oscillations. Combining these experimental results with observations of upward going muons made by Kamiokande, IMB and Baksan, and with the negative Frejus and NUSEX results [36] leads to the following range of neutrino oscillation parameters [37]

$$\Delta m^2_{\mu\tau} \approx 0.005 - 0.5 \text{ eV}^2, \quad \sin^2 2\theta_{\mu\tau} \approx 0.5$$

These recent analyses severely constrain the oscillation parameters, apparently excluding oscillations of $\nu_\mu$ to $\nu_\tau$ with maximal mixing, as expected in some theoretical models. However, the underlying uncertainties are still so large that it is unsafe to rule out maximal mixing with a high degree of confidence. Similar analyses have also been performed for the case of $\nu_\mu$ to $\nu_S$ as well as $\nu_\mu$ to $\nu_e$ channels, where matter effects play an important role [38].

Taken at face value, the above astrophysical and cosmological observations suggest an interesting theoretical puzzle, if one insists in accounting for all three observations on solar, dark matter and atmospheric neutrinos within a consistent theory. Indeed, it is difficult to reconcile these three observations simultaneously in the framework of the simplest seesaw model with just the three known neutrinos. The only possibility is if all three neutrinos are closely degenerate [40].
Figure 3: Allowed one sigma bands for $^7$Be and $^8$Be fluxes from all solar neutrino data.

Figure 4: Region of solar neutrino oscillation parameters allowed by experiment.
We now turn to model building. Can we reconcile the present hints from astrophysics and cosmology in the framework of a consistent elementary particle physics model?

It is known that the general seesaw models have two independent terms giving rise to the light neutrino masses. The first is an effective triplet vacuum expectation value \( [39] \) which is expected to be small in left-right symmetric models \( [3] \). Based on this fact one can in fact construct extended seesaw models where the main 2 eV or so contribution to the light neutrino masses is universal, due to a suitable horizontal symmetry, while the splittings between \( \nu_e \) and \( \nu_\mu \) explain the solar neutrino deficit and that between \( \nu_\mu \) and \( \nu_\tau \) explain the atmospheric neutrino anomaly \( [41] \).

The alternative way to fit all the data is to add a fourth neutrino species which, from the LEP data on the invisible Z width, we know must be of the sterile type, call it \( \nu_S \). Two basic schemes have been suggested in which the \( \nu_S \) either lies at the dark matter scale \( [42] \) or, alternatively, at the solar neutrino scale \( [43] \). In the first case the atmospheric neutrino puzzle is explained by \( \nu_\mu \) to \( \nu_S \) oscillations, while in the second it is explained by \( \nu_\mu \) to \( \nu_\tau \) oscillations. Correspondingly, the deficit of solar neutrinos is explained in the first case by \( \nu_e \) to \( \nu_\tau \) oscillations, while in the second it is explained by \( \nu_e \) to \( \nu_S \) oscillations. In both cases it is possible to fit all observations together. However, in the first case there is a clash with the bounds from big-bang nucleosynthesis while, in the latter case of where \( \nu_S \) is at the MSW scale these limits can be used to single out the nonadiabatic solution uniquely. Note however that, since the mixing angle characterizing the \( \nu_\mu \) to \( \nu_\tau \) oscillations is nearly maximal, the second solution is in apparent conflict with eq. \( [8] \). Another theoretical possibility is that all active neutrinos are very light but the sterile neutrino \( \nu_S \) is the single neutrino responsible for the dark matter \( [44] \).

In short, neutrino masses, besides being suggested by theory, seem to be required to fit present astrophysical and cosmological observations. The solid curves in fig. 5 show the regions of \( \nu_e \) to \( \nu_\mu \) and \( \nu_\mu \) to \( \nu_\tau \) oscillation parameters that are excluded by present accelerator and reactor experiments. The next generation of accelerator experiments at CERN may test for the existence of neutrino oscillations involving the \( \nu_\tau \). This is indicated by the dot-dashed line in figure 5. Finally, the regions suggested by present solar and atmospheric neutrino data are sketched, for comparison. Regions A and B are the allowed MSW solutions for solar neutrinos while the unlabeled regions are for atmospheric neutrinos. Similar plots can be made for the case of sterile neutrinos. Further progress will be achievable at the upcoming long
baseline experiments planned at BNL, Soudan, Icarus and Kamiokande (dashed lines). Underground experiments should also help to clarify whether or not solar neutrino conversions exist and also search for neutrinoless double beta decay with sensitivity enough to probe the quasidegenerate neutrino scenario outlined above.

In addition to neutrino oscillations, there are many other lepton flavour violating processes whose existence would be related to neutrino masses and neutrino properties beyond the standard model. These include $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$, $\mu \rightarrow e$ conversion in nuclei, $\tau \rightarrow e\pi^0$, $\tau \rightarrow e\gamma$, as well as two-body decays with the emission of a superweakly interacting majoron, e.g. $\mu \rightarrow e + J$ and $\tau \rightarrow e, \mu + J$. The underlying physics may also be probed at the high energies accessible at LEP, through related $Z$ decay processes. e.g. $Z \rightarrow N_\tau \nu_\tau$ or $Z \rightarrow \chi \tau$, where $N_\tau$ denotes a neutral heavy lepton, while $\chi$ denotes the lightest chargino. All of these processes may occur at levels consistent with present or planned experimental sensitivities, without violating any experimental data. For recent discussions see ref. [1, 45].

3 Invisible Higgs Decays

We now turn to a much less usual and less direct, but striking, possible manifestation of neutrino masses in the symmetry breaking sector of the electroweak theory.

In many models [46] neutrino masses are induced from the spontaneous violation of a global $U(1)$ lepton number symmetry by an $SU(2) \otimes U(1)$ singlet vacuum expectation value $\langle \sigma \rangle$, in such a way that $m_\nu \rightarrow 0$ as $\langle \sigma \rangle \rightarrow 0$. In contrast with the more usual seesaw majoron model [4], a low scale for the lepton number violation, close to the electroweak scale, is preferred in these models, since it is required in order to obtain small neutrino masses [46].

Another cosmological motivation for low-scale majoron models has been given in ref. [48].

In these models, although the majoron has very tiny couplings to matter, it can have significant couplings to the Higgs bosons. This implies that the Higgs boson may decay with a substantial branching ratio into the invisible mode

\[ h \rightarrow J + J \]  \hspace{1cm} (9)

where $J$ denotes the majoron. The presence of this invisible Higgs decay channel can affect the corresponding Higgs mass bounds in an important way, as well as lead to novel search strategies at higher energies.

The production and subsequent decay of any Higgs boson which may decay visibly or invisibly involves three independent parameters: the Higgs boson mass $M_H$, its coupling strength to the $Z$, normalized by that of the standard model, call this factor $e^2$, and the invisible Higgs boson decay branching ratio.

The results published by the LEP experiments on the searches for various exotic channels can be used in order to determine the regions in parameter space that are ruled out already. The procedure was described in [50, 51]. Basically it combines the results of the standard model Higgs boson searches with those one can obtain for the invisible decay. For each value of the Higgs mass, the lower bound on $e^2$ can be calculated as a function of the branching ratio $BR(H \rightarrow \text{visible})$, both this way as well as through the standard model Higgs search analyses techniques. The weakest of such bounds for $BR(H \rightarrow \text{visible})$ in the range between 0 and 1, provides the absolute bound on $e^2$. This procedure can be repeated for each value of $M_H$, thus providing an an exclusion contour in the plane $e^2$ vs. $M_H$, shown in Fig. (6), taken from ref. [50]. The region in $e^2$ vs. $M_H$ that is already excluded by the present LEP analyses holds irrespective of the mode of Higgs decay, visible or invisible. Finally, one can also determine the additional range of parameters that can be covered by LEP2 for a total integrated luminosity of 500 pb$^{-1}$ and centre-of-mass energies of 175 GeV and 190 GeV. This is shown as the dashed and dotted curves in Fig. (7).

The possibility of invisible Higgs decay is also very interesting from the point of view of a linear $e^+e^-$ collider at higher energy [52]. Heavier, intermediate-mass, invisibly decaying Higgs bosons can also be searched at high energy.

5 Another example is provided by the RPSUSY models [47].
Figure 6: Region in the $\epsilon^2$ vs. $m_H$ that can be excluded by the present LEP1 analyses, independent of the mode of Higgs decay, visible or invisible (solid curve). Also shown are the LEP2 extrapolations (dashed).

hadron supercolliders such as LHC/SSC [53]. The limits from LEP discussed above should serve as useful guidance for such future searches.

4 Conclusion

Present cosmological and astrophysical observations, as well as theory, suggest that neutrinos may be massive. Neutrino masses might even affect the electroweak symmetry breaking sector in a very important way.

Existing data do not preclude neutrinos from being responsible for a wide variety of measurable implications at the laboratory. These new phenomena would cover an impressive region of energy, from $\beta$ and double $\beta$ decays, to neutrino oscillations, to rare processes with lepton flavour violation, up to LEP energies. The next generation of neutrino oscillation searches sensitive to $\nu_\tau$ as dark matter (CHORUS/NOMAD/P803), $e^+e^-$ collisions from ARGUS/CLEO to tau-charm and B factories, as well as the experiments at LEP and the future LHC could all be sensitive to neutrino properties!

It is therefore worthwhile to keep pushing the underground experiments, for possible confirmation of neutrino masses. The neutrinoless $\beta\beta$ decay searches with enriched germanium could test the quasidegenerate neutrino scenario for the joint explanation of hot dark matter and solar and atmospheric neutrino anomalies. Further data from low energy pp neutrinos as well as from Superkamiokande, Borexino, and Sudbury will shed further light on the neutrino sector. The same can be said of the ongoing studies with atmospheric neutrinos.

Similarly, a new generation of experiments capable of more accurately measuring the cosmological temperature anisotropies at smaller angular scales than COBE, would be good probes of different models of structure formation, and presumably shed further light on the need for hot neutrino dark matter. All such endeavours should be gratifying!

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References
1. For a recent review see J. W. F. Valle, *Gauge Theories and the Physics of Neutrino Mass*, *Prog. Part. Nucl. Phys.* **26** (1991) 91-171 and references therein.

2. M Gell-Mann, P Ramond, R. Slansky, in *Supergravity*, ed. D. Freedman et al. (1979); T. Yanagida, in *KEK lectures*, ed. O. Sawada et al. (1979); R.N. Mohapatra and G. Senjanovic, *Phys. Rev.* **D23** (1981) 165; *Phys. Rev. Lett.* **44** (1980) 912

3. Y. Chikashige, R. Mohapatra, R. Peccei, *Phys. Rev. Lett.* **45** (1980) 1926

4. Particle Data Group, *Phys. Rev. D45* (1992) S1; J F Wilkerson, invited talk at XVI Int. Conference on Neutrino Physics and Astrophysics, *Nucl. Phys. B (Proc. Suppl.)* **31** (1993) 32.

5. J. Gomez-Cadenas, M. C. Gonzalez-Garcia, *Phys. Rev. D39* (1989) 1370; J. Gomez-Cadenas et al., *Phys. Rev. D41* (1990) 2179; and SLAC-PUB-5009, March 1990

6. J Schneps, *Neutrino Oscillations, past, present and future*, invited talk at XVI Int. Conference on Neutrino Physics and Astrophysics, *Nucl. Phys. B (Proc. Suppl.)* **31** (1993) 307.

7. F. Avignone, I. Kirpichnikov and H. V. Klapdor, Heidelberg-Moscow and IGEX collaboration talks at XVI Int. Conference on Neutrino Physics and Astrophysics, *Nucl. Phys. B (Proc. Suppl.)* **31** (1993) 72-79;

8. W. Haxton, talk at XVI Int. Conference on Neutrino Physics and Astrophysics, *Nucl. Phys. B (Proc. Suppl.)* **31** (1993) 88.

9. J. W. F. Valle, *Phys. Rev. D27* (1983) 1672 and references therein; L. Wolfenstein, *Nucl. Phys. B186* (1981) 147

10. J. Schechter and J. W. F. Valle, *Phys. Rev. D25* (1982) 2951

11. H. Georgi, S. Glashow, and S. Nussinov, *Phys. Lett. B193* (1981) 297

12. G. Gelmini and M. Roncadelli, *Phys. Lett. B99* (1981) 411

13. J. Steinberger, in *Electroweak Physics Beyond the Standard Model*, ed. J. W. F. Valle and J. Velasco (World Scientific, Singapore, 1992), p. 3.

14. Z. Berezhiani, A. Smirnov, and J. W. F. Valle, *Phys. Lett. B291* (1992) 99

15. C. P. Burgess and J. M. Cline, *Phys. Lett. B298* (1993) 141.

16. M. Beck et al., *Phys. Rev. Lett.* **70** (1993) 2853, A. Morales, H. V. Klapdor, E. Fiorini and M. Moe, private communication

17. G. Gerstein, Ya. B. Zeldovich, *Z. Eksp. Teor. Fiz. Pisma. Red.* **4** (1966) 174; R. Cowsik, J. McClelland, *Phys. Rev. Lett.* **29** (1972) 669; D. Dicus, et al., *Astrophys. J.* **221** (1978) 327; P Pal, *Nucl. Phys. B227* (1983) 237; E. Kolb, M. Turner, *The Early Universe*, Addison-Wesley, 1990.

18. J. W. F. Valle, *Phys. Lett. B131* (1983) 87; G. Gelmini, J. W. F. Valle, *Phys. Lett. B142* (1984) 181; M. C. Gonzalez-Garcia, J. W. F. Valle, *Phys. Lett. B216* (1989) 360. A. Joshipura, S. Rindani, PRL-TH/92-10.

19. P. Nogueira, J. C. Romão, J. W. F. Valle, *Phys. Lett. B251* (1990) 142; R. Barbieri, L. Hall, *Phys. Lett. B238* (1990) 86.
22. G. Steigman, M Turner, *Nucl. Phys.* B253 (1985) 375.
23. G. Steigman, private communication, H. S. Kang et al, preprint OSU-TA-5/93.
24. See, e.g. J Deutsch et al *Nucl. Phys.* A518 (1990) 149; *Particle World* 2 (1991) 81; A. Hime, *Nucl. Phys. B (Proc. Suppl.*) 31 (1993) 50.
25. G. F. Smoot et al., *Astrophys. J.* 396 (1992) L1-L5.
26. E.L. Wright et al., *Astrophys. J.* 396 (1992) L13; M. Davis, F.J. Summers, and D. Schagel, *Nature* 359 (1992) 393; A.N. Taylor and M. Rowan-Robinson, *ibid.* 359 (1992) 396; R.K. Schaefer and Q. Shafi, *Nature* 359 (1992) 199; J.A. Holtzman and J.R. Primack, *Astrophys. J.* 405 (1993) 428; A. Klypin et al., *Astrophys. J.* 416 (1993) 1.
27. CHORUS collaboration, preprint CERN-SPSC/90-42 (1992). NOMAD collaboration, preprint CERN-SPSLC/91-21 (1992). K. Kodama et al., FNAL preprint proposal P803 (1991).
28. J. R. Davis in *Proceedings of the 21th International Cosmic Ray Conference*, Vol. 12, ed. R. J. Protheroe (University of Adelaide Press, 1990) p. 293.
29. T. Kirsten, proceedings of the Erice School of Nuclear Physics, *Prog. Part. Nucl. Phys.* 32 (1994) ; GALLEX collaboration, *Phys. Lett.* B285 (1992) 376, *ibid.* B285 (1992) 390; *Phys. Lett.* B314 (1993) 445, preprint GX 44-1994.
30. A. Yu. Smirnov, private communication.
31. J. Bahcall, H. Bethe, *Phys. Rev.* D47 (1993) 1298, *Phys. Rev. Lett.* 65 (1990) 2233; V. Berezinsky, LNGS-93/86; S. Bludman, N. Hata, P. Langacker, UPR-0552-T X Shi, D Schramm, FERMILAB-PUB-92-322-A
32. M. Mikheyev, A. Smirnov, *Sov. J. Nucl. Phys.* 42 (1986) 913; L. Wolfenstein, *Phys. Rev.* D17 (1978) 2369; *ibid.* D20 (1979) 2634.
33. N. Hata, P. Langacker, preprint UPR-0592-T. This paper contains an extensive list of references.
34. S. Bludman, N. Hata, D. C.Kennedy, P. Langacker, *Phys. Rev.* D47 (1993) 2220; X. Shi, D. Schramm, J. Bahcall, *Phys. Rev.* 69 (1992) 717.
35. Kamiokande collaboration, *Phys. Lett.* B205 (1988) 416, *Phys. Lett.* B280 (1992) 146 and *Phys. Lett.* B283 (1992) 446 ; IMB collaboration, *Phys. Rev.* D46 (1992) 3720
36. M.M. Boliev et al. in Proceedings of the 3rd International Workshop on Neutrino Telescopes, Venice, Italy, 1991, edited by M. Baldo-Ceolin (INFN, Padova, 1991), p. 235; Ch. Berger et al., *Phys. Lett.* B (192) 45, 305, 1990; 227, 489 (1989); M. Aglietta et al., *Journal Europhys. Lett.* 15 (1991) 559.
37. See proceedings of *Int. Workshop on $\nu_\mu/\nu_e$ problem in atmospheric neutrinos* ed. V. Berezinsky and G Fiorentini, Gran Sasso, 1993.
38. E. Akhmedov, P. Lipari, and M. Lusignoli; *Phys. Lett.* B300 (1993) 128.
39. J. Schechter and J. W. F. Valle, *Phys. Rev.* D22 (1980) 2227; *Phys. Rev.* D25 (1982) 774
40. D.O. Caldwell and R.N. Mohapatra, *Phys. Rev.* D48 (1993) 3259; S. T. Petcov, A. Smirnov, preprint SISSA 113/93/EP; R. Mohapatra, talk at ICNAPP, Bangalore, India, January 1994
41. A. Ioannissyan, J.W.F. Valle, Valencia report FTUV/94-08, February 1994; D.O. Caldwell and R.N. Mohapatra, preprint UCSB-HEP-94-03, February 1994; B. Bamert, C.P. Burgess, preprint McGill-94/07, February 1994; A. S. Joshipura, preprint PRL-TH/93/20, December 1993; D.
Caldwell and R. N. Mohapatra, Maryland report, UMD-PP-94-90 (1994); D. G. Lee and R. N. Mohapatra, Maryland Report, UMD-PP-94-95 (1994).

42. J. T. Peltoniemi, D. Tommasini, and J. W. F. Valle, Phys. Lett. B298 (1993) 383
43. J. T. Peltoniemi, and J. W. F. Valle, Nucl. Phys. B406 (1993) 409;
   for another scheme, see E. Akhmedov, Z. Berezhiani, G. Senjanovic and Z. Tao, Phys. Rev. D47 (1993) 3245.
44. J. T. Peltoniemi, Mod. Phys. Lett. A38 (1993) 3593
45. P Depommier, proc. of the Second Tallin Workshop on Neutrino Physics, Estonia, ed. I. Ots, L. Paulgi, p. 148.
46. A. Joshipura and J. W. F. Valle, Nucl. Phys. B397 (1993) 105
47. J. C. Romao, F. de Campos, and J. W. F. Valle, Phys. Lett. B292 (1992) 329.
48. E. Akhmedov, Z. Berezhiani, R. Mohapatra, G. Senjanovic, Phys. Lett. B299 (1993) 90.
49. A. S. Joshipura, S. Rindani, Phys. Rev. Lett. 69 (1992) 3269; R. Barbieri, and L. Hall, Nucl. Phys. B364, 27 (1991).
   G. Jungman and M. Luty, Nucl. Phys. B361, 24 (1991).
   E. D. Carlson and L. B. Hall, Phys. Rev. D40, 3187 (1989)
50. A. Lopez-Fernandez, J. Romao, F. de Campos and J. W. F. Valle, Phys. Lett. B312 (1993) 240.
51. B. Brahmachari et al, Phys. Rev. D48 (1993) 4224.
52. O. Eboli, et al. Valencia report FTUV/93-50; Nucl. Phys. B (1994), in press; see also proc. of workshop on \(e^+e^-\) collisions at 500 GeV, the physics potential, ed. P. Zerwas et al, FTUV/93-42
53. J. D. Bjorken, SLAC Report, SLAC-PUB-5673 (1991); J. W. F. Valle, Physics at New Accelerators, Looking Beyond the Standard Model, invited talk at XV Int. Conference on Neutrino Physics and Astrophysics, Nucl. Phys. B (Proc. Suppl.) 31 (1993) 221-232; S. Frederiksen, N. Johnson, G. Kane, and J. Reid, SSCL-preprint-577 (1992); J. C. Romao, F. de Campos, L. Diaz-Cruz, and J. W. F. Valle, Mod. Phys. Lett. A (1994), in press; J. Gunion, Phys. Rev. Lett. 72 (1994) 199; D. Choudhury, D. P. Roy, TIFR/TH/93-64, to appear in Phys. Lett. B (1994).
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