Kaon to Pion Ratio in SU(3) PNJL Model

A. Friesen*, Yu. L. Kalinovsky*,b, and V. D. Toneev*

aJoint Institute for Nuclear Research, Dubna, Russia
bState University “Dubna”, Dubna, Russia
*e-mail: avfriesen@theor.jinr.ru

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Abstract—The article is devoted to investigation of the sharp peak in the $K^+/\pi^+$ ratio, known as a “horn”. We used the SU(3) PNJL model to show how the splitting of kaon mass can appear in dense matter and how it can affect the kaon to pion ratio. In the model we calculated the $K/\pi$ ratio of both positive and negative charged mesons, choosing temperature and baryonic chemical potential along the chiral phase transition line to show how matter properties can affect to the mesons ratio.

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1. INTRODUCTION

The study of matter formed during the heavy ion collision at high energies is on top of interest of high energy physics. Great interest still belongs to the search of the critical end point and phase transition in the hot and dense matter. Search for quark–gluon plasma (QGP) where hadrons dissolve into interacting gluons and quarks is difficult due to the short lifetime of the QGP phase. It is needed to find sensible probes for the transition to the QGP phase. A number of probes was proposed which could be a signal of the QGP formation: elliptic flows, jet quenching, quarkonia production and strangeness enhancement [1].

One of the possible signals is supposed to be a structure in the ratio of the positive charged kaon to the positive charged pion named “horn”.

First, a sharp peak in the $K^+/\pi^+$ ratio was presented by the NA49 Collaboration [2, 3] and later the picture was extended with the results of other collaborations (STAR, AGS) [4–7]. Now it is clear that the peak appears in the ratio of positive charged mesons in heavy-ion collisions when the center-of-mass energy is around 10 GeV. The $K^-/\pi^-$ ratio shows a continuous rise without a peak. The $K^+/\pi^-$ ratio is also smooth in the proton-proton collision. The recent results of the NA61/SHINE collaboration showed a strong dependence of the $K^+/\pi^+$ ratio on the size of the system and smoothing in the ratio for a light nucleus (Be+Be and Ar+Sc collisions) [8].

The strangeness enhancement in dense matter can be explained from the Pauli principle: high concentration of light quarks leads to that the Fermi energy for light quarks becomes higher than the $s\bar{s}$ mass and the creation of the last one is energy-efficient. Therefore, when in the heavy ion collision the quark-gluon plasma is created, the enhancement of the strangeness can be expected in comparison with the $p–p$ or $p–N$ collision.

The first good theoretical reproduction of the peak was obtained in the statistical model [9]. It was predicted in the SMES (the Statistical Model of Early Stages) that a jump in the ratio can appear as a result of the deconfinement transition. It was shown that after the deconfinement transition in the strange sector ($m_s$) the strangeness yield does not depend on energy. As a result, the $K/\pi$ ratio demonstrates a smooth behavior after jumping down [10, 11]. The microscopic transport model reproduced the experimental results after involving the partial restoration of the chiral symmetry at the early stages of the collision [12–14]. The authors showed the partial chiral symmetry restoration to be responsible for a quick increase in the $K^+/\pi^+$ ratio at low energies. They explained the decrease in the ratio after the maxima as a result of the chiral condensate destruction and QGP formation. Both SMES and PHSD predict a smoothing of the said ratio with decreasing system size for collision energies $\sqrt{s_{NN}} = 6–10$ GeV [11, 13, 15].

The present paper discusses of the phase transition and the strange matter in the frame of the SU(3) Nambu–Jona-Lasinio model with the Polyakov loop. The SU(3) PNJL model is capable of describing both the chiral phase transition and deconfinement transition [16–18]. The model describes the quarks developing quasipartical masses by propagating in the chiral condensate and are coupled at that time to a homogeneous background field presenting Polyakov loop...
dynamics. The chiral symmetry restores when the dynamically generated quark masses drop as a function of temperature and chemical potentials. Near this critical temperature, the binding energy for the pseudo-scalar meson bound states decreases and mesons dissociate changing their character to resonances with a finite lifetime (the Mott effect). At low density mesons are degenerated in the PNJL model with a standard parameter set. With increasing density splitting between kaons and antikaons occurs (as well for pions in the case when $u$, $d$-quark masses are not equal) [19–21]. This could explain the difference in the ratios of positive and negative charged mesons.

This paper is organized as follows: in Section 2, the formalism of the PNJL model and phase diagram structure will be discussed. The behavior of mesons and quarks at zero and finite density will be discussed in Section 3. In Section 4, the obtained results will be discussed.

2. SU(3) PNJL MODEL
AND PHASE DIAGRAM

The PNJL model is a practical tool for description of the nuclear matter at finite temperature and chemical potential. The Lagrangian of the model with three flavours and $U(1)$ anomaly has the form [16–18]:

$$
\mathcal{L} = \overline{q}(i\gamma^\mu D_\mu - \hat{m} - \gamma_5\mu)q + \frac{1}{2}g_5 \sum_{ab} \left[ (\overline{q}\gamma^\mu \lambda^a q)^2 + (\overline{q}\gamma^\mu \lambda^a \gamma^5 q)^2 \right] + g_D \{\det[\overline{q}(1 + \gamma_5)q] + \det[\overline{q}(1 - \gamma_5)q]\} - \mathcal{U}(\Phi, \overline{\Phi}; T),
$$

where $q = (u, d, s)$ is the quark field with three flavours, $N_f = 3$, and three colors, $N_c = 3$, $\lambda^a$ are the Gell–Mann matrices, $\lambda^0 = \frac{1}{2}I_3$ and $D_\mu = \partial_\mu - iA^\mu$, where $A^\mu$ is the gauge field with $A^0 = -iA_4$ and $A^\mu(x) = g_5 A_{\mu}^0 \frac{\lambda^a}{2}$ absorbs the strong interaction coupling. The global SU(3) $\otimes$ SU(3) chiral symmetry of the Lagrangian is obviously broken by introduction of the nonzero current quark masses $\hat{m} = \text{diag}(m_u, m_d, m_s)$, and the confinement/deconfinement properties ($Z_3$-symmetry) are described by the effective potential $\mathcal{U}(\Phi, \overline{\Phi}; T)$, that is constructed on the basis of the Lattice inputs in the pure gauge sector (see for details [18, 22]).

$$
\frac{\mathcal{U}(\Phi, \overline{\Phi}; T)}{T^4} = -\frac{b_2(T)}{2} \Phi \overline{\Phi} - \frac{b_6}{6} (\Phi^3 + \overline{\Phi}^3) + \frac{b_4}{4} (\Phi \overline{\Phi})^2,
$$

$$
b_2(T) = a_0 + a_1 \left( \frac{T}{T_c} \right) + a_2 \left( \frac{T}{T_c} \right)^2 + a_3 \left( \frac{T}{T_c} \right)^3,
$$

For the effective potential the following parameters were chosen: $T_0 = 0.19$ GeV, $a_0 = 6.75$, $a_1 = -1.95$, $a_2 = 2.625$, $a_3 = -7.44$, $b_3 = 0.75$, $b_4 = 7.5$ [23].

The grand potential density for the PNJL model in the mean-field approximation can be obtained from the Lagrangian density Eq. (1):

$$
\Omega = U(\Phi, \overline{\Phi}; T) + g_s \sum_{i=uds} \langle \overline{q} q_i \rangle^2 + 4g_D \langle \overline{q}_d q_d \rangle \langle \overline{q}_u q_u \rangle - 2N_c \sum_{i=ud,s} \int \frac{d^3p}{(2\pi)^3} E_i + 2T \sum_{i=ud,s} \int \frac{d^3p}{(2\pi)^3} (N_\Phi^+(E_i) + N_\Phi^-(E_i))
$$

with the functions

$$
N_\Phi^+(E_i) = \text{Tr}_c \left[ \ln(1 + L_i e^{-\beta (E_i - \mu)}) \right],
$$

$$
N_\Phi^-(E_i) = \text{Tr}_c \left[ \ln(1 + L_i e^{-\beta (E_i + \mu)}) \right],
$$

where $E_i^+ = E_i + \mu$, $\beta = \frac{1}{T}$, $E_i = \sqrt{p^2 + m_i^2}$ is the energy of quarks and $\langle \overline{q} q_i \rangle$ is the quark condensate.

To obtain equations for the order parameters, one needs to minimize the grand potential over the parameters

$$
\frac{\partial \Omega}{\partial \langle \overline{q} q_i \rangle} = 0, \quad \frac{\partial \Omega}{\partial \Phi} = 0, \quad \frac{\partial \Omega}{\partial \overline{\Phi}} = 0.
$$

The gap equation for quarks depends on the quark condensates:

$$
m_i = m_{ij} - 2g_s \langle \overline{q} q_j \rangle - 2g_D \langle \overline{q} q_j \rangle \langle \overline{q} q_k \rangle,
$$

where $i, j, k = u, d, s$ are chosen in cyclic order, $m_i$ are the constituent quark masses and the quark condensates are given by:

$$
\langle \overline{q} q_i \rangle = i \int \frac{dp}{(2\pi)^3} \text{Tr}_S(S(p_i)) = -2N_c \int \frac{d^3p}{(2\pi)^3} m_i \left( 1 - f_\Phi(E_i) - \frac{\beta}{E_i} \right),
$$

with the modified Fermi functions $f_\Phi(E_i)$:

$$
f_\Phi(E_i - \mu_i) = \frac{\Phi e^{-\beta (E_i - \mu_i)} + 2\Phi e^{-3\beta (E_i - \mu_i)} + e^{-3\beta (E_i - \mu_i)}}{1 + 3 \left( \Phi + \Phi e^{-3\beta (E_i - \mu_i)} e^{-\beta (E_i - \mu_i)} + e^{-3\beta (E_i - \mu_i)} \right)}.
$$

$$
f_\Phi(E_i + \mu_i) = \frac{\Phi e^{-\beta (E_i + \mu_i)} + 2\Phi e^{-\beta (E_i + \mu_i)} + e^{-3\beta (E_i + \mu_i)}}{1 + 3 \left( \Phi + \Phi e^{-\beta (E_i + \mu_i)} e^{-\beta (E_i + \mu_i)} + e^{-3\beta (E_i + \mu_i)} \right)},
$$

where $E_i = \sqrt{p^2 + m_i^2}$ is quark energy. So we take the temperature $T$, and the chemical potential of the i-quark ($\mu_i$) as independent state variables.
The model has a set of parameters fixed at zero T and \( \mu_B \): the current quark masses \( m_{u_d} \), \( m_0 = 5.5 \) MeV, \( m_0 = 0.131 \) GeV, the cut-off \( \Lambda = 0.652 \) GeV, couplings \( g_B = 89.9 \) GeV\(^{-2} \) and \( g_s = 4.3 \) GeV\(^{-2} \).

To use SU(3) NJL-like models, the strange quark chemical potential has to be defined. As a rule, the chemical potential of the strange quark is supposed to be zero \( \mu_s = 0 \) GeV. It is obvious, that this case is not effective for the K/\( \pi \) calculations. We consider the isospin symmetric matter with \( \mu_u = \mu_d \) and choose the strange quark chemical potential as \( \mu_s = \mu_u \) and \( \mu_s = 0.5 \mu_u \) (Case 1).

The phase diagram in the PNJL model is in agreement with the common picture of the QCD phase diagram (see Fig. 1, left panel): at high temperature and low density (chemical potential) the phase transition is soft (the chiral crossover). The crossover phase transition is defined as a local maximum of \( \chi_q = \partial^2 / \partial \mu^2 |_{T=\text{const}} \), which has a singularity at the critical end point (CEP) [24, 22]. We can see from Fig. 1 that the first order transition region ends in the vicinity of CEP (\( \mu_B,\text{CEP} = 0.993, T_{\text{CEP}} = 0.1 \)) in the case with \( \mu_s = \mu_u \) and CEP (\( \mu_B,\text{CEP} = 0.972, T_{\text{CEP}} = 0.11 \)) for the \( \mu_s = 0.5 \mu_u \). On the right panel of Fig. 1, the order parameters of the model are shown. The chiral symmetry restoration is characterized by melting the chiral condensate \( \langle \bar{q}q \rangle \rightarrow 0, m_q \rightarrow m_0 \). The PNJL model is characterized by the absence of the chiral symmetry restoration for a strange quark (when the quark mass (condensate) stays large up to relatively high temperatures), see, for example, [25].

Generally speaking, none of the ways described above can reproduce the medium in real heavy ion collisions. To match the experimental condition, the strangeness chemical potential is usually fixed to enforce the strangeness neutrality. For the PNJL model the strangeness neutrality can be introduced by an additional condition [26]:

\[
\frac{\partial \Omega}{\partial \mu_s} = 0, \quad \text{(10)}
\]

such constraint leads to \( \mu_s \) being the function of T and \( \mu_u \). The phase diagram and the value of the strange quark chemical potential \( \mu_s \) as a function of T and \( \mu_u \) are shown in Fig. 1. As can be seen in the right panel of Fig. 2, the value of the strange quark chemical potential has some "hill"-like structure as a function of temperature. The height of the "hill" depends on the value of \( \mu_u \), reaches maximum at \( \mu_u = 0.24 \) GeV and then becomes lower [26]. The introduction of the strangeness neutrality condition has no strong effect on the phase diagram structure: it is almost the same as for Case 1 (left panel of Fig. 2).

It can be seen from Figs. 1 and 2, that the critical temperature of the crossover transition at \( \mu_B = 0 \) GeV is higher in PNJL model than it was predicted by the Lattice QCD: \( T_c = 0.218 \) GeV vs. \( T_c = 154(9) \) GeV [27]. It was supposed that the reason of high \( T_c \) in the PNJL is the weak entanglement between gluons and quarks given only by \( D_\mu = \partial^\mu - iA^\mu \) [28, 29]. As the PNJL model includes extended \( Z_3 \)-symmetry [28], the reduction of the coupling constant \( g_5 \) could be the way of modeling of the \( Z_3 \)-symmetry breaking. It is possible to introduce a phenomenological dependence of \( g_5 \) on the Polyakov loop to increase the quark-gluon coupling:

\[
\tilde{g}_5(\Phi) = g_5(1 - \alpha_i \Phi \bar{\Phi} - \alpha_2 (\Phi^3 + \bar{\Phi}^3)), \quad \text{(11)}
\]

where the parameters \( \alpha_i = 0.2 \) and \( T_0 = 0.19 \) GeV are chosen to reproduce the LQCD data.
Such rescaling of the $g_S$ leads to rescaling of the critical temperature in the low-density region [28, 30], see Fig. 3.

The critical end point in the PNJL model appears at temperature much lower than in Lattice and at that time the $\mu_{CEP}$ is much higher. As a matter a fact, the first order transition region obtained in the PNJL model is smaller than it is expected to be. To explain it, we again refer to the parameter scheme of the model: the possible explanation of this fact is that the six-quark coupling constant $g_D$ is underestimated in the PNJL model. The coupling $g_D$ is fixed from the estimation of the $\eta^-$-meson mass which is higher than the cutoff parameter $\Lambda$ in the model [31]. On the other hand, the value of $g_D$ in medium has to differ from $g_D$ fixed in vacuum to move the $\mu_B$, where the first order transition occurs to lower values [25, 31, 32]. Following [25], we introduce $g_D = g_{D_0} \exp \left( \frac{\rho - \rho_0}{\rho_D} \right)^2$ where $\rho$ is the baryon density $\rho = (\rho_u + \rho_d + \rho_s) / 3$ with

$$\rho_i = 2 N_c \int \frac{d^3 p}{(2\pi)^3} \left( f^+_\phi(E_i) - f^-_\phi(E_i) \right). \quad (12)$$

In Fig. 3, left panel, all these Cases are shown: Case 1 with $\mu_s = 0.5 \mu_u$, the model with entanglement $g_S(\Phi)$ (Case 2), Case 3 where $g_S(\Phi)$ (11) and $g_D(\rho)$ (12) were combined. Cases 1–3 were considered with constraint $\mu_s = 0.5 \mu_u$. The right panel of Fig. 3 demonstrates the normalized condensates for light and strange quarks. Due to the entanglement of the quark-gluon coupling in Case 2, the phase transition temperature at the zero chemical potential significantly decreases. For Case 3, the phase diagram is more similar to the predicted by Lattice, than Case 1 and Case 2.

### 3. MESONS IN DENSE MATTER

The meson masses are defined by the Bethe–Salpeter equation at $P = 0$

$$1 - P_\pi \Pi_\pi(P_0 = M, P = 0) = 0, \quad (13)$$

where for non-diagonal pseudo-scalar mesons $\pi, K$:

$$P_\pi = g_S + g_D \langle \bar{q} \rho_s \rangle, \quad (14)$$

$$P_K = g_S + g_D \langle \bar{q} \rho_u \rangle. \quad (15)$$
and the polarization operator has the form

\[ \Pi^\mu_0 \left( P_0 \right) = 4 \left( \left( I^+_i + I_i^+ \right) - \left( P_0^2 - (m_0 - m_i)^2 \right) I^0_i \left( P_0 \right) \right). \]  

(16)

Integrals \( I_i^+ \) and \( I_i^0 \) are defined as:

\[ I_i^+ = i N_e \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 - m_i^2} = \frac{N_e}{4\pi^2} \Lambda \int_0^\Lambda p^2 dp, \]  

(17)

\[ I_i^0 \left( P_0 \right) = i N_e \int \frac{d^4 p}{(2\pi)^4} \frac{1}{(p^2 - m_0^2)(\left( (p^2 + P_0^2)^2 - m_i^2 \right)} . \]

When the meson mass exceeds the mass of its constituent \( P_0 > m_0 + m_i \), the meson turns into the resonance state and Mott transition occurs. In this case, the complex properties of the integrals have to be taken into account and the solution has to be defined in the form \( P_0 = M_M - \frac{1}{2} i \Gamma_M \). Equation (13) splits into two equations from which the meson mass \( M_M \) and the meson width \( \Gamma_M \) can be obtained. The mass spectra for the zero chemical potential \( (\mu_u = 0) \) are shown in Fig. 4, left panel. It is clear that charged multiplets are degenerated and scalar mesons in the PNJL model decay at close temperatures \((T_{\text{Mott}}^\pi = 0.232 \text{ GeV}, T_{\text{Mott}}^K = 0.23 \text{ GeV})\).

Though at the zero chemical potential the pions as well as the kaons are degenerated, their masses split with increasing density \([21, 33]\). At the nonzero chemical potential and low \( T \), the splitting of mass in charged multiplets is due to excitation of the Dirac sea modified by the presence of the medium (see Fig. 4, right panel). In dense baryon matter the concentration of light quarks is very high \([34]\). Therefore, the creation of a \( \bar{s}s \) pair dominates because of the Pauli principle: when the Fermi energy for light quarks is higher than the \( \bar{s}s \)-mass, the creation of the last one is energy-efficient. The increase in the \( K^+ \) (\( \bar{u}u \)) mass, with respect to that of \( K^- \) (\( \bar{d}d \)), is justified again by the Pauli blocking for s-quark (see for discussion \([19, 33]\)).

Technically, to describe the mesons in dense matter, it is needed to relate the chemical potential of quarks with the Fermi momentum \( \lambda_i \), \( \mu_i = \sqrt{\lambda_i^2 + m_i^2} \). The pions for the chosen cases are degenerate as the masses of light quarks are equal \((m_s = m_u)\).

All experimental data for the \( K/\pi \) ratio were obtained in midrapidity. For the effective models, it means that the ratio of the particle number can be calculated in terms of the ratio of the number densities of mesons \((K^+/\pi^+ = n_{K^+}/n_{\pi^+})\):

\[ n_{K^+} = \int_0^\infty \frac{p^2 dp}{e^{\sqrt{p^2 + m_{K^+}^2} / T} - 1}, \]  

(18)

\[ n_{\pi^+} = \int_0^\infty \frac{p^2 dp}{e^{\sqrt{p^2 + m_{\pi^+}^2} / T} - 1}. \]  

(19)

The chemical potential for pions is a phenomenological parameter and in this work it was chosen as a constant \( \mu_\pi = 0.135 \text{ GeV} \), following the works \([35–37]\). The chemical potential for kaons can be defined (see, for example, \([37, 38]\)) from \( \mu_q = B_q \mu_u + S_q \mu_s + I_q \mu_s \), and in the isospin symmetry case \((I_q = 0)\), the result is \( \mu_K = \mu_u - \mu_s \).

We should make some remarks to our calculations. First is that the PNJL model does not describe the dynamics of the collision of heavy ions and it can be said that it works till freeze-out. It makes impossible to take into account mesons created at the latest stage of collision appearing in the experiment; so the effective model results pretend only for qualitative reproduction of the data. The second is that in the effective model the collision energy \( E_{\text{coll}} \) never appears as a parameter. To avoid this, we used the fact that in the statistical model for each experimental energy of collision the temperature and the baryon chemical poten-
The potential of freeze-out can be found using parametrization suggested, for example, by J. Cleymans [39]:

\[ T(\mu_B) = a - b\mu_B^2 - c\mu_B^4, \quad (20) \]
\[ \mu_B(\sqrt{s}) = \frac{d}{1 + e\sqrt{s}}, \quad (21) \]

where \( a = 0.166 \pm 0.002 \text{ GeV}, b = 0.139 \pm 0.016 \text{ GeV}^{-1}, \) and \( c = 0.053 \pm 0.021 \text{ GeV}^{-3}, d = 1.308 \pm 0.028 \text{ GeV}, e = 0.273 \pm 0.008 \text{ GeV}^{-1}. \)

Using this fact, the \( K/\pi \) ratio can be considered as a function of a new variable \( T/\mu_B \) instead of \( \sqrt{s_{NN}} \), where \( (T, \mu_B) \) are taken along the freeze-out line using the parametrization (21). The result of rescaling is shown in Fig. 5, right panel. The third assumption was to decide which of two temperatures would correspond to the temperature of freeze-out in the PNJL-approach: the temperature of the chiral phase transition or the Mott temperature. It depends on the definition of the chemical freeze-out: if the freeze-out line on the phase diagram can be considered as a line which divides the phase where hadrons are still surviving and the phase without hadron degrees of freedom. Most likely, in the mean field approach the chiral phase transition line can play the role of freeze-out as in the PNJL model the chiral phase transition line divides the hadron phase and the quark-gluon phase.

The calculated ratios \( n_{K^+/K^-}/n_s \) are shown in Fig. 6 (left panel) as a function of the scaling variable \( T/\mu_B \), where values of \( T \) and \( \mu_B \) were chosen along the chiral phase transition line. It is clearly seen from the figure that in the region of high temperature and low density (high values of the \( T/\mu_B \)), the \( K^+/\pi^+ \) and \( K^-/\pi^- \) ratios tend to the same value. These results are in agreement...
with experimental results. In the PNJL model at high temperature and low density the difference between the mass of charged kaon multiplets decreases, their masses become equal to each other (kaons are degenerate at $T = 0$) and as can be seen, the difference in the ratios can also decrease.

The absence of the “horn” structure in the $K^+/\pi^+$-ratio is explained by the different sensibility of positive and negative charged kaons to the medium density. When the relative number of baryons decreases with increasing energy, the number of negative charged kaons does not change. The number of positive charged kaons must be balanced by strange baryons and, therefore, will decrease [40].

The quark matter properties in the frame of the PNJL model are formed by the choice of different assumptions for the environment: Case 1 with $\mu_s = \mu_u$, $\mu_s = a \cdot \mu_u$, where the constant $a$ can change or the case with the strangeness neutrality ($n_s = 0$). In Fig. 6, right panel, it can be seen how the enhancement in the $K^+/\pi^+$ ratio depends on the choice of the parameter $a$: in case when $\mu_K = 0$ ($\mu_u = \mu_s$, $\mu_S = 0$) the enhancement in the ratio disappears. The behavior of the $K^+/\pi^+$ ratio significantly differs for the strangeness neutrality case, where the “peak” structure in the ratio turns into fall from the high values in the low-energy region to the constant in the high-energy region.

Data for the $K^+/\pi^+$ ratio for three various phase diagrams (Cases 1–3) are shown in Fig. 5, right panel.

4. CONCLUSIONS

In this work, we have described the QCD matter in the framework of the SU(3) PNJL model. We paid special attention to the fact that the models able to describe the “horn” implied the phase transitions. In the work [13], the quick increase in the $K^+/\pi^+$ ratio and its decrease with further increasing energy was interpreted as a sequence of the chiral symmetry restoration and the deconfinement effect. The main idea of this work was to show that the reason of the appearance of the “horn” at energies $\sqrt{s_{NN}} = 8 - 10$ GeV may be a qualitative change in the state of the environment where kaons and pions are created.

If we assume that the detected particles can “remember” the properties of matter where they were created (the matter with the restored chiral symmetry or the deconfined matter), we can use the effective models to simulate the matter properties. In the PNJL model the picture is the following: when $T$ and $\mu_B$ are chosen along the phase transition line, the system is in the phase transition region and the chiral condensate is still not destroyed. The main difference between the choice of $(T, \mu_B)$ along the phase transition line is whether it is the crossover region or the 1st order transition region.

The high density matter in the PNJL model is characterized by the splitting of the masses for positive and negative mesons. The splitting decreases with density. This splitting can explain the difference in the behavior of the $K/\pi$ ratios for different charges in the high-density region and the fact that they tend to the same value at high temperatures (low densities), where kaons become degenerated (see Fig. 6, left panel).

From Figs. 3 and 7 it can be seen that the position of the peak pretends to depend on the position of the critical end point. This observation can be explained in a simple way using the Wroblewsky factor [42], which can be used to study whether strangeness has a maximum or not [43]:

$$\lambda_s(\sqrt{s_{NN}}) = \frac{2 \langle \bar{s}s \rangle}{\langle \bar{u}u \rangle + \langle \bar{d}d \rangle} = \frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle},$$

where the quantities in the angular brackets refer to the number of formed quark-antiquark pairs (i.e. the quark condensate) and the latest equality is correct for the PNJL model, where quark condensates of light quarks are equal. The slope of the tangent line for the smooth curve is the derivative at the point of tangent:

$$\frac{\partial \lambda_s(\sqrt{s_{NN}})}{\partial(\sqrt{s_{NN}})} = \frac{\partial \langle \bar{s}s \rangle}{\partial(\sqrt{s_{NN}})} \frac{1}{\langle \bar{s}s \rangle} = \frac{\partial \langle \bar{u}u \rangle}{\partial(\sqrt{s_{NN}})} \frac{1}{\langle \bar{u}u \rangle^2},$$

where $\sqrt{s_{NN}}$, $T$, $\mu$ can be associated via parametrization of the phase diagram, for example, using Eq. (21) with any parameters $a, b, c, d, e, f$. We can estimate the value of Eq. (23) in the assuming that we can detect the critical
end point at some energy. The values of \(\langle \bar{u}u \rangle\) and \(\langle \bar{s}s \rangle\)-condensates are finite, but \(\partial (\bar{u}u/\partial T) \rightarrow \infty\) (it is the selection condition for the CEP search, for example, in the PNJL model). Then the slope \(\tan \theta \rightarrow -\infty\) and \(\theta \rightarrow -90^\circ\). The points with negative slope can be found at energies where the \(K^+/pT\)-ratio falls.

According to Fig. 9 in the work [43], the \(\lambda_c(\sqrt{s_{NN}})\) behavior is enough smooth and there are no points with the slope \(\rightarrow -\infty\). But if we could describe the data on Fig. 5 using an arbitrary function \(F(\langle \bar{u}u \rangle, \langle \bar{s}s \rangle, \sqrt{s_{NN}})\), the point with the angle \(-90^\circ\) could be found near the maximum. This very simplified explanation cannot be considered as a proof of the CEP existence, but it is in agreement with our results.

The quark matter properties can be formed by the choice of the medium scenario (a medium with an equal baryon density, a medium with beta equilibrium, strange matter, or a medium with an equal number of protons of neutrons and hyperons). In Fig. 6, it was illustrated that the peak position depends on the choice of the strange chemical potential: in the case when \(\mu_u = \mu_d = \mu_s (\mu_K = 0 \text{ and } \mu_S = 0)\), there was no enhancement in the ratio [41]. It was also shown that the value of \(\mu_u/\mu_s\) affects the position and the height of the peak in the kaon-to-pion ratio.

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