Neutrino Cosmology after WMAP and LHC7

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Abstract

The gauge-extended $U(1)_C \times SU(2)_L \times U(1)_{I_R} \times U(1)_L$ model has the attractive property of elevating the two major global symmetries of the standard model (baryon number $B$ and lepton number $L$) to local gauge symmetries. The $U(1)_L$ symmetry prevents the generation of Majorana masses, leading to three superweakly interacting right-handed neutrinos. This also renders a $B-L$ symmetry non-anomalous. We show that the superweak interactions of these Dirac states (through their coupling to the TeV-scale $B-L$ gauge boson) permit right-handed neutrino decoupling just above the QCD phase transition: $175 \text{ MeV} \lesssim T_{\nu_R}^{\text{dec}} \lesssim 250 \text{ MeV}$. In this transitional region, the residual temperature ratio between $\nu_L$ and $\nu_R$ generates extra relativistic degrees of freedom at BBN and at the CMB epochs. Consistency (within 1$\sigma$) with both WMAP 7-year data and the most recent estimate of the primordial $^4\text{He}$ mass fraction is achieved for $3 \text{ TeV} < M_{B-L} < 6 \text{ TeV}$. The model is fully predictive, and can be confronted with dijet and dilepton data (or lack thereof) from LHC7 and, eventually, LHC14.
Heavy neutral vector gauge bosons ($Z'$s) are ubiquitous in extensions of the standard model (SM) \[1\], often including a gauged $B - L$ symmetry. This $U(1)$ symmetry is non-anomalous if the three left-handed Weyl neutrinos are accompanied by three right-handed neutrinos. The $Z'$ masses are \emph{a priori} open parameters – not determined by the low energy effective theory – but subject to recent experimental bounds ($M_{Z'} \gtrsim 3$ TeV) from searches of dilepton \[2\] and dijet \[3\] events in the 7 TeV run of the Large Hadron Collider (LHC14). In this Letter we re-examine some critical cosmological issues surrounding the presence of $Z'$ in our dynamical model which is coupled to $B - L$. These considerations, when viewed in the context of most recent data collected by the Wilkinson Microwave Anisotropy Probe (WMAP) \[5\], are found to constrain the mass of the $Z'$ to an interestingly narrow band, which will be directly probed by LHC14.

For a good part of the past two decades, big-bang nucleosynthesis (BBN) provided the best inference of the radiation content of the universe. The time-dependent quantity being the neutron abundance at $t \gtrsim \tau_n$, which regulates the primordial fraction of baryonic mass in $^4$He,

$$Y_p \simeq 0.251 + 0.014 \, \delta N_{\nu}^{\text{eff}} + 0.0002 \, \delta \tau_n + 0.009 \, \ln \left( \frac{\eta}{5 \times 10^{-10}} \right),$$

where $\delta N_{\nu}^{\text{eff}} = N_{\nu}^{\text{eff}} - 3$ is the effective number of extra (non-SM) light neutrino species, $\tau_n$ is the neutron half life, and $\eta$ is the ratio of the baryon number density to the photon number density \[6\]. The observationally-inferred primordial fractions of baryonic mass in $^4$He ($Y_p = 0.2472 \pm 0.0012$, $Y_p = 0.2516 \pm 0.0011$, $Y_p = 0.2477 \pm 0.0029$, and $Y_p = 0.240 \pm 0.006$ \[7\]) have been constantly favoring $N_{\nu}^{\text{eff}} \lesssim 3$ \[8\]. Out of the blue, two recent independent studies determined $Y_p = 0.2565 \pm 0.001(\text{stat}) \pm 0.005(\text{syst})$ and $Y_p = 0.2561 \pm 0.011$ \[9\]. For $\tau_n = 885.4 \pm 0.9$ s and $\tau_n = 878.5 \pm 0.8$ s, the updated effective number of light neutrino species is reported as $N_{\nu}^{\text{eff}} = 3.68^{+0.80}_{-0.70} (2\sigma)$ and $N_{\nu}^{\text{eff}} = 3.80^{+0.80}_{-0.70} (2\sigma)$, respectively.

Very recently, in support of these trends, observations of the cosmic microwave background (CMB) anisotropies and the large-scale structure distribution have allowed a probe of $N_{\nu}^{\text{eff}}$ at the CMB decoupling epoch with unprecedented precision. The relativistic particles that stream freely influence the CMB in two ways: (1) their energy density alters the matter-radiation equality epoch, and (2) their anisotropic stress acts as an additional source for the gravitational potential via Einstein’s equations. Hence, the number of light relativistic species becomes a function of the matter density ($\Omega_m h^2$) and the redshift of matter-radiation equality ($z_{eq}$),

$$1 + z_{eq} = \frac{\Omega_m h^2}{\Omega_r h^2} = \frac{\Omega_m h^2}{\Omega_r h^2} \left[ 1 + \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} N_{\nu}^{\text{eff}} \right]^{-1},$$

where $\Omega_r h^2 = 2.469 \times 10^{-5}$ is the present-day photon energy density (for $T_{\text{CMB}} = 2.725$ K) and the scaled Hubble parameter $h$ is defined by $H = 100 \, h \, \text{km s}^{-1} \, \text{Mpc}^{-1}$ \[10\]. The variation in $N_{\nu}^{\text{eff}}$ reads

$$\frac{\delta N_{\nu}^{\text{eff}}}{N_{\nu}^{\text{eff}}} \simeq 2.45 \frac{\delta(\Omega_m h^2)}{\Omega_m h^2} - 2.45 \frac{\delta z_{eq}}{1 + z_{eq}}.$$  

The equality redshift is one of the fundamental observables that one can extract from the CMB power spectrum. More specifically, WMAP data constrain $z_{eq}$ mainly from the height of the third acoustic peak relative to the first peak \[5\]. The fractional error in $\Omega_m h^2$ is
determined using external data: the latest distance measurements from the Baryon Acoustic Oscillations (BAO) in the distribution of galaxies [11] and precise measurements of the Hubble constant $H_0$ [12]. The parameter constraints from the combination of WMAP 7-year data, BAO, and $H_0$ lead to $N_{\nu}^{\text{eff}} = 4.34^{+0.66}_{-0.88} (68\% \text{CL})$ [5].

In summary, though uncertainties remain large, the most recent cosmological observations show a consistent preference for additional relativistic degrees of freedom (r.d.o.f.) during BBN and the CMB epochs. We take these hints as motivation for our analysis, which consists of the following tasks: (1) to present a model in which the additional r.d.o.f. are three flavors of light right-handed neutrinos which interact with the SM fermions via the exchange of heavy vector fields; (2) to suppress the six additional fermionic r.d.o.f. to levels in compliance with BBN and CMB. This is accomplished by imposing the decoupling of $\nu_R$’s from the plasma early enough so that they undergo incomplete reheating during the QCD phase transition; and late enough so as to leave an excess neutrino density suggested by the data [13]. These requirements strongly constrain the masses of the heavy vector fields. Together with the couplings, which are determined in accord with other considerations, the model is fully predictive, and can be confronted with dijet and dilepton data (or lack thereof) from LHC7 and, eventually, LHC14.

An economic choice of the model to implement the task outlined above is based on the gauge-extended sector $U(3)_C \times SU(2)_L \times U(1)_{I_R} \times U(1)_L$ [14]. The resulting $U(1)$ content gauges the baryon number $B$ [with $U(1)_B \subset U(3)_C$], the lepton number $L$, and a third additional abelian charge $I_R$ which acts as the third isospin component of an $SU(2)_R$. The usual electroweak hypercharge is a linear combination of these three $U(1)$ charges: $Y = \frac{1}{2}(B - L) + I_R$. The matter fields consist of six sets (labeled by an index $i = 1 - 6$) of Weyl fermion-antifermion pairs: $(U_R, D_R, L_L, E_R, Q_L, N_R)$. The field $N_R$ is the right-handed neutrino (and left-handed antineutrino) accompanying the fields in the set $E_R$, with mass $\sim 1 \, \text{eV}$. The gauging of lepton number precludes the presence of a seesaw for generating Majorana neutrino masses. In addition to the SM interactions, these fields experience two $U(1)$ gauge interactions mediated by two associated vector bosons ($Z'$ and $Z''$) whose masses lie well above a TeV.

The initially free parameters consist of three couplings $g_B, g_L, g_{I_R}$. These are augmented by three Euler angles to allow for a field rotation to coupling diagonal in hypercharge. This diagonalization fixes two of the angles and the orthogonal nature of the rotation introduces one constraint on the couplings $P(g_Y, g_B, g_L, g_{I_R}) = 0$. The baryon number coupling $g_B$ is fixed to be $\sqrt{3}/2$ of the non-abelian $SU(3)$ coupling at the scale of $U(N)$ unification, and is therefore determined at all energies through RG running. This leaves one free angle and two couplings with one constraint. The two remaining degrees of freedom allow a further rotation leaving $Z'$ to couple to $B$ at $90\%$ and $Z''$ to couple to $B - L$ at $99\%$. The $U(1)$ quantum numbers and the physical couplings of the $Z'$ and $Z''$ to the fermion fields are given in Table 1. These couplings, which have been computed elsewhere [14], are functions of the charge assignments, the $U(1)$ gauge couplings, and the mixing angles. All fields in a given set have a common $g', g''$ couplings.

The model as described enjoys distinct advantages: (1) Gauging of the anomalous $B$ and its cancellation by generalized Green-Schwarz mechanism (which leaves $B$ as a global

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1. Although not generally appreciated, it is important to note that a 100% coupling of the $Z'$ and $Z''$ to $B$ and $B - L$, respectively, is possible only if the $U(1)$ gauge coupling constants are equal.

2. Our couplings are consistent with the bounds presented in [15] from a variety of experimental constraints.
We consistently omit \( \nu \) where \( T \) is the actual (ideal Stefan-Bolzmann) entropy. Hence \( N(T_{\text{end}}) = 2 \) for each real vector field and \( N_F = 2 \) for each spin-\( \frac{1}{2} \) Weyl field. The coefficient \( r(T) \) is unity for the lepton and photon contributions, and is the ratio \( s(T)/s_{SB} \) for the quark-gluon plasma. Here \( s(T)(s_{SB}) \) is the actual (ideal Stefan-Bolzmann) entropy. Hence \( N(T_{\text{dec}}) = 37 r(T_{\text{dec}}) + 14.25 \). We take \( N(T_{\text{end}}) = 10.75 \) reflecting \( (e^-_L + e^-_R + e^+_L \nu_e_L + \bar{\nu}_e_L + \nu_{\mu_L} + \bar{\nu}_{\mu_L} + \nu_{\tau_L} + \bar{\nu}_{\tau_L} + \gamma_L + \gamma_R) \). We consistently omit \( \nu_R \) in considering the thermodynamics part of the discussion, but will include it when dealing with expansion. As stated in the introduction

\[
\delta N_{\nu}^{\text{eff}} = \begin{cases} 
0.68^{+0.40}_{-0.35} & \text{(1\sigma)} \quad \text{BBN} \\
1.34_{-0.88} & \text{(1\sigma)} \quad \text{WMAP + BAO+H_0} 
\end{cases}
\]
so the excess r.d.o.f. will lie within 1σ of the central value of each experiment if \(0.46 < \delta N_\nu^\text{eff} < 1.08\). From Eqs. (6) and (7), the allowable range for \(N\) is

\[
23 < N(T_{\text{dec}}) < 44. 
\]  
(8)

This is achieved for \(0.24 < r(T_{\text{dec}}) < 0.80\). By comparing to Fig. 8 in Ref. [16], this can be translated into temperature range

\[
175 \text{ MeV} < T_{\text{dec}} < 250 \text{ MeV}, 
\]  
(9)

with the lower temperature coinciding with the region of most rapid rise of the entropy. Thus, the data implies that the \(\nu_R\) decoupling takes place during QCD phase transition.

We now turn to use our model in conjunction with the decoupling condition to constrain its parameters. To this end we calculate the interaction rate \(\Gamma(T)\) for a right-handed neutrino and determine \(T_{\text{dec}}\) from the plasma via the prescription

\[
\Gamma(T_{\text{dec}}) = H(T_{\text{dec}}). 
\]  
(10)

Let \(f^i_L\) be a single species of Weyl fermion, representing the two r.d.o.f. \(\{f^i_L, \bar{f}^i_R\}\), where the superscript indicates bins \(i = 3, 5\). Similarly \(f^i_R \in \{f^i_R, \bar{f}^i_L\}\), for \(i = 1, 2, 4, 6\). Notice that the subscripts \(L, R\) denote the actual helicities of the massless particles in question, not the chirality of the fields. With this said, we may write the amplitude for \(f^i_L\) scattering

\[
\mathfrak{M} (\nu_R(p_1) f^i_L(p_2) \rightarrow \nu_R(p_3) \bar{f}^i_L(p_4)) = \frac{G_i}{\sqrt{2}} \left[ \bar{u}(p_3) \gamma^\mu (1 + \gamma_5) u(p_1) \right] \left[ \bar{u}(p_4) \gamma_\mu (1 - \gamma_5) u(p_2) \right]. 
\]  
(11)

The other 3 amplitudes are obtained by the crossing substitutions in the second square bracket; for scattering from

\[
\begin{align*}
\bar{f}^i_R &\rightarrow \bar{v}(p_2) \gamma_\mu (1 - \gamma_5) v(p_4) \\
f^i_R &\rightarrow \bar{u}(p_4) \gamma_\mu (1 + \gamma_5) u(p_2) \\
\bar{f}^i_L &\rightarrow \bar{v}(p_2) \gamma_\mu (1 + \gamma_5) v(p_4). 
\end{align*} 
\]  
(12)

The cross sections for the four scattering processes (no average over helicities) are

\[
\sigma (\nu_R f^i_L \rightarrow \nu_R f^i_L) = \frac{1}{3} \sigma (\nu_R \bar{f}^i_R \rightarrow \nu_R \bar{f}^i_R) = \frac{2 G_i^2 s}{3 \pi} \quad \text{(for bins } i = 3, 5) \]  
(13)

and

\[
\sigma (\nu_R \bar{f}^i_L \rightarrow \nu_R \bar{f}^i_L) = \frac{1}{3} \sigma (\nu_R f^i_R \rightarrow \nu_R f^i_R) = \frac{2 G_i^2 s}{3 \pi} \quad \text{(for bins } i = 1, 2, 4, 6). \]  
(14)

In addition to these scattering processes, the \(\nu_R\) interacts with the plasma through the annihilation processes: \(\nu_R \bar{v}_L \rightarrow f^i_L \bar{f}^i_R\), for bins \(i = 3, 5\), and \(\nu_R \bar{v}_L \rightarrow f^i_R \bar{f}^i_L\), for bins \(i = 1, 2, 4, 6\). These all yield cross sections \(2 G_i^2 s/(3\pi)\) due to forward and backward suppression. Assuming all chemical potentials to be zero, the plasma will have an equal number density \(n(T) = 0.0913 T^3\), for each fermion r.d.o.f. Thus,

\[
\Gamma^{\text{scat}}(T) = n(T) \left\langle \sum_{i=1}^{6} \sigma_i(s) v_M N_i \right\rangle, 
\]  
(15)
where \( v_M = 1 - \cos \theta_{12} \) is the Moller velocity, \( s = 2k_1k_2(1 - \cos \theta_{12}) \) is the square of the center-of-mass energy, and \( N_i \) is the multiplicity of Weyl fields in each bin (e.g., for \( i = 3 \), \( N_3 = 3 + 2 = 5 \)). The scattering cross section is given by

\[
\sigma_i^{\text{scat}} = \sigma(\nu_R f_L^i \to \nu_R f_L^i) + \sigma(\nu_R \bar{f}_R^i \to \nu_R \bar{f}_R^i) = \frac{4G_i^2s}{3\pi} \quad \text{for each } i = 1, \ldots, 6;
\]

similarly,

\[
\sigma_i^{\text{ann}}(s) = \sigma(\nu_R \bar{\nu}_L \to f_L^i \bar{f}_R^i + \bar{f}_R^i f_L^i) = \frac{1}{3}\frac{G_i^2s}{\pi} \quad \text{for each } i = 1, \ldots, 6.
\]

Since \( s = 2k_1k_2(1 - \cos \theta_{12}) \) and \( v_M = 1 - \cos \theta_{12} \), we perform an approximate angular average \( \langle (1 - \cos \theta_{12})^2 \rangle = 4/3 \), followed by a thermal averaging \( (2k_1k_2) = 2(3.15^2T^2) \) to give

\[
\Gamma^{\text{scat}}(T) = \left(\frac{4}{3}\right)^2 \frac{2}{\pi} 2(3.15T)^2(0.0919T^3) \left(\sum_{i=1}^{6} G_i^2N_i\right) \simeq 2.05G_i^2T^5.
\]

From (16), (17), and (18),

\[
\Gamma^{\text{ann}}(T) = \frac{1}{4}\Gamma^{\text{scat}}(T) \simeq 0.50G_i^2T^5.
\]

Each of the \( G_i \) is given by the sum of the contributions from \( Z' \) and \( Z'' \) exchange,

\[
\frac{4G_i}{\sqrt{2}} = \frac{g_6 g_8^i}{M_{Z'}^2} + \frac{g_6' g_8''^i}{M_{Z''}^2}.
\]

The Hubble expansion parameter during this time is

\[
H(T) = 1.66 \langle N(T) \rangle^{1/2} T^2/M_{Pl},
\]

where \( M_{Pl} \) is the Planck mass. Since the quark-gluon energy density in the plasma has a similar \( T \) dependence to that of the entropy (see Fig. 7 in [16]), we take \( N(T) = 37 \, r(T) + 19.5 \), so that \( H(T) = 0.82 \times 12.5 \, T^2/M_{Pl} \). The first factor provides an average for \( r(T) \) over the temperature region, and we have now included the six \( \nu_R \) r.d.o.f.) Since \( \Gamma \propto T^5 \) and \( H \sim T^2 \), it is clear that if at some temperature \( T_{\text{dec}} \), \( H(T_{\text{dec}}) = \Gamma_i(T_{\text{dec}}) \), the ratio \( \Gamma/H \) will fall rapidly on further cooling. Thus from (10) and (21) the equation determining \( T_{\text{dec}} \) depends on: (1) whether we need to preserve the absence of a chemical potential, or (2) whether we need simply to maintain physical equilibrium. The decoupling condition in these two cases is: (1) \( \Gamma^{\text{ann}}(T_{\text{dec}}) = H(T_{\text{dec}}) \) and (2) \( \Gamma^{\text{scat}}(T_{\text{dec}}) + \Gamma^{\text{ann}}(T_{\text{dec}}) = H(T_{\text{dec}}) \); or numerically: (1)

\[
0.50 \, G_i^2 \, T_{\text{dec}}^5 = 10.4 \, T_{\text{dec}}^2/M_{Pl} \Rightarrow T_{\text{dec}}^3 = 20.8 \, (G_i^2 M_{Pl})^{-1},
\]

and (2)

\[
2.50 \, G_i^2 \, T_{\text{dec}}^5 = 10.4 \, T_{\text{dec}}^2/M_{Pl} \Rightarrow T_{\text{dec}}^3 = 4.1 \, (G_i^2 M_{Pl})^{-1}.
\]

\( T_{\text{dec}} \) as determined from these equations must lie in the band [9].

Since all freedom of determining coupling constant and mixing angles has been exercised, there remains only constraints on the possible values of \( M_{Z'} \) and \( M_{Z''} \). Our results are encapsulated in Fig. [1] and along with other aspects of this work are summarized in these concluding remarks:
• In this Letter, we develop a dynamic explanation of recent hints that the relativistic component of the energy during the era of last scattering is equivalent to about 1 extra Weyl neutrino.

• We work within the context of a specific (string based) model with 3 $U(1)$ gauge symmetries, originally coupled to baryon number $B$, lepton number $L$, and a 3rd component of right-handed isospin $I_R$. We find that rotation of the gauge fields to a basis exactly diagonal in hypercharge $Y$, and very nearly diagonal in $B - L$ and $B$ fixes all the mixing angles and the gauge couplings. Of course, of most significance for this work, requiring that the $B - L$ current be anomaly free, implies the existence of 3 right-handed Weyl neutrinos.

• We then find that for certain ranges of $M_B$ and $M_{B-L}$ the decoupling of the $\nu_R$'s occurs during the course of the QCD phase transition, just so that they are only partially reheated compared to the $\nu_L$'s — the desired outcome.

• To carry out this program, we needed to make use of some high statistics lattice simulations of a QCD plasma in the hot phase, especially the behavior of the entropy during the changeover.

• Since our aim is to match the data, which has lower and upper bounds on the neutrino “excess”, we obtain corresponding upper and lower bounds on the gauge field masses. Roughly speaking, if decoupling requires a freezout of the annihilation channel (loss of chemical equilibrium), then $3 \text{ TeV} < M_{B-L} < 4 \text{ TeV}$. If thermal equilibrium via scattering is sufficient, then $4.5 \text{ TeV} < M_{B-L} < 6 \text{ TeV}$. These are ranges to be probed at LHC14.

• Finally, a remark about the model: the gauging of $B$ allows a global conservation of baryon number. The gauging of $L$ brings with it the loss of a heavy Majorana for the seesaw model, as well as for leptogenesis through the decay of this particle. Thus, along with all its companion fields, the neutrino it is a Dirac particle, with the small mass originating through small Yukawa.

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FIG. 1: Dark shaded areas show region allowed from decoupling requirements to accommodate WMAP and BBN data. Light shaded regions indicate the masses excluded by the LHC7 dijet searches. The dark shaded areas in the left and right figures pertain to chemical and thermal equilibrium, respectively. These two estimates should serve to bracket the size of the actual effect. The designation of $B$ corresponds to $Z'$ and $B - L$ to $Z''$. 

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