A modified Latin hypercube sampling based on prior information

Qing'an Cui 1, Huanjiao Duan1, Xueqing Wang 2,*

1School of Management Engineering, Zhengzhou University, 100 Science Avenue, Gaoxin District, Zhengzhou, China
2College of Management and Economics, Tianjin University, 92 Weijin Road, Nankai District, Tianjin, China

*Corresponding author e-mail: xueqing1012@tiu.edu.cn

Abstract. Latin hypercube sampling is widely used in industrial engineering. In the traditional Latin square sampling, engineers often arrange sample points in the feasible domain uniformly. However, in practice, engineers may have some prior information about the sub-domains where the response volatility is relatively large, named the interesting sub-domains. In order to make full use of these information, this paper employed the D-S evidence theory to fuse prior information from different sources/fields. Then we divide the feasible domain into different sub-domains and indicate the interesting sub-domains. For the sample placement, we put more points in these interesting sub-domains and less points in other sub-domains. Finally, we construct the model with the proposed sample points placement approach based on prior information. A case study was conducted to illustrate the proposed method. The case study shows that the proposed method performs better than the traditional model in MSE, MaxE and StdE.

1. Introduction

In manufacturing processes, there are multiple factors affecting the product quality. It is important to obtain efficient sample points to reflect the relationship with limited resources. Latin hypercube sampling (LHS), also called Latin hypercube design (LHD), is one of the most popular techniques in designed experiment, which is first developed by McKay et al. (1979)[1]. Since then it is widely used to collect samples points (Garud et. al, 2017)[2].

As the projection property of LHS is non-collapsing, it is widely and successfully used in many areas. But as Roshan Joseph(2016)[3]indicated, there are many possible sample placement choices for LHDs and not all of them are good as we expect. In some cases, the tendency of sample placement may be ascending and descending instead of irregular or uniform as the factors increase. It is really bad when all the sample points are placed on the diagonal line. In addition, the projection of one point may be very close to projection of the next point in the neighboring bins. Thus, Morris and Mitchell (1995)[4] proposed to choose an LHD that maximizes the minimum distance among the points. This guarantee the stability of the LHS space-filling capability. Damblin et al. (2013)[5] proposed the enhanced stochastic evolutionary method to optimize the Latin Hypercube Sampling’ sub-projection
properties. Aziz and Tayarani-N (2014)[6] employed the adaptive memetic particle swarm method for the LHS optimization. All these methods used optimization algorithms based on space filling principle which improve the convergence speed and the robustness of sample subproject.

The aforementioned researchers treated the whole feasible domain without distinction. In this case, the LHS may generate insufficient sample points in some subdomains where this response volatility is relatively large. Hereafter, we named these domains as the interesting domains. In complex manufacturing processes, we need to place enough sample points to reflect the true relationship. This is expensive if we put more sample points in each of sub-domains. In fact, engineers may have some prior information about how factors affect responses in some sub-domains. According to the prior information, large proportion of sample points should be allocated to the interesting sub-domains and the rest of sample points in the other sub-domains. Compared with the traditional Latin square sampling, we could obtain more useful information that may help engineers to build a more accuracy model.

In literature, to our best knowledge, there are few researches about prior information in optimization of LHS. Minasny and McBratney (2006)[7] proposed a conditioned Latin hypercube method based on prior information. Cui et al. (2014)[8] used the fuzzy comprehensive evaluation method to obtain sub-domains based on prior information, and constructed a model based on sample points from sub-domains. These researches emphasize the importance of prior information. However, in practice, there may be conflictive among prior information obtained from different aspects, and we should deal properly with all the conflicts. There are many ways to fuse prior information. Bayes information fusion (also called Bayesian information criterion) is dynamically adding information by posterior information changing into prior information iteratively. It needs much information to produce a prior distribution (Volinsky and Raftery, 2000)[9]. However, in complex manufacturing, there are not enough prior information to produce a prior distribution. The Fuzzy information fusion method employs a membership function to measure the fuzzy degree of objects. However, the information weigh is given artificially. So, it may be biased (Kwak and Pedrycz, 2005)[10]. The D-S evidence theory treats the information conflicts as the process of “evidence fusion”. The prerequisite for using D-S evidence theory is that the evidences (information) are independent of each other. In this paper, prior information is obtained from different sources. So, they meet the requirements of independence. It is employed in this paper for fusing the kinds of prior information. It is first developed by Dempster (1976)[11] and Shafer (1976)[12]. Then, it is widely used in artificial intelligence (Wang, et al. 2015)[13], information fusion (Wang et al. 2018)[14] etc. Cheng et al. (2017)[15] employed contour detection and D–S evidence theory for eye localization, which provide technological basis for fatigue driving. Yang et al. (2017)[16] used the D–S evidence theory to solve the conflicts between different feature evidences. The collected evidences indicate that the fusion results of D–S could keep the conflictive and consistent information effectively. Hereafter, we name the LHS based on prior information as the modified LHS. We could obtain sample points effectively from the modified LHS.

In order to compare the modified LHS with the traditional LHS, we use the generated sample points to fit models and compare the performance of the two models. In complex manufacturing processes, it is expensive and time-consuming to obtain enough sample points. In general, the size of sample points is small. Least Squares Support Vector Regression (LS-SVR) performs better in fitting and predictive with small sample size (Heddam and Kisi 2018)[17]. So the LSSVR may be suitable to construct a good model in the complex manufacturing process (Liao et al. 2011)[18].

In this paper, we proposed a modified Latin hypercube sampling method (MLHS) uses the D-S evidence theory to fuse the prior information and obtain the interesting domains according to the fusion results. Then, we place different sample points in the subdomains and get LS-SVR model. The rest of this paper is structured as follows. Section 2 describes the modified Latin hypercube sampling. Section 3 gives a case study.Finally, concluding remarks are made in Section 4.

2. The Modified Latin hypercube sampling

The Modified Latin hypercube sampling (MLHS) mainly includes two parts: fuse prior information and obtain the interesting sub-domain.
2.1. Fuse the prior information

In manufacturing process, there are always many prior information from different experts. To excavate effective information from prior information, we should consider different kinds of prior information. However, these information maybe conflicts with others. In this paper, we adopt the D-S evidence theory to fuse prior information. Firstly, we introduce the formulations of D-S evidence theory. Recognize space $\Theta$ is a finite set of possible subdomains, $\forall A \subseteq \Theta$. A is a coke which means a possible divided subdomain in this paper. $m(A)$ is basic trust function which shows the degree of trust for the subdomain $A$ occurrence. $m(A)$ satisfies the following equation.

\[
\begin{align*}
    m(\phi) &= 0 \\
    \sum_{A \subseteq \Theta} m(A) &= 1
\end{align*}
\]  

(1)

Assuming there are $n$ evidences namely $E_1, E_2, \ldots, E_n$ for the subdomain and their corresponding basic trust functions are $m_1, m_2, \ldots, m_n$.

\[
\begin{align*}
    E_1: & \quad m(A_{i1}), m(A_{i2}), \ldots, m(A_{i_n}) \\
    \vdots \\
    E_k: & \quad m(A_{k1}), m(A_{k2}), \ldots, m(A_{kn})
\end{align*}
\]  

(2)

$m(A_j)$ denotes $j$th subdomain in the $i$th evidence. The number of subdomains is not always constant for all evidences, that is, the $k$ values may be different in each subdomain. Then, fuse the evidence using Eq. (3). Eq. (3) is the most important formulation of D-S evidence theory (Li et al. 2017)[19].

\[
\begin{align*}
    m(A) &= \left\{ \begin{array}{ll}
        \frac{\sum_{i=1}^{n} \prod_{j=1}^{k} m_i(A_j)}{1 - K}, & A \neq \phi, 1 \leq i \leq n, 1 \leq j \leq k \\
        0, & A = \phi
    \end{array} \right.
\end{align*}
\]  

(3)

where $K = \prod_{i=1}^{n} m_i(A)$. D-S evidence theory fuses different kinds of prior information effectively. It also does not reduce the amount information from different prior information. However, fusion rules may lead to paradoxes such as 0-1 paradoxes, trust bias paradox, etc. (Jiao, Z. et al. 2018)[20]. In the complex manufacturing process, different kinds of prior information may be conflict with others which result in paradoxes (detailed in section 1). To cope with this conflict situation, we divide information fusion into two conditions. One is low conflict condition. Another is high conflict condition. We define the condition is low conflict when any of $m_i = 0, i = 1, 2, \ldots, n$ do not exist. Otherwise, the condition is high conflict. When the conflict is low, we use the traditional formulation (Eq. (3)). In order to use Eq. (3), we use the method in Wang and Yang (2007)[21]. Firstly, calculate the similarity coefficient (Eq. (4)) which form a similarity matrix according to subscripts. Assuming there are two evidences namely $E_i, E_j$.Cokes are respectively $A_i, B_j$.The similarity coefficient of two evidences is as follow. Secondly, sum the similarity coefficient by column. The sum indicates the support degree of all the evidences for each evidence. Thirdly, obtain the weight of each evidence after normalizing the sum result. Finally, fuse the weighted average evidence by Eq. (3).

\[
d_{ij} = \frac{\sum_{A \subseteq \Theta} m_i(A)m_j(B_j)}{\sqrt{\sum m_i^2(A) \left( \sum m_j^2(B_j) \right)}}
\]  

(4)

After collecting the evidences, we first calculate the conflict degree. Then select the fusion rules. Finally, we get the final result $m(A_1), m(A_2), \ldots, m(A_k)$. 


2.2. Obtain the interesting sub-domain

In the traditional LHS, engineers treat different subdomains as similar, and each subdomain has the same number of sample points. However, in fact, different subdomains may have different response volatility. If the response volatility is linear, two sample points is enough to approximate the relationship between the factors and response. If the response volatility is concave or convex, two sample points are prone to generate false relationships. Adding sample points in the interesting domains is more cost-effective than where this response volatility is relatively small. We should treat the interesting subdomains differently, and then optimize the sub-domains sample size purposely with limited resources. How do we select the interesting sub-domains? The values of prior information fusion are the support degree of all evidences for subdomains. The sum of the values is 1. So, we treat the value as the weight of an interesting subdomain. The bigger the value, the more likely it is to be regarded as an interesting subdomain. Compared with the traditional LHS, the modified LHS divided the feasible domain into subdomains. For generating sample points, each of subdomain will use the traditional LHS one time.

The procedure of the proposed method is following.

Step 1. Extract the main impact factors and set the number of factors as \( a \).

Step 2. Collect the prior information as evidences for one factor.

\[
E_i: m(A_{i1}), m(A_{i2}), m(A_{i3}) \cdots m(A_{i_k})
\]

\[
E_j: m(A_{j1}), m(A_{j2}), m(A_{j3}) \cdots m(A_{ja})
\]

Step 3. Calculate the conflict degree. According to the different conflict degree, the corresponding combined rules are selected to combine evidences in section 2.1. Result is as following.

\[
m(A_1), m(A_2), m(A_3) \cdots m(A_k)
\]

In formula (6), \( k \) contains the focal element of all the evidence in formula (5).

Step 4. Divide the feasible domain into different regions by the \( m \) values in the formula (6). When dividing a single factor, it may be merged if the basic trust functions are approximately equivalent and the factor feasible domain is continuous.

Step 5. According to step 2, 3, and 4, collect evidences for each of factors and then divide the feasible domain into different regions. The result of the divided of each of factors is following.

The first factor:

\[
E_i: m(A_{i1}), m(A_{i2}), m(A_{i3}) \cdots m(A_{i_k})
\]

The \( a \)th factor:

\[
E_j: m(A_{j1}), m(A_{j2}), m(A_{j3}) \cdots m(A_{ja})
\]

Step 6. Group regions of a factor to other factors freely according to the results of the division. The number of combinations is \( W \). The calculation formula is following.

\[
W = k_1 \times k_2 \times \cdots \times k_a
\]

Step 7. Collect the relevant prior information as evidences according to the obtained grouping result.

\[
E_i: m(A_{i1}), m(A_{i2}), m(A_{i3}) \cdots m(A_{ik})
\]

\[
E_j: m(A_{j1}), m(A_{j2}), m(A_{j3}) \cdots m(A_{ja})
\]

Where \( p \) represents the total number of evidences collected, and satisfies \( 1k, 2k, 3k \ldots pk \leq W \).

Step 8. According to the relevant combined rules, the evidences in (9) are combined to get the final fusion result:

\[
m(A_1), m(A_2), m(A_3) \cdots m(A_p)
\]
Step 9. Set the limited total sample size as $S$ according to the number of factors, feasible domain and experimental cost.

Step 10. Calculate sample size for combinations according to $S$ and formula (10).

$$S_i = S \times m(A_j)$$  

(11)

Where $i = 1,2,3…, w-1, w$.

Step 11. Use LHS method to generate sample points in each subdomain, and form sample set.

2.3. Performance comparison

This section employed Least Squares Support Vector Regression (LS-SVR) to build model for using sample points, which was proposed by Suykens et al. (1999, 2001)[22,23]. It is a modified SVR. In LS-SVR, the loss function is replaced by a classical squared loss function. It maps the inputs into high-dimension. In recent years, LSSVR have been extended to cope with various problems (Chang et al. 2013, Li et al. 2007, Thomas, 2016)[24,25,26]

Set the limited total sample size as $S$ according to the number of factors and experimental cost. Sample size of the subdomain is $S_i = S \times m(A_j), i = 1,2,3…, k-1, k$. Then, we get the training data set (sample points) $S = \{(x_i, y_i), (x_i, y_i), \cdots, (x_i, y_i), i = 1, \cdots, S\}$. The regression function is:

$$y = w \cdot \phi(x) + b$$  

(12)

where $y$ denotes the forecasting values, $\phi(x)$ is a nonlinear mapping function, and $w$ and $b$ are the coefficients to be adjusted.

LSSVR employs a least square to SVR by solving the regression problem. The optimization problem for LS-SVR can be denoted as follows:

$$J = \min_{w,b} \frac{1}{2} \|w\|^2 + C \sum_{i=1}^{S} e_i^2$$  

$$s.t. (w \cdot \phi(x_i) + b) - y_i = e_i, i = 1, 2, \cdots, S$$  

(13)

where $\frac{1}{2} \|w\|^2$ is the confidence interval which indicates the accuracy degree of the model. $C$ is the penalty parameter that makes a tradeoff between model complexity and approximation accuracy. $e_i$ is the error variable which determines the goodness of fit. In some cases, the parameter $w$ is infinite. Eq. (13) cannot be solved in this case. Therefore, the dual problem should be considered (see Suykens (2002) for detail). The Lagrange function is established by introducing the Lagrangian multipliers.

$$L(w, b, \xi, \alpha) = J(w, e) + \sum_{i=1}^{S} \alpha_i \left\{y_i - w^T \phi(x_i) - b + e_i \right\}$$  

(14)

Through solving the Lagrangian function, we finally get the LS-SVR model.

$$f(x) = \sum_{i=1}^{S} \alpha_i K(x_i, x_j) + b$$  

(15)

where $K(x_i, x_j)$ is a kernel function. It is always Gaussian functions (Radial Basis Function).

In this paper, we use the Mean Squared Error (MSE), MAX error (MaxE), standard error (StdE) to compare the performance of different models. As followings.

$$MSE = \sqrt{\frac{\sum_{i=1}^{S} (y_i - y_i)^2}{S}}$$  

(16)
\[ MaxE = \max_i e_i \] 
\[ e_i = \hat{y}_i - y_i \] 
\[ StdE = \frac{1}{n-1} \sum_{i=1}^{n} (e_i - \overline{e})^2 \] 
\[ \overline{e} = \frac{1}{n} \sum_{i=1}^{n} e_i \] 

where \( \hat{y}_i \) is predictive value, \( y_i \) is real value.

If all evidences for the same subdomain \( m(A_i) = 0 \), that is, all of the result values are also zero. As the above said, the subdomain will not be placed sample points. In fact, this value means that there is very little change in the subdomain. It needs to have sample points for constructing a global model.

3. Case study

This section presents an example for optimizing factors of the injection molding process based on the MLSH method (Fig.1). Quality of monitor panel is mainly measured by the warpage degree. The smaller Warpage degree, the better the panel quality. Three factors, holding pressure, holding time, and filling time are selected as important factors in the example (Table 1). Moldflow is a software which simulate the injection molding process. In this paper, we use the Moldflow to generate the samples. Temperature is a noise factor with three levels of 24°C, 25°C and 26°C. Temperature changes randomly at the three levels.

![Fig.1 Monitor panel](image)

**Table 1. Injection factors and feasible domain.**

| Factors          | Unit | Feasible domain |
|------------------|------|-----------------|
| holding pressure | MPa  | [120, 170]      |
| holding time     | s    | [8, 20]         |
| filling time     | s    | [8, 20]         |

Then, we use the procedure of the MLHS to optimize factors of the injection molding process.

1) Collect the relevant evidences and divide the holding pressure. Cokes are followed: A1 (120–125) B1 (125–130) C1 (130–135) D1 (135–140) E1 (140–145) F1 (145–150) G1 (150–155) H1 (155–160) I1 (160–165) J1 (165–170).

The fused result of the three evidences in Table 2 are got according to the MLHS method. The approximate value mean that the response volatility is consistent. So, we merge the first four and the next six, according to the results in the final row. Set as \( e \) and \( f \).

2) Collect the relevant evidences, and divide the holding time and filling time. The fused result of holding time are \( h \) (8–13) and \( i \) (14–20). The results of holding time are \( j \) (8–13), \( k \) (14–16) and \( l \) (17–20).
Table 2. Holding pressure evidences and fused result.

|   | A1 | B1 | C1 | D1 | E1 | F1 | G1 | H1 | I1 | J1 |
|---|----|----|----|----|----|----|----|----|----|----|
| \(E_1\) | 0.01 | 0.02 | 0.03 | 0.04 | 0.1 | 0.17 | 0.2 | 0.2 | 0.1 | 0.13 |
| \(E_2\) | 0.02 | 0.02 | 0.02 | 0.03 | 0.12 | 0.15 | 0.16 | 0.16 | 0.15 | 0.17 |
| \(E_3\) | 0.03 | 0.03 | 0.02 | 0.02 | 0.2 | 0.11 | 0.11 | 0.13 | 0.2 | 0.15 |
| Result | 0.0003 | 0.007 | 0.006 | 0.0011 | 0.124 | 0.145 | 0.182 | 0.215 | 0.154 | 0.171 |

The three factors combine freely and form 12 subdomains, and then the evidences are collected for the 12 subdomains (Table 3).

Table 3. Evidence fused results.

|   | \(E_1\) | \(E_2\) | \(E_3\) | Results | Sample size |
|---|---|---|---|---|---|
| \(ehj\) | 0.25 | 0.1 | 0.3 | 0.52245 | 27 |
| \(ehk\) | 0 | 0.08 | 0.1 | 0.008762 | 4 |
| \(ehl\) | 0.1 | 0.15 | 0.2 | 0.177315 | 12 |
| \(eij\) | 0.2 | 0.2 | 0.05 | 0.172794 | 12 |
| \(eik\) | 0.1 | 0.18 | 0 | 0.034901 | 4 |
| \(eil\) | 0.1 | 0.12 | 0 | 0.017556 | 3 |
| \(fhj\) | 0 | 0.06 | 0.1 | 0.006005 | 3 |
| \(fhk\) | 0.05 | 0 | 0.1 | 0.005015 | 3 |
| \(fhl\) | 0 | 0 | 0 | 0 | 2 |
| \(fij\) | 0.1 | 0.04 | 0 | 0.004358 | 3 |
| \(fik\) | 0.1 | 0.07 | 0.1 | 0.039724 | 5 |
| \(fil\) | 0 | 0 | 0.05 | 0.000114 | 2 |

3) Because of the high conflict degree, we first calculate the similarity matrix.

\[
S = \begin{bmatrix}
1 & 0.834 & 0.729 \\
0.834 & 1 & 0.590 \\
0.729 & 0.590 & 1
\end{bmatrix}
\]

The support degree is: \(\text{Sup}(m_1) = 2.563\) \(\text{Sup}(m_2) = 2.424\) \(\text{Sup}(m_3) = 2.319\)

The credibility degree is: \(\text{Crd}(m_1) = 0.3508\) \(\text{Crd}(m_2) = 0.3318\) \(\text{Crd}(m_3) = 0.317\)

Each subdomain has two sample points as basicline sample size. The total sample size is 80. The sample points are generated using the MLHS method in Table 4, and H1: filling time, H2: holding pressure, H3: holding time, H4: opt value.

Based on the 80 sample points (M is the MLHS method of sample points in Table 4), the LS-SVR model is built. The optimization warpage is 0.4180.

To compare with the above results, the traditional method is used to generate sample points with the same sample size (L represents the traditional method in Table 4). The results are shown in the following table (Table 5).

The noise factor value \(\hat{\sigma}\) of the process is 0.0205. It can be seen from Table 5 that the result is in a reasonable range of actual value \(\pm 3\hat{\sigma}\) obtained by the two methods. Optimization values of the MLHS method are smaller than the traditional methods. Although the difference is not obvious, this difference is enough to distinguish defective products and good products in the actual production process. Therefore, the case study once again proved the effectiveness of the MLHS method.

To compare with the above results, the traditional method is used to generate sample points with the same sample size (L represents the traditional method in Table 4). The results are shown in the following table (Table 5).
To compare with the above results, the traditional method is used to generate sample points with the same sample size (L represents the traditional method in Table 4). The results are shown in the following table (Table 5).

The noise factor value $\sigma$ of the process is 0.0205. It can be seen from Table 5 that the result is in a reasonable range of actual value $\pm 3\sigma$ obtained by the two methods. Optimization values of the MLHS
method are smaller than the traditional methods. Although the difference is not obvious, this difference is enough to distinguish defective products and good products in the actual production process. Therefore, the case study once again proved the effectiveness of the MLHS method.

**Table 5. Comparison of modeling optimal results.**

|     | filling time | holding pressure | holding time | opt value | actual value | $\pm 3 \hat{\sigma}$ |
|-----|--------------|------------------|--------------|-----------|--------------|----------------------|
| M.1 | 10.9         | 162.9            | 16.3         | 0.4206    | 0.3629       | [0.3014,0.4244]      |
| M.2 | 11.2         | 159.8            | 17.1         | 0.4208    | 0.3755       | [0.3140,0.4370]      |
| M.3 | 11.2         | 160.8            | 17.1         | 0.4180    | 0.3899       | [0.3284,0.4514]      |
| L.1 | 10.6         | 156.7            | 12.4         | 0.4354    | 0.4777       | [0.4162,0.5392]      |
| L.2 | 10.7         | 158.8            | 11.9         | 0.4101    | 0.4686       | [0.4071,0.5302]      |
| L.3 | 9.7          | 163.9            | 10.9         | 0.3994    | 0.4590       | [0.3975,0.5205]      |

4. **Conclusion**

In this paper, we propose a modified Latin hypercube sampling when the manufacturing has prior information. This study shows how to improve the product quality by using prior information. It collected prior information, and fused prior information based on D-S evidence theory. Then, the fused prior information was used to divide the feasible domain into several subdomains according to the response volatility. The sample points are arranged in the interesting domains, which overcomes the shortcoming of placing sample points blindly in feasible domain. In this way, the MLHS method enhances the model accuracy. The simulation shows that this proposed method can construct a better model using the same sample size than the traditional method.

5. **Acknowledgments**

This work was financially supported by the National Nature Science Foundation of China (Grants numbers 71571168) and supported by Support Plan for Scientific and Technological Innovation Talents in Colleges and Universities of Henan Province (Humanities and Social Sciences) (2019-cx-007) fund.

6. **References**

[1] Mckay, M. D. , Beckman, R. J. , & Conover, W. J. Comparison of three methods for selecting values of input variables in the analysis of output from a computer code. Technometrics. 1979, 21(2), 239-245.

[2] Garud, S. S., Karimi, I. A., & Kraft, M. Design of computer experiments: A review. Computers & Chemical Engineering. 2017,106, 71-95.

[3] V. Roshan Joseph. Space-filling designs for computer experiments: a review. Quality Engineering. 2016, 28(1), 28-35.

[4] Morris, M. D. & Mitchell, T. J. Exploratory designs for computational experiments. Journal of Statistical Planning & Inference. 1995, 43(3), 381-402.

[5] Damblin, G. Couplet, M. & Iooss, B.. Numerical studies of space filling designs: optimization of latin hypercube samples and subprojection properties. Journal of Simulation. 2013, 7(4), 276-289.

[6] Aziz, M., & Tayarani-N, M. H.. An adaptive memetic Particle Swarm Optimization algorithm for finding large-scale Latin hypercube designs. Engineering Applications of Artificial Intelligence. 2014, 36, 222-237.

[7] Minasny, B. & Mebratnay, A. B. . A conditioned latin hypercube method for sampling in the presence of ancillary information. Computers & Geosciences. 2006, 32(9), 1378-1388.

[8] Qingan Cui, Huihua Liu, Bo He.. A Study of Sub-regional Design of Experiments and Modeling Method Based on the Prior Information . Industrial Engineering and Management of China. 2014, 19 (3): 85-90.
[9] Volinsky, C. T. , & Raftery, A. E.. Bayesian information criterion for censored survival models. Biometrics. 2000, 56(1), 256-262.
[10] Kwak, K. C. , & Pedrycz, W. . Face recognition: a study in information fusion using fuzzy integral. Pattern Recognition Letters. 2005, 26(6), 719-733.
[11] Dempster A P.. Upper and low probabilities induced by a multi-valued mapping . Annals of Mathematical Statistics.1976, 38 (2): 325-339.
[12] Shafer G A.. A mathematical theory of evidence. Princeton, pan, USA: Princeton University Press,1976.
[13] Hongbo Wang, He Luo, Xinbao Liu, et al.. Evidential reasoning method based on isometric mapping in internet of vehicular Ad-hoc NETworks. Systems Engineering Theory & Practice of China. 2015, 35 (6): 1582-1594.
[14] Wang, H., Guo, L., Dou, Z., & Lin, Y.. A new method of cognitive signal recognition based on hybrid information entropy and d-s evidence theory. Mobile Networks & Applications. 2018, (4), 1-9.
[15] Cheng, X. , Zhao, X. , Zhou, J. , Zhang, H. , Cheng, X. , & Zhao, X. , et al. Eye localization method based on contour detection and d-s evidence theory. Signal Image & Video Processing. 2017, (2), 1-8.
[16] Yang, C., Pu, J., Dong, Y., Liu, Z., Liang, L., & Wang, X.. Salient object detection in complex scenes via DS evidence theory based region classification. The Visual Computer. 2017, 33(11), 1415-1428.
[17] Heddam, S., & Kisi, O.. Modelling daily dissolved oxygen concentration using least square support vector machine, multivariate adaptive regression splines and M5 model tree. Journal of Hydrology. 2018, 559, 499-509.
[18] Liao, R. , Zheng, H. , Grzybowski, S. , & Yang, L. . Particle swarm optimization-least squares support vector regression based forecasting model on dissolved gases in oil-filled power transformers. Electric Power Systems Research. 2011, 81(12), 2074-2080.
[19] Li, Y. , Zhu, Y. , & Wu, J. . Research on expert weighting method based on D-S evidence theory. Advanced Information Technology, Electronic & Automation Control Conference. IEEE,2017.
[20] Jiao, Z., Gong, H., & Wang, Y.. A DS evidence theory-based relay protection system hidden failures detection method in smart grid. IEEE Transactions on Smart Grid. 2018, 9(3), 2118-2126.
[21] Xiaoxia Wang, Fengbao Yang.. A Kind of Evidence Combination Rule in Conflict. Chinese Journal of Projectiles, Rockets, Missiles and Guidance. 2007, 27 (5): 255-257.
[22] Suykens, J. A., & Vandewalle, J.. Least squares support vector machine classifiers. Neural processing letters. 1999, 9(3), 293-300.
[23] Suykens, J. A., Vandewalle, J., & De Moor, B.. Optimal control by least squares support vector machines. Neural networks. 2001, 14(1), 23-35.
[24] Chang, P. T. , Hung, L. T. , Pai, P. F. , & Lin, K. P. . Improving project-profit prediction using a two-stage forecasting system. Computers & Industrial Engineering.2013, 66(4), 800-807.
[25] Li, Y., Shao, X., & Cai, W.. A consensus least squares support vector regression (ls-svr) for analysis of near-infrared spectra of plant samples. Talanta. 2007,72(1), 217-222.
[26] Thomas, S. , Pillai, G. N. , & Pal, K. . Prediction of peak ground acceleration using e-svr, v-svr and ls-svr algorithm. Geomatics, Natural Hazards and Risk. 2016, 1-17.