Role of spatial correlation in photon-counting integral imaging

SR Narravula\textsuperscript{1}, MM Hayat\textsuperscript{1}, and B Javidi\textsuperscript{2}

\textsuperscript{1} Center for High Technology Materials and Department of Electrical & Computer Engineering, The University of New Mexico, Albuquerque, NM 87131-1356
\textsuperscript{2} Department of Electrical & Computer Engineering, The University of Connecticut
Storrs, CT 06269-2157
E-mail: hayat@ece.unm.edu

Abstract. The method of photon-counting integral imaging (PCII) has been introduced recently for three-dimensional object sensing, visualization, recognition and classification of scenes under photon-starved conditions. This paper presents an information-theoretic model for the PCII method, thereby providing a rigorous foundation for our understanding of its demonstrated success in compressive imaging and classification.

1. Introduction

It is intriguing that 3D photon counting imagery captures much of the information or content in an image even under photon-starved conditions, as demonstrated by Yeom \textit{et al}. This feature has become particularly evident and useful in the context of image classification, where very good performance is observed even when very few photons per pixel are available, Yeom \textit{et al}.

If we consider the process of imaging by means of a photon-counting array, which we refer to as the method of photon-counting integral imaging (PCII) system, we can identify three main components in the system, the true intensity image or the “source,” the transformation rule or the “channel” and the “output” photon-counting array, which counts the photons impinging on each detector element. We observe that the mutual information between the source and the output is a measure of the change in the information present in an image prior to transmission and that present at the destination \cite{2,3}. In this paper we will use the normalized mutual information metric, the mutual information between the source and output normalized by the source’s entropy, applied to elemental images in the PCII process to investigate the role of spatial correlation in how well image-content is preserved.

2. Problem Formulation

Consider a stochastic column vector $\mathbf{X}$ whose entries, $X_i$, $i = 1, \ldots, n$, are discrete random variables in the interval $[0, 1]$ representing the reflectance of some unknown object or an unknown digital image. The number of photons detected per pixel in a unit integration time, $Y_i$ is a Poisson random variable with mean value $x_i \epsilon N_p$, where $\epsilon \in [0, 1]$ is an attenuation factor that we will use as a control parameter in this study and $N_p$ is the mean number of photons per pixel and per unit integration time.
Under photon starved conditions, the attenuation factor, $\epsilon$, is very small and the photon counts per pixel are either 0 or 1 with high probability. The probability mass function of the photon count $Y_i$, conditional on the pixel’s gray level $X_i = x_i$, takes the simpler form of [5]

$$P_{Y_i|X_i}(j|x_i) = (1 - j)e^{-N_p x_i \epsilon} + j(1 - e^{-N_p x_i \epsilon}), \quad j \in \{0, 1\}.$$  \hfill (1)

3. Results

In all of our calculations we have used Markov models for the source image; these are are very good probabilistic models for capturing spatial correlation. We have also adopted the hard-limited model (1) and formulated the normalized information metric, $\rho$, analytically.

In the one-dimensional case, correlation is introduced in terms of a parameter, $m$, such that as $m$ increases the spatial correlation increases between neighboring pixels. Figure 1(a) shows $\rho$ as a function of the transmission probability, $\epsilon$, parameterized by different spatial correlation indices, $m$. It is seen that performance metric, $\rho$, increases with spatial correlation in the source image. This suggests that the ability of the PCII approach to retain spatial information improves with the spatial correlation in the source image. Namely, the PCII approach seems to be inherently geared toward “images” rather than individual pixels.

![Graph](image)

Figure 1. Results for (a) the Markov chain model and (b) the Markov random field model.

We have also generated MRF images according to the generalized Ising model. We followed the Metropolis sampling algorithm [4] to generate 3-bit images of size $128 \times 128$ with varying spatial-correlation parameter, $\beta$. (The temperature parameter, $T$, is set to 3.) The algorithm is run for 1000 iterations for each image generated. We estimated the entropy and mutual information as in [3]. As in the case of the 1D Markov-chain model, we plot $\rho$ as a function of transmission probability, $\epsilon$, parameterized by $\beta$, as shown in Fig. 1(b). It is seen that $\rho$ increases monotonically with $\epsilon$. Moreover, for a fixed $\epsilon$, $\rho$ increases as $\beta$ is decreased (i.e., as correlation metric is increased). This confirms the above conclusion that the effectiveness of the imaging PCII approach is enhanced with the presence of spatial correlation in the source image.

References

[1] S. Yeom, B. Javidi, and E. Watson, “Three-dimensional distortion-tolerant object recognition using photon-counting integral imaging,” Optics Express 15, 1513–1533 (2007).
[2] T. M. Cover and J. A. Thomas, Elements of Information Theory (John Wiley & sons, 1991).
[3] E. Volden, G. Giraudon, and M. Berthod, “Information in Markov random fields and image redundancy,” in “Selected Papers from the 4th Canadian Workshop on Information Theory and Applications II,” (Springer-Verlag, London, UK, 1996), pp. 250–268.
[4] S. Geman and D. Geman, “Stochastic relaxation, gibbs distributions, and the bayesian restoration of images,” IEEE Trans. Pattern Analysis and Machine Intelligence 6, 721–741 (1984).
[5] S. R. Naravula, M. M. Hayat and B. Javidi, “Information theoretic view of photon-counting integral imaging,” submitted to Optics Express, (2009).