Data space reflectivity full waveform inversion

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Abstract. The full waveform inversion of seismic data aroused the hope to perform simultaneously and in automated way tomography and imaging by solving non-linear least-squares optimization problem. As it has been recognized early, brute force minimization by classical methods is hopeless if low time frequencies are absent in the data. The paper develops a reliable numerical technique for smooth velocity reconstruction via model space decomposition. We present realistic synthetic examples for validating presented algorithm.

1. Introduction

The velocity model building in the depth domain is necessary to guarantee the correct travel-times of wave propagation and therefore is a key element of the up-to-date seismic data processing. As early as the middle of 80th of the last century A. Tarantola introduced the Full Waveform Inversion (FWI) based on the matching the observed and the synthetic seismograms. The L2 norm is widely used for such matching, though other criteria are also considered. To minimize the misfit function and to find the elastic parameters of the subsurface, iterative gradient-based algorithms are usually applied. Such approach to FWI proposed originally by Lailly [1] andTarantola [2] has been developed and studied in a great number of publications (see [3] and the references therein). However, the straightforward application of FWI reconstructs reliably only the reflectivity component of the subsurface but fails to provide a smooth velocity model. The smooth component could not be recovered stably without the presence of extremely low time frequencies. The matter is the shape of the data misfit functional differs a lot with respect to various velocity components: it is nearly quadratic with respect to reflectors, but perturbations of the smooth velocity component (propagator) lead to a very complicated and non-linear behavior (see e.g. [4]). Heuristically it is explained by the so-called "cycle-skip" problem when phase shifts between the recorded and synthetic data produce local minima. In the paper [5] authors introduce some new inversion strategy based on the use of increasing time frequencies: they start inversion with the lowest available ones and at each subsequent iteration they increase the frequency and use the results from the previous step as the initial guess. But they perform this inversion in time domain, hence have to use at each step the low-pass band filtering of data in the time domain which reduces the efficiency of this technique. The next step in implementation of the frequency-domain FWI with increasing time frequencies was done by G. Pratt with co-authors in [6]. They use minimization in time frequency domain and very naturally proceed sequentially from low to high frequencies. The key moment in the successful implementation of this approach is effective solver for the stationary Helmholtz/Elastic waves equation. Unfortunately, this approach did not resolve the problem of stable recovery of a macrovelocity when there is a lack of low time frequencies in the data for both surface and well.
(Vertical Seismic Profile) acquisitions ([4],[7]). Of course, on this base they enhance the role of existing low time frequencies and in some sense improve stability of a propagator recovery, but cannot propose the stable solution of the problem. The alternative to this standard formulation was proposed in the paper [8]. The main message of this alternative is modification of the data misfit functional on the basis of the decomposition of a model space into two subspaces:

- subspace of smooth propagators (macrovelocity components);
- subspace of spatial reflectors, which, in turn, is the image of the subsurface.

This approach was modified for the frequency domain in [9] and was applied for reconstruction of well-known Marmousi model for realistic frequency bandwidth and offsets. In this paper, implementation of this approach for elastic Full Waveform Inversion is presented.

2. Method and theory

2.1. Data Space Reflectivity FWI reformulation

Seismic inverse problem is equivalent to solution of the nonlinear operator equation:

$$ F(m) = d, $$

where $F : M \rightarrow D$ is a nonlinear forward operator, associated with the elastic waves equation, which maps model space $M$ into data space $D$. As in acoustic MBTT case [8] the essence of this formulation is to decompose the velocity model into two constituents: smooth propagators $p$ and oscillating reflector $r$. Since we search for both $V_p$ and $V_s$ velocity models, in fact $p$ is a vector $p = (pV_p, pV_s)$, where $pV_p$ – $P$-velocity propagator and $pV_s$ – $S$-velocity propagator. In turn, reflector is treated as the result of true amplitude migration applied to the part of the data called data-space reflectivity (DSR) – preimage of the depth reflector in data space:

$$ m = p + r = M(p)s. $$

The key moment of this decomposition is propagator-reflector interrelation $r = M(p)s$ with operator $M(p)$ being some kind of true-amplitude prestack migration/linearized inversion. In particular, reweighted version of adjoint operator based migration:

$$ M(p)s = W \circ \Re\{DF^*s\} $$

Here $DF$ is Frechet derivative of the full nonlinear map $F$, $*$ means adjoint operator and $W$ is some linear operator providing true amplitudes imaging/migration. This kind of the model space decomposition leads to the following modified data misfit functional:

$$ E(p, s) = \|F(p + M(p)s) - d\|^2_D. $$

Minimization with respect to propagator $p$ and data-space reflectivity $s$ is performed independently and by turn. We start with admitting $s = d$ and do search for some intermediate value of propagator $p$. After stabilization of this process, the search is switched to data-space reflectivity $s$ and so on.

The search for propagator is implemented using Gauss-Newton method [6]. Since the dimension of propagator space is small enough with respect to the dimension of the whole model space $M$, we can calculate partial SVD (only highest singular vectors which correspond the highest singular values) of the approximate Hessian $H_a$ on the first iteration using matrix-free methods (such as Krylov-subspace based methods, for numerical implementation see [10]).
Then we use r-pseudoinverse of this operator \([11]\) as a preconditioner in Conjugate-Gradient method during propagator minimization:

\[
p_{k+1} = p_k + \mu_k S_k, \quad S_0 = \nabla_0,
\]

\[
S_k = -P\nabla_k - \frac{<P\nabla_k, \nabla_k - \nabla_{k-1}>_M}{<P\nabla_k, \nabla_{k-1}>_M} S_{k-1},
\]

where \(p_k = (pV_p, pV_s)\) – propagator on k-th iteration, \(P = (H_a)^\dagger\) – pseudoinverse of approximate Hessian. The gradient \(\nabla_k\) with respect to the propagator unknown is calculated as follows:

\[
\nabla_k = \Re\{DF^* \delta d_k + (D^2F(p_k)W^*DF^*\delta d_k)*s\}
\]

(7)

here \(\delta d_k = F(m_k) - d\) is data residual on current iteration, \(DF\) – first derivative of forward map calculated at point \(m_k = p_k + M(p_k)s\) and \(D^2F\) – second derivative calculated at point \(p_k\). Incorporation into inversion process Hessian \(H_a\) helps to mitigate crosstalk between \(V_p\) and \(V_s\) models. Minimization of the data-space reflectivity \(s\) is done via steepest descent method. The corresponding gradient \(\nabla_s\) is calculated as follows:

\[
\nabla_s = M^*(p)DF^*\delta d,
\]

(8)

3. Numerical results

3.1. Experiment 1

We use Gullfaks South [12] model to validate the approach based on propagator/reflector decomposition. 2D elastic model one can see in the Fig. 1 (left column). We would like to pay the attention that \(\gamma = \frac{V_p}{V_s}\) is not a constant, that is there no dependence between \(V_p\) and \(V_s\).

Input data are generated for the set of 19 uniformly sampled frequencies in the range \(5 - 20\) Hz. The acquisition system has 39 vertical point force sources and 200 2C-receivers located at surface \(z = 0m\) with a lateral spacing of 100m and 20m respectively. We start with depth dependent initial model (1, middle column) which is far from the real velocities. 2D B-Splines of order 3 were used as the basis of the smooth propagator space. Doing minimization, finally, we come to the result which fairly well coincides with original model for all three parameters: \(V_p, V_s\) and \(\gamma\) (1, right column).

The data comparison is demonstrated on Fig. 2. Synthetics are generated for the shot located at \(x = 700m\), source wavelet is Ricker with dominant frequency \(10\) Hz. As it can be clearly observed the reconstructed velocity model describes main part of the data for both vertical and horizontal receiver components.

3.2. Experiment 2

Finally, we consider the realistic example based on the Marmousi2 elastic synthetic velocity model [13]. Input data are synthesized in frequency domain for the set of twenty uniform frequencies in the range \(5 - 15Hz\). We choose the lowest temporal frequency equal to \(5Hz\) following some publications and discussions with colleagues dealing with real seismic data. The acquisition system has 46 volumetric sources and 460 receivers located at the top of the model with lateral spacing of 200m and 20m respectively. As initial guess for propagator, we use the vertically heterogeneous model (see Fig. 3). As an initial guess for the time reflectivity variable are used the observed data itself. The results of DSR FWI we present in Fig. 3. As in previous example, we search for the propagator in the space of 2D B-Splines functions of order 3. This guarantees the smoothness of the solution and reasonable size of the model space. To minimize
Figure 1. Gullfaks velocity model. Left column from top to bottom: original Vp, Vs and Vp/Vs models. Middle column: initial Vp, Vs and Vp/Vs models. Right column: reconstructed Vp, Vs and Vp/Vs velocity models after DSR FWI.

Figure 2. Synthetics generated for shot located at position $x = 700m$. Source wavelet Ricker with dominant frequency 10Hz. From the left to the right: vertical component for true model and for recovered model, horizontal component for true model and for recovered model.
the cost function we use the projected conjugate gradient method, with orthogonal $L_2$-projector onto smooth space (B-splines). To compute true-amplitude migration $M$ we apply the imaging procedure on the base of inverse generalized Radon transform with representation of the Greens function as decomposition of Gaussian beams. As one can see, local reconstruction of Marmousi velocity model is good enough.

Next, to evaluate the quality of the macrovelocity (propagator) reconstruction we generate common image gathers (CIGs) in the offset-depth domain (see Fig. 4). These CIGs are computed for the initial model and for the model reconstructed via DSR FWI. Reconstructed model significantly improve the flatness of the events in the CIGs with respect to the initial model CIGs. This proves the reliability of the velocity model reconstruction.

4. Conclusions
We introduce DSR FWI modified data misfit functional on the base of propagator/reflector decomposition of the model space. This decomposition provides alternative way to perform full waveform inversion. We do inversion separately for the search of smooth macrovelocity (reduced model dimension) and sharp reflectors (classical true amplitude prestack migration for known macrovelocity). By this approach we succeeded to reconstruct Gullfaks South model and the Marmousi2 model without low time frequencies we used frequencies from $5Hz$ to $20Hz$ only and do not use huge offsets.

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Figure 4. Common image gathers (CIGs) at positions X=[3.5:0.01:3.6] km from the initial guess (left) and model obtained via DSR FWI

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