COULOMB GAUGE IN ONE-LOOP QUANTUM COSMOLOGY

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Abstract. The well-known discrepancies between covariant and non-covariant formalisms in quantum field theory and quantum cosmology are analyzed by focusing on the Coulomb gauge for vacuum Maxwell theory. On studying a flat Euclidean background with boundaries, the corresponding mode-by-mode analysis of one-loop quantum amplitudes agrees with the results of the Schwinger-DeWitt technique and of mode-by-mode calculations in relativistic gauges.

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The quantization program for gauge fields and gravitation in the presence of boundaries is receiving careful consideration in the recent literature [1-12]. The motivations for this analysis come in part from the quantization of closed cosmologies [13-14], and in part from the need to understand the relation between different quantization techniques in field theory [15-16]. The choices to be made are as follows: (i) quantization technique; (ii) background 4-geometry; (iii) boundary 3-geometry; (iv) boundary conditions respecting BRST invariance and local supersymmetry; (v) gauge condition; (vi) regularization technique.

For a given choice of field theory, background and corresponding boundary 3-geometry, different quantization techniques and different gauge conditions have led to discrepancies for the semi-classical evaluation of quantum amplitudes [1-12]. This calculation has been performed within the framework of $\zeta$-function regularization, where the $\zeta(0)$ value yields both the scaling properties of the amplitudes and one-loop divergences of physical theories. On reducing a field theory with first-class constraints to its physical degrees of freedom before quantization [1-12], the resulting $\zeta(0)$ values disagree with the Schwinger-DeWitt $A_2$ coefficient [17-18]. This occurs both for compact Riemannian 4-manifolds without boundary [19-21] and for Riemannian backgrounds with boundary [1-12]. Moreover, further discrepancies have been found on studying the quantum theory of spin-$1/2$ fields at one-loop about background 4-geometries with boundaries [5,22-25].

It therefore seems that the discrepancies found in the literature have at least two origins, as follows.

(i) A geometrical source, owed to the singularity at the origin for manifolds with just one boundary. By this we mean that there is no regular vector field inside matching the normal at the boundary. Hence the normal and tangential components of physical fields inside are ill-defined, and it is impossible to achieve a consistent 3+1 split. Analogously, such a split cannot be obtained on compact 4-manifolds without boundary, if their Euler number does not vanish.

(ii) A field-theoretical source, i.e. non-physical degrees of freedom and ghost modes yield contributions to $\zeta(0)$ which do not cancel each other on considering curved backgrounds and/or the presence of boundaries.
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For example, for manifolds with just one boundary, the mode-by-mode analysis of Faddeev-Popov amplitudes, which relies on the expansion in harmonics of the potential (e.g. eqs. (4)-(5)) and hence on the eigenvalue equations for the modes multiplying each harmonic, disagrees with the Schwinger-DeWitt evaluation of the same amplitudes and is gauge-dependent. Moreover, on manifolds with two boundaries admitting a consistent 3+1 split, the physical degrees of freedom of vacuum Maxwell theory, i.e. the transverse part of the potential, do not enable one to recover the full $\zeta(0)$ value [26]. These calculations, when combined with a previous analysis of discrepancies for spin-$\frac{1}{2}$ fields [24-25], have motivated a mode-by-mode analysis of Euclidean Maxwell theory and linearized gravity [26-27] within the framework of Faddeev-Popov formalism [28-29] on manifolds with two boundaries. Remarkably, in this case the Schwinger-DeWitt and mode-by-mode formalisms for BRST-covariant Faddeev-Popov amplitudes are found to agree. While our previous analysis focused on relativistic gauges for such theories, it appears necessary to complete this investigation by considering non-relativistic gauges as well. Hence we here study the Coulomb gauge for vacuum Maxwell theory about flat Euclidean backgrounds with two boundaries, since such a gauge choice is more relevant for reduction to physical degrees of freedom and Hamiltonian formalism [30].

For this purpose, we use the version of $\zeta$-function technique [9,31] elaborated in refs. [6-8]. Following refs. [6-8], we write $f_n(M^2)$ for the function occurring in the equation obeyed by the eigenvalues by virtue of boundary conditions, and $d(n)$ for the degeneracy of the eigenvalues parametrized by the integer $n$. One then defines the function

$$ I(M^2, s) \equiv \sum_{n=n_0}^{\infty} d(n) \ n^{-2s} \ \log f_n(M^2) . $$

Such a function has an analytic continuation to the whole complex-$s$ plane as a meromorphic function, i.e.

$$ I(M^2, s) = \frac{I_{\text{pole}}(M^2)}{s} + I^R(M^2) + O(s) . $$

The $\zeta(0)$ value is then obtained as [6-8]

$$ \zeta(0) = I_{\log} + I_{\text{pole}}(\infty) - I_{\text{pole}}(0) , $$

(3)
where $I_{\text{log}}$ is the coefficient of $\log M$ from $I(M^2, s)$ as $M \to \infty$, and $I_{\text{pole}}(M^2)$ is the residue at $s = 0$.

To perform the one-loop analysis of vacuum Maxwell theory, we expand the normal and tangential components of the electromagnetic potential on a family of 3-spheres. With the notation of [2,9-10,26], one writes

$$A_0(x, \tau) = \sum_{n=1}^{\infty} R_n(\tau) \, Q^{(n)}(x) ,$$

$$A_k(x, \tau) = \sum_{n=2}^{\infty} \left[ f_n(\tau) \, S^{(n)}_k(x) + g_n(\tau) \, P^{(n)}_k(x) \right] .$$

Of course, $Q^{(n)}(x), S^{(n)}_k(x)$ and $P^{(n)}_k(x)$ are scalar, transverse and longitudinal harmonics on $S^3$ respectively [1-2,32].

Within the Faddeev-Popov quantization scheme [10,28-29], after adding to the Euclidean Lagrangian the Coulomb gauge-averaging term $\frac{1}{2\alpha} \tau^{-4} \left( A_i \right)^2$, one finds eigenvalue equations on taking variations of the total Euclidean action with respect to the modes $f_n, g_n, R_n$ of eqs. (4)-(5) [10,26]. If $\alpha = 1$, these equations take the following form (the decoupled mode will be treated separately):

$$\left( \frac{d^2}{d\tau^2} + \frac{1}{\tau} \frac{d}{d\tau} - \frac{n^2}{\tau^2} + \lambda_n \right)f_n(\tau) = 0 ,$$

$$\tilde{A}_n g_n(\tau) + \tilde{B}_n R_n(\tau) = 0 ,$$

$$\tilde{C}_n g_n(\tau) + \tilde{D}_n R_n(\tau) = 0 ,$$

where [33]

$$\tilde{A}_n \equiv \frac{d^2}{d\tau^2} + \frac{1}{\tau} \frac{d}{d\tau} - \frac{(n^2 - 1)}{\tau^2} + \lambda_n ,$$

$$\tilde{B}_n \equiv -(n^2 - 1) \left( \frac{d}{d\tau} + \frac{1}{\tau} \right) ,$$

$$\tilde{D}_n \equiv -(n^2 - 1) \left( \frac{d}{d\tau} - \frac{1}{\tau} \right) .$$
\[ \hat{C}_n \equiv \frac{1}{\tau^2} \frac{d}{d\tau} , \]
\[ \hat{D}_n \equiv -\left( \frac{n^2 - 1}{\tau^2} \right) + \lambda_n . \]

Moreover, in the Coulomb gauge, the ghost eigenvalue equation is found to be
\[ \frac{(n^2 - 1)}{\tau^2} \epsilon_n(\tau) = \lambda_n \epsilon_n(\tau) \quad \forall n \geq 1 , \]
where the modes \( \epsilon_n \) are the ones occurring in the expansion on a family of 3-spheres of the scalar field \( \epsilon(x, \tau) \) as \( \sum_{n=1}^{\infty} \epsilon_n(\tau) Q^n(x) \). Their contribution to \( \zeta(0) \) is finally multiplied by \(-2 \) \([9-10,26]\).

We here use magnetic boundary conditions \([3,10,26]\). Hence \( A_k(x, \tau) \) is set to zero at the 3-sphere boundaries as well as the Coulomb gauge \( \Phi_c(A) \equiv \tau^{-2} A_i \), and correspondingly the ghost modes \( \epsilon_n \) \([10,26]\).

The solution of eq. (6) is \( f_n = A_n I_n(M\tau) + B_n K_n(M\tau) \), where \( M^2 = -\lambda_n \). By virtue of our boundary conditions, such a solution should vanish at \( \tau = \tau_- \) and at \( \tau = \tau_+ \), where \( \tau_- , \tau_+ \) are the radii of the two concentric 3-sphere boundaries. Hence the corresponding eigenvalue condition can be written as
\[ I_n(M\tau_-)K_n(M\tau_+) - I_n(M\tau_+)K_n(M\tau_-) = 0 . \]

By using the technique described in refs. \([6-8]\) and outlined in eqs. (1)-(3), one can show that the contribution of transverse modes to \( \zeta(0) \) is \([26]\)
\[ \zeta_{\text{phys}}(0) = -\frac{1}{2} . \]

From eq. (13) one can see that the only non-vanishing contribution from ghost modes is obtained when \( n = 1 \), which implies that \( \lambda_n = 0 \). The corresponding eigenfunctions are arbitrary functions of \( \tau \) which obey homogeneous Dirichlet conditions at \( \tau = \tau_- \) and at \( \tau = \tau_+ \). The general eigenfunction can be written as
\[ \epsilon_1(\tau) = \sum_{k=1}^{\infty} c_k \sin \left( \frac{\pi k(\tau - \tau_-)}{\tau_+ - \tau_-} \right) . \]
Note that we here face an entirely new situation, in that we have to deal with an infinite number of zero-modes. The inclusion of zero-modes into the general expression for $\zeta(0)$ was studied in mathematical papers [34] and for the problems of quantum gravity as well [35-36]. It is known that when we have a finite number of zero-modes we have simply to add it to the $\zeta(0)$ value, however, we do not know a priori what should one do with an infinite number of such modes. Hence we need the appropriate regularization of an infinite number of our zero-modes. We try to achieve this being inspired by the ideas of $\zeta$-regularization technique. For this purpose, we point out that the eigenfunctions (15) belong to the space whose elements can be parametrized by the natural numbers $k = 1, 2, ...,$. All these eigenfunctions can be treated on equal footing. Thus, we are led to define the regularized dimension of this space as the regularized number of its basis elements, or, within the framework of $\zeta$-function technique (see below), as $\zeta_R(0) = -\frac{1}{2}$, where $\zeta_R(s) \equiv \sum_{n=1}^{\infty} n^{-s}$ is the usual Riemann $\zeta$-function, whose properties and values are well-known [37]. Thus

$$\zeta_{\text{ghost}}(0) = -\frac{1}{2}.$$  

(16)

Of course, infinitely many definitions of regularized dimension are possible on considering the zeta-functions

$$\zeta_{R,a}(s) \equiv \sum_{n=1}^{\infty} (n + a)^{-s},$$

where $a$ is a real parameter. Our choice corresponds to the value $a = 0$, and it appears plausible to take into account that the sine functions of eq. (15) are labelled by the integers $k = 1, 2, 3...$ only.

By virtue of our boundary conditions, coupled gauge modes vanish at the boundaries, i.e. $g_n(\tau_-) = g_n(\tau_+) = 0, R_n(\tau_-) = R_n(\tau_+) = 0$. The boundary conditions

$$R_n(\tau_+) = R_n(\tau_-) = 0 \quad \forall n \geq 1$$

can be obtained from a non-trivial application of gauge invariance. In other words, one may start from a relativistic gauge condition written in the form (cf. [33])

$$\Phi(A) \equiv \lambda A_0 \ Tr K + (3)^i A_i,$$

(17)
where $\lambda$ is a dimensionless parameter and \((3)\nabla_i A_i = \tau^2 A_i\). Now it is well-known that magnetic boundary conditions in relativistic gauges $\Phi_R$ imply that any such $\Phi_R$ should vanish at the boundaries [26]. When combined with homogeneous Dirichlet conditions on the tangential components of the electromagnetic potential, which require that

$$\begin{align*}
    f_n(\tau) = f_n(-\tau) = 0, \quad g_n(\tau) = g_n(-\tau) = 0, \quad \forall n \geq 2,
\end{align*}$$

this implies that $A_0(x, \tau) = A_0(x, -\tau) = 0$, i.e.

$$\begin{align*}
    R_n(\tau) = R_n(-\tau) = 0 \quad \forall n \geq 1.
\end{align*}$$

If gauge invariance is respected in quantum theory, this set of conditions holds for all values of $\lambda$. Hence, on taking the limit as $\lambda \to 0$ in (17), one recovers the Coulomb gauge we are interested in, subject to the same boundary conditions on $g$-modes and $R$-modes, providing gauge invariance holds. The legitimacy of this procedure depends crucially on a direct proof of gauge invariance in the presence of boundaries, not relying on the formal arguments frequently presented in the literature, and is the object of a paper in preparation by ourselves and other co-authors [33]. We are then able to obtain, in particular, boundary conditions on the decoupled mode $R_1$, which would otherwise remain totally arbitrary.

Moreover, eqs. (7)-(12) imply that

$$\begin{align*}
    R_n(\tau) = \frac{\dot{g}_n(\tau)}{\left[(n^2 - 1) + M^2 \tau^2\right]},
\end{align*}$$

where $g_n(\tau)$ obeys the second-order differential equation

$$\begin{align*}
    &\left[(n^2 - 1) + M^2 \tau^2\right] M^2 \tau^2 \frac{d^2 g_n}{d\tau^2} + \left[3(n^2 - 1) + M^2 \tau^2\right] M^2 \tau \frac{dg_n}{d\tau} \\
    &- \left[(n^2 - 1) + M^2 \tau^2\right]^2 \frac{(n^2 - 1)}{\tau^2} g_n \\
    &- M^2 \left[(n^2 - 1) + M^2 \tau^2\right] g_n = 0.
\end{align*}$$

Note that, if $M = 0$, the limiting form of (19) is

$$\begin{align*}
    \frac{(n^2 - 1)}{\tau^2} g_n = 0 \quad \forall n \geq 2,
\end{align*}$$
whose only solution is \( g_n(\tau) = 0, \forall n \geq 2 \) and \( \forall \tau \in [\tau_-, \tau_+] \). If \( M \) does not vanish, we may regard (19) as a second-order ordinary differential equation. Remarkably, one then deals with an overdetermined problem, in that both \( g_n \) and \( \dot{g}_n \) have to vanish at the boundaries in the light of boundary conditions and of eq. (18). Hence the only solution is the trivial one, i.e. \( g_n(\tau) = 0 \ \forall \tau \in [\tau_-, \tau_+] \) and \( \forall n \geq 2 \), and similarly for \( R_n(\tau), \forall n \geq 2 \). Thus, coupled gauge modes give a vanishing contribution to the full \( \zeta(0) \):

\[
\zeta_{\text{coupled}}(0) = 0
\]  

(20)

By contrast, the decoupled mode \( R_1(\tau) \) can be represented by an arbitrary function vanishing at \( \tau = \tau_- \) and at \( \tau = \tau_+ \). Hence, one can apply again the argument leading to eq. (16), which implies

\[
\zeta_{R_1}(0) = -\frac{1}{2}
\]  

(21)

By virtue of eqs. (14), (16), (20)-(21) one finds

\[
\zeta(0) = \zeta_{\text{phys}}(0) + \zeta_{\text{coupled}}(0) + \zeta_{R_1}(0) - 2\zeta_{\text{ghost}}(0) = 0
\]  

(22)

One can also consider another approach to boundary conditions for the electromagnetic field subject to the Coulomb gauge. For this purpose, we can view the normal component \( A_0 \) of the electromagnetic field as a Lagrange multiplier which should not be included in the gauge conditions and which should be integrated over not only inside the manifold under consideration, but also on its boundaries [30]. In this case the homogeneous mode \( R_1 \) should be excluded, since it does not correspond to any constraint (really, the constraint \( \nabla_i F^{0i} \) of the electromagnetic field has no homogeneous modes). Moreover, homogeneous ghost modes should also be discarded, because no homogeneous modes correspond to the Coulomb gauge condition (the problem of discarding ghost zero-modes was discussed in a slightly different framework in [38]).

Thus, in such an approach to the problem of boundary conditions and the treatment of Lagrange multipliers and ghost modes, there are no non-zero contributions to the full \( \zeta(0) \) from the decoupled mode and from ghost zero-modes (cf. eqs. (16), (21)). However, in this case we have a non-trivial contribution from the coupled gauge modes. Hence
our second-order equation (19) is no longer overdetermined, since there are no boundary conditions on $R_n$ (see (18)), and we have a Dirichlet boundary-value problem involving $g_n$-modes only. One can then show that the basis function of eq. (19) can be represented as

$$g_n(\tau) = w_\nu(M\tau) v_n(M\tau) x_n(M, \tau) ,$$

where $w_\nu$ is a linear combination of modified Bessel functions $I_\nu$ and $K_\nu$ with $\nu^2 \equiv 2(n^2-1)$. Then

$$v_n(M\tau) = \sqrt{(n^2-1) + M^2\tau^2} \over M^2\tau^2$$

and $x_n(M, \tau)$ obeys the equation

$$w_\nu \left[ \frac{\partial^2 x_n}{\partial \tau^2} + \frac{1}{\tau} \frac{\partial x_n}{\partial \tau} \right] + F(n, M, \tau) x_n + 2 \frac{d w_\nu}{d \tau} \frac{\partial x_n}{\partial \tau} = 0 ,$$

where

$$F(n, M, \tau) \equiv -\frac{(n^2-1)}{M^2\tau^4} \left[ (n^2-1) + \left(1 + \frac{(n^2-1)}{M^2\tau^2}\right)^{-1} \left( \frac{3(n^2-1) + M^2\tau^2}{(n^2-1) + M^2\tau^2} \right) \right]$$

$$- \frac{(n^2-1)}{M^2} \left(1 + \frac{(n^2-1)}{M^2\tau^2}\right)^{-1} \times$$

$$\times \left[ \frac{3}{\tau^4} + \frac{(n^2-1)}{M^2\tau^6} \left(1 + \frac{(n^2-1)}{M^2\tau^2}\right)^{-1} \right] .$$

The solution of (24) has the following asymptotic forms:

$$x_n(M, \tau) \sim A_n + \frac{B_n(\tau)}{M^2} \text{ as } M \to \infty ,$$

where $A_n$ is a constant, and

$$x_n(M, \tau) \sim \exp \left[ -\frac{(n^2-1)}{M\tau} \right] \text{ as } M \to 0 .$$

Hence one can show that the only function in the product (23) which can give a non-trivial contribution to $\zeta(0)$ is $w_\nu$, and its contribution can be easily calculated along the
lines described in refs. [6-8,24-27] and applied above to the calculation of the contribution of physical modes (eq. (14)). This leads to

$$\zeta_{\text{coupled}}(0) = \frac{1}{2}.$$  \hfill (26)

Combining eqs. (14) and (26) one has again

$$\zeta(0) = \zeta_{\text{phys}}(0) + \zeta_{\text{coupled}}(0) = 0 .$$

Thus, our result obtained with another treatment of boundary conditions coincides with the one resulting from eq. (22).

Interestingly, our result for the full $\zeta(0)$ agrees with previous calculations for relativistic gauges [26] and with the $A_2$ coefficients of the Schwinger-DeWitt technique [26]. For our manifold with two 3-sphere boundaries, the general formulae for the Schwinger-DeWitt $A_2$ coefficient [3,5,18] yield zero, since the contributions of the two boundaries cancel each other.

We should emphasize that part of our analysis relies on the Riemann-$\zeta$ regularization of the infinite dimension of the space of zero-modes for the decoupled component of $A_0$ and for the ghost field. We can only say that this procedure leads to the remarkable cancellation occurring in (22), but we cannot as yet put it on more rigorous grounds. Moreover, our boundary conditions in the Coulomb gauge result, in the first approach, from a non-trivial limiting procedure out of the well-established set of boundary conditions imposed on studying relativistic gauges. Such a limiting procedure leads to a non-vanishing contribution to $\zeta(0)$ resulting from ghost modes. As it is well-known, this non-vanishing contribution is absent in the case of flat space without boundaries. The ultimate justification of our approach can be obtained by proving in a direct way the gauge invariance of quantum amplitudes for problems with boundaries, rather than assuming it. A detailed analysis of such a problem can be found in ref. [33]. It seems encouraging, however, that boundary conditions for the Coulomb gauge derived from the familiar, non-relativistic framework, lead to a $\zeta(0)$ value equal to the one in (22).
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In agreement with recent work by the authors [26-27], our analysis of the Coulomb gauge adds evidence in favour of gauge- and ghost-modes being necessary to obtain gauge-invariant amplitudes for manifolds with boundaries. In general, there is no exact cancellation of such contributions on non-trivial backgrounds. Moreover, we should say that a very recent paper by Moss and Poletti [39], relying on previous work by Vassilevich [40], has corrected all previous Schwinger-DeWitt calculations for spin $>0$ for manifolds with boundaries. Hence they find full agreement between our investigations of the conformal anomalies for spin-$\frac{1}{2}$ and spin-1 fields [22-26], and their corrected analysis, even in the 1-boundary case. However, the problem remains of understanding why, in the 1-boundary case only, the mode-by-mode evaluation of one-loop quantum amplitudes is gauge-dependent, as shown in [26]. As far as we can see, this last remaining discrepancy seems to point out to some serious limitations which apply when the background 4-geometry does not admit a well-defined 3+1 decomposition.

The problem of the correct choice of technique for one-loop calculations in quantum cosmology is also important when one investigates the properties of the wave function of the universe [13-14]. These properties are crucial for studying the relation between quantum gravity, inflationary cosmology and particle physics [41-42].

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