Global phase diagram and quantum spin liquids in spin-$1/2$ triangular antiferromagnet

Shou-Shu Gong$^1$, W. Zhu$^2$, J.-X. Zhu$^{2,3}$, D. N. Sheng$^4$, and Kun Yang$^5$

$^1$National High Magnetic Field Laboratory, Florida State University, Tallahassee, FL 32310
$^2$Theoretical Division, T-4 and CNLS, Los Alamos National Laboratory, Los Alamos, NM 87545
$^3$Center for Integrated Nanotechnologies, Los Alamos National Laboratory, Los Alamos, NM 87545
$^4$Department of Physics and Astronomy, California State University, Northridge, CA 91330
$^5$National High Magnetic Field Laboratory and Department of Physics, Florida State University, Tallahassee, FL 32306

We study the spin-$1/2$ Heisenberg model on the triangular lattice with the nearest-neighbor $J_1 > 0$, the next-nearest-neighbor $J_2 > 0$ Heisenberg interactions, and the additional scalar chiral interaction $J_\chi (\vec{S}_i \times \vec{S}_j) \cdot \vec{S}_k$ for the three spins in all the triangles using large-scale density matrix renormalization group calculation on cylinder geometry. With increasing $J_2/(J_1 \lesssim 0.3)$ and $J_\chi (J_1/\chi \leq 1.0)$ interactions, we establish a quantum phase diagram with the magnetically ordered 120° phase, stripe phase, and non-coplanar tetrahedral phase. In between these magnetic order phases, we find a chiral spin liquid (CSL) phase, which is identified as a $\nu = 1/2$ bosonic fractional quantum Hall state with possible spontaneous rotational symmetry breaking. By switching on the chiral interaction, we find that the previously identified spin liquid in the $J_1 - J_2$ triangular model $(0.08 \lesssim J_2/J_1 \lesssim 0.15)$ shows a phase transition to the CSL phase at very small $J_\chi$. We also compute spin triplet gap in both spin liquid phases, and our finite-size results suggest large gap in the odd topological sector but small or vanishing gap in the even sector. We discuss the implications of our results to the nature of the spin liquid phases.

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I. INTRODUCTION

Quantum spin liquid (QSL) is one kind of long-range entangled states with fractionalized quasiparticles$^1$. Since the proposal by P. W. Anderson, the concept of QSL has been playing an important role for understanding strongly correlated materials and unconventional superconductors$^2$. Although QSLs have been pursued for more than two decades$^{3–9}$, only recently such novel states have been found in realistic spin models$^{10–23}$, in which geometric frustration and competing interactions play important roles for developing spin liquid states.

One of the most promising spin liquid candidates is the antiferromagnet on the corner-sharing kagome lattice. Experimentally, spin liquid-like behaviors have been observed in several kagome materials such as herbertsmithite$^{24–28}$. Theoretically, the most extensively studied kagome model is the spin-$1/2$ kagome Heisenberg model with the nearest-neighbor (NN) interaction. Thanks to the recent large-scale Density Matrix Renormalization Group (DMRG) simulations$^{29,30}$, conventional orders have been excluded, leading to a QSL ground state. However, the nature of this spin liquid is still in debate. DMRG calculations suggest a gapped spin liquid$^{30–31}$, seemingly consistent with a $Z_2$ topological order$^{30,31}$. Recent tensor network state simulations identify the $Z_2$ topological order of the obtained variational wavefunction$^{32}$, but so far the four degenerate ground states of the putative $Z_2$ QSL have not been found in exact diagonalization (ED)$^{33,34}$ and DMRG calculations, leaving this problem open. On the other hand, variational studies based on the fermionic parton wavefunctions find the gapless $U(1)$ Dirac spin liquid rather than the gapped $Z_2$ spin liquid with the optimized variational energy$^{35–37}$. Very recently, tensor renormalization group$^{38,39}$ and DMRG$^{40}$ calculations also suggest the gapless spin liquid as a strong candidate. Interestingly, studies on the modified kagome models$^{41–43}$ find that the kagome spin liquid emerges near the phase boundaries of several ordered phases, suggesting possible strong competitions of the different physics in the kagome spin liquid regime. In particular, a fully gapped chiral spin liquid (CSL)$^{41,45}$ is found by switching on small further-neighbor$^{15,16}$ or chiral interactions$^{15}$ on the NN kagome model.

Another promising spin liquid candidate is the antiferromagnet on the edge-sharing triangular lattice. Although frustration is present in the spin-$1/2$ NN triangular model, it turns out to still exhibit a 120° antiferromagnetic order$^{46,47}$. In the recent experiments on the triangular organic Mott insulators such as κ-(ET)$_2$Cu$_2$(CN)$_3$ and EtMg$_2$Sb[Pd(dmit)$_2$]$_2$56–58, spin liquid-like behaviors have been found. Theoretically, multi-spin exchange interactions, which can lead to the gapless spin Bose metal with a large spinon Fermi surface53–55 and the gapless spin liquid with a quadratic band touching56,57, depending on the strength of interaction, and the space anisotropic interaction58–61, have been suggested to understand the spin liquid behaviors in triangular materials.

Recently, a new spin liquid phase is found in the spin-$1/2$ triangular Heisenberg model with the NN $J_1$ and the next-nearest-neighbor (NNN) $J_2$ interactions for $0.08 \lesssim J_2/J_1 \lesssim 0.15$, which is sandwiched by a 120° magnetic phase and a stripe magnetic order phase$^{56,62–67}$. This frustrating $J_2$ interaction is considered as a possible mechanism to understand the spin liquid behaviors of the newly synthesized triangular materials YbMgGaO$_4$68 and Ba$_3$In$_2$O$_6$69. For this $J_1 - J_2$ model, DMRG calculations on cylinder system find the evidence of spin liquid including the two near-degenerate ground states in the even and odd topological sectors whose energy difference decays rapidly with growing cylinder width, and the fractionalized spin-$1/2$ quasiparticle revealed by inserting flux simulation and entanglement spectrum (ES)$^{64–66}$. On the finite-size DMRG calculations, the spin triplet gap measured above the overall ground state (in the odd sector) is big.
(\(\Delta_T \sim 0.3 J_1\)) seemingly consistent with a gapped spin liquid\(^{70,71}\). Nonetheless, the even and odd sectors show some distinct features in finite-size DMRG calculation. While the odd sector shows a short correlation length that could be consistent with the large gap, the even sector exhibits a much larger gap\(^{65,66}\), which may suggest smaller gap in the even sector. The low-lying entanglement spectrum in the even sector shows a Dirac node like structure, which is suggested as an implication of gapless spinon excitations\(^{66}\). The different DMRG results in the two sectors reasonably imply that either the putative gapped spin liquid is not yet well developed as the strong finite-size effects in numerical calculation, or a gapless spin liquid is possible. In the variational study, a U(1) Dirac gapless spin liquid indeed possesses the best variational energy\(^{67}\). The nature of this spin liquid remains an open question. To shed more light on this spin liquid phase, the modified \(J_1 - J_2\) triangular models have been investigated\(^{72-75}\). Interestingly, the variational\(^{71}\) and ED calculations\(^{74}\) suggest a possible CSL at the neighbor of the \(J_1 - J_2\) spin liquid, which seems to be similar to the situation in the kagome model and deserves more studies. Besides, the quantum phase transition between the two spin liquid phases is also far from clear.

In this article, we study the spin-1/2 \(J_1 - J_2\) Heisenberg model on the triangular lattice with additional time-reversal symmetry (TRS) breaking chiral interaction \(J_\chi\) using DMRG simulations. The model Hamiltonian is given as

\[
H = J_1 \sum_{(i,j)} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{(i,j)} \vec{S}_i \cdot \vec{S}_j + J_\chi \sum_{\triangle/\bigtriangledown} (\vec{S}_i \times \vec{S}_j) \cdot \vec{S}_k,
\]

where \(J_1\) and \(J_2\) denote the NN and the NNN interactions, respectively. The scalar chiral interaction \(J_\chi\) has the same magnitude for all the up (\(\triangle\)) and down (\(\bigtriangledown\)) triangles, and the three sites \(i, j, k\) for \(J_\chi\) follow the clockwise order in all the triangles as shown in Fig. 1(a). Physically, the scalar chiral interaction \(J_\chi\) term can be induced in the Hubbard model with large \(U\) in a magnetic field\(^{66,77}\). Starting from the Hubbard model, a \(t/U\) \((t\ and\ U\ are\ the\ hopping\ and\ interaction\ respectively)\) expansion to the second order at half filling gives the effective chiral interaction \(J_\chi (\vec{S}_i \times \vec{S}_j) \cdot \vec{S}_k\) with \(J_\chi \sim \Phi^3/U^2\), where \(\Phi\) is the magnetic flux enclosed by the triangle. We take \(J_1 = 1.0\) as the energy scale. Using DMRG simulation, we obtain a quantum phase diagram as shown in Fig. 1(d). Besides the 120° Neél phase, the stripe phase, and the time-reversal invariant spin liquid in the \(J_1 - J_2\) model (here we denote it as \(J_1 - J_2\ SL\)), we find a large regime of the non-coplanar tetrahedral order for large \(J_\chi\), whose spin configuration is shown in Fig. 1(c). Below the tetrahedral phase for \(J_2 \lesssim 0.25\), we identify a CSL as the \(\nu = 1/2\) bosonic fractional quantum Hall state by observing the gapless chiral edge mode. The strong nematic order of bond energy suggests a possible spontaneous lattice rotational symmetry breaking and implies an emergent nematic CSL. By studying the spin triplet gap and entanglement spectrum, we observe a transition from the \(J_1 - J_2\ SL\) to the CSL at small chiral interaction. While we find a large spin triplet gap above the overall ground state (in the odd sector) in the CSL phase, the small triplet gap in the even sector suggests that on our studied system size the topological nature in the even sector may not have been fully developed. A possible reason is that this CSL regime generated by increasing \(J_\chi\) is near the phase boundaries from the CSL to the neighboring phases. In the \(J_1 - J_2\) triangular model, the triplet gap in the even sector seems to be vanished, which could be consistent with the larger correlation length found in DMRG calculation\(^{65,66}\) and may suggest a possible gapless spin liquid\(^{67}\), which deserves more studies.

We study the system with cylinder geometry using DMRG\(^{78}\) with spin rotational SU(2) symmetry\(^{79}\). We choose two geometries that have one lattice direction parallel to either the \(x\) axis (XC) or the \(y\) axis (YC), as shown in Figs. 1(a)-(b). These cylinders are denoted as XC(YC)/\(L_y\)/\(L_x\), where \(L_y\) and \(L_x\) are the numbers of site along the two directions. To study the phase diagram and characterize the CSL phase, we perform calculations on the systems with \(L_y\) up to 8 and 10. We keep up to 4000 SU(2) states to obtain accurate results with the truncation error less than \(10^{-5}\) in most calculations.
II. TETRAHEDRAL ORDER AND 120° ORDER

For \( J_2 = 0.0 \), the triangular model has a coplanar 120° magnetic order at \( J_\chi = 0.0 \)\(^{46,47,80,81} \). In the large \( J_\chi \) limit, a classical spin analysis finds a tetrahedral magnetic state with the spins of the four sublattices pointing toward the corners of a tetrahedron\(^{82} \) (see Fig. 1(b)). In the classical picture, the 120° ordered state has the energy per site \( E_{120} = -3J_2/2 + 3J_2 \), the stripe ordered state has the energy \( E_{\text{stripe}} = -J_1 - J_2 \), and the tetrahedral ordered state has the energy \( E_{\text{tetra}} = -J_1 - J_2 - 8\sqrt{3}J_\chi/9 \). Thus we can get a classical phase diagram in the \( J_2 - J_\chi \) plane. In Fig. 1(d), the dotdashed line denotes the classical phase boundary between the 120° and the tetrahedral phase. For \( J_\chi = 0.0, J_2 > 0.125 \), the stripe state and the tetrahedral state have the degenerate energy. By switching on the chiral interaction, the tetrahedral state immediately gets the lower energy. In quantum model, we first investigate whether this tetrahedral order could survive for the spin-1/2 system with strong quantum fluctuations. In Figs. 2(a)-(b), we demonstrate the spin correlations \( \langle \vec{S}_i \cdot \vec{S}_j \rangle \) for \( J_2 = 0.0, J_\chi = 0.8 \) on both the XC8-24 and YC8-24 cylinders. The dashed diamonds denote the unit cell of the spin correlation, which is consistent with the four sublattice structure of the tetrahedral order. The spin correlation functions decay quite slowly in both systems, indicating an established magnetic order. In Figs. 2(c)-(d), we show the corresponding spin structure factor \( S(\vec{q}) \) on both cylinder geometries, which has the ordering peaks at \( \vec{q} = (0, \pi), (\pi, \pi/2) \) and \( (\pi/2, \pi) \) on the XC8 and YC8 cylinders, respectively.

For small \( J_\chi \) interaction we expect the 120° magnetic order. In Fig. 3, we show the spin structure factor for \( J_\chi = 0.0, 0.2 \) on both the XC8 and YC6 cylinders. For a finite \( J_\chi = 0.2 \), the characteristic peak of the 120° order is still very stable, indicating the dominant three-sublattice spin structure.

III. QUANTUM SPIN LIQUIDS

A. Chiral spin liquid

For showing our results of the chiral spin liquid phase, we choose the parameters with fixed \( J_2 = 0.1 \), where the system is in the \( J_1 - J_2 \) SL in the absence of the chiral interaction\(^{63-67} \) (we have also studied other \( J_2 \) such as \( J_2 = 0.125 \), which gives the same results). In our DMRG simulation of spin liquid phase on cylinder geometry, we control the even/odd parity of spinon flux in the ground state by removing or adding a spin-1/2 on each open edge of cylinder\(^{29,64,65} \).

We first exclude the conventional orders in the CSL phase. We show the spin correlations in Fig. 4(a), where the correla-
bulk energy per site

FIG. 4. Vanished magnetic order and non-zero chiral order in the CSL phase. (a) Log-linear plot of spin correlations for $J_2 = 0.1$ and different $J_K$ on the YC8-24 cylinder. (b) Distance dependence of the chiral order of triangle $\langle \chi_d \rangle$ from the open boundary to the bulk of cylinder for $J_2 = 0.1, J_K = 0.2$ for different systems.

FIG. 5. Characterizing the CSL phase through the entanglement spectrum. Entanglement spectra of the ground states in the even (a) and odd (b) sectors for $J_2 = 0.1, J_K = 0.2$ on the $L_y = 8$ cylinder. $\lambda_i$ is the eigenvalue of the reduced density matrix obtained by bipartitioning the cylinder system. The numbers denote the near degenerate pattern $\{1, 1, 2, 3, 5, 7, \cdots \}$ of the low-lying spectrum with different relative momentum quantum number $\Delta k_y$ in each spin-$S^z$ sector.

tions in the CSL phase decay faster than those in the $J_1 - J_2$ SL, indicating the vanished magnetic order. In Fig. 4(b), we plot the triangle chiral order $\langle \chi_d \rangle$ along the $x$ direction of cylinder. Different from the decayed chiral order in the $J_1 - J_2$ triangular model, here it rapidly converges to finite value and seems to be robust with increasing system width. For $L_y = 6, 8, 10$, the chiral orders in both sectors approach to each other, which agrees with the consistent local orders in different sectors of gapped spin liquid. Similar to the $J_1 - J_2$ SL, lattice translational symmetry is also preserved in the CSL phase, which we do not discuss in detail but show an example in Fig. 7.

Next, we characterize the CSL by identifying the conformal field theory (CFT) that describes the gapless edge excitations through entanglement spectrum. Since this strategy was proposed, the ES has been shown as a powerful tool to identify different topological orders with edge states. The nature of the CSLs in the kagome and honeycomb spin models have been characterized using ES. In Fig. 5, we show the ES of the reduced density matrix for half the cylinder in both sectors. By tracing out half of the degrees of freedom for the density matrix by bipartitioning the cylinder, we obtain the reduced density matrix and its eigenvalues $\lambda_i$. We focus on the leading eigenvalues by showing $-\ln \lambda_i$ in Fig. 5. The spectra are labeled by the quantum number total spin $S^z$ and relative momentum quantum number along the $y$ direction $\Delta k_y$. The leading spectra in both sectors have the degeneracy pattern $\{1, 1, 2, 3, 5, 7, \cdots \}$ with increasing $\Delta k_y$ in each $S^z$ sector, which follow the chiral $SU(2)_{1}$ Wess-Zumino-Witten CFT theory of the $\nu = 1/2$ fractional quantum Hall state. The spectra of the even and odd sectors are symmetric about $S^z = 0$ and $1/2$ respectively, indicating a spin-1/2 at each end of cylinder in the odd (spinon) sector. The similar degeneracy pattern of entanglement spectra have also been found in the CSLs in the kagome and honeycomb spin models. We further demonstrate the near-degenerate ground states in the CSL phase. In Fig. 6, we show the bulk energies in both the even and odd sectors, where the energy difference drops fast with increasing $L_y$. For $L_y = 10$, the energy difference is vanishing small, in consistent with the near-degenerate ground states in the two sectors.

For the $J_1 - J_2$ SL, DMRG calculations find the large lattice nematic order in the odd sector (defined as the energy difference between the zigzag and vertical bonds), suggesting a spin liquid with possible spontaneous rotational symmetry breaking. However, the nematic order in the even sector exhibits the opposite nematic pattern from the odd sector, which seems to approach vanishing with increasing $L_y = 6, 8, 10$. In the CSL phase, we also calculate the NN bond energy $\langle \vec{S}_i \cdot \vec{S}_j \rangle$ on the YC cylinder as shown in Figs. 7(a)-(b). On the YC8 cylinder, we also find the strong bond energy anisotropy and the nematic patterns are different in the two sectors. For studying the nematic order, we show the nematic order on different cylinders in Fig. 7(c) with a comparison to the data for $J_K = 0.1$. For $L_y = 6, 8, 10$, while the nematic order in the odd sector keeps growing with $L_y$; in the even sector it also appears to approach zero. The overall behaviors of the nematic order in the CSL are consistent with those in the $J_1 - J_2$ SL. We notice that the nematicity for the even sector in Fig. 7(c) shows a tendency to become...
positive with growing $L_y$. To shed more light on the nature of the nematicity, we calculate the bond energy for $L_y = 12$ by keeping the SU(2) states up to 6000. We find that while the nematic order in the odd sector shows a consistent behavior, the order in the even sector changes the pattern to that of the odd sector on the YC12 cylinder. Our results imply that in both the $J_1 - J_2$ SL and the CSL, the even sector may also host a nematic order in large size, suggesting possible nematic spin liquids. While CSL has been discovered in several spin models, the nematic CSL with coexisting topological order and nematic order has not been reported in a microscopic spin model as far as we know. In a strong-coupling perspective, a nematic FQH may be viewed as a partially melted solid, where the nematic FQH is proximate to the phase with broken translational and rotational symmetries. If the translational order is melted by tuning external parameter but nematic order is preserved, a nematic FQH might be obtained. Here, the CSL in the triangular model emerges at the neighbor of a stripe phase, which breaks translational and rotational symmetries. The possible nematicity of the CSL might be understood as a partially melted stripe order.

**B. Transition between the two spin liquids**

Now we study the quantum phase transition from the $J_1 - J_2$ SL to the CSL. We choose $J_2 = 0.1$ and switch on the chiral interaction $J_\chi$. In Fig. 8(a), we show the ground-state energy on the YC8 cylinder as well as on the $6 \times 6$ torus. The energy varies smoothly with growing $J_\chi$, and we notice the slight change of energy for $J_\chi \lesssim 0.02$. Then we compute the spin triplet gap $\Delta_T$ on the YC8 cylinder based on the ground state with the lowest energy, which is in the odd sector as shown in Fig. 6. The triplet gap is obtained by sweeping the total spin-1 sector in the bulk of long cylinder. We compare the obtained spin triplet gap by sweeping the spin-1 sector on the different system lengths, and we find the well converged triplet gap (one example can be found as the red square in Fig. 11(c)). In Fig. 8(b), we show the gap obtained by sweeping the middle $8 \times 16$ sites in the spin-1 sector based on the ground state in the odd sector on the YC8-24 cylinder. The triplet gap changes slightly for $J_\chi \lesssim 0.02$. Above $J_\chi \simeq 0.02$, the gap grows fast, consistent with the non-zero gap in the CSL phase. The $J_\chi$ dependence of energy and triplet gap imply a possible phase transition at small $J_\chi$.

Next, we study the entanglement spectrum. As shown in Figs. 9(a)-(b) for $J_\chi = 0.01, 0.02$ in the odd sector with total spin $S_z = 0$, the ES exhibit some features of the ES...
for $J_\chi = 0.066$, where four eigenvalues are found below the higher spectrum. We also notice that with increasing $J_\chi$, some eigenvalues in the higher spectrum are decreasing gradually as marked by red in Figs. 9(a)-(b). For $J_\chi = 0.05, 0.1$ as shown in Figs. 9(c)-(d), the decreasing eigenvalues seem to merge with the low-lying levels, which are separated by an ES gap from the higher spectrum. The ES levels below the gap exhibit the near degenerate pattern $\{1, 1, 2, 3\}$, which is consistent with the Laughlin CSL. The entanglement spectrum also suggests a phase transition at small $J_\chi$, which agrees with the transition suggested by energy and triplet gap in Fig. 8. In the even sector for $J_\chi = 0.0$, the low-lying part of the ES shows a deformed two-spinon continuum structure$^{66}$. By switching on the chiral interaction, the low-lying part of the ES quickly changes to the structure that looks like the one in the CSL, which are shown in Fig. 10 and may suggest a stronger tendency to the chiral state in the even sector.

We also compute the spin triplet gap. For a comparison, we demonstrate the results of the same calculation for the spin-$1/2 J_1 = J_2 = J_3$ kagome model. Here, we obtain the triplet gap by calculating the ground state on long cylinder first and then sweeping the bulk sites for the ground state in spin quantum number $S = 1$ sector for a given length $l_x$. (a) and (c) are the $1/l_x$ dependence of the gap on the YC8 cylinder in the $J_1-J_2$ SL ($J_2 = 0.1, J_\chi = 0.0$) and CSL ($J_2 = 0.1, J_\chi = 0.2$) for the triangular model. (b) and (d) are the $1/l_x$ dependence of the gap on the YC8 cylinder in the kagome spin liquid ($J_2 = J_3 = 0.0$) and CSL ($J_2 = J_3 = 0.5$) for the kagome model.
sector decreases fast with \( l_x \) and tends to vanish, seemly similar to the behaviors found in Fig. 11(a).

In the CSL phase as shown in Fig. 11(c), the overall triplet gap is also robust; but the gap in the even sector still decreases with \( l_x \). In the well established CSL phase, for example the CSL phase in the kagome model as shown in Fig. 11(d)\(^\text{15}\), one can find the robust triplet gap in both sectors. The decreasing gap of the triangular CSL in the even sector suggests that on our studied system size the topological nature in the even sector is not fully developed. A possible reason is that this CSL regime is very close to the phase boundaries from the CSL to the neighboring phases.

**IV. QUANTUM PHASE DIAGRAM**

First of all, we study the phase transition from the tetrahedral phase to the CSL phase by calculating magnetic structure factor. In Fig. 12, we show the \( J_x \) dependence of the tetrahedral structure factor peak on the YC8 cylinder, which is at \( \vec{q} = (\pi/2, \pi) \). With increasing \( J_x \), \( S(\pi/2, \pi) \) shows a jump that characterizes the phase transition. Above the transition \( J_{x_c} \), we find that the spin correlations decay quite slowly, consistent with a magnetic order developed. On the smaller YC6 and XC8 cylinders, the tetrahedral structure factor peak appears to increase smoothly, which may be owing to the finite-size effects. We show the phase boundary in Fig. 1 based on the results on the larger YC8 cylinder.

By increasing the chiral interaction \( J_y \) in the 120° phase for \( J_2 \lesssim 0.08 \), the magnetic order is suppressed, leading to a transition from the magnetic order to the non-magnetic spin liquid phase. On our studied system size, we do not find sharp features to characterize this transition. Thus we estimate a qualitative phase boundary as shown in Fig. 1(c) by comparing spin correlation function and magnitude of spin structure factor on the YC8 cylinder with the results at \( J_2 = 0.08, J_x = 0 \), where the system has the transition from the 120° phase to the \( J_1 - J_2 \) SL phase. We roughly take the parameter points which have the similar magnitudes of spin correlations and spin structure factor compared with those at \( J_2 = 0.08, J_x = 0 \) as the phase boundary. In Fig. 13, we show the spin structure factor \( S(\vec{q}) \), which characterizes the present and absent magnetic order in the two phases.

As shown in Fig. 1(c), the stripe phase goes to the tetrahedral phase with or without an intermediate CSL phase depending on \( J_2 \). We calculate the spin correlations and structure factor on the YC8 cylinder. In both the stripe and the tetrahedral phase, spin structure factor shows the peak at \( \vec{q} = (\pi/2, \pi) \). For the case with the intermediate CSL phase, we find that \( S(\pi/2, \pi) \) first decreases with growing \( J_x \) and then keeps small in the CSL phase; with further increasing \( J_x \), \( S(\pi/2, \pi) \) shows a jump at the transition to the tetrahedral phase, which is shown for \( J_2 = 0.225 \) in Fig. 14. On the other hand, for \( J_2 \gtrsim 0.25 \), \( S(\pi/2, \pi) \) decreases fast with \( J_x \) in the stripe
phase and changes slowly in the tetrahedral phase, showing a kink to characterize the phase transition. We remark that on the smaller YC6 cylinder for the $J_1 - J_2$ model, the magnetic order is weak at $0.15 \lesssim J_2 \lesssim 0.2$; however, on the larger YC and XC cylinders the stripe order is quite robust. We demonstrate our phase diagram based on the large-size results.

V. SUMMARY AND DISCUSSION

We have studied the competing quantum phases of the spin-1/2 triangular $J_1 - J_2$ Heisenberg model with the additional scalar chiral interaction $J_\chi$ for $0 \leq J_2 \leq 0.3$ and $0 \leq J_\chi \leq 1.0$ by DMRG simulations. As shown in Fig. 1(c), we find five phases: a non-coplanar tetrahedral magnetic order phase, a $120^\circ$ order phase, a stripe order phase, a $J_1 - J_2$ SL phase, and a chiral spin liquid (CSL) phase. The CSL is identified as the $\nu = 1/2$ bosonic fractional quantum Hall state, which seems to arise as a result of quantum fluctuations around the phase boundaries of the classical magnetic orders. In particular, the CSL state exhibits a strong bond anisotropy, strongly suggesting a nematic CSL with coexisting topological and nematic order. We argue that the nematicity can be understood as a partially melted stripe order. The emergent CSL induced from quantum fluctuations around classical phase boundaries has also been found in the kagome and honeycomb models, which may represent a common mechanism to generate novel quantum phases from frustration. We remark that while the analytical analyses suggest that the CSL is unlikely to emerge in the triangular model built out of weakly coupled chains with chiral interaction, DMRG results indicate that the CSL can emerge in the strong coupling regime.

By measuring the spin triplet gap and entanglement spectrum on the YC8 cylinder, we find a transition from the $J_1 - J_2$ SL to the CSL at small chiral coupling. We also compute the spin triplet gap on cylinder geometry. While the gap above the overall ground state (in the odd sector) is robust, the one in the even sector seems to be small in both spin liquid phases. In the CSL phase, the small gap suggests that the even sector may be not well developed on our studied system sizes. To find a robust CSL in finite-size calculations, other interactions may be needed besides the chiral interaction. In the $J_1 - J_2$ SL, the vanishing gap in the even sector also indicates that for a gapped spin liquid scenario the even sector is not well established. On the other hand, in the gapless spin liquid scenario, one finds it inconsistent with the odd sector exhibiting a large gap based on DMRG calculations ($\Delta_T \approx 0.35 J_1$ on the YC8 cylinder and the size scaling seems to suggest a large gap in the large-size limit). However, the spin gap measured in DMRG calculations may also have large finite-size effects, which requires further investigations. We hope that future studies will be able to resolve between these scenarios more clearly.

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Note Added.— While completing this work, we became aware of a related paper that also studies the robustness of the $J_1 - J_2$ spin liquid against the chiral interaction in the same $J_1 - J_2 - J_\chi$ triangular model. We find the overall agreement with Ref. 88.

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