A distributed localization method for mobile nodes

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Abstract. In Wireless Sensor Networks (WSNs), the location services are the basis of many application scenarios. However, for the range-based localization method, the localization accuracy and the system robustness of the distributed localization system are difficult to guarantee, due to the uncertainty of the distance estimation and position calculation are affected by the node state and communication uncertainty. In this paper, we propose the distributed localization method based on anchor node selection and Particle Filter optimization. In this method, we analyze the uncertainty of error propagation in the Least-squares method and find that there is a proportional relation between localization error and uncertainty propagation. According to this relationship, we propose the corresponding optimization criterion methods of anchor nodes. To optimize the initial localization results, we present the distributed localization method based on anchor node optimal selection and Particle Filter. Simulation results show that the methods we proposed could effectively improve the localization accuracy of the mobile nodes and the robustness of the system.

Keywords: Distributed localization, anchor node optimization, particle filter, Wireless Sensor Networks.

1. Introduction

Wireless Sensor Networks (WSNs) are extensively used in smart cities, disaster relief, and other fields. Presently, the most universal methods of providing location services are the Global Positioning System (GPS) and the Beidou system. Nevertheless, the localization accuracy is greatly reduced in buildings, indoors, or in canyons\textsuperscript{[1]}, which makes reliable localization information cannot be obtained.

For the range-based localization method, the accuracy of localization depends on the distance estimation of anchor nodes\textsuperscript{[2]}. Due to the complex transmission environment such as environmental interference, reflection, refraction, multi-path, and NLOS transmission\textsuperscript{[3]} in the process of wireless signal transmission, the distance estimation results have a different extent of the error. At the same time, the distributed localization method\textsuperscript{[4]} depend on the information exchange and coordination between the nodes. Wireless Sensor Networks have inevitable defects: the anchor nodes may have problems such as communication uncertainty\textsuperscript{[5]}, limited processing capacity, insufficient power
supply[6], etc, which cause the input error to enlarge. Therefore, it is necessary to select reliable anchor nodes for distance estimation and localization calculation in distributed mobile localization.

The rest of this paper is structured as follows: we review related works in Chapter 2; Chapter 3 illustrates the uncertainty propagation, and propose the MSDO-PF and MEPO-PF; In Chapter 4, we evaluate the performance of the proposed methods; Finally, we give the conclusion of our work.

2. Related works
The localization accuracy of WSN is affected by factors such as communication uncertainty, nodes' processing capabilities, etc. Scholars have considered different methods to improve the location results.

Some scholars have made improvements in localization calculation methods.[5] proposed an adaptive fading factor to compensate for the inconsistency and error of the estimation according to the communication uncertainty caused by the failures of sensor nodes and mobile targets; The application of Unscented Kalman Filter (UKF), and Particle Filter (PF)[7] in WSN are also studied. Some scholars put forward methods with considering the uncertainty of anchor nodes' position. Considering both Range-Based and Range-Free node localization methods with errors in anchor nodes, The algorithm proposed by [8] calculates the similarity between nodes according to the location information and hops of anchor nodes, and uses the K most similar anchor nodes to calculate the coordinates.

In the above methods, the improvement on localization accuracy by the improved location estimation methods is limited, and the analysis after error propagation is lacking. In this paper, we analyze the error propagation mechanism of the distributed mobile nodes, and propose the minimum standard deviation optimization(MSDO) criterion and the minimum error propagation optimization(MEPO) criterion. With using the Particle Filter algorithm, we propose the distributed location method based on anchor node optimal selection and particle filter(MSDO-PF and MEPO-PF).

3. Proposed algorithm

3.1. System structure
The framework of the distributed localization method based on anchor node selection and particle filter optimization is illustrated in Fig.1, which is composed of four sub-modules.

![Figure 1. The framework of MSDO-PF and MEPO-PF localization method.](image)

3.2. Uncertainty propagation analysis and Optimal selection of anchor nodes
We assume k known anchor nodes with corresponding coordinates: \((x_1, y_1), \ldots, (x_i, y_i), \ldots, (x_k, y_k)\), And the coordinate of the unknown node moving non-linearly is \((x, y)\), the corresponding distance estimated by anchor nodes are \(d = \{d_1, d_2, \ldots, d_i, \ldots, d_k\}\).

The localization equations can be formed as followed:

\[
\begin{align*}
\sqrt{(x-x_1)^2 + (y-y_1)^2} &= d_1 \\
\sqrt{(x-x_2)^2 + (y-y_2)^2} &= d_2 \\
&\vdots \\
\sqrt{(x-x_k)^2 + (y-y_k)^2} &= d_k
\end{align*}
\] (1)
In the form of matrix equality: \( AX = B \), and based on Least Square criterion, we can get the solution for the location equations:

\[
X = (A^T A)^{-1} A^T B
\]

(2)

When calculating the coordinates of moving nodes, one of the variables with uncertainty is the anchor node coordinates \((x_i, y_i)\) (i = 1, 2, …, k), whose size is the sum of the real value and a neighbourhood, and the neighbourhood is decomposed into \((\delta_x, \delta_y)\); The other uncertainty is the distance estimation \(d_i\) of the corresponding anchor node to the moving node, and the error is \(\delta_{d_i}\). The sensitivity coefficients of anchor node coordinate and distance estimation are defined as follows:

\[
S_{x_i} = \frac{\partial((A^T A)^{-1} A^T B)}{\partial x_i}, \quad S_{y_i} = \frac{\partial((A^T A)^{-1} A^T B)}{\partial y_i}, \quad S_{d_i} = \frac{\partial((A^T A)^{-1} A^T B)}{\partial d_i}, \quad (i = 1, 2, \ldots, k)
\]

(3)

According to the total differentiation formula (TDF), the positioning error can be obtained:

\[
\begin{bmatrix}
\delta x \\
\delta y \\
\vdots \\
\delta x_k
\end{bmatrix} = \begin{bmatrix}
S_{x_1} \delta x_1 + S_{y_1} \delta y_1 + S_{d_1} \delta d_1 + \ldots + S_{x_1} \delta x_k + S_{y_1} \delta y_k + S_{d_1} \delta d_k + \ldots + S_{x_k} \delta x_1 + S_{y_k} \delta y_1 + S_{d_k} \delta d_1 + \ldots + S_{x_k} \delta x_k + S_{y_k} \delta y_k + S_{d_k} \delta d_k
\end{bmatrix}
\]

(4)

We can get the standard deviation of localization result through (3) according to the square root rule:

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\vdots \\
\sigma_{x_k}
\end{bmatrix} = \sqrt{ \begin{bmatrix}
(2(A^T A)^{-1} A^T)^2 \sigma_{x_1}^2 + \ldots + (2(A^T A)^{-1} A^T)^2 \sigma_{y_1}^2 + \ldots + (2(A^T A)^{-1} A^T)^2 \sigma_{d_1}^2 \\
(2(A^T A)^{-1} A^T)^2 \sigma_{x_1}^2 + \ldots + (2(A^T A)^{-1} A^T)^2 \sigma_{y_1}^2 + \ldots + (2(A^T A)^{-1} A^T)^2 \sigma_{d_1}^2 \\
\vdots \\
(2(A^T A)^{-1} A^T)^2 \sigma_{x_1}^2 + \ldots + (2(A^T A)^{-1} A^T)^2 \sigma_{y_1}^2 + \ldots + (2(A^T A)^{-1} A^T)^2 \sigma_{d_1}^2
\end{bmatrix}}
\]

(5)

From (5), the error of the positioning result is proportional to the standard deviation of the distance estimation, the product of the estimated distance and the standard deviation of the corresponding estimation. Therefore, the smaller the range standard deviation and error propagation factor, the smaller the localization error. According to this relationship, we propose the minimum standard deviation optimization criteria (MSDO) and minimum error propagation optimization criteria (MEPO).

### 3.3. Improvement of the localization results with particle filter algorithm

In this section, the optimized coordinate is obtained by Particle Filter algorithm to track the motion state of the moving node. The MSDO-PF and MEPO-PF have been proposed, and the following is a detailed illustration.

Suppose that the state equation and observation equation are as follows:

\[
\begin{align*}
X_k &= f_k (X_{k-1}, \delta_k) \\
Y_k &= h_k (X_k, \gamma_k)
\end{align*}
\]

(6)

Where \(k\) denotes the motion time of mobile nodes, the random variable \(X_i\) denotes the predicted value of target location, and \(Y_k\) denotes the observed value of the target position. In this method, we suppose the \(Y_k = d_{-s} s = \{d_{-s} s, d_{-s} s, \ldots, d_{-s} s, \ldots, d_{-s} s\}\) is the preliminary result of positioning after the optimization of anchor nodes. \(f_k\) and \(h_k\) are Non-linear function. \(\delta_k\) denotes the system noise and \(\gamma_k\) denotes the observation noise, they are independent.

Construct a set \(\{X_k^{(i)}, W_k^{(i)}\}_{i=1}^{N}\) containing N particles, where \(X_k^{(i)}\) represents the state of the ith particle at the moment, \(W_k^{(i)}\) represents the weight of this particle, and the weights satisfied that...
\[ \sum_{i=1}^{N} W_{i}^{(i)} = 1 \]

\( f(X_i) \) denotes the real coordinates of the target at time \( k \), and \( p(X_i | Y_{i_k}) \) is the posterior probability density of \( X_k \) at this time. Then the final positioning result is expressed as:

\[ E[f(X_k)] = \int f(X_k) p(X_k | Y_{i_k}) dX_k \]  \hspace{0.5cm} (7)

In practical application, for avoiding directly extracting samples of the posterior probability distribution \( p(X_k | Y_{i_k}) \), the importance sampling is introduced to improve sampling efficiency. The importance sampling method extracts samples of the known importance sampling density \( q(X_k | Y_{i_k}) \). Then equation (7) can be expressed as:

\[ E[f(X_k)] = \int f(X_k) \frac{p(X_k | Y_{i_k})}{q(X_k | Y_{i_k})} q(X_k | Y_{i_k}) dX_k \]

\[ w(X_k) = \frac{p(X_k | Y_{i_k})}{q(X_k | Y_{i_k})} \] \hspace{0.5cm} (9)

According to the Sequential Importance Sampling (SIS), and corresponding steps, equation (7) can be expressed as:

\[ E[f(X_k)] = \sum_{i=1}^{N} W_{i}^{(i)} f(X_i) \] \hspace{0.5cm} (10)

Where the particle weight is:

\[ W_{i}^{(i)} = \frac{w(X_i^{(i)})}{\sum_{i=1}^{N} w(X_i^{(i)})} \] \hspace{0.5cm} (11)

According to the theory of particle filter, resampling is added to reduce the degradation of the particle filter. With the combination of Particle Filter, the MSDO-PF and MEPO-PF can get better localization results.

4. Simulation And Analysis

We compare the localization results of MSDO-PF and MEPO-PF, the trajectory tracking results are shown in Fig.2 and Fig.3 respectively.

![Figure 2. The localization results of MEPO using PF](image-url)
We independently conducted five times localization simulations, and statistically calculated the localization effects of Path 1 and Path 2 after using MSDO method and Particle Filter(MSDO-PF) or MEPO method and Particle Filter(MEPO-PF), the experiment dates are as shown in Table 1.

**Table 1.** The localization effects of using three anchor nodes selection methods and Particle Filter

| Methods          | Average positioning error $e_{MSE}$ (m) | Error variance $e_{VAR}$ (m²) |
|------------------|----------------------------------------|-------------------------------|
| Path1 + RS-PF    | 0.33                                    | 0.12                          |
| Path2 + RS-PF    | 1.34                                    | 0.51                          |
| Path1 +MSDO-PF   | 0.35                                    | 0.13                          |
| Path2 +MSDO-PF   | 1.26                                    | 0.47                          |
| Path1 +MEPO-PF   | 0.28                                    | 0.05                          |
| Path2 +MEPO-PF   | 0.05                                    | 0.48                          |

According to the statistical calculation, when the MSDO-PF is used to optimize the anchor node localization, compared with the RS-PF, the localization accuracy and the error variance of path 1 are equivalent, the localization accuracy of path 2 is improved by 5.9%, and the error variance is reduced by 7.8%. When MEPO-PF is used to optimize localization, compared with RS-PF, the localization accuracy of path 1 is improved by 14.8%, the error variance is reduced by 56.3%; the localization accuracy of path 2 is the same, but the error variance is reduced by 5.5%.

5. Conclusions
Considering the negative factors during distance estimation of anchor nodes, we comprehensively analyze the error propagation mechanism of the distributed mobile nodes through WSN, and propose the minimum standard deviation optimization(MSDO) criterion and the minimum error propagation optimization(MEPO) criterion. According to the utilization of the Particle Filter to optimize the localization results, we also present the distributed location method based on anchor node optimal selection and particle filtering(MSDO-PF and MEPO-PF). We experiment to evaluate the accuracy and robustness of our proposed method, and the experiment results illustrate that the proposed algorithm is accurate and stable.

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References

[1] A. Maddumabandara, H. Leung and M. Liu, "Experimental Evaluation of Indoor Localization Using Wireless Sensor Networks," in IEEE Sensors Journal, vol. 15, no. 9, pp. 5228-5237, Sept. 2015.

[2] X. Yan, Q. Luo, Y. Yang, S. Liu, H. Li and C. Hu, "ITL-MEPOSA: Improved Trilateration Localization With Minimum Uncertainty Propagation and Optimized Selection of Anchor Nodes for Wireless Sensor Networks," in IEEE Access, vol. 7, pp. 53136-53146, 2019.

[3] Vempaty A, Ozdemir O, Agrawak K, Chen H. Localization in wireless sensor networks: byzantines and mitigation techniques. IEEE Trans. on Signal Processing, 2013,61(6):1495–1508.

[4] P. Zhu and W. Ren, "Fully Distributed Joint Localization and Target Tracking With Mobile Robot Networks," in IEEE Transactions on Control Systems Technology, 2020.2991126.

[5] H. Zhu and M. Luo, "Hybrid Robust Sequential Fusion Estimation for WSN-Assisted Moving-Target Localization With Sensor-Node-Position Uncertainty," in IEEE Transactions on Instrumentation and Measurement, vol. 69, no. 9, pp. 6499-6508, Sept. 2020.

[6] M. I. M. Ismail, R. A. Dziyauddin, R. Ahmad, N. Ahmad, N. A. Ahmad and A. M. A. Hamid, "A Review of Energy Harvesting in Localization for Wireless Sensor Node Tracking," in IEEE Access, vol. 9, pp. 60108-60122, 2021.

[7] Pak, Jung Min et al. Improving Reliability of Particle Filter-Based Localization in Wireless Sensor Networks via Hybrid Particle/FIR Filtering[J]. IEEE transactions on industrial informatics, 2015, 11(5) : 1089-1098.

[8] H. WANG, K. LYU, H. JIANG, Y. WU, Q. YUE and Q. ZHAO, "Location Algorithm for WSNs with Mobile Anchor Node Based on Optimized K-Nearest Neighbors," 2019 Eleventh International Conference on Advanced Computational Intelligence (ICACI), 2019, pp. 190-195.