Skyrmion ↔ pseudoSkyrmion Transition

in Bilayer Quantum Hall States at $\nu = 1$

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Abstract

Bilayer quantum Hall states at $\nu = 1$ have been demonstrated to possess a distinguished state with interlayer phase coherence. The state has both excitations of Skyrmion with spin and pseudoSkyrmion with pseudospin. We show that Skyrmion ↔ pseudoSkyrmion transition arises in the state by changing imbalance between electron densities in both layers; PseudoSkyrmion is realized at balance point, while Skyrmion is realized at large imbalance. The transition can be seen by observing the dependence of activation energies on magnetic field parallel to the layers.

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It has recently been paid great interests to bilayer quantum Hall states \([1]\) at the filling factor \(\nu = 1\). Especially, interlayer phase coherence and Goldstone boson have been observed in recent experiments \([2,3]\). Their existence has been originally derived \([4–6]\) from the similarity between superconducting states and quantum Hall states; both of them are characterized as the condensed states of electron pairs or composite electrons \([7]\). Furthermore, Skyrmion excitations \([8]\) have been shown \([9]\) to exist in bilayer quantum Hall states at \(\nu = 2\). Before these observations, several measurements on the states have been performed \([10–12]\) by imposing parallel magnetic field or by changing imbalance between electron densities in both layers. As a result it have been shown that the states have various types of excitations.

In this paper we discuss excitations of Skyrmion and pseudoSkyrmion \([13]\) in the bilayer quantum Hall state with the interlayer phase coherence at \(\nu = 1\). Particularly, we show that pseudoSkyrmion is realized in the state with equal electron densities in two layers and that a transition from the pseudoSkyrmion to Skyrmion arises by changing imbalance of the electron densities.

Skyrmion excitation has real spin components, while pseudoSkyrmion excitation has pseudospin components ( pseudospin up and down components are given by the number of electrons in front-layer and in back-layer, respectively ). Suppose that the density, \(\rho_f\), in the front-layer is equal to the density, \(\rho_b\), in the back-layer ( balanced state ). With an appropriate interlayer distance \(d \simeq l\) ( \(l\) being magnetic length ), a state with interlayer phase coherence is realized at \(\nu = 1\). We assume that tunneling energy, \(\Delta_{sas} \sim O(K)\) between the two layers and Zeeman energy, \(g\mu B \sim O(K)\), are much smaller than a typical Coulomb energy, \(e^2/\epsilon l \sim 100K\), where \(g\) ( \(\mu\) ) is electron g factor ( Bohr magneton ) and \(\epsilon \simeq 13\) is the dielectric constant. Then, we can show that the pseudoSkyrmion is an excitation with the lowest energy in the balanced state. As making the imbalance, \(\sigma = (\rho_f - \rho_b)/(\rho_f + \rho_b)\), larger, the energy of the pseudoSkyrmion increases, while the energy of the Skyrmion does not change so much. Thus, at a critical point \(\sigma_c\) of the imbalance both energies become equal and the energy of the Skyrmion becomes lower than that of the pseudoSkyrmion at \(\sigma > \sigma_c\).
Therefore, at large imbalance e.g. $\sigma \simeq 1$, the Skyrmion excitation is realized. Actually, the Skyrmion have been observed \cite{9} in the case that all of electrons are involved only in one layer, i.e. $\sigma = 1$.

We can distinguish these two types of Skyrmion excitations by observing the dependence of the activation energy on parallel magnetic field. The energy of the pseudoSkyrmion decreases with the parallel magnetic field, while the energy of the Skyrmion increases. Thus, the distinction can be easily found.

First, we explain briefly Skyrmion excitations in single layer quantum Hall states by using the theory \cite{7} of bosonized electrons, which is described by Chern-Simons gauge theory. Hamiltonian of bosonized electrons with spin is given by \cite{4,7}

$$H = \int dx^2 \left\{ \sum_{j=u,d} \frac{1}{2m_e} |(\partial_z - ia_z + ieA_z)\Psi_j|^2 + \frac{1}{2} g\mu B(|\Psi_d|^2 - |\Psi_u|^2) \right\} + \frac{\omega N}{2}$$

(1)

with cyclotron frequency $\omega = eB/m_e$ and total number of electrons $N$; we use the notation, $\partial_z = \partial_1 - i\partial_2$ and $a_z = a_1 - ia_2$ ($A_z = A_1 - iA_2$), and use the unit of $\hbar = 1$ and light velocity $= 1$. $\Psi_j$ represents field of bosonized electrons with spin $j$ ( ‘$u$’ and ‘$d$’ denote spins parallel and anti-parallel to $\vec{B}$), respectively. $\vec{A}$ is a gauge potential of the magnetic field $\vec{B}$. $\vec{a}$ is a Chern-Simons gauge field satisfying a constraint, $\partial_1 a_2 - \partial_2 a_1 = 2\alpha(|\Psi_u|^2 + |\Psi_d|^2)$.

The constraint equation implies that the bosonized field represents fermion with the choice of $\alpha = \text{odd integer} \times \pi$: Attaching fictitious Chern-Simons flux of magnitude e.g. $2\pi$ to a boson makes the boson become a fermion.

We find from eq(1) that the states $|L> \text{ satisfying the projection equation, } (\partial_z - ia_z + ieA_z)\Psi_j|L> = 0 \text{ for all } j$, are the states with the energy $\omega/2$ in the lowest Landau level. Hereafter, we analyze quantum Hall states in semiclassical approximation. It means that we solve classically equations corresponding to these equations, that is, $(\partial_z - ia_z + ieA_z)\Psi_u,d(x) = 0 \text{ and } \partial_1 a_2 - \partial_2 a_1 = 2\alpha(|\Psi_u|^2 + |\Psi_d|^2)$.

Solutions of the equations are given by, $\Psi_u = w_u(z)\exp(-a(x))$ and $\Psi_d = w_d(z)\exp(-a(x))$ with $z = x_1 - ix_2$, where $a(x)$ is defined such as $a_i - eA_i = \epsilon_{ij}\partial_j a(x)$ ($\epsilon_{12} = 1$ and $\epsilon_{ij} = -\epsilon_{ji}$) and satisfies the equation, $-\partial^2 a(x) = 2\pi\{|w_u|^2 + |w_d|^2\} \exp(-2a(x)) - \rho$, 

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where \( w_u \) and \( w_d \) are arbitrary functions of \( z \). We have assumed \( \alpha = \pi \) since we discuss only the states at \( \nu = 1 \) in this paper.

A solution of ground state describing uniform density of electrons \( \rho \) is given by \( |w_u| = \sqrt{\rho} \) and \( |w_d| = a(x) = 0 \). It is obvious that the solutions, \( |w_u| = \sqrt{\rho_u}, \ |w_d| = \sqrt{\rho_d} \) and \( a(x) = 0 \) with arbitrary \( \rho_u \) and \( \rho_d \), are also possible ground states as far as \( \rho_u + \rho_d = \rho \) when Zeeman energy vanishes. It has, however, been argued that Coulomb interaction and Fermi statistics of electrons (namely, exchange interactions) force the ground state to be ferromagnetic (only the state of electrons with parallel spin component, \( |w_u| = \sqrt{\rho} \) and \( w_d = 0 \), is the true ground state even without Zeeman energy).

Note that there is spin SU(2) symmetry among the fields in the absence of Zeeman energy. Thus, this ferromagnetic state breaks spontaneously the symmetry, which gives rise to the phase coherence between up and down spin states, and solitons of Skyrmion.

A solution of such a Skyrmion is obtained by choosing the functions \( w_{u,d} \) such that \( w_u = \sqrt{\rho} \) \( z \) and \( w_d = \sqrt{\rho} \) c. Then, \( a(x) \) satisfies \(- \partial^2 a(x) = 2\pi \rho \{(r^2 + c^2) \exp(-2a(x)) - 1\}\), with \( r^2 = |z|^2 \). It turns out numerically that the parameter, \( c \), represents a typical scale of the spatial extension of the Skyrmion.

The solution of the Skyrmion must satisfy a boundary condition, \( |\Psi_u| \to \sqrt{\rho} \) as \( r \) goes infinity. This implies \( \exp(-a(x)) \to 1/r \) as \( r \to \infty \); we have found numerically that the function, \( \exp(-a(x)) \), is regular at \( r = 0 \) and a smooth decreasing function approximately given by \( \sqrt{1/(r^2 + c^2)} \) for \( c \gg l \). The electric charge of the Skyrmion, is determined by this boundary condition such that \( \int dx^2 e \{(|\Psi_u|^2 - \rho) + |\Psi_d|^2\} = -e > 0 \).

Zeeman energy of the Skyrmion is given by \(- (1/2) g \mu_B \int dx^2 \{(|\Psi_u|^2 - \rho) - |\Psi_d|^2\} = (1/2) g \mu_B + g \mu B \rho c^2 \int dx^2 \exp(-2a(x)) \). The Zeeman energy is proportional to the volume, \( c^2 \), of the Skyrmion and is logarithmically divergent since \( \exp(-a(x)) \to 1/r \) as \( r \to \infty \). But the integral is finite when we take into account finiteness of actual coherent length, \( \eta \), of the boson field \( \Psi_{u,d} \) \[13\]. The length is qualitatively given by the inverse of Zeeman energy, \((g \mu B)^{-1} \) since the presence of the nonvanishing Zeeman energy breaks the SU(2) symmetry explicitly and makes the coherent length finite.
The length scale $c$ of the Skyrmion is determined by minimizing the total energy of the Skyrmion, i.e. $h_1 e^2/4c + g\mu B (c/l)^2 I(c)/2\pi$ where $I(c) = \int dx^2 \exp(-2a(x))$ and $h_1 \simeq 3\pi^2/64$. The Coulomb energy is approximately correct for $c$ larger than $l = \sqrt{1/eB}$. The integral, $I(c)$, is almost constant in $c > l$; it behaves such as $I(c) \simeq 2\pi \log(\eta/c)$ for $c > l$. Thus, minimizing the energy in $c$, we find the length scale of the Skyrmion, $c_0 \sim l \{(e^2/\epsilon l)/(g\mu B)\}^{1/3}$ and the energy of the Skyrmion, $\sim (e^2/\epsilon l)\{(g\mu B)/(e^2/\epsilon l)\}^{1/3} > e^2/\epsilon l$.

We should mention that the Skyrmion has an additional energy associated with the exchange interaction which gives rise to the ferromagnetism. The energy is given by $\int dx^2 (\rho_s/2) \partial_k \vec{m} \partial_k \vec{m} = 4\pi \rho_s$, where $\vec{m}$ denotes unit vector of the spin of the Skyrmion, $\vec{m} \propto \Psi^\dagger \vec{\sigma} \Psi$ with $\Psi^\dagger = (\Psi^\dagger_u, \Psi^\dagger_d)$. $\rho_s = (1/16\sqrt{2}\pi)e^2/\epsilon l$ is the spin stiffness and $\vec{\sigma}$ are Pauli matrices.

We now proceed to discuss Skyrmion excitations in bilayer quantum Hall states at $\nu = 2\pi \rho/eB = 1$; $\rho = \rho_f + \rho_b$. In the bilayer system we take into account indices distinguishing electrons located in different layers along with the spin indices (u, d). We denote the field $\Psi_{fu}$ representing electrons with up spin located in the front-layer, $\Psi_{bu}$ representing electrons located in the back-layer, e.t.c..

All of these fields satisfy the following equations guaranteeing electrons lying in the lowest Landau level, that is, $(\partial_x - ia_x + ieA_x)\Psi(x) = 0$ with $\partial_1 a_2 - \partial_2 a_1 = 2\pi (|\Psi_{fu}|^2 + |\Psi_{fd}|^2 + |\Psi_{bu}|^2 + |\Psi_{bd}|^2)$, where $j$ denotes one of four states of electrons. We should stress that there exists a local gauge symmetry, $\Psi_j \to \Psi_j \exp(i\Lambda)$ and $a_k \to a_k + \partial_k \Lambda$ in this system.

Solutions of the equations are given such that $\Psi_j = w_j(z) \exp(-a(x))$ with arbitrary functions $w_j(z)$, where $a(x)$ is a solution of the equation, $-\partial^2 a(x) = 2\pi \{(|w_{fu}|^2 + |w_{fd}|^2 + |w_{bu}|^2 + |w_{bd}|^2) \exp(-2a(x)) - \rho\}$.

Ground states, as in the previous ferromagnetic case, are given such that $\Psi_{fu} = \sqrt{\rho_f} \exp(i\theta_f)$ and $\Psi_{bu} = \sqrt{\rho_b} \exp(i\theta_b)$ with any $\rho_l$ as far as $\rho = \rho_f + \rho_b$ ( $\Psi_{ld} = 0$ for $l = f, b$ ), where $\theta_l$ are arbitrary real parameters. This implies that quantum Hall states may exist for any $\sigma$. Actually, this result has been confirmed in the previous experiment [11].
and is one of the outstanding properties of the quantum Hall state with the interlayer phase coherence. We mention that the difference of two phases $\theta_f - \theta_b$ in the state is a physically relevant variable [4] just like one in Josephson junction, while the phase, $\theta_f + \theta_b$, is not physically relevant since the corresponding local gauge symmetry exists.

Now, we explain two types of Skyrmions [13]; Skyrmion and pseudoSkyrmion. We address to only spherical form of solutions; namely $a(x)$ depends only on $r$. First we show solutions of Skyrmions, $w_{fu} = \sqrt{\rho_f} z$, $w_{fd} = \sqrt{\rho_f} c$, $w_{bu} = \sqrt{\rho_b} z$, and $w_{bd} = \sqrt{\rho_b} c$. Then, $a(x)$ satisfies $-\partial^2 a(x) = 2\pi \rho \{ (r^2 + c^2) \exp(-2a(x)) - 1 \}$, with the boundary condition, $\exp(-a) \to 1/r$ as $r$ goes to infinity.

The Skyrmions have the same electric charge $-e$, Zeeman energy and exchange energy as Skyrmions in the single layer. Contrary to the previous ones, the charge is distributed over two layers, $Q_k = -e \rho_k / \rho$ ( $-e = Q_f + Q_b$ ) with $k = f, b$. Furthermore, the Skyrmions have tunneling energies whenever $\Delta_{sas}$ is nonvanishing, $E_t = -\frac{1}{2} \int dx x^2 \Delta_{sas} (\psi^\dagger_{fu} \psi_{bu} + \psi^\dagger_{fd} \psi_{bd} + c.c.) - 2\sqrt{\rho_f \rho_b} = \Delta_{sas} \frac{\sqrt{1 - \sigma^2}}{2}$.

Therefore, the energy of the Skyrmion with $c > l$ is given by

$$E_{sk} = 4\pi \rho_s + \frac{h_1 e^2}{2c} + \frac{g\mu B c^2}{2\pi l^2} I(c) + \frac{\Delta_{sas} \sqrt{1 - \sigma^2}}{2} + \frac{g\mu B}{2},$$

(2)

where we have neglected the small charging energy $\simeq 0.14 \sigma^2 (e^2 / \epsilon l) (l/c) (d/c)$, which does not play important roles in the present discussion.

By minimizing $E_{sk}(c)$ in $c$, we find that the spatial extension, $c_0$, of the Skyrmion and $E_{sk}(c_0)$ are approximately given by

$$c_0 \simeq l \left\{ \frac{h_1 e^2}{2c} \frac{g\mu B}{\eta / c} \right\}^{1/3},$$

$$E_{sk}(c_0) \simeq 4\pi \rho_s + \frac{3 h_1 e^2}{2c c_0} + \frac{\Delta_{sas} \sqrt{1 - \sigma^2}}{2} + \frac{g\mu B}{2},$$

(3)

where we have assumed $\log(\eta / c) \simeq 1$ for simplicity [13]. We note that $E_{sk}(c_0)$ increases monotonously with Zeeman energy.

The other type of Skyrmions is pseudoSkyrmion whose solutions are given such that $w_{fu} = \sqrt{\rho_f} (z + c_f)$, $w_{fd} = 0$, $w_{bu} = \sqrt{\rho_b} (z + c_b)$ and $w_{bd} = 0$, where $c_i$ are arbitrary
real parameters. We may choose the parameters $c_i$ without loss of generality such that $\rho_f c_f + \rho_b c_b = 0$ and $\rho_f c_f^2 + \rho_b c_b^2 = \rho c^2$. Then, $a(x)$ satisfies $-\partial^2 a(x) = 2\pi \rho \{ (r^2 + c^2) \exp(-2a(x)) - 1 \}$. We note that there are two centers in the solution ($z = -c_f, -c_b$) and the phase $\theta$ of $\Psi_{tu}$ changes by $2\pi$ when we go around the center at $z = -c_i$. Obviously, the parameter $c \propto |c_f - c_b|$ represents a length scale of the phase coherence of the solution.

The pseudoSkyrmion has no electrons with down spin component. Although it has no real spins, the pseudoSkyrmion possesses pseudospin. Thus, it has an exchange energy associated with the pseudospin, $\int dx^2 (\rho_{ss}/2) \partial_k m_s \partial_k m_s = 4\pi \rho_{ss}$, where $m_s$ denotes unit vector of the pseudospin; $m_s \propto \Psi_{ps}^\dagger \sigma \Psi_{ps}$ with $\Psi_{ps}^\dagger = (\Psi_{fu}^\dagger, \Psi_{bu}^\dagger)$. $\rho_{ss}$ is the pseudospin stiffness smaller than $\rho_s$, in general; e.g. $\rho_{ss} \simeq 0.14 \rho_s (1 - \sigma^2)$ for $d/l = 1.5$ \cite{14}.

Contrary to the case of the Skyrmion, the pseudoSkyrmion possesses a nontrivial charging energy because of its nontrivial pseudospins $s_3(x) \propto m_{s,3}$,

$$e^2 \frac{4\epsilon}{4\epsilon} \int dx^2 dy^2 (s_3(x) - s_3)V_-(x - y)(s_3(y) - s_3)$$

$$= e^2 \frac{4\epsilon}{4\epsilon} \int dx^2 dy^2 \{ \sigma^2 \rho_r(x) \rho_r(y) + (1 - \sigma^2) \rho_t(x) \rho_t(y) \cos(\theta_x) \cos(\theta_y) \} V_-(x - y).$$

(4)

with $s_3(x) = (|w_{fu}|^2 - |w_{bu}|^2) \exp(-2a(x))$, $s_3 = \rho_f - \rho_b$ and $V_- = 1/|x - y| - 1/\sqrt{(x - y)^2 + d^2}$, where the integral with the coefficient, $\sigma^2$, is finite but the integral with the coefficient, $1 - \sigma^2$, is logarithmically divergent; $\rho_r(x) = \rho \{ (r^2 + c^2) \exp(-2a(x)) - 1 \}$ and $\rho_t(x) = 2\rho c r \exp(-2a(x))$. This divergent integral results from non spherical distribution of the electric charge. Such charge distribution would be screened. We expect that the screening effect makes the energy of this term much smaller than that of the first term proportional to $\sigma^2$ and the exchange (Coulomb) energy $\sim e^2/\epsilon l$. This assumption is crucial in our argument but seems to hold in actual samples because previous observations \cite{10,12} are consistent with our results based on the assumption.

The pseudoSkyrmion also has the tunneling energy, $E_t = \Delta_{sas} \sqrt{1 - \sigma^2}/2 + \Delta_{sas} \sqrt{1 - \sigma^2} c^2 I(c)/2\pi l^2$. Therefore, the total energy of the pseudoSkyrmion with $c > l$ is given by.
\[ E_{\text{psk}} = 4\pi \rho_{ss} + \frac{h_1 e^2}{\epsilon c} + \frac{h_2 e^2 \sigma^2}{cl} \frac{d}{l^2} + \frac{\Delta_{sas} \sqrt{1 - \sigma^2}}{2\pi} \frac{c^2}{l^2} I(c) + \frac{\Delta_{sas} \sqrt{1 - \sigma^2}}{2} + \frac{g\mu B}{2}, \]  

with \( h_2 \simeq 0.4 \). We find that the charging energy is much larger than that of the Skyrmion, which makes us distinguish experimentally the Skyrmion from the pseudoSkyrmion.

By minimizing \( E_{\text{psk}} \) in the parameter \( c \), we find that the length scale \( c_0 \) and the energy of the pseudoSkyrmion are approximately given by

\[
c_0 \simeq l \left\{ \frac{h_1 e^2 / 2cl}{\Delta_{sas} \sqrt{1 - \sigma^2} + (h_2 e^2 \sigma^2 / cl)(d/l)} \right\}^{1/3},
\]

\[
E_{\text{psk}}(c_0) \simeq 4\pi \rho_{ss} + \frac{3h_1 e^2}{2\epsilon c_0} + \frac{\Delta_{sas} \sqrt{1 - \sigma^2}}{2} + \frac{g\mu B}{2}.
\]

Roughly speaking, \( E_{\text{psk}}(c_0) \) decreases with the tunneling energy, \( \Delta_{sas} \), for the small imbalance, \( \sigma^2 \ll 1 \), while it remains a constant for \( \sigma^2 \simeq 1 \).

We comment that the terms, \( \Delta_{sas} \sqrt{1 - \sigma^2}/2 + g\mu B/2 \) in both \( E_{sk} \) and \( E_{psk} \) are irrelevant in the observation of activation energies. A pair of Skyrmion and anti-Skyrmion is actually observed so that these terms cancel with corresponding ones of anti-Skyrmion, \(-\Delta_{sas} \sqrt{1 - \sigma^2}/2 + g\mu B/2 \). Thus, we ignore the terms in the subsequent discussions.

Here we should mention that the tunneling energy of both types of Skyrmions decreases by increasing parallel magnetic field, \( B_\parallel \), with keeping longitudinal component \( B_\perp \) ( \( \nu = 2\pi/eB_\perp = 1 \)),

\[
E_t^{B_\parallel}(\text{Sk}) = -\Delta_{sas}^Q \frac{\sqrt{1 - \sigma^2}}{2} \rho \int dx^2 \{ (r^2 + c^2) \exp(-2a(x)) - 1 \} \cos(Qx_1) \]

\[
E_t^{B_\parallel}(\text{pSk}) = E_t^{B_\parallel}(\text{Sk}) + 2\Delta_{sas}^Q \frac{\sqrt{1 - \sigma^2}}{2} \rho \int dx^2 c^2 \exp(-2a(x)) \cos(Qx_1)
\]

with \( Q = edB_\parallel \), where ‘Sk’ for Skyrmion (‘pSk’ for pseudoSkyrmion) and these replace the tunneling energies in eq(3) and eq(6). The tunneling strength, \( \Delta_{sas} \), is reduced to \( \Delta_{sas}^Q = \Delta_{sas} \exp\{-(d/2l)^2 \tan^2 \Theta \} \) owing to suppression of tunneling by the parallel magnetic field; \( \tan \Theta = B_\parallel / B_\perp \). We can see numerically that this tunneling energy becomes small with \( Q \) and vanishingly small when \( Q > c^{-1} \) owing to the factor, \( \cos(Qx_1) \). Note that \( c \) is determined for the energies of Skyrmions to be minimized. Especially in the case of the
pseudoSkyrmion at $\sigma = 0$, such $c(Q)$ increases with $Q$ since $c(Q)$ increases as $E_t^{B||}$ decreases and $E_t^{B||}$ decreases with $Q$. But, $c(Q)$ can not be larger than the coherent length $\eta \propto \Delta_{sas}^{-1}$ of the phase, $\theta_f - \theta_b$. Thus, there exist a critical point $Q_c \simeq c(Q_c)^{-1} = \eta^{-1} \propto \Delta_{sas}$, at which the tunneling energy becomes vanishingly small,

$$Q_c = edB_\parallel \propto \Delta_{sas} \text{ or } \tan \theta_c = h_0 \frac{l}{d} \frac{\Delta_{sas}}{e^2/l}$$  (9)

with $h_0$ being a numerical constant. Hence, the activation energy of the pseudoSkyrmion decreases with $Q$ until $Q$ is equal to $Q_c$. (The ground state has been argued to change from a commensurate state to an incommensurate state when $Q$ goes beyond the critical point $Q_c$. Thus, it is necessary to analyze the relevance of these solitons in the region of $Q > Q_c$ more carefully.) This estimation in eq(9) gives a rough agreement with previous measurements [10] of the activation energies. On the other hand, in the case of Skyrmion the scale $c_0 \propto (g\mu B)^{-1/3}$ does not depend on $E_t^{B||}$. Thus, the dependence of its energy on $B_\parallel$ is rather simple; there is no critical behavior in $Q$.

Finally, we wish to discuss which type of Skyrmion is realized when we change the imbalance parameter. We find that at $\sigma = 0$ the energy of the pseudoSkyrmion is smaller than that of the Skyrmion because $\rho_{ss} \ll \rho_s$, while at $\sigma = 1$ the energy of the Skyrmion is smaller than that of the pseudoSkyrmion. This is because the charging energy of the pseudoSkyrmion vanishes at $\sigma = 0$ but becomes maximal, i.e. $\sim (e^2/\epsilon l) d c_0^2/l^3$ at $\sigma = 1$. Thus there exists a critical point $\sigma_c$, at which a transition between the Skyrmion and the pseudoSkyrmion arises; e.g. $\sigma_c \simeq 0.4$, with a typical sample parameter, $\rho = 10^{11}/$cm$^2$, $d/l = 1.5$ and $\Delta_{sas} = 1 \sim 5K$. This agrees roughly with the experiment [12].

The transition between the Skyrmion and the pseudoSkyrmion can be seen by the measurement of the dependence of the activation energy on $B_\parallel$ or $\sigma$. Imposing the parallel magnetic field with fixing $\nu = 1$, we make Zeeman energy increase, but the tunneling energy, $E_t^{B||}$, decrease. Therefore, The energy of the Skyrmion increases monotonously with $B_\parallel$ such as $E_{sk}(c_0) \propto (g\mu B)^{1/3}$. On the other hand, the energy, $E_{psk}(c_0) \propto 1/c_0(Q)$, of the pseudoSkyrmion decreases with $B_\parallel$, but remains a constant when $B_\parallel > Q_c/ed$; $c_0(Q)$ can
not be larger than $\eta \sim \Delta_{ss}^{-1}$. Furthermore, we find that the energy of the pseudoSkyrmion increases with $\sigma$ for $\sigma < \sigma_c$, while that of the Skyrmion remains a constant for $\sigma > \sigma_c$; more precisely, owing to the presence of the charging energy, $\simeq 0.14\sigma^2(e^2/\epsilon l)(l/c)(d/c)$, of the Skyrmion, the total energy of the Skyrmion increases but much more slowly with $\sigma$ than that of the pseudoSkyrmion increases. Thus, the distinction of these two types of Skyrmions can be easily found. The dependence of the activation energies on $\sigma$ and $B_{||}$ has been partially measured [10–12] and is consistent with our present discussions.

Probably, Skyrmions and pseudoSkyrmions are relevant excitations in the bilayer quantum Hall states with the interlayer phase coherence at $\nu = 1$. In order to confirm our results, an experiment suggested in this paper is in progress.

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