Charm meson resonances in $D \to P\ell\nu$ decays

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Motivated by recent experimental results we reconsider semileptonic $D \to P\ell\nu$ decays within a model which combines heavy quark symmetry and properties of the chiral Lagrangian. We include excited charm meson states, some of them recently observed, in our Lagrangian and determine their impact on the charm meson semileptonic form factors. We find that the inclusion of excited charm meson states in the model leads to a rather good agreement with the experimental results on the $q^2$ shape of the $F_2(q^2)$ form factor. We also calculate branching ratios for all $D \to P\ell\nu$ decays.

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I. INTRODUCTION

The knowledge of the form factors which describe the weak $\text{heavy} \to \text{light}$ semileptonic transitions is very important for the accurate determination of the CKM parameters from the experimentally measured inclusive decay rates. A lack of precise information about the shapes of various form factors is thus still the main source of uncertainties especially in these processes.

In addition to many studies of exclusive $B$-meson semileptonic decays there are new interesting results on $D$-meson semileptonic decays. The CLEO and FOCUS collaborations have studied semileptonic decays $D^0 \to \pi^-\ell^+\nu$ and $D^0 \to K^-\ell^+\nu$ \cite{1,2}. Their data provide new information on the $D^0 \to \pi^-\ell^+\nu$ and $D^0 \to K^-\ell^+\nu$ form factors. Usually in $D$ semileptonic decays a simple pole parametrization has been used in the past. The results of Refs. \cite{1,2} for the single pole parameters required by the fit of their data, however, suggest pole masses, which are inconsistent with the physical masses of the lowest lying charm meson resonances. In their analyses they also utilized a modified pole fit as suggested in \cite{3} and their results indeed suggest the existence of contributions beyond the lowest lying charm meson resonances \cite{4}.

There exist many theoretical calculations describing semileptonic decays of heavy to light meson: quark models (QM) \cite{5,6,7,8,9}, QCD sum rules (SR) \cite{10,11,12,13,14,15,16,17}, lattice QCD \cite{18,19,20}, and a few attempts to use combined heavy meson and chiral Lagrangian theory (HM\chi T) \cite{21,22}. Most of the above methods have limited range of applicability. For example, QCD sum rules are suitable only for describing the low $q^2$ region while lattice QCD and HM\chi T give good results only for the high $q^2$ region. However, the quark models, which do provide the full $q^2$ range of the form factors, cannot easily be related to the QCD Lagrangian and require input parameters, which may not be of fundamental significance \cite{4}.

Recently new experimental studies of charm meson resonances have provided a lot of new information on the charm sector. Several experiments on open charm hadrons have reported very interesting discoveries: First BaBar \cite{23} announced a new, narrow meson $D_{sJ}(2317)^+$. This was confirmed by Focus \cite{24} and CLEO \cite{25} who also observed another narrow state, $D_{sJ}(2463)^+$. Both states were confirmed by Belle \cite{25} who also provided first evidence for two new, broad states $D_0'(2308)$ and $D_1'(2427)$, both ca. 350 – 400 MeV higher above the usual $D^0, D^*$ states and with opposite parity. Finally, Selex \cite{26} has announced a new, surprisingly narrow state $D_{sJ}^*(2632)$ with the spin parity assignment $1^-$. Both $D_{sJ}^*(2317)^+$ and $D_{sJ}^*(2463)^+$ have already been proposed as members of the $(0^+,1^+)$ spin doublet chiral partners of the heavy-light pseudoscalar and vector $D_s$ mesons \cite{23,27}, while the states $D_0'(2308)$ and $D_1'(2427)$ have also been proposed as chiral partners of the $D$-meson doublet \cite{28}. On the other hand, a proposal has been put forward for the $D_{sJ}^*(2632)$ state as the first radial excitation of the $D_0^*(2112)$ \cite{23,29}. While strong and electromagnetic transitions of these new states have already been studied \cite{23,29}, they have not yet been applied to the description of weak decays of charmed mesons. Specifically in the description of heavy to light weak transition form factors they are yet to be taken into account as possible transition resonances.

The purpose of this paper is to investigate contributions of the newly discovered and theoretically predicted charm mesons within an effective model based on HM\chi T by incorporating the newly discovered heavy meson fields into the HM\chi T Lagrangian. In this paper we only focus onto the $D \to P\ell\nu$ transitions, since available experimental data can be used in determination of all the required parameters. We restrain our discussion to the leading chiral and $1/m_H$ terms in the expansion, but we hope to capture the main physical features about the impact of the nearest poles in the $t$-channel to the $q^2$-dependence of the form factors.

Within the context of heavy to light transitions the HM\chi T is only valid at small recoil momentum (large $q^2$) and in order to predict the whole $q^2$ region depen-
dence, the form factors have to be modeled with a presumed pole ansatz. After including excited charm vector meson states we notice that their presence implies a natural appearance of the double pole shape for the $F_+$ form factor. Assuming such form factor shape we obtain good agreement with the observed $q^2$ dependence of $F_+$ in $D^0 \to K/\pi$ transitions as well as with the measured branching ratios for these decays.

We compare our results for the form factor $q^2$ dependence with existing experimental \[1, 2, 22\] and lattice QCD \[17\] results, as well as results of other theoretical studies \[4, 9\]. We complete our study by calculating branching ratios for all $D \to P\ell\nu$ ($P = \pi, K, \eta, \eta'$) transitions.

Sec. II describes the framework we use in our calculations: first we write down the HMYT Lagrangian for heavy and light mesons and extend it to incorporate new heavy meson fields. In Sec. III we study $q^2$ behavior of the form factors in $D \to P\ell\nu$ decays. Finally, a short summary of the results and comparison with experimental data is given in Sec. IV.

II. THE FRAMEWORK

A. Strong interactions

Interactions, relevant for our study, between odd parity heavy mesons and light pseudoscalar mesons are described by the leading order interaction Lagrangian, namely (see e.g. \[18, 19\])

$$\mathcal{L}_{\text{int}} = i g \langle H_b \gamma_\mu \gamma_5 A_\mu \bar{H}_a \rangle,$$

with $H = 1/2(1 + \hat{P})[P_\mu \gamma_\mu - P_5 \gamma_5]$, the matrix representation of the heavy meson fields, where $P_\mu$ and $P$ are creation operators for heavy-light vector and pseudo scalar mesons respectively. Light pseudoscalar meson fields are encoded in $\Sigma = \exp(2i\mathcal{M}/f)$ where $\mathcal{M}$ is the pseudogoldstone flavor matrix

$$\mathcal{M} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\eta + \pi^0) & \pi^+ & K^+ \\ \bar{\pi}^- & \frac{1}{\sqrt{2}}(\eta - \pi^0) & K^0 \\ \bar{K}^- & \bar{K}^0 & \eta_s \end{pmatrix}. \quad (2)$$

To describe $\eta$ $\eta'$ mixing we follow the work of Ref. \[30\], where $|\eta\rangle = |\eta_h\rangle \cos \phi - |\eta_s\rangle \sin \phi$ and $|\eta'\rangle = |\eta_h\rangle \sin \phi + |\eta_s\rangle \cos \phi$, and $\phi$ is the mixing angle between the flavor and mass eigenstates. The light pseudoscalar meson axial current is defined as $A_\mu = 1/2(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger)$, where $\xi^2 = \Sigma$ so that $A_\mu \approx (i/f) \partial_\mu \mathcal{M}$. Furthermore, $\langle \ldots \rangle$ indicate a trace over spin matrices and summation over light quark flavor indices.

In order to incorporate positive parity heavy meson states into the model, we introduce the scalar-axialvector fields doublet $G = 1/2(1 + \hat{P})[S_\mu \gamma_\mu \gamma_5 - S]$ representing axial-vector ($S_\mu$) and scalar ($S$) mesons and incorporate it into the interaction Lagrangian by adding an additional leading order interaction term between even and odd parity fields:

$$\mathcal{L}'_{\text{int}} = i h (G_b \gamma_\mu \gamma_5 A_\mu \bar{H}_a) + \text{h.c.}. \quad (3)$$

Finally we include the radially excited states into our discussion by introducing another odd parity heavy meson multiplet field $H' = 1/2(1 + \hat{P})[P'_\mu \gamma_\mu - P'_5 \gamma_5]$ containing the radial excitations of ground state pseudoscalar and vector mesons. Such excited states were predicted in Ref. \[31\]. The strong interactions between these fields and ground state meson fields $H$ can again be described by the lowest order interaction Lagrangian analogous to \[19\]

$$\mathcal{L}_{\text{int}} = i g \langle H_b^* \gamma_\mu \gamma_5 A_\mu \bar{H}_a \rangle + \text{h.c.}. \quad (4)$$

B. Weak interactions

For the semileptonic decays the weak Lagrangian can be given by the effective current-current Fermi interaction

$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} [\bar{\nu} \gamma_\mu (1 - \gamma^5) \nu \mathcal{J}_\mu],$$

where $G_F$ is the Fermi constant and $\mathcal{J}_\mu$ is the effective hadronic current. In heavy to light meson decays it can be written as $\mathcal{J}_\mu = K_a J_a^\mu$, where constants $K_a$ parametrize the $SU(3)$ flavor mixing, while the leading order weak current $J_a^\mu$ in $1/m_H$ (where $m_H$ is the heavy meson mass) and chiral expansion can be written as

$$J_a^\mu = \frac{1}{2} i \alpha (\gamma^\mu (1 - \gamma^5) H_b \xi_{ba}^\dagger) \quad (6)$$

for the $0^-$ and $1^-$ heavy mesons, while similarly for the $0^+$ and $1^+$ states we write \[19\]

$$J_a^\mu = \frac{1}{2} i \alpha' (\gamma^\mu (1 - \gamma^5) G_b \xi_{ba}^\dagger). \quad (7)$$

Finally for the radially excited pseudoscalar and vector fields $J_a$ can be written as:

$$J_a^\mu = \frac{1}{2} i \alpha'' (\gamma^\mu (1 - \gamma^5) H_b \xi_{ba}^\dagger). \quad (8)$$

III. FORM FACTOR CALCULATION

Now we turn to the discussion of the form factor $q^2$ distribution. The $H \to P$ current matrix element is usually parametrized as

$$\langle P(p_P) | (V - A)^\mu | H(p_H) \rangle = F_+(q^2) \left( (p_H + p_P)^\mu - \frac{m_H^2 - m_P^2}{q^2} q^\mu \right) + F_0(q^2) \frac{m_H^2 - m_P^2}{q^2} q^\mu, \quad (9)$$

$$F_+(q^2) = \int_0^{\infty} \frac{d^4 p}{(2\pi)^4} \frac{\epsilon^\mu p^\nu \mathcal{T}}{p^0 + m_H} \epsilon_{\nu\nu'}, \quad (10)$$

$$F_0(q^2) = \int_0^{\infty} \frac{d^4 p}{(2\pi)^4} \frac{\epsilon^\mu p^\nu \mathcal{T}}{p^0 - m_H}, \quad (11)$$

where $\mathcal{T}$ is the transition amplitude and $\epsilon$ is the polarization four vector of the final state quark.
where \((V - A)\) is the weak left-handed quark current and \(q = (p_H - p_F)\) is the exchanged momentum. Here \(F_+\) denotes the vector form factor and it is dominated by vector meson resonances, while \(F_0\) denotes the scalar form factor and is expected to be dominated by scalar meson resonance exchange \([3, 32]\). In order that these matrix elements are finite at \(q^2 = 0\), the form factors must also satisfy the relation

\[
F_+(0) = F_0(0). \tag{10}
\]

In Ref. \([32]\) it was pointed out that in the limit of a static heavy meson

\[
|H(v)\rangle_{\text{HQET}} = \lim_{m_H\to\infty} \frac{1}{\sqrt{m_H}}|H(p_H)\rangle \tag{11}
\]

one can use the following decomposition:

\[
\langle P(p_P)|H(v)\rangle_{\text{HQET}} = \left[\frac{\mu}{m_H}\right]^\alpha v^\mu f_p(v \cdot p_P) + v^\mu f_v(v \cdot p_P), \tag{12}
\]

where the form factors \(f_{p,v}\) are functions of the variable

\[
v \cdot p_P = \frac{m_H^2 + m_F^2 - q^2}{2m_H}. \tag{13}
\]

The form factors \(F_{+,0}\) given in \([9]\) and the form factors \(f_{p,v}(v \cdot p_P)\) are related to each other by matching QCD to Heavy quark effective theory (HQET) at the scale \(\mu \simeq m_c\) (see Eq. (14) of Ref. \([33]\)). As in \([33]\) we fix the matching constants to their tree level values. This approach immediately accounts for the \(F_{+,0}\) behavior at \(q_{\text{max}}^2\): At the leading order in heavy quark expansion, the two definitions are then related near zero recoil momentum \((q^2 \simeq q_{\text{max}}^2 = (m_H - m_F)^2\) or equivalently \(|\vec{p}_R| \simeq 0\) as

\[
F_0(q^2)|q^2=q_{\text{max}}^2 = \frac{1}{\sqrt{m_H}} f_v(v \cdot p_P) \tag{14a}
\]

\[
F_+(q^2)|q^2=q_{\text{max}}^2 = \frac{\sqrt{m_H}}{2} f_p(v \cdot p_P). \tag{14b}
\]

Note, however, that the \(F_{+,0}\) form factors might contain \(1/m_c\) corrections \([33]\), which we do not consider within present approach.

We use Feynman rules for HM\(X\)T coming from the strong and weak Lagrangians described in the previous section. For the heavy meson propagators we use \(i\delta_{ab}/2(v \cdot k - \Delta)\) and \(-i\delta_{ab}(q_{nm} - v^a v^b)/(2(v \cdot k - \Delta))\) for the pseudoscalar(scalar) and vector (axial) mesons respectively, where \(k^\mu = q^\mu - m_H v^\mu\) and \(\Delta = \Delta_R\) is the mass splitting between the heavy resonance meson \(R\) and the ground state heavy pseudoscalar meson (see e.g. Ref. \([19]\)). For the hadronic current matrix element we thus get

\[
\langle P(p_P)|J^\mu|H(v)\rangle = -\frac{\alpha}{f} v^\mu + \frac{\alpha}{g} \frac{|v|}{|p|} (v \cdot p_P - p_P^\mu) + \frac{\alpha}{g} \frac{|v|}{|p|} (v \cdot p_P + \Delta^2) + \frac{\alpha}{h} \frac{|v|}{|p|} v^\mu (v \cdot p_P + \Delta H), \tag{15}
\]

We apply the projectors \(v^\mu\) and \(v^\mu v \cdot p_P - p_P^\mu\) on eq. (15) and extract the form factors \(F_+(q^2)\) and \(F_0(q^2)\) at \(q_{\text{max}}^2\) using Eqs. (12), (14) and (16).

\[
F_+(q_{\text{max}}^2) = -\frac{\alpha}{\sqrt{m_H} f} \frac{m_H}{m_p + \Delta H} \tag{16}
\]

and

\[
F_0(q_{\text{max}}^2) = -\frac{\alpha}{\sqrt{m_H} f} + \frac{\alpha'}{h} \frac{m_H}{m_p + \Delta H}. \tag{17}
\]

If one uses directly relation (9) instead of this extraction of form factors at large \(q_{\text{max}}^2\), one ends up with the mixed leading \(\sqrt{m_H}\) terms and the subleading \(1/\sqrt{m_H}\) terms. Furthermore, the scalar meson contribution appears in the \(F_+\) form factor. The extraction of form factors we follow here \([32]\) gives a correct \(1/m_H\) behavior of the form factors and the contributions of \(1^-\) resonances enter in \(F_+\), while \(0^+\) resonances contribute to \(F_0\) as they must \([32]\). In previous theoretical studies of the \(D\) meson form factors \([17]\) a single pole parametrization has been used when extrapolating from \(F_+(q_{\text{max}}^2)\). In such calculations only the \(D^+\) resonance contributed, and a lower value of the \(g\) strong coupling was used. At that time only few decay rates were measured. In comparison with the present experimental data the predicted branching ratios were too large. The approach of Ref. \([34]\) was developed to treat \(D\) meson semileptonic decay within heavy light meson symmetries in the allowed kinematical region by using the full propagators. We find that this approach cannot reproduce the observed \(q^2\) shape of the \(F_+\) form factors.

In our study of form factors’ \(q^2\) distributions, we follow the analysis of Ref. \([2]\), where the \(F_+\) form factor is given as a sum of two pole contributions, while the \(F_0\) form factor is written as a single pole. This parametrization includes all known properties of form factors at large \(m_H\) and the QCD sum rules for low \(q^2\) region. Their proposal for the form factor parametrization is

\[
F_+(q^2) = c_B \left(\frac{1}{1-x} - \frac{a}{1-x/\gamma}\right) \tag{18a}
\]

\[
F_0(q^2) = \frac{c_B(1-a)}{1-bx}, \tag{18b}
\]

where \(x = q^2/m_H^2\). Using the relation which connects the form factors within large energy release approach \([35]\)
the authors in Ref. 3 then find $a = 1/\gamma$ and so obtain a simplified double pole function for the $F_+$ form factor

$$F_+(q^2) = \frac{F_+(0)}{(1-x)(1-ax)}$$

(19)

Although the $D$ mesons might not be considered heavy enough, we employ these formulas with the model matching condition at $q^2_{\text{max}}$. At the same time we fix the parameters $a$ and $b$ in Eqs. (18) and (19) by the next-to-nearest resonances. We use physical pole masses of excited state charmed mesons in this extrapolation, giving for the ratios $a = m_{H^*}/m_{H}$ and $b = m_{H}/m_{H^*}$. Although in the original idea the extra pole in $F_+$ parametrized all the neglected higher resonances, we are here saturating those by a single nearest resonance.

In our numerical analyses we use available experimental information and theoretical predictions on charm meson resonances. Particularly for the scalar resonance we use the $D_{sJ}(2317)^+\to \eta\pi^+$ state with mass $m_{D_{sJ}(2317)^+} = 2.317\ GeV$. For the radially excited vector resonance we then have the Selex $D_{sJ}^*(2632)^+$ state with mass $m_{D_{sJ}^*(2632)} = 2.632\ GeV$. It is important to note, however, that so-far the Selex discovery has not been confirmed by any other experimental results. In $D$ decays, the situation is similarly ambiguous. Although the vector $D^*$ resonance was discovered by Delphi with a mass of $m_{D^*} = 2.637\ GeV$ and spin-parity $1^-$, its existence was not confirmed by other searches 31. On the other hand recent theoretical studies 32, 33 indicate that both radially excited states of $D$ as well as $D_s$ should have slightly larger masses of $m_{D^*} \approx 2.7\ GeV$ and $m_{D_s^*} \approx 2.8\ GeV$ 31. We use these theoretically predicted values in our analysis. Even less clear is the situation with the scalar resonance ($D^0$), which was predicted in Ref. 31 but not yet confirmed by experiment. Here we use the theoretically predicted mass value of $m_{D^0} = 2.3\ GeV$ from Ref. 31.

In our calculations we use for the heavy meson weak current coupling $\alpha = f_H\sqrt{m_H}$ 34, which we calculate from the lattice QCD value of $f_H = 0.225\ GeV$ 35 and experimental $D$ meson mass $m_D = 1.87\ GeV$ 36, yielding $\alpha = 0.33\ GeV^{3/2}$. For light pseudoscalar mesons we use $f = 130\ MeV$, while for the $\eta$ coupling we use the value of $\phi \simeq 40\ GeV$ 36. For the $g$-coupling we use the experimentally determined value of $g = 0.59$ 40 while for the $h$ coupling we use the value of $h = -0.6$ from the global estimate of Ref. 41. We can also derive a constraint on the absolute value of $\hat{g}$ from the Selex bound on the $D_{sJ}(2632)\to D^0K^+$ decay rate of $\Gamma(D_{sJ}(2632)\to D^0K^+) < 17\ MeV$ 24. In HM$^X$T this decay rate can be derived directly from the interaction Lagrangian 4 and reads $\Gamma = 1/6\pi[\hat{g}/f^2][\hat{g}K]^{3}$ which gives for the coupling constant $|\hat{g}| < 0.21$.

Eqs. (18a) and (19) can be combined to obtain a theoretical estimate for the value of $\hat{g}$ in the limit of infinite heavy meson mass. By equating both terms in Eqs. (19a) and (19a) at $q^2_{\text{max}}$ and then imposing $a = 1/\gamma$ one obtains

$$\hat{g} \sim -\alpha g \frac{m_p + \Delta_{H^*}}{m_p + \Delta_{H^*}} \frac{(m_H - m_p)^2 - m_{H^*}^2}{(m_H - m_p)^2 - m_{H^*}^2}. \quad (20)$$

We apply this formula to the $D \to \pi$ transitions where the chiral symmetry breaking corrections are smallest and thus the results most reliable. This yields $\hat{g} \sim -0.15\ GeV^{3/2}$. On the other hand the $1/m_D$ and chiral corrections might still modify this result significantly. Such corrections were explicitly written out in Ref. 42, but they include additional parameters which cannot be fixed within this context.

Similarly, in this limit we can infer on the value of $\alpha'$. By applying equalities (10) and (20) to Eqs. (17) and (18) we obtain

$$\alpha' \sim \frac{\alpha}{h} \times \frac{m_p + \Delta_{H^*}}{m_p} \times \left[ 1 - \frac{g}{2} \times \frac{m_H}{m_p + \Delta_{H^*}} \left( 1 - \frac{m_{H^*}^2}{m_{H^*}^2} \right) \frac{1}{1 - \frac{m_{H^*}^2}{m_{H^*}^2}} \right], \quad (21)$$

which gives, when applied to the $D \to \pi$ transitions a value of $\alpha' \sim -0.69\ GeV^{3/2}$.

Alternatively the values of the new model parameters can be determined by fitting the model predictions to known experimental values of branching ratios $B(D^0 \to K^-\ell^+\nu)$, $B(D^+ \to K^0\ell^+\nu)$, $B(D^0 \to \pi^+\ell^+\nu)$, $B(D^+ \to \pi^+\ell^+\nu)$, $B(D_{sJ}^* \to \eta\ell^+\nu)$, $B(D_{sJ}^+ \to \eta\ell^+\nu)$ 32. In our decay width calculations we shall neglect the lepton mass, so the form factor $F_0$, which is proportional to $q^0$, does not contribute. For the decay width we get [34]:

$$\Gamma = \frac{G_F^2 m_y^2 |K_H|^2}{24\pi^3} \int_0^{y_m} dy |F_+(m_H^2 y)|^2|\hat{g}P(y)|^3, \quad (22)$$

where $y = q^2/m_H^2$, so that

$$y_m = \left( 1 - \frac{m_p}{m_H} \right)^2 \quad (23)$$
experimental and lattice data when extrapolated with a HM

From the plots it becomes apparent, that our model’s experimental results of a double pole fit from CLEO [1] lattice QCD double pole fit analysis [17], as well as the α

The equality (10), when applied to the same decays with corrections which are not included systematically into analyses. This discrepancy further increases if only the first resonance contribution is kept in the $F_+(q^2_{\text{max}})$ calculation for the single pole extrapolation within HM\_Y\_T as done in previous calculations [19]. Note also that the experimental fits on the single pole parametrization of the $F_+$ form factor in $D^0 \rightarrow \pi^- (D^0 \rightarrow K^-)$ transitions done in Refs. [1, 2] yielded effective pole masses which are somewhat lower than the physical masses of the $D^*(D^*)$ meson resonances used in this analysis.

We also compare our predictions for the $F_0$ scalar form factor $q^2$ dependence for the $D^0 \rightarrow K^-$ and $D^0 \rightarrow \pi^-$ transitions with those of quark model in Ref. [3] and with lattice QCD pole fit analysis of Ref. [17]. The results are depicted in FIG. 3 and FIG. 4. Note that without the scalar resonance, one only gets a contribution from the $\alpha/\sqrt{m_H f}$ term from Eq. (17). This gives for the $q^2$ dependence of $F_0$ a constant value $F_0(q^2) = 1.81$ for both $D^0 \rightarrow \pi^-$ and $D^0 \rightarrow K^-$ transitions, which largely disagrees with lattice QCD results as well as heavily violates relation (10).

Finally, using numerical values as explained above, we calculate the branching ratios for all the relevant $D \rightarrow P$ semileptonic decays and compare the predictions of our model with various other model predictions found in the literature, and with experimental data from PDG. The results are summarized in Table III. For comparison we also include the results for the rates obtained with our approach for $F_+(q^2_{\text{max}})$ (Eq. 16) but using a single pole fit. It is very interesting that our model extrapolated with a double pole gives branching ratios for $D \rightarrow P\ell\nu_\ell$ in rather good agreement with experimental results for the already measured decay rates, while the predictions for the unmeasured decay rates agree with results of existing approaches. It is also obvious that the single pole fit gives the rates up to a factor of two larger than the experimental results. Only for decays to $\eta$ and $\eta'$ as given in Table III an agreement with experiment of the double pole version of the model is not better but worse than for the single pole case.

We also calculate branching ratios within our model, when the value of $\tilde{\alpha}g$ is estimated from Eq. (20) rather than from the branching ratio fit. While the $D \rightarrow \pi$ decay rates remain in reasonable agreement with experimental values, predictions for other decay rates come out up to 30% lower. This again indicates large contributions from $1/m_D$ and especially chiral corrections in these decay amplitudes. We also attempt using experimental fits for the parameter $a$ in Eq. (16) instead of saturating the second pole by physical pole masses $a = m_H^2 / m_{\bar{m}}^2$. By using the experimentally fitted values of $a = 0.36(0.37)$ for $D \rightarrow K(D \rightarrow \pi)$ decays from Ref. [3] we notice that, while the fitted values of $\tilde{\alpha}g$ tend to shift towards larger negative values, the overall goodness of fit on the experimental branching ratios and consequently decay rate predictions of our model remain almost the same. This indicates that the actual positions of the poles (masses of excited charmed resonances) are not very volatile input

The constants $K_{HP}$ parametrize the flavor mixing relevant to a particular transition, and are given in Table I together with the pole mesons.

We calculate the result for $\tilde{g}\alpha$ by a weighted average of values obtained from all the measured decay rates taking account for the experimental uncertainties. The calculation yields $\tilde{\alpha}g = -0.0050 \text{ GeV}^{3/2}$, which is rather small compared to estimation given by [20]. This discrepancy can be attributed to the presence of $1/m_D$ and chiral corrections which are not included systematically into consideration here due too many new parameters [17] which cannot be fixed within this approach. However, we estimate the influence of such corrections on the fitted value of $\tilde{\alpha}g$ by varying the value of $\alpha g/\sqrt{f}$ in Eq. (10) by 10% [13] and inspecting the fit results. We obtain a range of $\tilde{\alpha}g \in [-0.05, 0.04] \text{ GeV}^{3/2}$.

Knowing $\tilde{\alpha}g$ we can further infer on the value of $\alpha'$. The equality (10) [11] when applied to the same decays with $\tilde{\alpha}g = -0.0050 \text{ GeV}^{3/2}$, gives an average value of $\alpha' = -0.47 \text{ GeV}^{3/2}$, which is close to the estimate from [21]. We use these values for $\tilde{\alpha}g$ and $\alpha'$ in all our subsequent analyses.

We next draw the $q^2$ dependence of the $F_+$ form factors for the $D^0 \rightarrow K^-$ and $D^0 \rightarrow \pi^-$ transitions and compare it with results of quark models in Refs. [1, 2], lattice QCD double pole fit analysis [17], as well as the experimental results of a double pole fit from CLEO [11] and FOCUS [2] with $F_+(0)$ values taken from Ref. [29]. The results are depicted in FIG. 1 and FIG. 2. For comparison we also plot results when single pole fit is used. Also in this case we calculate $F_+(q^2_{\text{max}})$ within HM\_Y\_T and take into account both resonances (Eq. 16).

From the plots it becomes apparent, that our model’s predictions for both $D^0 \rightarrow K^-$ and $D^0 \rightarrow \pi^-$ transition $F_+$ form factors are in good agreement with experimental and lattice data when extrapolated with a double pole, while single pole extrapolations are not in good agreement with experimental results. This discrepancy further increases if only the first resonance contribution is kept in the $F_+(q^2_{\text{max}})$ calculation for the single pole extrapolation within HM\_Y\_T as done in previous calculations [19].

and

$$|\tilde{P}_P(y)|^2 = \frac{[m_H^2(1-y) + m_{\bar{m}}^2]^2}{4m_H^2} - m_{\bar{m}}^2.$$ (24)

| $H$ | $P$ | $H^*$ | $H^{**}$ | $H_P$ | $K_{HP}$ |
|-----|-----|-----|-----|-----|-----|
| $D^0$ | $K^-$ | $D^+_s$ | $D^+_s$ | $D_{sJ}(2317)^+$ | $V_{cs}$ |
| $D^+$ | $P^0$ | $D^+_s$ | $D^+_s$ | $D_{sJ}(2317)^+$ | $V_{cs}$ |
| $D^+$ | $\eta^-$ | $D^+_s$ | $D^+_s$ | $D_{sJ}(2317)^+$ | $V_{cs}$ |
| $D^+$ | $\eta^-$ | $D^+_s$ | $D^+_s$ | $D_{sJ}(2317)^+$ | $V_{cs}$ |
| $D^+$ | $K^0$ | $D^+_s$ | $D^+_s$ | $D^+_s$ | $V_{cd}$ |
| $D^+$ | $\eta^0$ | $D^+_s$ | $D^+_s$ | $D^+_s$ | $V_{cd}$ |
| $D^+$ | $\eta^0$ | $D^+_s$ | $D^+_s$ | $D^+_s$ | $V_{cd}$ |
| $D^+$ | $K^0$ | $D^+_s$ | $D^+_s$ | $D^+_s$ | $V_{cd}$ |

TABLE I: The pole mesons and the flavor mixing constants $K_{HP}$ for the $D \rightarrow P$ semileptonic decays.
parameters of our model. On the other hand corrections of the order $1/m_D$, as well as chiral corrections in the case of $D \to K\ell\nu$ and $D_s \to \eta(\eta')\ell\nu$ might improve the agreement with the experimental data, but due to the presence of a large number of new couplings it is impossible to include them into the calculation within present framework. We expect that the errors in the predicted decays rates stemming from the uncertainties in the input parameters we used can be 30%.

IV. SUMMARY

We have investigated semileptonic form factors for $D \to P$ decays within an approach which combines heavy meson and chiral symmetry. The contributions of excited charm meson states are included into analysis. The double pole behavior of the $F_+$ form factors is a result of the presence of the two $1^-$ charm meson resonances. The obtained $q^2$ dependence of the form factors is in good agreement with recent experimental results and the lattice calculation for $D^0 \to K^- (\pi^-)\ell\nu$. The calculated branching ratios are close to the experimental ones.

Our study shows that the single physical pole parametrization cannot explain the $q^2$ distribution of the semileptonic form factors. The calculated rates for this form factor parametrization are not in good agreement with the experimental results.

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FIG. 1: Comparison of the $D^0 \to K^-$ transition $F_+$ form factor $q^2$ dependence of our model double pole extrapolation (thick solid (black) line), single pole extrapolation (thick dashed (black) line), quark model of Ref. [4] (thick dotted (magenta) line), quark model of Ref. [9] (thin dotted (purple) line), lattice QCD fitted to a double pole [17] (dot-dashed (blue) line) and experimental double pole fits [1, 2] (thin (green) solid and dashed lines).

FIG. 2: Comparison of $D^0 \to \pi^-$ transition $F_+$ form factor $q^2$ dependence of our model double pole extrapolation (thick solid (black) line), single pole extrapolation (thick dashed (black) line), quark model of Ref. [4] (thick dotted (magenta) line), quark model of Ref. [9] (thin dotted (purple) line), lattice QCD fitted to a double pole [17] (dot-dashed (blue) line) and experimental double pole fit [1] (thin solid (green) line).
FIG. 3: Comparison of the $D^0 \to K^-$ transition $F_0$ form factor $q^2$ dependence of our model (solid (black) line), quark model of Ref. [4] (dotted (magenta) line) and lattice QCD fitted to a pole [17] (dot-dashed (blue) line).

FIG. 4: Comparison of the $D^0 \to \pi^-$ transition $F_0$ form factor $q^2$ dependence of our model (solid (black) line), quark model of Ref. [4] (dotted (magenta) line) and lattice QCD fitted to a pole [17] (dot-dashed (blue) line).
TABLE II: The branching ratios for the \( D \to P \) semileptonic decays. Comparison of different model predictions with experiment as explained in the text.

| Decay          | \( \mathcal{B}[,\%] \) | Reference                  |
|----------------|--------------------------|----------------------------|
| \( D^0 \to K^- \) | 3.4                      | This work (double pole)    |
|                | 4.9                      | This work (single pole)    |
|                | 3.75 ± 1.16              | QM [8]                     |
|                | 4.0                      | QM [4]                     |
|                | 3.9 ± 1.2                | QM [9]                     |
|                | 2.7 ± 0.6                | SR [10]                    |
|                | 3.4 ± 1.2                | SR [12]                    |
|                | 3.7 ± 1.4                | SR [15]                    |
|                | 3.43 ± 0.14              | Expt.                      |
| \( D^0 \to \pi^- \) | 0.27                     | This work (double pole)    |
|                | 0.56                     | This work (single pole)    |
|                | 0.236 ± 0.034            | QM [8]                     |
|                | 0.39                     | QM [4]                     |
|                | 0.30 ± 0.09              | QM [9]                     |
|                | 0.16 ± 0.3               | SR [11]                    |
|                | 0.28 ± 0.09              | SR [12]                    |
|                | 0.27 ± 0.10              | SR [15]                    |
|                | 0.36 ± 0.06              | Expt.                      |
| \( D_s^+ \to \eta \) | 1.7                      | This work (double pole)    |
|                | 2.5                      | This work (single pole)    |
|                | 1.8 ± 0.6                | QM [8]                     |
|                | 2.45                     | QM [4]                     |
|                | 2.5 ± 0.7                | Expt.                      |
| \( D_s^+ \to \eta' \) | 0.61                     | This work (double pole)    |
|                | 0.74                     | This work (single pole)    |
|                | 0.93 ± 0.29              | QM [8]                     |
|                | 0.95                     | QM [4]                     |
|                | 0.89 ± 0.33              | Expt.                      |
| \( D^+ \to \bar{K}^0 \) | 8.4                      | This work (double pole)    |
|                | 12.4                     | This work (single pole)    |
|                | 6.8 ± 0.8                | Expt.                      |
| \( D^+ \to \pi^0 \) | 0.33                     | This work (double pole)    |
|                | 0.70                     | This work (single pole)    |
|                | 0.31 ± 0.15              | Expt.                      |
| \( D^+ \to \eta \) | 0.10                     | This work (double pole)    |
|                | 0.15                     | This work (single pole)    |
|                | < 0.5                    | Expt.                      |
| \( D^+ \to \eta' \) | 0.016                    | This work (double pole)    |
|                | 0.019                    | This work (single pole)    |
|                | < 1.1                    | Expt.                      |
| \( D_s^+ \to K^0 \) | 0.20                     | This work (double pole)    |
|                | 0.32                     | This work (single pole)    |
|                | 0.3                      | QM [4]                     |