Neutrino Masses in a Neutrinosphere

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Abstract

The energies of neutrinos in a neutrinosphere, the dense matter created after the gravitational collapse of a massive star, are estimated. Cubic equations for energy eigenvalues of neutrinos are used, with the effective masses found by taking the neutrinos at rest in neutrinosphere matter. Large differences in the effective mass of some neutrino species in a neutrinosphere compared to vacuum are found.

Keywords: Neutrino masses, neutrino potentials, neutrinosphere

1 Introduction

Neutrino energies and neutrino transport are very important for many aspects of astroparticle physics and cosmology. Cosmic microwave background (CMB) and other astrophysical studies have shown that approximately 23% of matter in the universe is dark matter. Recent measurements of CMB power spectrum favor the existence of a neutrino species in addition to the three standard neutrinos[1], which might be one or more sterile neutrinos associated with dark matter. An important aspect of neutrino transport is the neutrino emissivity during the first 20 s after the gravitational collapse of a massive star, leading to resulting neutron stars with large luminosity having large velocities, called the pulsar kick. One possible source or the pulsar kick is the emission of standard neutrinos after 10 s[2]. Recently it has been shown that using parameters for electron to sterile neutrino oscillations, obtained by experiments on active neutrino oscillations, the pulsar kicks can result from the emission of sterile neutrinos during the first 10 s, when the neutrinos are in a neutrinosphere[3].

Neutrino oscillations, in which one species of active neutrinos converts to another species, have been studied for tests of symmetry violations. The effects of matter on neutrino oscillations have been studied for many decades [4, 5]; and matter effects on neutrino oscillations for neutrinos traversing the earth have been estimated[6, 7]. See these references for references to earlier work. More recently possible tests of time reversal violation[8] and CP violation[9]
have been investigated. In all studies of neutrino oscillations the neutrino masses are very important, however, only mass differences are needed.

More than a decade ago, in preparation for studies of neutrino oscillations, energy eigenvalues of neutrinos in the earth were investigated by Kim and Sze[10] and Freund[11] using a cubic eigenvalue formalism. In our present study we use the formalism of Freund[11] to estimate the energies and effective masses, defined as the energies of neutrinos with no velocity, of neutrinos in a neutrinosphere.

2 Neutrino energy eigenvalues in a neutrinosphere

Using the method of Ref [11], the neutrino energy eigenvalues $E_i$, $H|\nu_i> = E_i|\nu_i>$, are found as eigenvalues of the matrix $M[11]$ obtained from the 3 x 3 matrices $U[11]$ and the Hamiltonian $H$:

$$M = \begin{pmatrix} s_{13}^2 + \hat{A} + \alpha c_{13}^2 & \alpha c_{13} s_{12} c_{13} & s_{13} c_{13} - \alpha c_{13} s_{13} s_{12}^2 \\ \alpha c_{13} s_{12} c_{13} & \alpha c_{12}^2 & -\alpha s_{13} s_{12} c_{12} \\ s_{13} c_{13} - \alpha c_{13} s_{13} s_{12}^2 & -\alpha s_{13} s_{12} c_{12} & c_{13}^2 + \alpha s_{13}^2 s_{12}^2 \end{pmatrix}$$

(1)

With $c_{ij} \equiv \cos(\theta_{ij})$, $s_{ij} \equiv \sin(\theta_{ij})$, and $\delta m^2_{rs} \equiv m^2_r - m^2_s$, the parameters in $M$ are:

$$c_{12}^2 = 0.69$$
$$c_{13}^2 = 0.9775 \simeq 1.0$$
$$\delta m^2_{12} = 7.59 \times 10^{-5} \text{eV}^2$$
$$\delta m^2_{13} = 2.45 \times 10^{-3}$$
$$\alpha \equiv \delta m^2_{12}/\delta m^2_{13} = 0.031$$
$$\hat{A} = 2E_\nu V/\delta m^2_{13},$$

with $V$ the potential for neutrino interaction in matter. It is well-known that $V = \sqrt{2} G_F n_e$, where $G_F$ is the weak interaction Fermi constant, and $n_e$ is the density of electrons in matter. See, e.g., Ref [6]. For neutrinos in earth $V \simeq 1.13 \times 10^{-13} eV$, $\hat{A} \ll 1.0$.

The eigenvalues of $M$ satisfy the cubic equation (see Eq(17) in Ref[11] with $a=-I_1$, $b = I_2$, $c = -I_3$):

$$\bar{E}_i^3 + a \bar{E}_i^2 + b \bar{E}_i + c = 0$$

$$a = -(1 + \hat{A} + \alpha)$$
$$b = \alpha + \hat{A} \alpha c_{12}^2 c_{13} + \hat{A} (c_{13}^2 + \alpha s_{13}^2)$$
$$c = -\alpha c_{12}^2 c_{13}$$

(3)

with dimensionless quantities $E_i = (E_i - E^0_1)/(E^0_3 - E^0_1)$, where $E^0_i$ are neutrino energy eigenvalues with $V=0$. We study neutrinos at rest, so $E_i \equiv m_i c^2 \equiv m_i$, with $i = 1, 2,$ and
\( E_i^0 \) are the neutrino masses in vacuum, and \( m_i \) are the effective masses of neutrinos in matter.

From the parameters \( c_{12}^2, c_{13}^2 \), and \( \alpha \) (Eq.(2)) one finds

\[
\begin{align*}
    a &= - (1.031 + \hat{A}) \\
    b &= 0.031 + \hat{A} \\
    c &= -0.0209 \hat{A} .
\end{align*}
\]  

(4)

We take \( E_\nu \simeq m_3 \simeq \sqrt{\Delta m_{13}^2} \), as \( m_1 \ll m_3 \), for which \( \hat{A} \) is maximum, resulting in the largest matter effect on neutrino eigenstates. First, we solve the cubic equations for neutrinos in vacuum. From Eqs.(3,4) for \( V=0 \) (\( \hat{A} = 0 \))

\[
\begin{align*}
    \bar{E}_1 &= 3.08 \times 10^{-12} \simeq 0 \\
    \bar{E}_2 &= 0.031 \\
    \bar{E}_3 &= 1.0 ,
\end{align*}
\]  

(5)

which is nearly the same as for neutrinos in earth (\( \hat{A} \ll 1.0 \)).

The density of nucleons in the neutrinosphere is approximately that of atomic nuclear matter, \( \rho_n = 4 \times 10^{17} \text{kgm/m}^3 \). Taking the ratio of the electron mass to the proton mass one finds for the electron density in the neutrinosphere \( \rho_e \approx 2 \times 10^{11} \text{gm/cc} \), giving the neutrino potential in the neutrinosphere \( V \approx 10^{-2} \text{eV} \). This gives \( \hat{A}_{ns} = 0.404 \), which is \( \hat{A} \) for neutrinos in a neutrinosphere. Solving Eq(3) with this dense matter potential one finds

\[
\begin{align*}
    \bar{E}^\text{ns}_1 &= 0.0208 \\
    \bar{E}^\text{ns}_2 &= 0.3998 \\
    \bar{E}^\text{ns}_3 &= 1.0144 ,
\end{align*}
\]  

(6)

Comparing Eq(6) with Eq (5), \( m_3 \simeq m_3 (V = 0) \), while \( m_2 - m_1 (V = 0) \simeq 0.4eV \simeq 13.0 \times (m_2 (V = 0) - m_1 (V = 0)) \). Therefore, the neutrino effective masses in the neutrinosphere are quite different than in earth or vacuum.

\section{3 Conclusions}

We find for neutrinos in a neutrinosphere during the approximately 10 sec after the collapse of a large star the matter potential is about \( 10^{11} \) times larger than in earth, and the effect on the effective masses of neutrinos is quite large, with the effective mass difference between \( m_2 \) and \( m_1 \) about 13 times larger in the neutrinosphere than in vacuum or earth. Since the neutrino potential for neutrinos in earth is very small, we find that the three neutrino effective masses are approximately the same in earth matter as in vacuum. The large effect of matter on neutrino oscillations arise from a large baseline and depend on the energy of the neutrino beam.
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References

[1] D. Hooper, F.S. Queiroz, and N.Y. Gnedin, arXiv:1111.6599/astro-ph(2012)
[2] E.M. Henley, M.B. Johnson and L.S. Kisslinger, Phys. Rev. D 76,125007 (2007)
[3] L. S. Kisslinger and M.B. Johnson, arXiv:1207.2798/ astro-ph; Mod. Phys. Lett. A 27, 1250215 (2012)
[4] L. Wolfenstein, Phys. Rev. D17, 2369 (1978)
[5] V. Barger, K. Whisnant, S. Pakvasa, and R.J.N. Phillips, Phys. Rev. D22, 2718 (1980)
[6] T. Ohlsson and H. Snellman, Phys. Lett. B474, 153 (2000); B480, 419(E) (2000)
[7] E. Akhmedov, P. Huber, M. Lindner, and T. Ohlsson, Nucl. Phys. B608, 394 (2001)
[8] E. M. Henley, M. B. Johnson, L. S. Kisslinger, Int. J. Mod. Phys. E 20, 2463 (2011)
[9] L. S. Kisslinger, E. M. Henley, M. B. Johnson, arXiv:1203.6613; Int. J. Mod. Phys. E 21, 1250065 (2012)
[10] C.W. Kim and W.K. Sze, Phys. Rev. D35, 1404 (1987)
[11] M. Freund, Phys. Rev. D64, 053003 (2001)