A Novel Full Boundary Element Formulation for Harmonic Analysis of Elastic Membranes Coupled to Acoustics Fluids

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Abstract. A novel full Boundary Element Formulation for the harmonic analysis of elastic membranes coupled to acoustics fluid is presented. The elastic membranes is modeled using the classical linear elastic pre-stretched membrane theory. The acoustic fluid is modeled using the acoustic-wave equation for homogeneous, isotropic, inviscid and irrotational fluids. Elastostatic fundamental solution is used in the boundary element formulation for the elastic membrane. The boundary element formulation for the acoustic fluid is based on the fundamental solution of three dimensional Poisson equation. Domain integrals related to inertial terms and those related with distributed pressure on membrane, were treated using the Dual Reciprocity Boundary Element Method. Fluid-structure coupling equations were established considering the continuity of the normal acceleration of the particles and dynamic pressure at fluid-structure interfaces. Results obtained shows the accuracy and efficiency of the proposed boundary element formulation.

Nomenclature
\begin{itemize}
\item \(\dddot{p}\) second time derivative for pressure
\item \(\dddot{w}\) second time derivative for displacement
\item \(\Gamma_f\) fluid boundary surface
\item \(\Gamma_f^p\) fluid boundary surface with stablished \(p\)
\item \(\Gamma_f^q\) fluid boundary surface with stablished \(q\)
\item \(\Gamma_s\) membrane boundary
\item \(\Gamma_s^f\) membrane boundary with stablished \(f\)
\item \(\Gamma_s^w\) membrane boundary with stablished \(w\)
\item \(\Gamma_{fs}\) fluid-structure interface
\item \(\hat{\dot{p}}_0(x)\) known initial first time derivative for pressure in \(\Omega_f\)
\item \(\hat{\dot{w}}_0(x)\) known initial velocity in \(\Omega_s\)
\item \(\hat{f}(x, t)\) known normal traction on \(\Gamma_s^f\)
\item \(\hat{p}(x, t)\) known pressure on \(\Gamma_f^p\)
\end{itemize}
\( \hat{p}_0(x) \) known initial pressure in \( \Omega_f \)

\( \hat{P}_j(x', x) \) particular solutions for pressure as a poisson type equation

\( \hat{q}(x, t) \) known normal pressure gradient on \( \Gamma_f \)

\( \hat{Q}_j(x', x) \) particular solutions for gradient pressures as a poisson type equation

\( \hat{T}_j(x', x) \) particular solutions for tractions as a poisson type equation

\( \hat{w}(x, t) \) known displacement on \( \Gamma_s \)

\( \hat{W}_j(x', x) \) particular solutions for displacements as a poisson type equation

\( \kappa \) bulk modulus

\( \Omega_f \) fluid opened cavity domain

\( \Omega_s \) flexible membrane domain

\( \rho_f \) fluid mass density

\( \rho_s \) membrane mass density

\( A_{ij} \) matrix of coefficients, obtained by taking the value of \( p(t) \) at different DRM points

\( B_{ij} \) matrix of coefficients, obtained by taking the value of \( \ddot{w}(t) \) at different DRM points

\( c(x') \) jump terms arising from the terms of \( O(1/r) \) in the respective kernel

\( c_f \) fluid wave propagation velocity

\( c_w \) membrane wave propagation velocity

\( f = f(x, t) \) normal membrane traction

\( F_{ij} \) matrix of coefficients, obtained by taking the value of \( \ddot{p}(t) \) at different DRM points

\( f_j(r) \) radial basis function

\( h \) membrane thickness

\( N \) number of boundary elements

\( n^w_i \) components of the outward normal vector

\( NDRF \) number of total DRM collocation points used in the fluid

\( NDRM \) number of total DRM collocation points used in the membrane

\( P(x', x) \) fundamentals solutions for pressure for three dimensional acoustic problems

\( p = p(x, t) \) fluid pressure

\( p_w \) distributed pressure applied over the membrane

\( p_{w, \alpha} \) pressure laplacian

\( p_{\text{max}} \) distributed time harmonic pressure load amplitude

\( Q(x', x) \) fundamentals solutions for gradient pressures for three dimensional acoustic problems

\( q = q(x, t) \) normal fluid pressure gradient

\( q_n \) pressure gradient acting on the fluid-structure interface

\( T(x', x) \) fundamentals solutions for tractions

\( T_0 \) unit length tension

\( W(x', x) \) fundamentals solutions for displacements

\( w = w(x, t) \) membrane transversal displacement

\( w_{,\alpha} \) displacement laplacian

\( \alpha_j \) coefficients that relate \( A_{ij} \) with \( p_w \)

\( \mathbf{A} \) fluid-membrane coupling sub-matrix of \( H \)
\( C_w \) and \( C_f \) connectivity matrices joining fluid and structural degree of freedom at fluid-structure interface

\( H \) and \( G \) global coupling influence matrices

\( M \) global coupling mass matrix

\( n_w \) outward normal vector

\( S \) fluid-membrane coupling sub-matrix of \( G \)

\( x \) field points

\( x' \) collocation points

\( \hat{\alpha}_{ij} \) coefficients that relate \( E_{ij} \) with \( \ddot{p} \)

\( \hat{\beta}_{ij} \) coefficients that relate \( B_{ij} \) with \( \ddot{w} \)

\( fG_{fs} \) and \( fG_{ff} \) sub-matrices of \( fG \) related with degrees of freedom defined on the interface \( \Gamma_{fs} \)

\( sB_{fs} \) sub-matrix of \( sB \) related with pressure terms defined in \( \Omega_s \)

\( t \) time

\( fH \) and \( fG \) influence matrices for the fluid

\( fM \) fluid mass matrix

\( sB \) influence matrix related with distributed pressure \( p_w \) applied over the membrane

\( sH \) and \( sG \) influence matrices for the membrane

\( sM \) membrane mass matrix

1. Introduction

Acoustic problems in real life applications involve considerations of structural and acoustic part together and thus calls for a coupled vibroacoustic treatment rather a pure acoustic approach. Thus, whenever an elastic structure is in contact with a fluid, the structural vibrations and the acoustic pressure field in the fluid are influenced by the mutual vibro-acoustic coupling interaction. The analysis of fluid-structure coupled systems is a challenging and complex task. In general, the use of experimentation or numerical methods represent the two uniques alternatives to obtain approximate solutions for these kind of problems. However, numerical methods based on domain discretization requires refined meshes for high frequency problems, since the length of the elements should be proportional to the size of the wavelength. This means a more time-consuming model. The boundary element method (BEM) is a modern numerical technique which has enjoyed increasing popularity over the last two decades, and is now an established alternative to traditional computational methods of engineering analysis [1, 2]. The main advantage of the BEM is its unique ability to provide a complete solution in terms of boundary values only, with substantial savings in modelling effort. Since consolidation of the boundary element method (BEM) as reliable numerical method for structural and fluid analysis, linear vibrations of structures coupled with an internal fluid has been carried out using hybrid BEM-FEM formulations. In these formulations BEM is used to model the fluid media and the FEM to model the structural response [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. The main advantage of such formulations lies in a substantial reduction in the number of degrees of freedom in the discretization of the fluid domain. However, in this formulations is necessary to discretize the entirely structure due to the use of the FEM. Few BEM-BEM coupled formulations for fluid structure-interaction has been published [14, 15, 16, 17]. However, despite the fact that the BEM has been used for dynamic analysis of membrane structures and for analyses of acoustic fluids, to the best of authors knowledge, these formulations do not have been used for the fluid-structure interaction problem analysis using a full boundary element formulation for such purpose. In this work, a new full boundary element formulation for the transient dynamic analysis of acoustic
fluids coupled to elastic membranes is presented. Membranes are modeled using a boundary element formulation based on the linear elastic membrane theory under small deflection. The acoustic fluid is modeled using a boundary element formulation for the three dimensional acoustic wave equation. Fluid-structure coupling equations were established considering the continuity of the normal acceleration of the particles at fluid-structure interfaces. Domain integrals on both, fluid and structure equations, were treated using the Dual Reciprocity Boundary Element Method. The developed formulation was used to study the linear vibration response of elastic membranes coupled to acoustic fluids.

2. Structure subjected to a fluid pressure loading
Consider a partially opened cavity $\Omega_f$ with rigid walls and a flexible elastic membrane $\Omega_s$ with mass density $\rho_s$ and thickness $h$ (see Fig. 1). Cavity contains an homogeneous and isotropic acoustic fluid with mass density $\rho_f$. The membrane vibrations and the acoustic pressure field in the fluid are influenced by the mutual vibro-acoustic coupling interaction. In this work, the vibro-acoustic coupling interaction is modeled using an eulerian formulation where the acoustic response is described by the pressure, while the membrane response is described by the transversal displacement field.

2.1. Acoustic wave equation
The dynamic pressure of an ideal inviscid fluid under small perturbations in a spatial region $\Omega_f$ confined by the boundary surface $\Gamma_f = \Gamma^p_f \cup \Gamma^q_f$, is governed by the wave equation (see Fig. 2) [18]:

$$p_{,\alpha\alpha} = \frac{1}{c_f^2} \ddot{p}$$

In this equation $p$ is the fluid pressure, $c_f^2 = \kappa/\rho_f$ stands for wave propagation velocity, $\kappa$ is the bulk modulus. Above equation can be modified to include the effect of presence of an acoustic source. Double dot represents a second time derivative. Indicial notation is used throughout this work. Greek indices vary from 1 to 2 and Roman indices from 1 to 3. The boundary and initial conditions for equation (1) in the time interval $[0, t^\ast]$ are:

$$p(\mathbf{x}, t) = \hat{p}(\mathbf{x}, t), \quad \mathbf{x} \in \Gamma^p_f, \quad t \in [0, t^\ast]$$
$$q(\mathbf{x}, t) = n_i^f p_{,i} = \hat{q}(\mathbf{x}, t), \quad \mathbf{x} \in \Gamma^q_f, \quad t \in [0, t^\ast]$$
$$p(\mathbf{x}, 0) = \hat{p}_0(\mathbf{x}), \quad \dot{p}(\mathbf{x}, 0) = \hat{\dot{p}}_0(\mathbf{x}), \quad \mathbf{x} \in \Omega_f$$

where $n_i^f$ are the components of the outward normal vector $\mathbf{n}_f$ at boundary. and $t$ denotes time.
2.2. Dynamic equation of an elastic membrane

Now consider a linear elastic membrane with thickness $h$ occupying the spatial domain $\Omega_w$ confined by the boundary $\Gamma_s = \Gamma_s^w \cup \Gamma_s^f$ (see Fig. 3). An initial tension $T_0$ is uniformly applied to the membrane. In this work, the small deflection elastic membrane theory is considered. Thus, differential equation describing the transversal displacement $w(x,t)$ of this membrane in the time interval $[0,t^*]$ is given by:

$$w_{\alpha\alpha} + \frac{p_w}{T_0} = \frac{1}{c_w^2} \ddot{w}$$

where $c_w^2 = T_0/\rho_s$ and $p_w(x,t)$ is a distributed pressure applied over the membrane. The boundary and initial conditions for this equation are given by:

$$w(x,t) = \dot{w}(x,t), \quad x \in \Gamma_s^w, \quad t \in [0,t^*]$$

$$f(x,t) = n^w_i w_i = \dot{f}(x,t), \quad x \in \Gamma_s^f, \quad t \in [0,t^*]$$

$$w(x,0) = \dot{w}_0(x), \quad \ddot{w}(x,0) = \dot{\dot{w}}_0(x), \quad x \in \Omega_s$$

In these expressions, $n^w_i$ are the components of the outward normal vector $n_w$ at boundary.

3. Boundary element equations

3.1. Boundary element equations for acoustic wave equation

The derivation of the integral formulation for equation (1) is based on application of the Boundary Element Method to the acoustic wave equation as presented in [2]. Thus, by using
the weighted residual method and making use of the Green’s identity, the following equation is obtained:

\[ c(x')p(x') + \int_{\Gamma_f} Q(x', x)p(x)d\Gamma_f = \int_{\Gamma_f} P(x', x)q(x)d\Gamma_f + \frac{1}{c_f^2} \int_{\Omega_f} P(x', X)\ddot{p}(X)d\Omega_f \]  

(5)

In this equation, \( x' \) and \( x \) represent collocation and field points, respectively; \( P(x', x) \) and \( Q(x', x) \) are fundamentals solutions for pressure and gradient pressures for three dimensional acoustic problems, respectively, as presented in [2]. The value of \( c(x') \) is equal to \( \frac{1}{2} \) when \( x' \) is located on a smooth boundary.

In order to threat the domain integral, the Dual Reciprocity Boundary Element Method (DRM) is used as presented in [19]. In this way, equation (5) can be re-written as:

\[ c(x')p(x') + \int_{\Gamma_f} Q(x', x)p(x)d\Gamma_f = \int_{\Gamma_f} P(x', x)q(x)d\Gamma_f + \frac{1}{c_f^2} \sum_{j=1}^{NDRF} \tilde{\alpha}_j(t) \left[ c_i \tilde{P}_{ij} + \int_{\Gamma_f} Q(x', x)\tilde{P}_j d\Gamma_f - \int_{\Gamma_f} P(x', x)\tilde{Q}_j d\Gamma_f \right] \]  

(6)

In this equation, \( N_{DRF} \) represents the number of total DRM collocations points used in the fluid; \( \tilde{P}_j(x', x) \) and \( \tilde{Q}_j(x', x) \) are the particular solutions to equivalent homogeneous equation (1). These particular solutions were obtained considering the function \( f_j(r) = 1 + r_j \) for the approximation of \( \ddot{p}(t) \), as presented in [19]. Coefficients \( \tilde{\alpha}_j \) are related to \( \ddot{p} \) through: \( \ddot{p} = F_{ij}\tilde{\alpha}_j(t) \), where \( F_{ij} \) is a matrix of coefficients, obtained by taking the value of \( \ddot{p}(t) \) at different DRM points.

In order to discretize boundary surfaces of the acoustic medium, \( N \) boundary quadrilateral elements were used and \( p(x) \) and \( q(x) \) were assumed to be constant over each element and equal to their values at the mid-element node. Thus, the discretised form of equation (6) is given by:

\[ c_i(x')p(x', t) + \sum_{k=1}^{N} \left[ \int_{\Gamma_k} Q(x', x)d\Gamma \right] p_k(t) - \sum_{k=1}^{N} \left[ \int_{\Gamma_k} P(x', x)d\Gamma \right] q_k(t) = \]  

\[ \frac{1}{c_f^2} \sum_{j=1}^{NDRF} \tilde{\alpha}_j \left[ c_i(x')\tilde{P}_{ij}(x', x) + \sum_{k=1}^{N} \int_{\Gamma_k} Q(x', x)\tilde{P}_j(x', x)d\Gamma \right. \]  

\[ \left. - \sum_{k=1}^{N} \int_{\Gamma_k} P(x', x)\tilde{Q}_j(x', x)d\Gamma \right] \]  

(7)

Applying this equation at each collocation point, the following linear system of equations is obtained:

\[ \hat{f}M\ddot{p} + \hat{f}H\ddot{p} = \hat{f}Gq \]  

(8)

where \( \hat{f}M \) is the fluid mass matrix, \( \hat{f}H \) and \( \hat{f}G \) are boundary element influence matrices; \( p \) and \( q \) are vectors of nodal pressures and normal derivative of pressure, respectively.

### 3.2. Boundary element equations for an elastic membrane

The derivation of the integral formulation for equation (3) is based on the application of the BEM to the membrane equation as presented in [2]. Thus, by using the weighted residual method, and making use of the Green’s identity, the integral formulation for equation (3) is given by:

\[ c(x')w(x', t) + \int_{\Gamma_s} T(x', x)w(x, t)d\Gamma_s - \int_{\Gamma_s} W(x', x)f(x, t)d\Gamma_s = -\frac{1}{t_0} \int_{\Omega_s} W(x', x)p_w(x, t)d\Omega_s \]  

\[ + \frac{1}{c_w} \int_{\Omega_s} W(x', x)\ddot{w}(x, t)d\Omega_s \]  

(9)
where \(x\) and \(x'\) are field and collocation points respectively, \(W(x',x)\) and \(T(x',x)\) are fundamental solutions for displacement and traction, respectively as given in [1]. \(c(x')\) is the jump term arising from the terms of \(O(1/r)\) in the kernel \(T(x',x)\).

In this work, the DRM was used to transform domain integrals related to inertial terms into boundary integrals. In this way, this equation (9) can be re-written as [19]:

\[
c_i(x')w(x',t) + \int_{\Gamma_s} T(x',x)w(x,t) d\Gamma_s - \int_{\Gamma_s} W(x',x)f(x,t) d\Gamma_s = -\frac{1}{T_0} \sum_{j=1}^{NDRM} \alpha_j(t) [c_i(x')\hat{W}_j(x,t) + \int_{\Gamma_s} T(x',x)\hat{W}_j(x',x) d\Gamma_s] + \frac{1}{c_{sw}} \sum_{j=1}^{MDRM} \beta_j(t) [c_i(x')\hat{W}_j(x,t) + \int_{\Gamma_s} T(x',x)\hat{W}_j(x',x) d\Gamma_s - \int_{\Gamma_s} W(x',x)\hat{T}_j(x',x) d\Gamma_s]
\]

where \(NDRM\) represent the total number of DRM collocation used in the membrane; \(\hat{T}_j(x',x)\) and \(\hat{W}_j(x',x)\) are the particular solutions to equivalent homogeneous equation (3). These particular solutions were obtained considering the function \(f_j(r) = 1 + r_j\) for the approximation of \(\ddot{w}(t)\) and \(p_w(t)\) terms, as presented in [19]. Coefficients \(\alpha_j\) are related to \(p_w(t)\) through: \(p_w = A_{ij}\alpha_j(t)\), where \(A_{ij}\) is a matrix of coefficients, obtained by taking the value of \(p(t)\) at different DRM points. Similarly, coefficients \(\beta_j\) are related to \(\ddot{w}\) through: \(\ddot{w} = B_{ij}\beta_j(t)\), where \(B_{ij}\) is a matrix of coefficients, obtained by taking the value of \(\ddot{w}(t)\) at different DRM points.

Applying this equation at each collocation point, the following linear system of equations is obtained:

\[
^sM\ddot{w} + ^sHw = ^sGf - ^sBp_w
\]

Here \(^sM\) is the membrane mass matrix, \(^sH\) and \(^sG\) are the influence matrices, \(^sB\) is the influence matrix related with distributed pressure \(p_w\) applied over the membrane.
4. Fluid-structure coupling equations

Fluid-structure coupling equations are given by compatibility considerations about normal pressure and dynamic pressure force acting at fluid-structure interface. Mathematically, these conditions can be written as follows [20]:

\[ n_f \cdot \nabla p = q_n \equiv -\rho_w C_w \ddot{w} \]  
\[ p_w \equiv -C_f p \]  
\[ n^{\text{f}} \cdot \nabla q_n = 0 \]

That is, pressure gradient acting on the fluid-structure interface \( \Gamma_{fs} \) are related to normal acceleration of the plate and the acoustic pressure is equilibrated with pressure on the membrane (see Fig. 4). In these equations, \( C_w \) and \( C_f \) represent connectivity matrices joining fluid and structural degree of freedom at fluid-structure interface.

Replacing equations (13) and (13) into equations (8) and (11) we obtain the coupled fluid-structure equation problem:

\[ \begin{bmatrix} s^M & 0 \\ S & \int M \end{bmatrix} \begin{bmatrix} \dddot{w} \\ \dddot{p} \end{bmatrix} + \begin{bmatrix} s^H & -A \\ 0 & \int H \end{bmatrix} \begin{bmatrix} w \\ p \end{bmatrix} = \begin{bmatrix} s^G & 0 \\ 0 & \int G_{ff} \end{bmatrix} \begin{bmatrix} f \\ q \end{bmatrix} \]

In this equation, the off-diagonal sub-matrices \( A = s^B_{fs} C_f \) and \( S = \rho_w \int G_{fs} C_w \) are fluid-membrane coupling matrices. \( \int G_{fs} \) and \( \int G_{ff} \) are sub-matrices of \( \int G \) related with degrees of freedom defined on the interface \( \Gamma_{fs} \). \( s^B_{fs} \) is a sub-matrix of \( s^B \) related with pressure terms defined in \( \Omega_s \).

Equations (14) can be rewritten in a general way as:

\[ M\dddot{u} + H u = G r \]

where, \( \dddot{u} = (\dddot{w}, \dddot{p})^T \), \( u = (w, p)^T \) and \( r = (f, q)^T \).

5. Numerical examples

5.1. Box-shaped structure containing an acoustic fluid coupled to a membrane

Consider a partially opened box-shaped structure with rigid walls and a flexible elastic membrane with \( c_w = 22.36 \text{ m/s} \) as presented in Fig. 5. The structure contains an acoustic fluid with mass density \( \rho_f = 100 \text{ kg/m}^3 \) and a bulk’s modulus \( \kappa = 2.5 \times 10^5 \text{ Pa} \). A distributed time harmonic pressure load \( p_{max} = 1.0 \text{ Pa} \) is applied over the face of the structure located at \( x_3 = 2 \text{ m} \). Pressure gradient \( q = n^f p_{s} = 0 \) is considered zero in the other faces.

Four meshes were used to show the convergence. Table 1 shows the respective meshes.

| Meshes | Elements | Collocation Points |
|--------|----------|--------------------|
| Mesh 1 | 54 (12)  | 125 (9)            |
| Mesh 2 | 150 (20) | 125 (25)           |
| Mesh 3 | 294 (28) | 343 (49)           |
| Mesh 4 | 486 (36) | 343 (81)           |

The collocation points are coincident with fluid collocation points located in the fluid-membrane interface. A membrane tensile load \( T_0 = 500 \text{ N/m} \) was considered in the analysis. The frequency-history response of the central deflection for frequency interval 0.039788 Hz. to 20.0 Hz. is shown in figure 6.

The collocation points are coincident with fluid collocation points located in the fluid-membrane interface. A membrane tensile load \( T_0 = 500 \text{ N/m} \) was considered in the analysis.
Figure 5. Discretized coupled problem using BEM.

Figure 6. Maximum displacement frequency history at point $A(0.25, -0.25, -1)$ using DR-BEM

Figure 7. Pressure frequency history at point $A(0.25, -0.25, -1)$ using DR-BEM
5.2. Rigid channel containing an acoustic fluid coupled to a membrane

In this problem, a 10 m. long channel containing an acoustic fluid under a distributed time harmonic pressure load and coupled with a homogeneous elastic membrane with $c_w = 22.36$ m/s and density $\mu_w = 1.0$ kg/m$^2$ is analyzed (see Fig. 10). The fluid has a density $\rho_f = 100$ kg/m$^3$ and a bulk’s modulus of $\kappa = 2.5 \times 10^5$ Pa. The membrane is inclined 20.56° with respect to the vertical axis. A distributed time harmonic pressure load $p_{\text{max}} = 1.0$ Pa is applied on the membrane. Pressure gradient $q = n^T \sigma_i$ is considered zero in the other faces.

Four meshes were used to show the convergence. Table 2 shows this meshes.

| Meshes | Elements | Collocation Points |
|--------|----------|--------------------|
| Mesh 1 | 162 (12) | 90 (9)             |
| Mesh 2 | 330 (20) | 90 (25)            |
| Mesh 3 | 658 (28) | 171 (49)           |
| Mesh 4 | 1098 (36)| 171 (81)          |

Figure 8. Maximum displacement frequency history at point $A(0.25, 0.25, -1)$ using DR-BEM

Figure 9. Pressure frequency history at point $A(0.25, 0.25, -1)$ using DR-BEM
Figure 10. Discretized coupled problem using BEM.

Figure 11. Maximum displacement frequency history at point A using DR-BEM

Figure 12. Pressure frequency history at point A using DR-BEM
Figure 13. Maximum displacement frequency history at point A using DR-BEM

Figure 14. Pressure frequency history at point A using DR-BEM
6. Conclusions
A new full boundary element formulation for the harmonic dynamic analysis of acoustic fluids coupled to elastic membranes is presented. Membranes were modeled using a boundary element formulation based on the linear elastic membrane theory under small deflection. The acoustic fluid was modeled using a boundary element formulation for the three dimensional acoustic wave equation. Fluid-structure coupling equations were established considering the continuity of the normal acceleration of the particles at fluid-structure interfaces. Domain integrals on both, fluid and structure equations, were treated using the Dual Reciprocity Boundary Element Method. Results show good agreement with those obtained from finite difference models, turning proposed formulation a reliable and an alternative numeral engineering tool for the dynamic analysis of acoustic fluids coupled to flexible elastic membranes.

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