Status of the Unitarity Triangle analysis in the Standard Model

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Abstract. We present the update of the Unitarity Triangle analysis in the Standard Model. Combining the direct measurements on sides and angles of the Unitarity Triangle, we determine the values of the CKM parameters \( \rho \) and \( \eta \). This set of information provides a very powerful test of the Standard Model in the fermion sector. Thanks to the fact that the fit is overconstrained, we can predict hadronic QCD parameters without relying on any theoretical calculation of hadronic elements.

1. Introduction

We present the status of the Unitarity Triangle analysis in the Standard Model using the updated determinations of experimental and theoretical parameters, following the method described in [1] and [2]. The basic (LEP-like) constraints come from the measurement of direct CP violation in the kaon sector (\( \epsilon_K \)), the \( B_d \) and \( B_s \) mixing (\( \Delta m_d, \frac{\Delta m_s}{\sqrt{2} m_s} \)) and the semileptonic \( B \) decays (|\( \frac{V_{ub}}{V_{cs}} \)|). With the \( B \)-factories advent, measurements of CP-violating quantities in the \( B \) sector have become possible. The (angles) constraints come from the measurements of \( \sin 2\beta, \alpha, \gamma, \cos 2\beta, \text{and} 2\beta + \gamma \). Basic constraint have also been improved and \( \Delta m_s \) measured fro Tevatron.

Nowadays the Unitarity Triangle analysis can in addition provide an accurate extraction of the hadronic matrix elements relevant for \( K-K \) and \( B_s-d-B_s-d \) mixing and of the leptonic decay constant \( f_B \), as shown in [3]. The values obtained this way can then be compared with the theoretical predictions, mainly from lattice QCD.

2. Standard Model Unitarity Triangle analysis

In figure 1, left plot, we show the determination of \( \rho \) and \( \eta \) from the basic (LEP-like) constraints (\( \epsilon_K, \Delta m_d, \frac{\Delta m_s}{\sqrt{2} m_s}, |\frac{V_{ub}}{V_{cs}}| \)), for which we use all the updated measurements from the \( B \)-factories and from the Tevatron.

In figure 1, center plot, we show the impact of the angles constraints on the \( \rho-\eta \) plane. The determination of \( \sin 2\beta \), now a precision measurement of the \( B \)-factories, comes from the time-dependent asymmetry measurement in \( B^0 \rightarrow J/\psi K_s \). The theoretical error, calculated in a data driven way [4], is taken into account.

The constraint on the \( \rho-\eta \) plane from \( \sin 2\beta \) would have an ambiguity. Measurements of time-dependent angular analysis \( B^0 \rightarrow J/\psi K_s^0 \) and Dalitz analysis of \( B^0 \rightarrow D^0 \pi^0 \) give a constraint on \( \beta \)-related quantities that, also if much less precise than the \( \sin 2\beta \) measurement, helps solving the ambiguity.
Time-dependent CP asymmetries and branching fractions on $B \to \pi \pi$ and $B \to \rho \rho$ channels allow the determination of the angle $\alpha$, through an isospin analysis. The constraint on $\alpha$ is extracted from the experimental measurements exploiting general reasonable arguments on the hadronic amplitudes [5]. The angle $\alpha$ is also constrained by $B^0 \to (\rho \pi)^0$ analysis on the Dalitz plot.

The angle $\gamma$ is measured from $B^+ \to D^{(*)0}K^\pm$ using several methods. The analysis of the time-dependent CP asymmetry of $B^0 \to D^{(*)}\pi$ and $B^0 \to D\rho$, gives constraints on $(2\beta + \gamma)$.

In figure 1, right plot, we show the constraint on the $\bar{\rho}$-$\bar{\eta}$ plane from the combination of all the measurements.

The determination of the 68% range for $\bar{\rho}$ and $\bar{\eta}$ from the LEP-like, angles and all constraints is summarized in table 1. The good agreement within the different determinations confirms the success of the CKM mechanism in describing the CP-violation in the Standard Model.

![Figure 1. Constraints on the $\bar{\rho}$-$\bar{\eta}$ plane from $\epsilon_K$, $\Delta m_d$, $|\Delta m_s|$, and $|V_{ub}|$ (left plot), from $\sin 2\beta$, $\alpha$, $\gamma$ and $2\beta + \gamma$ (center plot) and from the combination of all those measurements (right plot). The closed contours at 68% and 95% probability are shown. The full lines correspond to 95% probability regions for the different constraints.](image)

|       | LEP-like | angles | All         |
|-------|----------|--------|-------------|
| $\bar{\rho}$ | $0.191 \pm 0.036$ | $0.134 \pm 0.038$ | $0.164 \pm 0.028$ |
| $\bar{\eta}$ | $0.371 \pm 0.027$ | $0.327 \pm 0.020$ | $0.340 \pm 0.016$ |

Table 1. Determination of the 68% range for $\bar{\rho}$ and $\bar{\eta}$ from the LEP-like, angles and all constraints.

In such a scenario, new physics effects are expected to appear as small corrections to the CKM picture.

In figure 2, left plot, allowed regions for $(\bar{\rho}, \bar{\eta})$ (contours at 68%, 95%) as selected by the measurements of $|V_{ub}|$, $\Delta m_d$ and $\Delta m_s$ are compared to the bounds (at 95% probability) from the measurements of CP violating quantities in the kaon ($\epsilon_K$) and in the B ($\alpha, \beta, \gamma$ and $2\beta + \gamma$) sectors, showing a slight tension between the two determinations.

Looking at the data, one can see that this tension is actually between the $|V_{ub}|$ inclusive measurement and the rest of the fit. In the right plot of figure 2, we show the compatibility between direct and indirect determinations, given in terms of standard deviations (color code), as a function of the measured value and its experimental uncertainty for $|V_{ub}|$. The plot shows a $\sim 2\sigma$ deviation between the $|V_{ub}|$ inclusive measurement and the $|V_{ub}|$ indirect determination from the rest of the fit.
3. UT\textit{fit vs lattice QCD}

All the available measurements and theoretical inputs combined in the Unitarity Triangle analysis overconstrain the CKM parameters. For this reason it is possible to perform the Unitarity Triangle analysis without relying on any theoretical calculations of hadronic matrix elements and their extraction from this study is useful to understand where improvements of theoretical calculations are necessary [3]. The values of theoretical parameters from the Unitarity Triangle fit and from lattice QCD calculations is shown in table 2.

| Parameter     | UT\textit{angles} | UT\textit{angles}+V_{ub}/V_{cb} | lattice QCD       |
|---------------|--------------------|---------------------------------|-------------------|
| $B_K$         | 0.78 ± 0.09        | 0.75 ± 0.09                     | 0.75 ± 0.04 ± 0.08|
| $f_{B_d}/B_{B_s}$ [MeV] | 262 ± 6            | 261 ± 6                         | 262 ± 35          |
| $\xi$         | 1.25 ± 0.06        | 1.24 ± 0.08                     | 1.23 ± 0.06       |
| $f_{B_d}$ [MeV] | 186 ± 11           | 187 ± 13                        | 189 ± 27          |
| $f_{B_s}$ [MeV] | 232 ± 9            | 231 ± 9                         | 230 ± 30          |

Table 2. Values of some theoretical parameters as determined by the UT\textit{fit} analysis using the \textit{angles} measurements, \textit{angles} measurements combined with $|V_{ub}/V_{cb}|$ and as determined by lattice QCD calculations.

If we also use the information on the $B_d$ bag parameter, $B_{B_d} = 1.28 + 0.05 - 0.09$ [6], we can predict the value of the branching fraction for the process $B \rightarrow \tau\nu$ without relying on any lattice calculations for $f_B$. As can be observed in table 3 the predicted value depends upon the value used for $|V_{ub}|$. The same $|V_{ub}|$ dependency can be observed in the predicted value of $\Delta m_d$. This should be taken into account when those quantities are used in New Physics oriented analyses, as [7], [8].

4. Conclusions

We present the status of the Unitarity Triangle analysis in the Standard Model using the updated determinations of experimental and theoretical parameters, from the global fit, $\bar{\rho}$ and $\bar{\eta}$ are
determined with a precision of 5% and 17% respectively. There is a good agreement within the different determination of the CKM parameters, more studies are needed to investigate the $\sim 2\sigma$ between the $|V_{ub}|$ inclusive measurement and the $|V_{ub}|$ indirect determination from the rest of the fit.

Thanks to the fact that the fit is overconstraint, we can predict hadronic QCD parameters without relying on any theoretical calculation of hadronic elements. In addition we can predict the value of the branching fraction of $B \to \tau\nu$ and the value of $\Delta m_s$, crucial quantities for New Physics oriented analyses.

| Prediction                  | Exclusive $|V_{ub}|$ | Inclusive $|V_{ub}|$ | $V_{ub} - \text{All}$ |
|-----------------------------|---------------------|---------------------|-------------------------|
| $BR(B \to \tau\nu)$ [$10^{-4}$] | 0.76 ± 0.12         | 0.96 ± 0.20         | 0.90 ± 0.16             |
| $\Delta m_s$ [ps$^{-1}$]    | 21.7 ± 2.8          | 19.4 ± 2.5          | 20.9 ± 2.6              |

**Table 3.** Prediction of $BR(B \to \tau\nu)$ and $\Delta m_s$ not relying on any assumption on $f_B$.

**References**

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