Optimal Design and Competitive Spans of Timber Floor Joists Based on Multi-Parametric MINLP Optimization

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Abstract: This study investigates the optimization of the design of timber floor joists, taking into account the self-manufacturing costs and the discrete sizes of the structure. This non-linear and discrete class of optimization problem was solved with the multi-parametric mixed-integer non-linear programming (MINLP). An MINLP optimization model was developed. In the model, an accurate objective function of the material and labor costs of the structure was subjected to design, strength, vibration and deflection (in)equality constraints, defined according to Eurocode regulations. The optimal design of timber floor joists was investigated for different floor systems, different materials (sawn wood and glulam), different load sharing systems, different vertical imposed loads, different spans, and different alternatives of discrete cross-sections. For the above parameters, 380 individual MINLP optimizations were performed. Based on the results obtained, a recommended optimal design for timber floor joists was developed. Engineers can select from the recommendations the optimal design system for a given imposed load and span of the structure. Economically suitable spans for timber floor joists structures were found. The current knowledge of competitive spans for timber floor joists is extended based on cost optimization and Eurocode standards.

Keywords: structural optimization; cost optimization; discrete optimization; mixed-integer non-linear programming; MINLP; timber floor joists

1. Introduction

Timber floor joists consist of parallel timber beams and a structural sheathing, joined together by shear connectors (see Figure 1). Floors made of timber, along with those made of reinforced concrete and steel-concrete composites, are among the most common forms of construction used in the modern building industry [1]. The advantages of timber floors are: use of a natural building material; use of a sustainable building material [2]; low greenhouse gas emissions [3,4]; agreeable interior climate [5]; diverse architectural design options; good ratio between self-weight and load-carrying capacity [6]; fast-dry form of construction; and no scaffolding or props required for erection. Timber floor joists are easily adapted to various levels of loading, fire resistance and sound insulation [7]. This floor system has low self-weight and is highly flexible in terms of fitting out. The disadvantages of floor joists are that considerable structural depth is required and a limited range of solid timber sections is available [8]. The vibration responses of a timber floors have been assessed experimentally [9,10] and numerically [11], and it has been concluded that vibration serviceability of timber floors is increasingly relevant due to their increased long-span applications [12]. With increased floor span, sawn timber beams are replaced with glued laminated timber beams, and secondary timber beams are provided. Based on thermo-mechanical analysis and experimental testing of timber floor systems, design methods for calculating fire resistance have been developed [13].
Nowadays, structural efficiency, economy and environmental behavior are the dominant criteria in the structural design of floor systems [14]. In order to reduce manufacturing costs or mass, various floor structures have often been optimized. While the optimizations of concrete slabs and composite floors have been frequently performed and widely reported, such investigations on timber floors have been very rarely performed according to literature. Among the references in the last five years in the field of structural optimization of concrete slabs is the optimization of large-span prestressed concrete slabs by Kaveh et al. [15] using probabilistic particle swarm, cost optimization of reinforced concrete flat slabs by Aldwaik et al. [16] using the charged system search algorithm. Poitras et al. [17] reported the optimization of composite and steel floor structures using particle swarm optimization. Kaveh and Shakouri Mahmud Abadi [18] presented the cost optimization of a composite floor using the improved harmony search algorithm, while Omkar et al. [19] presented the multi-objective design optimization of composite structures using the artificial bee colony algorithm. Poitras et al. [20] reported the optimization of composite and steel floor systems using particle swarm optimization. Kaveh and Behnam [21] performed the cost optimization of a composite floor, waffle slab and concrete slab formwork using the charged system search algorithm, while Kaveh and Ahangaran [22] introduced the discrete cost optimization of a composite floor using the social harmony search model. Luo et al. [23] presented an optimal topology design of composite structures using a standard gradient-based search. Žula et al. [24] and Kravanja et al. [25] performed the optimization of a composite floor system using mixed-integer non-linear programming,
and Silva and Rodrigues [31] carried out the optimization of steel-concrete composite beams using sequential linear programming. In addition, Jelušić and Kravanja [32] reported the cost optimization of timber-concrete composite floors using multi-parametric mixed-integer non-linear programming, while Kaveh and Ghafari [33] optimized a steel floor system using enhanced colliding body optimization.

In the field of timber floor structures, Stanić et al. [34] and Brank et al. [35] optimized the cross laminated timber plates with ribs. They performed cost optimization of floors using a gradient-based algorithm for mechanical systems by Kegl and Butinar [36]. Mayencourt et al. [37] and Mayencourt and Mueller [38] studied the structural optimization of cross-laminated timber panels. They performed material consumption optimization of structures using the non-linear programming solver (fmincon), implemented in Matlab. Santos et al. [39] applied the cost optimization of cross-insulated timber panels. They also used Matlab software (fmincon solver). Finally, Nesheim et al. [40] reported the cost and ECO2 optimization of timber floor elements for adaptable buildings. For the optimization, they used the mixed-integer sequential linearization procedure.

The above references of timber floor structures do not indicate spans for which timber floor systems are economically viable. There is some information in the literature on the competitive spans of timber floors based on the classical calculations from engineering practice. Kolbitsch [41] reported that a timber floor system with timber beams and sheathing is suitable for spans ranging from 2 to 7 m. While Kolb [42] showed that this system is suitable for spans between 4 and 7.5 m, Cobb [43] proposed this floor system for spans between 1 and 6.5 m. For longer spans, PINE Manufacturers [44] showed that timber floors with glued beams are suitable for spans from 2 to 10 m, while APA [45] reported suitable spans of floors with glued beams from 2.44 m to 15.85 m.

2. Materials and Methods

2.1. Objectives and Framework of the Study

The novelty of this work is not only the calculation of the optimal design, but also the determination of the competitive spans of timber floor joists, based on the optimization performed and the Eurocode specifications. In contrast to the above-mentioned references in the field of timber floors, which reported individual cost/design optimizations of structures, this work deals with a multi-parametric cost optimization performed for different spans, loads and systems.

With this study, we aimed to verify the above-mentioned spans of timber floor joists and also to increase the knowledge about these spans based on cost optimization and Eurocode standards. In the paper, we propose to determine the competitive spans of two different timber floor systems (see Figure 1):

- Floor system A (SYS-A), which contains only timber beams and structural sheathing. However, two types of timber beams are used in SYS-A:
  - Sawn wood (SYS-A-SW) and
  - Glued laminated timber (SYS-A-Gl);
- Floor system B (SYS-B) made of glued laminated primary timber beams, sawn secondary timber beams and structural sheathing as the main components.

The cost optimizations of the timber floor structures were calculated using mixed-integer non-linear programming (MINLP). MINLP enables simultaneous continuous and discrete optimization. For this purpose, continuous and discrete binary 0–1 variables are defined. The continuous variables are used for optimization of sizes, inner forces, deflections, costs and mass, whilst the discrete variables are employed to calculate material grades and standard sizes. A Modified Outer-Approximation/Equality-Relaxation (OA/ER) algorithm was used to perform optimization of the discrete and non-linear class of the timber floor structure; see Kravanja and Grossmann [46] and Kravanja et al. [47].

We developed an MINLP model for the optimization of timber floor joists in which an objective function of the structure’s self-manufacturing costs is subjected to design,
resistance, vibration and deflection constraints. These dimensioning constraints satisfy
the conditions of ultimate and serviceability limit states according to Eurocode regula-
tions [48]. Optimal design of the timber floor joists was studied for different important
design parameters, such as different floor systems, spans, vertical imposed loads, timber
strengths and discrete dimensions of cross-sections. Since a number of individual MINLP
optimizations were calculated for the different defined parameters, a multi-parametric
MINLP optimization of the floor structure was performed.

Based on the results obtained, a recommended optimal design for timber floor joists
was developed. A sensitivity analysis was performed on the contribution of various
design parameters to the variability of the overall production costs of timber floor systems,
and economically suitable timber floor systems for various spans were found. Finally,
competitive spans were determined for the considered timber floor joists.

2.2. MINLP Optimization Model

In order to perform a cost optimization of timber floor joists, an MINLP optimization
model titled TIMBFJ (TIMBer Floor Joists) was developed. It was modelled in the
GAMS environment (General Algebraic Modeling System, commercial software) [49], and
it enables the MINLP optimization of a structure for different design parameters such as
different loads, spans, strengths and prices. The following assumptions were applied in the
optimization model and the calculation:

- two different floor systems were considered:
  - timber floor with primary beams (SYS-A),
  - timber floor system with primary and secondary beams (SYS-B);
- two different sharing systems for mechanically jointed timber floors were used:
  - system strength factor \( k_{sys} \),
  - Gamma method;
- depths (heights) of the beams were bounded to be less than or equal to the extreme
  values which are still, but very rarely, used in the practice:
  - 30 cm for sawn wood beams,
  - 150 cm for glued laminated timber beams;
- Eurocode 5 was used to define the dimensioning constraints;
- prices in Central Europe were considered in the study.

The diversity of different structure systems and joint treatments described above
enables us to make an economical comparison between sawn (SYS-A-SW) and glued
(SYS-A-Gl) laminated timber beams.

2.2.1. MINLP Model Formulation

The proposed cost and discrete sizing optimization of a timber floor structure is a
combined continuous, discrete and non-linear type of optimization. The optimization
model is modelled on the following MINLP formulation:

\[
\begin{align*}
\min \quad z &= f(x, y), \\
\text{subjected to: } \\
& \quad g_k(x, y) \leq 0, \quad k \in K \\
\quad x &\in X = \{x \in \mathbb{R}^n: x_{LO} \leq x \leq x_{UP}\} \\
\quad y &\in Y = \{0, 1\}^m
\end{align*}
\]

MINLP handles continuous variables \( x \) and discrete (binary) variables \( y \). Continuous
variables are used here to determine the structure costs, loads, loadings, resistances and
deflections, while discrete binary variables are employed for choosing the discrete sizes of
timber cross-sections. The cost objective function \( z \) is subject to the design, loading, stress,
resistance and deflection (in)equality constraints, \( g_k(x, y) \leq 0, k \in K \). These constraints represent
the calculation of inner forces and the dimensions of the structure, where the ultimate
and serviceability limit state conditions are defined according to Eurocode standards.
The proposed TIMBFJ optimization model includes input data (constants), variables, sets, parameters, the cost objective function of the structure, structural analysis and dimensioning constraints, and logical constraints. The main input data, continuous and discrete variables, sets and parameters are listed in Table 1. While the main input data include a structure span $L$ (m), an imposed load $q_k$ (kN/m$^2$), various material properties, factors and prices, the continuous variables include the production costs of the structure COST, dimensions, cross-section characteristics, stresses, deflections, masses, etc. Sets, discrete variables and parameters are defined to calculate/select the discrete/standard materials (characteristic bending strength of timber) and standard dimensions.

### Table 1. Input data, continuous and discrete variables.

| **Input data:** |  |
|------------------|---------------------------------|
| $a$              | allowable deflection at concentrated static force 1 kN |
| $l_{floor}$     | floor width |
| $c_{board}$     | basic price per unit of timber board |
| $c_{f,board}$   | price of sawn timber or glued laminated timber (C24) |
| $c_{L,board}$   | price of timber impregnation |
| $E$             | modulus of elasticity of timber |
| $f_{board}$     | characteristic bending strength of the timber board |
| $f_{board,sk}$  | characteristic bending strength of timber |
| $f_0$           | characteristic shear strength of timber |
| $f_k$           | characteristic shear strength of the timber board |
| $k_{amp}$       | amplification factor |
| $k_{def}$       | deformation factor |
| $k_{mod}$       | modification factor to strength values |
| $k_{sys}$       | loading sharing factor |
| $q_k$           | vertical imposed load |
| $L$             | span of floor beams |
| $\gamma_{board}$ | specific factors for permanent actions |
| $\gamma_{\xi}$ | specific weight of timber beam |
| $\gamma_M$      | partial factors for material properties |
| $\gamma_y$      | partial factors for imposed actions |
| $\gamma_1$      | composite action of the beam and timber board |
| $\gamma_2$      | composite action of the beam |
| $\xi$           | modal damping coefficient |
| $\psi$          | factor for the quasi-permanent value of the variable action |

| **Variables $x \in X$:** |  |
|------------------|---------------------------------|
| $b$              | width of the timber beam |
| $b_{pb}$         | width of the primary beam |
| $b_{sys}$        | velocity response constant |
| $c_{board}$      | basic price per unit of timber board |
| $d$              | thickness of the floorboard |
| $e$              | spacing between floor beams |
| $e_{pb}$         | spacing between floor primary beams |
| $f_{o,board}$    | design bending strength of the primary beams |
| $f_{o,board,pb}$ | design bending strength of the primary beams |
| $f_{o,sys}$      | design shear strength of the beam |
| $f_{o,sys,pb}$   | design shear strength of the primary beams |
| $f_{board,sk}$   | characteristic bending strength of the timber board |

| **Discrete binary variables $y = (y^{mat}, y^{st})$, $y \in Y$:** |  |
|------------------|---------------------------------|
| $y^{mat}$        | the sub-vector of the binary variables for bending strengths |
| $y^{st}$         | the sub-vector of the binary variables for standard dimensions |

| **Sets** |  |
|------------------|---------------------------------|
| $i, i \in I$    | alternatives of characteristic bending strengths of timber |
| $j, j \in J$    | alternatives of standard dimensions |
2.2.2. Cost Objective Function

For two different floor systems, SYS-A and SYS-B, the objective function of the self-manufacturing costs \( \text{COST} \) (EUR/m\(^2\)) of timber floor joists is defined by Equation (1):

\[
\begin{align*}
\text{min: } \text{COST}_{\text{SYS-A}} &= C_{M,t} + C_{M,\text{imp}} + C_{M,\text{board}} + C_{L,\text{board}} \\
\text{min: } \text{COST}_{\text{SYS-B}} &= C_{M,t} + C_{M,\text{imp}} + C_{M,\text{board}} + C_{L,\text{board}} + C_{M,t,\text{pb}} + C_{M,\text{imp},\text{pb}}.
\end{align*}
\] (1)

Different cost items of the objective function for systems SYS-A and SYS-B are presented in Table 2. While floor system SYS-A includes only cost items defined by Equations (2)–(5), system SYS-B also includes items for glulam primary beams determined by Equations (6) and (7). This study considered the cost of spruce wood (Picea Excelsa), one of the most commonly used materials in the construction industry.

| Table 2. Cost items of the objective function for floor systems SYS A and SYS B. |
|-----------------|-----------------|-----------------|
| **Floor System A (SYS A)** | **Additional Material Cost Items in Timber Floor System B (SYS B)** |
| Timber: \( C_{M,t} \) | Timber for primary beams: \( C_{M,t,\text{pb}} \) |
| \( C_{M,t} = \frac{c_{M,t} \cdot b \cdot b}{e} \) | \( C_{M,t,\text{pb}} = \frac{c_{M,t,\text{pb}} \cdot h \cdot b}{e} \) |
| Timber impregnation: \( C_{M,\text{imp}} \) | Timber impregnation for primary beams: \( C_{M,\text{imp},\text{pb}} \) |
| \( C_{M,\text{imp}} = \frac{c_{M,\text{imp}} \cdot h \cdot b}{e} \) | \( C_{M,\text{imp},\text{pb}} = \frac{c_{M,\text{imp},\text{pb}} \cdot h \cdot b}{e} \) |
| Floor boarding: \( C_{L,\text{board}} \) | |
| \( C_{L,\text{board}} = c_{L,\text{board}} \) | |
| Floor board placing: \( C_{L,\text{board}} \) | |
| \( c_{L,\text{board}} = c_{L,\text{board}} \) | |
| \( c_{M,t} \) | \( b \) width of the timber beam (m) |
| \( c_{M,\text{imp}} \) | \( h \) height of the timber beam (m) |
| \( c_{M,\text{board}} \) | \( e \) spacing between timber beams (m) |
| \( c_{L,\text{board}} \) | \( b_{\text{pb}} \) width of the timber primary beam (m) |
| \( c_{M,\text{pb}} \) | \( h_{\text{pb}} \) height of the timber primary beam (m) |
| \( c_{M,\text{imp},\text{pb}} \) | \( e_{\text{pb}} \) spacing between timber primary beams (m) |

In Equations (2)–(7), \( c_{M,t} \) (EUR/m\(^3\)) is the unit price of the timber, \( c_{M,\text{imp}} \) (EUR/m\(^3\)) is the unit price of timber impregnation, \( c_{M,\text{board}} \) (EUR/m\(^2\)) is the unit price of the prefabricated floor boards and \( c_{L,\text{board}} \) (EUR/m\(^3\)) is the unit price of floor board placing. Index \( \text{pb} \) is used to define the dimensions of the glulam primary beam in system SYS-B; see Equations (6) and (7).

On local markets, the unit prices of floor boards depend on their thicknesses. Floor boards with higher thickness are generally more expensive. The unit price of prefabricated timber boards \( c_{M,\text{board}} = f(d) \) is thus included in the cost objective function by applying an approximation function which was formulated by executing multiple regression using MS Excel software with Data Analysis Add-in. The price per unit of timber floor board \( c_{M,\text{board}} \) (EUR/m\(^2\)) is thus calculated to be dependent on the thickness; see Equation (8), where \( c_{\text{board}} \) (EUR/m\(^2\)) stands for the basic price per unit of timber board; the thickness of timber board \( d \) must be defined in mm.

\[
c_{M,\text{board}} = c_{\text{board}} \cdot (0.0762 \cdot d + 0.5238).
\] (8)

2.2.3. Structural Analysis and Dimensioning Constraints

The cost objective function is subjected to structural analysis and dimensioning constraints which are defined in accordance with Eurocode standards [50]; see Equations (9)–(58). Equations (10)–(43) in Table 3 stand for the floor system SYS-A (and for the secondary beams and floor boards of the system SYS B), and Equations (44)–(58) in Table 4 are defined for the glulam primary beams of the floor system SYS-B. In this paper only the most important equations are presented. For a better understanding of the symbols (constants and variables) used in the equations; see Table 1. The timber structure
is dimensioned to support its own weight $g_{k}$ (kN/m²), timber beams plus floor boards) and the uniformly distributed variable imposed load $q_{k}$ (kN/m²); see Equation (9):

$$\gamma_{k} \cdot g_{k} + \gamma_{q} \cdot q_{k}.$$

(9)

**Table 3.** Structural analysis and dimensioning constraints (SYS-A).

**Ultimate limit state constraints (ULS):**

- Design bending stress must be smaller than the design bending strength of timber beam: $\sigma_{m,y,d} \leq f_{m,y,d}$

  when the loading sharing factor $k_{sys}$ is used to calculate the design bending strength:

  $$\sigma_{m,y,d} = \left[ (\gamma_{k} \cdot g_{k} + \gamma_{q} \cdot q_{k}) \cdot e \cdot L^2 / 8 \right] / \left[ (b \cdot h^2 / 6) \right] \text{ where } g_{k} = b \cdot h \cdot \gamma_{k} / e + d \cdot \gamma_{board}$$

  $$f_{m,y,d} = k_{h} \cdot k_{sys} \cdot k_{mod} \cdot f_{mk} / \gamma_{M} \text{ where } k_{h} = \min[[150 / h]^{0.2}, 1.3]$$

  (11),(12)

  when the Gamma method $\gamma$ is used to calculate the effective stiffness of timber floor system:

  $$\sigma_{m,y,d} = (0.5 \cdot E \cdot (\gamma_{k} \cdot g_{k} + \gamma_{q} \cdot q_{k}) \cdot e \cdot L^2 / 8) / (EI)_{eff}$$

  (15)

  $$(EI)_{eff} = E_{board} \cdot I_{c,eff} + \gamma_{1} \cdot E_{board} \cdot A_{c,eff} \cdot a_{d,eff} \cdot E_{board} \cdot A_{tim} \cdot a_{d,lim}^2$$

  where:

  $$I_{c,eff} = b_{c,eff} \cdot d^3 / 12$$

  $$A_{c,eff} = b_{c,eff} \cdot d$$

  $$a_{d,eff} = \min(0.2 \cdot L, 3d)$$

  $$a_{d,lim} = \min(h + d / 2 - h / 2 - a_{d,lim})$$

  (16),(18),(19)

  $$(d + h) / [2 \cdot (\gamma_{1} \cdot E_{board} \cdot A_{c,eff} + \gamma_{2} \cdot E \cdot A_{tim})]$$

  where $A_{tim} = b \cdot h$

  $$l_{tim} = b \cdot h^3 / 12$$

  (20)

  (21),(22)

  (23)

- Design shear stress in timber beam must not exceed the design shear strength of the timber: $\tau_{max} \leq f_{cd}$

  when the loading sharing factor $k_{sys}$ is used:

  $$\tau_{max} = \left[ 1.5 \cdot (\gamma_{k} \cdot g_{k} + \gamma_{q} \cdot q_{k}) \cdot e \cdot L / 2 \right] / (b \cdot h)$$

  (24)

  $$f_{cd} = k_{sys} \cdot k_{mod} \cdot f_{ck} / \gamma_{M}$$

  (25)

  when the Gamma method $\gamma$ is used:

  $$\tau_{max} = \left[ 1.5 \cdot (\gamma_{k} \cdot g_{k} + \gamma_{q} \cdot q_{k}) \cdot e \cdot L / 2 \cdot 0.5 \cdot E \cdot b \cdot (h / 2 + a_{d,lim})^2 \right] / [b \cdot (EI)_{eff}]$$

  (27)

- Design bending stress in timber board must be smaller than the design bending strength of board: $\sigma_{board,y,d} \leq f_{board,y,d}$

  $$\sigma_{board,y,d} = \left[ (\gamma_{k} \cdot g_{board,k} + \gamma_{q} \cdot q_{k}) \cdot e^2 / 8 \right] / [1 \cdot d^2 / 6] \text{ where } g_{k,board} = d \cdot \gamma_{board}$$

  $$f_{board,y,d} = k_{h,board} \cdot k_{mod,board} \cdot f_{mod,board,k} / \gamma_{M} \text{ where } k_{h,board} = 1.3$$

  (29),(30)

  (31),(32)

- Design shear stress in timber boards must not exceed the design shear strength: $\tau_{board,max} \leq f_{board,cd}$

  $$\tau_{board,max} = \left[ 1.5 \cdot (\gamma_{k} \cdot g_{board,k} + \gamma_{q} \cdot q_{k}) \cdot e \cdot L / 2 \right] / [1 \cdot h]$$

  (33)

  $$f_{board,cd} = k_{sys} \cdot k_{mod,board,k} / \gamma_{M}$$

  (34)
**Table 3. Cont.**

Serviceability limit state constraints (SLS):

- Checking the instantaneous deflection of the timber beam: \( u_{inst} \leq L/300 \) when the loading sharing factor \( k_{sys} \) is used
  \[ u_{inst} = 5(g_k + q_k) \cdot e \cdot L^4/[384 \cdot E \cdot b \cdot h^3/12] + 12(g_k + q_k) \cdot e \cdot L^2/[5 \cdot E \cdot b \cdot h] \] (36)

- Checking the final deflection of the timber beam: \( u_{fin} \leq L/250 \)
  when the loading sharing factor \( k_{sys} \) is used
  \[ u_{fin} = [5g_k \cdot e \cdot L^4/(384 \cdot E \cdot b \cdot h^3/12)] + 12q_k \cdot e \cdot L^2/(5 \cdot E \cdot b \cdot h) \cdot (1 + k_{def}) + [5q_k \cdot e \cdot L^4/(384 \cdot E \cdot (EI)_{eff}) + 12q_k \cdot e \cdot L^2/(5 \cdot E \cdot b \cdot h) \cdot (1 + \psi_2 \cdot k_{def}) \] (41)

- Checking the instantaneous deflection of the timber boards between the timber beams: \( u_{inst,board} \leq \epsilon/300 \)
  \[ u_{inst,board} = 5(g_k + q_k) \cdot 1 \cdot e^4/[384 \cdot E \cdot 1 \cdot d^3/12] + 12(g_k + q_k) \cdot 1 \cdot e^2/[5 \cdot E \cdot 1 \cdot d] \] (39)

- Checking the final deflection of the timber beam: \( u_{fin} \leq L/250 \)
  when the loading sharing factor \( k_{sys} \) is used
  \[ u_{fin} = [5g_k \cdot e \cdot L^4/(384 \cdot E \cdot b \cdot h^3/12)] + 12q_k \cdot e \cdot L^2/(5 \cdot E \cdot b \cdot h) \cdot (1 + k_{def}) + [5q_k \cdot e \cdot L^4/(384 \cdot E \cdot (EI)_{eff}) + 12q_k \cdot e \cdot L^2/(5 \cdot E \cdot b \cdot h) \cdot (1 + \psi_2 \cdot k_{def}) \] (45)

- Proportion of the point load acting on a single joist must be greater than the acceptable value: \( k_{dist} \geq 0.3 \)
  \[ k_{dist} = k_{strat} \cdot (0.38 - 0.08 \cdot \ln((EI)_b/e^4)) \text{ where } (EI)_b = E \cdot 1 \cdot d^3/12 \] (47)

- Vertical deflection caused by concentrated static force \( F = 1.0 \text{ kN} \) must not exceed: \( w \leq a \)
  \[ w = 1000 \cdot k_{dist} \cdot L^3 \cdot k_{amp} /[48 \cdot (EI)_{joint} \text{ where } (EI)_{joint} = E \cdot b \cdot h^3/12 \] (50)
  \[ a = (1.8 \text{ for } L \leq 4000 \text{ mm}; 16,500/L^{1.1}; \text{ for } L > 4000 \text{ mm} \) (52)

- Fundamental frequency of vibration of a rectangular residential floor should exceed: \( f_1 \geq 8 \text{ Hz} \)
  \[ f_1 = \pi / (2 \cdot L^2) \sqrt{(EI)_b/ \text{mass}_b \text{ where } \text{mass}_b = g_k \cdot e \cdot 1000/9.81 \] (55)

- Unit impulse floor velocity must be lower than the permissible floor velocity: \( v \leq v_{allow} \)
  \[ v = 4 \cdot (0.4 + 0.6 \cdot n_{40}) / (\text{mass}_b \cdot b_{floor} / L + 200) \] (57)
  \[ v_{allow} = b_{vy} \cdot f_1 \cdot (1 - e^{-1}) \] (59)

- Checking the deflection of the timber board \( u_{fin,board} \leq \epsilon/250 \)
  \[ u_{fin,board} = [5g_k \cdot e \cdot L^4/(384 \cdot E \cdot 1 \cdot d^3/12)] + 12g_k \cdot 1 \cdot e^2/(5 \cdot E \cdot 1 \cdot d) \cdot (1 + k_{def}) + [5q_k \cdot e \cdot L^4/(384 \cdot E \cdot (EI)_{eff}) + 12q_k \cdot 1 \cdot e^2/(5 \cdot E \cdot 1 \cdot d) \cdot (1 + \psi_2 \cdot k_{def}) \] (44)

- Fundamental frequency of vibration of a rectangular residential floor should exceed: \( f_1 \geq 8 \text{ Hz} \)
  \[ f_1 = \pi / (2 \cdot L^2) \sqrt{(EI)_b/ \text{mass}_b \text{ where } \text{mass}_b = g_k \cdot e \cdot 1000/9.81 \] (55)

- Unit impulse floor velocity must be lower than the permissible floor velocity: \( v \leq v_{allow} \)
  \[ v = 4 \cdot (0.4 + 0.6 \cdot n_{40}) / (\text{mass}_b \cdot b_{floor} / L + 200) \] (57)
  \[ v_{allow} = b_{vy} \cdot f_1 \cdot (1 - e^{-1}) \] (59)

where: \( n_{40} = [(40/f_1)^2 - 1] \cdot (b_{floor} / L)^4 \cdot (EI)_b / (e \cdot (EI)_b)^{0.25} \) (60)

\( b_{vy} = 180 - 60 \cdot w \text{ for } w \leq 1 \text{ mm}; \quad b_{vy} = 180 - 40 \cdot w \text{ for } w > 1 \text{ mm} \) (61),(62)
- Design bending stress must be smaller than the design bending strength of the primary timber beam: \( \sigma_{m,y,d,\text{pb}} \leq f_{m,y,d,\text{pb}} \) (63)

\[
\sigma_{m,y,d,\text{pb}} = \frac{(\gamma_g \cdot \sigma_{\text{pl},\text{pb}} + \gamma_t \cdot \sigma_{\text{m},\text{pb}} \cdot L^2/8)}{(b_h \cdot h_{\text{f},\text{pb}})^2/6}
\]

(64)

\[
g_{\text{k},\text{pb}} = b_h \cdot q_e \cdot 1 + d \cdot f_{\text{mod}} \cdot b_h \cdot h_{\text{f},\text{pb}}
\]

(65)

\[
f_{m,y,d,\text{pb}} = k_b \cdot k_{\text{sys}} \cdot k_{\text{mod}} \cdot f_{\text{ak}} / \gamma_{M,\text{pb}} \text{where } k_b = \min(\{150/b_{\text{pb}}\}^{0.2}, 1.3)
\]

(66),(67)

- Design shear stress in timber beam must not exceed the design shear strength of the primary timber beam: \( \tau_{\text{max},\text{pb}} \leq f_{\text{sd},\text{pb}} \) (68)

\[
\tau_{\text{max},\text{pb}} = \frac{1}{2} \left( \frac{\gamma_g \cdot \tau_{\text{pl},\text{pb}} + \gamma_t \cdot \tau_{\text{m},\text{pb}} \cdot L/2}{[b_h \cdot h_{\text{f},\text{pb}}]} \right)
\]

(69)

\[
f_{\text{sd},\text{pb}} = k_{\text{sys}} \cdot k_{\text{mod}} \cdot f_{\text{ak}} / \gamma_{M,\text{pb}}
\]

(70)

Serviceability limit state constraints (SLS):

- Checking the instantaneous deflection of the primary timber beam: \( u_{\text{inst},\text{pb}} \leq L/300 \) (71)

\[
u_{\text{inst},\text{pb}} = 5 \left( g_{\text{k},\text{pb}} \cdot \sigma_{\text{pl},\text{pb}} + q_e \right) \cdot L^4 / [384 \cdot E_{\text{pb}} \cdot b_{\text{h},\text{pb}} \cdot h_{\text{f},\text{pb}}^3 / 12] + 12 (g_{\text{k},\text{pb}} \cdot \sigma_{\text{m},\text{pb}} \cdot L^2 / [5 \cdot E_{\text{pb}} \cdot b_h \cdot h_{\text{f},\text{pb}}]
\]

(72)

- Checking the net deflection of the primary timber beam: \( u_{\text{fin},\text{pb}} \leq L/250 \) (73)

\[
u_{\text{fin},\text{pb}} = 5 \left( g_{\text{k},\text{pb}} \cdot \sigma_{\text{pl},\text{pb}} \cdot L^4 / [384 \cdot E_{\text{pb}} \cdot b_{\text{h},\text{pb}} \cdot h_{\text{f},\text{pb}}^3 / 12] + 12 (g_{\text{k},\text{pb}} + q_e \cdot L^2 / (5 \cdot E_{\text{pb}} \cdot b_h \cdot h_{\text{f},\text{pb}}) \right) (1 + k_{\text{dyf}}) + [5 \cdot q_e \cdot \sigma_{\text{m},\text{pb}} \cdot L^4 / (384 \cdot E_{\text{pb}} \cdot b_h \cdot h_{\text{f},\text{pb}}^3 / 12) + 12 \cdot q_e \cdot \sigma_{\text{m},\text{pb}} \cdot L^2 / (5 \cdot E_{\text{pb}} \cdot b_h \cdot h_{\text{f},\text{pb}})] (1 + 2 \cdot k_{\text{dyf}})
\]

(74)

- Proportion of the point load acting on a single primary beam must be greater than the acceptable value: \( k_{\text{dist},\text{pb}} \geq 0.3 \) (75)

\[
k_{\text{dist},\text{pb}} = k_{\text{strat}} \cdot (0.38 - 0.08 \cdot \ln ((E_{\text{eff}})/\gamma_{\text{y}}))
\]

(76)

- Vertical deflection caused by concentrated static force \( F = 1.0 \text{ kN} \) must not exceed: \( w_{\text{pb}} \leq a \) (77)

\[
w_{\text{pb}} = 1000 \cdot k_{\text{dist},\text{pb}} \cdot L^4 \cdot k_{\text{strat}} / \left[ 48 \cdot (E_{\text{eff}}) / \gamma_{\text{y},\text{pb}} \right]
\]

where \((E_{\text{eff}}) / \gamma_{\text{y},\text{pb}} = E_{\text{pb}} \cdot b_h \cdot h_{\text{f},\text{pb}}^3 / 12\) (78),(79)

Partial factors for actions \( \gamma_g =1.35 \) and \( \gamma_t = 1.5 \) are defined at the ultimate limit state and \( \gamma_y = 1.0 \) and \( \gamma_y = 1.0 \) at the serviceability limit state.

The ultimate limit state condition, where the maximal design bending stress in timber beam \( \sigma_{m,y,d,\text{eff}} (\text{MPa}) \) must not exceed the design bending strength of the timber \( f_{m,y,d,\text{eff}} (\text{MPa}) \) is calculated by Equation (10). In Equations (11)–(14), the design bending strength is calculated by using a loading sharing factor \( k_{\text{sys}} \). In the equations, \( k_{\text{h}} \) is a depth factor, \( k_{\text{mod}} \) is a modification factor for duration of load and moisture content, \( f_{\text{mk}} \) is the characteristic bending strength of timber, \( \gamma_y \) is the specific weight of timber, \( \gamma_{y,\text{board}} \) is the specific weight of timber boards and \( \gamma_{M,\text{pb}} \) is a partial factor for material properties. The design bending stress in the above equation is calculated without considering the favorable action of timber boards. Therefore, the Gamma method is also proposed to calculate the effective stiffness of the timber floor system \((E_{\text{eff}})\). see Equations (15)–(23). In the equations, \( E \) and \( E_{\text{board}} \) are the modules of elasticity of timber and timber boards, respectively, and \( \gamma_1 \) and \( \gamma_2 \) determine the composite action of the beam and timber board. Equations (17)–(23) define substituted terms \( l_{\text{eff}}, A_{\text{eff}}, d_{\text{eff}}, d_{\text{eff},\text{eff}}, d_{\text{eff},\text{eff}}, l_{\text{eff}}, l_{\text{eff},\text{eff}}, \) and \( l_{\text{eff},\text{eff}} \) involved in \((E_{\text{eff}})\). Constraint Equation (24) determines that the calculated maximal design shear stress in timber beams \( \tau_{\text{max}} \) (the stress component perpendicular to the grain, MPa) must not exceed the design shear strength of the timber \( f_{\text{sd}} \) (MPa). \( \tau_{\text{max}} \) and \( f_{\text{sd}} \) are defined by Equations (25) and (26), respectively, where \( f_{\text{sd}} \) (MPa) stands for the shear strength of the timber beam. In the design methodology according to the Gamma method, the maximal design shear stress is expressed by Equation (27).

Equation (28) define the conditions under which the maximal design bending stress in timber board \( \sigma_{\text{board,y,d}} (\text{MPa}) \) must not exceed the design bending strength in board \( f_{\text{board,y,d}} (\text{MPa}) \). \( \sigma_{\text{board,y,d}} \) and \( f_{\text{board,y,d}} \) are defined according to Equations (29)–(32). In the equations, \( \sigma_{\text{k,board}} \) represents the self-weight (kN/m²) of the floor board, depth factor \( k_{\text{h,board}} \) is taken to be equal to 1.3 because of the small thickness of the board and \( f_{\text{board,y,gb}} (\text{MPa}) \) represents the characteristic bending strength of timber board. In addition, Equation (33) defines that the maximal design shear stress in the board \( \tau_{\text{board,max}} \) (the stress component perpendicular to the grain, MPa) must not exceed the design shear strength in the board \( f_{\text{board,y,d}} (\text{MPa}) \).
The serviceability limit state condition, where the instantaneous deflection of the timber beam \(u_{\text{inst}}\) (m) must be less than or equal to the recommended value, is defined by Equation (36). In the case when the system strength factor \(k_{\text{sys}}\) is used, \(u_{\text{inst}}\) is defined by Equation (37). In the case of the Gamma method design, the instantaneous deflection of the timber beam is expressed by Equation (38). The instantaneous deflection of the timber boards \(u_{\text{inst},\text{board}}\) (m) must also be less than or equal to the allowable value; see Equation (39). Deflection \(u_{\text{inst},\text{board}}\) is expressed by Equation (40). The calculated final deflection of the timber beam \(u_{\text{fin}}\) (m) must be less than or equal to the recommended value; see Equation (41). In the case of using the factor \(k_{\text{sys}}\), \(u_{\text{fin}}\) is defined by Equation (42), where \(\psi_2\) is the factor for the quasi-permanent value of the variable action and \(k_{\text{def}}\) is the deformation factor. In this way, the deflection of the structure resulting from the effect of long-term use and moisture is limited. The final deflection \(u_{\text{fin},\text{strut}}\), calculated by the Gamma method, is determined by Equation (43). The final deflection of the timber board \(u_{\text{fin,board}}\) (m), which must be less than the recommended value, is verified by Equation (44). The substituted final deflection \(u_{\text{fin,board}}\) is expressed by Equation (45).

The constraint Equation (46) defines that the proportion of the point load acting on a single joist \(k_{\text{dist}}\) must be greater than or equal to the acceptable value. \(k_{\text{dist}}\) is expressed by Equations (47) and (48), where \(EI_b\) (Nm\(^2\)/m) is the equivalent board/plate bending stiffness parallel to the beams and \(k_{\text{strut}}\) stands for the partial factor for the installed strutting. Equation (49) defines the condition, where the vertical deflection \(w\) (m) of the joist caused by the concentrated static force \(F = 1.0\) kN must not exceed the allowable deflection \(a\) (m). Terms \(w\) and \(a\) are expressed by Equations (50)–(52), where \(k_{\text{amp}}\) is the amplification factor that takes into account the shear deflection, and \((EI)_{\text{pluq}}\) is the stiffness of the joist, which in the case of the Gamma method used is equal to the effective stiffness of the timber floor system \((EI)_{\text{eff}}\). Equation (53).

According to Equations (54)–(56), the fundamental frequency of vibration of a rectangular residential floor \(f_1\) (Hz) should exceed 8 Hz, where \(EI_j\) (Nm\(^2\)/m) is the equivalent bending stiffness of the joists divided by the joist spacing in the beam direction and \(\text{mass}_v\) (kg/m\(^1\)) is the mass of the timber floor structure. Finally, the calculated unit impulse floor velocity \(v\) (1/Ns) must be less than or equal to the allowable floor velocity \(v_{\text{allow}}\) (1/Ns); see Equations (57)–(62), where \(n_40\) represents the number of first-order modes with natural frequencies less than \(40\) Hz, \(b_{\text{floor}}\) (m) represents the floor width, \(b_{\text{cr}}\) denotes the velocity response value, \(\xi\) is the modal damping coefficient, and the other symbols are defined as above.

The above equations are used for the floor system SYS-A and for the upper part of the floor system SYS-B (for the secondary beams and timber boards of the system SYS-B). Equations (63)–(82) are defined for the calculation of the primary glulam beams of the system SYS-B. These equations are nearly identical to those of the system SYS-A, so the terms in the equations are indexed \(pb\) (primary beam). Equations (63)–(67) define that the design bending stress in the primary beam \(\sigma_{\text{n,y,d,}\text{pb}}\) (MPa) must not exceed the design bending strength of the timber of the primary beam \(f_{\text{n,y,d,}\text{ pb}}\) (MPa), Equations (68)–(70) define that the design shear stress in the primary beam \(\tau_{\text{max,}\text{ pb}}\) (MPa) must not exceed the design shear strength of the timber of the primary beam \(f_{\text{vd,}\text{ pb}}\) (MPa), Equations (71) and (72) define that the instantaneous deflection of the primary beam \(u_{\text{inst,}\text{ pb}}\) (m) must be less than or equal to the recommended value, Equations (73) and (74) calculate that the final deflection of the primary beam \(u_{\text{fin,}\text{ pb}}\) (m) must be less than or equal to the recommended value. Equations (75)–(76) regulate that the proportion of the point load acting on a single primary beam \(k_{\text{dist,}\text{ pb}}\) must be greater than or equal to the acceptable value, and Equations (77)–(79) limit the vertical deflection of the primary beam \(w_{\text{ pb}}\) (m) caused by the concentrated static force \(F = 1.0\) kN.

Structural analysis and dimensioning (in)equality constraints also determine the timber floor joist dimensions and the material properties to be calculated inside the defined
limits. The characteristic shear strength of the timber beams $f_{vk}$ (MPa), the specific weight of the timber beams $\gamma_k$ (kN/m$^3$) and the modulus of elasticity of the timber $E$ (MPa) are substituted in the optimization model as functions of the characteristic bending strength of timber $f_{mk}$ (MPa); see Equations (80)–(82).

$$f_{vk} = 0.4311 + 0.0858 \cdot f_{mk}$$

$$\gamma_k = 2.3 + 0.05 \cdot f_{mk}$$

$$E = -3714.3 + 970.24 \cdot f_{mk} - 14.881 \cdot f_{mk}^2.$$  

(80)  

(81)  

(82)  

The glued laminated timber beams are usually manufactured with a height to width ratio smaller than or equal to 12, see Equation (83).

$$\frac{h}{b} \leq 12.$$  

(83)  

2.2.4. Logical Constraints

The objective function is subject to logical constraints which determine the discrete values for dimensions of cross-sections and the discrete values for standard materials (strengths). Each discrete dimension $d_{st}$ (or timber strength) is defined as a scalar product between the vector of $j$, $j \in J$, discrete dimension alternatives $q_j = \{q_1, q_2, q_3, \ldots, q_j\}$, and the vector of $j$ associated binary variables $y_{jst} = \{y_{1st}, y_{2st}, y_{3st}, \ldots, y_{jst}\}$; see Equation (84).

Only one discrete value is selected for the dimension since the sum of the binary variables $y_{jst}$ has to be equal to 1; see Equation (85).

$$d_{st} = \sum_{j \in J} q_j y_{jst}$$  

(84)  

$$\sum_{j \in J} y_{jst} = 1.$$  

(85)  

Figure 2 shows the flowchart of the optimal design of timber floor joists based on multi-parametric MINLP optimization.
2.3. Multi-Parametric MINLP Optimization

Multi-parametric MINLP optimization of the production costs of the timber floor joists was performed for combinations of different design parameters:

- Nineteen different structure spans: 2 m, 3 m, 4 m, 5 m, 6 m, 7 m, 8 m, 9 m, 10 m, 11 m, 12 m, 13 m, 14 m, 15 m, 16 m, 17 m, 18 m, 19 m and 20 m;
- Five different imposed loads: 1 kN/m$^2$, 2 kN/m$^2$, 3 kN/m$^2$, 4 kN/m$^2$ and 5 kN/m$^2$;
- For this purpose, 95 individual MINLP optimizations were executed. For each combination, two different load sharing systems were applied:
  - The system strength factor $k_{sys}$;
  - The Gamma method.

In the structural analysis, the following values were assigned: $k_{sys} = 1.1$ and $\gamma_1 = 0.2$. By assigning a corresponding value $\gamma_1$, different types of fastening can be used for the sheathing. The sheathing can be fastened with nails or screws at the appropriate distances as long as the gamma value $\gamma_1$ is guaranteed. The gamma factors $\gamma_1$ in case the sheathing is fastened with screws or nails can be calculated with the following equations:

$$\gamma_1 = \left(1 + \frac{\pi^2 E A s_i}{K_i L^2}\right)^{-1}$$

$$K_i = \frac{p_{m}^{1.5}}{23}$$ for screws

$$K_i = \frac{p_{m}^{1.5}}{23}$$ for nails (without pre-drilling),

where $p_{m}^{1.5}$ is the nominal load in kN/m$^2$.\n
---

**Figure 2.** Flowchart of optimal design of timber floor joists based on multi-parametric MINLP optimization.
where $K_i$ is the slip modulus, $\rho_m$ is the mean wood density, $\phi$ is the diameter of the fastener, $s_i$ is the spacing between the fasteners, $L$ is the span, and $A$ is the cross-sectional area of the sheathing.

Numbers of different discrete alternatives for dimensions of cross-sections and beam spacings were defined; see Table 5. These dimensions (available on the market) were discretized in order to obtain real structural solutions. Only the characteristic bending strength $f_{mk} = 24$ MPa is defined in the model because most companies provide just this timber strength. The combinatorics of the defined discrete alternatives in the SYS-A floor system yielded $d_{altern} \times h_{altern} \times e_{altern} \times f_{mk,altern} = 3 \times 16 \times 12 \times 1 = 6912$ different structure alternatives (for each different combination between the defined imposed load and structure span). In the SYS-B floor system with primary and secondary timber beams, the number of combinations between different discrete alternatives increased to $d_{altern} \times b_{altern} \times h_{altern} \times e_{altern} \times f_{mk,altern} \times b_{pb,altern} \times h_{pb,altern} \times e_{pb,altern} = 3 \times 16 \times 12 \times 13 \times 1 \times 12 \times 61 \times 31 = 169,917,696$ different structure alternatives. Only one is optimal (for a given load and span).

Table 5. Discrete alternatives of dimensions and strengths.

| Variable | Discrete Alternatives |
|----------|-----------------------|
| $d$ (mm) | 20, 25, 30 |
| $b$ (mm) | 50, 60, 70, 80, 90, 100, 120, 140, 160, 180, 200, 220, 240, 260, 280, 300 |
| $h$ (mm) | 80, 100, 120, 140, 160, 180, 200, 220, 240, 260, 280, 300 |
| $e$ (m) | 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2 |
| $f_{mk}$ (MPa) | 24 |
| $b_{pb}$ (mm) | 50, 60, 70, 80, 90, 100, 110, 120, 130, 140, 150, 160, 180 |
| $h_{pb}$ (mm) | 300, 320, 340, 360, 380, 400, 420, 440, 460, 480, 500, 520, 540, 560, 580 |
| $e_{pb}$ (m) | 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 3.0, 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, 3.9, 4.0 |

The MINLP optimization model TIMBFJ was used. The task of the optimizations was to calculate the minimal self-manufacturing costs of the timber floor joist (€/m$^2$), the structure mass (kg/m$^2$) and all the dimensions (design variables). The material unit prices (December 2021), labor costs and other data are shown in Table 6. Values from engineering practice in Central Europe were adopted.
Table 6. Material and labor costs and other data.

| Symbol | Description | Cost |
|--------|-------------|------|
| $c_{M,t}$ | Sawn timber or glued laminated timber (C24) | 250 €/$m^3$ or 500 €/$m^3$ |
| $c_{M,imp}$ | Timber impregnation | 125 €/$m^3$ |
| $c_{board}$ | Timber boards | 21 €/$m^2$ |
| $c_{L,board}$ | Price of floor board installation | 13 €/$m^2$ |
| $k_{sys}$ | Loading sharing factor | 1.1 |
| $k_{mod}$ | Modification factor for duration of load and moisture content | 0.8 |
| $k_{def}$ | Deformation factor | 0.8 |
| $\gamma_M$ | Partial factor for sawn timber or glued laminated timber | 1.3 or 1.25 |
| $\gamma_{board}$ | Specific weight of timber board | 5 kN$/m^3$ |
| $\psi_2$ | Factor for the quasi-permanent value of the variable action | 0.3 |
| $\gamma_g$ | Partial factor for permanent action | 1.35 |
| $\gamma_d$ | Partial factor for variable action | 1.50 |
| $k_{strut}$ | Partial factor for installed strutting | 1.0 |
| $k_{amp}$ | Amplification factor to account for shear deflection | 1.05 |
| $\zeta$ | Modal damping coefficient | 0.02 |
| $b_{floor}$ | Floor width | 10 m |
| $f_{board,y,k}$ | Characteristic bending strength of timber board | 24 MPa |
| $\gamma_1$ | Composite action of the beam and timber board | 0.2 |
| $\gamma_2$ | Composite action of the beam | 1.0 |

The Modified Outer-Approximation/Equality-Relaxation (OA/ER) algorithm by Kranja and Grossmann [46] and a two-phase MINLP strategy [47] were used. The OA/ER optimization algorithm solves an alternative sequence of non-linear programming (NLP) sub-problems and mixed-integer linear programming (MILP) master problems. The NLP corresponds to the continuous optimization of parameters and yields an upper bound towards the objective to be minimized. The MILP involves a global linear approximation to the superstructure of discrete alternatives in which new discrete sizes are identified. The optimization search is terminated when the predicted lower bound exceeds the upper bound. In cases of non-convex problems, the optimization process is terminated when the NLP solution can no longer be improved. The OA/ER algorithm guarantees global optimality of solutions for convex and quasi-convex optimization problems. The optimization is proposed to be performed sequentially in two different phases in order to accelerate the convergence of the OA/ER algorithm:

- The optimization is proposed to start with continuous NLP optimization of the structure (the initialization), where all variables are treated as continuous. The result represents a good starting point and accumulates an effective global linear approximation of the superstructure for the next discrete optimization;

- The optimization continues with MINLP discrete optimization (a sequence of NLP and MILP solutions) in the second phase until the optimal solution is found.

The optimizations were carried out by the MINLP computer package MIPSYN [50,51]. While GAMS/CONOPT (generalized reduced-gradient method, commercial software) by Drudd [52] was used to solve the NLP sub-problems, GAMS/CPLEX (branch and bound method, commercial software) [53] was employed to find solutions to the MILP master problems.

3. Results and Discussion

3.1. Recommended Optimal Design for Timber Floor Joists

The recommended optimal design for timber floor joists was determined on the basis of multi-parametric MINLP optimization; see Tables 7–9. The recommended design is shown depending on different variable imposed loads $q_k$ and different structure spans $L$. 

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Table 7. Recommended optimal design for timber floor joists constructed of sawn wood in SYS-A-SW (only sawn primary beams).

| Span L (m) | 2 m  | 3 m  | 4 m  | 5 m  | 6 m  | 7 m  | 8 m  | 9 m  | 2 m  | 3 m  | 4 m  | 5 m  | 6 m  | 7 m  | 8 m  | 9 m  |
|------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| d (mm)     | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   |
| b (mm)     | 50   | 50   | 50   | 50   | 100  | 100  | 100  | 100  | 100  | 100  | 100  | 100  | 100  | 100  | 100  | 100  |
| h (mm)     | 120  | 160  | 220  | 300  | 300  | 300  | 300  | 300  | 300  | 300  | 300  | 300  | 300  | 300  | 300  | 300  |
| e (m)      | 1.1  | 1.1  | 1.1  | 1.1  | 1.1  | 1.1  | 0.9  | 0.9  | 1.1  | 1.1  | 1.1  | 1.1  | 1.1  | 1.1  | 1.1  | 1.1  |
| f_{mk} (MPa) | 24   | 24   | 24   | 24   | 24   | 24   | 24   | 24   | 24   | 24   | 24   | 24   | 24   | 24   | 24   | 24   |
| mass (kg/m²) | 11.91 | 12.80 | 13.50 | 14.77 | 19.55 | 28.90 | 45.00 | 73.00 | 11.27 | 12.23 | 12.86 | 14.14 | 17.35 | 25.27 | 38.64 | 58.00 |
| COST (€/m²) | 36.05 | 37.00 | 37.75 | 39.11 | 39.11 | 39.11 | 39.11 | 39.11 | 39.11 | 39.11 | 39.11 | 39.11 | 39.11 | 39.11 | 39.11 | 39.11 |

Imposed Load $q_k = 1$ kN/m²

| Span L (m) | 2 m  | 3 m  | 4 m  | 5 m  | 6 m  | 7 m  | 8 m  | 9 m  | 2 m  | 3 m  | 4 m  | 5 m  | 6 m  | 7 m  | 8 m  | 9 m  |
|------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| d (mm)     | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   |
| b (mm)     | 50   | 50   | 50   | 50   | 100  | 100  | 100  | 100  | 100  | 100  | 100  | 100  | 100  | 100  | 100  | 100  |
| h (mm)     | 120  | 180  | 240  | 300  | 300  | 300  | 300  | 300  | 300  | 300  | 300  | 300  | 300  | 300  | 300  | 300  |
| e (m)      | 0.9  | 0.9  | 0.9  | 0.9  | 0.9  | 0.9  | 0.9  | 0.9  | 0.9  | 0.9  | 0.9  | 0.9  | 0.9  | 0.9  | 0.9  | 0.9  |
| f_{mk} (MPa) | 24   | 24   | 24   | 24   | 24   | 24   | 24   | 24   | 24   | 24   | 24   | 24   | 24   | 24   | 24   | 24   |
| mass (kg/m²) | 12.33 | 13.50 | 14.67 | 15.83 | 31.00 | 45.00 | 73.00 | 73.00 | 11.94 | 12.72 | 13.89 | 15.06 | 17.88 | 26.33 | 40.33 | 58.00 |
| COST (€/m²) | 36.50 | 37.75 | 39.00 | 40.25 | 40.25 | 40.25 | 40.25 | 40.25 | 36.08 | 36.92 | 38.17 | 39.42 | 42.44 | 51.50 | 66.50 | 90.75 |

Imposed Load $q_k = 2$ kN/m²

| Span L (m) | 2 m  | 3 m  | 4 m  | 5 m  | 6 m  | 7 m  | 8 m  | 9 m  | 2 m  | 3 m  | 4 m  | 5 m  | 6 m  | 7 m  | 8 m  | 9 m  |
|------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| d (mm)     | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   |
| b (mm)     | 50   | 50   | 50   | 50   | 70   | 100  | 160  | 280  | 280  | 300  | 300  | 300  | 300  | 300  | 300  | 300  |
| h (mm)     | 120  | 200  | 260  | 300  | 300  | 300  | 300  | 300  | 300  | 300  | 300  | 300  | 300  | 300  | 300  | 300  |
| e (m)      | 0.7  | 0.8  | 0.8  | 0.8  | 0.8  | 0.8  | 0.8  | 0.8  | 0.8  | 0.8  | 0.8  | 0.8  | 0.8  | 0.8  | 0.8  | 0.8  |
| f_{mk} (MPa) | 24   | 24   | 24   | 24   | 24   | 24   | 24   | 24   | 24   | 24   | 24   | 24   | 24   | 24   | 24   | 24   |
| mass (kg/m²) | 13.00 | 14.38 | 15.69 | 16.58 | 23.13 | 31.00 | 46.75 | 73   | 12.19 | 13.50 | 14.81 | 16.13 | 18.75 | 28.00 | 41.50 | 58.00 |
| COST (€/m²) | 37.21 | 36.69 | 40.09 | 43.19 | 48.06 | 56.50 | 73.38 | 101.5 | 36.34 | 37.75 | 39.16 | 40.56 | 43.38 | 53.29 | 67.75 | 90.75 |
Table 7. Cont.

| Span L: | 2 m | 3 m | 4 m | 5 m | 6 m | 7 m | 8 m | 9 m | 2 m | 3 m | 4 m | 5 m | 6 m | 7 m | 8 m | 9 m |
|--------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| d (mm) | 20  | 20  | 20  | 20  | 20  | 20  | 20  | 20  | 20  | 20  | 20  | 20  | 20  | 20  | 20  | 20  |
| b (mm) | 50  | 50  | 50  | 70  | 100 | 160 | 300 | 300 | 50  | 50  | 50  | 50  | 60  | 160 | 180 | 280 |
| h (mm) | 140 | 200 | 280 | 300 | 300 | 300 | 300 | 300 | 50  | 50  | 50  | 50  | 60  | 160 | 180 | 280 |
| e (m)  | 0.7 | 0.7 | 0.6 | 0.6 | 0.6 | 0.6 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 | 0.5 |
| $f_{mk}$ (MPa) | 24  | 24  | 24  | 24  | 24  | 24  | 24  | 24  | 24  | 24  | 24  | 24  | 24  | 24  | 24  | 24  |
| COST ($€/m^2$) | 37.75 | 39.36 | 42.13 | 46.25 | 52.75 | 64.00 | 82.21 | Dim. outside the defined limits |

For Imposed load $q_k = 4$ kN/m$^2$:

| Span L: | 2 m | 3 m | 4 m | 5 m | 6 m | 7 m | 8 m | 9 m |
|--------|-----|-----|-----|-----|-----|-----|-----|-----|
| d (mm) | 20  | 20  | 20  | 20  | 20  | 20  | 20  | 20  |
| b (mm) | 50  | 60  | 90  | 120 | 200 | 260 | 200 | 260 |
| h (mm) | 160 | 240 | 300 | 300 | 300 | 300 | 300 | 300 |
| e (m)  | 0.7 | 0.7 | 0.7 | 0.6 | 0.6 | 0.5 | 0.5 | 0.5 |
| $f_{mk}$ (MPa) | 24  | 24  | 24  | 24  | 24  | 24  | 24  | 24  |
| COST ($€/m^2$) | 14.00 | 16.00 | 20.08 | 23.50 | 31.00 | 45.00 | 64.60 |

For Imposed load $q_k = 5$ kN/m$^2$:

| Span L: | 2 m | 3 m | 4 m | 5 m | 6 m | 7 m | 8 m | 9 m |
|--------|-----|-----|-----|-----|-----|-----|-----|-----|
| d (mm) | 20  | 20  | 20  | 20  | 20  | 20  | 20  | 20  |
| b (mm) | 50  | 60  | 90  | 120 | 200 | 260 | 200 | 260 |
| h (mm) | 160 | 240 | 300 | 300 | 300 | 300 | 300 | 300 |
| e (m)  | 0.7 | 0.7 | 0.7 | 0.6 | 0.6 | 0.5 | 0.5 | 0.5 |
| $f_{mk}$ (MPa) | 24  | 24  | 24  | 24  | 24  | 24  | 24  | 24  |
| mass (kg/m$^2$) | 14.00 | 16.00 | 20.08 | 23.50 | 31.00 | 45.00 | 64.60 |
| COST ($€/m^2$) | 37.75 | 39.36 | 42.13 | 46.25 | 52.75 | 64.00 | 82.21 | 97.00 |

For Imposed load $q_k = 5$ kN/m$^2$:

| Span L: | 2 m | 3 m | 4 m | 5 m | 6 m | 7 m | 8 m | 9 m |
|--------|-----|-----|-----|-----|-----|-----|-----|-----|
| d (mm) | 20  | 20  | 20  | 20  | 20  | 20  | 20  | 20  |
| b (mm) | 50  | 60  | 90  | 120 | 200 | 260 | 200 | 260 |
| h (mm) | 160 | 240 | 300 | 300 | 300 | 300 | 300 | 300 |
| e (m)  | 0.7 | 0.7 | 0.7 | 0.6 | 0.6 | 0.5 | 0.5 | 0.5 |
| $f_{mk}$ (MPa) | 24  | 24  | 24  | 24  | 24  | 24  | 24  | 24  |
| COST ($€/m^2$) | 37.75 | 39.36 | 42.13 | 46.25 | 52.75 | 64.00 | 82.21 | 97.00 |

For Imposed load $q_k = 5$ kN/m$^2$:

| Span L: | 2 m | 3 m | 4 m | 5 m | 6 m | 7 m | 8 m | 9 m |
|--------|-----|-----|-----|-----|-----|-----|-----|-----|
| d (mm) | 20  | 20  | 20  | 20  | 20  | 20  | 20  | 20  |
| b (mm) | 50  | 60  | 90  | 120 | 200 | 260 | 200 | 260 |
| h (mm) | 160 | 240 | 300 | 300 | 300 | 300 | 300 | 300 |
| e (m)  | 0.7 | 0.7 | 0.7 | 0.6 | 0.6 | 0.5 | 0.5 | 0.5 |
| $f_{mk}$ (MPa) | 24  | 24  | 24  | 24  | 24  | 24  | 24  | 24  |
| COST ($€/m^2$) | 14.00 | 16.00 | 20.08 | 23.50 | 31.00 | 45.00 | 64.60 | 97.00 | 14.00 | 16.00 | 20.08 | 23.50 | 31.00 | 45.00 | 64.60 | 97.00 |

For Imposed load $q_k = 5$ kN/m$^2$:

| Span L: | 2 m | 3 m | 4 m | 5 m | 6 m | 7 m | 8 m | 9 m |
|--------|-----|-----|-----|-----|-----|-----|-----|-----|
| d (mm) | 20  | 20  | 20  | 20  | 20  | 20  | 20  | 20  |
| b (mm) | 50  | 60  | 90  | 120 | 200 | 260 | 200 | 260 |
| h (mm) | 160 | 240 | 300 | 300 | 300 | 300 | 300 | 300 |
| e (m)  | 0.7 | 0.7 | 0.7 | 0.6 | 0.6 | 0.5 | 0.5 | 0.5 |
| $f_{mk}$ (MPa) | 24  | 24  | 24  | 24  | 24  | 24  | 24  | 24  |
| COST ($€/m^2$) | 14.00 | 16.00 | 20.08 | 23.50 | 31.00 | 45.00 | 64.60 | 97.00 | 14.00 | 16.00 | 20.08 | 23.50 | 31.00 | 45.00 | 64.60 | 97.00 |
Table 8. Recommended optimal design for timber floor joists constructed of glued laminated timber beams in SYS-A-Gl (only glulam primary beams).

| System SYS-A-Gl (Including $\gamma_1 = 0.2$) |
|---------------------------------------------|
| Imposed Load $q_k = 1 \text{kN/m}^2$ |
| Span L (m): 5 m | 6 m | 7 m | 8 m | 9 m | 10 m | 11 m | 12 m | 13 m | 14 m | 15 m | 16 m | 17 m | 18 m | 19 m | 20 m |
| $d$ (mm) | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 |
| $b$ (mm) | 60 | 60 | 60 | 60 | 60 | 60 | 80 | 80 | 80 | 100 | 100 | 100 | 100 | 120 | 120 | 120 |
| $h$ (mm) | 280 | 360 | 440 | 540 | 620 | 720 | 740 | 840 | 940 | 960 | 1060 | 1140 | 1200 | 1260 | 1340 | 1440 |
| $e$ (m) | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.2 |
| $f_{\text{mod}}$ (MPa) | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 |
| mass (kg/m$^2$) | 15.3 | 16.9 | 18.4 | 20.3 | 21.8 | 23.7 | 28.8 | 31.4 | 33.9 | 40.5 | 43.7 | 50.7 | 58.1 | 59.4 | 62.9 |
| COST (€/m$^2$) | 43.5 | 46.3 | 49.0 | 52.4 | 55.1 | 58.5 | 67.6 | 72.2 | 76.7 | 88.5 | 94.2 | 98.8 | 110.2 | 119.9 | 125.8 | 132.0 |

| Imposed load $q_k = 2 \text{kN/m}^2$ |
|---------------------------------------------|
| Span L (m): 5 m | 6 m | 7 m | 8 m | 9 m | 10 m | 11 m | 12 m | 13 m | 14 m | 15 m | 16 m | 17 m | 18 m | 19 m | 20 m |
| $d$ (mm) | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 |
| $b$ (mm) | 60 | 60 | 60 | 60 | 60 | 60 | 80 | 80 | 80 | 100 | 100 | 100 | 100 | 120 | 120 | 120 |
| $h$ (mm) | 280 | 360 | 440 | 520 | 600 | 700 | 720 | 800 | 900 | 920 | 1000 | 1100 | 1180 | 1300 | 1420 |
| $e$ (m) | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 |
| $f_{\text{mod}}$ (MPa) | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 |
| mass (kg/m$^2$) | 16.5 | 18.4 | 20.3 | 22.1 | 23.7 | 26.3 | 32.4 | 34.9 | 35.4 | 41.8 | 47.5 | 51.0 | 52.5 | 64.6 | 69.0 | 81.3 |
| COST (€/m$^2$) | 45.7 | 49.0 | 52.3 | 55.7 | 59.0 | 63.2 | 74.0 | 78.4 | 82.9 | 94.3 | 105.7 | 110.8 | 117.0 | 138.5 | 147.5 | 168.3 |

| Imposed load $q_k = 3 \text{kN/m}^2$ |
|---------------------------------------------|
| Span L (m): 5 m | 6 m | 7 m | 8 m | 9 m | 10 m | 11 m | 12 m | 13 m | 14 m | 15 m | 16 m | 17 m | 18 m | 19 m | 20 m |
| $d$ (mm) | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 |
| $b$ (mm) | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 80 | 80 | 80 | 80 | 80 | 100 | 100 | 120 |
| $h$ (mm) | 320 | 380 | 460 | 520 | 600 | 680 | 700 | 780 | 880 | 920 | 960 | 1000 | 1060 | 1160 | 1260 |
| $e$ (m) | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 | 0.9 | 0.9 | 0.9 | 0.9 | 0.7 | 0.7 | 0.7 | 0.6 |
| $f_{\text{mod}}$ (MPa) | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 |
| mass (kg/m$^2$) | 18.4 | 20.0 | 22.1 | 24.0 | 26.3 | 32.4 | 34.9 | 35.4 | 41.8 | 47.5 | 51.0 | 52.5 | 64.6 | 69.0 | 81.3 |
| COST (€/m$^2$) | 49.0 | 51.8 | 55.6 | 58.4 | 62.1 | 65.9 | 77.8 | 85.3 | 94.0 | 101.1 | 118.6 | 128.1 | 144.6 | 153.6 | 207.5 |

| Imposed load $q_k = 4 \text{kN/m}^2$ |
|---------------------------------------------|
| Span L (m): 5 m | 6 m | 7 m | 8 m | 9 m | 10 m | 11 m | 12 m | 13 m | 14 m | 15 m | 16 m | 17 m | 18 m | 19 m | 20 m |
| $d$ (mm) | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 |
| $b$ (mm) | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 80 | 80 | 80 | 80 | 80 | 100 | 100 | 100 |
| $h$ (mm) | 340 | 420 | 500 | 580 | 660 | 680 | 680 | 720 | 780 | 920 | 920 | 940 |
| $e$ (m) | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 | 0.6 | 0.5 | 0.6 | 0.7 | 0.7 | 0.6 | 0.6 |
| $f_{\text{mod}}$ (MPa) | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 | 24 |
| mass (kg/m$^2$) | 20.2 | 22.6 | 25.0 | 27.4 | 29.8 | 33.8 | 38.6 | 46.1 | 48.9 | 51.8 | 57.9 | 69.8 |
| COST (€/m$^2$) | 52.2 | 56.5 | 60.8 | 65.1 | 69.4 | 76.5 | 85.0 | 102.0 | 107.0 | 115.7 | 126.7 | 147.9 |
### Table 8. Cont.

| System SYS-A-Gl (Including $\gamma_1 = 0.2$) | Imposed load $q_k = 5 \text{kN/m}^2$ |
|-------------------------------------------|-------------------------------------|
| $d$ (mm)                                  | 20 20 20 20 20 20 20 20 25 30 30 30 30 30 30 30 |
| $b$ (mm)                                  | 60 60 60 60 60 60 60 80 80 80 80 80 80 80 80 80 |
| $h$ (mm)                                  | 380 460 560 620 680 720 800 880 |
| $e$ (m)                                   | 0.7 0.7 0.7 0.6 0.5 0.5 0.6 0.6 0.6 0.6 0.6 0.6 0.6 0.6 0.6 0.6 |
| $f_{\text{sek}}$ (MPa)                    | 24 24 24 24 24 24 24 24 24 24 24 24 24 24 24 24 |
| mass (kg/m$^2$)                           | 21.4 23.8 26.8 31.0 36.0 38.6 46.1 52.3 56.1 |
| COST ($\varepsilon$/m$^2$)                | 54.4 58.6 64.0 71.5 80.5 85.0 102.0 116.7 123.3 |

| Dimension outside the defined limits       | Dimension outside the defined limits |
|-------------------------------------------|-------------------------------------|
| Dimension outside the defined limits      | Dimension outside the defined limits |
| Dimension outside the defined limits      | Dimension outside the defined limits |
| Dimension outside the defined limits      | Dimension outside the defined limits |
| Dimension outside the defined limits      | Dimension outside the defined limits |

### Table 9. Recommended optimal design for timber floor joists in system SYS-B (glulam primary beams plus sawn secondary beams).

| System SYS-B (Including $\gamma_1 = 0.2$) | Imposed Load $q_k = 1 \text{kN/m}^2$ |
|-------------------------------------------|-------------------------------------|
| $d$ (mm)                                  | 20 20 20 20 20 20 20 20 20 20 20 20 20 20 20 20 |
| $b$ (mm)                                  | 50 50 50 50 50 50 50 50 50 50 80 80 100 100 100 100 |
| $h$ (mm)                                  | 120 120 140 160 160 180 180 180 180 180 180 180 180 180 180 180 |
| $e$ (m)                                   | 1.1 1.1 1.1 1.1 1.1 1.1 1.1 1.1 1.1 1.1 1.1 1.1 1.1 1.1 1.1 1.1 |
| $f_{\text{sek}}$ (MPa)                    | 24 24 24 24 24 24 24 24 24 24 24 24 24 24 24 24 |
| $h_{\text{pb}}$ (mm)                      | 340 420 540 660 720 780 880 920 1040 1140 1200 1280 1400 1440 1480 1500 |
| $e_{\text{pb}}$ (m)                       | 2.8 2.8 3.2 3.6 3.3 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 |
| mass (kg/m$^2$)                           | 14.5 15.1 15.8 16.4 17.0 18.3 19.0 20.9 22.0 22.8 25.3 26.3 27.6 30.5 33.6 45.5 |
| COST ($\varepsilon$/m$^2$)                | 40.6 41.7 42.7 43.6 44.8 46.8 48.1 51.4 53.3 54.9 59.2 61.1 63.3 68.6 74.1 96.0 |

| Dimension outside the defined limits       | Dimension outside the defined limits |
|-------------------------------------------|-------------------------------------|
| Dimension outside the defined limits      | Dimension outside the defined limits |
| Dimension outside the defined limits      | Dimension outside the defined limits |
| Dimension outside the defined limits      | Dimension outside the defined limits |
| Dimension outside the defined limits      | Dimension outside the defined limits |

### Table 8. Cont.

| System SYS-A-Gl (Including $\gamma_1 = 0.2$) | Imposed load $q_k = 2 \text{kN/m}^2$ |
|-------------------------------------------|-------------------------------------|
| $d$ (mm)                                  | 20 20 20 20 20 20 20 20 20 20 20 20 20 20 20 20 |
| $b$ (mm)                                  | 50 50 50 50 50 50 50 50 50 50 50 50 50 50 50 50 |
| $h$ (mm)                                  | 120 120 120 100 180 160 160 180 180 180 180 180 180 180 180 180 |
| $e$ (m)                                   | 0.9 0.9 0.9 0.9 0.9 0.9 0.9 0.9 0.9 0.9 0.9 0.9 0.9 0.9 0.9 0.9 |
| $f_{\text{sek}}$ (MPa)                    | 24 24 24 24 24 24 24 24 24 24 24 24 24 24 24 24 |
| $h_{\text{pb}}$ (mm)                      | 480 580 700 720 960 960 1080 1160 1200 1300 1400 1440 1480 1500 1500 1500 |
| $e_{\text{pb}}$ (m)                       | 2.7 2.7 2.7 2.1 3.8 3.1 4 3.8 3.4 4 4 3.7 4 3.6 3.2 2.5 |
| mass (kg/m$^2$)                           | 16.1 16.8 17.8 19.1 20.6 21.4 23.3 24.2 25.5 27.5 28.6 29.8 32.0 33.9 36.0 45.9 |
| COST ($\varepsilon$/m$^2$)                | 43.2 44.6 46.2 48.9 50.4 52.4 55.0 56.8 59.4 62.5 64.4 66.9 70.5 74.2 78.297 96.5 |
Table 9. Cont.

**System SYS-B (Including $\gamma_1 = 0.2$)**

| Imposed load $q_k = 3 \text{kN/m}^2$ | Imposed load $q_k = 4 \text{kN/m}^2$ | Imposed load $q_k = 5 \text{kN/m}^2$ |
|-------------------------------------|-------------------------------------|-------------------------------------|
| **Span L:**                        | **Span L:**                        | **Span L:**                        |
| 5 m                                 | 5 m                                 | 5 m                                 |
| 6 m                                 | 6 m                                 | 6 m                                 |
| 7 m                                 | 7 m                                 | 7 m                                 |
| 8 m                                 | 8 m                                 | 8 m                                 |
| 9 m                                 | 9 m                                 | 9 m                                 |
| 10 m                                | 10 m                                | 10 m                                |
| 11 m                                | 11 m                                | 11 m                                |
| 12 m                                | 12 m                                | 12 m                                |
| 13 m                                | 13 m                                | 13 m                                |
| 14 m                                | 14 m                                | 14 m                                |
| 15 m                                | 15 m                                | 15 m                                |
| 16 m                                | 16 m                                | 16 m                                |
| 17 m                                | 17 m                                | 17 m                                |
| 18 m                                | 18 m                                | 18 m                                |
| 19 m                                | 19 m                                | 19 m                                |
| 20 m                                | 20 m                                | 20 m                                |
| **d (mm)**                          | **d (mm)**                          | **d (mm)**                          |
| 20                                  | 20                                  | 20                                  |
| 20                                  | 20                                  | 20                                  |
| 20                                  | 20                                  | 20                                  |
| 20                                  | 20                                  | 20                                  |
| 20                                  | 20                                  | 20                                  |
| 20                                  | 20                                  | 20                                  |
| 20                                  | 20                                  | 20                                  |
| 20                                  | 20                                  | 20                                  |
| 20                                  | 20                                  | 20                                  |
| 20                                  | 20                                  | 20                                  |
| 20                                  | 20                                  | 20                                  |
| **b (mm)**                          | **b (mm)**                          | **b (mm)**                          |
| 50                                  | 50                                  | 50                                  |
| 50                                  | 50                                  | 50                                  |
| 50                                  | 50                                  | 50                                  |
| 50                                  | 50                                  | 50                                  |
| 50                                  | 50                                  | 50                                  |
| 50                                  | 50                                  | 50                                  |
| 50                                  | 50                                  | 50                                  |
| 50                                  | 50                                  | 50                                  |
| 50                                  | 50                                  | 50                                  |
| 50                                  | 50                                  | 50                                  |
| **h (mm)**                          | **h (mm)**                          | **h (mm)**                          |
| 140                                 | 140                                 | 140                                 |
| 140                                 | 140                                 | 140                                 |
| 140                                 | 140                                 | 140                                 |
| 140                                 | 140                                 | 140                                 |
| 140                                 | 140                                 | 140                                 |
| 140                                 | 140                                 | 140                                 |
| 140                                 | 140                                 | 140                                 |
| 140                                 | 140                                 | 140                                 |
| 140                                 | 140                                 | 140                                 |
| 140                                 | 140                                 | 140                                 |
| **e (m)**                           | **e (m)**                           | **e (m)**                           |
| 0.8                                 | 0.8                                 | 0.8                                 |
| 0.8                                 | 0.8                                 | 0.8                                 |
| 0.8                                 | 0.8                                 | 0.8                                 |
| 0.8                                 | 0.8                                 | 0.8                                 |
| 0.8                                 | 0.8                                 | 0.8                                 |
| 0.8                                 | 0.8                                 | 0.8                                 |
| 0.8                                 | 0.8                                 | 0.8                                 |
| 0.8                                 | 0.8                                 | 0.8                                 |
| 0.8                                 | 0.8                                 | 0.8                                 |
| 0.8                                 | 0.8                                 | 0.8                                 |
| **f_{sk} (MPa)**                    | **f_{sk} (MPa)**                    | **f_{sk} (MPa)**                    |
| 24                                  | 24                                  | 24                                  |
| 24                                  | 24                                  | 24                                  |
| 24                                  | 24                                  | 24                                  |
| 24                                  | 24                                  | 24                                  |
| 24                                  | 24                                  | 24                                  |
| 24                                  | 24                                  | 24                                  |
| 24                                  | 24                                  | 24                                  |
| 24                                  | 24                                  | 24                                  |
| 24                                  | 24                                  | 24                                  |
| 24                                  | 24                                  | 24                                  |
| **h_{pb} (mm)**                     | **h_{pb} (mm)**                     | **h_{pb} (mm)**                     |
| 60                                  | 60                                  | 60                                  |
| 60                                  | 60                                  | 60                                  |
| 60                                  | 60                                  | 60                                  |
| 60                                  | 60                                  | 60                                  |
| 60                                  | 60                                  | 60                                  |
| 60                                  | 60                                  | 60                                  |
| 60                                  | 60                                  | 60                                  |
| 60                                  | 60                                  | 60                                  |
| 60                                  | 60                                  | 60                                  |
| 60                                  | 60                                  | 60                                  |
| **e_{pb} (m)**                      | **e_{pb} (m)**                      | **e_{pb} (m)**                      |
| 2.8                                 | 2.8                                 | 2.8                                 |
| 2.8                                 | 2.8                                 | 2.8                                 |
| 2.8                                 | 2.8                                 | 2.8                                 |
| 2.8                                 | 2.8                                 | 2.8                                 |
| 2.8                                 | 2.8                                 | 2.8                                 |
| 2.8                                 | 2.8                                 | 2.8                                 |
| 2.8                                 | 2.8                                 | 2.8                                 |
| 2.8                                 | 2.8                                 | 2.8                                 |
| 2.8                                 | 2.8                                 | 2.8                                 |
| 2.8                                 | 2.8                                 | 2.8                                 |
| **mass (kg/m²)**                    | **mass (kg/m²)**                    | **mass (kg/m²)**                    |
| 17.6                                | 17.6                                | 17.6                                |
| 17.6                                | 17.6                                | 17.6                                |
| 17.6                                | 17.6                                | 17.6                                |
| 17.6                                | 17.6                                | 17.6                                |
| 17.6                                | 17.6                                | 17.6                                |
| 17.6                                | 17.6                                | 17.6                                |
| 17.6                                | 17.6                                | 17.6                                |
| 17.6                                | 17.6                                | 17.6                                |
| 17.6                                | 17.6                                | 17.6                                |
| 17.6                                | 17.6                                | 17.6                                |
| **COST (€/m²)**                     | **COST (€/m²)**                     | **COST (€/m²)**                     |
| 45.3                                | 45.3                                | 45.3                                |
| 45.3                                | 45.3                                | 45.3                                |
| 45.3                                | 45.3                                | 45.3                                |
| 45.3                                | 45.3                                | 45.3                                |
| 45.3                                | 45.3                                | 45.3                                |
| 45.3                                | 45.3                                | 45.3                                |
| 45.3                                | 45.3                                | 45.3                                |
| 45.3                                | 45.3                                | 45.3                                |
| 45.3                                | 45.3                                | 45.3                                |
| 45.3                                | 45.3                                | 45.3                                |
Comparative analysis of the results shows that it is more economical to increase the timber beam height than to reduce the spacing between beams. If the Gamma method ($\gamma_1 = 0.2$) is used instead of the strength system factor ($k_{sys} = 1.1$), the costs can be reduced by up to 10%.

Among all the defined conditions relating to timber boards, the serviceability limit state was found to be decisive. This finding applies to all optimal designs of timber floors. The condition of the vertical deflection caused by a concentrated static force ($F = 1.0 \text{kN}$) is decisive for small loads up to 2 kN/m² for all considered spans and all timber floor systems (SYS-A-SW, SYS-A-Gl and SYS-B). In the SYS-A-SW system with spans ranging up to 4 m and for imposed loads larger than 2 kN/m², the main difference between the strength system factor ($k_{sys}$) method and the Gamma method ($\gamma$) is:

- If the strength system factor ($k_{sys}$) is used, the critical condition is the bending strength of the timber;
- While if the Gamma method is used, the critical condition is the final deflection of the timber floor.

In the SYS-A-Gl system, the bending strength of timber was found to be decisive for imposed loads larger than 3 kN/m² for all considered spans. In the SYS-B system, for spans longer than 5 m and imposed loads larger than 2 kN/m², the ultimate limit state is dominant for primary glued laminated timber beams, while the serviceability limit state is dominant for secondary sawn timber beams.

### 3.2. Economically Suitable Spans of Timber Floor Joists

Figure 3 shows diagrams of the calculated optimal self-manufacturing structure costs of nine different spans ranging from 2 m to 10 m. The diagrams were developed for five different variable imposed loads from 1 kN/m² to 5 kN/m², and clearly show that floor joists made of SYS-A-SW sawn timber beams are economically suitable for spans up to 6 m. For these spans, SYS-A-SW is the most appropriate system for all imposed loads. It should be noted that for imposed loads from 1 kN/m² to 3 kN/m² and spans longer than 8 m, nearly the same optimal solutions were calculated, because of the decisive vibration velocity condition.

![Figure 3](image)

**Figure 3.** Diagrams of the self-manufacturing costs of the SYS-A-SW timber floor joist system for different spans and imposed loads, (a) strength system factor $k_{sys} = 1.1$ and (b) Gamma method, $\gamma_1 = 0.2$.

Figure 4 presents diagrams of the calculated optimal self-manufacturing structure costs of the SYS-A-SW (primary sawn timber beams only) floor system and of the SYS-B (with primary glulam beams and secondary sawn timber beams) floor system. The diagrams are drawn for different spans (from 2 m to 10 m) and for different imposed loads (from 1 kN/m² to 5 kN/m²). A comparison between the diagrams clearly shows that the
SYS-B system is more economically suitable than the SYS-A-SW system for spans longer than 6 m.

Figure 4. Diagrams of the self-manufacturing costs of timber floor joists for different spans and imposed loads (Gamma method, $\gamma_1 = 0.2$), (a) comparison between SYS-A-SW and SYS-B systems, (b) comparison between SYS-A-Gl and SYS-B systems.

3.3. Calculation Example

In order to interpret the applicability of the comparative diagrams and the recommended optimal design for the timber floor joists, we present a calculation example that determines the most favorable joist structure for the given design parameters and two different load cases.

3.3.1. The structure Self-Weight

In this example, we determine the most competitive design for timber floor joists with a span of $L = 6$ m, subjected to the structure self-weight and the vertical variable imposed load of $q_k = 2$ kN/m$^2$. It should be noted that the Gamma method ($\gamma_1 = 0.2$) is applied.

Figure 3b shows that the optimal timber floor SYS-A-SW with a span of 6 m and a vertical imposed load of 2 kN/m$^2$ yields self-manufacturing costs of about 42 €/m$^2$. The recommended floor joists in Table 7 for a span of 6 m and imposed load of 2 kN/m$^2$ are comprised of: thickness of the floor board $d = 20$ mm, width of the timber beam $b = 60$ mm, height of the beam $h = 300$ mm, spacing between floor beams $e = 0.8$ m, characteristic bending strength of timber $f_{mk} = 24$ MPa, mass of the structure $mass = 17.88$ kg/m$^2$ and self-manufacturing costs of the structure $COST = 42.44$ €/m$^2$. The optimal design is shown...
in Figure 5a. For system SYS-B, Figure 4 shows optimal manufacturing costs for the given design parameter at less than €44.6/m². The recommended design for system SYS-B in Table 9 is comprised of: thickness of the floor board $d = 20$ mm; width of the secondary timber beam $b = 50$ mm; height of the secondary beam $h = 120$ mm; spacing between secondary floor beams $e = 0.9$ m; characteristic bending strength of timber $f_{mk} = 24$ MPa; width of the primary glulam beam $b_{gb} = 60$ mm; height of the primary glulam beam $h_{gb} = 580$ mm; spacing between primary beams $e_{gb} = 2.7$ m; mass of the structure $m = 16.8$ kg/m² and self-manufacturing costs of the structure $COST = 44.6$ €/m²; see Figure 5.

Figure 5. The optimal design of timber floor joists for a structure span $L = 6$ m and variable imposed load $q = 2$ kN/m², (a,b) floor system A (SYS-A), (c,d) floor system B (SYS-B).

3.3.2. The Structure Self-Weight Plus Additional Permanent Load

In the same example (timber floor joists with a span of $L = 6$ m, subjected to the structure self-weight and a vertical variable imposed load of $q = 2$ kN/m²), the load on the floor is added, which includes the floor finishing $0.50$ kN/m², the insulation between the beams $0.10$ kN/m² and the ceiling under the beams $0.30$ kN/m², which together gives $0.90$ kN/m² of additional permanent load $g_p$.

Since the comparative diagrams and recommendations are shown in dependence on the vertical imposed load only, the additional permanent load $g_p$ of $0.90$ kN/m² must be transformed into a variable load and added to the defined variable imposed load $q = 2$ kN/m². Taking into account Equation (9) and different partial factors for actions at the ultimate limit state for permanent and variable loads, the defined variable imposed load of $2$ kN/m² can be modified with the additional permanent load $g_p$ by Equation (89):

$$q_k = q_o + g_p \frac{\gamma_g}{\gamma_q} = 2.0 + 0.9 \frac{1.35}{1.5} = 2.81 \text{ kN/m}^2.$$  \hspace{1cm} (89)

The nearest upper integer value to the calculated modified imposed load $q_k$ is $3$ kN/m². Comparing the self-manufacturing costs plotted in Figures 3 and 4 for a span of $6$ m and a vertical imposed load of $3$ kN/m², the diagram in Figure 3b shows that the cheapest
timber floor floor system is SYS-A-SW at about 44 €/m². The recommended timber structure from Table 7 includes the thickness of floor boards of 20 mm, timber beam width of 50 mm, beam height of 300 mm, beam spacing of 0.6 m, characteristic bending strength of timber 24 MPa, structure mass 18.75 kg/m² and self-manufacturing structure costs of 43.38 €/m², see Figure 6.

Figure 6. The optimal design of timber floor joists for a structure span \( L = 6 \) m and variable imposed load \( q_k = 3 \) kN/m².

4. Conclusions

The optimization of timber floor joists takes into account the minimal self-manufacturing costs of a timber structure and the discrete sizes of each element’s cross-sections. Because the optimization problem is discrete and non-linear, mixed-integer non-linear programming (MINLP) was applied to develop the TIMBFJ optimization model, in which an accurate objective function of the structure’s material and labor costs is subjected to design and dimensioning (in)equality constraints, which were defined in accordance with the requirements of Eurocode standards. Two different structural floor systems were considered: a system which contains only timber beams and sheathing, and a system with primary and secondary beams and sheathing (the secondary beams are laid on the primary ones). Two different load sharing systems were applied, a load sharing factor and a multi-parametric MINLP optimization of the timber floor joists was performed. The optimization problems were solved using the Modified Outer-Approximation/Equality-Relaxation (OA/ER) algorithm and the MINLP computer package MIPSYN.

A comparative analysis of the calculated results shows that it is more economical to increase the timber beam height than to reduce the spacing between beams. If the Gamma method (\( \gamma_1 = 0.2 \)) is used instead of the strength system factor (\( k_{sys} = 1.1 \)), costs can be reduced by up to 10%. On the basis of the obtained results, a recommended optimal design for timber floor joists was developed. Using these recommendations, an engineer can choose the optimal structure system and material (sawn wood or glulam) for a defined vertical variable imposed load and structure span.

It was found that floor joists made of primary sawn timber beams only (SYS-A-SW) are economically suitable for spans up to 6 m for all defined imposed loads (up to 5 kN/m²). This system is the most appropriate floor system for shorter spans. For spans longer than 6 m, a floor system constructed of primary glulam beams and secondary sawn timber beams (SYS-B) is more economically suitable.

For imposed loads up to 5 kN/m², a floor system consisting of primary glulam beams only (SYS-A-Gl) is economically suitable for spans up to 10 m, while a floor system constructed of primary glulam beams and secondary sawn timber beams (SYS-B) is
economically suitable for spans up to 20 m. A floor system consisting of primary and secondary beams (SYS-B) is more appropriate than a system consisting of primary glulam beams only (SYS-A-Gl) for all the loads and spans studied. In this way, the competitive spans of the considered timber floor joists were found based on the cost optimization.

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