Bi-model RBDO process based on constructing SVM model by using adaptive support vector clamping method

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Abstract: Kriging model is an efficient method to solve reliability-based design optimization problem with black box constraints. However, the estimation process of the Kriging model must be based on all sample points, so there still needs a large evaluate time, especially when dealing with high dimensional constraints. To overcome this difficulty, a bi-model RBDO process is proposed in this paper. In the first stage, accurate Kriging model of the constraint is constructed. In the second stage, accurate SVM model is constructed based on the Kriging model by using adaptive support vector clamping method. And the optimal design point of the RBDO problem is calculated based on this SVM model. The results show that the proposed approach is more efficient with necessary accuracy when solving the RBDO problem.

1. Introduction
By constructing accurate surrogate model, Kriging model can be an effective method to reduce the calculation cost of the reliability-based design optimization (RBDO) problem, especially when the constraints are complex nonlinear functions or black box functions [1-2]. However, all sample points are essential in the estimation process, so the Kriging model requires a large storage when dealing with high dimensional problem, which will reduce the valuation efficiency significantly. On the other hand, for the double-loop procedure in RBDO needs a large number of invocations of the surrogate model, many strategies are proposed to improve the efficiency [3-6]. When the problem is high-dimensional, the situation will be worse [7].

To deal with this difficulty, a bi-model RBDO process is proposed in this paper to improve the efficiency of solving RBDO problem. In this process, the first stage is to build an accurate Kriging model. The second stage is to build an accurate SVM model to replace the Kriging model, and the subsequent RBDO process is carried out based on this SVM model. Because the SVM model is determined only by the support vectors, the estimation efficiency can be promoted effectively. A new adaptive support vector clamping method is proposed in this paper. With this method, high accuracy SVM model can be constructed efficiently to replace the Kriging model. In order to ensure the success of this approximation process, a modified complex method is proposed to find a feasible point based on a series of infeasible point.

2. Modified complex method
The complex method is an important direct solution to solve constraint optimization problems [8-9]. This method was presented and improved based on simplex method [10-11], so it is a constraint simplex method and is named complex method. The basic idea of the complex method in constraint optimization problem is to construct an initial complex which has some vertices in the feasible region.
The objective function value of all vertices are calculated and compared with each other, and the biggest one is called the worst point. Then, a new point is selected based on the searching method to make the objective function value smaller, and replace the worst point with this new point. Through this method, a new complex is constructed. As the shape changes in each iteration, the complex moves gradually to the optimal point, until it is close enough to the optimal point. In this process, the complex does not need to keep a regular shape. And there is no special requirement for the shape of objective function and constraint functions. So, this method has strong adaptability.

In the initial stage of some RBDO process, it may be difficult to have a feasible initial point for starting the optimization process and the support vector clamping method. To overcome this difficulty, the modified complex method is proposed to find a feasible point based on a series of infeasible point in RBDO problem. In traditional optimization problem, the complex method is used as an optimization method to find the optimum solution, so all the starting points in complex are feasible points. However, in the modified complex method, all the starting points in complex are infeasible points. Basic concepts and process of this method is presented followed.

2.1 The degree of constraint violation

Degree of constraint violation (DCV) is an essential foundation of the modified complex method, so the definition of DCV is introduced first. Usually, it is hard to estimate the distance from a sample point to the feasible region, especially when the complicated feasible region with high nonlinear constraint functions. To deal with this difficulty, a new concept of the DCV is proposed in this paper. By estimating the degree of constraint violation, a quantitative index can be obtained to show the infeasibility of a sample. The calculation procedure of the DCV is presented as followed.

A normalization process is performed at each sample point, and the normalization is formulated as follows:

\[ Y(x_i) = \frac{y(x_i)}{y_{\text{max}} - y_{\text{min}}} \quad i = 1, 2, \ldots, N \] (1)

where \(x_i\) is the ith sample point, \(N\) is the number of sample points, \(y(x_i)\) is the constraint function value at \(x_i\), \(y_{\text{max}}\) and \(y_{\text{min}}\) are the maximum and the minimum of all constraint function values of existing sample points, and \(Y(x_i)\) is the normalized constraint function value at \(x_i\).

For each sample point, it is necessary to determine whether it is a feasible point. If a sample point is a feasible one, it means that this sample point does not violate the constraint, and the normalized constraint function value \(Y\) will not contribute to the degree of constraint violation. So \(Y\) is defined as zero. This process can be expressed as follows:

\[ Y(x_i) = 0 \quad \text{if} \quad Y(x_i) < 0 \]

\(i = 1, 2, \ldots, N\) (2)

If there are many constraint functions in RBDO problem, it should calculate the sum of all constraint function values. This process can be expressed as follows:

\[ D(x_i) = \sum_{j=1}^{K} Y(x_{ij}) \]

\(i = 1, 2, \ldots, N; \quad j = 1, 2, \ldots, K\) (3)

where \(D(x_i)\) is the DCV of sample point \(x_i\), and \(K\) is the number of constraints.

2.2 The process of the modified complex method

To obtain a feasible initial point, the complex method is modified. The procedures are presented as follows:

Step1: Calculate the DCV of each infeasible point, and choose \(k\) starting samples with the minimum DCVs from existing sample points.
Step 2: Sort these k points based on DCV. The best point is named XL, the worst point is named XH, and the second worst point is named XG.

Step 3: Calculate the center of all vertices after point XH is removed. The center point XC is calculated as follows:

$$x_c = \frac{1}{k-1} \sum_{j=1}^{k-1} (x_j)$$  \hspace{1cm} (4)

Step 4: Evaluate point XC and determine whether it is a feasible point. If it is a feasible point, then end, otherwise, go to Step 5.

Step 5: Initialize the reflection coefficient, $\alpha = 1.3$.

Step 6: Calculate the value of XR by using the function as follows:

$$x_R = x_c + \alpha(x_c - x_H)$$  \hspace{1cm} (5)

Step 7: Evaluate point XR and determine whether it is a feasible point. If it is a feasible point, then end, otherwise, go to Step 8.

Step 8: If the DCV of XR is smaller than the DCV of XH, XH = XR, and go to Step 2; otherwise, go to Step 9.

Step 9: If the reflection coefficient is less than 0.1, go to Step 10, otherwise, go to Step 11.

Step 10: Change the value of reflection coefficient, $\alpha = 0.5\alpha$, and go to Step 6.

Step 11: Replace XH with XG, and go to Step 3.

3. Adaptive support vector clamping method

Support vector machine is a kind of machine learning method, which is used to construct supervised learning models. There are two kinds of SVM, including the support vector classification machine (SVC) [12-13] for classification problems and the support vector regression machine (SVR) [14] for regression problems. In the RBDO problem, the design space is divided into feasible and infeasible domains by the constraint functions, and each sample is endowed with a state of safety or failure. So, both kinds of SVM are applicable to the RBDO problem.

The purpose of the SVM classification model is to find an optimal classification hyperplane to divide samples into positive and negative points. Based on the kernel function, the SVM model can map low dimensional space into high dimensional space, so SVM model have been used in many studies to construct highly nonlinear explicit boundaries [15-16]. On the other hand, when dealing with high-dimensional problems, the SVM model can also maintain a good effect on both accuracy and efficiency [7].

Given N samples that are distributed within the design space, the SVM equation is given by [17]

$$s(x) = b + \sum_{i=1}^{N} \lambda_i y_i K(x_i, x)$$  \hspace{1cm} (6)

where $x_i$ is the training data; $b$ is the bias; $\lambda_i$ is the Lagrange multiplier; $y_i$ is the classification value; and $K$ is the kernel function. For a new sample $x$, the classification can be predicted by the sign of $s(x)$. Each training data has one of two classification values, -1 or +1. And the classification model is determined by the nearest opposite training data points, which are called support vectors. On both sides of the classification model, there are two hyperplanes passing through these support vectors. These two hyperplanes can form a margin that does not contain any data points.

A schematic of SVM is shown in Figure 1. According to $s(x) = 0$, the hyperplane of SVM decision function is located in the middle of opposite classed support vectors. Undoubtedly, when these opposite classed support vectors are closed enough, SVM will approximate the true limit state constraint boundary very accurately. In order to make the SVM to obtain good approximation effect, new opposite classed samples are expected to be located in the vicinity of the true limit state constraint boundary.
In this paper, a sequence sampling process is proposed to make the center of a pair of opposite classed samples approach the true limit state constraint boundary gradually. The procedures of this process are as below:

Step1: Find the nearest opposite classed sample point from the initial sample point, and calculate the center of these two points.

Step2: Select the center as a new sample point and add it into the sample set.

Step3: Evaluate the constraint function value at the new sample point, and update SVM model.

Step4: Determine which class the new sample point belongs to. When the class of the new sample is different from the class of the new sample in the last iteration, then end, otherwise, select the center as the initial sample point and go to Step1.

After this process, two adjacent support vectors are located beside both sides of the SVM decision function. The SVM model in the vicinity of this pair of support vectors can get a very good approximation effect. Because this pair of support vectors can control the SVM model like a clamp, so the proposed method is named support vector clamping process. This support vector clamping process is shown in Fig. 2.

If there is no feasible point in the beginning, the new sample obtained by the modified complex method will be the only feasible point, so this point must be a support vector according to $s(x) = 1$. And this feasible point is selected as the initial point to find the nearest opposite classed sample point from the sample set. This nearest sample will be a support vector according to $s(x) = -1$. As shown in Figure 2, the basic conditions of the support vector clamping process are constructed based on these two opposite classed support vectors. Because the initial support vectors have an unequal distribution on the both sides of the true limit state boundary, there exists a great bias from the SVM decision function to the true limit state. In order to improve the prediction of the SVM, new samples are hoped to be located in the margin.

Then, the center of these two points is calculated, and the SVM model is updated based on this center point. As shown in Figure 2(a), the center point will be the new support vector based on the
updated SVM model. Such, a new pair of opposite classed vectors is obtained, which is used in the next iteration. As can be seen clearly from Figure 2, with the selection of new center samples and the update of the SVM model, the margin between hyperplanes becomes narrow, and the SVM decision function approaches the true limit state boundary gradually. A smaller error will make the prediction of the SVM model more accurate.

In order to obtain an accurate RBDO result based on the SVM model efficiently, the pair of opposite classed support vectors should be selected reasonably in the RBDO process. An adaptive support vector clamping method is proposed in this section. The main idea of this method is locating the center of the pair of opposite classed support vectors at the place where the biggest approximation error may exist. To start the support vector clamping process adaptively, the new initial sample of the opposite classed support vectors are selected by the maximum curvature and minimum distance (MCMD) sampling criterion. This sampling criterion is shown as follows:

$$\text{max} : K(x) \cdot D$$

where $K(x)$ is the curvature of SVM model at point x, and $D$ is the minimal distance from the current sample point x to the existing centers of closest opposite support vectors after clamping procedures. And the value of $K(x)$ is calculated based on the eigenvalues of the Hessian matrix of the SVM model. The calculation formula is defined as follows:

$$K(x) = \sqrt{\sum_{i=1}^{p} k_i^2(x)}$$

where $k_1, k_2, \ldots, k_p$ are eigenvalues of the Hessian matrix. When the new initial sample is selected by equation (7), another sample point is selected as the closest sample from existing sample points.

Generally, surrogate model can get a good approximation effect in the linear area but be more likely to cause a big error in the nonlinear area. A large value of $K(x)$ means the SVM model might has a large error at this point, so this new point is added into the training points set. On the other hand, $D$ can ensure the new support vector clamping procedure perform in the unknown area, which will be helpful for the SVM model to be accurate over all given domain. By this combination method, the approximation quality of the SVM model is improved based on as few new samples as possible.

4. Procedures and flowchart of the bi-model RBDO

Based on the above methods, a new bi-model RBDO process is proposed by using Kriging model and SVM model together. The purpose of this method is reducing the calculation time in each iteration, so as to improve the efficiency of RBDO process. In this bi-model RBDO process, an accurate Kriging model is constructed to replace the origin constraint function. Then, the following procedures are applied based on the Kriging model, which can minimize the number of expensive valuations based on the original constraint function. Based on this precondition, the SVM model fitting process is performed, in which the adaptive support vector clamping method is integrated into the RBDO process. Based on the characteristics of SVM, the uncertainty analysis time can be reduced effectively. The flowchart of the proposed bi-model RBDO process is shown in Figure 3, and the procedures are as follows:

Step1: Initialize the sample set by using Latin hypercube sampling method.
Step2: Construct/update the Kriging model of the constraint function.
Step3: Assess every samples based on current Kriging model.
Step4: Determine whether the sample set contains feasible points. If there is no feasible point in the sample set, then go to Step5, otherwise, the feasible point with the minimum objective function value is selected as the initial point, then go to Step6.
Step5: Obtain a feasible point by using modified complex method. To carry out this step, new sample points are selected based on the Kriging model which is constructed in Step2. The new feasible point obtained by modified complex method is added into the sample set, and this point is set as the
new initial point which is used for starting the optimization in the following calculation process, then go to Step2.

Step6: Calculate the deterministic optimum point based on the deterministic optimization which ignores the influence of uncertainty factors of the given problem.

Step7: If two consecutive deterministic optimum points are close enough, stop the refinement process of the Kriging model, and go to Step 9; otherwise go to Step8.

Step8: Select a new sample and add it into the sample set, then go to Step2. In each iteration of the refinement process of the Kriging model, the new sample is selected by CBS method.

Step9: Construct the initial SVM model of the constraint function based on existing sample points.

Step10: Take all infeasible support vectors as initial sample points to perform the clamping process one by one.

Step11: Select a new sample point based on the MCMD sampling criterion.

Step12: Set the new sample point as the initial sample point to perform the clamping process.

Step13: Based on the current SVM model, calculate the design point by using SORA method.

Step14: Check the stopping criterion. If converged, then end, otherwise, go to Step 11.

Figure 3 The bi-model RBDO process

In general, the RBDO process contains two phases when the SORA method is used for obtaining the RBDO results. In the first phase, an accurate Kriging model of the constraint boundary is built. In order to ensure the accuracy can meet the requirement, the calling of the Kriging model is inevitable.
In the second phase, the SORA method is used to obtain the RBDO design point. In SORA method, the PMA problem is solved by iterative calculation. More iterations can get higher accurate calculation result, but the invocation counts of the Kriging model will also increase. So this process will also cost a lot of computing time, especially when the dimension of the problem is high. However, the characteristics of SVM can make it effectively avoid the influence of high dimensional factors. At the same time, the storage space of SVM model is smaller than that of Kriging model. So, the Kriging model can be replaced by an accurate SVM model. Based on the advantages of SVM, the calculation time can be saved, and the computing efficient can be improved greatly. On the other hand, the construction process of the SVM model requires very little computational cost. Because all samples selected by CBS method are located near the constraint boundary, these samples are also close to the Kriging model. Undoubtedly, these is no big margin between two types of support vectors. Therefore, in the clamping process, the number of times the Kriging model is called will be very small. It is clearly that the higher the dimension of the RBDO problem, the more time can be saved by this strategy. In this way, the RBDO problem solving efficiency can be significantly improved.

With a series of new samples, new clamping processes will make the SVM model more accurate. When two consecutive optimal RBDO design points are close enough, the bi-model RBDO ends. In this paper, the termination criterion is defined as follows:

$$\|d_i - d_{i-1}\| \leq \epsilon$$

where $\epsilon$ is a small positive number; $d_i$ is the optimal RBDO design point of the iteration $i$.

5. Example

This example is constructed based on the Haupt example [18], to exhibit the adaptive support vector clamping process clearly. This modified Haupt example has two random design variables and a nonlinear constraint. And the design region is shrunk to a smaller area. The RBDO problem of this example is formulated as follows:

\[
\begin{align*}
\text{find } & \quad d = [d_1, d_2] \\
\text{min } & \quad f(d) = (d_1 - 3.7)^2 + (d_2 - 4)^2 \\
\text{s.t.} & \quad \text{prob}[g_i(X) < 0] \leq \Phi(-\beta_i^f), \quad i = 1, 2 \\
& \quad g(X) = -X_1 \sin(4X_1) - 1.1X_2 \sin(2X_2) \\
& \quad 2.2 \leq d_i \leq 4.0, \quad 2.0 \leq d_2 \leq 4.0 \\
& \quad \beta_i^f = \beta_i^s = 2.0 \\
& \quad X_i \sim N(d_i, 0.1^2), \quad n = 1, 2
\end{align*}
\]

Figure 4 Graphical representation of the true constraint boundaries of modified Haupt example

In this example, all random variables are statistically independent and normal distributed. The
target reliability index and the standard deviation are $\beta = 2.0$ and $\sigma = 0.1$, respectively. As shown in Figure 4, the simple quadratic objective function decreases from lower left to upper right, and is denoted by dotted lines. The nonlinear function of the constraint is denoted by a black line. Based on the constraint, the feasible region is shown as the shaded region. The deterministic optimum point and the RBDO design point based on the analytical method that directly calls the true functions are also displaced in this figure. The optimal RBDO design point obtained by directly calling the true constraint function is located at (2.8243, 3.2686).

Based on the adaptive support vector clamping method, the process of the bi-model RBDO of this example is shown in Figure 5. As shown in Figure 5 (a), the accurate Kriging model is represented as the green line. And all feasible and infeasible samples are represented by red and blue points respectively. As shown in Figure 5 (b), the initial SVM model is constructed based on all support vectors. The SVM model and two hyperplane are represented as the black, red and blue lines, according to $s(x) = 0$, $s(x) = 1$ and $s(x) = -1$ respectively. And the support vector is represented as the point around by a circle. Then, the support vector clamping method is applied based on these initial support vectors. As shown in Figure 5 (c), a new SVM model is constructed after the support vector clamping process. This SVM model is similar to the Kriging model on the whole, but has a poor approximation effect when the curvature is large. To improve the accuracy of the SVM model, a new sample is selected by using MCMD sampling criterion. This new sample and the nearest opposite classified sample, which are represented as the points around by small rectangle in Figure 5 (d), are selected as two initial points to perform the clamping process. As shown in Figure 5 (e), the new clamping process has a great contribution to the refinement of the SVM model, especially at the place where the curvature is large. In the following iterations, new samples are added into the sample set, and the corresponding clamping processes are also performed. With the continuous refinement of the SVM model, the calculation result of the RBDO problem will be more accurate. As shown in Figure 5 (f), when the termination criterion is satisfied in the end, an accurate RBDO design point is obtained, and an accurate SVM model is constructed.
The comparison result for this example is shown in Table 1. In this table, the optimal design points obtained based on Kriging model and SVM model are very close to each other. Compared with Kriging model, there is only a slight increase in the error of the optimal design point obtained based on SVM model. However, the estimation time of SVM model is much more efficient than Kriging model. In general, based on the SVM model obtained by adaptive support vector clamping method, the RBDO process can get an accurate calculation result more efficiently. Undoubtedly, the proposed bi-model RBDO process can save the calculation time effectively and get an accurate optimal design point, especially when the problem is high-dimensional.

| Surrogate model | Kriging model | SVM model |
|-----------------|---------------|-----------|
| Optimal design point | (2.8206, 3.2734) | (2.8122, 3.2793) |
| Norm error      | 0.006         | 0.016     |
| Estimation time (sec) | 0.69          | 0.48      |

**6. Conclusion**

This paper presents a new bi-model RBDO process. After the accurate Kriging model has been built, the adaptive support vector clamping method is used to construct the accurate SVM model to replace the Kriging model. To ensure this process, the modified complex method is used to find a feasible
point when all initial samples are infeasible. And then the optimal design point of the RBDO problem is calculated based on this SVM model. The results show that this bi-model RBDO process can maintain the accuracy level similar to the situation when use Kriging model only, and reduce the evaluation time efficiently.

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