Interferometric 2D small angle generator for autocollimator calibration

Ville Heikkinen, Ville Byman, Ilkka Palosuo, Björn Hemming and Antti Lassila

VTT Technical Research Centre of Finland Ltd, Centre for Metrology MIKES, P.O. Box 1000, 02044 VTT Tekniikantie 1, 02150 Espoo, Finland

E-mail: ville.heikkinen@vtt.fi

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Abstract
Small angle generators are simple devices for providing small angles traceable to the definition of the SI-unit radian. The most accurate ones use a laser interferometer for measurement of angular displacement. Small angle generators used for autocollimator calibration usually create angles around a single axis. A two-directional angle generator would be preferable, as it could more efficiently reveal artefacts related to angular displacement across both axes, such as orthogonality of the autocollimator’s measuring axes. Characterizing errors depending on one axis only would also be easier, as the setup needs to be aligned only once for studying both axes of the autocollimator. We describe a novel interferometric two-directional small angle generator which we have built and tested for autocollimator calibration. The range is $\pm 1000''$ for both axes. The estimated standard uncertainty for full range is 0.0036" for the horizontal and 0.0053" for the vertical direction.

Keywords: autocollimator, interferometry, metrology, angle measurement

(Some figures may appear in colour only in the online journal)
with interferometric position measurement at one or both ends of the fixed length to provide traceable and precise displacement data. Only a few publications deal with small angle generators using an interferometric readout [10, 16]. Typical configurations include two corner cubes at the ends of a fixed length bar, measured by either two interferometers [19] or a differential interferometer [16].

Electronic autocollimators are used to measure the angular deviation of a mirror [1, 2]. These can be used for measurement of straightness, flatness, angular position and e.g. precise mapping of surface shapes [1–5, 14]. Typically measured angles are less than 1°, and accuracy specification is usually around 1″ to 0.1″ [17] for models with 1000″ class range. The front lens of an autocollimator projects a collimated beam from a point source onto the surface being measured. The beam is reflected back at an angle created by the surface tilt. The angle is converted by the front lens into a spatial deviation, which is detected and measured by the sensor (figure 1). The detection of reflected light was traditionally done through an eyepiece, but today it is usually electronic, allowing better precision and repeatability. Electronic autocollimators have sensors such as two orthogonal line CCDs or a 2D CCD cell that allows measuring position of back-reflected light over two axes several times a second. Here we refer to the two autocollimator measurement axes as follows: the x-axis is defined as the horizontal tilt or rotation around a vertical axis, and the y-axis is defined as the vertical tilt or rotation around a horizontal axis perpendicular to the viewing direction.

The autocollimator has several characteristic error sources, such as the light source or sensor being misaligned or out of focus, tilt or non-ideality of the lens, and non-orthogonality of the x- and y-axes of the CCD sensor (figure 1). Focusing and tilting errors can lead to the light beam being non-optimally focused on the sensor, causing sub-optimal behaviour of the device. A typical example is the curve in figure 1. Non-orthogonality or rotation of the sensor can lead to crosstalk between the x- and y-axes. There are also two error sources that are important at smaller scale. At small angles, multiple reflections cause a periodic error with a period of 10°–100° [13]. Also an error with smaller amplitude and period can be present due to finite pixel size of the CCD and interpolation of measured light intensity to results at subpixel resolution [13]. However, visibility of the latter effect depends on the autocollimator (CCD pixel size, focal length, light spot shape, interpolation method and repeatability). Recent findings of PTB and CMI researchers during the EMRP project Angles indicate that if better than 0.03° accuracy from the autocollimator is desired, possible sensitivity of the autocollimator to the refractive index of air needs to be studied and taken into account [18]. Under laboratory conditions at ~20 °C, the main parameter affecting the refractive index of air is pressure.

Additional autocollimator errors can be caused by flatness deviation, poor reflectivity or small size of the sample. Typically, a non-flat or poorly reflecting sample only causes a worse SNR, but it can also create a systematic error especially if the mirror is attached off centre to the rotation axis [19]. When measuring through an aperture, the measurement geometry usually creates small systematic deviations to the autocollimator measurement function, which should be calibrated out using exactly the same measurement geometry. Also, if measuring the angular deviation in one direction, a deviation in the orthogonal direction may cause a pyramid error which exaggerates the tilt at large angles [19].

Traditionally, autocollimator calibrations have been done with devices which allow tilt around one axis [1, 2, 6]. Small angle generators are well suited to autocollimator calibration, as they can provide accurate angular deviations at the correct angle scale [8, 9, 11, 15]. However, 2D calibration can reveal errors related to simultaneous engagement of both measuring axes. To our knowledge, there is only one traceable 2D angle generator [14], and no publications exist on traceable 2D autocollimator calibration devices based on interferometry. There are commercial measurement devices that measure 2 orthogonal angles using laser interferometers [20]. However, these are not designed to be traceable angle generators and thus have not been thoroughly characterized to link the measurement results to the definition of radian. The short distance between the interferometers also increases the uncertainty of the measured angles.

Figure 1. Schematic view of an electronic autocollimator (A) and example of observed amplification error (dX) as function of measured angle (X) (B).
The measurement of two angles simultaneously gives more information on the calibrated autocollimator. With two orthogonal axes, both axes of the autocollimator can be tested in one measurement that is not necessary much longer than a 1D scan if adjustment of the setup is taken into account. Orthogonality of the two axes of the autocollimator is easily seen in the results, and errors linked to engagement of both measuring axes can be characterized.

Addition of a second rotation axis complicates the design of the calibration setup significantly. 1D calibration devices typically move only around the vertical axis [10, 16]. For a 2D setup, the additional rotation axis makes the system more complicated [9]. Since there is a need for rotation around the horizontal axis, the setup cannot be made symmetrical and gravity causes bending moments. The bending effects are often dependent on the y-angle, and may thus increase the measurement uncertainty. The second axis also makes the setup potentially more flexible and can make vibrations more difficult to handle. Preferably, the crosstalk of the setup should be orders of magnitude smaller than that of the characterized autocollimator, so that orthogonality of the autocollimator can be easily defined with one measurement.

**Methods**

**Autocollimator calibration setup**

The VTT MIKES interferometric 2D small angle generator (I2D-SAG) setup is based on three heterodyne laser interferometers which allow measurement of tilt around both the vertical and horizontal axes of a position-controlled reference mirror (see figures 2 and 3). The laser head and interference counter cards used are Zygo ZMI 7702 and Zygo ZMI 1000, respectively. Laser interferometer beam paths are separated with beam splitters and adjusted parallel using adjustable beam benders. The dead paths are set as short as possible and as similar in length as possible, around 30 mm. The linear interferometer optics are Agilent 10702A and the
corner cubes of Keysight 10767A are used as reflectors. The interference signals are fed to the interference counters using fibre collimators and multimode fibres. The whole laser beam is shielded by a metal housing to minimize the effects of air movements and variations in beam position and direction due to the refractive index of air gradients and fluctuations along the beam path. The reference mirror and corner cubes of the interferometers are mounted on an aluminium reference plate. Typically, a high quality 50 mm round mirror with surface flatness of 10 nm peak-to-peak is used as the reference mirror. The position of one of the corner cubes can be fine-tuned to adjust the orthogonality.

The setup is connected to an optical table with a Kugler 118 mm diameter air bearing spindle. This provides the vertical rotation axis and holds the U-shaped aluminium support frame. The support frame is connected to a secondary plate with two air bearings providing rotation around the horizontal axis. The left bearing of the horizontal axis is a spherical air bearing (Nelson Air corp.), and the right bearing is a radial air bearing (New Wave) (figure 2). The horizontal rotation axis is set orthogonal to the vertical axis by adjusting the radial bearings support. The reference plate is kinematically mounted on a secondary plate so that the rotation axis of the air bearings is on the same plane as the reference plate. This construction was chosen so that minimum bending moments would be exerted on the reference plate. Also, possible hysteresis related to bearings or the drivetrain should not cause changes to the relative positions of the corner cubes and reference mirror, as they are connected to the same reference plate (figures 2 and 3). The reference plate was set to a position with closest parallelism of the interferometric measurement axis and rotation axes. Heat sources such as the laser head and computers are on a different side of the reference plate at a considerable distance from the calibrated autocollimator to eliminate errors due to heat. Also, thermal radiation shields are used around the I2D-SAG to improve stability. The whole setup is on a vibration-isolated optical table in an air-conditioned room with temperature fluctuations less than ±0.05 °C within a day and 0.1 °C long-term stability [21].

Rotation of the secondary plate is driven by two PI M-405 DG linear positioning stages with PI C-844 controller. Measurement software written in Python drives the controller using feedback from the laser interferometers. The positioning tolerance can be adjusted to allow faster positioning if positioning precision is not crucial. The resolution of the position stages limits the position accuracy to 0.01 ″. The first stage controls the rotation of the U-shaped aluminium support frame attached to the bottom bearing rotating the support and attached secondary plate around the vertical axis. The second stage rotates the secondary plate with respect to the U-shaped support around the horizontal bearing (figure 2).

The material temperature of the reference plate along both the horizontal and vertical measurement arms is monitored using two Pt100 sensors and a Keithley 2001 4-wire voltage meter with internal scanner. Variations caused to the distance between reflectors by thermal expansion of aluminium are compensated by the measurement program. The high thermal conductivity makes aluminium suitable for our application, as temperature gradients would cause bending of the plate which would be more difficult to compensate for than simple uniform thermal expansion. The reference plate is covered by an insulating 2 mm PET-aluminium sandwich plating to make temperature changes slower and temperature gradients inside the plate material smaller.

To compensate for the refractive index of air along the beam paths of the interferometers, the air temperature is monitored using one Pt100 sensor at each measurement path, once every 25 s. Air pressure and humidity are measured at a position close to the setup by a calibrated Vaisala PTU200. The modified Edlen equation by Bönsch and Potulski [22] is used to calculate refractive index of air.

The laser wavelength is periodically calibrated by a 633 nm iodine-stabilised laser traceable to the definition of the metre [7, 23]. The stability of the laser wavelength is better than 0.1 ppm within the calibration interval. The parallelism of the laser interferometers was defined to be better than 3 × 10⁻⁴ rad by measuring beam deviations over a distance of 3.5 m. Off-parallelism at this scale would cause a cosine error smaller than 4.5 × 10⁻⁸ × d in the measured distance (d), and a similar proportional error in the created angles.

The distances between corner cubes were measured using a Mitutoyo Quickvision Hyper Vision measurement machine at a standard uncertainty of 1 µm to confirm orthogonality. The measurements were repeated four times with 90° turns in between. The traceability of measurements was ensured using an interferometrically calibrated [24] Zerodur line scale of length 280 mm as substitution reference. The position of the apex point of the corner cubes was determined by measuring the 3D position and location of each of three edges of the corner cubes at different heights and extrapolating their intersection. The distances between corner cubes were 339 650.3 µm and 340 317.5 µm for the horizontal and vertical measurement arm, respectively. The difference from a 90° angle between corner cubes is (+9.2 ± 3.7) × 10⁻⁶ radians ((+1.8 ± 0.8)″) based on this characterization. This value is used to correct setup non-orthogonality from the results.

**Operation**

Prior to measurement, the reference mirror is adjusted parallel to the reference plate. This is done by measuring the angle of an auxiliary mirror attached straight to the reference plate with the autocollimator in two 180° rotated orientations of the auxiliary mirror. The average of the two measurements gives the angle of the reference plate. Using the autocollimator, the angle of the reference mirror is adjusted similarly to the average angle of the auxiliary mirror. Then the I2D-SAG is set to (0, 0) so that the reference plate is vertical and the beam paths of the laser interferometers have equal length. Horizontal alignment is done using a gauge block and vertical alignment using a level. These two steps aim to minimize the pyramid error in the measurements.

The axis of an autocollimator under calibration is adjusted to be orthogonal to the I2D-SAG reference mirror. The position of the autocollimator is set to the middle of the reference
mirror using a laser pointer attached to the autocollimator, and the x-axis of the autocollimator is set orthogonally to the y-axis of the I2D-SAG. Calibrations can be done at all distances up to 1 m with different apertures in front of the reference mirror.

The interferometers are read at the reference position 0, then the actuators are used to move the position to a set angular value \( s \). Environmental sensors are read continuously and the refractive index of air and thermal corrections of the reflector distances are calculated. The angular position readings are calculated using the following formulae:

\[
\theta_x = a \sin \left( \frac{d_h - d_c}{L_h} \right) = a \sin \left( \frac{\left( \frac{F_x - F_h}{n_x} - \frac{F_x - F_h}{n_h} \right)}{L_h(1 + (t_h - 20 \, ^\circ\mathrm{C})\alpha_L)} \right),
\]

\[
\theta_y = a \sin \left( \frac{d_v - d_c}{L_v} \right) = a \sin \left( \frac{\left( \frac{F_v - F_h}{n_v} - \frac{F_v - F_h}{n_h} \right)}{L_v(1 + (t_v - 20 \, ^\circ\mathrm{C})\alpha_L)} \right) + \delta C_{x-y},
\]

where \( \theta_x \) and \( \theta_y \) are the generated horizontal (x) and vertical (y) angles; \( d_h \), \( d_v \), and \( d_c \) are measured displacements in the horizontal, vertical, and common interferometers; \( L_h \) and \( L_v \) are the distance between the horizontal and vertical corner cubes, respectively; \( t_h \) and \( t_v \) are the same distances at 20 \(^\circ\mathrm{C} \), \( \alpha_L \) is the thermal expansion coefficient of the reference plate material; \( F_c \) is the common interferometer counter reading at the reference position, and \( F_x \) is the reading at set angular positions and respectively for horizontal and vertical interferometer readings, \( F_h \) and \( F_v \); \( \lambda \) is the vacuum wavelength; and \( \delta C_{x-y} \) is the correction due to crosstalk.

The angle readings are stored by the measurement software for each 0.7 s interval independently, and can be studied later in further data analysis. Environmental conditions are also stored, which allows any offline corrections to be made later, or e.g. measurements to be discarded or re-done if the environmental conditions change too much.

**Results**

**Characterisation of the setup**

Noise in the interferometer readout was evaluated by collecting lengthy datasets. This information was used to select the averaging times. Both autocollimator and interferometer values were recorded to see what part of the noise and drift is correlated with vibration of whole reference plate, and which is not. Based on these measurements a 0.7 s averaging time was selected.

For measurements with the selected 0.7 s averaging time, the noise level for x- and y-angles (x, y) was (0.0022°, 0.0064°) for the interferometer, (0.0053°, 0.0080°) for the autocollimator and (0.0047°, 0.0059°) for the difference between them defined as the standard deviation for sets of five repeats. This shows that there is some mechanical noise especially in the y-angle, but the changes in reference plate angle can be tracked by the interferometer and tested autocollimator, since the difference in the y-angle between them is less than the noise in each instrument. The larger noise in the y-angle was expected, because the y-interferometer is further from the base bearing supporting the reference plate via the support frame than the common interferometer or the x-interferometer. Periodic nonlinearity of interferometers was defined by moving the reference plate linearly using a piezo actuator driven by a signal generator and reading the interferometer data. The nonlinearity of each interferometer was defined to be 2.5 nm peak-to-peak or less.

The orthogonality of the I2D-SAG was tested by measuring scans along the x- and y-axes four times and rotating the autocollimator 90° between each set. This test was done without using the non-orthogonality correction calculated from corner cube position measurements. This allowed separating the non-orthogonality of the setup and the autocollimator. The deviation from 90° of the I2D-SAG was determined to be (+7.0 ± 2.6) \times 10^{-4} \, \text{radians} based on this test. This result is within the uncertainty limits of the results of the corner cube position measurements described in the methods section. This test was also repeated without interferometric feedback on the ‘non-changing’ axis to evaluate the non-orthogonality of the mechanical x- and y-rotation axes. The non-orthogonality of the rotation axis was determined to be (−4.0 ± 0.9) \times 10^{-4} \, \text{rad}. Accurate orthogonality of the rotation axes is not as important for accurate measurement results, but it does make precise angle positioning faster.

**Results of autocollimator calibration**

The I2D-SAG was used to calibrate an electronic autocollimator (Möller-Wedel Elcomat 3000) for the range ±900 x and y with 60° steps, and range ±45° x and y with 3° steps. Calibration of the x-axis of the autocollimator was done for the range ±1000 with 5° steps. The tests were performed with an autocollimator-to-mirror distance of 225 mm and full aperture. For each calibration, each position was measured with 5 \times 0.7 s averaging, and the whole calibration was measured six times with alternating direction between adjacent measurement sets to reduce the effect of drift.

The setup could measure both the tilt between the axes and smaller scale features in the measurement function of the autocollimator. The most easily visible feature in 2D calibration is the tilt of the axis (figure 4(a)). When measuring with short steps between adjacent angles, also a periodic error due to multiple reflections (−35° period for Elcomat 3000) is clearly visible (figure 4(b)). The measured non-orthogonality of the autocollimator x- and y-axes was (8.81 ± 0.06) \times 10^{-4} \, \text{rad}. The largest measured deviation on the x-scale was (±0.116 ± 0.006)° at the nominal angle of (900°, 900°) and on the y-scale (−0.839 ± 0.007)° at the nominal angle of (−900°, 900°), respectively. Periodic nonlinearity in the middle of the x-scale had a period of (34.07 ± 0.07)° and amplitude of (0.079 ± 0.006)°. Periodic nonlinearity in the middle of the y-scale had a period of (33.5 ± 0.3)° and amplitude of (0.076 ± 0.006)°.

In 1D calibration tests (figure 5), periodic nonlinearity was seen over an area from −150° to +150°, and the distinctive
S-curve amplification error of the autocollimator can be seen. Figure 5 also shows the repeatability of the setup, as two repeats of the same data were measured. The root mean square difference between the two measurement sets was $3.64 \times 10^{-3}^\circ$, and the maximum positive and negative deviations between the sets were between $(9.0 \pm 9.1) \times 10^{-3}^\circ$ and $(-15.9 \pm 8.5) \times 10^{-3}^\circ$.

**Uncertainty analysis**

The model for the measured angle difference ($\Delta R_x, \Delta R_y$) of the autocollimator reading and angle generator, with corrections for uncertainty calculation (equations (3) and (4)) was created according to [25]:

$$\Delta R_x = \frac{1}{k} \sum_{j=1}^{k} ((\alpha_{AC,x} + \delta\alpha_{AC,P}) - (\theta_x + \delta h_{parallel} + \delta B_{y \rightarrow x}))$$  \hspace{1cm} (3)

$$\Delta R_y = \frac{1}{k} \sum_{j=1}^{k} ((\alpha_{AC,y} + \delta\alpha_{AC,P}) - (\theta_y + \delta h_{parallel} + \delta B_{x \rightarrow y}))$$  \hspace{1cm} (4)

Where $\alpha_{AC}$ is the angle measured by the autocollimator, $\delta\alpha_{AC,P}$ is the correction for pressure sensitivity of the autocollimator,
Table 1. Uncertainty components of the measurement for calibration with a set of six repeats. Here error sources marked as type A are determined statistically and type B based on scientific judgment using all the relevant information available. Distribution N is normal distribution, R is rectangular/uniform distribution.

| Description of input quantity \( x_i \) | Symbol \( x_i \) | Type \( A \) or \( B \) | Distribution | Std. meas. uncertainty \( u(\alpha_i) \) of input quantity | Sensitivity coefficient \( \alpha = \frac{\partial\theta}{\partial x_i} \) | Std. meas. uncertainty contribution \( u_\theta(\delta) \) |
|-----------------------------------------|----------------|----------------|----------------|---------------------------------|---------------------------------|---------------------------------|
| Autocollimator reading, noise, \( k = 6 \) | \( \alpha_{AC} \) | A | N | 0.00083° | 1 | 0.00083° |
| Correction for pressure sensitivity of autocollimator | \( \delta_{OAC,P} \) | A | N | 1 hPa | 7.1 \times 10^{-7}/\text{hPa} \times \alpha | 7.1 \times 10^{-7} \times \alpha |
| Repeatability after re-adjustments of autocollimator | \( R \) | A | N | 0.00538° | 1 | 0.00538° |
| I2D-SAG reading, noise \( x \)-axis, \( k = 6 \) | \( \delta_x \) | A | N | 0.00023° | 1 | 0.00023° |
| I2D-SAG reading, noise \( y \)-axis, \( k = 6 \) | \( \delta_y \) | A | N | 0.00069° | 1 | 0.00069° |
| Periodic nonlinearity of interferometers | \( F_{x,0}, F_{y,0}, F_{h,0} \) | B | arcsine | 1.8 nm | 2/0.34 m | 0.0021° |
| Reflector distance | \( h_b, l_y \) | B | N | 1 \( \mu \text{m} \) | \( \alpha/0.34 \text{ m} \) | \( 2.9 \times 10^{-6} \times \alpha \) |
| Material temperature of reference plate deviation from 20 °C | \( h_b, -20 \text{ °C} \) | B | N | 0.020 °C | 22.2 \( \times 10^{-6}/\text{°C} \times \alpha \) | 4.4 \( \times 10^{-7} \times \alpha \) |
| Thermal expansion coefficient | \( \alpha_t \) | B | R | 1.2 \( \times 10^{-6}/\text{°C} \) | 0.1 °C \( \times \alpha \) | 1.2 \( \times 10^{-7} \times \alpha \) |
| Refractive index of air includes \( t_a, p, h \) | \( n \) | B | N | 1.0 \( \times 10^{-7} \times \alpha \) | \( 1.0 \times 10^{-7} \times \alpha \) |
| Laser vacuum wavelength | \( \lambda \) | B | N | 1.0 \( \times 10^{-7} \times \alpha \) | \( 1.0 \times 10^{-7} \times \alpha \) |
| Correction due to laser beam non-parallelism | \( \delta_{s0,\text{parall}} \) | B | R | 4.5 \( \times 10^{-8} \times \alpha \) | \( 4.5 \times 10^{-8} \times \alpha \) |
| Correction due to crosstalk to \( y \) rotation | \( \delta_{s0,\text{x0}} \) | B | N | \( 3.7 \times 10^{-6} \times \alpha_x \) | \( 3.7 \times 10^{-6} \times \alpha_x \) |
| Correction due to bending to \( x \)-axis due to \( y \) rotation | \( \delta_{s0,\text{y0}} \) | B | R | \( 4.0 \times 10^{-7} \times \alpha_y \) | \( 4.0 \times 10^{-7} \times \alpha_y \) |
| Correction due to bending to \( y \)-axis due to \( y \) rotation | \( \delta_{s0,\text{y0}} \) | B | R | \( 4.1 \times 10^{-7} \times \alpha_y \) | \( 4.1 \times 10^{-7} \times \alpha_y \) |

\( \theta \) is the angle measured by the I2D-SAG device (see equations (1) and (2)), \( k \) is the number of measurements; \( \delta_{s0,\text{parall}} \) is the correction for non-parallelism of the laser beams, and \( \delta_{s0,\text{x0}} \) and \( \delta_{s0,\text{y0}} \) are corrections for deformation with angular movement in the vertical axis. The input quantities of the calibration and their uncertainty contributions are listed in Table 1 and the uncertainty contributions are evaluated briefly in the following.

Autocollimator pressure sensitivity was defined to be \((7.1 \pm 0.8) \times 10^{-7}/\text{hPa}\). Typically, there may be 1 hPa class pressure changes within a measurement. Currently the calibration results are given without correcting the angle readings to normal pressure, and the ambient pressure is noted. Thus an uncertainty of \(7.1 \times 10^{-7}\) is estimated for \(\delta_{OAC,P}\). When the autocollimator is used at different pressures than where it was calibrated, the pressure sensitivity needs to be taken into account.

The repeatability after re-adjustments \( R \) includes uncertainty components related to realignment of the autocollimator to the SAG set-up. As the calibration uncertainty targeted here is orders of magnitude smaller than the specified accuracy of the autocollimator, there may be unexpected changes in this angular scale due to small changes in autocollimator alignment, distance, aperture position, attachment, tilt etc. All these contribute to the difference between two calibration results being greater than expected from other error sources.

The uncertainty contributions of the autocollimator angle \( \alpha_{AC,d} \) repeatability and I2D-SAG noise are estimated based on static noise tests and normal calibration measurements using the setup. Noise in the small angle generator readings explains the slightly larger noise in the calibration results than would be expected based on the autocollimator repeatability only, and it also explains the slightly larger noise for \( y \). This term contains uncertainty due to repeatability of the interferometer, high frequency changes of refractive indices, setup vibration, and possible other high frequency error sources. The uncertainty contribution of autocollimator noise to the 6 \( \times \) repeated calibration result is \(0.00038\), whereas contributions of SAG repeatability are \(0.00023\) for \( x \) and \(0.00069\) for \( y \). The periodic nonlinearity of interferometers \( F_{x,0}, F_{y,0}, F_{h,0}, F_{c,0} \) is used at different pressures than where it was calibrated, the sensitivity contribution \( F_{c,0} \) was defined to cause an uncertainty of \(0.0021°\).

The temperature of the reference plate \( h_b \) is monitored at two positions close to midpoints of the \( x(h) \) and \( y(v) \) reference arms every 25 s. The temperature on this time scale or slower is compensated for. The largest temperature difference measured between two positions in the aluminium is \(0.0030 \text{ °C}\), which makes temperature-induced bending negligible. Uncertainty
of the measured temperature inside the reference plate is estimated to be 0.020 °C, which causes an uncertainty of $0.44 \times 10^{-5}$ to the measured angle ($\theta$). The thermal expansion coefficient $\alpha_t$ is known with an uncertainty of $1.2 \times 10^{-6}$, which together with the estimated maximum temperature deviation of 0.1 °C from 20.0 °C has an uncertainty contribution of $0.12 \times 10^{-6}$. The standard uncertainty of determined length of the horizontal and vertical arms at 20 °C ($L_h$, $L_v$) is 1 μm, which causes a relative uncertainty of $2.9 \times 10^{-8}$ to the measured angles.

The air temperature, pressure and humidity sensors are calibrated regularly. The used Edlen equation has an uncertainty of $1 \times 10^{-8}$ under laboratory conditions. In total, the uncertainty in the used refractive index of air, $n$, is estimated to be $0.10 \times 10^{-6}$, which causes an uncertainty of $0.10 \times 10^{-6} \times \theta$ to the created angles. The stability of the laser vacuum wavelength ($\lambda$) is $1 \times 10^{-7}$, and uncertainty of the parallelism of the laser beams ($\delta_h_{\text{parall}}$) causes an uncertainty of $4.5 \times 10^{-8}$, both proportional to the created angle.

Bending of the reference plate of the interferometer could cause the reference mirror to be misaligned from the plane defined by the corner cubes. This could cause systematic errors in the measured angles of the reference mirror. The bending could be caused by moments, due to e.g. the weight of the reference plate or uneven temperature inside the plate. Thermal bending of the reference plate of the interferometer could cause the reference mirror to be misaligned from the plane defined by the corner cubes. This could cause systematic errors in the created angles. Both are proportional to the created angle.

Gravity effects are minimized by using an aluminium reference plate that is both stiff and relatively light. In FEM simulations, small bending was seen when the reference plate was tilted in the ‘Y’ direction. This caused less than a $2.5 \times 10^{-8}$ radian ($0.005\,16\,$′) deformation over the y-axis ($B_{y\rightarrow x}$), and less than a $2.4 \times 10^{-8}$ radian ($0.004\,95\,$′) deformation over the x-axis ($B_{x\rightarrow y}$) due to a y-angle tilt of 1°. Shortening of $x$ and $y$ prism distances due to bending was negligible. These contribute to a small standard uncertainty of $4.0 \times 10^{-7} \times \alpha_y$, on the $x$ axis and $4.1 \times 10^{-7} \times \alpha_x$ on the $y$ axis. Thermal bending effects are small thanks to the consistent temperature, high thermal conductivity of aluminium, and thermal shielding on the front and back surfaces of the plate.

The cross-talk due to non-orthogonal positioning of the corner cubes is corrected with an uncertainty of $3.7 \times 10^{-6}$ based on the corner cube position measurements.

The standard uncertainty of the I2D-SAG was determined to be $\sqrt{(0.002\,12\,\alpha_y)^2 + (2.9 \times 10^{-6} \alpha_x)^2 + (0.4 \times 10^{-6} \alpha_x)^2}$ for the x-angle and $\sqrt{(0.002\,22\,\alpha_y)^2 + (3.0 \times 10^{-6} \alpha_x)^2 + (3.7 \times 10^{-6} \alpha_x)^2}$ for the y-angle. This leads to a maximum x-axis uncertainty of 0.0036° and maximum y-uncertainty of 0.0053° within measuring ranges of ±1000′ for both axes. The total standard uncertainty of the 2D calibration $u_{\text{cal}}$ was determined to be $\sqrt{(0.005\,84\,\alpha_y)^2 + (3.0 \times 10^{-6} \alpha_x)^2 + (0.4 \times 10^{-6} \alpha_x)^2}$ for the x-angle and $\sqrt{(0.005\,88\,\alpha_y)^2 + (3.1 \times 10^{-6} \alpha_x)^2 + (3.7 \times 10^{-6} \alpha_x)^2}$ for the y-angle. This leads to a maximum x-axis uncertainty of 0.0066° and maximum y-uncertainty of 0.0076° within measuring ranges of ±1000′ for both axes. The most significant uncertainty sources are repeatability after re-adjustments of the autocollimator, interferometer periodic nonlinearity, uncertainty of distance between the reflectors, and cross-talk.

Conclusions

A two-directional interferometric small angle generator was implemented for autocollimator calibration. The generator was characterized to allow calibration of autocollimators in the range ±1000′. The estimated standard uncertainty for a generated 2D angle is (0.0021′, 0.0022′) for the smallest angles and (0.0036′, 0.0053′) for the largest angles within the range. During calibration of a high-quality autocollimator, the achieved standard uncertainty varied from (0.0058′, 0.0059′) to (0.0066′, 0.0076′).

The achieved uncertainty allows for clear characterization of the characteristic nonlinearities of the autocollimator: a large scale S-curve and periodic nonlinearity due to multiple reflections (30′ range). The typical autocollimator axe non-orthogonality can be characterized with just one positioning and measurement run.

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