We study the transconductance for two coupled one-dimensional wires or edge states described by Luttinger liquid models. The wires are assumed to interact over a finite segment. We find for the interaction parameter $g = 1/2$ that the drag rate is finite at zero temperature, which cannot occur in a Fermi-liquid system. The zero temperature drag is, however, cut off at low temperature due to the finite length of the wires. We also consider edge states in the fractional quantum Hall regime, and we suggest that the low temperature enhancement of the drag effect might be seen in the fractional quantum Hall regime.

One-dimensional (1D) systems have attracted much attention since the advances in lithographical fabrication techniques have made it possible to study systems such as e.g. quantum point contacts, quantum wires, quantum dots, and nano-tubes. Interacting 1D systems are particularly interesting, because they are believed to be Luttinger liquids (LLs) and thus exhibit non-Fermi liquid behavior. However, it is still not clear to what extent the non-Fermi liquid behavior can be seen in a transport experiment for clean systems, i.e. without impurity scattering. Several authors [1] have shown that the interactions do not influence the conductance, and the reason for a possible interaction induced effect observed [2] at finite temperature remains unexplained.

Another very interesting testing ground for LL behavior is edge states in the fractional quantum Hall (FQH) regime, where edge states can be described as chiral LLs [3]. Surprisingly, experiments have shown that the tunneling between two edge states follows the LL behavior [4] both for compressible and incompressible states. Theoretical descriptions in terms of 1D edge channels have been developed [5], and several authors [6,7] have recently extended this idea and calculated tunneling density of states which offers an explanation of the observed characteristics.

In this paper, another experiment which measures the interaction effects is suggested, namely a Coulomb drag experiment. Coulomb drag has during recent years proved to be a powerful tool for studying interaction and screening properties of coupled two-dimensional electron systems [8], including phonon-mediated interactions [9], coupled QH systems both in the integer regime [10] and in the fractional regime where recent experiments show Coulomb drag that saturates at the lowest temperatures at filling factor close to one half [11]. This type of behavior cannot be explained within the present weak coupling theories for bulk composite-Fermion drag [12].

1D drag between two Fermi liquids has been studied in the linear and the non-linear regimes in Ref. [13]. In Ref. [14] the non-linear transresistance for two LLs at zero temperature was studied and it was argued that absolute drag (equal currents in the two wires) is possible. Here we show a related effect for the linear conductance. We concentrate on the temperature dependence of the linear response. Utilizing a mapping to two decoupled LLs, it is shown that a LL description shows zero temperature drag for the case where the coupling parameter $g = 1/2$ [15]. Moreover, we find an interesting dependence of the length of the interaction region and a regime for $g < 1/2$, where the drag current is almost equal to the drive current. Our results are applied to edge states in coupled FQH systems with a narrow constriction, which is different from the situation of the Ref. [11].

In our model the two spin-less LLs are coupled by Coulomb interactions, and interwire tunneling is neglected. They are coupled in a finite region of length $L_I$. The wires have lengths $L_W$ and it is assumed that the intrawire interaction is constant in this region, see Fig. 1. Thus the results are valid for temperatures or voltages larger than the cut-off given by the energy of charge excitations with wavelengths of order $L_W$, i.e., $T > T_W = \hbar v_F / L_W$, where $v_F$ is a Fermi velocity.

The Hamiltonians for the separate systems are those of two LLs ($i = 1, 2$) (we use $\hbar = k_B = 1$)
\[
H_{0i} = \frac{v_i}{2} \int dx \left( |P_i(x)|^2 + \frac{1}{g_i^2} \partial_x \phi_i(x) |^2 \right),
\]  
(1)

where \(g_i, v_i\) are the interaction parameters and Fermi velocities, respectively, and where the densities, \(\rho_L\) and \(\rho_R\), of left and right movers respectively, enter as \(\partial_x \phi_i(x) = [\rho_{Li}(x) + \rho_{Ri}(x)]\sqrt{\pi}\) and \(P(x) = -[\rho_{Ri}(x) - \rho_{Li}(x)]\sqrt{\pi}\). The fields \(\phi_i\) and \(P_i\) form conjugate variables: \(\phi_i(x), P_i(x') = i\delta_{ij} \delta(x - x')\).

The Coulomb interaction between the two wires is given by
\[
H_{\text{int}} = \int dxdy U_{12}(x, y) \rho_1(x) \rho_2(y). \tag{2}
\]

Note that the interwire interaction \(U_{12}(x, y)\) only acts within a region of length \(L_I\). Here the subsystem densities are given by
\[
\rho_i(x) = \rho_{Li}(x) + \rho_{Ri}(x) + \Psi_{Li}^\dagger(x) \Psi_{Li}(x) + \Psi_{Ri}^\dagger(x) \Psi_{Ri}(x), \tag{3}
\]

where \(\Psi_{Li(R)}\) are Fermion operators corresponding to the left(right) movers. In the bosonization language these are given by \(\Psi_{L(R)}(x) = \exp( \pm ik_F x + 2 \pi \int_{-\infty}^x dy \rho_{L(R)}(y) ) / \sqrt{2\pi \alpha}\) \(\alpha\) is a cut-off parameter \(\left[17\right]\).

We separate the interwire interaction in 4 terms describing the different possibilities of forward and/or backward scattering. Note that since the interaction region is finite, momentum need not be conserved in the scattering process. We have
\[
H_{\text{int}} = \int dxdy U_{12}(x, y) \times
[B^a(x, y) + B^b(x, y) + B^c(x, y) + B^d(x, y)], \tag{4}
\]

where
\[
B^a(x, y) = A_1(x) A_1^\dagger(y) + \text{h.c.}, \tag{5a}
\]
\[
B^b(x, y) = A_1(x) A_2^\dagger(y) + \text{h.c.}, \tag{5b}
\]
\[
B^c(x, y) = \frac{\partial_x \phi_1(x) \partial_y \phi_2(y)}{\pi}, \tag{5c}
\]
\[
B^d(x, y) = \phi_1(x) \left[ A_2(y) + A_1^\dagger(y) \right] / \sqrt{\pi} + 1 \leftrightarrow 2. \tag{5d}
\]

Here \(A(x)\) is a "backscattering operator" defined as \(A(x) = \exp(2ik_F x + i2\sqrt{\pi} \phi(x) )/(2\pi \alpha)\).

The two terms \(B^b\) and \(B^d\) correspond to non-momentum-conserving scatterings processes. The terms \(B^c\) and \(B^d\) do not provide a mechanism for Coulomb drag (to any order in perturbation theory) and, furthermore, since the renormalization due to these terms is not important at low energies, they are omitted. We will make one further approximation which is valid when the relevant energy scale is smaller than \(v_F/L_I\). In this limit, the spatial dependence of \(\phi\)'s in the backscattering operator can be neglected. The final expression for the interaction now reads
\[
H_{\text{int}} = \frac{1}{2\pi^2 \alpha^2} \int dxdy U_{12}(x, y) \times \]
\[
\left\{ \cos[2(k_{F1}x - k_{F2}y) + 2\sqrt{\pi} (\phi_1(0) - \phi_2(0))] + \cos[2(k_{F1}x + k_{F2}y) + 2\sqrt{\pi} (\phi_1(0) + \phi_2(0))] \right\}. \tag{6}
\]

Now we transform the Hamiltonian to that of two (interacting) LLs scattering against a single impurity potential. Define the new field operators
\[
\Phi = \phi_1 + \phi_2, \quad P = (P_1 + P_2)/2, \tag{7a}
\]
\[
\Theta = \phi_1 - \phi_2, \quad \Pi = (P_1 - P_2)/2, \tag{7b}
\]

defined such that \(\Phi, \Theta, \Pi\) are conjugate pairs. The interwire interaction term becomes particularly simple in this basis and the Hamiltonian transforms to
\[
H = H_0 + H' + H_{\text{int}}, \tag{8}
\]

where
\[
H_0 = \frac{\bar{v}}{2} \int dx \left( |P(x)|^2 + \frac{1}{g^2} [\partial_x \Phi(x)]^2 \right)
+ \left( |\Pi(x)|^2 + \frac{1}{g^2} [\partial_x \Theta(x)]^2 \right), \tag{9}
\]

and where \(H'\) describes the interaction between the new field operators
\[
H' = \bar{v} \int dx \left[ aP(x) \Pi(x) + \frac{b}{g^2} \partial_x \Phi(x) \partial_x \Theta(x) \right], \tag{10}
\]

where \(\bar{v} = (v_1 + v_2), 1/g^2 = (v_1/g_1^2 + v_2/g_2^2)/4\bar{v}, a = (v_1 - v_2)/\bar{v}, \) and \(b = (v_1/g_1^2 - v_2/g_2^2)/(v_1/g_1^2 + v_2/g_2^2).\)

The current operator for the LL model is given by \(j_i = v_F P_i / \sqrt{\pi}\) and through the continuity equation, the current is expressed as \(j_i(x) = -\partial_t \phi_i(x)/\sqrt{\pi}\). Using the Kubo formula, we obtain the transconductance in terms of the new fields defined in Eq. \(\left[13\right]\) as
\[
G_{21}(\omega) = \frac{i\omega e^2}{4\pi} [D_\Phi^\dagger(x, x'; \omega) - D_\Phi(x, x'; \omega)], \tag{11}
\]

where the Green’s functions
\[
D_\Phi(t - t') = -i\Theta(t - t') \langle [\Phi(x, t'), \Phi(x', t)] \rangle, \tag{12a}
\]
\[
D_\Theta(t - t') = -i\Theta(t - t') \langle [\Theta(x, t), \Theta(x', t')] \rangle, \tag{12b}
\]

have been defined (note that \(D(x, x'; \omega)\) is independent of \(x, x'\) in the dc-limit).

For identical wires the part of the Hamiltonian which couples the \(\Phi\) and \(\Theta\) sector in Eq. \(\left[14\right]\) is equal to zero. The remaining Hamiltonian is equivalent to two LLs models scattering against single impurities, but with new interaction parameters. We can therefore use the results from this well-studied problem \(\left[18\right]\). The transconductance simplifies to
\[ G_{21} = \frac{1}{4} (G_{\text{Latt}}(V_1, 2g) - G_{\text{Latt}}(V_2, 2g)), \]  

where \( g \equiv g_1 = g_2 \) and where \( G_{\text{Latt}}(V, 2g) \) is the conductance of a LL with interaction parameter, \( 2g \), scattering against a single impurity with a backscattering amplitude \( V = V(2k_F) \). (Below it is shown that Eq. (13) is valid even when the velocities are different.) Here we have defined

\[ V_{1,2} = \frac{D}{2\pi v_F} \int dx \exp[2ik_F(x \pm y)], \]

where the small momentum cut-off is parametrized in terms of a high energy cut-off: \( D = v_F/\alpha \).

Now several conclusions follow. Firstly, it is seen that for a short interaction region \( L_1 k_F \ll 1 \), which implies \( V_1 \approx V_2 \), all momentum transfers have equal weights and hence there is no Coulomb drag [21]. More importantly, the scaling properties of the model can be read out [13,14]:

For \( g > 1/2 \), \( G_{\text{Latt}} \) goes to a constant as \( T \to 0 \), which means that \( G_{21} \) goes to zero at zero temperature. The corrections to the low temperature limit give the power law: \( G_{21} \sim T^{4g-2} \). For non-interacting wires the drag effect is thus quadratic in temperature; in contrast to the case where they interact throughout the wires, in which case the drag scales linearly with temperature [13].

For \( g < 1/2 \), both terms in Eq. (13) go to zero, because the model scales towards strong backscattering. The power laws are however the same but the prefactors will be different and we can conclude that \( G_{21} \sim T^{1/2g-2} \).

For \( g = 1/2 \) the problem maps to Fermi-liquids and we get a temperature independent \( G_{21} \) and hence only for \( g = 1/2 \) does the transconductance remain finite as temperature goes to zero.

Consider again the case of identical wires in the vicinity of \( g = 1/2 \), where the problem maps to that of two Fermi-liquids. We may use the perturbative renormalization method developed in Ref. [22] rather than the exact solution based on the Bethe ansatz [19], and we obtain for the transconductance [21]

\[ G_{21} = \frac{e^2}{4\pi} \frac{(|W_2|^2 - |W_1|^2)^\gamma}{[1 + |W_1|^2t^2][1 + |W_2|^2t^2]}, \]

where \( t = (T/D) \), \( W_i = V_i/v_F \), and \( \gamma = 4g - 2 \).

In Fig. 2, we show \( G_{21} \) expressed in Eq. (15) for different parameters. The interwire interaction \( U_{12} \) is calculated for typical parameters for GaAs quantum wires and the distance between the wires is chosen to be \( 2/k_F \). In accordance with the arguments given above, the transconductance peaks when \( g = 1/2 \) at low temperatures and the peak moves to smaller \( g \) values for higher temperatures. (Furthermore, the stronger the interwire interaction the closer is the peak position to \( g = 1/2 \).) Therefore for \( g < 1/2 \), \( G_{21} \) shows non-monotonic behavior as a function of temperature.

For a long interaction region, i.e. \( g_2 \ll 1 \), \( G_{21} \) approaches the value \( e^2/2h \) for small temperatures (but still larger than \( T^* = D|W_1|^{-2/\gamma} \), see inset of Fig. 2. In this regime, the diagonal conductance [23] also tends to \( e^2/2h \) and thus the currents in two wires become the same, which is similar to the absolute drag effects found in Ref. [14]. This interesting effect occurs because the momentum conserving backscattering increases with decreasing temperature (second term of Eq. (13) goes to zero). It saturates when the two currents are the same and the net momentum exchange hence is zero.

In the general case, where the wires have different \( g \)-values and velocities, there is a coupling term in the transformed Hamiltonian, Eq. (11). Since the conductance involves only the fields at \( x = 0 \), we integrate out all the \( x \neq 0 \) fields. The resulting action reads

\[ S = S_0 + S_{\text{int}}, \]

\[ S_0 = \frac{1}{\beta} \sum_{\omega_n} \frac{|\omega_n|}{g} (\Phi^*_\omega \Theta_\omega)^* \begin{pmatrix} k_+ & k_- \end{pmatrix} \begin{pmatrix} \Phi_\omega \\ \Theta_\omega \end{pmatrix}, \]

\[ S_{\text{int}} = \int_0^\beta d\tau \left\{ V_2 e^{2i\sqrt{\pi} \Theta(\tau)} + V_1 e^{2i\sqrt{\pi} \Phi(\tau)} + \text{c.c.} \right\}, \]

where \( k_{\pm} = \frac{1}{2} [(v_2 + v_1)(g_1 \pm g_2)/|g_1^2v_2 + g_2^2v_1|]^{1/2} \).

For the special case, when the two wires have the same interaction parameter (but unequal velocities) the two sectors separate and the solution is again given by Eq. (13). For the general situation, we perform a perturbative renormalization group calculation and find

\[ \frac{dV_i}{d\ln D} = (1 - (g_1 + g_2))V_i. \]
The flow is marginal when the sum of the $g$-constants is equal to one and the zero temperature drag predicted above occurs at this line and for $g_1 + g_2 < 1$ a strong enhancement of the Coulomb drag occurs.

Finally, we consider Coulomb drag of edge excitations in the FQH regime. Recently, there has been a large activity trying to understand the low energy edge excitations and the tunneling experiments \cite{4} in FQH states \cite{5,6}. These works show that the edge excitations can be described as unidirectional bosonic edge excitations, which were then used to form an electron charge creation operator as input in an independent boson model for the tunneling density of states.

Following these works, we write the Hamiltonian that governs the dynamics for (say) the left edge branch as $H_{\text{edge}, L} = \frac{\hbar}{2}\varepsilon_D \sum_{k > 0} \rho_L(k) \rho_L(-k)$, where $\rho_L$ is the left branch 1D charge operator and $[\rho_L(-k), \rho_L(k)] = \nu k L / 2 \pi$. Here $\nu$ is the filling factor, $\varepsilon_D$ is the drift velocity at the edge and $L$ a normalization length. Similar expression is obtained for the right branch, similar to the Tomonaga-Luttinger model. The operator which creates a single charge $e$ at point $x$, moving with velocity $\varepsilon_D$, in the left channel is $\Psi_L^\dagger \sim e^{-i k_D x} \exp \left( \frac{\hbar}{2\mu} \sum_k \rho_L(k) e^{i k x} / k \right)$. Notice that this operator does not fulfill the correct anticommutation relations for fermions. However, the backscattering operator $A = \Psi_R^\dagger \Psi_L$ does obey the correct commutation relations and below it is utilized to describe interedge tunnelings in a model Hamiltonian for coupled FQH systems.

The following experiment is proposed: the coupled QH systems are narrowed by a constriction such that edge states moving in opposite directions are coupled and backscattering can occur. This system can be modeled by writing the backscattering interaction in terms of the backscattering operator, $A$. After some simple algebra, we arrive at the Hamiltonian for two coupled LLs, Eqs. (1) and (4), with $g = \nu$ \cite{2} and conclusions similar to above therefore immediately follow from Eq. (13): In particular, we predict non-zero drag at zero temperature for half-filled Landau levels and furthermore near its maximum value the drag effect increases with decreasing temperature (or voltage). For long interaction region, the transconductance goes to a universal value $ve^2 / 2h$ at low temperature for $\nu < 1/2$.

In conclusion, we have calculated Coulomb drag for LLs and found interesting behavior near $g=1/2$ such as zero temperature drag and non-monotonic temperature dependence. These predictions can be tested for edge states in FQH systems.

The author acknowledges valuable discussions with Ben Hu and Antti-Pekka Jauho.

Recently, a related paper appeared \cite{2}. These authors discuss the conductance of crossed LLs coupled at a single point and find very similar results to ours.

\[1\] I. Safi and H. J. Schulz, Phys. Rev. B 52, R17040 (1996)
\[2\] D. I. Maslov and M. Stone, Phys. Rev. B 52, R5539 (1995; V. Ponomarenko, Phys. Rev. B 52, R8666 (1995;
\[3\] A. Kawabata, Phys. Soc. Jap. 65, 30 (1996); Y. Oreg and A. Finkel’stein, Phys. Rev. B 54, R14265 (1996). I. Safi, Phys. Rev. B 55 R7331 (1997).
\[4\] S. Tarucha, T. Honda, and T. Saku, Solid State Comm., 94 (1995) 413; K. Thomas et al., Phys. Rev. Lett. 77, 135 (1996); A. Yacoby et al., Phys. Rev. Lett. 77, 4612 (1996).
\[5\] X. Wen, Int. J. Mod. Phys. B 6, 1711 (1992).
\[6\] M. Grayson et al., Phys. Rev. Lett. 80, 1062 (1998), and references therein.
\[7\] I. Aleiner and L. Glazman, Phys. Rev. Lett. 72, 2935 (1995).
\[8\] S. Conti and G. Vignale, Phys. Rev. B 54, R14309 (1996), cond-mat/9709057; J. Han and D. Thouless, Phys. Rev. B 55, R1926 (1997); J. Han, Phys. Rev. B 56, 15806 (1997); U. Zülicke and A. H. MacDonald, cond-mat/9802019.
\[9\] D. V. Khveshchenko, cond-mat/971013.
\[10\] K. Flensberg and B. Y.-K. Hu, Phys. Rev. Lett. 73, 3572 (1994); N. P. R. Hill et al., Phys. Rev. Lett. 78, 2204 (1997).
\[11\] T. J. Gramila et al., Phys. Rev. Lett. 66, 1216 (1991); Phys. Rev. B 47, 12957 (1993), Physica B 197, 442 (1994); M.C. Bonsager et al., Phys. Rev. B, in press.
\[12\] N.P.R. Hill et al., J. Phys.: Condens. Matter 5, 5009 (1993); H. Rubel et al., Phys. Rev. Lett. 78, 1763 (1997); E. Shimshoni and S.L. Sondhi, Phys. Rev. B 49, 11484 (1994); M.C. Bonsager et al., Phys. Rev. Lett. 77, 1366 (1996), Phys. Rev. B 56, 10314 (1997).
\[13\] M.P. Lilly, J.P. Eisenstein, L.N. Pfeiffer and K.W. West, Phys. Rev. Lett. 80, 1714 (1998).
\[14\] S. Sakhi, Phys. Rev. B 56, 4098 (1997); I. Ussishkin and A. Stern, Phys. Rev. B 56, 4013 (1997); Y.B. Kim and A.J. Millis, cond-mat/9711123.
\[15\] B. Y.-K. Hu and K. Flensberg, in Hot Carriers in Semiconductors, edited by K. Hess (Plenum Press, New York, 1996), p. 943.
\[16\] Y. V. Nazarov and D. V. Averin, cond-mat/9705158. These authors consider the first term of Eq. (6) valid for long wires.

Note that since our effect evolves a transport experiment the zero temperature drag considered here is different from that of a closed system in studied in A. G. Rojo and G. D. Mahan, Phys. Rev. Lett. 68, 2074 (1992).
Be aware that in absence of scattering $G_{\text{Lutt}}(2g) = 2e^2/h$, because in this case $g$ should be set to one\[1\]. This is not the case for the FQH edge states.

The diagonal conductance is $G_{11}$ given by Eq. (13) with a plus instead of the minus and $G_{11}$ of course remains finite for $V_1 = V_2$.

D. Yue, L. Glazman, and K. Matveev, Phys. Rev. B 49, 1966 (1994).

These result will change when including interedge interactions, which lead to small logarithmic corrections. See Ref. \[2\] and K. Moon and S. M. Girvin, Phys. Rev. B 54, 4448 (1996); U. Zülicke and A. H. MacDonald, \textit{ibid} 54, R8349 (1996).

A. Komnik and R. Egger, Phys. Rev. Lett. 80, 2881 (1998).