Acoustic waves in a superheated liquid with a gas nuclei

U O Agisheva¹, I I Vdovenko² and M N Galimzyanov¹, ², ³
¹Mavlyutov Institute of Mechanics, UFRC RAS
²Bashkir State University
³Ufa State Aviation Technical University

E-mail: agisheva_u@mail.ru

Abstract. The propagation of weak perturbations in the superheated air-water bubbly medium is studied when in addition to water vapor bubbles contain inert gas (air, for example) does not participating in phase transitions. The effect of the initial overheating and the initial volume content of bubbles on the evolution of harmonic waves is analyzed. It is established that in the case of perturbations from the low-frequency region, an increase in the initial volumetric gas content leads to an increase in the damping coefficient, and the phase velocity decreases significantly in this case.

1. Introduction
Research interest in the problems of the propagation of small perturbations in a liquid with bubbles is known for a long time [1–9]. Expanding of experimental equipment capabilities and the need for more detailed study of the dynamics of bubbling media served as a beginning of a new round of studies, that results are presented in [10–11]. At the beginning of this century calculations in a two-dimensional formulation [12–13] were made. In [14–16] an overview of the theoretical and experimental works that has been published in recent years on the effect of nano and micro bubbles on the properties of a bubble liquid has been made.

In this paper we consider the features of sound propagation and the development of instability in a superheated liquid containing gas nuclei depending on the magnitude of its overheating defined as

$$\Delta T_0 = T_0 - T_s(p_0).$$

It should be noted that in the presence of gas nuclei in the system, its state is determined by four parameters (the value of the parameters $a_0$, $a_0$, $p_0$, $T_0$, for example), and if there are no gas nuclei in the liquid, the equilibrium state of the vapor-gas-liquid system is determined by three parameters, for example, volume content of bubbles $a_0$, their radius $a_0$ and pressure in the liquid phase $p_0$ (the latter parameter can be taken as the temperature $T_0$).

2. Governing equations
Suppose that a fluid with the temperature $T_0$ and pressure $p_0$ contains spherical bubbles with a radii $a_0$ filled with vapour and gas insoluble in the liquid phase. Let us consider the propagation of small perturbations in the described system in a planar one-dimensional and one-velocity approximation. Assuming that the liquid is acoustically compressible we can write the following linearized equation for pressure $p_1$, velocity, and bubble radius oscillations [8]:
\[
\frac{1 - \alpha_0 \frac{\partial p_t}{\partial t}}{C_l^0} + \rho_0^0 \frac{\partial \nu}{\partial x} - 3 \rho_0^0 \frac{\alpha_0}{a_0} \frac{\partial a}{a_0} = 0,
\]

here \( \nu \) is the velocity of the medium, \( a_0 \) is the radius of the bubbles, and \( C_l^0 \) is the frozen speed of sound in the liquid, \( \alpha_0 \) is volume content of the void fraction, \( \rho_0^0 \) are the density of phases in initial undisturbed state.

The momentum equation has the form:
\[
\rho_0^0 (1 - \alpha_0) \frac{\partial \nu}{\partial t} + \frac{\partial p_t}{\partial x} = 0.
\]

We assume that the perturbations of the pressure in the liquid \( p_l \) and in the bubbles \( p_g = p_o + p_a \) are connected by the Rayleigh-Lamb equation. After linearization with capillary forces taken into account it has the form
\[
\rho_0^0 a_0 \frac{\partial^2 a}{\partial t^2} + 4 \rho_0^0 \nu^{(\mu)} \frac{\partial a}{a_0} = p_g - p_l + \frac{2 \sigma}{a_0^2} a.
\]

Here \( \nu^{(\mu)} \) – kinematic viscosity of the liquid.

We write the equation of heat conduction and diffusion inside the bubbles as well as the heat equation in the liquid around the bubble to take into account the interfacial heat and mass transfer:
\[
\frac{\partial T_g}{\partial t} = \frac{\lambda_g}{r^2} \frac{\partial}{\partial r} \left( \lambda_g r^2 \frac{\partial T_g}{\partial r} \right) + \frac{\partial P_g}{\partial t} = \frac{D}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T_g}{\partial r} \right) (0 < r < a_0),
\]
\[
\rho_0^0 \frac{\partial T_l}{\partial t} = \frac{\lambda_l}{r^2} \frac{\partial}{\partial r} \left( \lambda_l r^2 \frac{\partial T_l}{\partial r} \right) (a_0 < r < a_0 a_0^{-1/3}).
\]

Here \( r \) is the radial coordinate counted from the centre of the bubble, \( k \) is mass concentration of vapour phase, \( T_g, T_l, c_g, c_l, \lambda_g, \lambda_l \) denote the temperature, specific heat and thermal conductivity of the gas and liquid phases respectively, \( D \) is a coefficient of diffusion.

On the interface of phases \( (r = a_0) \), we define the boundary conditions following from the conditions for the balance of heat and mass:
\[
T_g = T_l = T_{(a)}, \quad k = k_{(a)}, \quad \frac{\lambda_g}{r} \frac{\partial T_g}{\partial r} - \frac{\lambda_l}{r} \frac{\partial T_l}{\partial r} = j L \quad j = \frac{D}{1 - k_0} \left( \frac{\partial k}{\partial r} \right)_{r=a_0},
\]

where \( L \) is specific heat of water vapour formation, \( j \) is a mass transfer intensity, \( T_{(a)} \) and \( k_{(a)} \) are perturbations of temperature and vapour concentration on the bubble surface.

Для For the gas pressure inside the bubble we get:
\[
\frac{\partial p_g}{\partial t} = -3 \gamma \frac{p_g}{a_0} \frac{\partial a}{a_0} + 3(\gamma - 1) \frac{\lambda_g}{a_0} \left( \frac{\partial T_g}{\partial r} \right)_{r=a_0} + 3 \gamma \frac{c_v^0}{a_0} \frac{B_o}{(1 - k_0) B_o} \left( \frac{\partial k}{\partial r} \right)_{r=a_0},
\]

\( \gamma = c_v / (c_v - B_o) \), \( c_v = c_v k_o - c_v (1 - k_o) \).

For the initial undisturbed state we write the following equation:
\[
\frac{p_g(T_0)}{p_{g,0}} = \frac{B_o k_o}{B_o + (B_o - B_o) k_o},
\]

which uniquely relates to the mass concentration of vapour in the bubble with temperature \( T_0 \). To determine \( p_g(T) \) we use a formula \( p_g(T) = p_0 \exp(-T/T_0) \) the empirical parameters depend on the type of liquid.

3. Dispersive analysis

The solution of the system above will be searched in the form of a damped travelling wave:
\begin{align*}
p_I, \ p_g, \ \nu, \ \omega, \ a & \sim \exp\left[i(Kx - \omega t)\right], \ T_I = T_I(r) \exp\left[i(Kx - \omega t)\right], \ (i = g, \ell), \\
k = k(r) \exp\left[i(Kx - \omega t)\right], \ (K = k + i\delta, \ C_p = \omega/k, \ i = \sqrt{-1}),
\end{align*}

where \( K \) is the wave vector, \( \delta \) and \( C_p \) are the damping coefficient and the phase velocity of the wave, respectively. From the condition for the existence of a solution of this type, taking into account the acoustic discharge effects \([17]\) of bubbles, we obtain the dispersion equation:

\begin{equation}
\frac{K^2}{\omega^2} = 1 + \frac{3\rho_0^2\sigma_0}{\psi} - \frac{3\rho_0^2\sigma_0^2 a_0^2}{\xi},
\end{equation}

\begin{equation}
\psi = \frac{3\gamma p_g}{Q} - \frac{\rho_0^2\omega^2 a_0^2}{\xi} - 4i\rho_0^2v_{tr}\omega - \frac{2\sigma}{a_0},
\end{equation}

\begin{equation}
p_g = p_0 + \frac{2\sigma}{a_0}, \ \xi = 1 - io\alpha_A, \ t_A = \frac{a_0}{\sqrt{\alpha_0^2 C^2}},
\end{equation}

\begin{equation}
Q = 1 + \left(\frac{\gamma - 1}{k_0} H_k h_k(y_\ell) + \frac{\gamma}{1 - k_0} H_k h_k(z)\right)\left(\frac{H_a}{k_0} + \frac{\gamma kh(z)}{1 - k_0} / \beta h_k(y_\ell)\right)^{-1},
\end{equation}

\begin{equation}
kh(x) = 3(xth(x - 1)x^{-2}, \ \ h(x) = 3(1 + x(A_x xth(\alpha_x - 1)) - 1)(A_x x - th(\alpha_x - 1)))x^{-2},
\end{equation}

or \( h(x) = 3(1 + x)x^{-2}, \)

\begin{equation}
A_x = \alpha_0^{1/3}, \ \ y_\ell = \sqrt{-io\alpha_0^2 / v_{tr}}, \ \ z = \sqrt{-io\alpha_0^2 / D}, \ \ \beta = (\gamma - 1)\eta H_x \chi^2,
\end{equation}

\begin{equation}
\eta = \rho_0^0 C^2 / \rho_g^0 C_g, \ \ \chi = \frac{c_g T_0}{L}, \ \ H_v = \frac{B_v}{B_0}, \ \ H_a = \frac{B_a}{B_0}, \ \ H = H_v - H_a.
\end{equation}

4. Numerical results

On the basis of the above dispersion equation (1) numerical calculations for water with vapor-air bubbles were carried out. As values of physical and thermophysical parameters data from the reference book \([18]\) were used. This article presents the results of calculations for the value of the superheat value \( \Delta T = 1 \) K. For the value of the equilibrium bubble radius accepted \( a_0 = 10^{-6} \) m.

Fig. 1 shows the dependence of the phase velocity and damping coefficient on the disturbance frequency when the mixture is in a stable region for the case of the considered superheat at the initial void fraction content \( \alpha_0 = 10^{-3} \). The mass content of steam and the mass of the air nucleus \( k_0 = 0.355 \) and \( m_{g_0} = 7.0 \cdot 10^{-18} \) kg. For the low-frequency range \( \omega < \omega_g, \ \ \omega_g = a_0^1 \sqrt{3\gamma p_0 / \rho_0^0} / \beta \) is the Minnaert frequency of the natural oscillations of the bubbles), the dependence of the phase velocity on the frequency is constant. This feature in this case is associated with a sufficiently high mass concentration of inert gas in the bubbles. As a consequence the elasticity of the bubbles is mainly determined by the mass content of the gas. In consequence of this feature of bubbles the value of the damping decrement in the region of low-frequency perturbations increases and changes by 6 orders of magnitude, and the region of high-frequency perturbations remains constant.
Fig. 1. The dependence of the phase velocity (solid line) and the damping coefficient (dashed line) on the frequency of perturbations upon overheating $\Delta T = 1$ K.

Fig. 2. The dependence of the phase velocity (solid line) and the damping coefficient (dashed line) on the frequency of perturbations for different values of the volume content of bubbles $\alpha_0$: 1 – $10^{-4}$, 2 – $10^{-3}$ and 3 – $10^{-2}$.

Fig. 2 shows the dependence of the phase velocity (solid line) and the damping coefficient (dashed line) on the frequency of perturbations for different values of the initial volume content of bubbles. Lines 1, 2 and 3 correspond to the following values of the initial volume content of bubbles $\alpha_0 = 10^{-4}$, $10^{-3}$ and $10^{-2}$.
10^{-3}$ and $10^{-2}$. It can be seen from the diagrams that as the volume content decreases, the phase velocity in the low-frequency region ($\omega \leq \omega_r$) increases approximately in 5 times, but for this medium the phase velocity remains constant. This is due to the fact that at low frequencies the role of phase transitions increases due to which gas-vapor bubbles with a small mass content of inert gas become less elastic. For the consideration of the case, the elasticity of the bubbles is mainly determined by the mass content of the gas. In a zone where the external perturbation frequency is comparable to the Minnaert resonance of bubbles ($\omega > \omega_r$), a strong increase in the phase velocity occurs. Moreover, the larger the initial volume content of bubbles, the greater the change in phase velocity. Similar patterns are observed for the attenuation coefficient.

In Fig. 3 illustrates the character of the dispersion dependencies for a discrete change in the equilibrium radius $a_0$. Lines 1 and 2 correspond to the following values of the bubble radius $a_0 = 10^{-6}$ and $2.2 \cdot 10^{-5}$ m. Moreover, the largest value of the radius corresponds to the equilibrium radius located at the boundary of the stability region. From the presented graphs, a strong increase in the attenuation coefficient follows with approaching the value of the equilibrium radius $a_0$ to the value $2.2 \cdot 10^{-5}$ m. For a low-frequency region, an increase in the radius by a factor of two leads to an increase in the attenuation coefficient by 2-3 orders of magnitude. The phase velocity decreases several times.

![Fig. 3. The dependence of the phase velocity (solid line) and the damping coefficient (dashed line) on the initial bubble radius $a_0$: 1 - $10^{-6}$ m and 2 - $2.2 \cdot 10^{-5}$ m.](image)

**5. Conclusion**

The effect of superheating of a liquid on the value of the phase velocity and the damping coefficient is considered when the system is in a stable state. It is established that for the equilibrium radius $a_0$ m in the stability zone, the considered superheat does not significantly affect the phase velocity and attenuation coefficient, which is associated with a sufficiently high concentration of inert gas in the bubbles. The damping decrement in the stable region does not change more than twice.
An analysis of the dependence of the initial volume content of bubbles on the wave dynamics shows that, when passing through the critical frequency \( \omega_R = \frac{1}{a_0} \sqrt{\frac{3 \nu P_0}{\rho_0}} \) – the Minnaert frequency of the natural oscillations of the bubbles), a sharp increase in the phase velocity occurs with a subsequent exit to a certain stationary value. Moreover, the larger the volume content, the sharper the increase in the phase velocity.

The results obtained are consistent with [14–16] and are a continuation of the studies from [19].

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