Junior high school mathematics teachers’ pedagogical content knowledge in teaching of polyhedra

Ma'rufi, M Ilyas and Salwah
Universitas Cokroaminoto Palopo, Jl. Latammacelling No.19, Tompotika, Wara, Palopo91911, Indonesia
E-mail: marufi.ilyas@gmail.com

Abstract. A teacher plays an important role in developing students’ knowledge and skills in learning mathematics. This article focuses on the development of mathematics teachers’ Pedagogical Content Knowledge (PCK) in the teaching of polyhedra, particularly on cube and cuboid for Year 8 students. PCK is a special knowledge for teacher integrating content knowledge, pedagogical knowledge, and knowledge of students. Content knowledge is defined as knowledge on facts, concepts, relations among concepts, and procedures about mathematics, particularly the concept of polyhedra. Pedagogical knowledge refers to teacher’s knowledge in organizing teaching and learning process of polyhedra, while knowledge of students is defined as teacher’s knowledge on students’ difficulties and mistakes on polyhedra. Data were gathered by recording the teaching and learning process and interviewing the students. The data were analyzed by reduction, categorization, and interpretation. The result showed that the teacher tended to combine the conceptual and procedural explanation to correct the mistakes and address the difficulties of students on polyhedra by providing scaffolding and employing peer tutoring.

1. Introduction
The law on teachers and lecturers states that teachers in carrying out their duties must have four competencies: pedagogic, professional, personality, and social. The competencies indicate that the teacher should; (1) knowing the students well (2) master the subject and the teaching materials, (3) be able to organize educational learning, which includes the plan and implementation of the learning, the evaluation of learning process and outcomes, and follow-up for the improvement of learning, 4) develop the personality and professionalism in a sustainable manner. This is in line with the meaning of Pedagogical Content Knowledge (PCK) mentioning that teacher is one of the critical factors of students’ achievement. Teachers are required to master the content subject and an effective way to deliver it, as well as interpret the students' thinking so that they understand the subject matter well [1]. The development of teachers’ PCK can be done through training as professional development and training are important to improve teacher's PCK in mathematics learning [2,3].

“PCK is a teacher's method for understanding student cognition so that students can reduce misunderstandings, difficulties, and misconceptions” (p.9) [4]. Students’ difficulties in learning mathematics at schools are closely related to the mathematical and pedagogical knowledge of the teachers. “Pedagogical content knowledge was defined as teacher’s ability has to use their knowledge of mathematics to “unwrap” the mathematical topics and present the content in ways for students to successfully learn mathematics”[5]. A teacher should be able to use his or her mathematical
knowledge to examine mathematics topics, using various means and representations to strengthen students’ understanding, thus reducing student difficulties, errors, and misconceptions [6].

2. Method
This research aimed at developing a Mathematics Mentoring model based on Pedagogical Content Knowledge (MGM-PCK). The model developed in this research referred to the Plomp’s model [7] consisting of five phases, namely: (a) initial assessment phase, (b) design phase, (c) construction phase, (d) testing, evaluation, and revision phase, and (e) implementation phase. This article focuses on the development of PCK of mathematics teachers in the learning of polyhedra in Year 8 during the pilot study.

Data were collected through the recording of learning activities and interviews. Two lessons were recorded in this research. Researchers and five lecturers from the mathematics education program were directly involved in data collection. The instructional videos were repeatedly observed and then transcribed. Data were analyzed using the qualitative analysis including the reduction, categorization, interpretation and drawing conclusion. Components of PCK in this research were: (1) teacher's knowledge about polyhedra, (2) teacher’s pedagogical of knowledge polyhedra learning (3) teacher's knowledge in overcoming student's difficulties and mistakes on polyhedra.

3. Result
PCK of Year 8 mathematics teachers on polyhedra includes knowledge of polyhedra, pedagogy, and knowledge of students. The topic for lesson 1 is to discover the formula of the surface area of the cube and rectangular, while lesson 2 is to find the formula of the surface area of the prism. The learning outcomes of each session are described in the following section.

3.1. Lesson 1
3.1.1 Knowledge of subject matter
The teacher did not directly explain the facts related to cubes and rectangular at the beginning of the lesson. Instead, they were delivered through examples and questions that guided students to construct definitions of terms in cubes and cuboid, such as edge and face. Teachers also always related the concept of the cube/cuboid with the prerequisite material, i.e., plane geometry (square and rectangle). Besides, the link between concepts was explained by analogizing the geometric shape of a cube/cuboid with the form of concrete objects that are familiar to the students and can easily be found around them such as cabinet and shoe boxes. Procedurally, the teacher also asked about the formula of area of a square (\(A = \text{face}^2\)) and the area of a rectangle (\(A = l \times w\)) and then wrote it on the board correctly. Nevertheless, there was one term that was still difficult to understand by the students, the term of congruency, since it had not yet taught in Year 8. Therefore, the teacher used another congruent replacement term that is, two plane geometry of the same shape and size. On the other hand, the mathematical procedure was used to find out the formula of the surface area of the cube and the cuboid that had been written in the student worksheets but cannot be appropriately done by the students.

3.1.2 Knowledge of pedagogy
The steps to achieve the learning objectives include: the teacher conveying apperception on the area of plane geometry; presenting the lesson objectives; showing the cube and cuboid through video; distributing worksheets and media to each group of students; guiding each group to work on the student worksheet; asking a group to explain the work, and confirming the conclusion together.

The broad concept of cubes and cuboid was represented through visual media in the form of animated video consisting of the formation of cubes and cuboid from their nets and concrete media in the form of cubes and cuboid nets made of cardboard. The use of this two media aimed to show the students the process of opening a cube/cuboid into a net, and by calculating the total area of the net, it is expected that the student can understand that it is the process of discovery of the concept area of
polyhedra. In the implementation of learning, visual media was used at the stage of information delivery. Concrete media was used in the core learning activities, by sharing media with each group of students to encourage them to be active learners. Students started to be engaged when they hold the cube and cuboid, cut, and measured them. During the learning activities, the teacher did not explain the material/instructions on the board, but she/he visited each group. The teacher gave direction to each student to investigate the model of cube nets. However, not all students did so. Therefore, the teacher provided limited assistance with scaffolding techniques on difficult parts. Teacher’s role was still dominant so that the planned guided discovery did not work effectively since the students were not accustomed to working independently and waited for instructions from the teacher. This process took longer than expected and as a result, individual and group assessments could not be conducted due to the insufficient time.

3.1.3 Knowledge of students
The teacher explored students’ background knowledge by asking several questions related to the characteristics of cubes and cuboid including the number of edges, vertices, faces, the area of squares and rectangles, asking them to analogize objects resembling cubes/cuboid in their surroundings so that they can distinguish the characteristics of both. When a student had difficulty in answering the point mentioned previously, the teacher guided him/her by showing the cube and cuboid models through a concrete media and demonstrated how to open the cube/cuboid nets in each group. Besides, misconceptions also occurred when students referred to edges as faces. To overcome this, the teacher re-explained the difference of square as plane geometry and the cube as a polyhedron. For a square, the boundaries are called sides, while for a cube, the face is a plane bounded by four edges. Another challenging concept for students concerned the formula of each face of cuboid and cube into the formula of surface area. Students could only calculate the size of each edge and then multiplied it, but they were not successful in writing it into an algebraic form of \( A = 2lw + 2lh + 2wh \). Besides these challenges, there was also a misconception of students mentioned that the kite was a square and a triangle was a polyhedron. This occurred during the apperception. The teacher then gave a personal explanation to each student having this misconception. The teacher also always motivated students who seemed to be lethargic and less enthusiastic.

3.2 Lesson 2
3.2.1 Knowledge of subject matter
The facts, definitions, and symbols used in the lesson can be seen when the teacher drew and showed some plane geometry like square and triangle, but she/he were not consistent in naming the sides. Each side can be called with the letter/name of the corner facing the side. However, the teacher did not write it down. For the relations between concepts, the teacher associated the shape of the prism base with particular plane geometry. If the prism base is square, then it is called a rectangular/square prism, similar case for a triangular base. The teacher also wrote the general formula and the circumference of some plane geometry that can be the base of the prism such as the area and circumference of the square and the area and circumference of the triangle. To relate between concepts and formulas, the teacher provided explanation for some mathematical procedures associated with the invention of the surface area formula of the prism; the student first determined the base area and the area of the prism cover (top base) and then discovered the area of the vertical faces of prisms and summed the three.

3.2.2 Knowledge of pedagogy
Teachers’ learning steps were in line with the lesson design, starting with the apperception of several forms of plane geometry, area and circumference, conveying the learning objectives, briefly explaining the surface area of the prism, distributing student worksheets to each group, working with the students, asking one of the groups to present their work, and confirming the conclusion. In explaining the materials, the teacher tended to explain the contents and relevant information of the student worksheet directly.
The surface area of the prism was explained through cardboard of the net of triangular prisms. The teacher communicated the representation of a prism in the form of the net, determined the area of each net, and then summed the area of each face to obtain the total surface area of the prism. The teacher also gave some examples of plane geometry at the beginning of learning such as triangle, square, and how to determine the square area, determine the height of the triangle, and the triangle area contextually. The examples were written on the board, and then the teacher explained them to the students. In addition to the examples, teachers also assigned individual tasks for the students to solve two problems. To engage students in group work, the students were also encouraged to ask questions. In this case, teachers selected the student randomly to obtain some questions from them. Some students were also asked to help their friends who were struggling to understand the worksheet. In term of assessment, assessment instruments were not used, however, overall, teachers always appreciated the students by praising and giving applause for those who actively proposed questions and provided correct answers.

3.2.3 Knowledge of students
Teachers may find the initial knowledge of students through the apperception of the square area, the height of the triangle, and triangle area by asking. Not many students remembered them, so the teacher re-explained how to determine the square area, the height of the triangle, and the triangle area. When there were students facing difficulties, the teacher guided them by giving simple examples (scaffolding) and slowly reduced the amount of assistance until the students can independently solve the problem. Any possible difficulties, errors, and misconceptions described by the teacher during the planning, just as students would have difficulty composing algebraic forms from the surface area of the prism, the error in determining the sum of each faces in the prism net, and misconceptions about the prism base on the position of the prism. To overcome these issues, the teacher explained the correct concepts in rotation from one group to another. Because of the high enthusiasm and curiosity of students, the teacher needed to perform this process repeatedly, explaining the same thing to each group.

4. Discussion
4.1. Lesson 2
The knowledge of Year 8 high school teachers of polyhedra are divided into three categories: factual, conceptual and procedural knowledge. Factual knowledge is shown by facts and mathematical symbols associated with polyhedral presented accurately in the lesson. The knowledge of concepts and their relationships between mathematical concepts was explained accurately; the teacher properly connected the concept of plane geometry and polyhedra, mathematical formulas such as the area of a square, rectangle, and triangle were well defined; mathematical procedures related to polyhedra was explained in systematic steps. Knowledge of subject matter is referred to knowledge of facts and concepts of mathematics and their relationships [8]. Likewise, knowledge of subject matter is classified into two categories, namely conceptual knowledge and procedural knowledge [9]. Knowledge of teaching materials may include relating certain mathematical concepts to others and explaining the underlying reasons for certain mathematical procedures. Teachers should understand the rules, procedures, and facts of mathematics and the reasons underlying the procedures to be able to enhance students' conceptual understanding.

4.2. Lesson 2
The pedagogical knowledge of Year 8 teachers in learning polyhedra is divided into two categories: learning organization and assessment. Knowledge of mathematics learning strategies includes the ability of teachers to use appropriate activities in mathematics learning, real-life examples and analogies in mathematical learning as well as different learning strategies in presentation, and the ability to use different representations in mathematics learning [10]. Mastering the teaching material is
not enough to achieve the desired learning objective [11]. This indicates that teachers should be able to teach the material in a way to be easily understood by the students.

The learning organization of teachers was shown by the use of discovery method including the use manipulative and worksheets to guide students to discover the formula of the surface area of cubes, cuboid, and prisms; to explore students’ prerequisite knowledge by relating the lesson to the previous one. Furthermore, teachers involved students in learning by giving them the opportunity to find their formula of the surface area of cubes, cuboid, and prisms. Students’ active learning was created through teacher's explorative questions. The strategy used by teachers was to provide scaffolding to students during the learning. The teacher assessed the learning process through performance appraisal. Teachers need to have different knowledge and skills to plan and implement learning strategies that encourage students’ understanding and learning.

4.3 Knowledge of students

The teacher's knowledge of the students in learning polyhedra is divided into two categories: teacher's knowledge about student difficulties and how to overcome the difficulties. The teacher obtained the information about the student's difficulties based on the same questions from each group. The teacher explored the students’ initial knowledge by asking questions related to the topic to be studied. The teacher overcame the errors, difficulties, and misconceptions of students by giving a simple analogy and applying the scaffolding technique. It is essential for the teachers to recognize certain aspects of the topic that may cause problems for students, teachers should also have the ability to address the issues faced by the students regarding learning materials [12]. Teachers should not only confirm that a solution is correct but also explain why it is correct [13]. Teachers who have content knowledge of polyhedra and good pedagogical knowledge but are lack of the knowledge of students will have difficulties in formulating learning strategies that meet the characteristics of students as they do not have a clear picture of possible errors and student misconceptions.

5. Conclusion

This research shows that PCK is important and necessary in a learning, including learning polyhedra in Year 8. The PCK of Year 8 teachers of in polyhedra is the teacher's knowledge of the polyhedra integrated with the knowledge of students which have an impact on teacher pedagogical knowledge in the learning organization. Knowledge of students requires both the knowledge of teaching materials as well as the pedagogical knowledge.

References

[1] Ma'rufi 2016The 1st International Conference on Mathematics, Science, and Computer Science (ICMSC) (Balikpapan: American Institute of Physics) p 1
[2] Lee J 2010 Exploring kindergarten teachers’ pedagogical content knowledge of mathematics: Department of Curriculum and Instruction-EC4, College of Education and Health ProfesionalInternational Journal of Early Childhood4227
[3] Tanisli D and Kose N Y 2013Preservice Mathematics Teachers’ Knowledge of Students about the Algebraic ConceptsAustralian Journal of Teacher Education38
[4] Shulman L S 1986 Those who understand: Knowledge growth in teaching Educational Researcher15 4
[5] Ma L 1999Knowing and teaching elementary mathematics: Teachers’ understanding of fundamental mathematics in China and the United States(New York: Routledge)
[6] Ma'rufi 2017International Conference on Natural and Social Sciences (Makassar: Universitas Cikroaminoto Palopo) p 264
[7] Plomp T 1997 Educational and training system design. (The Netherlands: University of Twente)
[8] Kilic H 2011Turkish Online Journal of Qualitative Inquiry2
[9] Maat S M and Zakaria E 2014 Analyzing pedagogical content knowledge of algebra using confirmatory factor analysis *Indian Journal of Science and Technology* 7 249

[10] Guzel E B 2010 *Journal Scientific Research and Essay* 5 1872

[11] Isiskal M and Cakirogu E 2011 *Journal Math Teacher Education* 14 213

[12] Hawkins W J 2012 An investigation of primary teachers’ pedagogical content knowledge when teaching measurement three and four *12th International on Mathematics Education*

[13] Kleve B 2009 Aspects of a teacher's mathematical knowledge on a lesson on fractions *Proceeding of The British Society for Research into Learning Mathematics* vol 29 p 67