Identification and stochastic modelling of sources in copper ore crusher vibrations

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Abstract. A problem of rolling element bearings diagnostics for different machines is widely discussed in the literature. Most of the methods are based on the vibration signal analysis. However for some real signals the classical methods of damage detection are insufficient because of the specific nature of examined data. This specific nature is very often manifested through overlapping, mixing or interleaving of processes with different statistical properties and may be the result of different sources that have influence on the analysed signal. The problem of different sources identification and parametrisation of processes which are related to them is very challenging and requires advanced techniques. There are many methods which can be useful in this context however each signal should be analysed separately and there is no universal technique adequate to all possible time series. In this paper we propose a method of sources identification for vibration signal from the heavy duty crusher used in mineral processing plant. A crusher is a kind of machine which use a metal surface to crumble materials into small fractional pieces. During this process, as well as during entering material stream into the crusher, a lot of impacts appear. They are present in vibration signal acquired from bearings housing. Moreover, for some cases we also observe cyclic impulses which may be related to damage of rolling element bearings in the machine. The proposed sources identification method, especially useful for crushers vibrations, is based on the statistical analysis of examined data. Moreover by using advanced techniques of time series theory we propose a stochastic model that exhibits similar statistical properties as analysed signals. The introduced technique can be a starting point to damage detection of rolling element bearings of copper ore crushers.
1. Introduction
One of the main problems of rolling element bearings diagnostics is to extract information about damage. There are many methods that can be useful in this context. Most of them are based on the analysis of the vibration signal of a given machine in the time and/or frequency domain. Extraction of the information about damage is very often related to the detection of informative signal [12, 22, 23] and most of the methods are based on the impulsiveness of given time series [1, 2, 3, 5, 18, 25, 27]. However, for many real vibration signals the classical methods seem to be inadequate because of the very specific nature of examined data. One of the examples is frequency and amplitude modulated signal obtained from machine working in varying operating conditions [9] or non-Gaussian character of external noise (a critical case is when noise is impulsive) [4, 22, 24]. Therefore there is need to use more advanced techniques that can be useful in the problem of fault detection for such cases. The mentioned specific nature is often manifested through overlapping, mixing or interleaving of processes with different statistical properties and may be the result of different sources that have influence on the analysed signal. Therefore in the group of damage detection methods there are segmentation techniques which lead to identification of subsignals with homogeneous structure. In the literature one can find various methods of segmentation which can be used to different signals [13, 19, 30]. Some of them are based on the statistical properties of examined time series, [14]. As it was mentioned, the segmentation problem is strongly related to identification of different sources and parametrisation of processes which are related to them. This parametrisation can be expressed by the statistical or stochastic properties of extracted segments therefore the stochastic analysis and modelling of appropriate vibration signals occur more frequently in the literature [25, 20, 21, 31].

In this paper we propose a novel segmentation method applied to vibration signal from the heavy duty crusher used in mineral processing plant. The analysed vibration signal is a mixture of different processes therefore in the first step of fault detection it is necessary to identify the appropriate subsignals and analyse them by using stochastic methods. This can lead to sources identification and finally to damage detection. The proposed method consists of two steps. First, the primary segmentation is made in order to identify distributions of selected subsignals. In the second step we make a proper segmentation in order to identify the processes which have influence on the given signal. This method is based on the two regimes switching model for which we assume that the examined time series is a mixture of two processes with different distributions. The regime switching models (called also Hidden Markov models) were initially used as a segmentation method in speech recognition [26], but they have been also exploited in other contexts, including as diverse areas as software reliability [28], electromagnetic field measurements [6] or electricity markets [15].

The rest of the paper is structured as follows: in Section 2 we describe the investigated machine. In Section 3 we present the methodology, i.e. the two steps segmentation procedure. Next, we analyse the real copper ore crushers vibration signal. Last section contains conclusions.

2. Experiment description
The machine discussed here is a heavy duty crusher used in a mineral processing plant. The crusher is a kind of machine which use a metal surface to crumble materials into small fractional pieces. During this process, as well as during entering material stream into the crusher, a lot of impacts/shocks appear. They are present in vibration signal acquired from the bearings housing, see Figure 3. Several crushers were investigated: they were in different technical state and operated under different loading conditions. Vibration from bearings housing in horizontal and vertical direction and tacho signal have been measured using Endevco accelerometers BruelKjaer Laser probe, NI DAQ card and Labview Signal Express. Signals were acquired for idle mode as well as for different values of load (volume and granulation of material stream) with approx
120s duration each, however, for further investigation shorter signals were selected. Sampling frequency was set to 25kHz, [24]. In Figure 1 we present the machine.

![Figure 1. A crusher general view (note bearings with yellow housing).](image)

3. Methodology
In this section we present the new technique of identification of different processes related to sources that have influence on copper ore crusher vibration signal. This method is based on a two step procedure: the primary segmentation and stochastic analysis of separated parts and the main method based on the two regimes switching model. In the presented methodology we assume that the analysed signal is a mixture of processes with different statistical properties which are represented by different distributions. We intuitively assume that impacts related to crushing process are rather strongly non-Gaussian, while ”proper bearing vibration” will be rather Gaussian. Obviously, we will validate this assumptions. In the primary segmentation method we identify the observations adequate to different regimes and conclude on their distribution. This is a starting point to the main segmentation method for which the distribution of processes in separate regimes should be properly defined.

3.1. The primary segmentation method
The primary segmentation method of the signal $x_1, x_2, ..., x_n$ is based on positive and negative peaks finding in a fixed neighbourhood. More precisely, if in the copper ore crushers vibration signal we observe peaks, then they are treated as observations from different regime than the other data. We should mention here that the peaks we find in our procedure are identified in a given neighbourhood by the assumption that they exceed a given threshold, which we denote as $th_1$. In order to find the peaks, we use the procedure ‘findpeaks’ in MATLAB. However, in the vibration signal of copper ore crushers we observe behaviour adequate to three regimes, i.e. positive peaks (regime R1), negative peaks (regime R3) and signal that arises after their removing (regime R2). Therefore the mentioned two steps procedure of peaks finding is also applied in order to find observations corresponding to regime R3. More precisely, we apply the mentioned technique to a signal which is equal to the examined dataset with negative values. In this case the thresholds used in the procedure are denoted by $th_3$.

After three regimes identification, next we analyse them separately by using statistical methods, in order to identify the appropriate distributions. In order to identify the distribution of R2, we propose to use the advanced method presented in [8] which is based on the graphical interpretation of stability index of a given dataset. The complete description of this procedure can be found in [8].
The analysis of regimes R1 and R3 is based on the observation of the behaviour of their empirical right tails. More precisely, for observations \(x_1, x_2, \ldots, x_n\) we first calculate the empirical right tail, which is equal to \(1 - F_n(t)\), where \(F_n(t)\) is the empirical cumulative distribution function:

\[
F_n(t) = \frac{1}{n} \sum_{i=1}^{n} 1\{x_i \leq t\},
\]

where \(1\{A\}\) is the indicator of the set \(A\). Next, we compare the empirical tail with the power function \(t^{-\gamma}\). On the basis of this we can conclude on the distribution of a given vector of observations corresponding to regimes R1 and R3. The primary analysis shows that the fitted power function of empirical right tails of observations from regimes R1 and R3 is adequate to shifted Pareto distribution, [10], which is defined through its probability density function in the following way:

\[
f(t) = \frac{\gamma \lambda^\gamma}{\gamma t^{\gamma+1}}, \quad t > \lambda,
\]

where \(\gamma\) is a scale parameter and \(\lambda\) - a shift parameter. For the Pareto distribution the theoretical right tail is given by:

\[
1 - F(t) = \left(\frac{\lambda}{t}\right)^\gamma.
\]

The Pareto distribution was used in various different applications. We mention here the insurance [10], but also the machine diagnostics to description of diagnostic features [29]. At the end, we estimate the parameters of Pareto distribution. The shift parameter is equal to the minimum of a given vector of observations, while the scale parameter is estimated by fitting the theoretical tail (3) to the empirical one (1) by using least squares method. The analysis of the vibration signal of copper ore crushers indicates that the observations from regimes R1 and R3 can be treated as Pareto distributed with the similar parameters, while for regime R2 we have the Gaussian distribution. Therefore in the main segmentation method, presented in the next part of this section, we propose to use the regime switching model with two regimes: one represented by the peaks (both negative and positive) with Pareto distribution and second represented by signal after removing the peaks, which can be modeled by a Gaussian distribution.

### 3.2. The two regimes switching modelling

The main segmentation method is based on the assumption that the observed crusher signal can be modelled by a Markov regime switching model with two regimes. The intuitive idea behind such a modelling approach is to describe the observed signal by separate regimes with different statistical properties of the underlying processes. Since the observed crusher signal can be treated either as a peak, being an effect of the crushing process, or a "normal" behaviour resulting from "proper bearing vibration", it seems to be natural to use such a model.

Based on the results of the primary segmentation method, the values of the observed signal, \(x_i\), are assumed to be independent, identically distributed random variables following Gaussian distribution in the "normal" regime (denoted by R2 in Section 3.1) and symmetric Pareto in the peak regime (denoted by R1 and R3 in Section 3.1, note that here we call it regime R1). Precisely:

\[
x_i = \begin{cases} 
  x_{i,1} \sim sP(\gamma, x_m) & \text{if } R_i = 1, \\
  x_{i,2} \sim N(\mu, \sigma^2) & \text{if } R_i = 2,
\end{cases}
\]
where \( sP(\gamma, x_m) \) is the symmetric Pareto distribution with a probability density function

\[
f(t) = \frac{1}{2} \frac{\gamma \lambda^\gamma}{|t|^\gamma+1}, \quad |t| > \lambda,
\]

and \( R_i \) is the state (regime) variable for the \( i \)-th signal. Definition (4) implies that the statistical properties of the observed signal differ depending on the actual state \( R_i \). We assume that the state process \( R_i \) is an unobserved (latent) Markov chain. As a consequence, switching from one state to another is governed by a transition matrix \( P \) which contains the probabilities \( p_{ij} = P(R_{i+1} = j | R_i = l) \) of switching from regime \( l \) at time \( i \) to regime \( j \) at time \( i + 1 \). The switching mechanism is not observed, meaning that we do not have the information, if the recorded signal was driven the peak or the "normal" regime. However, as a by-product of the estimation algorithm of the assumed model, we obtain probabilities \( P(R_i = 1 | x_1, x_2, ..., x_n) \) and \( P(R_i = 2 | x_1, x_2, ..., x_n) \) that the \( i \)-th signal came from the peak (R1) or the "normal" (R2) regime, respectively. Hence, based on these probabilities, we, in fact, get a segmentation method.

The estimation of the model parameters is based on the expectation-maximization algorithm, initially proposed in [11] and later refined for the regime-switching models in [17]. It is a two-step – expectation (E-step) and maximisation (M-step)– iterative procedure. In each iteration, first, the inferences about the state values are made based on the previously calculated parameters, second, the new set of parameters is estimated based on these inferences. The iteration is repeated until the (local) maximum of the likelihood function is reached.

The procedure of the segmentation method presented in this section is illustrated in Figure 2.

![Figure 2. The scheme of the segmentation method.](image)

4. Application to real signal

In this section we present the results of the segmentation methods (primary and main) described in Section 3. In Figure 3 (left panel) we present the analysed vibration signal of copper ore crushers. According to the presented procedure, first the primary segmentation method is made. It is worth mentioning, that the presented methodology leads to the identification of large peaks which are related to crushing process. The small peaks (related to the bearing’s work) hidden in the noise are not detectable. We start with identifying the positive peaks (regime R1) for the analysed signal and the signal with negative values (negative peaks, regime R3), as well as
signal that arises after replacing the observations corresponding to R1 and R3 by the mean of the remaining observations. In our analysis we select the following thresholds $th_1 = 0.6$ and $th_3 = 0.6$. The procedure of estimating thresholds is based on the assumption of the Gaussian distribution of the R2 regime. The result of the primary segmentation is presented in Figure 3 (right panel).

![Figure 3](image1.png)

**Figure 3.** Left panel: the examined vibration signal of copper ore crusher. Right panel: the result of the primary segmentation method applied to vibration signal of copper ore crusher.

In the next step we examine separately the signals corresponding to the three identified regimes. First, we analyse the regimes R1 and R3. In Figure 4 we present the empirical tails corresponding to those regimes and the fitted power functions (in log-log scale). As we observe, we have a good agreement between theoretical and empirical distributions.

![Figure 4](image2.png)

**Figure 4.** The empirical right tails of the observations corresponding to R1 and R3 and the fitted appropriate power functions.

The estimated parameters of the Pareto distribution for the subsignals of the regimes R1 and R3 are as follows:

$$R1 : \gamma = 3.23, \lambda = 0.6, \quad R3 : \gamma = 3.21, \lambda = 0.6.$$

As we observe, the fitted Pareto distributions have similar parameters. Therefore, as it was mentioned earlier, we can treat the signals corresponding to those regimes like signals from the same distribution. At the end of the primary segmentation method we confirm the assumption that the signal corresponding to the regime R2 has a Gaussian distribution. Gaussianity is confirmed by using the graphical test presented in [8].
As a summary of the results of the primary segmentation method we can conclude that the examined vibration signal is a mixture of two processes. The first one, represented by positive and negative peaks has a Pareto distribution, while the second one is Gaussian. The primary results are the starting point for the main segmentation method based on the regime switching model, in which we assume two regimes – Pareto and Gaussian distributed. After applying the methodology presented in Section 3.2, we obtain the identification of the mentioned regimes. The result of the segmentation method based on the regime switching model is presented in Figure 5.

Figure 5. The result of the segmentation method based on the regime switching model with two regimes (left panel) and identified regimes R1+R3 and R2 (right panel).

5. Conclusions
In this paper we have proposed the segmentation method that can be applied to vibration signals with a specific nature. The proposed technique consists of two steps: the primary segmentation and the main one. By using the presented procedures we can identify the processes comprising the vibration signal and finally the sources that affect the signal. The proposed method is based on the characteristics of the signal in time domain and stochastic analysis of the appropriate subsignals. The main result of the primary segmentation method is the primary identification of the regimes and their distributions. This method is a starting point for the main technique based on the regime switching model with two regimes. This method is universal and can be applied to various signals with a complicated structure, like the vibration signal from copper ore crushers, where we observe a mixture of processes with different stochastic properties.

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