Application of sequential quadratic programming for obtaining optimal obstacle avoidance posture and end position of a 7-axis robotic manipulator

T Y Huang and W J Chen
Department of Industrial Education and Technology, National Changhua University of Education, Changhua, Taiwan

Email: wjong@cc.ncue.edu.tw

Abstract. This paper used sequential quadratic programming (SQP) method to obtain the optimal obstacle avoidance posture and end position of a 7-axis robotic manipulator. By applying restraining conditions, obstacle information, and finishing point coordinates, a homogeneous transformation matrix was firstly used to construct a space for limited movement for the first and sixth axes of the robotic manipulator. Subsequently, we determined whether the terminal point was within reach of the robotic manipulator and calculated the angle of each axis. Finally, the MATLAB software was used to conduct a simulation and verification. In this study, the SQP method was used to introduce inequality constraints into the computation and conducted the iterative method. The simulation results revealed that the method can obtain avoidance of obstacle postures and select optimal values from the iteration results of multiple initial values to avoid the local optimal problem. Overall, the research findings are that the functions and applicability of the 7-axis robotic manipulator can be extended in industry when applied in future intelligent manufacturing.

1. Introduction
With the advent of smart manufacturing (Industry 4.0) in recent years, robotic manipulators have become indispensable. Redundant manipulators with many degrees of freedom (DOF) can not only position and orientate themselves but also avoid obstacles and singularities. Redundant manipulators with six DOF are widely used in various industries, manufacturing, agriculture, and electronics. However, the use of a manipulator with six or fewer DOF on a production line will not offer any advantage in terms of space-saving. Moreover, workspaces can be limited and size contain obstacles. Therefore, several studies on optimizing the location of an end-effector have been conducted. Novitarini et al [1] used Denavit–Hartenberg (D–H) parameters to kinematically analyze a four-DOF manipulator. Their research highlighted the percentages of error of two different kinematics in the manual, simulation, and reality. Iliukin et al [2] attempted to control a five-DOF manipulator by using a brain–computer interface. They modeled the manipulator, solved the associated forward and inverse kinematics and simulated manipulator position in MATLAB. Filipovich et al [3] applied two kinematic schemes to a SCARA-type manipulator and analyzed the manipulator’s reachable space in each scheme as well as the modeling method and its characteristics. Sun et al [4] addressed the inverse kinematics of a six-DOF robot by using D–H parameter modeling, a homogeneous transformation matrix, and forward kinematics. Shehab Mohamed [5] analyzed each rotating axis of a seven-axis
serial robotic arm with forward and inverse kinematics; they analyzed the singularity configuration and workspace to control an end-effector to avoid obstacles and maintain joints within constraints while following straight trajectories. Busson et al [6] utilized a seven-DOF manipulator to maximize Cartesian rigidity. To find the stiffest configuration, they projected the gradient of the rigidity cost function onto the null space of the manipulator Jacobian applied for the redundant robot. Kim et al [7] proposed a rapidly exploring random tree path-planning algorithm for a seven-DOF robot arm. The occlusion-free and collision-avoiding path-planning constraints were determined through MATLAB and V-REP simulation, respectively. Cao et al [8] improved the rapidly exploring random tree algorithm to generate a collision-free path. A genetic algorithm was used to smooth out sharp points and reduce the redundant points in the path. These methods were applied to a six-DOF serial robot arm. Chao Mal et al [9] analyzed each angle of a robot’s joints with inverse kinematics. A spatial path-planning method based on the RRT algorithm was explored for a 6R serial manipulator. Mahdavian et al [10] presented a method for optimizing energy consumption and avoiding moving obstacles in trajectory generation for a four-DOF arm. Yi Cao et al [11] proposed a path-planning method for an obstacle-free workspace and a 3R manipulator. Computer graphics were used to depict the intersection between the workspace and an obstacle through the RRT method within the nonobstacle workspace to generate a collision-free path. Most aforementioned have achieved obstacle avoidance for only the end-effector. When a manipulator moves from a starting point to a terminal point, each link in the manipulator is likely to collide with an obstacle. Therefore, this study proposes a method for generating a collision-free end-posture for each link of a seven-DOF manipulator. First, the kinematic analysis of the manipulator with constraints is imposed on the first and sixth axes. The sequential quadratic programming (SQP) method in MATLAB applies equality and inequality constraints in the iterative computation of the angle of each joint of the robotic manipulator. The modeling, simulation, and verification of the optimized end-posture and obstacle avoidance strategies for the entire robotic manipulator are presented.

2. Modeling seven-DOF manipulator
As illustrated in Fig. 1, the design parameters of the seven-DOF serial manipulator are as follows: $\theta_1$–$\theta_7$ represent the angle of rotation of each joint of the manipulator, $\alpha_1$–$\alpha_7$ the difference in the rotation angle of each joint with respect to their former joints, $d_{i(1,3,4,7)}$ the distances along axis $Z_i$ between axes $X_i$ and axis $X_{i+1}$, and $l_i$ the length of each link. Here, $d_1 = 184$ mm, $l_2 = 96$ mm, $d_{34} = 200$ mm, $l_5 = 78$ mm, and $d_6 = 96$ mm; $d_{34}$ consists of $d_3$ and $d_4$. The limitations of each joint are as follows: $-180^\circ \leq \theta_1 \leq 180^\circ$, $-90^\circ \leq \theta_2 \leq 90^\circ$, $-90^\circ \leq \theta_3 \leq 90^\circ$, $-90^\circ \leq \theta_4 \leq 90^\circ$, $-90^\circ \leq \theta_5 \leq 90^\circ$, $-90^\circ \leq \theta_6 \leq 90^\circ$, and $-90^\circ \leq \theta_7 \leq 90^\circ$. The coordinate equations of the individual joints were derived using homogeneous transformation matrices and D–H parameters. To model the end-effector, the orientation of the manipulator, coordinates, and parameters of each axis listed in Table 1 are essential. The $4 \times 4$ homogeneous transformation matrix consists of a rotation matrix and the coordinates of the end-effector, $P = [P_x \ P_y \ P_z]^T$, in the world coordinate system.

| Table 1. D–H parameters |
|--------------------------|
| $\theta$ (Joint angle) | d (offset) | $a_i$ (Link length) | $\alpha_i$ (Twist angle) |
| Link1 | 0 | 0 | $\pi/2$ |
| Link2 | $\pi/2$ | 0 | 0 | 0 |
| Link3 | $-\pi/2$ | 0 | 0 | $-\pi/2$ |
| Link4 | 0 | $d_{34}$ | 0 | $\pi/2$ |
| Link5 | $\pi/2$ | 0 | $l_5$ | $-\pi/2$ |
| Link6 | $-\pi/2$ | 0 | 0 | $-\pi/2$ |
| Link7 | 0 | $d_6$ | 0 | 0 |
The following steps are used to determine the relationship between two sequential axes. First, rotate the $Z$-axis by an angle $\theta$. Second, translate vertically distance $d$ along the $Z$-axis. Third, translate a distance $a_i$ along the $X$-axis. Finally, rotate the $X$-axis by an angle $\alpha_i$ (in degrees). The homogeneous transformation matrix is expressed as follows:

$$
\begin{bmatrix}
cos \theta & -sin \theta cos \alpha_i & sin \theta sin \alpha_i & a_i \cdot cos \theta \\
sin \theta & cos \theta cos \alpha_i & -cos \theta sin \alpha_i & a_i \cdot sin \theta \\
0 & sin \alpha_i & cos \alpha_i & d \\
0 & 0 & 0 & 1
\end{bmatrix} = {^i\text{T}^{i-1}}_T
$$

The matrices $^{i-1}_iT$ ($i = 1, 2, 3, \ldots 7$) represent the homogeneous transformation matrices of each joint from the first axis to the seventh axis. Through substitution of all variables from the Table 1 into the matrix, each coordinate of each joint is obtainable.

$$
{^7T}_0 = \begin{bmatrix}
A_X & B_X & C_X & P_{7X} \\
A_Y & B_Y & C_Y & P_{7Y} \\
A_Z & B_Z & C_Z & P_{7Z} \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

In (2), $A_X$, $A_Y$, $A_Z$; $B_X$, $B_Y$, $B_Z$; and $C_X$, $C_Y$, $C_Z$ represent the coordinate system of the seventh axis ($O_7$, $X_7$, $Y_7$, $Z_7$) relative to the world coordinate system ($O_0$, $X_0$, $Y_0$, $Z_0$), as illustrated in Fig. 1(b). To simplify the relationship, assume that the world coordinate system ($O_0$, $X_0$, $Y_0$, $Z_0$) and the coordinate system of the first axis are the same. The coordinates of the end-effector of the seven-DOF manipulator are $P_{7X}$, $P_{7Y}$, and $P_{7Z}$.

3. Deriving constraints and limiting method

By applying the coordinates of the obstacle, inequality and equality constraints, restraining conditions, and terminal point coordinates, not only end-effectors but also other links of the manipulator can be made to avoid obstacles by attaining optimal posture.

3.1. First constraint

To avoid the area $OABC$ (as shown in Fig. 2), it is essential to define $\theta_1$, which is the permissible rotation angle of the first axis; $\theta_R$ denotes the angle from the $X_1$-axis to the shortest line ($\overline{OB}$) between the origin, coordinate $O$, and the center of the obstacle. $\theta_o$ is the angle between $\overline{OB}$ and the border of the obstacle ($\overline{OA}$ or $\overline{OC}$). $\theta_1$ is limited within two ranges of ($180^\circ$ to $(-\theta_R + \theta_o)$) and ($-180^\circ$ to $(-\theta_R - \theta_o)$), as depicted in yellow and green respectively in Fig. 2.
3.2. Second constraint

Constraining the coordinates of the sixth axis ($P_{6X}$, $P_{6Y}$, and $P_{6Z}$) in the movable area can limit the end-posture of the manipulator. The coordinates of the obstacle are $O_X$, $O_Y$, and $O_Z$. The radius of the obstacle is $r$. When $(P_{6X} - (O_X))^2 + (P_{6Y} - (O_Y))^2 > r^2$, the coordinates of the sixth axis (green arrow) cannot collide with the obstacle (red area represents the obstacle’s area). Because this constraint has no height limitation, the border conditions of the obstacle are only two-dimensional, as shown in Fig. 3. Adding the height constraint ($P_{6Z} \leq O_Z$) can limit the height of the sixth axis to be lower than that of the obstacle. The second constraint is expressed in (3).

\[
(P_{6X} - (O_X))^2 + (P_{6Y} - (O_Y))^2 - (rod + r)^2 > 0 \\
P_{6Z} \leq O_Z
\]

![Figure 3. The schematics of the second constraint](image)

3.3. Third constraint

![Figure 4. Position of third constraint of reference point](image)
By limiting the coordinates of the sixth axis within a certain region, we can avoid collisions between the sixth axis and area $FGH\bar{O}_{new}$, as depicted in Fig. 4. To satisfy this new constraint, a new coordinate system ($O_{new}, X_{new}, Y_{new}, Z_{new}$) is formed. The angle ($\theta_{6}$) is defined by the relationship between the new coordinate system and obstacle; that is, ($\theta_{6}$) is formed between the shortest line from the center of the obstacle to the destination ($\bar{O}_{new}$) and the $X_{new}$ axis. The angle $\theta_{6}$ is formed between the shortest line from the sixth axis to the destination ($\bar{O}_{new}$) and the $X_{new}$ axis. The angle $\theta_{6}$ is formed between the border ($\bar{O}_{new}$ or $\bar{H}_{new}$) of the obstacle and $\bar{O}_{new}$.

By utilizing the angles $\theta_{6}$ and $\theta_{06}$, we can limit $\theta_{6}$ to within the rotatable angle region ($180^\circ$ to ($\theta_{6} + \theta_{06}$)) and ($-180^\circ$ to ($\theta_{6} - \theta_{06}$)). The rotatable angle region of $\theta_{6}$ is ($\theta_{6} + \theta_{06}$) $\leq \theta_{6} \leq 180^\circ$ (light blue area) and ($\theta_{6} - \theta_{06}$) $\geq \theta_{6} \geq -180^\circ$ (purple area).

3.4. Fourth constraint

The Euler angles of the seventh coordinate system relative to the world coordinate system can be obtained from the matrix in (2). To realize this constraint, the end-effector and the coordinates of the sixth axis must be presented at the same height, as expressed by (4).

$$p_{6z} = p_{7z}$$ (4)

Utilizing the equation $\theta_{yaw} = 2\tan^{-1} \left( \frac{a_{x}}{a_{y}} \right)$ (from matrix in (2)), the yaw of the seventh coordinate system can be determined. This constraint is expressed as (5).

$$2\tan^{-1} \left( \frac{a_{x}}{a_{y}} \right) = \pm 90$$ (5)

4. SQP and karush-kuhn-tucker(KKT)

4.1. SQP

By utilizing the gradient and given inequality and equality equations in SQP, the minimum solution from objective functions can be acquired in this method. In this study, SQP was used to calculate the angle of each joint to optimize obstacle avoidance behaviour and the manipulator’s end-posture. $f(x)$ is the objective functions. $g_{u}(x)$ and $c_{n}(x)$ are the inequality and equality equations, as shown in equation(6).

$$\min f(x)$$

s.t. $g_{u}(x) = 0 \ u = 1, \ldots, v$  \hspace{1cm} (6)

$$c_{n}(x) \leq 0 \ n = 1, \ldots, k$$

4.2. Karush-Kuhn-Tucker condition

The Karush–Kuhn–Tucker (KKT) condition is essential for optimizing nonlinear programming in optimization theory. The KKT condition applies equality constraints with Lagrange multipliers to derive corresponding inequality equations. By applying Lagrange multipliers to the equality equation $g_{u}(x)$, we can calculate $\lambda_{1}$ and $x_{l}$.

$$f(x_{l}, \lambda_{1}) = \begin{bmatrix} \nabla f(x) - G(x)^{T} \lambda \\ g_{u}(x) \end{bmatrix}$$ (7)

$f(x)$ can be transformed to $f(x_{l}, \lambda_{1})$ by utilizing the Lagrange equation, as in (7). $G(x) = [\nabla g_{1}(x), \nabla g_{2}(x), \ldots \nabla g_{u}(x); f^{*}(x) = g_{u}(x) \cdot \lambda_{u}$. It can be written as (8) by substituting (7) into (6).
\[ f(x_i, \lambda_u, m_n) = f(x) + \sum_{i=1}^{p} \lambda_u g_u(x) + \sum_{n} m_n c_n(k) \] (8)

The Lagrange multiplier of the inequality equations in the KKT condition given in (6) is \( m_n \), as expressed in (8). To satisfy the constraint, \( m_n \) should be less than or equal to zero. Equation (9) can be obtained by expanding the objective functions with Taylor’s second-order expansion and expanding the equality and inequality equations with Taylor’s first-order expansion. Let \( P = x - x_i \), applying the Newton’s method, \( P = x_i - \frac{f(x_i)}{f'(x_i)} \).

\[ \min_{x} f(x_i) + \nabla f(x_i)^T (P) + \frac{1}{2} (P)^T \cdot \nabla^2 f(x_i, \lambda_u, m_n)(P) \]
\[ \text{s. t. } g_u(x_i) + \nabla g_u(x_i)^T P = 0 \]
\[ c_n(x_i) + \nabla c_n(x_i)^T P \leq 0 \] (9)

\( f(x_i) = [\theta_1, \theta_2, ..., \theta_7] \). By applying the homogeneous transformation matrix (\( T \)), the coordinates \([P_{6X}, P_{6Y}, P_{6Z}]^T\) can be obtained, as given in (2). The objective functions can be written as \( f_{P7X}, f_{P7Y}, f_{P7Z} \). The inequality and equality equations can be written as (10) and (11), respectively.

\[ c_n(x) = [+180^\circ \geq \theta_1 \geq (-\theta_R + \theta_o) \text{ and } -180^\circ \leq \theta_1 \leq (-\theta_R - \theta_o)], \]
\[ (P_{6X} - (O_X))^2 + (P_{6Y} - (O_Y))^2 - (rod + r)^2 > 0, \quad P_{6Z} \leq O_z, \]
\[ (\theta_{R6} + \theta_{O6}) \leq \theta_{E6} \leq 180^\circ \text{ and } (\theta_{R6} - \theta_{O6}) \geq \theta_{E6} \geq -180^\circ] \] (10)

\[ g_u(x) = [P_{6Z} = P_{7Z}, 2\tan^{-1}\left(\frac{x}{A_y}\right) = \pm 90^\circ] \] (11)

5. Research process
This research considers each rotation angle to be limited. By applying the given coordinates of the terminal point and four constraints, all joint angles corresponding to the optimal posture are calculated through iterative SQP computation. The process followed in this study is illustrated in Fig. 5.

6. Simulation results
This research optimizes posture for obstacle avoidance through SQP with four constraints. After obtaining the best solution from the iterative computation, a program verifies each angle by simulating the manipulator and obstacle. The first iterative result converges at the 48th iteration, \( 4.978343 \times 10^{-8} \text{ m} \), as illustrated in Fig. 6.
Table 2 showed the angles of the motors and the position of the end-effector. The result is the position error $\sqrt{X^2 + Y^2 + Z^2}$. The position errors in each dimension are $X_{\text{err}} = \pm 6.34796742948751 \times 10^{-7}$ cm, $Y_{\text{err}} = \pm 4.665374547796475 \times 10^{-6}$ cm, and $Z_{\text{err}} = 1.617160555467123 \times 10^{-6}$ cm. MATLAB was used to simulate the starting point of the end-effector $X = 35.068$ cm, $Y = 0$ cm, $Z = 30.899$ cm, as shown in Fig. 7(a). The blue cylinder represents an obstacle of height 26.5 cm and radius 2 cm, and the coordinates of the obstacle are $X = -10$ cm and $Y = 20$ cm. The widest radius of the manipulator is 5 cm (rod). The entire manipulator avoids the obstacle. The end-effector denoted by the gray lines reaches the terminal point $X = -18.9$ cm, $Y = 35.5$ cm, $Z = 12.5$ cm, as depicted in Fig. 7(b).

![Figure 6. Results of the 48th iteration](image)

**Table 2.** The results of the 48th iteration for position and rotational angles of end-effector

| Obtained angles | Terminal position (cm) | Obtained position (cm) |
|-----------------|------------------------|------------------------|
| $\theta_1$      |                         | $X$        | $Y$        | $Z$        |
| 85.24553689°    |                         | -18.9      | 35.5      | 12.5      |
| $\theta_2$      | -53.05491743°           |                         |           |           |
| $\theta_3$      | -82.12917794°           |                         |           |           |
| $\theta_4$      | -9.697441977°           |                         |           |           |
| $\theta_5$      | 64.024917°              | $X$        | $Y$        | $Z$        |
| $\theta_6$      | 69.28116133°            | -18.899993652033       | 35.4999953346255 | 12.500001617606 |
| $\theta_7$      | -70.39163178°           |                         |           |           |

(a) Starting point: $X = 35.068$ cm, $Y = 0$ cm, $Z = 30.899$ cm  
(b) Obtained position: $X = -18.899$ cm, $Y = 35.499$ cm, $Z = 12.500001$ cm

**Figure 7.** The position of the end-effector converging at the 48th iteration
The same terminal point is used in the second iterative computation, which converges at the 52nd iteration, 5.634721444921398×10⁻⁸ m, as illustrated in Fig. 8. Table 3 showed the angles of the motors and the position of the end-effector. The position errors in each dimensions are \( X_{\text{error}} = 1.168242738658343 \times 10^{-6} \) cm, \( Y_{\text{error}} = 3.68677750479077 \times 10^{-6} \) cm, and \( Z_{\text{error}} = 4.11433958510458 \times 10^{-6} \) cm. The starting point of the end-effector was \( X = 35.068 \) cm, \( Y = 0 \) cm, \( Z = 30.899 \) cm, as indicated in Fig. 9(a). The end-effector reaches the same terminal point \( X = -18.9 \) cm, \( Y = 35.5 \) cm, \( Z = 12.5 \) cm, as depicted in Fig. 9(b), in which the gray line denotes the trajectory of the end-effector.

**Table 3.** The results of the 52nd iteration for position and rotational angles of end-effector

| Obtained angles | Terminal position (cm) | Obtained position (cm) |
|-----------------|------------------------|------------------------|
| \( \theta_1 \)  | 161.95705129699° | \( X \) | \( Y \) | \( Z \) |
| \( \theta_2 \)  | -42.31190250384° | -18.9 | 35.5 | 12.5 |
| \( \theta_3 \)  | -90.00000000028° | | | |
| \( \theta_4 \)  | 75.887357775075° | | | |
| \( \theta_5 \)  | 79.21578562762° | | | |
| \( \theta_6 \)  | -4.080974708412° | -18.8999988317573 | 35.5000036686777 | 12.500004114134 |
| \( \theta_7 \)  | 44.028418816509° | | | |

(a) Starting point: \( X = 35.068 \) cm, \( Y = 0 \) cm, and \( Z = 30.899 \) cm

(b) Obtained position: \( X = -18.899 \) cm, \( Y = 35.500003 \) cm, and \( Z = 12.500004 \) cm

**Figure 9.** The position of the end-effector converging at the 52nd iteration
7. Conclusion
SQP with multiple initial values can improve local solutions. Sampling the optimal value from several local solutions avoids the possibility of the solution being trapped in one local optimum. By using the computational scheme and the constraints outlined in this study, the optimal angles for obstacle avoidance posture of a manipulator can be obtained. This paper discussed the application of the SOP algorithm with constraints on the first and sixth axes to a seven-DOF manipulator. Simulation results indicate that entire manipulator can avoid obstacles. Moreover, the end-effector can reach its destination precisely. Thus, convergence is achieved, with a three-dimensional position error smaller than 0.1 µm.

Acknowledgment
The authors gratefully acknowledge the financial support of the Ministry of Science and Technology of Taiwan under Grant No. MOST 107-2221-E-018-008-.

References
[1] Novitarini A, Aniroh Y, Anshori D Y and Budiprayitno S 2017 A Closed-Form Solution of Inverse Kinematic for 4 DOF Tetrix Manipulator Robot (Int. Conf. on Advanced Mechatronics, Intelligent Manufacture, and Industrial Automation, Surabaya, pp 25-29)
[2] Iliukhina V N, Mitkovskiib K B, Bizyanovaa D A and Akopyana A A 2017 Procedia Engineering 176 pp 498-505
[3] Filipovich O, Chalenkov N and Balakin A 2018 The analysis of the kinematic characteristics of the multifunctional module based on the SCARA type manipulator (Int. Multi-Conf. on Industrial Engineering and Modern Technologies, FarEastCon, pp 1-6)
[4] Sun J D, Cao G Z, Li W B, Liang Y X and Huang S D 2017 Analytical Inverse Kinematic Solution Using the D-H Method for a 6-DOF Robot (Int. Conf. on Ubiquitous Robots and Ambient Intelligence (URAI), Korea: Jeju, pp 714-716)
[5] Mohammed S, VT 2016 Kinematic Motion Planning for a 7-Axis Robotic Arm Master’s thesis (Sewden, Umeå University)
[6] Busson D, Bearee R and Olabi A 2017 Task-oriented rigidity optimization for 7 DOF redundant manipulators (Int. on Federation of Automatic Control (IFAC), France: Toulouse, pp 14588-14593)
[7] Kim Y J, Wang J H, Park S Y, Lee J Y, Kim J J and Lee J J 2014 A RRT-Based Collision-Free and Occlusion-Free Path Planning Method for a 7DOF Manipulator (Proc. Int. Conf. on IEEE Mechatronics and Automation, China: Tianjin, pp 1017-1021)
[8] Cao X, Zou X, Jia C, Chen M and Zeng Z 2019 Computers and Electronics in Agriculture 156 pp 105-118
[9] Ma C, Zhang Y, Zhao Q and Bai K 2016 6R Serial Manipulator Space Path Planning Based on RRT (Int. Conf. on Intelligent Human-Machine Systems and Cybernetics(IHMSC), China: Hangzhou, pp 99-102)
[10] Mahdavian M, Shariat-Panahi M, Yousefi-Koma A and Ghasemi-Toudeshki A 2015 Optimal Trajectory Generation for Energy Consumption Minimization and Moving Obstacle Avoidance of a 4DOF Robot Arm (3rd RSI Int. Conf. on Robotics and Mechatronics(ICROM), Iran: Tehran, pp 353-358)
[11] Cao Y, Guo M and Li Y 2017 Obstacle-free Workspace based Path Planning for Serial Manipulator (Proc. on Chinese Control Conference(CCC), China: Dalian, pp 6897-6900)