WMAP five-year constraints on lepton asymmetry and radiation energy density: implications for PLANCK

L A Popa and A Vasile

ISS Institute for Space Sciences Bucharest-Magurele, Ro-077125, Romania
E-mail: lpopa@venus.nipne.ro and avasile@venus.nipne.ro

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Abstract. In this paper we set bounds on the radiation content of the Universe and neutrino properties by using the WMAP (Wilkinson microwave anisotropy probe) five-year CMB (cosmic microwave background) measurements complemented with most of the existing CMB and LSS (large scale structure) data (WMAP5 + All), imposing also self-consistent BBN (big bang nucleosynthesis) constraints on the primordial helium abundance.

We consider lepton asymmetric cosmological models parametrized by the neutrino degeneracy parameter $\xi_\nu$ and the variation of the relativistic degrees of freedom, $\Delta N_{\text{eff}}^{\text{rel}}$, due to possible other physical processes occurring between BBN and structure formation epochs.

We get a mean value of the effective number of relativistic neutrino species of $N_{\text{eff}} = 2.98^{+3.60}_{-2.27}^{+1.65} 4.37$, providing an important improvement over the similar result obtained from WMAP5 + BAO + SN + HST (BAO: baryonic acoustic oscillations; SN: supernovae; HST: Hubble Space Telescope) data (Komatsu et al (WMAP Collaboration), 2008 Astrophys. J. Suppl. submitted [0803.0547]).

We also find a strong correlation between $\Omega_m h^2$ and $z_{\text{eq}}$, showing that we observe $N_{\text{eff}}$ mainly via the effect of $z_{\text{eq}}$, rather than via neutrino anisotropic stress as claimed by the WMAP team (Komatsu et al (WMAP Collaboration), 2008 Astrophys. J. Suppl. submitted [0803.0547]).

WMAP5 + All data provide a strong bound on the helium mass fraction of $Y_p = 0.2486 \pm 0.0085$ (68% CL), that rivals the bound on $Y_p$ obtained from the conservative analysis of the present data on helium abundance.

For the neutrino degeneracy parameter we find a bound of $-0.216 \leq \xi_\nu \leq 0.226$ (68% CL), which represents an important improvement over the similar result obtained by using the WMAP three-year data.
The inclusion in the analysis of LSS data reduces the upper limit of the neutrino mass to $m_\nu < 0.46$ eV (95% CL) with respect to the values obtained from the analysis from WMAP5-only data (Dunkley et al (WMAP Collaboration), 2008 Astrophys. J. Suppl. submitted [0803.0586]) and WMAP5 + BAO + SN + HST data (Komatsu et al (WMAP Collaboration), 2008 Astrophys. J. Suppl. submitted [0803.0547]).

We forecast that the CMB temperature and polarization measurements observed with high angular resolutions and sensitivities by the future PLANCK satellite will reduce the errors on $\xi_\nu$ and $Y_p$ down to $\sigma(\xi_\nu) \simeq 0.089$ (68% CL) and $\sigma(Y_p) = 0.013$ (68% CL) respectively, values fully consistent with the BBN bounds on these parameters.

This work has been done on behalf of PLANCK-LFI (low frequency instrument) activities.

Keywords: CMBR theory, dark matter, cosmological neutrinos, big bang nucleosynthesis
decoupling and the QED corrections indicates that the number of relativistic neutrino species is \(N_{\text{eff}} = 3.046\) [5]. Any departure of \(N_{\text{eff}}\) from this last value would be due to non-standard neutrino features or to the contribution of other relativistic relics.

The solar and atmospheric neutrino oscillation experiments [6,7] indicate the existence of non-zero neutrino masses in the eV range.

There are also indications of neutrino oscillations with larger mass-squared difference, coming from the short baseline oscillation experiments [8,9], that can be explained by adding one or two sterile neutrinos with eV scale mass to the standard scheme with three active neutrino flavours (see [10] for a recent analysis). Such results have an impact on cosmology because sterile neutrinos can contribute to the number of relativistic degrees of freedom in the big bang nucleosynthesis [11]. These models are subject to strong bounds on the sum of active neutrino masses from the combination of various cosmological data sets [12,13], ruling out a thermalized sterile neutrino component with eV mass [14,15].

However, there is the possibility to accommodate the cosmological observations with data from short baseline neutrino oscillation experiments by postulating the existence of a sterile neutrino with the mass of few keV having a phase-space distribution significantly suppressed relative to the thermal distribution.

For both non-resonant zero-lepton-number production and enhanced resonant production with initial cosmological lepton number, the keV mass sterile neutrino produced via small mixing angle oscillation conversion of thermal active neutrinos [16] provides a valuable dark matter (DM) candidate, alleviating the accumulating contradiction between the \(\Lambda\)CDM model predictions on small scales and observations by smearing out the small scale structure [17]–[20].

A recent analysis of x-ray and Lyman-\(\alpha\) data indicates that keV sterile neutrinos can be considered valuable DM candidates only if they are produced via resonant oscillations with non-zero lepton number or via other (non-oscillatory) production mechanisms [21]. On the other hand, the possible existence of new particles such as axions and gravitons, the time variation of the physical constants and other non-standard scenarios (see e.g. [22] and references therein) could contribute to the radiation energy density in the BBN epoch.

At the same time, more phenomenological extensions to the standard neutrino sector have been studied, the most natural being the consideration of leptonic asymmetry [23]–[25], parametrized by the neutrino degeneracy parameter \(\xi_\nu = \mu_\nu / T_{\nu_0}\) \((\mu_\nu\) is the neutrino chemical potential and \(T_{\nu_0}\) is the present temperature of the neutrino background, \(T_{\nu_0} / T_{\text{cmb}} = (4/11)^{1/3}\)).

Although the standard model of particle physics predicts a value of the leptonic asymmetry of the same order as the value of the baryonic asymmetry, \(B \sim 10^{-10}\), there are many particle physics scenarios in which a leptonic asymmetry much larger can be generated [26,27]. One of the cosmological implications of a larger leptonic asymmetry is the possibility of generating small baryonic asymmetry of the Universe through the non-perturbative (sphaleron) processes [28]–[30]. Therefore, distinguishing between a vanishing and non-vanishing \(\xi_\nu\) in the BBN epoch is a crucial test of the standard assumption that sphaleron effects equilibrate the cosmic lepton and baryon asymmetries.

The measured neutrino mixing parameters imply that neutrinos reach chemical equilibrium before BBN [31]–[33], so that all neutrino flavours are characterized by the same degeneracy parameter, \(\xi_\nu\), at this epoch. The most important impact of the leptonic asymmetry on BBN is the shift of the beta equilibrium between protons and neutrons and
the increase of the radiation energy density parametrized by

\[ \Delta N_{\text{eff}}(\xi_\nu) = 3 \left[ \frac{30}{7} \left( \frac{\xi_\nu}{\pi} \right)^2 + \frac{15}{7} \left( \frac{\xi_\nu}{\pi} \right)^4 \right]. \]  

(1)

The BBN constraints on \( N_{\text{eff}} \) have been recently reanalysed by comparing the theoretical predictions and experimental data on the primordial abundances of light elements, by using the baryon abundance derived from the WMAP three-year (WMAP3) CMB temperature and polarization measurements [34]–[36]: \( \eta_B = 6.14 \times 10^{-10}(1.00 \pm 0.04) \). In particular, the \(^4\)He abundance, \( Y_p \), is quite sensitive to the value of \( N_{\text{eff}} \). In the analysis of [37], the conservative error of the helium abundance, \( Y_p = 0.249 \pm 0.009 \) [38], yielded \( N_{\text{eff}} = 3.1^{+1.4}_{-1.2} \) (95% CL) in good agreement with the standard value, but still leaving some room for non-standard values, while more stringent error bars of helium abundance, \( Y_p = 0.2516 \pm 0.0011 \) [39], led to \( N_{\text{eff}} = 3.32^{+0.23}_{-0.24} \) (95% CL) [40].

The stronger constraints on the degeneracy parameter obtained from BBN [41] give \( -0.04 < \xi < 0.07 \) (69% CL), adopting the conservative error analysis of \( Y_p \) of [38], and \( \xi = 0.024 \pm 0.0092 \) (68% CL), adopting the more stringent error bars of \( Y_p \) of [42].

The CMB anisotropies and LSS matter density fluctuation power spectra carry the signature of the energy density of the Universe at the time of matter–radiation equality (energy density of order eV\(^4\)), making possible the measurement of \( N_{\text{eff}} \) through its effects on the growth of cosmological perturbations.

The number of relativistic neutrino species influences the CMB power spectrum by changing the time of matter–radiation equality that enhances the integrated Sachs–Wolfe effect, leading to a higher first acoustic Doppler peak amplitude. Also, the temperature anisotropy of the neutrino background (the anisotropic stress) acts as an additional source term for the gravitational potential [43,44], changing the CMB anisotropy power spectrum at the level of \( \sim 20\% \).

The delay of the epoch of matter–radiation equality shifts the LSS matter power spectrum turnover position toward larger angular scales, suppressing the power at small scales. In particular, the non-zero neutrino chemical potential leads to changes in neutrino free-streaming length and neutrino Jeans mass due to the increase of the neutrino velocity dispersion [45,46].

Since the WMAP3 data release there have been many works aiming to constrain \( N_{\text{eff}} \) from cosmological observations [14,34,37], [47]–[49]. Their results suggest large values for \( N_{\text{eff}} \) within the 95% CL interval, some of them not including the standard value 3.046 [14,34,37]. Recently [50] argued that the discrepancies are due to the treatment of the scale-dependent biasing in the galaxy power spectrum inferred from the main galaxy sample of the Sloan Digital Sky Survey data release 2 (SDSS-DR2) [51,52] and the large fluctuation amplitude reconstructed from the Lyman-\( \alpha \) forest data [53] relative to that inferred from WMAP3.

Discrepancies between BBN and cosmological data results on \( N_{\text{eff}} \) were interpreted as evidence of the fact that further relativistic species are produced by particle decay between BBN and structure formation [48,49]. Other theoretical scenarios include the violation of the spin statistics in the neutrino sector [54], the possibility of an extra interaction between the dark energy and radiation or dark matter, the existence of a Brans–Dicke field which could mimic the effect of adding extra relativistic energy density between BBN and structure formation epochs [55].
A lower limit to \( N_{\text{eff}} > 2.3 \) (95% CL) was recently obtained from the analysis of the WMAP five-year (WMAP5) data alone [56], while the combination of the WMAP5 data with distance information from baryonic acoustic oscillations (BAO), supernovae (SN) and the Hubble constant measured by Hubble Space Telescope (HST) led to \( N_{\text{eff}} = 4.4 \pm 1.5 \) (68% CL), fully consistent with the standard value [57].

The extra energy density can be split into two distinct uncorrelated contributions, first due to net lepton asymmetry of the neutrino background and second due to the extra contributions from other unknown processes:

\[
\Delta N_{\text{eff}} = \Delta N_{\text{eff}}(\xi) + \Delta N_{\text{eff}}^{\text{oth}}.
\]

The aim of this paper is to obtain bounds on the neutrino lepton asymmetry and on the extra radiation energy density by using WMAP5 data in combination with most of the existing CMB and LSS measurements and self-consistent BBN priors on \( Y_p \). We also compute the sensitivity of the future PLANCK experiment [58] for these parameters, testing the restrictions on cosmological models with extra relativistic degrees of freedom expected from high precision CMB temperature and polarization anisotropy measurements.

### 2. Leptonic asymmetric cosmological models

The density perturbations in leptonic asymmetric cosmological models have been discussed in the literature [59,60,45,46,63,64]. We applied them to modify the Boltzmann code for anisotropies in the microwave background (CAMB) [65]–[67], to compute the CMB temperature and polarization anisotropy power spectra and LSS matter density fluctuation power spectra for the case of three degenerate neutrinos/antineutrinos with the total mass \( m_\nu \) and degeneracy parameter \( \xi_\nu \). As neutrinos reach their approximate chemical potential equilibrium before the BBN epoch [31]–[33], we consider in our computation that all three flavours of neutrinos/antineutrinos have the same degeneracy parameter \( \xi_\nu \).

When the Universe was hot enough, neutrinos and antineutrinos of each flavour \( \nu_i \) behaved like relativistic particles with Fermi–Dirac phase-space distributions:

\[
\mathcal{F}_{\nu_i}(q) = \frac{1}{e^{E_{\nu_i}/T_\nu - \xi_\nu} + 1}, \quad \mathcal{F}_{\bar{\nu}_i}(q) = \frac{1}{e^{E_{\bar{\nu}_i}/T_\nu - \xi_\nu} + 1} \quad (i = e, \mu, \tau),
\]

where \( E_{\nu_i} = \sqrt{q^2 + a^2m_{\nu_i}} \) is one flavour neutrino/antineutrino energy and \( q = ap \) is the comoving momentum. Hereafter, \( a \) is the cosmological scale factor (\( a_0 = 1 \) today). The mean energy density and pressure of one flavour of massively degenerate neutrinos and antineutrinos can be written as

\[
\rho_{\nu_i} + \rho_{\bar{\nu}_i} = (k_BT_\nu)^4 \int_0^{\infty} \frac{d^3q}{(2\pi)^3} q^2 E_{\nu_i}(\mathcal{F}_{\nu_i}(q) + \mathcal{F}_{\bar{\nu}_i}(q)),
\]

\[
3(P_{\nu_i} + P_{\bar{\nu}_i}) = (k_BT_\nu)^4 \int_0^{\infty} \frac{d^3q}{(2\pi)^3} \frac{q^2}{E_{\nu_i}}(\mathcal{F}_{\nu_i}(q) + \mathcal{F}_{\bar{\nu}_i}(q)).
\]

We modify in CAMB the expressions for the energy density and the pressure in the relativistic and non-relativistic limits for the degenerate case [46] and follow the standard procedure for computing the perturbed quantities by expanding the

\[1\) http://camb.info
phase-space distribution function of neutrinos and antineutrinos into homogeneous and perturbed inhomogeneous components [67]–[69]. Since the gravitational source term in the Boltzmann equation is proportional to the logarithmic derivative of the neutrino distribution function with respect to comoving momentum, \( d \ln (F_\nu + F_{\bar{\nu}})/d \ln q \), we also modify this term to account for \( \xi_\nu \neq 0 \) [46, 63].

As mentioned before, the BBN theory gives strong constraints on \( N_{\text{eff}} \) and \( \xi_\nu \) from comparing the measured light element abundance with the theoretical predictions. The only free parameter is the baryon to photon ratio, \( \eta_B = n_b/n_\gamma \), that is obtained from the determination of \( \Omega_b h^2 \) from CMB measurements.

In particular, the \(^4\)He mass fraction, \( Y_p \), affects the CMB angular power spectra through its impact on different evolution phases of the ionization/recombination history [70].

As previously demonstrated [71]–[73], the impact of the self-consistent BBN prior on \( Y_p \) has a net impact on the parameter inference, improving the bounds on cosmological parameters compared to the analysis which treats \( Y_p \) as a constant or free parameter.

We use the public BBN code PArthEnoPE [74]\(^2\) to compute the dependence of \( Y_p \) on \( \Omega_b h^2 \), \( \Delta N_{\text{eff}} \) as given in equation (2) and \( \xi_\nu \). The accuracy for \( Y_p \) obtained by using the PArthEnoPE code is \( \sim 10^{-4} \), being only limited by the experimental uncertainty on the neutron lifetime [76]. We also modify the recombination routine Recfast v1.4 [75] of the CAMB code to account for \( Y_p \) dependence on \( \Omega_b h^2 \), \( \Delta N_{\text{eff}} \) and \( \xi_\nu \).

In our computation we implicitly assume that the value of \( N_{\text{eff}} \) is not changed between the epoch of BBN and matter–radiation equality.

3. Analysis

We use the CosmoMC Monte Carlo Markov chain (MCMC) public package [77]\(^3\) modified for our extended 6 + 3-parameter space to sample from the posterior distribution giving the following experimental data sets:

- The WMAP5 temperature and polarization CMB measurements [56, 57, 78] complemented with the CMB measurements from Boomerang [79, 80], ACBAR [81] and CBI [82] experiments.
- The LSS power spectrum of the matter density fluctuations inferred from the galaxy clustering data of the Sloan Digital Sky Survey (SDSS) [51, 52, 83, 84] and Two-Degree Field Galaxy Redshift Survey (2dFGRS) [85]. In particular, the luminous red galaxy (LRG) sample from SDSS data release 5 (SDSS-DR5) [83, 84] has more statistical significance than the spectrum retrieved from the SDSS main galaxy sample from data release 2 (SDSS-DR2) [51, 52], eliminating the existing tension between the power spectra from SDSS-DR2 and 2dFGRS. For this reason we include in our analysis the matter power spectra from SDSS-LRG and 2dFGRS. We consider SDSS-LRG data up to \( k_{\text{max}} \simeq 0.2h\ \text{Mpc}^{-1} \) and the 2dFGRS data up to \( k_{\text{max}} \simeq 0.14h\ \text{Mpc}^{-1} \) and apply the corrections due to the non-linearity behaviour and scale-dependent bias [83], connecting the linear matter power spectrum, \( P_{\text{lin}}(k) \), and the galaxy power

\(^2\) http://parthenope.na.infn.it
\(^3\) http://cosmologist.info/cosmomc
Table 1. The free parameters of our model, their fiducial values used to generate the Planck-like simulated power spectra and the prior ranges adopted in the analysis.

| Parameter                  | Symbol      | Fiducial value | Prior range          |
|----------------------------|-------------|----------------|----------------------|
| Baryon density             | $\Omega_b h^2$ | 0.022          | 0.005 → 0.04        |
| Dark matter density        | $\Omega_{\text{dm}} h^2$ | 0.11           | 0.01 → 0.5          |
| Hubble constant            | $H_0$       | 70             | 40 → 100             |
| Redshift of reionization   | $z_{\text{re}}$ | 11             | 3 → 20               |
| Scalar spectral index      | $n_s$       | 0.96           | 0.5 → 1.3           |
| Normalization              | $\ln[10^{10} A_s]$ | 3.264          | 2.7 → 4             |
| Neutrino density           | $\Omega_\nu h^2$ | 0.01           | 0 → 0.3            |
| Neutrino degeneracy parameter | $\xi_\nu$ | 0              | $\xi_\nu$            |
| Number of extra rel. d.o.f. | $\Delta N_{\text{eff}}^\text{oth}$ | 0.046 | $\Delta N_{\text{eff}}^\text{oth}$ |
| Helium mass fraction       | $Y_p$       | 0.248          | 0.07 → 0.6          |

spectrum, $P_{\text{gal}}(k)$, through

$$P_{\text{gal}}(k) = b^2 \frac{1 + Q_{\text{nl}} k^2}{1 + 4k^2} P_{\text{lin}}(k),$$

where the free parameters $b$ and $Q_{\text{nl}}$ are marginalized.

- The luminosity distance measurements of distant type Ia supernovae (SNIa) obtained by the Supernova Legacy Survey (SNLS) [86] and the Hubble Space Telescope [87].
- The BBN constraints on $Y_p$ as obtained from the PARthENOPE code, allowing $\Omega_b h^2 \Delta N_{\text{eff}}$ and $\xi_\nu$ to span the ranges indicated in table 1.

Hereafter, we will denote the WMAP5 + SDSS – DR5 + 2dFGRS + SNIa + BBN data set as WMAP5 + All.

We perform our analysis in the framework of the extended ΛCDM cosmological model described by 6 + 3 free parameters:

$$\Theta = (\Omega_b h^2, \Omega_{\text{dm}} h^2, H_0, z_{\text{re}}, n_s, A_s, \Omega_\nu h^2, \xi_\nu, \Delta N_{\text{eff}}^\text{oth}).$$

Here $\Omega_b h^2$ and $\Omega_{\text{dm}} h^2$ are the baryon and cold dark matter energy density parameters, $H_0$ is the Hubble expansion rate, $z_{\text{re}}$ is the redshift of reionization, $n_s$ is the scalar spectral index of the primordial density perturbation power spectrum and $A_s$ is its amplitude at the pivot scale $k_*=0.002h \text{ Mpc}^{-1}$. The additional three parameters denote the neutrino energy density $\Omega_\nu h^2$, the neutrino degeneracy parameter $\xi_\nu$ and the contribution of extra relativistic degrees of freedom from other unknown processes $\Delta N_{\text{eff}}^\text{oth}$. Table 1 presents the parameters of our model, their fiducial values used to generate the Planck-like simulated power spectra and the prior ranges adopted in the analysis.

For the forecast from PLANCK-like simulated data we use the CMB temperature ($TT$), the polarization ($EE$) and their cross-correlation ($TE$) power spectra obtained for our fiducial cosmological model for multipoles up to $l=2000$ and the expected experimental characteristics of the PLANCK frequency channels presented in Table 2 [58]. We assume a sky coverage of $f_{\text{sky}} = 0.8$. 


Table 2. The expected experimental characteristics for the PLANCK frequency channels considered in the paper. $\Delta_T$ and $\Delta_P$ are the sensitivities per pixel for temperature and polarization maps. We assume a sky coverage of $f_{\text{sky}} = 0.8$.

| $\nu$ (GHz) | FWHM (arcmin) | $\Delta_T$ ($\mu$K) | $\Delta_P$ ($\mu$K) |
|-------------|----------------|---------------------|---------------------|
| 100         | 9.5            | 6.8                 | 10.9                |
| 143         | 7.1            | 6.0                 | 11.4                |
| 217         | 5.0            | 13.1                | 26.7                |

Following the method described in [61,62], for each frequency channel we consider a homogeneous detector noise with the power spectrum given by

$$N^c_{l,c} = (\theta_b \Delta_c)^2 \exp(l(l+1)\theta_b^2/8 \ln 2) \quad c \in (T, P),$$

where $\nu$ is the frequency of the channel, $\theta_b$ is the FWHM of the beam and $\Delta_c$ are the corresponding sensitivities per pixel of temperature ($T$) and polarization ($P$) maps. The global noise of the experiment is obtained as

$$N^c_l = \left[ \sum_{\nu} (N^c_{l,\nu})^{-1} \right]^{-1}.$$  

3.1. Statistical inference

We use the marginalized posterior likelihood probability distributions, $\mathcal{L}(\theta)$, generated using the CosmoMC package to compute the mean values and the errors of the main cosmological parameters by using the central credible interval (CCI) statistical inference procedure.

We assume uniform prior probability on parameters $\theta$ (i.e. we assume that all values of parameters are equally probable) and compute the posterior mean value $\langle \theta \rangle = \int \theta \mathcal{L}(\theta) \, d\theta$ and the cumulative distribution function $C(\theta) = \int_{\theta_{\text{min}}}^{\theta} \mathcal{L}(\theta) \, d\theta / \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \mathcal{L}(\theta) \, d\theta$, quoting as upper and lower intervals at 68% (95%) CL the values at which $C(\theta)$ is 0.84 (0.975) and 0.16 (0.025) respectively.

For the case of neutrino mass, $m_\nu$, we quote only the upper limits at 68% and 95% CL.

To facilitate the comparison of our estimates on $N_{\text{eff}}$ with the results obtained by similar analysis, we also compute the credible regions in parameter space by using the minimal credible interval (MCI) inference methodology [50]. The minimum credible interval $[\theta_{\text{min}}, \theta_{\text{max}}]$ at 68% (95%) CL was chosen to minimize the difference $\theta_{\text{max}} - \theta_{\text{min}}$, while $\int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \mathcal{L}(\theta) \, d\theta = 0.68(0.95)$. The most probable parameter value is then given by $\hat{\theta} = \text{arg}[\max \mathcal{L}(\theta)]$.

Although CCI and MCI methodologies became identical in the Gaussian limit, MCI is shown to be more meaningful for inference from skewed distributions [50].

As the maximization methodology is used by the most of the recent studies of $N_{\text{eff}}$ and $m_\nu$ inference [12, 40, 48, 50, 60], we also present the maximization intervals at 68% and 95% CL obtained for the global best fit model.
Figure 1. The marginalized posterior likelihood probabilities of the main cosmological parameters obtained for: WMAP5 + All with $\Delta N_{\text{eff}}^\text{oth} = 0$ and $\xi_\nu = 0$ priors (black lines), WMAP5 + All with $\xi_\nu = 0$ prior (green lines), WMAP5 + All with no neutrino priors (blue lines) and PLANCK-like simulated data with no neutrino priors (red lines).

4. Results

We start by making a consistency check, verifying that by using WMAP5 + All data and imposing $\xi_\nu = 0$ and $\Delta N_{\text{eff}}^\text{oth} = 0$ priors we obtain results in agreement with the ones obtained by the WMAP collaboration [56, 57].

In order to understand how the extra relativistic energy density and the leptonic asymmetry affect the determination of other cosmological parameters, we compute first the likelihood functions for WMAP5 + All by imposing the $\xi_\nu = 0$ prior, then extending our computation over the whole parameter space for WMAP5 + All and PLANCK-like simulated data. In figure 1 we compare the marginalized likelihood probabilities obtained for the main cosmological parameters.

As neutrinos with eV mass decouple when they are still relativistic ($T_{\text{dec}} \sim 2$ MeV), the main effect of including $\Delta N_{\text{eff}}$ is the change of relativistic energy density. This changes the redshift of matter–radiation equality, $z_{\text{eq}}$, that affects the determination of $\Omega_m h^2$ from...
Figure 2. The marginalized posterior likelihood probabilities of the neutrino mass, $m_\nu$, degeneracy parameter, $\xi_\nu$, leptonic asymmetry, $L_\nu$, helium mass fraction, $Y_p$, and redshift of matter–radiation equality, $z_{eq}$, for: WMAP5 + All with $\Delta N_{\text{eff}}=0$ and $\xi_\nu = 0$ priors (black lines), WMAP5 + All with $\xi_\nu = 0$ prior (green lines), WMAP5 + All with no neutrino priors (blue lines) and PLANCK-like simulated data with no neutrino priors (red lines).

CMB measurements because of its linear dependence on $N_{\text{eff}}$ [57]:

$$1 + z_{eq} = \frac{\Omega_m h^2}{\Omega_b h^2} \frac{1}{1 + 0.2271 N_{\text{eff}}}.$$  \hspace{1cm} (10)

Here $\Omega_b h^2 = 2.469 \times 10^{-5}$ is the present photon energy density parameter for $T_{\text{cmb}} = 2.725$ K. As a consequence, $N_{\text{eff}}$ and $\Omega_m h^2$ are linearly correlated, with the width of the degeneracy line given by the uncertainty in the determination of $z_{eq}$.

In figure 2 we compare the marginalized posterior likelihood probabilities of neutrino parameters, $Y_p$ and $z_{eq}$ obtained in our models. The mean values of these parameters and the corresponding (68% CL) error bars are given in table 3.

The LSS measurements provide an independent constraint on $\Omega_m h^2$ which helps to reduce the degeneracy of this parameter and $z_{eq}$. From WMAP5 + All data with $\Delta N_{\text{eff}}=0$ and $\xi_\nu = 0$ priors we find a mean value of $z_{eq} = 3158 \pm 68$ (68% CL). One should note that the mean value of $z_{eq}$ for the standard $\Lambda$CDM model with $N_{\text{eff}} = 3.046$ is $z_{eq} = 3176^{+151}_{-150}$ (68% CL) from WMAP5 data only [57].

Figure 3 presents the joint two-dimensional marginalized distributions (68% and 95% CL) in the $\Omega_m h^2$--$N_{\text{eff}}$ plane (left panel) and the $\Omega_m h^2$--$z_{eq}$ plane (right panel). The thick solid lines in the left panel show the 68% and 95% CL limits calculated from the corresponding limits on $z_{eq}$ obtained from the WMAP5 + All data with the $\xi_\nu = 0$ prior by using equation (10).
Table 3. The table shows the mean values and the absolute errors on the main cosmological parameters obtained from the analysis of WMAP5 + All data and PLANCK-like simulated data by using the central credible interval (CCI) statistical inference procedure. For all parameters we quote the errors at 68% CL.

| Parameter       | Priors: ξν = 0; ΔNeff = 0 | Priors: ξν = 0 | No neutrino priors | No neutrino priors |
|-----------------|---------------------------|---------------|-------------------|-------------------|
|                 | WMAP5 + All               | WMAP5 + All   | WMAP5 + All       | PLANCK            |
| Ω_b h^2        | 0.02247 ± 0.00053         | 0.02245 ± 0.00059 | 0.02246±0.00063   | 0.02247 ± 0.00021 |
| Ω_dm h^2       | 0.1094 ± 0.0028           | 0.1086 ± 0.0078 | 0.1115±0.0089     | 0.1110 ± 0.0038   |
| H_0             | 70.5 ± 1.7                | 69.9 ± 3.4     | 70.2±3.8          | 70.1 ± 2.3        |
| z_re            | 11.02 ± 1.71              | 11.07 ± 2.01   | 11.31±1.92        | 11.01 ± 0.39      |
| n_s             | 0.966 ± 0.013             | 0.965 ± 0.019  | 0.965 ± 0.018     | 0.961 ± 0.008     |
| ln[10^{10} A_s]| 3.264 ± 0.036             | 3.264 ± 0.051  | 3.265 ± 0.056     | 3.264 ± 0.017     |
| ξν              | —                         | —              | 0.005 ± 0.221     | 0.021 ± 0.089     |
| L_ν             | —                         | —              | 0.001 ± 0.165     | 0.015 ± 0.073     |
| Y_p             | 0.2480 ± 0.0002           | 0.2486 ± 0.0085| 0.2487±0.0451     | 0.2477 ± 0.0133   |
| z_eq            | 3158 ± 68                 | 3167±103      | 3124±120          | 3145 ± 34         |

When we transform the Neff axis of the left panel to the z_eq axis from the right panel we observe a strong degeneracy of z_eq and Ω_m h^2. This is valid for both WMAP5 + All with the ξν = 0 prior and WMAP5 + All with no neutrino priors. For the last case, the non-zero neutrino chemical potential augments the value of Neff by ΔNeff(ξν) as given in equation (1). This implies a larger expansion rate of the Universe, an earlier weak process freeze-out with a higher value for the neutron to proton density ratio, and thus a larger value of Y_p. On the other hand, a non-zero value of the electron neutrino chemical potential shifts the neutron–proton–beta equilibrium, leading to a large variation of Y_p [72, 73].

The left panel from figure 4 presents the two-dimensional marginalized joint probability distributions (68% and 95% CL), showing the degeneracy of Y_p and Neff. The total effect of ξν ≠ 0 is a noticeable increase of the projected error on Neff.

As the anisotropic stress of neutrinos leaves distinct signatures in the CMB power spectrum which are not degenerate with Ω_m h^2, we conclude from this analysis that we observe a non-zero value of Neff from WMAP5 + All data mainly via the change of z_eq rather than by the effect of neutrino anisotropic stress.

Our conclusion is not in agreement with the claim of the WMAP team concerning the strong evidence of neutrino anisotropic stress from a similar analysis of the WMAP5 + BAO + SN + HST data [57].

In table 4 we compare the estimates of Neff obtained from different inference schemes. It is interesting to note that all data sets bring the marginal posterior mode and the MCI confidence intervals toward the posterior mean and CCI intervals.

Our best estimates are Neff = 2.98±2.27±1.65 from WMAP5 + All with ξν = 0 prior and Neff = 3.08±3.69±5.08 from WMAP5 + All with no neutrino priors, in good agreement with Neff = 2.9±3.9±4.9 obtained in [89] by using WMAP5 + All data.
The confidence intervals from maximization are larger, leading to $1.15 < N_{\text{eff}} < 4.92$ (68% CL) with 2.89 as the best fit, from WMAP5 + All data with the $\xi_\nu = 0$ prior.

These values provide an important improvement over the similar result obtained from WMAP5 + BAO + SN + HST data: $N_{\text{eff}} = 4.4 \pm 1.5$ (68% CL).

From PLANCK-like simulated data the 68% error is $\sigma(N_{\text{eff}}) \approx 0.4$ from MCI and $\sigma(N_{\text{eff}}) \approx 0.8$ from maximization.

The analysis of WMAP5 + All data with the $\xi_\nu = 0$ prior provides a strong bound on the helium mass fraction of $Y_p = 0.2486 \pm 0.0085$ (68% CL), that rivals the bound on $Y_p$ obtained from the conservative analysis of the present data on helium abundance [2].

Under the assumption of degenerate BBN this bound is weakened, leading to $Y_p = 0.2487^{+0.0451}_{-0.0484}$ (68% CL) from WMAP5 + All with no neutrino priors, which reflects the strong dependence of $Y_p$ on $\xi_\nu$. 

Table 4. Point estimates and the confidence intervals (at 68% and 95% CL) for $N_{\text{eff}}$ obtained by using minimum credible interval (MCI), central credible interval (CCI) and maximization statistical inference procedures.

| Data; priors                      | MCI       |             |             |             | CCI       |             |             |             | Maximization |
|----------------------------------|-----------|-------------|-------------|-------------|-----------|-------------|-------------|-------------|--------------|
|                                  | $N_{\text{eff}}$ | 68% 95%↑    | 68% 95%↓    | (N_{\text{eff}}) | 68% 95%↑    | 68% 95%↓    | N_{\text{like}} | 68% 95%↑    | 68% 95%↓    |
| WMAP5 + All; $\xi_\nu = 0$       | 2.98      | 3.69 4.37   | 2.27 1.65   | 2.99 3.61   | 4.39 2.32   | 4.17 1.74   | 2.89 4.92   | 0.99 2.89   | 1.15 0.86    |
| WMAP5 + All; no neutrino priors  | 3.08      | 3.69 5.08   | 2.51 1.75   | 2.97 3.77   | 1.39 1.99   | 2.91 7.29   | 1.28 0.94   | 1.28 0.94   | 2.27 2.17    |
| PLANCK; no neutrino priors       | 3.01      | 3.41 3.82   | 2.70 2.42   | 3.09 3.44   | 3.76 2.85   | 2.75 2.52   | 2.65 1.84   | 2.65 1.84   | 2.27 2.17    |
Figure 4. Two-dimensional marginalized joint probability distributions (68% and 95% CL) showing the degeneracy of: $Y_p$ and $N_{\text{eff}}$ (left panel) and $N_{\text{eff}}$ and $\xi_\nu$ (right panel). The contours show: WMAP5 + All data with $\xi_\nu$ = 0 prior (green lines), WMAP5 + All data with no neutrino priors (blue lines) and PLANCK-like simulated data with no neutrino priors (red lines).

From PLANCK-like simulated data the 68% error on $Y_p$ is $\sigma(Y_p) = 0.0133$, fully consistent with $Y_p$ bounds obtained from the conservative analysis of the present data on helium abundance.

We get for neutrino degeneracy parameter a bound of $-0.216 \leq \xi_\nu \leq 0.226$ (68% CL) that represents an important improvement over the similar result obtained by using the WMAP three-year data [45].

The CMB only is able to constrain $\xi_\nu$ through its contribution to the radiation energy density during radiation domination epoch and the BBN constraints on $Y_p$.

We find that the sensitivity of CMB to $Y_p$ from PLANCK-like simulated data will reduce the error on $\xi_\nu$ down to $\sigma(\xi_\nu) \simeq 0.089$ (68% CL), a value fully consistent with the BBN bounds.

In the right panel from figure 4 we compare the two-dimensional marginalized joint probability distributions (68% and 95% CL) of $N_{\text{eff}}$ and $\xi_\nu$, showing the potentiality of the future high sensitivity CMB measurements to reduce the degeneracy of these parameters.

Table 5 presents the upper limits of the neutrino mass (68% and 95% CL) obtained by using CCI and maximization inference schemes. The inclusion in the analysis of LSS data significantly reduces the upper limit of the neutrino mass to $m_\nu < 0.29$ eV (95% CL) from CCI and $m_\nu < 0.46$ eV (95% CL) from maximization, for WMAP5 + All data with $\Delta N_{\text{eff}} = 0$ and $\xi_\nu = 0$ priors. One should note that the analysis of WMAP5 + BAO + SN + HST led to $m_\nu < 0.61$ eV (95% CL) [57].

Under the assumption of degenerate BBN this bound is weakened due to the degeneracy of $N_{\text{eff}}$ and $\xi_\nu$. For WMAP5 + All with no neutrino priors we find $m_\nu < 0.54$ eV (95%CL) while for PLANCK-like simulated data with no neutrino priors we obtain $m_\nu < 0.44$ eV (95% CL), by using CCI inference.
Table 5. Upper limits of the neutrino mass (68% and 95% CL) obtained by using central credible interval (CCI) and maximization inference procedures.

| Data; priors                     | CCI   | Maximization |
|----------------------------------|-------|--------------|
|                                  | $\sum m_\nu \,(\text{eV})$ | $\sum m_\nu \,(\text{eV})$ |
|                                  | <68%  | <68%         |
| WMAP5 + All; $\xi_\nu = 0$, $\Delta N_{\text{eff}} = 0$ | 0.23  | 0.41         |
| WMAP5 + All; $\xi_\nu = 0$    | 0.24  | 0.39         |
| WMAP5 + All; no neutrino priors | 0.27  | 0.37         |
| PLANCK; no neutrino priors      | 0.29  | 0.45         |

5. Summary and conclusions

In this paper, we set bounds on the radiation content of the Universe and neutrino properties by using the WMAP five-year CMB measurements complemented with most of the existing CMB and LSS measurements (WMAP5 + All), imposing also self-consistent BBN constraints on the primordial helium abundance, which proved to be important in the estimation of cosmological parameters [72,73].

We consider lepton asymmetric cosmological models parametrized by the neutrino degeneracy parameter $\xi_\nu$ and the variation of the relativistic degrees of freedom, $\Delta N_{\text{eff}}$, due to possible other physical processes occurring between BBN and structure formation epochs.

From our analysis we get $N_{\text{eff}} = 2.98^{+0.437}_{-0.227}$ from WMAP5 + All data with the $\xi_\nu = 0$ prior, providing an important improvement over the similar result obtained from WMAP5 + BAO + SN + HST data [57].

Although the LSS measurements provide an independent constraint on $\Omega_m h^2$ which helps to reduce the degeneracy of this parameter and the redshift of matter radiation equality, $z_{\text{eq}}$, we find a strong correlation between $\Omega_m h^2$ and $z_{\text{eq}}$, showing that we observe a non-zero $N_{\text{eff}}$ value from WMAP5 + All data mainly due to the change of $z_{\text{eq}}$, rather than the neutrino anisotropic stress.

The analysis of WMAP5 + All data with the $\xi_\nu = 0$ prior provides a strong bound on the helium mass fraction of $Y_p = 0.2486 \pm 0.0085$ (68% CL), that rivals the bound on $Y_p$ obtained from the conservative analysis of the present data on helium abundance [2]. Under the assumption of degenerate BBN this bound is weakened to $Y_p = 0.2487^{+0.0451}_{-0.0484}$ (68% CL), reflecting the strong dependence of $Y_p$ on $\xi_\nu$.

From the analysis of WMAP5 + All data we get for the neutrino degeneracy parameter a bound of $-0.216 \leq \xi_\nu \leq 0.226$ (68%CL).

The inclusion in the analysis of LSS data reduces the upper limit of the neutrino mass to $m_\nu < 0.54$ eV (95% CL), from WMAP5 + All data with no neutrino priors, with respect to the values obtained from the analysis from WMAP5-only data [56] and WMAP5 + BAO + SN + HST data [57].

The analysis of WMAP5 + All measurements also provides an important improvement over the similar results obtained by using WMAP one-year measurements complemented with LSS data [60] and the WMAP three-year data alone [45, 88].
We forecast that the CMB temperature and polarization measurements observed with high angular resolutions and sensitivity by the future PLANCK satellite will reduce the errors on $\xi_0$ and $Y_p$ down to $\sigma(\xi_0) \simeq 0.089$ (68% CL) and $\sigma(Y_p) = 0.0133$ (68% CL) respectively, values fully consistent with the BBN bounds on these parameters [12].

Our forecast errors on the cosmological parameters from PLANCK-like simulated data are also consistent with those obtained in [72].

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