Anomalous conformal currents, shadow fields and massive AdS fields

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In the framework of gauge invariant approach involving Stueckelberg and auxiliary fields, totally symmetric arbitrary spin anomalous conformal current and shadow field in flat space-time of dimension greater than or equal to four are studied. Gauge invariant differential constraints for such anomalous conformal current and shadow field and realization of global conformal symmetries are obtained. Gauge invariant two-point vertex of the arbitrary spin anomalous shadow field is also obtained. In Stueckelberg gauge frame, the two-point gauge invariant vertex becomes the standard two-point vertex of CFT. Light-cone gauge two-point vertex of the arbitrary spin anomalous shadow field is derived. The AdS/CFT correspondence for arbitrary spin anomalous conformal current and shadow field and the respective normalizable and non-normalizable modes of massive arbitrary spin AdS field is studied. The AdS field is considered in modified de Donder gauge which simplifies considerably the study of AdS/CFT correspondence. We show that on-shell leftover gauge symmetries of bulk massive field are related to gauge symmetries of boundary anomalous conformal current and shadow field, while the modified de Donder gauge condition for bulk massive field is related to differential constraints for boundary anomalous conformal current and shadow field.

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I. INTRODUCTION

We start with a brief recall of some basic notions of CFT. In space-time of dimension \( d \geq 4 \), fields of CFT can be separated into two groups: conformal currents and shadow fields. This is to say that field having Lorentz algebra spin \( s \), \( s \geq 1 \), and conformal dimension \( \Delta = s + d - 2 \) is referred to as conformal current with canonical dimension, while field having Lorentz algebra spin \( s \), \( s \geq 1 \), and conformal dimension \( \Delta > s + d - 2 \) is referred to as anomalous conformal current. Accordingly, field having Lorentz algebra spin \( s \), \( s \geq 1 \), and conformal dimension \( \Delta = 2 - s \) is referred to as shadow field with canonical dimension\(^1\), while field having Lorentz algebra spin \( s \), \( s \geq 1 \), and conformal dimension \( \Delta < 2 - s \) is referred to as anomalous shadow field.

In Refs.[3, 4], we developed the gauge invariant formulation of the conformal currents and shadow fields having the canonical conformal dimensions. We recall that, in the framework of AdS/CFT correspondence, such conformal currents and shadow fields are related to massless AdS fields. In Ref. [5], we extended our approach to the case of low spin-\( s \), \( s = 1, 2 \), anomalous conformal currents and shadow fields. The purpose of this paper is to develop gauge invariant approach to bosonic totally symmetric arbitrary spin-\( s \) anomalous conformal current and shadow field. In the framework of AdS/CFT correspondence, spin-\( s \) anomalous conformal current and shadow field are related to spin-\( s \) massive AdS field. Massive totally symmetric spin-\( s \) AdS fields with even \( s \geq 4 \) form leading Regge trajectory of AdS string theory. Therefore, extension of our approach to the case of arbitrary \( s \) is important. Our approach to anomalous conformal current and shadow field is summarized as follows.

i) Starting with the field content of anomalous conformal current (and anomalous shadow field) in the standard CFT, we introduce Stueckelberg fields and auxiliary fields. In other words, we extend space of fields entering the standard CFT.

ii) On the extended space of fields entering our approach, we introduce differential constraints, gauge transformations, and conformal algebra transformations. The differential constraints are required to be invariant under the gauge transformations and the conformal algebra transformations.

iii) The gauge symmetries and the differential constraints allow us to match our approach and the standard CFT. Namely, by imposing gauge conditions to exclude the Stueckelberg fields and by solving differential constraints to exclude the auxiliary fields, we obtain formulation of anomalous conformal current and shadow field in the standard CFT.

Besides the gauge invariant approach, we discuss the anomalous conformal current and shadow field by using Stueckelberg gauge and light-cone gauge conditions. Reasons for discussing these two gauge conditions are as follows.

i) The Stueckelberg gauge reduces our approach to the standard formulation of CFT. This is to say that use of the Stueckelberg gauge allows us to demonstrate how the standard approach to anomalous conformal current and shadow field is connected with our gauge invariant approach.

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\(^1\) Shadow fields having the canonical dimension are used to build conformal invariant equations of motion and Lagrangian formulations for conformal fields in Refs.[1]. Interesting discussion of shadow field dualities may be found in Ref.[2].
ii) Studying of CFT in the light-cone gauge frame is motivated by the conjectured duality of the supersymmetric Yang-Mills theory and AdS superstring theory [6]. We expect, by analogy with flat space, that a quantization of the type IIB Green-Schwarz AdS superstring [7] will be straightforward only in the light-cone gauge [8–10]. Therefore we think that, from the stringy perspective of AdS/CFT correspondence, the light-cone approach to CFT deserves to be understood better. In this respect, we note that our approach provides quick access to the light-cone gauge formulation of CFT. This implies that our approach gives easy access to the studying of AdS/CFT correspondence in light-cone gauge. This seems to be important for future application of our approach to the studying of string/gauge theory duality.

We use our gauge invariant formulation of CFT for the studying AdS/CFT correspondence between arbitrary spin massive AdS field and the corresponding arbitrary spin boundary anomalous conformal current and shadow field. Namely, we show that non-normalizable modes of arbitrary spin-s massive AdS field are related to arbitrary spin-s anomalous shadow field, while normalizable modes of arbitrary spin-s massive AdS field are related to arbitrary spin-s anomalous conformal current. We recall that, in earlier literature, the AdS/CFT correspondence between non-normalizable modes of massive spin-1 and spin-2 AdS fields and the corresponding spin-1 and spin-2 anomalous shadow fields was studied in Refs. [11, 12]. The AdS/CFT correspondence for spin-s massive AdS field with s > 2 and the corresponding spin-s anomalous conformal current and shadow field has not been considered in the earlier literature. Our treatment of AdS/CFT correspondence is summarized as follows.

i) We exploit the CFT adapted gauge invariant approach to massive AdS fields and modified de Donder gauge obtained in Refs. [13, 14]. The modified de Donder gauge leads to the simple decoupled bulk equations of motion which are easily solved. We show that the two-point gauge invariant vertex for the arbitrary spin-s anomalous shadow field does indeed emerge from massive arbitrary spin-s AdS field action when it is evaluated on solution of the Dirichlet problem. Throughout this paper the AdS field action evaluated on the solution of the Dirichlet problem is referred to as effective action.

ii) The number of boundary gauge fields involved in our approach to the anomalous conformal current (or anomalous shadow field) coincides with the number of gauge fields involved in the CFT adapted formulation of massive AdS field in Ref. [14].

iii) The number of gauge transformation parameters involved in our approach to the anomalous conformal current (or anomalous shadow field) coincides with the number of gauge transformation parameters involved in the CFT adapted gauge invariant formulation of massive AdS field in Ref. [14].

iv) The modified de Donder gauge for massive bulk field is related to the differential constraint for boundary anomalous conformal current (or anomalous shadow field).

v) On-shell leftover gauge symmetries of massive bulk field are related to the gauge symmetries of boundary anomalous conformal current (or anomalous shadow field).

Our paper is organized as follows.

In Sec. [II] we summarize our notation and conventions.

Section [III] is devoted to gauge invariant formulation of arbitrary spin-s anomalous conformal current. We discuss gauge symmetries and realization of global conformal algebras symmetries on space of gauge fields we use for the description of the anomalous conformal current. We demonstrate how our gauge invariant approach is related to the standard CFT. Also, using our approach, we obtain light-cone gauge description of the arbitrary spin-s anomalous conformal current.

In Sec. [IV] we extend results in Sec. [III] to the case of arbitrary spin-s anomalous shadow field. Also, we find gauge invariant two-point vertex for the arbitrary spin-s anomalous shadow field. We discuss the two-point vertex in Stueckelberg gauge frame and in light-cone gauge frame.

In Sec. [V] we discuss the two-point current-shadow field interaction vertex.

In Sec. [VI] we review the CFT adapted gauge invariant approach to massive arbitrary spin AdS field. Because the use of the modified de Donder gauge makes our study of AdS/CFT correspondence for arbitrary spin fields similar to the one for scalar field, we briefly review the AdS/CFT correspondence for the scalar field.

Section [VII] is devoted to the study of AdS/CFT correspondence between normalizable modes of massive spin-s AdS field and spin-s anomalous conformal current, while, in Sec. [VIII] we study the AdS/CFT correspondence between non-normalizable modes of massive spin-s AdS field and spin-s anomalous shadow field.

Section [IX] summarizes our conclusions and suggests directions for future research.

In Appendix, we present some details of matching of the bulk and boundary conformal boost symmetries.

II. PRELIMINARIES

A. Notation

We use the following conventions. The Cartesian coordinates in d-dimensional flat space-time are denoted by $x^a$, while derivatives with respect to $x^a$ are denoted by $\partial_a, \partial_a \equiv \partial / \partial x^a$. The vector indices of the Lorentz algebra so(d–1, 1) take the values $a, b, c, e = 0, 1, \ldots, d - 1$. We use the mostly positive flat metric tensor $\eta^{ab}$ and, to simplify our expressions,
we drop \( \eta_{ab} \) in scalar products: \( X^a Y^a \equiv \eta_{ab} X^a Y^b \). Creation operators \( \alpha^a, \alpha^z, \zeta \) and the respective annihilation operators \( \bar{\alpha}^a, \bar{\alpha}^z, \bar{\zeta} \) are referred to as oscillators.\(^2\) Commutation relations of the oscillators, the vacuum \( |0\rangle \), and hermitian conjugation rules are defined as
\[
[a^a, a^b] = \eta^{ab}, \quad [\bar{a}^a, \bar{a}^z] = 1, \quad [\bar{\zeta}, \zeta] = 1, \quad (2.1)
\]
\[
\bar{a}^a |0\rangle = 0, \quad \bar{a}^z |0\rangle = 0, \quad \bar{\zeta} |0\rangle = 0, \quad (2.2)
\]
\[
\alpha^a \equiv \bar{\alpha}^a, \quad \alpha^z \equiv \bar{a}^z, \quad \zeta \equiv \bar{\zeta}, \quad (2.3)
\]
The oscillators \( \alpha^a, \bar{\alpha}^a \) and \( \zeta, \bar{\zeta}, \bar{\alpha}^z \) transform in the respective vector and scalar representations of the Lorentz algebra \( so(d - 1,1) \). Throughout this paper we use operators constructed out of the derivatives, coordinates, and the oscillators,
\[
\Box \equiv \partial^a \partial_a, \quad x \partial \equiv x^a \partial_a, \quad x^2 \equiv x^a x^a, \quad (2.4)
\]
\(^2\) We use oscillators to handle the many indices appearing for tensor fields (discussion of oscillator formulation may be found in Refs. [15, 16].)

\[
r_\zeta \equiv \left( \frac{(s + \frac{d-4}{2} - N_\zeta)(\kappa - s - \frac{d-4}{2} + N_\zeta)(\kappa + 1 + N_\zeta)}{2(s + \frac{d-4}{2} - N_\zeta - N_\zeta)(\kappa + N_\zeta - N_\zeta + 1)} \right)^{1/2},
\]
\[
r_z \equiv \left( \frac{(s + \frac{d-4}{2} - N_z)(\kappa + s + \frac{d-4}{2} - N_z)(\kappa - 1 - N_z)}{2(s + \frac{d-4}{2} - N_\zeta - N_\zeta)(\kappa + N_\zeta - N_\zeta + 1)} \right)^{1/2},
\]
where parameter \( \kappa \) appearing in (2.13) is defined below in (3.4). Throughout the paper the notation \( \lambda \in [n]_2 \) implies that
\[
\lambda \in [n]_2 \implies \lambda = -n, -n + 2, -n + 4, \ldots , n - 4, n - 2, n.
\]
Often, we use the following set of scalar, vector, and totally symmetric tensor fields of the Lorentz algebra \( so(d - 1,1) \):
\[
\phi_{x_{a_1 \ldots a_{s'}}}^{\lambda_{s'}}, \quad s' = 0, 1, \ldots , s, \quad \lambda \in [s - s']_2, \quad (2.15)
\]
where \( \phi_{x_{a_1 \ldots a_{s'}}}^{\lambda_{s'}} \) is rank- \( s' \) totally symmetric traceful tensor field of the Lorentz algebra \( so(d - 1,1) \). To illustrate the field content given in (2.15), we use shortcut \( \phi_{\lambda}^s \) for the field \( \phi_{x_{a_1 \ldots a_{s'}}}^{\lambda_{s'}} \) and note that fields in (2.15) can be represented as
\[
\phi_{0}^s \\
\phi_{-1}^s \quad \phi_{1}^s \\
\ldots \\
\phi_{-s}^s \quad \phi_{s}^s
\]
(2.16)

Our conventions for light-cone frame are as follows. The space-time coordinates are decomposed as \( x^a = x^+, x^-, x^i \), where the coordinates in \( \pm \) directions are defined as
\[x^\pm = \ldots \]
\[(x^{d-1} \pm x^0)/\sqrt{2} \text{ and } x^+ \text{ is taken to be a light-cone time.} \]

Vector indices of the \( so(d - 2) \) algebra take values \( i, j = 1, \ldots, d - 2 \). We use the following conventions for the derivatives:

\[\partial^i = \partial_i \equiv \partial/\partial x^i, \quad \partial^\pm = \partial/\partial x^\pm. \quad (2.17)\]

**B. Global conformal symmetries**

In \( d \)-dimensional flat space-time, the conformal algebra \( so(d, 2) \) consists of translation generators \( P^a \), dilatation generator \( D \), conformal boost generators \( K^a \), and generators of the \( so(d - 1, 1) \) Lorentz algebra \( J^{ab} \). We use the following nontrivial commutators of the conformal algebra:

\[ [D, P^a] = -P^a, \quad [P^a, J^{bc}] = \eta^{ab} P^c - \eta^{ac} P^b, \]

\[ [D, K^a] = K^a, \quad [K^a, J^{bc}] = \eta^{ab} K^c - \eta^{ac} K^b, \quad (2.18) \]

\[ [P^a, K^b] = \eta^{ab} D - J^{ab}, \]

\[ [J^{ab}, J^{ce}] = \eta^{bc} J^{ae} + 3 \text{ terms}. \]

Let \( \phi \) denotes anomalous conformal current (or anomalous shadow field) in the \( d \)-dimensional flat space-time. Under the action of conformal algebra, the \( \phi \) transforms as

\[ \delta_G \phi = \hat{G} \phi, \quad (2.19) \]

where the realization of the conformal algebra generators \( \hat{G} \) on space of \( \phi \) is given by

\[ P^a = \partial^a, \quad (2.20) \]

\[ J^{ab} = x^a \partial^b - x^b \partial^a + M^{ab}, \quad (2.21) \]

\[ D = x \partial + \Delta, \quad (2.22) \]

\[ K^a = K^a_{\Delta M} + R^a, \quad (2.23) \]

\[ K^a_{\Delta M} = -\frac{1}{2} x^2 \partial^a + x^a D + M^{ab} x^b. \quad (2.24) \]

In relations (2.21)-(2.23), \( \Delta \) is an operator of conformal dimension, while \( M^{ab} \) is a spin operator of the Lorentz algebra,

\[ [M^{ab}, M^{ce}] = \eta^{bc} M^{ae} + 3 \text{ terms}. \quad (2.25) \]

The spin operator of the Lorentz algebra is well known for arbitrary spin anomalous conformal current and shadow field (see (2.8)). In general, operator \( R^a \) appearing in (2.23) depends on the derivatives and does not depend on the space-time coordinates\(^3\). In the standard CFT, the operator \( R^a \) is equal to zero, while, in the gauge invariant approach to anomalous conformal current and shadow field we develop in this paper, the operator \( R^a \) is nontrivial. This is to say that, in the framework of our gauge invariant approach, the complete description of the conformal current and shadow field requires, among other things, finding the operator \( R^a \).

**III. ARBITRARY SPIN ANOMALOUS CONFORMAL CURRENT**

**A. Gauge invariant formulation**

**Field content.** To develop gauge invariant formulation of arbitrary spin-\( s \) anomalous conformal current in flat space of dimension \( d \geq 4 \) we use the following fields:

\[ \phi^{a_1 \ldots a_s'}_{\text{cur}, \lambda}, \quad s' = 0, 1, \ldots, s, \quad \lambda \in [s - s']_2. \quad (3.1) \]

We note that:

i) In (3.1), the fields \( \phi_{\text{cur}, \lambda} \) and \( \phi_{\text{cur}, \lambda}^{s, \lambda} \) are the respective scalar and vector fields of the Lorentz algebra, while the field \( \phi^{a_1 \ldots a_s'}_{\text{cur}, \lambda}, s' \geq 1 \), is rank-\( s' \) totally symmetric traceful tensor field of the Lorentz algebra \( so(d - 1, 1) \). Using shortcut \( \phi^{a_1 \ldots a_s'}_{\text{cur}, \lambda} \) for the field \( \phi^{a_1 \ldots a_s'}_{\text{cur}, \lambda} \), fields in (3.1) can be represented as in (2.16).

ii) The tensor fields \( \phi^{a_1 \ldots a_s'}_{\text{cur}, \lambda} \) with \( s' \geq 4 \) satisfy the double-tracelessness constraint

\[ \phi^{a_1 \ldots a_s'}_{\text{cur}, \lambda} = 0, \quad s' = 4, 5, \ldots, s. \quad (3.2) \]

iii) The fields \( \phi^{a_1 \ldots a_s'}_{\text{cur}, \lambda} \) have the following conformal dimensions:

\[ \Delta (\phi^{a_1 \ldots a_s'}_{\text{cur}, \lambda}) = \frac{d}{2} + \kappa + \lambda. \quad (3.3) \]

iv) In the framework of AdS/CFT correspondence, \( \kappa \) is related to the mass parameter \( m \) of spin-\( s \) massive field in \( AdS_{d+1} \) as

\[ \kappa \equiv \sqrt{m^2 + (s + \frac{d - 4}{2})^2}. \quad (3.4) \]

In order to obtain the gauge invariant description in an easy-to-use form we use the oscillators and introduce a ket-vector \( |\phi_{\text{cur}}\rangle \) defined by

\[ |\phi_{\text{cur}}\rangle = \sum_{s' = 0}^{s} |\phi^{s'}_{\text{cur}}\rangle, \quad (3.5) \]

this paper, the operator \( R^a \) is independent of the derivatives. Dependence of the operator \( R^a \) on the derivatives appears in the ordinary-derivative approach to conformal fields (see Refs. [13]-[19]).

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\(^3\) For the anomalous conformal currents and shadow fields considered in this paper, the operator \( R^a \) is independent of the derivatives. Dependence of the operator \( R^a \) on the derivatives appears in the ordinary-derivative approach to conformal fields (see Refs. [13]-[19]).
\[ |\phi_{\text{cur}}^{s'}\rangle \]
\[ = \sum_{\lambda \in [s-s']_{2}} \xi^{s-s'-\frac{d}{2}} \alpha_{s} \ldots \alpha_{s'} \phi_{\text{cur},\lambda} |0\rangle. \]

From (3.5), we see that the ket-vector \( |\phi_{\text{cur}}\rangle \) is degree-\( s \) homogeneous polynomial in the oscillators \( \alpha^{s}, \alpha^{z}, \zeta \), while the ket-vector \( |\phi_{\text{cur}}^{s'}\rangle \) is degree-\( s' \) homogeneous polynomial in the oscillators \( \alpha^{s'}, \zeta \), i.e., these ket-vectors satisfy the relations
\[ (N_{a} + N_{z} + N_{\zeta} - s)|\phi_{\text{cur}}\rangle = 0, \quad (N_{a} - s')|\phi_{\text{cur}}^{s'}\rangle = 0. \]

In terms of the ket-vector \( |\phi_{\text{cur}}\rangle \), double-tracelessness constraint (3.2) takes the form
\[ (\alpha^{2})^{2}|\phi_{\text{cur}}\rangle = 0. \quad (3.8) \]

**Differential constraint.** We find the following differential constraint for the anomalous conformal current:
\[ C_{\text{cur}}|\phi_{\text{cur}}\rangle = 0, \quad (3.9) \]
\[ C_{\text{cur}} = \tilde{e}\frac{\partial}{\partial \tilde{e}} - \frac{1}{2} \alpha \partial \alpha^{2} - \tilde{e}_{1,\text{cur}} \Pi^{[i]}\partial \alpha^{2} + \frac{1}{2} \epsilon_{1,\text{cur}} \alpha^{2}, \quad (3.10) \]
\[ e_{1,\text{cur}} = \tilde{r}\zeta \alpha^{1} + \alpha \zeta r_{z}, \quad \tilde{e}_{1,\text{cur}} = -r\tilde{r}\zeta - r_{z}\alpha^{2}, \quad (3.11) \]

where the operators \( \Pi^{[i]} \), \( r_{\zeta} \), \( r_{z} \) are defined in (2.9), (2.13). One can make sure that constraint (3.9) is invariant under gauge transformation and conformal algebra transformations which we discuss below.

**Gauge symmetries.** We now discuss gauge symmetries of the anomalous conformal current. To this end we introduce the following gauge transformation parameters:
\[ \zeta^{a_{1} \ldots a_{s'}}, \quad s' = 0, 1, \ldots, s - 1, \quad \lambda \in [s - 1 - s']_{2}. \quad (3.12) \]

We note that
\[ i) \text{In (3.12), the gauge transformation parameters } \zeta^{a_{1} \ldots a_{s'}} \text{ and } \zeta^{a_{1} \ldots a_{s'}} \text{ are the respective scalar and vector fields of the Lorentz algebra, while the gauge transformation parameter } \zeta^{a_{1} \ldots a_{s'}}, \quad s' > 1, \text{ is rank-}s' \text{ totally symmetric tensor field of the Lorentz algebra } so(d-1,1). \]

\[ \text{ii) The gauge transformation parameters } \zeta^{a_{1} \ldots a_{s'}} \text{ with } s' \geq 2 \text{ satisfy the tracelessness constraint,} \]
\[ \zeta^{a_{0}a_{1} \ldots a_{s'}}_{\text{cur},\lambda} = 0, \quad s' = 2, 3, \ldots, s - 1. \quad (3.13) \]

\[ \text{iii) The gauge transformation parameters } \zeta^{a_{1} \ldots a_{s'}} \text{ have the conformal dimensions} \]
\[ \Delta(\zeta^{a_{1} \ldots a_{s'}}) = \frac{d}{2} + \kappa + \lambda - 1. \quad (3.14) \]

Now, as usually, we collect the gauge transformation parameters in a ket-vector \( |\xi_{\text{cur}}\rangle \) defined by
\[ |\xi_{\text{cur}}\rangle = \sum_{s'=0}^{s-1} |\xi_{\text{cur}}^{s'}\rangle, \quad (3.15) \]
\[ |\xi_{\text{cur}}^{s'}\rangle = \sum_{\lambda \in [s-1-s']_{2}} \xi^{s-s'-\frac{d}{2}} \alpha_{s} \ldots \alpha_{s'} \phi_{\text{cur},\lambda} |0\rangle. \]

The ket-vectors \( |\xi_{\text{cur}}\rangle, |\xi_{\text{cur}}^{s'}\rangle \) satisfy the algebraic constraints
\[ (N_{a} + N_{z} + N_{\zeta} - s + 1)|\xi_{\text{cur}}\rangle = 0, \quad (\Delta \text{ of } s+1) \quad (3.16) \]
\[ (N_{a} - s')|\xi_{\text{cur}}^{s'}\rangle = 0, \quad (3.17) \]

which tell us that \( |\xi_{\text{cur}}\rangle \) is a degree-(\( s+1 \)) homogeneous polynomial in the oscillators \( \alpha^{s}, \alpha^{z}, \zeta \), while the ket-vector \( |\xi_{\text{cur}}^{s'}\rangle \) is degree-\( s' \) homogeneous polynomial in the oscillators \( \alpha^{s'} \). In terms of the ket-vector \( |\xi_{\text{cur}}\rangle \), tracelessness constraint (3.13) takes the form
\[ \tilde{a}^{2}|\xi_{\text{cur}}\rangle = 0. \quad (3.18) \]

Gauge transformation can entirely be written in terms of \( |\phi_{\text{cur}}\rangle \) and \( |\xi_{\text{cur}}\rangle \). This is to say that gauge transformation takes the form
\[ \delta|\phi_{\text{cur}}\rangle = G_{\text{cur}}|\xi_{\text{cur}}\rangle, \quad (3.19) \]
\[ G_{\text{cur}} \equiv \alpha \partial - \epsilon_{1,\text{cur}} - \alpha \frac{1}{2N_{a} + d - 2}\tilde{e}_{1,\text{cur}}, \quad (3.20) \]

where \( \epsilon_{1,\text{cur}}, \tilde{e}_{1,\text{cur}} \) are given in (3.11). As we have already said, constraint (3.9) is invariant under gauge transformation (3.19).

**Realization of conformal algebra symmetries.** To complete the gauge invariant formulation of the spin-\( s \) anomalous conformal current we provide realization of the conformal algebra symmetries on space of the ket-vector \( |\phi_{\text{cur}}\rangle \). All that is needed is to fix the operators \( M^{ab}, \Delta \), and \( R^{a} \) and insert then these operators into (2.22), (2.23). Realization of the spin operator \( M^{ab} \) on ket-vector \( |\phi_{\text{cur}}\rangle \) is given in (2.3), while realization of the operator \( \Delta \)
\[ \Delta_{\text{cur}} = \frac{d}{2} + \nu, \quad \nu \equiv \kappa + N_{\zeta} - N_{z}. \quad (3.21) \]

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4 In this paper we adapt the formulation in terms of the double traceless gauge fields [23]. To develop the gauge invariant approach one can use unconstrained gauge fields studied in Refs. [21]. Discussion of other gauge fields which seem to be most suitable for the theory of interacting fields may be found, e.g., in [22].
The operators can be read from (3.3). In the gauge invariant formulation, finding the operator $R^a$ provides the real difficulty. We find the following realization of the operator $R^a$ on space of $|\phi_{\text{cur}}\rangle$:

$$R^a_{\text{cur}} = -2\zeta r_z (\nu + 1)\tilde{C}^a - \tilde{C}_\alpha^a + \frac{1}{2N_\alpha + d - 2} \tilde{C}_\alpha^a r_z \tilde{C}^z ,$$

(3.22)

where the operators $\tilde{C}^a, \tilde{C}_\alpha^a$ are given in (2.11),(2.12), while the operators $r_z, r_z$ are defined in (2.13).

We derived the differential constraint, gauge transformation, and the realization of the operator $R^a_{\text{cur}}$ by generalizing our results for the spin-\(s\) conformal current with the canonical dimension which we obtained in Ref. [3]. It is worthwhile to note that results in this section can also be obtained by using the tractor approach in Refs. [23, 24]. Our constraint (3.9) and gauge transformation (3.19) can be matched with the ones in Ref. [23] by using appropriate field redefinitions. The basis of the fields we use in this paper turns out to be more convenient for the study of AdS/CFT correspondence. To summarize, our fields (3.1) can be written as a tractor rank-\(s\) tensor field subject to a Thomas-D divergence type constraint in Ref. [23]. A similar construction was used to describe spin-\(s\) massive bulk field in Ref. [23]. Note that, in our approach, we use our fields (3.1) for the discussion of spin-\(s\) anomalous conformal current.

### B. Stueckelberg gauge frame

We proceed with discussion of the spin-\(s\) anomalous conformal current in the Stueckelberg gauge frame. To this end we note that the Stueckelberg gauge frame is achieved through the use of differential constraint (3.9) and the Stueckelberg gauge condition. From (3.19), we see that a field defined by

$$\alpha^2 \Pi^{(1,2)} |\phi_{\text{cur}}\rangle$$

(3.23)

transforms as Stueckelberg field. Therefore this field can be gauged away via Stueckelberg gauge fixing,

$$\alpha^2 \Pi^{(1,2)} |\phi_{\text{cur}}\rangle = 0 ,$$

(3.24)

where $\Pi^{(1,2)}$ is given in (2.9). Using gauge condition (3.24) and differential constraint (3.9), we find the relations

$$\alpha^2 |\phi_{\text{cur}}\rangle = 0 ,$$

(3.25)

$$|\phi'_{\text{cur}}\rangle = X_{\text{cur}} \zeta^{s-s'} (\tilde{a} \partial)^{s-s'} |\phi_{\text{cur}}\rangle ,$$

(3.26)

Relation (3.25) tells us that all fields $\phi_{\text{cur},\Lambda}^{a_1...a_s}$ with $s' \geq 2$ become traceless. From relation (3.26), we learn that the fields $\phi_{\text{cur},\Lambda}^{a_1...a_{s'}}$ with $\lambda \neq s-s'$ become equal to zero, while the fields $\phi_{\text{cur},\Lambda}^{a_1...a_{s'}}$ with $\lambda = s-s'$ and $s' = 0, 1, \ldots s-1$ are expressed in terms of the rank-\(s\) traceless tensor field $\phi_{\text{cur},0}^{a_1...a_s}$.

$$\phi_{\text{cur},\Lambda}^{a_1...a_{s'}} = 0 , \quad \text{for} \quad \lambda \neq s-s' ,$$

(3.27)

$$\phi_{\text{cur},s-s'} = \sqrt{(s-s')!} X_{\text{cur}} \partial_{b_1} \ldots \partial_{b_{s-s'}} \phi_{\text{cur},0}^{b_1...b_{s-s'}a_1...a_s} .$$

(3.28)

We recall that, in the standard CFT, the spin-\(s\) anomalous conformal current is described by the traceless field $\phi_{\text{cur},0}^{a_1...a_s}$. Thus we see that making use of the gauge symmetry and differential constraint we reduce the field content of our approach to the one in the standard CFT. In other words, use of the gauge symmetry and differential constraint allows us to match our approach and the standard formulation of the spin-\(s\) anomalous conformal current. To summarize, our gauge invariant approach is equivalent to the standard one.

### C. Light-cone gauge frame

We now discuss the spin-\(s\) anomalous conformal current in the light-cone gauge frame. To this end we note that, for the anomalous conformal current, the light-cone gauge frame is achieved through the use of differential constraint (3.9) and light-cone gauge condition. Using gauge symmetry of the spin-\(s\) anomalous conformal current (3.19), we impose the light-cone gauge on the $|\phi_{\text{cur}}\rangle$,

$$\alpha^+ \Pi^{(1,2)} |\phi_{\text{cur}}\rangle = 0 ,$$

(3.29)

where $\Pi^{(1,2)}$ is given in (2.9). Using gauge (3.29) and differential constraint (3.9), we find

$$|\phi_{\text{cur}}\rangle = \exp \left( - \frac{\alpha^+}{\alpha^+} (\alpha \partial \partial - \bar{e}_{1 \text{ cur}}) \right) |\phi_{\text{cur}}^{1,c}\rangle ,$$

(3.30)

$$\alpha^+ |\phi_{\text{cur}}^{1,c}\rangle = 0 ,$$

(3.31)

where a light-cone ket-vector $|\phi_{\text{cur}}^{1,c}\rangle$ is obtained from $|\phi_{\text{cur}}\rangle$ by equating $\alpha^+ = \alpha^- = 0$,

$$|\phi_{\text{cur}}^{1,c}\rangle \equiv |\phi_{\text{cur}}\rangle |_{\alpha^+=\alpha^-=0} ,$$

(3.32)

---

5 In mathematical literature, interesting discussion of the tractor approach may be found in Ref. [28].

6 We note that, as in the standard CFT, our currents can be considered either as composite operators or as fundamental field degrees of freedom. At the group theoretical level, we study in this paper, this distinction does not matter. Methods for building conformal currents as composite operators are discussed in Refs. [29].
We see that we are left with light-cone fields
\[ \phi_{\text{cur},\lambda}^{i_1\ldots i_s}, \quad s' = 0, 1, \ldots, s, \quad \lambda \in [s-s']_2, \]  
which are traceless tensor fields of \( so(d-2) \) algebra, \( \phi_{\text{cur},\lambda}^{i_1\ldots i_s} = 0 \). These fields constitute the field content of the light-cone gauge frame. Note that, in contrast to the Stueckelberg gauge frame, all fields \( \phi_{\text{cur},\lambda}^{i_1\ldots i_s} \) are not equal to zero. Also note that, in contrast to the gauge invariant approach, the fields (3.33) are not subject to any differential constraint.

IV. ARBITRARY SPIN ANOMALOUS SHADOW FIELD

A. Gauge invariant formulation

Field content. To discuss gauge invariant formulation of arbitrary spin-\( s \) anomalous shadow field in flat space of dimension \( d \geq 4 \) we use the following fields:
\[ \phi_{s\lambda}^{a_1\ldots a_s}, \quad s' = 0, 1, \ldots, s, \quad \lambda \in [s-s']_2. \]  
We note that:

i) In (4.1), the fields \( \phi_{s\lambda}^{a_1\ldots a_s} \) are the respective scalar and vector fields of the Lorentz algebra, while the field \( \phi_{s\lambda}^{a_1\ldots a_s}, s' > 1 \), is rank-\( s' \) totally symmetric traceful tensor field of the Lorentz algebra \( so(d-1,1) \). Using shortcut \( \phi_{s\lambda}^{a_1\ldots a_s} \) for the field \( \phi_{s\lambda}^{a_1\ldots a_s} \), fields in (4.1) can be represented as in 2.16.

ii) The tensor fields \( \phi_{s\lambda}^{a_1\ldots a_s} \) with \( s' \geq 4 \) satisfy the double-tracelessness constraint
\[ \phi_{s\lambda}^{a_1\ldots a_s} = 0, \quad s' = 4, 5, \ldots, s. \]  

iii) The fields \( \phi_{s\lambda}^{a_1\ldots a_s} \) have the following conformal dimensions:
\[ \Delta(\phi_{s\lambda}^{a_1\ldots a_s}) = \frac{d}{2} - \kappa + \lambda, \]  
where \( \kappa \) is related to the mass parameter \( m \) of spin-\( s \) massive field in \( AdS_{d+1} \) as in (3.3).

In order to obtain the gauge invariant description in an easy-to-use form we use the oscillators and introduce a ket-vector \( |\phi_{s\lambda}\rangle \) defined by
\[ |\phi_{s\lambda}\rangle = \sum_{s'=0}^{s} |\phi_{s\lambda}^{s'}\rangle, \]  
\[ |\phi_{s\lambda}^{s'}\rangle = \sum_{\lambda \in [s-s']_2} \frac{\xi_{s\lambda}^{a_1\ldots a_{s'}}}{s'! \sqrt{(s-s')!((s-s'-\kappa+\lambda)!)} \phi_{s\lambda}^{a_1\ldots a_{s'}}|0\rangle, \]  
From (4.4), we see that the ket-vector \( |\phi_{s\lambda}\rangle \) is degree-\( s \) homogeneous polynomial in the oscillators \( \alpha, \alpha^2, \zeta \), while the ket-vector \( |\phi_{s\lambda}^{s'}\rangle \) is degree-\( s' \) homogeneous polynomial in the oscillators \( \alpha, \zeta \), i.e., these ket-vectors satisfy the relations
\[ (N_\alpha + N_\zeta - s)|\phi_{s\lambda}\rangle = 0, \]  
\[ (N_\alpha - s')|\phi_{s\lambda}^{s'}\rangle = 0. \]

In terms of the ket-vector \( |\phi_{s\lambda}\rangle \), double-tracelessness constraint (4.2) takes the form
\[ (\bar{\alpha}^2)^2|\phi_{s\lambda}\rangle = 0. \]  

Differential constraint. We find the following differential constraint for the anomalous shadow field:
\[ \tilde{C}_{s\lambda}|\phi_{s\lambda}\rangle = 0; \]  
\[ \tilde{C}_{s\lambda} = \tilde{\alpha} \tilde{\alpha} - \frac{1}{2} \alpha \bar{\alpha}^2 - \tilde{e}_{s\lambda} \Pi^{[1,2]} + \frac{1}{2} \tilde{e}_{s\lambda} \bar{\alpha}^2, \]  
where the operators \( \Pi^{[1,2]} \) are defined in (2.9), (2.13). One can make sure that constraint (4.8) is invariant under gauge transformation and conformal algebra transformations which we discuss below.

Gauge symmetries. We now discuss gauge symmetries of the anomalous shadow field. To this end we introduce the following gauge transformation parameters:
\[ \xi_{s\lambda}^{a_1\ldots a_{s'}}, \quad s' = 0, 1, \ldots, s - 1, \quad \lambda \in [s - 1 - s']_2. \]  
We note that

i) In (4.11), the gauge transformation parameters \( \xi_{s\lambda}^{a_1\ldots a_{s'}} \) are the respective scalar and vector fields of the Lorentz algebra, while the gauge transformation parameter \( \xi_{s\lambda}^{a_1\ldots a_{s'}}, s' > 1 \), is rank-\( s' \) totally symmetric tensor field of the Lorentz algebra \( so(d-1,1) \).

ii) The gauge transformation parameters \( \xi_{s\lambda}^{a_1\ldots a_{s'}} \) with \( s' \geq 2 \) satisfy the tracelessness constraint
\[ \xi_{s\lambda}^{a_1\ldots a_{s''}} = 0, \quad s' = 2, 3, \ldots, s - 1. \]  

iii) The gauge transformation parameters \( \xi_{s\lambda}^{a_1\ldots a_{s'}} \) have the conformal dimensions
\[ \Delta(\xi_{s\lambda}^{a_1\ldots a_{s'}}) = \frac{d}{2} - \kappa + \lambda - 1. \]

Now, as usually, we collect the gauge transformation parameters in a ket-vector \( |\xi_{s\lambda}\rangle \) defined by
\[ |\xi_{s\lambda}\rangle = \sum_{s'=0}^{s-1} |\xi_{s\lambda}^{s'}\rangle, \]
\[ |\xi_{sh}'\rangle = \sum_{\lambda \in \{1-s'\}} \frac{\xi_{\lambda} e_{\lambda} - \xi_{\lambda} e_{\lambda}}{s!\sqrt{(\xi_{\lambda} e_{\lambda} - \xi_{\lambda} e_{\lambda})!}} \psi_{sh,\lambda} |0\rangle. \]

The ket-vectors \(|\xi_{sh}\rangle, |\xi_{sh}'\rangle\) satisfy the algebraic constraints

\[
(N_\alpha + N_z + N_\xi - s + 1)|\xi_{sh}\rangle = 0, \quad (N_\alpha - s')|\xi_{sh}'\rangle = 0, \tag{4.15} \tag{4.16}
\]

which tell us that \(|\xi_{sh}\rangle\) is a degree-(s-1) homogeneous polynomial in the oscillators \(\alpha^a, \alpha^\xi, \zeta\), while the ket-vector \(|\xi_{sh}'\rangle\) is degree-\(s'\) homogeneous polynomial in the oscillators \(\alpha^a\).

In terms of the ket-vector \(|\xi_{sh}\rangle\), tracelessness constraint (4.12) takes the form

\[
\bar{a}^2|\xi_{sh}\rangle = 0. \tag{4.17}
\]

Gauge transformation can entirely be written in terms of \(|\phi_{sh}\rangle\) and \(|\xi_{sh}\rangle\). This is to say that gauge transformation takes the form

\[
\delta|\phi_{sh}\rangle = G_{sh}|\xi_{sh}\rangle, \tag{4.18}
\]

\[
G_{sh} = \alpha \partial - e_{1,sh} - \frac{1}{2N_\alpha + d - 2} \bar{e}_{1,sh}, \tag{4.19}
\]

where \(e_{1,sh}, \bar{e}_{1,sh}\) are given in (4.10).

**Realization of conformal algebra symmetries.** To complete the gauge invariant formulation of the spin-\(s\) anomalous shadow field we provide realization of the conformal algebra symmetries on space of the ket-vector \(|\phi_{sh}\rangle\). All that is required is to fix the operators \(M^{ab}, \Delta, \) and \(R^a\) and insert then these operators into (2.20)-(2.23). Realization of the spin operator \(M^{ab}\) on-ket-vector \(|\phi_{sh}\rangle\) (4.3) is given in (2.3), while realization of the operator \(\Delta\),

\[
\Delta_{sh} = \frac{d}{2} - \nu, \quad \nu = \kappa + N_\xi - N_z, \tag{4.20}
\]

can be read from (4.3). Realization of the operator \(R^a\) on space of \(|\phi_{sh}\rangle\), which we find, is given by

\[
R^a_{sh} = 2\alpha \bar{r}_z \left( (\nu - 1)\bar{a}^a + \bar{C}_+^a \right)
+ 2 \left( \nu \bar{C}_-^a - \alpha^2 \frac{1}{2N_\alpha + d - 2} \bar{C}_+^a \right) r_z \zeta, \tag{4.21}
\]

where the operators \(\bar{C}_+^a, \bar{C}_-^a\) are given in (2.11), while the operators \(r_z, z\) are defined in (2.13).

**Two-point gauge invariant vertex.** We now discuss two-point vertex for the spin-\(s\) anomalous shadow field. This is to say that we find the following gauge invariant two-point vertex:

\[
\Gamma = \int d^dx_1d^dx_2 \Gamma_{12}, \tag{4.22}
\]

\[
\Gamma_{12} = \frac{1}{2} \langle \phi_{sh}(x_1) | \mu f_\nu \rangle \langle \phi_{sh}(x_2) | \phi_{sh}(x_1) \rangle, \tag{4.23}
\]

\[
f_\nu = \frac{\Gamma(\nu + \frac{d}{2}) \Gamma(\nu + 1)}{4\kappa - \nu \Gamma(\kappa + \frac{d}{2}) \Gamma(\kappa + 1)}, \tag{4.24}
\]

\[
\nu = \kappa + N_\xi - N_z, \tag{4.25}
\]

\[
|x_{12}|^2 = x_1^a x_2^a, \quad x_1^a = x_1^a - x_2^a, \tag{4.26}
\]

where \(\mu\) is given in (2.10). Vertex \(\Gamma_{12}\) is invariant under gauge transformation of the anomalous shadow field \(|\phi_{sh}\rangle\) (4.18) provided this anomalous shadow field satisfies differential constraint (4.18). The vertex is obviously invariant under the Poincaré algebra and dilatation symmetries. We make sure that vertex (4.22) is invariant under the conformal boost transformations.

To illustrate structure of the vertex \(\Gamma_{12}\) we note that, in terms of the tensor fields \(\phi_{sh,\lambda}^{\alpha_1\ldots\alpha_{s'}},\) vertex \(\Gamma_{12}\) (4.23) can be represented as

\[
\Gamma_{12} = \sum_{s'=0}^s \sum_{\lambda \in \{1-s'\}} \Gamma_{12,\lambda}, \tag{4.27}
\]

\[
\Gamma_{12,\lambda} = \frac{\lambda}{2s! \lambda} \langle \phi_{sh,\lambda}^{\alpha_1\ldots\alpha_{s'}(x_1)} \phi_{sh,\lambda}^{\alpha_1\ldots\alpha_{s'}(x_2)} \}
- \frac{s'! (s' - 1)}{4} \phi_{sh,\lambda}^{\alpha_{s}a_1\ldots\alpha_{s'}(x_1)} \phi_{sh,\lambda}^{\alpha_{s}a_1\ldots\alpha_{s'}(x_2)}, \tag{4.28}
\]

\[
\lambda = \frac{\Gamma(\kappa - \lambda + \frac{d}{2}) \Gamma(\kappa - \lambda + 1)}{4\lambda \Gamma(\kappa + \frac{d}{2}) \Gamma(\kappa + 1)} \tag{4.29}
\]

We note that the kernel of the vertex \(\Gamma\) is connected with a two-point correlation function of the anomalous conformal current. In the framework of our approach, the anomalous conformal current is described by gauge fields (3.1) subject to differential constraints. In order to discuss the correlation function of the spin-\(s\) anomalous conformal current in a proper way, we can impose a gauge condition on the gauge fields given in (3.1). Recall that we have considered the spin-\(s\) anomalous conformal current by using the Stueckelberg and light-cone gauge frames. Obviously, the two-point correlation function of the anomalous conformal current in the Stueckelberg and light-cone gauge frames is obtained from the two-point vertex \(\Gamma\) taken in the respective Stueckelberg and light-cone gauge frames. To this end we proceed by discussing the anomalous shadow field in the Stueckelberg and light-cone gauge frames.
B. Stueckelberg gauge frame

We now discuss the spin-$s$ anomalous shadow field in the Stueckelberg gauge frame. We note that the Stueckelberg gauge frame can be achieved through the use of differential constraint given in (4.8) and the Stueckelberg gauge condition. From (4.18), we see that a field defined by

$$\zeta^{\Pi^{[1:2]}}|\phi_{sh}\rangle \quad (4.30)$$

transforms as Stueckelberg field. Therefore this field can be gauged away via Stueckelberg gauge fixing,

$$\zeta^{\Pi^{[1:2]}}|\phi_{sh}\rangle = 0 \quad (4.31)$$

Using this gauge condition and differential constraint (4.8), we find the following relations:

$$\alpha^2|\phi_{sh}'\rangle = 0 \quad (4.32)$$

$$|\phi_{sh}'\rangle = X_{sh}\alpha_{s-s'}(\tilde{\alpha}\tilde{\partial})^{s-s'}|\phi_{sh}\rangle \quad (4.33)$$

$$X_{sh} = \left(\frac{s-s'}{s-s'}\right)^{\frac{2-s-s'}{2}} \Gamma(\kappa+s-d+2)\Gamma(\kappa+1) \Gamma(\kappa+s-1+s')^{1/2} \quad (4.36)$$

Relation (4.32) tells us that all fields $\phi_{sh,\lambda}^{a_1\ldots a_s}$ with $s' \geq 2$ become traceless. From relation (4.33), we learn that the fields $\phi_{sh,\lambda}^{a_1\ldots a_s}$ with $\lambda = s-s'$ become equal to zero, while the fields $\phi_{sh,\lambda}^{a_1\ldots a_s'}$ with $\lambda = s-s'$ and $s' = 0, 1, \ldots, s-1$ are expressed in terms of the rank-$s$ traceless tensor field $\phi_{sh,0}^{a_1\ldots a_s}$,

$$\left(\phi_{sh,\lambda}^{a_1\ldots a_s'} = 0 \quad \text{for} \quad \lambda \neq s-s', \quad (4.34)\right)$$

$$\phi_{sh,s-s'}^{a_1\ldots a_s'} = \sqrt{(s-s')!}X_{sh}\phi_{sh,0}^{b_1\ldots b_{s-s'}}\phi_{sh,0}^{b_{s-s'}\ldots a_1\ldots a_s'} \quad (4.35)$$

We recall that, in the standard CFT, the spin-$s$ anomalous shadow field is described by the traceless rank-$s$ tensor field $\phi_{sh,0}^{a_1\ldots a_s}$. Thus we see that making use of the gauge symmetry and differential constraint we reduce the field content of our approach to the one in the standard CFT. In other words, use of the gauge symmetry and differential constraint allows us to match our approach and the standard formulation of the anomalous shadow field. To summarize, our gauge invariant approach is equivalent to the standard one.

We now discuss Stueckelberg gauge-fixed two-point vertex of the anomalous shadow field. In other words, we are going to connect our vertex (4.22) with the one in the standard CFT. To do that we note that vertex of the standard CFT is obtained from our gauge invariant vertex (4.22) by plugging solution to the differential constraint (4.33) into (4.22). Doing so, we find the following two-point density (up to total derivative) in the Stueckelberg gauge frame:

$$\Gamma_{12}^{\text{Stuck.g. frame}} = k_s\Gamma_{12}^{\text{stand}} \quad (4.36)$$

$$\Gamma_{12}^{\text{stand}} = s!\langle \phi_{sh}^s(x_1)|O_{12}^s|\phi_{sh}^s(x_2) \rangle \quad (4.37)$$

$$O_{12}^s = \sum_{n=0}^s \left((-1)^n(\alpha x_{12})^n(\bar{\alpha} x_{12})^n\right)_{|x_{12}|^{2s+4+2n}} \quad (4.38)$$

$$k_s = \frac{2\kappa + 2s + d - 2}{2s!(\kappa + 2s + d - 2)} \quad (4.39)$$

where $\alpha x_{12} = \alpha^a x_{12}^a$, $\bar{\alpha} x_{12} = \bar{\alpha}^a x_{12}^a$ and $\Gamma_{12}^{\text{stand}}$ in (4.36), (4.37) stands for the two-point vertex of the spin-$s$ anomalous shadow field in the standard CFT. Relation (4.37) provides oscillator representation for the $\Gamma_{12}^{\text{stand}}$. In terms of the tensor field $\phi_{sh,0}^{a_1\ldots a_s}$, vertex $\Gamma_{12}^{\text{stand}}$ (4.37) can be represented in the commonly used form,

$$\Gamma_{12}^{\text{stand}} = \phi_{sh,0}^{a_1\ldots a_s}(x_1)O_{12}^{a_1b_1}\ldots O_{12}^{a_sb_s}|\phi_{sh,0}^0(x_2) \rangle \quad (4.40)$$

$$O_{12}^{ab} = \eta^{ab} - \frac{2x_{12}^ax_{12}^b}{|x_{12}|^2} \quad (4.41)$$

From (4.36), we see that our gauge invariant vertex $\Gamma_{12}$ considered in the Stueckelberg gauge frame coincides, up to normalization factor $k_s$, with the two-point vertex in the standard CFT. In section III B, we have demonstrated that, in the Stueckelberg gauge frame, we are left with rank-$s$ traceless tensor field $\phi_{sh,0}^{a_1\ldots a_s}$. Two-point correlation function of this tensor field is defined by the kernel of vertex $\Gamma_{12}^{\text{stand}}$ (4.40).

C. Light-cone gauge frame

We proceed with discussion of the anomalous shadow field in the light-cone gauge frame. For the anomalous shadow field, the light-cone gauge frame can be achieved through the use of differential constraint (4.8) and light-cone gauge condition. Using gauge symmetry of the spin-$s$ anomalous shadow field (4.18), we impose the light-cone gauge rule on the $|\phi_{sh}\rangle$,

$$\tilde{\alpha}^+\Pi^{[1:2]}|\phi_{sh}\rangle = 0 \quad (4.42)$$

where $\Pi^{[1:2]}$ is given in (2.9). Using gauge condition (4.42) and differential constraint (4.8), we obtain

$$|\phi_{sh}\rangle = \exp\left(-\frac{\alpha^+}{\tilde{\alpha}^+}(\tilde{\alpha}^+\hat{\partial} - \tilde{e}_{1sh})|\phi_{sh}^{l.c.} \rangle \right) \quad (4.43)$$

$$\alpha^+|\phi_{sh}^{l.c.} \rangle = 0 \quad (4.44)$$

where a light-cone ket-vector $|\phi_{sh}^{l.c.} \rangle$ is obtained from $|\phi_{sh}\rangle$ (4.4) by equating $\alpha^+ = \alpha^- = 0$,

$$|\phi_{sh}^{l.c.} \rangle \equiv |\phi_{sh}\rangle \bigg|_{\alpha^+=\alpha^-=0} \quad (4.45)$$
We see that we are left with light-cone fields
\[ \phi_{sh,\lambda}^{i_1 \cdots i_s}, \quad s' = 0, 1, \ldots, s, \quad \lambda \in [s - s]'_2, \quad (4.46) \]
which are traceless tensor fields of \( so(d - 2) \) algebra, \( \phi_{sh,\lambda}^{i_1 \cdots i_s} = 0 \). These fields constitute the field content of the light-cone gauge frame. Note that, in contrast to the Stueckelberg gauge frame, all fields \( \phi_{sh,\lambda}^{i_1 \cdots i_s} \) are not equal to zero. Also note that, in contrast to the gauge invariant approach, fields \((4.46)\) are not subject to any differential constraint. Using \((4.43)\) in \((4.23)\) leads to light-cone gauge-fixed vertex
\[ \Gamma_{12}^{lc} = \frac{1}{2} \langle \phi_{sh,\lambda}^{i_1} (x_1) f_{\nu} \phi_{sh,\lambda}^{i_1} (x_2) \rangle , \quad (4.47) \]
where \( f_{\nu} \) is defined in \((4.24)\).

To illustrate the structure of vertex \( \Gamma_{12}^{lc} \) \((4.47)\) we note that, in terms of the fields \( \phi_{sh,\lambda}^{i_1 \cdots i_s} \), the vertex can be represented as
\[ \Gamma_{12}^{lc} = \sum_{s'=0}^s \sum_{\lambda \in [s-s]'_2} \Gamma_{12,\lambda}^{s',1}, \quad (4.48) \]
\[ \Gamma_{12,\lambda}^{s',1} = \frac{w_{\lambda}}{2s!} \langle \phi_{sh,\lambda}^{i_1 \cdots i_s} (x_1) \phi_{sh,\lambda}^{i_1 \cdots i_s} (x_2) \rangle , \quad (4.49) \]
where \( w_{\lambda} \) is given in \((4.29)\). We see that, as in the case of gauge invariant vertex, light-cone vertex \((4.48)\) is diagonal with respect to the light-cone fields \( \phi_{sh,\lambda}^{i_1 \cdots i_s} \). Note however that, in contrast to the gauge invariant vertex, the light-cone vertex is constructed out of the light-cone fields which are not subject to any differential constraints.

Thus, we see that our gauge invariant vertex does indeed provide easy and quick access to the light-cone gauge vertex. Namely, all that is needed to obtain light-cone gauge vertex \((4.48)\) is to remove traces of the tensor fields \( \phi_{sh,\lambda}^{i_1 \cdots i_s} \) and replace the \( so(d - 1, 1) \) Lorentz algebra vector indices appearing in gauge invariant vertex \((4.23)\) by the respective vector indices of the \( so(d - 2) \) algebra.

The kernel of the light-cone vertex gives the two-point correlation function of the spin-\( s \) anomalous conformal current taken to be in the light-cone gauge. Defining two-point correlation functions of the fields \( \phi_{\text{cur,}\lambda}^{i_1 \cdots i_s} \) as the second functional derivative of \( \Gamma \) with respect to the shadow fields \( \phi_{sh,\lambda}^{i_1 \cdots i_s} \), we obtain the following correlation functions:
\[ \langle \phi_{\text{cur,}\lambda}^{i_1 \cdots i_s} (x_1), \phi_{\text{cur,}\lambda}^{i_1 \cdots i_s} (x_2) \rangle \]
\[ = \frac{w_{\lambda}}{2s!} \Pi^{i_1 \cdots i_s, j_1 \cdots j_s} (x_1) , \quad (4.50) \]
where \( w_{\lambda} \) is defined in \((4.29)\) and \( \Pi^{i_1 \cdots i_s, j_1 \cdots j_s} \) stands for the projector on traceless rank-\( s' \) tensor field of the \( so(d - 2) \) algebra. Explicit form of the projector may be found, e.g., in Ref. \([27]\).

V. TWO-POINT CURRENT-SHADOW FIELD INTERACTION VERTEX

We now briefly discuss the two-point current-shadow field interaction vertex. In our approach, this interaction vertex is determined by requiring that:

\( i \) the vertex is invariant under both gauge transformations of anomalous conformal current and shadow field;

\( ii \) vertex is invariant under conformal algebra transformations.

We find the following vertex:
\[ \mathcal{L} = \langle \phi_{\text{cur}} | \mu | \phi_{\text{sh}} \rangle , \quad (5.1) \]
where \( \mu \) is given in \((2.10)\). We note that, under gauge transformation of the anomalous conformal current \((3.19)\), the variation of vertex \((5.1)\) takes the form (up to total derivative)
\[ \delta_{\xi_{\text{cur}}} \mathcal{L} = - \langle \xi_{\text{cur}} | \bar{C}_{\text{sh}} | \phi_{\text{cur}} \rangle . \quad (5.2) \]
From \((5.2)\), we see that the vertex \( \mathcal{L} \) is invariant under gauge transformation of the anomalous conformal current provided the anomalous shadow field satisfies differential constraint \((4.3)\). Next, we note that under gauge transformation of the anomalous shadow field \((4.18)\) the gauge variation of vertex \((5.1)\) takes the form (up to total derivative)
\[ \delta_{\xi_{\text{sh}}} \mathcal{L} = - \langle \xi_{\text{sh}} | \bar{C}_{\text{cur}} | \phi_{\text{cur}} \rangle . \quad (5.3) \]
From \((5.3)\), we see that the vertex \( \mathcal{L} \) is invariant under gauge transformation of the anomalous shadow field provided the anomalous conformal current satisfies differential constraint \((3.9)\).

Using the realization of the conformal algebra symmetries obtained in Sections \( IIIV \) we make sure that vertex \( \mathcal{L} \) \((5.1)\) is invariant under the conformal algebra transformations.

VI. ADS/CFT CORRESPONDENCE. PRELIMINARIES

We now study the AdS/CFT correspondence for free arbitrary spin massless AdS field and boundary arbitrary spin anomalous conformal current and shadow field. To study the AdS/CFT correspondence we use the gauge invariant CFT adapted formulation of massive AdS field and modified de Donder gauge condition found in Ref. \([14]\).\(^7\) We emphasize

\(^7\) Applications of the standard de Donder gauge to the various problems of massless fields may be found in Refs. \([28]\). Recent interesting discussion of modified de Donder gauge may be found in Ref. \([29]\). We believe that our modified de Donder gauge will also be useful for better understanding of various aspects of AdS/QCD correspondence which are discussed, e.g., in Ref. \([30]\).
that it is the use of our massive gauge fields and the modified de Donder gauge condition that leads to the decoupled gauge-fixed equations of motion and surprisingly simple Lagrangian\(^8\). The use of our massive gauge fields and the modified de Donder gauge condition makes the study of AdS/CFT correspondence for arbitrary spin-\(s\) massive AdS field similar to the one for spin-0 massive AdS field. Owing these properties of our massive gauge fields and the modified de Donder gauge condition, the computation of effective action is considerably simplified. Perhaps, this is the main advantage of our approach.

In our approach to the AdS/CFT correspondence, we have gauge symmetries not only at AdS side but also at the boundary CFT. Also, we note that the modified de Donder gauge condition turns out to be invariant under on-shell leftover gauge symmetries of massive AdS field. This is to say that, in the framework of our approach, the study of AdS/CFT correspondence implies the matching of:

i) modified de Donder gauge condition for bulk massive field and the corresponding differential constraint for boundary anomalous conformal current and shadow field;

ii) on-shell leftover gauge symmetries of bulk massive field and the corresponding gauge symmetries of boundary anomalous conformal current and shadow field;

iii) on-shell global symmetries of bulk massive field and the corresponding global symmetries of boundary anomalous conformal current and shadow field;

iv) an effective action evaluated on the solution of AdS massive field equations of motion with the Dirichlet problem corresponding to the boundary anomalous shadow field and the boundary two-point gauge invariant vertex for the anomalous shadow field.

As we have already said, to discuss the AdS/CFT correspondence for bulk arbitrary spin massive AdS field and boundary arbitrary spin anomalous conformal current and shadow field we use the CFT adapted gauge invariant Lagrangian and the modified de Donder gauge condition for the arbitrary spin massive AdS field found in Ref.\([14]\). We begin therefore with the presentation of our result in Ref.\([14]\).

### A. CFT adapted approach to massive arbitrary spin AdS field

In AdS\(_{d+1}\) space, massive spin-\(s\) field is described by the following scalar, vector, and totally symmetric tensor fields of the \(so(d)\) algebra:\(^9\)

\[
\phi^{a_1\ldots a_{s'}}_\lambda, \quad s' = 0, 1, \ldots, s, \quad \lambda \in [s - s']_2.
\] (6.1)

Using shortcut \(\phi^{\lambda}_{a_1\ldots a_s}\), fields in (6.1) can be represented as in (2.16). The fields \(\phi^{a_1\ldots a_{s'}}_\lambda\) with \(s' \geq 4\) are double-traceless,

\[
\phi^{aaabba\ldots a_{s'}}_\lambda = 0, \quad s' = 4, 5, \ldots, s.
\] (6.2)

In order to obtain the gauge invariant description in an easy–to–use form we use the oscillators and introduce a ket-vector \(|\phi\rangle\) defined by

\[
|\phi\rangle = \sum_{s' = 0}^s |\phi^{s'}\rangle, \quad (6.3)
\]

\[
|\phi^{s'}\rangle
\]

\[
= \sum_{\lambda \in [s - s']_2} \frac{\zeta_\lambda}{s!' \sqrt{((s' - 2)_+)!((s' - 2)_-)!}} \phi^{a_1\ldots a_{s'}}_\lambda |0\rangle.
\] (6.4)

From (6.3), we see that the ket-vector \(|\phi\rangle\) is degree-\(s\) homogeneous polynomial in the oscillators \(\alpha^a, \alpha^z, \zeta\), while the ket-vector \(|\phi^{s'}\rangle\) is degree-\(s'\) homogeneous polynomial in the oscillators \(\alpha^a\). In terms of the ket-vector \(|\phi\rangle\), double-tracelessness constraint (6.2) takes the form

\[
\langle \bar{\alpha}^2 \rangle^2 |\phi\rangle = 0.
\] (6.5)

Using the Poincaé parametrization of AdS\(_{d+1}\) space

\[
ds^2 = \frac{1}{z^2} (dx^a dx^a + dz dz),
\] (6.6)

we present CFT adapted gauge invariant action and Lagrangian \([14]\),

\[
S = \int d^d x dz L,
\] (6.7)

where we use the notation

\[
\bar{C} = \bar{\alpha} \bar{\alpha} - \frac{1}{2} \alpha \bar{\alpha}^2 - \bar{e}_1 \Pi^{(1,1)} + \frac{1}{2} e_1 \bar{\alpha}^2,
\] (6.8)

\[
- e_1 = \zeta \bar{\zeta} T_{-\nu} - \frac{1}{2} + \alpha^2 r_\nu T_{-\nu} - \frac{1}{2},
\] (6.9)

\[
- \bar{e}_1 = T_{\nu + \frac{1}{2}} r_\nu \bar{\zeta} + T_{-\nu + \frac{1}{2}} \bar{r}_\nu \bar{\alpha}^2,
\] (6.10)

\(^8\) Our massive gauge fields are obtained from gauge fields used in gauge invariant approach to massive fields in Ref.\([31]\) by the invertible transformation which is described in Appendix in Ref.\([14]\). Discussion of interesting methods for solving AdS field equations of motion without gauge fixing may be found in Refs.\([13]\).

\(^9\) From now on we use, unless otherwise specified, the Euclidian signature.
\[ \mathcal{T}_\nu = \partial_z + \frac{\nu}{z} , \quad (6.11) \]
\[ \nu = \kappa + N\zeta - N_z , \quad (6.12) \]
and \( \Omega^{[1,2]}, \mu, r_\zeta, r_z, \) and \( \kappa \) are given in (2.9), (2.10), (2.13), and (3.4) respectively.

To discuss gauge symmetries of Lagrangian (6.7) we introduce the gauge transformations parameters,
\[ \xi^{a_1 \ldots a_s'} , \quad s' = 0, 1, \ldots, s - 1 , \quad \lambda \in [s - 1 - s']_2 , \quad (6.13) \]
which are scalar, vector, and totally symmetric tensor fields of the \( so(d) \) algebra. The gauge transformation parameters with \( s' \geq 2 \) are traceless, \( \xi_{a_2 a_3 \ldots a_s'} = 0 \).

As usually, we collect the gauge transformation parameters in a ket-vector \( |\xi\rangle \) defined by
\[ |\xi\rangle = \sum_{s'=0}^{s-1} |\xi^{s'}\rangle , \quad (6.14) \]
and the operator \( \tilde{a}^{a} \) is given in (2.11).

**Modified de Donder gauge.** Gauge invariant equations of motion obtained from Lagrangian (6.7) take the form
\[ \mu \Box \nu |\phi\rangle - C \bar{C} |\phi\rangle = 0 , \quad (6.25) \]
\[ \Box \nu \equiv \Box + \partial_z^2 \frac{1}{2} (\nu^2 - \frac{1}{4}) , \quad (6.26) \]
\[ C \equiv \alpha \partial_z - \frac{1}{2} \alpha^2 \bar{\alpha} \partial_z - \frac{1}{2} \alpha \bar{\alpha} + \frac{1}{2} \tilde{e}_1 \alpha^2 , \quad (6.27) \]
where \( \bar{C} \) and \( \nu \) are given in (6.8) and (6.12). Note that, for the derivation of (6.25), we use the relations \( C^I = -\bar{C} \).

Modified de Donder gauge is defined to be
\[ \tilde{C} |\phi\rangle = 0 , \quad \text{modified de Donder gauge} , \quad (6.29) \]
where \( \tilde{C} \) is given in (6.8). Using this gauge in (6.25) leads to the simple gauge-fixed equations of motion,
\[ \Box \nu |\phi\rangle = 0 , \quad (6.30) \]
i.e., the gauge-fixed equations turn out to be decoupled.

We note that the modified de Donder gauge and gauge-fixed equations have on-shell leftover gauge symmetry. This is to say that modified de Donder gauge (6.29) and gauge-fixed equations (6.30) are invariant with respect to gauge transformation given in (6.13) provided the gauge transformation parameter satisfies the following equation:
\[ \Box \nu |\xi\rangle = 0 . \quad (6.31) \]

---

10 We note that, in our approach, only \( so(d-1,1) \) symmetries are realized manifestly. Symmetries of the \( so(d,2) \) algebra could be realized manifestly by using the framework of ambient space approach (see, e.g., Ref. [33--35]).
B. AdS/CFT correspondence for spin-0 field

As we have already said, the use of our massive gauge fields and the modified de Donder gauge makes the study of AdS/CFT correspondence for arbitrary spin-
$s$ massive AdS field similar to the one for spin-0 massive AdS field. Therefore, for the reader convenience, we now briefly recall the AdS/CFT correspondence for the scalar massive AdS field.

AdS/CFT correspondence for normalizable modes of spin-0 massive AdS field and spin-0 conformal current\textsuperscript{11}. The action of massive scalar field in $AdS_{d+1}$ background takes the form

\[ S = \int d^dx dz \mathcal{L}, \quad (6.32) \]

\[ \mathcal{L} = \frac{1}{2} \sqrt{|g|} \left( g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi + m^2 \phi^2 \right). \quad (6.33) \]

Using the canonically normalized field $\phi$ defined by relation $\phi = z^{-\frac{\nu}{2}} \phi$, we represent Lagrangian (6.33) as (up to total derivative)

\[ \mathcal{L} = \frac{1}{2} |d\phi|^2 + \frac{1}{2} |\nabla_z \phi|^2, \quad (6.34) \]

\[ \nabla_z \equiv \frac{\partial}{\partial z}, \quad \nu = \sqrt{m^2 + \frac{d^2}{4}}. \quad (6.35) \]

Note that only for spin-0 massive AdS field the $\nu$ takes the form given in (6.35). For spin-$s$ massive AdS field with $s > 0$ the $\nu$ is given in (6.12). The equation of motion obtained from Lagrangian (6.34) is given by

\[ \Box_z \phi = 0, \quad (6.36) \]

where $\Box_z$ is defined in (6.26). The normalizable solution of Eq. (6.36) takes the form

\[ \phi(x, z) = U^\nu_{\nu} \phi_{\text{cur}}(x), \quad (6.37) \]

\[ U^\nu_{\nu} \equiv h_\nu z q J_\nu(zq) q^{-\left(\nu + \frac{1}{2}\right)}, \quad (6.38) \]

\[ h_\nu \equiv 2^{\nu} \Gamma(\nu + 1), \quad q^2 \equiv \Box. \quad (6.39) \]

where $J_\nu$ stands for the Bessel function. The asymptotic behavior of solution (6.37) is given by

\[ \phi(x, z) \xrightarrow{z \to 0} z^{\nu + \frac{1}{2}} \phi_{\text{cur}}(x). \quad (6.40) \]

From (6.40), we see that the field $\phi_{\text{cur}}$ is indeed the asymptotic boundary value of the normalizable solution.

For the case of scalar field, there are no gauge symmetries and gauge conditions to be matched. All that is needed to complete the AdS/CFT correspondence is to match the bulk global symmetries of the AdS field $\phi(x, z)$ and the respective boundary global symmetries of the current $\phi_{\text{cur}}(x)$. Realization of the global symmetries on AdS side and CFT side is given in (6.17), (6.24) and (2.20), (2.23) respectively. Obviously, the Poincaré symmetries match automatically. Introducing the notation $D_A$ and $D_{\text{CFT}}$ for the respective realizations of $D$ symmetry on bulk fields (6.19) and boundary conformal currents (2.22), we get the relation

\[ D_A \phi(x, z) = U^\nu_{\nu} D_{\text{CFT}} \phi_{\text{cur}}(x), \quad (6.41) \]

where $D_{\text{CFT}}$ corresponding to the conformal spin-0 current $\phi_{\text{cur}}$ is obtained from (2.22) by using $\Delta = \frac{d}{2} + \nu$ with $\nu$ given in (6.35). From (6.41), we see that $D$ symmetries of $\phi(x, z)$ and $\phi_{\text{cur}}(x)$ also match. Finally, using $R^a \phi_{\text{cur}}(x) = 0$ and taking into account that, for spin-0 AdS field, $R_{\alpha(0)} \phi(x, z) = 0$, we see that the $R^a$ symmetries also match.

AdS/CFT correspondence for non-normalizable modes of spin-0 massive AdS field and spin-0 shadow field. As shown in Ref.\textsuperscript{[37]}, the non-normalizable solution of Eq. (6.36) with the Dirichlet problem corresponding to the boundary shadow field $\phi_{\text{sh}}(x)$ can be presented as

\[ \phi(x, z) = \sigma \int d^d y G_\nu(x - y, z) \phi_{\text{sh}}(y), \quad (6.42) \]

\[ G_\nu(x, z) = \frac{c_\nu z^{\nu + \frac{1}{2}}}{(z^2 + |x|^2)^{\nu + \frac{d}{2}}}, \quad (6.43) \]

\[ c_\nu \equiv \frac{\Gamma(\nu + \frac{d}{2})}{\pi^{d/2} \Gamma(\nu)}. \quad (6.44) \]

For the later use, we introduce the normalization factor $\sigma$ in relation (6.42). We recall that commonly used value of $\sigma$ is achieved by setting $\sigma = 1$. The asymptotic behaviors of Green function (6.43) and solution (6.42) are given by

\[ G_\nu(x, z) \xrightarrow{z \to 0} z^{-\nu + \frac{d}{2}} \delta^d(x), \quad (6.45) \]

\[ \phi(x, z) \xrightarrow{z \to 0} z^{-\nu + \frac{1}{2}} \sigma \phi_{\text{sh}}(x). \quad (6.46) \]

Relation (6.46) tells us that solution (6.42) has indeed asymptotic behavior corresponding to the shadow field. Taking into account (6.46) and (6.32), (6.34), we find the well-known expression for effective action\textsuperscript{12}

\[ -S_{\text{eff}} = \int d^dx L_{\text{eff}}|_{z \to 0}, \quad (6.47) \]

\[ \text{12 As usually, since solution of the Dirichlet problem (6.42) tends to zero as } z \to \infty, \text{ we ignore contribution to } S_{\text{eff}} \text{ when } z = \infty. \]
\[ L_{\text{eff}} = \frac{1}{2} \phi \mathcal{T}_{\nu} \phi , \]  
(6.48)

Using solution of the Dirichlet problem (6.42) in (6.47), (6.48), we get the effective action
\[ -S_{\text{eff}} = \nu c_{\nu} \sigma^{2} \int d^{d}x_{1} d^{d}x_{2} \frac{\phi_{\text{sh}}(x_{1})\phi_{\text{sh}}(x_{2})}{|x_{12}|^{2\nu+d}}. \]  
(6.49)

Plugging the commonly used value of \( \sigma, \sigma = 1 \), into (6.49), we get the properly normalized effective action found in Refs.[38, 39]. An interesting novelty of our computation of the effective action is that we use the Fourier transform of the Green function. For the details of our computation, see Appendix C in Ref.[4].

**VII. ADS/CFT CORRESPONDENCE FOR NORMALIZABLE MODES OF MASSIVE ADS FIELD AND ANOMALOUS CONFORMAL CURRENT**

We now ready to consider the AdS/CFT correspondence for the spin-\( s \) massive AdS field and spin-\( s \) anomalous conformal current. We begin with the discussion of the normalizable solution of Eq. (6.30). The normalizable solution of Eq. (6.30) is given by
\[ |\phi(x, z)\rangle = U_{\nu}|\phi_{\text{cur}}(x)\rangle, \]  
(7.1)
\[ U_{\nu} \equiv h_{\nu}(-)^{N_{s}} \sqrt{zq} J_{\nu}(zq)q^{-(\nu+\frac{e}{2})}, \]  
(7.2)
\[ h_{\nu} \equiv 2^{e} \Gamma(\kappa + 1), \]  
(7.3)
where we do not show explicitly the dependence of \( U_{\nu} \) on \( z, q \), and the parameter \( \kappa \) defined in (3.4). The asymptotic behavior of solution (7.1) takes the form
\[ |\phi(x, z)\rangle \xrightarrow{z \rightarrow 0} \begin{cases} z^{\nu+\frac{e}{2}} \Gamma(\kappa + 1)(-)^{N_{s}}|\phi_{\text{cur}}(x)\rangle. & \text{if } q \equiv \square, \\ z^{\nu+\frac{e}{2}} \Gamma(\kappa + 1)(-)^{N_{s}}|\phi_{\text{cur}}(x)\rangle. & \text{if } q \equiv \bigcirc. \end{cases} \]  
(7.4)

From (7.4), we see that \( |\phi_{\text{cur}}\rangle \) is indeed boundary value of the normalizable solution. In the right-hand side of relation (7.1), we use the notation \( |\phi_{\text{cur}}\rangle \) because we are going to demonstrate that this boundary value is indeed the gauge field appearing in the gauge invariant formulation of the spin-\( s \) anomalous conformal current in Sec.III. Namely, we are going to prove the following statements:

i) For normalizable solution (7.1), modified de Donder gauge condition (6.29) leads to the differential constraint of the spin-\( s \) anomalous conformal current given in (3.9).

ii) On-shell leftover gauge transformation (6.15) of normalizable solution (7.1) leads to the gauge transformation of the spin-\( s \) anomalous conformal current given in (3.10).

iii) On-shell bulk \( so(d, 2) \) symmetries of the normalizable solution (7.1) amount to boundary \( so(d, 2) \) conformal symmetries of the spin-\( s \) anomalous conformal current.

To prove these statements we use the following relations for the operator \( U_{\nu} \):
\[ T_{\nu} U_{\nu} = U_{\nu-1}, \]  
(7.5)
\[ T_{-\nu} U_{\nu} = -U_{\nu+1}, \]  
(7.6)
\[ T_{-\nu} U_{\nu}(zU_{\nu}) = -zU_{\nu+1} \square + 2U_{\nu}, \]  
(7.7)
\[ \square_{\nu}(zU_{\nu}) = 2U_{\nu}, \]  
(7.8)

which, in turn, can be derived by using the following textbook identities for the Bessel function:
\[ e_{1} U_{\nu} = U_{\nu} e_{1,\text{cur}}, \]  
(7.9)
acting with operator \( \tilde{C} \) (6.8) on solution \( |\phi\rangle \) (7.1) and using (7.10), we obtain the relation
\[ \tilde{C}|\phi(x, z)\rangle = U_{\nu}|\phi_{\text{cur}}(x)\rangle. \]  
(7.11)
where \( \tilde{C}_{\text{cur}} \) is given in (3.10). From (7.11), we see that our modified de Donder gauge condition (6.29) leads indeed to the differential constraint for the anomalous conformal current given in (3.9).

**Matching of bulk and boundary gauge symmetries.** We now show how gauge transformation of the anomalous conformal current (3.19) is related to the on-shell leftover gauge transformation of the massive AdS field (6.15). To this end we note that on-shell leftover gauge transformation of massive AdS field is obtained from (6.15) by plugging gauge transformation parameter that satisfies equation (6.31) into (6.15). The normalizable solution of equation for the gauge transformation parameter (6.31) takes the form
\[ |\xi(x, z)\rangle = U_{\nu}|\xi_{\text{cur}}(x)\rangle, \]  
(7.12)

Note that gauge transformation given in (6.15) is off-shell gauge transformation. On-shell leftover gauge transformation is obtained from gauge transformation (6.15) by using gauge transformation parameter which satisfies equation (6.31).
where $U_p$ is given in (7.2). On the one hand, plugging (7.12) into (6.15) and using (7.10), we find that bulk on-shell leftover gauge transformation takes the form

$$\delta |\phi(x, z)\rangle = U_p G_{\text{cur}} |\xi_{\text{cur}}(x)\rangle \, .$$

(7.13)

On the other hand, relation (7.1) leads to

$$\delta |\phi(x, z)\rangle = U_p \delta |\phi_{\text{cur}}(x)\rangle \, .$$

(7.14)

Comparing (7.13) and (7.14), we conclude that bulk and boundary gauge symmetries does indeed match.

**Matching of bulk and boundary global symmetries.**

Now we are going to demonstrate the matching of the so(d, 2) algebra generators for bulk massive AdS field in (6.17)–(6.20) and the ones for the boundary anomalous conformal current in (2.20)–(2.23). Representation for generators of the so(d, 2) algebra given in (6.17)–(6.20) is valid for the gauge invariant theory of AdS fields. Note however that the modified de Donder gauge respects the Poincaré and dilatation symmetries, but breaks the conformal boost symmetries ($K^a$ symmetries).

This implies that realization for generators $P^a$, $J^{ab}$ and $D$ given in (6.17)–(6.19) is still valid for the gauge-fixed AdS fields, while realization for the conformal boost generator $K^a$ given in (6.20) must be modified to restore $K^a$ symmetries of the gauge-fixed AdS fields. We begin with matching of the Poincaré and dilatation symmetries. Matching of the Poincaré symmetries is obvious: from (2.20), (2.21) and (6.17), (6.18), we see that bulk and boundary generators of the Poincaré algebra, $P^a$, $J^{ab}$, coincide. Next, we consider the dilatation generator $D$. To match the dilatation symmetries we need an explicit form of the solution for AdS field equations of motion in (7.1). Using the notation $D_{\text{AdS}}$ and $D_{\text{CFT}}$ for the respective bulk dilatation generator in (6.19) and boundary dilatation generator in (2.22), we get the relation

$$D_{\text{AdS}} |\phi(x, z)\rangle = U_p D_{\text{CFT}} |\phi_{\text{cur}}(x)\rangle \, ,$$

(7.15)

where $D_{\text{CFT}}$ corresponding to $|\phi_{\text{cur}}\rangle$ is obtainable from (2.22) and the conformal dimension operator given in (3.21). From (7.15), we see that the generators $D_{\text{AdS}}$ and $D_{\text{CFT}}$ match.

We now match the $K^a$ symmetries. As we noted above, our modified de Donder gauge breaks the $K^a$ symmetries. This can be seen as follows. Using realization of $K^a$ transformations in (6.20), we find that the gauge-fixed massive AdS field satisfies the relation

$$\bar{C} K^a |\phi\rangle = -2\bar{C}_{\perp}^a |\phi\rangle \, ,$$

(7.16)

which tells us that the modified de Donder gauge condition, $\bar{C} |\phi\rangle = 0$, is not invariant under $K^a$ symmetries that are described by generator $K^a$ in (6.20). Therefore, to restore the $K^a$ symmetries of the gauge-fixed AdS field theory, we should modify the generator $K^a$ in (6.20). We modify the generator $K^a$ by using the standard procedure. Namely, we add compensating gauge transformations to maintain the $K^a$ symmetries. This is to say that, in order to find improved $K^a_{\text{impr}}$ transformations we start with the generic global $K^a$ transformations (6.20) supplemented by the suitable compensating gauge transformation

$$K^a_{\text{impr}} |\phi\rangle = K^a |\phi\rangle + G |\xi_{K^a}\rangle \, ,$$

(7.17)

where $G$ is given in (6.16) and $|\xi_{K^a}\rangle$ stands for the parameter of the compensating gauge transformation. Using the relation

$$\bar{C} G |\xi_{K^a}\rangle = \Box_{\nu} |\xi_{K^a}\rangle \, ,$$

(7.18)

and (7.16), we find

$$\bar{C} K^a_{\text{impr}} |\phi\rangle = -2\bar{C}_{\perp}^a |\phi\rangle + \Box_{\nu} |\xi_{K^a}\rangle \, .$$

(7.19)

Requiring the improved $K^a_{\text{impr}}$ transformations to maintain the modified de Donder gauge condition,

$$\bar{C} K^a_{\text{impr}} |\phi\rangle = 0 \, ,$$

(7.20)

we get the equation for $|\xi_{K^a}\rangle$,

$$\Box_{\nu} |\xi_{K^a}\rangle - 2\bar{C}_{\perp}^a |\phi\rangle = 0 \, .$$

(7.21)

From (7.21), we see that the compensating gauge transformation parameter $|\xi_{K^a}\rangle$ should satisfy the nonhomogeneous second-order differential equation. Plugging normalized solution of equation (7.1) into (7.21), we see that equation (7.21) leads to the equation

$$\Box_{\nu} |\xi_{K^a}(x, z)\rangle = 2U_p \bar{C}_{\perp}^a |\phi_{\text{cur}}(x)\rangle \, .$$

(7.22)

Using identity (7.8), the solution to Eq. (7.22) is found to be

$$|\xi_{K^a}(x, z)\rangle = zU_{\nu+1} \bar{C}_{\perp}^a |\phi_{\text{cur}}(x)\rangle \, .$$

(7.23)

VIII. **AdS/CFT Correspondence for Non-Normalizable Modes of Massive AdS Field and Anomalous Shadow Field**

We now discuss the AdS/CFT correspondence for bulk spin-s massive AdS field and boundary spin-s anomalous shadow field. We begin with an analysis of the non-normalizable solution of Eq. (6.30). Because gauge-fixed
The equation of motion \((6.30)\) is similar to the one for the scalar AdS field \((6.36)\), we can simply apply result in Sec. VII. This is to say that solution of Eq. \((6.30)\) with the Dirichlet problem corresponding to the spin-\(s\) anomalous shadow field takes the form

\[
|\phi(x, z)| = \sigma_{\nu} \int d^d y G_{\nu}(x - y, z)|\phi_{sh}(y)|, \quad (8.1)
\]

\[
\sigma_{\nu} \equiv \frac{2^{\nu} \Gamma(\nu)}{2^{\nu} \Gamma(\nu)} (-)^{N_{\nu}}, \quad (8.2)
\]

where the Green function is given in \((6.45)\).

Using asymptotic behavior of the Green function \((6.43)\), we find the asymptotic behavior of our solution

\[
|\phi(x, z)| \overset{z \to 0}{\sim} z^{-\nu + \frac{1}{2}} \sigma_{\nu}|\phi_{sh}(x)|. \quad (8.3)
\]

From this expression, we see that solution \((8.1)\) has indeed asymptotic behavior corresponding to the spin-\(s\) anomalous shadow field.\(^\text{14}\) In the right-hand side of \((8.1)\) we use the notation \(|\phi_{sh}|\), because we are going to show that this boundary value is indeed the gauge field entering the gauge invariant formulation of the spin-\(s\) anomalous shadow field in Sec. IV. Namely, we are going to prove the following statements:

i) For solution \((8.1)\), modified de Donder gauge condition \((6.29)\) leads to differential constraint of the anomalous shadow field given in \((4.8)\).

ii) On-shell leftover gauge transformation \((6.15)\) of solution \((8.1)\) leads to gauge transformation of the spin-\(s\) anomalous shadow field given in \((4.18)\).

iii) On-shell global so\((d, 2)\) symmetries of solution \((8.1)\) become global so\((d, 2)\) conformal symmetries of the spin-\(s\) anomalous shadow field.

iv) an effective action evaluated on solution \((8.1)\) coincides, up to normalization factor, with boundary two-point gauge invariant vertex for the anomalous shadow field given in \((4.22)\).

Below we demonstrate how these statements can be proved by using the following relations for the Green function \(G_{\nu} \equiv G_{\nu}(x - y, z)\):

\[
T_{\nu + \frac{1}{2}} G_{\nu - 1} = -2(\nu - 1) G_{\nu}, \quad (8.4)
\]

\[
T_{\nu + \frac{1}{2}} G_{\nu + 1} = \frac{1}{2 \nu} \Box G_{\nu}, \quad (8.5)
\]

\[
\Box (z G_{\nu - 1}) = -4(\nu - 1) G_{\nu}. \quad (8.6)
\]

Matching of bulk modified de Donder gauge and boundary differential constraint. We note that it is choice of normalization factor \(\sigma_{\nu}\) in \((8.2)\) that allows us to match bulk modified de Donder gauge and boundary differential constraint. Namely, the normalization factor \(\sigma_{\nu}\) \((8.2)\) is uniquely determined by the following two requirements:

i) The factor \(\sigma_{\nu}\) is normalized to be

\[
\sigma_{\nu} = 1, \quad \text{for} \quad N_{\nu} = 0, \quad N_{\nu} = 0. \quad (8.7)
\]

ii) Modified de Donder gauge condition for AdS field \(|\phi\rangle\) \((6.29)\) should lead to the differential constraint for the shadow field \(|\phi_{sh}\rangle\) \((4.8)\).

We note that the choice of normalization condition \((8.7)\) is a matter of convenience. This commonly used condition implies the following normalization of asymptotic behavior of solution \((8.1)\) for the leading rank-\(s\) tensor field \(\phi_{0}^{a_{1}...a_{s}}\) in \((6.4)\):

\[
\phi_{0}^{a_{1}...a_{s}}(x, z) \overset{z \to 0}{\sim} z^{-\nu + \frac{1}{2}} \phi_{sh,0}^{a_{1}...a_{s}}(x), \quad (8.8)
\]

where \(\phi_{sh,0}^{a_{1}...a_{s}}\) is the leading rank-\(s\) tensor field in \((4.1)\).

Using \((8.4)\), \((8.5)\) we find the relations

\[
e_{1} \sigma_{\nu} G_{\nu} = \sigma_{\nu} G_{\nu} e_{1,sh}, \quad e_{1} \sigma_{\nu} G_{\nu} = \sigma_{\nu} G_{\nu} e_{1,sh}, \quad (8.9)
\]

where Laplace operator \(\Box\) appearing in \(e_{1,sh}, e_{1,sh}\) is acting on the Green function \(G_{\nu}\). Acting with operator \(\overline{C}\) \((8.8)\) on solution \((8.1)\) and using \((8.9)\), we find the relation

\[
\overline{C}|\phi\rangle = \sigma_{\nu} \int d^d y G_{\nu}(x - y, z) |\phi_{sh}(y)|. \quad (8.10)
\]

From this relation, we see that modified de Donder gauge for the spin-\(s\) massive AdS field \((6.29)\) and differential constraint for the spin-\(s\) anomalous shadow field given in \((4.8)\) match.

Matching of bulk and boundary gauge symmetries. We now show how gauge transformation of the anomalous shadow field \((4.18)\) is obtained from the on-shell leftover gauge transformation of the massive AdS field \((6.15)\). To this end we note that the corresponding on-shell leftover gauge transformation of massive AdS field is obtained from \((6.15)\) by plugging non-normalizable solution of equation for gauge transformation parameter \((6.31)\) into \((6.15)\). The non-normalizable solution of equation \((6.31)\) is given by

\[
|\xi(x, z)| = \sigma_{\nu} \int d^d y G_{\nu}(x - y, z)|\xi_{sh}(y)|, \quad (8.11)
\]

where \(\sigma_{\nu}\) is given in \((8.2)\). We now note that, on the one hand, plugging \((8.11)\) into \((6.15)\) and using \((8.9)\), we can cast the on-shell leftover gauge transformation of \(|\phi\rangle\) into the form

\[
\delta|\phi\rangle = \sigma_{\nu} \int d^d y G_{\nu}(x - y, z) G_{sh}|\xi_{sh}(y)|. \quad (8.12)
\]

On the other hand, making use of relation \((8.1)\), we get

\[
\delta|\phi\rangle = \sigma_{\nu} \int d^d y G_{\nu}(x - y, z) \delta|\phi_{sh}(y)|. \quad (8.13)
\]

\(^{14}\) Since solution \((8.1)\) has nonintegrable asymptotic behavior \((8.3)\), such solution is sometimes referred to as the non-normalizable solution.
Comparing (8.12) with (8.13), we conclude that the on-shell leftover gauge symmetries of solution of the Dirichlet problem for the spin-s massive AdS field are indeed related to gauge symmetries of the spin-s anomalous shadow field.

Matching of bulk and boundary global symmetries. Matching of global symmetries can be demonstrated by using the procedure we described for the spin-s anomalous conformal current in Sec.VII Therefore, to avoid repetition, let us briefly discuss only relevant details. The matching of bulk and boundary symmetries of the Poincaré algebra is straightforward. To match bulk and boundary dilatation symmetries all that we needed are solution for bulk field in (8.1), bulk dilatation operator (6.19), and conformal dimension operator for the spin-s anomalous shadow field given in (4.20). To match conformal boost symmetries we introduced improved bulk $K^a_{\text{impr}}$ transformations with compensating gauge transformation parameter that satisfies Eq.(7.21) with $|\phi|$ as in (8.1). Using relation (8.6), we find that the solution to Eq.(7.21) with $|\phi|$ as in (8.1) is given by

$$|\xi^{K^\alpha}(x,z)| = -z\sigma_{-1}\int d^4y G_{-1}(x-y,z)\bar{C}_{\perp}|\phi_{\text{sh}}(y)|.$$  

(8.14)

Using (8.1) and (8.14), we check that improved bulk $K^a_{\text{impr}}$ transformations lead to $K^\alpha$ transformations of the spin-s anomalous shadow field given in (2.23) and (4.21).

Matching of effective action and boundary two-point vertex. To find the effective action we follow the standard procedure. Namely, we plug non-normalizable solution of the bulk equation of motion with the Dirichlet problem corresponding to the boundary anomalous shadow field (8.1) into the bulk action (6.6). We proceed as follows. Using gauge invariant equation of motion (6.25) in (6.6), we get the following expression for the effective action:

$$S_{\text{eff}} = -\int d^dx L_{\text{eff}} \bigg|_{z\to0},$$ 

(8.15)

$$L_{\text{eff}} = \frac{1}{2}\langle\phi|\mu T_{-\frac{1}{2}}|\phi\rangle + \frac{1}{2}\langle\phi|Y\bar{C}|\phi\rangle,$$

$$Y \equiv \frac{1}{2}\alpha^2(r\bar{\zeta} + r_z\bar{\alpha}^2) - \zeta r_\zeta - \alpha^2 r_z.$$  

(8.16)

From (8.16), it is clear that the use of modified de Donder gauge condition (6.29) considerably simplifies the expression for $L_{\text{eff}}$.

$$L_{\text{eff}} |_{\bar{C}|\phi\rangle = 0} = \frac{1}{2}\langle\phi|\mu T_{-\frac{1}{2}}|\phi\rangle.$$  

(8.17)

Thus we see that it is the use of modified de Donder gauge that leads to $L_{\text{eff}}$ for massive arbitrary spin AdS field (8.17) which has the same form as $L_{\text{eff}}$ for scalar field (6.48). Therefore, to find $S_{\text{eff}}$ for the massive arbitrary spin AdS field we can use results for the scalar field. Namely, all that remains to obtain the effective action is to plug solution (8.1) into (8.15), (8.17) and use general formula given in (6.49). Doing so, we get

$$-S_{\text{eff}} = 2\kappa c_\nu\Gamma,$$  

(8.18)

where $\kappa$ and $c_\nu$ are given in (4.24), (4.24), respectively, while $\Gamma$ stands for gauge invariant two-point vertex of the spin-s anomalous shadow field given in (4.22), (4.23).

To summarize, using the modified de Donder gauge for the computation of the spin-s massive AdS field action on the solution of equations of motion with the Dirichlet problem corresponding to the boundary anomalous shadow field, we obtain the gauge invariant two-point vertex of the spin-s anomalous shadow field. It is the matching of the bulk leftover on-shell gauge symmetries of solution to the Dirichlet problem and bulk global symmetries and the respective boundary gauge symmetries of the anomalous shadow field and boundary global symmetries that explains why the effective action of the AdS massive field coincides, up to a normalization factor, with the gauge invariant two-point vertex for the boundary anomalous shadow field.

In the literature, the effective action is expressed in terms of the two-point vertex taken in the Stueckelberg gauge frame $\Gamma^{\text{stand}}$ (4.40). To express our result in terms of $\Gamma^{\text{stand}}$, we use relations (4.30), (4.39) to represent our result (8.18) as

$$-S_{\text{eff}} = \frac{\kappa(2\kappa + 2s + d - 2)}{s!(2s + d - 2)}c_\nu\Gamma^{\text{stand}}.$$  

(8.19)

The following remarks are in order:

i) For the particular values $s = 1$ and $s = 2$, our results for normalization factor in front of $\Gamma^{\text{stand}}$ (8.19) coincide with the ones obtained in Refs.[11] and [12] respectively. Thus, our results agree with the previously reported results for the particular values $s = 1, 2$ and give the normalization factor for arbitrary values of $s$.

ii) Using (8.19) and taking into account the expression for $\Gamma^{\text{stand}}$ given in (4.22), (4.40), we see that the AdS/CFT correspondence for massive spin-s AdS field leads to two-point correlation function of spin-s anomalous conformal current with the conformal dimension given by

$$\Delta = \frac{d}{2} + \sqrt{m^2 + \left(s + \frac{d - 4}{2}\right)^2}.$$  

(8.20)

According to the AdS/CFT correspondence this conformal dimension should be equal to lowest energy value $E_0$ for massive spin-s AdS field with mass parameter $m$. Comparing

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15 All that is needed for the derivation of (6.18) is to make the replacement $\sigma \rightarrow \sigma_{\nu}$ in formula for scalar field (6.49) and note the easily derived algebraic relation $\nu C_{\nu}^2 = \kappa c_\nu f_\nu$, where $f_\nu$ is defined in (4.24).
The fact that the effective action of massive AdS field is proportional to the two-point vertex of anomalous shadow field is expected because of the conformal symmetry. Note however that for the systematic study of AdS/CFT correspondence it is important to know the normalization factor in front of $\Gamma^{\text{stand}}(8.19)$.

iv) In the massless limit, $m \to 0$, our result for $S_{\text{eff}}(8.19)$ agrees with the previously reported results in literature. Computation of $S_{\text{eff}}$ for spin-1 and spin-2 massless fields may be found in the respective Ref. [39] and Refs. [41]. Computation of $S_{\text{eff}}$ for arbitrary spin-$s$ massless fields may be found in Ref. [4]. The study of AdS/CFT correspondence for massless fields in light-cone gauge frame may be found in Refs. [42]-[44] (see also Ref. [45]).

v) The effective action given in (8.18) is gauge invariant, while the effective action given in (8.19) is obtained from one in (8.18) by using the Stueckelberg gauge frame. One of advantages of our approach is that our approach gives the possibility to study the effective action by using other gauge conditions which might be preferable in various applications. For example, in the light-cone gauge frame, the effective action given in (8.18) takes the form

$$- S_{\text{eff}} = 2\kappa c_{\gamma} \Gamma^{(1.c.)}, \quad (8.21)$$

where light-cone gauge vertex $\Gamma^{(1.c.)}$ is given in (4.22), (4.47). It is the formula (8.21) that seems to be interesting for the studying the duality of light-cone gauge type IIB Green-Schwarz AdS superstring and the corresponding CFT.

IX. CONCLUSIONS

In the present paper, we extended our gauge invariant approach to CFT initiated in Ref. [13] to the studying of arbitrary spin anomalous conformal currents and shadow fields. We recall that, in the framework of string/gauge theory duality, the anomalous conformal currents and shadow fields are related to massive fields of AdS string theory. We note that all Lorentz covariant approaches to string field theory involve large amount of Stueckelberg fields (see, e.g., Ref. [43]). Because our approach to anomalous conformal currents and shadow fields also involves Stueckelberg fields we believe that our approach will be helpful to understand string/gauge theory duality better.

We obtained the gauge invariant vertex for anomalous shadow field which, in the framework of AdS/CFT correspondence, is related to AdS field action evaluated on solution of the Dirichlet problem. Our gauge invariant vertex provides quick and easy access to the light-cone gauge vertex. Because one expects that the quantization of AdS superstring is straightforward only in the light-cone gauge we believe that our light-cone gauge vertex will also be helpful in various studies of AdS/CFT duality. Our results should have a number of the following interesting applications and generalizations.

i) In this paper, we studied bosonic anomalous conformal currents and shadow fields. It would be interesting to extend our approach to the study of AdS/CFT correspondence for arbitrary spin fermionic anomalous conformal currents and shadow fields and related arbitrary spin massive fermionic fields [47].

ii) In this paper, we studied the AdS/CFT correspondence by using CFT adapted approach to massive AdS fields developed in Ref. [14]. In the last years, new interesting approaches to massive AdS fields were developed (see, e.g., Refs. [48]). It would be interesting to apply these new approaches to the study of AdS/CFT correspondence for massive AdS fields.

iii) An extension of our approach to the case of 3-point and 4-point gauge invariant vertices of anomalous shadow fields will give us the possibility to study various applications of our approach along the lines of Refs. [49].

iv) Idea of arranging $d$-dimensional conformal physics in $d+2$ dimensional multiplets was extensively studied in Refs. [50]. Obviously, the use of the methods developed in Refs. [50] may be very useful for the studying AdS/CFT correspondence.

v) The Becchi-Rouet-Stora-Tyutin (BRST) approach is one of powerful methods of modern quantum field theory (see, e.g., Refs. [51]). Obviously, an extension of BRST approach to the case of anomalous conformal currents and shadow fields will provide new interesting possibilities for the studying CFT.

vi) Mixed-symmetry fields have extensively been studied in the last years (see, e.g., Refs. [52]). Needless to say that generalization of our approach to the mixed-symmetry conformal currents and shadow fields could be of some interest.

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Appendix A: Matching of bulk and boundary conformal boost symmetries

In this appendix, we demonstrate matching of the improved $K_{\text{imp}}^a$ transformations of the normalizable modes of massive AdS field (7.17) and the $K^a$ transformations of boundary anomalous conformal current given in (2.19), (2.23) with op-
where $K$ in (2.23), (3.22). Fields can be demonstrated in a quite similar way.

We start with the realization of the improved $K^a_{\text{impr}}$ transformations on space of normalizable modes of massive gauge-fixed AdS field given by (see (7.17))

$$K^a_{\text{impr}}|\phi_{\text{norm}}\rangle = K^a_{\text{AdS}}|\phi_{\text{norm}}\rangle + G_{\text{AdS}}|\xi^K_{\text{norm}}\rangle ,$$  

(A1)

where, in this Appendix, the normalizable solution $|\phi\rangle$ in (7.1) is denoted by $|\phi_{\text{norm}}\rangle$, while the normalizable solution for the compensating gauge transformation parameter $|\xi^K_{\text{norm}}\rangle$ in (7.23) is denoted by $|\xi^K_{\text{norm}}\rangle$. Also, in this Appendix, the generic generator of $K^a$ symmetries in (6.20) is denoted by $K^a_{\text{AdS}}$, while the gauge transformation operator $G$ in (6.16) is denoted by $G_{\text{AdS}}$. Our purpose is to demonstrate that the improved $K^a_{\text{impr}}$ transformations of the normalizable modes of massive gauge-fixed AdS field become $K^a$ transformations of the anomalous conformal current. Namely, we are going to prove the relation

$$K^a_{\text{impr}}|\phi_{\text{norm}}\rangle = U_\nu R^a_{\text{cFT}}|\phi_{\text{cur}}\rangle ,$$  

(A2)

where $R^a_{\text{cFT}}$ stands for the realization of the conformal boost generator on space of the anomalous conformal current given in (2.23), (3.22).

In order to prove relation (A2) we represent the operator $K^a_{\text{AdS}}$ (6.20) as

$$K^a_{\text{AdS}} = K^a_{\Delta_{\text{AdS}}} + R^{a(1)} + M^{ab}b^b + R^a_{(0)} ,$$  

(A3)

$$K^a_{\Delta_{\text{AdS}}} = -\frac{1}{2} x^2 \partial^a + a^a D_{\text{AdS}} ,$$  

(A4)

where operators $D_{\text{AdS}}$, $R^{a(1)}$, and $R^a_{(0)}$ are given in (6.19), (6.23), and (6.24) respectively. Next, we note the relations

$$(K^a_{\Delta_{\text{AdS}}} + R^{a(1)}|\phi_{\text{norm}}\rangle = U_\nu K^a_{\Delta_{\text{cur}}}|\phi_{\text{cur}}\rangle ,$$  

(A5)

$$(M^{ab}b^b + R^a_{(0)}|\phi_{\text{norm}}\rangle + G_{\text{AdS}}|\xi^K_{\text{norm}}\rangle = U_\nu (M^{ab}b^b + R^a_{\text{cur}})|\phi_{\text{cur}}\rangle ,$$  

(A6)

where

$$K^a_{\Delta_{\text{cur}}} = \frac{1}{2} x^2 \partial^a + x^a D_{\text{cur}} ,$$  

(A7)

$$D_{\text{cur}} = x \partial + \Delta_{\text{cur}} ,$$  

(A8)

while $\Delta_{\text{cur}}$ and $R^a_{\text{cur}}$ are given in (3.21) and (3.22) respectively. Using (A5), (A6), we see that relation (A2) does indeed hold.

We now comment on the derivation of relations (A5), (A6). These relations are obtained by using the following general formulas

$$(K^a_{\Delta_{\text{AdS}}} + R^a_{(1)}) U_\nu = U_\nu (K^a_{\Delta_{\text{cur}}} + x^a \partial z)$$

$$- h_x (-)^N q^{-\frac{d}{2}} \partial^a (\partial q Z_\nu (q z)) z \partial z ,$$  

(A9)

$$(M^{ab}b^b + R^a_{(0)}) U_\nu + G_{\text{AdS}} (z U_{\nu + 1} \bar{C}^a_{\perp})$$

$$\approx U_\nu (M^{ab}b^b + R^a_{\text{cur}}) ,$$  

(A10)

where $Z_\nu (z) \equiv \sqrt{\xi} J_\nu (z)$, while $q$ is defined in (7.3). In (A10) and relation (A11) given below, the sign $\approx$ implies that relations (A9), (A10) are valid only on space of the anomalous conformal current $|\phi_{\text{cur}}\rangle$. Recall that $|\phi_{\text{cur}}\rangle$ is subject to differential constraint (3.9). We now see that, by applying relations (A9) and (A10) to the anomalous conformal current $|\phi_{\text{cur}}\rangle$, we obtain the respective relations (A5) and (A6).

For the reader convenience, we write down the helpful formulas to be used for the derivation of relation (A10),

$$M^{ab}b^b U_\nu \approx U_\nu M^{ab}b^b$$

$$+ z U_{\nu + 1} (- G_{\text{cur}} \bar{C}^a_{\perp} - e_{1,\text{cur}} \bar{\alpha}^a + \bar{C}^a \bar{e}_{1,\text{cur}}) ,$$  

(A11)

$$R^a_{(0)} U_\nu = z U_{\nu + 1} (\alpha^2 z \bar{\alpha}^a + \bar{C}^a \bar{\alpha}^2)$$

$$- z U_{\nu - 1} (\bar{\alpha} \bar{\alpha} \bar{C}^a + \bar{C}^a \bar{\alpha} \bar{C}^a) ,$$  

(A12)

$$G_{\text{AdS}} (z U_{\nu + 1} \bar{C}^a_{\perp}) = z U_{\nu + 1} G_{\text{cur}} \bar{C}^a_{\perp}$$

$$+ U_\nu (2 \bar{\alpha} \bar{C}^a - 2 a^2 \frac{1}{2 N_\alpha - d - 2} r z \bar{C}^a \bar{C}^a_{\perp}) ,$$  

(A13)

$$z U_{\nu - 1} + z \Box U_{\nu + 1} = 2 v U_\nu .$$  

(A14)

In turn, relation (A11) is obtained by using the identity

$$M^{ab}b^b + G_{\text{cur}} \bar{C}^a_{\perp}$$

$$= \alpha^a \bar{C}_{\text{cur}} - e_{1,\text{cur}} \bar{\alpha}^a + \bar{C}^a \bar{e}_{1,\text{cur}}$$  

(A15)

and differential constraint (3.9).

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