Multi-fluid studies of plasma shocks relevant to inertial confinement fusion

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Abstract. Results from inertial confinement fusion (ICF) experiments performed at the Omega laser facility suggest the potential role of kinetic effects in plasmas during implosion. Recent theoretical and numerical work has indicated the importance of diffusion effects in the presence of multiple ion species as well as the importance of ion viscosity. This provides the motivation to adequately develop multi-fluid plasma models capable of capturing kinetic physics including concentration diffusion and ion species separation driven by the ion concentration gradient, the ion pressure gradient, the electron and ion temperature gradients, and the electric field. Benchmarks between the newly developed code and analytical results are presented for multi-fluid plasma shocks.

1. Introduction
Recent results from inertial confinement fusion (ICF) experiments performed at the Omega laser facility suggest the potential role of kinetic effects in plasmas during implosion \cite{1, 2, 3}. In particular, inter-ion-species diffusion can degrade yields in an ICF implosion via fuel stratification in the implosion core. This has been supported by theoretical and numerical results which have highlighted the importance of inter-species transport diffusion effects as well as viscosity in the presence of multiple ion species \cite{4, 5, 6, 7, 8, 9, 10, 11, 12}. This motivates the development of multi-fluid plasma modeling capabilities that capture adequate kinetic physics unlike the majority of radiation-hydrodynamic codes that are traditionally used for ICF simulations. Concentration diffusion and ion species separation are driven by the ion concentration gradient, the ion pressure gradient, the electron and ion temperature gradients, and the electric field. Multi-fluid codes are a valuable tool to understand subtle physics phenomena such as fuel stratification that can significantly impact ICF implosion performance. The purpose of this paper is to benchmark the newly-developed code against a known physics-motivated solution. Diffusion is likely to affect many ICF-relevant phenomena such as mix at the interface. The benchmark demonstrated in this paper is motivated by the separation of the fuel constituents in the hot-spot by a shock wave hence, the simulations presented here focus on the structure of a plasma shock wave.

The WARPX (Washington Approximate Riemann Plasma) code \cite{13, 14} is used for the results presented here. WARPX is a two-fluid plasma code, the two fluids being ions and electrons. A second order MUSCL \cite{15} scheme is used for the results presented here with a third order accurate Runge-Kutta time-stepping scheme. In order to study multi-ion diffusion effects...
relevant to ICF, the WARPX code is updated to include multiple ion species and an electron species. The work presented in this paper focuses on a single ion and single electron species multi-fluid implementation with relevant kinetic terms included [16].

Multi-component plasmas can be modeled in two basic ways. The first is using center-of-mass dynamics along with an equation accounting for concentration evolution. The second approach involves evolving individual momentum equations for each species. Except for the viscous effect on the diffusive flux, the two approaches are equivalent [5]. Transport properties of multi-component plasmas ranging from weakly-coupled to strongly-coupled have been described in Refs. [8, 17, 12] respectively. The electrons are assumed to be massless. The electric field is determined from Ohm’s law using the electron pressure gradient and the ion-electron friction and thermal forces. These equations includes the ion kinetic and thermal energies whereas the electron energy \( \epsilon_e \) is solely the thermal energy due to the assumption of massless electrons. The electric field is determined from Ohm’s law using the electron pressure gradient and the ion-electron friction and thermal forces \( \mathbf{R} \). The ion viscosity, electron thermal conductivity, friction and thermal forces, and the thermal equilibration terms are described in Ref. [16]. The total ion energy \( \epsilon_i \) includes the ion kinetic and thermal energies whereas the electron energy \( \epsilon_e \) is solely the thermal energy due to the assumption of massless electrons. The electric field is determined from Ohm’s law using the electron pressure gradient and the ion-electron friction and thermal forces. These equations enforce quasineutrality and ambipolar flow. A separate set of continuity, momentum, and energy equations is solved for each ion species and \( \mathbf{R} \) is modified to include the summation of friction and thermal forces over all ion species present [5].

Simulations of stationary, two-fluid, collisional shocks are presented here and are compared to the semi-analytical model presented by Jaffrin and Probstein [18]. The purpose of the results presented here is two-fold. Firstly, the multi-fluid plasma model is benchmarked to known analytical results which is necessary prior to applying it to dynamic shock problems. Secondly, the two-fluid plasma shocks presented here go beyond the analytical work of Jaffrin and Probstein by including the ion-electron thermal forces that were absent in the analytical work. Any qualitative and quantitative differences are highlighted. In the presence of multiple ion species, the thermal force term has a significant impact on shock structure and species separation hence, it cannot be neglected for ICF applications.

2. Two-fluid shock results
Simulations are initialized with Hugoniot conditions for a two-fluid plasma shock at a plasma Mach number of 3. This corresponds to an ion Mach number of 4.2. The analytical results from the Jaffrin and Probstein model are reproduced here for arbitrary Mach numbers and are compared to the numerical results. For the electron collision frequency, an artificial ion-to-electron mass ratio of 100 is used to accelerate the computational time. The initial conditions
Figure 1. Comparing numerical and analytical [18] solutions of two-fluid electrostatic stationary shocks at a plasma Mach number of 3. $\xi$ refers to the spatial scale normalized to the mean-free-path downstream of the shock. The velocities and temperatures are normalized to their respective downstream values.

are then allowed to relax to the stationary shock solutions via collisional effects. The boundary conditions for density, velocity, and temperature are maintained throughout the simulation based on the prescribed Hugoniot conditions. The simulation domain is chosen so as to provide sufficient spatial extent so that the ion and electron temperatures equilibrate at both domain boundaries. The simulations are run to approximately 100 electron collision times to allow for adequate relaxation of the shock.

Figure 1 presents results of ion velocity, ion temperature, and electron temperature along with corresponding analytical solutions for a plasma Mach 3 shock. $\xi$ is the spatial scale normalized to the mean-free-path ($\lambda_{mfp}$) downstream of the shock. Consistent with theory, note the formation of three distinct regions in the shock: the pre-heat region, the ion viscous jump, and the post-shock region. Due to the electron thermal conductivity being much larger than the ion thermal conductivity, the electrons are heated in the pre-heat layer resulting in a shallow gradient of the electron temperature across the upstream region of the shock. The second region is the viscous jump layer that corresponds to the heating and compression of the ions which contributes to a sharp gradient of the ion temperature across a few mean-free-paths. The analytical results assume a flat electron temperature profile across this layer since there is insufficient time for ions and electrons to exchange energy in this region, but the numerical results contain a small gradient in the electron temperature across this region. In the third layer, the post-shocked region, the ion and electron temperatures equilibrate after approximately $\sqrt{m_i/m_e}\lambda_{mfp}$ due to the electron heat conduction and energy exchange between ions and electrons.

Slight quantitative differences are observed in the ion temperature profile. In the pre-heat region, the analytical solution gives rise to a shallow gradient in the ion temperature whereas the numerical solution has a fairly flat ion temperature profile across this region. In the viscous jump, the analytical solution reaches a higher ion temperature compared to the numerical results. This
is due to having a shallow temperature gradient in the pre-heat region that produces a higher ion temperature just upstream of the viscous jump for the analytical solution. Corresponding differences are also seen in the ion velocity profile in the viscous jump layer. The quantitative and qualitative differences between the numerical and analytical results may be attributed to the inclusion of the ion-electron thermal forces in the numerical solutions which were not present in the analytical solutions of Jaffrin and Probstein. Furthermore, the analytical work assumes a constant Coulomb logarithm since it is a very slow function of temperature and density. In the numerical simulations, the Coulomb logarithm is computed based on the density and temperature at the given spatial location so it is allowed to vary through the shock, however, the effect of this is expected to be relatively minor.

3. Summary

Results are presented for collisional two-fluid plasma shocks and comparisons are made to theoretical results. The simulation framework has been extended to multiple ion species and simulations involving two or more ions will be presented in future publications. Due to the need to resolve the mean-free-path within the shock and the need to resolve the electron collision frequency, explicit time-stepping is used for the results presented here. The choice of plasma Mach number of 3 used here is primarily due to the large computational effort involved when using higher Mach numbers. Higher Mach numbers correspond to larger mean-free-paths and longer collision times hence, solutions requiring higher Mach numbers increase the size of the simulation domain and the run-time significantly. Furthermore, at higher Mach numbers, the fluid description of plasma shock waves breaks down [19]. Accelerated time-stepping schemes and the use of emerging computing architectures will potentially allow access to higher Mach number regimes for multi-dimensional problems which will be explored in future work.

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