Virtual network as excitable medium

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Abstract. We simulated the spread of an activity in a virtual group using the model of excitable medium. We assumed that the structure of the virtual group corresponds to a scale-free network. In our simulation, the network consists of 100 nodes, the average degree of the nodes is 1.98. We considered the propagation of excitation both in a homogeneous and an inhomogeneous excitable medium. The simulation showed that the initial conditions have a little effect on the behaviour of the model. In inhomogeneous medium, fraction of the excited nodes increases, when permanent excited elements ('active' centres) appear in the network. The fraction of the excited nodes increases, when we increase the number of the permanent excited elements. Locations of the active centres do not affect at the level of excitation. External source of activator increases the fraction of the excited nodes in the scale-free network with distribution of parameters.

1. Introduction
Dissemination of ideas and opinions in virtual networks, e.g., in social networks, the Internet, academic networks, the blogosphere, etc. is of special interest [1]. A virtual network can be considered as the active medium [2, 3]. Model by Wiener and Rosenblueth is one of the possible approaches for describing such systems [4]. It is a cellular automaton model that allows explaining the main features of the processes in excitable media, such as electrical excitation’s wave propagation through the heart. Elements of the systems have three possible states, of rest, excitation, and refractoriness, and implement a series of successive transitions from one state to another. The elements of a cellular automaton may be connected with different number of the neighbours, so the structure of networks can be not only a regular lattice, but also arbitrarily complex network.

Recently, the spread of certain ideas in a professional virtual group was simulated using generalized Wiener-Rosenblueth model [3]. A particular case has been investigated when a structure of the virtual group corresponded to a complete graph and each member (site) had an individual set of parameters. The simulation showed that interest in the idea can vanish or fluctuate depending on the set of parameters in the virtual group.

We perform simulations of activity in a virtual group using the model of excitable medium. In contrast with [3], we suppose that the structure of the virtual group corresponds to a scale-free network. We considered the propagation of excitation in a homogeneous and an inhomogeneous excitable medium. We studied also impact of initial conditions, the effect of ‘active’ centre (permanent excited element) and external influences on the behaviour of the model.
2. Model
2.1. Structure of the networks
We considered the discrete systems, i.e. the networks. A scale-free network is a network whose degree distribution follows a power law \[5\]. There are some nodes in a scale-free network which have much more connections than others; such nodes are called ‘hubs’. Many networks have been reported to be scale-free \[5\]. We suppose that the structure of the virtual group corresponds to a scale-free network. In our model, scale-free network was generated using the function \textit{igraph barabasi game} from the library for analysis of social networks \textit{igraph C library}\(^1\).

2.2. Model of excitable medium
We used generalized Wiener-Rosenblueth model of excitable medium \[4, 6\]\(^2\). Each element of the network have three possible states, i.e. rest, excitation, refractoriness. Initially, elements are in the rest state. The \(i\)-th element becomes excited under the influence of an external excitation whose intensity is not lower than the threshold value, \(h_i\). The \(i\)-th element states in the excited state during the time \(\tau^{e}_i\), then it goes into the refractory state in which it states during the time \(\tau^{r}_i\), then it comes back to the state of the rest. A state of any element is specified by the integer phase, \(\Phi^n_i\), and the activator concentration, \(u^n_i\), where integer \(n\) indicates the discrete time step. Activator decays with time, \(g_i\), is the rate of decay.

Transitions between the states obey the following set of rules

\[
\Phi^{n+1}_i = \begin{cases} 
\Phi^n_i + 1, & \text{when } 0 < \Phi^n_i < \tau^{e}_i + \tau^{r}_i, \\
0, & \text{when } \Phi^n_i = \tau^{e}_i + \tau^{r}_i, \\
0, & \text{when } \Phi^n_i = 0 \text{ and } u^{n+1}_i < h_i, \\
1, & \text{when } \Phi^n_i = 0 \text{ and } u^{n+1}_i \geq h_i.
\end{cases}
\]

A site gets activator from its associated active neighbours of the network.

\[
u^{n+1}_i = g_i u^n_i + \sum_j I^n_j,
\]

where \(j\) is the number of the neighbour site \(i\),

\[
I^n_j = \begin{cases} 
1, & \text{when } 0 < \Phi^n_j \leq \tau^{e}_j, \\
0, & \text{when } \tau^{e}_j < \Phi^n_j \leq \tau^{e}_j + \tau^{r}_j \text{ or } \Phi^n_j = 0.
\end{cases}
\]

3. Results
3.1. Homogeneous excitable medium
In homogeneous excitable medium, all elements (sites) have the same parameters thereby the subscripts in the notations of all quantities may be omitted.

In our simulation, we observed the following modes.

- The excitation does not propagate, i.e. an initially excited site returns to the state of rest without passing excitation to any of its neighbours.
- Initial excitation propagates to a small area of the network and then vanishes.
- A solitary wave of excitation propagates through the entire network only once (Fig. 1 and Supplementary multimedia file Wave.gif).

\(^1\) http://igraph.org/c
\(^2\) Original description of the generalized Wiener–Rosenblueth model [6] is a bibliographic rarity. Nevertheless, it is described in several books, see, e.g. [7]
Figure 1. Excitation wave propagation in a homogeneous scale-free network: (a) step 0, (b) step 4, (c) step 9. Parameters of the model are $h = 0.75$, $g = 0.12$, $\tau_e = 1$, $\tau_r = 2$. Initial conditions are one node (hub) is in the excited state.

Figure 2. Travelling waves in a homogeneous scale-free network. Initial conditions are (a) one node (hub) is in the excited state, (b) one node with one connection is in the excited state. $E$ is the number of excited sites, $N$ is the total number of sites. Parameters of the model are $h = 1$, $g = 0.5$, $\tau_e = 1$, $\tau_r = 2$.

- Travelling waves propagate through the entire network one by one (Fig. 2 and Supplementary multimedia files Waves.gif, Waves2.gif).

Fig. 2 shows, that travelling waves appear for a given set of parameters regardless of the initial conditions, i.e. when the initial excitation is localized in a hub or in a node with a small number of bonds. Nevertheless, the excitation propagates with the higher speed, when the initial excitation is localized in a hub.

Fig. 3(a) shows how the fraction of the excited nodes changes with time in a scale-free network. The excitation starts with a single node (hub) and propagates throughout the network only once. When the node states always in an excited state then the travelling wave will propagate in the
Figure 3. Variation of the fraction of the excited nodes with time (a) in a scale-free network, (b) in a scale-free network with active centre (single node), (c) in a scale-free network with the external source of the activator. Parameters of the model are $h = 0.75$, $g = 0.12$, $\tau_e = 1$, $\tau_r = 2$.

Figure 4. Variation of the fraction of the excited nodes with time (a) in a scale-free network, (b) in a scale-free network with active centre (nodes with 4 connections), (c) in a scale-free network with the external source of activator. Parameters of the model are $h = 2$, $g = 0.75$, $\tau_e = 3$, $\tau_r = 3$.

network (Fig. 3(b)). External source of the activator leads to oscillations, whose amplitude depends on the initial fraction of the excited nodes (Fig. 3(c)).

The values of model parameters have a drastic influence on the spread of excitation over the network. For example, the excitation does not spread across the network when the parameters of the model are $h = 5$, $g = 0.5$, $\tau_e = 3$, $\tau_r = 4$ and a randomly selected node is initially excited. Activator produced only by one node is not enough to excite the neighbouring nodes (high threshold of excitation, $h$). In this case, either presence of an active centre or receiving the activator from the external permanent source at each step may not affect the propagation of the excitation. Nevertheless, the excitation will propagate when the parameter $g$ is greater, or when each node receives the amount of activator equals to 3 at each step. However, it is possible to increase the fraction of the nodes in the excited state with a help of the ‘active’ nodes or external influence (Fig. 4).

3.2. Inhomogeneous excitable medium

In inhomogeneous excitable medium, each member (site) has an individual set of parameters, i.e. each person (site) has their own temper and manner. Parameter $h$ is a real positive number. We suppose that the values of the parameter $h$ among the group have a lognormal distribution. $\sigma_h$ is the standard deviation of the distribution. Parameter $g$ is also a real positive number, 0 $\leqslant$ $g$ $\leqslant$ 1. In our simulation, we use normal distribution for this parameter. $\sigma_g$ is the standard deviation of the distribution. Parameters $\tau_e$, $\tau_r$ are positive integers. We used the Poisson
Figure 5. Variation of the fraction of the excited nodes with time in scale-free network. (a) Lognormal distribution of the parameter $h$. Parameters of the model are $h = 1, \sigma_h = 0.5, g = 0.9, \tau_e = 2, \tau_r = 2$. (b) Normal distribution of the parameter $g$. Parameters of the model are $h = 1, g = 0.5, \sigma_g = 0.1, \tau_e = 1, \tau_r = 2$.

distribution for these parameters. $\lambda_{\tau_e}, \lambda_{\tau_r}$ are parameters for the Poisson distribution. We shift the values, obtained by using the Poisson distribution, by one unit to avoid zero values for parameters $\tau_e, \tau_r$.

We examined the effect of distribution of each parameter individually, i.e. only values of the one parameter have a distribution. The biggest impact has distribution of parameter $h$, the smallest impact has the distribution of the parameter $g$. The fraction of the excited nodes decreases as the number of initially excited nodes decreases, when we use lognormal distribution of the parameter $h$ (Fig. 5(a)). Herewith the time of excitation propagation increases. In this case, there appears a mode in which only a fraction of the nodes is involved in the transmission of excitation through the network. Initial conditions can affect the amplitude of oscillation when we use normal distribution of the parameter $g$ (Fig. 5(b)). The initial conditions have a little effect on the behaviour of the model with the distribution of parameter $\tau_e$. Initial conditions have a little impact when we use distribution of all parameters.

We examined the effect of ‘active’ centre and external source of activator in scale-free network with a distribution of all parameters (Fig. 6). External source of the activator increases the fraction of the excited nodes (Fig. 6(b)). Increasing the amount of the permanent excited elements increases the fraction of the excited nodes. Location of the active centres does not affect at the level of excitation (Fig. 6(c)).

4. Conclusion
For the homogeneous scale-free network, we found that

- the initial conditions have a little effect on the behaviour of the model;
Figure 6. Inhomogeneous scale-free network. The effect of ‘active’ centre and external source of activator: (a) there is no external influence, (b) external source of the activator, (c) active centre (variable $m$ means a number of connections). Initial conditions are single node (hub) is in the excited state. Parameters of the model are $h = 1, \sigma_h = 0.5, g = 0.6, \sigma_g = 0.06, \lambda_{r_e} = 20, \lambda_{r_r} = 1$.

- behaviour of the model is more dependent on the parameters;
- in the cases, when the excitation does not propagate, active centre cannot change the behaviour of the model;
- external source of the activator leads to oscillations, whose amplitude depends on the initial fraction of the excited nodes.

For the inhomogeneous scale-free network, we found that

- the initial conditions have a little effect on the behaviour of the model;
- ‘active’ nodes can increase the fraction of the excited nodes by increasing the number of ‘active’ nodes, rather than by their location on the network;
- external source of the activator increases the fraction of the excited nodes.

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