On a local formalism for time evolution of dynamical systems

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Abstract

The formalism of local maximization for entropy gradient producing the evolution and dynamical equations for closed systems. It eliminates the inconsistency between the reversibility of time in dynamical equations and the strict direction of irreversible evolution for complex systems, causality contradictions and ambiguity of time flow in different systems. Independently it leads to basic principles of special relativity.

Keywords: time direction, irreversibility, entropy, dynamical equations

1 Introduction

1.1 Evolution & Dynamics

The conventional understanding of dynamics of any processes in the nature is the time dependance of state of a system.

Here, the primary concept of any formulation of dynamics is the state. A state is determined by the complete set of attributes - degrees of freedom (DoF) of the system.

As a process we understood the change of state in the time as a variable parameter. The dynamics is understood as a set of states for different values

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of the *time* variable, which are ordered by increasing values of time. It results as a solution of *dynamical equations*.

The state of a complex system is defined as a collectivity of single systems - components. For complex systems with a sufficiently high number of equal (or similar) components the description of state is formulated in terms of statistical functions (*statistical systems*), such as statistical sums, mean values, entropy, dispersion, fluctuations and further concepts derived from them.

The dynamics (or *evolution*) of its statistical states is formulated in terms of the same (universal) time, as for each component.

### 1.2 Dynamical equations - short review of conventional approaches

#### 1.2.1 Lagrangian formalism

The minimization condition for the *action functional*

\[ S = \int_{t_i}^{t_f} \mathcal{L} [q_i, \dot{q}_i, t] \, dt \rightarrow \text{min} \]

produces the *Euler-Lagrange* dynamical equations

\[ \frac{d}{dt} \frac{\partial}{\partial \dot{q}_i} \mathcal{L} = \frac{\partial}{\partial q_i} \mathcal{L} \]

for DoF’s *q*ᵢ, which are of second order in external time *t* and can be therefore invariant under time reversion *t* → −*t* [1]

The Lagrangian formalism reveals two obvious shortcomings:

- **causality**
  - the main drawback of this approach is its *global formulation*.

The *local* dynamical equations are obtained from the *integral* over the total evolution time. it means: in order to define its *local* behaviour - **the at a given time in a given state - state space point**- the system should ’know’ all possible states in all times in the past and the future → thus the **causality** is *a priori* disregarded in this formulation.
• the **time reversibility** does not reproduce the irreversible evolution of complex systems

### 1.2.2 Hamiltonian formalism

The state space is determined by the double set of DoF’s \( q_i, p_i \) by means of local scalar function

\[
H = H(q_i, p_i),
\]

the Hamiltonian function - the key object generating the evolution in the state space.

The time dependence - dynamics- is introduced externally by imposing the dynamical equations

\[
\frac{dq}{dt} = H_{p_i}; \quad \frac{dp}{dt} = -H_q
\]

- the **Hamilton equations**. The following main disadvantages of this approach should be mentioned here are

• redundant degrees of freedom (2n instead of n), although \( p_i \) are generally connected with time derivatives of \( q_i \);

• time is an extrinsic parameter - the time scale of the dynamics is not an intrinsic feature of the system and can be chosen arbitrary.

### 1.2.3 General shortcomings of conventional formulations

• **time irreversibility is not regarded**, that causes inconsistencies with generalisation for complex statistical systems;

• since the **causality is generally not assured**, it must be demanded externally by conditions for space-time interval (special relativity) \([2]\); it causes an ambiguity for definitions of time flow - time is subjected to Lorentz transformations (especially resulting in relativity of simultaneity)

Summarizing the above-mentioned weak points, several reasons for a partial revision of this historically formed views are in particular the following:
1.2.4 Time direction problem

According to the conventional belief, the time (as well as the space) is an *a priori* existent parameter, external for a considered system among all subsystems and observer (system of observation). The canonical set of equations, formulated in terms of such time, can possess a symmetry property, regarding the time (reversibility), e.g. if the equations are of an even order (as for the case of Newtonian mechanics).

On the other hand, a great macro-system consisting of a plurality of elementary time-reversible systems never possesses a reversibility. Instead of this, the evolution of a macro-system follows the *second law of thermodynamics*, according to which the entropy of the closed system, regarding the increasing time, cannot decrease.

Thus a conventional (e.g. the Newtonian or Lagrangian) formulation leads to the general inconsistency between the reversibility of time in dynamic equations and the strict direction of irreversible evolution of complex systems consisting of a plurality of elementary systems subjected to the time-reversible dynamic equations.

In other words, the time as an evolution parameter for elementary (reversible) systems and the global time of (irreversible) macroscopic evolution is apparently not one and the same. This evidence suggests probably, that the priority or hierarchy of basic principles for elementary systems and for macro-systems should be re-ordered, what (at least formally) eliminates this contradiction.

1.2.5 Causality problem

The causality is the time-ordering of states according to the sequence *cause* - *result* for elementary systems as well as for complex systems.

The relativity of time, resulting from the relativity theory, brings a certain ambiguity in the interpretation of the resulting dynamics, even in the classical level. Together with the time reversibility mentioned above; as well as a demand to distinguish two cases (e.g. advanced-retarded solution) additionally arising.

Furthermore in a quantum level, harder contradictions occur (e.g. Schroedinger’s cat, EPS-paradox)

Another issue for troubles with causality, especially in the classical mechanics and field theories, is the conventional formulation of component dy-
nematics in terms of Lagrangian formalism. As it was mentioned above the main drawback of this approach (fatal for causality) is the sufficiently global formulation - the key object, producing the local dynamic equations (action functional) - is the integral over the total configuration space and the evolution time. In simple words it means, in order to define its behaviour at the certain time in the certain configuration state, the system should "know" its behaviour in all possible states in all times in the past and future. Thus, the causality is disregarded in the formulation from the beginning on.

1.3 Re-definition of time

Time flows differently in different systems at different conditions. The ultimate solution for this problem would be to suppose that the "time" does not exists in general as a physically well-defined measure.

The possible revision of the time concept can be a considering of the time defined-as-measured instead of assuming the existence of time a priori (like the definition of a wave velocity [4]):

Observe the evolution of some subsystem S (as "system") of the total closed system. The "measurement of time" assigned to the each state of S means, being compared with the state of another subsystem C (as "clock") corresponding to the same global state of the total system. (Fig.1)

The "time direction" means, the states of the system C are ordered with respect to the entropy of the total system. And the "time evolution" of the subsystem S is the ordering of its states with respect to the states of subsystem C.

1.4 Entropy maximization principle

Finally, the ordering of the global states of the total closed system follows the increase of its entropy.

Thus, the basic postulate, generating the ordering (interpreted as evolution) of a closed sytem among all its subsystems is the ordering by entropy, or second law of thermodynamics applied inversely.

Finally, this principle can be re-interpreted in a local formulation: the "trajectory" of the total system in the state space obeys the condition of the entropy maximization, the state change occurs in the direction of maximal increasing entropy. Thus, this principle can be called entropy-gradient-maximization.
1.5 Local formulation

Summarizing the suggestions sketched above, the ”dynamics” of the system with arbitrary DoF’s results from following principles:

- the formulation is deterministic: all possible states and the structure of the state space for a closed system are determined by values of its DoF’s
- the trajectories are determined by initial points and the entropy gradient maximization principle with additional conditions
- the topological structure of the state space is determined by paths (trajectories) between points, where the starting points corresponds to the cause is the initial condition of path; the causality is then uniquely fixed;
- the ”time dynamics” is the ordering of the points of trajectory with respect to substates of a chosen subsystem (clock system);

The mathematical formalisation of these principles is developed in the next sections below, proceeding from the example of euclidean space with the euclidean norm, further generalization on the case of an arbitrary state space with an arbitrary ergodicity condition is formulated;

The resulting dynamical equations are considered first for a system with two DoF’s, with the resulting corollaries for time and causality restrictions.

The further generalization for a system with 3 DoF points the common way for further generalization of the formalism for more DoF’s. In particular, the issue of the time ambiguity or ”relativity” is explained. In the last section the argumentations and resulting implications are concluded.

2 Postulates of the formalism

2.1 Entropy-gradient-maximization principle of evolution

The problem of time direction and causality can be eliminated by rearranging the hierarchy of key principles:

- the primary (generating) principle is the second law of thermodynamics - entropy maximization - the ordering of global states of a total
closed system follows the increase of its entropy (in conventional approaches it should be fulfilled in a secondary priority);

- this formulation should be *local*: the state evolution in each point of the state space occurs in the direction of maximal increasing entropy;

- each solution for the 'trajectory' of the total system in the state space obeys therefore the condition of the entropy maximization, which is defined as *causality*;

### 2.2 Definition of time 'as-measured'

Observe the evolution of some subsystem S (as 'system') of the total closed system (Fig.1) Measurement procedure

- States of S are compared with states of a subsystem C ('clock'): each state of S corresponds to the state of a subsystem C in the same *global* state of the total system

- 'time evolution' of the subsystem S is the ordering of its states with respect to the states of subsystem C

- 'time direction' means that states of the system C are ordered with respect to the entropy increasing of the total system
2.3 Introducing example of gradient maximization - gradient of a scalar field

Consider the scalar field $\varphi(x_i)$, defined on the $n$-dimensional euclidean space $\vec{x} = \{x_i\}$

The property of gradient vector $\nabla \varphi(x_i)$

$$\nabla \varphi(x_i) := \left\{ \frac{\partial}{\partial x_i} \varphi \right\}$$

is, it points in the direction of maximal increasing of $\varphi$ [3]. It can be shown as follows: Let the point vector $\vec{x}_i$ gets a variation $x_i$ and one seeks its components $x_i$ such that the variation of $\varphi$ in the point $\vec{x}_i$ maximizes

$$\delta \varphi(\vec{x}_i + x_i) \to \max_{x_i}$$

To this end it is enough to consider the linear part of the variation as a function of the variation components

$$\delta \varphi(\vec{x}_i + x_i) = \left. \frac{\partial}{\partial x_i} \varphi(x_i) \right|_{\vec{x}_i} x_i := \nabla \varphi|_{\vec{x}_i} x_i,$$

which should be maximized with the normalization condition:

$$|\vec{x}|^2 = \sum_{i=0}^{n} x_i^2 = c^2 = \text{const}$$

The solution for maximum condition using the Lagrange multiplier method is:

i) vanishing of first derivatives (subscript $x_i$ denotes the partial derivative $\frac{\partial}{\partial x_i}$):

$$\{\delta \varphi\}_{x_i} = \left\{ \nabla \varphi(x_i)|_{\vec{x}_i} x_i + \lambda \left( \sum_{i=0}^{n} x_i^2 - c^2 \right) \right\}_{x_i} = 0$$

(n equations, $\lambda$ is a Lagrange multiplier)

ii) non-positivity of the second derivative matrix

$$\{\delta \varphi\}_{x_i x_k} = \left\{ \nabla \varphi|_{\vec{x}_i} x_i + \lambda \left( \sum_{i=0}^{n} x_i^2 - c^2 \right) \right\}_{x_i x_k}$$
The first requirement (i) provides
\[ \frac{\partial}{\partial x_i} \varphi + 2\lambda x_i := \varphi_{x_i} + 2\lambda x_i = 0, \quad \varphi_{x_i} = -2\lambda x_i, \quad i = 0 \ldots n \]

squared and summarized over \( i \) and taking into account the additional (normalization) condition:
\[ \lambda^2 = \frac{1}{4c^2} \sum_{i=0}^{n} \varphi_{x_i}^2; \quad \lambda = \pm \frac{1}{2c} \sqrt{\sum_{i=0}^{n} \varphi_{x_i}^2} \]

and the second requirement (ii) fixes the "+" sign of \( \lambda \):
\[ \varphi_{x_i x_k} = -2\lambda \delta_{ik} \]

, what results in the final solution:
\[ x_i = \frac{\varphi_{x_i}}{2\lambda} = c \frac{\varphi_{x_i}}{\sqrt{\sum_{i=0}^{n} \varphi_{x_i}^2}} = c \nabla \varphi(x) \frac{\sqrt{\sum_{i=0}^{n} \varphi_{x_i}^2}}{\sqrt{\sum_{i=0}^{n} \varphi_{x_i}^2}} \]

3 Formulation of the entropy-gradient-maximization

3.1 General Formulation - state ordering

Consider the total (closed) system with \( n + 1 \) degrees of freedom (DoF).
\[ q_i = \{q_1, q_2, ..., q_n\} \text{ and } \tau \]

which define the state of system completely (system defined by \( \{q_i, \tau\} \) is a closed system).

The states \( \{q_i, \tau\} \) are ordered with respect to the the increasing values of scalar function \( S(q, \tau) \) -entropy, which can be interpreted as a scalar field on the space \( \{q_i, \tau\} \)

3.1.1 Choice of time

The values of the DoF \( \tau \) are ordered so that:
\[ S(q, \tau_1) \leq S(q, \tau_2) \leq ... \leq S(q, \tau_n) \leq S(q, \tau_{n+1}) \leq ... \]
for $\tau_1 < \tau_2 < \ldots < \tau_n < \tau_{n+1} < \ldots$

Form the values $\tau_1 < \tau_2 < \ldots < \tau_n < \tau_{n+1}$ a continuum, is $S(q_i, \tau)$ a monotonic non-decreasing function of $\tau$:

$$\frac{\partial}{\partial \tau} S := S_\tau \geq 0 \text{ (time-eligibility condition for } \tau) \quad (1)$$

### 3.1.2 First-order-gradient formalism

Consider the first order variation of the entropy $S(q_i, \tau)$ in the point $\{q_i, \tau\}$:

$$\delta S(q_i, \tau) = S_\tau + \sum_i S_{q_i} q_i \quad (2)$$

(values of partial derivatives are taken in the point $\{q_i, \tau\}$)

The change of state occurs in the direction, where the entropy $\delta S(q_i, \tau)$ variation maximizes; with the $q_i, \tau$ - variation vector obeying the additional condition:

$$h(q_i, \tau) = \varepsilon_0 \quad (3)$$

- the *ergodicity* condition, which means usually some conservation law, e.g. for the total energy (a simple euclidean norm cannot be used, since the space $(q_i, \tau)$ is not euclidean).

It provides the equations: i)

$$\delta S_\tau = 0$$
$$\delta S_{q_i} = 0$$

trajectory direction

for first derivartives and ii)

$$\begin{bmatrix} \delta S_{\tau \tau} & \delta S_{\tau \tau} q_i \\ \delta S_{q_k \tau} & \delta S_{q_k \tau} \end{bmatrix} \{\leq 0\} \text{ matrix } [n+1 \times n+1] \text{ is non-positive} \quad (4)$$

- causality condition

### 4 Example of the 2 DoF system

Consider the system possessing 2 DoF’s denoted $q, \tau$ with the entropy $S(q, \tau)$ and the ergodicity condition $h(q, \tau) = \varepsilon_0$ and apply the conditions i) - ii) mentioned above.
The derivatives can be obtained avoiding the procedure of Lagrange multiplier for the sake of lucidity; it provides for \( \delta S = \delta S(q, \tau) \):

\[
d\delta S = S_q dq + S_\tau d\tau \\
dh(q, \tau) = h_q dq + h_\tau d\tau = 0
\]

from the latter we have:

\[
\dot{q} := \frac{dq}{d\tau} = \frac{h_\tau}{h_q} = -\frac{S_\tau}{S_q}, \text{ (and } \tau_q := \frac{d\tau}{dq} = -\frac{h_q}{h_\tau} \text{ respectively) (5)}
\]

substituted into the former and with the condition i) applied:

\[
\frac{d\delta S}{d\tau} = S_q \dot{q} + S_\tau = -\frac{h_\tau}{h_q} S_q + S_\tau = 0 \\
\frac{d\delta S}{d\tau} = S_q + S_\tau \tau_q = S_q - \frac{h_q}{h_\tau} S_\tau = 0,
\]

two equations which are the same. Thus the evolution trajectory in \( q, \tau \) is governed by a **first-order equation**

For example, the system obeys the ergodicity condition

\[ h(x, t) = \varepsilon_0 \]  \hspace{1cm} (6)

-some kind of energy conservation with the \( x \)-1D space coordinate, \( t \) value of ordered states of a clock. It results from the (5)

\[ -\dot{x} h_x = h_t \]

and the usual construction

\[ h(x, t) = T(\dot{x}) + U(x) = \frac{m}{2} \dot{x}^2 + U(x) \]

leads to

\[ -\dot{x} U_x = \dot{x} m \ddot{x} \]

providing the dynamical (newtonian) equation and the indentity. It indicates, that the time reversibility origins from the essential constraint, contained in the conventional form of energy (6). Additionaly, it imposes a requirement for special form of entropy by (5).
A detailed analysis of conventional dynamical systems is performed in the forthcoming investigation [5].

The second condition results in the matrix
\[
\frac{\partial^2 S}{\partial \delta = \left[ \begin{array}{cc} \delta S_{\tau \tau} & \delta S_{\tau q} \\ \delta S_{\tau q} & \delta S_{qq} \end{array} \right] = -S_{\tau} \frac{\alpha_q - \alpha_{\tau}}{\alpha^2} \left[ \begin{array}{cc} 1 & -\alpha \\ -\alpha & \alpha^2 \end{array} \right]
\]

with the notation \( \alpha := \frac{\hbar q}{\hbar r} = \frac{S_q}{S_r} \).

Since the matrix is explicitly non-negative, the condition results in:
\[
\alpha_q - \alpha_{\tau} > 0 \quad \text{or} \quad \frac{\dot{q} \hbar_q + \hbar_{\tau q}}{\dot{q} \hbar_q - \dot{q}^2 \hbar_{qq}} < 1, \quad \dot{q} = -\frac{\hbar_q}{\hbar q} = -\frac{S_q}{S_r}
\]

Thus, an existence of finite rate of DoF (corresponding to finite velocity for relativistic causality) is a direct corollary of the formalism.

5 Example of the 3 DoF system

5.1 Application of the formalism

Now, let the system possess 3 DoF’s \( \{q, r, \tau\} \).

The local entropy variation
\[
\delta S = S_q \dot{q} + S_r \dot{r} + S_{\tau \tau}
\]

minimizes with the ergodicity
\[
h(q, r, \tau) = \varepsilon
\]

The condition i) results in the set of conditions
\[
\begin{align*}
S_q \dot{q} + S_r \dot{r} + S_{\tau} &= 0 \\
S_q \dot{q}_r + S_r \dot{r}_r + S_r &= 0 \\
S_q + S_{\tau \tau} + S_r &= 0 \\
h_q \dot{q} + h_r \dot{r} + h_{\tau} &= 0 \\
h_q \dot{q}_r + h_r + h_{\tau} &= 0 \\
h_q + h_r r_q + h_{\tau \tau} &= 0
\end{align*}
\]
where only two are independent of each other.

The system is formally a linear system of 6 inhomogeneous equations:

\[
\begin{bmatrix}
S_q & S_r \\
S_r & S_r \\
q_r & q_r \\
h_q & h_r \\
h_q & h_r \\
h_r & h_r \\
\end{bmatrix}
\begin{bmatrix}
\dot{q} \\
\dot{r} \\
\dot{\tau} \\
\dot{q} \\
\dot{r} \\
\dot{\tau} \\
\end{bmatrix}
= -
\begin{bmatrix}
S_r \\
S_r \\
q_r = 1/\dot{r} \\
h_r \\
h_r \\
h_r \\
\end{bmatrix}
\begin{bmatrix}
S_q \\
S_q \\
s_r = 1/\dot{q} \\
h_q \\
h_q \\
h_q \\
\end{bmatrix}
\]  
(10)

with the solution:

\[
\dot{q} = -\alpha := -\frac{D_q}{D_r}, \quad \dot{r} = -\beta := -\frac{D_r}{D_r}, \quad q_r = \gamma := \frac{D_q}{D_r}
\]  
(11)

where the determinants are:

\[
D_q = \begin{vmatrix}
S_r & S_r \\
h_r & h_r \\
\end{vmatrix}, \quad D_r = \begin{vmatrix}
S_q & S_r \\
h_q & h_r \\
\end{vmatrix}, \quad D_{\tau} = \begin{vmatrix}
S_q & S_r \\
h_q & h_r \\
\end{vmatrix}.
\]  
(12)

The trajectory of the system in the state space \( \{q, r, \tau\} \) is defined by two equations:

\[
\frac{dq}{q(\tau)} = \frac{dr}{r(\tau)} = \frac{d\tau}{\tau}
\]  
(13)

where \( q, r, \tau \) are solutions of the system

\[
S_q(q, r, \tau) + S_r(q, r, \tau)\frac{1}{\gamma} + S_\tau(q, r, \tau)\frac{1}{\alpha} = 0 \\
h(q, r, \tau) = \varepsilon
\]

The causality matrix ii) consists of

\[
\delta S_{rr} = -S_q\alpha_r - S_r\beta_r \quad \delta S_{qq} = -S_r(1/\alpha)q - S_\tau(1/\gamma)q \quad \delta S_{rr} = -S_q\gamma_r - S_\tau(1/\beta)_r \\
\delta S_{qq} = -S_q\alpha_q - S_r\beta_q \quad \delta S_{qr} = -S_r(1/\alpha)_r - S_\tau(1/\gamma)_r \quad \delta S_{rr} = -S_q\gamma_r - S_\tau(1/\beta)_r
\]

Thus the causality condition consists of 3 inequalities for 3 determinants according to the Sylvester-Jacobi criterion of non-positivity (and generally n inequalities for n degrees of freedom).
5.2 The "time transformation"

In the (8,9) the DoF $\tau$ is chosen to be a "clock" subsystem. Since all DoF’s $(q, r, \tau)$ are equivalent in sense pari passu to be chosen as a "clock" e.g. the DoF $r$ can be considered as a "time" as well. Suppose furthermore, the DoF’s $\tau$ and $r$ are equal to be chosen as a 'clock':

$$S_\tau = S_r$$

(14)

then

$$\dot{q} = -\frac{S_\tau}{S_q} (\dot{r} + 1)$$

and the same equation reads also

$$S_\tau \left( \frac{1}{\dot{q}} + \frac{1}{q_r} \right) + S_q = 0$$

that means

$$\dot{q} = -\frac{S_\tau q_r}{S_q q_r + S_\tau}$$

(15)

that describes a transformation of a rate of the DoF $q$ (e.g. co-ordinate) with respect to the reference DoF $\tau$ under the change to the reference DoF $r$. - interpreted as an explicit manifestation of the 'time relativity'.

6 Conclusion

In the present note, basic principles of dynamics and derivation of dynamical equations are revised and re-ordered in an alternative formulation.

6.1 Short summary of outstanding inconsistencies to be solved by an alternative approach

- **Time direction and reversibility** -
  inconsistency between the reversibiltiy of time in dynamical equations and the strict direction of irreversible evolution for complex systems

- **Causality** -
  ambiguity of definition for cause - result sequence especially for relativistic and quantum systems
• **ambiguity of time definition** - time flows differently in different systems at different conditions

A possible way to remediation of these shortcomings could be a partial revision of our understanding of the dynamics and first of all, the phenomena of time appearing, measurement and interpretation.

These are the conclusions following from the time as defined by measurement:

• there is no universal time as a physically well defined measure;

• not states are ordered in time, but the time arises as a result of ordering;

• the basic measure for ordering is the entropy; thus the phenomenon of "time" has a pure statistical issue;

• to fix the local evolution direction, an additional condition (conditions) - ergodicity- should be kept; it is usually interpreted as a conservation law (laws);

• for a system with more than 2 DoF’s, the first order gradient with only one ergodicity condition provides not a unique equation: to obtain a uniqueness there are two possibilities
  1. consider more ergodicity equations (conservations)
  2. extend the entropy variation to higher order gradients

• the state evolution results form the principle of entropy gradient maximization in form of evolution equations; together with initial conditions is the evolution uniquely defined;

• as a time measure, a state sequence of arbitrary system (clock) can be taken and is therefore related to this subsystem especially, the relativity of time is a manifestation of this relation;

• the dynamic equations are the evolution equations with the separated DoF’s of the chosen clock system;

• change of the time variable from one clock-subsystem to another implies a transformation of time
These conclusions are illustrated and discussed on the known example of classical hamiltonian systems systems with discoupled time DoF, especially with a large number of DoF’s (stastistical systems) subjected to Boltzmann dynamical equations. [5]

It is worth noting that some usual conceptions are conversed in the philosophy of the present formalism, such as:

• solution of dynamical equations should obey the second thermodynamic law for global systems
  → the second thermodynamic law generates the dynamical equations

• the entropy increases in the direction of time
  → the direction of time is determined by increasing entropy

• conservation laws follow from the solutions of dynamical equations
  → the dynamical equations resut from conservation laws interpreted as ergodicity conditions

• clock ticks differently in different reference systems, because of the time relativity
  → the time is defined by clock ticking.

6.2 Technical advances of the formalism

• the formalism takes for a basis only one well justified governing principle

• the formulation is strongly deterministic:
  all possible states and the structure of the state space for a closed system are determined by its DoF’s completely

• the formulation is strongly local:
  a next state is determined completely by local maximization and initial conditions respectivelythe trajectories are determined by initial points and the entropy gradient maximized under additional condition(conditions)

• thus the topological structure of the state space is determined by paths (trajectories) between a starting point (cause)- the initial condition of path; the causality is then fixed uniquely;
• ‘time dynamics’ is the ordering of the points of trajectory with respect to substates of a chosen subsystem (clock system) of the same closed system;

It means, that the time is only defined with respect the reference subsystem (chosen as a clock) and its degrees of freedom. For example, a time interval between two events of a microscopic system in quantum mechanics is defined in fact in a macroscopic system of observation.

In a contrary, the term ‘age of the Universe’ is generally meaningless, since no reference system can be defined.

6.3 General advances resulting from the formalism

• the formalism contains only one (just existent and well known!) basic principle of a statistical issue - the entropy maximization

• the time reversibility is eliminated, since the evolution obeys a first-order conditions

• the causality is uniquely fixed as a solution of the first-order problem with initial condition

• postulates of the special relativity (existence of a finite rate, transformation of time), consequently the special relativity itself as well - need not to be postulated anymore since all these principles follow automatically from the formalism

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