Cherenkov radiation by neutrinos

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Abstract

We discuss the Cherenkov process $\nu \rightarrow \nu \gamma$ in the presence of a homogeneous magnetic field. The neutrinos are taken to be massless with only standard-model couplings. The magnetic field fulfills the dual purpose of inducing an effective neutrino-photon vertex and of modifying the photon dispersion relation such that the Cherenkov condition $\omega < |k|$ is fulfilled. For a field strength $B_{\text{crit}} = m_e^2/e = 4.41 \times 10^{13}$ Gauss and for $E = 2m_e$ the Cherenkov rate is about $6 \times 10^{-11}$ s$^{-1}$.

In many astrophysical environments the absorption, emission, or scattering of neutrinos occurs in dense media or in the presence of strong magnetic fields [1]. Of particular conceptual interest are those reactions which have no counterpart in vacuum, notably the decay $\gamma \rightarrow \bar{\nu} \nu$ and the Cherenkov process $\nu \rightarrow \nu \gamma$. These reactions do not occur in vacuum because they are kinematically forbidden and because neutrinos do not couple to photons. In the presence of a medium or $B$-field, neutrinos acquire an effective coupling to photons by virtue of intermediate charged particles. In addition, media or external fields modify the dispersion relations of all particles so that phase space is opened for neutrino-photon reactions of the type $1 \rightarrow 2 + 3$.

If neutrinos are exactly massless as we will always assume, and if medium-induced modifications of their dispersion relation can be neglected, the Cherenkov decay $\nu \rightarrow \nu \gamma$ is kinematically possible whenever the photon four momentum $k = (\omega, \mathbf{k})$ is space-like, i.e. $\mathbf{k}^2 - \omega^2 > 0$. Often the dispersion relation is expressed by $|\mathbf{k}| = n\omega$ in terms of the refractive index $n$. In this language the Cherenkov decay is kinematically possible whenever $n > 1$.

Around pulsars field strengths around the critical value $B_{\text{crit}} = m_e^2/e = 4.41 \times 10^{13}$ Gauss. The Cherenkov condition is satisfied for significant ranges of photon frequencies. In addition, the magnetic field itself causes an effective $\nu-\gamma$-vertex by standard-model neutrino couplings to virtual electrons and positrons. Therefore, we study the Cherenkov effect entirely within the particle-physics standard model.

This process has been calculated earlier in [2]. However, we do not agree with their results.

Our work is closely related to a recent series of papers [3] who studied the neutrino radiative decay $\nu \rightarrow \nu \gamma$ in the presence of magnetic fields.

Our work is also related to the process of photon splitting that may occur in magnetic fields as discussed, for example, in Refs. [4, 5].

Photons couple to neutrinos by the amplitudes shown in Figs. 1(a) and (b). We limit our discussion to field strengths not very much larger than $B_{\text{crit}} = m_e^2/e$. Therefore, we keep only electron in the loop. Moreover, we are interested in neutrino energies very much smaller than the $W$- and $Z$-boson masses, allowing us to use the limit of infinitely heavy gauge bosons and thus an effective four-fermion interaction (Fig. 1(c)). The matrix element has the form
where $\varepsilon$ is the photon polarization vector and $Z$ its wave-function renormalization factor. For the physical circumstances of interest to us, the photon refractive index will be very close to unity so that we will be able to use the vacuum approximation $Z = 1$. $g_V = 2\sin^2\theta_W - \frac{1}{2}$ and $g_A = -\frac{1}{2}$ for $\nu_e$, and $g_V = 2\sin^2\theta_W - \frac{1}{2}$ and $g_A = -\frac{1}{2}$ for $\nu_{\mu,\tau}$.

Following Refs. [4, 9] $\Pi^{\mu\nu}$ and $\Pi_5^{\mu\nu}$ are

$$
\Pi^{\mu\nu}(k) = \frac{e^3 B}{(4\pi)^2} \left[ (g^{\mu\nu}k^2 - k^\mu k^\nu)N_0 - (g^{\mu\nu}k_\perp^2 - k^\mu k^\nu)N_\parallel + (g^{\mu\nu}k_\parallel^2 - k^\mu k^\nu)N_\perp \right],
$$

$$
\Pi_5^{\mu\nu}(k) = \frac{e^3 (F_0 - N_0)}{(4\pi)^2 m_e^2} \left\{ -C_\parallel k_\perp^\nu (\tilde{F}k)^\mu + C_\perp \left[ k_\perp^\nu (\tilde{F}k)^\mu + k_\perp^\mu (\tilde{F}k)^\nu - k_\perp^\mu \tilde{F}^{\mu\nu} \right] \right\},
$$

where $\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$, where $F_{12} = -F_{21} = B$. The $\parallel$ and $\perp$ decomposition of the metric is $g_{\parallel} = \text{diag}(-1, 0, 0, +)$ and $g_{\perp} = g - g_{\parallel} = \text{diag}(0, +, +, 0)$. $k$ is the four momentum of the photon. $N_0, N_\perp, N_\parallel, C_\parallel$ and $C_\perp$ are functions on $B, k_\parallel^2$ and $k_\perp^2$. They are real for $\omega < 2m_e$, i.e. below the pair-production threshold.

The four-momenta conservation constrains the photon emission angle to have the value

$$
\cos \theta = \frac{1}{n} \left[ 1 + (n^2 - 1) \frac{\omega}{2E} \right],
$$

where $\theta$ is the angle between the emitted photon and incoming neutrino. It turns out that for all situations of practical interest we have $|n - 1| \ll 1$ [4, 9]. This reveals that the outgoing photon propagates parallel to the original neutrino direction.

It is easy to see that the parity-conserving part of the effective vertex ($\Pi^{\mu\nu}$) is proportional to the small parameter $(n-1)^2 \ll 1$ and the parity-violating part ($\Pi_5^{\mu\nu}$) is not. It is interesting to compare this finding with the standard plasma decay process $\gamma \rightarrow \bar{\nu}\nu$ which is dominated by the $\Pi^{\mu\nu}$. Therefore, in the approximation $\sin^2\theta_W = \frac{1}{2}$ only the electron flavor contributes to plasma decay. Here the Cherenkov rate is equal for (anti)neutrinos of all flavors.

We consider at first neutrino energies below the pair-production threshold $E < 2m_e$. For $\omega < 2m_e$ the photon refractive index always obeys the Cherenkov condition $n > 1$ [4, 9]. Further, it turns out that in the range $0 < \omega < 2m_e$ $C_\parallel, C_\perp$ depend only weakly on $\omega$ so that it is well approximated by its value at $\omega = 0$. For neutrinos which propagate perpendicular to the magnetic field, a Cherenkov emission rate can be written in the form

$$
\Gamma \approx \frac{4\alpha G_F^2 e^5}{135(4\pi)^4} \left( \frac{B}{B_{\text{crit}}} \right)^2 h(B) = 2.0 \times 10^{-9} \text{ s}^{-1} \left( \frac{E}{2m_e} \right)^5 \left( \frac{B}{B_{\text{crit}}} \right)^2 h(B),
$$
where
\[
    h(B) = \begin{cases} 
        (4/25)(B/B_{\text{crit}})^4 & \text{for } B \ll B_{\text{crit}}, \\
        1 & \text{for } B \gg B_{\text{crit}}.
    \end{cases}
\]  

(5)

Turning next to the case $E > 2m_e$ we note that in the presence of a magnetic field the electron and positron wavefunctions are Landau states so that the process $\nu \rightarrow \nu e^+ e^-$ becomes kinematically allowed. Therefore, neutrinos with such large energies will lose energy primarily by pair production rather than by Cherenkov radiation (for recent calculations see [10]).

The strongest magnetic fields known in nature are near pulsars. However, they have a spatial extent of only tens of kilometers. Therefore, even if the field strength is as large as the critical one, most neutrinos escaping from the pulsar or passing through its magnetosphere will not emit Cherenkov photons. Thus, the magnetosphere of a pulsar is quite transparent to neutrinos as one might have expected.

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