Intrinsic scatter of the luminosity relation, redshift distribution of the standard candles, and the constraining capability

Shi Qi

Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008, China
Key Laboratory of Dark Matter and Space Astronomy, Chinese Academy of Sciences, Nanjing 210008, China
Joint Center for Particle, Nuclear Physics and Cosmology, Nanjing University—Purple Mountain Observatory, Nanjing 210008, China and Kavli Institute for Theoretical Physics China, Chinese Academy of Sciences, Beijing 100190, China

Standard candles are one of the most important tools to study the universe. In this paper, the constraints of standards candles on the cosmological parameters are estimated for different cases. The dependence of the constraints on the intrinsic scatter of the luminosity relation and the redshift distribution of the standard candles is specifically investigated. The results, especially for the constraints on the components of the universe, clearly show that constraints from standard candles at different redshifts have different degeneracy orientations, thus standard candles with a wide redshift distribution can self break the degeneracy and improve the constraints significantly. As a result of this, even with the current level of tightness of known luminosity relations, gamma-ray bursts (GRBs) can give comparable tightness of constraint with type Ia supernovae (SNe Ia) on the components of the universe as long as the redshifts of the GRBs are diversifying enough. However, for a substantial constraint on the dark energy EOS, tighter luminosity relations for GRBs are needed, since the constraints on the dark energy from standard candles at high redshifts are very weak and are thus less helpful in the degeneracy breaking.

Standard candles are one of the most important tools to study the universe. Type Ia supernovae (SNe Ia) are currently the maturest standard candles on cosmological scales, studies on which lead to the discovery of cosmic acceleration \[1,2\]. Gamma-ray bursts (GRBs) have also attracted much attention as standard candles (see e.g. \[3\] and references therein). GRBs cover much wider redshift range than SNe Ia, but, on the other hand, have larger intrinsic scatters in their luminosity relations, which makes them less ideal as standard candles than SNe Ia. In this paper, the constraints of standards candles on the cosmological parameters are estimated for different cases. The dependence of the constraints on the intrinsic scatter of the luminosity relation and the redshift distribution of the standard candles is specifically investigated. The investigation is done by keeping in mind the current development of the GRBs as standard candles (see e.g. \[3\]).

In \[4\], a general procedure for estimating constraints of standard candles on cosmological parameters using mock data was discussed and, as a result, analytical formulae for the marginal likelihood of the cosmological parameters were derived. Consider a general luminosity relation of the form

\[
y = \log \left(4\pi d_L^2 \mathcal{F}\right),
\]

where \(d_L\) is the luminosity distance and \(\mathcal{F}\) may be any physical quantity that can be directly measured from observation. Define

\[
l(z, \theta, \theta_0) = 2 \log \frac{d_L(z, \theta)}{d_L(z, \theta_0)}
\]

and use \(l_i\) as the abbreviation for \(l(z_i, \theta, \theta_0)\). Ignoring the measurement uncertainties, we have the marginal likelihood of the cosmological parameters \(\theta\)

\[
\mathcal{L}(\theta) \propto \left(\sigma_{\text{int},0}^2 + \sigma_{\text{int}}^2\right)^{-\frac{N}{2}},
\]

where \(p\) is the number of the calibration parameters which include the coefficients \(c\) and the intrinsic scatter \(\sigma_{\text{int}}\). See \[4\] for more details.

From Eqs. \[3\] and \[4\], we can see that, to estimate the constraining capability of a sample of standard candles on the cosmological parameters (without considering the measurement uncertainties), we only need to input the information

1. about the luminosity relation: its intrinsic scatter and the number of luminosity indicators involved;
2. about the sample: the number of the standard candles and their redshifts.

No further detailed information is needed. This much simplifies the procedure and makes things transparent and clear. For example, one can immediately tell from Eq. \[4\] that the smaller the intrinsic scatter \(\sigma_{\text{int},0}\) is and/or the larger the sample size \(N\) is, the more sensitive the marginal likelihood \(\mathcal{L}(\theta)\) is to the variation of the cosmological parameters \(\theta\), which means that the constraint is tighter. The Hubble constant only contributes
to \( l(z, \theta, \theta_0) \) a constant that is same for all the standard candles and has no effect on \( \sigma_l \), so \( \mathcal{L}(\theta) \) is independent of the Hubble constant and we cannot directly constrain it in this way.

Here, utilizing Eqs. (3) and (4), the constraining capability of standard candles on cosmological parameters was estimated using mock data. The flat ΛCDM with \( \Omega_m = 0.3 \) was used as the fiducial model and \( p = 3 \) was assumed. The constraints on \( (\Omega_m, \Omega_\Lambda) \) for the ΛCDM model and on \( (\Omega_m, w) \) for the flat wCDM model were studied. The dependence of the constraints on the intrinsic scatter of the luminosity relation and the redshift distribution of the standard candles was specifically investigated. For the intrinsic scatter of the luminosity relation, \( \sigma_{\text{int}, 0} = 0.4, 0.3, 0.2, \) and 0.1 were considered. For the redshift distribution of the standard candles, five cases were considered, i.e., 500 standard candles uniformly distributing in the redshift range \([0, 1, 1]\), \([1, 2]\), \([2, 4]\), \([4, 7]\), and \([0, 1, 7]\). These values were chosen specifically by keeping in mind the current development of the GRBs as standard candles (see e.g. [3]). The results are presented in Figs. 1 and 2.

From Figs. 1 and 2 we can see that, as expected, the smaller the intrinsic scatter \( \sigma_{\text{int}, 0} \) is, the tighter the constraints are. For the dependence of the constraints on the redshift distribution of the standard candles, we can see, especially from Fig. 1 that the constraints from standard candles of different redshifts show different degeneracy orientations, thus a combination of standard candles from a wide redshift range can self break the degeneracy and improve the constraints significantly. This is because that \( \mathcal{L}(\theta) \) depends on \( l(z, \theta, \theta_0) \) through its variance along the redshift. Standard candles with a narrow redshift distribution can only reflect the local redshift variance of \( l(z, \theta, \theta_0) \), and the redshift variance of \( l(z, \theta, \theta_0) \) has different features at different redshifts, which leads to different degeneracy orientations of \( \mathcal{L}(\theta) \). In contrast, standard candles with a wide redshift distribution can reflect the global redshift variance of \( l(z, \theta, \theta_0) \), the derived \( \mathcal{L}(\theta) \) is more sensitive to the variation of the cosmological parameters \( \theta \), which means tighter constraints. Such a feature of standard candles was also shown in [3] by using strips of constant \( d_L \) at different redshifts in the cosmological parameter space.

The self degeneracy breaking feature of a wide redshift distribution of standard candles means that redshift distribution of standard candles can play a similar role as the intrinsic scatter of the luminosity relation in determining the tightness of the constraints. For a given intrinsic scatter, standard candles of same sample size with wider redshift distributions give tighter constraints. A loose luminosity relation combined with a wide redshift distribution of standard candles can have comparable tightness of constraint with a tight luminosity relation combined with a narrow redshift distribution of standard candles. For example, in Fig. 1 the contour plots of the top right \( (\sigma_{\text{int}, 0} = 0.4 \) and 500 standard candles uniformly distributing in the redshift range \([0, 1, 7]\)) and the bottom left \( (\sigma_{\text{int}, 0} = 0.1 \) and 500 standard candles uniformly distributing in the redshift range \([0, 1, 1]\)) give comparable tightness of constraint.

The self degeneracy breaking due to the redshift distribution of standard candles is less obvious in Fig. 2 than in Fig. 1. This is because that the constraints on the dark energy EOS \( w \) mainly come from standard candles at redshifts less than, say, about 2. The universe is matter dominated at high redshifts, where dark energy does not contribute much in determining the evolution of the universe, so the constraints on the dark energy from standard candles at high redshifts are very weak and are thus less helpful in the degeneracy breaking.

As mentioned earlier, the intrinsic scatter of the luminosity relation and the redshift distribution of the standard candles are chosen by keeping in mind the current development of GRBs as standard candles. From the results, we can conclude that, even with the current level of tightness of known luminosity relations (see e.g. [3]), GRBs can give comparable tightness of constraint with SNe Ia on the components of the universe as long as the redshifts of the GRBs are diversifying enough. However, for a substantial constraint on the dark energy EOS, we need tighter luminosity relations for GRBs.

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FIG. 1. 68.3% and 95.4% confidence regions in the \((\Omega_m, \Omega_\Lambda)\) plane for the ΛCDM model. The flat ΛCDM with \(\Omega_m = 0.3\) was used as the fiducial model and is represented by the black plus sign in the figures. The rows represent different intrinsic scatters of the luminosity relation for the standard candles. From top to bottom, \(\sigma_{\text{int},0} = 0.4, 0.3, 0.2, 0.1\). The columns represent different redshift distributions of standard candles. From left to right, 500 standard candles uniformly distributing in the redshift range \([0.1, 1], [1, 2], [2, 4], [4, 7], [0.1, 7]\) were used. The luminosity relation was assumed to have only one luminosity indicator involved. The upper left gray region in the figures represent the parameter space for which the universe does not experience a big bang in the past.
FIG. 2. 68.3% and 95.4% confidence regions in the \((\Omega_m, w)\) plane for the flat \(w\)CDM model. The flat \(\Lambda\)CDM with \(\Omega_m = 0.3\) was used as the fiducial model and is represented by the black plus sign in the figures. The rows represent different intrinsic scatters of the luminosity relation for the standard candles. From top to bottom, \(\sigma_{\text{int}, 0} = 0.4, 0.3, 0.2, 0.1\). The columns represent different redshift distributions of standard candles. From left to right, 500 standard candles uniformly distributing in the redshift range \([0.1, 1], [1, 2], [2, 4], [4, 7], [0.1, 7]\) were used. The luminosity relation was assumed to have only one luminosity indicator involved.