DIS AT LOW X, SATURATION SCALE, GLUON STRUCTURE FUNCTION AND VECTOR-MESON PRODUCTION

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Deep inelastic scattering at low $x$ can be described by essentially only two fitted parameters. The interpretation of $J/\psi$ photoproduction in terms of the gluon structure function is elaborated upon.

I will concentrate on the intimate connection between the $x$-dependence and the $Q^2$ dependence of the structure function $F_2(x,Q^2)$, and subsequently I will turn to vector-meson production, to $J/\psi$ production in particular.

In deep inelastic scattering (DIS) at low $x \simeq Q^2/W^2 \ll 0.1$, the photon fluctuates into a $q\bar{q}$ color-dipole state that in the virtual forward-Compton-scattering amplitude interacts via the generic structure of two-gluon exchange with the proton. The QCD gauge-theory structure implies diagonal and off-diagonal transitions in the masses of the color-dipole vector states, and accordingly it implies a dependence on the transverse three-momentum of the gluon, $\vec{l}_\perp$, that couples to the color dipole. The effective value of $\vec{l}_\perp$ introduces a novel scale, the saturation scale, relevant in low-$x$ DIS. In our approach, the saturation scale, $\Lambda_{sat}^2(W^2)$, depends on the energy, $W$, and

$$\Lambda_{sat}^2(W^2) = \frac{1}{6} \langle \vec{l}_\perp^2 \rangle \cong \frac{1}{6} \text{const} \left( \frac{W^2}{1 GeV^2} \right)^{c_2}.$$  

(1)

A fit to the total photoabsorption cross section by the power law (1) in the HERA energy range gave

$$2 GeV^2 \lesssim \Lambda_{sat}^2(W^2) \lesssim 7 GeV^2,$$

(2)

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where

\[ \text{const} = 0.340 \pm 0.063 \text{GeV}^2, \]
\[ C_2 = C_2^{\text{exp}} = 0.27 \pm 0.01. \] (3)

In addition to \( \Lambda_{\text{sat}}^2(W^2) \), the total (virtual) photoabsorption cross section depends on the cross section \( \sigma^{(\infty)} \) of hadronic size,

\[ \sigma^{(\infty)} = 48 \text{GeV}^{-2} = 18.7 \text{mb}, \] (4)

(for \( R_{e^+e^-} = 10/3, \) four flavours),

and is approximately given by

\[ \sigma_{\gamma^*p}(W^2, Q^2) \approx \frac{\alpha}{3\pi} R_{e^+e^-} \sigma^{(\infty)} \left\{ \begin{array}{ll}
\ln \eta^{-1}, & (\eta \ll 1), \\
\frac{1}{2} \eta^{-1}, & (\eta \geq 1),
\end{array} \right. \] (5)

with the scaling variable

\[ \eta(W^2, Q^2) = \frac{Q^2 + m_0^2}{\Lambda_{\text{sat}}^2(W^2)} \] (6)

and \( m_0^2 \approx 0.15 \text{GeV}^2 \). Apart from this threshold mass, the cross section (5), or equivalently \( F_2(W^2, Q^2) \), contains three adjusted parameters, the two parameters (3) determining the saturation scale and the cross section (4).

Application of DGLAP evolution in the region of \( Q^2 \gg \Lambda_{\text{sat}}^2(W^2) \), where appropriate, actually reduces the number of three to only two adjusted parameters, since evolution allows one to determine the exponent \( C_2 \) in (1). This will be pointed out next.

The representation (5) of the experimental data contains the assumption that the scattering amplitude for longitudinal, \( (q\bar{q})^J_L=1 \), (vector) states and for transverse ones, \( (q\bar{q})^J_T=1 \), be proportional to each other. In terms of the sea-quark, \( x\Sigma(x, Q^2) \), and the gluon distribution, \( xg(x, Q^2) \), and the proportionality constant \( r \), this proportionality reads

\[ x\Sigma(x, Q^2) = \frac{12}{R_{e^+e^-}} F_2(x, Q^2) = \frac{8}{3\pi} \left( r + \frac{1}{2} \right) \alpha_s(Q^2) xg(x, Q^2)|_{x=Q^2/W^2} \] (7)

The constant \( r \) also determines the ratio of the longitudinal to the transverse photoabsorption cross section,

\[ \frac{\sigma_{\gamma^*p}(W^2, Q^2)}{\sigma_{\gamma^*p}(W^2, Q^2)} = \frac{1}{2r}. \] (8)

The (successful) representation \( r=1 \) of the experimental data was based on \( r=1 \). With (5) and (7), the evolution equation (at low \( x \))

\[ \frac{\partial F_2(x, Q^2)}{\partial \ln Q^2} = \frac{R_{e^+e^-}}{9\pi} \alpha_s(Q^2) xg(x, Q^2) \] (9)

turns into an equation for \( \Lambda_{\text{sat}}^2(W^2) \). Inserting the power law (1), one finds a constraint on \( C_2 \) that is given by

\[ (2r + 1)2^{C_2}C_2 = 1. \] (10)

In Table 1, we show the relation between \( r \) and \( C_2 \) resulting from (10). The constant \( r \), according to (7), determines the relative magnitude of gluon to sea distribution. The dependence of the structure function \( F_2(W^2) = F_2(Q^2/x) \) for \( Q^2 \gg \Lambda_{\text{sat}}^2(W^2) \) follows from (5).
Table 1: Results for $C_2^{\text{theor.}}$ for different values of $r$ according to (10).

| $r$ | $C_2^{\text{theor.}}$ | $\alpha_s \cdot \text{glue}$ | $\sigma_g^*/\sigma_T^*$ | $F_2\left(\frac{Q^2}{x}\right)$ |
|-----|---------------------|-----------------|-------------------|------------------|
| $\rightarrow \infty$ | 0 | $< \text{sea}$ | 0 | $(Q^2/x)^0 = \text{const.}$ |
| 1 | 0.276 | $\approx \text{sea}$ | $\sim \frac{1}{2}$ | $(Q^2/x)^{0.276}$ |
| 0 | 0.65 | $> \text{sea}$ | $\infty$ | $(Q^2/x)^{0.65}$ |

We summarize:

i) The theoretical value of $C_2$ in Table 1 from (9) and (10) for $r = 1$ coincides with the experimental one (3) obtained for $r = 1$,

$$C_2^{\text{theor.}} \approx C_2^{\text{exp.}},$$

and thus the underlying ansatz for the dipole cross section is consistent with the evolution equations from QCD. A (strong) violation of (10) would have ruled out this ansatz, and in particular the underlying assumption of $W$ being the relevant variable to describe diffractive processes at low $x$.

ii) Essentially two parameters, the normalization of the saturation scale $\Lambda_{\text{sat}}^2(W^2)$ in (3) and the cross section of hadronic magnitude (4) are sufficient to determine the low-$x$ proton structure function including the photoproduction limit.

iii) The $Q^2$ and the $x$ dependence of $F_2(x,Q^2)$ are strongly correlated with each other and correlated with the relative magnitude of the gluon and sea contributions, compare Table 1.

iv) A sufficiently large gluon contribution implies a strong rise of $F_2(x,Q^2)$ with increasing $Q^2$ for constant $x$, and an equally strong rise with decreasing $x$ at fixed $Q^2$ (compare lines 2 and 3 in Table 1). This qualitative feature is experimentally realized, and theoretically it is a natural consequence of $W$ as the relevant variable that describes the scattering cross section of a color dipole on the proton (rather than $x$).

v) Since the relative magnitude of the gluon and the sea is correlated with $\sigma_g^*/\sigma_{T^*}$, direct measurements of this ratio are urgently needed. This allows one to investigate the limits of validity of the underlying assumed proportionality of sea and gluon distributions.

Turning to $J/\psi$ production, in figs. 1 and 2, I show our result of an absolute prediction based on the description of the inclusive DIS data I told you about. For details, I have to refer to the original publications.

I wish to mention one important point, however, related to the interpretation of $J/\psi$ photoproduction ($Q^2 = 0$) in terms of the gluon structure function. From (7), valid for sufficiently large $Q^2 \gg \Lambda_{\text{sat}}^2(W^2)$, we have

$$\alpha_s(Q^2)xg(x,Q^2)|_{x=Q^2/W^2} = \frac{1}{8\pi^2}\sigma^{(\infty)}\Lambda_{\text{sat}}^2(W^2 = Q^2/x).$$

(11)

According to (11), a determination of the energy dependence of $\Lambda_{\text{sat}}^2(W^2)$ at any $Q^2$, e.g. at $Q^2 = 0$ in $J/\psi$ photoproduction, yields the dependence of the gluon structure function on the left-hand side as a function of $x$ at $Q^2 \gg \Lambda_{\text{sat}}^2(W^2)$, where relation (11) becomes valid. Clearly, the measurement of $J/\psi$ photoproduction does not provide a measurement of the structure function for $Q^2 \lesssim m_c^2$, $\Lambda_{\text{sat}}^2(W^2)$, where (11) breaks down.
More generally, independent of our representation of the data on DIS, any unique prediction of $J/\psi$ photoproduction necessarily requires the left-hand side of (11) to only depend on $W^2$. Otherwise no unique prediction of $J/\psi$ photoproduction will emerge. This should be kept in mind, when predicting the energy dependence of vector meson photoabsorption, i.e. for any specific fit of the gluon structure function the left-hand side in (11) should be examined on whether it only depends on $W^2$ in good approximation at large $Q^2$.

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