Two-stage restoration strategies for power systems considering coordinated dispatch between plug-in electric vehicles and wind power units

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Abstract: With increased penetration of wind power units and plug-in electric vehicles (PEVs), their control flexibility and quick response potentially provide an alternative way to fulfil the need for rapid restoration. A two-stage restoration strategy optimisation approach is presented with the coordination of PEVs and wind power units considered. The optimisation model is to maximise the restored generation capability and minimise the fluctuation of cranking power. In the first stage, the aim is to provide reliable cranking power remotely for black start generators through coordinated dispatch of PEVs and wind power units and quadratic programming (QP) models for dispatching electric vehicle aggregators (EVAs) subject to wind power fluctuations, and for dispatching numerous PEVs within each EVA are developed. To ensure close coordination between these two dispatching procedures, bi-level programming-based hierarchical decomposition approach is used to solve the QP models in an iterative way. In the second stage, an integer linear programming model is proposed to optimise the restoration schedules through an effective transformation of the original non-linear formulation, so as to reduce the computing time and effort significantly. Finally, a case study is presented to demonstrate the effectiveness and essential features of the developed models and methods.

1 Introduction

The blackout happened on 30 July 2012 in India and many other blackout events imply that it is very difficult, if not impossible, to prevent blackouts from the occurrence, and hence the importance of the rapid system restoration after such an event cannot be overlooked [1–5]. Thus, power system restoration is still an important area deserving further and in-depth studies. In [6], an optimisation problem is formulated to optimally allocate black start units in a power system. In [7], a model predictive control based generator start-up optimisation strategy for power system restoration is proposed by utilising microgrids as black-start resources. Knowledge-based systems [8–10] have been developed to address the power restoration problem. However, it is difficult to maintain a large-scale knowledge base. Heuristic methods [11–13] are employed to solve the optimisation problems, but the attainment of the global optimal solution cannot be guaranteed.

In traditional power system restoration, a system operator has to engage dedicated generators with quick startup capability and sufficient cranking power at additional investment to black start other non-black-start units. The ever increasing wind power is playing a growing active role in system operation [14, 15]. Compared to conventional generators, wind power units have certain merits such as quick startup at almost zero cost and less demand for cranking power, which can potentially support quick system restoration from a blackout or large outage event. With a large number of wind power units well distributed in a wide area, the potential of using wind power to support system restoration is worthy of exploration. The major challenge facing such a proposition exists in the inherent intermittence of wind power output, which has to be mitigated by some advanced technologies such as dedicated battery storages at, however, high costs.

Along with the increased awareness of energy conservation and climate change, the ownership of plug-in electric vehicles (PEVs) gains rapid growth in recent years. PEVs can not only be charged from the power system but also be employed to implement the vehicle to grid function [16, 17], rendering perfect capabilities for smoothing wind power outputs and other grid support services. Thus, without additional investments for new storage devices and others, proper coordination of numerous PEVs and wind power units could potentially facilitate the system black start by providing initial cranking power at a significantly reduced cost, which motivates the research work in this paper.

In this paper, a hybrid resource pool formed with PEVs and wind farms is proposed to provide the cranking power to black start generators. A novel two-stage optimisation model is proposed to maximise the restored generation capability while minimising the fluctuation of cranking power supplied by the resource pool. In the first stage of the proposed model, bi-level programming based hierarchical decomposition approach, which is our previous work, is used to solve the quadratic programming (QP) models in an iterative way to provide reliable cranking power remotely for generators through coordinated dispatch of PEVs. In the second stage of the proposed model, a novel and effective transformation of the original non-linear formulation is proposed to generate the optimal restoration schedule for generating units with less computing time and effort.

The prerequisite of implementing PEVs and wind power units as the remote racking source is an optimisation model that can coordinate the two technologies subject to various constraints, which will be addressed in the paper. The proposed model involves a two-stage optimisation procedure to achieve coordinated dispatch of PEVs and wind power unit, and the optimal restoration schedule of generation units, respectively. The objective of the first stage is to ensure a reliable cranking power supply to the black-start units. This is achieved through two QP models to first dispatch electric vehicle aggregators (EVAs) considering the fluctuation of wind power outputs by the system operator, and subsequently, dispatch many PEVs by each EVA. To ensure close coordination between the two dispatches, bi-level programming based hierarchical decomposition approach is used to solve the QP models iteratively. The second stage optimises the restoration schedule of generating units with remote cranking power supplied by PEVs and wind power units through an integer linear programming (ILP) model. Notably, the classical restoration schedule model for generation
units is usually formulated as a non-linear optimisation one, which is hard to be solved especially for large-scale power systems. With an effective linear transformation for the non-linear optimisation model, the problem of developing the optimal restoration schedule is formulated as an ILP model in this paper, which results in significant improvements in the computational speed compared to other existing models. Finally, a case study is presented to demonstrate the effectiveness and essential features of the proposed method.

2 Optimisation model of restoration schedule

A blackout event can cause significant losses and even asset damages to both utilities and customers. Thus, quickly restoring generating units is demanded to minimise negative impacts. Different restoration schedules for generating units will lead to a different amount of generation capability, which determines the restored electric demands accordingly. Therefore, the restoration schedule optimisation of generating units is the key issue in the power system restoration after a blackout. In this paper, the restoration schedule is optimised to maximise the overall generation capability [8, 18] for the restoration period. The system generation capability \( E_{\text{gen}} \) is given by

\[
E_{\text{gen}} = \sum_{k = 1}^{N_k} \left[ \frac{1}{2} \left( p_{\text{max},k}^\text{nb} - p_{\text{max},k}^\text{ab} \right)^2 - \left( \frac{p_{\text{max},k}^\text{nb}}{R_{k,\text{nb}}} - \frac{p_{\text{max},k}^\text{ab} + \frac{p_{\text{max},k}^\text{max}}{R_{k,\text{ab}}} - \frac{p_{\text{max},k}^\text{nb}}{R_{k,\text{nb}}} \right) \right]
\]

(1)

where \( t_{k,\text{ab}}, T_{k,\text{ab}}, p_{\text{max},k}^\text{nb}, p_{\text{max},k}^\text{ab}, p_{\text{max},k}^\text{max}, R_{k,\text{nb}} \) and \( R_{k,\text{ab}} \) are the starting time, cranking time, start-up power, maximum output and ramping rate of non-black-start generator \( k \), respectively and \( T \) is the specified restoration period. The system generation capability \( E_{\text{gen}} \) is the sum of all the generators’ generation capability. The generation capability of a generator is equal to the area enclosed by the generator output curve and the horizontal axis. The area below the horizontal axis is negative and vice versa. The start-up characteristic of non-black-start generator \( k \) is shown in Fig. 1, where the generator is restarted at \( t_{k,\text{ab}} \) when it is supplied with the cranking power; the generator starts to output power at \( t_{k,\text{ab}} + T_{k,\text{ab}} \), and then increases the output gradually with the ramping rate until the maximum output \( P_{k,\text{ab}}^\text{max} \) is reached. As to the black-start generators, their start-up characteristic can be a special case of Fig. 1, where both \( t_{k,\text{ab}} \) and \( P_{k,\text{ab}}^\text{nb} \) are equal to zero.

Since \( t_{k,\text{ab}} \) is the only unknown variable in (1), the objective of maximising the generation capability can be transformed as

\[
\min \sum_{k = 1}^{N_k} \left( P_{k,\text{ab}}^\text{max} \right)
\]

(2)

It can be seen from Fig. 1 that the output \( P_{k,\text{ab}},t \) of generator \( k \) can be described as

\[
P_{k,\text{ab}},t = \begin{cases} 0 & \text{when } 0 \leq t < t_{k,\text{ab}} \\ -p_{\text{nb},k}^\text{ab} & \text{when } t_{k,\text{ab}} \leq t < t_{k,\text{ab}} + T_{k,\text{ab}} \\ R_{k,\text{ab}}(t - t_{k,\text{ab}} - T_{k,\text{ab}} - \frac{p_{\text{nb},k}^\text{ab}}{R_{k,\text{nb}}}) & \text{when } t_{k,\text{ab}} + T_{k,\text{ab}} < t < t_{k,\text{ab}} + T_{k,\text{ab}} + \frac{p_{\text{max},k}^\text{nb} + \frac{p_{\text{max},k}^\text{max}}{R_{k,\text{ab}}}}{R_{k,\text{ab}}} \\ P_{\text{max},k}^\text{nb} & \text{when } t \geq t_{k,\text{ab}} + T_{k,\text{ab}} + \frac{p_{\text{max},k}^\text{nb} + \frac{p_{\text{max},k}^\text{max}}{R_{k,\text{ab}}}}{R_{k,\text{ab}}} 
\end{cases}
\]

(3)

where \( k = 1, 2, \ldots, N_k \). If the generator needs cranking power from other ones to start up, the value of its output is negative when it is in the cranking time.

The generator cannot be restored until there is enough cranking power. Accordingly, the start-up power requirement can be formulated as

\[
\sum_{k = 1}^{N_k} P_{k,\text{ab}},t \geq 0
\]

(4)

For a generator with minimum critical interval or maximum critical interval for the startup, it should be restored in a certain period after the blackout, as described below

\[
t_{k,\text{ab}} \geq T_{k,\text{cold}}, k = 1, 2, \ldots, N_{\text{ab},\text{cold}}
\]

(5)

\[
t_{k,\text{ab}} \leq T_{k,\text{hot}}, k = 1, 2, \ldots, N_{\text{ab},\text{hot}}
\]

(6)

where \( T_{k,\text{cold}} \) is the critical minimum interval of generator \( k \), \( t_{k,\text{ab}} \) is the time when generator \( k \) obtains the cranking power and \( N_{\text{ab},\text{cold}} \) is the number of generators with the critical minimum interval. Generator \( k \) can be restored only after the interval. \( T_{k,\text{hot}} \) is the critical maximum interval of generator \( k \), \( t_{k,\text{ab}} \) is the time when generator \( k \) obtains the cranking power and \( N_{\text{ab},\text{hot}} \) is the number of generators with the critical maximum interval. If generator \( k \) can obtain the cranking power within the critical maximum interval, it will restart and supply power to the system quickly. Otherwise, it will be unavailable until after a longer time delay.

The formulations in (2)–(6) represent a typical non-linear combinatorial optimisation model. Methods such as the enumerative algorithm [19], dynamic programming [20] or two-stage algorithm [21] can be implemented to solve the model. However, the enumerative algorithm and dynamic programming are highly demanding in computation and therefore not applicable for solving practical restoration cases of systems with hundreds and even thousands of buses. On the other hand, the formulated model can be solved in a two-stage procedure, which however cannot guarantee the global optimality of the solution [18]. In this paper, the proposed non-linear combinatorial model for the system restoration is transformed into an ILP problem, which significantly facilitates finding the optimal solution at a very fast computational speed.
To transform the model, three binary decision variables \( w_{k, 1, t} \), \( w_{k, 2, t} \), and \( w_{k, 3, t} \) are introduced to the output function \( P_{k, ab, t} \) of (4) for attaining a quadratic form as described below

\[
P_{k, ab, t} = -w_{k, 1, t} P_{k, ab}^{\text{on}} + w_{k, 2, t} R_{k, ab} \left( V_T - T_{k, ab}^{\text{on}} - \frac{P_{k, ab}^{\text{on}}}{R_{k, ab}} \right) + w_{k, 3, t} P_{k, ab}^{\text{max}} \quad k = 1, 2, \ldots, N_{ab} \tag{7}
\]

A decision variable \( b_{k,d} \) is next introduced to transform \( t_{k, ab} \) into a form with the sum of products

\[
t_{k, ab} = 1 + \sum_{d=1}^{N_{d}} (1 - b_{k,d}) \quad k = 1, 2, \ldots, N_{ab} \tag{8}
\]

where \( b_{k,d} \) denotes the on-off status of generator \( k \) at time slot \( d \). Generator \( k \) is on/off when \( b_{k,d} \) equals to one/zero. \( N_{T} \) is the number of time slots in the time period \( T \). The variable \( b_{k,d} \) respects the following constraint:

\[
b_{k,d+1} \geq b_{k,d} \quad d = 1, 2, \ldots, N_{T} - 1 \tag{9}
\]

Considering \( N_{T} \) time slots in the period \( T \), (7) can be described as

\[
P_{k, ab, t} = -w_{k, 1, t} P_{k, ab}^{\text{on}} + w_{k, 2, t} R_{k, ab} \left( V_T - T_{k, ab}^{\text{on}} - \frac{P_{k, ab}^{\text{on}}}{R_{k, ab}} \right) - w_{k, 3, t} R_{k, ab} V_T + w_{k, 1, t} P_{k, ab}^{\text{max}} \tag{10}
\]

where \( V_T \) is the period of each time slot, \( k = 1, 2, \ldots, N_{ab} \), \( t = 1, 2, \ldots, N_{T}, t \in \{1, 2, \ldots, N_{T}\} \). Subsequently, three binary variables \( g_{k, 1, t, d} \), \( g_{k, 2, t, d} \), and \( g_{k, 3, t, d} \) are introduced to transform (10) from the quadratic form to the linear one as

\[
g_{k, j, t, d} = w_{k, j, t} b_{k,d} \quad j = 1, 2, 3 \tag{11}
\]

Since the variables in (11) are all binary ones, (11) is actually equivalent to the following three inequality equations

\[
\begin{align*}
g_{k, j, t, d} & \geq w_{k, j, t} + b_{k,d} - 1 \\
g_{k, j, t, d} & \geq w_{k, j, t} \\
g_{k, j, t, d} & \leq b_{k,d}
\end{align*} \tag{12}
\]

Thus, (10) can be transformed into the following form:

\[
P_{k, ab, t} = -w_{k, 1, t} P_{k, ab}^{\text{on}} + w_{k, 2, t} R_{k, ab} \left( V_T - T_{k, ab}^{\text{on}} - \frac{P_{k, ab}^{\text{on}}}{R_{k, ab}} \right) - w_{k, 2, t} R_{k, ab} V_T (1 + N_T) + R_{k, ab} V_T \sum_{d=1}^{N_T} g_{k, 2, t, d} + w_{k, 3, t} P_{k, ab}^{\text{max}} \tag{13}
\]

The variables \( w_{k, 1, t} \), \( w_{k, 2, t} \), and \( w_{k, 3, t} \) respect the following constraints:

\[
\begin{align*}
0 & \leq \sum_{j=1}^{3} w_{k, j, t} \leq 1 \\
\sum_{j=1}^{3} w_{k, j, t} & \geq 1 - \frac{t_{k, ab}}{t} \\
(1 + N_T) w_{k, 1, t} - \sum_{d=1}^{N_T} g_{k, 2, t, d} & \leq t \tag{14}
\end{align*}
\]

Thus, (10) can be transformed into the following form:

\[
0 \leq \sum_{j=1}^{3} w_{k, j, t} \leq 1 \\
\sum_{j=1}^{3} w_{k, j, t} \geq 1 - \frac{t_{k, ab}}{t} \tag{15}
\]
hierarchical and zone-based dispatching architecture via the concept of EV aggregator will be a feasible solution to reduce the communication and computation needs \([22-24]\). In this architecture, the PEVs in each distribution system or region are managed by an EVA, which serves as an intermediary managing the charging (discharging) operation on behalf of many PEV owners within the region \([22]\). The system operator communicates with the EVAs only to dispatch the total demand in an optimal manner. Thus, the computational scale and complexity of the dispatch can be reduced. Besides, the needed communication supports could be greatly reduced too. It should be noted that the communication system considered in the architecture has independent and reliable power supply via such as UPS, so that it can support the system restoration from blackout events.

In order to assist the system operator and EVAs to make reasonable decisions, the owners of the PEVs should report personal plans, such as the time when their PEVs will be plugged into and leave the system. In order to ensure PEV owners to report their time schedule properly, some attractive incentives can be adopted by, e.g., providing financial rebates to PEVs, which can well fulfill the charging schedule.

With the hierarchical dispatching architecture, each EVA can manage up to hundreds and even thousands of PEVs, which can still be challenging to implement the optimal charging/discharging dispatch. Thus, a grouping approach is introduced to decrease the scale of the problem. If the charging plans reported by different owners are similar, these PEVs can be clustered into the same group, and accordingly be managed as a whole by an EVA. This approach can greatly reduce the dimensionality of the model and decrease the difficulty of solving the dispatch problem for each EVA.

Accordingly, the steps for the system operator and EVAs making the dispatching schedule are shown as follows:

**Step 1:** The PEV owners send the necessary information to the affiliated EVAs according to their individual driving demands in the next day.

**Step 2:** The EVAs collect the reported information and cluster the PEVs into different groups by employing the grouping approach. In addition, the EVAs should calculate their schedulable capacities and send this information to the system operator.

**Step 3:** The system operator formulates the coordinated dispatching strategy according to the specific objective and then sends these instructions back to the EVAs.

**Step 4:** The EVAs formulate the coordinated dispatching strategy for PEVs so as to strictly follow the system operator’s instructions and then send back dispatching deviation to the system operator.

**Step 5:** According to the specific objective and the received dispatching deviation, the system operator adjusts instructions through a certain optimisation approach and then sends the newly optimised instructions to the EVAs.

**Step 6:** Repeat steps 4 and 5 until the pre-defined convergence or termination criterion is reached.

### 3.2 Bi-level programming method for coordinated dispatch of PEVs considering wind power

The energy management of PEVs in the proposed restoration model involves two levels of optimal dispatch for EVAs considering wind power, and for PEVs respectively to fulfil the restoration needs. The proposed model relies on the close coordination between twoQP-based dispatch models to eventually deliver the reliable cranking power to black-start units based on the hybrid resource pool including PEVs and wind power.

To ensure the coordination between the two optimal dispatch models, the bi-level programming \([25, 26]\), which is a special case of the multilevel programming \([27]\) and consists of two levels with a leader–follower structure, is used in this paper. The upper-level acts as the leader and the lower-level acts as the follower in the bi-level programming. The strategy made by the upper-level will influence the strategy made by the lower-level, and the lower-level strategy can be fed back to influence the upper-level one too. Through an iterative procedure, which involves timely information exchange between the two levels, close coordination between the two optimal dispatch models can be ensured, and the final optimal dispatch plans for EVAs and PEVs can be attained.

In the upper level, the system operator formulates the optimal instructions for EVAs in each time slot to mitigate the fluctuation of wind power. The objective of the upper level can be described as a QP model

\[
\min F = \frac{1}{H} \sum_{a=1}^{N_a} \left( P_{\text{wind},a} - \sum_{b=1}^{N_b} p_{\text{EV},a,b} - P_{\text{vac}} \right)^2 + C_{\text{ineq}} \sum_{a=1}^{N_a} \sum_{b=1}^{N_b} (p_{\text{EV},a,b} - p_{\text{EV}}) 
\]

where \(H\) is the number of time slots in the specified period; \(P_{\text{wind},a}\) is the output of the wind farm at time slot \(h\); \(P_{\text{vac}}\) is the average of the outputs of wind power and EVAs in \(H\) time slots; this average value is used to help assess the output fluctuation of the hybrid resource pool; \(p_{\text{EV},a}\) is the dispatching instruction for the \(a\)th EVA at time slot \(h\); the PEVs managed by the \(a\)th EVA are regarded as a whole and consume electricity from the system if \(p_{\text{EV},a} > 0\) and vice versa; \(N_a\) is the number of EVAs in the system; \(C_{\text{ineq}}\) is the penalty coefficient for the dispatching deviation between the two levels; \(p_{\text{EV}}\) is the dispatching instruction made by the system operator for all EVAs; \(p_{\text{EV},a,h}\) is the charging or discharging power of the \(m\)th PEV governed by the \(a\)th EVA at time slot \(h\). Equation (18) is the objective of the upper level, and (19)–(22) explain the variables in (18). Equation (19) gives the definition of the average of the outputs of wind power and EVAs in \(H\) time slots. Equation (20) means that the system operator makes the dispatching instruction for each EVA. Equation (21) shows that the instruction for each EVA consists of the instruction in each time slot. Equation (22) means that each EVA makes the dispatching instructions for each PEV in each time slot, and different PEV might give different dispatching instructions. The PEVs of an EVA act as a whole to meet the dispatching instruction for the EVA.

The dispatching instructions from the system operator are made according to the information collected from each EVA. The dispatching instruction for each EVA respects the dispatching capacity constraint for EVAs at each time interval, i.e.

\[
\sum_{a=1}^{N_a} p_{\text{EV},a,m,h} + b_{\text{EV},a,m,h} \leq p_{\text{EV},a,h} \leq \sum_{a=1}^{N_a} p_{\text{EV},a,m,h} 
\]

where \(p_{\text{EV},a,m,h}\) and \(b_{\text{EV},a,m,h}\) are the charging power, discharging power, connection status of the \(m\)th PEV managed by the \(a\)th EVA, respectively; \(b_{\text{EV},a,m,h} = 1\) means that this PEV is plugged into the system and \(b_{\text{EV},a,m,h} = 0\) means vice versa.

In the lower level, each EVA optimises the dispatching strategy for the managed PEVs to follow the instruction given by the system operator. The objective of the \(a\)th EVA can be described also as a QP model, i.e.

\[
\begin{align*}
&\min F = \frac{1}{H_a} \sum_{a=1}^{N_a} \left( P_{\text{wind},a} - \sum_{b=1}^{N_b} p_{\text{EV},a,b} - P_{\text{vac}} \right)^2 + C_{\text{ineq}} \sum_{a=1}^{N_a} \sum_{b=1}^{N_b} (p_{\text{EV},a,b} - p_{\text{EV}}) \\
\text{subject to:} \\
&\sum_{a=1}^{N_a} p_{\text{EV},a,m,h} + b_{\text{EV},a,m,h} \leq p_{\text{EV},a,h} \leq \sum_{a=1}^{N_a} p_{\text{EV},a,m,h} \\
&b_{\text{EV},a,m,h} = 1 \quad \text{if PEV is plugged into the system} \\
&b_{\text{EV},a,m,h} = 0 \quad \text{otherwise}
\end{align*}
\]
The two variables \( u_{a,m,h}^{ch} \) and \( u_{a,m,h}^{dis} \) have to respect the following constraint in order to reinforce PEVs at either charging, discharging, or neither charging or discharging status at each time interval

\[
0 \leq u_{a,m,h}^{ch} + u_{a,m,h}^{dis} \leq 1
\]  

(28)

Each PEV battery needs to observe the capacity limit as well, which can be represented by

\[
S_{\text{min}} \leq S_{a,m,h} \leq S_{\text{max}}
\]  

(29)

where \( S_{\text{min}} \) and \( S_{\text{max}} \) are the minimum and maximum SOCs for each PEV battery, respectively. In addition, the two introduced variables have to observe the following conditions when PEVs are unavailable to the power system concerned

\[
u_{a,m,h}^{ch} = 0 \quad h \not\in \{h_{a,m,s}, h_{a,m,e}\}
\]  

(30)

\[
u_{a,m,h}^{dis} = 0h \not\in \{h_{a,m,s}, h_{a,m,e}\}
\]  

(31)

where \( h_{a,m,s} \) and \( h_{a,m,e} \) are the time slots when the \( m \)th PEV of the \( a \)th EVA is plugged-in and plugged-out from the power grid.

### 3.3 Optimal energy management of PEVs by EVAs

In the proposed model, PEVs are considered forming a special kind of power pool, which can not only provide electricity to the power system but also consume power from the system. The charging posts at parking lots within a certain region can be considered as an integral system. In order to quickly support the system restoration, the electrical lines connecting the parking lots and wind farms should be kept connected after the blackout. PEVs are coordinated with wind power so as to form a reliable resource to support black-start of the system. The two-stage method is proposed to optimise the dispatch strategies for PEVs/EVAs, as well as the restoration scheduling for generators. By doing so, the scale and complexity of the optimisation problem can be effectively reduced. At the first stage, the bi-level programming approach is employed to optimise the PEVs/EVAs dispatching strategies to mitigate the fluctuation of wind power and ensure a reliable cranking power supply. At the second stage, the provided cranking power is effectively utilised to determine the optimal restoration schedule for generation units so as to maximise the available generation capacity during the system restoration.

Taking into account the cranking power provided by the hybrid resource pool, the constraint represented by (15) of the optimal restoration schedule model can be modified as (see (32)).

The flowchart of the overall proposed two-stage restoration model is shown in Fig. 2, where the two-stage approach involving restoration schedule optimisation, and optimal dispatch of EVAs by the system operator and optimal dispatch of PEVs by each EVA is clearly shown.

It should be noted that a large number of PEVs are considered in the proposed method. The number of PEVs should be large enough to mitigate power fluctuations of wind power plants providing cranking power in the early stage of system restoration through flexible charging and discharging operations. The cranking power provided by the hybrid resource pool should last until at least one of the non-black-start generators can provide the cranking power for other non-black-start generators. Then, the wind power and PEVs can be dismissed from the restoration task, which will then be handed over to those restarted non-black-start generator.

Since the energy stored by the PEVs should be large enough to provide reliable cranking power needed for restarting at least one
non-black generators, a large pool of PEVs should be considered in the proposed method.

4 Case study

4.1 Optimisation results of restoration schedule without considering PEVs and wind power

The New England 10-unit 39-bus power system shown in Fig. 3 is employed to compare the proposed method with some other methods. The ten units G1, G2, ..., G10 are located at ten different nodes shown in Fig. 3. Table 1 shows the generation information of the system, which can also be found in [18]. A blackout is assumed, and G10 serves as the black-start unit and restarts right after the blackout. The restoration actions are updated every 10 min. The proposed model is employed for optimising the generating units restoration schedule. The optimal restoration schedule of generating units obtained by the proposed method is shown in Table 2.

The same restoration case has been solved by several other methods including the enumerative method and dynamic programming method, and the corresponding computation time and the optimality of the solution are compared in Table 3, which echoes some results in [15].

Table 3 indicates that the method proposed in [18] is better than the other three methods in terms of the computing speed and solution quality. It can be seen from Table 3 that even though both the proposed method and the one in [18] can achieve the same optimal result, the computational speed of the proposed method is significantly faster than that in [18]. This is mainly because of the linear transformation implemented by (7)–(15), which is based on the introduction of the three binary decision variables $w_{k,1,t}$, $w_{k,2,t}$ and $w_{k,3,t}$. Thus the number of variables in the proposed method is reduced compared to the model in [18]. Three new variables $t_{i1}$, $t_{i2}$ and $t_{i3}$ are introduced in [18] to solve the problem. The numbers of $t_{i1}$, $t_{i2}$ and $t_{i3}$ are $N_{tb}N_T + N_{tb}N_T + N_{tb}N_T$ and $N_{tb}N_T$, respectively. The total number of the three variables is $3N_{tb}N_T$.

![Fig. 3 New England 10-unit 39-bus power system](image-url)

**Table 1** Data of the New England 10-unit 39-bus system

| Gen. | $T_{nb}, \text{ h}$ | $T_{cold}, \text{ h}$ | $T_{hot}, \text{ h}$ | $R_{nb}, \text{ MW/h}$ | $P_{nb}^{\text{max}}, \text{ MW}$ | $P_{nb}^{\text{cold}}, \text{ MW}$ | $P_{nb}^{\text{hot}}, \text{ MW}$ |
|------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| G1   | 0:35            | 0:40            | N/A             | 215             | 5.5             | 572.9           |
| G2   | 0:35            | N/A             | N/A             | 246             | 8               | 650             |
| G3   | 0:35            | N/A             | 2:00            | 236             | 7               | 632             |
| G4   | 0:35            | 1:10            | N/A             | 198             | 5               | 508             |
| G5   | 0:35            | N/A             | 1:00            | 244             | 8               | 650             |
| G6   | 0:35            | N/A             | N/A             | 214             | 6               | 560             |
| G7   | 0:35            | N/A             | N/A             | 210             | 6               | 540             |
| G8   | 0:35            | N/A             | N/A             | 346             | 13.2            | 830             |
| G9   | 0:35            | N/A             | N/A             | 284             | 15              | 1000            |
| G10  | 0:15            | N/A             | N/A             | 162             | 0               | 250             |

**Table 2** Optimal generating units restoration time

| Gen. | G1 | G2 | G3 | G4 | G5 | G6 | G7 | G8 | G9 |
|------|----|----|----|----|----|----|----|----|----|
| $T_{nb}^{\text{opt}}, \text{ h}$ | 0:50 | 0:30 | 0:20 | 1:10 | 0:40 | 0:20 | 0:30 | 0:30 | 0:40 |

**Table 3** Solution quality and computation speed of different methods

| Method          | Global optimality | Computational time |
|-----------------|-------------------|--------------------|
| enumeration     | Yes               | 1 h 53 min         |
| dynamic programming | No               | 55 min             |
| two-step        | No                | 4 min              |
| [15]            | Yes               | 8 s                |
| the proposed method | Yes              | 1 s                |
PEVs. A survey on the travel behaviour of the American public was conducted by the US Department of Transportation Federal Highway Administration (FHWA), and the results were released in 2009. The national household travel survey [29] is the authoritative source of US national data to characterise daily personal travel patterns across the country. The survey results are analysed by the probabilistic method to unveil the statistical patterns of driving behaviours. The time when the first trip starts and the time when the last trip ends in one day can be reasonably modelled with normal distributions, while the daily travel mileage can be modelled with a logarithmic normal distribution. The probabilistic models used in the driving data analysis are displayed in (33)–(35) [19].

$$f_d(x) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma_s} \exp\left(-\frac{(x - \mu_s)^2}{2\sigma_s^2}\right) & 0 < x \leq (\mu_s + 12) \\ \frac{1}{\sqrt{2\pi}\sigma_s} \exp\left(-\frac{(x - 24 - \mu_s)^2}{2\sigma_s^2}\right) & (\mu_s + 12) < x \leq 24 \end{cases}$$ (33)

$$f_d(x) = \frac{1}{\sqrt{2\pi}\sigma_f} \exp\left(-\frac{(x - 24 - \mu_f)^2}{2\sigma_f^2}\right)$$ (34)

$$f_d(x) = \frac{1}{\sqrt{2\pi}\sigma_f} \exp\left(-\frac{(x - \mu_f)^2}{2\sigma_f^2}\right)$$ (35)

where $$\mu_s = 8.92$$, $$\sigma_s = 3.24$$, $$\mu_f = 17.47$$, $$\sigma_f = 3.41$$, $$\mu_f = 2.98$$ and $$\sigma_f = 1.14$$; $$f_d(\cdot)$$ and $$f_d(\cdot)$$ represent the probability density functions of the time when the first trip starts and the time when the last trip ends, respectively, while $$f_d(\cdot)$$ denotes the probability density function of the daily travel mileage.

It is assumed that there are 20 000 PEVs in the testing system and the PEV quantity managed by each EVA is described in Table 4. In the case study, the Monte Carlo simulation method is employed to simulate the information reported by the PEV owners. The time when the PEVs plugged into the power network, the time when plugged out of the network and the daily travel mileage are generated by sampling from the probability density functions $$f_d(\cdot)$$ and $$f_d(\cdot)$$, respectively. $$f_d(\cdot)$$ is used to calculate the initial SOC of each PEV when integrated into the system.

The output curve of the wind farm and the output curve of the hybrid resource pool considering the wind farm and PEVs are depicted in Fig. 5. It can be seen from Fig. 5 that the fluctuation of the wind power is obviously mitigated by coordinated dispatch of PEVs. The final dispatching instructions of power output by the system operator and the actual optimisation results of power output by the EVAs are displayed in Table 5 and the coordinated dispatching strategy of the upper-level model in the first stage is depicted in Fig. 5. The values associated with the nodes in Fig. 5 denote the power provided by the PEVs to the system to mitigate the fluctuation of wind power. If the value is negative, it means that the PEVs managed by the EVAs as a whole draws electricity from the system, and vice versa. Briefly, the curves in Fig. 5 are drawn in the view of a generator, not a load.

It can be seen from Table 5 that the actual optimisation results of power output by the EVAs are similar to the dispatching instructions of power output by the system operator. The difference is very small so as to meet the purpose of the system operator for mitigating the fluctuation of the wind power. In some intervals, the output of the EVAs is positive, it means that PEVs managed by the EVAs discharge power into the system to offset the shortfall of the wind power output, and vice versa.

The PEV and wind power based hybrid resource pool is utilised to provide cranking power for the system after the blackout. The restoration schedule of the generators is optimised and shown in Table 6. The output power curve of the system is shown in Fig. 6, where each time interval is 10 min (same as the following figures).
Fig. 5  Output power curves of the wind farm and hybrid resource pool

Fig. 6  Coordinated dispatch instructions of power in the upper-level model at the first stage

### Table 5  Dispatching instructions of power by the system operator versus the actual optimisation results of power by the EVAs

| Time intervals | EVA(2), MW | EVA(9), MW | EVA(15), MW | EVA(18), MW | EVA(26), MW |
|----------------|------------|------------|-------------|-------------|-------------|
| 12:00–12:10    | 0.465      | 0.462      | 0.291       | 0.288       | 0.483       |
|                | 0.480      | 0.489      | 0.486       | 0.387       | 0.384       |
| 12:10–12:20    | 0.254      | 0.252      | 0.242       | 0.240       | 0.266       |
|                | 0.264      | 0.218      | 0.216       | 0.266       | 0.264       |
| 12:20–12:30    | 0.464      | 0.462      | 0.314       | 0.312       | 0.458       |
|                | 0.456      | 0.488      | 0.486       | 0.480       | 0.480       |
| 12:30–12:40    | −0.275     | −0.273     | −0.314      | −0.312      | −0.314      |
|                | −0.312     | −0.314     | −0.312      | −0.314      | −0.312      |
| 12:40–12:50    | −0.275     | −0.273     | −0.314      | −0.312      | −0.314      |
|                | −0.312     | −0.298     | −0.297      | −0.314      | −0.312      |
| 12:50–13:00    | −0.483     | −0.483     | −0.360      | −0.360      | −0.552      |
|                | −0.552     | −0.623     | −0.621      | −0.480      | −0.480      |
| 13:00–13:10    | −0.106     | −0.105     | −0.121      | −0.120      | −0.121      |
|                | −0.120     | −0.120     | −0.082      | −0.081      | −0.097      |
| 13:10–13:20    | −0.717     | −0.714     | −0.699      | −0.696      | −0.699      |
|                | −0.699     | −0.705     | −0.702      | −0.699      | −0.696      |
| 13:20–13:30    | −0.506     | −0.504     | −0.506      | −0.504      | −0.506      |
|                | −0.504     | −0.504     | −0.504      | −0.504      | −0.504      |
| 13:30–13:40    | 0.253      | 0.252      | 0.265       | 0.264       | 0.264       |
|                | 0.264      | 0.264      | 0.244       | 0.243       | 0.243       |
| 13:40–13:50    | 0.233      | 0.231      | 0.266       | 0.264       | 0.264       |
|                | 0.264      | 0.264      | 0.245       | 0.243       | 0.243       |
| 13:50–14:00    | 0.759      | 0.756      | 0.888       | 0.888       | 0.915       |
|                | 0.912      | 0.894      | 0.891       | 0.891       | 0.888       |

### Table 6  Optimal restoration schedule of the generators

| Gen. | G1 | G2 | G3 | G4 | G5 |
|------|----|----|----|----|----|
| T_{lab} | 12:50 | 12:30 | 12:30 | 13:10 | 13:30 |

| Gen. | G6 | G7 | G8 | G9 | G10 |
|------|----|----|----|----|-----|
| T_{lab} | 12:20 | 12:20 | 12:40 | 12:40 | 12:00 |
The occurrence of a blackout is hard to predict. It might take place at any time. In this paper, the time when a blackout takes place can influence the results of the restoration due to the fact that the wind power generation and the number of PEVs are time-varying and so as their capabilities of supporting the black-start power. Four scenarios described in Table 7 are designed to illustrate the impact of the time when a blackout occurs on the optimal dispatching results of the proposed model.

The output power curves of the system in four scenarios are shown in Fig. 7. It can be seen from Fig. 8 that the output power curve varies with the time when a blackout occurs. The objective of optimising the restoration schedule is to maximise the overall generation capability, which equals to the area enclosed by the power output curve and the horizontal axis. It can be seen from Fig. 7 that the area of scenario 3 is bigger than the other three scenarios. With more generation capability, more electricity of the load can be restored. Generally, the maximum power of the load is less than the sum of the maximum output of all the generators of the system. Different load levels can be drawn on Fig. 8 as several horizontal lines, and the bigger the overall generation capability is, the more quickly the system can be restored in the specific load level. The overall generation capability of scenario 3 is the biggest, the system can be most quickly restored with different levels of the load, which is usually less than the sum of the maximum output of all the generators. In scenario 3, the system is restored more quickly, because more cranking power is provided by the hybrid resource pool after the blackout. With more wind power and schedulable PEVs, the power system can achieve quicker restoration speed after a blackout.

5 Conclusion

Power system blackouts can hardly be prevented in reality. Owing to the fact that significant losses can be resulted, a quick restoration strategy is essential to minimise the consequence and recover the power supply as soon as possible. Traditional power system restoration has to engage power plants equipped with quick startup capability and sufficient cranking power at high investments. With increased penetration of wind power and PEVs in power systems, their control flexibility and quick response potentially provide an alternative way to fulfill the need for rapid restoration. In this paper, a two-stage optimisation model considering coordinated dispatch of PEVs and wind power for power system restoration is proposed. The first stage of the model makes effective use of a hybrid resource pool made of wind power and PEVs to provide reliable cranking power needed in system restoration. This is achieved through a bi-programming based approach to coordinate between the dispatch of EVAs by the system operator, and the dispatch of PEVs by each individual EVA. Subsequently, the restoration schedule of generating units is optimised at the second stage, where the original non-linear optimisation model is effectively transformed into a linear one, which significantly improves the computational speed as well as the quality of the solution. The case study using a modified New England 10-unit 39-node testing system with a wind farm and 5 EVAs was included, which validated the feasibility and superior performance of the developed model. Further, the impact of blackout time has also been examined through the case study. In principle, other renewable and storage technologies could also be integrated into the proposed method enabling broad applications.

It should be pointed out that the prerequisite of the proposed model involves advanced information and communication infrastructure to support effective energy management of PEVs and wind power. In addition, a proper incentive scheme should be designed to encourage the participation of PEVs owners. The method proposed in this paper can be regarded as the first yet preliminary attempt of its kind to explore the potential of renewable energy and PEVs for system restoration purposes in the context of smart grid development. Further researches are underway to investigate, e.g. restoration path optimisation and load restoration by extending the proposed model.

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