Pairwise velocities of dark matter haloes: a test for the $\Lambda$ cold dark matter model using the bullet cluster

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ABSTRACT

The existence of a bullet cluster (such as 1E 0657−56) poses a challenge to the concordance $\Lambda$ cold dark matter ($\Lambda$CDM) model. Here we investigate the velocity distribution of dark matter (DM) halo pairs in large $N$-body simulations with differing box sizes ($250 \, h^{-1} \, \text{Mpc} \sim 2 \, h^{-1} \, \text{Gpc}$) and resolutions. We examine various basic statistics such as the halo masses, pairwise halo velocities $(v_{12})$, collisional angles and pair separation distances. We then compare our results to the initial conditions required to reproduce the observational properties of 1E 0657−56 in non-cosmological hydrodynamical simulations.

We find that the high-velocity tail of the $v_{12}$ distribution extends to greater velocities as we increase the simulation box size. We also find that the number of high $v_{12}$ pairs increases as we increase the particle count and resolution with a fixed box size; however, this increase is mostly due to lower mass haloes which do not match the observed masses of 1E 0657−56. We find that the redshift evolution effect is not very strong for the $v_{12}$ distribution function between $z = 0.0$ and $z \sim 0.5$.

We identify some pairs whose $v_{12}$ resemble the required initial conditions, however, even the best candidates have either wrong halo mass ratios or too large separations. Our simulations suggest that it is very difficult to produce such initial conditions at $z = 0.0, 0.296$ and $0.489$ in comoving volumes as large as $(2 \, h^{-1} \, \text{Gpc})^3$. Based on the extrapolation of our cumulative $v_{12}$ function, we find that one needs a simulation with a comoving box size of $(4.48 \, h^{-1} \, \text{Gpc})^3$ and $2240^3$ DM particles in order to produce at least one pair of haloes that resembles the required $v_{12}$ and observed masses of 1E 0657−56. From our simulated $v_{12}$ probability distribution function, we find that the probability of finding a halo pair with $v_{12} \geq 3000 \, \text{km} \, \text{s}^{-1}$ and masses $\geq 10^{14} \, \text{M}_\odot$ to be $2.76 \times 10^{-8}$ at $z = 0.489$. We conclude that either 1E 0657−56 is incompatible with the concordance $\Lambda$CDM universe or the initial conditions suggested by the non-cosmological simulations must be revised to give a lower value of $v_{12}$.

Key words: galaxies: clusters: general – galaxies: evolution – galaxies: formation – cosmology: theory – dark matter.

1 INTRODUCTION

It is widely believed that the structure formation in our Universe is largely driven by the gravity of dark matter (DM). Therefore, it is worthwhile to probe DM dynamics through measurements of galaxy peculiar velocities and constrain our cosmological model by comparing against numerical simulations. In fact, there has been extensive work along these lines, recovering the local density field from the measured velocity field (Bertschinger & Dekel 1989; Davis, Nusser & Willick 1996; Willick et al. 1996). Unfortunately the observations of peculiar velocity fields contain large uncertainties, and accurate determination of the cosmological mass density parameter $\Omega_m$ turned out to be difficult using this method.

More recently, clusters of galaxies have been used to prove the existence of DM itself, thanks to accurate measurements of projected DM density using weak and strong lensing techniques. Some clusters show signs of a cluster–cluster merger, where the baryonic component and collisionless DM show different spatial distributions, strongly supporting the existence of DM. Furthermore, using the shock features seen in the gas, one can infer the collision velocity of two galaxy clusters (Clowe, Gonzalez & Markevitch 2004; Bradac et al. 2006; Clowe et al. 2006). These new observations have brought renewed interest to DM dynamics and using it to check the standard $\Lambda$ cold dark matter ($\Lambda$CDM) cosmological model

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In particular, the observations of the massive cluster of galaxies 1E0657–56 seem to suggest a much higher relative DM halo velocity than one would expect in the $\Lambda$CDM model. This system includes a massive subcluster (the `bullet') with $M_{\text{bullet}} \approx 1.5 \times 10^{14} \, \text{M}_\odot$ that has fallen through the parent cluster of $M_{\text{parent}} \approx 1.5 \times 10^{15} \, \text{M}_\odot$ roughly 150 million years ago, and is separated by $\approx 0.72$ Mpc on the sky at an observed redshift of $z = 0.296$ (Clowe et al. 2004, 2006; Bradac et al. 2006). The uniqueness of this system comes from the collision trajectory being almost perpendicular to our line of sight. This provides an opportunity to better study the dynamics of large cluster collisions. The Chandra observations revealed that the primary baryonic component had been stripped away in the collision and resided between the two clusters in the form of hot X-ray emitting gas (Markevitch 2006). This provides strong evidence for the existence of DM. As the two clusters passed through each other, the baryonic components interacted and slowed down due to ram pressure, while the DM component was allowed to move ahead of the gas since it only interacts through gravity without dissipation. One can infer the velocity of the bow shock preceding the `bullet' using the shock Mach number and a measurement of the pre-shock temperature. The inferred shock velocity was found to be $v_{\text{shock}} = 4740\pm 710 \, \text{km s}^{-1}$ (Markevitch 2006).

Hayashi & White (2006) examined the Millennium Run (Springel et al. 2005) in search for such a subcluster moving with a velocity relative to its parent cluster of $v_{\text{bullet}} = 4500^{+1100}_{-400} \, \text{km s}^{-1}$ (Markevitch et al. 2004). Due to the limited volume of the simulation ($500 h^{-1}$ Mpc)$^3$, few haloes had masses comparable to 1E0657–56. Still they estimated that about one in 100 have velocities comparable to the bullet cluster, and concluded that the event is not impossible within the current $\Lambda$CDM model.

It is often assumed that the inferred shock velocity is equal to the velocity of the DM `bullet' itself. Several groups have shown, however, that this is not necessarily true through the use of non-cosmological hydrodynamic simulations. Milosavljević et al. (2007) used two-dimensional simulations to find that the subcluster’s velocity differed from the shock velocity by about 16 per cent, bringing the relative velocity of DM haloes down to $\approx 3980 \, \text{km s}^{-1}$. They assumed a zero relative velocity at a separation distance of 4.6 Mpc for their initial conditions. They also emphasized that their conclusion is sensitive to the initial mass and gas density profile of the two clusters. Springel & Farrar (2007) was able to reproduce the inferred shock velocity through the use of an idealized three-dimensional hydrodynamic simulation with initial conditions that assumed a relative velocity of $2057 \, \text{km s}^{-1}$ at a separation distance of $3.37 \, \text{Mpc}$, and found that the subcluster was moving with a relative speed of only $\approx 2600 \, \text{km s}^{-1}$ just after the collision. Mastropietro & Burkert (2008) argued that Springel & Farrar (2007) failed to reproduce the observed displacement of X-ray peaks that represent an important indicator of the nature of the interaction. In their simulations they found that in order to reproduce the observational data of 1E0657–56, a relative halo infall velocity of $\approx 3000 \, \text{km s}^{-1}$ at an initial separation distance of 5 Mpc was required.

Similar to previous work by Hayashi & White (2006), Lee &Komatsu (2010) quantified the likelihood of finding bullet-like systems in the large cosmological N-body simulation MICE (Crocce et al. 2010). They examined DM haloes at $z = 0.0$ and 0.5, searching for a halo pair matching the initial conditions of Mastropietro & Burkert (2008). They concluded that $\Lambda$CDM is excluded by more than 99.91 per cent confidence level at $z = 0$. Their results at $z = 0.5$ are inconclusive due to limited statistics. However, by fitting their pairwise velocity probability distribution function (PDF) to a Gaussian distribution, they were able to estimate the probability of finding a pair with $v_{12} > 3000 \, \text{km s}^{-1}$ to be $3.6 \times 10^{-9}$ and $v_{12} > 2000 \, \text{km s}^{-1}$ to be $2.2 \times 10^{-3}$ at $z = 0.5$. They did warn that one must be careful about this approach since they are probing the tail of the distribution where their fits may not be accurate.

Most recently, Forero-Romero, Gottlöber & Yepes (2010) approached the problem from a different perspective. They studied data from the MareNostrum Universe (Gottlöber & Yepes 2007) which contain baryonic matter in addition to collisionless DM. Instead of examining the pairwise velocities of DM halo pairs, they concerned themselves with the physical separation between the dominant gas clump and its predominant DM structure. They argued that their approach provides a more robust comparison to observation; deriving the relative velocity from the observations includes statistical and systematic uncertainties, whereas the separation uncertainty is dominated by statistical errors in the measuring process. Additionally they point out that current simulations do not include the proper prescriptions for cooling, star formation or feedback, which implies that their predictions of the detailed X-ray properties of hot gas in massive haloes are not robust. Using their method, they found that large displacements between gas and DM are common in $\Lambda$CDM simulations. Therefore, 1E0657–56 should not be considered a challenge.

In this paper, we take a similar approach to that of Lee & Komatsu (2010), and examine large $\Lambda$CDM N-body simulations to see how common these high relative velocities are among massive DM haloes. One of the things that the earlier works have not performed is an examination of resolution and box size effect. Therefore, we first conduct a study to determine the effects of increasing resolution or varying box sizes on the parameters of interest. We then examine our largest simulation in search for a pair matching the initial conditions required by Mastropietro & Burkert (2008) to reproduce the observed properties of 1E0657–56.

The rest of the paper is organized as follows. Section 2 discusses simulation parameters, Section 3 shows the simulation results and examines the distribution of parameters relevant to this study, such as halo masses, pairwise velocity and pair separation distances. Section 4 examines the simulation results at earlier redshifts of $z = 0.296$ and 0.489, and how they relate to the bullet system. Finally, Section 5 contains concluding remarks and discussion of future prospects.
mostly changes the small-scale structures. All simulations contain only collisionless DM particles that interact solely through gravity.

Several simulations with varying particle counts and box sizes were run from $z = 100$ to $0$. The list of simulations along with other parameters can be found in Table 1. Starting with the L250N125 run, the box size and particle count were simultaneously increased (from $L_{\text{box}} = 250$ to $2016\ h^{-1}\ \text{Mpc}$, and from $N = 125^3$ to $1008^3$ particles) in order to maintain the same mass resolution and gravitational softening length up until the L2016N1008 run. The second set of simulations were ran to examine the resolution effect. We started with the original L250N125 simulation and increased the particle count and decreased the gravitational softening length while keeping the box size the same, up to the L250N500 run.

### Table 1. Summary of simulations.

| Run name   | Box size ($h^{-1}\ \text{Mpc}$) | Particle count | $M_{\text{dm}}$ ($h^{-1}\ M_\odot$) | $\epsilon$  | FOF LL ($h^{-1}\ \text{kpc}$) |
|------------|---------------------------------|----------------|-------------------------------------|-------------|--------------------------------|
| L250 N125  | 250                             | $125^3$        | $5.74 \times 10^{11}$              | 80          | 400                            |
| L500 N250  | 500                             | $250^3$        | $5.74 \times 10^{11}$              | 80          | 400                            |
| L1000 N500 | 1000                            | $500^3$        | $5.74 \times 10^{11}$              | 80          | 400                            |
| L2016 N1008| 2016                            | $1008^3$       | $5.74 \times 10^{11}$              | 80          | 400                            |

| Run name   | Box size ($h^{-1}\ \text{Mpc}$) | Particle count | $M_{\text{dm}}$ ($h^{-1}\ M_\odot$) | $\epsilon$  | FOF LL ($h^{-1}\ \text{kpc}$) |
|------------|---------------------------------|----------------|-------------------------------------|-------------|--------------------------------|
| Resolution effects |                                  |                |                                     |             |                                |
| L250 N125  | 250                             | $125^3$        | $5.74 \times 10^{11}$              | 80          | 400                            |
| L250 N165  | 250                             | $165^3$        | $2.50 \times 10^{11}$              | 60.6        | 303                            |
| L250 N250  | 250                             | $250^3$        | $7.18 \times 10^{10}$              | 40          | 200                            |
| L250 N500  | 250                             | $500^3$        | $8.97 \times 10^9$                 | 20          | 100                            |

*Note. Summary of simulations employed in this paper. $M_{\text{dm}}$ is the mass of each DM particle. $\epsilon$ is the comoving gravitational softening length, and FOF LL is the friends-of-friends linking length. The top four simulations explore the effects of increasing box size with fixed resolution, while the bottom four explore the effects of increasing resolution with a fixed box size.

#### 3 DATA ANALYSIS AND RESULTS

##### 3.1 Halo mass function

DM particles were grouped using a simplified version of the parallel friends-of-friends (FOF) group finder SUBFIND (Springel et al. 2001). The code groups the particles into DM haloes if they lie within a specified linking length (FOF LL). This linking length is a fraction of the initial mean interparticle separation, for which we adopt a standard value of $b = 0.2$. In order to be considered a halo it must also contain at least 32 particles.

Figs 1 and 2 show DM halo mass functions in our simulations. Both figures include the Sheth & Tormen (ST; 1999) mass function plotted as a black dotted line. Recent work by More et al. (2011) found that the commonly used value of $b = 0.2$ selects a significantly larger local overdensity ($\delta_{\text{FOF}}$) than previously thought. Normally it is assumed that $b = 0.2$ results in $\delta_{\text{FOF}} \approx 60$ (corresponding to the enclosed overdensity of $\delta \sim 180$), but their study finds that it results in $\delta_{\text{FOF}} \approx 80.61$, which is a $\sim 35$ per cent increase. We find that our mass function is slightly higher than the ST mass function on all mass scales. By regrouping the L1000N500 sim using $b = 0.1$ we underpredict the number density on all mass scales, as shown by the solid magenta line in Figs 1 and 2. Changes in $b$ certainly have a significant impact on the halo mass function.

Fig. 1 shows that the number of high-mass haloes increases by increasing the box size from 250 to $2016\ h^{-1}\ \text{Mpc}$ while maintaining the same resolution. The lowest mass halo in all simulations shown in Fig. 1 is $M_{\text{halo,min}} = 1.84 \times 10^{13}\ h^{-1}\ M_\odot$. The run with the largest box size (L2016) shows a slight shortage in the number of low-mass haloes around $M_{\text{halo}} \approx 10^{13.24 - 14.20}\ h^{-1}\ M_\odot$ when compared to the other three runs with smaller box sizes. The most likely explanation for this shortage is that the lower mass haloes were absorbed into higher mass haloes.

Higher resolution runs can resolve larger number of low-mass haloes as seen in Fig. 2. The least massive halo for the highest resolution simulation (L250N500) has $M_{\text{halo}} = 2.87 \times 10^{11}\ h^{-1}\ M_\odot$, which is roughly 2 orders of magnitude lower than the lowest mass haloes found in Fig. 1.

While searching for a bullet-like pair of haloes with masses on the order of $M_{\text{bullet}}$ and $M_{\text{parent}}$, Figs 1 and 2 indicate that it is possible to form such massive haloes in box sizes as small as $250\ h^{-1}\ \text{Mpc}$ at $z = 0$ but there will be a low number of them.

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3.2 Pairwise velocity function

In this section, we present the results on the pairwise velocity ($v_{12} = |v_1 - v_2|$) function, i.e. the number of halo pairs within a velocity bin per unit volume ($dn/dv_{12}$). Figs 3 and 4 show $dn/dv_{12}$ with four panels for different simulation runs, each panel containing three lines for halo pairs with a separation distance of less than $d_{12} = 2, 5$ and $10$ Mpc.

Fig. 3 shows that increasing the box size with a fixed resolution allows for a greater number of high $v_{12}$ pairs, but with greater separation distances. Doubling the box size from L250 to L500 yields only a small increase in high $v_{12}$ pairs. Doubling it again to L1000 gives us a considerable jump in high $v_{12}$ pairs with separation distances of $5 < d_{12} < 10$ Mpc, while the $2 < d_{12} < 5$ Mpc range only sees a moderate increase. Doubling the box size one final time to L2016, we again only see a moderate increase in $v_{12}$ similar to going from the L250 to L500 sim. The number of close halo pairs with $d_{12} < 2$ Mpc seems to remain fairly constant with relatively low $v_{12}$ throughout changes in the box size. This implies that increasing the box size does not increase $v_{12}$ for pairs within 2 Mpc of one another.

By increasing the resolution, the number of halo pairs with high $v_{12}$ increases (Fig. 4), but unlike the case of enlarging the box, this does not necessarily come at the cost of increased separation distances. Each increase in resolution gives us a larger number of low and high $v_{12}$ pairs on all distance scales. When compared to Fig. 3, the simulations shown in Fig. 4 are better at resolving smaller structures and length-scales, leading to larger values of $v_{12}$. Unfortunately, these data do not give us any information on the mass of the halo pairs, so increasing the resolution in order to increase the number of close high $v_{12}$ pairs may not be beneficial when searching for high-mass pairs such as 1E0657−56.

3.3 Relative halo velocity and halo mass

It is useful to study the effects of different box sizes and resolutions on the average mass of a halo pair versus $v_{12}$. Fig. 5 shows how increasing the box size with a constant resolution increases the
in the number of high-mass, high \( v_{12} \) pairs. Increasing the box size yields high \( v_{12} \) pairs with increasing mass, while increasing the resolution yields a larger number of high \( v_{12} \) pairs at the maximum halo mass allowed by the box.

### 3.4 Cumulative \( v_{12} \) function

To examine how the box size and resolution affect the actual number of high \( v_{12} \) halo pairs, we plot the cumulative \( v_{12} \) distribution function as shown in Figs 7 and 8. Changing the box size (Fig. 7) extends the curve to higher \( v_{12} \). The larger box and particle count

![Figure 7. Cumulative \( v_{12} \) function of DM haloes at \( z = 0 \). This figure shows how increasing the box size increases the number of high \( v_{12} \) pairs, extending the tail of the distribution.](https://academic.oup.com/mnras/article-abstract/419/4/3560/2908088)

The increase in the number of low-mass, high \( v_{12} \) halo pairs, along with increasing the number of high-mass, high \( v_{12} \) pairs to a lesser degree. As the box size increases, we are allowing for a greater number of rare high \( v_{12} \) halo pairs which probe the tail of the distribution.

Fig. 6 shows that an increase in the resolution results in a larger number of low-mass, high \( v_{12} \) pairs, and a less substantial increase in the number of high-mass, high \( v_{12} \) pairs. Increasing the box size yields high \( v_{12} \) pairs with increasing mass, while increasing the resolution yields a larger number of high \( v_{12} \) pairs at the maximum halo mass allowed by the box.

![Figure 8. Cumulative \( v_{12} \) function of DM haloes at \( z = 0 \). This figure shows the resolution effect. As the resolution increases, the normalization of the distribution increases due to a larger number of lower mass haloes with higher velocities.](https://academic.oup.com/mnras/article-abstract/419/4/3560/2908088)
result in better statistics, which allows us to probe the high-velocity tail of the $v_{12}$ distribution as mentioned in the previous section.

By increasing the resolution alone (Fig. 8), we see that the normalization of the cumulative $v_{12}$ distribution function becomes higher due to larger number of lower mass haloes. These figures suggest that by increasing the box size and/or resolution one would be able to produce a halo pair with a greater $v_{12}$; however, as we saw earlier in Figs 5 and 6, the majority of high $v_{12}$ pairs have lower average masses than 1E0657–56.

4 RESULTS AT EARLIER REDSHIFTS

To be fully consistent with the observations of 1E0657–56, comparing our simulations at the same redshift as 1E0657–56 would be ideal. Up until this point, we have examined only simulation data at $z = 0$, yet 1E0657–56 is observed at $z = 0.296$. This difference in time of ~3.31 billion years can have a considerable impact on the velocities, sizes and separation distances of the DM haloes contained in the simulation. Another problem arises when we consider how we group the DM particles. At $z = 0.296$ the separation between the two haloes of 1E0657–56 is $d_{12} \approx 0.72$ Mpc, which is larger than the linking lengths listed in Table 1 (0.1–0.4 Mpc) for each of our simulations. At first glance, it may appear that we could identify each halo independently within our sims, but when one considers their large masses, we find that this is not the case. The virial radius of each halo is found to be 1.42 and 3.06 Mpc for the ‘bullet’ ($M_{\text{bullet}} \approx 1.5 \times 10^{14} M_\odot$) and its ‘parent’ ($M_{\text{parent}} \approx 1.5 \times 10^{15} M_\odot$), respectively. When two haloes of this size are separated by $\sim 0.72$ Mpc, they will easily overlap, resulting in the FOF group finder identifying them as a single halo at the observed redshift of $z = 0.296$. If we assume the separation distance of 5 Mpc and infall velocity of 3000 km s$^{-1}$ as required by Mastropietro & Burkert (2008) to reproduce the observed quantities of 1E0657–56, then a halo pair in this initial configuration should be found at $z = 0.489$.

4.1 Peculiar velocities

Before we examine the simulation at $z = 0.489$, we first compare our simulations to the prediction of linear theory for further validation. Linear theory predicts that for an Einstein–de Sitter (EdS) universe the growing mode of the peculiar velocity field grows as $t^{1/3}$. The peculiar velocity of each mode in a non-EdS universe is given by (Peebles 1980)

$$v_{\text{pec}} = \frac{H(z)a^2}{4\pi} \frac{dD}{da},$$

where $H(z) = H_0E(z)$ is the Hubble parameter, $a$ is the scalefactor, $D$ is the growth factor for linear perturbations, and $E(z) = (\Omega_m(1 + z)^3 + (1 - \Omega_m - \Omega_k)(1 + z)^2 + \Omega_k)^{1/2}$.

The peculiar velocity of each halo in five of our runs was calculated and averaged up to $z = 10$, then compared against the normalized theory curve in Fig. 9. Our simulations agree well between $z = 6$ and 1.0, but start to deviate from the linear theory curve at $z < 1.0$, which is likely due to their virialization.

4.2 Pairwise velocity: linear theory

Juszkiewicz, Springel & Durrer (1999) proposed a simple closed-form expression relating the mean relative velocity of pairs of galaxies at a fixed separation to the two-point correlation function of mass density fluctuations:

$$-\frac{v_{12}}{Hr} \approx \frac{2}{3} f \xi \left[1 + a\xi \right],$$

where $H$ is the Hubble parameter, $r = ax$ is the proper separation, $f = \ln D/\ln a$, $a \approx 1.2 - 0.65 \gamma$, $\gamma$ is the logarithmic slope of the correlation function, $\xi = \xi / [1 + \xi]$, $\xi = 3x^{-3} \int_0^x \xi y^2 dy$, and $\xi$ is the two-point correlation function. At $z = 0$, the value of $f$ is $\approx 0.5$, and then it asymptotes to unity at $z \gg 8$.

To obtain theoretical results based on equation (2) that can be compared with our simulations, we calculate $\xi$ by correlating the centre of mass positions of haloes with a random data set and use the Landy & Szalay (1993) estimator:

$$\xi(r)_{\text{halo}} = \frac{DD - 2DR + RR}{RR},$$

where DD, DR and RR represent halo pair counts for data–data, data–random and random–random data sets at a given value of $r$. The result of $\xi(r)_{\text{halo}}$ for the L250N500 sim is plotted in Fig. 10. Higher values of $\xi_{\text{halo}}$ correspond to a larger probability that another halo lies at a separation of $r$. The value of $\xi_{\text{halo}}$ decreases with increasing $r$, implying that haloes tend to cluster more on smaller scales. The value of $\xi_{\text{halo}}$ also decreases with increasing redshift, meaning haloes are less clustered in the earlier universe.

To compare our simulation with equation (2), we calculated the average pairwise halo velocities ($v_{12}$) for pairs residing within physical shells of 1 Mpc thickness (±0.5 Mpc) around $r = 1$, 3, 5 and 10 Mpc for the L250N500 run. The results are shown in Fig. 11, where the solid curves represent simulation data, the dashed curves correspond to the theoretical predictions of equation (2) using $\xi$-values from Fig. 10, and the different colours distinguish between different values of $r$. Juszkiewicz et al. (1999) did not consider the effect of galaxy bias relative to DM, and without any correction, we find that ($v_{12}$) of haloes in our simulation are somewhat higher than those predicted by equation (2). Therefore, we invoke an ad hoc correction factor of ×1.5 to account for this effect, and the dashed lines in Fig. 11 include this multiplicative factor in the right-hand side of...
Figure 10. Autocorrelation function of DM haloes in the L250N500 run at \( z = 0-6 \). The vertical cyan dashed lines indicate \( r = 1, 3, 5 \), and 10 Mpc, where we measure the evolution of \( \xi \) as a function of redshift. Symbols lying along these dashed lines represent the \( \xi \)-values used in equation (2) for producing the dashed lines in Fig. 11. For comparison, we also show the dashed black line with a slope of \( \xi \propto r^{-1.8} \) — the result consistent with the \( z = 0 \) Sloan Digital Sky Survey (SDSS) galaxies (Zehavi et al. 2010).

Figure 11. Solid lines: average pairwise halo velocities \( \langle v_{12} \rangle \) from the 250MpcN500 run residing in physical shells of 1 Mpc thickness with the indicated radii. Dashed lines: theoretical \( \langle v_{12} \rangle \) curves given by equation (2) using the \( \xi \)-values from Fig. 10 at each corresponding radius. The dashed cyan line represents data from Fukushige & Suto (2001) at a separation distance of \( r = 1.52 \) Mpc. When these curves reside below unity the Hubble flow is greater than their pairwise velocities, at unity their physical separations remain constant, and above unity their pairwise velocities are greater than the Hubble flow.

4.3 In search of the ‘bullet’

Hereafter we will only be examining our largest simulation (L2016N1008) at redshifts of \( z = 0.0, 0.296 \), and 0.489. In Fig. 12, we show the redshift evolution of the pairwise velocity function \( (d_n/dv_{12}) \) from \( z = 0 \) to 0.489. Qualitatively this plot changes very little with redshift, except that there is a slight increase in the number of pairs at the highest end of the \( v_{12} \) distribution. Pairs within separation distances of \( d_{12} < 2 \) Mpc have maximum \( v_{12} \) on the order of \( \approx 1800 \) km s\(^{-1}\) at \( z = 0.296 \) and 0.489. For pairs with greater \( d_{12} \), the maximum \( v_{12} \) reaches as high as \( \approx 3300 \) km s\(^{-1}\).

In Fig. 13, we show the redshift evolution of the average halo mass versus their pairwise velocity. One can see the effect of halo mergers and the number of high-mass halo pairs with \( \langle M_{\text{halo}} \rangle > 10^{15} \) M\(_{\odot}\) are increasing from \( z = 0.489 \) to 0. The cyan dashed lines in the equation (2). After this correction, our simulation agrees with equation (2) very well for \( r = 3 \) and 5 Mpc, but there is some deviation from theory for the \( r = 1 \) and 10 Mpc results. The shape of the theory curve is determined by the competition between increasing \( H(z) \), decreasing \( \xi \) and increasing \( f \) with increasing redshift.

Fukushige & Suto (2001) examined the validity and limitations of the stable condition \( (-v_{12}/Hr = 1) \), which states that the mean physical separation \( r \) of galaxy pairs is constant on small scales. They found a significant time variation in the mean pairwise peculiar velocities and argued that this behaviour was not due to a numerical artefact, but a natural consequence of the continuous merging process. This irregular oscillatory behaviour could be reduced by averaging over cosmological volumes larger than 200 Mpc\(^3\), resulting in a more accurate estimate of the mean pairwise velocity. Our data are also consistent with Fukushige & Suto (2001) (dashed cyan line in Fig. 11), in that the oscillatory behaviour is suppressed due to our cosmological volume being greater than 200 Mpc\(^3\), and their result for \( r = 1.52 \) Mpc lies between our \( r = 1 \) and 3 Mpc curves.
$z = 0.489$ panel illustrate the average pair mass of $1E\,0657 + Gpc$ produces $0.0, 0.296$ and $0.489$ suggested by Mastropietro & Burkert (2008). Two pairs are found in our simulation near the region of interest, but their masses and velocities are still too low.

4.3.1 Candidate halo pairs

Table 2 lists the five halo pairs with highest $\langle M_{\text{halo}} \rangle$ for $z = 0, 0.296$ and 0.489. A simulation of this size (comoving $2\,h^{-1}\,Gpc$) produces many halo pairs massive enough to match that of $1E\,0657 - 56$ at the examined redshifts. While the separation distances of these pairs may be in the range we are interested in, the pairwise velocities are too low to match the required $v_{12} = 3000\,\text{km}\,\text{s}^{-1}$ by Mastropietro & Burkert (2008).

Table 3 lists the five halo pairs with the highest $v_{12}$ at the three examined redshifts. All halo pairs in this table match or exceed the required $v_{12}$ of $3000\,\text{km}\,\text{s}^{-1}$, but they miss the mark when it comes to the other observables of $1E\,0657 - 56$. All of the haloes in this table have masses 1 or 2 orders of magnitude lower than $M_{\text{bullet}}$ and $M_{\text{parent}}$. The mass ratios are also a bit high; the lowest being $\sim 0.3$ at $z = 0.489$ compared to 0.1 for $1E\,0657 - 56$ at $z = 0.296$. None of the collision angles is head-on, yet most are highly inclined. Lastly, the separation distance of each pair at $z = 0.489$ is somewhat large; Mastropietro & Burkert (2008) set their initial separation at proper $5\,\text{Mpc}$ while each pair in this table is separated by $>7.5\,\text{Mpc}$.

### Table 2. Highest mass pairs.

| Pair | $v_{12}$ | $\theta$ | $M_1$ | $M_2$ | Mass ratio | $d$ | $r_1$ virial | $r_2$ virial |
|------|--------|--------|------|------|-----------|----|-------------|-------------|
| $z = 0$ |        |        |      |      |           |    |             |             |
| 1    | 1670   | 165    | $5.71E+15$ | $5.02E+14$ | 0.088     | 8.70 | 5.67        | 5.67        |
| 2    | 1792   | 46     | $5.71E+15$ | $1.99E+14$ | 0.035     | 7.84 | 5.67        | 1.85        |
| 3    | 1767   | 75     | $5.71E+15$ | $1.01E+14$ | 0.018     | 7.63 | 5.67        | 1.48        |
| 4    | 1624   | 80     | $5.71E+15$ | $7.33E+13$ | 0.013     | 7.13 | 5.67        | 1.33        |
| 5    | 2316   | 72     | $5.71E+15$ | $7.04E+13$ | 0.012     | 6.20 | 5.67        | 1.31        |
| $z = 0.296$ |        |        |      |      |           |    |             |             |
| 6    | 1360   | 141    | $3.80E+15$ | $3.50E+14$ | 0.092     | 9.55 | 4.18        | 1.89        |
| 7    | 1533   | 44     | $3.80E+15$ | $2.61E+14$ | 0.069     | 6.23 | 4.18        | 1.71        |
| 8    | 1486   | 56     | $3.80E+15$ | $2.51E+14$ | 0.066     | 10.00 | 4.18 | 1.69       |
| 9    | 1425   | 129    | $3.80E+15$ | $2.13E+14$ | 0.056     | 6.20 | 4.18        | 1.60        |
| 10   | 2007   | 112    | $3.80E+15$ | $1.78E+14$ | 0.047     | 5.65 | 4.18        | 1.51        |
| $z = 0.489$ |        |        |      |      |           |    |             |             |
| 11   | 869    | 91     | $3.28E+15$ | $5.59E+14$ | 0.170     | 8.78 | 3.70        | 2.05        |
| 12   | 1277   | 111    | $2.64E+15$ | $1.07E+15$ | 0.405     | 8.11 | 3.44        | 2.55        |
| 13   | 1875   | 132    | $2.45E+15$ | $1.19E+15$ | 0.485     | 3.86 | 3.36        | 2.64        |
| 14   | 1257   | 108    | $2.45E+15$ | $1.08E+15$ | 0.440     | 4.83 | 3.36        | 2.55        |
| 15   | 1256   | 54     | $3.28E+15$ | $1.73E+14$ | 0.053     | 6.01 | 3.70        | 1.39        |

**Note.** Five halo pairs with the highest average halo mass from the L2016N1008 simulation at $z = 0, 0.296$ and 0.489. The values of $v_{12}$ are given in $\text{km}\,\text{s}^{-1}$, collision angles $\theta$ in degrees, masses $(M_1, M_2)$ in $\text{M}_\odot$, pair separation distances $(d_{12})$ and virial radius of each halo in physical Mpc. Although this simulation can produce massive pairs matching the observed mass of $1E\,0657 - 56$, these pairs have too low relative velocities, and too large separation distances.
Table 3. Highest velocity pairs.

| Pair | \(v_{12}\) | \(\theta\) | \(M_1\) | \(M_2\) | Mass ratio | \(d\) | \(r_1\) virial | \(r_2\) virial |
|------|-------------|----------|--------|--------|------------|------|-------------|-------------|
| \(z = 0\) |
| 31   | 3674        | 103      | 3.64E+13 | 2.71E+13 | 0.746      | 8.83 | 1.05        | 0.95        |
| 32   | 3199        | 151      | 2.14E+13 | 2.02E+13 | 0.946      | 8.20 | 0.88        | 0.86        |
| 33   | 3133        | 134      | 5.83E+13 | 2.60E+13 | 0.446      | 9.09 | 1.23        | 0.94        |
| 34   | 3095        | 113      | 8.20E+13 | 4.56E+13 | 0.556      | 9.21 | 1.38        | 1.13        |
| 35   | 3053        | 108      | 8.20E+13 | 2.14E+13 | 0.261      | 9.11 | 1.38        | 0.88        |
| \(z = 0.296\) |
| 36   | 3538        | 143      | 3.35E+13 | 1.96E+13 | 0.586      | 9.94 | 0.86        | 0.72        |
| 37   | 3282        | 125      | 4.96E+13 | 2.37E+13 | 0.477      | 9.39 | 0.98        | 0.77        |
| 38   | 3141        | 155      | 8.60E+13 | 3.41E+13 | 0.396      | 8.80 | 1.18        | 0.87        |
| 39   | 3089        | 170      | 6.93E+13 | 2.77E+13 | 0.400      | 5.27 | 1.10        | 0.81        |
| 40   | 3053        | 153      | 4.16E+13 | 2.48E+13 | 0.597      | 8.60 | 0.93        | 0.78        |
| \(z = 0.489\) |
| 41   | 3361        | 128      | 6.75E+13 | 2.60E+13 | 0.385      | 8.81 | 1.01        | 0.74        |
| 42   | 3312        | 148      | 5.66E+13 | 3.18E+13 | 0.561      | 8.03 | 0.96        | 0.79        |
| 43   | 3239        | 102      | 6.75E+13 | 2.37E+13 | 0.350      | 7.57 | 1.01        | 0.72        |
| 44   | 3109        | 146      | 2.94E+13 | 2.37E+13 | 0.804      | 9.57 | 0.77        | 0.72        |
| 45   | 3083        | 103      | 7.56E+13 | 2.37E+13 | 0.313      | 9.25 | 1.05        | 0.72        |

Note. Five halo pairs with highest \(v_{12}\) found in the L2016N1008 simulation at \(z = 0.0\), 0.296 and 0.489. The values of \(v_{12}\) are given in km s\(^{-1}\), collision angles \(\theta\) in degrees, masses \((M_1, M_2)\) in M\(_\odot\), pair separation distances \((d_{12})\) and virial radius of each halo in physical Mpc. None of these high velocity halo pairs is massive enough to match the observations of 1E 0657−56.

Figure 14. Comoving number density of halo pairs in the N2016N1008 run with masses above \(10^{14}\) M\(_\odot\) at \(z = 0.0\), 0.296 and 0.489. We also overplot a quadratic fit described in the text for \(z = 0.489\). The horizontal dashed line illustrates the number density of haloes with \(v_{12} = 3000\) km s\(^{-1}\) corresponding to a box size of \((4.48 h^{-1}\) Gpc\(^3\)) and 2240\(^3\) DM particles. The black filled circles represent the \(v_{12}\) values listed in Table 4.

The form \(y = y_0 + ax + bx^2\) was fitted to the \(z = 0.489\) curve between the values of \(v_{12} = 800\)–1500 km s\(^{-1}\), and we obtain the best-fitting values of \(y_0 = -3.97\), \(a = -3.31 \times 10^{-3}\) and \(b = 5.95 \times 10^{-7}\). Based on this fit, we estimate the minimum box sizes and particle counts (for the same resolution as the L2016N1008 run) required to produce at least one halo with the initial velocities given by Mastropietro & Burkert (2008) and Springel & Farrar (2007). The result is listed in Table 4.
The exact values of the required \( v_{12} \) and \( v_\text{binned} \), and the required initial \( v_{12} \) at \( z = 0.489 \) suggested by each of the authors. See text in Section 4.3.2 for more details.

Our result suggests that we would need a simulation box size of \((4.48 \pm 1) \text{ Gpc}^3\) and \([2240 \pm 3] \text{ DM particles}\) in order to produce at least one halo pair with an average mass greater than \(10^{14} \text{ M}_\odot\) and \( v_{12} > 3000 \text{ km s}^{-1} \) at \( z = 0.489 \). The exact values of the required box size and particle count is somewhat sensitive to the range of \( v_{12} \) used for the fit, therefore the values listed in Table 4 should be taken as a rough estimate. The required simulations are so large and they would take significant computational resources which is currently not feasible for us.

### 4.3.3 Probability of finding the ‘bullet’

We also examine the PDF of \( v_{12} \) for haloes with \( (M_{\text{halo}}) > 10^{14} \text{ M}_\odot\). We perform a least-square fit to the data using a skewed normal distribution (Azzalini & Capitanio 2010), and calculate the probability of finding a halo pair with \( v_{12} > 3000 \text{ km s}^{-1} \) at \( z = 0.489 \).

In Fig. 15, we show the binned PDF data with blue circles and the best-fitting skew normal distribution as the red curve. By integrating the PDF from \( v_{12} = 3000 \text{ km s}^{-1} \) to infinity, we calculate the probability of finding a halo pair with masses greater than \(10^{14} \text{ M}_\odot\) and \( v_{12} > 3000 \text{ km s}^{-1} \) to be \( P(>3000 \text{ km s}^{-1}) = 2.8 \times 10^{-8} \), which is roughly \(1\) order of magnitude higher than calculations done by Lee & Komatsu (2010) \( (P = 3.6 \times 10^{-9}) \). This very low probability corroborates our earlier finding that it is very difficult to produce a massive halo pair with a high \( v_{12} \) matching the required initial configuration suggested by Mastropietro & Burkert (2008).

### 5 Conclusions

We performed many \(N\)-body cosmological simulations with varying box sizes and resolutions in order to examine how changing these parameters affect the search for high \( v_{12} \) halo pairs comparable to the initial conditions required to reproduce the observed properties of the 1E0657-56 system in non-cosmological simulations. Using our largest L2016N1008 run, we examined the pairwise velocities, halo masses and halo separation distances at \( z = 0.0, 0.296 \) and 0.489.

We find that the high \( v_{12} \) tail of the distribution extends to a greater velocities as we increase the simulation box size. We also find that the number of high \( v_{12} \) pairs increased as we increase the particle count and resolution with a fixed box size; however, this increase is mostly due to lower mass haloes which do not correspond to the characteristics of 1E0657-56. We find that the redshift evolution effect is not very strong for the \( v_{12} \) distribution function.

As we show in Table 3, some of the halo pairs have a high relative velocity similar to the initial conditions required to reproduce the observational quantities of 1E0657-56 in non-cosmological simulations, but they are galaxy group-scale haloes \((10^{10} - 10^{11} \text{ M}_\odot)\) and much less massive than the observed estimates for 1E0657-56.

We find that, in \(N\)-body simulations with comoving volumes of less than \((2 \text{ h}^{-1} \text{ Gpc})^3\), it is very difficult to reproduce a system that resembles the initial conditions required to reproduce the observational properties of 1E0657-56. Based on the extrapolation of our cumulative \(v_{12}\) function, we find that one needs a simulation with a comoving box size of \((4.48 \pm 1) \text{ Gpc}^3\) and \([2240 \pm 3] \text{ DM particles}\) in order to produce at least one pair of haloes that resembles the initial conditions suggested by Mastropietro & Burkert (2008). In the future it would be useful to run larger simulations (e.g. with \(\sim 5 \text{ Gpc box and } \sim 2500 \text{ particles}\)) to improve the statistics of massive haloes.

From the simulated \( v_{12} \) PDF of haloes, we calculated the probability of finding a halo pair with \( v_{12} \geq 3000 \text{ km s}^{-1} \) and masses \( \geq 10^{14} \text{ M}_\odot \) to be \(2.76 \times 10^{-8}\), which is somewhat larger than previous work by Lee & Komatsu (2010). However, both probabilities are quite small and the difference is negligible. These results suggest that a system like 1E0657-56 is currently incompatible with the concordance \( \Lambda\text{CDM} \) universe, if its initial condition really requires an initial pairwise velocity of \( v_{12} \geq 3000 \text{ km s}^{-1} \). As Lee & Komatsu (2010) discussed in detail, there seems to be more systems like 1E0657-56 being observed already, which exacerbates the incompatibility in terms of probability. One other possibility is that there is something wrong with the referred non-cosmological simulations, and the suggested initial \( v_{12} \) must be revised to a lower value.

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**Table 4. Simulation requirements to produce a bullet.**

| Reference                  | \( v_{12} \) (km s\(^{-1}\)) | Box size \((h^{-1} \text{ Mpc})\) | Particle count |
|----------------------------|-------------------------------|-----------------------------------|----------------|
| Mastropietro & Burkert (2008) | 3000                          | 4480                              | 2240\(^3\)     |
| Springel & Farrar (2007)    | 2057                          | 2224                              | 1112\(^3\)     |

**Note:** Required box size and particle number needed to produce at least one halo pair with an average mass greater than \(10^{14} \text{ M}_\odot\) and a certain value of \( v_{12} \) at \( z = 0.489 \) suggested by each of the authors. See text in Section 4.3.2 for more details.

**Figure 15.** Pairwise velocity PDF for halo pairs with masses above \(10^{14} \text{ M}_\odot\) in our L2016N1008 run. The blue circles represent \( v_{12} \) binned PDF data, the blue curve is the linearly interpolated values, and the red curve is the best-fitting skew normal distribution (Azzalini & Capitanio 2010). Integrating the fit from \( v_{12} = 3000 \text{ km s}^{-1} \) to infinity gives \( P(>3000 \text{ km s}^{-1}) = 2.8 \times 10^{-8} \). This very low probability suggests that it is very difficult to produce a halo pair with high mass and high \( v_{12} \) as the observed 1E0657-56.
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