Collective Properties of X-ray Binary Populations of Galaxies III. 
The Low-mass X-ray Binary Luminosity Function

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ABSTRACT

Continuing our exploration of the collective properties of low-mass X-ray binaries (LMXBs) in the stellar fields (i.e., outside globular clusters) of normal galaxies, we compute in this paper (Paper III in the series) the expected X-ray luminosity function (XLF) of LMXBs, starting from the results of the previous paper in the series (Paper II). For this, we follow the evolution of the pre-LMXB population (described in Paper II) through Roche-lobe contact and the consequent LMXB turn-on, LMXB evolution through the accretion phase, and the eventual conclusion of accretion and LMXB turn-off. We treat separately two classes of LMXB evolution, the first (a) being close systems whose initial orbital periods are below the bifurcation period, wherein the companion is on the main sequence when Roche-lobe contact occurs, the subsequent evolution is driven by angular-momentum loss from the system, and the second (b) being wider systems whose initial orbital periods are above the bifurcation period, wherein the companion is on the giant branch when Roche-lobe contact occurs, and the evolution of these systems is driven by the nuclear evolution of the companion. We obtain model luminosity profiles $L(t)$ for individual LMXBs of both classes, showing that they are in general agreement with those in previous works in the subject. We point out that the basic features of the luminosity profile in the angular-momentum-loss driven case can be well-understood from scaling laws first pointed out by Patterson (1984) in a related context, which we call Patterson scaling. We then compute the LMXB XLF by “folding in” the inputs for the pre-LMXB collective properties and formation rates from Paper II with the above luminosity profiles. Because of the long timescale ($\sim 10^9$ yr) on which LMXBs evolve, this computation becomes more involved, as one needs to keep track of the evolution of the star-formation rate (SFR) on the same timescale,
and we use star-formation histories (SFH) given by canonical models. We compare the observed LMXB XLF with our computed one, keeping in mind that we have included only neutron-star systems in this work, so that there would be an unaccounted-for population of black-hole binaries at the high-luminosity end. We show that a qualitative similarity already exists between the two, and we discuss the role of the giant fraction, i.e., that fraction of all LMXBs which harbors a low-mass giant companion, on the shape of the XLF at the high-luminosity end.

*Subject headings:* binaries: close – stars: evolution – stars: neutron – stars: low-mass – supernovae: general – X-rays: binaries – X-rays: galaxies
1. Introduction

This is Paper III in our series of papers exploring the basic physics underlying the collective properties of accretion powered X-ray binaries. In Paper II, we described our study of the constraints on the formation of the so-called pre-low-mass X-ray binaries (pre-LMXBs), which is a relatively rare process, and our calculation of the collective properties (i.e., distributions of their essential parameters) and the rates of formation of these pre-LMXBs. In this paper, we use the results of Paper II to compute the LMXB X-ray luminosity function (XLF).

Our procedure here is to first follow the evolution of the pre-LMXB population through Roche-lobe contact and consequent LMXB turn-on, LMXB evolution through the accretion phase, and eventual conclusion of accretion and LMXB turn-off. To this end, we calculate the X-ray luminosity evolution of individual model systems in our computational scheme, and so construct luminosity profiles, \(L(t)\), for these systems. We treat separately the two well-known categories of LMXB evolution. The first is that for close systems whose initial orbital periods are below the so-called bifurcation period, wherein the companion is on the main sequence when Roche-lobe contact occurs and mass transfer starts. Evolution of such systems is driven by angular-momentum loss from the systems through gravitational radiation and magnetic braking, and their orbits shrink as mass transfer proceeds. The second is that for wider systems whose initial orbital periods are above the bifurcation period, wherein the companion has finished its main-sequence evolution and is on the giant branch when Roche-lobe contact occurs and mass transfer starts. Evolution of these systems is driven by nuclear evolution and expansion of the companion, and their orbits expand as mass transfer proceeds. We obtain model luminosity profiles for both categories, and show that they are in general agreement with those in previous works in the subject. We point out that the basic features of the luminosity profile in the angular-momentum-loss
driven category can be accounted for by scaling laws first pointed out by \textit{Patterson} (1984) in a related context, which we call \textit{Patterson scaling}.

We obtain the LMXB XLF by combining the above inputs. Because of the long timescale ($\sim 10^9$ yr) on which LMXBs evolve, the computation here becomes more involved, as one needs to keep track of the evolution of the star-formation rate (SFR) on the same timescale, since the rate of formation of primordial binaries, which evolve into LMXBs, is determined by the SFR. Accordingly, we construct a scheme for “folding in” the inputs from the pre-LMXB collective properties and formation rates, together with the star-formation history (SFH), \textit{i.e.}, the time-variation profile of the SFR, as given by canonical models in wide use. We thus obtain the theoretical LMXB XLF and study its properties and their variations with some parameters characterising the primordial binary distribution the supernova process.

We compare the observed LMXB XLF with our computed one, keeping in mind that we have included only neutron-star systems in this work, so that there would be an unaccounted-for population of black-hole binaries at the high-luminosity end. We show that a qualitative similarity already exists between the two, and we discuss the role of the \textit{giant fraction}, \textit{i.e.}, the fraction of all LMXB systems harboring a low-mass giant companion, on the shape of the XLF at the high-luminosity end.

The rest of the paper is organized as follows. In Sec.2 we describe the essential aspects of LMXB evolution that we need for our purposes here. We give the prescriptions for angular momentum loss by gravitational radiation and magnetic braking, and we describe the two categories of LMXB evolution in terms of the bifurcation period. We obtain the luminosity profile for angular-momentum-loss driven evolution, and introduce and discuss the Patterson scaling (1984) which accounts for it. We then obtain the luminosity profile for evolution driven by nuclear evolution of the companion. In Sec.3 we describe our
scheme for computing the XLF through the device of convolving the inputs on pre-LMXBs from Paper II with these luminosity profiles, together with the SFH for taking into account the simultaneous evolution of the SFH. We discuss the properties of our computed XLF, exploring the effects of some primordial-binary parameters and supernova characteristics on it. In Sec.4 we compare our computed XLF with observations, and discuss the results. Our conclusions are presented in Sec.4.2.

2. LMXB Evolution

A pre-LMXB system is initially detached. Mass transfer and accretion onto the neutron star is not possible in this state and becomes so only when the system evolves, its orbit shrinks due to loss of angular momentum, and Roche-lobe contact is achieved. The orbital angular momentum of the binary is given by

\[ J^2 = G \frac{M_c^2 M_{NS}^2}{M_t a} \]

Here \( M_c \) and \( M_{NS} \) are the masses of the companion and the neutron star respectively and \( M_t = M_{NS} + M_c \) is the total mass of the system. \( a \) is the binary separation.

We describe binary orbital evolution due to angular momentum loss by differentiating Eq.(1) with respect to time. We first note that \( \dot{M}_t = \dot{M}_{NS} + \dot{M}_c \). Since the companion is the mass donor, we can write \( \dot{M}_c = -\dot{M} \) where \( \dot{M} \) is the mass-transfer rate. Allowing for non-conservative mass transfer in general, we write \( \dot{M}_{NS} = \gamma \dot{M} \), i.e., a fraction \( \gamma \) of the mass transferred by the donor is accreted onto the neutron star and the rest of the mass is lost from the system, reducing its total mass at a rate \( \dot{M}_t = -(1 - \gamma)\dot{M} \). This mass loss implies an appropriate loss of angular momentum from the system. It is customary to represent this loss of angular momentum in terms of the specific angular momentum of the
system (Belczynski et al. 2008; Pfahl et al. 2003), expressing the result as \( \dot{J}/J = \mu \dot{M}_t/M_t \).

Here, \( \mu \) is typically of order unity, its exact value depending on the binary parameters as well as the evolutionary state of the donor. By inserting the expressions for the rates of change of various quantities as described above, the differential form of eqn. 1 can be rewritten as:

\[
\left( \frac{\dot{J}}{J} \right)_{NML} = \frac{\dot{M}}{M_c} \left[ \gamma q - 1 + \frac{(1 - \gamma)(1 + 2\mu)}{2} \frac{q}{1 + q} \right] + \frac{\dot{a}}{2a}
\]  

(2)

Here, \( (\dot{J}/J)_{NML} \) is the rate of loss of angular momentum from the system due to processes other than the above mass loss from the system, and \( q = M_c/M_{NS} \) is the mass ratio. We now consider such processes of angular momentum loss.

2.1. Mechanisms of angular momentum loss

Two mechanisms have been widely studied in the literature that cause the angular momentum loss in the pre-LMXB systems. These are gravitational radiation and magnetic braking. We now briefly review these two mechanisms and discuss their effects on the binary parameters.

It is well-known that two point masses orbiting around each other would emit gravitational waves according to the general theory of relativity (Peters & Mathews 1963; Peters 1964), and the rate of evolution of the orbital separation (as also the eccentricity in case of eccentric orbits) due to this effect is well-known. It was shown by Faulkner (1971) that this effect can be important in close binary systems like CVs and LMXBs. The timescale for the rate of angular momentum loss is comparable to typical LMXB evolutionary timescales, which makes it an essential effect in LMXB evolution. The rate of loss of the angular momentum due to this effect is given for a circular orbit by:
\[
\left( \frac{\dot{J}}{J} \right)_{GW} = -0.831 \frac{M_c M_{NS} M_t}{a^4}
\]  

We shall use the following system of units throughout this paper, unless stated otherwise. All the masses and the radii/distances are given in terms of the solar mass and the solar radius respectively, and time is in the units of Gyr. X-ray luminosity of the LMXBs will be given in units of \(10^{36}\) erg/s and will be denoted as \(L_{36}\).

We observe the strong dependence on the orbital separation in Eq. (3), which makes gravitational radiation completely ineffective at large separations. However, since pre-LMXB orbits are already very compact due to large contractions during the CE phase, this mechanism can be, and often is, the dominant one determining the orbit shrinkage that converts pre-LMXBs into LMXBs.

Magnetic braking was suggested as a possible mechanism for angular momentum loss by Verbunt & Zwaan (1981) in order to explain the observed high rates of the mass transfer in some LMXB systems. This mechanism depends upon the mass loss due to a magnetic wind from a tidally locked companion, working as follows. The magnetic field of the companion makes the matter in its coronal region co-rotate with the star, so that this matter has a large specific angular momentum, and consequently even a moderate mass-loss rate can lead to a large rate of angular momentum loss. Since the companion is tidally locked in pre-LMXBs and LMXBs, this loss of angular momentum will ultimately come from the orbital angular momentum of the binary, making it shrink. The rate of loss of angular momentum due to magnetic braking is given by

\[
\left( \frac{\dot{J}}{J} \right)_{MB} = -57.2 \eta \frac{M_c^2 R_c^4}{M_{NS} a^5}
\]  

Here \(R_c\) is the radius of the companion and \(\eta\) is a parameter of order unity which
can be empirically adjusted to fine-tune the strength of the magnetic braking. We note here that the magnetic braking is effective only when the star has a considerable magnetic activity. It has been suggested that magnetic activity would suddenly decrease drastically when the star becomes fully convective. This happens when the mass of the star falls below $\sim 0.3M_\odot$. Thus the magnetic braking is expected to turn off when the companion mass falls below this limit, and this is believed to be the reason for the observed period gap in CV systems.

Some authors have suggested lower strengths of magnetic braking than in the original formulation (see, e.g., van der Sluys et al. (2005)), implemented by either (a) reducing the braking by a fixed fraction ($\eta = 0.25$, say), or (b) using different functional forms in two different regions of angular velocity. It was also suggested by Podsiadlowski et al. (2002) that this reduction in strength may depend upon the mass of the convective envelope. The calculational scheme for collective properties of LMXBs which we will be describing in section 3 is capable of handling all such prescriptions of magnetic braking. However, we are interested here in determining the dependence of the collective properties of LMXBs on the very basic ingredients of the process of formation and evolution, and not on their details. We therefore assume the simple form given in eqn. 4 in this work, postponing the study of more complicated effects to future works.

A detached pre-LMXB will shrink through the loss of angular momentum which is caused by gravitational radiation and magnetic braking, described in Eq. (2) by $$(\dot{J}/J)_{NML} = (\dot{J}/J)_{GW} + (\dot{J}/J)_{MB},$$ together with Eqs. (3) and (4). We note that angular momentum loss due to gravitational waves as well as the magnetic braking depends on the instantaneous binary parameters, i.e., the two masses, the orbital separation and the radius of the companion for magnetic braking. Consequently, the evolution rate of the parameters of the system at any instant is completely determined by their current values. This fact
can be used to evolve the system in a straightforward scheme till the point of Roche lobe contact, which is given by \( R_c = a r_L \), where for the Roche lobe radius \( r_L \) we use the Eggleton prescription, as described in Paper II (Eggleton 1983).

2.2. The bifurcation period

Two main channels possible for LMXB evolution under the above conditions have been identified for a long time now. When the binary is sufficiently close to begin with, i.e., when the initial binary period is shorter than the so-called bifurcation period, the companion fills its Roche lobe while it is still on the main sequence, mass transfer starts, and the LMXB turns on. Further orbital evolution in this channel, which is followed by close LMXBs and CVs, is determined by angular momentum loss, and the orbital period decreases during this evolution. On the other hand, if the initial binary period is longer than the bifurcation period, the companion fills its Roche lobe after it finishes its main-sequence evolution and starts ascending the giant branch. In this channel, which is followed by the wider LMXBs, further orbital evolution is basically determined by nuclear evolution of the companion, and the orbital period increases during this evolution, which explains why it is called the bifurcation period.

Detailed calculations of this bifurcation period (which depends on the stellar masses and weakly on other parameters) were given by Pylyser & Savonije (1988, 1989), showing that this period lies in the range 14 -18 hours for the range of masses and parameters relevant for LMXBs. In the following subsections, we describe our calculations of LMXB evolution in the above two channels.
2.3. Evolution of LMXBs with main-sequence companions

We consider first the evolution of LMXBs with main-sequence companions and initial orbital periods below the bifurcation period. Once Roche-lobe contact is established, mass transfer begins from the low-mass main-sequence companion to the neutron star. Mass transferred through the \( L_1 \)-point carries a specific angular momentum comparable to that of the binary orbit. Therefore, this mass cannot directly accrete onto the neutron star but rather forms an accretion disk around it, from where it is slowly transferred to the neutron star on a viscous timescale. We in this work do not consider the detailed dynamics of this accretion process, as it is not relevant for our purposes. All we need for our calculations here is the fact that the average accretion rate onto the neutron star (i.e., averaged over any fluctuations caused by accretion-disk processes) is \( \gamma \dot{M} \), which determines the average X-ray luminosity.

The condition for Roche-lobe contact described in the previous section, i.e., \( R_c = a r_L \), assumes in effect an infinitely sharp boundary of the companion star at its formal radius, where the stellar density suddenly falls to zero beyond this radius. Such a strict condition is of course unphysical and can be relaxed to accommodate a stellar atmosphere of rapidly falling density. We here follow the prescription by Ritter (1988) of an exponentially decreasing mass transfer rate for larger separations, given by:

\[
\dot{M} = \dot{M}_0 \exp \left( \frac{a r_L - R_c}{H} \right) \tag{5}
\]

\( \dot{M}_0 \) here is the rate of mass transfer exactly at the start of full Roche-lobe overflow and \( H \) is the scale height of the atmosphere. We take \( H/R_c = 0.005 \) for main-sequence companions and \( H/R_c = 0.01 \) on the giant branch. Note that we set the modification factor to unity in case of a slight overfilling of the Roche lobe, in order to avoid the very high,
umphysical rates of mass transfer which would formally result from the above exponential prescription, and which merely reflect the fact that this prescription is not valid when the Roche lobe is overfilled. This method of handling the beginning of Roche-lobe overflow smoothly is a natural way of incorporating low-luminosity systems in our scheme. We discuss this further in Sec. 3.3.

After attaining full Roche lobe contact, mass transfer begins at its full rate from the companion to the neutron star. The response of the orbital separation to the mass transfer is dependent on the mass ratio $q$. It is a well-known fact that, if there is no mass loss (i.e., $\gamma = 1$), then for $q < 1$ the orbit expands whereas for $q > 1$ the orbit contracts. Thus the mass-transfer effects strengthen those of the angular momentum loss for $q > 1$, while for $q < 1$ the two effects oppose each other.

We also note that, for sustained mass transfer, the radius of the star must always equal the Roche-lobe radius. This condition is automatically self-sustained for $q < 1$, which can be easily seen in the following way. For Roche-lobe underfilling systems, since no mass is transferred, the orbit shrinks due to angular momentum loss until Roche-lobe contact is re-established, while for Roche-lobe overfilling systems, heavy mass transfer occurs and the orbit widens until exact Roche-lobe contact is re-established. For $q > 1$, the situation is more complicated, but still viable unless $q$ has a relatively large value (see below). For Roche-lobe underfilling systems, the same argument as above applies. But for Roche-lobe overfilling systems, mass transfer further shrinks the orbit and likely leads to runaway mass transfer and heavy mass loss from the companion. It is believed that this eventually brings the companion back into exact Roche-lobe contact. However, at sufficiently high values of $q$, even if the entire mass lost by the companion is lost from the system ($\gamma = 0$), the orbit does not expand, and the above argument fails. This situation is similar to the one described by Podsiadlowski et al. (2002) for companion masses $> 4M_\odot$. Since such situations do
not appear relevant for LMXBs, however, we do not consider them any further here, and
assume that the system is in constant Roche lobe contact. This condition is twofold. It
implies that (a) the radius of the companion is equal to its Roche lobe radius, and (b) the
rate of change of its radius also equals the rate of change of the Roche lobe radius. The
latter condition can be rewritten as

$$\frac{\dot{R}_c}{R_c} = \frac{\dot{a}}{a} + \frac{\dot{r}_L}{r_L}$$

For a complete description of the evolution, a mass-radius relation for the companion
is also required. Since we assume the companion to be on the main sequence in this channel
of LMXB evolution, we can take a simple power-law relation $R_c \propto M_c^n$. We note that the
mass-radius realtion for the companion could, in principle, evolve because of the nuclear
evolution of the low-mass companion as the evolution of a given LMXB system proceeds,
but this effect is unimportant for the following reason. In this channel, LMXB evolution
occurs on the timescale of angular-momentum loss from the system, which is shorter
than the nuclear timescale of the companion. Further, as mass transfer proceeds and the
compaion’s mass decreases, its nuclear-evolution timescale becomes longer and longer,
so that the extent of the companion’s nuclear evolution over the entire LMXB evolution
becomes quite tiny. Thus, nuclear evolution of the companion is in effect “frozen”, to use
the succint description of Pylyser & Savonije (1988), over the whole LMXB evolution of
interest to us here. Accordingly, we assume the above mass-radius relation to be static.
In the computations reported in this paper, we use $n = 1$, which is adequate for our first,
approximate study here, which is intended to serve as a proof of principle. Other, slightly
different, values of $n$ are also discussed later when appropriate occasions come up.

We emphasize here that the above assumption of a static mass-radius relation would,
of course, be completely inadequate for giant companions. We give an account of our
handling of the time dependence of the mass-radius relation on the giant branch in Sec. 2.4.
However, in this part of the work, this relation is static, so that the change in the radius
of the mass-transferring companion is only through the change in its mass, and for the
power-law mass-radius relation given above, we have \( \dot{R}_c / R_c = -n(M/M_c) \), remembering
the earlier relation between \( \dot{M} \) and \( \dot{M}_c \).

The rate of change of the Roche lobe radius can be computed by differentiating the
Eggleton expression for the effective Roche lobe radius given in Paper II. We obtain
\[
\frac{d}{dq} \ln r_L = \frac{-\dot{M}}{M_c} \frac{dr_L}{dq} q(1 + \gamma q)
\]
\[
\frac{dr_L}{dq} = \frac{r_L^2}{1.47p^4} \left( \frac{2\ln(1 + p)}{p} - \frac{1}{1 + p} \right)
\]
Here, \( p = q^{1/3} \). Equations (6) and (7) can be substituted in Eq.(2) with the above
mass-radius relation to obtain the relation between the rate of mass transfer and the rate
of angular momentum loss. This relation is:
\[
\left( \frac{J}{\dot{J}} \right)_{NML} = g(q; \gamma, \mu) \frac{\dot{M}}{M_c}
\]
\[
g(q; \gamma, \mu) = \left[ \gamma q - 1 + \frac{(1 - \gamma)(1 + 2\mu)}{2} \frac{q}{1 + q} - \frac{n}{2} + \frac{1 + \gamma q}{2} \frac{d\ln r_L}{d\ln q} \right]
\]
Equation(8) gives the rate of mass transfer in terms of the current system parameters.
\( \dot{M} \) and \( \dot{a} \) are also related to each other by Eq.(6), with the aid of the above mass-radius
relation. Thus the evolution of a LMXB with a main sequence companion is completely
specified by the above equations for given values of the parameters \( \gamma \) and \( \mu \).

We take \( \mu = 1 \) for main-sequence companions in this part of the work. A determination
of \( \gamma \) is not very straightforward, but we can proceed in the following way. First, we shall
assume in this work that the value of \( \gamma \) is as high as possible, subject to an absolute
upper limit of unity. This assumption effectively means that the system’s dynamics and
hydrodynamics are such that it prefers channeling the mass through the \( L_1 \) point rather
than sending it out of the system as far as possible, consistent with all laws of motion. This is, of course, a simple assumption of convenience in a first study, whose effects can be studied in later, detailed studies. The second constraint comes from the fact that there is an upper limit to the accretion rate on the neutron star, which corresponds to the Eddington luminosity for the mass of the neutron star. Finally, Eq. (8) requires $g(q, \gamma, \mu) < 0$. This poses an additional constraint on $\gamma$, given by

$$\gamma \left( q - \frac{1 + 2\mu}{2} \frac{q}{1 + q} + \frac{q^2}{2r_L} \frac{dr_L}{dq} \right) < 1 + \frac{n}{2} - \frac{1 + 2\mu}{2} \frac{q}{1 + q} - \frac{q}{2r_L} \frac{dr_L}{dq}$$

We apply the above constraints to obtain a consistent value of $\gamma$, which we use to calculate the evolution of the system with the aid of Eq. (8).

The essential features of the evolution of such LMXBs (and the related CV systems) with low-mass main-sequence companions, where angular-momentum loss from the system drives the mass transfer and the evolution, are well-known. Consider first the evolution of the LMXB orbit, as shown in Fig. 1 for a system with an initial companion mass of $M_s = 0.7M_\odot$ and an initial separation of $a_i = 3.0R_\odot$, which corresponds to an initial orbital period of $P_{i, orb} \approx 10$ hr, i.e., shorter than the bifurcation period. As time progresses, the LMXB orbit shrinks and the orbital period decreases, even as the companion mass decreases due to mass transfer. Magnetic braking turns off when the companion becomes fully convective at a mass $M_C \approx 0.3M_\odot$. Beyond this point, the system continues evolution due to angular-momentum loss through gravitational radiation alone, until hydrogen burning is extinguished in the companion below a critical mass $M_c \sim 0.1M_\odot$. Subsequently, the companion follows the mass-radius relation for degenerate stars, the orbit expands after passing through the so-called period minimum, and the final product is a close binary consisting of a recycled neutron star and a low-mass He white dwarf. For our purposes here, it is sufficient to follow the main-sequence phase of the companion, and accordingly we terminate our computations when the companion mass reaches $M_c = 0.1$. As a consequence,
we do not follow the system quite up to the period minimum in Fig.1, but otherwise the period evolution is very similar to that given in previous works (see, e.g., Fig.3 of Pylyser & Savonije 1988, Fig.3 of Podsiadlowski et al. 2002, top panel, blue curve), and the magnetic-breaking turn-off point shows up as a “kink” in this diagram (at $t \approx 1.5$ Gyr in Fig.1), as it does in the above diagram of Podsiadlowski et al.

![Diagram](image)

Fig. 1.— *Angular-momentum-loss driven orbital evolution of a typical LMXB with a main-sequence companion (see text).*

Crucial for us in this work is the evolution of the accretion luminosity of the LMXB, which we display in Fig.2 for the system whose orbital evolution is shown in Fig.1.

The origin on the time axis in this figure is the formation time of the pre-LMXB, as before. The system comes into Roche-lobe contact at $t \approx 0.4$ Gyr (which shows up as another kink in Fig.1), at which point the LMXB turns on, giving the initial luminosity.
Fig. 2.—Angular-momentum-loss driven luminosity evolution of a typical LMXB with a main-sequence companion (see text and Fig[7]).

spike in Fig[2]. As time progresses beyond this, there is a sharp decline in the luminosity until the magnetic braking turn-off point is reached at $t \approx 1.5 \text{ Gyr}$ (see above), at which time the accretion rate and so the luminosity drops dramatically, since the dominant source of angular-momentum loss is suddenly turned off. During the subsequent evolution, gravitational radiation is the only mechanism of angular-momentum loss, and the luminosity rises very slowly with time.

The above luminosity profile (i.e., $L - t$ relation) is in general agreement with those in previous works, an example of which is in the above Podsiadlowski et al. (2002) work (bottom panel of their Fig.3, blue curve). Note that, since time is plotted on a logarithmic scale by these authors, the initial rise in the luminosity is actually resolved in their figure, while in our Fig[2] where we have plotted time on a linear scale, it appears as a luminosity spike, as it does in other previous works with a linear time plot. The basic arguments which
determine the shape of this luminosity profile are interesting, and go back to two scaling
laws originally demonstrated by Patterson (1984) in a related but different context. We
briefly recount these arguments here, as their importance does not seem to have been fully
appreciated in the LMXB literature.

The $L - t$ profile, or equivalently the $\dot{M} - t$ profile, is computed by relating the relative
rate of change of the companion mass, $\dot{M}/M_c$, to the relative rate of change of angular
momentum, $\dot{J}/J$, as detailed in Sec[2]. For clarifying the basic scalings, we shall give a
simpler calculation here, wherein we shall use somewhat simpler prescriptions than in the
detailed computations of Sec[2] and obtain analytic results in the two regimes where angular
momentum loss by either gravitational radiation (the GR regime) or magnetic braking (the
MB regime) dominates.

We note first that in the limit $\gamma = 1$, i.e., no mass loss from the system (which is a
good approximation for the angular-momentum-loss driven systems with main-sequence
companions considered in this section, as our detailed computations have shown, as well as
previous works), Eq.(2) reduces to:

$$\left( \frac{\dot{J}}{J} \right)_{NML} = \dot{\gamma} M_c (q - 1) + \frac{\dot{a}}{2a}. \quad (10)$$

In the GR regime, the left-hand side of the above equation is given by Eq.(3), while in the
MB regime, it is given by Eq.(4). The second term on the right-hand side of this equation
is basically the relative rate of change of the orbital radius, which can be related to the
relative rate of change of the companion mass, $\dot{M}/M_c$, with the aid of two conditions. The first
is the Roche-lobe filling condition, whose differential form is given by Eq.(6), and the second
is the mass-radius relation, $R_c \propto M_c^n$, whose differential form, $(\dot{R}_c/R_c) = -n(\dot{M}/M_c)$ was
also given earlier. When this is done, we obtain

$$\left( \frac{\dot{J}}{J} \right)_{NML} = \frac{\dot{M}}{M_c} \left[ - \left( \frac{n}{2} + 1 \right) + \left\{ q + \frac{1 + q}{2} \frac{d \ln r_L}{d \ln q} \right\} \right], \quad (11)$$
which is, of course, the simplified version of Eq.(8) for $\gamma = 1$.

We now introduce a further simplification which makes the calculation transparent and simple for our purposes of scaling demonstration in this part of the paper. Instead of the more complicated, but more accurate, Eggleton prescription (see Sec.2.1 and Paper II), which we used throughout our detailed computations described earlier, we now use the simpler, original Paczynski (1971) prescription, which is still a good approximation for the low companion masses involved here, namely:

$$r_L \propto \left( \frac{q}{1 + q} \right)^{1/3} \tag{12}$$

This simplifies the calculation, yielding

$$\frac{d \ln r_L}{d \ln q} = \frac{1}{3} \cdot \frac{1}{1 + q}, \tag{13}$$

which immediately converts Eq.(11) into a very simple form

$$\left( \frac{\dot{J}}{\bar{J}} \right)_{NML} = - \frac{\dot{M}}{M_c} \left( \frac{5}{6} + \frac{n}{2} - q \right), \tag{14}$$

which is useful for our purposes here. If we take $n = 1$ for lower-main-sequence stars, the quantity within the parentheses on the right-hand side of the above equation reduces to $(4/3 - q)$.

We can now obtain our analytic approximations in the GR and MB regimes by equating the left-hand side of Eq.(14) to the right-hand sides of Eqs.(3) and (4) respectively, and re-expressing the stellar masses in terms of the total mass $M_t$ and the mass-ratio $q$ in the ensuing calculations (this is useful because $M_t$ remains constant in these approximate calculations due to our assumption of $\gamma = 1$ as given above). After some straightforward algebra, we get in the GR regime a profile:

$$L_{GR} \propto \dot{M}_{GR} \propto \frac{a^{10 - 12n}}{(1 + q) \left( 1 - \frac{6}{5 + 3n} \cdot q \right)}, \tag{15}$$
where the time dependence comes from the fact that both \( a \) and \( q \) in the above equation are time-dependent, both decreasing with time, and the \( a - t \) profile being given in Fig.1.

In the special case \( n = 1 \) (see above), the profile is:

\[
L_{GR} \propto \dot{M}_{GR} \propto \frac{a^{-1}}{(1 + q)(1 - 0.75q)}.
\]  

(16)

The profile in the MB regime is obtained in a similar way, and found to be

\[
L_{MB} \propto \dot{M}_{MB} \propto a^{\frac{12n}{3n-t}} \frac{(1 + q)^{5(n-1/3)}}{(1 - \frac{6}{5+3n}q)}.
\]  

(17)

In the special case \( n = 1 \) (see above), the profile is:

\[
L_{MB} \propto \dot{M}_{MB} \propto a^{6} \frac{(1 + q)^{10/3}}{(1 - 0.75q)}.
\]  

(18)

The basic physics underlying the luminosity profile in Fig.2 is clear now from the scalings evident in Eqs.(16) and (18). Both the GR and the MB profile factorize into \( a \)-dependent and \( q \)-dependent parts, and the profile is largely determined by the \( a \)-dependent part, as the \( q \)-dependent part changes relatively slowly (compared to the \( a \)-dependent part) as the low-mass companion loses mass and \( q \) decreases with ongoing mass transfer. Of course, since the \( a \)-dependence is not as strong for GR case as it is in the MB case, some modification does come in from the \( q \)-dependence, but the qualitative argument still holds. The sharp decline in \( L \) in the MB-dominated regime between the LMXB turn-on and the MB turn-off at \( t \approx 1.5 \) Gyr is basically due to the very strong \( a^{6} \)-dependence of the MB luminosity in a shrinking orbit, as seen in Eq.(18). Similarly, the slow rise in \( L \) in the GR-dominated regime beyond the MB turn-off is basically due to the \( a^{-1} \)-dependence of the GR luminosity in a shrinking orbit (duly modified by the \( q \)-dependence), as seen in Eq.(16).

These scalings were first pointed out by Patterson (1984) in a study of CV and LMXB evolution, for the purpose of comparing the expected \( \dot{M} - P_{\text{orb}} \) correlation with the data, largely on CVs. (The transformation between \( a \) and \( P_{\text{orb}} \) is, of course, readily obtained
through Kepler’s third law.) This amounts to looking at a “snapshot” at the current epoch of a collection of systems with various stellar masses and at various stages of evolution, and comparing this data with the theoretical scaling. As Patterson noted, a reasonable account of the data was indeed given by the above scaling, which was subsequently confirmed by Pylyser & Savonije (1988), by comparing the results of their detailed evolutionary computations with the same data. We have argued above that the same scaling must also be necessarily applicable to the evolution of a single system as time proceeds, and have demonstrated that this is indeed so. Accordingly, we shall call this scaling the Patterson scaling throughout this work.

We have given above a first-principles derivation of the Patterson scaling to emphasize its fundamental and transparent nature, and also because some details were different in the original Patterson work, e.g., this author used a slightly different magnetic-braking model. However, we find a close agreement between Patterson’s result and ours, showing the robustness of the scaling. In particular, for the specific value of $n = 0.88$ that Patterson adopted, our scaling give $\dot{M}_{GR} \propto a^{-0.34} \propto P_{\text{orb}}^{-0.23}$, while Patterson gives $\dot{M}_{GR} \propto P_{\text{orb}}^{-0.26}$.

Similarly, in the MB regime, our scaling gives $\dot{M}_{MB} \propto a^6 \propto P_{\text{orb}}^{4.33}$, while Patterson gives $\dot{M}_{MB} \propto P_{\text{orb}}^{4.55}$. It is clear that the Patterson scaling is at the heart of angular-momentum-loss driven luminosity evolution of LMXBs with main-sequence companions found in this work and numerous previous works in the subject.

### 2.4. Evolution of LMXBs with giant-branch companions

When the pre-LMXB is not sufficiently close according to the criterion described earlier, i.e., the binary period is longer than the bifurcation period, the companion finishes its main-sequence evolution before Roche lobe contact. In such a case, the LMXB phase turns on when the companion goes into the giant branch, starts expanding rapidly and
overfills the Roche lobe. The actual transition timescale from the main sequence to the giant branch is that of the traversal of the Hertzsprung gap in the HR diagram, which is extremely short compared to the typical timescale of LMXB evolution, and hence can be considered practically instantaneous for our purposes here. The orbit cannot expand in step with this very rapid process to accommodate the expanding giant companion, so that large mass losses occur from the system, until the Roche-lobe filling condition is satisfied again. We assume in this work that there is no accretion during this short phase, i.e., we set $\gamma = 0$ in it. The angular momentum loss due to the non-mass-loss mechanisms are also negligible due to the short duration of this phase. Consequently, the evolution of the system can be described by integrating Eq.(2) under these assumptions, which gives the condition:

$$
M_i^c \sqrt{\frac{a_i}{(M_i^c)^{1+2\mu}}} = M_f^c \sqrt{\frac{a_f}{(M_f^c)^{1+2\mu}}} \quad (19)
$$

Here, the superscripts $i$ and $f$ denote the initial and the final state respectively. This equation is to be used along with the condition that the orbital separation after this phase is just enough to fit a giant donor within its Roche lobe, i.e., $a_f = R_{GB}(M_f^c)/r_L(M_f^c/M_{NS})$.

In such LMXBs with giant companions, the mass-transfer rate is determined by the rate of expansion of the companion due to its nuclear evolution. We assume in this work that the condition of constant Roche-lobe filling is still valid on the giant branch and Eq.(6) holds for these systems as well. However, the assumption of a static mass-radius relation is not valid on the giant branch, and the time evolution of the companion radius needs to be determined in the following way. The radius of a star on the giant branch depends upon its mass as well as its luminosity. Now, the luminosity on the giant branch depends upon the core mass, which, in turn, depends upon the initial mass rather than the current mass of the companion. Thus the evolution of a LMXB with a giant donor depends upon the initial values of the parameters as well as their current values. With the aid of Eq.(48) of
Hurley et al. (2000), we can write the radius a star on the giant branch as:

\[ R_c = R_{GB} = 1.1 M_c^{-0.3} (L_c^{0.4} + 0.383 L_c^{0.76}). \]  

(20)

Here, \( L_c \) is the instantaneous luminosity of the companion. If we define \( f(L_c) \equiv (L_c^{0.4} + 0.383 L_c^{0.76}) \), rate of change of the stellar radius can be given as

\[ \frac{\dot{R}_c}{R_c} = 0.3 \frac{\dot{M}}{M_c} + \frac{1}{f} \frac{df}{dL_c} \dot{L}_c \]

(21)

Analytical fitting formulae for numerical stellar-evolution results provided by Hurley et al. can be used to calculate \( L_c \) and \( \dot{L}_c \), which, in turn, give the rate of expansion of the stellar radius. Eqs. (6) and (7) can now be inserted, along with the new mass-radius relation given by Eqs. (20) and (21), into Eq. (2) to obtain the equation governing the LMXB evolution for giant donors. We get:

\[ \left( \frac{\dot{J}}{J} \right)_{NML} = h(q; \gamma, \mu) \frac{\dot{M}}{M_c} + \frac{1}{2 f} \frac{df}{dL_c} \dot{L}_c 
\]

\[ h(q; \gamma, \mu) = \left[ \gamma q - 1 + \frac{(1 - \gamma)(1 + 2 \mu)}{2} \frac{q}{1 + q} + 0.15 + \frac{q(1 + \gamma q)}{2 r_L} \frac{dr_L}{dq} \right] \]

(22)

Values of \( \mu \) and \( \gamma \) may be different here from those for the main-sequence donor case. Whereas the value of \( \mu \) is completely unknown as per current understanding of the subject, it is customarily taken as unity in studies of populations of LMXBs (Podsiadlowski et al. 2002). We argue here, however, that the actual value in case of giant donors may well be less than this, since the outer envelope, which is the actual supplier of the transferred mass, is less tightly bound in the case of giants. We therefore adopt \( \mu = 0.75 \) in our calculations.

The value of \( \gamma \) is subject to similar constraints as in the case of main-sequence donors. However, we note that in case of giants some mass loss would be inevitable. Therefore the upper limit on the value of \( \gamma \) is unlikely to be unity in the case of giant companions. This value is generally taken to be 0.5 (Podsiadlowski et al. 2002; Belczynski et al. 2008), but
this is a somewhat arbitrary assumption, as noted by Podsiadlowski et al.. We therefore keep this as a free parameter and study its effects on the evolution of LMXB systems. Of course, the other two constraints described earlier, namely, (a) that due to the Eddington limit on the mass accreting onto the neutron star, and (b) that expressed by the condition $h(q; \gamma, \mu) < 0$, still apply. The latter condition can be written explicitly as:

$$\left( q - \frac{1 + 2\mu}{2} - \frac{q}{1 + q} + \frac{q^2}{2r_L} \frac{dr_L}{dq} \right) < 0.85 - \frac{1 + 2\mu}{2} - \frac{q}{1 + q} - \frac{q}{2r_L} \frac{dr_L}{dq}. \quad (23)$$

The value of $\gamma$ is obtained subject to the three conditions given above, and it is then used to compute the evolution of the LMXB, which is described by Eq. (22). We emphasize here that a knowledge of the current values of the system parameters is not sufficient for calculating LMXB evolution with giant-branch donors, which makes the computations more complicated.

LMXBs with the giant donors end their evolution when they run out of the supply of transferrable mass. This means that the mass transfer basically continues until the entire envelope of the companion is transferred to the neutron star, leaving only its core. We can write this condition as $M_c = M_{s,c}$ where $M_{s,c}$ is the core mass of the companion, which depends upon its initial mass. This He-core now contracts basically like an isolated He-star, so that Roche-lobe contact is lost and the LMXB is turned off. We stop our numerical computations at this point. The final product of such evolution is a wide, circular binary consisting of a recycled neutron star and a low-mass white dwarf with $M_{WD} \leq 0.45M_\odot$ or so.

Figure 4 depicts the luminosity evolution of such a prototype system, the corresponding orbital evolution being shown in Fig. 3. In this case, the initial secondary mass is $M_s = 1.3$ and the initial orbital separation is $a_i = 11$, corresponding to an initial orbital period of $P_{i,orb} \approx 2.6$ days. In sharp contrast to the earlier angular-momentum-loss driven case, here the orbit expands continually as evolution proceeds, as found in previous works (see, e.g.,
Fig. 3.— Nuclear-evolution driven orbital evolution of a typical LMXB with a giant companion (see text). Line-style of curves coded by the values of $\gamma_{\text{max}}$ as indicated.

Figs. 2b and 2c of Tauris & Savonije (1999), Fig. 11 of Podsiadlowski et al. (2002), Figs. 9 and 10 of Belczynski et al. (2008), which are StarTrack studies of the above 2b and 2c cases in the Tauris-Savonije work). In these figures, zero on the time axis corresponds to the formation of the pre-LMXB, as before, but since the system remains out of Roche-lobe contact during the secondary’s main sequence life, we have so shifted and rescaled the time axis as to show most clearly only the X-ray active part with non-zero LMXB luminosity, after the system reaches Roche-lobe contact at $t \approx 4.55$ Gyr. The evolutions of both the orbit and the luminosity found by us are rather similar to those found in previous works, when account is taken of the fact that the display methods in some of these previous works are different from ours, e.g., time plotted on a logarithmic scale in Podsiadlowski et al. (2002), and the decreasing companion mass instead of increasing time as the abscissa in some panels of Tauris & Savonije (1999). The major feature of the luminosity profile after
Fig. 4.— Nuclear-evolution driven luminosity evolution of a typical LMXB with a giant companion (see text and Fig. 3). Line-style of curves coded by the values of $\gamma_{\text{max}}$ as indicated.

Roche-lobe contact and LMXB turn-on is a sharp initial decline, followed by a slower decline and a plateau phase, and a very slight rise at the longest times in some cases. An explanation of this luminosity profile involves details of stellar evolution on the giant branch, and so is outside the scope of this paper, contrary to the basic, simple explanation in terms of Patterson scaling that was possible earlier for the angular-momentum-loss driven evolution of LMXBs with main-sequence companions.
3. X-ray Luminosity Function (XLF) Calculation Scheme

3.1. The partial XLF

The importance of the cosmic star formation history (SFH) and its effect on the pre-LMXB formation rate was noted in Paper II. It was explained there that the SFH over a few Gyrs previous to the current epoch plays a crucial role in determining the current population of LMXBs. This can also be seen from the two figures given in previous section depicting the evolution of a typical LMXB system with a main-sequence or giant-branch companion. Figures 2 and 4 show that, while typical evolutionary timescales as counted from the formation of the corresponding pre-LMXBs is rather similar for LMXBs with main-sequence and giant-branch companions, typically $\sim 3 - 5$ Gyrs, the timescales as counted from the turn-on points of the LMXBs are very different, typically $\sim 3$ Gyr for LMXBs with main-sequence companions, but typically $\sim 0.2$ Gyr, i.e., an order of magnitude shorter, for LMXBs with giant-branch companions. Therefore, if we observe two LMXB systems with the same luminosity in the current epoch, their turn-on epochs may differ by upto a few Gyrs, depending on the natures of their companions. In order to follow the evolution of LMXB populations, it is therefore essential to take into consideration evolutionary changes in the SFR. This problem was first considered by Ghosh & White (2001), who showed that the peak in the number of LMXBs lags appropriately behind the SFR peak. They considered typical timescales of LMXB evolution that are comparable to the timescales obtained from our calculations. With the more detailed evolutionary scenario considered in our scheme, we are able to deal with the various timescales more accurately, considering them as functions of the initial parameters of the binary. This enables us to go one step beyond the calculations of these authors, and explore the evolution in the distributions of the essential collective properties of LMXBs.

The evolution of a single LMXB detailed in Sec.2 gives the complete evolutionary
history of a given LMXB with a specified initial state, which is determined by the initial mass of the companion \( M_c^i = M_s \), see Paper II) and the initial orbital separation \( a^i \), for brevity we drop the superscript from here on, and call it just \( a \). In other words, the parameters of the system, e.g., luminosity, orbital period, instantaneous companion mass, and so on can be computed at any subsequent time. With this input, we can now calculate the X-ray luminosity function (XLF) of LMXBs in the following way, and we note in passing that the same method can, in principle, be applied to the distribution of any other collective property of LMXBs.

Consider first all pre-LMXB systems formed with the same initial parameters. The evolution of all these systems will be similar to one another and can be computed with our scheme, giving the luminosity as a function of time. This relation between luminosity and time can then be inverted, so that, given the luminosity, we can determine the time required to attain it. Thus, if a given LMXB with a luminosity \( L \) is observed at a current time \( t \), we know its formation time \( t_f = t - t_{lb} \), where \( t_{lb} \) is the look-back time, i.e., the time taken by the system since its formation to reach this value of luminosity. Now, the origin of time is set in these calculations at the formation of the pre-LMXB. Therefore, \( t_f \) is the formation time of the corresponding pre-LMXB. Thus, the number of systems with a luminosity \( L \), observed at present, is equal to the number of pre-LMXBs formed at time \( t_f \) before, as described in Paper II. Since the formation rate can also be computed for the given set of initial parameters, we can calculate the number of LMXBs at each value of luminosity. By suitable binning, then, we can numerically obtain the XLF of these systems. We note here that this XLF is calculated for the systems which had a specific value of \( M_s \) and \( a \) to start with. We therefore name this the partial XLF.
3.2. The full XLF

The partial XLF, as described in the previous subsection, can be obtained for any initial-value set \((M_s, a)\). The full, or integrated, XLF can then be obtained by integrating over \(M_s\) and \(a\), or, in case of discrete data, by summing over all allowed values of \(M_s\) and \(a\). We note here that the formation rate of pre-LMXBs given in Paper II already includes the distribution of \(M_s\) and \(a\), so that one does not require further weighting in the process of such summation/integration. The range of these variables relevant for the XLF calculation was also discussed in Paper II. We simply state these ranges here, and refer to Paper II for further details. The range of \(M_s\) is taken to be \([0.1, 2.5]\). We note here that this is the initial value of the companion mass, so that the instantaneous value of this mass (denoted in this work by \(M_c\)) will be lower than this at some intermediate time of evolution, after mass transfer starts. Systems with higher companion masses are expected to have higher accretion rates and lower values of \(\gamma\), implying significant mass loss in some systems. When the systems are actually observed as LMXBs, companions more massive than \(1M_\odot\) are rather unlikely to be found. Nevertheless, our scheme does allow higher-mass companions for completeness, so that systems like Her X-1 can also be included in the scheme.

The range of \(a\) is \([1.0, 20.0]\). Very few systems wider than this can come into Roche-lobe contact in a Hubble time. Many systems with companion masses at the lower end of the mass-range cannot come into Roche lobe contact at separations \(>10R_\odot\). We note here that the systems with very low-mass companions cannot reach the giant phase because their main sequence lifetime is longer than the Hubble time. This lower limit on the initial companion mass can be as high as \(0.9M_\odot\), depending upon the value of Hubble constant assumed. We note that these constraints do not need to be posed explicitly on the systems under consideration, as our scheme of computations is so designed as to automatically take care of such issues related to timescales.
The XLF thus obtained depends upon (a) system parameters at the pre-LMXB stage, which affect the results through the formation rate of these systems, and (b) various parameters that affect the further evolution of LMXBs. A complete parameter study is outside the scope of this work. We therefore fix the values of nearly all the parameters which we list now. At the pre-LMXB stage, we take the CE-parameter $\alpha \lambda = 1.0$, and the metallicity of the primary $z = 0.02$. To calculate the formation rate of the pre-LMXBs, we assume a Madau-profile for the SFR, which is a peak-type profile with $z_{\text{max}} = 0.39$ and $p = 4.6$ (see Blain et al. (1999) and also relevant references given in Paper II for the types of SFR profile and the parameters used to describe them). We study the final XLF for both values of $\beta$ introduced in Paper II, viz., the uniform and the falling-power-law distribution for the mass ratio, and also for the with kick and without kick scenarios (this latter scenario was shown in Paper II to give results almost identical to those for ECSN-type low kicks).

We note here that $z = 0.02$ may be an incorrect assumption, since many LMXBs are found in early-type galaxies, where the progenitor stars could have been metal-poor. However, we do not test this assumption at this stage, noting that the effect of changing the metallicity can be judged from the variation of the pre-LMXB PDF calculated in Paper II. Values of the other parameters adopted in the calculation of the evolution of the LMXBs are given in Secs. 2.3 and 2.4. We mention here again that we take $\mu$ and $\gamma_{\text{max}}$ as unity for the evolution with main-sequence companions, where $\gamma_{\text{max}}$ is the upper limit for $\gamma$. $\mu = 0.75$ is taken for the evolution with companions on the giant branch, and in this case we study two values of $\gamma_{\text{max}}$ which are 0.5 and 0.8. We realize that these particular choices allow us to explore only a limited section of the full parameter space. However, our aim here is to demonstrate that the straightforward scheme described in this work can be used to calculate the XLF, which can be directly compared with observations, i.e., a proof of principle. We thus concentrate in this work on an exploration of the relative importances of various physical processes on the LMXB XLF, deferring parameter studies to future works.
3.3. Properties of the calculated LMXB XLF

We first note that, due to the calculational procedure adopted here, the cumulative XLF is easier to calculate than the differential one. Therefore, we discuss various properties of the calculated cumulative XLF in this subsection. We first discuss the dependence of the computed XLF on the pre-LMXB parameters and then that on the LMXB-evolution parameters. We chose two pre-LMXB parameters to illustrate the essential points. The first is the nature of the primordial $q$-distribution, and the second is the SN-kick scenario. We had noted earlier that ECSN-type kicks with $\sigma = 26.5$ km/s give PDFs similar to the no-kick scenario, whereas ICCSN-type kicks with $\sigma = 265$ km/s result in a different PDF.

Fig. 5.— Dependence of cumulative LMXB XLF on pre-LMXB parameters. The four cases are coded by line-style according to the values of the parameters, as indicated.

Fig. 5 shows the cumulative XLF for various values of pre-LMXB parameters, the four cases shown being the same as those considered in Paper II. (For definiteness, the XLF has been normalised to a total of 100 systems here.) The following general features can
be seen. The shape of the XLF is much more complex than that of the HMXB XLF. It starts with a flat region at low luminosities, which extends upto \( L \approx 10^{36} \) erg/s. We discuss the XLF in this low-luminosity regime later in this section. A power-law like fall is seen at higher luminosities, which extends upto \( \sim 10^{38} \) erg/s. The XLF cuts off beyond this point, where the luminosity approaches the Eddington luminosity for neutron stars. We note here that, since the mass of the neutron star need not be exactly the same for all systems, the Eddington cut-off may also have a range in general.

It can be seen that the “with-kick” scenario does not produce a very different XLF from the “without-kick” scenario, except for the sharper drop near the cutoff when the kicks are included. By contrast, the two different \( q \)-distributions produce quite different XLFs in the falling-power-law regime, with a steeper fall for the distribution of \( q \) with \( \beta = -2.7 \).

The numerically computed XLFs in the luminosity range \( 3 \times 10^{36} - 10^{38} \) erg/s, as well as over other ranges, can be fitted by power-laws of the form \( N(> L) \propto L^n \). The results of such least-squares fits are given in Table 1 where the leftmost column shows the luminosity range considered, and the next columns give the values of \( n \) for the four different cases.

Relative contributions of main-sequence and giant-branch donors can influence the shape of the overall LMXB XLF, since many characteristics of these two donor classes differ from each other. First, the giants come into Roche-lobe contact later than the

| Luminosity range \((x10^{36} \text{ erg/s})\) | \(\beta = 0\) without kick | \(\beta = 0\) with kick | \(\beta = -2.7\) without kick | \(\beta = -2.7\) with kick |
|---|---|---|---|---|
| 3 - 10 | -1.18 | -1.16 | -1.65 | -1.51 |
| 10 - 100 | -1.41 | -1.44 | -1.7 | -1.71 |
| 3 - 100 | -1.31 | -1.28 | -1.59 | -1.53 |

Table 1: Best-fit power-law indices over different luminosity ranges for our computed LMXB XLF, for different values of pre-LMXB parameters as indicated.
main-sequence companions. Thus the giant population corresponds to an earlier population of primordial binaries. Since the Madau profile suggests a peak in SFR at $z = z_{\text{max}} = 0.39$, this effect can be important. Second, systems with giant companions are typically brighter, but have shorter lifetimes. This makes them much smaller in number, but situated at the high-$L$ end of the XLF. We study the effect of the fraction of systems with giant companions, which we name the *giant fraction*, on the XLF in the following way. We first evolve the LMXB systems in the way described in Sec. 2. We calculate the XLF obtained from this evolutionary scheme which includes main sequence as well as giant donors. We then modify our algorithm of evolution in such a way that, when a companion completes its main sequence life, the system is removed from the computations. This eliminates the possibility of having giant donors, keeping in our computation only those systems which reach Roche-lobe contact during their main-sequence life, and so ensuring that all LMXBs in our computation have *only* main-sequence companions. The XLF computed in such a way is thus that corresponding to main-sequence donors alone.

Figure 6 shows the XLF for only main-sequence donors, superposed on the XLF after the inclusion of the giants. We see that the total number of systems with giant companions is not more than a few percent of the total number, but the giant-companion systems all lie at the bright end of the XLF, making the XLF considerably flatter at this end (see Table 1) than that with main-sequence companions alone, which has a slope of $n \approx -1.94$ at this end. We note here that, though the computation presented here to illustrate this point is for $\beta = 0$ and the “with-kick” case, the qualitative result is the same for all other combinations of $\beta$-value and kick-scenario. Therefore the giant fraction, although numerically small, is a very important factor in deciding the shape of the LMXB XLF at the bright end.

The parameter $\gamma$ denotes the accretion rate onto the neutron star per unit rate of mass loss by the companion. This parameter is close to unity for main-sequence companions,
Fig. 6.— *Relative contributions of main-sequence and giant donors in the computed XLF. Cases encoded by line-style as indicated.*

but its value is expected to be lower for giant companions, and this value was taken to be $\gamma_{GB} = 0.8$ in previous calculations. A change in the value of $\gamma_{GB}$ results in a change of the luminosity of those systems with giant donors which are operating at a minimum mass loss from the system (see Sec.2.4). The importance of the giant fraction in determining the shape of the XLF has already been shown above, from which we expect that lowering the value of $\gamma_{GB}$ will lower the influence of the giant-companion systems on the total XLF (since the maximum allowed luminosity for these objects will be lowered), and so make the XLF fall more steeply at the bright end. Figure 7 shows that this is indeed the case, displaying the effect for two values of $\gamma_{GB}$. The first value is 0.8, which has been used throughout this work. The second value is 0.5, which has been suggested by some authors, as noted in Sec.2.4. The power-law index obtained by least-squares fit method in the luminosity range $L_{36} = 3 - 100$ is $n = -1.48$ in the second case, *i.e.*, steeper than in the first case (see Table 1), but not as steep as in the $\gamma_{GB} = 0.8$ case (see above).
Finally, we consider the nature of our computed XLF at very low luminosities ($L < 10^{36}$ erg/s). The XLF at these luminosities appears to be not very interesting for several reasons. First, it is difficult to observe such faint systems with the current sensitivity of the X-ray detectors, even in nearby galaxies. Thus the observed XLF will be strongly influenced by selection effects in this region. Although studies often attempt to account for incompleteness, there are necessarily large error-bars in observed XLFs in this regime, as a study of relevant works shows. Second, one can see from our calculations that the standard theory predicts that there would be very few systems in this range. In fact, if we exclude Roche-lobe underfilling companions, the minimum luminosity possible is $\approx 10^{36}$ erg/s, which corresponds to a binary of a $1.4M_\odot$ neutron star and a main-sequence companion with a mass just below $0.3M_\odot$, the critical point where the magnetic braking is turned off (corresponding roughly to the minimum luminosity in Fig[2]). Only with our inclusion of Roche-lobe underfilling companions and the Ritter recipe for atmospheric Roche-lobe
overflow (wherein the accretion rate drops exponentially for increasing disparity between the stellar radius and the Roche-lobe radius; see Eq. (5)), as detailed in Sec 2.3 do we account for systems at lower luminosity.

Fig. 8. — LMXB XLF at very low luminosities.

Figure 8 shows the details of the computed XLF in this luminosity regime, using the above Ritter recipe. Because of the very narrow range of the ordinate ($N(>L)$) involved in this regime, this part of the XLF appeared almost flat (i.e., constant $N(>L)$) in all previous XLFs shown in this paper, where the scale of $N(>L)$ was logarithmic. In this figure, we have displayed $N(>L)$ on a linear scale, and shown only that range of $N(>L)$ over which it varies in this regime, in order to show the nature of its variation. Apart from artifacts resulting from binning and numerical effects, the average trend is consistent with a logarithmic decrease in $N(>L)$ with increasing $L$-values.
4. Discussion

4.1. Comparison with observations

In recent years, LMXB XLFs have been obtained for many early-type galaxies from Chandra and XMM-Newton observations. Many such works have come to the conclusion that the observed LMXB XLF cannot be described by a simple, single-power-law distribution, unlike the HMXB XLF (Grimm et al. 2002; Humphrey & Buote 2004; Kim & Fabbiano 2004; Revnivtsev et al. 2008; Voss & Gilfanov 2006; Voss et al. 2009; Fragos et al. 2009). These works have argued that the LMXB XLF can be adequately described by a broken power-law with a high luminosity cut-off. It was suggested by Grimm et al. (2002) that the number of LMXBs in a galaxy would scale with the total stellar mass of that galaxy, so that, when the LMXB XLF is normalised by this stellar mass, a roughly universal LMXF XLF would emerge, similar to the situation for the HMXB XLF and its scaling with the current SFR in the particular galaxy, which was described in Paper I. However, this scaling was not found to be as precise as that in the case of HMXBs.

It was shown by Gilfanov (2004) that a doubly-broken power-law (i.e., a power law with two breaks) template gave an adequate fit to the observed universal LMXB XLF. The break points obtained by these authors were at $L_{36} = 19$ and $L_{36} = 500$, and the power-law exponents given by them for the differential XLF $dN/dL$ in the three regions separated by these breaks were $-1.0 \pm 0.13$, $-1.86 \pm 0.12$ and $-4.8 \pm 1.1$, going from low to high luminosities. We stress again that these authors provided fits to the differential XLF, i.e., $dN/dL$. Power-law indices in this description are obtained by subtracting 1 from the ones in the cumulative description, except in the special case of a power-law index of $-1$ in the differential description, where in the corresponding cumulative description $N(> L)$ is a logarithmically decreasing function of $L$. 
However, Kim & Fabbiano (2004) proposed a different template. These authors showed that the XLF of a single galaxy can be described adequately by a single power-law. With a sample of 14 E and S0 galaxies, these authors concluded that power-law exponents in the range ($-0.8, -1.2$) could explain the cumulative XLFs of all galaxies in their sample, and proposed further that a single power-law with an exponent $-1.1 \pm 0.1$ for the cumulative XLF would be consistent with the combined data. However, these authors also pointed out that a broken power-law would provide an improved fit. The break-point suggested by these authors roughly agrees with the second break-point of Gilfanov. The power-law slopes given by Kim & Fabbiano (2004) for the differential XLF were $-0.8 \pm 0.2$ and $-1.8 \pm 0.6$ respectively, below and above this break. Thus, a broken power-law (with one or two breaks) generally seems to account for all observations of LMXB XLFs to date, and we shall use this as the template for our discussion here.

However, comparing these observed XLFs with our computed XLF is not completely straightforward, due to the complex nature of the XLFs. Note first that observed XLFs include NS as well BH binaries, and so extend up to luminosities $\sim 10^{39}$ erg/s. Systems beyond the second break in the above template are clearly BH systems, whereas their contributions below the second break, which need not be zero, are unknown. By contrast, the calculations presented by us in Paper II and in this work include only NS systems, so that a strong cut-off is expected and seen at the neutron-star Eddington luminosity. The power-law regime in our computed XLF is in the range $L_{36} = (3, 100)$, which roughly overlaps with the middle region between the two breaks in the observed “universal” XLF, which is $L_{36} = (20, 500)$. We note from Table I that our $\beta = 0$ case gives a power-law slope relatively close to that of the observed XLF of Gilfanov (2004), whereas the $\beta = -2.7$ case gives an XLF considerably steeper than the observed one. Our results for the $\beta = 0$ case also match reasonably with the results of Kim & Fabbiano (2004) for the single power-law fit to individual galaxies.
On the whole, it appears at this stage of comparison between observed and computed XLFs that $\beta = 0$ provides a better match than $\beta = -2.7$. However, it remains to be explored if different appropriate choices of other parameters might not give a power-law slope in better agreement with observations even for the latter value of $\beta$.

### 4.2. Conclusions

We remind ourselves that, while the computed XLF presented in this paper depends closely on the computed pre-LMXB PDFs presented in Paper II, pre-LMXBs are, by and large, not observable. Accordingly, the LMXB XLF comparisons presented in this paper (and possibly such comparisons of calculable distributions of other observable collective properties of LMXBs) serve as a major check on the above pre-LMXB PDFs. To the extent that even a qualitative account of observed LMXB XLFs is possible by our simple scheme, a certain measure of confidence in the inferred pre-LMXB PDFs of Paper II is gained from these investigations, which has implications for studies of related types of X-ray binaries.

We saw in the previous section that the giant fraction is an important parameter in determining the shape of the XLF at the bright end. We emphasize that this fraction is not a free parameter in our calculational scheme, set by hand. Rather, it is decided by the shape of the pre-LMXB PDF. Thus, a pre-LMXB PDF which gives a larger giant fraction produces a shallower XLF. At one remove, this corresponds to the inclusion of a higher fraction of wider pre-LMXBs with higher-mass companions. The choices made by us for primordial-binary parameters and subsequent evlutionary parameters in Paper II determined this fraction entirely, leaving no free parameters to adjust at the stage of computing the XLF.

A primordial-binary parameter which seems to be very important for its ultimate
influence on the XLF shape is that which controls the shape of the $q$-distribution, *viz.*, the parameter $\beta$. The reason for this can be traced to the influence of $\beta$ on the pre-LMXB PDF for both $M_s$ and $a$, as shown in Paper II (see Figs. 6 and 11 of that paper). Between the $q$-distribution and the SN-kick scenario, the former has by far the dominant influence on the pre-LMXB PDF, when other parameters are held fixed. It comes as no surprise, therefore, that $\beta$ should have a strong influence on the XLF shape. What we do find interesting is that, of the two values of $\beta$ used in previous works on the subject, one fares so much better than the other by the XLF criterion. However, we must remind ourselves that this has been shown to be true in this work only for the chosen values of the other parameters. It would be premature to take the results of this work as a definite indication that a flat $q$-distribution in primordial-binary distribution is always preferred, until the phase-space of the other parameters is thoroughly studied. However, to the extent that the values of the latter parameters are representative of actual pre-LMXBs, our result does seem suggestive.

By contrast, the SN-kick scenario has little influence on the XLF, as expected from the above arguments, since it has little effect on the pre-LMXB PDF, as shown in Paper II. Indeed, the only case in which it has any significant effect is that of the $a$-distribution in the $\beta = 0$ case (see Fig.11 of Paper II). But even this has little final influence on the XLF, since the $M_s$-distribution is much more effective in controlling the XLF than the $a$-distribution.

The calculations of various rates of the mass transfer and the orbital shrinking with our prescription of the mass loss depend crucially upon the assumption of *constant* Roche lobe contact during mass transfer and loss (if any). We have given arguments in favor of this assumption at appropriate places in this work. However, we stress here that this assumption should be made with appropriate timescales in mind, as we now explain. The typical timescales of the LMXB evolution are $\sim 2 - 3$ Gyrs. A timestep of $10^{-2}$ Gyrs is therefore sufficiently short for keeping an accurate track of the evolution of these systems,
and we have, of course, verified that finer time-steps give essentially the same result. This is tantamount to assuming that the orbit readjusts itself to Roche-lobe contact within the timescale of the step size assumed, i.e., $\sim 10^7$ yr, which appears to be a safe assumption.

With the cautionary remark that, since the falling-power-law region in our calculated XLF does not exactly overlap with the power-law region suggested by empirical fits to the observations, we should be careful about drawing conclusions, it does seem significant that a uniform primordial $q$-distribution does consistently lead to a better agreement with observations than the falling-power-law $q$-distribution with a slope of -2.7. With due caution, we therefore suggest that our results do indicate that a $\beta = 0$ distribution for LMXB-progenitor primordial binaries seems to be favored by the XLF observations. It would be interesting to see if the inclusion of black-hole systems leads to an exact match with the observations, which would be a rather gratifying confirmation of our suggestion for the underlying primordial $q$-distribution. We note, however, that the other parameters held fixed in this study, e.g., the CE parameter, may also have significant roles in determining the slope in this region. The value of $\gamma_{max}$ on the giant branch is also uncertain. However, low values of this parameter around 0.5 would decrease the effect of the giant populations and hence are less likely. It may also be not impossible that this parameter gradually decreases with time as the companion star evolves along the giant branch. As we saw earlier, the value $\gamma_{max} = 0.8$ produces a better fit with the data. This may thus represent an average value of the $\gamma_{max}$ over this entire evolution.

The problems of a proper representation of the LMXB XLF at the lowest-luminosity end are many, not the least of which is the possibility that the accretion paradigm in this region may be entirely different, e.g., ADAF (see Narayan & McClintock (2008); Lasota (2008) for recent reviews) or related flow models. Incorporation of such models into our scheme is a very ambitious task, since the relations between the mass-transfer
rate, the mass-accretion rate, and the output X-ray luminosity are thought to be very different in some of these models from the simple accretion-disk paradigm we have used here. Accordingly, we defer the inclusions of both black-hole systems and such accretion flow models to future works. We conclude by re-emphasizing that our effort in this series of papers must be regarded as an attempt at a proof-of-principles exploration of whether observed collective properties of X-ray binaries can be accounted for by evolving canonical collective properties of primordial binaries through well-accepted scenarios of individual binary evolution. Considering the simplicity of our scheme (for both HMXBs and LMXBs), the agreement for HMXBs was remarkable, and that for LMXBs, although not as precise, is still qualitatively in the correct direction. Thus encouraged, we feel justified in attempting more elaborate future explorations, which would incorporate detailed stellar-evolution models.
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