Greybody factors for topological massless black holes

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We study the greybody factors, the reflection and transmission coefficients for a non-minimally coupled massive scalar field in a $d$-dimensional topological massless black hole background in the zero-frequency limit. We show that there is a range of modes contributing to the absorption cross section, contrary to the current results where the mode with lowest angular momentum contributes alone to the absorption cross section.

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I. INTRODUCTION

The Hawking radiation is an important quantum effect in black hole physics but, this one is enigmatic because at quantum level black holes are not 'black' completely since these emit radiation with a temperature given by $\frac{h}{8\pi k_B GM}$, contrary to the classical context where it is believed that anything can escape from them. The originated thermal radiation at the black hole event horizon is emitted into surrounding spacetime with the consequence that the semiclassical approach for a black hole exhibit that it slowly loose its mass and eventually evaporates. At the event horizon the Hawking radiation is in fact blackbody radiation. However, this radiation still has to traverse a non-trivial curved spacetime geometry before it reaches after all to an observer where it is detected. The surrounding spacetime thus works as a potential barrier for the radiation giving a deviation from the blackbody radiation spectrum, as it is detected by an asymptotic observer.

The greybody factors are the probabilities for outgoing waves in the $\omega$-mode to reach infinity, the horizon which filter the initially blackbody spectrum emanating from the horizon. If one integrates the greybody factors over all spectra, the total black hole emission rate is obtained. Moreover, if they are constant the black hole emission spectrum would be exactly that of a blackbody radiation. This is the non-triviality of the greybody factor which leads to deviations of blackbody emissions and the consequent greybody radiation. Essentially, such radiation possesses a thermal character and inevitably black holes slowly evaporate. Furthermore, in the understanding of the event horizon for a black hole, the Hawking radiation plays an important role providing clues about the quantum structure of GR. In order to study the Hawking radiation we need to allow quanta to fall into the hole. The absorption cross section for low energy particles in $3+1$ dimensions consider a particle to be a massless minimally coupled scalar field. The cross section equals the area of the black hole. Additionally, it was shown that for all spherically symmetric black holes the low energy cross section for massless minimally coupled scalar fields is always the area of the horizon. Nevertheless, contrary to the existing results, we show in this work that there is a range of modes which contribute to the absorption cross section in the zero-frequency limit.

A well known fact to be mentioned that in four dimensions the Einstein tensor is the only symmetric and conserved tensor depending on the metric and its derivatives, which is linear in the second derivatives of the metric. The invariant action that gives rise to these fields equations is the Einstein-Hilbert action with cosmological constant $\lambda$. Alike, in higher dimensions the Lanczos-Lovelock (LL) action which is non-linear in the Riemann tensor, gives rise

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also to second-order field equations. Hence, for describing higher dimensional black holes this type of action results useful. An exhaustive analysis for asymptotically local AdS black holes geometries for nontrivial topologies of the transverse section was performed in [10].

The purpose of this work is to compute the greybody factors, the reflection and transmission coefficients for topological massless black holes with nontrivial topology of the transverse section asymptotically AdS in d dimensions. The paper is organized as follows. In Sec. II we provide mathematical preliminaries and we describe the background spacetime that we will use along the work. In Sec. III, we study the scalar perturbation of d-dimensional topological massless black holes. We find the associated greybody factors and the reflection and transmission coefficients in Sec. IV. Also, in this section we evaluate numerically our results, specialized to the 4-dimensional case. We finish with some comments and discuss the relevance of our results in Sec. V.

II. GRAVITY IN HIGHER DIMENSIONS

The Lanczos-Lovelock (LL) action is the outstanding extension of general relativity in d-dimensional space-times that leads to second order field equations for the metric [9]. It is given by

$$S_{LL}[g_{\mu\nu}] = \kappa \int \sum_{p=0}^{d} c_p L^p,$$

where $L^p = \epsilon_{\alpha_1...\alpha_d} R^{\alpha_1\alpha_2}...R^{\alpha_{2p-1}\alpha_{2p}} e^{\alpha_{2p+1}}...e^{\alpha_d}$, and $e^{\alpha}$ and $R^{\alpha\beta}$ stand for the vielbein and the curvature two-form $(\alpha, \beta = 0, 1, \ldots, d - 1)$, and $c_p = \frac{\kappa^{2(p-k)}}{d-2p} \binom{d}{k}$ for $p \leq k$ and it vanishes for $p > k$, with $1 \leq k \leq \left\lfloor \frac{d-1}{2} \right\rfloor$ ($\lfloor x \rfloor$ denotes integer part of $x$). The constants $\kappa$ and $\Lambda$ are related to the gravitational constant $G_k$ and the cosmological constant $k$ through

$$\kappa = \frac{1}{2(d-2)!\Omega_{d-2}G_k},$$

$$\Lambda = -\frac{(d-1)(d-2)}{2}\Omega_{d-2},$$

where $\Omega_{d-2}$ corresponds to the volume of the $(d-2)$-dimensional sphere.

The static black hole-like geometries possessing topologically non-trivial AdS asymptotic behaviors admitting a unique global vacuum were found in [10]. These theories and their corresponding solutions were classified by the integer $k$ which corresponds to the highest power of curvature into the LL Lagrangian. Such solutions describe a non-trivial $(d-2)$-dimensional transverse spatial section, $\Sigma_\gamma$. These surfaces are labelled by the constant $\gamma = +1, -1, 0$, depending on the curvature of the transverse section associated to a spherical, hyperbolic or plane section, respectively. The set of solutions describing a black hole in a free torsion theory, given by [10]

$$ds^2 = -\left[\gamma + \frac{r^2}{l^2} - \alpha \left(\frac{2\mu G_k}{r^{d-2k-1}}\right)^{\frac{1}{k-1}}\right]dt^2 + \frac{dr^2}{\gamma + \frac{r^2}{l^2} - \alpha \left(\frac{2\mu G_k}{r^{d-2k-1}}\right)^{\frac{1}{k-1}}} + r^2d\sigma_\gamma^2,$$

where $\alpha = (\pm 1)^{k+1}$ and the constant $\mu$ is related to the black hole horizon $r_+$ through

$$\mu = \frac{r_+^{d-2k-1}}{2G_k} \left(\gamma + \frac{r_+^2}{l^2}\right)^{k},$$

possesses an asymptotic behavior which is locally AdS for any topology of $\Sigma_\gamma$. On the other hand, $\mu$ is also related to the black hole mass $M$ by $\mu = \frac{\Omega_{d-2}M}{2G_k}$,Here, $\Sigma_{d-2}$ denotes the volume of the transverse space. The conditions that the metric (4) must fulfill in order to have an appropriate black hole solution have been extensively discussed in [10, 11].

III. SCALAR PERTURBATION FOR A d-DIMENSIONAL TOPOLOGICAL MASSLESS BLACK HOLE

If we specialize the solution (4) for $\mu = 0$, the horizon geometry is described by a negative constant curvature with $\gamma = -1$. In consequence the metric (4) reads

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2d\sigma^2,$$
where \( f(r) = -1 + \frac{2}{r} \) and \( dr^2 \) is the line element of a \((d - 2)\)-dimensional surface, \( \Sigma_{d-2} \). Clearly, this metric has a horizon at \( r_+ = l \). It was shown in \([10]\) that this solution not necessarily describe a black hole. In fact, if the transverse section \( \Sigma \) has the topology \( R^{d-2} \), the metric \([6]\) does not represent a black hole. It could be the case provided suitable identifications are performed on \( \Sigma_{d-1} \) \([12, 13]\). As mentioned in the Introduction, to gain insight into the quantum nature of black holes the kinematical properties provide relevant clues about their semiclassical aspects. In this spirit, the scalar perturbations on a massless black hole are dictated by a massive non-minimally coupled scalar field, \( \phi \), propagating in the vicinity of the massless black hole. The action governing the dynamics of the fields is

\[
S[g_{\mu\nu}, \phi] = S_{\text{La}} + \int d^dx \sqrt{-g} \left( \frac{1}{2} \partial^\mu \phi \partial^\nu \phi + \frac{1}{2} m^2 \phi^2 + \frac{1}{12} \zeta R \phi^2 \right),
\]

where \( \zeta \) is a parameter from the non-minimal coupling. The corresponding equation of motion for the scalar field is

\[
(\Box - m_{\text{eff}}^2) \phi = 0,
\]

where \( \Box = \frac{1}{\sqrt{-g}} \partial_{\alpha}(\sqrt{-g} g^{\alpha\beta} \partial_{\beta}) \) is the Laplace-Beltrami operator associated with the metric \([6]\) and \( m_{\text{eff}}^2 = m^2 - \zeta \frac{d(d-2)}{4r^2} \) plays the role of an effective mass for \( \phi \) where \( m \) is the mass of the scalar field and \( R = -d(d-1)r^{-2} \) is the scalar curvature \([11]\). Hence, in this fashion the equation of motion \([8]\) resembles a field equation for a minimally coupled scalar field. By means of the following ansatz

\[
\phi = \frac{U(r)}{r} Y(\Sigma_{d-2}) e^{-i\omega t},
\]

the radial part of \([8]\), in four dimensions reduces to a Schrodinger-like equation for a central potential function. Here, \( Y = Y(\Sigma_{d-2}) \) is a normalizable harmonic function on \( \Sigma_{d-2} \) satisfying \( \nabla^2 Y = -QY \) where \( \nabla^2 \) is the Laplace operator and \( Q = \left( \frac{d^3}{4r^2} \right)^2 + \xi^2 \), and \( \xi \) is any real number \([14]\) and \( \omega \) is the frequency of the wave. The radial function \( U(r) \) satisfies

\[
f(r) \left\{ f(r) \frac{d^2}{dr^2} + f(r) \left[ \frac{d - 4}{r} + \frac{f'(r)}{f(r)} \right] \frac{d}{dr} - \frac{d - 4}{r^2} f(r) - \frac{f'(r)}{r} - \frac{Q}{r^2} - m_{\text{eff}}^2 \right\} U(r) + \omega^2 U(r) = 0,
\]

where \( f'(r) = \frac{df}{dr} \). By introducing the tortoise coordinate \( r_+ = r_+(r) \), given by \( dr_+ = \frac{df}{r f} \), the latter equation is rewritten as one-dimensional Schrodinger equation,

\[
\left[ \frac{d^2}{dr_+^2} + \omega^2 - V_{\text{eff}}(r) \right] U(r_+) = 0,
\]

where we can read off immediately the effective potential

\[
V_{\text{eff}}(r) = f(r) \left[ m_{\text{eff}}^2 + \frac{Q}{r^2} + \frac{f'(r)}{r} \right].
\]

This potential is depicted in Fig. \([11]\). In connection with \( f(r) \), explicitly the tortoise coordinate is given by \( r_+ = -l \arctanh \left( \frac{z}{l} \right) = -r_+ \arctanh \left( \frac{z}{r_+} \right) \). With order to solve analytically the wave equation, the change of variables, \( z = 1 - t^2/r^2 \) and \( t = lt \) result useful \([11]\). By using the ansatz \( \varphi = R(z) Y(\Sigma) e^{-i\omega t} \), the radial function obeys the following differential equation

\[
\left\{ z(1-z) \frac{d^2}{dz^2} + \left[ 1 + \left( \frac{d-5}{2} \right) z \right] \frac{d}{dz} + \left[ \frac{\omega^2}{4z} - \frac{Q}{4} - \frac{m_{\text{eff}}^2 z^2}{4(1-z)} \right] \right\} R(z) = 0.
\]

Assuming that \( R(z) = z^a(1-z)^b K(z) \), Eq. \([13]\) yields

\[
z(1-z)K''(z) + [c - (1+a+b)z]K'(z) - abK(z) = 0,
\]

whose solution is given in terms of hypergeometric functions \([15]\)

\[
K(z) = C_1 F(a, b, c, z) + C_2 z^{1-c} F(a - c + 1, b - c + 1, 2 - c, z),
\]

where \( F(a, b, c, z) \) is the hypergeometric function with parameters \( a, b, c \) and argument \( z \).
with $C_1$ and $C_2$ being constants. The hypergeometric coefficients, $a, b$ and $c$ are defined as follows

\[
  a = -\left(\frac{d-3}{4}\right) + \alpha + \beta \pm \frac{i}{2} \xi , \\
  b = -\left(\frac{d-3}{4}\right) + \alpha + \beta - \frac{i}{2} \xi , \\
  c = 1 + 2\alpha ,
\]

with $C$ being a non-integer, and

\[
  \alpha = \pm \frac{\omega}{2}, \quad \beta = \beta \pm \frac{1}{2} \sqrt{\left(\frac{d-1}{2}\right)^2 + m_{\text{eff}}^2 r^2} .
\]  

Without loss of generality we choose the negative sign for $\alpha$. One interesting feature to notice is that the function \[14\] has three regular singular points at $z = 0, z = 1$ and $z = \infty$. Then, the solution $R(z)$ is

\[
  R(z) = C_1 z^{\alpha} (1 - z)^{a} F_1(a, b, c; z) + C_2 z^{-\alpha} (1 - z)^{b} F_1(a - c + 1, b - c + 1, 2 - c; z) .
\]  

Notice that in the neighborhood of the horizon, $z = 0$, by using the property $F(a, b, c, 0) = 1$ \[15\], the function $R(z)$ acquires the form $R(z) = C_1 e^{\alpha \ln z} + C_2 e^{-\alpha \ln z}$. Therefore, the scalar field $\phi$ behaves as

\[
  \phi \sim C_1 e^{-i\omega \left(t + \frac{1}{2} \ln z\right)} + C_2 e^{-i\omega \left(t - \frac{1}{2} \ln z\right)} .
\]  

This expression is quite general as it follows from \[15\] and notice that $\phi$ represents both ingoing and outgoing waves. To be able to interpret to the scalar field as being only ingoing waves at the horizon the constant $C_2$ must be eliminated. The general radial solution with boundary conditions at the horizon can then be written as

\[
  R(z) = C_1 e^{-i\omega \ln z} (1 - z)^{a} F_1(a, b, c; z) .
\]  

In order the implement suitable boundary conditions at infinity ($z = 1$) for the solution \[18\] we find convenient to use the Kummer’s relation for the hypergeometric functions (see for example, \[13\]). The radial function is therefore given by

\[
  R(r) = C_1 \left[ \left(\frac{r+}{r^-}\right)^{2\beta} \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} + \left(\frac{r^-}{r+}\right)^{d-1-2\beta} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} \right] ,
\]

where we have used the fact that $1 - z = \frac{l^2}{r^2} = \frac{r^2}{r^2}$ besides the limit of $R(z)$ when $z \to 1$. 

![FIG. 1: $V_{\text{eff}}$ vs $r$; $d = 4$, $l = 1$ and $m_{\text{eff}} = 0$.](image_url)
Another way at looking at the radial solution when \( r \to \infty \) at the asymptotic region, is from the wave equation

\[
R''(r) + \frac{d}{r}R'(r) + \frac{l^2}{r^2} \left( \frac{\omega^2}{r^2} - \frac{Q}{r^2} - m_{\text{eff}}^2 \right) R(r) = 0 ,
\]

where we have used the asymptotic behavior of \( f(r) \) and \( f'(r) \), and the ansatz \( \phi = R(r)Y(\sum_{d-2})e^{-\omega t} \) with \( R'(r) = \frac{dR}{dr} \). The solution to this equation is given in terms of Bessel functions

\[
R(r) = \left( \frac{\sqrt{A}}{2r} \right)^{\frac{d-1}{2}} \left[ D_1 \Gamma(1 - C)J_{-C} \left( \frac{\sqrt{A}}{r} \right) + D_2 \Gamma(1 + C)J_C \left( \frac{\sqrt{A}}{r} \right) \right] ,
\]

where

\[
A = l^2(\omega^2 - Q) = r_+^2(\omega^2 - Q) ,
\]

\[
C = \frac{1}{2} \sqrt{(d-1)^2 + 4m_{\text{eff}}^2} ,
\]

where \( D_1 \) and \( D_2 \) are integration constants. It is straightforward to simplify the radial solution by using the expansion of the Bessel function for small arguments, namely

\[
J_n(x) = \frac{x^n}{2^n \Gamma(n + 1)} \left\{ 1 - \frac{x^2}{2(2n + 2)} + ... \right\} , \quad \text{for } x \ll 1 .
\]

A short calculation shows that the asymptotic radial solution exhibit the polynomial form

\[
R_{\text{asympt}}(r) = \tilde{D}_1 \left( \frac{1}{r} \right)^{\frac{d+1}{2} - C} + \tilde{D}_2 \left( \frac{1}{r} \right)^{\frac{d+1}{2} + C} ,
\]

where we have introduced the constants \( \tilde{D}_1 = D_1 \left( \frac{\sqrt{A}}{2} \right)^{\frac{d+1}{2} - C} \) and \( \tilde{D}_2 = D_2 \left( \frac{\sqrt{A}}{2} \right)^{\frac{d+1}{2} + C} \), also we used \( \sqrt{A} \ll 1 \). In Ref. [16], it was discussed that a scalar field with asymptotic behavior, similar that Eq. (22), generically leads to an unstable state (the so called, big crunch singularity) which is a clearly sign of nonlinear instability. However, in order to induce such instability, this modify the boundary conditions that scalar field must satisfy at infinity, Ref. [17]. Although the modified boundary conditions preserve the full set of asymptotic AdS symmetries, and allow for a finite conserved energy to be defined, this energy can be negative. We notice, that the imposition of regularity condition on the radial function (22) at the infinity implies \( \frac{d-1}{2} - C \geq 0 \) or \( \frac{-(d-1)^2}{4} \leq m_{\text{eff}}^2 l^2 \leq 0 \). This is in agreement with the condition for any effective mass in order to have a stable asymptotic AdS spacetime in \( d \) dimensions, \( m_{\text{eff}}^2 l^2 \geq \frac{-(d-1)^2}{4} \) [17] which sets requirements on the nonminimal coupling constant, once the bare mass of the scalar field and the dimensions are fixed. Besides, \( a + b - c = -C \), for \( \beta = \beta_+ \), and \( c - a - b = -C \), for \( \beta = \beta_- \). For this reason \( C \) can not be an integer, because the gamma function is singular at that point and the regularity conditions are not satisfied. We now take advantage of the inherent symmetry that the radial solution possesses in the asymptotic region. More specifically, we have the freedom to choose the form of the constant \( \beta \) since by changing \( \beta_+ \) to \( \beta_- \) this solution is unchanged. Comparison of Eqs. (19) and (22), regarding \( \beta = \beta_- \), allows us to immediately to read off the coefficients \( \tilde{D}_1 \) and \( \tilde{D}_2 \),

\[
\tilde{D}_1 = C_1 r_+^{2d - \frac{d-1}{2}} \frac{\Gamma(c) \Gamma(c - a - b)}{\Gamma(c - a) \Gamma(c - b)} , \quad \tilde{D}_2 = C_1 r_+^{2d - 2\beta_- + \frac{d-1}{2}} \frac{\Gamma(c) \Gamma(a + b - c) \Gamma(c)}{\Gamma(a) \Gamma(b)} .
\]

IV. REFLECTION AND ABSORPTION COEFFICIENTS. ABSORPTION CROSS SECTION IN A \( d \)-DIMENSIONAL TOPOLOGICAL MASSLESS BLACK HOLE

The reflection and absorption coefficients, \( \mathbb{R} \) and \( \mathbb{I} \), respectively, are defined by

\[
\mathbb{R} := \frac{F_{\text{out \ asympt}}}{{F_{\text{in \ asympt}}}} \quad \mathbb{I} := \frac{F_{\text{in \ asympt}}}{{F_{\text{in \ asympt}}}} ,
\]

where
where $F(r)$ is the conserved flux defined by \[ F = \frac{\sqrt{-g g^{rr}}}{2i} (R^* \partial_r R - R \partial_r R^*) \] (25)

$R$ being the radial solution of the wave equation \[ Eq. (8) \] and $i$ is the complex unity and $*$ stands for complex conjugation. According to our development the behavior of the flux $F(r)$ at the horizon is obtained by the introduction of Eq. (18) into Eq. (25). Thus, up to an irrelevant factor coming from angular part of the solution, the flux at the horizon is given by

$$F_{\text{hor}}^n = -|C_1|^2 \omega l^{d-3}. \quad (26)$$

Now, by inserting Eq. (22) into Eq. (25), a similar computation leads us to obtain the flux at the asymptotic region

$$F_{\text{asymp}} = -iC\left(\frac{1}{l^2} - \frac{1}{r^2}\right) \left(\hat{D}_1^2 \hat{D}_1 - \hat{D}_1^*_2 \hat{D}_2\right). \quad (27)$$

Nevertheless, the distinction between the ingoing and outgoing fluxes at the asymptotic region is a non trivial task because the spacetime is asymptotically AdS. In order to characterize the fluxes we find convenient to split up the coefficients $\hat{D}_1$ and $\hat{D}_2$ in terms of the incoming and outgoing coefficients, $D_{\text{in}}$ and $D_{\text{out}}$, respectively. Making the partition $\hat{D}_1 = D_{\text{in}} + D_{\text{out}}$ and $\hat{D}_2 = i\hbar(D_{\text{out}} - D_{\text{in}})$ with $\hbar$ being a dimensionless constant which will be assumed to be independent of the energy $\omega$. [19] [22]. If we claim physical meaning for the coefficients under study we need specific values for the parameter $\hbar$. We will come back at this point in the last section. In this way the asymptotic flux Eq. (27) becomes

$$F_{\text{asymp}} \approx \frac{2\hbar C}{l^2} \left(|D_{\text{in}}|^2 - |D_{\text{out}}|^2\right). \quad (28)$$

Therefore, the coefficients Eq. (24) are given by

$$\Re = \frac{|D_{\text{out}}|^2}{|D_{\text{in}}|^2}, \quad (29)$$

$$\Im = \frac{\omega |D_{\text{in}}|^2 C_1^2}{2 |\hbar C |D_{\text{in}}|^2}, \quad (30)$$

where the coefficients $D_{\text{in}}$ and $D_{\text{out}}$, are expressed as

$$D_{\text{in}} = \frac{C_1}{2} \left[ \frac{2\beta}{r^2} \Gamma(c)\Gamma(c-a-b) + \frac{i}{\hbar} \frac{\omega^{d-1-2\beta} \Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} \right], \quad (31)$$

$$D_{\text{out}} = \frac{C_1}{2} \left[ \frac{2\beta}{r^2} \Gamma(c)\Gamma(c-a-b) + \frac{i}{\hbar} \frac{\omega^{d-1-2\beta} \Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} \right]. \quad (32)$$

On the other hand, the absorption cross section, or greybody factor $\sigma_{\text{abs}}$, is given by

$$\sigma_{\text{abs}} = \frac{\Im}{\omega} = \frac{\omega^{d-1} |C_1|^2}{2 |\hbar C |D_{\text{in}}|^2}. \quad (33)$$

### A. 4-dimensional case

If we restrict our general results developed above to the 4-dimensional case, we minimize the amount of formalism. We focus mainly on the radial solutions for the wave equation \[ Eq. (8) \] in order to obtain the scalar perturbations on a 4-dimensional massless black hole. For such a case ($d = 4$), from \[ Eq. (19) \] and \[ Eq. (22) \] we have immediately the radial solutions with their corresponding behaviors

$$R(r) = C_1 \left[ \frac{r_+^{2\beta}}{r} \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} + \frac{r_+^{3-2\beta}}{r} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} \right], \quad (34)$$
FIG. 2: Reflection coefficient $v/s \omega$; $d = 4, m^2_{\text{eff}}l^2 = 0, l = 1$ and $\xi = 0$.

FIG. 3: Transmission coefficient $v/s \omega$; $d = 4, m^2_{\text{eff}}l^2 = 0, l = 1$ and $\xi = 0$.

FIG. 4: Absorption Cross Section $v/s \omega$; $d = 4, m^2_{\text{eff}}l^2 = 0, l = 1$, and $\xi = 0$. 
FIG. 5: Physical conditions $\Re + \Im$ is plotted against $\omega$; $d = 4$, $m^2_{\text{eff}}l^2 = 0$, $l = 1$, $\xi = 0$ and $h = -1, -2, -3, -4$. This figure shows us the physical requirement is satisfied for negative values of parameter $h$.

FIG. 6: Reflection coefficient v/s $\omega$; $d = 4$, $m^2_{\text{eff}}l^2 = 0$, $l = 1$ and $h = -1$.

FIG. 7: Transmission coefficient v/s $\omega$; $d = 4$, $m^2_{\text{eff}}l^2 = 0$, $l = 1$ and $h = -1$. 
FIG. 8: Absorption Cross Section $\nu/s$ $\omega$; $d = 4$, $m_{eff}^2 l^2 = 0$, $l = 1$ and $h = -1$.

FIG. 9: Reflection coefficient $\nu/s$ $\omega$; $m_{eff}^2 l^2 = -2, -1.5, -0.5, 0$, $l = 1$, $h = -1$, $\xi = 0$ and $d = 4$.

FIG. 10: Transmission coefficient $\nu/s$ $\omega$; $m_{eff}^2 l^2 = -2, -1.5, -0.5, 0$, $l = 1$, $h = -1$, $\xi = 0$ and $d = 4$. 
Absorption Cross Section

\[ \cdots \quad m^2_l=0 \]
\[ \cdots \quad m^2_l=-0.5 \]
\[ \cdots \quad m^2_l=-1.5 \]
\[ \cdots \quad m^2_l=-2 \]

FIG. 11: Absorption Cross Section v/s \( \omega \); \( m^2_l= -2, -1.5, -0.5, 0, l = 1, h = -1, \xi = 0 \) and \( d = 4 \).

\[ R_{\text{asymp}}(r) = \hat{D}_1 \left( \frac{1}{r} \right)^{\frac{3}{2} - C} + \hat{D}_2 \left( \frac{1}{r} \right)^{\frac{3}{2} + C}, \] (35)

Note that the radial function \( \hat{D}_1 \) satisfies the regularity condition at infinite if \( \frac{3}{2} - C \geq 0 \) or \( \frac{9}{4} \leq m^2_{nl} \leq 0 \). This condition is in agreement with the Breitenlohner-Freedman bound for the positivity of energy in global AdS\(_4\), \( \[23, 24\], \) \( m^2_{nl} \geq -\frac{9}{4} \), which sets requirements on the nonminimal coupling constant once the bare mass of the scalar field and the dimensions are fixed. Besides, \( a + b - c = -C \), for \( \beta = \beta_- \), and \( c - a - b = -C \), for \( \beta = \beta_+ \). This is the reason why \( C \) can not be an integer because the gamma function is singular at that point and the regularity conditions are not fulfilled. On the other hand, the solution given by Eq. (35) is symmetric by changing \( \beta_- \) to \( \beta_+ \). This leads us to consider \( \beta_- \) without loss of generality. The comparison between radial solutions becomes

\[ \hat{D}_1 = C_1 r_+^{2\beta_-} \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}, \quad \hat{D}_2 = C_1 r_+^{3-2\beta_-} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)}, \] (36)

with \( C_1 \) being an integration constant. Now, with respect to the reflection and absorption coefficients in four dimensions we have straightforwardly

\[ \Re = \frac{|D_{\text{out}}|^2}{|D_{\text{in}}|^2} \quad \text{and} \quad \Im = \frac{\omega l^3 |C_1|^2}{2 |h| C |D_{\text{in}}|^2}, \] (37)

where the coefficients are given by (31) and (32) specialized to \( d = 4 \). In closing this section, we have the corresponding greybody factor

\[ \sigma_{\text{abs}} = \frac{\Im}{\omega} = \frac{l^3 |C_1|^2}{2 |h| C |D_{\text{in}}|^2}. \] (38)

A numerical analysis of the coefficients is depicted in Figs. (2-11).

V. DISCUSSIONS AND COMMENTS

In this paper we have computed the greybody factors, the reflection and transmission coefficients for topological massless black holes in arbitrary dimensions. The involved physical content by the coefficients is more suitable viewed when we evaluate them numerically in four dimensions. Our numerical analysis needed of specific choices for the aforementioned constant \( h \). To this respect we made allowance for the currently accepted discussion for the selection of the parameter \( h \). On the one hand, the constant \( h \) can be chosen conveniently in such a manner that the absorption cross section can be expressed by the area of horizon in the zero-frequency limit \( [8, 19] \). On the other hand, it can be chosen also, in order to obtain the correct value of the Hawking temperature \( [20] \), besides that of completeness where is necessary to assure that the sum of the reflection and transmission coefficients becomes the unity \( [25] \). In
addition, it has been reported that this freedom in the choice of $h$ as a numerical factor is usually set up by imposing appropriate physical conditions [21]. Thus, according to our concern we proceeded to choose the parameter $h$ in such a way that either the values of the greybody factors, reflection and transmission coefficients represent an acceptable physical situation. In this sense we employed $h$ as a free parameter and we used it to plot the reflection coefficient (see Fig. (2)), transmission coefficient (see Fig. (3)) and the greybody factors (see Fig. (4)), for some values of $m_{\text{eff}}, l$ and $\xi$. We found that negative values for the parameter $h$ provide physical meaning whereas in the positive case some of the coefficients under study become divergent. We observed also that the parameter $h$ must be bounded. It results smaller than zero and greater than some other value, such that the absorption cross section or the greybody factor become real in the zero-frequency limit. Likewise, in this range the greybody factor is such that this coefficient is increasing if the parameter $h$ is increasing, (see Fig. (1)). Besides, for completeness in our description we have plotted the condition $\Re + i \xi$, Fig. (5), for $m_{\text{eff}}^2 l^2 = 0$, $l = 1$, $\xi = 0$ and $h = -1, -2, -3, -4$. This condition is satisfied being equal to the unity. On the other hand, as mentioned previously, in four dimensions, $-\frac{3}{4} \leq m_{\text{eff}}^2 l^2 \leq 0$ and $C$ can not be an integer or equivalently $m_{\text{eff}}^2 l^2 \neq -\frac{3}{4}, -\frac{5}{4}$, because for these values the regularity conditions are not fulfilled. Therefore, we consider without loss of generality, $m_{\text{eff}}^2 l^2 = 0$, $l = 1$ and $h = -1$, as a fixed parameter useful to analyze the behavior of all the coefficients in four dimensions. Along these lines of reasoning, for the values $\xi = 0, 1, 1.5, 2$ we have in Figs. (6), (7) and (8) the reflection and transmission coefficients as well as the greybody factors, respectively. We point out the existence of a minimum and maximum point for the reflection and transmission coefficients. We note further that for these coefficients we have two branches. In the reflection case, one of them is decreasing for low frequencies and the other one is increasing. For the transmission case the behavior is contrary to the latter, increasing and then decreasing. This offer to us with some valuable insight about the existence of one optimal frequency to transfer energy out of the bulk. Additionally, we found that negative values for the parameter $h$ provide physical meaning whereas in the positive case some of the coefficients under study become divergent. We observed also that the parameter $h$ must be bounded. It results smaller than zero and greater than some other value, such that the absorption cross section or the greybody factor become real in the zero-frequency limit. Likewise, in this range the greybody factor is such that this coefficient is increasing if the parameter $h$ is increasing, (see Fig. (1)). Besides, for completeness in our description we have plotted the condition $\Re + i \xi$, Fig. (5), for $m_{\text{eff}}^2 l^2 = 0$, $l = 1$, $\xi = 0$ and $h = -1, -2, -3, -4$. This condition is satisfied being equal to the unity. On the other hand, as mentioned previously, in four dimensions, $-\frac{3}{4} \leq m_{\text{eff}}^2 l^2 \leq 0$ and $C$ can not be an integer or equivalently $m_{\text{eff}}^2 l^2 \neq -\frac{3}{4}, -\frac{5}{4}$, because for these values the regularity conditions are not fulfilled. Therefore, we consider without loss of generality, $m_{\text{eff}}^2 l^2 = 0$, $l = 1$ and $h = -1$, as a fixed parameter useful to analyze the behavior of all the coefficients in four dimensions. Along these lines of reasoning, for the values $\xi = 0, 1, 1.5, 2$ we have in Figs. (6), (7) and (8) the reflection and transmission coefficients as well as the greybody factors, respectively. We point out the existence of a minimum and maximum point for the reflection and transmission coefficients. We note further that for these coefficients we have two branches. In the reflection case, one of them is decreasing for low frequencies and the other one is increasing. For the transmission case the behavior is contrary to the latter, increasing and then decreasing. This offer to us with some valuable insight about the existence of one optimal frequency to transfer energy out of the bulk. Additionally, we found that there is a range of values of $\xi$ that contribute to the greybody factor (see Fig. (8)) in the zero-frequency limit, contrary to the case studied by Dass, Gibbons and Matur [8], where the mode with lowest angular momentum contribute at the absorption cross section, in that limit. This reason does not allow to fix the value of $h$. We would like to mention further that we notice the effect of the $m_{\text{eff}}$ on the coefficients, (see Figs. (9), (10) and (11)), where the absorption cross section decreases in the zero-frequency limit when $m_{\text{eff}}$ increases. Finally, we mention that the case $\mu \neq 0$ is a rather involved computation and we are currently working on this point. This will be reported elsewhere. 

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