Hysteresis effect in $\nu = 1$ quantum Hall system under periodic electrostatic modulation

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The effect of a one-dimensional periodic electrostatic modulation on quantum Hall systems with filling factor $\nu = 1$ is studied. We propose that, either when the amplitude of the modulation potential or the tilt angle of the magnetic field is varied, the system can undergo a first-order phase transition from a fully spin-polarized homogeneous state to a partially spin-polarized charge-density-wave state, and show hysteresis behavior of the spin polarization. This is confirmed by our self-consistent numerical calculations within the Hartree-Fock approximation. Finally we suggest that the $\nu = 1/3$ fractional quantum Hall state may also show similar hysteresis behavior in the presence of a periodic potential modulation.

The discovery of the integer and fractional quantum Hall (QH) effects offered invaluable tools to study quantum phase transitions in low dimensions. For example, the translational-invariant QH phases can exhibit novel forms of two-dimensional ferromagnetism, and show interesting magnetic transitions. Recently, the hysteresis phenomena reminiscent of conventional ferromagnetic materials are discovered in many QH systems. For instance, in close proximity to the critical pressure necessary for the transition from the spin-polarized state to the spin-unpolarized state, a hysteretic evolution of the magnetoresistance is observed in the vicinity of even-numerator fractional quantum Hall states. Furthermore, by applying a gate bias, hysteresis behavior of the longitudinal resistivity is observed in a wide quantum well at even-integer QH states. The physical origin of these observed hysteresis may be associated with the crossing of Landau levels for electrons (or for composite fermions in the case of the fractional QH states) with different spin polarizations.

Besides the hydrostatic pressure and the gate bias, modern techniques allow us to introduce other external perturbations to QH systems, such as a lateral periodic electrostatic potential and/or a periodic magnetic field. QH systems under periodic modulations, either one-dimensional or two-dimensional, have been studied to a great extent. Recently, Manolescu and Gudmundsson have theoretically studied a QH system at non-integer filling factor $1/(\nu + 1)$ under periodic modulations. By varying either the amplitude of the modulation or the tilt angle of the magnetic field with its normal component $B_0$ being fixed, they found a hysteretic evolution of the ground state due to the combined effects of the external potential and the exchange interaction. It is interesting to study if similar hysteresis effects can be found in other situations.

In this paper, we consider the $\nu = 1$ QH system in a single layer under a one-dimensional periodic modulation. For the system without external periodic modulation, it is well-known that, even when the Zeeman splitting is negligible, because of the strong exchange interaction among electrons, the ground state is a homogeneous, fully spin-polarized incompressible liquid. However, Bychkov et al. found that, when the mixing between the Landau levels is neglected, the presence of a one-dimensional periodic modulation could diminish the excitation energy of spin excitons with finite momentum to zero. Due to these gapless spin excitons, one may expect that the system can transit from a fully spin-polarized homogeneous state to a partially spin-polarized charge-density-wave (CDW) state. While Bychkov et al. suggested this possibility, they did not pursue this issue further, because their approach is applicable only prior to the occurrence of this instability.

We notice that the proposed instability is very likely a first-order phase transition accompanying hysteresis phenomena. This can be understood as follows. First, this instability is related to the crossing of the single-particle Landau bands with different spin polarizations. Similar picture has been used to explain the observed hysteresis due to the application of the hydrostatic pressure or the gate bias. It indicates that the proposed transition could be first-order and show a hysteresis effect. Second, it is expected that, by varying the modulation amplitude, the evolution of the ground state could be history-dependent. If the system is originally in the uniform fully spin-polarized state, as the modulation amplitude increases, in order to minimize the external potential energy, electrons tend to accumulate at the local minima of the external modulation potential. However, this process must accompany with spin flipping and therefore will cost exchange energy among the spin-up electrons. When the cost in exchange energy is larger than the reduction in the external potential energy, the uniform fully spin-polarized state is maintained even under a modest modulation. The rigidity of the uniform phase breaks down only when the modulation strength exceeds a critical value, and a sudden transition to a partially spin-polarized state ensues. In contrast, once the system is in a partially spin-polarized state (now electrons with both spin orientations are present), when the modulation strength is reduced (the local minima become shallower), minority-spin (spin-down) electrons will be squeezed out of the potential minima one by one. Note that, in this
case, when a minority-spin electron is pushed back to the nearby unoccupied majority-spin (spin-up) state due to the reduction of the modulation amplitude, the gain in exchange energy among the spin-up electrons is roughly the same order as the cost among the spin-down electrons. That is, the effect of the exchange interaction is minor in the present case. Therefore, both the evolution of the ground state and the change of the population are mainly determined by the external modulation potential, and they change gradually as the modulation is varied slightly. Hence the fully spin-polarized state will not immediately be restored even though the modulation strength is decreased below the previous critical value. Only when the modulation strength is sufficiently small, such that the difference of the populations between the spin-up and spin-down states exceeds a critical amount, will the spin gap be abruptly amplified, and the fully polarized state be recovered. This explains why a hysteretic effect appears in the present system. The above argument is confirmed by our calculations within the Hartree-Fock approximation (HFA). We find that, either by changing the amplitude of the modulation potential or the tilt angle of the magnetic field, hysteresis behavior of the spin polarization will occur in the modulated $\nu = 1$ QH system.

By using the Landau gauge and neglecting the Landau level mixing, which is valid when the cyclotron energy $\hbar \omega_c$ is much larger than the typical Coulomb energy, the Hamiltonian of a modulated QH system under a strong perpendicular magnetic field $B_0$ can be expressed in the form

$$H = \sum_{X,\alpha} (-\frac{\alpha}{2} \Delta^{(0)}(0) - \mu) c^\dagger_{X,\alpha} c_{X,\alpha} + H_M + H_C,$$

$$H_M = \sum_{X,\alpha} V_0 e^{-g(|\alpha|)/2} \cos(G_0 X) c^\dagger_{X,\alpha} c_{X,\alpha},$$

where $c^\dagger_{X,\alpha}$ is the creation operator of an electron in the lowest Landau level (LLL) with a guiding center coordinate $X$ and spin $\alpha$ ($\alpha = \pm 1$). $\Delta^{(0)}(0)$, $\mu$, and $l \equiv \sqrt{\hbar/eB_0}$ are the Zeeman energy, the chemical potential, and the magnetic length, respectively. Here we take $\hbar \omega_c/2$ as the zero of energy. In the present investigation, we consider the simplest model of a one-dimensional electrostatic modulation $V(x) = V_0 \cos(G_0 x)$ with a period $a = 2\pi/G_0$. Finally, the Coulomb interaction gives a contribution $H_C$,

$$H_C = \frac{1}{2} \sum_{\{X_1,\alpha,\beta\}} \langle X_1, X_2 | X_3, X_4 \rangle c^\dagger_{X_1,\alpha} c^\dagger_{X_2,\beta} c_{X_3,\beta} c_{X_4,\alpha},$$

$$\langle X_1, X_2 | X_3, X_4 \rangle = \frac{1}{A} \sum_{\mathbf{q}} v(\mathbf{q}) e^{-(\mathbf{q})^2/2} e^{i\mathbf{q} \cdot (X_1 + X_2)} e^{-i\mathbf{q} \cdot (X_2 + X_3)}/2 \delta_{X_1, X_3 + \mathbf{q} \cdot \mathbf{r}} \delta_{X_2, X_3 - \mathbf{q} \cdot \mathbf{r}},$$

where $A$ is the area of the system and $v(\mathbf{q}) = 2\pi e^2/\kappa|\mathbf{q}|$ is the Fourier transform of the Coulomb potential with $\kappa$ being the dielectric constant.

In the spirit of the HFA, the model Hamiltonian becomes

$$H_{HF} = \sum_{X,\alpha} (\varepsilon_{X,\alpha} - \mu) c^\dagger_{X,\alpha} c_{X,\alpha},$$

where the single-particle energies under the HFA are

$$\varepsilon_{X,\alpha} = -\frac{\alpha}{2} \Delta^{(0)}(0) + V_0 e^{-g(|\alpha|)/2} \cos(G_0 X) + \sum_{G_j} W^\alpha_0(G_j) e^{-iG_j \cdot X},$$

with $\{G_j = jG_0, j = 0, \pm 1, \pm 2 \cdots \}$ being the reciprocal lattice vectors of the one-dimensional periodic structure. The Hartree-Fock effective interaction potential $W^\alpha_0(G)$ can be split into a direct $[H_0(G)]$ and an exchange $[X_0(G)]$ component:

$$W^\alpha_0(G) = \frac{e^2}{\mathbf{r}!} \sum_{\beta} [H_0(G) - \delta_{\alpha,\beta} X_0(G)] \left\langle \rho^\beta_0(-G) \right\rangle$$

with

$$H_0(G) = \frac{1}{|G|} e^{-(G_0)^2/2} \left(1 - \delta_{G,0}\right),$$

$$X_0(G) = \sqrt{\frac{\pi}{2}} e^{-(G_0)^2/4} I_0 \left[ \frac{(G_0)^2}{4} \right],$$

$$\left\langle \rho^\beta_0(-G) \right\rangle = \frac{1}{N_\varphi} \sum_X e^{iG X} \left\langle c^\dagger_{X,\beta} c_{X,\beta} \right\rangle,$$

where $I_0(x)$ is the modified Bessel function and $N_\varphi$ is the Landau level degeneracy. The factor $(1 - \delta_{G,0})$ in the expression of $H_0(G)$ is due to the neutralizing positive background. From Eq. (2), we find that the modulation lifts the degeneracy of the Landau levels and the resulting single-particle energies have a periodic structure, $\varepsilon_{X,\alpha} = \varepsilon_{X+\mathbf{r},\alpha}$. Within the HFA, the thermal expectation value $\left\langle c^\dagger_{X,\alpha} c_{X,\alpha} \right\rangle = f(\varepsilon_{X,\alpha} - \mu)$, where $f(x)$ is the Fermi-Dirac distribution function. The condition of $\nu = 1$ is given by $(1/N_\varphi) \sum_{X,\alpha} \left\langle c^\dagger_{X,\alpha} c_{X,\alpha} \right\rangle = 1$. Eqs. (3)-(4), together with this condition, form the basis of our self-consistent scheme.
Before starting the numerical calculations, we notice that, for ν = 1, the Hamiltonian within the LLL approximation [Eqs. (1)] is invariant under the following particle-hole transformation,

\[ c_{X,\alpha} \rightarrow c_{X+a/2,-\alpha} \quad \text{and} \quad c_{X,\alpha} \rightarrow c_{X+a/2,-\alpha}^\dagger. \]  

(11)

By using this particle-hole symmetry, one leads to an exact expression of µ at all temperatures, µ = −ε_e/2, where ε_e = \sqrt{π/2} (\epsilon^2/\kappa l) is the exchange energy per electron of the uniform fully polarized state. For simplicity, in the following, the units of length and energy are taken to be the magnetic length l and ε_e, respectively.

Thus the critical value of \( V_0 \) is given by the right-hand side of the above inequality. For \( \Delta_z^{(0)} = 0.03 \), the critical value is 0.568 (0.538) for \( a = 10 \) (\( a = 15.7 \)), which agrees with the result in Fig. 1. Moreover, from Fig. 1 we find that the larger the period \( a \) is, the smaller the hysteresis loop becomes. Thus our result implies that the hysteresis loop will no longer exist for a long enough period.

![FIG. 1. Evolution of the spin polarization for \( a = 10 \) (solid line) and \( a = 15.7 \) (dashed line) at \( T = 0 \).](image)

We consider two modulation periods, \( a = 10 \) and \( a = 15.7 \) (i.e., \( G_0 = 0.4 \)), with \( \Delta_z^{(0)} = 0.03 \). The period \( a = 15.7 \) is chosen in order to compare our result with that in Ref. 11. We begin with the potential amplitude \( V_0 = 0 \), and find the self-consistent solution by iteration, starting from the fully polarized state. Then the value of \( V_0 \) is increased slightly and a new self-consistent solution is obtained by using the previous solution as the initial try. The modulation amplitude is then changed again and this self-consistent scheme is repeated. As shown in Fig. 2 by increasing and then decreasing \( V_0 \), we obtain hysteresis behavior of the spin polarization, \((1/N_x)\sum_{X,\alpha} \alpha(c_{X,\alpha}^\dagger c_{X,\alpha})\). This indicates that the system undergoes a first-order phase transition from a fully polarized homogeneous state to a partially polarized CDW state (see the discussions below), which corresponds to the sudden drop of the spin polarization in Fig. 1. Within the HFA, the critical value of \( V_0 \) at which the instability occurs can be estimated as follows. For the uniform fully polarized state to be the self-consistent solution at \( T = 0 \), the maximal value of \( \varepsilon_{X,+1} \) for this state must be lower than the chemical potential \( \mu = −ε_e/2 \). From Eq. (11), this gives

\[ V_0 \leq \frac{1}{2} \left( \Delta_z^{(0)} + ε_e \right) e^{(G_0 l)^2/4}. \]  

(12)

![FIG. 2. The self-consistent solution of the energy spectra in the first Brillouin zone, \( 0 \leq X < a \) for \( a = 10 \). The modulation amplitude \( V_0 = 0.5 \) in Fig. 2(a) and \( V_0 = 0.6 \) in Fig. 2(b). The horizontal dashed lines denote the Fermi levels.](image)

To investigate the electron population in the ground state, energy spectra \( \varepsilon_{X,\alpha} \) for two different \( V_0 \)'s with \( a = 10 \) and \( \Delta_z^{(0)} = 0.03 \) are shown in Fig. 2. For \( V_0 \) smaller than the critical value [Fig. 3(a)], the spin splitting is amplified by the exchange energy \( ε_e \), and there is no overlap between the spin-split bands. Therefore, we have a spatially uniform ground state with fully-polarized spins even though the system is modulated. The energy spectrum keeps this structure until \( V_0 \) reaches the critical value, where the two Landau bands begin to touch each other at the energy of \( \mu = −ε_e/2 \). When \( V_0 \) is larger than the critical value so that the bands overlap, the population of the spin-down band becomes nonzero, and the spin splitting is no longer a constant for different X’s [Fig. 3(b)]. Thus a partially polarized CDW state is formed, where both the charge and the spin densities are periodically distributed. If \( V_0 \) is now decreased, the energy spectrum will keep similar structure as that in Fig. 3(b). Only when \( V_0 \) is sufficiently small, such that the difference of the populations between the spin-up and spin-down states exceeds a critical amount, will the spin gap be abruptly amplified. Hence the energy spectrum
again becomes similar to Fig. 2(a) and the fully polarized state is recovered. This history-dependent evolution of energy spectra is in fact implied in Fig. 1.

Instead of changing the modulation amplitude, the hysteresis loop can also appear by tilting the magnetic field. For a given large modulation amplitude (say, \(V_0 = 0.6\)), when the external magnetic field is tilted by an angle \(\phi\) but with a fixed normal component \(B_0\), the total field \(B = B_0 \cos \phi\) and therefore the Zeeman splitting \(\Delta_z = \Delta_z^{(0)}/\cos \phi\) are varied. By using the same numerical iterative procedure, we show in Fig. 3 the spin polarization for two temperatures (\(k_B T = 0\) and \(k_B T = 0.01\)) with \(a = 10\) and \(V_0 = 0.6\) to illustrate the effect of thermal fluctuation. We find that, just as conventional ferromagnets, the hysteresis loop shrinks as the temperature rises.

\[ \text{FIG. 3. Hysteresis loops for the spin polarization for } k_B T = 0 \text{ (solid line) and } k_B T = 0.01 \text{ (dashed line).} \]

In conclusion, within the HFA and the LLL approximation, we investigate hysteresis properties for the \(\nu = 1\) QH systems by tuning the periodic modulation amplitude and the tilted magnetic field. These results indicate that the intra-Landau-level exchange interaction plays a crucial role in forming the hysteresis. This hysteresis can be tested experimentally for modulated QH systems. For example, according to the composite-fermion theory, the physics of the \(\nu = 1/3\) fractional QH state should be similar to that of the composite fermion filling factor \(\nu_{\text{CF}} = 1\) QH state. Therefore, by translating the present discussion into the language of composite fermions, our results indicate that the \(\nu = 1/3\) fractional QH state can be another candidate for this modulation-induced hysteresis.

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12 However, the estimate based on the HFA will be somewhat larger than that obtained in Ref. [10], since the vertex correction of the Coulomb interaction, which accounts for the attraction between the electron and the hole of a spin exciton, is neglected in the HFA. For example, for the system parameters used in Ref. [10] (\(G_0 = 0.4\) and \(\Delta_z^{(0)} = 0.03\)), our critical value of \(V_0\) is 0.538, while the value obtained in Ref. [10] is 0.465.
13 For example, see J. K. Jain in Ref. [2]