Generation and characterization of resource state for nonlinear cubic phase gate

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Unitary non-Gaussian nonlinearity is one of the key components required for quantum computation and other developing applications of quantum information processing. Sufficient operation of this kind is still not available, but it can be approximatively implemented with help of a specifically engineered resource state constructed from individual photons. We present experimental realization and thorough analysis of such quantum resource state, and confirm that the state does indeed possess properties of a state produced by unitary dynamics driven by cubic nonlinearity.

In principle, to realize an arbitrary unitary operation of quantum harmonic oscillator, it is sufficient to have access to the quantum cubic nonlinearity 1, 20. Cubic nonlinearity is represented by Hamiltonian $\hat{H} \propto \hat{x}^3 \propto (\hat{a} + \hat{a}^\dagger)/\sqrt{2}$, where $\hat{x} = (\hat{a} + \hat{a}^\dagger)/\sqrt{2}$ is the position operator of the quantum harmonic oscillator ($\hat{a}$ is the annihilation operator). The evolution driven by this Hamiltonian preserves the behavior of $\hat{x}$, while changing the complementary momentum quadrature $\hat{p}$ by an additive term proportional to $\hat{x}^2$. As of now, neither quantum cubic nonlinearity, nor quantum states produced by it (cubic states), have been observed on any experimental platform. Beginning from a ground state, even the weak cubic interaction generates highly nonclassical states 21. However, nonclassicality of these states lies in the superposition of $|1\rangle$ and $|3\rangle$ ($|1&3\rangle$ for shorthand) and it is unfortunately masked by the superposition of $|1&3\rangle$ with the dominant ground state $|0\rangle$, especially considering its fragility with regards to damping of the oscillator. It is therefore challenging not only to generate and detect these states but also to understand and verify their nonclassical features.

The measurement induced approach towards implementing the demanding cubic nonlinearity on any input state relies on a highly squeezed version of the cubic state. From this resource state, the nonlinearity is pasted onto the target input state using the continuous-variable measurement and quantum feed-forward control 8, 21, 22. A preparation of such the high-cubic state is currently an impossible ordeal, as it requires the cubic nonlinearity which is inaccessible in the first place. A recent proposal therefore suggested use of an approximative weak cubic state, described as a superposition of Fock states
The first step towards understanding and realization of the cubic nonlinearity therefore lies in obtaining firm grasp of the nonclassical properties of the weak cubic state. In this Letter, we present the first experimental preparation of a non-Gaussian quantum state of light, with key features consistent with state produced by cubic nonlinearity.

The ideal cubic state, which can be used as a resource for the nonlinear cubic gate, can be expressed as \( \int e^{i\chi x^3} |x\rangle dx \). Note that normalization factors are omitted in this letter unless otherwise noted. The cubic state can be obtained by applying cubic nonlinear interaction \( \hat{U}(\chi_0) = \exp(i\chi_0 \hat{x}^3) \) to an infinitely squeezed state. This is a nonphysical state possessing strong nonlinear behavior, much akin to a state with infinitely large squeezing. Due to general inaccessibility of a cubic nonlinear operation, any physical realization of the state needs to be some kind of approximation. For weak cubic nonlinearity and finite energy, the state can be approximated by some kind of approximation. For weak cubic nonlinearity and finite energy, the state can be approximated by \( \hat{S}(-r)(1 + i\chi \hat{x}^3)|0\rangle \). Here, the cubic nonlinearity \( \chi \) is given by \( \chi = \chi_0 e^{i\alpha r} \), and \( \hat{S}(-r) = \exp[-(ir/2)(\hat{x}^2 + \hat{p}^2)] \) is a squeezing operation - a Gaussian operation, which can be considered feasible and well accessible in the contemporary experimental practice \([13, 14]\). The squeezing operation does not affect the cubic behavior of the state and can be therefore omitted in our first attempts to implement the cubic operation. The approximative weak cubic state can be then expressed in the Fock space as

\[
(1 + i\chi \hat{x}^3)|0\rangle = |0\rangle + i\frac{\sqrt{5}}{2\sqrt{2}} |1&3\rangle,
\]

where \( |1&3\rangle = (\sqrt{3}|1\rangle + \sqrt{2}|3\rangle)/\sqrt{5} \). It is a specific superposition of zero, one and three photons, but it can be also viewed as a superposition of vacuum \( |0\rangle \) with a state \( |1&3\rangle \), which in itself is an approximation of odd superposition of coherent states. The vacuum contribution results from the first term of the unitary evolution \( \hat{U}(\chi) \approx 1 + i\chi \hat{x}^3 \). It is an important term for the function of the deterministic cubic phase gate, but at the same time it masks the nonclassical features of the state \( |1&3\rangle \).

We attempt to generate the approximative weak cubic state of Eq. 1. An experimental scheme is the same as the one in the experiment of \([19]\) where superpositions of Fock states from zero to three are generated. A schematic of the experiment is shown in Fig. 1. The light source is a continuous wave Ti:Sapphire laser of 860 nm. With around 20 mW of pump beam of 430 nm, a two-mode squeezed vacuum is generated from a non-degenerate optical parametric oscillator (NOPO), which contains a periodically-poled KTiOPO_4 crystal as an optical nonlinear medium. The pump beam is generated by second harmonic generation of the fundamental beam, and frequency-shifted with an acousto-optic modulator by around 600 MHz (equal to free spectral range of NOPO, \( \Delta \omega \)). As a result, photon pairs of frequency \( \omega \) (signal) and \( \omega + \Delta \omega \) (idler) are obtained (\( \omega \) corresponds to the frequency of the fundamental beam). The output photons are spatially separated by a split cavity whose free spectral range is 2\( \Delta \omega \). The idler photons passing through the split cavity are sent to two frequency filtering cavities, and are split into three beams with beamSplitters. Each beam is interfered with displacement beams at mirrors of 99% reflectivity. Phase of the displacement is controlled by piezo electric transducers, and amplitude of the displacement is controlled by rotating half-wave plates followed by linear polarizers. The idler photons are detected by avalanche photo diodes (APDs). When APDs detect photons, they output electronic pulses which are combined into an AND circuit to get three-fold coincidence clicks. The signal beam is measured by homodyne detection with a local oscillator beam of 10 mW. The homodyne current is sent to an oscilloscope and stored every time of coincidence clicks. The density matrix and Wigner function of the output state are numerically reconstructed from a set of measured quadratures and phases of the local oscillator beam. The experimentally reconstructed density matrix and Wigner function are shown in Fig. 2 (a).

In the following we attempt to analyze nonlinearity inherent in the experimentally generated state. This is not straightforward because of similarity between the target and the vacuum states. This makes fidelity, which is usually used in state generation scenarios, unsuitable. We will therefore focus on confirming the presence of higher photon numbers in nontrivial superpositions.

We can start by applying a virtual single photon subtraction \( \hat{\rho}_{\text{exp}} \to \hat{\rho}_{\text{sub}} = \hat{\rho}_{\text{exp}} \hat{a}^\dagger/\text{Tr}[\hat{\rho}_{\text{exp}} \hat{a}^\dagger] \), where \( \rho_{\text{exp}} \)
represents the experimentally generated cubic state. For the ideal resource state, $(1 + i\sqrt{2})|0\rangle$, this should result in a superposition $|0\rangle + \sqrt{2}|2\rangle$, which is a state fairly similar to an even superposition of coherent states and, as such, it should possess several regions of negativity. Thus we can convert the cubic state into a state with well known properties, which can be easily tested. We can also remove the dominant influence of the vacuum state. Figure 2 (b) shows that the Wigner function and the density matrix of the numerically photon-subtracted experimental state. Notice that two distinctive regions of negativity are indeed present. Moreover, apart from considerations involving specific states, the areas of negativity sufficiently indicate nonclassical behavior of the initial state, as they would not appear if the state was only a mixture of coherent states $|\alpha\rangle\langle\alpha|$, where $|\alpha\rangle$ is a coherent state with amplitude $\alpha$. The probability of two photons $p_2' = 0.29$ is clearly dominating over $p_1' = 0.12$ and $p_3' = 0.03$, where $p_i' = \langle i|\hat{\rho}_{\text{sub}}|i\rangle$. To characterize the superposition of basis states $|0\rangle$ and $|\Phi\rangle$, we use the normalized off-diagonal element $R_{0,\Phi}(\hat{\rho}) = |\langle 0|\hat{\rho}_{\text{sub}}|\Phi\rangle|^2$, which characterizes quality of any unbalanced superposition. Since the subtraction preserves superposition of Fock states, $R_{0,2}(\hat{\rho}_{\text{sub}}) = 0.24$ after the subtraction proves the coherent superposition originating from the state $|1&3\rangle$. In a similar way we can confirm that the three-photon element is significantly dominant over the two- and four-photon elements. Two virtual photon subtractions transform the state $\hat{\rho}_{\text{exp}} \rightarrow \hat{\rho}_{\text{2sub}} = \hat{\rho}_{\text{exp}} \hat{a}^2/\text{Tr}[\hat{a}^2 \hat{\rho}_{\text{exp}} \hat{a}^{2\dagger}]$, where the single photon state is present with a probability of $p_i'' = |\langle i|\hat{\rho}_{\text{sub}}|i\rangle|^2 = 0.68$. In a generated single photon state this would be a sufficient confirmation the state cannot be emulated by a mixture of Gaussian states. In our case it is the argument for the strong presence of the three-photon element.

Our analysis confirms presence of the highly nonclassical superposition state $|1&3\rangle$, but we also need to demonstrate that the state appears in a superposition with the vacuum state, not just as a part of mixture. For this we look at the normalized off-diagonal element $R_{0,1&3}(\hat{\rho}_{\text{exp}})$ between the $|0\rangle$ and $|1&3\rangle$ for the original (not photon-subtracted) experimental state, which would attain the value of one for the ideal pure state, and value of zero for a complete mixture. In our case the value is $R_{0,1&3}(\hat{\rho}_{\text{exp}}) = 0.50$, so the superposition is present, even if it is not perfectly visible due to effects of noise. More importantly, the element is significantly larger than $R_{0,1&3}(\hat{\rho}_{\text{exp}})/2 = 0.11$, where $|1&3\rangle = (\sqrt{2}|1\rangle - \sqrt{3}|3\rangle)/\sqrt{5}$ is orthogonal to $|1&3\rangle$. This shows that the desired and theoretically expected superpositions are dominant.

What remains to be seen is whether the state also possesses sufficient cubic nonlinearity which can be used for cubic unitary transformation. The effect of such transformation may be already visible at a classical level. Cubic nonlinearity directly transforms the first moments of the input state’s quadratures $\hat{x}_{\text{in}}$ and $\hat{p}_{\text{in}}$ according to $\langle \hat{x}_{\text{out}} \rangle = \langle \hat{x}_{\text{in}} \rangle$, $\langle \hat{p}_{\text{out}} \rangle = \langle \hat{p}_{\text{in}} \rangle + 3\chi \langle \hat{x}_{\text{in}}^2 \rangle$. The first moment of $\hat{x}$ should be preserved, while the first moment of $\hat{p}$ should become linearly dependent on the second moment $\langle \hat{x}^2 \rangle = \text{var}(\hat{x}) + \langle \hat{x}^2 \rangle$. Note that $\text{var}(\hat{x})$ is a variance of $\hat{x}$. If we choose a set of input states with identical variances, there should be observable quadratic dependence of the first moment of $\hat{p}$ on the first moment of $\hat{x}$.

We can try implementing this nonlinearity by taking an imprint of the generated cubic state, in a very similar manner to how a single photon can be used to obtain a probabilistic map. As the set of target states we will consider coherent states $|\alpha\rangle$, where $0 \leq \alpha \leq 1$, with first moments $\langle \hat{x}_{\text{in}} \rangle = \sqrt{2}\alpha$ and $\langle \hat{p}_{\text{in}} \rangle = 0$. The operation, imprinting nonlinearity from the ancillary mixed state $\hat{\rho}_A$ to the target state $\hat{\rho}_m = |\alpha\rangle\langle\alpha|$ can be realized by map

$$\hat{\rho}_m = \text{Tr}_A[\hat{U}_{\text{BS}} \hat{\rho}_m \otimes \hat{\rho}_A \hat{U}_{\text{BS}}^\dagger|x = 0\rangle\langle x = 0|],$$

where $\hat{U}_{\text{BS}}$ is a unitary operator realizing transformation by a balanced beam splitter and $|x = 0\rangle_A$ is the zero value position eigenstate. It can be easily confirmed that this map fuses two states with wave functions $\psi_S(x_S)$ and $\psi_A(x_A)$ into a state with wave function $\psi_S(x_S/\sqrt{2})\psi_A(x_S/\sqrt{2})$. The factor $\sqrt{2}$ only introduces
linear scaling of the measured data and has no influence on any nonlinear properties. Since the imprinting operation uses only Gaussian tools, any non-Gaussian nonlinearity of the transformed state needs to originate in nonlinear properties of the ancillary state $\hat{\rho}_A$. The nonlinear behavior should manifest in the first moment of quadrature $P$, which we plot in Fig. 3. We can see that the dependence is distinctively quadratic. This behavior is actually in a very good match with that of the ideal cubic state (1) with $\chi = 0.090$. They only differ by a constant displacement, which has probably arisen due to experimental imperfections and which can be easily compensated. This showcases our ability to prepare a quantum state capable of imposing high-order nonlinearity in a different quantum state.

We can also attempt to observe the cubic nonlinearity directly, using density matrix in coordinate representation. In this picture, the continuous density matrix elements are defined as $\rho(x,x') = \langle x | \hat{\rho} | x' \rangle$. The cubic nonlinearity is best visible in the imaginary part of the main anti-diagonal: for the ideal state $(1+i\chi x^3)|0\rangle\langle 0| (1-i\chi x^3)$, the density matrix elements are $\text{Im}[\rho(x,-x)] = 2\chi^3 e^{-x^2}$ and the cubic nonlinearity is nicely visible. One problem in this picture is that the cubic nonlinearity can be concealed by other operations. The second order nonlinearity does not manifest in the imaginary part (no even order nonlinearities do), but a simple displacement can conceal the desired behavior. On the other hand, displacement can be quite straightforwardly compensated by performing a virtual operation on the data. The comparison of the ideal state, the generated state, and the displaced generated state can be seen in Fig. 4. We can see that although the cubic nonlinearity is not immediately apparent in the generated state, the suitable displacement can reveal it effectively. This nicely witnesses our ability to conditionally prepare quantum state equivalent to the outcome of the required higher-order nonlinearity.

We have generated a nonclassical non-Gaussian quantum state of light, which exhibits key features of a state produced by unitary dynamics driven by cubic quantum nonlinearity. Our experimental test has demonstrated the feasibility of conditional optical preparation of the ancillary resource state for the cubic measurement-induced nonlinearity. As this is the first state of this kind ever to be experimentally observed, our analysis has contributed to general understanding of quantum states produced by the higher-order quantum nonlinearities. This understanding is a crucial step towards physically implementing these nonlinearities as a part of quantum information processing and we expect first attempts in this direction to appear soon.

Acknowledgements - This work was partly supported by PDIS, GIA, G-COE, APSA, and FIRST commissioned by the MEXT of Japan, and ASCR-JSPS. K. M. acknowledges financial support from ALPS. P. M. and R. F. acknowledge the support of the Czech Ministry of Education under the grant LH13248.

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