Detecting Josephson effect in the excitonic condensate by coherent emission of light

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Coherent emission of light by a split excitonic Bose-Einstein condensate – excitonic Josephson junction – can be extremely sensitive to the Josephson phase established across the junction. As a result of this, the emission can be redistributed between different directions and even cancelled. The cases of the dipole- and of the quadrupole-active excitons are considered.

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Bose-Einstein condensation of low density excitons attract much of experimental and theoretical efforts \[1\], \[3\]. Depending on the nature of the excitons, the excitonic condensate may display a variety of properties specific for superfluids and superconductors. One of these is Josephson effect. In Ref. \[6\] it has been shown that an optical analog of the Josephson effect – optical Josephson effect – can be observed in the excitonic condensate. This effect can be revealed by the phase-sensitive transmutation of light \[10\]. In Ref. \[6\] it was suggested that the Josephson current can be detected in the case of the condensate of indirect excitons due to spatial separation of holes and electrons.

Reports on realization of the excitonic condensate \[3\] are based on observing a phonon assisted excitonic photoluminescence whose spectrum is proportional to the excitonic distribution in the energy space. Such a method does not test directly coherence of the excitonic cloud. In Ref. \[3\] the excitonic photoluminescence from the quantum wells subjected to magnetic field have exhibited large noise which was considered as an evidence of the excitonic condensation stimulated by the field and characterized by finite size domains. Recently, several proposals have been discussed \[3\] \[11\]. These rely on observing light produced by the condensate of optically active excitons. In Ref. \[3\], the two photon emission has been shown to be a unique property of the condensate. Statistical properties of the emitted radiation have been addressed in Refs. \[9\] \[10\]. The polarization of the coherent emission from the two component excitonic condensate has been discussed very recently \[11\]. It has been shown that it is sensitive to the relative phase between the components.

In this paper it is pointed out that the excitonic Josephson effect between two confined excitonic condensates may affect significantly the intensity of the light emitted by such condensates. Thus, observing this effect can be a direct proof of the excitonic condensation. Besides that, various properties of the excitonic Josephson junction can be tested by means of measuring the emission intensity.

As long as excitons can transform into light, the local polarization is proportional to the excitonic operator \(\Psi\). In the excitonic condensate the mean \(\psi = \langle \Psi \rangle\) becomes finite due to a formation of the off-diagonal long range order \[12\]. Accordingly, the mean electric polarization \(P \sim \langle \Psi \rangle = \psi \exp(-i\omega t)\) becomes finite, where \(\omega\) is the frequency of the excitonic transition. This implies that the emission becomes coherent, as opposed to the case when only thermal excitons emit \[10\].

Here a possibility of observing a dependence of the emission intensity on the relative phase \(\varphi\) between two excitonic condensates will be analyzed. A simple general analysis shows that this dependence should exist. Indeed, let us assume that two identical confined condensates are spatially separated by some displacement \(x_0\) and do not overlap significantly. Then, the two-condensate wavefunction can be represented as

\[
\psi(x) = \psi_1(x) + e^{iqx} \psi_2(x - x_0),
\]

where \(\psi_1(x)\) and \(e^{iqx} \psi_2(x - x_0)\) are the wave functions of the first and the second condensates, respectively. The electric and magnetic fields of the coherent radiation at the excitonic frequency \(\omega\) in the far zone are given by the polarization density which is proportional to \(\psi\). This leads to the differential intensity \(W(q) \sim |\psi_q|^2\) of emission of light with the wave vector \(q\), where \(\psi_q\) stands for the spatial Fourier component of \(\psi(x)\). Thus, from the representation \[6\] one finds

\[
W(q) = 2W_1(q)(1 + \cos(\varphi - qx_0)),
\]

where \(W_1(q)\) denotes the intensity due to one condensate only when the second is not present at all. Eq.(2) shows that the differential emission intensity of two condensates depends on their relative phase \(\varphi\).

If \(x_0\) is much smaller than the wavelength \(\lambda = 2\pi/q\) of the emitted light, one may neglect the term \(qx_0\) in Eq.(2). Then, the total intensity \(W^{(tot)} = \int d\mathbf{n} W(q)\) as an integral over the \(4\pi\)-solid angle \((\mathbf{n} = q/q)\) will be very sensitive to the relative phase. Specifically,

\[
W^{(tot)} = 2W_1^{(tot)}(1 + \cos \varphi)
\]
where $W^{(tot)}_I = \int d\mathbf{q} W_I(q)$. It can be seen that for $\varphi = \pm \pi$ the emission can be practically cancelled. This is an example of the *dark state* which can be realized in the excitonic Josephson junction.

However, if the spatial separation $x_0$ of the condensates is comparable with or larger than $\lambda$, the incoherence of the total solid angle may suppress the dependence on the phase $\varphi$ in Eq.(6). Nevertheless, as it will be seen below, the intensity of emission by two condensates of optically active excitons can remain strongly sensitive to the phase for large $x_0$. In contrast, in the case of the quadrupole-active excitons in $Cu_2O$, the dependence on the phase practically disappears for $x_0 \gg \lambda$.

First, let us analyze the case of *optically active* (OA) excitons confined inside, e.g., two identical spatially separated cubes with the side $2a$. The excitonic condensate is characterized by the density $n = |\psi_0|^2$ which is taken the same for both condensates. It is naturally to consider a limit when the excitonic healing length $l = 1/\sqrt{n a_s}$ is much smaller than the wavelength $\lambda$ of the emitted radiation. Indeed, for typical achievable densities $n \approx 10^{19} \text{cm}^{-3}$ [14] and the excitonic scattering length $a_s$ taken as a typical excitonic radius $\approx 10^{-7} \text{cm}$, one finds $l \approx 10^{-6} \text{cm}$. Under this condition the condensate density can be taken as a constant inside the cubes. To specify a relative position of the cubes, these are assumed to be identically oriented and displaced by $x_0 > 2a$ along one of their 4th-fold symmetry axis, which will be called the $x$-axis. The excitonic polarization vector $\mathbf{P}$ is directed along another 4th-fold symmetry axis, which will be called the $z$-axis. Thus, the polarization density vector $(0,0,P_z)$ can be represented as

$$P_z = d\Phi(0)\Psi,$$

where $d$ denotes the interband dipole matrix element; $\Psi$ stands for the operator of the excitonic center of mass motion, and $\Phi(\mathbf{r})$ is the wave function of the relative motion of the electron and the hole forming the exciton. In the condensate phase at the temperature $T = 0$ the operator $\Psi$ is to be replaced by the condensate wave function $\psi$ [4].

Given (3) for a pure condensate case, one finds the differential intensity $W_1(q)$ in Eq.(2) as

$$W_1(q) = W_0 \left( \frac{\sin(qz_1) \sin(qz_2) \sin(qz_3)}{q \sin \theta \cos \theta \sin \phi \cos \phi} \right)^2,$$

where within the orthogonal coordinate system $x, y, z$ the angles $\theta$ and $\phi$ give the direction of the vector $q$ so that the components of this vector are $q_x = q \cos \theta$, $q_y = q \sin \theta \cos \phi$ and $q_z = q \sin \theta \sin \phi$; the introduced constant is $W_0 = 8 c (d\Phi(0))^2 n/\pi$, where $c$ stands for the speed of light. A simple analysis of the angular dependence (3) shows that, when $qa > 1$, the emission pattern is characterized by a very anisotropic angular distribution. Almost all the radiation occurs inside 4 narrow cones of the angular area $\delta \approx 1/(qa)^2 << 1$. These cones are collinear with the $x$ and $y$ axes. Thus, employing Eq.(3) in Eq.(5), one can introduce four integrated intensities $W^{(tot)}_x, W^{(tot)}_{-x}, W^{(tot)}_y, W^{(tot)}_{-y}$ emitted into the corresponding directions $x > 0$, $x < 0$, $y > 0$, $y < 0$. It is important to note that the intensities $W^{(tot)}_y = W^{(tot)}_{-y}$, emitted perpendicular to the vector $x_0$, are practically insensitive to the phase $\varphi$. In contrast, the intensities $W^{(tot)}_x, W^{(tot)}_{-x}$, emitted collinear with $x_0$, are very sensitive to the phase $\varphi$. Specifically, one finds

$$W^{(tot)}_x = 2W^{(tot)}_y(1 + \cos(\varphi - x_0 q)), \quad W^{(tot)}_{-x} = 2W^{(tot)}_y(1 + \cos(\varphi + x_0 q)),$$

$$W^{(tot)}_y = W^{(tot)}_{-y} = 32 \pi c (d\Phi(0))^2 n a_s^2 \sin^4(qa)$$

in the limit $qa \gg 1$. Note that for $x_0 q = p\pi$ where $p$ is integer, the emission in the both directions $x > 0$, $x < 0$ can be completely suppressed by choosing $\varphi = 0$ or $\varphi = \pi$ depending on parity of $p$. For $x_0 q \neq p\pi$, the emission can be cancelled in either $x > 0$ or $x < 0$ direction. It is important to note that such a phase dependence of the emission is a direct consequence of the excitonic condensation. No such an effect will occur when no condensate is present. The phase difference can be induced by a difference of the chemical potential created by, e.g., some external field [5]. If the two condensates are connected by a constriction, an excitonic flux from one condensate to another will pass through this narrow bridge and exhibit a sudden change of the velocity. The velocity of the condensate is associated with the gradient of the phase $\varphi$. Therefore, the velocity change in the constriction will result in a jump of $\varphi$ between two condensates. As far as the total emission intensity (4) is very sensitive to the phase, the phase dynamics can be studied by means of measuring the total emission intensity.

A special consideration should be given to the case of excitons in $Cu_2O$. The paraexcitons have been reported to form the condensate [6]. In Ref. [6] the condensation of orthoexcitons has been reported as well. However, the orthoexcitons are characterized by fast Auger decay [7]. Thus, a possibility of achieving a coherence time long enough to observe the Josephson effect in a short living condensate of the orthoexcitons is questionable. In what follows the case of the long living paraexcitons will be considered. In the non-stressed crystal no direct radiation can be observed from the paraexcitons. The phonon assisted emission is inherently incoherent. However, if a mechanical
stress producing the deformation $u_{ij}$ is applied, a quadrupole and magnetic dipole radiation can be detected from the radiatively recombining paraexcitons \cite{14}. Below it will be shown that, if the geometry of the confinement of the two paraexcitonic condensates is properly chosen, the excitonic Josephson effect can be revealed in this case as well. For sake of simplicity, the cubic crystal of C$_{60}$O will be approximated by isotropic medium. Then, instead of Eq.(4), one may write for the electric polarization

$$P_1 = \sum_j Q_{ij} \Phi(0) \partial_j \Psi$$

(8)

where $Q_{ij}$ is the interband quadrupole tensor of the optical transition. In isotropic medium the second-rank tensor $Q_{ij}$ can be induced by the deformation $u_{ij}$ so that $Q_{ij} = gu_{ij}$ in the limit of small deformations, where $g$ stands for some constant, which can be determined from the experimental dependence of the intensity of the direct recombination emission as a function of the applied stress \cite{17}. Let the deformation be along the $z$ axis, so that the only non-vanishing component is $u_{zz}$. Thus, again, as in the case of the OA excitons, the polarization is along $z$-axis ($P_z = gu_{zz} \Phi(0) \partial_z \psi$). Now, however, this polarization is concentrated very close to the boundaries where the gradients of the excitonic condensate are the largest. Because of this, the emitted radiation becomes more isotropic than in the case of the OA excitons considered above. Consequently, the term $\Phi_x$ under the cos in Eq.(2) will practically erase the phase dependence unless $x_0$ is less than $q = 2\pi/\lambda$ (so that Eq.(3) can be employed). Thus, the considered above geometry of the two cubes is not appropriate for the paraexcitons. Instead, it is reasonable to consider a different arrangement.

Specifically, let us consider a slab whose width along the $z$-axis is $4z_0 << \lambda$, and whose base is a square with the side $2a >> \lambda$ oriented along the $x, y$ axes. This slab is divided in half by the plane perpendicular to the $z$-axis. The Josephson phase $\varphi$ is assumed to exist between these two halves. Given the representation (8), calculations of the emission intensity in this case show that the factor $W_1$ in Eq.(4) is to be replaced by

$$W_{1}^{(p)}(q) = W_{0}^{(p)} \left( \frac{\sin(aq_x) \sin(aq_y) \sin(qz_x)}{\sin \theta \sin \phi \cos \phi} \right)^2$$

(9)

where $W_{0}^{(p)} = (16/\pi)c|Q_{zz} \Phi(0)|^2 n$ and again $q_z = q \cos \theta, q_x = q \sin \theta \cos \phi$ and $q_y = q \sin \theta \sin \phi$. It can be seen that even in the limit $qa >> 1$ the radiation is no more concentrated in the plane $\theta = \pi/2$. Instead, it is spread with respect to the axial angle $\theta$ around the directions $\theta = \pi/4, 3\pi/4$. However, the radiation is still very anisotropic with respect to the polar angle $\phi$. It is equally emitted along the directions $\phi = 0, \pi/2$, and $-\pi/2$ into a very small angle $\delta \phi \approx 1/qa << 1$. Let us calculate the integrated intensity $W^{(p)}$ emitted into one of these directions (e.g. $\phi = 0$) by means of integrating Eq.(2) over $f_{-1}^{1} d \cos \theta \int_{-\delta \phi}^{\delta \phi} d \phi$, with the replacement $W_{1}^{(p)}(q) \rightarrow W_{1}(q)$ performed. This yields

$$W^{(p)} = 16\pi c n q^3 z_0^2 |Q_{zz} \Phi(0)|^2 (1 + \cos \varphi)$$

(10)

in the limit $z_0 << \lambda$.

Now let us discuss a role of the thermal component. The intensities (8), (9), (11) of the coherent emission are not proportional to the total volume of the condensates. In the case of the OA excitons the intensity (9) scales as $\sim a^2$ and for the paraexcitons the intensity (11) is $\sim a$ in the limit $a \rightarrow \infty$. In contrast to this, the intensity of the incoherent emission $W^{(in)}$ due to the thermal excitons should be proportional to the total volume occupied by the excitons. Specifically, $W^{(in)} \sim a^3$ for the OA excitons, and $W^{(in)} \sim z_0 a^2$ in the case of the paraexcitons. Thus, one may expect a masking of the coherent emission by the thermal component.

However, it should be noted that, while the incoherent radiation is almost isotropic, the coherent emission is very concentrated in certain directions. Therefore, a comparison should be performed between the intensities integrated over the small solid angles, to where the most of the coherent radiation is emitted. In the case of the OA condensate, the integrated incoherent emission intensity is $\sim a^3/(qa)^2 \sim a$, where it was taken into account that the coherent emission is concentrated into the cones of the angular area $\approx 1/(qa)^2$. Accordingly, the ratio of the incoherent to the coherent integrated intensities is $\sim 1/a \rightarrow 0$ in the limit $a \rightarrow \infty$. Thus, the coherent radiation should always be resolved for large enough values of $a$.

A different situation occurs for the paraexcitons. The angular area of the coherent emission "cones" is $\delta \phi \approx 1/qa$. Thus, the integrated incoherent intensity is $\sim a^2/qa \sim a$, and the ratio of it to the intensity (11) is independent of the largest size $a$ of the system. In what follows the intensity of the incoherent emission will be calculated for the case of the paraexcitons, and the condition insuring that the coherent effect can be resolved will be derived.

In order to find the incoherent intensity, one should employ the representation (8) for the polarization. At $T \neq 0$, the operator $\hat{\Psi} = \psi + \psi'$ contains the condensate part $\psi$ and the normal part $\psi'$. The differential intensity of the incoherent emission at the wave vector $q$ is $W^{(in)}(q) \sim \langle \psi'^{\dagger} \psi' \rangle$ where the averaging is performed over the thermal
ensemble. The average can be calculated by employing the Bogolubov method \[14\]. Specifically, the thermal part is represented as

$$\psi' = \sum_k (u_k a_k + v_k^* a_k^d), \quad (11)$$

where $u_k$ and $v_k$ stand for the Bogolubov amplitudes: $a_k$ destroys and $a_k^d$ creates a normal excitation at the state $k$ characterized by the energy $E_k$. Here the quasi-particles can be characterized by the quasi-momentum $q$ inside the slab. Thus, the index $k$ must be replaced by $q$ in Eq. (11). A direct conversion of the excitation with given $q$ into a photon with almost the same $q$ (in the limit $qa \gg 1$) is considered. In the contribution due to the normal component, the only non-zero means are $(a_k^d a_q)$ which are the population factors of the quasi-particles. Thus, calculating the emission intensity into a unit solid angle due to the thermal paraexciton, one finds

$$W^{(\text{in})}(q) = \frac{c}{2s} |Q_{zz} \Phi(0)|^2 V \sin^2(2\theta) \frac{q^6 (|u_q|^2 + |v_q|^2)}{\exp(E_q/T) - 1}, \quad (12)$$

where $V = 16 z_0 a^2$ stands for the total volume of the excitonic cloud; the quantum depletion term $\sim |v_q|^2$, which is finite at $T = 0$, has been neglected. In Eq. (12) the amplitudes and the energy are $\xi = g_T/T/\varepsilon_T < 1$.

In here $g_q = q^2/2m$ denotes kinetic energy of a free exciton, and $g_{ex} = 4\pi a_x/m$ stands for the two-body excitonic interaction constant expressed in terms of the scattering length $a_x$ (in units $\hbar = 1$). In Eq. (12) the main term proportional to the total volume of the system is selected. In the limit $n\lambda^3 >> 1$ and for the gas parameter $na_x << 1$, one can safely replace the Bose factor in Eq. (12) by $T/E_q$ unless $T$ is extremely low. Here it is assumed that $T$ is of the order of the temperature $T_c \approx 3.3n^{2/3}/m$ of the excitonic Bose-Einstein condensation (for one component excitons) for typical densities $\sim 10^{18}$cm$^{-3}$. Under these circumstances the expression (12) is weakly sensitive to the interaction constant $g_{ex}$ in the limit $E_q < T$. Specifically, it can be seen that $W^{(\text{in})}$ changes by only a factor of 2, while $g_{ex}$ varies from zero to such values that the excitonic spectrum becomes sound like $E_q = \sqrt{g_{ex}n/m \varepsilon_q}$. In the latter situation the $q$-dependent factor in Eq. (12) becomes $q^6 T/2\varepsilon_q$, that is independent of $g_{ex}$. In the opposite limit ($g_{ex} \to 0$), this factor is $q^6 T/\varepsilon_q$.

The total intensity of the incoherent radiation emitted into the narrow solid angle $\delta \phi$, to where the most part of the coherent radiation is collected, is $\approx W^{(\text{in})}(q)/qa$. Thus, the ratio of the integrated incoherent intensity to that given by Eq. (10) for $\varphi = 0$ is

$$\xi \approx 0.02 \frac{T}{T_c n^{1/3} z_0}. \quad (14)$$

The coherent effect can be well resolved if $\xi < 1$. It can be seen that for not very small $z_0$, one obtains $\xi < 1$ as long as $T < T_c$. Indeed, making an estimate of $\xi$ for the typical values of the wavelength $\lambda \approx 600$ nm, the density $n = 10^{18}$ cm$^{-3}$, the temperature $T = T_c/2$ and $2\pi z_0/\lambda = 1/3$, one finds $\xi \approx 10^{-2}$. Let us also estimate the absolute intensity of the coherent emission from the paraexciton condensate. To this end, the following maximum value of $Q_{zz}$ is chosen $Q_{zz} \approx ea_B^2$, where $e$ stands for the electron charge and $a_B = 0.5\lambda$ denotes Bohr’s radius. $|\Phi(0)|^2 = 1/|ea_B^2 z_0^{1/3}|$ is expressed in terms of the excitonic radius $a_{ex} \approx 10\lambda$. Thus, for the chosen above values, one arrives at the intensity (14) as $W^{(\text{in})} \sim 10^{-5}$W for $a/\lambda = 10$. It is worth noting that the intensity of the coherent emission from the condensates of the OA excitons is 10$^3$ times larger than the above estimate. Depending on the choice of $d$ which can be varied by external fields (18), this factor can enhance the above estimate by several orders of magnitude.

In summary, the coherent radiation from two excitonic condensates can be sensitive to their relative phase. In the case of the OA excitons, the emission can be totally suppressed or significantly redistributed in the opposite directions by means of imposing appropriate values of the phase. In the case of the paraexcitons in $Cu_2O$ under mechanical stress, the dependence of the emission on the phase can be also very distinct, if the two condensates form a double layer whose thickness is less than the wavelength of the emitted radiation, and if the mechanical stress is applied perpendicular to the plane of the layer.

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