Analytical Fragility Function for Seismic Damage Evaluation of Unreinforced Masonry Buildings in High Seismic Zone

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Abstract

The paper describes the evaluation of seismic capacity of the unreinforced masonry building in the high seismic zone by an analytical procedure, based on nonlinear static approach. In this procedure, each wall plane is modeled as some vertical strip walls separated by the openings and is characterized by two primary components such as vertical component pier and horizontal component spandrel. These walls are analysed considering the capacity of pier in conjunction with the coupling effect between pier and spandrel. However, capacity of spandrel is not considered in this case. The shear capacity of the wall is calculated based on lower bound theorem of plasticity without considering the tensile strength of masonry. Capacity curves of the walls are determined by three parameters as shear capacity, yield displacement and ultimate displacement at zero stiffness approaching a bilinear approximation and these capacity curves of the walls in a certain orthogonal direction are superposed to obtain capacity curve of the building in that direction. The smaller capacity of the building in the prevalent direction is then compared with seismic demand and a capacity-demand relationship is found and damages are introduced in this capacity-demand relationship to obtain vulnerability function. The vulnerability function of the representative masonry building models are used to construct fragility function of the building group and this fragility function shows the expected damage of the building group as a function of spectral displacement.

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1. Introduction

North-eastern region of India is the country’s most severe seismic hazard zone and experienced a number of large and great earthquakes such as the 1897 Shillong earthquake (Mw 8.1); the 1918 Srimangal earthquake (Ms 7.6); the 1947 Arunachal earthquake (M 7.5); the 1950 Assam earthquake (Mw 8.7) etc. in the past. Considering the distribution of epicenters, fault plane solution and geotectonic features, the region is mainly divided into five major seismotectonic zones as Eastern Himalayan collision zone, Indo-Myanmar subduction zone, Syntaxis zone of Himalayan arc and Burmese arc (Mishmi Hills), Plate boundary zone of Shillong Plateau and Assam Valley and Bengal Basin and Plate Boundary Zone of Tripura Mizoram Fold belt. In this context, this higher level and diffused seismicity of this region ensures the high probability of occurring large and great earthquakes in this region in future.

However, these earthquakes caused a wide casualties and destruction of properties. In case of housing, the buildings of this region are considerably unreinforced masonry (URM) buildings, primarily owing to its lesser cost and ease of construction. Moreover, a traditional manner without considering engineering input is employed for the construction process and the quality control and maintenance is dependent on monetary assistance of the owner. A wide irregularity in the spacing and distribution of walls and position of openings in the walls are frequently observed in such buildings. The codal provisions such as seismic bands and corner reinforcement are sometimes overlooked. Therefore, these buildings are very vulnerable during earthquakes and causes damages to a great extent. In such a state, potential seismic evaluation of URM buildings is very important before seismic risk assessment or seismic hazard strategic mitigation planning.

In case of non-availability of past damage statistics, the evaluation of seismic capacity is computed on various finite element (FE) models and other simplified models with the help of popular software packages. However, analysis of these models requires a higher amount of data and memory as well as a higher computational cost and analytical skills. Therefore, analytical procedures based on nonlinear static approach are a viable alternative tool in lieu of FE modeling techniques to evaluate seismic capacity of the building. Such analytical procedures are found in the works of Calvi [1], Lang [2], Restrepo-Velez and Magenes [3].

Generally, it is reported in the literature from past earthquake damage observations and laboratory tests that failure of masonry buildings with light and flexible roof is primarily due to the out-of-plane flexures whereas masonry building with rigid roof undergoes in-plane shear stresses ([4],[5]). The main aim of the present study reported in this paper is seismic vulnerability assessment of residential unreinforced masonry buildings with flat rigid roofs/floors primarily subjected to in-plane shear stresses. Analytical procedure proposed by Lang [2] is used to evaluate capacity curve and vulnerability function of the building and then probabilistic analysis of these obtained vulnerability functions are carried out to form analytical fragility curves. These analytical fragility curves are used to predict the level of damage that a building or a class of building may experience when it is subject to a certain ground acceleration level.

2. Analytical procedure for analysis of the URM buildings

The seismic capacity of the URM buildings can be evaluated by conducting nonlinear analyses on the nonlinear finite element models or equivalent frame models. However, these techniques consume a wider range of data and memory. Apart from these, analytical procedures based on nonlinear static approach can be used to find out the seismic capacity of the URM buildings. In this regard, an analytical solution proposed by Lang [2] based on nonlinear static approach is described sequentially into four parts: analytical modeling, capacity of the building, capacity-demand relationship, vulnerability function.

Analytical modeling considers the structural wall planes into a number of vertical strip walls of height of that plane. An example of analytical modeling of a structural wall plane is shown in Figure 1(a). The red outlined portion of dimension of length \( l_w \) and height \( H_{tot} \) is wall which is unit structural element of the building considered in the analysis. Pier is vertical element of the wall adjacent to opening and has dimension of length \( l_w \) and height \( h_p \) equal to the height of the adjacent opening and spandrel is a horizontal element exists between two vertically placed consecutive openings. Assembly of the walls in one plane connected by the floors and spandrels constitutes a wall plane.
Coupling effect is generally the interaction of cantilever walls or piers with spandrels. This effect is negligible in case of absence of spandrel and failure under pier mechanism while it is admissible for deep spandrels and failure under coupled mechanism. In this method, coupling effect is taken into account by a single parameter, the height of zero moment in the pier \( h_0 \) which can be determined as a function of the ratio of the flexural stiffness of the spandrels to the flexural stiffness of the piers, \((EI_{spandrel}/EI_{pier})(h_0/h_0)\). The variation of \((h_0/h_{st})\) with \((EI_{spandrel}/l_0)/(EIpier/h_{st})\) for different force distribution of two-storey frame is shown in Figure 1(b).

![Diagram](image1)

Fig. 1. (a) Terminology used in the analytical modeling; (b) The variation of \((h_0/h_{st})\) with \((EI_{spandrel}/l_0)/(EIpier/h_{st})\) for different force distribution of two-storey frame [2]

In order to represent the first mode response of the building by capacity curve, it is assumed that the building responds to a seismic input predominantly in its fundamental mode of vibration. Capacity curve of the walls are evaluated and these curves are superposed to obtain the capacity curve of the building considering that the displacements of the walls at the floor levels are equal and floors are completely rigid and torsional effects are neglected.

Capacity curve of a wall is expressed as bilinear capacity curve using three parameters i.e. the shear capacity of the wall \( V_m \), the nominal yield displacement at the top of the wall \( \Delta y \) and the nominal ultimate displacement at the top of the wall \( \Delta u \).

Shear capacity of the wall is evaluated by Eq. (1) considering lower bound theorem of plasticity without taking tensile strength of masonry into account.

Maximum shear capacity of wall, 
\[
V_m = \frac{f_{m,ty} t. N \cdot \tan \phi}{N + N \cdot (\tan \phi)^2} + 2 f_{m,ty} t. h_v \cdot \tan \phi
\]  
(1)

But, in no case violate the conditions of (i) stress criterion \((f_{\text{inclined}} \leq f_{uy} \text{or}, f_{\text{vertical}} \leq f_{ux} - f_{uy})\) and (ii) sliding criterion \((\tan \alpha \leq \tan \phi)\).

Otherwise \( V_m \) has to be manually set by a trial and error process until these conditions are satisfied.

Yield displacement of wall, 
\[
\Delta y = V_m H_{m,ty} \left( \frac{h_v \left(3h_v - h_p\right)}{6EI_{eff}} + \frac{\kappa}{GA_{eff}} \right)
\]  
(2)

taking into account bending, shear and coupling effect.

Ultimate displacement at the top of the wall, 
\[
\Delta u = \mu_{uy} \Delta y
\]  
(3)

Capacity curve of the building, 
\[
V_b (\Delta) = \sum V_j (\Delta)
\]  
(4)

Stiffness of the building, 
\[
k = \frac{V_m}{\Delta_y} = \sum k_{eff}
\]  
(5)
where $f_{uy}$ is compressive strength parallel to the mortar bed; $l_w$ is length of wall-pier; $t$ is thickness of wall-pier; $N$ is total vertical load on wall-pier; $\alpha$ is angle of inclination; $\tan \phi$ is angle of internal friction; $V_{OG}$ is shear force acting at the upper storey is taken as 0.67$V_m$; $f_{ux}$ is compressive strength orthogonal to the mortar bed; $c$ is cohesion; $f_{vertical}$ is vertical strut stress; $f_{inclined}$ is inclined strut stress; $N_{OG}$ is normal force acting at the upper storey; $EI_{eff}$ is effective flexural rigidity; $GA_{eff}$ is effective shear rigidity; $\kappa$ is 1.2 for rectangular cross-section; $\mu_w$ is displacement ductility of the wall; $j$ is wall index, $j=1…n$, $n$ being the total number of walls acting in one direction; $V_m$ is shear capacity; $A_{by}$ is nominal top yield displacement of the building.

If the conditions (i) and (ii) laid for shear capacity against Eq. (1) are satisfied, Eqs. (1) - (3) are used to obtain the bilinear capacity curve of the walls and these bilinear capacity curves of the walls in any orthogonal direction are superposed and thereby the capacity curve of the building is obtained and it is shown in Figure 2(a).

The capacity curve of the building so produced is required to compare with seismic demand curve. Seismic demand is the displacement response spectrum and is a plot of spectral displacement at the fundamental frequency of the building as shown in Figure 2(b). In order to apply the spectrum the MDOF system with lumped masses at the floors is suitably converted to an equilibrium SDOF system of equilibrium mass and stiffness having same fundamental frequency of the building. The displacement demand of the building is calculated taking into account the inelastic behaviour of the building by strength reduction factor and ductility demand and is given by the expression Eq. (6).

Displacement demand at the top of the building, $A_b = c_n \Gamma \varphi_n S_d \left( f_i \right)$  \hfill (6)

where $c_n$ is constant takes effect of nonlinear behaviour and expressed as a function of the strength reduction factor and ductility demand; $\varphi_n$ corresponds to the first mode displacement at the top storey of the MDOF system; $\Gamma$ is modal participation factor; and $S_d \left( f_i \right)$ is spectral displacement corresponding to SDOF system.

Displacement demand is found for every incremental value of spectral displacement using Eq. (6) and a relationship is formed between displacement demand and seismic demand which in turn known as capacity-demand relationship as shown in Figure 2(c). Damage grades as per classification of damage to building in EMS-1998 [7] are five DG1, DG2, DG3, DG4 and DG5 excluding no damage DG0 and these are listed in Table 1. Each damage grade conveys certain identification in respect of cracking or yielding of the wall. The cracking displacement is

| Damage grade | EMS-1998 | Identification |
|--------------|----------|----------------|
| DG 0         | No structural or non-structural damage | No Damage |
| DG 1         | Negligible to slight damage | The wall that cracks first |
| DG 2         | Moderate damage | Yield of the first wall. |
| DG 3         | Substantial to heavy damage | Yield of the last wall. |
| DG 4         | Very heavy damage | Failure of first wall |
| DG 5         | Destruction | Drop of the base shear of the building $V_b<0.67V_{iso}$ |

Fig. 2.  (a) Capacity curve of the walls and capacity curve of the entire building; (b) seismic demand curve; (c) capacity–demand relationship; and (d) vulnerability function.
calculated for each wall using Eqs. (7) and (8) [2] and based on these, damage grades are identified in the so obtained capacity-demand relationship as a function of spectral displacement demand for a certain level of ground motion. In this way, vulnerability function is formed as shown in Figure 2(d).

\[ V_{cr} = \frac{NH_{cr}}{6h_{cr}} \]  

\[ \Delta_{cr} = V_{cr}H_{cr}\left(\frac{h_{cr}^3}{6EI_{eff}} + \frac{\kappa}{GA_{eff}}\right) \]

3. Analysis of existing masonry buildings

The low rise URM and RCC buildings constitute a considerable portion of housing stocks in Agartala. Out of these buildings, URM buildings are in the consideration of the present study. These buildings possess a sufficient compressive strength and due to this, they withstand well under the vertical load but when an earthquake occurred, lateral inertial loads imposed on these buildings and causes flexural and shear stresses on the structural walls and these walls cannot withstand so developed stresses owing to very weak bond strength between brick and mortar. Therefore, all these URM buildings are to be analysed in order to obtain a comprehensive damage scenario of this building class of the concerned area. However, the entire process of analysis of all the buildings is a time consuming process and generates a huge data and requires a broader memory. In order to shorten the entire process of analysis, the buildings of this area are grouped based on the total floor area of the buildings which will, in turn, indicate the possible projected socio economic levels of this city. The groups are as, Group-I (Gr-I) for total floor area less than 56 sq.m., Group-II (Gr-II) for total floor area between 56-112 sq.m and Group-III (Gr-III) for total floor area greater than 112 sq.m. The construction practice and material properties of the buildings of same class in an area are seemed to be similar. Each group of the buildings has a representative case building whose total floor area is within the specified total floor area of the group. Group-II has buildings of first storey and double storey height within its specified floor area. Therefore, Group-II has been assigned with two representative case buildings as Gr-ISS and Gr-IIDS for single storey and double storey respectively. The parameters of representative case buildings are listed in Table 2. The plan view of the ground floor of the representative buildings are shown in Figure 3.

The material properties used in the analysis are taken from the Manual of Indian Society for Earthquake Technology [8] with 1:6 cement-sand mortars. Such property like shear modulus is considered 0.35E; friction coefficient is considered as the maximum limit and self-weight is taken from IS 875 [9] and these are tabulated in Table 3. The analytical modeling of the representative building is then carried out and the bilinear capacity curves of
the structural walls of the buildings in both the orthogonal directions are calculated using the analytical procedure. These bilinear capacity curves of the walls along a particular orthogonal direction are then superposed and the capacity curve of the building is obtained for that direction. The minimum shear capacity of the prevalent direction is considered as the governing direction for the buildings and the capacity curve along that governing direction is considered as the capacity curve of the building for further analysis. Capacity curves of the representative buildings are shown in Figure 4(a).

Table 2. Parameters of selected representative buildings of three groups

| Group  | Total floor area (sq.m.) | Fundamental frequency (Hz) | Wall density in x-direction (%) | Wall density in y-direction (%) |
|--------|------------------------|---------------------------|--------------------------------|-------------------------------|
| Gr-I   | 52.21                  | 7.70                      | 6.50                           | 9.49                          |
| Gr-IISS| 83.41                  | 10.00                     | 5.55                           | 7.06                          |
| Gr-IIDS| 104.44                 | 4.20                      | 6.50                           | 9.49                          |
| Gr-III | 147.02                 | 4.80                      | 6.73                           | 8.37                          |

Table 3. Material properties considered for representative buildings ([8], [9])

| Parameter                  | Value          | Parameter                  | Value          |
|----------------------------|----------------|----------------------------|----------------|
| Self-weight ($\gamma$)     | 18.85 KN/m$^3$| Young’s modulus ($E$)      | 2000 MPa       |
| Compressive strength ($f_u$)| 6 MPa          | Shear modulus ($G$)        | 700 MPa        |
| Tensile strength ($f_{tu}$)| 0.25 MPa       | Friction coefficient ($\mu$)| 0.8            |
| Shearing strength ($c$)    | 0.39 MPa       |

Fig. 4. (a) Capacity curves of the representative buildings, (b) displacement response spectrum for 5% damping for medium soil for zone V with maximum ground acceleration 0.36g [10]

Fig. 5. Vulnerability functions of the representative buildings
The capacity curve of the building is then compared to seismic demand which is taken as displacement demand spectrum for 5% damping for medium soil site for zone V with maximum ground acceleration 0.36g as shown in Figure 4(b) [10]. Thus, a capacity-demand relationship is established in which damage grades [7] are introduced as a function of spectral displacement and consequently, vulnerability function of the building is obtained as shown in Figure 5 and this function will predict the probable damage grade for respective top lateral displacement of the building.

4. Derivation of fragility function Results and Discussion

Single deterministic vulnerability function is merely relevant for a particular building. Therefore, vulnerability function with dispersion using probabilistic distribution will express the vulnerability function of the entire group of the building. In this purpose, lognormal distribution is carried out considering median value of the sample as mean and logarithmic dispersion value as dispersion. The cumulative distribution so obtained in this analysis is commonly known as fragility function. Fragility functions are the cumulative distribution functions which describe the probability of reaching or exceeding a particular damage grade given a spectral response such as spectral displacement.

Therefore, probability of reaching or exceeding damage grade DG, given the spectral displacement \( S_d \), is described by the following expression [11]:

\[
P[DG \geq DG_i | S_d] = \phi \left[ \frac{1}{\beta_{DG_i}} \ln \left( \frac{S_d}{\bar{S}_d, DG_i} \right) \right] \text{where, } \beta_{DG_i} = \sqrt{\text{CONV} \{\beta_c, \beta_D\}^2 + \beta_{T, DG_i}^2}
\]  

(9)

where \( \bar{S}_d, DG_i \) is the median value of spectral displacement at which the building reaches the damage grade threshold, \( DG_i \); \( \beta_{DG_i} \) is the standard deviation of the natural logarithm of spectral displacement for damage grade, \( DG_i \); \( \phi \) is the standard normal cumulative distribution function; \( \beta_c \) is the uncertainty related with building capacity; \( \beta_D \) is the uncertainty related with earthquake demand and \( \beta_{T, DG_i} \) is the uncertainty related with damage grade threshold. Total variability is influenced by the \( \beta_{T, DG_i} \) and the combined uncertainty of capacity and demand, \( \text{CONV} \{\beta_c, \beta_D\} \) and it is calculated by convolution process. In order to minimise the difficulty level of convolution process, the pre-calculated values of damage-grade beta from ‘Advanced Engineering Building Module’ (Table 6.5, [11]) are used. It has been considered that minor degradation corresponds to DG1 and DG2, major degradation corresponds to DG3, and extreme degradation corresponds to DG4 and DG5. Considering moderate conditions, the values corresponding to \( \beta_c = 0.3 \) and \( \beta_{T, DG_i} = 0.4 \) is taken for values of \( \beta_{DG_i} \) which are tabulated in Table 4.

Table 4. Values of standard deviations (\( \beta_{DG_i} \)) for damage states [11]

| Damage states | DG1 | DG2 | DG3 | DG4 | DG5 |
|---------------|-----|-----|-----|-----|-----|
| For \( \beta_c = 0.3 \) and \( \beta_{T, DG_i} = 0.4 \) | 0.80 | 0.80 | 0.95 | 1.05 | 1.05 |

5. Establishment of fragility relationship of the URM buildings

Fragility functions of the building groups are prepared by a probabilistic distribution considering dispersion of the vulnerability function of the single representative case building of the pertinent groups. The spectral displacement values of the damage grades are extracted from the vulnerability function of the building and these values are taken as median spectral displacement in conjunction with the dispersion values (Table 4) to develop fragility curves.

IS 1893 [10] has assigned hazard factor of MSK intensity of IX or above with 0.36g PGA for maximum considered earthquake for entire zone V. The demand spectrum of 0.36g PGA has already been used in the analysis.

Fragility curves for the representative buildings are shown in Figure 6. The discrete damage state probabilities can be calculated by taking the difference of the cumulative exceedance probabilities of successive damage states.
for a given spectral displacement demand. It has been observed that fragility curves of double storey buildings are flatter in collapse zone than that of single storey buildings. It means higher percentage of single storey buildings attain collapse state at the smaller spectral displacement. In case of spectral displacement at the MCE, the probability of exceeding complete destruction state i.e. DG5 is nearly 84% for all the groups of buildings and that of DG4 and DG3 is 88% and 98% respectively. The probability of DG2 and DG1 is nearly 100%.

6. Conclusion

North-eastern region of India is a seismotectonically active region. The region has already experienced a number of large and great earthquakes. In such a state, due to less costlier and easier construction, URM buildings are by and large a favourable construction practice in this region even though these buildings are not sufficiently capable to resist earthquake forces. Therefore, seismic evaluation of this building class becomes very essential. Although, FE models are more accurate to find out the seismic capacity of these building, but due to the limitation of time and memory, simple analytical solution based on the nonlinear static method is more capable to handle these shortcomings with satisfactory outputs. In this analytical procedure, the capacity curve of the wall is found by bilinear approximation and these capacity curves of walls are linearly added to obtain the capacity curve of the building. In order to compare capacity curve of the building with seismic demand, MDOF system is converted to an equivalent SDOF system of having equivalent dynamic properties. Damage grades are introduced into the so obtained capacity-demand relationship and the vulnerability function is formed. Probabilistic distribution with dispersion of this single representative vulnerability function is carried out to form the displacement based analytical fragility function. The fragility curves represent the damage scenarios of the building groups. Present study reveals that for URM buildings, probability of exceeding complete destruction under MCE level is 84%.

References

[1] Calvi G.M., A displacement-based approach for vulnerability evaluation of classes of buildings, J. Earthq. Eng. 3 (1999) 411-438.
[2] Lang K., Seismic vulnerability of existing buildings, Phd thesis, Institute of Structural Engineering, Zurich, Switzerland, Swiss Federal Institute of Technology, 2002.
[3] Restrepo-Velez L.F., Magenes G., Simplified procedure for the seismic risk assessment of unreinforced masonry buildings, In: Proceedings of the 13th World Conference on Earthquake Engineering, Vancouver, B.C., Canada, Paper No. 2561, 2004.
[4] Tomažević M., Lutman M., Petkovic L., Seismic behavior of masonry walls: experimental simulation, J. Struct. Eng. 122(9) (1996) 1040–1047.
[5] Magenes G., Calvi G.M., In-plane seismic response of brick masonry walls, Earthq. Eng. Struct. Dyn. 26 (1997) 1091–1112.
[6] Lang K., Bachmann H., On the seismic vulnerability of existing unreinforced masonry buildings, J. Earthq. Eng. 7(3) (2003) 407–426.
[7] Grünthal G. (ed.), European macroseismic scale 1998; Council of Europe, Luxembourg, 1998.
[8] ISET, A manual of earthquake resistant non-engineered construction, Indian Society of Earthquake Technology, Indian Institute of Technology, Roorkee, India, 2001.
[9] IS-875 (Part1), Code of practice for design loads (other than earthquake) for buildings and structures, Bureau of Indian Standards, New Delhi, 1987.
[10] IS-1893, Indian standard criteria for earthquake resistant design of structures, part 1- general provisions and buildings, Bureau of Indian Standards, New Delhi, 2002.
[11] FEMA, Earthquake loss estimation methodology-HAZUS®-MH MR5. Advanced Engineering Building Module-Technical and User’s Manual, Federal Emergency Management Agency, Washington, DC, 2002.