Accuracy and Efficiency of Updated FRF Coupling for the Dynamic Behaviour Investigation of an Assembled Structure

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Abstract. The frequency based substructuring (FBS) method allows the combination between the experimental and analytical frequency response function (FRF). The combined FRF is then used for calculation of the dynamic behaviour of an assembled structure which usually consists of a large number structural components or substructures. However, the accuracy of the dynamic behaviour calculated using the FBS method relies heavily on the quality of experimental FRF data of the interfaces on which, in practice, it is very problematic to be attained. Furthermore, in most cases, some parts of rotational FRF data are not included in the FBS method due to the complex measurement process. The exclusion of the rotational FRF data will usually lead to numerous errors in the combined FRF data. Therefore, this paper proposes a new frequency response function (FRF) coupling of the FBS method that may uniquely address the difficulties and improve the quality of predicted results of the FBS method. The proposed coupling was formulated based on the finite element method, model updating method and FRF synthesize method. The finite element model of the physical test substructure was developed and reconciled based on the mode shapes obtained from experimental modal analysis. The FRF synthesize method was performed on the updated finite element model to obtain a complete matrix with a full degree of freedom FRF data containing all the translational and rotational FRF data required. It was found that the proposed coupling has allowed generating full translational and rotational FRF data. The generation has improved significantly the FRF coupling process and the quality of the predicted dynamic behaviour of the FBS method.

1. Introduction

The frequency based substructuring method (FBS) is one of the most powerful dynamic substructuring methods and the FBS method theoretically allows the combination of numerical and experimentally measured frequency response function (FRF) data [1]. The FBS method has been widely used in high-profile engineering analyses to improve the computing efficiency, especially in the structural dynamics and acoustics domain as presented in [2–4]. However, the method has been reported to have often undergone incompatible coupling issues; mainly during the preparation of the experimental model [4–6].

One of the major problems encountered during the preparation of the experimental model for the FBS is the excitation and measurement of rotational FRF data. In the FBS method, the rotational FRF data is essential in order to obtain the full FRF matrix [4] at the coupling the interface. Theoretically, a complete matrix of a single node system is described as follows:

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\begin{equation}
\mathbf{X}(\omega) = \mathbf{G}^H(\omega) \mathbf{F}(\omega) \mathbf{G}(\omega)
\end{equation}
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where \(\mathbf{X}(\omega)\) is the FRF vector, \(\mathbf{G}(\omega)\) is the FRF basis matrix, and \(\mathbf{F}(\omega)\) is the FRF coefficient matrix.
\[
\begin{bmatrix}
    u_t \\
    u_\theta
\end{bmatrix} =
\begin{bmatrix}
    Y_{tt} & Y_{t\theta} \\
    Y_{\theta t} & Y_{\theta\theta}
\end{bmatrix}
\begin{bmatrix}
    f_t \\
    f_\theta
\end{bmatrix}
\] (1)

where \( u \) and \( f \) represent the displacement and force vector respectively. The abbreviation of \( Y \) is denoted as a transfer function. The translational and rotational data are denoted by the subscripts \( t \) and \( \theta \). The above presented matrix clearly indicates that 75% of a single node matrix is built based on the rotational data. Therefore, neglecting 75% of the matrix will give erroneous results on the coupled system because the incomplete matrix is unable to represent the interface flexibility of the physical model.

Recently, several approaches such as equivalent multipoint connection (EMPC) approach [7], coupled and decoupled scheme [8], transmission simulator method [9] and expanding the full FRF matrix from the measured rotational FRF data [10] have been introduced in order to indirectly obtain the rotational FRF data. However, it has been found that the performance of the methods is heavily dependent on the excellent quality of measured data and the high accuracy of FE model on which, in fact, they seem to be very difficult to accomplish [5]. The aim of this research is to introduce a different approach through which the rotational FRF data of the experimental model are obtained by using the FRF synthesis method based on the updated finite element model that may uniquely address the difficulties and improve the quality of predicted results of the FBS method.

This paper puts forward an approach of obtaining the rotational FRF data of the experimental model using the updated FE model based FRF synthesis method. The findings are used to develop an accurate and efficient FRF coupling of the FBS method. The proposed approach is formulated based on the finite element method, model updating technique and FRF synthesis method. The detailed explanation of model updating methods can be found in [11–13]. The proposed approach may offer a unique capability to address the difficulties encountered and is expected to improve the quality of the predicted dynamic behaviour of the FBS method. In addition, the high dependency on the experimental data on which the FBS method relies heavily, may be significantly reduced.

1.1. Description of the Test Assembled Structure

In this work, the applicability and the accuracy of the proposed approach for coupling the finite element model and experimental model were demonstrated by using an assembled structure with bolted joints. The assembled structure consists of two structural components namely an irregular plate and a simple beam as shown in Figure 1. The FRF of the beam was derived from the finite element model, while the FRF of the irregular plate was acquired from the experimental work and proposed method. The test assembled structure was made of steel, 500mm in length and 200mm in width. Both structural components were connected with two bolted joints, namely connection points 1 and 2.

![Figure 1. The test assembled structure](image)

The frequency of interest in this work was between 0Hz and 2000Hz. This is because there were several bending and torsional modes occurred at the connection interfaces within the frequency range which is essential for the study of the rotational effects. Prior to performing the FBS analysis using the proposed approach, the FRF of the test assembled structure was measured for validation purposes using
the experimental modal analysis (EMA) with free-free boundary conditions as presented in previous studies [4,10,14]. Upon the completion of the FRF measurement, the structure was then disassembled.

On the development of a finite element model of the beam, 3D elements were used and the final FE model is presented in Figure 2. The FE model was used to derive FRFs using the FRF synthesis method [cite a relevant paper covers the method]. For the experimental model (irregular plate), the FRFs were derived from a simplified finite element (SFE) model which is presented in the next subsection.

![Figure 2. The finite element model for the beam](image)

1.2. Development of the simplified and approximated finite element (SAFE) model for the experimental model

Previous works [4,5,15] reported that the rotational FRF data for the experimental model is normally obtained by performing the modal expansion method or deriving directly from several measured translational FRF data to form a full FRF coupling matrix. In addition, using a full FRF coupling matrix during the coupling process of structural components is proven to lead to a successful FBS analysis as presented in [10]. Therefore, instead of using the modal expansion method to obtain rotational FRF data, this study proposes a new FRF coupling approach developed using the updating method [cite paper here] in the light of experimental data where several mode shapes were used as updating parameters. The updated finite element model of the experimental model was then used to derive and extract the incomplete rotational FRF data required.

Constructing an accurate finite element model of a large complex structure is very difficult and time consuming [16]. Therefore, instead of spending enormous amount of time preparing a finite element model in great detail, this study introduces an approximate modelling approach in which the finite element model of the experimental model was simplified and developed approximately. Figure 3 shows the processes involved in the development of the simplified finite element model of the experimental model (irregular shape). The SFE model of the experimental model was developed using 2D shell elements with a simplified and approximate outline of the experimental model, giving enough details in representing the physical structural component. In this case, it is the irregular shape.

![Figure 3. The development and finite element modelling of the SFE model](image)

Theoretically, the developed SFE model allows its parameters to be updated by referring to the experimentally obtained mode shapes. By adopting the SFE method, the mode shapes, natural frequencies and FRF from the updated SFE model can be matched as close as possible to the actual experimental one. The updated SFE model was then used to derive a new FRF matrix which contained both translational and rotational FRFs.
2. Test Setup of the Experimental Model (Irregular Plate)
As mentioned earlier, the SFE model was reconciled based on the experimental natural frequencies and mode shapes of the experimental model of the irregular plate. The plate was tested by using experimental modal analysis to acquire its dynamic behaviour. To minimize any constraints, the experimental work was carried out by suspending the structure using soft springs in order to simulate free-free boundary conditions [17]. Figure 4 shows the test set-up of the irregular plate in which the translational FRF data at the connection interfaces were measured using two 100 mV/g tri-axial accelerometers. The measurement was performed in order to validate the synthesized FRF data of the SFE model.

![Diagram of test setup](image)

**Figure 4.** The Schematic diagram for the test setup

3. Model Updating of the SFE Model
The objective of finite element model updating of the SFE model is to reconcile the SFE model with the experimental model by improving the potential design parameters of the SFE model in the light of experimental data to an acceptable level of accuracy. To achieve it, the discrepancies between the measured and predicted responses which are modal data, are minimised [18–20]. The parameterisation can be performed via sensitivity analysis in the form of

$$ S = \Phi_i^T \begin{bmatrix} \frac{\delta K}{\delta \theta_j} - \omega_i \frac{\delta M}{\delta \theta_j} \end{bmatrix} \Phi_i $$

where matrix $S$ represents the sensitivity matrix, while $\Phi$, $\omega$ and $\theta$ represent the eigenvector, eigenvalue and parameter respectively. The $i$ indicates the $i$-th eigenvalue and $j$ is the $j$-th parameter. Generally, the sensitivity analysis is used to identify the most sensitive parameters to the predicted natural frequencies. Prior to performing the modal updating method, modal assurance criterion (MAC) analysis was performed to quantify the accuracy of the predicted mode shapes. The information about the MAC analysis can be found in [21]. Then, the objective function containing the natural frequencies and MAC number are defined in order to reduce the discrepancies between the finite element model and experimental data. In this study, 9 mode shapes were used in the updating process. Updating the parameters was performed by using MSC NASTRAN Solution 200 with the same objective function used in [22]. The comparisons between the initial and updated natural frequencies and MAC values of the SFE model are presented in Figure 5 and Table 1.
Figure 5. a) MAC results of the initial SFE model b) MAC values of the updated SFE model

From Figure 5 and Table 1, it was found that the MAC values of the 6th to 9th modes of the initial SFE model have recorded uncorrelated, very low and unacceptable values. This indicates that the mode shapes calculated from the initial SFE model are not in good agreement with the measured counterparts especially for the higher modes. The discrepancies occurred because the geometrical dimensions and material properties used for the development of the initial SFE model are based on the simplifications and initial assumptions. As a result, the initial SFE was incapable of representing the actual irregular plate. However, a significant improvement in the natural frequencies and the MAC values of all 9 modes were recorded after the initial SFE was updated. This achievement suggests that the updated SFE model has successfully been used in representing the dynamic behaviour (natural frequencies and mode shapes) of the actual irregular shape.

Table 1. The comparison of the dynamic behaviour between the initial and updated SFE model

| Mode   | Initial SFE Model | Updated SFE Model |
|--------|-------------------|-------------------|
| 1st    | FEA – 343.02 Hz   | FEA – 350.45 Hz   |
|        | EMA – 352.19 Hz   | EMA – 352.19 Hz   |
|        | MAC – 97.4%       | MAC – 98.5%       |
| 2nd    | FEA 482.63 Hz     | FEA 478.35 Hz     |
|        | EMA – 474.80 Hz   | EMA – 474.80 Hz   |
|        | MAC – 97.5%       | MAC – 98.7%       |
| 3rd    | FEA – 857.81 Hz   | FEA – 887.50 Hz   |
|        | EMA – 876.62 Hz   | EMA – 876.62 Hz   |
|        | MAC – 77.0%       | MAC – 81.7%       |
| 4th    | FEA – 1033.4 Hz   | FEA – 1038.3 Hz   |
|        | EMA – 1038.0 Hz   | EMA – 1038.0 Hz   |
|        | MAC – 89.3%       | MAC – 94.5%       |
| 5th    | FEA – 1111.0 Hz   | FEA – 1130.6 Hz   |
|        | EMA – 1128.0 Hz   | EMA – 1128.0 Hz   |
|        | MAC – 93.7%       | MAC – 94.7%       |
4. Results and Discussion
As stated in the introduction section, the FBS method requires the FRF data instead of modal model data during the coupling process. Before performing the FBS analysis, the translational FRF data of the SFE model were derived and compared with the measured translational FRF counterparts for validation purposes. Later, the rotational FRF data were derived and then used to form a new FRF coupling matrix with complete degrees of freedom.

4.1. Synthesised FRF Validation
The translational FRF data at the connection points 1 and 2 of the updated SFE model were derived numerically by using the FRF synthesis method based on calculated modes. For this method, the synthesized FRF matrix $H_{\text{syn}}(\omega_k)$ and mode shapes are expressed in the form of:

$$H_{\text{syn}}(\omega_k) = \sum_{i=1}^{N} \frac{\{\phi\}_i^T \{\phi\}_i^T}{(\omega_{n_i}^2 - \omega_k^2) + j2\xi_i\omega_k\omega_{n_i}}$$  \hspace{1cm} (3)

where $N$ is the number of calculated modes, $\{\phi\}_i$ is the $i$th mass normalised mode shapes, $\omega_{n_i}$ is $i$th natural frequency and $\xi_i$ is the $i$th modal damping ratio. The synthesized FRF data were compared with the measured counterparts. Only the FRF data in the $z$-axis were used in the comparison. This is because all modes of the irregular plate largely are present in the $z$-direction. Figures 6, 7 and 8 show the comparisons between the FRF data of the $z$-axis FE and that of EMA at connection points 1, 2 and the cross axis FRF between points 1 and 2.
From Figures 6, 7, and 8, it can be clearly seen that all the resonance peaks of the synthesised FRF data have shown a perfect match with the measured FRF counterparts, especially at the peaks at connection point 1. Another striking point that can be observed in the synthesised FRF data from Figures 7 and 8 is that the anti-resonance peaks have shown minor discrepancies, particularly, at the peaks within the frequency range of 1000-2000Hz. The discrepancies may be as a result of excluding the effect of structural damping in normal modes analysis of the SFE model. However, the accuracy of the synthesized FRF data obtained from the SFE model is within an acceptable level. This is because the predicted resonance peaks that represent the natural frequencies have shown a strong correlation. The rotational FRF data derived from the SFE model are presented in Figures 9 and 10. Since the rotational FRF data were not measured in this study [23], therefore, the accuracy of the data will be validated with the predicted results of the FBS method.
4.2. *FBS method using the developed SFE model*

The FBS method was employed to couple the FRF data derived from the finite element model of the beam and the updated SFE model in order to evaluate the accuracy and efficiency of the proposed FRF coupling matrix. The coupling process of the finite element model and experimental model was carried out using the proposed FRF coupling matrix containing both translational and rotational FRF data required. The validation of the proposed FRF coupling matrix is presented in Figure 11 in which the FRF data of the proposed FRF coupling matrix are compared with that of experimental modal analysis of the assembled structure.

**Figure 9.** The derived rotational FRF data at connection point 1

**Figure 10.** The derived rotational FRF data at connection point 2

**Figure 11.** The comparison of the coupled and experimental FRF of the assembled structure

One good point that can be seen from the direct comparison between the proposed coupling matrix and experimental modal analysis as shown in Figure 11 is that there is a strong correlation between both
FRF data within the frequency bandwidth of 0Hz and 2000Hz. The strong correlation in terms of resonance and anti-resonance peaks indicates that the SFE model could be used accurately and efficiently in developing the FRF coupling matrix containing rotational and translational FRF data. Therefore, it is worth noting that the proposed FRF coupling matrix using the SFE model has offered a great capability to accurately predict the dynamic behaviour of the jointed structure.

Another striking point from Figure 11 is that there are only minor discrepancies in the resonance and anti-resonance peaks for the higher modes. The discrepancies are suspected of link with the stiffness determined and the type of the coupling used in modelling the bolted joints, which may be inadequate to accurately represent the actual physical properties of the bolted joints in the assembled structure[24,25]. This is a typical phenomenon encountered in the field of structural dynamics because accurately modelling of joints is very problematic and challenging [17,26].

5. Conclusions
A new FRF coupling of the FBS method for the investigation of the dynamic behaviour of the assembled structure consisting of a beam and an irregular plate has been proposed and evaluated with the experimental results. The comparison of results between the proposed FRF coupling and experimental counterparts reveal that the proposed FRF coupling has a great capability to accurately predict the dynamic behaviour of the assembled structure. This achievement suggests that the proposed FRF coupling has significantly reduced the high dependency of the FBS method on the experimental FRF data which are very difficult to accurately measure. The proposed FRF coupling of the FBS method could be applied quite reliably to other complex assembled structures without a significant declination in accuracy and efficiency.

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References
[1] Law M and Ihlenfeldt S 2015 A frequency-based substructuring approach to efficiently model position-dependent dynamics in machine tools *Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-body Dynamics* 229 304–17
[2] Wang J, Zeng X and Gasparini D a. 2008 Dynamic response of high-speed rail foundations using linear hysteretic damping and frequency domain substructuring *Soil Dynamics and Earthquake Engineering* 28 258–76
[3] Noh H M 2016 Prediction and reduction in vehicle noise by frequency response function-based substructuring *Advances in Mechanical Engineering* 8 1–13
[4] Mottershead J E, Ghandchi Tehrani M, Stanciou D, James S and Shahverdi H 2006 Structural modification of a helicopter tailcone *Journal of Sound and Vibration* 298 366–84
[5] Nicgorski D and Avitabile P 2010 Experimental issues related to frequency response function measurements for frequency-based substructuring *Mechanical Systems and Signal Processing* 24 1324–37
[6] de Klerk D, Rixen D J, Voormeren S N and Pasteuning F 2008 Solving the RDoF Problem in Experimental Dynamic Substructuring *Proceedings of the 26th International Modal Analysis Conference - IMAC*
[7] Van der Seijs M, van den Bosch D, Rixen D and de Klerk D 2014 An Improved Methodology for the Virtual Point Transformation of Measured Frequency Response Functions in Dynamic Substructuring *Proceedings of the 4th International Conference on Computational Methods in Structural Dynamics and Earthquake Engineering (COMPDYN 2013)* 4334–47
[8] Law M, Rentzsch H, Ihlenfeldt S and Putz M 2016 Application of Substructure Decoupling
Techniques to Predict Mobile Machine Tool Dynamics: Numerical Investigations *Procedia CIRP* **46** 537–40

[9] Mayes R L, Hunter P S, Simmermacher T W and Allen M S 2008 Combining Experimental and Analytical Substructures with Multiple Connections *26th IMAC: Conference and Exposition on Structural Dynamics 2008: IMAC XXVI* (Orlando, Florida)

[10] Drozg A, Čepon G and Boltežar M 2018 Full-degrees-of-freedom frequency based substructuring *Mechanical Systems and Signal Processing* **98** 570–9

[11] Mohd Zin M S, Rani M N A, Yunus M A, Wan Iskandar Mirza W I I, Mat Isa A A and Mohamed Z 2017 Modal and FRF Based Updating Methods for the Investigation of the Dynamic Behaviour of a Plate *Journal of Mechanical Engineering* **4** 175–89

[12] Mottershead J E and Friswell M I 1993 Model Updating In Structural Dynamics: A Survey *Journal of Sound and Vibration* **167** 347–75

[13] Mottershead J E, Link M and Friswell M I 2011 The sensitivity method in finite element model updating: A tutorial *Mechanical Systems and Signal Processing* **25** 2275–96

[14] Williams A, Chipman C and Avitabile P 2008 Modal and frequency based substructuring using rotational DOF conidetations *26th International Modal Analysis Conference* (IMAC XXVI)

[15] Cuppens K, Sas P and Hermans L 2001 Evaluation of the FRF based substructuring and modal synthesis technique applied to vehicle FE data *Proceedings of ISMA 25* 1143–50

[16] Wan Iskandar Mirza W I I, Rani M N A, Othman M H, Kasolang S and Yunus M A 2016 Reduced order model for model updating of a jointed structure *Journal of Engineering and Applied Sciences* **11** 2383–6

[17] Omar R, Rani M N A, Yunus M A, Mirza W I I W I and Zin M S M 2018 Efficient finite element modelling for the investigation of the dynamic behaviour of a structure with bolted joints *AIP Conference Proceedings* **020082** 020082

[18] Rani M N A, Kasolang S, Othman M H, Yunus M A and Wan Iskandar Mirza W I I 2016 Finite Element Modelling and Modal Based Up- Dating for The Dynamic Behaviour of a Laser Spot Welded Structure *ICSV 2016 - 23rd International Congress on Sound and Vibration: From Ancient to Modern Acoustics* pp 1–8

[19] Abdullah N A Z, Sani M S M, Husain N A, Rahman M M and Zaman I 2017 Dynamics properties of a Go-kart chassis structure and its prediction improvement using model updating approach *International Journal of Automotive and Mechanical Engineering* **14** 3887–97

[20] Fouzi M S M, Jelani K M, Nazri N A and Sani M S M 2018 Finite Element Modelling and Updating of Welded Thin-Walled Beam *International Journal of Automotive and Mechanical Engineering* **15** 5874–89

[21] Pastor M, Binda M and Harčarik T 2012 Modal assurance criterion *Procedia Engineering* **48** 543–8

[22] Rani M N A, Ouyang H, Yunus M A and Aminudin B A 2013 Model Updating for a Thin Steel Sheet Welded Structure *20th International Congress on Sound and Vibration 2013 (ICSV 2013)* **2** 1138–45

[23] Wan Iskandar Mirza W I I, Rani M N A, Yunus M A, Ayub M A, Sani M S M and Mohd Zin M S 2019 Frequency Based Substructurings for Structure with Double Bolted Joints: A Case Study *International Journal of Automotive and Mechanical Engineering* **16** 6188–99

[24] Omar R, M N A, Wan Iskandar Mirza W I I, Yunus M A and Othman M H 2017 Finite Element Modelling and Updating for Bolted Lap Joints *Journal of Mechanical Engineering* **4** 202–22

[25] Omar R, Rani M N A, Yunus M A, Mat Isa A A, Wan Iskandar Mirza W I I., Mohd Zin M S and Roslan L 2018 Investigation of Mesh Size Effect on Dynamic Behaviour of an Assembled Structure with Bolted Jointusing Finite Element Method *International Journal of Automotive and Mechanical Engineering* **15** 5695–708

[26] Zahari S N, Sani M S M, Husain N A, Ishak M and Zaman I 2016 Dynamic analysis of friction stir welding joints in dissipable material plate structure *Jurnal Teknologi* **78** 57–65