Relic gravitational waves and cosmic accelerated expansion

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Abstract. The possibility of reconstructing the whole history of the scale factor of the Universe from the power spectrum of relic gravitational waves (RGWs) makes the study of these waves quite interesting. First, we explore the impact of a hypothetical era -right after reheating- dominated by mini black holes and radiation that may lower the spectrum several orders of magnitude. Next, we calculate the power spectrum of the RGWs taking into account the present stage of accelerated expansion and an hypothetical second dust era. Finally, we study the generalized second law of gravitational thermodynamics applied to the present era of accelerated expansion of the Universe.

INTRODUCTION

The future detection of relic gravitational waves is expected to provide us with invaluable information about the instant of their decoupling from other fields, i.e., about $10^{-43}$ seconds after the Big Bang. The relic gravitational waves (RGWs) are generated by parametric amplification of the quantum vacuum during the expansion of the Universe.

The equation governing the evolution of the RGWs is the so-called Lifshitz equation

$$\mu''(\eta) + \left(k^2 - \frac{a''(\eta)}{a(\eta)}\right)\mu(\eta) = 0,$$

which can be interpreted as the equation of an harmonic oscillator parametrically excited by the term $a''/a$ [1]. When $k^2 \gg a''/a$, Lifshitz equation becomes the equation of the simple pendulum and consequently the amplitude of the wave decreases adiabatically as $a^{-1}$ in an expanding universe. In the opposite regime, $k^2 \ll a''/a$, the dominant solution is proportional to the scale factor and the amplitude remains constant with the expansion of the universe. This phenomenon is called “supera-diabatic” or “parametric” amplification of gravitational waves [2]. Another approach to the RGWs amplification relies on the method of Bogoliubov coefficients and uses the adiabatic vacuum approximation (for details see, e.g., [3]). Here we evaluate the number of RGWs created in the adiabatic vacuum approximation in a universe which experiences three successive stages of evolution: an initial de Sitter stage followed by a stage dominated by the radiation and, finally, a stage dominated by the non-relativistic matter up to the present day. The power spectrum, defined from the energy density of the RGWs as $d\rho_g(\omega) = P(\omega)d\omega$, reads [3, 4]

$$P(\omega) \sim \begin{cases} 0 & (\omega(\eta_0) > 2\pi(a_1/a_0)H_1), \\ \omega^{-1}(\eta_0) & (2\pi(a_2/a_0)H_2 < \omega(\eta_0) < 2\pi(a_1/a_0)H_1), \\ \omega^{-3}(\eta_0) & (2\pi H_0 < \omega(\eta_0) < 2\pi(a_2/a_0)H_2). \end{cases}$$

(1)
As is well known, mini black holes (MBHs) can be created by quantum tunnelling from the hot radiation and coexist with it until their evaporation [5]. It is reasonable to expect that, at this point, a steady state in the very early Universe would be achieved where the total energy density is shared between the black holes and radiation whence $\rho = \rho_{BH} + \rho_R$ and, consequently, the total pressure is $p = p_R = (\gamma - 1)\rho$, where the constant $\gamma$ lies in the interval $1 \leq \gamma < 4/3$. If the density of MBHs is large enough to dominate the expansion of the Universe, then $\gamma \simeq 1$. In the opposite case, the Universe expansion is dominated by the radiation, $\gamma \simeq 4/3$. During the “MBHs+rad” era, one finds from the Einstein equations that $a(\eta) \propto \eta^l$, where $l = 2/(3\gamma - 2)$ and $1 < l \leq 2$ [6].

According to this, it seems reasonable to assume a four-stage model of universe, initially de Sitter, then dominated by a mixture of MBHs and radiation, then dominated by the radiation after the evaporation of the MBHs and finally dominated by dust until today. The only free parameters considered here are $l$, the duration of the “MBHs+rad” era, $\tau$, and the Hubble factor in the de Sitter era, $H_1$.

The power spectrum for this four-stage scenario is plotted in figure 1. The spectrum predicted for the three-stage model of the previous section is shown for comparison (labelled as $l = 1$). Parameters $\tau$ and $H_1$ are chosen assuming that each spectrum has the maximum value allowed by the CMB anisotropy data at the frequency $\omega = 2\pi H_0 = 2.24 \times 10^{-18} s^{-1}$. The predicted power spectrum of the four-stage model is lower than the three-stage one by several orders of magnitude. In the plot of the bottom-right panel the power spectrum for $l = 2$ is excluded as it predicts a RGWs energy density at the matter-radiation decoupling that generates via the Sach-Wolfe effect a CMB anisotropy larger than observed.

Some models of dark energy predict that the present accelerated phase of cosmic expansion governed by $a(\eta) \propto \eta^l$ with $l \leq -1$ is transitory and that the expansion, sooner or later, will be dominated by “dust” again [7]. This five-stage model of universe (de Sitter-radiation era-dust era-dark energy era-“second dust” era) predicts a current RGWs power spectrum that is not at variance with the one of the three-stage scenario but evolves differently. As the universe expands in the three-stage model the Hubble volume is continuously increasing and new RGWs are reentering it and contributing to the power spectrum. Meanwhile, in the dark energy scenario during the accelerated era the Hubble volume decreases, the RGWs begin to leave it and cease to contribute to the power spectrum [8]. Once the universe reaches the second dust era the Hubble radius begins to grow again and the RGWs reenter it in a $l$ dependent way. Figure 2 shows the evolution of the RGWs density parameter, defined as $\Omega_g = \rho_g/\rho_c$.

1 These models predict that the dark energy will evolve in a way that mimics the expansion of a universe dominated by non relativistic matter.
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During the present accelerated era of expansion the number density of RGWs is decreasing with time and, eventually, all the RGWs will leave the Hubble radius. It seems reasonable to assume that the entropy of the RGWs depends on their number present in the considered volume [10]. Thus, the entropy of the RGWs decreases with time. In the simplest assumption the entropy density of RGWs is proportional to their number density, i.e. \( s_g = A n_g \). The generalized second law of gravitational thermodynamics\(^2\) will be satisfied if the constant \( A \) is lower than certain bound, which is model dependent [9]. This is the only information at our disposal on this proportionality constant.

CONCLUSIONS

The MBH four-stage scenario predicts a much lower power spectrum than the conventional three-stage scenario for the same \( H_1 \). The free parameters of this scenario are constrained by the CMB anisotropy data. Although the current power spectrum of the RGWs in the dark energy four-accelerated scenario is not at variance with that of the three-stage scenario, the RGWs density parameter evolves differently. Its future evolution may also help discern between different dark energy decaying models. Assuming that the entropy density of the RGWs is proportional to their number density, the GSL is fulfilled provided a condition over the proportionality constant is met.

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\(^2\) According to the generalized second law (GSL) of gravitational thermodynamics, the entropy of the event horizon plus its surroundings (in our case, the entropy in the volume enclosed by the horizon) cannot decrease.
FIGURE 1. GWs spectrum for an expanding universe with a “MBHs+rad” era for certain values of $l$, $\tau$ and $H_1$.

FIGURE 2. Evolution of the density parameter $\Omega_g$ with the scale factor in the five-stage scenario (De Sitter inflation-radiation-dust-dark energy-second dust era) from the beginning of the dark energy era. The solid, dotted and dashed lines correspond to $l = -1$, $l = -1.5$ and $l = -2$, respectively.