Direct Messenger-Matter Interactions in Gauge-Mediated Supersymmetry Breaking Models

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Abstract

We categorize general messenger and matter interactions in gauge-mediated SUSY breaking models by an $R$-parity for the messengers and study their phenomenological consequences. The new interactions may induce baryon- and lepton-number violating processes as well as flavor-changing neutral currents. Bounds on the couplings from low-energy data are generally weak due to the large messenger mass suppression, except for the constraint from proton decay. The soft masses for the scalar particles receive negative corrections from the new interactions. Consequently, in certain region of SUSY parameter space the $\mu$-parameter is greatly reduced. The pattern of radiative electroweak symmetry breaking, SUSY particle mass spectrum and decay channels are also affected, leading to observable experimental signature at the current and future colliders.

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1 Introduction

A model with gauge-mediated supersymmetry breaking (GMSB) \([1]\) is a simple and well-motivated version of the minimal supersymmetric extension of the Standard Model (MSSM). In addition to the observable sector and a supersymmetry (SUSY) breaking hidden sector, the model also possesses messenger fields which mediate the SUSY breaking to the observable fields via the SM gauge interactions. The “minimal” model has a pair of messengers transforming under the SU(5) representation \(5 + \overline{5}\), decomposed as color triplets \((D + \overline{D})\) and weak doublets \((L + \overline{L})\). They couple to a gauge singlet field \(S\) through a superpotential

\[
W_{\text{minimal}} = \lambda (SD\overline{D} + SL\overline{L}),
\]

where \(S\) acquires non-zero vacuum expectation values for both its scalar component \((\langle S \rangle)\) and auxiliary component \((F)\).

There are several attractive features in this minimal model. First, all supersymmetric particle masses are determined by two parameters: the messenger scale \(M = \lambda \langle S \rangle\) (the messenger fermion mass) and the effective SUSY breaking scale \(\Lambda = F/\langle S \rangle\). The gaugino and scalar soft masses are given, at one- and two-loop level respectively, by

\[
M_i(M) \approx \frac{\alpha_i(M)}{4\pi} \Lambda, \quad i = 1, 2, 3
\]

\[
\tilde{m}^2(M) \approx 2 \sum_{i=1}^{3} C_i \left(\frac{\alpha_i(M)}{4\pi}\right)^2 \Lambda^2,
\]

where \(C_i\)'s are \(4/3, 3/4\) for the fundamental representations of SU(3), SU(2) and \(3Y^2/5\) for U\(_Y\)(1). If \(\Lambda \sim \mathcal{O}(10 - 100\text{ TeV})\), the SUSY particles (sparticles) can have a desirable mass spectrum of \(\mathcal{O}(100\text{ GeV})\). Second, since the scalar masses are degenerate in the family space, the flavor-changing neutral current (FCNC) and CP-violation in SUSY sector are generally small. Finally, the gravitino mass is typically of \(\mathcal{O}(\text{keV})\) (assuming only one SUSY breaking scale), so it is the lightest supersymmetric particle in this model. Consequently, the lightest neutralino promptly decays to a gravitino plus a photon via an enhanced gravitational interaction. This would have significant implications for collider phenomenology and particle cosmology \([3]\).

However, there may be cosmological problems in the messenger sector \([3]\). Due to the conserved “messenger number” in Eq. \([3]\), the lightest messenger particle (LMP) is stable. Although naturally neutral \([3]\) in most of the SUSY parameter space, the LMP would have to be lighter than a few TeV in order not to overclose the Universe \([3, 4]\) in the standard inflationary cosmology. On the other hand, direct searches have already excluded a scalar dark matter particle with a mass less than about 3 TeV at a 90% confidence level, assuming it accounts for more than about 30% of a galactic halo with local density \(0.3\ \text{GeV/cm}^3\) \([5]\). One would have to introduce the messenger-Higgs mixing, along with a gauge singlet, to evade the direct dark matter detection \([4]\). However, with those additional interactions, one may run into the \(\mu-B\mu\) problem \([1, 3, 4]\). Besides, such a low-mass LMP needs a certain degree of fine-tuning.

One possible solution to the problem is to abandon the messenger number conservation by introducing direct messenger-matter interactions. In fact, it is natural to consider this
possibility since the messengers intrinsically carry the SM gauge quantum numbers. In this paper, we study general interactions between the messengers and MSSM fields. In Sec. 2 we present the most general superpotential in the framework of the minimal GMSB model and categorize different terms with an $R$-parity for the messengers. We then derive the effective Lagrangian at low energies and examine the current experimental constraints on the couplings in Sec. 3. We also study the theoretical implications on the SUSY particle mass spectrum, the electroweak symmetry breaking (EWSB) and the $\mu$-parameter. In Sec. 4 we make some general remarks and draw our conclusion.

2 General Messenger-Matter Interactions

The most general superpotential with direct messenger-matter interactions allowed by the SM gauge symmetry is

$$W_{\text{mix}} = H_D L_4 \bar{E} + H_D Q \bar{D}_4 + LL_4 \bar{E} + Q L_4 \bar{D} + Q L_4 \bar{U} + E \bar{U} D_4 + L Q \bar{D}_4 + Q Q D_4 + U \bar{D} \bar{D}_4 + H_U L_4 + H_D \bar{L}_4 + L \bar{L}_4 + \bar{D} D_4 + Q L_4 \bar{D}_4,$$

where we have suppressed the Yukawa coupling constant for each term and the generation indices for the superfields. A subscript “4” has been introduced for the messenger fields in Eq. (1) as $(D_4, L_4)$ and $(\bar{D}_4, \bar{L}_4)$, in analogue to the three-generation matter fields.

At this superpotential level, the bilinear terms can be rotated away at no cost by properly redefining the superfields, so we will not consider them any further. We will also ignore the state mixing among the messengers and MSSM fields associated with those rotations. The last term in Eq. (3) is the only one which involves two messenger fields. When examining its physical consequences at low energies by integrating out the heavy messenger fields, the resulting operators would be more suppressed. Although this term respects the SM gauge symmetry, it does not naturally arise in an SU(5) unification theory. We will not discuss this term further.

To classify the remaining terms in Eq. (4), we recall that the superfields $(H_U, H_D)$ have a positive (matter) $R$-parity assignment while the others $(L, \bar{E}, Q, \bar{U}, \bar{D})$ are negative. It follows that the first two terms in Eq. (4) have different $R$-parity property from the rest. It is therefore convenient to categorize these two groups by their $R$-parities. If we formally require the $R$-parity conservation, we then can generalize the ordinary $R$-parity to the messengers. The two possibilities are: messenger superfields with a positive $R$-parity, which we will call the $M^+$-model, and messenger superfields with a negative $R$-parity, the $M^-$-model.

2.1 The $M^+$-model

If we assign the messenger superfields $(D_4, \bar{L}_4)$ and $(\bar{D}_4, L_4)$ with a positive $R$-parity, then $R$-parity invariance leads to the following interaction terms

$$W_{\text{mix}}^+ = y_{ij} \bar{E}_i L_j L_4 + y'_{ij} \bar{D}_i Q_j L_4 + y''_{ij} \bar{U}_i Q_j \bar{L}_4 + \lambda_{ij} L_i Q_j \bar{D}_4 + \frac{1}{2} \lambda'^{ij} Q_i Q_j D_4 + \lambda''_{ij} \bar{U}_i \bar{D}_j \bar{D}_4,$$
where $y_{ij}, y'_{ij}, y''_{ij}$ and $\lambda_{ij}, \lambda'_{ij}, \lambda''_{ij}$ are Yukawa couplings naturally of order one, with $i, j = 1, 2, 3$ as generation indices. Note that $\lambda''_{ij} = -\lambda'_{ij}$. According to the conventional $R$-parity assignment for component fields, $(-1)^{2S+3(B-L)}$, where $S$ is the particle spin, $B$ and $L$ the baryon- and lepton-number respectively, the $R$-parity so assigned corresponds to that assuming zero $B$- and $L$-numbers for the messenger superfields. The first three terms conserve $B$ and $L$, but generate FCNC processes in general. Although Eq. (3) preserves $R$-parity by definition, the $\lambda^l, \lambda'$ terms in (3) violate $B$ and $L$ by $\Delta B = 1/3$ and $\Delta L = 1$, and the $\lambda^q, \lambda''$ terms violate $B$ by $\Delta B = 2/3$. Simultaneous existence of $\lambda^l$ and $\lambda^q$ or $\lambda'$ and $\lambda''$ may induce abrupt proton decay. We will discuss this point in the next section. Note that the terms of $y_{ij}, \lambda'_{ij}, \lambda''_{ij}$ are the direct analogue to those $R$-parity violating interactions of $\lambda_{ijk}, \lambda_{ijk}', \lambda_{ijk}''$ in the MSSM.

### 2.2 The $M^-$-model

If we instead assign the messenger superfields with a negative parity, then $R$-parity invariance excludes the terms in Eq. (3) and we are left with the two terms involving Higgs superfields

$$W_{\text{mix}} = y_i H_D L_4 \bar{D}_i + y'_i H_D Q_i \bar{D}_4.$$  \hspace{1cm} (6)

This $R$-parity assignment is equivalent to assuming the messenger superfields $(\bar{D}_4, L_4)$ to carry the same $B$- and $L$-numbers as the MSSM superfields $(\bar{D}, L)$. Although these two terms do not induce any $B$- or $L$-violating processes, the messenger couplings to the MSSM fields will mediate FCNC processes if more than one $y_i$ (or $y'_i$) coexists.

It is interesting to note that in either $R$-parity assignment ($M^+$ or $M^-$), $R$-parity invariance forbids the $QL_4 \bar{D}_4$ term in Eq. (4).

### 3 Physical Implications

#### 3.1 Effective Lagrangian and Low-Energy Constraints

By integrating out heavy messengers, we can obtain low-energy effective Lagrangian in terms of $1/M^2$ expansion. For simplicity, we only examine the leading operators by assuming $\Lambda/M \ll 1$ and the particles and Higgs bosons to be much heavier than the energy scale considered.

In the $M^+$-model of Eq. (5), the first three terms ($y$-couplings) result in the following four-fermion operators

$$L_y^+ \quad = \quad \frac{y_{ij} y_{ij}'}{2M^2} \left( \bar{e}_{iR} \gamma^\mu e_{iR} \bar{e}_{jL} \gamma^\nu e_{jL} + \bar{e}_{iR} \gamma^\mu e_{iR} \bar{e}_{jL} \gamma^\nu e_{jL} \right)$$

$$\quad + \quad \frac{y'_{ij} y''_{ij}}{2M^2} \left( \bar{d}_{iR} \gamma^\mu d_{iR} \bar{d}_{jL} \gamma^\nu d_{jL} + \bar{d}_{iR} \gamma^\mu d_{iR} \bar{d}_{jL} \gamma^\nu d_{jL} \right)$$

---

1. In the direct-transmission model, the messengers are also generally charged under the dynamical group, the interaction can arise from higher dimensional operators involving the dynamical sector fields. As a result, the couplings would generically be much smaller.

2. One could instead assign the messengers with appropriate $B$- or $L$-numbers to eliminate terms inducing proton decay, regardless of the $R$-parity assignment. We thank X. Tata for a discussion on this point.
\[ + \left[ \frac{y_{ij} y'_{j'j}}{2M^2} \left( d_{iR} \sigma^\mu e_{j'jL} \gamma_\mu d_{jL} + d_{iR} \sigma^\mu e_{j'jR} \nu_{j'L} \gamma_\mu u_{jL} \right) + \text{h.c.} \right] \]
\[ + \frac{y_{ij} y'_{j'j}}{2M^2} \left( \frac{u_{iR} \sigma^\mu u_{j'jL} \gamma_\mu u_{jL}}{2M^2} + \frac{u_{iR} \sigma^\mu u_{j'jR} \gamma_\mu u_{j\gamma L}}{2M^2} \right), \]
\[ \text{where } \alpha, \beta \text{ are color indices. Similarly, we can obtain the effective four-fermion operators for the last four terms (} \lambda \text{-couplings) in Eq. (5) to leading order of } 1/M^2, \]
\[ \mathcal{L}_\lambda^+ = \frac{\lambda_{ij} \lambda_{ij}^*}{2M^2} \left( e_{iL} \gamma^\mu e_{j'jL} \gamma_\mu u_{jL} \right) - \left( \frac{\lambda_{ij} \lambda_{ij}^*}{2M^2} \left( e_{iL} \gamma^\mu u_{j'jL} \gamma_\mu d_{jL} \right) + \text{h.c.} \right) \]
\[ + \frac{\lambda_{ij} \lambda_{ij}^*}{2M^2} \left( e_{iL} \gamma^\mu u_{j'jL} \gamma_\mu d_{jL} \right) \]
\[ + \frac{\lambda_{ij} \lambda_{ij}^*}{2M^2} \left( u_{iR} \gamma_\mu u_{j'jL} \gamma_\mu d_{jL} \right) \]
\[ + \frac{\lambda_{ij} \lambda_{ij}^*}{2M^2} \left( u_{iR} \gamma_\mu u_{j'jL} \gamma_\mu d_{jL} \right) + \frac{\epsilon_{\alpha \beta \gamma} \lambda_{ij} \lambda_{ij}^*}{2M^2} \left( e_{iL} \gamma^\mu u_{j'jL} \gamma_\mu d_{jL} \right) \]
\[ - \frac{\lambda_{ij} \lambda_{ij}^*}{2M^2} \left( e_{iL} \gamma^\mu u_{j'jL} \gamma_\mu d_{jL} \right) + \text{h.c.} \right). \]
\[ \text{These effective operators are similar to those obtained from the } R \text{-parity violating interactions in the MSSM and may lead to very rich physics. The coupling coefficients (} y' \text{s and } \lambda \text{s) are considered to be naturally of order one. However, if we consider only one term at a time with given generation indices in Eq. (5), there is essentially no significant experimental constraint on them because of the suppression by the large messenger mass } M \approx \mathcal{O}(100 \text{ TeV}). \text{ For the same reason, none of the terms would lead to observable signature in the current and near-future experiments. Even when several terms with different generation indices coexist, most effects of those operators in Eqs. (7) and (8) are still rather weak in general. For example, the modifications on charged current universality and on various } \tau, D \text{ and } B \text{ decays are too small to be observable unless the couplings } y_{ij}, \lambda_{ij}, \lambda_{ij}^* > \mathcal{O}(100). \text{ On the other hand, there are processes from rare and SM forbidden decays and from neutral meson mixing that can be sensitive to test certain operators with the accuracy of current and near-future experiments. For instance, considering } \mu \to e\gamma \text{ with one-loop diagrams via the virtual messenger exchange, we obtain} \]
\[ |y_{ij} y_{j'j}| < 3 \left( \frac{M}{100 \text{ TeV}} \right)^2. \]
\[ \text{The most stringent bound on } y' \text{ comes from the } K_L-K_S \text{ mass difference:} \]
\[ |y'_{12} y'_{21}| < 5 \times 10^{-4} \left( \frac{M}{100 \text{ TeV}} \right)^2, \]
\[ \text{which is at a very interesting level to constrain the theory. The } D^0 \text{ mass difference constrains the operators } y'_{12} y'_{21}, \text{ while the } B^0 \text{ mass difference bounds the operators with the} \]
third generation index $y_{13} y'_{31}$. It is also interesting to note that the operators $y_{ij} y_{i'j'}$ contributing to $K_L \rightarrow \ell^+ \ell^-$ are of the nature of a pseudoscalar current, so there is no explicit lepton mass dependence in the decay width, unlike the $R$-parity violating interactions \[1\] where there is essentially no sensitivity to new physics for the decay $K_L \rightarrow e^+ e^-$. This feature may serve as a criterion to distinguish different models if a signal beyond the SM is observed.

The last three operators in Eq. (8) mediate proton decay such as $p \rightarrow e^+ \pi^0(K^0), \mu^+ \pi^0(K^0)$ and $\nu \pi^+(K^+)$. Requiring the proton lifetime to be larger than $10^{32}$ years puts very stringent bounds to them: products of two appropriate couplings are restricted at order of $10^{-21}$ for a 100 TeV messenger. It is therefore unlikely for the operators $\lambda^l, \lambda^q$ or $\lambda', \lambda''$ to coexist. In Table 1, we summarize the meaningful bounds on the products of two different couplings in Eqs. (7) and (8), along with the corresponding experimental data \[12\]. Future high precision measurements would explore the operators to a more significant level.

In the $M^R$-model, by integrating out the messenger and Higgs fields, one may obtain fermionic bilinear terms. However, they are not only suppressed by the heavy masses $M$ and $m_h$, but also by chirality. They are generally small and we will not discuss them.

| couplings | bounds | low-energy data |
|-----------|--------|-----------------|
| $|y_{1j} y_{2j}|, |y_{1j} y_{2j}|$ | 3 | $BR(\mu \rightarrow e\gamma) < 4.9 \times 10^{-11}$ |
| $|y_{1j} y_{2j}|, |y_{1j} y_{2j}|$ | 0.7 | $BR(\mu \rightarrow 3e) < 10^{-12}$ |
| $|y_{1j} y_{2j}|, |y_{1j} y_{2j}|$ | 10 | $\mu$ Ti(Pb) $\rightarrow e$ Ti(Pb) ($g^l_{\mu} < 4 \times 10^{-5}$) |
| $|y_{1j} y_{2j}|, |y_{1j} y_{2j}|$ | 0.05 | $BR(K_L \rightarrow \mu^+ e^-) < 3.3 \times 10^{-11}$ |
| $|y_{1j} y_{2j}|, |y_{1j} y_{2j}|$ | 0.06 | $BR(K_L \rightarrow e^+ e^-) < 4.1 \times 10^{-11}$ |
| $|y_{1j} y_{2j}|, |y_{1j} y_{2j}|$ | 0.8 | $BR(K_L \rightarrow \mu^+ \mu^-) < 7.2 \times 10^{-9}$ |
| $|y_{1j} y_{2j}|, |y_{1j} y_{2j}|$ | $5 \times 10^{-4}$ | $\Delta M_K = 3.491 \times 10^{-12}$ MeV |
| $|y_{1j} y_{2j}|, |y_{1j} y_{2j}|$ | 0.03 | $\Delta M_D < 1.38 \times 10^{-10}$ MeV |
| $|y_{1j} y_{2j}|, |y_{1j} y_{2j}|$ | 0.03 | $\Delta M_B = 3.12 \times 10^{-10}$ MeV |
| $|y_{1j} y_{2j}|, |y_{1j} y_{2j}|$ | $3 \times 10^{-6}$ | $|e_K| \approx 2.275 \times 10^{-3}$ |
| $|\lambda_{11}^{l1} \lambda_{12}^{l1}|, |\lambda_{11}^{l1} \lambda_{12}^{l1}|, |\lambda_{11}^{l1} \lambda_{12}^{l1}|, |\lambda_{11}^{l1} \lambda_{12}^{l1}|$ | 10 | $\mu$ Ti(Pb) $\rightarrow e$ Ti(Pb) ($g^l_{\mu} < 4 \times 10^{-5}$) |
| $|\lambda_{11}^{l1} \lambda_{12}^{l1}|, |\lambda_{11}^{l1} \lambda_{12}^{l1}|, |\lambda_{11}^{l1} \lambda_{12}^{l1}|, |\lambda_{11}^{l1} \lambda_{12}^{l1}|$ | 3 | $BR(\mu \rightarrow e\gamma) < 4.9 \times 10^{-11}$ |
| $|\lambda_{11}^{l1} \lambda_{12}^{l1}|, |\lambda_{11}^{l1} \lambda_{12}^{l1}|, |\lambda_{11}^{l1} \lambda_{12}^{l1}|, |\lambda_{11}^{l1} \lambda_{12}^{l1}|$ | $10^{-21}$ | $\tau(p \rightarrow e^+ \pi^0; e^+ K^0) > 10^{32}$ yr |
| $|\lambda_{11}^{l1} \lambda_{12}^{l1}|, |\lambda_{11}^{l1} \lambda_{12}^{l1}|, |\lambda_{11}^{l1} \lambda_{12}^{l1}|, |\lambda_{11}^{l1} \lambda_{12}^{l1}|$ | $10^{-21}$ | $\tau(p \rightarrow \mu^+ \pi^0; \mu^+ K^0) > 10^{32}$ yr |
| $|\lambda_{11}^{l1} \lambda_{12}^{l1}|, |\lambda_{11}^{l1} \lambda_{12}^{l1}|, |\lambda_{11}^{l1} \lambda_{12}^{l1}|, |\lambda_{11}^{l1} \lambda_{12}^{l1}|$ | $10^{-21}$ | $\tau(p \rightarrow \nu \pi^0; \nu K^0) > 10^{32}$ yr |

Table 1: Bounds on the couplings from low-energy experimental data \[12\] (and $\mu-e$ conversion from \[13\]), in units of $(M/100 \text{TeV})^2$.

### 3.2 Sparticle Mass Spectrum

While the masses of MSSM fermions are protected from the radiative corrections either by a chiral symmetry or by a continuous $R$-symmetry, the scalar masses squared receive large negative corrections from the messenger-matter interactions in Eqs. (3) and (8). They are
given by, for the $M^+$-model,

\[
\delta m_{E_{ij}}^2 = \frac{1}{16\pi^2} \sum_{k=1}^{3} y_{ki} y_{kj}^* M^2 u(\Lambda/M), \tag{11}
\]

\[
\delta m_{E_{ij}}^2 = \frac{1}{8\pi^2} \sum_{k=1}^{3} y_{ki} y_{kj}^* M^2 u(\Lambda/M), \tag{12}
\]

\[
\delta m_{Q_{ij}}^2 = \frac{1}{16\pi^2} \sum_{k=1}^{3} \left(y''_{ki} y''_{kj}^* + y''_{ki} y''_{kj}^*\right) M^2 u(\Lambda/M), \tag{13}
\]

\[
\delta m_{U_{ij}}^2 = \frac{1}{8\pi^2} \sum_{k=1}^{3} y''_{ik} y''_{jk}^* M^2 u(\Lambda/M), \tag{14}
\]

\[
\delta m_{D_{ij}}^2 = \frac{1}{8\pi^2} \sum_{k=1}^{3} y''_{ik} y''_{jk}^* M^2 u(\Lambda/M), \tag{15}
\]

where we have ignored the $\lambda$-couplings in Eq. (5), and for the $M^-$-model \cite{7, 9},

\[
\delta m_{E_{ij}}^2 = \frac{1}{8\pi^2} y_{i} y_{j}^* M^2 u(\Lambda/M), \tag{16}
\]

\[
\delta m_{Q_{ij}}^2 = \frac{1}{16\pi^2} y_{i} y_{j}^* M^2 u(\Lambda/M), \tag{17}
\]

\[
\delta m_{H_D}^2 = \frac{1}{16\pi^2} \sum_{i=1}^{3} \left(|y_i|^2 + 3|y_i'|^2\right) M^2 u(\Lambda/M), \tag{18}
\]

where the function

\[
u(x) = \ln(1 - x^2) + \frac{x}{2} \ln \frac{1 + x}{1 - x}. \tag{19}\]

For small $x$, the function has an expansion $-x^4/6$, and it monotonically decreases as $x$ goes to 1.

For $\Lambda/M \ll 1$ ($x \to 0$), the corrections are suppressed by $(\Lambda/M)^2$, so that there is no significant difference from the minimal model for sparticle spectrum. On the other hand, for $\Lambda/M \simeq 1$ ($x \to 1$), the function $u$ is very negative and the corrections to the mass can be very substantial. Significant upper limits on the Yukawa couplings may be obtained by requiring that these negative corrections do not change the sign of the scalar mass squared. In GMSB models, the slepton and Higgs soft masses are generically smaller than the squark soft masses, so the tightest bounds come from Eqs. (11), (12), (16) and (18). As an illustration, we choose

\[
\Lambda = 100 \text{ TeV}, \quad \tan \beta = 2 \quad \text{and} \quad \mu > 0. \tag{20}\]

By requiring the scalar mass squared to remain positive, we find that typical upper bounds on $\sum_{i=1}^{3} |y_i|^2$ in the $M^-$-model for several $M/\Lambda$ values are

\[
\frac{M}{\Lambda} = \begin{array}{cccc}
1.25 & 5 & 10 & 30 \\
\end{array} \\
\sum_{i=1}^{3} |y_i|^2 < \begin{array}{cccc}
10^{-3} & 0.03 & 0.15 & 1.2 \\
\end{array} \tag{21}\]
Similar constraints are also obtained for \( \sum_{i=1}^{3} |y'_i|^2 \) and for the couplings in the \( M^+ \)-model. Although the bounds obtained here depend on the model parameter \( M/\Lambda \), they are the only upper bounds available on individual couplings. They are therefore complementary to those extracted from the low-energy data in the previous section.

The negative corrections to the scalar masses squared can also induce miss-alignments of the fermion-sfermion mass matrices and as a result, the flavor-changing neutral currents. A study \[9\] found that bounds on the products of two couplings from \( \mu \rightarrow e\gamma \) and \( \mu - e \) conversion can be as strong as \( 10^{-5} \) for \( \Lambda/M \approx 1 \). However, the bounds obtained there depend again sensitively on the parameter choice, and they are much looser for \( \Lambda/M \ll 1 \).

### 3.3 Electroweak Symmetry Breaking and The \( \mu \)-Parameter

One of the most important features in SUSY theories is the radiative generation of the electroweak symmetry breaking (EWSB) \[14\]. At the scale \( M_{\text{SUSY}} \) where the EWSB is imposed, the Higgs soft mass squared \( m_{H_U}^2 \) is approximately given by the solution to the one-loop renormalization group equation

\[
m_{H_U}^2(M_{\text{SUSY}}) \simeq m_{H_U}^2(M) - \frac{3\lambda_t^2}{8\pi^2} (m_{\tilde{Q}_3}^2 + m_{\tilde{U}_3}^2) \ln \left( \frac{M}{M_{\text{SUSY}}} \right),
\]

where for simplicity \( \lambda_b \) term has been neglected. The large top-quark Yukawa coupling \( \lambda_t \) can drag \( m_{H_U}^2 \) at \( M_{\text{SUSY}} \) negative, thus triggers the EWSB.

\[
\tan\beta=2 \quad \Lambda=100 \text{ TeV} \quad M/\Lambda=1.25
\]

![Figure 1: Representative masses as functions of the messenger-matter coupling \( y'_{33} \). The hatched region does not have the right EWSB. Shown in the plots are the third generation squarks \( \tilde{q} \), the \( \mu \)-parameter, the lightest neutralino \( \tilde{\chi}_1^0 \) and the lightest CP-even and CP-odd Higgs bosons \( h, A \).](image)

However, the messenger-matter interactions give large negative corrections to the scalar masses squared, as seen in Eqs. \[11\]-\[13\] and Eqs. \[16\]-\[18\]. When the third generation squark masses become small, the Higgs mass squared is less negative, and the EWSB can be changed significantly or may not even occur. A more restrictive version of the
EWSB condition in a supersymmetric theory is usually expressed in the following (tree-level) equation which also determines the \( \mu \)-parameter

\[
\mu^2 = \frac{m_{H_D}^2 - \tan^2 \beta \ m_{H_U}^2}{\tan^2 \beta - 1} - \frac{M_Z^2}{2} \quad .
\] (23)

Requiring the model to yield a desirable pattern of EWSB would put additional constraints on the couplings. We consider the mass correction effects on EWSB in \( M^+ \)-model to Eqs. (22) and (23). We choose the SUSY parameters as in Eq. (20). To maximize the effects from messenger-matter interactions, we also choose that \( M/\Lambda = 1.25 \) and \( y_{33}' = y_{33}'' \). We have run the coupled two-loop renormalization group equations \[15\] of soft masses to the scale \( M_{SUSY} = m_{\tilde{q}}^{GM} + m_{\tilde{q}}^{mix} \), where \( m_{\tilde{q}}^{GM} \) and \( m_{\tilde{q}}^{mix} \) represent the contributions to the third generation squark masses from the minimal GMSB model (Eq. (3)) and messenger-matter interactions (Eqs. (13-15)) respectively. At \( M_{SUSY} \) we impose the EWSB condition and calculate all physical masses and the \( \mu \)-parameter consistently to the full one-loop order.

In Fig. 1 we show our results for the SUSY particle mass spectrum. The hatched region is where the EWSB does not occur. This can be anticipated from looking at the decreasing squark masses, which eventually become too small to drive the Higgs mass squared of Eq. (22) negative. The limits on the individual Yukawa couplings obtained here are similar to that in Eq. (21) and are complementary to those from the low-energy experiments discussed in the previous section. We note that the \( \mu \)-parameter decreases as the coupling increases.

\[
\tan \beta = 2 \quad \Lambda = 100 \text{ TeV} \quad M/\Lambda = 1.25
\]

Figure 2: Dimensionless quantities in Eq. (24) which describe the degree of fine-tuning as functions of \( y_{33}' \).

Models with direct messenger-matter interactions can display mass spectrum with very different characteristics from that of the minimal GMSB model. To demonstrate this point, we show a representative mass spectrum in Table 2, where we choose \( y_{33}'^2 = y_{33}''^2 = 0.032 \) and all other parameters are taken to be the same as Fig. 1. For comparison, we also show that for the minimal GMSB model with no direct messenger-matter interactions.
\( y'_{33} = y''_{33} = 0 \) and that for \( y'_{33} = y''_{33} = 0.032 \). Also shown in the Table are the \( \mu \)-parameter, the three gaugino soft masses \( M_1, M_2, M_3 \), branching ratios for \( \tilde{\chi}_0^1 \rightarrow \gamma \tilde{G} \) and \( \tilde{\chi}_0^1 \rightarrow h \tilde{G} \), and \( \tilde{\chi}_0^1 \) decay length.

Table 2: Representative masses for the two models: minimal GMSB with \( y'_{33} = y''_{33} = 0 \), and that for \( y'_{33} = y''_{33} = 0.032 \). Also shown in the Table are the \( \mu \)-parameter, the three gaugino soft masses \( M_1, M_2, M_3 \), branching ratios for \( \tilde{\chi}_0^1 \rightarrow \gamma \tilde{G} \) and \( \tilde{\chi}_0^1 \rightarrow h \tilde{G} \), and \( \tilde{\chi}_0^1 \) decay length.

The masses for the third generation squarks, the neutralino/chargino and especially the Higgs bosons are all significantly lighter than those for the minimal model. The lightest neutralino \( \tilde{\chi}_0^1 \) has a large Higgsino component, so it can decay to the light Higgs \( h \) with a fairly large branching ratio of 26%. The decay length of \( \tilde{\chi}_0^1 \) becomes somewhat shorter as well. These interesting features may lead to very distinctive experimental signature in the current and future collider experiments. The slepton masses can be decreased significantly as well depending on the choice of the couplings \( y' \)’s.

In the minimal GMSB model, because the squarks are much heavier than other sparticles, one finds that \( \sqrt{|m^2_{H_U}|} \) and therefore \( \mu \) are typically much larger than \( M_Z \). This renders a difficult balance of Eq. (23) and is usually referred to as the fine-tuning problem in GMSB models [16]. In the presence of messenger-matter interactions, one should expect that \( \mu \) is generally smaller (as seen from Fig. 1), and the fine-tuning problem should be less severe. We examine the dimensionless quantities as a measure of fine-tuning [17]:

\[
c(M_Z^2; \mu^2) = |\partial \ln M_Z^2 / \partial \ln \mu^2|, \quad \text{and} \quad c(M_Z^2; B_\mu) = |\partial \ln M_Z^2 / \partial \ln B_\mu|.
\]

(24)

where \( B_\mu \) is the bilinear soft Higgs mass parameter. The results are shown in Fig. 2. Indeed
the fine-tuning improves for a bigger coupling $y'_{33}$.

4 Discussions and Conclusion

Before we draw our conclusions, several remarks are in order. First, in constructing the low-energy effective Lagrangian in the previous section, we have ignored terms proportional to $\Lambda/M$. This is the case where the SUSY breaking effect in the messenger sector is much smaller than the messenger scale $M$ itself. The terms proportional to $\Lambda/M$ have essentially the same structure as those in Eqs. (7) and (8), with somewhat different combination of the Yukawa couplings. The physics implications are however very much similar.

Second, in principle, one can integrate out only the heavy messengers and obtain effective Lagrangians involving external sparticles. For example, terms in Eq. (5) could induce pair productions of $\tilde{l}_i\tilde{l}_j^*$ and $\tilde{q}_i\tilde{q}_j^*$ at lepton and hadron colliders, and those in Eq. (6) would give Higgs and Higgsino pair production. Although the strength of the new interactions is generically small, these distinctive processes may provide new experimental signature at future colliders.

Third, we have neglected the complication of the CKM matrix when deriving the couplings from Eq. (3) to Eqs. (7) and (8). It can be systematically included by performing the proper quark field rotation between the weak and mass eigenstates.

Finally, although we only concentrate on a pair of $\mathbf{5} + \overline{\mathbf{5}}$ in this paper, the analysis can also be readily carried out for the cases of several $\mathbf{5} + \overline{\mathbf{5}}$ pairs or a pair of $\mathbf{10} + \overline{\mathbf{10}}$. In the case where the messengers are a pair of $\mathbf{10} + \overline{\mathbf{10}}$, the LMP is charged and cannot be stable without causing problems in the standard inflationary cosmology. The direct messenger-matter interactions thus may necessarily occur. We should note that in these cases general mixing among messengers are also possible. Under certain assumptions, it is shown in Ref. [18] that hypercharge $D$-term contributions to the scalar particle masses can be generated at two-loop level, but these terms are generally much smaller than the one-loop contributions from the messenger and matter interactions.

In conclusion, we have constructed the direct messenger-matter interactions in the minimal GMSB model. The new interactions avoid the cosmological problem associated with the stable messenger particle, but they generally introduce $B$ and $L$ violating and FCNC processes. We obtain the low-energy effective Lagrangians by integrating out the heavy messenger fields as well as the sparticles. If we assume that the couplings are naturally of order one, we find that the constraints from the low-energy data are generally not very restrictive except for those leading to proton decay. On the other hand, certain combinations of the couplings may contribute to some flavor violating processes within the current and future experimental reach. We also show that the new interactions have negative contributions to the scalar particle masses. Consequently, one can generally reduce the value of the $\mu$-parameter and greatly change the pattern of EWSB. The fine-tuning problem associated with the $\mu$-parameter can be alleviated at most of the parameter space. The significantly lighter mass spectrum for the sparticles and Higgs bosons and the different sparticle decay pattern can result in distinctive experimental signature at the current and future colliders.

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References

[1] M. Dine and A. Nelson, Phys. Rev. D48 (1993) 1277; M. Dine, A. Nelson and Y. Shirman, Phys. Rev. D51 (1995) 1362; M. Dine, A. Nelson, Y. Nir and Y. Shirman, Phys. Rev. D53 (1996) 2658.

[2] For a recent review, see e. g., G.F. Giudice and R. Rattazzi, hep-ph/9801271, and references therein.

[3] S. Dimopoulos, G.F. Giudice and A. Pomarol, Phys. Lett. B389 (1996) 37.

[4] T. Han and R. Hempfling, Phys. Lett. B415 (1997) 161.

[5] M. Beck et al., Phys. Lett. B336 (1994) 141; E. Garcia et al., Phys. Rev. D51 (1995) 1458.

[6] G. Dvali, G.F. Giudice and A. Pomarol, Nucl. Phys. B478 (1996) 31.

[7] M. Dine, Y. Nir and Y. Shirman, Phys. Rev. D55 (1997) 1501.

[8] E. Poppitz and S. Trivedi, Phys. Rev. D55 (1997) 5508; N. Arkani-Hamed, J. March-Russell and H. Murayama, Nucl. Phys. B509 (1998) 3; H. Murayama, Phys. Rev. Lett. 79 (1997) 18; S. Dimopoulos, G. Dvali, R. Rattazzi and G.F. Giudice, Nucl. Phys. B510 (1998) 12.

[9] S.L. Dubovsky and D.S. Gorbunov, hep-ph/9706273.

[10] See, e. g., V. Barger, G.F. Giudice and T. Han, Phys. Rev. D40 (1989) 2987.

[11] G. Bhattacharyya and A. Raychaudhuri, hep-ph/9712245, and references therein.

[12] Particle Data Group, Review of Particle Physics, Phys. Rev. D54 (1996) 1.

[13] A. Czarnecki, W. J. Marciano and K. Melnikov, hep-ph/9801218.

[14] K. Inoue, A. Kakuto, H. Komatsu, and S. Takeshita, Prog. Theor. Phys. 689 (1982) 927; L. Alvarez-Gaumé, J. Polchinski and M.B. Wise, Nucl. Phys. B221 (1983) 495; J. Ellis, J.S. Hagelin, D.V. Nanopoulos, and K. Tamvakis, Phys. Lett. B125 (1983) 275; L.E. Ibáñez and C. Lopez, Nucl. Phys. B233 (1984) 511; L.E. Ibáñez, C. Lopez, and C. Muñoz, Nucl. Phys. B256 (1985) 218.

[15] K.S. Babu, C. Kolda and F. Wilczek, Phys. Rev. Lett. 77 (1996) 3070; S. Dimopoulos, S. Thomas and J.D. Wells, Nucl. Phys. B488 (1997) 39; J.A. Bagger, K.T. Matchev, D.M. Pierce and R.-J. Zhang, Phys. Rev. D55 (1997) 3188.

[16] P. Ciafaloni and A. Strumia, Nucl. Phys. B494 (1997) 41; G. Bhattacharyya and A. Romanino, Phys. Rev. D55 (1997) 7015; K. Agashe and M. Graesser, Nucl. Phys. B507 (1997) 3.
[17] R. Barbieri and G.F. Giudice, Nucl. Phys. B\textbf{306} (1988) 63.

[18] S. Dimopoulos and G.F. Giudice, Phys. Lett. B\textbf{393} (1997) 72.