Adiabatic quantum computation with Cooper pairs

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Abstract

We propose a new variant of the controlled-NOT quantum logic gate based on adiabatic level-crossing dynamics of the q-bits. The gate has a natural implementation in terms of the Cooper pair transport in arrays of small Josephson tunnel junctions. An important advantage of the adiabatic approach is that the gate dynamics is insensitive to the unavoidable spread of junction parameters.

An invention of quantum algorithms (for a review see, e.g., [1]) changed the foundations of the theoretical computer science by demonstrating that the information is processed differently by quantum and classical systems. In an ideal world, a quantum algorithm implemented on a quantum computer can radically outperform the classical algorithm by making use of quantum parallelism inherent in entangled quantum states. Examples of problems which can be efficiently solved with quantum algorithms include factorization of large numbers [2] and database search [3].

Practical realization of a quantum computer requires, however, very precise and reversible time evolution of complex quantum mechanical systems, the fact that gives rise to serious doubts [4] as to whether even the simplest version of a quantum computer will ever become a reality. It is therefore important to look into various possible ways of implementing simple elements of quantum computer – quantum logic gates in order to find the optimal approach to building such a computer. Generally speaking, a quantum gate should satisfy two contradictory requirements: being isolated from the outside world in order to maintain...
quantum coherence, and interacting with other q-bits, read-out system, etc., in order to
perform a meaningful computation. Existing quantum gate proposals use various systems
including ion traps [5,6], electrodynamic cavities [7], semiconductor quantum dots [8,9],
NMR spectroscopy [10], quantum flux dynamics in SQUIDs [11]. Some of these proposals, for
instance, ion-trap or NMR, are characterized by potentially very long relaxation times, since
the gates in these proposals are well isolated physically from the outside world. However, due
to the very same reason, these gates can not be combined easily to form larger computing
systems. For other gates, for instance, based on semiconductor quantum dots, the situation
is the opposite. In principle, it is not too difficult to integrate them into larger structures,
but there seems to be very little hope of reducing decoherence rates to a level acceptable for
quantum computation.

The aim of this work is to suggest another possible realization of quantum gates which
is based on manipulation of the charge states of small Josephson tunnel junctions. This
approach combines both the potential for relatively long relaxation times and large degree
of design flexibility, and probably represents one of the best, if not the best, hope for
realization of a quantum computer of medium complexity. Such a computer, while not
being sufficient for factorization of large numbers of practical interest, could be sufficiently
complex to perform privacy amplification [12,13] in quantum communication channels.

The basic universal set of quantum logic gates consists of the one q-bit gates and two q-bit
controlled-NOT (CN) gate. In practically any implementation, including the one discussed
below, the dynamics of the two q-bit gates contains all elements of the one q-bit operation,
and therefore, we can concentrate on the two q-bit CN gate. The operation of this gate
can be described simply as inversion of the target q-bit states when the control q-bit is
the “1” state. The state of the control q-bit should be unchanged during this operation.
In the standard approach, the CN-operation is achieved through the use of the ac-driven
Rabi transition between the q-bit states [5,6,10]. We propose another general scheme of the
CN gate which uses adiabatic transitions between the q-bit states [12,13].
The main idea of the adiabatic CN-gate is as follows. Interaction between the control and target q-bit makes the energy difference between the two states of the target q-bit dependent on the state of the control q-bit. If the control q-bit is in the state “1” of the computational basis, the energy difference is smaller and under application of the time-dependent bias the target q-bit passes through the level-crossing point, where the energies of its two states are equal – see Fig. 1. If the rate of the bias increase is sufficiently small, the two states of the target q-bit exchange their occupation probabilities. When the control q-bit is in the “0” state, the energy difference is larger and the same bias pulse does not drive the target q-bit through the level-crossing point. In this case, the occupation probabilities remain the same. The tunnel coupling between the states of the control q-bit is suppressed during the entire process so that their occupation probabilities do not change in either case. This time evolution realizes CN-gate operation provided that the parameters of the two q-bits are chosen in such a way that the dynamic phases accumulated in the system evolution along all four “paths” are equal.

To implement this dynamics of the two q-bit system we need to control both the energy difference $\varepsilon_j(t)$ between the two states of a q-bit in the computational basis $|0\rangle$ and $|1\rangle$, and the tunneling amplitude $\Omega_j(t)$ in this basis, i.e., the Hamiltonian of the system should be:

$$H = \sum_{j=1,2} (\varepsilon_j(t) \sigma_{zj} + \Omega_j(t) \sigma_{xj}) + \eta \sigma_{z1} \sigma_{z2},$$

where $\sigma_j$ are the Pauli matrices for the $j$th q-bit, and $\eta$ is the energy of interaction between the q-bits. Although the basic gate dynamics does not require the modulation of the interaction strength $\eta$, this interaction does not allow to make the energies of all four gate states equal after the gate operation. This means that the relative phases of these states will continue to evolve at a rate on the order of $\eta/\hbar$ and we need to be able to manipulate the gate on a time scale much shorter than $\hbar/\eta$, which in its turn should be much shorter than the time scale of the adiabatic dynamics. This long hierarchy of time scales presents a serious problem that exists for other proposals of quantum gates as well. A better solution is to design a gate in a way that allows to switch the interaction on and off, despite the fact...
that this makes the design appreciably more complicated.

If the interaction energy \( \eta \) in the Hamiltonian (1) can be controlled, we can separate the gate dynamics into three steps. At first, the two q-bits are brought into contact by switching on \( \eta \) and \( \Omega_2 \) (\( \Omega_1 \) is completely suppressed throughout the gate operation). Simultaneously the energy difference \( \varepsilon_2 \) between the states of the target q-bit is set to some nonvanishing initial value. Then this energy difference is increased while all other energies are kept constant. This step is the central “level-crossing” part of the gate dynamics. During the final third step both \( \eta \) and \( \Omega_2 \) are suppressed back to zero so that the two q-bits are effectively separated and \( \varepsilon_2 \) can also be reduced to zero.

The precise functional dependence of \( \Omega_2, \varepsilon_2, \) and \( \eta \) on time does not qualitatively affect the gate dynamics, as long as all these parameters are changed gradually. The limitation on the rate of the parameter variations is associated with the unwanted transitions between the instantaneous energy eigenstates of the system which are brought about by these variations. These transitions violate the correct adiabatic dynamics which assumes that the system remains at all times in the same eigenstate it occupied initially. Adopting a simple model time dependence of the energy difference \( \varepsilon_2(t) \):

\[
\varepsilon_2(t) = \varepsilon + u \tanh(t/\tau),
\]

and using the standard quasiclassical approach [14] we can calculate explicitly the probability \( p \) that the system makes an unwanted transition during the central second step of the gate operation. This simple calculation confirms the expected result that the probability \( p \) reaches maximum when the system passes through the level crossing-point and is given then by the standard Landau-Zener expression:

\[
p_{LZ} = \exp\left\{-\frac{\pi \tau \Omega_2^2}{\hbar u}\right\}.
\]

Here \( \Omega \) is the magnitude of the tunnel amplitude \( \Omega_2(t) \) that is kept constant during this step of the gate operation. Thus, the condition \( p_{LZ} \ll 1 \), i.e., \( \tau \gg \hbar u/\Omega^2 \), ensures the correct adiabatic dynamics of the gate.
This implies that dynamics of the occupation probabilities of the states of adiabatic CN gate is not sensitive to the precise values of the parameters in the Hamiltonian (II) provided that they satisfy several constrains which ensure the gate operation shown in Fig. 1:

\[\eta + u - \varepsilon \gg \Omega, \quad \eta - u - \varepsilon \ll -\Omega, \quad u - \eta - \varepsilon \ll -\Omega.\quad (4)\]

If all these conditions are satisfied, the evolution of the absolute values of the occupation amplitudes \(\alpha_{ij}\) of the four gate states corresponds to the correct CN operation:

\[|\alpha_{0j}| \to |\alpha_{0j}|, \quad |\alpha_{10}| \to |\alpha_{11}|, \quad |\alpha_{11}| \to |\alpha_{10}|.\]

(The indices \(i\) and \(j\) denote the states of the control and target q-bit respectively.) Besides this time evolution of the absolute values of \(\alpha_{ij}\), the correct gate dynamics requires also that phases of the four states accumulated in the process of the gate evolution are equal modulo \(2\pi\). This can be achieved by adjusting the bias \(\varepsilon_{1,2}\) of the two q-bits and the energy splitting \(\Omega_2\) during the gate operation. The bias \(\varepsilon_1\) controls the relative phases of the pairs of states evolving from the 0 and 1 state of the control q-bit, while \(\varepsilon_2\) and \(\Omega_2\) control the phases within each pair. With such an adjustment of the phases, the adiabatic time evolution of the coupled q-bits represents correctly the CN quantum logic gate.

This gate can be naturally implemented in systems of small Josephson tunnel junctions in the Coulomb blockade regime – see, e.g., [15,16]. The energy diagram of an elementary building block of such a system, a single junction, is shown in Fig. 2. The dominant contribution to the junction energy is given by the charging energy \(U(n)\) of the junction as a capacitor:

\[U(n) = \frac{(2en - Q_0)^2}{2C},\]

where \(C\) is the junction capacitance, \(n\) is the number of Cooper pairs transferred across the junction, and \(Q_0\) is the charge induced by the external bias voltage \(V_0\) across the junction, \(Q_0 = V_0C\). In general, the states with different \(n\)’s are separated by large energy gaps on the order of elementary charging energy \(E_C = e^2/2C\). However, when the external voltage
\( V_0 \) induces the charge of approximately one electron on the junction capacitance, \( Q_0 \approx e \), the two state, \( n = 0 \) and \( n = 1 \) are nearly degenerate and are separated from all other states by the large energy gaps – see Fig. 2. In this regime the junction behaves effectively as a two-level system. The energy difference \( \varepsilon \) between the level of this two-level system is controlled by the external voltage \( \varepsilon = 2e(e - Q_0)/C \), while the amplitude \( \Omega \) of tunneling between them is determined by the Josephson coupling energy \( E_J \) of the junction, \( \Omega = E_J/2 \).

The Josephson coupling energy depends on the tunnel resistance \( R_T \) of the insulator barrier between the electrodes, and for the electrodes with equal superconducting energy gaps \( \Delta \) is equal to \( \pi \hbar \Delta/4e^2R_T \) – see, e.g., [16].

Thus, the appropriately biased small Josephson tunnel junction is a macroscopic two-level system, with the two states represented by the position of a single Cooper pair on the left or right electrode of the junction. In principle, this system can be used as a q-bit of the quantum logic gates. However, if q-bits are represented with single junctions, neither the tunneling amplitude \( E_J/2 \) nor the coupling strength of the two q-bits which is determined by the coupling capacitance between the junction electrodes can be modulated in time as required by the design of the adiabatic CN gate. In particular, to realize adiabatic dynamics it should be possible to suppress both the tunneling amplitude and interaction strength to zero between the active cycles of the gate operation. This problem can be circumvented if q-bits are represented not with individual junctions but with the one-dimensional arrays of junctions. In an array, the tunneling amplitude \( \Omega \) between the two islands of the array can be effectively modulated by the gate voltages applied to the islands of the array, and the interaction energy \( \eta \) of charges in the array decreases exponentially with the distance between them.

To make a q-bit out of a uniform array, all islands should have individual gate electrodes supplying the gate voltages \( V_j \) (Fig. 3a,b), and two internal islands of the array should be biased with the voltages \( \pm e/C_t \), where \( C_t = (C_0^2 + 4CC_0)^{1/2} \) is the total capacitance of an internal island in the array – see, e.g., [15], and \( C, C_0 \) are, respectively, the junction capacitance, and the capacitance between each island and its gate electrode (Fig. 3b). These
voltages induce the charges $e$ and $-e$ on the two islands, so that the two charge configurations of the array: one with no Cooper pair transferred across any junction and another one with a Cooper pair transferred between the two biased islands, from $e$ to $-e$, have the same energy. This means that if the bias conditions do not deviate strongly from these conditions, all other charge configurations of the array have much larger energies and the array dynamics is equivalent to the two-state dynamics that can be described in terms of the tunneling of a single Cooper pair between the two islands. In this regime the array can be viewed as a q-bit with the two positions of the Cooper pair on one or another island representing the two states of the computational basis of this q-bit.

If the two islands containing the q-bit states are separated by $m$ junctions, the amplitude of tunneling $\Omega$ between them depends exponentially on the separation $m$. The dominant contribution to $\Omega$ comes from the process in which the Cooper pair is transferred sequentially through the junctions separating the islands, and can be written as:

$$\Omega = \frac{E_J}{2} \prod_{k=1}^{m-1} \frac{E_J}{2E_k},$$

where $E_k$ are the energies of the intermediate charge configurations resulting from the Cooper pair transfer through the first $k$ junctions. These energies are controlled by the gate voltages applied to the intermediate islands.

The most important feature of the Cooper pair states forming q-bit basis is that they can be moved along the array by the adiabatic level-crossing transitions similar to those discussed above. A Cooper pair is transferred between the two adjacent islands when a gate voltage of the initially occupied island is increased/decreased while the gate voltage of the neighboring island is decreased/increased adiabatically past $e/C_t$. The adjacent islands are coupled by the tunneling amplitude $E_J/2$, and the Cooper pair is transferred with the probability exponentially approaching one if the rate of change of the gate voltages is small on the scale of this amplitude. Similar manipulation of the gate voltages also shifts the empty state of the q-bit by one island. In this way it is possible to move the q-bit states around, either shifting both states along the array, or changing the separation $m$ between
the two states.

This dynamics is analogous to the one used in the so-called single-electron [17] and single Cooper pair [18] pump, or single-electron parametron [19]. It allows to implement the general scheme of the adiabatic CN gate with the two coupled arrays representing the two q-bits of the gate (Fig. 3c). As a first step of the gate operation, the q-bit states in both arrays are moved towards the ends of the arrays where they can interact via the coupling capacitance $C_i$. The states of the controlled q-bit in the first array have sufficiently large separation $m$ so that their tunnel coupling $\Omega_1$ is negligible. By contrast, the states of the target q-bit in the second array are put on the adjacent islands in order to maximize their tunnel coupling, $\Omega_2 = E_J/2$. Then a pulse of the bias voltage is applied to the first junction of the target q-bit array. If the control q-bit is in the “1” state, a Cooper pair occupies the island of the first array closest to the second array and creates additional potential drop $\delta V$ across the junction of the target q-bit:

$$\delta V = \frac{8eC_i}{(C_0 + C_t + 2C)(C_0 + C_t + 4C_i)}.$$  \hspace{1cm} (6)

In this case the bias pulse drives the target q-bit through the level-crossing point so that the occupation probabilities of its states are interchanged. When the control q-bit is in the “0” state, the Cooper pair of this q-bit is inside the array and does not produce extra voltage across the target q-bit junction, which then does not reach the level-crossing point, and the occupation probabilities of its states remain the same. During the last step of the gate operation it is returned to its initial configuration, i.e., the separation of the states of the target q-bit is increased to suppress the tunnel amplitude $\Omega_2$ to zero, and the states of the both q-bits are shifted inside the arrays. Then the interaction of the q-bit states becomes negligible due to screening by the gate electrodes, which is known to lead to the exponential suppression of the interaction energy $\eta$ between two Cooper pairs separated by $m$ junctions of one array [13]:

$$\eta = \frac{(2e)^2}{C_t} \lambda^m, \quad \lambda = \frac{2C}{2C + C_0 + C_t}.$$  \hspace{1cm} (7)
This implementation of the CN quantum gate can only be practical if it is stable against deviations of the real gate structure from the idealized model used above. Such deviations are fundamentally unavoidable in all macroscopic realizations of quantum gates. For instance, the real electrostatics of the Josephson junction gate is much more complicated that the model characterized by the two nearest-neighbor capacitances \( C \) and \( C_0 \). It involves full capacitance matrix \( C_{ij} \) in which even remote islands interact with each other, and should also describe small fluctuations of the nearest-neighbor capacitances around their average values. An important advantage of the adiabatic approach is that these complications can be compensated for by the adjustment of the bias voltages and do not change qualitatively the gate dynamics. Indeed, the adiabatic transfer of a Cooper pair depends only on the resonance condition that the energies of all Cooper pair states along the array are the same, which ensures correct transfer of the occupation probabilities of the gate states. The bias voltages can always be tuned to satisfy the resonance condition regardless of the form of the capacitance matrix. A practical proof of this statement is provided by the experimentally demonstrated operation of a similar system, single-electron pump, with accuracy better than \( 10^{-6} \) \cite{20}.

The only instance when the gate dynamic relies heavily on the simplified model of the array electrostatic is in the assumption of the exponential screening of the electrostatic interaction inside the array. In the realistic model of electrostatics, interaction at large distances depends on the external environment of the array. The exponential screening of the interaction can still be obtained even in this case, but requires that the array is placed between the two conducting ground planes.

These considerations show that dynamics of the occupation probabilities of the gate states is indeed insensitive to the week disorder in the gate parameters. However, the proper dynamics of the system as quantum logic gate requires also that the phases of the occupation amplitudes accumulated during the gate operation are all equal modulo \( 2\pi \). In this respect, fluctuations of the junction parameters do present a problem since they make the dynamic phases of the gate states unpredictable. This problem can be resolved if the
disorder in the parameters is static on a sufficiently long time scale. In this case, the phases can be measured and compensated for by the fine-tuning of the gate voltages.

In order to measure the phases, we need to transform them into the occupation probabilities of the gate states which in their turn can be measured with a single-electron electrometer (see, e.g., [21], Chapter 9). An electrometer measures an average charge of the island and therefore gives information about the occupation probabilities of the gate state, but is insensitive to the phase of the occupation amplitudes. Suppose that as a result of a prior measurements, we know that the occupation probabilities of the two q-bit states are $p_1$ and $p_2$. The two states are decoupled (the corresponding tunneling amplitude $\Omega$ is zero) and their energies are equal, so that there is some stationary phase difference $\phi$ between their occupation amplitudes. The phase $\phi$ can be transformed into the occupation probability by rotation $\hat{U}$ of the q-bit states in the Hilbert state, $\hat{U} = \exp\{i\pi\sigma_z/4\}$. This rotation is achieved if the barrier between the states is reduced temporarily in such a way that

$$\int dt \Omega(t) = \pi\hbar/4.$$ 

The resulting occupation probabilities

$$q_{1,2} = 1/2 \mp (p_1p_2)^{1/2} \sin \phi,$$

depend on the phase $\phi$, and by measuring them we can measure $\phi$. After the phase is known it can be compensated for by adding an extra voltage pulses at the end of the gate operation. With this fine-tuning, the gate dynamics becomes effectively independent of the static disorder in the gate parameters.

The above discussion assumes that the energy relaxation and associated with it time-dependent fluctuations of the phase are negligible. There are several dissipation and dephasing mechanisms in the Josephson tunnel junction systems. Some of them are well understood and can be controlled within certain limits. One of these mechanisms is the quasiparticle tunneling. In general, it coexists with the Cooper pair tunneling and makes junction dynamics irreversible. However, if both the temperature $T$ and charging energy $E_C$
of the junctions are much smaller than the superconducting energy gap $\Delta$, the quasiparticle tunneling is suppressed by the parity effects \cite{22, 24} to a level where it can be negligible on the macroscopic time scales \cite{25, 26}. Another dissipation mechanism is coupling to the electromagnetic excitations supported by the system of superconducting electrodes. A Cooper pair oscillating between the two islands creates oscillating currents in the islands and electric fields around the islands which couple to these modes. The power $P$ lost to electromagnetic modes depends on the specific geometry of the islands and connecting them tunnel junctions. Part of the losses comes from the direct dipole radiation from the junctions and can be estimated as radiation of a dipole of length equal to the length $d$ of the junction electrodes:

$$P_d \simeq \frac{e^2 \omega^4 d^2}{4\pi \epsilon_0 c^3}.$$  \(8\)

The radiated power is not exponentially small, nevertheless it decreases sufficiently rapidly with decreasing ratio of the island size to the radiation wavelength $\lambda \simeq c/\omega$ at frequency $\omega \simeq E_J/\hbar$. Therefore, to keep this type of radiation losses small the islands of the junction arrays should be much smaller than the wavelength at frequency $E_J/\hbar$, the condition that is always satisfied in small junctions.

The crucial contribution to radiation losses comes from the coupling to electromagnetic modes supported by essentially “infinite” external gate electrodes supplying bias voltages to the islands. In the relevant regime with $C_0 \ll C$, the power dissipated into these modes can be estimated in terms of the wave impedance $\rho$ of the gate electrodes as

$$P_l \simeq \left(\frac{e C_0}{C}\right)^2 \omega^2 \rho.$$  \(9\)

We see that this dissipation mechanism limits the magnitude of the island capacitance to the gate electrodes $C_0$. In the simple model of the gate electrostatics, $C_0$ determines also the number of islands of the junction array that are polarized by a single Cooper pair, and restriction on $C_0$ translates into the limitation on how small the number of junctions in the arrays can be. If however, one introduces ground planes which give rise to extra stray capacitances of the array islands, these two limitations becomes uncoupled. In any
case, for realistic values of the parameters (see the estimates below) the losses in the external electrodes should give the dominant contribution to decoherence for the Cooper pair tunneling.

Besides these “controllable” mechanisms of dissipation that depend on the gate geometry, the Cooper pair tunneling in the junction arrays is affected also by the “internal” dissipation in all elements of the arrays. The most important source of noise and dissipation of this kind is the $1/f$ charge noise in the insulators surrounding the junctions: substrate and tunnel barriers. The strength of the noise is material dependent and can not be estimated from first principles. Experimentally, characteristic time scale of the charge noise varies from millisecond range [27] to seconds and hours [28], and is much longer than characteristic time of the Cooper pair tunneling $\hbar/E_J$ which determines the rate of the gate operation. Therefore, the gate can go through the large number of cycles of operation before the decoherence due to the charge noise starts to affect its dynamics.

Before concluding, we summarize the conditions that should be satisfied by junction arrays in order to operate as quantum logic gates. The first set of conditions is given by the following string of inequalities:

\[ T \ll E_J \ll E_C \ll \Delta. \]  

(10)

The two limiting energy scales in this relations, temperature $T$ and energy gap $\Delta$, are practically constrained by the available refrigeration technology and superconducting materials. The lower limit is set by the typical electron temperature attainable in experiments with the dilution refrigerator and is on the order of 30 mK. The upper limit can not be much larger than the energy gap of niobium, or its compounds, i.e., about 20 K. The ratio of the Josephson coupling energy $E_J$ to the charging energy $E_C$ can not be varied arbitrarily because of the technological limitations on the critical current density that can be obtained while preserving the quality of the tunnel junction. Conditions (10) are satisfied if we take, for instance, $E_J \simeq 1$ K, and $E_C \simeq 3$ K. This value of $E_C$ corresponds to the junction capacitance $C \simeq 0.5$ fF, which for a typical specific capacitance of a tunnel junction, 0.1
pF/µm², requires the junction area of about 70 × 70 nm². With this area, the cited $E_J$ value corresponds to the critical current density $j_c$ about 10 µA/µm², and the total critical current $I_c = 2eE_J/h \simeq 50$ nA. Experimentally, this value of $j_c$ is within the range of current densities that can be achieved without the degradation of the tunnel junction quality [29].

Another condition on the junction array as a CN gate is that the number $N$ of junctions in it is much larger than its screening length:

$$N \gg (C/C^*)^{1/2}. $$

Here $C^*$ is the total stray capacitance of the array islands which include capacitance $C_0$ to the gate electrodes and capacitance to the ground planes. This condition does not represent a serious obstacle to realization of a CN gate. Specific values of $N$ and $C^*$ are dictated by the convenience of fabrication of either longer arrays or larger capacitances to the ground.

The most difficult is the condition that the probability $\alpha$ of the decoherence-induced error during one cycle of the gate operation is small. Estimating the period of this cycle roughly as $\hbar/E_J$ we obtain from eq. (9) that the lower bound on $\alpha$ is:

$$\alpha \simeq \frac{(C_0/C)^2 e^2 \rho}{\hbar}. $$

(11)

The values of parameters that are typical for existing experiments (in which no effort was maid to decrease $\alpha$) are $C_0/C \simeq 0.1$, and $\rho \simeq 300$ Ohm [30]. (The latter value corresponds to a narrow, about 1µm, electrode.) In this case $\alpha \simeq 10^{-3}$. The error probability can be substantially reduced by making coupling capacitance $C_0$ smaller, and gate electrodes wider thus decreasing $\rho$. Although only experiments can tell what is the limit to decrease in decoherence rate, it is reasonable to expect that $\alpha$ can be reduced further by a few orders of magnitude to a value about $10^{-6}$.

In summary, we proposed a new design of the controlled-NOT quantum logic gate based on the adiabatic level-crossing dynamics of the coupled q-bits. The design is suitable for implementation in systems of small tunnel Josephson junctions and has the advantage of being insensitive to spread of the junction parameters. The level of decoherence in the small
tunnel junction systems is estimated and appears to be sufficiently small for medium-scale quantum computation.

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FIGURES

Time evolution of the energy levels of a controlled-NOT quantum logic gate based on the adiabatic level-crossing dynamics.

Energy diagram of a tunnel Josephson junction in the Coulomb blockade regime biased with the external voltage that induces the charge $Q_0 \simeq e$ on the junction capacitance. The two states $n = 0, 1$ are nearly-degenerate and the junction behaves effectively as a macroscopic two-level system.

Schematic layout (a) and equivalent electrostatic circuit (b) of an array of small Josephson junctions representing one q-bit. (c) The controlled-NOT gate obtained by coupling of the two arrays.