INTRODUCTION

Hip endoprosthesis are considered the greatest stride forward in orthopedic surgery in the last 100 years. Every year carried over 1,000,000 operations, of which only 300,000 in the United States. Hip arthroplasty is an operation involving the replacement of a diseased hip joint with an artificial one. During the operation, the damaged femoral head and the inside of the acetabulum are removed and replaced with artificial elements. Each endoprosthesis consists of an acetabulum, acetabular insert, stem and metal head. The artificial cup is mounted in place of the natural acetabulum. Most often, it is made of titanium, and inside it, there is an insert made of polyethylene or ceramics. The endoprosthesis mandrel is fixed in the femur. It is also made of titanium and a metal or ceramic head is placed on it. In this way, the metal head and acetabular insert form a new joint, capable of making movements.

A very important issue when designing, implanting and using endoprosthesis is the assessment of contact pressure depending on the load, diameter and the size of the gap (radial clearance) in the joint. Unfortunately, the literature lacks justified assessments using appropriate calculation methods based on classic methods of contact mechanics. In a number of papers, solutions made by FEM numerical simulations...
are given [1, 2, 3, 5-10]. Due to the fact that these methods are approximate, they do not give a clear answer. In [11] shows the approximate calculation method for estimating the contact pressures. It should be emphasized that there are no analytical methods in the literature concerning the results of endoprosthesis research as a system of spherical bodies. [3] is the only paper presenting a simplified and at the same time advanced to implement calculation method based on the old work of [4]. Lin W. et al. show that “... the accuracy of FEM predictions depends on the input from laboratory experiments” [7], which appropriately justifies that mathematical modeling can be used to estimate contact parameters in endoprosthesis.

Estimating the amount of surface pressure is a very important issue from the point of view of materials used today to build hip joint prostheses. Biocompatible polymers are commonly used, which, after exceeding their limit pressures, undergo plastic deformation. These in turn cause an increase in radial clearances and, consequently, loosening of the hip prosthesis.

Loosening of the hip is one of the most serious complications of joint arthroplasty. The result of this phenomenon is that a new artificial hip joint ceases to perform its function. In extreme cases, when radial clearance is too large, the head of the prosthesis could disconnect from the acetabulum of the polymer and loose support of the human body, causing immobilization of an individual. Another negative consequence of the loosening of the endoprosthesis joint due to exceeding the surface pressure is the formation of inflammation, which causes further degradation of human bone material as well as accelerated wear of the tribological pair.

The hip endoprosthesis is a hip ball joint of a limited deviation angle in a sliding movement at a single-track working load. This paper presents the results of numerical analysis of hip joint endoprosthesis with the use of the proprietary method of solving the contact problems of the theory of elasticity. This method was used to estimate the contact parameters of joints of bodies with a circular cross-section. These bodies are of similar diameters and are in internal contact [12-15]. The aim of the study is to show that the author’s empirical method also gives the correct solution to the problem taking into account the joint load, the diameter of the endoprosthesis head and its geometric parameters.

**THE STUDY OF CONTACT PRESSURE**

In order to use the indicated method of testing flat contact problems, this type of endoprosthesis (3D system) (Fig. 1a) was modeled (Fig. 1b) with the diameter of the head with a cylindrical joint with the cylindrical joint with a fixed socket (3D system) by introducing a model radius (contractual). This method was previously used to test the load capacity of resting joints, as well as initial contact pressures in the moving systems: slide bearings, reciprocating cylindrical guides and pendulum joints [16-18].

The tested 3D system was transformed into a 2D flat system reducing the total compressive load N (Fig. 1c) per head of the prosthesis to one unit of its length (head diameter), i.e. N'=N/D. It is assumed that R₁ ≈ R₂ = R, where, respectively R₁, R₂ – the radius of the bushing (bearing shells) 1 and the radius of the disk 2. There is a small radial clearance (aperture) in the hip e = R₁ − R₂ ≥ 0 << R. Under the influence of load, there is contact of the system elements in the zone defined by the angle and arises contact pressure. The elastic characteristics of the bodies are not the same. In the state of unidirectional pendulum motion (partial rotation of the endoprosthesis head towards its cup during gait in the loading phase), we will model this system with a plain bearing (Fig. 1c).

**SOLUTION METHOD**

The solution method consists of determining the maximum contact pressure, the contact angle and distribution of pressures in the contact sphere. The equation of contact pressures pₒ in the case of symmetric contact against the eternal loading of the components in the layout shall be in the form [12–14]:

\[
c_1 \int_{a_0}^{a_0} \cot \left( \frac{\alpha - \theta}{2} \right) p_\theta d\theta = c_2 p_\alpha + c_3 \int_{a_0}^{a_0} p_\alpha d\alpha + c_4 \cot \left( \frac{\alpha - \theta}{2} \right) p_\alpha d\alpha + \frac{e}{r^2}
\]

where: \( p'_\theta = \frac{d\theta}{db} \); \( \alpha \) – polar angle; \( 0 \leq \alpha \leq \theta; \)

\[
c_1 = \frac{1}{N' \pi R} \left( \frac{1 + k_1}{G_1 R_1} + \frac{1 + k_2}{G_2 R_2} \right); \quad c_2 = \frac{1}{4} \left( \frac{1 - k_1}{G_1 R_1} - \frac{1 - k_2}{G_2 R_2} \right)
\]

\[
c_3 = \frac{1 + k_1}{N' \pi G_1 R_1^2}; \quad c_4 = \frac{1}{2\pi} \left( \frac{k_1}{G_1 R_1^2} + \frac{1}{G_2 R_2^2} \right)
\]

\( G_1, G_2 \) – the modulus of material elasticity of all the components in the layout

\( v_1, v_2 \) – Poisson’s ratio; \( \kappa = 3 - 4v \) – the state of flat deformation.
The approximate solution of the equation (1) is achieved by the collocation method. The function of contact pressures $p_\alpha$ is presented in the form of [13-16]:

\[ p_\alpha \approx E_0 \varepsilon \sqrt{\tan^2 \frac{\alpha_0}{2} - \tan^2 \frac{\alpha}{2}} \] (2)

where:

\[ E_0 = \left( \frac{e_4}{R_2} \right) \cos^2 \left( \frac{\alpha_0}{4} \right) \] – simplified version,

\[ E_0 = \left( \frac{e_4}{R_2} \right) \cos^2 \left( \frac{\alpha_0}{4} \right) - e_1 \sqrt{\tan^2 \left( \frac{\alpha_0}{2} \right) - \tan^2 \left( \frac{\alpha_0}{4} \right)} - 
\left. \frac{1}{4} \right. \sin^2 \frac{\alpha_0}{4} \left( e_2 \cos^{-1} \left( \frac{\alpha_0}{2} \right) + 2e_3 \cos \left( \frac{\alpha_0}{2} \right) \right)^{-1}
\] – detailed version,

\[ e_1 = \frac{2}{2} \left[ (1 - \kappa_1)(1 + \mu_1)E_2 - (1 - \kappa_2)(1 + \mu_2)E_1 \right]; \]

\[ e_2 = \frac{2}{2} (1 + \kappa_1)(1 + \mu_2)E_2; \]

\[ e_3 = \frac{4}{2} \kappa_1 (1 + \mu_1)E_2 + (1 + \mu_2)E_1; \]

\[ e_4 = \frac{4E_1 E_2}{2}; \]

\[ e_5 = \frac{4E_1 E_2}{2}; \]

\[ Z = (1 + \kappa_1)(1 + \nu_1)E_2 + (1 + \kappa_2)(1 + \nu_2)E_1; \]

\[ E = \frac{2G}{(1 + \nu)} \] – Young’s modulus.

Maximum contact pressures $p_0$ are obtained when $\alpha = 0$. Then

\[ p_0 \approx E_0 \varepsilon \tan \frac{\alpha_0}{2} \] (3)

The balanced forces exerted on the second target determine an unknown half-angle $\alpha_0$.

\[ N' = R \int_{-\alpha_0}^{\alpha_0} p_\alpha \cos \alpha \, d\alpha = 4\pi E_0 \varepsilon sin^2 \frac{\alpha_0}{4} \] (4)

**LOADING CONDITIONS IN THE HIP JOINT**

The total force $N$ on the femoral head is determined based on various literature data [17]. It is a geometric sum of the two forces – body weight $K$ as well as muscular strength $M$ (Fig. 2).

In the cycle of movement, the value of $N$ changes substantially – $1.45K \leq N \leq 4.4K$ (Fig. 2). Therefore, in normal conditions of walking in two peaks, we observed pressure of $3.0K$ and $4.4K$. In extreme cases, it reached up to $9K$. Therefore, taking all the data into consideration, the average
value of the compression force $N_{avg}$ registers about 1900 N (assuming $K = 700$ N).

**NUMERICAL SOLUTION OF THE PROBLEM**

Numerical solution of the problem was conducted for the examined layout when $\varepsilon > 0$. To calculate the parameters of the contact in the joint the following data were selected: $N_{max} = 2900$ N, $N_{avg} = 1900$ N, $N_{min} = 1000$ N; $N' = N/D$; $D_2 = 28, 48$ and $58$ mm; $\varepsilon = 0.02$–0.2 mm; for calculations were adopted accordingly (Table 1):

A simplified version was applied in case of $E_0$. In the calculation of the model cylindrical joint, a conventional radius $R' = 0.5\sqrt{(R_1 + \varepsilon)R_2}$ was introduced. This resulted in a replacement plane system for the prosthesis as the spatial system with $R_1$ and $R_2$ rays interacting elements.

**Table 1. Input data for calculations**

| $D_2$ | $N_{max}$ | $N_{avg}$ | $N_{min}$ |
|-------|-----------|-----------|-----------|
| 28 mm | 103.6 N/mm| 68 N/mm   | 35.7 N/mm |
| 48 mm | 60.4 N/mm | 39.6 N/mm | 20.8 N/mm |
| 58 mm | 50 N/mm   | 32.8 N/mm | 17.2 N/mm |

**Table 2. Input data for calculations**

| $D_2$ | $N_{max}$ | $N_{avg}$ | $N_{min}$ |
|-------|-----------|-----------|-----------|
| 28 mm | 2900 N    | 68 N/mm   | 35.7 N/mm |
| 58 mm | 1900 N    | 32.8 N/mm | 17.2 N/mm |

| $\varepsilon$ [mm] | 0.049 | 0.05 | 0.1 | 0.2 | 0.032 | 0.05 | 0.1 | 0.2 | 0.017 | 0.05 | 0.1 | 0.2 |
|--------------------|------|------|-----|-----|-------|------|-----|-----|-------|------|-----|-----|
| $p_0$ [MPa]        | 1.978| 1.98 | 2.6 | 3.56| 1.30  | 1.52 | 2.06| 2.86| 0.48  | 1.06 | 1.44| 2.0 |
| $2\alpha$ [']      | 158.8| 157.6| 106.6| 74  | 148.8 | 126.6| 85.8| 60  | 158.8 | 85.8 | 60  | 42.2|

| $\varepsilon$ [mm] | 0.1023| 0.2  | 0.0671| 0.1  | 0.2  | 0.035| 0.05 | 0.1 | 0.2 |
|--------------------|------|------|-------|------|-----|------|------|-----|-----|
| $p_0$ [MPa]        | 8.48 | 10.92| 5.56  | 6.41 | 8.64| 2.92 | 3.26 | 4.4 | 6.1 |
| $2\alpha$ [']      | 158.8| 108.4| 158.8 | 128.2| 86.8| 158.8| 128.2| 88.2| 61.8|
Materials used for the endoprosthesis: head 2 – NitridedGRADE 2 (TDN) [10], for which $E_2 = 112$ GPa, $v_2 = 0.32$ (GRADE 2 - titanium); acetabulum 1 – polyethylene PE-UHMW, for which $E_1 = 0.625$ GPa (37 °C), $v_1 = 0.46$. The results of the solution are given in Figures 3 and 4 as well as in Table 2. In Figure 3 the ratio correlation of maximum contact pressures $p_0$ of radial clearance $\varepsilon$ is presented with the head diameter $D_2$ of endoprosthesis and the reduced compression force $N'$.

Analysis of the results allows us to draw conclusions from linear relationship of increasing $p_0$ from $\varepsilon$ within the scope of $0.05 \leq \varepsilon \leq 0.2$ mm. When $\varepsilon \leq 0.05$ mm the above-mentioned correlation becomes non-linear. The intensity of growth $p_0$ depends on the size of head diameter $D_2$. In Table 2 results of $p_0$ and $2a_0$ are presented.

For the maximum and average value of compression force and the examined radial clearances, the relationship between maximum contact pressures $p_0$ with the head diameter of endoprosthesis $D_2$ was presented in Figure 4.

The results of the calculation indicate that by increasing $D_2$ by 2.07 causes the reduction of $p_0$ by...
2.96 \div 4.24, depending on the value of N as well as \( \varepsilon \). According to the study [8]

\[
p_0 = a_1 + a_2 N^{a_3} + a_3 \varepsilon^{a_4} + a_4 N^{a_3} \varepsilon^{a_5}
\]

(5)

If \( \varepsilon \geq 0; a_1, a_2, \ldots \) factors of approximation.

And according to [11]:

\[
p_0 = c_0 \frac{E_0}{R},
\]

(6)

where: \( E_2 = \infty, E_1 = 0.625 \) GPa, \( c_0 \) - collocation rate depends on \( \alpha_0 \).

In our case \( E_2/E_1 = 112/0.625 = 179.2 \) (GRADE2/PE-UHMW). Formally, it may be stated that the head of endoprosthesis is very rigid in relation to the bearing and deformation does not occur due to the contact force.

In terms of the flat contact formula, the maximum contact pressure \( p_0 \) was also estimated according to the Hertz equation for the contact disc with aperture:

\[
p_0 = 0.564 \sqrt[\frac{N' R_1 - R_2}{\beta R_1 R_2}}
\]

(7)

where: \( \beta = \frac{(1-\nu_1^2)}{E_1} + \frac{(1-\nu_2^2)}{E_2} \) - shear modulus,

\[
R_1 - R_2 = \varepsilon, R_1 = R_2 + \varepsilon.
\]

Hertz formula for the internal ball connection of slightly different radii is not possible to use within the examined issue of flat contact strength of theory of elasticity due to the two-dimensional (circle shape) area of contact. Table 3 and Figures 5 and 6 show the results of the calculations \( p_0 \) according to the certain methods and their relative change for the layout where radial clearance \( \varepsilon = 0.1 \) and \( \varepsilon = 0.2 \) mm occurs.

### CONCLUSIONS

The developed method enables the effective assessment of maximum contact pressures in endoprosthesis in case of the occurrence of radial clearance. Maximum contact pressures \( p_0 \) depend on the loading force \( N \) against the

### Table 3. Results of the calculations

| Param. | Formula | \( D_2 = 58 \) mm | \( N = 2900 \)N | \( N' = 50 \)N/mm (Aut.) | \( (6) \) | \( (7) \) | \( (5) \) |
|--------|----------|-------------------|-----------------|---------------------|---------|---------|---------|
| \( \varepsilon \) [mm] | 0.1 | 0.2 | 0.1 | 0.2 | 0.1 | 0.2 | 0.1 | 0.2 |
| \( p_0 \) [MPa] | 2.59 | 3.56 | 2.74 | 3.81 | 1.22 | 1.73 | 3.0 | 4.0 |
| \( 2\alpha_1 \) [°] | 106.6 | 74.0 | 106.6 | 74.0 | - | - | - | - |
| \( p_0/p_0 \) | 1.0 | 1.0 | 1.06 | 1.07 | 0.47 | 0.485 | 1.16 | 1.12 |

Figure 5. Alteration of maximum contact pressures for radial clearance \( \varepsilon = 0.1 \)mm and \( \varepsilon = 0.2 \)mm.
Authors - the results of the authors’ research, Panasyuk [11] - solution by method (6) [11], Hertz (7) - solution by Hertz equation (7), Grushko [9] - solution by method (5) [9]
endoprosthesis head, its diameter \(D_2\) and radial clearance \(\varepsilon\) in the layout (Fig. 3, 4). Their values increase almost linearly with the increase of radial clearance in the range of \(0.05 \leq \varepsilon \leq 0.2\) mm (Fig. 3). At \(\varepsilon \leq 0.05\) mm, their change slightly differs from the linear one. A comparative analysis of results regarding the assessment of \(p_0\) according to the author’s method is in accordance with other methods (Fig. 5 and 6).

Due to the method [10] certain pressures \(p_0\) of slightly higher value (1.07) were determined and according to the Hertz formula the result is inappropriate and according to the method [8] – higher than the original method up to 1.16 times. According to the data established in this paper [5] the solution for the endoprosthesis problem was conducted CoCr – CoCr, when \(N = 3200\) N, \(D_2 = 58.6\) mm, \(\varepsilon = 0.05\) mm. It was established that the maximum contact pressures reached 24.05 MPa by the original method and 22.0 MPa according to FEM [5] that means they are 1.093 times higher. However, according to the scholarly work [8] and the original authors method for the endoprosthesis CoCrMo – PE-UHMW, if \(N = 2500\) N, \(D_2 = 32\) mm, \(\varepsilon = 0.098\) mm the pressures are identical – 10.2 MPa. According to the work [3] for Steel – PE-UHMW endoprosthesis, when \(N = 2500\) N, \(D_2 = 28\) mm, \(\varepsilon = 0.25\) mm, the maximum contact pressures are 14.94 MPa, and according to the author’s method – 13.44 MPa, i.e. they are lower by 1.11 times.

From the practical point of view, the results of the examined issue indicate that the reduction of radial clearance below 0.05 mm does not seem to be deliberate.

Producers of hip endoprosthesis anticipate the initial radial clearance \(\varepsilon = 0.05 \pm 0.25\) mm [8]. As a practical matter, the assessment of maximum contact pressures outside the scope is not targeted. Nonetheless, reduction \(\varepsilon\) of the above-mentioned \(\varepsilon_{\text{min}}\), even more hypothetically factored to zero does not provide any substantial benefits in an increase of the contact surface \(2 \alpha_0\) of all the components of endoprosthesis and the reduction of level \(p_0\).

The results of the calculation (Table 3) show that there are certain values of clearance \(\varepsilon\) in each examined case where the angle of the contact \(2 \alpha_0\) reaches the limit value of 160 degrees. It is caused by the increased deformation of the bearing material (PE-UHMW) upon the contact pressures. In Figure 4 those threshold values \(\varepsilon\) were established for which the contact angle for this method will be verging. Especially in cases \(D_2 = 28\) mm and \(N_{\text{max}} = 2900\) N, \(N_{\text{av}} = 1900\) N appears at \(\varepsilon > 0.05\) mm, and for \(D_2 = 58\) mm, \(N_{\text{max}} = 2900\) N, when \(\varepsilon > \varepsilon_{\text{min}} = 0.05\) mm.

The reduction of the initial radial clearance in endoprosthesis of the examined type below 0.05 mm aims at the reduction of the initial contact pressures is not justified since those pressures will be reduced while acetabulum wear. It is also known that at acetabulum wear the size of
its diameter approaches the size of the diameter of rigid head, which technically is hardwearing. Therefore, the construction clearance declines which results in the reduction of maximum contact pressures.

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