Basis invariant conditions for supersymmetry in the two-Higgs-doublet model

P. M. Ferreira,1,2 Howard E. Haber,3 and João P. Silva1,4
1Instituto Superior de Engenharia de Lisboa, Rua Conselheiro Emídio Navarro, 1900 Lisboa, Portugal
2Centro de Física Teórica e Computacional, Faculdade de Ciências, Universidade de Lisboa, Av. Prof. Gama Pinto 2, 1649-003 Lisboa, Portugal
3Santa Cruz Institute for Particle Physics, University of California, Santa Cruz, California 95064, USA
4Centro de Física Teórica de Partículas, Instituto Superior Técnico, P-1049-001 Lisboa, Portugal

(Dated: June 15, 2010)

The minimal supersymmetric standard model involves a rather restrictive Higgs potential with two Higgs fields. Recently, the full set of classes of symmetries allowed in the most general two Higgs doublet model was identified; these classes do not include the supersymmetric limit as a particular class. Thus, a physically meaningful definition of the supersymmetric limit must involve the interaction of the Higgs sector with other sectors of the theory. Here we show how one can construct basis invariant probes of supersymmetry involving both the Higgs sector and the gaugino-higgsino-Higgs interactions.

PACS numbers: 11.30.Er, 12.60.Fr, 14.80.Cp, 11.30.Ly

I. INTRODUCTION

The Standard Model (SM) of electroweak interactions has provided an extraordinarily successful description of currently observed particle physics phenomena. Nevertheless, there are strong reasons to expect that new physics beyond the Standard Model must emerge, ranging from the hierarchy problem and the unification of all coupling constants, to baryogenesis and dark matter. One of the leading candidates for physics beyond the SM incorporates supersymmetry near the scale of electroweak symmetry breaking in order to provide a natural explanation for the existence of the Higgs boson. Much attention has been devoted to the minimal supersymmetric extension of the standard model (MSSM), which requires two complex Higgs doublets and superpartners for all Standard Model particles [1].

In general, there is no fundamental reason why the SM should possess only one complex Higgs doublet. The most well-studied extended Higgs sector is that of the two-Higgs-doublet model (THDM) [2]. The scalar potential of the most general THDM involves 14 parameters. Of these parameters, only eleven combinations are physical, as three degrees of freedom can be absorbed into a redefinition of the Higgs fields [3, 4]. This number may be further reduced by imposing some symmetry requirements on the Higgs Lagrangian. But, identifying such symmetries is complicated by the fact that one may perform a basis transformation of the Higgs fields. A symmetry that looks simple in one basis may be completely obscured in another basis. Hence, it is important to develop basis-invariant signals of such symmetries, which can identify the physically meaningful and experimentally accessible parameters in the theory. The need to seek basis invariant observables in models with many Higgs bosons was pointed out by Lavoura and Silva [5], and by Botella and Silva [6], stressing applications to CP violation. Refs. [6, 7] indicate how to construct basis invariant quantities in a systematic fashion for any model, including multi-Higgs-doublet models. A number of recent articles concerning symmetries and/or basis invariance in the THDM include Refs. [4, 8–19].

It is remarkable that there are exactly six classes of symmetries that one may impose on the scalar sector of the most general THDM. This was shown by Ivanov [15] and expanded upon by us in Ref. [19]. Since the Higgs sector of the MSSM is a particular case of the THDM, one would expect that the constraints satisfied by the Higgs sector of the MSSM would correspond to one of the six classes of symmetries identified in the scalar sector of the THDM. This is not the case. The correct conclusion is that a physically meaningful definition of the supersymmetric limit must involve the interaction of the Higgs sector with other sectors of the supersymmetric theory. In this article we construct basis-invariant probes of supersymmetry involving both the Higgs sector and the gaugino-higgsino-Higgs interactions.

This article is organized as follows. In section II we introduce our notation. In section III we construct the basis invariant quantities that identify the supersymmetric limit of the scalar sector of the THDM. We draw our conclusions in section IV.
II. THE SCALAR SECTOR OF THE THDM

A. The scalar potential

Let us consider a $SU(2) \otimes U(1)$ gauge theory with two hypercharge-one Higgs-doublets, denoted by $\Phi_a$, where $a = 1, 2$. The scalar potential may be written as

$$V_H = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[ m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right]$$

$$\quad + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)$$

$$\quad + \left[ \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \lambda_6 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_7 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_2) + \text{h.c.} \right],$$

(1)

where $m_{11}^2$, $m_{22}^2$, and $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are real parameters, and $m_{12}^2, \lambda_5, \lambda_6$ and $\lambda_7$ are potentially complex.

An alternative notation, useful for the construction of invariants and championed by Botella and Silva [6] is

$$V_H = Y_{ab} (\Phi_a^\dagger \Phi_b) + \frac{1}{2} Z_{ab,cd} (\Phi_a^\dagger \Phi_b)(\Phi_c^\dagger \Phi_d),$$

(2)

where Hermiticity implies

$$Y_{ab} = Y_{ba}^*, \quad Z_{ab,cd} = Z_{cd,ab}^*.$$  

(3)

One should be very careful when comparing Eqs. (1) and (2) among different authors, since the same symbol may be used for quantities that differ by signs, factors of two, or complex conjugation. Here we follow the definitions of Davidson and Haber [4]. With these definitions:

$$Y_{11} = m_{11}^2, \quad Y_{12} = -m_{12}^2, \quad Y_{21} = -(m_{12}^2)^*, \quad Y_{22} = m_{22}^2,$$

(4)

and

$$Z_{11,11} = \lambda_1, \quad Z_{22,22} = \lambda_2,$$

$$Z_{11,22} = Z_{22,11} = \lambda_3, \quad Z_{12,21} = Z_{21,12} = \lambda_4,$$

$$Z_{12,12} = \lambda_5, \quad Z_{21,21} = \lambda_6^*,$$

$$Z_{11,12} = Z_{12,11} = \lambda_6, \quad Z_{11,21} = Z_{21,11} = \lambda_6^*,$$

$$Z_{22,12} = Z_{22,21} = \lambda_7, \quad Z_{22,22} = Z_{21,21} = \lambda_7^*.$$  

(5)

B. Basis transformations

The scalar potential can be rewritten in terms of new fields $\Phi'^a$, obtained from the original ones by a simple (global) basis transformation

$$\Phi_a \rightarrow \Phi'^a = U_{ab} \Phi_b,$$

(6)

where $U \in U(2)$ is a $2 \times 2$ unitary matrix. Under this unitary basis transformation, the gauge-kinetic terms are unchanged, but the coefficients $Y_{ab}$ and $Z_{ab,cd}$ are transformed as

$$Y_{ab} \rightarrow Y'^{ab} = U_{aa} Y_{a\beta} U_{b\beta}^*,$$

(7)

$$Z_{ab,cd} \rightarrow Z'^{ab,cd} = U_{aa} U_{c\gamma} Z_{a\beta,\gamma\delta} U_{b\beta}^* U_{d\delta}^*.$$  

(8)

Thus, the basis transformations $U$ may be utilized in order to absorb some of the degrees of freedom of $Y$ and/or $Z$, which implies that not all parameters of Eq. (2) have physical significance.
C. The six classes of symmetries in the THDM

Symmetries leaving the scalar Lagrangian unchanged may be of two types. On the one hand, one may relate \( \Phi_a \) with some unitary transformation of \( \Phi_b \):

\[
\Phi_a \rightarrow \Phi_a^S = S_{ab} \Phi_b,
\]

where \( S \) is a unitary matrix. These are known as Higgs Family symmetries, or HF symmetries. As a result of this symmetry,

\[
Y_{ab} = S_{aa} Y_{a\beta} S_{b\beta}^*,
\]

\[
Z_{ab,cd} = S_{aa} S_{c\gamma} S_{d\delta}^* Y_{a\gamma} Y_{b\delta}.
\]

On the other hand, one may relate \( \Phi_a \) with some unitary transformation of \( \Phi^*_a \):

\[
\Phi_a \rightarrow \Phi_a^{GCP} = X_{aa} \Phi_a^*,
\]

where \( X \) is an arbitrary unitary matrix. These are known as generalized CP symmetries, or GCP symmetries. The potential is invariant under this symmetry if and only if

\[
Y_{ab}^* = X_{aa}^* Y_{a\beta} X_{b\beta}^*
\]

\[
Z_{ab,cd}^* = X_{aa}^* X_{c\gamma}^* Z_{a\gamma,b\delta} X_{b\delta} X_{d\delta}.
\]

Under the basis transformation of Eq. (6), the specific forms of the HF and GCP symmetries are altered, respectively, as follows:

\[
S' = USU^\dagger,
\]

\[
X' = UXU^\top.
\]

Hence, a basis-invariant treatment is critical for distinguishing between two potentially different symmetries.

Of course, one may combine several HF symmetries and/or GCP symmetries. Ivanov \[15\] has proved that, whatever combination one chooses, one will end up in one of six distinct classes of symmetries. In a recent article we have clarified this issue showing how to construct such classes with simple examples \[19\]. The result is shown in Table I.

| symmetry | \( m_{11}^2 \) | \( m_{12}^2 \) | \( m_{13}^2 \) | \( \lambda_1 \) | \( \lambda_2 \) | \( \lambda_3 \) | \( \lambda_4 \) | \( \lambda_5 \) | \( \lambda_6 \) | \( \lambda_7 \) |
|----------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \( Z_2 \) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| \( U(1) \) | 0 | 0 | \( \lambda_1 \) | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| \( U(2) \) | \( m_{11}^2 \) | 0 | \( \lambda_1 \) | \( \lambda_1 - \lambda_3 \) | 0 | 0 | 0 | 0 |
| CP1 | real | real | real | real | real | real | real | real | real | real |
| CP2 | \( m_{11}^2 \) | 0 | \( \lambda_1 \) | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| CP3 | \( m_{11}^2 \) | 0 | \( \lambda_1 \) | \( \lambda_1 - \lambda_3 - \lambda_4 \) (real) | 0 | 0 | 0 | 0 | 0 | 0 |

1 The space coordinates of the fields, which we have suppressed, are inverted by a generalized CP transformation.
Five of the symmetry classes may be imposed by the following single requirements:

\[ Z_2 : \quad S = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \]

\[ U(1) : \quad S = \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & e^{-i\alpha} \end{pmatrix}, \alpha \neq \pi/2, \]

\[ CP_1 : \quad X = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \]

\[ CP_2 : \quad X = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \]

\[ CP_3 : \quad X = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}. \]

Here \( 0 < \alpha < \pi \) (but \( \alpha \neq \pi/2 \), since the case of \( \alpha = \pi/2 \) corresponds to the \( Z_2 \) symmetry), and \( 0 < \theta < \pi/2 \).

Invariance under the full \( U(2) \) global symmetry is obtained by requiring the invariance of the scalar potential under Eq. (10), for all unitary matrices \( S \).

III. BASIS INVARIANT PROBES OF THE MSSM

A. The Higgs sector of the MSSM

The Higgs potential of the MSSM (prior to including soft-supersymmetry-breaking dimension-two squared-mass terms) is a particular case of Eq. (1), with

\[ m_{11}^2 = m_{22}^2, \]

\[ m_{12}^2 = 0, \]

\[ \lambda_1 = \lambda_2 = \frac{1}{4}(g^2 + g'^2), \]

\[ \lambda_3 = \frac{1}{4}(g^2 - g'^2), \]

\[ \lambda_4 = -\frac{1}{2}g^2, \]

\[ \lambda_5 = \lambda_6 = \lambda_7 = 0, \]

where \( g \) and \( g' \) are the \( SU(2)_L \) and \( U(1)_Y \) gauge coupling constants, respectively. In this case, Eqs. (14) and (15) become

\[ Y_{11} = Y_{22}, \quad Y_{12} = Y_{21} = 0, \]

and

\[ Z_{11,11} = \lambda_1, \quad Z_{22,22} = \lambda_1, \]

\[ Z_{11,22} = Z_{22,11} = \lambda_3, \quad Z_{12,21} = Z_{21,12} = -\lambda_1 - \lambda_3, \]

with \( \lambda_1 \) given by Eq. (23), \( \lambda_3 \) given by Eq. (24), and all other components of the \( Z \) tensor equal to zero.

Comparing Eqs. (21)–(26) with Table I we see that these requirements are almost the same as in the THDM with the full \( U(2) \) flavor symmetry. The difference is that the \( U(2) \)-symmetric case implies \( \lambda_4 = \lambda_1 - \lambda_3 \), while the supersymmetry limit implies \( \lambda_4 = -\lambda_1 - \lambda_3 \). As shown by Ivanov \[15\] and by us \[19\], the former relation can come from a symmetry requirement that exclusively involves the Higgs potential, while the latter relation cannot. In particular, there are no basis changes one can perform on the THDM to obtain, from one of the six symmetries listed in Table I, the SUSY condition \( \lambda_4 = -\lambda_1 - \lambda_3 \).

This can also be seen by examining the renormalization group equations that control the evolution of the \( \lambda_i \). Here, we focus only on those terms arising from the Higgs potential and the gauge couplings.\(^2\) The relevant expressions can

\(^2\)In order to include fermions in the analysis, one would have to investigate the constraints of the THDM symmetries on the Higgs-fermion Yukawa couplings.
be found, for example, in Refs. [22–25]. Using \( \lambda_1 = \lambda_2 \) and \( \lambda_5 = \lambda_6 = \lambda_7 = 0 \), we find

\[
\mathcal{D}_1 = 6\lambda_1^2 + 2\lambda_2^2 + \lambda_3^2 + 2\lambda_3\lambda_4 - \frac{1}{2}\left(9g^2 + 3g'^2\right)\lambda_1 + \frac{1}{6}\left(9g^4 + 6g^2g'^2 + 3g'^4\right),
\]

\[
\mathcal{D}_3 = 2\lambda_3^2 + \lambda_4^2 + 2\lambda_1(3\lambda_3 + \lambda_4) - \frac{1}{2}\left(9g^2 + 3g'^2\right)\lambda_3 + \frac{1}{6}\left(9g^4 - 6g^2g'^2 + 3g'^4\right),
\]

\[
\mathcal{D}_4 = 2\lambda_4^2 + 2\lambda_1\lambda_4 + 4\lambda_3\lambda_4 - \frac{1}{2}\left(9g^2 + 3g'^2\right)\lambda_4 + \frac{2}{9}g^2g'^2,
\]

(29)

where \( \mathcal{D} = 16\pi^2\mu(d/d\mu) \), and \( \mathcal{D}_5 = \mathcal{D}_6 = \mathcal{D}_7 = 0 \). Hence, given the constraints \( \lambda_1 = \lambda_2 \) and \( \lambda_5 = \lambda_6 = \lambda_7 = 0 \),

\[
\mathcal{D}(\lambda_1 + \lambda_3 - \lambda_1) = \frac{1}{2}((\lambda_1 + \lambda_3 - \lambda_1)(12\lambda_1 + 4\lambda_4 - 9g^2 - 3g'^2))
\]

(30)

\[
\mathcal{D}(\lambda_1 + \lambda_3 + \lambda_1) = 2(3\lambda_1^2 + (3\lambda_3 + 2\lambda_4)\lambda_1 + 2\lambda_3^2 + 2\lambda_3\lambda_4)
\]

\[-\frac{1}{2}\left(9g^2 + 3g'^2\right)(\lambda_4 + \lambda_3 + \lambda_1) + \frac{1}{6}\left(9g^4 + 6g^2g'^2 + 3g'^4\right).
\]

(31)

The first equation vanishes if \( \lambda_4 = \lambda_1 - \lambda_3 \); the second does not vanish if \( \lambda_4 = -\lambda_1 - \lambda_3 \). That is, the condition \( \lambda_1 = \lambda_1 - \lambda_3 \) is renormalization group invariant, whereas the condition \( \lambda_4 = -\lambda_1 - \lambda_3 \) is not. Note that we have not yet imposed the specific relations between the \( \lambda \) and the gauge couplings required by the MSSM. If we impose the MSSM constraints specified by Eqs. (23)–(25) on the right hand side of Eq. (31), we obtain

\[
\mathcal{D}(\lambda_1 + \lambda_3 + \lambda_1) = 3g^4 + 2g^2g'^2 + g'^4,
\]

(32)

i.e., \( \lambda_1 = -\lambda_3 - \lambda_4 \) is still not RGE invariant.

The latter result is not unexpected. After all, the gauge boson–Higgs boson sector considered by itself can never be supersymmetric, as the corresponding superpartners are not included. Consequently, the SUSY limit of the gauge boson–Higgs boson sector can only be defined in a manner invariant under Higgs basis changes if the corresponding gaugino and higgsino superpartners are taken into account. The gaugino and higgsino interactions generate additional terms on the right hand side of Eq. (29). In the supersymmetric limit, these effects yield [22]

\[
\delta_{\text{SUSY}}(\mathcal{D}_1) = -\frac{5}{2}g^4 - g^2g'^2 - \frac{1}{2}g'^4,
\]

\[
\delta_{\text{SUSY}}(\mathcal{D}_3) = -\frac{5}{2}g^4 + g^2g'^2 - \frac{1}{2}g'^4,
\]

\[
\delta_{\text{SUSY}}(\mathcal{D}_4) = 2g^4 - 2g^2g'^2.
\]

(33)

Hence,

\[
\delta_{\text{SUSY}}(\mathcal{D}(\lambda_1 + \lambda_3 + \lambda_1)) = -(3g^4 + 2g^2g'^2 + g'^4).
\]

Indeed, when the latter is added to Eq. (32), we see that \( \mathcal{D}(\lambda_1 + \lambda_3 + \lambda_1) = 0 \) as expected. Thus, the SUSY relation \( \lambda_1 = -\lambda_3 - \lambda_4 = -\frac{1}{4}(g^2 + g'^2) \) is renormalization group invariant when all the Higgs/higgsino/gauge/gaugino interactions are included.

### B. The gaugino-higgsino-Higgs interactions

In the MSSM, the tree-level Lagrangian describing the interactions of the gauginos with the Higgs-doublets may be written as

\[
\mathcal{L}_{\text{MSSM}}^{\text{gaugino-Higgs}} = \mu c_1 \psi_{H_u}^j \psi_{H_u}^j + \frac{ig}{\sqrt{2}} \lambda^\alpha \lambda^\alpha \left( \psi_{H_u}^j \Phi_{H_u}^{j\dagger} + e^{ik} \psi_{H_D}^j \Phi_{H_D}^{j\dagger} \right) + \frac{ig'}{\sqrt{2}} \lambda' \left( \psi_{H_u}^j \Phi_{H_u}^{j\dagger} - e^{ik} \psi_{H_D}^j \Phi_{H_D}^{j\dagger} \right) + \text{h.c.},
\]

(34)

where \( \lambda^\alpha \) and \( \lambda' \) are the two-component spinor gaugino fields that are superpartners to the SU(2) and U(1)-hypercharge gauge bosons, and \( \psi_{H_D} \) and \( \psi_{H_U} \) are, respectively, the hypercharge \(-1\) and hypercharge \(+1\) weak doublet two-component spinor higgsino fields. The indices \( i, j \) and \( k \) label the two components of the weak doublet, and the index \( \alpha \) is the adjoint index of the SU(2) gaugino field. We have included a supersymmetric Majorana mass term for the two-component higgsino fields, which defines the parameter \( \mu \). As usual, \( e^{12} = -e^{21} = +1 \) and \( e^{11} = e^{22} = 0 \).

If we relax the constraints imposed by supersymmetry, the coupling strengths of the gaugino-higgsino-Higgs interaction above are no longer constrained to be gauge couplings as in Eq. (34). Moreover, four additional dimension-four
interaction terms are possible, consistent with SU(2)×U(1) gauge invariance. These terms are the so-called “wrong-Higgs” couplings of Ref. [26], and are obtained from those of Eq. (34) by interchanging Φ₁ and Φ₂. In our analysis below, we consider the most general dimension-four gauge invariant couplings between the gaugino, higgsino and Higgs fields. We shall write these couplings in a form that is manifestly independent of the choice of basis for the Higgs fields:

\[
\mathcal{L}_{\text{gaugino-Higgs}} = \frac{i}{\sqrt{2}} \lambda^a_{ij} \epsilon_{ij} \left( \bar{\psi}_H^j f_U^i \Phi_i^a + \epsilon^{ik} \psi^i_H f_D^i \Phi^k_a \right) + \frac{i}{\sqrt{2}} \lambda' \left( \bar{\psi}_H^j f_U^i \Phi_i^a - \epsilon^{ik} \psi^i_H f_D^i \Phi^k_a \right) + \text{h.c.} ,
\]

where the couplings \( f_U^i, f_D^i, f_U^a, \) and \( f_D^a \) transform covariantly under a Higgs basis U(2)-transformation,\(^3\)

\[
f_U^a \rightarrow U_{ab} f_U^b, \quad f_D^a \rightarrow U_{ab} f_D^b, \quad f_U^a \rightarrow f_U^a, \quad f_D^a \rightarrow f_D^a.
\]

In the supersymmetric limit, there is a natural choice of basis for the Higgs fields, henceforth called the SUSY basis, in which:

\[
f_U^a = g \delta^{a2}, \quad f_D^a = g' \delta^{a1}, \quad f_U^a = g' \delta^{a1}, \quad f_D^a = g' \delta^{a1}.
\]

In particular, in the SUSY basis, the so-called “wrong-Higgs interactions” of Ref. [26] are absent in the supersymmetric limit. However, under a general Higgs basis transformation, the supersymmetric gaugino-higgsino-Higgs Lagrangian will transform into a linear combination of supersymmetric and wrong-Higgs interaction terms. Thus, in a generic basis choice for the Higgs fields, the supersymmetry is not manifest. One of the goals of this section is to determine a set of basis-independent conditions that guarantees the existence of a basis choice in which Eq. (37) is satisfied. Such basis-independent conditions would constitute an invariant signal for manifest supersymmetric Higgs interactions.

The couplings \( f_U^i, f_D^i, f_U^a, \) and \( f_D^a \) are complex vectors that live in the two-dimensional Higgs flavor space. It is convenient to define the corresponding unit vectors, \( f^a = f^a / |f|, \) where \( |f| = (f^a \cdot f^a)^{1/2} \) is the length of the complex vector \( f^a. \) Next, we introduce vectors that are orthogonal to \( f_U^a, f_D^a, f_U^a, \) and \( f_D^a \), respectively,

\[
\hat{g}_U^a \equiv f_U^a \epsilon^{ba}, \quad \hat{g}_D^a \equiv f_D^a \epsilon^{ba}, \quad \hat{g}_U^a \equiv f_U^a \epsilon^{ba}, \quad \hat{g}_D^a \equiv f_D^a \epsilon^{ba}.
\]

These are pseudo-vectors with respect to U(2) Higgs basis transformations,\(^4\)

\[
\hat{g}_U^a \rightarrow (\det U)^{-1} U_{ab} \hat{g}_U^b, \quad \hat{g}_D^a \rightarrow (\det U)^{-1} U_{ab} \hat{g}_D^b.
\]

due to the appearance of the complex phase, \( \det U. \)

We now define U(2)-invariant, hypercharge-one Higgs fields as follows:

\[
H_U \equiv \hat{f}_U^a \Phi^a, \quad H_U \equiv \hat{f}_U^a \Phi^a, \quad H_D \equiv \hat{f}_D^a \Phi^a, \quad H_D \equiv \hat{f}_D^a \Phi^a.
\]

One can also define a corresponding set of hypercharge \(-1\) fields, e.g.,

\[
H_D^i \equiv \epsilon^{ij} H_D^j, \quad H_D^i \equiv \epsilon^{ij} H_D^j.
\]

It is also convenient to define U(2) pseudo-invariant Higgs fields (denoted by calligraphic fonts),

\[
\mathcal{H}_U \equiv \hat{g}_U^a \Phi^a, \quad \mathcal{H}_U \equiv \hat{g}_U^a \Phi^a, \quad \mathcal{H}_D \equiv \hat{g}_D^a \Phi^a, \quad \mathcal{H}_D \equiv \hat{g}_D^a \Phi^a.
\]

\(^3\) Note that the global U(1) Higgs flavor transformation corresponding to \( \Phi_a \rightarrow e^{i \chi} \Phi_a \) (\( a = 1, 2 \)) is distinguished from the global U(1) hypercharge transformation, since the higgsino fields do not transform under the rephasing of the Higgs fields.

\(^4\) Starting from the transformation law \( f^a \rightarrow U_{ab} f^b, \) where \( U = U^{-1} \), it follows that \( \hat{g}^a \rightarrow U_{ab} g^b e^{adbc} \). If we now recognize that \( U_{ab} e^{ad} = \det(U^{-1}) U_{cd} e^{bc} \), the results of Eq. (40) easily follow.
It then follows that:

\[ \Phi_a = \hat{f}^a_U H_U + \hat{g}^a_U H_U = \hat{f}^a_D \tilde{H}_D + \hat{g}^a_D \tilde{H}_D \]

(46)

\[ = \hat{f}_U^{a} H'_U + \hat{g}_U^{a} H'_U = \hat{f}_D^{a} \tilde{H}'_D + \hat{g}_D^{a} \tilde{H}'_D , \]

(47)

after using \( \hat{f}^a_{U,D} \hat{g}^a_{U,D} = \hat{g}^a_{U,D} \hat{f}^a_{U,D} = 1 \) and \( \hat{g}^a_{U,D} \hat{g}^a_{U,D} = 0 \).

There is some motivation for this proliferation of Higgs field definitions. In particular, as we show later in Eqs. (55)–(58), the choices of

\[ \{ H_U, H_U^t \}, \{ H'_U, H'_U^t \}, \{ \tilde{H}_D, \tilde{H}_D^t \}, \text{ and } \{ \tilde{H}'_D, \tilde{H}'_D^t \} , \]

(48)

correspond to four different basis choices for the hypercharge-one Higgs doublet fields.

One can express the gaugino-higgsino-Higgs interaction Lagrangian in a manifestly U(2)-invariant form. For example, using the definitions of the invariant Higgs fields \( H_U, H_D, H'_U \) and \( H'_D \) [defined by Eqs. (41) and (43)], Eq. (35) can be rewritten as:

\[ \mathcal{L}_{\text{gaugino-Higgs}} = \frac{i}{\sqrt{2}} \lambda^\alpha \tau^\alpha_{ij} \left( |f_U| \psi^j_{H_U} H_U^{i\dagger} + |f_D| \psi^j_{H_D} H_D^{i\dagger} \right) + \frac{i}{\sqrt{2}} \gamma \left( |f_U'| \psi^j_{H_U'} H_U'^{i\dagger} - |f_D'| \psi^j_{H_D'} H_D'^{i\dagger} \right) + \text{h.c.} \]

(49)

However, this form is not particularly useful outside of the supersymmetric limit, since \( \{ H_U, \tilde{H}_D \} \) and \( \{ H'_U, \tilde{H}'_D \} \) are not orthogonal pairs of hypercharge-one Higgs doublet fields in the general case. Of course, one can always rewrite Eq. (35) in terms of one of the four basis choices of hypercharge one doublet Higgs fields listed in Eq. (48) by inserting the appropriate form for \( \Phi_a \) given in Eqs. (46) and (47) into Eq. (35).

C. Basis-invariant probes of the supersymmetric Higgs interactions

Supersymmetry imposes strong constraints on the scalar Higgs potential and the gaugino-higgsino-Higgs interactions. These constraints must involve basis-independent combinations of the scalar potential parameters \( Y_{ab}, Z_{abcd} \), and the gaugino-higgsino-Higgs couplings \( f_{U,D}^a, f_{U,D}^{a*} \). It is straightforward to find the necessary relations. First we exhibit the basis invariant relations that enforce supersymmetric gaugino-higgsino-Higgs couplings:

\[ f_U^a f_{U}^{a*} = 0, \quad f_U^a f_{D}^{a*} = 0, \]

\[ f_D^a f_{D}^{a*} = 0, \quad f_D^a f_{U}^{a*} = g^2, \]

\[ f_U^a f_{U}^{a*} = g g', \quad f_D^a f_{D}^{a*} = g g'. \]

(50)

To establish U(2)-invariant conditions that enforce a supersymmetric scalar Higgs potential, we first construct basis-independent quantities that involve both the scalar potential parameters and the gaugino-higgsino-Higgs couplings. For example,

\[ \mathcal{Y}_{DD} = \hat{f}_D f_{D}^{a*} Y_{ab}, \quad \mathcal{Y}_{UU} = \hat{f}_U f_{U}^{a*} Y_{ab}, \]

\[ \mathcal{Y}_{DU} = \hat{f}_D f_{U}^{a*} Y_{ab}, \quad \mathcal{Y}_{UD} = \hat{f}_U f_{D}^{a*} Y_{ab}, \]

(51)

provide basis-invariant quantities involving the quadratic coefficients of the Higgs potential. Likewise,

\[ Z_{\alpha \beta \gamma \delta} = \hat{f}^{a} \hat{f}_{D}^{b*} \hat{f}_{U}^{c*} \hat{f}_{D}^{d*} Z_{ab,cd}, \]

(52)

where the indices \( \alpha, \beta, \gamma, \) and \( \delta \) can take the values \( D \) or \( U \), provide basis-invariant quartic coefficients for the Higgs potential.

Evaluating the invariant quantities introduced in Eqs. (51) and (52) in the supersymmetric basis defined by Eqs. (27), (28) and (37), it follows that

\[ \mathcal{Y}_{DD} = \mathcal{Y}_{UU}, \quad \mathcal{Y}_{DU} = \mathcal{Y}_{UD} = 0, \]

(53)

5 Assuming Eq. (30) is satisfied, it is not necessary to construct additional invariants that involve \( f_U^a \) and \( f'_D \).
Since these equations are basis invariant, they must hold in any theory made up of the MSSM fields with exact supersymmetry, regardless of the exact basis choice made for the Higgs fields.

That is, independently of the choice of basis for the Higgs fields, the supersymmetric limit of the Higgs/higgsino/gauge/gaugino sectors holds if and only if Eqs. (50), (55) and (54) hold. If Eqs. (50) and (54) hold but Eq. (53) does not, then supersymmetry is softly broken due to the quadratic terms of the Higgs potential. The above results fully resolves the question of the basis-invariant form for the supersymmetric limit of the THDM.

D. A preferred basis in the MSSM

The gaugino-higgsino-Higgs interactions provide a means for defining a *quasi-physical* choice of basis. In this context, a quasi-physical basis is one in which the Higgs fields are invariant, up to a possible rephasing of one of the Higgs fields, under U(2) transformations. That is, the coefficients of the Higgs potential are either U(2)-invariants or pseudo-invariants.

There are four possible independent quasi-physical bases, corresponding to the four normalized gaugino-higgsino-Higgs couplings, \( f_U^a, f_D^a, f_U^a \) and \( f_D^a \). Each basis is defined by imposing the condition that one of the two components of the corresponding coupling vanishes, while setting the non-vanishing component to unity. This defines the quasi-physical basis up to an arbitrary rephasing of the Higgs field that lies in the direction of the vanishing component of \( f \). The latter is a quasi-invariant field, whereas the Higgs field that lies in the direction of the non-vanishing component of \( f \) is a U(2)-invariant field.\(^6\)

The four quasi-physical bases and their corresponding Higgs fields are:

\[ B_U : \text{ defined by } f_U^a = (0, 1), \quad \text{Higgs fields} : (H_U, H_U) \],

\[ B_D : \text{ defined by } f_D^a = (1, 0), \quad \text{Higgs fields} : (H_D, H_D) \],

\[ B_U' : \text{ defined by } f_U'^a = (0, 1), \quad \text{Higgs fields} : (H_U', H_U') \],

\[ B_D' : \text{ defined by } f_D'^a = (1, 0), \quad \text{Higgs fields} : (H_D', H_D') \],

where the fields denoted by (calligraphic) Roman fonts are (pseudo-)invariant with respect to U(2) basis transformations. The Higgs fields with a \( U \) (\( D \)) subscript are hypercharge \(+1\) (\(-1\)) fields. The coefficients of the scalar potential in the quasi-physical basis are easily constructed. For example, in basis \( B_D \),

\[
\begin{array}{l}
Y_{D1} \equiv \frac{\alpha_{ab}}{D} f_D^a \tilde{f}_D^b Y_{ab}, \quad Y_{D2} \equiv \frac{\alpha_{ab}}{D} g_D^a Y_{ab}, \\
Y_{D3} \equiv \frac{\alpha_{ab}}{D} g_D^a Y_{ab}, \quad Z_{D1} \equiv \frac{\alpha_{ab}}{D} \tilde{f}_D^a \tilde{f}_D^b Z_{abcd}, \\
Z_{D2} \equiv \frac{\alpha_{ab}}{D} g_D^a g_D^b Z_{abcd}, \quad Z_{D3} \equiv \frac{\alpha_{ab}}{D} \tilde{f}_D^a g_D^b g_D^d Z_{abcd}, \\
Z_{D4} \equiv \frac{\alpha_{ab}}{D} \tilde{f}_D^a \tilde{f}_D^b \tilde{z}_{abcd}, \quad Z_{D5} \equiv \frac{\alpha_{ab}}{D} \tilde{f}_D^a \tilde{f}_D^b \tilde{z}_{abcd}, \\
Z_{D6} \equiv \frac{\alpha_{ab}}{D} \tilde{f}_D^a \tilde{f}_D^b \tilde{z}_{abcd}, \quad Z_{D7} \equiv \frac{\alpha_{ab}}{D} \tilde{f}_D^a g_D^b g_D^d \tilde{z}_{abcd}.
\end{array}
\]

That is, starting in a generic basis with a Higgs potential given by Eq. (10), one can always transform to the basis \( B_D \)

\(^6\) In the most general 2HDM after spontaneous symmetry breaking, the so-called Higgs basis in which \( \tilde{\phi} = (1, 0) \) is an example of a quasi-physical basis. The Higgs fields in this basis, \( H_3 \equiv \bar{\psi}_3 \Phi_3 \) and \( H_2 \equiv \epsilon_{ab} \bar{\psi}_a \Phi_3 \), are respectively invariant and pseudo-invariant with respect to U(2) basis transformations. See Refs. 4, 6, 11, 12 for details.
with a Higgs potential given by:

\[ V_H = Y_D^a H_D H_D + Y_{D_2}^a H_{D_2} H_{D_2} + \left[ Y_{D_3}^a H_D^a H_D + \text{h.c.} \right] \]

\[ + \frac{1}{2} Z_{D_4}(H_D^a H_D)^2 + \frac{1}{2} Z_{D_2}(H_D^a H_D)^2 + Z_{D_1}(H_D^a H_D)(H_D^a H_D) + Z_{D_5}(H_D H_D)(H_D^a H_D) + Z_{D_6}(H_D H_D)(H_D^a H_D) + \text{h.c.} \].

(64)

The parameters \( Y_{D_1}, Y_{D_2}, Z_{D_1}, Z_{D_2}, Z_{D_3} \) and \( Z_{D_4} \) are manifestly real \( U(2) \)-invariants, whereas the parameters \( Y_{D_2}, Z_{D_5}, Z_{D_6} \) are (potentially) complex pseudo-invariants with respect to \( U(2) \) basis transformations. The physical Higgs couplings and masses of the theory can be expressed in terms of the invariant parameters and invariant combinations of the pseudo-invariant parameters.

The discussion above is completely general. But, suppose we impose supersymmetry on the Higgs-higgsino-gaugino system. Supersymmetry imposes a very strong constraint on the quasi-physical bases,

\[ \mathcal{B}_D = \mathcal{B}_U = \mathcal{B}'_D = \mathcal{B}'_U. \]

(65)

That is, supersymmetry picks out a physically meaningful basis, namely the SUSY basis, in which the gaugino-higgsino-Higgs couplings take the form given by Eq. (37). Eq. (65) impose covariant relations among the normalized gaugino-higgsino-Higgs couplings:

\[ \tilde{g}_D^a = g_D^a = e^{i\eta} \tilde{f}_D^a = e^{i\eta} \tilde{f}_U^a, \quad \tilde{g}_U^a = g_U^a = -e^{i\eta} \tilde{f}_D^a = -e^{i\eta} \tilde{f}_U^a, \]

(66)

where \( e^{i\eta} \) is a pseudo-invariant quantity [i.e., \( e^{i\eta} \to (\det U)^{-1} e^{i\eta} \) under a \( U(2) \) transformation] that is equal to 1 in the SUSY basis. Eq. (66) implies that the invariant and pseudo-invariant Higgs fields defined in Eqs. (11)–(15) are related in the supersymmetric limit,

\[ H_U = H'_U = e^{i\eta} \tilde{H}_D = e^{i\eta} \tilde{H}'_D, \]

(67)

\[ H_D = H'_D = e^{-i\eta} \tilde{H}_U = e^{-i\eta} \tilde{H}'_U, \]

(68)

for the hypercharge +1 and hypercharge −1 MSSM Higgs fields, respectively. If we now insert Eq. (66) into Eqs. (69)–(73), we find that the coefficients of the Higgs potential in the SUSY basis are \( \gamma_{DD}, \gamma_{UU}, \gamma_{UD}, \gamma_{UU,DD}, \gamma_{DD,DD}, \gamma_{DD,DU}, \gamma_{DU,UD}, e^{i\eta} \tilde{Z}_{DU,DU}, e^{i\eta} \tilde{Z}_{DD,DU}, e^{i\eta} \tilde{Z}_{UU,UD} \), subject to the conditions of Eqs. (53) and (54). Note that when the conditions of Eqs. (53) and (54) are imposed, the phase \( \eta \) drops out completely. Indeed, the nonzero Higgs potential parameters in the SUSY basis, \( \gamma_{DD}, \gamma_{UU}, \gamma_{DU,DU}, \gamma_{DD,DD}, \gamma_{DD,DU}, \gamma_{DU,UD} \) are physical observables that are real and invariant with respect to \( U(2) \)-basis transformations.

The existence of a physical SUSY basis is not surprising. Consider the higgsino Majorana mass term, which arises from a supersymmetric term in the MSSM superpotential,

\[ \mu \epsilon_{ij} \psi_{HD}^i \psi_{HD}^j. \]

(69)

A THDM basis transformation mixes \( \Phi_1 \) and \( \Phi_2 \), which corresponds to a transformation between the MSSM Higgs fields, \( H_U \) with \( H_D^a \). But the Higgs fields belong to Higgs superfields whose scalar and fermionic components are \( (H_U, \psi_{H_U}) \) and \( (H_D, \psi_{H_D}) \). As a result, one cannot apply a general \( U(2) \)-basis transformation to the full Higgs supermultiplets consistent with supersymmetry. Thus, Eq. (69) effectively defines a preferred basis for \( \Phi_1 \) and \( \Phi_2 \) within the MSSM. This is why the SUSY basis has a physical significance.

E. Invariant \( \tan \beta \)-like parameters

Until this subsection, we have made no assumptions about the nature of the Higgs vacuum. In the supersymmetric limit of the MSSM, the minimum of the tree-level potential resides at zero field and there is no electroweak symmetry breaking. After dimension-two soft-supersymmetry-breaking terms are included, it becomes possible to break the SU(2) \( \times \) U(1) symmetry of the scalar potential down to U(1)_{EM}. In the generic basis, we define vacuum expectation values,

\[ \langle \Phi_a^0 \rangle = v_a \equiv v \tilde{v}_a, \]

(70)
where $\hat{v}_a$ is a complex unit vector in the two-dimensional Higgs flavor space and $v \approx 246$ GeV. In the MSSM, the parameter $\tan \beta$ is defined in the SUSY basis,\(^7\)

\[ \tan \beta \equiv \frac{v_U}{v_D}. \tag{71} \]

However, in the most general THDM, $\tan \beta$ is a basis-dependent concept that does not correspond to any physical observable. In Ref. [12], a number of invariant parameters connected to the Higgs-fermion Yukawa couplings are introduced that play the role of $\tan \beta$ in constrained THDMs. Here, we shall indicate how to define $\tan \beta$-like parameters associated with the gaugino-higgsino-Higgs interactions.

The basic idea is to define $\tan \beta$ as the ratio of vacuum expectation values in the quasi-physical bases introduced in Eqs. (55)–(58). One can then define four invariant $\tan \beta$-like parameters,\(^8\)

\[
\begin{align*}
\tan \beta_U &\equiv -e^{-i\eta} \frac{\langle H_U^0 \rangle}{\langle H_D^0 \rangle} = -e^{-i\eta} \frac{\hat{f}_U^a \hat{v}_a}{\hat{g}_U^a \hat{v}_a}, & \tan \beta_D &\equiv e^{-i\eta} \frac{\langle H_D^0 \rangle}{\langle H_U^0 \rangle} = e^{-i\eta} \frac{\hat{g}_D^a \hat{v}_a^*}{\hat{f}_D^a \hat{v}_a^*}, \tag{72} \\
\tan \beta'_U &\equiv -e^{-i\eta} \frac{\langle H_U^{0*} \rangle}{\langle H_U^0 \rangle} = -e^{-i\eta} \frac{\hat{f}_U^{a*} \hat{v}_a}{\hat{g}_U^{a*} \hat{v}_a}, & \tan \beta'_D &\equiv e^{-i\eta} \frac{\langle H_U^{0*} \rangle}{\langle H_D^0 \rangle} = e^{-i\eta} \frac{\hat{g}_U^{a*} \hat{v}_a^*}{\hat{f}_D^a \hat{v}_a^*}. \tag{73}
\end{align*}
\]

Without the factors of $e^{i\eta}$, the corresponding $\tan \beta$-like parameters are complex pseudo-invariants, whose magnitudes are basis-independent. The interpretation of these $\tan \beta$-like parameters is simplest in the Higgs basis, which is defined by $\hat{v} = (1, 0)$. In the Higgs basis, the parameters in Eqs. (72) and (73) are (up to an overall phase) simply the ratios of gaugino-higgsino-Higgs couplings. One can now investigate the limit in which all dimension-four couplings respect supersymmetry. In this case, the invariant and pseudo-invariant Higgs fields are constrained by Eqs. (67) and (68), in which case,

\[ \tan \beta = \frac{\langle H_U^0 \rangle}{\langle H_D^0 \rangle} = \tan \beta_U = (\tan \beta_D)^* = \tan \beta'_U = (\tan \beta'_D)^*, \quad [\text{SUSY limit}]. \tag{74} \]

Corrections to the above relations at the few percent level are expected at the one-loop level due to supersymmetry-breaking effects that enter into the loops. In principle, one could extract the $\tan \beta$ parameters of Eqs. (72) and (73) from collider data (assuming gaugino-higgsino-Higgs couplings could be measured with sufficient accuracy), and check to see whether Eq. (74) holds.

**IV. CONCLUSIONS**

The THDM includes the Higgs sector of the MSSM as a particular case. In the THDM only quantities that are invariant under a unitary basis change, $\Phi_a \to U_{ab} \Phi_b$ (where $a, b = 1, 2$ and $U$ is an arbitrary unitary matrix) can have physical meaning. Thus, it is natural to look for basis invariant definitions of the supersymmetric limit. We have shown that there is no basis-invariant definition of a supersymmetric THDM based on invariants defined exclusively in terms of parameters of the scalar Higgs potential. We have constructed basis invariant probes of the supersymmetric limit and soft-supersymmetry-breaking by employing both the Higgs potential and the gaugino-higgsino-Higgs interactions. We also observe that the usual basis choice of $H_U$ and $H_D$ in the MSSM does have a physical significance, due to the $\mu$-term of the MSSM superpotential [cf. Eq. (39)].

Finally, we addressed the physical significance of the parameter $\tan \beta$. In the MSSM, the tree-level parameter $\tan \beta$ is well-defined because it is defined in terms of vacuum expectation values of the neutral Higgs fields, which are given in a physical basis. However, in the most general THDM Higgs sector coupled to new particles with electroweak quantum numbers that coincide with the gauginos and higgsinos of the MSSM, the parameter $\tan \beta$, which is defined in terms of a ratio of Higgs vacuum expectation values, is a basis dependent quantity and hence unphysical. We have shown that four invariant $\tan \beta$-like parameters can be defined that are basis-independent and hence physical. One can verify that in the supersymmetric limit, these four invariant parameters coincide (at tree-level) and are equal to the $\tan \beta$ parameter of the MSSM.

---

\(^7\) In the SUSY basis, the overall phase of the Higgs fields can be chosen such that $v_U$ and $v_D$ are real and positive.

\(^8\) The signs in Eqs. (72) and (73) have been conveniently chosen so that no extraneous signs appear in the SUSY limit [cf. Eq. (74)].
Acknowledgments

The work of P.M.F. is supported in part by the Portuguese Fundação para a Ciência e a Tecnologia (FCT) under contract PTDC/FIS/70156/2006. The work of H.E.H. is supported in part by the U.S. Department of Energy, under grant number DE-FG02-04ER41268. The work of J.P.S. is supported in part by FCT under contract CFTP-Plurianual (U777).

H.E.H. is most grateful for the kind hospitality and support of the Centro de Física Teórica e Computacional at Universidade de Lisboa during his visit to Lisbon. He also acknowledges conversations with Sacha Davidson and John Mason, which inspired a number of ideas presented in this paper.

[1] For a review and references to the literature, see e.g., H.E. Haber, “Supersymmetry, Part 1 (Theory),” in C. Amsler et al. [Particle Data Group Collaboration], Review of Particle Physics, Phys. Lett. B667 (2008) 1211 and 2009 partial update for the 2010 edition.
[2] T.D. Lee, Phys. Rev. D 8 1226 (1973); Phys. Rep. 9, 143 (1974); N.G. Deshpande and E. Ma, Phys. Rev. D 18 (1978) 2574; H.E. Haber, G.L. Kane and T. Sterling, Nucl. Phys. B 161, 493 (1979); J.F. Donoghue and L.F. Li, Phys. Rev. D 19, 945 (1979).
[3] L. Lavoura, Phys. Rev. D 50, 7089 (1994).
[4] S. Davidson and H. E. Haber, Phys. Rev. D 72, 055004 (2005) [Erratum-ibid D 72, 099902 (2005)].
[5] L. Lavoura, J. P. Silva, Phys. Rev. D 50, 4619 (1994).
[6] F. J. Botella and J. P. Silva, Phys. Rev. D 51, 3870 (1995).
[7] G. C. Branco, L. Lavoura, and J. P. Silva, CP Violation (Oxford University Press, Oxford, UK, 1999).
[8] J. F. Gunion, talk given at the CPNSH, CERN, Switzerland, December (2004).
[9] G. C. Branco, M. N. Rebelo, and J. I. Silva-Marcos, Phys. Lett. B 614, 187 (2005).
[10] I. F. Ginzburg and M. Krawczyk, Phys. Rev. D 72, 115013 (2005).
[11] J. F. Gunion and H. E. Haber, Phys. Rev. D 72, 095002 (2005); H. E. Haber and D. O’Neil, Phys. Rev. D 74, 015018 (2006) [Erratum-ibid. D 74, 059905 (2006)].
[12] C. C. Nishi, Phys. Rev. D 74, 036003 (2006); Phys. Rev. D 76, 055013 (2007); Phys. Rev. D 77, 055009 (2008).
[13] M. Maniatis, A. von Manteuffel, O. Nachtmann, and F. Nagel, Eur. Phys. J. C 48, 805 (2006); M. Maniatis, A. von Manteuffel, and O. Nachtmann, Eur. Phys. J. C 49, 1067 (2007); M. Maniatis, A. von Manteuffel, and O. Nachtmann, Eur. Phys. J. C 57, 719 (2008); M. Maniatis, A. von Manteuffel, and O. Nachtmann, Eur. Phys. J. C 57, 739 (2008); M. Maniatis and O. Nachtmann, JHEP 0905, 028 (2009).
[14] I. P. Ivanov, Phys. Rev. D 77, 015017 (2008).
[15] I. P. Ivanov, Phys. Lett. B 632, 360 (2006); Phys. Rev. D 75, 035001 (2007); ibid. 76, 039902(E) (2007).
[16] J.-M. Gerard, and M. Herquet, Phys. Rev. Lett. 98, 251802 (2007); S. Visscher, J.-M. Gerard, M. Herquet, V. Lemaitre, and F. Maltoni, JHEP 0908, 042 (2009).
[17] P. M. Ferreira and J. P. Silva, Phys. Rev. D 78, 116007 (2008).
[18] P. M. Ferreira, H. E. Haber, and J. P. Silva, Phys. Rev. D 79, 116004 (2009).
[19] For early work on GCP see, for example, G. Ecker, W. Grimus, and W. Konetschny, Nucl. Phys. B191, 465 (1981); G. Ecker, W. Grimus, and H. Neufeld, Nucl. Phys. B247, 70 (1984); H. Neufeld, W. Grimus, and G. Ecker, Int. J. Mod. Phys. A 3, 603 (1988). For more recent treatments, see also Refs. [20] and [21].
[20] For early work on GCP see, for example, G. Ecker, W. Grimus, and H. Neufeld, J. Phys. A20, L807 (1987).
[21] T. P. Cheng, E. Eichten, and L. F. Li, Phys. Rev. D 9, 2259 (1974).
[22] H. E. Haber and R. Hempfling, Phys. Rev. D 48, 4280 (1993).
[23] W. Grimus and L. Lavoura, Eur. Phys. J. C 39, 219 (2005).
[24] P. M. Ferreira and D. R. T. Jones, JHEP 0908, 069 (2009).
[25] H. E. Haber and J. D. Mason, Phys. Rev. D 77, 115011 (2008).
Basis invariant conditions for supersymmetry in the two-Higgs-doublet model

P. M. Ferreira,1,2 Howard E. Haber,3 and João P. Silva1,4

1Instituto Superior de Engenharia de Lisboa, Rua Conselheiro Emídio Navarro, 1900 Lisboa, Portugal
2Centro de Física Teórica e Computacional, Faculdade de Ciências, Universidade de Lisboa, Av. Prof. Gama Pinto 2, 1649-003 Lisboa, Portugal
3Santa Cruz Institute for Particle Physics, University of California, Santa Cruz, California 95064, USA
4Centro de Física Teórica de Partículas, Instituto Superior Técnico, P-1049-001 Lisboa, Portugal

(Dated: June 15, 2010)

The minimal supersymmetric standard model involves a rather restrictive Higgs potential with two Higgs fields. Recently, the full set of classes of symmetries allowed in the most general two Higgs doublet model was identified; these classes do not include the supersymmetric limit as a particular class. Thus, a physically meaningful definition of the supersymmetric limit must involve the interaction of the Higgs sector with other sectors of the theory. Here we show how one can construct basis invariant probes of supersymmetry involving both the Higgs sector and the gaugino-higgsino-Higgs interactions.

PACS numbers: 11.30.Er, 12.60.Fr, 14.80.Cp, 11.30.Ly

I. INTRODUCTION

The Standard Model (SM) of electroweak interactions has provided an extraordinarily successful description of currently observed particle physics phenomena. Nevertheless, there are strong reasons to expect that new physics beyond the Standard Model must emerge, ranging from the hierarchy problem and the unification of all coupling constants, to baryogenesis and dark matter. One of the leading candidates for physics beyond the SM incorporates supersymmetry near the scale of electroweak symmetry breaking in order to provide a natural explanation for the existence of the Higgs boson. Much attention has been devoted to the minimal supersymmetric extension of the standard model (MSSM), which requires two complex Higgs doublets and superpartners for all Standard Model particles.

In general, there is no fundamental reason why the SM should possess only one complex Higgs doublet. The most well-studied extended Higgs sector is that of the two-Higgs-doublet model (THDM). The scalar potential of the most general THDM involves 14 parameters. Of these parameters, only eleven combinations are physical, as three degrees of freedom can be absorbed into a redefinition of the Higgs fields. This number may be further reduced by imposing some symmetry requirements on the Higgs Lagrangian. But, identifying such symmetries is complicated by the fact that one may perform a basis transformation of the Higgs fields. A symmetry that looks simple in one basis may be completely obscured in another basis. Hence, it is important to develop basis-invariant signals of such symmetries, which can identify the physically meaningful and experimentally accessible parameters in the theory. The need to seek basis invariant observables in models with many Higgs bosons was pointed out by Lavoura and Silva, and by Botella and Silva, stressing applications to CP violation. Refs. indicate how to construct basis invariant quantities in a systematic fashion for any model, including multi-Higgs-doublet models. A number of recent articles concerning symmetries and/or basis invariance in the THDM include Refs.

It is remarkable that there are exactly six classes of symmetries that one may impose on the scalar sector of the most general THDM. This was shown by Ivanov and expanded upon by us in Ref. Since the Higgs sector of the MSSM is a particular case of the THDM, one would expect that the constraints satisfied by the Higgs sector of the MSSM would correspond to one of the six classes of symmetries identified in the scalar sector of the THDM. This is not the case. The correct conclusion is that a physically meaningful definition of the supersymmetric limit must involve the interaction of the Higgs sector with other sectors of the supersymmetric theory. In this article we construct basis-invariant probes of supersymmetry involving both the Higgs sector and the gaugino-higgsino-Higgs interactions.

This article is organized as follows. In section we introduce our notation. In section we construct the basis invariant quantities that identify the supersymmetric limit of the scalar sector of the THDM. We draw our conclusions in section.
II. THE SCALAR SECTOR OF THE THDM

A. The scalar potential

Let us consider a $SU(2) \otimes U(1)$ gauge theory with two hypercharge-one Higgs-doublets, denoted by $\Phi_a$, where $a = 1, 2$. The scalar potential may be written as

$$V_H = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[ m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right]$$

$$+ \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1)$$

$$+ \left[ \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \lambda_6 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_7 (\Phi_2^\dagger \Phi_2) (\Phi_2^\dagger \Phi_2) + \text{h.c.} \right],$$

where $m_{11}^2$, $m_{22}^2$, and $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are real parameters, and $m_{12}^2$, $\lambda_5$, $\lambda_6$ and $\lambda_7$ are potentially complex.

An alternative notation, useful for the construction of invariants and championed by Botella and Silva [6] is

$$V_H = Y_{ab} (\Phi_a^\dagger \Phi_b) + \frac{1}{2} Z_{ab,cd} (\Phi_a^\dagger \Phi_b) (\Phi_c^\dagger \Phi_d),$$

where Hermiticity implies

$$Y_{ab} = Y_{ba}^*, \quad Z_{ab,cd} = Z_{cd,ab}^*.$$  

(3)

One should be very careful when comparing Eqs. (1) and (2) among different authors, since the same symbol may be used for quantities that differ by signs, factors of two, or complex conjugation. Here we follow the definitions of Davidson and Haber [4]. With these definitions:

$$Y_{11} = m_{11}^2, \quad Y_{12} = -m_{12}^2, \quad Y_{21} = -(m_{12}^2)^*, \quad Y_{22} = m_{22}^2,$$

(4)

and

$$Z_{11,11} = \lambda_1, \quad Z_{22,22} = \lambda_2, \quad Z_{11,22} = \lambda_3, \quad Z_{22,11} = \lambda_4, \quad Z_{12,12} = \lambda_5, \quad Z_{21,21} = \lambda_6^*, \quad Z_{11,12} = Z_{22,11} = \lambda_6, \quad Z_{12,22} = \lambda_7, \quad Z_{21,21} = Z_{22,22} = \lambda_7^*.$$  

(5)

B. Basis transformations

The scalar potential can be rewritten in terms of new fields $\Phi'_a$, obtained from the original ones by a simple (global) basis transformation

$$\Phi_a \rightarrow \Phi'_a = U_{ab} \Phi_b.$$  

(6)

where $U \in U(2)$ is a $2 \times 2$ unitary matrix. Under this unitary basis transformation, the gauge-kinetic terms are unchanged, but the coefficients $Y_{ab}$ and $Z_{ab,cd}$ are transformed as

$$Y_{ab} \rightarrow Y'_{ab} = U_{aa} Y_{a\beta} U_{b\delta}^*, \quad Z_{ab,cd} \rightarrow Z'_{ab,cd} = U_{aa} U_{c\gamma} Z_{a\beta,\gamma\delta} U_{b\delta}^* U_{d\delta}^*.$$  

(7)

Thus, the basis transformations $U$ may be utilized in order to absorb some of the degrees of freedom of $Y$ and/or $Z$, which implies that not all parameters of Eq. (2) have physical significance.
C. The six classes of symmetries in the THDM

Symmetries leaving the scalar Lagrangian unchanged may be of two types. On the one hand, one may relate \( \Phi_a \) with some unitary transformation of \( \Phi_b \):

\[
\Phi_a \rightarrow \Phi_a^S = S_{ab} \Phi_b,
\]

where \( S \) is a unitary matrix. These are known as Higgs Family symmetries, or HF symmetries. As a result of this symmetry,

\[
Y_{ab} = S_{aa} Y_{a\beta} S_{b\beta}^* \tag{10}
\]

\[
Z_{ab,cd} = S_{aa} S_{c\gamma} Z_{a\beta,\gamma\delta} S_{b\beta}^* S_{d\delta}^* \tag{11}
\]

On the other hand, one may relate \( \Phi_a \) with some unitary transformation of \( \Phi_a^* \):

\[
\Phi_a \rightarrow \Phi_a^{\text{GCP}} = X_{aa} \Phi_a^* \tag{12}
\]

where \( X \) is an arbitrary unitary matrix. These are known as generalized CP symmetries, or GCP symmetries \[20, 21\]. The potential is invariant under this symmetry if and only if

\[
Y_{ab}^* = X_{aa}^* Y_{a\beta} X_{b\beta}^* \tag{13}
\]

\[
Z_{ab,cd}^* = X_{aa}^* X_{c\gamma}^* Z_{a\beta,\gamma\delta} X_{b\beta}^* X_{d\delta}^* \tag{13}
\]

Under the basis transformation of Eq. (6), the specific forms of the HF and GCP symmetries are altered, respectively, as follows:

\[
S' = USU^\dagger \tag{14}
\]

\[
X' = UXU^\top \tag{15}
\]

Hence, a basis-invariant treatment is critical for distinguishing between two potentially different symmetries.

Of course, one may combine several HF symmetries and/or GCP symmetries. Ivanov \[15\] has proved that, whatever combination one chooses, one will end up in one of six distinct classes of symmetries. In a recent article we have clarified this issue showing how to construct such classes with simple examples \[19\]. The result is shown in Table I.

| symmetry | \( m_{11}^2 \) | \( m_{12}^2 \) | \( m_{22}^2 \) | \( \lambda_1 \) | \( \lambda_2 \) | \( \lambda_3 \) | \( \lambda_4 \) | \( \lambda_5 \) | \( \lambda_6 \) | \( \lambda_7 \) |
|----------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \( Z_2 \) | 0              | 0              | 0              | 0              | 0              | 0              | 0              | 0              | 0              | 0              |
| \( U(1) \) | 0              | 0              | 0              | 0              | 0              | 0              | 0              | 0              | 0              | 0              |
| \( SO(3) \) | \( m_{11}^2 \) | \( m_{12}^2 \) | \( m_{22}^2 \) | \( \lambda_1 \) | \( \lambda_1 - \lambda_3 \) | 0              | 0              | 0              | 0              | 0              |
| CP1      | real           | real           | real           | real           | real           | real           | real           | real           | real           | real           |
| CP2      | \( m_{11}^2 \) | \( m_{12}^2 \) | \( m_{22}^2 \) | \( \lambda_1 \) | \( \lambda_1 - \lambda_3 \) | 0              | \( -\lambda_6 \) | 0              | 0              | 0              |
| CP3      | \( m_{11}^2 \) | \( m_{12}^2 \) | \( m_{22}^2 \) | \( \lambda_1 \) | \( \lambda_1 - \lambda_3 - \lambda_4 \) (real) | 0              | 0              | 0              | 0              | 0              |

\[1\] The space coordinates of the fields, which we have suppressed, are inverted by a generalized CP transformation.
Five of the symmetry classes may be imposed by the following single requirements:

\[ Z_2 : \quad S = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \]  
(16)

\[ U(1) : \quad S = \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & e^{-i\alpha} \end{pmatrix}, \quad \alpha \neq \pi/2, \]  
(17)

\[ CP1 : \quad X = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \]  
(18)

\[ CP2 : \quad X = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \]  
(19)

\[ CP3 : \quad X = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}. \]  
(20)

Here \( 0 < \alpha < \pi \) (but \( \alpha \neq \pi/2 \), since the case of \( \alpha = \pi/2 \) corresponds to the \( Z_2 \) symmetry), and \( 0 < \theta < \pi/2 \).

Invariance under the full \( U(2) \) global symmetry is obtained by requiring the invariance of the scalar potential under Eq. (9), for all unitary matrices \( S \).

III. BASIS INVARIANT PROBES OF THE MSSM

A. The Higgs sector of the MSSM

The Higgs potential of the MSSM (prior to including soft-supersymmetry-breaking dimension-two squared-mass terms) is a particular case of Eq. (1), with

\[ m^2_{11} = m^2_{22}, \]  
(21)

\[ m^2_{12} = 0, \]  
(22)

\[ \lambda_1 = \lambda_2 = \frac{1}{4}(g^2 + g'^2), \]  
(23)

\[ \lambda_3 = \frac{1}{4}(g^2 - g'^2), \]  
(24)

\[ \lambda_4 = -\frac{1}{2}g^2, \]  
(25)

\[ \lambda_5 = \lambda_6 = \lambda_7 = 0, \]  
(26)

where \( g \) and \( g' \) are the \( SU(2)_L \) and \( U(1)_Y \) gauge coupling constants, respectively. In this case, Eqs. (4) and (5) become

\[ Y_{11} = Y_{22}, \quad Y_{12} = Y_{21} = 0, \]  
(27)

and

\[ Z_{11,11} = \lambda_1, \quad Z_{22,22} = \lambda_1, \quad Z_{12,21} = Z_{21,12} = -\lambda_1 - \lambda_3, \]  
(28)

with \( \lambda_1 \) given by Eq. (23), \( \lambda_3 \) given by Eq. (24), and all other components of the \( Z \) tensor equal to zero.

Comparing Eqs. (21)–(26) with Table I, we see that these requirements are almost the same as in the THDM with the full \( U(2) \) flavor symmetry. The difference is that the \( U(2) \)-symmetric case implies \( \lambda_4 = \lambda_1 - \lambda_3 \), while the supersymmetry limit implies \( \lambda_4 = -\lambda_1 - \lambda_3 \). As shown by Ivanov [15] and by us [19], the former relation can come from a symmetry requirement that exclusively involves the Higgs potential, while the latter relation cannot. In

\[ \text{SO}(3) \text{ Higgs flavor symmetry listed in Table I is orthogonal to the } U(1)_Y \text{ hypercharge invariance (under which the THDM potential is always invariant). In the SO}(3)\text{-symmetric case, the full Higgs flavor symmetry group is } U(2) \cong \text{SO}(3) \otimes U(1)_Y. \]
particular, there are no basis changes one can perform on the THDM to obtain, from one of the six symmetries listed in table I, the SUSY condition \( \lambda_4 = -\lambda_1 - \lambda_3 \).

This can also be seen by examining the renormalization group equations that control the evolution of the \( \lambda_i \). Here, we focus only on those terms arising from the Higgs potential and the gauge couplings.\(^3\) The relevant expressions can be found, for example, in Refs. \[22–25\]. Using \( \lambda_1 = \lambda_2 \) and \( \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = 0 \), we find

\[
D\lambda_1 = 6\lambda_1^2 + 2\lambda_3^2 + \lambda_2^2 + 2\lambda_3\lambda_4 - \frac{5}{3} \left(9g^2 + 3g'^2\right) \lambda_1 + \frac{1}{9} \left(9g^4 + 6g^2g'^2 + 3g'^4\right),
\]
\[
D\lambda_3 = 2\lambda_3^2 + \lambda_2^2 + 2\lambda_1(3\lambda_3 + \lambda_4) - \frac{1}{3} \left(9g^2 + 3g'^2\right) \lambda_3 + \frac{2}{9} \left(9g^4 - 6g^2g'^2 + 3g'^4\right),
\]
\[
D\lambda_4 = 2\lambda_4^2 + 2\lambda_1\lambda_4 + 4\lambda_3\lambda_4 - \frac{1}{3} \left(9g^2 + 3g'^2\right) \lambda_4 + \frac{4}{9}g^2g'^2,
\]

where \( D = 16\pi^2\mu(d/d\mu) \), and \( D\lambda_5 = D\lambda_6 = D\lambda_7 = 0 \). Hence, given the constraints \( \lambda_1 = \lambda_2 \) and \( \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = 0 \),

\[
D(\lambda_4 + \lambda_3 - \lambda_1) = \frac{1}{2} (\lambda_4 + \lambda_3 - \lambda_1)(12\lambda_1 + 4\lambda_4 - 9g^2 - 3g'^2)
\]
\[
D(\lambda_4 + \lambda_3 + \lambda_1) = 2 \left(3\lambda_1^2 + 3\lambda_3 + 2\lambda_4\right) \lambda_1 + 2\lambda_3^2 + 3\lambda_3\lambda_4
\]
\[
- \frac{1}{2} \left(9g^2 + 3g'^2\right) (\lambda_4 + \lambda_3 + \lambda_1) + \frac{1}{4} \left(9g^4 + 6g^2g'^2 + 3g'^4\right).
\]

The first equation vanishes if \( \lambda_4 = \lambda_1 - \lambda_3 \); the second does not vanish if \( \lambda_4 = -\lambda_1 - \lambda_3 \). That is, the condition \( \lambda_4 = \lambda_1 - \lambda_3 \) is renormalization group invariant, whereas the condition \( \lambda_4 = -\lambda_1 - \lambda_3 \) is not. Note that we have not yet imposed the specific relations between the \( \lambda_i \) and the gauge couplings required by the MSSM. If we impose the MSSM constraints specified by Eqs. \[23\]–\[26\] on the right hand side of Eq. \[31\], we obtain

\[
D(\lambda_4 + \lambda_3 + \lambda_1) = 3g^4 + 2g^2g'^2 + g'^4,
\]

i.e., \( \lambda_1 = -\lambda_3 - \lambda_4 \) is still not RGE invariant.

The latter result is not unexpected. After all, the gauge boson–Higgs boson sector considered by itself can never be supersymmetric, as the corresponding superpartners are not included. Consequently, the SUSY limit of the gauge boson–Higgs boson sector can only be defined in a manner invariant under Higgs basis changes if the corresponding gaugino and higgsino superpartners are taken into account. The gaugino and higgsino interactions generate additional terms on the right hand side of Eq. \[29\]. In the supersymmetric limit, these effects yield \[22\]

\[
\delta_{\text{SUSY}}(D\lambda_1) = -\frac{5}{2}g^4 - g^2g'^2 - \frac{1}{2}g'^4,
\]
\[
\delta_{\text{SUSY}}(D\lambda_3) = -\frac{5}{2}g^4 + g^2g'^2 - \frac{1}{2}g'^4,
\]
\[
\delta_{\text{SUSY}}(D\lambda_4) = 2g^4 - 2g^2g'^2.
\]

Hence,

\[
\delta_{\text{SUSY}}\{D(\lambda_4 + \lambda_3 + \lambda_1)\} = -(3g^4 + 2g^2g'^2 + g'^4).
\]

Indeed, when the latter is added to Eq. \[32\], we see that \( D(\lambda_4 + \lambda_3 + \lambda_1) = 0 \) as expected. Thus, the SUSY relation \( \lambda_1 = -\lambda_3 - \lambda_4 = \frac{1}{3}(g^2 + g'^2) \) is renormalization group invariant when all the Higgs/higgsino/gauge/gaugino interactions are included.

## B. Basis-independent conditions for the MSSM Higgs potential that are necessary but not sufficient

Based on the arguments of section III A, no basis-invariant conditions constructed solely from the \( Y_{ab} \) and \( Z_{ab,cd} \) exist that can guarantee that Eqs. \[21\]–\[26\] are satisfied for some choice of basis. Nevertheless, we can establish a weaker invariant condition that is necessary (although not sufficient) for the existence of a supersymmetric THDM scalar potential.\(^4\)

\(^3\) In order to include fermions in the analysis, one would have to investigate the constraints of the THDM symmetries on the Higgs-fermion Yukawa couplings.

\(^4\) We thank the anonymous referee for encouraging us to consider the necessary symmetry constraints that govern the MSSM Higgs scalar potential.
Consider a basis choice in which:

\[ m_{11}^2 = m_{22}^2, \quad \lambda_1 = \lambda_2, \quad \text{and} \quad m_{12}^2 = \lambda_5 = \lambda_6 = \lambda_7 = 0. \tag{34} \]

Clearly Eq. (34) is satisfied by the MSSM Higgs potential in the standard MSSM basis choice for the Higgs fields. However, Eq. (34) is not sufficient since the condition \( \lambda_4 = -\lambda - \lambda_3 \) is imposed, nor are the conditions relating the quartic Higgs couplings to gauge couplings imposed. Nevertheless, suppose that one could establish a basis-independent condition that was equivalent to the statement that a basis exists in which Eq. (34) is satisfied. Such a basis-independent condition would then be a necessary condition for a supersymmetric Higgs sector.\(^5\)

The conditions of Eq. (34) do not match any single condition listed in Table I. Nevertheless, given a scalar potential whose parameters satisfy Eq. (34), one can always transform to a new basis that explicitly satisfies the CP3-symmetry conditions of Table I and vice versa. That is, a basis-invariant characterization of a CP3-symmetric THDM would guarantee that some basis exists in which Eq. (34) holds. It follows that the Higgs sector of the MSSM is a CP3-symmetric THDM.

The proof of the assertions above have been given in Ref. 19. For completeness, we review the arguments here. We first define the discrete flavor symmetry \( \Pi_2 \) which corresponds to a THDM scalar potential that is symmetric under the interchange of the two Higgs fields. In this case, the scalar potential parameters satisfy:

\[ m_{11}^2 = m_{22}^2, \quad m_{12}^2 \text{ real}, \quad \lambda_1 = \lambda_2, \quad \lambda_5 \text{ real}, \quad \text{and} \quad \lambda_6 = \lambda_7. \tag{35} \]

In Ref. 19, we showed that given a \( \Pi_2 \)-symmetric scalar potential, there exists another basis which is \( Z_2 \)-symmetric. Consequently, we did not list \( \Pi_2 \) as a separate symmetry in Table I.\(^6\) However, if we simultaneously impose \( \Pi_2 \) and \( U(1) \) in the same basis, then one easily sees that the conditions of Eq. (34) are satisfied. The same conclusion also follows if we simultaneously impose \( U(1) \) and CP2 in the same basis. Moreover, starting from a scalar potential in which the conditions of Eq. (34) are satisfied, a basis transformation can be found (see Ref. 19 for the details) such that \( \lambda_5 = \lambda_1 - \lambda_3 - \lambda_4 \) in the new basis, as required in the CP3-symmetric THDM (cf. Table I).

Finally, we indicate the basis-independent conditions that guarantee that a basis exists in which the CP3-symmetric conditions are satisfied. Defining \( Z_{ab}^{(1)} \equiv Z_{ac,cb} \), we first require that \[ Y_{ab} = m_{11}^2 \delta_{ab} \quad \text{and} \quad \text{Tr}[Z^{(1)}]^2 = \frac{1}{2}(\text{Tr} Z^{(1)})^2. \tag{36} \]

If Eq. (36) is satisfied, then

\[ m_{11}^2 = m_{22}^2, \quad m_{12}^2 = 0, \quad \lambda_1 = \lambda_2, \quad \lambda_7 = -\lambda_6, \tag{37} \]

must hold in all basis choices. This is the so-called exceptional region of parameter space (ERPS) of the THDM. In Ref. 19, we then constructed a second invariant quantity \( D \) built out of the \( Y_{ab} \) and \( Z_{ab,cd} \) with the following property: if \( D = 0 \) in the ERPS, then there exists a basis for the scalar fields such that the CP3-conditions of Table I are satisfied.\(^6\) Thus, the conditions for the ERPS plus \( D = 0 \) provide necessary (although not sufficient) invariant conditions that must be satisfied by the MSSM Higgs scalar potential.

### C. The gaugino-higgsino-Higgs interactions

In the MSSM, the tree-level Lagrangian describing the interactions of the gauginos with the Higgs-doublets may be written as

\[
L_{\text{gaugino-Higgs}}^{\text{MSSM}} = \mu \epsilon_{ij} \psi_{H_D}^i \psi_{H_U}^j + \frac{i g}{\sqrt{2}} \lambda^a \epsilon_{ij} \left( \psi_{H_D}^i \Phi_{2}^{i \dagger} + \epsilon^{ik} \psi_{H_D}^j \Phi_{1}^{k} \right) + \frac{i g'}{\sqrt{2}} \lambda' \left( \psi_{H_U}^i \Phi_{2}^{i \dagger} - \epsilon^{ik} \psi_{H_U}^j \Phi_{1}^{k} \right) + \text{h.c.}, \tag{38} \]

where \( \lambda^a \) and \( \lambda' \) are the two-component spinor gaugino fields that are superpartners to the SU(2) and U(1)-hypercharge gauge bosons, and \( \psi_{H_D} \) and \( \psi_{H_U} \) are, respectively, the hypercharge \(-1\) and hypercharge \(+1\) weak doublet two-component spinor higgsino fields. The indices \( i, j \) and \( k \) label the two components of the weak doublet, and the

\(^5\) The additional conditions necessary to establish necessary and sufficient basis-independent probes of supersymmetry require consideration of the gaugino-higgsino-Higgs interactions. This is the subject of section III C that follows.

\(^6\) The explicit expression for \( D \) is rather complicated and can be found in eqs. (39)–(41) and (44) of Ref. 19.
index \( \alpha \) is the adjoint index of the SU(2) gaugino field. We have included a supersymmetric Majorana mass term for the two-component higgsino fields, which defines the parameter \( \mu \). As usual, \( \epsilon^{12} = -\epsilon^{21} = +1 \) and \( \epsilon^{11} = \epsilon^{22} = 0 \).

If we relax the constraints imposed by supersymmetry, the coupling strengths of the gaugino-higgsino-Higgs interaction above are no longer constrained to be gauge couplings as in Eq. (38). Moreover, four additional dimension-four interaction terms are possible, consistent with SU(2) \( \times \) U(1) gauge invariance. These terms are the so-called “wrong-Higgs” couplings of Ref. 26, and are obtained from those of Eq. (38) by interchanging \( \Phi_1 \) and \( \Phi_2 \). In our analysis below, we consider the most general dimension-four gauge invariant couplings between the gaugino, higgsino and Higgs fields. We shall write these couplings in a form that is manifestly independent of the choice of basis for the Higgs fields:

\[
\mathcal{L}_{\text{gaugino-Higgs}} = \frac{i}{\sqrt{2}} \lambda^a \tau^a_{ij} \left( \psi^j_H f^a_{ij} \Phi^\dagger_a + \epsilon^{ik} \psi^k_H f^a_{D} \Phi^\dagger_k \right) + \frac{i}{\sqrt{2}} \lambda' \left( \psi^j_H f^\prime a_{ij} \Phi^\dagger a + \epsilon^{ik} \psi^k_H f^\prime a_D \Phi^\dagger k \right) + \text{h.c.},
\]

where the couplings \( f^a_{ij}, f^\prime a_{ij}, \) and \( f^a_D, f^\prime a_D \) transform covariantly under a Higgs basis U(2)-transformation,\(^7\)

\[
f^a_{ij} \rightarrow U_{ij} f^b_D, \quad f^\prime a_{ij} \rightarrow U_{ij} f^b_D, \quad f^a_D \rightarrow U_{ab} f^b_D.
\]

In the supersymmetric limit, there is a natural choice of basis for the Higgs fields, henceforth called the SUSY basis, in which:

\[
f^a_0 = g^a_{\sigma^2}, \quad f^\star a_D = g^{\sigma^1}, \quad f^{\prime a}_0 = g^{\sigma^1}, \quad f^{\prime a}_D = g^{\sigma^1}.
\]

In particular, in the SUSY basis, the so-called “wrong-Higgs interactions” of Ref. 26 are absent in the supersymmetric limit. However, under a general Higgs basis transformation, the supersymmetric gaugino-higgsino-Higgs Lagrangian will transform into a linear combination of supersymmetric and wrong-Higgs interaction terms. Thus, in a generic basis choice for the Higgs fields, the supersymmetry is not manifest. One of the goals of this section is to determine a set of basis-independent conditions that guarantees the existence of a basis choice in which Eq. (41) is satisfied. Such basis-independent conditions would constitute an invariant signal for manifestly supersymmetric Higgs interactions.

The couplings \( f^a, f^\prime a, f^a_D, \) and \( f^{\prime a}_D \) are complex vectors that live in the two-dimensional Higgs flavor space. It is convenient to define the corresponding unit vectors, \( f^a \equiv f^a/|f|, \) where \(|f| \equiv (f^a \ast f^a)^{1/2} \) is the length of the complex vector \( f^a \). Next, we introduce vectors that are orthogonal to \( f^a, f^\prime a, f^a_D, \) and \( f^{\prime a}_D \), respectively,

\[
\tilde{g}^a_U = f^{\star b}_{U} \epsilon^b, \quad \tilde{g}^a_D = f^{\star b}_{D} \epsilon^b, \quad \tilde{g}^{\prime a}_U = f^{\prime \star b}_U \epsilon^b, \quad \tilde{g}^{\prime a}_D = f^{\prime \star b}_D \epsilon^b.
\]

These are pseudo-vectors with respect to U(2) Higgs basis transformations,\(^8\)

\[
\tilde{g}^a_{U,D} \rightarrow (\det U)^{-1} U_{ab} \tilde{g}^b_{U,D}, \quad \tilde{g}^{\prime a}_{U,D} \rightarrow (\det U)^{-1} U_{ab} \tilde{g}^{\prime b}_{U,D},
\]

due to the appearance of the complex phase, \( \det U \).

We now define U(2)-invariant, hypercharge-one Higgs fields as follows:

\[
H_U \equiv f^{\star a}_U \Phi^a, \quad H_D \equiv f^{\star a}_D \Phi^a.
\]

One can also define a corresponding set of hypercharge \(-1\) fields, e.g.,

\[
H_D^i \equiv \epsilon^{ij} H_D^{ij}, \quad H_D^{\prime i} \equiv \epsilon^{ij} H_D^{\prime ij}.
\]

It is also convenient to define U(2) pseudo-invariant Higgs fields (denoted by calligraphic fonts),

\[
\tilde{H}_U \equiv \tilde{g}^{\prime a}_U \Phi^a, \quad \tilde{H}_D \equiv \tilde{g}^{\prime a}_D \Phi^a.
\]

---

\(^7\) Note that the global U(1) Higgs flavor transformation corresponding to \( \Phi_a \rightarrow e^{i \chi} \Phi_a \) \((\alpha = 1,2)\) is distinguished from the global U(1) hypercharge transformation, since the higgsino fields do not transform under the rephasing of the Higgs fields.

\(^8\) Starting from the transformation law \( \tilde{g}^a \rightarrow U_{ab} \tilde{g}^b \), where \( U^\dagger = U^{-1} \), it follows that \( \tilde{g}^a \rightarrow U_{ab} \tilde{g}^b g^{\epsilon a b c} \). If we now recognize that \( U_{ab}^{-1} \epsilon^{a d} = \det(U^{-1}) U_{cd} \epsilon^{d c} \), the results of Eq. (41) easily follow.
It then follows that:

\[ \Phi_a = \hat{f}_U^a H_U + \hat{g}_U^a H_U = \hat{f}_D^a H_D + \hat{g}_D^a H_D \]  
(50)

\[ = \hat{f}_{U}^{\dagger a} H'_U + \hat{g}_{U}^{\dagger a} H'_U = \hat{f}_{D}^{\dagger a} H'_D + \hat{g}_{D}^{\dagger a} H'_D, \]  
(51)

after using \( \hat{f}_{U,D}^a \hat{f}_{U,D}^{a*} = \hat{g}_{U,D}^a \hat{g}_{U,D}^{a*} = 1 \) and \( \hat{g}_{U,D}^a \hat{H}_{U,D}^{a*} = \hat{f}_{U,D}^a \hat{H}_{U,D}^{a*} = 0 \).

There is some motivation for this proliferation of Higgs field definitions. In particular, as we show later in Eqs. \( \text{(59–62)} \), the choices of

\[ \{ H_U, H_D \}, \{ H'_U, H'_D \}, \{ \tilde{H}_D, \tilde{H}_D \}, \text{ and } \{ \tilde{H}'_D, \tilde{H}'_D \}, \]  
(52)

correspond to four different basis choices for the hypercharge-one Higgs doublet fields.

One can express the gaugino-higgsino-Higgs interaction Lagrangian in a manifestly \( U(2) \)-invariant form. For example, using the definitions of the invariant Higgs fields \( H_U, H_D, H'_U \) and \( H'_D \) [defined by Eqs. \( \text{(45)} \) and \( \text{(47)} \)], Eq. \( \text{(39)} \) can be rewritten as:

\[ \mathcal{L}_{\text{gaugino-Higgs}} = \frac{i}{\sqrt{2}} \lambda^i \tau^a \left( |f_U|^2 \psi_D^i H_U^i + |f_D|^2 \psi_D^i H_D^i \right) + \frac{i}{\sqrt{2}} \mathcal{V}' \left( |f_U|^2 \psi_D^i H_U^i - |f_D|^2 \psi_D^i H_D^i \right) + \text{h.c.} \]  
(53)

However, this form is not particularly useful outside of the supersymmetric limit, since \( \{ H_U, \tilde{H}_D \} \) and \( \{ H'_U, \tilde{H}'_D \} \) are not orthogonal pairs of hypercharge-one Higgs doublet fields in the general case. Of course, one can always rewrite Eq. \( \text{(39)} \) in terms of one of the four basis choices of hyper-charge one doublet Higgs fields listed in Eq. \( \text{(52)} \) by inserting the appropriate form for \( \Phi_a \) given in Eqs. \( \text{(50)} \) and \( \text{(51)} \) into Eq. \( \text{(39)} \).

D. Basis-invariant probes of the supersymmetric Higgs interactions

Supersymmetry imposes strong constraints on the scalar Higgs potential and the gaugino-higgsino-Higgs interactions. These constraints must involve basis-independent combinations of the scalar potential parameters \( Y_{ab}, Z_{abcd} \), and the gaugino-higgsino-Higgs couplings \( f_U^a, f_D^a, f_U^{a*}, f_D^{a*} \). It is straightforward to find the necessary relations. First we exhibit the basis invariant relations that enforce supersymmetric gaugino-higgsino-Higgs couplings:

\[ f_U^a f_D^{a*} = 0, \quad f_U^{a*} f_D^a = 0, \]
\[ f_U^a f_D^{a*} = 0, \quad f_U^{a*} f_D^a = 0, \]
\[ f_U f_D^{a*} = f_D^a f_U^{a*} = g^2, \quad f_U^a f_D^{a*} = f_D^a f_U^{a*} = g^2, \]
\[ f_U^a f_D^{a*} = g g', \quad f_D^a f_U^{a*} = g g'. \]  
(54)

To establish \( U(2) \)-invariant conditions that enforce a supersymmetric scalar Higgs potential, we first construct basis-independent quantities that involve both the scalar potential parameters and the gaugino-higgsino-Higgs couplings. For example,\(^9\)

\[ \mathcal{Y}_{DD} = \hat{f}_D^b \hat{f}_D^b Y_{ab}, \quad \mathcal{Y}_{UU} = \hat{f}_U^a \hat{f}_U^a Y_{ab}, \]
\[ \mathcal{Y}_{DU} = \hat{f}_D^a \hat{f}_D^b Y_{ab}, \quad \mathcal{Y}_{UD} = \hat{f}_U^a \hat{f}_U^b Y_{ab}, \]  
(55)

provide basis-invariant quantities involving the quadratic coefficients of the Higgs potential. Likewise,

\[ Z_{\alpha\beta, \gamma\delta} = \hat{f}_a \hat{f}_b \hat{f}_c \hat{f}_d Z_{ab,cd}, \]  
(56)

where the indices \( \alpha, \beta, \gamma, \) and \( \delta \) can take the values \( D \) or \( U \), provide basis-invariant quartic coefficients for the Higgs potential.

Evaluating the invariant quantities introduced in Eqs. \( \text{(55)} \) and \( \text{(56)} \) in the supersymmetric basis defined by Eqs. \( \text{(27)}, \text{(28)} \) and \( \text{(41)} \), it follows that

\[ \mathcal{Y}_{DD} = \mathcal{Y}_{UU}, \quad \mathcal{Y}_{DU} = \mathcal{Y}_{UD} = 0, \]  
(57)

\(^9\) Assuming Eq. \( \text{(54)} \) is satisfied, it is not necessary to construct additional invariants that involve \( f_U^a \) and \( f_D^a \).
In the most general 2HDM after spontaneous symmetry breaking, the so-called Higgs basis in which $\hat{f}$ component of $f$ pseudo-invariants.

In the quasi-physical basis, a quasi-physical basis is one in which the Higgs fields are invariant, up to a possible rephasing of one of the physical basis up to an arbitrary rephasing of the Higgs field that lies in the direction of the vanishing component of the corresponding coupling vanishes, while setting the non-vanishing component to unity. This defines the quasi-physical basis.

That is, starting in a generic basis with a Higgs potential given by Eq. (1), one can always transform to the basis

\[ Z_{DD,DD} = Z_{UU,UU} = \frac{1}{4} [f_U^{a*} f_U^a + f_D^{a*} f_D^a], \]
\[ Z_{DD,UU} = Z_{UU,DD} = \frac{1}{4} [f_U^{a*} f_U^a - f_U^{a*} f_U^a], \]
\[ Z_{DU,UD} = Z_{UD,DU} = -Z_{DD,DD} - Z_{DD,UU}, \]
\[ Z_{DU,DU} = Z_{UD,UD} = 0, \]
\[ Z_{UD,DD} = Z_{DU,DD} = Z_{DD,UD} = Z_{DD,DU} = 0, \]
\[ Z_{DU,UU} = Z_{UD,UU} = Z_{UU,DU} = Z_{UU,UD} = 0. \]

(58)

Since these equations are basis invariant, they must hold in any theory made up of the MSSM fields with exact supersymmetry, regardless of the exact basis choice made for the Higgs fields.

That is, independently of the choice of basis for the Higgs fields, the supersymmetric limit of the Higgs/higgsino/gauge/gaugino sectors holds if and only if Eqs. (54), (57) and (58) hold. If Eqs. (54) and (58) hold but Eq. (57) does not, then supersymmetry is softly broken due to the quadratic terms of the Higgs potential. The above results fully resolves the question of the basis-invariant form for the supersymmetric limit of the THDM.

E. A preferred basis in the MSSM

The gaugino-higgsino-Higgs interactions provide a means for defining a quasi-physical choice of basis. In this context, a quasi-physical basis is one in which the Higgs fields are invariant, up to a possible rephasing of one of the Higgs fields, under U(2) transformations. That is, the coefficients of the Higgs potential are either U(2)-invariant or pseudo-invariant.

There are four independent quasi-physical bases, corresponding to the four normalized gaugino-higgsino-Higgs couplings, $\hat{f}_U^a$, $\hat{f}_D^a$, $\hat{f}_U^a$ and $\hat{f}_D^a$. Each basis is defined by imposing the condition that one of the two components of the corresponding coupling vanishes, while setting the non-vanishing component to unity. This defines the quasi-physical basis up to an arbitrary rephasing of the Higgs field that lies in the direction of the vanishing component of $f$. The latter is a pseudo-invariant field, whereas the Higgs field that lies in the direction of the non-vanishing component of $f$ is a U(2)-invariant field.\(^\text{10}\)

The four quasi-physical bases and their corresponding Higgs fields are:

\[ \mathcal{B}_U : \text{defined by } \hat{f}_U^a = (0, 1), \quad \text{Higgs fields: } (H_U, H_U), \]
\[ \mathcal{B}_D : \text{defined by } \hat{f}_D^a = (1, 0), \quad \text{Higgs fields: } (H_D, H_D), \]
\[ \mathcal{B}_U' : \text{defined by } \hat{f}_U'^a = (0, 1), \quad \text{Higgs fields: } (H'_U, H'_U), \]
\[ \mathcal{B}_D' : \text{defined by } \hat{f}_D'^a = (1, 0), \quad \text{Higgs fields: } (H'_D, H'_D), \]

(59) \quad (60) \quad (61) \quad (62)

where the fields denoted by (calligraphic) Roman fonts are (pseudo-)invariant with respect to U(2) basis transformations. The Higgs fields with a $U (D)$ subscript are hypercharge +1 (−1) fields. The coefficients of the scalar potential in the quasi-physical basis are easily constructed. For example, in basis $\mathcal{B}_D$,

\[ Y_{D1} \equiv f_D^{ab} \phi_Y Y_{ab}, \quad Y_{D2} \equiv g_{D}^{ab} \phi_{Y}^* Y_{ab}, \]
\[ Y_{D3} \equiv f_D^{abc} \phi_{Y}^* Y_{abc}, \quad Z_{D1} \equiv f_D^{abc} \phi_Y g_{Y} g_{Y} g_{Y} Z_{abcd}, \]
\[ Z_{D2} \equiv g_{D}^{abc} \phi_{Y}^* g_{Y} g_{Y} g_{Y} Z_{abcd}, \quad Z_{D3} \equiv f_D^{abc} \phi_{Y} g_{Y} g_{Y} g_{Y} Z_{abcd}, \]
\[ Z_{D4} \equiv g_{D}^{abc} \phi_{Y} g_{Y} g_{Y} g_{Y} Z_{abcd}, \quad Z_{D5} \equiv f_D^{abc} \phi_Y^* g_{Y} g_{Y} g_{Y} Z_{abcd}, \]
\[ Z_{D6} \equiv f_D^{abc} \phi_{Y}^* g_{Y} g_{Y} g_{Y} Z_{abcd}, \quad Z_{D7} \equiv f_D^{abc} \phi_{Y} g_{Y} g_{Y} g_{Y} Z_{abcd}. \]

(63) \quad (64) \quad (65) \quad (66) \quad (67)

That is, starting in a generic basis with a Higgs potential given by Eq. (1), one can always transform to the basis $\mathcal{B}_D$

\(^{10}\) In the most general 2HDM after spontaneous symmetry breaking, the so-called Higgs basis in which $\phi^a = (1, 0)$ is an example of a quasi-physical basis. The Higgs fields in this basis, $H_1 \equiv \phi^a \Phi_a$ and $H_2 \equiv \epsilon^a \phi^a \Phi_a$, are respectively invariant and pseudo-invariant with respect to U(2) basis transformations. See Refs. 4, 6, 11, 12 for details.
with a Higgs potential given by:

$$V_H = Y_D H_D^* H_D + Y_{D^c} H_{D^c}^* H_D + \left[ Y_{D^c} H_{D^c}^* H_D + \text{h.c.} \right]$$

$$+ \frac{1}{2} Z_{D_1} (H_D^* H_D)^2 + \frac{1}{2} Z_{D_2} (H_{D^c}^* H_{D^c})^2 + Z_{D_3} (H_D^* H_{D^c}) (H_{D^c}^* H_D) + Z_{D_4} (H_D^* H_D) (H_{D^c}^* H_{D^c}) + \text{h.c.}$$

(68)

The parameters $Y_{D_1}, Y_{D_2}, Z_{D_1}, Z_{D_2}, Z_{D_3}$ and $Z_{D_4}$ are manifestly real U(2)-invariants, whereas the parameters $Y_{D_3}, Z_{D_5}, Z_{D_6}$ and $Z_{D_7}$ are (potentially) complex pseudo-invariants with respect to U(2) basis transformations. The physical Higgs couplings and masses of the theory can be expressed in terms of the invariant parameters and invariant combinations of the pseudo-invariant parameters.

The discussion above is completely general. But, suppose we impose supersymmetry on the Higgs-higgsino-gaugino system. Supersymmetry imposes a very strong constraint on the quasi-physical bases, namely the SUSY basis. Eq. (70) implies that the invariant and pseudo-invariant Higgs fields defined in Eqs. (45)–(49) are

$$Y_i \eta$$

The parameters $Y_i$ and $\eta$ are pseudo-invariant quantities [i.e., $e^{i \eta} \to (\det U)^{-1} e^{i \eta}$ under a U(2) transformation] that is equal to 1 in the SUSY basis. Eq. (70) implies that the invariant and pseudo-invariant Higgs fields defined in Eqs. (45)–(49) are related in the supersymmetric limit,

$$H_U = H'_U = e^{i \eta} H_D = e^{-i \eta} H_D^*,$$

(71)

$$H_D = H'_D = e^{-i \eta} H_U = e^{i \eta} H_U^*.$$  

(72)

for the hypercharge +1 and hypercharge −1 MSSM Higgs fields, respectively. If we now insert Eq. (70) into Eqs. (63)–(67), we find that the coefficients of the Higgs potential in the SUSY basis are \(Y_U, Y_{D^c}, Z_{UU}, Z_{DD}, Z_{DD,DU}, Z_{DD,UU}, Z_{DU,UD}, e^{i \eta} Z_{DU,DU}, e^{i \eta} Z_{DD,DU}, e^{i \eta} Z_{DU,UD}, e^{i \eta} Z_{UU,UD}, \) subject to the conditions of Eqs. (57) and (58). Note that when the conditions of Eqs. (57) and (58) are imposed, the phase \(\eta\) drops out completely. Indeed, the nonzero Higgs potential parameters in the SUSY basis, \(Y_U, Y_{D^c}, Z_{UU}, Z_{DD}, Z_{DD,DU}, Z_{DD,UU}, Z_{DU,UD}, Z_{DU,UD} \) are physical observables that are real and invariant with respect to U(2)-basis transformations.

The existence of a physical SUSY basis is not surprising. Consider the higgsino Majorana mass term, which arises from a supersymmetric term in the MSSM superpotential,

$$\mu \epsilon_{ij} \psi_{H_D}^* \psi_{H_{U^c}}^i.$$  

(73)

A THDM basis transformation mixes \(\Phi_1\) and \(\Phi_2\), which corresponds to a transformation between the MSSM Higgs fields, \(H_U\) with \(H_{U^c}^*\). But the Higgs fields belong to Higgs superfields whose scalar and fermionic components are \((H_U, \psi_{H_U})\) and \((H_{U^c}, \psi_{H_{U^c}})\). As a result, one cannot apply a general U(2)-basis transformation to the full Higgs supermultiplets consistent with supersymmetry. Thus, Eq. (73) effectively defines a preferred basis for \(\Phi_1\) and \(\Phi_2\) within the MSSM. This is why the SUSY basis has a physical significance.

F. Invariant tanβ-like parameters

Until this subsection, we have made no assumptions about the nature of the Higgs vacuum. In the supersymmetric limit of the MSSM, the minimum of the tree-level potential resides at zero field and there is no electroweak symmetry breaking. After dimension-two soft-supersymmetry-breaking terms are included, it becomes possible to break the SU(2)×U(1) symmetry of the scalar potential down to U(1)_{EM}. In the generic basis, we define vacuum expectation values,

$$\langle \Phi_a^0 \rangle = v_a = \bar{v}_a,$$

(74)
where $\hat{v}_a$ is a complex unit vector in the two-dimensional Higgs flavor space and $v \approx 246$ GeV. In the MSSM, the parameter $\tan \beta$ is defined in the SUSY basis,

$$
\tan \beta \equiv \frac{v_U}{v_D}.
$$

(75)

However, in the most general THDM, $\tan \beta$ is a basis-dependent concept that does not correspond to any physical observable. In Ref. 12, a number of invariant parameters connected to the Higgs-fermion Yukawa couplings are introduced that play the role of $\tan \beta$ in constrained THDMs. Here, we shall indicate how to define $\tan \beta$-like parameters associated with the gaugino-higgsino-Higgs interactions.

The basic idea is to define $\tan \beta$ as the ratio of vacuum expectation values in the quasi-physical bases introduced in Eqs. (59)–(62). One can then define four invariant $\tan \beta$-like parameters,

$$
\tan \beta_U \equiv e^{-i\eta} \frac{\langle H_U^0 \rangle}{\langle H_U^0 \rangle} = -e^{-i\eta} \frac{\hat{f}^{aa}_U \hat{v}_a}{\hat{g}^{aa}_U \hat{v}_a}, \quad \tan \beta_D \equiv e^{-i\eta} \frac{\langle H_D^0 \rangle}{\langle H_D^0 \rangle} = e^{-i\eta} \frac{\hat{g}^{a}_D \hat{v}_a^*}{\hat{f}^{aa}_D \hat{v}_a^*},
$$

(76)

$$
\tan \beta'_U \equiv e^{-i\eta} \frac{\langle H_U^0 \rangle}{\langle H_U^0 \rangle} = -e^{-i\eta} \frac{\hat{f}^{aa}_U \hat{v}_a}{\hat{g}^{aa}_U \hat{v}_a}, \quad \tan \beta'_D \equiv e^{-i\eta} \frac{\langle H_D^0 \rangle}{\langle H_D^0 \rangle} = e^{-i\eta} \frac{\hat{g}^{a}_D \hat{v}_a^*}{\hat{f}^{aa}_D \hat{v}_a^*}.
$$

(77)

Without the factors of $e^{i\eta}$, the corresponding $\tan \beta$-like parameters are complex pseudo-invariants, whose magnitudes are basis-independent. The interpretation of these $\tan \beta$-like parameters is simplest in the Higgs basis, which is defined by $\hat{v} = (1, 0)$. In the Higgs basis, the parameters in Eqs. (76) and (77) are (up to an overall phase) simply the ratios of gaugino-higgsino-Higgs couplings. One can now investigate the limit in which all dimension-four couplings respect supersymmetry. In this case, the invariant and pseudo-invariant Higgs fields are constrained by Eqs. (71) and (72), in which case,

$$
\tan \beta \equiv \frac{\langle H_U^0 \rangle}{\langle H_D^0 \rangle} = \tan \beta_U = (\tan \beta_D)^* = \tan \beta'_U = (\tan \beta'_D)^*, \quad [\text{SUSY limit}].
$$

(78)

Corrections to the above relations at the few percent level are expected at the one-loop level due to supersymmetry-breaking effects that enter into the loops. In principle, one could extract the $\tan \beta$ parameters of Eqs. (76) and (77) from collider data (assuming gaugino-higgsino-Higgs couplings could be measured with sufficient accuracy), and check to see whether Eq. (78) holds.

### IV. CONCLUSIONS

The THDM includes the Higgs sector of the MSSM as a particular case. In the THDM only quantities that are invariant under a unitary basis change, $\Phi_a \rightarrow U_{ab} \Phi_b$ (where $a, b = 1, 2$ and $U$ is an arbitrary unitary matrix) can have physical meaning. Thus, it is natural to look for basis invariant definitions of the supersymmetric limit. We have shown that there is no basis-invariant definition of a supersymmetric THDM based on invariants defined exclusively in terms of parameters of the scalar Higgs potential. We have constructed basis invariant probes of the supersymmetric limit and soft-supersymmetry-breaking by employing both the Higgs potential and the gaugino-higgsino-Higgs interactions. We also observe that the usual basis choice of $H_U$ and $H_D$ in the MSSM does have a physical significance, due to the $\mu$-term of the MSSM superpotential [cf. Eq. (73)].

Finally, we addressed the physical significance of the parameter $\tan \beta$. In the MSSM, the tree-level parameter $\tan \beta$ is well-defined because it is defined in terms of vacuum expectation values of the neutral Higgs fields, which are given in a physical basis. However, in the most general THDM Higgs sector coupled to new particles with electroweak quantum numbers that coincide with the gauginos and higgsinos of the MSSM, the parameter $\tan \beta$, which is defined in terms of a ratio of Higgs vacuum expectation values, is a basis dependent quantity and hence unphysical. We have shown that four invariant $\tan \beta$-like parameters can be defined that are basis-independent and hence physical. One can verify that in the supersymmetric limit, these four invariant parameters coincide (at tree-level) and are equal to the $\tan \beta$ parameter of the MSSM.

---

11 In the SUSY basis, the overall phase of the Higgs fields can be chosen such that $v_U$ and $v_D$ are real and positive.

12 The signs in Eqs. (60) and (67) have been conveniently chosen so that no extraneous signs appear in the SUSY limit [cf. Eq. (78)].
Acknowledgments

The work of P.M.F. is supported in part by the Portuguese Fundação para a Ciência e a Tecnologia (FCT) under contract PTDC/FIS/70156/2006. The work of H.E.H. is supported in part by the U.S. Department of Energy, under grant number DE-FG02-04ER41268. The work of J.P.S. is supported in part by FCT under contract CFTP-Plurianual (U777) and under project CERN/FP/109305/2009, and by the EU RTN project Marie Curie: MRTN-CT-2006-035505. H.E.H. is most grateful for the kind hospitality and support of the Centro de Física Teórica e Computacional at Universidade de Lisboa during his visit to Lisbon. He also acknowledges conversations with Sacha Davidson and John Mason, which inspired a number of ideas presented in this paper.

[1] For a review and references to the literature, see e.g., H.E. Haber, “Supersymmetry, Part 1 (Theory),” in C. Amsler et al. [Particle Data Group Collaboration], Review of Particle Physics, Phys. Lett. B667 (2008) 1211 and 2009 partial update for the 2010 edition.

[2] T.D. Lee, Phys. Rev. D 8 1226 (1973); Phys. Rep. 9, 143 (1974); N.G. Deshpande and E. Ma, Phys. Rev. D 18 (1978) 2574; H. Georgi, Hadronic J. 1, 155 (1978); H.E. Haber, G.L. Kane and T. Sterling, Nucl. Phys. B 161, 493 (1979); J.F. Donoghue and L.F. Li, Phys. Rev. D 19, 945 (1979); E. Golowich and T.C. Yang, Phys. Lett. B 80, 245 (1979).

[3] L. Lavoura, Phys. Rev. D 50, 7089 (1994).

[4] S. Davidson and H. E. Haber, Phys. Rev. D 72, 035004 (2005) [Erratum-ibid D 72, 099902 (2005)].

[5] L. Lavoura, J. P. Silva, Phys. Rev. D 50, 4619 (1994).

[6] F. J. Botella and J. P. Silva, Phys. Rev. D 51, 3870 (1995).

[7] G. C. Branco, L. Lavoura, and J. P. Silva, CP Violation (Oxford University Press, Oxford, UK, 1999).

[8] J. F. Gunion, talk given at the CPNSH, CERN, Switzerland, December (2004).

[9] G. C. Branco, M. N. Rebelo, and J. I. Silva-Marcos, Phys. Lett. B 614, 187 (2005).

[10] I. F. Ginzburg and M. Krawczyk, Phys. Rev. D 72, 115013 (2005).

[11] J. F. Gunion and H. E. Haber, Phys. Rev. D 72, 095002 (2005);

[12] H. E. Haber and D. O’Neil, Phys. Rev. D 74, 015018 (2006) [Erratum-ibid. D 74, 059905 (2006)].

[13] C. C. Nishi, Phys. Rev. D 74, 036003 (2006); Phys. Rev. D 76, 055013 (2007); Phys. Rev. D 77, 055009 (2008).

[14] M. Maniatis, A. von Manteuffel, O. Nachtmann, and F. Nagel, Eur. Phys. J. C 48, 805 (2006); M. Maniatis, A. von Manteuffel, and O. Nachtmann, Eur. Phys. J. C 49, 1067 (2007); M. Maniatis, A. von Manteuffel, and O. Nachtmann, Eur. Phys. J. C 57, 719 (2008); M. Maniatis, A. von Manteuffel, and O. Nachtmann, Eur. Phys. J. C 57, 739 (2008); M. Maniatis and O. Nachtmann, JHEP 0905, 028 (2009).

[15] I. P. Ivanov, Phys. Rev. D 77, 015017 (2008).

[16] I. P. Ivanov, Phys. Lett. B 632, 360 (2006); Phys. Rev. D 75, 035001 (2007); ibid. 76, 039902(E) (2007).

[17] J.-M. Gerard, and M. Herquet, Phys. Rev. Lett. 98, 251802 (2007); S. Visscher, J.-M. Gerard, M. Herquet, V. Lemaître, and F. Maltoni, JHEP 0908, 042 (2009).

[18] P. M. Ferreira and J. P. Silva, Phys. Rev. D 78, 116007 (2008).

[19] P. M. Ferreira, H. E. Haber, and J. P. Silva, Phys. Rev. D 79, 116004 (2009).

[20] For early work on GCP see, for example, G. Ecker, W. Grimus, and W. Konetschny, Nucl. Phys. B191, 465 (1981); G. Ecker, W. Grimus, and H. Neufeld, Nucl. Phys. B247, 70 (1984); H. Neufeld, W. Grimus, and G. Ecker, Int. J. Mod. Phys. A 3, 603 (1988). For more recent treatments, see also Refs. [2, 3].

[21] G. Ecker, W. Grimus, and H. Neufeld, J. Phys. A20, L807 (1987).

[22] T. P. Cheng, E. Eichten, and L. F. Li, Phys. Rev. D 9, 2259 (1974).

[23] H. E. Haber and R. Hempfling, Phys. Rev. D 48, 4280 (1993).

[24] W. Grimus and L. Lavoura, Eur. Phys. J. C 39, 219 (2005).

[25] P. M. Ferreira and D. R. T. Jones, JHEP 0908, 069 (2009).

[26] H. E. Haber and J. D. Mason, Phys. Rev. D 77, 115011 (2008).