Entropic destruction of heavy quarkonium in quark-gluon plasma with gluon condensate

Zi-qiang Zhang \textsuperscript{a,}\textsuperscript{*}, De-fu Hou \textsuperscript{b}

\textsuperscript{a} School of Mathematics and Physics, China University of Geosciences, Wuhan 430074, China
\textsuperscript{b} Institute of Particle Physics, Central China Normal University, Wuhan 430079, China

A R T I C L E   I N F O
Article history:
Received 24 January 2020
Received in revised form 11 February 2020
Accepted 12 February 2020
Available online 17 February 2020

A B S T R A C T

It has been argued that the peak of the quarkonium entropy at the deconfinement transition is related to the entropic force which induces dissociation of quarkonium states. In this paper, we study the effect of the gluon condensate on the entropic force by using the AdS/CFT correspondence. It is shown that the dropping gluon condensate near $T_c$ (the deconfinement temperature) increases the entropic force thus enhancing the quarkonium dissociation.

© 2020 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/). Funded by SCOAP\textsuperscript{3}.

1. Introduction

In heavy ion collisions at Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC), one of the main experimental signatures for strongly coupled quark-gluon plasma (QGP) \cite{1-3} is dissociation of heavy quarkonium \cite{4}. They are expected to create during the initial stages of the collision and give us significant information about the entire evolution of QGP. For instance, it was suggested that the quarkonium is suppressed due to the Debye screening induced by the high density of color charges in the hot plasma. However, recent experimental studies of charmonium ($c\bar{c}$) show a puzzle: the $c\bar{c}$ suppression at RHIC (lower energy density) appeared to be stronger than that at LHC (higher energy density) \cite{5,6}. Evidently, this is in contradiction with both the Debye screening \cite{4} and the thermal activation through the impact of gluons \cite{7,8}. In explaining how, some scholars suggested \cite{9,10} that the recombination of the produced charm quarks into charmonium would be one possible solution. Specifically, if a region of deconfined quarks and gluons is formed, the quarkonium could be formed from a quark and an anti-quark which were originally created in separate incoherent interactions.

However, recently D. Kharzeev argued \cite{11} that an anomalously strong suppression of $c\bar{c}$ at RHIC is not related to the deconfinement transition, but a consequence of the nature of deconfinement. In particular, the peak of the quarkonium entropy at the deconfinement transition can be related to the entropic force which induces the melting of $c\bar{c}$. This argument is based upon the Lattice QCD results \cite{12-15} which indicate that there is a large amount of entropy associated with the heavy quark-antiquark pair around the crossover region of QGP. The proposal of \cite{11} is that the entropy $S$ gives rise to the entropic force as

$$F = -\frac{\partial S}{\partial L},$$

with $T$ the temperature of the plasma and $L$ the inter-quark distance. Note that this force does not describe other fundamental interactions; instead, it is an emergent force that originates from multiple interactions, driving the system toward the state with a larger entropy. The entropic force was originally introduced in \cite{16} to explain the elasticity of polymer strands in rubber and recently argued in \cite{17} to be responsible for gravity. Here we will not discuss these points and restrict ourselves to its application in dissociating of quarkonium in QGP.

The AdS/CFT correspondence \cite{18-20} provides a new method for studying various aspects of QGP (see \cite{21} for a good review). In this approach, K. Hashimoto and D. Kharzeev have first carried out the entropic force for $\mathcal{N} = 4$ SYM plasma \cite{22}. It is found that the entropy growing with the inter-distance can yield the entropic force. Later, this idea has been extended to various cases. For example, the entropic force of moving quarkonium was studied in \cite{23}. The chemical potential effect on this force was addressed in \cite{24}. Moreover, this force has been discussed from AdS/QCD \cite{25}.

In this paper, we are interested in studying the effect of the gluon condensate on the entropic force. The gluon condensate was introduced \cite{28} as a measure for nonperturbative physics (at zero temperature) and considered \cite{29-31} as an order parameter for (de)confinement. Furthermore, it has found applications in QGP \cite{32}. On the other hand, lattice results indicate that the value of
the gluon condensate shows a drastic change near \( T_c \) regardless of the number of quark flavors \([33,34]\). Due to the above reasons, it would be interesting to study the possible effects that the gluon condensate might cause on various physical observables. Already, there have been some research in this field. For instance, the effect of the gluon condensate on the heavy quark potential was addressed in \([35]\) and the results show that the potential becomes deeper as the value of the gluon condensate decreases in the deconfined phase, indicating the quarkonium mass drops above the deconfinement transition. Moreover, the effect of the gluon condensate on the quark energy loss was considered in \([36,37]\) and the results indicate that the energy loss decreases as the gluon condensate decreases near \( T_c \). In the present work, we would like to study the effect of the gluon condensate on the entropic force mainly because the gluon condensate drastically changes near \( T_c \) while the entropy peaks around \( T_c \). Our goal is to understand how gluon condensate modifies the entropic force. One step further, how gluon condensate affects the quarkonium dissociation, especially near \( T_c \).

The structure of the paper is as follows. In the next section, we briefly review the deformed AdS background with back-reaction due to the gluon condensate given in \([38]\). In section 3, we investigate the entropic force in the dilaton black hole background and analyze how gluon condensate influences it as well as the quarkonium dissociation. Finally, we provide a concluding discussion in section 4.

### 2. Background geometry

The action of the gravity with a dilaton coupled is \([39]\)

\[
l = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g}(\mathcal{R} + \frac{12}{R^2} - \frac{1}{2} \partial_M \phi \partial^M \phi),
\]

where \( k^2 \) is the 5-dimensional Newtonian constant. \( \mathcal{R} \) is the Ricci scalar. \( R \) denotes the AdS radius (for convenience hereafter we set \( R = 1 \)). \( \phi \) represents the dilaton which couples to the gluon operator.

By solving the Einstein’s equations and the dilaton equation of motion, one could obtain two relevant solutions. The first is the dilaton-wall solution \([40,41]\)

\[
ds^2 = \frac{1}{z^2}(\sqrt{1 - c^2 z^2}(d\vec{x}^2 + dt^2) + dz^2),
\]

with the dilaton profile

\[
\phi(z) = \frac{1}{2} \log \left( \frac{1 + c z^2}{1 - c z^2} \right) + \phi_0,
\]

where \( \vec{x} = x_1, x_2, x_3 \) are the boundary coordinates, \( z \) is the coordinate of the 5th dimension with \( z = 0 \) the boundary. \( \phi_0 \) is a constant. \( c \) is nothing but the gluon condensate. The second is the dilaton black hole solution \([38,42]\)

\[
ds^2 = \frac{1}{z^2}[H(z)d\vec{x}^2 - P(z)dt^2 + dz^2],
\]

with

\[
\phi(z) = \frac{C}{f} \sqrt{\frac{3}{2}} \log \left( \frac{1 + f z^4}{1 - f z^4} \right) + \phi_0,
\]

and

\[
H(z) = (1 + f z^4)^{(f-1)/2f} (1 - f z^4)^{(f-3)/2f},
\]

\[
P(z) = (1 + f z^4)^{(f-3)/2f} (1 - f z^4)^{(f+3)/2f},
\]

\[
f = \alpha^2 + \beta^2,
\]

where \( \alpha \) is related to the temperature by \( a = (\pi T)^4/4 \). \( f \) determines the position of the singularity with \( f^{-1/4} = z_f \) (implying the above solution is well defined only in the range \( 0 < z < z_f \)), where \( z_f \) is regarded as an IR cutoff. Notice that for \( a = 0 \), \( f \) reduces to (3). While for \( c = 0 \), it returns to the Schwarzschild black hole solution. Also, there exists a Hawking-Page transition between (3) and (5) at some critical value of \( a \). Therefore, the dilaton-wall background is for the confined phase while the dilaton black hole describes the deconfined phase. For more details on them, refer to \([38]\).

### 3. Entropic force in the dilaton black hole background

Following the holographic prescription given in \([22]\), in this section we calculate the entropic force for the dilaton black hole background and study the effect of the gluon condensate on it. For comparison’s sake, we will work with \( r = r/2 \) as the radial coordinate, following the notation of \([22]\).

The Nambu-Goto action is

\[
S_{NG} = -\frac{1}{2\pi\alpha'} \int d^4x \sqrt{-g} \mathcal{L} = -\frac{1}{2\pi\alpha'} \int d^4x \sqrt{-g} g_{\alpha\beta} \frac{\partial X^\alpha}{\partial \sigma} \frac{\partial X^\beta}{\partial \sigma}.
\]

with

\[
\mathcal{L} = 1/2 \frac{\partial X^\alpha}{\partial \sigma} \frac{\partial X^\beta}{\partial \sigma}.
\]

where \( g_{\alpha\beta} \) is the induced metric and parameterized by \( (\tau, \sigma) \) on the string world-sheet. \( g_{\mu\nu} \) denotes the metric, \( X^\mu \) represents the target space coordinate.

For our purpose, we take the static gauge

\[
t = \tau, \quad x_1 = \sigma,
\]

and suppose \( r \) depends only on \( \sigma \).

\[
r = r(\sigma).
\]

Given that, the Lagrangian density reads

\[
\mathcal{L} = \sqrt{r^2 H(r) P(r)e^{\phi(r)}} + P(r)e^{\phi(r)} r^2,
\]

with \( ir = i x_1 \), \( H(r) \equiv H(x_1)(x_1 = 1/r) \), etc.

Now that \( \mathcal{L} \) does not depend on \( \sigma \) explicitly, one obtains a conserved quantity,

\[
\frac{\partial \mathcal{L}}{\partial \dot{r}} = \text{constant}.
\]

Imposing the boundary condition at \( \sigma = 0 \),

\[
\dot{r} = 0, \quad r = r_c \quad (r_f < r_c),
\]

the conserved quantity becomes

\[
\frac{r^4 H(r)P(r)e^{\phi(r)}}{\sqrt{r^2 H(r)P(r)e^{\phi(r)}} + P(r)e^{\phi(r)} r^2} = \frac{r_c^4 H(r_c)P(r_c)e^{\phi(r_c)}}{r_c^2 H(r_c)P(r_c)e^{\phi(r_c)}}.
\]

with \( r_f \equiv f^{1/4} \). \( H(r) \equiv H(r)_{r=r_f}, \) etc.

Rewriting the conserved quantity, one gets

\[
\frac{dr}{ds} = \frac{r^2 H(r)P(r)e^{\phi(r)} - r^4 H(r)P(r)e^{\phi(r)} r^2}{r_c^2 H(r_c)P(r_c)e^{\phi(r_c)} r^2}.
\]

Integrating (16), the inter-distance of the quark-antiquark pair is obtained as

\[
L = 2 \int_{r_f}^{\infty} dr \frac{r_c^2 H(r_c)P(r) e^{\phi(r_c)}}{r^4 H(r)P(r)e^{\phi(r)} - r^2 H(r)P(r)e^{\phi(r)}}.
\]
The next task is to compute the entropy \( S \), given by
\[
S = -\frac{\partial F}{\partial T},
\]
where \( F \) is the free energy of the quark-antiquark pair. This quantity has been holographically calculated at zero temperature [43] and finite temperature [44,45], respectively. Generally, there are two cases.

1. If \( L > \frac{c}{r} \) (where \( c \) represents the maximum value of \( LT \)), one needs to consider some new configurations [46] and then the choice of the free energy is not unique [47]. Here we select a configuration of two disconnected trailing drag strings [48,49], the corresponding free energy reads
\[
F(1) = \frac{1}{\pi a'} \int_{r_{c}}^{\infty} dr \sqrt{A(r)B(r)} - \frac{\pi}{2} T \theta(L - \frac{c}{T}),
\]
where \( \theta(L - \frac{c}{T}) \) denotes the Heaviside step function.

2. If \( x < \frac{c}{r} \), the fundamental string is connected. For this case, the free energy could be derived from the on-shell action of the fundamental string in the dual geometry, that is
\[
F(2) = \frac{1}{\pi a'} \int_{r_{c}}^{\infty} dr \sqrt{A(r)B(r)} - \frac{\pi}{2} T \theta(c - \frac{r_{c}}{T}),
\]
where \( A(r) = r^{4}H(r)P(r)e^{\phi(r)} \), \( B(r) = P(r)e^{\phi(r)} \), \( A(r_{c}) = r_{c}^{4}H(r_{c})P(r_{c})e^{\phi(r_{c})} \).

Since \( a \) is related to \( T \), one could rewrite Eq. (18) as
\[
S = -\frac{\partial F}{\partial T} = -\frac{\partial F}{\partial a} \frac{\partial a}{\partial T} = -\pi^{3} T^{3} \frac{\partial F}{\partial a}.
\]
Then one gets
\[
S^{(2)} = -\frac{\partial F^{(2)}}{\partial T} = -\pi T^{3} \int_{r_{c}}^{\infty} dr \sqrt{A(r)B(r)[A(r)-A(r_{c})][B(r)-B(r_{c})]}.
\]
with
\[
A'(r) = r^{4}e^{\phi'(r)}[H'(r)P(r)+H(r)P'(r)+H(r)P(r)\phi'(r)],
\]
\[
B'(r) = P'(r)e^{\phi'(r)}+P(r)e^{\phi(r)}\phi'(r),
\]
\[
H'(r) = r^{-4}f'(1+f^{-4})\frac{4}{2f}r^{-2}f(1-f^{-4})\frac{1}{r_{c}^{4}}r^{-2},
\]
\[
P'(r) = r^{-4}f'[1-\frac{3a}{2f}+\frac{1}{1-f^{-4}}(1-f^{-4})+\frac{2}{2f}(1-f^{-4})\frac{4}{2f}r^{-2}f(1-f^{-4})],
\]
\[
\phi'(r) = c\sqrt{\frac{3}{2}f'\frac{2r^{-4}(1+f^{-4})(1+f^{-4})\log(1+f^{-4})}{(1-f^{-4})f^{2}}},
\]
and \( A'(r_{c}) = A'(r)_{r=r_{c}} \), etc., where the derivatives are with respect to \( a \). It seems very hard to evaluate (24) analytically, but it is possible numerically. Before numerical calculation, we discuss the values of some parameters, i.e., \( T \) and \( c \). Since lattice calculations [15] show that the entropy (associated with the heavy quark pair) peaks around \( T_{c} \) and essentially vanishes above \( 1.5T_{c} \), we take \( T \leq 1.5T_{c} \). Moreover, we set \( 0 < c < 0.9 \text{GeV}^{4} \), similar to [35–37].

Let’s discuss results. First, we analyze how gluon condensate affects the entropic force. To this end, we plot \( S^{(2)}/\sqrt{\lambda} \) as a function of \( LT \) for different values of \( c \) in the left and right panel of Fig. 1, where the left is for \( T = 170 \text{ MeV} \) while the right \( T = 250 \text{ MeV} \) (here we have used the relation \( a' = 1/\sqrt{\lambda} \)). In all of the plots from top to bottom \( c = 0, 0.2, 0.9 \text{GeV}^{4} \). From these figures, one finds that increasing \( c \) leads to smaller entropy at small distances. As we know, the entropic force is related to the growth of the entropy with the distance (see Eq. (1)) and responsible for dissociating the quarkonia. Therefore, one concludes that the inclusion of the gluon condensate decreases the entropic force and then decreases the quarkonia dissociation. However, lattice results show [33,34] that the value of \( a' \) drastically drops near \( T_{c} \), so one infers that the dropping gluon condensate near \( T_{c} \) increases the entropic force thus enhancing the quarkonia dissociation.

Also, we would like to investigate the temperature dependence of the entropic force. But, as far as we know, there is no clear conclusion on the temperature dependence of the gluon condensate for full QCD. Here we consider the gluon condensate in the lattice.
gauge theory and suppose that the temperature dependence of the gluon condensate is tuned to describe the lattice data given in [34]. Then we plot $S^{(2)}/\sqrt{\lambda}$ versus $LT$ for various cases in Fig. 2. From these figures, one finds as $T$ increases (meanwhile $c$ decreases), the entropic force increases. But at high temperatures, the differences are not significant. Therefore, one concludes that under the influence of the gluon condensate, the temperature leads to increasing the entropic force thus making the quarkonium melting easier. The physical significance of the results will be discussed in the next section.

4. Conclusion

Recently it has been argued that the entropic force may represent a mechanism for melting the heavy quarkonium. In this paper, we studied the effect of the gluon condensate on the entropic force in a dilaton black hole background using AdS/CFT correspondence. It is shown that the dropping gluon condensate near $T_c$ increases the entropic force thus enhancing the quarkonium dissociation. Also, we analyzed the temperature dependence of the entropic force and found under the influence of the gluon condensate, the entropic force increases with the temperature.

There are, of course, some problems in this research. First, one should bear in mind that the holographic model is not real QCD, which may lead to somewhat different results, e.g., Ref. [15] shows that the entropy essentially vanishes above $1.5T_c$ but we got a different result from Eq. (24) (Ref. [22] also faces the same problem). Moreover, when analyzing the temperature dependence of the entropic force, we have considered the gluon condensate from lattice gauge theory [34] which may differ from real QCD.

One may wonder how gluon condensate affects the dissociation of other quarkonium states, e.g., bottomonium? We would like to make the following comment. Most of the bottomonium states have smaller sizes, which are much less affected by the entropic force [11], but this does not mean the gluon condensate has no effect on the bottomonium dissociation. It would be significant to check whether gluon condensate has an effect on Debye screening or thermal gluon activation. We hope to report our progress in this regard in the near future.

Acknowledgements

This work is supported by the NSFC under Grant Nos. 11705166, 11735007 and the Fundamental Research Funds for the Central Universities, China University of Geosciences (Wuhan) (No. CUGL180402).

References

[1] J. Adams, et al., STAR Collaboration, Nucl. Phys. A 757 (2005) 102.
[2] K. Adcox, et al., PHENIX Collaboration, Nucl. Phys. A 757 (2005) 184.
[3] E.V. Shuryak, Nucl. Phys. A 750 (2005) 64.
[4] T. Matsu, H. Satz, Phys. Lett. B 178 (1986) 416.
[5] A. Adare, et al., PHENIX Collaboration, Phys. Rev. Lett. 98 (2007) 223201.
[6] B.B. Abelev, et al., ALICE Collaboration, Phys. Lett. B 734 (2014) 314.
[7] D. Kharzeev, H. Satz, Phys. Lett. B 334 (1994) 155.
[8] E.V. Shuryak, Phys. Lett. B 78 (1978) 150.
[9] P. Braun-Munzinger, J. Stachel, Phys. Lett. B 490 (2000) 196.
[10] R.L. Thews, M. Schroedter, J. Rafelski, Phys. Rev. C 63 (2001) 549013.
[11] D.E. Kharzeev, Phys. Rev. D 90 (2014) 074007.
[12] O. Kaczmarek, F. Karsch, P. Petreczky, F. Zantow, Phys. Lett. B 543 (2002) 41.
[13] O. Kaczmarek, F. Zantow, arXiv:hep-lat/0506019.
[14] P. Petreczky, K. Petrov, Phys. Rev. D 70 (2004) 054503.
[15] O. Kaczmarek, F. Zantow, PoS LAT (2005) 192. arXiv:hep-lat/0510094, 2006.
[16] K.H. Meyer, G. Sussich, E. Valk, Kolloid Z. 59 (1932) 208.
[17] E.P. Verlinde, J. High Energy Phys. 04 (2011) 029.
[18] J.M. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231.
[19] S.S. Gubser, I.R. Klebanov, A.M. Polyakov, Phys. Lett. B 428 (1998) 105.
[20] E. Witten, Adv. Theor. Math. Phys. 2 (1998) 253.
[21] J.C. Solana, H. Liu, D. Mateos, K. Rajagopal, U.A. Wiedemann, arXiv:1101.0618 [hep-th].
[22] K. Hashimoto, D.E. Kharzeev, Phys. Rev. D 90 (2014) 125012.
[23] K.B. Fadafan, S.K. Tabatabaei, Phys. Rev. D 94 (2016) 026007.
[24] Y. Zhang, D.F. Hou, G. Chen, Phys. Lett. B 768 (2017) 180.
[25] I. Iatrakis, D.E. Kharzeev, Phys. Rev. D 93 (2016) 086009.
[26] D.E. Kharzeev, Nucl. Part. Proc. Phys. 276–279 (2016) 2013–2016.
[27] Z. Zhang, Zj. Luo, D.F. Hou, Ann. Phys. 391 (2018) 47–55.
[28] M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B 147 (1979) 385.
[29] S.H. Lee, Phys. Rev. D 40 (1989) 2484.
[30] M.D. Elia, A.D. Giacomo, E. Meggiolaro, Phys. Rev. D 67 (2003) 114504.
[31] D.E. Miller, Phys. Rep. 443 (2007) 55.
[32] C.E. Brown, J.W. Holt, C.-H. Lee, M. Rho, Phys. Rev. D 529 (2000) 161.
[33] G. Boyd, et al., Nucl. Phys. B 469 (1996) 419.
[34] G. Boyd, D.E. Miller, arXiv:hep-ph/9608482.
[35] Y. Kim, B.-H. Lee, C. Park, S.-J. Sin, Phys. Rev. D 80 (2009) 105016.
[36] Zq. Zhang, X.R. Zha, Eur. Phys. J. C 79 (2019) 107.
[37] Zq. Zhang, Eur. Phys. J. C 79 (2019) 992.
[38] Y. Kim, B.-H. Lee, C. Park, S.-J. Sin, J. High Energy Phys. 09 (2007) 105.
[39] S. Nojiri, S.D. Odintsov, Phys. Lett. B 449 (1999) 39.
[40] A. Kehagias, K. Sfetsos, Phys. Lett. B 454 (1999) 270.
[41] C. Csaki, M. Reece, J. High Energy Phys. 07 (2005) 062.
[42] D. Bak, M. Gutperle, S. Hirano, N. Ohta, Phys. Rev. D 70 (2004) 086004.
[43] J.M. Maldacena, Phys. Rev. Lett. 80 (1998) 4859.
[44] A. Brandhuber, N. Itzhaki, J. Sonnenschein, S. Yankielowicz, Phys. Lett. B 434 (1998) 36.
[45] S.J. Rey, S. Theisen, J.T. Yee, Nucl. Phys. B 527 (1998) 171.
[46] D. Bak, A. Karch, L.G. Yaffe, J. High Energy Phys. 0708 (2007) 049.
[47] M. Chernicoff, J.A. Garcia, A. Guijosa, J. High Energy Phys. 0609 (2006) 068.
[48] C.P. Herzog, A. Karch, P. Kovtun, C. Rzeczka, L.G. Yaffe, J. High Energy Phys. 0607 (2006) 013.
[49] S.S. Gubser, Phys. Rev. D 74 (2006) 126005.

Fig. 2. $S^{(2)}/\sqrt{\lambda}$ versus $LT$ for different $T$ and $c$. From top to bottom, ($T = 250$ MeV, $c = 0.001$ GeV$^4$), ($T = 170$ MeV, $c = 0.007$ GeV$^4$), ($T = 150$ MeV, $c = 0.01$ GeV$^4$), respectively.