Three-body charmed baryon Decays with SU(3) flavor symmetry

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Abstract

We study the three-body anti-triplet $B_c \to B_n M M'$ decays with the SU(3) flavor (SU(3)$_f$) symmetry, where $B_c$ denotes the charmed baryon anti-triplet of ($\Xi_c^0, -\Xi_c^+, \Lambda_c^+$), and $B_n$ and $M(M')$ represent baryon and meson octets, respectively. By considering only the S-wave $MM'$-pair contributions without resonance effects, the decays of $B_c \to B_n M M'$ can be decomposed into irreducible forms with 11 parameters under SU(3)$_f$, which are fitted by the 14 existing data, resulting in a reasonable value of $\chi^2/d.o.f = 2.8$ for the fit. Consequently, we find that the triangle sum rule of $A(\Lambda_c^+ \to nK^0\pi^+) - A(\Lambda_c^+ \to pK^+\pi^-) - \sqrt{2}A(\Lambda_c^+ \to p\bar{K}^0\pi^0) = 0$ given by the isospin symmetry holds under SU(3)$_f$, where $A$ stands for the decay amplitude. In addition, we predict that $B(\Lambda_c^+ \to n\pi^+\bar{K}^0) = (0.9 \pm 0.8) \times 10^{-2}$, which is 3–4 times smaller than the BESIII observation, indicating the existence of the resonant states. For the to-be-observed $B_c \to B_n M M'$ decays, we compute the branching fractions with the SU(3)$_f$ amplitudes to be compared to the BESIII and LHCb measurements in the future.
I. INTRODUCTION

The three-body charmed baryon $\mathbf{B}_c \rightarrow \mathbf{B}_n M M'$ decays have been recently searched by the experimental Collaborations of BELLE, BESIII and LHCb, where $\mathbf{B}_c \equiv (\Xi^0_c, -\Xi^+_c, \Lambda^+_c)$ denotes the charmed baryon anti-triplet, and $\mathbf{B}_n$ and $M(M')$ correspond to the baryon and meson octets, respectively. For example, the decay of $\Lambda^+_c \rightarrow pK^-\pi^+$ has been observed with high precision by BELLE and BESIII [1, 2], which improves the accuracy of the $\Lambda_b$ decays with $\Lambda^+_c$ as one of the final states [5]. Besides, the crucial information on the higher wave baryon resonances like $\Lambda(1405)$ has been extracted from the $\Sigma \pi \pi$ invariant mass spectra of the $\Lambda^+_c \rightarrow \Sigma \pi \pi$ decays [3]. The other interest comes from the test of the theoretical approach. For example, the first observation of $\Lambda^+_c \rightarrow nK^0 \pi^+$ has been used to examine the isospin relation [4], that is, $R(\Delta) \equiv A(\Lambda^+_c \rightarrow n\bar{K}^0\pi^+) + A(\Lambda^+_c \rightarrow pK^-\pi^+) + \sqrt{2}A(\Lambda^+_c \rightarrow p\bar{K}^0\pi^0) = 0$ [6, 7].

Since the $\Lambda^+_c \rightarrow pK^-\pi^+$ decay shares the similar diagrams as the doubly Cabibbo-suppressed $\Lambda^+_c \rightarrow pK^+\pi^-$ one, we have the ratio of $\mathcal{B}(\Lambda^+_c \rightarrow pK^+\pi^-)/\mathcal{B}(\Lambda^+_c \rightarrow pK^-\pi^+) = R_{K\pi} \tan^4 \theta_c$ with $R_{K\pi} \sim 1.0$ and $\theta_c$ the Cabibbo angle should hold. Nonetheless, the values of $R_{K\pi} = 0.82 \pm 0.12$ [9] and $0.58 \pm 0.06$ [10] have been measured by BELLE and LHCb, respectively, showing a possible deviation caused by an additional $W$-exchange amplitude for $\Lambda^+_c \rightarrow pK^-\pi^+$. As a result, the $\mathbf{B}_c \rightarrow \mathbf{B}_n M M'$ decays are important for achieving a deeper insight into the hadronization of particle interactions.

In contrast with the abundant observations, there rarely exist systematic theoretical studies on the $\mathbf{B}_c \rightarrow \mathbf{B}_n M M'$ decays, apart from those based on the isospin symmetry [6, 7]. This is due to the fact that the scale of the charm quark mass ($m_c$) is too large for the flavor SU(3) ($SU(3)_f$) symmetry, but the theories based on the heavy quark expansion may not be valid as $m_c$ is not large enough. In addition, the factorization fails to work well in the charmed hadron decays [6, 11], whereas it is successfully used in the beauty hadron ones [12–14]. The alternative approaches for the charmed hadron decays have been shown in Refs. [15–20], which take into account the non-factorizable effects. On the other hand, the $SU(3)_f$ symmetry has been tested as a useful tool both in the beauty and charmed

\footnote{To calculate the decay amplitude of $A$, we use the conventions of $|\pi^+\rangle = -|11\rangle$ and $|\bar{K}^0\rangle = -|\frac{1}{2}\frac{1}{2}\rangle$, whereas $|\pi^-\rangle = |11\rangle$ and $|K^0\rangle = |\frac{1}{2}\frac{1}{2}\rangle$ are taken in Refs. [6, 7], resulting in the relation to be $R(\Delta) \equiv A(\Lambda^+_c \rightarrow n\bar{K}^0\pi^+) - A(\Lambda^+_c \rightarrow pK^-\pi^+) - \sqrt{2}A(\Lambda^+_c \rightarrow p\bar{K}^0\pi^0) = 0$. However, the different signs in $R(\Delta)$ and other similar relations do not affect the physical consequences of these relations due to the arbitrariness of the phase of the amplitude.}
hadron decays \[21–28\], particularly, the two-body \( B_c \to B_nM \) decays \[6, 29–38\]. It is hence expected that the same symmetry can be applied to the three-body \( B_c \to B_nMM' \) decays. In this paper, we will relate the possible \( B_c \to B_nMM' \) decay processes with the \( SU(3)_f \) parameters \[29\], by which the systematic numerical analysis can be performed for the first time. Under the \( SU(3)_f \) symmetry, we will derive the relation of \( R(\Delta) = 0 \), and examine the value of \( R_{K\pi} \) from the ratio of \( B(\Lambda_c^+ \to pK^+\pi^-)/B(\Lambda_c^+ \to pK^-\pi^+) \).

Our paper is organized as follows. We give the formalism in Sec. II, where the amplitudes for the three-body charmed baryon decays under the \( SU(3)_f \) symmetry are presented. In Sec. III, we show our numerical results and discussions. Our conclusions are in Sec. IV.

II. FORMALISM

The three-body \( B_c \to B_nMM' \) decays can proceed through the charm quark decays of \( c \to su\bar{d}, c \to udd(uss) \) and \( c \to dus \), where \( B_{c,n} \) and \( M' \) denote the baryon and meson states, respectively. Accordingly, the tree-level effective Hamiltonian is given by \[39\]

\[
\mathcal{H}_{\text{eff}} = \sum_{i=-,+,0} \frac{G_F}{\sqrt{2}} c_i \left[ V_{cs} V_{ud} O_i + V_{cd} V_{ud} O_i^\dagger + V_{cd} V_{us} O_i' \right],
\]

where \( G_F \) is the Fermi constant, \( c_{\pm} \) represent the Wilson coefficients and \( V_{ij} \) correspond to the CKM matrix elements, while \( O_{\mp}, O_{\mp}^\dagger \) and \( O_{\mp}' \) are the four-quark operators, written as

\[
O_{\mp} = \frac{1}{2} \left[ (\bar{u}d)(\bar{s}c) \mp (\bar{d}u)(\bar{c}s) \right],
\]

\[
O_{\mp}^\dagger = \frac{1}{2} \left[ (\bar{u}d)(\bar{d}c) \mp (\bar{d}u)(\bar{c}s) \right] - \frac{1}{2} \left[ (\bar{u}s)(\bar{s}c) \mp (\bar{s}u)(\bar{c}c) \right],
\]

\[
O_{\mp}' = \frac{1}{2} \left[ (\bar{u}s)(\bar{d}c) \mp (\bar{d}s)(\bar{u}c) \right],
\]

with \( (\bar{q}_1 q_2)(\bar{q}_3 c) \equiv \bar{q}_1 \gamma_\mu(1 - \gamma_5)q_2 \bar{q}_3 \gamma^\mu(1 - \gamma_5)c \). Here, the relation of \( V_{cs} V_{us} = -V_{cd} V_{ud} \) has been used for \( O_{\mp}^\dagger \) to combine the \( c \to udd \) and \( c \to uss \) transitions. By means of the Cabibbo angle \( \theta_c \), it is given that \( (V_{cs}V_{ud}, V_{cd}V_{ud}, V_{cd}V_{us}) = c_c^2 (1, -t_c, -t_c^2) \), where \( (c_c, t_c) \equiv (\cos \theta_c, \tan \theta_c) \), such that the decays with \( O_{\mp}, O_{\mp}^\dagger \) and \( O_{\mp}' \) are classified as the Cabibbo-favored (CF), Cabibbo-suppressed (CS), and doubly Cabibbo-suppressed (DCS) processes, respectively.

In Eq. (2), \( (\bar{q}_1 q_2)(\bar{q}_3 c) \) can be rewritten as \( (\bar{q}\gamma_\mu q)(\bar{c}c) \) with \( q_i = (u, d, s) \) the triplet of 3 under the \( SU(3)_f \) symmetry, by suppressing the Dirac and Lorentz indices. Furthermore,
since \((q_i^q q_k q_j^q)c\) can be decomposed as the irreducible forms of \((3 \times 3 \times 3)c = (\bar{3} + \bar{3}' + 6 + \bar{15})c\), one derives that \[29\]

\[
O_{- (+)} \simeq \mathcal{O}_{6(\bar{15})} = \frac{1}{2}(\bar{u} d \bar{s} \mp s d \bar{u})c,
\]
\[
O_{- (+)}^\dagger \simeq \mathcal{O}_{6(\bar{15})}^\dagger = \frac{1}{2}(\bar{u} d d \bar{u} - d d \bar{u})c - \frac{1}{2}(\bar{u} s s \bar{s} - s s \bar{u})c,
\]
\[
O_{- (+)}' \simeq \mathcal{O}_{6(\bar{15})}' = \frac{1}{2}(\bar{u} s d \mp d s \bar{u})c.
\]

Subsequently, the effective Hamiltonian in Eq. (1) has the expression under the \(SU(3)_f\) symmetry, given by \[32–35\]

\[
\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} c \frac{\epsilon^{ijl}}{2} H(6)_{lk} + c_+ H(\bar{15})_{ikj} \biggr] c,
\]

(4)

where \(H(6, \bar{15})\) are presented as the tensor forms of \((\mathcal{O}_{6}^{(t,r)}, \mathcal{O}_{15}^{(t,r)})\) in Eq. (3). Their non-zero entries are given by \[29, 30\]

\[
H_{22}(6) = 2, H_{13}^{12}(\bar{15}) = H_{23}^{31}(\bar{15}) = 1,
\]
\[
H_{23}(6) = H_{32}(6) = -2 t_c, H_{12}^{21}(\bar{15}) = H_{21}^{21}(\bar{15}) = t_c,
\]
\[
H_{33}(6) = 2 t_c^2, H_{31}^{12}(\bar{15}) = H_{32}^{21}(\bar{15}) = -t_c^2,
\]

(5)

with \((i, j, k)\) for the quark indices. Correspondingly, the three lowest-lying charmed baryon states of \(B_c\) form an anti-triplet of \(\bar{3}\) to consist of \((d s - s d)c, (u s - s u)c\) and \((u d - d u)c\), and \(B_n(M)\) belongs to the baryon (meson) octet of 8, which are written as

\[
B_c = (\Xi^0_c, -\Xi^+_c, \Lambda^+_c),
\]
\[
B_n = \left(\begin{array}{ccc}
\frac{1}{\sqrt{6}} \Lambda^0 + \frac{1}{\sqrt{2}} \Sigma^0 & \Sigma^- & \Xi^- \\
\Sigma^+ & \frac{1}{\sqrt{6}} \Lambda^0 - \frac{1}{\sqrt{2}} \Sigma^0 & \Xi^0 \\
p & n & -\sqrt{\frac{2}{3}} \Lambda^0
\end{array} \right),
\]
\[
M = \left(\begin{array}{ccc}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^- & K^- \\
\pi^+ & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & K^0 \\
K^+ & K^0 & -\sqrt{\frac{2}{3}} \eta
\end{array} \right),
\]

(6)

respectively.

Now, one is able to connect the octets of \((B_n, M)^j\) and anti-triplet of \((B_c)^i\) to \((\epsilon^{ijl} H(6)_{lk}, H(\bar{15})_{ikj}^{ij})\) in \(\mathcal{H}_{\text{eff}}\) of Eq. (4) to get the \(SU(3)_f\) amplitudes. Since the Wilson coefficients are scale-dependent, in the NDR scheme it is calculated that \((c_-, c_+) = (1.78, 0.76)\) at the scale
The value of \((c_-/c_+)^2 \approx 5.5\) implies the suppressed branching ratios associated with \(H(15)\). Hence, we follow Refs. \[6, 32, 37\] to ignore the amplitudes from \(H(15)\). By means of \(\mathcal{A}(B_c \to B_nMM') \equiv (G_F/\sqrt{2})T(B_c \to B_nMM')\), the T-amplitude of \(B_c \to B_nMM'\) can be derived as \[29\]

\[
T(B_c \to B_nMM) = a_1(\bar{B_n})^k_i(M)^m_j(M')^l_mH(6)_{jk}T^{ij} + a_2(\bar{B_n})^k_i(M)^m_j(M')^l_mH(6)_{jl}T^{ij} + a_3(\bar{B_n})^k_i(M)^m_j(M')^l_mH(6)_{im}T^{ij} + a_4(\bar{B_n})^k_i(M)^m_j(M')^l_mH(6)_{im}T^{ij} + a_5(\bar{B_n})^k_i(M)^m_j(M')^l_mH(6)_{im}T^{ij} + a_6(\bar{B_n})^k_i(M)^m_j(M')^l_mH(6)_{im}T^{ij}, \tag{7}
\]

with \(T^{ij} = (B_c)_{ae}^{aij}\), where \(c_2^2\) and \(c_-\) in \(\mathcal{H}_{eff}\) have been absorbed into the \(SU(3)_f\) parameters \(a_i\) \((i = 1, 2, ..., 6)\). While there exists the relative orbital angular momentum \(L\) between the two-meson states, we have assumed the S-wave \(MM'\)-pair \((L = 0)\) in the dominant amplitudes in Eq. \[7\], whereas the P-wave one \((L = 1)\) is neglected. However, there are some cases in which the S-wave contributions vanish, but P-wave ones are dominant, resulting in the other set of amplitudes to be studied elsewhere. For example, the decay of \(\Lambda_c^+ \to \Lambda\pi^+\pi^0\) with the measured branching ratio around 7.1\% is mainly from the P-wave contribution.

The integration over the phase space of the three-body decay relies on the equation of \[5\]

\[
\Gamma = \left|\mathcal{A}(B_c \to B_nMM')\right|^2 \left(\frac{m_{12}^2}{2\pi^3}\cdot\frac{32m_{B_c}^3}{3m_{12}^2m_{23}^2}\right)dm_{12}^2dm_{23}^2, \tag{8}
\]

where \(m_{12} = p_M + p_{M'}\), \(m_{23} = p_{M'} + p_{B_n}\) and \(\mathcal{A}(B_c \to B_nMM')\) is related to \(T(B_c \to B_nMM')\) given in Eq. \[7\]. In Tables I, II and III we show the full expansions of \(T(\Lambda_c^+ \to B_nMM'), T(\Xi_c^+ \to B_nMM')\) and \(T(\Xi_c^0 \to B_nMM')\), respectively. In general, the \(SU(3)_f\) parameters depend on \(m_{12}\) and \(m_{23}\). However, all structures in the Dalitz plots come from the dynamical effects, such as those from the resonant states. Clearly, the squared amplitude in the Dalitz plot is almost structureless for the decay without the resonance. As a result, we treat our decay amplitudes as constants without energy dependences so that they can be factored out from the integrals as an approximation.
TABLE I. T-amplitudes of $\Lambda^+_c \rightarrow B_n M M'$.

| CF mode | T-amp | | CS mode | T-amp/t.c. | | DCS mode | T-amp/t_c^2 |
|---------|-------| | | | | | |
| $\Sigma^+\pi^0\bar{\eta}^0$ | $4a_1 + 2a_2 + 2a_3 + 2a_4 - 2a_5$ | | $\Sigma^+\pi^0K^0$ | $\sqrt{2}a_2 + \sqrt{2}a_3 + 2\sqrt{2}a_4$ | | $\Sigma^+K^0\bar{\eta}^0$ | $4a_4$ |
| $\Sigma^+\pi^0\pi^+$ | $4a_1 + 2a_2 + 2a_3 - 2a_5 - 2a_6$ | | $\Sigma^+\pi^+ - K^+$ | $-2a_2 - 2a_3 + 2a_6$ | | $\Sigma^+\pi^0\pi^+$ | $2\sqrt{2}a_4$ |
| $\Sigma^+\pi^0\bar{\eta}^0$ | $4a_1 + 2a_3 + 2a_4 + 2a_5 - 2a_6$ | | $\Sigma^+\bar{\eta}^0\pi^0$ | $\sqrt{2}a_2 - \sqrt{2}a_3 + 2\sqrt{2}a_4$ | | $\Sigma^+\pi^+\bar{K}$ | $-4a_4$ |
| $\Sigma^+\pi^0\bar{K}$ | $4a_1 + 2a_2 + 2a_3$ | | $\Sigma^+\bar{\eta}^0\pi^0$ | $2\sqrt{2}a_2 - 2\sqrt{2}a_3 + 2\sqrt{2}a_4$ | | $\Sigma^+\pi^+\eta^0$ | $\sqrt{2}a_2 - \sqrt{2}a_3 + 2\sqrt{2}a_4$ |
| $\Sigma^+\pi^0\pi^+$ | $4a_1 - 2a_3 - 2a_5$ | | $\Sigma^+\pi^0\pi^+$ | $-2a_2 - 2a_3 + 2a_5$ | | $\Sigma^+\bar{K}^0\bar{\eta}^0$ | $-\sqrt{2}a_2$ |
| $\Sigma^+\pi^0\eta^0$ | $-2a_2 - 2a_5$ | | $\Sigma^+\pi^+ - K^+$ | | | |
| $\Sigma^+\pi^0\bar{K}$ | $\sqrt{2}a_2 + \sqrt{2}a_3 + 2\sqrt{2}a_4$ | | $\Sigma^+\pi^0\pi^+$ | | | |
| $\Sigma^+\pi^0\pi^+$ | $4a_4 + 2a_6$ | | $\Sigma^+\bar{\eta}^0\pi^0$ | | | |

TABLE II. T-amplitudes of $\Xi^+_c \rightarrow B_n M M'$.

| CF mode | T-amp | | CS mode | T-amp/t.c. | | DCS mode | T-amp/t_c^2 |
|---------|-------| | | | | | |
| $\Sigma^+\pi^0\bar{\eta}^0$ | $-\sqrt{2}a_2 - \sqrt{2}a_3$ | | $\Sigma^+\pi^0\pi^+$ | $-2a_1 - 2a_3 + 2a_5$ | | $\Sigma^+\pi^0\pi^+$ | $2a_4 - 2a_6$ |
| $\Sigma^+\pi^0\pi^+$ | $2a_2$ | | $\Sigma^+\pi^0\pi^0$ | $-2\sqrt{2}a_2 + 2\sqrt{2}a_3 + 2\sqrt{2}a_4$ | | $\Sigma^+\pi^0\pi^+$ | $-\sqrt{2}a_2$ |
| $\Sigma^+\pi^0\pi^+$ | $\sqrt{2}a_2 - \sqrt{2}a_3 + 2\sqrt{2}a_4$ | | $\Sigma^+\pi^0\pi^0$ | $-2\sqrt{2}a_2 + 2\sqrt{2}a_3 + 2\sqrt{2}a_4$ | | $\Sigma^+\pi^0\pi^+$ | $2a_4 - 2a_6$ |
| $\Sigma^+\pi^0\pi^+$ | $-2a_2 - 2a_3 + 2a_5$ | | $\Sigma^+\pi^+ - K^+$ | | | |
| $\Sigma^+\pi^0\pi^+$ | $4a_1 + 2a_2 + 2a_3 - 2a_5 - 2a_6$ | | $\Sigma^+\pi^0\pi^+$ | | | |
| $\Sigma^+\pi^0\pi^+$ | $4a_1 + 2a_2 + 2a_3 - 2a_5 - 2a_6$ | | $\Sigma^+\pi^0\pi^+$ | | | |
| $\Sigma^+\pi^0\pi^+$ | $-2a_2 - 2a_3 + 2a_5$ | | $\Sigma^+\pi^0\pi^+$ | | | |
| $\Sigma^+\pi^0\pi^+$ | $-2a_2 - 2a_3 + 2a_5$ | | $\Sigma^+\pi^0\pi^+$ | | | |
| $\Sigma^+\pi^0\pi^+$ | $2a_4 + 2a_5 + 2a_6$ | | $\Sigma^+\pi^0\pi^+$ | | | |
| $\Sigma^+\pi^0\pi^+$ | $2a_4 + 2a_5 + 2a_6$ | | $\Sigma^+\pi^0\pi^+$ | | | |
| $\Sigma^+\pi^0\pi^+$ | $2a_4 + 2a_5 + 2a_6$ | | $\Sigma^+\pi^0\pi^+$ | | | |
| $\Sigma^+\pi^0\pi^+$ | $2a_4 + 2a_5 + 2a_6$ | | $\Sigma^+\pi^0\pi^+$ | | | |
TABLE III. T-amplitudes of $\Xi_0^0 \rightarrow B_n MM'$.

| CF mode | T-amp | CS mode | T-amp/t_{ic} | DCS mode | T-amp/t_{ic}^2 |
|----------|-------|---------|--------------|-----------|-----------------|
| $\Sigma^+ \pi^0 K^-$ | $\sqrt{2}a_5$ | $\Sigma^+ \pi^0$ | $-\sqrt{2}a_5$ | $\Sigma^+ \pi^- K^0$ | $-2a_6$ |
| $\Sigma^+ \pi^- K^0$ | $2a_5 + 2a_6$ | $\Sigma^+ \pi^- \eta^0$ | $2\sqrt{2}a_5 + \sqrt{2}a_6$ | $\Sigma^+ \pi^- K^0$ | $a_3 - 2a_6$ |
| $\Sigma^+ K^- \eta^0$ | $-\sqrt{2}a_5$ | $\Sigma^+ K^- \eta^0$ | $2a_5$ | $\Sigma^+ K^- \eta^0$ | $-\sqrt{2}a_5$ |
| $\Sigma^0 \pi^0 K^0$ | $a_2 + a_4 + a_5 + 2a_6$ | $\Sigma^0 \pi^0 \eta^0$ | $\sqrt{2}a_1 + \sqrt{2}a_3 - \sqrt{2}a_5 - 2\sqrt{2}a_6$ | $\Sigma^0 \pi^0 K^0$ | $a_3 - 2a_6$ |
| $\Sigma^0 K^- \eta^0$ | $\sqrt{2}a_2 - \sqrt{2}a_5$ | $\Sigma^0 K^- \eta^0$ | $\sqrt{2}a_3 + \sqrt{2}a_4 + \sqrt{2}a_6$ | $\Sigma^0 K^- \eta^0$ | $\sqrt{2}a_3$ |
| $\Sigma^0 \pi^0 K^0$ | $\sqrt{2}a_2 - \sqrt{2}a_5$ | $\Sigma^0 \pi^0 K^0$ | $2\sqrt{2}a_1 + \sqrt{2}a_3 - \sqrt{2}a_5 + \sqrt{2}a_6$ | $\Sigma^0 K^- \eta^0$ | $2a_3 - 2a_6$ |
| $\Sigma^0 K^- \eta^0$ | $\sqrt{2}a_2 - \sqrt{2}a_5$ | $\Sigma^0 K^- \eta^0$ | $2\sqrt{2}a_1 + \sqrt{2}a_3 - \sqrt{2}a_5 + \sqrt{2}a_6$ | $\Sigma^0 K^- \eta^0$ | $2a_3 - 2a_6$ |
| $\Sigma^0 \pi^- K^0$ | $\sqrt{2}(a_1 + a_2 + a_3 + a_5)$ | $\Sigma^0 K^- \eta^0$ | $-\sqrt{2}a_6$ | $\Sigma^0 K^0$ | $-2a_6$ |
| $\Sigma^0 K^- \eta^0$ | $-2\sqrt{2}a_2$ | $\Sigma^0 K^- \eta^0$ | $-2\sqrt{2}a_2 + 2\sqrt{2}a_6 + \sqrt{2}a_6$ | $\Sigma^0 K^- \eta^0$ | $-2a_6$ |
| $\Sigma^0 K^- \eta^0$ | $-2\sqrt{2}a_2 + 2\sqrt{2}a_6 + \sqrt{2}a_6$ | $\Sigma^0 K^- \eta^0$ | $-2\sqrt{2}a_2 + 2\sqrt{2}a_6 + \sqrt{2}a_6$ | $\Sigma^0 K^- \eta^0$ | $-2a_6$ |
| $\Sigma^0 K^- \eta^0$ | $-2\sqrt{2}a_2 + 2\sqrt{2}a_6 + \sqrt{2}a_6$ | $\Sigma^0 K^- \eta^0$ | $-2\sqrt{2}a_2 + 2\sqrt{2}a_6 + \sqrt{2}a_6$ | $\Sigma^0 K^- \eta^0$ | $-2a_6$ |
| $\Xi^0_\pi^- \pi^0$ | $-a_5$ | $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ | $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ |
| $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ | $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ | $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ |
| $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ | $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ | $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ |
| $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ | $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ | $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ |
| $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ | $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ | $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ |
| $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ | $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ | $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ |
| $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ | $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ | $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ |
| $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ | $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ | $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ |
| $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ | $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ | $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ |
| $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ | $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ | $\Xi^0_\pi^- \pi^0$ | $-a_5 - a_6$ |
TABLE IV. The data of $\mathcal{B}(\Lambda_c^+ \to B_n MM)$ from the PDG [5], except for $\mathcal{B}(\Lambda_c^+ \to \Sigma^+\pi^0\pi^0, pK^+\pi^-)$ [3, 10].

|                | data          | our results |                | data          | our results |
|----------------|---------------|-------------|----------------|---------------|-------------|
| $10^2\mathcal{B}(\Lambda_c^+ \to pK^-\pi^+)$ | 3.4 ± 0.4     | 3.3 ± 1.0   | $10^2\mathcal{B}(\Lambda_c^+ \to \Xi^-K^+\pi^+)$ | 6.2 ± 0.6     | 6.3 ± 0.6   |
| $10^2\mathcal{B}(\Lambda_c^+ \to p\bar{K}^0\eta)$ | 1.6 ± 0.4     | 0.9 ± 0.1   | $10^2\mathcal{B}(\Xi_c^+ \to \Xi^-\pi^+\pi^+)$ | 6.1 ± 3.1     | 7.2 ± 2.0   |
| $10^2\mathcal{B}(\Lambda_c^+ \to \Lambda^0K^+\bar{K}^0)$ | 5.6 ± 1.1     | 5.7 ± 1.1   | $10^2\mathcal{B}(\Lambda_c^+ \to p\pi^-\pi^+)$ | 4.2 ± 0.4     | 4.7 ± 1.6   |
| $10^2\mathcal{B}(\Lambda_c^+ \to \Lambda^0\pi^+\eta)$ | 2.2 ± 0.5     | 2.1 ± 0.9   | $10^2\mathcal{B}(\Lambda_c^+ \to pK^-K^+)$ | 5.2 ± 1.2     | 5.1 ± 2.1   |
| $10^2\mathcal{B}(\Lambda_c^+ \to \Sigma^+\pi^+\pi^-)$ | 4.4 ± 0.3     | 4.4 ± 3.5   | $10^2\mathcal{B}(\Lambda_c^+ \to pK^+\pi^-)$ | 1.0 ± 0.1     | 1.0 ± 0.1   |
| $10^2\mathcal{B}(\Lambda_c^+ \to \Sigma^-\pi^+\pi^+)$ | 1.9 ± 0.2     | 1.9 ± 1.3   |                 |               |             |
| $10^2\mathcal{B}(\Lambda_c^+ \to \Sigma^0\pi^+\pi^0)$ | 2.2 ± 0.8     | 1.0 ± 0.8   |                 |               |             |
| $10^2\mathcal{B}(\Lambda_c^+ \to \Sigma^+\pi^0\pi^0)$ | 1.3 ± 0.1     | 1.3 ± 1.3   |                 |               |             |
| $10^2\mathcal{B}(\Lambda_c^+ \to \Sigma^+K^+\pi^-)$ | 2.1 ± 0.6     | 3.0 ± 0.4   |                 |               |             |

III. NUMERICAL RESULTS AND DISCUSSIONS

In the numerical analysis, we perform the minimum $\chi^2$ fit to examine if the $SU(3)_f$ symmetry is valid in the $B_c \to B_n MM'$ decays. The equation of the $\chi^2$ fit is given by

$$\chi^2 = \sum_i \left( \frac{B_{th}^i - B_{ex}^i}{\sigma_{ex}^i} \right)^2,$$

(9)

where $B_{th}$ as $B(B_c \to B_n MM')$ is calculated by the $SU(3)_f$ parameters, and $B_{ex}$ the experimental value in Table IV, with $\sigma$ the experimental error. With $\sin \theta_c = 0.2248$ [5], one obtains that $t_c = 0.2307$ as the input in Eq. [5]. The $SU(3)_f$ parameters are written as

$$a_1, a_2 e^{i\delta_{a2}}, a_3 e^{i\delta_{a3}}, a_4 e^{i\delta_{a4}}, a_5 e^{i\delta_{a5}}, a_6 e^{i\delta_{a6}},$$

(10)

where the phases $\delta_{a2,3,...,6}$ are due to the nature of complex numbers associated with $a_i$, while $a_1$ can be relatively real. This leads to the reduced 11 parameters to be extracted with 14 data inputs in Table IV, where the fitting values of $a_i$ and $\delta_{a_i}$ are shown in Table V. We find that $\chi^2/d.o.f = 8.4/3 = 2.8$ with $d.o.f$ representing the degree of freedom, and we reproduce the branching ratios in the third column of Table IV in order to be compared to the data. Note that in calculating the decay branching ratios, we have treated our $SU(3)_f$ parameters as independent ones, which may result in overestimated error ranges in our results.
TABLE V. Fitting results for $a_i$ and $\delta a_i$.

| $a_i$ result (GeV$^2$) | $\delta a_i$ | result   |
|------------------------|-------------|----------|
| $a_1$                  | $9.1 \pm 0.6$ | $-$ $-$   |
| $a_2$                  | $4.6 \pm 0.2$ | $\delta a_2$ $164^\circ \pm 5^\circ$ |
| $a_3$                  | $8.2 \pm 0.3$ | $\delta a_3$ $135^\circ \pm 5^\circ$ |
| $a_4$                  | $2.9 \pm 0.4$ | $\delta a_4$ $-30^\circ \pm 13^\circ$ |
| $a_5$                  | $15.4 \pm 1.4$ | $\delta a_5$ $24^\circ \pm 3^\circ$ |
| $a_6$                  | $4.2 \pm 0.2$ | $\delta a_6$ $120^\circ \pm 10^\circ$ |

To determine the $SU(3)_f$ parameters, we use the non-resonant parts of $\Lambda^+_c \to pK^-\pi^+$ from the PDG [3]. Note that the resonant $\Lambda^+_c \to p(K^{*0} \to K^-\pi^+)$, $K^- (\Delta(1232)^{++} \to p\pi^+)$ and $\pi^+(\Lambda(1520) \to pK^-)$ contributions are separated from its total branching ratio. In addition, the decay of $\Lambda^+_c \to pK^-K^+$ is free from the resonant one of $\Lambda^+_c \to p(\phi \to K^-K^+)$. For the other $\Lambda^+_c$ decays in Table IV, some of their resonant parts might be present, but taken to be small, such as $B(\Lambda^+_c \to \Sigma^+(\rho^0 \to \pi^+\pi^-)) < 1.7\%$ [5], which should be insensitive to the fit. We hence use their total branching ratios, instead of excluding the resonant contributions. The $\Xi^0_c \to B_n M M'$ decays are partially observed, such that we can barely use their data. Nonetheless, in terms of $T(\Lambda^+_c \to \Xi^- K^+\pi^+) = 1/(-2t_c)T(\Xi^+_c \to \Sigma^-\pi^+\pi^+) = -2a_6$ and the data of $B(\Lambda^+_c \to \Xi^- K^+\pi^+)$, we obtain $B(\Xi^+_c \to \Sigma^-\pi^+\pi^+) = (1.1 \pm 0.1) \times 10^{-2}$, by which the observed ratio of $B(\Xi^+_c \to \Sigma^-\pi^+\pi^+)/B(\Xi^+_c \to \Xi^-\pi^+\pi^+) = 0.18 \pm 0.09$ and it leads to $B(\Xi^+_c \to \Xi^-\pi^+\pi^+) = (6.1 \pm 3.1) \times 10^{-2}$ as given in Table IV.

With $\chi^2/d.o.f$ being 2.8 in Eq. (V), it turns out to be a reasonable fit, so that the $SU(3)_f$ symmetry with the reduced parameters can be used to explain the three-body $B_c \to B_n M M'$ decays. The relations of $T(\Lambda^+_c \to n\bar{K}^0\pi^+) = T(\Lambda^+_c \to \Sigma^0\pi^0\pi^+)$ and $T(\Lambda^+_c \to \Sigma^0\pi^0\pi^+) = T(\Lambda^+_c \to \Sigma^-\pi^+\pi^+)/2$ yield

$$B(\Lambda^+_c \to n\bar{K}^0\pi^+) \simeq B(\Lambda^+_c \to \Sigma^0\pi^0\pi^+) \simeq \frac{1}{2}B(\Lambda^+_c \to \Sigma^-\pi^+\pi^+),$$

which agrees with our numerical analysis. Note that the calculation of $B(\Lambda^+_c \to \Sigma^-\pi^+\pi^+)$ needs an additional pre-factor of $1/2$ to $T(\Lambda^+_c \to \Sigma^-\pi^+\pi^+)$ due to the fact that the $\pi^+\pi^+$ meson-pair involves two identical bosons.

From $B(\Lambda^+_c \to pK^+\pi^-)/B(\Lambda^+_c \to pK^-\pi^+) = \tan^4 \theta_c$, we find that $R_{K\pi} = 1.1 \pm 0.3$ in the fit without the resonant part. The ratio of $R_{K\pi} \sim 1$ is related to the same topological
diagrams. Note that the experimental data of $\mathcal{R}_{K\pi}^{Exp} = 0.58 \pm 0.06$ by LHCb \cite{10} has been obtained by including the resonant contributions in $\Lambda_c^+ \rightarrow pK^-\pi^+$. As the prediction from the lowest-wave contributions, $\mathcal{B}(\Lambda_c^+ \rightarrow n\pi^+\bar{K}^0, p\bar{K}^0\pi^0) = (0.9 \pm 0.8, 2.8 \pm 0.6) \times 10^{-2}$ are smaller than the data of $(3.6 \pm 0.6, 4.0 \pm 0.3) \times 10^{-2}$ \cite{4, 5}, which indicate that the resonant and/or high-wave contributions have not been clearly identified yet.

There exist the sum rules for the T-amplitudes in Table I. In particular, by taking the CF $\Lambda_c^+$ decay modes as an example, we obtain

$$R(\Delta) \equiv T(\Lambda_c^+ \rightarrow n\bar{K}^0\pi^+) - T(\Lambda_c^+ \rightarrow pK^-\pi^+) - \sqrt{2}T(\Lambda_c^+ \rightarrow p\bar{K}^0\pi^0) = 0 ,$$

$$T(\Lambda_c^+ \rightarrow \Sigma^+\bar{\pi}^0\pi^0) - T(\Lambda_c^+ \rightarrow \Sigma^+\pi^+\pi^+) + \frac{1}{2}T(\Lambda_c^+ \rightarrow \Sigma^-\pi^+\pi^+) = 0 ,$$

$$T(\Lambda_c^+ \rightarrow \Sigma^+K^0\bar{K}^0) - T(\Lambda_c^+ \rightarrow \Sigma^+K^+K^-) - \sqrt{2}T(\Lambda_c^+ \rightarrow \Sigma^0K^+\bar{K}^0) = 0 ,$$

$$T(\Lambda_c^+ \rightarrow \Xi^0\pi^+K^0) - T(\Lambda_c^+ \rightarrow \Xi^-\pi^+K^+) - \sqrt{2}T(\Lambda_c^+ \rightarrow \Xi^0\pi^0K^+) = 0 . \quad (12)$$

Note that the first relation of $R(\Delta)$ in Eq. (12), which has been used in Ref. \cite{4} to reveal the broken isospin symmetry, is also derived by the isospin symmetry in Refs. \cite{6, 7} with some different signs in the relation due to the conventions of the $\pi^+$ and $\bar{K}^0$ states. In addition, the second relation in Eq. (12) can be identified as the special case in Ref. \cite{7}, given by

$$T(\Lambda_c^+ \rightarrow \Sigma^+\bar{\pi}^0\pi^0) - T_{sym}(\Lambda_c^+ \rightarrow \Sigma^+\pi^+\pi^+) + \frac{1}{2}T(\Lambda_c^+ \rightarrow \Sigma^-\pi^+\pi^+) = 0 , \quad (13)$$

with the symmetrized amplitude of

$$T_{sym}(\Lambda_c^+ \rightarrow \Sigma^+\pi^+\pi^-) = \frac{1}{2} \left[T'(\Lambda_c^+ \rightarrow \Sigma^+\pi^+\pi^-) + T'(\Lambda_c^+ \rightarrow \Sigma^+\pi^-\pi^+) \right] \quad (14)$$

where $T'(\Lambda_c^+ \rightarrow \Sigma^+\pi^+\pi^-)$ are the amplitudes calculated by the isospin analysis in Ref. \cite{7}. Likewise, one can take the relations in Eq. (12) to explore the broken $SU(3)_f$ symmetry.

There are other relations and sum rules obtained from the U-spin symmetry, which is also a subgroup of $SU(3)_f$ \cite{8}.

The not-yet-observed $\mathcal{B}(\Lambda_c^+ \rightarrow B_nMM')$ can be calculated by the $SU(3)_f$ parameters, which are given in Table VII. The branching ratios of the three-body $\Xi_{c^+}^{0}$ decays are partially observed, such that we predict $\mathcal{B}(\Xi_{c^+}^{0} \rightarrow B_nMM')$ in Tables VII and VIII, respectively, to be compared to the upcoming data.

\footnote{There is also a sign issue for the U-spin quantum state in Ref. \cite{8}.}
TABLE VI. Numerical results for the branching ratios of $\Lambda_c^+ \to B_n MM'$, where $B_{BnMM'} \equiv \mathcal{B}(\Lambda_c^+ \to B_n MM')$.

| CF mode | our result | CS mode | our result | DCS mode | our result |
|---------|------------|---------|------------|----------|------------|
| $10^2 B_{\Sigma^+ \pi^0 \eta^0}$ | $3.5 \pm 0.8$ | $10^4 B_{\Sigma^+ \pi^0 K^0}$ | $8.6 \pm 2.6$ | $10^6 B_{\Sigma^+ K^0 \eta^0}$ | $2.0 \pm 0.5$ |
| $10^3 B_{\Sigma^+ K^0 \bar{K}^0}$ | $5.2 \pm 1.2$ | $10^5 B_{\Sigma^+ K^0 \eta^0}$ | $3.5 \pm 0.4$ | $10^6 B_{\Sigma^0 K^0 \eta^0}$ | $2.0 \pm 0.6$ |
| $10^3 B_{\Sigma^+ + K^+ K^-}$ | $3.0 \pm 0.7$ | $10^5 B_{\Sigma^+ K^0 \eta^0}$ | $1.2 \pm 0.3$ | $10^6 B_{\Sigma^+ K^0 \eta^0}$ | $2.0 \pm 0.5$ |
| $10^7 B_{\Sigma^+ \rho^0 \eta^0}$ | $2.8 \pm 0.6$ | $10^4 B_{\Sigma^0 K^+ K^-}$ | $8.3 \pm 2.5$ | $10^5 B_{\pi^0 \eta^0 \bar{K}^0}$ | $5.0 \pm 0.5$ |
| $10^6 B_{\Sigma^0 \pi^+ K^-}$ | $3.4 \pm 0.8$ | $10^5 B_{\Sigma^0 K^0 \eta^0}$ | $1.8 \pm 0.2$ | $10^5 B_{\pi^0 \eta^0 \bar{K}^0}$ | $5.0 \pm 0.5$ |
| $10^6 B_{\Sigma^0 \pi^+ K^-}$ | $0.5 \pm 0.1$ | $10^4 B_{\Sigma^- \pi^+ K^-}$ | $3.3 \pm 2.3$ | $10^4 B_{\pi^+ \eta^0 \bar{K}^0}$ | $1.0 \pm 0.1$ |
| $10^6 B_{\bar{K}^0 K^0}$ | $4.5 \pm 0.8$ | $10^6 B_{\bar{K}^0 \eta^0 \bar{K}^0}$ | $2.4 \pm 0.8$ | $10^6 B_{\bar{K}^0 \eta^0 \bar{K}^0}$ | $2.4 \pm 0.8$ |
| $10^6 B_{\bar{K}^0 \pi^0 K^0}$ | $8.7 \pm 1.7$ | $10^6 B_{\bar{K}^0 \pi^0 K^0}$ | $3.7 \pm 0.9$ | $10^6 B_{\bar{K}^0 \pi^0 K^0}$ | $3.7 \pm 0.9$ |
| $10^6 B_{\rho^0 \pi^0 \bar{K}^0}$ | $2.8 \pm 0.6$ | $10^6 B_{\rho^0 \pi^0 \bar{K}^0}$ | $4.3 \pm 1.0$ | $10^6 B_{\rho^0 \pi^0 \bar{K}^0}$ | $4.3 \pm 1.0$ |
| $10^6 B_{\pi^+ \eta^0 \bar{K}^0}$ | $0.9 \pm 0.8$ | $10^6 B_{\pi^+ \eta^0 \bar{K}^0}$ | $4.7 \pm 1.0$ | $10^6 B_{\pi^+ \eta^0 \bar{K}^0}$ | $4.7 \pm 1.0$ |
| $10^6 B_{\eta^0 \pi^0 K^0}$ | $7.3 \pm 1.8$ | $10^6 B_{\eta^0 \pi^0 K^0}$ | $5.9 \pm 1.3$ | $10^6 B_{\eta^0 \pi^0 K^0}$ | $5.9 \pm 1.3$ |
| $10^6 B_{\eta^0 \pi^0 K^0}$ | $4.5 \pm 0.8$ | $10^6 B_{\eta^0 \pi^0 K^0}$ | $8.8 \pm 1.5$ | $10^6 B_{\eta^0 \pi^0 K^0}$ | $8.8 \pm 1.5$ |
| $10^6 B_{\Lambda^0 \pi^0 K^0}$ | $1.9 \pm 0.6$ | $10^6 B_{\Lambda^0 \pi^0 K^0}$ | $1.9 \pm 0.6$ | $10^6 B_{\Lambda^0 \pi^0 K^0}$ | $1.9 \pm 0.6$ |

IV. CONCLUSIONS

We have studied the three-body anti-triplet $B_c \to B_n MM'$ decays in the approach of the $SU(3)_f$ symmetry. In our analysis, we have only concentrated on the S-wave $MM'$-pair contributions, so that the decays of $B_c \to B_n MM'$ can be decomposed into irreducible forms with 11 parameters under $SU(3)_f$. With the minimum $\chi^2$ fit to the 14 existing data points, we have obtained a reasonable value of $\chi^2/d.o.f = 2.8$. With our numerical results, we have shown the same triangle relation of $\mathcal{A}(\Lambda_c^+ \to n \bar{K}^0 \pi^0) - \mathcal{A}(\Lambda_c^+ \to p K^- \pi^+) - \sqrt{2} \mathcal{A}(\Lambda_c^+ \to p \bar{K}^0 \pi^0) = 0$ under $SU(3)_f$ as that based on the isospin symmetry. In addition, for the CF decays, we have obtained the sum rules of $\mathcal{A}(\Lambda_c^+ \to \Sigma^+ \pi^0 \pi^0) - \mathcal{A}(\Lambda_c^+ \to \Sigma^+ \pi^0 \pi^+) + 1/2 \mathcal{A}(\Lambda_c^+ \to \Sigma^- \pi^+ \pi^+) = 0$, $\mathcal{A}(\Lambda_c^+ \to \Sigma^+ K^0 \bar{K}^0) - \mathcal{A}(\Lambda_c^+ \to \Sigma^+ K^+ K^-) - \sqrt{2} \mathcal{A}(\Lambda_c^+ \to \Sigma^0 K^+ \bar{K}^0) = 0$ and $\mathcal{A}(\Lambda_c^+ \to \Xi^0 \pi^0 K^0) - \mathcal{A}(\Lambda_c^+ \to \Xi^- \pi^+ K^+) - \sqrt{2} \mathcal{A}(\Lambda_c^+ \to \Xi^0 \pi^0 K^+) = 0$.  

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TABLE VII. Numerical results for the branching ratios of $\Xi_c^+ \to B_nMM'$, where $B_{B_nMM'} \equiv B(\Xi_c^+ \to B_nMM')$.

| CF mode  | our result | CS mode  | our result | DCS mode | our result |
|----------|------------|----------|------------|----------|------------|
| $10^3 B_{\Sigma^{+}0\bar{K}^0}$ | 5.4 ± 4.0 | $10^3 B_{\Sigma^{+}0\eta^0}$ | 9.6 ± 1.8 | $10^4 B_{\Sigma^{+}0K^0}$ | 2.6 ± 0.2 |
| $10^2 B_{\Sigma^{+}0\bar{K}^0}$ | 6.1 ± 0.6 | $10^3 B_{\Sigma^{+}0\pi^-}$ | 5.1 ± 2.0 | $10^4 B_{\Sigma^{+}0\pi^+}$ | 1.4 ± 0.3 |
| $10^2 B_{\Sigma^{+}0\pi^-}$ | 4.6 ± 0.6 | $10^3 B_{\Sigma^{+}K^0K^0}$ | 5.4 ± 1.3 | $10^6 B_{\Sigma^{+}K^0\eta^0}$ | 2.0 ± 1.4 |
| $10^3 B_{\Sigma^0\pi^+\bar{K}^0}$ | 1.2 ± 0.3 | $10^3 B_{\Sigma^0\pi^+\bar{K}^0}$ | 1.0 ± 0.4 | $10^6 B_{\Sigma^0\pi^0\bar{K}^0}$ | 7.6 ± 5.9 |
| $10^3 B_{\Xi^{0}0\eta^0}$ | 1.9 ± 0.5 | $10^3 B_{\Xi^{0}0\eta^0}$ | 1.8 ± 1.0 | $10^4 B_{\Xi^{0}0\pi^0\eta^0}$ | 2.5 ± 0.2 |
| $10^3 B_{\Xi^{0}0\pi^0\eta^0}$ | 1.0 ± 0.2 | $10^3 B_{\Xi^{0}0\pi^0\eta^0}$ | 5.6 ± 0.5 | $10^6 B_{\Xi^{0}0\pi^0\eta^0}$ | 1.0 ± 0.7 |
| $10^3 B_{\Xi^{0}0\pi^0\eta^0}$ | 4.9 ± 0.5 | $10^3 B_{\Xi^{0}0\pi^0\eta^0}$ | 9.4 ± 1.8 | $10^4 B_{\Xi^{0}0\pi^0\eta^0}$ | 1.3 ± 0.1 |
| $10^3 B_{\Xi^{0}0\pi^0\eta^0}$ | 4.3 ± 1.2 | $10^3 B_{\Xi^{0}0\pi^0\eta^0}$ | 4.4 ± 0.9 | $10^6 B_{\Xi^{0}0\pi^0\eta^0}$ | 3.0 ± 1.9 |
| $10^3 B_{\Lambda^{0}0\pi^0\bar{K}^0}$ | 4.6 ± 1.2 | $10^3 B_{\Lambda^{0}0\pi^0\bar{K}^0}$ | 1.1 ± 0.1 | $10^6 B_{\Xi^{0}0\pi^0\eta^0}$ | 5.7 ± 3.2 |
| $10^3 B_{\Xi^{0}0\pi^0\eta^0}$ | 6.4 ± 1.6 | $10^3 B_{\Xi^{0}0\pi^0\eta^0}$ | 7.2 ± 1.8 | $10^4 B_{\Xi^{0}0\pi^0\eta^0}$ | 1.1 ± 0.2 |
| $10^3 B_{\Xi^{0}0\pi^0\eta^0}$ | 1.9 ± 0.4 | $10^3 B_{\Xi^{0}0\pi^0\eta^0}$ | 1.3 ± 0.3 | $10^3 B_{\Xi^{0}0\pi^0\eta^0}$ | 1.4 ± 0.4 |
| $10^3 B_{\Xi^{0}0\pi^0\eta^0}$ | 1.3 ± 0.3 | $10^3 B_{\Xi^{0}0\pi^0\eta^0}$ | 8.3 ± 5.3 | $10^4 B_{\Xi^{0}0\pi^0\eta^0}$ | 7.7 ± 1.7 |
| $10^3 B_{\Xi^{0}0\pi^0\eta^0}$ | 2.4 ± 0.2 | $10^3 B_{\Xi^{0}0\pi^0\eta^0}$ | 1.6 ± 1.2 | $10^4 B_{\Xi^{0}0\pi^0\eta^0}$ | 9.3 ± 4.5 |
| $10^3 B_{\Xi^{0}0\pi^0\eta^0}$ | 2.4 ± 0.3 | $10^3 B_{\Xi^{0}0\pi^0\eta^0}$ | 2.1 ± 0.4 | $10^3 B_{\Xi^{0}0\pi^0\eta^0}$ | 1.6 ± 0.3 |
| $10^3 B_{\Xi^{0}0\pi^0\eta^0}$ | 5.5 ± 0.5 | $10^3 B_{\Xi^{0}0\pi^0\eta^0}$ | 4.7 ± 1.0 | $10^3 B_{\Xi^{0}0\pi^0\eta^0}$ | 5.0 ± 1.0 |
| $10^3 B_{\Xi^{0}0\pi^0\eta^0}$ | 1.7 ± 0.3 | $10^3 B_{\Xi^{0}0\pi^0\eta^0}$ | 9.7 ± 2.0 | $10^3 B_{\Xi^{0}0\pi^0\eta^0}$ | 9.0 ± 2.2 |
TABLE VIII. Numerical results for the branching ratios of $\Xi_c^0 \to B_n M M'$, where $B_{B_n M M'} \equiv B(\Xi_c^0 \to B_n M M')$.

| CF mode          | our result | CS mode          | our result | DCS mode | our result |
|------------------|------------|------------------|------------|----------|------------|
| $10^2 B_{\Sigma^+ \pi^0 K^-}$ | 8.8 ± 1.5  | $10^4 B_{\Sigma^+ \pi^0 \pi^-}$ | 7.2 ± 0.7  | $10^6 B_{\Sigma^+ \pi^- K^0}$ | 3.4 ± 0.3  |
| $10^1 B_{\Sigma^+ \pi^- K^0}$ | 1.8 ± 0.3  | $10^3 B_{\Sigma^+ \pi^- \eta}$ | 5.7 ± 0.9  | $10^6 B_{\Sigma^0 \pi^- K^+}$ | 6.5 ± 0.5  |
| $10^3 B_{\Sigma^+ K^- \eta}$ | 5.2 ± 0.9  | $10^3 B_{\Sigma^+ K^0 K^+}$ | 2.4 ± 0.4  | $10^7 B_{\Sigma^0 K^0 \eta}$ | 2.6 ± 1.7  |
| $10^2 B_{\Sigma^0 \pi^0 K^0}$ | 4.4 ± 1.1  | $10^3 B_{\Sigma^0 \pi^0 \pi^0}$ | 1.3 ± 0.3  | $10^5 B_{\Sigma^- \pi^0 K^+}$ | 6.4 ± 0.5  |
| $10^2 B_{\Sigma^0 \pi^+ +}$ | 5.4 ± 1.2  | $10^3 B_{\Sigma^0 \pi^0 \eta}$ | 1.9 ± 0.4  | $10^5 B_{\Sigma^- \pi^+ K^0}$ | 3.4 ± 0.7  |
| $10^3 B_{\Sigma^0 K^0 \eta}$ | 1.4 ± 0.3  | $10^4 B_{\Sigma^- \pi^0 \pi^+}$ | 9.7 ± 1.7  | $10^7 B_{\Sigma^- K^+ \eta}$ | 5.1 ± 3.4  |
| $10^2 B_{\Xi^0 \pi^0 \pi^0}$ | 8.1 ± 1.9  | $10^5 B_{\Xi^0 \pi^0 \eta}$ | 2.3 ± 1.2  | $10^6 B_{\Xi^0 \pi^0 K^0}$ | 1.5 ± 1.1  |
| $10^2 B_{\Xi^- \pi^0 \pi}$ | 1.2 ± 0.2  | $10^4 B_{\Xi^- \pi^0 \pi^+}$ | 7.1 ± 0.6  | $10^7 B_{\Xi^- K^0 \pi^0}$ | 7.1 ± 6.7  |
| $10^1 B_{\Xi^0 \pi^0 \pi^-}$ | 1.3 ± 0.3  | $10^4 B_{\Xi^- \pi^- \pi^0}$ | 6.3 ± 2.0  | $10^4 B_{\rho \pi^- \eta}$ | 5.4 ± 0.9  |
| $10^3 B_{\Xi^0 K^+ K^-}$ | 3.6 ± 0.9  | $10^4 B_{\Xi^- K^- K^0}$ | 2.9 ± 0.6  | $10^4 B_{\rho K^0 \pi^-}$ | 4.2 ± 0.7  |
| $10^4 B_{\Xi^0 \rho \rho \eta}$ | 2.2 ± 0.9  | $10^3 B_{\Xi^0 \rho \rho K^0}$ | 3.0 ± 0.7  | $10^4 B_{n \pi^0 \eta}$ | 1.8 ± 0.5  |
| $10^3 B_{\Xi^- \pi^0 \pi^-}$ | 4.6 ± 1.2  | $10^3 B_{\Xi^- \pi^- \pi^0}$ | 4.8 ± 0.9  | $10^4 B_{n \pi^0 \eta}$ | 2.7 ± 0.5  |
| $10^2 B_{\Xi^- \pi^- \pi^-}$ | 1.1 ± 0.1  | $10^4 B_{\Xi^- \pi^- \pi^0}$ | 6.2 ± 1.3  | $10^4 B_{n \pi^+ \pi^-}$ | 3.6 ± 0.9  |
| $10^2 B_{\rho \pi^- K^0}$ | 1.2 ± 0.1  | $10^4 B_{\Xi^- \pi^- \pi^0}$ | 7.2 ± 1.5  | $10^5 B_{n K^0 \pi^0}$ | 3.9 ± 2.9  |
| $10^3 B_{n K^0 \eta}$ | 6.4 ± 6.3  | $10^3 B_{\rho \pi \rho K^-}$ | 9.5 ± 1.6  | $10^4 B_{n K^+ K^-}$ | 2.0 ± 0.5  |
| $10^2 B_{\Lambda^0 \pi^0 K^0}$ | 2.0 ± 0.6  | $10^2 B_{\rho \pi \rho K^0}$ | 1.9 ± 0.3  | $10^5 B_{n \pi \rho \eta}$ | 2.4 ± 1.2  |
| $10^2 B_{\Lambda^0 \pi^+ K^-}$ | 5.9 ± 0.8  | $10^3 B_{\rho \pi \rho K^-}$ | 1.8 ± 0.3  | $10^4 B_{\Lambda^0 \pi^0 \pi^0}$ | 1.3 ± 0.3  |
| $10^3 B_{n \pi \rho \eta}$ | 5.2 ± 1.3  | $10^4 B_{\Lambda^0 \pi^0 K^0}$ | 1.3 ± 0.3  | $10^3 B_{\Lambda^0 \pi^- K^+}$ | 2.5 ± 0.5  |
| $10^2 B_{n \pi^+ K^-}$ | 1.5 ± 0.3  | $10^5 B_{\Lambda^0 K^0 \eta}$ | 2.3 ± 0.6  | $10^3 B_{n \pi \rho \eta}$ | 1.9 ± 0.6  |
| $10^3 B_{\Lambda^0 \pi^0 \eta}$ | 5.3 ± 1.5  | $10^3 B_{\Lambda^0 \pi^0 \pi}$ | 2.2 ± 0.4  | $10^2 B_{\Lambda^0 \pi^0 \pi^0}$ | 1.1 ± 0.3  |
| $10^3 B_{\Lambda^0 \pi^+ \pi^-}$ | 3.0 ± 2.5  | $10^4 B_{\Lambda^0 K^+ K^0}$ | 2.4 ± 1.4  | $10^4 B_{\Lambda^0 \rho \rho \eta}$ | 2.4 ± 1.4  |
Furthermore, we have predicted that $\mathcal{B}(\Lambda_c^+ \rightarrow n\pi^+K^0) = (0.9 \pm 0.8) \times 10^{-2}$, which is (3-4) times smaller than the BESIII observation of $(3.6 \pm 0.6) \times 10^{-2}$. This indicates that there are some contributions from the resonant and/or P-wave states. For the to-be-observed $\Lambda_c^+ \rightarrow B_nMM'$ and the partial observed $\Xi_c^0 \rightarrow B_nMM'$ decays, the branching ratios have been calculated with the $SU(3)_f$ amplitudes, to be compared to the future measurements by BESIII and LHCb.

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