Examples of $D = 11$ S-supersymmetric actions for point-like dynamical systems.

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Abstract

A non standard super extensions of the Poincare algebra (S-algebra [1,2]), which seems to be relevant for construction of various $D = 11$ models, are studied. We present two examples of actions for point-like dynamical systems, which are invariant under off-shell closed realization of the S-algebra as well as under local fermionic $\kappa$-symmetry. On this ground, an explicit form of the S-algebra is advocated.

1 Introduction

The construction of higher-dimensional ($D > 10$) SYM [3,4] and superstring [5,6,2,7,8] models, which might be interesting in the M-theory context (see [9-14] and references therein), is under intensive investigation at present. It is known that consistency of the super Poincare and local symmetry transformations imply rigid restrictions on possible dimensions of

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1 We mean gauge transformations for the SYM-theory and local $\kappa$-symmetry transformations for the case of superstring.
space-time where these models can be formulated \([15,9]\). In particular, the standard methods can not be directly applied in \(D > 10\) to construct the above mentioned super Poincare invariant models. One possibility to avoid these restrictions is to consider some different super extensions of the Poincare algebra. In recent works \([1-4,6,8,16,17]\) a relevant higher-dimensional superalgebra was discussed. It includes the Poincare generators as well as generators \(Q\) of new supertranslations with commutator may be written in the form

\[
\{Q_\alpha, Q_\beta\} \sim \Gamma^{\mu\nu} P_\mu n_\nu, \tag{1}
\]

where \(\Gamma^{\mu\nu}\) is antisymmetric product of \(D = 11\) \(\gamma\)-matrices (we use \(\gamma\)-matrix conventions from \([8]\)). It is known as S-algebra previously discussed in the M-theory context \([1]\) (see \([18]\) for discussion of a general case). For \(D = 11\) case it can be realized in a superspace as follows:

\[
\delta \theta = \epsilon, \quad \delta x^\mu = i(\bar{\epsilon} \Gamma^{\mu\nu} \theta) n_\nu, \quad \delta n^\mu = 0. \tag{2}
\]

The appearance of a new variable \(n^\mu\) seems to be an essential property of the construction (see discussion in \([7,8]\)). In this relation it is interesting to clarify the role of the variable \(n^\mu\) from the dynamical point of view, in particular, to present some examples of Lagrangian systems with \(n^\mu\) incorporated on equal footing with other variables. Only in this case the corresponding theory can be actually SO(1,10) invariant.

It was also pointed out \([2,8]\) that after substitution \(n^\mu = (0, \cdots 0, 1)\) (which breaks SO(1,10) covariance up to SO(1,9) one) the transformations

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2 In recent works \([5]\) \(D = 11\) superstring action with second-class constraints simulating a gauge fixation for the \(\kappa\)-symmetry was suggested. The action was constructed by adding of an appropriately chosen terms to the GS action written in \(D = 11\). Supersymmetry of quantum state spectrum for the model is under investigation now.

3 As it will be demonstrated below (see also Ref.\([2,8]\)) an explicit form of the algebra is \(\{Q, Q\} \sim \Gamma^{\mu\nu} Z_{\mu\nu}\), with some additional bosonic generators \(Z_{\mu\nu}\).
(2) reduce to
\[ \delta \theta^\alpha = \epsilon^\alpha, \quad \delta \bar{\theta}_\alpha = \bar{\epsilon}_\alpha, \]
\[ \delta x^{\bar{\mu}} = -i \bar{\epsilon}_\alpha \bar{\Gamma}^{\bar{\mu}}{}_{\alpha\beta} \bar{\theta}^\beta - i \epsilon^\alpha \Gamma_{\alpha\beta} \theta^\beta, \quad \delta x^{10} = 0, \]
where \( \theta = (\bar{\theta}_\alpha, \theta^\alpha), \mu = (\bar{\mu}, 10), \bar{\mu} = 0, 1, \ldots, 9, \alpha = 1 \ldots 16. \) One can see that (3) coincides exactly with the standard \( D = 10, \) type IIA supersymmetry transformations. In this sense the latter can be rewritten in a manifestly SO(1,10) covariant notations (2). Thus, it is naturally to ask about possibility of lifting the known \( D = 10 \) type IIA theories up to SO(1,10) invariant form. From the present discussion it is clear that the requirement of S-invariance instead of the super Poincare invariance might be a natural framework for construction of such a kind \( D = 11 \) formulations.

In this letter we present two examples of \( D = 11 \) finite-dimensional systems based on the S-algebra of global symmetries. For the first model the variable \( n^{\mu} \) survives in the sector of physical degrees of freedom, while for the second one it turns out to be a nondynamical variable, which may be killed by a proper gauge fixing. It will be also demonstrated, that local \( \kappa \)-symmetry is consistent with global S-invariance in both cases.

The first example which we are going to study is in fact zero-tension limit of the \( D = 11 \) superstring action suggested in [8]. Physical degrees of freedom for the mechanical model may be considered as describing a composite system, the latter consists of a free moving particle and a super-particle (see also Refs.[6,16,17]). We present a Lagrangian action, which is invariant under local \( \kappa \)-symmetry as well as under off-shell closed realization of the S-algebra of global symmetries. The advantage of the present formulation (in comparison with [3,4,6,16,17]) is that an explicit Lagrangian action, with all the variables treated on equal footing is given. In particular, global symmetry transformations of the action form a super-
algebra in the usual sense, without appearance of nonlinear in generators terms in the right hand side of Eq.(1). In the result, a model-independent form of the S-algebra is presented.

From the discussion related to (2),(3) it is clear that a formulation where one may impose the gauge \( n^\mu = (0, \cdots, 0, 1) \) would be at most preferable. As a second example, we present S-invariant model, which admits such a gauge, and which describes the propagation of a superparticle only. We hope that a similar construction may work for the case of \( D = 11 \) superstring as well.

The work is organized as follows. In the Sec.2 we present and discuss a \( D = 11 \) Poincare invariant action for the above mentioned composite system. In the Sec.3, a bosonic action which contains the nondynamical variable \( n^\mu(\tau) \) related to S-symmetry is proposed. It is shown that the action describes a free propagating massless particle. On the base of this action S-supersymmetric version in \( D = 11 \) space-time is constructed in Sec.4. The latter action is invariant also under local fermionic \( \kappa \)-symmetry. Similarly to the Casalbuoni-Brink-Schwarz superparticle [19-21] it provides a free character of the dynamics for the physical sector variables.

2 D=11 composite system of a particle and a super-particle.

Let us consider the following \( D = 11 \) Lagrangian action

\[
S = \int d\tau \left\{ v_\mu \Pi^\mu - \frac{1}{2} e v^2 + n_\mu \dot{z}^\mu - \frac{1}{2} \phi (n^2 + 1) \right\},
\]

\[
\Pi^\mu \equiv \dot{x}^\mu - i (\bar{\theta} \Gamma^\mu \theta) n^\nu - \xi n^\mu,
\]

where \( x^\mu, v^\mu, z^\mu, n^\mu, e, \phi, \xi \) are Grassmann even and \( \theta^\alpha \) are Grassmann odd variables, dependent on the evolution parameter \( \tau \). The action is a direct
mechanical analog of the $D = 11$ superstring suggested in [8]. Note [8] that eliminating the variable $v^\mu$ we can rewrite (4) in the second-order form relative to $x^\mu$. Global bosonic symmetries of the action are both $D = 11$ Poincare transformations (with the variable $n^\mu$ being inert under the Poincare shifts) and the following transformations

$$
\delta_b x^\mu = b^\mu_\nu n^\nu, \quad \delta_b z^\mu = -b^\mu_\nu v^\nu,
$$

(5)

with antisymmetric parameters $b^{\mu\nu} = -b^{\nu\mu}$. There is also a global symmetry with fermionic parameters $\epsilon^\alpha$,

$$
\delta_\epsilon \theta = \epsilon, \quad \delta_\epsilon x^\mu = -i(\bar{\theta} \Gamma^{\mu\nu} \epsilon)n^\nu, \quad \delta_\epsilon z^\mu = i(\bar{\theta} \Gamma^{\mu\nu} \epsilon)v^\nu.
$$

(6)

The algebra of the corresponding commutators turns out to be off-shell closed. Thus, the S-algebra consist of Poincare subalgebra $(M^{\mu\nu}, P^\mu)$ and includes generators of new supertranslations $Q_\alpha$ as well as second-rank Lorentz tensor $Z^{\mu\nu}$. The nonzero commutators of the new generators are

$$
\{Q_\alpha, Q_\beta\} = 2i(\Gamma^{\mu\nu})_{\alpha\beta}Z^{\mu\nu}.
$$

(7)

Their commutators with the Poincare transformations have the standard form. Note, that it is not a modification of the super Poincare algebra but essentially different one, since the commutator of the supertranslations leads to $Z$-transformation instead of the Poincare shift.

The action (4) is also invariant under the local $\kappa$-symmetry transformations,

$$
\delta \theta = v^\mu \Gamma^\mu_\kappa, \quad \delta \xi = -2i(\bar{\theta} \delta \theta), \quad \delta e = 4i(\bar{\theta} \Gamma^\mu_\kappa)n^\mu.
$$

$$
\delta x^\mu = i(\bar{\theta} \Gamma^{\mu\nu} \delta \theta)n^\nu, \quad \delta z^\mu = -i(\bar{\theta} \Gamma^{\mu\nu} \delta \theta)v^\nu,
$$

(8)

This fact turns out to be crucial to verify that physical sector variables obey free equations of motion. Let us present the corresponding analysis

\footnote{S-algebra can be off-shell closed also for the action (4) written in the second order form [8]}
in the Hamiltonian framework [22,23]. One finds the following trivial pairs of second-class constraints: \( p_\mu^\mu = 0, \ p_\nu^\mu - n^\mu = 0; \ p_\nu^\mu = 0, \ p^\mu - v^\mu = 0, \) among primary constraints of the theory (\( p_\mu \) is conjugated momenta for \( x^\mu, \) and momenta, conjugated to all the other configuration space variables \( q^i \) are denoted as \( p_{qi} \)). Then the canonical pairs \((n^\mu, p_{n\mu}), (v^\mu, p_{v\mu})\) can be omitted after introducing the associated Dirac bracket. Dirac brackets for the remaining variables coincide with Poisson ones [23] and the total Hamiltonian have the form

\[
H^{(1)} = \frac{1}{2} e p^2 + \xi (p p_z) + \frac{1}{2} (p_z^2 + 1) + \lambda_e p_e + \lambda_\phi p_\phi + \lambda_\xi p_\xi + (\bar{p}_\theta - i \bar{\theta} \Gamma^{\mu\nu} p_\mu p_{z\nu}) \lambda_\theta,
\]

where Lagrange multipliers corresponding to primary constraints are denoted as \( \lambda_* \). The complete set of constraints can be written in the following form:

\[
\begin{align*}
p_e &= 0, \quad p_\phi = 0, \quad p_\xi = 0; \quad (10.a) \\
p_z^2 &= -1, \quad (p p_z) = 0, \quad p^2 = 0; \quad (10.b) \\
L_\alpha &\equiv \bar{p}_{\theta\alpha} - i (\bar{\theta}' \Gamma^\mu)_{\alpha\mu} p_\mu = 0; \quad (10.c)
\end{align*}
\]

where \( \theta' \equiv p_{z\mu} \Gamma^\mu \theta \). The matrix of the Poisson brackets of the fermionic constraints

\[
\{L_\alpha, L_\beta\} = 2i (C \Gamma^{\mu\nu})_{\alpha\beta} p_\mu p_{z\nu}, \quad (11)
\]

is degenerated on the constraint surface as a consequence of the identity

\[
(\Gamma^{\mu\nu} p_\mu p_{z\nu})^2 = 4[(p p_z) - p^2 p_z^2] 1 = 0. \]

It means that half of the constraints are first-class. From the condition \( \{L_\alpha, H^{(1)}\} = 0 \) one finds equation which determine \( \lambda_\theta \)-multipliers,

\[
p_\mu \Gamma^\mu \lambda_\theta' = 0, \quad \lambda_\theta' \equiv p_{z\mu} \Gamma^\mu \lambda_\theta. \quad (12)
\]

Imposing the gauge conditions \( e = 1, \phi = 1, \xi = 0 \) to the first-class constraints (10.a), one can omits the canonical pairs \((e, p_e), (\phi, p_\phi), (\xi, p_\xi)\) from
the consideration. The dynamics of the remaining variables is governed by the equations

$$\dot{z}^\mu = p_z^\mu + i(\bar{\theta} \Gamma^{\mu\nu} \lambda_\theta) p_{\nu}, \quad \dot{p}_z^\mu = 0; \quad (13.a)$$

$$\dot{x}^\mu = p^\mu - i(\bar{\theta} \Gamma^{\mu\nu} \lambda_\theta) p_{z\nu}, \quad \dot{p}^\mu = 0; \quad (13.b)$$

$$\dot{\theta}^\alpha = -\lambda_\theta^\alpha, \quad \dot{\bar{p}}_{\theta\alpha} = 0. \quad (13.c)$$

As the next step we impose gauge conditions

$$\Gamma^\pm \theta' = 0 \quad (14)$$

to the first-class constraints which follow from the equations (10.c). By virtue of (12),(13.c) all $\lambda_\theta$-multipliers can be determined, $\lambda_\theta = 0$, and (13.a-c) are reduced to free equations of motion.

The resulting picture corresponds to zero-tension limit of the $D = 11$ superstring action from [8]. Physical degrees of freedom for the model (4) may be considered as describing a composite system. It consists of the bosonic $z^\mu$-particle (13.a) and the superparticle (13.b), (13.c), subject to the constraints (10.b). Both of them propagate freely except the kinematic constraint $(pp_z) = 0$, which means that the superparticle lives on $D = 10$ hyperplane orthogonal to the direction of motion of $z^\mu$-particle.

A few comments are in order. In the model considered variables $(z^\mu, p_z^\mu)$ describe a tachyon $p_z^2 = -1$. To avoid the problem, it was suggested in [6,16,17] to consider a target space of a non-standard signature $(2,9)$ with the metric $\eta^{\mu\nu} = (+, - \cdot \cdot \cdot, +)$. In such a space there is no of tachyon, but negative norm states appear in the model. Actually, four constraints are necessary to gauge out the undesirable components $x^0, x^{10}, z^0, z^{10}$. However, it is impossible to form four Poincare covariant constraints using only the variables $p^\mu, p_z^\mu$, which are in our disposal. This situation can be improved by considering of a modified action which describes a superparticle

Note that it make no of special problem for the case of $D = 11$ superstring [8]
and a pair of particles $z_i^\mu, i = 1, 2$. Using the corresponding conjugate momenta $p_\mu, p_{i\mu}$ six constraints can be formed, which allow one to gauge out the six components $x^0, x^{10} \equiv x^0_i, z_i^{10}$. We will not discuss such a construction in a more details, since our example considered in the next Sections do not have such problems.

3 \ SO(1,D-1)\times\SO(D-2)-invariant formulation for the bosonic particle.

In this Section we construct a free propagating bosonic particle action, which will be appropriate for our aims of supersymmetrization. Namely, it contains an auxiliary space-like variable $\pi_D^\mu$, for which the gauge $\pi_D^\mu = (0, \ldots, 0, 1)$ is possible. We start from the Poincare invariant action which describes D particles in D-dimensional space-time

$$S_0 = \int d\tau \left\{ \pi_{\bar{a}\mu} \dot{x}_{\bar{a}}^\mu - \frac{1}{2} \sum_{\bar{a} = 0}^{D-1} \phi_{\bar{a}} (\pi_{\bar{a}\mu} \pi_{\bar{a}}^\mu + c_{\bar{a}}^2) \right\}, \quad (15)$$

where $x_{\bar{a}}^\mu = (x_0^\mu \equiv x^\mu, x_{a}^\mu), a = 1, 2, \ldots, D - 1$, and the number $c_{\bar{a}}$ determines the mass of a particle with the index $\bar{a}$. Let us consider the problem of reducing a number of physical degrees of freedom for the model by means of a localization of a part of global symmetries presented in the action. First, we note that the following transformation (without sum on $\bar{a}, \bar{b}$):

$$\delta_\lambda x_{\bar{a}}^\mu = \lambda_{\bar{a}\bar{b}} \pi_{\bar{b}}^\mu, \quad \delta_\lambda x_{\bar{b}}^\mu = \lambda_{\bar{b}\bar{a}} \pi_{\bar{a}}^\mu, \quad \lambda_{\bar{a}\bar{b}} \equiv \lambda_{\bar{b}\bar{a}}, \quad (16)$$

is a global symmetry of the action for any fixed pair of indices $\bar{a} \neq \bar{b}$ (note that for $\bar{a} = \bar{b}$ the symmetry is the local one, with the variable $\phi_{\bar{a}}$ being a corresponding gauge field). In order to localize this transformation it is sufficient to covariantize the time derivatives: $\dot{x}_{\bar{a}}^\mu \rightarrow \dot{x}_{\bar{a}}^\mu - \frac{1}{2} \phi_{\bar{a}\bar{b}} \pi_{\bar{a}}^\mu, \quad \dot{x}_{\bar{b}}^\mu \rightarrow \dot{x}_{\bar{b}}^\mu - \frac{1}{2} \phi_{\bar{b}\bar{a}} \pi_{\bar{a}}^\mu$, where $\phi_{\bar{a}\bar{b}} \equiv \phi_{\bar{b}\bar{a}}$ is the corresponding “gauge field” with the
transformation low $\delta \lambda \phi_{\bar{a}\bar{b}} = \dot{\lambda}_{\bar{a}\bar{b}}$. It is useful to write the resulting locally invariant action in the form

$$S_1 = \int d\tau \left\{ \pi_{\bar{a}\mu} \dot{x}_{\bar{a}}^\mu - \frac{1}{2} \sum' \phi_{\bar{a}\bar{b}} (\pi_{\bar{a}\mu} \pi_{\bar{b}}^\mu + c_{\bar{a}}^2 \delta_{\bar{a}\bar{b}}) \right\}, \quad (17)$$

where the touch means that the sum includes those pairs of indices for which the corresponding symmetry was localized. In particular, if all the symmetries are localized, one has $D(D + 1)/2$ constraints and a number of physical degree of freedom for the model is equal to $D(D - 1)/2$. Note, that it coincides exactly with the number of Lorentz symmetry generators. Further reduction of the physical degree of freedom can be achieved by a localization of the Lorentz symmetry transformations,

$$\delta x_{\bar{a}}^\mu = \omega_{\mu\nu} x_{\bar{a}}^\nu, \quad \delta \pi_{\bar{a}}^\mu = \omega_{\mu\nu} \pi_{\bar{a}}^\nu. \quad (18)$$

By the covariantization of the derivatives, $\dot{x}_{\bar{a}}^\mu \rightarrow Dx_{\bar{a}}^\mu \equiv \dot{x}_{\bar{a}}^\mu - A_{\mu\nu} x_{\bar{a}}^\nu$, where $\delta A_{\mu\nu} = \dot{\omega}_{\mu\nu} + \omega_{\mu\rho} A_{\rho\nu} - A_{\mu\rho} \omega_{\rho\nu}$, one obtains the action

$$S_2 = \int d\tau \left\{ \pi_{\bar{a}\mu} Dx_{\bar{a}}^\mu - \frac{1}{2} \sum' \phi_{\bar{a}\bar{b}} (\pi_{\bar{a}\mu} \pi_{\bar{b}}^\mu + c_{\bar{a}}^2 \delta_{\bar{a}\bar{b}}) \right\}, \quad (19)$$

which does not contain of physical degree of freedom if the sum runs over all indices. To get a model with nontrivial dynamics, let us retain nonlocalized a part of symmetries (16). The following action will be appropriate for our aims

$$S_3 = \int d\tau \left\{ \pi_{\mu} Dx^\mu - \frac{1}{2} e \pi^2 - \xi (\pi_{\mu} \pi_D^\mu - 1) + \pi_{a\mu} Dx_{\bar{a}}^\mu - \frac{1}{2} \sum_{a,b=1}^{D-1} \phi_{ab} (\pi_{a\mu} \pi_{\bar{b}}^\mu + \delta_{ab}) \right\}. \quad (20)$$

Here in addition to the local $SO(1, D - 1)$ symmetry there is also a global symmetry $SO(D - 2)$, acting on the indices $a, b = 1, 2, \cdots, D - 2$. Let us demonstrate that the action (20) describes the propagation of a free massless particle. A straightforward Hamiltonian analysis reveal the following
first-class constraints:

\[ L_{ab} \equiv p_{a\mu}p_{b}^{\mu} + \delta_{ab} = 0, \]
\[ L^{\mu\nu} \equiv x^{[\mu}p^{\nu]} + \sum_{a=1}^{D-1} x^{[\mu}_{a}p^{\nu]}_{a} = 0; \]  
\[ p_{\mu}p^{\mu} = 0, \quad p_{\mu}p^{\mu}_{D-1} = 0, \]  \( 21 \)

Then the equations

\[ x^{\mu}_{a} = \tau \delta_{a}^{\mu}, \quad \mu \geq a; \quad p^{\mu}_{a} = 0, \quad \mu < a, \]  \( 23 \)

turn out to be a gauge fixation for the constraints \( 21 \). Then the unique solution of \( 21,23 \) is

\[ x^{\mu}_{a} = \tau \delta_{a}^{\mu}, \quad p^{\mu}_{a} = \delta_{a}^{\mu}, \quad a = 1, \cdots, D - 1. \]  \( 24 \)

In particular, in this gauge, the variable \( \pi^{\mu}_{D-1} \approx p^{\mu}_{D-1} \) acquires the desired form

\[ \pi^{\mu}_{D-1} \approx p^{\mu}_{D-1} = (0, \cdots, 0, 1). \]  \( 25 \)

The dynamics of the remaining variables \( (x^{\mu}, p_{\mu}) \) is governed now by the free equations

\[ \dot{x}^{\mu} = p^{\mu}, \quad \dot{p}^{\mu} = 0, \]  \( 26 \)

which is accompanied by the constraints \( 22 \).

The \( SO(1, 9) \)-covariance of the resulting system \( 26,22 \) can be considered as a residual symmetry of the initial formulation \( 20 \), surviving in the gauge \( 23 \). Namely, one can see that the combination of \( SO(1, 10), SO(9) \) and \( \lambda \)-transformations, which do not violates the gauge \( 23 \), are \( SO(1, 9) \) Lorentz transformations. As to the translation invariance, let us note that the action \( 20 \) is invariant also under transformations \( \delta x^{\mu} = f^{\mu} \) with covariantly constant functions \( f^{\mu} \), \( Df^{\mu} = 0 \). The general solution of this equation \( f^{\mu}(a^{\mu}) \) consists of an arbitrary numbers \( a^{\mu} \), which are parameters of the global symmetry. In the gauge \( 23 \) this symmetry reduces to the standard Poincare shifts.
4 S-invariant action for the eleven-dimensional superparticle.

In this Section we present a supersymmetric version of the bosonic action (20) for the case \( D = 11 \). It will be shown that global symmetry transformations for the model is a realization of \( N = 1, D = 11 \) S-algebra (7). These transformations are reduced to \( N = 2, D = 10 \) super Poincare one in the gauge (23)-(25). The action is also invariant under the local fermionic \( \kappa \)-symmetry which reduces a number of fermionic degree of freedom by one half. Similarly to the CBS superparticle it provides a free dynamics for the physical sector variables. Besides, the present action describes a superparticle only, in contrast to the example of Sec.2, where a composite system was considered.

The action under consideration is

\[
S = \int d\tau \left\{ \pi_\mu \left[ Dx^\mu - i(\bar{\theta}\Gamma^{\mu\nu}D\theta)\pi_{10\nu} - \xi\pi_{10\mu} \right] - \frac{1}{2}e\pi^2 + \pi_{a\mu}D_{a}x^\mu - \frac{1}{2}\phi_{ab}(\pi_{a\mu}\pi^\mu_{b} + \delta_{ab}) \right\},
\]

(27)

where \( a = 1, 2, \cdots, 10 \), and

\[
D_{a}x^\mu_a \equiv \dot{x}_{a}^\mu - A_{\nu}^\mu x_{a}^\nu, \quad D\theta \equiv \dot{\theta} + \frac{1}{4}A_{\mu\nu}\Gamma^{\mu\nu}\theta.
\]

(28)

The local bosonic symmetries for the action are both \( SO(1,10) \) transformations,

\[
\delta x_{a}^\mu = \omega_{\nu}^\mu x_{a}^\nu, \quad \delta \pi_{a}^\mu = \omega_{\nu}^\mu \pi_{a}^\nu,
\]

\[
\delta \theta = -\frac{1}{4}\omega_{\mu\nu}\Gamma^{\mu\nu}\theta, \quad \delta A_{\nu}^\mu = \dot{\omega}_{\nu}^\mu + \omega_{\rho}^\mu A_{\nu}^\rho - A_{\rho}^\mu \omega_{\rho}^\nu,
\]

(29)

and the transformations (without sum on \( a, b \)),

\[
\delta x^\mu = \alpha\pi^\mu, \quad \delta e = \dot{\alpha};
\]

\[
\delta x_{10}^\mu = \lambda\pi_{10}^\mu, \quad \delta x_{10}^\mu = \lambda\pi^\mu, \quad \delta \xi = \dot{\lambda};
\]
\[ \delta x^\mu_a = \lambda_{ab} \pi^\mu_b, \quad \delta x^\mu_b = \lambda_{ba} \pi^\mu_a, \quad \delta \phi_{ab} = \dot{\lambda}_{ab}, \quad \lambda_{ab} \equiv \lambda_{ba}. \] (30)

There are also local fermionic \( \kappa \)-symmetry transformations with the parameter \( \kappa^\alpha \) being \( SO(1, 10) \) Majorana spinor,

\[ \delta \theta = \pi^\mu \Gamma^\mu \kappa, \]
\[ \delta x^\mu = i(\bar{\theta} \Gamma^{\mu\nu} \delta \theta) \pi_{10}^\nu, \]
\[ \delta x^\mu_{10} = -i(\bar{\theta} \Gamma^{\mu\nu} \delta \theta) \pi_\nu, \]
\[ \delta e = 4i(\bar{D} \Gamma^\mu \kappa) \pi_{10}^\mu, \]
\[ \delta \xi = -2i(\bar{D} \Gamma^\mu \kappa) \pi_\mu. \] (31)

The global new supersymmetry transformations are realized as follows:

\[ \delta \epsilon^\alpha = f^\alpha(\epsilon), \]
\[ \delta \epsilon x^\mu = i(f \Gamma^{\mu\nu} \epsilon) \pi_{10}^\nu, \]
\[ \delta \epsilon x^\mu_{10} = -i(f \Gamma^{\mu\nu} \epsilon) \pi_\nu, \] (32)

with covariantly constant odd functions \( f^\alpha(\epsilon), \) \( D f^\alpha = 0. \) The general solution of this equation consists of arbitrary constants \( \epsilon^\alpha, \) which are parameters of global symmetry (32). Besides, there is global bosonic symmetry with the parameters \( b^{\mu\nu} = -b^{\nu\mu}, \)

\[ \delta_b x^\mu = f^{\mu}_{\ \nu}(b) \pi_{10}^\nu, \]
\[ \delta x^\mu_{10} = -f^{\mu}_{\ \nu}(b) \pi_\nu. \] (33)

Note that \( \delta A^{\mu\nu} = 0 \) under these transformations, and there are no of derivatives in (32), (33). As a consequence, the algebra of the generators \( Q_\alpha, Z_{\mu\nu}, \) corresponding to the transformations (32), (33), coincides with (7). Thus, (32), (33) is a realization of the S-algebra for the model under consideration.

Let us study the dynamics of the model in the Hamiltonian framework. The total Hamiltonian is

\[ H^{(1)} = \frac{1}{2} \epsilon p^2 + \xi p_{\mu} \pi^\mu_{10} + \frac{1}{2} \phi_{ab} L_{ab} + A_{\mu\nu} L^{\mu\nu} + \lambda_{x\dot{a}} (p^\mu_{\dot{a}} - \pi^\mu_{\dot{a}}) + \lambda_c p_c + \lambda_\phi p_\phi + \lambda_{\pi\dot{a}} p^\mu_{\pi\dot{a}} + \lambda^\mu_{\pi\dot{a}} p_{\pi\mu} + L_\alpha \lambda^\alpha, \] (34)

where \( p_{\dot{a} \mu} \equiv (p_\mu, p_{a\mu}), \) \( p_{\pi\dot{a}} = (p_{\pi\mu}, p_{\pi\dot{a}}) \) are momenta conjugated to the variables \( x^\mu_{\dot{a}} \equiv (x^\mu, x^\mu_{\dot{a}}), \) \( \pi^\mu_{\dot{a}} \equiv (\pi^\mu, \pi^\mu_{\dot{a}}). \) The complete set of constraints can
be written in the form

\[ p^\mu_{\pi a} = 0, \quad p^\mu_a - \pi^\mu_{\bar{a}} = 0; \quad (35.a) \]

\[ p_e = 0, \quad p_\xi = 0, \quad p_{\phi ab} = 0, \quad p_{A\mu\nu} = 0; \quad (35.b) \]

\[ p^2 = 0, \quad pp_{10} = 0; \quad (35.c) \]

\[ L_{ab} \equiv p_{a\mu} p^\mu_b + \delta_{ab} = 0, \quad L^{\mu\nu} \equiv x^{[\mu}_{\bar{a}} \bar{p}^{\nu]}_{\bar{a}} - \frac{1}{4} \bar{p}_\theta \Gamma^{\mu\nu} \theta = 0; \quad (35.d) \]

\[ L_\alpha \equiv \bar{p}_\theta \alpha - i (\bar{\theta} \Gamma^{\mu\nu})_\alpha p_{\mu} p_{10\nu} = 0. \quad (35.e) \]

Besides, some equations for the Lagrange multipliers can be determined in the course of Dirac procedure,

\[ \lambda^\mu_{x\bar{a}} = \delta_{\bar{a}, 0} e p^\mu + \delta_{\bar{a}, 10} \xi p^\mu_{10} - \phi_{\bar{a}\bar{b}} p^\mu_b - A^\mu_{\nu} x_{\bar{a}\bar{b}}^\nu, \quad \lambda^\mu_{\pi \bar{a}} = A^\mu_{\nu} p^\nu_{\bar{a}}, \quad (36) \]

Imposing the gauge conditions

\[ e = 1, \quad \xi = 0, \quad \phi_{ab} = \delta_{ab}, \quad A^{\mu\nu} = 0, \quad (37) \]

to the first-class constraints (35.b) and taking into account the second-class constraints (35.a), one can eliminate the canonical pairs \((e, p_e), (\xi, p_\xi), (\phi_{ab}, p_{\phi ab}), (A^{\mu\nu}, p_{A\mu\nu}), (\pi^\mu_{\bar{a}}, p^\mu_{\pi \bar{a}})\) from the consideration. The constraints (35.d,e) obey the following algebra:

\[
\begin{align*}
\{L^{\mu\nu}, L^{\rho\delta}\} &= \eta^{\mu\rho} \eta^{\nu\delta} + (\text{permutations } \mu\nu\rho\delta) \approx 0, \\
\{L^{\mu\nu}, L_\alpha\} &= -\frac{1}{4} (\Gamma^{\mu\nu} L)_\alpha \approx 0, \\
\{L_\alpha, L_\beta\} &= 2i (\bar{C} \Gamma^{\mu\nu})_{\alpha\beta} p_{\mu} p_{10\nu} \quad (38)
\end{align*}
\]

whereas all other Poisson brackets vanish identically. It follows from the last equation (38) and from the identity \((\Gamma^{\mu\nu} p_{\mu} p_{10\nu})^2 = 4 \left[(pp_{10}) - p^2 p_{10}^2\right] = 0\) that half of the constraints \(L_\alpha = 0\) are first-class. They correspond to the local \(\kappa\)-symmetry (31). The next step is to impose the gauge conditions (23) for the first-class constraints \(L_{ab} = 0, L^{\mu\nu} = 0\) and the gauge condition
$x^{10} = 0$ for the equation $(pp_{10}) = 0$. Then, in particular, $p^\mu_{10} = (0, \ldots, 0, 1)$, which breaks the manifest $D = 11$ S-invariance (32), (33) up to $D = 10$, type IIA super Poincare one. It is useful on this stage to introduce SO(1,9) notations for the SO(1,10) objects [8],

$$
\Gamma^\mu = (\Gamma^\bar{\mu}, \Gamma^{10}) = \left( \left( \begin{array}{cc} 0 & \Gamma^\bar{\mu} \\ \Gamma^{\bar{\mu}} & 0 \end{array} \right), \left( \begin{array}{cc} 1_{16} & 0 \\ 0 & -1_{16} \end{array} \right) \right),
$$

$$
\theta = (\bar{\theta}_\alpha, \theta^\alpha), \quad \bar{p}_\theta = (\bar{p}_{\bar{\theta}}, p_{\theta\alpha}),
$$

$$
\bar{\mu} = 0, 1, \ldots, 9, \quad \alpha = 1, \ldots, 16,
$$

(39)

where $\bar{\theta}_\alpha, \theta^\alpha$ are SO(1,9) Majorana-Weyl spinors of opposite chirality. In such notations equations of motion for the remaining variables can be written as

$$
\dot{x}^{\bar{\mu}} = \bar{p}^{\bar{\mu}} + i \theta \Gamma^{\bar{\mu}} \lambda_\theta + i \bar{\theta}_{\bar{\mu}} \lambda_{\bar{\theta}}, \quad \dot{p}^{\bar{\mu}} = 0;
$$

$$
\dot{\theta}^\alpha = -\lambda_\theta^\alpha, \quad \dot{\bar{\theta}}_\alpha = -\bar{\lambda}_{\bar{\theta}^\alpha},
$$

(40)

while for the remaining constraints one finds the expressions

$$
p^2 = 0, \quad (41.a)
$$

$$
p_{\theta\alpha} + i \theta_\beta \Gamma^{\bar{\mu}}_{\beta\alpha} p^{\bar{\mu}} = 0, \quad \bar{p}^{\bar{\alpha}}_\theta + i \theta_\beta \bar{\Gamma}^{\bar{\mu}}_{\beta\alpha} p^{\bar{\mu}} = 0. \quad (41.b)
$$

To get the final form of the dynamics, we pass to the light-cone coordinates $x^{\bar{\mu}} \to (x^+, x^-, x^i)$, $i = 1, 2, \ldots, 8$, and to SO(8) notations for spinors, $\bar{\theta}_\alpha = (\theta_\alpha, \bar{\theta}_{\bar{\alpha}})$, $\theta^\alpha = (\theta'_a, \bar{\theta}_{\bar{\alpha}})$, $\bar{p}_{\bar{\theta}} = (p_{\theta\alpha}, \bar{p}_{\bar{\theta}\bar{\alpha}})$, $p_{\theta\alpha} = (p_{\theta\alpha}, \bar{p}_{\bar{\theta}\bar{\alpha}})$, $a, \bar{\alpha} = 1, 2, \ldots, 8$. It allows one to write an equivalent to (41.b) set of constraints, which is explicitly classified as first- and second-class respectively,

$$
\sqrt{2} p^+ p'^+ + \bar{p}_{\theta} \gamma^i p^i = 0, \quad \sqrt{2} p^+ \bar{p}_{\theta} - p_{\theta} \gamma^i p^i = 0; \quad (42.a)
$$

$$
\bar{p}_{\theta} + i \sqrt{2} p^+ \bar{\theta} - i \theta'_\gamma \gamma^i p^i = 0, \quad p_\theta - i \sqrt{2} p^+ \theta - i \bar{\theta}' \gamma^i p^i = 0. \quad (42.b)
$$

Then the equations $\theta'_a = 0, \bar{\theta}'_{\bar{\alpha}} = 0$ (or, equivalently, $\Gamma^{+}_{32} \theta_{32} = 0$) are the gauge conditions for the first-class constraints (42.a). It follows from (36),
that $\lambda_\theta = 0$ in this gauge. Thus the dynamics of the physical variables is described by the equations
\begin{align*}
\dot{x}^{\bar{\mu}} &= p^{\bar{\mu}}, \\
p^{\bar{\mu}} &= 0, \\
p^2 &= 0; \\
\dot{\theta}_a &= 0, \\
\dot{\bar{\theta}}_a &= 0. \\
\end{align*}
Using the same arguments as in Sec.3, one can prove that $D = 10$ super Poincare symmetry transformations for (43) are some combinations of the symmetries (29)-(33), which do not spoil the gauge chosen. Besides the S-algebra (7) reduces to the type IIA supersymmetry algebra in this gauge.

5 Summary.

In the present paper we have constructed explicitly several Lagrangian actions for $D = 11$ S-invariant mechanical models. In particular, it was shown that $D = 10$ type IIA superparticle (40), (41) can be presented in the S-invariant formulation (27). In course of the consideration an explicit form of the S-algebra (7) was obtained. Being model-independent, it may be used as a basis for a systematic construction of various $D = 11$ models. In particular, it follows from the consideration of Sec.4 that there may exist a more transparent algebraic formulation for the $D = 11$ superparticle in terms of the Lorentz-harmonic variables [24-28]. We consider these models as a preliminary step towards a construction of $D = 11$ S-invariant formulations for SYM and superstring actions, which might contribute to a better understanding of the uncompactified M-theory [10-13].

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