Parametric Identification of Bouc-Wen Model and Its Application in Mild Steel Damper Modeling

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Abstract

Bouc-Wen model is one of the most widely used parametric models of hysteresis in mechanics. In this paper, the normalized Bouc-Wen model is primarily introduced, while its parameters are illustrated by its physical meaning. Also the sensitivity of the parameters is investigated through a series of numerical simulation. For a better understanding, the Bouc-Wen model is made a contrast with bilinear model. Then an approach for identifying the parameters of the Bouc-Wen model is proposed, which is based on the least square method. And the mild steel damper is modeled by Bouc-Wen model, whose parameters are identified through the proposed approach. The accuracy and effectiveness of the approach have been demonstrated by using experimental data. Comparing the simulation and test, it is concluded that the Bouc-Wen model is capable of describing the nonlinear behavior of mild steel damper.

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1. Introduction

Hysteresis is a system property that is fundamental to a range of engineering applications as the components of systems with hysteresis are able to react differently to different forces applied to them. Mild steel damper, which can dissipate substantial energy during an earthquake, exhibits stable hysteretic behavior. Several hysteretic models have been developed for mild steel damper. These can be broadly classified into two types, polygonal hysteretic model and smooth hysteretic model (Sivaselvan and Reinhorn 2000). Models based on piecewise linear behavior are polygonal hysteretic model, such as bilinear model. And the bilinear model has a vast application in engineering in the past few decades for its simple expression and obvious physical meaning. Yet, the bilinear model has sharp change which is contradictory with the reality. In the paper (Kasai et al. 2009), steel damper is not accurately predicted by using bilinear model. On the other hand, Bouc-Wen model, belonging to smooth hysteretic model, refers to model with continuous change of stiffness due to yielding. As the Bouc-Wen model expressed by differential equation, it is possible to generate a large variety of different shapes of the hysteresis loops. However, due to its differential expression, it is hard to identify the parameters and understand the physical meaning of the Bouc-Wen model.

As the issues discussed above, the objectives of this study are mainly two parts: illustrating the characteristic of Bouc-Wen model; identifying the parameters of the model.

2. Characteristic of Bouc-Wen Model

Bouc-Wen model was proposed initially by Bouc early in 1967 (Wen 1967) and subsequently generalized by Wen in 1976 (Wen 1976). Its typical equations are expressed as follows:

\[ R = \alpha k x + (1 - \alpha) k z \]  

(1)

\[ \dot{z} = A \dot{x} - \beta |\dot{x}|^{n-1} z - \gamma |\dot{x}| z'' \]  

(2)

Where \( R \) is the restoring force, \( k \) is the stiffness coefficient, \( \alpha \) is the ratio of post-yield to pre-yield stiffness, \( z \) is the hysteresis displacement. \( A, n, \beta, \gamma \) are parameters that control the hysteresis shape.

2.1 The normalized Bouc-Wen model

Analyzing the Equation (1) and Equation (2), it is found that there are redundant parameters in Bouc-Wen model (Ma et al. 2000). A drawback to this property is that identification procedures that use input-output data cannot determine the parameters of the Bouc–Wen model. To cope with this problem, users of the Bouc–Wen model often fix some parameters to arbitrary values. In order to eliminate the redundant parameters of the model, Bouc–Wen model needs to be normalized (Ikhouane and Rodellar 2007; Ismail et al. 2009). The normalized equations are defined as follows:

\[ R = k_v x + k_w w \]  

(3)

\[ \dot{w} = \rho (\dot{x} - \sigma |\dot{x}|^{n-1} w + (\sigma - 1) |\dot{x}| w'' \)  

(4)
Where:

\[
z_0 = \sqrt{\frac{A}{\beta + \gamma}}, \quad \rho = \frac{A}{z_0}, \quad \sigma = \frac{\beta}{\beta + \gamma}
\]

\[
k_x = \alpha k, \quad k_w = (1 - \alpha)kz_0, \quad w = z / z_0
\]

(5)

Where \(z_0\) is a normalized coefficient; \(k_x\) reflects post-yield stiffness; \(k_w\) reflects initial stiffness of hysteresis component; \(w\) is the internal hysteresis variable; \(\rho\) reflects the initial stiffness of \(w\); \(\sigma\) is the parameter controlling the hysteresis shape. The dimension of \(k_x\), \(k_w\) and \(\rho\) are ‘force/displacement’, ‘force’ and ‘1/displacement’ respectively, while \(z_0\), \(w\), \(\sigma\) are non-dimensional numbers. And the normalized form has the advantage of having only five parameters to identify instead of the six parameters for the typical equation.

2.2 Relationship with bilinear model

Bilinear model, due to its clear physical meaning, was always used to model the mild damper in past few days. However, it is difficult to determine the yield displacement since the actual load-deformation curve is smooth. As the Bouc-Wen model maps smooth hysteresis curve well, establishing the relationship between Bouc-Wen and bilinear model can help understand the physical meaning of the Bouc-Wen model. And it is feasible to establishing the relationship from the discussion in section 2.1. For example, while give a Bouc-Wen model with the parameters \(k_x\), \(k_w\), \(\rho\), \(\sigma\), \(n\) are 2kN/mm, 98kN, 1/mm, 0.5, 1 respectively. The parameters of corresponding bilinear model would be calculated as follows. The initial stiffness could be \(k_x + \rho k_w = 100\)kN/mm; the ratio of post-yield to pre-yield stiffness could be \(k_x / \rho k_w = 0.02\); and the yield displacement is \(1 / \rho = 1\)mm. Comparing to the Bouc-Wen model, the yield force is about 1.29 times of corresponding restoring force \(R\). The two model’s hysteresis curves are shown as figure 1. When the parameters \(n\) tend to be infinite, the Bouc-Wen model inclines to bilinear model.

![Figure 1: Hysteresis curves of Bouc-Wen and bilinear model](image-url)
2.3 Parameter Sensitivity Analysis

The sensitivity of a model with respect to an input parameter is the degree to which the parameter affects the model output. Sensitivity analysis is the study of how changes in the output of a model can be apportioned, qualitatively or quantitatively, to variations in different input parameters. The simplest way to conduct sensitivity analysis is to repeatedly vary one parameter at a time while holding the others fixed at chosen nominal values (Ma et al. 2004). In this paper, the restoring force is defined as output, while the $k_x$, $k_w$, $\rho$, $\sigma$, $n$ are defined as input. As a typical example, the base values, excepting $n$ changed to 2, are selected identically to the values mentioned previously in section 2.2. With the region of loading displacement between -5mm to 5mm, each of these five parameters is then varied, one at a time, by up to 50% from its base value while holding all other parameters at the base position. The results are shown as figure 2.

Figure 2: Spider diagram generated by the one-factor-at-a-time method

It can be indicated that $k_w$ is most sensitive, while $n$ is least sensitive to restoring force $R$ from figure 2. Same conclusion can be obtained when change the region of loading displacement. However $k_x$ become more sensitive with the region of loading displacement enlarged.

3. Approach Of Identification

The main idea of the approach of identification proposed in this paper is to utilize the characteristic of Bouc-Wen model and identify the parameters step by step. Through mathematical transform, the nonlinear parameter identification problem is transferred into linear problem. Least square method is used for solution. The details of the approach are explained as follows.

The loading displacement $x$ is defined as input, while the restoring force $R$ is defined as output. They are all measurable. $k_x$, $k_w$, $\rho$, $\sigma$, $n$ are the parameters to be identified. The first term $k_x x$ on the right of Equation (3) is considered to be non-hysteretic component. In order to reduce the difficulty of identification, $k_x$ is identified firstly. Ikhouane (Ikhouane and Rodellar 2007) proposed a method that two loading displacements are kept with a constant distance to identify $k_x$. Yet the low cycle tests of mild steel damper, done in past few years, universally dissatisfy the requirements of this method. Through
the analysis of Equation (4), it should be noted that the tangency of the hysteresis variable $w$ is less than 2x10^{-4}, when the loading displacement is five times of the yield displacement. This means the increment of restoring force $R$ is mainly controlled by the first term of Equation (3) when the loading displacement is great than five times of yield displacement. Consequently the $k_x$ can be determined by Equation (3). And the other four parameters can be identified through least square method. Denoting $y$ as observed valuable, there is:

$$y(t) = R(t) - R(t-1) - k_x (x(t) - x(t-1)) = k_w (w(t) - w(t-1)) \quad t = 1, 2, \ldots$$

(6)

On the basis of first order of difference, there is:

$$\dot{w} = (w(t) - w(t-1)) / \Delta t \quad \dot{x} = (x(t) - x(t-1)) / \Delta t$$

(7)

Equation (3), (4) and (7) can be combined through Eq. (6). And there is:

$$y = \Phi \theta$$

(8)

Where $\theta$ is the vector to be identified. $n$ can be limited in a certain region of natural number since it is not sensitive to restoring force $R$ from the discussion of section 2.3. Where:

$$\theta = [ \rho k_w - \rho \sigma / k_w^{(n-1)} \quad \rho (\sigma - 1) / k_w^{n-1} ]$$

(9)

$$\Phi = [(x(t) - x(t-1)) \quad \|R(t) - k_x (x(t))\|^n - (R(t) - k_x (x(t)) \quad (x(t) - x(t-1)) \|R(t) - k_x (x(t))\|^n]$$

(10)

And $\theta$ can be determined by Eq. (8) through least square method:

$$\hat{\theta} = (\Phi^T \Phi)^{-1} \Phi^T y$$

(11)

Since $\hat{\theta}$ varies with $n$, $n$ is determined by minimizing the error. Consequently, $\theta$ is determined. Then $k_w$, $\rho$, $\sigma$ are calculated from the Equation (9).

4. Application

A series of studies and tests of mild steel damper have been conducted in Tongji university (Xu 2008). The loading equipments are shown as figure 3. The loading mode is based on displacement control, while a triangle signal whose frequency is 0.01HZ was used as actuating signal. The force and the relative displacement of the mild steel damper were measured in the test. Bouc-Wen model is used to model the mild steel damper. Based on the previously mentioned approach, the parameters are identified from the test results. The experimental versus simulated load displacement results are shown as figure 4. The
parameters $k_x$, $k_w$, $\rho$, $\sigma$, $n$ are identified as 0.992KN/mm, 64.83kN, 1.06/mm, 0.8217, 1. The identified error of restoring force is about 0.932kN. It shows that the simulation successfully fit the experiment results.

Figure 3: Loading equipments

Figure 4: Experimental versus simulated load displacement results

5. Conclusions

From the studies described in this paper, we can draw the following conclusions:

1) The illustration of the characteristic of Bouc-Wen model is helpful for its application.

2) The approach proposed in this paper gives a way to identify the parameters of Bouc-Wen model. And the identified Bouc-Wen model can fit the load displacement curve well.

3) The Bouc-Wen model used in this paper does not consider stiffness and strength degradation, which need further study.

4) The load frequency is 0.01HZ, which does not accord with the actual earthquake excitation. And the effect of dynamic load should be considered in the numerical simulation in the future.
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