Precise stellar surface gravities from the time scales of convectively driven brightness variations

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A significant part of the intrinsic brightness variations in cool stars of low and intermediate mass arises from surface convection (seen as granulation) and acoustic oscillations (p-mode pulsations). The characteristics of these phenomena are largely determined by the stars’ surface gravity. Detailed photometric measurements of either signal can yield an accurate value of $g$. However, even with ultraprecise photometry from NASA’s Kepler mission, many stars are too faint for current methods or only moderate accuracy can be achieved in a limited range of stellar evolutionary stages. This means that many of the stars in the Kepler sample, including exoplanet hosts, are not sufficiently characterized to fully describe the sample and exoplanet properties. We present a novel way to measure surface gravities with accuracies of about 4%. Our technique exploits the tight relation between $g$ and the characteristic time scale of the combined granulation and p-mode oscillation signal. It is applicable to all stars with a convective envelope, including active stars. It can measure $g$ in stars for which no other analysis is now possible. Because it depends on the time scale (and no other properties) of the signal, our technique is largely independent of the type of measurement (for example, photometry or radial velocity measurements) and the calibration of the instrumentation used. However, the oscillation signal must be temporally resolved; thus, it cannot be applied to dwarf stars observed by Kepler in its long-cadence mode.

INTRODUCTION

For spherical bodies such as stars, surface gravity $g$ is proportional to mass divided by radius squared. Usually expressed as $\log g$ (where $\log$ denotes the decadic logarithm), it plays a key role in many aspects of astrophysics, from knowing stellar properties such as mass and radius to knowing the size of an exoplanet and whether it orbits in its star’s habitable zone. Usually, $g$ is difficult to measure accurately, but a tight relation ($1 - 3$) between $g$ and the properties of surface convection should allow for a precise inference of surface gravity for stars with a convective envelope. Gas in a convective cell travels a vertical distance proportional to the pressure scale height ($H_p$) at the speed of sound ($c_s$), where $H_p$ is the characteristic length scale of the variation of pressure. For an ideal gas of uniform chemical composition in a hydrostatic equilibrium, $H_p = P/(\rho g)^{1/2} \propto T_g^{-1}$, where $P$, $\rho$, and $T$ are pressure, density, and temperature, respectively. In an adiabatic environment, $c_s \propto T_g^{1/2}$, so the typical time scale of convection (just like the acoustic cutoff frequency for pulsations ($4$)) is expected to be proportional to $T_g^{-1/2}$ ($1, 2$). This time scale is in turn associated with the properties of granulation (the visible signature of convection on a star’s surface) and also sets the typical time scale of acoustic oscillations (resonantly driven by the turbulent motions in the convective envelope).

In stars with a convective envelope, brightness variations arising from a combination of phenomena that are gravity-dependent [granulation and acoustic oscillations ($5, 6$)] and gravity-independent [for example, rotation and magnetic activity ($7$)] are usually observed. Granulation and oscillations are both tied to convection and thus act on similar scales. They can, however, be disentangled ($3$) to measure their individual properties, from which $g$ can be inferred in various ways ($3, 8$). Most precise surface gravities can be obtained by the analysis of a star’s global oscillations (that is, asteroseismology), which can yield $g$ with typical uncertainties better than 2% ($8$). Although powerful, asteroseismology is now limited to relatively bright stars and underlying scaling relations about which there remain unanswered questions ($2, 3$).

Determination of more reliable surface gravities in a larger sample of stars calls for new strategies. An elegant method to estimate $g$ from Kepler photometry ($9$) was recently established by directly measuring the amplitude of the brightness variations due to granulation and acoustic oscillations in the light curves ($10$). The advantage here is that instead of disentangling granulation from oscillations and measuring their individual characteristics, the method uses the combined signal, which can be up to four times as strong as the oscillation signal alone ($3$). This “8-hour flicker amplitude” method (using the root mean square of variations at time scales shorter than 8 hours) was introduced to separate power from $g$-dependent and $g$-independent sources. When applied to a large sample of Kepler targets, the flicker method produced a large set of $g$ estimates; however, it has drawbacks:

a) Granulation and oscillation amplitudes are not determined solely by $g$ ($2, 3$).

b) The fixed frequency limit of the 8-hour filter supresses an increasing fraction of stellar signal as stars evolve up the giant branch, where the variability time scales rapidly increase. For stars with oscillation power peaking at 35 $\mu$Hz, which corresponds to a time scale of about 8 hours (that is, basically all stars that are more luminous than about 30 to 40 times the Sun), most of the brightness variations are filtered out. For even more evolved stars, the flicker method eventually breaks down because essentially only photon noise remains to be measured.

c) Toward the Zero Age Main Sequence, the 8-hour filter admits more power from gravity-independent sources such as rotation and activity, which adds significant uncertainty.

d) Always part of the 8-hour flicker amplitude is photon noise, which needs to be evaluated on a case-by-case basis.

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Drawbacks (b) and (c) exclude fast rotators (that is, young dwarf stars but also M dwarfs of a wide range of ages) and a large number of red giants (basically all stars with log $g < 2.5$). Even within the valid evolutionary range, the method gives surface gravities of only moderate accuracy (about 25%). Drawback (d) can be mitigated to some extent for Kepler targets, but the sensitivity to photon noise means that the flicker method will not work (without major modifications) for very faint stars (for example, M dwarfs, where the intrinsic brightness variations are typically much smaller than the photon noise) or on most of the targets from missions such as Microvariability and Oscillations of STars (MOST) (11) and BRight Target Explorer (BRITE) (12), where it is more difficult to separate the contribution of photon noise from those of other sources of instrumental noise.

RESULTS

The time scale technique

Here, we present a method to determine surface gravity, which avoids the above drawbacks, based not on the amplitude of brightness variations in a star but on the typical time scale of those variations. The method applies a self-adapting filter to time-series data to isolate the gravity-dependent signal (that is, granulation and oscillations) from slower gravity-independent stellar (that is, activity and rotation) and instrumental effects. The typical time scale of the residual signal can be determined directly from the autocorrelation function (ACF) of the filtered time-series data (see Fig. 1 for three examples of time-series and ACF analysis results). The ACF quantifies the temporal correlation of the observed brightness fluctuations and therefore also reveals the intrinsic nature of the signal. In case of coherent harmonic variations from a classical pulsator such as a δ Scuti star (13), the ACF is periodic, even for multiperiodic variations. The damped and stochastically re-excited signal that originates from granulation and solar-like oscillations, however, results in an ACF that approximates a sinc function (that is, approaches zero correlation toward larger time lags; Fig. 1, insets to lower panels). These quite different ACF shapes allow us to identify stars with both granulation and solar-like oscillation signals. The typical ACF time scale is then defined as the width of the sinc function fitted to the ACF of the filtered time series (Supplementary Materials).

Fig. 1. Examples of the Kepler light curves (top) of stars at different evolutionary stages (from left to right: upper main sequence, lower giant branch, and upper giant branch) and the corresponding power density spectra (bottom). (Top) Gray symbols represent relative intensity measurements as a function of time, and the red curves represent boxcar smoothings of the data. (Insets) Shorter segments of the light curves (as solid points) after high-pass filtering, where the black curves represent three-point running means and the spacings of the vertical red lines represent the dominant variability time scales measured from the ACF. (Bottom) The gray and black curves represent the power density spectra of the original and filtered data, respectively. The solid colored curves represent global model fits to the original spectra, and the dashed curves represent the components of these models (3). A long arrow points to the typical ACF frequency ($\nu_{ACF} = 1/\tau_{ACF}$) in each panel. Each inset shows the squared ACF of the filtered time series (solid points) along with the fit (red curve) from which we estimate the typical ACF time scale $\tau_{ACF}$ (vertical dashed line).
The ACF time scale represents the characteristic time scale of the combined granulation and oscillation variability and therefore should tightly scale with surface gravity. To calibrate the relation, we measured ACF time scales of about 1300 stars from their Kepler corrected light curves (14) and correlated them (Fig. 2) with surface gravities derived from an asteroseismic analysis of their acoustic oscillations (3, 8). The sample covers a wide range of stellar masses and evolutionary stages (including about five active stars with a strong rotationally modulated signal) [details of the observations may be found in Kallinger et al. (3)]. The residuals from a low-order polynomial fit (Supplementary Materials) show that a measure of the ACF time scale gives $g$ with an average precision of about 4%. The improvement in precision by a factor of about 6 over the flicker method is partly attributable to the fact that the use of a self-adapting filter in the time scale technique reliably separates the gravity-dependent signal from the magnetic/rotation component, minimizing contamination of the astrophysical relation between $g$ and the ACF time scale. Other contributing factors are as follows: (i) the time scales of granulation and oscillation variability are inherently better correlated with surface gravity than their amplitudes, and (ii) the ACF is relatively insensitive to uncorrelated noise (and instrumental perturbations) in the time-series data, which does contribute to the 8-hour flicker amplitude.

**Limitations of the time scale technique**

A disadvantage of the time scale method is that the observations require sufficient temporal resolution. The flicker method is designed to estimate the surface gravity $g$ of main sequence stars and subgiant stars (with the oscillations acting in a time scale range from a few minutes to tens of minutes) from their Kepler long-cadence observations (15). For the time scale method to be effective, it must temporally resolve the oscillations, and hence the high-frequency tail of the granulation signal, even though the signals can be below the formal detection limit in the Fourier spectrum. This implies that we can measure $g$ for main sequence stars only from their Kepler short-cadence data. This is, however, specific to Kepler and not relevant to future missions such as Transiting Exoplanet Survey Satellite (TESS) (16) and PLANetary Transits and Oscillations of stars (PLATO) (17) with planned sampling intervals of minutes and seconds. In fact, the time scale method will make it possible to characterize a much larger sample of stars observed by those missions than will be possible by any other method.

**Surface gravities from noisy light curves**

The time scale technique delivers results of high accuracy for all our test stars. It gives results of reasonable accuracy even for very noisy light curves, where asteroseismology is not possible [that is, the oscillation modes are barely visible (or undetectable) in the Fourier spectrum of the time series]. Mathematically, the ACF of a time series is equivalent to the Fourier transform of its Fourier transform or, put simply, the ACF averages the signal shape over time and frequency. Given that and the fact that the combined granulation and oscillation signal is up to four times larger than the oscillation signal alone (3), it is not surprising that the time scale technique is able to reconstruct $g$ for stars whose oscillations are not detectable in the Fourier spectrum. To demonstrate this, we apply the time scale technique to the ~1300-day-long time series of the planet-hosting star Kepler 22 (18), where we have removed the transit signal. In these data, the oscillation signal is buried in the Fourier noise (Fig. 3) such that asteroseismic measurement of the surface gravity is unreliable. Despite the high Fourier noise, we extract a clear ACF signal corresponding to $g = 4.39 \pm 0.04$, consistent with the spectroscopic determination for this star.

Tests on Kepler data (Supplementary Materials) suggest that reliable results are possible for red giants as faint as $V \sim 14$ and for main sequence stars as faint as $V \sim 12.5$, even with light curves as short as 2 months. The ACF time scales measured from the space-based light curve and ground-based radial velocity (RV) curve of e Ophiuchi (the only pulsating red giant for which both types of sufficiently precise time series exist (19, 20)) are comparable (Fig. 2, upper right inset). This suggests that the time scale technique will give accurate measurements of surface gravity when applied to RV data from ground-based networks such as the Stellar Observations Network Group (SONG) (21).

**Surface gravities of active stars**

Another advantage of the time scale technique is that it is largely independent of the activity level of a star. To demonstrate this, we show in Fig. 4 the light curve and power density spectrum of KIC 6508366—
one of the stars in our main sequence sample exhibiting a strong low-frequency signal, presumably resulting from rotational modulation due to spots. The dominant signal (frequency, ~3 mHz; period, ~4 days) in the light curve of this bright F6 IV star is roughly 10^5 times stronger (in power) than the granulation signal and leaks significant power into the frequency range where the flicker amplitude is measured. As a result, the flicker amplitude is significantly overestimated, leading to a log \( g \) estimate of about 2.94, which is considerably lower than the asteroseismic reference value (3.880 ± 0.005). The time scale technique, on the other hand, filters the light curve at a frequency (~200 mHz) high enough to suppress any rotational signal and recovers log \( g \) = 3.864 ± 0.015, consistent with the asteroseismic measurement.

**DISCUSSION**

We have demonstrated that the typical time scale of the combined granulation and oscillation variability is a reliable tracer of stellar surface gravity for stars with masses 0.8 to 3 times the mass of the Sun across a wide evolutionary range—from main sequence stars with granulation time scales of minutes to hours to red giants with granulation time scales of days, including luminous red giants with time scales of weeks. We have tested this for a well-defined subsample of the Kepler catalog and found it to maintain a high accuracy, about six times better than that of the flicker method. In addition, it is more noise-tolerant than asteroseismology and gives a reasonably accurate surface gravity \( g \) for stars that are too faint for a reliable asteroseismic analysis. Therefore, the time scale technique makes it possible to study otherwise poorly understood stars, which will lead to better characterization of exoplanetary systems both individually and statistically.

The methodology established here is based on Kepler photometry, for which clear oscillation signatures were detected in the data. We will, however, extend our tests to the 1500 or more targets from the short-cadence survey phase with no measureable oscillation signal and to K2 targets (in particular, short-cadence observations of known exoplanet host stars) (22). Other potential applications include the still incomplete characterization of all CONvection ROtation and planetary Transits (CoRoT) (23) and MOST targets and future TESS mission data, for which the shorter time series and the smaller signal-to-noise ratios relative to Kepler will require a new generation of improved techniques. The technique we present here is the first of that new generation.

**MATERIALS AND METHODS**

This work was based on data from the Kepler space telescope, which was launched in March 2009 with the primary goal of searching for...
transiting Earth-sized planets in and near the habitable zones of Sun-like stars. Kepler houses a 95-cm aperture telescope that points at a single field in the constellations Cygnus and Lyra, feeding a photometer with a 115-deg$^2$ field of view to continuously monitor the brightness of more than 145,000 stars. Kepler observations are subdivided into quarters, starting with the initial commissioning run (10 days, Q0) then followed by a short first quarter (34 days, Q1) and the subsequent full quarters 90 days long. Photometry of most of the stars is conducted at a long cadence of 29.42 min, but a subset of up to 512 stars can be observed at a short cadence of 58.82 s.

Our studies were based on long-cadence data for red giants spanning from Q0 to Q13 (~1140 days of continuous observations) and on short-cadence data obtained between Q5 and Q8 spanning at least one full quarter of observations for subgiants and main sequence stars. The Kepler raw data were reduced in the manner of Kallinger et al. (3) and García et al. (14) and subsequently smoothed with a triangular filter to suppress residual instrumental long-term trends with time scales longer than about 40 days. To calibrate the time scale technique, we used the Bayesian inference tool MULTINEST (24). All fits and more details about the methods are given in the Supplementary Materials.

SUPPLEMENTARY MATERIALS

Supplementary material for this article is available at http://advances.sciencemag.org/cgi/content/full/2/1/e1500654/DC1

Materials and Methods

Fig. S1. Improving filter frequency.

Fig. S2. Normalized squared ACFs for about 250 stars (gray dots) in the sample.

Fig. S3. Robustness of the ACF time scale ($\tau_{ACF}$) with respect to filter frequency ($f_{filter}$).

Fig. S4. Relation between asteroseismic surface gravity and ACF time scale, including stars with suppressed dipole modes (black circles).

Fig. S5. Performance of the time scale method with different lengths of and noise levels in time series.

Table S1. Reference (log $g_{ACF}$ and time scale technique gravities (log $g_{ACF}$) for our independent calibration sample.

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