Approximation of mean stress relaxation by numerical simulation using the Jiang model and extrapolation of results

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Abstract

Local mean stresses have a significant influence on the fatigue strength of many components. Often only the initial values of the mean stresses are known. Typically the quasi-stable values are lower because residual stresses as well as the absolute value of mean stresses in notch roots tend to be reduced during cyclic loading.

Some simple, but coarse empirical estimations of the amount of mean stress relaxation have been proposed in literature. More complex material models like the Jiang model are able to give a better description of that process, but calculating a sufficient number of loading cycles for a real component using such a model is very time-consuming. In this paper an approach combining both methods is presented: The first loading cycles are simulated through FE analyses using Jiang’s material model. The result of this simulation is extrapolated using equations according to the empirical estimation methods proposed by Landgraf, Kodama and Maxwell. Various extrapolation approaches are discussed and the results for sheet steel ZStE 500 and steel 1070 are shown.

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1. Introduction

Stability of mean stresses is an important issue in fatigue evaluation. Typically the mean stress values after half of the fatigue life (N/2) are considered in fatigue strength estimation. Mean stress relaxation occurs in many technical components if local cyclic plasticity is present. This process depends on material, component geometry and external loading.

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A very suitable material model for relaxation analyses is the model proposed by Jiang [1], which offers user-defined parameters to fit the relaxation behaviour to experimental data. Nevertheless for technical components it is practically impossible to derive a mean stress value at N/2 only by an elastic-plastic FE analysis. As an alternative, many empirically derived equations to estimate the amount of relaxed mean stresses can be found in literature, e.g. [2, 3, 4, 5, 6]. They are easier to use, but unable to take into account component related effects like stress equilibrium. In the following, a combination of both approaches which is supposed to represent a compromise between numerical effort and accuracy of results will be discussed for constant amplitude loading. Only a relatively small number of cycles are simulated in a nonlinear FE analysis, the remaining relaxation process is extrapolated using an appropriate approximation function. This approach takes advantage of the fact, that mean stress relaxation proceeds fastest during the first cycles. The main challenge is to find the appropriate extrapolation function. A function based on the differential equation system of the Jiang model is hard to derive and would not be adequate for extrapolation because the number of fitting constants would be too high. Established approximation functions for mean stress relaxation might be more useful for extrapolation. Some well-chosen examples are given in table 1. In the literature referenced in this table, parameters for the given function types are suggested. However, in the following these parameters are treated as fitting constants.
Table 1: Established extrapolation function types for mean stress relaxation

| Function Type | Concepts Using This Function Type |
|---------------|-----------------------------------|
| \( \sigma_m(n) = \sigma_m(n = 1) \cdot n^r \) | Landgraf [5], Jhansale & Topper [4] |
| \( \log(\frac{\sigma_m(n)}{\sigma_m(n = 1)}) = \frac{1}{1-m} \cdot \log(nBE(m-1) \cdot (\sigma_m(n = 1))^{m-1} + 1) \) | Maxwell-Arcari [6] |
| \( \sigma_m(n) = A + M \cdot \log(n) \) | Kodama [3], Morrow & Sinclair [2] |

Since mean stress relaxation in Jiang’s material model does not depend on cycle number \( n \) but on accumulated plastic strain \( \varepsilon \), \( n \) is replaced by \( \varepsilon \) in the functions given in Table 1 for the following investigations.

2. Numerical Investigations

In order to evaluate the suitability of the extrapolation function types, computations of uniaxial stress strain behaviour using the Jiang model have been performed. Different strain amplitudes, strain ratios and materials have been regarded. In addition, 2D FE analyses of two component-like geometries, a notched specimen under tensile loading (Fig. 1, left hand side) and an unnotched bending bar (Fig. 1, right hand side), have been carried out. Regarding these models interaction of differently loaded model regions by stress equilibrium is of particular interest. In all cases, 10,000 cycles have been computed for comparison of numerical simulation and the proposed extrapolations. Hence, the FE models are kept as simple as possible. The parameters used for the Jiang model for sheet steel ZStE 500 are given in Table 2. They have been derived from uniaxial strain controlled constant amplitude fatigue tests using 8 backstresses. Parameters for 1070 steel are taken from literature [1]. The numerical simulation results for all evaluated cases are shown in Fig. 2.

For extrapolation, the parameters of the extrapolation functions are optimized with respect to the result of the simulation with Jiang’s model in the interval \( 1 \leq n \leq n_s \) in order to minimize the expression given in Eq. (1)

\[
\Delta = \sum_n (| \sigma_m,\text{extr.}(n) - \sigma_m,\text{Jiang}(n) |)
\]

Additionally, a side condition compels the extrapolation functions to be equal to the numerical result at the location \( n_s \) in order to avoid a difference at the beginning of extrapolation. Fig. 3 shows an example of a result of a simulation with Jiang’s model and the three different extrapolations with \( n_s = 100 \).

Fig. 1: Geometry of the notched tensile specimen (left hand side) and of the bending bar (right hand side).
Table 2: Jiang model parameters for ZStE 500 sheet steel.

| i   | E [MPa] | v [-] | k1 [MPa] | a1 [-] | b1 [1/MPa] | a2 [-] | b2 [1/MPa] | cM [-] |
|-----|---------|-------|----------|--------|------------|--------|------------|--------|
| 1   | 216,000 | 0.3   | 86.5     | 0      | -          | 0.86   | -0.011     | 1000   |
| 2   |         |       |          |        |            |        |            |        |
| 3   |         |       |          |        |            |        |            |        |
| 4   |         |       |          |        |            |        |            |        |
| 5   |         |       |          |        |            |        |            |        |
| 6   |         |       |          |        |            |        |            |        |
| 7   |         |       |          |        |            |        |            |        |
| 8   |         |       |          |        |            |        |            |        |


| i   | c_i^0 [-] | r_i^0 [MPa] | a_i^0 [-] | b_i^0 [-] | Q_i^0 [-] |
|-----|-----------|-------------|-----------|-----------|-----------|
| 1   | 2361      | 44.54       | 1.123     | 0.9648    | 4.25      |
| 2   | 1099      | 22.72       | 2.851     | 13.42     | 4.63      |
| 3   | 674.7     | 22.60       | 1.616     | 1.728     | 4.63      |
| 4   | 335.4     | 22.51       | 2.845     | 18.5169   | 4.63      |
| 5   | 145.5     | 21.83       | 3.772     | 12.53     | 4.63      |
| 6   | 79.15     | 26.19       | 5.682     | 20.37     | 4.63      |
| 7   | 1000      | 37.44       | 5.519     | 31.68     | 4.63      |
| 8   |           | 97.00       | 3.287     | 0.6546    | 4.63      |

Fig. 2: Mean stresses calculated using Jiang material model. (BB: Bending Bar, NS: Notched Specimen)

\[ b \] preliminary value
3. Discussion of results

For evaluation of the extrapolation quality the difference in mean stress at $n_t = 10^4$ related to the initial mean stress is compared, eq. (2).

$$\Delta \sigma(n_t) = |\sigma_{m, extr}(n_t, n = 10^4) - \sigma_{m, Jiang}(n = 10^4)|/\sigma_{m, Jiang}(n = 1) \quad (2)$$

The deviation is related to the initial mean stress in order to achieve a similar weighting for all cases. Since the relative deviation $\Delta \sigma$ is a function of $n_s$, it is important which number of load cycles has to be computed in order to get a satisfactory low deviation. For this reason the function $\Delta \sigma(n_s)$ is computed for all cases shown in fig. 2. Fig. 4 shows the mean and the maximum deviations of all these deviation functions depending on $n_s$ using the three extrapolation function types. The achieved deviations are the lowest for all values of $n_s \leq 60$ when using the Maxwell-Arcari function type.

Fig. 3: Mean stress relaxation simulation result derived applying Jiang’s material model to the bending bar and extrapolation based on the first 100 cycles with three different function types ($R_i = 0, \varepsilon_i = 0.3\%$).

Fig. 4: Mean (left hand side) and maximum (right hand side) relative deviation of mean stresses using different extrapolation function types in dependency on $n_s$ for three different extrapolation function types.
Even simulating only 10 cycles, the relative deviation with this combined approach is always below 10%, and in most cases below 4%. Nevertheless since FE analysis is more precise than the extrapolation it is desirable to simulate numerically at least 50% of the mean stress relaxation process. A suggestion for the choice of \( n_s \) based on the experience of these investigations is given in eq. (3).

\[
\frac{1}{2} \leq \frac{\log(n_s)/\log(n_t)}{2} \leq \frac{1}{2}
\]  

(3)

The reference mean stress is typically at \( n_c = N/2 \), but \( N \) is the result of a fatigue analysis and therefore not known before the mean stress estimation. A preliminary value \( N^- \) can be derived neglecting mean stresses in order to define \( n_s \). Using the mean stress \( \sigma_{m}(n_t(N^-)) \) the number of cycles to failure can be computed recursively.

4. Conclusion

It is possible to extrapolate mean stress relaxation results computed with the Jiang model by using simple extrapolation functions. In the cases presented in this paper the extrapolation functions of Landgraf and Maxwell-Arcari type both result in good approximations, while Kodama function type is less precise. With \( n_s = 10,000 \) cycles less than 1% of the cycles had to be computed numerically in order to achieve a mean relative deviation below 3% for the presented cases using the Maxwell-Arcari function type. Investigations with other materials should be carried out to verify this result. Analyses using the Jiang model are considered as benchmark since they take into account interactions between several material and component properties, but the quality of the Jiang models parameterization process, which is not topic in this paper, has a high impact on the accuracy of results.

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