A METHOD TO EXTRACT THE ANGULAR POWER SPECTRUM OF THE EPOCH OF REIONIZATION FROM LOW-FREQUENCY RADIO INTERFEROMETERS

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ABSTRACT

The redshifted 21 cm signal of neutral hydrogen from the epoch of reionization (EoR) is extremely weak and its first detection is therefore expected to be statistical with first-generation low-frequency radio interferometers. In this Letter, we propose a method to extract the angular power spectrum of the EoR from the visibility correlation coefficients $p_{ij}(u, v)$, instead of the visibilities $V_{ij}(u, v)$ measured directly by radio interferometers in conventional algorithm. The visibility correlation coefficients are defined as $p_{ij}(u, v) = V_{ij}(u, v)/\sqrt{|V_{ii}||V_{jj}|}$ by introducing the autocorrelation terms $V_{ii}$ and $V_{jj}$ such that the angular power spectrum $C_\ell$ can be obtained through $C_\ell = T_0^2 (|p_{ij}(u, v)|^2)$, independently of the primary beams of antennas. This also partially removes the influence of receiver gains in the measurement of $C_\ell$ because the amplitudes of the gains cancel each other out in the statistical average operation of $(|p_{ij}(u, v)|^2)$. We use the average system temperature $T_0$ as a calibrator of $C_\ell$, which is dominated by the Milky Way and extragalactic sources in the frequency range that we are interested in, below 200 MHz. Finally, we demonstrate the feasibility of this novel method using the simulated sky maps as targets and the 21 CentiMeter Array (21CMA) as interferometer.

Key words: cosmology: theory – diffuse radiation – intergalactic medium – methods: data analysis – techniques: interferometric

1. INTRODUCTION

While the redshifted 21 cm emission/absorption of neutral hydrogen provides a unique cosmological probe of the epoch of reionization (EoR), the last frontier of observational cosmology, there are two primary challenges in measuring the EoR signatures with most of the dedicated radio facilities (e.g., 21CMA, LOFAR, LWA, MWA, PAPER, SKA, etc.). First, the cosmic signal from the EoR is deeply buried under the extremely bright foreground dominated by our Galaxy, extragalactic sources, and telescope noise, and an unprecedented level of foreground removals down to five orders of magnitude should be required in order to detect the cosmic signal (e.g., Madau et al. 1997; Zaldarriaga et al. 2004). Second, low-frequency radio interferometric measurements in current 21 cm experiments suffer from various instrumental contaminations in addition to man-made radio-frequency interference. Outstanding among these are the frequency-dependent point-spread function and field of view (also known as “mode-mixing”), complexity of calibration, and bright source subtraction (e.g., Morales et al. 2006; Liu et al. 2009; Bowman et al. 2009; Datta et al. 2009, 2010; Bernardi et al. 2010; Petrovic & Oh 2011). Yet, it is generally agreed among the 21 cm cosmology community that the advent of many sophisticated techniques and algorithms in recent years helps overcome these observational and technical hurdles, allowing us to reach the desired detection sensitivity with the first generation of radio interferometers (for recent reviews see Furlanetto et al. 2006; Morales & Wytse 2010; Pritchard & Loeb 2012; Zaroubi 2012).

The theoretically predicted brightness temperature of the 21 cm signal from the EoR is only $\sim 10$ mK, and first capture of such extremely weak signal is therefore expected to be statistical (Zaldarriaga et al. 2004). Most of the current 21 cm experiments aiming to detect the EoR signal are based on the radio interferometric technique, which provides a direct measure of the Fourier component of the sky brightness $I(s)$ convolved with the primary beam of the antennas $B_j(s)$ toward direction $s$, often known as the visibility $V_{ij}(u)$ at a given baseline $u$ in units of wavelength:

$$V_{ij}(u) = g_i g_j^* \int B_{ij}(s) I(s) e^{-2\pi i u \cdot s} d^2 s, \quad (1)$$

where $g_i$ and $g_j$ are the complex gain factors of antenna pair $i$ and $j$, respectively. It can be easily shown that the angular power spectrum ($C_\ell$) of the sky brightness distribution $I(s)$ can be constructed using the average value of the square of $V_{ij}(u)$ with the Fourier wavenumber $\ell = 2\pi u$ (White et al. 1999). In particular, under the assumption that the angular power spectrum varies rather slowly with scale relative to the Fourier component of the primary beam $B_{ij}(s) = \mathcal{F}[B_{ij}(s)]$, we obtain the commonly used formula in the estimation of the angular power spectrum of low-frequency sky at a given frequency (Bharadwaj & Sethi 2001; Zaldarriaga et al. 2004; Bharadwaj & Ali 2005; Santos et al. 2005; Ali et al. 2008; Pen et al. 2009; Paciga et al. 2011; Ghosh et al. 2011a, 2011b):

$$\langle |V_{ij}(u)|^2 \rangle \approx C_\ell 2 \pi u |g_i|^2 |g_j|^2 \int d^2 s' |\tilde{B}_{ij}(u - u')|^2. \quad (2)$$

However, most of the theoretical studies in literature implicitly assume that perfect calibrations are being made for radio interferometers, and gains and primary beam introduce no spectral and spatial structures to destroy the reconstruction of the EoR angular power spectrum. But in reality, to perform precise calibration in the existence of ionospheric turbulence and to maintain high stability of radio instruments for a rather long integration time are extremely difficult for current 21 cm experiments. This motivates us in this Letter to explore a possible remedy to overcome some of these shortcomings by using the visibility correlation coefficients instead of the visibilities. With this novel method it will be possible to statistically extract the EoR angular power spectrum from radio interferometric measurements, independently of the primary beam of the antennas.
The method also allows us to remove partially the influence of receiver gains in the statistical measurement of \( C_\ell \) because the amplitudes of the gains cancel each other out.

2. FORMALISM

We begin with the autocorrelation of the electric field measured at each antenna, which corresponds to the visibility with baseline \( \mathbf{u} = 0 \):

\[
|V_{il}|^2 = |g_i|^2 I_0^2 \int d^2 \mathbf{u} |\tilde{B}_{il}(\mathbf{u})|^2,
\]

where \( I_0 = \sqrt{C_0} \) is the monopole or mean sky brightness if receiver noise is neglected since the system temperature is dominated by sky contribution for low-frequency observation. The above integral makes no difference if \( B_i(s) \) is used to replace its Fourier component \( \tilde{B}_i(\mathbf{u})/2\pi \) in terms of Parseval’s theorem. Now we define the visibility correlation coefficient such that

\[
p_{ij}(\mathbf{u}) = \frac{V_{ij}(\mathbf{u})}{\sqrt{|V_{il}| |V_{jl}|}}.
\]

With this definition the amplitudes of the complex gains cancel each other out but the signal coherence remains. Following Equation (2) we take the average value of the square of \( p_{ij}(\mathbf{u}) \),

\[
\langle |p_{ij}(\mathbf{u})|^2 \rangle \approx \frac{C_{\ell=2\pi u}}{I_0^2} \frac{\int d^2 \mathbf{u} |\tilde{B}_{ij}(\mathbf{u} - \mathbf{u}')|^2}{\int d^2 \mathbf{u} |\tilde{B}_{ij}(\mathbf{u})|^2 \int d^2 \mathbf{u} |\tilde{B}_{jj}(\mathbf{u}')|^2}.
\]

Using the “frequency shift” theorem of the Fourier transform, we can rewrite the Fourier component of the primary beam \( \tilde{B}_{ij}(\mathbf{u} - \mathbf{u}') \) as \( \tilde{B}_{ij}(\mathbf{u} - \mathbf{u}') = \mathcal{F}(B_{ij}(-s) e^{-2\pi i s \cdot u}) \). According to Parseval’s theorem, the total power in the \( s \) domain or the \( \mathbf{u}' \) domain should be the same. This yields

\[
\frac{1}{4\pi^2} \int d^2 \mathbf{u} |\tilde{B}_{ij}(\mathbf{u} - \mathbf{u}')|^2 = \int d^2 s |B_{ij}(-s) e^{-2\pi i s \cdot u}|^2
\]

\[
= \int d^2 s |B_{ij}(-s)|^2.
\]

The most crucial point of this equation, however, is that the power \( \int d^2 \mathbf{u} |\tilde{B}_{ij}(\mathbf{u} - \mathbf{u}')|^2 \) is actually independent of \( \mathbf{u} \). Namely, in the conventional estimate of angular power spectrum from Equation (2) the primary beam term \( \int d^2 \mathbf{u} |\tilde{B}_{ij}(\mathbf{u} - \mathbf{u}')|^2 \) does not alter the shape of the power spectrum. This arises, of course, from the presumption that the primary beam varies rather slowly with angular scale relative to the cosmic signal, a necessary prerequisite for taking out the \( C_{\ell=2\pi u} \) term from the integral in Equation (2). Furthermore, the symmetric feature of the primary beam for interferometric array element in all the current 21 cm experiments suggests that \( B_i(s) \) can be actually treated as an even function, implying \( B_j(-s) = B_j(s) \). Now, we apply the Cauchy–Schwarz inequality to the integral \( \int d^2 s |B_{ij}(s)|^2 \) by noting that \( B_{ij}(s) = B_i(s) B^*_j(s) \), \( |B_i(s)|^2 = |B_i(s)|, \) and \(|B_j(s)|^2 = |B_j(s)|^2\), and obtain

\[
\left| \int d^2 s |B_{ij}(s)|^2 \right|^2 \leq \int d^2 s |B_i(s)|^2 \int d^2 s |B_j(s)|^2.
\]

The equals sign holds when all antenna elements have identical primary beam \( B_i(s) = B_j(s) \). Finally, replacing the integral \( \int d^2 \mathbf{u} |\tilde{B}_{ij}(\mathbf{u} - \mathbf{u}')|^2 \) in Equation (5) by Equations (6) and (7) and using again Parseval’s theorem we find

\[
\frac{\int d^2 \mathbf{u} |\tilde{B}_{ij}(\mathbf{u} - \mathbf{u}')|^2}{\sqrt{\int d^2 \mathbf{u} |\tilde{B}_{ij}(\mathbf{u})|^2 \int d^2 \mathbf{u} |\tilde{B}_{jj}(\mathbf{u}')|^2}} = 1.
\]

This allows us to evaluate the angular power spectrum \( C_\ell \) relative to the global sky brightness \( I_0 \) simply through

\[
C_{\ell=2\pi u} = I_0^2 \langle |p_{ij}(\mathbf{u})|^2 \rangle.
\]

Consequently, we may use the average sky brightness over the primary beam of the antenna as the reference or calibration. While many sophisticated models have been constructed for low-frequency sky for the purpose of foreground removal in 21 cm experiments (e.g., de Oliveira-Costa et al. 2008; Jelic et al. 2008; Wilman et al. 2008; Vernstrom et al. 2011; Kogut 2012), for our purpose a simple power law as the global sky brightness temperature model should suffice to calibrate the angular power spectrum at each frequency \( \nu : T_0 = 260 \text{K}(\nu/150 \text{MHz})^{-2.85} \). This is the best-fit mean sky temperature over a field of view of \( 5^\circ \times 5^\circ \) at the frequency range 50–200 MHz based on the numerical simulations by Jelic et al. (2008). A combination of various radio measurements at different frequencies in literature gives essentially similar result (Holder 2012). Note that \( C_\ell \) will represent the angular power spectrum of the sky brightness temperature distribution if \( I_0 \) is replaced by \( T_0 \) in Equation (9): \( C_{\ell=2\pi u} = T_0^2 \langle |p_{ij}(\mathbf{u})|^2 \rangle \).

3. SIMULATION TEST

In order to test the feasibility of extracting the angular power spectrum of the low-frequency sky with this novel method, we use the simulated sky maps by Wilman et al. (2008) as the input and the 21 CentiMeter Array (21CMA) as the radio interferometer. Following the same algorithm in Zheng et al. (2012), we generate a set of sky maps of \( 20^\circ \times 20^\circ \) for different frequencies, containing five distinct radio source types out to a redshift of \( z = 20 \). We estimate the flux of each radio source at different observing frequencies between 100 MHz and 200 MHz using a running power law in frequency: \( S \propto \nu^{-\alpha} \), where \( \nu_s \) is the characteristic frequency and is taken to be 150 MHz in this study. Finally, we exclude all the bright sources with fluxes \( S_{150 \text{MHz}} > 10 \text{mJy} \) at 150 MHz which are assumed to be resolvable by current radio interferometers and can therefore be excited with existing algorithms. We fix a 4096\(^2\) grid for each of the simulated images, which gives rise to an angular resolution of 0.3\(^\circ\) (see Figure 1).

21CMA, in Western China, is a ground-based meter-wave array operating at frequencies from 50 MHz to 200 MHz designed to probe the EoR. The array consists of 80 pods (or stations), each with 127 log-period antennas, which are deployed in two perpendicular arms along the east–west and north–south directions, respectively. Spacings between pods are integral multiples of 20 m, with a maximum baseline of 2740 m along each baseline. The 21CMA redundancy is used for the purposes of not only calibrations but also statistical measurement of the angular power spectrum of the EoR at specific modes. In this work, we choose the 40 pods of the east–west arm to generate \( \nu \mathbf{u} \) sampling toward the north celestial pole region, which significantly reduces
the computing complexity in visibilities $V_{ij}(u, v)$ since only a two-dimensional Fourier transform is involved for such a configuration. The corresponding baseline (i.e., $uv$ sampling) distribution of the 780 pod pairs as interferometers for the 21CMA east–west arm is shown in Figure 2, among which there are only 127 independent baselines.

The simulated sky map is convolved with the 21CMA primary beam $B_i(s)$, which is identical for all the pods and approximately takes a Gaussian function with FWHM = $4.26(v/100$ MHz)$^{-1}$. We perform the Fourier transform of the simulated sky map modulated by $B_i(s)$ to produce the $uv$ map in terms of Equation (1) by setting $g_i = g_j = 1$. The $uv$ map is further sampled by the 21CMA east–west baselines shown in Figure 2. We now calculate the visibility correlation coefficients $p_{ij}(u, v)$ at each frequency channel for all the 780 baselines. This yields a set of $p_{ij}(u, v)$ measured at 127 independent Fourier modes $\ell = 2\pi \sqrt{u^2 + v^2}$. Finally, the angular power spectra at these specific modes can be obtained using Equation (9), $C_{\ell=2\pi u} = T_0^2 \langle |p_{ij}(u, v)|^2 \rangle$, in which the average sky brightness temperature for our case is determined by the unresolved extragalactic sources $T_0 = 13.7$ K$(v/150$ MHz)$^{-2.74}$ because the Galactic foreground has not been included in the simulation and the bright sources have been already removed.

Figure 3 shows the angular power spectra of the simulated sky maps at four frequencies ranging from 120 MHz to 180 MHz, together with the recovered ones at specific modes sampled by the 21CMA east–west baselines. The former are constructed directly using the Fourier transform of the simulated sky images without inclusion of any instrumental effects, while the latter are the reconstructed results from the 21CMA “observations” based on the novel algorithm of Equation (9). It appears that the two results show remarkably good agreement. We have also demonstrated the measurement errors assuming an integration time of 300 days, an observing bandwidth of 1 MHz, and system noise of 300 K. In particular, the noise level at $C_\ell$ has been suppressed by a factor of $1/\sqrt{N_t}$ for the redundant baseline of $N_t$ equally spaced pods. Large error bars at small- and large-$\ell$ ends can be attributed to the arcminute-scale angular resolution due to the short baselines and the cosmic variance due to the small field of view of the 21CMA, respectively. For comparison the reconstructed angular power spectra, constructed directly from Figure 1, are also plotted in Figure 3. It appears that the Gaussian beam alters only the amplitude rather than the shape of the angular power spectrum in terms of Equations (2) and (6). As is shown in Figure 3, such an amplitude effect has been corrected for when the visibility correlation coefficients $p_{ij}(u, v)$ are used. Yet, in practice, the primary beam can hardly be modeled by a perfect Gaussian function or other form of simple analytical function. A careful calibration of the primary beam of antennas to a high degree of precision must be made. Imperfect and inaccurate calibrations of both spatial and spectral properties of the primary beam may lead to significant errors in reconstruction of the power spectrum of the EoR for 21 cm experiments. Employment of the visibility correlation coefficients in the statistical study of the EoR allows us to eliminate concerns about the calibration of the primary beam. While with our new algorithm we have successfully recovered the angular power spectra of the simulated radio foregrounds, the foregrounds should be eventually suppressed to a level below 10 mK, the minimum requirement for statistically extracting the signatures of EoR. This can be achieved, for example, using the foreground removal technique suggested recently by Cho et al. (2012), which works straightforwardly with the angular power spectrum. We have tested the technique and found that the foregrounds can indeed be subtracted to the level below 10 mK.

4. DISCUSSION AND CONCLUSIONS

Instead of directly employing the visibilities in conventional interferometric measurements of the statistical fluctuations of the EoR suggested in literature, we propose to work with the visibility correlation coefficients defined by $p_{ij}(u, v) = V_{ij}(u, v)/\sqrt{|V_{ii}| |V_{jj}|}$. This allows us to eliminate the effect of primary beams of antennas and also partially reduce the influence of receiver gains on the statistical extraction of the angular power spectrum of the low-frequency sky: $C_{\ell=2\pi u} = T_0^2 \langle |p_{ij}(u, v)|^2 \rangle$. Yet, we need to calibrate the power spectrum using the system noise $T_0$ dominated by the Milky Way and
extragalactic sources for low frequencies below 200 MHz. Observationally, $T_0$ has been determined so far to a degree of satisfaction at least for our purpose (e.g., de Oliveira-Costa et al. 2008; Kogut 2012; Holder 2012).

Introduction of the visibility correlation coefficient $p_{ij}(u, v)$ does not change the coherence of the original signal. Hence, the phase correction such as self-calibration and/or redundant calibration should still be made before combining $p_{ij}(u, v)$ data. Furthermore, bright sources in the field of view have to be removed to reduce the Poisson noise in computation of the angular power spectrum. This also implies that the sidelobes of bright sources still remain as troublesome for reconstruction of angular power spectrum whether $V_{ij}(u, v)$ or $p_{ij}(u, v)$ is used. Another reason that bright sources should be excised before recovery of the angular power spectrum is the requirement of the uniformity assumption to take out the angular power spectrum term $C_\ell$ in Equation (2) from the integral.

We have tested the feasibility of this novel method using the simulated sky maps of $20^\circ \times 20^\circ$ for extragalactic sources in low frequencies as the targets and 21CMA as the radio interferometer. We have successfully recovered the angular power spectra of the foregrounds at specific Fourier modes sampled by the 127 independent baselines of the 21CMA east–west arm, after the bright sources with fluxes of $S_{150 \text{MHz}} \geq 10$ mJy are removed. While we have not included the gain fluctuations in sampling the visibilities, this new method does allow us to remove the effect of the spatial response of the 21CMA antennas on the reconstruction of the angular power spectrum of the low-frequency sky. In combination with various sophisticated foreground removal techniques developed in recent years especially in the power spectrum domain (e.g., Cho et al. 2012), we should be able to subtract the foreground to the level needed for statistical detection of the 21 cm signal from EoR.

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REFERENCES

Ali, S. S., Bharadwaj, S., & Chengalur, J. N. 2008, MNRAS, 385, 2166
Bernardi, G., de Bruyn, A. G., Harker, G., et al. 2010, A&A, 522, A67
Bharadwaj, S., & Ali, S. S. 2005, MNRAS, 356, 1519
Bharadwaj, S., & Sethi, S. K. 2001, JA&A, 22, 293
Bowman, J. D., Morales, M. F., & Hewitt, J. N. 2009, ApJ, 695, 183
Cho, J., Lazarian, A., & Timbie, P. T. 2012, ApJ, 749, 164
Datta, A., Bhatnagar, S., & Carilli, C. L. 2009, ApJ, 703, 1851
Datta, A., Bowman, J. D., & Carilli, C. L. 2010, ApJ, 724, 526
de Oliveira-Costa, A., Tegmark, M., Gaensler, B. M., et al. 2008, MNRAS, 388, 247

Figure 3. Angular power spectra of the simulated sky maps at four frequencies ranging from 120 MHz to 180 MHz, represented by $\delta T = [\ell (2\ell + 1) C_\ell / 4\pi]^{1/2}$. The solid lines are the results derived from the simulated maps without any observational and instrumental effects; blue circles are the reconstructed $\delta T$, in terms of the visibility correlation coefficients, measured at the 21CMA east–west independent baselines. For comparison, the angular power spectra constructed directly from Figure 1 are also shown (red circles), in which the Gaussian beam results in a decrease of the amplitude but does not alter the overall shape of $\delta T$. 
Furlanetto, S. R., Oh, S. P., & Briggs, F. H. 2006, *Phys. Rep.*, 433, 181
Ghosh, A., Bharadwaj, S., Ali, S. S., & Chengalur, J. N. 2011a, *MNRAS*, 411, 2426
Ghosh, A., Bharadwaj, S., Ali, S. S., & Chengalur, J. N. 2011b, *MNRAS*, 418, 2584
Holder, G. P. 2012, arXiv:1207.0856
Jelić, V., Zaroubi, S., Labropoulos, P., et al. 2008, *MNRAS*, 389, 1319
Kogut, A. 2012, *ApJ*, 753, 110
Liu, A., Tegmark, M., Bowman, J., Hewitt, J., & Zaldarriaga, M. 2009, *MNRAS*, 398, 401
Madau, P., Meiksin, A., & Rees, M. J. 1997, *ApJ*, 475, 429
Morales, M. F., Bowmann, J. D., & Hewitt, J. N. 2006, *ApJ*, 648, 767
Morales, M. F., & Wyithe, J. S. B. 2010, *ARA&A*, 48, 127
Paciga, G., Chang, T.-C., Gupta, Y., et al. 2011, *MNRAS*, 413, 1174
Pen, U.-L., Chang, T.-C., Hirata, C. M., et al. 2009, *MNRAS*, 399, 181
Petrovic, N., & Oh, S. P. 2011, *MNRAS*, 413, 2103
Pritchard, J. R., & Loeb, A. 2012, *Rep. Prog. Phys.*, 75, 086901
Santos, M. G., Cooray, A., & Knox, L. 2005, *ApJ*, 625, 575
Vernstrom, T., Scott, D., & Wall, J. V. 2011, *MNRAS*, 415, 3641
White, M., Carlstrom, J. E., Dragovan, M., & Holzapfel, W. L. 1999, *ApJ*, 514, 12
Wilman, R. J., Miller, L., Jarvis, M. J., et al. 2008, *MNRAS*, 388, 1335
Zaldarriaga, M., Furlanetto, S. R., & Hernquist, L. 2004, *ApJ*, 608, 622
Zaroubi, S. 2012, in the First Galaxies—Theoretical Predictions and Observational Clues, ed. T. Wiklind, B. Mobasher, & V. Bromm (Berlin: Springer), in press (arXiv:1206.0267)
Zheng, Q., Wu, X.-P., Gu, J.-H., Wang, J., & Xu, H. 2012, *MNRAS*, 424, 2562