Cutoff parameter versus Ginzburg-Landau coherence length in the mixed state of high-$\kappa$ superconductors with impurities: quasiclassical approach

P. Belova$^{1,2}$, I. Zakharchuk$^{1,3}$, K. B. Traito$^1$, E. Lähderanta$^1$

$^1$ Lappeenranta University of Technology, P.O.Box 20, FI-53851, Lappeenranta, Finland
$^2$ Petrozavodsk State University, Lenin str. 33, RU-185640, Petrozavodsk, Russia
$^3$ Saint-Petersburg Electrotechnical University, Popov str. 5, RU-197376, St.Petersburg, Russia

E-mail: Polina.Belova@lut.fi

Abstract. The influence of the impurities on the ratio of the cutoff parameter, $\xi_h$, and the Ginzburg-Landau coherence length, $\xi_{c2}$, in the mixed state of high-$\kappa$ s-wave superconductors is investigated in framework of the quasiclassical nonlocal Eilenberger theory. Quasiparticle scattering by impurities and lowering of the temperature reduce the value of $\xi_h$ to values much less than $\xi_{c2}$. This is different from the prediction of the local Ginzburg-Landau theory where $\xi_h$ is scaled by $\xi_{c2}$. Detailed comparison with the behavior of the order parameter coherence length $\xi_1$ is done. It is found that impurities influence by different way on $\xi_h$ and $\xi_1$. The curve $\xi_h/\xi_{c2}(B/B_{c2})$ shifts downward with increasing of impurity scattering rate while $\xi_1/\xi_{c2}(B/B_{c2})$ curve shifts upward in this case.

The detailed properties of the vortex structure, such as its field dependence, attract much attention both in the conventional s-wave superconductors and in high-$T_c$ superconductors. Several important means to probe the vortex structure are available experimentally in various superconductors (see review in Ref. [1]). The vortex core size is determined from the $\mu$SR measurements by fitting it into a theoretical function for $B(r)$ that includes a cutoff function $F(G, \xi_h)$, where $G$ refers to the reciprocal lattice vectors. The parameter $\xi_h$ is the cutoff $G_{\text{max}}$ of the sum over the reciprocal lattice. This takes away the divergence of the sum over the $G$ in the expression for the field distribution in the London approach. The cutoff cannot be improved within the London theory; to this end, one should use a theory which is able to handle the core structure properly. The functional form of $F(G, \xi_h)$ depends on the spatial dependence of the superconducting order parameter $\Delta(r)$ in the core region. Cutoff function $F(G, \xi_h)$ was obtained in the variational approach of the Ginzburg-Landau (GL) equations [2] (the Hao-Clem theory (HC), the analytical GL theory (AGL)) and the field dependence of $\xi_h$ was calculated ($\xi_v$ in a notation of the AGL theory). In this model, $\xi_h/\xi_{c2}$ is a universal function of $B/B_{c2}$. Here, $\xi_{c2}$ is determined from the relation $B_{c2} = \Phi_0/2\pi\xi_{c2}^2$, where $B_{c2}$ is an upper critical field and $\Phi_0$ is a flux quantum. Analyzed with this method, $\mu$SR experimental results in $V_3Si$, $Nb_3Sn$, $NbSe_2$, $YNi_2BC$ and $LuNi_2B_2C$ in intermediate magnetic fields and low temperatures showed that $\xi_h/\xi_{c2} \ll 1$ [1], which needs explanations. In the AGL theory, the equality $\xi_h = \xi_1$ is suggested. Here, the order characteristic length $\xi_1$ is determined as $1/\xi_1 = (\partial|\Delta(r)|/\partial r)_{r=0}/|\Delta_{NN}|$, where $|\Delta_{NN}|$ is the maximum value of the order parameter along the nearest-neighbor direction which is the direction of taking the derivative.
The microscopical theory valid in the whole temperature range is the quasiclassical Eilenberger theory. The cutoff parameter can be found from the fitting of the calculated magnetic field distribution obtained from the Eilenberger equations to the Hao-Clem type field distribution [3]

\[ h_{EHC}(r) = \frac{\Phi_0}{S} \sum_G \frac{F(G)e^{iGr}}{1 + \lambda^2 G^2}, \]

where \( F(G) = uK_1(u) \), \( K_1(u) \) is the modified Bessel function, \( u = \xi_h G \) and \( S \) is the area of the vortex lattice unit cell. In Eq. (1), \( \lambda(T) \) is calculated from microscopical theory and renormalized by nonmagnetic impurity scattering.

We solve the quasiclassical self-consistent Eilenberger equations for triangular FLL and s-wave pairing symmetry. Quasiclassical Green functions \( f \) and \( g \) can be parameterized with the Riccati transformation of the Eilenberger equations via functions \( a \) and \( b \) [4]

\[ \tilde{f} = \frac{2a}{1 + ab}, \quad \tilde{f}^\dagger = \frac{2b}{1 + ab}, \quad g = \frac{1 - ab}{1 + ab}, \]

satisfying the nonlinear Riccati equations. In Born approximation for the nonmagnetic impurity scattering we have

\[ n \cdot \nabla a = -a[2(\omega_n + G) + in \cdot A] + (\Delta + F) - a^2(\Delta^* + F^*), \]

\[ n \cdot \nabla b = b[2(\omega_n + G) + in \cdot A] - (\Delta^* + F^*) + b^2(\Delta + F), \]

where \( \omega_n = \pi T(2n + 1) \), \( F = 2\pi \langle f \rangle \cdot \Gamma \) and \( G = \pi n_i N_F |u|^2 \) is the impurity scattering rate (\( u \) is impurity scattering amplitude) and \( n \) is a unit vector of the Fermi velocity. The impurity renormalization correction in Eqs. (3) and (4) are averaged over Fermi surface and can be reduced to averages over the polar angle \( \theta \), i.e. \( \langle \ldots \rangle = (1/2\pi) \int \ldots d\theta \). To take into account the influence of screening, the vector potential \( A(r) \) in Eqs. (3) and (4) is obtained from the equation \( \nabla \times \nabla \times A_E = \frac{i}{\hbar} J \), where the supercurrent \( J(r) \) is given in terms of \( g(\omega_n, \theta, r) \) by

\[ J(r) = 2\pi T \sum_{\omega_n > 0} \int_0^{\omega_c} \frac{d\theta}{2\pi} \frac{k}{v_F} g(\omega_n, \theta, r). \]

Here \( A \) and \( J \) are measured in units of \( \phi_0/2\pi\xi_0 \) and \( 2e v_F N_0 T_c \), respectively. The self-consistent condition for the pairing potential \( \Delta(r) \) is given by

\[ \Delta(r) = V^{SC} 2\pi T \sum_{\omega_n > 0} \int_0^{\omega_c} \frac{d\theta}{2\pi} f(\omega_n, \theta, r), \]

where \( V^{SC} \) is the superconducting coupling constant and \( \omega_c \) is the ultraviolet cutoff frequency determining \( T_c \) [3]. All over our paper, the energy, the temperature, and the length are measured in units of \( T_c \) and the characteristic length \( \xi_0 = \nu_F /\Delta_0 \). Here \( v_F = \nu_F /\pi \Delta_0 \). where \( v_F \) is the Fermi velocity and \( \Delta_0 \) is temperature dependent uniform gap. The magnetic field \( B \) is given in units of \( \phi_0 /2\pi\xi_0^2 \). The impurity scattering rates are in units of \( 2\pi T_c \). In computations the ratio \( \kappa = \lambda_0 /\rho_0 = 10 \) is used. It corresponds to \( \kappa_{GL} = 43.3 \) [4]. The Riccati equations are solved by the Fast Fourier Transform (FFT) method [3, 5].

A strong decrease in \( \xi_h/\xi_2 \) with a decreasing temperature is clearly visible in Fig. 1 at \( \Gamma = 0.5 \). This can be explained by the Kramer-Pesch effect [6]. This effect was observed in the \( \mu \)SR investigation of the \( NbSe_2 \) single crystal [1]. The change of the shape of the \( \xi_h(B) \) curve in different fields with an increasing scattering rate \( \Gamma \) is shown in detail in Fig. 2 at \( T/T_c = 0.5 \).
Strong suppression of the $\xi_h/\xi_{c2}$ to values much lower than 1 at an increasing $\Gamma$ is also visible from this figure. This is different from the prediction of the local Usadel theory where $\xi_h$ is scaled by $\xi_{c2}$ and the ratio $\xi_h/\xi_{c2}$ is not dependent on impurity scattering [7, 8]. It means that the nonlocal effects are important for the description of the vortex core even in the "dirty" limit. This is the main result of our paper. The low $\xi_h/\xi_{c2}$ values obtained are consistent with the experimental observation in some high-$\kappa$ low-$T_c$ superconductors.

In Figs. 1 and 2, the normalization constant $\xi_{c2}$ depends on the impurity scattering rate $\Gamma$. It is well known that at a high $\Gamma$, the $\xi_{c2} \sim \sqrt{1/\Gamma} \sim \sqrt{l}$, where $l$ is the mean-free path. Therefore, the decreasing of the ratio $\xi_h/\xi_{c2}$ with $\Gamma$ implies a strong dependence of $\xi_h$ on $l$. It is found that $\xi_h$ in dirty superconductors can be scaled with the relaxation time $\tau$, $\xi_h(B, T, \tau) = \xi_{pure}(B, T)/(1 + \tau_0(B, T))$, where $\xi_{pure}(B, T)$ is the effective coherence length in clean superconductors [3] and $\tau_0$ is a characteristic relaxation time. This results in $\xi_h \sim l$ dependence at a high $\Gamma$ similar to the behavior of the nonlocality radius resulting in the decrease of $\xi_h/\xi_{c2}$ versus $B/B_{c2}$ at a high $\Gamma$, as shown in Fig. 2.

Such a rapid decrease of $\xi_h$ can be compared with the behavior of the another characteristic length $\xi_1$. It has been found that at low temperatures impurity scattering suppresses Kramer-
Pesch effect in $\xi_1(T)$ dependence, resulting in the nonmonotonous behavior of $\xi_1(\Gamma)$. This can be seen from Fig. 3, where the normalization constant $\xi_{BCS}$ is used ($\xi_{BCS}$ is not dependent on $\Gamma$). It is apparent that $\xi_h$ monotonously decreases with $1/\Gamma$ (Fig. 3 (a)) in contrast to the nonmonotonous behavior of $\xi_1$ for a single vortex obtained from Ref. [9] (Fig. 3 (b)). The different behavior of $\xi_h$ and $\xi_1$ is also visible when using the $\Gamma$-dependent normalization constant $\xi_{c2}$. This is shown in Fig. 4, which presents the curves $\xi_h/\xi_{c2}(B/B_{c2})$ at $T/T_{c0} = 0.15$ (Fig. 4 (a)) and $\xi_1/\xi_{c2}(B/B_{c2})$ at $T/T_{c0} = 0.1$ (Fig. 4 (b)) obtained from Ref. [4] at different scattering rates. The curve $\xi_h/\xi_{c2}(B/B_{c2})$ shifts downward with an increasing impurity scattering rate, while the $\xi_1/\xi_{c2}(B/B_{c2})$ curve shifts upward in this case. The high scattering limit for $\xi_1(B)$ dependence (Fig. 4 (b)) looks similar to Usadel theory prediction [7]. However, the different behavior of $\xi_h(B)$ at a high $\Gamma$ (Fig. 4 (a)) means that the local limit [8] is achieved more slowly for $\xi_h$ than for $\xi_1$ or it is not achieved at all. The high scattering limit of the Eilenberger theory for $\xi_1$ was investigated in Ref. [9] for the single vortex problem, and in addition, the “dirty” limit was not observed clearly.

![Figure 4](image_url)

**Figure 4.** (a) The magnetic field dependence of $\xi_h/\xi_{c2}$ at $T/T_{c0} = 0.15$ at different impurity scattering rates $\Gamma$. (b) The magnetic field dependence of $\xi_1/\xi_{c2}$ at $T/T_{c0} = 0.1$ at different impurity scattering rates $\Gamma$ from Ref. [4].

To conclude, the field distribution of the mixed state in dirty $s$-wave superconductors in a wide temperature and field range is investigated in the framework of the nonlocal Eilenberger theory. The normalized magnetic field dependences of the cutoff parameter $\xi_h/\xi_{c2}$ for $\xi_{c2}$ responsible for the line shape of the $\mu$SR resonance are obtained. It is found that this dependence is nonuniversal and depends on the impurity scattering rate $\Gamma$ and the temperature. At high enough values of $\Gamma/2\pi T_{c0} \geq 0.5$, the dependence plateaus in the intermediate field range and the low temperatures, and $\xi_h(B)/\xi_{c2}$ is of the order of 0.25. The strong suppression of $\xi_h/\xi_{c2}$ with $\Gamma$ can explain the experimental results in many low-temperature superconductors ($V_3Si$, $NbSe_{2}$ and $LuNi_{2}B_2C$).

This work was supported by Finnish Cultural Foundation.

References

[1] Sonier J E 2007 *Rep. Prog. Phys.* 70 1717
[2] Hao Z, Clem J R, McElfresh M W, Civale L, Malozemoff A P and Holtzberg F 1991 *Phys. Rev. B* 43 2844
[3] Laiho R, Safonchik M and Traito K B 2008 *Phys. Rev. B* 78 064521
[4] Miranović P, Ichioka M and Machida K 2004 *Phys. Rev. B* 70 104510
[5] Belova P, Traito K B and Lähderanta E 2011 *J. Appl. Phys.* 110 033911
[6] Nakai N, Miranović P, Ichioka M and Machida K 2006 *Phys. Rev. B* 73 172501
[7] Golubov A A and Hartmann U 1994 *Phys. Rev. Lett.* 72 3602
[8] Sonier J E, Kied R F, Brewer J H, Chakhalian J, Dunsiger S R, MacFarlane W A, Miller R I, Wong A, Luke G M and Brill J W 1997 *Phys. Rev. Lett.* 79 1742
[9] Hayashi N, Kato Y and Sigrist M 1995 *J. Low Temp. Phys.* 139 79