Gluon confinement and quantum censorship*

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The dynamical Maxwell-cut, a degeneracy is shown to be a precursor of condensate in the $\phi^4$ and the sine-Gordon models. The difference of the way the Maxwell-cut is obtained is pointed out and quantum censorship, the generation of semiclassically looking phenomenon by loop-corrections is conjectured in the sine-Gordon model. It is argued that quantum censorship and gluon confinement exclude each other.

I. INTRODUCTION

The peculiarity of a condensate arises from its macroscopic occupation. The order of magnitude of the typical quantum fluctuations is $O(\hbar^{1/2})$ in a trivial, perturbative vacuum. When a condensate is present then the field is supposed to display an $\hbar$-independent, $O(\hbar^0)$ expectation value. The expectation value of the square of the field in a state with $n$ particles is $\hbar O(n)$ thus an occupation number $O(\hbar^{-1})$ is needed to make up the condensate.

The macroscopically high occupation number is possible in the absence of strong repulsive forces acting between the particles only. In other words, the presence of a condensate suggests a high degree of degeneracy in the vacuum, a necessary condition of the semiclassical approximation. A dynamical generalization of the Maxwell-cut was found by inspecting the semiclassical contributions to the functional renormalization group equations in theories with condensate[1]. It turned out that the loop contributions may change this picture and quantum censorship was proposed as a mechanism which reduces a semiclassical degeneracy to an approximate degeneracy driven by quantum fluctuations[2, 3].

The high degree of degeneracy indicating the emerging condensate influences the asymptotic sector of a theory in a fundamental manner. When created semiclassically then the degeneracy consists of localized modes. In the quantum censorship scenario the loop contributions are handled in the extended plane wave basis and their way of realizing the degeneracy is based on extended modes. The Yang-Mills vacuum contains a condensate, as well, and the structure of its soft is obviously important for confinement. It is argued below that gluon confinement excludes the building up of the quantum censorship.

II. TREE-LEVEL CONDENSATES

We consider condensate in a weakly coupled Euclidean field theory for a real scalar field $\phi(x)$ where the sharp UV cutoff in momentum space will be lowered by the recursive equation

$$e^{-\frac{\hbar}{2}S_k-\Delta_k[\phi]} = \int D[\phi']e^{-\frac{\hbar}{2}S_k-\Delta_k[\phi+\phi']}.$$  (1)

Here $\phi(x)$ and $\phi'(x)$ denote fields with support in $0 < p < k - \Delta k$ and $k - dk < p < k$, respectively. We follow for simplicity the local potential approximation where the form

$$S_k[\phi] = \int dx \left[ \frac{1}{2} (\partial \phi)^2 + U_k(\phi) \right]$$  (2)

is assumed. Most of the examples mentioned below correspond to the initial condition $U_\Lambda(\phi) = m^2_\phi^2/2 + g_\phi \phi^4/4!$ in four space-time dimensions. The functional integral will be evaluated in the loop-expansion and the Wegner-Houghton equation[4]

$$\partial \kappa U_k(\phi) = -\frac{\hbar k^3}{16\pi^2} \ln[k^2 + U_k''(\phi)],$$  (3)

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follows by assuming trivial saddle points. This is an exact equation because the small parameter of the loop-expansion is actually $\hbar \Delta k/k$ because each loop integral is restricted to a shell of thickness $\Delta k$ in momentum space. But what happens when nontrivial saddle points are encountered?

The simplest strategy to find an answer is to ignore the loop corrections altogether and consider the tree-level evolution only. It turns out that the nontrivial saddle point of the blocking relation (1) is the hallmark of spontaneous symmetry breaking, a condensate in the vacuum. There is no evolution on the tree-level as long as the action reaches its minimum at $\phi'(x) = 0$ within the shell $k - \Delta k < p < k$. But if the potential has negative curvature at the initial condition at $k = \Lambda$ say at $\phi = 0$ then the gradual decrease of the gliding cutoff $k$ leads to nontrivial saddle points at some finite scale $k = k_{cr}$. The evolution from this scale was followed numerically by setting up an iterative process where the cutoff was decreased by a small but finite amount, $k \rightarrow k - \Delta k$ and for each value of the homogeneous field configuration $\Phi$ a saddle point $\phi_{cl}'$ was sought within the family of plane waves with momentum $k$ and the new potential $V \nu_k = S_k[\Phi + \phi_{cl}]$ was calculated. The potential evolved in an interval around zero, $0 < \phi_k$ where $\Phi_k$ increases from zero as $k$ decreases from $k_{cr}$ and the form

$$V_{k}^{\text{inst}}(\phi) = -\frac{k^2}{2}\phi^2$$

was found as shown in Fig. 1.

Such a potential makes the action degenerate at the scale $p = k$ by canceling the kinetic term in. What happened numerically was that the action lost convexity at $k = k_{cr}$ and in the next step, $k \rightarrow k_{cr} - \Delta k$ a shallow, $O(\Delta k/k)$ minimum was found in the action as the function of the plane wave amplitude. This saddle point made the action $S_{k_{cr} - \Delta k}(\phi)$ degenerate for modes with $p = k - \Delta k$ with a slightly larger amplitude. This scenario repeated itself during the iteration, namely the saddle point, found in a shallow minimum made the new action degenerate again. The best description of the dynamics of a mode with a given momentum $p$ is provided by $S_k[\phi]$ with $k = p$, hence the degeneracy of the blocked actions at their cutoff can be interpreted as the indication within the renormalization group scheme that the dynamics is degenerate for all modes with small enough amplitude and momentum $p < k_{cr}$. This is a dynamical Maxwell cut, a precursor of the condensate, and leads to the traditional flattening of the effective potential at $k = 0$, $U_{eff}(\Phi) = V_{k=0}(\Phi)$ between the vacuum expectation values $\langle 0|\phi(x)|0 \rangle = \pm \Phi_{\text{vac}}$. It is natural to interpret the plane wave saddle points as domain walls. There is a saddle point for each scale $p < k_{cr}$ and the functional integration over their soft zero modes, arising from their breaking of the space-time symmetries makes the system strongly coupled as expected by analogies with the mixed phase of first order phase transitions.
FIG. 2: Dependence of the dimensionless degeneracy for vanishing field amplitude, $P_k(0)$ of the action on the scale $k$ for $g_B = 0.8$, the bare mass square values are shown in the figure [2].

Though these results are simple and reasonable, they raise some doubt and concern. The first question concerns the role of loop-correction. Can they alter the conclusion in a qualitative manner? The procedure is based on the sharp cutoff which prevents us from including higher orders of the gradient expansion in the action [2]. Will the general picture remain valid when these terms are taken into account in some manner? The idea of the renormalization group is the construction of the renormalized, full action by integrating a differential equation. The emergence of a saddle point usually signals the appearance of non-analytic terms. Will the evolution equation remain integrable in $k$ at $k_{cr}$?

As of the trouble spots, the first is related to the shallowness of the minimum of the action around the saddle point because it removes the factor $\Delta k/k$ from the small parameter of the loop expansion and we fall back on the $\hbar$-expansion. To make things worse, we even lose the $\hbar$-expansion in the degenerate limit, $\Delta k \to 0$, having a flat integrand. The evolution equation (3) requires the availability of the loop expansion even if the higher orders are resummed in the differential equation limit $\Delta k \to 0$. In general, we have neither analytical nor numerical method at hand to tackle constant integrands, the ultimate strong coupling limit.

Another troubling aspect of the soft zero mode dynamics of the domain walls is that it renders the physics of the mixed phase, $\langle 0|\phi(x)|0\rangle < \Phi_{vac}$ non-perturbative and opens the possibility of having new free, relevant parameter in the scalar model, such as the value of the condensate[13]. The true vacuum lies at the common point of the stable region $\langle 0|\phi(x)|0\rangle > \Phi_{vac}$ and the mixed phase therefore “half” of the quantum fluctuations which tend to decrease $\phi(x)$ might well be non-perturbative.

III. LOOPS-CORRECTIONS

The first of the previous questions is taken up in this section only, the eventual modification of the dynamical Maxwell-cut by including the loop corrections within the ansatz [2]. The potential is non-analytical according to the tree-level analysis therefore the numerical integration of Eq. (4) must be done without assuming a polynomial representation. This is possible by using the spline representation for the potential [2]. It is interesting to follow the evolution of the dimensionless curvature of the action (4) for a mode $p = k$, $P_k(\Phi) = 1 + \partial_k^2 U_k(\Phi)/k^2$, depicted in Fig. 2. It shows a sudden drop at a finite scale followed by the stop of the program. The numerical algorithm makes dynamical adjustment of the precision needed during the integration and stops because the system of linear equations for the spline coefficients becomes nearly singular and requires extreme precision. The value of $k$ where this happens depends slightly on the accuracy but seemed to be not moving significantly up to our limit, several thousand splines. The potential approaches the form (4) at the last steps as can clearly be seen from Fig. 2. The lesson of such a failed attempt to reach the infrared end point suggests a full or nearly complete dynamical Maxwell-cut.

The possible problems of the loop-expansion for a nearly degenerate action is avoided when the evolution of the effective action is followed [3,7], an algorithm with $\Delta k/k$ as the only small parameter. Similar, sudden drop of the
The curvature of the action was observed at finite scale in this scheme when smooth cutoff was used\cite{2}, leaving room for the eventual inclusion of higher orders of the gradient expansion in the ansatz \cite{2}. Naturally the difficulties of the degenerate bare action remain in this scheme in the disguise of the problem of justifying any usable ansatz for the effective action.

Can we decide whether the action is exactly or only nearly degenerate? The analytical efforts presented so far\cite{8–11} leave room for reaching true singularities during integrating the evolution equation in the form of degenerate action involving non-analytical terms\cite{3}. Numerical methods, based on finite amount of computer power can give no satisfactory answer neither. I believe that the following options are left open:

- The evolution equation driven by loop-contributions leads to a truly degenerate action as in the tree-level case. We have no analytical or numerical methods to tackle such models.

- The loop-contributions manage to keep the action regular and generate an approximative dynamical Maxwell-cut. Such a shielding of the semiclassical singularity by quantum fluctuations which make up a similar effect is called Quantum Censorship.

Quantum Censorship seems to be realized in the two-dimensional sine-Gordon model\cite{3} defined by the bare dimensionless potential $\tilde{U}_{k=A}(\phi) = k^{-2}U_B(\phi) = \tilde{u}_B \cos(\sqrt{8\pi} \beta_r \phi)$. The effective potential must be constant, being the only convex periodic function, but this naturally does prevent the model to display highly nontrivial dynamics in the infrared. The tree-level evolution gives saddle points and produces degenerate action\cite{12} for $k < k_{cr} \neq 0$ in the phase $\beta_r < 1$. When the loop-contributions are added then we observe a sudden drop of the curvature of the action, characteristic of the dynamical Maxwell-cut followed by a surprising stabilization of the curvature at very small values\cite{3}.

\section*{IV. YANG-MILLS THEORIES}

We now leave the territory of well established results and make an attempt to interpret the difference of Figs. 3 and 4 and to conjecture about its relevance for Yang-Mills theories.

It seems reasonable to assume that the increase of the accuracy of the numerical algorithm would find a plateau in Fig. 4 for $\beta_r < 0.6$, too. When this state of affairs is compared with Fig. 3 which follows the evolution of models from nearly at the critical point to deep into the symmetry broken phase with the same numerical accuracy then one has the impression that Quantum Censorship is not observed in the non-periodic $\phi^4$ model.

Accepting this interpretation one wonders about the possible source of this difference between the two models. I believe that it is to be found in the non-propagating nature of the excitations in the mixed phase of the $\phi^4$ model. The domain walls formed in a vacuum with $|\langle 0|\phi(x)|0\rangle| < \Phi_{vac}$ can be deformed with small energy investment. As
mentioned before, such a deformation represent the Goldstone modes of the broken space-time symmetries. These modes restore the homogeneity of the vacuum like in a liquid but leave an important imprint on the dynamics by generating dissipation. In the language of solid state physics the sound wave of the mixed phase is damped by the reflection and the traverse of a domain walls and the velocity of sound is reduced to zero. In particle physics this situation is interpreted as having no asymptotic particle states. The domain walls in the small $\beta_r$ phase of two dimensional sine-Gordon model where the periodic symmetry of the theory is dynamically broken\cite{12} are kinks, stable propagating particles. As a result, elementary plane wave excitations have more chance to propagate and the long distance structure of the theory can be reconstructed by means of plane waves. Once this is possible the gradual turning on the plane waves during the evolution may lead us to the correct vacuum.

Let us no turn to Yang-Mills theories. The vacuum of the scalar $\phi^4$ model lies at the boundary of the non-perturbative mixed phase and can safely be approached from the stable region. This is not the case in Yang-Mills theories where the vacuum is within the mixed phase. In fact, the strong chromo-magnetic attraction among gluons is supposed to generated a liquid vacuum filled with condensate\cite{14} which is inhomogeneous\cite{15, 16}, an analogy of the mixed phase of the $\phi^4$ model. The conjecture that the integration of the evolution equation of Yang-Mills theories in the plane wave basis would run into a “naked singularity” seems reasonable when gluons are confinement. In fact, otherwise there should be colored asymptotic states. Though confinement excludes strictly particle-like asymptotic gluon states only, non-particle like asymptotic states are not expected to exists neither according to the general view. In other words, color confinement and Quantum Censorship are exclusive properties. The verification of this conjecture is a challenging question because I think that we lack some technical elements related to gauge symmetry, Wick rotation and the ansatz for the effective action.

There have been impressive advances achieved in applying the functional renormalization group for Yang-Mills theories\cite{17} by relying on gauge fixing and modified Slavnov-Taylor identities. A different method which avoids gauge fixing altogether or is explicitly independent of it would be useful to assure that the singularities arising from the degeneracy of the unphysical sector are properly separated.

Another issue awaiting for careful consideration is the return to real time and Minkowski space-time. The renormalization group studies of the Euclidean theory can shed light on the way the contributions of the off-shell modes pile up as the long distance physics is approached. But confinement of color is beyond this issue, it concerns the dynamics of modes on the mas-shell. A basic element of the renormalization group idea is the successive dealing with the degrees of freedom. The order of their elimination is in principle arbitrary but it is advised to start with simple, perturbative modes and finish with soft, large amplitude fluctuations. In fact, the piling up the informations gained during the elimination process makes the effective dynamics better prepared to deal with the non-perturbative modes at a later stage of the elimination process. Recall the renormalization group approach to fermions at finite density where the blocking zooms into the Fermi sphere in the Brioullin zone instead of the zero momentum point as for bosonic particles. In order to address the confinement problem in Yang-Mills theory with the renormalization group
method we have to zoom into the mass-shell which is possible in Minkowski space-time only. There is reason to suspect that mass-shell singularities are stronger in the Yang-Mills vacuum than for non-confining models. For instance the perturbative collinear divergences are stronger for non-Abelian gauge theories and the view of confinement as an Anderson localization in space-time suggests that pinch-singularities may arise, too. The liquid models of the vacuum indicate the presence of an unusually large number of soft modes which enhance the dressing, as well.

Finally, the truncation of the ansatz used in solving the evolution equation may be critical for models with condensate. The (approximate) dynamical Maxwell-cut can not be obtained in the scalar $\phi^4$ theory or the sine-Gordon model when the loop corrections are collected in a truncated power or Fourier series representation of the local potential. In a similar manner one expects the need of more flexible ansatz than those what has been used so far in Yang-Mills models to address the issue of degeneracy.

V. SUMMARY

Seemingly disparate points are related in this work. The precursor of the formation of a condensate, a large degree of degeneracy when the vacuum is constructed by the successive turning on the modes in the plane wave basis is claimed to be related to gluon confinement. Even if correct, this view does not add much to our understanding of color confinement, it rather orient our attention to some difficulties waiting us along the road. Gauge fixing is an ever returning problem for non-perturbative methods for Yang-Mills models. The Wick rotation and the choice of the ansatz are important problems of the functional renormalization group method independently of their possible role in Yang-Mills theories. Their improvement would be a gain for other domains, too.

Finally, another subjective remark about the functional renormalization group. I think that it is a promising method whose limitation is not yet in sight. It can conveniently interpolate between numerical and more intuitive, analytically based schemes to solve strongly coupled theories. As such, it should ultimately integrate into itself the experiences gained in lattice gauge theory and use them where this latter is not reliable, in real time scattering processes and finite particle density.

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