On the 4D effective theory in warped compactifications with fluxes and branes

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Abstract

We present a systematic way to derive the four-dimensional effective theories for warped compactifications with fluxes and branes in the ten-dimensional type IIB supergravity. The ten-dimensional equations of motion are solved using the gradient expansion method and the effective four-dimensional equations of motions are derived by imposing the consistency condition that the total derivative terms with respect to the six-dimensional internal coordinates vanish when integrated over the internal manifold. By solving the effective four-dimensional equations, we can find the gravitational backreaction to the warped geometry due to the dynamics of moduli fields, branes and fluxes.

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1 Introduction

Warped string compactifications with fluxes and branes have provided a novel approach to long standing problems of particle physics and cosmology. Inspired by the earlier work by Randall and Sundrum \cite{1}, Giddings et al (GKP) \cite{2} showed that, in type IIB string theory, it is possible to realize the warped compactifications that can accommodate the large hierarchy between the electroweak scale and the Planck scale. In the GKP model, the warping of the

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extra-dimensions is generated by the fluxes and the presence of D-branes and orientifold planes. All moduli fields are stabilized due to fluxes except for the universal Kähler modulus. The warped compactifications also provide a promising background for cosmological inflation. An inflaton can be identified with the internal coordinate of a mobile D3 brane moving in a warped throat region [3].

So far, most works are essentially based on effective 4D theories derived by a dimensional reduction. However, it is a non-trivial problem to derive the 4D effective theories including the warping and branes. In fact there have been debates on the validity of the derivation of the 4D effective theories. In a conventional Kaluza-Klein theory, the effective theory can be derived by assuming a factorizable ansatz for the higher-dimensional fields. This approach has been applied even in the presence of the warping. For example, the 10D metric ansatz of the form

$$ds_{10}^2 = h^{-1/2}(y) e^{-6u(x)} g_{\mu\nu}(x) dx^\mu dx^\nu + h^{1/2}(y) e^{2u(x)} \gamma_{mn} dy^m dy^n,$$

is often used where $x$ denotes the coordinates of the 4D non-compact spacetime and $y$ denotes the coordinates of the 6D compact manifold. The function $u(x)$ is identified with the universal Kähler modulus. However, it has been criticized that the higher-dimensional dynamics do not satisfy this ansatz [4]. Thus it is required to derive the 4D effective theory by starting from a correct ansatz and consistently solving the 10D equations of motion.

These attempts were initiated in Refs. [5,6,7]. Ref [5] derived an exact time-dependent 10D solution that describes the instability of the warped compactification due to the non-stabilized Kähler modulus. It was shown that this dynamical solution cannot be described by the metric ansatz (1). Ref. [6] derived the potential for the moduli fields by consistently solving the 10D equations of motion using the metric ansatz that is consistent with the dynamical solutions in the 10D theory. It was found that the universal Kähler modulus is not a simple scaling of the internal metric as is assumed in Eq. (1). A similar consistent ansatz was proposed in Ref [7] and the 4D effective theory without potentials was derived.

In this letter, we build on these works to present a systematic way to derive the 4D effective theory by consistently solving the 10D equations of motion. We exploit the so-called gradient expansion method, which has been shown to be a powerful method to derive the 4D effective theory in the context of 5D brane world models [8]. Our method also provides a scheme to study the gravitational backreaction to the warped geometry due to the moduli dynamics, branes and fluxes.
2 10D equations of motion

Let us start by describing the type IIB supergravity based on Ref. [2]. In the Einstein frame, the bosonic part of the action for the type IIB supergravity is given by

\[
S_{IIB} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left( R - \frac{\partial_M \tau \partial^M \bar{\tau}}{2 (\text{Im}\tau)^2} - \frac{G(3) \cdot \bar{G}(3)}{12 \text{Im}\tau} - \frac{\tilde{F}^2(5)}{4 \cdot 5!} \right) - \frac{i}{8\kappa_{10}^2} \int \frac{C(4) \wedge G(3) \wedge \bar{G}(3)}{\text{Im}\tau},
\]

(2)

where the combined 3-form flux is \( G(3) = F(3) - \tau H(3) \) with \( F(3) = dC(2) \), \( H(3) = dB(2) \) and

\[
\tau = C(0) + ie^{-\phi},
\]

(3)

where \( \phi \) is the so-called dilaton, \( C(j) \) is the Ramond-Ramond potential of rank \( j \) and \( B(2) \) is the NS-NS potential. The 5-form field is given by

\[
\tilde{F}(5) = F(5) - \frac{1}{2} C(2) \wedge H(3) + \frac{1}{2} B(2) \wedge F(3),
\]

(4)

with \( F(5) = dC(4) \). The total action in our model is

\[
S = S_{IIB} + S_{loc},
\]

(5)

where the term \( S_{loc} \) is the action for the localized sources such as D3-branes and O3 planes;

\[
S_{loc} = \sum_j \left( -\mu_3 \int d^4x \sqrt{-g_j} + T_3 \int d^4xC_4 \right),
\]

(6)

where the integrals are calculated over the 4D non-compact space at the point \( j \) in the compact space and \( g_j \) is the determinant of the induced metric on a brane at the point \( j \) (\( \mu_3 \) is positive/negative for D3-branes/O3 planes).

We can derive the equations of motion for the fields from the action (2). The trace reversed Einstein equations are given by

\[
R_{AB} = \frac{\text{Re} \left( \partial_A \bar{\tau} \partial_B \tau \right)}{2 (\text{Im}\tau)^2} + \frac{1}{4 \text{Im}\tau} \left[ \text{Re} \left( G_{ACD} \bar{G}_B^{CD} \right) - \frac{1}{12} G_{CDE} \bar{G}^{CDE(10)} g_{AB} \right] + \frac{1}{96} \tilde{F}_{AP_1...P_4} \bar{F}_B^{P_1...P_4} + \kappa_{10}^2 \left( T_{AB}^{loc} - \frac{1}{8} g_{AB} T^{loc} \right),
\]

(7)

where \( T_{AB}^{loc} \) is the energy-momentum tensor for the localized sources defined
by

\[ T_{AB}^{\text{loc}} = -\frac{2}{\sqrt{\delta^{(10)}g}} \frac{\delta S_{\text{loc}}^{(10)}}{\delta g^{AB}}, \tag{8} \]

and \( g_{AB} \) is the metric for the 10D spacetime.

Following Refs. [6,7], we take the 10D metric as

\[ dS_{10}^2 = h^{-\frac{1}{2}}(x, y) g_{\mu\nu}(x, y) dx^\mu dx^\nu + h^{\frac{1}{2}}(x, y) \gamma_{mn}(y) dy^m dy^n, \tag{9} \]

with

\[ h(x, y) = h_1(y) + h_0(x), \tag{10} \]

where upper case latin indices run from 0 to 9, greek indices run from 0 to 3 (non-compact dimensions) and lower case latin indices run from 4 to 9 (compact dimensions). In the following, all indices are raised by \( g_{\mu\nu} \) and \( \gamma_{mn} \).

The function \( h_0(x) \) is the so-called universal Kähler modulus. It should be emphasized that this is not a simple scaling of the internal metric.

Using our metric ansatz, we can calculate the 10D Ricci tensor. The mixed component is calculated as

\[ R_{\mu\nu} = -g^{\alpha\beta} K_{\mu\beta p|\alpha} + K_{p,\mu} - \frac{1}{2} h^{-1} h_{,\mu} K_p, \tag{11} \]

where we defined

\[ K_{\mu\nu} \equiv -\frac{1}{2} g_{\mu\nu,p}, \quad K_p \equiv g^{\mu\nu} K_{\mu\nu p}, \tag{12} \]

and | denotes the covariant derivative with respect to \( g_{\mu\nu} \), that is,

\[ K_{\mu\delta p|\alpha} \equiv K_{\mu\delta p,\alpha} - \Gamma_{\mu\alpha}^{c} K_{\nu c p} - \Gamma_{\delta\alpha}^{c} K_{\mu c p}, \tag{13} \]

where \( \Gamma_{\mu\alpha}^{c} \) is the Christoffel symbol constructed from \( g_{\mu\nu} \). The non-compact components are given by

\[ R_{\mu\nu} = \Gamma_{\mu\nu}(g) - h^{-1} h_{,\mu\nu} + \frac{1}{4} h^{-2} g_{\mu\nu} h|^{a}_{|\alpha} + \frac{1}{4} h^{-1} g_{\mu\nu} h_{,\alpha}^{a} - \frac{1}{4} h^{-3} h_{,\alpha} h_{,a} h^{,a} g_{\mu\nu} \]

\[ + h^{-1} K_{\mu\nu}^{;b} - \frac{1}{4} h^{-2} h^{,b} K_b g_{\mu\nu} - h^{-1} K_{\mu\nu}^{,b} K_b + 2 h^{-1} K_{\mu\nu}^{,b} K_{\nu c}^{,b}, \tag{14} \]

where ; denotes covariant derivative with respect to \( \gamma_{ab} \), that is,

\[ K_{\mu\nu b,a} \equiv K_{\mu\nu b,a} - \Gamma_{\mu\nu b}^{c} K_{\mu\nu c}, \tag{15} \]

where \( \Gamma_{\mu\nu}^{c} \) is the Christoffel symbol constructed from \( \gamma_{ab} \). The compact components of the Ricci tensor are
\[ R_{ab} = (6) R_{ab}(\gamma) + \frac{1}{4} h^{-2} h_{cd} h^d \gamma_{ab} - \frac{1}{4} h^{-1} h^d \gamma_{ab} - \frac{1}{4} h^{-1} h_{cd} \gamma_{ab} - \frac{1}{2} h^{-2} h_{ca} h_{cb} \]
\[ + K_{ab} - \frac{1}{2} h^{-1} (h_b K_a + h_a K_b) + \frac{1}{4} h^{-1} h^c K_c \gamma_{ab} - K_{ab}^\alpha \gamma^\alpha. \] \tag{16}

Note that the self duality of the 5-form field must be imposed by hand
\[ \tilde{F}_5 = \ast \tilde{F}_5. \] \tag{17}

We shall take the self-dual 5-form field in the form
\[ \tilde{F}_5 = (1 + \ast) \sqrt{-g} d y \alpha(x, y) \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3, \] \tag{18}

Then the self duality condition (17) is automatically satisfied and the only non-zero components of \( \tilde{F}_5 \) are
\[ \tilde{F}_{(5)\mu_0 \cdots \mu_5} = \alpha^{(4)}_{\mu_0 \cdots \mu_5}, \] \tag{19}
\[ \tilde{F}_{(5)n_1 \cdots n_5} = - h^2 \alpha^{(6)}_{n_1 \cdots n_5}, \] \tag{20}

where \( (4) \varepsilon_{\mu_0 \cdots \mu_3} \) denotes the Levi-Civita tensor with respect to \( g_{\mu \nu} \) and \( (6) \varepsilon_{c n_1 \cdots n_5} \) denotes the Levi-Civita tensor with respect to \( \gamma_{mn} \).

### 3 Gradient expansion method

In this section, we will use the gradient expansion method to solve the 10D Einstein equations (7).

#### 3.1 Gradient expansion

The gradient expansion is based on the assumption that \( x \) derivatives are suppressed compared with \( y \) derivatives
\[ \partial^2_x \ll \partial^2_y. \] \tag{21}

Using this assumption, we can reduce the partial differential equations with respect to \( x \) and \( y \) to a set of ordinary differential equations with respect to \( x \). We expand the metric as
\[ g_{\mu \nu}(x, y) = g^{(0)}_{\mu \nu}(x, y) + g^{(1)}_{\mu \nu}(x, y) + \cdots, \] \tag{22}

where the first order quantities are of the order \( \partial^2_x / \partial^2_y \). Accordingly, \( K_{\mu \nu} \) is also expanded as
\[ K_{\mu \nu} = K^{(0)}_{\mu \nu} + K^{(1)}_{\mu \nu} + \cdots. \] \tag{23}
3.2 First order equations

We assume that at zeroth-order, the axion/dilaton is constant and the 3-form field only has non-zero components in the compact space, i.e.

\[ G_{(3)} = \frac{1}{3!} G_{abc}(y) dy^a \wedge dy^b \wedge dy^c. \]  

(24)

Furthermore, we assume that the zeroth-order metric is independent of the compact coordinates

\[ g^{(0)}_{\mu \nu}(x, y) = g^{(0)}_{\mu \nu}(x). \]  

(25)

The Bianchi identity/equation of motion for the 5-form field becomes

\[ \alpha^{c}_{;c} + 2h^{-1} h_{;c} \alpha^{c} = i h^{-2} \frac{G_{pqr} *_{6} \tilde{G}_{pqr}}{12 \text{Im} \tau} + 2 \kappa_{10}^2 h^{-2} T_{3} \rho_{3}^{loc}, \]  

(26)

at zeroth order where \(*_{6}\) is the Hodge dual with respect to \(g_{ab}\) and we defined a rescaled D3 charge density \(\rho_{3}^{loc}\) which does not depend on \(h\). We define \(\omega\) as

\[ \omega = \alpha - h^{-1}. \]  

(27)

Then the Bianchi identity is rewritten as

\[ -h^{\mu}_{;c} = -h^2 \omega^{\mu}_{;c} - 2hh^{\mu} \omega_{;c} + i \frac{G_{pqr} *_{6} \tilde{G}_{pqr}}{12 \text{Im} \tau} + 2 \kappa_{10}^2 T_{3} \rho_{3}^{loc}. \]  

(28)

The non-compact components of the Einstein equations (7) can be rewritten as

\[ R^{(4)}_{\mu \nu} - h^{-1} \left( h_{[\mu} h^{[a]}_{\nu]} - \frac{1}{4} g^{(0)}_{\mu \nu} h^{[a]}_{[\mu} \right) + h^{-1} K^{b}_{\mu b} - \frac{1}{4} h^{-2} h^{b}_{;b} K^{(0)}_{b \mu \nu} + \frac{1}{4} \omega^{c}_{;c} g^{(0)}_{\mu \nu} \]

\[ = - \frac{h^{-2}}{96 \text{Im} \tau} g^{(0)}_{\mu \nu} \left| iG_{(3)} - *_{6} G_{(3)} \right|^2 + \frac{\kappa_{10}^2}{2} h^{-2} g^{(0)}_{\mu \nu} \left( T_{3} \rho_{3}^{loc} - \mu_{3}(y) \right), \]  

(29)

where \(R^{(4)}_{\mu \nu}\) denotes the Ricci tensor constructed from \(g_{\mu \nu}\), \(\mu_{3}(y) = \mu_{3} \delta(y - y_i)/\sqrt{\gamma}\) which is independent of \(h\) and we dropped the non-linear term in \(\omega\). In the same way, the compact equations can be rewritten as
\( (6) R_{ab} - \frac{1}{4} h^{[\delta}_{\gamma} \gamma_{ab} + K^{(1)}_{\delta;b} - \frac{1}{2} h^{-1} \left( h_{b} K^{(1)}_{a} + h_{a} K^{(1)}_{b} \right) + \frac{1}{4} h^{-1} h^{c} K^{(1)}_{c} \gamma_{ab} \)

\[ = \frac{1}{4} h \omega^{c}_{\gamma} \gamma_{ab} + \frac{1}{2} \left( h_{a} \omega_{b} + h_{b} \omega_{a} \right) + \frac{h^{-1}}{4 \text{Im} \tau} \left[ \text{Re} \left( G_{acd} G_{bd} \right) - \frac{1}{12} G_{cde} G_{\gamma_{ab}} - \frac{i}{12} G_{pqr} *_{6} G_{pqr} \gamma_{ab} \right] \]

\[ + \kappa^{2}_{10} \left( T^{\text{loc}}_{ab} - \frac{1}{8} h^{\gamma}_{\gamma} T^{\text{loc}} - \frac{1}{2} h^{-1} T_{3 \rho_{3} \gamma_{ab}} \right), \quad (30) \]

where \( (6) R_{ab} \) denotes the Ricci tensor constructed from \( \gamma_{ab} \).

The GKP solution is obtained by taking

\[ *_{6} G_{(3)} = i G_{(3)}, \quad \omega = 0, \quad (31) \]

with local sources that satisfy \( \mu_{3}(y) = T_{3 \rho_{3} \gamma_{ab}} \). With these conditions, it is straightforward to show that the 10D equations motion are satisfied. Then the warp factor is determined by

\[ -h_{\gamma}^{\gamma} = \frac{1}{12 \text{Im} \tau} G_{pqr} G^{pqr} + 2 \kappa^{2}_{10} T_{3 \rho_{3} \gamma_{ab}}. \quad (32) \]

Our strategy is to assume that \( \omega, (6) R_{ab} \) and the right hand sides of Eqs. (29) and (30) are first order in the gradient expansion and take into account these contributions as a source for the 4D dynamics of \( g_{\mu\nu}(x) \) and \( h_{0}(x) \).

\section{4D Effective equations}

In this section we solve the 10D Einstein equations to get the 4D effective Einstein equations. Hereafter we omit the superscript \( (i) \) that denotes the order of the gradient expansion. A key point is the consistency condition that implies that the integration of the total derivative term over the 6D internal dimension vanishes

\[ \int d^{6} y \sqrt{\gamma} v_{c} = 0, \quad (33) \]

for an arbitrary \( v \). This condition provides the boundary conditions for the 10D gravitational fields and yields the 4D effective equations.

First let us take the trace of Eq. (29);

\[ (4) R + \left( h^{-1} K_{c} + \omega_{c} \right)^{c} = -\frac{h^{-2}}{24 \text{Im} \tau} \left| i G_{(3)} - *_{6} G_{(3)} \right|^{2} + 2 \kappa^{2}_{10} h^{-2} \left( T_{3 \rho_{3} \gamma_{ab}} - \mu_{3}(y) \right). \quad (34) \]

Then integrating Eq. (34) over the internal manifold, we get
\[
(4) \, R = -\frac{1}{24V(6)\operatorname{Im} \tau} \int d^6 y \sqrt{\gamma} h^{-2} \left| iG(3) - \ast_6 G(3) \right|^2
+ 2 \kappa_{10}^2 V(6) \int d^6 y \sqrt{\gamma} h^{-2} \left( T_3 \rho_{3}^{\text{loc}} - \mu_3(y) \right),
\]

where \( V(6) \) is the volume of the internal space

\[
V(6) \equiv \int d^6 y \sqrt{\gamma}.
\]

On the other hand, by combining Eq. (29) with Eq. (34), we obtain the traceless part of the equation

\[
(4) \, R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} (4) \, R - h^{-1} \left( h_{|\mu\nu} - \frac{1}{4} g_{\mu\nu} h|_\delta^\delta \right) + h^{-1} \left( K_{\mu\nu} - \frac{1}{4} g_{\mu\nu} K_b \right)^b = 0.
\]

Integrating this over the compact space we obtain

\[
(4) \, R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} (4) \, R = H^{-1} \left( h_{|\mu\nu} - \frac{1}{4} g_{\mu\nu} h|_\delta^\delta \right),
\]

where the function \( H(x) \) is defined as

\[
H(x) \equiv h_0(x) + C,
\]

where

\[
C \equiv \frac{1}{V(6)} \int d^6 y \sqrt{\gamma} h_1(y).
\]

Here \( h_1(y) \) is obtained by solving Eq. (32). Finally, we need an equation that determines the dynamics of \( h_0(x) \). Let us calculate the trace of Eq. (30)

\[
(6) \, R - \frac{3}{2} h_{|\delta}^\delta + K^{\alpha}_a + \frac{1}{2} h^{-1} h^{\alpha} K_a = \frac{3}{2} h \left( \omega_c^{ic} + \frac{2}{3} h^{-1} h_{,c} \omega_c^{ic} \right)
+ \frac{h^{-1}}{16\operatorname{Im} \tau} \left| iG(3) - \ast_6 G(3) \right|^2 + 3 \kappa_{10}^2 h^{-1} \left( \mu_3(y) - T_3 \rho_{3}^{\text{loc}} \right).
\]

Combining Eq. (41) with Eq. (34) we get

\[
(6) \, R + \frac{1}{2} h^{(4)} - \frac{3}{2} h_{|\mu}^\mu + \left( \frac{3}{2} K_c - h \omega_c \right)^{ic} = \frac{h^{-1}}{24\operatorname{Im} \tau} \left| iG(3) - \ast_6 G(3) \right|^2
+ 2 \kappa_{10}^2 h^{-1} \left( \mu_3 - T_3 \rho_{3}^{\text{loc}} \right).
\]

Then integrating this over the compact space we obtain the equation of motion for \( H(x) \).
\[ H^{[\delta} = \frac{H}{3}^{(4) R} + \frac{2}{3V(6)} \int d^6 y \sqrt{-\gamma} R \]
\[ - \frac{1}{36V(6) \text{Im} \tau} \int d^6 y \sqrt{-\gamma} h^{-1} \left| iG_{(3)} - *_6G_{(3)} \right|^2 \]
\[ - \frac{4 \kappa_{10}^2}{3V(6)} \int d^6 y \sqrt{-\gamma} h^{-1} \left( \mu_3(y) - T_3\rho_3^{\text{loc}} \right). \] (43)

5 The 4D effective theory

The effective 4D equations are summarized as
\[ ^{(4)}G_{\mu\nu} = H^{-1} \left( H_{[\mu\nu} - g_{\mu\nu}H^{[\delta} - V g_{\mu\nu} \right), \] (44)
\[ H^{[\delta} = - \frac{4}{3} V + \frac{2}{3} H \frac{dV}{dH}, \] (45)
where the potential \( V(H) \) is given by
\[ V(H) = - \frac{1}{2V(6)} \int d^6 y \sqrt{-\gamma} R + \frac{1}{48V(6) \text{Im} \tau} \int d^6 y \sqrt{-\gamma} h^{-1} \left| iG_{(3)} - *_6G_{(3)} \right|^2 \]
\[ + \frac{\kappa_{10}^2}{V(6)} \int d^6 y \sqrt{-\gamma} h^{-1} \left( \mu_3(y) - T_3\rho_3^{\text{loc}} \right). \] (46)

They can be deduced from the following 4D effective action
\[ S_{\text{eff}} = \frac{1}{2\kappa_4^2} \int d^4 x \sqrt{-g} \left[ H^{(4) R(g)} - 2V(H) \right], \] (47)
where \( \kappa_4^2 = \kappa_{10}^2 / V(6) \), which can be determined by integrating the 10D action over the six internal dimensions.

Performing the conformal transformation \( g_{\mu\nu} = H^{-1} f_{\mu\nu} \) we can write the previous 4D action in the 4D Einstein frame as
\[ S_E = \frac{1}{2\kappa_4^2} \int d^4 x \sqrt{-f} \left[ R(f_{\mu\nu}) - \frac{3}{2} \left( \nabla \ln H \right)^2 - 2V(H)H^{-2} \right], \] (48)
where now \( \nabla \) denotes the covariant derivative with respect to the Einstein frame metric \( f_{\mu\nu} \). The 4D effective equations of motion in the Einstein frame are given by
\[ R_{\mu\nu}(f) = \frac{3}{2} \left( \nabla_\mu \ln H \right) \left( \nabla_\nu \ln H \right) + V(H)H^{-2}f_{\mu\nu}, \] (49)
\[
\n\nabla_\alpha \nabla^\alpha \ln H = -\frac{4}{3} V(H)H^{-2} + \frac{2}{3} \frac{dV}{dH} H^{-1}. \tag{50}
\]

By defining \( \rho(x) = iH(x) \), the kinetic term can be rewritten into a familiar form

\[
S_{E,kin} = \frac{1}{2 \kappa_4^2} \int d^4x \sqrt{-f} \left[ R - 6 \left( \frac{\partial_\mu \rho \partial^\mu \bar{\rho}}{|\rho - \bar{\rho}|^2} \right) \right]. \tag{51}
\]

We would find the same result for the kinetic term for the universal Kähler modulus even if the wrong metric ansatz Eq. (1) was used to perform a dimensional reduction where \( \rho(x) = i e^{4u(x)} \). In a region where the warping is negligible \( h_1(y) \ll h_0(x) \), \( H(x) \) can be identified as \( e^{4u(x)} \), which is the simple scaling of the internal metric. However, in a region where the warping is not negligible, the original 10D dynamics is completely different between (1) and (9) [7].

The potential associated with the 3-form agrees with the result obtained in Ref. [6]. It should be emphasized that this potential is positive-definite. Nevertheless, the 3-form contribution to the 4D Ricci scalar \( (4)R \) is negative definite [4] in accordance with the no-go theorem for getting de Sitter spacetime [9]. The resolution is the kinetic term of the modulus \( H(x) \), that is, \( (4)R \) is not directly related to \( V \) [6]. In fact the 4D Ricci scalar is related to the potential as

\[
(4)R = H^{-1} \left( 4V + 3H|\delta H|_\delta \right) = 2 \frac{dV}{dH}. \tag{52}
\]

At the minimum of the potential we have \( (4)R = V = dV/dH = 0 \), but, if we move away from the minimum and the modulus \( H(x) \) is moving, the potential cannot be read off from \( (4)R \).

We also notice that D3 branes with \( \mu_3(y) = T_3 \rho_3^{loc} \) do not give any gravitational energy in the 4D effective theory. On the other hand, anti-D3 branes with \( \mu_3(y) = -T_3 \rho_3^{loc} \) give a potential energy. This was used to realize de Sitter vacuum [10].

6 Discussions

In this letter, we presented a systematic way to derive the 4D effective theory by consistently solving the type IIB supergravity equations of motion in warped compactifications with fluxes and branes. We used the gradient expansion method to solve the 10D equations of motion. The consistency condition that the integration of the total derivative terms over the internal 6D space vanishes gives the boundary conditions for the 10D gravitational fields. These boundary conditions give the 4D effective equations. Once the solutions for the 4D effective equations are obtained, we can determine the backreaction to the 10D geometry by solving Eqs. (34), (37) and (42).
In this letter, we did not introduce the stabilization mechanism for the modulus $H(x)$. It is essential to stabilize this universal Kähler modulus to get a viable phenomenology [10]. Usually, non-perturbative effects are assumed to give a potential to this moduli. The non-perturbative effects will modify the 10D dynamics and the 4D effective potential have to be consistent with this 10D dynamics. Most works so far introduce the non-perturbative potentials for the modulus filed directly in the 4D effective theory and it is not clear this is consistent with the original 10D dynamics. It is desirable to derive the potential for the moduli fields by consistently solving the 10D Einstein equation with non-perturbative corrections. In addition, a mobile D3 brane plays a central role to realize inflation [3]. The D3 brane with $\mu_3 = T_3$ probes of a no-scale compactification and the brane can sit at any point of the compact space with no energy cost. In fact, the D3 brane does not give any gravitational energy in the 4D effective theory. However, once the stabilization mechanism is included, this is no longer true. This is in fact an important effect which generally yields a potential for the D3 brane that is not enough flat for slow roll. Recently, it was pointed out that the gravitational backreaction of the D3 brane is essential to calculate the corrections to the potential [11].

Our method can be extended to include the non-perturbative effects by introducing an effective 10D energy-momentum tensor in the 10D Einstein equations. It is also possible to include a moving D3 brane in our scheme along the line of Ref. [12], which studied the dynamics of D-branes with self-gravity in a 5D toy model. Then we can calculate the potential for a mobile D3 brane by taking into account the stabilization and the backreaction of the D3 brane. In addition, it is essential to study how the motion of the D3 brane is coupled to 4D gravity in order to address the dynamics of inflation. We come back to this issue in a future publication.

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