ANALYSIS OF THE Z(4430) AS THE FIRST RADIAL EXCITATION OF THE Zc(3900)

Zhi-Gang Wang
Department of Physics, North China Electric Power University, Baoding 071003, P. R. China

Abstract

In this article, we take the Zc(3900) and Z(4430) as the ground state and the first radial excited state of the axial-vector tetraquark states with JPC = 1+−, respectively, and study their masses and pole residues with the QCD sum rules by calculating the contributions of the vacuum condensates up to dimension-10 in a consistent way in the operator product expansion. The numerical result favors assigning the Zc(3900) and Z(4430) as the ground state and first radial excited state of the axial-vector tetraquark states, respectively.

PACS number: 12.39.Mk, 12.38.Lg
Key words: Tetraquark state, QCD sum rules

1 Introduction

In 2007, the Belle collaboration observed a distinct peak in the \( \pi^+\psi' \) invariant mass distribution in the \( B \to K\pi^+\psi' \) decays with the statistical significance of 6.5\( \sigma \), the mass and width are \( M = (4433 \pm 4 \pm 2) \) MeV and \( \Gamma = (45_{-13}^{+18}^{+30}) \) MeV, respectively [1]. In 2009, the Belle collaboration observed a signal for the decay \( Z(4430)^+ \to \pi^+\psi' \) with a mass \( M = (4443_{-12}^{+15}^{+19}) \) MeV and a width \( \Gamma = (107_{-43}^{+36}^{+56}) \) MeV with a significance of 6.4\( \sigma \) from a Dalitz plot analysis of the decays \( B \to K\pi^+\psi' \) [2]. In 2013, the Belle collaboration performed a full amplitude analysis of the \( B^0 \to \psi'K\pi^- \) decays to constrain the spin and parity of the \( Z(4430)^- \), and observed the \( J^P = 1^+ \) hypothesis is favored over the 0−, 1−, 2− and 2+ hypotheses at the levels of 3.4\( \sigma \), 3.7\( \sigma \), 4.7\( \sigma \) and 5.1\( \sigma \), respectively [3]. Recently, the LHCb collaboration analyzed the \( B^0 \to \psi'\pi^-K^- \) decays by performing a four-dimensional fit of the decay amplitude using pp collision data corresponding to 3fb\(^{-1}\) collected with the LHCb detector, and provided the first independent confirmation of the existence of the \( Z(4430)^- \) resonance and established its spin-parity to be 1+. The measured mass and width are \( M = (4475 \pm 7_{-25}^{+15}) \) MeV and \( \Gamma = (172 \pm 13_{-32}^{+54}) \) MeV, respectively [4]. There have been several tentative assignments of the \( Z(4430) \), such as the threshold effect [5], molecular state [6], tetraquark state [7, 8, 9], baryonium [10], hadro-charmonium state [11], etc.

In 2013, the BESIII collaboration studied the process \( e^+e^- \to \pi^+\pi^-J/\psi \) and observed a structure \( Z_c(3900) \) in the \( \pi^+J/\psi \) mass spectrum with a mass of \( (3899.0 \pm 3.6 \pm 4.9) \) MeV and a width of \( (46\pm10\pm20) \) MeV [12]. Then the structure \( Z_c(3900) \) was confirmed by the Belle and CLEO collaborations [13, 14]. R. Facini et al tentatively identify the \( Z_c(3900) \) as the negative charge conjugation partner of the X(3872) [15], other assignments, such as molecular state [16], tetraquark state [17], hadro-charmonium [18], rescattering effect [19], are also suggested. In Ref. [20], L. Maiani et al take the \( Z(4430) \) as the first radial excitation of the \( Z_c(3900) \) according to the analogous decays,

\[
\begin{align*}
Z_c(3900)_{\pm} & \to J/\psi\pi^\pm, \\
Z(4430)_{\pm} & \to \psi'\pi^\pm.
\end{align*}
\]

The mass differences are \( M_{Z(4430)} - M_{Z_c(3900)} = 576 \) MeV and \( M_{\psi'} - M_{J/\psi} = 589 \) MeV, so it is natural to take the \( Z(4430) \) as the first radial excitation of the \( Z_c(3900) \) [21].

The QCD sum rules is a powerful nonperturbative theoretical tool in studying the ground state hadrons [22, 23]. In Refs. [24, 25], we focus on the scenario of tetraquark states, calculate the vacuum condensates up to dimension-10 in the operator product expansion, study the diquark-antidiquark type scalar, vector, axial-vector, tensor hidden charmed tetraquark states and axial-vector hidden 1E-mail: zgwang@aliyun.com.
bottom tetraquark states systematically with the QCD sum rules, and make reasonable assignments of the X(3872), Z_c(3900), Z_c(3885), Z_c(4020), Z_c(4025), Z(4050), Z(4250), Y(4360), Y(4630), Y(4660), Z_b(10610) and Z_b(10650). In Ref. [23], we focus on the scenario of molecular states, calculate the vacuum condensates up to dimension-10 in the operator product expansion, study the scalar, axial-vector and tensor hadronic molecular states with the QCD sum rules, and make tentative assignments of the X(3872), Z_c(3900), Y(3940), Y(4140), Z_c(4020), Z_c(4025), Z_b(10610) and Z_b(10650). In Refs. [24, 25, 26], we explore the energy scale dependence of the hidden charmed (bottom) tetraquark states and molecular states in details for the first time, and suggest a formula with the effective masses $M_Q$ to determine the energy scales of the QCD spectral densities in the QCD sum rules, which works very well.

In this article, we extend our previous work on the X(3872), Z_c(3900), Z_c(3885) [24], focus on the scenario of tetraquark states, take the Z(4430) as the ground state and first radial excited state of the axial-vector tetraquark states with the symbolic quark structure $[u][d][S][S]$, and study them with the QCD sum rules. The article is arranged as follows: we derive the QCD sum rules for the masses and pole residues of the (4430) in section 2; in section 3, we present the numerical results and discussions; section 4 is reserved for our conclusion.

2 QCD sum rules for the $J^{PC} = 1^{++}$ tetraquark states

In the following, we write down the two-point correlation function $\Pi_{\mu\nu}(p)$ in the QCD sum rules,

$$\Pi_{\mu\nu}(p) = i \int d^4x e^{ipx} \langle 0 | T \{ J_\mu(x) J^\dagger_\nu(0) \} | 0 \rangle,$$

$$J_\mu(x) = \frac{e^{ijk\epsilon nmn}}{\sqrt{2}} \{ u^T(x) C \gamma_5 e^k(x) \bar{d}^m(x) \gamma_\mu C \bar{c}^n(x) - u^T(x) C \gamma_\mu e^k(x) \bar{d}^m(x) \gamma_5 C \bar{c}^n(x) \},$$

the $i, j, k, m, n$ are color indexes, the $C$ is the charge conjugation matrix. We choose the current $J_\mu(x)$ to interpolate the $J^{PC} = 1^{++}$ diquark-antidiquark type tetraquark states $Z_c(3900)$ and $Z(4430)$. Under charge conjugation transform $\hat{C}$, the current $J_\mu(x)$ has the property,

$$\hat{C} J_\mu(x) \hat{C}^{-1} = -J_\mu(x) \big|_{u\leftrightarrow d},$$

which originates from the charge conjugation properties of the scalar and axial-vector diquark states,

$$\hat{C} \left[ \epsilon^{ijk} q^i C \gamma_5 c^k \right] \hat{C}^{-1} = \epsilon^{ijk} \bar{q}^i \gamma_5 c^k,$$

$$\hat{C} \left[ \epsilon^{ijk} q^i C \gamma_\mu c^k \right] \hat{C}^{-1} = \epsilon^{ijk} \bar{q}^i \gamma_\mu c^k.$$

We can insert a complete set of intermediate hadronic states with the same quantum numbers as the current operator $J_\mu(x)$ into the correlation function $\Pi_{\mu\nu}(p)$ to obtain the hadronic representation [22, 23]. After isolating the ground state and the first radial excited state contributions from the pole terms, which are supposed to be the tetraquark states $Z_c(3900)$ and $Z(4430)$, we get the following results,

$$\Pi_{\mu\nu}(p) = \left[ \frac{\lambda^2_{Z_c(3900)}}{M^2_{Z_c(3900)} - p^2} + \frac{\lambda^2_{Z(4430)}}{M^2_{Z(4430)} - p^2} \right] \left( -g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \right) + \cdots,$$

$$= \Pi(p^2) \left( -g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \right) + \cdots,$$
where the pole residues $\lambda_Z$ are defined by

$$\langle 0| J_\mu(0)| Z(p) \rangle = \lambda_Z \varepsilon_\mu,$$

(9)

the $\varepsilon_\mu$ are the polarization vectors of the axial-vector mesons $Z_c(3900)$ and $Z_c(4430)$. The current $J_\mu(x)$ has the $J^{PC} = 1^{+}-$, the $Z_c(3900)$ and $Z_c(4430)$ also have the $J^{PC} = 1^{+}-$ according to the decays $Z_c(3900) \rightarrow J/\psi \pi^\pm$ and $Z_c(4430) \rightarrow J/\psi' \pi^\pm$. The final states $J/\psi \pi^\pm$ and $J/\psi' \pi^\pm$ indicate that the $Z_c(3900)$ and $Z_c(4430)$ must have some $c\bar{c}u\bar{d}$ or $c\bar{c}d\bar{u}$ components at the quark level. The current $J_\mu(x)$ couples potentially to the $Z_c(3900)$ and $Z_c(4430)$. On the other hand, the current $J_\mu(x)$ has non-vanishing couplings with the scattering states $DD^*$, $J/\psi \pi$, $J/\psi \rho$, $\cdots$ [27]. The coupling to the intermediate scattering states $DD^*$, $J/\psi \pi$, $J/\psi \rho$, $\cdots$ modifies the hadronic states $Z_c(3900)$ and $Z_c(4430)$ through self-energy corrections [24]. The renormalized self-energies contribute a finite imaginary part to modify the dispersion relation [24].

$$\Pi(p^2) = -\frac{\lambda^2_{Z_c(3900)}}{p^2 - M^2_{Z_c(3900)} + i \sqrt{p^2 \Gamma_{Z_c(3900)}(p^2)}} - \frac{\lambda^2_{Z_c(4430)}}{p^2 - M^2_{Z_c(4430)} + i \sqrt{p^2 \Gamma_{Z_c(4430)}(p^2)}} + \cdots,$$

(10)

where the physical widths $\Gamma_{Z_c(3900)}(M^2_{Z_c(3900)}) = (46 \pm 10 \pm 20) \text{ MeV}$ and $\Gamma_{Z_c(4430)}(M^2_{Z_c(4430)}) = (172 \pm 13 \pm 37) \text{ MeV}$ are not very large, the zero width approximation in the hadronic spectral densities works [28].

We carry out the operator product expansion to the vacuum condensates up to dimension-10 and take the assumption of vacuum saturation for the higher dimension vacuum condensates. The condensates $\langle \bar{q}q \rangle \langle \bar{q}q \rangle$, $\langle \bar{q}q \rangle^2 \langle \bar{q}q \rangle$, $\langle \bar{q}q \rangle^3 \langle \bar{q}q \rangle$, $\langle \bar{q}q \rangle^2 \langle \bar{q}q \rangle \langle \bar{q}q \rangle \langle \bar{q}q \rangle$ are the vacuum expectations of the operators of the order $O(\alpha_s)$. The condensates $\langle \bar{q}q \rangle^2 \langle \bar{q}q \rangle$, $\langle \bar{q}q \rangle^3 \langle \bar{q}q \rangle$, $\langle \bar{q}q \rangle^2 \langle \bar{q}q \rangle \langle \bar{q}q \rangle \langle \bar{q}q \rangle$ have the dimensions 6, 8, 9 respectively, but they are the vacuum expectations of the operators of the order $O(\alpha_s^3)$, $O(\alpha_s^2)$, $O(\alpha_s^3)$ respectively, and discarded. We take the truncations $n \leq 10$ and $k \leq 1$ in a consistent way, the operators of the orders $O(\alpha_s^k)$ with $k > 1$ are discarded. Furthermore, the values of the condensates $\langle \bar{q}q \rangle^2 \langle \bar{q}q \rangle$, $\langle \bar{q}q \rangle^3 \langle \bar{q}q \rangle$, $\langle \bar{q}q \rangle^2 \langle \bar{q}q \rangle \langle \bar{q}q \rangle \langle \bar{q}q \rangle$ are very small, they can be neglected safely. For the technical details, one can consult Ref. [24].

Once the QCD spectral densities are obtained, we can take the quark-hadron duality below the continuum threshold $s_0$ and perform Borel transform with respect to the variable $P^2 = -p^2$ to obtain the following QCD sum rule:

$$\Pi(T^2) = \lambda^2_{Z_c(3900)} \exp \left( -\frac{M^2_{Z_c(3900)}}{T^2} \right) + \lambda^2_{Z_c(4430)} \exp \left( -\frac{M^2_{Z_c(4430)}}{T^2} \right),$$

(11)

where the effective $c$-quark mass $M_c = 1.8 \text{ GeV}$ [24, 25, 26], to overcome the shortcoming [29].

In Ref. [29], M. S. Maior de Sousa and R. Rodrigues da Silva introduce a new approach to calculate the masses and decay constants of the ground state and first radial excited state with the QCD sum rules. Furthermore, they study the masses and decay constants of the $\rho(1S, 2S)$, $\psi(1S, 2S)$, $\Upsilon(1S, 2S)$ as an application, and observe that the ground state masses are smaller than the experimental values, which is explained as a shortcoming of this approach. In this article, we apply the new approach to study the heavy tetraquark systems, and resort to the energy scale formula

$$\mu = \sqrt{M^2_{\Lambda/\Sigma} - (2M_c)^2},$$

(12)

where the effective $c$-quark mass $M_c = 1.8 \text{ GeV}$ [24, 25, 26], to overcome the shortcoming [29].
ground state \((Z,(3900))\) and the first excited state \(Z(4430)\), respectively, then the QCD sum rule can be written as

\[
\lambda_{i}^{2} \exp(-\tau M_{i}^{2}) + \lambda_{j}^{2} \exp(-\tau M_{j}^{2}) = \Pi_{QCD}(\tau). \tag{13}
\]

We differentiate the QCD sum rule with respect to \(\tau\) to obtain

\[
\lambda_{i}^{2} M_{i}^{2} \exp(-\tau M_{i}^{2}) + \lambda_{j}^{2} M_{j}^{2} \exp(-\tau M_{j}^{2}) = D\Pi_{QCD}(\tau). \tag{14}
\]

Now we have two equations, it is easy to obtain the sum rules,

\[
\lambda_{i}^{2} \exp(-\tau M_{i}^{2}) = \frac{(D - M_{i}^{2}) \Pi_{QCD}(\tau)}{M_{i}^{2} - M_{j}^{2}}, \tag{15}
\]

where \(i \neq j\). Again we differentiate above QCD sum rules with respect to \(\tau\) to obtain

\[
M_{i}^{2} = \frac{(D^2 - M_{j}^{2} D) \Pi_{QCD}(\tau)}{(D - M_{j}^{2}) \Pi_{QCD}(\tau)}, \tag{16}
\]

\[
M_{i}^{4} = \frac{(D^{3} - M_{j}^{2} D^2) \Pi_{QCD}(\tau)}{(D - M_{j}^{2}) \Pi_{QCD}(\tau)}. \tag{18}
\]

The squared masses \(M_{i}^{2}\) satisfy the following equation,

\[
M_{i}^{4} - bM_{i}^{2} + c = 0, \tag{17}
\]

where

\[
b = \frac{D^{3} \otimes D^{0} - D^{2} \otimes D}{D^{2} \otimes D^{0} - D \otimes D},
\]

\[
c = \frac{D^{3} \otimes D - D^{2} \otimes D^{2}}{D^{2} \otimes D^{0} - D \otimes D},
\]

\[
D^{j} \otimes D^{k} = D^{j} \Pi_{QCD}(\tau) D^{k} \Pi_{QCD}(\tau), \tag{18}
\]

\(i = 1, 2, j, k = 0, 1, 2, 3\). The solutions are

\[
M_{1}^{2} = \frac{b - \sqrt{b^{2} - 4c}}{2},
\]

\[
M_{2}^{2} = \frac{b + \sqrt{b^{2} - 4c}}{2}. \tag{19}
\]

### 3 Numerical results and discussions

The input parameters are taken to be the standard values \(\langle \bar{q}q \rangle = -0.24 \pm 0.01 \text{ GeV}^{3}\), \(\langle \bar{q}q, \sigma Gq \rangle = m_{0}^{2} \langle \bar{q}q \rangle\), \(m_{0}^{2} = (0.8 \pm 0.1) \text{ GeV}^{2}\), \(\langle \sigma_{QG} \rangle = (0.33 \text{ GeV})^{4}\) at the energy scale \(\mu = 1 \text{ GeV}\) \cite{22,23,29,31}. The quark condensate and mixed quark condensate evolve with the renormalization group equation,

\[
\langle \bar{q}q \rangle(\mu^{2}) = \langle \bar{q}q \rangle(Q^{2}) \left[ \frac{\alpha_{s}(Q^{2})}{\alpha_{s}(\mu^{2})} \right]^{\frac{3}{2}},
\]

\(\langle \bar{q}q, \sigma Gq \rangle(\mu^{2}) = \langle \bar{q}q, \sigma Gq \rangle(Q^{2}) \left[ \frac{\alpha_{s}(Q^{2})}{\alpha_{s}(\mu^{2})} \right]^{\frac{3}{2}}\).

In the article, we take the \(\overline{MS}\) mass \(m_{c}(m_{c}^{2}) = (1.275 \pm 0.025) \text{ GeV}\) from the Particle Data Group \cite{27}, and take into account the energy-scale dependence of the \(\overline{MS}\) mass from the renormalization group equation,

\[
m_{c}(\mu^{2}) = m_{c}(m_{c}^{2}) \left[ \frac{\alpha_{s}(\mu)}{\alpha_{s}(m_{c})} \right]^{\frac{3}{2}},
\]

\[
\alpha_{s}(\mu) = \frac{1}{b_{0}t} \left[ 1 - \frac{b_{1} \log t}{b_{0}^{2} t} + \frac{b_{2} (\log^{2} t - \log t - 1) + b_{0} b_{2}}{b_{0}^{4} t^{2}} \right]. \tag{20}
\]
where \( t = \log \frac{\mu^2}{\Lambda^2} \), \( b_0 = \frac{33-2n_f}{12\pi}, \) \( b_1 = \frac{153-19n_f}{24\pi}, \) \( b_2 = \frac{2857-3533n_f+324n_f^2}{128\pi^3}, \) \( \Lambda = 213 \text{ MeV}, \) 296 \text{ MeV} and 339 MeV for the flavors \( n_f = 5, 4 \) and 3, respectively \[27].

The mass and width from the LHCb collaboration are \( M_{Z(4430)} = (4475 \pm 7^{+15}_{-25}) \) MeV and \( \Gamma_{Z(4430)} = (172 \pm 13^{+37}_{-34}) \) MeV, respectively \[1\]. We take the continuum threshold parameter as \( \sqrt{s_0} = (4.7 - 4.9) \) GeV tentatively to avoid the contaminations from the higher resonances and continuum states, here we have assumed that the energy gap between the first radial excited state and the second radial excited state is about \( (0.3 \pm 0.1) \) GeV, which is smaller than the energy gap \( (0.5 \pm 0.1) \) GeV between the ground state and the first radial excited state. If we take the Borel parameter as \( T^2 = (2.7 - 3.3) \) GeV\(^2\), the pole contribution is \( (55 - 80)\% ((64 - 86)\%) \) at the typical energy scale \( \mu = 1.5 \) GeV \( (\mu = 2.7 \) GeV\). In Ref.\[24\], we take the parameters \( \sqrt{s_0} = (4.3-4.5) \) GeV, \( T^2 = (2.2 - 2.8) \) GeV\(^2\) and \( \mu = 1.5 \) GeV to study the ground state \( Z_c(3900) \), the two criteria (pole dominance and convergence of the operator product expansion) of the QCD sum rules are fully satisfied. In the present case, if we take the parameters \( \sqrt{s_0} = (4.7 - 4.9) \) GeV, \( T^2 = (2.4 - 3.8) \) GeV\(^2\) and \( \mu = 1.5 \) GeV, the two criteria of the QCD sum rules are also satisfied.

Firstly, we choose the continuum threshold parameter as \( \sqrt{s_0} = (4.7 - 4.9) \) GeV and the Borel parameter as \( T^2 = (2.4 - 3.8) \) GeV\(^2\), take the masses \( M_{Z_c(3900)} = 3899 \) MeV and \( M_{Z(4430)} = 4475 \) MeV as input parameters, fit the pole residues \( \lambda_{Z_c(3900)} \) and \( \lambda_{Z(4430)} \) as free parameters with the MINUIT, and obtain the results,

\[
\lambda_{Z_c(3900)} = (1.9977 \pm 0.0856) \times 10^{-2} \text{ GeV}^5, \\
\lambda_{Z(4430)} = (3.6186 \pm 0.2248) \times 10^{-2} \text{ GeV}^5, \tag{21}
\]

at the energy scale \( \mu = 1.5 \) GeV and

\[
\lambda_{Z_c(3900)} = (3.5125 \pm 0.4098) \times 10^{-2} \text{ GeV}^5, \\
\lambda_{Z(4430)} = (3.3554 \pm 1.9965) \times 10^{-2} \text{ GeV}^5, \tag{22}
\]

at the energy scale \( \mu = 2.7 \) GeV.

In Ref.\[24\], we obtain the mass and pole residue of the \( Z_c(3900) \) with the single pole QCD sum rules,

\[
M_{Z_c(3900)} = 3.91^{+0.11}_{-0.09} \text{ GeV}, \\
\lambda_{Z_c(3900)} = 2.20^{+0.36}_{-0.23} \times 10^{-2} \text{ GeV}^5. \tag{23}
\]

The value of the pole residue \( \lambda_{Z_c(3900)} \) obtained in the present work at the energy scale \( \mu = 1.5 \) GeV is compatible with that of Ref.\[24\]. In Fig.1, we plot the central values of the Borel transformed correlation function \( \Pi(T^2) \) at both the QCD side and the hadron side at the energy scale \( \mu = 1.5 \) GeV. From the figure, we can see that the two curves coincide, the fitting is excellent, on the other hand, the corresponding two curves also coincide at the energy scale \( \mu = 2.7 \) GeV. At the two typical energy scales \( \mu = 1.5 \) GeV and \( 2.7 \) GeV, we can take the masses of the \( Z_c(3900) \) and \( Z(4430) \) as basic input parameters, and choose suitable pole residues to reproduce the Borel transformed correlation function \( \Pi(T^2) \) at the QCD side. The QCD sum rules favor assigning the \( Z(4430) \) as the first radial excitation of the \( Z_c(3900) \) with the \( J^{PC} = 1^{+} \).

In Refs.\[24\] \[25\] \[26\], we calculate the vacuum condensates up to dimension-10 in the operator product expansion, study the hidden charmed (bottom) tetraquark states and molecular states systematically with the QCD sum rules, and explore the energy scale dependence of the hidden charmed (bottom) tetraquark states and molecular states in details for the first time, and suggest a formula

\[
\mu = \sqrt{M_{X/Y/Z}^2 - (2M_Q)^2}, \tag{24}
\]

to determine the energy scales of the QCD spectral densities. In the present case, if we resort to the formulae in Eqs.(15-19) to study the masses and pole residues of the \( Z_c(3900) \) and \( Z(4430) \) as
the ground state and the first radial excited state of the $J^{PC} = 1^{+-}$ tetraquark states, respectively, the optimal energy scales are $\mu = 1.5 \text{ GeV}$ and $2.7 \text{ GeV}$ for the QCD sum rules of the $Z_c(3900)$ and $Z(4430)$, respectively, the shortcoming in Ref.[29] is overcome. At the energy scale $\mu = 1.5 \text{ GeV}$ ($2.7 \text{ GeV}$), we can obtain the physical value $M_{Z_c(3900)}$ ($M_{Z(4430)}$), the associate value $M_{Z_c(3900)}$ ($M_{Z_c(3900)}$) from the coupled Eqs.(18-19) is not necessary the physical value, and is discarded.

Now we take into account the uncertainties and obtain the values of the masses and pole residues of the $Z_c(3900)$ and $Z(4430)$,

$$M_{Z_c(3900)} = 3.91^{+0.21}_{-0.17} \text{ GeV}, \quad \text{Experimental value } 3899.0 \pm 3.6 \pm 4.9 \text{ MeV [12]},$$
$$M_{Z(4430)} = 4.70^{+0.98}_{-0.17} \text{ GeV},$$
$$\lambda_{Z_c(3900)} = 2.23^{+1.02}_{-0.58} \times 10^{-2} \text{ GeV}^5,$$
$$\lambda_{Z(4430)} = 4.19^{+3.83}_{-0.76} \times 10^{-2} \text{ GeV}^5,$$

at the energy scale $\mu = 1.5 \text{ GeV}$ and

$$M_{Z_c(3900)} = 3.58^{+0.16}_{-0.11} \text{ GeV},$$
$$M_{Z(4430)} = 4.51^{+0.17}_{-0.09} \text{ GeV}, \quad \text{Experimental value } 4475 \pm 7^{+15}_{-25} \text{ MeV [4]},$$
$$\lambda_{Z_c(3900)} = 1.95^{+0.61}_{-0.26} \times 10^{-2} \text{ GeV}^5,$$
$$\lambda_{Z(4430)} = 5.75^{+0.98}_{-0.78} \times 10^{-2} \text{ GeV}^5,$$

at the energy scale $\mu = 2.7 \text{ GeV}$. Then we take the central values of the masses and pole residues, and obtain the corresponding pole contributions,

$$\text{pole}_{Z_c(3900)} = (38 - 62)\%,$$
$$\text{pole}_{Z(4430)} = (17 - 18)\%,$$

at the energy scale $\mu = 1.5 \text{ GeV}$ and

$$\text{pole}_{Z_c(3900)} = (34 - 56)\%,$$
$$\text{pole}_{Z(4430)} = 30\%,$$
at the energy scale $\mu = 2.7$ GeV. The pole contribution of the $Z_c(3900)$ ($Z(4430)$) at the energy scale $\mu = 1.5$ GeV (2.7 GeV) is a larger than that at the energy scale $\mu = 2.7$ GeV (1.5 GeV), we prefer to extract the mass and pole residue of the $Z_c(3900)$ ($Z(4430)$) at the energy scale $\mu = 1.5$ GeV (2.7 GeV) and discard the ones at the energy scale $\mu = 2.7$ GeV (1.5 GeV), and refer to the values $M_{Z_c(3900)} = 3.91^{+0.21}_{-0.17}$ GeV, $M_{Z(4430)} = 4.51^{+0.17}_{-0.09}$ GeV, $\lambda_{Z_c(3900)} = 2.23^{+1.02}_{-0.58} \times 10^{-2}$ GeV$^5$, $\lambda_{Z(4430)} = 5.75^{+0.98}_{-0.75} \times 10^{-2}$ GeV$^5$ as the physical values, which are shown explicitly in Figs.8-3. The predicted masses $M_{Z_c(3900)} = 3.91^{+0.21}_{-0.17}$ GeV and $M_{Z(4430)} = 4.51^{+0.17}_{-0.09}$ GeV satisfy the energy scale formula in Eq.(24).

The predicted masses $M_{Z_c(3900)} = 3.91^{+0.21}_{-0.17}$ GeV and $M_{Z(4430)} = 4.51^{+0.17}_{-0.09}$ GeV are in excellent agreement with the experimental data, the present calculations favor assigning the $Z(4430)$ as the first radial excited state of the $Z_c(3900)$. At the energy scale $\mu = 1.5$ GeV, the values of the pole residue $\lambda_{Z_c(3900)}$ from the numerical fitting, the single-pole QCD sum rules [24] and the double-pole QCD sum rules are consistent with each other. At the energy scale $\mu = 2.7$ GeV, the values of the pole residue $\lambda_{Z(4430)}$ from the numerical fitting and the double-pole QCD sum rules are not consistent, as the pole residue $\lambda_{Z(4430)}$ is sensitive to the mass $M_{Z(4430)}$.

The parameters $M_{Z_c(3900)}$, $M_{Z(4430)}$, $\lambda_{Z_c(3900)}$, $\lambda_{Z(4430)}$ are not independent, they correlate with each other. For example, at the neighborhood of the values $M_{Z_c(3900)} = 3.899$ GeV, $M_{Z(4430)} = 4.475$ GeV, $\lambda_{Z_c(3900)} = 1.9977 \times 10^{-2}$ GeV$^5$, $\lambda_{Z(4430)} = 3.6186 \times 10^{-2}$ GeV$^5$, we can obtain the relations,

$$
M_{Z_c(3900)} \uparrow \quad \rightarrow \quad M_{Z(4430)} \downarrow, \quad \lambda_{Z_c(3900)} \downarrow, \\
M_{Z_c(3900)} \downarrow \quad \rightarrow \quad M_{Z(4430)} \uparrow, \quad \lambda_{Z(4430)} \uparrow, \\
\lambda_{Z_c(3900)} \uparrow \quad \rightarrow \quad M_{Z(4430)} \uparrow, \quad \lambda_{Z_c(3900)} \uparrow, \\
\lambda_{Z_c(3900)} \downarrow \quad \rightarrow \quad M_{Z(4430)} \downarrow, \quad \lambda_{Z(4430)} \downarrow,
$$

(29)

from the QCD sum rule in Eq.(11) at the energy scale $\mu = 1.5$ GeV, the small variations of the $M_{Z_c(3900)}$ and $\lambda_{Z_c(3900)}$ can lead to rather large changes of the $M_{Z(4430)}$ and $\lambda_{Z(4430)}$. In Eqs.(21-22), we take the experimental values $M_{Z_c(3900)} = 3899$ MeV and $M_{Z(4430)} = 4475$ MeV as input parameters, so the fitted parameters $\lambda_{Z_c(3900)}$ and $\lambda_{Z(4430)}$ are not as robust as the ones from the QCD sum rules in Eqs.(15-19).

We may expect to calculate the masses and pole residues of the ground state and the first radial excited state of the $1^{++}$ tetraquark states at the same energy scale. In Fig.4, the masses of the ground state and the first radial excited state are plotted with variations of the Borel parameters $T^2$ and energy scales $\mu$. From the figure, we can see that the masses decrease monotonously with increase of the energy scales, it is impossible to reproduce the experimental values $M_{Z_c(3900)} = (3899.0 \pm 3.6 \pm 4.9)$ MeV and $M_{Z(4430)} = (4475 \pm 7 \pm 15)$ MeV at the same energy scale, just as in the case of the $\rho(1S,2S)$, $\psi(1S,2S)$ and $\Upsilon(1S,2S)$ [29].

In this article, we take the threshold parameters as $\sqrt{s_0} = (4.7 - 4.9)$ GeV and Borel parameters as $T^2 = (2.7 - 3.3)$ GeV$^2$, then

$$
\exp \left( -\frac{s_0}{T^2} \right) = e^{-8.9} \sim e^{-6.7},
$$

(30)

the continuum states are greatly depressed. The predictions are not sensitive to the continuum threshold parameters, although the masses and pole residues increase with increase of the threshold parameters. At the Borel window $T^2 = (2.7 - 3.3)$ GeV$^2$, the masses and pole residues are rather stable with variations of the Borel parameters, platforms appear, so the predictions are reasonable.

Now we perform Fierz re-arrangement to the current $J_\mu$ both in the color and Dirac-spinor spaces and obtain the following result,

$$
J^\mu = \frac{1}{2\sqrt{2}} \left\{ i\bar{c}i\gamma_5c\bar{d}\gamma^\mu u - i\bar{c}\gamma^\mu c\bar{d}\gamma_5 u + \bar{c}u\bar{d}\gamma^\mu\gamma_5 c - \bar{c}\gamma^\mu\gamma_5 u\bar{d}c \\
- i\bar{c}\gamma^\mu\gamma_5 c\bar{d}\sigma^{\mu\nu} u + i\bar{c}\sigma^{\mu\nu} c\bar{d}\gamma_5 u - i\bar{c}\sigma^{\mu\nu} c\bar{d}\gamma_5 u + i\bar{c}\gamma_5 u\bar{d}\sigma^{\mu\nu} \gamma_5 c \right\},
$$

(31)
Figure 2: The masses of the ground state and the first radial excited state of the $1^{++}$ tetraquark states with variations of the Borel parameters $T^2$.

Figure 3: The pole residues of the ground state and the first radial excited state of the $1^{++}$ tetraquark states with variations of the Borel parameters $T^2$. 
the components such as $\bar{c}i\gamma_5 c\bar{d}\gamma^\mu u$, $\bar{c}\gamma^\mu c\bar{d}\gamma_5 u$, etc couple to the meson-meson pairs, the strong decays

\begin{align}
Z^\pm_c(3900)(1^{+-}) &\rightarrow h_c(1P)\pi^\pm, J/\psi\pi^\pm, \eta_c\rho^\pm, \eta_c(\pi\pi)_p^\pm, \\
Z^\pm(4430)(1^{+-}) &\rightarrow h_c(2P)\pi^\pm, \psi'\pi^\pm, \eta'_c\rho^\pm, \eta'_c(\pi\pi)_p^\pm, h_c(1P)\pi^\pm, J/\psi\pi^\pm, \eta_c\rho^\pm, \\
&\eta_c(\pi\pi)_p^\pm, \eta_c h_1(1170)^\pm, (D_0^* (2400) D)^\pm, (D^* D^*)^\pm, 
\end{align}

are Okubo-Zweig-Iizuka (OZI) super-allowed, we take the decays to the $\pm(\pi\pi)$ final states as OZI super-allowed according to the decays $\rho \rightarrow \pi\pi$. We can search for the $Z^\pm_c(3900)(1^{+-})$ and $Z^\pm(4430)(1^{+-})$ in those strong decays.

4 Conclusion

In this article, we take the $Z_c(3900)$ and $Z(4430)$ as the ground state and the first radial excited state of the axial-vector tetraquark states respectively with the symbolic quark structure $[cu]_{s=1} [\bar{c}\bar{d}]_{s=0} - [cu]_{s=0} [\bar{c}\bar{d}]_{s=1}$, and study their masses and pole residues with the QCD sum rules by calculating the contributions of the vacuum condensates up to dimension-10 in a consistent way in the operator product expansion. The numerical result favors assigning the $Z_c(3900)$ and $Z(4430)$ as the ground state and the first radial excited axial-vector tetraquark states, respectively. We can search for the $Z_c(3900)$ and $Z(4430)$ in the OZI super-allowed decays listed in Sect.3 in the future.

Acknowledgements

This work is supported by National Natural Science Foundation, Grant Number 11375063, and Natural Science Foundation of Hebei province, Grant Number A2014502017.
References

[1] S. K. Choi et al, Phys. Rev. Lett. **100** (2008) 142001.
[2] R. Mizuk et al, Phys. Rev. **D80** (2009) 031104.
[3] K. Chilikin et al, Phys. Rev. **D88** (2013) 074026.
[4] R. Aaij et al, Phys. Rev. Lett. **112** (2014) 222002.
[5] J. L. Rosner, Phys. Rev. **D76** (2007) 114002.
[6] C. Meng and K. T. Chao, [arXiv:0708.4222](http://arxiv.org/abs/0708.4222); S. H. Lee, A. Mihara, F. S. Navarra and M. Nielsen, Phys. Lett. **B661** (2008) 28; X. Liu, Y. R. Liu, W. Z. Deng and S. L. Zhu, Phys. Rev. **D77** (2008) 034003; G. J. Ding, [arXiv:0711.148](http://arxiv.org/abs/0711.148); E. Braaten and M. Lu, Phys. Rev. **D79** (2009) 051503; T. Branz, T. Gutsche and V. E. Lyubovitskij, Phys. Rev. **D82** (2010) 054025.
[7] K. M. Cheung, W. Y. Keung and T. C. Yuan, Phys. Rev. **D76** (2007) 117501; Y. Li, C. D. Lu and W. Wang, Phys. Rev. **D77** (2008) 054001; X. H. Liu, Q. Zhao and F. E. Close, Phys. Rev. **D77** (2008) 094005; D. Ebert, R. N. Faustov and V. O. Galkin, Eur. Phys. J. **C58** (2008) 399; M. E. Bracco, S. H. Lee, M. Nielsen and R. Rodrigues da Silva, Phys. Lett. **B671** (2009) 240.
[8] L. Maiani, A. D. Polosa and V. Riquer, New J. Phys. **10** (2008) 073004.
[9] Z. G. Wang, Eur. Phys. J. **C70** (2010) 139.
[10] C. F. Qiao, J. Phys. **G35** (2008) 075008.
[11] S. Dubynskiy and M. B. Voloshin, Phys. Lett. **B666** (2008) 344.
[12] M. Ablikim et al, Phys. Rev. Lett. **110** (2013) 252001.
[13] Z. Q. Liu et al, Phys. Rev. Lett. **110** (2013) 252002.
[14] T. Xiao, S. Dobbs, A. Tomaradze and K. K. Seth, Phys. Lett. **B727** (2013) 366.
[15] R. Faccini, L. Maiani, F. Piccinini, A. Pilloni, A. D. Polosa and V. Riquer, Phys. Rev. **D87** (2013) 111102(R).
[16] F. K. Guo, C. Hidalgo-Duque, J. Nieves and M. P. Valderrama, Phys. Rev. **D88** (2013) 054007; J. R. Zhang, Phys. Rev. **D87** (2013) 116004; Y. Dong, A. Faessler, T. Gutsche and V. E. Lyubovitskij, Phys. Rev. **D88** (2013) 014030; H. W. Ke, Z. T. Wei and X. Q. Li, Eur. Phys. J. **C73** (2013) 2561; S. Prelovsek and L. Leskovec, Phys. Lett. **B727** (2013) 172; C. Y. Cui, Y. L. Liu, W. B. Chen and M. Q. Huang, J. Phys. **G41** (2014) 075003.
[17] M. Karliner and S. Nussinov, JHEP **1307** (2013) 153; N. Mahajan, [arXiv:1304.1301](http://arxiv.org/abs/1304.1301); J. M. Dias, F. S. Navarra, M. Nielsen and C. M. Zanetti, Phys. Rev. **D88** (2013) 016004; E. Braaten, Phys. Rev. Lett. **111** (2013) 162003.
[18] M. B. Voloshin, Phys. Rev. **D87** (2013) 091501.
[19] D. Y. Chen, X. Liu and T. Matsuki, Phys. Rev. Lett. **110** (2013) 232001; Q. Wang, C. Hanhart and Q. Zhao, Phys. Rev. Lett. **111** (2013) 132003; Q. Wang, C. Hanhart and Q. Zhao, Phys. Lett. **B725** (2013) 106; X. H. Liu and G. Li, Phys. Rev. **D88** (2013) 014013.
[20] L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. **D89** (2014) 114010.
[21] M. Nielsen and F. S. Navarra, Mod. Phys. Lett. **A29** (2014) 1430005.
[22] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B147 (1979) 385; Nucl. Phys. B147 (1979) 448.

[23] L. J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rept. 127 (1985) 1.

[24] Z. G. Wang and T. Huang, Phys. Rev. D89 (2014) 054019.

[25] Z. G. Wang, Eur. Phys. J. C74 (2014) 2874; Z. G. Wang, arXiv:1312.1537; Z. G. Wang and T. Huang, Nucl. Phys. A930 (2014) 63.

[26] Z. G. Wang and T. Huang, Eur. Phys. J. C74 (2014) 2891; Z. G. Wang, Eur. Phys. J. C74 (2014) 2963.

[27] K. A. Olive et al, Chin. Phys. C38 (2014) 090001.

[28] Z. G. Wang, Int. J. Theor. Phys. 51 (2012) 507.

[29] M. S. Maior de Sousa and R. Rodrigues da Silva, arXiv:1205.6793.

[30] B. L. Ioffe, Prog. Part. Nucl. Phys. 56 (2006) 232.

[31] P. Colangelo and A. Khodjamirian, hep-ph/0010175.