An $SU(2)_L \times U(1)_Y \times S_3 \times D$ model
for atmospheric and solar neutrino deficits

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Abstract

Motivated by the recent Super-Kamiokande experiment on atmospheric and solar neutrino oscillation we propose a see-saw model of three generations of neutrinos based on the gauge group $SU(2)_L \times U(1)_Y$ with discrete symmetries ($S_3 \times D$) and three right handed singlet neutrinos so that this model can accommodate the recent Super-Kamiokande data on atmospheric and solar neutrino oscillations. The model predicts maximal mixing between $\nu_\mu$ and $\nu_\tau$ with $\sin^2 2\theta_{\mu\tau} = 1$ as required by the atmospheric neutrino data and small mixing between $\nu_e$ and $\nu_\mu$ with $\sin^2 2\theta_{e\mu} \sim (10^{-2} - 10^{-3})$ as a possible explanation of the solar neutrino deficit through the MSW mechanism. The model admits two mass scales of which one breaks the electroweak symmetry and the other is responsible for the breaking of the lepton number symmetry at GUT scale leading to small Majorana mass of the left handed doublet neutrinos.

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1 Introduction

Recent results from the Super-Kamiokande experiment on atmospheric neutrino anomaly [1] and solar neutrino deficit [2] have supported neutrino flavour oscillation as possible explanation of these effects. Atmospheric neutrino data are consistent with $\nu_\mu \rightarrow \nu_\tau$ oscillations with $\Delta m_{\mu\tau}^2 = (0.5 - 6) \times 10^{-3} eV^2$ and a nearly maximal mixing $\sin^2 2\theta_{\mu\tau} \geq 0.82$. The data are equally consistent if $\nu_\tau$ is replaced by a sterile neutrino and several authors have considered models with an extra light singlet neutrino, in addition to the usual three heavy right handed singlet neutrinos [3].

Furthermore, Super-Kamiokande data on solar neutrino deficit allow a solution in terms of $\nu_e$ disappearance vacuum oscillation [4] with $\Delta m_{e\mu}^2 \sim 10^{-10} eV^2$ and a nearly maximal mixing angle $\sin^2 2\theta_{e\mu} \sim 1.0$. Also consistent with these data are the small angle matter enhanced solution with $\Delta m_{e\mu}^2 \sim (0.5 - 1) \times 10^{-5} eV^2$ and $\sin^2 2\theta_{e\mu} \sim 10^{-2} - 10^{-3}$ [5] as well as matter enhanced large angle solution with $\Delta m_{e\mu}^2 \sim (10^{-5} - 10^{-4}) eV^2$ and $\sin^2 2\theta_{e\mu} \sim 1.0$ [6] although the small angle is more likely at the moment. The recent CHOOZ [7] reactor result excludes the large mixing angle neutrino oscillation of $\nu_\mu \rightarrow \nu_e$ as far as $\Delta m_{\mu e}^2 \geq 9 \times 10^{-4} eV^2$. While the results of CHORUS [8] and NOMAD [9] on $\nu_\mu$ oscillations are awaited and the LSND [10] data, which are disfavoured by the KARMEN [11] experiment, needs confirmation from future experiments, several authors have proposed the strategy of fitting the Super-Kamiokande data on atmospheric and solar neutrinos discarding the LSND data to discuss the problem of neutrino flavour oscillations within the framework of three generations of neutrinos. Such studies have shown that two mass squared differences $\Delta m_{solar}^2$ and $\Delta m_{atmos}^2$ are relevant and the hierarchy of light neutrino masses such as $m_1, m_2 >> m_3$ or $m_1, m_2 << m_3$ is adequate to explain the recent Super-Kamiokande data on neutrino oscillations. Another type of study [12] investigating the impact of neutrino oscillation on the texture of neutrino mass matrices in a model independent way led to maximal mixing and appropriate mass squared difference, which are necessary to explain the Super-Kamiokande data. Recently several authors [13,14] have studied see-saw models based on the extension of the standard model including two singlet neutrinos with an extra $U(1)'$ gauge group corresponding to a newly defined gauge charge, the gauge charge being $(B - 3L_e)$ for Ref.[13] and $(B - 3/2[L_\mu + L_\tau])$ for Ref.[14]. Assuming the $U(1)'$ symmetry breaking scale to be of the order of ($\sim 10^{12} - 10^{16}$)
GeV, the model naturally accounts for the large(small) mixing solutions to the atmospheric (solar) neutrino oscillations and explains the hierarchy of the left handed light neutrino masses. This has motivated us to propose an $SU(2)_L \times U(1)_Y$ model with $(S_3 \times D)$ discrete symmetries and three right handed singlet neutrinos to explain the results of Super-Kamiokande experiment on atmospheric and solar neutrinos. Our model differs from those in Ref.[13] and Ref.[14] in not having extra gauge symmetry and in the choice of discrete symmetries. The present model contains three right handed singlet neutrinos $\nu_{eR}, \nu_{\mu R}, \nu_{\tau R}$ along with three doublet Higgs fields $\phi_1, \phi_2, \phi_3$ and three singlet Higgs fields $\chi_1, \chi_2, \chi_3$. The doublet Higgs fields are responsible for generating the Dirac mass terms of the neutrinos where as right handed singlet neutrinos acquire their heavy masses through the singlet Higgs fields. We have imposed an extra global symmetry in our model to start with to ensure lepton number conservation by making appropriate lepton number assignments for the Higgs fields. In our model each singlet Higgs field is assigned with a (-2) lepton number. As soon as $\chi_1, \chi_2, \chi_3$ develop their non-zero vevs, the lepton number symmetry is broken spontaneously at the GUT scale ($\sim 10^{16}$) GeV yielding three heavy right handed Majorana neutrino masses $(M_1, M_2, M_3)$ through the well-known see-saw mechanism, which induce small hierarchical masses to the left handed doublet neutrinos ($\nu_e, \nu_\mu, \nu_\tau$). The $(S_3 \times D)$ discrete symmetry facilitates appropriate mixing between the left handed neutrino flavours in order to explain atmospheric and solar neutrino data.

We organise our paper as follows. Section II contains the model. Section III describes the Higgs potential, the neutrino mass matrix and its analysis. Conclusions are presented in Section IV.
2 The Model

In our present model the lepton and the Higgs fields have the following representation contents under the gauge group $SU(2)_L \times U(1)_Y$

$$l_i L (2, -1)_1, e_i R (1, -2)_1, \nu_{eR}(1, 0)_1, \nu_{\mu R}(1, 0)_1, \nu_{\tau R}(1, 0)_1$$ (1)

where $i = 1, 2, 3$.

$$\phi_i (2, 1)_0, \chi_i (1, 0)_{-2}$$ (2)

The subscripts in the parenthesis correspond to the lepton number $L (= L_e + L_\mu + L_\tau)$.

We consider the following VEVs of the Higgs fields:

$$< \phi > = \begin{pmatrix} < \phi_i^+ > \\ < \phi_i^0 > \end{pmatrix} = \begin{pmatrix} 0 \\ v_i \end{pmatrix},$$

$$< \chi_i^0 > = k_i$$ (3)

The following discrete symmetries have been incorporated in the model in order to generate the required pattern of Majorana neutrino mass matrix as well as the mixing:

(i) $S_3$- symmetry:

$$l_1 L \rightarrow 1, l_2 L \rightarrow 1, l_3 L \rightarrow 1, (\nu_{eR}, \nu_{\mu R}) \rightarrow 2, \mu_R \rightarrow 1, (\nu_{eR}, \nu_{\tau R}) \rightarrow 2, \nu_{\mu R} \rightarrow 1,$$

$$\phi_1, \phi_3 \rightarrow 2, \phi_2 \rightarrow 1, \chi_1 \rightarrow 1, \chi_2 \rightarrow 1, \chi_3 \rightarrow 1$$ (4)

(ii) D- symmetry:

$$l_1 L \rightarrow i\omega^* l_1 L, l_2 L \rightarrow -i\omega^* l_2 L, l_3 L \rightarrow -i\omega^* l_3 L, e_R \rightarrow -i\omega^* e_R, \mu_R \rightarrow \mu_R,$$

$$\nu_{eR} \rightarrow \nu_{eR}, \nu_{\mu R} \rightarrow \nu_{\mu R}, \nu_{\tau R} \rightarrow \nu_{\tau R},$$

$$\phi_1 \rightarrow -i\phi_1, \phi_2 \rightarrow i\omega \phi_2, \phi_3 \rightarrow i\phi_3, \chi_1 \rightarrow -\omega^2 \chi_1, \chi_2 \rightarrow \chi_2, \chi_3 \rightarrow \omega^2 \chi_3$$ (5)
where \( \omega = e^{\frac{2\pi i}{3}} \).

The choice of the discrete symmetries \( S_3 \times D \) allow \( \nu_{eL} \) to couple with \( \nu_{\tau R} \) only through \( \phi_1 \) Higgs field. But similar couplings of \( \nu_{eL} \) with other two singlet neutrinos are prohibited. However, \( \nu_{\mu L} \) and \( \nu_{\tau L} \) couple with the singlet neutrinos through the Higgs fields \( \phi_2 \) and \( \phi_3 \). This facilitates small mixing between \( \nu_e \) and \( \nu_{\mu} \) as required for explanation of the solar neutrino deficit and maximal mixing between \( \nu_{\mu} \) and \( \nu_{\tau} \) in order to explain atmospheric neutrino anomaly with appropriate choice of the vev’s of the Higgs fields: \(< \phi_1 > \sim 1 GeV, < \phi_2 > = < \phi_3 > \sim 10^2 GeV \). The small vev of the Higgs field \( \phi_1 \) is achieved by choosing positive mass term of the \( \phi_1 \) Higgs field in the Higgs potential. The non-zero but small vev of \( \phi_1 \) arises due to the presence of quartic coupling term in the Higgs potential [15].

3 Higgs Potential and Neutrino Mass Matrix

We focus our attention on the relevant terms of the Higgs potential, which yield small value of the vev of \( \phi_1 \) and, hence, write explicitly the following \( \phi_1 \) dependent terms in the potential.

\[
V_{\phi_1} = m_1^2(\phi_1^\dagger \phi_1) + \lambda_1(\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + \lambda_2(\phi_1^\dagger \phi_1)(\phi_3^\dagger \phi_3) + \lambda_3(\phi_1^\dagger \phi_1)(\chi_1^* \chi_1) \\
+ \lambda_4(\phi_1^\dagger \phi_1)(\chi_2^* \chi_2) + \lambda_5(\phi_1^\dagger \phi_1)(\chi_3^* \chi_3) + \lambda'(\phi_1^\dagger \phi_3 \chi_1^* \chi_3 \\
+ \chi_3^* \chi_1 \phi_3^\dagger \phi_1)
\] (6)

After substituting the vev’s of the Higgs fields in the above potential and minimising with respect to \(< \phi_1 > = v_1 \), we obtain,

\[
v_1 = -\frac{\lambda' v_3 k_1 k_3}{M_{\phi_1}^2}
\] (7)

where

\[
M_{\phi_1}^2 = m_1^2 + \lambda_1 v_2^2 + \lambda_2 v_3^2 + \lambda_3 k_1^2 + \lambda_4 k_2^2 + \lambda_5 k_3^2
\] (8)

is the physical mass of the \( \phi_1 \) Higgs field.
Assuming $m_1^2$ to be much greater than all other terms in Eqn.(8) and putting $M_{\phi_1}^2 \sim m_1^2$, which does not affect the essential results obtained in our model, we get,

$$v_1 = \frac{-\lambda'v_3k_1k_3}{m_1^2}. \quad (9)$$

We require the value of the vev of $< \phi_1 > \sim 1\text{GeV}$. This would correspond to $\lambda' \sim 1, v_3 \sim 10^2 \text{GeV} k_1 \sim k_3$ and $m_1 \sim 10k_3$.

Turning to the neutrino sector, the most general discrete symmetry invariant Yukawa interaction is given by

$$L_\nu^y = f_1 \bar{\nu}_1 \nu_\tau R \tilde{\phi}_1 + f_2 \bar{\nu}_2 \nu_\mu R \tilde{\phi}_2 + f_3 \bar{\nu}_3 \nu_\tau R \tilde{\phi}_3 + f_4 \bar{\nu}_4 \nu_\mu R \tilde{\phi}_2 \\
+ f_1 \bar{l}_3 \nu_\tau R \tilde{\phi}_3 + g_1 \nu^c_\tau e R \nu_\tau R \chi_1^0 + g_2 \nu^c_\mu e R \nu_{eR} \chi_2^0 + g_3 \nu^c_\tau e R \nu_\tau R \chi_3^0 + h.c. \quad (10)$$

Substituting the vev’s of the Higgs fields in Eqn.(10) we obtain the following form of (6x6) Majorana neutrino mass matrix in the basis $(\nu_eL, \nu_\mu L, \nu_\tau L, \nu^c_\tau eR, \nu^c_\mu R, \nu^c_\tau R)$:

$$M_\nu = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & c_3 \\
0 & 0 & 0 & a_2 & a_3 & 0 \\
0 & 0 & 0 & b_2 & b_3 & 0 \\
0 & 0 & 0 & M_1 & 0 & 0 \\
0 & a_2 & b_2 & 0 & M_2 & 0 \\
c_3 & a_3 & b_3 & 0 & 0 & M_3
\end{pmatrix} \quad (11)$$

where $D$ and $M$ denote the (3x3) Dirac and Majorana neutrino mass matrices respectively. The matrix elements of Eqn.(11) are given by

$$c_3 = f_1v_1, a_2 = f_2v_2, a_3 = f_3v_3, b_2 = f_4v_2, \\
b_3 = f_5v_3, M_1 = g_1k_1, M_2 = g_2k_2, M_3 = g_3k_3. \quad (12)$$

We see from the (6x6) mass matrix that the right handed Majorana neutrino mass matrix $M$ becomes diagonal with mass eigenvalues $M_1, M_2$ and $M_3$ due to the imposition of the $(S_3 \times D)$ discrete symmetries. While all the heavy right handed neutrinos acquire their masses at the lepton number
symmetry breaking scale which is of the order of $\sim 10^{16}$ GeV, we assume the following hierarchy of masses

$$\frac{M_2}{M_3} = \frac{1}{10} \quad (13)$$

in order to arrive at the required mass ratio for the left handed doublet neutrinos.

The induced (3x3) mass matrix for the left handed neutrinos ($\nu_e, \nu_\mu, \nu_\tau$) can be calculated from Eqn.(11) using the see-saw formula in the basis of a diagonal Majorana mass matrix as

$$m_{ij} = \frac{D_{1i}D_{1j}}{M_1} + \frac{D_{2i}D_{2j}}{M_2} + \frac{D_{3i}D_{3j}}{M_3} \quad (14)$$

where i and j denote the three neutrino flavours, and $D_{1i}, D_{2i}, \text{and } D_{3i}$ refer to the respective Dirac mass matrix elements in Eqn.(11). Thus we get

$$m_\nu = \begin{pmatrix}
\frac{c_3^2}{M_3} & \frac{c_1a_3}{M_3} & \frac{c_1b_3}{M_3} \\
\frac{c_1a_3}{M_3} & \frac{a_3^2}{M_3} + \frac{a_2b_3}{M_2} + \frac{a_1b_3}{M_3} \\
\frac{c_1b_3}{M_3} & \frac{a_2b_3}{M_2} + \frac{a_1b_3}{M_3} & \frac{b_3^2}{M_3}
\end{pmatrix} \quad (15)$$

With the redefinition of the matrix elements as

$$\frac{c_3}{\sqrt{M_3}} \rightarrow C_3, \frac{a_3}{\sqrt{M_3}} \rightarrow A_3, \frac{b_3}{\sqrt{M_3}} \rightarrow B_3, \frac{a_2}{\sqrt{M_2}} \rightarrow A_2, \frac{b_2}{\sqrt{M_2}} \rightarrow B_2$$

Eqn.(15) is reduced to the following simple form

$$m_\nu = \begin{pmatrix}
C_3^2 & C_3A_3 & C_3B_3 \\
C_3A_3 & A_3^2 + A_3^2 & A_2B_2 + A_3B_3 \\
C_3B_3 & A_2B_2 + A_3B_3 & B_2^2 + B_3^2
\end{pmatrix} \quad (16)$$

We now proceed to calculate the masses and mixing angles of the three light left handed neutrinos by diagonalising the mass matrix in Eqn.(16). It can be shown that the determinant of $m_\nu$ is zero implying that one of the eigen values of $m_\nu$ is zero.
We then assume the approximation, $A_2, B_2 \gg A_3, B_3 \gg C_3$, which will be true for our parameter space of interest. After diagonalisation of the $(3 \times 3)$ induced neutrino mass matrix we get the following mass eigenvalues:

$$m_1 \sim A_2^2 + B_2^2$$

$$m_2 \sim \frac{(A_2 B_3 - A_3 B_2)^2}{A_2^2 + B_2^2}$$

$$m_3 = 0$$

where $\frac{m_2}{m_1} \sim \frac{M_2}{M_3}$.

Finally, we investigate the mixing matrix connecting the three flavour eigenstates ($\nu_e, \nu_\mu, \nu_\tau$) to the mass eigen states ($\nu_3, \nu_2, \nu_1$) in increasing order of mass:

$$
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
= \begin{pmatrix}
1 & -C_3 \sqrt{(A_2^2 + B_2^2)} & -1 \\
-C_3 B_3 - A_4 B_2 & A_2 B_3 - A_3 B_2 & A_2 A_3 B_2 - B_3 B_2 \\
-A_2 B_3 - A_4 B_2 & A_2 B_3 - A_3 B_2 & A_2 A_3 B_2 - B_3 B_2
\end{pmatrix}
\begin{pmatrix}
\nu_3 \\
\nu_2 \\
\nu_1
\end{pmatrix}
$$

$$\text{(18)}$$

In our model the mixing angle between $\nu_\mu$ and $\nu_\tau$ is given by ratio of the two Yukawa couplings as follows:

$$\tan \theta_{\mu\tau} \sim \frac{A_2}{B_2} \sim \frac{f_2}{f_4}$$

$$\text{(19)}$$

Assuming these Yukawa couplings to be equal, we obtain $\theta_{\mu\tau} = 45^0$ ie, $\sin^2 2\theta_{\mu\tau} = 1$. This agrees with the Super-Kamiokande data on atmospheric neutrinos, which suggest maximal mixing between $\nu_\mu$ and $\nu_\tau$. Now, for the solar neutrino oscillation $\nu_e \rightarrow \nu_\mu$, the mixing angle between $\nu_e$ and $\nu_\mu$ in our model is given by

$$\sin \theta_{e\mu} \sim \frac{C_3 \sqrt{(A_2^2 + B_2^2)}}{A_2 B_3 - A_3 B_2} \sim \frac{f_1 v_1}{f_3 v_3}$$

$$\text{(20)}$$
If the couplings are assumed to be of the same order, we get

$$\sin \theta_{e\mu} \sim \frac{v_1}{v_3} \sim 10^{-2}. \quad (21)$$

This is in good agreement with the recent Super-Kamiokande solar neutrino data.

We shall now estimate the two nonzero light neutrino masses in our model. Assuming $v_3 = 1.5 \times 10^2$ GeV and $M_2 \sim 10^{15}$ GeV and $M_3 \sim 10^{16}$ GeV, we get $m_{\nu_\mu} \sim 0.0023$ eV and $m_{\nu_e} \sim 0.045$ eV. Thus our model leads to $\Delta m_{\mu\tau}^2 \sim 2 \times 10^{-3} eV^2$ and $\Delta m_{e\mu}^2 \sim 0.53 \times 10^{-5} eV^2$, which are consistent with the experimental results of Super-Kamiokande.

4 Conclusions

In the present paper we have focused our attention on the Super-Kamiokande data on atmospheric and solar neutrinos without taking into account the LSND result. We demonstrate that the evidence of atmospheric and solar neutrino oscillations provided by Super-Kamiokande experiments can be accommodated in an extension of the standard electroweak gauge model based on the gauge group $SU(2)_L \times U(1)_Y$ with appropriate discrete symmetries ($S_3 \times D$) and Higgs fields along with the three right handed singlet neutrinos. The model can simultaneously reconcile the maximal mixing between $\nu_\mu$ and $\nu_\tau$ as required by the atmospheric neutrino data as well as matter enhanced MSW solution to solar neutrino problem with small mixing angle between $\nu_e$ and $\nu_\mu$. The desired small masses of the left handed neutrinos are generated by the well known see-saw mechanism with reasonable choice of the model parameters. Apart from the electroweak symmetry breaking scale, the model admits a lepton number symmetry breaking scale at $\sim 10^{16}$ GeV.

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