CONSTRAINING COSMOLOGICAL MODELS BY THE CLUSTER MASS FUNCTION

NURUR RAHMAN AND SERGEI F. SHANDARIN
Department of Physics and Astronomy, University of Kansas, Lawrence, KS 66045; nurur@kusmos.phsx.ukans.edu, sergei@ukans.edu

Received 2000 December 18; accepted 2001 February 16; published 2001 March 23

ABSTRACT

We present a comparison between two observational mass functions and three theoretical mass functions for eight cosmological models suggested by the data from the recently completed BOOMERANG-98 and MAXIMA-1 cosmic microwave background (CMB) anisotropy experiments as well as from peculiar velocities (PVs) and Type Ia supernova (SN) observations. The cosmological models have been proposed as the best-fit models by several groups. We show that no model is in agreement with the abundances of X-ray clusters at \$10^{14.7} M_{\odot}$, On the other hand, we find that the BOOM + MAX + COBE: I, Refined Concordance, and ΛDM models are in a good agreement with the abundances of optical clusters. The P11 model and especially the Concordance model predict slightly lower abundances than observed at \$10^{14.6} M_{\odot}$, The BOOM + MAX + COBE: II and PV + CMB + SN models predict slightly higher abundances than observed at \$10^{14.9} M_{\odot}$, The nonflat MAXIMA-1 is in a fatal conflict with the observational cluster abundances and can be safely ruled out.

Subject headings: cosmology: observations — cosmology: theory — galaxies: clusters: general — large-scale structure of universe

1. INTRODUCTION

Recently, certain cosmological models have received a fairly strong observational boost. Several groups have used the new cosmic microwave background (CMB) data from the BOOMERANG-98 (the 1998 Balloon Observations Of Millimetric Extragalactic Radiation And Geophysics; de Bernardis et al. 2000) and MAXIMA-1 (the first overnight flight of the Millimeter Anisotropy eXperiment IMaging Array; Hanany et al. 2000) experiments to constrain cosmological parameters. Other groups combined the constraints from the CMB data with cosmological nucleosynthesis data, peculiar velocities (PVs), and Type Ia supernova (SN) observations. The values of the cosmological parameters vary from one set to the next, but all of these models are in reasonable agreement with a flat cold dark matter (CDM) universe \((\Omega_m + \Omega_{\Lambda} = 1)\) dominated by vacuum energy, except for the MAXIMA-1 model with a matter density \(\Omega_m = 0.68\) and a vacuum energy density \(\Omega_{\Lambda} = 0.23\) (Balbi et al. 2000).

In this Letter, we compare the abundances of clusters of galaxies predicted by some popular cosmological models with observed cluster abundances. The abundance of clusters has been shown to be one of the simplest but most effective cosmological tools for constraining the models of structure formation. It can place strong constraints on the parameters of cosmological models (Kaiser 1986; Peebles, Daly, & Juszkiewicz 1989; Simakov & Shandarin 1989), including the mass density in the universe \((\Omega_m)\) and the amplitude of the mass density fluctuations \((\sigma_8)\) or, equivalently, the bias factor \((b = 1/\sigma_8)\) (Evrard 1989; Frenk et al. 1990; Henry & Arnaud 1991; Bahcall & Cen 1992; Lilje 1992; Oukbir & Blanchard 1992; Kofman, Gnedin, & Bahcall 1993; White, Efstathiou, & Frenk 1993; Bond & Myers 1996; Eke, Cole, & Frenk 1996; Mo, Jing, & White 1996; Viana & Liddle 1996; Borgani et al. 1997; Henry 1997; Pen 1998; Postman 1999; Verde et al. 2001; Pierpaoli, Scott, & White 2001).

The abundance of clusters and their evolution are quantified by the mass distribution function. The theoretical derivation of the mass function of gravitationally bound objects has been pioneered by Press & Schechter (1974, hereafter PS). Despite the various modifications that have been suggested recently (Cavaliere, Colafrancesco, & Scaramella 1991; Blanchard, Valls-Gabaud, & Mamon 1992; Monaco 1997a, 1997b; Audit, Teyssier, & Alimi 1997; Lee & Shandarin 1998, hereafter LS; Sheth, Mo, & Tormen 1999, hereafter SMT), it remains a viable model of the mass function and is widely used.

In this Letter, we make use of three theoretical models suggested for the cosmological mass function: (1) the original PS mass function \(n_{\nu_\alpha}\) assuming the spherically symmetric collapse, (2) the mass function \(n_{\nu_\beta}\) that incorporates the anisotropic collapse as it is described by the Zeldovich approximation (LS), and (3) the mass function \(n_{\nu_\gamma}\) suggested by Sheth & Tormen (1999, hereafter ST) and later derived by SMT that takes into account both the anisotropic collapse and some nonlocal effects. Recently, Jenkins et al. (2001) suggested fits to mass functions obtained in the “Hubble Volume” N-body simulations of some cosmological models. We have found that using the fits suggested by Jenkins et al. (2001) does not change the conclusions of this Letter. For comparison with observations, we use the mass functions obtained for cluster virial masses by Girardi et al. (1998) and Reiprich, Böhringer, & Schuecker (2000).

Here we report the results for eight cosmological models. Among these, seven have recently been claimed as the best-fit models satisfying the data from CMB anisotropy experiments \((COBE)\) Differential Microwave Radiometer, BOOMERANG-98, and MAXIMA-1) as well as from nucleosynthesis, large-scale structure, and Type Ia SN observations. These models are labeled P11, BOOM + MAX + COBE: I, BOOM + MAX + COBE: II, PV + CMB + SN, Refined Concordance, MAXIMA-1, and ΛDM. We have included the Concordance model as a reference model since it is often referred to as the standard ΛCDM model.

None of the proponents of the best-fit models in our list have mentioned the explicit cluster abundance test. Rather some of them claimed that their models satisfy one of the many \(\sigma_8\Omega_m\) relations reported in the literature; others even did not apply this test at all. We have noticed more than a dozen predictions of the \(\sigma_8\Omega_m\) relation in the literature, some of which are in conflict with the others. We believe our approach here to present the result of the cluster abundance test is more explicit.

This Letter is organized as follows: in § 2 we briefly sum-
n_{\bar{s}}(M) = \frac{25\sqrt{5}}{24\sqrt{2}\pi} F(\tilde{\rho}, \sigma_M) \lambda \left[ -20 \lambda \exp \left( -\frac{9\lambda^2}{2} \right) + \sqrt{2\pi}(20\lambda^2 - 1) \exp \left( -\frac{5\lambda^2}{2} \right) \text{erfc} \left( \sqrt{2}\lambda \right) + 3\sqrt{3}\pi \exp \left( -\frac{15\lambda^2}{4} \right) \text{erfc} \left( \frac{3\lambda}{2} \right) \right]. \quad (2)

The parameters $A = 0.322$, $a = 0.707$, and $q = 0.3$ chosen by ST have been determined empirically from an $N$-body simulation. At $A = \frac{1}{2}$, $a = 1.0$, and $q = 0$, one finds $n_{\text{ST}}(M) = n_{\text{PS}}$. The cosmological parameters enter the cosmological mass function via the shape and normalization of the linear power spectrum. One of the most accurate approximations of a power spectrum fitting formula incorporating baryon density was developed by Eisenstein & Hu (1998). Their formula has an accuracy of better than 5% for a baryon fraction $\Omega_\Lambda/\Omega_0$ of less than 30%. The cosmological models discussed here predict a baryon fraction of less than 20%; therefore, we have used the Eisenstein & Hu fits for the power spectrum.

3. OBSERVATIONAL MASS FUNCTIONS

The predictions of the theoretical models have been tested against the measurements of the virial mass functions in the $N$-body simulations (see, e.g., ST and references therein). Therefore, the theoretical mass functions must be compared with the observational virial mass functions.

Girardi et al. (1998) provided the cumulative mass functions estimating the virial masses of clusters of richness $R \geq 1$ and $R \geq 1$. Both practically coincide for $M > 10^{14.6} h^{-1} M_\odot$ (see Fig. 2 in Girardi et al. 1998). This mass function is shown by filled circles in Figures 1 and 2. Reiprich et al. (2000) determined the cmf using an X-ray flux-limited sample from the ROSAT All-Sky Survey. They determined the masses from measured gas temperatures based on ASCA observations. In this Letter, we use the mass function corresponding to $\frac{2}{5}\sigma_\text{DM}$, that is usually referred to as the virial radius (open squares, Figs. 1 and 2). At $M < 10^{14.8} h^{-1} M_\odot$, the Girardi et al. mass function is significantly higher than that of Reiprich et al.

It should be mentioned that the estimation of the masses is not a simple problem. For further discussion, see, e.g., Girardi et al. (1998), Reiprich et al. (2000), Pierpaoli et al. (2001), and references therein. In addition, there is no one-to-one corre-
have chosen only the above-mentioned one for our comparison. The cosmological parameters have been obtained from observational data through likelihood analysis with various prior assumptions. These parameters ($\Omega_0$, $\Omega_{\text{dm}}$, $\Omega_{\Lambda}$, $n_s$, $h$, and $\sigma_8$) from different models are presented in Table 1. In our notation, $\Omega_0 = \Omega_\Lambda + \Omega_{\text{dm}}$ and spectral index $n = n_s + n$. In this Letter, we have taken a zero gravity wave contribution, i.e., $n_s = 0$, with zero reionization. Among these models, P11, BOOM + MAX + COBE: I, BOOM + MAX + COBE: II, Concordance, and MAXIMA-1 are COBE-normalized following the prescription of Bunn & White (1997). For other models, we have followed the normalization suggested by the authors.

5. SUMMARY

We have compared the theoretical predictions of cluster abundances by several cosmological models with the observational mass functions determined by Girardi et al. (1998; Figs. 1 and 2, filled circles) and Reiprich & Böhringer (2000; Figs. 1 and 2, open squares). In this Letter, we make use of three theoretical mass functions: $n_{0\text{st}}$, $n_s$, and $n_{0\text{st}}$. It is worth stressing that in the range of masses ($4 \times 10^{14} h^{-1} M_\odot \leq M \leq 3 \times 10^{15} h^{-1} M_\odot$), the theoretical models differ one from another roughly less or similar to the error bars of both observational mass functions.

At $M \leq 10^{14.5} h^{-1} M_\odot$, no model can be reconciled with the Reiprich et al. (2000) X-ray mass function. On the other hand, almost all models are in much better agreement with the Girardi et al. (1998) optical mass function. Thus, the resolution/explanation of the discrepancies between optical and X-ray mass functions becomes crucial for the well being of all models in question.

As far as the optical mass function is concerned, the Refined Concordance, BOOM + MAX + COBE: I, and AMDM models show a reasonable agreement with observations. The P11 model and especially the Concordance model predict slightly lower abundances than observed at $\sim 10^{14.9} h^{-1} M_\odot$. On the other hand, the BOOM + MAX + COBE: II and PV + CMB + SN models predict slightly higher abundances than observed at $\sim 10^{14.9} h^{-1} M_\odot$. The MAXIMA-1 model seems to be safely ruled out by the data on cluster abundances.

A similar comparison using the sharp k-space filter for evaluation of $\sigma_{0\text{st}}$ which is better justified for the PS mass function (Bond et al. 1991), shows that all three theoretical mass functions are systematically higher than that for the top-hat filter. The sharp k-space filter approach improves the agreement with observations for the P11 and Concordance models and makes it worse for the BOOM + MAX + COBE: II and PV + CMB + SN models. The conclusions for the other models did not change much.

We thank the referee for useful comments and acknowledge the comments made by B. Novosyadlyj. N. R. thanks Hume

![Figure 2](image-url)
Feldman, Patrick Gorman, and Surujhdeo Seunarine for their helpful discussions and especially A. Jenkins for his help in developing the numerical code. We acknowledge the support of grant GRF 2001 from the University of Kansas.

REFERENCES

Audit, E., Teyssier, R., & Alimi, J. M. 1997, A&A, 325, 439
Bahcall, N. A., & Cen, R. 1992, ApJ, 398, L81
Balbi, A., et al. 2000, ApJ, 545, L1
Blanchard, A., Valls-Gabaud, D., & Mamon, G. A. 1992, A&A, 264, 365
Bond, J. R., Cole, S., Efstathiou, G., & Kaiser, N. 1991, ApJ, 379, 440
Bond, J. R., & Myers, S. T. 1996, ApJS, 103, 1
Borgani, S., Gardini, A., Girardi, M., & Gottlöber, S. 1997, NewA, 2, 119
Bridle, S. L., Zehavi, I., Dekel, A., Lahav, O., Hobson, M. P., & Lasenby, A. N. 2001, MNRAS, 321, 333
Bunn, E. F., & White, M. 1997, ApJ, 480, 6
Cavaliere, A., Colafrancesco, S., & Scaramella, R. 1991, ApJ, 380, 15
de Bernardis, P., et al. 2000, Nature, 404, 955
Durrer, R., & Novosyadlyj, B. 2001, MNRAS, in press (astro-ph/0009057)
Eisenstein, D. J., & Hu, W. 1998, ApJ, 496, 605
Eke, V. R., Cole, S., & Frenk, C. S. 1996, MNRAS, 282, 263
Evrard, A. E. 1989, ApJ, 341, L71
Frenk, C. S., White, S. D. M., Efstathiou, G., & Davis, G. 1990, ApJ, 351, 10
Girardi, M., Borgani, S., Giuricin, G., Mardirossian, F., & Mezzetti, M. 1998, ApJ, 506, 45
Hanany, S., et al. 2000, ApJ, 545, L5
Henry, J. P. 1997, ApJ, 489, L1
Henry, J. P., & Arnaud, K. A. 1991, ApJ, 372, 410
Hu, W., Fukugita, M., Zaldarriaga, M., & Tegmark, M. 2001, ApJ, 549, 669
Jaffe, A. H., et al. 2000, Phys. Rev. Lett., submitted (astro-ph/0007333)
Jenkins, A., Frenk, C. S., White, S. D. M., Colberg, J. M., Cole, S., Evrard, A. E., Couchman, H. M. P., & Yoshida, N. 2001, MNRAS, 321, 372
Kaiser, N. 1986, MNRAS, 222, 323
Kofman, D. A., Nieder, N. Y., & Bahcall, N. A. 1993, ApJ, 413, 1
Lange, A. E., et al. 2001, Phys. Rev. D, 63, 042001
Lee, J., & Shandarin, S. F. 1998, ApJ, 500, 14 (LS)
———. 1999, ApJ, 517, L5
Lilje, P. B. 1992, ApJ, 386, L33
Mo, H. J., Jing, Y. P., & White, S. D. M. 1996, MNRAS, 282, 1096
Monaco, P. 1997a, MNRAS, 287, 753
———. 1997b, MNRAS, 290, 439
Ostriker, J., & Steinhardt, P. 1995, Nature, 377, 600
Oukbir, J., & Blanchard, A. 1992, A&A, 262, L21
Peebles, P. J. E., Daly, R. A., & Juszkiewicz, R. 1989, ApJ, 347, 563
Pen, U. L. 1998, ApJ, 498, 60
Pierpaoli, E., Scott, D., & White, M. 2001, MNRAS, in press (astro-ph/0010039)
Postman, M. 1999, in Evolution of Large Scale Structure: From Recombination to Garching, ed. A. J. Banday, R. K. Sheth, & L. N. da Costa (Einschede: PrintPartners Ipskamp), 270
Press, W., & Schechter, P. 1974, ApJ, 187, 425 (PS)
Primack, J. R., & Gross, M. A. K. 2001, in Current Aspects of Neutrino Physics, ed. D. O. Caldwell (New York: Springer), in press (astro-ph/0007165)
Reiprich, T., & Böhringer, H. 2000, in Proc. 19th Texas Symp. on Relativistic Astrophysics and Cosmology, ed. E. Aubourg et al. (Amsterdam: North-Holland), in press (astro-ph/9908357)
Reiprich, T. H., Böhringer, H., & Schuecker, P. 2000, in Proc. Workshop on X-Ray Astronomy, ed. R. Giacconi, L. Stella, & S. Serio (San Francisco: ASP), in press
Sheth, R., Mo, J. H., & Tormen, G. 1999, MNRAS, 307, 203 (SMT)
Sheth, R., & Tormen, G. 1999, MNRAS, 308, 119 (ST)
Simakov, S. G., & Shandarin, S. F. 1989, Astrophysics, 30, 33
Tegmark, M., Zaldarriaga, M., & Hamilton, A. J. S. 2001, Phys. Rev. D, 63, 043007
Valdarnini, R., Kahnashvili, T., & Novosyadlyj, B. 1998, A&A, 336, 11
Verde, L., Kamionkowski, M., Mohr, J. J., & Benson, A. J. 2001, MNRAS, 321, L7
Viana, P. T. P., & Liddle, A. R. 1996, MNRAS, 281, 323
White, S. D. M., Efstathiou, G., & Frenk, C. S. 1993, MNRAS, 262, 1023