Researches on small amplitude dynamic pressure measurement method by laser interferometer based on refractive index

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Abstract
Traceable measurement problems have always been difficult in dynamic pressure calibration. The dynamic pressure measurement method by laser interferometer based on refractive index is studied for micro sinusoidal pressure calibration. The linear relationship between the pressure and the refractive index is verified by the refractive index model of water and air and the static pressure experiments. Based on the in-situ calibration method and the optical multiplier the measurement method is preliminary applied on the micro sinusoidal pressure calibration device with air. The main influencing factors such as the temperature’s change and the glass window’s movement were quantified, the advantages and disadvantages of water and air used as the pressure medium were compared, and the follow-up research direction is further clarified.

Keywords: traceable measurement; dynamic pressure calibration; laser interferometer; refractive index

1. Introduction
The dynamic pressure measurement is widely used in engine trials, wind tunnel tests, high-speed rail trains, high-rise buildings, medical diagnosis and other fields\cite{1}. Silicon pressure sensors are commonly used for these dynamic pressure measurements with piezoelectric pressure sensors. Researchers have also developed various step pressure, sinusoidal pressure, and pulse pressure devices for dynamic calibration of pressure sensors or test systems\cite{2}.

But traceable dynamic pressure measurement is the primary problem faced by dynamic pressure metrology. Due to the time-varying characteristics of dynamic pressure, the application of Traditional static pressure traceable measurement methods such as piston pressure gauge and liquid column height method in dynamic pressure standards have great limitations. Therefore, researchers have studied various traceable dynamic pressure measurement methods based on the macro definition of pressure or the micro interpretation of pressure\cite{3}, such as the liquid impulse pressure standard based on inertial force\cite{4}\cite{5}, the step pressure standard based on shock wave theory, and the micro sinusoidal pressure/sound pressure standard based on the gas state equation\cite{6}. New methods such as the absorption spectrum of gas molecules have also been explored. Laser interferometry is a basic information acquisition method in many measurement methods due to its advantages of high dynamics and high resolution. The dynamic pressure measurement by laser interferometry based on fluid refractive index is one of the hotspots in the research of dynamic pressure standard\cite{7}\cite{9}. This method is studied here in the small amplitude dynamic pressure measurement.
2. Measurement principle and method

2.1. Theoretical basis

According to the Lorentz-Lorenz equation, there is a relation in the medium's refractive index $n$ and the density $\rho$. The equation can be written in the form:

$$\frac{n^2 - 1}{n^2 + 2} = \rho k,$$

with

$$k = \frac{N_A \alpha}{3 \epsilon_0 M_mol}$$

(1)

Where, $M_mol$ and $N_A$ are the molar mass and the Avogadro constant, $\alpha$ and $\epsilon_0$ are the molecular polarizability and the vacuum permittivity. $k$ is a function of the optical frequency $f$ or the wavelength $\lambda$.

And according to the fluid state equation, the density's change synchronized with the pressure. So the pressure's change causes the refractive index's change along with the optical path length's change when no temperature and no optical measurement length's changes are considered, which is showed in figure 1. The optical path length's change can be measured by the laser interferometer with high displacement resolution and high frequency range.

Figure 1. The optical path length’s change caused by the pressure’ change

The Edlen formula and IAPWS[10] are used to analyse the relationship between the pressure and the refractive index in air or water respectively, and there is a good linear relationship between the pressure and the refractive index of water or air in a certain range.

For deionized water from 0.1MPa to 10MPa at 20°C, the following formula is obtained by linear fitting:

$$\Delta n_w \approx 1.666 \times 10^{-4} \Delta p$$

(2)

For air from 0MPa to 6MPa at 20°C:

$$\Delta n_a \approx 2.696 \times 10^{-3} \Delta p$$

(3)

Here, the ratio of the refractive index with the pressure variation is defined as the refractive index sensitivity of pressure. It can be seen that the refractive index sensitivity with air is much greater than the water.

2.2. Static pressure experiment verification

In order to further confirm the refractive index model of water and air, in particular the linear relationship, static pressure experiments were performed. The experiment site is shown in Figure 2. In order to reduce the interference of air fluctuations, windows movement, etc., a relatively long pressure chamber ($L = 170$ mm) is used.

Figure 2. Static pressure measurement experiments with laser interferometer

The $n-p$ curve of air and the $l-p$ curve of water obtained by the static pressure experiments are shown in Figure 3, and Fig. 4. The refractive index model of air or water has good linearity, and the sensitivities are also close to the formula (2), and formula (3).
Therefore, when the physical path length of the laser through the measured medium is constant, a linear relationship can be established between the optical path and the pressure, and the ratio is called the optical sensitivity of the pressure $s_o$.

2.3. Dynamic measurement traceability

To perform the dynamic measurement, first, $s_o$ should be gained. However, due to the following influences, it is not advisable to use the sensitivity $s_o$ obtained by theory.

1) Internal length of measuring cavity $L$ is difficult to be installed with high repeatability and accurately measured.

2) The physical properties of the medium are difficult to guarantee consistency according to the environmental temperature and the composition change of the medium.

So $s_o$ can be obtained by in-situ static calibration in actual operation. Figure 5 shows the use of the in-situ calibrated sensitivity in dynamic measurement.

3. Application in micro sinusoidal pressure calibration

3.1. Micro sinusoidal pressure generator

The structure of the micro gas sinusoidal pressure generator is shown in the figure 6, which uses the vibrating table excitation to becomes the integrated sinusoidal pressure generator.

The dynamic pressure generated by the sinusoidal pressure generator is as follows:

1) Static pressure: $(30^{\circ}-230)$ kPa.

2) Dynamic pressure (peak-peak): $(0.1^{\circ}-10)$ kPa.

3) Frequency range: $(1^{\circ}-1000)$ Hz.
3.2. Micro sinusoidal pressure measured by laser

Considering that the distribution of dynamic pressure is uneven in the cavity, the pressure chamber cannot be large. Here the optical measurement length in the pressure chamber used in the experiment is about 10 mm. Thus, according to the front air refractive index model, the optical path varies from 3 nm when the pressure amplitude is 0.1 kPa. When the pressure amplitude is up to 10 kPa, the optical path change is still less than 300 nm. It can be seen that the laser measurement output signal is too small for a small dynamic pressure measurement.

One optical multiplier was designed to significantly increase the sensitivity so in the case where the pressure chamber is not significantly increased. The schematic of the new pressure chamber is showed in Figure 7. After using this new structure, the optical sensitivity so can increase approximately 10 times.

![Figure 7. Schematic of the new pressure chamber with optical path](image)

Micro sinusoidal pressure measurement experiments with laser interferometer is showed in figure 8. One high-frequency piezoresistive pressure sensor was used to measure the dynamic pressure and verify the data measured by laser.

![Figure 8. Micro sinusoidal pressure measurement experiments with laser interferometer](image)
The initial experimental data shows that when the pressure amplitude is relatively large (generally not less than 1 kPa), the curve of the laser measurement is similar to the curve measured by the pressure sensor. Figure 3 shows the measurement curves of 10 Hz air sinusoidal pressure with an amplitude of about 10 kPa by laser and pressure sensor. The waveforms of the two are relatively consistent.

![Figure 3: Measurement curves of 10 Hz air sinusoidal pressure](image)

Figure 3. 10 Hz micro sinusoidal pressure measurement by laser interferometer

However, when the pressure amplitude is relatively small, the noise and distortion of the laser measurement curve are large, which is very inconsistent with the sensor measurement curve.

4. Main problems analysis

In addition to the above noise and distortion problems, there are still the following serious problems in the experiments.

1) In the in-situ calibrations, the optical path sensitivity of pressure so shows a certain degree of nonlinearity.

2) As the frequency of the measured sinusoidal pressure increases, the consistency and repeatability of the measurement results by laser and sensor may become worse.

The following aspects are analysed.

4.1. Impact of physical path changes

In order to analyse these problems, according to the schematic diagram of the laser interferometric measurement path as shown in the figure 10, a unified measurement model (4) is established:

![Figure 10: Schematic diagram of laser path for pressure measurement](image)

Figure 10. Schematic diagram of laser path for pressure measurement

\[ l = n_w L_w + n_a L_a + n_g L_g \] (4)

The error formula (5) can be obtained after decomposition and simplifying the formula (4).

\[ \Delta l \approx K_m L_{m_0} p + (n_{m_0} - n_q) WS(p) \] (5)

Where \( WS(p) \) is the movement of glass window. The experiments showed that \( WS(p) \) cannot be ignored, and it may exceed 1 µm. For water the influence of the optical windows’ movement is more serious, due to the large refractive index deviation between water and air, and the amplitude measurement error caused by it may be greater than 100%.

We can also conduct further analysis from the above model.

1) The laser measures the average pressure change along the path of the laser, not just the pressure on one point, which may cause the measured object’s inconsistency between laser and sensor.
2) When using a commercial laser interferometer, the laser inevitably passes through a section of the outside atmosphere, then the optical path change caused by the atmospheric pressure fluctuation and noise are also reflected in the measurement result.

3) The vibration of the laser measurement partner and the interferometer in the measurement direction will be directly introduced into the measurement result.

4) The pressure change in the pressure chamber will cause the deformation and displacement of the optical glass, and therefore cause the change of the optical path length of the pressure medium, which will affect the measurement result.

4.2. Signal-to-noise ratio

When the amplitude of sinusoidal pressure is too small, the signal-to-noise ratio of the measured waveform by laser is very poor, or the real signal even can't be recognized. The main sources of laser measurement noise include: mechanical vibration and environmental noise. The direction of future improvement includes:

1) Increase the pressure amplitude;
2) Vibration isolation of the laser interference measurement system;
3) Shield the air disturbance and noise in laser measurement path;
4) Reduce the light path exposed to the air $L_a$;
5) Increase the pressure sensing optical path $L_m$.

4.3. Influence of temperature change

Equation (2) and (3) are assumed that the temperature of the medium is unchanged. However, in a dynamic pressure measurement, the rapid changes in pressure will inevitably accompany the temperature change.

By analysing the water’s insulation process, the measurement error of (1 ~ 10) MPa dynamic pressure at initial temperature ($15 ~ 25$)$^\circ$C caused by the temperature change is between (-0.43 ~ 1.16)$\%$. It can be seen that the effect of temperature change on laser measurement is not very significant for of water. For Air, the adiabatic process is described by:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

(6)

Combined with the Lorentz-Lorenz equation, the laser measurement error caused by the change in air temperature can be obtained at around 30%, as shown in Table 1.

| Table 1. Effect of temperature change of adiabatic process on pressure laser measurement |
|-----------------|-------------------|-----------------|-----------------|-----------------|-----------------|
| P1(kPa) | P2(kPa) | T1(K) | T2(T1(K) | Diff of $\Delta n$(%) |
| 100 | 110 | 296.15 | 8.18 | 30.37 |
| 200 | 101 | 296.15 | 0.84 | 28.75 |
| 80 | 81 | 296.15 | 2.51 | 28.00 |
| 60 | 61 | 296.15 | 1.40 | 28.88 |
| 40 | 41 | 296.15 | 2.10 | 29.03 |
| 20 | 21 | 296.15 | 4.16 | 29.48 |

However, there is still a more complex problem for the gas. Due to heat transfer, as the pressure frequency is lowered, the gas state will be closer to the isothermal process, and the actual process is difficult to determine.

4.4. Summary of problems

The problems are further analysed combining the two different application scenarios.

1) As the frequency of measuring pressure increases, the wavelength decreases, and the possibility of pressure inconsistency caused by the spatial distance between the laser and the sensor increases. Since the sound velocity of water is obviously higher than that of air, the high-frequency application of laser interferometry in water has advantages over air.

2) Because the refractive index change rate of water is much smaller than that of air, it is more difficult
to measure dynamic pressure with small amplitude.
3) Increasing the length of the physical path of the laser through the measured pressure medium is an effective way to improve the signal-to-noise ratio, but it is not good for increasing the pressure frequency.
4) For the laser measurement of gas dynamic pressure, the temperature will change with pressure and cause the change of refractive index. This problem is usually corrected based on the adiabatic model. But whether the assumption of the adiabatic model is still satisfied at low frequency needs further study.
5) Due to the obvious difference between the refractive index of water and air, the error caused by the deformation and displacement of the optical glass in the water dynamic pressure measurement is more significant than that of gas.

5. Conclusion and outlook
The basic model of dynamic pressure measurement by laser interferometer based on refractive index is established by theoretical analysis and static pressure experiment. The gas micro sinusoidal pressure measurement experiment preliminates the feasibility of this method in small amplitude dynamic pressure calibration applications. By analyzing the optimal measurement error sources such as optical windows, environmental noise interference and temperature variation, the error formation mechanism and optimization improvement direction are studied. At the same time, the comparative analysis of water and air indicates that the air has a significant advantage in sensitivity and window movement relative to water, but the temperature change impact is an important issue to be solved.

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