Nuclear binding, correlations and the origin of EMC effect

Omar Benhar\textsuperscript{1,2} and Ingo Sick\textsuperscript{3}
\textsuperscript{1}INFN, Sezione di Roma, I-00185 Roma, Italy
\textsuperscript{2}Dipartimento di Fisica, “Sapienza” Università di Roma, I-00185 Roma, Italy
\textsuperscript{3}Dept. für Physik, Universität Basel, CH-4056 Basel, Switzerland
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Recent data for the slope of the EMC-ratio in the intermediate $x$-region for light nuclei, with $3 \leq A \leq 12$, have the potential to shed new light on the origin of the EMC effect. Here we study the role of nuclear binding using the scaling variable $\tilde{y}$, best suited to take into account this effect, and the understanding of the average nucleon removal energies, $\bar{E}$, provided by state-of-the-art calculations based on nuclear many body theory. We find an excellent correlation between the new EMC data at $x \sim 0.5$ and $\bar{E}$ for nuclei with $A$ from 3 to $\infty$, indicating that in this $x$ region binding is an important ingredient to explain the EMC effect. The role played by nucleon-nucleon correlations in this context is also discussed.

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The so called EMC effect was, in 1983, a surprising discovery\textsuperscript{1}. The ratio $R(x)$ between the Deep Inelastic Scattering (DIS) cross sections of leptons on iron and the deuteron was found to sizably differ from unity. At low values of the Bjorken scaling variable $x$ the ratio is $<1$, at medium values of $x$ $R$ drops from $\sim 1$ to values as low as 0.8 and at large $x$ it reaches values larger than 1. While the latter feature can be quantitatively explained by the smearing of the parton distribution functions arising from the momentum distribution of nucleons in nuclei, the former one is accounted for by including the effect of nuclear shadowing. The reduction of $R(x)$ at intermediate $x$ was not easily explained, although many different models have been proposed (for reviews of the extensive literature see, e.g., Refs.\textsuperscript{2,3}). Often, but not always, these models do involve the role of the binding of nucleons in the nucleus. In general, however, the effect of binding alone is not large enough to reproduce the EMC effect, and is responsible in particular for the role of nuclear high-momentum components that increase away from the low-$x$ region, where the potential contributions of pions, coherence effects and final state interactions complicate matters.

In order to provide data least affected by experimental errors, the authors of Ref.\textsuperscript{7} have studied in particular the slope $dR/dx$, which is insensitive to normalization errors. The correlation between $dR/dx$ for $A=3,4,9,12$ and the average density $\bar{n}$ has been studied in order to find a phenomenological relation that could further elucidate the EMC-effect at mid-$x$: no unambiguous ($\pm$linear) correlation has been found. In particular, the $dR/dx$ value for $^9$Be does not follow the tendency observed for the other light nuclei. The relation to quantities such as nuclear binding energy or nucleon separation energy also has produced no further insight. A correlation with the inclusive quasi-elastic cross section for electron scattering at $x > 1$ has been found\textsuperscript{8,9}, and attributed to the role of nuclear high-momentum components that increase with increasing $A$. However, in the standard treatment of Fermi motion, folding the nucleon parton distributions with nuclear momentum distributions alone would lead to values of the EMC-ratios $>1$ and a mid-$x$ slope $dR/dx$ of the wrong sign.

In this paper, we argue that DIS data are best analyzed in terms of the scaling variable $\tilde{y}$, widely employed in studies of a variety of scattering processes involving composite targets. We explore the correlation of $dR/d\tilde{y}$ with nuclear binding, which in many approaches is advocated as an important element for the explanation of the EMC effect, and is responsible in particular for the decrease of the EMC-ratios to values $<1$ at intermediate values of the scaling variable.
In general, DIS cross sections are studied in the framework of the parton model, which explains the occurrence of scaling and provides the basis for the determination of the parton distribution functions [11]. The constituents the lepton scatters from are assumed to be point-like spin one half particles carrying, in the Infinite Momentum Frame (IMF), a fraction \( x \) of the nucleon four-momentum. This quantity is identified with the Bjorken scaling variable \( x = Q^2/(2M\nu) \), where \( Q^2 \) \( M \) and \( \nu \) are the squared four-momentum transfer, the nucleon mass and the lepton energy loss, respectively. In the IMF interaction effects are expected to vanish [12], and the partons in the nucleon are on-shell. Within this approach the binding (off-shellness) of nucleons in nuclei is conceptually difficult to introduce.

In order to avoid this problem, we here employ a somewhat different approach, first developed to analyze quasi-elastic electron-nucleus scattering, a process perfectly analogous to DIS. Scaling of the quasi-elastic scattering cross sections has been studied in terms of the scaling variable \( y \) [13] which is derived from the kinematics of the underlying process of elastic scattering from nucleons bound in nuclei.

Energy-momentum conservation of elastic scattering from a constituent of mass \( m \) \( k \) yields
\[
\nu = \sqrt{|k + q|^2 + m^2} - m \tag{1}
\]
with \( q \) being the 3-momentum transfer. and \( m \) being, for quasi-elastic scattering, the mass of the nucleon, \( i.e. m = M \). Note that in this approach the initially bound nucleon is assumed to be off-shell. Off-shell means that there is no \( -\sqrt{|k|^2 + m^2} \) term on the right-hand side, only a term \( m \); while the nucleon has momentum \( k \) it is assumed to be bound with total energy \( =0 \). As usual, the interaction energy of the knocked-out nucleon with momentum \( k + q \) is neglected, which at large \( |k + q| \) in general is a good approximation. An additional term \( E > 0 \), accounting for the average nucleon removal energy, is normally added on the right-hand side of Eq. (1).

Neglecting the component of \( k \) perpendicular to \( q \), which is justified in the limit of large \( |q| = q \), yields the standard scaling variable introduced in Ref. [13]
\[
y = \sqrt{\nu^2 + 2m\nu - q} \tag{2}
\]
The physical meaning of \( y \) is straightforward: it is the component of the momentum of the initially bound nucleon parallel to \( q \) in the rest frame of the nucleus.

Here, we specialize \( y \) to the conditions appropriate for DIS on the nucleon, \( i.e. \) scattering from (basically) up- and down-quarks, the rest masses of which, \( m = M_u, M_d <10 \text{ MeV} \), can be safely neglected at the energies relevant to DIS. In this case, the expression for \( y \) of Eq. (2) simplifies to
\[
\tilde{y} = \nu - q \tag{3}
\]
where a ’tilde’ has been added to remind ourselves that \( \tilde{y} \) corresponds to the \( m \rightarrow 0 \) limit [14]. The physical meaning of \( \tilde{y} \) is analogous to the one of \( y \) in quasi-elastic electron-nucleus scattering: \( \tilde{y} \) is the component of the \( u/d \)-quark momentum parallel to \( q \) in the rest frame of the nucleon.

Due to the large energy and momentum transfer in DIS on nuclei, the system of hit quark plus remaining debris of an initially bound nucleon will leave the nucleus with high energy and momentum, and the interaction with the \( (A–1) \)-nucleus is weak. The removal of this system costs an energy equal to the nucleon mean removal energy \( \bar{E} \) which the scattered lepton has to provide. For the DIS process on the nucleon proper only the energy \( \nu' = \nu - \bar{E} \) is available. This leads to
\[
\tilde{y} = \nu' - q = \nu - \bar{E} - q \tag{4}
\]
As both the energy of the quark in the nucleon and \( \bar{E} \) are well defined quantities in the rest frame of the nucleon (nucleus) they can be added without conceptual difficulties.

For the isolated nucleon, \( i.e. \) for \( \bar{E} = 0 \), using the variable \( \tilde{y} \) is known to leads to a scaling of the DIS cross section which is even better in quality than the scaling observed in terms of Bjorken \( x \). This can be shown easily, as \( \tilde{y} \) is trivially related to the Nachtmann variable \( \xi \) [15]. While being usually defined using the much less transparent equation
\[
\xi = 2x/(1 + \sqrt{1 + 4M^2x^2/Q^2}) \tag{5}
\]
Nachtmann’s \( \xi \) can easily be shown to reduce to \( (q - \nu)/M \), implying \( \tilde{y} = -\xi M \). Using Nachtmann’s \( \xi \), which becomes identical to \( x \) in the \( Q^2/\nu^2 \rightarrow 0 \) limit, is known to extend the scaling property of DIS to lower \( \nu \).

As compared to \( x \) or \( \xi \), \( \tilde{y} \) has a well defined physical meaning [30] in the nucleon rest frame, the coordinate system where the DIS experiments are usually done and where theoretical studies, such as lattice QCD calculations, hope to provide a quantitative understanding of the nucleon structure functions.

In order to calculate \( \bar{E} \) directly one would need to know the spectral function \( S(k, E) \), giving the probability to find in the nucleus a nucleon with momentum \( k \) and removal energy \( E \).

Theoretical calculations of the spectral function require the knowledge of both the \( A \)-nucleon ground state and the full spectrum of eigenstates of the \( (A–1) \)-nucleon system. In addition, they involve an intrinsic degree of complexity, associated with the evaluation of non diagonal nuclear matrix elements, that rapidly increases with \( A \).

Studies based on nuclear many body theory and realistic hamiltonians have been carried out for \( A = 3 \) [16 18] and for isospin symmetric nuclear matter [19 20], while in the case of nuclei with \( A > 12 \) approximate spectral
functions obtained using the local density approximation are available \cite{21}. In the present work, aimed at studying the correlation between binding effects and the EMC effect for \(3 \leq A \leq 12\), we exploit Koltun’s sum rule \cite{22,24} to obtain the average removal energies from the results of GFMC calculations.

In the absence of three-nucleon interactions, Koltun’s sum rule states that
\[
E_0/A = \frac{1}{2} \left[ \bar{T} A - 2 A - E \right],
\]
where \(E_0/A\) is the nuclear binding energy per particle obtained from nuclear masses and includes a (small) correction for the Coulomb energy,
\[
\bar{T} = \int d^3 k dE \frac{k^2}{2M} S(k, E),
\]
and
\[
\bar{E} = \int d^3 k dE E S(k, E).
\]

The small contribution of the three-nucleon potential \(V_3\), which is known to be needed to achieve a precise determination of \(E_0\) for \(A \geq 3\), can be taken into account \cite{24} by adding a term \((V_3)/A\), where \((\cdots)\) denotes the ground state expectation value, to the right hand side of Eq. \(6\).

It has to be pointed out that the Koltun sum rule is an exact result, although its experimental verification involves severe difficulties. The results of the analysis of the \((e, e'p)\) reaction carried out by Bernheim et al. \cite{23}, suggesting that the sum rule is badly violated, are due to the limited kinematical range covered by the experiment, which does not include the contribution of correlation effects, leading to the appearance of tails of \(S(k, E)\) extending to large energy and momentum \cite{23}.

The GFMC approach allows one to obtain essentially exact binding energies for the ground states and very good estimates of the energies of low-lying excited states. The wave functions resulting from GFMC calculations have also been employed to obtain density and momentum distributions, electromagnetic form factors and spectroscopic factors, as well as to compute many electroweak processes of astrophysical interest. These studies have clearly shown that the \textit{ab initio} approach based on the numerical solution of the many body Schrödinger equation and modern nucleon-nucleon interactions, fitted to NN scattering data, is capable to describe the full complexity of nuclear structure, including single particle properties, correlations and clustering.

For all the light nuclei \(A \leq 12\) of interest here, GFMC calculations have been carried out [for the lighter nuclei, exact solutions of the Schrödinger equation are also available from simpler approaches, such as Variational Monte Carlo (VMC)]. From the resulting binding energies, momentum distributions and (small) 3-body contributions we have calculated the average removal energies \(\bar{E}\) which are listed in the following table.

| nucleus | removal energy | method   | reference |
|---------|----------------|----------|-----------|
| \(^2\)H | 2.2 MeV        | GFMC     | \cite{26} |
| \(^3\)He | 14.6 MeV       | GFMC     | \cite{27} |
| \(^4\)He | 35.8 MeV       | VMC      | \cite{27} |
| \(^9\)Be | 43.8 MeV       | GFMC     | \cite{28} |
| \(^{12}\)C | 52.2 MeV       | GFMC     | \cite{28} |
| NM     | 70.5 MeV       | FHNC     | \cite{29} |

The values of \(\bar{E}\) are significantly larger than the typical values that have been used in previous studies of the EMC effect \cite{37}, studies which often found that binding effects alone were not large enough to explain the EMC data. For \(^{12}\)C, for example, the average removal energy one would derive from the centroids of the s- and p-shell removal energies as measured by \((e, e'p)\) experiments \cite{30} is of the order 25 MeV. A similar value is obtained from mean-field calculations, such as Hartree-Fock. The difference to the value listed in Table 1, 52 MeV, is due to the fact that the s- and p-shell peaks \cite{30} account only for (the \(\sim 75\%\) of) nucleons occupying mean-field orbitals \cite{31}. The short-range correlations between nucleons, induced by the strong short-range components of the NN interaction (both central and tensor) lead to the appearance of nucleons in states of high momentum and high removal energy; the corresponding strength, which is thinly spread over a large range of initial momenta \(k\) and removal energies \(E\), gives an important contribution to \(\bar{E}\). This picture has been confirmed by \((e, e'p)\) experiments designed to provide a measurement of the spectral function \(S(k, E)\) at large values of \(k\) and \(E\) \cite{23}.

Coming back to the EMC-effect, we show in Fig. 1 the correlation between the average removal energy \(\bar{E}\) and the derivatives \(dR/dy\) determined from the \(dR/dx\) values of Ref. \cite{7,38}. As Fig. 1 shows, there is an excellent linear correlation between \(\bar{E}\) and \(dR/dy\). This correlation is much better than the one of \(dR/dy\) (\(dR/dx\)) with other quantities (referred to above). This indicates that the driving quantity for the EMC-effect at mid-\(x\) values is indeed the binding of nucleons, as described by the mean removal energy.

To the extent that quasi-elastic electron-nucleus scattering at large \(x\) could be identified with high-momentum components, the correlation with \(a_2\) found in \cite{4,10} actually would be based on the same physics as discussed here. There is a strong correlation between \(\bar{E}\) and \(\bar{T}\) (for \(^{12}\)C, for instance, \(\bar{E}=25.2\) MeV, \(\bar{T}=31.2\) MeV, the difference leading to a comparatively small \(E_0/A=6.1\) MeV). But while high-\(k\) nucleons alone lead to EMC-ratios \(R > 1\) and a positive slope near \(x \sim 0.5\), the binding
leads to a much larger effect in the opposite direction, producing overall the $R < 1$ and $dR/dx < 1$ as found by experiment.

In Figure 1 we have also included the point corresponding to uniform, isospin symmetric, nuclear matter (NM). The average removal energy has been determined as discussed above from the variational results of Ref. [29], obtained using the Fermi Hyper-Netted Chain (FHNC) summation scheme. The slope $dR/d\tilde{y}$ has been extracted from Ref. [32, 39].

As the determination of $R(x)$ for nuclear matter involved a fit to the world EMC-data for all nuclei with mass number $A \geq 12$, the NM point is indicative of the fact that the excellent correlation between $dR/d\tilde{y}$ and $\bar{E}$ is valid for all nuclei.

We note that the correlation between $dR/dx$ and $\tilde{E}$ (not shown) is very similar to the one observed in Fig. 1. Only the NM data point, which on average corresponds to larger $Q^2$ than the data of [2], would be slightly shifted due to a different conversion factor between $x$ and $\tilde{y}$. While numerically the difference between $x$ and $\tilde{y}$ is small, these quantities differ radically in their physical interpretation.

In principle, the approach based on many body theory employed in our work, while not including some of the mechanisms which are believed to determine the low-$x$ behavior of EMC ratio, may be used to obtain theoretical estimates of its slope at mid $x$. However, to achieve the level of accuracy required for a meaningful comparison with the data, one would need spectral functions computed using the Green’s function Monte Carlo technique and including the full set of eigenstates of the recoiling nucleus, which are not yet available.

In the case of infinite nuclear matter, $S(k, E)$ has been computed within the FHNC/SOC summation scheme, including the contributions of one hole and two hole-one particle states [19]. However, compared to the calculation of the ground state expectation value of the hamiltonian discussed in Ref. [29], the work of Ref. [19] was based on an oversimplified treatment of the three-nucleon interactions and involved a number of additional technical difficulties (e.g. the orthogonalization of correlated states), leading to a larger theoretical uncertainty. As a result, the value of the average removal energy obtained from the spectral function of Ref. [19], $\bar{E} \approx 61$ MeV, appreciably differs from the one reported in Table 1.

In conclusion, we have shown that there is a strong correlation between the EMC-effect at mid-$x$ and the average nucleon removal energy. This correlation covers all nuclei, from $^3$He all the way to infinite nuclear matter. This confirms that the binding of the nucleons in the nucleus is very important for the EMC effect at $x \sim 0.5$.

As binding plays an important role, our study of the data was done in terms of the scaling variable $\tilde{y}$. Being derived as a property of initially off-shell quarks in the Lab frame, $\tilde{y}$ can be generalized to take into account the additional off-shellness due to nuclear binding without conceptual difficulties. It should also be emphasized that, while $\tilde{y}$ is particularly suited to discuss binding effects, it may also be preferable to Bjorken-$x$ in general as a scaling variable. Not only does $\tilde{y}$ yield better scaling, it also has a more intuitive physics interpretation as the momentum component of the $u, d$-quarks parallel to $q$ in the rest frame of the nucleon, and allows for a unified description of inclusive scattering. The analogous occurrence of $y$-scaling, and its straightforward interpretation, have been exploited to extract valuable information from the analysis of a variety of scattering processes, ranging from photon scattering from electrons bound in atoms to neutron scattering from quantum liquids and quasi-elastic electron-nucleus scattering [33].

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* Electronic address: benhar@roma1.infn.it
† Electronic address: ingo.sick@unibas.ch

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[37] Early exceptions are Refs. [5, 34].
[38] $\xi = -\tilde{y}/M$ differs by less than 5% from $x$ and is calculated using e.g. eq. [5] and $\tilde{Q}^2$ from $\tilde{f}$. We have added Coulomb corrections in [22]; they change the slope by $\sim 2\%$. 
