Ensembles of driven or motile bodies moving along opposite directions are generically reported to self-organize into strongly anisotropic lanes. Here, building on a minimal model of self-propelled bodies targeting opposite directions, we first evidence a critical phase transition between a mingled state and a phase-separated lane state specific to active particles. We then demonstrate that the mingled state displays algebraic structural correlations also found in driven binary mixtures. Finally, constructing a hydrodynamic theory, we single out the physical mechanisms responsible for these universal long-range correlations typical of ensembles of oppositely moving bodies.

Should you want to mix two groups of pedestrians, or two ensembles of colloidal beads, one of the worst possible strategies would be pushing them towards each other. Both experiments and numerical simulations have demonstrated the segregation of oppositely driven Brownian particles into parallel lanes [1–5]. Even the tiniest drive results in the formation of finite slender lanes which exponentially grow with the driving strength [5]. The same qualitative phenomenology is consistently observed in pedestrian counterflows [6–10]. From our daily observation of urban traffic to laboratory experiments, the emergence of counter propagating lanes is one of the most robust phenomena in population dynamics, and has been at the very origin of the early description of pedestrians as granular materials [11, 12]. However, a description as isotropic grains is usually not sufficient to account for the dynamics of interacting motile bodies [13–15]. From motility-induced phase separation [15], to giant density fluctuations in flocks [13, 16, 17], to pedestrian scattering [18, 19], the most significant collective phenomena in active matter stem from the interplay between their position and orientation degrees of freedom.

In this communication, we address the phase behavior of a binary mixture of active particles targeting opposite directions. Building on a prototypical model of self-propelled bodies with repulsive interactions, we numerically evidence two nonequilibrium steady states: (i) a lane state where the two populations maximize their flux and phase separate, and (ii) a mixed state where all motile particles mingle homogeneously. We show that these two distinct states are separated by a genuine critical phase transition. In addition, we demonstrate algebraic density correlations in the homogeneous phase, akin to that recently reported for oppositely driven Brownian particles [20]. Finally, we construct a hydrodynamic description to elucidate these long-range structural correlations, and conclude that they are universal to both active and driven ensembles of oppositely-moving bodies.

RESULTS

A minimal model of active binary mixtures

We consider an ensemble of $N$ self-propelled particles characterized by their instantaneous positions $\mathbf{r}_i(t)$ and orientations $\hat{p}_i(t) = (\cos \theta_i, \sin \theta_i)$, where $i = 1, \ldots, N$ (in all that follows $\hat{x}$ stands for $x/|x|$). Each particle moves along its orientation vector at constant speed ($|\mathbf{r}_i| = 1$). We separate the particle ensemble into two groups of equal size following either the direction $\Theta_i = 0$ (right movers) or $\pi$ (left movers) according to a harmonic angular potential $\mathcal{V}(\theta_i) = \frac{B}{2}(\theta_i - \Theta_i)^2$. Their equations of motion take the simple form:

$$\dot{\mathbf{r}}_i = \hat{p}_i, \quad (1)$$

$$\dot{\theta}_i = -\partial_{\theta_i} \mathcal{V}(\theta_i) + \sum_j T_{ij}, \quad (2)$$

In principle, oriented particles can interact via both forces and torques. We here focus on the impact of orientational couplings and consider that neighbouring particles interact solely through pairwise additive torques $T_{ij}$. This type of model has been successfully used to describe a number of seemingly different active systems, starting from bird flocks, fish schools and bacteria colonies to synthetic active matter made of self-propelled colloids or polymeric biofilaments [13, 21–27]. We here elaborate on a minimal construction where the particles interact only via repulsive torques. In practical terms, we choose the standard form $T_{ij} = -\partial_{\theta_j} \mathcal{E}_{ij}$, where the effective angular energy simply reads $\mathcal{E}_{ij} = -B(r_{ij}) \hat{p}_i \cdot \hat{r}_{ij}$. As sketched in Fig. 1a, this interaction promotes the orientation of $\hat{p}_i$ along the direction of the center-to-center vector $\mathbf{r}_{ij} = (\mathbf{r}_i - \mathbf{r}_j)$: as they interact particles turn their back to each other (see also e.g. [24, 28–30]). The spatial decay of the interactions is given by: $B(r_{ij}) = B(1 - r_{ij}/(a_i + a_j))$, where $B$ is a finite constant if $r_{ij} < (a_i + a_j)$ and 0 otherwise. In all that follows we focus on the regime where repulsion overcomes alignment along the preferred direction ($B > 1$). The interaction ranges $a_i$ are chosen to be polydisperse in or-
FIG. 1. Phase behavior. (a) Trajectories of two particles interacting solely via a repulsive torque as defined in Eq. (2) with $B = 5$. The post-collision orientations $\mathbf{p}_i(t)$ are along the center to center axis $r_{ij}$. (b) and (d) Snapshots of a square window at the center of the simulation box ($L_x = 168, N = 1973, \pi \rho a^2 = 0.65$), respectively in the lane ($B = 2$) and the homogeneous ($B = 5$) states. The arrows indicate the instantaneous position and orientation of the particles. Dark blue: right movers. Light blue: left movers. (c) Phase diagram. $\pi \rho a^2$ is the particle area fraction. Filled symbols: homogeneous state. Open symbols: lanes. (e) Probability distribution of the density difference $\hat{\rho} = \rho_r - \rho_l$. Light orange line: $B = 2$, $\pi \rho a^2 = 0.65$. Dark blue line: $B = 5$, $\pi \rho a^2 = 0.65$. Dashed line: best Gaussian fit. (f) P.d.f. of the orientational fluctuations around the preferred orientation (lin-log plot). Same parameters and colors as in (e). Inset: orientational diffusivity $D_\theta$ in the homogeneous state at a fixed repulsion magnitude ($B = 5$) and different particle area fractions $\pi \rho a^2$. $D_\theta$ is defined as the decorrelation time of the particle orientation. In the mingled state, the velocity autocorrelation decays exponentially at short time, $D_\theta$ is therefore defined without ambiguity, see also Supplementary Note 1 for a full description of the numerical computation of $D_\theta$. Dashed line: best linear fit.

Critical mingling

Starting from random initial conditions, we numerically solve Eqs. (1) and (2) using forward Euler integration with a time step of $10^{-2}$, and a sweep-and-prune algorithm for neighbour summation. We use a rectangular simulation box of aspect ratio $L_x = 2L_y$ with periodic boundary conditions in both directions. We also restrain our analysis to $H = 1$, leaving two control parameters that are the repulsion strength $B$ and the overall density $\bar{\rho}$. The following results correspond to simulations with $N$ comprised between 493 and 197,300 particles.

We observe two clearly distinct stationary states illustrated in Figs. 1b and 1d. At low density and/or weak repulsion the system quickly phase separates. Computing the local density difference between the right and left movers $\bar{\rho}(r, t) = \rho_r(r, t) - \rho_l(r, t)$, we show that this dynamical state is characterized by a strongly bimodal density distribution, see Fig. 1e. The left and right movers quickly self-organize into counter-propagating lanes separated by a sharp interface, Fig. 1b. In each stream, virtually no particle interact and most of the interactions occur at the interface. As a result the particle orientations are very narrowly distributed around their mean value, Fig. 1f. In stark contrast, at high density and/or strong repulsion, the motile particles do not phase separate. Instead, the two populations mingle and continuously interact to form a homogeneous liquid phase with Gaussian density fluctuations, and much broader orientational fluctuations, Figs. 1d, 1e, and 1f. This behavior is summarized by the phase diagram in Fig. 1c. Although phase separation is most often synonymous of first order transition in equilibrium liquids, we now argue that the lane and the mingled states are two genuine non-equilibrium phases separated by a critical line in the $(B, \bar{\rho})$ plane. To do so, we first introduce the following orientational order parameter:

$$\langle W \rangle = \langle 1 - \cos(\theta_i - \Theta_i) \rangle_i.$$  

(3)

$\langle W \rangle$ vanishes in the lane phase where on average all particles follow their preferred direction, and takes a non zero value otherwise. We show in Fig. 2a how $\langle W \rangle$ increases with the repulsion strength $B$ at constant $\bar{\rho}$. For $\pi \rho a^2 = 0.65$ the order parameter averages to zero below $B_c = 2.17 \pm 0.02$, while above $B_c$ it sharply increases as $W \sim |B - B_c|^\beta$, with $\beta = 0.33 \pm 0.07$, Fig. 2b. This scaling law suggests a genuine critical behavior. We further
confirm this hypothesis in Fig. 2c, showing that the fluctuations of the order parameter diverge as $|B - B_c|^{-\gamma}$, with $\gamma = 0.64 \pm 0.07$. Deep in the homogeneous phase the fluctuations plateau to a constant value of the order of $1/N$. Finally, the criticality hypothesis is unambiguously ascertained by Fig. 2d, which shows the power-law divergence of the correlation time of $\langle W \rangle (t)$: $\tau_W \sim |B - B_c|^{-z\nu}$ with $z\nu = 1.27 \pm 0.16$.

We do not have a quantitative explanation for this critical behavior. However, we can gain some insight from the counterintuitive two-body scattering between active particles. In the overdamped limit, the collision between two passive colloids driven by an external field would at most shift their position over an interaction diameter $[31]$. Here these transverse displacements are not bounded by the range of the repulsive interactions. For a finite set of impact parameters, collisions between self-propelled particles result in persistent deviations transverse to their preferred trajectories illustrated in Fig. 3 and Supplementary Note 2. This persistent scattering stems from the competition between repulsion and alignment. When these two contributions compare, bound pairs of oppositely moving particles can even form and steadily propel along the transverse direction $y$. Figs. 3a and 3b. We stress that this behavior is not peculiar to this two-body setting: persistent transverse motion of bound pairs is clearly observed in simulations at the onset of laning. We therefore strongly suspect the resulting enhanced mixing to be at the origin of the sharp melting of the lanes and the emergence of the mingled state.

**Long-range correlations in mingled liquids**

We now evidence long-range structural correlations in this novel active-liquid phase, and analytically demonstrate their universality. The overall pair correlation function of the active liquid, $g(r)$, is plotted in Fig. 4a. At a first glance, deep in the homogeneous phase, the few visible oscillations would suggest a simple anisotropic liquid structure. However, denoting $\alpha$ and $\beta$ the preferred direction of the populations (left or right), we find that the asymptotic behaviors of all pair correlation functions $g_{\alpha\beta}(x, y = 0)$ decay algebraically as $|1 - g_{\alpha\beta}(x, 0)| \sim x^{-\nu_x}$ with $\nu_x \sim 1.5$, Fig. 4b. This power-law behavior is very close to that reported in numerical simulations $[4]$.
and fluctuating density functional theories of oppositely driven colloids at finite temperature [20].

Hydrodynamic description

In order to explain the robustness of these long-range correlations, we provide a hydrodynamic description of the mingled state, and compute its structural response to random fluctuations. We first observe that the orientational diffusivity of the particles increases linearly with the average density \( \rho \) in Fig. 1f inset. This behaviour indicates that binary collisions set the fluctuations of this active liquid, and hence suggests using a Boltzmann kinetic-theory framework, see e.g [32, 33] for an active-matter perspective. In the large \( B \) limit, the microscopic interactions are accounted for by a simplified scattering rule anticipated from Eq. (2) and confirmed by the inspection of typical trajectories (see Fig. 1a). Upon binary collisions the self-propelled particles align their orientation with the center-to-center axis regardless of their initial orientation and external drive. Assuming molecular chaos and binary collisions only, the time evolution of the one-point distribution functions \( \psi_\alpha(r, \theta, t) \) reads:

\[
\partial_t \psi_\alpha + \nabla \cdot [\hat{\mathbf{p}}_\alpha \psi_\alpha] + \partial_\theta \left[ \hat{\mathbf{p}}_\alpha \cdot \hat{\mathbf{h}}_\alpha \right] \psi_\alpha = \mathcal{I}^{\text{coll}}_\alpha.
\]

The convective term on the l.h.s stems from self-propulsion, the third term accounts for alignment with the preferred direction \( \hat{\mathbf{h}}_\alpha = \hat{\mathbf{x}} \) (resp. \( -\hat{\mathbf{x}} \)) for the right (resp. left) movers. Using the simplified scattering rule to express the so-called collision integral on the r.h.s, we can establish the dynamical equations for the density fluctuations \( \delta \rho_\alpha \) around the average homogeneous state (see Methods for technical details). Within a linear response approximation, they take the compact form:

\[
\partial_t \delta \rho_\alpha(r, t) + \nabla \cdot (J_\alpha + \tilde{J}) = 0, \tag{5}
\]

where \( J_\alpha \) describes the convection and the collision-induced diffusion of the \( \alpha \) species, and \( \tilde{J} \) is the coupling term, crucial to the anomalous fluctuations of the active liquid:

\[
J_\alpha = v_0 \hat{\mathbf{h}}_\alpha \delta \rho_\alpha - \mathbf{D} \cdot \nabla \delta \rho_\alpha, \tag{6}
\]

\[
\tilde{J} = -\hat{\mathbf{h}}_\alpha \cdot \mathbf{D} \cdot \nabla \mathbf{p}. \tag{7}
\]

The two anisotropic diffusion tensors \( \mathbf{D} \) and \( \tilde{\mathbf{D}} \) are diagonal and their expression is provided in Supplementary Note 3 together with all the hydrodynamic coefficients. \( \tilde{J} \) is a particle current stemming from the fluctuations of the other species and has two origins. The first term arises from the competition between alignment along the driving direction \( \hat{\mathbf{h}}_\alpha \) and orientational diffusion caused by the collisions: the higher the local density \( \mathbf{p} \), the smaller the longitudinal current. The second term originates from the pressure term \( \propto \nabla \mathbf{p} \): a local density gradient results in a net flow of both species (see Methods for details). This diffusive coupling is therefore generic and enters the description of any binary compressible fluid. Two additional comments are in order. Firstly, this prediction is not specific to the small-density regime and is expected to be robust to the microscopic details of the interactions. As a matter of fact, the above hydrodynamic description is not only valid in the limit of strong repulsion and small densities discussed above but also in the opposite limit, where the particle density is very large while the repulsion remains finite as detailed in the Supplementary Note 5. Secondly, the robustness of this hydrodynamic description could have been anticipated using conservation laws and symmetry considerations, as done e. g. in [16] for active flocks. Here the situation is simpler, momentum is not conserved and no soft mode is associated to any spontaneous symmetry breaking. As a result the only two hydrodynamic variables are the coupled...
(self-advected) densities of the two populations [34]. The associated mass currents are constructed from the only two vectors that can be formed in this homogeneous but anisotropic setting: \( \mathbf{h}_a \) and \( \nabla \delta \rho_a \). These simple observations are enough to set the functional form of Eqs. (5), (6) and (7).

By construction the above hydrodynamic description alone cannot account for any structural correlation. In order to go beyond this mean-field picture we classically account for fluctuations by adding a conserved noise source to Eqs. (5) and compute the resulting density-fluctuation spectrum [13]. At the linear response level, without loss of generality, we can restrain ourselves to two vectors that can be formed in this homogeneous but anisotropic setting (regardless of the associated noise anisotropy, see [35] and Supplementary 4). The structure factor is non-analytic as \( q \to 0 \), and the density correlations algebraic, only when \( a \neq b \). Inspecting Eq. (8), we readily see that this condition is generically fulfilled as soon as the coupling current \( \mathbf{J} \) is non zero. In other words, as soon as the collisions between the particles either modify their transverse diffusion \( (\mathbf{D} \cdot \mathbf{v}) \delta \rho \) or their longitudinal advection \( (\partial \mathbf{h}_a / \partial t) \delta \rho \). Both ingredients are present in our model of active particles (see Eqs. (5)) and, based on symmetry considerations, should be generic to any driven binary mixtures with local interactions. Another simple physical explanation can be provided to account for the variations of the pair correlations in the transverse direction shown Figs. 4c and 4d and also reported in simulations of driven particles [20]. Self-propulsion causes the particles to move, on average, at constant speed along the \( x \)-direction while frontal collisions induce their transverse diffusion. As a result the \( x \)-position of the particles increase linearly with time, and their transverse position increases as \( \sim t^{1/2} \). We therefore expect the longitudinal and transverse correlations to be related by a homogeneous function of \( y/x^{1/2} \) in steady state as observed in simulations of both active and driven particles. Altogether these observations confirm the universality of the long-range structural correlations found in both classes of non-equilibrium mixtures.

In conclusion, we have demonstrated that the interplay between orientational and translational degrees of freedom, inherent to motile bodies, can result in a critical transition between a phase separated and a mingled state in binary active mixtures. In addition we have singled out the very mechanisms responsible for long-range structural correlations in any ensemble of particles driven towards opposite directions, should they be passive colloids or self-propelled agents.

METHODS

Let us summarize the main steps of the kinetic theory employed to establish Eqs. (5) (6), (7). The so-called collision integral on the r.h.s of Eq. (4) includes two contributions which translate the behavior illustrated in Fig. 1a:

\[
\mathcal{I}^\alpha = \mathcal{D}_{in} \rho_a(\mathbf{r}) \mathcal{P}(\mathbf{r} - 2 \mathbf{a} \mathbf{p}) - \mathcal{D}_{out} \mathcal{P}(\mathbf{r}) \psi_a(\mathbf{r}, \theta). \tag{9}
\]

The first term indicates that a collision with any particle located at \( (\mathbf{r} - 2 \mathbf{a} \mathbf{p}) \) reorients the \( \alpha \) particles along \( \mathbf{p}(\theta) \) at a rate \( \mathcal{D}_{in} \). The second term accounts for the random reorientation, at a rate \( \mathcal{D}_{out} \), of a particle aligned with \( \mathbf{p}(\theta) \) upon collision with any other particle. Within a two-fluid
picture, the velocity and nematic texture of the α particles are given by \( \mathbf{v}_\alpha = \rho_\alpha^{-1} (p\hat{\mathbf{p}}) \) and \( Q_\alpha = \rho_\alpha^{-1} (\hat{\mathbf{p}}\hat{\mathbf{p}} - \frac{1}{2} I) \). The mass conservation relation, \( \partial_t \rho_\alpha + \nabla \cdot (\rho_\alpha \mathbf{v}_\alpha) = 0 \), is obtained by integrating Eq. (4) with respect to \( \theta \) and constrains \( (2\pi D_{\text{in}}) = D_{\text{out}} \equiv D \). The time evolution of the velocity field is also readily obtained from Eq. (4):

\[
\partial_t (\rho_\alpha \mathbf{v}_\alpha) + \nabla \cdot \left[ \rho_\alpha \left( \frac{1}{2} + Q_\alpha \right) \right] = \mathbf{F}_\alpha ,
\]

where the second term on the l.h.s is a convective term stemming from self-propulsion. The force field \( \mathbf{F}_\alpha \) on the r.h.s of Eq. (10) reads: \( \mathbf{F}_\alpha = \rho_\alpha \left( \frac{1}{2} - Q_\alpha \right) \cdot \mathbf{h}_\alpha - (aD \rho_\alpha) \nabla \hat{p} - (D \hat{p}) \rho_\alpha \mathbf{v}_\alpha . \) The first term originates from the alignment of particles along the \( \mathbf{h}_\alpha \) direction, the second term is a repulsion-induced pressure, and the third one echoes the collision-induced rotational diffusivity of the particles. An additional closure relation between \( Q_\alpha \), \( \mathbf{v}_\alpha \) and \( \rho_\alpha \) is required to yield a self-consistent hydrodynamic description. Deep in the homogeneous phase, we make a wrapped Gaussian approximation for the orientational fluctuations in each population [24, 36]. This hypothesis is equivalent to setting \( Q_\alpha = \langle \mathbf{v}_\alpha \rangle^2 (\hat{\mathbf{v}}_\alpha \hat{\mathbf{v}}_\alpha - \frac{1}{2} I) \) [24, 37]. As momentum is not conserved, the velocity field is not a hydrodynamic variable; in the long wavelength limit the velocity modes relax much faster than the (conserved) density modes. We therefore ignore the temporal variations in Eq. (10) and use this simplified equation to eliminate \( \mathbf{v}_\alpha \) in the mass-conservation relation, leading to the mass conservation equation Eq. (5).

**DATA AVAILABILITY**

The data that support the findings of this study are available from the corresponding author upon request.

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**AUTHOR CONTRIBUTIONS**

D. B. Designed the research. B.N. performed the numerical simulations. D. B. and N. B. performed the theory, discussed the results and wrote the paper.

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