Systematic analysis of strange single heavy baryons

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Based on the Godfrey-Isgur (GI) quark model, we investigate the mass spectra of strange single heavy baryons systematically. In the calculation of the Hamiltonian matrix elements, the Infinitesimally shifted Gaussian (ISG) basis functions are employed. Our results show that the $\lambda$-modes appear lower in energy than other excited modes for a given state $nL(J^P)$. Considering this feature, we perform a systematic study of the mass spectra of the $\Xi_c$ ($\Xi'_c$) and $\Xi_b$ ($\Xi'_b$) families. It is shown that the experimental data can be well reproduced by the predicted masses. In addition, the mass spectra allow us to successfully construct the Regge trajectories in the $(J, M^2)$ plane. For the excited states with quantum numbers up to $n = 2$ and $L = 2$, we also calculate their strong decay widths by the $^3P_0$ model. Based on the calculated mass spectra and the corresponding strong decay properties, we analyze the recently observed baryons, including $\Xi_c(3055)$, $\Xi_c(3080)$, $\Xi_c(2930)$, $\Xi_c(2923)$, $\Xi_c(2939)$, $\Xi_c(2970)$, $\Xi_c(3123)$, $\Xi_b(6100)$, $\Xi_b(6227)$, $\Xi_b(6327)$ and $\Xi_b(6333)$. At last, the shell structure of the strange single heavy baryon spectra is shown, from which one could get a bird’s-eye view of the mass spectra. Accordingly, we predict several new baryons that might be observed in forthcoming experiments.

Key words: Single heavy baryons, Mass spectra, GI model, $^3P_0$ model.

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I. Introduction

In recent years, many single heavy baryons have been observed in experiments, and the mass spectra of single heavy baryon families have become more and more abundant \cite{1,31}. Under this background, searching for new heavy baryons and completing the heavy baryons spectra provoked many interests in the field of hadron physics. At the same time, such a wealth of experimental data gives theorists...
an opportunity to test the validity of current theoretical frameworks. Additionally, this is also a good time to carry out a systematic and precise calculation with some theoretical methods/models, so as to promote the consistency between the experiments and theories.

As an important part of the heavy baryons spectra, the strange single heavy baryon (Ξ_Q) families including Ξ_c (Ξ_c') [1–15] and Ξ_b (Ξ_b') [17–27], are being established step by step with the cooperative efforts from both experimentalists and theorists. Up to now, more than a dozen of Ξ_Q baryons have been recorded in the latest particle data group (PDG) [30], even though the quantum numbers J^P of some members are still not confirmed, for instance, Ξ_c(3055), Ξ_c(3080) and Ξ_c(6227). In addition, several new Ξ_Q baryons have been observed in experiment, such as Ξ_c(3123) [3], Ξ_c(2930) [13], Ξ_c(2923), Ξ_c(2939), Ξ_c(2964) [14], Ξ_b(6327) and Ξ_b(6333) [27]. Once their J^P quantum numbers are identified, they will be accommodated in the Ξ_Q families as well. Accordingly, there have been a lot of theoretical studies on these baryons in recent years, such as Ξ_c(3055) [32–36], Ξ_c(3080) [36–39], Ξ_c(2923)^0 (including Ξ_c(2939)^0 and Ξ_c(2965)^0) [40], Ξ_c(2970) [42–47], Ξ_c(3123) [48], Ξ_b(6227) [49, 50], Ξ_b(6100) [51, 52], and Ξ_b(6327) (Ξ_b(6333)) [53–55]. It is believed that more strange single heavy baryons are to be discovered in the near future. In order to identify their quantum numbers and assign them suitable positions in the mass spectra, it is necessary to systematically investigate their spectroscopies and strong decay properties.

For a definite state of baryons, its mass and strong decay behavior are commonly studied by the following theoretical methods: The quark potential model in the heavy quark-light diquark picture [48, 57–60], relativistic quark model [61, 62], harmonic oscillator quark model [40], constituent quark model [63–66], chiral quark model [32, 41, 49, 56], chiral perturbation theory [67–71], relativistic flux tube model [42, 72], Bethe-Salpeter formalism [73], effective Lagrangian approach [50, 3^P_0 decay model [74–81], lattice QCD [82–85], bound state picture [86], light cone QCD sum rules [87–96], and QCD sum rules method [97–106].

In the 1980’s, Godfrey and Isgur developed a relativistic quark model, the so-called Godfrey-Isgur (GI) model, by which they studied the mass spectra of mesons [107] and baryons [61] preliminarily. It is shown the GI model is effective in the study of hadron spectra. Actually, to our knowledge, the GI model has become a widely used relativistic quark model with chromodynamics so far. In the GI model, the Hamiltonian contains almost all of the interactions between quarks, which is expected to give accurate calculations for heavy baryons spectra.

In recent decades, the Gaussian expansion method(GEM) and the infinitesimally-shifted Gaussian(ISG) basis functions [108] have been successfully applied to few-body systems in nuclear physics. They were first introduced in the study of heavy baryons [63] in 2015 and then applied to analyze furthermore heavy baryons [64, 65], tetraquarks [109–111] and pentaquark resonances [112]. In the present calculations, we try to combine the GI model with the ISG method, and apply them to the
study of single heavy baryons spectra. The main goal of this work is to calculate the mass spectra of excited single heavy baryons accurately.

For a further understanding of these excited baryons, it is useful to perform a systematic study of their strong decays. The $^3P_0$ decay model or the quark pair creation (QPC) model, which was built by Micu [113] and further developed by Yaouanc et al. [114–118], has been extensively applied to study the Okubo-Zweig-Iizuka (OZI)-allowed strong decays of hadrons [119–129]. Based on the mass spectra obtained with the GI model, we can systematically calculate the corresponding decay widths of excited baryons by the $^3P_0$ model.

This paper is organized as follows. In Sect.II, we briefly describe the methods used in the theoretical calculations, mainly including the GI model, the GEM(ISG) method and the $^3P_0$ model; In Sect.III, we present the complete mass spectra of the $\Xi_Q$ baryons, construct the Regge trajectories and calculate the corresponding strong decay widths. On these bases, we perform a detailed analysis of the baryons that have been of interest recently; And Sect.IV is reserved for our conclusions.

II. Phenomenological methods adopted in this work

In our series of papers on the heavy baryons, we apply the GI model to investigate the mass spectra, so as to obtain the allowed quantum states and their masses. Then, we calculate the corresponding strong decay widths with the $^3P_0$ model. The calculated masses and decay widths for certain quantum states are helpful in the identification of the related baryons. The relevant technical details can be found in our previous paper [62]. Therefore, in this section, we only give an outline of the phenomenological methods.

2.1 GI model and Jacobi coordinates

The GI model is a relativized quark model proposed by Godfrey and Isgur [61, 104]. This model has been extensively adopted to study the properties of conventional hadrons and it may give a unified description of different flavor sectors. Briefly, this model is based on the hypothesis that baryons may be approximately described in terms of center-of-mass (CM) frame valence-quark configurations, the dynamics of which are governed by a Hamiltonian with a one-gluon exchange dominant component at short distances and with a confinement implemented by a flavor-independent Lorentz-scalar interaction [61]. For a three-quark system the Hamiltonian reads,

$$H = H_0 + V,$$  (1)
\[ H_0 = \sum_{i=1}^{3} (p_i^2 + m_i^2)^{1/2}, \]  
\[ V = \sum_{i<j} (\tilde{H}_{ij}^{\text{conf}} + \tilde{H}_{ij}^{\text{so}} + \tilde{H}_{ij}^{\text{hyp}}), \]

where \( \tilde{H}_{ij}^{\text{conf}} \), \( \tilde{H}_{ij}^{\text{so}} \) and \( \tilde{H}_{ij}^{\text{hyp}} \) are the confinement, spin-orbit and hyperfine interactions, respectively. The confinement item includes one-gluon exchange potentials and the linear confined potentials. Due to the relativistic effect, the interactions should be modified with CM momentum-dependent factors. It is worth noting that the forms of the interactions in this paper have been rearranged for ease of use \[109, 130]. \]

The interactions are decomposed as follows:

\[ \tilde{H}_{ij}^{\text{conf}} = G'_{ij}(r) + \tilde{S}_{ij}(r), \]  
\[ \tilde{H}_{ij}^{\text{so}} = \tilde{H}_{ij}^{\text{so}(v)} + \tilde{H}_{ij}^{\text{so}(s)}, \]  
\[ \tilde{H}_{ij}^{\text{hyp}} = \tilde{H}_{ij}^{\text{tensor}} + \tilde{H}_{ij}^{c}, \]

with

\[ \tilde{H}_{ij}^{\text{so}(v)} = \frac{S_i \cdot L_{ij}}{2m_i^2r_{ij}} \partial \tilde{G}^{\text{so}(v)} \left( \frac{m_i}{E_i} \right) \partial_{r_{ij}} + \frac{S_j \cdot L_{ij}}{2m_j^2r_{ij}} \partial \tilde{G}^{\text{so}(v)} \left( \frac{m_j}{E_j} \right) \partial_{r_{ij}}, \]  
\[ \tilde{H}_{ij}^{\text{so}(s)} = \frac{S_i \cdot L_{ij}}{2m_i^2r_{ij}} \partial \tilde{S}^{\text{so}(s)} \left( \frac{m_i}{E_i} \right) \partial_{r_{ij}} - \frac{S_j \cdot L_{ij}}{2m_j^2r_{ij}} \partial \tilde{S}^{\text{so}(s)} \left( \frac{m_j}{E_j} \right) \partial_{r_{ij}}, \]  
\[ \tilde{H}_{ij}^{\text{tensor}} = -\frac{S_i \cdot r_{ij} S_j \cdot r_{ij}}{m_i m_j} \frac{r_{ij}^2 - \frac{4}{3} S_i \cdot S_j}{r_{ij}^3} \times \left( \frac{\partial^2}{r_{ij}^2} - \frac{1}{r_{ij}^3} \frac{\partial}{\partial r_{ij}} \right) \tilde{G}^{t}, \]  
\[ \tilde{H}_{ij}^{c} = \frac{2S_i \cdot S_j}{3m_i^2 m_j} \nabla^2 \tilde{G}_ij^c. \]

The modified terms in Eqs. (4), (7), (8), (9) and (10) read,

\[ G'_{ij} = (1 + \frac{p_i^2}{E_i})^{\frac{1}{2}} \tilde{G}_{ij} (r_{ij}) (1 + \frac{p_j^2}{E_j})^{\frac{1}{2}}, \]  
\[ \tilde{G}^{\text{so}(v)} = \left( \frac{m_i m_j}{E_i E_j} \right)^{\frac{1}{2}} \tilde{G}^{\text{so}(v)} \left( \frac{m_i}{E_i} \right)^{\frac{1}{2}} \tilde{G}^{\text{so}(v)} \left( \frac{m_j}{E_j} \right)^{\frac{1}{2}}, \]  
\[ \tilde{S}^{\text{so}(s)} = \left( \frac{m_i m_j}{E_i E_j} \right)^{\frac{1}{2}} \tilde{S}^{\text{so}(s)} \left( \frac{m_i}{E_i} \right)^{\frac{1}{2}} \tilde{S}^{\text{so}(s)} \left( \frac{m_j}{E_j} \right)^{\frac{1}{2}}, \]  
\[ \tilde{G}^{t} = \left( \frac{m_i m_j}{E_i E_j} \right)^{\frac{1}{2}} \tilde{G}^{t} \left( \frac{m_i}{E_i} \right)^{\frac{1}{2}} \tilde{G}^{t} \left( \frac{m_j}{E_j} \right)^{\frac{1}{2}}, \]  
\[ \tilde{G}^{c} = \left( \frac{m_i m_j}{E_i E_j} \right)^{\frac{1}{2}} \tilde{G}^{c} \left( \frac{m_i}{E_i} \right)^{\frac{1}{2}} \tilde{G}^{c} \left( \frac{m_j}{E_j} \right)^{\frac{1}{2}}, \]

where \( E_i = \sqrt{m_i^2 + p_i^2} \) is the relativistic kinetic energy, and \( p_{ij} \) is the momentum magnitude of either of the quarks in the CM frame of the \( ij \) quark subsystem \[109]. \]

\( \tilde{G}_{ij}(r_{ij}) \) and \( \tilde{S}_{ij}(r_{ij}) \) are obtained by the smearing transformations of the one-gluon exchange potential \( G(r) = -\frac{4\alpha_s(r)}{3r^2} \) and linear confinement potential \( S(r) = br + c \), respectively,

\[ \tilde{G}_{ij}(r_{ij}) = \mathbf{F}_i \cdot \mathbf{F}_j \sum_{k=1}^{3} \frac{2\alpha_k}{\sqrt{\pi} r_{ij}} \int_{0}^{r_{ij}} e^{-x^2} dx, \]
\[ \tilde{S}_{ij}(r_{ij}) = -\frac{3}{4} \mathbf{F}_i \cdot \mathbf{F}_j \{ b_{ij} \frac{e^{-\sigma_{ij}^2 r_{ij}^2}}{\sqrt{\pi \sigma_{ij} r_{ij}}} + (1 + \frac{1}{2\sigma_{ij}^2 r_{ij}^2}) \frac{2}{\sqrt{\pi}} \int_0^{r_{ij}} e^{-x^2} dx \} + c, \]  

(17)

with

\[ \tau_{ki} = \frac{1}{\sqrt{\frac{1}{\sigma_{ij}^2} + \frac{1}{\gamma_k^2}}}, \]

(18)

\[ \sigma_{ij} = \sqrt{\frac{s^2 (2m_i m_j)}{(m_i + m_j)^2} + \frac{\sigma_0^2}{4} \left( \frac{4m_i m_j}{(m_i + m_j)^2} \right)^4 + \frac{1}{2}}, \]

(19)

Here \( \alpha_k \) and \( \gamma_k \) are constants. \( \mathbf{F}_i \cdot \mathbf{F}_j \) stands for the inner product of the color matrices of quarks \( i \) and \( j \). \( \mathbf{F} \) includes 8 components (the so-called Gellmann matrices), which can be written as

\[ F_n = \begin{cases} \frac{\hat{\lambda}_n}{2}, & \text{for quarks,} \\ -\frac{\hat{\lambda}_n^*}{2}, & \text{for antiquarks}, \end{cases} \]

(20)

with \( n = 1, \cdots, 8 \). All of the parameters in these formulas have been adjusted to the best values [61, 62].

To represent the internal motion of quarks in a few-body system, one commonly introduces the Jacobi coordinates. As shown in Fig.1, there are totally three channels of Jacobi coordinates for the three-body system. The corresponding Jacobi coordinates are defined as

\[ \rho_i = r_j - r_k, \]

(21)

\[ \lambda_i = r_i - \frac{m_i r_j + m_k r_k}{m_j + m_k}, \]

(22)

where \( i, j, k = 1, 2, 3 \) (or replace their positions in turn). \( r_i \) and \( m_i \) denote the position vector and the mass of the \( i \)th quark, respectively.

FIG. 1: (Color online) Jacobi coordinates for the three-body system. We denote the heavy quark as the 3rd particle in the case of single heavy baryons.

In principle, the three channels are equivalent. We perform our calculations based on the channel 3. In this case, the 3rd quark is just the heavy quark, which is consistent with the heavy quark limit [131, 132]. What is more, \( l_{\rho 3} \) (denoted in short as \( l_\rho \)) is clearly defined as the orbital angular momentum between the light quarks, and \( l_{\lambda 3} \) (denoted in short as \( l_\lambda \)) represents the one between
the heavy quark and the light-quark pair. $l_\rho$ and $l_\lambda$ will be selected as good quantum numbers to define the states in the next subsections. Based on the transformation of Jacobi coordinates, we can calculate all the matrix elements in channel 3 [62].

### 2.2 The heavy quark limit and wave function

In the heavy quark limit [131, 132], the heavy quark within the heavy baryon system is decoupled from the two light quarks. With the requirement of the flavor SU(3) subgroups for the light quarks, the baryons belong to either a sextet ($6_F$) of flavor symmetric states $\Xi'_Q$ with the total spin of the light quarks $s = 1$, or an antitriplet ($\bar{3}_F$) of the flavor antisymmetric states $\Xi_Q$ with $s = 0$. This guarantees the antisymmetry of the total wave function, because the color wave function is always asymmetric for a baryon. The flavor wave functions of strange single heavy baryons are written as,

$$\Xi'_Q = \frac{1}{\sqrt{2}} (qq_s + q^*_s q)Q,$$

$$\Xi_Q = \frac{1}{\sqrt{2}} (qq_s - q^*_s q)Q. \tag{23}$$

Here $q$ denotes light quark $u$ or $d$, and $Q$ is $c$ or $b$ quark. $q_s$ is $s$ quark. For a definite state of baryons, the spatial wave function is combined with the spin function as follow.

$$|l_\rho l_\lambda L s j J M_J\rangle = \sum_{m_{s1}=-1/2}^{1/2} \sum_{m_{j1}=-j}^{j} \sum_{M_L=-L}^{L} \sum_{m_s=-s}^{s} \sum_{m_{s2}=-1/2}^{1/2} \sum_{m_{\rho}=-l_{\rho}}^{l_{\rho}} \sum_{m_{\lambda}=-l_{\lambda}}^{l_{\lambda}}$$

$$\times \langle j m_j s_3 m_{s3} | j s_3 J M_J \rangle \times \langle L M_L s m_s | L s j m_j \rangle$$

$$\times \langle s_1 m_{s1} s_2 m_{s2} | s_1 s_2 s m_s \rangle \times \langle l_{\rho} m_{\rho} l_{\lambda} m_{\lambda} | l_{\rho} l_{\lambda} L M_L \rangle$$

$$\times |l_{\rho} m_{\rho}\rangle \otimes |l_{\lambda} m_{\lambda}\rangle \otimes |s_1 m_{s1}\rangle \otimes |s_2 m_{s2}\rangle \otimes |s_3 m_{s3}\rangle, \tag{24}$$

with $J = j + s_3$, $j = L + s$, $s = s_1 + s_2$, $L = l_{\rho} + l_{\lambda}$, $l_{\rho}$, $l_{\lambda}$, $L$, $s$, $j$, $J$ and $M_J$ are the quantum numbers which characterize a given quantum state of baryons.

### 2.3 GEM and ISG

In calculations, the spatial wave function $|l_{\rho} m_{\rho}\rangle \otimes |l_{\lambda} m_{\lambda}\rangle$ in formula (24) should be expanded in a set of basis functions. Naturally, one of the candidates is the simple harmonic oscillator(SHO) basis for its good orthogonality. However, the completeness of the SHO is not rigorous in calculations because a truncated set has to be used [61, 107]. One can commonly enlarge the harmonic-oscillator-based space to asymptotically satisfy the completeness. Nevertheless, the computation time will increase significantly. Compared to the SHO basis functions, the advantage of the Gaussian basis functions is that they can form an approximately complete set in a finite coordinate space.
Following formula (24), the spatial wave function is expanded in terms of a set of Gaussian basis functions,

\[ |l_{p}m_{p} \rangle \otimes |l_{\lambda}m_{\lambda} \rangle = \sum_{n_{p}=1}^{n_{\text{max}}} \sum_{n_{\lambda}=1}^{n_{\text{max}}} c_{n_{p},n_{\lambda}} |n_{p}l_{p}m_{p} \rangle^{G} \otimes |n_{\lambda}l_{\lambda}m_{\lambda} \rangle^{G}, \]

(25)

where the Gaussian basis function \( |nlm \rangle^{G} \) is commonly written in position space as \[108\]

\[ \phi_{nlm}^{G}(r) = \phi_{nl}^{G}(r) Y_{lm}(\hat{r}), \]

\[ \phi_{nl}^{G}(r) = N_{nl} r^{l} e^{-\nu_{n} r^{2}}, \]

(26)

or in momentum space as

\[ \phi_{nlm}'(p) = \phi_{nl}'(p) Y_{lm}(\hat{p}), \]

\[ \phi_{nl}'(p) = N_{nl}' p^{l} e^{-\frac{p^{2}}{4\nu_{n}}}, \]

(27)

with

\[ \nu_{n} = \frac{1}{r_{n}^{2}}, \]

\[ r_{n} = r_{1} a^{n-1} \quad (n = 1, 2, ..., n_{\text{max}}). \]

\( r_{1}, a, \) and \( n_{\text{max}} \) are the Gaussian size parameters in the geometric progression for numerical calculations, and the final results are stable and independent of these parameters within an approximately complete set in a sufficiently large space.

The Gaussian basis functions are non-orthogonal, which leads to a generalized matrix eigenvalue problem,

\[ \sum_{\kappa,\kappa'=1}^{\kappa_{\text{max}}} [H_{\kappa \kappa'} - E \tilde{N}_{\kappa \kappa'}] c_{\kappa'} = 0, \]

(29)

with

\[ \tilde{N}_{\kappa \kappa'} = \langle \phi_{n_{p},l_{p},m_{p}}^{G} | \phi_{n_{\lambda},l_{\lambda},m_{\lambda}}^{G} \rangle \times \langle \phi_{n_{\lambda},l_{\lambda},m_{\lambda}}^{G} | \phi_{n_{p},l_{p},m_{p}}^{G} \rangle \times \frac{2 \sqrt{\nu_{n_{p}} \nu_{n_{\lambda}}}}{\nu_{n_{p}} + \nu_{n_{\lambda}}} l_{p}^{l_{p}+3/2} \times \frac{2 \sqrt{\nu_{n_{p}} \nu_{n_{\lambda}}}}{\nu_{n_{p}} + \nu_{n_{\lambda}}} l_{\lambda}^{l_{\lambda}+3/2}, \]

(30)

where \( \kappa = 1, 2, ..., \kappa_{\text{max}}, \kappa_{\text{max}} = n_{\text{max}} \times n_{\text{max}} \) and \( c_{\kappa} = c_{n_{p},n_{\lambda}}. \) \( H \) and \( E \) denote the Hamiltonian and the eigenvalue, respectively.

In the calculation of Hamiltonian matrix elements of three-body systems, particularly when complicated interactions are employed, integrations over all of the radial and angular coordinates become
laborious even with the Gaussian basis functions. This process can be simplified by introducing the ISG basis functions. The technical details can be found in references 62, 108.

Note that there are two key issues in our mass spectrum calculations. The first is the Hamiltonian calculation which starts with the two-body interactions, and is different from that of the quark potential model in the heavy quark-light diquark picture 58. The second is the definition of a baryon calculation which starts with the two-body interactions, and is different from that of the quark potential model by the $|\rho_i l\lambda L s j J M_J\rangle$ as shown in formula (24). In fact, there exists another scheme written as $|\rho_i l\lambda L s J M_J\rangle(L-S$ coupling), which may be transformed into $|\rho_i l\lambda L s J M_J\rangle(j-s$ coupling) by means of Racah coefficients. We select the $j-s$ coupling scheme just for the spirit of the heavy quark limit. In fact, the $j-s$ coupling scheme has also been widely employed in the strong decay calculations of heavy baryons by the $^3P_0$ model 78.

2.4 $^3P_0$ model

For baryon decays, one quark from the initial baryon regroups with the created antiquark to form a meson, and the remaining two quarks regroup with the created quark to form a daughter baryon (see Fig. 2). This mechanism is described by the $^3P_0$ model for the OZI-allowed strong decays. The decay width $\Gamma$ is

$$\Gamma = \frac{\pi^2 |p|^2}{m_A^2 2J_A + 1} \sum_{M_J A, M_J B, M_J C} |\mathcal{M}^{M_J A, M_J B, M_J C}|^2,$$

(31)

where $p$ is the momentum of the daughter baryon in $A$’s CM frame,

$$|p| = \frac{\sqrt{m_A^2 - (m_B - m_C)^2} [m_A^2 - (m_B + m_C)^2]}{2m_A}.$$

(32)

$\mathcal{M}^{M_J A, M_J B, M_J C}$ is the helicity amplitude, which reads

$$\mathcal{M}^{M_J A, M_J B, M_J C} = - F\gamma \sqrt{8E_A E_B E_C} \sum_{m_{\rho A} M_{LA}} \sum_{m_{\rho B} M_{LB}} \sum_{m_{\rho C} M_{LC}} \sum_{m_{s 1} m_{s 2} m_{s 3}} \sum_{m_{s 4}} \sum_{m_{s 5}} \langle j A m_{j A} s_{s 3} m_{s 3} | J A M_{J A} \rangle
$$

$$\times \langle l_{A M_{LA}} s_{A m_{s 3}} | j_{A M_{j A}} \rangle \langle l_{p A m_{p A} l_{\rho A} m_{\rho A}} | l_{A M_{LA}} \rangle \langle s_{1 m_{s 1}} s_{2 m_{s 2}} s_{A m_{s A}} \rangle
$$

$$\times \langle j_{B M_{LB}} s_{3 m_{s 3}} | J_{B M_{j B}} \rangle \langle l_{B M_{LB}} s_{B m_{s B}} | J_{B M_{j B}} \rangle \langle l_{p B m_{p B} l_{\rho B} m_{\rho B}} | l_{B M_{LB}} \rangle
$$

$$\times \langle s_{1 m_{s 1}} s_{4 m_{s 4}} s_{B m_{s B}} | s_{B M_{LB}} s_{C m_{s C}} | J_{C M_{LC}} \rangle
$$

$$\times \langle s_{2 m_{s 2}} s_{5 m_{s 5}} | s_{C m_{s C}} | 1 m_{1} - m_{0} | 00 \rangle \langle \varphi_{B}^{1.4.3.2.5}, \varphi_{C}^{1.2.3.4.5}, \varphi_{A}^{M_{LA}}, m_{L_{AB}, M_{LC}} \rangle |p|,$$

(33)

where $\gamma$ is a dimensionless parameter representing the pair-creation strength in the vacuum. $F$ is a factor of 2 when each of the two quarks in $C$ has isospin $\frac{1}{2}$, and $F = 1$ when one of the two quarks in $C$ has isospin 0. $\langle \varphi_{B}^{1.4.3.2.5}, \varphi_{C}^{1.2.3.4.5}, \varphi_{A}^{M_{LA}}, m_{L_{AB}, M_{LC}} \rangle |p|$ is the flavor matrix,

$$\langle \varphi_{B}^{1.4.3.2.5}, \varphi_{C}^{1.2.3.4.5}, \varphi_{A}^{M_{LA}}, m_{L_{AB}, M_{LC}} \rangle = F^{(I_{A}: I_{B}, I_{C})} (I_{B} I_{C} I_{A})^{3} (I_{A} I_{B})^{3}.$$

(34)
FIG. 2: (Color online) The strong decay process $A \rightarrow B + C$ in the $^3P_0$ model. According to the arrangement, there are three kinds of processes. The process here is called $(14)_3$ process. And the other two processes are $(24)_3$ and $(14)_2$. The heavy quark is numbered as $3$.

with

$$F(I_A; I_B I_C) = f \cdot (-1)^{I_{12} + I_C + I_A + I_3} \left\{ \begin{array}{c} I_{12} \quad I_B \quad I_4 \\ I_C \quad I_3 \quad I_A \end{array} \right\} \left( \frac{(2I_C + 1)(2I_B + 1)}{2} \right)^{1/2} \delta^3(p_1 + p_2 + p_3 - p_A) \delta^3(p_4 + p_5) \delta^3(p_1 + p_4 + p_3 - p_B) \delta^3(p_2 + p_5 - p_C) \delta^3(p_1 + p_2 + p_3 - p_A) \delta^3(p_4 + p_5) \delta^3(p_1 + p_4 + p_3 - p_B) \delta^3(p_2 + p_5 - p_C),$$

where $\Psi$ = $|n_{\rho}\rho_{n_{\rho}} \otimes n_{\lambda}\lambda_{n_{\lambda}}$ for baryons and $y_{lm}(p) \equiv |p|^l Y_{lm}(\hat{p})$. We choose the SHO wave functions in the momentum representation to carry out the integration. The final results with the $^3P_0$ model depend on some input parameters such as the quark pair $(q\bar{q})$ creation strength $\gamma$, the SHO wave function scale parameter $R$, and the masses of the hadrons. The detailed values of these parameters can be found in our previous work [62].

III. Numerical results and discussions

3.1 Numerical stabilities and $\lambda$-modes

In order to obtain stable numerical solutions of the generalized matrix eigenvalue problem, the Gaussian size parameters set $\{n_{max}, r_1, r_{n_{max}}\}$ should be optimized. For the Gaussian functions which are a set of non-orthogonal bases in a finite coordinate space, the number of the bases should be located within a reasonable range. As shown in Fig.3, the numerical stability is achieved when the
dimension parameter \( n_{\text{max}} \) falls in the range of \( 9 \sim 14 \), with \( r_1 = 0.18 \text{GeV}^{-1} \) and \( r_{n_{\text{max}}} = 15 \text{GeV}^{-1} \). \( n_{\text{max}} = 10 \) is finally adopted in this work, with which both the computation efficiency and accuracy are actually satisfied [62].

\[
nL(J^P) \text{ is usually used to describe a baryon state in experiment, where } l_\rho \text{ and } l_\lambda \text{ are not observables. However, } l_\rho \text{ and } l_\lambda \text{ are good quantum numbers in our theoretical framework, i.e. } |l_\rho l_\lambda LsjJM_J \rangle \text{ is a well defined quantum state. If angular momentum } L \neq 0, \text{ there exist several } |l_\rho l_\lambda LsjJM_J \rangle \text{ states under the condition of } L = l_\rho + l_\lambda. \text{ They may be divided into the following three modes: (1) The } \rho \text{-mode with } l_\rho \neq 0 \text{ and } l_\lambda = 0; (2) The } \lambda \text{-mode with } l_\rho = 0 \text{ and } l_\lambda \neq 0; (3) The } \lambda-\rho \text{ mixing mode with } l_\rho \neq 0 \text{ and } l_\lambda \neq 0. \text{ In our theoretical framework, the states with the same } L \text{ are not degenerate and independent of each other actually. The most likely quantum states to be observed experimentally should be those with lower energies.}

\]n

In this subsection, the excitation energies of the \( 1P(\frac{1}{2}^-, \frac{3}{2}^-)_{j=1} \) states of \( \bar{3}_F \) as functions of \( m_Q \) are investigated, where the dependence of excitation energies on \( m_Q \) of the \( \lambda \)-mode is compared with that of the \( \rho \)-mode. As shown in Fig.4, the \( \lambda \)-mode and the \( \rho \)-mode are clearly separated when \( m_Q \) increases from 0.2 GeV to 5.0 GeV. In general, the excitation energies of the \( \lambda \)-mode are lower than those of the \( \rho \)-mode. In the case of \( 6_F \), we come to the same conclusion when \( m_Q > 1.5 \text{GeV} \) as shown in Fig.5.

Generally, the \( \lambda \)-modes appear lower in energy than the other two modes and they do not mix with each other in the heavy quark limit for single heavy baryons [62, 63]. Actually, the \( \lambda \)-modes have been applied widely in the study of heavy baryon strong decays [27, 35, 54, 132]. Therefore, we only calculate the \( \lambda \)-modes in \( \Xi_Q \) baryons. Additionally, the excitation energies of the \( \frac{1}{2}^- \) and \( \frac{3}{2}^- \) states are closer to each other with increasing \( m_Q \) as shown in Figs. 4 and 5, which is consistent with the heavy quark spin symmetry [133].

FIG. 3: (Color online) Numerical stability of \( \Xi_c 1S(\frac{1}{2}^+) \) mass with respect to the dimension parameter \( n_{\text{max}} \).
3.2 Mass spectra

In this subsection, the mass spectra of strange single heavy baryons are presented. For convenience, the relevant experimental data are given together. The detailed results are listed in Tables I-VI. There are a total of four families for Ξ_Q baryons in this work, namely Ξ_c, Ξ′_c, Ξ_b, and Ξ′_b. The mass spectra of excited states with quantum numbers up to n = 4 and L = 4 are displayed. Ebert et al. have studied the heavy baryon spectra with a quark potential model in the heavy quark-light diquark picture [58]. As an important reference, their numerical results are also placed in the tables. And the predictions from a number of other models are also shown there.

Through the analysis of these calculated results, some general features of the mass spectra are summarized as follows:

1. The GI model gives the same mass for Ξ^+_c and Ξ^0_c, due to the isospin symmetry of u and d quarks in this model.

2. Ξ_Q is lower than Ξ′_Q in energy. This feature has been recognized in light baryons where the highly orbitally excited states have an antisymmetric structure which minimizes the energy [134].

3. For the spin-doublet states, the energy of the J = j + \( \frac{1}{2} \) state is higher than that of the J = j − \( \frac{1}{2} \) state. This is also a general rule in hadronic physics.

4. The mass splitting of spin-doublet states becomes smaller with increasing L. For example, Table I shows the mass differences of the spin-doublets of 1P-, 1D-, 1F-, and 1G-wave are 30 MeV, 13 MeV, 5 MeV, and 1 MeV, respectively.
(5) For the same $L$, the mass splitting hardly changes with the increase of $j$. For example, Tables II and III show the mass differences of $1D$ doublets with $j = 1, 2, 3$ are 10 MeV, 10 MeV and 13 MeV, respectively.

(6) The mass difference between the two adjacent radial excited states gradually decreases with increasing $n$, which is clearly different from that given by Ebert et al.

(7) The above six features are roughly the same for $\Xi_c (\Xi'_c)$ and $\Xi_b (\Xi'_b)$ families, which is consistent with the heavy quark flavor symmetry [133].

We first pay attention to the well determined $\Xi_c (\Xi'_c)$ baryons in the PDG. As shown in Table I, $\Xi^+_c$ and $\Xi^0_c$ as an isospin doublet, can be assigned to be the $\Xi_c$ $1S(\frac{1}{2}^+)$ state. Similarly, $\Xi^{'+,0}_c$ are likely to be the $1S(\frac{3}{2}^+)$ state of $\Xi'_c$ (see Table II). In the same way, $\Xi_c(2645)^+,0$ should correspond to the $1S(\frac{3}{2}^+)$ state of $\Xi'_c$ in Table II. $\Xi_c(2790)^+,0$ and $\Xi_c(2815)^+,0$ are relatively well-determined $P$-wave $\Xi_c$ baryons with quantum numbers $J^P = \frac{1}{2}^-$ and $\frac{3}{2}^-$, respectively, as shown in Table I. By the assignment of these charm-strange baryons, we find the measured masses can be well reproduced in our calculations and the deviation is usually less than 14 MeV. Additionally, it needs to be noted that $\Xi_c(2645)$ should be a member of $\Xi'_c$ family. So it ought to be labelled with $\Xi'_c(2645)$.

$\Xi_c(2970)$, earlier known as $\Xi_c(2980)$, was first observed by the Belle [4] in 2006. Now the quantum numbers of $\Xi_c(2970)$ are determined as $\frac{1}{2}^+$ in the latest PDG. In our calculations, there are two candidates with the same $J^P$ for that baryon, namely $2S(\frac{1}{2}^+)$ of $\Xi_c$ and $2S(\frac{3}{2}^+)$ of $\Xi'_c$, as shown in Tables I and II. However, the predicted masses do not match with the experimental data. On the other hand, the predicted mass of $\Xi'_c(1P(\frac{3}{2}^-))$ in Table II is 2964 MeV which is in agreement with that of $\Xi_c(2970)$, but its quantum numbers disagree with the observation. At last, $\Xi_c(3055)$ and $\Xi_c(3080)$ were observed by the BABAR [3] and the Belle [11, 16]. As shown in the PDG, the spin and parity values of them have not yet been clear so far. According to the measured masses of $\Xi_c(3055)$ and $\Xi_c(3080)$, they are likely to be the $1D$ doublet ($\frac{3}{2}^+, \frac{5}{2}^+$) of $\Xi_c$ in Table I or the $2S$ doublet ($\frac{1}{2}^+, \frac{3}{2}^+$) of $\Xi'_c$ in Table II.

Outside of the PDG data, the Belle and the LHCb observed four charm-strange baryons, namely $\Xi_c(2930)$ [13], $\Xi_c(2923)$, $\Xi_c(2939)$, and $\Xi_c(2964)$ [14], whose masses are very close to each other. By the predicted masses in Tables I and II, the above four baryons can be assigned to be the first orbital ($1P$) excitation of $\Xi'_c$ or the first radial ($2S$) excitation of $\Xi_c$. At present, we can not determine their quantum numbers accurately.

The calculated mass spectra of $\Xi_b$ and $\Xi'_b$ families are listed in Tables IV-VI where a total of six bottom-strange baryons with determined quantum numbers in the PDG have been assigned to the possible states. The mass and $J^P$ values of $\Xi_b^{-,0}$ are consistent with those of the $\Xi_b$ $1S(\frac{1}{2}^+)$ state as shown in Table IV. Similarly, $\Xi'_b(5935)^-$ should correspond to the $\Xi'_b$ $1S(\frac{1}{2}^+)$ state in Table V. $\Xi_b(5945)^0$ and its isospin partner $\Xi_b(5955)^-$ can be assigned to the $\Xi'_b$ $1S(\frac{3}{2}^+)$ state in Table V. In
2021, AMS collaboration determined the $\Xi_b(6100)$ with the quantum numbers $J^P = \frac{3}{2}^-$ by measuring the typical decay chain of $\Xi_b(6100)^- \rightarrow \Xi_b^0 \pi^- \rightarrow \Xi_b^- \pi^+ \pi^-$. Very recently, the values of $J^P = \frac{3}{2}^-$ of $\Xi_b(6100)$ were written into the PDG data. Table IV shows the mass of the $\Xi_b(6100)$ is very close to that of the $\Xi_b 1P(\frac{3}{2}^-)$ state. So, the $\Xi_b(6100)$ is most likely to be the $1P(\frac{3}{2}^-)$ state of $\Xi_b$. From Tables IV and V, we find the experimental data can be well reproduced by our theoretical calculations. In addition, it should be pointed out $\Xi_b(5945)^0$ and $\Xi_b(5955)^-$ belong to the $\Xi_b'$ family in our calculations. So, they ought to be labelled with $\Xi_b'(5945)^0$ and $\Xi_b'(5955)^-$.

The last two $\Xi_b$ baryons in the PDG, $\Xi_b(6227)^-$ and $\Xi_b(6227)^0$, were reported by the LHCb Collaboration in 2018 [25]. But their spin and parity values are still not confirmed. In Tables IV and V, there are six states (one $2S$ state of $\Xi_b$ and five $1P$ states of $\Xi_b'$) whose masses range from 6224 MeV to 6243 MeV. Each of them could be considered as a possible assignment to the $\Xi_b(6227)$.

In 2021, two bottom-strange baryons $\Xi_b(6327)$ and $\Xi_b(6333)$ were reported by the LHCb Collaboration. Very recently, the LHCb implied in experiment that they should belong to the $\Xi_b 1D(\frac{3}{2}^+, \frac{5}{2}^+)$ doublet [27]. In Table IV, one can see the predicted masses of the 1D doublet ($\frac{3}{2}^+, \frac{5}{2}^+$) of $\Xi_b$ indeed match with the experimental data of the $\Xi_b(6327)$ and $\Xi_b(6333)$.

From the above analyses and discussion, we find the masses from the experimentally determined $\Xi_Q$ baryons can be reproduced nicely in this work. The accuracy of the predicted masses is impressive. Therefore, the calculated mass spectra of the $\Xi_Q$ families can be used as important and reliable reference data. Of course, only the mass information could not give the full description of the baryon states. We will further investigate the nature of the above baryon states with the help of their strong decay information in subsection 3.4.

### 3.3 Regge trajectories

Regge theory is based on Lorentz invariance, unitarity and analyticity of the scattering matrix and serves as a fundamental theory of strong interactions at very high energies and still an indispensable tool in phenomenological studies [137, 138] due to its generality although the Regge’s original work does not involve quarks and gluons and even a confining potential. Chew and Frautschi applied the theory to the case of strong interactions and found mesons and baryons lie on linear trajectories of the $(J, M^2)$ plane [139, 140]. In 2002, more general linear Regge trajectories were proposed in evaluating the semiclassical expansion of an effective string theory about a classical rotating string solution for long distance QCD, where the linear relationship between $n$ and $M^2$ appeared [141]. In 2011, Ebert et al. constructed the heavy baryon Regge trajectories in both the $(J, M^2)$ and the $(n, M^2)$ planes, based on the heavy-quark light-diquark picture in the framework of the QCD-motivated relativistic quark model [58].
TABLE I: Masses of the $\Xi_c$ family (in MeV)

| $l_x l_y L s j$ | $nL(J^P)$ | *our* | *exp* | $[58]$ | $[135]$ | $[136]$ |
|-----------------|-----------|-------|-------|--------|--------|--------|
| 0 0 0 0 0       | $1S(\frac{3}{2}^+)$ | 2479  | $\Xi_c^+$ | 2467.71(0.23) | 2476  | 2496  | 2469  |
|                 | $\Xi_c^0$ | 2470.44(0.28) |       |        |        |        |
| 0 1 1 0 1       | $1P(\frac{3}{2}^-)$ | 2789  | $2791.9(0.5)$ | 2792  | 2749  | 2769  |
|                 | $2793.9(0.5)$ |       |        |        |        |        |
| 0 1 1 0 1       | $2P(\frac{3}{2}^-)$ | 3176  | 3179  | 3201  | 3519  | 3804  |
|                 | $3P(\frac{3}{2}^-)$ | 3412  | 3678  |        |        |        |
|                 | $4P(\frac{3}{2}^-)$ | 3508  | 3945  |        |        |        |
| 0 2 2 0 2       | $1D(\frac{3}{2}^+)$ | 3063  | 3055?  | 3059  | 2951  |        |
|                 | $3060_{-130}^{+110}$ | $[38]$ |        |        |        |        |
|                 | $3040_{-150}^{+150}$ | $[36]$ |        |        |        |        |
| 0 2 2 0 2       | $2D(\frac{3}{2}^+)$ | 3406  | 33888 |        |        |        |
|                 | $3D(\frac{3}{2}^+)$ | 3617  | 3678  |        |        |        |
|                 | $4D(\frac{3}{2}^+)$ | 3676  | 3945  |        |        |        |
| 0 3 3 0 3       | $1F(\frac{5}{2}^-)$ | 3289  | 3278  |        |        |        |
|                 | $2F(\frac{5}{2}^-)$ | 3613  | 3572  |        |        |        |
|                 | $3F(\frac{5}{2}^-)$ | 3817  | 3845  |        |        |        |
|                 | $4F(\frac{5}{2}^-)$ | 3861  | 4098  |        |        |        |
| 0 3 3 0 3       | $1F(\frac{7}{2}^-)$ | 3294  | 3292  |        |        |        |
|                 | $2F(\frac{7}{2}^-)$ | 3619  | 3592  |        |        |        |
|                 | $3F(\frac{7}{2}^-)$ | 3821  | 3865  |        |        |        |
|                 | $4F(\frac{7}{2}^-)$ | 3871  | 4120  |        |        |        |
| 0 4 4 0 4       | $1G(\frac{9}{2}^+)$ | 3486  | 3469  |        |        |        |
|                 | $2G(\frac{9}{2}^+)$ | 3798  | 3745  |        |        |        |
|                 | $3G(\frac{9}{2}^+)$ | 4000  |        |        |        |        |
|                 | $4G(\frac{9}{2}^+)$ | 4054  |        |        |        |        |
| 0 4 4 0 4       | $1G(\frac{9}{2}^-)$ | 3487  | 3483  |        |        |        |
|                 | $2G(\frac{9}{2}^-)$ | 3799  | 3763  |        |        |        |
|                 | $3G(\frac{9}{2}^-)$ | 4001  |        |        |        |        |
|                 | $4G(\frac{9}{2}^-)$ | 4064  |        |        |        |        |
TABLE II: Masses of the $\Xi_c$ family (in MeV) (Part I)

| $l_p$ $l_s$ $L$ $s$ $j$ | $nL(J^P)$ | our | exp[30] | [58] | [135] |
|--------------------------|----------|------|---------|-----|-----|
| 1S($\frac{1}{2}^+$)     | 2590     |      | $\Xi_c^+$ 2578.2(0.5) | 2579 | 2574 |
|                          |          |      | $\Xi_c^0$ 2578.7(0.5) |      |      |
| 0 0 0 1 1                | 2S($\frac{1}{2}^+$) | 3046 | 3055? | 2983 |      |
|                          | 3S($\frac{3}{2}^+$) | 3201 |      | 3377 |      |
|                          | 4S($\frac{1}{2}^+$) | 3425 |      | 3695 |      |
| 0 0 0 1 1                | 1S($\frac{3}{2}^+$) | 2658 | 2645.10(0.30) | 2654 | 2633 |
|                          |          |      | 2646.16(0.25) |      |      |
| 0 1 1 1 0                | 1P($\frac{1}{2}^-$) | 2952 |      | 2936 | 2829 |
|                          | 2P($\frac{1}{2}^-$) | 3326 |      | 3313 |      |
|                          | 3P($\frac{1}{2}^-$) | 3469 |      | 3630 |      |
|                          | 4P($\frac{1}{2}^-$) | 3636 |      | 3912 |      |
| 0 1 1 1 1                | 1P($\frac{3}{2}^-$) | 2958 |      | 2935 | 2829 |
|                          | 2P($\frac{3}{2}^-$) | 3331 |      | 3311 |      |
|                          | 3P($\frac{3}{2}^-$) | 3473 |      | 3628 |      |
|                          | 4P($\frac{3}{2}^-$) | 3640 |      | 3911 |      |
| 0 1 1 1 1                | 1P($\frac{5}{2}^-$) | 2934 | 2930? [13] | 2912 |      |
|                          | 2P($\frac{5}{2}^-$) | 3310 |      | 3293 |      |
|                          | 3P($\frac{5}{2}^-$) | 3456 |      | 3613 |      |
|                          | 4P($\frac{5}{2}^-$) | 3624 |      | 3898 |      |
| 0 1 1 1 2                | 1P($\frac{3}{2}^-$) | 2964 | 2970? | 2929 |      |
| 0 1 1 2                  | 1P($\frac{5}{2}^-$) | 3335 |      | 3303 |      |
|                          | 3P($\frac{5}{2}^-$) | 3477 |      | 3619 |      |
|                          | 4P($\frac{5}{2}^-$) | 3644 |      | 3902 |      |
| 0 2 2 1 1                | 1D($\frac{1}{2}^+$) | 3201 |      | 3163 |      |
|                          | 2D($\frac{1}{2}^+$) | 3541 |      | 3505 |      |
|                          | 3D($\frac{1}{2}^+$) | 3676 |      | 3619 |      |
|                          | 4D($\frac{1}{2}^+$) | 3816 |      | 3902 |      |
| 0 2 2 1 1                | 1D($\frac{3}{2}^+$) | 3211 |      | 3167 |      |
|                          | 2D($\frac{3}{2}^+$) | 3550 |      | 3506 |      |
|                          | 3D($\frac{3}{2}^+$) | 3684 |      | 3619 |      |
|                          | 4D($\frac{3}{2}^+$) | 3827 |      |      |      |
In this subsection, we investigate the Regge trajectories in the \((J, M^2)\) plane based on our calculated mass spectra of the strange single heavy baryons. The states in a baryon family can be classified according to the following parities and angular momenta: (1) Natural \(P = (-1)^{J+1/2}\) and unnatural \(P = (-1)^{J-1/2}\) parities (written in short as \(NP\) and \(UP\), respectively)\(^{14}\); (2) \(J = j + 1/2\) and \(J = j - 1/2\) (written in short as \(NJ\) and \(UJ\)). Thus, the states in the \(\Xi_c\) or \(\Xi_b\) family are divided into two groups, and the states in the \(\Xi'_c\) or \(\Xi'_b\) family are divided into six groups. In this paper, we use the following definition for the \((J, M^2)\) Regge trajectories,

\[
M^2 = \alpha J + \beta, \tag{37}
\]

where \(\alpha\) and \(\beta\) are the slope and intercept. In Figs. 6-9, we plot the Regge trajectories in the \((J, M^2)\) plane with our calculated mass spectra. The three lines in each figure correspond to the radial quantum number \(n = 1, 2, 3\), respectively. The fitted slopes and intercepts of the Regge trajectories are given in Tables VII and VIII.

It is shown that the linear trajectories appear clearly in the \((J, M^2)\) plane. All the data points fall on the trajectory lines. This indicates that the Regge trajectory has a strong universality and our theoretical calculations are reliable. These trajectories are almost parallel, but not equidistant, which is the first difference between our mass spectra and those in reference \(^{58}\). Additionally, there are six groups of Regge trajectories for the \(\Xi'_c\) or \(\Xi'_b\) family as shown in Fig.7 or Fig.9. However, the Fig.4 in reference \(^{58}\) shows that the \(\Xi'_c\) family has only two groups of Regge trajectories. This is the second difference between our mass spectra and those in reference \(^{58}\).

In this paper, we do not show the Regge trajectories in the \((n, M^2)\) plane. In fact, the linear trajectories in the \((n, M^2)\) plane can not be constructed from our predicted masses, due to the first difference mentioned above. As will be mentioned in subsection 3.5, the observation of heavy baryons in forthcoming experiments will touch the \(3S\) sub shell, which might be a good time to check the \((n, M^2)\) Regge trajectories, we may then judge whether single heavy baryons are a three-quark system or a quark-diquark system.

![Regge Trajectories](image_url)

FIG. 6: (Color online) \((J, M^2)\) Regge trajectories for the \(\Xi_c\) family and \(M^2\) is in GeV\(^2\).
| $l_{\rho}$ $l_{\lambda}$ $L$ $s$ $j$ | $nL(J^P)$ | our | [58] | $l_{\rho}$ $l_{\lambda}$ $L$ $s$ $j$ | $nL(J^P)$ | our | [58] |
|---|---|---|---|---|---|---|---|
| 0 2 2 1 2 | $1D(\frac{3}{2}^-)$ | 3201 | 3160 | 0 2 2 1 2 | $1F(\frac{7}{2}^-)$ | 3423 | 3373 |
| | $2D(\frac{5}{2}^-)$ | 3541 | 3497 | | | $2F(\frac{5}{2}^-)$ | 3744 |
| | $3D(\frac{3}{2}^-)$ | 3676 | | | | $3F(\frac{7}{2}^-)$ | 3872 |
| | $4D(\frac{5}{2}^-)$ | 3816 | | | | $4F(\frac{7}{2}^-)$ | 4009 |
| 0 2 2 1 2 | $1D(\frac{3}{2}^-)$ | 3211 | 3166 | 0 3 3 1 4 | $1F(\frac{7}{2}^-)$ | 3428 | 3357 |
| | $2D(\frac{5}{2}^-)$ | 3551 | 3504 | | | $2F(\frac{5}{2}^-)$ | 3740 |
| | $3D(\frac{3}{2}^-)$ | 3685 | | | | $3F(\frac{7}{2}^-)$ | 3876 |
| | $4D(\frac{5}{2}^-)$ | 3828 | | | | $4F(\frac{7}{2}^-)$ | 4021 |
| 0 2 2 1 3 | $1D(\frac{3}{2}^-)$ | 3200 | 3153 | 0 4 4 1 3 | $1G(\frac{5}{2}^-)$ | 3621 | 3623 |
| | $2D(\frac{5}{2}^-)$ | 3540 | 3493 | | | $2G(\frac{5}{2}^-)$ | 3925 |
| | $3D(\frac{3}{2}^-)$ | 3676 | | | | $3G(\frac{5}{2}^-)$ | 4052 |
| | $4D(\frac{5}{2}^-)$ | 3815 | | | | $4G(\frac{5}{2}^-)$ | 4212 |
| 0 2 2 1 3 | $1D(\frac{3}{2}^-)$ | 3213 | 3147 | 0 4 4 1 3 | $1G(\frac{3}{2}^-)$ | 3622 | 3608 |
| | $2D(\frac{5}{2}^-)$ | 3552 | 3486 | | | $2G(\frac{3}{2}^-)$ | 3925 |
| | $3D(\frac{3}{2}^-)$ | 3686 | | | | $3G(\frac{3}{2}^-)$ | 4053 |
| | $4D(\frac{5}{2}^-)$ | 3829 | | | | $4G(\frac{3}{2}^-)$ | 4222 |
| 0 3 3 1 2 | $1F(\frac{7}{2}^-)$ | 3424 | 3418 | 0 4 4 1 4 | $1G(\frac{7}{2}^-)$ | 3621 | 3584 |
| | $2F(\frac{9}{2}^-)$ | 3744 | | | | $2G(\frac{7}{2}^-)$ | 3925 |
| | $3F(\frac{7}{2}^-)$ | 3872 | | | | $3G(\frac{7}{2}^-)$ | 4052 |
| | $4F(\frac{9}{2}^-)$ | 4010 | | | | $4G(\frac{7}{2}^-)$ | 4212 |
| 0 3 3 1 2 | $1F(\frac{7}{2}^-)$ | 3428 | 3408 | 0 4 4 1 4 | $1G(\frac{7}{2}^-)$ | 3622 | 3582 |
| | $2F(\frac{9}{2}^-)$ | 3748 | | | | $2G(\frac{7}{2}^-)$ | 3925 |
| | $3F(\frac{7}{2}^-)$ | 3876 | | | | $3G(\frac{7}{2}^-)$ | 4053 |
| | $4F(\frac{9}{2}^-)$ | 4020 | | | | $4G(\frac{7}{2}^-)$ | 4222 |
| 0 3 3 1 3 | $1F(\frac{7}{2}^-)$ | 3424 | 3394 | 0 4 4 1 5 | $1G(\frac{11}{2}^-)$ | 3621 | 3558 |
| | $2F(\frac{9}{2}^-)$ | 3744 | | | | $2G(\frac{11}{2}^-)$ | 3924 |
| | $3F(\frac{7}{2}^-)$ | 3872 | | | | $3G(\frac{11}{2}^-)$ | 4052 |
| | $4F(\frac{9}{2}^-)$ | 4009 | | | | $4G(\frac{11}{2}^-)$ | 4212 |
| 0 3 3 1 3 | $1F(\frac{7}{2}^-)$ | 3428 | 3393 | 0 4 4 1 5 | $1G(\frac{11}{2}^-)$ | 3622 | 3536 |
| | $2F(\frac{9}{2}^-)$ | 3748 | | | | $2G(\frac{11}{2}^-)$ | 3925 |
| | $3F(\frac{7}{2}^-)$ | 3876 | | | | $3G(\frac{11}{2}^-)$ | 4053 |
| | $4F(\frac{9}{2}^-)$ | 4021 | | | | $4G(\frac{11}{2}^-)$ | 4223 |
| $l_\nu$ | $l_\lambda$ | $L$ | $s$ | $j$ | $nL(J^P)$ | $\text{our}$ | $\text{exp}$ | $\text{[58]}$ | $\text{[135]}$ | $\text{[50]}$ | $\text{[36]}$ |
|-------|--------|-----|-----|-----|-------|---------|-----------|---------|---------|---------|---------|
| 0 0 0 0 | 0 0 0 0 | 0 0 0 0 | 0 0 0 0 | 0 0 0 0 | 0 0 0 0 | 0 0 0 0 | 0 0 0 0 | 0 0 0 0 | 0 0 0 0 | 0 0 0 0 | 0 0 0 0 |
| 1S($\frac{1}{2}^-$) | 5806 | $\Xi_b^-$ | 5797.0(0.6) | 5803 | 5825 | 5791.9 |
| 2S($\frac{1}{2}^+$) | 6224 | $\Xi_b^0$ | 5791.9(0.5) | 6266 | 6201 | 6013 |
| 4S($\frac{1}{2}^+$) | 6568 | 6913 | 6913 | 6913 | 6913 | 6913 |
| 1P($\frac{1}{2}^-$) | 6084 | 6120 | 6076 | 6120 | 6076 | 6120 |
| 2P($\frac{1}{2}^-$) | 6421 | 6496 | 6496 | 6496 | 6496 | 6496 |
| 4P($\frac{1}{2}^-$) | 6732 | 7068 | 7068 | 7068 | 7068 | 7068 |
| 1P($\frac{3}{2}^-$) | 6097 | 6100.3(0.6) | 6130 | 6076 | 6100.3(0.6) | 6130 |
| 2P($\frac{3}{2}^-$) | 6432 | 6502 | 6502 | 6502 | 6502 | 6502 |
| 4P($\frac{3}{2}^-$) | 6739 | 7073 | 7073 | 7073 | 7073 | 7073 |
| 1D($\frac{3}{2}^+$) | 6320 | 6366 | 6190$^{+100}_{-120}$ | 6366 | 6366 | 6366 |
| 2D($\frac{3}{2}^+$) | 6613 | 6690 | 6690 | 6690 | 6690 | 6690 |
| 4D($\frac{3}{2}^+$) | 6890 | 7208 | 7208 | 7208 | 7208 | 7208 |
| 1F($\frac{5}{2}^-$) | 6518 | 6577 | 6190$^{+100}_{-120}$ | 6577 | 6577 | 6577 |
| 2F($\frac{5}{2}^-$) | 6795 | 6863 | 6863 | 6863 | 6863 | 6863 |
| 4F($\frac{5}{2}^-$) | 7057 | 7339 | 7339 | 7339 | 7339 | 7339 |
| 0 3 3 0 3 | 0 3 3 0 3 | 0 3 3 0 3 | 0 3 3 0 3 | 0 3 3 0 3 | 0 3 3 0 3 | 0 3 3 0 3 | 0 3 3 0 3 | 0 3 3 0 3 | 0 3 3 0 3 | 0 3 3 0 3 | 0 3 3 0 3 |
| 1F($\frac{7}{2}^-$) | 6523 | 6581 | 6190$^{+100}_{-120}$ | 6581 | 6581 | 6581 |
| 2F($\frac{7}{2}^-$) | 6801 | 6867 | 6867 | 6867 | 6867 | 6867 |
| 4F($\frac{7}{2}^-$) | 7060 | 7342 | 7342 | 7342 | 7342 | 7342 |
| 0 4 4 0 4 | 0 4 4 0 4 | 0 4 4 0 4 | 0 4 4 0 4 | 0 4 4 0 4 | 0 4 4 0 4 | 0 4 4 0 4 | 0 4 4 0 4 | 0 4 4 0 4 | 0 4 4 0 4 | 0 4 4 0 4 | 0 4 4 0 4 |
| 1G($\frac{9}{2}^-$) | 6692 | 6760 | 6760 | 6760 | 6760 | 6760 |
| 2G($\frac{9}{2}^+$) | 6970 | 7020 | 7020 | 7020 | 7020 | 7020 |
| 3G($\frac{9}{2}^+$) | 7167 | 7167 | 7167 | 7167 | 7167 | 7167 |
| 4G($\frac{9}{2}^+$) | 7214 | 7214 | 7214 | 7214 | 7214 | 7214 |
| $l_\rho$ $l_\Lambda$ $L$ $s$ $j$ | $nL(J^P)$ | our | exp[30] | [58] | [135] |
|---|---|---|---|---|---|
| 0 0 0 1 1 | $1S\left(\frac{1}{2}^+\right)$ | 5943 | 5935.02(0.05) | 5936 |
| | $2S\left(\frac{1}{2}^+\right)$ | 6350 | 6329 |
| | $3S\left(\frac{1}{2}^+\right)$ | 6535 | 6687 |
| | $4S\left(\frac{1}{2}^+\right)$ | 6691 | 6978 |
| 0 0 0 1 1 | $1S\left(\frac{3}{2}^+\right)$ | 5971 | 5952.3(0.6) | 5963 | 5967 |
| | $2S\left(\frac{3}{2}^+\right)$ | 6370 | 6329 |
| | $3S\left(\frac{3}{2}^+\right)$ | 6554 | 6695 |
| | $4S\left(\frac{3}{2}^+\right)$ | 6705 | 6984 |
| 0 1 1 1 0 | $1P\left(\frac{1}{2}^\pm\right)$ | 6238 | 6233 |
| | $2P\left(\frac{1}{2}^-\right)$ | 6569 | 6611 |
| | $3P\left(\frac{1}{2}^-\right)$ | 6758 | 6915 |
| | $4P\left(\frac{1}{2}^-\right)$ | 6866 | 7174 |
| 0 1 1 1 1 | $1P\left(\frac{3}{2}^\pm\right)$ | 6232 | 6227 |
| | $2P\left(\frac{3}{2}^-\right)$ | 6564 | 6604 |
| | $3P\left(\frac{3}{2}^-\right)$ | 6754 | 6904 |
| | $4P\left(\frac{3}{2}^-\right)$ | 6863 | 7164 |
| 0 1 1 1 1 | $1P\left(\frac{5}{2}^-\right)$ | 6240 | 6234 |
| | $2P\left(\frac{5}{2}^-\right)$ | 6572 | 6605 |
| | $3P\left(\frac{5}{2}^-\right)$ | 6760 | 6905 |
| | $4P\left(\frac{5}{2}^-\right)$ | 6868 | 7163 |
| 0 1 1 1 2 | $1P\left(\frac{3}{2}^-\right)$ | 6229 | 6227.9(0.9)? | 6224 |
| | $2P\left(\frac{3}{2}^-\right)$ | 6562 | 6598 |
| | $3P\left(\frac{3}{2}^-\right)$ | 6752 | 6900 |
| | $4P\left(\frac{3}{2}^-\right)$ | 6861 | 7159 |
| 0 1 1 1 2 | $1P\left(\frac{5}{2}^-\right)$ | 6243 | 6226 |
| | $2P\left(\frac{5}{2}^-\right)$ | 6574 | 6596 |
| | $3P\left(\frac{5}{2}^-\right)$ | 6762 | 6897 |
| | $4P\left(\frac{5}{2}^-\right)$ | 6869 | 7156 |
| 0 2 2 1 1 | $1D\left(\frac{1}{2}^\pm\right)$ | 6461 | 6447 |
| | $2D\left(\frac{1}{2}^+\right)$ | 6757 | 6767 |
| | $3D\left(\frac{1}{2}^+\right)$ | 6941 |
| | $4D\left(\frac{1}{2}^+\right)$ | 7017 |
| 0 2 2 1 1 | $1D\left(\frac{3}{2}^+\right)$ | 6466 | 6459 |
| | $2D\left(\frac{3}{2}^+\right)$ | 6763 | 6775 |
| | $3D\left(\frac{3}{2}^+\right)$ | 6946 |
| | $4D\left(\frac{3}{2}^+\right)$ | 7020 |
| $l_{\rho}$ $l_{\lambda}$ $L$ $s$ $j$ | $nL(J^P)$ | our [58] | $l_{\rho}$ $l_{\lambda}$ $L$ $s$ $j$ | $nL(J^P)$ | our [58] |
|-----------------|----------|---------|-----------------|----------|---------|
| 0 2 2 1 2       | 1$D(^3\frac{3}{2}^+)$ | 6460   | 6431            | 0 3 3 1 4 | 1$F(^1\frac{3}{2}^-)$ | 6657   | 6619 |
|                 | 2$D(^3\frac{3}{2}^+)$ | 6758   | 6751            |           | 2$F(^1\frac{3}{2}^-)$ | 6942   |     |
|                 | 3$D(^3\frac{3}{2}^+)$ | 6941   |                 |           | 3$F(^1\frac{3}{2}^-)$ | 7110   |     |
|                 | 4$D(^3\frac{3}{2}^+)$ | 7017   |                 |           | 4$F(^1\frac{3}{2}^-)$ | 7162   |     |
| 0 2 2 1 2       | 1$D(^3\frac{5}{2}^+)$ | 6466   | 6432            | 0 3 3 1 4 | 1$F(^1\frac{5}{2}^-)$ | 6661   | 6610 |
|                 | 2$D(^3\frac{5}{2}^+)$ | 6764   | 6751            |           | 2$F(^1\frac{5}{2}^-)$ | 6947   |     |
|                 | 3$D(^3\frac{5}{2}^+)$ | 6946   |                 |           | 3$F(^1\frac{5}{2}^-)$ | 7114   |     |
|                 | 4$D(^3\frac{5}{2}^+)$ | 7021   |                 |           | 4$F(^1\frac{5}{2}^-)$ | 7165   |     |
| 0 2 2 1 3       | 1$D(^5\frac{3}{2}^+)$ | 6460   | 6420            | 0 4 4 1 3 | 1$G(^3\frac{3}{2}^+)$ | 6831   | 6867 |
|                 | 2$D(^5\frac{3}{2}^+)$ | 6757   | 6740            |           | 2$G(^3\frac{3}{2}^+)$ | 7119   |     |
|                 | 3$D(^5\frac{3}{2}^+)$ | 6941   |                 |           | 3$G(^3\frac{3}{2}^+)$ | 7266   |     |
|                 | 4$D(^5\frac{3}{2}^+)$ | 7021   |                 |           | 4$G(^3\frac{3}{2}^+)$ | 7302   |     |
| 0 2 2 1 3       | 1$D(^7\frac{3}{2}^+)$ | 6467   | 6414            | 0 4 4 1 3 | 1$G(^5\frac{3}{2}^+)$ | 6833   | 6876 |
|                 | 2$D(^7\frac{3}{2}^+)$ | 6755   | 6736            |           | 2$G(^5\frac{3}{2}^+)$ | 7122   |     |
|                 | 3$D(^7\frac{3}{2}^+)$ | 6946   |                 |           | 3$G(^5\frac{3}{2}^+)$ | 7268   |     |
|                 | 4$D(^7\frac{3}{2}^+)$ | 7021   |                 |           | 4$G(^5\frac{3}{2}^+)$ | 7304   |     |
| 0 3 3 1 2       | 1$F(^1\frac{5}{2}^-)$ | 6657   | 6675            | 0 4 4 1 4 | 1$G(^3\frac{5}{2}^+)$ | 6831   | 6822 |
|                 | 2$F(^1\frac{5}{2}^-)$ | 6942   |                 |           | 2$G(^3\frac{5}{2}^+)$ | 7119   |     |
|                 | 3$F(^1\frac{5}{2}^-)$ | 7110   |                 |           | 3$G(^3\frac{5}{2}^+)$ | 7266   |     |
|                 | 4$F(^1\frac{5}{2}^-)$ | 7162   |                 |           | 4$G(^3\frac{5}{2}^+)$ | 7302   |     |
| 0 3 3 1 2       | 1$F(^1\frac{7}{2}^-)$ | 6660   | 6686            | 0 4 4 1 4 | 1$G(^5\frac{7}{2}^+)$ | 6833   | 6821 |
|                 | 2$F(^1\frac{7}{2}^-)$ | 6946   |                 |           | 2$G(^5\frac{7}{2}^+)$ | 7122   |     |
|                 | 3$F(^1\frac{7}{2}^-)$ | 7114   |                 |           | 3$G(^5\frac{7}{2}^+)$ | 7268   |     |
|                 | 4$F(^1\frac{7}{2}^-)$ | 7164   |                 |           | 4$G(^5\frac{7}{2}^+)$ | 7304   |     |
| 0 3 3 1 3       | 1$F(^1\frac{11}{2}^-)$ | 6657   | 6640            | 0 4 4 1 5 | 1$G(^7\frac{11}{2}^+)$ | 6831   | 6792 |
|                 | 2$F(^1\frac{11}{2}^-)$ | 6942   |                 |           | 2$G(^7\frac{11}{2}^+)$ | 7119   |     |
|                 | 3$F(^1\frac{11}{2}^-)$ | 7110   |                 |           | 3$G(^7\frac{11}{2}^+)$ | 7266   |     |
|                 | 4$F(^1\frac{11}{2}^-)$ | 7162   |                 |           | 4$G(^7\frac{11}{2}^+)$ | 7301   |     |
| 0 3 3 1 3       | 1$F(^1\frac{13}{2}^-)$ | 6660   | 6641            | 0 4 4 1 5 | 1$G(^9\frac{13}{2}^+)$ | 6833   | 6782 |
|                 | 2$F(^1\frac{13}{2}^-)$ | 6947   |                 |           | 2$G(^9\frac{13}{2}^+)$ | 7123   |     |
|                 | 3$F(^1\frac{13}{2}^-)$ | 7114   |                 |           | 3$G(^9\frac{13}{2}^+)$ | 7268   |     |
|                 | 4$F(^1\frac{13}{2}^-)$ | 7165   |                 |           | 4$G(^9\frac{13}{2}^+)$ | 7304   |     |
FIG. 7: (Color online) (J, M^2) Regge trajectories for the Ξ' family and M^2 is in GeV^2.

FIG. 8: (Color online) (J, M^2) Regge trajectories for the Ξ family and M^2 is in GeV^2.
TABLE VII: Fitted values for the slope and intercept of the Regge trajectories for the Ξ family and $M^2$ is in GeV$^2$.

| Trajectory | $n = 1$ |          | $n = 2$ |          | $n = 3$ |          |
|------------|---------|----------|---------|----------|---------|----------|
|            | $\alpha$(GeV$^2$) | $\beta$(GeV) | $\alpha$(GeV$^2$) | $\beta$(GeV) | $\alpha$(GeV$^2$) | $\beta$(GeV) |
| $3_F(NP)(NJ)$ | 1.493 ± 0.056 5.580 ± 0.160 | 1.433 ± 0.022 8.047 ± 0.063 | 1.507 ± 0.032 9.305 ± 0.091 |
| $3_F(UP)(UJ)$ | 1.456 ± 0.043 7.122 ± 0.098 | 1.446 ± 0.023 9.399 ± 0.052 | 1.501 ± 0.026 10.784 ± 0.059 |
| $6_F(NP)(UJ)$ | 1.592 ± 0.058 6.098 ± 0.166 | 1.530 ± 0.033 8.611 ± 0.096 | 1.539 ± 0.031 9.574 ± 0.089 |
| $6_F(UJ)(UJ)$ | 1.487 ± 0.033 7.959 ± 0.076 | 1.473 ± 0.026 10.292 ± 0.059 | 1.482 ± 0.018 11.260 ± 0.041 |
| $6_F(NP)(UJ)$ | 1.433 ± 0.026 9.545 ± 0.044 | 1.433 ± 0.026 11.837 ± 0.045 | 1.453 ± 0.015 12.795 ± 0.026 |
| $6_F(UJ)(NJ)$ | 1.507 ± 0.040 4.933 ± 0.153 | 1.459 ± 0.022 7.451 ± 0.082 | 1.474 ± 0.019 8.371 ± 0.071 |
| $6_F(NJ)(NJ)$ | 1.455 ± 0.031 6.618 ± 0.098 | 1.437 ± 0.025 8.980 ± 0.079 | 1.454 ± 0.018 9.911 ± 0.058 |
| $6_F(UJ)(NJ)$ | 1.463 ± 0.034 8.041 ± 0.078 | 1.444 ± 0.025 10.383 ± 0.057 | 1.460 ± 0.019 11.336 ± 0.043 |

FIG. 9: (Color online)($J, M^2$) Regge trajectories for the Ξ family and $M^2$ is in GeV$^2$. 

TABLE VII: Fitted values for the slope and intercept of the Regge trajectories for the Ξ and Ξ’ families.
TABLE VIII: Fitted values for the slope and intercept of the Regge trajectories for the \( \Xi \) and \( \Xi' \) families.

| Trajectory       | \( n = 1 \)   | \( n = 2 \)   | \( n = 3 \)   |
|------------------|---------------|---------------|---------------|
|                  | \( \alpha \) (GeV\(^2\)) | \( \beta \) (GeV\(^2\)) | \( \alpha \) (GeV\(^2\)) | \( \beta \) (GeV\(^2\)) | \( \alpha \) (GeV\(^2\)) | \( \beta \) (GeV\(^2\)) |
| \( 3F(NP)(NJ) \) | 2.760 ± 0.134 | 32.756 ± 0.386 | 2.470 ± 0.027 | 37.595 ± 0.077 | 2.341 ± 0.122 | 41.187 ± 0.350 |
| \( 3F'(UP)(UJ) \) | 2.582 ± 0.099 | 35.891 ± 0.226 | 2.449 ± 0.014 | 40.029 ± 0.033 | 2.190 ± 0.114 | 43.86 ± 0.262  |
| \( 6F(NP)(UJ) \) | 2.820 ± 0.129 | 34.316 ± 0.371 | 2.592 ± 0.032 | 39.116 ± 0.092 | 2.518 ± 0.076  | 41.681 ± 0.219 |
| \( 6F(UP)(UJ) \) | 2.605 ± 0.087 | 37.677 ± 0.199 | 2.529 ± 0.014 | 41.849 ± 0.033 | 2.388 ± 0.051  | 44.509 ± 0.116 |
| \( 6F(NP)(NJ) \) | 2.465 ± 0.068 | 40.538 ± 0.117 | 2.512 ± 0.013 | 44.409 ± 0.022 | 2.309 ± 0.038  | 47.045 ± 0.065 |
| \( 6F(UP)(NJ) \) | 2.747 ± 0.113 | 31.888 ± 0.428 | 2.536 ± 0.019 | 36.834 ± 0.072 | 2.462 ± 0.062  | 39.454 ± 0.235 |
| \( 6F(NP)(NJ) \) | 2.580 ± 0.085 | 35.207 ± 0.273 | 2.510 ± 0.016 | 39.450 ± 0.050 | 2.374 ± 0.053  | 42.222 ± 0.170 |
| \( 6F(UP)(NJ) \) | 2.588 ± 0.089 | 37.766 ± 0.205 | 2.522 ± 0.018 | 41.921 ± 0.041 | 2.382 ± 0.057  | 44.573 ± 0.131 |

3.4 Strong decay behaviors

In subsection 3.2 we have obtained abundant information of mass spectra for the \( \Xi_Q \) families. For some baryons whose spin and parity numbers are still not confirmed in experiment, we attempted to assign them to reasonable states. The strong decay information of baryons could be of help for a full description of baryon states.

In this subsection, we investigate the strong decay behaviors of the \( \Xi_Q \) excited states with quantum numbers up to \( n = 2 \) and \( L = 2 \), based on the mass spectra listed in subsection 3.2. First, the labels of various excited states need to be defined for convenience. For example, \( (\Xi_Q^L, \Xi_Q^{L*}) \) denote the \( 1L \)-wave doublet states of \( 6F \) with \( J = j - 1/2 \) and \( J = j + 1/2 \), respectively. For the first radial excited \( (2L) \) states, there are two types of excitations with \( (n_p, n_{\lambda}) = (1, 0) \) and \( (0, 1) \), respectively. Accordingly, we can use \( \Xi_Q^L \) (or \( \Xi_Q^{L*} \)) to represent the \( \rho \)-type (or \( \lambda \)-type) excited states of \( 3F \). Thus, every state can be labelled exactly in this work. The calculated strong decay widths with the \( 3P_0 \) model have been listed in Tables IX-XXIV. For the convenience of comparison, we also collect some experimental data in Table XXV.

The reliability of our theoretical calculations should be evaluated with the available baryon data which have been experimentally confirmed. They are listed in the top half of Table XXV. Accordingly, we have four states for comparison, and the calculated total widths are listed in the bottom half of Table XXV. The \( \Xi_c(2645)^+ \) corresponds to our \( \Xi_{c1}^{0+}(2658) \), the \( \Xi_c(2790)^+ \) corresponds to our \( \Xi_{c1}^{1+}(2789) \), and so on. By comparison, we find that the experimental total widths are smaller than the predicted values. However, the differences are regular. For \( 3F \) states, our calculation values are about 10 ~ 25 times larger than the experimental data. For \( 6F \) states, our values are about 5 ~ 8 times larger than the experimental ones. According to this feature, the corrected widths (\( \Gamma_{Corr} \)) are
given in the last row of Tables IX-XXIV for easy of reference.

In subsection 3.2, we have preliminarily assigned some baryons. In this subsection, by using the predicted mass spectra and taking into account the strong decay behaviors, we shall identify these baryons exactly. Our previous analyses show there are two sets of candidates for the \( \Xi_c(3055) \) and \( \Xi_c(3080) \), namely, the \( \Xi_c \) 1D doublet and the \( \Xi_c \) 2S doublet. In reference \[16\], the \( \Xi_c(3055)^0 \) was observed in the \( \Lambda D^0 \) mode; the \( \Xi_c(3055)^+ \) and \( \Xi_c(3080)^+ \) were observed in the \( \Lambda D^+ \) mode. According to this fact, the 1D doublet states of the \( \Xi_c \) are excluded, because the \( \Lambda D^+ \) mode is forbidden in our calculation as shown in Table X. So, the 2S doublet states of the \( \Xi_c \) in the model prediction are the better candidates for the \( \Xi_c(3055) \) and \( \Xi_c(3080) \), their predicted decay behaviors are consistent with the observation qualitatively.

Very recently, the Belle collaboration determined the spin and parity of the \( \Xi_c(2970) \) to be \( \frac{1}{2}^+ \) by analyzing the decay angle distributions in the chain \( \Xi_c(2970)^+ \rightarrow \Xi_c^{*0}\pi^+ \rightarrow \Xi_c^+\pi^-\pi^+ \). And the measured ratio of the decay branching fractions \( R = \frac{B[\Xi_c(2970)^+ \rightarrow \Xi_c^{*0}\pi^+]}{B[\Xi_c(2970)^+ \rightarrow \Xi_c^0\pi^+] \} \) is about 1.67. In our mass spectra, there are four states whose masses are close to that of the \( \Xi_c(2970) \), namely, the \( \Xi_{c0}^0 \) 2S(\( \frac{1}{2}^+ \)), \( \Xi_{c1}^1 \) 1P(\( \frac{3}{2}^- \)), \( \Xi_{c1}^{1*} \) 1P(\( \frac{3}{2}^- \)) and \( \Xi_{c2}^{1*} \) 1P(\( \frac{5}{2}^- \)). Firstly, the \( \Xi_{c0}^{1*} \) 1P(\( \frac{5}{2}^- \)) in Table XIII is excluded because the \( \Lambda^{++}\bar{K} \) mode is allowed in our prediction, which is not consistent with the observation (see the PDG). Secondly, the \( \Xi_{c0}^{1*} \) 1P(\( \frac{3}{2}^- \)) in Table XI should also be denied because the decay channel \( \Xi_c^{*0}\pi^+ \) is forbidden in our theoretical calculation, which disagrees with the experiment. Thirdly, the predicted ratio \( R \) mentioned above is about 126 for the \( \Xi_{c1}^{1*} \) 1P(\( \frac{3}{2}^- \)) as shown in Table XII, which is much bigger than the experimental value. For the remaining possible candidate, the \( \Xi_c \) 2S(\( \frac{1}{2}^+ \)) state, the calculated ratio \( R \) is about 0.92 which is roughly compatible with the experimental data. But the decay channel \( \Sigma^{++}\bar{K} \) of the 2S(\( \frac{1}{2}^+ \)) state in Table IX is not allowed in our calculation, in which the mass of this state is chosen to be 2949 MeV. When we restudy the strong decay behavior of the 2S(\( \frac{1}{2}^+ \)) state by adjusting its mass to be 2964 MeV, the decay channels \( \Sigma^{++}\bar{K} \) and \( \Sigma^{++}\bar{K} \) are both open in theory, and the corresponding widths are 0.15 MeV and 0.47 MeV, respectively, which are in agreement with the observation (see the PDG). So, the 2S(\( \frac{1}{2}^+ \)) state of \( \Xi_c \) might be a good candidate of the \( \Xi_c(2970) \).

Recently, the \( \Xi_c(2923)^0 \), \( \Xi_c(2939)^0 \) and \( \Xi_c(2965)^0 \) were observed in the same experiment \[14\], where the signal of the \( \Xi_c(2930)^0 \) was absent. In reference \[13\], however, the \( \Xi_c(2930)^+ \) was observed and the product branching fraction related to the decay mode \( \Lambda^{++}\bar{K} \) was measured. Reference \[14\] speculated that the broad bump of the \( \Xi_c(2930) \) observed in \[13\] might be due to the overlap of two narrower states, such as the \( \Xi_c(2923)^0 \) and \( \Xi_c(2939)^0 \) baryons. In addition, reference \[14\] implied that the \( \Xi_c(2965) \) state is in the vicinity of the known \( \Xi_c(2970) \) baryon, however, their masses and natural widths differ significantly.

We find that there are six states with very close masses (from 2934 MeV to 2964 MeV) as shown in
Tables I and II. They can be divided into the following four groups: the $\Xi_{c0}^0$ 2S state, $\Xi_{c0}^1$ 1P singlet state, $\Xi_{c0}^{\prime}1P$ doublet states, and $\Xi_{c0}^{\prime\prime}1P$ doublet states. Now that the $\Xi_c(2970)$ has been assigned to the 2S($\frac{1}{2}^-$) state of $\Xi_c$, there are only five states left. As shown in Table XII, the decay channel $\Lambda_c^+ K$ is forbidden for the 1P doublet states with $j = 1$, which is not consistent with the observation. Therefore, only three states remain there, they are the 1P singlet state and the 1P doublet states with $j = 2$. According to the speculation of reference [14], if the $\Xi_c(2930)^0$ is the overlap of the $\Xi_c(2923)^0$ and $\Xi_c(2939)^0$ baryons, there are only three baryons left and we assign them as follows. The $\Xi_c(2965)^0$ is likely to be the $\Xi_c^\prime 1P(\frac{3}{2}^-)_{j=2}$ state. The $\Xi_c(2923)^0$ with a lighter mass should be assigned to be the $\Xi_c^\prime 1P(\frac{1}{2}^-)_{j=2}$ state, and the $\Xi_c(2939)^0$ might be the $\Xi_c^\prime 1P(\frac{1}{2}^-)$ singlet state.

The $\Xi_c(3123)$ was observed by the BABAR Collaboration [5]. However, it has not yet appeared in the PDG [30] so far. In our calculations, the predicted masses of the $\Xi_c 3S(\frac{1}{2}^+)$ and $\Xi_c^\prime 2S(\frac{3}{2}^+)$ are 3155 MeV and 3095 MeV, respectively, which are relatively closer to the measured value of the $\Xi_c(3123)$ than those of other states. The $\Xi_c^\prime 2S(\frac{1}{2}^+)$ has been considered as the candidate of the $\Xi_c(3080)$. Then, the $\Xi_c(3123)$ is likely to be the $\Xi_c 3S(\frac{1}{2}^+)$ state. According to reference [24], the $\Xi_c(3123)^+$ signals were observed in the $\Xi_c(2455)^{++} K^-$ and $\Sigma_c(2520)^{++} K^-$ intermediate-resonant decays. We investigate the decay behaviors of the $\Xi_c 3S(\frac{1}{2}^+)$ state and find that these two decay modes are allowed in our theoretical calculations. The corresponding decay widths are 1.4 MeV and 1.3 MeV, respectively, which are in agreement with the measured values.

As mentioned in subsection 3.2, the $\Xi_b(6227)$ should belong to the 2S-wave of $\Xi_b$ or the 1P-wave of $\Xi_b^\prime$. The $\Xi_b^\prime 1P(\frac{3}{2}^-)$ as the partner of the $\Xi_b^\prime 1P(\frac{3}{2}^-)$ in Table V, has been eliminated due to its greater mass. Here, we would reconsider it with the help of the calculated results by the $3P_0$ model. The measured decay modes of the $\Xi_b(6227)^-$ are the $\Xi_b^0\pi^-$ and $\Lambda_b^0 K^-$ as shown in Table XXV. According to the calculated decay widths, the $\Xi_b 2S(\frac{1}{2}^+)$, $\Xi_b^\prime 1P(\frac{1}{2}^-)$ and $\Xi_b^\prime 1P(\frac{3}{2}^-)$ are ruled out because the decay mode of $\Lambda_b^0 K^-$ is forbidden for them (see Tables XVII and XX). As shown in Table XIX, the $\Xi_b^\prime 1P(\frac{1}{2}^-)$ can not be an ideal candidate due to its large decay width. As a result, the $\Xi_b^\prime 1P(\frac{3}{2}^-)$ state is the only choice, and the comparison of the theoretical total width with the experimental data shows a better consistence (see Table XXI).

In 2021, the AMS collaboration determined the $\Xi_b(6100)$ with the spin and parity quantum numbers $J^P = \frac{3}{2}^-$, by measuring the typical decay chain of $\Xi_b(6100)^- \to \Xi_b^{*0} \pi^- \to \Xi_b^- \pi^+ \pi^- [26]$. And this result has been written in the PDG. In Table XVII, one can see the $\Xi_b(6100)$ is close to the $\Xi_b 1P(\frac{3}{2}^-)$ in mass. And the related decay width(0.8 ~ 2.2 MeV) of the $\Xi_b 1P(\frac{3}{2}^-)$ is in agreement with the experimental data(< 1.9 MeV). So, the $\Xi_b(6100)$ is very likely to be the $\Xi_b 1P(\frac{3}{2}^-)_{j=1}$ state.

At last, we focus on the assignment of the $\Xi_b(6327)$ and $\Xi_b(6333)$. In subsection 3.2, we thought the $\Xi_b 1D(\frac{3}{2}^+, \frac{5}{2}^+)$ doublet should be the best candidates. The experimental total widths of the $\Xi_b(6327)$ and $\Xi_b(6333)$ are less than 2.20 MeV and 1.55 MeV, respectively [27] as shown in Table XXV. In
Table XVIII, the predicted total widths of the Ξ_b 1D(\(3/2^+\), \(5/2^+\)) doublet are both no more than 1.1 MeV, which is consistent with the experimental result. So, we can confirm this point again that the Ξ_b(6327) and Ξ_b(6333) belong to the Ξ_b 1D doublet states.

| Assignment | Ξ_{c0}^{0+} | Ξ_{c0}^{0-} | Ξ_{c1}^{1+} | Ξ_{c1}^{1-} | Ξ_{c1}^{1++} | Ξ_{c1}^{1--} |
|------------|-------------|-------------|-------------|-------------|-------------|-------------|
| nL(J^P)    | 2S(\(1/2^+\)) | 2S(\(1/2^+\)) | 1P(\(3/2^-\)) | 2P(\(3/2^-\)) | 2P(\(3/2^-\)) | 2P(\(3/2^-\)) |
| (n_\lambda) | (0.1) | (0.1) | (0.1) | (0.1) | (0.1) | (0.1) |
| Mass       | 2949       | 2949       | 2789       | 3176       | 3176       | 2819       |
| Corr       | 61.84      | 15.69      | 101.86     | 10.76      | 54.63      | 60.19       |

3.5 Shell structure of the mass spectra

To get a clear outline of the baryon spectra, the mass spectra of the Ξ_c (Ξ_c') and Ξ_b (Ξ_b') baryons are mapped to the planes with column and row indices nL and J as shown in Tables XXVI and XXVII, respectively. In these tables, those states which have been experimentally well established are marked in boldface. Those states are marked with a check which need a further conformation in experiment and in theory. While those states with a rather large theoretic decay width (\(\Gamma_{Corr} > 40\text{MeV}\)) should
obvious shell structure, which is analogous to the energy level structure in a nucleus. It is shown that the mass distribution presents an obvious shell structure, which is analogous to the energy level structure in a nucleus. The situations for the $\Xi_c$ family (on the top half) and the $\bar{\Xi}_c$ family (on the bottom half). The masses are arranged in ascending order from smallest to largest. It is shown that the mass distribution presents an obvious shell structure, which is analogous to the energy level structure in a nucleus.

From these two tables, we could get a bird’s-eye view of the mass spectra. Firstly, the baryon spectra of the $\Xi_c$ ($\Sigma_c^*$) and $\Xi_{b}$ ($\Sigma_{b}^*$) almost have the same shell structure. Secondly, the baryons with lighter masses were discovered earlier in experiment. As shown in Table XXVI, the observed baryons in experiment occupy the lowest four sub shells one by one. In the case of $\Xi_c$, the lowest two sub shells are almost fully occupied. The situations for the $\Xi_{b}$ and $\bar{\Xi}_c$ families are similar to the above one. Thirdly, the states which are possibly observed in experiment can easily be predicted by this method. As shown in Table XXVI, the $1D$ doublet states of $\Xi_c$ are most likely to be discovered. However,
their decay behaviors and masses in the model calculations are very similar to those of the 2S doublet states of Ξ_c, which makes difficult to confirm them in experiment. On the other hand, it would be also difficult to observe the Ξ_c 1P(\frac{1}{2}^+, \frac{3}{2}^-) states because their theoretical decay widths are too large. So, the charm-strange baryons which are most likely to be found in experiment might be located in the 3S and 2P sub shells of Ξ_c, and the predicted masses range from 3155 MeV to 3199 MeV.

For the Ξ_b family, the Ξ_b 1P(\frac{1}{2}^-) is likely to be found first in experiment, because its partner Ξ_b 1P(\frac{3}{2}^-) has been confirmed by the AMS collaboration, namely, the Ξ_b(6100). According to our calculated results in subsection 3.4, the typical decay mode of the Ξ_b 1P(\frac{1}{2}^-) state is the Ξ_bπ which is significantly different from the decay mode of the Ξ_b(6100) baryon (see Table XVII). Besides, the Ξ_b 1P(\frac{5}{2}^-) as the partner of the Ξ_b(6227), its strong decay behavior is almost identical to that of the Ξ_b(6227) as shown in Table XXI. So, the Ξ_b 1P(\frac{5}{2}^-) might be observed together with the Ξ_b(6227)
TABLE XII: Decay widths (MeV) of the $\Xi_{c}^{+}(6\mathfrak{p})$ baryons (Part II).

| Assignment | $\Xi_{c}^{1+}$ | $\Xi_{c}^{1\lambda+}$ | $\Xi_{c}^{1\rho+}$ | $\Xi_{c}^{1++}$ | $\Xi_{c}^{1++}$ | $\Xi_{c}^{1++}$ |
|------------|---------------|---------------------|-----------------|----------------|----------------|----------------|
| $nL(J^P)$  | $1P(\frac{1}{2}^-)$ | $2P(\frac{1}{2}^-)$ | $2P(\frac{3}{2}^-)$ | $1P(\frac{3}{2}^-)$ | $2P(\frac{3}{2}^-)$ | $2P(\frac{3}{2}^-)$ |
| $(n_c, n_b)$ | (0, 0) | (0, 1) | (1, 0) | (0, 0) | (0, 1) | (1, 0) |
| Mass       | 2941         | 3315               | 3315            | 2958           | 3331           | 3331           |

| $\Xi_{c}^{0\pi^+}$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ |
| $\Xi_{c}^{0\pi^+}$ | 325.81   | 11.80    | $2.45 \times 10^{-3}$ | 2.18 | 0.65 | 19.90 |
| $\Xi_{c}^{*0\pi^+}$ | 0.88     | 0.61     | 27.55           | 273.76 | 12.14 | 16.95 |
| $\Xi_{c}^{0\pi^0}$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ |
| $\Xi_{c}^{*0\pi^0}$ | 162.95   | 5.90     | $3.09 \times 10^{-3}$ | 1.11 | 0.33 | 9.97 |
| $\Xi_{c}^{*0\pi^0}$ | 0.47     | 0.31     | 13.89           | 137.79 | 6.06 | 8.46 |

| $\Lambda_{c}^{+}K^0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ |
| $\Lambda_{c}^{+}K^{*0}$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ |
| $\Sigma_{c}^{*+}K^0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ |
| $\Sigma_{c}^{*+}K^{*0}$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ |
| $\Sigma_{c}^{*0}K^0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ |
| $\Sigma_{c}^{*0}K^{*0}$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ |
| $\Sigma_{c}^{*0}K^{*0}$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ |

| $\Lambda D^+$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ |
| $\Sigma D^0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ |
| $\Gamma_{Total}$ | 490.11 | 35.38 | 68.23 | 414.84 | 26.16 | 84.93 |
| $\Gamma_{Carr}$ | 61.2 $\sim$ 98.1 | 4.4 $\sim$ 7.1 | 8.5 $\sim$ 13.7 | 51.8 $\sim$ 83.0 | 3.2 $\sim$ 5.3 | 10.6 $\sim$ 17.0 |

in the same experiment and this prediction is in fact consistent with other theoretic analyses [51].

IV. Conclusions

In this work, based on the GI quark model, we investigate the strange single heavy baryon spectra systematically. In the calculations we employed the ISG method. According to the feature that the $\lambda$-modes appear lower in energy for definite states $nL(J^P)$, we obtain the complete mass spectra of the $\Xi_c$, $\Xi'_c$, $\Xi_b$ and $\Xi'_b$ families. For the well established baryons, our predicted masses can nicely reproduce the experimental data. In addition, the predicted mass spectra help us to successfully construct the Regge trajectories in the $(J, M^2)$ plane. Nevertheless, we can not currently construct the linear trajectories in the $(n, M^2)$ plane. We find that there are two main differences between our
might be the $P$ to the $\Xi$ c.

The $\Xi$ c families, we conclude that: (1) The $\Xi_c(3055)$ and $\Xi_c(3080)$ should belong to the $\Xi_c^* 2S(1^+ , 3^+)$ spin-doublet states; (2) The $\Xi_c(2970)$ is likely to be the $2S(1^+)$ state of $\Xi_c$; (3) In this work, the $\Xi_c(2930)^0$ is considered as an overlap of the $\Xi_c(2923)^0$ and $\Xi_c(2939)^0$ states, the $\Xi_c(2923)^0$ and $\Xi_c(2965)^0$ should belong to the $1P$-doublet states of $\Xi_c$ with $j = 2$, the $\Xi_c(2939)^0$ might be the $1P$-singlet state of $\Xi_c^*$; (4) The $\Xi_c(3123)$ is likely to be the $\Xi_c 3S(1^+)$ state.

Second, for the $\Xi_b$ and $\Xi_b^*$ families, we conclude that: (1) The $\Xi_b(6227)$ should be the $\Xi_b^* 1P(0^-)_{j=2}$

## Table XIII: Decay widths (MeV) of the $\Xi_c^+(6\pi)$ (Part III).

| Assignment | $\Xi_{c}^{1+}$ | $\Xi_{c}^{3+}$ | $\Xi_{c}^{1+0}$ | $\Xi_{c}^{3+0}$ | $\Xi_{c}^{1+1}$ | $\Xi_{c}^{3+1}$ |
|------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $nL(J^P)$  | $1P(3/2^+)$     | $2P(1/2^+)$     | $2P(3/2^+)$     | $1P(5/2^-)$     | $2P(1/2^-)$     | $2P(5/2^-)$     |
| $(n_\pi, n_\lambda)$ | (0, 0) | (0, 1) | (1, 0) | (0, 0) | (0, 1) | (1, 0) |
| Mass       | 2934            | 3310            | 3310            | 2964            | 3335            | 3335            |
| $\Xi_0^{0+}$ | 15.93           | 3.00            | 61.42           | 22.37           | 3.62            | 65.15           |
| $\Xi_0^{0+}$ | 2.64            | 0.97            | 32.94           | 1.92            | 0.54            | 16.16           |
| $\Xi_0^{0+}$ | 0.67            | 0.52            | 24.15           | 2.03            | 1.04            | 42.72           |
| $\Xi_0^{0+}$ | 8.35            | 1.54            | 30.95           | 11.67           | 1.85            | 32.79           |
| $\Xi_0^{0+}$ | 1.35            | 0.49            | 16.51           | 0.98            | 0.27            | 8.10            |
| $\Xi_0^{0+}$ | 0.36            | 0.27            | 12.18           | 1.07            | 0.53            | 21.52           |

For the excited states with the quantum numbers up to $n = 2$ and $L = 2$, we also analyze their strong decay widths by the $3P_0$ model. Combining the mass spectra and the strong decay information, we can systematically analyze the baryons observed in experiment up to now.

First, for the $\Xi_c$ and $\Xi_c^*$ families, we conclude that: (1) The $\Xi_c(3055)$ and $\Xi_c(3080)$ should belong to the $\Xi_c^* 2S(1^+ , 3^+)$ spin-doublet states; (2) The $\Xi_c(2970)$ is likely to be the $2S(1^+)$ state of $\Xi_c$; (3) In this work, the $\Xi_c(2930)^0$ is considered as an overlap of the $\Xi_c(2923)^0$ and $\Xi_c(2939)^0$ states, the $\Xi_c(2923)^0$ and $\Xi_c(2965)^0$ should belong to the $1P$-doublet states of $\Xi_c$ with $j = 2$, the $\Xi_c(2939)^0$ might be the $1P$-singlet state of $\Xi_c^*$; (4) The $\Xi_c(3123)$ is likely to be the $\Xi_c 3S(1^+)$ state.

Second, for the $\Xi_b$ and $\Xi_b^*$ families, we conclude that: (1) The $\Xi_b(6227)$ should be the $\Xi_b^* 1P(0^-)_{j=2}$
The $\Xi_b$ column and row indices $n_L$ and $S$ might be observed in the forthcoming experiments as follows: (1) The $\Xi_b$ and $\bar{\Xi}_c$ are very likely to be the $\Xi_b^*$ state; (2) The $\Xi_b$ state and $2\pi^0$ are likely to be observed experimentally, and the predicted mass is 6084 MeV, is likely to be observed experimentally, and the typical decay mode is $\Xi_b^0\pi$; (3) The $\Xi_b^*$ might be observed together with the $\Xi_b(6227)$ in the same experiment, and the predicted mass is 6243 MeV.

As mentioned in subsection 3.3, the predicted masses of the states in the $3S$ sub shell can be used

| Assignment | $\Xi_{c1}^{2^+}$ | $\Xi_{c1}^{2\lambda^+}$ | $\Xi_{c1}^{2\rho^+}$ | $\Xi_{c1}^{2\pi^+}$ | $\Xi_{c1}^{2\lambda^+}$ | $\Xi_{c1}^{2\rho^+}$ |
|------------|-------------------|-------------------------|----------------------|----------------------|------------------------|----------------------|
| $nL(J^P)$  | $1D(\frac{3}{2}^+)$ | $2D(\frac{1}{2}^+)$    | $2D(\frac{3}{2}^+)$ | $1D(\frac{3}{2}^+)$ | $2D(\frac{3}{2}^+)$ | $2D(\frac{3}{2}^+)$ |
| $(n_\pi, n_\lambda)$ | (0, 0) | (0.1) | (1.0) | (0.0) | (0.1) | (1.0) |
| Mass       | 3201             | 3541                    | 3541                 | 3211                 | 3550                   | 3550                 |

TABLE XIV: Decay widths (MeV) of the $\Xi_b^*(6\gamma)$ baryons (Part IV).
Based on the analyses and discussions in this paper, one can find the predicted masses of the $\Xi_Q$ baryons by the GI model are reliable. So, the calculated mass spectra of the $\Xi_Q$ families may be the important and valuable reference data. In addition, the calculated strong decay widths by the $^3P_0$ model are also of great value as references.
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| Assignment | $\Xi_{c3}^{2+}$ | $\Xi_{c3}^{2\lambda+}$ | $\Xi_{c3}^{2\rho+}$ | $\Xi_{c3}^{2\sigma+}$ | $\Xi_{c3}^{2\lambda+}$ | $\Xi_{c3}^{2\rho+}$ |
|------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| nL(JP)     | $1D(\frac{5}{2}^+)$ | $2D(\frac{5}{2}^+)$ | $2D(\frac{5}{2}^+)$ | $1D(\frac{5}{2}^+)$ | $2D(\frac{5}{2}^+)$ | $2D(\frac{7}{2}^+)$ |
| (n_p, n_K) | (0.0)           | (0.1)           | (1.0)           | (0.0)           | (0.1)           | (1.0)           |
| Mass       | 3200            | 3540            | 3540            | 3213            | 3552            | 3552            |
| $\Xi_{c3}^0\pi^+$ | 5.65            | 13.11           | 3.89            | 6.28            | 13.90           | 3.85            |
| $\Xi_{c3}^0\pi^+$ | 1.27            | 4.61            | 2.18            | 0.82            | 2.78            | 1.25            |
| $\Xi_{c3}^{+}\pi^0$ | 0.74            | 3.90            | 2.36            | 1.18            | 5.69            | 3.30            |
| $\Xi_{c3}^{+}\pi^0$ | 2.91            | 6.66            | 1.94            | 3.23            | 7.05            | 1.92            |
| $\Xi_{c3}^{+}\pi^0$ | 0.64            | 2.31            | 1.09            | 0.41            | 1.39            | 0.62            |
| $\Xi_{c3}^{+}\pi^0$ | 0.38            | 1.97            | 1.18            | 0.60            | 2.87            | 1.65            |
| $\Sigma_{c3}^+K^0$ | 0.09            | 0.59            | 0.29            | 0.06            | 0.36            | 0.17            |
| $\Sigma_{c3}^{++}\bar{K}^0$ | 0.04            | 0.49            | 0.32            | 0.07            | 0.73            | 0.44            |
| $\Sigma_{c3}^{++}\bar{K}^0$ | –              | 2.91            | 1.89            | –              | 0.04            | 0.11            |
| $\Sigma_{c3}^{++}\bar{K}^0$ | –              | 0.59            | 0.60            | –              | 2.84            | 2.58            |
| $\Lambda_{c3}^+\bar{K}^0$ | 1.00            | 2.28            | 0.48            | 1.11            | 2.41            | 0.47            |
| $\Lambda_{c3}^+\bar{K}^{*0}$ | 2.66 × 10⁻⁴ | 0.84            | 1.26            | 1.42 × 10⁻³ | 0.94            | 1.33            |
| $\Sigma_{c3}^0K^+$ | 0.19            | 1.20            | 0.59            | 0.13            | 0.73            | 0.34            |
| $\Sigma_{c3}^{++}\bar{K}^-$ | 0.08            | 1.00            | 0.63            | 0.14            | 1.47            | 0.89            |
| $\Sigma_{c3}^{++}\bar{K}^-$ | –              | 5.86            | 3.75            | –              | 0.09            | 0.23            |
| $\Sigma_{c3}^{++}\bar{K}^-$ | –              | 1.19            | 1.21            | –              | 5.75            | 5.17            |
| $\Lambda D^+$ | 0.17            | 1.15            | 6.23 × 10⁻³ | 0.21            | 1.22            | 9.04 × 10⁻³ |
| $\Sigma D^0$ | 0.07            | 1.03            | 3.71 × 10⁻⁴ | 0.05            | 0.62            | 5.47 × 10⁻⁴ |
| $\Gamma_{Total}$ | 13.23          | 51.69           | 23.66           | 14.29           | 50.88           | 24.32           |
| $\Gamma_{Carr}$ | 1.6 ~ 2.7      | 6.4 ~ 10.4      | 2.9 ~ 4.8       | 1.7 ~ 2.9       | 6.3 ~ 10.2      | 3.0 ~ 4.9       |
Leading Innovation Project (Grant No. LC 192209000701).

| Assignment | \( nL(J^P) \) | \( \Xi_0^0 \) | \( \Xi_0^+ \) | \( \Xi_0^- \) | \( \Xi_0^{*0} \) | \( \Xi_0^{*+} \) | \( \Xi_0^{*-} \) |
|------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \( n\Lambda \) | \( (n_\Lambda, n_\Delta) \) | \( (0, 1) \) | \( (1, 0) \) | \( (0, 0) \) | \( (0, 1) \) | \( (1, 0) \) | \( (0, 1) \) |
| Mass | \( 6224 \) | \( 6224 \) | \( 6084 \) | \( 6421 \) | \( 6421 \) | \( 6097 \) | \( 6432 \) |
| \( \Sigma^0 \) | \( 0 \) | \( 0 \) | \( 0 \) | \( 0 \) | \( 0 \) | \( 0 \) | \( 0 \) |
| \( \Xi_0^0 \) | \( \Xi_0^+ \) | \( \Xi_0^- \) | \( \Xi_0^{*0} \) | \( \Xi_0^{*+} \) | \( \Xi_0^{*-} \) |
| \( \Sigma^0 \) | \( 4.56 \) | \( 1.41 \) | \( 11.26 \) | \( 2.44 \) | \( 8.64 \) | \( 1.66 \times 10^{-3} \) | \( 0.06 \) | \( 5.34 \) |
| \( \Sigma_0^+ \) | \( 6.49 \) | \( 2.19 \) | \( - \) | \( 0.08 \) | \( 7.77 \) | \( 7.58 \) | \( 2.42 \) | \( 13.51 \) |
| \( \Xi_0^+ \) | \( 8.89 \) | \( 2.77 \) | \( 19.26 \) | \( 4.87 \) | \( 17.46 \) | \( 2.33 \times 10^{-3} \) | \( 0.12 \) | \( 10.59 \) |
| \( \Xi_0^{*-} \) | \( 13.30 \) | \( 4.46 \) | \( - \) | \( 0.16 \) | \( 15.88 \) | \( 14.51 \) | \( 4.88 \) | \( 27.04 \) |
| \( \Xi_0^0 \) | \( \Xi_0^+ \) | \( \Xi_0^- \) | \( \Xi_0^{*0} \) | \( \Xi_0^{*+} \) | \( \Xi_0^{*-} \) |
| \( \Sigma^0 \) | \( 3.19 \times 10^{-3} \) | \( 0.56 \) | \( - \) | \( 0.59 \) | \( 5.61 \) |
| \( \Sigma_0^+ \) | \( \Xi_0^+ \) | \( \Xi_0^- \) | \( \Xi_0^{*0} \) | \( \Xi_0^{*+} \) | \( \Xi_0^{*-} \) |
| \( \Lambda_0^0 \) | \( \Lambda_0^+ \) | \( \Lambda_0^- \) | \( \Lambda_0^{*0} \) | \( \Lambda_0^{*+} \) | \( \Lambda_0^{*-} \) |
| \( \Sigma_0^+ \) | \( 1.25 \) | \( 10.02 \) | \( - \) | \( 8.62 \times 10^{-3} \) | \( 1.20 \) |
| \( \Sigma_0^- \) | \( \Xi_0^+ \) | \( \Xi_0^- \) | \( \Xi_0^{*0} \) | \( \Xi_0^{*+} \) | \( \Xi_0^{*-} \) |
| \( \Sigma_0^0 \) | \( \Sigma_0^+ \) | \( \Sigma_0^- \) | \( \Sigma_0^{*0} \) | \( \Sigma_0^{*+} \) | \( \Sigma_0^{*-} \) |
| \( \Lambda B^0 \) | \( \Lambda B^- \) | \( \Lambda B^{*0} \) | \( \Lambda B^{*-} \) | \( \Lambda B^{*+} \) | \( \Lambda B^{*0} \) |

| | \( \Gamma_{Total} \) | \( 33.24 \) | \( 10.83 \) | \( 30.52 \) | \( 9.42 \) | \( 66.62 \) | \( 22.09 \) | \( 9.27 \) | \( 74.99 \) |
| | \( \Gamma_{Corr} \) | \( 1.3 \sim 3.4 \) | \( 0.4 \sim 1.1 \) | \( 1.2 \sim 3.1 \) | \( 0.3 \sim 1.0 \) | \( 2.6 \sim 6.7 \) | \( 0.8 \sim 2.2 \) | \( 0.3 \sim 1.0 \) | \( 2.9 \sim 7.5 \) |

TABLE XVII: Decay widths (MeV) of the \( \Xi_0(3\rho) \) baryons (Part I).
### TABLE XVIII: Decay widths (MeV) of the $\Xi_b$ ($3\bar{p}$) baryons (Part II).

| Assignment | $\Xi_{b2}^0 \pi^0$ | $\Xi_{b2}^o \pi^0$ | $\Xi_{b2}^0 \pi^+$ | $\Xi_{b2}^- \pi^+$ | $\Xi_{b2}^* \pi^+$ |
|------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| $nL(J^P)$  | $1D(\frac{3}{2}^-)$ | $2D(\frac{3}{2}^-)$ | $2D(\frac{5}{2}^-)$ | $1D(\frac{5}{2}^-)$ | $2D(\frac{5}{2}^-)$ |
| $(n_\rho, n_\lambda)$ | $(0, 0)$ | $(0, 1)$ | $(1, 0)$ | $(0, 0)$ | $(0, 1)$ |
| Mass       | 6320               | 6613                | 6613                | 6327                | 6621                |

| $\Sigma_b^0 \bar{K}^0$ | 0.04 | 0.96 | 0.10 | $1.75 \times 10^{-5}$ | 0.13 | 0.15 |
| $\Sigma_b^0 \bar{K}^0$ | 0.35 | 0.23 | 1.20 | 0.24 |
| $\Sigma_b^0 \bar{K}^{*0}$ | 0 | 0 | 0 | 0 |
| $\Sigma_b^0 \bar{K}^{*0}$ | 0 | 0 | 0 | 0 |

| $\Lambda_b^0 \bar{K}^0$ | 0 | 0 | 0 | 0 | 0 |
| $\Lambda_b^0 \bar{K}^{*0}$ | 0.72 | 0.78 | 0.79 | 0.82 |

| $\Sigma_b^+ \bar{K}^-$ | 0.11 | 1.91 | 0.20 | $5.69 \times 10^{-5}$ | 0.27 | 0.30 |
| $\Sigma_b^+ \bar{K}^-$ | 0.71 | 0.46 | 0.01 | 2.40 | 0.47 |
| $\Sigma_b^* \bar{K}^{*-}$ | 0 | 0 | 0 | 0 |
| $\Sigma_b^* \bar{K}^{*-}$ | 0 | 0 | 0 | 0 |

| $\Lambda B^0$ | 0 | 0 | 0 | 0 | 0 |
| $\Sigma B^-$ | 0.23 | 0.14 | 0.11 | 4.76 $\times 10^{-3}$ |

| $\Gamma_{Total}$ | 10.38 | 21.71 | 5.57 | 9.61 | 19.77 | 6.15 |
| $\Gamma_{Total}$ | 0.4 $\sim$ 1.1 | 0.8 $\sim$ 2.2 | 0.2 $\sim$ 0.6 | 0.3 $\sim$ 1.0 | 0.7 $\sim$ 2.0 | 0.2 $\sim$ 0.7 |
TABLE XIX: Decay widths (MeV) of the $\Xi(6_J^P)$ baryons (Part I).

| Assignment | $\Xi_0^{\pm} \pi^0$ | $\Xi_0^{\mp} \pi^0$ | $\Xi_0^{++} \pi^0$ | $\Xi_0^{0+} \pi^0$ | $\Xi_0^{00} \pi^- \pi^+$ | $\Xi_0^{1+} \pi^- \pi^+$ | $\Sigma_0^0 K^0$ | $\Sigma_0^0 K^{+}$ | $\Sigma_0^{00} K^0$ | $\Sigma_0^{00} K^+$ |
|------------|----------------------|----------------------|----------------------|----------------------|--------------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| Mass       | 5943                 | 6350                 | 6350                 | 5971                 | 6370                     | 6370                 | 6238                 | 6569                 | 6569                 | 6569                 |
| $nL(J^P)$  | 1S$^0(\frac{1}{2}^+)$ | 2S$^1(\frac{1}{2}^+)$ | 2S$^1(\frac{3}{2}^+)$ | 1S$^0(\frac{3}{2}^+)$ | 2S$^1(\frac{3}{2}^+)$   | 2S$^1(\frac{5}{2}^-)$ | 1P$^0(\frac{1}{2}^-)$ | 2P$^1(\frac{1}{2}^-)$ | 2P$^1(\frac{3}{2}^-)$ | 2P$^1(\frac{3}{2}^-)$ |
| $(n_\pi^+, n_\Lambda)$ | (0, 0)               | (0, 1)               | (1, 0)               | (0, 0)               | (0, 1)                    | (1, 0)               | (0, 0)               | (0, 1)               | (1, 0)               | (1, 0)               |
| $\Xi_0^{\pm} \pi^0$ | 0.02                 | 67.13                | 1.29                 | 3.09                 | 76.30                     | 0.82                 | 312.33               | 9.25                 | 7.03                 |
| $\Xi_0^{\mp} \pi^0$ | –                    | 28.83                | 4.11                 | –                    | 34.80                     | 4.16                 | ~ 0                  | ~ 0                  | ~ 0                  |
| $\Xi_0^{++} \pi^0$  | –                    | 11.81                | 1.97                 | –                    | 14.43                     | 2.05                 | ~ 0                  | ~ 0                  | ~ 0                  |
| $\Xi_0^{0+} \pi^0$  | 0.01                 | 129.29               | 2.88                 | 4.27                 | 147.21                    | 1.91                 | 62.31                | 18.68                | 12.27                |
| $\Xi_0^{00} \pi^- \pi^+$ | 0.01                 | 57.08                | 8.22                 | –                    | 68.98                     | 8.34                 | ~ 0                  | ~ 0                  | ~ 0                  |
| $\Xi_0^{0+} \pi^- \pi^+$ | 24.08                | 3.96                 | –                    | 29.40                | 4.13                      | ~ 0                  | ~ 0                  | ~ 0                  | ~ 0                  |
| $\Sigma_0^0 K^0$     | –                    | 1.42                 | 0.53                 | –                    | 2.68                      | 0.84                 | ~ 0                  | ~ 0                  | ~ 0                  |
| $\Sigma_0^{00} K^+$  | –                    | 0.26                 | 0.11                 | –                    | 0.71                      | 0.26                 | ~ 0                  | ~ 0                  | ~ 0                  |
| $\Sigma_0^{00} K^-$  | –                    | –                    | –                    | –                    | –                         | –                    | –                    | –                    | –                    |
| $\Sigma_0^{00} K^{+}$ | –                    | –                    | –                    | –                    | –                         | –                    | –                    | –                    | –                    |
| $\Lambda^b B^0$      | –                    | 19.88                | 0.55                 | –                    | 23.17                     | 0.34                 | 111.11               | 2.51                 | 3.79                 |
| $\Sigma^b B^0$       | –                    | –                    | –                    | –                    | –                         | ~ 0                  | ~ 0                  | ~ 0                  | ~ 0                  |
| $\Sigma^b B^-$       | –                    | 3.13                 | 1.13                 | –                    | 5.69                      | 1.76                 | ~ 0                  | ~ 0                  | ~ 0                  |
| $\Lambda^b B^-$      | –                    | 0.65                 | 0.27                 | –                    | 1.62                      | 0.58                 | ~ 0                  | ~ 0                  | ~ 0                  |
| $\Sigma^b B^+$       | –                    | –                    | –                    | –                    | –                         | –                    | –                    | –                    | –                    |
| $\Sigma^b B^+$       | –                    | –                    | –                    | –                    | –                         | –                    | –                    | –                    | –                    |
| $\Lambda B^0$        | –                    | –                    | –                    | –                    | –                         | –                    | –                    | ~ 0                  | ~ 0                  |
| $\Gamma_{Total}$     | 0.03                 | 343.56               | 25.02                | 7.36                 | 404.99                    | 25.19                | 485.75               | 41.07                | 24.95                |
| $\Gamma_{Corr}$      | < 0.06               | 42.9  68.8  3.1  5.0  | 9.0  15.0  50.6  81.0  | 3.1  5.1  60.7  97.2  | 5.1  13.0  3.1  15.0  |
TABLE XX: Decay widths (MeV) of the $\Xi'_0(6\rho)$ baryons (Part II).

| Assignment | $\Xi^{1+}_{b1}$ | $\Xi^{1\lambda+}_{b1}$ | $\Xi^{1\rho+}_{b1}$ | $\Xi^{1+\nu}_{b1}$ | $\Xi^{1+\lambda+}_{b1}$ | $\Xi^{1+\rho+}_{b1}$ |
|------------|------------------|----------------------|-------------------|------------------|----------------------|------------------|
| $\eta L(J^P)$ | $1P(\frac{1}{2}^-)$ | $2P(\frac{1}{2}^-)$ | $2P(\frac{1}{2}^-)$ | $1P(\frac{3}{2}^-)$ | $2P(\frac{3}{2}^-)$ | $2P(\frac{3}{2}^-)$ |
| $(n_\rho, n_\lambda)$ | (0, 0) | (0, 1) | (1, 0) | (0, 0) | (0, 1) | (1, 0) |
| Mass | 6232 | 6564 | 6564 | 6240 | 6572 | 6572 |
| $\Xi^{0}_{b0} \pi^0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ |
| $\Xi^{0}_{b0} \pi^0$ | 139.69 | 6.46 | 21.7 | 0.35 | 0.18 | 7.84 |
| $\Xi^{0}_{b0} \pi^0$ | 0.36 | 0.26 | 13.20 | 131.36 | 6.53 | 10.00 |
| $\Xi^{0}_{b0} \pi^+$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ |
| $\Xi^{0}_{b0} \pi^+$ | 278.49 | 12.92 | 4.48 | 0.68 | 0.35 | 15.62 |
| $\Xi^{0}_{b0} \pi^+$ | 0.75 | 0.53 | 26.79 | 26.00 | 13.10 | 19.89 |
| $\Sigma^{0}_{b0} \partial^0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ |
| $\Sigma^{0}_{b0} \partial^0$ | 1.93 | 2.30 | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ |
| $\Sigma^{0}_{b0} \partial^0$ | 0.04 | 2.77 | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ |
| $\Sigma^{0}_{b0} \partial^0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ |
| $\Xi^{0}_{b0} \rho^+$ | 419.29 | 26.82 | 84.97 | 398.39 | 26.96 | 94.70 |
| $\Gamma_{Total}$ | 52.4 $\sim 83.9$ | 3.3 $\sim 5.4$ | 10.6 $\sim 17.0$ | 49.7 $\sim 79.7$ | 3.3 $\sim 5.4$ | 11.8 $\sim 19.0$ |

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### TABLE XXI: Decay widths (MeV) of the $\Xi_c(6\bar{q}q)$ baryons (Part III).

| Assignment | $\Xi_c^{1\lambda}_{4/3}$ | $\Xi_c^{1\lambda}_{2/3}$ | $\Xi_c^{1\lambda}_{2}$ | $\Xi_c^{1\lambda}_{1/3}$ | $\Xi_c^{1\lambda}_{0}$ | $\Xi_c^{1\lambda}_{-1/3}$ |
|------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| $nL(J^P)$   | $1P(\frac{3}{2}^-$)     | $2P(\frac{3}{2}^-$)     | $2P(\frac{3}{2}^-$)     | $1P(\frac{5}{2}^-)$     | $2P(\frac{5}{2}^-)$     | $2P(\frac{5}{2}^-)$     |
| $(n_\rho, n_\Lambda)$ | (0, 0) | (0, 1) | (1, 0) | (0, 0) | (0, 1) | (1, 0) |
| Mass        | 6229                    | 6562                    | 6562                    | 6243                    | 6574                    | 6574                    |

- $\Xi_c^0\pi^0$: 8.03, 1.57, 34.00, 9.64, 1.75, 35.27
- $\Xi_c^0\eta^0$: 0.49, 0.29, 13.29, 0.30, 0.15, 6.34
- $\Xi_c^0\eta^0$: 0.30, 0.23, 11.72, 0.67, 0.41, 19.70
- $\Xi_c^0\pi^+$: 14.80, 2.99, 66.82, 17.83, 0.33, 69.42
- $\Xi_c^0\pi^+$: 0.95, 0.57, 26.48, 0.58, 0.29, 12.64
- $\Xi_c^0\pi^+$: 0.62, 0.47, 23.79, 1.38, 0.84, 40.00

| $\Lambda_c^0\bar{K}^0$ | 1.16 | 0.59 | 10.75 | 1.60 | 0.66 | 11.08 |
|--------------------------|------|------|-------|------|------|-------|
| $\Lambda_c^+\bar{K}^0$ | $6.98 \times 10^{-4}$ | 0.77 | $1.50 \times 10^{-3}$ | 1.28 |

- $\Sigma_c^+\bar{K}^-$: 0.10 | 5.95 | - | 0.05 | 2.92 |
- $\Sigma_c^+\bar{K}^-$: 0.07 | 5.00 | - | 0.14 | 8.70 |
- $\Sigma_c^+_c\bar{K}^*$: - | - | - | - | - |
- $\Sigma_c^0\bar{K}^*$: - | - | - | - | - |

- $\Lambda B^0$: 0.27 | 1.01 | - | 0.33 | 1.12 |
- $\Sigma B^-$: 0.10 | 0.65 | - | 0.06 | 0.36 |

| $\Gamma_{Total}$ | 26.35 | 7.34 | 205.61 | 32.00 | 5.11 | 214.54 |
| $\Gamma_{Corr}$ | 3.2 ~ 5.3 | 0.9 ~ 1.5 | 25.7 ~ 41.2 | 4.0 ~ 6.4 | 0.6 ~ 1.1 | 26.8 ~ 42.9 |

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TABLE XXII: Decay widths (MeV) of the $\Xi_{b}(6\rho)$ baryons (Part IV).

| Assignment | $\Xi_{b}^{0}$ | $\Xi_{b}^{\pm}$ | $\Xi_{b}^{0,\pm}$ | $\Xi_{b}^{0,\alpha}$ | $\Xi_{b}^{0,\lambda}$ | $\Xi_{b}^{0,\rho}$ |
|------------|--------------|----------------|------------------|------------------|------------------|------------------|
| $n\ell(J^{P})$ | $1D(\frac{1}{2}^{+})$ | $2D(\frac{1}{2}^{+})$ | $2D(\frac{1}{2}^{+})$ | $1D(\frac{3}{2}^{+})$ | $2D(\frac{3}{2}^{+})$ | $2D(\frac{3}{2}^{+})$ |
| $(n_{\rho}, n_{\lambda})$ | (0, 0) | (0, 1) | (0, 1) | (0, 0) | (0, 1) | (1, 0) |
| Mass | 6460 | 6757 | 6757 | 6466 | 6763 | 6763 |
| $\Xi_{b}^{0}K^{0}$ | 19.53 | 5.09 | 1.66 | 19.64 | 4.97 | 1.78 |
| $\Xi_{b}^{0}K^{0}$ | 4.55 | 2.38 | 0.02 | 1.17 | 0.59 | 7.31 $\times$ 10$^{-3}$ |
| $\Xi_{b}^{0}K^{0}$ | 2.09 | 1.21 | 2.14 $\times$ 10$^{-3}$ | 5.36 | 3.00 | 9.71 $\times$ 10$^{-3}$ |
| $\Xi_{b}^{0}K^{0}$ | 38.85 | 10.42 | 3.13 | 39.11 | 10.18 | 3.35 |
| $\Xi_{b}^{0}K^{0}$ | 9.08 | 4.77 | 0.04 | 2.33 | 1.19 | 0.01 |
| $\Xi_{b}^{0}K^{0}$ | 4.22 | 2.41 | 5.32 $\times$ 10$^{-3}$ | 10.83 | 6.01 | 0.02 |
| $\Sigma_{b}^{\pm}K^{0}$ | 1.01 | 0.69 | 2.12 $\times$ 10$^{-3}$ | 0.27 | 0.17 | 9.37 $\times$ 10$^{-4}$ |
| $\Sigma_{b}^{\pm}K^{0}$ | 0.42 | 0.35 | 4.36 $\times$ 10$^{-6}$ | 1.12 | 0.88 | 1.77 $\times$ 10$^{-4}$ |
| $\Sigma_{b}^{\pm}K^{0}$ | – | 0.09 | 0.13 | – | 0.08 | 0.13 |
| $\Sigma_{b}^{\pm}K^{0}$ | – | 0.07 | 0.12 | – | 0.11 | 0.17 |
| $\Lambda_{b}^{0}K^{0}$ | 5.98 | 1.06 | 0.76 | 5.99 | 1.02 | 0.79 |
| $\Lambda_{b}^{0}K^{0}$ | – | 1.08 | 0.21 | – | 1.09 | 0.19 |
| $\Sigma_{b}^{\pm}K^{-}$ | 2.05 | 1.38 | 4.83 $\times$ 10$^{-3}$ | 0.54 | 0.34 | 2.06 $\times$ 10$^{-3}$ |
| $\Sigma_{b}^{\pm}K^{-}$ | 0.87 | 0.70 | 2.67 $\times$ 10$^{-6}$ | 2.30 | 1.75 | 7.66 $\times$ 10$^{-4}$ |
| $\Sigma_{b}^{\pm}K^{-}$ | – | 0.18 | 0.25 | – | 0.16 | 0.25 |
| $\Sigma_{b}^{\pm}K^{-}$ | – | 0.15 | 0.23 | – | 0.22 | 0.33 |
| $\Lambda B^{0}$ | 2.17 | 2.83 | 1.20 | 2.42 | 2.80 | 1.23 |
| $\Sigma B^{0}$ | – | 2.11 | 0.58 | – | 0.52 | 0.15 |
| $\Sigma Total$ | 90.82 | 36.97 | 8.33 | 91.08 | 35.08 | 8.40 |
| $\Gamma_{Total}$ | 11.3 $\sim$ 18.2 | 4.6 $\sim$ 7.4 | 1.0 $\sim$ 1.7 | 11.3 $\sim$ 18.3 | 4.3 $\sim$ 7.1 | 1.0 $\sim$ 1.7 |

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### TABLE XXIII: Decay widths (MeV) of the $\Xi_{q}(6_{p})$ baryons (Part V).

| Assignment | $\Xi_{q}^{0}$ | $\Xi_{q}^{2+}$ | $\Xi_{q}^{2+}$ | $\Xi_{q}^{2+}$ | $\Xi_{q}^{2+}$ | $\Xi_{q}^{2+}$ | $\Xi_{q}^{2+}$ |
|------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $nL(J^{P})$ | $1D(\frac{3}{2}^{+})$ | $2D(\frac{3}{2}^{+})$ | $2D(\frac{3}{2}^{+})$ | $1D(\frac{3}{2}^{+})$ | $2D(\frac{3}{2}^{+})$ | $2D(\frac{3}{2}^{+})$ | $2D(\frac{3}{2}^{+})$ |
| $(n_{\pi}, n_{\lambda})$ | (0, 0) | (0, 1) | (1, 0) | (0, 0) | (0, 1) | (1, 0) | |
| Mass | 6460 | 6758 | 6758 | 6466 | 6764 | 6764 | 6764 |
| $\Xi_{q}^{0} \pi^{0}$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ |
| $\Xi_{q}^{0} \pi^{0}$ | 10.24 | 5.41 | 0.05 | 0.30 | 1.50 | 0.92 | |
| $\Xi_{q}^{0} \pi^{0}$ | 2.25 | 3.30 | 1.51 | 11.76 | 7.51 | 0.71 | |
| $\Xi_{q}^{-} \pi^{0}$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ | $\sim 0$ |
| $\Xi_{q}^{-} \pi^{0}$ | 20.43 | 10.71 | 0.10 | 0.58 | 2.99 | 1.85 | |
| $\Xi_{q}^{-} \pi^{0}$ | 4.55 | 6.70 | 3.06 | 23.77 | 15.06 | 1.45 | |
| $\Sigma_{q}^{0} K^{0}$ | 22.78 | 1.56 | $5.29 \times 10^{-3}$ | 0.03 | 0.39 | 0.25 | |
| $\Sigma_{q}^{0} K^{0}$ | 0.41 | 0.88 | 0.41 | 2.44 | 2.16 | 0.19 | |
| $\Sigma_{q}^{0} K^{0}$ | $\sim 0$ | 0.26 | 0.36 | $\sim 0$ | 0.03 | 0.04 | |
| $\Sigma_{q}^{0} K^{0}$ | $\sim 0$ | 0.05 | 0.08 | $\sim 0$ | 0.22 | 0.33 | |
| $\Lambda_{q}^{0} K^{0}$ | $\sim 0$ | 0.33 | 1.09 | $\sim 0$ | 2.37 | 1.09 | |
| $\Lambda_{q}^{0} K^{0}$ | $\sim 0$ | 3.11 | 0.01 | 0.05 | 0.79 | 0.51 | |
| $\Sigma_{q}^{+} K^{0}$ | 0.84 | 1.78 | 0.82 | 4.99 | 4.32 | 0.38 | |
| $\Sigma_{q}^{+} K^{0}$ | $\sim 0$ | 0.51 | 0.70 | $\sim 0$ | 0.05 | 0.08 | |
| $\Sigma_{q}^{+} K^{0}$ | $\sim 0$ | 0.10 | 0.15 | $\sim 0$ | 0.45 | 0.66 | |
| $\Lambda B^{0}$ | $\sim 0$ | 0.33 | 1.09 | $\sim 0$ | 2.37 | 1.09 | |
| $\Sigma B^{0}$ | $\sim 0$ | 4.74 | 1.31 | $\sim 0$ | 0.59 | $1.90 \times 10^{-4}$ | |
| $\Gamma_{Total}$ | 66.11 | 41.44 | 9.65 | 43.92 | 38.43 | 8.46 | |
| $\Gamma_{Corr}$ | 8.2 $\sim$ 13.3 | 5.1 $\sim$ 8.3 | 1.2 $\sim$ 2.0 | 5.4 $\sim$ 8.8 | 4.8 $\sim$ 7.7 | 1.0 $\sim$ 1.7 | |

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TABLE XXIV: Decay widths (MeV) of the $\Xi(6\rho)$ baryons (Part VI).

| Assignment | $\Xi^{−}_{3/2}$ | $\Xi^{+}_{3/2}$ | $\Xi^{0}_{3/2}$ | $\Xi^{+}_{3/2}$ | $\Xi^{0}_{3/2}$ | $\Xi^{++}_{3/2}$ | $\Xi^{0}_{3/2}$ | $\Xi^{+}_{3/2}$ |
|------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $nL(J^P)$  | $2D(3^+)$     | $2D(3^+)$     | $2D(3^+)$     | $2D(3^+)$     | $2D(3^+)$     | $2D(3^+)$     | $2D(3^+)$     | $2D(3^+)$     |
| $(n_\rho, n_\lambda)$ | (0, 0) | (0, 1) | (1, 0) | (0, 0) | (0, 1) | (1, 0) |  |
| Mass       | 6460           | 6757           | 6757           | 6467           | 6765           | 6765           | 6765           |  |
| $\Xi^{0}_{3/2}\pi^0$ | 2.76 | 7.05 | 2.27 | 2.96 | 7.38 | 2.27 |  |
| $\Xi^{0}_{3/2}\pi^0$ | 0.31 | 1.63 | 1.03 | 0.19 | 0.97 | 0.60 |  |
| $\Xi^{0}_{3/2}\pi^0$ | 0.29 | 1.74 | 1.19 | 0.43 | 2.51 | 1.67 |  |
| $\Xi^{+}_{3/2}\pi^0$ | 5.22 | 13.65 | 4.53 | 5.59 | 14.30 | 4.54 |  |
| $\Xi^{+}_{3/2}\pi^0$ | 0.61 | 3.25 | 2.06 | 0.38 | 1.94 | 1.19 |  |
| $\Xi^{+}_{3/2}\pi^0$ | 0.60 | 3.55 | 2.41 | 0.90 | 5.12 | 3.36 |  |
| $\Sigma^0_3\bar{K}^0$ | 0.03 | 0.42 | 0.28 | 0.02 | 0.25 | 0.16 |  |
| $\Sigma^0_3\bar{K}^0$ | 0.02 | 0.44 | 0.32 | 0.03 | 0.64 | 0.45 |  |
| $\Sigma^0_3\bar{K}^0$ | – | 1.04 | 1.41 | – | $1.58 \times 10^{-3}$ | $5.76 \times 10^{-3}$ |  |
| $\Sigma^0_3\bar{K}^0$ | – | 0.16 | 0.25 | – | 0.97 | 1.41 |  |
| $\Lambda^0_4\bar{K}^0$ | 0.94 | 2.62 | 0.60 | 1.02 | 2.74 | 0.58 |  |
| $\Lambda^0_4\bar{K}^0$ | – | 0.54 | 1.01 | – | 0.60 | 1.09 |  |
| $\Sigma^+_4\bar{K}^-$ | 0.05 | 0.85 | 0.57 | 0.04 | 0.51 | 0.33 |  |
| $\Sigma^+_4\bar{K}^-$ | 0.04 | 0.90 | 0.65 | 0.07 | 1.30 | 0.91 |  |
| $\Sigma^+_4\bar{K}^-$ | – | 2.03 | 2.78 | – | $3.00 \times 10^{-3}$ | 0.01 |  |
| $\Sigma^+_4\bar{K}^-$ | – | 0.32 | 0.49 | – | 1.93 | 2.81 |  |
| $\Lambda^0\bar{B}^0$ | $8.62 \times 10^{-3}$ | 0.89 | $2.20 \times 10^{-3}$ | 0.01 | 0.95 | $3.44 \times 10^{-3}$ |  |
| $\Sigma^+\bar{B}^-$ | – | 0.63 | $4.75 \times 10^{-4}$ | – | 0.38 | $1.06 \times 10^{-4}$ |  |
| $\Gamma_{Total}$ | 10.87 | 41.71 | 21.85 | 11.64 | 42.49 | 21.38 |  |
| $\Gamma_{Corr}$ | $1.3 \sim 2.2$ | $5.2 \sim 8.4$ | $2.7 \sim 4.4$ | $1.4 \sim 2.4$ | $5.3 \sim 8.5$ | $2.6 \sim 4.3$ |  |

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TABLE XXV: Total widths (MeV) of some related baryons from experiments.

| State            | Mass        | $I(J^P)$         | Total width  | typical decay modes     |
|------------------|-------------|------------------|--------------|-------------------------|
| $\Xi_c(2645)^+$ | 2645.10 ± 0.30 | $\frac{1}{2}(-\frac{3}{2})$ | 2.14 ± 0.19  | $\Xi_0^0\pi^+$          |
| $\Xi_c(2645)\Omega$ | 2646.16 ± 0.25 | $\frac{1}{2}(-\frac{3}{2})$ | 2.35 ± 0.22  | $\Xi_0^+\pi^-$          |
| $\Xi_c(2790)^+$ | 2791.9 ± 0.5  | $\frac{1}{2}(-\frac{3}{2})$ | 8.9 ± 1.0    | $\Xi_c^0\pi$            |
| $\Xi_c(2790)\Omega$ | 2793.9 ± 0.5  | $\frac{1}{2}(-\frac{3}{2})$ | 10.0 ± 1.1   | $\Xi_c^+\pi$            |
| $\Xi_c(2815)^+$ | 2816.51 ± 0.25 | $\frac{1}{2}(-\frac{3}{2})$ | 2.43 ± 0.26  | $\Xi_0^+\pi$, $\Xi_c(2645)\pi$ |
| $\Xi_c(2815)\Omega$ | 2819.79 ± 0.30 | $\frac{1}{2}(-\frac{3}{2})$ | 2.54 ± 0.25  | $\Xi_c^0\pi$, $\Xi_c(2645)\pi$ |
| $\Xi_c(5945)\Omega$ | 5952.3 ± 0.6  | $\frac{3}{2}^+$    | 0.90 ± 0.18  | $\Xi_0^0\pi^+$          |
| $\Xi_c(5955)\Omega$ | 5953.3 ± 0.6  | $\frac{3}{2}^+$    | 1.65 ± 0.33  | $\Xi_0^0\pi^-$          |
| $\Xi_c(2923)\Omega$ | 2923.04 ± 0.59 | (?(?))          | 7.1 ± 2.6    | $\Lambda_c^+K^-$         |
| $\Xi_c(2930)^+$ | 2942.3 ± 5.9  | (?(?))          | 14.8 ± 11.3  | $\Lambda_c^+K^0$         |
| $\Xi_c(2939)\Omega$ | 2938.55 ± 0.52 | (?(?))          | 10.2 ± 1.9   | $\Lambda_c^+K^-$         |
| $\Xi_c(2965)\Omega$ | 2964.88 ± 0.54 | (?(?))          | 14.1 ± 2.2   | $\Lambda_c^+K^-$         |
| $\Xi_c(2970)^+$ | 2964.3 ± 1.5  | $\frac{1}{2}(-\frac{3}{2})$ | 20.9 ± 2.4   | $\Sigma_c^0K$, $\Sigma_c^+K$, $\Xi_c^0\pi$ |
| $\Xi_c(3055)\Omega$ | 3055.9 ± 0.4  | (?(?))          | 7.8 ± 1.9    | $\Sigma_c^+K^0, \Lambda D^+$ |
| $\Xi_c(3080)\Omega$ | 3077.2 ± 0.4  | $\frac{1}{2}(-\frac{3}{2})$ | 3.6 ± 1.1    | $\Sigma_c^0K$, $\Sigma_c^+K^0, \Lambda D^+$ |
| $\Xi_c(3123)^+$ | 3122.9 ± 1.6  | (?(?))          | 4.4 ± 3.4 ± 1.7 | $\Sigma_c(2520)^{++}K^+$ |
| $\Xi_c(6227)\Omega$ | 6227.9 ± 0.9  | (?(?))          | 19.9 ± 2.6   | $\Xi_c^0\pi^-, \Lambda_c^0K^-$ |
| $\Xi_c(6227)\Omega$ | 6226.8 ± 1.6  | (?(?))          | 19 ± 5 ± 4   | $\Xi_c^0\pi^+$          |
| $\Xi_c(6100)\Omega$ | 6100.3 ± 0.9  | $\frac{3}{2}^+$  | < 1.9        | $\Xi_c^0\pi^-$          |
| $\Xi_c(6327)\Omega$ | 6327.28 ± 0.50 | $\frac{3}{2}^+$  | < 2.20 ± 2.56 | $\Lambda_c^0K^+\pi^-$ |
| $\Xi_c(6333)\Omega$ | 6332.69 ± 0.42 | $\frac{3}{2}^+$  | < 1.55 ± 1.85 | $\Lambda_c^0K^+\pi^-$ |

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TABLE XXVI: Mass spectrum shells of the $\Xi_c'$ (top half) and $\Xi_c$ (bottom half) families and masses are measured in MeV.

| nL | $J = 1/2$ | $J = 3/2$ | $J = 5/2$ | $J = 7/2$ | $J = 9/2$ |
|----|-----------|-----------|-----------|-----------|-----------|
| 4S | 3425      | 3456      |           | 3423      | 3428      |
| 1F |           | 3424      |           | 3428      |           |
| 2P | 3315      | 3331      | 3310      | 3335      |           |
| 3S | 3201      | 3244      | 3200      | 3213      |           |
| 1D |           |           | 3201      | 3211      |           |
| 2S | 3046✓     | 3095✓     |           |           |           |
| 1P | 2941♦     | 2958♦     | 2934✓     | 2964✓     |           |
| 1S | 2590      | 2658      |           |           | 3486      | 3487      |
| 1G |           |           |           |           |           |
| 2D |           |           |           |           |           |
| 3P | 3390      | 3412      |           |           |           |
| 4S |           |           | 3318      |           |           |
| 1F |           |           |           | 3289      | 3294      |
| 2P | 3176      | 3199      |           |           |           |
| 3S | 3155      |           |           |           |           |
| 1D |           |           | 3063?     | 3076?     |           |
| 2S | 2949✓     |           |           |           |           |
| 1P | 2789      | 2819      |           |           |           |
| 1S | 2479      |           |           |           |           |

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TABLE XXVII: Mass spectrum shells of the $\Xi'_b$ (top half) and $\Xi_b$ (bottom half) families and masses are measured in MeV.

| nL | $J = 1/2$ | $J = 3/2$ | $J = 5/2$ | $J = 7/2$ | $J = 9/2$ |
|----|-----------|-----------|-----------|-----------|-----------|
| 4S | 6691      | 6705      |           |           |           |
| 1F |           |           |           | 6657      | 6661      |
|    |           |           | 6657      | 6660      |           |
| 2P | 6564      | 6572      | 6562      | 6574      |           |
|    |           |           |           |           |           |
| 3S | 6535      | 6554      |           |           |           |
| 1D |           | 6460      | 6460      | 6466      |           |
|    | 6460      |           | 6466      |           |           |
| 2S | 6350♦     | 6370♦     |           |           |           |
| 1P | 6232♦     | 6240♦     |           |           | 6229      |
|    | 6238♦     |           |           | 6243      |           |
| 1S | 5943      | 5971      |           |           |           |
| 3P | 6690      | 6700      |           |           |           |
| 1G |           |           |           | 6692      | 6695      |
| 2D |           |           |           | 6613      | 6621      |
| 4S | 6568      |           |           |           |           |
| 1F |           |           | 6518      | 6523      |           |
| 3S | 6480      |           |           |           |           |
| 2P | 6421      | 6432      |           |           |           |
| 1D |           |           | 6320✓     | 6327✓     |           |
| 2S | 6224      |           |           |           |           |
| 1P | 6084      |           |           |           |           |
| 1S | 5806      |           |           |           |           |

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