SMEFT Corrections to $Z$ Boson Decays

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We compute the one-loop corrections to $Z$ decay properties from dimension-6 operators in the Standard Model Effective Field Theory (SMEFT) that contribute also to anomalous 3-gauge boson couplings and examine the relative sensitivity of the two processes to the anomalous couplings. The size of the contributions is of order a few percent, of the same size as Standard Model electroweak corrections. This is part of a program of computing electroweak quantities to one-loop in the SMEFT: these calculations are needed for a future global fit to limit the coefficients of the dimension-six Wilson coefficients consistently at one loop.

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## Contents

I. Introduction 8

II. SMEFT at one-loop 5

III. Results 5
   A. Effective Z Vertices 6

IV. Conclusions 8

   Acknowledgements 8

   A. UV poles 8

   B. 2-point functions 9

   C. Vertex functions 16

   D. LEP and LHC measurements 19

   References 20
I. INTRODUCTION

The development of the precision electroweak program at the LHC is a major task for the coming decade. At present, the interactions of the Higgs boson and the electroweak gauge bosons appear to have approximately Standard Model (SM) like interactions and there is no sign of new massive particles. These points together imply that deviations from the SM can be analyzed in an effective field theory framework [1, 2].

In the Standard Model Effective Field Theory (SMEFT), deviations from the SM are parameterized in terms of a tower of higher dimension operators, $O^d_k$, 

$$\mathcal{L} = \mathcal{L}_{SM} + \sum d \sum k C^d_k \frac{O^d_k}{\Lambda^{d-k}},$$  

where the operators, $O^d_k$, contain only SM fields and are invariant under $SU(3) \times SU(2) \times U(1)$. The complete set of dimension-6 operators was first compiled in Refs. [3, 4] and the Feynman rules in this basis ("Warsaw Basis") are conveniently given in Ref. [5]. The new physics is completely contained in the coefficient functions, $C^d_k$. The scale of the assumed UV complete theory is $\Lambda$, and we assume $\Lambda \gg v = 246$ GeV. For a weakly coupled theory, the corrections to SM predictions are dominated by the dimension-6 contributions.

Predictions for Higgs production and decay, along with $VV$ ($W^\pm, Z, \gamma$) interactions are well known at tree level in the SMEFT [1, 2, 6, 7]. Including also contributions to the oblique parameters, limits on the allowed sizes of the SMEFT coefficients can be extracted in a global fit to Higgs signal rates and gauge boson pair production [8–11]. A precision Higgs and electroweak physics program, however, requires SMEFT calculations beyond the leading order if matching between the experimental results and theory is to be eventually done at the few percent level.

The program of calculating SMEFT quantities beyond leading order is in its infancy. One-loop calculations exist for $H \rightarrow \gamma\gamma$ [12–14], $H \rightarrow b\bar{b}$ [15] and the unphysical $H \rightarrow ZZ$ and $H \rightarrow W^+W^−$ processes [17, 18]. The one-loop Yukawa, $y_t$, and $\lambda = \frac{M_H^2}{2v^2}$ contributions to $Z$ decays are also known [19]. In addition to effects in the electroweak sector, one-loop contributions from top-quark operators can significantly affect Higgs production rates at the LHC [20, 21].

In this paper, we compute the 1-loop corrections to the partial $Z$ decay widths due to the dimension-6 operators that contribute to $q\bar{q} \rightarrow W^+W^−$ [23, 28] and to $Z \rightarrow f\bar{f}$. These operators are particularly interesting because for transverse gauge boson production they contribute to different helicity amplitudes [22, 23], such that their interference with the SM does not grow with energy unless decays or higher order corrections are considered [24, 25]. Along with anomalous 3-gauge boson couplings, we inculde in our calculation the shifts in the $Z$ decay widths due to anomalous fermion couplings, which have important contributions not only to the $Z$ widths [26], but also to gauge boson pair production [23, 27, 28].

In this work, we consider modifications of the $Zf\bar{f}$ and $W^+W^−V$ ($V = Z, \gamma$) vertices. We consider only operators that contribute to both $q\bar{q} \rightarrow W^+W^−$ [23, 28] and to $Z \rightarrow f\bar{f}$.

II. SMEFT AT ONE-LOOP

In this work we consider modifications of the $Zf\bar{f}$ and $W^+W^−V$ ($V = Z, \gamma$) vertices. We consider only operators that contribute to both $q\bar{q} \rightarrow W^+W^−$ and to $Z \rightarrow f\bar{f}$.

The fermion vertices can be parameterized as,

$$\mathcal{L}_f = g_Z Z_{\mu} \left[ \left( g_{Zf}^{\mu} + \delta g_{Zf}^{\mu} \right) J_L^{\gamma\mu} f_L + \left[ g_{Rf}^{\mu} + \delta g_{Rf}^{\mu} \right] J_R^{\gamma\mu} f_R + (f \rightarrow f') \right]$$

In Section [II], we review the basics of the one-loop SMEFT calculation and in Section [III] the calculation of $Z \rightarrow f\bar{f}$ in the SMEFT is summarized, with analytic formulae presented in a series of appendices. Numerical results are given in Section [III].
\[ + \frac{g}{\sqrt{2}} \left\{ W_\mu \left[ (1 + \delta g_W^L) T_L \gamma_\mu f_L^\dagger + \delta g_W^R T_R \gamma_\mu f_R^\dagger \right] + h.c. \right\}, \]

where \( g_Z \equiv e/(c_W s_W) = g/c_W \) and \( f (f') \) denotes up-type (down-type) quarks. The SM fermion couplings are:

\[ g_Z^L = -s_W^2 Q_f \quad \text{and} \quad g_Z^R = T_f^\dagger - s_W^2 Q_f, \]

where \( T_f^\dagger = \pm \frac{1}{2} \) and \( Q_f \) are the weak isospin and electric charge of the fermions, respectively.

Assuming CP conservation, the most general Lorentz invariant 3-gauge boson couplings can be written as

\[ L_V = -ig_{WWV} \left( g_1^V \left( W_\mu^+ W_\nu^- V^\mu - W_\mu^- W_\nu^+ V^\mu \right) + \kappa^V W_\mu^+ W_\nu^- V^{\mu\nu} - \frac{\Lambda^V}{M^2_W} W_\mu^+ W_\nu^- V^{\mu\nu} \right), \]

where \( g_{WWV} = e \) and \( g_{WWZ} = g_{cW} \). For the 3-gauge boson couplings we define \( g_1^V = 1 + \delta g_1^V, \kappa^V = 1 + \delta \kappa^V, \) and in the SM, \( \delta g_1^V = \delta \kappa^V = \lambda^V = 0. \) Because of gauge invariance we always have \( \delta g_1^V = 0. \) We assume \( SU(2) \) invariance, which implies the coefficients are related by,

\[
\begin{align*}
\delta g_L^W &= \delta g_Z^f - \delta g_Z^{f'}, \\
\delta g_L^Z &= \delta \kappa^Z + s_W^2 \delta \kappa^\gamma, \\
\lambda^\gamma &= \lambda^Z,
\end{align*}
\]

leaving three independent effective couplings.

We work in the Warsaw basis and the dimension-6 operators contributing to the 3-gauge boson vertices are,

\[
\begin{align*}
O_W &= \epsilon^{abc} W_\mu^a W_\nu^b W_\rho^c, \\
O_{HWB} &= \Phi^\dagger \sigma^a \Phi W_\mu^a B^{\mu\nu}, \\
O_{HW} &= (\Phi^\dagger \Phi) W_\mu^a W_\nu^a, \\
O_{WB} &= (\Phi^\dagger \Phi) B_{\mu\nu},
\end{align*}
\]

where \( D_\mu \Phi = (\partial_\mu - i \frac{g}{2} \sigma^a W_\mu^a - i \frac{g'}{2} B_\mu) \Phi, \) \( W_\mu^{a} = D_\mu W_\mu^a - \partial_\mu W_\mu^a + g_{cW} \epsilon^{abc} W_\rho^b W_\nu^c, \) and \( \Phi \) is the Higgs doublet field with a vacuum expectation value \( \langle \Phi \rangle = (0, v/\sqrt{2}) \Gamma. \) In the mapping from EFT operators to anomalous couplings we have to take into account the EFT shift \( s_W^2 \rightarrow s_W^2 + \delta s_W^2 \) in the definition of the model input parameters for the gauge couplings, as well as for \( s_W, \) so that we get back to canonically normalized gauge fields. We take as our input parameters \( M_W, M_Z \) and \( G_\mu. \) The only EFT shift involving \( C_{HWB} \) is \( s_W^2 \rightarrow s_W^2 + \delta s_W^2, \) where

\[ \delta s_W^2 = \frac{s_{WCW} - s_W^2}{s_W^2} \left( \frac{v^2}{\Lambda^2} C_{HWB} \right), \]

where at this order we can use the tree level relations, \( c_W = \frac{M_W}{\sqrt{2}}. \) We find the following mappings between the SMEFT coefficient, \( C_{HWB}, \) and the effective couplings,

\[
\begin{align*}
\delta g_L^Z &= \frac{\delta s_W^2}{c_W}, \\
\delta \kappa^Z &= -2 \delta s_W^2, \\
\delta \kappa^\gamma &= \frac{c_W^2 - s_W^2}{s_W^2} \delta s_W^2, \\
\lambda^V &= \frac{v}{\Lambda^2} 3M_W C_W, \\
\delta g_L^W &= \delta s_W^2, \\
\delta g_R^W &= 0, \\
\delta g_R^Z &= Q_f \delta s_W^2, \quad (8)
\end{align*}
\]

The shifts including all SMEFT operators can be found in Refs. [27, 35].
III. RESULTS

At tree level, the decay amplitude for $Z \rightarrow f(p)f'(p')$ in the SMEFT is, (including only those terms that contribute also to 3- gauge boson vertices),

$$M_0 = 2M_{Z0}\sqrt{2G_F}\rho(1 - \frac{M_{W0}^2}{M_{Z0}^2}) + Q_f\frac{M_{W0}}{M_{Z0}}\sqrt{1 - \frac{M_{W0}^2}{M_{Z0}^2}v^2C_{HWB}}\bar{u}(p)\gamma^\mu(p + p')v(p'), \quad (9)$$

where the subscript "0" indicates the unrenormalized tree level value, and in the $C_{HWB}$ term we can take $v^2 = \frac{1}{\sqrt{2}G_F}$.

At one loop, there are contributions from corrections to the input parameters and fields to $M_0$, $Z - \gamma$ mixing, and the one-particle irreducible loop corrections to the decay, $M_1$. The virtual decay amplitude is,

$$M_{1-loop} = \left(1 + \frac{\partial C_{HWB}}{\partial C_{HWB}} + \frac{\partial G_\mu}{\partial G_\mu} + \frac{\partial M_Z^2}{\partial M_Z^2} + \frac{\partial M_W^2}{\partial M_W^2} + \frac{1}{2}\delta Z_{\mu} + \delta Z_f \right) M_0
- M_{\gamma} \frac{\Pi_{\gamma Z}(M_H^2)}{M_Z^2} + M_1. \quad (10)$$

Here $M_{\gamma}$ is the amplitude for $\gamma \rightarrow f(p)f'(p')$, which is

$$M_{\gamma} = 2M_W\sqrt{2G_F}Q_f\left(1 - \frac{M_{W0}^2}{M_{Z0}^2}M_W\frac{\partial C_{HWB}}{\partial C_{HWB}}\right)\bar{u}(p)\gamma^\mu(p + p')v(p'), \quad (11)$$

in the SMEFT. In Eq. [10] $Z_f$ and $Z_Z$ are the wavefunction renormalizations of the external $Z$ boson and the fermions. We use on-shell renormalization for all quantities, except for the Wilson coefficients which are renormalized using $\overline{MS}$ subtraction. In general, the coefficients are renormalized as [37, 38],

$$C_i(\mu) = C_{0,i} = \frac{1}{32\pi^2}\gamma_{ij}C_{ij}, \quad (12)$$

where $\mu$ is the renormalization scale, $\gamma_{ij}$ is the one-loop anomalous dimension and $\gamma^{-1} = \epsilon^{-1} - \gamma_E + \log(4\pi)$ is related to the regulator $\epsilon$ for integrals evaluated in $d = 4 - 2\epsilon$ dimensions.

The renormalization of $G_\mu$ in the SMEFT, including both logarithms and constant contributions, can be found in the appendix of Ref. [17]. The shifts in the SM input parameters as well as the external field wave function renormalizations follow from the 2-point functions in Appendix [3].

We calculate the contributions to Eq. [10] to $\mathcal{O}(\frac{1}{\Lambda^2})$, neglecting higher order terms whose impact would be expected to be comparable to that of dimension-8 operators. The 1PI loop amplitude $M_1$ is given in Appendix [C]. We use FeynArts [39] and FeynCalc [40, 41] to calculate loop amplitudes with the SMEFT package for FeynRules [42, 43]. Explicit analytic expressions for the loop integrals have been computed using the FeynHelpers interface [44] between FeynCalc and Package-X [45]. As a check of our calculation, we demonstrate that the UV poles in Eq. [10] cancel completely in Appendix [A].

There are IR divergences arising from loops with massless photons, appearing in the fermion wave function renormalization and $M_1$. We regulate these divergences with a photon mass, $M_Z\beta$. We find,

$$Re\left(Z_fM_0 + M_1\right) = \sqrt{G_\mu}\frac{M_ZM_W^2}{\sqrt{2\pi^2}}Q_f^2\log(3 + \log(\beta))\sqrt{1 - \frac{M_W^2}{M_Z^2}}\left[-G_\mu\sqrt{1 - \frac{M_W^2}{M_Z^2}}\left[T_f^3 - Q_f\left(1 - \frac{M_W^2}{M_Z^2}\right)\right]\right]. \quad (13)$$

The above divergences give $\beta$-dependent terms in the decay width, which are in turn canceled by soft real photon emission. The SMEFT calculation of $Z \rightarrow f\bar{f}\gamma$ proceeds analogously to the well-known SM result [46] with additional terms proportional to $C_{HWB}$ from the tree level shifts of the input parameters. The result is

$$|M(Z \rightarrow f\bar{f}\gamma)|^2 = \frac{4G_F^2M_W^2}{\pi^2}Q_f^2\left(T_f^3 - Q_f\sum_{H_W}\left[-\log(\beta)(3 + \log(\beta)) + \frac{3E_0^2}{2M_Z^2}\right]\right).$$
\[ -\log(\theta_0) \left( \frac{3E_0}{M_Z} + \log \left( \frac{M_Z}{2E_0} - 1 \right) - \frac{3}{4} \right) + \frac{5\pi^2}{12} - \frac{87}{16} \cdot \left[ -s_W^2 \left( T_3^f - Q_f s_W^2 \right) \right] \]
\[ + \frac{2c_W s_W v^2}{\Lambda^2} C_{HWB} (T_3^f - 2Q_f s_W^2) \right\} , \tag{14} \]

where \(s_W^2 = 1 - \frac{M_W^2}{M_Z^2}, c_W = \frac{M_W}{M_Z} \) in Eq. 14 and \(\theta_0\) and \(E_0\) are the angular and energy cutoff for observing the photon.

After summing Eq. 13 and the contributions from virtual and real photon emission, taking into account the fermion wave function renormalization, there is no \(\beta\) dependence. In our numerical results below, we take \(\theta_0 = 1^\circ\) and \(E_0 = 1\) GeV.

### A. Effective Z Vertices

From Eq. [10] we obtain the contribution to the \(Z \to f \bar{f}\) decay width from \(C_{HWB}, C_{HW}, C_{HB}\) and \(C_W\), still working to \(O\left(\frac{1}{\Lambda^2}\right)\). We write our result in terms of effective fermion couplings as

\[ \Gamma(Z \to f \bar{f}) = \frac{G_\mu M_Z^2}{6\sqrt{2}\pi} N_c (g_f^f)^2 \tag{15} \]

where \(i = L, R\) indicates the fermion helicity and we neglect fermion masses. For a fermion with charge \(Q_f\) and weak isospin \(T_3^f\), the effective coupling is

\[ g_f^f = (g'_f)_{SM} \left[ 1 + \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2 \right] \left[ -0.23 C_{HWB} Q_f^4 + 1.5 T_3^f C_{HWB} Q_f^2 \right. \]
\[ \left. + ( -1.9 C_{HWB} T_3^f)^2 + 0.15 C_{HB} + 0.15 C_{HW} + 11.0 C_{HWB} + 0.19 C_W \right] Q_f^2 \]
\[ + T_3^f (-0.67 C_{HB} - 0.69 C_{HW} - 49.0 C_{HWB} - 0.85 C_W) Q_f + (T_3^f)^2 (0.0084 C_{HB} + 0.029 C_{HW} - 0.23 C_{HWB} + 0.032 C_W) \right\} , \tag{16} \]

where

\[ D(Q_f, T_3^f) = Q_f^4 - 8.7 T_3^f Q_f^2 + \left( 17 (T_3^f)^2 - 76 \right) Q_f^2 + 660 T_3^f Q_f - 1400 (T_3^f)^2 . \tag{17} \]

The relatively large size of the \(C_{HWB}\) coefficients is due to the fact that they contribute at tree level. For our numerical results we use,

\[
\begin{align*}
G_\mu &= 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2} \\
M_Z &= 91.1876 \pm 0.0021 \text{ GeV} \\
M_W &= 80.385 \pm 0.015 \text{ GeV} \\
M_H &= 125.09 \pm 0.21 \pm 0.11 \text{ GeV} \\
M_T &= 173.1 \pm 0.6 \text{ GeV} .
\end{align*}
\]
The coefficients of all other operators are set to zero. For comparison, we show bounds on each of the operators from planes of the coefficients of these operators along with that of \( O \) constrain all of the operators in Eq. 18, we focus on the implications of our calculation for the operators \( LEP \) measurements of the quantities \( (g) \) on the \( Z \) contributions of \( \Delta g \) contribute at tree level are shown for comparison.

In particular, the SM fermion vertex couplings are

\[
\begin{align*}
g_L^g &= (g_L^g)_{SM} \left[ 1 + \delta g_L^{Z^u} \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2 \right] \left\{ -6.0 \cdot 10^{-6} C_{HB} - 2.1 \cdot 10^{-5} C_{HW} + 1.6 \cdot 10^{-4} C_{HWB} - 2.3 \cdot 10^{-5} C_W \right\} \\
g_L^z &= (g_L^z)_{SM} \left[ 1 + \delta g_L^{Z^x} \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2 \right] \left\{ 0.0019 C_{HB} + 0.0019 C_{HW} + 0.043 C_{HWB} + 0.0023 C_W \right\} \\
g_R^g &= (g_R^g)_{SM} \left[ 1 + \delta g_R^{Z^u} \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2 \right] \left\{ -0.0020 C_{HB} - 0.0020 C_{HW} - 0.033 C_{HWB} - 0.0025 C_W \right\} \\
g_R^z &= (g_R^z)_{SM} \left[ 1 + \delta g_R^{Z^x} \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2 \right] \left\{ 9.3 \cdot 10^{-4} C_{HB} + 9.3 \cdot 10^{-4} C_{HW} + 0.021 C_{HWB} + 0.0011 C_W \right\} \\
g_L^b &= (g_L^b)_{SM} \left[ 1 + \delta g_L^{Z^d} \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2 \right] \left\{ 3.7 \cdot 10^{-4} C_{HB} + 3.6 \cdot 10^{-4} C_{HW} + 0.0080 C_{HWB} + 4.5 \cdot 10^{-4} C_W \right\} \\
g_R^b &= (g_R^b)_{SM} \left[ 1 + \delta g_R^{Z^d} \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2 \right] \left\{ -0.0020 C_{HB} - 0.0020 C_{HW} - 0.034 C_{HWB} - 0.0025 C_W \right\} \\
\end{align*}
\]

For \( b_L \), the coefficient in front of \( C_W \) is \( 2.7 \cdot 10^{-4} \) rather than \( 4.5 \cdot 10^{-4} \) because of top mass effects. The tree level contributions of \( C_{HWB} \) are contained in the \( \delta g_{L,R}^{Z^f} \) contributions as given in Eq. 18.

These effective couplings are bounded by LEP measurements at the Z pole. We proceed to take the limits of [17] on the Z-fermion couplings to constrain the SMEFT operators. We minimize a \( \chi^2 \) function constructed using the LEP measurements of the quantities \( (g_L^g, g_L^z, g_R^g, g_R^z, g_L^b, g_R^b) \) and their correlations. While Z pole measurements constrain all of the operators in Eq. 18, we focus on the implications of our calculation for the operators \( O_W, O_{HW} \) and \( O_{HWB} \), which do not contribute to Z decay at tree level. In Fig. 1, we show the resulting 90% CL limits in 2-dimensional planes of the coefficients of these operators along with that of \( O_{HWB} \), which affects electroweak couplings at tree level. The coefficients of all other operators are set to zero. For comparison, we show bounds on each of the operators from

FIG. 1: Limits on SMEFT operators that contribute to Z decay at one loop. The left (right) plot shows bounds in the \( O_W - O_{HWB} \) (\( O_{HB} - O_{HW} \)) plane, with all other operators set to zero. Bounds from processes to which the operators contribute at tree level are shown for comparison.
processes to which they contribute at tree level. For \( \mathcal{O}_W \), we use the limits of \([23]\) obtained by using 8 TeV LHC gauge boson pair production in leptonic final states \([15, 51]\). For \( \mathcal{O}_{HB} \) and \( \mathcal{O}_{HW} \), we use limits \([15]\) from the calculation of \( H \rightarrow \gamma \gamma \) \([12, 14, 18]\) in the SMEFT, as compared to measurements of \( H \rightarrow \gamma \gamma \) at Run 1 and 2 of the LHC \([52, 54]\). \( \mathcal{O}_{HW} \) corresponds to the oblique parameter \( S \) \([53, 56]\), whose limit we take from the Glitter collaboration \([57]\). The existing bounds in Fig. 1 are stronger than those that we obtain directly from \( Z \) pole measurements. Nevertheless, they provide complementary information, and in particular in the left panel the interplay between the limits on \( C_W \) and \( C_{HWB} \) demonstrates the power of electroweak precision measurements to constrain couplings that only contribute at loop level. In the case of the operators \( C_{HB} \) and \( C_{HW} \) which directly affect \( H \rightarrow \gamma \gamma \), Higgs precision is already significantly more effective than \( Z \) pole measurements in setting limits, due to the loop suppression of these operators’ contributions to \( Z \) decay. We have included further details of our fit procedure as well as the numerical values of the existing experimental bounds in Appendix D.

IV. CONCLUSIONS

Precision measurements of electroweak physics will eventually necessitate higher order calculations of BSM contributions. The SMEFT framework takes a general approach to potential new UV physics by parametrizing its effects in terms of higher dimension operators involving the SM fields. In this work, we have furthered the applicability of the SMEFT to probe new physics by considering the one loop corrections to \( Z \) decay from operators which contribute to gauge boson production.

While the contributions of the operators \( \mathcal{O}_W \), \( \mathcal{O}_{HB} \) and \( \mathcal{O}_{HW} \) are small relative to those of the operators that modify the \( Z \) coupling to fermions at tree level, the relative size of all of the SMEFT operators is fixed by the new physics. In particular, integrating out a heavy SM singlet scalar could naturally give these operators without changing the leading \( Z \) couplings to the fermions \([53]\). In such a scenario, it would be essential to have the higher order contributions of the BSM physics to all possible processes. In this regard our calculation provides a useful prediction, relating the effects of new physics in \( Z \) decay to those in other electroweak processes provided the states responsible for deviations from the SM are heavy enough to be integrated out.

A full calculation of \( Z \) decay at one loop in the SMEFT would provide even more complete information about the influence of higher dimensional operators on \( Z \) physics. With this as well as other higher order calculations of electroweak processes, in the future a global fit at NLO in the SMEFT could be performed to bound the sizes of all possible dimension-6 SMEFT operators.

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Appendix A: UV poles

The cancellation of UV poles follows from the individual contributions: Numerically with \( \Lambda = 1 \) TeV, the pieces are as follows.

\[
\frac{\partial M_0}{\partial C_{HWB}} \delta C_{HWB} = \frac{1}{\epsilon} \left\{ Q_f \left( (2.7 \times 10^{-5}) C_{HB} + (2.7 \times 10^{-5}) C_{HW} + (4.6 \times 10^{-4}) C_{HWB} + (2.6 \times 10^{-5}) C_W \right) \right\} + \mathcal{O}(\epsilon^0)
\]

\[
\frac{\partial M_0}{\partial G_\mu} \delta G_\mu = \frac{1}{\epsilon} \left\{ (2.1 \times 10^{-5}) \left( Q_f (C_{HWB} + 8.8) - 40 T^f_3 \right) (\xi - 5.5) \right\} + \mathcal{O}(\epsilon^0)
\]

\[
\frac{\partial M_0}{\partial M^2_Z} \delta M^2_Z = \frac{1}{\epsilon} \left\{ Q_f \left( (-1.4 \times 10^{-4}) \xi C_W - (3.2 \times 10^{-4}) C_{HW} - (6.7 \times 10^{-4}) C_{HWB} + (7.7 \times 10^{-4}) C_W \right) \right\} + \mathcal{O}(\epsilon^0)
\]
\[
\frac{\partial M_0}{\partial M_W} \delta M_W = \frac{Q_f}{\epsilon} \left\{ \begin{array}{l}
(3.7 \times 10^{-5}) \xi \left[ C_{HW} - (2.3 \times 10^{-4}) C_{HW B} + (1.9 \times 10^{-4}) C_W + C_{HW} \left( (3.5 \times 10^{-5}) \xi - 5.2 \times 10^{-5} \right) \\
- (1.7 \times 10^{-4}) \xi + 1.3 \times 10^{-3} \right] + \mathcal{O}(\epsilon^0)
\end{array} \right.
\]

The sum vanishes for any given fermion.

Appendix B: 2-point functions

In this appendix, we show the two-point functions in \( R_{\xi} \) gauge due to the SMEFT operators that also contribute to gauge boson pair production. Previous results for the gauge boson two-point functions in other operator bases appear in \[ 51, 59].

In \( D = 4 - 2\epsilon \) dimensions, the two-point function for a massless fermion with weak isospin \( T^I_3 \) and charge \( Q_f \) is

\[
\Sigma(p) = \frac{1}{8\pi^2} \left\{ \frac{2\sqrt{2} A_0(M_W^2) G_{\mu T^I_3} \left( (D - 2) M_W^2 - p^2 \right)}{p^2} + 2\sqrt{2} A_0(M_W^2) \xi G_{\mu T^I_3} \right. \\
+ \frac{1}{M_Z^2 p^2} \left( A_0(M_Z^2) \left( (D - 2) M_Z^2 - p^2 \right) (M_W^2 Q_f + M_W^2 (T^I_3 - Q_f)) \left( \sqrt{2} G_{\mu} \left( M_W^2 Q_f + M_W^2 (T^I_3 - Q_f) \right) \right) \\
+ \frac{2M_W Q_f C_{HW B} \sqrt{M_Z^2 - M_W^2}}{M_Z^2 p^2} \\
- \frac{A_0(M_Z^2 \beta) (D - 2) M_W^2 Q_f}{M_Z^2 p^2} \left( \sqrt{2} G_{\mu} (M_W - M_Z) (M_W + M_Z) + 2M_W C_{HW B} \sqrt{M_Z^2 - M_W^2} \right) \right. \\
\left. + \frac{A_0(M_Z^2 \xi) \left( M_W^2 Q_f + M_W^2 (T^I_3 - Q_f) \right)}{M_Z^2 p^2} \left( \sqrt{2} G_{\mu} \left( M_W^2 Q_f + M_W^2 (T^I_3 - Q_f) \right) + 2M_W Q_f C_{HW B} \sqrt{M_Z^2 - M_W^2} \right) \right\}
\]
We have regulated IR divergences with a photon of mass $M$, and use standard FeynCalc notation [40] for the Passarino-Veltman functions.

This leads to the wave function renormalization

$$Z_f = \frac{1}{8\pi^2\epsilon} \left( \frac{1}{M_Z} \left( Q_f^2 \left( \sqrt{2G_{\mu}(M_W - M_Z)(M_W + M_Z) + 2M_WC_{\text{HWB}}\sqrt{M_Z^2 - M_W^2}} \right) (2\xi \epsilon(M_W^2 - \log(\beta) - \log(\xi)) + M_Z^2 \log(M_Z^2/\mu^2) + \log(\pi\xi)) + 2M_W^2(\xi - 1) + M_Z^2((2(\gamma - 1)\xi + 3)\epsilon - 2\xi) \right) 
+ 2Q_fT_3^f \left( \sqrt{2G_{\mu}(M_W - M_Z)(M_W + M_Z) + 2M_WC_{\text{HWB}}\sqrt{M_Z^2 - M_W^2}} \right) (2\epsilon(2 \log(M_Z^2/\mu^2) + \log(\pi\xi)) - 2\xi
+(2(\gamma - 1)\xi + 3)\epsilon) - \sqrt{2G_{\mu}(M_W - M_Z)(M_W + M_Z) + 2M_WC_{\text{HWB}}\sqrt{M_Z^2 - M_W^2}} \right) (2\epsilon(2 \log(M_Z^2/\mu^2) + \log(\pi\xi)) + 2\xi((\gamma - 1)\epsilon - 1) + 3\epsilon) + 4M_W^2\xi\epsilon \log(M_W^2/\mu^2)
+ 2M_Z^2\xi\epsilon \log(M_Z^2/\mu^2)) \right)
$$

(B2)

For the $b_L$, there are corrections proportional to the top mass, leading to an additional wave function renormalization which in Feynman gauge is

$$Z_{\chi_L} = Z_f \left( Q_f = -\frac{1}{3}, T_3^f = -\frac{1}{2} \right) + \frac{G_{\mu}M_T^2}{16\pi^2\epsilon} \left( 2\epsilon \log \left( \frac{m_t^2}{\mu^2} \right) - 2\gamma\epsilon + \epsilon - 2\epsilon \log \pi + 2 \right)
$$

(B3)

The transverse $W$ two-point function is

$$\Pi_{WW}^T(p^2) = \frac{1}{16p^2\pi^2} \left( -\frac{18\sqrt{2}B_0(p^2, 0, 0)(D - 2)G_{\mu}M_W^4p^4}{D - 1} - \frac{1}{(D - 1)M_Z^2} \left( C_0(0, p^2, p^2, 0, 0, M_W^2) \left( M_W^2 - p^2 \right) \right) \right) \times \left( \sqrt{2G_{\mu}(M_W - M_Z)(M_W + M_Z) + 2M_W\sqrt{M_Z^2 - M_W^2}} \right) (\xi - 1) \left( p^2 - M_W^2 \right)\right)^2
$$

$$= \frac{6\sqrt{2}A_0(M_T^2)G_{\mu}M_W^2(M_T^2 - (D - 2)p^2) + 6\sqrt{2}B_0(p^2, 0, M_T^2)G_{\mu}M_W^4(M_T - p)(M_T + p)(M_T^2 + (D - 2)p^2)}{D - 1} \left( B_0(p^2, M_W^2, M_W^2)M_W^2 \left( 8(D - 1) \left( -M_W^2 + M_T^2 + p^2 \right) \right) \right)
$$
\[
+ \sqrt{2}G_{\mu} \left( p^4 - 2 (M_{W}^2 + (3 - 2D)M_{W}^3) p^2 + (M_{W}^2 - M_{W}^3)^2 \right) \right) + \frac{1}{D - 1} \left( A_0(M_{W}^2) \left( \sqrt{2}G_{\mu}M_{W}^2 (M_{W}^2 - M_{W}^3) \right) \right)
- (D - 2)p^2 - 2(D - 1)p^2 (4M_{W}^2 + p^2) C_{HWB} \right) - \frac{1}{D - 1} \left( B_0(p^2, M_{W}^4, M_{W}^2)M_{W}^2 (M_{W}^2 + M_{Z} - p)(M_{W} + M_{Z} + p) \right) \left( (M_{W} - M_{Z})^2 - p^2 \right) \left( 36\sqrt{2}(D - 2)\sqrt{G_{\mu}}M_{W}^2M_{Z}^2 C_{W} p^2 + \sqrt{2}G_{\mu} M_{W} (M_{W}^4 + 2(D - 3) (M_{Z}^2 + p^2) M_{W}^2 + p^4 + (2D - 3) M_{Z}^2 p^2) C_{HWB} \right) \right) + \frac{1}{(D - 1)M_{Z}^2} \left( C_0(0, p^2, p^2, 0, M_{W}^2) (M_{W}^2 - p^2)^2 (M_{W}^4 + 2(D - 3) p^2 M_{W}^2 + p^4) \left( \sqrt{2}G_{\mu} (M_{Z}^2 - M_{W}^2) - 2M_{W} \sqrt{M_{Z}^2 - M_{W}^2} C_{HWB} \right) (\xi - 1) \right)
+ \frac{1}{(D - 1)M_{Z}^2} \left( B_0(p^2, M_{W}^2, M_{Z}^2)M_{W}^2 (M_{W}^2 - p^4) \left( \sqrt{2}G_{\mu} M_{W}^2 + 2\sqrt{M_{Z}^2 - M_{W}^2} C_{HWB} \right) \times \left( p^4 - 2 (M_{W}^2 + M_{Z}^2) \xi p^2 + (M_{W}^2 - M_{Z}^2)^2 \xi^2 \right) \right) + \frac{1}{(D - 1)M_{Z}^2} \left( B_0(p^2, M_{W}^2, M_{Z}^2) M_{W}^2 (M_{W}^2 - p^2)^2 \times \left( \sqrt{2}G_{\mu} M_{W}^2 + 2\sqrt{M_{Z}^2 - M_{W}^2} C_{HWB} \right) \left( M_{W}^4 + ((4D - 6)p^2 - 2M_{Z}^2) \xi M_{W}^2 + (p^2 - M_{Z}^2) \xi^2 \right) \right) - \frac{1}{D - 1} \left( A_0(M_{W}^2) \xi \left( 2 \left( (2D - 1) C_{HWB} p^4 + M_{W} \sqrt{M_{Z}^2 - M_{W}^2} C_{HWB} p^2 - M_{W} \sqrt{M_{Z}^2 - M_{W}^2} C_{HWB} \right) \right) - \frac{1}{(D - 1)M_{Z}^2} \left( 2B_0(p^2, 0, M_{W}^2) \times \left( 18\sqrt{2}(D - 2)\sqrt{G_{\mu}} M_{W} (M_{W} - M_{Z}) (M_{W} + M_{Z}) (M_{W} - p)^2 C_{W} (M_{W} + p) \right) \right.

\left. + \sqrt{2}G_{\mu} (M_{W} - M_{Z}) (M_{W} + M_{Z}) (M_{W}^4 + p^2) \left( (2D + \xi - 4) M_{W}^4 - 2p^2 (4D + \xi - 6) M_{W}^2 + p^4 (2D + \xi - 4) \right) \right)

+ M_{W} \sqrt{M_{Z}^2 - M_{W}^2} (M_{W}^2 + p^2) C_{HWB} \left( (2D + 2\xi - 5) M_{W}^4 - 2p^2 (6D + 2\xi - 9) M_{W}^2 + p^4 (2D + 2\xi - 5) \right) \right)

- \frac{1}{(D - 1)M_{Z}^2} \left( 2B_0(p^2, 0, M_{W}^2) \xi (M_{W} - p)^2 (\xi^2 + 2\xi + 2D - 3) M_{W}^2 - p^4 (2D + \xi - 4) \right) - M_{W} \sqrt{M_{Z}^2 - M_{W}^2} C_{HWB} \left( (\xi - 2) \xi M_{W}^2 + 2p^2 (\xi^2 + D(\xi + 2) - 3) M_{W}^4 + p^4 (2D + 2\xi - 5) \right) \right)

+ \frac{1}{(D - 1)M_{Z}^2} \left( A_0(M_{W}^2) \left( \sqrt{2}G_{\mu} ((\xi + 1) M_{W}^4 + (4D - 2)p^2 - 2M_{Z}^2) \xi M_{W}^2 \right) \right. \left. + 2M_{W}^2 (2D - 1) M_{Z}^2 \left( 0, p^2, p^2, 0, M_{W}^2 \right) \left( \sqrt{2}G_{\mu} M_{W}^2 (M_{W}^2 - p^4) C_{W} p^4 \right) + 2M_{W} \left( 3 - 2D \right) \sqrt{M_{Z}^2 - M_{W}^2} C_{HWB} M_{W}^2 + (4D - 1)p^2 C_{HWB} M_{W} + (3 - 2D) \sqrt{M_{Z}^2 - M_{W}^2} p^2 C_{HWB} \right) \left. + \sqrt{2}G_{\mu} (\xi + 1) M_{W}^4 - M_{H} M_{W}^2 (M_{Z}^2 + 2p^2 (2D - 3) D + \xi + 3) \right) M_{W}^2 + p^4 (4D + \xi - 7) \right) \right)

- \frac{1}{(D - 1)M_{Z}^2} \left( A_0(M_{W}^2) \left( 36\sqrt{2}(D - 2)\sqrt{G_{\mu}} M_{W}^2 (M_{W}^2 - M_{Z}^2) C_{W} p^2 + \sqrt{2}G_{\mu} M_{W}^2 (\xi + 1) M_{W}^4 + 2 \left( 2D - 1 \right) M_{Z}^2 + p^2) M_{Z}^2 \xi M_{W}^2 + \left( (9 - 4D) M_{W}^2 + 2p^2 (2D - 2)^2 + \xi \right) M_{Z}^2 - p^4 (4D + \xi - 7) \right) M_{W}^4 \right)
- \frac{1}{(D - 1)M_{Z}^2} \left( (\xi + 1) M_{W}^4 - M_{H} M_{W}^2 (M_{Z}^2 + 2p^2 (2D - 3) D + \xi + 3) \right) M_{W}^2 + p^4 (4D + \xi - 7) \right) \right)

- \frac{1}{(D - 1)M_{Z}^2} \left( A_0(M_{W}^2) \left( 36\sqrt{2}(D - 2)\sqrt{G_{\mu}} M_{W}^2 (M_{W}^2 - M_{Z}^2) C_{W} p^2 + \sqrt{2}G_{\mu} M_{W}^2 (\xi + 1) M_{W}^4 + 2 \left( 2D - 1 \right) M_{Z}^2 + p^2) M_{Z}^2 \xi M_{W}^2 + \left( (9 - 4D) M_{W}^2 + 2p^2 (2D - 2)^2 + \xi \right) M_{Z}^2 - p^4 (4D + \xi - 7) \right) M_{W}^4 \right)
- \frac{1}{(D - 1)M_{Z}^2} \left( (\xi + 1) M_{W}^4 - M_{H} M_{W}^2 (M_{Z}^2 + 2p^2 (2D - 3) D + \xi + 3) \right) M_{W}^2 + p^4 (4D + \xi - 7) \right) \right)
\[\Pi_{zz}^\gamma(p^2) = \frac{1}{48\pi^2} \left( -\frac{1}{(D-1)M_Z^2} (B_0(p^2, M_\gamma^2, M_{\gamma}^2, \xi) M_W (M_W - p)(M_W + p) - \sqrt{G}_\mu M_W (M_W^2 - 2M_Z^2 + p^2) (M_Z^2 + (4D - 6)p^2 - 2M_W^2) (M_Z^2 + (p^2 - M_{\gamma}^2)^2) - 2\sqrt{M_Z^2 - M_W^2} C_{HB}(\xi^2 M_W^2 - \xi (M_Z^2 (\xi + 2) - p^2 (\xi - 2)) M_W^4 + ((2\xi + 1)M_Z^2 + 2p^2 (-\xi + D(\xi + 2) - 3)M_Z^2 + p^4 (1 - 2\xi)) M_W^2 - M_Z^2 + p^6 + (2D - 5)M_Z^2 p^4 + 3(3 - 2D)M_Z^2 p^2) \right) \right) \]
\[
\delta M_Z^2 = \Pi_{ZZ}^T(M_Z^2) = \dfrac{1}{48\pi^2}\left( -\dfrac{1}{D-1}\left(3A_0(M_H^2)\left(\sqrt{2}G_\mu \left( M_H^2 - (D-1)M_Z^2\right) - 10(D-1)\left(M_W^{CHWB}\sqrt{M_Z^2 - M_W^2} + M_W^2(C_{HW} - C_{HB}) + M_Z^2CHWB\right)\right) - \dfrac{1}{(D-1)M_Z^2}\left(2A_0(M_H^2)(D-2)\left(\sqrt{2}G_\mu \left(32M_W^2 - 40M_W^2M_Z^2 + 17M_Z^2\right) + 8M_W^{CHWB}\left(8M_Z^2 - 5M_W^2\right)\sqrt{M_Z^2 - M_W^2} - M_W^2\right) - \dfrac{1}{(D-1)M_Z^2}\left(6A_0(M_H^2)\left(36\sqrt{2}(D-2)\sqrt{C_\mu M_W^2M_Z^2C_W + \sqrt{2}G_\mu (2M_Z^2)\xi - 2p^2(2D-3)(D + \xi + 3)M_W^2 + (-\xi - 1)M_Z^2 - 2p^2M_Z^2 + p^2(4D + \xi - 7)) M_W^2 + M_Z^2p^2\right)\right) + 3A_0(M_H^2)\left(\sqrt{2}G_\mu \left(M_H^2 - 2M_Z^2\right) + 8(D-1)\left(M_W^{CHWB}\sqrt{M_Z^2 - M_W^2} + M_W^2(C_{HW} - C_{HB}) + M_Z^2CHWB\right)\right)\right)\right) + \dfrac{1}{D-1}\left(3A_0(M_H^2)\left(\sqrt{2}G_\mu \left( M_H^2 - 2M_Z^2\right) + 8(D-1)\left(M_W^{CHWB}\sqrt{M_Z^2 - M_W^2} + M_W^2(C_{HW} - C_{HB}) + M_Z^2CHWB\right)\right)\right)\right) + B_0(M_Z^2, 0, 0)(D-2)\left(\sqrt{2}G_\mu \left(160M_W^2 - 200M_W^2M_Z^2 + 103M_Z^2\right) + 40M_W^{CHWB}\left(8M_W^2 - 5M_Z^2\right)\sqrt{M_Z^2 - M_W^2}\right) + \dfrac{1}{D-1}\left(3A_0(M_H^2)\left(\sqrt{2}G_\mu \left(M_H^2 - 2M_Z^2\right) + 8(D-1)\left(M_W^{CHWB}\sqrt{M_Z^2 - M_W^2} + M_W^2(C_{HW} - C_{HB}) + M_Z^2CHWB\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)
\]

Which yields the mass shift

\[
\delta M_Z^2 = \Pi_{ZZ}^T(M_Z^2)
\]
\[ \delta Z = -\frac{\partial \tilde{Z}_Z(p^2)}{p^2} \bigg|_{p^2 = M_Z^2} = \frac{1}{48\pi^2} \left( 3\sqrt{2}A_0(M_Z^2)G_\mu(M_Z^2 - M_H^2) + 6A_0(M_Z^2\xi) \right) \left( (C_{HW} - C_{HB})M_W + \sqrt{M_Z^2 - M_H^2}C_{HWB}M_W + M_Z^2C_{HB} \right) M_Z^2 \]

\[ + \frac{1}{(D-1)M_Z^2} (3b_0(M_Z^2, M_H^2, M_\tau^2)) \left( \sqrt{2}G_\mu \left( 2(64M_W^2 - 80M_H^2M_W^2 + (34 - 9D)M_H^2) M_H^2 + (D - 2)M_Z^2 (32M_W^2 - 40M_H^2M_W^2 + 17M_Z^2) \right) + 8M_W (8M_W^2 - 5M_Z^2) \sqrt{M_Z^2 - M_H^2} (4M_T^2 + (D - 2)M_Z^2) C_{HWB} \right) \]

\[ + \frac{1}{(D-1)M_Z^2} \left( 3a_0(M_Z^2) \left( \sqrt{2}G_\mu (M_H - M_Z) (M_H + M_Z) + 2(D-1) \left( (C_{HW} - C_{HB})M_W \right) \right) \sqrt{M_Z^2 - M_H^2}C_{HWB}M_W + M_Z^2C_{HB} \right) \]

\[ + \frac{1}{(D-1)M_Z^2} \left( \sqrt{M_Z^2 - M_H^2}C_{HWB}M_W + M_Z^2C_{HB} \right) \bigg) + \frac{1}{D-1} \left( 3b_0(M_Z^2, M_H^2, M_\tau^2) (8(D-1)M_Z^2 (C_{HW} - C_{HB})M_W^2 \right) \]

\[ + \frac{1}{(D-1)M_Z^2} \left( \sqrt{2}G_\mu (M_H - 4M_Z^2 M_H^2 + 4(D-1)M_Z^2) - 8(D-1) (M_H - 2M_Z^2) \left( (C_{HW} - C_{HB})M_H^2 \right) \right) \]

\[ + \frac{1}{(D-1)M_Z^2} \left( \sqrt{2}G_\mu (2M_W^2 - 2M_H^2) C_W M_W^2 + 4\sqrt{M_Z^2 - M_H^2} (2D-1)M_W^2 + (2D - 3)M_Z^2) C_{HWB}M_W \right) \]

\[ + \frac{1}{(D-1)M_Z^2} \left( \sqrt{2}G_\mu (4(D-1)M_W^4 + 4(2D - 3)M_Z^2 M_W^2 + M_Z^2) \right) \bigg) + \frac{1}{D-1} \left( 3b_0(M_Z^2, M_W^2, M_\tau^2) (M_Z^2 - 4M_W^2) \right) \]

\[ + \frac{1}{(D-1)M_Z^2} \left( 12b_0(M_Z^2, M_W^2, M_\tau^2) \left( \sqrt{2}G_\mu (M_W - M_Z)(M_W + M_Z) ((\xi - 1)^2 M_W^2 + 2M_Z^2(2D - \xi - 3)M_W^2 \right) \right) \]
\[
\Pi_{\gamma Z}(p^2) = \frac{1}{48\pi^2} \left( \frac{3\sqrt{2}B_0(p^2, M_{\gamma W}^2, M_{Z}^2)G_\mu \sqrt{M_{Z}^2 - M_{\gamma W}^2} (4M_{\gamma W}^2 - p^2)}{(D - 1)M_{Z}^2} \right) M_{Z}^2
+ \frac{1}{(D - 1)M_{Z}^2} \left( 12A_0(M_{\gamma W}^2)\sqrt{M_{Z}^2 - M_{\gamma W}^2} (5M_{Z}^2 - 8M_{\gamma W}^2) \right) M_{Z}^2 \frac{3A_0(M_{Z}^2)C_{HB} - C_{HW}}{M_{Z}^2 - M_{\gamma W}^2} M_{Z}^2 \left( 2C_{HB}M_{Z}^2 + 2\sqrt{M_{Z}^2 - M_{\gamma W}^2} (C_{HB} - C_{HW})M_{W} - M_{Z}^2 C_{HW}B \right) \right. \\
+ \left. \frac{1}{(D - 1)M_{Z}^2} \left( 20B_0(p^2, 0, 0)(D - 2)M_{W} \right) \left( \sqrt{2}G_\mu \sqrt{M_{Z}^2 - M_{\gamma W}^2} \right) \left( 2C_{HB}M_{Z}^2 + 2\sqrt{M_{Z}^2 - M_{\gamma W}^2} (C_{HB} - C_{HW})M_{W} - M_{Z}^2 C_{HW}B \right) \right)
+ \frac{1}{(D - 1)M_{Z}^2} \left( 6A_0(M_{Z}^2) \right) \left( 2C_{HB}M_{Z}^2 + 2\sqrt{M_{Z}^2 - M_{\gamma W}^2} (C_{HB} - C_{HW})M_{W} + M_{Z}^2 C_{HW}B \right)
+ \frac{1}{(D - 1)M_{Z}^2} \left( 3A_0(M_{Z}^2) \right) \left( 2C_{HB}M_{Z}^2 + 2\sqrt{M_{Z}^2 - M_{\gamma W}^2} (C_{HB} - C_{HW})M_{W} + M_{Z}^2 C_{HW}B \right)
+ \frac{1}{(D - 1)M_{Z}^2} \left( 4B_0(p^2, M_{Z}^2, M_{Z}^2)M_{W} \right) \left( \sqrt{2}G_\mu \sqrt{M_{Z}^2 - M_{\gamma W}^2} \right)
+ \frac{1}{(D - 1)M_{Z}^2} \left( 8A_0(M_{Z}^2)(D - 2)M_{W} \right) \left( \sqrt{2}G_\mu \sqrt{M_{Z}^2 - M_{\gamma W}^2} \right)
+ \frac{1}{(D - 1)M_{Z}^2} \left( 2B_0(p^2, M_{Z}^2, M_{Z}^2) \right) \left( 3B_0(p^2, M_{Z}^2, M_{Z}^2) \right) \left( 4M_{\gamma W}^2 - p^2 \right)
+ \frac{1}{(D - 1)M_{Z}^2} \left( 36\sqrt{2}(D - 2)\sqrt{G_\mu \sqrt{M_{Z}^2 - M_{\gamma W}^2}} \right) \left( 2C_{HB}M_{Z}^2 + 2\sqrt{M_{Z}^2 - M_{\gamma W}^2} (C_{HB} - C_{HW})M_{W} - \sqrt{2}G_\mu \sqrt{M_{Z}^2 - M_{\gamma W}^2} \right)
+ \frac{1}{(D - 1)M_{Z}^2} \left( 3B_0(p^2, M_{Z}^2, M_{Z}^2) \right) \left( 4M_{\gamma W}^2 - p^2 \right)
+ \frac{1}{(D - 1)M_{Z}^2} \left( 6B_0(p^2, M_{Z}^2, M_{Z}^2) \right) \left( 6A_0(M_{\gamma W}^2) \right) \left( 2C_{HB}(\xi + 1)M_{W} + (3 - 2D)p^4 \right)
+ \frac{1}{(D - 1)M_{Z}^2} \left( 6B_0(p^2, M_{Z}^2, M_{Z}^2) \right) \left( 6A_0(M_{\gamma W}^2) \right) \left( 2C_{HB}(\xi + 1)M_{W} + (3 - 2D)p^4 \right)
\right)
\]

The \( \gamma - Z \) two-point function is

\[
\Pi_{\gamma Z}(p^2) = \frac{1}{48\pi^2} \left( \frac{3\sqrt{2}B_0(p^2, M_{\gamma W}^2, M_{Z}^2)G_\mu \sqrt{M_{Z}^2 - M_{\gamma W}^2} (4M_{\gamma W}^2 - p^2)}{(D - 1)M_{Z}^2} \right) M_{Z}^2
+ \frac{1}{(D - 1)M_{Z}^2} \left( 12A_0(M_{\gamma W}^2)\sqrt{M_{Z}^2 - M_{\gamma W}^2} (5M_{Z}^2 - 8M_{\gamma W}^2) \right) M_{Z}^2 \frac{3A_0(M_{Z}^2)C_{HB} - C_{HW}}{M_{Z}^2 - M_{\gamma W}^2} M_{Z}^2 \left( 2C_{HB}M_{Z}^2 + 2\sqrt{M_{Z}^2 - M_{\gamma W}^2} (C_{HB} - C_{HW})M_{W} - M_{Z}^2 C_{HW}B \right) \right)
+ \frac{1}{(D - 1)M_{Z}^2} \left( 20B_0(p^2, 0, 0)(D - 2)M_{W} \right) \left( \sqrt{2}G_\mu \sqrt{M_{Z}^2 - M_{\gamma W}^2} \right) \left( 2C_{HB}M_{Z}^2 + 2\sqrt{M_{Z}^2 - M_{\gamma W}^2} (C_{HB} - C_{HW})M_{W} - M_{Z}^2 C_{HW}B \right)
+ \frac{1}{(D - 1)M_{Z}^2} \left( 6A_0(M_{Z}^2) \right) \left( 2C_{HB}M_{Z}^2 + 2\sqrt{M_{Z}^2 - M_{\gamma W}^2} (C_{HB} - C_{HW})M_{W} + M_{Z}^2 C_{HW}B \right)
+ \frac{1}{(D - 1)M_{Z}^2} \left( 3A_0(M_{Z}^2) \right) \left( 2C_{HB}M_{Z}^2 + 2\sqrt{M_{Z}^2 - M_{\gamma W}^2} (C_{HB} - C_{HW})M_{W} + M_{Z}^2 C_{HW}B \right)
+ \frac{1}{(D - 1)M_{Z}^2} \left( 4B_0(p^2, M_{Z}^2, M_{Z}^2)M_{W} \right) \left( \sqrt{2}G_\mu \sqrt{M_{Z}^2 - M_{\gamma W}^2} \right)
+ \frac{1}{(D - 1)M_{Z}^2} \left( 8A_0(M_{Z}^2)(D - 2)M_{W} \right) \left( \sqrt{2}G_\mu \sqrt{M_{Z}^2 - M_{\gamma W}^2} \right)
+ \frac{1}{(D - 1)M_{Z}^2} \left( 2B_0(p^2, M_{Z}^2, M_{Z}^2) \right) \left( 3B_0(p^2, M_{Z}^2, M_{Z}^2) \right) \left( 4M_{\gamma W}^2 - p^2 \right)
+ \frac{1}{(D - 1)M_{Z}^2} \left( 36\sqrt{2}(D - 2)\sqrt{G_\mu \sqrt{M_{Z}^2 - M_{\gamma W}^2}} \right) \left( 2C_{HB}M_{Z}^2 + 2\sqrt{M_{Z}^2 - M_{\gamma W}^2} (C_{HB} - C_{HW})M_{W} - \sqrt{2}G_\mu \sqrt{M_{Z}^2 - M_{\gamma W}^2} \right)
+ \frac{1}{(D - 1)M_{Z}^2} \left( 3B_0(p^2, M_{Z}^2, M_{Z}^2) \right) \left( 4M_{\gamma W}^2 - p^2 \right)
+ \frac{1}{(D - 1)M_{Z}^2} \left( 6B_0(p^2, M_{Z}^2, M_{Z}^2) \right) \left( 6A_0(M_{\gamma W}^2) \right) \left( 2C_{HB}(\xi + 1)M_{W} + (3 - 2D)p^4 \right)
+ \frac{1}{(D - 1)M_{Z}^2} \left( 6B_0(p^2, M_{Z}^2, M_{Z}^2) \right) \left( 6A_0(M_{\gamma W}^2) \right) \left( 2C_{HB}(\xi + 1)M_{W} + (3 - 2D)p^4 \right)
\right)
\]
\[(4D + \xi - 7)p^4 - M_Z^2p^2 + M_W^2 (M_Z^2(\xi - 1) - 2p^2(2(D - 3)D + \xi + 3)) + C_{HWB}(2p^2(4(D - 3)D + \xi + 7)
- M_Z^2(\xi - 1))M_W^4 - p^2 ((2(D^2 - 5) + \xi + 4)M_Z^2 + 2(2D - 3)p^2) M_W^4 + (2D - 3)M_Z^2p^4))\)
\]

which yields the on-shell mixing

\[\Pi_{\pi Z}(M_Z^2) = \frac{1}{48\pi^2} \left( -3\sqrt{2} B_0(M_Z^2, M_W^2, M_W^2, M_W^2, M_W, M_W, M_Z^2, M_Z^2, M_Z^2, M_Z^2) \rule{0pt}{2ex} \right) \]


\[\ \times \left( 2C_{HWB} M_W^2 + 2\sqrt{M_Z^2 - M_W^2}(C_{HB} - C_{HW}) M_W - M_Z^2 C_{HWB} \right) + 3A_0(M_Z^2) \left( 2C_{HWB} M_W^2 \right)
+ 2\sqrt{M_Z^2 - M_W^2}(C_{HB} - C_{HW}) M_W - M_Z^2 C_{HWB} + 6A_0(M_Z^2) \left( -2C_{HWB} M_W^2 \right)
+ 2\sqrt{M_Z^2 - M_W^2}(C_{HB} - C_{HW}) M_W + M_Z^2 C_{HWB} + 6B_0(M_Z^2, M_W^2, M_Z^2) \left( M_Z^2 - 2M_W^2 \right)
+ 2\sqrt{M_Z^2 - M_W^2}(C_{HB} - C_{HW}) M_W + M_Z^2 C_{HWB} + \frac{1}{\sqrt{D-1}} \left( 20B_0(M_Z^2, 0, 0)(D - 2)M_W \right)
\times \left( \sqrt{2} G_\mu \sqrt{M_Z^2 - M_W^2}(5M_Z^2 - 8M_W^2) + M_W(16M_W^2 - 13M_Z^2) C_{HWB} \right) \]

\[+ \frac{1}{\sqrt{D-1}M_Z^2} \left( 4B_0(M_Z^2, M_W^2, M_Z^2, M_W^2, M_W)(4M_W^2 + (D - 2)M_Z^2) \right) \left( \sqrt{2} G_\mu \sqrt{M_Z^2 - M_W^2}(5M_Z^2 - 8M_W^2) \right)
+ M_W(16M_W^2 - 13M_Z^2) C_{HWB} \right) + \frac{1}{\sqrt{D-1}M_Z^2} \left( 3B_0(M_Z^2, M_W^2, M_Z^2) \right) \left( 4M_W^2 - 2M_Z^2 \right)
\times \left( 36\sqrt{2}(D - 2)\sqrt{M_Z^2 - M_W^2}(12C_{HW}C_{HWB}M_W - 2(2M_W^2 - M_Z^2)(2(D - 1)M_W + (2D - 3)M_Z^2) C_{HWB}M_W \right.
\left. + \sqrt{2} G_\mu \sqrt{M_Z^2 - M_W^2}(4(D - 1)M_W + 4(2D - 3)M_Z^2 M_W^2 + M_Z^2) \right) \)
\]

\[+ \frac{1}{\sqrt{D-1}M_W M_Z^2} \left( 6B_0(M_Z^2, M_W^2, M_W, M_W)(M_W - M_Z)(M_W + M_Z) \right)(M_W C_{HWB}((\xi - 1)^2 M_W^2
+ 2(D - 2)M_Z^2(\xi + 1)M_W^2 + (3 - 2D)M_Z^2) - \sqrt{2} G_\mu \sqrt{M_Z^2 - M_W^2}(\xi - 1)^2 M_W^4 + 2M_Z^2(2D - \xi - 3)M_W^4
+ M_Z^4) \right) - \frac{1}{\sqrt{D-1}M_Z^2} \left( 6A_0(M_W^2)(-36\sqrt{2}(D - 2)\sqrt{M_Z^2 - M_W^2}(12C_{HW}C_{HWB}M_W - 2(2M_W^2 - M_Z^2)(2(D - 1)M_W + (2D - 3)M_Z^2) C_{HWB}M_W \right.
\left. + \sqrt{2} G_\mu M_W \sqrt{M_Z^2 - M_W^2}(M_W^4(4(D - 3)D + \xi + 7) - M_Z^4(4D + \xi + 8)) - C_{HWB}((8(D - 3)D + \xi + 15)M_W^4
- M_Z^4(4D^2 - 6D + \xi - 2) M_W^4 + (2D - 3)M_Z^4) \right) \right) + \frac{1}{\sqrt{D-1}M_Z^2} \left( 6A_0(M_W^2) \right) \left( \sqrt{2} G_\mu M_W \sqrt{M_Z^2 - M_W^2}(M_W^4(\xi - 1) - M_Z^4(2D + \xi - 4)) + (C_{HWB} - C_{HWB} \xi) M_W^4
+ M_Z^2 C_{HWB}(2D + \xi - 4)M_W^2 + 2(D - 1)M_Z^2 \sqrt{M_Z^2 - M_W^2}(C_{HB} - C_{HW}) M_W - (D - 2)M_Z^2 C_{HWB} \right) \right) \)

Appendix C: Vertex functions

The one loop amplitude for \( Z(p + p') \rightarrow f(p) \bar{f}(p') \), the decay of a Z boson to a pair of massless fermions with weak isospin \( T^f_3 \) and charge \( Q_f \), is

\[ \mathcal{M}_f = V \bar{u}(p) \gamma^8 (p + p') u(p') \]

\[ (C1) \]
where the vertex function is

\[
V = \frac{\sqrt{\gamma}}{4 \cdot 2^{3/4} M_W^3 \pi^2} \left( \sqrt{2} B_0(M_Z^2, M_W^2, \xi) M_W^3 T_3^f (M_Z^2 - 4 M_W^2 \xi) \frac{M_Z^2}{D - 1} + 2 \sqrt{2} C_0(M_Z^2, 0, 0, 0, M_Z^2)(D - 8)(Q_f M_W^2) + M_Z^2 (T_3^f - Q_f))^2 \right) \cdot (2 G_\mu \left( Q_f M_W^2 + M_Z^2 (T_3^f - Q_f) \right) + 3 \sqrt{2} M_W \sqrt{M_Z^2 - M_W^2} Q_f C_{HWB} M_Z^2 + 4 C_0(M_Z^2, 0, 0, 0, M_Z^2) Q_f^2 M_Z^2 \left( \sqrt{2} G_\mu (M_W - M_Z)(M_W + M_Z) \left( Q_f M_W^2 + M_Z^2 (T_3^f - Q_f) \right) \right)
\]

\[
+ M_W \sqrt{M_Z^2 - M_W^2} \left( 3 Q_f M_W^2 + M_Z^2 (2 T_3^f - 3 Q_f) \right) C_{HWB} \left( \beta (-D + 2 \beta + 8) + 2 M_Z^2 + 8 \sqrt{2} B_0(0, 0, M_Z^2)(Q_f M_W^2 + M_Z^2 (T_3^f - Q_f)) \right)
\]

\[
- 4 \sqrt{2} C_0(0, M_Z^2, 0, M_W^2, 0, 0, 0)(D - 2) \left( Q_f M_W^2 + M_Z^2 (T_3^f - Q_f) \right) + 3 \sqrt{2} M_W \sqrt{M_Z^2 - M_W^2} Q_f C_{HWB} M_Z^2 + 8 C_0(M_Z^2, 0, 0, 0, M_W^2) (2 M_W^2 - (D - 8) M_Z^2 M_W^2 + 2 M_Z^2 M_W^2) T_3^f + \left( \sqrt{2} G_\mu \left( M_Z^2 (Q_f - T_3^f) - M_W^2 (Q_f - 2 T_3^f) \right) - M_W \sqrt{M_Z^2 - M_W^2} (Q_f - 2 T_3^f) C_{HWB} \right) M_Z^2
\]

\[
+ \frac{1}{D - 2} \left( 4 C_0(M_Z^2, 0, 0, M_W^2, 0, 0, M_W^2) M_Z^2 T_3^f \left( 3 \sqrt{2}(3D - 8) \sqrt{G_\mu} M_W^2 C_{W_M} M_Z^2 + 2 \sqrt{2}(D - 2) G_\mu M_W \left( M_W^2 + 2 M_Z^2 \right) - (2D - 2) \sqrt{M_Z^2 - M_W^2} \left( M_W^2 + 2 M_Z^2 \right) \right) + \frac{1}{D - 2} \left( 4 \sqrt{2}(D - 3) \sqrt{G_\mu} M_W^2 C_{W_M} M_Z^2 + 2 \sqrt{2}(D - 2) G_\mu M_W \left( M_W^2 + 2 M_Z^2 \right) - (2D - 2) \sqrt{M_Z^2 - M_W^2} \left( M_W^2 + 2 M_Z^2 \right) \right) \times \left( \left( 2 Q_f - T_3^f \right) T_3^f + 1 \right) M_W^2 + \frac{1}{D - 2} \left( (2D - 2) \left( 2 M_W^2 - (D - 2) M_Z^2 M_W^2 + 2 M_Z^2 M_W^2 + (D - 3) M_Z^2 M_W^2 \right) - 16 B_0(0, 0, M_Z^2, M_W^2, 0, 0, 0) M_Z^2 T_3^f \left( \sqrt{2} G_\mu (M_W - M_Z)(M_W + M_Z) \left( Q_f M_W^2 + M_Z^2 (T_3^f - Q_f) \right) \right) + M_W \sqrt{M_Z^2 - M_W^2} \left( 3 Q_f M_W^2 + M_Z^2 (2 T_3^f - 3 Q_f) \right) C_{HWB} \left( \beta + 1 \right) M_Z^2 - 4 B_0(0, 0, M_Z^2, M_W^2, 0, 0, 0) M_Z^2 T_3^f \left( \sqrt{2} G_\mu (M_W - M_Z)(M_W + M_Z) \left( Q_f M_W^2 + M_Z^2 (T_3^f - Q_f) \right) \right) \times \left( \left( 3 Q_f M_W^2 + M_Z^2 (2 T_3^f - 3 Q_f) \right) C_{HWB} \right) \left( \xi - 1 \right) M_Z^2
\]

\[
+ \frac{1}{D - 1} \left( 2 B_0(M_Z^2, M_W^2, M_W^2, \xi) T_3^f \left( \sqrt{2} G_\mu (M_W - M_Z)(M_W + M_Z) \left( (\xi - 1)^2 M_W^2 + 2 M_Z^2 (2D - \xi - 3) M_W^2 + M_Z^2 \right) \right) + M_W \sqrt{M_Z^2 - M_W^2} C_{HWB} \left( (\xi - 1)^2 M_W^2 + 2 M_Z^2 (2D - \xi - 3) M_W^2 + M_Z^2 \right)
\]

\[
+ \left( 3 - 2D \right) M_Z^2 \right) \right) \right) + \frac{1}{D - 1} \left( 2 A_0(M_Z^2, M_W^2, \xi) T_3^f \left( M_W \sqrt{M_Z^2 - M_W^2} C_{HWB} \left( M_Z^2 - (D - 1) T_3^f + (4 - D) Q_f T_3^f \right) + 2D - 3 \right) - M_W^2 \left( (\xi - 1)^2 M_W^2 + 2 M_Z^2 (2D - \xi - 3) M_W^2 + M_Z^2 \right)
\]

\[
+ M_W \sqrt{M_Z^2 - M_W^2} (Q_f - 2 T_3^f) C_{HWB} \right) + \frac{M_W}{(D - 2)(D - 1)} \left( - 6 \sqrt{2}(D - 4) \sqrt{G_\mu} M_W^2 C_{W_M} M_Z^2 + (D - 2) M_Z^2 M_W^2 \left( \xi - 1 \right) M_Z^2
\]

\[
- (D - 2) \sqrt{M_Z^2 - M_W^2} C_{HWB} \left( (4D - (D - 3) M_W^2 + (2D - 3) M_Z^2) - \sqrt{2}(D - 2) G_\mu M_W \left( (4D - \xi - 3) M_W^2 \right) \right)
\]
\[
\begin{align*}
V &= \frac{\sqrt{G_\mu}}{216 \cdot 2^{3/4} \pi^{7/2} m^2} \bigg( -108 M_2^2 \left( 2 \sqrt{M_Z^2 - M_W^2} C_{\text{HWB}} C_{0} \left( M_2^2, 0, 0, M_2^2, M_2^2, M_2^2, M_2^2 \right) M_2^2 \\
+ 2 \sqrt{G_\mu} \left( C_0 \left( M_2^2, 0, 0, M_2^2, M_2^2, M_2^2, M_2^2 \right), M_2^2, M_2^2, M_2^2, M_2^2 \right) M_2^2 + B_0 \left( 0, M_2^2, M_2^2, M_2^2, M_2^2 \right) - M_2^2 C_1 \left( 0, M_2^2, 0, M_2^2, M_2^2, M_2^2 \right) \\
+ (D - 2) C_{00} \left( 0, M_2^2, 0, M_2^2, M_2^2, M_2^2, M_2^2 \right) M_2^2 + 2 \sqrt{M_Z^2 - M_W^2} C_{\text{HWB}} B_0 \left( 0, M_2^2, M_2^2 \right) \\
+ 2 D \sqrt{M_Z^2 - M_W^2} C_{\text{HWB}} C_{00} \left( 0, M_2^2, 0, M_2^2, M_2^2, M_2^2, M_2^2 \right) - 4 \sqrt{M_Z^2 - M_W^2} C_{\text{HWB}} C_{00} \left( 0, M_2^2, 0, M_2^2, M_2^2, M_2^2 \right) \\
+ 3 \sqrt{2} G_\mu M_2^2 C_W \left( C_0 \left( M_2^2, 0, 0, M_2^2, M_2^2, M_2^2, M_2^2 \right), M_2^2, M_2^2, M_2^2, M_2^2 \right) M_2^2 + B_0 \left( 0, M_2^2, M_2^2 \right) \\
+ (M_2^2 - 2 M_2^2) C_1 \left( 0, M_2^2, 0, M_2^2, M_2^2, M_2^2, M_2^2 \right) + (D - 4) C_{00} \left( 0, M_2^2, 0, M_2^2, M_2^2, M_2^2, M_2^2 \right) \right) M_2^2 \\
+ 18 M_2^2 \left( 4 \sqrt{2 (D - 6)} G_\mu C_0 \left( M_2^2, 0, 0, M_2^2, M_2^2, M_2^2, M_2^2 \right) M_2^2 + 4 D \sqrt{M_Z^2 - M_W^2} C_{\text{HWB}} C_{00} \left( M_2^2, 0, 0, M_2^2, M_2^2, M_2^2, M_2^2 \right) M_2^2 \\
- 24 \sqrt{M_Z^2 - M_W^2} C_{\text{HWB}} C_{0} \left( M_2^2, 0, 0, M_2^2, M_2^2, M_2^2, M_2^2 \right) M_2^2 \\
- \sqrt{G_\mu} \left( (4 (D - 6) M_Z^2 + (D + 2) M_Z^2) C_0 \left( M_2^2, 0, 0, M_2^2, M_2^2, M_2^2, M_2^2 \right) + 8 (D - 2) C_{00} \left( 0, M_2^2, 0, M_2^2, M_2^2, M_2^2, M_2^2 \right) \right) M_2^2 \\
- 4 D M_2^2 \sqrt{M_Z^2 - M_W^2} C_{\text{HWB}} C_{0} \left( M_2^2, 0, 0, M_2^2, M_2^2, M_2^2, M_2^2 \right) M_2^2 \\
+ 24 M_2^2 \sqrt{M_Z^2 - M_W^2} C_{\text{HWB}} C_{0} \left( M_2^2, 0, 0, M_2^2, M_2^2, M_2^2, M_2^2 \right) M_2^2 \\
- 8 M_2^2 \sqrt{M_Z^2 - M_W^2} C_{\text{HWB}} C_{0} \left( M_2^2, 0, 0, M_2^2, M_2^2, M_2^2, M_2^2 \right) M_2^2 \\
- 8 D \sqrt{M_Z^2 - M_W^2} C_{\text{HWB}} C_{00} \left( 0, M_2^2, 0, M_2^2, M_2^2, M_2^2, M_2^2 \right) M_2^2 + 16 \sqrt{M_Z^2 - M_W^2} C_{\text{HWB}} C_{00} \left( 0, M_2^2, 0, M_2^2, M_2^2, M_2^2, M_2^2 \right) M_2^2 \\
+ 4 \left( \sqrt{2} G_\mu \left( 4 M_2^2 - M_2^2 \right) + 4 M_2^2 \sqrt{M_Z^2 - M_W^2} C_{\text{HWB}} \right) B_0 \left( 0, M_2^2, M_2^2 \right) + (D - 6) \left( \sqrt{2} G_\mu \left( 4 M_2^2 - M_Z^2 \right) \right) \\
+ 4 M_2^2 \sqrt{M_Z^2 - M_W^2} C_{\text{HWB}} \right) B_0 \left( M_2^2, M_2^2, M_2^2 \right) + 2 \sqrt{2 G_\mu M_2^2 \left( 2 (D - 3) M_Z^2 + M_2^2 \right) C_0 \left( M_2^2, 0, 0, M_2^2, M_2^2, M_2^2, M_2^2 \right) \right) M_2^2 + 4 B_0 \left( 0, 0, M_2^2 \right) \\
+ (D - 2) C_{00} \left( 0, M_2^2, 0, M_2^2, M_2^2, M_2^2, M_2^2 \right) \left) \right) M_2^2 + 4 \left( 6 \sqrt{M_Z^2 - M_W^2} C_{\text{HWB}} M_2^4 \right) \\
+ \sqrt{2} G_\mu \left( M_W - M_Z \right) \left( M_W + M_Z \right) \left( 2 M_2^2 + M_2^2 \right) \right) \left( ( (D - 6) \beta - 2) C_0 \left( M_2^2, 0, 0, 0, M_2^2, \beta \right) M_2^2 + 4 B_0 \left( 0, 0, M_2^2 \right) \right) \\
+ (D - 6) B_0 \left( M_2^2, 0, 0, 0, 0, 0, 0 \right) \right) M_2^2 + 108 M_2^2 M_2^2 \left( 2 \sqrt{2} G_\mu M_W \left( M_W - M_Z \right) \left( M_W + M_Z \right) \right) \\
\end{align*}
\]

For the $b_L$, there are also top mass effects, and the vertex function in Feynman gauge is
TABLE I: LEP and SLD measurements of the effective $Z$ couplings to fermions.

| Parameter | SM prediction | Measurement | Correlations |
|-----------|---------------|-------------|--------------|
| $g^L_L$   | $0.50199 \pm 0.00020$ | $0.50075 \pm 0.00077$ | $1.00$      |
| $g^L_R$   | $-0.26919 \pm 0.00020$ | $-0.26939 \pm 0.00022$ | $-0.32$     |
| $g^L_t$   | $-0.42114 \pm 0.00045$ | $-0.4182 \pm 0.0015$ | $0.05$      |
| $g^L_b$   | $0.34674 \pm 0.00017$ | $0.3453 \pm 0.0036$ | $-0.02$     |
| $g^R_L$   | $0.23208 \pm 0.00018$ | $0.23186 \pm 0.00023$ | $0.25$      |
| $g^R_R$   | $0.077420 \pm 0.000052$ | $0.0962 \pm 0.0063$ | $0.00$      |
| $g^R_t$   | $-0.15470 \pm 0.00011$ | $-0.1580 \pm 0.0051$ | $0.00$      |

In this appendix, we show the measurements used in Section III to produce Figure 1.

LEP and SLD [17] measured the effective fermion couplings and correlations in Table I. We seek to minimize the quantity

$$ (x^2)_{\text{LEP}} = (\vec{g}_{\text{SMEFT}} - \vec{g}_{\text{exp}})^T V^{-1} (\vec{g}_{\text{SMEFT}} - \vec{g}_{\text{exp}}) $$

where $\vec{g} = (g^L_L, g^L_R, g^L_t, g^L_b, g^R_L, g^R_R, g^R_t)$ and $V$ is the covariance matrix constructed from the errors and correlations above. We use Eq. 18 together with the SM predictions of Table I to calculate $\vec{g}_{\text{SMEFT}}$. Since we set light fermion masses to zero in our SMEFT analysis, the effective couplings for the down (up) quark apply equally to the $b$ ($c$) quark, with the exception of the $b_L$, for which top quark corrections apply as specified below Eq. 18.

We compare our results to processes in which the SMEFT operators contribute at tree level. The limit of [28], set using LHC Run I data [48–51], is converted in our notation to

$$ -0.17 < C_W \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2 < 0.18 $$

We also show limits [18] from the SMEFT calculation of $H \rightarrow \gamma \gamma$ and current LHC Higgs measurements [52–54],

$$ \left| C_{\text{HH}} \left( \frac{1 \text{ TeV}}{\Lambda} \right) \right| < 0.001 $$

$$ \left| C_{\text{HW}} \left( \frac{1 \text{ TeV}}{\Lambda} \right) \right| < 0.02 $$
Finally, for $C_{HWB}$ we also use the Gfitter limit \[57\] of $S = 0.04 \pm 0.11$ to set the $2\sigma$ bound

\[-0.004 < C_{HWB} < 0.006\]  

(D4)
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