Dynamics of a Single Degree of Freedom Clutch Damper System with Multiple Discontinuous Nonlinearities

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Abstract. In this work, the dynamics of a clutch damper system with multiple nonlinearities modeled as a single degree freedom system is investigated numerically. The experimentally obtained torque displacement diagram shows the presence of preload, hysteresis and piecewise nonlinearities in the system. All the above said nonlinearities are discontinuous in nature and special solution techniques are required to solve the same. In this paper, the discontinuous functions are replaced by smooth functions to avoid the computational difficulties. The dynamics is investigated by plotting bifurcation diagrams with excitation frequency as the parameter. In the presence of multiple nonlinearities, it is found that the system exhibits complex dynamics in the form of periodic, quasiperiodic and chaotic solutions for different parameter values. It is found that the addition of mean load changes the symmetry associated with the system. The study provides better understanding on the dynamics of the system which is essential for a safe design.

1. Introduction

Modern design practices need weightless, high speed machineries. During the design process and operation, nonlinearities in different forms will manifest. Nonlinearities associated with systems can be classified into continuous or discontinuous. Discontinuous nonlinear systems include systems with preload, hysteresis, friction, backlash and impact. Turbomachinery bladed systems with friction dampers [6], loss of contact in gear systems [7], rub between the rotor and stator etc. are examples for discontinuous nonlinear systems. Modeling and analysis of discontinuous nonlinear systems is a challenging task as the state variables shift from one region in state space to other regions as the time unwinds. Modeling of these systems and development of a computational method for analysis is a challenging task.

Duan and Singh [1] investigated the dynamics of a single degree of freedom system which is subjected to a preload nonlinearity. They proposed an indirect multi-term harmonic balance technique to counter the computational issues in the conventional harmonic balance method. Yoon [2] modelled a multi-stage clutch damper system as a single degree of freedom system with multiple nonlinearities. They have considered the hysteresis and asymmetric stiffness in the system. Harmonic balance method was used to find the periodic solutions of the system. Yoon and Singh [3] studied the torsional driveline system of a front-engine front wheel type of vehicle which is modelled by a reduced order of the manual transmission based on the modal
characteristics of the system. The multi-stage clutch damper is modelled with asymmetric transition angles and preloads in the system. Methods to solve real-life clutch damper systems was also discussed. Bipin and Devarajan [4] investigated the dynamics of a friction oscillator in the presence of geometric nonlinearity. Analytical solutions for the response at different regimes were derived using the averaging method. Filippov based framework is used to solve for the dynamics of discontinuous oscillator [5]. The event based and the switch model based framework are efficient to study the dynamics of discontinuous nonlinear systems.

This work is in line with the previous work on single degree of freedom clutch damper by Yoon et al. [2]. In their study, the dynamics of a clutch damper system is investigated in the presence of hysteresis and piecewise nonlinear stiffness. From the experimental torque displacement diagram, it is understood that apart from the above two nonlinearities, the preload nonlinearity is also significant. In this paper the dynamics of a single degree of freedom clutch damper system in the presence of preload, hysteresis and piecewise linear stiffness is investigated. The equations of motion are formulated and integrated numerically. The discontinuous signum functions in the equations of motion are replaced by hyperbolic tangent functions to reduce the complexity of the problem as well as to reduce the computational effort. For better understanding, the dynamics of a harmonically excited single degree of freedom system with preload, hysteresis and piecewise linear stiffness are investigated separately. The dynamics is understood through time histories, phase plots, Poincare maps and bifurcation diagrams. The different types of motion exhibited by the system are classified. An asymmetry in the form of a constant mean load is also considered. It is understood that the addition of mean load the symmetry associated with the system will be lost.

2. Model and equations of motion

Clutch damper system is an integral part of automobile driveline system whose model is shown in Figure 1 [3] in which flywheel (f), clutch hub (h), input shaft (ie), output shaft (OG), unloaded gear (ou) are modeled as individual subsystems.

Figure 1. Driveline system

Figure 2. Model of clutch

Figure 2 [2] shows the simplified model of the clutch damper system with $I_f$ the moment of inertia of the disc, $C_f$ the viscous damping coefficient and $f_{nl}(\theta_f, \dot{\theta}_f)$ stands for the nonlinear forces present in the system. The experimentally obtained torque displacement diagram is shown in Figure 3 [2]. From the figure it is clear that nonlinearities in different forms are embedded in the system. The nonlinearities can be classified as preload, hysteresis and piecewise linear stiffness. The variation of the forces due to the different nonlinearities are plotted in Figure 4 to 6.
The piecewise linear stiffness is due to the variable stiffness present between the friction plates and pressure plates, preload due to initial compression in the springs and hysteresis effect due to the friction present at the interface. The nondimensional equation of motion for the system assuming a harmonic input excitation is given by

\[ \ddot{\theta}_f + 2\zeta\dot{\theta}_f + \theta_f + f_{nl} = T_e(t) \]  

(1)

where \( T_e(t) = f_m + f_e \cos(\Omega t) \) in which \( f_m \) is the mean load, \( \Omega \) and \( f_e \) are the nondimensional frequency and amplitude of the external excitation, \( \zeta = \frac{C_f}{f_i} \omega_n \) is the damping ratio and \( \omega_n \) is the natural frequency of the system. The form of the nonlinear function \( f_{nl} \) depends on the type of nonlinearity considered and is given below.

The force displacement relationship for a preload nonlinearity is shown in Figure 4. It can be represented mathematically as

\[ f_{nl} = T_{pr} \]  

(2)

\[ = \frac{1}{2}T_{pr1}[\tanh(\sigma\theta_f) + 1] + \frac{1}{2}T_{pr2}[\tanh(\sigma\theta_f) + 1] \]  

(3)

where \( T_{pr} \) is the total torque induced by preload, \( T_{pr1} \) and \( T_{pr2} \) are the the positive and negative torque induced by preload, \( \theta_f \) is the displacement of the system and \( \sigma \) is the smoothening factor.

For the case of nonlinearity due to asymmetric stiffness, the force displacement relationship shown in Figure 5 can be mathematically represented as

\[ f_{nl} = T_s \]  

(4)
\[
  f_{nl} = \frac{1}{2}(K_{c2} - K_{c1})(D_{sp} - D_{sn})
\]

(5)

\[
  D_{sp} = (\theta_f - \phi_{p1}).(\tanh(\sigma_c(\theta_f - \phi_{p1})) + 1)
\]

(6)

\[
  D_{sn} = (\theta_f + \phi_{n1}).(\tanh(\sigma_c(\theta_f + \phi_{n1})) - 1)
\]

(7)

where \(K_{c1}\) and \(K_{c2}\) are the stiffness of the clutch at first and second stage, \(\phi_{p1}\) and \(\phi_{n1}\) are the transition angles on the positive and negative side.

Similarly for the case of hysteresis nonlinearity, the force displacement relationship shown in Figure 6 can be mathematically represented as

\[
  f_{nl} = T_h
\]

\[
  = \frac{1}{2}(H_1 \tanh(\sigma_h \dot{\theta}_f)) + \frac{1}{4}(H_2 - H_1).(D_{hp} + D_{hn})
\]

(8)

\[
  D_{hp} = (\tanh(\sigma_c(\theta_f - \phi_{p1})) + 1).\left(1 + \tanh(\sigma_h \dot{\theta}_f)\right)
\]

(9)

\[
  D_{hn} = (\tanh(\sigma_c(\theta_f + \phi_{n1})) + 1).\left(1 - \tanh(\sigma_h \dot{\theta}_f)\right)
\]

(10)

(11)

where \(H_1\) and \(H_2\) are the hysteresis values at first and second stage and \(\sigma_h\) is the smoothening factor used.

The influence of multiple discontinuous nonlinearities on the dynamics of a single degree of freedom clutch damper system with harmonic excitation is considered in this paper. As a preliminary study, the dynamics of the system is investigated with preload, hysteresis and piecewise linear stiffness nonlinearities. The combined effect of all these nonlinearities is further investigated to understand the complex dynamics associated with the system.

3. Results and discussion

In this section, the dynamics is investigated by numerically integrating the equation of motion (1) for different conditions of \(F_{nl}\).

3.1. Dynamics under preload nonlinearity

The equation of motion (1) is integrated by considering \(f_{nl}\) as given in equation (3). The parameter values used are \(\zeta = 1.59\), \(f_e = 1.2\), \(f_m = 1\) (if mean load is considered else \(f_m = 0\)), \(T_{pr1} = 10\), \(T_{pr2} = -10\) and \(\sigma_c = 1000\).

\textbf{Figure 7.} Bifurcation plot without mean load

\textbf{Figure 8.} Bifurcation plot with mean load

Bifurcation diagrams are generated with \(\Omega\) as the parameter in the presence and absence of a mean load is shown in Figure 7 and 8. The bifurcation diagram shows chaotic regions separated...
by periodic windows in the lower frequency region. The time histories and phase plane plots for different parameter values are shown in Figure 9 to 16.

**Figure 9.** Phase plane plot for \( \Omega = 0.4 \) without mean load

**Figure 10.** Time history for \( \Omega = 0.4 \) without mean load

**Figure 11.** Phase plane plot for \( \Omega = 0.4 \) with mean load

**Figure 12.** Time history for \( \Omega = 0.4 \) with mean load

**Figure 13.** Phase plane plot for \( \Omega = 0.94 \) without mean load

**Figure 14.** Time history for \( \Omega = 0.94 \) without mean load
From the phase plane plots and time histories, one can find that the solutions are periodic as well as chaotic for different parameter values. The phase plane plots for the chaotic solutions are shown in Figure 13 and 15. The Poincare points shown in red color resembles a strange attractor thus confirming chaos. It can also observed from Figure 9 and 11 that the addition of mean load breaks the symmetry associated with the periodic solution. The periodic solution which was symmetric in Figure 9 changes to an asymmetric solution as shown in Figure 11.

3.2. Dynamics under piecewise nonlinearity

The equation of motion (1) is integrated numerically assuming \( f_{nl} \) as given by equation (5). The parameter values used are \( \zeta = 1.59, f_c = 1.2, f_m = 0, K_{c1} = 10.1, K_{c2} = 61.8, \phi_{pl} = 0.05, \phi_{nl} = 0.04 \) and \( \sigma_c = 1000 \). Parametric study with \( \Omega \) as the parameter is shown in Figure 17. The Poincare points are plotted as a function of \( \Omega \).

The phase plane and time histories are obtained from the bifurcation diagram for different values of \( \Omega \) are shown in Figure 18 to 23. The Poincare points reveal that the solution is either periodic as in Figure 18 or quasi-periodic as in Figure 20. The corresponding time histories are also shown in Figure 19 and 21.
3.3. Dynamics under hysteresis nonlinearity

The equation of motion (1) along with $f_{nl}$ given by equation (9) are integrated numerically to understand the dynamics of the single degree of freedom system to hysteresis nonlinearity. The parameter values considered are $\zeta = 1.59$, $f_c = 1.2$, $f_m = 0$, $H_1 = 0.98$, $H_2 = 1.96$, $\phi_{p1} = 0.05$, $\phi_{n1} = 0.04$, $\sigma_h = 1000$ and $\sigma_c = 1000$. The Poincare points are plotted as a function of $\Omega$ and is shown in Figure 22. The time
histories and phase plane plots for lower and higher values of Omega are shown in Figure 23 to 26.

For Ω = 0.4, the response shows stick slip phenomenon similar to the dry friction oscillator. For larger value of Ω = 1.02, the system behavior is quasi periodic.

3.4. Dynamics of a clutch damper system under multiple discontinuous nonlinearities

In the previous section, the dynamics of a harmonically excited single degree of freedom system to preload, hysteresis and piecewise stiffness nonlinearities were investigated numerically. From the study it is understood that the system is capable of exhibiting complex dynamics in the form of periodic, quasiperiodic and chaotic solutions. In this section, the dynamics of a clutch damper system modeled as a single degree of freedom system is investigated in the presence of all the above said nonlinearities. The equations of motion are integrated numerically and the dynamics is understood through bifurcations diagrams, time histories and phase plane plots.

The equation of motion (1) with $f_{nl}$ defined as a combination of (3),(5) and (9) are considered in the analysis. The parameter values considered are $\zeta = 1.59$, $f_c = 1.2$, $f_m = 0$, $T_{pr1} = 10$, $T_{pr2} = -10$, $K_{cl} = 10.1$, $K_{c2} = 61.8$, $H_1 = 0.98$, $H_2 = 1.96$, $\phi_{p1} = 0.05$, $\phi_{n1} = 0.04$, $\sigma_h = 1000$ and $\sigma_c = 1000$.

The Poincare points are plotted as a function of Ω and is shown in Figure 27.
The nonlinearity in the stiffness causes the peak amplitude to be offset from $\Omega = 1$ to $\Omega = 1.72$. This is due to the nonlinearity in stiffness.

**Figure 27.** Bifurcation plot

**Figure 28.** Phase plane plot for $\Omega = 0.4$

**Figure 29.** Time history for $\Omega = 0.4$

**Figure 30.** Phase plane plot for $\Omega = 0.8$

**Figure 31.** Time history for $\Omega = 0.8$
The time histories and phase plane plots for different values of $\Omega$ are shown in Figure 28 to 30. The Poincare points reveal that the solution can be periodic as in Figure 28, chaotic as in Figure 30 or quasi-periodic as in Figure 32. A kink appears in all the phase plane plots at $x = 0$ and $\dot{x} = 0$ which is due to the hyperbolic tangent approximation used while modelling the system. The time histories shown in Figure 31 and 33 shows stick - slip motion which is due to the presence of nonlinearity in hysteresis.

4. Conclusion

Discontinuous nonlinearities in different forms exist in physical systems. When these systems are subjected to external excitation, they exhibit rich dynamic behavior ranging from periodic, quasi-periodic and chaotic solutions. In this work, a single degree of freedom model of a clutch damper system is investigated using time domain method. The system has inherent discontinuous nonlinearities in the form of preload, hysteresis and piecewise linear stiffness. It is understood that the presence of these nonlinearities drastically modify the dynamic response of the system. Understanding these behaviors are essential for the safe design of the clutch damper system.

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