Gravitational Waves in ECSK theory: Robustness of mergers as standard sirens and nonvanishing torsion

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Abstract

The amplitude propagation of gravitational waves in an Einstein-Cartan-Sciamma-Kibble (ECSK) theory is studied by assuming a dark matter spin tensor sourcing for spacetime torsion at cosmological scales. The analysis focuses on a “weak-torsion regime,” such that gravitational wave emission, at leading and subleading orders, does not deviate from standard General Relativity. We show that, in principle, the background torsion induced by an eventual dark matter spin component could lead to an anomalous dampening or amplification of the gravitational wave amplitude, after going across a long cosmological distance. The importance of this torsion-induced anomalous propagation of amplitude for binary black hole mergers is assessed. For realistic late-universe astrophysical scenarios, the effect is tiny and falls below detection thresholds, even for near-future interferometers such as LISA. To detect this effect may not be impossible, but it is still beyond our technological capabilities.
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1 Introduction

We will here assess the issue of whether unaccounted torsion effects could introduce systematic errors when using binary black hole mergers as standard sirens.

This problem is crucial for understanding the universe’s evolution and exploring the nature of dark energy and dark matter. Both, dark energy and dark matter, give rise to significant physical effects, such as the accelerated expansion of the Universe [1–4], gravitational lensing around galaxy clusters [5], and the anomalous velocity profile of stars orbiting spiral galaxies [6], among several other effects.

The best observational evidence for dark energy is the so-called distance-redshift relation, coming from the observation of Type Ia supernovae (SNe). They seem robust standard candles, and it is possible to calibrate their observed brightness and luminosity distance. However, the absence of a solid theoretical description still leaves open the possibility of having, for instance, evolutionary processes in SN brightness leading to unknown systematic errors and consequently threatening the confidence in the estimation of the cosmological parameters [7].

In contrast, gravitational waves (GW) and multi-messenger astronomy [8] promise an era of high-precision cosmology. In particular, the GW-driven spiraling dynamics of binary black holes (BBH) may provide a way for high-accuracy measurements of luminosity distances, $\delta D_L/D_L \sim 1 - 10\%$. Moreover, in a multi-messenger event, the electromagnetic counterpart measurement of the redshift [9] could allow us to reduce this error even further to $0.5 - 1\%$. For this reason, BHH mergers as standard sirens are essential for developing high-precision cosmology [10].

All this is particularly true for high redshifts. BBH merger events should follow the mergers of galaxies and pregalactic structures at high redshift [11]. Though the merger rate is poorly understood, LISA should measure at least several events over its mission, especially considering its sensitivity [12]. For this reason, it is crucial to examine any source of systematic error that could arise when using BBH GW as standard sirens.

This article assesses whether a possible source of systematic error could arise from not considering torsion when studying the propagation of GWs. Torsion is not such a far-fetched possibility; for instance, dark matter could be a source of torsion, and torsion could be a dark matter component [13–18].

Of course, there are multiple proposals for dark matter candidates, such as weakly interacting massive particles, sterile neutrinos, axions, cold massive halo objects, and primordial black holes [19–21], among others. Moreover, due to the standard cosmology hassles for satisfactorily describing dark matter (see, for instance, [22]), there is an active trend for studying modified gravity theories [23–35].

In general, many of these dark matter candidates can give rise to torsion. For instance, when considering modified gravity theories, non-minimal couplings and second derivatives in the Lagrangian are actual sources of torsion [36]. Furthermore, when considering the dark matter as new particles, it is important to stress that fermions give rise to torsion. In particular, this article will focus on particle dark matter giving rise to torsion, and the consequences of this torsion on GW propagation (and BBH as standard sirens).

To analyze dark matter particles as a source of torsion, we use Einstein-Cartan-Sciama-Kibble (ECSK) gravity [37]. In ECSK, quantum mechanical spin acts as the source of torsion.\footnote{It is important to stress that with spin, we refer only to intrinsic quantum mechanical spin, and we should not confuse this with the angular momentum density. This natural confusion has already led to some mistakes in}

\[ \text{ECSK} \]

\[ \text{quantum mechanical spin acts as the source of torsion} \]

\[ \text{1}\]
in the same way as energy is a source of curvature \([39, 40]\). Consequently, torsion could have been relevant in the early universe because of its extremely high fermion densities \([41–51]\). Moreover, in standard ECSK the torsion does not propagate in a vacuum, and it interacts very weakly with Standard Model fermions (see Chap. 8.4 of Ref. \([52]\) and Refs. \([53–55]\)). Thus, torsion might potentially be a component of dark matter \([13–18]\).

Another interesting observation is that in a theory based on Riemann-Cartan geometry, the Einstein-Hilbert term does not change the dispersion relation, i.e. the speed of GWs does not change. However, torsion may influence the propagation of the GW amplitude and polarization (see ref. \([36]\)). In principle, these effects directly impact the reliability of mergers as standard sirens.

The current article argues that the anomalous amplitude propagation effect (due to ECSK torsion) is too weak to affect the use of mergers as standard sirens. Consequently, we can expect that results of standard torsionless GR will still be valid regarding amplitude propagation.

We show the explicit computation for the case of ECSK, supposing that dark matter has a nonvanishing spin density giving rise to torsion at cosmological scales. Furthermore, we assume a weak torsional background scenario, in which GW emission occurs as in GR, with torsional effects negligible compared with the subleading order in the eikonal limit (at the moment of emission). In this scenario, torsional effects can accumulate when GW propagates over a cosmological distance and dampen or reinforce the amplitude. In principle, we could expect that if we did not consider torsional effects, it could affect our ability to use mergers as standard sirens.

This article has the following structure: In section 2, we briefly review the essential aspects of i) how to properly implement wave operators over Riemann-Cartan geometries, ii) linear and second-order perturbations of fields and their potential contributions at leading and sub-leading order in the eikonal approximation\(^2\). Then, in section 3, we study GW propagation in ECSK theory, and we explicitly compute the anomalous propagation of the amplitude and polarization of GWs in the eikonal approximation. In section 4, we discuss the possibility of using GWs for studying the implications of torsion at cosmological scales, and we constrain different ansatz used in literature based on observational data. Finally, section (5) contains the summary and conclusions of the paper.

## 2 Waves in Riemann-Cartan geometry

### 2.1 Notation

When describing a theory on a Riemann-Cartan geometry (i.e., without imposing the vanishing torsion condition), there are several alternatives regarding mathematical language and notation. The two most common alternatives are (1) differential forms on an orthonormal basis, i.e., describing geometry in terms of the vielbein 1-form \(e^a = e^a_\mu dx^\mu\) and the spin connection 1-form \(\omega^{ab} = \omega^{ab}_\mu dx^\mu\), and (2) the standard tensorial language, describing the geometry in terms of the metric \(g_{\mu\nu}\) and the affine connection \(\Gamma^\lambda_{\mu\nu}\) on the coordinate basis.

\(^2\)There have been other approaches for studying GWs beyond Riemannian geometry (See for instance \([56, 57]\) and references therein). The advantage in our analysis relies on the fact that, given a proper definition of wave operators acting on Riemann-Cartan spaces, enable us to carry out the wave propagation analysis not only to leading but to subleading order in the eikonal approximation.
Of course, the choice of language and basis is physically and mathematically irrelevant. Nevertheless, a particular basis can be more advantageous/traditional to express some ideas than others. For this reason, we will use the conciseness of the differential form language when referring to the general mathematical properties of the wave operator on a Riemann-Cartan geometry (Sec. 2). However, we will use traditional tensor components in the standard coordinate basis when analyzing gravitational waves’ propagation on a cosmological background (Sec. 3).

We consider a 4-dimensional spacetime manifold \( M \) with \((-,-,+,+\)) signature. Let us use lowercase Greek characters to denote elements of the coordinate basis of vectors \( \{ \partial_\mu \} \) and 1-forms \( \{ dx^\mu \} \), and lowercase Latin characters to denote elements of the orthonormal basis of vectors \( \{ e_a = e^a_\mu \partial_\mu \} \) and 1-forms \( \{ e^a = e^a_\mu dx^\mu \} \).

To map between both basis, we use the standard relations

\[
\begin{align*}
g_{\mu\nu} &= \eta_{ab} e^a_\mu e^b_\nu, \\
g^{\mu\nu} &= \eta^{ab} e_a^\mu e_b^\nu,
\end{align*}
\]

and

\[
\partial_\mu e^a_\nu + \omega^a_\mu e^b_\nu - \Gamma^a_\lambda \mu \nu e^\lambda_\nu = 0,
\]

where \( \eta_{ab} \) denotes Minkowski metric components. The Lorentz curvature 2-form corresponds to

\[
R^{ab} = d\omega^{ab} + \omega^{a}_c \wedge \omega^{cb},
\]

and the torsion 2-form to

\[
T^a = De^a = de^a + \omega^a_\mu e^\mu_\nu, \]

where \( d : \Omega^p(M) \to \Omega^{p+1}(M) \) denotes the exterior derivative and \( D = d + \omega \) denotes the Lorentz-covariant derivative.

A circle above an entity denotes the torsionless version of it. For instance, it is possible to write the spin connection 1-form as

\[
\hat{\omega}^{ab} = \omega^{ab} + \kappa^{ab},
\]

where

\[
de^a + \hat{\omega}^a_\mu e^\mu_\nu = 0,
\]

and \( \kappa^{ab} \) defines the contorsion (or contortion) 1-form,

\[
T^a = \kappa^a_\mu e^\mu_\nu.
\]

In terms of \( \kappa^{ab} \) the Lorentz curvature splits as

\[
R^{ab} = \bar{R}^{ab} + \bar{D}\kappa^{ab} + \kappa^a_\mu \wedge \kappa^b_\nu.
\]

It is possible to write the \( \bar{R}^{ab} \) components as

\[
\bar{R}^{ab} = \frac{1}{2} e^a_\mu e^b_\nu \bar{R}^{\mu\nu}_\rho\sigma dx^\rho \wedge dx^\sigma,
\]

where we have that \( \bar{R}^{\mu\nu}_\rho\sigma \) correspond to the coordinate components of the Riemann tensor.
2.2 Lichnerowicz-DeRham wave operator on a Riemann-Cartan geometry

Ref. [58] provides a formal mathematical definition of the Lichnerowicz-DeRham wave operator on a Riemann-Cartan geometry, and in Refs. [36,59] we find examples of using it in theories with nonvanishing torsion. We refer to these former articles for a deeper treatment, but nevertheless this section briefly reviews the wave operators definitions and properties in terms of the differential form language. Of course, it is possible to proceed without introducing this mathematical toolkit, but it greatly simplifies calculations and makes results much more transparent.

In terms of the Hodge-dual operator $\ast : \Omega^p(M) \rightarrow \Omega^{4-p}(M)$, we define the operator $I_{a_1\cdots a_q} : \Omega^p(M) \rightarrow \Omega^{p-q}(M)$, on the 4-dimensional spacetime manifold $M$ as

$$I_{a_1\cdots a_q} = -(-1)^{p(p-q)} \ast (e_{a_1} \wedge \cdots \wedge e_{a_q}) \wedge \ast .$$

At this point, it is useful to define two new derivatives on Riemann-Cartan geometries in terms of the operator $I_a$ and the Lorentz covariant derivative $D : \Omega^p(M) \rightarrow \Omega^{p+1}(M)$. The first of them is the generalized covariant coderivative $D_\parallel : \Omega^p(M) \rightarrow \Omega^{p-1}(M)$ as

$$D_\parallel = -I^a DI_a .$$

The second one corresponds to $D_a : \Omega^p(M) \rightarrow \Omega^p(M)$ defined by

$$D_a = I_a D + DI_a .$$

The operator $D_\parallel$ is the Riemann-Cartan generalization of the standard coderivative operator $d^\parallel = \ast d \ast$ of Riemannian geometry. When torsion vanishes, $D_\parallel$ and $d^\parallel$ coincide. The derivative $D_a$ satisfies the Leibniz rule, and it generalizes the standard coordinate definition $\nabla = \partial + \Gamma$ in such a way that $-D_a D^a$ defines the generalized (torsionfull) Beltrami wave operator. In fact, it is possible to prove that

$$D_a = e_a^\mu \nabla_\mu + I_a T^b \wedge I_b ,$$

and therefore their vanishing torsion counterparts $\bar{D}_a$ and $\bar{\nabla}_\mu$ are equivalent. A practical feature of operators $I_a, D,$ and $D_a$ is that they obey the Leibniz rule, and they span an open superalgebra satisfying the super Jacobi identity (see Ref. [58]):

$$\{I_a, D\} = D_a ,$$

$$\{I_a, I_b\} = 0 ,$$

$$\{D, D\} = 2D^2$$

$$[I_a, D_b] = -T_{ab}^c I_c ,$$

$$[D, D_a] = D^2 I_a - I_a D^2 ,$$

$$[D_a, D_b] = I_{ab} D^2 + D^2 I_{ab} + I_a D^2 I_b - I_b D^2 I_a - (DT_{ab}^c \wedge I_c + T_{ab}^c D_c) .$$

These properties greatly simplify algebraic calculations.

In terms of $D_\parallel$, the Lichnerowicz-DeRham wave operator corresponds to $\Box = D_\parallel D + DD_\parallel$ on Riemann-Cartan geometries. From a mathematical point of view, it is the right definition because it satisfies a generalized Weitzenböck identity (see ref. [58])

$$D_\parallel D + DD_\parallel = -D^a D_a + I_a D^2 T^a ,$$

6
where the second term gives rise to the Lorentz curvature via the Bianchi identities. From a physical and phenomenological point of view, Eq. (2.9) is also the appropriate wave operator definition. It is because $D^\dagger D + DD^\dagger$ (or equivalently, the generalized Beltrami operator $-D^aD_a$) is the wave operator that arises from perturbations of the Einstein-Hilbert term in the case of nonvanishing torsion. To make this point clear, in the following sections we briefly review perturbations of the Riemann-Cartan geometry and the high frequency perturbations of the Einstein-Hilbert term in this case.

### 2.3 Generic perturbations of Riemann-Cartan geometry

The ref. [60] considered general perturbations of a Riemann-Cartan geometry. Here we briefly review these results to apply them to gravitational waves in the Einstein-Cartan theory.

In a Riemann-Cartan geometry, the vierbein and the spin connection correspond to independent degrees of freedom, and in consequence, their perturbations are, too. It proves convenient to write these independent perturbations as

$$e^a \mapsto \bar{e}^a = e^a + \frac{1}{2} H^a,$$

$$\omega^{ab} \mapsto \bar{\omega}^{ab} = \omega^{ab} + U^{ab}(H) + V^{ab}.$$  \hspace{1cm} (2.10)

Here, the $H^a = H^a_e e^b$ 1-form maps the same degrees of freedom as the standard metric perturbation $h_{\mu\nu}$ through eq. (2.1). The term $U^{ab}(H)$ describes the connection piece that depends on $H^a$ and its derivatives through

$$U^{ab}(H) = U^{ab}_1 + U^{ab}_2 + U^{ab}_3 + \cdots$$

where

$$U^{ab}_1 = -\frac{1}{2} \left( \Gamma^a_d H^b - \Gamma^b_d H^a \right),$$

$$U^{ab}_2 = \frac{1}{8} \Gamma^{bc}_d \left( DH_c \wedge H^c \right) - \frac{1}{2} \left[ I^a \left( U^{bc}_1 \wedge H_c \right) - I^b \left( U^{ac}_1 \wedge H_c \right) \right],$$

and (1), (2) label linear and quadratic terms in $H^a$. For a theory with propagating torsion, the contorsional perturbation term $V^{ab} = V^{ab}_e e^c$ 1-form is an independent degree of freedom, and it describes a ‘roton’ or ‘torsionon’ (see ref. [55, 60, 61]). For non-propagating torsion theories (as Einstein-Cartan theory), it is possible to solve $V^{ab}$ in terms of $H^a$ and $T^a$, in such a way that in a region where the background torsion vanishes, the perturbation $V^{ab}$ also does. Up to second order, the perturbations of Lorentz curvature and torsion read

$$T_a \mapsto \bar{T}_a = T_a + T_a^{(1)} + T_a^{(2)},$$

$$R^{ab} \mapsto \bar{R}^{ab} = R^{ab} + R^{ab}_1 + R^{ab}_2,$$
where

\[ T^a_{(1)} = V^a_b \wedge e^b - \frac{1}{2} I^a \left( H^b \wedge T_b \right), \]  
\[ T^a_{(2)} = \frac{1}{2} V^a_b \wedge H^b + \frac{1}{4} I^a \left[ H^b \wedge I_b (H^c \wedge T_c) \right], \]
\[ R_{(1)}^{ab} = DU_{(1)}^{ab} + DV^{ab}, \]
\[ R_{(2)}^{ab} = DU_{(2)}^{ab} + \left( U^{a}_{(1)c} + V^{a}_{c} \right) \wedge \left( U^{c}_{(1)} + V^{c}_{b} \right). \]  

In order to consider these perturbations a proper gravitational wave, they must satisfy the eikonal condition, i.e., they must correspond to variations in a characteristic scale \( \lambda \) much shorter than the characteristic scale \( L \) of changes on the background geometry. This condition defines the eikonal parameter

\[ \epsilon = \frac{\lambda}{L} \ll 1. \]

Furthermore, it must be possible to write the perturbations \( H^a \) and \( V^{ab} \) as

\[ H^a = e^{i\theta} \sum_{p=0}^{\infty} H^a_{(p)}, \]
\[ V^{ab} = e^{i\theta} \sum_{p=0}^{\infty} V^{ab}_{(p)}, \]

where \( \theta \) corresponds to a real, rapidly-changing phase, and its derivative defines the wavefront 1-form \( k = k_\mu dx^\mu = d\theta \) as the dual of the wave vector. In the summation, the terms \( H^a_{(0)} \) and \( V^{ab}_{(0)} \) correspond to leading, \( \lambda \)-independent terms, and \( H^a_{(p)} \) and \( V^{ab}_{(p)} \) correspond to deviations of order \( \epsilon^p \) to the geometric optics. The leading term \( H^a_{(0)} \) defines the polarization \( P^a = P^a_b e^b \in \mathbb{C} \) and amplitude \( \varphi \in \mathbb{R} \) of the gravitational wave as

\[ H^a_{(0)} = \varphi P^a. \]

We can always choose to normalize the polarization as

\[ P_{ab} P^{ab} = 1 \]  

without losing generality.\(^3\) Regardless of the theory, it is always possible to use the local Lorentz invariance and the Lie derivative to prove that the components \( H_{ab} \) must be symmetric, \( H_{ab} = H_{ba} \) and to impose the Lorenz gauge on a Riemann-Cartan geometry as (see \[36\] for further details)

\[ D_a H^a - \frac{1}{2} dl_a H^a = 0. \]

However, for a generic nonvanishing torsion theory, it is impossible to make further gauge fixing, and the six possible gravitational polarization modes could be present in the components

\(^3\)The same should be done with \( V^{ab} \) when dealing with propagating torsion. In the simple ECSK case, torsion does not propagate, and this decomposition of \( V^{a}_{b} \) as amplitude and polarization is unnecessary.
of $H_{ab}$. For instance, it is possible to write the polarization of a gravitational wave propagating in the third direction of the orthonormal frame as

$$P_{ab} = p^{(+)ab} + p^{(x)ab} + p^{(b)ab} + p^{(l)ab} + p^{(x)ab} + p^{(y)ab},$$

with the orthonormal polarization basis

$$p^{(+)ab} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad p^{(x)ab} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$p^{(b)ab} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad p^{(l)ab} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$p^{(x)ab} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad p^{(y)ab} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

and where

$$\tilde{p}^{(+)p(+)} + \tilde{p}^{(x)p(x)} + \tilde{p}^{(b)p(b)} + \tilde{p}^{(l)p(l)} + \tilde{p}^{(x)p(x)} + \tilde{p}^{(y)p(y)} = 1.$$  \hspace{1cm} (2.20)

It is essential to remember that, even in the case of standard torsionless GR, further gauge fixing (as the transverse traceless gauge) is only approximate on a generic background geometry, and it is only valid at leading and subleading orders in the eikonal expansion. For a generic geometry background, this means that in standard torsionless GR, some polarization components dominate over others,

$$\tilde{p}^{(+)p(+)} + \tilde{p}^{(x)p(x)} \approx 1, \quad \tilde{p}^{(b)p(b)} + \tilde{p}^{(l)p(l)} + \tilde{p}^{(x)p(x)} + \tilde{p}^{(y)p(y)} \leq \epsilon^2. \hspace{1cm} (2.21)$$

This hierarchy of polarization modes can break down in a generic torsional background, and in principle, the modes beyond $(+)$ and $(\times)$ could become significant.

### 3 ECSK theory as a playground to study GW propagation

The current article focuses on ECSK theory for several reasons. First of all, the great observational success of standard GR makes it attractive to analyze theories close to its dynamics
(as ECSK). Second, ECSK is the simplest theory with nonvanishing torsion, but despite having a non-propagating torsion, it can affect the propagation of gravitational waves in a non-trivial way. For these reasons, ECSK provides an excellent playground to study the phenomenology of GW propagation before jumping to consider more complex Lagrangians. For other theories, the explicit form of some subleading couplings will vary (see Sec. 3.2), but the general procedure also applies to those cases.

In terms of the vierbein and the spin connection, the ECSK Lagrangian 4-form reads

\[ \mathcal{L}_{\text{EH}} = \frac{1}{\kappa_4} \left( \frac{1}{4} \epsilon_{abcd} R^{ab} \wedge e^c \wedge e^d - \frac{\Lambda}{4!} \epsilon_{abcd} e^a \wedge e^b \wedge e^c \wedge e^d \right) + \mathcal{L}_M (e, \omega, \psi) , \]

where \( \mathcal{L}_M \) corresponds to the matter Lagrangian 4-form. The equations of motion provided by independent variations of the vierbein and the spin connection correspond to

\[ \frac{1}{2} \epsilon_{abcd} R^{ab} \wedge e^c - \frac{\Lambda}{3!} \epsilon_{abcd} e^a \wedge e^b \wedge e^c \wedge e^d = \kappa_4 \ast T_n , \]

\[ \epsilon_{abcd} T^c \wedge e^d = \kappa_4 \ast \sigma_{ab} , \]

where the variations of \( \mathcal{L}^{(4)}_M \) define the stress-energy 1-form \( T_n = T_{mn} e^m \) and the spin density 1-form \( \sigma^{ab} = \sigma^{c a b} e^c \) as

\[ \delta_e \mathcal{L}_M = - \ast T_d \wedge \delta e^d , \]

\[ \delta_\omega \mathcal{L}_M = - \frac{1}{2} \delta \omega^{ab} \wedge \ast \sigma_{ab} . \]

On the coordinate basis, these field equations read

\[ R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R = \kappa_4 T_{\mu \nu} , \]

\[ T^\lambda \mu \nu - \delta^\lambda \mu T^\sigma \sigma \nu + \delta^\lambda \nu T^\sigma \sigma \mu = \kappa_4 \sigma^\lambda \mu \nu , \]

where \( R_{\mu \nu} \) corresponds to the generalized Ricci tensor constructed with the complete Lorentz curvature components \( R^{ab} = \frac{1}{2} e^a \mu e^b \nu R_{\mu \nu \rho \sigma} dx^{\rho} \wedge dx^{\sigma} \) instead of only the Riemannian ones. The generalized Ricci tensor \( R_{\mu \nu} \) and stress-energy tensor \( T_{\mu \nu} \) also have an antisymmetric part; it is possible to prove that they depend on the derivatives of the spin tensor.

It is straightforward to observe that the torsion components are algebraically related to matter’s spin tensor, eq. (3.4). Therefore, in a vacuum, torsion identically vanishes and cannot propagate. When besides this, we consider the Standard Model as the matter Lagrangian, torsion would seem to be doomed to irrelevance in the context of ECSK. In the Standard Model, only fermions\(^4\) are a source of torsion (they have a nonvanishing spin tensor) and are affected by it \([39, 52]\). However, the expected effect is so weak that there are no realistic particle physics

\(^4\)In the physics literature, it is trendy to introduce boson-torsion coupling by following the recipe of changing partial derivatives on Minkowski space by covariant derivatives. For instance, the electromagnetic field components become \( \nabla_\mu A_\nu - \nabla_\nu A_\mu \) with \( \nabla = \partial + \Gamma \) through this mechanism, creating an electromagnetism-torsion coupling. However, it is unnecessary and unnatural from a mathematical point of view. Gauge fields correspond to curvatures \( F = dA + \frac{1}{2} [A, A] \) on principal bundles \([62]\), with \( d \) an exterior derivative and nothing else. For instance, from the principal bundle point of view, the electromagnetic field strength corresponds only to the 2-form \( F = dA \). Introducing the affine connection \( \Gamma \) in its definition seems like putting fibers on top of the fibers of a fiber bundle.
experiments that could detect torsion in the foreseeable future, and some references even jokingly advise against betting for a detection of this kind (see the end Chapter 8 of ref. [52]). Even more, Standard Model fermions interact forming localized structures. Since any torsional effect they create cannot propagate in a vacuum, it would seem that torsion cannot play a role in the late evolution of the universe\(^5\).

However, it is necessary to take into account dark matter before dismissing torsion. It is not clear yet whether dark matter carries spin and we don’t know whether its spin density vanishes or not. Standard cosmological models generally assume a vanishing spin density for simplicity, but there is no physical reasons behind this hypothesis: the observational constraints on torsion are poor. At the contrary, assuming a nonvanishing dark matter spin density provides a simple explanation for phenomena as the Hubble parameter tension, see [18].

It opens an intriguing possibility. As we shall see, a torsion background affects GW propagation of its amplitude and polarization. It means that, in principle, even a weak torsion background could affect a gravitational wave propagating over a long cosmological distance. Even worst, it may seem that an unaccounted anomalous propagation of GW amplitude could hinder our efforts to use mergers as standard sirens, making them appear farther or closer than they are. In the current article, we prove that it is not the case and that an unrealistically strong torsion background would be necessary to affect amplitude propagation in an observable way. The following section analyzes GWs on an ECSK theory with a nonvanishing torsional background.

### 3.1 Propagation of GWs in ECSK theory

When studying GWs (in any theory), we must separate high-frequency and low-frequency terms (up to second order in perturbations). The high-frequency piece describes how the wave propagates on the background geometry, while the low-frequency piece describes how the wave may affect the background geometry, creating an effective stress-energy tensor and spin tensor. Chapters 1 and 4 of ref. [64] provide an excellent example of how to perform this separation in the context of GR.

In this article, we will focus only on the propagation of the GW (i.e., high-frequency effects). In this high-frequency piece, not all the terms are equally important. Therefore, this article will consider only terms contributing to the leading and subleading orders in the eikonal approximation. This way, we start by studying the perturbation of all terms of eq.(3.1). However, proceeding as in standard GR (see ref. [36]), it is straightforward to prove that from eq.(3.1) the only perturbation terms contributing to GW propagation in this approximation are inside of the expression

\[
\frac{1}{2} \varepsilon_{abcd} R_{(1)}^{ab} \wedge \varepsilon^c = 0,
\]

where \( R_{(1)}^{ab} \) corresponds to the linear perturbations of Lorentz curvature, eq.(3.1)

---

\(^5\)The situation is contrary when considering extremely dense fermion plasma, as in the very early universe or during a black hole collapse. In this case, torsion can avoid the singularity, provide an alternative to inflation models and give origin to Big Bounce models, see refs. [41, 48–51]. Regarding late universe evolution, it is interesting to observe that including torsional effects in vacuum fluctuations reduces the cosmological constant problem from 122 orders of magnitude to just 8, see ref. [63].
It is possible to prove (see Appendix A) of ref. [36]) that

$$\frac{1}{2}\epsilon_{a b c n}R_{(1)}^{a b} \wedge e^c = \left(W_{mn} - \frac{1}{2}\eta_{mn}W_p^p\right) \ast e^m, \quad (3.5)$$

where after some algebra and using the Lorenz gauge eq.(2.3), we have

$$W_{mn} = -\frac{1}{2}(I_nD_aD^aH_m - I_n[D_a, D_m]H^a) + (I_nD_a - D_nI_a)V^a_m. \quad (3.6)$$

Therefore, the eq. (3.5) is equivalent to just $W_{mn} = 0$. Observe that the wave operator corresponds to the generalized Beltrami operator, already found in eq.(2.9).

### 3.2 Eikonal analysis

Up to this point, we have preferred the conciseness of differential forms on the orthonormal basis to study the general properties of the wave operator and ECSK theory. However, in the following sections, we must analyze the GWs’ eikonal limit and their propagation on a cosmological background. It is much more friendly for most readers to carry out this work in terms of tensors on a coordinate basis, so we will move to this description in what follows.

Firstly, it is convenient to replace the open superalgebra eqs. (2.3)-(2.8) in eq. (3.6), preserving only leading ($O(\epsilon^{-2})$) and subleading ($O(\epsilon^{-1})$) terms in eq. (3.6), and move to the coordinate basis using eq. (2.2). The result of this is

$$W^\mu_\nu|_{\text{lead.+sublead.}} = -\left[\frac{1}{2}\nabla_\lambda\nabla^\lambda H^\mu_\nu + T_{\sigma\rho\nu}\nabla^\sigma H^\rho_\mu + \frac{1}{2}T_{\rho\sigma\mu}\nabla^\rho H^\sigma_\nu\right] + \nabla^\lambda V^\mu_\nu + \nabla_\nu V^\lambda_\mu.$$

This equation has a symmetric and an antisymmetric piece, and they must independently vanish,

$$W^\mu_\nu|_{\text{lead.+sublead.}} = 0,$$

$$W^-^\mu_\nu|_{\text{lead.+sublead.}} = 0.$$

The vanishing of $W^-^\mu_\nu$ allow us to solve $V_{\alpha\beta\gamma}$ in terms of $H^\mu_\nu$ and $T^\lambda_\mu\nu$ as

$$V_{\alpha\beta\gamma} = \frac{1}{4}\left([T^\sigma_\alpha\beta H^\rho_\alpha - T^\rho_\alpha\beta H^\rho_\alpha]g_{\beta\gamma} - [T^\sigma_\alpha\beta H^\rho_\beta - T^\rho_\beta\beta H^\rho_\alpha]g_{\alpha\gamma}\right) +
- \left(T^\rho_{\beta\gamma} + \frac{1}{4}[T_{\alpha\beta\rho} - T_{\beta\alpha\rho}]\right)H^\rho_\gamma + \frac{1}{4}\left((T_{\beta\rho\gamma} + T_{\gamma\rho\beta})H^\rho_\alpha - (T_{\alpha\rho\gamma} + T_{\gamma\rho\alpha})H^\rho_\beta\right).$$

Here we can see that a vacuum $T^\lambda_\mu\nu = 0$ also implies $V_{\alpha\beta\gamma} = 0$, as it should be expected for non-propagating torsion.

Replacing it back in $W^\mu_\nu|_{\text{lead.+sublead.}}$, we obtain

$$W^\mu_\nu|_{\text{lead.+sublead.}} = 0.$$
From this expression, it is clear that torsion affects the propagation of GWs at subleading order, and when torsion vanishes, we recover the standard GR wave equation in terms of the standard Beltrami operator.

To perform the eikonal analysis using eqs. (2.16)-(2.17) and to study its implications at leading and subleading order, let us omit the (0) at the leading term of the summation eqs. (2.16)-(2.17) and let us call $H_{\mu\nu} = H_{\mu\nu}^{(0)}$ to simplify the notation. At leading order $O(\epsilon^{-2})$, the dispersion relation remains unchanged,

$$k^\mu k_\mu = 0,$$  
(3.7)

implying GW propagation at the speed of light. Even more, taking into account that $k_\mu = \partial_\mu \theta$ and taking the derivative of the dispersion relation, we have that

$$k^\mu \nabla_\mu k^\lambda = 0.$$  
(3.8)

These relations reveal an essential fact. Despite the nonvanishing background torsion, GWs move along null, torsionless geodesics. It may seem counterintuitive because, besides standard torsionless geodesics, a Riemann-Cartan geometry allows defining auto-parallels. Auto-parallels are curves given by

$$\frac{d^2 X^\lambda}{d\tau^2} + \Gamma^\lambda_{\mu\nu} \frac{dX^\mu}{d\tau} \frac{dX^\nu}{d\tau} = 0,$$  
using the full connection $\Gamma^\lambda_{\mu\nu}$ instead of only the Christoffel piece $\Gamma^\lambda_{\mu\nu}$. This simple fact is crucial from an observational point of view. The multimessenger observation GW170817/GRB170817 implies that GWs and electromagnetic waves travel at the same speed, and on the same kind of trajectories. If electromagnetic waves moved on null torsionless geodesics and GWs on null auto-parallels, it still could have caused a delay even if both waves traveled at the same speed.

The subleading order $O(\epsilon^{-1})$ provide us with the relation

$$\frac{1}{2} \nabla_\lambda k^\lambda H_{\mu\nu} + k^\lambda \left( \nabla_\lambda H_{\mu\nu} + \frac{1}{2} M^+_{\lambda\mu\nu} \right) = 0,$$  
(3.9)

with

$$M^+_{\lambda\mu\nu} = \left( 2T_{\sigma\lambda\nu} + \frac{1}{2} \left[ T_{\lambda\sigma\nu} + T_{\nu\sigma\lambda} \right] \right) H^\sigma_{\mu} + \left( 2T_{\sigma\lambda\mu} + \frac{1}{2} \left[ T_{\lambda\sigma\mu} + T_{\mu\sigma\lambda} \right] \right) H^\sigma_{\nu} +$$  
$$+ \frac{1}{2} g_{\mu\nu} T_{\rho\sigma\lambda} H^\rho_{\lambda} - \frac{1}{4} [g_{\mu\nu} T^\sigma_{\sigma\lambda} - (T_{\mu\nu} + T_{\nu\mu})] H.$$  
(3.10)

Notice that eq. (3.9) codifies the amplitude $\varphi$ and polarization $P_{\mu\nu}$ propagation eq. (2.18). The time-honored way of getting this information is by defining the ‘number of rays’ current density $J_\mu = \varphi^2 k_\mu$ (also known as ‘number of photons’ current density in standard optics). In the standard Riemannian case $J_\mu$ is conserved, but it is no longer the case for nonvanishing torsion. In our case, after replacing eq. (2.18) and eq. (2.19) in eq. (3.9) (see Appendix A), we get

$$\nabla_\lambda J^\lambda = (\Pi^{\mu\nu} - g^{\mu\nu}) T_{\mu\nu\lambda} J^\lambda,$$  
(3.12)

with $\Pi^{\mu\nu}$ given by

$$\Pi^{\mu\nu} = \frac{1}{2} \left[ \begin{array}{c} \left( P^\mu_{\sigma\sigma} P_\nu^\nu + P^\mu_{\sigma\sigma} P_\nu^\nu \right) - (\tilde{P} P^\mu_{\mu} + \tilde{P} P^\mu_{\nu} P) + \frac{1}{2} \tilde{P} P g^{\mu\nu} \right],$$  
(3.13)
and \( P = P^\lambda_\lambda \) denoting the trace of the polarization.

The polarization also propagates anomalously on a torsional background. Replacing eq.\((3.12)\) in eq. \((3.9)\), we find after some algebra (see Appendix A) the relation
\[
k^\lambda \bar{\nabla}_\lambda P_{\mu\nu} = -\frac{1}{2}k^\lambda \left[ \Pi^{\rho\sigma} T_{\rho\sigma\lambda} P_{\mu\nu} + \left( T_{\sigma\lambda\nu} - \frac{1}{2} [T_{\lambda\sigma\nu} + T_{\nu\sigma\lambda}] \right) P^{\sigma\mu} + \left( T_{\sigma\lambda\mu} - \frac{1}{2} [T_{\lambda\sigma\mu} + T_{\mu\sigma\lambda}] \right) P^{\sigma\nu} + \frac{1}{2} g_{\mu\nu} T^{\rho\sigma}_{\rho\sigma\lambda} P^{\rho\sigma} - \frac{1}{4} [g_{\mu\nu} T^{\sigma\sigma}_{\sigma\lambda} - (T_{\mu\nu\lambda} + T_{\nu\mu\lambda})] P \right].
\] (3.14)

The equations \((3.12)\) and \((3.14)\) show that torsion, even if it does not propagate, can affect the propagation of the GWs’ amplitude and polarization.

Some comments are in order. Regarding eqs. \((3.12)-(3.14)\), the explicit form of the right-hand side of eqs. \((3.12)-(3.14)\) depends on the ECSK Lagrangian structure, but it is a generic feature of theories with nonvanishing torsion to create an anomalous propagation of amplitude and polarization. This anomalous propagation is an intrinsic feature of the wave operator on Riemann-Cartan geometries, and changing of theory would only change coefficients weights in eqs. \((3.12)-(3.14)\). For instance, even if we start with just the operator\( \mathcal{D}_\mu D^\mu H_m = 0 \) instead of vanishing the whole eq. \((3.6)\), it will still lead to anomalous propagation, see ref. [58].

### 3.3 Propagation of amplitude in a weak torsion scenario

The enormous observational success of GR make it clear that torsional effects, if they are present, should be weak. For instance, the GWs profile emitted by mergers fits GR predictions nicely. Torsion, if present (for instance, being created by a possible dark matter spin density), seems not to have observable effects in the process of emission. There are some recent works supporting this argument. In fact, ref. [65] studied the process of mergers’ GWs emission in ECSK theory. It showed that only extremely high spin densities (as the ones in the very early universe) could give rise to important departures from GR in the emission process. For mergers in current astrophysical conditions, emission should be as the one of standard GR.

It makes interesting to consider a “weak torsion scenario”, where a torsional background may be present, but it is too weak to affect the GWs emission process. In particular, only the polarization modes \((+)\) and \((\times)\) are relevant, and the other four modes are at least \(\epsilon\) times weaker (see eq. 2.21). However, this GW has to travel over a long cosmological distance before reaching our detectors. In such case, it is non-trivial to decide whether or not the cumulative effect of the torsional background eqs. \((3.12)-(3.14)\) could be observed at the moment of detection.

The scrambling of polarization modes along the geodesic eq. \((3.14)\) could make grow the other polarization modes while the GW propagates. Furthermore, an unaccounted anomalous propagation of GW amplitude eq. \((3.12)\) could hinder our efforts to use mergers as standard sirens, making them appear farther or closer than they are. In this article we focus our attention in the amplitude problem; the details of polarization propagation are left for future work.

Let us consider a GW propagating along the null torsionless geodesic eq. \((3.8)\), with a \( \frac{d\mathbf{x}^\mu}{d\eta} \propto k^\mu \) tangent vector and an affine parameter \( \eta \), and let us call \( \eta_0 \) the affine parameter at
the moment of emission. Let \( \varphi (\eta) \) be the amplitude of the gravitational wave at \( \eta \), and \( \dot{\varphi} (\eta) \) the amplitude predicted at \( \eta \) by standard GR. Then, we can define

\[
A (\eta) = \ln \frac{\varphi}{\dot{\varphi}},
\]

as a parameter describing the anomalous propagation of amplitude. The weak torsion scenario implies that \( \varphi (\eta_0) = \dot{\varphi} (\eta_0) \), and therefore \( A (\eta_0) = 0 \). In this context, let us assume a generic theory with non-vanishing torsion, which lead us to a breaking of the conservation of \( J_\mu \) in some generalized form of eq. (3.12),

\[
\nabla_\mu J^\mu = N_\mu J^\mu.
\]

Since \( \varphi = e^A \dot{\varphi} \), it is possible to write this last equation as

\[
\partial_\mu e^{2A} \dot{\varphi}^2 k^\mu + e^{2A} \nabla_\mu \dot{J}^\mu = N_\mu e^{2A} \dot{\varphi}^2 k^\mu,
\]

where \( \dot{J}^\mu = \dot{\varphi}^2 k^\mu \). Furthermore, using the fact that \( \nabla_\mu \dot{J}^\mu = 0 \), after a bit of algebra one finds

\[
\frac{dA}{d\eta} = \frac{1}{2} N_\mu \frac{dX^\mu}{d\eta}.
\]

In this way, we have that

\[
A = \frac{1}{2} \int_{\eta_0}^\eta d\tilde{\eta} N_\mu (\tilde{\eta}) \frac{dX^\mu}{d\eta},
\]

and

\[
\varphi (\eta) = e^{\frac{1}{2} \int_{\eta_0}^\eta d\tilde{\eta} N_\lambda (\tilde{\eta}) \frac{dX^\lambda}{d\eta}} \dot{\varphi} (\eta),
\]

(3.15)

where

\[
N_\lambda = (\Pi^{\mu\nu} - g^{\mu\nu}) T_{\mu\nu\lambda},
\]

in the case of ECSK theory. Consequently, in the case of mergers, the mismatch between \( \varphi (\eta) \) and \( \dot{\varphi} (\eta) \) could lead in principle to a wrong assessment of the luminosity distance relation.

To be more precise, let us observe that the predicted amplitude for mergers in standard GR has the form

\[
\dot{\varphi} = \frac{1}{D_L} F_{GW},
\]

where \( D_L \) is the luminosity distance, and \( F_{GW} \) is a complicated, time-dependent function of the masses and angular momentum of the merger (see chap. 4 of ref. [64]).

A torsional background introduces the extra correction factor \( e^A \) to the amplitude. Therefore, an observer modeling the GWs profile by standard GR, would assign the wrong luminosity distance \( \dot{D}_L \) to the source

\[
\varphi = \frac{1}{\dot{D}_L} F_{GW} = e^A \frac{1}{D_L} F_{GW},
\]

i.e.,

\[
\frac{D_L}{\dot{D}_L} = e^A.
\]

(3.16)

In the weak torsion scenario we consider here, the effect would be noticeable only when \( |A| > 0 \), i.e., when the GW propagates over long cosmological distances. For this reason and, in order to constrain any possible observable effect, in the following section we will integrate out these relations, considering different cosmological scenarios involving non-vanishing torsion.
4 Torsion and cosmological symmetries

A Riemann-Cartan geometry satisfying the Copernican symmetries of homogeneity and isotropy on a flat spatial section has a metric and torsion tensor of the form

$$ds^2 = -c^2 dt^2 + a^2(t) (dx^2 + dy^2 + dz^2),$$  \hfill (4.1)

$$T_{\mu\nu\lambda} = -\frac{1}{c^2} \left[ \nu_+(t) (g_{\mu\lambda} g_{\nu\rho} - g_{\mu\nu} g_{\lambda\rho}) + 2\sqrt{|g|} \nu_-(t) \epsilon_{\lambda\mu\nu\rho} \right] U^\rho.$$ \hfill (4.2)

This way, to describe this geometry, we need the two functions $\nu_+(t)$ and $\nu_-(t)$ besides the common scale factor $a(t)$. The symbols ± refer to the parity of the associated torsion component. Using the ansatz eq. (4.2), one finds

$$N_\lambda = (\Pi^{\mu\nu} - g^{\mu\nu}) T_{\mu\nu\lambda},$$

$$= -\frac{1}{c} \nu_+ (t) (\Pi_{\lambda0} - g_{\lambda0} - (\Pi^\sigma_\sigma - 4) g_{\lambda0}).$$

From eq. (4.2), we have that $\Pi^\sigma_\sigma = 3$, and therefore

$$N_\lambda = -\frac{1}{4c} \nu_+ (t) \bar{P} P g_{\lambda0}.$$  

From this expression for $N_\lambda$, we have that

$$A(t) = \frac{1}{4} \int_{t_0}^t dt' \nu_+ (t') \frac{1}{2} |P|^2.$$ \hfill (4.3)

In this way, to solve $A(t)$ we need $P(t)$. Tracing eq. (3.14) we get

$$k^\lambda \nabla_\lambda P = \frac{1}{2c} \nu_+ (t) k^\lambda \left( \frac{1}{4} \bar{P} P - \frac{1}{2} \right) P g_{\lambda0},$$ \hfill (4.4)

and since along the geodesic $k^\lambda \propto \frac{dX^\lambda}{dt}$,

$$\frac{dP}{dt} = -\frac{1}{2} \nu_+ (t) \left( \frac{1}{4} \bar{P} P - 1 \right) \frac{1}{2} P.$$ \hfill (4.5)

From here, we have that

$$\frac{d}{dt} \left( \frac{1}{2} |P|^2 \right) = -\frac{1}{2} \nu_+ (t) \left( \frac{1}{2} |P|^2 - 1 \right) \frac{1}{2} |P|^2.$$  

It is simple to integrate this equation as

$$\frac{1}{2} |P|^2 (t) = \frac{1}{\left( \frac{1}{2} |P|^2 (t_0) - 1 \right) e^{-\frac{1}{2} \Theta^+} + 1},$$

with

$$\Theta^+ = \int_{t_0}^t dt' \nu_+ (t').$$  \hfill (4.6)
With this expression, one integrates eq. (4.3) as

\[
e^{A(t)} = \sqrt{\frac{1 - \frac{1}{2} |P|^2(t_0)}{1 - \frac{1}{2} |P|^2(t)}} ,
\]  

(4.7)

\[
e^{A(t)} = \sqrt{1 + \frac{1}{2} |P|^2(t_0) \left[ e^{\frac{1}{2}\Theta^+} - 1 \right]} .
\]  

(4.8)

Some comments are in order. First, the anomaly in amplitude propagation only depends on the \(\nu_+\) component and not on \(\nu_-\). It is not strange when we consider the role of torsion in cosmic evolution; see sec. 4. In the weak torsion scenario, \(\frac{1}{2} |P|^2(t_0) \sim \epsilon^2\) is a tiny positive number (see eq. 2.21), since in astrophysical situations \(\epsilon \leq 10^{-20}\). Therefore, the only chance to have an observable deviation of GR is if \(\Theta^+ \gg 0\) and big enough to compensate for the smallness of \(\frac{1}{2} |P|^2(t_0)\). A \(\Theta^+ \leq 0\) stands no chance of producing an observable effect in \(e^{A(t)}\). It means that, in principle, ECSK gravity could make mergers to appear closer than they really are \((e^{A(t)} > 1\) in eq. (3.16)) but not further than they are.

However, when using mergers as standard candles, there will always be some uncertainty \(\delta \bar{D}_L\) in determining the luminosity distance \(D_L\), see the ref. [10]. Therefore, to have an observable torsion anomaly in the determination of luminosity distances, it is necessary that \(D_L > \bar{D}_L + \delta \bar{D}_L\), i.e., that

\[
e^A > 1 + \frac{\delta \bar{D}_L}{D_L} .
\]  

(4.9)

Considering eq. (4.8), we have

\[
\sqrt{1 + \frac{1}{2} |P|^2(t_0) \left[ e^{\frac{1}{2}\Theta^+} - 1 \right]} > 1 + \frac{\delta \bar{D}_L}{D_L} ,
\]  

(4.10)

and therefore

\[
\Theta^+ > 2 \ln \left[ \left( 1 + \frac{\delta \bar{D}_L}{D_L} \right)^2 - 1 + 1 \right] .
\]  

(4.11)

Since at most \(\frac{1}{2} |P|^2(t_0) \sim \epsilon^2\), the minimal \(\Theta^+_{\text{min}}\) that could produce a torsional anomaly above the detection threshold corresponds to

\[
\Theta^+_{\text{min}} \approx 2 \ln \left[ \left( \frac{\epsilon}{2} \right)^2 \frac{\delta \bar{D}_L}{D_L} \right] .
\]  

(4.12)

Using the performance estimates for LISA (see ref. [10]), we have that in the best scenario

\[
\frac{\delta \bar{D}_L}{D_L} \sim 10^{-3} ,
\]  

(4.13)

(which would be an astounding feat). Considering that \(\epsilon < 10^{-20}\), we have that

\[
\Theta^+_{\text{min}} \sim 170 .
\]  

(4.14)
As we shall see in the following sections, meeting these conditions seems physically unfeasible for realistic ECSK models: reaching the minimal detectable value of $\Theta_{\text{min}}^+$ would require, to the best of our knowledge, conditions that cosmological observations rule out. For this reason, we can conclude that mergers are reliable standard sirens even if there are unaccounted torsional ECSK effects at play. The mergers’ anomalous amplitude propagation that ECSK torsion could create falls below the LISA detection threshold.

### 4.1 Evaluating $\Theta^+$ or different ECSK cosmological models

It is possible to express $\Theta^+$ in terms of the redshift $z = \frac{a_0}{a} - 1$ considering that

$$ dt = -\frac{dz}{(z+1)H}, $$

where $H = \frac{\dot{a}}{a}$ corresponds to the Hubble parameter. For a merger at redshift $z$ the eq. (4.6) becomes

$$ \Theta^+ (z) = -\int_0^z d\tilde{z} \frac{\nu^+ (\tilde{z})}{(\tilde{z} + 1) H (\tilde{z})}. \quad (4.15) $$

For $z < 1$ it is possible to attempt an estimation of this integral through observational data; for $z > 1$ we can use ECSK cosmological models that agree with observations.

### 4.2 Estimation for merger at $z < 1$

For $z < 1$, we have

$$ \frac{1}{H (z)} = \frac{1}{H_0} [1 - z (1 + q_0)] + \cdots, $$

where $q_0$ corresponds to the deceleration parameter. In terms of this expression, we have

$$ \Theta^+ (z) \approx -\frac{1}{H_0} \int_z^0 d\tilde{z} \frac{\nu^+ (\tilde{z})}{\tilde{z} + 1} [1 - z (1 + q_0)]. $$

To integrate this last equation, in rigor we need a particular cosmological model of torsion for $\nu^+ (\tilde{z})$. However, for an estimation we will just introduce some “representative value” $\langle \nu^+ \rangle$,

$$ \Theta^+ (z) \approx -\frac{\langle \nu^+ \rangle}{H_0} \int_z^0 d\tilde{z} \frac{1 - (1 + q_0) z}{1 + \tilde{z}}, $$

$$ = -\frac{\langle \nu^+ \rangle}{H_0} [(1 + q_0) z - (2 + q_0) \ln (1 + z)]. $$

Using the observational estimations of $q_0 \sim -0.6$, we conclude that when $z < 1$ then

$$ \Theta^+ (z) < \frac{\langle \nu^+ \rangle}{H_0} - 0.6, $$

and therefore, to reach the minimal value of eq. (4.14) it is necessary something as

$$ \langle \nu^+ \rangle \sim 100 H_0. \quad (4.16) $$

The estimates for $\nu_0^+$ in the literature are model-dependent, but $\nu_0^+ = 100 H_0$ is orders of magnitude above the even most optimistic estimates for ECSK cosmology, see Refs. [43,66,67]. In the following sections we will briefly review ECSK cosmology models to make $\Theta^+ (z)$ estimations for $z > 1$. 

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4.3 Review of ECSK cosmology

Let us consider the Copernican ansatz of eqs. (4.1)-(4.2), and replace it in the field eqs. (3.3)-(3.4), for a universe with cosmological constant, dark matter and Standard Model matter. The result is

$$3\dot{H}^2 = \kappa_4 c^2 \rho, \quad (4.17)$$
$$2\dot{H} - 3\dot{H}^2 + 2\nu_+ \dot{H} = -\kappa_4 c^2 p, \quad (4.18)$$

with

$$\mathcal{H} = H - \nu_+, \quad (4.19)$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter. The total density and pressure are given by

$$\rho = \frac{\Lambda}{\kappa_4} + \rho_{SM} + \rho_{DM} + \frac{3}{c^2 \kappa_4} \nu_-^2,$$
$$p = -\frac{\Lambda}{\kappa_4} + p_{SM} + p_{DM} - \frac{1}{c^2 \kappa_4} \nu_-^2.$$

Assuming dark matter as a source for spin and consequently for torsion, implies that $\nu_+$ and $\nu_-$ must be functions of $\rho_{DM}$. However, both functions play a completely different role in eqs. (4.17)-(4.18). It is clear that $\nu_-$ just play the role of an extra dark matter density and pressure, while $\nu_+$ plays a different role in the dynamics. For this reason, it is natural to assume a “barotropic” ansatz for $\nu_-$ such that

$$\nu_-^2 = \alpha_-^2 \frac{c^2 \kappa_4}{3} \rho_{DM}, \quad (4.20)$$

where $\alpha_-$ plays the role of a “barotropic” parameter, providing us with an effective dark matter density $\rho_{eff}$ and pressure $p_{eff}$ given by

$$\rho_{eff} = \left(1 + \alpha_-^2\right) \rho_{DM},$$
$$p_{eff} = \omega_{eff} \rho_{eff},$$

with

$$\omega_{eff} = \frac{1}{1 + \alpha_-^2} \omega_{DM} = -\frac{1}{3 1 + \frac{1}{\alpha_-^2}},$$

where have assumed a barotropic relation $p_{DM} = \omega_{DM} \rho_{DM}$. We can see that the net effect of $\nu_-$ was only to shift the barotropic constant. In a cold DM case, $\omega_{DM} = 0$, we are left with $-\frac{1}{3} < \omega_{eff} \leq 0$ which in fact could explain the Hubble parameter tension with values as small as $\omega_{eff} \sim -10^{-2}$, see ref. [18].

The eqs. (4.17,4.18) can be rewritten as

$$3 (H - \nu_+)^2 = \kappa_4 c^2 \rho, \quad (4.21)$$
$$\dot{\rho} + 3H (\rho + p) - \nu_+^2 (\rho + 3p) = 0, \quad (4.22)$$

with

$$\rho = \frac{\Lambda}{\kappa_4} + \rho_{SM} + \rho_{eff},$$
$$p = -\frac{\Lambda}{\kappa_4} + p_{SM} + p_{eff}.$$
In terms of the redshift, eq. (4.22) becomes

\[- (1 + z) \frac{d\rho}{dz} + 3 (\rho + p) - \frac{\nu^+}{H} (\rho + 3p) = 0. \tag{4.23}\]

To solve these equations, we need to know how \(\nu^+\) as a function of the dark matter density. This is precisely the information that should provide the field equation (3.4). However, since we do not have a dark matter lagrangian, we have no information a priori on its possible spin tensor and the field equation (3.4) becomes useless.

For this reason, when considering ECSK cosmologies, we must propose a “reasonable” ansatz in order to model the dependence of \(\nu^+\) on dark matter density. In the following sections we briefly review a couple of ansatz that agree with observations, and we will see how the GW amplitude propagate on them at mergers at \(z > 1\).

### 4.4 Ansatz \(\nu^+ \propto H \rho_{\text{eff}}^n\)

Let us consider an ansatz of the form

\[\nu^+ = CH \left(\frac{\rho_{\text{eff}}}{\rho_c}\right)^n, \tag{4.24}\]

where \(H\) is the Hubble parameter, \(\rho_c\) corresponds to the current critical density

\[\rho_c = \frac{3H_0^2}{c^2\kappa_4}, \tag{4.25}\]

and \(C\) is just a proportionality constant. In general, torsional effects accelerate the expansion of the universe and may play the role of dark energy. Therefore, let assume a toymodel with \(\Lambda = 0\) and where \(\rho_{\text{SM}}\) is negligible when compared to \(\rho_{\text{eff}}\). In this case and considering the ansatz eq. (4.24), the eq. (4.23) becomes

\[- (1 + z) \frac{d\rho_{\text{eff}}}{dz} - \frac{C}{\rho_c^n} (1 + 3\omega_{\text{eff}}) \rho_{\text{eff}}^{n+1} + 3 (1 + \omega_{\text{eff}}) \rho_{\text{eff}} = 0. \tag{4.26}\]

The solution of this Bernoulli ODE is (see ref. [66,68])

\[\rho_{\text{eff}} (z) = \rho_c \left[ \frac{3 \frac{1 + \omega_{\text{eff}}}{\kappa_4} \frac{1 + 3\omega_{\text{eff}}}{\Omega_0} - \frac{1}{(1 + z)^{rac{3}{2}}}} {\left( \frac{3 \frac{1 + \omega_{\text{eff}}}{\kappa_4} \frac{1 + 3\omega_{\text{eff}}}{\Omega_0} - 1}{(1 + z)^{3n(1 + \omega_{\text{eff}})}} - 1 \right)^{\frac{1}{2}}} \right]^{\frac{1}{2}} \]

with

\[\Omega_0 = \frac{\rho_{\text{eff}0}}{\rho_c}. \]

Using eqs. (4.17) and (4.24), it is straightforward to find

\[\frac{H}{H_0} = \sqrt{\frac{\rho_{\text{eff}}}{\rho_c}} \frac{1}{1 - C \left(\frac{\rho_{\text{eff}}}{\rho_c}\right)^n}. \tag{4.27}\]
and for \( \frac{\nu^+}{H} \) we have

\[
\frac{\nu^+}{H} = C \left( \frac{\rho_{\text{eff}}}{\rho_c} \right)^n, \tag{4.28}
\]

\[
= \frac{\frac{3(1 + \omega_{\text{eff}})}{4 + 3\omega_{\text{eff}}}}{\left( \frac{3}{4 + 3\omega_{\text{eff}}} \Omega_0 - 1 \right) (1 + z)^{-\frac{3}{n}(1 + \omega_{\text{eff}}) - 1}}. \tag{4.29}
\]

The ref. \([66]\) shows that the observations allow for the following ranges of parameters for this model:

| Table 1: Observational Parameters |
|-----------------------------------|
| \( \Omega_0 \) | \( C \) | \( n \) | \( H_0 \) (\( \text{km/s/Mpc} \)) |
| 0.31\( ^{+0.11}_{-0.12} \) | 0.28\( ^{+0.28}_{-0.24} \) | -0.47\( ^{+0.26}_{-0.36} \) | 68.8\( ^{+3.0}_{-3.1} \) |

Inserting eq. (4.29) into eq. (4.15), with the approximation \( \omega_{\text{eff}} \approx 0 \) (cold dark matter), we find

\[
\Theta^+(z) = -\int_z^0 \frac{d\tilde{z}}{1 + \tilde{z}} \frac{1}{\left( \frac{3}{4 \sqrt{\Omega_0}} - 1 \right) (1 + \tilde{z})^{-\frac{3}{n}} + 1}. \tag{4.30}
\]

From the allowed observational parameters Table 1, we can make an estimation for \( \Theta^+(z) \). In fact, this function grow too slowly to compensate for the smallness of \( \frac{1}{2} |P_0|^2 \) in eq. (4.8). To see this, it suffices to make a plot and to compare it with the estimates of eq. (4.14).

Figure 1: \( \Theta^+(z) \) as a function of redshift \( z \). Blue lines contour the upper and lower deviation from the mean values showed in red.
4.5 Ansatz \( \nu_+ \propto H \)

The steady-state torsion ansatz, i.e.,

\[
\nu_+ = -\alpha H, \tag{4.31}
\]

with \( \alpha \) constant, is very popular among the ECSK cosmology models refs. [67]. The main reason for this is that eq. (4.23) becomes easily integrable under this ansatz. However, it has a severe physical drawback: from the field equation (3.4) we expect that \( \nu_+ \) should depend on some dark matter feature, e.g., its density, and not on a geometrical feature as \( H \). Regardless of whether or not we should consider \( \nu_+ = -\alpha H \) as a realistic astrophysical model, let us assess its capacity to give rise to some observable effect on GWs’ amplitude propagation. In this case, we have that eq. (4.15) becomes

\[
\Theta^+(z) = \alpha \int_z^0 \frac{d\tilde{z}}{1 + \tilde{z}} = -\alpha \ln (1 + z). \tag{4.33}
\]

According to ref. [67], the observations constrain this model allowing for a small positive value for \( \alpha \),

\[
\alpha = 0.086^{+0.094}_{-0.095}. \tag{4.34}
\]

![Image of \( \Theta^+(z) \) as a function of redshift \( z \).](image.png)

Figure 2: \( \Theta^+(z) \) as a function of redshift \( z \).

This negative (and slow-evolving) \( \Theta^+(z) \) stands no chance to compensate for the smallness of \( \frac{1}{2} |P_0|^2 \) in eq. (4.8), this \( \Theta^+(z) \) is well below the minimal value of eq. (4.14).

5 Conclusions and Outlook

After performing a general analysis of gravitational waves on Riemann-Cartan geometries and their associated features (reviewed in Sec. 2 and Refs. [36, 58, 59]), it becomes clear that a
torsional background would generically produce an anomalous propagation of a GW amplitude and polarization, as given by Eqs. (3.12)-(3.14). In particular, in the present article we have focused our attention on mergers’ GW amplitude propagation in the context of an ECSK theory, where a nonvanishing dark matter spin tensor could behave as a possible torsion source on cosmological scales.

Given the solid observational success of standard GR, we consider a “weak-torsion scenario.” This means that the background torsion is set to be weak enough to make the mergers GW emission process indistinguishable (at least at leading and subleading orders) from the one predicted by standard GR.

This leads, in particular, to a hierarchy of polarization modes, where at the moment of emission, polarization modes \((b), (l), (x),\) and \((y)\) are at least \(\epsilon\) times weaker than the modes \((+)\) and \((\times)\) (see Eqs. (2.20)-(2.21)).

However, the anomalous propagation of polarization in Eq. (3.14) implies (at least in principle) that torsion could slowly amplify these weaker polarization modes along the geodesic. In the same way, and also in principle, background torsion could lead to an anomalous dampening (or amplification) of the amplitude, Eqs. (3.12)-(3.15), after going across a long cosmological distance.

Since the merger masses and angular momentum correlate to the GW frequency evolution, an anomalous amplitude propagation could lead to a wrong luminosity distance assessment \(\delta D_L\) (see eq. (3.16)), potentially disabling the use of mergers as standard candles.

The main result in this paper is the conclusion, following from our analysis, that these worries are actually unfounded, so that mergers remain to be reliable standard candles, both with or without torsion (at least for ECSK theories). In Secs. 3 and 4 the strength of this anomalous amplitude propagation has been calculated for a generic ECSK theory. The conclusion is that the effect of torsion under these conditions is so tiny that it remains always below detection thresholds, even thinking in near-future interferometers, such as LISA. In particular, for the ECSK models considered in Sec. 4, to detect a possible anomalous amplitude propagation due to torsion one would need to measure mergers luminosity distances at \(z = 1\) with a precision of

\[
\frac{\delta D_L}{D_L} \leq 10^{-41}.
\] (5.1)

To detect such an effect may still be possible, but it is as of now beyond technological capabilities.

Elaborating further on this point, eq. (4.8) shows why this is the case. Even if the anomalous amplitude effect accumulates over a long cosmological distance in the integral \(\Theta^+\), the smallness of the GW trace modes \((b)\) and \((l)\) at the moment of emission renders the whole integrated effect still negligible. In turn, the smallness of \(\frac{1}{2} |P_0|^2\) is a direct consequence of the weak-torsion scenario, where torsion should be kept weak enough so that the GW emission process still happens as in the well checked standard GR.

This point opens a critical issue for further research. First, it is crucial to notice that the weak-torsion hypothesis is reasonable, but for the late universe only. In contrast, at the high spin densities of the very early universe, torsion could actually have been relevant \([41,48-51]\), and the GW emission process could have significantly departed, at that epoch, from the one predicted by GR \([65]\). In this case, it is not clear yet whether \(\frac{1}{2} |P_0|^2\) in Eq. (4.8) should still be considered negligible. Therefore, anomalous propagation of amplitude and polarization, Eqs. (3.12)-(3.14),
produced by torsion could, in principle, leave a detectable fingerprint in the cosmic gravitational wave background. This promising topic will be covered elsewhere.

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A Anomalous propagation of amplitude and polarization

Since
\[ \hat{\nabla}_\mu J^\mu = \nabla_\mu J^\mu - T^\mu_{\mu\lambda} J^\lambda. \]
using eq. (2.18), eq. (2.19) in eq. (3.9), with \( J^\mu = k^\mu \varphi^2 \), the ray current conservation breaks according to
\[ \hat{\nabla}_\lambda J^\lambda = (\Pi^{\mu\nu} - g^{\mu\nu}) T_{\mu\nu\lambda} J^\lambda, \]
where \( \Pi^{\mu\nu} \) is given in eq. (3.13).

Recall eq. (3.9) written in the form
\[ k^\lambda \nabla_\lambda H_{\mu\nu} + \frac{1}{2} \left( \nabla_\lambda (k^\lambda H_{\mu\nu} + k^\lambda M^+_{\lambda\mu\nu}) \right) = 0. \]
Inserting eq. (2.18) and \( J^a = \varphi^2 k^a \) one finds
\[ k^\lambda \nabla_\lambda P_{\mu\nu} + \frac{1}{2\varphi^2} (T_{\mu\nu\lambda} - T_{\nu\rho\lambda} + T_{\lambda\nu\rho}) P^\rho\mu + \frac{1}{2} k^\lambda M^+_{\lambda\mu\nu} = 0. \]
Using the fact that \( \nabla_\lambda J^\lambda = \Pi^{\mu\nu} T_{\mu\nu,\lambda} J^\lambda \) direct calculation shows
\[ \nabla_\lambda P_{\mu\nu} = \hat{\nabla}_\lambda P_{\mu\nu} + \frac{1}{2} (T_{\rho\nu\lambda} - T_{\nu\rho\lambda} + T_{\lambda\rho\nu}) P^\rho\mu + \frac{1}{2} (T_{\rho\mu\lambda} - T_{\mu\rho\lambda} + T_{\lambda\mu\rho}) P^\rho\nu \]
Using this result we can finally solve for \( k^\lambda \hat{\nabla}_\lambda P_{\mu\nu} \) as
\[ k^\lambda \hat{\nabla}_\lambda P_{\mu\nu} = -\frac{1}{2} k^\lambda \left[ \Pi^{\rho\sigma} T_{\rho\sigma,\lambda} P_{\mu\nu} + \left( T_{\sigma\lambda\nu} - \frac{1}{2} (T_{\lambda\sigma\nu} + T_{\nu\sigma\lambda}) \right) P^\rho\mu + \left( T_{\sigma\lambda\mu} - \frac{1}{2} (T_{\lambda\sigma\mu} + T_{\mu\sigma\lambda}) \right) P^\rho\nu + \frac{1}{2} g_{\mu\nu} T_{\rho\sigma\lambda} P^{\rho\sigma} - \frac{1}{4} [g_{\mu\nu} T_{\sigma\lambda} - (T_{\mu\sigma\lambda} + T_{\nu\mu\lambda})] P \right] \]
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