Abstract: A new probe which utilises a dual-ridged waveguide to provide broadband, non-destructive (ND) material characterisation measurements of a perfect electric conductor (PEC)-backed material is introduced. The new probe possesses a bandwidth similar to existing coaxial probes and is structurally robust like rectangular waveguide probes. The combinations of these two qualities make it especially attractive for ND inspection/evaluation applications in the field. The theoretical development of the dual-ridged-waveguide probe is discussed. A magnetic field integral equation is derived by applying Love’s equivalence theorem and enforcing the continuity of transverse fields at the dual-ridged-waveguide aperture. The magnetic field integral equation is then solved for the theoretical reflection coefficient using the method of moments. The permittivity and permeability of the material under test are found by minimising the root-mean-square difference between the theoretical and measured reflection coefficients using non-linear least squares. To validate the new probe, experimental results are presented of a magnetic absorbing material comparing results obtained using the new probe with those obtained using a traditional, destructive technique. The probe’s sensitivity to sample thickness, flange-plate thickness, cutoff wavenumber and measured S-parameter uncertainties is also investigated.

1 Introduction

Waveguide probes have been used for non-destructive (ND) material characterisation applications with great success for over 30 years. Although some two-port waveguide probes have been developed [1–5], a majority of the published waveguide probe research deals with one-port or single-probe geometries because of their more widespread applicability [6–24]. Both coaxial [7, 10–12, 15, 18, 21, 24] and rectangular/circular [6, 8, 9, 13, 14, 16, 17, 20, 22, 23] waveguide probes have been developed. The main advantage of using a coaxial probe is the potential for broadband material characterisation measurements; however, utilising coaxial probes for this purpose is difficult. In general, small aperture coaxial probes are most effective when used to characterise high-loss materials. Biological materials are often discussed in the literature, whose loss is driven by water content [19, 25–30]. On the other hand, large aperture probes are required to measure low-loss materials [10, 11]. This implies that broadband measurements using coaxial probes are only possible when the material under test (MUT) is known a priori to possess high loss. Rectangular/circular waveguide probes do not suffer from the coaxial probe problem just described and are more structurally robust than their coaxial counterparts making them especially useful for ND measurements in the field; however, they are naturally bandlimited.

In this paper, a new one-port waveguide probe is introduced which utilises a dual-ridged waveguide (DRWG) to provide ND, broadband (much like coaxial probes) material characterisation measurements of PEC-backed MUTs while maintaining the structural robustness of rectangular/circular waveguide probes. A schematic of the measurement geometry is shown in Fig. 1 – a DRWG attached to an infinite PEC flange plate is placed in contact with a PEC-backed magnetic material of unknown complex relative permittivity $\varepsilon_r$ and permeability $\mu_r$.

In the next section, the theoretical expression for the reflection coefficient $S_{11}^{th}$, necessary to characterise the MUT, is derived. This is achieved by replacing the DRWG aperture with an equivalent magnetic current which maintains the fields in the parallel-plate/MUT region in accordance with Love’s equivalence theorem [31–34]. Enforcing the continuity of the transverse magnetic fields at the DRWG aperture results in a magnetic field integral equation (MFIE), which when solved using the method of moments (MoM) [34, 35], yields $S_{11}^{th}$. The $\varepsilon_r$ and $\mu_r$ of the MUT are then found by minimising the root-mean-square difference between the theoretical $S_{11}^{th}$ and measured $S_{11}^{meas}$ reflection coefficients using the trust-region-reflective method [36] subject to the constraints for passive materials, viz.
where least two independent reflection methods – most notably, two-layer method [16, 23, 37–39], two-iris method [40], frequency-varying method [16, 41], short/free-space-backed method [15, 23, 37] and two-thickness method [9, 16, 22, 42]. Of these, the two-layer and two-iris methods are the most universally applicable techniques for the ND characterisation of PEC backed MUTs. However, because of the analytical complexities involved in using the two-layer method (Green’s function with off-diagonal elements combined with complex DRWG field expressions) and the additional hardware required to use the two-iris method, two-thickness method is utilised in this research. Note that this choice is only for convenience. Each of the techniques listed above can be used with the single DRWG probe presented in this paper.

Last, to experimentally verify the new probe, broadband material characterisation results of a magnetic absorbing material are presented and compared with those obtained using the traditional, destructive Nicolson–Ross–Weir (NRW) [43, 44] technique. The new probe’s sensitivity to sample thickness, flange-plate thickness, cutoff wavenumber and measured $S_{11}$ uncertainties is also presented.

2 Methodology

In this section, an expression for $S_{11}^{hy}$ is derived for the single DRWG probe shown in Fig. 1. The forms of the fields in the DRWG and parallel-plate/MUT regions of the probe are detailed first. A MFIE is derived by enforcing the continuity of the transverse DRWG and parallel-plate/MUT region magnetic fields at the DRWG aperture, that is, $z = 0$. This MFIE is subsequently solved using the MoM.

2.1 DRWG and parallel-plate region field distributions

The fields in the DRWG region of Fig. 1 are found using the technique outlined by Montgomery [45] and later by Elliot [46]. First, the electric and magnetic fields in each DRWG subregion (i.e. the gap subregion, $|y| < \Delta x \cap |y| < \Delta y$; and the two trough subregions, $\Delta x < |y| < a/2 \cap |y| < b/2$) are expanded in a set of TE$^z$ and, if applicable, TM$^z$ modes [32]. Note that only the dominant DRWG mode, commonly termed a TE$_{01}$ hybrid mode [45, 47], is considered in this research. Thus, only the TE$^z$ mode development is reported here.

The mode-matching technique [48] is then used to enforce the continuity of the transverse electric and magnetic fields at $x = -\Delta x$ and $\Delta x$ producing the following homogeneous matrix equation

$$A(k_\gamma) \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} A_{11}(k_\gamma) & A_{12}(k_\gamma) \\ A_{21}(k_\gamma) & A_{22}(k_\gamma) \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = 0 \quad (2)$$

where $k_\gamma$ is the cutoff wavenumber and $A_{11}, A_{12}, A_{21}$ and $A_{22}$ are $N \times N$ submatrices whose $ith$ row and $ith$ column entries are given by

$$A_{1m}(k_\gamma) = A_{2m}(k_\gamma) = \frac{\tan(k_{mn}^e \Delta x) - \cot(k_{mn}^e \Delta x)}{2k_{mn}^e \Delta x} \delta_{mn} - \sum_{m'} \phi_{mn} \psi_{mm'} \cot(k_{mm'}^e (a/2 - \Delta x))$$

respectively.

$$A_{2m}(k_\gamma) = A_{1m}^*(k_\gamma) = \frac{\tan(k_{mn}^e \Delta x) - \cot(k_{mn}^e \Delta x)}{2k_{mn}^e \Delta x} \delta_{mn}$$

and the $ith$ row and $ith$ column entries are given by

$$A_{1m}(k_\gamma) = A_{2m}(k_\gamma) = \frac{\tan(k_{mn}^e \Delta x) - \cot(k_{mn}^e \Delta x)}{2k_{mn}^e \Delta x} \delta_{mn}$$

$$A_{2m}(k_\gamma) = A_{1m}^*(k_\gamma) = \frac{\tan(k_{mn}^e \Delta x) - \cot(k_{mn}^e \Delta x)}{2k_{mn}^e \Delta x} \delta_{mn}$$

$$A_{1m}(k_\gamma) = A_{2m}(k_\gamma) = \frac{\tan(k_{mn}^e \Delta x) - \cot(k_{mn}^e \Delta x)}{2k_{mn}^e \Delta x} \delta_{mn}$$

and

$$A_{2m}(k_\gamma) = A_{1m}^*(k_\gamma) = \frac{\tan(k_{mn}^e \Delta x) - \cot(k_{mn}^e \Delta x)}{2k_{mn}^e \Delta x} \delta_{mn}$$

respectively. Here, $n$ and $\hat{n}$ represent basis and testing indices, respectively; $\delta_{ij}$ is the Kronecker delta; $k_{mn}^e = m\pi/b$ and $k_{mn}^h = n\pi/(2\Delta y)$ are the $y$-directed DRWG wavenumbers in the trough and gap subregions, respectively; $k_{mn}^r = \sqrt{k_\gamma^2 - (k_{mn}^e)^2}$ and $k_{mn}^g = \sqrt{k_\gamma^2 - (k_{mn}^h)^2}$ are the unknown $x$-directed DRWG wavenumbers in the trough and gap subregions, respectively; $\alpha_n$ and $\beta_n$ are the unknown complex TE$^z$ modal amplitudes; and

$$\psi_{mn} = \frac{1}{2} \left[ \frac{1}{k_{mm}^h - k_{mn}^h} - \frac{1}{k_{mm}^h + k_{mn}^h} \right] \left\{ \sin \left[ \frac{k_{mn}^h b}{2} + \Delta y \right] \right\} - \left( -1 \right)^n \sin \left[ \frac{k_{mn}^h b}{2} + \Delta y \right]$$

The cutoff wavenumber $k_c$ is found by forcing an eigenvalue
of \(A(k_z)\) to zero via numerical root search. The vector containing the complex modal amplitudes, \((\alpha \beta)\), is the associated eigenvector of that zero eigenvalue. There are an infinite number of wavenumbers which satisfy (2) each corresponding to a distinct TE\(_1\) DRWG mode. The \(k_z\) which corresponds to the first zero of (2) is the dominant DRWG mode cutoff wavenumber.

After \(k_z\) and \((\alpha \beta)\) have been found, expressions for the TE\(_1\) transverse DRWG fields can be obtained. The transverse fields in the DRWG region \((z < 0)\) are

\[
E_i = \begin{cases} 
\left(e^{-jk_z z} + \Gamma e^{jk_z z}\right) e_{i}'(\rho) & |\rho| < \Delta x \\
\left(e^{-jk_z z} + \Gamma e^{jk_z z}\right) e_{i}'(\rho) & x < -\Delta x \\
\left(e^{-jk_z z} + \Gamma e^{jk_z z}\right) e_{i}'(\rho) & x > \Delta x
\end{cases}
\]

\[
H_i = \begin{cases} 
\left(e^{-jk_z z} - \Gamma e^{jk_z z}\right) h_{i}'(\rho) & |\rho| < \Delta x \\
\left(e^{-jk_z z} - \Gamma e^{jk_z z}\right) h_{i}'(\rho) & x < -\Delta x \\
\left(e^{-jk_z z} - \Gamma e^{jk_z z}\right) h_{i}'(\rho) & x > \Delta x
\end{cases}
\]

(5)

Here \(k_z = \sqrt{k_0^2 - k_{D}^2}\), where \(k_0 = 2\pi/\sqrt{\varepsilon_0\mu_0}\), \(\Gamma = S_{11}^{\text{th}}\) is the desired theoretical reflection coefficient; and \(e_{i}'\), \(h_{i}'\), \(e_{i}'\text{trans}\), \(h_{i}'\text{trans}\), \(h_{i}'\text{em}\), and \(h_{i}'\text{em}\) are the DRWG dominant-mode transverse electric and magnetic field distributions in the gap and trough subregions, respectively. The analytical forms for these field distributions can be found in [4] and are not provided here for the sake of brevity.

The transverse magnetic field in the parallel-plate/MUT region of Fig. 1 is found by replacing the DRWG aperture with an equivalent transverse magnetic current \(\mathcal{M}\) in accordance with Love’s equivalence theorem [31–34]. The transverse magnetic field is given by

\[
H_{pp}^{\text{tr}}(\rho, z) = \frac{1}{j\omega\mu_0} \left(\mathcal{I}, k^2 + \nabla \nabla\right) \cdot F(\rho, z)
\]

(6)

where the electric vector potential \(F\) is

\[
F(\rho, z) = \int_S \mathcal{G}(\rho, z | \rho', 0) \cdot \varepsilon \mathcal{M}(\rho') \, dS'
\]

(7)

where \(\mathcal{G}\) is the dyadic magnetic-current-excited parallel-plate Green’s function [49], \(\mathcal{I} = \hat{x} x + \hat{y} y\), \(\varepsilon = \hat{x} (\partial/\partial x) + \hat{y} (\partial/\partial y)\), \(\rho = \hat{x} x + \hat{y} y\) is the observation vector, \(\rho' = \hat{x} x' + \hat{y} y'\) is the source vector, \(S\) represents the DRWG aperture cross section and \(k = 2\pi f/\sqrt{\varepsilon\mu}\).

2.2 MFIE and MoM solution

A MFIE can be derived by enforcing the continuity of the transverse magnetic fields in the DRWG and parallel-plate/MUT regions at \(z = 0\), namely

\[
\frac{1}{j\omega\mu_0} \left(\mathcal{I}, k^2 + \nabla \nabla\right) \cdot F(\rho, 0) = \left(1 - \Gamma\right) S_{11}^{\text{th}}
\]

(8)

where the unknowns in the above MFIE are \(\mathcal{M}\) and \(\Gamma\). Expanding \(\mathcal{M}\) using the transverse DRWG electric field distribution given in (5) and testing the resulting expression with the transverse DRWG magnetic field distribution also given in (5) yields the desired \(S_{11}^{\text{th}}\).

It is possible to calculate the convolution integral in (7) directly. This approach requires the numerical evaluation of four integrals – two basis and two testing integrals. It is numerically advantageous to apply the convolution theorem and perform the required integrations in the spectral domain [39]. This approach permits all the basis and testing integrals to be computed in closed form yielding spectral domain integrals which, in the worst computational cases, are given by

\[
\sum_{n,m} e_{n,m}^p e_{n,m}^q \int_{-\infty}^{\infty} \left(k^2 - \xi^2\right) f_{m,n}(\xi) \left(\xi + p\sinh(\xi)\right) d\xi
\]

(9)

where \(p = \sqrt{\xi^2 + \eta^2 - k^2}\) is the spectral domain wavenumber and

\[
f_{m,n}(\xi) = \int_{-\frac{\Delta x}{2}}^{\frac{\Delta x}{2}} \sin \left(k_{m,n} \left(x - \frac{\Delta x}{2}\right)\right) \cos (\xi x) \, dx
\]

\[
\times \int_{-\frac{\Delta y}{2}}^{\frac{\Delta y}{2}} \sin \left(k_{m,n} \left(y - \frac{\Delta y}{2}\right)\right) \cos (\xi y) \, dy
\]

\[
\times \int_{-\frac{\Delta x}{2}}^{\frac{\Delta x}{2}} \sin \left(k_{m,n} \left(x - \frac{\Delta x}{2}\right)\right) \cos (\xi x) \, dx
\]

\[
\times \int_{-\frac{\Delta y}{2}}^{\frac{\Delta y}{2}} \sin \left(k_{m,n} \left(y - \frac{\Delta y}{2}\right)\right) \cos (\xi y) \, dy
\]

The \(\eta\) integral can be evaluated using complex-plane analysis yielding a pole-series representation. The remaining \(\xi\) integral contains irremovable branch cuts and is therefore most easily computed numerically [3, 5, 22].

It should be noted that because of the summations in the basis and testing functions, special care must be taken when computing the spectral domain integrals. For optimal computational efficiency, it is best to bring the summations inside the \(\xi\) integral, evaluate the \(\xi\) integral via complex-plane analysis, evaluate the resulting summations and lastly, calculate the \(\eta\) integral numerically. Note that there are a total of 16 distinct spectral domain integrals which must be evaluated.

3 Experimental verification

3.1 Apparatus description and experimental procedure

To validate the new ND broadband probe, material measurements were made of ECCOSORB® SF-3 (\(d = 1.85\) mm) [50] using an Agilent E8362B vector network analyzer (VNA) [51]. The data were collected from 6 to 18 GHz using the apparatus shown in Fig. 2. The DRWG probe apparatus consisted of a single Microwave Engineering Corporation WRD650 DRWG (\(a = 18.29\) mm, \(b = 8.15\) mm, \(\Delta x = 2.20\) mm and \(\Delta y = 1.28\) mm) [52] connected via screws to a 15.24 cm × 15.24 cm × 0.95 cm aluminium flange plate. On the side of the flange plate which connected to the DRWG, 3.18 mm alignment holes were machined to ensure precision alignment between the DRWG and the flange-plate apertures.

Before the material measurements were made, the apparatus was calibrated using a thru-reflect-line (TRL) calibration [53]. Two custom made DRWG line standards
were used in the calibration – one 6.98 mm thick to cover 6–12 GHz and one 3.40 mm thick to cover 12–18 GHz. These line standards can be seen in the photograph in Fig. 2. The TRL calibration placed the port 1 calibration plane at the DRWG aperture. This calibration plane was then phase shifted to the front face of the MUT by

\[ S_{\text{meas}}^{11} = S_{\text{TRL}}^{11} e^{j\Delta k_h} \]  

(11)

The \( \epsilon_r \) and \( \mu_r \) of SF-3 were found by solving (1) to within a tolerance of \( 10^{-6} \) using the trust-region-reflective method [36]. As discussed above, two-thickness method [9, 16, 22, 42] was utilised to provide the second independent \( S_{11}^{11} \) measurement necessary to find \( \epsilon_r \) and \( \mu_r \) unambiguously. This method was chosen for analytical and computational convenience.

The new probe’s sensitivity to measurement errors was also investigated. Uncertainty analysis was performed on the extracted \( \epsilon_r \) and \( \mu_r \) values taking into account errors in \( S_{11}^{11} \) (\( \sigma_{\epsilon r} \) given in [51]), flange-plate thickness \( h \) (\( \sigma_h = 0.05 \) mm), MUT thickness \( d \) (\( \sigma_d = \sigma_{\Delta} = 0.05 \) mm) and cutoff wavenumber \( k_c \) (\( \sigma_{k_c} = 0.03 k_c \)). Note that \( \sigma_{k_c} \) arises mainly from uncertainties in DRWG aperture dimensions, in particular, errors because of gap width \( \Delta x \) and gap height \( \Delta y \) as well as errors because of rounded aperture corners [54]. Recent work has shown that the value of \( k_c \) varies by approximately 3% when these factors are considered [54]. To the authors’ knowledge, this is the first time that the effect of this error on the \( \epsilon_r \) and \( \mu_r \) extracted using a waveguide probe has been quantified.

One error source which was not considered in this analysis was probe lift-off or air-gap error [37]. To minimise the impact of this error, four vise-grip clamps were used to securely hold the flange plate, MUT and PEC-backing plate together. The clamps were placed surrounding and as close as possible to the DRWG aperture where the field concentration and therefore the impact of possible air gaps was the strongest. Quantifying probe lift-off error requires the two-layer magnetic-current-excited parallel-plane Green’s function, which when combined with the complex analytical form of the DRWG fields, represents a significant theoretical and computational challenge beyond the scope of the work presented here. Quantifying probe lift-off error is therefore left to future work.

The following expression was used to calculate the measurement uncertainty in the real part of \( \epsilon_r \) [55]

\[ \sigma_{\epsilon r}^2 = \left( \frac{\partial \epsilon_r}{\partial S_{11,1}^{11}} \sigma_{S_{11,1}^{11}} \right)^2 + \left( \frac{\partial \epsilon_r}{\partial S_{11,2}^{11}} \sigma_{S_{11,2}^{11}} \right)^2 + \left( \frac{\partial \epsilon_r}{\partial \epsilon_r} \sigma_{\epsilon_r} \right)^2 
\]

(12)

where the superscripts ‘r’ and ‘i’ denote the real and imaginary parts and ‘11’, ‘1’ and ‘11’, ‘2’ denote the first and second \( S_{11}^{11} \) measurements (i.e. thickness 1 and thickness 2 mm), respectively. The partial derivatives in the above expression were estimated using the forward difference approximation. The values for \( \sigma_{\epsilon r} \), \( \sigma_{\epsilon r} \) and \( \sigma_{\mu r} \) were calculated in a similar manner as above. Note that the error values provided by (12) are worst case estimates [55].

3.2 ECCOSORB® SF-3 results

Fig. 3 shows the SF-3 \( \epsilon_r \) (Fig. 3a) and \( \mu_r \) (Fig. 3b) results using the single DRWG probe (blue bars) introduced in this work. The solid black traces are the traditional, destructive NRW technique results using an SF-3 sample which uniformly fills the cross section of the DRWG. These results are provided to serve as a reference. Additional information about these DRWG NRW measurements, including descriptions of measurement procedures and sources of error, can be found in [54]. The widths of the blue bars in the plots represent the errors in \( \epsilon_r \) and \( \mu_r \), that is, \( \pm 2\sigma_{\epsilon r} \) and \( \pm 2\sigma_{\mu r} \), respectively, because of the measurement errors discussed above.

Overall, the SF-3 \( \epsilon_r \) and \( \mu_r \) reference results generally lie within the margins of error of the DRWG probe results. The major discrepancy between the two occurs in the \( \mu_r \) results, where the DRWG probe overestimates the amount of magnetic loss. Although the quality of the DRWG probe \( \mu_r \) estimate presented here is poor, it is consistent with those reported in the literature for similar ND single probe measurement geometries [15, 16, 22, 23, 38, 40–42].

3.3 Flange-plate size

Before concluding, it is worth discussing flange-plate size as this directly affects the probe’s applicability for field measurements. Because of the possibility that reflections from the edges of the flange plate would corrupt the reflection measurements (recall that in the theoretical development, the flange plate is assumed to be infinite in extent), waveguide probes have traditionally been applied to lossy materials. In these cases, the flange plate needs to be large enough such that the unwanted edge reflected wave is attenuated to a degree less than the noise floor of the VNA. This condition is easily met for SF-3 and the 15.24 cm × 15.24 cm flange plate used in the experimental results discussed above. The traditional approach to determining
must possess sufficient time-domain gating the measured reflections can be resolved and thus removed via time-domain gating. Since a vast majority of materials at microwave frequencies possess slower phase velocities than \( c \), (13) can still be used to determine an acceptable flange plate size even if \( \varepsilon_r \) and \( \mu_r \) are unknown by assuming the MUT is free space. Applying this criterion for the DRWG probe presented here results in a minimum flange-plate radius \( \rho = 1.25 \) cm. This is significantly smaller than that stipulated by the traditional flange-size calculation, in which the minimum flange-plate radius is approximately 3 m.

4 Conclusion

In this paper, a new waveguide probe which utilised a DRWG to provide broadband, ND material characterisation results of PEC-backed materials was introduced. The new probe possessed two qualities – broad bandwidth similar to coaxial probes and structure robustness characteristic of rectangular waveguide probes – which made it especially attractive for NDI/NDE applications in the field. The theoretical development of the DRWG probe was discussed in Section 2. This involved the derivation of the theoretical reflection coefficient necessary to characterise the MUT. This was achieved by utilising Love’s equivalence theorem to replace the DRWG aperture with an equivalent magnetic current. The continuity of transverse magnetic fields at the DRWG aperture was then enforced yielding a MFIE, which was subsequently solved for the theoretical reflection coefficient using the MoM. The \( \varepsilon_r \) and \( \mu_r \) of the MUT were then found by minimising the root-mean-square difference between the theoretical and measured reflection coefficients using the trust-region-reflective method. To experimentally verify the new probe, material characterisation results of ECCOSORB® SF-3 were presented and compared with those obtained using the traditional, destructive NRW technique. The probe’s sensitivity to sample thickness, flange-plate thickness, cutoff wavenumber and measured reflection coefficient uncertainties was also discussed. It was observed that the \( \varepsilon_r \), \( \varepsilon_i \) and \( \mu_r \) values returned by the probe were consistent with the reference results; however, there was a significant discrepancy between the probe and reference \( \mu_r \) results. Although the new probe performed poorly in this regard, \( \mu_r \) results of this quality are reported elsewhere in the literature for similar ND single probe measurement geometries. It should be noted that in the analysis presented here, only the contribution from the dominant DRWG mode was considered because of the theoretical and computational complexity of the problem. Incorporating higher-order DRWG modes into the analysis is future work.

5 Acknowledgments

The views expressed in this paper are those of the authors and do not reflect the official policy or position of the U.S. Air Force, the Department of Defense, or the U.S. Government.

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