Chern-Simons-fermion model of quarks

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We propose an extension of the standard model where quarks are viewed as fermions with a “bare” integer (weak) hypercharge which is normalized with a fractional part created by a quantized topological Chern-Simons configuration of the weak gauge fields. Consistency with hypercharge patterns not included in the standard model is shown.

It is well known that the standard model of the strong, weak and electromagnetic interactions [1], in spite of its spectacular success, cannot be complete. The recent experimental indications for nonzero neutrinos masses [2] can be viewed as prescriptions for the charge structures of confined quarks, instead of leptons, the quark hypercharge, as shown in Eq. (1), can be viewed as the sum of an integer part, which is equal to that of its lepton “partner”, and a fractional part which appears as a global hypercharge independent of flavor and handness, and conserved by strong and electroweak interactions. Moreover, the equality between the integer part of quark hypercharges and lepton hypercharges may reflect a discrete electroweak $\mathbb{Z}_2$ symmetry between the quark and lepton families, but at a subquark or “bare” level. Such patterns are not explained or included in the standard model, so that an extended model is demanded.

Preon models of quarks and leptons have been proposed at the level of accounting for the pattern of Eq. (1) [3]. However, to understand Eq. (1), it would mean to assume that the electroweak symmetry below the compositite scale is $SU(2)_L \times U(1)_Y$, with fundamental electroweak gauge bosons and a new superstrong binding force implying proliferation of preons, exotic fermions, and composite particles.

The purpose of this Letter is to show that the above features exhibited by quarks may be understood alternatively in terms of Chern-Simons-fermions, in the sense that a quark may be pictured as a “bare” fermion with integer hypercharge, a sort of predecessor or pre-state of the quark that we will refer to as the prequark [4], “dressed” or normalized with a fractional hypercharge created by a topological Chern-Simons configuration of the weak gauge fields [5], which is introduced to cancel the gauge anomalies produced by the prequark current. In this model, the prequark has the spin, color charge, and flavor of the quark, without existence of new strong interactions as in preon models of quarks and leptons. Leptons are assumed to be absolutely elementary. Thus, aside from the quark hypercharge split, all the aspects of the standard model are retained.

To introduce the Chern-Simons-fermions properly, we follow a series of steps which combines prequarks, leptons, $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge theory, and Chern-Simons field theory. We start by postulating that the primary fermionic constituents of matter are prequarks, which here will be denoted by hats, and leptons. Table 1 gives a classification of their first generation according to the representations they furnish of the standard gauge group, in which right-handed fermions have been replaced by their left-handed charge-conjugate partners. A $\mathbb{Z}_2$-symmetry between prequarks and leptons in the electroweak sector is explicitly exhibited.

The assignment of integer hypercharges for prequarks, however, introduces triangle gauge anomalies of the $U(1) \times SU(2)_L \times U(1)_Y$ type. Instead of adding exotic fermions to cancel these anomalies, as it is usually done [6], we propose an approach with anomaly compensation
terms involving topological Chern-Simons configurations of gauge fields. These local counterterms would restore gauge symmetry and current conservation via quark formalism. The prequark hypercharges given in Table I are then “bare” values. What can we gain with this approach? Firstly, an unpleasant proliferation of basic fermions is avoided. Secondly, these anomaly cancellation counterterms may bring about the required topological fractional hypercharges to construct quarks from prequarks. And thirdly, a fundamental discrete symmetry between prequarks and leptons may be revealed in the electroweak sector. We will show that this is actually the case and that the standard model is then consistently obtained.

Following the standard work [1], we find that the U(1) gauge current

\[ \hat{J}_Y = \tilde{f}_{qL} Y^\mu \frac{g}{2} f_{qL} + \tilde{f}_{\ell L} Y^\mu \frac{g}{2} f_{\ell L}, \]  

(3)

with \( \tilde{f}_{qL} \) and \( f_{\ell L} \) uniting all the left-handed prequarks and leptons of one generation, respectively, exhibits the U(1)x[SU(2)]\(^2\) and [U(1)]\(^3\) anomalies

\[ \partial_\mu \hat{J}_Y = -\frac{g^2}{32\pi^2} \left( \sum_{\tilde{q}_L, \tilde{\ell}_L} \frac{Y}{2} \right) \text{tr} W_{\mu\nu} \tilde{W}^{\mu\nu} \]

\[ -\frac{g^2}{48\pi^2} \left( \sum_{\tilde{f}_L} \left( \frac{Y}{2} \right)^3 \right) F_{\mu\nu} \tilde{F}^{\mu\nu}, \]  

(4)

where

\[ W_{\mu\nu} = \tau^a W^a_{\mu\nu}, \]

\[ \tilde{W}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\lambda\rho} \tau^a W^a_{\lambda\rho}, \]

\[ W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g e^{abc} W^b_\mu W^c_\nu, \]

\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \]

\[ \tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\lambda\rho} F^{\lambda\rho}, \]  

and \( g \) and \( g' \) are the SU(2) and U(1) gauge coupling constants, respectively. The first sum of Eq. (4) runs over the fermions in the doublets, with the hypercharge \( Y \) in units of \( g' \); the second one does over all prequarks and leptons. The anomalies are introduced because these sums do not vanish in each generation:

\[ \sum_{\tilde{q}_L, \tilde{\ell}_L} Y = -8, \quad \sum_{\tilde{f}_L} Y^3 = 24. \]  

(6)

As frequently noted in standard references [1], the terms on the right-handed side of Eq. (6) are divergences of gauge-dependent currents:

\[ \partial_\mu \tilde{J}_Y = -\left( \sum_{\tilde{q}_L, \tilde{\ell}_L} \frac{Y}{2} \right) \frac{\partial_\mu K_\mu}{2} \]

\[ \quad - \left( \sum_{\tilde{f}_L} \frac{Y}{2} \right)^3 \frac{\partial_\mu L_\mu}{2} \]  

(7)

with

\[ K_\mu = \frac{g^2}{8\pi^2} e^{\mu\nu\lambda\rho} \text{tr}(W_\nu \partial_\lambda W_\rho - \frac{2}{3} i g W_\nu W_\lambda W_\rho), \]

\[ L_\mu = \frac{g^2}{12\pi^2} e^{\mu\nu\lambda\rho} A_\nu \partial_\lambda A_\rho. \]

(8)

These quantities, often referred to as Chern-Simons classes or topological currents, are not invariant under SU(2) and U(1) gauge transformations, respectively. Nevertheless, they can be combined with the anomalous fermionic current \( \tilde{J}_Y \) to define a new current

\[ J_\mu^o = \tilde{J}_Y^o + \left( \sum_{\tilde{q}_L, \tilde{\ell}_L} \frac{Y}{2} \right) \frac{K_\mu}{2} + \left( \sum_{\tilde{f}_L} \frac{Y}{2} \right)^3 \frac{L_\mu}{2}, \]  

(9)

which is conserved and free of anomalies. We remark that the bosonic fields are not beset by anomalies, but serve to remove the anomalies from fermionic currents. Also, the local counterterms to be added to the Lagrangian of prequarks and leptons, needed to obtain the anomaly-free current of Eq. (9), are

\[ \Delta \mathcal{L} = \frac{g^2}{2} \left( \sum_{\tilde{q}_L, \tilde{\ell}_L} \right) K_\mu A_\mu, \]  

(10)

where only the non-Abelian Chern-Simons current of Eq. (9) is required; the counterterms with the Abelian current vanish because of the antisymmetry of \( e^{\mu\nu\lambda\rho} \), so that one can always introduce this current in Eqs. (4) and (9).

The charge corresponding to the current of Eq. (9) is

\[ Q_Y = \int d^3 x J_\mu^o = \int d^3 x \left[ \tilde{J}_Y^o + \left( \sum_{\tilde{q}_L, \tilde{\ell}_L} \frac{Y}{2} \right) \frac{K_\mu}{2} \right. \]

\[ \quad + \left( \sum_{\tilde{f}_L} \frac{Y}{2} \right)^3 \frac{L_\mu}{2} \]  

(11)
This charge is not gauge invariant because of the existence of the topological charge associated with the non-Abelian gauge fields; in the case of the Abelian fields, a gauge transformation causes a surface integral that vanishes at infinity. Moreover, it is not conserved. This can be seen by integrating $\partial_\mu J_\mu^V$ over Euclidean 4-space to obtain a change that can be presented in the form

$$ Q_Y(\infty) = Q_Y(-\infty) - \left( \sum_{q_L,\ell_L} \frac{Y}{2} \right) \frac{Q_K}{2}, \quad (12) $$

where

$$ Q_K = \int \partial_\mu K^\mu d^4x = \frac{g^2}{16\pi^2} \int tr(W_{\mu\nu}\tilde{W}^{\mu\nu})d^4x, \quad (13) $$

is the so-called topological charge which can have nonzero values in Euclidean space. In fact, it holds that

$$ Q_K = n, \quad (14) $$

where the integer number $n$ is named the Pontryagin index or winding number and it is defined by the global characteristics of the gauge field. Thus, the current of Eq. (6) is really not conserved. However, quark formation solves this serious problem as we now show.

The anomaly-free current $J_Y$ of Eq. (1) can be separated into the prequark and lepton parts, as follows:

$$ J_{q\ell}^\mu = \frac{f_{qL}}{2} \gamma^{\mu} \tilde{Y} \frac{f_{\ell L}}{2} + \left( \sum_{q_L,\ell_L} \frac{Y}{2} \right) K^\mu \frac{L}{2} + \left( \sum_{f_L} \left( \frac{Y}{2} \right)^3 \right) \frac{L}{2}, \quad (15) $$

so that the prequark current concentrates all the gauge field contributions and the lepton one retains its standard form, as assumed from the beginning. We note that the modified prequark current is a collective quantity that involves prequarks, leptons, and gauge fields. The change of total prequark hypercharge is obtained from Eqs. (12) and (14)

$$ Y_q(\infty) = Y_q(-\infty) - \left( \sum_{q_L,\ell_L} Y \right) \frac{n}{2}, \quad (16) $$

We next define a final hypercharge for each left-handed prequark according to

$$ Y_q = \hat{Y}_q - \left( \sum_{q_L,\ell_L} \frac{Y}{2} N_q \right), \quad \hat{Y}_q = \frac{n}{3}, \quad (17) $$

where we have divided the nonzero value of the whole hypercharge variation by the total number of left-handed prequarks, $N_q = 12$, in each generation. Here, since there is no other restriction to impose, we have advocated the principle of equality or democracy at the local level for all the prequarks of the system. In a sense, it appears that the original integer-charged prequarks effectively “swallow” equal fractions of the global topological charge, in this manner transforming themselves into final prequarks with a greater fractional charge.

Now, working backwards, the extra hypercharge $\Delta Y = n/3$ should be counted as itself a part of the original prequark hypercharge. To make the calculation self-consistent, the assumed hypercharge should already contain this topological piece. In other words, the final values obtained from Eq. (17) are the actual hypercharges that we should apply. Going back to the above Eqs. (4), (7), (9), (10), (11), (12), (13), (14), and (15), we see that the final hypercharges automatically cancel the gauge anomalies of the current and makes it to be gauge invariant and conserved if the Pontryagin index for the non-Abelian gauge fields turns to be

$$ n = 4. \quad (18) $$

In fact, changing the original “bare” hypercharge $\hat{Y}_q$ of prequarks at the distant past by the final physical ones $Y_q$ at the distant future, we have

$$ \sum_{q_L,\ell_L} Y = 0, \quad \sum_{f_L} Y^3 = 0, \quad (19) $$

instead of the values given in Eq. (17), and the physical current is then

$$ J_{q\ell}^\mu = \frac{f_{qL}}{2} \gamma^{\mu} \frac{Y_q}{2} f_{\ell L}, \quad (20) $$

where $f_q$ describes final prequarks. We should note that actually all anomalies cancel within each generation of prequarks and leptons for the above value of $n$. Explicit solutions for pure gauge configurations giving $n \neq 0$ are known from the physics of instantons (14).

At this point, and according to our initial discussion, we are led to associate the final local hypercharges $Y_q$ of Eq. (17) with $n = 4$ as well as the current $J_Y$ of Eq. (20) with standard quarks. It is important here to observe that in our model prequarks and quarks have the same quantum numbers, except hypercharge values. The connection is consistent with the constitutive relations of Eq. (1), the value of $n$ being independent of the hypercharge normalization used (15). The outcome is that initial “bare” prequarks transform into final “dressed” or normalized quarks, which are therefore the genuine physical particles of the system. But prequarks are an essential part of the system because they are primary fermions and the electroweak prequark-lepton symmetry, as inferred from Eqs. (1) and (2), is in the first place. This discrete symmetry is broken by the vacuum of the non-Abelian gauge theory, owing to the existence of topological charge which interpolates (in Euclidean time) between prequarks (at the distant past) and quarks (at the distant future).

On the other hand, in order to make a complete scheme for the transition from prequarks to quarks, and so effectively have the standard model, we must add to the final
hypercharge current of Eq. (20) similar relations applying
to the weak and strong currents. Consequently, we
make the following identifications in the weak sector
\[ J^a_{qL} = \bar{q}_L \gamma^a \frac{\sigma^a}{2} q_L = \bar{q}_L \gamma^a \frac{\sigma^a}{2} q_L = j^a_{qL}, \] 
while in the color sector we identify
\[ J^{a\mu} = \bar{q}_L \gamma^a \frac{\lambda^a}{2} q = \bar{q}_L \gamma^a \frac{\lambda^a}{2} q = j^{a\mu}, \] 
where \( \sigma^a \) and \( \lambda^a \) are the usual 2x2 Pauli and 3x3 Gell-
Mann matrices, respectively. These identifications may
be justified by noting that prequarks and quarks have
the same flavor and color quantum numbers. Also, we
include their connections in the Yukawa sector:
\[ G^{(q)d}_{nm} \bar{q}_{nL} \phi d_{mR} = G^{(q)d}_{nm} \bar{q}_{nL} \phi d_{mR}, \]
\[ C^{(q)u}_{nm} \bar{q}_{nL} \phi^c u_{mR} = G^{(q)u}_{nm} \bar{q}_{nL} \phi^c u_{mR}, \]
where \( m = 1,2,3 \) for the three generations, regardless of
the fact that, as in the standard model, it is not known
yet how to compute Yukawa coupling constants from first principles.
In conclusion, we have constructed step by step a
Chern-Simons-fermion model for quarks motivated by
the observation that their (weak) hypercharges are
connected with leptons by precise relations, not considered
in the standard model, that may reflect a new discrete
symmetry in the electroweak sector. The pattern shows
that the quark fractional hypercharge may be viewed as
the sum of an integer “bare” hypercharge (being equal
to the hypercharge of its lepton partner) and a “dressing”
or normalizing fractional hypercharge \( Y = 4/3 \) which,
remarkably, is independent of the quark flavor and hand-
ness and conserved by strong and electroweak interactions. \[ \] This fractional part was associated with a
topological Chern-Simons configuration of the weak
gauge fields that was introduced in the first place to can-
cel the gauge anomalies of the fermionic currents of the
model. The integer component of the quark hypercharge
was associated with the so-called prequark which carries
the spin, flavor, and color of the quark. We can either
gregard prequarks as bare quarks or quarks as dressed pre-
quarks. Original prequarks lead to an anomalous theory
while final quarks make the theory anomaly-free. In other
words, one may conclude that the complicated collection
of prequarks and topological configurations of gauge
fields behaves consistently as single confined particles:
quarks. In some sense, we find surprising the similiar-
ity that exists between our Chern-Simons-fermion model
for quarks and the fermionic Chern-Simons or composite-
fermion theory to describe the fractional quantum Hall
effect, where electrons with local fractional electric charge
are generated from the binding of electrons of intrinsic in-
teger charge with a number of Chern-Simons flux quanta
which screen the electron charge. \[ \] It shows at least
how wide the applications of gauge field theory combined
with Chern-Simons theory can be.

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\item[10] For example, if the normalization \( Q = T_3 - Y \) is used, \[ \]
leptons in the doublets have a fractional hypercharge \( 1/2 \),
the prequark screening hypercharge becomes \( Y = -2/3 \), but
the Pontryagin index is still \( n = 4 \).
\item[11] Another possibility is to look for a quark-antilepton sym-
metry (instead of a quark-lepton symmetry as done in this
Letter) and so have a normalizing fractional hypercharge
\( Y = -2/3 \), with \( Q(u) = 1 \) and \( Q(d) = 0 \). The Pontryagin
index is the same \( (n = 4) \). We have insisted on the particle
content of the standard model.
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