Natural frequency of skew FGM plates using finite element method

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Abstract. In the present work aims the free vibration analysis of functionally graded material (FGM) skew plates. Kinematics equations are based on the first order shear deformation theory and a four noded rectangular element is used to mesh the plate geometry. Properties of FGM (Stainless steel and Alumina) material constituents of the plate are considered to be as position and assumed to vary along the thickness direction according to a simple power law. Rectangle plate displacement components are rennovated into skew plate geometry by suitable transformation rule. The effects of power law index and boundary condition on natural frequency parameters of FGM skew plates are reported based on FOSD. The related results are discussed briefly. Results are generated for vibration analysis of the FGM skew plate, which may be implemented in the future research involving similar kind of problems.

1. Introduction

The functionally graded materials (FGM) are microscopically heterogeneous and created from isotropic substances inclusive of metals and ceramics. These are a new class of advanced composite materials in which the material properties arrangement constantly from one bottom surface to other surface. The skew FGM plates have attained significant importance to researchers from long time because of its epitome performance which include excessive heat resistance, mechanical energy and strength. Skew plates have many engineering applications such as aeronautical, parallelogram slabs in civil structures, skew bridges and ship hulls in marine. Because of realistic significance and its mathematical complexity, many efforts were excited to the vibrational analyses of FGM skew plates using both an analytical or numerical response procedure. The numerical method like the finite element method (FEM) is used to solve the vibration analysis of FGM skew plates.

The boundary situations of skew plate issues was studied by Nair and Durvasula [1]. They have been approached via the variational method of Ritz, a double series of beam feature functions being the use of the proper combination of various boundary situations. Mizusawa et.al. [2] employed the vibrations of skew plates with the aid of the Rayleigh-Ritz technique with B-spline features as coordinate functions. Liew et al. [3] studied the vibrational analysis of thick skew plate the use of Mindlin shear deformation concept. Sengupta [4] studied the skew rhombic plates in transverse bending the use of a simple finite detail technique. Liew and Han [5] presented the bending evaluation of a simply supported thick skew plate based on the first-order shear deformation Reissner/Mindlin principle. Woo et.al. [6] used integrals of Legendre polynomials on p-version finite element approach to obtained natural frequencies and mode shapes of skew plates with and without cut-outs. Eftekhari
and Jafari [7] proposed the vibration analysis of rectangular and skew Mindlin plates with specific boundary situations through combined finite element-differential quadrature method. Combination of these approaches is easier than the individually carried out by the FEM or DQM. Wang et al. [8] studied the differential quadrature method (DQM) for an accurate free vibration evaluation of skew plates. An advanced the analytical solutions for skewed thick plates on elastic foundation were presented by Pang-jo and Yun [9]. The free vibration analysis of isotropic and laminated composite skew plates with finite element methods have been studied by Srinivasa et al. [10].

FGM suggests hastily growing the region of research that consists of material technological expertise, mechanics, and dynamics. The finite-element models had been presented to study concerning problem involving of free vibration. Da-Guang Zhang and You-He Zhou [11] examined the theoretical evaluation of FGM (functionally graded materials) thin plates primarily based on the physical neutral surface. Lucia Della Croce and Paolo Venini [12] determined governing equations of square plate’s behaviour fabricated from functionally graded substances (FGM) through Reissner–Mindlin plate concept. Modal analysis of FGM plates was studied by Ramu and Mohanty [13, 14].

The finite element method (FEM) is considerably analysed as a powerful numerical approximate modelling technique, which shows the advantages of the coupled system of equations, smooth convergence rate, and easy modelling. The objective of this work is to derive the governing equation from the skew co-ordinate system with right co-ordinate modifications. The existing numerical experiment solutions were in comparison with the referenced outcomes. Finally, the accuracy and the convergence characteristics of non-dimensional frequency parameters are investigated via analysing skew Mindlin plates with various skew angles, power index and numerous boundary situations.

2. Mathematical formulation
2.1 Functionally graded material plates

The material configuration of an FGM changes steadily through-the-thickness. The graphical representation Vc and Vm (volume fraction of ceramic and metal) of FGM through along the thickness direction of the example as shown in fig.1.

![Fig. 1 FGM plate neutral surface](image)

2.2 Simple power law

More common in the analysis for functionally graded materials with two constituent materials the variations through the thickness of material properties P can be expressed as

\[ P(z) = P_m + (P_c - P_m)V_c \]

(1)

Here P can represent Young’s modulus E, Poisson ratio μ, and the mass density ρ, and Vc (z) is the volume fraction variation of the ceramic material, and it is assumed to follow a simple power-law distribution as

\[ V_c = \left( \frac{1}{2} + \frac{z}{h} \right)^\nu \]

(2)
Where \(-h/2 \leq z \leq h/2\) is coordinate through the thickness from the middle surface to ceramic and metal sides, and \(n\) is a power law index. Working range of design requirements in this case is based on a power law indexed as shown in the figure 2.

![Variation of Young's modulus along the thickness of the FGM plate](image)

Fig. 2 Variation of young's modulus along the thickness of the FGM plate

2.3 Physical neutral surface of the FGM plate

The variation of the material composition along thickness the neutral plane does not coincide with the geometrical mid-plane of the plate. The distance of the neutral plane (\(d\)) from the geometric mid plane may be expressed as

\[
d = \frac{1}{h} \int_{-h/2}^{h/2} zE(z)dz
\]

(3)

2.4 Oblique boundary transformation

The skewness of the plate edges may not be parallel to global axes \(x\) and \(y\), so it is in terms of the displacements \(w\), \(\theta_x\) and \(\theta_y\). The position plane edge translations \(w_i\), \(\theta_{xi}\) and \(\theta_{yi}\), these are tangential and normal to the oblique edge, at such edges to require the boundary conditions. Where, \(\theta_{x}\) and \(\theta_{y}\) represent the middling rotations of the standard to thereference plane, tangential and normal to the askew edge. So it is necessary to transform the element matrices corresponding to axes \((x, y)\) along which the boundary conditions are specified. The displacement transformation for a node \(n^{th}\) on the oblique boundary is expressed as

\[
\begin{bmatrix}
w \\
\theta_x \\
\theta_y
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \alpha & -\sin \alpha \\
0 & \sin \alpha & \cos \alpha
\end{bmatrix}
\begin{bmatrix}
w \\
\theta_x \\
\theta_y
\end{bmatrix}
\]

(4)

This relationship transformation can be represented

\[
u_n = T_r \bar{u}
\]

where \(u_n\) and \(\bar{u}\) are the generalized displacement vectors in the global and local edge coordinate systems. This matrix is only for three degrees of freedom per node. Thus, for an \(n\)-noded boundary element, the element transformation matrix can be expressed as

\[
[T_r] = \begin{bmatrix}
T_x & 0 & 0 \\
0 & T_x & 0 \\
0 & 0 & T_x
\end{bmatrix}
\]

(6)
3. Governing equation of motion

The governing equation of motion of skew plate may be derived from the Hamilton’s principle, which requires the energy functions to satisfy the condition.

\[ \delta \int_{t_1}^{t_2} \Pi = \delta \int_{t_1}^{t_2} (U - T) dt \]

\[ = \delta \left[ \frac{1}{2} \int_{h_1}^{h_2} e^T \left[ D \right] e dv dt - \int_{h_1}^{h_2} \rho \dot{w}^2 dv dt \right] = 0 \]

where \( \delta \) is the variation operator and the displacement field over an element can be defined by \( w = Nw^e \) where \( w^e \) is the vector of nodal displacements and nodeless coefficients, and \( N \) is the shape function. After differentiating above Eq. (7) with respect to time, the final energy equation expressed in matrix form can be obtained after substituting the strain–displacement and constitutive relationships.

\[ \Pi_e = \left\{ \begin{array}{l}
\frac{1}{2} \int_{v_1}^{v_2} [w^e]^T [T_r]^T [B]^T [D][B][T_r][w_e] dv \\
- \frac{1}{2} \int_{v_1}^{v_2} \frac{d}{dt} [T_r]^T [N]^T \rho [N][T_r] \frac{dw_e}{dt} dv 
\end{array} \right\} \]

The minimization of energy function with respect to the nodal displacement \( w_e \) for an element results in

\[ \frac{\partial \Pi_e}{\partial w_e} = \left\{ \begin{array}{l}
\int_{v_1}^{v_2} [T_r]^T [B]^T [D][B][T_r] w_e dv \\
- \int_{v_1}^{v_2} \frac{dw_e}{dt} [T_r]^T [N]^T \rho [N][T_r] \frac{dw_e}{dt} dv 
\end{array} \right\} \]

The element stiffness and the mass matrices are then arrived at after mapping into the standard (r, s) co-ordinate system as

\[ K_e = \left[ [T_r]^T [B]^T [D][B][T_r] dv \right] = \int_{\alpha_1}^{\alpha_2} \int_{\beta_1}^{\beta_2} [T_r]^T [B]^T [D][B][T_r] \det J ds dr \]

\[ M_e = \left[ [T_r]^T [N]^T \rho [N][T_r] dv \right] = \int_{\alpha_1}^{\alpha_2} \int_{\beta_1}^{\beta_2} [T_r]^T [N]^T \rho [N][T_r] \det J ds dr \]
here \( \det J dsdr = dx dy, I = \rho h \), also \( N \) is the shape functions given as:

\[
N = \begin{bmatrix}
N_1 & 0 & 0 & N_4 & 0 & 0 & N_2 & 0 & 0 \\
0 & N_1 & 0 & 0 & N_4 & 0 & 0 & N_2 & 0 \\
0 & 0 & N_1 & 0 & 0 & N_4 & 0 & 0 & N_2 \\
0 & 0 & 0 & N_1 & 0 & 0 & N_4 & 0 & 0 \\
\end{bmatrix}
\]  

(11)

The free vibrational analysis of skew plate governing equation of motion can be expressed as:

\[
[K] - \omega^2 [M] = 0
\]

(12)

where \( \omega \) natural frequencies of the skew plate.

4. Results and discussion

The results of the present numerical model has been validate in tables in terms of non-dimensional frequency parameter and critical buckling parameter for skew plates with different boundary conditions. The isotropic material properties were considered for comparison with results available in the literature [3] and good agreement between the results was found in table 1.

### Table 1 Comparison of frequency parameters, \( \lambda \) of skew plates having different boundary condition and \( W/L=1, h=0.1 \) m, poisons ratio 0.3.

| Degrees | SSSS | 1   | 2   | 3   | Mode sequence number | 4   | 5   | 6   | 7   | 8   |
|---------|------|-----|-----|-----|----------------------|-----|-----|-----|-----|-----|
| 0       | Ref.[3] | 1.931 | 4.605 | 4.605 | 7.064 | 8.605 | 8.605 | 10.793 | 10.793 |
|         | Present | 1.952 | 4.699 | 4.699 | 7.182 | 8.862 | 8.862 | 11.038 | 11.038 |
| 15      | Ref.[3] | 2.037 | 4.506 | 5.184 | 7.071 | 9.007 | 9.374 | 10.227 | 11.894 |
|         | Present | 2.089 | 4.560 | 5.247 | 7.141 | 9.068 | 9.634 | 10.402 | 12.033 |
| 30      | Ref.[3] | 2.419 | 4.888 | 6.489 | 7.453 | 10.398 | 10.398 | 11.665 | 13.611 |
|         | Present | 2.620 | 4.668 | 6.505 | 7.220 | 9.487 | 10.053 | 11.713 | 13.165 |
| 45      | Ref.[3] | 3.354 | 6.034 | 8.733 | 9.304 | 11.677 | 13.548 | 14.656 | 16.795 |
|         | Present | 4.062 | 5.810 | 8.537 | 9.991 | 11.753 | 12.01 | 15.232 | 15.975 |
| SCSC    | 0     | Ref.[3] | 2.699 | 4.971 | 5.990 | 7.973 | 8.787 | 10.250 | 11.338 | 12.024 |
|         | Present | 2.735 | 5.04 | 6.095 | 8.085 | 8.971 | 10.485 | 11.523 | 12.245 |
| 15      | Ref.[3] | 2.848 | 5.122 | 6.395 | 7.968 | 9.444 | 10.867 | 11.070 | 12.860 |
|         | Present | 2.940 | 5.221 | 6.738 | 8.293 | 9.702 | 11.613 | 11.728 | 13.551 |
| 30      | Ref.[3] | 3.370 | 5.708 | 7.738 | 8.444 | 11.174 | 11.373 | 12.994 | 14.564 |
|         | Present | 3.361 | 5.322 | 7.933 | 8.301 | 10.424 | 11.417 | 13.776 | 14.774 |
| 45      | Ref.[3] | 4.596 | 7.152 | 9.953 | 10.701 | 12.885 | 14.627 | 15.801 | 18.061 |
|         | Present | 4.412 | 5.772 | 8.257 | 10.447 | 11.334 | 12.149 | 14.405 | 15.409 |
| CCCC    | 0     | Ref.[3] | 3.292 | 6.276 | 6.276 | 8.793 | 10.357 | 10.456 | 12.524 | 12.524 |
|         | Present | 3.339 | 6.385 | 6.385 | 8.929 | 10.594 | 10.694 | 12.759 | 12.759 |
| 15      | Ref.[3] | 3.474 | 6.223 | 6.959 | 8.870 | 10.818 | 11.282 | 12.037 | 13.643 |
|         | Present | 3.470 | 6.275 | 7.010 | 8.972 | 10.961 | 11.575 | 12.290 | 13.881 |
| 30      | Ref.[3] | 3.892 | 6.554 | 8.325 | 9.388 | 11.843 | 12.590 | 14.031 | 15.737 |
|         | Present | 3.604 | 8.477 | 9.471 | 11.785 | 14.104 | 15.872 | 16.989 | 19.059 |
| 45      | Ref.[3] | 5.311 | 8.113 | 11.992 | 12.069 | 15.165 | 16.315 | 20.674 | 21.089 |

For this analysis, the FGM plate consists of steel (SUS304) and alumina (Al2O3). Boundary conditions should be noted that in all tables and figs, S represents simply supported and C denotes clamped. In figure 4 shows the skew angles 0, 15, 30, 45 and 60 degrees. Variation of the frequency parameter of a skew FGM plate versus power law index with various skew angles and simply supported and clamped cases as shown in Fig. 4 and 5. The increase of power law index makes to reduction of the ceramic fraction of volume. The reduced ceramic content cause to decrease the stiffness of FGM plate, it may cause reduce the natural frequencies of the plate. Similarly, it can be observed that the figure 4 and 5.
the frequency parameter rapidly changes the index value in between the 0 to 2. The increased index value causes to reduce the parameter of skew plate natural frequencies. The skewness also increases the frequency parameter of the plate with simply supported and clamped boundary condition cases.

5. Conclusions
The free vibration analysis of FGM skew plate with different boundary conditions is studied in this work. The material properties are assumed to vary according to a simple power law, the properties vary along the thickness direction only. An efficient finite element modal which is based on the first order shear deformation theory is used for this study. The present method results are compared with available literature results for the efficiency. The natural frequency of the FGM skew plate effects with skew angle and power law index are studied in detail. From the numerical experiments the frequency parameter reduces by increase the power law index. Similarly with increase the skew angle frequency parameter increases. The vibration characteristics of FGM skew plate with different boundary conditions is studied with different index value and skew angle.

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