Multiplicity dependence of correlation functions in $\bar{p}p$ reactions at $\sqrt{s} = 630$ GeV

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Abstract

Discussions about Bose-Einstein correlations between decay products of coproduced W-bosons again raise the question about the behaviour of correlations if several strings are produced. This is studied by the multiplicity dependence of correlation functions of particle pairs with like-sign and opposite-sign charge in $\bar{p}p$ reactions at $\sqrt{s} = 630$ GeV.

1 Introduction

Recently, there has been much discussion regarding the possibility that Bose-Einstein correlations and other interconnection effects between decay products of different strings could affect the measurement of the $W$ mass [1]. Since measurements are hampered by low statistics and experimental difficulties [2–4], the question arises whether and where effects of the superposition of several strings can be tested independently. Within the Dual Parton Model [5] for hadron-hadron reactions, the charged-particle multiplicity $N$ is expected to rise with the number of strings. The decrease in the observed correlation strength $\lambda$ of the Bose-Einstein effect as function of multiplicity [6,7] may be explained in terms of products of different strings, where each string symmetrizes separately [8,9]. To test this idea further, it is desirable to investigate quantitatively the multiplicity dependence of like-sign particle correlations.

Improvements in experimental analysis techniques [10] and larger data samples make it possible to repeat and extend Bose-Einstein analyses with an advanced strategy. [11] In this Letter, we investigate correlation functions of
particle pairs with like-sign (ℓs) and opposite-sign (os) charges at different total charged-particle multiplicities with the same model-independent strategy and good statistics. The bias introduced by selecting events of a given overall multiplicity is eliminated with the use of “internal cumulants” [10].

2 Data sample and normalized density correlation functions

The data sample consists of 1,200,000 non-single-diffractive \( \bar{p}p \) reactions at \( \sqrt{s} = 630 \text{ GeV} \) measured by the UA1 central detector [12]. Only vertex-associated charged tracks with transverse momentum \( p_T \geq 0.15 \text{ GeV/c} \), \( |\eta| \leq 3 \), good measurement quality and fitted length \( \geq 30 \text{cm} \) have been used. To avoid acceptance problems, we restrict the azimuthal angle to \( 45^\circ \leq |\phi| \leq 135^\circ \) (“good azimuth”). Since, however, the multiplicity for the entire azimuthal range is the physically relevant quantity, we select events according to their uncorrected all-azimuth charged-particle multiplicity \( N \). The corrected multiplicity density is then estimated as twice \( n \), the charged-particle multiplicity in good azimuth: \( (dN_c/d\eta) \simeq 2(dn/d\eta) \).

All quantities measured are defined in the notation of correlation integrals [13],

\[
    r_2(Q) = \frac{\rho_2(Q)}{\rho_1 \otimes \rho_1(Q)} = \frac{\int_\Omega d^3p_1 d^3p_2 \rho_2(p_1, p_2) \delta [Q - q(p_1, p_2)]}{\int_\Omega d^3p_1 d^3p_2 \rho_1(p_1) \rho_1(p_2) \delta [Q - q(p_1, p_2)]},
\]

with \( p_i \) the three-momenta, \( p_i \) the corresponding four-momenta, and \( q \equiv \sqrt{-(p_1 - p_2)^2} \). As usual, \( \rho_i \) denotes the joint particle density of order \( i \) which in integrated form is the appropriate factorial moment. The integration region \( \Omega \) is identical with our experimental cuts as specified above and specifically refers to the good-azimuth region. All particles have been assumed to be pions.

In \( \bar{p}p \) reactions and in full phase space, the number of positive and negative particles are equal, as are the corresponding one-and two-particle densities \( \rho_1 \), \( \rho_2 \). Given the charged particle density \( \rho_1(p) = \rho_1^+(p) + \rho_1^-(p) \), we hence assume that \( \rho_1^+(p) = \rho_1^-(p) = \frac{1}{2}\rho_1(p) \) and therefore also \( \rho_1^+ \otimes \rho_1^-(Q) = \frac{1}{4}\rho_1 \otimes \rho_1(Q) \) etc. The like-sign and opposite-sign normalised two-particle densities become, respectively,

\[
    r_2^{\ell_s}(Q) = \frac{\rho_2^{\ell_s}(Q)}{\rho_1 \otimes \rho_1^{\ell_s}(Q)} = \frac{\rho_2^{++}(Q)}{\rho_1^+ \otimes \rho_1^+(Q)} = \frac{\rho_2^{--}(Q)}{\rho_1^- \otimes \rho_1^-(Q)} \simeq \frac{\rho_2^{++}(Q) + \rho_2^{--}(Q)}{\frac{1}{2}\rho_1 \otimes \rho_1(Q)},
\]

\[
    r_2^{os}(Q) = \frac{\rho_2^{os}(Q)}{\rho_1 \otimes \rho_1^{os}(Q)} = \frac{\rho_2^{+-}(Q)}{\rho_1^+ \otimes \rho_1^-(Q)} = \frac{\rho_2^{-+}(Q)}{\rho_1^- \otimes \rho_1^+(Q)} \simeq \frac{\rho_2^{+-}(Q) + \rho_2^{-+}(Q)}{\frac{1}{2}\rho_1 \otimes \rho_1(Q)}.
\]
Fig. 1 shows the normalized density correlation functions $r_2$ for pairs of like-sign ($\ell s$) charge and for opposite-sign ($os$) charge separately. Restricting the total uncorrected charged-particle multiplicity $N$ in $|\eta| \leq 3$ to the windows $5 \leq N \leq 9$ (Fig. 1a) and $28 \leq N \leq 35$ (Fig. 1b), one obtains for the corrected particle density in the central rapidity region $dN_c/d\eta = 1.22 \pm 0.09$ and $dN_c/d\eta = 5.32 \pm 0.17$ respectively. Both the like-sign and opposite-sign correlation densities show a strong dependence on multiplicity.

3 Cumulants for fixed multiplicity and multiplicity ranges

While the multiplicity dependence of $r_2$ is of much interest, a quantitative analysis must both remove combinatorial background by calculating the cumulants and also correct for the bias introduced by working at fixed multiplicity. This bias arises because at fixed total multiplicity $N$, standard second-order factorial cumulants, defined by

$$\kappa^2(P_1, P_2 | N) = \rho_2(P_1, P_2 | N) - \rho_1(P_1 | N) \rho_1(P_2 | N), \quad (2)$$

are nonzero even when particles are completely uncorrelated; for example, for purely uncorrelated multinomially-distributed events, $\kappa^\text{mult}(P_1, P_2 | N) = -(1/N)\rho_1(P_1 | N) \rho_1(P_2 | N) \neq 0$. Because these correlations result solely from the restriction of the sample to a fixed multiplicity, they are termed “external” and should be removed. The “internal cumulants” of Ref. [10]

$$\kappa^I_2(P_1, P_2 | N) \equiv \kappa_2(P_1, P_2 | N) - \kappa_2^\text{mult}(P_1, P_2 | N)$$

$$= \rho_2(P_1, P_2 | N) - \frac{N(N-1)}{N^2} \rho_1(P_1 | N) \rho_1(P_2 | N) \quad (3)$$

correct this bias exactly: they are zero whenever the $N$ particles behave multinomially.

The following arguments lead to a unique choice of normalization $\rho_2^{\text{norm}}$. The correctly normalized internal cumulants $K^I_2(P_1, P_2 | N) = \kappa^I_2(P_1, P_2 | N)/\rho_2^{\text{norm}}$ should be independent of multiplicity $N$ whenever $\rho_1(P | N)$ and $\rho_2(P_1, P_2 | N)$ depend on $N$ only globally i.e. have the same shapes (in terms of the momenta) for different $N$. Such “shape constancy”,

1 The usual Bose-Einstein analysis assumes that $r_2^{\ell s}$ tends to a constant for large $Q$, i.e. $r_2^{\ell s}_B(Q \geq 1) = \text{constant}$. It should be clear from Figure 1 that no such constancy exists for limited-multiplicity windows.
\[
\rho_1(p|N') = \frac{N'}{N} \rho_1(p|N),
\]

(4)

\[
\rho_2(p_1, p_2|N') = \frac{N'(N'-1)}{N(N-1)} \rho_2(p_1, p_2|N),
\]

(5)
coupled to the requirement that

\[
K_2^I(p_1, p_2|N') = K_2^I(p_1, p_2|N),
\]

(6)

fixes the appropriate normalisation to be

\[
\rho_2^{\text{norm}}(N) = \frac{N(N-1)}{N^2} \rho_2(p_1, p_2|N),
\]

(7)

the quantity to which \( \rho_2 \) defaults when \( p_1 \) and \( p_2 \) become statistically independent. The correctly normalized internal cumulant at fixed multiplicity consequently reads

\[
K_2^I(p_1, p_2|N) = \frac{N^2}{N(N-1)} \frac{\rho_2(p_1, p_2|N)}{\rho_1(p_1|N) \rho_1(p_2|N)} - 1.
\]

(8)

Extending these arguments to multiplicity ranges \( N \in [A, B] \), specifying them for different charge combinations and adopting the correlation integral Eq. (1), we arrive at our measurement prescription for the normalized internal cumulants for \( \ell s \) and \( os \) pairs,

\[
\overline{K}_2^{I\ell s}(Q|AB) = \frac{\overline{n}_{\ell s}^2}{n(n-1)_{\ell s}} r_{2\ell s}(Q|AB) - 1,
\]

(9)

\[
\overline{K}_2^{Ios}(Q|AB) = \frac{\overline{n}_+ \overline{n}_-}{n_+ n_-} r_{2os}(Q|AB) - 1,
\]

(10)

where \( \overline{n}_{\ell s} (= \overline{n}_+ = \overline{n}_-) \), \( n_+ n_- \) and \( n(n-1)_{\ell s} \) are the mean numbers of positive or negative particles, \( os \) pairs and \( ++ \) (or \( -- \)) pairs respectively in the whole interval \( \Omega \) and in the multiplicity range \( [A, B] \). Eqs. (9) and (10) are obtained \([14]\) by assuming shape constancy for all averaging procedures\(^2\).

Since all quantities shown here and below are to be understood as mean values in multiplicity ranges, we henceforth (and in Fig. 1) omit the bar on the symbols. Fig. 2 shows both like- and opposite-sign internal cumulants for two selections of \( dN_c/d\eta \). Three features are apparent:

\(^2\) The renormalization factors in front of \( r_2 \) were used in Ref \([15]\) and elsewhere. Here, they are derived from Eqs. (4), (5) and the requirement (6).
I. The importance of changing from $r_2$ to $K_2^I$ lies in the fact that the latter demarcate clearly the “line of no correlation” which, due to the fixed-multiplicity conditioning, is not equal to unity for $r_2$ in Fig. 1a.

II. Internal cumulants integrate to zero over the entire phase space; hence the positive part of $K_2^I$ at small $Q$ is compensated by a negative part at larger $Q$. Physically, this means that particles like to cluster, so that there is a surfeit of pairs at small $Q$ and a dearth of pairs at large $Q$ compared to the uncorrelated case.

III. The dependencies of the like- and opposite-sign cumulants on multiplicity are rather similar in that both decrease markedly with $dN_c/d\eta$. This is discussed more fully below.

4 Multiplicity dependence of normalized cumulants

The behaviour of $K_2^{I\ell s}$ and $K_2^{I\ell os}$ as a function of $dN_c/d\eta$ suggests that both could have approximately the same functional dependence $C(dN_c/d\eta)$ on multiplicity density. Under this hypothesis, where both depend on $dN_c/d\eta$ with the same functional form\[3\],

\[
K_2^{I\ell s}(Q|N_c) = Y^{\ell s}(Q) \, C(N_c, Q),
\]
\[
K_2^{I\ell os}(Q|N_c) = Y^{\ell os}(Q) \, C(N_c, Q),
\]

the quotient of the cumulants should be independent of multiplicity,

\[
\frac{K_2^{I\ell s}(Q|N_c)}{K_2^{I\ell os}(Q|N_c)} = \frac{Y^{\ell s}(Q)}{Y^{\ell os}(Q)} = \text{(constant in } N_c). \tag{12}
\]

Figure 3a shows that (12) holds approximately. (In the region where both cumulants are near zero, no meaningful quotients can be formed.)

Having shown that like- and unlike-sign internal cumulants behave approximately in the same way as functions of $dN_c/d\eta$, we now look for an appropriate functional form for this dependence. A first hypothesis is that $K_2^I$ depends inversely on $N_c$,

\[
K_2^{Ia}(Q|N_c) = Y^a(Q) \, C(N_c, Q) = Y^a(Q) \, N_c^{-1} \quad a = \ell s, os. \tag{13}
\]

This can be motivated theoretically by

\[3\] For simplicity we write $N_c$ instead of $dN_c/d\eta$ here and below.
a) **Resonances:** If the unnormalized cumulants $\kappa^{I\text{os}}_2$ and $\kappa^{I\ells}_2$ were wholly the result of resonance decays and if the number of resonances were proportional to the multiplicity $N_c$, then $\kappa^{I}_2 \propto N_c$. Assuming shape constancy $\rho_1(p|N_c) \propto N_c \rho_1(p)$ gives $\rho_1 \otimes \rho_1 \propto N_c^2$, and hence after normalization, the resonance-inspired guess is

$$K^{I}_{2} = \frac{\kappa^{I\text{res}}_2}{\rho_1 \otimes \rho_1} \propto \frac{1}{N_c}.$$ 

b) **Independent superposition** in momentum space of $\nu$ equal strings would also lead to $K^{I}_{2} = \nu \kappa^{I\text{string}}_2/(\rho_1 \otimes \rho_1) \propto (1/\nu) \propto (1/N_c)$. Deviations can occur if the strings are unequal or if there is no strong proportionality between $\nu$ and $N_c$.

Eq. (13) implies that $K^{Ia}_{2}(Q|N_1)/K^{Ia}_{2}(Q|N_2) = (N_2/N_1) = (\text{constant in } Q)$ for two multiplicities $N_1$ and $N_2$ for the same $a (= \ells$ or $\text{os}$). In Fig. 3b, we show the quotient of cumulants for two multiplicities. Surprisingly, we find not one but two regions of approximate constancy in $Q$, one for small $Q \lesssim 0.4 \text{ GeV}$, one for large $Q \gtrsim 2 \text{ GeV}$, where only the latter corresponds to the value $(N_2/N_1)$ expected from (13), shown as the dotted line. At small $Q$, one must clearly look for other functional forms for $C(N_c, Q)$. Some phenomenological guesses are as follows.

c) In the **Quantum Statistical approach** of Bose-Einstein correlations [16], no multiplicity dependence is expected with our renormalization of $r_2$ in Eq. (9).

d) A mixture of processes a) and c) could result in a dependence

$$K^{I\ells}_{2}(Q|N_c) \approx a(Q) + \frac{b(Q)}{N_c}.$$ 

(14)

However, no comparable picture is available for the unlike-sign case.

In order to test the above ideas, we plot the cumulants against $(dN_c/d\eta)^{-1}$ as follows. To avoid local statistical fluctuations, the normalized cumulants $K^{I\ells}_{2}(Q)$ and $K^{I\text{os}}_{2}(Q)$ are fitted with suitable functions in restricted $Q$-ranges for each $dN_c/d\eta$ (not shown). The best-fit values at small $Q$ (0.1 GeV/c) and at large $Q$ (7 GeV/c) are plotted in Figs. 4a and 4b respectively. The $K^{I\ells}_{2}(Q)$ have also been fitted to an exponential parametrization for $Q < 1 \text{ GeV/c}$,

$$K^{I\ells}_{2}(Q) = a + \lambda e^{-RQ},$$ 

(15)

since the reported increase [6,7] of the radius $R$ in case of $\ells$ (Bose-Einstein) functions could cause part of the decrease with $dN_c/d\eta$. The $\lambda$ values obtained are thus corrected for effects of varying radii, but still indicate a pronounced multiplicity dependence (crosses in Fig. 4a) similar to the model-
independent cumulants (filled circles). Fig. 4 shows that, as in Fig. 3b, the 
\(1/N_c\)-dependence is satisfied only for large but not for small \(Q\). The \(a+b/N_c\) 
dependence in Fig. 4a (solid line) provides a possible, but hardly unique, ex-
planation.

The important region around \(Q \approx 1 \text{ GeV}\), where the phase space contributes 
maximally [19] is unfortunately difficult to investigate, because there the \(K^{I_{os}}\) 
are decreasing rapidly with increasing \(Q\) while the \(K^{I_{\ell s}}\) are already small 
(Fig. 2). A \(1/N_c\)-dependence (due to resonances such as \(\rho^0\)) in this dominant 
region around 1 GeV/c could presumably cause the large-\(Q\) region to follow 
suit via missing pairs.

5 Summary

The multiplicity dependence of like-sign and opposite-sign two-body correlation 
functions have been studied with the same model-independent strategy. The bias introduced by selecting events of a given overall multiplicity is elim-
inated by measuring internal cumulants. We observe that

- the like-sign and opposite-sign cumulants have very similar multiplicity de-
pendence,
- there exist two regions, one at small \(Q\), where the multiplicity dependence 
of both is weaker than \(1/N_c\), and one at large \(Q\) (\(\geq 2 \text{ GeV/c}\)), where the 
cumulants are negative and follow roughly an \(1/N_c\) law,
- a third region around \(Q = 1 \text{ GeV/c}\) shows small and rapidly changing 
cumulants.

The decrease of \(\ell_s\) functions at small \(Q\) with increasing multiplicity favours 
an interpretation in terms of a suppression of Bose-Einstein correlations be-
tween products of different strings. In the Dual Parton Model approach [5], 
multiparton collisions corresponding to multipomeron exchange are expected 
to contribute to the inelastic cross section. The observed energy dependence of 
multiplicity distributions supports this view [20]. Up to 2-3 pomeron exchanges 
might occur at our highest multiplicities. This could explain quantitatively the 
the corresponding suppression of \(\ell_s\) (Bose-Einstein) functions in Fig. 4a. But this 
interpretation is not unique. Adopting e.g. the “core-halo picture” [21], one 
could explain the observations by assuming that long-lived resonances (such

\footnote{The values of \(R\) from the fit to Eq. (15) increase by about 30\% over the range 
of \(dN_c/dy\) considered. A stronger dependence of \(R\) on multiplicity and strongly 
decreasing \(\lambda\) values have been observed when fitting Gaussian functions [6] and 
extending the fit range to 2 GeV/c. This indicates the dependence of the results on 
the choice of fit functions and regions. Here, we are emphasizing the region of small 
\(Q\) where Gaussian fits fail completely [17,18].}
as the $\omega$ and $\eta$) are produced more frequently at larger $N_c$ thereby increase the relative strength of the halo [22] and hence do not contribute to Bose-Einstein correlations. Resonance decays (Regge terms) should arguably also contribute to $\ell s$-functions [23]. One could therefore try to explain alternatively the decrease of $\ell s$ functions with $N_c$ by a mixture of Bose-Einstein correlations (assumed to be constant in $N_c$) with resonance production.

All this leaves unexplained, however, the similar behaviour of $os$ and $\ell s$ functions. Theoretical work [5,8,9,24] is challenged by the above experimental results. Together with the results of other reactions, they represent a new piece of information [25] in the colourful puzzle of multiparticle production.

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Fig. 1. Normalised moments $r_{ls}^2$ versus $Q$ for like-sign pairs (filled circles) and $r_{os}^2$ versus $Q$ for opposite-sign pairs (open circles) at a) $dN_c/d\eta = 1.2$ and b) $dN_c/d\eta = 5.3$.

Fig. 2. Internal cumulants Eqs. (9)–(10) for $ls$ pairs (filled circles) and $os$ pairs (open circles) for multiplicity densities a) $dN_c/d\eta = 1.2$ and b) $dN_c/d\eta = 5.3$. 
Fig. 3. a) The ratio Eq. (12), shown for two selections of $dN_c/d\eta$. b) Ratio of two like-sign and two opposite-sign cumulants at different multiplicities $dN_c/d\eta$. The region around $Q = 1$ GeV has been omitted because both numerators and denominators are near zero.

Fig. 4. a) Multiplicity dependence of $K_{2}^{lls}$ (filled circles) and $K_{2}^{los}$ (open circles), both at $Q = 0.1$ GeV, plus $\lambda$ values (crosses) of exponential fit (15). b) as in a) but for the large $Q = 7$ GeV-region. Note from Fig. 2 that the cumulants are negative at large $Q$. For better comparison of the respective dependencies on $dN_c/d\eta$, the absolute values have been scaled by constant factors. Solid lines are best fits to the $\lambda$'s using Eq. (15).