Multi-messenger Emission from Tidal Waves in Neutron Star Oceans

Andrew G. Sullivan,1★ Lucas M. B. Alves,1 Georgina O. Spence,2 Isabella P. Leite,3 Doğa Veske,1 Imre Bartos,4 Zsuzsa Márka,5 Szabolcs Márka1
1 Department of Physics, Columbia University in the City of New York, New York, NY 10027, USA
2 Department of Mathematics, Barnard College of Columbia University in the City of New York, New York, NY 10027, USA
3 Department of Biomedical Engineering, Columbia University in the City of New York, New York, NY 10027, USA
4 Department of Physics, University of Florida, Gainesville, FL 32611-8440, USA
5 Columbia Astrophysics Laboratory, Columbia University in the City of New York, New York, NY 10027, USA

Accepted 2023 January 31. Received 2023 January 9; in original form 2022 June 3

ABSTRACT
Neutron stars in astrophysical binary systems represent exciting sources for multi-messenger astrophysics. A potential source of electromagnetic transients from compact binary systems is the neutron star ocean, the external fluid layer encasing a neutron star. We present a groundwork study into tidal waves in neutron star oceans and their consequences. Specifically, we investigate how oscillation modes in neutron star oceans can be tidally excited during compact binary inspirals and parabolic encounters. We find that neutron star oceans can sustain tidal waves with frequencies between $0.01 - 20$ Hz. Our results suggest that tidally resonant neutron star ocean waves may serve as a never-before studied source of precursor electromagnetic emission prior to neutron star-black hole and binary neutron star mergers. If accompanied by electromagnetic flares, tidally resonant neutron star ocean waves, whose energy budget can reach $10^{46}$ erg, may serve as early warning signs ($\gtrsim$ 1 minute before merger) for compact binary mergers. Similarly, excited ocean tidal waves will coincide with neutron star parabolic encounters. Depending on the neutron star ocean model and a flare emission scenario, tidally resonant ocean flares may be detectable by Fermi and NuSTAR out to $\gtrsim$ 100 Mpc with detection rates as high as $\sim 7$ yr$^{-1}$ for binary neutron stars and $\sim 0.6$ yr$^{-1}$ for neutron star-black hole binaries. Observations of emission from neutron star ocean tidal waves along with gravitational waves will provide insight into the equation of state at the neutron star surface, the composition of neutron star oceans and crusts, and neutron star geophysics.

Key words: (transients:) black hole - neutron star mergers -- (transients:) neutron star mergers -- stars: oscillations -- gravitational waves -- X-rays: bursts

1 INTRODUCTION

With the recent detections of gravitational waves (GWs) from compact binary systems by GW detectors such as LIGO, Virgo, and KAGRA (Acernese et al. 2015; LIGO Scientific Collaboration 2015; LIGO Scientific Collaboration & Virgo Collaboration 2017; Akutsu et al. 2019; The LIGO Scientific Collaboration & the Virgo Collaboration 2021; LIGO Scientific Collaboration et al. 2021), binary systems that include neutron stars have come to the forefront of high energy astrophysics. Neutron stars represent a unique class of stellar objects in that, though very dense, they emit light, making them a candidate for combined GW-electromagnetic multi-messenger astrophysical searches (Rosswog 2015; Abbott et al. 2018). Neutron stars are thought to consist of three distinct layers: a very dense fluid core, a solid crust, and an external fluid ocean (Lattimer & Prakash 2001). The respective properties of each of these layers largely depend on the neutron star equation of state (Lattimer & Prakash 2001), whose details remain an active problem in nuclear physics and astrophysics. Detections of X-ray bursts (Bildsten & Cutler 1995; Strøhmayer & Mahmoodifar 2014; Chambers & Watts 2020), gamma ray bursts, (Tsang et al. 2012; Tsang 2013; Suvorov & Kokkotas 2020), ejecta from compact binary inspirals (Metzger et al. 2010; Metzger & Fernández 2014; Gerosa et al. 2017; Metzger 2017; Radice et al. 2018; Metzger 2019; Soares-Santos et al. 2017; Cowperthwaite et al. 2017; Nicholl et al. 2017; Chornock et al. 2017; Bartos & Márka 2019), and GWs (Andersson & Kokkotas 1998; Ferrari 2010; Suvorov 2018; Chatziioannou 2020) may probe this structure.

In recent years, GW astrophysics has become a unique observational tool to study neutron star physics. The answers to a number of open questions concerning the properties of neutron stars may lie in the rich capabilities of multi-messenger astrophysics with GWs. Works have investigated the possibility of mountains on the surfaces of spinning neutron stars, whose asymmetries could gener-
ate detectable continuous GWs (Ushomirsky et al. 2000; Osborne & Jones 2020; Gittins et al. 2021; Gittins & Andersson 2021). Searches for continuous GWs potentially originating from spinning neutron stars have been undertaken (Aasi et al. 2015; Papa et al. 2020; The LIGO Scientific Collaboration et al. 2021c; Abbott et al. 2022; The LIGO Scientific Collaboration et al. 2022), and may study neutron star geophysical structure and seismology (Gerozannis et al. 2017; Yang et al. 2018; Suvorov 2018; Andersson 2021).

Neutron stars exhibit a variety of pulsational modes (McDermott et al. 1988; Reisenegger & Goldreich 1994; Lai 1994; Passamonti et al. 2006; Samuelsson et al. 2007; Passamonti & Andersson 2012). These oscillation modes are associated with the restoring forces and structure of the star. Modes include the fundamental mode or $f$-mode (Lau et al. 2010; LIGO Scientific Collaboration & Virgo Collaboration 2017; Wen et al. 2019; LIGO Scientific Collaboration et al. 2021), pressure modes or $p$-modes (Bandari 2014), gravity modes or $g$-modes (McDermott et al. 1988; Bildsten & Cutler 1995; Bildsten et al. 1996; Deibel 2016; Andersson & Prigorouas 2020; Passamonti et al. 2021; Kuan et al. 2021b), $r$-modes in rotating neutron stars (Haskell 2015; Mitidis 2015; Ma et al. 2021; Chambers & Watts 2020), and interface modes or $i$-modes (McDermott et al. 1988; Passamonti & Andersson 2012). Oscillation modes may be excited during accretion (Reisenegger & Goldreich 1994; Deibel 2016) or by tides (Lai 1994; Ho & Lai 1999; Gittins et al. 2021).

Neutron star oscillations have been studied in connection with emission of electromagnetic radiation. The prospect of observing neutron star ocean oscillations induced by accretion, in particular, has been considered in many previous works (Bildsten & Cutler 1995; Bildsten et al. 1996; Heyl 2004; Deibel 2016; Chambers & Watts 2020; van Baal et al. 2020). Thermonuclear burning on neutron star surfaces during accretion can excite oscillation modes, which could represent the oscillations in type-I X-ray burst light curves (Hansen & van Horn 1975; Woosley & Taam 1976; Maraschi & Cavaliere 1977; Bildsten & Cutler 1995; Spitkovsky et al. 2002; Lee 2004; Piro & Bildsten 2005b; Chambers et al. 2018; Chambers & Watts 2020). Observed thermonuclear X-ray bursts on neutron stars show signs of ocean mode oscillation (Galloway et al. 2008; Bilous & Watts 2019; Roy et al. 2021; Bult et al. 2021).

Because neutron stars can exist in binaries, tidal deformations play a role in neutron star physics as well. A neutron star’s response to tidal forces largely depends on its internal properties, including its oscillation modes (Lai 1994). Observations of tidally excited oscillation modes would probe the composition of neutron stars.

In this work, we analyze neutron star ocean oscillations generated by the dynamical tide during interactions with other compact objects. We principally consider ocean tidal waves in compact binary inspirals, where tidal forces become resonant with neutron star oceans. We also investigate tidal waves from unbound neutron star encounters. We present models for neutron star oceans and investigate the size of tidal waves sustainable in these oceans. Ultimately, we consider astrophysical emission that tidally excited neutron star oceans might produce, including electromagnetic flares and GWs. We perform all of our analysis using Newtonian theory due to the exploratory and phenomenological nature of this study.

We divide the paper into the following sections. In section 2, we present the background neutron star model used, as well as introduce the three neutron star ocean models investigated. In section 3, we discuss the equations of motion for neutron star oscillations and determine the neutron star ocean oscillation modes for our models. In section 4, we discuss the tidal interaction and compute tidal wave properties for each of the oceans and orbital configurations considered. In section 5, we discuss our results and their consequences, including potential emission produced by neutron star ocean tidal waves. In section 6, we conclude.

2 BACKGROUND NEUTRON STAR AND OCEAN MODEL

To focus on the properties of the neutron star ocean, we use a simple background neutron star model with a rigid crust. We will later extrapolate our results with this model to the case where the neutron star crust is elastic rather than rigid.

To solve for the star’s background density $\rho$ and pressure $p$, we use the classical equilibrium equations for a spherically symmetric fluid

$$\frac{dp}{dr} = -\rho g,$$

$$\frac{dM}{dr} = 4\pi G r^2 \rho,$$

where $M(r)$ is the mass enclosed at a given radius, $G$ is Newton’s gravitational constant, and $g = \frac{GM(r)}{r^2}$ (Chandrasekhar 1957). Given an equation of state, these equations can be solved and provide the star’s background pressure and density. In this work, we use a polytropic equation of state (Ferrari et al. 2010)

$$p = K \rho^\Gamma,$$

where $K$ is a proportionality constant. Choosing $\Gamma = 2$ yields an analytic solution for the mass density when $r < R_*$

$$\rho(r) = \rho_c \frac{\sin \frac{4\pi_G r}{\sqrt{4\pi_G K}}}{\frac{4\pi_G r}{\sqrt{4\pi_G K}}},$$

where $\rho_c$ is the density at the center of the star and $R_*$ is the radius of the neutron star (Chandrasekhar 1957). When $r > R_*$, we have $\rho(r) = 0$. Note that the radius of the star is completely specified by the constant $K$.

For this study, we assume our neutron star is non-rotating and has no magnetic field. The effects of rotation and magnetization, if small enough, will serve as perturbations to the oscillation mode structures and frequencies without changing the physics (Kuan et al. 2021c,d; Krüger et al. 2021). Because we are interested in early inspirals, the effects of general relativity should not play a role in spinning up the rotation of neutron stars. While we expect effects such as tidal locking to also spin up neutron stars, we do not consider them in our study. We leave consideration of rotating and magnetized neutron stars to future work.

2.1 Neutron Star Ocean Depth

The depth of the neutron star ocean depends on the density at which the neutron star crust melts. The top of the crust is typically considered to be a body-centered cubic (bcc) Coulomb crystal (Bildsten & Cutler 1995; Haensel et al. 2007; Horowitz & Kaduw 2009; Baiko & Chugunov 2018; Gittins et al. 2020). In a Coulomb crystal, the ions which compose the lattice interact exclusively by the Coulomb interaction (Chambers et al. 2018) because the electron screening in the outer crust is weak (Chanel & Haensel 2008). The Coulomb crystal undergoes a phase transition when the thermal energy exceeds the electric binding energy of the material by some critical
factor $\gamma$ (Farouki & Hamaguchi 1993). The crust melts when the following condition is met:
\[
k_B T \geq \frac{1}{\gamma} \frac{1}{4\pi\epsilon_0} Z^2 e^2 d.
\]
where $k_B$ is Boltzmann’s constant, $T$ is the temperature, $\epsilon_0$ is the permittivity of free space, $Z$ is the proton number of atomic nuclei in the lattice, $e$ is electron charge, and $d$ is the mean spacing between nuclei. Molecular dynamics studies have found $\gamma \approx 173$ (Farouki & Hamaguchi 1993). Assuming that the ion number density is $n_i = \left(\frac{3}{4\pi d^3}\right)^{-1}$, the mass density at which the crust melts and the ocean forms is
\[
\rho_o = Amn_n_i = \frac{3}{4\pi\rho_c} A m_n \left(\frac{4\pi\epsilon_0 k_B T}{Z^2 e^2}\right)^3 \approx 2.705 \times 10^{10} \text{g cm}^{-3} \left(\frac{A}{16}\right) \left(\frac{8}{Z}\right) \left(\frac{T}{10^8 \text{K}}\right)^3.
\]
where $A$ is the atomic mass of the nuclei in the lattice, $m_n$ is nucleon mass, and we have used the condition in equation 4 for $d$ at the transition between the neutron star crust and ocean. Equation 5 shows the melting density’s strong dependence on temperature and ion atomic number. More proton-rich nuclei will reduce the density at which the ocean begins.

By plugging equation 5 into the left-hand side of equation 3, we determine the radius at which the ocean begins and by extension the depth of the ocean as a function of $A$, $Z$, and $T$ when $\Gamma = 2$. Since the ocean is very shallow compared to the neutron star radius (Bildsten & Cutler 1995; Bildsten et al. 1996; Urpin 2004; Deibel 2016; van Baal et al. 2020), we approximate $\Gamma$ in the denominator of equation 3 as the stellar radius $R_* = \left(\frac{2\pi G M}{3c^2}\right)^{\frac{3}{2}}$. The radius at which the neutron star ocean begins for $\Gamma = 2$ polytropic equation of state is
\[
r_o = \sqrt{\frac{2K}{4\pi G}} \arccos \left(\frac{3}{8} \frac{Am_n}{\rho_c} \left(\frac{4\pi\epsilon_0 k_B T}{Z^2 e^2}\right)^3\right) + \frac{\pi}{2}.
\]
We also obtain an approximate ocean depth $h_o$ for a general polytropic equation of state in terms of $\rho_o$. Differentiating equation 2 gives
\[
\frac{dp}{dr} = \Gamma \rho \Gamma^{-1} \frac{d\rho}{dr}.
\]
Combining equations 1a and 7 provides a differential equation for $\rho$ and $r$
\[
\frac{dp}{dr} = -\frac{g}{\Gamma K} \rho^{2-\Gamma}.
\]
Integrating equation 8 from the ocean floor to the surface assuming constant $g = \frac{GM}{R_c^2}$ gives
\[
h_o = R_* - r_o = \frac{\Gamma K}{\Gamma - 1} \frac{E_0^{\Gamma - 1}}{\Gamma - 1} g.
\]
Any choice of $K$ and $\Gamma$ in the ocean can therefore give an approximate $h_o$.

In this work, we consider three model crusts respectively made up of four elements thought to be found in neutron star surfaces due to their production by r-processes (Meisel et al. 2018): carbon with $Z = 6$ and $A = 12$, oxygen with $Z = 8$ and $A = 16$, and iron with $Z = 26$ and $A = 56$. For referential convenience, we refer to the three oceans corresponding to these differently composed crusts as carbon, oxygen, or iron oceans. Neutron star crust temperatures are typically $T \sim 10^8$ K when the crust is in thermal equilibrium with the core (Brown et al. 1998; Brown & Cumming 2009). Accretion can raise the temperature of the neutron star ocean floor to $T \sim 10^8$ K (Fujimoto et al. 1984; Haensel & Zdunik 1990, 2003, 2008). The temperature decreases through the ocean to $10^8$ K at the surface (Miralda-Escude et al. 1990; Chamel & Haensel 2008). In our study, we neglect effects of the ocean temperature gradient. We discuss our choice for crust temperature in section 3.4.1.

### 3 NEUTRON STAR OCEAN OSCILLATION MODES

We solve for the dynamical response of the neutron star ocean. To do this, we use the formalism of Lagrangian perturbation theory for fluids (Friedman & Schutz 1978). Using the Newtonian formalism typically used, we solve for the oscillation modes of the neutron star ocean (Dziembowski 1971; Ledoux 1974; McDermott et al. 1988; Passamonti & Andersson 2012), so that we may study the dynamical response to tidal forces (Lai 1994; Reisenegger & Goldreich 1994; Tsang et al. 2012; Tsang 2013; Passamonti et al. 2021).

#### 3.1 Equations of Motion

The equation of motion for Lagrangian perturbative displacements is the perturbed Euler equation
\[
\ddot{\epsilon}_x^2 \tilde{\xi} + \frac{\nabla \delta p}{\rho} - \frac{\delta \rho}{\rho^2} \nabla p + \nabla \delta \phi - \frac{1}{\rho} \nabla \cdot \sigma = -\nabla \chi,
\]
where $\tilde{\xi}$ is the Lagrangian displacement vector, $\rho$ is the background fluid density, $p$ is the background fluid pressure, $\delta \rho$ is the Eulerian perturbation of the density, $\delta \phi$ is the Eulerian perturbation of the pressure, $\sigma = \sigma_{ij}$ is the elastic stress tensor, and $\chi$ is an unspecified (for now) external potential that drives the system. The elastic stress tensor is defined as
\[
\sigma_{ij} = \bar{\mu} \left(\nabla_i \epsilon_j + \nabla_j \epsilon_i\right) - \frac{2}{3} \bar{\mu} \delta_{ij} \left(\nabla \cdot \tilde{\xi}\right),
\]
where $\bar{\mu}$ is the shear modulus and $\delta_{ij}$ is the Kronecker delta. In a fluid, $\bar{\mu} = 0$.

The Lagrangian perturbation for density can be written as
\[
\Delta \rho = \delta \rho + \tilde{\xi} \cdot \nabla \rho = -\rho \nabla \cdot \tilde{\xi},
\]
where the first equality is the definition of the Lagrangian perturbation in terms of the Eulerian perturbation and the second equality arises from conservation of mass (Friedman & Schutz 1978). If the oscillations are adiabatic, the Lagrangian perturbations for pressure and density are related by
\[
\frac{\Delta \rho}{\rho} = \frac{1}{\Gamma_1} \frac{\Delta p}{p},
\]
where $\Gamma_1$ is the adiabatic index. We note that $\Gamma_1$ does not necessarily equal $\Gamma$. When $\Gamma_1 \neq \Gamma$, the neutron star is stratified and can sustain internal g-modes (McDermott et al. 1988; Bildsten & Cutler 1995; Bildsten et al. 1996; Andersson & Psigouras 2020; Passamonti et al. 2021).

The final equation of motion that governs this system is the perturbative form of the Poisson equation
\[
\nabla^2 \delta \phi = 4\pi G \delta \rho.
\]
Since the ocean is the uppermost layer of the star and typically very shallow, the perturbation of the gravitational potential $\delta \phi$ and its gradient $\nabla \delta \phi$ must be very small compared to the background gravitational potential $\phi$ and the background gravitational acceleration.
We use equations 12 and 13 to obtain the other equation we need to solve this system. From the definition of the Lagrangian perturbation (Friedman & Schutz 1978), we have
\[ \Delta p = \delta p + \tilde{\xi} \cdot \nabla p = \rho \delta \tilde{\mu} + \hat{\xi}_r \frac{dp}{dr}, \] (17)
where \( \hat{\xi}_r \) is the radial component of \( \tilde{\xi} \) and the second equality comes from equation 15 and spherical symmetry (i.e. \( \nabla p = \frac{dp}{dr} \)). Substituting equations 17 and 12 into equation 13, we obtain
\[ \nabla \cdot \tilde{\xi} = -\frac{1}{\Gamma_1 \rho} \left( \rho \delta \tilde{\mu} + \hat{\xi}_r \frac{dp}{dr} \right). \] (18)

Equations 16 and 18 are a system of partial differential equations which can be solved using a clever ansatz for \( \tilde{\xi} \). We decompose \( \tilde{\xi} \) into normal modes (Dziembowski 1971; Ledoux 1974; McDermott et al. 1988; Passamonti & Andersson 2012)
\[ \tilde{\xi} = \sum_n \xi_n e^{i\omega_n t}. \] (19)
where \( \omega_n \) is the angular frequency of a resonant mode and \( \tilde{\xi}_n \) is the eigenfunction that solves the equation
\[ (L - \rho \omega^2) \tilde{\xi} = 0, \] (20)
where \( L \) is an operator defined such that \( L \tilde{\xi} = \rho \nabla \delta \tilde{\mu} - \nabla \cdot \sigma \) (Press & Teukolsky 1977; Passamonti et al. 2021). The index \( n \) denotes the mode. The orthogonality of these modes requires (Press & Teukolsky 1977; Lai 1994; Passamonti et al. 2021)
\[ \int \tilde{\xi}_m \cdot \tilde{\xi}_n \delta m \nu = A_n^2 \delta_{nm}, \] (21)
where the integral is over the volume of the star, \( \tilde{\xi}_n^* \) is the complex conjugate of \( \tilde{\xi}_n \), and \( A_n^2 \) is the normalization factor. The spherical symmetry of the problem allows us to write \( \tilde{\xi}_n \) as (Dziembowski 1971; Ledoux 1974; McDermott et al. 1988; Passamonti & Andersson 2012)
\[ \tilde{\xi}_n = \left( U(r)Y_{lm}(\theta, \phi), V(r)\partial_\theta Y_{lm}(\theta, \phi), \frac{V(r)}{\sin \theta} \partial_\phi Y_{lm}(\theta, \phi) \right), \] (22)
where \( Y_{lm}(\theta, \phi) \) are the spherical harmonic functions (Jackson 1962), \( \partial_\theta \) is the partial derivative with respect to the variable \( x \), and \( U(r) \) and \( V(r) \) are functions of the radial coordinate that must be solved for. The spherical symmetry also allows us to write \( \delta \tilde{\mu} \) as \( \delta \tilde{\mu} = \delta \mu(r)Y_{lm}(\theta, \phi)e^{i\omega t}. \)

### 3.1.1 Fluid Ocean

In the fluid components of the neutron star where \( \tilde{\mu} = 0 \), equation 16 simplifies considerably. With spherical symmetry and \( \tilde{\mu} = 0 \), equations 16 and 18 become first order ordinary differential equations in the radial coordinate \( r \)
\[ -\omega^2 U + \frac{d}{dr} \delta \tilde{\mu} = 0, \] (23a)
\[ -\omega^2 V + \frac{\delta \tilde{\mu}}{r} = 0, \] (23b)
\[ \frac{dU}{dr} + \frac{2}{r} U - \frac{l(l+1)}{r^2 \omega^2} \delta \tilde{\mu} = 0, \] (23c)
\[ \frac{dV}{dr} + \frac{2}{r} V - \frac{l(l+1)}{r^2 \omega^2} \delta \tilde{\mu} = 0. \] (23d)

These equations are valid in the fluid neutron star ocean. Rearranging equations 23a and 23c and using the relationship in equation 23b, this system reduces to two ordinary differential equations
\[ \frac{d}{dr} \tilde{\mu} = \omega^2 U. \] (24a)
\[ \frac{dU}{dr} = -\left( \frac{2}{r} + \frac{1}{\Gamma_1 \rho} \frac{dp}{dr} \right) U + \left( \frac{\rho}{\Gamma_1 \rho} + \frac{l(l+1)}{r^2 \omega^2} \right) \delta \tilde{\mu}. \] (24b)

We write these equations in terms of the dimensionless variables \( y_1 = \frac{r}{\Gamma_1} \) and \( y_2 = \frac{\delta \tilde{\mu}}{\rho} \), where \( \rho \) is the background gravitational field as a function of radius. We obtain
\[ \frac{dy_1}{dr} = -\left( \frac{2}{r} + \frac{1}{\Gamma_1 \rho} \frac{dp}{dr} \right) y_1 + \left( \frac{1}{r} + \frac{l(l+1)}{r^2 \omega^2} \right) y_2. \] (25a)
\[ \frac{dy_2}{dr} = \frac{\omega^2}{\rho} y_1 - \left( \frac{2}{r} + \frac{1}{\Gamma_1 \rho} \frac{dp}{dr} \right) y_2. \] (25b)

In the single fluid limit and the Cowling approximation, the equations in section A1 of Passamonti & Andersson (2012) reduce to equations 25. Given boundary conditions at the ocean-crust interface and ocean surface, a value for \( \Gamma_1 \) (which is not constant in general), and a value of \( l \), we may solve these equations as a boundary value problem.

#### 3.1.2 Elastic Crust

If the crust is elastic, the ocean oscillation modes may penetrate into the crust, requiring one to solve equations 16 when \( \tilde{\mu} \neq 0 \) (Piro & Bildsten 2005a). In this work, rather than solving equations 16 and 18 in the elastic crust, we will solve for modes in the ocean assuming a rigid crust and extrapolate our results from the rigid to the elastic case, focusing particularly on the consequences for the neutron star tide.

### 3.2 Boundary Conditions

In this work, we solve equations 23 assuming the mode is entirely confined to the ocean. For this simple case, oscillations do not penetrate into the crust. At the ocean floor we apply the condition
\[ y_1(r_o) = 0. \] (26)

This is the same condition applied by Bildsten & Cutler (1995) to solve for deep ocean g-modes. At the surface of the ocean we apply
the condition $\Delta p = 0$ (McDermott et al. 1988; Lai 1994). In our variables, this becomes

$$0 = y_1(R_*) - y_2(R_*) .$$

(27)

Additionally, so that we can find the functional form of the oscillation modes, we apply a normalization condition at the surface of the ocean,

$$y_1(R_*) = 1 .$$

(28)

With these three boundary conditions, our system is closed and solvable. We note that equation 26 only preserves continuity of the radial displacement if there is no radial displacement in the crust. This is not necessarily a suitable boundary condition for an elastic crust as the true ocean-crust junction condition is the continuity of the radial displacement and traction variables (McDermott et al. 1988; Passamonti & Andersson 2012; Passamonti et al. 2021). For a more detailed treatment including the mode’s penetration into the crust, one must impose these conditions.

### 3.3 Semi-Analytic Ocean Modes and Tidal Resonance

We now provide a simple analytic argument to demonstrate the existence of ocean modes and estimate how the ocean mode frequency scales with model parameters in both the rigid crust and elastic crust cases. This will also give the time of tidal resonance as a function of model parameters.

#### 3.3.1 Shallow Ocean Surface Wave Model

Treating the neutron star ocean as an incompressible shallow ocean, we analytically estimate the neutron star ocean mode frequencies. For waves in a shallow ocean, one solves for the height of the wave

$$\text{Combining equations 31 and 32 gives the wave equation for the height of the wave } \eta$$

$$\partial_t^2 \eta - g h_o \nabla_H^2 \eta = 0 .$$

(33)

At this point, we reintroduce the spherical nature of this problem by assuming $\eta = \eta(t) l_m$. In spherical coordinates, this problem becomes that of an incompressible fluid shell surrounding a sphere of radius $R_*$, similar to a neutron star ocean. While we have previously been working in Cartesian coordinates, the wave equation for $\eta$ holds in spherical coordinates if $\eta$ is not a function of $r$. Expanding $\eta$ in spherical harmonics, equation 33 becomes

$$\partial_t^2 \eta + \frac{l(l+1)}{R_*^2} g h_o \eta = 0 .$$

(34)

We arrive at ocean mode frequencies in an incompressible fluid

$$\omega_i \sim \frac{1}{R_*} \sqrt{(l+1)K \Gamma - 1} \rho_o^{-1} .$$

(36)

where the $i$ subscript refers to incompressibility. To obtain intuition about the functional dependence of the real ocean mode frequencies on our model parameters, we substitute equation 9 for $h_o$ in equation 35 and obtain

$$\omega_i \sim \frac{1}{R_*} \sqrt{(l+1)K \Gamma - 1} \left( \frac{3}{4\pi} m_n \right)^{\frac{1}{2}} \frac{4\pi n_0 k_B}{e^2} \left( \frac{\frac{1}{2}}{A} \right)^{\frac{2}{3}} \left( \frac{\frac{1}{2}}{Z^3} \right)^{\frac{1}{3}} T^{-\frac{1}{4}} .$$

(37)

Equation 37 estimates the mode frequency when the crust is taken to be rigid. One can see that the mode frequency increases as a function of $T$ and $A$ and decreases as a function of $Z$ for $\Gamma > 1$.

Piro & Bildsten (2005a) showed that if the pressure at the crust-ocean interface $p_o$ exceeds the shear modulus $\mu$, one cannot treat this mode as purely a shallow ocean surface wave, but rather as an interface mode with a nonzero $\xi$ in the crust. The interface mode frequency will be the shallow ocean mode frequency scaled by $\sqrt{\frac{Z}{T}}$.

$$\omega \sim \frac{\mu}{p_o} \frac{1}{R_*} \sqrt{(l+1)K \Gamma - 1} \left( \frac{3}{4\pi} m_n \right)^{\frac{1}{2}} \frac{4\pi n_0 k_B}{e^2} \left( \frac{\frac{1}{2}}{\mu} \right)^{\frac{2}{3}} \left( \frac{\frac{1}{2}}{Z^3} \right)^{\frac{1}{3}} T^{-\frac{1}{4}} .$$

(38)

Consequently, when the ocean has an elastic crust, equation 38 approximates the ocean mode frequency.

These expressions show the functional dependence of the mode parameters on the model. Such modes have been shown to exist in non-homogenous atmospheres as well (Taylor 1936). This analysis demonstrates the capacity of oceans to sustain modes with lower frequencies than the neutron star $f$-mode (McDermott et al. 1988; Passamonti et al. 2021, eg) regardless of ocean stratification.

#### 3.3.2 Tidal Resonance Estimates

Since we are interested in tidal resonances during compact binary inspirals, we estimate the time before compact binary merger of an
ocean tidal resonance. Tidal resonances should occur when
\[ \Phi = \frac{n}{m} \in \mathbb{Z} \] (39)
where \( \Phi \) is the orbital frequency of the compact binary and \( m \) is the spherical harmonic index. For circular binaries, we have
\[ \Phi = \sqrt{\frac{G(M + M_d)}{D^3}} \] (40)
where \( M_d \) is the mass of the companion object and \( D \) is orbital separation. The time to merger for a given orbital separation \( D \) is (Peters 1964)
\[ t_m = \frac{D^4}{4\beta}. \] (41)
where \( \beta \) is
\[ \beta = \frac{64G^2M^2M_d(M + M_d)}{\pi^2c^3}. \] (42)
where \( c \) is the speed of light. Combining equations 37, 39, 40, and 41 gives an expression for the time before merger when resonance occurs (hereafter resonance time) in the rigid crust case as
\[ t_r \sim \frac{1}{4\beta} \left( \frac{R_0^2m^2G(M + M_d)(\Gamma - 1)}{I/(l + 1)KT} \right)^{\frac{7}{4} - 4\delta} \times \left( \frac{8\pi^2\hbar k_B}{e^2} \right)^{4 - 4\delta} A \frac{Z^8}{8^8} \frac{1}{7^4}. \] (43)
Combining equations 38, 39, 40, and 41 gives an expression for the resonance time in the elastic crust case
\[ t_r \sim \frac{1}{4\beta} \left( \frac{p_o}{\rho \beta} \right) \left( \frac{R_0^2m^2G(M + M_d)(\Gamma - 1)}{I/(l + 1)KT} \right)^{\frac{7}{4} - 4\delta} \times \left( \frac{8\pi^2\hbar k_B}{e^2} \right)^{4 - 4\delta} A \frac{Z^8}{8^8} \frac{1}{7^4}. \] (44)

These analytical estimates for the mode frequency and resonance time will allow for frequency extraction, should tidal resonances from these modes be observed.

3.4 Ocean Mode Results

We now discuss the numerical values we choose for model parameters and present the computed mode results.

3.4.1 Neutron Star Model Parameters

Our neutron star model has a central density \( \rho_c = 10^{15} \text{ g cm}^{-3} \). We choose \( \Gamma = 2 \) as was done by Passamonti et al. (2021). The value of \( K \) that we use is \( K = 6.67 \times 10^4 \text{ cm}^3 \text{ g}^{-1} \text{ s}^{-2} \). These choices yield a neutron star that has radius \( R_\star = 12.5 \text{ km} \) and a mass \( M = 1.25 M_\odot \). This is just smaller than the peak mass of the galactic neutron star population 1.39 \( M_\odot \) (Antoniadis et al. 2016; Alsing et al. 2018). For our computations, we fix the temperature \( T = 10^8 \) K at the crust-ocean interface, so that \( t_r < 100 \text{ yr} \) for all scenarios considered. A longer resonance time would be practically too long for the coincident detection of tidal resonances with compact binary mergers. We note that to get temperatures as hot as \( T = 10^8 \) K in the crust, one typically needs heating due to accretion (Fujimoto et al. 1984; Haensel & Zdunik 1990, 2003, 2008). For this simple study, we do not account for accretion when computing the tidal wave amplitudes or energies.

The ratio of the ocean floor depth to the neutron star radius is independent of the choice of \( K \) in the equation of state. From equation 6, we find that the ocean floor depths of the three oceans are \( h_{o,c} = 1.14 \times 10^{-5} R_\star \), \( h_{o,o} = 2.71 \times 10^{-5} R_\star \), and \( h_{o,i} = 8.03 \times 10^{-6} R_\star \) for carbon, oxygen, and iron, respectively.

The carbon and oxygen oceans form below the electron capture density of those elements, so the bottoms of these oceans would be a dense plasma of ions and electrons (Bildsten & Cutler 1995). For simplicity, we neglect the effect of the ocean having distinct layers and leave this to future work.

3.4.2 Neutron Star Ocean Modes

We solve equations 25 using a four-stage Runge-Kutta scheme. We use a shooting method (Press et al. 1986) to obtain the frequencies of each mode. The unphysical nature of our neutron star model at the surface (i.e. that both \( \rho = 0 \) and \( \rho = 0 \) at \( R_\star \)) causes a divergence in equation 25. To avoid this divergence, we must choose a coordinate \( r \) just below \( R_\star \) at which to impose the surface boundary condition equation 27. This ensures that our neutron star ocean model is well behaved throughout the region in which we solve the hydrodynamic equations. Bildsten & Cutler (1995) and Piro & Bildsten (2005a) each address this, with Bildsten & Cutler (1995) choosing to apply the boundary condition at density \( \rho = 10^7 \text{ g cm}^{-3} \) and Piro & Bildsten (2005a) choosing to apply the boundary conditions at column depth \( 10^7 \text{ g cm}^{-2} \). In the present work, we apply the surface boundary condition at the radial coordinate corresponding to \( p = 0.05p_o \). Our mode frequency calculations are robust in the following sense: applying the surface boundary condition for five different cutoffs \( (p = 0.05p_o, p = 0.01p_o, p = 0.005p_o, p = 0.001p_o, p = 0.0001p_o) \), we find that the computed mode frequencies change by order unity. Present limitations in the theory of neutron star oceans and atmospheres prevent achieving mode frequency calculations more accurate than within an order of magnitude.

Because tidal forces correspond to \( l \geq 2 \) spherical harmonics, \( l = 0 \) and \( l = 1 \) modes remain practically unaffected by tidal forces, so we do not solve for them. We only solve for \( l = 2 \) modes as those are the modes most likely to be excited tidally. As previously mentioned, we assume the neutron star is barotropic so that \( \Gamma_1 = \Gamma_2 = 2 \). As such, our ocean is unstratified and cannot sustain \( g \)-modes in the traditional sense (i.e. where the equation of state of the perturbed fluid differs from the background equation of state).

For each ocean model, we find that the ocean can sustain one \( l = 2 \) mode with a frequency below the orbital frequency at which two neutron stars merge \((\gtrsim 10^{15} \text{ Hz})\) (Abbott et al. 2019). As previously mentioned, the modes we find are not the surface \( g \)-modes found by McDermott et al. (1988) and Passamonti et al. (2021). Instead these modes are interface modes or \( i \)-modes associated with the crust-ocean interface and ocean surface. These modes resemble shallow ocean surface waves due to the fixed crust-ocean boundary and free ocean surface (Piro & Bildsten 2005a). We note that stratified models can produce \( g \)-modes with frequencies of order \( \sim 1 \text{ Hz} \) (Bildsten & Cutler 1995). Table 1 shows the densities at the ocean floor, the depths, and the mode frequencies of the neutron star ocean models, as well as integrals computed later in the paper.

The mode frequency increases with the square root of ocean depth as predicted by equation 35. Carbon oceans have the highest mode frequency at 16.7 Hz, while iron oceans have a mode frequency of 0.44 Hz. We note that the fully computed mode fre-
where \(\omega(t)\) is an amplitude that scales the eigenfunction and encodes all time dependence of \(\tilde{\chi}\). Using the operator \(\mathcal{L}\tilde{\chi} = \rho \nabla \delta \tilde{\mu}\) defined in section 3.1 with \(\tilde{\mu} = 0\), equation 47 becomes

\[
(p\tilde{\mu}^2 + \mathcal{L})\tilde{\chi} = -\rho \nabla \chi. \tag{49}
\]

Substituting our ansatz for \(\tilde{\chi}\) gives

\[
-\rho \nabla \chi = \sum_n \rho \tilde{a}_n(t) \tilde{\chi}_n + \tilde{a}_n(t) \mathcal{L}\tilde{\chi}_n = \sum_n (\tilde{a}_n(t) + \omega_n^2 \tilde{a}_n(t)) \rho \tilde{\chi}_n. \tag{50}
\]

where the last equality follows from equation 20. We use the orthogonality condition in equation 21 to isolate an equation for \(\tilde{a}_n(t)\). Applying orthogonality yields

\[
\tilde{a}_n(t) + \omega_n^2 \tilde{a}_n(t) = -\frac{1}{A_n^2} \int \rho \tilde{\chi}_n \cdot \nabla \chi dV. \tag{51}
\]

Inputting the tidal potential from equation 45, equation 51 becomes

\[
\tilde{a}_n(t) + \omega_n^2 \tilde{a}_n(t) = \frac{G M_1 D(t)^{1/3}}{D(t)^{1/3}} W_{lm} \frac{Q_n}{A_n^2} e^{-i m \Phi(t)}. \tag{52}
\]

where \(Q_n\) is the overlap integral defined by (Press & Teukolsky 1977; Lai 1994; Tsang et al. 2012; Tsang 2013; Andersson & Pinoughas 2020; Passamonti et al. 2021)

\[
Q_n = \int \rho \tilde{\chi}_n \cdot \nabla (r^l Y_{lm}(\theta, \phi)) dV = l \int \rho (U + V(l + 1)) r^{l+1} dr, \tag{53}
\]

where we have used equation 22 to obtain the last equality. Note that the overlap integral is entirely a property of the mode and quantifies how strongly the mode gets excited by tidal forces. We define a normalized overlap integral, dimensionless for the \(l = 2\) modes, as

\[
\hat{Q}_n = \frac{Q_n}{A_n^2}. \tag{54}
\]

In Table 1, we report the normalized overlap integrals for each of the three ocean modes. We must also estimate the overlap integrals for modes which penetrate into the elastic crust. Piro & Bildsten (2005a) determine that the mode energy is principally confined to the ocean,
Figure 1. The shallow ocean surface mode for the three neutron star ocean models we study in this paper: a) the carbon ocean, b) the oxygen ocean, and c) the iron ocean. The left-hand plots show the dimensionless function $\frac{U(r)}{r}$ as a function of distance from the ocean floor while the right hand plots show the dimensionless function $\frac{V(r)}{r}$ as a function of distance from the ocean floor. Note that horizontal axis in c) is in mm because the iron ocean is only 1 mm deep. The mode frequency of each ocean model is shown in the legend of the right hand plot.
even when the mode penetrates into the crust. Furthermore, while the radial displacement has a node in the ocean with an elastic crust and not with a rigid crust, the tangential displacement of Piro & Bildsten (2005a) is multiple orders of magnitude larger than the radial displacement. Because our computed rigid crust modes have this same property, the large tangential displacement in the ocean will dominate the overlap integral in both cases. Consequently, we use our computed rigid crust overlap integrals to estimate the overlap integrals of $i$-modes which penetrate into the crust.

Following the analysis of Lai (1994), we perform a change of variables to solve equation 52 where
\[ a(t) = GM_{i} \tilde{Q}_{m} \tilde{W}_{m} b(t) e^{-im\Phi(t)} \] (55)
and $b(t)$ is the new function to solve for. In terms of $b(t)$, equation 52 becomes
\[ \ddot{b} - 2im\Phi \dot{b} + (\omega^2 - m^2 \Phi^2 - Im\Phi)b = \frac{1}{D(t)^{i+1}}. \] (56)
If we decompose $b$ into a real part $b^r$ and an imaginary part $b^i$, equation 56 becomes the following two equations:
\[ \ddot{b}^r + 2m\Phi \dot{b}^r + m^2 \Phi \dot{b}^r + (\omega^2 - m^2 \Phi^2 \dot{b}^r = \frac{1}{D(t)^{i+1}}. \] (57a)
\[ \ddot{b}^i - 2m\Phi \dot{b}^i + m^2 \Phi \dot{b}^i + (\omega^2 - m^2 \Phi^2 \dot{b}^i = 0. \] (57b)
Given an orbital trajectory for a companion celestial body, equations 57a and 57b can be solved. By plugging solutions to equations 57a and 57b back into equation 55, the tidal wave amplitude $a(t)$ in the neutron star ocean can be found.

4.1 Tidal Interaction Scenarios
We consider three tidal interaction scenarios: a BNS inspiral in a circular orbit, a neutron star-black hole binary inspiral (NSBH) in an unbound parabolic encounter between two neutron stars (NSPE). While NSPEs are expected to be fairly rare (Tsang 2013) due to the low presence of neutron stars predicted in stellar clusters (Bae et al. 2014; Belczynski et al. 2018; Ye et al. 2020; Mandel & Broekgaarden 2022), tidal interactions from these events remain relatively unexplored beyond Tsang (2013), so we consider them in this work. In the following subsections, we enumerate the initial conditions and orbital parameters in each of these scenarios.

4.1.1 Neutron Star Binary and Neutron Star-Black Hole Binary
The initial conditions and orbital motion of BNSs and NSBHs are largely the same when the orbital separation well exceeds the diameter of stellar mass black holes. Due to the lower mode frequencies of the three neutron star resonances, we will consider both BNSs and NSBHs at earlier times.

For an inspiraling circular binary, the time derivative of the true anomaly $\Phi$ is just the orbital frequency
\[ \dot{\Phi} = \sqrt{\frac{G(M + M_{s})}{D(t)^{3}}}. \] (58)
where $G$ is the gravitational constant, $M$ is the mass of the neutron star that is tidally perturbed, $M_{s}$ is the mass of the companion object, and $D(t)$ is the orbital separation as a function of time. The second derivative of the true anomaly is
\[ \ddot{\Phi} = \frac{3}{2} \dot{\Phi} \frac{D}{D^{'}}. \] (59)
where $D$ is the time derivative of $D$. Due to the emission of GWs, the binary loses energy and $D(t)$ decreases over time. The separation as a function of time $D(t)$ for an inspiraling circular binary is given by (Peters 1964)
\[ D(t) = \left( \frac{D_{0}^{4} - 256G^{2}MML(M + M_{s})^{2}}{5c^{2}} \right)^{1/4}, \] (60)
where $D_{0}$ is the orbital separation at time $t = 0$, and $c$ is the speed of light. We have neglected the effects of energy transfer to the neutron star ocean on the orbital motion because, as will be discussed in section 4.2.3, the orbital energy will far exceed the energy transmitted to the ocean mode.

To numerically solve equations 57a and 57b, we must choose initial values for $b^r$, $b^i$, $b_r^r$, and $b_i^i$. We use the same initial conditions for circular binary inspirals used by Lai (1994) and start our integration at a time when the binary is very far from merging. These conditions are
\[ b^r(0) = \frac{1}{D^{2+1}(\omega^2 - m^2 \Phi^2)}. \] (61a)
\[ \dot{b}^r(0) = \left[ -(l + 1) \frac{D}{D^{'}} + \frac{2m^2 \Phi \dot{\Phi}}{\omega^2 - m^2 \Phi} \right] b^r(0), \] (61b)
\[ \dot{b}^i(0) = \frac{1}{(\omega^2 - m^2 \Phi^2)} (2m \Phi \dot{b}^r(0) + m \Phi \dot{b}^i(0)). \] (61c)
\[ \ddot{b}^i(0) \approx 0. \] (61d)
We compute $b$ for the $l = 2, m = 2$ cases, since $m = 2$ and $m = -2$ modes will be equally excited (Lai 1994). The $m = 0$ binary inspiral cases will be small compared to the $m = 2$ resonant case. The $m = 0$ case corresponds to static deformations of the neutron star, rather than the larger amplitude resonant oscillations. Resonance of the ocean mode with the tidal force is likely to occur in any isolated binary system containing a neutron star because the system’s orbital frequency continuously evolves. We do not compute the $m = \pm 1$ case since $Y_{2\pm 1} = 0$.

4.1.2 Neutron Star Parabolic Encounter
We consider close encounters of neutron stars whose minimum distance of approach is a distance $s$. Since parabolic orbits correspond to those with an orbital eccentricity of $e = 1$, the orbital separation as a function of radius is
\[ D(t) = \frac{2s}{1 + \cos \Phi(t)}. \] (62)
where $\Phi(t)$ here is the true anomaly for a parabolic orbit. Using conservation of angular momentum, we obtain a differential equation for the true anomaly as a function of time
\[ \dot{\Phi}(t) = \frac{1}{4} \sqrt{\frac{G(M + M_{s})}{s^{3}}} (1 + \cos \Phi(t))^{2}. \] (63)
We also obtain the second derivative of the true anomaly $\dot{\Phi}$ by taking the derivative of $\dot{\Phi}$
\[ \ddot{\Phi}(t) = \frac{1}{2} \sqrt{\frac{G(M + M_{s})}{s^{3}}} (1 + \cos \Phi(t)) \sin \Phi(t) \Phi(t). \] (64)

MNRAS 000, 1–16 (2023)
Solving equation 63 gives the true anomaly as a function of time for a parabolic orbit.

Again, we must choose appropriate initial conditions for $b^r$, $b^l$, $b^i$, and $b^\theta$ to solve equations 57a and 57b. For a parabolic orbit, the two bodies begin infinitely far away from one another with no speed. Thus, when the companion object is far away from the neutron star, we have $\Phi(t) \approx 0$ and $\dot{\Phi}(t) \approx 0$. When this is the case, $D(t)$ is approximately constant, so we obtain a first approximation to $b$ at large distances

$$b \approx \frac{1}{\omega^2 D(t)^{i+1}}.$$  

(65)

The time derivative $\dot{b}$ becomes

$$\dot{b} \approx -(l+1) \frac{D(t)}{D(t)} b,$$  

(66)

where $D(t)$ is the time derivative of orbital separation. Taking the time derivative of equation 66 gives

$$\ddot{b} \approx -\frac{1}{D^2(t)} \left(\frac{D(t)}{D(t)} \right)^2 b,$$  

(67)

where $\dot{D}$ is the second time derivative of orbital separation. We may plug equations 66 and 67 into equation 56 and obtain an expression for $b$ containing initial conditions for both $b^r$ and $b^l$

$$b \approx \frac{1}{D^2(t)} \left(\frac{\omega^2 - m^2 \Phi^2 + i(m \Phi - 2m(l + 1) \Phi b^r)}{\omega^4} \right).$$  

(68)

where we have kept only the largest terms. Separating equation 68 into a real and an imaginary part, we get initial conditions valid when $D(t) \gg s$

$$b^r(0) = \frac{1}{\omega^2 D^2(t)},$$  

(69a)

$$\dot{b}^r(0) = -(l+1) \frac{D(t)}{D(t)} b^r(0),$$  

(69b)

$$b^l(0) = \frac{1}{\omega^2 (2m \Phi b^r(0) + m \Phi b^r(0))},$$  

(69c)

$$\dot{b}^l(0) = 0.$$  

(69d)

We solve for $b$ in the cases where $l = 2$ and $m = 0$. The $m = 2$ NSPE tidal amplitude will be significantly weaker than the $m = 0$ amplitude as resonant oscillations during NSPEs require very specific initial conditions on the neutron star trajectories, making them less likely to be found in nature.

### 4.2 Tidal Results

We report our results for the BNS, NSBH, and NSPE cases. $a(t)$ is computed by numerically solving equations equations 57a and 57b for $b(t)$ and substituting $b(t)$ into equation 55. The companion mass used in the BNS and NSPE is 1.25 $M_\odot$ and the companion mass used in the NSBH is 20 $M_\odot$. We show results for the NSPE when $m = 0$ and the binary inspirals when $m = 2$. We report one tidal response for each possible combination of ocean and companion orbit. Table 2 contains the main quantitative results of this paper.

#### 4.2.1 Resonant Tidal Waves in Binary Inspirals

Both BNS and NSBH inspirals will resonantly excite the ocean modes of their component neutron stars. It is when resonance occurs that the tidal wave achieves its maximum amplitude.

In figure 2, we show the magnitude of the tidal wave amplitudes for both BNSs and NSBHs in the times surrounding resonance. The general evolution of the tidal amplitudes of all three oceans is similar between both BNSs and NSBHs. In the minutes leading up to resonance, the amplitudes of the tidal waves increase by a full order of magnitude. We have not considered any damping mechanisms, although possible mechanisms which can decrease the tidal wave amplitudes include diffusion (Kraiv et al. 2021; Dommes & Gusakov 2021), heating (Beloborodov & Li 2016), and GW emission (Lioutas & Stergioulas 2018). Without damping, the tidal wave continues to pulsate with the same amplitude following the resonance time. Carbon and oxygen oceans possess tidal wave amplitudes of similar size. The overlap integrals and mode frequencies of these modes are less than an order of magnitude different, with carbon oceans having larger amplitudes. In contrast, the amplitudes in the iron ocean are about a factor of 100 less than those in the oxygen ocean. These differences result from the different overlap integrals calculated for each ocean.

Slight differences between the BNS and NSBH case are apparent. The BNS cases generate higher amplitudes than the NSBH cases because resonance during a BNS occurs when the two bodies are roughly twice as close as during an NSBH. Additionally, the evolution of the tidal wave amplitude and frequency is noticeably slower in the BNS cases. This is a direct consequence of the slower frequency evolution in BNSs.

We determine how long before merger these resonances occur from equation 60. We make $D_0$ the separation at resonance time, set $D(t) = 0$, and solve for $t$. In BNSs with rigid crusts, the carbon ocean reaches resonance with the tidal force $\approx 5$ minutes before merger, the oxygen ocean reaches resonance $\approx 40$ minutes before merger, and the iron ocean reaches resonance $\approx 60$ days before merger. When scaling these results for elastic crusts, the carbon ocean reaches resonance $\approx 40$ hours before merger, the oxygen ocean reaches resonance $\approx 10$ days before merger, and the iron ocean reaches resonance $\approx 70$ years before merger. In NSBHs with rigid crusts, the carbon ocean reaches resonance $\approx 1$ minute before merger, the oxygen ocean reaches resonance $\approx 5$ minutes before merger, and the iron ocean reaches resonance $\approx 7$ days before merger. Scaling these results for elastic crusts gives resonance times $\approx 5$ hours before merger in carbon oceans, $\approx 30$ hours before merger in oxygen oceans, and $\approx 10$ years before merger in iron oceans. Thus any emission from the tidally resonant oceans would well precede corresponding compact binary mergers.

#### 4.2.2 Tidal Waves Excited by Parabolic Encounters

NSPEs will excite tidal waves in neutron star oceans at periapsis. When this occurs, the tidal force of the companion star provides an impulse to the ocean, causing it to pulsate. In this paper, we quote results when the closest distance of approach is $s = 3.4 \times 10^6$ cm. This is the distance of closest approach where the carbon ocean tidal wave amplitude $a(t)$ in an NSPE is approximately equal to that of a BNS. For different values of $s$, the amplitudes of the excited tidal waves will scale our results by a factor of $(s/3.4 \times 10^6)^{-3}$. Figure 3 shows the tidal wave amplitudes during an NSPE for the three oceans.

After the NSPE occurs, the tidal waves will oscillate with the...
Multi-messenger Emission from Neutron Star Oceans

| Ocean          | Carbon (Rigid) | Oxygen (Rigid) | Iron (Rigid) | Carbon (Elastic) | Oxygen (Elastic) | Iron (Elastic) |
|----------------|----------------|----------------|--------------|------------------|------------------|----------------|
| Energy deposited (erg) | $8.6 \times 10^{46}$ | $3.8 \times 10^{45}$ | $1.3 \times 10^{40}$ | $8.6 \times 10^{44}$ | $3.8 \times 10^{43}$ | $1.3 \times 10^{38}$ |
| Time before BNS merger (min) | 5.33 | 35.33 | 8.3 | $2.5 \times 10^{4}$ | 1.6 | $3.9 \times 10^{7}$ |
| Energy deposited in NSBH (erg) | $3.9 \times 10^{46}$ | $1.7 \times 10^{45}$ | $5.8 \times 10^{39}$ | $3.9 \times 10^{44}$ | $1.7 \times 10^{43}$ | $5.8 \times 10^{37}$ |
| Time before NSBH merger (min) | 0.67 | 4.5 | 1.1 | $310$ | 2.1 | $4.9 \times 10^{6}$ |
| Energy deposited in NSPE (erg) | $4.3 \times 10^{46}$ | $2.5 \times 10^{45}$ | $2.3 \times 10^{40}$ | $4.3 \times 10^{44}$ | $2.5 \times 10^{43}$ | $2.3 \times 10^{38}$ |
| Time before NSPE (s) | 0 | 0 | 0 | 0 | 0 | 0 |

Table 2. The main quantitative results of this paper for the three neutron star oceans and three tidal scenarios considered. This table includes the energy deposited into each ocean due to the tide and the time at which this energy is deposited. The energy reported for the NSPE corresponds to $s = 3.4 \times 10^6$ cm.

Figure 2. The magnitudes of the dimensionless resonant ocean tidal wave amplitude $|\alpha(t)|$ during compact binary inspirals as a function of time without damping. The horizontal axis shows the time from resonance in hours and the vertical axis shows $|\alpha(t)|$. The top row shows tidal wave amplitudes during a BNS (a) in a carbon ocean, (b) in an oxygen ocean, and (c) in an iron ocean. The bottom row shows tidal wave amplitudes during an NSBH (d) in a carbon ocean, (e) in an oxygen ocean, and (f) in an iron ocean.

Figure 3. The real part of the dimensionless ocean tidal wave amplitude $\alpha(t)$ during an NSPE as a function of time without damping. The horizontal axis shows the time from resonance in seconds and the vertical axis shows $\alpha(t)$. The sharp increase in the tidal wave height at time $t = 0$ is due to the impulse from the neutron stars reaching their smallest orbital separation. (a) shows tidal wave amplitudes in a carbon ocean, (b) shows the tidal wave amplitude in an oxygen ocean, and (c) shows the tidal wave amplitude in an iron ocean.
mode frequency of the ocean mode. The amplitudes are approximately the same order for each of the three oceans we consider. As in the binary inspiral case, the iron ocean has the smallest amplitude.

The distance of closest approach in this NSPE is about an order of magnitude smaller than the resonance distance in the binary inspiral case. NSPEs require closer encounters than NSBHs and BNSs to produce sizable ocean tidal waves.

We estimate the event rate for NSPEs within this nominal encounter distance inside a globular cluster. NSPE event rates have been computed in previous works (Kocsis et al. 2006; Tsang 2013), but not for these very small encounter distances. We estimate the event rate of NSPEs in a globular cluster as

\[ \nu_{PE} = \frac{1}{2} N n_\bullet v_0 \sigma_{PE}, \]

where \( N \) is the number of neutron stars in a globular cluster, \( n_\bullet \) is the number density of neutron stars in a globular cluster, \( v_0 = [G(M + M_\bullet)]^{1/2} \) is the relative speed of neutron stars in an NSPE at periastron, and \( \sigma_{PE} \) is the cross-section of NSPEs. Note that in this expression for event rate, we use the relative velocity between neutron stars at periastron, while Kocsis et al. (2006) use the relative velocity at infinite separation \( v_0 \). We use the velocity at periastron because we are considering parabolic orbits where \( v_\infty = 0 \). The cross section will be

\[ \sigma_{PE} = \pi s^2, \]

for the encounter distance \( s \). The event rate for NSPEs within a distance \( s \) becomes

\[ \nu_{PE} = \frac{1}{2} \frac{N^2}{2\pi R_{GC}^3} \sqrt{G(M + M_\bullet)s^3}, \]

where we have assumed the number density of neutron stars in a globular cluster is uniform such that \( n_\bullet = \frac{N}{2\pi R_{GC}^3} \). Using \( N = 500 \), \( R_{GC} = 1 \) pc as was done by Kocsis et al. (2006), and \( M = M_\bullet = 1.25 M_\odot \), we find \( \nu_{PE} = 5.5 \times 10^{-21} \) yr\(^{-1}\). Close NSPEs are therefore extremely rare.

Despite the rarity of these events, their tidal waves are generated in exact coincidence with the time of the closest passage of the neutron stars, so observation of emission from such tides can exactly demarcate the time of periastron.

### 4.2.3 Energetics of Ocean Tidal Waves

The energy of an oscillation mode is divided into potential and kinetic energy. The kinetic energy and potential energy are (Lai 1994)

\[ E_k = \frac{1}{2} \int \rho \frac{\partial \xi}{\partial t} \frac{\partial \xi}{\partial t} dV = \frac{1}{2} |a(t)|^2 A_n^2, \]

and

\[ E_p = \frac{1}{2} \int \rho \omega_n^2 \xi \cdot \ddot{\xi} dV = \frac{1}{2} \omega_n^2 |a(t)|^2 A_n^2. \]

After tidal resonance in binary inspirals, the maximum kinetic and potential energies should be equal. Additionally, both the \( m = 2 \) and \( m = -2 \) modes contribute to the energy equally. Therefore, the tidal interaction will deposit a total energy into each mode (Lai 1994)

\[ E = \omega_n^2 |a_{n,max}|^2 A_n^2, \]

where \( |a_{n,max}| \) is the maximum amplitude of the tidal wave. For the NSPE \( m = 0 \) case, only one mode contributes to the deposited energy. The NSPE total energy will be half of the energy of a binary inspiral of the same amplitude (Lai 1994).

We compute the energy deposited into the shallow ocean surface mode after tidal resonance during a BNS inspiral to be \( \sim 8.6 \times 10^{46} \) erg in a carbon ocean, \( \sim 3.8 \times 10^{45} \) erg in an oxygen ocean, and \( \sim 1.3 \times 10^{40} \) erg in an iron ocean. Similarly, we compute the energy deposited into the ocean after tidal resonance during an NSBH inspiral to be \( \sim 3.9 \times 10^{46} \) erg in a carbon ocean, \( \sim 1.7 \times 10^{45} \) erg in an oxygen ocean, and \( 5.8 \times 10^{39} \) erg in an iron ocean. The orbital energy at the time of resonance is \( \gtrsim 10^{50} \) erg, justifying our assumption that the orbital motion remains unaffected.

After an NSPE whose distance of closest approach is \( s = 3.4 \times 10^6 \) cm, we compute the energy deposited into the shallow ocean surface mode to be \( 4.3 \times 10^{46} \) erg for carbon oceans, \( 2.5 \times 10^{45} \) erg for oxygen oceans, and \( 2.3 \times 10^{46} \) erg for iron oceans. For different values of \( s \), these energy results will scale by \( (s/3.4 \times 10^6 \text{ cm})^{-6} \).

The mode energy has dependence \( E \propto \omega_n^2 Q_n^2 \). Because the mode frequency in the elastic case is the shallow ocean surface mode frequency scaled by \( \frac{\nu}{\nu_\infty} \), the energy deposited into the crust-penetrating \( i \)-modes should be the energy deposited into the corresponding shallow ocean surface modes scaled by a factor of \( \frac{\nu}{\nu_\infty} \approx 0.01 \). (Piro & Bildsten 2005a). Consequently, our energy results for the elastic crust cases are the energies reported above, reduced by a factor of 100. We also report these values in table 1.

### 5 DISCUSSION

We have determined that tidal waves in neutron star oceans can be generated during BNS inspirals, NSBH inspirals, and NSPEs, and quantitatively estimated their amplitudes, energies, and timing. The tidal waves in each of these systems have unique properties. In binary inspirals, the neutron star ocean mode becomes resonant with the tidal force of the companion minutes to days before coalescence if the crust is rigid, and hours to years if the crust is elastic. Conversely, the impulsive tidal force during an NSPE excites the ocean mode at the moment of closest approach. The impulse generates simple continuous oscillatory tidal waves with the frequency of the neutron star ocean mode. The implications of these results extend to multimessenger astronomy and neutron star geophysics.

### 5.1 Ocean Tidal Waves as Compact Binary Merger Precursor Flares and Parabolic Encounter Multi-Messenger Sources

Dynamical activity in neutron star oceans may emit neutrino and electromagnetic radiation (Reisenegger & Goldreich 1994; Heyl 2004; Deibel 2016; Wang et al. 2021). Additionally, mode oscillations have been observed during electromagnetic bursts (Strohmayer & Mahmoodifar 2014). Therefore, the tidal waves in neutron star oceans during a binary inspiral might correspond to multi-messenger emission. We hypothesize that tidally resonant ocean waves in neutron stars may be a new source of compact binary merger precursor emission.

The energies deposited into the ocean modes after resonance (computed in section 4.2.3) represent estimates of the energy available for these flares. Thus, \( \sim 10^{37} - 10^{46} \) erg are available to source tidally resonant ocean flares. The energy deposited into the carbon and oxygen oceans during NSBHs and BNSs is comparable to the breaking energy of neutron star crusts, which ranges from
$10^{44} - 10^{46}$ erg (Tsang et al. 2012; Baiko & Chugunov 2018). Consequently, the energy imparted to the ocean may affect the neutron star crust. If the deposited energy exceeds the breaking energy, the crust may either crack or melt. Past work on crust breaking by resonant $i$-modes has mostly focused on the crust-core $i$-mode (Tsang et al. 2012; Passamonti et al. 2021). Our results show that the crust-ocean $i$-mode may have the ability to break the crust from the top, leading to interesting physics within the ocean. Additionally, while we have neglected the presence of magnetic fields, the interaction between the excited ocean and the surface magnetic field could generate electromagnetic emission. Particularly, if the neutron star crust breaks, subsequent magnetic reconnection of the surface magnetic field may cause large electromagnetic flares (Lander et al. 2015; Kaspi & Beloborodov 2017, e.g.). Because neutron star surfaces also emit thermal neutrinos (Yakovlev & Pethick 2004, e.g.), it is possible that this emission is manifest in neutrinos. In the remainder of this paper, we will limit our discussion to accompanying electromagnetic emission.

Pre-existing mechanisms for producing compact binary merger precursor flares include interactions of neutron star magnetospheres in BNSs (Ascenzioni et al. 2021), orbital motion of a weakly magnetized companion and a highly magnetized neutron star in either BNSs or NSBHs (Vietri 1996; Hansen & Lyutikov 2001; McWilliams & Levin 2011; Lai 2012; Piro 2012; Sridhar et al. 2021), and tidally induced cracking of a neutron star crust during high frequency mode tidal resonances in either BNSs or NSBHs (Suvorov & Kokotos 2020; Gittins et al. 2020; Passamonti et al. 2021; Kuan et al. 2021c,a). Precursor flares from previously considered channels are only expected just before a merger ($\lesssim 10^8$ s) (Mathews & Wilson 1997; Sridhar et al. 2021; Passamonti et al. 2021).

In contrast to these other mechanisms, precursor flares associated with tidally resonant neutron star ocean waves could be excited minutes to even years before the merger. Tidally resonant ocean flares can therefore be early warning signs of compact binary mergers involving neutron stars. Notably, NSBHs should have less trouble emitting early flares since the black hole will be farther from the neutron star than in other scenarios and should not absorb all the emission.

Early warning precursor flares can be additional messengers for studying neutron stars and compact binary systems. The time before merger will provide information about both the type of merger and the material in neutron star oceans. In fact, the delay between flare and merger can distinguish these qualities. Simply observing a flare within 100 years of a corresponding merger significantly constrains the parameter space and provides limits on the crust temperature. Because a crust temperature of $T \sim 10^8$ K is needed to use our considered scenarios have resonance times of less than 100 years, a successful flare observation could suggest a higher crust temperature and consequently provide information about surface heating and accretion during compact binary inspirals.

Observing these flares in practice will likely require retroactive searches for electromagnetic data coincident in sky localization with compact binary mergers observed by GW detectors. The use of space-based GW detectors such as LISA (Amaro-Seoane et al. 2017) may assist in identifying flares in advance of mergers, as space-based detectors will detect GWs from compact binary inspirals well before mergers at galactic distances (LISA Study Team et al. 2000; Robson et al. 2019). Observations of tidally resonant ocean flares during compact binary inspirals would complement multi-messenger efforts to study these exotic systems and their oceans.

NSPEs could generate flares as well. The ignition of the tidal wave would precisely coincide with the NSPE. As such, coincident detections of the broadband GW bursts generated by the orbital motion (Turner 1977a; Kovacs & Thorne 1978; Kocsis et al. 2006; De Vittori et al. 2012) and tidally induced electromagnetic flares can allow for the multi-messenger study of NSPEs and their constituent neutron stars.

5.2 Detection of Electromagnetic Flares from Neutron Star Ocean Tidal Waves

We posit two possible scenarios for electromagnetic flares originating from neutron star ocean tidal waves and qualitatively discuss their detection. Since the mode frequencies of the oceans studied are $\sim 1 - 100$ Hz, the electromagnetic radiation from neutron star ocean tides may be ultra low frequency. As of this paper’s writing, detection of ultra low frequency electromagnetic radiation on geophysical scales has been considered (Grimm 2002; Grimm et al. 2009; Korazhikiewicz et al. 2016), but no astronomical electromagnetic instrument capable of tapping frequencies below $\sim 0.001$ MHz has been proposed (Bergman et al. 2009; Saks et al. 2010; Blott et al. 2013; Boonstra et al. 2016; Cecconi et al. 2018; Belov et al. 2018; Prinsloo et al. 2018; Rajan et al. 2016; Bentum et al. 2020). Therefore, it would be extremely difficult to detect $\lesssim 100$ Hz radiation from neutron star ocean tidal waves.

However, due to complicated micro-physics, the large amount of energy deposited into the neutron star ocean, and the potential for magnetic reconnection, we propose that neutron star ocean tides may produce high-energy electromagnetic radiation in the gamma ray or X-ray regime with spectra and time-scales similar to that of soft-gamma repeaters (SGRs) or type-I X-ray bursts, perhaps through interactions between the surface magnetic field and the ocean. The hot temperatures of neutron star surfaces make thermal X-rays a particularly compelling manifestation of this emission. Since we have considered neutron stars with $T = 10^8$ K at the crust, these neutron stars may already be accreting and emitting X-rays thermally. The tidal resonance will impart additional energy into the ocean, which we suppose may increase the flux of photons on timescales comparable to the period of the computed ocean mode. We note that accretion often requires a non-compact companion to supply material. We have neglected the effects of such additional companions for simplicity.

Taking our high-energy flare conjecture at face-value and assuming the energy deposited into the ocean from the tide is isotropically expelled as either gamma rays or X-rays, we estimate how far away a resonant neutron star ocean tidal flare can be detected by the gamma ray detector Fermi (Atwood et al. 2009) and X-ray telescopic array NuSTAR (Harrison et al. 2013).

We estimate the photon flux from such a flare by assuming all energy deposited into the mode is radiated away as either X-rays or gamma rays. Taking $R$ to be the distance between a detector and the source, we approximate the photon flux at the detector as

$$F_\gamma \approx \frac{E_\gamma}{4\pi R^2},$$

where $E_\gamma$ is the energy of the ocean tidal wave, $E_\gamma$ is the energy of a photon, and $\omega$ is the mode frequency. We have assumed that all energy is radiated on a timescale comparable to the inverse of the mode frequency $\omega$ as it is the only short timescale we have.

For Fermi, we estimate the furthest distance at which such a
flares could be observed as

\[ R \simeq \sqrt{\frac{E \cdot \omega}{E_Y \cdot 4\pi F_I}}, \]  

(76)

where \( F_I \) is the photon flux threshold for Fermi. The photon flux threshold of Fermi is 0.74 photons cm\(^{-2}\) s\(^{-1}\) in the range of 8 keV to 40 MeV (Atwood et al. 2009).

For short duration X-ray sources, NuSTAR’s sensitivity is limited by photon statistics. The signal to noise ratio (SNR) for a short-duration flare assuming all photons are at the same energy is

\[ K = \sqrt{F_Y T/A}, \]  

(77)

where \( T \) is the duration of the flare and \( A \) is the effective area of NuSTAR. Substituting our estimate for \( F_Y \) and taking \( T \approx \omega^{-1} \) gives

\[ K \simeq \sqrt{E \cdot A / E_Y \cdot 4\pi R^2}. \]  

(78)

We estimate the furthest distance at which a flare can be observed by NuSTAR as

\[ R \approx \frac{1}{K} \sqrt{\frac{E \cdot A}{E_Y \cdot 4\pi}}, \]  

(79)

for some SNR threshold \( K \). Notice that \( \omega \) has dropped out of equation 79, so this estimate is independent of the precise timescale of the flare as long as it is short-duration (Harrison et al. 2013). The effective area of NuSTAR is approximately 800 cm\(^2\) for photons energies of 6 – 10 keV and 300 cm\(^2\) for photon energies of 10 – 30 keV (Harrison et al. 2013). We set a putative SNR threshold of \( K = 5 \).

We compute the distances for each ocean and binary inspiral case under four detection scenarios: tidally resonant ocean flare photons are 1) gamma rays with \( E_Y = 40 \) MeV detected by Fermi, 2) X-rays with \( E_Y = 8 \) keV detected by Fermi, 3) higher energy X-rays with \( E_Y = 20 \) keV detected by NuSTAR, and 4) lower energy X-rays with \( E_Y = 8 \) keV detected by NuSTAR. Note that \( R \approx E^2 / \gamma \) for both Fermi and NuSTAR detections. Since the elastic crust energy estimates are the rigid crust energy scaled by \( \mu / \mu_0 \), we scale the distances from their rigid crust values by a factor of \(( \mu / \mu_0 )^{1/2} \approx 0.1 \) to extrapolate the distance results for elastic crust case. We quote our results in table 3.

We find that if the emission from tidally resonant ocean flares is in the gamma ray spectrum or if crusts are composed of iron, Fermi and NuSTAR will have almost no capability to detect flares of extralactic compact binaries, the main sources of interest for Fermi and NuSTAR detections. Since the elastic crust energy estimates are the rigid crust energy scaled by \( \mu / \mu_0 \), we scale the distances from their rigid crust values by a factor of \(( \mu / \mu_0 )^{1/2} \approx 0.1 \) to extrapolate the distance results for elastic crust case. We quote our results in table 3.

The 90% credible interval for the merger rates is reported as \( 10 – 1700 \) Gpc\(^{-3}\) yr\(^{-1}\) for BNSs and \( 7.8 – 140 \) Gpc\(^{-3}\) yr\(^{-1}\) for NSBHs (The LIGO Scientific Collaboration et al. 2021b; Mandel & Broekgaarden 2022). Assuming a flare detectable out to \( 1 – 100 \) Mpc, we estimate the event rates for detectable tidally resonant ocean flares by multiplying spherical volumes with radii 1 Mpc and 100 Mpc by the lower and upper limits on the quoted merger rates, respectively. The event rates would be \( \sim 4 \times 10^{-8} – 7 \) yr\(^{-1}\) for BNSs and \( \sim 3 \times 10^{-8} – 0.6 \) yr\(^{-1}\) for NSBHs. Depending on the details of the crust, ocean, and flare, precursor flares associated with tidally resonant ocean waves in compact binary inspirals may be detectable.

### 5.3 Gravitational Waves from Neutron Star Ocean Tidal Waves

The time dependent mass density perturbations of tidal pulsations in compact stars should also generate GWs (Turner 1977b). We now investigate the GWs produced by neutron star ocean tidal waves. The GW metric \( h^{TT} \) (not to be confused with the ocean depth \( h_\omega \)) can be written as a multipole expansion (Turner 1977b)

\[ h_{ij}^{TT} = \frac{G}{Rc^2} \sum_{l,m} b_{lm} \left( 1 - \frac{R}{c} \right)^l Y_{lm}(\theta, \phi), \]  

(80)

where \( G \) is the gravitational constant, \( c \) is the speed of light, \( b_{lm} \) is a time-dependent amplitude evaluated at the retarded time with dimensions of the second time derivative of the mass quadrupole moment, and \( T_{lm} \) are transverse-traceless tensor spherical harmonics (Turner 1977b,a). As we have done throughout this work, we restrict ourselves to the \( l = 2 \) harmonic. For an oscillation mode that generates small perturbations in the mass density, \( B_{2m} \) is (Turner 1977b)

\[ B_{2m}(t) = \frac{16\pi}{5\sqrt{3}} \frac{d^2}{dt^2} \int \delta \rho \cdot (\rho \ddot{\mathbf{r}}) = -a(t) \left( \frac{dU}{dr} + \rho \left( \frac{dU}{dr} - \frac{2U}{r} - l(l+1) \frac{V}{r} \right) \right) \mathbf{r} \cdot \mathbf{Y}_{2m}. \]  

(82)

Substituting equation 82 into equation 81 gives

\[ B_{2m}(t) = \frac{16\pi}{5\sqrt{3}} \tilde{a}(t) H_n, \]  

(83)

where we have defined an integral \( H_n \) as

\[ H_n = - \int \left( \frac{dU}{dr} + \rho \left( \frac{dU}{dr} + \frac{2U}{r} - l(l+1) \frac{V}{r} \right) \right) r^2 dr, \]  

(84)

which quantifies an oscillation mode’s ability to generate GWs. Note that the only time dependence in equation 83 arises from \( \tilde{a} \). We obtain results for the integral \( H_n \) for each of our three ocean models. These are displayed in table 1 in units of g cm\(^{-2}\). Like other integrals computed, \( H_n \) is largest in carbon oceans because the carbon ocean is the largest.

We approximate the GW strain \( h(t) \) from neutron star ocean tidal waves as

\[ h(t) \approx \frac{G}{Rc^2} \frac{16\pi}{5\sqrt{3}} \tilde{a}(t) H_n. \]  

(85)
We determine at what distance $R$ there would be GW signals with amplitudes $h \sim 10^{-20}$. This is approximately the smallest amplitude detectable with current space-based GW detector technology (LISA Study Team et al. 2000; Robson et al. 2019). We find that GWs from none of the configurations considered will be able to escape the immediate vicinity of the neutron star. The values we report are for the rigid crust models. The configuration which generates the largest GWs is a carbon ocean during a BNS inspiral. The distance from the ocean at which the GWs have an amplitude of $\sim 10^{-20}$ is $\sim 10$ au. In contrast, the smallest GWs are generated in an iron ocean during an NSBH inspiral. In this case, the GW amplitude is $\sim 10^{-20}$ only $\sim 9$ km away. This makes detecting GWs from neutron star ocean tides virtually impossible. While these GWs will serve as a source of extremely weak damping, we find that the damping timescales are $\gtrsim 10^7$ yr and will not impact neutron star ocean tides on relevant timescales.

While ocean tidal wave GWs may be undetectable, the orbital motion of these binary systems generates sizable GWs. During the early inspirals of BNSs and NSBHs, GWs will be detectable by LISA (LISA Study Team et al. 2000; Robson et al. 2019). GWs from BNS and NSBH mergers are already detected by ground-based GW detectors (LIGO Scientific Collaboration & Virgo Collaboration 2017; LIGO Scientific Collaboration et al. 2021). Consequently, joint detection of GWs with tidally resonant ocean flares remains a possibility for multi-messenger astrophysics.

### Table 3

| Ocean                              | Carbon (Rigid) | Oxygen (Rigid) | Iron (Rigid) | Carbon (Elastic) | Oxygen (Elastic) | Iron (Elastic) |
|------------------------------------|----------------|----------------|--------------|-----------------|-----------------|---------------|
| BNS Gamma Ray with Fermi (Mpc)     | 3.9            | 0.82           | 0.0015       | 0.39            | 0.082           | 0.00015       |
| BNS X-Ray with Fermi (Mpc)         | 280            | 58             | 0.11         | 28              | 5.8             | 0.011         |
| BNS Higher energy X-Ray with NuSTAR (Mpc) | 520           | 110            | 0.20         | 52              | 11              | 0.020         |
| BNS Lower energy X-Ray with NuSTAR (Mpc) | 1300          | 280            | 0.52         | 130             | 28              | 0.052         |
| NSBH Gamma Ray with Fermi (Mpc)    | 2.6            | 0.55           | 0.0010       | 0.26            | 2.055           | 0.00010       |
| NSBH X-Ray with Fermi (Mpc)        | 190            | 39             | 0.0071       | 19              | 3.9             | 0.0071        |
| NSBH Higher energy X-Ray with NuSTAR (Mpc) | 350           | 74             | 0.13         | 35              | 7.4             | 0.013         |
| NSBH Lower energy X-Ray with NuSTAR (Mpc) | 900           | 190            | 0.34         | 90              | 19              | 0.034         |

We find that tidally resonant neutron star ocean flares, if in the X-ray band, may be detected at distances of $1-1000$ Mpc with Fermi and NuSTAR in most cases, comparable to the distances of observed BNS and NSBH mergers. We find that X-ray emission could have detection rates as high as $\sim 7$ yr$^{-1}$ for BNSs and $\sim 0.6$ yr$^{-1}$ for NSBHs. Neutron star ocean tides are consequently a possible source of emission which can accompany observable GWs. Subsequent work may involve reviewing past NuSTAR and Fermi data for X-ray bursts in coincident angular locations of observed BNS and NSBH mergers.

Neutron star ocean tides and oscillations may contribute to future multi-messenger observations of astrophysical compact binary mergers and neutron stars. Future studies into ocean tidal waves on top of crustal mountains (Gittins et al. 2021; Gittins & Andersson 2021) and resultant neutron star ocean tsunamis may yield interesting results. More exotic systems including collisions between neutron stars and planets may also produce ocean activity that results in multi-messenger emission. Multi-messenger emission from neutron stars, including emission from ocean tidal waves, will provide new knowledge about the enigmatic but rich physics of neutron stars.

## 6 CONCLUSION

Neutron star oceans can sustain resonant tides. Though rather small in size, the tidal waves excited in compact binary inspirals and in parabolic encounters possess large amounts of energy, ranging of $10^{37} - 10^{46}$ erg, depending on the properties of the neutron star crust. This energy, coupled with the rotational and magnetic energy of a real neutron star, has the potential to break neutron star crusts and fuel electromagnetic flares. Such electromagnetic flares could become early warning signs of merging NSBHs and BNS systems, preceding mergers by $\gtrsim 1$ minute if neutron star crusts are rigid and $\gtrsim 1$ hour if the neutron star crusts are elastic. Observations of these flares could shed light on neutron star ocean and crust properties. Their timing relative to compact binary mergers, as well as their duration and oscillation periods may serve as distinct signatures of these flares. Nevertheless, more work is needed to understand the physical mechanisms which can release the energy for flares as well as the effects of rotation and magnetization.

## ACKNOWLEDGMENTS

The authors are grateful to Nils Andersson, Fabian Gittins, Péter Petreczky, Charles Hailey, and Benjamin Owen for very helpful discussions as well as reviewing the manuscript and providing constructive feedback. The authors thank Columbia University in the City of New York and the University of Florida for their generous support. The Columbia Experimental Gravity group is grateful for the generous support of Columbia University. A.S. is grateful for the support of the Columbia College Science Research Fellows program and the Heinrich, CC Summer Research Fellowship. L.M.B.A. is grateful for the Columbia Undergraduate Scholars Program Summer Enhancement Fellowship and the Columbia Center for Career Education Summer Funding Program. G.S. is grateful for the generous support of the Columbia University Department of Mathematics Research Experience for Undergraduates program. I.L. is grateful for the generous support of the Columbia University Amgen Scholars program. I.B. acknowledges the support of the Alfred P. Sloan Foundation and NSF grants PHY-1911796 and PHY-2110060.
Multi-messenger Emission from Neutron Star Oceans

Metzger B. D., 2019, Living Reviews in Relativity, 23, 1
Metzger B. D., Fernández R., 2014, MNRAS, 441, 3444
Metzger B. D., et al., 2010, MNRAS, 406, 2650
Miralda-Escude J., Haensel P., Paczynski B., 1990, ApJ, 362, 572
Mitos A., 2015, PhD thesis, University of Florida
Nicholl M., et al., 2017, ApJ, 848, L18
Osborne E. L., Jones D. I., 2020, MNRAS, 494, 2839
Papa M. A., et al., 2020, ApJ, 897, 22
Passamonti A., Andersson N., 2012, MNRAS, 419, 638
Passamonti A., Bruni M., Gualtieri L., Nagar A., Sopuerta C. F., 2006, Phys. Rev. D, 73, 084010
Passamonti A., Andersson N., Pniouras P., 2021, MNRAS, 504, 1273
Peters P. C., 1964, Physical Review, 136, 1224
Piro A. L., 2012, ApJ, 755, 80
Piro A. L., Bildsten L., 2005a, ApJ, 619, 1054
Piro A. L., Bildsten L., 2005b, ApJ, 629, 438
Press W. H., Teukolsky S. A., 1977, ApJ, 213, 183
Press W. H., Vetterling W. T., Teukolsky S. A., Flannery B. P., 1986, Numerical recipes. Vol. 818, Cambridge university press Cambridge
Prinsloo D., et al., 2018, in 12th European Conference on Antennas and Propagation (EuCAP 2018). pp 1–4
Radice D., Perea A., Hotokezaka K., Fromm S. A., Bernardi S., Roberts L. F., 2018, ApJ, 869, 130
Rajan R. T., Boonstra A.-J., Bentum M., Klein-Wolt M., Belien F., Arts M., Saks N., van der Veen A.-J., 2016, Experimental Astronomy, 41, 271
Randall D. A., 2006.
Reisenegger A., Goldreich P., 1994, ApJ, 426, 688
Robson T., Cornish N. J., Liu C., 2019, Classical and Quantum Gravity, 36, 105011
Rosswog S., 2015, International Journal of Modern Physics D, 24, 1530012
Roy P., Beri A., Bhattacharyya S., 2021, MNRAS, 508, 2123
Saks N., Boonstra A.-J., Rajan R. T., Bentum M., Belien F., van’t Klooster K., 2010, in The 4S Symposium, Small Satellites Systems and Services.
Samuelsson L., Andersson N., Maniopoulou A., 2007, Classical and Quantum Gravity, 24, 4147
Soares-Santos M., et al., 2017, ApJ, 848, L16
Spytkowski A., Levin Y., Usmanovskiy G., 2002, ApJ, 566, 1018
Srivastava N., Zinke J., Metzger B. D., Sironi L., Giannios D., 2021, MNRAS, 510, 3184
Strohmayer T., Mahmodifar S., 2014, ApJ, 793, L38
Suvorov A. G., 2018, MNRAS, 478, 167
Suvorov A. G., Kokkotas K. D., 2020, Phys. Rev. D, 101, 083002
Taylor G. I., 1936, Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences, 156, 318
The LIGO Scientific Collaboration the Virgo Collaboration 2021, arXiv e-prints, p. arXiv:2108.01045
The LIGO Scientific Collaboration the Virgo Collaboration the KAGRA Collaboration 2021a, arXiv e-prints, p. arXiv:2112.010990
The LIGO Scientific Collaboration the Virgo Collaboration the KAGRA Collaboration 2021b, arXiv e-prints, p. arXiv:2111.03634
The LIGO Scientific Collaboration the Virgo Collaboration the KAGRA Collaboration 2021c, arXiv e-prints, p. arXiv:2112.010990
The LIGO Scientific Collaboration the Virgo Collaboration the KAGRA Collaboration 2022, arXiv e-prints, p. arXiv:2201.00697
Tsang D., 2013, ApJ, 777, 103
Tsang D., Read J. S., Hinderer T., Piro A. L., Bondarescu R., 2012, Phys. Rev. Lett., 108, 011102
Turner M., 1977a, ApJ, 216, 610
Turner M., 1977b, ApJ, 216, 914
Urpin V., 2004, A&A, 421, L5
Usmanovskiy G., Cutler C., Bildsten L., 2000, Monthly Notices of the Royal Astronomical Society, 319, 902
Vietri M., 1996, ApJ, 471, L95
Wang L.-J., Tan L., Li Z., Misch G. W., Sun Y., 2021, arXiv e-prints, p. arXiv:2102.06010
Wen D.-H., Li B.-A., Chen H.-Y., Zhang N.-B., 2019, Phys. Rev. C, 99, 045806
Woosley S. E., Taam R. E., 1976, Nature, 263, 101

MNRAS 000, 1–16 (2023)