Calculation of Single-Row Foundations for Loads Acting in a Plane Perpendicular to the Row Plane

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Abstract. Basements and foundations are designed following the current regulatory documents. In the case of a faulty design, even a flawlessly executed structure will no longer meet the operational requirements. Global construction experience shows that most of the accidents in constructed structures are caused by errors made during the construction of foundations. This paper describes the methodology for calculating single-row foundations for loads acting in a plane perpendicular to the row plane. In each specific case, a large number of alternative decisions can be made by the designer. The calculation of foundations using this technique will ensure the reliability and quality of the design and construction of buildings and structures.

1. Introduction
The research results were shown on an example of a pile foundation with a rigid grillage. Due to foundations being symmetrical with the load action plane and its grillage, it can be considered as infinitely rigid [1]. In this case, the limiting force \( N \), the shear force \( H \) and the bending moment \( M \), acting in the upper section of each pile element, are determined assuming that the external load is equally distributed between all foundation piles [2-4].

2. Experimental part
Loads \( N \), \( H \) and \( M \) are considered positive when the specified forces acting from the grillage on pile heads are directed downward, to the right and clockwise, respectively (Fig. 1).
Horizontal displacement \( a \) of grillage base and angle \( \beta \) of its rotation are determined by the formulas:

\[
a = Y + \varphi_0 l_0 + \frac{l_0^2}{6EI} (3M + 2Hl_o); \\
\beta = \varphi_0 + \frac{l_0}{2EI} (2M + Hl_0),
\]

where \( Y_0 \) and \( \varphi_0 \) — horizontal displacement in m and angle of rotation in degrees of the pile element's cross-section at the calculated soil surface level (with a grillage located above the ground), or the grillage base (if buried in the ground) are positive when the section is displaced to the right and rotated clockwise; \( l_0 \) — the length of a pile section located above the calculated soil surface (with a grillage buried in the ground is taken \( l_0 = 0 \)); \( EI \) — bending stiffness of the pile cross-section.

Displacements \( a \) and \( \beta \) are positive when the grillage base is displaced to the right and turned clockwise [5,6].

Displacements

\[
\begin{align*}
Y &= H_1 \delta_{\text{nh}} + M_1 \delta_{\text{nh}}; \\
\varphi &= H_2 \delta_{\text{nm}} + M_1 \delta_{\text{mm}},
\end{align*}
\]

where \( H_1 \) and \( M_1 \) - transverse force in kN, and bending moment in kN·m, acting in the pile element cross-section located above the ground level or above the grillage base level (when buried in the ground), which are positive when the force and moment transmitted from the upper part of the pile to the lower are directed to the right and clockwise, respectively;

\( \delta_{\text{nh}} \) - pile horizontal displacement (with a free upper end) at the calculated soil surface level (with a grillage located above the ground) or the grillage base level (when buried in the ground) from force \( H_1 = 1 \) applied at the same level, m (Figure 2a), equal to 5 mm horizontal displacement of this cross-section from moment \( M_1 = 1 \) (Figure 2b);

\( \delta_{\text{nm}} \) - rotation angle for the specified cross-section from moment \( M_1 = 1 \) (Figure 2b).
Figure 2. Pile ground displacement patterns.

Internal loads

\[ H_I = H; \quad M_I = M + Hl_0. \]

Displacements.

\[
\begin{align*}
\delta_{\text{HN}} &= \frac{1}{a^2 E I} A_0 \\
\delta_{\text{HM}} &= \frac{1}{a^2 E I} B_0 \\
\delta_{\text{MN}} &= \frac{1}{a^2 E I} C_0
\end{align*}
\]

where \( \alpha_c \) — pile element deformation coefficient determined by the formula (6); \( A_0, B_0 \) and \( C_0 \) - dimensionless coefficients taken according to Table 1, depending on whether the lower end of the pile is seating on non-rocky soil, seating on a rock (or embedded in it), as well as on the reduced (dimensionless) pile depth \( \check{h} \) in the ground [7-9].

\[
\alpha_c = \sqrt{\frac{K h_w}{E I}}
\]

Table 1. Dimensionless coefficients.

| \( \check{h} \) | Seating on non-rocky ground | Seating on rock | Embedded in rock |
|-----------------|-----------------------------|-----------------|------------------|
|                 | \( A_0 \)                   | \( B_0 \)       | \( C_0 \)       |
| 0.5             | 72.0038                     | 192.026         | 576.243         |
| 0.7             | 36.745                      | 70.0228         | 150.278         |
| 0.9             | 22.2442                     | 33.0076         | 55.3068         |
| 1.1             | 14.9161                     | 18.1597         | 25.1225         |
| 1.3             | 10.7170                     | 11.1028         | 13.2354         |
| 1.5             | 8.10139                     | 7.34874         | 7.83820         |
| 1.7             | 6.37548                     | 5.18938         | 5.13287         |
| 1.9             | 5.19043                     | 3.87760         | 3.67920         |
| 2.2             | 4.03194                     | 2.75504         | 2.58096         |
| 2.4             | 3.52575                     | 2.32685         | 2.22092         |
| 2.6             | 3.16284                     | 2.04829         | 2.01293         |
| 2.8             | 2.90543                     | 1.86946         | 1.88660         |
| 3.0             | 2.7266                      | 1.7575          | 1.818           |

\( \check{h} \) is the reduced pile depth. 

\( A_0, B_0, C_0 \) are dimensionless coefficients obtained from Table 1, depending on whether the lower end of the pile is seating on non-rocky soil, seating on a rock (or embedded in it), as well as on the reduced (dimensionless) pile depth \( \check{h} \) in the ground [7-9].
Depth \( \hat{h} \) is calculated using the formula (7) and rounded to the nearest table value. At \( h > 4 \), \( h \) can be taken equal to 4.

\[
\hat{h} = \alpha_c h,
\]

where \( \alpha_c \) — pile deformation coefficient determined by the formula (6); \( h \) — pile depth in the ground.

Support top horizontal displacement \( a' \) in m is calculated by the formula:

\[
a' = a + \beta h_{\text{on}} + \Delta_1
\]

where \( a \) and \( \beta \) — horizontal displacement of the grillage base in m, and the angle of its rotation in degrees, determined by the formula (1) and (2) or (9);

\( h_{\text{sup}} \) — distance from the grillage base to the support top in m;

\( \Delta_1 \) — horizontal displacement of the support top due to the deformation of its part located above the foundation.

At \( h > 2.6 \) for non-rocky ground, and when seating on rock at \( h > 4 \), the horizontal displacement of the grillage base \( a \) and rotation angle \( \beta \) can be calculated by formulas:

\[
a = \frac{l_m}{6EI}(3M + 2HL_m)
\]

\[
\beta = \frac{l_m}{2EI}(2M + HL_m)
\]

where \( l_m \) — pile element bending length in meters determined by the formula (10); \( K_2 \) — coefficient determined according to the diagram (Figure 2) depending on the reduced (dimensionless) pile depth in the ground; the remaining quantities are the same as in formulas (1) and (2).

![Figure 3. Graph for determining coefficient \( K_2 \)](image)

Pile element bending length, m:

\[
l_m = l_0 \frac{K_2}{\alpha_c}
\]

Bending moment \( M_z \) and lateral force \( Q_z \) in the pile element cross-section, as well as pressure \( \sigma_z \) on the ground conveyed by the pile lateral surface at a depth \( z \), is determined by the formulas:

\[
M_z = \alpha_c^2 EI Y_0 A_3 - \alpha_c EL \varphi_0 B_3 + M_1 C_2 + \frac{H_1}{\alpha_c} D_3;
\]

\[
Q_z = \alpha_c^3 EI Y_0 A_4 - \alpha_c EL \varphi_0 B_4 + M_4 C_4 + H_1 D_4;
\]

\[
\sigma_z = \frac{K}{\alpha_c} z;
\]

where \( \alpha_c \) — pile deformation coefficient calculated by the formula (6); \( K \) — proportionality factor (Table 2); \( A_1, B_1, C_1, D_1, A_3, B_3, C_3, D_3 \) — influence function values according to Table 3 depending on the reduced (dimensionless) depth \( \hat{z} = \alpha_c z \) with internal efforts at this depth \( M_z \) and \( Q_z \), as well as pressure \( \sigma_z \).
The element's cross-sectional moment of inertia can be determined by the formula:

\[ I = \frac{b h^3}{12} \]

and with a cement-aggregate ratio of 0.6.

\[ q = \frac{M}{h} \]

\[ \n \]

where \( q \) is the bending moment, \( M \) is the load, \( h \) is the depth.

\[ h = \frac{M}{q} \]

3. Conclusions

By substituting influence function values corresponding to a given depth \( z = h = z_0 \) into formulas (11) and (12), we obtain bending moment \( M_z \) and lateral force \( Q_z \) values, used to check drilled pile embedding strength in rocky ground.

If a pile is installed in a pre-drilled well by filling and compaction of soil within the active layer, and with a cement-sand grillage below it, then the strength and fracture toughness of a drill pile shall be additionally checked in case the bending moment value (kN-m) equals:

\[ M^* = H_h q + M_z, \]  

(14)

where \( h_0 \) – active layer thickness, m.

Maximum bending moment \( M_{\text{max}} \), acting across the pile element's cross-section in the area located underground can be determined by the formula:

\[ M_{\text{max}} = M_1 + H_1 \frac{K_3}{a_c} \]  

(15)

where \( K_3 \) — coefficient, which depends on the reduced (dimensionless) pile depth in the ground: at \( h=2.6 \) \( K_3=0.65 \), at \( h=3.0 \) \( K_3=0.70 \), at \( h=3.5 \) \( K_3=0.75 \).

Quantities \( M_1 \) and \( H_1 \) are explained in the formula (4). Pressure on the soil along the pile element lateral surface \( \sigma_z \), acting at depth \( z = z_0 \), can be determined by the formula:

\[ \sigma_z = \frac{4M_z + 10H_1 z}{9b_p z^2} - \xi_3, \]  

(16)
where $\xi_1$ is the coefficient, which at $h \geq 4.0$ equals 0.7, and at $2.6 < h < 4.0$ is determined by the formula $\xi_1 = 1.5 - 0.2h$.

4. References

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