Collective Modes of Trapped Fermi Gases in the Normal Phase

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We consider the collective mode spectrum of a normal Fermi gas in a spherical harmonic trap. Using a self-consistent random-phase-approximation, we systematically examine the effects of the two-body interactions on the modes of various symmetries. For weak coupling where the spectrum is shifted very little away from the ideal gas case, a sum-rule approach is shown to work well. For stronger coupling, the interplay between the single particle and the collective excitations causes effects such as mode splitting and Landau damping. A finite low frequency response present at stronger coupling is predicted for the quadrupole mode. Finally, we briefly discuss the effect of a finite temperature on the spectrum and the excitation of higher collective modes.

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I. INTRODUCTION

Trapped Fermi gases is rapidly emerging as a new important area of research in the field of ultracold atomic gases. Impressive experimental results have been presented with the demonstration of trapping and cooling of fermionic alkalis using a magnetic trap for $^{40}$K [1] and $^6$Li [2], and an optical trap for $^6$Li [3]. These systems provide a large degree of experimental control: One can manipulate the effective particle-particle interaction by tuning an external magnetic field [4], the geometry of the system can be changed, the number of internal degrees of freedom of the constituent particles is given by the number of trapped hyperfine states, and the fundamental quantum statistics can be altered by trapping alkali atoms of Bose, Fermi (or both) statistics. Ultracold atomic Fermi gases therefore provide a very rich system to study with useful concepts coming from atomic, condensed matter, nuclear, and statistical physics and several theoretical papers have already been published focusing on various aspects of this subject [5–7].

The high precision spectroscopy of collective modes possible in these systems combined with the ability to manipulate the interaction strength makes the study of collective modes for trapped Fermi gases a promising subject. Indeed, such studies have proven to be very useful in the field of Bose-Einstein condensation [8]. The collective modes which can be observed as shape oscillations of the atomic cloud correspond to sound waves in a homogeneous gas, but for a confined system their spectrum is discrete. It is well known that there in general are two regimes in which to study these modes [9]: When the lifetime $\tau$ of the quasiparticles is much longer than the characteristic period of motion (i.e. $\omega T \tau \gg 1$ for atoms in a harmonic trap of frequency $\omega_T$), there are few scattering events per sound oscillation, and the restoring force is due to the self-consistent mean field of the gas. Wave motion encountered in this limit is designated ”zero sound”. For the hydrodynamic regime $\omega_T \tau \ll 1$, on the other hand, collisions ensure local thermodynamic equilibrium. Hydrodynamic sound waves are sometimes called “first sound”. Due to the large variations in the effective interaction between the relevant alkali atoms in different hyperfine states, both regimes could be experimentally relevant and they have thus both been studied theoretically [10,11].

We will in this paper examine the spectrum of collective modes in the collisionless regime for a gas in a spherical harmonic trap. By using a formalism well-known from nuclear physics, we systematically study the effect of the interactions on various aspects of the collective mode spectrum for modes of monopole, dipole, and quadrupole symmetry when the gas is in the normal phase. We show that for weak coupling, the spectrum is characterized by a large degree of collectivity (giant resonances). For such weak interactions where the shift of the collective mode spectrum away from the ideal gas result is very small, a sum rule calculation of the collective mode spectrum assuming a non-interacting ground state is shown to work well. With increasing coupling however, the effect of the interactions on the ground state become significant thus complicating a sum rule calculation. For stronger coupling, we demonstrate that effects such as Landau damping and mode splitting become important for some modes. The magnitude of these effects, which depends on the overlap between the single particle and the collective response, is shown to be different for the various modes considered. Finally, we briefly discuss the effects of a finite temperature on the spectrum, and higher modes relevant for the thermodynamics of the system are considered.

II. FORMALISM

In this section, we outline the basic theoretical framework used to calculate the collective mode spectrum of a trapped gas of interacting fermions. We consider a gas of fermionic atoms of mass $m$, confined by a potential $U_0(r)$, with an equal number of atoms $N_\sigma$ in each of two hyperfine states, $|\sigma = \uparrow, \downarrow\rangle$. Two fermions in the same internal state
\(\sigma\) must have odd relative orbital angular momentum (minimally \(p\)-wave), and at low temperatures \(T\) the centrifugal barrier suppresses their mutual interaction [13]. Thus, we assume the interaction to be effective only between atoms in different hyperfine states, and to be dominated by the low-energy \(s\)-wave scattering characterized by the scattering length \(a\). Within mean field (Hartree Fock) theory, the gas is then described by the Hamiltonian:

\[
\hat{H} = \sum_{\sigma} \int d^3r \psi_\sigma^\dagger(r) \left[ \frac{-\hbar^2}{2m} \nabla^2 + \frac{1}{2}m\omega_r^2 r^2 - \mu_F \right] \psi_\sigma(r) + g \sum_{\sigma} \int d^3r \langle \psi_\sigma^\dagger(r) \psi_\sigma(r) \rangle \psi_\sigma^\dagger(r) \psi_\sigma(r) + \hat{F}(t),
\]

(1)

where \(\langle \ldots \rangle\) denotes the thermal average, and the field operators \(\psi_\sigma(r)\) obey the usual fermionic anticommutation rules describing the annihilation of a fermion at position \(r\) in the hyperfine state \(|\sigma\rangle\). The trapping potential is for simplicity assumed to be an isotropic harmonic oscillator \(U_0(r) = \frac{1}{2}m\omega_r^2 r^2\), \(\mu_F\) is the chemical potential, and \(g = 4\pi a^2/m\). \(\hat{F}(t)\) is an external perturbation with which we probe the system. The subject of this paper is the effects of the interaction on the collective mode spectrum of the gas in the normal phase and we are therefore neglecting pairing terms describing a possible superfluid transition.

When the atom-atom interaction is rather weak such that the life-time \(\tau\) of the quasiparticles is much larger than \(1/\omega_r\), the appropriate way to calculate the spectrum is the self-consistent random phase approximation (RPA) [13]. The problem of collective modes of a spatially confined two-component Fermi system has been studied extensively in the field of nuclear physics [14]. Thus, the basic formalism used in this paper is well-known from this field.

As we are interested in modes corresponding to density fluctuations excited by perturbations of the kind \(\hat{F}(t) \propto \exp(i\omega t) \sum_{\sigma} \int d^3r F_{\sigma}(r) \hat{\rho}_\sigma(r)\), we consider the retarded density-density correlation function \(\langle \langle \hat{\rho}_\sigma(r) \hat{\rho}_{\sigma'}(r') \rangle \rangle(\omega)\). Here, \(\hat{\rho}_\sigma(r) = \psi_\sigma^\dagger(r) \psi_\sigma(r)\) is the density operator for atoms in the hyperfine state \(|\sigma\rangle\) and \(\langle \langle AB \rangle \rangle(\omega)\) is the Fourier transform of the retarded function \(-i\theta(t-t') \langle AB \rangle\) [13]. Within RPA, we have

\[
\bar{\Pi}(\omega) = [1 - \bar{g}\bar{\Pi}_0(\omega)]^{-1} \bar{\Pi}_0(\omega)
\]

(2)

and

\[
\bar{\Pi}_0(\omega) = \left\{ \langle \langle \hat{\rho}_\sigma(r) \hat{\rho}_{\sigma'}(r') \rangle \rangle_0(\omega) \begin{array}{ccc} 0 & \delta(r-r') & 0 \\ \delta(r-r') & 0 & \end{array} \right\}
\]

(3)

(4)

\[
\bar{g} = g \frac{\hbar}{\xi} \left\{ \begin{array}{ccc} 0 & \delta(r-r') & 0 \\ \delta(r-r') & 0 & \end{array} \right\}.
\]

(5)

Here the matrix products denote integrals: \(\bar{A}\bar{B} \equiv \int d^3r'' A(r,r'') B(r'',r')\). The independent particle correlation functions \(\langle \langle \ldots \rangle \rangle_0\) are calculated within the time-independent Hartree-Fock (HF) approximation. In order to describe the effect of the interactions on the collective mode spectrum correctly, it is crucial that one uses the self-consistent static HF states to calculate the independent particle correlation functions \(\langle \langle \ldots \rangle \rangle_0\). If one simply uses the non-interacting trap states, the effect of the interactions on the single particle excitations are ignored and the predicted collective mode spectrum from Eq. (3) will be incorrect.

For a spherically symmetric trap, the correlation functions split into terms describing the various multipole modes. Indeed, we have

\[
\langle \langle \hat{\rho}_\sigma(r) \hat{\rho}_{\sigma'}(r') \rangle \rangle(\omega) = \sum_{LM} \rho \rho(r,\sigma',r',\omega) Y_{LM}(\theta,\phi) Y_{LM}(\theta',\phi')^*.
\]

(6)

with \(r\) denoting the radial distance to the center of the trap and the \(Y_{LM}(\theta,\phi)\) being the usual spherical harmonics.

For the independent particle correlation functions, we obtain for the radial part [13]:

\[
\rho \rho(r,\sigma,r',\sigma',\omega) = \sum_{n\ell \ell' \ell''} \frac{\delta_{\ell,\ell'} \xi_{\ell'} \hbar (2\ell + 1)(2\ell' + 1)}{4\pi(2\ell + 1)r^2 2^{2\ell^2}} \langle L' | L 0 | 0 \rangle \frac{2 \xi_{\ell' \ell''} (\ell' + 1) \xi_{\ell' \ell''}}{\xi_{\ell' \ell''} - \hbar \omega - i\delta} (f_{\ell' \ell''} - f_{\ell''}^*).
\]

(7)

The quasiparticle wave functions \(u_{n\ell}(r)\) are obtained from a self-consistent solution to mean field Hamiltonian in Eq.(1) with \(\hat{F}(t) = 0\) writing the HF wave functions on the form \(u_{n\ell}(r) = u_{n\ell} Y_{LM}(\theta,\phi)/r\) [13]. The quasiparticles
on the fact that for weak interactions the ground state can be well approximated by an ideal Fermi gas, the lowest spectrum obtained by solving the RPA equations for varying coupling strengths.

The strength function is the main object of the present paper. We will calculate it for perturbing operators of the form $\hat{Z}$, which has the form

$$\hat{Z} = \sum_{n,m} e^{-\beta E_n} E_n |n| \langle m | \hat{F} | m \rangle | \delta(h \omega + E_n - E_m) .$$

(8)

Here $Z$ is the grand partition function and $|n\rangle$ is an eigenstate of the Hamiltonian with energy $E_n$. For operators of the form $\hat{F}(t) \propto \int d^3 r F_\sigma(r) \hat{\rho}_\sigma(r)$, it can be obtained as

$$S(F, \omega) = -\frac{1}{\hbar \omega} \sum_{\sigma, \sigma'} \int d^3 r d^3 r' F_\sigma(r) \overline{F}_{\sigma'}(r') \text{Im}[\Pi_{\sigma, \sigma'}(r, r', \omega)].$$

(9)

The strength function is the main object of the present paper. We will calculate it for perturbing operators of the form $F_\sigma(r) = F_\sigma(r) Y_{LM}(\theta, \pi)$ excitation the monopole ($L = 0$), dipole ($L = 1$), and quadrupole ($L = 2$) modes. We take $F_\gamma(r) = F_\gamma^0(r)$ for the monopole and quadrupole modes thereby exciting total density fluctuations of the given symmetries. For the dipole symmetry, we take $F_\gamma(r) = \pm F_\gamma^0(r)$ exciting both the dipole, and the spin-dipole mode. For the dipole mode, we have $\omega = \omega_T$ independent of the interactions since it simply corresponds to a center-of-mass oscillation of the cloud. We calculate it to check the consistency of our numerics.

If one defines the moments $m_k \equiv \int d\omega S(F, \omega) \omega^k$, the frequency $\tilde{\omega}$ of the mode excited by $\hat{F}$ can be calculated as

$$\tilde{\omega} = \sqrt{\frac{m_{k+2}}{m_k}}$$

(10)

if the operator only excites one mode (giant resonance). This method is very handy since the two moments can be calculated as $m_1 = \langle 0 | [\hat{F}, [\hat{H}, \hat{F}]] | 0 \rangle$ and $m_3 = \langle 0 | [\hat{F}, [\hat{H}, [\hat{H}, \hat{F}]]] | 0 \rangle$; i.e. one only needs information about the ground state of the system to calculate the excitation energy.

An important rule coming from particle conservation is the $\tilde{f}$-sum rule.

$$m_1 = \int d\omega S(F, \omega) \omega = \frac{1}{m} \sum_{\sigma} \int d^3 r |\nabla F_\sigma(r)|^2 \rho_\sigma(r)$$

(11)

with $\rho_\sigma(r) = \langle \hat{\rho}_\sigma(r) \rangle$ being the equilibrium density. This identity provides another important check on the reliability of our RPA calculations as the equilibrium density is easily obtained from the static HF solution. For our numerical RPA results, this sum rule turns out to be obeyed to within $\sim 3\%$ indicating the accuracy of our calculations.

### III. Observables and Sum Rules

A quantity of great physical interest is the strength function directly related to the net transitions per unit time with energy $\hbar \omega$ induced by the operator $\hat{F}$:

$$S(F, \omega) = \sum_{n,m} e^{-\beta E_n} E_n |n| \langle m | \hat{F} | m \rangle | \delta(h \omega + E_n - E_m).$$

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### IV. Results

In this section, we present some typical results concerning the effect of the interactions on the collective mode spectrum obtained by solving the RPA equations for varying coupling strengths.

As outlined above, if the perturbing operator only excites one state, then a sum-rule calculation based on Eq. (10) should give the correct energy of the excited state if an accurate ground state is used. Based on this assumption and on the fact that for weak interactions the ground state can be well approximated by an ideal Fermi gas, the lowest
collective mode of monopole, (spin)-dipole, and quadrupole symmetry was calculated in Ref. [10]. Several effects can make such a calculation invalid: The interactions can change the ground state significantly away from the ideal gas limit although the operator $\hat{F}$ still excites a single collective mode (giant resonance). For stronger couplings, the mode can be fragmented into several modes and/or an incoherent excitation of several single particle states. This latter effect is known as Landau damping. We will now consider the importance of these effects for varying coupling strengths.

For very weak interactions, we expect the sum-rule approach to give an accurate description of the lowest collective modes. We have checked this by comparing our RPA results for small coupling strengths with the prediction based on a sum rule calculation. For simplicity, we here only show results for the monopole mode $[F_\sigma(r) = F(r)Y_{00}(\theta, \phi)]$. Completely equivalent results are obtained for the spin-dipole and the quadrupole modes. Using a local density approximation for the $T = 0$ ground state of an ideal gas, a sum rule calculation predicts the frequency of the lowest monopole mode to be $\omega_M \approx 2\omega_T(1 + 0.3 \times k_F a)^{1/2}$ with $k_F = \sqrt{2m_F/\hbar^2}$ being the Fermi vector in the center of the trap [10]. Figure (1) depicts a typical result obtained from a RPA calculation. We plot $\omega_T$ from (10) to be

\[
\omega_T \approx \sqrt{\frac{4}{\pi}} \frac{\hbar^2}{m} \left( \frac{\sigma}{\sqrt{\lambda}} \right) ^{1/2} \times \left( \frac{a}{\sqrt{\lambda}} \right) ^{1/2} \times \left( \frac{\sigma}{\sqrt{\lambda}} \right) ^{1/2}.
\]

for the monopole, spin-dipole, and the quadrupole modes and compared with the predictions based on the sum-rule approximation to the ground state. We conclude that the discrepancy in fig. (1) is due to the fact that the simple non-interacting approximation to the ground state is inadequate in a sum-rule calculation for these coupling strengths.

We have performed calculations with varying number of particles trapped and different coupling strengths for both the monopole, spin-dipole, and quadrupole modes and compared with the predictions based on the sum-rule approach as given in Ref. [11]. The general conclusion is, that the simple sum-rule results work well for $k_F a \lesssim 0.2$ where the frequency shift away from the ideal gas result is very small for all these modes. For stronger interactions, one still has a giant resonance for the lowest monopole, spin-dipole, and quadrupole modes. However, to get an accurate prediction for the eigenfrequencies for these coupling strengths based on a sum-rule calculation, one would need to take into account the effect of the interactions on the ground state. Such a calculation could involve determining the ground state using a static Hartree-Fock theory and then calculating the moments $m_k$.

We now consider the collective mode spectrum for more strongly interacting systems where the eigenmodes differ significantly from the ideal gas result. In this regime, the results based on the simple sum-rule calculation are invalid. This limit is especially relevant for experiments on $^6$Li, where large an negative scattering lengths between certain hyperfine states have been predicted theoretically [11] and recently measured experimentally [3]. In fig. (1)-(3), we plot the response $S(F, \omega)$ for the monopole, spin-dipole, and quadrupole symmetries excited by $F_\sigma(r) = r^2$, $F_\sigma = -F_\sigma = r$, and $F_\sigma(r) = r^2$, $F_\sigma = -F_\sigma = r$, respectively. We also plot the dipole response excited by $F_\sigma = r$ to further establish the accuracy of the calculations. There are $\sim 1.6 \times 10^4$ particles trapped, $k_F a \approx -0.6$, $\Gamma = 0.05\hbar\omega_T$, and $T = 0$. The symbols × again indicate the sum-rule predictions for the collective modes [10]. We also plot the single particle response $S_0(F, \omega)$ based on using $\Pi_0(\omega)$ instead of $\Pi(\omega)$ in Eq. (10). The collective response is indicated by solid lines whereas the single particle response is indicated by dotted lines.

First, we note from fig. (2) that the dipole response is very accurately peaked at $\omega = \omega_T$ as it should be. This provides another confirmation of the accuracy of our calculations. Also, we see that the single particle response (dashed lines) in general is moved to higher frequencies as compared to the ideal gas result for all modes. This is because the mean field is attractive for $a < 0$ and concentrated in the center of the trap. Since the spatial extend of the trap states in general increases with their energy, the decrease in the corresponding eigen-energies due to the mean field is larger for the lower trap states compared to the higher states. The gap between the harmonic oscillator bands thus increases compared to the non-interacting limit resulting in a shift of the single particle response to higher frequencies [12]. Correspondingly, for a repulsive interaction the shift in the single particle response would have been to lower frequencies. We see that for all modes (except, of course, for the dipole mode), there is a significant disagreement between the sum-rule results and the RPA prediction as expected.

Figure (2) indicates that the monopole mode is still characterized by a single giant resonance excited by the operator
$r^2$. This is because contrary to the spin-dipole and quadrupole modes, the collective response for the monopole mode is moved to lower frequencies compared to the ideal gas result ($\omega_M = 2\omega_T$) due to the interactions. As can be seen from fig. (2), the overlap between the collective response and the single particle response (which is moved to frequencies $\omega > \omega_T$) is therefore insignificant prohibiting a decay of the collective mode into single particle excitations. The monopole mode is therefore well-defined.

The strength of spin-dipole mode depicted in fig. (3) is now distributed over a large frequency region. Both the collective and the single particle response is shifted to $\omega > \omega_T$ resulting in a significant overlap between the collective mode and the single particle excitations. The collective mode can therefore decay into an incoherent motion of individual quasi particles (Landau damping). We also note from fig. (3) that this overlap with the single particle spectrum gives rise to a second small resonance located at $\omega \approx 1.15\omega_T$. Obviously, such behaviour cannot be described within a sum-rule approach. The general shift to a higher frequency for the spin-dipole collective response as compared to the ideal case ($\omega = \omega_T$) directly follows from the fact that for $a < 0$, the mean field provides an extra attractive force for the two hyperfine species oscillating in anti-phase and the spring constant for this motion is thus enhanced.

For the quadrupole mode, there is a very low frequency response in the region $0 \leq \omega \lesssim 0.2\omega_T$ as is highlighted in the inset in fig. (3). This low frequency response is straightforward to understand: The quadrupole mode is characterized by single particle excitations with angular momentum change $\Delta l = \pm 2$. The non-interacting spherical harmonic oscillator energies are given by the bands $E_{nl} = (n + 3/2)\hbar \omega$ with $l = 0, 2, \ldots, n$ for $n$ even and $l = 1, 3, \ldots n - 1$ for $n$ odd. Hence, the low frequency part of the quadrupole response is simply due to quasi particle excitations within such an energy band. If the gas was ideal, the frequency of such intra-band transitions would be $0$ and there would be no net transitions. Due to mean field effects there is a slight dispersion within each band resulting in a finite response at small $\omega$. For the monopole and spin-dipole modes, we have $\Delta l = 0$ and $\Delta l = \pm 1$ and there are no intra-band transitions excited. These two modes therefore do not have any low frequency response until the interactions are so strong as to make each harmonic oscillator band overlap such that inter-band transitions have a low frequency. This low frequency response should have a significant effect on the quadrupole compressibility of the gas since from simple 1. order perturbation theory one easily sees that the effect of a static quadrupole perturbation on the shape of the cloud is determined by the low energy excitations.

The gas is in principle superfluid for $T = 0$ for an attractive interaction, and $k_B T_c \simeq 2.8 \hbar \omega_T$ for the parameters given above. The presence of superfluidity could change significantly the low frequency response of the gas and this effect will be examined in a future publication. Since the gas strictly is superfluid for $T = 0$ for an attractive interaction, we need for consistency to examine whether the effects described above relevant for a gas in the normal phase are still present for $T > T_c$. In Fig. (3), we therefore also plot the collective response for $k_B T = 3\hbar \omega_T > k_B T_c$ indicated as a dotted line. We see that the effect of a finite temperature on the spectrum is as expected to smear the response out somewhat due to the thermal occupation of excited states. Furthermore, as a finite $T$ tends to make the gas more dilute and thus diminish the effect of the interactions, the spectrum is shifted slightly towards the ideal gas limit. However, the effect is quite small for the low $T$ considered here and the main features of the $T = 0$ spectrum remain. Importantly, we note that even though $k_B T = 3\hbar \omega_T$, the low frequency response ($\omega \lesssim 0.2\omega_T$) is not completely suppressed. Obviously, for increasing $T$ the response the gas eventually becomes ideal.

In order to excite higher modes of the gas, one in general needs perturbations containing higher powers of $r$ as can be seen if one writes these operators in terms of the raising and lowering operators for the harmonic oscillator. This is illustrated in fig. (4), where we plot the response to a perturbation given by $F_\sigma (r) = r^4$, $F_\tau = -F_\sigma = r^3$, and $F_\tau (r) = r^4$ for the monopole, spin-dipole and quadrupole respectively. We have chosen $\sim 1.6 \times 10^4$ particles trapped, $k_F a \simeq -0.6$, $\Gamma = 0.1\hbar \omega$, and $T = 0$. We see that for such perturbations, higher modes are also excited as expected. However, the strength of these higher excitations is rather low compared to the lowest modes. This is due to the fact that these modes correspond to additional nodes in the density fluctuation profiles and their overlap with the perturbing operator $F_\sigma (r)$ is therefore smaller than for the lowest modes. In order to increase the excitation rate of these higher modes, one could introduce a perturbation with more such radial nodes. In general, we believe that the excitation of these higher modes is somewhat more complicated than the excitation of the lowest modes, as one needs to experimentally generate spatial perturbations of a non-harmonic kind.

V. CONCLUSION

We have systematically examined the effects of the particle-particle interactions on the collective mode spectrum of a normal trapped Fermi gas within a self-consistent RPA scheme. For weak interactions where the shift in the eigenfrequencies away from the ideal case is very small, it was shown that a simple sum-rule approach to calculate the frequencies of the lowest modes works well. With increasing interaction strength however, such a calculation was
demonstrated to be complicated by the fact that it is necessary to include the effects of the particle-particle scattering on the ground state to achieve accurate results. As the coupling strengths increases further, we demonstrated the emergence of effects such as mode splitting and Landau damping coming from the interplay between the collective and the single particle excitations. The importance of such effects was shown to depend on the symmetry of the mode. For the quadrupole mode, a very low frequency response was predicted with possible implications on the quadrupole compressibility of the gas. The temperature dependence of the modes was considered and we finally briefly discussed the excitation of some of the higher collective modes. Due to the possibility of experimentally manipulating the effective interaction between alkali atoms and the high precision spectroscopy typically obtainable in such systems, we believe that some of the effects described in this paper should be observable using present day experimental technology.

VI. ACKNOWLEDGEMENTS

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[1] B. DeMarco and D. S. Jin, Science 285, 1703 (1999)
[2] F. Schreck et al., cond-mat/0011291
[3] K. M. O’Hara et al., Phys. Rev. Lett. 85, 2092 (2000)
[4] M. Houbiers et al., Phys. Rev. A 57, R1497 (1998)
[5] See e.g. D. A. Butts and D. S. Rokhsar, Phys. Rev. A 55, 4346 (1997); J. Schneider and H. Wallis, Phys. Rev. A 57, 1253 (1998); P. Törnma and P. Zoller, Phys. Rev. Lett. 85, 487 (2000); F. Weig and W. Zwerger, Europhys. Lett. 49, 282 (2000); W. Zhang, C.A. Sackett, and R.G. Hulet, Phys. Rev. A 60, 504 (1999); J. Ruostekoski, Phys. Rev. A 60, R1775 (1999)
[6] G. M. Bruun and K. Burnett, Phys. Rev. A 58, 2427 (1998)
[7] H. T. C. Stoof, M. Houbiers, C. A. Sackett, and R. G. Hulet, Phys. Rev. Lett. 76, 10 (1996); G. M. Bruun, Y. Castin, R. Dum, and K. Burnett, Eur. Phys. J. D 7, 433 (1999); M. A. Baranov and D. S. Petrov, Phys. Rev. A 58, R801 (1998)
[8] See e.g. F. Dalfovo, S. Giorgini, L. P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. 71, 463 (1999); the Georgia Southern University BEC Home Page, amo.phy.gasou.edu/bec.html
[9] D. Pines and P. Nozières, The Theory of Quantum Liquids (W. A. Benjamin, New York, 1966), Vol. 1
[10] L. Vichi and S. Stringari, Phys. Rev. A 60, 4734 (1999)
[11] G. M. Bruun and C. W. Clark, Phys. Rev. Lett. 83, 5415 (1999); M. Amoruso et al., Eur. Phys. J. D 8, 361 (2000); M. Amoruso et al., cond-mat/9907370
[12] B. DeMarco and D. S. Jin, Phys. Rev. A 58, 4267 (1998)
[13] D. Bohm and D. Pines, Phys. Rev. 92, 609 (1953); D. Pines, Phys. Rev. 92, 626 (1953)
[14] Aa. Bohr and B. R. Mottelson, Nuclear Structure (World Scientific, Singapore, 1998), Vol. 2
[15] A. L. Fetter and J. D. Walecka, Quantum Theory of Many-Particle Systems (McGraw-Hill, New York, 1971)
[16] G. Bertsch, Computer Phys. Comm. 60, 247 (1990); G. F. Bertsch and R. A. Broglia, Oscillations in Finite Quantum Systems (Cambridge 1994)
[17] J. P. Blaizot and G. Ripka, Quantum Theory of Finite Systems (MIT 1986)
[18] E. Lipparini and S. Stringari, Phys. Rep. 175, 103 (1989)
[19] G. M. Bruun and C. W. Clark, J. Phys. B. 33, 3953 (2000)
FIG. 1. The $T = 0$ collective response $S(F, \omega)$ in units of $l_n^4/\hbar \omega_T$ for the monopole mode excited by $F(r) = r^2$ for various coupling strengths. The sum-rule results are connected with the corresponding RPA results by an arrow. The inset shows $S(F, \omega)$ for $k_F|a| \simeq 0.32$ over a larger frequency range.

FIG. 2. The $T = 0$ collective (solid line) and single particle (dashed line) response for the monopole (in units of $l_n^4/\hbar \omega_T$) and dipole mode (in units of $l_n^2/\hbar \omega_T$) excited by $F_\sigma(r) = r^2$ and $F_\sigma(r) = r$ respectively. The coupling strength is $k_F|a| \simeq 0.6$. 
FIG. 3. The $T = 0$ collective (solid line) and single particle (dashed line) response for the spin-dipole (in units of $\hbar^2/\hbar\omega_T$) and the quadrupole mode (in units of $\hbar^4/\hbar\omega_T$) excited by $F_σ(r) = -F_\bar{σ}(r) = r$ and $F_σ(r) = r^2$ respectively. The coupling strength is $k_F|a| \simeq 0.6$. For the quadrupole mode, the dotted line depicts the collective response for $k_B T = 3\hbar\omega_T$. The inset shows in more detail the low frequency collective quadrupole response.

FIG. 4. The $T = 0$ collective (solid line) and single particle (dashed line) response for the monopole (in units of $\hbar^8/\hbar\omega_T$), spin-dipole (in units of $\hbar^6/\hbar\omega_T$), and quadrupole modes (in units of $\hbar^8/\hbar\omega_T$) excited by $F_σ(r) = r^4$, $F_σ(r) = -F_\bar{σ}(r) = r^3$ and $F_σ(r) = r^4$ respectively.