Dynamical Masses and Confinement in Graphene

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Abstract. Dynamical Mass Generation and Confinement are non-perturbative phenomena with large impact on the study of hadron and condensed matter physics. We study these phenomena in the light of the Schwinger-Dyson equations in various field theories, like Quantum Electrodynamics (QED) and Quantum Electrodynamics in the plane (QED3). Our aim is to describe the impact of these phenomena for the semi-metal-insulator transition in graphene, which is modeled by a peculiar field theory that inherits properties of both QED and QED3.

1. Introduction
Graphene, is a novel material which consists of a single layer of carbon atoms packed in a hexagonal (honeycomb) lattice. The low energy excitations near the Dirac points $K$ and $K'$ of the first Brillouin zone yields a gapless linear spectrum \cite{1}, and thus correspond to ultrarelativistic fermions or “graphinos” \cite{2}. Graphene differs from most conventional three-dimensional materials. Intrinsic graphene is a semi-metal or zero-gap semiconductor. Understanding its electronic structure is the starting point for finding the band structure of graphite. In this work, we study the gap equation for graphinos and its similarities to the gap equation for electrons in QED3 and QED4 in the Rainbow Approximation. This is the basis of the work \cite{3}.

2. Rainbow Approximation
Schwinger-Dyson equations (SDEs) are general relations among the Green functions of a given Quantum Field Theory. These correspond to Euler-Lagrange equations of quantum field theories, since they are the equations of motion for Green functions. In QED, the SDE for the electron propagator is coupled to that of the photon propagator and the electron-photon vertex. Therefore, in order to find the electron propagator self-consistently, one must assume a particular form of the photon propagator and electron-photon vertex. In this work we are interested in the Rainbow Approximation, in which we consider the tree level contribution for both these Green functions. The diagram for the SDE in this approximation is

\begin{equation}
S^{-1}(p) = S_0^{-1}(p) - ie^2 \int \frac{d^dk}{(2\pi)^d} \gamma^\nu S(k)\gamma^\mu \Delta^0_{\mu\nu}(q) . \tag{1}
\end{equation}

and corresponds to the expression

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\end{equation}
Here, $e^2$ is the electromagnetic coupling, which in dimensions other than 4 is dimensionful, and $q = k - p$. The bare photon propagator is

$$\Delta^0_{\mu\nu}(q) = \frac{1}{q^2} \left( g_{\mu\nu} + (\xi - 1) \frac{q_{\mu}q_{\nu}}{q^2} \right),$$

and $S(p)$ is the electron propagator, which we prefer to write in its most general form as

$$S(p) = \frac{F(p^2)}{p - M(p^2)} = \sigma_v(p^2) p + \sigma_s(p^2)$$

(2)

where $F(p^2)$ and $M(p^2)$ are the electron wavefunction renormalization and mass function. The bare propagator $S_0(p)$ corresponds to $F(p^2) = 1$ and $M(p^2) = m$. Equivalently, $\sigma_v(p^2)$ and $\sigma_s(p^2)$ are the vector and scalar parts of the propagator, respectively.

A standard procedure reveals that in the Landau gauge ($\xi = 0$), $F(p^2) = 1$, and thus the gap equation of QED in arbitrary space-time dimensions reads [3]

$$M(p^2) = m - ie^2 \int \frac{d^dk}{(2\pi)^d} \frac{M(k^2)}{k^2 + M^2(k^2)} q^2 (k^2 + M^2(k^2)),$$

(3)

where $m$ is the electron bare mass, which we set to zero from now onward. $M(p^2)$ is the electron mass function. A non trivial solution $M(p^2) \neq 0$ to this expression corresponds to chiral symmetry breaking due to self-interactions. This means that we can have electron mass even if we started with $m = 0$. Below we discuss the nontrivial solutions to the above equation in 3 and 4 space-time dimensions and their role in the description of confinement.

**Figure 1.** Scaling Law for QED, QED3 and Graphene.

**Figure 2.** Confinement Test QED.

### 3. Gap equation in QED

In this section we work in the 4 dimensional space-time of ordinary QED. In order to solve Eq.(1), we perform a Wick rotation ($p^0 \rightarrow ip^0$) to Euclidean space. Then, the gap equation is

$$M(p^2) = \frac{3\alpha}{2\pi^2} \int_0^{\Lambda^2} dk^2k^2 \frac{M(k^2)}{k^2 + M^2(k^2)} \int_0^{\pi} d\psi \frac{\sin^2 \psi}{q^2},$$

(4)

where $\Lambda$ is an ultraviolet cut-off. DMG in QED arises when the coupling constant reaches a critical value ($\alpha_c = \pi/3$) as can be seen by solving numerically Eq.(2). In a vicinity of the
critical coupling $\alpha_c$, the generated mass $m_d = M(0)$ obeys the Miransky scaling law [3]

$$\Lambda \frac{m_d}{m_d} = \exp \left( \frac{A}{\sqrt{\alpha - 1} - B \sqrt{\alpha}} \right).$$

(5)

In order to corroborate this scaling relation, we solve numerically the Eq.(2) for several values of the coupling. Results are shown in Fig. 1. The curve for QED agrees with the Miransky law with $A \simeq \pi$ and $B \simeq 2$.

Another important non-perturbative aspect to analyze is confinement. It is said that a theory is confined if we need an infinite amount of energy to take apart a two-particle bound state. To perform our confinement test, we invoke the violation of reflection positivity in the following way: If the vector part of the propagator

$$\sigma_v(p^2) = \frac{1}{p^2 + M^2(p^2)}$$

presents a inflection point, then we have confinement; the inflection point guarantees that the Fourier transform of $\sigma_v$ flips sign in coordinate space, hence breeching reflection positivity. In Fig. 2 we plot the derivate of the $\sigma_v$. We can note from the absence of local extrema that we do not have confinement as expected for a theory with a static electromagnetic potential which behaves like $1/r$.

4. Gap equations in QED3 & Graphene

4.1. QED3

For QED3, the gap equation is

$$M(p) = \frac{\alpha}{\pi^2} \int \frac{d^3k}{q^2} \frac{M(k)}{(k^2 + M^2(k))}.$$ (6)

By means of a numerical analysis of Eq.(4), we can verify that there is not critical coupling for DMG to take place, opposely to QED. This means that if have DMG for one value of $\alpha$, we have it for all values of the coupling. Moreover, we observe that the mass function scales like

$$\alpha M(p; \alpha) = M(p/\alpha; 1),$$

(7)

such that the scaling of the dynamically generated mass is lineal with the coupling, as can be seen in Fig. 1.

On the other hand, performing the same confinement test as in QED, we notice that this theory is confined as we expected for a planar theory with a logarithmic potential (Fig. 3).

4.2. Graphene

The low energy effective theory of graphene corresponds to a peculiar form of QED in which the charge carriers, the graphinos, are restricted to move in a plane, while the photons are freely moving in space. The question now is: What differences and similarities can we find between these theories and the dynamical generation of masses and confinement in graphene? This question is closely related to the semi-metal–insulator transition in graphene. To answer this, we start from the gap equation proposed by Gusynin et al. [4]

$$M(p) = \frac{2\lambda}{\pi} \int_0^{\Lambda} \frac{kM(k) \, dk}{\sqrt{k^2 + M^2(k)}} K(p, k), \quad \lambda = \frac{\alpha}{2(1 + \pi \alpha/2)},$$

(8)
where the lineal kernel is
\[
\mathcal{K} = \frac{\pi}{2} \left( \frac{\theta(p - k)}{p} + \frac{\theta(k - p)}{k} \right).
\]
This equation takes into account the one-loop corrections to the Coulomb interaction and can be solved by similar techniques. A numerical analysis shows that, just as in QED, there is a critical coupling \( \lambda_c \sim 1/4 \) above which masses are dynamically generated. Interestingly, the generated mass follows a Miransky-type scaling law. This means that the scenario of mass generation in graphene is similar to that of QED.

On the other hand, we perform the confinement on the solution to the gap equation and observe confinement just, like in QED3 [3].

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{QED3_confinement.png}
\includegraphics[width=0.4\textwidth]{Graphene_confinement.png}
\caption{Confinement Test QED3. Confinement Test Graphene.}
\end{figure}

5. Conclusions

In this work, we have explored similarities and difference of the gap equation in QED, QED3 and the low-energy effective theory of graphene regarding how dynamical mass generation and confinement are linked. We have found that for graphene, the scenario for confinement is similar to QED3. However, considering that photons are not restricted to the plane, the theory requires a critical value for coupling to generate masses, just like in QED. Thanks to this critical value a gap is opened and the graphene goes from a semimetal to a insulator.

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