Mathematical model of heart sounds

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Abstract. The article describes a mathematical model of physiological heart sounds created with due account for the pseudorandom generation parameters of component harmonic. The model uses the sine-cosine Fourier transform with probabilistic elements that are included to impart stochastic properties. These elements determine the frequency, origination time, life cycle and amplitude of harmonics. It allows synthesizing a phonocardiogram that quite accurately reflects the individual features of heart sounds within every systolic cycle. The article describes the spectral differences of reference and synthesized signals. In the authors’ opinion, these differences are conditional to distortions typical of the traditional microphone-based sensors.

1. Introduction

The developers of up-to-date cardiologic diagnostic equipment often face the problem of verifying their software via real-life physiological and pathological signals. In any case, they have to achieve the smallest incoherence in the assessment of pathological conditions between expert diagnostic conclusions and results provided by the diagnostic software module of developed devices. However the confident detection of pathologies requires long-term and labor-intensive research studies under real-life conditions with reference and experimental groups consisting of several thousand people. This problem is especially relevant to rare diseases as well as spontaneous and short-term pathological manifestations. In this case, the most logical option is to create a hardware and software system that will authentically imitate registered cardiologic signals, including phonocardiographic signals. Such a system should be based on the harmonized mathematical models of normal physiological signals and applied signals representing pathological deviations. In addition, it is necessary to make provisions for synthesizing pathological signals only, which is required to train diagnostic systems with the use of artificial intelligence elements.

Despite the availability of a significant number of mathematical models of different phonocardiograms (e.g., [1, 2]), practically all of these models are created based on statistical signal parameters or the idea of invariant frequency and phase parameters of specific heart sounds with constant individual features. Meanwhile, thoroughly planned research studies have revealed that every individual heart sound has its own time-and-frequency and phase profile [3] that is most likely connected with the stochastic nature of changes in the concentration of ions [Ca++] inside cardiomyocytes [4]. Heart sounds demonstrate the most pronounced individuality during the active phase of the cardiac cycle, i.e. systole that is accompanied by first heart sound S1. Diagnostic heart sound S2 appears to be less prone to random changes in its individual characteristics, as it reflects the passive processes of the cardiac cycle accompanied by a reduction in the concentration of [Ca++] in cardiomyoplasm, as it is transferred to the endoplasmic reticulum and excreted into the intercellular space [5].
In this connection, the authors set a goal to develop a basic mathematical model of physiological heart sounds with due account for random processes at the moments of emergence, maximization and attenuation of the harmonic components of heart sounds S1 and S2 within the range of 16-78 Hz as well as their amplitude. Higher harmonics are not taken into account due to their low priority in the signal spectrum – the total share of these harmonics amounts to no more than 2% of the energy of combined signal S1+S2. To detect the above-mentioned signal parameters, the authors analyzed a considerable amount of phonocardiograms with high time resolution (sampling frequency of 192 kHz), which allowed them to specify the problem formulation and modeling parameters [3].

2. Materials and methods

The model was created using the Scilab 6.0.1 computing mathematics system with the Linux Debian 10.2 operating system. The system was run on a workstation with the Intel Core i5-3550 processor and 32GB RAM. The model represents a set of interconnected software code files (scenarios) and data tables. Data tables contain the key point parameters of enveloping signal harmonics with due account for the confidence interval obtained in the course of research studies.

The stochastic nature of the model was realized via changing the position of these key points in time and changing the amplitudes of signals within acceptable ranges. Two random number generators – with normal and tapered exponential distribution – were used to realize the stochastic nature: the expert group of cardiologists considered this combination of systolic sound characteristics created by means of these generations to be most realistic.

3. Mathematical model description

The sine-cosine Fourier transform was chosen as a basic model. Probabilistic elements $\delta_{L,k}$ were introduced into the transform to assign stochastic properties:

$$ x(k) = \sum_{n=1}^{N} \sum_{k=0}^{\infty} \delta_{A,k}A_k \sin(\delta \omega_1 n \omega_n,k + \delta \phi_1 n \phi_n,k) + \delta_{B,k}B_k \cos(\delta \omega_2 n \omega_n,k + \delta \phi_2 n \phi_n,k), \quad (1) $$

where $x(k)$ – value of the generated signal, $k$ – set of positive integers that correspond to the readings of several values; $A_k, B_k$ – coefficients determining the amplitudes of harmonics ($A$ – sinusoidal harmonic, $B$ – cosinusoidal harmonic), $\omega_k$ – frequency of harmonics for value $k$, $\phi_k$ – $k$-th harmonic phase, $\delta_{L,k}$ – coefficients determining the input and/or presence of the specified components in generated output signals for the sinusoidal ($\omega_1, \phi_1$) and cosinusoidal ($\omega_2, \phi_2$) $n$-th harmonics at moment $k$. All the coefficients of this model were calculated based on experimental data.

This approach can be described with the following elementary functions utilizing damped sinusoids with randomly generated parameters that comply with a simple law (2):

$$ x(t) = \sin(\delta_1 \omega t + \delta_2 \omega) \exp(\delta_3 - t \cdot \delta_4), \quad (2) $$

where $\delta_i$ – randomly generated parameters of the sinusoidal function. Resulting signals were generated according to (1).

3.1. Description of individual harmonics

The description of each harmonic represented a line of the two-dimensional array of double-precision numbers. The columns of this array corresponded to the specific parameters of the harmonic model. The set of harmonics forming specific heart sounds was controlled by one more random number generator with a downward sloping exponential characteristic. With that, over eight individual harmonics of grade 3+ were randomly chosen at the same time, while the first (16 Hz) and the second (18 Hz) harmonics were present in signal S1 on a permanent basis.

To synthesize first heart sound S1, the authors used signal components within the frequency range from 16 Hz (1$\omega$) to 64 Hz (4$\omega$) with an increment of 2 Hz (2$\omega$/8), in total, 24 harmonics representing a “standard” range. If the authors had to represent the frequency of harmonics as an integer, they
performed downward rounding to the closest harmonic frequency. In practice, it resulted in the maximum additional error of no more than 3%.

As for diastolic heart sound S2 perceived as a higher sound, the range of harmonics lied within 30-78 Hz, which involved 24 harmonics as well. The first (30 Hz) and the second (32 Hz) harmonics of generated signal S2 were also present on a permanent basis, but the set of randomly added harmonics was increased to 12 components, which, according to the medical experts, made the sound more natural [7].

3.2. Method of calculating individual harmonics
The sequence of generated values was calculated based on the current task as well, therefore the sampling frequency of synthesized signals was from 45 µs (corresponds to the quality of 48 kHz compact disk audio) to 1,000 µs (data storage systems). The data were saved as RAW files or more common WAVE files. This sampling period could be technically decreased to 5.21 µs, which would correspond to the Bluray audio standard, but the authors considered this option unnecessary.

A double-type vector (one-dimensional array) appeared to be an optimal solution to represent the structure of data bound to specific harmonic frequencies. Representing the ordered set of such vectors as a two-dimensional array was a basis for calculations. The data vector fields listed sequentially based on an increase in the index value had the following meaning:

- Numerical order of the harmonic.
- Harmonic frequency, rad/s.
- Mean origination time of the harmonic relatively to the emergence of the heart sound, µs.
- Confidence interval for the harmonic origination time, µs.
- Mean time to achieve the maximum amplitude of the enveloping harmonic, µs.
- Confidence interval for the time to achieve the maximum amplitude of the enveloping harmonic, µs.
- Relative maximum amplitude of the harmonic; the mean amplitude of the enveloping harmonic with the frequency of 125 rad/s (20 Hz) was taken as 1.
- Confidence interval for the maximum amplitude of the harmonic.
- Mean time to reduce the amplitude of the enveloping harmonic to 0.05 of the maximum value, µs.
- Confidence interval for the time to reduce the amplitude of the enveloping harmonic to 0.05 of the maximum value, µs.

As a result, the numeric value of the parameter was represented as a mean value and its confidence interval setting the limits for potential changes for the random number generator with normal distribution, and the final value of the parameter was the sum of these numbers with due account for their mathematical sign. If the output value of the random number generator was higher than the tabular probability value, the latter was used for calculations. Thanks to this approach, the look of the synthesized heart sounds, like their individual spectral characteristics and tonality (in the opinion of the invited medical experts), appeared to be practically identical to those of the heart sounds available from experiments.

Necessary numerical parameters were represented as a row matrix, which allowed making required alterations. The operating mode of this matrix was a binary image that significantly speeded up data loading and simplified data processing. Before binary matrix loading, its control sum was calculated, which guaranteed the absence of potential errors.

3.3. Method of calculating amplitude envelopes
The harmonic envelope was considered asymmetric and described as a \(\lambda\)-shaped curve with different time constants. The spline interpolation, Weibull distribution and polyexponential envelope representation were checked to calculate the intermediate values of harmonic amplitudes. The shapes of type 3 and 4 spline interpolations appeared to be far from that of the expected \(\lambda\)-shaped curve. For this reason, the authors decided to reject them. The Weibull distribution, a family of two parametric
continuous functions, did not allow selecting parameters that would approximate the envelope amplitudes with an acceptable error of less than 5% to experimental data. Taking into account the physical processes of changes in the concentration of Ca++ in mycoplasma, the authors initially tried approximating harmonic amplitudes via four exponents (migration of calcium ions through the cellular membrane and sarcoplasmic reticulum membrane) [5]. However they subsequently used a well-known two-exponent model with compromise coefficient values. This model allowed significantly simplifying the calculations (3).

\[ x(t) = k_0 e^{k_1 t} + k_2 \left(1 - e^{k_3 t}\right), \quad (3) \]

Exponent approximation parameters were calculated in two stages as a biexponential model consisting of a sum of descending and saturation exponents typical of many biological processes. The authors used the least squares criterion. This method can be considered a modification of the Prony method for the sum of same-type (descending or saturation) exponents [6].

The instant values of harmonic amplitudes were calculated dynamically based on one-time values of time parameters (mean time value plus random deviation). For this reason, the probability of repeating harmonics for the synthesis of neighboring systolic heart sounds S1 was quite low. Figure 1 shows the shapes of the envelopes created based on the biexponential model (3) with due account for tabular data for the 20 Hz and 36 Hz harmonics and outputs of random number generators. These harmonic envelopes were generated without regard to the time of origination (phase lag). They are used only to demonstrate the potential of this approach.

Following the synthesis of harmonic envelopes, the development of the harmonic model came down to multiplying the instant value of its frequency by the instant value of the envelope with a pre-set time sampling interval according to the rule (4)

\[ H(t) = harm(n, t) \ast expn(t), \quad (4) \]

where harm(n,t) – generation function of the n-th harmonic, expn(t) – function simulating the harmonic envelope. The output of this operation is shown in figure 2.

Biexponential envelope: k1=0.20, k2=0.08, a=1.50, b=3.00  
Biexponential envelope: k1=0.50, k2=0.90, a=1.00, b=1.00

![Figure 1](image1.png)  
**Figure 1.** Amplitude envelopes of the 20 Hz (A) and 36 Hz (B) harmonics synthesized with due account for stochastic influence. The amplitude of the envelopes is shown in conditional units.
3.4. Formation of heart sounds

It was possible to start forming heart sounds upon completion of forming the component harmonics of heart sounds S1 and S2. As it has already been mentioned, the 16Hz and 18Hz harmonics were present in systolic heart sound S1 on a permanent basis, while the presence of other harmonics and their parameters (time of origination, amplitude, etc.) were determined by the random number generators. The look of the S1 harmonics synthesized in this way is shown in figure 3. The first image (figure 3A) shows the synthesis output with a focus on low-frequency components (from Hz to 48 Hz). The second image (figure 3B) shows the entire frequency range (from 16 Hz to 64 Hz).

Second diastolic heart sound S2 is less prone to random changes in parameters. For this reason, the authors assumed that the parameters of its component harmonics were changing within significantly narrower limits. This hypothesis was confirmed in practice, first of all, in relation to the amplitude and life cycle of the harmonics. The formation of diastolic heart sound S2 was identical to the heart sound S1 formation algorithm with the only difference: the authors’ own table with the parameters of high-resolution signals available from experiments was used as signal parameters. The range of generated signals lied within the interval of 30-72 Hz with an increment of 2 Hz. The number of randomly added harmonics was increased to 12 signal components. The look of the S2 harmonics synthesized in this way is shown in figure 4.
Second sound synthesis N1

Second sound synthesis N4

Figure 4. Synthesis of second (diastolic) heart sound S2 signals under different initial conditions within the harmonic range of up to 60 Hz (A) and up to 72 Hz (B).

The first image (figure 4A) shows the synthesis output with a focus on low-frequency components (from 30 Hz to 60 Hz). The second image (figure 4B) shows the entire frequency range (from 30 Hz to 72 Hz).

3.5. Phonocardiogram synthesis

The systolic phonocardiogram cycle was synthesized by summing signals S1 and S2 with due account for the origination time of second heart sound S2. The time interval between the heart sounds was created by delaying the generation of heart sound S2 vs. the generation of heart sound S1 by 320 ms [8]. The overall look of heart sounds S1+S2 is shown in figure 5. Figure 5A demonstrates the synthesis output with the complete range of harmonics, while figure 5B demonstrates the half range of odd harmonics. Low-amplitude sounds typical of the interval between the heart sounds were not generated.

Figure 5. Synthesis of heart sounds. A: With the complete range of even and odd harmonics. B: With a half range of odd harmonics.

Short time sequences containing 3-5 systolic cycles were generated to assess the quality of the synthesized phonocardiograms.
4. Discussion of the results

The mathematical model was verified through the comparison of the synthesized signal and phonocardiograms from available phonocardiographic libraries created by independent third-party experts [9, 10]. The selected libraries contain MP3 files compressed with a loss of quality. However the bitrate of the files in these libraries is 320 kbps, which corresponds to the generally accepted high fidelity (hi-fi) musical audio category [11]. Cardiologists with general music education (specialized in piano) were invited as medical experts.

The reference and synthesized phonocardiograms are acoustically identical. However, the authors believe that an objective criterion is qualitatively coinciding spectral characteristics of the signals under study. The spectral characteristics of the randomly chosen phonocardiographic signals containing three systolic cycles are shown in figure 6.

![Amplitude Spectrum](image)

**Figure 6.** Spectrums of verified and synthesized signals. A. Comparison of signal spectrums: solid line – synthesized signal, dashed line – reference signal. B. Difference curve of spectrums.

The authors believe that the differences observed in the spectrums of the synthesized and reference signals (Phonocardiogram Library [9]) can be explained with the use of different sensors. The Littmann® electronic stethoscopes [12] utilize the classical audio registration scheme with an elastic membrane and electret microphone as a sensor. This scheme allows intensifying acoustic oscillations in proportion to the ratio between the areas of the elastic membrane and electret microphone membrane. However it has critical drawbacks connected with its design features.

First of all, this scheme introduces distortions into the frequency response of signals, as it serves as kind of a frequency filter where parameters are determined by the geometric pattern of the stethoscope receiver cavity and elastic membrane material. The situation definitely gets worse if the acoustic sensor is moved from the stethoscope receiver to the device case: in this situation, the distortions caused by the receiver are combined with the acoustic limitations of the sound-transmitting tube determined by its geometric parameters, materials and other factors that are hard to consider. Contact acoustic detectors do not have this peculiarity [14].

Second, electret microphones, even those from the category of measuring sensors, demonstrate low sensitivity to low- and very-low frequencies, and their frequency response is not standardized for the frequencies below 20 Hz: the reference microphone Panasonic WM-61 can be used as an example. It is kind of a standard option for precision measurement systems [13]. Piezoelectric sensors and systems with these sensors (e.g., acoustic MEMS) have the lower frequency limit of detected oscillations being several centihertz in the worst-case scenario [14].

The authors believe that the above-mentioned factors are determinant to introduce frequency distortions into the spectrum of phonocardiograms. In this connection, the lower threshold of the frequency spectrum of physiological phonocardiograms is limited to 26-28 Hz in many reputable
guidelines, and this frequency is typical of the electrodynamic microphones that were widely used in phonocardiographs prior to the early 1990s [8]. The data on the frequency response of the phonocardiograms obtained in that period of time are replicated without any changes from one specialized guidebook to another. They practically represent the axiom for cardiology study books.

In the authors’ opinion, contact acoustic sensors based on piezoelectric films made of piezoceramics with a high coefficient of mechanoelectrical conversion allow solving the above-mentioned challenges with the low-frequency limitation of registered phonocardiographic signals and potential distortions caused by the procedure of hardware-assisted auscultation. Using proprietary experimental data as a basis for a mathematical phonocardiogram synthesis model will help develop a hardware-and-software system to correctly imitate different types of both physiological and pathological phonocardiograms.

Therefore, the sine-cosine signal decomposition into the Fourier transform allowed the authors to create a functional model and synthesize heart sounds that are comparable with normal phonocardiographic signals. The authors solved the task of selecting probable parameters of harmonic components to ensure the maximum reliability of the model.

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