Chiral Low-Energy Constants*

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Abstract

The progress in determining the coupling constants of mesonic chiral Lagrangians is reviewed, with emphasis on the work performed in three successive European Networks (Eurodaphne I and II, Euridice). Reliable estimates of those constants are essential for making full use of next-to-next-to-leading-order calculations in chiral perturbation theory. The precision in the values of the strong coupling constants of $O(p^4)$ has been increasing steadily over the years. The situation is less satisfactory in the nonleptonic weak sector where further phenomenological input and more theoretical work are needed. A lot of progress has recently been achieved for electromagnetic coupling constants occurring in radiative corrections for mesonic processes at low energies.

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1 Introduction

Chiral low-energy constants (LECs) are the coupling constants of effective chiral Lagrangians. They are independent of the light quark masses by construction and they describe the influence of all “heavy” degrees of freedom that are not contained explicitly in the effective Lagrangians. The construction of effective Lagrangians is based on symmetry considerations only so that a lot of information is lost in going from the underlying Standard Model to the effective theory. As a consequence, effective Lagrangians contain many LECs, especially at higher orders in the chiral expansion. Progress in chiral perturbation theory (CHPT) [1–3] depends on realistic estimates of chiral LECs.

My task in this talk was to review the progress in determining or estimating chiral LECs since 1993 when the first Eurodaphne Network got started. Most of this progress is in fact due to work performed in the three European Networks Eurodaphne I, Eurodaphne II and Euridice. I will only consider the meson sector here. The corresponding effective chiral Lagrangian is given in Table I. For a recent review including the baryon sector see Ref. [4].

| $\mathcal{L}$ | chiral order | (# of LECs) | loop order |
|-------------|--------------|----------------|-------------|
| $\mathcal{L}_{p^2}$ | (2) | | $L = 0$ |
| $\mathcal{L}_{pr^3}$ | (0) | | $L = 0$ |
| $\mathcal{L}_{G_{p^2}p^2}$ | (2) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (1) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (1) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (23) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (21) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (22) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (23) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (24) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (25) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (26) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (27) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (28) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (29) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (30) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (31) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (32) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (33) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (34) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (35) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (36) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (37) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (38) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (39) | | $L = 0$ |
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| $\mathcal{L}_{G_{e^2p^2}}$ | (50) | | $L = 0$ |
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| $\mathcal{L}_{G_{e^2p^2}}$ | (58) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (59) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (60) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (61) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (62) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (63) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (64) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (65) | | $L = 0$ |
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| $\mathcal{L}_{G_{e^2p^2}}$ | (69) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (70) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (71) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (72) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (73) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (74) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (75) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (76) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (77) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (78) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (79) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (80) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (81) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (82) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (83) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (84) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (85) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (86) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (87) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (88) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (89) | | $L = 0$ |
| $\mathcal{L}_{G_{e^2p^2}}$ | (90) | | $L = 0$ |

Table 1: Effective chiral Lagrangian in the meson sector. The numbers in brackets refer to the number of LECs for chiral $SU(3)$.

Information on chiral LECs is obtained either from phenomenology or with additional input from theory.
Extraction from experiment

Some LECs are associated with terms in the Lagrangian that contribute to amplitudes even in the chiral limit. They govern the momentum dependence of amplitudes and are at least in principle accessible experimentally. The other class involves chiral symmetry breaking terms that specify the quark mass dependence of amplitudes. The related LECs are much more difficult to determine phenomenologically but they are accessible in lattice simulations.

Input from theory

- Large-$N_c$ methods match CHPT with QCD by bridging the gap $M_K \lesssim E \lesssim 1.5$ GeV with resonance exchange.
- Lattice QCD.

2 Strong interactions

At lowest order in the chiral expansion, there are only two LECs $B$ and $F$. $B$ depends on the QCD renormalization scale and always appears multiplied by quark masses in CHPT amplitudes. The products $Bm_q$ and the constant $F$ can be expressed in terms of meson masses and of the pion decay constant $F_\pi$. Those relations involve LECs of $O(p^4)$ or higher to be discussed subsequently.

The strong chiral Lagrangians of $O(p^4)$ contain 7 measurable LECs $l_i$ for chiral $SU(2)$ [2] and 10 LECs $L_i$ for chiral $SU(3)$ [3]. The current phenomenological values are based on calculations to $O(p^6)$ in most cases, sometimes supplemented by dispersive methods. Although I restrict the discussion here to $SU(3)$, the most precise determinations have been obtained for the $SU(2)$ LECs $l_1, l_2, l_4$ by combining CHPT to $O(p^6)$ with Roy equations [5]. This information can also be used for some of the $SU(3)$ LECs. The relations between the $l_i$ and the $L_i$ are however only known [3] to $O(p^4)$, which is not sufficient for the present purpose. Work in progress by the Bern group [6] will soon provide those relations to $p^6$ accuracy.

The present values of the renormalized $SU(3)$ LECs $L_i(M_\rho)$ are shown in the second column of Table 2. The first column contains the values originally obtained in Ref. [3]. No drastic changes have occurred although the mean values have generally decreased in absolute magnitude. For $L_1, L_2, L_3$ the information from $\pi\pi$ scattering [5] will be very useful once the relations between the $SU(3)$ and $SU(2)$ LECs will be available at the $p^6$ level [6].
Table 2: Phenomenological values and theoretical estimates for the $SU(3)$ LECs $L_i(M_\rho)$ in units of $10^{-3}$. The first column shows the original values of Ref. [3], the second displays the present values taken from Ref. [7] and references therein. The third column is based on an analysis of $\pi K$ scattering [8] (only experimental errors are shown). The fourth column contains recent lattice results from the MILC Collaboration [9]. The fifth column shows the resonance saturation results of Ref. [10] and the last column reproduces a systematic estimate of resonance contributions to lowest order in $1/N_c$ [15]. The entries marked with † were taken as input in Ref. [10].

| i | $O(p^4)$ | $O(p^6)$ | $\pi K$ | lattice | Ref. [10] | Ref. [15] |
|---|----------|----------|--------|---------|-----------|-----------|
| 1 | 0.7 ± 0.3 | 0.43 ± 0.12 | 1.05 ± 0.12 | 0.6 | 0.9 |
| 2 | 1.3 ± 0.7 | 0.73 ± 0.12 | 1.32 ± 0.03 | 1.2 | 1.8 |
| 3 | −4.4 ± 2.5 | −2.35 ± 0.37 | −4.53 ± 0.14 | −3.0 | −4.3 |
| 4 | −0.3 ± 0.5 | ∼ 0.2 | 0.53 ± 0.39 | −0.2 ± 0.4 | 0 | 0 |
| 5 | 1.4 ± 0.5 | 0.97 ± 0.11 | 1.2 ± 0.4 | 1.4† | 2.2 |
| 6 | −0.2 ± 0.3 | ∼ 0.0 | 0.1 ± 0.2 | 0 | 0 |
| 7 | −0.4 ± 0.2 | −0.31 ± 0.14 | | −0.3 | −0.3 |
| 8 | 0.9 ± 0.3 | 0.60 ± 0.18 | 0.7 ± 0.2 | 0.9† | 0.8 |
| 9 | 6.9 ± 0.7 | 5.93 ± 0.43 | | 6.9† | 7.2 |
| 10 | −5.5 ± 0.7 | −5.09 ± 0.47 | | −6.0 | −5.4 |
The LECs $L_1, \ldots, L_4$ have also been extracted from $\pi K$ scattering, based on a dispersive analysis [8] applied to a CHPT calculation of $O(p^4)$. The results are displayed in the third column in Table 2 where only experimental errors are shown.

With most of the chiral LECs of $O(p^4)$ reasonably well known, can we understand the specific values with additional theory input? Lattice QCD has come a long way to determine some of the constants directly from QCD. The fourth column in Table 2 shows the most recent results of the MILC Collaboration [9] with three dynamical light (staggered) quarks. The agreement with the phenomenological values in column 2 is indeed “staggering”.

A different approach makes use of the properties of QCD at large $N_c$ where amplitudes can be expressed in terms of (stable) resonance exchange. To illustrate the main features of this approach [10,11], let us consider elastic meson-meson scattering, specializing to a channel with $s \leftrightarrow u$ symmetry (e.g.: $\pi^+ \pi^0 \rightarrow \pi^+ \pi^0$). From axiomatic field theory (Froissart theorem) we know that the scattering amplitude $A(\nu, t)$ satisfies a once-subtracted forward dispersion relation in $\nu = (s - u)/2$:

$$A(\nu, t = 0) = A(0, 0) + \frac{\nu^2}{\pi} \int_0^{\infty} d\nu' 2 A(\nu', 0) \frac{Abs A(\nu', 0)}{\nu^2 (\nu^2 - \nu'^2)} . \quad (1)$$

Exchange of a resonance $R$ generates the absorptive part

$$Abs A(\nu, 0) = \pi c_R M_R^4 \delta(\nu^2 - M_R^4) \quad (2)$$

where the constant $c_R$ is related to the partial decay width $\Gamma(R \rightarrow \pi \pi)$ in this case. Therefore, Eq. (1) gives rise to

$$A(\nu, 0) = A(0, 0) + \frac{c_R \nu^2}{\nu^2 - M_R^4} . \quad (3)$$

On the other hand, resonance exchange on the basis of a chiral resonance Lagrangian produces an amplitude of the general form

$$A_R(\nu, 0) = \frac{P_R(\nu^2)}{\nu^2 - M_R^4} , \quad (4)$$

with a polynomial $P_R(\nu^2)$ satisfying the on-shell condition $P_R(M_R^4) = c_R M_R^4$. Decomposing the polynomial $P_R(\nu^2)$ as

$$P_R(\nu^2) = P_R(M_R^4) + (\nu^2 - M_R^4) T_R(\nu^2) , \quad (5)$$
the condition \( A_R(\nu, 0) = A(\nu, 0) \) requires \( \mathcal{P}_R(\nu^2) \) to be a constant,

\[
\mathcal{P}_R(\nu^2) = A(0, 0) + c_R ,
\]

which will not be the case for a general resonance Lagrangian. Therefore, the short-distance constraint embodied in the once-subtracted dispersion relation \( \Pi \) demands that in general an appropriate polynomial \( P_c(\nu^2) \) be added to \( A_R(\nu, 0) \):

\[
A_R(\nu, 0) = P_c(\nu^2) + \mathcal{P}_R(\nu^2) + \frac{P_R(M_R^4)}{\nu^2 - M_R^4} .
\]

The counterterm polynomial \( P_c(\nu^2) \) is fixed by the short-distance constraint to satisfy

\[
P_c(\nu^2) + \mathcal{P}_R(\nu^2) = A(0, 0) + c_R ,
\]

ensuring at the same time the correct low-energy behaviour of the resonance exchange amplitude:

\[
A_R(\nu, 0) = A(\nu, 0) = A(0, 0) - \frac{c_R}{M_R^4} \nu^2 + O(p^8) .
\]

The coefficient of \( \nu^2 \) depends only on the mass and on the partial decay width of the resonance and it defines the resonance contribution to a certain combination of the \( L_i \).

With the proper choice of resonance fields, such counterterm polynomials are not needed at \( O(p^4) \) for the exchange of \( V(1^{--}) \), \( A(1^{++}) \), \( S(0^{++}) \) and \( P(0^{-+}) \) mesons [12], but they are unavoidable for \( T(2^{++}) \) and \( A(1^{+-}) \) exchange [13, 14].

The example of elastic meson meson scattering raises the legitimate question why one should bother at all with resonance Lagrangians? It may seem like an unnecessary detour to use the couplings of a resonance Lagrangian that have to be corrected by short-distance constraints after all. The alternative is to study Green functions directly with a large-\( N_c \) inspired ansatz in the first place. The main advantages of a Lagrangian approach are first of all that chiral symmetry is automatically guaranteed for the generated Green functions and amplitudes and there is no need to impose chiral Ward identities. At least as important from a practical point of view is the possibility to integrate out the resonances once and for all in the generating functional of Green functions (always to leading order in \( 1/N_c \)), thereby generating all LECs of a given order. Of course, the short-distance analysis still remains to be done.
The fifth column in Table 2 shows the original resonance estimates of Ref. [10]. The last column contains more recent systematic estimates of resonance contributions to lowest order in $1/N_c$ [15]. Remembering that the renormalization scale is not fixed at leading order in $1/N_c$, the agreement between the resonance exchange contributions and the phenomenological values in Table 2 is more than satisfactory. Attempts to include corrections of next-to-leading order in $1/N_c$ have also been made [16,17].

To improve the precision of LECs of $O(p^4)$, realistic estimates for some LECs of $O(p^6)$ are also needed. Several results have already been obtained by members of the Euridice Collaboration [18–21]. All possible resonance contributions of the standard variety $V(1^{--}), A(1^{++}), S(0^{++}), P(0^{--})$ have recently been presented in Ref. [22]. The short-distance analysis remains to be done in many cases of interest.

The odd-intrinsic-parity Lagrangian of $O(p^4)$ is given by the Wess-Zumino-Witten Lagrangian $\mathcal{L}_{p4}^{\text{odd}}(0)$ [23] in Table 1. After several conflicting results in the literature there is now a consensus that the corresponding Lagrangian of $O(p^6)$ has 23 LECs [24]. Only partial results are available for the numerical values of those constants, but the most promising approach is again based on a short-distance analysis with or without chiral resonance Lagrangians [18,25].

3 Nonleptonic weak interactions

The chiral Lagrangian of lowest order, $O(G_F p^2)$, contains two LECs $g_8, g_{27}$ to describe nonleptonic weak decays of kaons [26]. Especially the value of the octet coupling $g_8$ is very sensitive to chiral corrections [27]. Isospin breaking corrections are potentially important for the 27-plet coupling constant $g_{27}$. The present status is presented in Table 3. Although different isospin breaking contributions are sizeable the overall corrections are small for both LECs.

The LECs of $O(G_F p^4)$ (22 couplings $N_i$ in the octet and 28 couplings $D_i$ in the 27-plet Lagrangians) are much less known than their strong counterparts at $O(p^4)$. The most recent phenomenological analysis of those combinations that occur in the dominant $K \to 2\pi, 3\pi$ decays can be found in Ref. [29]. Many more LECs appear in rare $K$ decays and a phenomenological update is definitely needed here.
Resonance saturation of weak LECs [30] suffers from the obvious drawbacks that the weak resonance couplings are unknown and that short-distance constraints are missing. Nevertheless, resonance saturation provides at least a possible parametrization of the LECs. The most systematic approach is based on factorization (valid to leading order in $1/N_c$) [28, 31] but higher-order corrections in $1/N_c$ may well be sizeable.

If the situation of the LECs of $O(G_F p^4)$ is already unsatisfactory, even less is known about higher orders. However, the leading (double) chiral logs of $O(G_F p^6)$ are known [32].

### 4 Dynamical photons

Radiative corrections at low energies involve the effective Lagrangians in Table 1 with superscripts $em$, $emweak$ or $leptons$ (semileptonic decays).

In the presence of photons as dynamical degrees of freedom the chiral counting is different from the purely strong or nonleptonic weak cases. The lowest-order Lagrangian for electromagnetic corrections to strong processes is of $O(e^2 p^0)$ with a single LEC [10] that can be determined from the $\pi^+ - \pi^0$ mass difference. The next-to-leading-order Lagrangian of $O(e^2 p^2)$ with 13 LECs $K_i$ was constructed by Urech [33]. By convoluting pure QCD $n$-point functions ($n \leq 4$) with the photon propagator, sum rule representations were derived for all the $K_i$ [34]. Numerical estimates for the $K_i$ are obtained by saturating the sum rules with resonance exchange. Since the LECs $K_i$ are difficult to determine phenomenologically, the systematic work of Ref. [34] is especially important for controlling radiative corrections to strong processes at low energies.

The situation is much less favourable for electromagnetic corrections to nonleptonic weak processes. The single LEC of lowest order, $O(G_S e^2 p^0)$,
related to the electromagnetic penguin contribution \[35\], is reasonably well known. However, the 14 additional LECs of $O(G_8 e^2 p^2)$ \[36\] are only known to leading order in $1/N_c$ (factorization). In this way, the LECs can be expressed in terms of Wilson coefficients, the strong LECs $L_5$, $L_8$ and the electromagnetic LECs $K_i$ \[28, 31\].

5 Dynamical photons and leptons

Radiative corrections for semileptonic weak decays require the incorporation of leptons as dynamical degrees of freedom. The leading-order Lagrangian $\mathcal{L}_{e^{+}p}^{\text{leptons}}(5)$ in Table \[\text{I}\] contains five LECs $X_i$ \[37\].

With a two-step matching procedure (Standard Model $\leftrightarrow$ Fermi theory $\leftrightarrow$ CHPT), Descotes-Genon and Moussallam have recently established integral representations for all the $X_i$ \[38\]. One important application is in $K_{l3}$ decays, still the best source for extracting the CKM matrix element $V_{us}$. As a consistency check, the isospin violating ratio

$$r_{+0} := \left( \frac{2 \Gamma(K_{e3(\gamma)}^+)}{\Gamma(K_{e3(\gamma)}^0)} \frac{M_{K_0}^5}{M_{K^0}^5} \frac{I_{K_0}}{I_{K^0}} \right)^{1/2} = \left| \frac{f_{K^+}^{K^+\pi^0}(0)}{f_{K^0}^{K^0\pi^+}(0)} \right|$$

has been considered \[39\] that depends essentially only on $X_1$. With the result for $X_1$ from Ref. \[38\], the theoretical prediction $r_{+0} = 1.024 \pm 0.003$ is now in perfect agreement with the most recent $K_{l3}$ data \[40\]. The agreement also indicates that higher-order corrections to the theoretical prediction for $r_{+0}$ of $O((m_u - m_d)p^4, e^2 p^4)$ behave as expected from chiral power counting.

6 Outlook

Since 1993, when Eurodaphne got started, substantial progress has been made in the understanding of low-energy constants, both from phenomenology and from theory (lattice QCD and large-$N_c$ approaches).

In the strong sector, most of the machinery is now ready for a precision determination of the LECs of $O(p^4)$. This endeavour involves also LECs of $O(p^6)$ where we are still in the exploratory stage. We need reliable estimates for some of those LECs to make full use of next-to-next-to-leading-order calculations. In the nonleptonic sector, improvements both in phenomenology
and in theory are needed. The most impressive progress in recent years has happened for electromagnetic LECs. As a consequence, radiative corrections in the meson sector at low energies are now under control. Semileptonic $K_{l3}$ decays are one prime example of phenomenological relevance.

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