The genetic decomposition of students about infinite series through the ethnomathematics of Bengkulu, Indonesia

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Abstract. The cognitive theory views individuals as active information processors, so that individuals were able to represent each information according to the level of knowledge they have. Student representation can be seen as its genetic decomposition. Ethnomathematics was a vehicle for unlimited series learning. The purpose of this study was to describe the genetical decomposition of students about infinite series through the Bengkulu ethnomathematics. This was the initial research from a series of development research. This stage we interviewed in depth 10 high school students in Bengkulu, Indonesia. The research instrument was the researchers themselves who were guided by interview guides about understanding concepts and the principle of infinite series. Interviews were conducted during and after Bengkulu's ethnomathematics learning. Data were analyzed through fixed comparison techniques. The results of this study were found that students can coordinate two or more actions about convergence of sequences, but not for convergence of infinite series. Conversely, there were students who can describe a particular object about converging an infinite series but not coordinated with the processes that were built for converging sequences. Also, there were students who can coordinate the action-process related objects so that a schema of converging sequence was formed, but for infinite series only in the form of separate actions or processes. The conclusion of this study was the genetic decomposition of students in understanding the infinite series through ethnomathematics learning at the intermediate level (inter level). The student before ethnomathematics learning was at the lower level (intra and pre-intra levels).

1. Introduction
Mathematics was a difficult subject for high school students[1][2]. One difficult concept was an infinite series. Students were often wrong in determining their convergence. This obstacle occurs due to structuralist mathematical learning. Students tend to memorize and only act mechanically. This was less realistic [3]. Whereas in learning, the potential and activeness of students were the main elements that influence their learning achievement [4]. Students were expected to be able to process information, storage, and recall of knowledge from the brain. This was collection of mental and physical activities was a genetic decomposition [5][6][7].

The genetic decomposition was a structured collection of mental activities carried out by someone to describe how mathematical concepts and principles can be developed in his mind [4]. This can be analyzed through action-process-object-scheme activities [8][9][10]. Action was a procedural activity
through physical repetition or mental manipulation to transform objects in several ways [11][12]. As an indication was to do a reaction to the external stimuli received, in the form of a detailed expression of the steps that must be done [4]. Then the actions were interiorized into a process. Interiorization was a change of activity from a procedural activity to be able to carry out activities in imagining some of the meanings that influence the conditions produced (this was a form of action in the process) [13][4]. Processes were encapsulated on an object. As an indication was to do mental transformation (in the form of cognitive coordination) of a process on a cognitive object [14]. If the process itself was transformed by several actions, there will be encapsulation that matches an object [4]. Finally the thematization was done to construct the scheme by linking separate actions, processes, or objects to a particular object [10][6]. This results in a scheme that was the formation of a total entity from different objects constructed through cognitive coordination [15][4]. Students who were able to achieve complete genetic decomposition will achieve a mature scheme [11]. The mature scheme of a mathematical object was a system that was coherent from actions, processes, objects, and other schemes that have been built before [4]. Everything was synthesized by individuals in the form of structures used to deal with certain problem situations [14].

Based on task-based interviews challenge convergence of lines and infinite series, there were subjects who think that an infinite number of positive numbers must have a value [4]. This was an error which was the principle of an infinite number of positive numbers. Also, a subject was found which states that the last terms of the sequence \(\cos (\pi/k)\) were constant [16]. It turns out that this was a result of mechanistic and structuralistic learning [17]. Therefore, learning needs to be close to students' minds [18]. Also close to culture in daily life. The ethnomathematics approach was more realistic for him [1][2].

The term ethnomathematics was used to express the relationship between culture and mathematics [19]. Ethnomathematics studies aspects of mathematical culture. It presented the concept of school mathematics by linking it with student experience and everyday culture. It enhances their ability to describe connections that were meaningful and deepen their understanding of mathematics [20]. Ethnomathematics emphasizes the competence of students developed in different cultural groups in their daily lives. The notion of mathematical literacy mainly focuses on mathematical and social requirements for student competence. Therefore, sociomathematics was an analytical concept for the subject areas, namely cognitive, affective, and social relations with mathematics in society [21]. Mathematics teachers were challenged to handle the cultural diversity of people that occur in each classroom. Ethnomathematics has an important role in the curriculum, being meaningful in exploring various aspects of mathematical literacy [22].

The results of the study of Herawaty, et al., students solved mathematical problems through an ethnomathematics mathematical process. Students were aware that ethnomathematics was the starting point of horizontal mathematical activity. Just like traditional homes, culture was a real problem to achieve geometric concepts, such as geometric figures in 2 dimensions and 3 dimensions. In particular, students can find out about the surface area and volume of pyramids, prisms, rectangular prisms, and cubes [2]. Mathematical understanding of students learning ethnomathematics-oriented material was higher than the material studied which was not ethnomathematics oriented (realistic mathematics learning applied to both groups) [1]. There were significant differences in students' mathematical problem solving abilities between before and after being given ethnomathematics learning with outdoor learning models. The mathematical problem solving abilities of students after being given ethnomathematics with outdoor learning models were higher than before being given the learning models [23]. The group of students who were given material oriented to ethnomathematics, the ability to understand mathematical concepts from students taught with realistic mathematical learning approaches was higher than those taught with direct instruction. Conversely, for groups of students who were given mathematical material oriented to non-ethnomathematics, the ability to understand mathematical concepts from students who learned using
realistic mathematics learning approaches was lower than students who studied with direct instruction [24].

The implementation of the realistic mathematics education with starting point ethnomathematics was to improve the mathematical communication abilities [25]. It was a mathematical learning approach that was close to the thoughts and culture of students. Ethnomathematics learning can improve students' mathematical abilities in building thematic relationships between actions, processes, objects, and other schemes. This scheme can be used to solve mathematical problems and related characters. Also, it was used to classify objects selectively [12]. Through ethnomathematics learning, and considering both the need for meaningful contexts in mathematics learning will be able to develop mathematical ideas in the direction of abstraction and generalization (in a flexible sense, not to be confused with decontextualization) [26]. The ethnomathematics was a vehicle to explore real-world mathematics applications in global communities, and represents resourcefulness, inventiveness, wisdom grounded in the past, and hope for the future [27]. Knowledge was dynamic, and because of objectivity and subjectivity, and reflection and action were connected dialectically, educators must consider culture and context daily habits, language and ideology as inseparable from the practice of learning mathematics [28]. Thus, ethnomathematics was an important role in mathematics learning. It was an effort to make it easier for students to carry out abstraction processes and generalize mathematical concepts. It was the thinking process of students. According to Widada, et.al., characteristics of students' mathematical thinking responses can be explored during mathematics learning [29].

Understanding concepts was the first step in learning mathematics. A concept can stimulate phenomena in processing information relating to complex things, or closely related to personal information can be represented in memory. It was a knowledge representation structure. The structure of knowledge representation was a role internally in information processing [30]. It was a genetic decomposition that can be analyzed through task-based interviews during ethnomathematics learning. Thus, we realize that ethnomathematics learning was a vehicle for exploring students' cognitive processes. One of the difficulties of students was learning about infinite series. According to Widada, et. al., infinite series was one of the difficult calculus materials for students. It was necessary for students to make it easier for teachers to arrange learning plans [31]. The teacher prepares a lesson plan by applying the ethnomathematics approach. During the implementation of ethnomathematics learning, students' cognitive processes were traced through their genetic decomposition. It was a characteristic of students, which can be used as a guide to reflect on their learning processes and outcomes. Finally, we can reveal the genetic decomposition of students about infinite series through the ethnomathematics.

2. Methods

This study was part of phases of Plomp’s developmental research model [32]. Itu adalah tahap Preliminary research. In this phase we emphasize the needs assessment, which produces student characteristics. We were conduct literature and project reviews, past and/or now. This produces guidelines for the framework and the first blueprint for intervention. The subjects of this study were ten high school students in Bengkulu, Indonesia. The research subjects followed classroom learning with the ethnomathematics approach. They were interviewed based on assignments during and after the learning. The tasks given were as follows:

"Know row \( (u_k) \), with \( u_k = \cos \left( \frac{\pi}{k} \right) \). a) Investigate, was \( (u_k) \) convergent? If yes, specify \( \lim_{k \to \infty} \cos \left( \frac{\pi}{k} \right) \). Then determine, was it convergent? Show the truth of your conclusions!"

The research instrument was the researchers themselves who were guided by interview guides about understanding concepts and the principle of infinite series. Interviews were conducted during and after Bengkulu’s ethnomathematics learning. Data were analyzed through fixed comparison techniques. That was applying the APOS Theory. APOS theory was a constructivist theory of how the possibility of
learning takes place in a mathematical concept. APOS theory was an elaboration of mental constructs of actions, processes, objects and schemes. In examining how students learn mathematical concepts was an essential element that must be given by researchers in an analysis of concepts in that particular construct. The description generated from this analysis was referred to as the genetic decomposition of the concept[10].

3. Results and Discussion

Based on task-based interview data during and after ethnomathematics learning in Bengkulu. This data analysis aims to reveal the special mental construction (genetic decomposition) that students do in learning the infinite series concept. In this case the role of teaching treatment was to gain students' understanding in making mental construction and use it to construct concepts and apply them in mathematical and non-mathematical situations. Genetic decomposition analysis of convergence of infinite series was done separately, namely schematic behavior about converging sequences, and schematic behavior about converging infinite series. The following will be presented interview footage and analysis. Let interviewer (=P), and research subject (=KY).

3.1. Part 1

Q: Please complete the infinite series convergence problem on the available sheet ...

KY 2.01: Yes ... alright ... [...... shut up .......] [KY nodded and then solved the problem with paper and board about 10 minutes]

P: Okay! Please explain about the results of your work? ... part a) first!

KY 2.02: I specify that the limit of \( \cos \frac{\pi}{k} \) for \( k \) close to infinity, it was not equal to \( \cos 0 = 1 \). This was based on the limit theorem where the limit was from \( \frac{\pi}{k} \) for \( k \) towards infinity equals \( 0 \), so that the sequence \( (u_k) \) converges to 1. [KY shows the results of its work in accordance with KY 2.02]

Q: Where do you conclude that the sequence was convergent?

KY 2.03: Here see that limit \( u_k \) it exists, namely 1 ... only this ... [KY shows that the limit of \( \cos \frac{\pi}{k} \) for \( k \) towards infinity equals \( 0 = 1 \)]

Based on Exhibit 1, KY can coordinate between limits or lines with their convergence, so that a scheme was formed about converging sequences and can conclude that the sequence \( u_k \) converge with \( \lim_{k \to \infty} u_k = 1 \).

3.2. Part 2

P: Well ... then, now that was b) Try your little one and finish it!

KY 2.04: Okay ... I'll investigate first ...... [KY Trying to investigate about 5 minutes]

P: Okay! Explain what you have produced!

KY 2.05: I tried to explain that \( \sum \cos \left( \frac{\pi}{k} \right) = \cos \left( \frac{\pi}{1} \right) + \cos \left( \frac{\pi}{2} \right) + \cos \left( \frac{\pi}{3} \right) + \cos \left( \frac{\pi}{4} \right) + \ldots + \cos \left( \frac{\pi}{5} \right) \) was greater than the last term 1 was obtained, so [The sequence is] was equal to \(-1 + 0 + \frac{1}{2} + \frac{1}{2} \sqrt{2} + \ldots + 1\] Meaning \( \sum \cos \left( \frac{\pi}{2} \right) \) there was [That was \( 0 + \frac{1}{2} + \frac{1}{2} \sqrt{2} + \ldots \)] there were numbers.[KY shows its work like the expression KY 2.05]

P: Why were you substituting \( \sum \cos \left( \frac{\pi}{2} \right) = 1 \)?

KY 2.06: Because [based on a] \( \left( \frac{\pi}{2} \right) \) converge to 0, so \( \cos \left( \frac{\pi}{2} \right) \) close to 1.

P: What was your conclusion
KY 2.07: Number of desires so that the series $\sum \cos \frac{\pi}{k}$ convergent.

P: Try showing the truth that there were numbers!

KY 2.08: [...] here I can divide into three parts [i.e.] $\cos \left( \frac{\pi}{1} \right) + \cos \left( \frac{\pi}{2} \right) + \cos \left( \frac{\pi}{3} \right) + \ldots + \cos \left( \frac{\pi}{5} \right) = (-1) + \left( \frac{1}{2} \sqrt{2} + \ldots + 1 \right) = \frac{1}{2} + \frac{1}{2} \sqrt{2} + \ldots$. There will be more value. But with an integral difficulty test [As I have done this] But as above, what was clear $\sum \cos \frac{\pi}{k}$ there is. And I don’t know the other way. [KY shows the work on paper and shows that he has tried with an integral test but difficulty].

KY carries out an action about an infinite series by describing an infinite series $u_k$ and declare the final term with 1 (see KY 2.05 and KY 2.06 so KY concludes that the number was there which means the series was convergent (See KY 2.07). Another action taken by KY was to calculate the number of series see KY 2.08, and see Figure 1.

![Figure 1. Solution from KY](image)

Based on the analysis of genetic decomposition during and after learning with the ethnomathematics approach, separately the development of the convergence scheme of lines and the convergence of infinite series, it can be determined as follows: 1) Genetic decomposition of converging sequences, KY can coordinate objects about the definition of convergence of rows and limit theorems, so that a coherent sequence scheme was formed. It states that KY was included in the Trans Level. 2) Genetic decomposition of infinite series convergence, KY only carries out actions about the given series, because KY only does a reaction about infinite series terms, but it was not interiorized, because KY cannot imagine how the condition of the series ends. From this description put KY in the Intra Level.
Based on the description of KY, while resolving the problem of convergence of infinite series, he coordinated objects about the definition of convergence of lines and terrorism limits, so that a coherent sequence scheme was formed. KY tries to coordinate it with convergence of infinite series, but it actually results in errors. That was the result of KY only taking actions about the given series. The subject only performs an action about infinite series terms, but it was not interiorized. KY cannot imagine the condition of the end of the series. Also, it resulted in errors in drawing conclusions about the convergence of the infinite series. Overall, KY was at the Inter Level.

The results of this study support previous studies such as Widada, et. al. subjects can coordinate other objects and processes, so that the scheme was formed about the convergence of infinite series and sequences. The subject was a trance level characteristic[31]. Students at the inter level can carry out actions that were interiorized into processes, even though the resulting object was not right[33]. Some barriers to inter-level students in applying concepts and derivative characteristics in an analytical manner include, tangents almost parallel to the y axis, cusp points, second derivatives, extreme points, contradictions, flat asymptotes, and conceptual notions[30]. According to Dubinsky & McDonald, it has shown how to observe students' success in making or not making mental constructs proposed by the theory and using these observations to analyze data can regulate our thinking about learning mathematical concepts, giving explanations about student difficulties and predicting success or failure in understanding mathematical concepts. There were many mathematical concepts that can and have been applied by APOS Theory and this theory was used as a language to communicate ideas about learning[10].

In many studies it was also found that genetic decomposition was an analysis of students' mathematical abilities during and after learning. Asiala, et.al., describe certain mental constructions for learning mathematics, including actions, processes, objects, and schemas, and the relationships among these constructions. Under instructional treatment, we describe the components of the ACE teaching cycle (activities, class discussion, and exercises), cooperative learning and the use of a mathematical programming language. Finally, we describe the methodology used in data collection and analysis[34].According to Maharaj, the APOS theory was "genetic decomposition" of concepts that students must learn in terms of the mental construction required by learning. Genetic decomposition focuses on the mental constructs needed for student success during the initial conceptual stages of learning related level problems. Decomposition was made using the author's knowledge of the subject. Genetic decomposition was used to build the teaching cycle of Action - Discussion - Exercise (ACE) which was then tested in two groups of students. Finally, students were asked to solve price related problems during individual interviews with the author. Data from student involvement in the ACE cycle and their work during the interview process were then used to suggest changes to genetic decomposition and the ACE cycle[35]. Thus, we believe that the cognitive process of students after taking ethnomathematics learning increases their cognitive level.

4. Conclusion
Based on the results of the genetic decomposition during and after learning with the ethnomathematics approach, we can conclude that the subject of research was at the inter level. The character was through ethnomathematics learning, students can solve the problem of converging infinite series. He coordinated objects about the definition of convergence of lines and terrorism limits, so that a coherent sequence scheme was formed. However, he experienced an error, a result of actions which only carried out actions about the series given. Also, unable to imagine how the condition of the end of the series. Finally, students experience errors in drawing conclusions about converging infinite series. This level was an increase from the level before participating in learning with the ethnomathematics approach. We suggest applying the ethnomathematics approach in each mathematics learning.
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