The eLSM at nonzero density

Francesco Giacosa

Institute of Physics, Jan Kochanowski University, ul. Swietokrzyska 15, 25-406 Kielce, Poland
and Institute for Theoretical Physics, J. W. Goethe University, Max-von-Laue-Str. 1, 60438
Frankfurt am Main, Germany
E-mail: fgiacosa@ujk.edu.pl

Abstract. The extended Linear Sigma Model (eLSM) is an effective model of QCD which
includes in the mesonic sector (pseudo)scalar and (axial-)vector quarkonia mesons as well as
one dilaton/glueball field and in the baryonic sector the nucleon doublet and its chiral partner
in the mirror assignment. The chiral partner of the pion turns out to be the resonance $f_0(1370)$,
which is then predominantly a quarkonium state. As a consequence, $f_0(500)$ is predominately
not a quarkonium state but a four-quark object and is at first not part of the model. Yet,
$f_0(500)$ is important in the baryonic sector and affects nuclear matter saturation, the high-
density behavior, and nucleon-nucleon scattering. In these proceedings, we show how to enlarge
the two-flavour version of the eLSM in order to include the four-quark field $f_0(500)$ in a chirally
invariant manner. We then discuss homogeneous and inhomogeneous chiral restoration in a
dense medium.

1. Introduction

Linear Sigma Models (LSMs) are effective models of QCD which contains hadrons (mesons and
baryons) as degrees of freedom and which are based on the linear realization of chiral symmetry
[1]. As a consequence of the spontaneous breaking of chiral symmetry, in these models the pions
emerge as quasi-Goldstone bosons (a relatively small mass is present because of the explicit
breaking of chiral symmetry). The chiral partner of the pion, denoted as $\sigma_N$, is also an explicit
d.o.f. of such models. As various studies show, this meson corresponds to the scalar resonance
$f_0(1370)$: this state is then predominantly a quark-antiquark state. Hence, the lightest scalar
state listed in the PDG [2], the resonance $f_0(500)$, must be something else: its substructure
corresponds to a four-quark state, either as diquark-antidiquark or pion-pion enhancement (see
e.g. the recent review on the subject [3]). As such, this resonance should not be (at first) part
of a LSM. Yet, as we shall describe later on, $f_0(500)$ is important at nonzero density since it
describes a necessary middle-range attraction between nucleons (see also Ref. [4]).

Extensions of LSMS toward the inclusion of (axial-)vector degrees of freedom were performed
in Ref. [5]. Quite recently, an as complete as possible LSM, called extended Linear Sigma Model
eLSM), was developed. The eLSM contains from the very beginning (axial-)vector fields and
embodies both chiral symmetry and dilatation invariance. Spontaneous and explicit breaking of
the former as well as anomalous breaking of the latter are present. As a consequence of chiral
symmetry and the dilation invariance, the eLSM contains a finite number of terms. The eLSM
was first presented for $N_f = 2$ in Refs. [6], then it was enlarged to $N_f = 3$ in Refs. [7] (this
is the first version of a chiral model with $N_f = 3$ containing (axial-)vector d.o.f.), and lately it
was also studied for charmed mesons ($N_f = 4$, Ref. [8]).
In the baryonic sector, the eLSM was investigated for $N_f = 2$ in Refs. [9] (a first step toward the eLSM at $N_f = 3$ was performed in Ref. [12]). In the eLSM, the mirror assignment, which allows for chirally invariant mass term, is used [10, 11]. As mentioned in Ref. [9], a four-quark field $\chi$ corresponding to $f_0(500)$ can be coupled to the eLSM in a chirally invariant way, see Sec. 2. The eLSM with $f_0(500)$ has been investigated at nonzero density in Ref. [14], where the chiral phase transition has been investigated, and later on in Ref. [15], in which inhomogeneous condensation has been studied; these results are here summarized in Sec. 3. The role of the additional, non-conventional meson $f_0(500)$ turns out to be important: it makes a description of the properties of nuclear matter possible (both saturation and compressibility are in agreement with data) and it strongly affects the properties of nuclear matter at high density. Quite interestingly, the resonance $f_0(500)$ was recently investigated in Ref. [16] in the framework of nucleon-nucleon scattering: also in this case, its presence is necessary for a correct description of data.

2. The eLSM

2.1. eLSM without $f_0(500)$

We briefly present the eLSM for $N_f = 2$. (Pseudo)scalar mesons are contained in $\Phi = (\sigma_N + i\eta_N)\rho^0 + (\bar{q}_i + i\tau)\vec{t}$, where $\rho^0 = 1_2/2$, $\vec{t} = \vec{\sigma}/2$, $\sigma_i$ are the Pauli matrices. In Table 1 we report the identification of the fields with resonances of the PDG [2]. [Note: $\eta_N \equiv (u \bar{u} + d \bar{d})/\sqrt{2}$ reads $\eta_N = \cos \varphi_P \eta - \sin \varphi_P \eta'$ with $\varphi_P \approx -44^\circ$ [7].] Because of spontaneous symmetry breaking, $\sigma_N \rightarrow \sigma + \phi$, and $\eta_N \rightarrow \eta + \phi'$, where $\phi$ is the chiral condensate. Under $U(2)_R \times U(2)_L$ chiral transformations: $\Phi \rightarrow U_L \Phi U_R^\dagger$.

The left-handed and right-handed fields $L^\mu$ and $R^\mu$ contain the vector states $\omega^\mu$ and $\rho^\mu$ and the axial–vector states $f_1^\mu$ and $a_1^\mu$: $L^\mu = (\omega^\mu + f_1^\mu)\rho^0 + (\rho^\mu + a_1^\mu)\vec{t}$, $R^\mu = (\omega^\mu - f_1^\mu)\rho^0 + (\rho^\mu - a_1^\mu)\vec{t}$, see Table 1. Under chiral transformations: $L^\mu \rightarrow U_L \Phi U_R^\dagger$ and $R^\mu \rightarrow U_R \Phi U_L^\dagger$.

Table 1: Correspondence of eLSM $q\bar{q}$ fields to PDG [2].

| Field  | PDG     | Quark content | $I$   | $J^{PC}$ | Mass (GeV) |
|--------|---------|---------------|-------|----------|------------|
| $\pi^+, \pi^-, \pi^0$ | $\pi$ | $ud, \bar{d}\bar{u}, \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$ | 1 | 0$^+$ | 0.13957 |
| $\eta$ | $\eta(547)$ | $\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}\cos \varphi_P - s\bar{s}\sin \varphi_P$ | 0 | 0$^-$ | 0.54786 |
| $\eta'$ | $\eta'(958)$ | $\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}\sin \varphi_P + s\bar{s}\cos \varphi_P$ | 0 | 0$^-$ | 0.95778 |
| $a_0^+, a_0^-$, $a_0^0$ | $a_0(1450)$ | $ud, \bar{d}\bar{u}, \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$ | 1 | 0$^{++}$ | 1.474 |
| $\sigma_N$ | $f_0(1370)$ | $\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$ | 0 | 0$^{++}$ | 1.350 |
| $\rho^+, \rho^-, \rho^0$ | $\rho(770)$ | $ud, \bar{d}\bar{u}, \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$ | 1 | 1$^{--}$ | 0.77526 |
| $\omega_N$ | $\omega(782)$ | $\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$ | 0 | 1$^{--}$ | 0.78265 |
| $a_1^+, a_1^-, a_1^0$ | $a_1(1230)$ | $ud, \bar{d}\bar{u}, \frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$ | 1 | 1$^{++}$ | 1.230 |
| $f_{1,N}$ | $f_1(1285)$ | $\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$ | 0 | 1$^{++}$ | 1.2819 |

The mesonic part of the Lagrangian reads

$$
L_{\text{meso}} = \text{Tr} \left[ (D^\mu \Phi)^\dagger (D^\mu \Phi) - \mu_0^2 \text{Tr}[\Phi^\dagger \Phi - \lambda_0 \text{Tr}[(\Phi^\dagger \Phi)^2] - \frac{1}{4} \text{Tr}[(L^\mu)^2 + (R^\mu)^2] + \frac{m^2}{2} \text{Tr} \left[ (L^\mu + R^\mu)^2 \right] + \text{Tr}[H(\Phi + \Phi^\dagger)] + h_2 \text{Tr} \left[ \Phi^\dagger L^\mu \Phi + \Phi^\dagger R^\mu \Phi^\dagger \Phi \right] + 2h_3 \text{Tr} \left[ \Phi^\dagger R^\mu \Phi^\dagger L^\mu \right] \right] \ldots,
$$

(1)

where dots refer to large-$N_c$ suppressed terms (including the chiral anomaly). In Refs. [6, 7] it was shown that, thanks to the inclusion of (axial-)vector d.o.f., the eLSM provides a good
description of meson phenomenology. An interesting consequence is that the quark-antiquark field \( \sigma_N \), which represents chiral partner of the pion, is associated to \( f_0(1370) \), in agreement with previous phenomenological studies [13]. Hence, \( f_0(500) \) must be something else [3].

We now turn to the baryonic sector. For \( N_f = 2 \) one starts from two nucleon fields \( \Psi_1 \) and \( \Psi_2 \) with opposite parity which transform mirror-like under chiral transformations: \( \Psi_{1,2}(R)(L) = \bar{U}(2)(R)(L) \Psi_{1,2}(R)(L) \). The eLSM Lagrangian is [9]:

\[
\mathcal{L}_{\text{eLSM}}^{\text{baryons}} = \Psi_{1L}i\gamma_\mu D_{1L}^\mu \Psi_{1L} + \bar{\Psi}_{1R}i\gamma_\mu D_{1R}^\mu \Psi_{1R} + \bar{\Psi}_{2L}i\gamma_\mu D_{2L}^\mu \Psi_{2L} + \bar{\Psi}_{2R}i\gamma_\mu D_{2R}^\mu \Psi_{2R} - m_0(\bar{\Psi}_{1L}\Psi_{2R} - \bar{\Psi}_{1R}\Psi_{2L} - \bar{\Psi}_{2L}\Psi_{1R} + \bar{\Psi}_{2R}\Psi_{1L}) ,
\]

where \( D_{a(2)(R)L}^{\mu} = \partial^\mu - ic_{1(2)}(R)L)^{\mu} \). The fields \( \Psi_1 \) and \( \Psi_2 \) mix due to the \( m_0 \)-term and are related to the physical states \( N \) and its chiral partner \( N^* \) via:

\[
\Psi_1 = \frac{1}{\sqrt{2\cosh \delta}} \left( Ne^{\delta/2} + \gamma_5 N^*e^{-\delta/2} \right), \quad \Psi_2 = \frac{1}{\sqrt{2\cosh \delta}} \left( \gamma_5 N^*e^{-\delta/2} - N^*e^{\delta/2} \right),
\]

where \( \cosh \delta = \frac{m_{N+N^*}}{2m_0} \). The field \( N \) corresponds to the nucleon \( N(939) \) and \( N^* \) to \( N(1535) \) or \( N(1650) \). For the purposes of the present work, the assignment of \( N^* \) is marginal, see however [9, 12]. The parameter \( m_0 \) represents a chirally invariant mass, which was first discussed in Ref. [10] and further investigated in Refs. [9, 11]. The masses of the nucleon \( N \) and its chiral partner \( N^* \) are given by:

\[
m_{N,N^*} = \sqrt{m_0^2 + \frac{(g_1 + g_2)^2}{16}} \phi^2 \pm \frac{1}{4}(\hat{g}_1 - \hat{g}_2)\phi .
\]

In the limit \( m_0 \rightarrow 0 \) one obtains the result \( m_N = \frac{\hat{g}_1}{\phi} \), i.e., the nucleon mass is solely generated by the chiral condensate [as in the original Linear Sigma Model [1]]. The parameters of the model were determined in Ref. [9], to which we refer for details.

2.2. Inclusion of \( f_0(500) \) in the eLSM

We now introduce \( \chi \equiv f_0(500) \) with quantum numbers \( I(J^{PC}) = 0(0^{++}) \) and mass \( m_\chi = (0.475 \pm 0.25) \) GeV [2] into the eLSM. This state is regarded as an admixture of \( \pi \pi \) and \([u, d][\bar{u}, \bar{d}] \) configurations. For \( N_f = 2 \) it is a singlet under chiral transformations (\( \chi \rightarrow \chi \)). The coupling of \( \chi \) to baryons is obtained by modifying the \( m_0 \)-term as:

\[
m_0(\bar{\Psi}_{1L}\Psi_{2R} - \bar{\Psi}_{1R}\Psi_{2L} - \bar{\Psi}_{2L}\Psi_{1R} + \bar{\Psi}_{2R}\Psi_{1L}) = a_\chi(\bar{\Psi}_{1L}\Psi_{2R} - \bar{\Psi}_{1R}\Psi_{2L} - \bar{\Psi}_{2L}\Psi_{1R} + \bar{\Psi}_{2R}\Psi_{1L}) ,
\]

where \( a \) is now a dimensionless constant. Then, the mass parameter \( m_0 \) emerges as a condensation of the four-quark field \( \chi \): \( m_0 = a_{\chi_0} \). In the mesonic sector, one has to add

\[
\mathcal{L}_{\text{eLSM}}^{\text{meson}} \to \mathcal{L}_{\text{eLSM}}^{\text{meson}} + \frac{1}{2} \left( (\partial_\mu \chi)^2 - m_\chi^2 \chi^2 \right) + g_\chi \Phi Tr[\Phi^\dagger \Phi] + g_\Lambda \Phi Tr[L^\mu R^\mu + R^\mu L^\mu] + ... ,
\]

where dots refer to large-\( N_c \) suppressed terms. As a consequence, the condensate \( \chi_0 \) takes the form \( \chi_0 = g_\chi \phi^2 / m_\chi^2 \).

3. Results

3.1. Homogenous condensation

First, we study nuclear matter at nonzero density under the assumption that the condensates are homogenous. Two scalar fields condense: \( \langle \sigma_N \rangle = \phi(\mu) \) and \( \langle \chi \rangle = \bar{\chi}(\mu) \), where \( \mu \) is the nuclear
Figure 1: Left: homogenous condensation. The condensates $\phi$ and $\bar{\chi}$ drop to almost zero at $\mu_{\text{hom}}^c$. Right: inhomogeneous condensation. At $\mu_{\text{inhom}}^c$ a transition to the inhomogeneous condensation of Eq. (7) takes place. This configuration is more favorable than the homogenous case of the left panel.

### Chemical potential

The results are obtained by minimizing at a given $\mu$ the thermodynamical potential $\Omega$ w.r.t. $\phi$, $\bar{\chi}$, as well as $\langle \omega^0 \rangle$. The vacuum’s relation $\chi_0 = g_\chi \phi^2 / m_\chi^2$ holds approximately also at nonzero density: $\phi$ slowly decreases as function of $\mu$ together with $\bar{\chi}$. Then, at a critical $\mu_{\text{hom}}^c \sim 1 \text{ GeV}$ (corresponding to a density $\rho / \rho_0 \sim 2.7$, where $\rho_0$ is the nuclear matter saturation density) a first-order phase transition takes place: $\phi$ and $\bar{\chi}$ drop to very small (but nonzero) values. Chiral symmetry is restored (see Fig. 1, left panel). In terms of density, there is a long mixed phase between $2.7 \rho_0$ - $10 \rho_0$. The compressibility $K$ lies in the range 200-250 MeV and is therefore in good agreement with the experiment (200-300 MeV). The mass of the nucleon and that of the partner drop to almost zero in the chirally restored phase. For details, see Ref. [14].

### 3.2. Inhomogeneous condensation

The previous results were obtained under the assumption that the $\phi$ and $\bar{\chi}$ are homogenous, i.e. are not space-dependent. Yet, various studies have shown (see Ref. [17] and refs. therein) that an inhomogeneous condensation can be favoured. In Ref. [15] the so-called chiral-spiral [18] was investigated:

$$\phi_\mu(z) = \tilde{\phi} \cos(2fz), \quad \langle \pi^3 = 0 \rangle = \tilde{\phi} \sin(2fz)$$

(7)

with $\tilde{\phi} \equiv \tilde{\phi}(\mu)$ and $f \equiv f(\mu)$. The homogenous case corresponds to $f = 0$. For $f \neq 0$ one has a condensation of the neutral pion field, which corresponds to a spontaneous breaking of parity at nonzero density. At a given $\mu$, the thermodynamical potential is now minimized for $\tilde{\phi}$, $f$, $\bar{\chi}$, and $\langle \omega^0 \rangle$. The results show that below a certain critical chemical potential $\mu_{\text{inhom}}^c \lesssim 1 \text{ GeV}$ one has $f = 0$: homogenous condensation is realized, just as before. However, at $\mu_{\text{inhom}}^c$ a phase transition takes place: $f$ jumps to a finite value (of about 400 MeV, which then slowly increases with $\mu$), and $\tilde{\phi}$ to a lower but finite value, see Fig. 2. Interestingly, for a given parameter set, it turns out that $\mu_{\text{inhom}}^c < \mu_{\text{hom}}^c$: the homogenous chiral phase transition does not occur, but is only a local minimum. Chiral symmetry is only partially restored.

### 4. Conclusions

The resonance $f_0(500)$ is the lightest scalar listed in the PDG and hence is potentially interesting in hadron phenomenology. Its role has to be investigated case by case. While its condensate may
be relevant at nonzero temperature [19], its effect on thermal models turns out to be negligible because of a very subtle and interesting cancellation with the repulsive isotensor channel: for practical purposes, $f_0(500)$ can be neglected in thermal models of heavy ion collisions [20].

At nonzero density, $f_0(500)$ plays indeed a significant role because it mediates a sizable attraction between nucleons: in these proceedings, we have incorporated $f_0(500)$ into the eLSM and reviewed the properties at finite chemical potential. We have shown that inhomogeneous condensation of the chiral-spiral type is favored w.r.t. the homogeneous one. In the future, one should go beyond the chiral-spiral Ansatz and test arbitrary types of inhomogeneous condensation by using the numerical procedure put forward in Ref. [21].

Acknowledgments
The author thanks all the members of the chiral group in Frankfurt for the many valuable cooperations and discussions.

References
[1] B.W. Lee, Chiral Dynamics (Gordon and Breach, New York, 1972); S. Gasiorowicz and D.A. Geffen, Rev. Mod. Phys. 41 (1969) 531; V. Koch, nucl-th/9512029.
[2] K.A. Olive et al. (Particle Data Group), Chin. Phys. C 38, 090001 (2014).
[3] J.R. Pelaez, arXiv:1510.00653 [hep-ph].
[4] R. Machleidt, Phys. Rev. C 63 (2001) 024001 [nucl-th/0006014].
[5] P. Ko and S. Rudaz, Phys. Rev. D 50 (1994) 6877; M. Urban, M. Buballa and J. Wambach, Nucl. Phys. A 697 (2002) 338 [hep-ph/0102260].
[6] D. Parganlija, F. Giacosa and D.H. Rischke, Phys. Rev. D 82 (2010) 054024 [arXiv:1003.4934 [hep-ph]]; S. Janowski, D. Parganlija, F. Giacosa and D.H. Rischke, Phys. Rev. D 84 (2011) 054007 [arXiv:1103.3238 [hep-ph]].
[7] D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa and D. H. Rischke, Phys. Rev. D 87, 014011 (2013) arXiv:1208.0585 [hep-ph]; S. Janowski, F. Giacosa and D. H. Rischke, Phys. Rev. D 90 (2014) 114005 [arXiv:1408.4921 [hep-ph]].
[8] W.I. Eshraim, F. Giacosa and D.H. Rischke, Eur. Phys. J. A 51 (2015) 9, 112 [arXiv:1405.5861 [hep-ph]].
[9] S. Gallas, F. Giacosa and D.H. Rischke, Phys. Rev. D 82 (2010) 014004 [arXiv:0907.5084 [hep-ph]]; S. Gallas and F. Giacosa, Int. J. Mod. Phys. A 29 (2014) 17, 1450098 [arXiv:1308.4817 [hep-ph]].
[10] C.E. Detar and T. Kunihiro, Phys. Rev. D 39 (1989) 2805.
[11] D. Zschiesche et al, Phys. Rev. C 75 (2007) 055202 [nucl-th/0608044]; D. Jido, M. Oka, and A. Hosaka, Prog. Theor. Phys. 106 (2001) 873 [hep-ph/0110005]; D. Jido, Y. Nemoto, M. Oka and A. Hosaka, Nucl. Phys. A 671 (2000) 471 [hep-ph/9805306]; A. Hosaka, D. Jido and M. Oka, Prog. Theor. Phys. Suppl. 149 (2003) 203 [hep-ph/0305258]; M. Harada, M. Rho and C. Sasaki, Phys. Rev. D 70 (2004) 074002 [hep-ph/0312182]; C. Sasaki, I. Mishustin and K. Redlich, Phys. Rev. D 89 (2014) no.1, 014031 [arXiv:1308.3635 [hep-ph]].
[12] L. Olbrich, M. Zétényi, F. Giacosa and D. H. Rischke, Phys. Rev. D 93 (2016) no.3, 034021 [arXiv:1511.05035 [hep-ph]].
[13] C. Amsler and F. E. Close, Phys. Lett. B 353 (1995) 385; W. J. Lee and D. Weingarten, Phys. Rev. D 61 (2000) 014015 [arXiv:hep-lat/9910008]; F. E. Close and A. Kirk, Eur. Phys. J. C 21 (2001) 531 [arXiv:hep-ph/0103173]; H. Y. Cheng, C. K. Chua and K. F. Liu, Phys. Rev. D 74 (2006) 094005 [arXiv:hep-ph/0607206]; F. Giacosa, T. Gutsche, V. E. Lyubovitskij and A. Faessler, Phys. Rev. D 72, 094006 (2005) [arXiv:hep-ph/0509247].
[14] S. Gallas, F. Giacosa and G. Pagliara, Nucl. Phys. A 872 (2011) 13 [arXiv:1105.5003 [hep-ph]].
[15] A. Heinz, F. Giacosa and D.H. Rischke, Nucl. Phys. A 933 (2015) 34 [arXiv:1312.3244 [nucl-th]].
[16] W. Deinert, K. Teibl, F. Giacosa and D. H. Rischke, arXiv:1603.04312 [nucl-th].
[17] M. Buballa and S. Carignano, Prog. Part. Nucl. Phys. 81 (2015) 39 [arXiv:1406.1367 [hep-ph]].
[18] T. Kojo, Y. Hidaka, L. McLerran and R. D. Pisarski, Nucl. Phys. A 843 (2010) 37 [arXiv:0912.3800 [hep-ph]].
[19] A. Heinz, S. Struber, F. Giacosa and D.H. Rischke, Phys. Rev. D 79 (2009) 037502 [arXiv:0805.1134 [hep-ph]].
[20] W. Broniowski, F. Giacosa and V. Begun, Phys. Rev. C 92 (2015) no.3, 034905 [arXiv:1506.01260 [nucl-th]].
[21] A. Heinz, F. Giacosa, M. Wagner and D. H. Rischke, Phys. Rev. D 93 (2016) no.1, 014007 [arXiv:1508.06057 [hep-ph]].