Self-Constructing Fuzzy-Neural-Network-Imitating Sliding-Mode Control for Parallel-Inverter System in Grid-Connected Microgrid

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\textbf{ABSTRACT} This study mainly develops a self-constructing fuzzy neural network (SFNN) with the structure and parameter self-learning abilities to imitate a sliding-mode control (SMC), and implements the grid-connected current tracking control for a parallel-inverter system in a grid-connected microgrid (MG) with a master-slave current sharing strategy. In the proposed SFNN-imitating SMC (SFNNISMC) scheme, the initial nodes of the input layer are determined by the number of the grid-connected inverter units, and the rules of the membership layer are self-generated online from null online according to the instantaneous inputs based on the dynamic rule-generating scheme. Moreover, a dynamic Petri net is introduced to implement the pruning mechanism, and is utilized to recall the rules corresponding to the reconnected slave inverters. Only the parameters of favorable rules fired by the Petri net are updated online instead of all the parameters, which can significantly alleviate the computational burden of parameter learning. In addition, the projection algorithm and the Lyapunov stability theorem are adopted to ensure the convergence of the parameter adaptation and the grid-connected current-tracking errors. Furthermore, the rule evolutions of the proposed SFNNISMC in the structure self-learning process are illustrated in numerical simulations. The superiority of the proposed SFNNISMC framework is further validated by experimental comparisons with a proportional-integral control (PIC) strategy, an SMC scheme and an adaptive FNN-imitating SMC (AFNNISMC) framework with a fixed network structure from the previous research to be carried out on a parallel-inverter system with two single inverters.

\textbf{INDEX TERMS} Parallel-inverter system, microgrid, fuzzy neural network, self-constructing, adaptive control.

\section{I. INTRODUCTION}
Grid-connected inverters, which usually operate in parallel to the point of common coupling (PCC), are required for distributed generation (DG) systems to construct a microgrid (MG), and then provide electric energy to the utility grid (UG) [1]. As an interface device, the function of the grid-connected parallel-inverter system is to output high-quality sinusoidal current to the UG. Thus, the investigation of a high-performance control method for a grid-connected parallel-inverter system is of great importance for guaranteeing the safe operation of an MG and providing strong support to the UG [2].

For the high application of modern communication technology in a smart MG [3]-[4], superior performance in power/current sharing, and insensitivity to line-impedance parameters, master-slave control for parallel-inverter systems has been broadly applied in commercial MG applications. In general, the master inverter is controlled to track a current/power command, and the current controller for slave inverters tracks the current/power of the master inverter through a current sharing bus [5].

Proportional integral (PI) [6], proportional resonance (PR) [7], and hysteresis control [8] are frequently used in direct-current-controlled grid-connected inverters because of their fast dynamic response. However, under the consideration of nonlinear characteristics and system uncertainties, the robustness performance cannot be guaranteed by traditional PI, PR or hysteresis controllers. Total sliding-mode control (TSMC) can guarantee the global robustness of the system without the reaching phase in conventional sliding-mode control (SMC), which has been successfully applied to various practical systems [9]-[10] under the occurrence of...
uncertainties and disturbances in system dynamics. Unfortunately, the designed control law in TSMC requires detailed system dynamic information to ensure control performance. On the other hand, the control chattering phenomenon is an inherent defect of the SMC due to the selection of a large control gain used to cope with the bounds of system uncertainties.

To suppress the chattering phenomenon and enhance the robustness performance of the conventional SMC, the super-twisting SMC has been proven to be an effective solution [11]-[12]. On the other hand, a disturbance observer working as the compensator for the baseline-design SMC has been adopted to estimate and compensate system uncertainties in the conventional SMC law. Hou et al. [11]-[12] presented two kinds of composite control systems, including a disturbance observer and a super-twisting SMC, for the speed regulation of a permanent synchronous motor (PMSM). Although the speed-tracking precision and the disturbance-rejection performance can be significantly enhanced with a smaller switch gain, the disturbance observer is developed based on the model of the PMSM speed regulation system in [12], and the observation accuracy is susceptible to system uncertainties. Thus, model-free intelligent-based control methodologies have been studied in recent years [13]-[16].

A Fuzzy neural network (FNN) possesses the ability to approximate an arbitrarily continuous function, with a transparent internal decision-making process for incorporating the high-level fuzzy reasoning into a neural network. Without the need for the exact dynamic model of the controlled plant, it is friendly for the nonlinear systems with uncertainties and unknown disturbances whose accurate mathematical model is hard to develop. Therefore, much attention has been focused on the combination of FNN and traditional nonlinear control schemes, e.g., SMC. FNNs in the intelligent-based control framework are generally used for the observation of lumped uncertainty bound [13], the estimation of unknown system dynamics [14]-[15] or the approximation of model-based control laws [16]. For example, the upper bounds of unknown functions, including actuator faults and model uncertainties, in the SMC law for fault-tolerant aircraft control were estimated by self-constructing fuzzy neural networks (SCFNNs) in [13] to ensure the finite-time stability of the control system under fault situations. Zhu and Fei [14] introduced an FNN into a global fast terminal SMC to estimate the system uncertainties online to replace the sign function for a photovoltaic grid-connected system. Chu et al. [15] proposed a double hidden-layer recurrent neural network (DHLRNN) controller to estimate the unknown function in an ideal global SMC law, and the corresponding control performance was verified on an active power filter (APF). However, the chattering problem seems to be inevitable because of the sign function involved in the designed DHLRNN-based adaptive global SMC law. Cheng et al. [16] utilized a FNN controller to mimic the baseline control law of the SMC for power converters without the requirement of the exact model of the power converter. Nevertheless, in the aforementioned literature [14]-[16], whether the FNN is used as an estimator for the unknown part of an SMC law or the alternative of an ideal control law, a compensation controller is always necessary to compensate for the estimation error, which will increase the complexity of the control system.

Recently, more investigations have been performed on the applications of FNNs to imitate nonlinear control laws for relaxing the requirements of the system dynamic model and the compensation controller [17]-[19]. The control framework involving an adaptive FNN control (AFNCC) to imitate a TSMC law without auxiliary compensated controllers was developed in [19] for a parallel-inverter system in an islanded MG to realize high performance in voltage tracking and current sharing. In [19], the designed AFNCC was independent of detailed system dynamics, and realistic experiments validated that the chattering phenomena were alleviated effectively because of the unnecessary sign function in the AFNCC. Although FNNs in [14]-[19] can dramatically improve the control performance of traditional nonlinear controllers, these FNNs with fixed network structures in advance only have the ability of parameter learning (i.e., the mean and standard deviation of the Gaussian membership function and the connected weights), and the size of the network and initial values of network parameters are generally selected by expert knowledge or trial-and-error processes. It is not a trivial task to balance the network size and the control performance, especially for the systems with variable inputs. A large number of hidden neurons will increase the online computational time and delay the system response time, which is not appropriate for the real-time control applications. However, a network with fewer neurons may not achieve the desired control performance. On the other hand, the generalization capability of an FNN with a fixed network structure has difficulty adapting to frequently changing and complex actual operation conditions, especially for unpredictable conditions without any historical occurrences [20], because predefined static rules may not cover the whole variable input space. Thus, the capability of dynamic self-learning not only in the network parameters but also in the network structure is significant for an FNN to strike a balance between the control performance and the network complexity.

Currently, self-organizing FNNs (SFNNs) have attracted widespread attention [20]-[26]. In contrast to the conventional FNN with a fixed network structure, the SFNN has dynamic self-learning ability, not only in the network parameters but also in the network structure. Thus, the SFNN can balance the control performance and the network complexity. A data-driven-based online adaptive algorithm was developed in [20] to adjust the structure of a radial-basis-function neural network (RBFNN) to work as the error compensation model of an FNN controller for the antimony flotation process. In [20], a new node was added for the corresponding input signal if all the hyperspheres of the input data in the RBF node antecedents were greater than predefined thresholds. Although the RBF compensator has a self-organized learning ability, the growth of new rules
unboundedly would entail a large network in the absence of a pruning technique, which is difficult to realize in practical applications.

Many strategies for rule-growing and rule-pruning mechanisms have been presented [21]-[23]. Wang and Er [21] proposed the growing and pruning mechanism for a self-constructing FNN (SCFNN) approximator in a surface vehicle tracking control system. In [21], the generalized distances between the input signals and the existing membership functions were used to measure the novelty of the current inputs and the significance of the membership functions for generating and pruning fuzzy rules. Han et al. [22] proposed a self-organizing FNN (SOFNN) to imitate the unknown nonlinearity part of the control law and considered both the rule-growing and pruning algorithm in the design of SOFNN based on the structural risk minimization strategy. In [23], the firing strength of a rule for each input determined whether the generation of a new layer was required, and a MIN-MAX method was used for layer pruning.

In recent years, meta-cognitive algorithms have attracted much attention to deal with optimization problems. The meta-cognitive fuzzy neural network control framework can be constructed by combining the meta-cognitive algorithm with FNN control to realize the learning process and network structural adaptation. A meta-cognitive sequential learning algorithm for a neuro-fuzzy inference system was investigated in [24], in which the meta-cognitive component was taken as a self-regulatory learning mechanism to control the learning process of a Takagi–Sugeno–Kang type-0 neuro-fuzzy system, and the instantaneous error of the sample and spherical potential of the rule antecedents were used to select the best training strategy for the current inputs. A novel meta-cognitive fuzzy neural model was developed in [25] to approximate unknown nonlinear functions in a backstepping control law for a class of uncertain nonlinear systems, and the from-scratch evolution of the rule base and recursive adaptation of parameters were achieved. Hou et al. [26] introduced a meta-cognitive fuzzy neural network (MCFNN) framework to estimate the modeling uncertainties in the global SMC law and verified the control performance of the proposed controllers by simulation and experimental investigations on an active power filter system.

The aforementioned self-organizing FNN frameworks used in [20]-[26] possess structure and parameter self-learning abilities according to the input signals and the working conditions of the systems, which fully exert the generalization ability of the FNN and reduce the complexity and computational burden of the control system on the premise of effectively improving the control performance. Because the thresholds utilized in the growing and pruning algorithms should be predefined as constants by the trial-and-error process, and it is not easy to determine the corresponding reasonable values even for experienced designers. To solve this problem, Zhou et al. [27] designed a self-organizing FNN with a hierarchical pruning scheme to avoid the performance degeneration caused by the improper pruning threshold in the traditional error-reduction-ratio (ERR) method, and the effectiveness of the proposed scheme was verified in a wastewater treatment process. However, the accumulative density value over several sampling periods was used to measure the importance of the existing rules, which is not suitable for a real control system with a small time constant. The maximum hidden-layer size (MHLs) was introduced for presetting the growing and pruning thresholds in [28] to be meaningful for the user compared with the trial-and-error method. However, the selection of the MHLs is a trade-off between the controller accuracy and the computational burden.

To reduce the difficulties of parameter selection in controller design, variable thresholds have been widely used for the selection of useful neurons or the deletion of unsuitable neurons [29]-[32]. Wai and Lin [29] introduced a dynamic Petri net into a recurrent FNN (DPRFNN) for the moving-target tracking control of a vision-based mobile robot to preserve the robust control performance and efficiently shorten the computational burden. Lin et al. [30] developed a wavelet Petri layer with a dynamic threshold to reduce the number of fuzzy rules for efficient computation in the voltage stabilization control of MG. The dynamic threshold inversely correlated with the tracking errors was adopted in a recurrent feature-selection neural network (RFSNN) to adjust the structure and parameters of the neural network in [31]-[32].

For a traditional FNN control system with a fixed network structure for a parallel-inverter system with the master-slave current sharing strategy in [19], the input nodes and the size of the network are usually designed with the maximum size of parallel units. Moreover, the number of membership functions for each input is predetermined as large as possible to improve the control performance and robustness, which will increase the complexity of the network structure and the computational burden in the adaptation of the parameters. In addition, the unsuitable predefined fuzzy rules may not cover the whole input space, which would significantly reduce the dynamic control performance under the occurrence of unknown system uncertainties (e.g., the fluctuation of the direct current (DC) voltage, the variation of circuit parameters and the system structure alteration). Besides, the parallel-inverter system in an MG often suffers from variations of the connected structure because the partial inverter units disconnect from or reconnect to the parallel-inverter system. While the network parameters concerning the partial slave inverters disconnected from the parallel-inverter system are still updated online, the adaptations for those rules that do not contribute to the network output are meaningless and time-consuming. The motivation of this study is to deal with the drawbacks of chattering phenomena and model-based design in SMC and relax the computational burden of fixed-network-structure FNN. Thus, a model-free self-constructing FNN scheme with dynamic rule-generating and rule-pruning mechanisms is designed to imitate an SMC for grid-connected current tracking and current sharing control of the parallel-inverter system in a grid-connected MG. The proposed self-constructing fuzzy-neural-network-imitating sliding-mode control (SFNNSMC) scheme is
expected to possess both the ability of parameter self-adaptation and structure self-constructing, such that the optimal balance between the robust control performance and the computational time can be realized according to the state of the parallel-inverter system. The major advantages of this study are summarized as follows:

1) A model-free self-constructing FNN is developed to imitate an SMC law to relax the requirement of detailed dynamic information and the auxiliary compensation controller, as well as remedy the chattering phenomena in the conventional SMC. Moreover, the proposed SFNNISMC could be more appropriate for systems with unknown uncertainties and frequently changing operational conditions due to structure learning and parameter adaptation. In addition, it can realize the best balance between the control performance and the computational time in comparison to an FNN with a fixed network structure.

2) The proposed SFNNISMC can generate new rules and prune insignificant rules. Moreover, a variable threshold scheme is introduced into the generating/pruning algorithm in the self-learning process to match the requirement of the control accuracy and avoid mistaken deletion or rule redundancy. Therefore, it does not need any initial parameters to be predefined, which simplifies the debugging process of the controller implementation and enhances the generalization of the network. In addition, the input nodes can be added dynamically according to the number of inverters in the parallel-inverter system.

3) Only the parameters of favorable rules fired by the automatic Petri net are adjusted online, which can effectively reduce the computational burden. Meanwhile, the dynamic Petri net is also responsible for recalling the previous insignificant rules associated with the disconnected inverters when they are reconnected to the parallel system. Then, the transient adjustment process can be shortened.

4) The projection algorithm is adopted to avoid the singularity problem if the coefficients of the membership functions might be updated beyond the universe of discourse, and the Lyapunov stability theorem is also adopted to ensure the convergence of the parameter adaptation and the current-tracking errors.

There are five sections organized in this study. Following the introduction, the descriptions and mathematical model of a parallel-inverter system in a grid-connected MG are introduced in Section II. The detailed design process of the proposed SFNNISMC for grid-connected control is investigated in Section III. The effectiveness of the proposed SFNNISMC framework is validated by numerical simulations and experimental results in Section IV. Finally, some conclusions are drawn in Section V.

II. DESCRIPTIONS OF PARALLEL-INVERTER SYSTEM IN GRID-CONNECTED MICROGRID

A master inverter and \( n-1 \) slave inverters of a parallel-inverter system are integrated into the point of common coupling (PCC) and then connected to the utility grid by an autotransformer as shown in Fig. 1, where \( V_{\text{dcB}} \mid_{k=1,2,\ldots,n} \) and \( V_{\text{dA}} \mid_{k=1,2,\ldots,n} \) are the voltages of DC buses at different distributed generations, the voltages of parallel-connected inverters, and the voltages across filter inductors \( L_k \mid_{k=1,2,\ldots,n} \) in the \( k \)th inverter terminal, respectively; \( i_{\text{dA}} \mid_{k=1,2,\ldots,n} \) and \( i_{\text{dB}} \mid_{k=1,2,\ldots,n} \) denote the inductor currents in the \( k \)th inverter, and \( v_1 \) and \( v_2 \) denote the input current and the input voltage of the autotransformer connected to the utility grid (UG), respectively.

Ignoring the equivalent resistors of filter inductors, the mathematical model of the parallel-inverter system of the grid-connected microgrid (MG) in Fig. 1 can be derived as follows:

\[
\begin{bmatrix}
i_{1,1} \\
i_{1,2} \\
i_{1,n}
\end{bmatrix} = \text{diag}(K_{\text{PWM1}} / L_1, \ldots, K_{\text{PWMm}} / L_n) \begin{bmatrix}
v_{\text{cont}} \\
v_{\text{cond}} \\
v_{\text{com}}
\end{bmatrix} - \begin{bmatrix}
v_1 \\
v_2
\end{bmatrix}
\]

where \( K_{\text{PWM}} = V_{\text{dcB}} / V_{\text{in}} \mid_{k=1,2,\ldots,n} \) denotes the gain of the \( k \)th inverter, in which \( V_{\text{in}} \) represents the amplitude of the triangular-carrier signal; \( v_{\text{cond}} \) is a modulation signal to be designed; and \( \text{diag}(\cdot) \) denotes a diagonal matrix, in which \( K_{\text{PWM}} / L_k \mid_{k=1,2,\ldots,n} \) are the elements on the diagonal of the matrix.

By considering system uncertainties, the dynamic model in (1) can be rewritten as

\[
\dot{x}(t) = B_p(u(t) + D_p f(t) + P_g(t))
\]

where \( x(t) = [i_{1,1} \cdots i_{1,n} \cdots i_{n,1} \cdots i_{n,n}]^T \in \mathbb{R}^{n^2}, u(t) = [v_{\text{cont}} \cdots v_{\text{cont}} \cdots v_{\text{cont}}]^T \in \mathbb{R}^{n^2}, f(t) = v_1, B_p = \text{diag}(K_{\text{PWM1}} / L_{\text{dc}}, \ldots, K_{\text{PWMm}} / L_{\text{dc}}), D_p = [-1/L_{\text{dc}}, \cdots, -1/L_{\text{dc}}, \cdots, -1/L_{\text{dc}}]^T \in \mathbb{R}^{n^2} \) are nominal matrices with nominal filter inductor values to be represented as \( L_{\text{dc}} \mid_{k=1,2,\ldots,n} \). The lumped uncertainty vector \( P_g(t) \) in (2) can be expressed as

\[
P_g(t) = \Delta B_p u(t) + \Delta D_p f(t) + g(t)
\]

where \( \Delta B_p = \text{diag}(K_{\text{PWM1}} / \Delta L_1, \cdots, K_{\text{PWMm}} / \Delta L_n) \in \mathbb{R}^{n^2} \) and \( \Delta D_p = [-1/\Delta L_1, \cdots, -1/\Delta L_n, \cdots, -1/\Delta L_n]^T \in \mathbb{R}^{n^2} \) are uncertain matrices, in which \( \Delta L_k \mid_{k=1,2,\ldots,n} \) are the differences between real and nominal filter inductor values.

**Assumption 1:** The boundary value of the current lumped

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FIGURE 1. Framework of parallel-inverter system in grid-connected microgrid with master-slave current sharing.

uncertainty vector is assumed to be

\[ \| \rho_g \| < \| \rho_g \| \]  

where \( \| \| \) denotes the 1-norm operator, and \( \rho_g = [\rho_{g1} \cdots \rho_{gl} \cdots \rho_{gl}]^T \), in which \( \rho_{gl} \) are given positive constants.

III. SELF-CONSTRUCTING FUZZY-NEURAL-NETWORK-IMITATING SLIDING-MODE CONTROL

A. SMC DESIGN

Under the master-slave current sharing strategy, the control objective is to regulate the output current of the master inverter \( i_{L1} \) to track a sinusoidal current command \( i_{Lref} \), in which the sinusoidal signal of the command is designed by a phase-locked loop (PLL). Moreover, the currents of slave inverters \( i_{Lk} \) are controlled to track the current command of the \( k \)th inverter defined as \( i_{Lrefk} = [i_{Lref1} \cdots i_{Lrefk} \cdots i_{Lrefn}]^T \), in which \( k_{gl} \) are given positive constants. Design a sliding-surface vector as

\[ s_g(t) = e_g(t) - e_g(0) + K_g \int_0^t e_g(t) dt \]  

where \( s_g = [s_{g1} \cdots s_{gn}]^T \in \mathbb{R}^{n+1} \); \( e_g = [e_{g1} \cdots e_{gn}]^T \); \( e_{g0} = i_{Lref} - i_{L3} \); \( e_g(0) \) is the initial value vector of \( e_g(t) \); and \( K_g = \text{diag}[k_{g1} \cdots k_{gn}] \), in which \( k_{gl} \) are given positive constants.

Theorem 1: If the parallel-inverter system in a grid-connected microgrid (MG) shown in (2) is controlled by the sliding-mode control (SMC) law in (6), the high-quality grid-connected current and the current sharing can be obtained, and the system stability can be ensured even in the presence of system uncertainties.

\[ u_{SMC} = B_{pm}^{-1}[-D_{pm}f(t) + x_{ref} + K_g e_g + K_{pg} \text{sgn}(s_g(t))] \]  

where \( \text{sgn}(\cdot) \) is the sign function operator; \( K_{pg} = \text{diag}(k_{pg1} \cdots k_{pgk} \cdots k_{pgn}) \) is the control gain matrix of
the curbing control law, in which \( k_{pgk} \) are designed to be positive constants, and \( \|K_{pg}\| > \|P_{g}\| (t) \).

**Proof:** By defining the first Lyapunov function candidate as \( V_{SMC} = 0.5s_{g}^{T}s_{g} \), its derivative can be obtained as

\[
\dot{V}_{SMC} = s_{g}^{T}\dot{s}_{g} = s_{g}^{T}[-K_{pg} \text{sgn}(s_{g}) - P_{g}] \\
\leq -(\|K_{pg}\| - \|P_{g}\|)s_{g}^{T} \leq 0
\]

Equation (7) will hold under the condition of

\[
\|K_{pg}\| > \|P_{g}\|,
\]

which means that the first Lyapunov function \( V_{SMC} > \) 0 and its derivative \( \dot{V}_{SMC} \leq 0 \). Thus, the stability of the designed SMC system in (6) can be guaranteed. The proof of Theorem 1 is finished.

As seen from (6), the detailed model information of the parallel-inverter system is necessary for the implementation of the SMC. Moreover, the conservative selection of \( K_{pg} \) is always used to cover the bound of the system uncertainties, which may cause chattering phenomena by the sign term \( \text{sgn}(s_{g}) \) in (6).

In this study, a self-constructing fuzzy-neural-network-imitating SMC (SFNNISM C) framework shown in Fig. 2(a)
is proposed for a parallel-inverter system in a grid-connected MG to mimic the SMC law in (6) without an auxiliary compensator and detailed system information. Moreover, the nodes in the proposed SFNNISMC framework are generated adaptively so that it can simplify the initial parameter design compared to the FNN with a fixed network structure used in [19]. Moreover, control rules and the corresponding adaptive laws can also be used to significantly reduce the computational burden in comparison with [19].

**B. SFNNISMC DESIGN**

The proposed SFNNISMC scheme with a five-layer self-constructing FNN (SFNN) structure is depicted in Fig. 2(b). A detailed description of the construction of signals, the self-constructing scheme and the corresponding adaptation laws used in the proposed SFNNISMC framework are explained as follows.

1) The elements \( s_{i} [n] \) in the sliding-surface vector \( \{ s_{i} \} \) are chosen as the input variables \( q_{i} [n] \) to simplify the input space dimension, which are transmitted to the next layer directly. Each node in the input layer corresponds to one inverter unit.

2) Gaussian membership functions are used to map the input variables to fuzzy sets, which can be represented as

\[
\mu_{i}^{j}(q_{i}) = \exp[-(q_{i} - m_{i}^{j})^2/2c_{i}^{j2}]_{i=1,...,N_{pi}} \tag{8}
\]

where \( \exp[\cdot] \) is the exponential function; and \( m_{i}^{j} \) and \( c_{i}^{j} \) are the mean and standard deviation of the \( j \)th Gaussian function for the \( i \)th input, respectively. All the means and standard deviations are collected into vectors \( \mathbf{m}=[m_{1}^{1} \cdots m_{1}^{N_{pi}}]^{T} \in \mathbb{R}^{N_{pi} \times 1} \) and \( \mathbf{c}=[c_{1}^{1} \cdots c_{1}^{N_{pi}}]^{T} \in \mathbb{R}^{N_{pi} \times 1} \), in which \( m_{i}^{j} = [m_{i1}^{j} \cdots m_{iN_{pi}}^{j}]^{T} \in \mathbb{R}^{N_{pi} \times 1} \) and \( c_{i}^{j} = [c_{i1}^{j} \cdots c_{iN_{pi}}^{j}]^{T} \in \mathbb{R}^{N_{pi} \times 1} \), are the means and standard deviations for the \( i \)th input, respectively; \( N_{pi} \) is the number of the membership functions for the \( i \)th input; and \( N_{pi} = \sum_{i=1}^{N_{s}} N_{pi} \) denotes the total number of nodes in the membership layer.

In general, the values of network parameters are initialized by expert knowledge in the traditional FNN with a fixed network structure, and network parameters are adjusted by online adaptation laws. However, a small-size network structure leads to inaccurate identification; nevertheless, a large-size network structure tends to affect the generalization ability and increase the computational burden. For the parameter and structure uncertainties of the parallel-inverter system in a grid-connected MG, a variable-structure FNN framework with a lower computational burden for better transient performance is urgently required.

Distinguished from a conventional FNN with a fixed network structure, in the proposed SFNN, the initial number of nodes in the input layer is determined by the grid-connected inverter units. Moreover, the nodes in the membership layer and the rule layer can start from null according to the dynamic input signals, which are generated dynamically in the online learning process by performing the self-learning algorithm.

To fulfill the objectives of self-constructing according to the previous network and dynamic input signals, the rule-generating mechanism is explained as follows:

The novelty of an input signal in the feature space of (8) can be measured by the spherical potential [24]-[26] by considering the Gaussian membership functions in the second layer, which can be defined as

\[
\psi_{i} = \frac{2}{N_{pi}} \sum_{j=1}^{N_{pi}} \mu_{i}^{j}(q_{i}, m_{i}^{j}, c_{i}^{j}) \tag{9}
\]

where \( | \cdot | \) is the operator of an absolute value.

The novelty represents the similarity between the input and the previous fuzzy sets. If the value of \( \psi_{i} \) is less than a threshold value \( (g_{th}) \), it means that the previous fuzzy space cannot cover the \( i \)th input, and a new node is required to generate in the membership layer. To circumvent the difficulty in choosing a reasonable generating threshold, the dynamic threshold value \( g_{th} \) is tuned according to the following equation [29]:

\[
g_{th} = \alpha_{g} \exp(-\beta_{g} s_{g}) / (1 + \exp(-\beta_{g} s_{g})) \tag{10}
\]

where \( \alpha_{g} \) and \( \beta_{g} \) are positive constants; \( s_{g} \) is the defined sliding surface of the \( i \)th input in (5). Note that, the dynamic threshold value \( (g_{th} [n] = s_{g} [n]) \) varies with the tracking error of the inverter. A larger tracking error will make a smaller threshold value, which means that more nodes will be generated in the circumstance of larger tracking errors to guarantee the control accuracy.

Then, new means and standard deviations of Gaussian membership functions for the corresponding inputs \( (q_{i} [n] = s_{g} [n]) \) are generated, and the initial values for new means and standard deviations can be represented as follows:

\[
m_{i}^{N_{pi}+1} = q_{i} [n]_{i=1,...,N_{s}} \tag{11a}
\]

\[
c_{i}^{N_{pi}+1} = \lambda \min \| q_{i} - m_{i}^{N_{pi}} \|_{i=1,...,N_{s}} \tag{11b}
\]

where \( \lambda \) is the overlap between the antecedents related to the generalization ability. The initial weights for the corresponding rules are zeros.

3) The output of the rule layer is defined as the product operation of the membership function of different input signals, and the \( i \)th rule can be expressed as

\[
l_{i} = \prod_{j=1}^{N_{pi}} \mu_{i}^{j}(q_{i}) \tag{12}
\]
where $N_l$ denotes the total number of the rule layer. The vector of $I = [I_1 \cdots I_{N_l}] \in R^{|N_l|}$ is used to collect the values of $l_h|_{h=1,\ldots,N_l}$.

4) If the tracking errors are small when the system is operated in the steady state, some control rules with low contributions to the layer output are meaningless. In this case, only significant rules in the rule base should be transferred to the next layer to simplify the network structure and alleviate the computational burden. To accomplish this, the idea of a dynamic Petri layer [29] is introduced into the SCFNN to fire important rules by the following competition learning laws:

$$p_h(l_h) = I_h \max \left\{ \sum_{l=1}^{N_l} q_{hl} w_h^l \right\}|_{l=1,\ldots,N_l, h=1,\ldots,p} \quad (13)$$

$$t_h = \begin{cases} 1 & \text{if } p_h \geq f_{th} \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

where $\max (\cdot)$ is the maximum operator; $p_h$ is the rule contribution degree for the $h$th rule in (13); and $w_h^o$ is the connecting weight of the $o$th output in the Petri layer that will be described later in association with the $h$th rule $l_h$. Any node with a contribution degree larger than the threshold value ($f_{th}$) in the rule layer will be fired by the Petri net. Moreover, in the proposed SFNNISMC framework, a dynamic threshold value is tuned by the following equation:

$$f_{th} = \frac{\alpha_f \exp[-\beta_f (\frac{1}{2} s^T s')] + \exp[-\beta_f (\frac{1}{2} s^T s')]}{1 + \exp[-\beta_f (\frac{1}{2} s^T s')] } \quad (15)$$

where $\alpha_f$ and $\beta_f$ are positive constants. If tracking errors become large, the threshold values will be decreased to fire more control rules and vice versa.

The rule-pruning and refining mechanism used in this study is explained as follows. If the $h$th rule is unfired due to small tracking errors in the steady state, the rule will be pruned because of the low contribution to the output control effort. Meanwhile, the corresponding parameters and connections with other nodes are also removed, and their network parameters are not adjusted. On the other hand, if the fault inverter unit disconnects from the MG, the network parameters are not adjusted. On the other hand, if the rules with respect to the disconnected units are deleted, these rules will be regenerated step by step with initial parameters when the inverter is reconnected to the parallel-inverter system. This way will incur a cost in the training time and slow the transient response. Therefore, in this study, it is unnecessary to update the corresponding adaptive laws for those redundant control rules, while the corresponding parameters and connections are reserved. These rules will be refined as the inverter units to be reconnected to the MG.

Due to the compromise of the tracking accuracy and the computational burden, the error criterion with certain threshold values ($e_{gfl} \mid_{l=1,\ldots,g}$ and $e_{phl} \mid_{l=1,\ldots,p}$) is also adopted to activate the self-constructing ability of the network. When the condition of $|q_l|_{l=1,\ldots,p} > e_{gfl}$ holds, the rule-generating mechanism is implemented. On the other hand, the rule-pruning mechanism is activated when the condition of $|q_l|_{l=1,\ldots,p} < e_{phl}$ is satisfied.

5) The final output of the SFNN can be expressed as follows:

$$y_g = \sum_{h=1}^{N_l} w_h^g t_h \quad (16)$$

where $w_h^o$ is the weight between the output layer and the Petri layer. The following weight matrix ($W$) is used to collect the weights:

$$W_g = \begin{bmatrix} w_{g1} & w_{g2} & \cdots & w_{gn} \\ \vdots & \vdots & \ddots & \vdots \\ w_{gN_1} & w_{gN_2} & \cdots & w_{gN_l} \end{bmatrix} = [w_g^1 \cdots w_g^o \cdots w_g^n]^T \quad (17)$$

where $w_g$ is the weight between the output layer and the Petri layer. The following weight matrix ($W$) is used to collect the weights:

$$W_g = \begin{bmatrix} w_{g1} & w_{g2} & \cdots & w_{gn} \\ \vdots & \vdots & \ddots & \vdots \\ w_{gN_1} & w_{gN_2} & \cdots & w_{gN_l} \end{bmatrix} = [w_g^1 \cdots w_g^o \cdots w_g^n]^T \quad (17)$$

where $w_g^o = [w_g^o \cdots w_g^2 \cdots w_g^n] \in R^{1 \times N_l}$ . Thus, the control effort generated by the proposed SFNNISMC can be represented as

$$u_{SFNNISMC} (s, W, m, e) = Wl(s, m, c) \quad (18)$$

### C. Parameters Adjustment Algorithms

Aside from the self-learning ability in the structure of the proposed SFNN in the above description, the network parameter adjustment algorithms are also derived and comprised in the proposed SFNNISMC framework. To ensure the network convergence and the stability of the designed control system, the derivation processes of the network parameter adjustment algorithms are developed based on the Lyapunov stability theorem [33] and the projection algorithm [34].

**Assumption 2**: There is an optimal SFNNISMC law ($u_{SFNNISMC}$) with the optimal weight matrix ($W_g^*$), mean vectors ($m^*$), standard deviation vectors ($c^*$) and rule output vectors ($l^*$) to approximate the SMC law in (6), which can be written as

$$u_{SMC} = W_g^* l^*(s, m^*, c^*) + e_g^* ,$$

where $e_g^*$ is defined as the minimum reconstructed-error vector between $u_{SFNNISMC}^*$ and $u_{SMC}$. The actual control law of the proposed SFNNISMC can be expressed as

$$u_{SFNNISMC} (s, \hat{W}_g, \hat{m}, \hat{c}) = \hat{W}_g \hat{l}(s, \hat{m}, \hat{c}) \quad (19)$$

where $\hat{W}_g$, $\hat{m}$, $\hat{c}$ and $\hat{l}$ are the estimated vectors of $W_g^*$, $m^*$, $c^*$ and $l^*$, respectively. The approximation error ($\hat{u}_g$)
between the SMC law \((u_{SMC})\) and the optimal SFNNISMC law \((u_{SFNNISMC})\) can be expressed as

\[
\tilde{u}_g = u_{SFNNISMC}^* - \hat{u}_{SFNNISMC} + e_g = W_g^T (s_g^*, m^*, c^*) - W_g^T \hat{\beta}(s_g, \hat{m}, \hat{c}) + e_g \tag{20}
\]

In this study, the Taylor series expansion is utilized to transform the approximation error \((\tilde{u}_g)\) into partially linear forms for later stability analyses, and the approximation error \((\tilde{u}_g)\) in (20) can be represented as

\[
\tilde{u}_g = u_{SMC}^* - \hat{u}_{SFNNISMC} + e_g = Y_w \tilde{w}_g + U_{lg} \hat{\beta} + v_g \tag{21}
\]

where \(Y_w = \text{diag}(y_{\omega_1}, \ldots, y_{\omega_N}) \in \mathbb{R}^{N_y \times N_y}\), in which

\[
y_{\omega_i} = \frac{\partial u_{SFNNISMC}}{\partial \omega_{\gamma_i}} |_{\omega = \omega_{\gamma_i}}, \quad \omega_{\gamma_i} = (w_{\gamma_1}, \ldots, w_{\gamma_N})^T 
\]

\(\in \mathbb{R}^{N_y \times N_y}\), in which \(u_{\gamma_i} = \frac{\partial u_{SFNNISMC}}{\partial \gamma_{\gamma_i}} |_{\gamma = \gamma_{\gamma_i}}, \quad \gamma_{\gamma_i} = (\gamma_{\gamma_1}, \ldots, \gamma_{\gamma_N})^T \in \mathbb{R}^{N_y \times N_y}\);

\(\tilde{w}_g = w_g^* - \hat{w}_g = [\tilde{w}_g^T \cdots \tilde{w}_g^{(r)} \cdots \tilde{w}_g^{(N_y - 1)}]^T \in \mathbb{R}^{N_y \times 1}\), in which \(w_g^*\) is the optimum vectors of \(w_g\), \(\hat{w}_g\) is the estimated vectors of \(w_g\), \(\tilde{w}_g \in \mathbb{R}^{N_y \times 1}\) and \(N_y = N \times N_y\); \(\hat{\beta} = \hat{\beta} - \hat{\beta} \in \mathbb{R}^{N_y \times 1}\);

\(\hat{v}_g = h_{wg}^* + h_g + e_g\), in which \(h_{wg}\) are higher-order terms in the Taylor series. Moreover, the linear form of \(\hat{\beta}\) can be expressed as

\[
\hat{\beta}(s_g, m, c) = l_{mg} m + l_{cg} c + h_{mg}\tag{22}
\]

\[
\hat{\beta}(s_g, m, c) = l_{mg} m + l_{cg} c + h_{mg}\tag{22}
\]

where \(l_{mg} = \left[\frac{\partial l_{\gamma_1}}{\partial m}, \ldots, \frac{\partial l_{\gamma_N}}{\partial m}\right]^T \in \mathbb{R}^{N_y \times N_y}\); \(l_{cg} = \left[\frac{\partial l_{\gamma_1}}{\partial c}, \ldots, \frac{\partial l_{\gamma_N}}{\partial c}\right]^T \in \mathbb{R}^{N_y \times N_y}\);

\(c^* - \hat{c}\), and \(h_{mg} \in \mathbb{R}^{N_y \times 1}\) is the summation of the vectors of the higher-order term in the Taylor series of \(l_{mg}\) and \(l_{cg}\). By substituting (22) into (21), the approximation error \((\tilde{u}_g)\) can be rewritten as

\[
\tilde{u}_g = Y_w \tilde{w}_g^* + U_{lg} m + U_{lg} c + y_g^* \tag{23}
\]

where \(y_g^* = v_g^* + U_{lg} h_{mg}\).

**Theorem 2:** Consider the system dynamic model of the parallel-inverter system in the grid-connected MG as shown in (2) and utilize the projection algorithm to avoid the network parameters from being updated beyond the universe of discourse. If the control law of the proposed SFNNISMC is formulated as \((18)\) and the parameters of the designed SFNN are adjusted by the adaptation laws designed as \((24)-(26)\), the convergence of the SFNN parameters and the stability of the proposed SFNNISMC system can be assured.

\[
\text{If}(\|\hat{w}_g\| < b_{wg}) \text{ or } (\|\hat{w}_g\| = b_{wg} \text{ and } s_g^T Y_{wg} \hat{w}_g^* \leq 0) \tag{24a}
\]

\[
\dot{\hat{w}}_g = \eta_w (s_g^T Y_{wg})^T \tag{24b}
\]

\[
\text{If}(\|\hat{w}_g\| = b_{wg} \text{ and } s_g^T Y_{wg} \hat{w}_g^* > 0) \tag{24c}
\]

\[
\dot{\hat{m}} = \eta_m [s_g^T Y_{wg} - (s_g^T Y_{wg} \hat{m}^T / \|\hat{m}\|^2)]^T \tag{24d}
\]

\[
\text{If}(\|\hat{m}\| < b_{mg}) \text{ or } (\|\hat{m}\| = b_{mg} \text{ and } s_g^T U_{lg} \hat{m} \leq 0) \tag{25a}
\]

\[
\dot{\hat{c}} = \eta_c (s_g^T U_{lg} \hat{c})^T \tag{25b}
\]

\[
\text{If}(\|\hat{c}\| = b_{cg} \text{ and } s_g^T U_{lg} \hat{c} > 0) \tag{26a}
\]

\[
\dot{\hat{c}} = \eta_c [s_g^T U_{lg} \hat{c} - (s_g^T U_{lg} \hat{c}^T / \|\hat{c}\|^2)]^T \tag{26b}
\]

where \(\|\|\) denotes the Euclidean norm operator; \(b_{wg}\), \(b_{mg}\), and \(b_{cg}\) are given positive bound values; \(\eta_w\), \(\eta_m\), and \(\eta_c\) are given positive learning rates. In Fig. 2, \(\eta_w = \eta_m = \eta_c \) is a given positive learning-rate vector, and \(b_{wg} = b_{mg} = b_{cg} \) is a given positive bound vector.

**Proof:** Define a second Lyapunov function as

\[
V_{SFNNISMC}(s_g, \tilde{w}_g, \tilde{m}, \tilde{c}) = \frac{1}{2} s_g^T s_g + \frac{\tilde{w}_g^T \tilde{w}_g}{2 \eta_{wg}} + \frac{\tilde{m}^T \tilde{m} + \tilde{c}^T \tilde{c}}{2 \eta_{mg} + 2 \eta_{cg}} \tag{27}
\]

The derivative of \(V_{SFNNISMC}\) with respect to time can be expressed as

\[
\dot{V}_{SFNNISMC} = s_g^T [-K_{pg} \text{sgn}(s_g) - P_g + y_g^*] + V_{wg} + V_{mg} + V_{cg} \leq -\left(\|K_{pg}\| - \|y_g^* - P_g\|\right) s_g^T \|s_g\| \leq 0 \tag{28}
\]

where \(V_{wg} = s_g^T Y_{wg} \hat{w}_g^* - \frac{\tilde{w}_g^T \tilde{w}_g}{\eta_{wg}}\); \(V_{mg} = s_g^T U_{lg} \hat{m} - \frac{\tilde{m}^T \tilde{m}}{\eta_{mg}}\);

\(V_{cg} = s_g^T U_{lg} \hat{c} - \frac{\tilde{c}^T \tilde{c}}{\eta_{cg}}\). If the gain condition of \(\|K_{pg}\| > \|y_g^* - P_g\|\) can be satisfied, the result of
\( \dot{V}_{\text{SFNNISMC}}(s, \dot{w}, \ddot{m}, \dddot{c}) \leq 0 \) can be obtained. The detailed derivations of (28) are given in the Appendix. From (28), the derivative of the second Lyapunov function is a negative semi-definite function, i.e.,
\[
V_{\text{SFNNISMC}}(s(t), \dot{w}, \ddot{m}, \dddot{c}) \leq V_{\text{SFNNISMC}}(s(0), \dot{w}, \ddot{m}, \dddot{c}),
\]
and \( s(t), \dot{w}, \ddot{m}, \) and \( \dddot{c} \) are bounded functions.

Define a function \( G_g(t) \) as
\[
G_g(t) = \left( \|K_p \| \|y'_g - P_g(t)\| \|s'_g(t)\| \right)
\leq -V_{\text{SFNNISMC}}(s(t), \dot{w}, \ddot{m}, \dddot{c})
\]
Integrate the function of \( G_g(t) \) with respect to time as
\[
\int_0^t G_g(\tau) \, d\tau \leq V_{\text{SFNNISMC}}(s(0), \dot{w}, \ddot{m}, \dddot{c})
\]
Since \( V_{\text{SFNNISMC}}(s(0), \dot{w}, \ddot{m}, \dddot{c}) \) is a bounded function, and \( V_{\text{SFNNISMC}}(s(t), \dot{w}, \ddot{m}, \dddot{c}) \) is a nonincreasing and bounded function, it can conclude that \( \lim_{t \to \infty} \int_0^t G_g(\tau) \, d\tau < \infty \), and \( \dot{G}_g(t) \) is bounded. By Barbalat’s Lemma [33], it can be implied that \( s(t), \dot{w}, \ddot{m}, \) and \( \dddot{c} \) will converge to zero as \( t \to \infty \). Thus, the stability of the proposed SFNNISMC system shown in Fig. 2 and the convergence of the self-adaptation for the network parameters can be guaranteed. This finishes the proof of Theorem 2.

IV. NUMERICAL SIMULATIONS AND EXPERIMENTAL VERIFICATION

In this section, the effectiveness of the proposed self-constructing fuzzy-neural-network-imitating sliding-mode control (SFNNISMC) scheme is demonstrated by numerical simulations and experimental examinations on a parallel-inverter system in a grid-connected microgrid (MG) with two single-inverter units. The nominal values of circuit parameters in this parallel-inverter system are summarized in Table I.

| Circuit parameters | Master Inverter | Slave Inverter |
|--------------------|-----------------|-----------------|
| DC source voltage  | 200 V           | 200 V           |
| Grid-connected current command (RMS) | 10 A | 10 A |
| Utility grid voltage (RMS) | 110 V | 110 V |
| Filter inductor    | 2 mH            | 2 mH            |
| Fundamental Frequency | 50 Hz | 50 Hz |
| Switching Frequency | 20 kHz | 10 kHz |

For SFNN construction, the control parameters in the dynamic threshold values of the rule-generating and rule-firing mechanism in the Petri layer are given as
\[
\alpha_{g1} = 0.05, \quad \beta_{g1} = 1.02 \times 10^{-3},
\]
\[
\alpha_{g2} = 0.5, \quad \beta_{g2} = 5.2 \times 10^{-3},
\]
\[
\alpha_f = 0.1, \quad \beta_f = 350
\]

The better range of the threshold values for the generation of a new rule and the pruning of an insignificant rule can be tuned automatically according to (10) and (15). Thus, the parameters \( (\alpha_{g1}, \beta_{g1}, \alpha_{g2}, \beta_{g2}, \alpha_f, \text{and} \beta_f) \) in the dynamic threshold values (as shown in (10) and (15)) of the proposed SFNNISMC scheme can be roughly determined as (32). Moreover, the threshold values of \( \epsilon_{g1} = \epsilon_{g2} = 0.5 \) for the implementation of the rule-generating mechanism and the threshold values of \( \epsilon_{g1} = \epsilon_{g2} = 0.15 \) for the implementation of the rule-pruning mechanism are selected according to the compromise of the tracking accuracy and the computational burden. In addition, the value of \( \lambda = 0.8 \) in (11b) is chosen for a better generalization ability of the network, and the corresponding initial weights of new rules are initially set as zero.

For the performance comparison, an adaptive fuzzy-neural-network-imitating sliding-mode control (AFNNISMIC) strategy with a fixed network structure in [19] is also implemented and compared with the proposed SFNNISMC scheme. To ensure fairness, the AFNNISMIC in [19] is examined with the same derivation process of parameter-adaptation laws, and the control parameters and the learning rates are set the same as (31) to obtain a similar control performance with that of the proposed SFNNISMC in the nominal case. Two input signals \( (s_{g1} \text{ and } s_{g2}, n = 2) \) in the AFNNISMIC are equally divided into three feature spaces by the Gaussian membership function \( (N_{p1} = N_{p2} = 3) \), the initial
mean vectors \( \mathbf{m}_1 \) and \( \mathbf{m}_2 \) are all set as \([-35 0 35]\), the initial standard deviation vectors \( \mathbf{c}_1 \) and \( \mathbf{c}_2 \) are all selected as \([35 35 35]\), and the initial weight vectors \( \mathbf{w}_{g1} \) and \( \mathbf{w}_{g2} \) are set as zero. Those initial parameters are roughly selected by a trial-and-error process. The rule number in the rule layer is nine \( (N_l = 9) \), and there are two control efforts in the output layer for the master inverter and the slave inverter \( (N_y = 2) \), respectively. Thus, there are 2, 6, 9, and 2 nodes in the AFNNISMC structure.

A conventional proportional-integral control (PIC) method for the grid-connected parallel-inverter system is also implemented in the experiments to demonstrate the benefit brought by the proposed SFNNISMC framework. The formulas of the PIC for the parallel-inverter system with two inverter units can be represented as

\[
G_{PI1} = k_{p1} + \frac{k_{i1}}{s},
\]

\[
G_{PI2} = k_{p2} + \frac{k_{i2}}{s},
\]

(33)

where \( k_{p1}, k_{p2}, k_{i1}, \) and \( k_{i2} \) are the proportional and integral gains of the PIC framework for the master inverter and the slave inverter, respectively. The corresponding parameters of the PIC can be determined as \( k_{p1}=55, k_{i1}=6.8\times10^4, k_{p2}=55 \) and \( k_{i2}=6.8\times10^4 \) according to the Bode-plot stability analysis.

**FIGURE 3.** Implementation flowchart of proposed SFNNISMC scheme.
The normalized-mean-square-error (NMSE) value of the current tracking is defined in (34) to record the respective performance of the examined control schemes.

\[
\text{NMSE}(x) = \frac{1}{x_{\text{max}} T} \sum_{n=1}^{T} x^2(n)
\]  

(34)

where \(x\) is the grid-connected current-tracking error (\(e_i\)), i.e., the inductor-current-tracking error of the master inverter (\(e_{i1}\)) or the inductor-current-tracking error of the salve inverter (\(e_{i2}\)); \(x_{\text{max}}\) is the maximum value of the sinusoidal current command; and \(T\) is the sampling interval.

The implementation flowchart of the proposed SFNNISM C scheme is depicted in Fig. 3. Numerical simulations are carried out via the MATLAB software (Version R2020a, MathWorks, USA) on an Intel(R) Core(TM) i5-8250U personal computer (PC). In the experiments, the corresponding C codes are composed and debugged in the Code Composer Studio (CCS) (Version CCS10.3.0) Integrated Development Environment (IDE), and the corresponding programs are implemented in a TMS320F28335 32-bit floating-point digital signal processor (DSP).

A. NUMERICAL SIMULATIONS

The numerical simulation model of a parallel-inverter system in a grid-connected microgrid (MG) is built with MATLAB software. Comparative simulations with the AFNNISM C in [19] are implemented for the parallel-inverter system in the grid-connected MG to evaluate the superiority in the control accuracy and computational burden by the proposed SFNNISM C scheme with the self-learning ability. Note that, to exhibit the learning process and the superiority of the proposed SFNNISM C system, the pretrained procedure of the FNN is absent in the following numerical simulations.

Numerical simulations of the parallel-inverter system with only the master inverter action by the AFNNISM C in [19] and the proposed SFNNISM C are depicted in Fig. 4. There is only a master inverter to be operated at the beginning, and the current of the master inverter \((i_{sref})\) is controlled to track the grid-connected reference current \((i_{sref})\). The proposed SFNNISM C constructs its network from one input node, and the grid-connected current can track the reference current with an NMSE value of 0.0839 and a total harmonic distortion (THD) value of 0.16% by the structure and parameter self-learning processes. For the fixed network structure of the AFNNISM C in [19], degenerate performance of NMSE \((e_i) = 0.1562\) and THD \((e_i) = 0.19%\) are resulted due to only parameter adaptation. This is because that the means and standard deviations of the membership functions are generated according to the present input signals in the proposed SFNNISM C. However, to enhance the robustness, large initial means and standard deviations are selected to cover the input signals under system uncertainties in AFNNISM C. Thus, both the overshoot and the adjustment time in the proposed SFNNISM C are smaller than those in the AFNNISM C without the pretrained procedure. At similar tracking performance in the steady state, fewer rule numbers \((N_l = 9)\) are required in the proposed SFNNISM C framework to be superior to the AFNNISM C framework with the rule numbers \((N_l = 2)\).

![FIGURE 5. Simulated rule evolution in SFNNISM C for parallel-inverter system with only master inverter action.](image)

The corresponding process of the rule evolution in the proposed SFNNISM C, as shown in Fig. 5(a), is presented in Fig. 5. As seen from Fig. 5, new neurons are generated at points A and B when the spherical potential \((\psi_i)\) of the input signal \((\psi_{i1})\) corresponding to the master inverter under the previous rule base is lower than the dynamic threshold value \((g_{th})\). After a transparent self-organizing process, the rule base finally reaches a stable state with \(N_{p1} = 2\) and \(N_l = 2\), without the requirement of parameter initialization. In contrast, for the fixed-network-structure AFNNISM C for two inverter units with \(n = 2\), \(N_{p1} = N_{p2} = 3\) and \(N_l = 9\). The initial mean and standard deviation vectors are required to be predetermined to obtain remarkable performance in rapidity and accuracy by expert knowledge and the trial-and-error process. There are fewer membership functions and rules for the input with faster response and more accurate current tracking performance.
tracking by the proposed SFNNISMC in comparison with the AFNNISMC in [19]. Moreover, the computational burden imposed by the redundant rules corresponding to the unconnected slave inverter is avoided effectively by the self-organization ability of the proposed SFNNISMC according to the dynamic input signals.

To further validate the control performance and the rule evolution process of the proposed SFNNISMC, extra simulated conditions under the structural variation are considered. 1) The connection of the slave inverter to the master inverter at 0.075 s is examined to validate the rule generating and pruning mechanism and the current sharing performance. For this condition, the master inverter and the slave inverter will share the grid-connected reference current ($i_{sref}$); 2) To verify the rule-firing mechanism, the slave inverter is disconnected at 0.145 s; 3) The rule refiring mechanism is proven by the transient response at 0.215 s when the slave inverter reconnects to the parallel-inverter system.

Numerical simulations of the parallel-inverter system in the grid-connected MG under the structure variations by the proposed SFNNISMC are depicted in Fig. 6. Figure 6(a) shows the currents of the master inverter ($i_{L1}$) and the slave inverter ($i_{L2}$), the grid-connected current ($i_s$), and the grid-connected reference current ($i_{sref}$). A new input node is generated in the SFNNISMC when the slave inverter connects to the parallel-inverter system. The reference current for the master inverter is half of the grid-connected reference current ($i_{sref}$), and the current of the slave inverter ($i_{L2}$) is controlled to track the master inverter ($i_{L1}$). The spherical potential ($\varphi_1$) falls below the dynamic threshold value ($g_{th1}$), which means that the current input ($s_{g1}$) is novel for the rule base in the previous operation condition. Thus, new membership functions will be generated, and the stable number is 3, as shown in Fig. 6(b). When the new input signal ($s_{g2}$) is received in the network, the initial value of the spherical potential ($\varphi_2$) is zero, and new membership functions will be self-generated with the rule generating mechanism to reach a stable number of 3. Snapshots at 0.075 s are depicted in Fig. 6(b) and 6(c) to clearly illustrate the rule-generating process. As seen from the snapshots in Fig. 6(b) and 6(c), the proposed dynamic threshold values in the rule-generating mechanism can avoid the rules with close range being generated effectively, while it is hard to choose a proper constant threshold under the circumstance of the quite small difference between the two consecutive sampling points during the transient process. Moreover, the rule number in the rule layer is $3 \times 3 = 9$ in the fixed network structure. Fortunately, the redundant rules will be pruned by the proposed SFNNISMC if the corresponding rule contribution degree is lower than the dynamic threshold value with the proposed rule-pruning mechanism. As shown in Fig. 6(d), the rule number is stable in 7 from 9 in the fixed network structure. Fortunately, the redundant rules will be pruned by the proposed SFNNISMC if the corresponding rule contribution degree is lower than the dynamic threshold value with the proposed rule-pruning mechanism. As shown in Fig. 6(d), the rule number is stable in 7 from 9 in the fixed network structure. Therefore, the redundant rules will be pruned by the proposed SFNNISMC if the corresponding rule contribution degree is lower than the dynamic threshold value with the proposed rule-pruning mechanism. As shown in Fig. 6(d), the rule number is stable in 7 from 9 in the fixed network structure. Fortunately, the redundant rules will be pruned by the proposed SFNNISMC if the corresponding rule contribution degree is lower than the dynamic threshold value with the proposed rule-pruning mechanism. As shown in Fig. 6(d), the rule number is stable in 7 from 9 in the fixed network structure. Fortunately, the redundant rules will be pruned by the proposed SFNNISMC if the corresponding rule contribution degree is lower than the dynamic threshold value with the proposed rule-pruning mechanism. As shown in Fig. 6(d), the rule number is stable in 7 from 9 in the fixed network structure.
the rule number is reduced from 7 to 3 in this operational condition, as shown in Fig. 6(d). Meanwhile, the related network parameters (means, standard deviations, and connection weights) will not be adjusted. Therefore, the computational burden can be significantly reduced. Different from self-generating rules for the second input signal at 0.075 s, nodes and network parameters corresponding to the disconnected slave inverter are stored instead of pruning, and the network can refine the corresponding rules when the slave inverter is reconnected into the parallel-inverter system at 0.215 s. The rule number transiently returns to 8 and is finally stable in 7, as shown in Fig. 6(d), without generating new membership functions. It is obvious that the overshoots of the currents are fairly smaller in the transient response compared with those at 0.075 s, as shown in Fig. 6(a).

To verify the robustness of the proposed SFNNISMC against the system-parameter uncertainties, the root mean square (RMS) value of the grid-connected current reference changes from 10 A to 5 A at 0.305 s in Fig. 7, the DC input voltage of the slave inverter fluctuates from 200 V to 180 V at 0.305 s in Fig. 8, and the inductance of the slave inverter varies from the nominal value 2 mH to 1.5 mH at 0.305 s in Fig. 9, respectively. The simulated result in Fig. 7(a) shows remarkable transient performance with the stable rule number \( N_l = 8 \) in Fig. 7(b). As seen from Fig. 8(a) and Fig. 9(a), the control performance is scarcely influenced by the DC input voltage fluctuation and the inductance variation in the slave inverter. In addition, more rules are pruned with the reduction of current-tracking errors in the steady state (\( N_l = 7 \)), as shown in Fig. 8(b) and Fig. 9(b).

**FIGURE 7.** Numerical simulations of parallel-inverter system in grid-connected MG under grid-connected power variations by SFNNISMC: (a) Currents of master inverter \( i_{L1} \) and slave inverter \( i_{L2} \), grid-connected current \( i_s \) and grid-connected reference current \( i_{sref} \); (b) Rule evolution.

**FIGURE 8.** Numerical simulations of parallel-inverter system in grid-connected MG under input voltage fluctuation of slave inverter \( V_{dc1}=200 \text{ V}, V_{dc2} \text{ varied from } 200 \text{ V to } 180 \text{ V} \) by SFNNISMC: (a) Currents of master inverter \( i_{L1} \), slave inverter \( i_{L2} \), and grid-connected current \( i_s \); (b) Rule evolution.

**FIGURE 9.** Numerical simulations of parallel-inverter system in grid-connected MG under inductance variations by SFNNISMC: (a) Current of master inverter \( i_{L1} \), slave inverter \( i_{L2} \) and grid-connected current \( i_s \); (b) Rule evolution.
B. EXPERIMENTAL VERIFICATION

An experimental prototype with two parallel inverters in Fig. 10 is constructed to further demonstrate the functionality of the proposed SFNNISMC strategy in practical applications. The main circuit is full-bridge inverters composed of four FQA24N50F power MOSFETs, and the driver signals for power switches are provided by the driving circuit based on an IC chip of TLP250. The detailed circuit parameters are summarized in Table I. Moreover, the sampling circuit is designed to sense the utility voltage in real time by the series Hall voltage sensors (LV 25-P) for the unit grid-connected current command combined with a digital phase-locked-loop (PLL) scheme, and the inductor currents inside two inverters are measured for the feedback circuit by the series Hall current sensors (LA 25-NP). In addition, the designed control algorithms are carried out via a TMS320F28335 series digital signal processor (DSP) with a sampling and switching frequency of 20 kHz, and the control signals sent to the driving circuit are generated by the pulse-width-modulation (PWM) module in the DSP with a dead time of 0.5 μs between the two signals for the MOSFETs in the same bridge. Furthermore, the current from the point of common coupling (PCC) is connected to the utility grid through an autotransformer manufactured by the Chint Electrics company. Differential voltage probes and Hall current probes are utilized to measure the waveforms of the voltage of the utility grid, the inductor currents of the two inverters, and the grid-connected current, which are displayed on a four-channel digital oscilloscope manufactured by the Agilent company.

The master inverter and the slave inverter are designed with the same capacity in the experimentation. Moreover, the grid-connected current command is given for unity power factor control, and the parallel-inverter system in the grid-connected MG is controlled to achieve the tracking objective of the grid-connected current command and current sharing with the master-slave current sharing strategy. The experimental results of four control strategies, including the conventional proportional-integral control (PIC), the sliding-mode control (SMC), the fixed-network-structure AFNNISMC in [19] and the proposed SFNNISMC are recorded, and the parameters of the PIC, the SMC, the AFNNISMC in [19] are chosen appropriately to achieve a control performance similar to that of the proposed SFNNISMC in the nominal case for a fair comparison.

1) PERFORMANCE VERIFICATION IN STEADY STATE

The steady-state experimental results of the parallel-inverter system in the grid-connected MG are depicted in Fig. 11. Although the utility voltage ($v_s$) is notably distorted, the grid-connected current can still be controlled with a low THD, and a high grid-connected power factor (PF) for the grid-connected current command ($i_{sref}$) with a sinusoidal waveform is generated by a digital PLL. From Fig. 11(a), the grid-connected current can be stably controlled to the reference with THD ($i_1$) = 2.17%, NMSE ($e_i$) = 0.0303, and PF = 0.9903 by the PIC. Moreover, The THD and NMSE of the grid-connected current are 1.82% and 0.0223, respectively; and the PF value is measured to be 0.9928 by the SMC in Fig. 11(b). The THD and NMSE values recorded in Fig. 11(c) are respectively reduced to 1.55% and 0.0185 by the AFNNISMC in [19]; and the PF value is improved to be 0.9956. For the experimental result by the proposed SFNNISMC in Fig. 11(d), the THD and NMSE values are 1.43% and 0.0163, respectively, which are lower than those by the PIC in (33), the SMC in (6) and the AFNNISMC in [19]; and the PF value is controlled to be the best performance of 0.9977 among the four control strategies.

![FIGURE 10. Experimental prototype with two parallel inverters.](image-url)

![FIGURE 11. Steady state experimental results of parallel-inverter system in grid-connected MG: (a) PIC; (b) SMC; (c) AFNNISMC in [19]; (d) Proposed SFNNISMC.](image-url)
schemes. Although the THD of the grid-connected current of the parallel-inverter system can be stably controlled to less than 3%, and the grid-connected PF values are higher than 0.99 by all four control strategies, the proposed SFNNISMC framework yields superior grid-connected power supply quality with smaller THD and NMSE values and higher grid-connected PF in the steady state with the nominal system parameters.

2) DYNAMIC PERFORMANCE VERIFICATION

To verify the superiority of the proposed SFNNISMC against power variations, Figures 12 and 13 present the experimental results of the parallel-inverter system in the grid-connected MG by four control strategies under grid-connected power variations from 1 kW to 0.5 kW and from 0.5 kW to 1 kW, respectively.

![FIGURE 12. Experimental results of parallel-inverter system in grid-connected MG under grid-connected power variations from 1 kW to 0.5 kW: (a) PIC; (b) SMC; (c) AFNNISMC in [19]; (d) Proposed SFNNISMC.](image)

By observing Figs. 12(a) and 13(a), the currents of the inductive currents in the master and slave inverters oscillate noticeably at the moment of power variations, although the grid-connect current can be stabilized after a transient time of approximately 5 ms. It is obvious that the PIC is inefficient to resist power variations in the grid-connect current control for the parallel-inverter system. The parallel-inverter system can be stably controlled with a very short transition process by the SMC shown in Figs. 12(b) and 13(b), in which the NMSE values of the current tracking by the SMC in (6) are 0.0308 and 0.0302, which are better than the NMSE values of 0.0375 and 0.0407, respectively, in Figs. 12(a) and 13(a) by the PIC. Moreover, the transient oscillation caused by the PIC can be decreased effectively by the SMC under the occurrence of grid-connected power variations at its peak value. In addition, the grid-connected current performance by the AFNNISMC in [19] with NMSE \( (e_i) = 0.0245 \) in Fig. 12(c) and NMSE \( (e_i) = 0.0230 \) in Fig. 13(c) are better than those of the SMC in (6). In addition, the proposed SFNNSMC can provide at least 47.73%, 36.36% and 16.96% current tracking improvement with the NMSE values of 0.0196 in Fig. 12(d) and 0.0191 in Fig. 13(d) than the ones by the PIC, the SMC in (6) and the AFNNISMC in [19], respectively. As seen from Figs. 12 and 13, it can be concluded that the parallel-inverter system with the proposed SFNNISMC possesses excellent robust control performance against grid-connected power variations.

![FIGURE 13. Experimental results of parallel-inverter system in grid-connected MG under grid-connected power variations from 0.5 kW to 1 kW: (a) PIC; (b) SMC; (c) AFNNISMC in [19]; (d) Proposed SFNNISMC.](image)
varying from 1:1 to 2:1 and from 2:1 to 1:1 is investigated. In Figs. 14 and 15, the command of the inductor current in the slave inverter is set as \( i_{L2}^{ref} = 0.5 \times i_{L1} \). From Figs. 14 and 15, it can be seen that the proportional current sharing between the master and slave inverters can be accurately performed as the current sharing ratio command by the four control methods. The NMSE values (0.0385 and 0.0382) by the PIC in Figs. 14(a) and 15(a) are reduced to (0.0298 and 0.0291) by the SMC in Figs. 14(b) and 15(b), respectively, and the transient oscillation caused by the PIC can be significantly attenuated by the SMC. Moreover, the transient process in the change of the current sharing ratio for the master and slave inverters is nearly instantaneous by the SMC to be better than the requirement of 5 ms transient time in the PIC. In addition, the total grid-connected current is scarcely perturbed, as seen from Figs. 14(c) and 15(c) by the AFNNISMC in [19] and Figs. 14(d) and 15(d) by the proposed SFNNISMC. More importantly, the parallel-inverter system with the proposed SFNNISMC scheme yields a superior quality of the grid-connected current with the NMSE values of 0.0189 and 0.0185 in Figs. 14(d) and 15(d), respectively, which is better than the NMSE values of 0.0232 and 0.0227 in Figs. 14(c) and 15(c) by the AFNNISMC in [19].

For a practical parallel-inverter system in a grid-connected MG, a faulty inverter should be removed from the system and reconnected into the system again after the reparation. The dynamic experimental responses of the parallel-inverter system in a grid-connected MG as the slave inverter to be disconnected from and reconnected to the utility system are illustrated in Figs. 16 and 17, respectively. When the slave inverter is abruptly disconnected or reconnected in Figs. 16(a) and 17(a), it results in the current oscillation during the transient process, and the control performance by the PIC deteriorates markedly with the structure variation of the parallel-inverter system. Although favorable current tracking performance may be obtained if the control gains of the PIC are retuned manually, it is time-consuming. Fortunately, the SMC, the AFNNISMC in [19] and the proposed SFNNISMC have more preferable robustness against the structure variation compared with the traditional PIC. As seen from Figs. 16(b)-16(d), the remaining master inverter suddenly doubles the output current to ensure that the total grid-connected current can continuously track the grid-connected current command when the slave inverter is removed from the system. By observing Figs. 17(b)-17(d), when the repaired slave inverter is reconnected into the parallel-inverter system, the slave inverter can be controlled to achieve the current sharing with the master inverter during a very short transient process.
Moreover, the NMSE values (0.0301 and 0.0312) by the SMC in Figs. 16(b) and 17(b), respectively, can be reduced to 0.0231 and 0.0235 by the AFNNISMC in Figs. 16(c) and 17(c), respectively. In addition, the grid-connected current controlled by the proposed SFNNISMC is least sensitive to the disconnection and reconnection of the slave inverter, with the NMSE values of 0.0189 and 0.0193 in Figs. 16(d) and 17(d), respectively.

**Remark 1:** From the comparisons of the dynamic performances in Figs. 12-17, it is obvious that the proposed SFNNISMC strategy indeed yields more favorable robustness characteristics against grid-connected power variations, current sharing ratio changes, and the structure variation of the parallel-inverter system. Moreover, the proposed SFNNISMC strategy also obtains better grid-connected current tracking and current sharing performance with smaller NMSE values. In addition, the transient response time in the proposed SFNNISMC strategy is the shortest, even for the dynamic changes that occurred around the peak value of the grid-connected current. It effectively improves the reliability of the parallel-inverter system in a grid-connected MG under the occurrence of system uncertainties.

3) **ROBUSTNESS TO PARAMETER VARIATIONS**

The DC-bus voltages for master and slave inverters in a grid-connected MG often come from different distributed generations (DGs), so that the DC-bus voltages are not the same as each other. To exhibit the superior robustness of the proposed SFNNISMC under DC-bus voltage variations, Figure 18 illustrates the experimental result of the parallel-inverter system in a grid-connected MG with the nominal DC-bus voltage ($V_{dc1} = 200$ V) for the master inverter and a lower DC-bus voltage ($V_{dc2} = 180$ V) for the slave inverter controlled by the proposed SFNNISMC. Moreover, by considering the variation of the filter inductor from its nominal value, the experimental result of the parallel-inverter system in a grid-connected MG controlled by the proposed SFNNISMC under the asymmetrical inductance parameter condition ($L_1 = 2$ mH, $L_2 = 1.5$ mH) is depicted in Fig. 19.

The THD values are measured to be 1.45% and 1.48%, the grid-connected PF values are measured to be 0.9965 and 0.9970, and the NMSE values of the grid-connected current tracking are 0.0168 and 0.0167 in Figs. 18 and 19, respectively. By comparing Figs. 18-19 with Fig. 11(c), a 10% reduction in the DC bus voltage and 25% reduction in the filter inductance of the slave inverter only result in 1.4% and 3.4% increases in the THD value, and 3.0% and 2.5% increases in the NMSE value, respectively. Moreover, the corresponding PF values are almost unchanged. It can be concluded that the grid-connected power supply quality, grid-connected tracking and current sharing performance by
the proposed model-free SFNNISMC are less sensitive to the variations in the DC-bus voltage and the filter inductance.

FIGURE 18. Experimental results of parallel-inverter system in grid-connected MG under different DC-bus voltages condition ($V_{dc1} = 200V$, $V_{dc2} = 180V$).

FIGURE 19. Experimental results of parallel-inverter system in grid-connected MG under asymmetrical inductances condition ($L_1 = 2$ mH, $L_2 = 1.5$ mH).

C. COMPUTATIONAL BURDEN EVALUATION

The computational burden and performance comparisons of the AFNNISMC in [19] and proposed SFNNISMC system for the parallel-inverter system under four structure conditions are summarized in Table II.

TABLE II

| Control systems                  | Performance | AFNNISMC in [19] | Proposed SFNNISMC |
|----------------------------------|-------------|------------------|-------------------|
| Only master inverter action      | NMSE ($e_i$)| 0.015            | 0.0132            |
|                                  | Rule Number ($N$) | 9               | 2                 |
|                                  | Computational time | 42.6 $\mu$s     | 19.7 $\mu$s       |
| Two inverters operation in parallel | NMSE ($e_i$) | 0.0185           | 0.0163            |
|                                  | Rule Number ($N$) | 9               | 7                 |
|                                  | Computational time | 42.6 $\mu$s     | 36.4 $\mu$s       |
| Disconnection of slave inverter | NMSE ($e_i$) | 0.0231           | 0.0189            |
|                                  | Rule Number ($N$) | 9               | 3                 |
|                                  | Computational time | 42.6 $\mu$s     | 27.3 $\mu$s       |
| Reconnection of slave inverter  | NMSE ($e_i$) | 0.0235           | 0.0193            |
|                                  | Rule Number ($N$) | 9               | 7                 |
|                                  | Computational time | 42.6 $\mu$s     | 36.4 $\mu$s       |

The proposed SFNNISMC scheme can obtain better grid-current tracking performance by utilizing merely 2 rules with NMSE($e_i$) = 0.0132 than the fixed-network-structure FFNISMIC method in [19] with an NMSE value of 0.015. There is 77.7% rule number reduction and 53.75% computational time saving by the rule-generating algorithm compared with the rule number of 9 for the AFNNISMIC method in [19]. Moreover, the Petri net in the proposed SFNNISMC system combines the rule-pruning mechanism to remove the redundant rules. Thus, only 7 control rules are involved in the steady state when two inverters operate in parallel. The proposed SFNNISMC system achieves better control performance with fewer rules and parameters to be adjusted online, and the computational time can be reduced from 42.6 $\mu$s to 36.4 $\mu$s. When the fault slave-inverter is disconnected from the MG, the control rules corresponding to the disconnected slave inverter are in the dormant state and no longer participate in the control decision, and the corresponding network parameters are not updated online. As a result, the proposed SFNNISMC system only utilizes 33.3% of the complete rules and saves 35.92% computational time compared with the fixed-network-structure AFNNISMIC method in [19] on the premise of guaranteeing the control performance with a smaller NMSE value of 0.0189. The dormant rules can be recalled by the Petri net as the reconnection of the slave inverter. Note that the time consumption on the rule-generating mechanism can be saved by recalling the dormant rules compared with the regeneration process step by step with initial parameters. Meanwhile, to implement the proposed SFNNISMC framework easily, the function approximation or look-up table method can be used to carry out Gaussian membership functions instead of calling math-function codes, which is helpful to decrease the computational burden significantly.

V. CONCLUSION

This study has successfully utilized a self-constructing fuzzy neural network (SFNN) with structure and parameter self-learning abilities to imitate sliding-mode control (SMC) and implemented the formed SFNN-imitating SMC (SFNNISMIC) system to manipulate grid-connected current tracking for a parallel-inverter system in a grid-connected microgrid (MG) with a master-slave current sharing strategy. First, a sliding-surface vector is designed for the parallel-inverter system containing a master inverter and $n$-$1$ slave inverters. Then, the elements of the sliding-surface vector are taken as the inputs of the designed SFNN to imitate the SMC law, and the rules of the SFNN can be generated online from null according to the instantaneous inputs based on the rule-generating mechanism with the dynamic threshold. Moreover, the pruning mechanism is carried by a dynamic Petri net to fire the significant rules, which is also utilized to recall the rules corresponding to the reconnected slave inverters. In addition, only the parameters of favorable rules fired by the dynamic Petri net are updated online instead of all the parameters in the adaptive fuzzy-neural-network-imitating sliding-mode control (AFNNISMIC) framework with a fixed structure network in [19] for tuning. Furthermore, the stability of the proposed SFNNISMIC system and the convergence of the network parameters can be guaranteed by the projection algorithm and the Lyapunov stability theorem. The rule evolution of the proposed SFNN in the structure self-learning process is illustrated in numerical simulations, and the experimental verification demonstrates the superiority of the proposed SFNNISMIC system.
TABLE III

| Performance                   | Control systems | PIC        | SMC        | AFNNISMC in [19] | Proposed SFNNISMC |
|-------------------------------|-----------------|------------|------------|------------------|------------------|
| Steady-state performance      | Grid-connected power 1kW | THD ($i_e$) | 2.17%      | 1.82%            | 1.55%            | 1.43%            |
|                               |                  | PF         | 0.9903     | 0.9928           | 0.9956           | 0.9977           |
|                               |                  | NMSE ($e_i$) | 0.0303     | 0.0223           | 0.0185           | 0.0163           |
| Dynamic performance           | Grid-connected power variations from 1kW to 500W | NMSE ($e_i$) | 0.0375     | 0.0308           | 0.0245           | 0.0196           |
|                               | Grid-connected power variations from 500W to 1kW | NMSE ($e_i$) | 0.0407     | 0.0302           | 0.023            | 0.0191           |
|                               | current sharing ratio changes from 1:1 to 2:1 | NMSE ($e_i$) | 0.0385     | 0.0298           | 0.0232           | 0.0189           |
|                               | current sharing ratio changes from 2:1 to 1:1 | NMSE ($e_i$) | 0.0382     | 0.0291           | 0.0227           | 0.0185           |
|                               | Disconnection of slave inverter | NMSE ($e_i$) | 0.0443     | 0.0301           | 0.0231           | 0.0189           |
|                               | Reconnection of slave inverter | NMSE ($e_i$) | 0.0415     | 0.0312           | 0.0235           | 0.0193           |
| Requirement of system information | None       | High       | None       | None             | None             |
| Robustness                    | Poor            | Little     | Good       | Favorable        |                  |
| Network parameter learning ability | No         | No         | Yes        | Yes              |                  |
| Network structure learning ability | No         | No         | No         | Yes              |                  |
| Computational time            | Only master inverter action | 9.5 μs   | 12.3 μs    | 42.6 μs          | 19.7 μs          |
|                               | Two inverters action | 9.5 μs   | 12.3 μs    | 42.6 μs          | 36.4 μs          |
| Computational burden          | Low             | Low        | High       | Middle           |                  |

Experimental control performance comparisons and analytic results including the PIC method, the SMC scheme in (6), the AFNNISMC framework in [19], and the proposed SFNNISMC system under different operating conditions are summarized in Table III. As seen from the comparisons in Table III, the traditional PIC method brings larger NMSE values of the grid-connected current tracking under the occurrence of the system uncertainties. To improve the control performance of the PIC, the corresponding control gains should be retuned manually according to different operation conditions. Moreover, the SMC scheme has an acceptable control performance for the grid-connected current quality in the steady state, and is robust against system uncertainties. However, the dynamic information of the parallel-inverter system is required to guarantee the control performance, and a large gain for the sign function in the SMC law needs to be selected to cope with different system uncertainties such that chattering phenomena will be inevitable. In addition, the AFNNISICM framework with a fixed network structure in [19] possesses smaller THD and NMSE values due to the ability of reasoning and parameter self-learning without the requirement of dynamic mathematical models. Although the initial values of the network parameters can be roughly chosen based on expert knowledge, and favorable parameters can be obtained after several pretraining procedures, redundant rules in the steady state will result in a high computational burden, especially under the disconnection of the partial slave inverter from the parallel inverter system. Since the rule-generating mechanism is designed based on the dynamic inputs and the simplified-rule-firing mechanism is operated with less computational burden, the proposed SFNNISMC system with lower THD value and NMSE values possesses superiority static performance and stronger robustness against system uncertainties due to both the structure and parameters self-learning ability.

The proposed SFNNISMC scheme can provide 46.2%, 26.91% and 11.89% current tracking improvement, 34.1%, 21.43% and 7.74% THD value improvement in steady state, and at least 47.74%, 36.36% and 16.96% current tracking improvement under system uncertainties in comparisons with the PIC, the SMC and the AFNNISMC in [19], respectively. Although the computational costs of the proposed SFNNISMC scheme are higher than those of the PIC and the SMC, it can reduce the computational times by 53.76% and 14.6% for the parallel-inverter system with only the master inverter action and with two inverters action, respectively, compared to the AFNNISMC in [19]. Thus, the proposed SFNNISMC system provides better control performance than the PIC, the SMC and the AFNNISMC in [19].

The neurons in each layer of the proposed SFNNISMC framework only accept the outputs of the neurons in the previous layer, and the internal information of the SFNN is not adequately used. To enhance the dynamic imitating ability of the SFNN, a recurrent framework with internal feedback loops can be adopted to imitate the SMC law in...
future research. Moreover, the communication delay in the process of transmitting the currents or control signals may cause stability issues with the master-slave current sharing strategy. The design of an adaptive observer combined with an SMC can be further investigated for the time-delay compensation and the enhancement of the current control performance for the grid-connected current control system of the parallel-inverter system in a grid-connected MG.

APPENDIX

By differentiating (27), one can obtain

\[ \dot{V}_{\text{SFNNISMC}}(s_g, \dot{w}_g, \dot{\hat{m}}_g, \dot{\hat{c}}_g) = s_g^T V_{\text{w}} - \frac{\dot{w}_g \dot{\hat{w}}_g}{\eta_{wg}} - \frac{\dot{\hat{m}}_g \dot{\hat{m}}_g}{\eta_{mg}} - \frac{\dot{\hat{c}}_g \dot{\hat{c}}_g}{\eta_{cg}} \]  

(A1)

To rewrite (19) by replacing \( u_{\text{SMC}} \) with (6) and \( \ddot{u}_g \) with (23), the proposed self-constructing fuzzy-neural-network-imitating sliding-mode control (SFNNISMC) law can be formulated as follows:

\[ \dot{u}_{\text{SFNNISMC}} = u_{\text{SMC}} - \ddot{u}_g = B_{pg}^{-1} [-D_{pg} f(t) + x_{\text{cov}} + K_{pg} \text{sgn}(s_g(t))] - (Y_{wg} \dot{w}_g + U_{lg} \dot{m}_g + U_{cg} \dot{c}_g + y_g) \]  

(A2)

By substituting \( \dot{u}_{\text{SFNNISMC}} \) in (A2) into the control effort (u), the differential of the sliding-surface vector in (5) can be rewritten as

\[ \dot{s}_g = -K_{pg} \text{sgn}(s_g) - P_g + (Y_{wg} \dot{w}_g + U_{lg} \dot{m}_g + U_{cg} \dot{c}_g + y_g) \]  

(A3)

By substituting (A3) into (A1), one can obtain

\[ \dot{V}_{\text{SFNNISMC}} = s_g^T [-K_{pg} \text{sgn}(s_g) - P_g] + s_g^T (Y_{wg} \dot{w}_g + U_{lg} \dot{m}_g + U_{cg} \dot{c}_g + y_g) - \dot{w}_g \dot{\hat{w}}_g / \eta_{wg} - \dot{\hat{m}}_g \dot{\hat{m}}_g / \eta_{mg} - \dot{\hat{c}}_g \dot{\hat{c}}_g / \eta_{cg} = s_g^T [-K_{pg} \text{sgn}(s_g) - P_g + y_g] + (s_g^T Y_{wg} \dot{w}_g - \dot{\hat{w}}_g \dot{\hat{w}}_g / \eta_{wg}) + (s_g^T U_{lg} \dot{m}_g - \dot{\hat{m}}_g \dot{\hat{m}}_g / \eta_{mg}) + (s_g^T U_{cg} \dot{c}_g - \dot{\hat{c}}_g \dot{\hat{c}}_g / \eta_{cg}) = s_g^T [-K_{pg} \text{sgn}(s_g) - P_g + y_g] + V_{wg} + V_{mg} + V_{cg} \]

where

\[
V_{wg} = s_g^T Y_{wg} \dot{w}_g - \dot{\hat{w}}_g \dot{\hat{w}}_g / \eta_{wg}, \quad V_{mg} = s_g^T U_{lg} \dot{m}_g - \dot{\hat{m}}_g \dot{\hat{m}}_g / \eta_{mg}, \quad \text{and} \quad V_{cg} = s_g^T U_{cg} \dot{c}_g - \dot{\hat{c}}_g \dot{\hat{c}}_g / \eta_{cg}.
\]

If the adaptation law for the output weights in the SFNN is designed as (24), \( V_{wg} \) can be represented as

By (24a)

\[
V_{wg} = s_g^T Y_{wg} \dot{w}_g - \frac{\dot{\hat{w}}_g \dot{\hat{w}}_g}{\eta_{wg}} = s_g^T Y_{wg} \dot{w}_g - (\eta_{wg} s_g^T Y_{wg} \dot{w}_g / \eta_{wg}) = 0 \]  

(A5)

By (24b)

\[
V_{wg} = s_g^T Y_{wg} \dot{w}_g - \frac{\dot{\hat{w}}_g \dot{\hat{w}}_g}{\eta_{wg}} = s_g^T Y_{wg} \dot{w}_g - \eta_{wg} [s_g^T Y_{wg} \dot{w}_g - \frac{(s_g^T Y_{wg} \dot{w}_g / \eta_{wg})}{\eta_{wg}} = 0 \]  

(A6)

If the conditions of \( \| \dot{w}_g \| = b_{wg} \) and \( s_g^T Y_{wg} \dot{w}_g > 0 \) are satisfied, the result of \( \| \dot{\hat{w}}_g \| - \frac{(s_g^T Y_{wg} \dot{w}_g / \eta_{wg})}{\eta_{wg}} = 0 \)

(A7)

If the gain condition of \( \| K_{pg} \| > \| y_g - P_g \| \) is designed, the result of \( \dot{V}_{\text{SFNNISMC}} \leq 0 \) can be satisfied.

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