Performance Analysis of SWIPT System with Imperfect Channel State Information

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Abstract. This paper studies the system performance of the multiple-input-single-output (MISO) system using simultaneous wireless information and power transfer (SWIPT) technology under imperfect channel state information. A power split strategy is used in the receiver, which divides the received radio frequency (RF) signal into two parts, one part is directly used for signal decoding, and the other part is used as energy harvesting. This paper studies the optimization of system channel capacity under the constraints of harvested energy and the transmission rate. The resulting optimization problem is a two-layer non-convex optimization one, which is simplified by Cauchy-Schwarz inequality. Through the linear search of the energy division ratio in its value range, it becomes a one-dimensional optimization problem only related to the transmission power, which can be solved by CVX. The simulation results verify the effectiveness of the algorithm.

1. Introduction

With the rapid development of technologies such as the Internet of Things and the continuous expansion of the scale of wireless communication services, the demand for energy in communication services has also increased sharply. Ensuring sufficient and stable energy supply has become an urgent need in current communications. The current communication system mainly relies on power grid as the main power supply means. However, grid power has many limitations. For instance, in harsh environments, the use of grid power may lack stability, which has great negative effects on the performance of the communication system. At the same time, the establishment of the grid is costly. Therefore, energy harvesting from the surrounding environment by the communication node has become a new focus of current research, that is, the use of green energy in wireless communication [1]. To meet the requirements of green communication, except the use of green energy as mentioned above, researchers in recent years have begun to consider the use of energy harvesting technology in communication systems [2]. Energy harvesting technology is a technology that enables nodes to achieve energy harvesting functions, thereby improving the working efficiency of wireless communication nodes [3]. Currently, energy harvesting can be divided into the following four types according to the harvested energy source: solar radiation, wind, mechanical movement and electromagnetic radiation. Energy obtained from the environment, for instance, solar energy, wind energy, etc., has the advantage that the source of energy is green and environmentally friendly. However, the above-mentioned energy sources also have their obvious limitations, that is, these...
energy sources have great instability. In mobile communications, radio frequency signals are a more effective source of energy. Its advantages are as follows: First, radio frequency signals are hardly affected by the environment. Secondly, the radio frequency signal can be used as a carrier to propagate the signal, which can greatly improve the efficiency of the system. Therefore, simultaneous wireless information and power transfer (SWIPT) technology is a promising energy harvesting method in the current communication system and has practical research significance. So it is a popular research direction currently. The original SWIPT system was proposed in the point-to-point system model [4], and subsequent researches extended to the power split strategy in SWIPT [5]. SWIPT in multiple-input-single-output (MISO) systems was researched in [6]. Furthermore, the researchers conducted research on the use of SWIPT system and power splitting strategies in multiple-input-multiple-output (MIMO) system [7]. On the basis of previous researches, researchers in [8] carried on performance analysis on MIMO and MISO time division duplex wireless links using SWIPT. Optimization of system energy efficiency under the condition of ideal channel state information (CSI) is studied in [9]. However, in actual situations, it is hard to obtain perfect CSI, and the impact of imperfect channel information on system performance is actually considerable. Channel information errors are mainly affected by factors such as channel estimation and quantization errors.

Therefore, this paper considers the SWIPT system transmission performance under the condition of imperfect CSI. According to the constraints of the maximum transmission power and minimum energy harvest in the system, a non-convex optimization problem of maximizing channel capacity is formed, and the problem is solved with the help of two-layer iteration and CVX toolkit [10]. The structure of this article is as follows: The second part introduces the system model and problem formation. The third part gives the formation process of the optimization problem and the proposed algorithm. The fourth and fifth parts give the simulation results and the summary of the article, respectively.

2. System model and problem formation

A single-cell downlink system is considered in this paper. There is one base station in the cell, and the base station has \( \hat{N}_t \) transmission antenna, \( \hat{N}_t > 1 \). In each time slot, only one active user is assumed to be selected by the cell, and the number of receiving antennas of the user is 1. Assuming that the channel is frequency flat fading, and the wireless channel between the base station and the user can be expressed as a \( N \times \hat{N}_t \) matrix as follows:

\[
\hat{H} = \sqrt{c d^{-\beta}} s W
\]  

(1)

Where \( c d^{-\beta} \) is the path loss, \( d \) means the distance between the user and the base station, \( \beta \) is the path loss index, and \( c \) is the transmission constant. \( s \) represents log-normal shadow distribution, and \( W \) represents small-scale fading [11]. Due to the existence of channel estimation errors, it is difficult to achieve perfect channel information estimation in practical applications. This article considers the channel model of uncertainty, which can be formulated by the following formula:

\[
\hat{H} = \hat{H} + \Delta h
\]  

(2)

Where \( \hat{H} \in C^{\hat{N}_t \times N} \) represents the channel state matrix, \( \Delta h \) is the channel error matrix, and \( \alpha \) is the range of \( \| \Delta h \| \), \( \| \Delta h \| \leq \alpha \) [12].

Then the discrete-time baseband channel of the user can be expressed as:

\[
y = (\sqrt{c d^{-\beta}} s W + \Delta h)x + z
\]  

(3)

Where \( y \in C^{\hat{N}_t \times 1} \) represents the user's received signal, \( x \in C^{N \times 1} \) is the signal sent by the base station to the user, and \( z \) is additive white Gaussian noise.

In order to facilitate information processing at the receiving end, zero-forcing precoding
technology is used at the transmitting end. Suppose the information symbol sent to the user is $s$, and the zero-forcing precoding matrix is $\hat{T}$, then the formula (3) can be written as:

$$y = \hat{H}Ts + z$$

(4)

According to the maximum ratio transmission strategy, the zero-forcing precoding matrix is expressed as [11]:

$$T = \sqrt{P_t} \hat{H}^H$$

(5)

Where $P_t$ is the signal transmission power.

The received signal $y$ is divided into two independent parts, one for decoding and one for energy harvesting. Suppose the energy partition coefficient is $\rho$, that is, $\rho$ part of the signal is divided for decoding, and the remaining $1 - \rho$ part of the signal is used for energy harvesting. In actual situations, the energy harvesting device cannot achieve full energy conversion efficiency. So we set $\eta$ as its conversion efficiency, whose value ranges between 0 and 1.

Therefore, the energy harvested per unit time is:

$$Q_n = \eta(1 - \rho)[\hat{H}^H + \sigma_s^2]$$

(6)

The channel capacity can be expressed as:

$$R = W \ln(1 + \gamma)$$

(7)

$$\gamma = \frac{\rho |\hat{H}^H|^2}{\rho \sigma_s^2 + \sigma_a^2}$$

(8)

Where $W$ is the channel bandwidth, $\sigma_s^2$ is the additive white Gaussian noise power, and $\sigma_a^2$ is the antenna noise power.

![Energy splitting receiver model](image)

**Fig. 1** Energy splitting receiver model.

The channel capacity optimization problem formed in this paper is expressed as follows:
where \( P_{\text{max}} \) is the maximum transmission power, and \( P_{\text{min}} \) represents the minimum power that meets the energy harvesting requirements. Constraint one means that the system must meet the minimum energy harvesting requirements and constraint two makes sure that the transmission power must be less than \( P_{\text{max}} \), and the last constraint means that the energy split ratio is between 0 and 1.

### 3. Problem transformation and proposed algorithm

Under the condition of imperfect channel information, the optimization problem given in equation (9) can be specifically written as the following equation:

\[
\max_{\rho, P} \quad R = (\rho, P) \\
\text{s.t.} \quad Q_s(\rho, P) \geq P_{\text{min}} \\
0 \leq P \leq P_{\text{max}} \\
0 \leq \rho \leq 1
\]  

(9)

Due to the uncertainty of channel information, the multivariate optimization problem in (10) has an infinite number of constraints. Therefore, we need to simplify the problem and reduce the infinite constraints to finite constraints to make this problem solvable. Therefore, this paper considers the use of the nature of the norm-bounded matrix to reduce constraints. From the nature of the matrix, we can transform the quadratic expression to simplify the problem, namely:

\[
\left\| \hat{H} + \Delta h \hat{H}^H \right\|^2 = \hat{H} \left( \hat{h} + \Delta m \right) \hat{H}^H, 
\]

(11)

where \( \hat{h} = \hat{H}^H \hat{H} \) is the covariance matrix of the channel state information, and \( \Delta m \) is the uncertainty of the covariance matrix, which is expressed as \( \Delta m = \hat{H}^H \Delta h + \Delta h^H \hat{H} + \hat{H}^H \Delta h \). In order to facilitate the calculation, \( \| \Delta m \| \) needs to be simplified. According to the triangle inequality, the above formula can construct the following inequality relationship with the sum of the three terms constituting \( \Delta m \):

\[
\| \Delta m \| \leq \| \hat{H}^H \Delta h \| + \| \Delta h^H \hat{H} \| + \| \Delta h^H \Delta h \|
\]

(12)

According to the Cauchy-Schwarz inequality, the above formula can be further transformed into:

\[
\| \Delta m \| \leq \| \hat{H}^H \| \| \Delta h \| + \| \hat{H}^H \| \| \hat{H} \| + \| \Delta h^H \| \| \Delta h \| \leq \alpha^2 + 2\alpha \| \hat{H} \|
\]

(13)

Therefore, we can see that \( \| \Delta m \| \) bounded, let its upper limit be \( \tau \), then \( \tau = \alpha^2 + 2\alpha \| \hat{H} \| \). From this equation, the equation (11) can be further simplified:
\[
\|(\hat{H} + \Delta h)\hat{H}^\dagger\|^2 = \text{Tr}[(\hat{h} + \Delta m)\hat{h}] \tag{14}
\]

So formula (10) can be transformed into:

\[
\max_{\rho, P}\ R = W \ln(1 + \frac{\rho P \text{Tr}[(\hat{h} + \Delta m)\hat{h}]/\|\hat{h}\|^2}{\rho \sigma_s^2 + \sigma_a^2})
\]  
\[
\text{st} \quad \eta(1 - \rho)(P \text{Tr}[(\hat{h} + \Delta m)\hat{h}]/\sigma_a^2) \geq P_{\min}
\]
\[
0 \leq P \leq P_{\max}
\]
\[
0 \leq \rho \leq 1
\tag{15}
\]

Due to the boundedness of \(\|\Delta m\|\), the numerical value of \(\|\Delta m\|\) can be scaled to simplify the optimization problem and facilitate the solution. According to the upper limit set above, the following conversions can be made:

\[
\|\Delta m\| \leq \tau \Rightarrow -\tau I_{\Delta t} \leq \Delta m \leq \tau I_{\Delta t}
\tag{16}
\]

Then

\[
\min_{\|\Delta m\| \leq \tau} \text{Tr}[(\hat{h} + \Delta m)\hat{h}] = \text{Tr}[(\hat{h} - \tau I_{\Delta t})\hat{h}] \tag{17}
\]

In this formula, \(I_{\Delta t}\) is a unit vector of order \(\Delta t\). After the above transformation, the original optimization problem can be expressed by the following formula:

\[
\max_{\rho, P} \ R = W \ln(1 + \frac{\rho P \text{Tr}[(\hat{h} - \tau I_{\Delta t})\hat{h}]/\|\hat{h}\|^2}{\rho \sigma_s^2 + \sigma_a^2})
\]  
\[
\text{st} \quad \eta(1 - \rho)(P \text{Tr}[(\hat{h} - \tau I_{\Delta t})\hat{h}] + \sigma_a^2) \geq P_{\min}
\]
\[
0 \leq P \leq P_{\max}
\]
\[
0 \leq \rho \leq 1
\tag{18}
\]

It can be seen from equation (18) that the two variables of the optimization problem are \(\rho\) and \(P\), and the optimization problem is solved through two-level iteration. In the actual circuit, due to the digital effect of the circuit, the energy division coefficient \(\rho\) presents a discrete value, so it is reasonable to perform a discrete search on \(\rho\). Since the value of \(\rho\) ranges from 0 to 1, we consider to perform a linear search on the entire domain of \(\rho\). In the case of a given \(\rho\), the original optimization problem is simplified to a one-dimensional convex optimization problem, by the use of CVX, this problem can be solved. Table 1 below shows the specific flow of this algorithm.
**Tab. 1** Algorithm for channel capacity optimization

**Capacity-Max algorithm**

1. Initialize the empty set $G$ to store the maximum channel capacity obtained after each iteration.

2. Loop $\rho$ in the interval from 0 to 1 with the set step length.
   Use CVX to solve the optimization problem and get the optimal solution: $(G', P_t', \rho')$
   Put $(G', P_t', \rho')$ into the set $S$.
   End loop.

3. Find the optimal $(G', P_t', \rho')$.

**Tab. 2** Simulation parameters

| Simulation parameters                          | values   |
|-----------------------------------------------|----------|
| Channel bandwidth $W$                        | 15 KHZ   |
| Minimum harvested power requested $P_{\text{min}}$| 20 dBm  |
| Energy conversion efficiency $\eta$          | 0.8      |
| Power of signal processing noise $\sigma_i^2$| -95 dBm  |
| Power of antenna noise $\sigma_i^2$          | -98 dbm  |

4. **Numerical results**
This section presents the numerical simulation results of the channel capacity optimization problem under the constraints of different variables. Simulation parameter settings are shown in Table 2.

![Fig. 2 Channel capacity when changing $\rho$](image1)

![Fig. 3 Channel capacity when changing cell radius $D$.](image2)

It can be seen from Figure 2 that when only the energy split ratio is changed, the channel capacity increases with the energy split ratio, and when the minimum energy harvest requirement cannot be met, the curve drops sharply. The higher the signal transmission power, the higher the maximum value that
the energy division ratio can meet.

![Fig. 4 Channel capacity when changing maximum transmission power](image1)

![Fig. 5 Channel capacity when changing channel error.](image2)

Figure 3 shows that when only the distance between the base station and the user itself is changed, the maximum channel capacity decreases as the distance increases. The reason for the sharp drop in the curve is that the system cannot meet the minimum energy harvest requirement at this time. Figure 4 shows the change in channel capacity when the maximum transmit power is changed and the radius of the cell is given. It can be seen from the figure that when the distance from the base station to the user is set to 20 meters, when the maximum transmission power is set to 30 dBm, when the maximum transmission power increases, the maximum channel capacity also increases. When the cell radius is set to be 40 meters and the maximum transmission rate set to be 35dBm, similar to the previous situation, as the maximum transmission rate increases, the maximum channel capacity also increases.

Figure 5 shows the influence of channel error on the maximum channel capacity. It can be seen from the figure that, regardless of whether there is channel error and the size of the channel error, the system capacity increases with the increase of the maximum transmission rate. When $\alpha=0$, that is, when the transmitter can obtain perfect channel information, the transmitter can always obtain the best system performance. In the case of channel error, the channel capacity is always less than the perfect channel state, and when the channel error is smaller, that is, the channel is closer to the perfect channel, the curve of the maximum channel capacity is closer to the curve when $\alpha=0$.

5. Conclusion
This paper studied the channel capacity optimization problem of SWIPT system with imperfect CSI. The problem is transformed by Cauchy Schwartz inequality and the nature of the norm-bounded matrix, and the binary non-convex optimization problem is solved. A two-layer iterative optimization algorithm is proposed for the optimization problem of transformation. Simulation results show that CSI has an important impact on system performance. The more accurate the channel state information, the better the performance of the system.

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