Nonlinear Interaction of a Shock Wave with an Anisotropic Entropy Perturbation Field

K E Gorodnichev¹, S E Kuratov¹ and E E Gorodnichev²

¹Dukhov Research Institute of Automatics, Sushchevskaya st. 22, Moscow, 127055, Russia
²National Research Nuclear University MEPhI, Kashirskoe Shosse 31, Moscow, 115409, Russia
E-mail: cyrgo85@gmail.com

Abstract. The problem of the interaction of a shock wave with an anisotropic entropy perturbation field has been solved including second-order corrections to hydrodynamic quantities. It has been shown that nonlinear interactions between acoustic waves result in the localization of acoustic perturbations behind the shock front. This effect is observed when sound attenuation is absent in the linear approximation. The problem of the propagation of the shock wave in an incident sample, where the spatially anisotropic density perturbation field initially exists, has been numerically solved in application to the collision of two plates. Numerical calculations confirm the results of the theoretical analysis.

1. Introduction
The interaction of planar shock waves with various perturbation fields in a medium has been studied in the last half century in numerous theoretical and experimental works (see, e.g., [1–12]). In particular, in the linear approximation, the interaction of a shock wave with acoustic waves was considered in [2, 11], with an isotropic turbulent vorticity field in [8], and with the perturbation of the density in [9, 12]. In all these cases, acoustic and entropy–vortex perturbations appear behind the shock front in addition to the initial fields. The interaction of the shock wave with the anisotropic density perturbation field was considered in the linear approximation in [13]. It was shown that the regime of propagation of sound behind the shock front depends on the angle between the axis of anisotropy of the perturbation field and the velocity of the shock wave. In the case of relatively small angles \( \theta_0 \) between the axis of anisotropy and the inward normal to the shock front \( \mathbf{n}_0 \), the projection of the velocity of an acoustic wave \( c_x \) on the \( OX \parallel \mathbf{n}_0 \) axis is positive. With an increase in the angle \( \theta_0 \), the projection \( c_x \) decreases and, at \( \theta_0 = \theta_0^{**} \) (\( \theta_0^{**} \) is determined by the parameters of the medium behind and ahead of the shock front [13]), becomes equal to the velocity of the medium. In the angular interval \( \theta_0^{**} < \theta_0 \), the acoustic wave moves behind the shock wave. When the angle \( \theta_0 \) reaches the threshold value \( \theta_0^* \), the velocity projection \( c_x \) becomes equal to the velocity of the shock wave. With a further increase in the angle \( \theta_0 \) (\( \theta_0 > \theta_0^* \)), acoustic perturbations behind the shock wave are described by surface acoustic waves [2]. As follows from the described results [13], when \( \theta_0 = \theta_0^* \), the velocity projection \( c_x \) is equal to the velocity of the shock wave. A spatial decay of acoustic perturbations at \( \theta_0 = \theta_0^* \) is absent also. Consequently, the amplitude of these perturbations behind the shock front should be zero. The spatial structure of the acoustic field under these conditions can be correctly described only beyond the linear approximation.
Below, the problem of the interaction of the shock wave with the anisotropic density perturbation field is solved including second-order corrections to hydrodynamic quantities. In this approximation, hydrodynamic quantities satisfy inhomogeneous wave equations [14]. The application of standard perturbation theory to the solution of the equation for the pressure results in the appearance of secular (growing in time) terms. In this work, we propose a new analytical method for solving such equations. This method generalizes the approach used in the theory of nonlinear ordinary differential equations [15] and involves the introduction of an additional phase into a linear solution. Using this method, it is shown that damping acoustic perturbations exist behind the shock front at $\theta = \theta^*_0$. The problem of the propagation of the shock wave in a incident sample, where the spatially anisotropic density perturbation field initially exists, is numerically solved in application to the collision of two plates. The results of the calculations confirm qualitative conclusions made in the quadratic approximation concerning the character of propagation of sound in the immediate vicinity of the angle $\theta^*_0$ ($\theta^*_0 - \theta_0 \ll \theta^*_0$). It is found that the width of the region where acoustic perturbations are observed increases with the time. It is also shown that the damping acoustic field behind the shock front is observed not only in the immediate vicinity of the angle $\theta^*_0$, but also in the entire range $\theta^*_0 > \theta_0 \geq \theta^*_0$.

2. Interaction of the shock wave with the density perturbation field. Linear approximation

We consider the collision of two semi-infinite plates one of which is at rest (see Fig. 1). A spatially anisotropic density perturbation field initially exists in the incident plate (impactor). Collision results in the appearance of shock waves diverging in both plates from the contact discontinuity (see Fig. 1). It is assumed that the velocity of the impactor is quite high (about 5 km/s). For this reason, the elastoplastic properties of the colliding plates can be neglected [16] and the interaction of the shock wave with the density perturbation field in the impactor can be described by the equations of hydrodynamics

$$\rho \frac{d\mathbf{v}}{dt} + \nabla P = 0, \quad (1)$$
$$\frac{dp}{dt} + \rho \nabla \mathbf{v} = 0, \quad (2)$$
$$\frac{d\epsilon}{dt} - P \frac{dp}{dt} \rho^2 \frac{d\rho}{dt} = 0. \quad (3)$$

Equation (3) implies that the motion of the medium is isentropic.

The system of Eqs. (1)–(3) should be supplemented by the Rankine–Hugoniot conditions at
the shock front [17]:

\[ v_{1\tau} = v_{2\tau}, \quad v_{1n} - v_{2n} = \sqrt{(P_2 - P_1) \left( \frac{1}{\rho_1} - \frac{1}{\rho_2} \right)}, \]

\[ j^2 = \rho_1 \rho_2 \frac{P_2 - P_1}{\rho_2 - \rho_1}, \quad \varepsilon_1 - \varepsilon_2 = \frac{1}{2} \left( \frac{1}{\rho_2} - \frac{1}{\rho_1} \right) (P_1 + P_2), \]

where \( v_{1\tau} (v_{1n}) \) and \( v_{2\tau} (v_{2n}) \) are the tangential (normal) components of the velocity ahead of and behind the front, respectively; \( j = v_1 \rho_1 = v_2 \rho_2 \) is the mass flux; and \( \varepsilon_1 \) and \( \varepsilon_2 \) are the specific internal energies on both sides of the shock wave.

Following [18], we represent the specific internal energy \( \varepsilon \) in the form of the sum \( \varepsilon = \varepsilon_{el} + \zeta \), where \( \varepsilon_{el} \) is the elastic component of the internal energy and \( \zeta \) is the specific thermal energy. In this case, Eq. (3) can be written in the form

\[ \frac{d\zeta}{dt} - \frac{\Gamma \zeta}{\rho} \frac{dp}{dt} = 0. \]

(6)

The equation of state of the medium is used in the form of the Mie–Grüneisen equation [18]:

\[ P = \frac{\rho_0 c_0^2}{n} \left( \left( \frac{\rho}{\rho_0} \right)^n - 1 \right) + \Gamma \rho \zeta, \]

(7)

where \( \rho_0 \) is the density of the uncompressed (“cold”) medium, \( c_0 \) is the speed of sound in this medium, \( \rho \) is the current density, \( \Gamma \) is the Grüneisen coefficient, and \( n \) is a constant (e.g., for Fe – \( \rho_0 = 7.885 \) g/cm\(^3\), \( c_0 = 3.887 \) km/s, \( n = 4.3 \), \( \Gamma = 1.6 \) and for Al – \( \rho_0 = 2.710 \) g/cm\(^3\), \( c_0 = 5.333 \) km/s, \( n = 3.5 \), \( \Gamma = 2.13 \)).

Let the initial density perturbation field in the impactor be described by the expression

\[ \delta \rho_1 = A \delta_\rho_1 \exp \left( ik_{0x} x + ik_{0y} y \right). \]

(8)

The axis of anisotropy for such perturbation is directed along the vector \( k_0 \) (see Fig. 1). The shock wave moves from the right to the perturbation at the velocity \( (d_1 - v_1 < 0) \). The angle between the vector \( k_0 \) and the \( Ox \) axis is \( \theta_0 \) (\( \tan \theta_0 = k_{0y}/k_{0x} \)). At the interaction of the density perturbation field with the shock wave, acoustic and entropy–vortex waves are formed behind the shock front [1]. In the linear approximation, the system of Eqs. (1)–(3) behind the shock front has the form (see, e.g., [13])

\[ \frac{\partial \delta v_{2x}}{\partial t} + v_2 \frac{\partial \delta v_{2x}}{\partial x} = - \frac{1}{\rho_2} \frac{\partial \delta P_2}{\partial x}, \]

\[ \frac{\partial \delta v_{2y}}{\partial t} + v_2 \frac{\partial \delta v_{2y}}{\partial x} = - \frac{1}{\rho_2} \frac{\partial \delta P_2}{\partial y}, \]

\[ \frac{\partial \delta \rho_2}{\partial t} + v_2 \frac{\partial \delta \rho_2}{\partial x} + \rho_2 \left( \frac{\partial \delta v_{2x}}{\partial x} + \frac{\partial \delta v_{2y}}{\partial y} \right) = 0, \]

\[ \frac{\partial \delta \zeta_2}{\partial t} + v_2 \frac{\partial \delta \zeta_2}{\partial x} - \frac{\Gamma_1 \zeta_2}{\rho_2} \left( \frac{\partial \delta \rho_2}{\partial t} + v_2 \frac{\partial \delta \rho_2}{\partial x} \right) = 0, \]

(9)–(12)

where \( \delta v_2, \delta P_2, \) and \( \delta \zeta_2 \) are the first-order corrections to the initial values of the velocity, pressure, and internal energy, respectively. The system of Eqs. (9)-(12) should be supplemented by the Rankine–Hugoniot conditions written in the linear approximation (4), (5) (details see, e.g., in [13]).
It follows from Eqs. (9)–(12) that the $x$ projection of the wave vector of acoustic waves is given by the expression

$$
(k_x)_\pm = \frac{k_{0x}(d_1 - v_1)(v_2 - d_1) \pm c_2 \sqrt{k_{0y}^2 (v_1 - d_1)^2 - (c_2^2 - (v_2 - d_1)^2) k_{0y}^2}}{c_2^2 - (v_2 - d_1)^2}.
$$

According to Eq. (13), at angles of incidence in the interval

$$
0 < \theta_0 < \theta_0^{**}, \theta_0^{**} = \arctan \left( \frac{v_1 - d_1}{c_2} \right),
$$

the solution $(k_x)_-$ corresponds to the incidence of acoustic waves from the right on the shock front. Sound sources behind the shock front that can be responsible for the appearance of such waves are absent. For this reason, we below consider the acoustic field with the projection of the wave vector $(k_x)_+ \equiv k_x$ and the frequency

$$
\omega = k_x v_2 + c_2 \sqrt{(k_x)^2 + k_{0y}^2}.
$$

If $\theta_0 = \theta_0^{**}$, the projection of the wave vector $k_x$ is equal to zero ($k_x = 0$) and acoustic waves propagate parallel to the shock front.

At large angles of incidence in the interval

$$
\theta_0^* < \theta_0 < \frac{\pi}{2},
$$

the projection of the wave vector $k_x$ has a positive imaginary part and acoustic perturbations are described by surface waves [2].

The most interesting angular range is

$$
\theta_0^{**} < \theta_0 < \theta_0^*, \theta_0^* = \arctan \left( \frac{v_1 - d_1}{\sqrt{c_2^2 - (v_2 - d_1)^2}} \right).
$$

In this range, the projection $k_x$ has a negative real part and zero imaginary part and sound, in contrast to angles of incidence in interval (14), should propagate at the velocity

$$
c_{2x} = v_2 + k_x c_2 / \sqrt{k_x^2 + k_{0y}^2} < 0
$$

towards the shock front [13]. This is inconsistent with the condition of the absence of sound sources behind the shock front. This contradictory can be resolved by introducing a virtual source. The amplitude of the virtual wave at $x = c_{2x} t$ should be equal in absolute value to the amplitude of the actual wave and should damp as approaching the shock front. For the correct introduction of the virtual source into Eqs. (9)–(12), it is necessary to go beyond the linear approximation and to take into account nonlinear interactions between "behind-front" perturbations of hydrodynamic quantities.

In the next section, a solution of the system of Eqs. (1), (2), and (4)-(6) is found in the quadratic approximation when only the interaction between acoustic waves is taken into account.
3. Nonlinear interaction between acoustic waves

3.1. Additional-phase method

Inhomogeneous equations for second-order corrections to hydrodynamic quantities can be obtained from the system of Eqs. (1), (2), and (4)-(6) by standard perturbation theory [14]. In the general case, these equations include three types of sources. The first type corresponds to the interaction between acoustic waves. The second and third types describe the interaction of entropy–vortex waves with each other and with acoustic fields. The analysis shows that the main second-order contribution to the acoustic perturbation field comes from the sound–sound interaction. The equations for this field behind the shock front have the form

\[
\frac{\partial \delta'' v_{2x}}{\partial t} + v_2 \frac{\partial \delta'' v_{2x}}{\partial x} = -\frac{1}{\rho_2} \frac{\partial \delta'' P_2}{\partial x},
\]

(19)

\[
\frac{\partial \delta'' v_{2y}}{\partial t} + v_2 \frac{\partial \delta'' v_{2y}}{\partial x} = -\frac{1}{\rho_2} \frac{\partial \delta'' P_2}{\partial y},
\]

(20)

\[
\frac{\partial \delta'' \rho_2}{\partial t} + v_2 \frac{\partial \delta'' \rho_2}{\partial x} + \rho_2 \left( \frac{\partial \delta'' v_{2x}}{\partial x} + \frac{\partial \delta'' v_{2y}}{\partial y} \right) = -2i(\omega - v_2 k_x) \frac{(\delta \rho_2)^2}{\rho_2},
\]

(21)

\[
\frac{\partial \delta'' \gamma_2}{\partial t} + v_2 \frac{\partial \delta'' \gamma_2}{\partial x} - \frac{\Gamma_1 \gamma_2}{\rho_2} \left( \frac{\partial \delta'' \rho_2}{\partial t} + v_2 \frac{\partial \delta'' \rho_2}{\partial x} \right) = i\Gamma_1 \gamma_2 (\omega - v_2 k_x) (1 - \Gamma_1) \left( \frac{\delta \rho_2}{\rho_2} \right)^2,
\]

(22)

where \(\delta'' v_2, \delta'' \rho_2,\) and \(\delta'' \gamma_2\) are the second-order corrections to the velocity, density, and specific internal energy, respectively. On the right-hand sides of Eqs. (21) and (22), it is assumed that perturbations of hydrodynamic quantities in the linear approximation are plane waves [14].

According to Eqs. (19)-(22), the closed equation for the second-order correction to the pressure \(\delta'' P_2\) has the form

\[
\hat{F}_1 \delta'' P_2 = \psi \left( \frac{\delta \rho_2}{\rho_2} \right)^2,
\]

(23)

where \(\delta \rho_2 = \delta P_2/c_s^2\),

\[
\psi = -4(\omega - k_x v_2)^2 \left[ \frac{n + 1}{2} \frac{\Gamma_2}{2c_s^2} (\Gamma + 1)(\Gamma + 1 - n) \right] \rho_2,
\]

\[
\hat{F}_1 = \frac{1}{c_s^2} \frac{\partial^2}{\partial t^2} - \left( 1 - \left( \frac{v_2}{c_s} \right)^2 \right) \frac{\partial^2}{\partial x^2} + 2 \frac{v_2}{c_s^2} \frac{\partial^2}{\partial x \partial t} - \frac{\partial^2}{\partial y^2}.
\]

The density \(\rho_2\) is indirectly related to the pressure \(P_2\) through the equation of state (7) and the right-hand side of Eq. (23) coincides in order of magnitude with the ratio \((\delta P_2/P_2)^2\).

If, according to standard perturbation theory [14], the solution of Eq. (23) is sought in the form of a plane wave, secular (growing in time) terms appear in \(\delta'' P_2\) [14]. A similar problem arises when standard perturbation theory is applied to nonlinear ordinary differential equations [15]. A method that makes it possible to avoid the appearance of secular terms in such equations has been known for a long time (see, e.g., [15]). We generalize this method for solving Eq. (23).
We represent the perturbation of the pressure behind the shock front \( \Delta P_2 \) in the form of the sum
\[
\Delta P_2 = \delta \tilde{P}_2 + \delta'' \tilde{P}_2. \tag{24}
\]
The first term on the right-hand side of Eq. (24) differs from the first-order correction to the pressure
\[
\delta P_2 = A_\delta P_2 \exp (\varphi(x, y, t)) \quad \varphi(x, y, t) = ik_2 x + ik_0 y - i\omega t \quad \left( \hat{F}_1 \delta P_2 = 0 \right) \tag{25}
\]
in the phase factor
\[
\delta \tilde{P}_2 = \delta P_2 \exp (g(x, y, t)). \tag{26}
\]
The frequency \( \omega \) in Eq. (25) is given by dispersion relation (15) and the amplitude \( A_\delta P_2 \) is determined from the solution of the linearized system of equations of hydrodynamics (1), (2), and (4)–(6) [13]. The quantity \( g \) is chosen such that the function \( \delta \tilde{P}_2 \) satisfies inhomogeneous equation (23); i.e.,
\[
\hat{F}_1 \delta \tilde{P}_2 = \psi (\delta \rho_2 / \rho_2)^2. \tag{27}
\]
The function \( \delta'' \tilde{P}_2 \) in this case satisfies the homogeneous wave equation
\[
\hat{F}_1 \delta'' \tilde{P}_2 = 0, \tag{28}
\]
and secular terms in the second-order correction to the pressure do not appear.

Following [15], we assume that the additional phase \( g \) is much smaller than the quantity \( \varphi \) (26), \( |g| \ll |\varphi| \). As a result, according to Eqs. (26) and (27), the function \( g \) in the first approximation satisfies the linear equation
\[
\left( \hat{F}_1 + \hat{F}_2 \right) g(x, y, t) = \frac{\psi}{\rho_2 c_2^2} \frac{\delta P_2}{\rho_2}, \tag{29}
\]
where
\[
\hat{F}_2 = -2i \left\{ \frac{\omega}{c_2^2} \frac{\partial}{\partial t} + k_x \left( 1 - \left( \frac{v_2}{c_2} \right)^2 \right) \frac{\partial}{\partial x} + \frac{v_2}{c_2} \left( \frac{\omega}{c_2} - k_x \frac{\partial}{\partial t} \right) + k_0 y \frac{\partial}{\partial y} \right\}. 
\]
The boundary condition for Eq. (29) at the shock front has the form
\[
g(d_1 t, y, t) = 0. \tag{30}
\]
To determine the second-order corrections to the velocity, density, and specific energy (as well as the quantity \( \delta'' \tilde{P}_2 \)), the system of Eqs. (19)–(22) should be supplemented by equations taking into account the interaction of entropy–vortex waves with each other and with acoustic waves. The solutions of these inhomogeneous equations do not contain secular terms. For this reason, the desired hydrodynamic quantities can be calculated within standard perturbation theory [14].

3.2. Additional phase near the threshold of appearance of surface waves
We now study the spatial structure of the pressure perturbation field behind the shock front near the threshold of appearance of surface waves, \( \theta_0 - \theta_0 \ll \theta_0^s \). This range is of particular
interest because the projection of the speed of sound \( c_{2x} \) at \( \theta'_0 - \theta_0 \ll \theta'_0 \) is close to the velocity of the shock wave

\[
c_{2x} = d_1 + \frac{\delta \kappa c_2 k_{0y}}{(\kappa^2 + k_{0y}^2)^{3/2}},
\]

where

\[
\kappa \equiv k_x(\theta'_0) = \frac{(d_1 - v_2) k_{0y}}{\sqrt{c_2^2 - (d_1 - v_2)^2}},
\]

\[
\delta \kappa = k_x - \kappa = \frac{c_2^2 (v_1 - d_1) k_{0y}}{(c_2^2 - (v_2 - d_1)^2)^{3/2}} \sqrt{2 \cot \theta'_0 (\theta'_0 - \theta_0)}.
\]

In Eqs. (31) and (32), it is assumed that the angle \( \theta_0 \) varies only because of an increase in the \( x \) projection of the initial wave vector \( k_{ax} \) (\( k_{0y} = \text{const} \)).

According (31), for the angles \( \theta_0 \) in the immediate vicinity of the angle \( \theta'_0 \), the front of the acoustic wave \( x = c_{2x} t \) almost coincides with the shock front, \(|d_1 - c_{2x}| t \ll |d_1| t\).

We introduce the function

\[
f(x, y, t) = \nu(x - d_1 t) + \left( \eta (x - d_1 t) + \beta (x - d_1 t)^2 \right) \delta P_2.
\]

The action of the operator \( \hat{F}_1 + \hat{F}_2 \) on the function \( f \) (33) gives the result

\[
(\hat{F}_1 + \hat{F}_2) f = \frac{2 c_2^2}{c_2^2} \left[ c_2^2 - (d_1 - v_2)^2 \right] (i \nu \delta \kappa - \delta P_2 [2i \eta \delta \kappa - \beta (1 + i \delta \kappa (x - d_1 t))])
\]

We assume that the quantity \( \nu \) is small,

\[
|\nu| \ll |\kappa|,
\]

and the coefficients \( \eta \) and \( \beta \) are given by the expressions

\[
\eta = 0, \quad \beta = -\frac{\psi}{2 \nu_2^2 c_2^2 |c_2^2 - (d_1 - v_2)^2|}.
\]

In this case, the function \( f \) in neglect of quantities of the order of \(|\nu \delta \kappa/\kappa^2| \ll 1\) satisfies Eq. (29) with boundary condition (30) and coincides with the function \( g \), \( f \approx g \). The condition of applicability of the perturbation theory

\[
|g| \ll |\varphi|.
\]

is also satisfied.

Thus, the additional phase \( g \) in the angular range \( \theta'_0 - \theta_0 \ll \theta'_0 \) can be approximated by the expression

\[
g \approx \nu(x - d_1 t) + \beta (x - d_1 t)^2 \delta P_2.
\]

The coefficient \( \nu \) is generally complex:

\[
\nu = \nu_1 + i \nu_2.
\]

The real and imaginary parts of \( \nu \) should be negative (\( \nu_1 < 0 \)) and positive (\( \nu_2 > 0 \)), respectively. The first condition is due to the requirement of damping of the acoustic wave behind the shock
front. The second condition appears because the absolute value of the projection of the speed of sound $c_{2x}$ at $\theta_0 = \theta_0^*$

$$c_{2x}^* = d_1 + \frac{\nu_2 c_2 k_{0y}^2}{(\kappa^2 + k_{0y}^2)^{3/2}}$$

(40)
cannot exceed the velocity of the shock wave ($d_1 < 0$).

The quantities $\nu_1$ and $\nu_2$ depend on the conditions of appearance of the shock wave (see below) and cannot be determined in the used approximation.

At relatively small times

$$t \ll \left( \frac{1}{\kappa c_2} \right) \left( \frac{\rho_2}{\delta \rho_2} \right),$$

(41)
the front of the acoustic perturbation is weakly behind the shock wave and the second term in Eq. (38) can be neglected. As a result, the additional phase is given by the simple formula

$$g \approx (\nu_1 + i\nu_2)(x - d_1 t).$$

(42)

Formula (42) is the main result of this section. According to this formula, at angles of incidence $\theta_0^* - \theta_0 \ll \theta_0^*$ of the shock wave on the anisotropic density perturbation field, the acoustic field following the shock wave is formed behind the shock front because of the nonlinear sound–sound interaction. The amplitude of this field decreases with an increase in the distance from the front $x = d_1 t$ (Re $g = -|\nu_1|(x - d_1 t) < 0$).

4. Results of the numerical calculations

Although formula (42) for the additional phase $g$ was obtained under the assumption of the smallness of the amplitude of initial-density perturbation (see (41)), it indicates the behavior of the acoustic field for perturbations of a finite amplitude [14]. In order to confirm the conclusion made in the preceding section concerning the existence of the damping acoustic field behind the shock front, the equations of hydrodynamics (1)-(5) were numerically solved for the case of collision of the iron (impactor) and aluminum (target) plates (see Fig. 1). Figure 2 shows the numerically calculated spatial profiles of the perturbation of the pressure behind the shock front for the angles $\theta_0$ in the range $\theta_0^* < \theta_0 \leq \theta_0^*$ ($\theta_0^* = 0.83\theta_0^*$) (the calculations were performed with the code from [19]). According to Fig. 2, the decay of the acoustic field behind the shock front is observed not only at $\theta_0 = \theta_0^*$, but also at smaller angles of incidence. The width of the region of acoustic perturbations increases with the time.

The damping ratio depends on time because Eq. (42) is valid at times $t \ll 300 \text{ ns}$ and, therefore, is inapplicable for the times $t = 250$ and $750 \text{ ns}$ used in the numerical calculation. However, using the results of the preceding section, we can write an approximate formula providing a qualitatively appropriate description of the time dependence of the quantity $\nu_1(t)$. This formula can be most simply obtained for the angles $\theta_0 < \theta_0^*$ (Figs. 4 c – 4 f). In this case, $\nu_1(t)$ can be estimated as

$$|\nu_1| \simeq \frac{1}{|d_1 - c_{2x}| t}.$$  

(43)

In view of Eq. (31), Eq. (43) can be represented in the form

$$|\nu_1| \simeq \frac{\left( \kappa^2 + k_{0y}^2 \right)^{3/2}}{\delta \kappa c_2 k_{0y}^2 \nu_2 t}.$$  

(44)

According to the table ($\theta_0 = 53^\circ$, $\delta \kappa = 15.2 \text{ mm}^{-1}$; $\theta_0 = 48^\circ$, $\delta \kappa = 39.7 \text{ mm}^{-1}$), the results of calculations of the damping ratio by Eq. (44) are in qualitative agreement with the data obtained from the analysis of the numerical solution. The best agreement is observed at $\theta_0 = 0.89\theta_0^*$. 


Figure 2. Perturbation of the pressure $\delta P_2$ versus the distance from the shock front in the impactor at (a) $\theta_0 = \theta_0^*$, $t = 250$ ns; (b) $\theta_0 = \theta_0^*$, $t = 750$ ns; (c) $\theta_0 = 0.98\theta^*$, $t = 250$ ns; (d) $\theta_0 = 0.98\theta_0^*$, $t = 750$ ns; (e) $\theta_0 = 0.89\theta_0^*$, $t = 250$ ns; and (f) $\theta_0 = 0.89\theta_0^*$, $t = 750$ ns. The velocity of the impactor before the collision is $v_1 = 5$ km/s, the amplitude of the initial perturbation of the density is 5% of the initial density, $\theta_0^* = 53.9^\circ$, and $\theta_0^{**} = 44.5^\circ$. The minimum value of the coordinate $x$ corresponds to the position of the shock front.

The damping ratio was calculated by Eq. (44) with the value $c_2 = 6.44$ km/s for the speed of sound behind the front of the acoustic wave. This value was obtained from the numerical solution of Eqs. (1)–(3) and (7). The projection of the wave vector is $k_0y = 51.1mm^{-1}$.

At $\theta_0 = \theta_0^* = 53.9^\circ$, the speed of sound becomes equal to the velocity of the shock wave and estimate (43) is inapplicable. However, the analysis of the results of the numerical calculations shows that the product $t|\nu_1(t)|$ weakly depends on time (see table), $t|\nu_1(t)| \approx \text{const}$, and the
damping ratio decreases with an increase in the time $t$ as $|\nu_1| \sim 1/t$.

5. Conclusions

The interaction of the shock wave with the anisotropic density perturbation field has been studied in the quadratic approximation. To solve inhomogeneous equations appearing in this approximation, a new analytical method has been proposed to avoid the appearance of secular (growing in time) terms. This method generalizes the approach used in the theory of nonlinear ordinary differential equations [15] and involves the introduction of an additional phase into a solution obtained in the linear approximation. Using this method, we have shown that a damping acoustic field, which is not described by surface waves, appears behind the shock front incident at certain angles on the spatially anisotropic density perturbation field. The width of the region where acoustic perturbations are observed increases with the time. The problem of the propagation of the shock wave in the incident sample, where the spatially anisotropic density perturbation field initially exists, has been numerically solved in application to the collision of two plates. The results of the numerical solution of the equations of hydrodynamics confirm qualitative conclusions made in the quadratic approximation concerning the spatial structure of the acoustic field behind the shock front.

Acknowledgments

We are grateful to P.P. Zakharov for assistance in the numerical calculations.

References

[1] Diakov S P 1954 Zhur. Exp. i Teor. Fiz. 27 288 [in Russian]
[2] Kontorovich V M 1959 Sov. Phys. Acoustics 5 320
[3] Kuznetsov N M 1989 Sov. Phys. Usp. 159 993
[4] Keller J and Merzkirch W 1990 Experiments in Fluids 8 241
[5] Yang Y, Zhang Q and Sharp D H 1994 Phys. Fluids 6 1856
[6] Wouchuk J G, Huete Ruiz de Lira C and Velikovich A L 2009 Phys. Rev. E 79 066315
[7] Wouchuk J G 2001 Phys. Rev. E. 63 056303
[8] Wouchuk J G, Huete Ruiz de Lira C and Velikovich A L 2009 Phys. Rev. E 79 066315
[9] Huete Ruiz de Lira C, Velikovich A L and Wouchuk J G 2011 Phys. Rev. E 83 056320
[10] Nishihara K, Wouchuk J G, Matsuoka C, Ishizaki R and Zhakhovsky V V 2010 Phil. Trans. R. Soc. A 368 1769
[11] Huete Ruiz de Lira C, Wouchuk J G and Velikovich A L 2012 Phys. Rev. E 85 026312
[12] Huete Ruiz de Lira C, Wouchuk J G, Canaud B and Velikovich A L 2012 Journal of Fluid Mechanics 700 214
[13] Gorodnicehev K E and Kuratov S E 2013 VANT ser. Mat. Mod. Fiz. Protos. 2 37 [in Russian]
[14] Isakovich M A 1973 General Acoustics (Moscow: Nauka) [in Russian]
[15] Naife A H 2004 Perturbation Methods (Weinheim: WILEY-VCH Verlag GmbH and CO. KGaA)
[16] Zel’dovich Ya B, Raizer Yu P 2002 Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena (New York: Dover Publication Inc. Mineola)
[17] Landau L D and Lifshitz E M 1987 Fluid Mechanics (New York: Pergamon)
[18] Zababakhin E I 1997 Some issues of gasdynamics of explosion (Snezhinsk) [in Russian]
[19] Menshov I S and Zakharov P P 2014 Int.j. for Num. Meth. in Fluids 76 109