Quantum superconductor–insulator transition: implications of BKT critical behavior

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Abstract

We explore the implications of Berezinskii–Kosterlitz–Thouless (BKT) critical behavior on the two-dimensional (2D) quantum superconductor–insulator (QSI) transition driven by the tuning parameter $x$. Concentrating on the sheet resistance $R(x, T)$ BKT behavior implies: an explicit quantum scaling function for $R(x, T)$ along the superconducting branch ending at the nonuniversal critical value $R_c = R(x_c)$; a BKT-transition line $T_c(x) \propto (x - x_c)^{\nu}$, where $\nu$ is the dynamic exponent and $\nu$ the exponent of the zero-temperature correlation length; independent estimates of $\nu$, $z$, and $\nu$ from the $x$ dependence of the nonuniversal parameters entering the BKT expression for the sheet resistance. To illustrate the potential and the implications of this scenario we analyze the data of Bollinger et al (2011 Nature 472 458) taken on gate voltage tuned epitaxial films of La$_2$-xSr$_x$CuO$_4$ that are one unit cell in thickness. The resulting estimates, $\nu \simeq 3.1$ and $\nu \simeq 0.52$, indicate a clean 2D-QSI critical point where hyperscaling, the proportionality between $d/\lambda^z(0)$ and $T_c$, and the correspondence between the quantum phase transitions in $D$ dimensions and the classical ones in $(D + z)$ dimensions are violated. (Some figures may appear in colour only in the online journal)

1. Introduction

A variety of different materials undergo a quantum superconductor–insulator (QSI) transition in the limit of two dimensions (2D) and zero temperature by variation of a tuning parameter such as film thickness, disorder, applied magnetic field, and gate voltage [1–4]. A widespread observable to study this behavior is the temperature dependence of the sheet resistance $R(x, T)$ taken at various values of the tuning parameter $x$. The curves of $R(T)$ at a fixed temperature independent separatrix between the superconducting and insulating phase with a sheet resistance $R_c = R(x = x_c, T \approx 0)$. This behavior implies a crossing point of the isotherms $R(x)$ at different temperatures at $x_c$, which is a characteristic feature of a quantum phase transition and in the present case of a QSI transition. Traditionally, the interpretation of experimental data taken close to the 2D-QSI transition is based on the quantum scaling relation $R(x, T) = R_c G(y)$ with $y = c(x - x_c)/T^{1/\nu}$ [5–8], $\nu$ is the dynamic exponent and $\nu$ the critical exponent of the zero-temperature correlation length and $c$ a nonuniversal coefficient of proportionality [1–3]. Given sheet resistance data, fits to this scaling form yield estimates for the critical value of the tuning parameter $x_c$ and the exponent product $\nu \nu$, properties which are insufficient to distinguish different models, to fix the universality class to which the QSI transition belongs, or to clarify the relevance of disorder.

The nature of the 2D-QSI transition has been intensely debated [1–4]. The scenarios can be grouped into two classes, fermionic and bosonic. In the fermionic case the reduction of $T_c$ and the magnitude of the order parameter is attributed to a combination of a reduced density of states, enhanced Coulomb interaction and depairing due to an increase of the inelastic electron–electron scattering rate [9, 10]. The bosonic approach assumes that the fermionic degrees of freedom can be integrated out, the mean square of the order parameter does
not vanish at \( T_c \), phase fluctuations dominate and the reduction of \( T_c \) is attributable to quantum fluctuations (and in disordered systems to randomness in addition) [6, 7, 11]. This scenario is closely related to the suppression of ferroelectricity [12], e.g. in \( \text{SrTiO}_3 \) [13].

Here we adopt the bosonic scenario and concentrate on systems that undergo a normal state to superconductor transition at finite temperature with Berezinskii–Kosterlitz–Thouless (BKT) critical behavior [14, 15], originally derived for the 2D \( xy \) model with a two-component order parameter and short-range interactions. BKT behavior was also observed in two-dimensional bosonic mixtures of ultracold atoms [16]. The occurrence of BKT criticality in 2D superconductors also implies that the mean square of the order parameter does not vanish at \( T_c \) and that there are, in analogy to \( ^4\text{He} \), condensed pairs (bosons) below and uncondensed pairs above \( T_c \). In \( ^4\text{He} \) and superconductors the order parameter is a complex scalar corresponding to the components in the \( xy \) model. Supposing that in superconductors the interaction of Cooper pairs is short ranged and their effective charge is sufficiently small, the critical properties at finite temperature are then those of the 3D-\( xy \) (bulk) and 2D-\( xy \) (thin films) models [17, 18], reminiscent of the lambda transition in bulk \( ^4\text{He} \) and the BKT transition in thin \( ^4\text{He} \) films [20–22, 24]. In this context it is important to recognize that the existence of the BKT transition (vortex–antivortex dissociation instability) in \( ^4\text{He} \) films is intimately connected with the fact that the interaction energy between vortex pairs depends logarithmically on the separation between them. As shown by Pearl [25], vortex pairs in thin superconducting films (charged superfluid) have a logarithmic interaction energy out to the characteristic length \( \lambda_{2D} = \lambda^2/d \), beyond which the interaction energy falls off as \( 1/R \). Here \( \lambda \) is the magnetic penetration depth and \( d \) the film thickness. As \( \lambda_{2D} \) increases on approaching \( T_c \), the diamagnetism of the superconductor becomes less important and the vortices in a clean and thin superconducting film become progressively like those in \( ^4\text{He} \) films [26, 27].

The occurrence of a 2D-QSI transition implies a line \( T_c(x) \) of BKT-transition temperatures ending at the quantum critical point at \( x = x_c \), where \( T_c(x = x_c) = 0 \). It separates the superconducting from the insulating ground state. The BKT transition is rather special because the correlation length diverges above \( T_c \) as \( \xi(T) \propto \exp((b_R(x)/2\lambda_{2D}^2)^{1/2}(T/T_c(x) - 1)^{-1/2}) \) and the sheet resistance tends to zero according to \( R(x, T) = R_0(x) \exp(-((b_R(x)/\lambda_{2D}^2)^{1/2}(T/T_c(x) - 1)^{-1/2}) \) [26–28]. Approaching the 2D-QSI transition, quantum phase fluctuations renormalize \( R_0(x) \), \( b_R(x) \), and \( T_c(x) \). Indeed the BKT-transition line approaches the 2D-QSI transition as \( T_c(x = x_c) \propto (c(x - x_c)/\xi_c)^z \) because the quantum scaling form \( G(y) \) exhibits a finite-temperature singularity at a universal value \( \xi_c \) of the scaling argument [6, 8, 17]. Noting that the amplitude of the BKT correlation length \( \xi_0(x) \) should match the divergence of its quantum counterpart, \( \xi(T = 0) \propto (x - x_c)^{-\tau} \propto T_c^{-1/\nu}(x) \), the exponents \( z \) and \( \tau \) should emerge from the amplitude \( R_0(x) \) in terms of \( R_0(x) = R_c \propto \xi_0^{-2}(x) \propto \xi^{-2}(T = 0) \propto (x - x_c)^{2\tau} \propto T_c^{2/\nu} \) [28]. Accordingly, the nonuniversal functions \( T_c(x) \) and \( R_0(x) \) entering the BKT expression for the sheet resistance close to the QSI transition exhibit quantum critical properties disclosing the quantum critical exponents \( z \), \( \tau \), and \( \nu \). The exponents \( z \) and \( \tau \) are characteristic properties of the universality class to which the QSI transition belongs. In addition, given their values, the relevance of disorder and the equivalence between quantum phase transitions in systems with \( D \) spatial dimensions and those of classical phase transitions in \( (D + z) \) dimensions can be checked [5, 6, 17]. The occurrence of a BKT-transition line also implies: a nonuniversal critical sheet resistance \( R_c = R_0(x_c) \) because \( R_0(x) \) is nonuniversal; an explicit form of the superconductor branch of the quantum scaling function \( G(c(x - x_c)/T_c^{1/\nu}) \).

Even though BKT critical behavior is not affected by short-range correlated and uncorrelated disorder [29, 30] the observation of this behavior requires sufficiently homogeneous films and a tuning parameter which does not affect the disorder. Noting that sample inhomogeneity and vortex pinning are relevant in thickness and perpendicular magnetic field tuned transitions, electrostatic tuning of the 2D-QSI transition using the electric field effect appears to be more promising [28, 31–34]. Indeed electrostatic tuning is not expected to alter physical or chemical disorder, but changes the mobile carrier density.

In section 2 we sketch the theoretical background. To illustrate the potential and the implications of finite-temperature BKT criticality on the 2D-QSI transition we analyze in section 3 the sheet resistance data of Bollinger et al [32] taken on gate voltage tuned epitaxial films of \( \text{La}_{2-x}\text{Sr}_x\text{CuO}_4 \) that are one unit cell in thickness. To detect that the BKT regime is attained we perform a finite size scaling analysis, revealing that electrostatic tuning changes not only the carrier density but also the inhomogeneity landscape. Commenting on the difficulties in observing the BKT features in the magnetic penetration depth we consider the data of Bert et al [34] taken on the superconducting \( \text{LaAlO}_3/\text{SrTiO}_3 \) interface.

## 2. Theoretical background

Continuous quantum phase transitions (QPT) are transitions at zero temperature in which the ground state of a system is changed by varying a parameter of the Hamiltonian [5, 17, 18]. The quantum superconductor–insulator transitions (QSI) in two-dimensional (2D) systems tuned by disorder, film thickness, magnetic field or with the electrostatic field effect are believed to be such transitions [1–3, 17, 18, 28, 32]. Traditionally, the interpretation of experimental data taken close to the 2D-QSI transition is based on the quantum scaling relation [5–8],

\[
\frac{R(x, T)}{R_c} = G(y), \quad y = c \frac{|x - x_c|}{T^{1/\nu}} \tag{1}
\]

where \( R \) is the resistance per square, \( R_c \) the limiting \( x \to x_c \) and \( T \to 0 \) resistance, \( c \) the tuning parameter and \( c \) a nonuniversal coefficient of proportionality. \( G(y) \) is a universal scaling function of its argument such that \( G(y = 0) = 1 \). In
addition, $z$ is the dynamic exponent and $\nu$ the critical exponent of the correlation length, supposed to diverge as
$$
\xi(T = 0) = \xi_0 |x - x_c|^{-\nu}.
$$
(2)
The critical sheet resistance $R_c$ separating the superconducting and insulating ground states is determined from the isothermal sheet resistance at the crossing point in $R(T)$ versus the tuning parameter $x$ at $x_c$. The existence of such a crossing point, remaining temperature dependence in the zero-temperature limit, is the signature of a QPT. The data for $R(x, T)$ plotted versus $|x - x_c|/T^{1/\nu}$ should then collapse onto two branches joining at $R_c$. The lower branch stems from the superconducting ($x - x_c > 0$) and the upper one from the insulating phase ($x - x_c < 0$). This scaling form follows by noting that the divergence of the zero-temperature correlation length, $\xi(T = 0) = \xi_0 |x - x_c|^{-\nu}$, is at finite temperature limited by the length $L_T \propto T^{-1/\nu}$ [5]. Thus $G(y)$ is a finite size scaling function because $y \propto [T/T_c(T = 0)]^{1/\nu} \propto |x - x_c|/T^{1/\nu}$. Supposing that there is a line of finite-temperature phase transitions $T_c(x)$ ending at the quantum critical point $T_c(x = x_c) = 0$, the quantum scaling form (1) exhibits a finite-temperature singularity at the universal value $\nu_y$ of the scaling [6, 8, 17]. The phase transition line is then fixed by
$$
T_c = \left(\frac{c|x - x_c|}{\nu_y}\right)^{1/\nu}.
$$
(3)
Otherwise one expects that sufficiently homogeneous 2D superconductors at the superconductor to normal state transition exhibit BKT critical behavior [28, 14, 15]. Note that the Harris criterion [29] states that short-range correlated and uncorrelated disorder is irrelevant at the unperturbed critical transition. With the system and uncorrelated disorder is irrelevant at the unperturbed critical transition, the sheet resistance scales for $T \gtrsim T_c(x) \geq 0$ as [26–28]
$$
\frac{R(x, T)}{R_0(x)} = \exp\left(-\frac{b_R(x)}{T_c^{1/2}(x)(T/T_c(x) - 1)^{1/2}}\right),
$$
(4)
allowing one to probe the characteristic BKT correlation length [14, 15, 27, 28]
$$
\frac{\xi(x, T)}{\xi_0(x)} = \exp\left(-\frac{b_R(x)}{2T_c^{1/2}(x)(T/T_c(x) - 1)^{1/2}}\right).
$$
(5)
While $R_0(x)$, $b_R(x)$, and $T_c(x)$ depend on the tuning parameter and are subject to quantum fluctuations the characteristic BKT form of the correlation length and that of the sheet resistance applies for any $T \gtrsim T_c(x) \geq 0$. Through standard arguments (see, e.g., [35]) quantum mechanics does not modify universal finite temperature properties. $b_R(x)$ is given by [15, 28]
$$
b_R(x) = \tilde{b}_R T_c^{-1/2},
$$
(6)
with $\tilde{b}_R = 4\pi/b$. The parameter $b$ is expected to remain constant in the low-$T_c$ regime [28]. It is related to the vortex core energy $E_c$ in terms of $b = f(E_c/k_B T_c)$ [36] and below the Nelson–Kosterlitz jump controls the temperature dependence of the magnetic penetration depth in terms of $(\lambda(T_c)/\lambda(T))^2 = 1 + (b/4)(T/T_c - 1)^{1/2}$ [37]. The amplitude of the BKT correlation length $\xi_0(x)$ is proportional to the vortex core radius [20], known to increase with reduced $T_c$ [21, 24]. Indeed, the zero-temperature correlation length $\xi(T = 0)$ diverges as $\xi(T = 0) \propto (x - x_c)^{-\nu}$ (equation (2)) and combined with $T_c \propto (x - x_c)^{-\nu}$ (equation (3)) we obtain $\xi(T = 0) \propto (x - x_c)^{-\nu} T_c^{-1/\nu}$. Noting that $R_0(x)$ approaches $R_c$ from above, the scaling relation
$$
R_0(x) - R_c \propto \xi^{-2}(T = 0) \propto (x - x_c)^{2\nu} T_c^{2/\nu},
$$
(7)
should apply [28], making the determination of the exponents $\nu$ and $z$ possible. Consistency requires that the resulting $\xi^{2\nu}$ agrees with the estimate obtained from the critical BKT line $T_c(x) \propto (x - x_c)^{-\nu}$. Other implications concern the universality class of the 2D-QSI transition and the relevance of disorder. Given estimates for $\nu$ and $\nu_y$ the equivalence between quantum phase transitions in clean systems with $D$ spatial dimensions and those of classical phase transitions in $(D + 2)$ dimensions can be checked. The fate of a clean critical point under the influence of disorder is controlled by the Harris criterion [29]: if the zero-temperature correlation length critical exponent in the absence of disorder fulfils the Harris inequality $\nu \geq 2/D$ the disorder does not affect the critical behavior. Conversely, if the Harris criterion is violated then $\nu < 2/D$ and disorder is present the generic result is a new critical point with $\nu \geq 2D$, as shown by Chayes et al. [30]. Finally, a BKT-transition line expression (4) the sheet resistance given by equation (5) transforms with equation (6) and the scaling variable $y = c|x - x_c|/T^{1/\nu}$ to
$$
\ln\left(\frac{R(x, T)}{R_0(x)}\right) = -\tilde{b}_R\left(\left(\frac{y_c}{y}\right)^{2\nu} - 1\right)^{-1/2},
$$
(8)
valid for $y \leq y_c$. Close to quantum criticality, where $R_0(x) \simeq R_c$, this reduces to
$$
\ln\left(\frac{R(x, T)}{R_c}\right) = -\tilde{b}_R\left(\left(\frac{y_c}{y}\right)^{\nu} - 1\right)^{-1/2} = \ln(G(y)).
$$
(9)
These relations are explicit forms of the quantum scaling function $G(y)$ applicable to the superconductor branch. They reveal that the critical sheet resistance $R_0(x \rightarrow x_c) = R_c$ is the endpoint of a nonuniversal function and accordingly nonuniversal. Another implication concerns the universality class of the 2D-QSI transition. Supposing that the equivalence between quantum phase transitions in clean systems with $D$ spatial dimensions and those of classical phase transitions in $(D + 2)$ dimensions applies, the 2D-QSI transition at the endpoint of a BKT line $T_c(x)$ should belong to the finite-temperature $(2 + 2) - xy$ universality class. $xy$ denotes an order parameter with two components, including the complex scalar, $\Psi = |\Psi|\exp(i\phi)$, of a superconductor [5, 17].

The BKT theory of thermally excited vortex–antivortex pairs also predicts a superconductor-to-normal state phase
transition marked by the Nelson–Kosterlitz jump, a discontinuous drop in superfluid density from \[ T_c \] of \[ 0.3 \text{ to } 1 \text{ K} \] by Crowell et al. taken on epitaxial films of La\(_4\)Sr\(_4\)CuO\(_4\) using calorimetry techniques. The investigation focused on the onset of the superfluid–insulator transition marked by the Nelson–Kosterlitz jump, a discontinuous drop in superfluid density from \[ \rho_s \text{ at } T = 0 \text{ K} \text{ to } 0 \text{ K} \text{ by Crowell et al.} \]

\[ \frac{d}{\lambda^2 (T_c)} = \frac{32\pi^2 k_B T_c}{\Phi_0^2} \approx 1.02 T_c, \]  

(10)

to zero. The numerical relationship applies for \[ d/\lambda^2(T_c) \] in cm\(^{-1}\) and \( T_c \) in K. \( d \) denotes the thickness of the 2D system, \( \lambda \) the in-plane magnetic penetration depth, and \( \Phi_0 = hc/2e \). In addition there is the prediction that \[ d/\lambda^2(T = 0) \text{, a measure of the phase stiffness, scales near the endpoint of the BKT-transition line as} [6, 8, 17, 18] \]

\[ \frac{d}{\lambda^2 (T = 0)} = \frac{16\pi^3 k_B T_c}{\Phi_0^2} Q_2 \approx 1.6 Q_2 T_c, \]  

(11)

provided that \( D + z \) is below the upper critical dimension \( D_u \) where hyperscaling holds. Since \( D_u = 4 \) in the D-x-model, the validity of equation (11) is in \( D = 2 \) restricted to \( z < 2 \). Relation (11) is the quantum counterpart of the Nelson–Kosterlitz relation (8). \( Q_2 \) is a dimensionless critical amplitude with the lower bound \[ Q_2 \geq 2/\pi, \]  

dictated by the characteristic temperature dependence of \( d/\lambda^2(T) \) below the Nelson–Kosterlitz jump (equation (10)) [17, 18]. Combining equations (10) and (11) we obtain

\[ \left( \frac{\lambda(T = 0)}{\lambda(T_c)} \right)^2 = \frac{\rho_s(T = 0)}{\rho_s(T_c)} = \frac{\pi}{2} Q_2, \]  

(13)

where \( \rho_s \) is the superfluid density. The superfluid transition temperature \( T_c \) as a function of the superfluid density \( \rho_s(T = 0) \) has been measured in \( ^4 \)He films for transition temperatures ranging from 0.3 to 1 K by Crowell et al [22]. They studied \( ^4 \)He films adsorbed in two porous glasses, aerogel and Vycor, using high-precision torsional oscillator and dc calorimetry techniques. The investigation focused on the onset of superfluidity at low temperatures as the \( ^4 \)He coverage is increased. Their data yields \( \rho_s(T = 0) \approx 15.3 T_c \) with \( \rho_s \) in \( \mu \text{mol m}^{-2} \) and \( T_c \) in K. Combined with the BKT-transition line \( \rho_s(T_c = 0) = 8.73 T_c \) we obtain \( Q_2 \geq 1.12 \). In this context it should be kept in mind that in 1D the \( T = 0 \) superconductor–insulator transition has been studied in terms of a model of electrons with a BCS attractive interaction moving in a random potential [23]. It was found that the critical behavior of this \( T = 0 \) phase transition is in the same universality class as the superfluid–insulator transition in a model of repulsively interacting bosons, corresponding to the Cooper pairs, moving in a random potential. Known to be true precisely in \( D = 1 \) it should remain true in its neighborhood as well, and therefore near the superconductor–insulator transition in thin films.

3. Comparison with experiment

To illustrate the potential and the implications of the outlined BKT scenario we analyze next the data of Bollinger et al [32] taken on epitaxial films of La\(_{2-x}\)Sr\(_x\)CuO\(_4\) that are one unit cell in thickness. Very large electric fields and the associated changes in surface carrier density enabled shifts in the midpoint transition temperature \( T_e \) by up to 30 K. Hundreds of resistivity measurements were recorded and shown to collapse onto a single function, as the quantum scaling form (equation (1)) for a 2D-QSI transition predicts. The observed critical resistance is close to the quantum resistance for pairs, \( R_Q = h/4e^2 \approx 6.45 \text{ k}\Omega \). Our starting point is the temperature dependence of the sheet resistance taken at various gate voltages \( V_g \) where a BKT transition is expected to occur. As an example we depict in figure 1(a) the data for \( V_g = -1.5 \text{ V} \). The observation of BKT behavior requires that the data extend considerably below the midpoint transition temperature \( T_{c0} \). We estimated it with the aid of the Aslamasov–Larkin (AL) expression [38] for the conductivity, \( \sigma(T, V_g) = \sigma_0(V_g) + \sigma_0(T/T_{c0} - 1) \), with \( \sigma_0 = \pi e^2/8h \approx 1.52 \times 10^{-5} \Omega^{-1} \), where Gaussian fluctuations are taken into account and \( T_{c0} \) is the mean-field transition temperature. The resulting temperature dependence is included in figure 1(a). It clearly reveals that the data extend considerably below \( T_{c0} \approx 12.5 \text{ K} \). To establish and characterize BKT behavior below \( T_{c0} \) we invoke equation (4) and

\[ \left( \frac{d \ln R}{d T} \right)^{-2/3} = \left( \frac{2}{3} \right) \left( T - T_c(x) \right) \].  

(14)

As indicated in figure 1(b) this relation is used to fix \( h_B(T_c) \) and \( T_c(x) \) while \( R_0(x) \) is estimated by adjusting equation (4) with the given \( h_B(T_c) \) and \( T_c(x) \) to the sheet resistance data. Comparing the data with the respective lines we observe that the BKT regime is attained and that at the BKT \( T_c \) is almost an order of magnitude lower than its mean-field counterpart. This uncovers a BKT transition from uncondensed to condensed Cooper pairs driven by strong phase fluctuations. On the other hand, it is important to recognize that agreement with BKT criticality is established in a temperature window only. Its upper bound reflects the crossover from BKT- to AL-behavior while the lower bound stems from the rounded BKT transition. Precursors of this phenomenon are clearly visible in figure 1 around 7 K. Here the correlation length is prevented from growing beyond a limiting length \( L \), i.e. the linear extent of the homogeneous domains. As a result, a finite size effect and with that a rounded transition occurs [28]. Because the BKT correlation length does not exhibit the usual power law divergence of the correlation length as \( T_c \) is approached, it is particularly susceptible to such a finite size effect. For this reason we invoke a finite scaling analysis [39]. Supposing that the correlation length cannot grow beyond the limiting length \( L \), the sheet resistance adopts the finite size scaling form

\[ R(T, L) = R(T, L = \infty) f(x), \]  

(15)

where \( R(T, L) \) is the measured sheet resistance, \( R(T, L = \infty) \) the sheet resistance of the homogeneous system (equation (4)), and \( f(x) \) denotes the finite size scaling function, where

\[ x = \frac{g(L)}{R(T, L = \infty)} \propto g(L) \xi^2 (T, L = \infty). \]  

(16)
Figure 1. (a) Sheet resistance $R(T)$ for various $V_g$ taken from [32]. The dashed curve is a fit to the Aslamosov–Larkin expression, yielding $T_\nu \approx 12.5$ K. The other lines mark the BKT behavior according to equation (4), with the $R_0$, $T_c$, and $b_k$ values shown in figures 2 and 3. (b) $(\mathrm{d} \ln(R)/\mathrm{d} T)^{-2/3}$ versus $T$ derived from the data shown in figure 1(a). The lines are (equation (13)) with the $T_c$ and $b_k$ values shown in figures 2 and 3. (c) $R(T, L)/R(T, L = \infty)$ versus $1/R(T, L = \infty)$. The solid line is $R(T, L)/R(T, L = \infty) = 1$ and the other lines are $R(T, L) = g(L)$, characterizing the finite size dominated regime. The inset shows $1/g(L)$ versus $V_g$.

Its limiting behavior is

$$f(x) = \begin{cases} 1: x \to 0; \xi(T, L = \infty) < L \\ x: x \to \infty; \xi(T, L = \infty) > L. \end{cases} \quad (17)$$

BKT behavior occurs as long as $f(x) \approx 1$, while for $\xi(T, \infty) > L$ the scaling function approaches $f(x) = x$, and thus $R(T, L)$ saturates because $R(T, L) = g(L)$. Figure 1(c) shows $R(T, L)/R(T, L = \infty)$ versus $1/R(T, L = \infty)$ for various gate voltages, clearly revealing that in spite of the fact that the data does not extend below $T = 4$ K, and therefore close to $T_c$, the BKT regime is attained. This deficiency also implies that the finite size dominated regime is fully attained for $V_g = -2$ V only where $T_c \approx 3.5$ K. The gate voltage dependence of $1/g(L)$ reveals that electrostatic tuning changes not only the carrier density but also the inhomogeneity landscape. Note that standard finite size scaling implies $g(L) \propto L^{-2}$, which neglects the multiplicative logarithmic corrections associated with BKT critical behavior [40].

To explore the quantum critical behavior latent in $T_{\nu}(V_g)$, $R_0(V_g)$ and $b_k(T_c)$ we performed this analysis of the temperature dependence of the sheet resistance for additional gate voltages. However, the inset in figure 1(c) reveals that $g$ increases with reduced $T_c$. It implies reduced homogeneity with decreasing $T_c$ and shrinkage of the BKT regime. Nevertheless, our analysis reveals that the data considered here attain the BKT regime and thus permit one to estimate the quantities of interest. To fix the critical gate voltage $V_{gc}$ we used the empirical gate voltage dependence of the number of mobile holes $x$ per one formula unit of Bollinger et al [32], yielding $x(V_g) = x_c + 0.012(V_{gc} - V_g)$ down to $V_g = -2$ V, with $x_c = 0.0605$ and $V_{gc} \approx -0.7$ V. The results $T_{\nu}(V_g)$, $R_0(V_g)$ are shown in figures 2(a) and (b) while $b_k(T_c)$ is depicted in figure 3. As can be seen in figure 2(a) the BKT-transition line differs substantially from the so-called superconducting dome behavior observed in bulk cuprate superconductors. It is approximately given by $T_{\nu}(x)/T_{\nu,\text{max}} = 1 - 82.6(x - x_m)^2$, where $T_{\nu,\text{max}}(x_m = 0.16)$ is the maximum $T_{\nu}$ [41]. Close to the QSI transition, where even bulk cuprate superconductors become essentially 2D [18], it reduces to $T_{\nu}(x)/T_{\nu,\text{max}} \approx 10.3(x - x_c)$, with $x_c \approx 0.049$, and suggests $\bar{\nu} \approx 1$.

In any case it differs substantially from the BKT line shown in figure 2(a), yielding the estimates

$$z\bar{\nu} = 1.46 \pm 0.04, \quad \bar{\nu} = 1.51 \pm 0.05. \quad (18)$$

To obtain $z\bar{\nu} \approx 1.46$ all data were taken into account, while for $\bar{\nu} = 1.51$ they were restricted to the three data points closest to the critical gate voltage. Both estimates are in agreement with the value $z\bar{\nu} = 1.5 \pm 0.1$ derived by Bollinger et al [32] using the quantum scaling approach. However, our estimates and with that the corresponding fits included in figure 2(a) depend on the considered range of the gate voltages. It implies that the asymptotic critical quantum regime does not extend down to $V_g = -2$ V. For this reason we invoke the same procedure using equation (7) to derive the critical exponents $z$ and $\bar{\nu}$ from the nonuniversal parameters $R_0(V_g)$ and $R_0(T_c)$.
and the three data points closest to \( V_g \)

\[
\begin{align*}
\tau &= 0.63 \pm 0.03, \\
\zeta &= 2.34 \pm 0.17, \\
\end{align*}
\]

(19)

and the three data points closest to \( V_g \)

\[
\begin{align*}
\tau &= 0.52 \pm 0.02, \\
\zeta &= 3.1 \pm 0.1, \\
\end{align*}
\]

(20)

The corresponding fits are depicted in figure 2(b). They clearly confirm that the asymptotic critical regime does not extend down to \( V_g = -2 \) V. As a result, the estimates given in equation (19) correspond to effective exponents while those listed in equation (20) are more reliable because they stem from the data given closest to criticality.

As these exponents satisfy the inequality \((2 + \nu)\tau \geq 2\) we use the correct scaling argument, because \( \delta = (\mu - \mu_c) \propto (x - x_c) \propto (V_{gc} - V_g) \), where \( \mu \) denotes the chemical potential [6]. The agreement between these \( \tau \) values confirms the applicability of the scaling relation (7). Similarly, the nonuniversal parameter \( b_R(\nu) \) according to figure 3 exhibits the expected \( T_c \) dependence (equation (6)). Noting that \( D + z \simeq 2 + z \simeq 5.1 \) exceeds the upper critical dimension \( D_u = 4 \), hyperscaling does not apply and the exponent of the zero-temperature correlation length \( \tau \) should adopt its mean-field value \( \tau = 1/2 \) in the absence of disorder. Indeed, \( \tau = 1/2 \) violates the Harris criterion \( \tau \geq 2/D = 1 \) [29] and the inequality \( \tau \geq 2/D = 1 \) for the correlation length exponent in a disordered system [30]. Accordingly, our estimate \( \tau = 0.52 \pm 0.02 \), consistent with \( \tau = 1/2 \), then reveals a clean (absence of disorder) 2D-QSI transition with \( \zeta = 3.1 \pm 0.1 \). It violates the equivalence between quantum phase transitions in systems with \( D \) spatial dimensions and those of classical phase transitions in \((D + z)\) dimensions. In addition the proportionality between \( d/\lambda^2(0) \) and \( T_c \) (equation (11)), valid below the upper critical dimension \( D_u = 4 \), no longer holds because \((D + z) \simeq 5.1\) is above \( D_u = 4 \). In fact, the magnetic penetration depth measurements taken on underdoped high-quality YBa\(_2\)Cu\(_3\)O\(_{6+x}\) single crystals revealed \( T_c \propto (d/\lambda^2(0))^{0.61} \) [42].

To complete the analysis of the data of Bollinger et al [32] we depict in figure 4 the plot \( R(V_g, T)/R_0(V_g) \) versus \((T_c(V_g)/T)^{2/3}\) corresponding to the quantum scaling function \( G(y) \) (equation (8)) in terms of \( y/v_c = (T_c(V_g)/T)^{2/3} \). Apparently, the data does not fall completely on the BKT curve indicated by the dashed line. It corresponds to the
superconductor branch of the quantum scaling function. Instead we observe a flow to and away from the universal characteristics. As $T_c/T$ decreases for fixed $T_c$ the crossover to AL-behavior sets in, while the rounding of the transition with increasing $T_c/T$ leads to a flow away from criticality. Thus, the important lesson is that the quality of the data collapse on a single curve depends heavily on the temperature range of the data entering the plot. Another striking feature is the extended scaling regime. Within the BKT scenario it simply follows from the fact that the scaling form (4) applies along the extended scaling regime. Within the BKT scenario it simply follows from the fact that the scaling form (4) applies along the BKT-transition line irrespective of the distance from the QSI transition. In the quantum scaling approach this property remains hidden and merely suggests an extended quantum critical regime. The excellent quality of the piecewise data collapse also reveals that the observance of the substantial variation of $R_0(V_g)$ (see figure 2(b)) is essential, while in the quantum scaling approach it is fixed by the critical sheet resistance. To demonstrate these features even more compellingly we depict in figure 5(a) $R/R_c$ versus $(V_{gc} - V_g)/T^{2/3}$. Apparently the data do not fall even piecewise on a single curve. However, this is not surprising because the quantum scaling form (1) holds only close to the critical sheet resistance and $R_0(V_g)$ varies substantially in the gate voltage regime considered here (see figure 2(b)). As shown in figures 5(b) and 6, this behavior allows one to estimate $R_0(V_g)$ from $R/R_c$ versus $(V_{gc} - V_g)/T^{2/3}$ by rescaling $R_c$ in terms of $b(V_g)$ as $b(V_g)$ is shown in figure 6 and agrees well with $R_0(V_g)$. This derived from the BKT behavior of the sheet resistance (see figure 2(b)).

A BKT line with a QSI transition at its endpoint was also explored in some detail at the interface between the insulating oxides LaAlO$_3$ and SrTiO$_3$, exhibiting a superconducting 2D electron system that can be modulated by a gate voltage [28, 34, 43, 44]. BKT behavior and with that a 2D electron system was established as follows:

![Figure 4](image1.png)  
**Figure 4.** $R(V_g, T)/R_0(V_g)$ versus $(T_c(V_g)/T)^{2/3} = y/y_c$ corresponding to the BKT quantum scaling function $G(y)$ (equation (8)). $R(V_g, T)$ is taken from [32] in the temperature range from 4.4 to 100 K. The respective values for $R_0(x), T_c(V_g)$ and $R_R$ are shown in figures 2 and 3. The dashed line is the critical BKT behavior given by equation (4), with $b_R = b_R/T_c^{1/2} = 4.55$.

![Figure 5](image2.png)  
**Figure 5.** (a) $R/R_c$ versus $(V_{gc} - V_g)/T^{2/3}$, with $R_c = 6.45 \, \Omega$ and $V_{gc} = -0.7 \, \text{V}$, derived from the temperature dependence of the sheet resistance of Bollinger et al [32]. The data cover the range from 4 to 100 K. (b) $R/b(V_g)R_c$ versus $(V_{gc} - V_g)/T^{2/3}$ with $b(V_g) = 1$ and $b(V_g)$ adjusted to achieve a piecewise collapse of the data.

![Figure 6](image3.png)  
**Figure 6.** $b(V_g)$ and $R_0(V_g)$, with $R_0(V_g) = -1 \, \text{V}$. The solid line is $R_0(V_g)/R_0(V_g = -1 \, \text{V}) = (R_c + 15.9(V_{gc} - V_g)^{1.26})/R_0(V_g = -1 \, \text{V})$ with $R_c = 6.45 \, \Omega$, $V_{gc} = -0.7$ and $R_0(V_g = -1 \, \text{V}) = 10.7 \, \Omega$ taken from figure 2(b).
the current–voltage characteristics [43] at the BKT-transition temperature $T_c$ revealed the characteristic BKT form $V \propto I^a$ with $a = 3$ [27]. Consistency with the characteristic temperature dependence of the sheet resistance (equation (4)) was established [28, 43, 44]. It was also shown that the effective thickness of the superconducting 2D system can be extracted from the magnetic field dependence of the conductivity at $T_c$ [45]. The gate voltage tuned BKT phase transition line, $T_c(V_g)$, derived from the temperature dependence of the sheet resistance at various gate voltages with equation (4) revealed consistency with $T_c(V_g) = 8.9 \times 10^{-3} (V_g - V_{g_c})^{2/3}$ K, indicating quantum critical behavior (equation (3)) with $\gamma = 2/3$ and the critical sheet resistance $R_c = 2.7 \Omega$ [28, 44]. Furthermore the estimates $\nu \approx 2/3$ and $\gamma \approx 1$, confirming $\gamma = 2/3$, have been derived from $R_0(V_g)$ [28]. This suggests that the gate voltage tuned QSI transition of the 2D electron system at the LaAlO$_3$/SrTiO$_3$ interface belongs to the $2+\gamma$ universality class where hyperscaling and, thus, proportionality between $d/\lambda^2(0)$ and $T_c$ (equation (11)) applies. On the other hand because $\nu \approx 2/3 < 2/D = 1$ then according to the Harris theorem disorder is relevant as well [29].

To comment on the difficulties in observing the BKT features in the magnetic penetration depth as well as the relation between $d/\lambda^2(0)$ and $T_c$, we reproduce in figure 7 the data of Bert et al [34] for a gate voltage tuned superconducting LaAlO$_3$/SrTiO$_3$ interface in terms of $T_c$ versus $d/\lambda^2(T = 0.04$ K). $T_c$ is defined here as the temperature at which the diamagnetic screening drops below the noise level corresponding to a detectable $d/\lambda^2$ of 0.10–0.34 cm$^{-1}$. A glance at figure 7 reveals that this is just the regime where the universal quantum behavior applies, indicated by the dashed and solid lines, corresponding to the lower bound (12) and the behavior derived from the $^4$He data of Crowell et al [22], respectively. Nevertheless the data reveal the flow to the 2D-QSI transition, which is attained at much lower $T_c$s. Otherwise the data points resemble the outline of a fly’s wing [18], remarkably similar to the $T_c$ versus $1/\lambda^2_0(T = 0)$ plots of the bulk superconductors $Y_{0.8}\text{CuO}_2$-123, Ti-1212 [46], and Ti-2201 [47], covering nearly the doping regime of the so-called superconducting dome extending from the underdoped to the overdoped limit. According to the generic plot $T_c/T_c(x_m)$ versus $\gamma(x_m)/\gamma(T_c)$, where $\gamma = \kappa_{ab}/\kappa_c$ is the anisotropy and $\kappa_{ab,c}$ denote the in-plane and c-axis correlation length, these cuprates become nearly 2D in the underdoped limit [18]. In any case, figure 7 shows that in this plot the universal QSI behavior is attained at comparatively low $T_c$ values only. Accordingly, in the QSI regime of interest the Nelson–Kosterlitz jump given by equation (10) becomes very small and appears to be beyond present experimental resolution [34]. In contrast, in the temperature dependence of the sheet resistance the BKT critical regime is accessible because $T_c/T_{c0} \approx 0.75$ [45], even though small compared to that in the La$_{2-x}$Sr$_x$CuO$_4$ films, where $T_c/T_{c0} \approx 0.1$.

4. Summary and discussion

In summary, we sketched and explored the implications of Berezinskii–Kosterlitz–Thouless (BKT) critical behavior on the quantum critical properties of a two-dimensional (2D) quantum superconductor–insulator transition (QSI) driven by the tuning parameter $x$. It was shown that the finite-temperature BKT scenario implies, in terms of the characteristic temperature dependence of the BKT correlation length, an explicit quantum scaling function for the sheet resistance $R(x, T)$ along the superconducting branch ending at the nonuniversal critical value $R_c = R_0(x_c)$. This scaling form fixes the BKT-transition line $T_c(x)$ and provides estimates for the quantum critical exponent product $\gamma \nu$. In addition, independent estimates of $\gamma \nu$, $\nu$ and $\gamma$ follow from the $x$ dependence of the nonuniversal parameters $T_c(x)$ and $R_0(x)$ entering the characteristic BKT expression for the sheet resistance $R(x, T)$. This requires that the BKT critical regime where phase fluctuations dominate is attained and the finite-temperature BKT relation for the sheet resistance applies for any $T \geq T_c(x) > 0$. The last condition is satisfied because the BKT expression for the sheet resistance is simply related to the characteristic temperature dependence of the BKT correlation length. Quantum fluctuations enter via the nonuniversal parameters $T_c(x)$ and $R_0(x)$ disclosing the respective quantum critical behavior close to $x_c$. Even though BKT critical behavior is not affected by short-range correlated and uncorrelated disorder [29], the observation of this requires sufficiently homogeneous films and a tuning parameter which does not affect the disorder. Noting that in the magnetic field tuned case there is no BKT line, thickness and gate voltage tuned 2D-QSI transitions appear to be promising candidates. In any case the scenario outlined here requires a line of BKT-transitions $T_c(x)$ with a quantum critical endpoint $T_c(x_c) = 0$ and sheet resistance data which attain the BKT critical regime.

To illustrate the potential and the implications of this scenario we analyzed the data of Bollinger et al [32] taken.
on gate voltage tuned epitaxial films of La$_{2-x}$Sr$_x$CuO$_4$ that are one unit cell in thickness. Invoking finite size scaling, evidence for the dominant phase fluctuations and BKT critical behavior was established in terms of the temperature dependence of the sheet resistance, revealing a large critical regime extending substantially above the lowest attained temperature $T = 4$ K. From the nonuniversal parameters $T_c(x)$ and $R_0(x)$, disclosing the respective quantum critical properties, we derived the estimates for the quantum critical exponents: $\nu \approx 1.51$ from $T_c(x)$, $\nu \approx 0.52$ and $z \approx 3.1$ from $R_0(x)$, yielding $\nu \approx 1.61$. Thus, in contrast to the standard quantum scaling approach, providing an estimate for $\nu$ only, the BKT scenario uncovered $\nu$, $\nu$ and $z$ from the quantum critical behavior disclosed in $T_c(x)$ and $R_0(x)$. Additional evidence for $\nu \approx 1.5$ was established from the comparison of the scaled data with the explicit scaling BKT scaling form of the superconductor branch. We observed that the scaled data does not fail entirely on the BKT curve. Instead a flow to and away from the universal characteristics occurred. As $T_c(x)/T$ decreases for fixed $T_c$ a crossover to AL-behavior sets in, while the rounding of the transition leads with increasing $T_c(x)/T$ to a flow away from criticality. The important lesson then is that the quality of the data collapse on a single curve depends heavily on the temperature range of the data entering the plot. Another striking feature is the extended scaling regime. Within the BKT scenario it follows from the fact that the explicit scaling form of the sheet resistance applies along the entire BKT-transition line irrespective of the distance from the QSI transition. In the quantum scaling approach this property remains hidden and merely suggests an extended quantum critical regime. The piecewise excellent quality of the data collapse also reveals that the provision of the substantial variation of $R_0(x)$ is essential, while in the quantum scaling approach it is fixed by the critical sheet resistance. Supposing that the equivalence between quantum phase transitions with $D$ spatial dimensions and those of classical phase transitions in $(D + z)$ dimensions applies, the 2D-QSI transition at the endpoint of a BKT line $T_c(x)$ should belong to the finite-temperature $(2 + z) - xy$ universality class. However, our estimate $z \approx 3.1$, and with that $D = 5.1 - xy$, exceeds the upper critical dimension $D_\text{uc} = 4$ where hyperscaling no longer applies but the correlation length exponent adopts its mean-field value $\nu = 1/2$. On the other hand $\nu = 1/2$ and our estimate $\nu \approx 0.52$ violate the inequality [30] $\nu \geq 2/D = 1$ for disordered systems and the Harris inequality $\nu \geq 2/D = 1$ [29], revealing its relevance in the presence of disorder. Accordingly our estimate $\nu \approx 0.52$ for the correlation length exponent points clearly to the absence of disorder in the attained critical regime. The resulting clean 2D-QSI transition with $\nu \approx 0.52$ and $z \approx 3.1$ then violates the equivalence between quantum phase transitions in systems with $D$ spatial dimensions and those of classical phase transitions in $(D + z)$ dimensions. In addition the proportionality between $d/\lambda^2(0)$ and $T_c$ (equation (11)), valid below the upper critical dimension $D_\text{uc} = 4$, no longer holds because $(D + z) \approx 5.1$ is above $D_\text{uc} = 4$. In fact magnetic penetration depth measurements taken on underdoped high quality YBa$_{2-x}$Cu$_x$O$_{6+\delta}$ single crystals revealed $T_c \propto (d/\lambda^2(0))^{0.61}$ [42].

To comment on the BKT features in the temperature dependence of the magnetic penetration depth we considered the data of Bert et al [34] for a gate voltage tuned superconducting LaAlO$_x$/SrTiO$_3$ interface in terms of $T_c$ versus $d/\lambda^2(T = 0.04$ K). The data reveals the flow to the universal relationship (11), but much a lower $T_c$ must be attained to reach the quantum critical regime. However in this low-$T_c$ regime the Nelson–Kosterlitz jump given by equation (10) is very small and appears to be beyond present experimental resolution [34]. Otherwise the data points resemble the outline of a fly’s wing [18], remarkably similar to the $T_c$ versus $1/\lambda^2(T = 0)$ plots of the bulk superconductors Y$_{0.8}$Cu$_{0.2}$-123, TI-1212 [46], and TI-2201 [47], covering nearly the entire doping regime in the so-called superconducting dome extending from the underdoped to the overdoped limit.

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