A state-of-the-art review on low-frequency nonlinear vibration isolation with electromagnetic mechanisms

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Abstract Vibration isolation is one of the most efficient approaches to protecting host structures from harmful vibrations, especially in aerospace, mechanical, and architectural engineering, etc. Traditional linear vibration isolation is hard to meet the requirements of the loading capacity and isolation band simultaneously, which limits further engineering application, especially in the low-frequency range. In recent twenty years, the nonlinear vibration isolation technology has been widely investigated to broaden the vibration isolation band by exploiting beneficial nonlinearities. One of the most widely studied objects is the “three-spring” configured quasi-zero-stiffness (QZS) vibration isolator, which can realize the negative stiffness and high-static-low-dynamic stiffness (HSLDS) characteristics. The nonlinear vibration isolation with QZS can overcome the drawbacks of the linear one to achieve a better broadband vibration isolation performance. Due to the characteristics of fast response, strong stroke, nonlinearities, easy control, and low-cost, the nonlinear vibration with electromagnetic mechanisms has attracted attention. In this review, we focus on the basic theory, design methodology, nonlinear damping mechanism, and active control of electromagnetic QZS vibration isolators. Furthermore, we provide perspectives for further studies with electromagnetic devices to realize high-efficiency vibration isolation.

Key words quasi-zero-stiffness (QZS), nonlinear vibration isolation, low-frequency, electromagnetic vibration isolation, bistable

Chinese Library Classification O322, O328

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1 Introduction

Structural vibration widely exists in aerospace, mechanical, optical, submariner, and architectural engineering, etc., which would lead to the loss of safety and reliability and the production of unexpected results in devices and structures. As one of the most effective approaches, vibration isolation is widely applied in the above-mentioned fields to reduce these unwanted vibrations[1]. The earliest investigated traditional vibration isolator is composed of

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a linear spring and a damping element, which begins to isolate vibration at $\sqrt{2}\omega_n^2$, where $\omega_n$ denotes the natural frequency of the isolation systems. Damping can improve the vibration suppression performance in the resonance region but decrease the vibration isolation performance in the isolation region\cite{3}.

Considering the limitation of linear vibration isolators in static supporting capability and isolation band, nonlinear vibration isolators with high-static-low-dynamic stiffness (HSLDS) characteristics have become increasingly concerned, since Alabuzhev\cite{4} first proposed nonlinear springs with quasi-zero-stiffness (QZS) to the lower natural frequency. Platus\cite{5} presented negative stiffness mechanisms to cancel the stiffness of a spring, which could lower the natural frequency to 0.2 Hz. Then, after 16 years of studies, Ibrahim\cite{6} comprehensively reviewed the advances in nonlinear passive vibration isolators, which was a pioneering work and accelerated the development of nonlinear vibration isolation. Carrella et al.\cite{7} used one vertical spring and two oblique springs to construct a QZS isolator, where the vertical spring provided the positive stiffness to maintain the stability of both the static and dynamic states, and the two oblique springs provided the negative stiffness to decrease the dynamic stiffness\cite{8}. Figure 1 presents the elastic restoring force comparison between the linear and QZS vibration isolators. The stiffness of the nonlinear system is close to zero near the equilibrium point, while the stiffness of the linear isolators is a constant. It indicates that the QZS isolators outperform the linear system when the system parameters are properly chosen\cite{9}. It is noted that two oblique springs could be horizontal\cite{9}. Kovacic et al.\cite{10} found that a static force could dramatically change the dynamic behavior of a QZS system. Liu et al.\cite{11} and Huang et al.\cite{12} used the Euler buckled beam instead of two oblique or horizontal springs\cite{9} to construct a negative stiffness corrector, which could dramatically improve the vibration isolation performance of QZS isolators. Ding and Chen\cite{13} discussed the QZS characteristics of curved beams. An adjustment mechanism could also be added into QZS isolators to reduce the influence from the loads and self-weight\cite{14}. Lu et al.\cite{15} combined two QZS isolators in series to realize a two-stage vibration isolator. Zhou et al.\cite{16} investigated the QZS effects of the cam-roller-spring mechanisms.

In principle, the above-mentioned “three-spring” QZS vibration isolator was realized with geometric nonlinearities\cite{6}. Sun et al.\cite{17} designed a scissor-like structured platform, which possessed the considerable nonlinear stiffness and damping characteristics, and thus superior vibration isolation performance. Thereafter, Jing and his group presented a class of bio-inspired structures for the low-frequency and broadband vibration isolation\cite{18-22}. These results demonstrated that beneficial nonlinearities of bio-inspired structures could produce QZS and HSLDS characteristics to improve the vibration isolation performance. Deng et al.\cite{23} proposed a bio-inspired vibration isolation structure by mimicking the bird multi-layer neck, which could expand the effective range of dynamic displacement. Yan et al. designed a large stroke QZS vibration isolator by mimicking cats\cite{24} and toe\cite{25}. Then, Yan et al.\cite{26} presented a systemic review on bio-inspired vibration isolation, which pointed out that the bio-inspired structure
would be a research spot in this decade.

Apart from the three-spring and bio-inspired nonlinear vibration isolators, the low-frequency vibration isolators with electromagnetic mechanisms have advantages due to the characteristics of strong stroke and nonlinearities, fast response, contactless, easy to control, and low-cost\(^2\). Recently, the use of electromagnetic mechanisms to design low-frequency nonlinear vibration isolators and damping has attracted attention\(^{28-30}\). The electromagnetics mainly contains permanent magnets (PMs) and electromagnetic coupling devices. The former can be easily used to design nonlinear stiffness elements\(^3\), and the other can be used to design damping and controller to improve the vibration isolation performance\(^\)\(^{\)\(^{32}\). Furthermore, owing to the strong electromechanical coupling characteristics, the electromagnetic vibration isolators can be used bifunctionally to simultaneously reduce vibrations and harvest these waste energy\(^3\). Among these research aspects, the basic problems are how to use PMs to design passive low-frequency vibration isolators and to further improve the corresponding vibration isolation performance with damping and active control approaches, which has yet not been discussed comprehensively.

Consequently, this state-of-the-art review systematically presents the inspiration of electromagnetic mechanisms for low-frequency vibration isolation, including single and multiple direction electromagnetic QZS vibration isolators, electromagnetic bistable vibration isolation mechanisms, nonlinear damping mechanisms, and semi-active and active control of electromagnetic QZS vibration isolators in recent years. Some perspectives are also discussed in this review.

2 Inspiration of electromagnetic mechanisms for highly-efficient low-frequency vibration isolation

2.1 Low-frequency vibration isolation mechanisms with electromagnetics

As to an arbitrary vibration isolation system subject to external disturbances, the equation of motion is written as

\[ m\ddot{x} = F - F_R - F_D, \]

where \( m \) is the mass of an isolator. \( F \) is the generalized external force, which can be base acceleration excitation or external force excitation. \( F_R \) and \( F_D \) are the elastic restoring force and damping force of the isolation system, respectively. Specifically, when the isolation system is linear, Eq. (1) will be

\[ m\ddot{x} + c\dot{x} + kx = F, \]

where \( c \) and \( k \) are the damping coefficient and stiffness, respectively. Then, the elastic restoring force \( F_R \) is proportional to the displacement \( x \), and the damping force is also linear. The electromagnetics can be used to tune and optimize the mass and stiffness and damping coefficient parameters to realize highly-efficient low-frequency vibration isolation.

\( F_R \) of QZS vibration isolators is nonlinear\(^7\), which is represented with a polynomial as

\[ F_R = \sum_{n=1}^{N} k_i x^n, \]

where \( k_i \) denotes the \( i \)th order stiffness coefficient. \( N \) is the number of the polynomials. For the traditional three-spring typed QZS vibration isolator, \( N \) is 3 for simplicity of calculation\(^9\). Then, the equation of motion will be a standard Duffing equation. One can use the harmonic balance method (HBM) or perturbation method to derive the amplitude-frequency response\(^{33-34}\). This model is widely used to lower the natural frequency to broaden the isolation band.

One can also use nonlinear damping to improve the vibration isolation in the isolation region to overcome the drawback of the linear one. Mofidian and Bardawee\(^35\) investigated the effect
of cubic damping on the isolation performance of a QZS isolator. The damping force is expressed by

\[ F_D = c_1 \dot{x} + c_3 \dot{x}^3, \tag{3} \]

where \( c_3 \) is the nonlinear damping coefficient, which can reduce the response in the resonant region without influencing the isolation performance in the isolation region\(^{[36]}\).

### 2.2 QZS characteristics with PMs

Most of the investigated and applied QZS vibration isolators in recent years are based on nonlinear stiffness\(^{[6]}\). Magnetic force is a kind of nonlinear force that can produce negative stiffness and nonlinear stiffness when the PMs are in opposite direction configuration\(^{[25]}\). The nonlinear magnetic force is dependent on the geometry, magnetization, and relative position characteristics of PMs, and thus the modeling process is complicated. A PM can be theoretically simplified with the current model or charge model\(^{[37]}\). For a ring, a cylinder, or a rectangular PM, the corresponding models are different from each other. Despite the geometry of a uniformly magnetized PM, the volume current density is \( J_V = \nabla \times M = 0 \), and the surface current density is \( J_s = M \times r \). The nonlinear magnetic force between two magnets (PM\(^1\) and PM\(^2\)) can be derived according to the Ampere current model as

\[ F_{PM^1,PM^2} = \int J_{s1} ds \times B_{PM^1,PM^2}, \tag{4} \]

where \( J_{s1} \) is the surface current density of one of the PMs, and \( B_{PM^1,PM^2} \) is the magnetic flux density at the point 1 on the PM\(^1\) generated by PM\(^2\). If the axes of the two magnets are the same, Eq. (4) can be used directly to calculate the nonlinear magnetic force. Otherwise, the coordinate transformation matrix between the two magnets should be fully taken into consideration. If there are more than two magnets, one should consider all magnetic forces and project them along the \( x \)-, \( y \)-, and \( z \)-directions for simplicity of calculation.

Due to the complicated expression, nonlinear magnetic force can be approximated with a polynomial. If the working range is small, a third order is enough\(^{[30]}\). Otherwise, a higher order is essential, e.g., fifth order\(^{[38–39]}\) or seventh order\(^{[27]}\). However, a higher order polynomial would increase the difficulties of calculation.

Another method to predict the nonlinear magnetic force is the charge model, the process of which is similar to the current model. One can refer to Ref. [37]. The charge and current models can theoretically predict the nonlinear force. Generally, one can employ the finite element method and theoretical tests to verify the theoretical model\(^{[38]}\).

### 2.3 Electromagnetic damping

The interaction between PMs and coils produces the Ampere force which is also known as the Lorenz force and has been proved to be a kind of damping force\(^{[40–42]}\). For a single coil and magnet pair, the induced damping force can be calculated by

\[ F_D = \int NI(t) dl \times B = C_m I x, \tag{5} \]

where \( N \) is the turns of the coil, \( I \) is the induced current, and \( C_m \) is the electromechanical coupling coefficient. According to Faraday’s law, the electromotive force (EMF) is written as

\[ V_e = \int N(\dot{x} \times B) dl = C_e \dot{x}. \tag{6} \]

In general, \( C_e \) equals \( C_m \)^\(^{[43]}\). \( C_e \) is defined as the electromagnetic coupling coefficient. When the displacement is small, \( C_e \) can be regarded as a constant, which implies that the damping is linear\(^{[44]}\). Otherwise, \( C_e \) will be nonlinear due to the fact that the interaction between the
coil and magnets is nonlinear. Then, one can design the relative position between the coil and magnets to realize the nonlinear electromagnetic coupling coefficient \( C_e = \sum c_{e,i}x^i \). (7)

The order of the polynomial depends on the geometry of the coil and PMs.

### 2.4 Evaluation of vibration isolation performance

For vibration isolation systems, transmissibility is used to evaluate the vibration isolation performance. In general, the displacement and force transmissibilities are used. \(^{[45]}\)

When the QZS vibration isolator is subject to the base displacement excitation, the displacement transmissibility can be calculated directly by \(^{[17]}\)

\[
T_d = \left| \frac{x}{z_b} \right| = \sqrt{1 + \left( \frac{r}{z_b} \right)^2 + 2 \left( \frac{r}{z_b} \right) \cos \theta}, \tag{8}
\]

where \( r \) is the response amplitude, and \( z_b \) is the excitation amplitude. In most circumstances, the base acceleration excitation is widely used, and one can also use Eq. (8) to calculate the acceleration transmissibility.

When the QZS vibration isolator is subject to force excitation, the force transmissibility can be calculated by \(^{[46]}\)

\[
T_f = \frac{1}{F} \sqrt{\left( \frac{3}{4} \gamma r^3 \right)^2 + (2 \Omega r)^2}, \tag{9}
\]

where \( \Omega \) is the excitation frequency, and \( F \) is the amplitude of the external force.

### 3 Electromagnetic QZS vibration isolation mechanisms

This section presents the possible design of electromagnetic mechanisms to realize QZS vibration isolators for low-frequency vibration isolation, especially for one direction and multi-direction vibration isolation, and also bistable vibration isolation.

#### 3.1 Electromagnetic QZS vibration isolator

##### 3.1.1 Magnetic levitation

Early in 2002, Puppin and Fratello \(^{[47]}\) described a vibration isolation apparatus, where four couples of repelling magnets ensured the loading force to a suspended table as shown in Fig. 2(a). The vibration isolation efficiency of the repelling magnet devices was excellent and had the advantages that no external supply of energy or compressed air was needed. Carrella et al. \(^{[31]}\) utilized three PMs arranged in an attracting configuration to construct an HSLDS isolator shown in Fig. 2(b), where the central magnet served as the isolated mass, and the other two magnets were fixed. Such a configuration could produce negative stiffness to cancel the positive stiffness, and could reduce the natural frequency from 14 Hz to 7 Hz. However, there was an unstable region for the attracting configuration when the displacement exceeded the corresponding limits. Moreover, one could obtain bistable characteristics for a properly designed parameters, which will be discussed below. \(^{[33]}\)

Robertson et al. \(^{[48]}\) also investigated the levitated magnets, i.e., a pair of fixed cubic magnets supporting a mass against gravity by repelling the mass from below and attracting it from above, such that the configuration differs from that in Ref. [31], and could achieve low stiffness and reduce the stiffness in all three transitional degrees of freedom (DOFs). After that, Wu et al. \(^{[49]}\) presented a magnetic levitated QZS vibration isolator consisting of three cuboidal magnets configured in repulsive interaction. Although the central magnet was levitated by the other two, this configuration shown in Fig. 2(d) differs from the previous models for the levitated magnets moves perpendicular to the axis while those in Figs. 2(a)–2(c) are along their axis directions. Zheng et al. \(^{[29]}\) used two ring magnets that
magnetized along the radial direction to construct a QZS vibration isolator. Considering the
difficulty of such kind of magnetization, the tile magnets were employed to obtain negative
stiffness shown in Fig.2(e), which was proven to be able to improve the vibration isolation
performance\textsuperscript{[50]}. Sun et al.\textsuperscript{[51]} used two pairs of magnets in repulsive configuration to design
a levitated QZS vibration isolator, where the central two magnets were fixed on the tip of
a cantilever beam as shown in Fig.2(f). In summary, the QZS with the magnetic levitation
mechanism was firstly and mostly investigated in recent years, which can broaden the isolation
band and realize highly-efficient isolation performance due to negative stiffness and beneficial
nonlinear stiffness characteristics.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{magnets.png}
\caption{QZS vibration isolators with magnetic levitation mechanisms: (a) vibration isolation apparatus with two repelling magnets\textsuperscript{[47]}, (b) three magnets in levitated configuration with attractive configuration\textsuperscript{[31]}, (c) three cubic magnets configuration where two fixed magnets support a mass against gravity by repelling the mass from below and attracting it from above\textsuperscript{[48]}, (d) three cuboidal magnets configured in repulsive interaction\textsuperscript{[49]}, (e) two ring magnets with radial magnetization\textsuperscript{[29]}, and (f) two pairs of magnets in repulsive configuration\textsuperscript{[51]} (color online)}
\end{figure}

3.1.2 QZS isolators with magnetic spring

Apart from the magnetic levitation mechanism, some other devices were also designed to
realize QZS characteristics. Xu et al.\textsuperscript{[28]} used two pairs of repulsive magnets instead of oblique
or horizontal springs in traditional three-spring typed QZS isolators to develop a new nonlinear
magnetic QZS vibration isolator shown in Fig.3(a). The force transmissibility was derived,
and the results showed wonderful performance. Yan et al.\textsuperscript{[30]} designed a vari-stiffness nonlinear
vibration isolator with six ring magnets, where three magnets were fixed on the base plate, and
the others were assembled at the moving axis shown in Fig.3(b). This configuration could easily
tune the nonlinear magnetic stiffness through changing the relation position among magnets.
3.2 Multi-directional vibration isolation with electromagnetics

In most circumstances, host structures suffer from multi-directional disturbances or single excitation source may induce multi-direction responses after several amplification\(^{[52–53]}\). Zhou et al.\(^{[54]}\) designed a 6-DOF vibration isolator with the control algorithms for stabilizing the passively unstable magnetic levitation. As shown in Fig. 4(a), the isolator applies magnetic levitation as the payload support mechanism, which realizes inherent QZS levitation in the vertical direction and zero stiffness in the other five DOFs. While providing near zero stiffness in multiple DOFs, the design is also able to generate static magnetic forces to support the payload weight. Dong et al.\(^{[55]}\) also designed a 6-DOF QZS vibration isolator based on the magnetic levitation and spatial pendulum as shown in Fig. 4(b). This 6-DOF mount has the characteristics of weak dynamic coupling that the responses in different directions do not interact with each other. Then, Zheng et al.\(^{[56]}\) introduced the QZS isolator shown in Fig. 2(e) into a Stewart platform to form a 6-DOF vibration isolator shown in Fig. 4(c). The results showed that the Stewart isolator could isolate vibrations in all six directions.

3.3 Bistable vibration isolators with electromagnetics

We found that the QZS or HSLDS vibration isolator is a kind of monostable systems after analyzing the existing literature, which has only one equilibrium position. There are two stable equilibria for bistable systems\(^{[57]}\). Most of the studies on bistable systems focused on using the large intra-well motion to realize high-efficiency energy harvesting\(^{[58–59]}\). In recent years, vibration isolation with bistable structures has become increasingly conserved. Different from the traditional nonlinear vibration isolators, the bistable structure possesses a unique snap-through phenomenon\(^{[60]}\). Yang et al.\(^{[61–62]}\) discussed a bistable dual-stage isolator consisting of a bistable isolator and a linear oscillator. The bistable suspension avoids inducing the detrimental resonance adjacent to the roll-off frequency band that otherwise compromises the safety of a comparable linear suspension. Yan et al.\(^{[27]}\) proposed a bi-state nonlinear vibration isolator consisting of a linear mass-spring-damper and five ring magnets shown in Fig. 5(a), and the working state of the isolator could be monostable or bistable, depending on the relative position of the PMs. The monostable state behaves like a QZS vibration isolator that can dramatically improve vibration isolation performance, while the bistable state can produce negative stiffness to reduce the peak frequency. Thereafter, Yan et al.\(^{[38]}\) designed another electromagnetic bistable vibration isolator with six ring magnets as shown in Fig. 5(b), a configuration similar to that shown in Fig. 3(b) except the magnetization of some magnets. The results demonstrated that when the bistable vibration isolator was in the interwell oscillation, two peaks appeared in the transmissibility curve, and the transmissibility could be smaller than 1
Fig. 4 Multi-direction QZS isolators: (a) 6-DOF magnetic levitation vibration isolators\textsuperscript{[54]}, (b) 6-DOF QZS vibration isolators based on spatial pendulum\textsuperscript{[55]}, and (c) Stewart isolators\textsuperscript{[56]} (color online).

Fig. 5 Electromagnetic bistable vibration isolators: (a) bi-state nonlinear vibration isolator with axially symmetric magnets\textsuperscript{[27]}, (b) bistable vibration isolator with six axially asymmetric magnets\textsuperscript{[38]}, and (c) shock isolation with bistable vibration isolators\textsuperscript{[64]} (color online).
between the two peaks. In principle, the bistable vibration isolator can produce softening stiffness characteristics to broaden the isolation band, and a shallower potential barrier is beneficial to vibration isolation. However, the snap-through action is harmful to small displacement conditions. Therefore, one should control the motion type of the isolator to interwall for a better isolation performance\textsuperscript{[63]}. Considering that the traditional isolators are hard to reduce the first amplitude under shocks, Yan et al.\textsuperscript{[64]} applied the electromagnetic bistable vibration isolation for the shock isolation of space antennas, and the results demonstrated that the bistable vibration isolator could dramatically reduce the first shock under larger strokes, which provides an alternative approach for shock isolation.

4 Nonlinear damping mechanisms for QZS vibration isolators

The previous literature mainly used nonlinear stiffness characteristics to produce negative stiffness and hardening or softening stiffness (beneficial nonlinear stiffness) to lower natural frequency, and thus to improve the vibration isolation performance. As demonstrated in Subsection 2.3, damping in a dynamic system could also improve the vibration suppression performance in the resonance region. Eddy current damping is an effective damping treatment to passively dampen vibrations\textsuperscript{[65]}. Yan et al.\textsuperscript{[30]} considered the eddy current damping in a vari-stiffness nonlinear vibration isolator as shown in Fig. 3(b). The results showed that the eddy current damping could provide considerable damping to lower the peak response. Thereafter, Yan et al.\textsuperscript{[66]} used a lever structure to amplify the eddy current damping to further improve the vibration suppression performance in the resonance region. Although the eddy current damping shown in Refs.\textsuperscript{[30] and [66] was nonlinear, it was dependent very much on the relative distance between the conductive plate and magnets. Consequently, nonlinear damping is needed to improve the vibration isolation performance in a relative wide range. Lu et al.\textsuperscript{[67]} embedded linear damping into the “three-spring” QZS isolator to achieve nonlinear damping characteristics, and the results demonstrated that both the linear and nonlinear damping could reduce the transmissibility around resonance. Although some work discussed the benefits of nonlinear damping\textsuperscript{[35,68–73]}, the characteristics were only analyzed theoretically and numerically. Based on this limitation, Yan et al.\textsuperscript{[32]} used two coils connected in series but winding in opposite directions together with two repulsive magnets to realize nonlinear damping as shown in Fig. 6(a). A negative resistance shunt circuit is connected to the terminals of coils to improve the damping performance. The electromagnetic coupling coefficient $C_e$ is nonlinear, which approximates to zero at the equilibrium but increases far away from the equilibrium position. This nonlinear damping reduces the vibration peak in the resonance region without influencing that in the isolation region. Then, Ma et al.\textsuperscript{[74]} presented two types of nonlinear damping. Figure 6(b) shows a kind of configuration of nonlinear damping where the distance between two repulsive magnets is larger than the width of the coil. Then, the magnetic flux density will change dramatically when vibrating. $C_e$ is symmetric about the equilibrium and is minimum at the equilibrium. Figure 6(c) shows another damping where two coils are separately connected to two coils that are located in the central position of the two repulsive magnets.

Based on these nonlinear damping design, we introduced the damping shown in Fig. 6(a) into an electromagnetic QZS vibration isolator shown in Fig. 3(b), and found that the nonlinear electromagnetic shunt damping could reduce the peak transmissibility in the resonance region without weakening the isolation performance in the isolation region\textsuperscript{[75]}. Considering that the previous study only utilized negative resistance shunt to improve damping performance\textsuperscript{[32,74–75]}, we also proposed the hybrid shunts (negative inductance, negative resistance, and negative resistance positive inductance) and utilized them for the vibration isolation performance of QZS vibration isolators\textsuperscript{[76]}. The hybrid shunts could realize mass and damping effects to simul-
taneously decrease the peak frequency and improve the vibration isolation performance. In summary, it is concluded that a reasonable designed nonlinear damping with semi-active approaches is able to improve the vibration control performance in a relatively broad band.

5 Tunable QZS isolators with electromagnetics

5.1 Semi-active approach

The above-mentioned QZS isolators mainly use magnetic spring passively to improve the vibration isolation performance. However, the vibration performance is limited due to the fact that the conventional QZS system could not change the stiffness easily. To overcome this drawback, Zhou and Liu[77] designed an HSLDS vibration isolator with a magnetic spring and a pair of electromagnetics shown in Fig. 7(a), which could be passive and semi-active. The magnetic spring was produced by electromagnetics, which could also be positive or negative, depending on the polarity of the current. Meng et al.[78] developed a vibration isolator consisting of ring magnets and electromagnetic coils as shown in Fig. 7(b). The stiffness could be controlled by manipulating the negative stiffness-based current in a system with a positive and a negative stiffness in parallel. Pu et al.[79] designed a compact and contactless multi-layer electromag-
netic spring with tunable negative stiffness as shown in Fig. 7(c). The negative stiffness was generated by the electromagnetic force between the coils and the magnets and could be tuned online by controlling the current. Then, Sun et al.\cite{80} proposed an HSLDS isolator with tunable electromagnetic mechanisms as shown in Fig. 7(d), where the stiffness could be tuned by regulating the current. When the current is zero, the HSLDS isolator would degenerate into a passive isolator. Compared to the passive one, the semi-active tunable system could further widen the bandwidth of vibration isolation and improve the isolation performance near the natural frequency. Jiang et al.\cite{81} designed a magnetic-air hybrid QZS vibration isolator shown in Fig. 7(e). The air spring was used to adjust positive stiffness, which provides a stable and variable support force. Electromagnetic spring was adopted to adjust negative stiffness.

Semi-active method can provide an alterable approach to tuning the nonlinear and negative stiffness to further improve the isolation performance. Specifically, when the external load is changeable, one can use the semi-active approach to adjust the working range instead of the passive one.

![Fig. 7 Tunable semi-active QZS isolators with electromagnetics: (a) tunable HSLDS isolator with a magnetic spring and a pair of electromagnetics\cite{77}, (b) tunable vibration isolator consisting of ring magnets and electromagnetic coils\cite{78}, (c) multi-layer electromagnetic spring with tunable negative stiffness\cite{79}, (d) HSLDS isolator with tunable electromagnetic mechanisms\cite{80}, and (e) magnetic-air hybrid QZS vibration isolator\cite{81} (color online)](image)

### 5.2 Active approach

Active control method can provide robust and highly-efficient vibration isolation performance. Early in 2000, Yoshioka et al.\cite{82} achieved active micro-vibration isolation with voice coil motors and pneumatic and piezoelectric actuators. Sun et al.\cite{83} used the time-delayed active control to improve the vibration isolation performance of a three-spring typed QZS vibration isolator. Huang et al.\cite{84} investigated the delayed feedback control of a tristable vibration isolation system under the stochastic parametric excitation. For electromagnetic QZS systems, Zhang et al.\cite{85} presented an active-passive hybrid vibration isolator with magnetic negative stiffness as shown in Fig. 8(a). The acceleration proportional feedback method could
reduce 80% compared to the passive method. Yuan et al. [86] combined a linear electromagnetic spring in parallel with a conventional linear isolation system as shown in Fig. 8(b). The electromagnetic spring contains three toroidal coils arranged coaxially with a ring magnet. By controlling the current excited to the coils, the electromagnetic spring could generate a linear negative stiffness that balances the positive stiffness of the conventional system, thereby achieving a quasi-zero stiffness over the long stroke. Kamaruzaman et al. [87] applied the active control technique in a 6-DOF QZS magnetic spring system as shown in Fig. 8(c). It is found that in the case of passively stable DOFs, the isolation bandwidth is reduced due to the relatively high constant stiffness, compared to the sufficiently small active stiffness provided by the optimal control in the passively unstable DOFs. Wang et al. [88] proposed an adjustable electromagnetic negative stiffness Stewart platform as shown in Fig. 8(d), which could realize good stiffness match conveniently and has high vibration isolation performance in all 6 DOFs. Yuan et al. [89] proposed a tunable negative stiffness mechanism that combines the advantages of HSLDS isolators to expand the isolation frequency band and online variable stiffness isolators to suppress resonance as shown in Fig. 8(e). The active control method can control the control current to tune the nonlinear stiffness to online control the vibration isolation performance of QZS isolators.

Fig. 8 Active control of QZS vibration isolators: (a) active-passive hybrid vibration isolator with magnetic negative stiffness [85], (b) linear electromagnetic spring in parallel with a conventional linear isolation system [86], (c) active 6-DOF QZS magnetic spring system [87], (d) electromagnetic negative stiffness Stewart platform [88], and (e) tunable HSLDS vibration isolator [89] (color online)

6 Conclusions and perspectives

Electromagnetic devices have the advantages of fast response, strong stroke, nonlinearities, easy control, and low-cost. Considering the recent advances in vibration isolation with beneficial nonlinearities, this study presents a state-of-the-art review on the low-frequency vibration isolation with electromagnetic mechanisms. Because a dynamic system contains the mass, stiffness, and damping elements, the inspiration and basic theory for tuning stiffness, damping, and
evaluation of vibration isolation performance of electromagnetic mechanisms for highly-efficient low-frequency vibration isolation are summarized briefly. The single QZS vibration isolation with magnetic levitation and magnetic spring, multi-direction isolation, and also bistable vibration isolation mechanisms are discussed. Then, semi-active nonlinear damping for improving the vibration isolation performance is reviewed. Owing to the complicated dynamic environment, tunable QZS approaches with semi-active and active methods are also discussed to further improve the working range, stability, and also efficiency of vibration isolation systems. This review focuses on the nonlinear vibration isolation with electromagnetic mechanisms and briefly summarizes the recent advances covering the stiffness and damping regulation treatments.

In future, the electromagnetic mechanisms have great potential in these vibration related fields.

(I) Design of high-efficiency and compactable electromagnetic QZS isolators

The reported electromagnetic nonlinear isolators mainly employed magnetic levitation and magnetic spring to realize QZS characteristics\(^{[31,49]}\). Although several configurations were developed\(^{[30]}\), they seems large in size and are hard to achieve multi-directional isolation. Hence, the design methodology of low-frequency electromagnetic devices needs to be further investigated.

(II) Realization of nonlinear damping with electromagnetics

In existing QZS systems, only electromagnetic shunt damping was employed to experimentally achieve nonlinear damping\(^{[32,74–76]}\), and negative resistance shunt was used to improve the damping performance. However, these approaches depend on the relative position. Under the small displacement case, it will be linear and ineffective under micro-vibration conditions. Consequently, novel damping treatments should be considered in further study.

(III) Active approach to tuning the working range of QZS isolators

Early in 2010, a tunable method has been proposed to improve the isolation performance\(^{[77]}\). In recent several years, many researchers used current control method to tune or adjust the working range of QZS or HSLDS vibration isolators\(^{[79–80,86,88]}\). This method has advantages of easy to implement and control; however, the robust under complex condition is still a challenge. Therefore, active or intelligent approach seems to be a better choice in further study\(^{[85]}\).

(IV) Bi-functional vibration isolation and energy harvesting with QZS devices

Electromagnetic devices can transform vibrational energy into electricity according to Faraday's law. QZS characteristics make the dynamic system work in a low frequency range, which is beneficial for low-frequency energy harvesting. Then, the harvested energy can serve as an alterable source to power up low energy consumption sensors in the Internet of Things\(^{[3]}\).

(V) QZS metamaterials

Recently, Wang et al.\(^{[8,90]}\) have introduced a negative stiffness mechanism into a traditional linear resonator to form an HSLDS resonator to create a low-frequency band gap, which has been proven to be effective for the design of novel metamaterials. Therefore, one should consider the low-frequency metamaterials with electromagnetic mechanisms and the corresponding energy harvesting aspect\(^{[91]}\).

Although this review concentrates on the methods of vibration isolation in electromagnetic QZS isolators, the hybrid control methods are also needed for QZS vibration isolation. It should be pointed out that the complex control methods for active approaches are still not completely revealed. The further research of electromagnetic QZS vibration isolation will expand to multi-stable, multi-DOF, and metamaterials structure. In addition, the application of active control methods in complex QZS systems should be investigated. Overall, we believe that electromagnetic QZS devices deserve further investigation in more science and engineering fields.
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