Study of convective structures and phase transition induced in a water layer by non-stationary boundary conditions

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Abstract. The problem of reconstructing interference and Hilbert structures from a numerical model of the evolution of the thermal field of convective flows in a water layer bounded by flat heat-exchanging surfaces under unsteady boundary conditions in the monotonic cooling mode and taking into account density inversion at a temperature of +4 °C was solved. The simulation of the thermal fields of convective flows in the form of the dynamic structure of isotherms was carried out taking into account the nonlinear dependence of thermal conductivity and water density on temperature. The field of the phase function, its Hilbert image and the interference field, which are compared with the results of the interference and Hilbert visualization of the fields of phase optical density obtained in the experiment, were reconstructed from the isotherms field supplemented by calculating the velocity field and of the temperature gradients field. The presented films illustrate the qualitative adequacy of the coevolution of numerical models and real processes.

1. Introduction

Interest in the study of convective flows is associated with the special importance of convection in geodynamics, physics of the atmosphere and ocean, in hydrodynamic and thermophysical processes associated with the formation and growth of crystals [1, 2]. The relevance of this kind of research has recently increased due to the observed anomalies in the formation and melting of ice in the Arctic and Antarctic regions, as well as the intensification of the development of offshore oil and gas-bearing areas of the Arctic. Convective structures in a vertical water layer bounded by flat heat exchange surfaces under unsteady boundary conditions are visualized in [3] by methods of Hilbert optics and shear interferometry. Numerical simulation of temperature field as of field of isotherms in the mode of monotonic cooling of vertical walls is performed. The inverse problem of reconstruction of interferograms and hilbertograms from the numerical model of the temperature field is solved fragmentally.

In the development of these studies raises the problem of the reproduction of the co-evolution of the reconstructed interference fields and the Hilbert structures of the convective flows in comparison...
with the results obtained in the experiments. The novelty of the problem is due to the almost complete lack of data on the development of convective flows in the conditions of monotonic temperature changes on the vertical walls, taking into account the inversion of the water density in the vicinity of the isotherm (+4 °C) and consists in a comprehensive approach combining the results of a physical and numerical experiment in which the simulation of the isotherm field is supplemented by the calculation of the velocity field and temperature gradients.

2. Method
A sketch of a research complex containing an optical diagnostic system and an experimental stand is shown in figure 1 [3].

Figure 1. Sketch of the research complex.

The complex contains a lighting module consisting of a light source (1), a condenser lens (2) and a slit diaphragm (3) placed in the front focal plane of the lens (4), which forms a probing field in the test medium. Such an environment is water filling a rectangular cavity (5) with transparent glasses of optical quality.

The experimental stand is a rectangular cavity filled with water, with transparent optical-quality glasses. The internal dimensions of the cavity are 136×86×30 mm. Side walls are formed by parallel arranged cooled flat copper plates (6) and (7). On the outer side of the plates there are cavities into which coolant is supplied from the cryostat and thermostat (8) and (9). The temperature of one of the plates by pumping coolant from the cryostat can reach minus 28 °C. The temperature of the other plate is maintained in the range from room temperature to +8 °C. Thermostats are controlled by computer (10) according to a given program. The tubes supplying the refrigerant to the heat exchange side plates are designated as (11)–(14), the outflow tubes are (15), (16). The arrangement of the feed tubes and outflow tubes may vary according to the conditions of the experiment. The distance between the tubes is 80 mm. The lens (17) forms in the frequency plane the Fourier spectrum of the probing light field perturbed by the medium under study. The Hilbert filter (18) is placed in the Fourier plane of the lens (17). The frequency axis $K_x$ of the Hilbert filter is orthogonal to the direction of the image of the slit diaphragm of the light source ($\psi = 45^\circ$). Lens (19) performs the inverse Fourier transform of the filtered spatial-frequency spectrum. The Hilbert image of the phase structure of the light field perturbed by the medium under investigation is recorded by a digital video camera (20) connected to
the computer (10). The optical diagnostics system was implemented on the basis of the serial IAB-463M shadow device [4], in which the following modules were modified: the light source module, the Fourier filtering module of the optical signal, the image recording module of the phase perturbations of the light field induced by the medium under study and module of shear interferometry. The probing field, formed by the lens (4), passes through the test medium (water), in which the temperature boundary conditions \(T_1\) and \(T_2\), \(T_1 < T_2\) are set according to a given program by lateral thermostatted surfaces (6) and (7).

The coherent transfer function of the spatial-frequency filter 18 that performs the one-dimensional Hilbert transform is described by the expression:

\[
H(K_x, K_y) = \exp(-i\phi) \sigma(K_x) + \exp(i\phi) \sigma(-K_x)\sigma(K_y) + +[\exp(i\phi)\sigma(K_x) + \exp(-i\phi)\sigma(-K_x)]\sigma(-K_y) = \cos \phi - i \sin \phi \text{sgn} K_x,
\]

where \(K_x\) and \(K_y\) are spatial frequencies; \(\sigma(\pm K_x)\) and \(\sigma(\mp K_y)\) are Heaviside functions; \(\phi\) is the phase shift defined by the corresponding quadrant of the spatial-frequency filter. A filter with a coherent transfer function (1) performs a one-dimensional Foucault-Hilbert transform. In the Fourier plane \(H(K_x, K_y)\) the spatial frequency axis \(K_x\) is orthogonal to the image of the slot light source. Fourier spectrum of the light field directly after the filter:

\[
s(K_x, K_y)H(K_x, K_y) = s(K_x, K_y)\cos \phi + \hat{s}_x(K_x, K_y) \sin \phi
\]

(2)

Here \(s(K_x, K_y)\) is the spatial-frequency Fourier spectrum of the light field perturbed by the medium under study; \(\hat{s}_x(K_x, K_y) = -i \text{sgn}(K_x)s(K_x, K_y)\) is the Fourier spectrum of the light field subjected to a one-dimensional Hilbert transform along the axis \(K_x\). The phase shift \(\phi\) is a function of the wavelength \(\lambda\) of the probing light field: \(\phi = \phi(\lambda)\). When the wavelength \(\lambda = \lambda_0\), satisfies the condition \(\phi(\lambda) = \pi/2\), the Fourier filter (18) performs a one-dimensional Hilbert transform:

\[
\hat{s}_x(K_x, K_y) = -i \text{sgn}(K_x)s(K_x, K_y)
\]

(3)

Convective structures appear in the space between the thermostated surfaces, which are manifested as perturbations of the optical phase density fields. These structures are induced by convection complicated by water density anomaly in the temperature range \(0\,\text{oC}\). The optical diagnostics system by the methods of Hilbert optics and shear interferometry performs visualization of gradients of perturbations of the phase optical density induced in the water layer by non-stationary temperature boundary conditions on vertical heat exchange surfaces limiting the medium under study.

Near the temperature-controlled surface, as the temperature gradient grows, water turns into a supercooled liquid, passing into a state of unstable equilibrium. In such a medium, the transition from liquid phase to solid-crystalline state occurs. This is a phase transition of the first kind. It becomes apparent through the appearance of a crystallization wave and is accompanied by an energy release. In its turn the release affects the dynamic distribution of the optical phase density gradient in supercooled water and induces phase perturbations in the probing light field, the Fourier spectrum of which forms in the frequency plane of the lens (17).

Figure 2 shows a frame from the video-film attached to the article, which illustrates the results of visualization of the phase density field by Hilbert-optics (a) and shear interferometry (b). As a verification of the results obtained in [4], numerical simulation of the temperature fields of convective flows in the form of a dynamic structure of isotherms is performed. From these structures, shear interferograms and hilbertograms were reconstructed, which were compared with the results obtained in the experiment.

Convective heat transfer in a liquid in a two-dimensional formulation was described by a dimensionless system of Navier-Stokes equations, energy and continuity in the approximation Boussinesq. Conductive heat transfer in massive horizontal walls of Plexiglas was modeled using the heat transfer equation.

\[\text{doi:10.1088/1742-6596/1421/1/012047}\]
Figure 2. Visualization of the phase density field by Hilbert-optics (a) and shear interferometry (b).

The equations were solved numerically by the finite element method in the conjugate formulation. Dependences of density and coefficient of volumetric thermal expansion on temperature were taken into account. At the solid boundaries for the velocity (and, accordingly, for the current function), the conditions of adhesion and non-flow through were assigned. The boundary condition for the vortex is obtained from the field of the function of current using the method of conjugate results [3]. At the interface of the liquid with solid walls the condition of continuity of temperature and heat flow was set.

An uneven triangular grid, thickened bunched to varying degrees to all the boundaries of the computational domain, with the number of nodes $\approx 4 \times 10^4$ was used. Linear basis functions be given on the elements. A cellular step-by-step algorithm for the maximum angle was used to construct the triangulation [5]. The program implements an iterative process, in which, if necessary, the necessary values of the variables from the previous steps and the calculated values of the coefficients on the parameters are substituted.

The problem is solved in a non-stationary situation: the initial temperature of the system is $+20$ °C, the left vertical wall is monotonically cooled to $+8$ °C for 2500 seconds. The right wall was cooled to $0$ °C for 2500 seconds. The outer surfaces of the horizontal walls are adiabatic. In this paper, the simulation of the isotherms field is supplemented by the calculation of the velocity field and temperature gradients.

Temperature fields are reconstructed from the numerically models of isotherms (figure 2 (a)). The graphs are constructed in relative units, (in the figures the real size of the cuvette width equal to 86 mm is taken as a scale in the transition to dimensionless equations and boundary conditions).

Temperature boundary conditions on the left and right walls are indicated, respectively, $T_1$ and $T_2$. The visualized hilbertograms represent quasi-gradiented structure of the optical phase field $\Delta \varphi(x, y, T)$, induced in the under investigation environmental by the temperature boundary conditions:

$$\Delta \varphi(x, y, T) = k [n(x, y, T) - n_0] l,$$

where $l$ is the thickness of the liquid layer, $k = 2\pi/\lambda$ is the wave number of the light field, $n(x, y, T)$ is the refractive index of the liquid as a function of the spatial coordinates $x, y$ and temperature $T$.

This function is determined by the temperature dependence of the specific refraction of water. Temperature dependence of the refractive index of distilled water (figure 4) was calculated, according to [6, 7], on the density and specific refraction at a wavelength $\lambda = 589.3$ nm at normal atmospheric pressure.
The optical phase field was calculated from the spatial distribution of the refractive index (figure 5). The Hilbert transformation of this field was performed and the shear interferograms was reconstructed.

Figure 4. The dependence of the refractive index of distilled water on temperature from −2 °C to +21 °C [5,6].

Figure 5. Field of refractive index of distilled water (x, y axis).
Figure 6. Reconstruction of Hilbert fields of phase optical density.

Figure 7 shows a frame from another video attached to the article.

Figure 7. Reconstruction of shear interferograms of the optical density field.

At the top left a simulation of evolution of the fields of isotherms, temperature gradients and velocity fields is show, at the bottom left – shear interferograms reconstructed from the numerically obtained field of isotherms. On the right, the figure shows an experimentally obtained interference field that displays the structure of the convective flow induced by nonstationary boundary conditions. The film illustrates the qualitative agreement between the results of numerical simulation and experiment.
Figure 8. Convective structures induced by unsteady boundary conditions in the horizontal layer bounded by heat exchange surfaces: (a) – cooling; (b) – under the ice surface; ice formation; (c) – phase transition.

In development of the results obtained in [8], the convective structures and phase transition induced by unsteady boundary conditions in a horizontal water layer bounded by heat exchange surfaces are investigated. The fields of the phase optical density of convective structures are visualized by methods of the Hilbert optics. The dynamics of the crystallization wave front induced by a temperature gradient on the upper boundary of a horizontal layer of water bounded by thermostatted flat surfaces is studied. The dynamic profile of the isothermal surface bounding the spatial form of the solid phase on the water-ice interface is visualized. Video frames in figure 8 illustrate the evolution of convective structures in the cooling process (a), under the ice surface (b) and phase transition (c). The results obtained in the present study illustrate the efficiency of optical diagnostic methods based on the Hilbert optics. They are of interest for the development of physical models of phase transitions, studying ice formation processes, and various applications.

3. Conclusion

Methods the Hilbert-optics, shearing interferometry and numerical simulations are performed to study the evolution of convective structures induced by unsteady boundary conditions in the vertical layer of water bounded by flat heat exchange surfaces, in conditions of monotonic cooling. Solved the problem of the reconstruction of the Hilbert-images and shear interferograms fields of the phase optical density from numerically obtained fields of evolution of isotherms and velocity. The evolution of convective structures and phase transition induced by non-stationary boundary conditions in the horizontal water layer between the heat exchange surfaces is studied. Studies of convective flows induced by unsteady boundary conditions are relevant for solving various fundamental and applied problems, an example of
which is the development of new technologies for crystal growth, understanding of convective processes in the water column in the study of the Arctic and Antarctic regions of the world ocean.

**Acknowledgments**
The work was performed within the state task of IT SB RAS (State. reg. AAA–AH17–117030310010–9 and AAA–A17–117022850021–3).

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