SO(4,C)-covariant Ashtekar–Barbero gravity and the Immirzi parameter

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Abstract

An so(4,C)-covariant hamiltonian formulation of a family of generalized Hilbert–Palatini actions depending on a parameter (the so called Immirzi parameter) is developed. It encompasses the Ashtekar–Barbero gravity which serves as a basis of quantum loop gravity. Dirac quantization of this system is constructed. Next we study dependence of the quantum system on the Immirzi parameter. The path integral quantization shows no dependence on it. A way to modify the loop approach in the accordance with the formalism developed here is briefly outlined.

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1 Introduction

The construction of the complete theory of quantum gravity is still an open problem. There are several approaches to quantization of general relativity and to understanding what the quantum spacetime is. One of the most promising approaches is loop quantum gravity [1], [2], [3]. It is mathematically well-defined and explicitly background independent. In this framework a set of remarkable physical results has been obtained such as the discrete spectra of the area and volume operators [4], [5] and a derivation of the Bekenstein–Hawking formula for the black hole entropy [6].

However, there are still several important problems. One of them is the so called Immirzi parameter problem. This problem arises due to the results obtained for the spectra of the geometrical operators and the black hole entropy are proportional to an unphysical parameter. It is called ”Immirzi parameter” [7] and appears as a parameter of a canonical transformation in classical gravity [8], [9]. So at least at the classical level this parameter should be unphysical. The problem is whether the quantum theory can nevertheless depend on it and, if not, why we observe this dependence for the physical quantities. While this problem is not resolved, it does not allow to interpret the discrete spectra of the area and volume as evidence for a discrete structure of spacetime. Several interpretations [7] of this dependence have been proposed but there is no any acceptable explanation yet.

In this paper we suggest a new strategy to tackle the Immirzi parameter problem. It is based on the use of a larger symmetry group 1. Namely, our aim is to develop the

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1 The idea that the problem of the Immirzi parameter requires a group larger than SU(2) has been stressed by Immirzi himself in the work where the problem has been discovered [9].
canonical formalism for gravity with the full Lorentz gauge group in the tangent space. In contrast to the standard approach we do not impose any gauge fixing like “time gauge” used to obtain the Ashtekar–Barbero formulation [8], which lies in the ground of quantum loop gravity. However, we are still able to use a connection as a canonical variable, but it turns out to be \( \text{so}(4, \mathbb{C}) \) connection rather than \( \text{su}(2) \). It is possible since the formalism can be put in an \( \text{so}(4, \mathbb{C}) \) covariant form so that all calculations are carried out in a nice and elegant way.

Then this covariant representation is used for the investigation of dependence of the quantum theory on the Immirzi parameter. In this paper we give only some preliminary considerations in the frameworks of the path integral quantization and a modified loop approach. They can gain an insight on the Immirzi parameter problem. However, more elaborated technics is needed to set these considerations on a solid ground.

The paper is organized as follows. In the next section the 3+1 decomposition of the generalized Hilbert–Palatini action is obtained using the results from Ashtekar gravity [10], [11]. The decomposed action is presented in an \( \text{so}(4, \mathbb{C}) \) covariant form. In section 3 the hamiltonian formalism is constructed and the canonical commutation relations are obtained. Section 4 is devoted to application of the developed formalism to the investigation of dependence of the quantum theory on the Immirzi parameter. In the first subsection the path integral for gravity described by the generalized Hilbert–Palatini action is shown to be independent on the Immirzi parameter in the so called Yang–Mills gauge. The second one is intended to present some ideas how this formalism can be put in the ground of the loop approach and how it can cure the Immirzi parameter problem. Some concluding remarks are placed in section 5. The appendices contain some general formulas and examples.

We use the following notations for indices. The indices \( i, j, \ldots \) from the middle of the alphabet label the space coordinates. The latin indices \( a, b, \ldots \) from the beginning of the alphabet are the \( \text{su}(2) \) indices, whereas the capital letters \( X, Y, \ldots \) from the end of the alphabet are the \( \text{so}(3, 1) \) or \( \text{so}(4, \mathbb{C}) \) indices.

## 2 Generalized Hilbert–Palatini action

We start with the generalized Hilbert–Palatini action suggested by Holst

\[
S_{(\beta)} = \frac{1}{2} \int \varepsilon_{\alpha\beta\gamma\delta} e^\alpha \wedge e^\beta \wedge (\Omega^{\gamma\delta} + \frac{1}{\beta} \star \Omega^{\gamma\delta}).
\]

(1)

Here the star operator is defined as \( \star \omega^{\alpha\beta} = \frac{1}{2} \varepsilon^{\alpha\beta\gamma\delta} \omega_{\gamma\delta} \) and \( \Omega^{\alpha\beta} \) is the curvature of the spin-connection \( \omega^{\alpha\beta} \). In the work [12] Holst has shown that in the “time” gauge this action reproduces Barbero’s formulation [8] and the parameter \( \beta \) plays the role of the Immirzi parameter. Our aim is to investigate the action (1) without imposing any gauge fixing. As we shall see the hamiltonian formulation of the theory described by this action allows a remarkable covariant representation.

To this end, let us do the 3+1 decomposition. It easily can be obtained from the decomposition of the self-dual Hilbert–Palatini action \( (\beta = i) \) leading to Ashtekar gravity. Such decomposition in suitable variables without a gauge fixing has been obtained in [13] and looks like

\[
S_A = 2 \int dt \, d^4x (P^{(i)a}_\alpha A_{(i)a}^\alpha + A_{(0)a}^\alpha \mathcal{G}_a^{(A)} + \mathcal{N}_D i H_i^{(A)} + \mathcal{N} H^{(A)}),
\]

(2)
Here we split the 6-dimensional index $X$ structure constants shown the fields form multiplets in the adjoint representation of $\text{so}(4,\mathbb{C})$ with the following relations between the triad multiplets are presented in Appendix A. As it will be shown, the fields form multiplets in the adjoint representation of $\text{so}(4,\mathbb{C})$ with the following structure constants:

\[ f_{AB}^C = 0, \quad f_{AB}^C = \varepsilon_{ABC}, \quad f_{AB}^C = 0, \quad f_{AB}^C = \varepsilon_{ABC}. \]

Here we split the 6-dimensional index $X$ into a pair of 3-dimensional indices, $X = (A, B)$, so that $A, B = 1, 2, 3$. $\varepsilon$ is the Levi–Civita symbol, $\varepsilon^{123} = 1$. 

\[ \mathcal{G}_a = \nabla_i P_{i(a)}^a = \partial_i P_{i(a)}^a - \varepsilon_{ab} \varepsilon_i^b A_i^c P_{i(a)}^c, \]
\[ H_i^A = -P_{i(a)}^k F_{i}^{(A)k}, \]
\[ H^A = -\frac{1}{2} P_{i(a)}^i P_{i(b)}^j \varepsilon_{ab} F_{ij}^{(A)}, \]

where

\[ F_{ij}^{(A)} = \partial_i A_j^{(A)} - \partial_j A_i^{(A)} - \varepsilon^{abc} A_i^{(A)b} A_j^{(A)c}, \]
\[ A_i^{(A)} = \xi_i^a - i \varepsilon_i^a = \frac{1}{2} \varepsilon_{abc} \omega_i^a - i \omega_i^a, \]
\[ P_i^{(A)} = \varepsilon_{abc} E_b^a \chi_c + i \tilde{E}_a^i. \]

The label $(i)$ refers to the value of $\beta$. The Lagrange multipliers $\mathcal{N}_D, \mathcal{N}$, triad $\tilde{E}_a^i$ and field $\chi_a$ arise from the decomposition of the tetrad:

\[ \varepsilon^0 = \mathcal{N} dt + \chi_a E_i^a dx^i, \quad \varepsilon^a = E_i^a dx^i + E_i^a N_i^a dt, \]
\[ \tilde{E}_a^i = h_i^{1/2} E_a^i, \quad \mathcal{N} = h^{-1/2} N, \quad \sqrt{h} = \text{det} E_i^a, \]
\[ N_i^a = \mathcal{N}_D + \tilde{E}_a^i \chi^a \mathcal{N}, \quad \mathcal{N} = \mathcal{N}_D + E_i^a \chi_a N_D. \]

Here $E_i^a$ is the inverse of $E_a^i$. The field $\chi_a$ describes deviation of the normal to the spacelike hypersurface $(t = 0)$ from the time direction.

Let us return to the generalized Hilbert–Palatini action (1). It is related to the Ashtekar action by

\[ S_{(\beta)} = \text{Re} S_A - \frac{1}{\beta} \text{Im} S_A. \]

It is convenient to combine pairs of the $\text{su}(2)$ indices $a$ into a single $\text{so}(3,1)$ index $X$. Accordingly we can introduce the $\text{so}(3,1)$ multiplets:

\[ \mathcal{G}_X = (\mathcal{L}_a, \mathcal{G}_a) = (\text{Im} \mathcal{G}_a^{(A)}, \text{Re} \mathcal{G}_a^{(A)}) \quad \text{– constraint multiplet,} \]
\[ A_X = (\xi^a, \xi_a) \quad \text{– connection multiplet,} \]
\[ \tilde{P}_X^a = (\tilde{E}_a^i, \varepsilon_{abc} \tilde{E}_b^a \chi_c) \quad \text{– first triad multiplet,} \]
\[ \tilde{Q}_X^a = (-\varepsilon_{abc} \tilde{E}_b^a \chi c, \tilde{E}_a^i) \quad \text{– second triad multiplet,} \]
\[ \tilde{P}^{(i)}_X = \tilde{P}_X^a - \frac{i}{\beta} \tilde{Q}_X^a \quad \text{– dynamical triad multiplet.} \]

The relations between the triad multiplets are presented in Appendix A. As it will be shown, the fields form multiplets in the adjoint representation of $\text{so}(4,\mathbb{C})$ with the following structure constants:

\[ f_{AB}^C = 0, \quad f_{AB}^C = \varepsilon_{ABC}, \quad f_{AB}^C = 0, \quad f_{AB}^C = \varepsilon_{ABC}. \]
With these multiplets using (5) and (2) the action (1) can be represented in the form:

\[
S(\beta) = \int dt \ d^3x (\tilde{P}_{(\beta)}^i X \partial A_i X + N_G X G_X + N_{p} H_i + N H), \tag{8}
\]

\[
G_X = \partial_i \tilde{P}_{(\beta)}^i X + f_{XY} Z A_i \tilde{P}_{(\beta)}^i Z, \quad H_i = -\tilde{P}_{(\beta)}^i X F_{ij}, \quad H = -\frac{1}{2} \tilde{P}_{(\beta)}^i X \tilde{P}_Y Z f_{XY} Z F_{ij},
\]

\[
F_{ij}^X = \partial_i A_j^X - \partial_j A_i^X + f_{XY} Z A_i^Y A_j^Z,
\]

where we have used the Killing form to raise and low indices \( f_{XY}^Z = g^{XX'} g^{YY'} g_{ZZ'} f_{XY}^Z \):

\[
g_{XY} = f_{XZ} f_{YZ}, \quad g^{XY} = (g^{-1})^{XY}, \quad g_{XY} = \begin{pmatrix} \delta_{ab} & 0 \\ 0 & -\delta_{ab} \end{pmatrix}.
\tag{9}
\]

As a result we have represented the 3+1 decomposed action in the so(4,C) covariant form. Moreover, it is covariant under arbitrary transformations of the basis of the adjoint representation. If we change

\[
G_X \rightarrow (U^{-1})^X Y G_Y, \quad \tilde{E}_X^i \rightarrow (U^{-1})^X Y \tilde{E}_Y^i, \quad A_i^X \rightarrow U Y A_i^Y,
\tag{10}
\]

where \( U^X_Y \) is an arbitrary invertible matrix and \( \tilde{E}_X^i \) denotes any triad multiplet, the representation (8) is unchanged. An example of a formulation in such rotated basis, which appears naturally for the action (1) rewritten in terms of curvatures only without the star operator, is presented in Appendix B. From now on \( G_X, \tilde{E}_X^i, A_i^X \) will denote multiplets in an arbitrary basis.

### 3 Covariant canonical formulation

Let us construct the canonical formalism for the action (8). From the beginning \( A_i^X \) and \( \tilde{P}_{(\beta)}^i X \) are canonical variables, i.e.,

\[
\{ A_i^X, \tilde{P}_{(\beta)}^j Y \} = \delta_i^j \delta^X_Y, \quad \{ A_i^X, Q_{(\beta)}^j Y \} = \delta_i^j (\Lambda^{-1})^X_Y.
\tag{11}
\]

However, there are constraints on the momenta. In the covariant form they can be represented as

\[
\phi^{ij} = \Pi^{XY} \tilde{Q}_X^i \tilde{Q}_Y^j. \tag{12}
\]

The matrices \( \Pi \) and \( \Lambda \) appearing in the formulas above are introduced in Appendix A (41), (42). \( \phi^{ij} \) is symmetric, so there are only six independent constraints. It is clear that \( \{ G_X, \phi^{ij} \} = 0 \) and \( \{ H_k, \phi^{ij} \} \sim \phi^{ij} \). The only nontrivial bracket is with the hamiltonian constraint. Using (8), (44), (45), (46) we obtain

\[
\{ H, \phi^{ij} \} = 2 \tilde{Q}_X^a \tilde{Q}_Y^{ij} f_{XYZ} \left( \partial_n \tilde{Q}_Z^{ij} + f_{T}^{S} A_n X S \tilde{Q}_T^{ij} \right)
= \psi^{ij} + 2\phi^{ij} \tilde{P}_Z^a A_n^Z, \tag{13}
\]

where

\[
\psi^{ij} = 2 f_{XYZ} \tilde{Q}_X^a \tilde{Q}_Y^{ij} \partial_n \tilde{Q}_Z^{ij} - 2(\tilde{Q} \tilde{Q})^{[ij]} \tilde{Q}_Z^a A_n^Z, \tag{14}
\]

\[
(\tilde{Q} \tilde{Q})^{ij} = g^{XY} \tilde{Q}_X^i \tilde{Q}_Y^j \tag{15}
\]
and symmetrization is taken with the weight $1/\beta$, whereas antisymmetrization does not include any weight. It is remarkable that the second class constraints (12) and (14) don’t depend on $\beta$ that proves consistency of the constraints in different formulations.

One can calculate

$$D^{(ij)(kl)}_1 = \{\phi^{ij}, \psi^{kl}\} = \frac{4\beta^2}{1 + \beta^2} (\tilde{Q}\tilde{Q})^{ij} (\tilde{Q}\tilde{Q})^{kl},$$

$$D^{(ij)(kl)}_2 = \{\psi^{ij}, \psi^{kl}\} \approx \frac{8\beta^2}{1 + \beta^2} \left[ (\tilde{Q}\tilde{Q})^{ik} f_{XYZ} \tilde{Q}_X^{ij} \partial_n \tilde{P}_Z^n + (\tilde{Q}\tilde{Q})^{ij} f_{XYZ} \tilde{P}_X^n \tilde{Q}_Y^{k} \partial_n \tilde{Q}_Z^n - (\tilde{Q}\tilde{Q})^{kl} f_{XYZ} \tilde{P}_X^n \tilde{Q}_Y^{(i} \partial_n \tilde{Q}_Z^{j)} \right]$$

where in the second equality we used the Gauss constraint $\mathcal{G}_X$ and both second class constraints (12) and (14).

Let us redefine the constraints $\Phi_\alpha = (\mathcal{G}_X, H_i, H)$:

$$\tilde{\Phi}_\alpha = \Phi_\alpha - \phi^{ij} (D^{-1}_1)_{(ij)(kl)} \{\Phi_\alpha, \psi^{kl}\}.$$ (18)

Then $\tilde{\Phi}_\alpha$ are first class constraints with the algebra presented in Appendix C. The remaining constraints are second class. They form the matrix of commutators:

$$\Delta = \begin{pmatrix} 0 & D_1 \\ -D_1 & D_2 \end{pmatrix}, \quad \Delta^{-1} = \begin{pmatrix} D_1^{-1} & D_2 D_1^{-1} \\ D_1^{-1} & 0 \end{pmatrix}. $$ (19)

It gives rise to the Dirac bracket [15]

$$\{K, L\}_D = \{K, L\} - \{K, \varphi_r\} (\Delta^{-1})_{rr'} \{\varphi_{r'}, L\},$$ (20)

where $\varphi_r = (\phi^{ij}, \psi^{ij})$. However, it is simplified when one of the functions coincides with the first class constraint $\Phi_\alpha$ (or $\tilde{\Phi}_\alpha$). Then

$$\{\Phi_\alpha, L\}_D \approx \{\Phi_\alpha, L\} - \{\Phi_\alpha, \psi^{ij}\} (D^{-1}_1)_{(ij)(kl)} \{\phi^{kl}, L\}. $$ (21)

From (13) and the Jacoby identity one can see that $\{\mathcal{G}_X, \psi^{ij}\} = 0$ and $\{H_k, \psi^{ij}\} \sim \psi^{ij}$. Thus the last term in (21) survives only in the case when $\Phi_\alpha$ is the hamiltonian constraint and $L$ depends on the connection variables. In all other cases the Dirac bracket coincides with the ordinary one.

This fact is the reason for the remarkable relations between the brackets in different formulations, which can be easily checked by a direct calculation:

$$\{\Phi^{(\beta)}_\alpha, \tilde{P}_{(\beta)X}^i\}_D^{(\beta)} = \{\Phi^{(\beta)}_\alpha, \tilde{P}_{(\beta)X}^i\}_D^{(\beta)},$$

$$\{\Phi^{(\beta)}_\mu, \tilde{A}_X^i\}_D^{(\beta)} = \{\Phi^{(\beta)}_\mu, \tilde{A}_X^i\}_D^{(\beta)}.$$ (22)

Here the label $(\beta)$ indicates the formulation which a given object are taken from. The last equality is not valid for the hamiltonian constraint so that $\Phi_\mu = (\mathcal{G}_X, H_i)$.

Using the coincidence of the Dirac and Poisson brackets (21) the transformation laws of the multiplets can easily be found:

$$\{\mathcal{G}_X, \mathcal{G}_Y\}_D = f_{XY}^Z \mathcal{G}_Z,$$

$$\{\mathcal{G}_X, \tilde{A}_Y^i\}_D = \delta_X^Y \partial_i - f_{XZ}^Y \tilde{A}_Z^i,$$

$$\{\mathcal{G}_X, \tilde{E}_Y^i\}_D = f_{XY}^Z \tilde{E}_Z^i.$$ (23)

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3If we defined the secondary second class constraint as the full r.h.s. of (13), the Poisson bracket $D_2$ would vanish whereas $D_1$ would remain the same. This could be advantageous for calculation of the Dirac bracket defined below.
As it was declared they form the adjoint representation of so(4,C). $A_1^X$ is a true so(4,C)-
connection.

To be sure that we do have the so(4,C) gauge algebra rather than its real form only, one should check that complex gauge transformations are allowed. The criterion is reality of the 3-dimensional metric and its evolution:

$$g^{ij} = g^{XY} \tilde{P}_X^i \tilde{P}_Y^j = \frac{\beta^2}{1 + \beta^2} g^{XY} \tilde{P}_{(\beta)X}^i \tilde{P}_{(\beta)Y}^j.$$  \hspace{1cm} (24)

Since $\{G_X, g^{ij}\}_D = 0$, the complex gauge transformations do not destroy reality of the metric. Similarly, the time evolution remains real due to

$$\{G_X, \frac{d}{dt}g^{ij}\}_D = \{G_X, \{f d^3x \left( N_0^X G_Y + N_1^i H_i + N_{\beta} H \right), g^{ij}\}_D \}_D$$

$$= \{\{G(n), f d^3x \left( N_0^X G_X + N_1^i H_i + N_{\beta} H \right)\}_D, g^{ij}\}_D = 0.$$  \hspace{1cm} (25)

To complete the construction of the canonical formalism, we should find $D_1^{-1}$. To this end, introduce the inverse triad multiplets:

$$P_i^X = \left( \begin{array}{c} \delta^a_b - \chi^a\chi^b E^b_i \\ 1 - \chi^2 \end{array} \right),$$

$$Q_i^X = \left( \begin{array}{c} \varepsilon^{abc} E^a_i \chi^c \\ 1 - \chi^2 \end{array} \right).$$  \hspace{1cm} (26)

They obey the following equations:

$$\{G_X, Q_i^Y\} = -f_{YZ}^X Q_i^Z, \quad \{G_X, P_i^Y\} = -f_{YZ}^X P_i^Z,$$

$$\tilde{Q}_i^X \tilde{Q}_j^X = \delta_i^j, \quad \tilde{P}_i^X \tilde{P}_j^X = \delta_i^j,$$

$$\tilde{Q}_i^X P_j^X = \tilde{P}_i^X Q_j^X = 0.$$  \hspace{1cm} (27)

In the basis (6) one can obtain

$$I_{(P),X}^Y \equiv \tilde{P}_X^i P_i^Y = \left( \begin{array}{cc} \frac{\delta^b_a - \chi^a\chi^b}{1 - \chi^2} & \frac{\varepsilon^{abc} \chi^c}{1 - \chi^2} \\ \frac{\varepsilon^{abc} \chi^c}{1 - \chi^2} & \frac{\delta^b_a - \chi^a\chi^b}{1 - \chi^2} \end{array} \right),$$

$$I_{(Q),X}^Y \equiv \tilde{Q}_X^i Q_i^Y = \left( \begin{array}{cc} -\frac{\delta^b_a - \chi^a\chi^b}{1 - \chi^2} & -\frac{\varepsilon^{abc} \chi^c}{1 - \chi^2} \\ -\frac{\varepsilon^{abc} \chi^c}{1 - \chi^2} & -\frac{\delta^b_a - \chi^a\chi^b}{1 - \chi^2} \end{array} \right).$$  \hspace{1cm} (28)

Despite their complicated form the relations (28) have a simple interpretation. The matrices $I_{(P),X}^Y$ and $I_{(Q),X}^Y$ are projectors on $P$ and $Q$-multiplets in the linear space spanned by these vectors. Indeed, the following equalities are fulfilled due to the second class constraints $\phi^{ij}$:

$$I_{(P),X}^Y I_{(P),X}^Z = I_{(P),X}^Y, \quad I_{(Q),X}^Y I_{(Q),X}^Z = I_{(Q),X}^Y,$$

$$I_{(P),X}^Y + I_{(Q),X}^Y = \delta^Y_X,$$

$$I_{(P),X}^Y \tilde{P}_X^i = \tilde{P}_X^i, \quad I_{(P),X}^Y \tilde{Q}_X^i = \tilde{Q}_X^i, \quad I_{(Q),X}^Y \tilde{Q}_X^i = \tilde{Q}_X^i, \quad I_{(Q),X}^Y \tilde{P}_X^i = \tilde{P}_X^i.$$  \hspace{1cm} (29)
As a result one can check that
\[
(D_1^{-1})_{(kl)(mn)} = \frac{1}{8} \left(1 + \frac{1}{\beta^2}\right)((QQ)_{kl}(QQ)_{mn} - (QQ)_{km}(QQ)_{ln} - (QQ)_{kn}(QQ)_{lm}) \tag{30}
\]
gives \( D_{(ij)(kl)}^{-1}(D_1^{-1})_{(kl)(mn)} = \delta^{ij}_{mn} \). The Dirac brackets of the canonical variables take the form:
\[
\{\tilde{P}_{(\beta)}^i_X, \tilde{P}_{(\beta)}^j_Y\}_D = 0, \\
\{A_i^X, \tilde{P}_{(\beta)}^j_Y\}_D &= \delta^i_j \delta^X_Y - \frac{1}{2} \left( g^{XZ} - \frac{1}{\beta} \Pi^{XZ} \right) \left( \tilde{Q}_{\gamma}^i Q_i^W + \delta^i_j I_{(\gamma)\beta}^W \right) g_{WY}, \tag{31} \\
\{A_i^X, A_j^Y\}_D &= -\{A_i^X, \phi^{kl}\}(D_1^{-1})_{(kl)(mn)}\{\psi_{mn}, A_j^Y\}_D \\
&\quad -\{A_i^X, \tilde{P}_{(\beta)}^r\}_D\{A_r^Z, \psi_{mn}\}(D_1^{-1})_{(mn)(kl)}\{\phi^{kl}, A_j^Y\}.
\]

The Dirac brackets (31) represent the commutation algebra which should be used at the quantum level.

4 Notes on the Immirzi parameter problem

4.1 Path integral quantization

In this section we are going to compare the path integrals constructed for the formulations with different values of the Immirzi parameter. Consider the path integral for the theory with the action (8). Choose the gauge fixing condition in the following form:
\[
f^\alpha(\tilde{P}, A) + g^\alpha(\mathcal{N}) = 0. \tag{32}
\]

Here \( \mathcal{N}^\alpha = (\mathcal{N}_D^X, \mathcal{N}_D^\gamma, \mathcal{N}) \) is the set of the Lagrange multipliers. Let us restrict ourselves to a definite class of gauges, which are the Yang–Mills (YM) gauges introduced in the work [13]. They are described by the gauge fixing functions (32) with two additional conditions: (a) \( g^\alpha \) don’t depend on \( \mathcal{N}_D^X \), (b) \( f^\alpha \) don’t depend on \( A_i^X \). Thus we are not allowed to fix the Lagrange multipliers for the Gauss constraint and impose gauge conditions on the connection. Due to these restrictions the multihost interaction terms do not appear in the effective action and the path integral is given by the ordinary phase space path integral [16]:
\[
Z[j, \tilde{J}] = \int \mathcal{D}A_i^X \mathcal{D}\tilde{P}_{(\beta)}^i_X \mathcal{D}\mathcal{N}^\alpha \mathcal{D}\mathcal{C}^\alpha \mathcal{D}\bar{c}_\alpha \sqrt{\Delta} \delta(\phi^{ij}) \delta(\psi^{ij}) \delta(f^\alpha + g^\alpha) \\
\quad \times \exp \left[ i \int dt \left( L'_{\text{eff}} + j_a^i A_i^a + J_a^i \tilde{P}_{(\beta)}^i \right) \right], \tag{33}
\]

where
\[
L'_{\text{eff}} = L_\beta - i\bar{c}_\beta \left( \frac{\partial g^\beta}{\partial \mathcal{N}^\alpha} \partial_{a}^\alpha - \frac{\partial g^\beta}{\partial \mathcal{N}^\gamma} C_{a,\gamma}^\delta \mathcal{N}^\delta + \{\Phi_\alpha^\beta, f^\beta(\tilde{P})\}_D \right) e^\alpha. \tag{34}
\]

\( \Delta \) is taken from (19). We introduced the sources \( J \) for the fields \( \tilde{P} \) rather than \( \tilde{P}_{(\beta)} \) to simplify the comparison of the path integrals for different \( \beta \)'s. It does not change their sense, since \( \tilde{P}_{(\beta)} \) is expressed unambiguously through \( \tilde{P} \) due to (41), (42). In addition,
all physical operators (as, e.g., the area operator) should not depend on $\beta$ and so they are expressed more naturally through $\tilde{P}$.

Let us investigate the dependence of the path integral (33) on the Immirzi parameter $\beta$. We shall try to rewrite the path integral (33) in terms of variables corresponding to $\beta = \infty$ ($\tilde{P}^{(\infty)} = \tilde{P}$) and thus independent on the parameter. For this each $\beta$-dependent contribution will be extracted and discussed. There are several sources of such contributions.

The first source is the delta function $\delta(G(\beta)X)$ appearing after integration over $N X_G$. (We can perform this integration due to the first condition on the YM gauge.) Since $G(\beta)X = -\Lambda X \Pi Z G^{(\infty)}Z$, it gives the multiplier

$$m_1 = \det^{-1}(\Lambda \Pi) = \prod_{x,t} \left(1 + \frac{1}{\beta^2}\right)^{-3}.$$ \hspace{1cm} (35)

The second place where $\beta$-dependent terms arise is the action. However, in [13] it was established that the imaginary part of the Ashtekar action (2) vanishes on the surface of the second class constraints and the Gauss and Lorentz constraints. But it is just that part of the action (1), which introduces the $\beta$-dependence, what can be seen from (5). In addition, the path integral (33) contains the delta functions of the second class constraints $\phi^{ij}$ and $\psi^{ij}$ as well as the delta functions of the constraint $G^{(\beta)}X$. Thus $S_\beta$ can be reduced to $\text{Re} S_A = S^{(\infty)}$ and does not introduce any $\beta$-dependence.

The measure gives three contributions. The first one comes from $D\tilde{P}^{(\beta)}_X = \det^3(\Lambda \Pi) D\tilde{P}^i_X$. Thus

$$m_2 = \det^3(\Lambda \Pi) = \prod_{x,t} \left(1 + \frac{1}{\beta^2}\right)^9.$$ \hspace{1cm} (36)

The second contribution is given by $\sqrt{|\Delta|} = \det(D_1)$. Since $\beta$ enters in (16) in the common multiplier only we obtain

$$m_3 = \det\left(\frac{\beta^2}{1 + \beta^2} \mathbb{I}_{6 \times 6}\right) = \prod_{x,t} \left(1 + \frac{1}{\beta^2}\right)^{-6}.$$ \hspace{1cm} (37)

Finally, the last contribution can arise from the Faddeev-Popov determinant. However, it turns out that the determinants in the formulations with different $\beta$’s coincide. The reason for this is the $\beta$-independence of the structure constants of the constraint algebra (61) and the remarkable relations (22) between the Dirac brackets. Since the YM gauge does not allow for $f^\alpha$ to depend on the connection, one can replace the bracket in (34) by the bracket independent on $\beta$: $\{\Phi^{(\infty)}_\alpha, f^\beta(\tilde{P})\}^{(\infty)}_D$. Thus the Immirzi parameter does not appear in the second term of the effective action (34) which produces the Faddeev-Popov determinant.

As a result the dependence of the path integral on $\beta$ is contained in three multipliers (35), (36) and (37) which cancel each other. Thus we arrive to the conclusion that at least at the formal level the path integral for quantum gravity is independent on the Immirzi parameter.

4.2 Loop approach

Relying on the formalism developed in section 3 one can try to work out an alternative loop approach. The key point is that although $A_X^3$ is noncommutative due to (31) it is
transformed as a true connection under the gauge transformations (23). Thus the Wilson loop operator can be constructed

\[ U_\alpha(a, b) = \mathcal{P} \exp \left( \int_a^b dx^i A^X_i T_X \right), \]  

(38)

where \( \alpha \) is a path between two points \( a \) and \( b \), \( T_X \) is a gauge generator. Using these operators one can try to construct the full Hilbert space of quantum gravity in the same way as it is done in the standard loop approach. However, we encounter the serious obstacle on this way since the simple canonical commutation relations are changed now by the complicated commutators (31). Due to this the operators like (38) fail to form the loop algebra. Whether there is another algebra with an explicit geometric interpretation, which substitutes the loop algebra, is the crucial question for the formalism. Only the existence of this algebra will provide a solid ground for these speculations. So far we have not been able to do it because of a very complicated structure of the last relation in (31).

Even if such an algebra is found there are two most serious difficulties arising owing to the new commutation relations. The first one is connected with the non-compactness of the Lorentz group, which is suggested to be used to define a scalar product in this approach as SU(2) does. However, with the other hand it can open a possibility to tackle the problem of time in canonical quantum gravity. Also a necessity to deal with a non-compact gauge group is stressed in the paper [17].

The second difficulty is that the connection representation is not applicable in this framework due to the noncommutativity of \( A^X_i \). Nevertheless one may hope to extract some physical results relying on algebraic relations only or even to develop the \( \hat{E} \)-representation. For example, one can try to obtain the spectrum of the area operator

\[ A_S = \int_S d^2s \sqrt{\eta_{ij} g^{ij}} \]  

(39)

from its commutators with the Wilson loops, if the vacuum state is an eigenstate of the area operator. Here the metric \( g^{ij} \) is taken from (24). These commutators should be calculated using the quantum version of the commutation relations (31).

In this connection we can observe that

\[ \{ A^X_k, g^{ij} \}_D = g^{XY} (\delta^j_k \hat{P}^i_Y + \delta^i_k \hat{P}^j_Y), \]  

(40)

i.e., the additional contribution of the Dirac bracket cancels the dependence on \( \beta \). It is not clear how this fact reflects in the spectrum, but it shows in what way the Immirzi parameter can disappear from it. Remind, however, that all this will have a sense only after a substitute for the loop algebra is found.

It is worth to notice the crucial difference between the covariant formalism, which is outlined here, and the conventional loop approach with the Ashtekar–Barbero gravity in the ground. In the covariant formalism we have a unique connection for all values of \( \beta \) instead of a one-parameter family of connections in the Ashtekar–Barbero gravity. Since we did not solve the second class constraints, nothing can be added to \( A^X_i \) to obtain a true so(4,C)-connection. Indeed one can construct several quantities from \( \bar{Q} \) and \( Q \) multiplets which are transformed either homogeneously or as so(4,C)-connection. They are similar to the Christoffel connection. So one could use them to obtain a family of so(4,C)-connections. However, it turns out that all of them are transformed in a ”wrong” way under the diffeomorphism constraint. Thus there is only one connection with all right transformation laws.
This provides us with a new look at the generalized Wick rotation connecting the formulations with different \( \beta \)'s \[14\]. From (6) and (8) we observe that in our approach they differ by a shift of the dynamical triad multiplet (and may be a choice of the basis of the adjoint representation). This moves the accent from the connection onto the triad. Let us remind that in the Barbero approach based on the SU(2) gauge subgroup the Wick transformation changes the connection rather than the triad. In our case the connection is unique but there are two triad multiplets. Just this allows to form a one-parameter family of triad multiplets rather than connections. However, we have not succeeded so far in representing this shift by a canonical operator.

5 Conclusions

In this paper we have suggested a new hamiltonian formulation of general relativity based on the full SO(3,1) gauge group. Without any preliminary gauge fixing we have constructed the hamiltonian formalism for the generalized Hilbert-Palatini gravity which encompasses the Ashtekar–Barbero gravity \[8\]. It turns out to be covariant under \( \text{so}(4,C) \) transformations, and the set of the canonical variables forms multiplets in the adjoint representation of this algebra.

Then the developed formalism has been applied to the investigation of dependence of the quantum gravity on the Immirzi parameter. It has been shown in the framework of the formal path integral quantization that the formulations with different values of \( \beta \) should be all equivalent. We also speculate on possible extension of the loop approach to the theory developed above without giving, however, any rigorous results. To be able to develop a loop quantization of the suggested formalism one should overcome a number of difficulties. The main one is to find a loop representation of the algebra of the obtained Dirac brackets. It is quite nontrivial due to the noncommutativity of the connection and since the inverse triad multiplets are involved. It is not obvious whether this is possible at all. But correct transformation properties with respect to the full Lorentz group of the connection entering the Wilson loop operator give the hope that a success may be achieved in this way.

The noncommutativity of the connection is the main technical difficulty of the approach. But at the same time it may indicate the appearance of the noncommutative geometry in the framework. However, there is no a direct analogy between the observed noncommutativity and the one arising in the modern superstring theories, for instance. The main difference is that the former appears already at the classical level. Nevertheless it would be interesting to see how the methods of the noncommutative geometry, if it actually appears here, work in canonical quantum gravity.

Finally, the new realization of the generalized Wick rotation suggested in the end of section 4 allows to review the question how this transformation is implemented in the quantum theory \[7\]. It is very desirable to understand how the fact that this transformation is not canonical even at the classical level combines with the equivalence of the formulations with different \( \beta \)'s established using path integral.

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A Matrix algebra

Introduce the matrices connecting different triad multiplets:

\[
\begin{align*}
\tilde{P}^i_X &= \Pi^i_X \tilde{Q}^i_Y, \\
\Pi^Y_X &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \delta^b_a,
\end{align*}
\]

(41)

\[
\begin{align*}
\tilde{P}^i_{(\beta)X} &= \Lambda^i_X \tilde{Q}^i_Y, \\
\Lambda^Y_X &= \begin{pmatrix} -\frac{1}{\beta} & 1 \\ -1 & -\frac{1}{\beta} \end{pmatrix} \delta^b_a.
\end{align*}
\]

(42)

Although the matrices \(\Pi^Y_X\) and \(\Lambda^Y_X\) do not possess any symmetry the matrices \(\Pi^{XY} = g^{XZ} \Pi^Y_Z\) and \(\Lambda^{XY} = g^{XZ} \Lambda^Y_Z\) turn out to be symmetric. Besides, the following relations are fulfilled:

\[
\begin{align*}
\Lambda^Y_X &= \Pi^Y_X - \frac{1}{\beta} \delta^Y_X, \\
(\Pi^{-1})^Y_X &= -\Pi^Y_X, \\
(\Lambda^{-1})^Y_X &= -\frac{\Lambda^Y_X + \frac{1}{\beta} \delta^Y_X}{1 + \frac{1}{\beta^2}} = -\frac{\Pi^Y_X + \frac{1}{\beta} \delta^Y_X}{1 + \frac{1}{\beta^2}}.
\end{align*}
\]

(43)

Due to these relations \(\Pi, \Lambda\) and their inverse commute with each other. Furthermore, they commute with the structure constants in the following sense:

\[
f^{XYZ} \Pi^Z_{Y'} = f^{XY'Z} \Pi^Y_{Y'}.\]

(45)

One more useful relation is

\[
f_{XY}^T f_{WZ}^W = -g_{XZ} \delta^W_Y + g_{YZ} \delta^W_X + \Pi_{XZ} \Pi^W_Y - \Pi_{YZ} \Pi^W_X.
\]

(46)

Being established in the basis (6), all these relations are valid in an arbitrary basis.

B Dual representation

There is a special choice of the basis of the adjoint representation of \(so(4,\mathbb{C})\) which is closely connected with the variables used in Barbero’s formulation [8]. Let us express the action (1) in terms of a connection reduced in the “time” gauge to the Barbero connection and without the star operator. However, in contrast to the self-dual case it can be done only using two connections. Since there is no way to decide which connection should contain \(\beta\), we define them in the symmetric way:

\[
A_{(1)}^{\alpha\beta} = \frac{1}{2}(\omega^{\alpha\beta} - \beta_1 \star \omega^{\alpha\beta}), \quad A_{(2)}^{\alpha\beta} = \frac{1}{2}(\omega^{\alpha\beta} - \beta_2 \star \omega^{\alpha\beta}).
\]

(47)

The field strength two-forms are

\[
\begin{align*}
\mathcal{F}_{(1)}^{\alpha\beta} &= dA_{(1)}^{\alpha\beta} + A_{(1)}^{\alpha} \wedge A_{(1)}^{\beta} = \frac{1}{2}(\Omega^{\alpha\beta} - \beta_1 \star \Omega^{\alpha\beta}) - \frac{1}{4}(1 + \beta_1^2) \omega^{\alpha} \wedge \omega^{\beta}, \\
\mathcal{F}_{(2)}^{\alpha\beta} &= dA_{(2)}^{\alpha\beta} + A_{(2)}^{\alpha} \wedge A_{(2)}^{\beta} = \frac{1}{2}(\Omega^{\alpha\beta} - \beta_2 \star \Omega^{\alpha\beta}) - \frac{1}{4}(1 + \beta_2^2) \omega^{\alpha} \wedge \omega^{\beta}.
\end{align*}
\]

(48)

(49)

They obey the relation

\[
\frac{1}{\beta_1^2 - \beta_2^2} \left( (1 + \beta_1^2) \mathcal{F}_{(1)}^{\alpha\beta} - (1 + \beta_2^2) \mathcal{F}_{(2)}^{\alpha\beta} \right) = \frac{1}{2} \Omega^{\alpha\beta} - \frac{1}{2(\beta_1 + \beta_2)} \star \Omega^{\alpha\beta}.
\]

(50)
It means that we can rewrite (1) in the required form if we set \( \beta = \frac{\beta_1 + \beta_2}{1 - \beta_1 \beta_2} \):

\[
S(\beta) = -\frac{1}{\beta_1^2 - \beta_2^2} \int \epsilon_{\alpha\beta\gamma\delta} e^\gamma \wedge e^\delta \wedge ((1 + \beta_2^2) F_{(1)}^{\alpha\beta} - (1 + \beta_1^2) F_{(2)}^{\alpha\beta}).
\]  

(51)

The action \( S(\beta) \) is invariant under the "duality" transformation \( A_{(1)} \longleftrightarrow A_{(2)} \), \( \beta_1 \longleftrightarrow \beta_2 \). It is just a generalization of the selfduality leading to the Ashtekar action, which can be obtained from \( S(\beta) \) in the limit \( \beta_1 = i \) (or \( \beta_2 = i \)). Another useful limit is \( \beta_2 = 0, \beta_1 \rightarrow \infty \), which together with the redefinition of the connection \( A_{(1)} \rightarrow \frac{1}{\beta_1} A_{(1)} \) leads to the Hilbert–Palatini gravity.

Of course, the observed duality is trivial in the sense it is only a change of variables. Besides, indeed the theory depends on one parameter only. So one of \( \beta \)'s can be fixed in a way. For example, we can set \( \beta_2 = 0 \). But we shall keep them to be arbitrary to retain the duality.

The 3+1 decomposition of the action (51) can be obtained using the definitions (4). The result is represented in the covariant form (8), but the natural choice of the multiplets looks as

\[
G_X = \left( \begin{array}{c} \frac{1 + \beta_2^2}{\beta_2^2 - \beta_1^2} G^{(1)}_a, \ \frac{1 + \beta_2^2}{\beta_2^2 - \beta_1^2} G^{(2)}_a, \end{array} \right),
\]

\[
A_i^X = \left( \begin{array}{c} \frac{1 + \beta_2^2}{\beta_2^2 - \beta_1^2} A^{(1)}_i^a, \ \frac{1 + \beta_2^2}{\beta_2^2 - \beta_1^2} A^{(2)}_i^a, \end{array} \right),
\]

\[
\tilde{P}^{(1)}_{(\beta)X} = \left( \begin{array}{c} \frac{1 + \beta_2^2}{\beta_2^2 - \beta_1^2} P^{(1)}_a, \ \frac{1 + \beta_2^2}{\beta_2^2 - \beta_1^2} P^{(2)}_a, \end{array} \right),
\]

(52)

where we introduced the following fields:

\[
P^{(1)}_{(1)a} = \epsilon_a^{\quad bc} \tilde{E}_b^i X_c^i + \beta_1 \tilde{E}_b^i, \quad P^{(2)}_{(2)a} = \epsilon_a^{\quad bc} \tilde{E}_b^i X_c^i + \beta_2 \tilde{E}_b^i,
\]

\[
A^{(1)}_i^a = \xi_i^a - \beta_1 \zeta_i^a, \quad A^{(2)}_i^a = \xi_i^a - \beta_2 \zeta_i^a,
\]

\[
G^{(1)}_a = \partial_i P^{(1)}_{(1)c} - \epsilon_{abc} A^{(1)}_i^c \chi^{(1)}_c + \frac{1 + \beta_2^2}{(\beta_1 - \beta_2)^2} \epsilon_{abc} (A^{(1)}_i^c - A^{(2)}_i^c) (P^{(1)}_c - P^{(2)}_c),
\]

\[
G^{(2)}_a = \partial_i P^{(2)}_{(2)c} - \epsilon_{abc} A^{(2)}_i^c \chi^{(2)}_c + \frac{1 + \beta_2^2}{(\beta_1 - \beta_2)^2} \epsilon_{abc} (A^{(1)}_i^c - A^{(2)}_i^c) (P^{(1)}_c - P^{(2)}_c).
\]

(53-56)

These multiplets are related with the multiplets (6) by the following matrix:

\[
U^Y_X = \left( \begin{array}{cc} -\beta_1 \left( \frac{1 + \beta_2^2}{\beta_2^2 - \beta_1^2} \right)^{1/2} & -\beta_2 \left( \frac{1 + \beta_2^2}{\beta_2^2 - \beta_1^2} \right)^{1/2} \end{array} \right) \delta^b_a.
\]

(57)

In this new basis the structure constants become more complicated. They are obtained using (55) and (56):

\[
f_{A_1 A_2}^{A_3} = \frac{1 + 2 \beta_1 \beta_2 - \beta_1^2 - \beta_2^2}{(\beta_1 - \beta_2)^2} \left( \frac{\beta_2^2 - \beta_1^2}{1 + \beta_1^2} \right)^{1/2} \epsilon A_1 A_2 A_3, \quad f_{B_1 B_2}^{B_3} = \frac{1 + 2 \beta_1 \beta_2 - \beta_1^2 - \beta_2^2}{(\beta_1 - \beta_2)^2} \left( \frac{\beta_2^2 - \beta_1^2}{1 + \beta_1^2} \right)^{1/2} \epsilon B_1 B_2 B_3,
\]

\[
f_{A_1 A_2}^{B_3} = -\frac{(1 + \beta_1^2)(\beta_2^2 - \beta_1^2)^{1/2}}{(\beta_1 - \beta_2)^2} \epsilon A_1 B_2 A_3, \quad f_{A_1 A_2}^{B_3} = -\frac{(1 + \beta_1^2)(\beta_2^2 - \beta_1^2)^{1/2}}{(\beta_1 - \beta_2)^2} \epsilon A_1 B_2 B_3,
\]

(58)
Notice, that in this basis the hamiltonian constraint can be expressed through the
dynamical multiplet only

\begin{equation}
H = \frac{1}{2} \tilde{P}_{(\beta)}^i X_{Y} \tilde{P}_{(\gamma)}^j Y_{Z} F_{ij}^{YZ}. \tag{59}
\end{equation}

As it is easy to see the indices in (59) are contracted with help of the unit matrix. It
gives an invariant expression due to the existence of the unit invariant form. This is
a consequence of the full antisymmetry of the structure constants (58). However, such
representation is not covariant under change of the basis.

\section{Constraint algebra}

Define the smeared constraints:

\begin{align}
\mathcal{G}(n) &= \int d^3 x n^X \mathcal{G}_X, \quad H(\vec{N}) = \int d^3 x \vec{N} H, \\
\mathcal{D}(\vec{N}) &= \int d^3 x N^i (H_i + A_i^X \mathcal{G}_X). \tag{60}
\end{align}

They obey the following algebra:

\begin{align}
\{\mathcal{G}(n), \mathcal{G}(m)\}_D &= \mathcal{G}(n \times m), \\
\{\mathcal{D}(\vec{N}), \mathcal{D}(\vec{M})\}_D &= -\mathcal{D}([\vec{N}, \vec{M}]), \\
\{\mathcal{D}(\vec{N}), \mathcal{G}(n)\}_D &= -\mathcal{G}(N_i \partial_i n), \\
\{H(\vec{N}), \mathcal{G}(n)\}_D &= 0, \tag{61} \\
\{\mathcal{D}(\vec{N}), H(\vec{N})\}_D &= -H(\mathcal{L}_{\vec{N}} \vec{N}), \\
\{H(\vec{N}), H(\vec{M})\}_D &= \mathcal{D}(\vec{K}) - \mathcal{G}(K^j A_j),
\end{align}

where

\begin{align}
(n \times m)^X &= f_{YZ}^X n^Y m^Z, \quad \mathcal{L}_{\vec{N}} \vec{N} = N^i \partial_i \vec{N} - \vec{N} \partial_i N^i, \\
[N, \vec{M}]^i &= N^k \partial_k M^i - M^k \partial_k N_i, \tag{62} \\
K^j &= (\vec{N} \partial_i M_i - M_i \partial_i \vec{N}) \vec{Q}_X \vec{Q}_Y g^{XY}.
\end{align}

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