Insights into the origins and growth of supermassive black holes

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ABSTRACT

We present a well-tested, theoretically supported empirical relation that helps decipher the origins, growth, and properties of supermassive black holes (SMBHs). Based on theoretical considerations and analysis of mass (MBH) versus age (t) distribution of 93 high-redshift (>5.6) SMBHs, we get $M_{BH} = M_s \exp [14.6(t-100)/t(\text{Myr})]$, which gives the SMBH’s seed mass $M_s$, and its derivative gives the instantaneous mass-accretion rate. It yields seeds of $\sim 20$–$420 M_\odot$ (solar masses) for the recently discovered SMBHs GNz11, CEERS$_{1019}$, and UHZ1, and $3 \times 10^4 M_\odot$ for the largest ($1.24 \times 10^{10} M_\odot$) high-z SMBH. It is applied to 132446 SMBHs at $z<2.4$ cataloged by Kozlowski. The resultant seeds are classified based on size and likely formation mechanism: $\sim 58\%$ are light ($<400 M_\odot$) deemed to be Pop III remnants; $\sim 39\%$ are intermediate ($400-3 \times 10^3 M_\odot$) and $<3\%$ heavier seeds ($3 \times 10^3 M_\odot-3 \times 10^4 M_\odot$), both of which formed possibly by mergers of Pop III remnants. The DCBH mechanism is not required but not excluded. Furthermore, the results show the following. The mass-accretion rate increases exponentially from the seed’s inception ($z\sim 30$), reaches a broad plateau at $z\sim 8.5-6$ coincident with the epoch of reionization, and decreases monotonically. Sub-Eddington accretion is the norm, except during the first $\sim 150$ Myrs, SMBHs experienced super-Eddington accretion, or the radiative efficiency was $<0.1$. The largest seed can potentially grow via luminous accretion to $(6.6+2.2) \times 10^{10} M_\odot$, consistent with a theoretical limit of $\sim 5 \times 10^{10} M_\odot$ by King. The Eddington ratio decreases and radiative efficiency increases as $z$ decreases, consistent with recent findings.

Key words: black hole physics – galaxies: fundamental parameters – galaxies: nuclei – quasars: supermassive black holes – stars: black holes.
1. INTRODUCTION

It is generally accepted that supermassive black holes (SMBHs) grew from progenitor seeds formed or assembled in the early universe. However, it remains unclear how massive these seeds were. Were they predominantly light or heavy, or somewhere in between? The origins of the black holes (BH) or the mechanisms by which they may have formed are intimately related to seed size. Proposed mechanisms by which seeds may have formed have been extensively reviewed, among others, by Latif and Ferrara (2016) and Volonteri et al. (2022) and can broadly be divided into three categories depending on the size of the seeds. Light seeds with a typical mass of $\sim 100 \, M_\odot$ formed from the collapse of massive metal-free first stars (Madau and Rees, 2001; Johnson and. Bromm, 2007) dubbed as Pop III remnants. Formation of heavy seeds $10^4 M_\odot$-$10^6 M_\odot$ from the collapse of pristine gas clouds in massive dark matter halos (Bromm and Loeb, 2003; Begeleman et al., 2006; Lodato and Natarajan, 2006; Shang et al., 2010) dubbed as DCBHs, or by hierarchical growth of BHs in dense stellar clusters (Davies et al., 2011). And the formation of intermediate-size ($\sim 10^2 M_\odot$) seeds via runaway collisions of stars in dense stellar clusters (Portegies et al., 2004; Freitag et al., 2006; Mapelli, 2016) or by hierarchical merger of BHs in stellar clusters (Davies et al., 2011; Lupi et al., 2014). Simulations attempting to understand the properties of the seeds formed by such mechanisms, drawbacks, and conditions necessary for their formation have also been extensively reviewed (see Latif and Ferrara, 2016; Volonteri et al., 2022). It is safe to conclude that there is no consensus as to which of the proposed mechanisms may have played a dominant role and presumably cannot be ascertained without knowing the size distribution of the seeds formed.

Furthermore, black hole seeds are thought to have formed at $z \sim 20$–30 (Barkana & Loeb 2001), and the recent discovery of the AGN GNz11 at $z=10.6$ (Maiolino et al., 2024) supports such an assumption. The existence of active galactic nuclei (AGN) exceeding $10^6 M_\odot$ (solar masses) less than a billion years old (e.g., Wu et al., 2015) has, however, defied comprehension as to how these seeds grew so massive in such short times. This dilemma is best illustrated by the following case studies of three recently discovered SMBHs at $z \sim 8.7$. Larson et al.(2023) concluded that CEERS_1019 at $z \sim 8.7$ and $M_{BH} \sim 9 \times 10^8 M_\odot$ requires super-Eddington accretion from stellar-sized seeds or Eddington limited from massive BHs seeds. Schneider et al. (2023) and Maiolino et al. (2024) concluded that GNz11 at $z=10.6$ and $M_{BH} \sim 1.5 \times 10^9 M_\odot$ is accreting respectively at Eddington ratio $\lambda$ of $\sim 2.5$ and 5.5. In contrast, using simulations, Bhatt et al. (2024) found that the probability of observing a BH at $z \sim 10-11$ accreting with $\lambda \sim 5.5$ in the volume surveyed by JWST is $<0.2\%$. And for the third AGN UHZ1 at $z \sim 10.1$ with an estimated $M_{BH} \sim 4 \times 10^7 M_\odot$, Natarajan et al. (2024) concluded that it grew from a heavy seed ($\sim 10^4 M_\odot$) formed at $z \sim 14$ that was probably a DCBH. Again, it is safe to conclude that there is no consensus or clear understanding of how SMBHs, starting from progenitor seeds orders of
magnitude smaller, grew to their observed sizes in several hundred million to less than a billion years.

While studies of individual AGNs provide valuable insights into the properties of that BH and may support a particular theory of the formation or growth of BH seeds, they cannot rule out other theories or channels of seed formation (Volenteri et al., 2022); nor do such studies provide measures of the growth rates of SMBHs. Presumably, solving the dual interrelated enigma of the origins and growth of SMBHs requires insights and constraints derived from large sets of observational data. The JWST may provide such data in years to come. Meanwhile, a large corpus of publicly available data, comprising known properties (redshift, mass $M_{\text{BH}}$, Eddington ratio $\lambda$) of over a hundred thousand AGNs at various redshifts, remains largely unexploited. More precisely, this corpus consists of about 93 high-z AGNs at $z>5.6$ whose masses are well constrained and range over four orders of magnitude and ~560 Myr in age, and 132,446 lower-z AGNs at $z<2.4$ and $M_{\text{BH}} \geq 10^7 M_{\odot}$ determined using MgII lines by Kowalski (2017). Fragione and Pacucci (2023) have attempted to constrain the distribution of BH seeds using a catalog of 113 high redshift ($z>6$) AGNs compiled by Fan et al. (2023). Applying Bayesian analysis to these AGNs, they concluded that light and heavy seeds are required, whose distribution can be modeled by combining a power law and a lognormal function. Furthermore, they obtained mean values for the Eddington ratio, duty cycle, and radiative efficiency of the high-z AGNs. Here, prompted by new insights garnered from a deconstruction of a commonly used theoretical formula for the growth of a black hole (BH), we analyze the mass versus age distribution of the 93 high redshift AGNs and derive an empirical relation expressing a BH’s mass $M_{\text{BH}}$ as a function of its age $t$ or redshift $z$ and the mass $M_s$ of its seed, from which the seed mass $M_s$ can be ascertained knowing the BH’s mass $M_{\text{BH}}$ and redshift. The derivative of this primary relation gives a BH’s accretion rate at any instant of its active life. Furthermore, using a BH’s instantaneous accretion rate thus derived, its radiative efficiency can be obtained from its bolometric luminosity. Together, these relations comprise a set of powerful tools to decipher the origins, growth, and properties of SMBHs.

We begin by providing the theoretical basis of the primary relation and then derive the primary empirical relation using observational data. It is first applied to the 93 high-z BHs. The resulting distributions of seed size versus BH mass and BH age are analyzed, and conclusions are drawn regarding the range of seed sizes necessary to account for the masses of the high-z SMBHs observed thus far. In particular, the seed mass required to account for the mass of each of three recently discovered ultra-high $z$ AGNs, namely GNz11, CEERS-19, and UHZ1, is determined. The primary relation is then applied to the 132,446 AGNs at $z<2.4$ to test its universal applicability and the validity of the conclusions drawn from its application to the high-z AGNs. More importantly, the resulting mass distribution of the 132,446 seeds is analyzed and compared to simulations of mass functions of the first massive stars by Hirano et al. (2014) and intermediate-size BHs formed via runaway collisions in nuclear star clusters by Devecchi et al.
Thus, and nondimensional in and Therefore, the accretion rate, the Eddington ratio, and the radiative efficiency as functions of redshift and find the role if any super-Eddington accretion plays in the growth of SMBHs. We end with a summary of the principal conclusions.

2. THEORETICAL BASIS

Conventionally, a BH’s exponential growth is defined by Eq. 1, where $M_{BH}$ is the black hole mass, $t$ its age (Myr), $M_s$ its seed’s mass, $t_s$ the inception time, 45 Myr the Salpeter time scale ($t_{Sal}$) for a BH accreting at the Eddington limit, and $\gamma$ a non-dimensional parameter defined in Eq.2 and dubbed the “growth efficiency factor” by Zybovas and King (2022).

$$M_{BH} (t) = M_s \exp \left[ \frac{\gamma (t- t_s)}{45 \text{(Myr)}} \right]$$

$$\gamma = [0.11 \delta \lambda (1-\epsilon) / \epsilon]$$

In Eq.2, $\lambda$ is the Eddington ratio, $\epsilon$ the radiative efficiency, and $\delta$ the duty cycle, all averaged over a BH’s active life span ($t$-$t_s$). Note that for $\delta=1$, $\epsilon=0.1$, and $\lambda=1$, $\gamma=1$, and $t_{Sal}=45$Myr and Eq.1 reduces to the conventional Salpeter relation for a BH accreting at the Eddington limit. By definition, $\lambda= L_{bol}/L_{Edd}$, and the bolometric luminosity $L_{bol} \propto (\dot{M} \epsilon) / (1- \epsilon)$ where $\dot{M}$ is the accretion rate, and the Eddington luminosity $L_{Edd} \propto M_{BH}$. Hence, $\gamma \propto \delta(\dot{M}/M_{BH})$ or the accretion rate per unit BH mass. Ostensibly, the accretion rate or $\gamma$ cannot be a constant because a BH of say $10^{10}M_\odot$ at say $t=10^9$ years would grow untenably by ~100 orders of magnitude by $t=10^{10}$ years. On the contrary, evidence shows that $\delta$ decreases as $z$ decreases (Shankar et al., 2010), and more recently Aggarwal (2024) has demonstrated that $\lambda$ decreases as $z$ decreases, but the radiative efficiency $\epsilon$ increases as $z$ decreases, and hence $(1-\epsilon)/\epsilon$ decreases as BH ages. Therefore, $\gamma$ or the accretion rate per unit BH mass must be an inverse function of a BH’s age $t$, and its value averaged over the lifespan would invariably be greater than its value at any instant in the life of a BH. And since $\gamma$ is nondimensional, we can simply define $\gamma = \beta 45/t$, where $\beta$ is a nondimensional proportionality constant and $t$ in Myr. and 45 Myr is the standard e-folding time. Thus, Eq.1 reduces to Eq.3, where $M_{BH}$ is a function of its known age $t$ and three free parameters: $M_s$, $\beta$, and $t_s$.

$$M_{BH} (t) = M_s \exp \left[ \beta \frac{(t- t_s)}{t} \right]$$
The seed mass $M_s$ of a BH can be determined using Eq.3 if the values of $\beta$ and $t_s$ were known. To obtain the values of the constants $\beta$ and $t_s$ in Eq.3, we need three or more BHs of different ages but similar, albeit unknown, seed masses. In the next sections, we determine the optimum value of $\beta$ and seek constraints on inception time $t_s$ using publicly available data for SMBHs younger than a billion years.

3. OBSERVATIONAL DATABASE

We searched the literature for SMBHs younger than a billion years ($z > 5.65$) whose masses are reliably known. Table A1 (Appendix) lists 59 SMBHs discovered until the end of 2022 with references for data sources. The average reported 1$\sigma$ uncertainty in the $M_{BH}$ of the 59 BHs is $\sim$ 0.11dex. In addition, Shen et al. (2019) list 50 SMBHs within a narrow $z$ range straddling $z=6$, of which the $M_{BH}$ of 12 are not well constrained, and 6 are duplicates of those in Table A1. The remaining 32 BHs extracted from Table 3 in Shen et al. (2019) are identified in the Appendix. This compilation of 91 BHs is comparable to that of Fan et al. (2023) for 113 BHs at $z > 5.3$, except that we indicate the uncertainty in $M_{BH}$ of each BH and have excluded from the list those BHs whose $M_{BH}$ are poorly constrained. In addition, there are two more recently discovered BHs at $z > 8.6$. These are GNz11 at $z=10.6$ with $M_{BH} \sim 1.5 \times 10^8 M_\odot$ and CEERS_101911 at $z \sim 8.7$ with $M_{BH} \sim 9 \times 10^6 M_\odot$ for a total of 93. The UHZ1 AGN is omitted because its mass is estimated assuming $\lambda=1$ (Natarajan et al., 2024) and hence uncertain. At lower redshifts, Kozlowski (2017) lists $\sim$280,000 AGN at $z < 2.4$, almost all of which have $M_{BH} \geq 10^7 M_\odot$. Of these 280,000 AGN, $\sim$132,446 have $M_{BH}$ determined using the more reliable MgII lines and Eddington ratios based on a weighted average of bolometric luminosities derived using two or more AGN luminosities. The high-$z$ sample of 93 AGN appears to be skewed in favor of the larger BHs since $\sim$ 75% have $M_{BH} \geq 10^8 M_\odot$; whereas the lower-$z$ sample appears not to be because it has $\sim$63% in the $10^6$-10$^7 M_\odot$ range, $\sim$27% $\geq 10^8 M_\odot$, and the rest $\sim$10% $< 10^8 M_\odot$. We note that there are probably a few hundred SMBHs at intermediate redshifts of $2.5 < z < 5.5$ that we did not compile because their inclusion would not alter the results of this study. Note that $z$ is converted into $t$ using the Hubble constant ($H_0=67.4$ km/s/Mpc) and matter density parameter ($\Omega_m =0.315$) from the Planck group (2020). An SMBH is defined as $\geq 10^6 M_\odot$. Throughout this paper, $M_{BH}$ and $M_s$ are in solar masses ($M_\odot$) and age $t$ in Myr.

4. EMPIRICAL RELATIONS

Figure 1 shows the mass $M_{BH}$ versus age $t$ distribution of 91 of the 93 SMBHs with density contours. The two recently discovered at $t < 600 $Myr fall outside the bounds of Fig.1. Depending on their mass, the BHs in Fig.1 are denoted by three different symbols. Those in blue squares have similar masses within a factor of $\sim$2 of $2.5 \times 10^8 M_\odot$ but different ages covering the entire age spectrum in Fig.1; those in red triangles nominally have $M_{BH} > 5 \times 10^8 M_\odot$, and those in red circles
have $M_{\text{BH}} < 10^9 M_\odot$. First, we seek to constrain the value of $\beta$ and understand to what extent $M_{\text{BH}}$ depends on age $t$. For this purpose, we will tentatively assume that the seed inception time $t_s=100 \text{Myr}$ in Eq.3. Equation 3 indicates that $M_{\text{BH}}$ is directly proportional to seed mass $M_s$ and increases with age $t$. The most massive BHs are the most likely candidates for the heaviest seeds. If so, Eq.3 requires $\beta \leq 15.85$ for $M_s \geq 10^4 M_\odot$ for the largest high-z BH (#31, Table A1). As for the dependence of $M_{\text{BH}}$ on $t$, the ages of the 91 BHs range from 676-1000 Myr and Eq.3 indicates that a BH’s $M_{\text{BH}}$ increases by only a factor of $\sim 2$ from $t=676 \text{Myr}$ to $t=1000 \text{ Myr}$. For any likely value of $\beta \leq 15.85$, compared to the orders of magnitude differences in the $M_{\text{BH}}$ of the BHs in Fig.1. Hence, within this age range, a BH’s $M_{\text{BH}}$ depends primarily on its seed mass $M_s$ and relatively little of a BH’s age $t$.

![Black Hole Mass vs Black Hole Age](image)

Fig.1 Mass $M_{\text{BH}}$ versus age $t$ for 91 SMBHs listed in the Appendix. Density contours are shown. The average reported 1σ uncertainty in $M_{\text{BH}}$ is $\sim 0.11 \text{dex}$. The 60 BHs denoted by blue squares share a common feature: their masses are within a factor of $\sim 2$ of $2.5 \times 10^9 M_\odot$, but their ages span $\sim 325 \text{ Myr}$ of cosmic time. Those in red triangles nominally have $M_{\text{BH}} > 5 \times 10^9 M_\odot$, and those in red circles have $M_{\text{BH}} < 10^9 M_\odot$. 

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Figure 1 reveals that ~60 BHs identified by blue squares have similar masses within a factor of 2 of $2.5 \times 10^9 M_\odot$ but different ages spanning ~325 Myr of cosmic time. The inescapable conclusion is that these 60 BHs had similar-sized seeds. Hence, for each of the 60 BHs, we can write Eq.3 with one variable (t) and three free parameters (Ms, ts, $\beta$). We used the “SANN” method (Belisle, 1992), a Monte Carlo technique of solving optimization problems, with 10 million iterations to simultaneously solve the equations and optimize parameter values. Several sets of values for the parameters were generated using all 60 equations and subsets, allowing all three parameters to be free, assuming ts to be 100, 150 and 200 Myr, and varying the time window within which the BHs had similar masses, the motivation being to obtain the most likely optimum value for $\beta$. Computations with ts=150 and 200 gave the largest RMS residuals and were rejected. A covariance was noted between ts and $\beta$, in that an earlier ts gave a somewhat lower $\beta$ and vice versa. As suspected, subsets comprising BHs with masses within a narrower range of the mean value of $2.5 \times 10^9 M_\odot$ or a narrower age range produced lower residuals. The ensemble of the solutions indicated that the most likely value for ts was neither much less nor much more than 100 Myr. Finally, ts =100 Myr ($z\sim30$) and the corresponding $\beta \approx14.6$ were adopted as the most likely optimum values.

Substituting these values of $\beta$ and ts in Eq.3, we get Eq.4A where age t is in Myr. And using the approximation $1/t \propto (1+z)3/2$ for high $z$ (Bergström and Goober, 2006), we can rewrite Eq.4A expressing a BH’s $M_{BH}$ as a function of $z$ as in Eq.4B. Note that Eq. 4A and 4B may yield slightly different results because of the approximation. Note also that Eq.4 does not depend on any material assumption or ad-hoc data selection.

$$M_{BH} = Ms \exp [14.6(t-100)/t]$$  \hspace{1cm} (4A)

$$M_{BH} = Ms \exp 14.6 [1 - (1 + z)^{3/2} / (1+30)^{3/2}]$$  \hspace{1cm} (4B)

5. APPLICATION TO HIGH-Z SMBHs: LIMITS ON SEED SIZE

Figure 2 shows the seed mass Ms versus age t distribution for the 91 BHs in Fig.1 predicted using Eq.4. Figures 1&2 share identical $M_{BH}$ symbols. Remarkably, the predicted Ms distribution in Fig.2 closely mimics the observed $M_{BH}$ distribution in Fig.1. The 60 blue squares having similar $M_{BH}$ have markedly similar Ms; the red circles having smaller $M_{BH}$ have correspondingly smaller Ms; the red triangles with the largest $M_{BH}$ have the largest Ms; and the BHs range in $M_{BH}$ over ~2.5 orders of magnitude and so do their Ms. Figure 3 shows the predicted seed mass Ms versus BH mass $M_{BH}$ for all 93 BHs including GNz11 and CEERS+1019, the two red dots in the lower left corner of the figure. The symbols are the same as those in Figs. 1&2..The largest BH
in Fig.3 (#31 in Table A1) requires a seed mass \( M_s = (3 \pm 1) \times 10^4 M_\odot \) for a 2\( \sigma \) uncertainty in its mass. And those with masses \( 1 - 5 \times 10^6 M_\odot \) (blue squares in Fig.3) require seeds \( > 10^3 M_\odot \) but \( < 10^4 M_\odot \). Of the 93 BHs, GNz11 and CEERS_1019 are the smallest, requiring \( M_s \sim 20 M_\odot \) and \( \sim 53 M_\odot \), respectively, or a few to several tens of solar masses. Additionally, Eq.4 predicts that a seed of \( \sim 418 M_\odot \) formed at \( t_s = 100 \) Myr accounts for the mass of UHZ1 (\( z \sim 10.1 \)) estimated to be \( \sim 4 \times 10^7 M_\odot \) by Natraj et al. (2024), who concluded that a heavy seed of \( \sim 10^4 M_\odot \), probably a DCBH, formed at \( t_s = 300 \) Myr best fits its spectral data. If so, the value of \( \beta \) in Eq.3 for UHZ1 would be \( \sim 22.9 \) and lead to the unacceptable conclusion that UHZ1 would have grown to \( \sim 9 \times 10^{10} M_\odot \) by the age of a billion years, well exceeding the mass of the largest high-z SMBH observed so far. We conclude that none of the three recently discovered AGN at \( z \geq 8.7 \) require heavy seeds. Instead, their masses are accounted for with seeds formed at \( z = 30 \) ranging in \( M_s \) from a few tens to a few hundred solar masses.

Furthermore, we can place upper and lower limits on the size of seeds that formed at \( z \sim 30 \). The largest high-z BH (#31, Table A1) was discovered \( \sim 10 \) years ago, and the second largest (#59) more than two decades ago. In all likelihood, these BHs represent an upper limit on the size of SMBHs in the early universe. If so, we can conclude that the largest seeds formed at \( z \sim 30 \) did not exceed \( (3 \pm 1) \times 10^4 M_\odot \), the seed mass required for the largest of the 93 SMBHs. Conversely, while we cannot place a strict lower limit on the size of seeds formed at \( z = 30 \), we can conclude that it has to be \( < 20 M_\odot \) based on the \( M_s \) required for GNz11, the smallest of the 93 high-z SMBHs. Lastly, we note that Eq.4A predicts that the maximum size a BH can achieve via luminous accretion depends on its seed mass amounting to \( M_s \) times \( \text{Exp} 14.6 \) or \( \sim 2.2 \times 10^9 M_\odot \), which translates into \( (6.6 \pm 2.2) \times 10^{10} M_\odot \) for the empirically determined upper limit of \( M_s = (3 \pm 1) \times 10^4 M_\odot \), in excellent agreement with a probable theoretical limit of \( \sim 5 \times 10^{10} M_\odot \) proposed by King (2016).
6. APPLICATION TO LOWER-Z SMBHs: SEED CLASSIFICATION

We applied Eq.4B to the 132,446 AGN at z < 2.4 (Kozlowski, 2027) to test the universality of its applicability and derive the size distribution of the seeds. The resulting Ms are sorted into narrow bins, and the number in each mass bin is shown in Table 1. In most cases, the uncertainty in $M_{\text{BH}}$ is unknown but is of little importance except when there are relatively few seeds in a bin. Of the total population of BHs, 540 BHs have $\geq 10^{10}M_\odot$, and none of their predicted Ms exceed the preceding empirically established upper limit of $(3\pm1)\times10^4M_\odot$ except possibly in 5 cases (Table 1) and that too by a factor of < 1.7 in the worst case. However, the uncertainty in the $M_{\text{BH}}$ of these 5 BHs is unknown. In particular, we note that the predicted Ms for TON 618 at z = 2.219 and 4.07$x10^{10}M_\odot$ (Xue Ge et al., 2019), which is often cited as the most massive BH observed to date, is identical to that of the largest high-z BH (#31). At the lower end of the Ms spectrum, the results (Table 1) show that 88 BHs have predicted Ms between 5-10$M_\odot$ and ~1000 between
10-20$M_\odot$ in agreement with the lower limit of $M_s$ expected from the high-z data. This striking agreement on the upper and lower limits on the size of seeds deduced from 2 entirely different sets of data covering different cosmic periods cannot simply be fortuitous and testifies to the validity and universal applicability of Eq.4.

![Graph](image)

Fig.3. Seed mass $M_s$ versus black hole mass $M_{BH}$ on a log-log scale for the 91 BHs in Fig.1 and GNz11 and CEERS_1019 ($M_{BH} < 1e+07M_\odot$, bottom left corner). The symbols are identical to those in Figs.1&2. The $M_s$ for GNz11 and CEERS_1019 are respectively $\sim20M_\odot$ and $\sim53M_\odot$. The $M_s$ for BHs with $M_{BH}$ 1-5x$10^9M_\odot$ (blue squares) have $10^3M_\odot < M_s < 10^4M_\odot$, and the $M_s$ for the largest BH is $\sim3x10^4M_\odot$. 
Fig. 4. Histogram showing the number of seeds in bins of $50M_\odot$ (solar masses) for 90% of 132,446 SMBHs at $z < 2.4$ predicted by Eq. 4B. Each bar is plotted at the central mass value of the bin. The distribution of the rest at $M_\ast > 1500M_\odot$, not plotted because of their small numbers, is given in Table 1. Note the initial increase in seed counts before the monotonic decrease, the significance of which is discussed in the text.

The seed counts in Table 1 indicate that ~90% of the seeds have $M_\ast \leq 1.5 \times 10^3M_\odot$. The histogram in Fig. 4 shows their mass distribution, where each bin has the same size of $50M_\odot$ and is identified by the central value of $M_\ast$ in the bin. The remaining 10% are not plotted because of their small numbers but follow the same pattern, decreasing asymptotically towards zero at $M_\ast \sim 3 \times 10^4M_\odot$, the upper limit of seed mass. Table 1, however, gives the number of seeds in different bin sizes for the remaining 10% of AGN that amount to less than the number in the single bin of 50-100$M_\odot$ centered at $M_\ast = 75M_\odot$ in Fig. 4. The seeds in Fig. 4 range in $M_\ast$ from a low of $\sim 5M_\odot$ to a high of $\sim 1.5 \times 10^4M_\odot$ and can be designated as light to intermediate-size seeds based on their universally accepted size classification. They constitute an overwhelming majority of the seeds. In contrast, seeds $\geq 10^4M_\odot$ or heavy seeds constitute a small number totaling ~210 or ~0.16% of the seed population (see Table 1). Furthermore, the distribution in Fig. 4 shows a monotonic systematic decrease in the number of seeds without any hiatus or break, preceded by an initial increase in seed counts.
Table 1: Seed counts

| Ms     | Count | Ms    | Count | Ms    | Count | Ms    | Count |
|--------|-------|-------|-------|-------|-------|-------|-------|
| 5-10   | 88    | 200-250 | 8,473 | 800-850 | 1,963 | 1400-1450 | 745 |
| 10-20  | 1014  | 250-300 | 8,473 | 850-900 | 1,913 | 1450-1500 | 703 |
| 20-30  | 2394  | 300-350 | 6,063 | 900-950 | 1,721 | 1500-1800 | 3383 |
| 30-40  | 3332  | 350-400 | 5,241 | 950-1000 | 1,528 | 1800-2200 | 2775 |
| 40-50  | 3605  | 400-450 | 4,700 | 1000-1050 | 1,419 | 2200-2700 | 2217 |
| 50-60  | 3595  | 450-500 | 4,064 | 1050-1100 | 1,231 | 2700-3400 | 1747 |
| 60-70  | 3481  | 500-550 | 3,728 | 1100-1150 | 1,161 | 3400-4200 | 1051 |
| 70-80  | 3261  | 550-600 | 3,226 | 1150-1200 | 1,081 | 4200-6400 | 1097 |
| 80-90  | 3110  | 600-650 | 2,910 | 1200-1250 | 944 | 6400-10000 | 462 |
| 90-100 | 2979  | 650-700 | 2,672 | 1250-1300 | 926 | 1000014000 | 144 |
| 100-150| 12,739 | 700-750 | 2,385 | 1300-1350 | 843 | 1400030000 | 61 |
| 150-200| 10,159 | 750-800 | 2,217 | 1350-1400 | 829 | >30000 | 5 |

Ms seed mass (solar mass). The count is the number of seeds in a bin. Bin size varies.

Rather than reviewing the numerous simulations of BH seed formation reported in the literature, we shall attempt to identify those that best match the distribution of seeds in Fig.4 and Table 1. Light seeds are thought to have formed from the collapse of massive metal-free first or Pop III stars (Madau and Rees, 2001; Johnson and. Bromm, 2007). Our finding that the seeds formed around z=30 is consistent with the notion that the first stars formed at z~30 (e.g., Couchman and Rees,1986). Their stellar mass functions derived from simulations range from $<10M_\odot$ to $\sim 1000M_\odot$ (e.g., Hirano et al., 2014; Hosokawa et al., 2015; Stacy et al., 2016). These simulations have been reviewed by Latif and Ferrara (2016), who concluded that the results suggest that the typical mass of Pop III stars is $\sim 100M_\odot$ except for a few cases of $1000M_\odot$. As noted earlier, Fig.4 shows that the number of seeds decreases systematically as Ms increases except at the beginning or low end of the Ms spectrum. A closer examination reveals that at the
low end of the Ms spectrum, the number of seeds increases exponentially and reaches a plateau before decreasing monotonically. The counts in Table 1 show that the number of seeds increases from 88 having Ms ≤ 10M☉ to 1014 and 2395 in the successive bins of 10-20M☉ and 20-30M☉ and reaches a plateau of ~3600 in the bins of 40-50M☉ and 50-60M☉; after which the counts decrease slowly but monotonically. This predicted mass distribution of light seeds resembles the stellar-mass distribution shown in Fig.5 of Hirano et al.(2014) derived from hydrodynamic simulations. Their histogram of 100 first stars accreting at lower rates, identified by red and blue colors in Fig.5, shows that the number of stars increases >10-fold from 1 with a mass <10M☉, to 11 with a mass of ~40M☉, after which the number of stars decreases almost monotonically to 1 at ~1000M☉. However, as noted by them, most stars have masses from a few tens to a few hundred solar masses, consistent with the distribution of seeds in Table 1 and Fig.4 that show that ~60% of the seeds with Ms <1000M☉ have masses between 10M☉ and 300M☉ and ~70% have <400M☉. Thus, there is a good agreement between the mass distribution of the first stars derived from simulations and the empirically derived mass distribution of seeds that presumably formed from the collapse of the first stars. In the simulation, however, there is a marked low in the number of stars immediately following the maximum, whereas in Fig.4, there is no such hiatus. Later, we provide an explanation reconciling this dichotomy.

Most likely, there is an overlap in the sizes of light and intermediate seeds. There is, however, no decipherable change, hiatus, or break in the asymptotic decrease in the number of seeds as Ms increases in Fig.4. Thus, it is difficult, if not impossible, to define a strict Ms boundary in Fig.4 between the light and intermediate seeds. In contrast to light seeds, intermediate-size seeds are thought to have formed either via runaway collisions of stars in dense stellar clusters (Portegies et al., 2004; Freitag et al., 2006; Mapelli, 2016) or by the hierarchical merger of BHs in stellar clusters (Davies et al., 2011; Lupi et al., 2014). Devecchi et al. (2012) performed simulations simultaneously investigating the formation of Pop III remnants and BHs via runaway collisions in nuclear star clusters. The results in their Fig.4 show that the BHs formed via runaway collisions decrease in number by a factor of ~5 as BH mass increases from ~400M☉ to ~3000M☉, qualitatively in agreement with the decrease in the number of seeds as Ms increases in Fig.4 and Table 1 of this study. Note that ~97% of the seeds in Table 1 have Ms < 3000M☉. Hence, intermediate-size seeds formed via runaway collisions of stars and light seeds formed by the collapse of the first stars could together account for the entire population of seeds in Table 1 except ~3%. There is, however, a potential problem. The seeds in Table 1 were deduced to have formed at z~30, whereas the intermediate-size BHs via runaway collisions of stars in the simulations formed at z~15, a delay of ~150 Myr.

On the other hand, Lupi et al. (2014) explored gas-induced runaway merger of BHs dubbed the GIRM model, following the Davies et al. (2011) prescription of hierarchical growth of BHs in dense stellar clusters. The results in their Fig.7 also show that the number of BH seeds formed
via GIRM decreases dramatically as the seed mass increases from \( \sim 400\, M_\odot \) to \( \sim 2000\, M_\odot \). Most of the BHs in their simulations also formed at \( z < 20 \). They, however, point out that GIRM requires some degree of metal pollution of the intergalactic medium from the explosions of the massive Pop III stars and that the “GIRM channel does not pose any constraint on the level of the metallicity of the parent halo, and hence on the time of formation, provided Pop III stars have enhanced the metallicity above a threshold.” If so, this mechanism and that for the formation of light seeds could together also account for at least 94% of the seed population in Table 1. This mechanism has the advantage that the Pop III remnants could provide the degree of metal pollution required in the simulations. Alternatively, the formation of intermediate-size BHs could have resulted from the merger of Pop III remnants, but whether this is a realistic possibility can be ascertained by simulations, a task beyond the scope of this paper. If possible, it would explain why there is no hiatus or break in the mass distribution of seeds in Fig.4 and Table 1 and why intermediate-size follow the same asymptotically decreasing trend as the light seeds. Furthermore, the probability of forming larger seeds from a given population of stellar seeds would systematically decrease as seed mass increases, consistent with the systematic decrease in seed counts as Ms increases observed in Fig.4 and Table 1.

It is remarkable that in the preceding simulations, whether involving runaway collisions of stars or runaway mergers of BHs, the resulting mass spectrum of intermediate-size seeds is strikingly similar, ranging from \( \sim 400\, M_\odot \) to \( \sim 2000\, M_\odot \) or \( 3000\, M_\odot \). Thus, based on the results of these simulations and the preceding discussion on the distribution of seeds in Fig.4 in the context of the results of the simulations by Hirano et al. (2014), we can classify seeds \(< 400\, M_\odot \) as light seeds predominantly formed from the collapse of first stars and those between \( 400\, M_\odot \) and \( 3000\, M_\odot \) as predominantly intermediate size. Based on this working classification, we conclude that of the 132,446 seeds in Table 1, \( \sim 58\% \) are light, and \( \sim 39.4\% \) are intermediate-size. The remaining \(< 3\% \) or 3681 have Ms between \( \sim 3 \times 10^4\, M_\odot \) and \( 3 \times 10^4\, M_\odot \) (dubbed heavier seeds in contrast to heavy), of which only 210 have Ms \( \geq 10^6\, M_\odot \) that could strictly be classified as heavy seeds. Moreover, these relatively few heavy seeds have Ms \( \leq 3 \times 10^4\, M_\odot \) or at the lower end of the expected mass spectrum of \( 10^4\, M_\odot \) (see Volonteri et al., 2022) for heavy seeds thought to have formed by the DCBH mechanism. The implication is that the DCBH mechanism did not play an essential role in seed formation. Moreover, the “number of “heavier” seeds decreases as Ms increases (see Table 1), following the same pattern as the light to intermediate-size seeds in Fig.4, which suggests that the same mechanism responsible for forming intermediate-size seeds may also account for the rare formation of heavy seeds. Apropos, Davies et al. (2011) have proposed that seeds as large as \( 10^5\, M_\odot \) can be formed by hierarchical growth of BHs in dense stellar clusters. We conclude, therefore, that the DCBH mechanism is not required while not closing the possibility that a relatively small number may have formed by the DCBH mechanism.
7. ACCRETION RATE, EDDINGTON RATIO, RADIATIVE EFFICIENCY

Having established the universal applicability of Eq. 4, we can use its derivative to gain insights into the growth history of SMBHs. Differentiating $M_{\text{BH}}$ (d$M_{\text{BH}}$ / dt) in Eq.4A, we get Eq.5 expressing a BH’s instantaneous accretion rate $\dot{M}$ (M$_{\odot}$/year) as a function of $M_{\text{BH}}$ and $t$ (Myr) or as a function of $z$ using the approximation $1/t \propto (1+z)^{3/2}$ (Bergström, and Goober, 2006)

$$\dot{M} (M_{\odot}/\text{yr}) = 14.6 \times 10^{-4} M_{\text{BH}} / t^2 \approx 4.96 \times 10^{-12} M_{\text{BH}} (1+z)^3 \quad (5)$$

Note that in Eq.5, the instantaneous accretion rate per unit BH mass $\dot{M}/M_{\text{BH}}$ is directly proportional to $(1+z)^3$ and hence decreases as $z$ decreases, fulfilling the requirement in the section on the theoretical basis that $\dot{M}/M_{\text{BH}}$ decrease as $z$ decreases. Furthermore, in the Standard Cosmological Model, the ambient gas density scales as $(1+z)^3$, and hence, the accretion rate $\dot{M}$ scales as the product of a BH’s mass $M_{\text{BH}}$ and the ambient gas density.

Eq.5 can be applied to any actively accreting BH to ascertain its instantaneous accretion rate $\dot{M}$. For example, Eq.5 yields $\dot{M} \sim 0.012 M_{\odot}/\text{yr}$ for GNz11 at $z=10.6$, $\sim 24 M_{\odot}/\text{yr}$ for the largest high-z SMBH at $z=6.3$ (#31), and $\sim 6.6 M_{\odot}/\text{yr}$ for TON 618 at $z=2.219$. Moreover, by substituting $M_{\text{BH}}$ in Eq.5 with its expression in Eq.4B, we get Eq.6 expressing $\dot{M}$ as a function of a BH’s seed mass $M_s$ and redshift $z$.

$$\dot{M} = 4.96 \times 10^{-12} M_s (1+z)^3 \exp 14.6 [1 - (1+z)^{3/2} / (1+30)^{3/2}] \quad (6)$$

Using Eq.6, one can infer the history of a BH’s accretion rate from its inception at $z=30$ to any redshift $z$ by plugging in Eq.6 the BH’s seed mass $M_s$ inferred from Eq.4B. Figure 5 shows the accretion rate history of a seed of unit solar mass. Initially, $\dot{M}$ increases exponentially and reaches a broad peak between $z=8.5$ and 6, beyond which it decreases slowly but monotonically towards $z=0$. Note that the broad peak from $z=8.5$-6, during which a BH has the highest accretion rate and hence likely the highest luminosity, incidentally coincides with the epoch during which reionization is thought to have occurred (e.g., Grazian et al., 2018). However, exploring to what extent BHs during this period (the mass function of which can be inferred using the results in Table 1 and Eq.4) may have contributed to the reionization is beyond the current scope of this paper. Notwithstanding this sidebar, we note that two competing factors, namely the increase in a BH’s mass or gravitational reach as it ages and the decrease in the ambient gas density as $z$ decreases, determine a BH’s $\dot{M}$ at any instant of its life. Hence, the results in Fig.5 suggest that initially the BH’s mass or its gravitational reach increases faster than the decrease in the ambient gas density. The two competing factors reach a parity near $z\sim 7$, after which the decline in gas density dominates over the gradual increase in the BH’s mass and $\dot{M}$.
steadily decreases. For example, Eq.4B predicts that the seed of TON 618, one of the largest SMBHs, grew from \( M_s = 3.03 \times 10^8 M_\odot \) by \( \sim 5.5 \) orders of magnitude to \( \sim 9.8 \times 10^9 M_\odot \) by \( z=7 \) and after that by a factor of \( \sim 4 \) until its present \( z=2.219 \). And Eq.6 predicts that its accretion rate changed from \( 0.08 M_\odot /yr \) at \( z=25 \) to \( 24.6 M_\odot /yr \) at \( z=7 \) and \( 6.6 M_\odot /yr \) at \( z=2.219 \).

Using Eq.5, we can also define the Eddington ratio \( \lambda \), or the ratio of a BH’s bolometric luminosity \( L_{bol} \) to its Eddington luminosity \( (L_{EDD}) \), as a function of \( z \) and radiative efficiency \( \epsilon \). \[
L_{bol} = (\dot{M} c^2) \epsilon / (1 - \epsilon)
\]
where \( c \) is the velocity of light and \( \dot{M} \) the accretion rate; and \( L_{EDD} = 1.3 \times 10^{38} M_{BH} \) in ergs/s with \( M_{BH} \) in solar mass. Radiative efficiency \( \epsilon \) is conventionally defined relative to the mass inflow rate, such as the Bondi rate, and a BH’s \( \dot{M} \) is smaller by \( (1 - \epsilon) \). Thus, by Substituting \( \dot{M} \) from Eq.5 into the definition of \( L_{bol} \), we get Eq.7 expressing the Eddington ratio \( \lambda \) as a function of a BH’s redshift and radiative efficiency.

\[
\lambda = 2.18 \times 10^{-3} (1 + z)^3 \epsilon / (1 - \epsilon)
\]

Fig.5: Log of instantaneous accretion rate \( \dot{M} \) for a BH seed of unit (solar) mass \( M_\odot \) as a function of redshift \( z \) starting at \( z\sim 30 \) based on Eq.6. Initially \( \dot{M} \) increases exponentially, reaches a broad peak between \( z \sim 8.5 \) and 6 with a maximum at \( z \sim 7 \), and steadily decreases after that towards \( z=0 \). Note that the timing of the broad peak from \( z=85-6 \) coincides with the epoch during which reionization is thought to have occurred.
Equation 7 implies that $\lambda$ decreases as $z$ decreases, irrespective of whether $\epsilon$ depends on $z$. This implication is validated by empirical evidence. Using Kozlowski’s data for the tens of thousands of AGNs at $z<2.4$, Aggarwal (2024) unambiguously showed that $\lambda$ decreases as $z$ decreases. Moreover, Eq.7 implies that of 2 BHs at similar redshifts, the BH with a higher $\lambda$ is less efficient (higher $\epsilon$) in accreting gases than the one with a lower $\lambda$. This implication is also consistent with the finding by Aggarwal (2024) that larger BHs have lower $\lambda$ and are more efficient than smaller BHs. And, using the Bondi prescription for spherically symmetric accretion and observational data for temperature and density profiles near BHs in galaxies M87, NGC 3115, and NGC 1600, Aggarwal (2024) derived a scaling relation for $\lambda$ identical in form to the above Eq.7. Thus the substantiations of the implications of Eq.7 and the similarity between scaling relations for $\lambda$ derived from 2 different prescriptions and entirely different data sets are further evidence of the validity and universal applicability of Eq.4 from which Eq.7 is derived.

A BH’s radiative efficiency $\epsilon$ can be inferred from Eq.7, knowing its $\lambda$ or bolometric luminosity. Unfortunately, determining $\lambda$ is prone to multiple errors arising from uncertainties in the BH’s mass, its luminosity, and the correction factor used to get its bolometric luminosity. However, by applying Eq.7 to large groups of similarly situated BHs at markedly different redshifts, we can garner insights into the dependence of $\epsilon$ on $z$. For example, Shen et al. (2019) list 50 BHs, most of which have $1-4 \times 10^9 M_\odot$ with redshifts close to 6 and a mean $\lambda \sim 0.32$, whereas a large group of similar-size BHs at $z \sim 1$ in Kozlowski’s catalog have a mean $\lambda=0.03$. Applying Eq.7 to the two groups, one gets $\epsilon \sim 0.63$ for the lower and $\epsilon \sim 0.3$ for the higher-$z$ group, a decrease by a factor of 2 despite a $\sim$ ten-fold increase in $\lambda$ from $z \sim 1$ to $z \sim 6$. The implication is that $\epsilon$ decreases as $z$ increases, consistent with a similar finding by Aggarwal (2024).

The vast majority of the values of $\lambda$ for the BHs in Table A1 and reported by Shen et al. (2019) and Kozlowski (2017) are <1, all of which are at $z < 7.7$. GNz11 is a notable exception that deserves special attention because it is the highest-$z$ AGN observed so far and is inferred to be accreting by as much as 5.5 times the Eddington rate (Maiolino et al., 2024). If so, Eq.7 predicts a radiative efficiency $\epsilon$ of $\sim 0.59$ that ostensibly is exceptionally high, especially since $\epsilon$ has been shown to decrease as $z$ increases. And, it would imply that GNz11 is a relatively poor accretor, accreting only $\sim 41\%$ of the gas inflow. Interestingly, a lower $\lambda$ would make GNz11 a more efficient accretor. For example, $\lambda = 1$ would yield $\epsilon = 0.227$, making GNz11 $\sim 2.6$ times more efficient than if its $\lambda=5.5$. Its $\lambda$ is probably highly overestimated. We note that Schneider et al. (2023) estimated a significantly lower $\lambda$ of 2-3 for it, and Bhatt et al. (2024) found that the probability of observing a BH at $z \sim 10-11$ accreting with $\lambda \sim 5.5$ in the volume surveyed by JWST is $< 0.2\%$.

Given that $\lambda$ is prone to large uncertainties and hence a poor predictor of whether a BH is accreting above the Eddington limit, we propose instead the following. As noted earlier, the
growth efficiency parameter $\mathcal{V}$ defined by Eq.2 has a value of 1 for a BH accreting at the Eddington limit ($\lambda=1$) with a radiative efficiency $\varepsilon=0.1$ (its canonical value) and a duty cycle $\delta=1$. For $\delta=1$, a value of $\mathcal{V} > 1$ implies that the BH is either accreting above the Eddington limit ($\lambda>1$) or that its $\varepsilon < 0.1$. Substituting $\lambda$ from Eq.7 into Eq.2 (the definition of $\mathcal{V}$), one gets $\mathcal{V} \sim 2.4 \times 10^{-6} (1 + z)^3$ for the instantaneous value of $\mathcal{V}$ as opposed to its value averaged over a BH’s life span in Eq.1. Shankar et al. (2010) found that in their sample of AGNs, the duty cycle $\delta$ increased with $z$ reaching $\sim 0.9$ at $z=6$. It is likely, therefore, that $\delta \sim 1$ at even higher redshifts. Hence, at very high redshifts, the instantaneous value of $\mathcal{V} \sim 2.4 \times 10^{-6} (1 + z)^3$ is solely a function of the gas density. For $z > 15$, $\mathcal{V} > 1$, implying that either $\lambda > 1$ or $\varepsilon < 0.1$ at those high redshifts. Therefore, it is likely that during the first $\sim 150$ Myr ($z \geq 15$) of its life, a BH experienced super-Eddington accretion or its radiative efficiency was $< 0.1$. This finding is consistent with the suggestion by Wythe and Loeb (2012) and Pacucci et al. (2015) that super-Eddington accretion is possible when a BH is embedded in sufficiently dense gas that renders the radiation pressure less effective. For example, in the Standard Model, the ambient gas density at $z=20$ is 27 times greater than at $z=6$.

8. CONCLUSIONS

Prompted by insights derived from a deconstruction of the so-called Salpeter relation (Eqs.1-3), we analyzed the mass versus age distribution of 91 high-z SMBHs (Fig.1) that resulted in the formulation of Eq.4, the foundation on which the findings and conclusions of this paper are based. It describes a BH’s mass $M_{\text{BH}}$ as a function of its age $t$ or redshift $z$, from which the BH’s seed mass $M_s$ can be determined. It was extensively tested throughout the paper by verifying its implications and predictions. It, together with its derivates (Eqs.5-7), comprises a set of powerful tools to decipher the origins, growth, and properties of SMBHs.

We applied Eq.4 to 93 high-z ($>5.6$) and 132,446 AGNs at $z<2.4$ listed by Kozlowski (2017). The resulting mass distributions of seeds (Figs. 2, 3, 4, and Table 1) show that the masses of the smallest to the largest actively accreting SMBHs observed to date are accounted for by seeds formed at $z \sim 30$ ranging in $M_s$ from a low of $\sim 5M_\odot$ to a maximum of $(3\pm 1)x10^4M_\odot$. In particular, the $M_{\text{BH}}$ of GNz11, CEERS_1019, and UHZ1), the three highest redshift ($z=8.7-10.6$) AGNs discovered recently, are accounted for by stellar-mass seeds ranging from a few tens to a few hundred solar masses. Specifically, the results exclude the possibility that the seed of UHZ1 was heavy, presumably a DCBH, as postulated by Natrajan et al. (2024). Equation. 4A places an upper limit of $\sim 2.2x10^6M_\odot$ on the mass a seed can accrete via luminous accretion, which translates into $(6.6\pm 2.2)x10^{10}M_\odot$ for $M_s = (3\pm 1)x10^4M_\odot$ in agreement with a theoretical limit proposed by King (2016) and with the size of the largest SMBHs observed to date.
The mass distribution of seeds shown in Fig.4 and Table 1 was analyzed and compared with the simulated mass functions of first stars and intermediate-size BHs reported in the literature. Based on this comparative analysis, we classified the seed population into three broad categories depending on seed size and the likely mechanism for its formation, fully recognizing that there is probably overlap between the categories. Seeds $\lesssim 400M_\odot$ were classified as light seeds predominantly formed from the collapse of massive metal-free first stars (Madau and Rees, 2001; Johnson and Bromm, 2007). Their observed mass distribution in Fig.4 and Table 1 resembles the mass function in simulations of first stars by Hirano et al. (2014). Seeds ranging from 400M_\odot-3x10^3M_\odot were classified as intermediate size formed either by runaway collisions of stars in dense stellar clusters (Portegies et al., 2004; Freitag et al., 2006) or by the hierarchical growth via runaway mergers of BHs (Davies et al., 2011; Lupi et al., 2014). Their observed mass distribution (Fig.4 and Table 1) resembles the mass functions of BHs in simulations of runaway collisions of stars by Devecchi et al. (2012) and in gas-induced runaway merger of BHs by Lupi et al. (2014). Light seeds constitute $\sim$58% and intermediate size $\sim$ 39.4% of the population in Table 1. The remaining <3% raging in mass from $\sim$ 3x10^3M_\odot to $\sim$3x10^4M_\odot are dubbed heavier seeds in contrast to the classical heavy seeds ($10^6M_\odot$) thought to be DCBHs.

Of the 2 mechanisms for intermediate-size seeds, the hierarchical growth of BHs via runaway mergers is the more likely. In the Devecchi et al. (2012) simulations, the BHs formed at $z\sim$15 or significantly later than at $z\sim$30; whereas there is no such time restriction for the runaway merger of BHs as pointed out by Lupi et al. (2024). Furthermore, if the hierarchical growth occurred through the merger of Pop III remnants instead of new BHs formed at a later time, it would explain why light and intermediate-size seeds formed almost concurrently at $z\sim$30, why the two follow the same asymptotically declining trend as seed mass increases with no hiatus in the distribution of seed sizes as observed in Fig.4, and why light seeds outnumber intermediate-size seeds. Of the heavier seeds, only a minuscule number (210) have $>10^6M_\odot$ and that too at the lower end of the presumed sizes of DCBHs, which led us to conclude that the DCBH mechanism did not play a significant role and propose that the heavier seeds could also have formed via the merger of light to intermediate-size seeds. Apropos, Davies et al. (2011) postulated that BHs as large as $10^8M_\odot$ can form via runaway merger of BHs. In summary, the entire population of seeds in Table 1 could be Pop III remnants and larger BHs resulting from their hierarchical growth via runaway mergers under appropriate conditions. One such condition could be the inflow of gases as in the simulations by Lupi et al. (2014).

Equation 5 gives a BH’s instantaneous accretion rate as a function of its mass $M_{ BH}$ and age $t$ or redshift $z$. Eq.6 gives the same in terms of its seed mass $M_s$ (inferred from Eq.4) and $z$. For example, Eq. 5 predicts an accretion rate of $\sim 0.012M_\odot$/yr for GNz11, the smallest ($\sim$1.5x10^6M_\odot) high-z (10.6) AGN; $\sim$24M_\odot/yr for the largest ($\sim$1.24x10^{10}M_\odot) high-z (6.3) SMBH; and 6.6M_\odot/yr for TON 618 at $z\sim$2.22 arguably the largest (4.07x10^{10}M_\odot) AGN observed to date.
Figure 5 illustrates the change in a BH’s $\dot{M}$ with $z$ from its inception as a seed at $z\sim30$ to the present. Initially, $\dot{M}$ increases exponentially and reaches a broad plateau between $z=8.5-6$, after which it decreases monotonically. Two factors, namely the increase in a BH’s gravitational reach as its mass increases and the decrease in gas density as $z$ decreases, modulate the accretion rate as the BH ages. A BH’s mass increases by $\sim$6 orders of magnitude in the first billion years and only by a factor of $\sim$4 in the next $\sim$12.8 billion years. Also, we noted that the timing ($z=8.5-6$) of the maximum accretion rate in a BH’s history coincides with the epoch when reionization is thought to have occurred.

Equation 7 expresses the Eddington ratio $\lambda$ as a function of a BH’s $z$ and radiative efficiency $\varepsilon$. It implies that $\lambda$ decreases as $z$ decreases, an implication substantiated by unambiguous empirical evidence (see Aggarwal, 2024). Furthermore, a BH’s radiative efficiency $\varepsilon$ can be determined using Eq.7 from its $\lambda$. We, however, stressed that $\lambda$ is prone to large errors resulting from numerous uncertainties (see text), and hence Eq.7 should be used with caution. Nevertheless, using $\lambda$ data for two large groups of similar-size BHs at two different redshifts, we showed that $\varepsilon$ is significantly lower for the higher-$z$ group even though its $\lambda$ is substantially higher. The implication is that $\varepsilon$ increases as $z$ decreases or that a BH becomes less efficient in accreting gases as it ages, which suggests that $\varepsilon$ is an inverse function of the ambient gas density consistent with the Wythe and Loeb (2012) and Pacucci et al. (2015) suggestion that the radiation pressure is less effective when a BH is embedded in dense gas. Finally, applying Eq.7 to Eq.2, we inferred that at redshifts $>15$, the radiative efficiency is significantly $<0.1$ its canonical value or that $\lambda>1$, which suggests that SMBH’s may have experienced super-Eddington accretion for a short period of $\sim$150 Myr from the inception of their seeds at $z\sim30$.

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Table A1 gives the data for 59 of the 91 SMBHs in Figs.1 and 2 with references for the data sources. The BHs are listed in order of their redshift $z$ from the highest to the lowest. The first reference # is for BH’s discovery paper, and the second # is for BH’s mass estimate. The remaining 32 high-$z$ SMBHs are in Table 3 of Shen et al. (2019) in the order they appear.

J0002+2550; J0008-0626; J0810+5105; J0835+3217; J0836+0054; J0840+5624; J0841+2905; J0842+1218; J0850+3246; J1044-0125; J1137+3549; J1143+3808; J1148+5251; J1207+0630; J1243+2529; J1250+3130; J1257+6349; J1403+0902; J1425+3254; J1427+3312; J1436+5007; J1545+6028; J1602+4228; 1609+3041; J1623+3112; J1630+4012; P000+26; P060+24; P210+27; P228+21; and P333+26

| BH # | Black Hole Name                  | BH Mass MBH $(M_{\odot})$ $(\pm 1\sigma)$ | $z$ | Age (Myr) | Ref. |
|------|---------------------------------|---------------------------------|-----|-----------|------|
| 1    | J0313-1806                      | $1.6 \times 10^9$ $(+0.4/-0.4)$  | 7.64 | 676       | 1    |
| 2    | ULAS J1342+0928                 | $9.1 \times 10^8$ $(+1.3/-1.4)$ | 7.541 | 688       | 2    |
| 3    | J100758.264+211529.207          | $1.5 \times 10^9$ $(+0.2/-0.2)$  | 7.52 | 690       | 3    |
| 4    | ULAS J1120+0641                 | $2.0 \times 10^9$ $(+1.5/-0.7)$  | 7.085 | 747       | 4    |
| 5    | J124353.93+010038.5             | $3.3 \times 10^8$ $(+2.0/-2.0)$  | 7.07 | 749       | 5    |
| 6    | J0038-1527                      | $1.33 \times 10^8$ $(+0.25/-0.25)$ | 7.021 | 756       | 6    |
| 7    | DES J025216.64–050331.8         | $1.39 \times 10^8$ $(+0.16/-0.16)$ | 7   | 759       | 7    |
| 8    | ULAS J2348-3054                 | $2.1 \times 10^9$ $(+0.5/-0.5)$  | 6.886 | 775       | 8    |
| 9    | VDES J0020-3653                 | $1.67 \times 10^9$ $(0.32/-0.32)$ | 6.834 | 783       | 9    |
| 10   | PSO J172.3556+18.7734           | $2.9 \times 10^8$ $(+0.7/-0.6)$  | 6.823 | 784       | 10   |
| 11   | ULAS J0109-3047                 | $1.5 \times 10^8$ $(+0.4/-0.4)$  | 6.745 | 796       | 8    |
| 12   | HSC J1205-0000                  | $2.9 \times 10^8$ $(+0.4/-0.4)$  | 6.73  | 799       | 11,12|
| 13   | VDES J0244-5008                 | $1.15 \times 10^9$ $(+0.39/-0.39)$ | 6.724 | 800       | 9    |
|   |          |               |     |     |    |
|---|----------|---------------|-----|-----|----|
| 14| PSO J338.2298 | 3.7 x 10^9 (+1.3/-1.0) | 6.658 | 810 | 13 |
| 15| ULAS J0305-3150 | 1.0 x 10^9 (+0.1/-0.1) | 6.604 | 819 | 8  |
| 16| PSO J323.1382 | 1.39 x 10^9 (+0.32/-0.51) | 6.592 | 821 | 14 |
| 17| PSO J231.6575 | 3.05 x 10^9 (+0.44/-2.24) | 6.587 | 820 | 14 |
| 18| PSO J036.5078 | 3 x 10^9 (+0.92/-0.77) | 6.527 | 831 | 13,14 |
| 19| VDES J0224-4711 | 2.12 x 10^9 (+0.42/-0.42) | 6.526 | 831 | 9  |
| 20| PSO J167.6415 | 3 x 10^9 (+0.08/-0.12) | 6.508 | 834 | 13,14 |
| 21| PSO J261+19 | 6.7 x 10^9 (+0.21/-0.21) | 6.483 | 839 | 15 |
| 22| PSO J247.2970 | 5.2 x 10^9 (+0.22/-0.25) | 6.476 | 840 | 14 |
| 23| PSO J011+09 | 1.20 x 10^9 (+0.51/-0.51) | 6.458 | 843 | 15 |
| 24| CFHQS J0210-0456 | 8 x 10^7 (+5.5/-4.0) | 6.438 | 846 | 16 |
| 25| CFHQS J2329-0301 | 2.5 x 10^9 (+0.4/-0.4) | 6.417 | 850 | 16 |
| 26| SDSS J1148+5251 | 2.7 x 10^9 (+0.4/-0.4) | 6.41 | 851 | 17,18 |
| 27| HSC J0859+0022 | 3.8 x 10^7 (+0.1/-0.18) | 6.388 | 855 | 11,19 |
| 28| HSC J1152+0055 | 6.3 x 10^6 (+0.8/-1.2) | 6.36 | 860 | 11,19 |
| 29| SDSS J1148+0702 | 1.26 x 10^9 (+0.14/-0.14) | 6.339 | 863 | 20 |
| 30| SDSS J1030+0524 | 1.0 x 10^9 (+0.2/-0.2) | 6.3 | 870 | 21,22 |
| 31| SDSS J0100+2802 | 1.24 x 10^9 (+0.19/-0.19) | 6.3 | 870 | 23 |
| 32| CFHQS J0050+3445 | 2.6 x 10^9 (+0.50/-0.4) | 6.253 | 879 | 16 |
| 33| HSC J2239+0207 | 1.1 x 10^9 (+3/-2) | 6.245 | 880 | 19 |
| 34| VDES J0330–4025 | 5.87 x 10^9 (+0.89/-0.89) | 6.239 | 881 | 15 |
| 35| VDES J0323–4701 | 5.5 x 10^9 (+1.26/-1.26) | 6.238 | 881 | 15 |
| No. | Source | Name               | Magnitude | Redshift | Distance | Type  |
|-----|--------|--------------------|-----------|----------|----------|-------|
| 36  | SDSS   | J1623+3112         | 1.5 x 10^9 (+0.3/-0.3) | 6.211    | 886      | 21    |
| 37  | SDSS   | J1048+4637         | 3.9 x 10^9 (+2.1/-2.1)  | 6.198    | 889      | 24    |
| 38  | PSO    | J359–06            | 1.66 x 10^9 (+0.21/-0.21) | 6.164    | 895      | 15    |
| 39  | CFHQS  | J0221-0802         | 7 x 10^9   (+7.5/-4.7)    | 6.161    | 896      | 16    |
| 40  | HSC    | J1208-0200         | 7.1 x 10^9 (+2.4/-5.2)   | 6.144    | 899      | 19    |
| 41  | ULAS   | J1319+0950         | 2.7 x 10^9 (+0.6/-0.6)   | 6.131    | 902      | 25,26 |
| 42  | CFHQS  | J1509-1749         | 3 x 10^9   (+0.3/-0.3)    | 6.121    | 903      | 16    |
| 43  | PSO    | J239–07            | 3.63 x 10^9 (+0.20/-0.20) | 6.114    | 905      | 15    |
| 44  | HSC    | J2216-0016         | 7 x 10^9   (+1.4/-2.3)    | 6.109    | 906      | 19    |
| 45  | CFHQS  | J2100-1715         | 3.37 x 10^9 (+0.64/-0.64) | 6.087    | 910      | 16    |
| 46  | SDSS   | J0303-0019         | 3 x 10^9   (+2.0/-2.0)    | 6.079    | 911      | 24    |
| 47  | SDSS   | J0353+0104         | 1.4 x 10^9 (+1.0/-1.0)    | 6.072    | 913      | 24    |
| 48  | SDSS   | J0842+1218         | 1.7 x 10^9 (+1.2/-1.2)    | 6.069    | 913      | 24    |
| 49  | SDSS   | J1630+4012         | 9 x 10^9   (+0.8/-0.8)    | 6.058    | 915      | 24    |
| 50  | PSO    | J158–14            | 2.15 x 10^9 (+0.25/-0.25) | 6.057    | 916      | 15    |
| 51  | CFHQS  | J1641+3755         | 2.4 x 10^9 (+1.0/-0.8)    | 6.047    | 918      | 16    |
| 52  | SDSS   | J1306+0356         | 1.1 x 10^9 (+0.1/-0.1)    | 6.017    | 923      | 21    |
| 53  | SDSS   | J2310+1855         | 2.8 x 10^9 (+0.6/-0.6)    | 6.003    | 926      | 27, 19|
| 54  | CFHQS  | J0055+0146         | 2.4 x 10^9 (+0.9/-0.07)   | 5.983    | 930      | 16    |
| 55  | PSO    | J056–16            | 7.5 x 10^9 (+0.07/-0.07)  | 5.975    | 932      | 15    |
| 56  | SDSS   | J1411+1217         | 1.1 x 10^9 (+0.1/-0.01)   | 5.931    | 941      | 28, 22|
to the lowest. The first reference # is for the BH discovery paper, and the second # is for the BH mass estimate.

**References in Table A1**

|   | Parameters of SMBHs at $z > 5.7$ | Notes: BHs are listed in order of their redshift from the highest to the lowest. |   |
|---|----------------------------------|---------------------------------|---|
| 57 | SDSS J0005-0006                  | 3 x 10^9 (+0.1/-01)             | 5.85 957 28,22 |
| 58 | SDSS J0836+0054                  | 2.7 x 10^9 (+0.6/-0.6)          | 5.82 964 28, 22 |
| 59 | SDSS J1044-0125                  | 1.05 x 10^10 (+0.16/-0.16)      | 5.784 971 29, 21 |

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