SPONTANEOUSLY BROKEN SU(5) SYMMETRIES
AND ONE-LOOP EFFECTS
IN THE EARLY UNIVERSE*

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Abstract. This paper studies one-loop effective potential and spontaneous-symmetry-breaking pattern for SU(5) gauge theory in De Sitter space-time. Curvature effects modify the flat-space effective potential by means of a very complicated special function previously derived in the literature. An algebraic technique already developed by the first author to study spontaneous symmetry breaking of SU(n) for renormalizable polynomial potentials is here generalized, for SU(5), to the much harder case of a De Sitter background. A detailed algebraic and numerical analysis provides a better derivation of the stability of the extrema in the maximal subgroups SU(4) × U(1), SU(3) × SU(2) × U(1), SU(3) × U(1) × U(1) × R_{311}, SU(2) × SU(2) × U(1) × U(1) × R_{2211}, where R_{311} and R_{2211} discrete symmetries select
Spontaneously broken $SU(5)$ symmetries and one-loop effects in the early universe

particular directions in the corresponding two-dimensional strata. One thus obtains a
deeper understanding of the result, previously found with a different numerical analysis,
predicting the slide of the inflationary universe into either the $SU(3) \times SU(2) \times U(1)$ or
$SU(4) \times U(1)$ extremum. Interestingly, using this approach, one can easily generalize all
previous results to a more complete $SU(5)$ tree-level potential also containing cubic terms.

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1. Introduction

In the cosmological standard model [1], one assumes that gravity is described by Einstein’s general relativity, and that the observed universe is spatially homogeneous and isotropic. Moreover, if the energy-momentum tensor takes a perfect-fluid form, Einstein’s equations lead in particular to the following differential equation governing the time evolution of the cosmic scale factor $a(t)$:

$$
\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \rho ,
$$

(1.1)

where $k = +1, 0, -1$ respectively for a closed, flat or open universe, $G$ is Newton’s constant, and $\rho$ is the energy density. In the matter-dominated era $\rho$ is proportional to $a^{-3}$, and in the radiation-dominated era $\rho$ is proportional to $a^{-4}$.

The model here outlined, however, leads to a paradox: the universe would contain about $10^{84}$ regions causally disconnected, although its large-scale properties are described by the Friedmann-Robertson-Walker geometry. Moreover, denoting by $\rho_{cr}$ the energy-density value separating an open from a closed universe, one would find

$$
\left|\frac{\rho - \rho_{cr}}{\rho}\right| < 10^{-55} .
$$

(1.2)

This is a severe fine-tuning problem, since condition (1.2) does not seem to arise by virtue of general principles, and appears as an *ad hoc* extra assumption.

However, as shown in [2], one might hope to solve these problems (cf. [3,4]) if the cosmic scale factor $a(t)$ grows exponentially in the early universe, rather than following the
Spontaneously broken SU(5) symmetries and one-loop effects in the early universe

t\gamma\text{-behaviour of the cosmological standard model. This can be achieved if the right-hand side of Eq. (1.1) is constant, since this implies

\[ a(t) = a_0 \exp\left(\sqrt{\frac{8\pi}{3}} G \rho_0 t \right), \quad t \in [t_0, t_a[ , \tag{1.3} \]

provided the effect of \( \frac{k}{a^2} \) can be neglected in the interval \( [t_0, t_a[ \). One can then show that causally disconnected regions would no longer occur (although severe inhomogeneities can be shown to remain [5]). For this purpose, we need at least a (massive [5,6], or massless self-interacting [7]) scalar field, or a more complete theory of matter fields providing a large vacuum-energy density (\( \gg M_W \)) which drives inflation, i.e. the evolution of \( a(t) \) described by Eq. (1.3). If Eq. (1.3) holds, the corresponding geometry is the one of De Sitter space-time, the Lorentzian four-manifold with \( R \times S^3 \) topology and constant positive scalar curvature.

The naturally occurring candidates for a very fundamental theory which provides at the same time the unification of electro-weak and strong interactions, and a suitably large vacuum energy (see above) for symmetry-breaking are the GUTs [5,8]. Although the minimal SU(5) theory [9] has been ruled out by proton-decay experiments [10,11], the study of this SU(5) model may be very instructive. Moreover, it is worth bearing in mind that SU(5) is contained in SO(10) and \( E_6 \) [8].

We here study the one-loop effective potential to determine the phase to which the early universe eventually evolves [12,13]. Since we are interested in quantum-field-theory calculations, we use the Wick-rotated path-integral approach, and we work on the real, Riemannian section of the corresponding complex space-time manifold. Note that this
rotation does not affect the effective potential, while making the perturbative theory well-defined (see below). We are thus interested in the Riemannian version of the De Sitter manifold, with $S^4$ topology. Its metric is smooth and positive-definite, and the action of the non-abelian Yang-Mills-Higgs theory here studied involves elliptic, self-adjoint, positive-definite differential operators leading to Gaussian integrals, so that the corresponding one-loop calculations are well-defined, even though the full quantum theory via path integrals does not seem to have rigorous mathematical foundations. Note that we are not quantizing gravity, but we study quantized matter fields in a fixed, curved, Riemannian background geometry via Wick-rotated path integrals and perturbation theory.

Our paper is thus organized as follows. Sect. 2 describes the minimal $SU(5)$ model in De Sitter space and the corresponding results for the one-loop effective potential [13]. Sect. 3 presents the basic results about the tree-level Higgs potential for $SU(5)$ gauge theory in flat space [14]. The special function $\mathcal{A}$ occurring in the corresponding one-loop calculation in a De Sitter background is then studied in detail. Sect. 4 provides the generalization of the technique used in [14] to a De Sitter background. Absolute minima are derived using both analytic and numerical calculations, improving the understanding obtained in [13]. Exact, approximate and asymptotic formulae for the one-loop effective potential are shown to shed new light on the $SU(5)$ symmetry-breaking pattern. Finally, the concluding remarks are presented in Sect. 5.
2. **SU(5) model in De Sitter space**

Following the introduction and [13], the bare Lagrangian $L_0$ and the renormalizable tree potential of our $SU(5)$ Yang-Mills-Higgs theory in De Sitter space are taken to be respectively (after analytic continuation to the Riemannian manifold with $S^4$ topology)

$$L_0 = \frac{1}{4} Tr \left( F_{\mu \nu} F^{\mu \nu} \right) + \frac{1}{2} Tr \left[ \left( D_\mu \Phi \right) \left( D^\mu \Phi \right)^{\dagger} \right] + V_0(\Phi), \quad (2.1)$$

$$V_0(\Phi) = \frac{\xi}{2} RT \left( \Phi^2 \right) + \Lambda_2 \left( Tr \Phi^2 \right)^2 + \Lambda_4 \left( Tr \Phi^4 \right), \quad (2.2)$$

where $F_{\mu \nu} \equiv \nabla_\mu A_\nu - \nabla_\nu A_\mu - ig \left[ A_\mu, A_\nu \right]$, and $D_\mu \Phi \equiv \partial_\mu \Phi - ig \left[ A_\mu, \Phi \right]$. Note that the covariant derivative $\nabla_\mu$ differs from $\partial_\mu$ for terms involving Christoffel symbols [1], and $V_0(\Phi)$ is assumed to obey the symmetry $V_0(\Phi) = V_0(-\Phi)$. Moreover, as usual, $g$ is the dimensionless coupling constant and $R = \frac{12}{r^2}$ is the scalar curvature of De Sitter space ($r$ being the four-sphere radius).

The Higgs scalar field $\Phi$ is assumed to be in the adjoint representation of $SU(5)$ [13]. The presence in the minimal $SU(5)$ model of an additional representation $(5)$ of scalar fields $H$, necessary to break the symmetry down to $SU(3)_C \times U(1)_Q$, is irrelevant for the inflationary scheme, due to the smaller mass value $M_H \approx M_W$.

The background-field method is now applied to obtain the one-loop form of the potential, writing the Higgs field as $\Phi_0 + \tilde{\Phi}$, where $\Phi_0$ is a constant background field and $\tilde{\Phi}$
Spontaneously broken SU(5) symmetries and one-loop effects in the early universe

a fluctuation around $\Phi_0$ (and similarly for $A^\mu$). As explained in [13], it is convenient to choose t’Hooft’s gauge-fixing term

$$L_{G.F.} = \frac{\alpha}{2} Tr \left( \nabla_\mu A^\mu - i g \alpha^{-1} [\Phi_0, \Phi] \right)^2,$$

and Coleman-Weinberg’s theory can be used to neglect the contribution of all scalar-field loop diagrams. This implies that only gauge-field loop diagrams are relevant. A very convenient form of the one-loop potential is obtained using the gauge invariance of the theory which enables one to diagonalize the scalar field $\Phi$. The corresponding diagonal form of $\Phi$ is here denoted by $\hat{\Phi} = \text{diag}(\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5)$, where $\sum_{i=1}^{5} \varphi_i = 0$. Thus, denoting by $\psi(t)$ the special function $\frac{\Gamma'(t)}{\Gamma(t)}$, and defining

$$A(z) = \frac{z^2}{4} + \frac{z}{3} - \int_{2}^{t} \frac{\psi(t)}{(t - \frac{3}{2})(t - 3)\psi(t)} dt$$

$$- \int_{1}^{\frac{3}{2}} \frac{\psi(t)}{(t - \frac{3}{2})(t - 3)\psi(t)} dt,$$

the one-loop effective potential for the minimal SU(5) model is found to be [13]

$$V(\hat{\Phi}) = \frac{15}{64\pi^2} \left\{ Q + \frac{1}{3} \left( 1 - \log(r^2 M_X^2) \right) R g^2 \| \hat{\Phi} \|\right\}$$

$$+ \left\{ \frac{9}{128\pi^2} \left( 1 - \log(r^2 M_X^2) \right) - \frac{21}{320\pi^2} \Lambda \right\} g^4 \| \hat{\Phi} \|^2$$

$$+ \frac{15}{128\pi^2} \left\{ \frac{12}{5} \Lambda + \left( 1 - \log(r^2 M_X^2) \right) \right\} g^4 \sum_{i=1}^{5} \varphi_i^4$$

$$- \frac{3}{16\pi^2 r^4} \sum_{i,j=1}^{5} A \left[ \frac{r^2 g^2}{2} (\varphi_i - \varphi_j)^2 \right],$$

(2.5)
Spontaneously broken $SU(5)$ symmetries and one-loop effects in the early universe

where [13]

\[
Q \equiv \frac{32\pi^2}{15g^2} \left[ \xi - \frac{8}{5g^2} \left( \Lambda_2 + \frac{7}{30} \Lambda_4 \right) \right],
\]

\[
\Lambda \equiv \frac{64\pi^2}{15g^4} \left( \frac{3}{5} \Lambda_4 - \Lambda_2 \right),
\]

\[
\| \Phi \| \equiv \sum_{i=1}^{5} \varphi_i^2,
\]

and $M_X$ is related to the dimensional parameter $\mu$ appearing in the (regularized) one-loop amplitudes [12]. Moreover, if $\xi = \frac{1}{6}$, the Higgs field is conformally coupled to gravity.

The one-loop potential $V(\Phi)$ is then used to determine broken-symmetry phases and curved-space phase diagrams as shown in [13]. As a result of his numerical analysis, the author of [13] found what follows:

1. In the $SU(5)$ theory, the universe, in addition to the right $SU(3) \times SU(2) \times U(1)$ direction, is also likely to end up in the wrong $SU(4) \times U(1)$ phase;

2. The $SU(2) \times SU(2) \times U(1) \times U(1) \times R_{2211}$ and $SU(3) \times U(1) \times U(1) \times R_{311}$ phases are unstable for any values of the parameters appearing in the model.

As we said in the introduction, the aim of this paper is to provide a better understanding of the results obtained in [13]. For this purpose, we recall some basic results about spontaneous symmetry breaking of $SU(n)$ [14], and about the $A$ function [13] defined in Eq. (2.4).
3. Polynomial potentials and the $A$ function

To study the spontaneous-symmetry-breaking directions of the potential in Eq. (2.5), it is convenient to define the variables

$$a_i \equiv \frac{g r}{\sqrt{2}} \varphi_i . \quad (3.1)$$

For our purpose, it is not strictly needed to study the part of the potential depending on the norm of the $a$ field: $\| a \| \equiv \sum_{i=1}^{5} a_i^2$. The relevant part of the potential is instead given by (up to the multiplicative constant $\frac{3r^{-4}}{16\pi^2}$)

$$V_M \equiv b \sum_{i=1}^{5} a_i^4 - \sum_{i,j=1}^{5} A[(a_i - a_j)^2] , \quad (3.2)$$

$$b \equiv 6\Lambda + \frac{5}{2} \left( 1 - \log(r^2 M_X^2) \right) . \quad (3.3)$$

As a first step, it is useful to recall the exact results [14] holding for a theory where $V_M$ is only given by the first term on the right-hand side of Eq. (3.2). In that case, since $\sum_{i=1}^{5} a_i$ is set to zero, and $\sum_{i=1}^{5} a_i^2$ equals $\| a \|$ by definition, the Lagrange-multipliers technique can be used to study the third-order algebraic equations leading to the calculation of the minima [14].

The corresponding results yield, for the minima with the residual symmetry :

$$a = a_{321} \equiv \frac{\| a \|^\frac{7}{2}}{\sqrt{30}} \text{diag}(2, 2, 2, -3, -3) , \quad (SU(3) \times SU(2) \times U(1)) . \quad (3.4)$$
Spontaneously broken SU(5) symmetries and one-loop effects in the early universe

\[ a = a_{2211} \equiv \frac{\|a\|}{2} \; \text{diag}(1,1,0,-1,-1) \; , \; (SU(2) \times SU(2) \times U(1) \times U(1) \times R_{2211}) \; , \quad (3.5) \]

\[ a = a_{311} \equiv \frac{\|a\|}{\sqrt{2}} \; \text{diag}(0,0,1,-1) \; , \; (SU(3) \times U(1) \times U(1) \times R_{311}) \; , \quad (3.6) \]

\[ a = a_{41} \equiv \frac{\|a\|}{\sqrt{20}} \; \text{diag}(1,1,1,-4) \; , \; (SU(4) \times U(1)) \; , \quad (3.7) \]

the hierarchy

\[ V_M(a_{41}) > V_M(a_{311}) > V_M(a_{2211}) > V_M(a_{321}) \quad (3.8) \]

if \( b > 0 \), and the reversed inequalities if \( b < 0 \). Thus, when the Higgs field is in the adjoint representation, the SU(5) symmetry breaking leads only to the SU(4) × U(1) or SU(3) × SU(2) × U(1) symmetric minima.

Since the complete \( V_M \) potential is in our case given by Eqs. (3.2-3), we need to study in detail the contribution of the \( \mathcal{A} \) function. While performing this analysis, it is useful to supplement definition (2.4) by the Taylor expansion of \( \mathcal{A} \) as \( z \to 0 \), and its asymptotic expansion as \( z \to \infty \), which are given respectively by [13]

\[ \mathcal{A}(z) = 2 \left( \gamma - \frac{1}{3} \right) z + \frac{(\gamma - 1)}{2} z^2 + \frac{z^3}{6} \left( -5 + 4\zeta(3) \right) \]

\[ + \frac{z^4}{24} \left( -36 + 30\zeta(3) \right) + O(z^5) \; , \quad (3.9) \]

\[ \mathcal{A}(z) \sim -\left( \frac{z^2}{4} + z + \frac{19}{30} \right) \log(z) + \frac{3}{8} z^2 + z \; , \quad (3.10) \]
where \( \gamma \) is Euler’s constant and \( \zeta \) is the Riemann zeta-function [12]. Using Eqs. (2.4) and (3.9-10), we have found inequalities analogous to (3.8). In other words, defining

\[
A(a_{41}) \equiv -\sum_{i,j=1}^{5} A\left[(a_i - a_j)^2\right]_{a=a_{41}} ,
\]

and similarly for the other phases, one finds

\[
A(a_{41}) > A(a_{311}) > A(a_{2211}) > A(a_{321}) ,
\]

where

\[
A(a_{41}) = -8A\left(\frac{5}{4}||a||\right) ,
\]

\[
A(a_{311}) = -12A\left(\frac{||a||}{2}\right) - 2A\left(2||a||\right) ,
\]

\[
A(a_{2211}) = -8A\left(\frac{||a||}{4}\right) - 8A\left(||a||\right) ,
\]

\[
A(a_{321}) = -12A\left(\frac{5}{6}||a||\right) .
\]

The inequalities appearing in Eq. (3.12) are illustrated in Figures 1-3.

4. Absolute minima

For fixed values of the bare parameters \( \xi, \Lambda_2, \Lambda_4 \) and \( M_X \) (cf. Eqs. (2.6,7) and (3.3)), \( b \) depends on \( r \) as shown in Eq. (3.3). Thus in the early universe, at small values of \( r \), i.e.
when the scalar curvature is very large, $b$ is positive, whereas it may become negative as $r$ increases.

As shown in Sect. 3, when $b > 0$, the two terms of the $V_M$ potential in Eq. (3.2) follow the inequalities (3.8) and (3.12). This implies that in the very early universe the only possible phase transition is $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$.

By contrast, for suitably large values of $r$, $b$ becomes negative, and the polynomial part of the $V_M$ potential is then dominant. In this case the analysis in [14] holds, and the phase transition occurs in the $SU(4) \times U(1)$ direction (i.e. the previous hierarchy is inverted).

A more detailed analysis is however in order when $b < 0$ but $|b|$ is not too large. For this purpose, using the Taylor expansion (3.9) up to third-order, we begin by studying the range of validity of the inequalities

\begin{align}
V_M(a_{41}) > V_M(a_{311}) > V_M(a_{2211}) > V_M(a_{321}) \quad .
\end{align}

(4.1)

Thus, defining $\Omega \equiv \left( \frac{-5+4\zeta(3)}{6} \right) < 0$, one finds

\begin{align}
\left[ V_M(a_{41}) - V_M(a_{321}) \right] > 0 \iff 12b + 60(1 - \gamma) > -250 | \Omega | \parallel a \parallel ,
\end{align}

(4.2)

\begin{align}
\left[ V_M(a_{311}) - V_M(a_{321}) \right] > 0 \iff 12b + 60(1 - \gamma) > -475 | \Omega | \parallel a \parallel ,
\end{align}

(4.3)

\begin{align}
\left[ V_M(a_{2211}) - V_M(a_{321}) \right] > 0 \iff 12b + 60(1 - \gamma) > -850 | \Omega | \parallel a \parallel ,
\end{align}

(4.4)

\begin{align}
\left[ V_M(a_{41}) - V_M(a_{311}) \right] > 0 \iff 12b + 60(1 - \gamma) > 150 | \Omega | \parallel a \parallel ,
\end{align}

(4.5)

\begin{align}
\left[ V_M(a_{41}) - V_M(a_{2211}) \right] > 0 \iff 12b + 60(1 - \gamma) > -225 | \Omega | \parallel a \parallel ,
\end{align}

(4.6)
Spontaneously broken SU(5) symmetries and one-loop effects in the early universe

\[ \left[ V_M(a_{311}) - V_M(a_{2211}) \right] > 0 \iff 12b + 60(1 - \gamma) > -450 |\Omega| \parallel a \parallel . \]  

(4.7)

In light of Eqs. (4.2)-(4.7), if Eq. (4.5) is satisfied, this ensures that all remaining conditions hold. One thus obtains the inequality

\[ b > 5(\gamma - 1) + \left[ \frac{25}{2} |\Omega| \parallel a \parallel + O(\parallel a \parallel^2) \right] = b_0 , \]  

(4.8)

which is a necessary and sufficient condition for the validity of Eq. (4.1) when the Taylor expansion (3.9) is a good approximation. Note that the term in square brackets on the r.h.s. of Eq. (4.8) is a small correction of the value \( 5(\gamma - 1) < 0 \) provided \( \parallel a \parallel \to 0 \), as one would expect when the Taylor expansion makes sense. Interestingly, the inequalities (4.1) still hold for negative values of \( b \) provided Eq. (4.8) is satisfied, whereas the flat-space tree-level potential \( \hat{V}_M = \hat{b} \sum_{i=1}^{5} a_i^4 \) used in [14] leads to the value \( b_0 = 0 \).

Moreover, the reversed hierarchy (cf. (4.1))

\[ V_M(a_{321}) > V_M(a_{2211}) > V_M(a_{311}) > V_M(a_{41}) \]  

(4.9)

holds provided the following necessary and sufficient condition is satisfied (cf. (4.4)):

\[ b < 5(\gamma - 1) + \left[ -\frac{425}{6} |\Omega| \parallel a \parallel + O(\parallel a \parallel^2) \right] = b_1 . \]  

(4.10)

Again, the De Sitter background leads to a value \( b_1 \neq 0 \) with respect to the flat-space tree-level-potential result \( b_1 = b_0 = 0 \).

This preliminary analysis should be supplemented by a more detailed numerical study. The aim of this investigation is to prove that, for \textit{all} values of \( \parallel a \parallel \) and \( b \), the phase
transition occurs only in the $SU(3) \times SU(2) \times U(1)$ or $SU(4) \times U(1)$ directions. From our previous discussion (see also Figures 4-6), when $b \to +\infty$ the absolute minimum is in the $SU(3) \times SU(2) \times U(1)$ direction. However, if we compute for fixed $\|a\|$ the negative $b^0, b^1, b^2$ values of $b$ such that

$$V_M \left( b^0, a_{321} \right) = V_M \left( b^0, a_{41} \right), \quad (4.11)$$

$$V_M \left( b^1, a_{321} \right) = V_M \left( b^1, a_{2211} \right), \quad (4.12)$$

$$V_M \left( b^2, a_{321} \right) = V_M \left( b^2, a_{311} \right), \quad (4.13)$$

we find $b^0 > b^1$ and $b^0 > b^2$, $\forall \|a\|$. This means that the continuous transition to (4.9) leads to the interchanging of the $SU(3) \times SU(2) \times U(1)$ with the $SU(4) \times U(1)$ absolute minimum. Of course, similar interchanges also occur for the relative minima, but they do not affect the phase transition of the universe.

Defining

$$V_M^{(P)} \equiv \sum_{i=1}^{5} a^4_i , \quad (4.14)$$

and using Eqs. (3.13)-(3.16), it is useful to bear in mind the formulae for $b^0, b^1$ and $b^2$ obtained from Eqs. (4.11)-(4.13):

$$b^0 = \frac{\left[ A(a_{321}) - A(a_{41}) \right]}{\left[ V_M^{(P)}(a_{41}) - V_M^{(P)}(a_{321}) \right]} , \quad (4.15)$$
Spontaneously broken $SU(5)$ symmetries and one-loop effects in the early universe

$$b^1 = \frac{[A(a_{321}) - A(a_{2211})]}{[V_M^{(P)}(a_{2211}) - V_M^{(P)}(a_{321})]} , \quad (4.16)$$

$$b^2 = \frac{[A(a_{321}) - A(a_{311})]}{[V_M^{(P)}(a_{311}) - V_M^{(P)}(a_{321})]} . \quad (4.17)$$

The differences $(b^0 - b^1)$ and $(b^0 - b^2)$ are plotted in Figures 4-6 as functions of $\|a\|^2$ using Eqs. (4.15)-(4.17).

5. Concluding remarks

This paper has shown that the results in [14] about the $SU(n)$ symmetry breaking in flat space may be generalized to a curved, cosmological background such as De Sitter space.

The results in [13] have been thus re-obtained, by virtue of the properties of the $A$ function (Eq. (2.4) and Figures 1-3). They confirm that the absolute minimum of the complete one-loop potential lies either in the $SU(3) \times SU(2) \times U(1)$ or in the $SU(4) \times U(1)$ direction. This provides a better understanding (cf. [13]) of the instability of the $SU(3) \times U(1) \times U(1) \times R_{311}$ and $SU(2) \times SU(2) \times U(1) \times U(1) \times R_{2211}$ extrema, since very simple and basic algebraic and numerical techniques have been used (cf. Sect. 4).

Interestingly, we can extend all our results to the most general and renormalizable tree-level potential also containing cubic terms, since the tree-level potential does not affect the one-loop contribution within the Coleman-Weinberg approach [12,13,15], and
Spontaneously broken SU(5) symmetries and one-loop effects in the early universe

the presence of an additional cubic term in $V_M^{(P)}$ (see (4.14)) favours the directions $a_{41}$ and $a_{321}$ (for which $V_M^{(P)} < 1$) with respect to $a_{311}$ and $a_{2211}$ (for which $V_M^{(P)} = 0$). The $SU(n)$ symmetry-breaking pattern for this more general class of potentials in flat space can be found in [16], where the author extends and confirms the results obtained in [14]. The approach considered above might be used to discuss the general case of arbitrary directions in the adjoint representation of $SU(5)$; one expects, however, that even in this more general case the absolute minimum will be in the directions found by limiting the analysis to the one-dimensional orbits.

The method here described may be applied to other GUT theories, e.g. with $SO(10)$ or $E_6$ gauge groups, in De Sitter space [13]. These models appear as more realistic candidates for a unified theory of non-gravitational interactions [8]. One would then obtain a physically more relevant application of the techniques used in this paper.
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Figure captions

**Figure 1.** Differences of $\mathcal{A}$ values corresponding to (a) full curve $[\mathcal{A}(a_{41}) - \mathcal{A}(a_{321})]$, (b) broken curve $[\mathcal{A}(a_{311}) - \mathcal{A}(a_{321})]$ and (c) dotted curve $[\mathcal{A}(a_{2211}) - \mathcal{A}(a_{321})]$. They are evaluated using the Taylor expansion (3.9).

**Figure 2.** Differences of logarithms $\log_{10}$ of $\mathcal{A}$ values corresponding to

(a) full curve $\log_{10}[\mathcal{A}(a_{41})/\mathcal{A}(a_{321})]$,

(b) broken curve $\log_{10}[\mathcal{A}(a_{311})/\mathcal{A}(a_{321})]$ and

(c) dotted curve $\log_{10}[\mathcal{A}(a_{2211})/\mathcal{A}(a_{321})]$.

They are obtained using the exact formula (2.4) defining $\mathcal{A}(z)$.

**Figure 3.** Differences of logarithms $\log_{10}$ of $\mathcal{A}$ values corresponding to

(a) full curve $\log_{10}[\mathcal{A}(a_{41})/\mathcal{A}(a_{321})]$,

(b) broken curve $\log_{10}[\mathcal{A}(a_{311})/\mathcal{A}(a_{321})]$ and

(c) dotted curve $\log_{10}[\mathcal{A}(a_{2211})/\mathcal{A}(a_{321})]$.

The asymptotic expansion (3.10) is here applied.

**Figure 4.** The dotted curve corresponds to the difference $(\bar{b}^0 - \bar{b}^1)$, and the full curve corresponds to the difference $(\bar{b}^0 - \bar{b}^2)$, where $\bar{b}^0$, $\bar{b}^1$ and $\bar{b}^2$ have been obtained in Eqs. (4.15)-(4.17). The Taylor expansion (3.9) is here used for $\mathcal{A}(z)$.

**Figure 5.** Dotted and full curve have the same meaning as in Figure 4. The exact formula (2.4) is here used for $\mathcal{A}(z)$. 
Figure 6. Dotted and full curve are defined as in Figures 4 and 5. The asymptotic expansion (3.10) of $A(z)$ is here applied.