Quenched QCD at finite temperature with overlap Fermions

R. V. Gavai, Sourendu Gupta
Department of Theoretical Physics, Tata Institute of Fundamental Research,
Homi Bhabha Road, Mumbai 400005, India.

R. Lacaze
Service de Physique Theorique, CEA Saclay,
F-91191 Gif-sur-Yvette Cedex, France.

We study quenched QCD just above the phase transition temperature using overlap Fermions. Exact zero modes of the overlap operator are localized. Chiral symmetry is restored, as indicated by the behavior of the chiral condensate after subtracting the effects of zero modes. The vector and pseudo-scalar screening masses are close to ideal gas values.

In QCD for $T > T_c$ screening correlators in the vector, and axial-vector channels are saturated by nearly non-interacting quark anti-quark pairs in the medium. On the other hand, the scalar and pseudo-scalar screening masses show more complicated behavior—strong deviations from the ideal Fermi gas, and a strong temperature dependence. This puzzling behavior has been seen in quenched and dynamical simulations with two and four flavors of staggered quarks, as well as with Wilson quarks close to $T_c$. The new technique we bring to bear on this problem is the use of overlap Fermions. It has the advantage of preserving chiral symmetry on the lattice for any number of massless flavors of quarks, and hence having the correct number of pions.

The overlap Dirac operator ($D$) can be defined in terms of the negative Wilson-Dirac operator ($D_w$) by the relation

$$D = 1 - D_w (D_w^+ D_w)^{-1/2}.$$  \hspace{1cm} (1)

The computation of $D^{-1}$ needs a nested series of two matrix inversions for its evaluation; each step in the numerical inversion of $D$ involves the inversion of $D_w^+ D_w$. Details of the computation of the inverse square root of the matrix can be found in [1]. The study was performed on quenched QCD configurations at temperatures of $T/T_c = 1.25, 1.5$ and 2 on $4 \times 8^3$ and $4 \times 12^3$ lattices. The corresponding couplings are respectively $\beta = 5.8$, $5.8941$ and $6.0625$.

A massive overlap operator is defined by

$$D(ma) = ma + (1 - ma/2)D = G^{-1}(ma),$$ \hspace{1cm} (2)

where $m$ is the bare quark mass, $a$ the lattice spacing, and $D$ is defined in (1). We computed the quark propagator, $G$, on 12 point sources for 10 quark masses from $ma=0.001$ to 0.5 using a multimass inversion of $D^+D$. The Wilson mass term in $D_w$, which is an irrelevant regulator, was set to 1.8. The tolerance was $\epsilon = 10^{-6}$ in the inner CG and $10^{-4}$ in the outer CG. This meant that the Ginsparg-Wilson relation was satisfied to an accuracy of $10^{-9}$, and a chiral Ward identity to an accuracy of $10^{-5}$.

Seperately, a rough computation of the eigenvalues of $D^+D$, $\mu^2$, was made on each configuration with a Lanczos method in each chiral sector. Whenever $\mu^2 \simeq O(10^{-5})$ was obtained, the few lowest eigenvalues and eigenvectors were refined to a precision of about $10^{-8}$ by a Ritz functional minimization. As shown in Figure 1, for most configurations we found that the lowest $\mu^2$ was well away from zero. However, for some configurations we found zero and near-zero modes with $\mu^2 < 10^{-4}$ clearly seperated by a gap from the non-zero modes with $0.1 < \mu^2$. The non-zero modes clearly came in degenerate pairs of opposite chiralities. The zero modes were all less than
$10^{-7}$ and of definite chirality. There seems to be a spectral gap between these and the near zero modes (which were seen only at $T = 1.25T_c$). It remains to be seen whether the gap scales with lattice volume.

**Figure 1.** Scatter plot of the eigenvalue, $\mu^2$, and localisation, $\sigma$ for the smallest eigenvalues on $4 \times 12^3$ lattices at $T = 1.25T_c$. The pluses and crosses denote chiral positive and negative sectors respectively.

We constructed a gauge-invariant measure of localization, $\sigma$, which varies from unity for an eigenmode localized at just one site to $1/V$ for an eigenmode spread uniformly on a lattice of volume $V$. As shown in Figure 1, zero and near-zero modes tend to be more localized than the non-zero modes.

Our measurement of $\langle \bar{\psi} \psi \rangle$ comes from the diagonal part of $G(\mu a)$ on the 12 sources used for each configuration. Writing $G$ in terms of the eigenvectors $\Phi_\alpha$ of $D^\dagger D$ with eigenvalue $\mu^2$ and chirality $\alpha$, the contribution of a zero mode can be easily read off from the equation above and is seen to be proportional to $\Phi / \mu a$. Since the eigenvector corresponding to the zero mode is localized, its contribution to the condensate depends strongly on the spatial position of the source vector. The remaining modes are delocalized and closely spaced; so the overlap of the eigenvector on the source is averaged out. After subtracting the zero mode contribution, $\langle \bar{\psi} \psi \rangle$ is strikingly identical to that seen in the sample without zero modes, at all the couplings and lattice sizes studied. We found that $\langle \bar{\psi} \psi \rangle$ varies linearly with $\mu a$ and goes to zero as $\mu a \to 0$, with a value of $\langle \bar{\psi} \psi \rangle / \mu a$ which is independent of the lattice volume. This is how chiral symmetry restoration manifests itself in quenched QCD.

**Figure 2.** $a^3 \langle \bar{\psi} \psi \rangle / \mu a$ as a function of $\mu a$ at $T = 1.25T_c$ (circle), $1.5T_c$ (diamond) and $2T_c$ (square) on $4 \times 12^3$ lattices. Zero and near-zero mode contributions are subtracted. Data for $1.5T_c$ and $2T_c$ are displaced in $\mu a$ for visibility.

The following identities hold for overlap Fermions in the chirally symmetric phase as $\mu a \to 0$,

$$C_S(z) = -C_{PS}(z) \quad \text{and} \quad C_V(z) = C_{AV}(z). \quad (3)$$

Here $C$ is the screening correlation function in the spatial $z$ direction of an operator summed in the other three directions. The subscripts PS refer to a pseudo-scalar operator, $S$ to a scalar, $V$ to a vector and $AV$ to an axial-vector. We find that the $V$ and $AV$ correlators indeed agree at all $z$ and at all temperatures we studied. The $S$ and $PS$ obey this relation after the zero modes have been
subtracted. In addition, $C_V$ is described well by an ideal gas of overlap quarks on the same lattice, while $C_{PS}$ gives a screening mass within 10% of the ideal has result (see Figure 3 for an example).

In conclusion, working with chiral (overlap) Fermions, we have found several new results and a consistent picture of the high temperature phase of quenched QCD. The quenched thermal ensemble contains gauge fields which give rise to Fermion zero modes of definite chirality. When the effect of these modes is subtracted, $\langle \bar{\psi}\psi \rangle$ vanishes in the zero quark mass limit, showing that chiral symmetry is restored. Simultaneously, parity doubling is seen in the spectrum of screening masses, which are close to those expected in an ideal Fermi gas, even for the S/PS sector. Since some of these results are not obtained with staggered quarks, it is an interesting question whether the two flavor QCD phase transition is properly described by such a representation of quarks.

Some interesting problems remain to be solved. At $T \leq 1.25 T_c$, there are near-zero modes. It cannot be ruled out that these modes shift the quenched chiral symmetry restoration point away from $T_c$. However, this question is crucially related to the evolution of near-zero modes with lattice volume and spacing. Hence the nature of these complications, and the question of whether they are quenched artifacts or remain in full QCD, will only become clear with further studies which are underway.

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