Superembeddings, Partial Supersymmetry Breaking and Superbranes

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Abstract

It is advocated that the superembedding approach is a generic covariant method for the description of superbranes as models of (partial) spontaneous supersymmetry breaking. As an illustration we construct (in the framework of superembeddings) an $n=1, d=3$ worldvolume superfield action for a supermembrane propagating in $N=1, D=4,5,7$ and 11–dimensional supergravity backgrounds. We then show how in the case of an $N=1, D=4$ target superspace gauge fixing local worldvolume superdiffeomorphisms in the covariant supermembrane action results in an effective $N=2, d=3$ supersymmetric field theory with $N=2$ supersymmetry being spontaneously broken down to $N=1$. The broken part of $N=2, d=3$ supersymmetry is nonlinearly realized when acting on Goldstone $N=1, d=3$ superfields, which describe physical degrees of freedom of the model. As an introduction to the formalism, the procedure of getting effective field theories with partially broken supersymmetry by gauge fixing covariant superbrane actions is also demonstrated with a simpler example of a massive $N=2, D=2$ superparticle.

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1 Introduction

Superbranes are extended relativistic objects which arise as solitons of supersymmetric field theories. The dynamics of brane fluctuations can in turn be effectively described by quantum field theories on the worldvolumes of the superbranes. This is a manifestation of various kinds of dualities which have been found and extensively exploited in string/M–theory to gain deeper insight into its nonperturbative quantum structure. Interest in effective field theories on branes is also caused by their relevance to brane–world realizations of the Universe considered recently. For this it is useful to have an explicit form of brane effective actions, which generically are supersymmetric.

Single branes usually preserve half of target–space supersymmetry associated with supertranslations along the worldvolume. So the fluctuations of such superbranes are described by supersymmetric worldvolume sigma–models. In the standard ”Green–Schwarz” formulation of superbrane dynamics supersymmetry on the brane arises upon gauge fixing worldvolume diffeomorphisms and a local worldvolume fermionic κ–symmetry. Such gauge fixing breaks target superspace covariance, and supersymmetric transformations associated with the directions transverse to the brane become nonlinearly realized on the physical modes of brane fluctuations. These brane fluctuations can be interpreted as Goldstone modes of the spontaneously broken (nonlinearly realized) supertranslation symmetries of the target superspace. Note that from the point of view of the observer “living” on the brane it is half of the worldvolume (space–time) supersymmetry which is spontaneously broken. Thus, superbranes provide us with a mechanism of partial spontaneous breaking of space–time supersymmetry [1, 2], the resulting supersymmetric worldvolume nonlinear sigma–models are known to be of the Volkov–Akulov type [3].

Superbranes as models of (partial) spontaneous supersymmetry breaking have been under study for more than a dozen of years. One of the methods used for their description [4]–[12] has been the group–theoretical (coset space) method of nonlinear realizations of spontaneously broken symmetries [13, 14]. In this formulation the superbrane dynamics is from the beginning described in a physical (or “static”) gauge where all pure gauge degrees of freedom are eliminated and only worldvolume fields corresponding to the brane physical modes remain. The physical modes form a supermultiplet of unbroken worldvolume supersymmetry, and thus the dynamics of these modes can be formulated in terms of worldvolume (Goldstone) superfields at least on the mass shell. In some cases, such as an $N = 2$, $D = 4$ Dirichlet 3-brane [4] and an $N=1$, $D=4$ supermembrane [8], one can also construct worldvolume superfield actions describing their off–shell dynamics. We should note that in the method of nonlinear realizations a systematic way of constructing gauge fixed superbrane actions with the use of Goldstone superfields is still lacking, though the actions written in the components of the Goldstone supermultiplet are well known. These are the Green–Schwarz–type brane actions in the physical gauge. To obtain the superfield action for the field theory with partially broken supersymmetry one passes from the method of nonlinear realizations to a method which can be conventionally called the method of “linear” realization [8].

1The number $N$ of the supersymmetries stands for the number of irreducible spinor supercharges.
of spontaneously broken supersymmetry. Different, though related, recipes have been proposed to construct superfield actions in the framework of the latter approach \([4, 7]\) (see also \([13]\) for relevant duality–symmetric constructions).

To describe models with partial supersymmetry breaking the methods of nonlinear and linear realizations operate with \textit{a priori} different Goldstone superfields. These superfields are usually related to each other through complicated expressions (see \([8]\) for the example of the N=1, D=4 supermembrane), so that even if a superfield action is known in the linear realization method, in general, it is difficult to rewrite it in terms of the Goldstone superfields of the nonlinear realization approach, which upon integration over the Grassmann–odd coordinates should directly yield the gauge fixed Green–Schwarz action. As a result a direct relationship of the existing Goldstone superfield actions with the Green–Schwarz formulation of superbranes has not been established yet. Such a relationship has only been checked for the bosonic sectors of the actions, which were shown to coincide either with the gauge fixed Nambu–Goto or Dirac–Born–Infeld action depending on the type of the superbrane considered \([8, 4]\), while the fermionic sectors of different formulations can in general be related by a highly nontrivial redefinition of the fermionic fields.

A limitation of the methods of partial spontaneous supersymmetry breaking is that they are suitable for the description of superbranes propagating in superbackgrounds invariant under global supersymmetry (or, in other words, in target superspaces with isometries). The Goldstone superfield actions proposed by now describe superbranes in flat superbackgrounds with the isometries generated by super–Poincare algebras. Goldstone superfield actions for superbranes in more complicated superbackgrounds with isometries, such as supersymmetric anti–de–Sitter configurations of multidimensional supergravities have not been constructed yet, though component gauge fixed superbrane actions in AdS superbackgrounds have been intensively studied \([17]–[20]\) in connection with the AdS/CFT correspondence conjecture \([22]\).

A geometrical approach which describes the dynamics of the superbranes in arbitrary supergravity backgrounds is the method of superembeddings. This is a generic method for formulating the theory of superbranes. Other known superbrane formulations (including the method of nonlinear realizations) follow from the superembedding approach (see \([23]\) for a recent review).

Superembedding is an elegant and geometrically profound formulation which is based on a supersymmetric extension of the classical surface theory applied to the description of superbrane dynamics by means of embedding worldvolume supersurfaces into target superspaces \([24, 25, 26]\). Thus, this approach is manifestly supersymmetric and covariant both on the superworldvolume and in target superspace. The fermionic \(\kappa\)–symmetry of the Green–Schwarz formulation has its origin in local worldvolume supersymmetry \([27]\).

For superembedding to be relevant to the description of superbranes it should be specified by imposing an appropriate embedding condition. This condition has a clear geometrical meaning. Let us consider a supersurface \(\mathcal{M}\) parametrized by \(d = p + 1\) bosonic coordinates \(\xi^m\) and \(n\)

\footnote{The \(N = 1\) supersymmetric Dirac–Born–Infeld action was first constructed in \([16]\).}
fermionic coordinates $\eta^\mu$, which we will collectively call

$$z^M = (\xi^m, \eta^\mu), \quad m = 0, 1, \ldots, p, \quad \mu = 1, \ldots, n.$$  \hfill (1.1)

The geometry of the supersurface is described in a superdiffeomorphism invariant way by a set of supervielbein one–forms

$$e^A(z) = dz^M e^A_M = (e^a(\xi, \eta), e^\alpha(\xi, \eta)),$$  \hfill (1.2)

which form a local basis in the cotangent space of $M$. The indices $a$ and $\alpha$ are, respectively, the indices of the vector and a spinor representation of the group $SO(1,p)$ of local rotations in the cotangent space. The indices $\alpha$ are (in general) cumulative in the sense that they also include indices of a group $SO(D−p−1)$ which is the group of internal automorphisms of the Grassman–odd subspace of $M$ possessing $n = D − p − 1$ extended supersymmetry.

Let us now embed this supersurface into a curved target superspace $\mathcal{M}$ parametrized by $D$ bosonic coordinates $X^\underline{m}$ and $2n$ fermionic coordinates $\Theta^\underline{\mu}$, which we will collectively call

$$Z^\underline{M} = (X^\underline{m}, \Theta^\underline{\mu}), \quad m = 0, 1, \ldots, D − 1, \quad \mu = 1, \ldots, 2n.$$  \hfill (1.3)

Note that for embedding we have chosen a supersurface with the number of Grassmann–odd directions being half the number of target–superspace Grassmann–odd directions. This is for being able to identify $n$ local worldvolume supersymmetries with $n$ independent fermionic $\kappa$–symmetries of the standard (Green–Schwarz) formulation of superbrane dynamics. In this paper we shall also deal with supersurfaces with a less number of fermionic coordinates.

The geometry of the target superspace is described in a superdiffeomorphism invariant way by a set of supervielbein one–forms

$$E^{\underline{A}}(Z) = dZ^{\underline{M}} E^{\underline{A}}_{\underline{M}} = (E^a(X, \Theta), E^\alpha(X, \Theta)),$$  \hfill (1.4)

which form a local frame in the cotangent space of the target superspace. The indices $a$ and $\alpha$ are, respectively, the indices of the vector and a spinor representation of the group $SO(1,D − 1)$ of local rotations in the $\mathcal{M}$ cotangent space.

Superembedding is a map of $M$ into $\mathcal{M}$ which is locally described by $X^\underline{m}$ and $\Theta^\underline{\mu}$ as functions of the supersurface coordinates

$$z^M \rightarrow Z^{\underline{M}}(z) = (X^\underline{m}(\xi, \eta), \Theta^\underline{\mu}(\xi, \eta)).$$  \hfill (1.5)

The map induces the pullback onto the supersurface of the target superspace one–form (1.4). In particular, the vector supervielbein $E^a$ pullback is a one–superform on the supersurface. It has the following decomposition in the local basis (1.3) on $M$

$$E^\underline{a}(z) = e^a(z)E^a_{\underline{a}}(Z(z)) + e^\alpha(z)E^\alpha_{\underline{a}}(Z(z)).$$  \hfill (1.6)

The superembedding condition we are interested in is the vanishing of the worldvolume spinor components of $E^\underline{a}(z)$

$$E^\underline{a}_{\alpha}(Z(z)) = 0.$$  \hfill (1.7)
In other words eq. (1.7) is a superfield constraint on (1.5) which singles out the superembeddings such that the pullback of the supervielbein $E^a$ has non–zero components only along vector directions of the supersurface. It can be shown that (1.7) sets an induced supergeometry on the embedded supersurface [24], i.e. that the worldvolume supervielbein (1.2) is completely determined in terms of the components of the target space supervielbein pullback (1.4). This is in accordance with a well known fact that no supergravity propagates on the superbrane.

Thus, in the superembedding approach superbrane dynamics is described in the framework of a worldvolume superfield formalism.

Eq. (1.7) is the basic superembedding condition for the description of all superbranes. In some cases the superembedding condition produces only “kinematic” constraints (such as, for instance, Virasoro conditions) and does not put superbrane dynamics on the mass shell. (Examples are $N = 1, D = 2, 3, 4, 6, 10$ superparticles [27]–[33] and heterotic superstrings [34]–[38]). In these cases several methods have been developed [27, 35, 39, 32, 38] for constructing worldvolume superfield actions which produce dynamical equations of motion of the superbranes. Alternatively, the dynamical equations of motion can be obtained from a supersymmetric generalization of the condition of minimal area embedding imposed on the second fundamental form of the supersurface [24].

In other cases, such as the M–theory branes (a $D = 11$ supermembrane [28, 24] and a super-5–brane [26]), the superembedding condition contains all information about the classical dynamics of the superbranes (i.e. the constraints and the equations of motion). In these cases the worldvolume superfield actions have not been found, and one should instead deal with generalized action functionals [40, 41], or conventional Green–Schwarz–like actions.

Thus, the superembedding approach provides systematic geometrical methods for getting worldvolume superfield equations of motion, and for constructing worldvolume superfield brane actions when the superembedding condition is off the mass shell.

The knowledge of worldvolume superfield actions for superbranes in the covariant superembedding approach can be used to derive corresponding gauge fixed superbrane actions in terms of Goldstone superfields, which describe field theories with partial spontaneous supersymmetry breaking in the method of nonlinear realizations. The general procedure is as follows. One chooses the superbackground to be a superspace with isometries and studies spontaneous breaking of the isometries when a superbrane propagates in this superbackground.

The simplest case is when the target superspace (1.3), (1.4) is flat. Then one deals with a global $N = 2n$ supersymmetry in the target superspace broken down to its $n$-extended subsupergroup. This subsupergroup is associated with $n$ worldvolume superdiffeomorphisms which reduce, upon imposing a physical gauge condition, to an $n$-extended (unbroken) global supersymmetry on the superworldvolume. The physical gauge condition identifies the supercoordinates (1.1) of the superworldvolume with a part of the target superspace coordinates (1.3), (1.4)

$$X^m(\xi, \eta) = \xi^m, \quad \Theta^\mu(\xi, \eta) = \eta^\mu, \quad m = 0, 1, \ldots, p, \quad \mu = 1, \ldots, n.$$  

(1.8)

Using the worldvolume superdiffeomorphisms $z^M \rightarrow \tilde{z}^M(z)$ it is always possible, at least locally,
to make such a choice of the target superspace coordinates on the superbrane.

The remaining worldvolume superfields \((1.5)\)

\[
X^i(\xi, \eta), \quad \Theta^\mu(\xi, \eta), \quad i' = p + 1, \ldots, D - 1, \quad \mu' = n + 1, \ldots, 2n
\]  

(1.9)

are the Goldstone superfields associated with spontaneously broken supertranslations of the superbackground along the bosonic and fermionic directions normal to the brane superworldvolume. They describe the transverse fluctuations of the superbrane and transform nonlinearly under broken supersymmetry.

The superfields \((1.3)\) are not independent. They are related to each other by the superembedding condition \((1.7)\) which now plays the role of a condition ensuring a so called “inverse Higgs” effect \([12]\), i.e. when the number of Goldstone fields gets reduced by making some of them dependent on the other ones. The similarity of the superembedding condition and the inverse Higgs constraint has been known for a long time \([43, 29]\). Though, as we have discussed above, the former has a much more general and profound geometrical meaning.

In this paper we illustrate the general procedure of passing from the superembedding approach to the method of nonlinear realizations with instructive examples of a massive superparticle in \(N = 2, D = 2\) superspace and of a supermembrane in \(N = 1, D = 4\) superspace. The paper may be regarded as an up–to–date revision and generalization of the results of the study of the \(D = 2\) superparticle and the \(D = 4\) supermembrane considered in references \([2, 30, 31, 41, 8]\).

In Section 3 we present a new simple, worldvolume and target space supersymmetric, form of the action which describes the dynamics of a supermembrane in a superbackground of any dimension where the supermembrane is allowed to propagate by the brane scan \([14]\), for instance, in backgrounds of four- and eleven–dimensional supergravity. This action can be regarded as a dynamical realization of the superembedding approach. It is constructed with the use of the worldvolume superfields \((1.2)\) and \((1.5)\), and pullbacks of differential forms describing corresponding supergravity backgrounds. The action possesses interesting features. For instance, its main term is a superworldvolume integral of the co-dimension two component of a Wess–Zumino three–form, and it is invariant under super Weyl transformations of the worldvolume supervielbein \((1.2)\). Remember that, in contrast to strings, the Howe–Tucker–Polyakov formulation of membrane dynamics is not invariant under Weyl rescaling of the intrinsic worldvolume metric. In our case the super Weyl symmetry is required for the superembedding condition to identify intrinsic worldvolume supergeometry with supergeometry induced by embedding.

We shall demonstrate how the superembedding action is related to the Green–Schwarz–type formulation of \([15]\) and \([11]\), and how in the case of an \(N = 1, D = 4\) flat target superspace it reduces, in the physical gauge \((1.8)\), to a superfield generalization of the component action of \([2]\). The Goldstone superfield action thus obtained describes an \(N = 2, d = 3\) dimensional supersymmetric theory of a self–interacting scalar supermultiplet with one linearly realized supersymmetry and another one being spontaneously broken. The latter is realized as a nonlinear

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3This similarity was pointed out to authors of \([43, 29]\) by I. Bandos.
transformation of a single $N = 1, d = 3$ Goldstone scalar superfield. In this way we get the action
for the Goldstone superfield in the method of nonlinear realizations, which is thus directly related
to the superembedding approach and to the Green–Schwarz formulation. We also demonstrate
the relationship of this supermembrane action to the action constructed within the framework of
“linear” realizations.

Section 2 of the paper is devoted to a detailed consideration of a simpler example of a one–
dimensional sigma–model with partially broken $N = 2$ supersymmetry which is obtained from
the dynamics of a massive superparticle in an $N = 2, D = 2$ target superspace. This section
may be regarded as an introduction into the formalism and as an illustration of the links between
the superembedding approach and the methods of spontaneous supersymmetry breaking. This
should simplify understanding the example of the supermembrane considered in Section 3. In
Conclusion we discuss open problems and outlook.

2 The massive $N = 2, D = 2$ superparticle

In the framework of the superembedding approach the massive superparticle in an $N = 2, D = 2$
target superspace has been studied in [30, 31, 33]. ($N = 2$ here stands for the number of
one–component $D = 2$ Majorana–Weyl spinors which form a two–component Majorana spinor.)
We start the consideration from an action considered in [30, 31], and then generalize it in an
appropriate way for being able to impose the physical gauge (1.8) discussed in the Introduction.

In the case of a flat target superspace the action has the following form:

$$S = \int d\tau d\eta \left[ iP_{\alpha}(DX^\alpha - iD\bar{\Theta}\Gamma^2 \Theta) - mD\bar{\Theta}\Gamma^2 \Theta \right],$$

(2.1)

where the superworldline $M$ of a particle of mass $m$ is parametrized by the bosonic time variable
$\tau$ and the real fermionic variable $\eta$, and

$$D = \frac{\partial}{\partial \eta} + i\eta \partial_\tau \quad D^2 = \frac{1}{2}\{D, D\} = i\partial_\tau$$

(2.2)

is a ‘flat’ Grassmann covariant derivative. The image of $M$ in the target superspace is described by
the scalar worldvolume superfields $X^\alpha(\tau, \eta)$, ($\alpha = 0, 1$) and $\Theta^\alpha(\tau, \eta)$, ($\alpha = 1, 2$), which transform
as a vector and a Majorana spinor ($\bar{\Theta} = \Theta^T C$) under the action of the $D = 2$ Lorentz group
$SO(1, 1)$. The $D = 2$ Dirac matrices $\Gamma^2$, $\Gamma^2$ and the charge conjugation matrix $C_{\alpha\beta}$ are chosen
to be in a Majorana representation

$$\Gamma^0 = \left( \begin{array}{cc} 0 & -1 \\ 1 & 0 \end{array} \right), \quad \Gamma^1 = \left( \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right), \quad\Gamma^2 = \Gamma^0 \Gamma^1 = \left( \begin{array}{cc} 0 & -1 \\ 1 & 0 \end{array} \right).$$

(2.3)

$$C_{\alpha\beta} = C_{\beta\alpha} = \left( \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right).$$

(2.4)

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4One can compare this action with the standard massive superparticle action presented in eq. (A.11) of the Appendix.
The $\eta = 0$ components of the superfields $P_{\alpha}$, $X^\alpha$ and $\Theta^\alpha$ correspond to the variables $p_\alpha$, $x^\alpha$ and $\theta^\alpha$ of (A.11).

7
\[
\Gamma^0_{\alpha\beta} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \Gamma^1_{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \Gamma^2_{\alpha\beta} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}.
\]

(2.5)

The rules of raising and lowering the spinor indices are \( \Theta^\alpha = C^\alpha_\beta \Theta^\beta \), \( \Theta^\alpha = \Theta^\beta C^{\beta\alpha} \).

The action (2.1) is invariant (up to a total derivative) under the global supersymmetry transformations in the target superspace (note that the second term in (2.1) is of a Wess–Zumino type)

\[
\delta \Theta^\alpha = \epsilon^\alpha, \quad \delta X^a = i \bar{\Theta} \Gamma^a \delta \Theta,
\]

(2.6)

and under local transformations of the worldvolume coordinates of the following form

\[
\tau' - \tau = \delta \tau = 2 \Lambda(\tau, \eta) - i \eta D, \\
\eta' - \eta = \delta \eta = -i D \Lambda, \\
D' - D = \delta D = -\partial_{\tau} \Lambda D,
\]

(2.7)

where \( \Lambda(\tau, \eta) = a(\tau) + i \eta \alpha(\tau) \) is the superreparametrization parameter which contains the worldline bosonic reparametrization parameter \( a(\tau) \) and the local supersymmetry parameter \( \alpha(\tau) \).

Under (2.7) the superfields \( X^a(\tau, \eta) \) and \( \Theta^\alpha(\tau, \eta) \) transform as scalars, and the superfield \( P_a(\tau, \eta) \) in the first term of (2.1) transforms in an appropriate way to ensure the invariance of the action. Note that to ensure the invariance of the action (2.1) under (2.7) it was not necessary to introduce the worldvolume supervielbein (1.2). However, we shall need it later for promoting local worldvolume supersymmetry (2.7) to general superdiffeomorphisms, which is required for imposing the physical gauge condition (1.8).

\( P_a \) is the Lagrange multiplier\(^5\) whose variation in (2.1) produces the superembedding condition

\[
DX^a - iD\bar{\Theta}\Gamma^a \Theta = 0.
\]

(2.8)

Eq. (2.8) is a flat target space counterpart of the condition (1.7) where now \( E^a = dX^a - i d\bar{\Theta} \Gamma^a \Theta \) and \( e^a \) and \( e^\alpha \) are, respectively, \( d\tau - id\eta \) and \( d\eta \).

In the case under consideration eq. (2.8) is a constraint which relates the superfields \( X^a(\tau, \eta) \) and \( \Theta^\alpha(\tau, \eta) \). It produces the relativistic energy–momentum condition \( P_a P^a|_{\eta = 0} = m^2 \), but does not contain dynamical equations of motion \( [27, 30, 31] \). The latter are derived by varying the action (2.1) with respect to \( X^a \) and \( \Theta^\alpha \).

Our goal is to gauge fix the local superreparametrizations of the superworldline, to solve eq. (2.8) explicitly in terms of an independent superfield and to substitute this solution into the second (Wess–Zumino) term of the action (2.1). The resulting action will describe a one–dimensional supersymmetric nonlinear sigma–model with partially broken \( N = 2 \) supersymmetry.

We would like to relate the superembedding formulation of the superparticle to the Goldstone superfield action of [3], which up to a normalization is

\[
S = im \int d\tau d\eta \frac{\partial_{\tau} \Phi D \Phi}{1 + \sqrt{1 - (\partial_{\tau} \Phi)^2}}.
\]

(2.9)

\(^5\)The leading component \( P_a|_{\eta = 0} \) of \( P_a(\tau, \eta) \) is the particle canonical momentum.
\( \Phi(\tau, \eta) \) is a scalar superfield which describes the superparticle physical degrees of freedom in the physical gauge. Hence, we should impose the condition \((1.8)\). In the case under consideration it takes the form

\[
X^0(\tau, \eta) = \tau, \quad \Theta^1(\tau, \eta) = \eta. \tag{2.10}
\]

To be able to impose this condition which gauge fixes two superfield variables \(X^0\) and \(\Theta^1\) one must have worldvolume superreparametrizations with two independent superfield parameters. However, in the form \((2.1)\) the superparticle action is invariant under the one–parameter transformations \((2.7)\). As we show in the Appendix, the one–parameter superreparametrizations can be used to impose a light–cone gauge condition \(X^-(\tau, \eta) \equiv (X^0 - X^1) = \tau\), but not the conditions \((2.10)\). Therefore, we should modify the action \((2.1)\) in such away that it becomes invariant under general superdiffeomorphisms of the worldline supersurface

\[
\tau' = \tau'(\tau, \eta), \quad \eta' = \eta'(\tau, \eta) \tag{2.11}
\]

characterized by two independent superfunctions. For this we should “covariantize” the action \((2.1)\), i.e. couple it to worldline supergravity by introducing the worldline supervielbein \((1.2)\)

\[
e^A = dz^M e^A_M, \quad z^M = (\tau, \eta). \tag{2.12}
\]

It has been proved convenient to choose the matrix \(e^A_M\) in the form

\[
e^A_M = \begin{pmatrix} e & -if \tilde{e} \\ if & \tilde{e} \end{pmatrix}, \tag{2.13}
\]

then its inverse is

\[
e^A_M = \begin{pmatrix} e^{-1}(1 + \frac{e^I}{\tilde{e}}) & \frac{if}{\tilde{e}} \\ -\frac{if}{\tilde{e}} & \tilde{e}^{-1}(1 + \frac{e^f}{\tilde{e}}) \end{pmatrix}, \tag{2.14}
\]

where \(e(\tau, \eta)\) and \(\tilde{e}(\tau, \eta)\) are bosonic and \(f(\tau, \eta)\) and \(\tilde{f}(\tau, \eta)\) are fermionic worldvolume superfields.

It is straightforward to “covariantize” the first term of the action \((2.1)\) by replacing the flat covariant derivative \(D\) with its curved counterpart

\[
D = e^M_M \partial_M = \frac{1}{\tilde{e}} \left(1 + \frac{e^f}{\tilde{e}}\right) \partial_\eta + if \partial_\tau, \tag{2.15}
\]

where \(e^M_M\) is the second column of the supervielbein matrix \((2.14)\).

However, as far as the second term in \((2.1)\) is concerned, its generalization is more subtle. It must not spoil the property of this term to be of the Wess–Zumino type, i.e to be invariant under target–space supersymmetry \((2.4)\) up to a total derivative.

To find the appropriate generalization of the Wess–Zumino term we first consider the superembedding action for a massless superparticle in an \(N = 1, D = 3\) superspace \([27]\) in the form invariant under \((2.11)\) and then perform its dimensional reduction to the massive \(N = 2, D = 2\) superparticle action (as in \([33]\)).
The $N = 1, D = 3$ massless superparticle action in question has the following form

$$S = i \int d\tau d\eta P_{\hat{a}}(D X^{\hat{a}} - i D \hat{\Theta} \Gamma^{\hat{a}} \Theta)$$

$$= i \int d\tau d\eta P_{\hat{a}}(D X^{\hat{a}} - i D \hat{\Theta} \Gamma^{\hat{a}} \Theta) + i \int d\tau d\eta P_{2}(D X^{2} - i D \hat{\Theta} \Gamma^{2} \Theta),$$  \hspace{1cm} (2.16)

where $X^{\hat{a}} (\hat{a} = 0, 1, 2)$ are bosonic coordinates of the $D = 3$ superspace, and the covariant fermionic derivative $D$ has been introduced in (2.15).

By construction the action (2.16) is invariant under the target–space supersymmetry transformations

$$\delta \Theta^{\hat{a}} = \epsilon^{\hat{a}}, \quad \delta X^{\hat{a}} = i \hat{\Theta} \Gamma^{\hat{a}} \delta \Theta,$$  \hspace{1cm} (2.17)

and under the worldvolume superdiffeomorphisms (2.11) provided the Lagrange multiplier superfield $P_{\hat{a}}(\tau, \eta)$ (whose leading component is associated with the superparticle canonical momentum [27]) transforms in an appropriate way.

In addition, since only half of the components of the inverse supervielbein (2.14) enter the eq. (2.16) through the covariant derivative $D$, the action is invariant under super–Weyl transformations

$$\delta f e^{M} = \phi_{f}(\tau, \eta) e_{M}, \quad \delta f e^{M} = 0, \quad \delta f P_{\hat{a}} = 0;$$

$$\delta b e^{M} = \phi_{b}(\eta, \tau) e_{M}, \quad \delta b e^{M} = 0, \quad \delta b P_{\hat{a}} = -\phi_{b}(\eta, \tau) P_{\hat{a}},$$  \hspace{1cm} (2.18)

where $e_{M}$ are, respectively, the first and the second column of (2.14), and $\phi_{f}(\tau, \eta)$ and $\phi_{b}(\tau, \eta)$ are a bosonic and fermionic parameter of the super–Weyl transformations.

The transformations (2.18) can be used to put $\hat{f} = 0$ and $\hat{e} = 1$ in (2.13) and (2.14), which then reduce to

$$e^{A}_{M} = \begin{pmatrix} e & -ife \\ 0 & 1 \end{pmatrix},$$  \hspace{1cm} (2.19)

$$e^{M}_{A} = \begin{pmatrix} e^{-1} & if \\ 0 & 1 \end{pmatrix},$$  \hspace{1cm} (2.20)

and the covariant derivative in (2.16) takes the form

$$D = \partial_{\eta} + if(\tau, \eta) \partial_{\tau}, \quad D^{2} = i(D f) \partial_{\tau}.$$  \hspace{1cm} (2.21)

The choice of the worldvolume supervielbein in the form (2.19)–(2.21) fixes the super–Weyl invariance of the action (2.14), and in what follows we shall work in this gauge.

The dimensional reduction of the action (2.16) down to the $N = 2, D = 2$ superparticle action is carried out in the following way. The space dimension associated with the coordinate $X^{2}$ is assumed to be compactified on a circle, and the superparticle is restricted to move along the circle with a constant momentum whose value determines the particle mass in an effective (uncompactified) two–dimensional space–time. Technically this is done by solving for the equation of motion of $X^{2}(\tau, \eta)$, which is

$$D P_{2} + iP_{2} \partial_{\tau} f = 0,$$  \hspace{1cm} (2.22)
and substituting the solution back into the action (2.16).

The general solution of (2.22) is

$$P_2 = -\frac{m}{Df},$$

(2.23)

which can be checked using the properties of the covariant derivative (2.21). In (2.23) $m$ is a constant mass parameter.

Substituting (2.23) into eq. (2.16) and noticing that the term with $\mathcal{D}X^2$ becomes a total derivative, we arrive at the desired form of the $N=2$, $D=2$ superparticle action

$$S = i \int d\tau d\eta P_2 (\mathcal{D}X^2 - i\mathcal{D}\Theta \mathcal{D}\Theta) - i \int d\tau d\eta \frac{m}{Df} \mathcal{D}\Theta \mathcal{D}^2 \Theta. $$

(2.24)

We are now in a position to use the worldvolume superdiffeomorphisms (2.11) for imposing the physical gauge (2.10). Then the time and space component of the superembedding condition (2.8) reduce, respectively, to the following equations (which are obtained using the explicit form of the Dirac matrices (2.3))

$$if(\tau, \eta) = i\eta + i\varphi \partial \eta \varphi,$$

(2.25)

$$\mathcal{D} \Phi = 2i \varphi,$$

(2.26)

where

$$\varphi(\tau, \eta) \equiv \Theta^2, \quad \Phi(\tau, \eta) \equiv X^1 + i\eta \varphi.$$  

(2.27)

From the equation (2.23) we find the expression for the worldvolume supervielbein component $f(\tau, \eta)$ in terms of the “matter” superfield $\varphi(\tau, \eta)$

$$f = \eta + \frac{\varphi \partial \eta \varphi}{1 + i\varphi \partial \eta \varphi} = \eta + \varphi \partial \eta \varphi, \quad D = \partial \eta + i\eta \partial \tau.$$  

(2.28)

Then we can substitute (2.28) into (2.23) and solve this equation using a nice trick of Bagger and Galperin [4] (which we describe in the Appendix). We thus find the expression for $\varphi(\tau, \eta)$ in terms of the unconstrained superfield $\Phi(\tau, \eta) = \varphi(\tau) + i\eta \varphi(\tau)$, which describes the physical bosonic and fermionic degrees of freedom of the superparticle

$$\varphi = -\frac{iD\Phi}{1 + \sqrt{1 - (\partial \tau \Phi)^2}}.$$  

(2.29)

From the form of eqs. (2.26) and (2.28) one can see why the physical gauge (1.8) was inadmissible in the case of the superparticle action (2.1), where $f = \eta$. Putting $f = \eta$ already fixes the Grassmann–odd part of the diffeomorphisms (2.11). So if in addition we put also an extra condition $\eta = \Theta^1$, from (2.28) it would follow that $\varphi \partial \eta \varphi = 0$ and $\varphi = D\Phi$. Then, as can be easily checked, the equation of motion of $\varphi$ derived from (2.1) would reduce to $D\varphi = 0$, and thus result in $\partial \tau \Phi = 0$, which is too restrictive, since it describes a “static” particle (recall that particle motion in space is governed by second order differential equations).

Since we have explicitly solved the superembedding condition (2.8) in the physical gauge (2.10), the action (2.24), which now contains only the second (Wess–Zumino) term, reduces to the following form

$$S = \int d\tau d\eta \frac{m}{Df} (\eta - \varphi \partial \eta \varphi).$$

(2.30)
Upon some algebraic manipulations one can show that
\[
\frac{1}{Df} = 1 - D \frac{\Psi D \Psi}{1 + (D \Psi)^2} = 1 + \frac{i}{2} D \frac{D \Phi \partial_r \Phi}{1 + \sqrt{1 - (\partial_r \Phi)^2}} \tag{2.31}
\]
and
\[
\Psi D \Psi = -i \frac{D \Phi \partial_r \Phi}{(1 + \sqrt{1 - (\partial_r \Phi)^2})^2} \tag{2.32}
\]
Substituting (2.31) and (2.32) into (2.30) and integrating by parts we finally arrive at the action which coincides with eq. (2.9) up to the “cosmological” term \(\int d\tau m\), which was skipped in [2] in order to normalize to zero the energy of the “ground” state \(D \Phi(\tau, \eta) = 0\).

Let us now analyze the symmetries of the action (2.9). The gauge conditions (2.10) remain invariant under the combination of the \(N = 2, D = 2\) target space supersymmetry (2.6) and a global relic of the worldline superdiffeomorphisms (2.11) which must be related to (2.6) as follows
\[
\delta \eta = \epsilon^1, \quad \delta \tau = i \eta^1 + i \Psi(\eta, \tau) \epsilon^2. \tag{2.33}
\]
Under the \(\epsilon^1\)-transformations the superfield \(\Phi(\tau, \eta)\) entering the action (2.9) varies as the scalar superfield
\[
\delta \Phi = -\delta_{(\epsilon^1)} z^M \partial_M \Phi. \tag{2.34}
\]
Hence, the action (2.9) is manifestly invariant under \(N = 1\) global supersymmetry transformations in the worldline superspace \((\tau, \eta)\) associated with the parameter \(\epsilon^1\). This is the supersymmetry which remains unbroken.

The action is also invariant under the second, nonlinearly realized (and hence spontaneously broken) supersymmetry associated with the \(\epsilon^2\)-shifts (2.6) of the superfield \(\Psi(\tau, \eta) = \Theta^2\). Under the \(\epsilon^2\)-transformations the Goldstone fermion \(\Psi\) and its bosonic Goldstone partner \(\Phi\) (which is associated with spontaneously broken translations along the space direction \(X^1\)) vary in a nonlinear way
\[
\delta \Psi = \epsilon^2(1 + i \Psi \partial_r \Psi), \quad \delta \Phi = -i \epsilon^2(2 \eta - \Psi \partial_r \Phi) = -i \epsilon^2(2 \eta - L), \quad L = \Psi \partial_r \Phi = \frac{\partial_r \Phi D \Phi}{1 + \sqrt{1 - (\partial_r \Phi)^2}}, \tag{2.35}
\]
where \(L\) is the Lagrangian density of the action (2.9).

The transformations (2.35) can be easily derived from the definition (2.27) and (2.28) of \(\Phi\) and \(\Psi\), and using their variation properties with respect to the combination of target space (2.6) and worldline (2.33) supersymmetry transformations with the parameter \(\epsilon^2\).

The superfield transformations (2.35) have been obtained in [10] using somewhat different reasoning course.

We have thus demonstrated how the \(N = 2, D = 2\) massive superparticle action (2.24) in the doubly supersymmetric superembedding approach reduces (upon an appropriate gauge fixing of the local worldvolume superdiffeomorphisms (2.11)) to the one–dimensional nonlinear sigma–model (2.9) exhibiting partial breaking of \(N = 2\) global supersymmetry.

In the next section we proceed to the consideration of a more complicated and interesting example of a three–dimensional field theory with partially broken supersymmetry describing supermembrane fluctuations in target superspace.
3 The supermembrane

In the framework of the superembedding approach the supermembrane has been studied in [46, 24, 40, 41]. In [28, 24] it has been shown that in a $D = 11$ supergravity background the superembedding condition puts the dynamics of the supermembrane on the mass shell (i.e. contains the supermembrane equations of motion) if the worldvolume supersurface (1.1) with $p = 2$ has $n = 16$ Grassmann–odd directions, i.e. $N = 8$ supersymmetry in $d = 3$. In this case the superfield action of the type (2.1) cannot be constructed, since the Lagrange multipliers $P_a$ propagate redundant degrees of freedom. Instead one can deal with a generalized action functional [40] which, though being not a fully fledged superworldvolume action, allows one to derive the superembedding condition and, as a consequence, the full set of superfield equations of motion of the $D = 11$ supermembrane.

The superembedding condition can be relaxed if the worldvolume supersurface associated with the supermembrane has a less number of Grassmann–odd directions, for instance, $n = 2$. Such an $N = 1$, $d = 3$ supersurface is then parametrized by the supercoordinates

$$z^M = (\xi^m, \eta^\mu), \quad m = 0, 1, 2, \quad \mu = 1, 2.$$  \hspace{1cm} (3.1)

If we embed the supersurface (3.1) into a $D = 4, 5, 7, 11$ target superspace (with 4,8,16 and 32 Grassmann–odd directions, respectively) it can be shown, making the analysis described in [24] [11], that the superembedding condition (1.7) does not contain the supermembrane equations of motion. Hence, in this case an $N = 1$, $d = 3$ superworldvolume action can be constructed for a supermembrane propagating in a $D = 4, 5, 7$ and $11$ supergravity background (remember that these backgrounds fit into the brane scan [44]). Below we give the form of this action.

We should note that the embedding of the supersurface (3.1) with only two Grassmann–odd directions into a $D = 5, 7$ or $D = 11$ superbackground does not allow to trade all $\kappa$–symmetries of the standard formulation [45] (e.g. 16 in $D = 11$) for only two supersymmetries of the superworldvolume (3.1). In such a formulation a part of the $\kappa$–transformations remains as a hidden symmetry. The match of the number of the supersymmetries of $\mathcal{M}$ (3.1) and the number of $\kappa$–symmetries takes place when $\mathcal{M}$ is embedded into an $N = 1$, $D = 4$ superspace with four real Grassmann–odd directions. This last case will be of our main interest in view of the relationship of the superembedding approach and the methods of spontaneous supersymmetry breaking. However, until a certain point we shall not specify the dimension of the target superspace (1.3), (1.4).

3.1 The conventional form of the supermembrane action

The Green–Schwarz–type action for a supermembrane propagating in an $N = 1$, $D = 4, 5, 7$ or $11$ supergravity background has the following form [13]

$$S_{M2} = -\int_{\mathcal{M}_3} d^3 \xi \sqrt{-\det g_{mn}} \frac{1}{2} \int_{\mathcal{M}_3} d^3 \xi \varepsilon^{mnp} \partial_m Z^L \partial_n Z^M \partial_p Z^N A_{NML}(Z), \hspace{1cm} (3.2)$$

\[6\] The number $N$ of the supersymmetries stands for the number of Majorana spinor supercharges.
where \( g_{mn}(\xi) = \partial_m Z^M \partial_n Z^N(\xi) E_a^M(\xi) E_a^N(\xi) \) is a worldvolume metric induced by embedding the bosonic surface \( M_3 \) (parametrized by \( \xi^m \)) into a curved target superspace \((1.3), (1.4)\).

The supermembrane also minimally couples to a background three-form superfield \( A_{NML}(Z) \). In \( D = 11 \) its leading component \( A_{nm}(X) = A_{nm}(Z)| \Theta = 0 \) is the gauge field of \( D = 11 \) supergravity, and in \( D = 5 \) it is Hodge dual to a scalar component of the \( N = 1, D = 5 \) supergravity multiplet.

In \( D = 4 \) \( A_{nm}(X) \) does not have any local dynamical degrees of freedom, since its field strength \( F_{pnm} = 4! \partial_p A_{nm}(X) \) is constant on the mass shell

\[
D_p F_{pnm} = 0 \rightarrow F_{pnm} = c \varepsilon_{pnm}
\]

(3.3)

Though being locally non–dynamical \( F_{pnm} \) has a positive energy density and, hence, contributes to the cosmological constant value. This mechanism of the dynamical generation of the cosmological constant has been studied \cite{47, 48, 49} as a possible way of solving the zero cosmological constant problem. We see that in \( D = 4 \) the supermembrane naturally couples to such a “cosmological” field.

The action (3.2) is invariant under target–space superdiffeomorphisms

\[
Z^\prime M = Z^M(\xi),
\]

(3.4)

local worldvolume diffeomorphisms

\[
\xi^m = \xi^m(\xi)
\]

(3.5)

and local fermionic \( \kappa \)–symmetry transformations

\[
\delta_\kappa Z^M E_\alpha^M = 0, \quad \delta_\kappa Z^M E_a^M = (1 + \tilde{\Gamma})^\alpha_\beta \kappa^\beta(\xi),
\]

(3.6)

where

\[
\tilde{\Gamma} = \frac{1}{6 \sqrt{-g}} \varepsilon^{mnp} \Gamma_{mnp}, \quad \tilde{\Gamma}^2 \equiv 1
\]

(3.7)

and hence \( 1 + \tilde{\Gamma} \) is a spinor projection matrix. \( \Gamma_{mnp} \) is an antisymmetric product of the target–space gamma–matrices \((\Gamma_\alpha)\) pulled back on to the worldvolume, i.e \( \Gamma_m \equiv \partial_m Z^M E_a^M \frac{\delta}{\delta Z^M} \).

The appearance of the spinor projector in the \( \kappa \)–transformations reflects the fact that the presence of the supermembrane in the target superspace breaks half the \( 2n \) supersymmetries of a \( D \)–dimensional supergravity vacuum, the unbroken supersymmetries being associated with those Grassmann coordinates \( \Theta^\alpha \) which can be eliminated by \( \kappa \)–symmetry transformations, while remaining \( n \) \( \Theta^\alpha \) are worldvolume Goldstone fermions of the spontaneously broken supersymmetries and describe physical fermionic modes of supermembrane fluctuations. Thus \( \kappa \)–symmetry plays the same role as the worldvolume supersymmetry of the superembedding approach (as we have discussed in Introduction and Section 1). The exact form of the relationship between the \( \kappa \)–symmetry and the superdiffeomorphisms of the superworldvolume of the supermembrane the reader may find in \cite{23}.

An important requirement for the \( \kappa \)–transformations (3.6) to be a symmetry of the membrane action (3.2) is that the target–space supervielbeins \( E^a(Z), E^\alpha(Z) \), superconnections \( \Omega_a^\alpha(Z) \) and
the gauge superfield $A^{(3)}$ satisfy supergravity constraints. The essential constraints are the torsion constraint
\[ T^a = dE^a + E^d \Omega_2^d a = -i \bar{E}_2 (\Gamma^a)^d a \frac{E^d}{2}, \] (3.8)
and the field–strength $F^{(4)} = dA^{(3)}$ constraint
\[ F^{(4)} = \frac{i}{2} E^a E^b \bar{E}_2 (\Gamma_{ab})^d a \frac{E^d}{2} + \frac{1}{4!} E^a E^b E^c E^d F_{abcd}. \] (3.9)
Other constraints are either conventional or can be obtained from (3.8) and (3.9) by considering their Bianchi identities. For example, the gauge field–strength and components of torsion are related to each other by a constraint which in $D = 11$ has the form
\[ T^a = \frac{1}{288} F_{b_1 \ldots b_4} F^a \left( \Gamma_{b_1 \ldots b_4} - 8 \delta_{b_1}^{[b_2} \right) \right] \frac{a}{b} E^a. \] (3.10)
In $D = 4$ the first term on the right hand side of (3.10) disappears because the antisymmetrized product of five gamma–matrices is identically zero in $D = 4$.

We shall assume that the supergravity constraints are imposed on the target–superspace background also in the superembedding description of supermembrane dynamics. Then the integrability of the superembedding condition (1.7) requires that the geometry (1.2) of the $d = 3$ supersurface satisfies (analogous) worldvolume supergravity constraints, and vice versa [24, 25, 23]. This ensures the consistency of the superembedding. We choose the superworldvolume torsion constraints to be that of $N = 1, d = 3$ supergravity [50]
\[ T^a = \nabla e^a = de^a + e^b \omega_b^a = -i \gamma_{a\beta}^a e^a \wedge e^\beta + e^b e_{c\delta} \eta^{d} R(z), \] (3.11)
\[ T^a = \nabla e^a = \frac{1}{2} e^a \wedge e^b T_{(b}^a + \frac{i}{2} e^a \wedge e^b \gamma_{a\beta}^a R(z), \] (3.12)
where $\omega_b^a$ is a $d = 3$ spin connection, $R(z)$ is an unconstrained superfield, $\gamma_{a\beta}^a$ are the $d = 3$ Dirac matrices defined in (2.5) and $\eta^{d} = \text{diag}(-, +, +)$.

### 3.2 The supermembrane action in the superembedding approach

Let us associate with the supermembrane worldvolume in D-dimensional target superspace an $N = 1, d = 3$ supersurface $\mathcal{M}$ (1.1) parametrized by three bosonic $\xi^m$ and two real fermionic (Majorana–spinor) coordinates $\eta^\mu$. It has been shown in [41] that the condition (1.7) of embedding this supersurface into an $N = 1, D = 4$ target superspace does not contain dynamical equations of motion of the supermembrane. This is also so for the embeddings of $\mathcal{M}$ into $N = 1, D = 5, 11$ superspaces, which can be verified in the same way as described in [24, 41, 23]. Thus, for all these cases (1.7) is an off–shell constraint and one can construct an $N = 1, d = 3$ worldvolume superfield action describing the dynamics of the supermembrane in $N = 1, D = 4, 5, 7, 11$ supergravity backgrounds [1].

\[ \text{Recall that } N \text{ stands for the number of Majorana spinors in } d = 3 \text{ and } D = 4, 11 \text{ or Dirac spinors in } D = 5, 7. \]
Recall that when we deal with the $N = 1$, $d = 3$ supersurface, the $\kappa$–symmetry (3.6) is completely replaced by the worldvolume superdiffeomorphisms only in the $N = 1$, $D = 4$ target superspace. In the higher space–time dimensions a part of the $\kappa$–transformations remains a hidden symmetry (the form of these residual $\kappa$–transformations in the superembedding formulation of superparticles and superstrings has been reviewed in [23]). As we have already discussed, to replace all the $\kappa$–transformations with local worldvolume supersymmetry we should consider $N$–extended $d = 3$ supersurface (with $N = 8$ in the case of embedding into the $D = 11$ target superspace), but then the superembedding condition puts the theory on the mass shell and the worldvolume superfield action cannot be constructed. Since we are interested in constructing the action we choose the supermembrane worldvolume to be the $N = 1$, $d = 3$ supersurface.

A worldvolume superfield form of superbrane actions can be constructed using a generic prescription first proposed in [38] for superstrings. For the supermembrane the action of this type was constructed and analyzed in [46] (see also [41]). It has the following form

$$S = \int d^3\xi d^2\eta P_\alpha^a E_{\alpha a} + \int d^3\xi d^2\eta P^{MNP}[\bar{A}_{MNP} - (dQ)_{MNP}], \quad (3.13)$$

where the first term ensures the superembedding condition (1.7) as the equation of motion of the Lagrange multiplier $P_{\alpha}^a (\xi, \eta)$, and in the second term $P^{MNP} (\xi, \eta)$ is a Lagrange multiplier, $dz^N dz^M Q_{MN}(\xi, \eta)$ is a superworldvolume 2–form and $\bar{A}_{MNP}$ is a kind of the pullback on to the supersurface of the following combination of $A^{(3)}$ and $F^{(4)} = dA^{(3)}$

$$\bar{A}^{(3)} = A^{(3)} + \frac{1}{12} e^a \wedge e^b \wedge e^c \gamma_a^{\alpha\beta} E_{\alpha}^A E_{\beta}^B E_{\gamma}^C e^D F_{DCBA}. \quad (3.14)$$

$\gamma_a^{\alpha\beta}$ are $d = 3$ worldvolume Dirac matrices in the Majorana representation defined in (2.3)–(2.5). The worldvolume form $\bar{A}^{(3)}$ (3.14) is constructed in such a way that it is closed ($d\bar{A}^{(3)} = 0$) modulo the superembedding condition.

The action (3.13) is classically equivalent to the supermembrane action (3.2). For the superworldvolume with $N > 1$ Grassmann spinor coordinates the proof was given in [46]. For the $N = 1$ case under consideration we demonstrate the equivalence in the next subsection.

For our purposes to arrive at a worldvolume superfield action for a supermembrane in the physical gauge the action in the form (3.13) is too general, since it is invariant under a huge group of local transformations associated with the presence of the Lagrange multipliers $P_{\alpha}^a$, $P^{MNP}$ and the auxiliary two–form field $Q_{MN}$ (see [38, 46]). We should gauge fix at least a part of these symmetries. A possible gauge fixing condition is

$$P^{MNP} = \frac{1}{3!} \text{sdet}(e_L^A) e_{\alpha}^M e_{\alpha}^N e_{\beta}^P \gamma^{\alpha\beta}, \quad (3.15)$$

where $e_{\alpha}^M(z)$ is inverse of the worldvolume supervielbein matrix (1.2). Substituting (3.15) into (3.13) we reduce the action to the following form

$$S = \int d^3\xi d^2\eta P_{\alpha}^a E_{\alpha a} + \frac{1}{3!} \int d^3\xi d^2\eta \ \text{sdet} \ e^{\alpha\beta} A_{\alpha\beta} \quad (3.16)$$
where
\[ A_{\alpha \beta a} = E^C_{\alpha} E^B_{\beta} E^A_{a} A_{ABC} = 3E^\alpha_{\alpha} E^\beta_{\beta} E^\gamma_{\gamma} A_{\alpha \beta \gamma} + E^\alpha_{\alpha} E^\beta_{\beta} E^\gamma_{\gamma} A_{\alpha \beta \gamma} + \cdots \] (3.17)
The dots in (3.17) stand for the terms containing the \( E^\alpha_{\alpha}(z) \) components of the pullback of the target–space supervielbein \( E^a \) (1.6). These terms contribute to the first term of (3.16) (which simply results in the redefinition of \( P^a_{\alpha} \)) and hence can be ignored.

The action (3.16) is simpler and looks much more attractive than (3.13). Its second term (which actually produces the dynamical equations of motion of the supermembrane) does not contain Lagrange multipliers and resembles the Wess–Zumino term of the action (3.2). Indeed, upon integrating over the Grassmann–odd coordinates and eliminating the auxiliary fields (by the use of the superembedding condition incorporated in the first term), the second term of (3.16) produces both, the Nambu–Goto and the Wess–Zumino part of (3.2). We may also note that because of dimensional reasons the choice of the co–dimension two component of the pullback of \( A^{(3)} \) for the construction of the action is unique. For superstrings a similar form of the superembedding action was proposed in [35].

By construction the action (3.16) is manifestly invariant under the worldvolume and target space superdiffeomorphisms and local \( SO(1,2) \) rotations in the superworldvolume tangent superspace. In addition it is also invariant under the following super–Weyl transformations of the components of the worldvolume supervielbein
\[ e'^{a} = W^2(z)e^a, \quad e'^{\alpha} = W(z)e^{\alpha} - ie^a \gamma^{\alpha \beta} D_{\beta} W, \] (3.18)
and its inverse
\[ e'^{M}_{a} = W^{-2}e^{M}_{a} - W^{-3}D_{a}W^{\alpha \beta}e^{M}_{\alpha \beta}, \quad e'^{M}_{\alpha} = W^{-1}e^{M}_{\alpha}, \] (3.19)
where \( D_{a} = e^{M}_{a} \partial_{M} \). Note that the super–Weyl transformations (3.18) leave intact the torsion constraint (3.11) (i.e. \( T^{a}_{\alpha \beta} = -2i\gamma^{\alpha \beta} \)).

The invariance of (3.16) under (3.18), (3.19) can be easily verified using the following form of the superdeterminant
\[ sdet e^{A} = sdet^{-1} e^{M} = det^{-1}[e^{m}_{\alpha} - e^{\mu}_{\alpha}(e^{\alpha}_{\mu})^{-1} e^{m}_{\alpha}] det e^{\mu}, \] (3.20)
(where \( (e^{\alpha}_{\mu})^{-1} \) is inverse of \( e^{\alpha}_{\mu} \)) from which it follows that under (3.18), (3.19) rescales as
\[ sdet e' = W^4 sdet e. \] (3.21)

The super–Weyl variation of the Lagrangian density \( \gamma^{\alpha \beta} A_{\alpha \beta a} \) of (3.16) is
\[ \gamma^{\alpha \beta} A'_{\alpha \beta a} = W^{-4}\gamma^{\alpha \beta} A_{\alpha \beta a} + W^{-3}W_{\gamma \delta \gamma} \gamma^{\alpha \beta a} E^{\alpha \beta}_{\gamma} E^{\beta}_{\gamma} A_{\alpha \beta a} \cdots. \] (3.22)
The second term in (3.22) vanishes due to the \( d = 3 \) gamma–matrix cyclic identity
\[ \gamma^{\alpha \beta} \gamma^{\alpha \beta} + \gamma^{\delta \beta} \gamma^{\alpha \gamma} + \gamma^{\delta \alpha} \gamma^{\alpha \beta} = 0, \] (3.23)
and dots stand for a term proportional to \( E^{\alpha}_{\alpha} \) which can be canceled by an appropriate variation of the Lagrange multiplier \( P^a_{\alpha} \) in (3.16).
We have thus demonstrated that the supermembrane action (3.16) is invariant under the super–Weyl transformations (3.18), (3.19) of the superworldvolume. Note that conventional (super)membrane actions do not have such symmetry.

The local $SO(1,2)$ rotations and the super–Weyl transformations can be used to put the components $e_\alpha^\mu$ of the inverse supervielbein matrix $e_A^M$ to be equal to the unit matrix

$$e_A^M = \begin{pmatrix} e_\alpha^m & e_\alpha^\mu \\ e_\alpha^m & \delta_\alpha^\mu \end{pmatrix}, \quad (3.24)$$

then the superdeterminant (3.20) reduces to

$$s\text{det} e_A^M = \text{det}^{-1}[e_\alpha^m - e_\alpha^a e_\alpha^m]. \quad (3.25)$$

We shall further work in the gauge (3.24), (3.25).

### 3.3 Relationship with the conventional formulation

To establish the relationship we should consider the second term of (3.16), since the first term only serves for producing the superembedding constraint which relates the superworldvolume geometry with that of the target superspace, in other words, which ensures the components of the worldvolume supergravity multiplet to be pure auxiliary fields. In this sense the local worldvolume supersymmetric action (3.16) is an example of how a ‘no–go’ theorem [51] of the (non)existence of local worldvolume supersymmetric extensions of the Dirac membrane action (and, in particular, of its Howe–Tucker form [52]) can be overcome.

It is well known that integration over the Grassmann–odd variables is equivalent to differentiation. So (taking into account the superembedding condition (1.7) and the superworldvolume constraints (3.11) and (3.12)) we rewrite

$$S_A = \frac{1}{3!} \int d^3\xi d^2\eta \ s\text{det} e \gamma^{\alpha\beta} A_{\alpha\beta a} = \frac{1}{2i3!} \int d^3\xi \partial^\gamma \partial_\gamma (s\text{det} e \gamma^{\alpha\beta} A_{\alpha\beta a})$$

$$= \frac{1}{2i3!} \int d^3\xi \ s\text{det} e \nabla^\gamma \nabla_\gamma \gamma^{\alpha\beta} A_{\alpha\beta a} \big|_{\eta=0} \quad (3.26)$$

Upon some manipulations with the use of the supergravity constraints (3.8)–(3.10) and (3.11), and the superembedding condition (1.7), one finds that the second–order covariant derivative in (3.26) is

$$S_A = -\frac{1}{3!} \int d^3\xi \ s\text{det} e \left[ \frac{1}{2} \varepsilon^{abc} \gamma_{c}^{\alpha\beta} E_{\alpha}^{a} E_{\beta}^{b} E_{\gamma}^{c} F_{\alpha\beta\gamma} + \frac{1}{2} \varepsilon^{abc} A_{cda} \right] \big|_{\eta=0}$$

$$= - \int d^3\xi \ s\text{det} e \left[ \frac{1}{2 \cdot 3!} \varepsilon^{abc} \gamma_{c}^{\alpha\beta} E_{\alpha}^{a} E_{\beta}^{b} E_{\gamma}^{c} \Gamma_{\alpha\beta\gamma}^{a} \right] \big|_{\eta=0} - \frac{1}{2 \cdot 3!} \int d^3\xi \ s\text{det} e \varepsilon^{abc} A_{cda} \big|_{\eta=0}. \quad (3.27)$$

To reduce the action (3.27) to the Green–Schwarz action we choose a Wess–Zumino gauge such that

$$e_\alpha^\mu \big|_{\eta=0} = 0. \quad (3.28)$$
Then the leading component of the superdeterminant \((3.25)\) reduces to \(\det^{-1} e^m_a(\xi)\) and one can easily see that the second term of \((3.27)\) coincides with the Wess–Zumino term of \((3.2)\).

To show that the first term of \((3.27)\) is equivalent to the Nambu–Goto term we use the harmonic technics of the superembedding approach \([24, 41, 23]\). In the case of the supermembrane in \(N = 1, D = 4, 5, 7\) and \(11\), with our choice \((3.11)\) of the worldvolume supergeometry constraints, the superembedding condition \((1.7)\) allows us to choose the pullback of the target–space supervielbein \((1.4)\) to be

\[
E^a = e^a E^a_a = (1 + h^\beta h_\beta) e^p u^P_a \tag{3.29}
\]

\[
E^a = e^a E^a_a + h^\beta(z) \tilde{v}^\alpha_\alpha = e^P v^P_a n^p + h^\beta(z) \tilde{v}^\alpha_\alpha + e^a E^a_a, \tag{3.30}
\]

where \(u^a_a(z)\), and \(v^\alpha_\alpha(z)\) are, respectively, vector and spinor harmonics parametrizing the coset space \(SO(1, D - 1)/[SO(1, 2) \times SO(D - 3)]\) with indices \((a, \alpha)\) being associated with the vector and spinor representation of \(SO(1, 2)\) and the indices \((p, \tilde{q})\) corresponding to (in general different) \((D - 2)\)–dimensional spinor representations of \(SO(D - 3)\). \(n^p\) is a constant unit–norm spinor \((n^p n^p = 1)\) and \(h^\beta(z)\) is an unconstrained worldvolume superfield.

The harmonics have the following properties (see \([24, 23]\) for a review)

\[
u^\beta_\alpha v^\gamma_\beta C_{\alpha \beta} = \epsilon_{\alpha \beta \gamma} \delta_{pq}, \quad \tilde{v}^\alpha_\alpha \tilde{v}^\beta_\beta C_{\alpha \beta} = 0, \tag{3.31}
\]

\[
\langle v^\alpha_\alpha, v^\beta_\beta, \tilde{v}^\gamma_\gamma \rangle = \Gamma^\beta_\gamma v^\alpha_\alpha, \tag{3.32}
\]

Using \((3.28)\), \((3.29)\) and \((3.31)\) one finds that the induced metric is related to the bosonic vielbein matrix \(e_m^a\), as follows

\[
g_{mn}(\xi) = (1 + h^\beta h_\beta)^2 e^m_a e_n^a|_{\eta=0}, \tag{3.33}
\]

and hence

\[
\sdet e|_{\eta=0} = \det e_m^a(\xi) = \frac{1}{(1 + h^\beta h_\beta)^3} \sqrt{\det g_{mn}}. \tag{3.34}
\]

Finally, using the expressions of \(E^a_a\) and \(E^a_a\) in terms of the harmonics and of the superfield \(h^\beta(z)\) \((3.29)\), \((3.30)\) and the relations \((3.31)\) and \((3.32)\), as well as the gamma–matrix identity \((\gamma^a \gamma^b \gamma^c)^a_\beta = \delta^a_\beta \delta^{abc}\), one reduces the action \((3.27)\) to

\[
S = \int d^3 \xi \sqrt{-\det g_{mn}} \frac{1 - h^\beta h_\beta}{1 + h^\beta h_\beta} - \frac{1}{2} \int_{\mathcal{M}_4} d^3 \xi \varepsilon^{mnp} \partial_m Z^L \partial_n Z^M \partial_p Z^N A_{NML}(Z). \tag{3.35}
\]

Varying eq. \((3.35)\) with respect to \(h^\beta\) we find that its algebraic equation of motion implies \(h^\beta = 0\), and thus \((3.35)\) reduces to the conventional supermembrane action \((3.2)\).

Note that in the \(D = 4\) target–superspace the group \(SO(D - 3)\) gets trivialized so in this case we have only one scalar superfield \(h(z)\), and the supermembrane action \((3.35)\) coincides with the one constructed in \([41]\).

---

8This is so, if we assume that the induced metric is non–degenerate. Otherwise we would get a tensionless (null) supermembrane.
3.4 $D = 4$ supermembrane in the physical gauge and spontaneous supersymmetry breaking

We now proceed to the consideration of the dynamics of a supermembrane propagating in an $N = 1$, $D = 4$ flat target superspace in the physical gauge (1.8).

In the flat target superspace of dimension $D = 4$, 5, 7 and 11 the three–form field is

$$A^{(3)} = i \bar{\Theta} \Gamma_{ab} d\Theta (\mathcal{E}^a \mathcal{E}^b - i \mathcal{E}^a \bar{\Theta} \Gamma^b d\Theta - \frac{1}{3} \bar{\Theta} \Gamma^a d\Theta \bar{\Theta} \Gamma^b d\Theta),$$  

where

$$\mathcal{E}^a = dX^a - i d \bar{\Theta} \Gamma^a \Theta.$$  

The action (3.16) takes the form

$$S = \int d^3 \xi d^2 \eta P^a \mathcal{E}^a + \frac{1}{3!} \int d^3 \xi d^2 \eta sdet \epsilon^{\gamma \alpha \beta} [i \bar{\Theta} \Gamma_{ab} \Theta (i \bar{\Theta} \Gamma^a D_\beta \Theta \mathcal{E}^b_a + \bar{\Theta} \Gamma^a D_\beta \Theta \bar{\Theta} \Gamma^b D_\alpha \Theta] + \frac{2}{3} \bar{\Theta} \Gamma^a D_\alpha \Theta \bar{\Theta} \Gamma^b D_\beta \Theta],$$  

where the components of (3.36) containing $\mathcal{E}^a$ have been included into the first term, and $D_a = e_a^M \partial_M$. (Recall that $\mathcal{E}^a = 0$ is the superembedding condition.)

Using the cyclic identity for gamma–matrices in $D = 4$, 5, 7 and 11

$$\Gamma_a^{(\alpha \beta \gamma \delta)} \epsilon^{ab} = 0$$

(where $(\cdot)$ denotes the symmetrization of the spinor indices) we can reduce the second ‘Wess–Zumino’ term of the action (3.38) to

$$S_A = \frac{i}{3!} \int d^3 \xi d^2 \eta sdet \epsilon^{\gamma \alpha \beta} \bar{\Theta} \Gamma_{ab} D_\alpha \Theta (i \bar{\Theta} \Gamma^a D_\beta \Theta \mathcal{E}^b_a + \bar{\Theta} \Gamma^a D_\beta \Theta \bar{\Theta} \Gamma^b D_\alpha \Theta).$$  

In $D = 4$ the supermembrane action can be further simplified due to the one more cyclic gamma–matrix identity similar to (3.23). In particular, we find that

$$\bar{\Theta} \Gamma_{ab} d\Theta \bar{\Theta} \Gamma^b d\Theta = - \frac{1}{2} (\bar{\Theta} \Theta) d\bar{\Theta} \Gamma_a d\Theta,$$  

and, hence,

$$A^{(3)} = i \bar{\Theta} \Gamma_{ab} d\Theta \mathcal{E}^a \mathcal{E}^b + \frac{i}{2} (\bar{\Theta} \Theta) d\bar{\Theta} \Gamma_a d\Theta \mathcal{E}^a.$$  

Substituting the $A_{a;\beta a}$ component of the superworldvolume pullback of (3.42) into the action (3.16) we get the $D = 4$ supermembrane action in the form

$$S = \int d^3 \xi d^2 \eta P^a \mathcal{E}^a + \frac{i}{3!} \int d^3 \xi d^2 \eta sdet \epsilon^{\gamma \alpha \beta} (\bar{\Theta} \Theta) D_a \bar{\Theta} \Gamma_a \Theta \mathcal{E}^a.$$  

The form (3.43) of the supermembrane action and of the superdeterminant (3.25) prompt us that it is invariant under the following variation of supervielbein components (3.24)

$$e'_a^\mu = e_a^\mu + f_a^\mu (z), \quad e'_a^m = e_a^m + f_a^\alpha e_\alpha^m,$$
accompanied by an appropriate variation of the Lagrange multiplier \( P_\alpha \). This allows us to put \( e_a^\mu = 0 \), and thus reduce (3.24) and (3.25) to

\[
e_A^M = \left( e_a^m \\ e_a^m \delta_a^\mu \right), \quad sdet e_M^A = det^{-1}(e_a^m).
\] (3.45)

We can also notice that the integrability of the superembedding condition

\[
\mathcal{E}_\alpha^a = D_\alpha X^a - iD_\alpha \bar{\Theta} \Gamma^a \Theta = 0
\] (3.46)

requires that

\[
\gamma_{\alpha \beta}^a \mathcal{E}_\alpha^a = \gamma_{\alpha \beta}^a(D_\alpha X^a - iD_\alpha \bar{\Theta} \Gamma^a \Theta) = D_\alpha \bar{\Theta} \Gamma^a D_\beta \Theta.
\] (3.47)

Eq. (3.47) is obtained from (3.46) by hitting its right hand side with \( \bar{\nabla} = D_\beta + \omega_\beta \), symmetrizing the result with respect to indices \( \alpha, \beta \) and taking into account that due to the torsion constraint (3.11)

\[
\{\nabla_\alpha, \nabla_\beta\} = 2i\gamma_{\alpha \beta}^a \nabla_a - T_{\alpha \beta}^a \nabla_a + R_{\alpha \beta},
\] (3.48)

where \( R_{\alpha \beta}^A(z) \) are components of the superworldvolume curvature.

Using eq. (3.47) we can rewrite the action (3.43) in even simpler form

\[
S = \int d^3 \xi d^2 \eta P_\alpha^a \mathcal{E}_\alpha^a - \frac{i}{3!} \int d^3 \xi d^2 \eta \ det^{-1}(e_a^m) \ (\Theta \Theta) \mathcal{E}_a^A \mathcal{E}_b^B \eta_{ab}.
\] (3.49)

\( \eta_{ab} = diag (-, +, +, +) \).

Note that in the form (3.49) the action resembles the Howe–Tucker–Polyakov term of the supermembrane action.

We now use the worldvolume superdiffeomorphisms to impose the physical gauge (1.8). To this end we choose the following ‘\( d = 3 \) adapted’ Majorana representation of the \( D = 4 \) Dirac matrices \( \Gamma^a = (\Gamma^a, \Gamma^3) \) (\( a = 0, 1, 2 \))

\[
\Gamma_{\alpha \beta}^a = \left( \begin{array}{cc} \gamma_{\alpha \beta}^a & 0 \\ 0 & (\gamma^a)_{\alpha \beta} \end{array} \right), \quad \Gamma_{\alpha \beta}^3 = \left( \begin{array}{cc} 0 & \delta_{\beta}^\alpha \\ \delta_{\alpha}^\beta & 0 \end{array} \right), \quad C_{\alpha \beta} = \left( \begin{array}{cc} \epsilon_{\alpha \beta} & 0 \\ 0 & -\epsilon_{\alpha \beta} \end{array} \right),
\] (3.50)

where \( \gamma_{\alpha \beta}^a \) are the same as defined in (2.3).

With respect to (3.50) the Majorana spinor \( \Theta^\alpha (\alpha = 1, \cdots, 4) \) splits as

\[
\Theta^\alpha(z) = \begin{pmatrix} \theta^\alpha \\ \psi_\beta \end{pmatrix}.
\] (3.51)

In the physical gauge we identify target superspace coordinates \( X^a \) and \( \theta^a \) with superworldvolume coordinates

\[
X^a = \xi^a, \quad \theta^a = \eta^a.
\] (3.52)

\(^9\)Of course the variation (3.44) changes conventional worldvolume supergravity constraints in (3.11) and (3.12), but the essential constraint \( T_{\alpha \beta}^a = -2i\gamma_{\alpha \beta}^a \) remains unchanged.
Upon this identification there is no distinction between worldvolume indices \((m, \mu)\) and tangent superspace indices \((a, \alpha)\). The remaining superfields \(X^3(z)\) and \(\Psi_\alpha(z)\) are Goldstones of spontaneously broken space–time translations in the direction transverse to the membrane and of two supersymmetry transformations. In the gauge (3.52) the superembedding condition splits into the part parallel to the membrane (remember that the worldvolume supervielbein matrix has the form (3.45))

\[
e^m_\alpha - i\gamma^m_\alpha \eta^\beta - iD_\alpha \bar{\Psi} \gamma^m \Psi = 0
\]

and the transverse part

\[
D_\alpha \Phi(z) = 2i\Psi_\alpha(z),
\]

where

\[
\Phi = X^3 + i\eta^\alpha \Psi_\alpha, \quad D_\alpha = \partial_\alpha + e^m_\alpha \partial_m.
\]

The integrability condition (3.47) splits as follows. The parallel part is

\[
e^m_a - iD_a \bar{\Psi} \gamma^m \Psi = \delta^m_a - \frac{1}{2} \gamma^{\alpha\beta}_a D_\alpha \bar{\Psi} \gamma^m D_\beta \Psi
\]

and the transverse part is

\[
D_a \Phi = -\gamma^{\alpha\beta}_a D_\alpha \Psi_\beta, \quad D_a = e^m_a \partial_m.
\]

Now eqs. (3.53) and (3.56) can be rewritten in the form

\[
e^n_a (\delta^m_n - i\partial_n \bar{\Psi} \gamma^m \Psi) = i\gamma^m_\alpha \eta^\beta + i\partial_\alpha \bar{\Psi} \gamma^m \Psi.
\]

The inverse of the matrix \(M^m_n = \delta^m_n - i\partial_n \Psi \gamma^m \Psi\) is

\[
(M^{-1})^m_n = \delta^m_n + i\partial_n \bar{\Psi} \gamma^m \Psi - \partial_n \bar{\Psi} \gamma^b \Psi \partial_b \bar{\Psi} \gamma^m \Psi.
\]

Then from (3.58) we get the expression for the worldvolume supervielbein components \(e^n_A(z)\) in terms of the Goldstone fermion \(\Psi_\alpha(z)\)

\[
e^n_\alpha = i\gamma^m_\alpha \eta^\beta + iD_\alpha \bar{\Psi} \gamma^m \Psi - D_\alpha \bar{\Psi} \gamma^b \Psi \partial_b \bar{\Psi} \gamma^n \Psi
\]

\[
= i\gamma^m_\alpha \eta^\beta + iD_\alpha \bar{\Psi} \gamma^b \Psi (\delta^m_n + i\partial_n \bar{\Psi} \gamma^n \Psi),
\]

\[
e^n_a = (\delta^m_a - \frac{1}{2} \gamma^{\alpha\beta}_a D_\alpha \bar{\Psi} \gamma^b D_\beta \Psi)(\delta^m_n + i\partial_n \bar{\Psi} \gamma^n \Psi - \partial_n \bar{\Psi} \gamma^m \Psi \partial_m \bar{\Psi} \gamma^m \Psi),
\]

where

\[
D_a = \frac{\partial}{\partial \eta^a} + i\eta^b \gamma^a_{\beta} \frac{\partial}{\partial \xi^\beta}, \quad \{D_\alpha, D_\beta\} = 2i\gamma^a_{\alpha\beta} \frac{\partial}{\partial \xi^a}
\]

are covariant derivatives in a flat \(N = 1, d = 3\) superspace.

\(^{10}\)Up to a normalization our definition of (3.59) and \(D_a\) (3.54) is the same as in [8], and the definition of (3.60) and \(D_a\) (3.57) is related to that in [8] by the linear transformation with the matrix \((\delta^m_a - \frac{1}{2} \gamma^{\alpha\beta}_a D_\alpha \bar{\Psi} \gamma^b D_\beta \Psi)\)
To complete the list of expressions for the $e_A^m$ components we also give the equation relating $e_a^m$ and $e_a^m$

$$e_a^m = \frac{i}{2} \alpha^\alpha_\beta \mathcal{D}_\beta e_a^m,$$  \hspace{1cm} (3.62)

which can be easily obtained by taking the $\mathcal{D}_\beta$-derivative of (3.53), symmetrizing the result with respect to $\alpha, \beta$ and comparing it with eq. (3.56).

We have thus expressed all components of the worldvolume supervielbein (3.45) in terms of the Goldstone superfield, which implies that the geometry (supergravity) in the superworldvolume is indeed induced by its embedding into the target superspace.

Consider now the transverse part (3.54) of the superembedding condition. In view of (3.59) it can be presented in the following form

$$\Psi_\alpha = -\frac{i}{2} D_a \Phi + \frac{1}{2} \bar{D}_a \bar{\Psi} \gamma^b \Psi (\delta^m_b + i \partial_b \bar{\Psi} \gamma^n \Psi) \partial_n \Phi.$$  \hspace{1cm} (3.63)

This is an implicit expression of the Goldstone fermion $\Psi_\alpha (\xi, \eta)$ in terms of the independent Goldstone boson superfield $\Phi (\xi, \eta)$. Eq. (3.63) is exactly solvable. However, the solution looks rather cumbersome and we only present its general structure

$$\Psi_\alpha = i F_{\alpha}^\beta D_\beta \Phi + \bar{F}_\alpha D^\beta \Phi D_\beta \Phi,$$  \hspace{1cm} (3.64)

where $F_{\alpha}^\beta = F_1 \delta_{\alpha}^\beta + F_2 (\partial_m \Phi \gamma^a)_{\alpha}^\beta$ and $\bar{F}_\alpha$ are known though complicated functions of $\partial_m \Phi$ and $D_\alpha \Phi$.

To find a form of the supermembrane action (3.49) in the physical gauge we should calculate $\mathcal{E}_a^b \mathcal{E} b^a \eta_{ab}$ and $\text{det}(e_a^m)$ using the expressions (3.54), (3.55), (3.57) and (3.60). To this end it is convenient to rewrite the matrix (3.56) in the following form

$$L_a^m = (\delta^m_a - \frac{1}{2} \alpha^\alpha_\beta D_a \bar{\Psi} \gamma^m D_\beta \Psi) = \delta^m_a + \frac{1}{4} \alpha^m_a [(D_a \Psi^\alpha)^2 + D^c \Phi D_c \Phi] - \frac{1}{2} D_a \Phi D^m \Phi + \frac{1}{2} \sum c \eta_{ac} D_c \Phi (D_a \Psi^\alpha).$$  \hspace{1cm} (3.65)

Then one finds that

$$\mathcal{E}_a^b \mathcal{E} \mathcal{E} b^a \eta_{ab} = 3 [1 + \frac{1}{4} (D_a \Psi^\alpha)^2 + \frac{1}{2} D^c \Phi D_c \Phi]^2 - \frac{3}{4} (D_a \Psi^\alpha)^2 D^a \Phi D_a \Phi,$$  \hspace{1cm} (3.66)

$$\text{det}(e_a^m) = \text{det}(L_a^m) \text{det}^{-1}(\delta^b_m - i \partial_m \bar{\Psi} \gamma^b \Psi)$$

$$= \frac{1}{3} \mathcal{E}_a^b \mathcal{E} b^a \eta_{ab} \left[1 + \frac{1}{4} (D_a \Psi^\alpha)^2 - \frac{1}{4} D^c \Phi D_c \Phi \right] \text{det}^{-1}(\delta^b_m - i \partial_m \bar{\Psi} \gamma^b \Psi),$$  \hspace{1cm} (3.67)

and

$$\text{det}(\delta^b_m - i \partial_m \bar{\Psi} \gamma^b \Psi) = 1 - i \partial_m \bar{\Psi} \gamma^a \Psi + \frac{1}{2} \alpha^m_a \alpha^b (D_m \bar{\Psi} \gamma^a \Psi)^2,$$  \hspace{1cm} (3.68)

(where $(\Psi)^2 = \Psi^\alpha \Psi_\alpha$), and the action (3.49) takes the form

$$S = \frac{i}{2} \int d^3 \xi d^2 \eta \left(\eta^2 - (\Psi)^2\right) \frac{\text{det}(\delta^b_m - i \partial_m \bar{\Psi} \gamma^b \Psi)}{1 + \frac{1}{4} (D_a \Psi^\alpha)^2 - \frac{1}{4} D^c \Phi D_c \Phi},$$  \hspace{1cm} (3.69)

Upon some algebraic manipulations with the use of eqs. (3.59) the denominator of (3.69) can be represented as follows
\[1 + \frac{1}{4}(D_\alpha \Psi^\alpha)^2 - \frac{1}{4} D^c \Phi D_c \Phi =
\]
\[= 1 + \frac{1}{4} \left[ (D_\alpha \Psi^\alpha)^2 - (\bar{D}_\gamma \phi)(\bar{D} \gamma^a \Psi) \right] \text{det} \left( \delta_m^b - i \partial_m \bar{\Psi} \gamma^b \Psi \right). \quad (3.70)\]

Note also that
\[(D_\alpha \Psi^\alpha)^2 - (\bar{D}_\gamma \phi)(\bar{D} \gamma^a \Psi) = 2 D_\alpha \Psi^\beta D^\alpha \Psi^\beta = -D^2 \Psi^2 + 2 \Psi^\alpha D^2 \Psi^\alpha, \quad D^2 \equiv D^\alpha D_\alpha. \quad (3.71)\]

As we have already observed in the case of the superparticles (see eqs. (2.30) and (2.31)), the manifest worldvolume and target–space supersymmetry of the original action (3.16), and dimensional reasons requires the fractional factor in the action (3.69) to be of the form
\[L \equiv \text{det} \left( \delta^b_m - i \partial_m \bar{\Psi} \gamma^b \Psi \right) \frac{1}{1 + \frac{1}{4} (D_\alpha \Psi^\alpha)^2 - \frac{1}{4} D^c \Phi D_c \Phi} = D_\alpha D^\alpha \frac{\Psi^2}{Y(\xi, \eta)} + \partial_\alpha (\Psi^2 Y^a) + 1, \quad (3.72)\]

where \(Y(\xi, \eta)\) and \(Y^a(\xi, \eta)\) are superfields, and 1 reflects the fact that the energy density of the “ground” state (\(\Phi = \text{const}, \ \Psi = 0\)) of the supermembrane is normalized to be one (in tension unites), which is in accordance with the value of the energy density of a (non–fluctuating) supermembrane ground state in the Green–Schwarz formulation (3.2).

Note that because of its form the vector derivative term of (3.72) appears in the action (3.69) only as a total derivative, and, therefore, can be omitted.

We now show how to determine the form of the superfield \(Y(\xi, \eta)\). To this end we take the part \(\frac{1}{2} \int d^3 \xi d^2 \eta \Psi^2 L\) of the action (3.69). Because of the relations (3.70)–(3.71)
\[\frac{i}{2} \int d^3 \xi d^2 \eta \Psi^2 L = \frac{i}{2} \int d^3 \xi d^2 \eta \frac{\Psi^2}{1 - \frac{1}{4} D^2 \Psi^2}. \quad (3.73)\]

On the other hand from (3.72) we get (up to a total derivative) that
\[\frac{i}{2} \int d^3 \xi d^2 \eta \Psi^2 L = \frac{i}{2} \int d^3 \xi d^2 \eta \left[ (D^2 \Psi^2) \frac{\Psi^2}{Y(\xi, \eta)} + \Psi^2 \right]. \quad (3.74)\]

Comparing (3.73) with (3.74) we find that
\[Y = 4 \left(1 - \frac{1}{4} D^2 \Psi^2\right), \]
and
\[L = 1 + \frac{1}{4} D^2 \frac{\Psi^2}{1 - \frac{1}{4} D^2 \Psi^2}. \quad (3.75)\]

Substituting (3.73) into (3.69) we get the following \(N = 1, D = 3\) Goldstone superfield action
\[S = i \int d^3 \xi d^2 \eta \frac{\Psi^2}{1 - \frac{1}{4} D^2 \Psi^2} + \int d^3 \xi \cdot 1, \quad (3.76)\]
where the Goldstone fermion \(\Psi_\alpha\) depends on the Goldstone scalar \(\Phi\) (3.54), (3.64).
Finally we present the action in terms of the independent Goldstone scalar superfield \( \Phi(\xi, \eta) \), which is obtained from eq. (3.76) with the use of the expression (3.64),

\[
S = -\frac{i}{2} \int d^3 \xi d^2 \eta \frac{D^a \Phi D_a \Phi}{1 - \frac{1}{8} (D^2 \Phi)^2 + \sqrt{1 + \partial_a \Phi \partial^a \Phi(1 - \frac{1}{16} (D^2 \Phi)^2)}} + \int d^3 \xi \cdot 1 .
\] (3.77)

One can easily check that in the bosonic limit, when the fermionic \( D_\alpha \Phi |_{\eta=0} \) and auxiliary field \( D^2 \Phi |_{\eta=0} \) degrees of freedom are zero, the action (3.77) reduces to the gauge–fixed Nambu–Goto action for a membrane in \( D = 4 \)

\[
S = \int d^3 \xi \sqrt{1 + \partial_a \phi \partial^a \phi},
\]

where \( \phi(\xi) = \Phi(\xi, \eta)|_{\eta=0} \).

### 3.5 Supersymmetry properties of the \( d = 3 \) field theory

As in the superparticle case of Section 2, the physical gauge conditions (3.52), and therefore the action (3.69), are invariant under the following combination of the target–superspace global supersymmetry transformations (2.17) and the worldvolume superdiffeomorphisms

\[
\delta \eta^\alpha = \epsilon^1 \alpha, \quad \delta \xi^m = i \tilde{\eta} \gamma^m \epsilon^1 + i \tilde{\Psi}(\xi, \eta) \gamma^m \epsilon^2,
\] (3.78)

where \( \epsilon^a = (\epsilon^1, \epsilon^2) \) are two constant parameters of target–space supersymmetry (2.17) which is seen by the superworldvolume observer as \( N = 2, d = 3 \) supersymmetry.

The superfields \( \Psi_\alpha(\xi, \eta) \) and \( \Phi(\xi, \eta) \) (3.51), (3.55) transform under (3.78) in the following way

\[
\delta \Psi_\alpha = -\epsilon^1 \rho_\alpha \Phi + \epsilon^a \gamma^m \Psi \partial_m \Psi_\alpha,
\] (3.79)

\[
\delta \Phi = -\epsilon^1 \rho_\alpha \Phi + 2 i \eta^a \epsilon_\alpha + i \tilde{\eta} \gamma^m \Psi \partial_m \Phi.
\] (3.80)

We see that under the \( \epsilon^2 \)–supersymmetry transformations \( \Psi_\alpha(\xi, \eta) \) and \( \Phi(\xi, \eta) \) indeed vary in a nonlinear manner as Goldstone fields, while under \( \epsilon^1 \)–supersymmetry they transform as ordinary scalar superfields. Hence, the \( N = 2, d = 3 \) supersymmetry of the superfield action (3.76) is spontaneously broken down to \( N = 1 \).

### 3.6 Relationship to the Goldstone superfield action of [8]

The form of the gauge–fixed supermembrane action (3.76), (3.77) differs from the Goldstone superfield action constructed in [8], because the fields involved in the construction of these actions are different. The Goldstone superfields of the former action correspond to the nonlinear realization of spontaneously broken supersymmetry, while the latter is constructed with a Goldstone superfield of a ‘linear’ realization.

The Goldstone superfields of the two realizations are related as follows [8]

\[
\Psi_\alpha = \frac{\zeta_\alpha}{1 + D^2 F}, \quad \zeta_\alpha = D_\alpha \rho(\xi, \eta),
\] (3.81)
where $\rho(\xi, \eta)$ is a scalar superfield and

$$\mathcal{F} = \frac{1}{2} \frac{\zeta^2}{1 + \sqrt{1 + D^2 \zeta^2}}.$$  \hspace{1cm} (3.82)

From (3.81) and (3.82) we get

$$\Psi^2 = 4 \zeta^2 \frac{1 + \sqrt{1 + D^2 \zeta^2}}{(1 + \sqrt{1 + D^2 \zeta^2})^2}, \quad D^2 \Psi^2 = 4 \frac{D^2 \zeta^2}{1 + \sqrt{1 + D^2 \zeta^2}} + \cdots,$$  \hspace{1cm} (3.83)

where dots stand for irrelevant terms proportional to $\zeta$ and $\zeta^2$.

Substituting (3.83) into (3.76) we arrive at the action of $[8]$

$$S = 2i \int d^3 \xi d^2 \eta \frac{\zeta^2}{1 + \sqrt{1 + D^2 \zeta^2}} + \int d^3 \xi \cdot 1.$$  

This concludes the relationship of the superembedding description with other formulations of supermembrane dynamics.

4 Conclusion

In this paper, with the examples of the massive $N = 2$, $D = 2$ superparticle and the $N = 1$, $D = 4$ supermembrane, we have demonstrated how starting from the superembedding formulation of superbrane dynamics one arrives, upon gauge fixing worldvolume superdiffeomorphisms, at an effective nonlinear field theory on the brane superworldvolume with partially broken global supersymmetry. The latter is non–linearly realized on the superfields composed of supermultiplets transforming linearly under unbroken supersymmetry.

When the superembedding condition does not put the superbrane theory on the mass shell there is a generic prescription of how to construct superbrane actions of the form (3.13) in the superembedding approach $[38, 46, 41]$ using a corresponding Wess–Zumino–like form, which is closed modulo the superembedding condition (1.7). In the case of the $N = 1$, $D = 4$ supermembrane we have shown how by consecutive gauge fixing local worldvolume symmetries, eliminating auxiliary worldvolume superfields and solving for the superembedding condition one reduces this generic covariant superfield action to the $N = 1$, $d = 3$ Goldstone superfield action exhibiting the mechanism of partial breaking of $N = 2$ global supersymmetry. By construction this action is directly related to the conventional supermembrane component action in the physical gauge $[2]$, the physical degrees of freedom forming a Goldstone scalar supermultiplet. We have also demonstrated how the superembedding action is related to the Goldstone superfield action of $[8]$.

The superembedding approach thus provides us with a systematic way of deriving superfield actions for Goldstone superfields of the method of nonlinear realizations, which have so far been unknown, and establishes their direct link to the superbrane actions. As we have mentioned in the Introduction, the actions with partial supersymmetry breaking have been constructed for a different type of Goldstone superfields which appear in the method of ‘linear’ realizations $[4–9]$. 

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The relation of fermionic sectors of these actions with the fermionic sectors of corresponding
gauge fixed superbrane actions in general still remains an open problem.

It should be possible to extend the methods of this paper to more physically interesting
cases, in particular, to the construction of the covariant superembedding action for a space–filling
Dirichlet 3–brane in an \( N = 2, D = 4 \) target superspace, whose dynamics is also described
by the off–shell superembedding condition. A gauge fixed version of this action should be an
action for a \( D = 4 \) supersymmetric Dirac–Born–Infeld field theory with partially broken \( N = 2 \)
supersymmetry described in terms of Goldstone superfields of the method of nonlinear realizations
\(^4\).

The methods of superembedding and nonlinear realizations are also applicable to the description
of superbranes in AdS–superbackgrounds and to the derivation of actions for effective field
theories on the AdS boundary whose simple form is still lacking.

Another direction of research can be connected with studying partial breaking of local supersymmetry in supergravity theories. The embedding of curved supersurfaces into curved supergravity backgrounds governed by the superembedding condition (1.7) seems to be a natural basis for studying mechanisms of local supersymmetry breaking, which can also be related to the problem of finding supersymmetric versions of brane world scenarii.

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Appendix: The \( N = 2, D = 2 \) massive superparticle in the light–cone gauge

Below we demonstrate that when the \( N = 2, D = 2 \) massive superparticle action is chosen in the
form \((2.1)\), the worldline local superreparametrization \((2.7)\) allows one to impose in this action
the light–cone gauge condition instead of the static gauge condition \((2.10)\).

Remember that the action \((2.1)\) is obtained from a more general action \((2.24)\) upon gauge
fixing one of the independent superdiffeomorphisms \((2.11)\) of the latter by putting the worldline
supervielbein component \( f(\tau, \eta) = \eta \). This

\[ (\eta(\tau, \eta) = \eta). \]

This reduces the superdiffeomorphism group \((2.11)\) down to \((2.7)\). To gauge fix the latter we impose
the light–cone gauge condition

\[ X^– = (X^0 - X^1) = \tau. \] \((A.1)\)

Then, in view of the Majorana form of the Gamma–matrices \((2.3)\), the superembedding condition
(2.8) reduces to the following two equations

\[ DX^- = i\eta = iD(\Theta^1 - \Theta^2)(\Theta^1 - \Theta^2), \] (A.2)

\[ DX^+ = D(X^0 + X^1) = iD(\Theta^1 + \Theta^2)(\Theta^1 + \Theta^2), \] (A.3)

Solving for eq. (A.2) we get the light–cone gauge condition for the Grassmann–odd coordinates

\[ \Theta^1 - \Theta^2 \equiv \Theta^- = \pm \eta. \] (A.4)

The equation (A.3) can be solved for \( \Psi^+ \equiv \Theta^1 + \Theta^2 \) using the Bagger–Galperin trick [4]. From (A.4) we get

\[ \Psi^+ = -i\frac{DX^+}{D\Psi^+}. \] (A.5)

Now take the D–derivative of (A.5)

\[ D\Psi^+ = -\partial\tau X^+ \frac{D\Psi^+}{(D\Psi^+)^2}. \] (A.6)

Examining the eq. (A.6) we find that because \( DX^+ \) is Grassmann–odd \( (DX^+)^2 \equiv 0 \) the second term of the right hand side of (A.6) does not contribute to the right hand side of (A.3) and to the first term of (A.6), when we substitute \( D\Psi^+ \) in these terms with its recursive relation (A.6). This allows us to write down “effective” relations

\[ (D\Psi^+)_{eff} = -\frac{\partial\tau X^+}{(D\Psi^+)_{eff}}, \quad \Psi^+ = -i\frac{DX^+}{(D\Psi^+)_{eff}}. \] (A.7)

From the first equation in (A.7) we get (up to an irrelevant sign) \( (D\Psi^+)_{eff} = \sqrt{\partial\tau X^+} \), then the second equation takes the form

\[ \Psi^+ = -i\frac{DX^+}{\sqrt{\partial\tau X^+}}, \] (A.8)

which expresses the superfield \( \Psi^+ \) in terms of \( X^+ \).

We have thus explicitly solved the superembedding constraints (A.2), (A.3) in the light–cone gauge (A.1), (A.4). As a result (up to a total derivative) the superfield action (2.1) reduces to

\[ S = m \int d\tau d\eta \Psi^+ = -im \int d\tau d\eta \frac{DX^+}{\sqrt{\partial\tau X^+}}. \] (A.9)

One can easily verify that eq. (A.9) is the \( N = 1 \) worldline superfield form of the component action which one obtains by imposing the light–cone gauge in the standard component action for the \( N = 2, D = 2 \) superparticle

\[ S = m \int d\tau \left[ \sqrt{(\partial\tau x^a - i\partial\tau \bar{\theta} \Gamma^a \theta)(\partial\tau x^a - i\partial\tau \bar{\theta} \Gamma^a \theta) - i\partial\tau \bar{\theta} \Gamma^2 \theta} \right], \] (A.10)

or in the first–order form

\[ S = \int d\tau \left[ p_a (\partial\tau x^a - i\partial\tau \bar{\theta} \Gamma^a \theta) - \frac{e(\tau)}{2} (p_a p^a - m^2) - i\partial\tau \bar{\theta} \Gamma^2 \theta \right] \] (A.11)

The superfield \( X^+(\tau, \eta) \) is composed from the light–cone coordinates \( x^+(\tau) \) and \( \theta^+(\tau) \) of (A.10) as follows

\[ X^+(\tau, \eta) = x^+(\tau) + i\eta \theta^+ \sqrt{\partial\tau x^+}. \]
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