Learning trajectory of modeling situation problems utilizing tables and diagrams for elementary school students

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Abstract. This research investigated the learning obstacles found in algebra learning and utilized them in the development of learning trajectory on algebraic thinking. Students have experienced algebraic difficulties due to several obstacles. The efforts to overcome difficulties require appropriate learning stages. Learning stages are designed based on the analysis of learning difficulties called learning trajectories. Learning trajectories are composed of systematic topic about tables and diagrams for elementary schools. Problem modeling is represented through tables and diagrams from concrete to abstract levels. The process of analysis results in an alternative didactic situation that can overcome obstacles in algebraic thinking. This study has used qualitative research with hermeneutics and phenomenology paradigms. The result of this research is a learning trajectory which is a part of the lesson design in algebraic thinking in the elementary level.

1. Introduction

Unlike algebraic thinking, algebra is actually learned after arithmetic and this only starts from the junior high school level [1]. Algebraic reasoning is the manipulation of numerals and signs (e. g. X + 5 = 12 - 4) to solve for an unknown [2]. Another opinion, that algebraic thinking is the ability to solve a problem that is not known to be resolved through the right example. Algebraic thinking is one of the skills that must be mastered in the era of industrial revolution 4.0. Several studies found that junior high school’s students have difficulties in algebraic thinking, hence it is crucial to introduce algebraic thinking in the elementary school [3-6]. Therefore, the mathematics curriculum in elementary and secondary levels should include algebra in order to tackle students’ difficulties in algebraic thinking [7]. The trends of implementing algebraic thinking as early as elementary school level have emerged lately [8,9].

A report indicated that the development of algebraic thingking may be fostered in the fourth grade [10]. This may be initiated with expressing generalizations of numerical forms using several representations. Non-symbolic algebra may also be introduced to seven to eights year-old students [11-14]. Some other algebraic thinking that can be developed in the early stage comprise of patterning, generalizing, and problem solving.
Mathematics learning process covers two important aspects, namely students-topic relationship and student-teacher relationship. These aspects may lead to a complex didactic-epididogical situation, requiring ample teachers’ preparation that enables students to acquire correct understanding of mathematics concepts and principles. In addition, it takes into account the exact sequence of topic or is known as the term trajectory [15]. Hypothetical Learning Trajectory (HLT) is an estimate of student learning trajectory. The allegations are further examined day by day during the research based on plans in the form of learning activities [16]. The researcher conducts a theory study and makes the learning trajectory a preliminary design. The components of the hypothetical learning trajectory consist of learning objectives for students, plans for learning activities, and expectations of the learning process in the classroom.

When compiling the allegations of the learning process in the classroom, researchers need to predict the development of mathematical knowledge in the classroom and the understanding or strategies of students who might emerge as they did during the actual learning activities [17]. During the initial design implementation, if that happen outside of planning, the teacher must prepare didactic anticipations made in conjunction with HLT. Anticipation is not rigid, if there are things that are out of control in the field, then the researcher applies flexibly to provide actions beyond the anticipated didactic anticipation. The spontaneous treatment carried out by researchers became a research note for the development of advanced designs.

Therefore, the purpose of this research is to investigate the learning obstacles found in algebra learning and utilized them in the development of learning trajectory on algebraic thinking. Students’ difficulties were overcome by applying appropriate learning stages. These stages were designed based on the analysis of learning difficulties called learning trajectories composed of systematic topic about tables and diagrams for elementary schools. Problem modeling was represented through tables and diagrams from concrete to abstract levels. The process of analysis results in an alternative didactic situation that may overcome students’ obstacles in algebraic thinking.

2. Methods

This research is preliminary qualitative study with design didactical research (DDR). This research in which hermeneutics and phenomenology paradigms were used to analyze learning obstacles. The subjects of this study were 40 students of fifth and sixth grade of elementary school in Garut, and whom 9 of them were interviewed after the test. Instruments are created based on 3-5 level algebraic thinking indicators consisting of learning obstacles, interviews, book and learning analysis. Test questions were used to uncover students' errors in algebraic thingking test. After that, the causes of error were investigated through interviews, analysis of instructional topics and learning analysis.

3. Result and Discussion

3.1 Results of Analysis of Learning Trajectory

3.1.1. Results of Interviews with teachers. Based on the results of interviews conducted on elementary school teachers obtained information that the topic regarding tables and diagrams is in the subtopics of data processing in fifth grade. The topic is contained in basic competencies of organizing and presenting data relating to students themselves and comparing with data from the surrounding environment in the form of lists, tables, pictogram (pictograms), bar charts, or line charts. Through observation and discussion activities students to collect data and represent data in the form of tables, make diagrams, bar charts and line charts.

The results of interviews conducted on elementary school teachers obtained information that the topic regarding tables and diagrams is in the subtopics of data processing in fifth grade. The topic is contained in basic competencies of organizing and presenting data relating to students by themselves and comparing with data from the surrounding environment in the form of lists, tables, pictogram (pictograms), bar charts, or line charts. These basic competencies are clarified into the form of learning objectives. Through observation, observation and discussion activities students collect data and represent data in the form of tables, diagrams, bar charts and line charts.
3.1.2. Results of interviews with students. Generally, the results ways of thinking and the ways of understanding of fifth grade students in elementary school. Interviews are conducted based on the results of the problem analysis and obstacle learning. The didactic situations that are raised must contain data representation activities not only in the form of tables and diagrams, but also contain problem solving activities related to tables and diagrams. Therefore, every lesson design developed contains action activities or mental acts in the form of activities that make students actively take action. Formulations in the form of activities to find steps to make a table or diagram, validation contained in the form of confirmation and mutual agreement from the results of group discussions. Institutionalization which is facilitated by the process of solving open ended questions or problem solving. Other things that are considered in designing learning trajectories are unity, flexibility and coherence that pay attention to the relationship between the teacher, students, and the topic itself. In addition, pay attention to the continuous didactic flow.

3.1.3. The results of the textbook analysis. The core topic that will be analyzed is the sequence of displaying data in the form of tables and diagrams. Learning trajectories are developed by taking into account the interrelationship of the topic before the core topic and the topic thereafter. The following is a series of topic arranged into a learning path. As shown in the picture below:

![Figure 1 Hipotetical Learning Trajectory of Table and Diagrams](image)

Figure 1 shows that the components contained in the list of tables, pictogram, bar charts, and line charts, namely, columns, rows, lines, Cartesian coordinates, construct square or rectangular flat, description of data, numbers, and number lines. Before learning data processing 1, the topic that must be mastered by students are the concepts of integers and number lines, vertical and horizontal lines, and constructing a two-dimensional figure and pattern.

3.2. Discussion

The first prerequisite topic is integers and number lines. Integers consist of counts (0, 1, 2, 3, ...) and negatives (-1, -2, -3, ...; -0 are equal to 0 so they are no longer entered separately). Integers can be written without decimal or fraction components. In addition, you must understand the meaning of rows and columns so that they are not confused. The row is an elongated part from top to bottom in the table. The column is a sideways part from left to right in the table.
Additionally, students must know about number lines. The number line in basic mathematics is a straight line where each point is assumed to represent a real number and every real number refers to a particular point. The above line is divided into two symmetrical halves by the origin, which is symbolizing zero numbers.

After mastering the two concepts above students must also master the concepts of vertical and horizontal lines in the Cartesius diagram. Bar charts are closely related to the Cartesius coordinate system. In connection with the bar chart there is also the concept of two dimensional figure. A rectangle is a two-dimensional figure formed by two pairs of ribs, each of which is equal in length and parallel to the pair and has four angles, all of which are the angles of my elbows. The properties of a rectangle can occupy the frame correctly through four ways, having parallel sides that are parallel and equal in length, have diagonals that are equal in length and bisect each other in the same length, and have four right angles [18].

After students master it well, learning about tables and diagrams can begin with the topic of data collection. The collection technique can be done directly or indirectly. Direct data collection can be done by directly collecting data on the object to be captured data. This can be done by gathering data on the immediate environment with students. After students understand the meaning of data collection, students are asked to collect more data. Give students experience to collect data directly, so students find a more effective way. After going through a process of discussion to find a solution effective way, students collecting data using questionnaires or questionnaire. After taking data, students recapitulate the data obtained.

After students read all the collected data, the teacher and students discuss about effective ways to read data more effectively. After organizing the data, students present data in the form of lists, tables, pictogram (pictograms), bar charts, or line charts. Then, students organize the data in the form of a list of tables. The teacher displays several sample tables, the student listens carefully to the parts of the table. Students both individually and in groups are given the task of finding tables in the printed media that have been provided. It aims to get students to experience directly finding tables in real life. Students try to find ways to read tables based on their initial knowledge. Next, students compare reading data directly and read data that has been contained in the table. Students do exercises to read data on the table repeatedly. Then, reading and interpreting are not much different. In interpreting students must be able to find implied data. For example, the highest, lowest, etc. This information is contained in the table but is not explicit.

Data collected either directly or indirectly is transformed into a table. Students make tables based on the knowledge that has been gained in previous learning. Students discuss the results of their work with their classmates. The teacher guides students to formulate steps to make a table. The steps to create a table, namely, create a table title, determine the number of columns and rows, name columns and rows, enter data and re-check all the steps performed. The teacher guides students to solve problems based on open ended steps. Then, students and teachers discuss other alternatives in representing data. In this case the expected data display is more representative to read. Students are directed to organize data and present in the form of pictogram (pictograms). Image diagram is a diagram that uses images or other symbols in stating the amount of data.

The main purpose of bar charts is that data is arranged regularly and displayed correctly, so that everyone can easily read the data [18]. In addition, to presenting in the form of tables, a set of data can also be presented in the form of a diagram. The teacher shows an example bar chart to students. Students analyze the bar chart sections. Then conclude together that the bar chart is a diagram that is described as several rectangles with certain comparisons that correspond to the data in question [18]. Students try to find ways to read bar charts based on their initial knowledge. Students discuss how to read the correct bar chart with their friends. Each group presented how to read bar charts based on the results of group discussions. The teacher and students discuss how to read the correct bar chart. Through question and answer the teacher asks for some of the information contained in the bar chart. After that discuss the parts contained in the bar chart and its functions. Students compare reading data directly and reading data that has been contained in a bar chart. Students do exercises reading data on a bar chart repeatedly.
After that, interpreting the custom contained in the bar chart is done as well as interpreting the data in table form.

Furthermore, the data collected either directly or indirectly is transformed into a table. After that, it is presented in the form of a bar chart. Then condition students to solve problems individually. Then, student solves the problem individually. After that, students present the results of problem solving in front of the class, guide class discussions and guide students to find a variety of correct answers, record other different answers based on the results of the discussion and provide reinforcement on the most efficient way to solve the problem given. The last, students continued organizing and presenting data in the form of line charts.

The learning trajectory consisted of prerequisite topics, core topic and subsequent related topic [16]. The topics included integers and number lines, vertical and horizontal lines, and building a two-dimensional figure, as well as patterns. These topics have to be mastered before the lesson design may be implemented. The core topic for developing the ability to think algebra on table topic and diagrams is the representation of data using tables, pictogram, bar charts, and line charts.

Representation stage was carried out starting from the semi-abstract form followed with the image diagram. The process of transferring one representation to another is combined with problem solving related to the topic. This process is labelled as institutionalization process in didactic situation theory. The related topic thereafter includes pie charts, mean, median, and other statistical introductions. It is important to note that the topic before and after the core topic should be interconnected in the whole learning. Connections that are intertwined between topics form a knowledge that is highly beneficial for students.

4. Conclusion
Learning trajectories are designed as a basis for in the development of algebraic thinking’s didactical design. The learning trajectory is designed consisting of topics that are prerequisites, core topic and subsequent related topic. Learning between topics should be interrelated and relevant, hence the learning trajectory may be arranged comprehensively and meaningfully. Various aspects in assembling learning trajectories may later be developed into a didactic design of algebraic thinking. The learning trajectories designed comprises of prerequisite, core and subsequent related topics.

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