Particle-Particle-String Vertex\textsuperscript{1}

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ABSTRACT

We study a theory of particles interacting with strings. Considering such a theory for Type IIA superstring will give some clue about M-theory. As a first step toward such a theory, we construct the particle-particle-string interaction vertex generalizing the D-particle boundary state.

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1 Introduction

Recent developments in string theory dynamics strongly suggest the existence of M-theory whose low energy effective theory coincides with $D = 11$ supergravity [1]. Knowing what M-theory really is is very important considering its usefulness in various aspects of string dynamics. M-theory might be a theory of some extended objects which yields $D = 11$ supergravity in the low energy limit. Many people expect that the theory of supermembranes [2] will give M-theory but quantization of supermembranes is still very difficult.

Here we would like to take an alternative point of view. In order to do so, let us notice the following facts. M-theory can be realized as the strong coupling limit of Type IIA string theory [3]. In Type IIA superstring theory one can consider BPS saturated point-like objects with R-R charges, which can now be identified with D-particles [4]. The masses of these particles approach zero in the strong coupling limit and the theory looks like an eleven dimensional theory compactified on $S^1$ with large radius. The D-particles can be considered as the Kaluza-Klein modes.

Therefore if one can construct a theory of particles (which have the same quantum numbers as the Kaluza-Klein modes) interacting with Type IIA strings like D-particles do, it may at least yield $D = 11$ supergravity in the low energy limit. Since we are familiar with the perturbation theory of particles and strings, it is easy to quantize it perturbatively and see if the ultraviolet divergences cancel or not. Constructing such a theory will give us at least some clue about M-theory.

In string theory, we always consider Feynmann graphs consisting of one kind of worldsheet which is described by a two dimensional quantum field theory. This is in contrast to particle theory where there are many kinds of particles (e.g. gauge particles, fermions, etc.) and the worldlines of these particles correspond to different quantum mechanics. Therefore it may be possible to generalize string theory so that it includes various kinds of worldsheets or even a theory in which various kinds of strings and particles are interacting with each other. What we would like to do is to construct one of the simplest of such generalizations.

As a first step toward such a theory, we need to construct the vertex describing the interaction of such particles and strings. The most well-known interaction vertices of strings and particles are described by the “vertex operators”. They represent string-string-particle interaction. However the particles which appear in such vertices are included in the spectrum of strings and these are not what we need. The only vertices that are known so far and describe the interaction of strings with something other than strings are given by the D-brane boundary states. They represent D-brane-D-brane-string interactions in the special case where D-branes are classical and at rest. The vertex we need is a generalization of it.

What we will do in this note, is to construct particle-particle-string vertices starting from this D-brane boundary state. We here consider the interaction between bosonic strings and scalar particles for simplicity. It is straightforward to generalize it to
the interaction between superstrings and scalar particles. The interaction between superstrings and higher spin particles in the same supermultiplet may be obtained by using the space-time supersymmetry.

2 Particle-Particle-String Vertex

The first quantized action of a scalar particle is

\[ I = \frac{1}{2} \int dt \left[ \frac{\dot{x}^2}{e} + em^2 \right]. \]  

(1)

Here \( e \) is the einbein on the worldline and \( m \) is the mass of the particle. This gives a scalar field with Klein-Gordon type kinetic term. The vertex we will construct describes interactions between this type of particles and strings.

Before considering the string case, let us notice how scalar particles are coupled with gauge particles. The gauge invariant interaction between a vector particle \( A_\mu \) and the scalar particle is expressed by the Wilson line integral

\[ \int dt A_\mu(x) \dot{x}^\mu, \]  

(2)

over the worldline. This is of course gauge invariant classically.

In quantizing, one usually fixes the reparametrization invariance by putting \( e = 1 \), and \( \dot{x}^\mu \) becomes the momentum \( p^\mu \). Therefore one should be careful about the ordering of the operators in eq.(2). One usually takes the ordering so that the scalar-scalar-vector vertex becomes \( \epsilon \cdot J \equiv \epsilon \cdot (p + k/2) \), where \( p, k \) are as in Fig. 1. \( J_\mu = p_\mu + k_\mu/2 \) can be identified with the current \( i\phi^\dagger \partial_\mu \phi - i\partial_\mu \phi^\dagger \phi \) in the second quantized formalism. \( J_\mu \) corresponds to \( \dot{x}_\mu/e \) in the vertex in eq.(2). If the polarization \( \epsilon^\mu \) of the vector field is proportional to its momentum \( k^\mu \), this vertex yields

\[ k \cdot (p + \frac{k}{2}) = \frac{1}{2} [(p + k)^2 - m^2] - \frac{1}{2} (k^2 - m^2). \]  

(3)

This quantity vanishes if the particles involved are on-shell (i.e. \( (p + k)^2 = k^2 = m^2 \)). If the particles are not on-shell as in the case where the vertex appears in a Feynmann graph, this quantity cancels the scalar particle propagators attached to the vertex. Therefore choosing an appropriate sea gull term, one can show the amplitudes with such a polarization vanish which means they are gauge invariant. Eq.(3) corresponds to the current conservation equation \( \partial_\mu J^\mu = 0 \) in the second-quantized formalism.

Classically, interaction with graviton \( h_{\mu\nu} \) can be described by the vertex

\[ \int dt e h_{\mu\nu}(x) \frac{\dot{x}^\mu \dot{x}^\nu}{e}. \]  

(4)

\(^2e \) appears in this expression to make it reparametrization invariant.
This is gauge invariant up to the equation of motion

$$\partial_t \left(\frac{\dot{x}^\nu}{e}\right) = 0.$$  \hspace{1cm} (5)

The quantum mechanical treatment of this vertex is much more subtle than the vector particle case\cite{5}. The gauge variation of the quantum vertex again yields the inverse propagators of the particles attached to the vertex and can be cancelled by choosing appropriate contact interaction terms.

Since string includes various gauge fields, what we would like to construct is a generalization of eq.\((2)(4)\). We will present it in the following form:

$$\int dt e|V\rangle,$$  \hspace{1cm} (6)

where \(|V\rangle\) is a string state. This vertex represents the couplings between infinitely many particles in the string spectrum with the scalar particle in the way that the vertex of a particle corresponding to the string state \(|i\rangle\) is

$$\int dt e\langle i|V\rangle.$$  \hspace{1cm} (7)

In such a representation, the condition of gauge invariance of the vertex becomes as follows. The vertex of a null external state is

$$\int dt e\langle |Q_B|V\rangle.$$  \hspace{1cm} (8)

Therefore, in order for the vertex to be gauge invariant classically, \(Q_B|V\rangle\) should vanish up to the equations of motion i.e.

$$\partial_t \left(\frac{\dot{x}^\nu}{e}\right) = 0,$$

$$\frac{(e)^2 - \frac{\dot{x}^2}{m^2}}{m^2} = 0.$$  \hspace{1cm} (9)

Roughly speaking, when the scalar particle is quantized, these equations of motion yield terms which may be cancelled by choosing contact interaction terms. In quantizing, we should define the ordering of the operators so that \(Q_B|V\rangle\) is written as a sum of terms proportional to the inverse propagators. Then by choosing appropriate contact interaction terms, the gauge invariance will be shown. We will not pursue these contact interaction terms in this note.

Here we will first construct the classical vertex and then consider it quantum mechanically. We will construct this vertex starting from the following assumptions.

- \(|V\rangle\) depends on the informations of the particles only through \(J_\mu = \frac{\dot{x}^\mu}{e}\).
- For the particle at rest \(|V\rangle\) coincides with the D-particle boundary state.
The first assumption is reasonable because up to the equations of motion $J_\mu$ is the only independent quantity which is reparametrization invariant.

With the above assumptions, it is possible to obtain $|V\rangle$ uniquely up to the equation of motion. In order to do so, let us first rewrite the D-particle boundary state in terms of $J_\mu$. In that case, the particle is at rest and the gauge $t = x^0$ is taken. Therefore

$$\vec{J} = (m, 0, \cdots, 0).$$

The D-particle boundary state is

$$\exp\left[-\sum_{n>0} \frac{1}{n} \alpha_{-n} \tilde{\alpha}_{-n} - \sum_{n>0,i} \frac{1}{n} \alpha_{-n} \tilde{\alpha}_{-n}^i \right] |k\rangle \otimes |B\rangle_{gh},$$

where $|B\rangle_{gh}$ is the ghost boundary state:

$$|B\rangle_{gh} = \exp\left[-\sum_{n>0} (b_{-n} \tilde{c}_{-n} + \tilde{b}_{-n} c_{-n}) \right] (c_0 + \tilde{c}_0) c_1 \tilde{c}_1 |0\rangle_{gh}.$$ (12)

$|k\rangle$ here is the Fock vacuum with momentum $k$, ($k^0 = 0$). We have made a momentum eigenstate out of the usual boundary states with Dirichlet boundary conditions. This boundary state is BRST invariant and represents the interaction vertex of the (classical) particle at rest with a string with momentum $k$. This boundary state can be rewritten in terms of $J_\mu$ in eq.(10) as

$$\exp\left[-2\sum_{n>0} \frac{1}{n} \alpha_{-n} \tilde{\alpha}_{-n} - \sum_{n>0,i} \frac{1}{n} \alpha_{-n} \tilde{\alpha}_{-n}^i \right] |k\rangle \otimes |B\rangle_{gh} + \sum_{n>0} \frac{1}{n} \alpha_{-n} \tilde{\alpha}_{-n} |k\rangle \otimes |B\rangle_{gh} + J \cdot k |\rangle + (J^2 - m^2) |\rangle',$$ (13)

This is the only Lorentz invariant way to do so, up to the equations of motion. The condition $k^0 = 0$ corresponds to $J \cdot k = 0$. This boundary state is BRST invariant for any $J$ satisfying $J \cdot k = 0$, $J^2 = m^2$. Indeed

$$e|V\rangle(J, k),$$

with $e = \sqrt{\frac{\alpha}{m}}$ coincides with the boundary state for the D-particle with constant velocity.

Now let us construct $|V\rangle(J, k)$ with $J, k$ not necessarily satisfying $J \cdot k = 0$, $J^2 = m^2$. Since $|V\rangle(J, k)$ is given as eq.(13) when $J \cdot k = 0$, $J^2 = m^2$, it can be expressed as

$$\exp\left[-2\sum_{n>0} \frac{1}{n} \alpha_{-n} \tilde{\alpha}_{-n} - \sum_{n>0,i} \frac{1}{n} \alpha_{-n} \tilde{\alpha}_{-n}^i \right] |k\rangle \otimes |B\rangle_{gh} + J \cdot k |\rangle + (J^2 - m^2) |\rangle',$$ (15)

using some states $|\rangle$ and $|\rangle'$. However, since $J \cdot k$ and $J^2 - m^2$ are proportional to the equations of motion, eq.(13) holds up to equations of motion for general $J, k$.

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3Here we assume that $|V\rangle(J, k)$ depends on $J, k$ analytically.

4$J \cdot k$ is proportional to the equation of motion $\partial_t (\dot{x}^\mu/e)$ after the integration by parts in eq.(8).
Next thing we should do is to prove that this vertex yields a gauge invariant vertex. \( |V\rangle(J, k) \) in eq. (13) is BRST closed provided \( J \cdot k = 0, \; J^2 = m^2 \). Hence \( Q_B |V\rangle(J, k) \) is written as a sum of terms proportional to \( J \cdot k \) and \( J^2 - m^2 \), i.e. the equations of motion. Therefore \( |V\rangle(J, k) \) in eq. (13) gives the classical particle-particle-string vertex which is gauge invariant.

In order to make the vertex gauge invariant quantum mechanically, we should be careful about the ordering of the operators \( \dot{x}^\mu \) and \( x^\mu \). One way to do so is to substitute \( e = \sqrt{x^2}/m \) and \( \dot{x}_\mu = p_\mu + k_\mu/2 \) into eq. (13) (\( p, k \) are as in Fig. 2):

\[
|V\rangle(J, k) = \exp\left[-\frac{2}{n>0} \frac{1}{n} \alpha_{-n} \cdot (p + k/2) \bar{\alpha}_{-n} \cdot (p + k/2) (p + k/2)^2 + \sum_{n>0} \frac{1}{n} \alpha_{-n} \cdot \bar{\alpha}_{-n}\right]|k\rangle \otimes |B\rangle_{gh}.
\]

This makes \( Q_B |V\rangle(J, k) \) a sum of terms proportional to the inverse propagators of the scalar particles attached to the vertex.

### 3 Discussions

Now that we have constructed the interaction vertex, we can calculate various amplitudes. Of course, we should not forget to add the particle-particle-multi-string vertices to preserve the gauge symmetry. It is an intriguing problem to examine if such amplitudes diverges or not. Since the vertex in eq. (13) is BRST invariant only up to the inverse propagators of the scalar particle, we are not sure if the string miracles of divergence cancellation occur in such amplitudes. Also it is possible for interaction vertices involving only particles to exist. If such vertices exist for higher spin particles, the theory becomes nonrenormalizable in general. Consistency of the theory will fix if such vertices are necessary or not. All these problems will be studied elsewhere.

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Figure Captions

Fig. 1 Scalar-scalar-vector vertex.

Fig. 2 Scalar-scalar-string vertex.