The distribution of mass ratios in compact object binaries

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ABSTRACT

Using the StarTrack population synthesis code we compute the distribution of masses of merging compact object (black hole or neutron star) binaries. The shape of the mass distribution is sensitive to some of the parameters governing the stellar binary evolution. We discuss the possibility of constraining stellar evolution models using mass measurements obtained from the detection of compact object inspiral with the upcoming gravitational-wave observatories.

Keywords: compact objects, binaries, gravitational waves, stellar evolution

1. INTRODUCTION

The operational phase of the gravitational wave detectors LIGO\textsuperscript{1} and VIRGO\textsuperscript{2} is quickly approaching, and in a few years we anticipate the operation of the more sensitive advanced LIGO detector. Mergers of compact objects in binaries are the most promising sources of gravitational waves to be detected by these detectors. The expected rates of such mergers have been a subject of intense research. Recent analyses\textsuperscript{3–5} have shown there is a little chance that LIGO should see any mergers of compact object binaries, while LIGO II could detect a substantial number, up to hundreds or thousands of such events. These estimated rates however, do vary strongly with the model of stellar binary evolution.

Once we begin to detect such events, the data analysis should yield information about each system that merged. The merging of compact object binaries proceeds in three phases: inspiral, merger, and ringdown. It is the inspiral phase that will be used for detection of the stellar masses involved. The analysis of the frequency change with time of the event should yield the masses of the merging objects.\textsuperscript{6} However, determining their type, i.e. a neutron star or a black hole will be difficult\textsuperscript{7} with the measurement during this phase. The modifications of the gravitational wave signal due to the mass distribution of the neutron star are small and may amount to one in $10^4$ oscillations.

The analysis of the observation of merger phase may lead to determination of the type of the compact object i.e. will be different for mergers involving neutron star, and may even lead to constraints on the neutron star equation of state. Also, if the neutron stars are rotating there is a chance of exciting stellar oscillations by the orbital motion which may modify the gravitational waveform.\textsuperscript{8}

It is then important to ask what is the expected two dimensional distribution of masses of compact object binaries that merge, and how does this distribution depend on the assumed model of stellar binary evolution. In §\textsuperscript{2} we present the dependence of the mass ratio distribution on the particular stellar evolution model. Next, §\textsuperscript{3} is devoted to the potential observations. The main observable derived from the gravitational-wave signal is the chirp mass. We show the dependence of the distribution of the observed chirp masses on the stellar evolution parameters and estimate the possibility of distinguishing between the models using the amount of data that we expect to be obtained. Finally in §\textsuperscript{4} we summarize our results.

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Figure 1. The distributions of intrinsic mass ratio in compact objects produced in the standard model simulation (left panel), and the contributions of different types of binaries (BH-BH thick line, NS-NS thin line, and BH-NS dotted line) to this distribution. The distributions have been calculated using a total of 106307 compact object binaries and the bin width is $dq = 0.01$. Each distribution is normalized to unity.

2. THE DISTRIBUTION OF MASS RATIO IN COMPACT OBJECT BINARIES.

In order to calculate theoretical distributions of the mass ratio of compact object merging events we use the StarTrack population synthesis code. The standard evolutionary scenario of the code uses modified formulae of Ref. 9 to evolve single stars along with their proposed wind mass loss rates. The code takes into account different types of binary interactions: dynamically stable and unstable mass transfer phases. The dynamically stable events are modeled using the formalism of Ref. 10, while unstable mass transfers are treated following Ref. 11. Compact object (black holes, neutron stars) formation in supernova (SN) explosion/core collapse events, is described in the code following the results of hydrodynamical calculations of Ref. 12. Neutron stars receive natal kicks from the distribution of Ref. 13; intermediate mass black holes are formed through partial fallback and in attenuated SN explosions receiving smaller kicks than neutron stars; while most massive black holes are formed through direct collapse of their immediate progenitors with no accompanying SN explosion and they do not receive any kicks at their formation.

The distribution of masses in compact object binaries can be described by the probability density $\rho(m_1, m_2)$. The population synthesis code provides a numerical representation of this distribution

$$\rho(m_1, m_2) = \frac{1}{N} \sum_i \delta(m^i_1)\delta(m^i_2)$$

where $\delta$ is the Dirac’s function, $N$ is the number of binaries obtained in a simulation.

We present the mass ratios distributions

$$\frac{dp}{dq} = \int dm_1 \int dm_2 \rho(m_1, m_2)\delta \left(q - \frac{m_1}{m_2}\right) = \frac{1}{N} \sum_i \delta(q_i)$$

of different types of compact object binaries in the right panel of Figure 1, and in the left panel we show the total distribution summed over all the objects. Note that the total distribution is dominated by double NS binaries. They are more numerous than the binaries containing black holes simply because of the steep fall of the initial mass function. The distributions are not flat and exhibit distinctive features which are not numerical artifacts; the statistical fluctuation in the left panel of Figure 1 amount to less than 1%.

The StarTrack code allows to test different models of stellar binary evolution. We have run a set of simulations with different models to show the dependence of the observed distribution of mass ratio on a given evolutionary model. We list the models and their description in Table 1.
Table 1. Description of different population synthesis models for which the distributions of mass ratios have been calculated. For more details see Ref. 3.

| Model | Description |
|-------|-------------|
| A     | standard model described in Ref. 2. |
| B1,4,6,9 | zero kicks, single Maxwellian with $\sigma = 30, 50, 300, \text{km s}^{-1}$, |
| C     | no hyper–critical accretion onto NS/BH in CEs |
| D1–2  | maximum NS mass: $M_{\text{max,NS}} = 2, 1.5 M_\odot$ |
| E1–3  | $\alpha_{CE} \times \lambda = 0.1, 0.5, 2$ |
| F1–2  | mass fraction accreted: $f_a = 0.1, 1$ |
| G1–2  | wind changed by $f_{\text{wind}} = 0.5, 2$ |
| H1–2  | Convective Helium giants: $M_{\text{conv}} = 4.0, 0.0 M_\odot$ |
| I     | burst–like star formation history |
| J     | primary mass: $\propto M_1^{-2.35}$ |
| L1–2  | angular momentum of material lost in MT: $j = 0.5, 2.0$ |
| M1–2  | initial mass ratio distribution: $\Phi(q) \propto q^{-2.7}, q^3$ |
| N     | no helium giant radial evolution |
| O     | partial fall back for 5.0 < $M_{\text{CO}} < 14.0 M_\odot$ |

We present the distributions of the mass ratios for a range of stellar evolutionary models in Figure 2. The distributions vary significantly with the particular choice of the model. The kick velocities (models B) change the relative strength of the peaks in the distribution. These peaks are related to the neutron star in binaries, see Figure 1. Various parameters lead to specific changes in the distribution. In particular several parameters influence the shape of the distribution very significantly: variation of the common envelope efficiency $\alpha_{CE} \lambda$ (models E), changes of the stellar wind strength (models G) etc. The distribution of masses, and consequently mass ratios of compact object binaries does carry significant information about the evolution of the massive binaries. It is therefore important to ask the question: Can we constrain observationally this distribution? Is it possible to learn about the evolution of massive stars by measuring masses of merging binaries?

3. WHAT CAN BE OBSERVED BY GW DETECTORS?

Mergers of compact object binaries will be observed by the gravitational wave interferometers during the inspiral phase, just minutes prior to the merger. The data train will allow us to measure at least the chirp mass:

$$M(m_1, m_2) = (m_1 + m_2) \left( \frac{m_1}{m_2} + \frac{m_2}{m_1} \right)^{-3/5} = m_2(1 + q) (q + q^{-1})^{-3/5}$$

(3)

The individual masses of merging compact objects, $m_1$ and $m_2$, will be measured if higher order correction due to general relativity are taken into account. To find the distribution of the observed chirp masses we need to take into account also the different strength of the gravitational wave signal from sources with different mass.

The strength of the gravitational wave signal from coalescing compact sources has been estimated in Refs 15, 16. In the inspiral phase the signal to noise $S/N$ ratio for LIGO interferometers is proportional to $M^{5/6}$. In particular:

$$\left( \frac{S}{N} \right) = k \left( \frac{1 \text{Gpc}}{D} \right) \left( \frac{4 \mu}{M} \right)^{1/2} \left( \frac{(1 + z)M}{18 M_\odot} \right)^{5/6}$$

(4)

where $D$ is the distance, $M$ is the total mass of the system, $\mu$ is the reduced mass, $k = 0.58$ for the initial LIGO, and $k = 15.3$ for the advanced LIGO configuration. Therefore the sampling distance for a constant signal to noise ratio is proportional to $D \propto (4\mu/M)^{1/2} M^{5/6}$, and the sampling volume $V \propto D^3$. Thus the observed
number of sources observed with given masses $m_1$ and $m_2$ is

$$dN = V(m_1, m_2)\rho(m_1, m_2)dm_1dm_2. \quad (5)$$

and the total number of sources to be observed is $N_{tot} = \int dN$. The normalized distribution of the observed chirp masses is

$$p(M) = \frac{1}{N_{tot}} \frac{dN}{dM} = \frac{1}{N_{tot}} \int dm_1 \int dm_2 V(m_1, m_2)\rho(m_1, m_2)\delta(M - M(m_1, m_2)). \quad (6)$$

The population synthesis code provides us with a numerical representation of the $\rho(m_1, m_2)$ distribution from which we obtain the numerical representation of the distribution of the mass ratios $p(M)$, by inserting Equation 1 into Equation 6, and replacing integrals with sums.
Figure 3. The expected distributions of the observed chirp mass in selected models with varying kick velocity distributions (B1, B4, B6, B9), with different common envelope efficiency $\alpha_{CE}$ (E1, E2, E3) and without the hypercritical accretion onto the compact object (C). In each panel we present for comparison the distribution of model A.
Figure 4. The expected distributions of the observed chirp mass in models D,F,G,H. In each panel we present for comparison the distribution of model A.
Figure 5. The expected distributions of the observed chirp mass in models I, L, M, N, O. In each panel we present for comparison the distribution of model A.
We present the expected distributions of observed chirp masses in Figures 3, 4, and 5. The dependence of the observed distribution of mass ratios on the kick velocity is presented in the top panels of Figure 3. The observed distribution is dominated by the BH-BH mergers, and BHs do not receive substantial kicks, especially for the massive BH systems for which the sampling volume is the largest. However, the contribution of binaries containing neutron stars at $M \approx 1.5 \, M_\odot$ changes their number, see the strength of the peak at $M \approx 1 - 2 \, M_\odot$.

The bottom panels of Figure 3 show the dependence of the observed distribution of chirp masses ratios on the common envelope efficiency parameter $\alpha_{CE\lambda}$. Turning off the hypercritical accretion onto compact sources (model C), is shown in the bottom right panel of Figure 3.

Models D1 and D2 shown in the top panels of Figure 4 serve as self consistency checks of the calculations. In these models the maximum mass of a NS is decreased, and since the character of the gravitational wave in the inspiral phase does not depend on the nature (BH or NS) of a given compact object they should not differ from the standard model A. In models F1 and F2, shown also in Figure 4, we vary the mass fraction accreted onto a compact object in mass transfer events. While the statistic for this models is poor we can clearly say that the observed chirp mass distributions vary strongly. Models G1 and G2, shown in Figure 4 correspond to the increasing and decreasing of the stellar winds by a factor of two. The decreased wind strength shifts the distribution to the higher values and decreases the number of small chirp mass ratio mergers, while increasing the stellar winds makes NS-NS mergers dominate the observed sample, and the entire distribution shifts to the lower values. Models H1 and H2, where we test different values of $M_{\text{cone}}$ of helium giants affect only the properties of the neutron stars and therefore do not strongly change the overall shape of the distribution, save for the peak at $M \approx 1 - 2 \, M_\odot$.

Models L, presented in Figure 5 correspond to variation of the angular momentum fraction lost in mass transfer events, and affect strongly the shape of the observed distribution of chirp masses. Models M, where we vary the shape of the initial mass ratio distribution do not make a significant difference. This is because the compact object binaries are formed from binaries which initially had rather well defined properties (like the initial mass ratio), the number of such binaries does change for different initial mass ratio distributions but their properties do not. Model N where we neglect the helium giant radial evolution affects the properties of neutron stars and does not change significantly the distribution of observed chirp masses. Finally in model O we change the prescription for masses of the newly formed compact objects, which has a strong influence on the shape of the distribution of observed chirp masses.

4. CONSTRAINING STELLAR EVOLUTIONARY PARAMETERS

In this section we discuss a way of quantifying the results presented above. In order to test the usefulness of observations of chirp masses for constraining stellar evolution models we performed the a Monte Carlo simulation of such observations. We have assumed that the underlying stellar population is described by a given model chosen from Table 1. We estimated if one can reject a hypothesis that the stellar population is described by the standard model A with an observation of 20, 100, and 500 mergers. We chose ten thousand random realizations of the observed sample of mergers. Each such realization was compared with the distribution of model A using the Kolmogorov Smirnov (KS) test. We then examined the distribution of the KS-test probabilities and found the probability that appeared in less than 1% of simulations. A high value of this probability means that the given model can not be distinguished from model A, while a low value means that the two models can easily be distinguished. We present the results in Table 2. We first notice that models D do not differ from model A, just as expected. In the case of models that affect the distribution of masses and especially the range of their masses like e.g. models G and O, already a small sample of several tens of mergers will be sufficient to recognize them. With a larger sample of a hundred mergers one should be able to constrain the common envelope parameter $\alpha_{CE\lambda}$ (models E). Also it should be possible to determine the parameters describing mass transfer events: the mass fraction accreted in mass transfer events (models F), and the angular momentum of the material lost in mass transfers. However, a larger sample of mergers will be required to constrain the kick velocity distribution: even with 500 detections it is going to be possible to put only rough constraints on the width of the kick velocity distribution.
Table 2. Results of the simulations of observation of 20, 100 and 500 mergers. We list KS-test probabilities that appeared in less than 1% of tries. The lower the probability the easier can a given model be distinguished.

| Model | 20 mergers | 100 mergers | 500 mergers |
|-------|------------|-------------|-------------|
| B1    | 0.943      | 0.169       | 2.19 × 10⁻³ |
| B2    | 0.945      | 0.261       | 3.56 × 10⁻⁹ |
| B3    | 0.966      | 0.483       | 1.19 × 10⁻⁶ |
| B4    | 0.956      | 0.523       | 1.11 × 10⁻⁶ |
| B5    | 0.971      | 0.464       | 4.44 × 10⁻⁵ |
| B6    | 0.980      | 0.327       | 9.20 × 0⁻⁷  |
| B7    | 0.990      | 0.891       | 5.28 × 10⁻² |
| B8    | 0.991      | 0.977       | 0.625       |
| B9    | 0.991      | 0.989       | 0.823       |
| B10   | 0.988      | 0.886       | 0.138       |
| B11   | 0.984      | 0.580       | 2.45 × 10⁻⁵ |
| B12   | 0.973      | 0.286       | 1.77 × 10⁻⁶ |
| B13   | 0.995      | 0.990       | 0.870       |
| C     | 0.966      | 0.135       | 1.48 × 10⁻⁶ |
| D1    | 0.992      | 0.993       | 0.750       |
| D2    | 0.993      | 0.987       | 0.909       |
| E1    | 0.010      | 1.0 × 10⁻¹² | 0.0        |
| E2    | 0.990      | 0.8248529   | 7.25 × 10⁻³ |
| E3    | 0.915      | 0.0236      | 1.71 × 10⁻¹⁴ |
| F1    | 0.148      | 1.15 × 10⁻⁹ | 0.0        |
| F2    | 0.401      | 1.07 × 10⁻⁴ | 3.25 × 10⁻²³ |
| G1    | 0.001      | 6.64 × 10⁻²⁶ | 0.0     |
| G2    | 1.42 × 10⁻¹⁷ | 0.0       | 0.0         |
| H1    | 0.996      | 0.9941702   | 0.990      |
| H2    | 0.990      | 0.9725315   | 0.420      |
| I     | 0.989      | 0.9746811   | 0.592      |
| J     | 0.992      | 0.9898050   | 0.953      |
| L1    | 0.461      | 5.59 × 10⁻⁵ | 2.69 × 10⁻²⁷ |
| L2    | 0.188      | 4.95 × 10⁻⁹ | 0.0        |
| M1    | 0.995      | 0.9549476   | 0.111      |
| M2    | 0.985      | 0.9328853   | 0.405      |
| N     | 0.995      | 0.9818770   | 0.365      |
| O     | 6.16 × 10⁻¹² | 0.00      | 0.0        |

5. CONCLUSIONS

Using the StarTrack population synthesis code we show that the distribution of masses of merging compact objects carries useful information about the ways massive stars evolve. In particular the stellar wind strength, common envelope efficiency, loss of angular momentum efficiency, and the fraction of mass accreted in mass transfer events leave distinct tracks on the distribution of observed chirp masses. The Monte Carlo simulations show that analysis of this distribution shall yield useful constraints on the evolutionary parameters of binaries containing massive stars. The advanced LIGO configuration will be able to gather sufficient number of merging events to perform such analysis.
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