Generation of noncircular gears for variable motion of the crank-slider mechanism

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Abstract. The paper proposes a modified kinematics for the crank-slider mechanism of a nails machine. The variable rotational motion of the driven gear allows to slow down the velocity of the slider in the head forming phase and increases the period for the forming forces to be applied, improving the quality of the final product. The noncircular gears are designed based on a hybrid function for the gear transmission ratio whose parameters enable multiple variations of the noncircular driven gears and crank-slider mechanism kinematics, respectively. The AutoCAD graphical and programming facilities are used (i) to analyse and optimize the slider-crank mechanism output functions, in correlation with the predefined noncircular gears transmission ratio, (ii) to generate the noncircular centrodes using the kinematics hypothesis, (iii) to generate the variable geometry of the gear teeth profiles, based on the rolling method, and (iv) to produce the gears solid virtual models. The study highlights the benefits/limits that the noncircular gears transmission ratio defining hybrid functions have on both crank-slider mechanism kinematics and gears geometry.

1. Introduction

Compared to linkages, noncircular gear wheels provides a number of design advantages such as accurate transmission, ease of balancing and compact size. Furthermore, they are quite versatile due to the great flexibility of the desired transmission function chosen [1]. Their diffusion, however, is still limited, since, with the exception of elliptical gears, only recent CAD-CAM technologies allow the implementation of a fully integrated design and manufacturing process [2].

Non-uniform rotation mechanism are required in many applications: textile industry, packaging machines, quick-return mechanisms, pumps, flow meters, flying shear and instruments. As regards the new applications that have been reported, Emura et. al. proposed a new steering mechanism using noncircular gears [3]. This mechanism has the capability of turning a carrier with a small radius. Johnson designed a pedal powered gear transmission mechanism applicable to velocipedes, exercise machines or winches [4]. The resulting change in mechanical advantage partially compensates for the inherent disadvantage at and near dead centre pedal positions. Sugiyama used a pair of noncircular gears interposed between the crank-shaft and the ring gear of an engine [5]. The variable speed of the second noncircular gear can cancel the change in torque of the crankshaft, so that it is not transmitted to the starter engine. As a result, the starter operates at constant current all times.

The modified crank-slider mechanism has been subject of few applications. Mundo et. al. propose a modified crank-slider mechanism of a pressing machine, whose ram is driven, according to an optimized law motion, by a pair of noncircular gears [2]. At first, the pitch curves synthesis is
performed by means of an inverse kinematical analysis of the system. The teeth profiles are then generated by assuming the pressure angle to be constant for each tooth. Quintero et. al. present a novel modified crank-slider mechanism of an internal combustion engine, by introducing a noncircular gear pair [1]. The noncircular tooth bodies enable to adjust the piston speed throughout the entire cycle, so that the performance of the engine can be improved. Yokoyahama et.al. use a pair of non-circular gear, as transmission gears secured between a power shaft and a crank shaft at a powder compacting press [6]. The purpose is to get a long powder feeding time and possibly to have a longer time for compacting.

The paper is focused on the design of a noncircular gear pair that, attached to the crank-slider mechanism of the MCC337 nails machine, would optimize its kinematics. For this purpose:

(i) a modified kinematics of the slider is proposed, in order to improve the nail head forming process. The modified kinematics consists in a reduced velocity during the nail head forming phase and a longer time for the forming forces to be applied, compared to classical nails machines. These enable to improve the quality of the final product, as too much work-hardening of the nail head is avoided, and high capacity, high quality, production of nails with enlarged head, long tooling life, process stability and low-noise production are expected;

(ii) the slider kinematics is defined in correlation with the output motion of a noncircular gear pair. To enable an optimization of the mechanisms kinematics, the noncircular gear transmission ratio is defined as a hybrid trigonometric function with multiple parameters whose variations obey both the noncircular gear design and the nail-machine technological requirements;

(iii) the generation of the noncircular gears that provide the desire variable motion to be applied to the nail machine crank-slider mechanism is developed by analytical procedures. The noncircular gears centrodes are modelled using the basic algorithm that includes the gear transmission ratio as initial data [7] and the generation of the gears teeth profiles is based on the rolling method [7], [8].

2. Modified kinematics of the crank-slider mechanism
In case of a conventional crank-slider mechanism, the slider uniform translational motion output parameters, i.e. the law of motion and the slider kinematics, are related to the crank rotational angle. The instantaneous position of the slider could be defined as follows:

$$s(\varphi_2) = -r \cdot \cos \varphi_2 + \sqrt{l^2 - r^2 \cdot \sin^2 \varphi_2}$$

where $r$, $l$ are the crank and the connection rod lengths, respectively; $\varphi_2$ – the crank rotational angle.

![Figure 1. Conventional crank-slider mechanism geometry.](image)

If coupled to a spur gear trains (figure 1), with a constant transmission ratio equal to one, the crank rotational angle, as the driven noncircular gear rotational angle, could be replaced by the pinion rotational angle, as input data. The displacement and the kinematics of the slider, related to the spur driving gear uniform rotational motion, are described by symmetric functions (figure 2).
Influence of the noncircular gears transmission ratio on the crank-slider mechanism kinematics

Attempts to modify these functions, as industrial applications often require, are frequently based on electrical devices, but also mechanisms, as noncircular gears, are mentioned in literature [1], [2], [9], [10], [11]. In this paper, the authors want to achieve a similar law of output motion as that proposed by Doege [11] and Mundo [2] for the driving mechanism of pressing machines, also using noncircular gears. Therefore, the authors introduce a hybrid multiparameter function to define the noncircular gear transmission ratio, that enables the variation of the output crank speed to be chosen and correlation to the nail machine kinematics to be made, allowing a more flexible way to describe the mechanism variable kinematics. Dividing the period \( T = 2\pi \) of the crank and driven gear rotational motions, respectively, into a working phase and a free-running phase, by means of a variable \( \phi_0 \) angle, the gear transmission ratio is defined by the authors, as follows:

\[
m_{21}(\phi_1) = \begin{cases} 
0.5 \cdot (a + b) - 0.5 \cdot (b - a) \cdot \cos(\pi \phi_1/\phi_0), & \phi_1 \in [0, \phi_0] \\
0.5 \cdot (a + b) + 0.5 \cdot (b - a) \cdot \cos[\pi(\phi_1 - \phi_0)/(2\pi - \phi_0)], & \phi_1 \in (\phi_0, 2\pi] 
\end{cases}
\]

(2)

where \( a, b \) are the minimum and maximum values of the gear transmission ratio, respectively; \( \phi_1 \) - the driving gear rotational angle. The above expression was chosen in order to ensure a positive, continuous, derivative and periodical function for the gear transmission ratio, as required by the theory of noncircular gears. The driven gear rotational angle:

\[
\phi_2 = \int_0^{\phi_0} m_{21}(\phi)d\phi
\]

(3)

is also a positive, continuous, derivative and monotonically increasing function, as requested. To model closed centroides, it is necessary that \( \phi_2(2\pi) = 2\pi \); from equations (2, 3), this results in:

\[
a + b = 2
\]

(4)

condition that limits the noncircular gear transmission ratio variation.

As a consequence, the variation of the noncircular gears transmission ratio depends on two parameters, the functioning cycle dividing angle, \( \phi_0 \), and one of the extreme value of the transmission ratio; while its minimum value, \( a \), is important for the working phase, it is chosen as input data. Figure 3 illustrates the variation of the noncircular gears transmission ratio, curves that are similar to those of Doege and Mundo [11, 2]. As shown in figure 3a, as the functioning cycle dividing angle is reduced, the gears transmission ratio highly decreases, with similar consequence on the crank-slider mechanism kinematics - not a desirable modification; as the minimum value of the transmission ratio is reduced (figure 3b), the limits of its variation are highly enlarged at high rate, too.
The main objective of the paper is to analyze the influence of the variable motion of the noncircular driven gear and crank, respectively, on the slider kinematics. Using equations (1) and (3), the displacement and the relative speed of the slider vary as illustrated in figures 4 and 5. It can be noticed that reducing the working cycle dividing angle (figures 4a, 5a), the working phase lasts longer than the conventional one (half of a motion period) and the speed decreases in a variable manner. Reducing the minimum transmission ratio (figures 4b, 5b), the working phase is also prolonged and important variations in the slider relative speed are recorded, with higher values at the beginning and at the end of the functioning cycle.
2.2. Comparative analysis of the slider kinematics
Considering the influence of the variable rotational motion of the noncircular driven gear/crank on the slider kinematics (figures 4, 5), it was concluded that, in the attempt of the slider velocity modification by a pattern not so different from the conventional one, but assuring the best conditions for the nail head forming process that starts at the end of the slider working phase, the functioning cycles should be divided by an angle $\varphi_0$ close to the $\pi$ value and the minimum transmission ratio of the noncircular gears should be not chosen at reduced values. Initial design data as $\varphi_0 = 8\pi/9$ and $a = 0.4$ is considered to be the one that provides a low velocity, with a reduced variation that lasts as it is necessary for the nail head forming process; for the other sequences of the functioning cycle, the velocity of the slider is higher than for the conventional case, but this is not regarded as a limit/disadvantage.

Figures 6 and 7 illustrate a comparison made for the kinematics of a $\phi 4 \times 50$ nails machine type, for a $\phi 13$ mm head diameter. It takes into account the conventional kinematics of the crank-slider mechanism versus the modified kinematics, introduced by a noncircular gears train with the transmission ratio defined by equation (2), with the following design parameters: $\varphi_0 = 8\pi/9$, and $a = 0.4$.

![Figure 6: The conventional and the optimized law of the slider displacement.](image1)

![Figure 7: The conventional and the optimized law of the slider relative velocity.](image2)

During the nails head forming phase, the slider is translated along the distance $s' = 16$ mm (figure 6); this is happening for a rotational angle of the crank of $d\varphi_1' = 22^\circ$ in case of the conventional machine and $d\varphi_1'' = 50, 5^\circ$ in case of a nail machine with modified kinematics, respectively. It results in a reduced forming relative velocity from $v' = ds'/d\varphi_1' = 0.31$ mm/rad to $v'' = ds'/d\varphi_1'' = 0.12$ mm/rad.
and in an important decrease of the relative acceleration, from \( \frac{d^2v}{d\phi_1} = 0.807 \, \text{mm/rad}^2 \) to \( \frac{d^2v}{d\phi_1} = 0.136 \, \text{mm/rad}^2 \), with benefits of the nail head forming process, as mentioned in introduction.

3. Non-circular gears generation
The main steps required within the noncircular gears generation process is (i) the modelling of the mating noncircular centrodes/pitch curves and (ii) the teeth generation.

3.1. Noncircular pitch curves modelling
Defining the gears center distance, \( D \), the gears pitch curves that assure the transmission ratio defined by equation (2), have the following polar equations \[8\]:

\[
r_1(\phi_1) = \frac{D}{1 + m_{21}(\phi_1)} \quad (5)
\]

\[
r_2(\phi_2(\phi_1)) = D \frac{m_{21}(\phi_1)}{1 + m_{21}(\phi_1)} \quad (6)
\]

where \( \phi_1 \) is the driving centrode polar angle, uniformly increased within the limits \([0...2\pi]\); \( \phi_2 \) – the driven centrode polar angle, varying in accordance with equations (3), (2).

Figure 8 illustrates the noncircular pitch curves modelled as required by the above considered application, for the initial data: the functioning cycle diving angle \( \psi_0 = 8\pi/9 \), the variation of the transmission ratio is defined by the equation (2), within the limits \( a = 0.4 \) and \( b = 1.6 \), and the gear center distance is \( D = 174 \, \text{mm} \). The pitch curves generation is developed in AutoCAD environment, using an AutoLISP code.

![Figure 8](image)

**Figure 8.** Pitch curves of noncircular gears for the nail machine.

As shown in figure 8, the chosen initial data for the noncircular gears generation confirms a favourable geometry for the pitch curves; generally convex arcs are composing the pitch curves, there is just a small part of the driving centrode that exhibits a concave shape, with a permissive curvature as regards the possibility of the further tooth flanks generation.

3.2. Gears teeth generation
For the gear teeth flanks generation, an analytical procedure is applied considering the local geometry of the noncircular pitch curve and the profile of a “single tooth rack cutter” that rolls over the pitch curve. The tooth flank is defined as the set of the intersection points between the current line of action and the rack tooth flank.

Figure 9 illustrates the geometry and kinematics of an arbitrary driving gear tooth operating flank generation. The reference positioning point for the tooth flank is point \( P_i \) on the pitch curve, defined by
the polar angle \( \varphi \) and the radius \( r(\varphi) \), respectively (figure 9a). For the noncircular pitch curve, the local tangent line (tg) is considered, inclined by \( \mu \) angle with respect to the local polar radius and the line of action (la), inclined by the constant standard \( \alpha \) angle relative to the tangent line. The first point of the tooth profile is the point \( F_i \), as the intersection point between the line of action and the operating rack cutter tooth flank; in this particular position, \( F_i = P_i \).

To generate the tooth operating flank, all the motions required by the pure rolling are transferred to the rack tooth, as follows: it is moved from \( P_i \) to current point \( P_{ij} \), it is rotated around \( P_{ij} \), to align its pitch line with the current tangent (tg) \( ij \), and it is translated along the tangent, by the rolling distance \( s_{ij} \) equal to the length of the current pitch arc \( P_iP_{ij} \). The point of the gear tooth flank is picked up at the intersection of the current line of action (la) \( ij \) and the rack tooth operating flank. Figure 9b illustrates the generation of the gear tooth flank at addendum zone and enables to express the coordinates of the tooth flank generated points, related to the fixed coordinate system, \( O_1x_1y_1 \):

\[
x_{1ij} = r_{1ij}(\varphi_{1ij}) \cdot \cos \varphi_{2ij} \pm s_{ij} \cdot \cos \alpha \cdot \cos(\mu_{ij} + \alpha + \varphi_{2ij})
\]

\[
y_{1ij} = r_{1ij}(\varphi_{1ij}) \cdot \sin \varphi_{2ij} \pm s_{ij} \cdot \cos \alpha \cdot \sin(\mu_{ij} + \alpha + \varphi_{2ij})
\]

where \( r_{1ij} \), \( \varphi_{1ij} \) are the polar coordinates of the instantaneous center of rotation \( P_{ij} \), \( s_{ij} \) – the rolling distance along the current tangent line (tg) \( ij \), \( \mu_{ij} \) – the angle of the current tangent relative to the positioning vector \( O_1P_{ij} \); \( \alpha \) - the standard pressure angle (20°); (-) sign is for the gear tooth flank addendum points and (+) sign is for addendum zone points, when the rolling is counter clockwise performed [8].

Similar equations are written for the driving gear tooth opposite flank points. Based on the driving gear tooth flanks profiles, the driven gear tooth conjugate profiles are analytically expressed considering the gears meshing [8]. Using the coordinate transformation, the driven gear conjugate tooth is expressed, in its fixed coordinate system, as:

\[
\begin{bmatrix}
    x_{2ij} \\
    y_{2ij}
\end{bmatrix} =
\begin{bmatrix}
    \cos \varphi_{2ij} & \sin \varphi_{2ij} \\
    -\sin \varphi_{2ij} & \cos \varphi_{2ij}
\end{bmatrix}
\begin{bmatrix}
    -D \\
    0
\end{bmatrix}
+ \begin{bmatrix}
    \cos \varphi_{1ij} & \sin \varphi_{1ij} \\
    -\sin \varphi_{1ij} & \cos \varphi_{1ij}
\end{bmatrix}
\begin{bmatrix}
    x_{1ij} \\
    y_{1ij}
\end{bmatrix}
\]

(8)

where \( \varphi_{2ij} \) is the driven gear corresponding rotational angle, calculated by equation (3).

An AutoLISP code automatically generates the non-circular gears tooth flanks profiles within the predefined rolling angles/distances \( s_{ij} \). Further editing operations are completing the noncircular driving gear section and enables the gears solid models to be produced (figure 10).
4. Conclusions
The paper presents a modified kinematics of the crank-slider mechanism of a classical nails machine for the purpose of improving the nail head forming phase; the modified kinematics is achieved from a noncircular gear train whose transmission ratio is defined as a hybrid function with multiple parameters, in order to increase the flexibility of the gears design and to consider the nail machine technological requirements. At first, the influence of the transmission ratio parameters on the slider displacement and relative velocity have been studied in order to optimize the slider kinematics; these optimal parameters were introduced into the further noncircular gear design, i.e the pitch curves and tooth flanks profiles, respectively. Original AutoLISP codes enabled the slider kinematics to be analyzed and the noncircular gears to be generated in AutoCAD environment. Further studies will introduce new modifications for the slider kinematics in order to minimize the slider speed variation, specific to the functioning cycle free-running phase.

5. References
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