Erratum to “Measurement of the $\pi^-\pi^+$ cross section between 600 and 900 MeV using initial state radiation”
R. X. Yang, S. L. Yang, S. L. Yang, Y. H. Yang, Y. X. Yang, Yifan Yang, Zhi Yang, M. Ye, M. H. Ye, J. H. Yin, Z. Y. You, B. X. Yu, C. X. Yu, G. Yu, J. S. Yu, T. Yu, C. Z. Yuan, L. Yuan, W. Yuan, X. Q. Yuan, Z. Y. Yuan, C. X. Yue, A. Yuncu, A. A. Zafar, Y. Zeng, B. X. Zhang, Guangyi Zhang, H. Zhang, H. H. Zhang, H. Y. Zhang, J. J. Zhang, J. L. Zhang, J. Q. Zhang, J. W. Zhang, J. Y. Zhang, J. Z. Zhang, Jiayu Zhang, Jiawei Zhang, Lei Zhang, S. Zhang, Shulei Zhang, X. D. Zhang, Y. Zhang, Y. H. Zhang, Y. T. Zhang, Yan Zhang, Yao Zhang, Yi Zhang, Z. H. Zhang, Z. Y. Zhang, G. Zhao, J. Zhao, J. Y. Zhao, J. Z. Zhao, Lei Zhao, Ling Zhao, M. G. Zhao, Q. Zhao, S. J. Zhao, Y. B. Zhao, Y. X. Zhao, Z. G. Zhao, Zhemchugov, B. Zheng, J. P. Zheng, Y. Zheng, Y. H. Zheng, B. Zhong, C. Zhong, L. P. Zhou, Q. Zhou, X. Zhou, X. K. Zhou, X. R. Zhou, A. N. Zhu, J. Zhu, K. Zhu, K. J. Zhu, S. H. Zhu, T. J. Zhu, W. J. Zhu, X. L. Zhu, Y. C. Zhu, Z. A. Zhu, B. S. Zou, J. H. Zou

(BESIII Collaboration)

1 Institute of High Energy Physics, Beijing 100049, People’s Republic of China
2 Beihang University, Beijing 100191, People’s Republic of China
3 Beijing Institute of Petrochemical Technology, Beijing 102617, People’s Republic of China
4 Bochum Ruhr-University, D-44780 Bochum, Germany
5 Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA
6 Central China Normal University, Wuhan 430079, People’s Republic of China
7 China Center of Advanced Science and Technology, Beijing 100190, People’s Republic of China
8 COMSATS University Islamabad, Lahore Campus, Defence Road, Off Raiwind Road, 54000 Lahore, Pakistan
9 Fudan University, Shanghai 200443, People’s Republic of China
10 G.I. Budker Institute of Nuclear Physics SB RAS (BINP), Novosibirsk 630090, Russia
11 GSI Helmholtzcentre for Heavy Ion Research GmbH, D-64291 Darmstadt, Germany
12 Guangxi Normal University, Guilin 541004, People’s Republic of China
13 Guangxi University, Nanning 530004, People’s Republic of China
14 Hangzhou Normal University, Hangzhou 310036, People’s Republic of China
15 Helmholtz Institute Mainz, Johann-Joachim-Becher-Weg 45, D-55099 Mainz, Germany
16 Henan Normal University, Xinxiang 453007, People’s Republic of China
17 Henan University of Science and Technology, Luoyang 471003, People’s Republic of China
18 Huangshan College, Huangshan 245000, People’s Republic of China
19 Hunan Normal University, Changsha 410081, People’s Republic of China
20 Hunan University, Changsha 410082, People’s Republic of China
21 Indian Institute of Technology Madras, Chennai 600036, India
22 Indiana University, Bloomington, Indiana 47405, USA
23 INFN Laboratori Nazionali di Frascati, (A)INFN Laboratori Nazionali di Frascati, I-00044, Frascati, Italy
24 INFN Sezione di Perugia, I-06100, Perugia, Italy
25 INFN Sezione di Ferrara, INFN Sezione di Ferrara, I-44122, Ferrara, Italy
26 Institute of Modern Physics, Lanzhou 730000, People’s Republic of China
27 Institute of Physics and Technology, Peace Ave. 54B, Ulubambar, 13330, Mongolia
28 Johannes Gutenberg University of Mainz, Johann-Joachim-Becher-Weg 45, D-55099 Mainz, Germany
29 Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia
30 Justus-Liebig-Universitaet Giessen, II. Physikalisches Institut, Heinrich-Buff-Ring 16, D-35392 Giessen, Germany
31 KVI-CART, University of Groningen, NL-9747 AA Groningen, The Netherlands
32 Lanzhou University, Lanzhou 730000, People’s Republic of China
33 Liaoning Normal University, Dalian 116029, People’s Republic of China
34 Liaoning University, Shenyang 110036, People’s Republic of China
35 Nanjing Normal University, Nanjing 210023, People’s Republic of China
36 Nanjing University, Nanjing 210093, People’s Republic of China
37 Nankai University, Tianjin 300071, People’s Republic of China
38 North China Electric Power University, Beijing 102206, People’s Republic of China
39 Peking University, Beijing 100871, People’s Republic of China
40 Qufu Normal University, Qufu 273165, People’s Republic of China

(2)
Abstract
In Ref. [1] the BESIII collaboration published a cross section measurement of the process $e^+e^- \rightarrow \pi^+\pi^-$ in the energy range between 600 and 900 MeV. In this erratum we report a corrected evaluation of the statistical errors in terms of a fully propagated covariance matrix. The correction also yields a reduced statistical uncertainty for the hadronic vacuum polarization contribution to the anomalous magnetic moment of the muon, which now reads as $a_{\mu}^{\pi\pi,LO}(600 - 900 \text{ MeV}) = (368.2 \pm 1.5_{\text{stat}} \pm 3.3_{\text{syst}}) \times 10^{-10}$. The central values of the cross section measurement and of $a_{\mu}^{\pi\pi,LO}$, as well as the systematic uncertainties remain unchanged.

Keywords: Hadronic cross section, Muon anomaly, Initial state radiation, Pion form factor, Covariance matrix, BEPCII, BESIII

1. Introduction
Previously, we reported [1] a measurement of the cross section $\sigma_{\text{bare}}(e^+e^- \rightarrow \pi^+\pi^-)$ and the pion form factor $|F_{\pi}|^2$ in the energy range between 600 MeV and 900 MeV. As pointed out in Refs. [2] and [3], there exists a difference between the statistical uncertainties of the tabulated cross section of Ref. [1] and the covariance matrix which is documented as a supplemental material to the publication. Furthermore, when including the covariance matrix, it is not possible to reproduce the fit of the form factor presented in Ref. [1].

In scrutinizing the published analysis, we realized that the covariance matrix had not been properly propagated through the final analysis procedure. In this work, the statistical uncertainties are reevaluated and an updated value of the uncertainty of the two-pion contribution to the hadronic vacuum polarization contribution of the anomalous magnetic moment of the muon, $a_{\mu}^{\pi\pi,LO}$, is calculated.

2. Reevaluation of the Statistical Covariance Matrix
The covariance matrix results from the unfolding procedure, which is applied at the level of the event yield to compensate for mass resolution effects of the detector. The underlying algorithm of the procedure is based on singular value decomposition [4]. In an initial state radiation (ISR) measurement, the dressed cross section $\sigma_{\text{dressed}}(e^+e^- \rightarrow \pi^+\pi^-)$ is calculated from the unfolded event yield $N_{\text{unf}}$ of $\pi^+\pi^-\gamma_{\text{ISR}}$ events according to

$$\sigma_{\text{dressed}}(e^+e^- \rightarrow \pi^+\pi^-) = \frac{N_{\text{unf}}}{\epsilon_{\pi\pi}\cdot L_{\text{int}} \cdot H(s,s') \cdot (1 + \delta_{\pi\pi\gamma \text{FSR}})}, \quad (1)$$

where $\epsilon_{\pi\pi}$ is the reconstruction efficiency, $L_{\text{int}}$ is the integrated luminosity, and $H(s,s')$ is the radiator function, where the implementation of Ref. [5] is considered. The correction $(1 + \delta_{\pi\pi\gamma \text{FSR}})$ denotes the final state radiation (FSR) corrections on the level of radiative $\pi^+\pi^-\gamma$ events [6].

The bare cross section is obtained from the dressed cross section by applying mass-dependent corrections for vacuum polarization $\delta_{\text{VP}}$ [6] and by adding back effects of FSR on the level of the non-radiative $\pi^+\pi^-$ cross sections as parametrized within scalar QED in the Schwinger term $1 + \eta(s')\frac{\alpha}{\pi}$ [7]. The final formula for the bare cross section reads as:

$$\sigma_{\text{bare}}(e^+e^- \rightarrow \pi^+\pi^- (\gamma_{\text{FSR}})) = \sigma_{\text{dressed}}(e^+e^- \rightarrow \pi^+\pi^-) \frac{1 + \eta(s')\frac{\alpha}{\pi}}{\delta_{\text{VP}}(s')}, \quad (2)$$

where $s'$ denotes the two-pion invariant mass squared.

Since all the above mentioned values remain unchanged compared to the original work [1], the central value of the cross section does not change.

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1In Eq. 1 of Ref. [1], the factor $(1 + \delta_{\pi\pi\gamma})$ should be read as $\left[1 + \frac{1 + \eta(s')\frac{\alpha}{\pi}}{1 + \delta_{\text{FSR}}(s')}\right]$, contrary to the description in Section 6.3 therein.
The covariance matrix of the bare cross section is given by

\[
C^{\text{bare}} = \sum_{k \in \{N, e, \mathcal{L}_{\text{int}}, H, (1+\delta_{\text{FSR}})\}} (J^T)^k C^k J^k,
\]

with \( J^k_{ij} = \frac{\partial \sigma_{\text{bare}}}{\partial \delta_{ij}} \) being the Jacobian matrix of the bare cross section with respect to the contribution \( k \) to the statistical uncertainty, according to generalized Gaussian error propagation. Here, it is assumed that the contributing quantities are not correlated.

Since the time integrated luminosity is a single scalar value, its covariance matrix is simply given by the squared statistical uncertainty of the time integrated luminosity: \( C^{\mathcal{L}_{\text{int}}} = (\Delta \mathcal{L}_{\text{int}})^2 \).

It is assumed that the reconstruction efficiency, the time integrated luminosity, the radiator function, as well as the final state radiation correction term are completely uncorrelated. The respective diagonal elements of the covariance matrices are given by the square of the uncertainties. The contribution of the Schwinger correction term is neglected, since as a QED calculation, it is assumed to be exact. In the original work, the uncertainty of the vacuum polarization effect is considered to be purely systematic. Hence, it is also neglected in the calculation of the statistical covariance matrix. Consequently, the covariance matrix of the bare cross section has the form:

\[
C^{\text{bare}}_{ij} = \frac{C^{\mathcal{L}_{\text{int}}}}{L_{\text{int}}^2} \cdot \left( \frac{1 + \eta(s')\frac{\pi}{2}}{\pi_{\text{global}} \cdot H(s, s') \cdot (1 + \delta_{\text{FSR}})} \right)^2 \cdot \left( \frac{1 + \eta(s')\frac{\pi}{2}}{\pi_{\text{global}} \cdot H(s, s') \cdot (1 + \delta_{\text{FSR}})} \cdot \delta_{ij} \right)
\]

In the original publication, the error propagation of the covariance matrix had not been carried out properly. As a result, the statistical uncertainties of the published cross section do not reflect the information of the unfolding. Figure 1 shows a comparison of the relative statistical errors of the bare cross sections calculated as the diagonal uncertainties of this work (red crosses) and the uncertainties published in Ref. [1] (black circles). The values of the diagonal errors are listed in Tab. 1.

![Figure 1](image_url)

Figure 1: (Color online) Relative uncertainty of the bare cross section \( \sigma_{\text{bare}}(e^+e^+ \rightarrow \pi^+\pi^- (\gamma_{\text{FSR}})) \) of this work (red crosses) compared to the results of Ref. [1] (black circles). The uncertainties of the cross section of this work are the square roots of the diagonal elements of the matrix.

It must be stressed that only the statistical uncertainties of the measurements of \( \sigma_{\text{bare}}(e^+e^+ \rightarrow \pi^+\pi^- (\gamma_{\text{FSR}})) \) and of \( |F_{\pi}|^2 \) have been reevaluated. Thus, the systematic uncertainty of 0.9% evaluated in Ref. [1] is unchanged.
The BESIII collaboration has approved new data taking at 3.773 GeV in 2021-2022, aiming at a total data set of 20 fb$^{-1}$\cite{ref:BESIII}. In addition to a significant reduction of the statistical uncertainty, the new data will also allow for the alternative normalization scheme for $\sigma^{\text{bare}}(e^+e^+ \to \pi^+\pi^- (\gamma_{\text{FSR}}))$, discussed in Eq. 3 of Ref. \cite{ref:JLAB}, in which the dominating systematic uncertainties cancel. A total uncertainty of 0.6% can be expected.

Table 1: Results for the bare cross section $\sigma^{\text{bare}}$ and the pion form factor together with their statistical uncertainties. The systematical uncertainties are given by 0.9%\cite{ref:JLAB}.

| $\sqrt{s}$ [MeV] | $\sigma^{\text{bare}}_{\pi^+\pi^- (\gamma_{\text{FSR}})}$ [nb] | $|F_\pi|^2$ | $\sqrt{s}$ [MeV] | $\sigma^{\text{bare}}_{\pi^+\pi^- (\gamma_{\text{FSR}})}$ | $|F_\pi|^2$ |
|-----------------|---------------------------------|-----------|-----------------|---------------------------------|-----------|
| 602.5           | 288.3 ± 11.4                    | 6.9 ± 0.3 | 752.5           | 1276.1 ± 31.2                   | 41.8 ± 1.0 |
| 607.5           | 306.6 ± 10.8                    | 7.4 ± 0.3 | 757.5           | 1315.9 ± 31.4                   | 43.6 ± 1.0 |
| 612.5           | 332.8 ± 11.8                    | 8.2 ± 0.3 | 762.5           | 1339.3 ± 29.0                   | 44.8 ± 1.0 |
| 617.5           | 352.5 ± 12.4                    | 8.7 ± 0.3 | 767.5           | 1331.9 ± 30.0                   | 45.0 ± 1.0 |
| 622.5           | 367.7 ± 12.1                    | 9.2 ± 0.3 | 772.5           | 1327.0 ± 29.6                   | 45.2 ± 1.0 |
| 627.5           | 390.1 ± 12.7                    | 9.8 ± 0.3 | 777.5           | 1272.7 ± 28.3                   | 43.7 ± 1.0 |
| 632.5           | 408.0 ± 13.6                    | 10.4 ± 0.3| 782.5           | 1031.5 ± 26.8                   | 37.1 ± 1.0 |
| 637.5           | 426.6 ± 13.5                    | 11.0 ± 0.3| 787.5           | 810.7 ± 23.7                    | 30.3 ± 0.9 |
| 642.5           | 453.5 ± 14.6                    | 11.8 ± 0.4| 792.5           | 819.7 ± 21.8                    | 30.6 ± 0.8 |
| 647.5           | 477.7 ± 14.2                    | 12.5 ± 0.4| 797.5           | 803.1 ± 20.3                    | 30.1 ± 0.8 |
| 652.5           | 497.4 ± 15.9                    | 13.2 ± 0.4| 802.5           | 732.4 ± 21.1                    | 27.7 ± 0.8 |
| 657.5           | 509.2 ± 15.8                    | 13.6 ± 0.4| 807.5           | 679.9 ± 18.8                    | 25.9 ± 0.7 |
| 662.5           | 534.3 ± 16.6                    | 14.7 ± 0.4| 812.5           | 663.6 ± 17.1                    | 25.5 ± 0.7 |
| 667.5           | 558.0 ± 16.5                    | 16.0 ± 0.4| 817.5           | 622.2 ± 17.3                    | 24.1 ± 0.7 |
| 672.5           | 642.7 ± 17.6                    | 17.7 ± 0.5| 822.5           | 585.0 ± 16.1                    | 22.9 ± 0.6 |
| 677.5           | 640.5 ± 16.5                    | 17.8 ± 0.5| 827.5           | 540.8 ± 14.8                    | 21.4 ± 0.6 |
| 682.5           | 668.0 ± 18.4                    | 18.8 ± 0.5| 832.5           | 496.4 ± 14.8                    | 19.8 ± 0.6 |
| 687.5           | 724.4 ± 19.1                    | 20.6 ± 0.5| 837.5           | 450.4 ± 13.2                    | 18.1 ± 0.5 |
| 692.5           | 783.5 ± 18.9                    | 22.5 ± 0.5| 842.5           | 404.7 ± 13.2                    | 16.4 ± 0.5 |
| 697.5           | 858.6 ± 20.4                    | 24.9 ± 0.6| 847.5           | 391.3 ± 12.8                    | 16.0 ± 0.5 |
| 702.5           | 893.8 ± 20.3                    | 26.2 ± 0.6| 852.5           | 364.0 ± 11.8                    | 15.0 ± 0.5 |
| 707.5           | 897.8 ± 21.4                    | 26.6 ± 0.6| 857.5           | 339.6 ± 11.9                    | 14.2 ± 0.5 |
| 712.5           | 978.6 ± 22.9                    | 29.3 ± 0.7| 862.5           | 310.0 ± 11.5                    | 13.0 ± 0.5 |
| 717.5           | 1059.1 ± 23.6                   | 32.0 ± 0.7| 867.5           | 283.8 ± 9.8                     | 12.1 ± 0.4 |
| 722.5           | 1086.0 ± 25.2                   | 33.2 ± 0.8| 872.5           | 256.5 ± 9.2                     | 11.0 ± 0.4 |
| 727.5           | 1088.4 ± 25.3                   | 33.6 ± 0.8| 877.5           | 237.3 ± 9.2                     | 10.3 ± 0.4 |
| 732.5           | 1158.8 ± 23.7                   | 36.2 ± 0.7| 882.5           | 229.7 ± 8.6                     | 10.0 ± 0.4 |
| 737.5           | 1206.5 ± 25.1                   | 38.2 ± 0.8| 887.5           | 224.0 ± 8.1                     | 9.9 ± 0.4 |
| 742.5           | 1229.9 ± 25.9                   | 39.3 ± 0.8| 892.5           | 196.1 ± 8.0                     | 8.7 ± 0.4 |
| 747.5           | 1263.3 ± 27.6                   | 40.9 ± 0.9| 897.5           | 175.9 ± 7.6                     | 7.9 ± 0.3 |

3. Gounaris-Sakurai Fit of the Pion Form Factor

The pion form factor $|F_\pi|^2$ is defined as

$$|F_\pi|^2 = \frac{3s'}{\pi\alpha^2 m^2_{\pi}} \cdot \sigma^{\text{dressed}}(e^+e^- \to \pi^+\pi^-),$$  \hspace{1cm} (4)

where $\beta_\pi = \sqrt{1 - 4m^2_{\pi}/s'}$ denotes the pion velocity. The factor $\frac{3s'}{\pi\alpha^2 m^2_{\pi}}$ from pure QED calculations is considered to be exact. Thus, the statistical error-covariance matrix of the pion form factor is constructed analogously to Eq.\cite{ref:BESIII}. The diagonal elements of the matrix are presented as updated statistical uncertainties of the pion form factor in Tab.\cite{ref:BESIII}.
In the original work, a fit of the Gounaris-Sakurai parametrization \[9\] to the pion form factor is used to compare the BESIII measurement to previous publications. In the fit, the statistical covariance matrix is not considered. Instead, the uncertainties before having applied the unfolding procedure are considered. These are assumed to implicitly take into account all correlations. A good fit quality is achieved, but cannot be reproduced using the originally published covariance matrix in Ref. \[2\].

In this erratum, we have refitted the spectrum using the newly derived covariance matrix. As in the original work, the width of the $\omega$ meson is fixed to the PDG value \[10\], and the masses and widths of the higher $\rho$ states $\rho(1450)$, $\rho(1700)$, and $\rho(2150)$ are fixed to the values obtained by the BaBar collaboration \[11\]. The updated fit result is illustrated with a red line in the left panel of Fig. 2 and compared to the original fit result. The updated fit yields a reduced $\chi^2$ value of $\chi^2/\text{n.d.f.} = 70.70/56$. The right panel of Fig. 2 shows the individual contributions of the bins of the covariance matrix to the total $\chi^2$ value. Large fluctuations, as reported by Colangelo et al. \[2\] are not observed. The largest contribution to $\chi^2$ stems from the mass region between 600 and 615 MeV, where there is a systematic difference between the data and the Gounaris-Sakurai parametrization. The fit results are summarized in Tab. 2.

Table 2: Fit results together with the statistical uncertainties from this work (BESIII), the original work (BESIII 16 \[1\]), the BaBar measurement \[11\], and the PDG values \[10\].

| Parameter | BESIII | BESIII 16 | BaBar | PDG |
|-----------|--------|-----------|-------|-----|
| $m_\rho$ [MeV] | 776.58±0.42 | 776.0±0.4 | 775.02±0.31 | 775.26±0.25 |
| $\Gamma_\rho$ [MeV] | 152.05±0.65 | 151.7±0.7 | 149.59±0.67 | 147.8±0.9 |
| $m_\omega$ [MeV] | 782.69±0.34 | 782.2±0.6 | 781.91±0.18 | 782.65±0.12 |
| $|c_\omega| [10^{-3}]$ | 1.92±0.16 | 1.7±0.2 | 1.64±0.061 | – |
| $\phi_\omega$ [rad] | 0.15±0.11 | 0.04±0.13 | -0.011±0.037 | – |
| $\chi^2/\text{n.d.f.}$ | 70.70 / 56 | 49.1 / 56 | – | – |

By comparing the resulting parameters one finds a significant improvement of the uncertainty of the $\omega$ mass. The results obtained for other parameters agree well with the original work. The deviations between the fit results of BESIII and BaBar are on the level of $3\sigma$ or less, which might be well covered by systematic effects that are neglected at this point. The precise determination of resonance parameters is not the purpose of this erratum.
4. Reevaluation of $a_{\mu}^{\pi\pi,\text{LO}}(600 - 900 \text{ MeV})$

The hadronic vacuum polarization (HVP) contribution to the muon anomalous magnetic moment $a_{\mu}$ can be connected to the cross section $\sigma(e^+e^- \rightarrow \text{hadrons})$ using the optical theorem \[12\]

$$a_{\mu}^{\text{HVP}} = \frac{1}{4\pi^3} \int_{m_{\pi}^2}^{\infty} ds' K(s') \sigma(e^+e^- \rightarrow \text{hadrons}) \ ,$$

(5)

where $K(s')$ is a kernel function.

Following this formalism, the two-pion contribution to $a_{\mu}$ in the mass range of the $\rho$--$\omega$ interference is given by

$$a_{\mu}^{\pi\pi,\text{LO}}(600 - 900 \text{ MeV}) = \frac{1}{4\pi^3} \int_{(600 \text{ MeV})^2}^{(900 \text{ MeV})^2} ds' K(s') \sigma_{\text{bare}}^{\text{bare}}(e^+e^- \rightarrow \pi^+\pi^-(\gamma\text{FSR})) \ ,$$

(6)

which, due to the binned representation of the cross section, simplifies to

$$a_{\mu}^{\pi\pi,\text{LO}}(600 - 900 \text{ MeV}) = \frac{1}{4\pi^3} \sum_{i \in \text{bins}} \sigma_{\text{bare}}^{\text{bare},i} \int \Delta_i ds' K(s') \ ,$$

(7)

where the constraints of a given bin $i$ are represented by $\Delta_i$. Using Gaussian error propagation, the uncertainty is given by

$$\Delta a_{\mu}^{\pi\pi,\text{LO}}(600 - 900 \text{ MeV}) = \frac{1}{4\pi^3} \sqrt{\sum_{ij} \left(\int \Delta_i ds' K(s')\right) \cdot \left(\int \Delta_j ds' K(s')\right) C_{ij}^{\text{bare}}} \ .$$

The systematical uncertainty is 0.9% \[1\]. Thus, the BESIII result on the hadronic vacuum polarization now reads as $a_{\mu}^{\pi\pi,\text{LO}}(600 - 900 \text{ MeV}) = (368.2 \pm 1.5_{\text{stat}} \pm 3.3_{\text{syst}}) \times 10^{-10}$.

![Figure 3: Comparison of the updated calculation of the leading-order (LO) hadronic vacuum polarization contribution to $(g-2)_\mu$ due to $\pi^+\pi^-$ in the energy range 600 - 900 MeV from BESIII and the corresponding results from CMD-2 \[13\] [14], SND \[15\], BaBar \[11\], BESIII 16 \[1\], CLEO \[16\], and KLOE \[17\]. The respective values are taken from the white paper of the Muon g-2 Theory Initiative \[18\] \[19\] \[20\] \[21\] \[22\]. The yellow band indicates the $1\sigma$ range of the updated BESIII result.](image)

Figure 3 shows the results of the calculation compared to previous measurements. The statistical uncertainty is reduced by 40% compared to the original work. The result lines up well with the KLOE results, while the $1.7\sigma$ discrepancy between the BESII and BaBar results remains.
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References

[1] M. Ablikim, et al., Measurement of the $e^+e^- \to \pi^+\pi^-$ cross section between 600 and 900 MeV using initial state radiation, Phys. Lett. B 753 (2016) 629-638. arXiv:1507.08188 doi:10.1016/j.physletb.2015.11.043
[2] G. Colangelo, M. Hoferichter, P. Stoffer, Two-pion contribution to hadronic vacuum polarization, JHEP 02 (2019) 006. arXiv:1810.00007 doi:10.1007/JHEP02(2019)006
[3] M. Davier, A. Hoecker, B. Malaescu, Z. Zhang, A new evaluation of the hadronic vacuum polarisation contributions to the muon anomalous magnetic moment and to $a(m_{\mu}^2)$, Eur. Phys. J. C 80 (3) (2020) 241. arXiv:1908.00921 doi:10.1140/epjc/s10052-020-7792-2
[4] A. Hoecker, V. Kartvelishvili, SVD approach to data unfolding, Nucl. Instrum. Meth. A372 (1996) 469–481. arXiv:hep-ph/9605307
[5] G. Rodrigo, H. Czyz, J. H. Kuhn, M. Szopa, Radiative return at NLO and the measurement of the hadronic cross-section in electron positron annihilation, Eur. Phys. J. C 24 (2002) 71–82. arXiv:hep-ph/0112184 doi:10.1007/s100520200919
[6] F. Jegerlehner, Hadronic Contributions to Electroweak Parameter Shifts: A Detailed Analysis, Z. Phys. C32 (1986) 195, doi:10.1007/BF01552495
[7] J. S. Schwinger, PARTICLES, SOURCES, AND FIELDS. VOL. 3, Perseus, 1989.
[8] M. Ablikim, et al., Future Physics Programme of BESIII, Chin. Phys. C 44 (4) (2020) 040001. arXiv:1912.05893 doi:10.1088/1674-1137/44/4/040001
[9] G. Gounaris, J. Sakurai, Finite width corrections to the vector meson dominance prediction for $\rho \to e^+e^-$, Phys. Rev. Lett. 21 (1968) 244–247. doi:10.1103/PhysRevLett.21.244
[10] P. Zyla, et al., Review of Particle Physics, PTEP 2020 (8) (2020) 083C01. doi:10.1093/ptep/ptaa104
[11] J. P. Lees, et al., Precise Measurement of the $e^+e^- \to \pi^+\pi^-\gamma(\gamma)$ Cross Section with the Initial-State Radiation Method at BABAR, Phys. Rev. D86 (2012) 032013. arXiv:1205.2228 doi:10.1103/PhysRevD.86.032013
[12] S. Eidelman, F. Jegerlehner, Hadronic contributions to $(g-2)$ of the leptons and to the effective fine structure constant $\alpha(M_Z^2)$, Z. Phys. C67 (1995) 585–602. doi:10.1007/hep-ph/9502298
[13] R. R. Akhmetshin, et al., Reanalysis of hadronic cross-section measurements at CMD-2, Phys. Lett. B648 (2007) 28–38. arXiv:hep-ex/0610021 doi:10.1016/j.physletb.2007.03.033
[14] R. R. Akhmetshin, et al., High-statistics measurement of the pion form factor in the rho-meson energy range with the CMD-2 detector, Phys. Lett. B648 (2007) 28–38. arXiv:hep-ex/0610021 doi:10.1016/j.physletb.2007.01.073
[15] M. N. Achasov, et al., Update of the $e^+e^- \to \pi^+\pi^-$ cross-section measured by SND detector in the energy region 400 MeV $< \sqrt{s} < 1000$ MeV, J. Exp. Theor. Phys. 103 (2006) 380–384. [Zh. Eksp. Teor. Fiz. 130, 437 (2006)]. arXiv:hep-ex/0605013 doi:10.1134/S1063776106050007X
[16] T. Xiao, S. Dobbs, A. Tomaradze, K. K. Seth, G. Bonvicini, Precision Measurement of the Hadronic Contribution to the Muon Anomalous Magnetic Moment, Phys. Rev. D97 (3) (2018) 032012. arXiv:1712.04530 doi:10.1103/PhysRevD.97.032012
[17] A. Anastasi, et al., Combination of KLOE $\sigma(e^+e^- \to \pi^+\pi^-\gamma(\gamma))$ measurements and determination of $\sigma_{\gamma}^{\pi^+\pi^-}$ in the energy range $0.10 < s < 0.95$ GeV$^2$, JHEP 03 (2018) 173. arXiv:1711.03085 doi:10.1007/JHEP03(2018)173
[18] T. Aoyama, et al., The anomalous magnetic moment of the muon in the Standard Model (6 2020). arXiv:2006.04822
[19] M. Davier, A. Hoecker, B. Malaescu, Z. Zhang, Reevaluation of the hadronic vacuum polarization contributions to the Standard Model predictions of the muon $g-2$ and $a(m_{\mu}^2)$ using newest hadronic cross-section data, Eur. Phys. J. C77 (12) (2017) 827. arXiv:1706.09436 doi:10.1140/epjc/s10052-017-5161-6

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[20] A. Keshavarzi, D. Nomura, T. Teubner, Muon $g - 2$ and $\alpha(M_Z^2)$: a new data-based analysis, Phys. Rev. D97 (11) (2018) 114025. arXiv:1802.02995, doi:10.1103/PhysRevD.97.114025.

[21] M. Hoferichter, B.-L. Hold, B. Kubis, Three-pion contribution to hadronic vacuum polarization, JHEP 08 (2019) 137. arXiv:1907.01556, doi:10.1007/JHEP08(2019)137.

[22] A. Keshavarzi, D. Nomura, T. Teubner, The $g - 2$ of charged leptons, $\alpha(M_Z^2)$ and the hyperfine splitting of muonium, Phys. Rev. D101 (2020) 014029. arXiv:1911.00367, doi:10.1103/PhysRevD.101.014029.