CONFINING FIELDS IN LATTICE $SU(2)$

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We review some difficulties of the standard picture of confining fields which are viewed usually as ‘mild’, or quasiclassical field configurations. The paradoxes are naturally resolved by recent lattice observations of selftuned fluctuations which exhibit both ultraviolet and infrared scales. The ultraviolet scale is provided by the lattice spacing $a$ and is manifested in the non-Abelian action density associated with the fluctuations while the physical scale is exhibited by their geometrical characteristics. The data might suggest existence of a dual description of Yang-Mills theories on the fundamental level.

1. Introduction

Vacuum fields are responsible for the confinement. This dogma is rarely put in doubt. However, much less is known specifically on the corresponding field configurations. Still the common belief is that the confining fields are non-perturbative $^a$ and ‘mild’, see, e.g., a review $^2$ and references therein. The evidence is, however, indirect. Moreover, in the Minkowski space it is difficult actually to formulate what ‘soft’ field means because the vacuum should look the same in all the inertial frames.

The problem seems to be much more tractable in the Euclidean space. One is inclined to think of a single size characterizing one or another fluctuation. Moreover, there are two intrinsic scales in QCD, ultraviolet and infrared. We will have in mind the lattice regularization and by the ultraviolet cut off will understand, therefore, the lattice spacing $a$. The infrared scale is provided by $\Lambda_{QCD}$.

There are well known examples which demonstrate relevance of these scales to the vacuum fluctuations. First, consider zero-point fluctuations. They have no intrinsic scale built in and the typical size is determined by

$^a$See, however, $^1$. 

an external probe. For example, the vacuum expectation value of the gluon condensate (that is, a point-like probe) is given by:

\[ \langle 0 | (G_{\mu\nu}^a)^2 | 0 \rangle \approx \frac{(N_c^2 - 1)}{a^4}, \quad (1) \]

where \( G_{\mu\nu}^a \) is the gluonic field strength tensor and \( N_c \) is the number of colors, and is clearly ultraviolet-dominated. The very fact of the ultraviolet divergence in the matrix element (1) is not specific, of course, for the lattice and well known from calculations of the vacuum energy density in any formalism. The lattice allows to unambiguously fix (1).

On the other hand, the standard image for non-perturbative fluctuations is provided by instantons. The typical size of the instantons is of order \( (\Lambda_{QCD})^{-1} \). Moreover there exist detailed models of the instanton liquid developed along these lines

Thus, the standard picture is that at short distances zero-point fluctuations dominate while the confinement is due to soft quasiclassical fields, something like instantons. This picture can be probed, for example, through the sum-rule technique and its generalizations and claims many phenomenological successes, see, e.g., and references therein.

However, there has been accumulating evidence that this standard picture misses sometimes even large, leading effects. We will briefly review the issue in Section 3. In Section 4 we argue that the problems of the standard picture are resolved if one takes into account the newly discovered branes. The branes are two-dimensional surfaces whose total area scales in the physical units while the action density is of perturbative order (1). The branes represent a new kind of selftuned fluctuations which unify the scales \( a \) and \( \Lambda_{QCD} \). This, written version of the lectures presents only an overview of the actual content of the lectures given at the Summer Institute. Further details and references can be found in other talks of the author.

2. Difficulties of the standard picture

Confinement

We are considering pure Yang-Mills theory and the criterion of confinement is the existence of a linear potential at large distances for heavy external quarks. The lattice data can indeed be fitted by a very simple form,

\[ V_{QQ}(R) \approx -\frac{\text{const}}{R} + \sigma \cdot R, \quad (2) \]

at all the distances.
The point crucial for our discussion here is that the instanton liquid model does not reproduce the linear piece at all. Thus, the successes of the model in other cases turn into a problem for the standard picture. Indeed, the model seems to be the best realization of the standard picture but still misses the confinement.

**Current correlators**

A standard way of probing vacuum fields is provided by measuring current correlators, \( \langle 0 | j(x), j(0)|0 \rangle \) where \( j(x) \) are local currents constructed from the quark and gluon fields, see, e.g., [4,3]. The strategy is to start from short distances, \( x \ll \Lambda_{\text{QCD}}^{-1} \) and approach 'intermediate' distances where non-perturbative effects become sizable. In the most simplified form the theoretical predictions look as:

\[
\begin{align*}
\langle 0 | j(x), j(0)|0 \rangle & \approx (\text{parton model}) \left( 1 + c_j x^4 \right) \cdot <0|(G_{\mu\nu}^a)^2|0>_{\text{soft}} \ldots ,
\end{align*}
\]

(3)

where the coefficient \( c_j \) depends on the current considered and calculable while \( <0|(G_{\mu\nu}^a)^2|0>_{\text{soft}} \) stands for the contribution of the soft non-perturbative fields, \( <0|(G_{\mu\nu}^a)^2|0>_{\text{soft}} \sim \Lambda_{\text{QCD}}^2 \). In actual applications, there can be other vacuum condensates representing the non-perturbative fields. Another variation of (3) is an explicit calculation of a single-instanton contribution [3]. Also, one commonly keeps the first-order perturbative contribution of order \( \alpha_s(x) \) as well.

In many cases, the approximations like (3) work well. However, there are cases when (3) fails, see in particular [5]. Moreover, there is accumulating evidence that in cases when the Born (parton-model) approximation is dominated by gluon propagator the leading correction is of order

\[
\begin{align*}
\langle 0 | j(x), j(0)|0 \rangle & \approx (\text{parton model}) \left( 1 + b_j \Lambda_{\text{QCD}}^2 \cdot x^2 \ldots \right) ,
\end{align*}
\]

(4)

for the latest example of this kind and references see [9].

There is no model-independent way to relate the coefficients \( b_j \) in (4) in various channels since the quadratic correction is not captured by the operator product expansion. Phenomenologically, however, the model with a non-vanishing short-distance gluon mass \( m_g \) (see second paper in Ref [5]) turns to be successful. Within this model one modifies the propagator of a gluon by replacing

\[
1/q^2 \to 1/q^2 + |m_g^2|/q^4 ,
\]

(5)

where \( q \) is the momentum of the gluon, \( q^2 \gg m_g^2 \). It is worth emphasizing that the model (5) applies in the Born approximation when higher order corrections in \( \alpha_s \) are ignored. (Modification of the Born approximation
valid at all \( q^2 \) has been introduced in \(^{10}\)). In particular, the Cornell potential (2) at short distances exhibits a correction induced by (5). Indeed, the Born approximation in this case is given by one-gluon exchange and this is just the case when one expects the dominance of the quadratic correction.

3. Long perturbative series

3.1. Expectations

The difference between ‘long’ and ‘truncated’ perturbative series is crucial in view of the theorem \(^{11}\) that terms of order \( x^2 \cdot \Lambda^{2}_{QCD} \) are calculable perturbatively. Let us explain this point in more detail.

Generic perturbative expansion for a matrix element of a local operator looks as:

\[
\langle O \rangle = \langle \text{parton model} \rangle \cdot (1 + \sum_{n=1}^{\infty} a_n \alpha_s^n ) ,
\]

where we normalized the anomalous dimension of the operator \( O \) to zero. Note also that \( \alpha_s \) is the bare coupling, \( \alpha_s \ll 1 \).

In fact, expansions (6) are only formal since the coefficients \( a_n \) grow factorially at large \( n \):

\[
|a_n| \sim c_i^n \cdot n! ,
\]

where \( c_i \) are constants. Moreover, there are a few sources of the growth (7) and, respectively, \( c_i \) can take on various values, for review see, e.g., \(^{12}\). The factorial growth of \( a_n \) implies that the expansion (6) is asymptotic at best. Which means, in turn, that (6) cannot approximate a physical quantity to accuracy better than

\[
\Delta \sim \exp \left( - \frac{1}{c_i \alpha_s} \right) \sim \left( \Lambda^{2}_{QCD} \cdot a^2 \right)^{b_0/c_i} ,
\]

where \( b_0 \) is the first coefficient in the \( \beta \)-function. To compensate for these intrinsic uncertainties one modifies the original expansion (6) by adding the corresponding power corrections with unknown coefficients.

In case of the gluon condensate the theoretical expectations can be summarized as:

\[
\langle 0 | -\frac{\beta(\alpha_s)}{\alpha_s b_0} (G_{\mu\nu}^a)^2 | 0 \rangle \approx \alpha_s \frac{(N_c^2 - 1)}{a^4} \left( 1 + \sum_{n=1}^{\sim N_c} a_n \alpha_s^n + \text{(const)} a^4 \cdot \Lambda^{4}_{QCD} \right) ,
\]

(9)
where

\[ N_{ir} \approx \frac{2}{b_0 \alpha_s} \]

and terms proportional to \( \Lambda_{QCD}^4 \) correspond to \( <0|(G_{\mu\nu}^a)^2|0>_{soft}, \) see (3). It is the latter quantity which enters the QCD sum rules, see 4.

A conspicuous feature of the prediction (9) is the absence of a quadratic correction, compare (4). However, Eqs (4) and (9) are not necessarily in contradiction with each other. The point is that the phenomenological analysis relies on short or even very short truncated perturbative series plus a power-like correction while the theorem (9) keeps a long perturbative series (plus the first power-like correction).

### 3.2. Numerical results

Numerically, this problem was studied in greatest detail in case of the gluon condensate on the lattice 13. In terms of the lattice formulation the gluon condensate is nothing else but the average plaquette action. The result can be summarized in the following way. Represent the plaquette action \( \langle P \rangle \) as:

\[
\langle P \rangle \approx P_{pert}^N + b_N a^2 \Lambda_{QCD}^2 + c_N a^4 \Lambda_{QCD}^4 ,
\]

(10)

where the average plaquette action \( \langle P \rangle \) is measurable directly on the lattice and is known to high accuracy, \( P_{pert}^N \) is the perturbative contribution calculated up to order \( N \):

\[
P_{pert}^N \equiv 1 - \sum_{n=1}^{n=N} p_n g^{2n} ,
\]

(11)

and, finally coefficients \( b_N, c_N \) are fitting parameters whose value depends on the number of loops \( N \). Moreover, the form of the fitting function (10) is rather suggested by the data than imposed because of theoretical considerations.

The conclusion is that up to ten loops, \( N = 10 \) it is the quadratic correction which is seen on the plots while \( c_N \) are consistent with zero. However, the value of \( b_N \) decreases monotonically with growing \( N \) 13. The factorial divergence (7) is not seen yet and perturbative series reproduces the measured plaquette action at the level of \( 10^{-3} \). Finally, at the level \( 10^{-4} \) the \( \Lambda_{QCD}^4 \) term seems to emerge 13.
4. Branes

4.1. Coexistence of two scales

We have already mentioned that instantons do not confine. A natural question is then, what are the confining configurations. Actually the answer is also more or less known. Namely it is commonly believed (for a recent review and references see 14) that it is the monopoles and central vortices that confine. The next question is then, why we did not account for these fluctuations, say in (9) or (3). The answer is pure technical, at first sight. Namely, the monopoles and vortices are defined in terms of projected, not original non-Abelian fields. The projection, in turn, is determined non-locally and the track to the original Yang-Mills fields is lost. However, the general expectation seems to be that the monopoles and vortices correspond to soft, bulky fields and as far as dependence on $\Lambda_{QCD}$ is concerned are not much distinguishable from the instantons.

These expectations turned to be not true. Namely both the monopoles, see 15 and references therein, and vortices 6 appear to be associated with an excess of the non-Abelian action which is divergent in the ultraviolet:

$$\langle S_{\text{mon}} \rangle \sim \ln 7 \cdot \frac{L}{a}, \quad \langle S_{\text{vort}} \rangle \approx 0.54 \cdot \frac{A}{a^2},$$

where $L$ is the length of the monopole trajectory, $A$ is the area of the vortex.

It is most remarkable that the infrared scale is also relevant to the branes. Namely, it has been known since some time (for review see 14) that the densities of monopoles and vortices scale in physical units. The corresponding densities are defined as:

$$L_{\text{perc}} \equiv 4 \rho_{\text{perc}} V_4, \quad A_{\text{vort}} \equiv 6 \rho_{\text{vort}} V_4,$$

where $V_4$ is the volume of the lattice and $L_{\text{perc}}$ is the total length of the percolating cluster while $A_{\text{vort}}$ is the area of the vortices. The densities (13) scale in physical units and are independent on the lattice spacing. According to the latest measurements:

$$\rho_{\text{perc}} = 7.70(8) \, fm^{-3}, \quad A_{\text{vort}} \approx 4.0(2) \, fm^{-2},$$

see 16 and 6, respectively. Moreover, the monopole trajectories lie on the P-vortices 17,6.

4.2. Selftuning

Naively, one would expect that monopoles and vortices with action (12) propagate only very short distances, $L \sim a, A \sim a^2$. However, both
monopoles and vortices form clusters which percolate through the whole of the lattice volume $V_4$. This implies cancellation of the ultraviolet divergences between action and entropy. This cancellation is easy to quantify in case of monopoles (particles). Namely, the propagating mass, $m$ of the monopole is in fact not the radiative mass $M(a)$, where $M(a) \equiv S_{mon}/L$, see (12). The relation between the two masses is as follows:

$$m_{mon}^2 = \frac{\text{const}}{a} \left( M(a) - \frac{\ln 7}{a} \right).$$

(15)

The $\ln 7$ term here is due to the entropy (see, e.g., 18). Observation (14) implies cancellation between the action and entropy. Moreover, there is no parameter to tune to ensure this cancellation. Thus, the monopoles are rather selftuned. The same is true for the vortices. Both the tension and entropy are ultraviolet divergent 6.

4.3. Quadratic correction to the gluon condensate

Monopoles and vortices are defined, for a given configuration of the vacuum fields, for the whole of the lattice. Thus, they are seen as a nonlocal structure. Moreover, they are manifestly non-perturbative. Indeed, the probability $\theta(plaq)$ for a particular plaquette to belong to a brane has been found to be proportional to:

$$\theta(plaq) \approx (\text{const}) \exp\left( -\frac{1}{b_0 g^2(a)} \right) \sim (a \cdot \Lambda_{QCD})^2.$$  

(16)

On the other hand, the branes have an ultraviolet divergent tension which assumes a kind of locality.

To make contact with the continuum theory it is useful to evaluate contribution of the branes into local or quasi-local matrix elements. The gluon condensate turns to be the easiest case. Indeed, combining Eqs (14) and (12) one gets for the contribution of the vortices to the gluon condensate:

$$\langle (G_{\mu\nu}^a)^2 \rangle_{vort} \approx 0.3 \text{ GeV}^2 a^{-2}.$$  

(17)

We see that the mysterious $a^2 \cdot \Lambda_{QCD}^2$ correction, see Sect. 3.2., gets its explanation in terms of the branes. The mixture of the two scales, $a$ and $\Lambda_{QCD}$, exhibited by this correction appears to be a manifestation of selftuning of the branes.
5. Status of theory

5.1. Why branes?

The lattice data on the monopoles and vortices have been accumulating since long. However, they were analyzed almost exclusively within the confinement problem. Discovery of the ultraviolet divergences, see (12), makes the challenge to the theory much more direct. Indeed in an asymptotically free theory one should be able to understand short distances from first principles. At present, however, the phenomenology is by far ahead of the theory.

Observation of the branes is gratifying from the theoretical point of view. Indeed, the results (12) imply that at least at presently available lattices the size of the monopoles is not resolved. Moreover, gluons are already (approximately) free particles at such distances. On the other hand, it is well known that there exists no consistent field theory with both electric (color) and magnetic dynamical charges. Existence of the branes implies that the magnetic and electric (color) charges are separated in space. The color charges live in the bulk while the magnetic charges live on the two-dimensional branes (which percolate through the four-dimensional Euclidean space).

5.2. Constraints from asymptotic freedom

The asymptotic freedom implies that at short distances the only degrees of freedom relevant are those of free gluons. How do we count the degrees of freedom? Usually through ultraviolet divergences. The best known example is the $\beta$-function which counts logarithmic divergences. The logarithmic divergences are singled out since they are independent on details of the cut off. However, on the lattice the power divergences are also uniquely defined. And they are much easier to study. In particular, if we think in terms a scalar complex field $\phi_M$ describing monopoles then we can introduce a vacuum expectation value $\langle |\phi_M|^2 \rangle$. For an elementary monopoles we would have $\langle |\phi_M|^2 \rangle \sim a^{-2}$. This is not allowed since we may not have new particles at short distances. What is allowed is

$$\langle |\phi_M|^2 \rangle \sim \Lambda_{QCD}^2,$$

(18)

and this is a constraint from the asymptotic freedom.

\textsuperscript{b}The condensate can be thought of as non-perturbative part of the gauge invariant condensate of dimension two, see \textsuperscript{19}. 


Remarkably enough, this constraint can be rewritten in terms of the total length of the monopole trajectories (in (13) we considered only the percolating cluster while now we are including finite clusters as well):
\[
\rho_{\text{mon}}^{\text{tot}} \leq (\text{const}) \frac{\Lambda_{QCD}^2}{a} .
\] (19)
This constraint turns to be satisfied by the data. On the other hand, from pure geometrical point of view (19) implies that the monopoles actually percolate on a two-dimensional surface, not on the whole of the four-dimensional space. Which means branes.

In other words the data on the monopole action, see (12) plus asymptotic freedom of the original YM theory imply existence of branes.

5.3. Manifestations of the duality

As we mentioned above, the original formulation of the YM theory does not explain the selftuning of the monopole action, compare (12) and (15). Indeed the geometrical factor \( \ln 7 \) depends on the type of the lattice (cubic) and is meaningless in the continuum. Imagine, however, that there were a theorem proving existence of the dual formulation in the continuum limit. Then the selftuning would be derived since it is a necessary condition for the monopoles to survive in the continuum limit.

Also, compare the contribution to the gluon condensate from instantons and monopoles. The instantons are added to the perturbative expansion, see (9). The contribution of the branes, to the contrary, cannot be added to the perturbation theory but is, instead, dual to the perturbative series, see Sects. 3.2, 4.3. The reason is that the branes ‘belong’ to the dual world.

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\(^*\)For further applications of the percolation theory see, in particular, 20,16.
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