Synthesized magnetic field of a sawtooth superradiance lattice in Bose–Einstein condensates

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Ultracold atoms have become one of the most exciting platforms to synthesize novel condensed matter physics. Here we realize a sawtooth superradiance lattice in Bose–Einstein condensates and investigate its chiral edge currents. Based on one-dimensional superradiance lattice (SL) in standing wave-coupled electromagnetically induced transparency, a far-detuned standing-wave field is introduced to synthesize a magnetic field. The relative spatial phase between the two standing-wave coupling fields introduce a magnetic flux in the sawtooth loop transitions of the lattice. This flux determines the moving direction of excitations created in the SL and results in nonsymmetric reflectivities when the SL is probed in two opposite directions. Our work demonstrates an in situ technique to synthesize and detect artificial gauge field in cold atoms.

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INTRODUCTION

Ultracold atoms have been a highly controllable system for investigating condensed matter physics,1 quantum optics and quantum information processing.2 In particular, optical lattices are widely used to manipulate ultracold atoms and simulate many-body quantum physics in solid state systems. Recently, ultracold atoms in optical lattices3 have been realized in honeycomb,4 checkerboard,6 Kagome geometries5 as well as in bichromatic superlattices, which are double-well lattices consisting of a fundamental lattice and a frequency-doubled lattice.7,8 Incommensurate bichromatic lattice may generate a one-dimensional quasi-periodic lattice to study Anderson localization9 and effective magnetic fields.10,11 To introduce boundaries in lattices and study topological physics on edges, artificial dimensions12 have been synthesized by using the inner degrees of freedom of the atoms, such as spin,13,14 momentum,15,16 eigenstates in a harmonic trap.17 Chiral edge currents have been observed in ultracold13,14,16,18 and hot atoms.19

Recently, one-dimensional superradiance lattice (1D SL)20 of a Bose–Einstein condensate (BEC) based on standing wave-coupled electromagnetically induced transparency (EIT) was realized experimentally.24 The band structure was investigated by measuring the directional emissions of one of the superradiant excited state in the 1D SL. Toward realizing higher dimensional SLs where topological phenomena can be studied,25 in this paper, we investigate chiral edge dynamics of a quasi-1D sawtooth SL. The relative spatial phase between the two standing-wave coupling fields can be mapped to an Aharonov-Bohm (AB) phase in the standing wave component of the far-detuned coupling field. The relative spatial phase between the far-detuned and the near-resonant standing-wave coupling fields is the bosonic annihilation and creation operators of the atom in the state |m⟩, and k1 and k2 are the wave vectors of the two counter-propagating plane waves in the near-resonant standing-wave coupling fields. In the theoretical analysis, we assume that the wave vectors of the near-resonant and far-detuned standing waves be the same. The variation of θ along the atomic ensemble is taken into account in fitting the experimental data (see “Flux average” section).

RESULT

Artificial gauge field

We start with a brief description of our experimental method for generating the synthetic uniform magnetic field in a sawtooth SL (see details in “Methods” section). The original SL20,24 involves with three atomic levels: a ground state |g⟩, an excited state |e⟩ and a metastable state |m⟩. A set of timed-Dicke states26 in the momentum space are coupled by a near-resonant standing wave, forming a 1D tight-binding lattice.20 Based on that, we introduce a fourth atomic level |d⟩ and couple it to |m⟩ with a far-detuned standing-wave field, as shown in Fig. 1a. The key effect of this far-detuned standing-wave field is to induce second order transitions between timed-Dicke states that have a momentum difference of two light moments, as shown by the effective Hamiltonian.15

\[ H_f = \sum_q \kappa e^{i\theta} b^\dagger_m(q - k_{c1} + k_{c2}) b^\dagger_m(q) + h.c. \]

(1)

where \( \kappa = \Omega^2 / \Delta \) with \( \Omega \) being the Rabi frequency of each plane wave component of the far-detuned standing-wave coupling field, \( \theta \) is the relative spatial phase between the far-detuned and the near-resonant standing-wave coupling fields, \( b_m(q) \) and \( b_m^\dagger(q) \) are the bosonic annihilation and creation operators of the atom in the state |m⟩, and \( k_{c1} \) and \( k_{c2} \) are the wave vectors of the two counter-propagating plane waves in the near-resonant standing-wave coupling fields. In the theoretical analysis, we assume that the wave vectors of the near-resonant and far-detuned standing waves be the same. The variation of \( \theta \) along the atomic ensemble is taken into account in fitting the experimental data (see “Flux average” section).
The total Hamiltonian of single excitations can be written in a tight-binding form

\[ H_i = \sum_{\pm} \Delta_\pm \langle \ket{m_i \pm} \ket{e_i} \rangle \langle e_i \ket{m_i \pm} + \frac{1}{2} \langle \Omega \ket{e_i} \rangle \langle e_i \ket{m_i \pm} + \frac{1}{2} \langle \Omega \ket{m_i \pm} \rangle \langle e_i \ket{m_i \pm} + h.c. \right) \]  

(2)

where \( \Delta_+ = \delta + 2\chi \) with \( \delta = \nu_c - \omega_{em} \), being the detuning between near-adjacent standing-wave frequency \( \nu_c \), and the atomic transition frequency \( \omega_{em} \). \( \Omega \) being the Rabi frequency of the near-adjacent fields driving the transition between \( \ket{m_i \pm} \) and \( \ket{e_i} \). \( \langle \ket{m_i \pm} \ket{e_i} \rangle \approx [1, k_i] e^{-i\Theta} [1, 0] \) denotes the state of \( N - 1 \) atoms with zero momentum in the ground state \( \ket{g} \) and one atom with momentum \( k_i \) in the excited state \( \ket{e_i} \). \( \langle \ket{m_i \pm} \rangle \approx [1, k_i - k_{i-1}] / [1, 0] \) is similarly defined as \( \langle \ket{e_i} \rangle \), and \( k_i = k_{i-1} - \ell (k_{i-1} - k_{j-2}) \) with \( k_{i-1} \) being the wave vector of the probe field and \( \ell \) being an integer. In this lattice, the states \( \ket{e_i} \) with \( \ket{k_i} \neq \ket{k_{i-1}} \) can be coupled to the ground state by a vacuum mode via a directional superradiant enhancement in \( k_i \) mode. A superradiant enhancement is absent when \( \ket{k_i} = \ket{k_{i-1}} \), because the phase matching condition cannot be met. The kinetic energy due to the recoil can be neglected. The phase 2\( \Theta \) carried by the next-nearest-neighbor (NNN) coupling introduces the effective magnetic flux \( \Phi \) in the triangular transition loops in the lattice, as shown in Fig. 2a. By tuning the relative spatial phase \( \Theta \) between the two standing waves, we can control the magnitude and sign of the synthetic magnetic field.

The Hamiltionian of the chiral superradiance lattice in real space can be written as

\[ H = \hbar n \cdot a + \hbar \Omega l, \]

(3)

where \( a = \sum_{p=x,y,z} a_p \) is the vector of the Pauli matrices in the pseudo-spin basis \( \ket{\pm} \) (spin-up) and \( \ket{m} \) (spin-down), Bloch vector \( \mathbf{n} = (h_x x + h_z z) / h \), with \( h_x = 2\Omega \cos (k_x x + 2\Theta) \) and \( h_z = 2 \Delta_\pm \), \( h_\pm = \cos (2k_x x + 2\Theta) \) and \( \Theta = \hbar \sqrt{h_x^2 + h_z^2} \), and \( l \) is the \( 2 \times 2 \) unit matrix. The dispersion relation (eigenenergies as functions of positions) of the two bands are \( E_\pm = \pm \hbar \Omega + \hbar \Theta \) with the eigenstates \( \ket{\psi_\pm} = \cos \eta / 2 \ket{\pm} + \sin \eta / 2 \ket{\mp} \) and \( \ket{\psi_-} = - \sin \eta / 2 \ket{\pm} + \cos \eta / 2 \ket{\mp} \), the \( \eta \) is the polar angle of \( \mathbf{n} \). The band structure is plotted in Fig. 2c, with the spin texture \( \sigma_z \) being denoted by the color. When the flux \( \Phi \) equals to \( \pi / 2 \), we notice that the eigenstates that have large \( \ket{m} \) state component have a positive dispersion. On the other hand, the eigenstates that have large \( \ket{\pm} \) state component have a negative dispersion. When the flux equals to \(-\pi / 2 \), the dispersions are reversed (see more details about the \( \mathbf{n} \) and \( \Theta \) in "Methods" section). In Fig. 2c, the non-zero \( \Delta_\pm \) breaks the symmetry \( \hbar \Theta + \pi / 2 \rightarrow -\hbar \Theta \), leaving dispersionless band appears only for \( \Theta = \pi / 2 \).

The dispersion relation determines the direction of the edge currents, i.e., \( \partial p / \partial \Theta = -\hbar e / \hbar c \) with \( p \) the momentum of the excitations. The chiral current is defined as \( j_\ell = \sum_{\pm} \langle d\Omega \delta E_\ell - E \rangle \psi_\ell \rangle^2 \partial E / \partial x \rangle^2 \). For positive dispersion of the \( \ket{\pm} \) edge, i.e., red lines in Fig. 2b, the momentum decreases with time. In the steady state of the atoms being pumped into the state \( \ket{e_0} \), the probability of state \( \ket{e_\pm} \) is larger than that of \( \ket{e_\pm} \). As a result, the superradiant emission of Scattering 1 when the atoms are
pumped by Probe 1 is larger than the one of Scattering 2 when the atoms are pumped by Probe 2.

All the optical fields, including the coupling and probe lasers, illuminate atoms simultaneously for 80 μs. In order to obtain the direction of the chiral edge current, we measure the difference between Scattering 1 and 2 in Fig. 1b. Required by the phase matching of the superradiant emission, the angle between the probe light and the scattering light is 124°. The Scattering 1 and 2 are also in the opposite direction. In order to obtain the dark background and high signal-noise ratio for detecting the superradiant emission, the intersecting angle between the plane of the two coupling beams and the plane of the probe-scattering beams is 11°.24 The superradiant emission is measured with EMCCD.

First we show the results with only the coupling fields that couple |e⟩ and |m⟩. In this case, an ordinary 1D SL is obtained.24 The weak probe field pumps the ground state BEC into the state |e0⟩, which is further coupled to other states in the SL. We sweep the frequency of the probe field and keep the frequency of the coupling fields fixed. The spectrum is characterized by two narrow peaks, which is a feature of the density of states of the 1D tight-binding lattice, as shown in Fig. 3a. The spectra of Scattering 1 and 2 are the same when the BEC is probed in two opposite directions. The asymmetry of the two peaks is induced by the phase mismatch δk = |k - k2| - |k1 - k0| of the wave mixing process.34

When both standing-wave coupling fields are applied to the BEC, the scattering spectra of Probe 1 and 2 are different depending on the spatial relative phases between the two

![Graph](image)

Fig. 3 Superradiance emission for the SL probed in two opposite directions as a function of the frequency detuning of the probe light. a, b The results for SLs with only the on-resonant standing-wave coupling fields. (a1)–(a5) and (b1)–(b5) are experimental data and numerical simulations with both standing-wave coupling fields for different phases θ = −π/2, −π/4, 0, π/4, π/2, respectively. The red and blue colors are for the results in two opposite probe directions.

![Graph](image)

Fig. 4 Superradiance emission as a function of θ. The laser power of the near-resonant coupling field is fixed at 200 μW and that of the far-detuned one is 10 μW (a1), 20 μW (a2), 40 μW (a3), 60 μW (a4), 80 μW (a5), and 100 μW (a6), respectively. The red and blue colors are for the results in two opposite probe directions. The frequency of the probe light is set at zero detuning. In the numerical simulation, coupling strengths Ω = 2π × 15 MHz, κ = 2π × 2.25 MHz, detuning Δs = 2κ, and phase mismatch δk = 0.01κ.
pump a single excitation to the SL, we can measure the dynamics of the single excitation. We can also prepare many excitations. They behave the same with the single excitation provided that interaction is absent. Each excitation emits a photon in a probabilistic way. The lattice is gradually destroyed until all the photons are emitted. The time-dependent signal (rather than the steady state response in this work) is expected to oscillate, which is similar with the signals in ref. 18.

In the current work, the optical signals reveal the dynamics only on the sites \( |e_1 \rangle \) and \( |e_\perp \rangle \). However, it can be generalized to more complex lattice structure, as well as more superradiant sites. For instance, we can tune the far-detuned standing wave (lattice 2) on resonance. The corresponding diamond-shape SL is presented in Fig. 5. By pumping site \( |e_0 \rangle \), we can measure the emission from \( |e_\perp \rangle \) and \( |d_0 \rangle \) and distinguish them by the frequencies. The two signals indicate the dynamics along and perpendicular with the lattice, which may be used to characterize Hall-like effect.

In addition, due to the inevitable spontaneous emission of atomic level \( |e \rangle \), the Hamiltonian of SL is naturally non-Hermitian with different decay rates for \( |e \rangle \) and \( |m \rangle \) sites. It can be used to study the topological physics in non-Hermitian systems. 31,32 Another remarkable feature is that one energy band becomes completely flat when \( \phi = \pi \), as shown in Fig. 2b. The flatband is not observed in current scheme due to the flux averaging (see “Methods” section). In future, we can design a polarization dependent coupling scheme (see “Methods” section) to make the two standing waves have comparable wavelengths and the synthetic magnetic flux is constant along the atomic ensemble. In that case, the optical signature of the flatband is a sharp peak without asymmetry for the signals in the two detectors. The flatband is preferred for a strong many-body interaction, which can be realized by introducing Rydberg interaction. It can be used to study the interplay between the many-body interactions and artificial gauge field in the SL.

**METHODS**

**Bloch vector**

Bloch vectors \( \mathbf{n} \) are plotted in Fig. 6. We notice that the Bloch vector is polarized in \( z \)-axis in the Bloch sphere on the positive (negative) slope of \( h \) when \( \theta = \pi/4 \) \((3\pi/4)\) (the gray areas). It results in a non-zero \( J_+ \) in Fig. 2b.

**Experimental setup**

In our experiment, we prepared a pure BEC with typically \( 5 \times 10^5 \) \(^{87}\)Rb atoms in the \( |g \rangle \equiv |F = 2, m_F = 2 \rangle \) hyperfine ground state sublevel confined in a cross-beam dipole trap at a wavelength near 1064 nm. The geometric mean of trapping frequencies is \( \Omega \approx 2 \pi \times 80 \text{ Hz} \). The atomic size is estimated in the Thomas-Fermi regime to be 20 \( \mu \text{m} \) according to the scattering length for \( |g \rangle \) state at zero magnetic field with about 100\( \mu \text{m} \). A homogeneous bias magnetic field along the \( z \)-axis (gravity direction) is provided with \( B_\parallel = 2 \text{ G} \) by a pair of coils operating in the Helmholtz configuration. We choose the \( D_2 \) line (around 795 nm) of \(^{87}\)Rb atom with a simple three-level \( A \)-type model as shown in Fig. 1a. The probe laser couples the transition between the ground state and \( |e \rangle \equiv |F' = 1, m_{F'} = 1 \rangle \). A pair of strong laser beams with the intersecting angle \( \varphi = 56^\circ \) couple the transition between \( |m \rangle \equiv |F = 1, m_F = 1 \rangle \) and \( |e \rangle \), as shown in Fig. 1a. The coupling laser beams have the waist (1/e\(^2\)) radius about 280 \( \mu \text{m} \) at the BEC position. The weak probe light used to pump the atoms from \( |g \rangle \) to \( |e \rangle \) has a waist about 600 \( \mu \text{m} \). The frequencies of the coupling and probe laser are locked, which is described in our previous work. 24 Another strong laser couples the \( D_2 \) transition (around 780 nm) between \( |m \rangle \) and \( |d \rangle \equiv |F' = 2 \rangle \) with a blue detuning of 200 MHz. It induces a periodic dynamic Stark shift for the state \( |m \rangle \), as shown in Fig. 1a. In the experiment, we simultaneously apply the standing waves and the probe field for 80 microseconds, at the same time the superradiant light is collected for the full 80 ms. The BEC is quickly heated once the superradiant process occurs. After each run, the BEC is depleted. Then we reload the BEC to repeat the experiment to collect the data of the spectra. The spatial coherence of the atoms in the ground state remains. In Fig. 7, we show the 45 ms TOF image of the level \( |g \rangle \) after applying the lattices 1, 2 and probe fields for 20 \( \mu \text{s} \) with phase \( \theta = \pi/2 \). Each pair of nearest site is separated by 2\( \hbar k_F \sin(\varphi/2) \). The zeroth momentum state is indicated by the white circle.

**Phase adjustment**

The 780 nm lasers are combined with the 795 nm coupling laser beams by the beam splitters, and these laser beams are coupled simultaneously into the polarization maintaining single-mode fibers in order to obtain a perfect relative spatial overlap. The combined beams are split into two beams, and intersect at the position of the atoms to form standing waves. We change the displacement of the wedge to increase \( \theta \) for seeing the response return to the same pattern. Thus, we obtain the periodic curves for two opposite direction scattering, then label the phase according to the symmetry of two opposite direction scattering. For example, there are equal scattering strength for \( \theta = 0 \).
Fig. 8 The schematic of atomic level to avoid the phase averaging.

Kinetic energy

In the experiment, the ratio between the hopping strength and atomic decay rate $\Gamma_0/g_0/2 = 15 MHz/3 MHz = 5$. Within the life time of atoms in SL, the maximum momentum is about 10k, the corresponding maximum recoil energy is in the order of $100 \mu eV$, which is much smaller than relevant experimental parameters. $k_F$ and $E_F$ are the single photon recoil momentum, recoil energy of $^{87}Rb$ D1 line.

Flux average

The magnitudes of the wave vectors of the two coupling standing-wave laser fields are $k_1 = 2\pi \times \sin(\varphi/2)/\lambda_1$ and $k_2 = 2\pi \times \sin(\varphi/2)/\lambda_2$, where $\lambda_1 = 795 \text{ nm}$, $\lambda_2 = 780 \text{ nm}$, $\varphi = 56^\circ$ is the intersecting angle between the two standing-wave laser beams. Due to the difference between $\lambda_1$ and $\lambda_2$, there is a long beating wavelength between the two standing waves, $\lambda_0 = \lambda_1 \lambda_2/(2(\lambda_1 - \lambda_2) \sin(\varphi/2))$. In our experiment, $\lambda_0 \approx 44 \mu m$ is larger than twice the length of the BEC in the trap. The relative phase $\varphi$ varies for about $\pi/2$ over the BEC and the mean phase can be controlled by adjusting the insertion depth of a wedge into one arm.

In order to take into account this variation in the numerical simulation, we split the atomic ensemble into many slices and each slice is much smaller than $\lambda_0$ but includes large number of atoms. Since the relative phase $\varphi$ in each slice is fixed, we treat each of them with independent atomic ensemble and calculate the transmission matrix $M(\varphi)$. The total reflection is obtained by $M_r = \Pi_0 M(\varphi)$. An interesting observation is that the asymmetry between the Scattering 1 and 2 is robust over the phase average, in Figs. 3 and 4.

To fix the phase, we propose a polarization dependent coupling scheme in Fig. 8. A bias magnetic filed is applied to split the spin states. BEC is prepared in the level $|g\rangle \equiv |5^2S_{1/2}, F = 1, m = 1\rangle$. A $\sigma$ polarized standing wave is used to couple to level $|e\rangle \equiv |5^2P_{1/2}, F = 2, m = 1\rangle$ and $|m\rangle \equiv |5^2S_{1/2}, F = 2, m = 2\rangle$ resonantly, and a $\pi$ polarized standing-wave couples level $|d\rangle \equiv |5^2P_{1/2}, F = 2, m = 2\rangle$ and $|m\rangle$ with blue detuning. The beating wavelength between the two standing wave is in the order of centimeters, which is much larger than the typical length of the BEC. Therefore the phase can be regarded as constant.

DATA AVAILABILITY

All data generated or analysed during this study are included in this published article. Additional data are also available from the corresponding authors upon reasonable request.

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AUTHOR CONTRIBUTIONS
J.Z. designed the research. J.Z. and S.Y.Z. supervised the research. P.W., L.C., C.M., Z.M., L.H., K.N. and J.Z. performed the experiments. H.C. and D.W.W., performed the simulation. J.Z., H.C. and D.W.W. wrote the manuscript. All authors interpreted the results and reviewed the manuscript.

COMPETING INTERESTS
The authors declare no competing interests.