Abstract

We present a complete order $\alpha_s$ analysis of the process $pp \rightarrow \gamma\gamma + X$ with polarized initial states, previously studied by us with leading order structure functions. We include in our calculation new sets of parton densities evolved in NLO QCD, such as those of Gehrmann and Stirling and of Glück et al., which incorporate the new anomalous dimensions of Mertig and Van Neerven in the evolution equation. A detailed phenomenological analysis is also given which includes the photon isolation. Our results indicate that the asymmetries, although not very large, should be substantial enough at RHIC energies to be measurable in future planned experiments.
I. INTRODUCTION

The study of the spin structure of the nucleons is a fascinating subject which has attracted a lot of attention both from the theoretical and from the experimental side in recent years. From a phenomenological viewpoint, for instance, the EMC result has called for a reanalysis of the connections between the traditional parton model and the QCD description of polarized DIS, based on leading-power factorization theorems and on applications of the Operator Product Expansion.

There seems to be more agreement now in the literature on the interpretation of the EMC and of other more recent results than before, and it seems obvious that even in a perturbative framework (such as the OPE), subtle issues connected to the renormalization of anomalous operators, their scale dependence etc., had been overlooked in the past. Sum rules, largely inspired by the naive quark model, have also provided a bridge between the theory and the experiments.

It is expected in few years time that the information gathered from DIS studies of the polarized parton distributions will be supplemented and even extended by new results obtained from proton-proton colliders, such as the BNL Relativistic Heavy Ion Collider, RHIC. In particular, in these experiments a direct gluon coupling will allow direct measurements of the contributions of the gluons to the spin of the nucleon.

Recently, new sets of polarized parton distributions have been generated by various groups which incorporate the effects of the evolution equation up to $O(\alpha_s)$. This has been possible thanks to the ground-breaking calculation of Mertig and Van Neerven, later verified by Vogelsang [5] who have derived the $O(\alpha_s)$ radiative corrections to the anomalous dimensions of the evolution.

We have presented a study of the process $p p \rightarrow \gamma \gamma + X$ to next-to-leading order containing a brief phenomenological analysis of our results [17]. In this work we are going to extend this analysis for the same process by incorporating, for the first time, the new NLO sets of structure functions. Here most of our discussion, therefore, will be purely
phenomenological and we refer to our previous study for all the technical details and for a complete presentation of the methods involved in it.

This work is organized as follows. In section II, in order to make our discussion self contained, we briefly review the structure of the evolution equations for the structure functions to order $\alpha_s$, and discuss in general terms the various parametrizations. We then move to a detailed phenomenological study of double prompt photon production, and examine in particular whether it will be possible to get any new information on the polarized gluon distributions $\Delta G$ from a study of this process at RHIC. We will also discuss the uncertainties and the scale dependence of NLO results, and we make predictions of the large transverse momentum ($p_T$) behaviour of the cross section at RHIC. Our conclusions are presented in section IV.

II. THE NLO EVOLUTION

Different sets of polarized structure functions to NLO have been proposed recently by Gehrmann and Sterling (G-S) [7] and by Glück et al. (GRSV) [8]. For the G-S sets, three different parametrizations are considered, while for GRSV two possible scenarios have been analyzed. In order to render our treatment self contained, we briefly summarize these new results. We start from the usual framework of a DIS (longitudinal) polarized scattering.

In the QCD parton model we define

$$q = q^\uparrow + q^\downarrow$$
$$\Delta q = q^\uparrow - q^\downarrow$$

(2.1)

to be the unpolarized and polarized distributions respectively. $q^\uparrow$ and $q^\downarrow$ are the distributions which describe the probability for finding quarks with spin parallel or antiparallel to the longitudinally polarized nucleon. For instance, if we use values for the factorization scale and the renormalization scale both equal to $Q^2$, we can write down the relation between the polarized structure function $g_1$ and the polarized quark ($\Delta q(x)$) and gluon distributions ($\Delta G(x)$) by
\[g_1(x, Q^2) = \int_0^1 \frac{dy}{y} \left(C_q^S(x/y, \alpha_s(t)) \Delta \Sigma(y, t) + C_q^{NS}(x/y, \alpha_s(t)) \Delta q^{NS}(y, t)\right) + 2n_f C_g(x/y, \alpha_s(t)) \Delta G(y, t),\]

where

\[\Delta \Sigma(x, t) = \sum_{i=1}^{n_f} \left(\Delta q_i(x, t) + \Delta q_i(x, t)\right),\]

\[\Delta q_{NS} = \sum_{i=1}^{n_f} \frac{e_i^2 - \langle e^2 \rangle}{\langle e^2 \rangle} (\Delta q_i(x, t) + \Delta q_i(x, t)).\]

are respectively the singlet and the non-singlet quark distributions. \(n_f\) is the number of flavors \((i)\), each of charge \(e_i\), \(\langle e^2 \rangle = \sum e_i^2 / n_f\) and \(t = \log(Q^2/\Lambda^2)\).

The singlet part of the Altarelli-Parisi evolution is given by

\[\frac{d}{dt} \Delta \Sigma(x, t) = \frac{\alpha_s(t)}{2\pi} \int_x^1 \frac{dy}{y} \left[P_{qg}^S(x/y, \alpha_s(t)) \Delta \Sigma(y, t) + 2n_f P_{qg}(x/y, \alpha_s(t)) \Delta G(y, t)\right],\]

\[\frac{d}{dt} \Delta G(x, t) = \frac{\alpha_s(t)}{2\pi} \int_x^1 \frac{dy}{y} \left[P_{gq}^S(x/y, \alpha_s(t)) \Delta \Sigma(y, t) + 2n_f P_{gq}(x/y, \alpha_s(t)) \Delta G(y, t)\right].\]

while the non-singlet part evolves independently

\[\frac{d}{dt} \Delta q_{NS}(x, t) = \frac{\alpha_s(t)}{2\pi} \int_x^1 \frac{dy}{y} P_{qq}^{NS}(x/y, \alpha_s(t)) \Delta q^{NS}(y, t).\]

At NLO the expansion in power of \(\alpha_s\) of the coefficient functions \((C)\) and of the splitting functions \((P)\) reads

\[C(x, \alpha_s) = C^{(0)}(x) + \frac{\alpha_s}{2\pi} C^{(1)}(x) + O(\alpha^2_s),\]

\[P(x, \alpha_s) = P^{(0)}(x) + \frac{\alpha_s}{2\pi} P^{(1)}(x) + O(\alpha^2_s).\]

The first moment of \(g_1(x, Q^2)\) measures the expectation value of a combination of octet and singlet axial vector currents

\[\int_0^1 g_1(x) \, dx = \frac{1}{12} a_3 + \frac{1}{36} a_8 + \frac{1}{9} a_0,\]

(2.7)
where

\[ a_8 = 3F - D \]
\[ = \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} - 2(\Delta s + \Delta \bar{s}) = 0.579 \pm 0.025; \]  

(2.8)

\[ a_3 = g_A = F + D \]
\[ = \Delta u + \Delta \bar{u} - \Delta d - \Delta \bar{d} = 1.2573 \pm 0.0028, \]  

(2.9)

\[ a_0 = \sum q \Delta_q + \Delta \bar{q} = \Delta q_8 + 3(\Delta s + \Delta \bar{s}). \]  

(2.10)

\[ \Delta q \] denotes the first moment of the corresponding distributions \( \Delta q(x) \). Under suitable assumptions [3], an analysis done in 1983 gives for the matrix elements controlling the \( \beta \)-decay of the spin 1/2 hyperons

\[ F = 0.477 \pm 0.012 \quad D = 0.756 \pm 0.01. \]  

(2.11)

In a more recent analysis (1988) [4]

\[ F = 0.46 \pm 0.01 \quad D = 0.79 \pm 0.001, \]  

(2.12)

which implies \( a_0 = 0.06 \pm 0.12 \pm 0.17 \) if the EMC data \( (\Gamma^p_[Q^2] = 10.7) = 0.128 \pm 0.013 \pm 0.019 \) or the SMC data \( (\Gamma^p_[Q^2] = 10) = 0.136 \pm 0.011 \pm 0.011 \) for the first moment of \( g_1 \) are used. This value is smaller than the value \( a_0 = 0.188 \pm 0.004 \) expected on the basis of the Ellis-Jaffe sum rule. The values of \( a_3 \) and \( a_8 \) are constraints equations which have been used in most of the LO analysis of the polarized structure functions done so far. These two constraints are obtained under different assumptions. For instance (2.9) requires \( SU(2)_f \) symmetry between the matrix elements of the charged and neutral axial currents, while (2.8) requires an \( SU(3)_f \) symmetry between matrix elements of hyperon decays of
charged and neutral weak axial currents. This last assumption has been criticized by Lipkin [13]. He has argued that the $\beta$ decay of hyperons only fix the total helicity of the valence quarks. Therefore eqs. (2.8) and (2.9), in this scenario, should be replaced by

$$a_8 = \Delta u_v + \Delta d_v$$  \hspace{1cm} (2.13)

and

$$a_3 = \Delta u_v - \Delta d_v$$  \hspace{1cm} (2.14)

respectively. These two scenarios, the standard scenario and the valence one, generate two different sets of parton distributions GRSV1 and GRSV2 in ref. [8]. The expression of these two parametrizations can be found in the original work. Notice that these two scenarios, of course, require different assumptions on the contribution to the polarization coming from the sea quarks.

In fact, the two expressions for the first moment of $g_1$, assuming the validity of the Bjorken sum rule for $g_A$ (eq. 2.9) and of $SU(3)_f$ flavour symmetry are given by,

$$\Gamma_1^p = \left[ \frac{1}{12} (F + D) + \frac{5}{36} (3F - D) + \frac{1}{3} (\Delta s + \Delta \bar{s}) \right] \left( 1 - \frac{\alpha_s(Q^2)}{\pi} \right)$$  \hspace{1cm} (2.15)

for the standard scenario, and

$$\Gamma_1^p = \left[ \frac{1}{12} (F + D) + \frac{5}{36} (3F - D) + \frac{1}{18} (10\Delta q + \Delta s + \Delta \bar{s}) \right] \left( 1 - \frac{\alpha_s(Q^2)}{\pi} \right)$$  \hspace{1cm} (2.16)

for the valence scenario. Notice that in the standard scenario (eq. 2.15), a negative $\Delta s$ has to be required in order to bring down $\Gamma_1^p$ to the experimental EMC or SMC result. As we just mentioned, the condition $\Delta s = \Delta \bar{s} = 0$ is in fact still too high to match the data.

In the valence scenario of Lipkin (eq. 2.16) - even with zero strange quark contribution - a negative $\Delta \bar{q}$ is needed in order to obtain the same reduction. In general, a finite $\Delta s(Q^2)$ is generated by the evolution equations due to the non vanishing of the NLO anomalous dimension $\Delta \gamma_{qq}$. This happens for $Q^2 > \mu^2$, where $\mu$ is the renormalization scale to NLO where the evolution starts. In GRSV $\mu^2 = 0.34$ GeV$^2$. 

6
Let’s remark, at this point, that our understanding of higher twist effects and renormalon effects at small $Q^2$, which influence the sum rules and might affect the evolution, are not yet under control from a theoretical perspective. It is estimated, though, that at $Q^2 = 2$ Gev$^2$, for instance, higher twist contributions should be of the order of 10% [14].

It should be pointed out that it is quite common in the literature to find different expressions for $\Gamma_1^p$ depending over whether the anomalous contribution is included or not in the definition of $a_0$. Notice that this amount to a redefinition of $\Delta\Sigma$ of the form

$$\Delta\Sigma \to \Delta\Sigma - \frac{\alpha_s(Q^2)}{2\pi} \Delta G$$

which modifies the contribution to $g_1$ which can be attributed to the net helicity of the quarks by an anomalous gluonic part. However, from a practical viewpoint, it is convenient to remove it from the LO expression of $\Gamma_1$ and let it reappear through the evolution equation (in the $\overline{MS}$ scheme), now known to order $\alpha_s$, as an additional term in the splitting functions. The calculation of the NLO anomalous dimensions of ref. [4] are performed in the $\overline{MS}$ scheme.

Generically, the ansatz chosen by GRSV for the polarized parton distributions is of the form

$$\Delta q(x, Q^2) \sim x^\alpha (1 - x)^\beta q(x, Q^2)$$

and a similar form for the gluon content, where the unpolarized distributions $q(x, Q^2)$ are taken from [11]. The gluon distribution is only weakly constrained since, in LO, it doesn’t appear in the expression of the asymmetries. Various arguments, based on Regge theory or, as in ref. [13], arguments based on coherence effects, allow to estimate a behaviour of the form

$$\frac{\Delta G}{G} \bigg|_{x \to 0} \sim x.$$  

Notice that the extrapolation in $x$ of the various distribution is a remarkable source of complexity in spin physics, given the fact that the measurements are performed at different values of $Q^2$ at each bin $x$ and an intermediate AP evolution to common $Q^2$ is required.
The G-S distributions in ref. [7] are generated by global fits of the form

\[ x \Delta Q = \eta A x^a (1 - x)^b (1 + \gamma x + \rho \sqrt{x}) \]  

(2.20)

where three different parametrizations are given (here denoted G-S \( a \), G-S \( b \), G-S \( c \)). They differ in the parametrization adopted for the polarized gluon distributions. Specifically

\[
\begin{align*}
G - S \ a & \quad \gamma_G = 0 \quad \rho_G = 0, \\
G - S \ b & \quad \gamma_G = -1 \quad \rho_G = 2, \\
G - S \ c & \quad \gamma_G = 0 \quad \rho_G = -3.
\end{align*}
\]  

(2.21)

We refer the reader to the original work [7] for further details. In the next section we are going to discuss the implications of these parametrizations for the measurements of the double photon cross section at RHIC.

III. NUMERICAL RESULTS

We now turn to the numerical results for polarized isolated double photon production at RHIC. It is still not clear what isolation parameters will be used in the experiments, but preliminary studies [8] seem to indicate that an energy resolution parameter of \( \epsilon = 0.5 \) GeV/\( p_T^\gamma \) will be possible. We assume a cone size of \( R = 0.5 \), where \( R \) is defined as the radius of a cone in the pseudorapidity-azimuthal angle plane via the relation

\[ R = \sqrt{(\Delta y)^2 + (\Delta \phi)^2}. \]  

(3.1)

If a parton with energy fraction greater than \( \epsilon \) times that of the photon falls into the cone, the event is rejected.

As discussed in section II, we make use of the recent polarized proton distributions evolved in NLO by Gehrmann and Stirling [4] and Glück et al [5]. For the unpolarized proton distributions, we use the CTEQ3M set [10] after checking that the GRV [11] distributions give very similar results. We also use fragmentation functions for the photon evolved in NLO.
from ref. [12]. The value of $\Lambda_{QCD}$ used is chosen to correspond with the parton distribution set used. For the electromagnetic coupling constant we use $\alpha_{em} = 1/137$ and we use the two-loop expression for $\alpha_s$. Unless otherwise stated we set all renormalization/factorization scales to $\mu^2 = ((p_{T1}^\gamma)^2 + (p_{T2}^\gamma)^2)/2$, where $p_{T1}^\gamma$ and $p_{T2}^\gamma$ are the transverse momenta of the first and second photons respectively.

In Figs.1a and 1b we compare the different parametrizations of $\Delta G(x, Q^2)$ as a function of $x$ at $Q^2 = 100 \text{ GeV}^2$. The two GRSV gluons are practically identical so we would not expect very different predictions for any cross section sensitive to $\Delta G$ from these two parametrizations. The three G-S distributions on the other hand, show much larger differences, and are thus likely to give different predictions for processes depending on $\Delta G$.

At the moment, it is proposed that RHIC will run at various centre-of-mass energies between $\sqrt{S} = 50$ to 500 GeV, thus we perform calculations for two values, $\sqrt{S} = 200$ and 500 GeV. It turns out that the NLO parametrizations for the polarized distributions do not include charm quark distributions, thus in our study we neglect the contribution for charm quarks in both the polarized and unpolarized cases. It is expected that at RHIC cms energies, the charm contribution will not be very large.

In keeping with the convention started by the WA-70 collaboration [16], and which has been used in all subsequent analyses, we place asymmetric cuts on the two photon $p_T$-s. We require that the photon with the highest transverse momentum has $p_T \geq 10 \text{ GeV}$ while the other is required to have $p_T \geq 9 \text{ GeV}$. This asymmetric cut ensures that the two-body contributions to the cross section, such as the Born contribution, are not favored over the three-body contributions. This differs from the way we calculated the cross section in ref. [17], where we looked at the $p_T$ distribution of one of the photons while placing cuts on the other. In the present way of calculating the cross section, each photon which passes the cut contributes to the cross section. This means we have two entries for each event. Latest indications are that the maximum rapidity coverage achievable at RHIC will be in the range $-2 \leq y \leq 2$, we thus restrict ourselves to this range. It is also suggested that they will not be able to reliably detect photons with $p_T \leq 10 \text{ GeV}$.
For comparison, in Fig.2a we show the $p_T$ distribution for the unpolarized case as predicted using the GRV and CTEQ3M parton distributions at $\sqrt{S} = 200$ GeV. They clearly give very similar results in the region tested, hence, in all subsequent discussions we will use only the CTEQ3M distributions. In Figs.2b and 2c we display the corresponding polarized cross sections. The results indicate that it will be difficult to measure this cross section beyond about $p_T = 20$ GeV, even with the planned high luminosities at RHIC. More interestingly, Fig.2b suggests, as we expected, that GRSV distributions give very similar results and cannot be separated by this process. Fig2c suggests, on the other hand, that the three G-S distributions give substantially different predictions, and will probably be distinguishable. The G-S $a$ parametrization gives similar results to the two GRSV versions, as may be expected from Fig1a.

In Figs.3a and 3b we show the longitudinal asymmetries as predicted by the various parametrizations. The asymmetry, $A_{LL}$, is defined by the relation

$$A_{LL} = \frac{d\Delta\sigma}{dp_T}$$

the ratio of the polarized to the unpolarized cross section, and gives a measure of the spin dependence or spin sensitivity of the process. Again the two GRSV distributions, as expected, give similar results, while the G-S ones give clearly distinguishable results. The predictions also indicate that the asymmetries are not very large in the measurable region of the cross section, varying between 5 and 8% for the GRSV and 4 and 6% for the G-S $a$ parametrization. It will require very high statistics measurements to measure this asymmetry, but this may be achievable at RHIC, as long as the cross section can be measured, given the planned detectors.

At $\sqrt{S} = 500$ GeV the situation improves somewhat with regards to the size of the cross section. Fig.4a compares the $p_T$ distributions at $\sqrt{S} = 500$ and 200 GeV for the unpolarized cross section. As expected, at higher cms energies the cross section is substantially larger. In Fig.4b we show the polarized cross section at $\sqrt{S} = 500$ GeV. On the figure we also show the full prediction given by the G-S $a$ distribution as well as the contribution from $qg$ initiated
subprocesses. In the case of G-S a the contribution for the $q\bar{q}$ initiated subprocess (not shown) is negative. Corresponding distributions are shown for the GRSV 1 parametrization. In this case the $q\bar{q}$ process gives a positive contribution but it is substantially smaller than the $qg$ one. The G-S a and GRSV 1 distributions predict similar cross sections, but the relative importance of the subprocesses is clearly very different. The most interesting aspect of this result from the point of view of sensitivity to $\Delta G$ is the fact that in both cases, the $qg$ initiated process dominates. We should mention that in this calculation we do not include contributions for the higher order process $(O(\alpha_s^2)) \, gg \rightarrow \gamma\gamma$, preferring to keep consistently to $O(\alpha_s)$.

Figs.4c and 4d show the asymmetries as predicted by the G-S a and GRSV polarized distributions. Included also are the contributions to the asymmetries from the $qg$ and $q\bar{q}$ initiated processes. In Fig.4d we also include the asymmetry in LO where we have used the LO counterpart of the GRSV 1 parametrization. The LO cross section consists only of the process $q\bar{q} \rightarrow \gamma\gamma$ plus the fragmentation processes $q\bar{q} \rightarrow g\gamma$ plus $qg \rightarrow q\gamma$. It can be said that the asymmetry is fairly stable under the higher order corrections, but the corrections are clearly significant. As we expected from the results shown in Fig.4b, the NLO asymmetry is dominated by the $qg$ initiated process. this is the case when either distributions is used. On the other hand, the size of the asymmetry is not very different from that at $\sqrt{S} = 200$ GeV, varying between 3 and 8% over the measurable range.

As an indication of the stability of our predictions we show the scale dependence of the unpolarized cross section in Fig.5. The cuts and distributions are the same as for the solid curve in Fig.4a. By varying all renormalization and factorization scales in the range $1/2 \leq n \leq 2$, where $\mu = n((p_T^1)^2 + (p_T^2)^2)/2$, we can vary the cross section by as much as 20% in the region $10 \leq p_T \leq 35$. This is still a substantial uncertainty in the predictions, but it is a significant reduction over the corresponding figure of 60% for the LO predictions.

Finally in Fig.6 we show the K-factor as a function of $p_T$ where,

$$ K = \frac{\frac{d\sigma(LO)}{dp_T}}{\frac{d\sigma(NLO)}{dp_T}}. \quad (3.3) $$
at $\sqrt{S} = 500$ GeV, for the unpolarized cross section. This confirms that the NLO corrections are indeed quite significant at the $p_T$ values relevant at RHIC.

IV. CONCLUSIONS

An $O(\alpha_s)$ NLO calculation of the process $pp \to \gamma\gamma + X$ was presented, where for the first time polarized parton distributions evolved in NLO QCD is used. The two photons were also isolated using plausible isolation parameters. We found that the cross section for the process will not be very large at RHIC, particularly at the lower CMS energies, but it will nevertheless still be measurable in the lower $p_T^\gamma$ region. The new polarized parton distributions predict an asymmetry of between 3 and 8% in the measurable region for the process. While the asymmetry cannot be said to be large, given sufficiently good statistics, it should still be measurable. Given this possibility, we found that a discrimination between extreme parametrizations of the polarized gluon distribution $\Delta G$ should be possible. This is due in particular to the dominance of the subprocess $qg \to \gamma\gamma q$ over the $q\bar{q}$ annihilation process. We thus conclude that this process, if studied at RHIC could prove useful for supplementing information on the polarized distributions obtained from other sources.

V. ACKNOWLEDGEMENTS

We would like to thank Bob Bailey for help with the programing aspects of the Monte Carlo method used in this calculation. We thank G. Ramsey, A. Yokosawa for discussions. C.C. is grateful to Sang Hyeon Chang for helpful discussions. This work supported in part by the U.S. Department of Energy, Division of High Energy Physics, Contract W-31-109-ENG-38 and DEFG05-86-ER-40272.
REFERENCES

[1] J. Ashman et al., EMC collaboration, Phys. Lett. B206 (1988) 364, Nucl. Phys. B328 (1989) 1.

[2] SMC Collaboration, B Adeva et al. Phys. Let. B302 (1993) 533, ibidem B320 (1994) 400; SMC Collaboration, D. Adams et al., Phys. Lett. B329 (1994) 399; E143 Collaboration, K. Abe et al., SLAC-PUB-6508 (1994) preprint; E143 Collaboration, R. Arnold et al., presented at ICHEP94, Glasgow, August 1994.

[3] WA2 Collaboration, Z. Phys. C 21 (1983), 27; Particle Data Group, Phys. Lett. B111 1982.

[4] S. Y. Hsu et al., Phys. Rev. D 38 2056 (1988).

[5] R. Mertig and W. L. Van Neerven NIKHEF-H/95-031. W. Vogelsang, Rutherford Preprint, RAL-TR-95-071.

[6] A. Yokosawa, Private Communication.

[7] T. Gehrmann and W. J. Stirling, Durham Preprint DTP/95/82.

[8] M. Glück, E. Reya, M. Stratmann and W. Vogelsang, Dortmund Preprint DO-TH 95/13 and Rutherford Preprint RAL-TR-95-042.

[9] G. Altarelli and G. Parisi, Nucl. Phys. B 126 (1977) 298.

[10] H. L. Lai et al., CTEQ Collaboration, Phys. Rev. D51, 4763 (1995).

[11] M. Glück, E. Reya and A. Vogt, Z. Phys. C 67 433 (1995).

[12] M. Glück, E. Reya and A. Vogt, Phys. Rev. D48, 116 (1993).

[13] H.J. Lipkin, Phys. Lett. B 256, 284 (1991); B 337, 157 (1994).

[14] X. Ji, MIT-CTP-2411 hep-ph/9502288.
[15] S. J. Brodsky and I. Schmidt, Phys. Lett. B 234, 144, (1990); S. J. Brodsky, M. Burkhardt and I. Schmidt, Nucl. Phys. B441, 197 (1990).

[16] E. Bovin et al., Z. Phys. C41, 591 (1989).

[17] C. Corianò and L. E. Gordon Argonne Preprint, ANL-HEP-PR-95-84 and IFT-UFL-95-28.
Figure Captions

[1] (a) The polarized gluon distribution in NLO as a function of $x$ at $Q^2 = 100$ GeV$^2$ for the G-S $a$ (solid line) GRSV1 (dashed line) and the GRSV2 (dotted line). (b) Same as (a) but for the G-S $a$, G-S $b$ and G-S $c$ parametrizations.

[2] (a) The unpolarized cross section $d\sigma/dp_T^\gamma$ for $-2 \leq y^\gamma \leq 2$ as a function for $p_T^\gamma$ at $\sqrt{S} = 200$ GeV as predicted by the GRV (solid line) and CTEQ3M (dashed line) parametrizations of the proton distribution functions. (b) Same as (a) but for the polarized cross section as given by the GRSV1 (solid line) and GRSV2 (dashed line) parametrizations of the polarized parton distributions. (c) Same as (b) but for the G-S $a$ (solid line) G-S $b$ (dashed line) and G-S $c$ (dot dashed) line. The GRSV1 (dotted line) is included for comparison.

[3] (a) The longitudinal asymmetries for the cross sections displayed in Fig.2b. (b) Same as (a) but for the G-S polarized distributions. The unpolarized distributions used is CTEQ3M.

[4] (a) Comparison of the unpolarized cross section $d\sigma/dp_T^\gamma$ as a function of $p_T^\gamma$ for $-2 \leq y^\gamma \leq 2$ using the CTEQ3M proton distributions, at $\sqrt{S} = 200$ (dashed line) and $\sqrt{S} = 500$ GeV (solid line). (b) The polarized cross section at $\sqrt{S} = 500$ GeV using the G-S $a$ (solid line) and GRSV1 (dot dashed line). Included for comparison are the contributions from the $qg$ initiated process. In the case of G-S $a$ it is the dashed line and for GRSV1 the dotted line. (c) The asymmetry predicted by the G-S $a$ parametrization for the cross section given in (b). Included for comparison are the asymmetries as given by the $qg$ (dashed line) and $q\bar{q}$ (dotted line) initiated processes. (d) Same as (c) but for the GRSV1 distributions. The LO prediction for the asymmetry (dot dashed line) using the LO version of the GRSV1 distribution is included.
[5] The renormalization/factorization scale ($\mu$) dependence for the unpolarized cross section given in Fig.4a, for three different choices of scale, $\mu = n((p_{T1}^2)^2 + (p_{T2}^2)^2)/2$: 0.5, 1.0, and 2.

[6] The K-factor for the unpolarized cross section given in Fig.4a at $\sqrt{S} = 500$ GeV, where $K = d\sigma(LO)/dp_T^\gamma/d\sigma(NLO)/dp_T^\gamma$. 

$Q^2 = 100 \text{ GeV}^2$

- G–S a
- GRSV 1
- GRSV 2

$\Delta G(x, Q^2)$

$x$

Fig. 1a
$Q^2 = 100 \text{ GeV}^2$

$\Delta G(x, Q^2)$

$0.005 \ 0.01 \ 0.05 \ 0.1 \ 0.5$

Fig. 1b
$d\sigma/dp_T^\gamma$ (pb/GeV)

$-2 < y^\gamma < 2$
unpolarized

Fig. 2a
$-2 < y^\gamma < 2$

d$p_{T} (\text{GeV})$

d$\sigma / dp_{T} (\text{pb} / \text{GeV})$

GRSV 1
GRSV 2

Fig. 2b

polarized
Fig. 2c

-2 < y^\gamma < 2
polarized
\[ \frac{d\sigma}{dp_T^\gamma} \text{ (pb/GeV)} \]

-2 < y^\gamma < 2
unpolarized

\[ \sqrt{S} = 500 \text{ GeV} \]
\[ \sqrt{S} = 200 \text{ GeV} \]

Fig. 4a
$\sqrt{s} = 500$ GeV

$\frac{d\sigma}{dp_T^\gamma}$ (pb/GeV)

-2 < $y^\gamma$ < 2 polarized

Fig. 4b
\[ \sqrt{S} = 500 \text{ GeV} \]

- **full (G–S a)**
- **q \overline{q}**
- **q g**

![Graph](image-url)

*Fig. 4c*
\[ \sqrt{S} = 500 \text{ GeV} \]

- full (GRSV 1)
- \( q \bar{q} \)
- \( qg \)
- \( \text{LO} \)

Fig. 4d
\( \sqrt{S} = 500 \text{ GeV} \)

- \( n = 1 \)
- \( n = 2 \)
- \( n = 1/2 \)

\(-2 < y^\gamma < 2\)

unpolarized

\( \frac{d\sigma}{dp_T^\gamma} \) (pb/GeV)

\( p_T^\gamma \) (GeV)

Fig. 5
$\sqrt{S} = 500$ GeV

$-2 < y^\gamma < 2$

unpolarized