Planning and managing resources of transport enterprises

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Abstract. The paper considers a new approach to planning and forming a stock of spare parts necessary for the smooth functioning of transport enterprises. The basis of the author's approach is the developed simulation model of the warehouse adapted for any type of enterprise that serves and (or) operates road transport. The need for spare parts in this model is defined as a mixture of probability distributions of demand of various kinds, which makes it universal. The model was based on the new patterns of influence of following factors on the size of the stock revealed by the authors: the statistical indicators of the need for spare parts, the type and location of the enterprise, the target level of reliability of the supply system, etc. The proposed approach allows connecting the main indicators of the resource management effectiveness: the shortage rate, the cost of stock, the level of system reliability. It allows planning and forming a stock of spare parts reasonably. It is expected that the proposed approach will increase the efficiency of transport enterprises and the degree of satisfaction of consumers of their services.

1 Introduction

The main reason that does not allow finally solving the task of managing warehouses of spare car parts is an open question of forecasting demand for them. Known methods of forecasting on the basis of building of trends, taking into account seasonal fluctuations and other factors, do not lead to satisfactory results in this case. Methodologies based on the theory of reliability are very complex and time-consuming. And, as a rule, it is impossible to collect enough statistical data to use them in practice. The analysis of numerous data showed that the consumption of spare parts in enterprises serving or operating road transport in most cases cannot be described using known distribution laws of random variables. This is associated with the statistical heterogeneity of data on the consumption of spare parts. Depending on the type of enterprise, demand for them can represent a mixture of different numbers of probability distributions [1]. Figure 1 shows the typical dynamics of consumption of one position of spare parts at the enterprise of the automobile dealer of the “KamAZ” in Krasnoyarsk. As can be seen from the graph, not only one or two parts but also any other number up to 50 units can be sold from the warehouse in one day. This indicates the existence of different categories of consumers. But the task of managing stocks cannot be considered solved by formalizing the process of forecasting the need for

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car parts. Having received quantitative estimates of demand, it remains to determine how much and when to order spare parts for the warehouse in order to always have the optimal stock size. This optimization problem would successfully solve the revealed functional dependence between the efficiency indicators of warehouse management: the average size of the warehouse stock and the level of service. In practice, it would make it possible to determine the necessary stock sizes to achieve the required level of service without recourse to complex calculations or modeling [5].

Fig. 1. Dynamics of sales of sump gaskets of the KamAZ.

2 Methods

A schematic diagram of the modeling of the management process of the spare parts warehouse is shown in Fig. 2. The array of raw data (gray blocks in Figure 2) is: statistics of the consumption of stocks from the warehouse; time of filling an order for replenishment of stocks \( t \); the allowable level of shortage or size of the stock, and the level of significance \( \alpha \) for these parameters.

![Diagram](image)

Fig. 2. Scheme of simulating the management process of the spare parts warehouse.
Based on the results of the ABC-analysis, a method for forecasting the need for spare parts is chosen. For spare car parts belonging to group A (giving 80% of turnover), the demand is expressed as the density equation for the mixture of probability distributions:

\[ f(x) = p_1 f_1(x) + p_2 f_2(x) + \ldots + p_n f_n(x), \]  

(1)

where \( p_1, p_2, \ldots, p_n \) — a priori probability of occurrence of an event of a certain component of the mixture (consumer class);

\( f_1(x), f_2(x), \ldots, f_n(x) \) — distribution density function of the components of the mixture;

\( n \) — number of components of the mixture.

In turn, \( p_1, p_2, \ldots, p_n \) can also obey a distribution law of random variables.

The scope of use of the proposed method is limited by the identifiability of the components in the mixture of distributions. For this problem, the most suitable discrete probability distribution is the Poisson distribution. Its use in practice is limited by the conditions for the occurrence of a Poisson stream of random events [1].

![Fig. 3. Distribution of demand for spare parts for different consumer groups.](image)

If the random events of the emergence of the consumers of spare parts for a certain period of time obey the Poisson distribution law with the parameter \( \lambda \), and the number of items that they buy by themselves obeys the Poisson distribution law with the parameter \( \lambda' \), then the probability of occurrence of \( x \) consumers for each group is determined by the formulas:

\[ p_1(x) = \frac{e^{-\lambda_1} \cdot \lambda_1^x}{x!} , \quad p_2(x) = \frac{e^{-\lambda_2} \cdot \lambda_2^x}{x!} , \quad \ldots , \quad p_k(x) = \frac{e^{-\lambda_k} \cdot \lambda_k^x}{x!} , \]  

(2)

where \( \lambda_1, \lambda_2, \ldots, \lambda_k \) — parameters of probability distribution functions of the emergence of consumers of the first, second, ..., k-th group, respectively;
x — number of consumers.

And the probability of buying y items in one transaction by the consumers of the first, second, ..., k-th group: \( p_1(y), p_2(y), \ldots, p_k(y) \), are respectively equal to:

\[
p_1(y) = \frac{e^{-\lambda_1} \cdot \lambda_1^y}{y!}, \quad p_2(y) = \frac{e^{-\lambda_2} \cdot \lambda_2^y}{y!}, \quad \ldots, \quad p_k(y) = \frac{e^{-\lambda_k} \cdot \lambda_k^y}{y!},
\]

where \( \lambda_1, \lambda_2, \ldots, \lambda_k \) — parameters of distribution functions for the number of purchased spare parts by consumers of the first, second, ..., k-th group, respectively;

\( y \) — number of purchased parts by one consumer in one time.

Then the density function of the mixture of probability distributions of the need for spare parts can be represented in the form:

\[
f(x) = \sum_{q=1}^{m} \left[ \prod_{j=1}^{k} \prod_{i=1}^{n} p(n_{ij}) \cdot p(S_{ij}) \right]_q,
\]

where \( m \) — the number of all possible variants of sets of values of \( S_{ij} \);

\( p_j \) — the probability of appearance of only the i-th number of consumers of the j-th group for a certain period of time;

\( p(S_{ij}) \) — the probability of buying S items by the i-th user of the j-th group at a time.

Taking into account that the probability of the appearance of a certain number of consumers and the buying of a certain number of parts by them is a subject to Poisson's law, expression (4) can be represented in the form:

\[
f(x) = \sum_{q=1}^{m} \left( \prod_{j=1}^{k} \prod_{i=1}^{n} \frac{e^{-\lambda_j} \cdot \lambda_j^{z_j}}{z_j!} \cdot f(S_j) \right)_q,
\]

where \( z_j \) — number of consumers of the j-th group;

\( \lambda_j \) — parameter of the Poisson's probability distribution of the appearance of a certain number of consumers of the j-th group for a certain period of time.

A detailed proof of the formula (5) was considered by the authors in \[1\].

Determination of the values of the parameters of the mixture function of the distribution of random values of demand occurs as follows: the range of permissible values for each of the parameters \( \lambda_1, \lambda_2, \ldots, \lambda_k, \lambda_1', \lambda_2', ..., \lambda_k' \) is specified. Using a sequential search with the iteration step \( h \) in the range of admissible values \( \lambda_1 \) and \( \lambda_1' \), taking \( \lambda_2 = 0, \lambda_3 = 0, \ldots, \lambda_k = 0, \lambda_2' = 0, \lambda_3' = 0, \ldots, \lambda_k' = 0 \), the initial approximations \( \lambda_1 \) and \( \lambda_1' \) at which the deviation between the modeled and the actual distribution of demand will be minimal are obtained. In the same way, the approximate values of the remaining parameters are sequentially obtained. Reducing the range of parameters and the step in half, so that the obtained approximate values of the parameters become the middle of new intervals; further, we specify \( \lambda_1, \lambda_2, \ldots, \lambda_k, \lambda_1', \lambda_2', \ldots, \lambda_k' \) until the deviation reaches the specified value of the accuracy \( \zeta \).

Having obtained the necessary values of the parameters of the demand function, we generate random values of demand from the obtained theoretical function by the inverse transformation method and obtain the forecast of the need for spare parts.

Next, the problem arises of comparing simulated demand values and actual demand data. To do this, we will use a reliable approach to comparison of the simulated values of the actual data described in \[3\]. We choose \( m \) independent sets of data on the sale of spare parts and \( n \)
independent generated demand forecast values from the function of the mixture of probability distributions. Let $X_j$ be the average value of the observations in the warehouse dataset, and $Y_j$ - the average value in the model dataset. In addition, $X_j$ are independent and identically distributed quantities with mean value $x = E(X_j)$, and $Y_j$ are independent and identically distributed quantities with mean value $y = E(Y_j)$. We will compare the model with the real system, creating a confidence interval for $\zeta = x - y$. The reasons why the creation of a confidence interval for $\zeta$ is more preferable than testing the null hypothesis $H_0 (x = y)$ is also given in [3].

After this, a 100 $(1 - \alpha)$-percent confidence interval for $\zeta$ is constructed using the two-sided $t$-test method. Let $l(\alpha)$ and $u(\alpha)$ be the upper and lower endpoints of the confidence interval, respectively. If $0 \in [l(\alpha), u(\alpha)]$, any observed difference will not be statistically significant at the level $\alpha$ and can be explained by selective fluctuations.

The theoretical need for spare parts obtained in the manner described above requires verification of the consistency of the chosen distribution with the empirical one. To implement the procedure for assessing the consistency of the selected distribution with the empirical distribution in the automatic mode, one can use the Kolmogorov-Smirnov fitting criterion. The problem is that these requirements for spare parts are not continuous random variables, as are their distribution laws. The solution to this problem is seen in the use of the Smirnov’s transformation method and the method of transition from the discrete distribution function of the discrete values of the need for spare parts to continuous one [4]. This method consists in having the distribution function of some discrete random variable $F(x)$ and assuming that $Y_1, Y_2, ..., Y_n$ — $n$ independent realizations of a random variable uniformly distributed on $[0, 1]$.

It is also possible to check the degree of consistency of empirical and theoretical distributions by the Kolmogorov-Smirnov criterion by using the operator “kstest2” in the MATLAB program.

In the case of not a refutation of the hypothesis of fitting at the required level of significance, random values of the demand for spare parts are generated from the selected theoretical distribution - a demand forecast is compiled. For those details to which it is not possible to select the theoretical demand function with a given accuracy, forecasting techniques using known methods for approximating statistical data or the empirical distribution of demand can be used.

### 3 Results

During the implementation of the simulation process described above for the functioning of the spare parts warehouse, the dependencies between the indicators of its management effectiveness were obtained: the shortage level and the average stock size. Figure 4 presents the results of an imitation experiment with the following management system parameters: the time of filling an order for replenishment of stocks $t = 7$ days, the significance level $\alpha = 0.03$. Since the input data of the model - the demand is a stochastic value, then the output data are estimates of management effectiveness indicators. Comparing the given limit for these indicators with the average values of their estimates would not be correct, since the variance can be very significant. Therefore, under the size of the allowable shortage or the size of the stock, we mean some of their values, which with a certain probability will not be exceeded. To this end, for the values of the management effectiveness indicators (shortage and stock size) obtained during the simulation, a $(100-2\alpha)$-percent confidence interval is determined, where $\alpha$ is the level of significance. Thus, using the obtained optimal values of the parameters of the warehouse management system, the shortage (or the size of the stock) does not exceed the set value at the level of significance $\alpha$. 
The following relationship between the average stock (n) and shortage (d) was established experimentally:

\[ n = a \cdot \exp(b \cdot d) \]  

(6)

where \( a \) and \( b \) — coefficients that depend on the demand for a particular part and the parameters of the warehouse management system.

In this case, it was found that the coefficient \( b \) for spare car parts within the error margin can be taken as \(-0.033\). Coefficient \( a \) is a function of the average demand for the spare part \( u \), the standard deviation of demand \( \sigma \), the time of filling the order for replenishment of the stock \( t \), and the given level of significance \( \alpha \): \( a = f(u, \sigma, t, \alpha) \).

Thus, the relationship between the average stock and the shortage for spare parts can be represented in the form \( n = \exp(-0.033 \cdot d) \cdot 100\% \), if the parameters \( n \) and \( d \) are expressed in percentages (as shown in Figure 4) or in the form \( n = \exp(-3.3 \cdot d) \), if \( n \) and \( d \) are expressed in fractions.

An example of the results of simulation modeling at the time of filling the order for replenishment of stocks \( t = 7 \) days and significance level \( \alpha = 0.01 \) is presented in Table 1.

As can be seen from Table. 1, at identical values of \( t \) and \( \alpha \), spare parts with approximately equal variances of demand have coefficients a proportional to the average value of demand \( u \). At the same time, for spare parts with approximately equal average demand values \( u \), the values of the coefficient \( a \) turned out to be proportional to the squared coefficient of variation in demand \( v^2 \). Dividing the coefficient \( a \) by \( (u \cdot v^2) \), we get a new parameter that depends only on the parameters of the warehouse management system \( t \) and \( \alpha \). In order to simplify the use of the developed methodology in practice, it is proposed to introduce the coefficient \( K_{t,\alpha} = a/(u \cdot v^2) \) determined from the values of time of filling the order for replenishment of the stock \( t \) and the given level of significance \( \alpha \). The values of the coefficient \( K_{t,\alpha} \) obtained in the course of the simulation experiment are given in Table 2.

In other words, the obtained functional dependence links financial investments in the spare parts warehouse and the level of satisfaction of the demand for them. According to the statistics
of consumption of spare parts, the above technique allows receiving estimates of the parameters of the effectiveness of warehouse management.

Having selected the appropriate coefficient $K_{t,0.05} = 3.6$ from Table 2 based on the average sales values per day $u$ and their variation coefficients $v$, we determine the value of the parameter $a$:

$$a = K_{t,\alpha} \cdot u \cdot v^2,$$

(7)

Table 1. Results of the simulation experiment at the time of filling the order to replenish the stock $t = 7$ days and level of significance $\alpha = 0.01$.

| N  | Name of the KamAZ parts                        | a    | b                | The average value of sales per day, pcs. | Mean square deviation of demand | Coefficient of variation in demand $v$ | Coefficient $K_{t,\alpha} = a/(u \cdot v^2)$ |
|----|------------------------------------------------|------|------------------|--------------------------------------|----------------------------------|--------------------------------------|---------------------------------------------|
| 1  | Shock absorber P-3205                           | 70,4 | -0,034           | 1,083                                | 3,972                            | 3,667                                | 4,833                                       |
| 2  | Front shock absorber “GZAA” of cab              | 29,9 | -0,029           | 0,690                                | 2,065                            | 2,992                                | 4,840                                       |
| 3  | Bolt fastening the drive shaft of the front axle| 352  | -0,033           | 1,073                                | 8,973                            | 8,359                                | 4,693                                       |
| 4  | Clutch master cylinder and union assembly        | 35,9 | -0,032           | 0,617                                | 2,193                            | 3,555                                | 4,604                                       |
| 5  | EURO-1.2.3 head cylinder assembly                | 57   | -0,031           | 0,844                                | 3,227                            | 3,822                                | 4,621                                       |
| 6  | Clutch driven plate assembly                     | 47,8 | -0,031           | 1,156                                | 3,421                            | 2,960                                | 4,720                                       |
| 7  | Clutch pressure plate with casing assembly       | 20,2 | -0,033           | 0,483                                | 1,434                            | 2,969                                | 4,743                                       |
| 8  | Bypass valve of high-pressure fuel pump          | 108,2| -0,034           | 1,284                                | 5,352                            | 4,167                                | 4,850                                       |
| 9  | Brake shoe                                      | 87,5 | -0,034           | 0,721                                | 3,631                            | 5,037                                | 4,785                                       |

Table 2. Values of the coefficient $K_{t,\alpha}$.

| $\alpha$ \ $t$ | 3 days | 5 days | 7 days | 9 days | 11 days |
|----------------|--------|--------|--------|--------|---------|
| 0.01           | 3.5    | 4.2    | 4.7    | 5.4    | 5.7     |
| 0.03           | 3.0    | 3.6    | 4.0    | 4.6    | 4.9     |
| 0.05           | 2.7    | 3.3    | 3.6    | 4.1    | 4.4     |
| 0.1            | 2.5    | 3.0    | 3.3    | 3.8    | 4.0     |
| 0.2            | 2.1    | 2.5    | 2.8    | 3.2    | 3.4     |

Examples of calculating the coefficient $a$ according to the statistics of sales of spare parts of the Krasnoyarsk automobile dealer of the KamAZ are shown in Table. 3.

The method for calculating the parameters of the spare parts warehouse management system is shown in Fig. 5. Let us suppose that it is necessary to determine with 95% reliability level, what is the average stock of “Bypass valves of high-pressure fuel pump”, so that their shortage in the warehouse does not exceed 2%.

From Tables 2 and 3 by the formula (7), we obtain:

$$a = 3.6 \cdot 1,284 \cdot 4,167^2 = 80.3.$$

Then, according to the formula (6), we find:

$$n = 80.3 \cdot exp(-0.033 \cdot 2) = 75.$$
Table 3. An example of calculating the coefficient $a$ from sales data ($t = 7$ days and $\alpha = 0.01$).

| N | Name of the KamAZ parts | 2.01 | 3.01 | 4.01 | 5.01 | 8.01 | 9.01 | 10.01 | ... | 29.12 | u | v | a |
|---|-------------------------|------|------|------|------|------|------|-------|-----|-------|---|---|---|
| 1 | Bypass valve of high-pressure fuel pump | 0 | 1 | 5 | 0 | 0 | 2 | 0 | ... | 1 | 1.284 | 4.167 | 100.2 |
| 2 | Clutch release sleeve | 2 | 1 | 1 | 0 | 2 | 0 | 1 | ... | 1 | 0.58 | 3.49 | 86.2 |
| 3 | Water pump assembly | 0 | 1 | 0 | 2 | 0 | 1 | 0 | ... | 0 | 6.23 | 3.78 | 19.2 |
| 4 | Spray nozzle “YAZDA” | 0 | 0 | 40 | 0 | 4 | 8 | 10 | ... | 2 | 424.0 | 19.2 |

Fig. 5. Calculation of the efficiency parameters of the management system of the spare car parts warehouse.

By integrating the values obtained for each item of stored spare parts and materials into the entire stock list, it is possible to calculate the cost and quantity of the entire stock for the given system parameters.

In practice, the opposite task often arises - knowing the financial capabilities of the company, determine the minimum achievable shortage level for a particular cost of stocks. For example, by setting the average number of details of the “Water pump assembly” in the warehouse $n = 15$, we will determine the minimum achievable shortage level. Let us suppose that the time of filling the order for replenishment of stocks $t = 9$ and the level of significance $\alpha = 0.01$. Then, $K_{9,0.01} = 5.4$, and $a = 5.4 \cdot 0.58 \cdot 2.66^2 = 22.2$.

Next, we define $d$ by expressing it from formula (6):
Thus, it is possible to state with a probability of 99% that having an average of 15 “Water pumps assembly” in stock at the given demand indicators (u=0.58, v=2.66) and the time of filling the order for replenishment of stocks of 9 days, the minimum achievable level of the shortage is 11.9%.

4 Conclusions

The paper describes the methodology for forecasting the need for spare parts at transport enterprises on the basis of the function of a mixture of probability distributions of demand for them.

This methodology is implemented in the simulation model of warehouse to reveal a new pattern between the size of the stock and the following parameters: statistical indicators of the need for spare parts, type and location of the enterprise, a given level of reliability of the supply system (or level of significance), etc.

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