Lepton polarization in $B \to K_1 \ell^+ \ell^-$ Decays

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Abstract

We study the single and double lepton polarization asymmetries in the semileptonic $B$ meson decays $B \to K_1(1270)\ell^+\ell^- \ell \equiv e, \mu, \tau$, where the strange $P$-wave meson, $K_1(1270)$, is the mixtures of the $K_{1A}$ and $K_{1B}$, which are the $1^3P_1$ and $1^1P_1$ states, respectively. The lepton polarization asymmetries show relatively strong dependency in the various region of dileptonic invariant mass. The lepton polarization asymmetries can also be used for determining the $K_1(1270)$–$K_1(1400)$ mixing angle, $\theta_{K_1}$ and new physics effects. Furthermore, it is shown that these asymmetries in $B \to K_1(1270)\ell^+\ell^-$ decay compared with those of $B \to K^* \ell^+\ell^-$ decay are more sensitive to the dileptonic invariant mass.

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1 Introduction

The rare flavor-changing neutral-current (FCNC) processes are used to test the predictions of Standard Model (SM) at loop level and for searching new-physics (NP). In this regard, $b \to s (d)$ and $\mu \to e$ transitions have been studied to check predictions of SM at loop level and to look at the NP via their indirect effects where the direct productions are not accessible at present collider experiments. Semileptonic and radiative $B$ decays involving a vector or axial vector meson have been observed by BABAR, BELLE and CLEO. For $B \to K^* \ell^+ \ell^-$ decays, the forward-backward asymmetry has been measured by BABAR [1] and BELLE [2]. Recently, BABAR [3, 4, 5] has reported the measurements for the longitudinal polarization fraction and forward-backward asymmetry (FBA) of $B \to K^*(892) \ell^+ \ell^-$, and for the isospin asymmetry of $B^0 \to K^{*0}(892) \ell^+ \ell^-$ and $B^\pm \to K^{*\pm}(892) \ell^+ \ell^-$ channels. The data may challenge the signs of Wilson coefficients, for instance, $C_7^{eff}$. In order to extract the magnitudes and arguments of the effective Wilson coefficients, one may measure various observables in various inclusive and exclusive rare processes. In this regard, the studies of asymmetries, which are less sensitive to the hadronic uncertainties than the branching ratio, are favored. The studies of inclusive and exclusive rare processes as well as various asymmetries should be considerably improved at LHCb. The radiative $B$ decay involving the $K_1(1270)$, the orbitally excited ($P$-wave) state, is recently observed by Belle and other radiative and semileptonic decay modes involving $K_1(1270)$ and $K_1(1400)$ are hopefully expected to be observed soon. Some studies for $B \to K_1 \ell^+ \ell^-$ involving formfactors, branching ratio and forward-backward (FB) asymmetries of semileptonic decay modes have been made recently [6, 7, 8, 9]. In present work, we study the single and double lepton polarization asymmetries in the $B \to K_1(1270) \ell^+ \ell^-$ decays. These studies are complimentary to the studies of branching ratio and FB asymmetries. Note that, just like $B \to K^*(892) \ell^+ \ell^-$ decays [10, 11, 12, 13, 14, 15, 16, 17, 18, 19], $B \to K_1 \ell^+ \ell^-$ decays can be studied for the NP effects, however, these are much more sophisticated due to the mixing of the $K_1A$ and $K_{1B}$, which are the $1^3P_1$ and $1^1P_1$ states, respectively. The physical $K_1$ mesons are $K_1(1270)$ and $K_1(1400)$, described by [9]

$$
\begin{pmatrix}
|K_1(1270)\rangle \\
|K_1(1400)\rangle
\end{pmatrix} = M \begin{pmatrix}
|K_{1A}\rangle \\
|K_{1B}\rangle
\end{pmatrix}, \quad M = \begin{pmatrix}
\sin \theta_{K_1} & \cos \theta_{K_1} \\
\cos \theta_{K_1} & -\sin \theta_{K_1}
\end{pmatrix}. \tag{1}
$$

The mixing angle $\theta_{K_1}$ was estimated to be $|\theta_{K_1}| \approx 34^\circ \vee 57^\circ$ in Ref. [20], $35^\circ \leq |\theta_{K_1}| \leq 55^\circ$ in Ref. [21], $|\theta_{K_1}| = 37^\circ \vee 58^\circ$ in Ref. [22], and $\theta_{K_1} = -(34 \pm 13)^\circ$ in [9, 23]. In this study we will use the results of Ref. [9, 23] for numerical calculations.

The paper includes 5 sections: In section 2, we recall the effective Hamiltonian for $B \to K_1(1270) \ell^+ \ell^-$ decays. In section 3 we recall the calculations of effective Hamiltonian. In section 4, single and double lepton polarization asymmetries are derived, respectively. In section 5, we examine the sensitivity of these physical observable to the invariant dileptonic mass and our conclusion.

2 The effective Hamiltonian

Using the QCD corrected effective Hamiltonian, the matrix element $b \to s \ell^+ \ell^-$ can be written as:
\[ M(b \rightarrow s\ell^+\ell^-) = \frac{G_F\alpha}{\sqrt{2\pi}} V_{tb} V_{td}^* \left\{ \begin{array}{c} c_9^{ij} [d\gamma_\mu Lb] [\bar{\ell}\gamma^\mu \ell] \\ + c_{10} [d\gamma_\mu Lb] [\bar{\ell}\gamma^\mu\gamma^5 \ell] \\ - 2m_c c_i^{ij} [d\sigma_{\mu\nu} F_5^\mu Rb] [\bar{\ell}\gamma^\mu \ell] \end{array} \right\} \]

(2)

where \( c_i \) are Wilson coefficients calculated in naive dimensional regularization (NDR) scheme at the leading order (LO), next to leading order (NLO) and next-to-next leading order (NNLO) in the SM[25]–[32]. \( c_9^{\text{eff}}(\hat{s}) = c_9 + Y(\hat{s}) \), where \( Y(\hat{s}) = Y_{\text{pert}}(\hat{s}) + Y_{\text{LD}} \) contains both the perturbative part \( Y_{\text{pert}}(\hat{s}) \) and long-distance part \( Y_{\text{LD}}(\hat{s}) \). \( Y(\hat{s})_{\text{pert}} \) is given by [25]

\[
Y_{\text{pert}}(\hat{s}) = g(\hat{m}_c, \hat{s}) c_0 \\
\quad - \frac{1}{2} g(1, \hat{s})(4\bar{c}_3 + 4\bar{c}_4 + 3\bar{c}_5 + \bar{c}_6) - \frac{1}{2} g(0, \hat{s})(\bar{c}_3 + 3\bar{c}_4) \\
\quad + \frac{1}{2} (3\bar{c}_3 + \bar{c}_4 + 3\bar{c}_5 + \bar{c}_6),
\]

(3)

with \( c_0 \equiv \bar{c}_1 + 3\bar{c}_2 + 3\bar{c}_3 + \bar{c}_4 + 3\bar{c}_5 + \bar{c}_6 \),

(4)

and the function \( g(x, y) \) defined in [25]. Here, \( \bar{c}_1 - \bar{c}_6 \) are the Wilson coefficients in the leading logarithmic approximation. The relevant Wilson coefficients are given in Refs. [10]. \( Y(\hat{s})_{\text{LD}} \) involves \( B \rightarrow K_1 V(\bar{c}c) \) resonances [26, 33, 34], where \( V(\bar{c}c) \) are the vector charmonium states. We follow Refs. [26, 33] and set

\[
Y_{\text{LD}}(\hat{s}) = - \frac{3\pi}{\alpha_{\text{em}}^2} c_0 \sum_{V = \psi(1s), \ldots} \kappa_V \hat{m}_V B(V \rightarrow \ell^+\ell^-) \hat{\Gamma}_V^{\text{tot}} \]

(5)

where \( \hat{\Gamma}_V^{\text{tot}} \equiv \Gamma_V^{\text{tot}}/m_B \) and \( \kappa_V \) takes different value for different exclusive semileptonic decay. This phenomenological parameters \( \kappa_V \) can be fixed for \( B \rightarrow K^*\ell^+\ell^- \) decay by equating the naive factorization estimate of the \( B \rightarrow K^*V \) rate and the experimental measured results[10]. Except for the branching ratio of \( B \rightarrow J/\Psi K_1(1270) \)[24], there is no experimental results on \( B \rightarrow K_1 V(\bar{c}c) \). Thus, we will use the results of \( B \rightarrow K^*V \) to estimate the values of \( \kappa_V \). We assume that the effect of substituting \( K^* \) with \( K_1 \) is identical in the radiative and in the non leptonic decay, in other words that each form factor for the \( B \rightarrow K_1 \) transition is given by the corresponding form factor for \( B \rightarrow K^* \) multiplied by the same factor \( y \), which is define as follows[35]:

\[
y = \frac{f_{B \rightarrow K_1(0)}}{f_{B \rightarrow K^*(0)}} \approx 1.06
\]

(6)

once the change of parity between the two strange mesons is taken into account. We predict that

\[
\kappa_V(B \rightarrow K_1) \approx 1.06 \kappa_V(B \rightarrow K^*)
\]

(7)

. Using the above equation and the results for \( \kappa_V \) obtained for \( B \rightarrow K^* \) transition[10]. We find \( \kappa_V = 1.75 \) for \( J/\Psi(1S) \) and \( \kappa_V = 2.43 \) for \( \Psi(2S) \), respectively. The relevant properties of vector charmonium states are summarized in Table 1.
Table 1: Masses, total decay widths and branching fractions of dilepton decays of vector charmonium states [24].

| V          | Mass [GeV] | $\Gamma_{\text{tot}}$ [MeV] | $\mathcal{B}(V \rightarrow \ell^+\ell^-)$  |
|------------|------------|-----------------------------|--------------------------------------------|
| $J/\Psi(1S)$ | 3.097      | 0.093                       | $5.9 \times 10^{-2}$ for $\ell = e, \mu$  |
| $\Psi(2S)$ | 3.686      | 0.327                       | $7.4 \times 10^{-3}$ for $\ell = e, \mu$  |
| $\Psi(3770)$ | 3.772      | 25.2                        | $9.8 \times 10^{-6}$ for $\ell = e$       |
| $\Psi(4040)$ | 4.040      | 80                          | $1.1 \times 10^{-5}$ for $\ell = e$       |
| $\Psi(4160)$ | 4.153      | 103                         | $8.1 \times 10^{-6}$ for $\ell = e$       |
| $\Psi(4415)$ | 4.421      | 62                          | $9.4 \times 10^{-6}$ for $\ell = e$       |

The matrix element for the exclusive decay can be obtained by sandwiching Eq. (2) between initial hadron state $B(p_B)$ and final hadron state $K_1$ in terms of formfactors. The $\mathcal{B}(p_B) \rightarrow K_1(p_{K_1}, \lambda)$ formfactors are defined as (see [9])

\[
\langle K_1(p_{K_1}, \lambda)|\bar{s}\gamma_\mu (1-\gamma_5)b|B(p_B)\rangle = -\frac{i}{m_B + m_{K_1}} \epsilon_{\mu
u\rho\sigma} \epsilon^{(\nu)}(b) p_{\rho} p_{K_1} A_{K_1}(q^2)
\]

\[
= -\left[(m_B + m_{K_1})\epsilon^{(\nu)}_\mu V_{1K_1}(q^2) - (p_B + p_{K_1})_\mu (\epsilon^{(\nu)}(b) \cdot p_B) \frac{V_{2K_1}(q^2)}{m_B + m_{K_1}}\right]
\]

\[
+ 2m_{K_1} \frac{\epsilon^{(\nu)}(b) \cdot p_B}{q^2} q_\mu \left[V_{3K_1}(q^2) - V_{0K_1}(q^2)\right],
\]

(8)

\[
\langle K_1(p_{K_1}, \lambda)|\bar{s}\sigma_{\mu\nu}q^\nu (1+\gamma_5)b|B(p_B)\rangle = 2T_{K_1}^1(q^2) \epsilon_{\nu\rho\sigma} \epsilon^{(\nu)}(b) p_{\rho} p_{K_1}
\]

\[
- iT_{K_1}^2(q^2) \left[(m_B^2 - m_{K_1}^2) \epsilon^{(\nu)}_\mu - (\epsilon^{(\nu)}_\lambda \cdot q)(p_B + p_{K_1})_\mu\right]
\]

\[
- iT_{K_1}^3(q^2) (\epsilon^{(\nu)}_\mu \cdot q) \left[q_\mu - \frac{q^2}{m_B^2 - m_{K_1}^2} (p_{K_1} + p_B)_\mu\right],
\]

(9)

where $q \equiv p_B - p_{K_1} = p_{\ell^+} + p_{\ell^-}$. In order to ensure finiteness at $q^2 = 0$, it is required

\[
V_{3K_1}(0) = V_{0K_1}(0), \quad T_{K_1}^1(0) = T_{K_1}^2(0),
\]

\[
V_{3K_1}(q^2) = \frac{m_B + m_{K_1}}{2m_{K_1}} V_{1K_1}(q^2) - \frac{m_B - m_{K_1}}{2m_{K_1}} V_{2K_1}(q^2).
\]

(10)

The formfactors of $B \rightarrow K_1(1270)$ and $B \rightarrow K_1(1400)$ can be expressed in terms of $B \rightarrow K_A$ and $B \rightarrow K_B$ as follows (see [9]):

\[
\left(\frac{\langle K_1(1270)|\bar{s}\gamma_\mu (1-\gamma_5)b|B\rangle}{\langle K_1(1400)|\bar{s}\gamma_\mu (1-\gamma_5)b|B\rangle}\right) = M \left(\frac{\langle K_{1A}||\bar{s}\gamma_\mu (1-\gamma_5)b|B\rangle}{\langle K_{1B}||\bar{s}\gamma_\mu (1-\gamma_5)b|B\rangle}\right),
\]

(11)

\[
\left(\frac{\langle K_1(1270)|\bar{s}\sigma_\mu q^\mu (1+\gamma_5)b|B\rangle}{\langle K_1(1400)|\bar{s}\sigma_\mu q^\mu (1+\gamma_5)b|B\rangle}\right) = M \left(\frac{\langle K_{1A}||\bar{s}\sigma_\mu q^\mu (1+\gamma_5)b|B\rangle}{\langle K_{1B}||\bar{s}\sigma_\mu q^\mu (1+\gamma_5)b|B\rangle}\right),
\]

(12)
using the mixing matrix $M$ being given in Eq. (1) the formfactors $A^{K_1}, V_{0,1,2}^{K_1}$ and $T_{1,2,3}^{K_1}$ can be written as follows:

\[
\begin{align*}
(A^{K_1(1270)}/(m_B + m_{K_1(1270)}) & = M (A^{K_1A}/(m_B + m_{K_1A})), \\
(A^{K_1(1400)}/(m_B + m_{K_1(1400)}) & = M (A^{K_1B}/(m_B + m_{K_1B})), \\
(m_B + m_{K_1(1270)})V_1^{K_1(1270)} & = M (m_B + m_{K_1A})V_1^{K_1A}, \\
(m_B + m_{K_1(1400)})V_1^{K_1(1400)} & = M (m_B + m_{K_1A})V_1^{K_1A}, \\
V_2^{K_1(1270)} & = M V_2^{K_1A}, \\
V_2^{K_1(1400)} & = M V_2^{K_1A}, \\
(m_{K_1(1270)}V_0^{K_1(1270)}) & = M (m_{K_1A}V_0^{K_1A}), \\
(T_{1}^{K_1(1270)}) & = M (T_{1}^{K_1A}), \\
(T_{2}^{K_1(1400)}) & = M (T_{2}^{K_1B}), \\
(T_{3}^{K_1(1270)}) & = M (T_{3}^{K_1A}), \\
(T_{3}^{K_1(1400)}) & = M (T_{3}^{K_1B}),
\end{align*}
\]

where it is supposed that $p_{K_1(1270),K_1(1400)}^\mu \simeq p_{K_1A}^\mu \simeq p_{K_1B}^\mu$ [9]. These formfactors within light-cone sum rule (LCSR) are estimated in [36]. The momentum dependence of all formfactors is parameterized as:

\[
F(q^2) = \frac{F(0)}{1 - a(q^2/m_B^2) + b(q^2/m_B^2)^2}.
\]

The values of $F(0)$, $a$ and $b$ parameters are exhibited in Table 2.

Thus, the matrix element for $B \rightarrow K_1 \ell^+ \ell^-$ in terms of formfactors is given by

\[
\mathcal{M} = \frac{G_F \alpha_{em}}{2\sqrt{2\pi}} V_{ts} V_{tb} m_B \cdot (-i) \left[ T_{\mu(K_1)}^{(1)} \bar{\ell} \gamma^\mu \ell + T_{\mu(K_1)}^{(2)} \bar{\ell} \gamma^\mu \gamma_5 \ell \right],
\]

where

\[
\begin{align*}
T_{\mu(K_1)}^{(1)} & = \mathcal{A}^{K_1}(\hat{s})\epsilon_{\mu\nu\rho\sigma}\varepsilon^\nu\hat{p}_B\hat{p}_K - i \mathcal{B}^{K_1}(\hat{s})\varepsilon^\mu, \\
+ i\mathcal{C}^{K_1}(\hat{s})(\varepsilon^* \hat{p}_B)\hat{p}_\mu + i \mathcal{D}^{K_1}(\hat{s})(\varepsilon^* \hat{q}_B)\hat{q}_\mu, \\
T_{\mu(K_1)}^{(2)} & = \mathcal{E}^{K_1}(\hat{s})\epsilon_{\mu\nu\rho\sigma}\varepsilon^\nu\hat{p}_B\hat{p}_K - i \mathcal{F}^{K_1}(\hat{s})\varepsilon^\mu, \\
+ i\mathcal{G}^{K_1}(\hat{s})(\varepsilon^* \hat{p}_B)\hat{p}_\mu + i \mathcal{H}^{K_1}(\hat{s})(\varepsilon^* \hat{q}_B)\hat{q}_\mu,
\end{align*}
\]

with $\hat{p} = p/m_B$, $\hat{p}_B = p_B/m_B$, $\hat{q} = q/m_B$ and $p = p_B + p_{K_1}$, $q = p_B - p_{K_1} = p_{\ell^+} + p_{\ell^-}$.

Here $\mathcal{A}^{K_1}(-\hat{s}), \cdots, \mathcal{H}^{K_1}(-\hat{s})$ are defined by

\[
\mathcal{A}^{K_1}(\hat{s}) = \frac{2}{1 + \sqrt{r_{K_1}}} c^{eff}_{\hat{s}} A^{K_1}(\hat{s}) + \frac{\tilde{m}_b}{s} c^{eff}_{7} T^{K_1}_{1}(\hat{s}),
\]

\[
\mathcal{B}^{K_1}(\hat{s}) = \left( 1 + \sqrt{r_{K_1}} \right) \left[ c^{eff}_{9}(\hat{s}) V^{K_1}_{1}(\hat{s}) + \frac{2\tilde{m}_b}{s} (1 - \sqrt{r_{K_1}}) c^{eff}_{7} T^{K_1}_{2}(\hat{s}) \right],
\]
Table 2: Formfactors for $B \to K_{1A}, K_{1B}$ transitions obtained in the LCSR calculation [36] are fitted to the 3-parameter form in Eq. (20).

| $F$     | $F(0)$ | $a$   | $b$   | $F$     | $F(0)$ | $a$   | $b$   |
|---------|--------|-------|-------|---------|--------|-------|-------|
| $V_{1BK1A}^*$ | 0.34 ± 0.07 | 0.635 | 0.211 | $V_{1BK1B}^*$ | −0.29$^{+0.08}_{-0.05}$ | 0.729 | 0.074 |
| $V_{2BK1A}^*$ | 0.41 ± 0.08 | 1.51  | 1.18  | $V_{2BK1B}^*$ | −0.17$^{+0.05}_{-0.03}$ | 0.919 | 0.855 |
| $V_{0BK1A}^*$ | 0.22 ± 0.04 | 2.40  | 1.78  | $V_{0BK1B}^*$ | −0.45$^{+0.12}_{-0.08}$ | 1.34  | 0.690 |
| $A_{BK1A}^*$   | 0.45 ± 0.09 | 1.60  | 0.974 | $A_{BK1B}^*$   | −0.37$^{+0.10}_{-0.06}$ | 1.72  | 0.912 |
| $T_{1BK1A}^*$  | 0.31$^{+0.09}_{-0.05}$ | 2.01  | 1.50  | $T_{1BK1B}^*$  | −0.25$^{+0.06}_{-0.07}$ | 1.59  | 0.790 |
| $T_{2BK1A}^*$  | 0.31$^{+0.09}_{-0.05}$ | 0.629 | 0.387 | $T_{2BK1B}^*$  | −0.25$^{+0.06}_{-0.07}$ | 0.378 | −0.755 |
| $T_{3BK1A}^*$  | 0.28$^{+0.08}_{-0.05}$ | 1.36  | 0.720 | $T_{3BK1B}^*$  | −0.11 ± 0.02 | −1.61 | 10.2  |

\[
C^{K_1}(\hat{s}) = \frac{1}{1 - \hat{r}_{K_1}} \left[ (1 - \sqrt{\hat{r}_{K_1}}) c_0^{\text{eff}}(\hat{s}) V_{2K_1}^*(\hat{s}) + 2\hat{m}_b c_7^{\text{eff}} \left( T_{3K_1}^*(\hat{s}) + \frac{1 - \sqrt{\hat{r}_{K_1}^2}}{\hat{s}} T_{2K_1}^*(\hat{s}) \right) \right],
\]

\[
D^{K_1}(\hat{s}) = \frac{1}{\hat{s}} \left[ c_0^{\text{eff}}(\hat{s}) \left( (1 + \sqrt{\hat{r}_{K_1}}) V_{1K_1}^*(\hat{s}) - (1 - \sqrt{\hat{r}_{K_1}}) V_{2K_1}^*(\hat{s}) - 2\sqrt{\hat{r}_{K_1}} V_{0K_1}^*(\hat{s}) \right) \right.
\]

\[
+ 2\hat{m}_b c_7^{\text{eff}} T_{3K_1}(\hat{s}) \right],
\]

\[
E^{K_1}(\hat{s}) = \frac{2}{1 + \sqrt{\hat{r}_{K_1}}} c_{10} A_{K_1}^*(\hat{s}),
\]

\[
F^{K_1}(\hat{s}) = (1 + \sqrt{\hat{r}_{K_1}}) c_{10} V_{1K_1}^*(\hat{s}),
\]

\[
G^{K_1}(\hat{s}) = \frac{1}{1 + \sqrt{\hat{r}_{K_1}}} c_{10} V_{2K_1}^*(\hat{s}),
\]

\[
H^{K_1}(\hat{s}) = \frac{1}{\hat{s}} c_{10} \left[ (1 + \sqrt{\hat{r}_{K_1}}) V_{1K_1}^*(\hat{s}) - (1 - \sqrt{\hat{r}_{K_1}}) V_{2K_1}^*(\hat{s}) - 2\sqrt{\hat{r}_{K_1}} V_{0K_1}^*(\hat{s}) \right],
\]

with $\hat{r}_{K_1} = m_{K_1}^2 / m_B^2$, $\hat{m}_t = m_t / m_B$ and $\hat{s} = q^2 / m_B^2$. The differential decay spectrum can be obtained from the decay amplitude

\[
\frac{d\Gamma(B \to K_1 \ell^+ \ell^-)}{ds} = \frac{G_F^2 G_{\text{em}}^2 m_B^5}{28\pi^5} |V_{tb}V_{ts}^*|^2 \sqrt{\lambda} \Delta(\hat{s})
\]

\[
\Delta = \frac{8 \Re[F H^*] \hat{m}_t^2 \lambda}{\hat{r}_{K_1}} + \frac{8 \Re[G H^*] \hat{m}_t^2 (-1 + \hat{r}_{K_1}) \lambda}{\hat{r}_{K_1}} - \frac{8|H|^2 \hat{m}_t^2 \hat{s} \lambda}{\hat{r}_{K_1}}
\]

\[
- \frac{2 \Re[B C^*] (-1 + \hat{r}_{K_1} + \hat{s})(3 + 3\hat{r}_{K_1}^2 - 6\hat{s} + 3\hat{s}^2 - 6\hat{r}_{K_1}(1 + \hat{s}) - v^2\lambda)}{3\hat{r}_{K_1}}
\]

\[
- \frac{|C|^2 \lambda (3 + 3\hat{r}_{K_1}^2 - 6\hat{s} + 3\hat{s}^2 - 6\hat{r}_{K_1}(1 + \hat{s}) - v^2\lambda)}{3\hat{r}_{K_1}}
\]

\[
- \frac{|G|^2 \lambda (3 + 3\hat{r}_{K_1}^2 + 12\hat{m}_t^2 (2 + 2\hat{r}_{K_1} - \hat{s}) - 6\hat{s} + 3\hat{s}^2 - 6\hat{r}_{K_1}(1 + \hat{s}) - v^2\lambda)}{3\hat{r}_{K_1}}
\]
3 Lepton polarization asymmetries

In order to calculate the polarization asymmetries of the leptons, we must first define the orthogonal vectors $S$ in the rest frame of $\ell^-$ and $W$ in the rest frame of $\ell^+$ (where these vectors are the polarization vectors of the leptons). Note that, we will use the subscripts $L$, $N$ and $T$ to correspond to the leptons which are polarized along with the longitudinal, normal and transverse polarization of leptons, respectively \([34, 37]\).

\[
S^\mu_L \equiv (0, e_L) = \left(0, \frac{p_-}{|p_-|}\right),
\]

\[
S^\mu_N \equiv (0, e_N) = \left(0, \frac{p_K \times p_-}{|p_K \times p_-|}\right),
\]

\[
S^\mu_T \equiv (0, e_T) = \left(0, e_N \times e_L\right),
\]

\[
W^\mu_L \equiv (0, w_L) = \left(0, \frac{p_+}{|p_+|}\right),
\]

\[
W^\mu_N \equiv (0, w_N) = \left(0, \frac{p_K \times p_+}{|p_K \times p_+|}\right),
\]

\[
W^\mu_T \equiv (0, w_T) = \left(0, w_N \times w_L\right),
\]

where $p_+$, $p_-$ and $p_{K_1}$ are the three momenta of the $\ell^+$, $\ell^-$ and $K_1$ particles, respectively. On boosting the vectors defined by Eqs. (33,34) to the CM frame of the $\ell^- \ell^+$ system only the longitudinal vector will be boosted, while the other two remain the same. The longitudinal vectors in the CM frame of the $\ell^- \ell^+$ system become;

\[
S^\mu_L = \left(\frac{|p_-|}{m_\ell}, \frac{E_\ell p_-}{m_\ell |p_-|}\right),
\]

\[
W^\mu_L = \left(\frac{|p_-|}{m_\ell}, -\frac{E_\ell p_-}{m_\ell |p_-|}\right).
\]

The polarization asymmetries can now be calculated using the spin projector $\frac{1}{2}(1 + \gamma_5 S)$ for $\ell^-$ and the spin projector $\frac{1}{2}(1 + \gamma_5 W)$ for $\ell^+$. The single and double–lepton polarization
asymmetries \( P_{ij} \) are defined, respectively, as [37]

\[
P_i = \frac{dt(s^+ = \xi, s^- = \eta)}{ds} - \frac{dt(s^+ = -\xi, s^- = -\eta)}{ds} \tag{36}
\]

and

\[
P_{ij} = \frac{d^2t(s^+ = \xi, s^- = \eta)}{ds^2} - \frac{d^2t(s^+ = -\xi, s^- = -\eta)}{ds^2} + \frac{d^2t(s^+ = \xi, s^- = -\eta)}{ds^2} - \frac{d^2t(s^+ = -\xi, s^- = \eta)}{ds^2} \tag{37}
\]

where \( \hat{i} = L, N, T \) and \( \hat{j} = L, N, T \) are unit vectors.

Equipped with these definitions, we evaluate the single and double lepton polarization asymmetries and obtain the following results:

\[
P_L = \frac{-2Re[B\gamma^*](\hat{r}_K, + \hat{s} - 1)v}{3\hat{r}_K} \left( 3\hat{r}_K^2 - 6(\hat{s} + 1)\hat{r}_K + 3(\hat{s} - 1)^2 - \lambda \right)
\]

\[
- \frac{2Re[C\gamma^*](\hat{r}_K, + \hat{s} - 1)v}{3\hat{r}_K} \left( 3\hat{r}_K^2 - 6(\hat{s} + 1)\hat{r}_K + 3(\hat{s} - 1)^2 - \lambda \right)
\]

\[
- \frac{2Re[C\gamma^*]v\lambda}{3\hat{r}_K} \left( 3\hat{r}_K^2 - 6(\hat{s} + 1)\hat{r}_K + 3(\hat{s} - 1)^2 - \lambda \right)
\]

\[
\frac{2}{3}Re[A\epsilon^*]\hat{s}v \left( 3\hat{r}_K^2 - 6(\hat{s} + 1)\hat{r}_K + 3(\hat{s} - 1)^2 + \lambda \right)
\]

\[
+ \frac{2Re[B\gamma^*]v}{3\hat{r}_K} \left( \lambda - 3(\hat{r}_K^2 + (6\hat{s} - 2)\hat{r}_K + (\hat{s} - 1)^2) \right) \tag{38}
\]

\[
P_T = \frac{\pi \hat{m}_t}{\lambda} \sqrt{\hat{s}} \left\{ - \frac{Re[C\gamma^*]\sqrt{\hat{s}}\lambda}{\hat{r}_K} + \frac{Re[C\gamma^*]\lambda}{\hat{r}_K \sqrt{\hat{s}}} + \frac{Re[C\gamma^*](\hat{r}_K - 1)\lambda}{\hat{r}_K \sqrt{\hat{s}}} \right\}
\]

\[
- \frac{Re[B\gamma^*]\sqrt{\hat{s}}(\hat{r}_K + \hat{s} - 1)}{\hat{r}_K} + \frac{Re[B\gamma^*](\hat{r}_K + \hat{s} - 1)}{\hat{r}_K \sqrt{\hat{s}}}
\]

\[
+ \frac{Re[B\gamma^*](\hat{r}_K - 1)(\hat{r}_K + \hat{s} - 1)}{\hat{r}_K \sqrt{\hat{s}}} - 4Re[A\beta^*] \sqrt{\hat{s}} \right\}
\]

\[
P_N = \frac{i \pi \hat{m}_t}{\lambda} \sqrt{\hat{s}} \left\{ \frac{Im[G\gamma^*]\sqrt{\hat{s}}\lambda}{\hat{r}_K} + \frac{Im[G\gamma^*]\lambda}{\hat{r}_K \sqrt{\hat{s}}} \right\}
\]

\[
+ \frac{Im[F\gamma^*]\sqrt{\hat{s}}(\hat{r}_K + \hat{s} - 1)}{\hat{r}_K} - 2Im[A\epsilon^*] \sqrt{\hat{s}} - 2Im[A\epsilon^*] \sqrt{\hat{s}} \right\} \tag{39}
\]

\[
P_{LL} = \frac{4Re[F\gamma^*](2\hat{m}_t^2 + \hat{s}) \lambda}{\hat{r}_K} + \frac{4Re[G\gamma^*](-1 + \hat{r}_K)(2\hat{m}_t^2 + \hat{s}) \lambda}{\hat{r}_K} - \frac{2|\gamma|^2(2\hat{m}_t^2 + \hat{s}) \lambda}{\hat{r}_K}
\]

\[
- \frac{|\gamma|^2}{6\hat{m}_t^2 \hat{r}_K \hat{s}} \left( \hat{s}^2(3\hat{r}_K^2 + 3(-1 + \hat{s})^2 - 6\hat{r}_K(1 + \hat{s}) - \lambda) \right)
\]
\[
\begin{align*}
&+ 8\hat{m}_t^4(6 + 6\hat{r}_{K_1}^2 + 3(-2 + \hat{s})\hat{s} - 6\hat{r}_{K_1}(2 + \hat{s}) - \lambda) \\
&- 6\hat{m}_t^2\hat{s}(1 + \hat{r}_{K_1}^2 + 3(-2 + \hat{s})\hat{s} - 2(\hat{r}_{K_1} + 3\hat{r}_{K_1}\hat{s}) - \lambda)\lambda \\
&- |E|^2(3(8\hat{m}_t^4 + 2\hat{m}_t^2\hat{s} - \hat{s}^2)\lambda + (8\hat{m}_t^4 - 6\hat{m}_t^2\hat{s} + \hat{s}^2)\lambda) \\
&+ \frac{6\hat{m}_t^2}{3\hat{r}_{K_1}\hat{s}} \Re[BC^*](-1 + \hat{r}_{K_1} + \hat{s})(3\hat{s}(2\hat{m}_t^2)\lambda - (8\hat{m}_t^4 - 3\hat{m}_t^2\hat{s} + \hat{s}^2)\lambda) \\
&+ \frac{|C|^2\lambda(3\hat{s}(2\hat{m}_t^2 + \hat{s})\lambda - (8\hat{m}_t^4 - 2\hat{m}_t^2\hat{s} + \hat{s}^2))\lambda + (8\hat{m}_t^4 - 2\hat{m}_t^2\hat{s} + \hat{s}^2))\lambda}{6\hat{m}_t^2} \\
&+ \frac{|\mathcal{F}|^2}{6\hat{m}_t^2\hat{r}_{K_1}\hat{s}} \left(6\hat{m}_t^2\hat{s}(\hat{r}_{K_1}^2 + (-1 + \hat{s})^2 + \hat{r}_{K_1}(-2 + 6\hat{s}) - \lambda) \right) \\
&+ \frac{|\mathcal{B}|^2}{6\hat{m}_t^2\hat{r}_{K_1}\hat{s}} \left(\hat{s}^2(3\hat{r}_{K_1} + 3(-1 + \hat{s})^2 - 6\hat{r}_{K_1}(1 + \hat{s}) - \lambda) \right) \\
&- 8\hat{m}_t^4(12\hat{r}_{K_1}\hat{s} + \lambda) + 2\hat{m}_t^2\hat{s}(3(\hat{r}_{K_1}^2 + (-1 + \hat{s})^2 + 2\hat{r}_{K_1}(-1 + 7\hat{s}) + \lambda) \right) \\
&- \frac{\Re[\mathcal{F}\mathcal{G}^*]}{6\hat{m}_t^2\hat{r}_{K_1}\hat{s}} \left(\hat{s}^2(-1 + \hat{r}_{K_1} + \hat{s})(3\hat{r}_{K_1}^2 + 3(-1 + \hat{s})^2 - 6\hat{r}_{K_1}(1 + \hat{s}) - \lambda) \right) \\
&+ 8\hat{m}_t^4(6\hat{r}_{K_1}^2 - 9\hat{r}_{K_1}(2 + \hat{s}) + (-1 + \hat{s})(6 + 3(-3 + \hat{s})\hat{s} - \lambda) \\
&- \hat{r}_{K_1}(-18 + 6\hat{s} + \lambda) - 6\hat{m}_t^2\hat{s}(\hat{r}_{K_1}^2 + \hat{r}_{K_1}(-3 + \hat{s}) + (1 + \hat{s})(1 + \hat{s})(-4 + 3\hat{s}) - \lambda) \\
&- \hat{r}_{K_1}(-3 + \hat{s}(6 + 5\hat{s}) + \lambda)) \\n\end{align*}
\]

\[
\begin{align*}
\mathcal{P}_{TT} &= \frac{|E|^2}{3} \left((4\hat{m}_t^2 - \hat{s})(3 + 3\hat{r}_{K_1}^2 - 6\hat{s} + 3\hat{s}^2 - 6\hat{s}(1 + \hat{s}) - 5\lambda) \\
&- \frac{8\Re[\mathcal{F}\mathcal{H}^*]\hat{m}_t^2\lambda}{\hat{r}_{K_1}} - \frac{8\Re[\mathcal{G}\mathcal{H}^*]\hat{m}_t^2(-1 + \hat{r}_{K_1})\lambda}{\hat{r}_{K_1}} + \frac{4|\mathcal{H}|^2\hat{m}_t^2\hat{s}\lambda}{\hat{r}_{K_1}} \right) \\
&- \frac{2\Re[BC^*](-1 + \hat{r}_{K_1} + \hat{s})}{3\hat{r}_{K_1}\hat{s}} \left(-6(1 + \hat{r}_{K_1})\hat{s}^2 + 3\hat{s}^2 + \hat{s}(3 - 6\hat{r}_{K_1} + 3\hat{r}_{K_1}^2 - 5\lambda) + 4\hat{m}_t^2\lambda \right) \\
&- \frac{|C|^2\lambda(-6(1 + \hat{r}_{K_1})\hat{s}^2 + 3\hat{s}^2 + \hat{s}(3 - 6\hat{r}_{K_1} + 3\hat{r}_{K_1}^2 - 5\lambda) + 4\hat{m}_t^2\lambda}{3\hat{r}_{K_1}\hat{s}} \\
&+ \frac{|\mathcal{F}|^2(-6(1 + \hat{r}_{K_1})\hat{s}^2 + 3\hat{s}^2 + \hat{s}(3 - 6\hat{r}_{K_1} + 3\hat{r}_{K_1}^2 - 5\lambda) + 20\hat{m}_t^2\lambda}{3\hat{r}_{K_1}\hat{s}} \\
&+ \frac{|\mathcal{G}|^2\lambda(6(1 + 2\hat{m}_t^2 + \hat{r}_{K_1})\hat{s}^2 + 3\hat{s}^3}{3\hat{r}_{K_1}\hat{s}} \\
\end{align*}
\]
\[\begin{align*}
&+ \hat{s}(3 - 6\hat{r}_{K_1} + 3\hat{r}_{K_1}^2 + 24\hat{m}_t(1 + \hat{r}_{K_1}) - 5\lambda) + 20\hat{m}_t^2\lambda \\
&+ |A|^2\left(\frac{\hat{s}}{3} - 3\hat{r}_{K_1}^2 - 6\hat{s} + 3\hat{s}^2 - 6\hat{r}_{K_1}(1 + \hat{s}) - 5\lambda\right) \\
&+ \hat{m}_t^2\left(-4 - 4\hat{r}_{K_1}^2 + 8\hat{s} - 4\hat{s}^2 + 8\hat{r}_{K_1}(1 + \hat{s}) + \frac{4\lambda}{3}\right) \\
&+ |B|^2\left(6(1 + \hat{r}_{K_1})\hat{s}^2 - 3\hat{s}^3 - 4\hat{m}_t^2\lambda + \hat{s}(3 + (6 - 48\hat{m}_t^2)(\hat{r}_{K_1} - 3\hat{r}_{K_1}^2 + 5\lambda))\right) \\
&+ \frac{|B|}{3\hat{r}_{K_1}\hat{s}}(3(3 - 4\hat{m}_t^2 + \hat{r}_{K_1})\hat{s}^3 - 3\hat{s}^4 + \hat{s}(1 + 4\hat{m}_t^2 - \hat{r}_{K_1})(3 - 6\hat{r}_{K_1} + 3\hat{r}_{K_1}^2 - 5\lambda) \\
&- 20\hat{m}_t^2(-1 + \hat{r}_{K_1})\lambda + \hat{s}^2(-9 + 6\hat{r}_{K_1} + 3\hat{r}_{K_1}^2 - 24\hat{m}_t^2(1 + \hat{r}_{K_1}) + 5\lambda))
\end{align*}\]

\[P_{LT} = -2\text{Re}[AF^* + BE^*]\pi\hat{m}_t\sqrt{s}\lambda - \frac{|F|^2\pi\hat{m}_t((-1 + \hat{r}_{K_1} + \hat{s})\sqrt{s})}{\sqrt{\hat{r}_{K_1}\hat{s}}} \\
+ \frac{\text{Re}[FH^*]\pi\hat{m}_t((-1 + \hat{r}_{K_1} + \hat{s})\sqrt{s})}{\sqrt{\hat{r}_{K_1}\hat{s}}} + \frac{\text{Re}[FG^*]\pi\hat{m}_t((-2 - 2\hat{r}_{K_1}^2 + 3\hat{s} - \hat{s}^2 + \hat{r}_{K_1}(4 + \hat{s}))\sqrt{s})}{\sqrt{\hat{r}_{K_1}\hat{s}}} \\
- \frac{|G|^2\pi\hat{m}_t((-1 + \hat{r}_{K_1})\lambda\hat{s})}{\sqrt{\hat{r}_{K_1}\hat{s}}} + \frac{\text{Re}[GH^*]\pi\hat{m}_t\lambda\sqrt{s}}{\sqrt{\hat{r}_{K_1}\hat{s}}} \\
\]

\[P_{TL} = 2\text{Re}[AF^* + BE^*]\pi\hat{m}_t\sqrt{s}\lambda - \frac{|F|^2\pi\hat{m}_t((-1 + \hat{r}_{K_1} + \hat{s})\sqrt{s})}{\sqrt{\hat{r}_{K_1}\hat{s}}} \\
+ \frac{\text{Re}[FH^*]\pi\hat{m}_t((-1 + \hat{r}_{K_1} + \hat{s})\sqrt{s})}{\sqrt{\hat{r}_{K_1}\hat{s}}} + \frac{\text{Re}[FG^*]\pi\hat{m}_t((-2 - 2\hat{r}_{K_1}^2 + 3\hat{s} - \hat{s}^2 + \hat{r}_{K_1}(4 + \hat{s}))\sqrt{s})}{\sqrt{\hat{r}_{K_1}\hat{s}}} \\
- \frac{|G|^2\pi\hat{m}_t((-1 + \hat{r}_{K_1})\lambda\hat{s})}{\sqrt{\hat{r}_{K_1}\hat{s}}} + \frac{\text{Re}[GH^*]\pi\hat{m}_t\lambda\sqrt{s}}{\sqrt{\hat{r}_{K_1}\hat{s}}} \\
\]

\[P_{NN} = \frac{1}{3}(|A|^2 - |E|^2)(4\hat{m}_t^2 - \hat{s})(3 + 3\hat{r}_{K_1}^2 - 6\hat{s} + 3\hat{s}^2 - 6(1 - \hat{s}) - \lambda) + \frac{8\text{Re}[FH^*]\hat{m}_t^2\lambda}{\hat{r}_{K_1}} \\
+ \frac{8\text{Re}[GH^*]\hat{m}_t^2(-1 + \hat{r}_{K_1})\lambda}{\hat{r}_{K_1}} - \frac{4|H|^2\hat{m}_t^2\hat{s}\lambda}{3\hat{s}\hat{r}_{K_1}} \\
+ \frac{2\text{Re}[BC^*](-1 + \hat{r}_{K_1} + \hat{s})(-6(1 + \hat{r}_{K_1})\hat{s}^2 + 3\hat{s}^2 + \hat{s}(3 - 6\hat{r}_{K_1} + 3\hat{r}_{K_1}^2 - \lambda) + 4\hat{m}_t^2\lambda}{3\hat{s}\hat{r}_{K_1}} \\
+ \frac{|C|^2 - |F|^2)|\lambda(-6(1 + \hat{r}_{K_1})\hat{s}^2 + 3\hat{s}^2 + \hat{s}(3 - 6\hat{r}_{K_1} + 3\hat{r}_{K_1}^2 - \lambda) - 4\hat{m}_t^2\lambda}{3\hat{s}\hat{r}_{K_1}} \\
+ \frac{|B|^2(-6(1 + \hat{r}_{K_1})\hat{s}^2 + 3\hat{s}^2 + \hat{s}(3 + 6(-1 + 8\hat{m}_t^2)(\hat{r}_{K_1} + 3\hat{r}_{K_1}^2 - \lambda) + 4\hat{m}_t^2\lambda}{3\hat{s}\hat{r}_{K_1}} \\
- \frac{|G|^2\lambda(-6(1 + \hat{r}_{K_1} + 2\hat{m}_t^2)\hat{s}^2 + 3\hat{s}^2 + \hat{s}(3 - 6\hat{r}_{K_1} + 3\hat{r}_{K_1}^2 + 24\hat{m}_t^2(1 + \hat{r}_{K_1}) - \lambda) - 4\hat{m}_t^2\lambda}{3\hat{s}\hat{r}_{K_1}}
\]
Having the explicit expressions for the physically measurable quantities, in this section, we will study the dependence of these quantities on the dileptonic invariant mass \( q^2 \). We will use the parameters given in Tables 2 and 4 in our numerical analysis.
We present the dependence of the differential single and double lepton polarization for the $B \rightarrow K_1(1272)\ell^+\ell^-$, where $\ell = \mu, \tau$ decay on $q^2$ as well as its dependence on $q^2$ due to short distance effects ($\kappa_V \neq 0$ case). The phenomenological factors $\kappa_V$ for the $B \rightarrow K(K^*)\ell^+\ell^-$ decay can be determined from matching the experimental and theoretical results where they supposed to reproduce correct branching ratio relation

$$\mathcal{B}(B \rightarrow J/\psi K(K^*) \rightarrow K(K^*)\ell^+\ell^-) = \mathcal{B}(B \rightarrow J/\psi K(K^*)) \mathcal{B}(J/\psi \rightarrow \ell^+\ell^-),$$

where the right–hand side is determined from experiments. Using the experimental values of the branching ratios for the $B \rightarrow V_i K(K^*)$ and $V_i \rightarrow \ell^+\ell^-$ decays, for the lowest two $J/\psi$ and $\psi'$ resonances, the factor $\kappa_V$ takes the values: $\kappa_V = 2.7$, $\kappa_V = 3.51$ (for $K$ meson), and $\kappa_V = 1.65$, $\kappa_V = 2.36$ (for $K^*$ meson). The values of $\kappa_V$ used for higher resonances are usually the average of the values obtained for the $J/\psi$ and $\psi'$ resonances. Using Eq. (7) and the results for $\kappa_V$ obtained for $B \rightarrow K^*$ transition[10]. We find $\kappa_V = 1.75$ for $J/\Psi(1S)$ and $\kappa_V = 2.43$ for $\Psi(2S)$, respectively.

It is also experimentally useful to consider the averaged values of these asymmetries. Therefore, we shall calculate the averaged values of the polarization asymmetries using the averaging procedure defined as;

$$\langle P \rangle = \frac{\int_{4\hat{s}_0^2}^{(1-\sqrt{\hat{r}_{K_1})^2}} \mathcal{P} \frac{d\mathcal{B}}{d\hat{s}} d\hat{s}}{\int_{4\hat{s}_0^2}^{(1-\sqrt{\hat{r}_{K_1})^2}} d\mathcal{B} d\hat{s}},$$

, where $\mathcal{B}$ is the branching ratio. The results for averaged value of single and double lepton polarization asymmetries are presented in table 4. Some of these asymmetries in $B \rightarrow K_1(1270)\ell^+\ell^-$ decay(i.e., $P_{LL}$, $P_{NN}$ and $P_{TT}$ ) are larger than corresponding asymmetries in $B \rightarrow K^*\ell^+\ell^-$ decay presented in Ref. [39].

Figs. (1)-(14) show dependence on $q^2$ when considering the theoretical uncertainties among the formfactors. Note that, $P_N$, $P_{NL}$, $P_{LN}$, $P_{NT}$ and $P_{TN}$ for $\mu$ and $\tau$ channels are negligible for all values of $q^2$. Hence, we do not present their predictions in the figures.

From these figures, we deduce the following results:
- $P_L$ is plotted in Figs. (1) and (2) for muon and tau, respectively. It is decreasing for both of muon and and tau channels. Also, its magnitude is much larger for muon channel than tau one. Moreover, there is rather weak dependency on the theoretical uncertainties among the formfactors for tau channel.

- While $P_T$ is decreasing for $q^2 \leq 1.2\text{GeV}^2$ region it is increasing for $q^2 \geq 1.2\text{GeV}^2$ region for muon channel(see fig. (3)). Its local minimum at the point $q^2 \leq 1.2\text{GeV}^2$ is about $-0.15$. $P_T$ is increasing in terms of $q^2$ for tau channel(see fig. (4)). Also, $P_T$ vanishes at the end of kinematical region for both muon and tau channels.

- $P_{LL}$ takes both negative and positive values depending on $q^2$. Its zero position occurs at $q^2 \simeq 5\text{GeV}^2$. The measurement of the sign of $P_{LL}$ at $q^2 \leq 8\text{GeV}^2$, which is the nonresonance region, can be used as a good tool to either check the SM prediction or to search for new physics. $P_{LL}$ is quasi uniformly decreasing function of $q^2$ for tau channel.(see figs. (5) and (6)). Moreover, there is rather weak dependency on the theoretical uncertainties among the formfactors for tau channel.

- $P_{LT}$ is decreasing for $q^2 \leq 0.8(14.5)\text{GeV}^2$ region but increasing for $q^2 \geq 0.8(14.5)\text{GeV}^2$ region for muon(tau) channel (see figs. (7) and (8)). Its local minimum at the point $q^2 \leq 0.8(14.5)\text{GeV}^2$ is about $-0.2(0.22)$ for muon(tau) channel, respectively. Also, $P_{LT}$ vanishes at the end of kinematical region for both muon and tau channels. Moreover, there is rather weak dependency on the theoretical uncertainties among the formfactors for tau channel(see fig. (8)).

- $P_{NN}$ and $P_{TT}$ without resonance contributions are negligible at $q^2 \geq 8\text{GeV}^2$ region for muon channel(see figs. (9) and (11)). $P_{TT}$ takes much larger values in the high $q^2$ region than the low $q^2$ region for tau channel (see fig. (12)).

- $P_{TL}$ is decreasing for $q^2 \leq 0.6\text{GeV}^2$ region but increasing for $q^2 \geq 0.6\text{GeV}^2$ region for muon channel, (see fig. (13)). Its local minimum at the point $q^2 \leq 0.6\text{GeV}^2$ is about $-0.25$. $P_{TL}$ is negligible for all values of $q^2$ for tau lepton (see fig. (14)). Also, $P_{LT}$ vanishes at the end of kinematical region for both muon and tau channels.

Finally, the quantitative estimation about the accessibility to measure the various physical observables are in order. An observation of a $3\sigma$ signal for asymmetry of the order of the 1% requires about $\sim 10^{12}$ $\bar{B}B$ pairs. The number of $b\bar{b}$ pairs that are produced at B–factories and LHC are about $\sim 5 \times 10^8$ and $10^{12}$, respectively. As a result, $q^2$ dependence of the polarization asymmetries shown by figs. (1)-(13) as well as averaged values of the same asymmetries presented in table 4 can be detectable at LHC. Note that, the ratio of physical observables (for instance, CP , forward–backward and single or double lepton polarization asymmetries) less suffers from the uncertainty among the formfactors where large parts of the uncertainties partially cancel out.

In conclusion, the single and double lepton polarization asymmetries for exclusive dilepton rare B decays of $B \to K_1(1272)\ell^+\ell^-$ are studied. We have shown that while some components of lepton polarizations are almost zero, some other components are sizable to be measured at the future experiments. Moreover, we show that some of these asymmetries
Table 4: Averaged values of single and double lepton polarizations

| $\langle P_{ij} \rangle$ | $B \rightarrow K_1(1272)\mu^+\mu^-$ | $B \rightarrow K_1(1272)\tau^+\tau^-$ |
|------------------------|---------------------------------|---------------------------------|
| $\langle P_L \rangle$  | $-0.91 \pm 0.006$               | $-0.43 \pm 0.001$               |
| $\langle P_T \rangle$  | $-0.016 \pm 0.001$              | $-0.05 \pm 0.004$               |
| $\langle P_N \rangle$  | $0.001 \pm 0.001$               | $0.01 \pm 0.001$                |
| $\langle P_{LL} \rangle$ | $-0.34 \pm 0.0053$ | $-0.06 \pm 0.000$               |
| $\langle P_{LN} \rangle$ | $-0.001 \pm 0.000$ | $-0.03 \pm 0.001$               |
| $\langle P_{NL} \rangle$ | $0.001 \pm 0.000$   | $0.03 \pm 0.001$                |
| $\langle P_{LT} \rangle$ | $-0.06 \pm 0.003$   | $-0.17 \pm 0.000$               |
| $\langle P_{TL} \rangle$ | $-0.03 \pm 0.003$   | $-0.01 \pm 0.000$               |
| $\langle P_{TT} \rangle$ | $0.015 \pm 0.002$   | $0.11 \pm 0.002$                |
| $\langle P_{NN} \rangle$ | $0.01 \pm 0.004$    | $-0.17 \pm 0.001$               |
| $\langle P_{NT} \rangle$ | $-0.006 \pm 0.001$  | $0.001 \pm 0.001$               |
| $\langle P_{TN} \rangle$ | $0.006 \pm 0.001$    | $0.001 \pm 0.001$               |

in $B \rightarrow K_1(1270)\ell^+\ell^-$ decay (i.e., $P_{LL}$, $P_{NN}$ and $P_{TT}$) are larger than corresponding asymmetries in $B \rightarrow K^*\ell^+\ell^-$ decay. The study of the magnitude and the size of these physical observables can be used either to probe the predictions of SM or to search for new physics effects.

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Figure captions

Fig. (1) The dependence of the $P_L$ on $q^2$ for $B \rightarrow K_1(1270)\mu^+\mu^-$ decay, where the colored region shows the variation when theoretical uncertainties among the form factors take into account.

Fig. (2) The same as in Fig. (1), but for the $\tau$ lepton.

Fig. (3) The dependence of the $P_T$ on $q^2$ for $B \rightarrow K_1(1270)\mu^+\mu^-$ decay, where the colored region shows the variation when theoretical uncertainties among the form factors take into account.

Fig. (4) The same as in Fig. (3), but for the $\tau$ lepton.

Fig. (5) The dependence of the $P_{LL}$ on $q^2$ for $B \rightarrow K_1(1270)\mu^+\mu^-$ decay, where the colored region shows the variation when theoretical uncertainties among the form factors take into account.

Fig. (6) The same as in Fig. (5), but for the $\tau$ lepton.

Fig. (7) The dependence of the $P_{LT}$ on $q^2$ for $B \rightarrow K_1(1270)\mu^+\mu^-$ decay, where the colored region shows the variation when theoretical uncertainties among the form factors take into account.

Fig. (8) The same as in Fig. (7), but for the $\tau$ lepton.

Fig. (9) The dependence of the $P_{NN}$ on $q^2$ for $B \rightarrow K_1(1270)\mu^+\mu^-$ decay, where the colored region shows the variation when theoretical uncertainties among the form factors take into account.

Fig. (10) The same as in Fig. (9), but for the $\tau$ lepton.

Fig. (11) The dependence of the $P_{TT}$ on $q^2$ for $B \rightarrow K_1(1270)\mu^+\mu^-$ decay, where the colored region shows the variation when theoretical uncertainties among the form factors take into account.

Fig. (12) The same as in Fig. (11), but for the $\tau$ lepton.

Fig. (13) The dependence of the $P_{TL}$ on $q^2$ for $B \rightarrow K_1(1270)\mu^+\mu^-$ decay, where the colored region shows the variation when theoretical uncertainties among the form factors take into account.

Fig. (14) The same as in Fig. (13), but for the $\tau$ lepton.
Figure 3:

Figure 4:
Figure 7:

Figure 8:
Figure 11:

Figure 12:
