Hawking radiation from Trojan states in muonic Hydrogen in strong laser field

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Abstract – We show that the Unruh-Davis effect is measurable from Trojan wavepackets in muonic Hydrogen as the acceleration on the first muonic Bohr orbit reaches $10^{25}$ of the earth acceleration. It is the biggest acceleration achievable in the laboratory environment which have been ever predicted for the cyclotronic configuration. We calculate the ratio between the power of Larmor radiation and the power of Hawking radiation. The Hawking radiation is measurable even for Rydberg quantum numbers of the muon due to suppression of spontaneous emission in Trojan Hydrogen.

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1. INTRODUCTION

The possibility to observe the Unruh-Davis effect using single electron DeWitt detectors has been discussed quite long time ago [1, 2, 3, 4]. The measurements of depolarization of electrons in storage ring have been classified confirming of spin states heat up by the field vacuum fluctuation [5]. The perturbation of Thomas precession was shown to be weak as the time of the full de-excitation of the spin state turned out to exceed the age of the Universe.

However very recently microscopic cyclotrons with optical cyclotronic frequency in circularly polarized electromagnetic field have been predicted theoretically and confirmed experimentally in single atoms when well confined electron on circular orbit in moving almost eternally without dispersion in strong circularly polarized (CP) electromagnetic field [6, 7]. It has been also suggested recently that the Unruh-Davis effect can be observed in radiation of driven electrons in ultra strong laser field [8]. The laser field strength necessary for the successful experiment was proposed to be of the order of $10 TeV/cm$. The collaborative effect of Coulomb fields is known to cause enhanced ionization and the Coulomb ignition of atomic Van der Waals clusters [9]. Nuclear fusion was observed due to strain Coulomb field acceleration of energetic ions from atomic cluster [10]. The laser dissociation of muonic molecules have been recently also suggested [11]. In this paper we suggest an alternative method of observing Hawking radiation in strong laser field. In contrast to ultrastrong laser method it is mainly the Coulomb field which provides the gigantic acceleration of the Davis detector and much weaker laser field prepares the quantum particle to be classical. It is also the centrifugal acceleration which extends the time of observation practically to infinity on the scale of physical times involved. One may notice that for classical (Trojan) nonrelativistic Coulomb problem of muonic Hydrogen the centrifugal acceleration of classical electron on first
Bohr orbit is
\[ a = \frac{1}{4\pi\epsilon_0 \mu a^2_{fi}} = 1.90 \times 10^{-24} \text{g} \tag{1} \]

where \( a_\mu \) is muonic Bohr radium \( a_\mu = a_0 \mu / m_e \), respectively the estimated Davies temperature on \( n \)-th Rydberg orbit is
\[ T = \frac{\hbar a_n}{2\pi kc} = 75830K/n^4 \tag{2} \]

where \( n \) is the muonic Rydberg quantum number and \( a_n \) is centrifugal acceleration on \( n \)-the Rydberg orbit. It is 1000K for \( n = 3 \) and therefore should be easily observable.

2. TROJAN WAVEPACKETS

We start our detailed analysis from the harmonic theory of Trojan wavepackets in muonic Hydrogen \[6, 13\]. We neglect the spin for simplification. Note that even through the lifetime of the muon is \( 2.1970 \times 10^{-6} \text{s} \) the packet can make thousands of revolutions for the laser optical frequency, more then the lifetime due to its electromagnetic resonant nature.

Since the ratio between the electron velocity on the first Bohr orbit to the speed of light is always \( \alpha = 1/137 \) for arbitrary mass of quantum particle the system still can be described by the Schrödinger equation. The Schrödinger equation for the Trojan wavepacket in circularly polarized field in the Coulomb field is
\[
\left( \frac{p^2}{2\mu} - \frac{1}{r} - e\mathcal{E}_f + \omega L_z \right)\psi = i\hbar \dot{\psi} \tag{3}
\]

where \( \mu \) is the muon mass \( \mu = 206.7683m_e \). The standard stability analysis \[6, 13\] of the trajectory in rotating frame extended to relativistic case gives the following harmonic Gaussian wavefunction
\[
\psi_0 = Ne^{i\mu\omega x_0/h} e^{-iE_0t/h} \times e^{-\mu[A(x-x_0)^2+By^2+2iC(x-x_0)+Dz^2]/2h} \tag{4}
\]

with parameters \( A, B, C, D \). The stationary factor is the eigenfunction of the harmonic Hamiltonian
\[
H_h = \hbar \omega + a_+ a_+ - \hbar \omega a_- a_- + h \omega a_+ a_- + \text{const} \tag{5}
\]

where \[13\]
\[
\omega = \omega \sqrt{2 - q \pm 9q^2 - 8q/\sqrt{2}} \tag{6}
\]

\[
\omega = \omega \sqrt{q} \tag{7}
\]

end the parameter \( x_0 \) is the classical center of the wavepacket give implicitly
\[
- \hbar \omega x_0^2 + \hbar \mathcal{E}_f + \mu \omega \omega x_0 = 0 \tag{8}
\]

Note that it is not exactly the Hamiltonian of three harmonic oscillators because of negative sign of \( a_+ a_- \) but rather Hamiltonian predicting vacuum collapse if coupled to electromagnetic field. The parameters \( A, B \) and \( C \) are respectively functions of the dimensionless parameter \( q \) only \[13, 15\]
\[
A(q) = \sqrt{(1 + 2q)[4f(q) - 9q^2]/3q} \tag{9}
\]
\[
B(q) = \sqrt{(1 - q)[4f(q) - 9q^2]/3q} \tag{10}
\]
\[
C(q) = f(q)/3q, \tag{11}
\]
\[
D(q) = \sqrt{q} \tag{12}
\]

where
\[
f(q) = 2 + q - 2\sqrt{(1 - q)(1 + 2q)} \tag{13}
\]

3. HAWKING RADIATION

The probability of the Hawking photon emission by the non-relativistic Trojan wavepacket can be calculated similarly to the spontaneous emission rate \[15, 14\] as
\[
\Gamma_{UD} = \frac{e^2}{\hbar^2} \sum_{i,j} \langle \psi_0 | x_i | \psi_1 \rangle < \psi_1 | x_j | \psi_0 \rangle \times G_{ij} \left[ \frac{E_1 - E_0}{\hbar} \right] \tag{14}
\]
where \( G_{i,j}(\omega) \) is the Fourier transform field-field correlation function for the accelerated observer on the circular orbit

\[
G_{ij}(\Omega) = \int_{-\infty}^{\infty} e^{-i\omega \tau} G_{i,j}(\tau)
\]

(12)

\[
G_{ij}(\tau) = 0 < E_i(\tau_1)E_j(\tau_2)0 >
\]

(13)

The Wightman function (electromagnetic field tensor - tensor correlation function of the electromagnetic field in the Minkowski vacuum is

\[
< 0|F_{\mu\nu}(x)F_{\rho\sigma}(x')|0 >
\]

(14)

\[
= \frac{4\hbar c}{\epsilon_0 \pi} \Big\{ (x-x')^{-6} (g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})
\]

\[
- 2[(x-x')_{\mu}(x-x')_{\nu}g_{\sigma\rho}]
\]

\[
- (x-x')_{\mu}(x-x')_{\rho}g_{\mu\sigma}
\]

\[
+ (x-x')_{\nu}(x-x')_{\rho}g_{\mu\sigma}
\]

\[
\}
\]

The DeWitt detector response is due to delay of the proper time on the noninertial trajectory. The space-time trajectory parameterized by the proper time \( \tau \) is for Trojan atom

\[
x = (ct\gamma, R\cos(\omega t\gamma), R\sin(\omega t\gamma), 0)
\]

where

\[
\beta = \frac{v}{c}
\]

(15)

\[
\gamma = \frac{1}{\sqrt{1-\beta^2}}
\]

(16)

The field-field correlation function for the observer moving on the circular trajectory can be estimated \([16]\) \((\omega \tau \ll 1)\) as

\[
G_{ij}(\tilde{\tau} - \tilde{\tau}') = \frac{1}{\epsilon_0} \frac{4\hbar c}{\pi} \frac{1}{\gamma^4} \frac{1}{(\tilde{\tau} - \tilde{\tau}')^4} \delta_{ij}
\]

(17)

The only nonvanishing matrix elements between the first deexcited Trojan state and the Trojan state are those of \( x \) and \( y \) coordinates \([16]\) and can be expressed by their fluctuations. Using the result of \([15]\) one gets

\[
< \psi_1|x|\psi_0 > = \alpha \lambda < \psi_0|x^2|\psi_0 >
\]

\[
= \frac{\alpha \lambda h}{2Am\omega}
\]

\[
< \psi_1|y|\psi_0 > = \alpha < \psi_0|y^2|\psi_0 >
\]

\[
= \frac{\alpha h}{2Bm\omega}
\]

(18)

where \([15]\)

\[
\alpha = \frac{\sqrt{2\mu\omega}/\hbar}{\sqrt{\lambda^2/A + 1/B}} = \tilde{\alpha} \sqrt{2\mu\omega}/\hbar
\]

(19)

\[
\lambda = (1 + C)/(A + \omega_-/\omega).
\]

(20)

The energy difference between the Trojan and first deexcited Trojan state is

\[
(E_1 - E_0) = \hbar \omega_-
\]

(21)

The probability of emission of Hawking photon (the decay rate of muonic Trojan wavepacket due to the Unruh-Davies effect) is therefore

\[
\Gamma_{UD} = \frac{1}{3} \sqrt{\frac{\pi}{2}} \frac{e^2 \omega^2}{\epsilon_0 \mu c^3} \tilde{\alpha}^2 \left( \frac{\lambda^2}{A^2} + \frac{1}{B^2} \right)^3
\]

(22)

with the dimensionless scaling function

\[
\theta(q) = \sqrt{2 - q - \sqrt{(9q^2 - 8q)/2}} = \omega_-/\omega
\]

(23)

In order the effect to be observable it must be not much less the decay rate of the emission which is qualitatively the same as due to the Larmor radiation of classical charged particle on the circular orbit \([15]\).

The decay rate of the spontaneous emission of the nonrelativistic Trojan wavepacket is given by \([15]\)

\[
\Gamma_{SP} = \frac{e^2}{3\epsilon_0 \mu c^3} \frac{\omega^2}{(\mu/A - 1/B)^2}(\mu^2/A + 1/B)(1 + \theta)^3
\]

(24)

Fig.1 shows both the Hawking decay rate and the spontaneous emission rate as functions of the scaled electric field \( \tilde{E} = E_f \omega^{-4/3} = (1-q)/q^{1/3} \).
Figure 1: The relative spontaneous emission rate (for the reference only, see [13]) and Hawking emission rate due to Unruh-Davies effect as functions of scaled electric field for hypothetical $n = 1$. Both rates scale like $1/n^3$ with the resonant Rydberg number $n$. Note that singularities at stability border values are nonphysical but the Hawking decay rate goes to zero when there is no electric field present.

The ratio between $\Gamma_{UD}/\Gamma_{SP}$ is therefore dimensionless parameter depending only on parameter $q$ of the harmonic theory of the Trojan wavepacket. The Davis effect is observable when power of spontaneous (Larmor) radiation is comparable with the power of Hawking radiation. Note that the relativistic parameter $\gamma$ from the relativistic Wightman function within the theory limit presented here should be approximated by 1 for consistency ($\beta = 1/137$) since we neither use the Klein-Gordon or Dirac equation for the theory of the Trojan electron itself. Fig.2 shows the ratio between the Hawking decay rate and the spontaneous emission rate as function of the scaled electric field $\mathcal{E}$. For the best confined wavepacket we get $\Gamma_{UD} = 1.11784 \times 10^6 s^{-1}$ already for $n = 12$, almost twice faster then the decay of the muon itself, well sufficient to observe decay of the wavepacket before the decay of the muon and still during thousands of the packet revolutions. It corresponds to the laser intensity $I = 5.89 \times 10^{14} W/cm^2$ and the wavelength $\lambda = 380.782 nm$. Since the spontaneous emission is the Larmor radiation in the classical limit [15] the Unruh-Davies emission should be readily observable as the correction to the only mechanism of radiation. Note that the Unruh-Davies radiation vanishes in the limit of vanishing electric CP field which means that stationary circular states do not emit Hawking radiation at all as they are not localized DeWitt detectors.

5. CONCLUSIONS

As quantum detectors accelerate they heat up due to the vacuum fluctuations of the quantum
electromagnetic field. No detectors of classical mass can reach the accelerations necessary to observe the effect. Trojan atoms as the smallest cyclotrons ever predicted theoretically seem to be the best experimental systems to observe the effect. We show that the Unruh-Davies contribution to the decay of Trojan states is as significant as spontaneous emission and therefore readily observable effect as equivalent to the Larmor radiation of cyclotronic electrons in the classical limit.

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References

[1] J. S. Bell, and J. M. Leinaas, 1983, Nucl. Phys. B, 212, 131.

[2] O. Levin, Y. Peleg, and A. Peres, 1993, J. Phys. A, 26, 3001.

[3] J. Audretsch, R. Müller, and M. Holtzmann, 1995, Class. Quantum. Grav., 12, 2927.

[4] T. H. Boyer, 1980, Phys. Rev. D, 21, 2137.

[5] J. S. Bell, and J. M. Leinaas, 1987, Nucl. Phys. B, 284, 488.

[6] I. Bialynicki-Birula, M. Kalinski, and J. H. Eberly, 1994, Phys. Rev. Lett., 73, 1777

[7] J. Zakrzewski, D. Delande, and A. Buchleitner, 1995, Phys. Rev. Lett., 75, 1995; A. F. Brunello, T. Uzer, and D. Farrelly, 1996, Phys. Rev. Lett., 76, 4015; H. Maeda, and T. E. Gallagher, 2004, Phys. Rev. Lett., 92, 133004-1.

[8] P. Chen, and T. Tajima, 1999, Phys. Rev. Lett., 83, 256.

[9] C. Rose-Petruck, K. J. Schafer, K. R. Wilson, and C. P. Barty, 1997, Phys. Rev. A, 55, 1181.

[10] J. Zweiback, R. A. Smith, T. E. Cowan, G. Hays, K. B. Wharton, V. P. Yanovsky, and T. Ditmire, 2000, Phys. Rev. Lett., 84, 2634.

[11] S. Chelkowski, A. D. Bandrauk, and P. B. Corkum, 2004, Phys. Rev. Lett., 93, 083602-1.

[12] P. Chen, and T. Tajima, 1999, Phys. Rev. Lett., 83, 256.

[13] M. Kalinski, and J. H. Eberly, 1996, Phys. Rev. A, 53, 1715.

[14] H. Terashima, 1999, Phys. Rev. D, 60, 084001-1.

[15] Z. Bialynicka-Birula, and I. Bialynicki-Birula, 1997, Phys. Rev. A, 56, 3623.

[16] The exact general expressions are complicated and contain normal trigonometric factors of both types but they greatly simplify in almost static detector limit \( \omega \tau \ll 1 \) where the resonant factors are dominant.

[17] Nondivergent results can be obtained using expansion of Trojan states in hydrogenic basis as Stark states for paramagnetic Kepler problem \[13\].