Production and sequential decay of charmed hyperons

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Abstract

We investigate production and decay of the $\Lambda_c^+$ hyperon. The production considered is through the $e^+e^-$ annihilation channel, $e^+e^- \rightarrow \Lambda_c^+\bar{\Lambda}_c^-$, with summation over the $\bar{\Lambda}_c^-$ anti-hyperon spin directions. It is in this situation that the $\Lambda_c^+$ decay chain is identified. Two kinds of sequential decays are studied. The first one is the doubly weak decay $B_1 \rightarrow B_2 M_2$, followed by $B_2 \rightarrow B_3 M_3$. The other one is the mixed weak-electromagnetic decay $B_1 \rightarrow B_2 M_2$, followed by $B_2 \rightarrow B_3 \gamma$. In both schemes $B$ denotes baryons and $M$ mesons. We should also mention that the initial state of the $\Lambda_c^+$ hyperon is polarized.

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I. INTRODUCTION

We shall investigate properties of certain sequential decays of the \( \Lambda^+_c \) hyperon, but in order to do so we first need to produce them. To this end we consider the reaction \( e^+ e^- \rightarrow \Lambda^+_c \bar{\Lambda}^-_c \), which is analyzed in detail in Refs.\[1, 2\]. In order to describe such an annihilation process two hadronic form factors are needed. They can be parametrized by two parameters, \( \alpha \) and \( \Delta \Phi \), with \(-1 \leq \alpha \leq 1\). For their precise definitions we refer to Ref.\ [2].

The general cross-section distribution of this annihilation reaction depends on six structure functions which themselves are functions of \( \alpha \), \( \Delta \Phi \), and \( \theta \), the scattering angle. In our application, however, we sum over the decay products of the anti-hyperon \( \bar{\Lambda}^-_c \), but identify the decay chain of the hyperon \( \Lambda^+_c \), so called single tag events. In this simplified case only two structure functions are relevant,

\[
R = 1 + \alpha \cos^2 \theta, \\
S = \sqrt{1 - \alpha^2} \sin \theta \cos \theta \sin \Delta \Phi.
\]

The scattering distribution function for the \( \Lambda^+_c \) hyperon production becomes, according to Refs.\[2\], proportional to

\[
W(n) = R + S N \cdot n,
\]

where \( n \) is the direction of the hyperon spin vector in the hyperon rest system, \( N \) the normal to the scattering plane,

\[
N = \frac{1}{\sin \theta} \hat{p} \times \hat{k},
\]

and \( \cos \theta = \hat{p} \cdot \hat{k} \). The momenta \( k \) and \( p \) are the relative momenta in the initial and final states, in the center of momentum (c.m.) system. The meaning of the spin vector \( n \) is explained in Ref.\[3\].

From Eq.\( (I.3) \) we deduce for the spin distribution function,

\[
S(P) = 1 + P \cdot n
\]

and \( P \) the hyperon polarization,

\[
P = (S/R)N,
\]

subject to the restriction \(|P| \leq 1\). For an unpolarized initial state hyperon \( P = 0 \).
II. WEAK HYPERON DECAYS

The weak hyperon decay $c \rightarrow d\pi$, of which $\Lambda \rightarrow p\pi^-$ is an example, is described by two amplitudes, one S-wave and one P-wave amplitude. The decay distribution is commonly described by three parameters, denoted $\alpha\beta\gamma$. They are not independent but fulfill the relation

$$\alpha^2 + \beta^2 + \gamma^2 = 1.$$  \hspace{1cm} (II.7)

The parametrization of this hyperon decay is discussed in detail in Ref.[4] and also in Ref.[2].

We denote by $G_c(c, d)$ the distribution function describing the weak hyperon decay $c \rightarrow d\pi$, given the spin vectors $n_c$ and $n_d$,

$$G_c(c, d) = 1 + \alpha c n_c \cdot l_d + \alpha d n_d \cdot l_d + n_c \cdot L_c(n_d, l_d),$$  \hspace{1cm} (II.8)

with

$$L_c(n_d, l_d) = \gamma c n_d + (1 - \gamma c) n_d \cdot l_d l_d + \beta d n_d \times l_d.$$  \hspace{1cm} (II.9)

The vector $l_d$ is a unit vector in the direction of motion of the decay baryon $d$ in the rest system of baryon $c$. The indices on the $\alpha\beta\gamma$ parameters remind us they characterize hyperon $c$.

Since the spin of baryon $d$ is often not measured, the relevant decay density is obtained by averaging over the spin directions $n_d$,

$$W_c(n_c; l_d) = \left\langle G_c(c, d) \right\rangle_d$$

$$= U_c + n_c \cdot V_c,$$  \hspace{1cm} (II.10)

with

$$U_c = 1, \hspace{1cm} V_c = \alpha c l_d.$$  \hspace{1cm} (II.11)

Hyperons we study are produced in some reaction, and their states are described by some spin distribution function, Eq.(I.5),

$$S_c(P_c) = 1 + P_c \cdot n_c.$$  \hspace{1cm} (II.12)

The final-state distribution in a production reaction followed by decay is obtained by a folding, pertaining to the intermediate and final hyperon spin directions $n_c$ and $n_d$,

$$W_c(P_c; l_d) = \left\langle S_c(P_c)G_c(c, d) \right\rangle_{cd}$$

$$= 1 + P_c \cdot V_c,$$  \hspace{1cm} (II.13)
where \( V_c = \alpha_c l_d \), from Eq.(III.11).

The folding over intermediate spin directions follows the prescription of Ref.[1],

\[
\langle 1 \rangle_n = 1, \quad \langle n \rangle_n = 0, \quad \langle n \cdot k n \cdot l \rangle_n = k \cdot l. \quad \text{(II.14)}
\]

From Eq.(II.13) it is clear that if the polarization is known the asymmetry parameter \( \alpha_c \) can be measured, but not the \( \beta_c \) or \( \gamma_c \) parameters. For that to be possible we must measure the polarization of the decay baryon \( d \). If hyperon \( c \) is produced within a \( c\bar{c} \) pair in \( e^+e^- \) annihilation then the polarization can be determined from the cross-section distribution.

### III. ELECTROMAGNETIC HYPERON TRANSITIONS

Electromagnetic transitions such as \( \Sigma^0 \to \Lambda \gamma \) and \( \Xi^0 \to \Lambda \gamma \) can also be studied in \( \Lambda^+_c \) decays.

An electromagnetic transition \( c \to d\gamma \) is described by a transition distribution function similar to that of the weak decay, Eq.(II.8). However, the special feature of the electromagnetic interaction is the photon helicity which takes only two values, \( \lambda_\gamma = \pm 1 \).

The electromagnetic transition distribution function corresponding to Eq.(II.8) is

\[
G_\gamma(c, d; \lambda_\gamma) = (1 - n_c \cdot l_d l_d \cdot n_d) - \lambda_\gamma(n_c \cdot l_d - n_d \cdot l_d), \quad \text{(III.15)}
\]

where \( l_d \) is a unit vector in the direction of motion of hyperon \( d \) in the rest system of hyperon \( c \).

Averaging over photon polarizations the transition distribution takes a very simple form,

\[
G_\gamma(c, d) = 1 - n_c \cdot l_d l_d \cdot n_d. \quad \text{(III.16)}
\]

We notice that when both hadron spins are parallel or anti-parallel to the photon momentum, then the transition probability vanishes, a property of angular-momentum conservation. We also notice that expression (III.16) cannot be written in the \( \alpha\beta\gamma \) representation of Eq.(II.8).

### IV. TWO-STEP WEAK HYPERON DECAY

Now, we apply the above technique to hyperons decaying in two steps, such as \( b \to c \to d \), accompanied by pions. An example of this decay mode is \( \Lambda^+_c \to \Lambda\pi^+ \) followed by \( \Lambda \to p\pi^- \).
We denote by \( G_b(b, c) \) the distribution function describing the hyperon decay \( b \to c \pi \) pertaining to spin vectors \( n_b \) and \( n_c \),

\[
G_b(b, c) = 1 + \alpha_b n_b \cdot l_c + \alpha_b n_c \cdot l_b + n_b \cdot L_b(n_c, l_c),
\]

(IV.17)

with

\[
L_b(n_c, l_c) = \frac{\gamma b n_c}{1 - \gamma b} + \left[ (1 - \gamma b) n_c \cdot l_c \right] l_c + \beta_b n_c \times l_c.
\]

(IV.18)

The vector \( l_c \) is a unit vector in the direction of motion of baryon \( c \) in the rest system of baryon \( b \).

Folding together the distribution functions \( G_b(b, c) \) and \( G_c(c, d) \), averaging over spin vectors \( n_c \) and \( n_d \) following the folding prescription (II.14), we get the decay density distribution function

\[
W_b(n_b; l_c, l_d) = \left\langle G_b(b, c)G_c(c, d) \right\rangle_{cd} = U_b + n_b \cdot V_b,
\]

(IV.19)

with

\[
U_b = 1 + \alpha_b \alpha_c l_c \cdot l_d,
\]

(IV.20)

\[
V_b = \alpha_b l_c + \alpha_c L_b(l_d, l_c).
\]

(IV.21)

The result is interesting. In many cases the asymmetry parameter \( \alpha_c \) for the \( c \) hyperon and the polarization \( P_b \) for the initial-state \( b \) hyperon are known. Then, just as in the single-step case of Eq.(II.12), the initial state is described by a spin distribution function

\[
S_b(P_b) = 1 + P_b \cdot n_b.
\]

(IV.22)

For the decay distribution of a polarized hyperon, we obtain

\[
W_b(P_b; l_c, l_d) = \left\langle S_b(P_b)G_b(b, c)G_c(c, d) \right\rangle_{bcd} = U_b + P_b \cdot V_b.
\]

(IV.23)

This is equivalent to making the replacement \( n_b \to P_b \) in Eq.(IV.19).

We conclude that by determining \( U_b \) and \( V_b \) of Eqs.(IV.20) and (IV.21), we should be able to determine all three decay parameters \( \alpha_b \), \( \beta_b \), and \( \gamma_b \), for the \( b \) hyperon, and \( \alpha_c \) for the \( c \) hyperon.
It is now clear how to get the cross-section distribution for production of $\Lambda^+_c$ in $e^+e^-$ annihilation and its subsequent decay $\Lambda^+_c \rightarrow \Lambda \pi^+$ and $\Lambda \rightarrow p\pi^-$. Starting from the expressions for the scattering distribution function, Eq.(I.3), and the polarization, Eq.(I.6), we obtain

$$d\sigma \propto \left[ RU_{\Lambda_c} + SN \cdot V_{\Lambda_c} \right] d\Omega_{\Lambda_c} d\Omega_{\Lambda} d\Omega_p,$$

(IV.24)

with $N$, Eq.(I.4), the normal to the scattering plane. The functions $R$ and $S$ are defined in Eqs.(I.1) and (I.2) and depend among other things on the $\Lambda^+_c$ scattering angle $\theta = \theta_{\Lambda_c}$. In Eqs.(IV.20) and (IV.21) indices are interpreted as; $b = \Lambda^+_c$, $c = \Lambda$, $d = p$.

When integrating over the decay angles $\Omega_\Lambda$ and $\Omega_p$ in Eq.(IV.24) we observe that the term involving the polarization $N \cdot V_{\Lambda_c}$ vanishes, as does the term involving the angular dependent part of $U_{\Lambda_c}$. This results in the cross-section distribution

$$d\sigma \propto \left[ 1 + \alpha \cos^2\theta_{\Lambda_c} \right] d\Omega_{\Lambda_c},$$

(IV.25)

describing the annihilation reaction $e^+e^- \rightarrow \Lambda^+_c \bar{\Lambda}^-_c$.

It is more interesting to perform a partial integration. Let us integrate over the angles $\Omega_\Lambda$ and $\Omega_p$ keeping $\cos \theta_{\Lambda p}$ of

$$\cos \theta_{\Lambda p} = l_\Lambda \cdot l_p$$

(IV.26)
constant. Also in this case does the contribution involving the polarization vanish. We are left with

$$d\sigma \propto \left[ 1 + \alpha \cos^2\theta_{\Lambda_c} \right] \left[ 1 + \alpha_{\Lambda_c} \alpha \cos \theta_{\Lambda p} \right] d(\cos \theta_{\Lambda_c})d(\cos \theta_{\Lambda p}).$$

(IV.27)

The cross-section distribution of Eq.(IV.24) applies also to the decay chain, $\Lambda^+_c \rightarrow \Sigma^+\pi^0$ and $\Sigma^+ \rightarrow p\pi^0$, with the corresponding identification of indices $b$, $c$, and $d$.

V. DIFFERENTIAL DISTRIBUTIONS

The cross-section distribution (IV.24) is a function of two unit vectors $l_1 = l_\Lambda$, the direction of motion of the Lambda hyperon in the rest system of the charmed-Lambda hyperon, and $l_2 = l_p$ the direction of motion of the proton in the rest system of the Lambda hyperon. In order to handle these vectors we need a common coordinate system which we define as follows.

The scattering plane of the reaction $e^+e^- \rightarrow \Lambda_c\bar{\Lambda}_c$ is spanned by the unit vectors $\hat{p} = l_{\Lambda_c}$ and $\hat{k} = l_{c^+}$, as measured in the c.m. system. We assume the scattering to be to the left,
with scattering angle $\theta \geq 0$. If the scattering is to the right we rotate such an event 180° around the $k$-axis, so that the scattering appears to be to the left. The scattering plane makes up the $xz$-plane, with the $y$-axis along the normal to the scattering plane. We choose a right-handed coordinate system with basis vectors

\[
\begin{align*}
    e_z &= \hat{p}, \\
    e_y &= \frac{1}{\sin \theta} (\hat{p} \times \hat{k}), \\
    e_x &= \frac{1}{\sin \theta} (\hat{p} \times \hat{k}) \times \hat{p}.
\end{align*}
\]

Expressed in terms of them the initial-state momentum

\[
\hat{k} = \sin \theta e_x + \cos \theta e_z.
\]

This coordinate system is used for defining the directional angles of the Lambda and the proton. The directional angles of the Lambda hyperon in the charmed-Lambda hyperon rest system are,

\[
\mathbf{l}_1 = (\cos \phi_1 \sin \theta_1, \sin \phi_1 \sin \theta_1, \cos \theta_1),
\]

whereas the directional angles of the proton in the Lambda hyperon rest system are

\[
\mathbf{l}_2 = (\cos \phi_2 \sin \theta_2, \sin \phi_2 \sin \theta_2, \cos \theta_2).
\]

An event of the reaction $e^+e^- \rightarrow \bar{\Lambda}_c \Lambda_c (\rightarrow \Lambda (\rightarrow p\pi)\pi)$ is specified by the five dimensional vector $\xi = (\theta, \Omega_1, \Omega_2)$, and the differential-cross-section distribution as summarized by Eq.(IV.24) reads,

\[
d\sigma \propto W(\xi) \, d\cos \theta \, d\Omega_1 \, d\Omega_2.
\]

At the moment, we are not interested in absolute normalizations. The differential-distribution function $W(\xi)$ is obtained from Eqs.(I.1, I.2, IV.20, IV.21, IV.24) and can be expressed as,

\[
W(\xi) = F_0(\xi) + \alpha F_1(\xi) + \alpha_1 \alpha_2 \left( F_2(\xi) + \alpha F_3(\xi) \right) 
+ \sqrt{1 - \alpha^2} \cos(\Delta \Phi) \left( F_7(\xi) + \alpha_1 F_4(\xi) + \beta_1 F_6(\xi) 
+ \gamma_1 (F_5(\xi) - F_7(\xi)) \right),
\]
using a set of eight angular functions \( F_k(\xi) \) defined as:

\[
\begin{align*}
F_0(\xi) &= 1, \\
F_1(\xi) &= \cos^2 \theta, \\
F_2(\xi) &= \sin \theta_1 \sin \theta_2 \cos(\phi_1 - \phi_2) + \cos \theta_1 \cos \theta_2, \\
F_3(\xi) &= \cos^2 \theta \, F_2(\xi), \\
F_4(\xi) &= \sin \theta \cos \theta \sin \theta_1 \cos \phi_1, \\
F_5(\xi) &= \sin \theta \cos \theta \sin \theta_2 \sin \phi_2, \\
F_6(\xi) &= \sin \theta \cos \theta \sin \theta_1 \sin \phi_1 \, F_2(\xi), \\
F_7(\xi) &= \sin \theta \cos \theta \sin \theta_1 \sin \phi_1 \, F_2(\xi).
\end{align*}
\] (V.35)

The differential distribution of Eq. (V.34) involves two parameters related to the \( e^+e^- \rightarrow \Lambda_c \bar{\Lambda}_c \) reaction that can be determined by data: the ratio of form factors \( \alpha \), and the relative phase of form factors \( \Delta \Phi \). In addition, the distribution function \( \mathcal{W}(\xi) \) depends on the weak-decay parameters \( \alpha_1 \beta_1 \gamma_1 \) of the charmed-hyperon decay \( \Lambda_c \rightarrow \Lambda \pi \), and on the weak-decay parameters \( \alpha_2 \beta_2 \gamma_2 \) of the hyperon decay \( \Lambda \rightarrow p \pi^\mp \). However, the dependency on \( \beta_2 \) and \( \gamma_2 \) drops out. Similarly, integrating over \( d\Omega \) we get

\[
\begin{align*}
d\sigma &\propto \left[ 1 + \alpha \cos^2 \theta \\
&+ \alpha_1 \sqrt{1 - \alpha^2} \cos(\Delta \Phi) \sin \theta \cos \theta \sin \theta_1 \sin \phi_1 \right] d\Omega_1 \, d\Omega_2,
\end{align*}
\] (V.36)

where now the dependency on \( \beta_1 \) and \( \gamma_1 \) also drops out. The last term in this equation originates with the scalar \( P_{\Lambda_c} \cdot \mathbf{N} \). The charmed-hypern polarization vanishes at \( \theta = 0^\circ, 90^\circ \) and \( 180^\circ \).

The distributions presented here will hopefully be of value in the analysis of BESIII data.

VI. MIXED WEAK-ELECTROMAGNETIC HYPERON DECAY

Now, we extend the formalism to hyperons decaying in two steps, with one being electromagnetic. An example of such a decay chain is \( \Lambda_c^+ \rightarrow \Sigma^0 \pi^+ \) followed by \( \Sigma^0 \rightarrow \Lambda \gamma \). As before we employ indices \( b, c, \) and \( d \) for variables belonging to \( \Lambda_c^+, \Sigma^0, \) and \( \Lambda \).
The distribution functions for the weak and electromagnetic transitions are given in Eqs. (IV.17) and (III.16),

\[ G_b(b, c) = 1 + \alpha_b \mathbf{n}_b \cdot \mathbf{l}_c + \alpha_b \mathbf{n}_c \cdot \mathbf{l}_c + \mathbf{n}_b \cdot \mathbf{L}_b(\mathbf{n}_c, \mathbf{l}_c), \]  
(VI.37)

\[ G_\gamma(c, d) = 1 - \mathbf{n}_c \cdot \mathbf{l}_d \mathbf{l}_d \cdot \mathbf{n}_d. \]  
(VI.38)

Performing a folding of the product of the distribution functions \( G_b(b, c) \) and \( G_\gamma(c, d) \), i.e. averaging over spin vectors \( \mathbf{n}_c \) and \( \mathbf{n}_d \) following the folding prescription (II.14), we get

\[ W_b(\mathbf{n}_b; \mathbf{l}_c, \mathbf{l}_d) = \left\langle G_b(b, c)G_\gamma(c, d) \right\rangle_{cd} = U_b + \mathbf{n}_b \cdot \mathbf{V}_b, \]  
(VI.39)

with

\[ U_b = 1, \quad \mathbf{V}_b = \alpha_b \mathbf{l}_c. \]  
(VI.40)

These expressions for \( U_b \) and \( \mathbf{V}_b \) are noteworthy. They are in fact the same as those of a one-step \( b \rightarrow c\pi \) decay, Eq. (II.11). Hence, the electromagnetic decay does not add any structure, Eqs. (VI.40) are independent of \( \mathbf{l}_d \).

The initial state spin distribution function for hyperon \( b \) produced in \( e^+e^- \) annihilation is as above, Eq. (IV.22),

\[ S_b(\mathbf{P}_b) = 1 + \mathbf{P}_b \cdot \mathbf{n}_b, \]  
(VI.41)

Folding this distribution function with the decay distribution function of Eq. (VI.39), we obtain

\[ W_b(\mathbf{P}_b; \mathbf{l}_c, \mathbf{l}_d) = \left\langle S_b(\mathbf{P}_b)G_b(b, c)G_\gamma(c, d) \right\rangle_{bed} = U_b + \mathbf{P}_b \cdot \mathbf{V}_b. \]  
(VI.42)

As noted earlier this is equivalent to making the replacement \( \mathbf{n}_b \rightarrow \mathbf{P}_b \) in Eq. (VI.39). We also notice if we manage to determine \( U_b \) and \( \mathbf{V}_b \) of Eqs. (VI.42), the only parameter that can be fixed is \( \alpha_b \), a meager return.

The expression for the cross-section distribution for \( \Lambda_c^+ \) production and subsequent decays \( \Lambda_c^+ \rightarrow \Sigma^0 \pi^+ \) and \( \Sigma^0 \rightarrow \Lambda\gamma \) is

\[ d\sigma \propto \left[ \mathcal{R}U_{\Lambda_c} + \mathbf{S} \mathbf{N} \cdot \mathbf{V}_{\Lambda_c} \right] d\Omega_{\Lambda_c} d\Omega_\Sigma d\Omega_\Lambda, \]  
(VI.43)
with \( \mathbf{N} \), Eq.(I.4), the normal to the scattering plane, and \( U_{\Lambda_c} = 1 \), \( \mathbf{V}_{\Lambda_c} = \alpha_{\Lambda_c} \mathbf{l}_\Sigma \), from Eq.(VI.42). The functions \( \mathcal{R} \) and \( \mathcal{S} \) are defined in Eqs.(I.1) and (I.2) and depend among other things on the \( \Lambda_c^+ \) scattering angle \( \theta \).

Finally, we mention that it is possible to expand the \( \Lambda_c^+ \) decay chain by adding the decay \( \Lambda \rightarrow p\pi^- \).

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**Appendix**

In this Appendix we detail the angular integration leading to Eq.(IV.27).

Consider two unit vectors \( \mathbf{l}_c \) and \( \mathbf{l}_d \). We want to integrate over the angles \( \Omega_c \) and \( \Omega_d \) keeping \( \cos \theta_{cd} = \mathbf{l}_c \cdot \mathbf{l}_d \) fixed. To this end we put the vectors in the \( xy \)-plane of the coordinate system \( \mathrm{O}' \),

\[
\mathbf{l}_c = (1, 0, 0), \quad \mathbf{l}_d = (\cos \theta_{cd}, \sin \theta_{cd}, 0), \quad \mathbf{l}_c \times \mathbf{l}_d = (0, 0, \sin \theta_{cd}).
\]

We then rotate the coordinate system \( \mathrm{O}' \) with respect to the space fixed coordinate system \( \mathrm{O} \), where the normal to the scattering plane is along the \( Z \)-direction. The rotation matrix which transforms the column vector \( \vec{r}_b \) in \( \mathrm{O}' \) into the column vector \( \vec{r}_s \) in \( \mathrm{O} \) is the matrix

\[
R^{-1}(\alpha, \beta, \gamma) = \begin{pmatrix}
\cos \alpha \cos \beta \cos \gamma - \sin \alpha \sin \gamma & \cos \gamma \cos \beta \sin \alpha + \sin \gamma \cos \alpha & -\sin \beta \cos \gamma \\
-\sin \gamma \cos \beta \cos \alpha - \cos \gamma \sin \alpha & \sin \gamma \cos \beta \sin \alpha + \cos \gamma \cos \alpha & \sin \beta \sin \gamma \\
\sin \alpha \sin \beta & \sin \gamma \sin \beta & \cos \beta
\end{pmatrix}.
\]

with \( \vec{r}_s = R^{-1}(\alpha, \beta, \gamma) \vec{r}_b \) and \( \alpha \beta \gamma \) the Euler angles.

The angular integrations can be expressed in terms of the Euler angles, as

\[
d\Omega_c \, d\Omega_d = d(\cos \theta_{cd}) \, d\alpha \, d(\cos \beta) \, d\gamma.
\]
The expression to be integrated, Eq. (IV.21), reads

\[ U_b + P \cdot V_b = 1 + \alpha_b \alpha_c l_c \cdot l_d + \alpha_b P \cdot l_c + \gamma_b P \cdot l_d \\
+ (1 - \gamma_b)P \cdot l_d \cdot l_c + \beta_b P \cdot (l_d \times l_c), \]  

(VI.49)

with \( P \) along the Z-direction.

Now, we note that terms proportional to \( P \cdot l_c \) or \( P \cdot l_d \) vanish upon integration over angles \( \alpha \) or \( \gamma \). Therefore,

\[
\int d\Omega_c d\Omega_d (U_b + P \cdot V_b) =
\]

\[
= 4\pi^2 \int d(\cos \theta_{cd})d(\cos \beta) \left( 1 + \alpha_b \alpha_c \cos \theta_{cd} \right) \]

\[
- \beta_b \alpha_c P \sin \theta_{cd} \cos \beta \right) \]

\[
= 8\pi^2 \int d(\cos \theta_{cd}) \left( 1 + \alpha_b \alpha_c \cos \theta_{cd} \right). \]  

(VI.50)

This result leads to Eq. (IV.27).

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