Probing the equation of state of asymmetric nuclear matter with isospin diffusion and stopping in heavy-ion collisions

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Abstract. Results from the study of isospin asymmetric Sn+Sn collisions at E/A=35 MeV and 50 MeV, collected with the LASSA and the Chimera detectors at the NSCL of MSU, respectively, are shown. The diffusion of neutrons and protons between projectiles and targets with different N/Z-asymmetries is studied by means of imbalance ratios measurements. The obtained results and their comparison to transport model simulations provide constraints on the density dependence of the symmetry energy. The systematic study of these phenomena at different impact parameters also allows us to explore isospin transparency and stopping phenomena in more central collisions, with important implications about the attainment of chemical equilibrium in central collisions at 35 MeV/nucleon.

1. Introduction
Studying the density dependence of the symmetry energy in heavy-ion collisions has become a priority due to its relevance to both properties of neutron rich nuclei and neutron star physics [1-3]. The
interest in this field has recently increased due to the availability of radioactive beam facilities capable of producing energetic beams of nuclei with a large neutron/proton number asymmetry. Large N/Z asymmetries are key to amplify the effects of the asymmetry term in heavy-ion collision dynamics. In order to better quantify these effects the equation of state is commonly parameterized as $E(\rho/\rho_0, \delta) = E(\rho/\rho_0, \delta = 0) + E_{\text{sym}}(\rho/\rho_0) \cdot \delta^2$, with $\delta = (N-Z)/(N+Z)$, $\rho$ and $\delta$ being, respectively, the N/Z-asymmetry and the density of nuclear matter ($\rho_0 = 0.16 \text{ fm}^{-3}$ is the density at saturation). While the equation of state for symmetric nuclear matter, $E(\rho/\rho_0, \delta = 0)$, has been extensively studied in the last decades [4], the symmetry term is still largely unknown. Knowledge of the density dependence of the symmetry term, $E_{\text{sym}}(\rho)$, is key to study nucleosynthesis during pre-supernova evolution of massive stars and the cooling of proto-neutron stars [2]. Although the nuclear symmetry energy at normal nuclear matter density is known to be around 30 MeV from the empirical liquid-drop mass formula, its values at other densities, especially at supra-normal densities, are poorly known. Due to the small magnitude of the effects of the symmetry energy $E_{\text{sym}}(\rho)$ in heavy-ion collisions it is important to study systems with large $\delta$-asymmetries in order to amplify and isolate them. Furthermore, since the symmetry potential is repulsive for neutrons and attractive for protons, a number of observables that have been suggested as being sensitive to the symmetry energy consist of measuring relative neutron/proton quantities or, similarly, neutron-rich/proton-rich isotope yields such as $^4\text{He}$, $^7\text{Li}$/$^7\text{Be}$, etc. [1]. The density dependence of the symmetry energy has important effects on isospin transport consisting of the diffusion and drift of neutrons and protons during the collision between two heavy-ions in close contact [5,6]. In particular isospin diffusion has recently become one of the most sensitive observables to probe the density dependence of the symmetry energy in heavy-ion collisions [7-10]. Furthermore isospin diffusion can be used to study the degree of equilibration achieved during heavy-ion collisions providing probes for the expected hierarchical time evolution of different degrees of freedom [11].

In this contribution we will show results from the study of isospin diffusion in collisions between different Sn isotopes, $^{112}\text{Sn+}^{112}\text{Sn}$, $^{112}\text{Sn+}^{124}\text{Sn}$, $^{124}\text{Sn+}^{112}\text{Sn}$, $^{124}\text{Sn+}^{124}\text{Sn}$, at incident energies of E/A=35 and 50 MeV. It is shown that a substantial amount of transparency is observed even at the lower incident energy explored in these experiments. The rapidity dependence of imbalance ratios is used to probe the density dependence of the symmetry energy.

2. Isospin diffusion and drift in heavy-ion collisions

When two heavy ions with different N/Z asymmetries undergo a peripheral collision at intermediate energies (E/A=20-50 MeV) a drift and diffusion of neutrons and protons through the low-density region connecting the interacting partners is expected to occur [5]. The drift phenomenon is due to the density gradients existing between the low-density “neck” region and the two projectile-like and target-like partners that remain mostly at saturation density [5,6]. If the N/Z asymmetries of the two partners are different then the so-called diffusion phenomenon occurs: the projectile- and target-like systems exchange neutrons and protons towards a status of globally uniform N/Z distribution [5,6]. The actual attainment of this condition depends on the interaction and contact time between the reaction partners. Therefore it is intuitive to expect that a larger N/Z equilibration is achieved for lower incident energies and smaller impact parameters. A recent study performed by the INDRA collaboration has indeed used the net energy loss as a probe of the projectile and target interaction time, showing that the N/Z sharing between the partners proceeds from transparency to equilibration as the energy loss decreases [10]. Figure 1 shows a schematic view of how isospin diffusion can lead to complete N/Z equilibration or to N/Z translucency, respectively, for long and short interaction times.
Figure 1. Schematic view of isospin diffusion in a peripheral reaction between two heavy-ions with different N/Z asymmetries (indicated by different colours). See text for the details.

In Refs. [5,6] it was explained how both isospin drift and isospin diffusion are affected by the density dependence of the symmetry energy in the sub-saturation density region, $\rho < \rho_0$. In particular at these low densities neutron/proton drift is mostly driven by the derivative with respect to density while diffusion is driven mostly by the magnitude of the symmetry energy at $\rho < \rho_0$. A soft density dependence functional of $E_{\text{sym}}(\rho)$ is characterized by higher symmetry energy values at low density [1,5,6]. Therefore isospin diffusion and the chance of achieving a N/Z equilibration between the interacting systems are enhanced with a soft density dependence of the symmetry energy [1,5-10]. These considerations are meant to schematically explain how the density dependence of the symmetry energy affects isospin phenomena in mid-peripheral collisions. Indeed both drift and diffusion are complex phenomena and a clear understanding of the whole density dependence of the symmetry energy can clearly explain its role in affecting the N/Z sharing between the quasi-projectile, quasi-target and the neck region in heavy-ion collisions. Indeed, Refs. [7-10] have shown that a detailed study of N/Z sharing between the quasi-projectile (QP*) and quasi-target (QT*) provides strong probes of the density dependence of the symmetry energy [9,10].

3. Experimental results

$^{112}\text{Sn}+^{112}\text{Sn}$, $^{112}\text{Sn}+^{124}\text{Sn}$, $^{124}\text{Sn}+^{112}\text{Sn}$ and $^{124}\text{Sn}+^{124}\text{Sn}$ collisions at E/A=50 and 35 MeV have been studied at the National Superconducting Cyclotron Laboratory (NSCL) with the LASSA array [12] and at the INFN-Laboratori Nazionali del Sud (LNS) with the Chimera 4$\pi$ array [13], respectively. In both experiments the violence of the collision was probed by means of the multiplicity of charged particles detected in each event. This technique provides an estimate of the reduced impact parameter of the reaction, $b = b/b_{\text{max}}$, where $b_{\text{max}} = R_{\text{proj}} + R_{\text{ targ}}$ and $R_{\text{proj}}$ and $R_{\text{targ}}$ are, respectively, the projectile and target nuclei radii. The results obtained at E/A=50 MeV have been already extensively reported in the literature [7-9]. The main goal of those investigations consisted of studying isospin diffusion between projectile and target in peripheral reactions. As already discussed in describing Fig. 1, isospin diffusion can be observed by probing the isotopic composition of the quasi-projectile (QP) or quasi-target (QT) resulting from the collision process. If complete N/Z-equilibration occurs, QT and QP will have similar N/Z asymmetries. If a partial N/Z-transparency remains, then the N/Z of the QT will keep memory of the N/Z of the ancestor projectile (target) nucleus in the entrance channel of the reaction. It is therefore enough to focus on the isotopic composition of the quasi-projectile fragments in order to probe the attainment of N/Z equilibration or transparency and therefore study isospin
diffusion. In the case of Refs. [7-9] isospin diffusion between $^{112}$Sn target and $^{124}$Sn projectile nuclei has been studied by means of the so-called imbalance ratios

$$R_i = \frac{2x_i - x_{A+A} - x_{B+B}}{x_{A+A} - x_{B+B}}$$  \hspace{1cm} (1)$$

where $x$ is an isospin sensitive observable which is a linear function of the asymmetry $[\delta=(\rho_p-\rho_n)/2(\rho_p+\rho_n)]$ of a specific emitting source. By studying peripheral collisions, $\hat{b} \approx 0.8$, Refs. [7-9] have focused on the decay of the quasi-projectile QP and $x$ in Eq. (1) was chosen to be an observable that is sensitive to the $\delta$-asymmetry of the QP. Indeed, experimentally one does not have direct access to the $\delta$-asymmetry of the emitting source. In the experiment described in Refs. [7-9] the $x$-observable was constructed by using the slope of the isoscaling analysis of measured isotopic distributions [7] or the $^7$Li/$^7$Be isobaric yield ratios [8,9]. All these $x$-observables simply reflect the N/Z composition of the QP produced in the studied peripheral collision events. As it is shown on Fig. 1, isospin diffusion affects the extent to which the quasi-projectile keep memory or the original N/Z-asymmetry of beam nuclei. The attainment of N/Z-equilibration or transparency is then studied within the QP emitting source with the mixed N/Z-reaction systems, $^{112}$Sn+$^{112}$Sn, $^{112}$Sn+$^{124}$Sn, $^{124}$Sn+$^{112}$Sn and $^{124}$Sn+$^{124}$Sn. For the two symmetric systems $^{124}$Sn+$^{124}$Sn and $^{112}$Sn+$^{112}$Sn, $R_i$ is by construction normalized to $+1$ and $-1$, respectively. The occurrence of isospin equilibration or transparency is then reflected in the value of $R_i$ in the case of mixed N/Z-reaction systems, $^{112}$Sn+$^{124}$Sn (A+B) and $^{124}$Sn+$^{112}$Sn (B+A). In the limit of isospin equilibration, where isospin diffusion continues all the way down to completely mix the N/Z of QP and QT, one would observe $R_i=0$ for both A+B and B+A. In contrast, in the case of isospin transparency, with the QP that keeps memory of the original N/Z of the projectile nucleus, one would expect to observe $R_i=+1$ for $^{124}$Sn+$^{112}$Sn and $R_i=-1$ for $^{112}$Sn+$^{124}$Sn reactions at projectile rapidity [7]. The same considerations can be made by studying the target rapidity region where, in the case of isospin transparency, $R_i=-1$ for $^{124}$Sn+$^{112}$Sn and $R_i=+1$ for $^{112}$Sn+$^{124}$Sn, showing that the N/Z of the QT keeps memory of the original N/Z of the target nucleus. The measurements of Ref. [7], focused on an observable $x$ constructed with QP fragments (at projectile rapidity), showed an imbalance ratio of $R_i=0.5$ in the case of $^{124}$Sn+$^{112}$Sn at projectile rapidity [7,8]. This result is half-way between complete isospin mixing, $R_i=0$, and transparency, $R_i=+1$. This indicates a non-attainment of isospin equilibration and rather a certain degree of isospin transparency in peripheral reactions with a QP that keeps a considerable memory of the N/Z of the original projectile nucleus. In Ref. [8] the investigations was also done by looking at the quasi-target rapidity region where the results equivalent to those observed for the projectile-like source, i.e. the quasi-target QT keeps memory of the N/Z of the original target nucleus. The results of Refs. [7,8] indicated also that the reaction time is not long enough to lead to a complete N/Z equilibration in the di-nuclear system. Comparisons of measured imbalance ratios to those calculated by means of the Improved Quantum Molecular Dynamics (ImQMD) model [9] have also provided important constrains on the density dependence of the symmetry term. In ImQMD model the density dependence of the potential contribution to the symmetry energy is parameterized as

$$E_{sym}(\rho / \rho_0) = C_{sp}/2 \cdot (\rho / \rho_0)^\gamma$$

where $C_{sp}$ is a constant and $\gamma$ is the exponential parameter describing the stiffness of the density dependence. Comparisons to experimental data have provided a value of $\gamma=0.6-0.8$ [9].

The observed transparency of N/Z asymmetry in $^{112,124}$Sn+$^{112,124}$Sn peripheral collisions at E/A=50 MeV can be expected as a reasonable consequence of the relatively short interaction time between projectile and target nuclei. One may also expect that at lower incident energies longer interaction times lead to a larger degree of isospin equilibration and a smaller N/Z transparency effect. Imbalance
ratios techniques have therefore been used to study isospin diffusion phenomena at E/A=35 MeV with the Chimera array. In this case the $\delta$-asymmetry sensitive observable $x$ in Eq. (1) was the isobaric $^7\text{Li}/^7\text{Be}$ yield ratio constructed with fragments emitted at different rapidities and impact parameters. Once the $x$-observable has been chosen the imbalance ratio, $R_\gamma$, was constructed by using Eq. (1). Similarly to the $R$ quantity of Eq. (1), the $R_\gamma$ observable is constructed in each reaction system $^{112}\text{Sn}+^{124}\text{Sn}$, $^{112}\text{Sn}+^{124}\text{Sn}$, $^{124}\text{Sn}+^{112}\text{Sn}$ and $^{124}\text{Sn}+^{124}\text{Sn}$ and by construction it has the trivial values $R_\gamma=+1$ for $^{124}\text{Sn}+^{124}\text{Sn}$ reactions and $R_\gamma=-1$ for $^{112}\text{Sn}+^{112}\text{Sn}$ reactions. Then the whole investigation on isospin diffusion occurrence between projectile and target nuclei can be studied by inspecting the imbalance ratios measured for the mixed reaction systems $^{112}\text{Sn}+^{124}\text{Sn}$ and $^{124}\text{Sn}+^{112}\text{Sn}$. The obtained results are shown in Fig. 2 for the mixed reaction system $^{112}\text{Sn}+^{124}\text{Sn}$. In order to increase the statistical significance of the results, due to the symmetry of the studied reaction systems, ratios measured in rapidity regions symmetric with respect to mid-rapidity have been combined together. As already mentioned above, the effects of isospin diffusion can be observed by focusing on the QP. If isospin equilibration occurs, then the projectile rapidity region should display an imbalance ratio equal to zero, indicating complete isospin mixing between projectile and target. On the other extreme, if isospin transparency occurs, the imbalance ratio at projectile rapidities should stay close to unity, indicating that the QP maintain the same isospin asymmetry of the original projectile nucleus.

The plot of the left side shows the measured imbalance ratio, $R_\gamma$, constructed with the ratio $x=Y(^7\text{Li})/Y(^7\text{Be})$ measured with isobars $^7\text{Li}$ and $^7\text{Be}$ detected at rapidities larger than projectile rapidity. This large rapidity gate is applied in order to better isolate fragments emitted by quasi-projectile sources that are needed in order to probe the attainment of isospin diffusion phenomena. It is observed that the imbalance ratio never approaches zero, indicating that the QP keeps memory of the N/Z of the projectile nucleus. Even at the smallest impact parameters it is observed that the projectile rapidity region is characterized by an N/Z-asymmetry that remains at finite values around $R_\gamma$~0.35. This observation shows that even at a lower beam energy of E/A=35 MeV, the N/Z of the projectile and target nuclei never mix completely leading to isospin equilibration. Instead a large degree of isospin transparency exists. These results are consistent with observations made by the Indra collaboration on the study of collisions between different isotopic combinations of Xe projectiles and
Sn targets at E/A=32 MeV [14]. As already mentioned above, the extent to which isospin equilibration can occur depends on the contact and interaction time between projectile and target. At larger impact parameters the interaction time is shorter and isospin diffusion lasts for a short time. As a consequence the R7 value increases approaching values close to unity.

The data points on the right side of Fig. 2 show the rapidity dependence of R7 imbalance ratios measured at reduced impact parameters b/b\textsubscript{max}<0.6. In particular the data symbols correspond to measurements at reduced impact parameters of 0.1 (open circle), 0.23(open square), 0.34(open diamond), 0.46 (open triangle) and 0.58 (open star). It is clearly seen that only at mid-rapidity (y/y\textsubscript{proj}=0.5) and in central collisions isospin stopping and equilibration is achieved. Furthermore a weak impact parameter dependence is observed. The lines on Fig. 2 describe the results of simulations performed with the ImQMD model for a soft (γ=0.5, left panel) and a stiff (γ=2.0, right panel) density dependence of the symmetry energy. The different line types correspond to different impact parameters. The ImQMD model results confirm the observed impact parameter independence. Furthermore the simulations are capable of describing the experimental results in case of a soft symmetry energy, consistently with previous results obtained at high incident energies [9]. A very stiff symmetry energy with γ=2.0 seams not to be consistent with experimental data on isospin diffusion.

4. Conclusions and outlook

In conclusion, isospin diffusion has been studied in \(^{112,124}\text{Sn}^{+}\(^{112,124}\text{Sn} collisions at E/A=50 and 35 MeV with the LASSA and the Chimera arrays. In both experiments a considerable amount of isospin transparency is observed: N/Z equilibrium is not attained in dinuclear system produced in peripheral collisions, consistent with results from other experiments and transport model simulations. The obtained results also provide tools to probe the density dependence of the symmetry energy. Comparisons to quantum molecular dynamics transport simulations are consistent with a relatively soft density dependence of the symmetry energy, both at E/A=50 and E/A=35 MeV. The study of isospin diffusion phenomena and their links to the density dependence of the symmetry energy has been shown to provide results consistent with those extracted from similar investigations performed with other probes of the symmetry energy [9]. The density dependence of the symmetry energy can be described by means of its magnitude at saturation density, \(S=\text{E}\text{sym}(\rho_0)\), and by its first and second derivatives at \(\rho=\rho_0\), often indicated, respectively, as \(L\) and \(K_{\text{sym}}\) in the literature [9]. The use of these S, L and K\text{sym} parameters allows one to compare results obtained from heavy-ion collision experiments with those obtained from other experimental probes such as pygmy dipole resonances [15], nuclear surface symmetry energy [16], charge exchange spin-dipole resonances [17] and giant dipole resonances [18,19]. The results obtained from the data shown on Fig. 2 are consistent with those reported in Ref. [9]. The constrained density dependence of the symmetry energy and the values of the parameters S, L and K\text{sym} parameters have important implication also for important physical properties of neutron stars [20–22]. For instance the slope parameter, L is related to \(p_0\), i.e. the pressure from the symmetry energy for pure-neutron matter at saturation density. The symmetry pressure, \(p_0\), provides the dominant baryonic contribution to the pressure in neutron stars at saturation density [1,20–22].

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