Coherent amplification of classical pion fields during the cooling of droplets of quark plasma

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In the framework of the linear sigma model, we study the time evolution of a system of classical $\sigma$ and pion fields coupled to quarks. For this purpose we solve numerically the classical transport equation for relativistic quarks coupled to the nonlinear Klein-Gordon equations for the meson fields. We examine evolution starting from variety of initial conditions corresponding to spherical droplets of hot quark matter, which might mimic the behaviour of a quark plasma produced in high-energy nucleus-nucleus collisions. For large droplets we find a strong amplification of the pion field that oscillates in time. This leads to a coherent production of pions with a particular isospin and so would have similar observable effects to a disoriented chiral condensate which various authors have suggested might be a signal of the chiral phase transition. The mechanism for amplification of the pion field found here does not rely on this phase transition and is better thought of as a “pion laser” which is driven by large oscillations of the $\sigma$ field.

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I. INTRODUCTION

One of the main features of QCD, the underlying theory of strong interactions, is the spontaneous breaking of its approximate SU(2)$\times$SU(2) chiral symmetry. Spontaneous breaking of this approximate symmetry explains the very small pion masses; in the limit of exact chiral symmetry these particles would be massless Goldstone bosons. Another manifestation of this is the presence of a non-vanishing quark condensate in the vacuum. It is believed that at very high temperature a quark-gluon plasma is formed in which chiral symmetry is restored, and much effort is being made to explore such a phase transition by means of high-energy hadron or heavy-ion collisions.

To describe aspects of QCD related to this symmetry it is convenient to introduce a chiral four-vector of fields $(\sigma, \pi)$, where $\sigma$ represents the quark condensate and the three $\pi_i$ are the pion fields. In the physical vacuum, $(\sigma, \pi)$ points in the $\sigma$ direction. If the chiral symmetry were exact then there would be a “chiral circle” of states degenerate with this vacuum states. In practice the symmetry is explicitly broken by the current quark masses and so there is a unique vacuum.

Because of this circle of nearly degenerate field configurations, as the chirally restored plasma cools and returns to the normal phase the system could form regions in which the chiral fields are misaligned, that is chirally rotated from their usual orientation along the $\sigma$ direction. There has been much recent interest in this phenomenon, which is known as a disoriented chiral condensate (DCC). If such a state were formed, it would lead to anomalously large event-by-event fluctuations in the ratio of charged to neutral pions. Since the emergence of this idea [1–3], a lot of theoretical work has gone into developing suitable methods for modelling the phenomenon and in exploring how it could be used as a signal of a phase transition in high-energy nucleus-nucleus or hadron collisions; reviews on the subject can be found in Refs. [4,5].

A region of DCC can also be thought of as a coherent state of low-momentum pions. In order to produce such a state the hot plasma must evolve far from equilibrium and in particular it must reach an unstable configuration in which long-wavelength pion modes grow exponentially [3]. However it is not clear that the chiral phase transition is the only mechanism that could produce such a coherent pion excitation. Moreover if there were others ways of generating such a state, then its characteristic distribution of pions could not be regarded as a signal for formation of a quark-gluon plasma. In the present work we examine whether such states can form during the cooling and expansion of hot droplets of quark plasma coupled to chiral fields.

Since a prerequisite for DCC formation is that the system should evolve far from equilibrium, questions of whether a DCC forms and how it evolves cannot be addressed in the framework of equilibrium thermodynamics but require either a transport theory or a hydrodynamical approach. Techniques for applying QCD directly to such situations do not exist at present. Hence most authors who have investigated this subject have worked within the framework of the
linear sigma model \( [6] \). Some of them have made approximations that allow them to construct analytical solutions \( [7] \). Others have solved the evolution equations for the chiral fields numerically, using a variety of approaches and initial conditions \( [8–18] \). Of these works on DCC’s, the only one that, like ours, includes explicit quark degrees of freedom is that of Csernai and Mishustin \( [12] \). Those authors studied infinite quark matter undergoing a boost-invariant expansion in one direction. This allowed them to look for the onset of instability with respect to fluctuations of the chiral fields. In contrast we study here the full evolution starting from finite-sized droplets of quark matter.

We work within the framework of the linear sigma model \( [6] \). This is commonly used as a model for the QCD phase transition because it respects the \( \text{SU}(2) \times \text{SU}(2) \) chiral symmetry of QCD with two light flavours of quark and it contains a scalar field \( (\sigma) \) that has the same chiral properties as the quark condensate. The \( \sigma \) field can thus be used to represent the order parameter for the chiral phase transition. Moreover it has been argued that the phase transition of the linear sigma model (without quarks) should lie in the same universality class as that of QCD \( [19] \) (see also \( [20] \)).

Nonetheless universality does not provide a sufficient justification for the linear sigma model at temperatures that are well below the phase transition or in situations that are far from thermal equilibrium. The use of the model to discuss the possibility of forming a DCC thus remains an act of faith, based on the fact that the model possesses chiral symmetry and contains the most important degrees of freedom at low energies. If it is extended to include quarks (and if necessary gluons), the model can be used to describe the degrees of freedom relevant at high temperatures. At intermediate temperatures the quarks can also provide a rough model for the effects of massive hadrons (such as vector mesons and nucleons). An alternative way to include the effects of these particles would be to add terms involving higher derivatives of the chiral fields to the Lagrangian, as is done in chiral perturbation theory \( [20] \). However we do not expect such contact interactions to provide a good description of the effects of heavy hadrons at the energies of interest for the question of DCC formation and we believe that inclusion of explicit quarks provides more appropriate way to extend the model in this context.

A final comment that we need to make concerns the fact that the quarks in our model are unconfined and can escape to infinity, whereas in a more realistic model they should be converted into hadrons. We assume here that the hadronisation of the fast-moving quarks occurs well outside the original droplet and so does not affect the subsequent evolution of the chiral fields inside the droplet.

Our basic approach is very similar to that of Ref. \( [12] \) in that we assume a rapid quench that leaves the quarks out of thermal equilibrium with the chiral fields. The subsequent evolution of the system is then described by the classical Euler-Lagrange equations for the fields coupled to a transport equation for the quarks. The evolution of the quark density is described by the relativistic transport equation for fermions in the presence of chiral fields which has been derived by Shin and Rafelski \( [21] \) and Zhuang and Heinz \( [12] \) (see also \( [23] \)). Like the authors of Ref. \( [12] \) we work in the classical limit where this equation has the form of a relativistic Vlasov equation for the distribution of the quarks in phase space. The scalar and pseudoscalar quark densities provide source terms in the Euler-Lagrange equations for the chiral fields. The classical system of coupled Vlasov and field equations is solved using a test-particle method \( [24–26] \).

In using the classical Vlasov equation we are neglecting the nonlocalities that would be present in a fully quantum mechanical treatment of the quarks. Our chiral fields do display strong oscillations in both space and time, but in general they do so in regions where the quark density is small. Hence it is unlikely that serious errors are introduced by the classical treatment of the quarks. We have further neglected collision terms in the transport equations that could give rise to dissipative effects. Although their inclusion in a transport theory for quarks poses significant problems \( [27] \) that go beyond the scope of our current investigation, it is important that our approach be extended to include collision terms since these could significantly alter the free streaming of the quarks in our present treatment.

We study here evolution starting from a spherically symmetric droplet of quark plasma within which the quarks are taken to have a uniform density and a thermal momentum distribution at some temperature. The \( \sigma \) field inside the droplet is taken to have the value determined self-consistently from the scalar density of these quarks. We add a small pionic perturbation to this configuration and then examine whether the pion field grows as the system evolves. Since our initial conditions do not include a realistic distribution of thermal pions we cannot estimate what ratio of coherent to incoherent pions is produced by the decay of such a droplet. Our purpose here is to study whether such a system can generate coherent pions, and if so, by what mechanism.

Our results show that the finite size of the system plays a crucial role in determining its behaviour. In our numerical calculations the quarks rapidly escape from the original region in which they were placed \( (\text{cf. \( [25–28] \)}) \). This leaves the chiral fields in an unstable configuration. We find that the fields always “roll” away from this configuration in the \( \sigma \) direction, that is towards the true vacuum which surrounds the droplet. This demonstrates the importance of surface effects. In contrast to the picture suggested on the basis of bulk systems, there is no indication that such a droplet can form a region of more-or-less constant fields corresponding to a misaligned vacuum. Nonetheless we do find that, for large droplets, there can be a coherent amplification of any initial pion field. These pion fields display strong oscillations in space and time and so the behaviour we find is closer to that in Refs. \( [11–13] \) than that proposed...
The mechanism that produces this amplification involves the strong oscillations of the $\sigma$ field that pump energy into oscillations of the pion field. Similar behaviour has also been seen in Refs. [13,14]. Such a system is thus more like a “pion laser” than the traditional picture of a DCC. A further difference from a DCC is that a chiral phase transition is not required to produce these enhanced pion fields.

The paper is organized as follows. In Sec. II we present the linear sigma model that we use together with the classical transport and field equations. The method of solution and initial conditions are described in Secs. III and IV respectively. Our results are described in Sec. V. Their implications are discussed in Sec. VI and ways in which our approach could be improved are outlined.

II. THEORETICAL FRAMEWORK

A. Linear sigma model

A simple model that embodies the main features of QCD associated with chiral symmetry is the linear sigma model [3]. This model respects partial conservation of the axial current (PCAC) and includes a scalar field that can model changes in the quark condensate corresponding to restoration of chiral symmetry. The Lagrangian for the two-flavour version of this model is

$$\mathcal{L} = \bar{\psi}[i\partial - g(\sigma + i\pi \cdot \tau \gamma_5)]\psi + \frac{1}{2}(\partial_{\mu}\sigma\partial^\mu\sigma + \partial_{\mu}\pi\partial^\mu\pi) - U(\sigma, \pi),$$

where $\sigma$ and $\pi$ are the scalar meson and pion fields mentioned above. Although colour plays no dynamical role in our calculations, the quark field $\psi$ that we use describes quarks that come in three colours as well as two flavours.

The interactions among the meson fields are described by the potential $U$, which we take to be of the form

$$U(\sigma, \pi) = \frac{\lambda^2}{4}(\sigma^2 + \pi^2 - \nu^2)^2 - f_\pi^2 m_\pi^2 \sigma,$$

where $f_\pi = 93$ MeV is the pion decay constant and the parameter $\nu^2$ is given by $\nu^2 = f_\pi^2 - m_\pi^2/\lambda^2$. Apart from the final term in $U$ the Lagrangian is symmetric under $SU(2) \times SU(2)$ chiral symmetry. The “Mexican-hat” form of this potential leads to spontaneous breaking of this symmetry. In the vacuum the scalar field has a nonzero expectation value $\sigma = f_\pi$, which corresponds to the quark condensate of the QCD vacuum. The pions would be massless Goldstone bosons if the final term in $U$ were not present. In contrast the scalar mesons have a large mass, related to the coupling $\lambda^2$ by

$$m_\sigma^2 = 2\lambda^2 f_\pi^2 + m_\pi^2.$$  

The mass of the $\sigma$ meson is often taken to be around 500 MeV, since the attractive force between nucleons can be described by exchange of a $\sigma$ meson with such a mass. However one should remember that there are important contributions from two-pion exchange in this channel and hence the mass of this “effective” $\sigma$ meson should not be interpreted as the mass of the underlying $q\overline{q}$ state. That state may be better identified with the $f_0(1370)$ of the particle data tables [38], although Au, Morgan and Pennington [31] have suggested that this may be part of a much broader structure in $\pi\pi$ scattering at around 1000 MeV. For most of our work we have taken $m_\sigma = 1000$ MeV, but we have also considered values in the range 600–1000 MeV.

The quark-meson coupling constant $g$ is more conveniently specified in terms of the dynamical mass of the quarks in the vacuum with spontaneously broken chiral symmetry,

$$M_q = gf_\pi.$$  

The values for $M_q$ that we use lie between 300 and 500 MeV, corresponding to $3.23 < g < 5.38$.

In our studies we consider configurations in which only one component of the pion field is nonvanishing. As a further simplification in our numerical work we neglect the isospin dependence of the quark-pion coupling. In fact this coupling is unimportant for the systems we consider because the quark density is close to zero by the time that the pion field becomes important. In practice therefore we are working with the simplified Lagrangian

$$\mathcal{L} = \bar{\psi}[i\partial - g(\sigma + i\pi \gamma_5)]\psi + \frac{1}{2}(\partial_{\mu}\sigma\partial^\mu\sigma + \partial_{\mu}\pi\partial^\mu\pi) - U(\sigma, \pi),$$

in which the meson fields of an O(2) linear sigma model are coupled to two flavours of quark. The extension of our approach to the O(4) case is straightforward.

3
B. Equations of motion

As in the treatment of Csernai and Mishustin [22], we assume that the expanding droplet of quark matter undergoes a rapid quench which leaves the quarks out of equilibrium with the chiral fields. We treat the subsequent evolution of the system classically, ignoring possible effects of quantum fluctuations.

The classical equations of motion for the meson fields can be derived straightforwardly from the Lagrangian (4). These nonlinear Klein-Gordon equations take the forms
\[ \partial^\mu \partial_\mu \sigma = -\lambda^2 (\sigma^2 + \pi^2 - \nu^2) \sigma + f_\pi m_\pi^2 - g \langle \bar{\psi} \psi \rangle, \]
\[ \partial^\mu \partial_\mu \pi = -\lambda^2 (\sigma^2 + \pi^2 - \nu^2) \pi - g \langle \bar{\psi} i \gamma_5 \psi \rangle. \]

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The quarks are described using a relativistic transport theory for fermions in the presence of scalar and pseudoscalar fields [21–23]. We work in the classical limit (zeroth order in $\hbar$) where the distributions of quarks and antiquarks in phase space satisfy equations of Vlasov form. The evolution of the quark distribution in phase space $f(t, r, p)$ is determined by the equation
\[ \frac{\partial}{\partial t} + \frac{\mathbf{p}}{E(t, r, \mathbf{p})} \cdot \nabla_x - \left( \nabla_x E(t, r, \mathbf{p}) \right) \cdot \nabla_p f(t, r, \mathbf{p}) = 0, \]
where $E(x, p)$ is the energy of a relativistic quark at the space-time point $x = (t, r)$,
\[ E(x, p) = \sqrt{\mathbf{p}^2 + M^2(x)}, \]
and $M(x)$ is its mass which is related to the meson fields by
\[ M(x) = g\sqrt{\sigma^2(x) + \pi^2(x)}. \]

The equation for the antiquark distribution, denoted by $\tilde{f}(t, r, \mathbf{p})$, is identical in form to (8) since no vector fields are present in our model.

The Vlasov equation (6) can be obtained as the classical limit of an equation for the equal-time Wigner function of the quarks [22,23]. In fact it describes freely streaming classical quarks and antiquarks, each obeying the relativistic single-particle equations of motion:
\[ \dot{\mathbf{r}}(t) = \frac{\mathbf{p}(t)}{E(t, r, \mathbf{p}(t))}, \]
\[ \dot{\mathbf{p}}(t) = -\nabla_x E(t, r(t), \mathbf{p}(t)), \]

where the dots denote time derivatives and the dependence of the particle’s energy on its position and momentum is given by (9,10). Instead of solving (6) as a partial differential equation in seven dimensions, one can replace the smooth distributions $f(t, r, \mathbf{p})$ and $\tilde{f}(t, r, \mathbf{p})$ by a set of classical particles obeying the equations of motion (11,12). This is the basis for the test-particle method [21,22] which we use to construct approximate numerical solutions to (6), as described in the next section.

The couplings of a classical quark to the $\sigma$ and pion fields can be obtained by differentiating its energy (3) with respect to each of those fields. The resulting scalar and pseudoscalar quark densities are related to the quark and antiquark distributions by
\[ \langle \bar{\psi} \psi(x) \rangle = g\sigma(x) \int d^3 \mathbf{p} \frac{f(x, \mathbf{p}) + \tilde{f}(x, \mathbf{p})}{E(x, \mathbf{p})}, \]
\[ \langle \bar{\psi} i \gamma_5 \psi(x) \rangle = g\pi(x) \int d^3 \mathbf{p} \frac{f(x, \mathbf{p}) + \tilde{f}(x, \mathbf{p})}{E(x, \mathbf{p})}. \]

These can also be obtained from the classical limit of the Wigner function, as in Refs. [21,22]. Note that both scalar and pseudoscalar densities vanish for massless quarks ($\sigma = \pi = 0$). Using these expressions in the source terms for the fields, the field equations (6,7) can be rewritten in the form
\( \left( \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \sigma(t, r) = - \left[ \lambda^2 \left( \sigma^2(t, r) + \pi^2(t, r) - \nu^2 \right) + g^2 \int d^3 p \frac{f(t, r, p) + \tilde{f}(t, r, p)}{E(t, r, p)} \right] \sigma(t, r) + f_\pi m_\pi^2 \), \quad (15) \\
\( \left( \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \pi(t, r) = - \left[ \lambda^2 \left( \sigma^2(t, r) + \pi^2(t, r) - \nu^2 \right) + g^2 \int d^3 p \frac{f(t, r, p) + \tilde{f}(t, r, p)}{E(t, r, p)} \right] \pi(t, r). \) \quad (16) 

The evolution of our system is obtained by solving self-consistently the set of differential equations (15,16) subject to an appropriate set of initial conditions (to be described in Sec. III).

### III. Method of Solution

Solutions of the set of coupled equations (15,16) can be obtained only numerically. The methods we adopt are very similar to those employed in Ref. [22] to solve the similar set of equations that arise in a soliton bag model.

The Vlasov equation (8) is conveniently solved using the test-particle method which has been widely used in the context of intermediate-energy heavy-ion collisions [24,25], \( NN \) scattering [26] and evolution of a quark plasma [27]. The method consists of replacing the quark and antiquark distributions by a swarm of test particles. The smooth distributions \( f(t, r, p) \) and \( \tilde{f}(t, r, p) \) are thus approximated by

\[
f(t, r, p) = w \sum_{n=1}^{N} \delta^3(r - r_n(t)) \delta^3(p - p_n(t)) \tag{17}
\]

\[
\tilde{f}(t, r, p) = w \sum_{n=1}^{S} \delta^3(r - \tilde{r}_n(t)) \delta^3(p - \tilde{p}_n(t)) \tag{18}
\]

Each test particle follows a classical trajectory \( r_n(t) \), \( p_n(t) \) determined by the relativistic single-particle equations of motion

\[
\dot{r}_n(t) = \frac{p_n(t)}{E(t, r_n(t), p_n(t))}, \tag{19}
\]

\[
p_n(t) = -\nabla_r E(t, r_n(t), p_n(t)). \tag{20}
\]

Provided enough test particles are used, their distributions provide very good approximations to the smooth quark and antiquark distributions described by (8). In general the number of test particles is very much larger than the actual numbers of quarks and antiquarks we wish to describe. To account for this, the distributions must be multiplied by a normalization constant \( w \). If the actual numbers of quarks and antiquarks are denoted by \( A \) and \( \tilde{A} \) respectively then the numbers of test particles are related to these by

\[
A = \int d^3r d^3p f(t, r, p) = wN, \tag{21}
\]

\[
\tilde{A} = \int d^3r d^3p \tilde{f}(t, r, p) = w\tilde{N}. \tag{22}
\]

The nonlinear equations for the meson fields (15,16) are second-order in both space and time derivatives. To solve these we work on a discrete mesh of points in space. In the present work we confine our attention to spherically symmetric initial conditions. We further require that the symmetry is maintained during the evolution, by angle-averaging the source terms in the field equations. In this case, the field equations can be written

\[
\ddot{\sigma} = \frac{1}{r} \frac{\partial^2}{\partial r^2} (r\sigma) - \left[ \lambda^2 \left( \sigma^2(t, r) + \pi^2(t, r) - \nu^2 \right) + g^2 S_\xi(t, r) \right] \sigma(t, r) + f_\pi m_\pi^2, \tag{23}
\]
\begin{equation}
\dddot{\pi} = \frac{1}{r} \frac{\partial^2}{\partial r^2} (r \pi) - \left[ \lambda^2 \left( \sigma^2(t, r) + \pi^2(t, r) - \nu^2 \right) + g^2 S_q(t, r) \right] \pi(t, r),
\end{equation}

where the angle-averaged source term is

\begin{equation}
S_q(t, r) = \frac{w}{4\pi} \left( \sum_{n=1}^{N} \frac{1}{r_n^2} \delta(r - r_n(t)) + \sum_{n=1}^{N} \frac{1}{r_n^2} \delta(r - \tilde{r}_n(t)) \right).
\end{equation}

The restriction to spherically symmetric configurations considerably lessens the computational effort required to solve these equations. In addition the angle average in (23) reduces the number of test particles required to accurately map out the solution of the Vlasov equation. As a check on the consistency of this restriction we have run some small-scale three-dimensional simulations and have found no evidence for instability against non-spherical perturbations.

The field equations are solved on a radial mesh with spacing \( \Delta r = 0.05 \) fm. In most of our runs we have used a mesh of 280 points, corresponding to a region of radius \( R = 14 \) fm. At the outer edge of the lattice we impose the boundary condition that the pion field vanishes while the \( \sigma \) field is set equal to \( f_\pi \). This means that we do not lose energy from these fields when the radiated meson waves reach the edge, which is useful since we wish to calculate the energy radiated in particular modes of the pion field. However the fact that meson waves are reflected by the boundary means that our results become unphysical once those reflected waves return to the central region. For an initial droplet of size \( r_0 = 4 \) fm and a lattice of size \( R = 14 \) fm this occurs after about 18–20 fm/c. To describe evolution over longer times we need to use lattices with larger radii or to work with absorbing boundary conditions.

The discrete nature of the source term due to the test particles requires that the \( \delta \)-functions in (24) be smeared out in some way. In most of our calculations we have used a parabolic form,

\begin{equation}
\delta(r - r_n) \to \frac{3}{4d^3} \left( d^2 - (r - r_n)^2 \right), \quad |r - r_n| \leq d.
\end{equation}

In order that all test particles contribute to the source, the thickness \( d \) should be greater than half the mesh size: \( d \geq \Delta r/2 \). In practice \( d \) should be somewhat larger than this to ensure a reasonably smooth source. The results shown in this paper correspond to \( d = 2 \Delta r \). We have checked that our results are not sensitive to the precise value of \( d \) in this region, nor do they change if Eq. (24) is replaced by a Gaussian smearing function.

For the initial conditions that we study we use an initial density of test particles of about 400–500 fm\(^{-3}\). For a region of radius 4 fm this requires a total of about 125000 test particles. We have checked that our results are stable with respect to a further increase in the number of test particles.

The coupled equations are integrated using a leapfrog method with a time step of \( \Delta t = 0.01 \) fm/c. We have also compared our method with the more complicated staggered leapfrog method [24] described in the Appendix of Ref. [24], and checked that the two agree. Since the coupled equations corresponds to a conservative system, conservation of the total energy provides a good test of the accuracy of our numerical algorithm. We find that the energy changes by less than 1% over a time of 40 fm/c (4000 time steps).

IV. INITIAL CONDITIONS

Our initial configuration consists of a spherical droplet of hot quark matter. We take the quark density and temperature to be uniform within the droplet of radius \( r_0 \). The initial distribution of the quarks in phase space is thus

\begin{equation}
f(0, \mathbf{r}, \mathbf{p}) = \frac{2N_cN_f}{(2\pi)^3} \Theta(r_0 - r) \eta(E_0(\mathbf{p}), T, \mu)
\end{equation}

where \( N_c = 3 \) and \( N_f = 2 \) are the numbers of colours and flavours, respectively. \( \Theta \) denotes the step function and \( \eta(E, T, \mu) \) is the Fermi distribution

\begin{equation}
\eta(E, T, \mu) = \frac{1}{1 + e^{(E - \mu)/T}}.
\end{equation}

For the antiquark distribution, \( \eta \) should be replaced by

\begin{equation}
\tilde{\eta}(E, T, \mu) = \eta(E, T, -\mu).
\end{equation}
The quark energy appearing in these distributions has the form \( E_0(p) = \sqrt{p^2 + M_0^2} \) where the quark mass \( M_0 \) is obtained by solving self-consistently the equations for the constant \( \sigma \) field in the presence of the (anti-)quark distributions \( \sigma \). For a uniform quark density the equation \( \beta \) for the \( \sigma \) field takes the form

\[
\lambda \left( \sigma_0^2 - \nu^2 \right) + g^2 \frac{2N_cN_f}{(2\pi)^3} \int d^3p \frac{\tilde{g}(E_0, T, \mu) + \tilde{g}(E_0, T, \mu)}{E_0} \sigma_0 - f_\pi m_\pi^2 = 0, \tag{30}
\]

where \( E_0(p) \) implicitly depends on \( \sigma_0 \) since \( M_0 = g\sigma_0 \). In the chiral limit \((m_\pi^2 = 0)\) this equation always has a trivial solution \( \sigma_0 = 0 \), corresponding to a vacuum in which chiral symmetry is restored. For high enough temperatures or densities this is the only solution to \( \beta \). At low temperature and density there is also a nontrivial solution with lower energy which describes a vacuum in which chiral symmetry is spontaneously broken.

Nonetheless we can make use of the existence of this temperature to explore initial states in which chiral symmetry is restored and the quarks are nearly massless. For the initial \( \sigma \) field, we pick a form that interpolates smoothly between ones with small, but still nonzero, values of \( \sigma_0 \). Nonetheless, for a symmetry breaking strength that gives a realistic pion mass, this crossover behaviour occurs rapidly and one can still usefully define a critical temperature above which the approximate chiral symmetry is restored and the quarks are nearly massless.

For the initial \( \sigma \) field, we pick a form that interpolates smoothly between \( \sigma_0 \) inside the droplet and \( f_\pi \) outside:

\[
\sigma(t = 0, r) = \sigma_0 + (f_\pi - \sigma_0) \Theta_\alpha(r - r_0), \tag{31}
\]

where \( \Theta_\alpha \) is a suitably smoothed step function. In this work we have chosen to take

\[
\Theta_\alpha(r - r_0) = \begin{cases} (e^{ar^2} - 1)/2(e^{ar_0^2} - 1) & r < r_0 \\ (1 + \tanh \alpha(r - r_0))/2 & r \geq r_0, \end{cases} \tag{32}
\]

where \( a \) is determined by requiring the radial derivative of \( \Theta_\alpha \) to be continuous at \( r = r_0 \). This is a convenient parametrisation whose shape is close to the self-consistent solution of the field equation for \( \sigma \) in the presence of a static spherical quark distribution of the form \( \Theta_\alpha \). The constant \( \alpha \) which describes the inverse of the surface thickness should be proportional to \( m_\sigma \).

We take the initial pion field to be of the form

\[
\pi(t = 0, r) = \pi_0 \left( 1 - \Theta_\alpha(r - r_0) \right), \tag{33}
\]

where \( \pi_0 \) is a small, arbitrary amplitude corresponding to a fluctuation away from the self-consistent solution of \( \Theta_\alpha \). A nonzero initial value for either the pion field or \( \dot{\pi}(0, r) \) is needed since otherwise our system will never develop a pion field, as can be seen from the fact that all the terms in the field equation \( \beta \) contain a factor of \( \pi(t, r) \). Ultimately we would hope to take the initial fluctuations in both \( \sigma \) and pion fields from a thermal distribution, using an extension of the method described in \( \Theta \). However, for our current investigations we use the simple ansatz \( \Theta_\alpha \) in order to examine whether a system of this type can lead to a coherent pion field.

The equations of motion of the meson fields \( \dot{\sigma}, \dot{\pi} \) are of second order in time derivatives and so we need to specify the initial field velocities as well as the fields. In most of our calculations we have used static initial configurations, \( \dot{\sigma}(0, r) = \dot{\pi}(0, r) = 0 \). However, we have also considered more general cases, and in particular ones corresponding to droplets that are expanding with velocity \( v \):

\[
\dot{\sigma}(0, r) = -v \frac{\partial \sigma}{\partial r}(0, r), \tag{34}
\]

\[
\dot{\pi}(0, r) = -v \frac{\partial \pi}{\partial r}(0, r). \tag{35}
\]
V. RESULTS

In this section, we present the results of our simulations corresponding to various initial conditions of the system. Most of our results are for our “standard” parameter set, \( m_\sigma = 1000 \text{ MeV}, m_\pi = 139 \text{ MeV} \) and \( M_q = 300 \text{ MeV} \). For these \( \sigma \) and quark masses, in the chiral limit, chiral symmetry restoration occurs at the temperature \( T_0 \approx 235 \text{ MeV} \), as discussed above.

For these parameters we have examined the evolution of droplets of various sizes, with an initial temperature of 250 MeV (above \( T_0 \)) and zero chemical potential (zero baryon density). The initial energy density in this case is about 3.5 GeV/fm\(^3\), comparable to the energy densities that are expected to be reached in ultrarelativistic heavy-ion collisions. This energy density is dominantly due to the quarks, the chiral fields providing only about 0.14 GeV/fm\(^3\). Inside the droplets the initial value of the sigma field, determined from the gap equation (30) with explicit symmetry breaking, is \( \sigma_0 = 10 \text{ MeV} \). In order to study whether initial pionic fields can be amplified during the evolution we add a pionic perturbation to this configuration of magnitude \( \pi_0 = 0.05\pi_\sigma \).

We examine first the evolution of a small initial droplet of radius \( r_0 = 1.1 \text{ fm} \), plotting in Fig. 1 the \( \sigma \) and pion fields at several radii as a function of time. This shows the fields rolling away from the initial configuration towards the physical vacuum surrounding the initial droplet. Both fields then oscillate around their vacuum values, their amplitudes decaying as energy is carried away (mainly by long-wavelength sigma waves). Fig. 1 shows no indication of any enhancement of the pion field. We find similar results for small droplets with \( r_0 < 2 \text{ fm} \).

In contrast, for larger initial droplets we find significant amplification of the pion field, with strong oscillations in space and time. This is shown in Fig. 2 by the time evolution of the \( \sigma \) and pion fields in the case of a droplet of radius \( r_0 = 4.0 \text{ fm} \), with otherwise the same initial conditions as in Fig. 1. (The fluctuations that can be seen in the fields at \( r = 0.1 \text{ fm} \) for \( t < 4 \text{ fm}/c \) are numerical noise, due to the fact that the radial density of test quarks is much smaller at small radii.) Another view of the same system is provided in Fig. 3, where we show the \( \sigma \) and pion fields as a function of \( r \) at successive times. A larger initial volume means that the energy stored initially in the meson fields is larger, and this initial energy is crucial to determining whether or not the pion field is amplified.

Also from Fig. 2 we see that the strong oscillations of the fields start only after 4–6 fm/c. This is the time needed by the quarks to escape the initial region. Indeed we find that for our standard choice of parameters almost all the quarks rapidly stream out of the initial region. This leaves the chiral fields in an unstable configuration which collapses towards the true vacuum. This behaviour can be seen in Fig. 4 where we plot the net density of quarks and antiquarks: \( \int d^3p (f(t, r, p) + f(t, r, -p)) \) times \( r^2 \) as a function of \( r \) at successive times.

In order to investigate the spectrum of the pion modes that are excited we Fourier analyse the pion field at successive times. From the spatial Fourier transform of the pion field, \( \tilde{\pi}(t, k) \), and its time derivative, \( \dot{\tilde{\pi}}(t, k) \), we calculate the corresponding intensity in momentum space:

\[
\mathcal{E}_\pi(t, k) = \frac{1}{2} \left( |\dot{\tilde{\pi}}(t, k)|^2 + \omega_k^2 |\tilde{\pi}(t, k)|^2 \right),
\]

where \( \omega_k = \sqrt{k^2 + m_\pi^2} \). If the pion fields are sufficiently weak that they are well described by the linearised version of the equation of motion (36), then (37) is simply the energy density of the pion field in momentum space. At large times it thus gives the energy radiated in the form of pions as a function of momentum. The total energy of the pion field in such a regime is

\[
E_\pi(t) = \int \frac{d^3k}{(2\pi)^3} \mathcal{E}_\pi(t, k),
\]

and the corresponding number of pions is given by

\[
N_\pi(t) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_k} \mathcal{E}_\pi(t, k).
\]

However, one should note that while the system is still evolving nonlinearly, (38) has no such interpretation.

Fig. 5 shows the behaviour of the intensity (38) at successive times as a function of momentum. (These momenta are the discrete values \( k_n = n\pi/R, n \geq 1 \), consistent with our boundary condition at \( r = R \).) One can see from this plot that during the period when the \( \sigma \) field is undergoing violent oscillations (4–8 fm/c) the pion modes with momenta \( k \sim 0.5–2 \text{ fm}^{-1} \) are significantly amplified. There is also a smaller amplification for modes with larger momenta, up to \( \sim 4 \text{ fm}^{-1} \). This enhancement of pion modes with momenta less than \( \sim 2 \text{ fm}^{-1} \) is very similar to the behaviour found by Randrup [17], in spite of the use of a different approach.

In Fig. 6 we plot \( \mathcal{E}_\pi(t, k) \) for the most strongly amplified modes as a function of time. This shows that very little further energy is radiated in these modes after 8 fm/c. Similarly the total number of pions, calculated using (38) and
plotted in Fig. 7, shows very little change after this time. In this case the number of pions produced is 20 times larger than the number corresponding to the initial pionic perturbation.

Another related way to study the pionic modes is provided by the Fourier transform of the correlation function, whose time dependence was plotted by the authors of Refs. [3,4]. This quantity is equal to $|\hat{\pi}(t, k)|^2$ and so can be thought of as the second term of (36) divided by $\omega_k^2$. Once the pion field is in the regime where it satisfies a linear equation and oscillates harmonically, $|\hat{\pi}(t, k)|^2$ oscillates with frequency $2\omega_k$. These oscillations are smoothed out in (55) by the presence of the time-derivative term. Although we do not present results for the correlation function here, we have looked at it to check that the enhanced modes shown in Fig. 6 do indeed show the expected oscillations for radiated pions which are in the harmonic regime. The small remaining oscillations of the pion field in the central region that can be seen at times after 8 fm/c in Fig. 2 do not contribute significantly to the pion radiation. This can also be seen from the behaviour of the total number of pions in Fig. 7.

To explore the robustness of this mechanism for generating enhanced pion fields we have considered other initial conditions. These include examples with nonzero baryon density ($\mu \neq 0$) or expanding initial droplets ($v \neq 0$ in Eqs. (34,35)) and initial pionic perturbations with different magnitudes or spatial extents. In all these cases we obtained results that are qualitatively similar to those shown here. They are also qualitatively the same if we work in the chiral limit with massless pions. Our results are however sensitive to other parameters of the model. In particular smaller $\sigma$ masses correspond to smaller initial energy densities for the chiral fields. For example, with $m_\sigma = 600$ MeV a droplet of radius 4 fm leads to much less amplification than the case shown in Figs. 2–7. The effect of taking a larger quark-meson coupling is discussed below.

All of the cases discussed so far correspond to evolution starting from the phase with restored (approximate) chiral symmetry. One can also ask about evolution from an initial state where chiral symmetry is already spontaneously broken. For this purpose we have considered initial conditions where the temperature is $T = 200$ MeV ($< T_0$) but with other parameters as above. In this case, the initial hot quarks are “dressed” and have a dynamical mass of $g_0 = 206$ MeV where $\sigma_0$ is the nontrivial solution of (31). We take the radius of the initial droplet and the initial pionic perturbation to be the same as in the previous example. The time evolution of the fields at several radii are shown in Fig. 8 for this case. The qualitative behaviour is similar to that of the example with $T > T_0$ shown in Fig. 2. Again we see that $\sigma$ field oscillates strongly although for a rather shorter period: in this case 4–6 fm/c. During these oscillations there is a significant amplification of the pion field. This shows that there is nothing about this mechanism for generating a coherent pion field that requires a phase transition. What is essential is that the initial chiral fields have sufficient energy.

Csernai and Mishustin [12] have used the same model to study the simpler case of infinite quark matter undergoing a boost-invariant expansion in one direction. They found a similar coherent amplification of the pion field. In addition they pointed out that a more complete treatment of the model would be expected to lead to formation of clusters of quarks and antiquarks surrounded by domains of the coherent chiral fields. In the cases discussed above we did not find any such clusters (cf. Fig. 4). However if we increase the strength of the quark-meson coupling we do obtain such behaviour. This can be illustrated by the parameter set $M_q = 500$ MeV ($g = 5.38$) and $m_\sigma = 1$ GeV, in the chiral limit. In this case symmetry restoration occurs at $T_0 \simeq 170$ MeV. We consider a droplet of radius $r_0 = 2.75$ fm, with initial temperature $T = 250$ MeV and chemical potential $\mu = 100$ MeV. In Fig. 9 we show the radial dependence of the density of quarks and antiquarks times $r^2$ at successive times. In contrast to the case shown in Fig. 4, the quarks with smaller momenta are reflected from the surface region at around 3 fm and remain trapped within the droplet.

The behaviour of the chiral fields shown in Fig. 10 is quite different from the previous examples. The $\sigma$ field in the central region continues to oscillate wildly around values that are much less than $f_\sigma$. The quark cluster is obviously created in a highly excited configuration and as it settles down a significant fraction of its excitation energy is radiated in the form of pions, as can be seen from the fact that very large amplitude oscillations of the pion field continue for times up to 20 fm/c (and well beyond). In this particular example the quarks are left in a shell-like distribution with a radius $\sim 2.5$ fm, which is basically a “SLAC bag” soliton [36]. Other initial conditions that we have studied lead to more uniform distributions of quarks trapped inside shallow “bags” in the $\sigma$ field. In all cases the resulting excited quark clusters form efficient radiators of coherent pions.

VI. SUMMARY AND DISCUSSION

We have investigated the expansion of hot droplets of quark plasma coupled to the sigma and the pion fields of the linear sigma model, addressing in particular the question of whether such droplets can lead to the production of a coherent classical pion field or a disoriented chiral condensate (DCC). It has been suggested that the formation of a DCC could be used a signal for the chiral phase transition in high-energy nucleus-nucleus or hadron collisions [2–4]. We have therefore examined whether the phase transition is the only mechanism that could produce a coherent
pion field.

We treat the evolution of the system classically and solve numerically the Vlasov equation for the quarks coupled to the nonlinear Klein-Gordon equations for the chiral fields. These equations have previously been studied by Csernai and Mishustin [12], but only for the case of infinite quark matter undergoing a boost-invariant expansion. Our results show that finite-size effects play an important role in the evolution of such systems.

Our starting point is a spherical droplet containing a uniform density of quarks and antiquarks in thermal equilibrium at some temperature $T$. Inside this region the sigma field is taken to be constant. In using this type of initial configuration we are assuming a rapid quench that leaves the quarks out of thermal equilibrium with the meson fields. We add to this a small pionic perturbation, to act as a “seed” for possible formation of a coherent pion field. The fluctuations of the initial meson fields away from their self-consistent values ought to be taken from a thermal distribution. We plan to do this in future work using an extension of the method proposed by Randrup [33]. In the present paper we have used a uniform initial perturbation in order to study whether such droplets can lead to amplification of a coherent pion field.

We find that the quarks rapidly escape from the original region in which they were placed, as in Ref. [28,29]. This leaves the chiral fields in an unstable configuration. Starting from the surface of the droplet, the chiral fields relax towards the physical vacuum which surrounds the droplet ($\sigma = f_\pi, \pi = 0$). The behavior is rather different from the picture which has been suggested on the basis of a uniform system, namely, that the symmetry of the potential could allow different regions to relax towards vacuum configurations with different chiral orientations. In the finite systems that we have studied, the surface effects ensure that the fields always “roll” away from their unstable initial configuration towards the physical vacuum.

The departure of the quarks leaves the $\sigma$ field in a configuration with significant potential energy. As a result it undergoes rather violent oscillations, during which this energy is radiated in the form of meson waves, before settling down to its vacuum value. If an initial pion field is present then some of this energy can be converted into pions. This leads to a coherent production of pions with a particular isospin and so would have similar observable effects to a DCC. However unlike the picture of a DCC consisting regions of differently oriented vacuum, the pion fields oscillate strongly in space and time. Hence the behaviour we find is closer to that in Refs. [34,35,36] than that proposed in Ref. [3]. For large initial droplets that start in the chirally restored phase, we find significant amplification of the number of pions, typically by factors of the order of 20 or more. The mechanism is robust against changes in the initial conditions as described in Sec. V.

In some cases we find that the pion oscillations contain a strong component with half the frequency of the $\sigma$ oscillations. Similar oscillations of pion fields driven by the $\sigma$ field have also been seen in the homogeneous systems of $\sigma$ and pion fields (without quarks) studied by Boyanovsky et al. [13] and Mrowczynski and Müller [15]. Analogous behaviour is also seen in models of inflationary universes where an additional scalar field is coupled to the field that drives the inflation [37,38]. Mathematically one can describe this as a parametric resonance driven by the oscillations of the $\sigma$-dependent mass term in the pion field equation. More physically one can view it as a “pion laser” where energy is pumped into the pion field by the process $\sigma \rightarrow \pi \pi$. The larger the initial energy in the $\sigma$ field, the more effectively this mechanism operates. Hence we find the greatest amplification for the largest droplets, in cases with the largest $\sigma$ mass. Smaller droplets, with radii less than 2 fm, do not lead to significant pion production. Also the enhancement factors are much smaller for cases with $m_\sigma = 600$ MeV compared to those where we took $m_\sigma = 1000$ MeV.

If the quark-meson coupling is taken to be strong enough, then some of the quarks remain trapped inside the droplet and eventually form a cold multiquark cluster, as suggested by Csernai and Mishustin [12]. These clusters are produced in highly excited configurations that can act as extremely efficient radiators of coherent pions. However one should remember that the cold quark matter of the linear sigma model is not expected to be a very good approximation to hadronic matter. Hence one should not take this version of the mechanism for coherent pion production too seriously before more realistic models have been explored.

The nature of the mechanism for the enhancement of the pion field found in our work makes no obvious reference to the chiral phase transition. As explained in Sec. V the model possesses a temperature $T_0$ below which chiral symmetry is spontaneously broken. This enables us to study evolution from states in which the symmetry is partially restored, that is, where the temperature, although less than $T_0$, is still high enough that the $\sigma$ field is significantly reduced from its vacuum value. The matter in these states consists of dressed quarks with finite dynamical masses, and can be thought of as a crude model for hot hadronic matter in which hadrons with higher masses than pions are present. We find that large enough droplets with initial temperatures below $T_0$, behave in a similar way to ones with initially restored chiral symmetry. In particular they also lead to amplification of coherent pions. This is a further indication that the mechanism involved is that of a “pion laser” which is driven by the oscillating $\sigma$ field and which does not rely on the phase transition for its operation.

Like the DCC’s and other coherent pion fields discussed in the literature, the pions produced by this mechanism will have the same characteristic $1/\sqrt{T}$ distribution for the fraction $f$ of neutral pions. However to determine whether
this signal will be experimentally observable over the background of incoherently produced pions we need to know the actual number of coherent pions. This will require the use of more realistic initial conditions. In particular, as already mentioned, we need to take the initial pionic fluctuations from a thermal distribution. For this purpose we will need to extend the method described in [35] to our model which has explicit quark degrees of freedom. In addition we are studying other geometries since, especially for ultrarelativistic collisions, a cylindrical system undergoing a boost-invariant expansion is expected to be more relevant than a spherical one. Finally we should also point out the need to extend our approach to include both quantum effects and collision terms in the transport equations.

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Figure captions

Fig. 1: The $\sigma$ (a) and pion (b) fields as functions of time for various radii. The parameters of the model are: $m_\sigma = 1$ GeV, $m_\pi = 139$ MeV, and $M_q = 300$ MeV. The initial conditions are: $T = 250$ MeV, $\mu = 0$, and $r_0 = 1.1$ fm.

Fig. 2: As in Fig. 1 except for the initial radius: $r_0 = 4$ fm.

Fig. 3: The $\sigma$ (a,b) and pion (c,d) fields as functions of radius at successive times. The parameters of the model as well as the initial conditions are the same as in Fig. 2.

Fig. 4: The net density of quarks and antiquarks weighted with $r^2$ as a function of radius. The parameters of the model as well as the initial conditions are the same as in Fig. 2.

Fig. 5: The energy density of the pion field in momentum space $E_\pi$ as a function of momentum at successive times. The parameters of the model as well as the initial conditions are the same as in Fig. 2.

Fig. 6: The time evolution of $E_\pi$ for the relevant modes. The parameters of the model as well as the initial conditions are the same as in Fig. 2.

Fig. 7: The number of pions as defined in Eq. (38) as a function of time. The parameters of the model as well as the initial conditions are the same as in Fig. 2.

Fig. 8: The $\sigma$ (a) and pion (b) fields as functions of time for various radii. The parameters of the model are the same as in Fig. 1 while the initial conditions are: $T = 200$ MeV, $\mu = 0$, $r_0 = 4$ fm.

Fig. 9: The net density of quarks and antiquarks weighted with $r^2$ as a function of radius, in the case where $m_\sigma = 1$ GeV, $m_\pi = 0$, and $M_q = 500$ MeV. The initial conditions are: $T = 250$ MeV, $\mu = 100$, and $r_0 = 2.75$ fm.

Fig. 10: The $\sigma$ (a) and pion (b) fields as functions of time for various radii. The parameters of the model as well as the initial conditions are the same as in Fig. 9.
Abada/Birse, Fig. 1, part 1 of 2.
Abada/Birse, Fig. 2, part 1 of 2.

(a)
Abada/Birse, Fig. 2, part 2 of 2.
Abada/Birse, Fig. 3, part 1 of 4.
Abada/Birse, Fig. 3, part 3 of 4.
Abada/Birse, Fig. 5, part 1 of 2.

$E_{\pi}(k, t) \, (fm^2)$

- $t=1 \, fm/c$
- $t=3$
- $t=5$
- $t=7$
- $t=9$

$k \, (fm^{-1})$
Abada/Birse, Fig. 5, part 2 of 2.
Abada/Birse, Fig. 6.
Abada/Birse, Fig. 7.
Abada/Birse, Fig. 8, part 2 of 2.
Abada/Birse, Fig. 9, part 1 of 2.

(a)

\(r \text{ (fm)}\)

\(\text{(fm}^{-1}\)\)

- Solid line: \(t = 0 \text{ fm/c}\)
- Dashed line: \(t = 2\)
- Dash-dotted line: \(t = 4\)
- Dash-dash line: \(t = 6\)
- Triangles: \(t = 8\)
Abada/Birse, Fig. 9, part 2 of 2.
Abada/Birse, Fig. 10, part 1 of 2.
Abada/Birse, Fig. 10, part 2 of 2.