Proposal of a non linear static analysis procedure for bridges: the Incremental Modal Pushover Analysis for bridges (IMPAβ)

Alessandro Vittorio Bergami 1,*, Camillo Nuti 1,2, Davide Lavorato 1, Gabriele Fiorentino 1 and Bruno Briseghella 2

1 Department of Architecture, Roma Tre University, 00152 Rome, Italy; alessandro.bergami@uniroma3.it (A.V.B.); camillo.nuti@uniroma3.it (C.N.); davide.lavorato@uniroma3.it (D.L.); gabriele.fiorentino@uniroma3.it (G.F.)
2 College of Civil Engineering, Fuzhou University; bruno@fzu.edu.cn
* Correspondence: alessandro.bergami@uniroma3.it; Tel.: +39-(0)6-57332907

Abstract: A large number of bridges are designed and built without considering seismic actions and, differently from buildings, there are currently no comprehensive guidelines to evaluate existing bridges without performing, as in the well known incremental dynamic analysis (IDA), complex non linear dynamic analyses (RHA). Bridges are structurally very different from building but, at the same time, are sensitive to higher modes as well as many multi-storey buildings that inspired innovative pushover procedures such as the well known modal pushover analysis (MPA). In the present study the incremental modal pushover analysis (IMPA), a pushover based approach already proposed and applied on buildings by the same authors, is revised and proposed for bridges (IMPAβ). IMPAβ accounts for the effects of higher modes in order to accurately estimate the seismic response of bridges; the effect of higher modes is considered by introducing a suitable number of modes to ensure the participation of a predefined total effective modal mass. The efficiency of the proposed method is demonstrated by conducting a study on two bridges, one regular and one irregular, and the IDA analysis is employed as reference solution. Numerical results indicate good accuracy of the proposed method in assessing the seismic response and a very good accuracy if compared to other available pushover procedures available in the literature.

Keywords: Non-linear static (Pushover) analysis, modal pushover, Non-linear time-history analysis, incremental analysis, bridges, assessment of bridges, seismic response of bridges.

1. Introduction

The present study aims to develop a pushover-based procedure for performing a comprehensive assessment of the behavior of bridges under seismic loads. This innovative procedure, presented herein and named IMPAβ, is alternative to what is currently considered the most reliable procedure: the incremental dynamic analysis (IDA) [1, 2].

The idea of IMPAβ originates from the intent of extending the experience made by the same authors, dealing with seismic behavior of building structures sensitive to higher modes, and which led to the development of a well performing procedure named Incremental Modal Pushover Analysis (IMPA) [3, 4].

Therefore, being bridges structures wherein higher modes usually play a more critical role than in buildings, developing a modal pushover procedure for such structures is even more interesting than the case of buildings; this is true especially if we refer to countries where the most of the building
stock is composed by low to mid rise building (low sensitive to higher modes) but the presence of bridges and viaducts is widely diffuse (e.g. Europe)

IMPA is a methodology to investigate the seismic response of buildings, for assessment or design scope, that was proposed to substitute the several nonlinear dynamic analyses required by IDA, with static non-linear static analyses (pushover) producing results comparable with IDA; IMPAβ has the same approach proposed in IMPA but it is specialized for bridges. Indeed, in both IMPAβ and IDA, an explicit relationship between a scalar intensity measure (e.g. seismic intensity) versus the structural response (measured by an engineering demand parameter e.g. the displacement of a control point, the shear action etc.) is obtained.

Among the main advantages of preforming IMPAβ instead of IDA, in addition to the simplest and fastest characteristics of the static versus dynamic non-linear analysis, is that IMPAβ allows to easily manage the seismic input (e.g. selecting appropriate spectral shapes for each specific seismic intensity); in principle, this is possible in IDA too, and it should be done, but eventually it may be frequently very difficult (e.g. given the few records available in databases for high intensities). In this work, for simplicity, in the applications presented the spectrum shape used for designing the bridges was conserved for each intensity level explored but the authors consider the definition of a suitable seismic input as an essential issue that will be discussed in a specific future work.

As detailed in [3] and summarized in section 2.4, IMPA requires the execution of the Modal Pushover Analysis (MPA) [6] but, in order to validate it and verify the best pushover procedure to be proposed considering the bridge case, developing IMPAβ, other pushover methods have been considered too namely: pushover with a load pattern proportional to the first mode (SPA) or uniform (UPA) (this two load pattern are commonly adopted in pushover analysis as suggested in FEMA-356 [7] and Eurocode 8 [8-9]).

The cited procedure was applied, and consequently calibrated, for one regular bridge (RB) and one irregular bridge (IB); the regularity has been defined according to Eurocode (a detailed description of the case studies is in Section 3) and the IDA was used as a reference method for the proposed IMPAβ procedure. Analysis results and discussion of this preliminary validation of the proposed procedure are reported in Section 4.

2. Non linear static analysis for bridges

2.1 State of the art of non linear static procedures

NSPs are a very effective alternative to nonlinear response history analysis, but these are strongly influenced by the lateral force distribution selected; existing guideline for load pattern does not cover all possible cases and the specific case of bridges is not considered.

Well known methods to calculate target displacement for a given seismic input, given in FEMA 356 and ATC 40 [10], do not consider the higher mode participation assuming that the response of a MDOF system is directly proportional to that of a SDOF system. This approximation is likely to yield adequate predictions of the element deformation demands for low to medium-rise buildings, where the behaviour is dominated by a single mode. However, pushover analysis can be grossly inaccurate for buildings with irregularity and in general cases where the contributions from higher modes are significant. Therefore the scientific community, in the last two decades, is intensively working on this topic [11-24]. This studies have demonstrated that, “traditional” pushover analysis, can be an extremely useful tool if some conditions are respected (structural regularity, a single dominant modal shape, the fundamental mode doesn’t vary significantly in nonlinear stage) but the relevance of innovating emerge with the aim of expanding the field of applicability, of these procedures, also to buildings characterized by greater complexity (there are currently severe limitations for non-regular structures) or to different structural systems such as bridges as discussed herein.

Indeed, more recently, many studies aimed to define innovative methodologies [25-29] alternative to what nowadays considered the most reliable approach: the Incremental Dynamic Analysis (IDA). In fact, IDA requires to perform a set of nonlinear response history analysis, on a detailed three-dimensional numerical model of the structure, for a set of ground motions, each scaled
for various intensity levels, selected to cover a wide range of structural responses [8,9,30]. As recognized even by the developers of IDA, the computation of many RHAs can result extremely demanding and therefore the definition of a simplified method can be useful [9].

This simplification was the scope of the IMPA (detailed in the next section), developed for buildings and mainly finalized to obtain correlation between seismic demand and a damage index [8,31,32]; with IMPA the IDA’s RHAs are replaced by a set of modal pushover analyses (MPAs) conserving the conceptual simplicity and computational attractiveness of standard non-linear static analysis (NSA). Extending this argumentation, from building to other structures, non-linear static analysis appears as an interesting approach also for bridges offering an innovative contribute to a research topic, the applicability of NSA to seismic assessment of bridges, that has been scrutinized in the recent past testing and proposing several nonlinear static procedures. The use of NSA for bridges is not straightforward and an accurate model of the inelastic behaviour of the structure is necessary but not sufficient; as discussed in the next section of this paper, an accurate selection of the horizontal load distribution, and of the monitoring point (MP) is needed.

The evolution of the “standard” non-linear static analysis that consider higher modes effects, and in particular the IMPA, suggested a well promising way for an extension of NSA to structures that are strongly influenced by higher modes such as bridges.

Few applications of NSA to bridges are available in the recent scientific literature and in some codes. Pinho et. al. [33] discussed a non-linear analysis for continuous multi-span bridge and proposed a single-run approach currently at a preliminary study stage. Muljati and Warnitchai [34] applied the Modal Pushover Analysis to a bridge with a continuous decks highlighting that, in this application, the non-linear and linear range have a similar tendency and therefore the modal pushover could be considered efficient; such condition is generally true if the deck is elastic and if the bridge is not characterized by relevant irregularities as is discussed in section 3 dealing with the case studies selected for this study. Kappos et al. [35-36] applies a multi-modal pushover procedure, generally similar to the MPA of Chopra and Goel, to some case studies and compared results with RHA. Dealing with codes, in the Eurocode 8 a pushover procedure, based on the well-known N2 method [37], is proposed to assess the seismic behaviour of regular buildings and only some specifications for bridges, not thorough, are reported in parts 1 and 2; in particular the methodology to determine the capacity curve (loading directions and distribution pattern, monitoring point selection) are discussed.

2.2 From NSA and RHA to IMPA and IDA: aim of proposing IMPA/

Non-linear response history analysis (RHA) is widely considered to be the most accurate method for assessing the response of structures subjected to earthquake action but it is also well known that performing RHA implicates difficulties or drawbacks: particularly for what concerns the interpretation of results or even, e.g. in design office environment, developing the structural numerical model. Indeed the operator, being researcher or professional, has to face the following main difficulties.

The most relevant one is that it is a computationally demanding analysis, especially when numerical models with distributed inelasticity are adopted; this issue is extremely relevant if an iterative procedure (e.g. for modelling errors, designing a new structure or retrofitting an existing one) is performed or merely because the set of GMs commonly selected have 7 or more time histories. Furthermore, as already mentioned, results interpretation is not straightforward and therefore, in order to verify the reliability of the numerical model, frequently a simpler analyses (e.g. pushover analysis, modal analysis) are performed as a preliminary check of the model: the examination of the modes of vibration or the story/base shear distribution from a pushover analysis are frequently adopted to quickly assess the model.

It is worth also remembering that, for seismic assessment of structures by means of RHA, a set of ground motions (GM) compatible with the seismic hazard spectrum for the site must be generated; this, as described by Bommer and Acevedo [38], is a complex task since there are no clear procedures to generate artificial spectrum-compatible records or select appropriate suites of real GM.
Differently, NSAs in spite of being simplified method, if compared with RHA, can provide all the relevant information required for investigating structural response. With a pushover is very simple the identification of critical regions, prediction of the sequence of yielding and/or failure of structural members. Moreover, many fundamental data for structural analysis are explicit and summarized in the capacity curve (base shear vs. displacement): stiffness, strength, ductility, energy balance. According to what previously mentioned is extremely interesting and useful to continue developing and innovating in the field of NSPs. It is therefore intuitive why the authors of this work decided to improve and extend the NSA and in particular the IMPA proposing a new non-linear analysis procedure for bridges: the IMPAβ.

It is opinion of the authors that this approach, as demonstrated by the preliminary application conducted (section 3), is a reliable and useful tool when employed either as a replacement to time-history analysis in the seismic assessment of structures or as a complement to dynamic analysis of highly complex structures.

2.3 Application of pushover analysis: from building to bridges:

Bridges are structures that are frequently sensitive to higher modes and therefore the use of procedures based on MPA, such as IMPA, seems to be a promising choice but some relevant differences between building and bridges should be considered. What is probably the main difference between building and bridges, performing a pushover, is the selection of the monitoring point (MP) used to identify the displacement of the structure and therefore plot the conventional capacity curve (several capacity curves, one for each mode, performing the MPA); the selection of the monitoring point affects the shape of the capacity curve and therefore, in some cases, a wrong selection can compromise the analysis (e.g. if the selected point is characterized by null or quasi-null displacement, whereas the structures is correctly pushed and deformed, it can produce the impossibility to determine a performance point). Therefore, especially performing MPA where a monitoring point suitable with different loading patterns (several modal shapes) is required, this is not a simple choice that can be generalized without considering the specific structural typology selected.

For buildings, according to many years of experience, guide lines and codes, the monitoring point is the center mass at the top level and this solution is generally suitable with any load pattern (according to a specific rule or proportional to one or more modes) considered.

Differently, considering a bridge structure, the selection of a univocal monitoring point suitable with any modal shape considered, is not ordinary. For bridges, natural selections for the monitoring point are: the mass center of the deck, the top of the pier nearest to the mass center (FEMA 356, Eurocode 8), the top of the pier corresponding to the maximum deck displacement [1] or a point of the deck determined according to the properties of the bridge [39-43]. Considering that selecting a monitoring point corresponding, for a specific modal shape (e.g. in case of a twisting modal shape), to a null or quasi-null displacement value or characterized by an opposite direction of displacement if compared with the global direction of incremental deformation of the bridge (the loading component of the loading pattern is inverted if compared to the resulting pushover direction), numerical problems or even null displacements (impossibility to plot the capacity curve an finalize the analysis) can compromise the analysis process. Therefore, according to those remarks and to the experience reached in this work, the monitoring point can be selected according to any of the procedures previously listed but effecting a selection, analyzing the modal shapes selected for the MPA, oriented to avoid the undesired null analysis.

2.4 Incremental modal pushover analysis for bridges

The originally developed incremental modal pushover analysis (IMPA) is a pushover-based procedure that requires the execution of MPA and an evaluation of structural performance within a range of different seismic actions and intensity. The IMPA procedure is described in a convenient step-by-step form in Bergami et al. (2017) [3]; therefore in this paper the IMPA procedure is briefly reported and only the specifications for bridges, required to describe the proposed IMPAβ, are reported and discussed. Remembering that the IMPA requires the execution of several MPAs, the
seismic demand due to individual terms in the modal expansion of the effective earthquake forces is
determined by a non-linear static analysis using the inertia force distribution \( s_n \) for each mode:

\[
s_n = \Gamma_n M \phi_n = \Gamma_n \begin{bmatrix} m \phi_{xn} \\ m \phi_{yn} \end{bmatrix}
\]

(1)

\[
\Gamma_n = \frac{L_n}{M_n}, \quad M_n = \phi_n^T M \phi_n, \quad L_n = \begin{bmatrix} \phi_{xn} \end{bmatrix} \begin{bmatrix} m \end{bmatrix} \begin{bmatrix} \phi_{xn} \\ \phi_{yn} \end{bmatrix}
\]

for direction X

(2)

for direction Y

where \( \Gamma_n \) is the \( n \)th modal participation factor; \( M \) is a diagonal mass matrix of order \( 2n \), including the diagonal submatrices \( m \), and \( I \): \( m \) is a diagonal matrix with \( m_{jj} = m_{ij} \), the mass lumped mass barycenter of the \( j \)th pier; \( q_n \) is the \( n \)th natural vibration mode of the structure consisting of three sub-vectors: \( r_n \), \( y_n \) and \( \theta_n \); the \( n \times 1 \) vector \( I \) is equal to unit.

The IMPA procedure can be summarized in the following steps:

1. compute the natural frequencies, \( \omega_n \) and modes, \( n \) for the linear elastic vibration of the bridge.
   The modal properties of the bridge model are obtained from the linear dynamic modal analysis and the relevant modes of the bridge are selected;
2. define the seismic demand in term of response spectra (RS) for a defined range of intensity levels;
3. for the intensity level \( i \), represented by peak ground motion acceleration (PGA) the performance point (P.P.) for the selected (predominant) modes can be determined (Figure 1a);
4. using a combination rule to combine the P.P. corresponding to each mode for each intensity \( i \), the “multimodal performance point” (P.P.m,i) can be determined (Figure 1b). The P.P.m,i is expressed in terms of monitoring point displacement \( u_{rmi} \), and corresponding global base shear \( V_{bi} \), for each intensity level considered: being \( u_{rmi} \) the modal displacements of the monitoring point: in this paper the transverse direction has been considered (Figure 2).

![Figure 1](image)

**Figure 1.** a) Evaluation of the performance points (P.P.) for each capacity curve that belongs to the pushover analysis: proportional to Mode 1..Mode n. b) evaluation of the P.P. for each capacity curve: e.g. via C.S.M. c) capacity curve plotted in the \( u_r \)-intensity plane (\( u_r \) is the displacement of the monitoring point along the investigated direction e.g. the transverse direction)
Data resulting from MPA application within an identified range of seismic intensity provides all necessary information to estimate the seismic response for different intensity levels. Therefore IMPA allows to develop a multimodal capacity curve (Figure 1c) relating a control parameter with the seismic demand intensity. In the procedure, for each seismic intensity level, the corresponding Performance Point (P.P.) for the multi-degree-of-freedom (MDOF) is determined and the corresponding deformed configuration of the structure (the bridge in this specific case) is derived: the deformed configuration of the bridge is the deformed configuration of each monitored station (usually the top of the piers) at the P.P. determined: $u_{r1}, \ldots, u_{rn}$.

![Figure 2](image1.png)

**Figure 2.** Degree of freedom of the bridge: performing the NSA the displacement $u_r$ of the monitoring point is controlled. In this work $u_r$ is the transversal displacement.

In the application of the MPA presented in this paper, the P.Ps have been determined through the application of the Capacity Spectrum Method (ATC 40) [10] and P.P.s obtained for each significant modal shape have been combined using the Square Root of the Sum of Squares rule (3) to obtain a multimodal performance point (P.P.mi) for each specific seismic intensity level.

$$u_{rnni} = \left( \sum_{n} u_{rn}^2 \right)^{1/2},$$

(3)

![Figure 3](image2.png)

**Figure 3.** Application of the Square Root of the Sum of Squares rule to a bridge

From the applications performed on the two case study (RB and IR presented in section 3 and 4) is demonstrated that, specifically for the case of an irregular bridges, the extension of IMPA to bridges could be better performed if some additional analysis are conducted. Indeed, from the analysis conducted on the irregular bridge, the use of MPA and UPA offer complementary results: the use of both modal and uniform pushover, similarly to what is commonly required for buildings where a double load pattern is usually required (e.g. EC8, Italian technical code NTC 2018 [44]), results as a better performing procedure for bridges (this is the main difference between IMPA and IMPA$\beta$). Therefore in IMPA$\beta$ both MPA and UPA are performed, at each step of the incremental procedure, to obtain final results as an envelope. The flowchart of this new procedure is in Figure 4.

In the procedure, the seismic demand is expressed in terms of Response Spectrums (RS) that are defined for all the intensity level; the RS selection can be performed according to different approaches. The RS can be selected according to the design code specification and then it can be linearly scaled to cover the desired intensity range. Otherwise, the scaling procedure, can be performed acting on the return period (Tr) or moreover the RS considered can be derived from a set of ground motions (GM), generated or selected investigating the local seismicity, for example considering the spectrum of each GM (or the median spectrum of each set) for each intensity level. In this paper, in the applications described in section 4, the median spectrum (RSm) of a set of GM, generated from the bridge design spectrum, was used and the RSm has been linearly scaled using a scale factor (SF) from 0.5 to 2.0; a more detailed description of this step is reported in section 3.
3. Application to two case studies

3.1. Case studies

The two case studies selected are both straight bridge with four equal spans (span length 50m) and total length equal to 200m (Figure 5). In both cases the deck consists of a 14m wide pre-cast concrete box girders supported by piers consisting of circular cross-section with diameter of 2.5m; the heights of the piers are variable and differentiate the two case study: a regular bridge (RB) and an irregular bridge (IB). In the irregular bridge the height of the 3 piers varies between 7 and 21m (P1=14 m, P2=7m, P3=21m) instead for the regular one the height of the 3 piers varies between 14 and 21m (P1=14 m, P2=21m, P3=14m). In both cases the deck rests on its two abutments through elastomeric bearings (movement in the longitudinal direction is allowed at the abutments, but transverse displacements are restrained) and it is supported on the concrete pier-head through bearing locked in the transverse direction. The details of this bridge are described in Figure 6 and Figure 7; the concrete class used was C20/25 (characteristic compressive cylinder strength $f_{ck}=20$MPa) while B450C steel (characteristic yield strength $f_{yk}= 450$MPa) reinforcement was used throughout the structure. The two bridges have been designed according to Eurocode 8: design peak ground acceleration was 0.35g and the structural behaviour (coefficient $q$) was 3.5 for the regular one and 3.0 for the irregular one. The design loads are summarized in Table 1.
Figure 5. Layout of the bridges: Regular bridge (RB) and irregular bridge (IB)

Figure 6. (a) Regular bridge: pier sections; (b) Irregular bridge: pier sections
Table 1. Loads and actions

| Load                  | kN/m | kN |
|-----------------------|------|----|
| Dead Self-weight      | 200  | -  |
| Live Vehicle loads (Qik) | -   | 1200 |
| Live Distributed load (qik) | 54.5 | -  |

The response of the bridge model is estimated through the employment of non-linear static and dynamic analysis (SPA, UPA, MPA and RHA) and incremental static and dynamic analysis (IMPA/β and IDA). The dynamic analyses have been performed adopting a set of 7 ground motions (GM) generated from the response spectra (RS) used to design the bridge; from the set of ground motions a median response spectra (RSm) has been defined. To generate the set of GM, the spectrum match for a relevant period range has been adopted; this approach is considered sufficient to describe seismic behavior of an individual structure [45,46].

The RS has been defined according to the Italian technical code (NTC 2018) being: a) the location is Reggio Calabria (a region of high seismic hazard in the south of Italy), b) the soil is type B (very dense sand or gravel or very stiff clay, 360 ≤ Vs30 ≤ 800); c) and the return period considered is Tₗ = 949 years (life safety limit state: Pₚ;=10%; aₓ=0.35g; Tₓ=0.172s, Tₓ=0.516s, T₉=0.305s; F₀=2.464; Sₜ=1.0). Given this target spectrum, with the software Rexel [47], a set of 7 unscaled records, compatible in the average with them, and with the minimum dispersion of individual spectra, have been selected: the average response spectrum matches the target spectrum at a specified period range that includes all the periods considered relevant (participating mass > 1% along the transverse direction of the bridge.

The 7 ground motions selected are listed in Table 2, and in Figure 8 are shown the 5% damped response spectra of the transverse component of the ground motions. The 7 GM were used to perform the RHA as well as the median spectrum was used as design spectrum for purposes of evaluating the pushover-based procedure. The spectra and the ground motions have been scaled to multiple levels of intensity to perform the incremental analysis. To perform IDA the 7 ground motions selected were scaled by a factor from 0.5 to 2. The median response spectra (RSm) was used in the pushover procedures, and therefore in IMPA/β, to determine the seismic demand.

Table 2: list of the selected ground motions.

| Etq ID | Earthquake Name       | Waveform | Date         | PGA [g] |
|--------|------------------------|----------|--------------|---------|
| 1635   | South Iceland          | 4674-xa  | 17/06/2000   | 31.176  |
| 1635   | South Iceland          | 4674-ya  | 17/06/2000   | 31.176  |
| 2309   | Bingol                 | 7142-xa  | 01/05/2003   | 50.514  |
| 2309   | Bingol                 | 7142-ya  | 01/05/2003   | 50.514  |
| 2142   | South Iceland (aftershock) | 6349-xa  | 21/06/2000   | 72.947  |
| 2142   | South Iceland (aftershock) | 6332-ya  | 21/06/2000   | 51.881  |
| 1635   | South Iceland          | 6277-ya  | 17/06/2000   | 35.251  |
|        | Mean                   |          |              | 4.62    |

In this paper (section 4) analysis results are presented in terms of the bridge capacity curve, i.e. monitoring point displacement versus seismic intensity, configuration of the deck drift profile, plastic hinges rotations and bending moments. Following Eurocode 8 recommendations, the independent damage parameter selected as reference is the displacement of the node at the center of mass of each pile (P₁, P₂ and P₃); the selection criteria is discussed in section 3.
Figure 7. Individual response spectra $\zeta = 5\%$, “component” of the ground motion records (transverse direction of bridge models), for the 7 unscaled ground motions (RS1,…,RS7) and their median response spectrum (RSm) being RS is the code elastic spectrum.

3.2. Numerical models

The Bridges were modelled using the finite element (FE) software SAP2000 NL, v. 21; the 3D numerical models are shown in Figure 9 and 10. The models ideally represent the mass distribution, strength, stiffness and deformability. Piers and girders supporting deck are modelled by 3D frame elements. The girder-pier joints are modelled by giving end-offsets to the frame elements, to obtain the bending moments and forces at the beam and column faces. The girder-pier joints are assumed to be rigid and the pier end at foundation was considered as fixed. All the pier elements are modelled with nonlinear properties at the possible yield locations (plastic regions according to Eurocode 8).

![Figure 8](image)

**Figure 8.** (a) FE model of the regular bridge; (b) FE model of the irregular bridge

In the present study, a lumped plasticity approach is considered for modelling nonlinearity; the plastic hinges are assumed to be concentrated at a specific portion of the pier; hinges have been modelled with fiber (P-M2-M3) hinges. In this study the fiber hinges are defined by moment-rotation curves calculated using a fiber-based model of the cross-section according to the reinforcement details at the hinge locations.

3.3 Modal properties

Modal properties of the bridge model were obtained from the linear dynamic modal analysis.

Table 3 and 4 show the details of the “important” modes, in transverse direction, for both the bridge considered: the RB and the IB respectively. For the RB, the participating mass ratio of the first two relevant modes (mass ratio ≥3%) are respectively 78% and 12%; the cumulative mass participating ratio for first three modes is 90% (the participating mass ratio of any other single mode was less than 1%). Whereas Table 4 shows that, for the IB, the participating mass ratio of the first three relevant modes are respectively 16.9%, 71.3% and 4.5% (mode 4 is therefore almost insignificant in the elastic phase but this condition, as discussed later, will change in the nonlinear phase); the cumulative mass participating ratio for the first three modes is 92.7% (the participating mass ratio of the other modes was less than 1%). Therefore the higher mode participation in the response of both
RB and IB is significant and respectively the first two or three mode shapes in the transverse direction must be considered according to the MPA procedure; the dominant modes of the two cases are mode 2 (T2=1.02s) for the RB and mode 3 (T3=0.53s) for the IB.

### Table 3: Regular Bridge - Modal properties

| Mode | Period | Participating Mass |
|------|--------|--------------------|
| N⁰  | Sec    | %                  |
| 2    | 1,02   | 78,0               |
| 4    | 0,33   | 12,0               |

![Figure 9: Regular Bridge - Mode shapes (transverse direction)](image)

### Table 4: Irregular Bridge - Modal properties

| Mode | Period | Participating Mass |
|------|--------|--------------------|
| N⁰  | Sec    | %                  |
| 1    | 0,65   | 16,9               |
| 3    | 0,53   | 71,3               |
| 4    | 0,13   | 4,5                |

![Figure 10: Irregular Bridge - Mode shapes (transverse direction).](image)

### 4. Non-linear analyses

Fundamental mode based (SPA), as well as uniform loading (UPA) and modal pushover analysis (MPA) pushover analyses were first performed for assessing the inelastic response of the bridges; results of these analyses are presented and evaluated, by the RHA having a 2% Rayleigh damping assigned to the two modes with the highest effective modal mass, for both cases: RB and IB. Performing the pushover analysis the monitoring point selection is one of the fundamental steps.

Remembering that, according to the capacity spectrum method (CSM, ATC 40 [10]), the base shear forces and the corresponding displacements in each pushover curve (being \( V_{bn} \) and \( u_{bn} \) the pushover curve of the multi degree of freedom) are converted to ADRS format using the following relationships (being \( S_a \) and \( S_d \) the spectral accelerations and spectral displacements of an equivalent single degree of freedom (SDOF) system):

\[
\begin{align*}
V_{bn} &= S_a \\
u_{bn} &= S_d 
\end{align*}
\]
\[ S_a = \frac{V_{bn}}{M_n^*} \]  \hspace{1cm} (4)

\[ S_d = \frac{u_{rn}}{\Gamma_n \varphi_{rn}} \]  \hspace{1cm} (5)

wherein \( \varphi_m \) is the value of \( \varphi_n \) at the monitoring point, \( M_n^* = L_n \Gamma_n \) is the effective modal mass, \( L_n = \varphi_n^T m I \), \( \Gamma_n = L_n / M_n \), and \( M_n = \varphi_n^T m \varphi_n \) is the generalized mass, for the natural mode \( n \).

As discussed in (Kappos et al, 2010), if the structure remains elastic for the given earthquake intensity the spectral displacement \( S_d \) and the product \( \Gamma_n \cdot \varphi_n \) will be independent of the selection of the monitoring point; this means that deck displacements are independent from the location of the monitoring point.

On the contrary, in some applications of the same author, it was found that deck displacements derived with respect to different monitoring points, for inelastic behaviour of the structure, are not identical, but rather the estimated deformed shape of the bridge depends on the monitoring point selected for drawing the pushover curve for each mode.

For inelastic behaviour, the conversion of the displacement demand of the \( n \)-mode SDOF system to the peak displacement of the monitoring point, \( u_{rn} \) of the bridge, gives a different value not only because of the deviation of the elastic mode shape \( \varphi_n \) from the actual deformed shape of the structure, but also due to the fact that the spectral displacement \( S_d \) is dependent on the selection of monitoring point if the structure exhibits inelastic behaviour (due to the bilinearization of the capacity curve). For the applications conducted in this work (RB and IB) it is noted that the approximations involved in the capacity-demand spectra procedure, deriving deck displacements with respect to different monitoring points, are negligible and results are deemed acceptable, analogously to what has been concluded by other authors (Paraskeva et al, 2006 - 2010), for all practical purposes. Figures 11 and 12 illustrate the deck displacements, of the RB and the IB, derived using pushover analysis for each mode independently, as well as the MPA, considering the monitoring point at the different positions: top of the pier 1 (P1), pier 2 (P2) and pier 3 (P3). The results illustrated in those figures highlight that, for the bridges considered, results are not affected by the monitoring point (the deformed shape derived considering M.P.P1 or M.P.P2 or M.P.P3 are substantially one over the other).

**Figure 11**: Regular Bridge - Deck displacements derived performing pushover analysis (\( \text{ag}=0.7\text{g} \)), with a load pattern proportional to the main mode shapes, with respect to different monitoring points (M.P.). Pushover with Mode 2 (a) or Mode 4 (b).
According to what discussed in section 2.3, the monitoring point selection, instead the same results can be finally obtained selecting one or the other, the choice of the monitoring point in Pier 2 should be discarded because, in this case, in order to achieve a complete analysis much many iterations and a more complex analysis step calibration are required. Therefore, the selection of a monitoring point corresponding, for a specific mode shape, to a null or quasi-null modal coefficient or with an opposite sign if compared with the main bridge incremental deformation (e.g. similarly to Pier 2 @ Mode 1 of the IB – Figure 11 and Pier 2 @ Mode 4 of both the IB and RB – Figure 10), can cause numerical difficulties performing the pushover analysis. According to this and considering all the relevant mode shapes selected for the modal pushover, the selection of the top of Pier 1 as the most suitable monitoring point was adopted for both cases.

4.1 Analysis of the regular bridge (RB)

The bridge RB was analyzed at several seismic intensity levels: from PGA=0.175g to PGA=0.7g being PGA=0.35g the design value.

Since the response of this bridge is governed by one dominant mode (Mode 2), being Mode 2 and Mode 4 the modes considered in the multimodal approach, and since the mode shape is not changing significantly (it is not considerably influenced by the occurrence of a new hinges), the estimated response is practically the same regardless the method used (MPA, SPA or UPA) in the analysis (Figure 13). The estimated response coincides quite well with the results of RHA for PGA = 0.175g (SF=0.5) and PGA = 0.35g (SF=1.0).

Figure 12: Irregular Bridge - Deck displacements derived performing pushover analysis (ag=0.7g), with a load pattern proportional to the main mode shapes, with respect to different monitoring points (M.P.). Pushover with Mode 1 (a), Mode 3 (b) or Mode 4 (c).

Figure 13: Regular Bridge - Deck displacements derived performing non linear dynamic analysis and pushover analysis according to the approaches considered (PGA from 0.175g to 0.7g).
Figure 14: Regular Bridge - Deck displacements derived performing non linear dynamic analysis and pushover analysis according to the approaches considered (PGA from 0.175g to 0.7g).

Figure 15: Regular Bridge - Pier drift (top displ/height) derived performing non linear dynamic analysis and pushover analysis according to the approaches considered (PGA from 0.175g to 0.7g).

Figure 16: Regular Bridge - Deck drift (relative displacement/span length) derived performing non linear dynamic analysis and pushover analysis according to the approaches considered (PGA 0.175g - 0.7g)
Figure 17: Regular Bridge – bending moment ($M_b$) and curvatures ($\theta_b$) at the hinge location derived performing non linear dynamic analysis and modal pushover according to each single mode (PGA from 0.175g to 0.7g)

Analyzing in Figure 19 the capacity curves, or rather the incremental capacity curves, obtained by applying the IMPAβ and comparing them with what can be obtained through IDA, it is confirmed that the use of the incremental modal pushover analysis allows to describe the structural response with excellent accuracy, and without relevant estimation errors, inside the intensity range for which the bridge was designed. Indeed, the use of the second pushover approach (UPA) offers conservative results in terms of deck displacements related to the seismic intensity considered but it underestimates the base shear if the capacity curve is considered. Therefore, concluding, considering IMPAβ as an envelope of modal and uniform pushover based procedures, according to the general considerations that will be discussed presenting the other case study (IB), the results obtained are always conservative.
Figure 18: Regular Bridge – bending moment ($M_b$) and curvatures ($\psi_b$) at the hinge location derived performing non linear dynamic analysis and pushover analysis (PGA from 0.175g to 0.7g)

Figure 19: Regular Bridge – incremental curve (displacement-intensity) and capacity curves derived with IDA (maximum registered values of $u_r$ and $V_{b,x}$) or IMPA/$\beta$ (the design PGA is 0.35g corresponding to a transversal base shear of $V_{b,x} \approx 5200$ kN)
4.2 Analysis of the irregular bridge (IB)

The same analysis performed on the RB were repeated for the IB. All relevant modes (mode 1, 3 and 4) have been taken into account performing the procedure (Figure 11). From Figures 20 and 21, it can be observed that the results of the “standard” pushover methods (SPA and UPA) differ, even qualitatively, from the results of the RHA and the envelope of MPA and UPA emerges as the best approach that, for an intensity level corresponding to the design intensity (0.35g), offers results similar to RHA. In general, every pushover procedure considered overestimate the response at the stiffer side (Pier 1) whereas at the center (Pier 2) and at the flexible side (Pier 3) the response is underestimated. According to the results presented herein, also confirmed from the scientific literature (other applications on this type of bridges - Kappos et al.), bridges with asymmetric modes (Mode 1 and 4) and other relevant modes translational (Mode 3, the most relevant one) crucial is the occurrence of the first plastic hinges; in the IB when the hinge occurred in the central column the mode shape drastically changed and should be recognized that even the multimode method can’t reflect these sudden and substantial changes of the dynamic properties of the structure.

On the other hand the MPA works relatively fine even in the case of the design earthquake, when the hinge in the central column occurs. This, however, is not surprising, since the response is predominantly influenced by one mode dictated by the superstructure. Results are reported in the following figures considering the incremental range. All relevant modes (mode 1, 3 and 4) have been taken into account performing each MPA.

![Hinge Yielding](image)

**Figure 20:** Irregular Bridge - Deck displacements derived performing non-linear dynamic analysis and pushover analysis according to the approaches considered (PGA from 0.175g to 0.7g)

As can be observed also from Figure 20-22 (in Figure 20, for each intensity step, the curves MPA-UPAenv are compared with the RHA results), MPA coincides quite well with the results of RHA up to the design intensity (from PGA 0.175g to PGA 0.35g) but, to achieve a better estimation, the envelope of MPA and UPA should be considered (MPA_UPAenv).

For higher intensities (scale factor 1.5 and 2.0: PGA over 0.525g) a good estimation of displacement can be observed. Figure 20 illustrates how the first plastic hinges are in Pier 2, first, and Pier 1 after (at a PGA of 0.175g that is lower than the design intensity), whereas in Pier 3 the first hinge emerges only at a very high intensity level (greater than PGA 0.525g). It was observed that a limit state (ultimate limit state at the base of Pier 2) is reached when the deformed shape is close to PGA 0.525g; therefore, for the scope of the procedure, investigate higher intensities is useless.

In terms of hinge curvatures (Figure 24) results reflects the same behaviour previously discussed. From Figure 25, in terms of bending moments underestimation, previously highlighted at Pier 2 and 3, is irrelevant being the bridge in a plastic stage. In terms of deck drift (Figure 23), for the relevant range of intensities (PGA<0.525g) previously indicated, the pushover procedures are all well performing and the MPA-UPAenv results the better solution being the most conservative up to the design intensity and well performing for higher intensities.

The capacity curves of the structure have been determined performing both the incremental multimodal pushover analysis IMPA and the incremental dynamic analysis IDA; the curves are plotted in Figure 26 and 27. Figure 26 demonstrates that the capacity curve, plotted considering
different monitoring points, provides similar results in all cases; the IDA and IMPA curves are comparable with a moderate difference in the elastic-plastic transition. Comparing IMPA with a “traditional” pushover approach (Figure 27) based on the incremental SPA (ISPA), the reliability of IMPA is confirmed. The “real” behavior plotted with IDA is included between IMPA and IUPA; therefore the IMPA/β (envelope of IMPA and IUPA) is confirmed as conservative and better performing approach. In conclusion, also investigating the relation between seismic intensity and shear action, the IMPA/β curve can be considered well performing, also in terms of base shear, if compared with other pushover procedures confirming it as a valid approach.

Figure 21: Irregular Bridge - Deck displacements derived performing non linear dynamic analysis and pushover analysis according to the approaches considered (PGA from 0.175g to 0.7g)

Figure 22: Irregular Bridge - Pier drift (top displ/height) derived performing non-linear dynamic analysis and pushover analysis according to the approaches considered (PGA from 0.175g to 0.7g)
Figure 23: Irregular Bridge - Deck drift (relative displacement/span length) derived performing non linear dynamic analysis and pushover analysis according to the approaches considered (PGA from 0.175g to 0.7g)
Figure 24: Irregular Bridge – bending moment ($M_b$) and curvatures ($\theta_b$) at the hinge location derived performing non linear dynamic analysis and modal pushover according to each single mode (PGA from 0.175g to 0.7g)
5 Conclusions and future developments

In this paper a study finalized to develop a pushover-based procedure for performing a comprehensive assessment of the behavior of bridges under seismic loads has been presented. This innovative procedure, named IMPA/β, is alternative to what is currently considered the most reliable procedure, the incremental dynamic analysis (IDA), and its reliability has been demonstrated through the application on two case study of bridges defined, according to Eurocode, the regular bridge (RB) and the irregular bridge (IB).

The application performed allowed to develop this procedure starting from the previous experience of the same authors developing IMPA that is an incremental modal pushover analysis conceived and tested for building structures. Therefore the main differences between analyzing...
buildings or bridges, with a pushover-based procedure, were preliminary discussed; a fundamental issue is the monitoring point (MP) selection that is quite straightforward for buildings whereas, for bridges, required the discussion proposed in this work. Dealing with a bridge structure, the selection of a univocal monitoring point suitable with any mode shape considered performing a pushover analysis, being a standard pushover analysis (SPA) or a multimodal pushover (MPA), is not ordinary; the selection of the monitoring point affects the shape of the capacity curve and therefore, in some cases, a wrong selection can compromise the non-linear static analysis. According to existing literature or codes for bridges, the MP can be located at: the mass center of the deck or the top of the pier nearest to the mass center (FEMA 356, Eurocode 8) or the top of the pier corresponding to the maximum deck displacement or a point of the deck determined according to the properties of the bridge. In this work it has been concluded that the MP can be selected according to any of the cited criteria but avoiding locations characterized by, for a specific mode shape (e.g. in case of a twisting mode shape), a null or quasi-null displacement value or an opposite direction if compared with the main bridge deformation (the displacement direction that characterizes the capacity curve in the post elastic range).

The case studies allowed to define the procedure finally tested and compared with IDA; from the analysis performed on both the RB and the IB emerges that the IMPA\(\beta\) procedures must be performed considering a double pushover approach. It means that, differently from IMPA, both modal pushover and uniform pushover analysis are required in order to obtain reliable and conservative results (for both the case study considered and also for a wider range of intensities).

In particular, from the application on the IB, results (expressed in terms of capacity curve or seismic intensity vs deck displacement) obtained performing MPA only can be considered well performing up to the design intensity of the bridge whereas, for higher intensities and therefore after the activation of one or more plastic hinges, the MPA became inaccurate. Differently, if a uniform loading profile (UPA) is additionally performed and the MPA and UPA are enveloped, for each intensity level, results strongly improve: the curves derived with IMPA (incremental modal pushover analysis based on MPA) and IUPA (incremental pushover analysis based on UPA) include, as an upper and a lower bounds, the curves derived with IDA. Therefore the IMPA\(\beta\), envelope of IMPA and IUPA, if compared with IDA, is a well performing procedure and in particular excellently performing up to the bridge design intensity.

Differently from the IB, results from the RB were not affected by the pushover procedure adopted performing the incremental analysis being the difference among IMPA, IUPA and ISPA negligible. Therefore the curves, derived monitoring any parameter (base shear vs deck displacement or seismic intensity vs a damage parameter) with IMPA, IUPA and ISPA, hence adopting IMPA\(\beta\) (envelope of IMPA and IUPA according what obtained working on the IB), are substantially identical confirming that the procedure calibrated for the IB can be conserved and generalized to both the case study. Discussing the specific case of the RB, if compared with IDA, IMPA\(\beta\) results excellently performing up to the design intensity and well performing (giving conservative results) for higher seismic intensities.

Concluding, in this paper an incremental pushover-based approach, named IMPA\(\beta\), where both MPA and UPA are conducted in order to derive an envelope, is reported and applied; the activity herein presented is the first step of a more extensive study that will imply a wider discussion of other interesting issues [48, 54] dealing with bridges: different configurations (deck shape and sections, constrains etc.), variable level of irregularity (also as a consequence of a post-earthquake retrofitting), a more detailed definition of the seismic input and non-synchronous seismic input. The results obtained, as demonstrated by the numerical study presented, can be considered encouraging for this study confirming IMPA\(\beta\) as a well performing procedure for bridges.

**Author Contributions:** Conceptualization, A.V.B., C.N.; methodology, A.V.B., C.N., D.L.; Resources, A.V.B, C.N. and B.B., software-analysis, A.V.B.; writing—original draft preparation, A.V.B.; writing—review and editing, A.V.B., G.F. C.N., D.L. and B.B. All authors have contributed substantially to the work reported. All authors have read and agreed to the published version of the manuscript.
Funding: The authors gratefully acknowledge the funding received by The Laboratories University Network of Seismic Engineering (ReLUIS): research project ReLUIS/DPC 2019-2021 Reinforced Concrete Existing Structures and the National Natural Science Foundation of China [grant No. 51778148].

Conflicts of Interest: The authors declare no conflict of interest.

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