Populating the Galaxy with pulsars – II. Galactic dynamics

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ABSTRACT

Pulsar observations provide a suite of tests to which stellar and binary evolutionary theory may compare. Importantly, the number of pulsar systems found from recent surveys has increased the statistical significance of pulsar population synthesis results. To take advantage of this, we are in the process of developing a complete pulsar population synthesis code that accounts for isolated and binary pulsar evolution, Galactic spatial evolution and pulsar survey selection effects. In a recent paper, we described the first component of this code and explored how uncertainties in the parameters of binary and pulsar evolution affected the appearance of the pulsar population in terms of magnetic field and spin period. We now describe the second component which focuses on following the orbits of the pulsars within the Galactic potential. In combination with the first component, we produce synthetic populations of pulsars within our Galaxy and calculate the resulting scaleheights as well as the radial and space velocity distributions of the pulsars. Correlations between the binary and kinematic evolution of pulsars are also examined. Results are presented for isolated pulsars, binary pulsars and millisecond pulsars. We also test the robustness of the outcomes to variations in the assumed form of the Galactic potential, the birth distribution of binary positions and the strength of the velocity kick given to neutron stars at birth. We find that isolated pulsars have a greater scaleheight than binary pulsars. This is also true when restricted to millisecond pulsars unless we allow for low-mass stars to be ablated by radiation from their pulsar companion in which case the isolated and binary scaleheights are comparable. Double neutron stars are found to have a large variety of space velocities; in particular, some systems have speeds similar to the Sun. We look in detail at the predicted Galactic population of millisecond pulsars with black hole companions, including their formation pathways, and show where the short-period systems reside in the Galaxy. Some of our population predictions are compared in a limited way to observations but the full potential of this aspect will be realized in the near future when we complete our population synthesis code with the selection effects component.

Key words: binaries: close – stars: evolution – stars: neutron – pulsars: general – Galaxy: kinematics and dynamics – Galaxy: stellar content.

1 INTRODUCTION

Detailed examinations of the stellar populations within the Galaxy, and indeed the greater Universe, have unearthed many fascinating objects. Examples of such systems are gamma-ray bursts (Klebesadel, Strong & Olson 1973; Paczynski 1986; Bogomazov, Lipunov & Tutukov 2008), coalescing double neutron stars (NSs) (Rantsiou et al. 2008), X-ray binaries (Schreier et al. 1972; Liu, van Paradijs & van den Heuvel 2007; Galloway et al. 2008), microquasars (Abell & Margon 1979; Margon et al. 1979; Combi, Albacete-Colombo & Marti 2008) and millisecond pulsars (MSPs) (Backer et al. 1982; Manchester et al. 2005). Current belief suggests that all the aforementioned systems arise from binary stars in which both stars are close enough to experience a strong gravitational interaction with their companion. This may lead to mass transfer, and depending upon the mass ratio, types and ages of the two stars a plethora of different stellar and binary evolutionary phases may occur. Recent surveys spanning a large range of observed wavelengths now make it possible to monitor and analyse the combined properties of this variety of stellar populations. This in turn can place further constraints upon our theoretical understanding of stellar and binary evolution. For example, with the quickly increasing number of known low-mass stellar X-ray binary systems (187; Liu et al. 2008).
2007), high-mass X-ray binary systems (114; Liu, van Paradijs & van den Heuvel 2006) and pulsars (> 1600; Manchester et al. 2005), it is possible to constrain features of NS and black hole (BH) formation from their relative Galactic scaleheights. These have been examined for NS and BH low-mass X-ray binaries (via observations; Jonker & Nelemans (2004)) showing evidence that (contrary to previous belief) BHs may receive momentum from their formation mechanism – the supernova (SN) event.

In this work, our focus is on the Galactic population of pulsars. Observations of such objects occur primarily at radio (centimetre) wavelengths. These systems have intrinsically weak signals (typically measured in mJy; Lorimer 2008) and are observed as a regular series of pulses in time. Due to the frequency of the pulses, we know these systems are compact (Hewish et al. 1968), while the period derivative allows evolutionary models of these systems to be developed (e.g. Goldreich & Julian 1969; Ostriker & Gunn 1969; Gunn & Ostriker 1970; Bisnovatyi-Kogan & Komberg 1974; van den Heuvel 1984; Kulkarni & Narayan 1988; Chen & Ruderman 1993) and tested (Dewey & Cordes 1987; Tauris & Bailes 1996; Dewi, Podsiadlowski & Pols 2005; Kiel et al. 2008, hereafter Paper I). Finally, distances may be estimated from the dispersion of the pulse due to the electron density distribution within the Galaxy (Cordes & Lazio 2002). From these observations, we now believe that pulsars are magnetic rotating NSs and that the radio signal arises from the magnetosphere and is well collimated. Timing of these radio pulses gives the spin period $P$ and the spin period derivative $\dot{P}$ from which the magnetic field and characteristic age of the pulsar may be inferred (see Paper I, and references therein for further details).

Knowledge of pulsar space velocities provides constraints on the effects of SNe on the NSs they give rise to. This may, in turn, place constraints on the SN mechanism and NS structure. The birth velocities of pulsars (Lyne & Lorimer 1994) combined with the work of Gott, Gunn & Ostriker (1970), Cordes, Romani & Lundgren (1993), Dewey & Cordes (1987) and Bailes (1989) demonstrated the necessity of asymmetric SN velocity kicks imparted on NSs by observing large isolated pulsar space velocities of the order of 1000 km s$^{-1}$. Later studies also found similar conclusions, again because observations suggest pulsar space velocities in excess of the mean for normal field stars (Fryer & Kalogera 2001). Evidence for asymmetric SNe is also provided by the misalignment of binary pulsar spin vectors to the orbital angular momentum vector (e.g. Kaspi et al. 1996). The vectors would normally be expected to be coupled before the violent SNe because of tidal effects and any occurrence of mass transfer (Bhattacharya & van den Heuvel 1991; Hurley, Tout & Pols 2002, hereafter HTP02). There is observational evidence of this misalignment for a number of pulsar binary systems including double NS systems (e.g. Kramer 1998).

Another method in which we are able to constrain the effects of SNe and to test the validity of our evolutionary assumptions (including the average magnitude of any kick delivered by a SN) is to compare, with observations, the kinematics of model pulsar populations within a model Galaxy (previous works include Dewey & Cordes 1987; Bailes 1989; Lorimer et al. 1993; Lyne et al. 1998; Sun & Han 2004). There are two obvious pulsar populations we may recognize, those that are isolated pulsars and those pulsars within binary systems. We may make further distinction with regard to the spin of a pulsar – those of a ‘standard’ spin period ($P > 1$ s), those of millisecond spin periods ($P < 0.1$ s; known as MSPs) while those between these two period ranges are known as partially spun-up or partially recycled pulsars. The final pulsar population which we wish to point out here are those pulsars which reside in double compact binaries, in particular double NS binaries (such as PSR 1913 + 16; Hulse & Taylor 1975) and, within that population, double pulsar systems (PSR J0737 – 3039A&B; Burgay et al. 2003; Lyne et al. 2004).

From a simple evolutionary analysis of these systems (see Paper I) we may expect there to be distinct scaleheights for each pulsar population within the Galaxy. This is assuming that all pulsars are given an asymmetric SN kick velocity drawn from the same distribution (i.e. ignoring the possible electron capture SN scenario which may confuse this issue; see Paper I; Podsiadlowski et al. 2004; Ivanova et al. 2008). Isolated pulsars – it is reasonable to presume – would have a greater scaleheight than those pulsars within binary systems. This is because binaries are on average heavier than single stars and also the binary orbit is able to absorb energy from the kick (as detailed in Hills 1983; Tauris & Takens 1998, hereafter T98). Having been previously ejected from a binary or having always been a single star would allow the kick velocity to have greater effect on the stellar space velocity. Isolated pulsars that have felt two SN kicks during their lifetime (one indirect and one direct) would be expected to have the largest scaleheight of any pulsar population (Bailes 1989). Say, for instance, that the first of two massive stars disrupts a binary system, giving momentum to both stars out of the Galactic plane, then the second massive star (now isolated) undergoes a SN and receives further momentum. Pulsars of different spin types, for example MSPs, can also be expected to have differing Galactic scaleheights dependent upon the nature of their evolutionary histories. If one assumes an MSP forms via Roche lobe overflow mass transfer from a low-mass companion the system only passes through one SN event, and this will occur in a close binary which can overcome larger velocity kicks to stay bound, say, a standard pulsar in a wider (pre-SN) orbit (e.g. Bailes 1989; Portegies Zwart & Yungelson 1998). If, however, some MSPs are formed via wind accretion from a high-mass companion (as discussed in Paper I), which may itself form a NS, a fraction of the MSP population may feel two kicks. This may lead to an increase of the MSP Galactic scaleheight and also produce a method for isolated MSP production (as described by Narayan, Piran & Shemi 1991 and Paper I). Finally, we may expect double pulsar systems to have a greater Galactic scaleheight than the single pulsar binary systems because they feel two SNe. However, the fact that the binary system had to survive these two kicks requires the kick velocity imparted from both events to be small enough (or well directed) for the binary to survive. Therefore, although two kicks occur it seems plausible that double pulsar systems may not attain a greater scaleheight than their isolated cousins (Dewi, Podsiadlowski & Pols 2005; Pfahl, Podsiadlowski & Rappaport 2005). Further to this simple analysis, the relative ages of each system type will also play an important role in the observable scaleheight, because older systems will have had more time to relax (outwards) in the Galactic gravitational potential – the distributions diffuse over time. For a more in-depth review of pulsar evolution and kinematics, see Lorimer (2008).

The work described in this paper allows us to address these issues and to predict and compare the Galactic spatial distributions of pulsar populations. We follow on from our pulsar population synthesis in Paper I to not only evolve the stellar evolution of pulsars but also move them within the Galactic gravitational potential. In other words, we follow Galactic stellar, binary and kinematic evolution. As introduced in Paper I, we are developing a code comprising three modules: BINPOP (binary evolution), BINKIN (Galactic kinematics)
and BINSFX (synthetic survey simulations). An upcoming paper will describe the third module, BINSFX, where we impose selection effects on the simulations, thus giving simulated data that can be compared directly to observations.

Section 2 outlines our binary evolution code (BINPOP) and details a necessary update to follow the evolution of a system that is disrupted owing to an asymmetric SN velocity kick. In Section 3, we describe the Galaxy kinematic code (BINKIN), which integrates the positions of pulsars – both in binary systems and isolated – forward in time within the Galaxy. This includes details of how the initial conditions for the Galactic population are chosen and the parameters involved. Results are given in Section 4 where, assuming a favoured binary and stellar evolutionary model of Paper I, we examine the pulsar population scaleheights and velocities that arise from different BINKIN model assumptions. This includes a detailed examination of the MSP–BH binary population. In particular, we explore the formation and evolution of these systems. This is followed by a discussion of our findings and the main uncertainties involved in Section 5.

2 RAPID BINARY EVOLUTION

The first module, BINPOP, was described in detail in Paper I. Below, we give an overview and also address the necessary modifications to BINPOP in order to correctly follow the Galactic positions of both members of a disrupted binary system.

2.1 BINPOP

BINPOP is a stellar/binary population synthesis package which involves the binary stellar evolution (BSE) code of HTP02 with realistic initial stellar and binary parameter distributions (as developed in Kiel & Hurley 2006 and Paper I). Stellar evolution is included according to the formulae presented in Hurley, Pols & Tout (2000). Meanwhile, BSE attempts to account for all important binary evolutionary processes. These include tidal evolution, mass transfer, common envelope (CE) evolution, stellar mergers, magnetic braking, orbital gravitational radiation and SNe velocity kicks. In Paper I, extensive additions were made to BSE, in terms of NS physics, so that pulsar evolution could be followed in detail. This means that aspects such as magnetic field decay, accretion-induced field decay and spin-up, propeller evolution and pulsar death lines are now included. Inherent uncertainties in the variety of binary and pulsar evolutionary processes require a host of parametrized prescriptions to be incorporated into BSE. For example, our lack of understanding of CE evolution is expressed as a parameter, $\alpha_{CE}$, often referred to as the efficiency parameter. In terms of pulsar evolution, there is the magnetic field decay time-scale, $t_B$, and the accretion-induced decay time-scale, $k$, for example, which are uncertain. Over time, the uncertainty in many parameters has decreased – albeit slightly – due to a large array of simulations and increasingly detailed observations (e.g. Lyne et al. 1998; Portegies Zwart & Yungelson 1998; HTP02; Belczynski, Kalogera & Bulik 2002b; Podsiadlowski, Rappaport & Han 2003; Sun & Han 2004; Yusifov & Kucuk 2004; Hobbs et al. 2005; Kiel & Hurley 2006; Cordes et al. 2006; Lorimer et al. 2006; Ferrario & Wickramasinghe 2007; Liu et al. 2007; O’Shaughnessy et al. 2008; Belczynski et al. 2008 and Paper I).

2.2 Binary evolution SN kick update

When following the kinematic evolution of a binary system within the Galaxy, we require the knowledge of the Galactic gravitational potential – the acceleration felt on the binary centre of mass (CoM) owing to the Galaxy – as well as any internal sources of momentum that arise. The primary stellar evolutionary phase that can perturb an orbit or disrupt a binary system is a SN. If the SN occurs in a binary and enough material is ejected from the system during the event (more than half of the total binary mass), the binary may disrupt (Hills 1983). Along with the assumed instantaneous mass-loss, if there is any asymmetry in the explosion (which is arguable in the case of BHs; see Podsiadlowski et al. 2003 but note Pfahl et al. 2005), the newly formed compact star will receive a velocity kick which the binary CoM will feel (Shklovskii 1970; Lyne & Lorimer 1994; TT98; HTP02). Depending upon the velocity kick direction and magnitude this may disrupt a binary or save it from mass-loss-induced dissipation (Hills 1983; Kalogera 1998; Pfahl, Rappaport & Podsiadlowski 2003).

The algorithm described by HTP02 allows realistic orbital evolution modelling if the binary system survives the blast and stays gravitationally bound (eccentricity, $e < 1$). However, because binary systems cease to exist once they become unbound, HTP02 were not troubled with calculating the recoil escape velocities of the two disassociated stars travelling on hyperbolic orbits with $e > 1$. Now that the Galactic spatial kinematics of both binary systems and isolated stars are a concern, the knowledge of all associated velocity changes is required. To this end, we formulate a disruption model (in Section 2.2.1) and also describe similar methods derived by other groups (in Section 2.2.2). All methods considered here are generalized to allow for initially eccentric systems.

2.2.1 BSE disruption model

Before one can calculate the final velocities of the disrupted stars, the binary system must be known to disrupt. We start by considering what is already within the capabilities of the rapid binary evolution code (cf. appendix A of HTP02) which we outline here. This assumes a reference frame in which the pre-SN CoM is at rest, $V_z = 0$, and the secondary star (the star not exploding) is at the origin (as shown in Fig. 1). The magnitude of the pre-SN relative

![Figure 1. HTP02 orbital geometry as asymmetric SN occurs. Taken from fig. A1 of HTP02.](https://academic.oup.com/mnras/article-abstract/395/4/2326/972484)
Prior to the SN, our chosen coordinate system had the orbital angular momentum vector directed along the z-axis but this will no longer be true for the post-SN system (unless \(V_{\text{kick}} = 0\)). However, to simplify our post-SN calculations it is desirable to realign the vector with the z-axis. This realignment in the x-z plane, owing to the SN, is performed by a rotation, \(R_\phi\), around the y-axis such that the post-SN unaligned orbital specific angular momentum, \(h' = r \times V' = [0, r, 0] \times [V_x, V_y, V_z]\), becomes \(h'' = [0, h_{\text{y}}']\). Note that we are using the symbol prime (′) to denote the frame immediately after the SN and double prime (″) when referring to the coordinate system of the frame after rotation. Here, the rotation is guided by \(\nu\), the angle between the pre- and post-SN angular momentum vectors (see equation A13 of HTP02). This rotation matrix allows us to map our final velocities back on to the original coordinate system.

Now, we consider the post-SN motion of the two stars which is governed by a hyperbolic conic section. We can no longer assume that the separation vector between the stars is aligned along the y-axis because the system is not necessarily at periastron. To account for this possible shift in coordinate system around the \(z\)-axis, we calculate where in the orbit each star resides. Here, we have

\[
\cos \psi = \frac{1}{e' \sqrt{r}} \left( \sqrt{r} - 1 \right) = \frac{1}{e} \left( \frac{(h')^2}{GM_{\text{NS}}} - 1 \right)
\]

with \(\psi\) defined to be positive in the positive \(y\)-direction and zero along the positive \(y\)-axis. There are two possible hyperbolic orbits for which each star may travel along – one in the positive \(y\)-region, the other in the negative \(y\)-region – which is governed by the direction of the \(y\)-component of the new relative velocity, that is, \(r \cdot V'\). The sign of \(\psi\) depends upon the sign of the \(r \cdot V'\) value, where \(\psi < 0\) when \(r \cdot V' > 0\). The post-SN binary mass must also be updated: \(M'_{\text{NS}} = M_{\text{NS}} + M_2\) with \(M_{\text{NS}}\) being the primary star mass. Using this and the new semimajor axis, we may now calculate the final velocities of the two stars in the pre-SN CoF reference frame. Assuming a velocity at infinity, \(V_\infty\), which is directed along an asymptote of the hyperbolic orbit we have final velocities for the two stars of

\[
V_{1f} = V'_1 - \frac{M_2}{M_\text{NS}} V_\infty
\]

and

\[
V_{2f} = V'_1 + \frac{M_2}{M_\text{NS}} V_\infty
\]

A simple calculation gives the magnitude

\[
V_\infty = \sqrt{\frac{GM_{\text{NS}}}{a'}}
\]

The angle of the hyperbolic asymptote may be calculated from the angle \(\sigma\) (as shown from Fig. 2 which describes the post-SN coordinate system) where \(\cos \sigma = 1/e'\) (as in T&T98) and \(\sigma\) is always positive. This restricts \(\sigma\) to range from \(0 \rightarrow \pi/2\). With our two angles we define the difference angle \(\gamma = \sigma - \psi\) which is used in rotating the coordinate system around the \(z\)-axis. Separating \(V_{1f}\) and \(V_{2f}\) into component form gives us

\[
V_{1x} = \frac{M_{\text{NS}}}{M_\text{NS}} V_{\text{kick}} \cos \omega \cos \phi + \frac{\Delta M M_2}{M_\text{NS} M_\text{NS}} V_{\text{orb}} \sin \beta - V_\infty \cos \nu \sin \gamma,
\]

\[
V_{1y} = \frac{M_{\text{NS}}}{M_\text{NS}} V_{\text{kick}} \sin \omega \cos \phi + \frac{\Delta M M_2}{M_\text{NS} M_\text{NS}} V_{\text{orb}} \cos \beta - V_\infty \cos \gamma,
\]

where \(\Delta M = M_1 - M_2\).
P. D. Kiel and J. R. Hurley developed a relatively sophisticated model. The major difference between our model and the TT98 model is the coordinate system scheme, and the latter’s assumption that the companion star may have momentum imparted directly on to it from the SN blast wave (as in Dewey & Cordes 1987; Belczynski et al. 2008). The companion star may also have some fraction of mass stripped off it and/or ablated owing to the impact of the shell of material ejected from the primary. To include the possibility of investigating the effect of these additional considerations, we have worked through the TT98 demonstration and implemented it as an option in \texttt{bse}, generalized to eccentric orbits. However, we do not exercise this option in this work.

2.2.3 Disruption model illustration

To illustrate the effect the binary orbit has on the runaway velocities of disrupted stars, we produce a simple population of binary systems. For this population, the primary mass, $M_1$, is randomly selected from a flat distribution ranging from 10 to 20$M_{\odot}$, the secondary mass, $M_2$, from a flat distribution ranging from 0.1 to 20$M_{\odot}$, and the orbital separation is selected randomly from a flat distribution from 1 to 10,000$R_{\odot}$. All systems are initially circular to simplify the analysis. The radius of the secondary star is linked to the distance of closest approach: if $R_2 > a'$ ($a' = a$) we assume the stars coalesce (see also TT98; Belczynski et al. 2008) and the merger outcome has the final CofM velocity, $V'$, and the mass of the NS and companion is combined (see HTP02 for further details of merger outcomes).

Equations (15) to (20) are the velocities calculated within the \texttt{bse} kick routine and are communicated into \texttt{BINKIN}. Before adding these to the Galactic binary CofM velocity at the time of SN, it is first necessary to perform a random orientation of the pre-SN binary orbital plane, which until now has been fixed in the Galactic $xy$-plane. We randomly choose three Euler angles $\alpha_E$, $\beta_E$ and $\gamma_E$, within the ranges $0 \leq \alpha_E, \gamma_E \leq 2\pi$ and $0 \leq \beta_E < \pi$, to give a three dimensional (3D) rotation of the velocities in equations (12) and (13). The post-SN velocities (pre-SN CofM velocity plus the rotated disruption velocities) are then used within the kinematic routine to calculate the subsequent velocities and positions within the Galaxy of the pulsar and its former companion.

2.2.2 Related disruption models

There are two other groups who have independently developed models for deriving the run-away velocities of stars from a disrupted binary. The method published recently by Belczynski et al. (2008) is similar to our demonstration and is also generalized for arbitrary eccentricity. The main difference is that it allows a velocity $v_{\text{imp}}$ to be imparted to the secondary from the expanding shell of the SN. In our work, we essentially assume that $v_{\text{imp}} = 0$ which has minimal effect except in some cases of small pre-SN orbital separation (Kalogera 1996; Belczynski et al. 2008). TT98 have

\begin{align*}
V_{1x} &= \frac{M_{NS}}{M_1} V_{\text{kick}} \sin \phi, \\
V_{2x} &= \frac{M_{NS}}{M_2} V_{\text{kick}} \cos \omega \cos \phi + \frac{\Delta M M_2}{M_1 M_b} \omega \sin \beta \\
&\quad + V_{\infty} \cos \nu \sin \gamma, \\
V_{2y} &= \frac{M_{NS}}{M_2} V_{\text{kick}} \cos \omega \cos \phi + \frac{\Delta M M_2}{M_1 M_b} \omega \cos \beta \\
&\quad + V_{\infty} \cos \nu \gamma, \\
\text{and} \\
V_{2z} &= \frac{M_{NS}}{M_2} V_{\text{kick}} \sin \phi.
\end{align*}

We also account for coalescence of the two stars if the newly formed compact star velocity kick is directed towards the companion. Coalescence occurs if the companion radius, $R_2$, is greater than periastron or the distance of closest approach: if $R_2 > a'$ ($a' = a$) we assume the stars coalesce (see also TT98; Belczynski et al. 2008) and the merger outcome has the final CofM velocity, $V'$, and the mass of the NS and companion is combined (see HTP02 for further details of merger outcomes).

Equations (15) to (20) are the velocities calculated within the \texttt{bse} kick routine and are communicated into \texttt{BINKIN}. Before adding these to the Galactic binary CofM velocity at the time of SN, it is first necessary to perform a random orientation of the pre-SN binary orbital plane, which until now has been fixed in the Galactic $xy$-plane. We randomly choose three Euler angles $\alpha_E$, $\beta_E$ and $\gamma_E$, within the ranges $0 \leq \alpha_E, \gamma_E \leq 2\pi$ and $0 \leq \beta_E < \pi$, to give a three dimensional (3D) rotation of the velocities in equations (12) and (13). The post-SN velocities (pre-SN CofM velocity plus the rotated disruption velocities) are then used within the kinematic routine to calculate the subsequent velocities and positions within the Galaxy of the pulsar and its former companion.

![Figure 2. Orbital geometry of our disruption method after an asymmetric SNe.](https://example.com/figure2.png)

To illustrate the effect the binary orbit has on the runaway velocities of disrupted stars, we produce a simple population of binary systems. For this population, the primary mass, $M_1$, is randomly selected from a flat distribution ranging from 10 to 20$M_{\odot}$, the secondary mass, $M_2$, from a flat distribution ranging from 0.1 to 20$M_{\odot}$, and the orbital separation is selected randomly from a flat distribution from 1 to 10,000$R_{\odot}$. All systems are initially circular to simplify the analysis. The radius of the secondary star is linked to its mass by $R_2 = 1.3M_2^{\frac{1}{3}}$ if $M_2$ is greater than unity and by $R_2 = M_2$ otherwise. We make sure the system is not in contact at birth. We then let the primary undergo a SN that leaves a NS with $M_{NS} = 1.4M_{\odot}$ and assume the remnant is given a kick from equation (7) with $V_{\infty} = 190$ km s$^{-1}$ (in accordance with Hansen & Phinney 1997). The post-SN velocities for disrupted stars are calculated using the \texttt{bse} method detailed above.

After generating a million systems we find that the majority (99 per cent) become unbound and the incidence of coalescence is negligible. In Fig. 3, we see the distributions of NS and companion recoil speed.

![Figure 3. The speeds of the two stars following the SNe. The thick line represents the NS recoil speed distribution while the thin line is the companion recoil speed distribution. Included is the assumed asymmetric SN kick distribution assuming a dispersion of 190 km s$^{-1}$ (dashed line).](https://example.com/figure3.png)
star recoil speed and compare this to the \( V_{\text{kick}} \) distribution, i.e. the distribution for a population of standard single NSs. The first item we wish to note is the difference in the typical velocities received by both stars: the NS, which directly experiences the additional momentum imparted from the asymmetric SN, will most likely depart the binary system with a greater velocity than the companion star (relative to the CoM). The second point of interest is the similarity between the NS recoil speed distribution and the kick distribution. Clearly, not all of the momentum imparted on to the NS goes into the NS recoil velocity; some of the momentum is instead transported into the CoM momentum, consumed by the disruption of the binary system and converted into additional velocity of the companion star. Therefore, although the shape of the NS recoil speed distribution is consistent with the kick distribution, the NS distribution is shifted somewhat to lower values. In this regard, observational pulsar velocity studies that do not account for the possibility of a fraction of the sample being disrupted from binary systems may underestimate the underlying SN kick distribution. It also suggests a possible mechanism for any bimodality found in the velocity structure of pulsar observations – similar to that found by Arzoumanian et al. (2002) from which they concluded a bimodal asymmetric SN kick distribution, or possibly binary disruption effects, could cause such detected velocities. We note that the form of the underlying orbital period (or separation) distribution of the model binaries will affect the distribution of recoil speeds – with an increased proportion of short-period systems leading to an increased difference between the NS recoil speed and kick distributions – and we have not explored this aspect in detail here.

3 GALACTIC KINEMATICS

3.1 Galactic gravitational potentials

Much work over the years has led to estimates of the Galactic gravitational potential. Miyamoto & Nagai (1975) generalized the work of Toomre (1963) who calculated flattened Plummer (1911) models for the Galaxy. Since then, further observations have led to estimates by Carlberg & Innanen (1987) of disc–halo, bulge and nucleus potentials which in turn have been updated by Kuijken & Gilmore (1989; hereafter KG89). KG89 used more extensive observations of Galactic stellar densities, which allows the mapping to an assumed (to first order) smooth time-independent Galactic gravitational potential.

The KG89 model potential is

\[
\Phi_{\text{KG}} = \Phi_{\text{disc/halo}}^{\text{KG}} + \Phi_{\text{nuc}}^{\text{KG}} + \Phi_{\text{bulge}}^{\text{KG}}
\]

where

\[
\Phi_{\text{disc/halo}}^{\text{KG}} = -\frac{GM_{\text{disc}}}{\sqrt{\left(a + \sum_{i=1}^{3} \beta_i \sqrt{z^2 + h_i^2}\right)^2 + b^2 + r^2}}
\]

\[
\Phi_{\text{nuc}}^{\text{KG}} = -\frac{GM_{\text{nuc}}}{\sqrt{b^2 + r^2}}
\]

\[
\Phi_{\text{bulge}}^{\text{KG}} = -\frac{GM_{\text{bulge}}}{\sqrt{b^2 + r^2}}
\]

and the parameter values for each region are:

- disc/halo: \( \beta_1 = 0.4, \beta_2 = 0.5, \beta_3 = 0.1, h_1 = 0.325 \text{kpc}, h_2 = 0.090 \text{kpc}, h_3 = 0.125 \text{kpc}, a = 2.4 \text{kpc}, b = 5.5 \text{kpc}, M_{\text{disc}} = 1.45 \times 10^{11} \text{M}_\odot \)

- bulge: \( b = 1.5 \text{kpc}, M_{\text{bulge}} = 1.0 \times 10^{10} \text{M}_\odot \)

nucleus: \( b = 0.25 \text{kpc}, M_{\text{nuc}} = 9.3 \times 10^9 \text{M}_\odot \).

If necessary, the use of the KG89 model allows us to compare results to previous binary pulsar population synthesis works such as Lorimer et al. (1993). While now considered somewhat outdated, the KG89 potential is still in use within recent observational works, such as Freire, Ransom & Guptier (2007), who use it in their calculation of the pulsar spin period when accounting for observational effects of the acceleration between the Solar system Barycentre and NGC 1851 – the globular cluster in which their observed pulsar resides. However, a recent appraisal of the most promising Galactic gravitational potentials to use was completed by Sun & Han (2004). Their favoured method for ease of implementation is that given by Paczynski (1990, hereafter P90; see below). Sun & Han (2004) also commented favourably on the work of Dehnen & Binney (1998) who fit a multi-parameter mass model to kinematic data of the Milky Way. Sun & Han (2004) found that the Dehnen & Binney (1998) model is overly complicated to set up and manipulate, while the simpler P90 model is as accurate as the Dehnen & Binney (1998) model. As such, we also include the P90 model in our work.

The P90 model, like the KG89 model, follows the potential of Miyamoto & Nagai (1975) for the disc \( (i = 1) \) and spherical components \( (i = 2) \). Equation (9) of P90 is

\[
\Phi_i^P(R, z) = -\frac{GM_i}{\left(R^2 + [a_i + (z^2 + b_i^2)^{1/2}]^2\right)^{1/2}},
\]

where \( R = \sqrt{x^2 + y^2}. \) The P90 model, however, differs from the KG89 model not only with the assumed constant values used within equation (25) (for the disc potential) but also with the assumed form of the halo potential,

\[
\Phi_h^P(r) = -\frac{GM_h}{r_h} \left[\frac{1}{2} \ln \left(1 + \frac{r^2}{r_h^2}\right) + \frac{r}{r_h} \tan \left(\frac{r}{r_h}\right)\right],
\]

where \( r^2 = R^2 + z^2 \) is used to simplify the equation. The parameters being

- disc \( (i = 1) \): \( a_1 = 0 \text{kpc}, b_1 = 0.277 \text{kpc}, M_1 = 1.12 \times 10^{10} \text{M}_\odot \)
- spheroid \( (i = 2) \): \( a_2 = 3.7 \text{kpc}, b_2 = 0.20 \text{kpc}, M_2 = 8.07 \times 10^{10} \text{M}_\odot \)
- halo \( (i = h) \): \( r_h = 6.0 \text{kpc}, M_h = 5.0 \times 10^{10} \text{M}_\odot \).

The KG89 and P90 models are both based on old observations of the stellar neighbourhood. As such, these models are only considered accurate out to a radius of \( \sim 12 \text{kpc}. \) More recent observations completed in the Sloan Digital Sky Survey (SDSS; York et al. 2000), within the Sloan Extension for Galactic Understanding and Exploration (SEGUE; Lee et al. 2008) program, have allowed the Galactic gravitational potential to be measured out to a radii of \( \sim 60 \text{kpc} \) (Xue et al. 2008, hereafter X08). To do this, X08 have made line-of-sight velocity measurements of \( \sim 2500 \) blue horizontal branch stars which are converted into circular velocity estimates of the Milky Way. Ultimately, the work of X08 is completed to probe the halo of our Galaxy, and thus they do not examine in any detail the inner Galactic potential. However, at this stage we consider their complete assumed Galactic gravitational potential as an option in our work. The X08 model makes use of the following exponential disc, Hernquist (1990) bulge and Navarro, Frenk & White (1996; NFW) halo potentials, respectively:

\[
\Phi_{\text{disc}}^{\text{X}}(r) = -\frac{GM_{\text{disc}}}{r} \left[1 - \exp^{-r/h}\right],
\]
Figure 4. The three Galactic gravitational models are depicted in two different manners here to illustrate their properties and differences. The top panel (a) shows the circular velocity for a range of Galactocentric radial positions in the plane of the Galactic potential. The bottom panel (b) depicts modulus acceleration in increasing height from the plane.

\[
\Phi^X_{\text{bulge}}(r) = - \frac{GM_{\text{bulge}}}{r + C_0} \tag{28}
\]

and

\[
\Phi^X_{\text{NFW}}(r) = \frac{GM_{\text{vir}}}{r_C} \ln \left( 1 + \frac{r}{r_C} \right). \tag{29}
\]

Here \(C_0 = (1 + C_x) - C_x/(1 + C_x)\), while the values used within X08 vary depending upon the assumed halo description. We also note here that the form of \(\Phi^X_{\text{NFW}}(r)\) is exactly that of Smith et al. (2007), who provide differing values for the virial mass \(M_{\text{vir}}\), radius \(r_{\text{vir}}\) and concentration \(C_x\). X08 match their observed circular Galactic stellar velocity estimates to smooth particle hydrodynamical simulations from which they provide values for \(M_{\text{vir}}, r_{\text{vir}}\) and \(C_x\). The values from X08 used in our work are

- disc: \(M^X_{\text{disc}} = 5 \times 10^{10} M_\odot\), \(b = 4\) kpc,
- bulge: \(M^X_{\text{bulge}} = 1.5 \times 10^{10} M_\odot\), \(C_0 = 0.6\) kpc,
- NFW: \(M^X_{\text{vir}} = 1.03 \times 10^{12} M_\odot\), \(r_{\text{vir}} = 278\) kpc, \(C_x = 11.8\).

These Galactic gravitational potential models are now a part of BINKIN, updating the original algorithm based on Lorimer, Bailes & Harrison (1997), and extending upon similar population synthesis models such as that of Faucher-Giguere & Kaspi (2006).

Fig. 4 depicts the three assumed Galactic gravitational models. In particular, we wish to point out the inner region of the X08 model, which contains a smaller restoring force than the other two models. We return to this later in Section 4. The KG89 gravitational potential model decays faster than the other two models beyond the central Galactic region and the P90 model rotational curve follows the KG89 model in the inner region of the Galaxy while beyond a Galactic radius of \(\sim 20\) kpc the rotational curve flattens off similarly to the X08 model.

### 3.2 Initial conditions and integration method

Once we have our assumed Galactic potential, which is in cylindrical coordinates, \(\Phi(r, \phi, z)\), the progenitor pulsar systems must be given some initial Galactic position, \(R_{\text{init}}\), selected randomly from a given distribution. The most straightforward distribution is to assume a thin disc with some maximum height and radius. This simple method allows the system to relax over time into a similar distribution to the observed stellar number density distribution in height with respect to the plane (see distributions given in Sun & Han 2004). However, as shown in Section 4.1, this overestimates the number of systems (in this case pulsars) found in the central region – the deficiency of observed pulsars within the central region is believed to be a real lack of pulsars and not caused solely by observation selection effects (Lorimer et al. 2006). However, Lorimer et al. (2006) caution readers that more observations are required to provide a definitive result.

To combat this over density within the central regions, a preferred option is to use the distribution of SN remnants developed in P90:

\[
P(R_{\text{init}}) \, dR = a_R \left( \frac{R_{\text{init}}}{R_{\text{exp}}} \right) \exp \left( - \frac{R_{\text{init}}}{R_{\text{exp}}} \right) \, dR. \tag{30}
\]

where \(R_{\text{exp}} = 4.5\) is simply an exponential scalelength of the radial distribution and \(a_R\) is a constant of integration equal to 1.0683 over \(R = 0 \rightarrow 20\) kpc. A third option is to use the distribution derived by Yusifov & Kucuk (2004; their equation 17) from observations of OB type Population I stars:

\[
P(R_{\text{init}}) \, dR = a_R \left( \frac{R_{\text{init}}}{R_\odot} \right)^4 \exp \left( -b \frac{R_{\text{init}}}{R_\odot} \right) \, dR. \tag{31}
\]

If OB stars are assumed to be the progenitors of NSs, then this can be taken as the Galactic pulsar progenitor birth radial distribution. Here, \(a_R \sim b^4/24 \sim 606\) (for \(0 \geq R_{\text{init}} \geq 20\) kpc). We utilize all three radial distributions – thin disc, P90 and Yusifov & Kucuk (2004) – in our models to generate birth locations. However, because the P90 distribution has had much use in the past we assume this to be our standard description.

In terms of the initial distribution of systems in height from the plane, we simply assume a uniform distribution with maximum height of \(|z_{\text{max}}|\). Systems over time relax outwards in \(|z|\) and such a simple initial \(|z|\) distribution compares well to those favoured in Sun & Han (2004). Furthermore, according to P90 as long as the systems are born relatively close to the Galactic plane (a few hundred parsec), the initial distribution in \(|z|\) for energetic populations is redundant. In the future, the initial spatial distribution will contain spiral arms, similar to that completed within Faucher-Giguere & Kaspi (2006) who suggest that because Galactic arm structure is visible in large observational surveys it is necessary for any realistic pulsar population synthesis simulation to model this structure.

Once a position is found for a system, an initial velocity is simply calculated from the (estimated) Galactic gravitational potential at that point. With knowledge of the position and space velocity, the next step is to solve four coupled equations of motion to evolve the system position forward in time. These are (P90)

\[
\begin{align*}
\frac{dR}{dt} &= V_R, \\
\frac{dz}{dt} &= V_z, \\
\frac{dV_R}{dt} &= - \frac{\partial \Phi}{\partial R} + \frac{L^2}{R^3}, \\
\frac{dV_z}{dt} &= - \frac{\partial \Phi}{\partial z}.
\end{align*}
\tag{32}
\]

These four equations are found by calculating the acceleration in \(R, \phi, z\) and \(V_R\) and \(V_z\) are velocities in \(R\) and \(z\), respectively) induced on to a test particle by the gravitational potential. Assuming an axisymmetric potential around \(z\) produces constant angular momentum, \(L\), felt by a test particle. To integrate forward in time...
a fourth-order Runge–Kutta integration routine is used, similar to that used by P90 and Lorimer et al. (1993).

It is now possible for us to evolve the complete Galactic orbital evolution of a system of interest. If a SN event occurs and the system is not disrupted, the velocity injected into the system by a SN, calculated within BSE, is simply vectorially added to the known Galactic velocity of the system CoM at the time of the SN. If the system is disrupted, the run-away velocities of the two stars – as calculated in Section 2.2 – are, again, vectorially added to their previous Galactic velocity (that of their system of origin). In this way, we are able to follow the complete Galactic orbital history of a system (star or binary), even if it passes through two SNe and with (or without) binary disruption. Because we assume no interaction between orbiting systems, we are able to evolve each system separately, one after another (or in parallel). A beneficial consequence resulting from this assumption is that BINKIN is faster to run than other dynamical codes, such as typical N-body codes (McGlynn 1984). Of course, if one is dealing with compact stellar clusters, dynamical interactions between the stellar components are very important to follow.

4 PULSAR POPULATION STATISTICS

We now describe the results of a series of population synthesis calculations that utilize our BINPOP and BINKIN modules to follow the stellar/binary and kinematic evolution of a population of binary stars to produce artificial Galactic pulsar populations. The primary aim of this section is to predict scaleheights and other kinematic characteristics for populations of pulsars, first assuming that all pulsars can be detected. We also compare the kinematics of our model pulsar populations with available kinematic tracers found from pulsar survey observations (such as those produced in Yusifov & Kucuk 2004; Hobbs et al. 2005; Lorimer et al. 2006). For now, we hold back from making direct statistically significant comparisons as this requires modelling of selection effects which will be covered in detail in a companion paper (the BINSEX component as mentioned in Section 1). Here, we focus more on showing how modifying certain parameters affects the final pulsar population kinematics, in terms of scaleheights and space velocity distributions.

The first step is to evolve a population of binaries within BINPOP. For this, we proceed using our favoured model from Paper I (Model Fd). This sets choices for BINPOP stellar and binary evolutionary parameters of: solar metallicity $Z = 0.02$; a maximum possible NS mass of $3M_{\odot}$ and $a_{CE} = 3$. It also sets the following parameters governing pulsar evolution: $\tau_B = 2000$ Myr; $k = 3000$; no propeller evolution; the initial pulsar period and magnetic field (McGlynn 1984). Of course, if one is dealing with compact stellar clusters, dynamical interactions between the stellar components are very important to follow.

The next step is to take each of the BINPOP binaries and follow their corresponding kinematic evolution in BINKIN. For each binary, a random birth age is assigned and the evolution followed from this age up to the age of the Galaxy. For this, we begin by defining a standard model which we will call Model A. This uses the P90 distribution for setting the initial Galactic radial positions of the binaries (see equation 30) with a maximum initial height off the plane of $|z_{max}| = 75$ pc. It also assumes the P90 form of the Galactic potential (see equations 25 and 26) and sets $V_{\|} = 190$ km s$^{-1}$ (used within BINPOP) as the dispersion of the SN velocity kick distribution. Further models arise due to variations of these choices and are listed in Table 1.

For each model, we examine the scaleheights for a range of pulsar systems. These are given in Table 2. We take the scaleheight to be that distance in $|z|$ for which the number of stars within that distance is 63 per cent (approximately twice the $e$-folding distance) of the entire population. The most prolifically observed pulsar system is what is known as a standard pulsar. Here, we define a standard pulsar as one which satisfies

$$\log B \geq -2.5 \times \log P + 8.1. \tag{33}$$

This equation artificially divides the ‘standard’ pulsar ‘island’ from all other radio pulsars in the $B - P$ diagram (see Paper I). We define an MSP to be a pulsar spinning more rapidly than $P = 0.02$ s. All other pulsars bridge these two pulsar types – islands within the $B - P$ plane (see also the description given in Paper I). We also distinguish between binary and isolated pulsars. It is possible to compare our model results in a limited manner to observations. To do this, we make use of the ATNF Pulsar Catalogue which provides us with $\sim 1610$ Galactic plane pulsars (we ignore pulsars from the catalogue that have any association with another object, for example with a globular cluster or external galaxy). Approximately 1550 of these are isolated. Only 15 standard pulsars are found in binary systems within the Galactic disc. Of the total observed pulsars, there are 65 MSPs of which 19 are isolated. We show the scaleheights of the catalogue pulsars within Table 3.

The radial distributions of Galactic pulsars resulting from our set of models are shown in Fig. 5 (left-hand panels). We also compare a subset of the models in more detail in Fig. 6 and include a comparison to the initial distributions used in the models and also the pulse distribution suggested by Yusifov & Kucuk (2004) which is based

| Model | $R_{bin}$ distribution | $\Phi$ | $|z_{max}|$ (kpc) | $V_{\|}$ (km s$^{-1}$) |
|-------|-----------------------|-------|------------------|------------------|
| A     | Paczynski P90         | 10    | 190              |
| B     | Flat P90              | 10    | 190              |
| C     | Yusifov & Kucuk P90   | 10    | 190              |
| C     | Yusifov & Kucuk P90   | 2     | 190              |
| C     | Yusifov & Kucuk P90   | 20    | 190              |
| D     | Paczynski KG89        | 10    | 190              |
| E     | Paczynski X08         | 10    | 190              |
| F     | Paczynski P90         | 10    | 550              |
| G     | Paczynski P90         | 10    | 265              |
Table 2. Model scaleheights (in kpc) for a range of pulsar types in the Galaxy.

| Type       | Model | A    | B    | C    | D    | E    | F    | C2   | C3   |
|------------|-------|------|------|------|------|------|------|------|------|
| Both       | All   | 1.39 | 1.49 | 1.25 | 1.33 | 2.58 | 1.93 | 0.58 | 1.40 |
| Isolated   | All   | 1.43 | 1.53 | 1.30 | 1.37 | 2.64 | 1.93 | 0.59 | 1.46 |
| Binary     | All   | 0.96 | 1.10 | 0.79 | 0.91 | 1.98 | 2.00 | 0.46 | 0.83 |
| Both       | Isolated | 1.43 | 1.53 | 1.30 | 1.36 | 2.63 | 1.93 | 0.59 | 1.45 |
| Isolated   | Isolated | 1.43 | 1.53 | 1.30 | 1.37 | 2.64 | 1.93 | 0.59 | 1.46 |
| Binary     | Isolated | 0.73 | 0.85 | 0.59 | 0.71 | 1.68 | 1.58 | 0.38 | 0.61 |
| Both       | All MSPs | 1.00 | 1.15 | 0.82 | 0.95 | 2.04 | 2.02 | 0.47 | 0.88 |
| Isolated   | All MSPs | 1.76 | 1.67 | 1.76 | 1.49 | 2.90 | 2.33 | 0.60 | 2.21 |
| Binary     | All MSPs | 0.99 | 1.14 | 0.82 | 0.94 | 2.03 | 2.02 | 0.47 | 0.87 |

Table 3. Observed scaleheights (in kpc) for different types of pulsars. We have not taken account of selection effects when calculating these values. Standard pulsars are those pulsars which satisfy equation (33), while MSPs are those pulsars which have spin periods \( P \leq 0.02 \) s. We note that there is uncertainty in some of these numbers owing to small number statistics and the clumpy distribution of the pulsars. For example, there are only 19 isolated MSPs and the difference in height between the 12th and 13th most distant (in terms of height from the plane) is 0.07 kpc while the average distance between the first 12 is 0.02 kpc.

| Type       | All    | Standard | MSP    |
|------------|--------|----------|--------|
| Both       | 0.40   | 0.39     | 0.41   |
| Isolated   | 0.41   | 0.39     | 0.26   |
| Binary     | 0.38   | 0.17     | 0.48   |

on observations. We also explore the pulsar population 3D space velocity distributions. These are shown in the right-hand panels of Fig. 5 for the models in Table 1.

Our results are analysed in more detail in four parts. In Section 4.1, we examine the effect of varying the assumed pulsar birth radial distribution. This analysis makes use of Models A, B and C. Within Section 4.1, we also consider how modifying the target area considered (the ‘observable’ Galactic area) in our scaleheight calculations affects the scaleheight values of pulsars produced in Model C. This is completed with the use of Models C2 and C3. Section 4.2 analyses different forms of the Galactic gravitational potential by comparing Models A, D and E pulsar scaleheights, final radial distributions and final velocity distributions. We then consider the effect of varying the assumed SN velocity kick distribution with Models A, F and G in Section 4.3. Finally, after examining differences in bulk pulsar properties, we explore in detail the MSP population of Model C in Section 4.4. We make use of our MSP analysis to further investigate the effect of model assumptions such as the initial scaleheight, Galactic age and the number of systems evolved.

4.1 Initial distributions and target area

To begin with, we focus on Model A. We note that unless otherwise specified, when calculating scaleheights we consider only pulsars located within the Galactic region defined by \( |z| \leq |z_{\text{max}}| \) kpc and \( r = \sqrt{x^2 + y^2} \leq 30 \) kpc. First comparing the scaleheights of the isolated and binary pulsars we see that as expected (see Section 1) it is the isolated pulsars which have the greatest scaleheight when considering all pulsars. It is also no surprise that the scaleheight of the complete population (top row of Table 2) is closely aligned with the isolated pulsar scaleheight. This is because 91 per cent of pulsars in Model A are isolated at the end of the simulation (even though all stars are initially in binaries). When considering only standard pulsars, the domination of the isolated component is even greater: 99 per cent of standard pulsars are isolated. However, the tables are turned when we look at the MSP population: the binary MSP population makes up 99 per cent of all MSPs within Model A (a greater percentage than what is observed; however, see Section 4.4.1 for further discussion on this point). These relative numbers explain why in Table 2 the scaleheight of standard pulsars is insensitive to standard binary pulsars, and the same can be said for the total MSP scaleheight compared to isolated MSPs.

For the binary pulsars, we find that the standard pulsar population has a lower scaleheight than for the MSP population. This suggests that on average binary MSPs receive greater post-SN recoil velocities than their standard binary pulsar counterparts. The reasons for this were alluded to in Section 1 but we reiterate them here (see also the findings of Stollman & van den Heuvel 1986; Bailes 1989). Basically, the very existence of an MSP relies on the occurrence of mass transfer on to the NS which in turn requires a close binary. Such a binary will have a greater binding energy than the equivalent standard (wider) binary pulsar and can therefore survive a greater SN velocity kick as there is more energy to overcome for disruption. As a result, it is possible for the proto-binary MSP system to be given a faster recoil velocity (with respect to the systems initial CoM). The isolated MSP population scaleheight also increases compared to the entire isolated pulsar population. Again, this is not surprising given the model isolated MSP formation scenario as addressed in Paper I and within Section 1. Owing to the SN that produced the NS progenitor of the MSP, and the SN of the companion star that disrupted the system, isolated MSPs can receive greater recoil velocities than any other pulsar population – hence the largest population scaleheight (this formation mechanism is also discussed further in Section 4.4).

To measure what effect the initial Galactic birth distribution has on the final model pulsar population, we compare Models A, B and C. These models assume the birth distribution of P90 (from the observed distribution of SN remnants), a uniform thin disc and the clumpy distribution of the pulsars. We have not taken account of selection effects when calculating these values. Standard pulsars are those pulsars which satisfy equation (33), while MSPs are those pulsars which have spin periods \( P \leq 0.02 \) s. We note that there is uncertainty in some of these numbers owing to small number statistics and the clumpy distribution of the pulsars. For example, there are only 19 isolated MSPs and the difference in height between the 12th and 13th most distant (in terms of height from the plane) is 0.07 kpc while the average distance between the first 12 is 0.02 kpc.
shallower and the distribution is more extended than Model C. We note that the number of pulsar systems ‘observed’ in Model C is higher than in Model A (by a factor of \(\sim 1.1\)). For Model B, we find that the pulsar radial distribution peaks at the Galactic centre and then decays with increasing radius. The space velocity distributions for the pulsars in the three models are compared in the upper-right panel of Fig. 5, and we see that there is no discernible difference.

The relation of the final pulsar radial distribution to the assumed birth distribution of binaries can be seen in Fig. 6 for Models A and C. Here, all distributions are normalized to unity to aid comparison of the peak position and distribution shapes. We see that the birth distribution of Model A is broader than for Model C, and this is reflected in their final shapes. However, the width of the distribution increases with time in both cases, while the peak of the distribution moves in towards the Galactic centre, which is a typical effect of Galactic potentials (Sun & Han 2004). Initially, Model A peaks at a radius of 4.5 kpc which moves inwards to a radius of 3.9 kpc at 10 Gyr. For Model C, the peak moves from 5.0 to 4.5 kpc.

In Fig. 6, we also compare the model distributions to the distribution of an observed sample of pulsars presented by Yusifov & Kucuk (2004). We note that at this stage we are not including selection effects in our models so a direct comparison with observations is not possible. However, comparison with the Yusifov & Kucuk (2004) sample, which includes selection effects somewhat by being limited to pulsars with \(P > 10^{-17}\) s\(^{-1}\), can still provide a meaningful guide to discerning between our models. Although not included in Fig. 6, we can see immediately that Model B is an unrealistic model of the Galactic pulsar population. The apparent deficit of observed pulsars in the inner region of the Galaxy cannot be reproduced by assuming all binaries are born in a uniform thin disc – we require a paucity of pulsars to be born in the central region of the Galaxy when attempting to match observations (similar to P90; Sun & Han 2004). This lack of observed inner Galactic pulsars may be due to the high electron density in this region of the Galaxy and therefore larger scattering of the pulse signal, however, the latest observations do suggest an intrinsic scarcity of central pulsars (Lorimer et al. 2006). Model A provides a good comparison to the observed radial pulsar distribution for the inner regions of the Galaxy but has too many pulsars and is too extended beyond \(\sim 4\) kpc. In terms of shape, Model C best represents the observations. However, Model C peaks further from the Galactic centre by 1–1.5 kpc. This suggests that the ideal initial distribution would of the form derived by Yusifov & Kucuk (2004) from observations of OB stars but scaled so that the distribution peaked at a radius of \(\sim 4\) kpc.

The scaleheights for Models A, B and C can be compared in Table 2. We see that Model B has systematically the largest scaleheights, while Model C has the smallest. However, the trends observed for Model A – the relative scaleheights of the various pulsar populations – are consistent across the models.

We next demonstrate what effect modifying the Galactic region of interest has on Model C by considering pulsars only out to \(|z| = 2\) kpc in Model C2 and out to \(|z| = 20\) kpc for Model C3 (as opposed to \(|z| = 10\) kpc for Model C). We find that reducing the height of our ‘Galaxy’ by a factor of 5 approximately halves the calculated scaleheights for all pulsar populations. On the other hand, doubling the height considered does not significantly affect the calculated scaleheights (except perhaps the isolated MSP population which suffers from small number statistics). However, it does not appear to greatly affect the relative scaleheights of pulsar sub-populations and certainly does not switch any trends noted in the models. Beyond this limit, only the results of highly energetic systems may still vary.
Therefore, factors which limit the region of the Galaxy observed (or considered), such as the numerous selection effects which occur in radio pulsar observations, can modify the underlying pulsar population scaleheights within $|z| < 10$ kpc of the Galactic plane (as discussed by many works including Taylor & Manchester 1977 and Narayan & Ostriker 1990). We note that in terms of Galactic pulsar observations there is the limit of $\sim 1.75$ kpc beyond which dispersion measure distance estimates of pulsars break down (see Manchester et al. 2005).

We now have Model C, a suitable model for which we may compare pulsar scaleheights to observations (the latter values are given in Table 3). Model C2 is roughly consistent with the observed scaleheights, although on average the model values are greater than the observed values. The trends when considering all pulsars are similar but this breaks down for the MSP population where the model predicts a greater scaleheight for isolated MSPs than for binary MSPs which is opposite to the observed MSP scaleheights (although see Section 4.4.1). This is, however, consistent with our previous simple analysis in Section 1 from the binary disruption formation mechanism of isolated MSPs. Another difference between Model C2 and observations is the relative number of isolated to binary MSP systems. In Model C2 $\sim 99$ per cent of MSPs are found within a binary system while a direct observational comparison shows $\sim 70$ per cent of Galactic disc MSPs in binary systems. These two differences between our model and observations indicate that our mechanism for producing isolated MSPs – binary disruption in a SN event – cannot be the sole (or even dominant) production mechanism. We explore this line of thought further in Section 4.4.

4.2 Model Galactic gravitational potentials

We now explore what effect modifying the assumed Galactic gravitational potential has on the pulsar scaleheights and final distributions (radial and space velocity). Model D assumes a potential of the form described by the KG89 model (equation 21). The scaleheights of Model D are all slightly less than their Model A counterparts which used the P90 model. This suggests that over time stellar systems may diffuse less efficiently in Model D than in Model A. Fig. 4 gives some indication of the cause of this difference. The lower panel provides the model Galactic gravitational force towards the plane with respect to the height above the plane. We see that above a height of $|z| \sim 1$ kpc, the KG89 model has a slightly greater attractive force than the P90 model. However, for the inner regions the situation is reversed and indeed if we calculate the scaleheights for pulsars with $|z| < 1$ kpc, we find that the scaleheight behaviour for Model D relative to Model A is also reversed. As shown in Figs 5 and 6, the radial distribution does not differ greatly between Models A and D: small differences to note are that the distribution of Model D decays more rapidly than Model A while Model D contains less pulsar systems within 10 kpc of the Galactic plane. The velocity curve of Model D does not change greatly from Model A as there is only a small increase in systems with space velocities at the lower end of the distribution, perhaps reflecting the rapid decay in circular velocity of Model D with respect to Model A (see Fig. 4).

Model E assumes the Galactic gravitational potential of X08 (equations 27 to 29). This model shows a large increase in scaleheight values compared to both Models A and D. The reason for this can once again be seen from Fig. 4 which shows that the X08 Galactic gravitational model cannot retard the movement of systems out of the Galactic plane as effectively as the P90 (Model A) or KG89 (Model D) models. Fig. 5 shows that the number of systems retained by Model E within $|z| \leq 10$ kpc is less than within both Models A and D. The 3D space velocity curves of Models A, D and E all peak at roughly the same value but the Model E distribution possesses a much steeper slope on either side of its peak value. The flatness of the rotation curve assumed in Model E, depicted in Fig. 4, causes such a narrow distribution in velocity space. Model E highlights the importance of the assumed Galactic gravitational potential in modelling Galactic population kinematics (as also discussed by KG89; Dehnen & Binney 1998; Sun & Han 2004).

4.3 Kicks

Considering the uncertainty involved in the true form of the distribution of kick speeds given to NSs at birth (as mentioned in Section 2.2.1), we next investigate how changing the dispersion of the assumed Maxwellian distribution affects our results. Recalling that Model A used $V_{\alpha} = 190$ km s$^{-1}$, we first compare with Model F which uses $V_{\alpha} = 550$ km s$^{-1}$ as an extreme illustration. We see from Table 2 that the scaleheights in Model F are much greater than in Model A, with increases by as much as a factor of 2. While it is expected that pulsars can move further from the galactic plane in Model F, it also means that more objects escape from the Galaxy: the number of pulsars in Model F within 10 kpc of the plane decreases by a factor of 10 compared to Model A. This decrease in numbers can skew the expected outcomes and is evident when looking at the radial distributions in Fig. 5. For example, less binary systems are kicked out of the $|z| \leq 10$ kpc Galactic region than isolated pulsars – owing to binaries being heavier on average and the binary orbit absorbing a fraction of the energy injected by the kick – so we find in Model F that binary pulsars have a greater scaleheight than isolated pulsars (the opposite to Model A). This is not the case for MSPs although the difference between the isolated and binary MSP scaleheights has decreased compared to Model A. We note that the ratio of standard isolated pulsars to standard binary pulsars is an order of magnitude greater for Model F than for Model A owing to a greater incidence of binary disruptions in Model F. Therefore, as established in previous works (e.g. Taylor & Manchester 1977; Hills 1983), the assumed SN kick velocity is an extremely important factor, especially when comparing model pulsar kinematics to observations.

When examining the resultant radial distribution in Fig. 5, we find that Model F is much broader than Model A – an intuitive result owing to the increased distance that the pulsars move – and certainly Model F is not a good representation of the observed distribution (not that it was expected to be). Comparing the space velocity distributions of Models A and F in Fig. 5, it is surprising to see the relatively high number of pulsars in Model F that are travelling at the distribution peak speed ($\sim 250$ km s$^{-1}$). This population with similar velocities includes isolated and binary pulsars. We remind the reader that when discussing the velocity distribution here we mean the actual velocity each system has with respect to the Galactic centre – we do not account for the local standard of rest (LSR; solar motion).

In Fig. 5, we also compare the 3D space velocity distributions of Models A and F with the 3D space velocity distribution derived by Hobbs et al. (2005) from pulsar observations. An important distinction to make is that the Hobbs et al. (2005) sample was restricted to pulsars with characteristic ages $< 3$ Myr. Thus, it is intended to be a distribution of pulsar birth velocities and was used by Hobbs et al. (2005) to suggest that $V_{\alpha} = 265$ km s$^{-1}$ in the SN velocity kick distribution. By comparing this with our models, we can gauge the effect that the Galactic potential and binary have
on the form of the pulsar velocity distribution as the population evolves.

Comparing the Model A pulsar velocity distribution (at a population age of 10 Gyr) with the Hobbs et al. (2005) birth velocity distribution shows changes in the peak (shifted to lower velocity in the model) and shape. The shift of the peak can be mostly attributed to the difference in the average age of the two pulsar populations and the low dispersion value assumed in the Model A velocity kick distribution. The age difference allows fast moving systems in Model A to have time to leave the Galaxy and thus be culled from the final velocity distribution. Also, over time, the pulsar velocities are retarded by the Galactic potential which shifts the final pulsar velocity distribution to lower velocities. In terms of shape, we find that Model A is a more focused distribution – the model peak is more acute and the distribution as a whole is narrower. This difference is likely due, in part, to the binding energy of the host binaries impinging on the SN velocity kick (as discussed in Hills 1983; Bailes 1989) and causing a greater number of systems to have similar final space velocities than otherwise. Disruptions triggered primarily by mass-loss will act to increase the proportion of low-velocity pulsars while the absorption of large kick velocities by the binary binding energy may also skew the distribution to smaller final pulsar velocities. This narrowing of the model pulsar velocity distribution compared to the observed distribution has been found in other works, most notably that of Dewey & Cordes (1987). They attributed the difference to errors in pulsar distance measurements, which will broaden the distribution, and that non-Maxwellian processes may be more important in producing pulsar velocities than their models assume (i.e. nascent NS receiving a kick selected from a Maxwellian distribution with \( V = 90 \text{ km s}^{-1} \)). It appears that the difference between the velocity distribution of models and observations results from a combination of the selected pulsar population used to derive the observed pulsar velocity distribution (see Hobbs et al. 2005), errors in pulsar velocity and distance measurements, and the binding energy of host binary systems affecting the resultant pulsar run-away velocity.

To remove the binary orbit effect and highlight the effect of age evolution on the pulsar velocity distribution, we have created Model G which evolves a population of single stars according to the Galactic setup described for Model A but with \( V' = 265 \text{ km s}^{-1} \). With every system evolved within Model G being isolated from birth, we are now able to compare the resultant velocity distribution of a population of pulsars which receive uninhibited SN kick velocities drawn directly from the suggested Hobbs et al. (2005) SN kick distribution. We now see in Fig. 5 that the distribution closely resembles the Hobbs et al. (2005) distribution in shape but is shifted to lower velocities. The final pulsar distribution is best fit by a Maxwellian distribution with \( V = 140 \text{ km s}^{-1} \).

4.4 Millisecond pulsars

In Paper I, we were primarily interested in the production of MSPs within the \( P-P \) diagram. We now continue our exploration of the MSP population by examining in more detail the Galactic MSP distributions and in particular focusing on the behaviour of isolated MSPs and those with MS star, WD, NS or BH companions. In doing this, we focus solely on Model C. To begin, we extend our evaluation of scaleheights in Table 2 with those of the MSP populations (given in Table 4 and discussed in Section 4.4.1). The scaleheights in Table 4 are supplemented by Fig. 7 which provides the scaleheight for each MSP population as a function of Galactocentric radius. Also shown is the Galactic \( x \) and \( z \) parameter space of MSPs: for all MSPs (see Fig. 8) and those that reside above a magnetic field cut-off (see Fig. 9). The population of MSP–BH binaries is then discussed in detail within Section 4.4.2. We then look at the MSP population recoil velocities and space velocities in Section 4.4.3 and make some limited comparisons to previous work and observations. To further our investigation into how different model assumptions affect our pulsar population kinematics, we modify Model C, our favoured model thus far, to account for a greater birth \( |\varepsilon|_{\text{max}}\) range (Model C in Section 4.4.4); a greater age of the Galaxy (Model C’ in Section 4.4.5) and a higher resolution sample (in Section 4.4.6).

4.4.1 Model C MSP scaleheights and Galactic spatial properties

Looking at the scaleheight values in Table 4 for Model C, we see that as expected the binary MSP population with the greatest scaleheight is the MSP–NS systems, in which two SNk kicks occur. These doublecompact systems, however, are much rarer than the MSP–MS or MSP–WD systems, and therefore the results are less statistically significant. The relative number of MSP–NSs compared to...
MSP–MS systems (the second most numerous MSP population), the relative number is 0.003. For MSP–MS systems (the second most numerous MSP population), the relative number is 0.044 per MSP–WD system. We find similar scaleheights for the MSP–MS and MSP–WD systems, although the former are systematically smaller owing to the population being younger on average. Recently, there have been suggestions that asymmetric mass-loss during the asymptotic giant branch phase (Spruit 1998) gives rise to WD recoil velocities of the order of a few km s$^{-1}$ (Fellhauer et al. 2003). Such kick velocities have been raised for possible explanations of the apparent deficit of WDs in open clusters (Fellhauer et al. 2003) and the radial distributions of WDs in globular clusters (Heyl 2007; Davis et al. 2008). Currently, we do not include this possibility in our models but note that it would presumably lead to a modest increase in the MSP–WD population scaleheight.

MSP–BH systems are found to have a small Galactic scaleheight. This arises due to the orbital parameters required in order to form these systems which we examine in further detail below (see Section 4.4.2). Also, we remind the reader that we currently assume BHs do not receive kicks during their formation. As shown in Tables 2 and 4, the isolated MSP population has a scaleheight of 1.76 kpc. These MSPs emerge from disrupted binary systems, and although the kick at the time of disruption may be large it is not the MSP which is exploding at that point. Therefore, the MSP is considered by our kick routine to be the secondary star which, as shown in Section 2.2.3, receives (on average) only a small increase in momentum. This results in the lower scaleheight of isolated MSPs compared to MSP–NS binary systems (albeit only slightly less than the MSP–NS value). Furthermore, we note that for the binary system to survive the first SNe, allowing mass transfer on to the progenitor MSP, the resultant velocity kick at this point must be relatively small (we find $V_{\text{kick}}$ of approximately 80 km s$^{-1}$ or less). This is in accordance with many other population synthesis works, including Stollman & van den Heuvel (1986), Iben, Tutukov & Yungelson (1995) and Ramachandran & Bhattacharya (1997).

Previous results shown in Section 4.1 placed doubt on isolated MSPs formed via the disruption of binary systems being the sole ‘type’ of isolated MSPs – there must be another formation mechanism. One such mechanism that exists in the literature is the ablation model (Eichler & Levinson 1988; Ruderman et al. 1989) based on observations such as those of van Paradijs et al. (1988). Here, the assumption is that the MSP is produced as a result of mass transfer from a MS companion in what would be a low-mass X-ray binary. Then at some point the mass of the MS star becomes low enough that it is destroyed, or ablated, by the highly energetic radiation flowing from the rapidly spinning pulsar (van Paradijs et al. 1988; Tavani 1992). We calculate that the time-scale for the destruction of the MS companion star in this manner should take of the order of $\sim$5 Myr once the companion is below a mass of $\sim$0.02 $M_\odot$ (see Appendix A). Thus, we propose a simple model to belatedly estimate the impact of ablation on our results where we assume that any MSP with a MS companion of mass less than 0.008 $M_\odot$ (to be on the safe side) is in fact an isolated MSP. With the inclusion of ablation, we find that the percentage of isolated MSPs increases from 1 to 36 per cent. This new value is in rough agreement with observations where it is estimated that one-third of the MSPs are isolated (Huang & Becker 2007). Iben et al. (1995) similarly found good agreement with observations for binary to isolated ratios when assuming ablation of MSP companions. We see from the last two rows in Table 4 that the isolated and binary MSP populations now have comparable scaleheight values (in fact, the isolated scaleheight is now slightly the lower of the two). Therefore, the kinematics of the binary and isolated MSP populations are now similar. This last point is actually consistent with the observations of MSPs, which via statistical arguments show no difference in binary and isolated MSP kinematics (Lorimer et al. 2007). From this simple test, we see that the low-mass companions to MSPs do occur and that the ablation process deserves serious consideration in future models.
In Fig. 7, we show how the scaleheights of the MSP populations vary with Galactocentric radius. We note that the region of the Galaxy where the populations are most numerous is between 4 and 6 kpc from the Galactic centre. The top panel of Fig. 7 depicts the similarity of MSP–MS and MSP–WD kinematics. It is only out beyond ~13 kpc that the two populations diverge, and this is only due to low number statistics which begin to plague the MSP–MS results. Low number statistics have a much greater influence in the middle panel of Fig. 7. For example, the highest number of systems in a radial bin (1 kpc in width) for the MSP–NS population contains 39 systems while the lowest only three. The MSP–NS and isolated MSP systems have similar scaleheights throughout the majority of the Galaxy (after accounting for statistical uncertainty) and systems can be found far from the plane. The MSP–BH systems on the other hand are all found close to the Galactic plane. When accounting for the ablation of MSP companions, we find a very similar distribution of binary and isolated MSPs throughout the entire Galaxy.

It is also interesting to compare the spatial Galactic x – z distributions of the MSP populations. This is shown in Fig. 8 for the Galactic x-z-plane and emphasizes what we have already seen in Table 4 and Fig. 7: isolated MSPs and MSP–NS binaries have quite extended distributions (relative to their numbers), MSP–BH systems reside close to the plane and the majority of MSPs are found with WD companions (MSP population numbers relative to MSP–WD systems are given in the lower-right corner of each panel). What is surprising in Fig. 8 is the large number of MSP–WD systems out to |z| = 10 kpc. This suggests that there may be many MSP–WD systems lost from – but surrounding – the Galaxy. We next investigate the result of imposing a limited selection effect on the MSP population where we only consider pulsars that have $B > 6 \times 10^7$ G. This magnetic field value is a suggested limit (Zhang & Kojima 2006) of the required field strength to turn on (or off) the pulse mechanism (see Paper I). Fig. 9 shows the field strength limited MSP population, and the result in comparison to Fig. 8 is dramatic. The entire MSP–BH population now disappears, which is also almost the case for the MSP–MS population where only two systems are left. The relative numbers of both isolated MSPs and MSP–NSs have now increased compared to the MSP–WD systems (see values in Figs 8 and 9). Clearly, many MSPs in Model C accrete enough mass to cause a large decay in the magnetic field. In particular, every pulsar within an MSP–BH system has accreted more than ~0.04 $M_\odot$ of material which is the typical amount of mass it takes for our model pulsar magnetic fields to decay below $B = 6 \times 10^7$ G (see Paper I). This is compared to other works which assume $\Delta M > 0.1 M_\odot$ is required for MSP production (e.g. Willems & Kolb 2002).

4.4.2 Model C MSPs and BHs

Although other works have detailed BH and pulsar binary evolution in varying detail (Narayan et al. 1991; Lipunov et al. 1994; Pfahl et al. 2005; Lipunov, Bogomazov & Abukhzerov 2005), we evaluate the accretion history of MSP–BHs and why these systems reside close to the Galactic plane. To place this into context we explore the initial orbital period and initial primary mass (the MSP progenitor and initially the less massive star) parameter space in Fig. 10. This figure is also designed to show which systems reside in a range of observable orbital periods (that is orbital periods which would be observed now; if the age of the Galaxy is 10 Gyr). Fig. 10 also gives the secondary mass (BH progenitor and initially the less massive star) range for each binary system depicted. What we find in

![Figure 10: MSP–BH initial population parameter space of Model C, in particular the initial orbital period and zero-age main-sequence primary star (MSP progenitor) mass. Provided are ranges of the initial companion mass (BH progenitor). Those systems with initial orbital periods, $P_{\text{orb}} > 100$ d end with final orbital periods $P_{\text{orb}} > 10000$ d. Those with $P_{\text{orb}} < 10$ d end with $20 < P_{\text{orb}} < 10000$ d and those with $10 < P_{\text{orb}} < 100$ d end with $P_{\text{orb}} < 20$ d.](https://example.com/figure10.png)
with the primary as a naked helium star with a mass of about $10\,M_\odot$. During the phase the secondary accretes approximately 80 per cent of the transferred material with the remainder lost from the system. The orbital separation at this point is typically $200\,R_\odot$ and subsequently increases further owing to winds from the helium star and the now massive secondary. At a system time of $\sim 5\,\text{Myr}$, the primary undergoes a SN explosion and becomes a NS. We find that velocity kick magnitudes of $V_{\text{kick}} \leq 80\,\text{km}\,\text{s}^{-1}$ allow the system to remain bound. Beyond the first SN, the secondary evolves quickly and loses a large proportion of its matter in a wind, some of which is accreted by the NS companion. The secondary evolves via a naked helium star phase to explode as a SN and leaves a BH remnant. At this point, we have an eccentric MSP–BH system which has received one mild SN velocity kick in its lifetime and has typical component masses of 2 and 10–13 $M_\odot$, for the NS and BH, respectively. The orbital separation is in the range of $1000–4000\,R_\odot$ (depending on the precise details of the kick velocity and the mass-loss history).

Those MSP–BH systems with $10 < P_{\text{orb}} < 100\,\text{d}$ as seen in Fig. 10 end their lives with a large range of BH masses extending from $3\,M_\odot$ through to $11\,M_\odot$ in tight orbits around their MSP companion ($P_{\text{orb}} < 20\,\text{d}$). It is this population of MSP–BHs which is most likely to coalesce at and around the age of the Galaxy (similar to Pfahl et al. 2005). Initial primary masses are in the $18 < M_1/\,M_\odot < 30$ range and secondary masses are typically $10 < M_2/\,M_\odot < 20$. The initial orbital separation ranges from 100 to 300 $R_\odot$. Early evolution proceeds similarly to that of the previous group; non-conservative mass transfer from the primary to the secondary accompanied by an increase in the orbital separation and ending with the primary as a naked helium star. The primary then undergoes a SN and becomes a NS at a system time of about 8 Myr. We find that generally these systems can survive slightly larger SN velocity kicks than the systems described in the previous group. The companion is now a massive MS star ($\sim 30\,M_\odot$) and subsequently fills its Roche lobe while crossing the Hertzsprung Gap. This initiates dynamical-time-scale mass transfer leading to a CE phase and the creation of a tight binary comprising the NS primary ($\sim 2\,M_\odot$) and a naked helium star secondary ($\sim 10\,M_\odot$). We note that systems in the first group avoid this second Roche lobe filling event because the secondary is more massive and loses mass in a wind at a greater rate leading to more substantial orbit expansion after NS formation. After emerging from the CE, the NS then accretes material from the wind of the companion to become an MSP. This ends when the companion becomes a BH. The final MSP–BH binary will have an orbital separation of less than $10\,R_\odot$ and systems such as this may coalesce within a Hubble time.

The MSP–BH systems with $P_{\text{orb}} > 100\,\text{d}$ with orbital periods of 1000 d or more (see Fig. 11 for the final orbital period range). We note that the smallest primary and secondary masses belong to this group. Once again mass transfer occurs prior to the first SN event but owing to the wider orbit this is initiated later ($\sim 15\,\text{Myr}$) than in the previous cases and when the primary is a giant star. The orbital separation when the primary undergoes a SN (to become a NS) is typically $2000–3000\,R_\odot$ which means that relatively smaller kicks are required if the system is to remain bound and proceed to become an MSP–BH binary. We find that kicks of the order of $20\,\text{km}\,\text{s}^{-1}$ or less are necessary (slightly larger if the kick is well directed). After NS formation, the secondary is a MS star with mass of approximately $20\,M_\odot$. The secondary then evolves off the MS and transfers some material to the NS before ending its life as a BH of mass less than $5\,M_\odot$.

The above analysis shows that the most likely MSP–BH systems to be created are those in which the first SN – the only one assumed to impart a velocity kick on to the compact remnant – produces a small velocity kick, which is why these systems are found to hug the Galactic plane as suggested by Narayan et al. (1991). In fact, compared to the other MSP binary populations the MSP–BH systems effectively represent a different kick distribution, in that the distribution of kicks given to systems that remain bound is distinct. As touched on in the evolutionary descriptions, this is also true internal to the MSP–BH population, where the effective kick distribution for systems that remain bound is different for each of the three period groupings we identified in Fig. 10. This is depicted indirectly in Fig. 11. Here, we see the MSP–BH height from the Galactic plane and the populations is designated by their grouping in the final orbital period parameter space. Each population has a different scatter in $|z|$, which can be used as an indicator for the average strength of the SN velocity kick. We see that the majority of those small orbital period MSP–BH systems are further off the plane than the extremely long-period MSP–BH systems, suggesting that as expected from Bailes (1989), the close binary systems can survive larger kick velocities than the larger binary systems (which was outlined in the evolutionary examples). Only three MSP–BH systems are found beyond 10 kpc from the Galactic plane.

The MSP–BH orbital period distributions as shown in Fig. 11 are remarkably distinct and perhaps surprisingly not smeared out by our use of random birth ages. This is due to the vast orbital period differences between these populations and the time-scales these populations evolve on. The orbit of MSP–BH binary systems, after the formation of the BH, can only shrink in time owing to gravitational radiation (Landau & Lifshitz 1951; Hulse & Taylor 1975; HTP02). However, the time-scale on which this decrease occurs is greatly dependent on the size and eccentricity of the orbit. Long-period binary systems have very large time-scales for orbital parameter change and thus remain as long-period systems over a Hubble time. The very close systems (separation $\lesssim 10\,R_\odot$) will
shrink more rapidly and may even coalesce within a Hubble time. Therefore, the long-period systems stay long and the short-period systems only get shorter, and as a result the MSP–BH systems stay within their orbital period groups as they evolve throughout the Galaxy. Thus, we observe three distinct MSP–BH groups, a result differing some what from the orbital period distribution of Pfahl et al. (2005) who find that most MSP–BHs have orbital periods of 1–6 h.

4.4.3 Model C MSP recoil and 3D space velocities

We now examine the MSP population in velocity and orbital period parameter space. In terms of velocity, we consider both the recoil velocity and the space velocity of the binary systems. The recoil velocity is defined as the change in velocity of the binary centre-of-mass owing to the SN explosion that created the NS (that went on to become the MSP). The space velocity is the velocity of the binary within the Galaxy at the time when the Galactic age is 10 Gyr. In calculating the final space velocities, the solar motion around the Galactic centre is accounted for by removing the LSR velocity of \(\sim 220 \text{ km s}^{-1}\) (Dehnen & Binney 1998). The orbital period is taken as the final orbital period at a Galactic age of 10 Gyr. In a similar vein to Section 4.4.1, we examine the parameter space when considering all pulsars and then examine it again after limiting the sample population to MSPs that have \(B \geq 6 \times 10^{-7} \text{ G}\). The results are shown in Fig. 12.

The recoil velocities for all MSP binaries are shown in Fig. 12(a). The first item to note is the MSP–BH systems which all have low recoil velocities but cover a large range of final orbital periods. Such a distribution is not surprising given our detailed analysis of such systems in Section 4.4.2. Also not surprising is the rather large recoil velocity range of double-NS systems. The typical total recoil velocity incident on such systems is greater than 200 km s\(^{-1}\). We can also see from Fig. 12(a) that these systems are likely to be eccentric rather than circular. For MSP–NS systems that receive large recoil velocities (\(>450 \text{ km s}^{-1}\)), there appears to be a lower limit to the possible final orbital period. The initial orbital period of these systems is very important in determining the evolution outcomes and the appearance of the final parameter space (Tauris & Bailes 1996). Also, in a related manner and as discussed for MSP–BH systems in Section 4.4.2, the onset of mass transfer and the details of the CE phase are crucial factors. What we find is that a significant proportion of the double-NS population end up with extremely small periods (and a range of eccentricities) and coalesce rapidly (within a few Myr after double-NS formation) similar to that found by Belczynski, Bulik & Rudak (2002a). This leads to the orbital period gap observed in Fig. 12(a). We leave further discussion on these systems for future work (Kiel, Hurley & Bailes 2009). Turning to the MSP–WD systems, we see that these typically receive rather low recoil velocities with the average value being less than 100 km s\(^{-1}\) (much less than \(\sqrt{2} V_\sigma\)) and in accordance to previous population synthesis results of Ramachandran & Bhattacharya (1997), Phinne & Kulkarni (1994), Lyne et al. (1998) and Sun & Han (2004). We also see that a similar but opposite trend occurs for MSP–WDs as did for the MSP–NSs in that for large recoil velocity values there appears to be an upper limit to the possible final orbital period. Again, this is related to the orbital evolution and in particular whether a system enters CE evolution (and survives without coalescence) or not.

Fig. 12(c) shows the recoil velocity and final orbital period parameter space for the magnetic-field-limited MSP population. We see that the population has been significantly thinned out. In particular, the entire MSP–BH population has been removed as have the low-period MSP–WD systems. It is also possible to compare our findings to the results of Tauris & Bailes (1996: see their fig. 2c) who followed the formation of MSPs using stellar and binary evolution algorithms that were quite advanced for their time. Compared to Tauris & Bailes (1996), we find fewer systems with orbital periods greater than a day. However, for the MSP–WD population, we observe a similar trend of orbital period to recoil velocity: the smaller the orbital period the greater the range in possible recoil velocity of the system.

In Fig. 12(b), we look at the final space velocities and orbital periods for all MSP binaries. There is much similarity between the velocities given to each system (their recoil velocities) and their LSR Galactic motion. The form of this parameter space is therefore governed by the same evolutionary phases that dictated the appearance of Fig. 12(a). In Fig. 12(d), we show the space velocity-orbital period parameter space distribution of MSPs with \(B \geq 6 \times 10^{-7} \text{ G}\). Included for comparison are pulsar proper motion observations which, convolved with distance estimates, give rise to observed transverse velocities. From the ATNF pulsar catalogue (Manchester et al. 2005), there are at present 28 Galactic disc pulsars with spin periods less than 0.02 s that have measured orbital periods and estimated transverse velocities. Although we do not directly compare total model space velocities to observed pulsar proper motions, useful information can still be gleaned from simple comparisons between the two noting that the transverse velocities are a lower limit to the true space motion (although measurement errors not included within Fig. 12, especially in distance calculations, cloud this picture slightly). First, once accounting for the LSR, we can see that many of the model MSP systems travel with speeds within the typical stellar velocity range of approximately \(\pm 16 \text{ km s}^{-1}\) (as given by Dehnen & Binney 1998). However, there appears to be an overabundance of model MSP binaries at low velocities. The model also fails to produce enough of the fast-moving MSPs with large orbital periods. One reason for this may be the \(V_\sigma = 190 \text{ km s}^{-1}\) assumed in Model C which is lower than the value suggested from observations (\(V_\sigma = 265 \text{ km s}^{-1}\); Hobbs et al. 2005). On the other hand, Fig. 12(d) shows a large range of space velocities for MSP–NS systems. Perhaps surprisingly, some of these systems even have velocities close to the LSR, although not so surprising according to Dewi et al. (2005) who suggest that double-NSs receive small kicks. This is of particular interest for understanding the nature of the double pulsar \(J0737 – 3039\) (Burgay et al. 2003) which is observed to have a transverse velocity of \(30 \text{ km s}^{-1}\) or less (Kramer et al. 2006). Considering that the system will have experienced two SN events, this has been taken as evidence for little or no velocity kicks within this system. However, Kalogera et al. (2007) have described models which show that kick velocities of \(100 \text{ km s}^{-1}\) or more are still possible. Our results agree with this in that it is not necessary to make any unusual assumptions regarding kicks in binary systems to explain the observed velocities of systems such as \(J0737 – 3039\) (Deller, Bailes & Tingay 2009).

4.4.4 Effects of the assumed initial scaleheight

Until now, we have set a maximum of \(|z_{\text{max}}| = 75 \text{ pc}\) to the initial birth height distribution of binaries, effectively modelling a thin disc. We now examine the effect this has on the scaleheight calculations by extending it to \(|z_{\text{max}}| = 150 \text{ pc}\) in Model C. The results are compared to Model C in Table 4. We find that there is no significant change in the calculated scaleheights. This agrees with previous
works, such as P90 or Sun & Han (2004), who have suggested that for such kinematically active systems as pulsars the initial height above the plane does not greatly affect the outcome.

4.4.5 Effects of the assumed Galactic age

The age of the Galaxy is an important assumption, especially when populations of systems with large differences in life times are modelled together. To address this, we have Model C" which assumes the Galactic age is 15 Gyr rather than 10 Gyr for Model C. It is the isolated MSPs and MSP–NSs whose scaleheights change the most appreciably in this new model compared to Model C. This is primarily owing to low number statistics (see Fig. 8 for the relative numbers of the populations). Otherwise, it does not appear that the increase in Galaxy age has a significant effect on the kinematics of the MSPs.

4.4.6 Sufficient model resolution?

Finally, to test whether our previous models have a fine enough resolution in the initial parameter space to faithfully represent the entire Galactic pulsar population, we have extended Model C to include $10^9$ binary systems (a factor of 100 increase). The only systems for which the scaleheight changed noticeably was the MSP–NS systems, which are relatively rare and kinematically energetic systems. In all other respects, it appears that the results for $10^7$ systems scale reliably to larger populations. We note here that modelling $10^9$ binary systems is equivalent to modelling $\sim 10$ per cent the mass of the Galaxy, assuming binary systems are of interest. To run a model of this size takes roughly 4500 CPU hours and even when farming the model out to 100 processors (on the Swinburne supercomputer\(^2\)) it takes almost 2 d to complete. Thus, it is obviously an advantage when examining a variety of evolutionary assumptions a single model at a time to be able to represent the Galaxy faithfully with fewer systems.

5 DISCUSSION

Using our newly developed BINKIN module for integrating the positions of stars and binaries within the Galaxy, we have worked through a series of models in order to understand how various options available in the module affect the outcomes. This has allowed us to develop a favoured model – our Model C. In doing this, we have used pulsar populations as our yardstick, computing scaleheights, radial and velocity distributions, and orbital characteristics (in the case of binary systems) with limited comparison to observations. What we have done is to make predictions in all these areas about

\(^2\) http://astronomy.swin.edu.au/supercomputing/
the particulars of the Galactic pulsar population, assuming that all pulsar systems can be observed. Of course, this is not the case in reality and our model results cannot truly be confronted by observations until we include selection effects in our modelling. This will be completed when we add our next and final module \textsc{binpop} to our population synthesis code. As such, we leave a discussion of the necessary selection effects that need to be considered and their treatment to an upcoming paper focused on the \textsc{binpop} module (Kiel, Bailes & Hurley, in preparation). This paper will include features such as the predicted pulsar \( P \sim P \) diagram for distinct regions in the Galaxy (following on from our investigation of this diagram in Paper I in terms of binary evolution parameters). Below, we discuss future additions to the \textsc{binpop} and \textsc{binkin} modules in relation to pulsar evolution as well as caveats to our current findings.

5.1 Accretion-induced collapse formation of neutron stars

Further analysis of our MSP populations shows that some of the NSs which go on to become MSPs in our models are formed from the accretion-induced collapse (AIC) of WDs (Canal & Schatzman 1976; Nomoto & Kondo 1991). In the scenario of Nomoto & Kondo (1991), an O–Ne–Mg WD accretes enough material to reach the Chandrasekhar mass, the maximum mass possible for a WD to support itself, and collapses to form a NS. To date, we have allowed NSs formed in this way to receive velocity kicks in the same manner as for NSs formed in core-collapse SNe. Generally, if the binary system remains bound an AIC NS will continue to accrete material after the SN, a SN which induced an eccentricity into the orbit. This formation pathway produces a substantial number of MSP–WD and MSP–MS systems with eccentricities greater than 0.1, that reside within the Galaxy. These systems highlight the importance of a correct mass-transfer treatment for eccentric binaries (see Paper I and Bonačić Marinović, Glebbeek & Pols 2008). This is something which is not currently accounted for in our models (we make use of equations which assume the orbit is circular; see HTP02) and it most likely will affect the production and visibility of these systems. Of course, if the AIC systems were not given any velocity kick (as has been modelled previously; HTP02), or a much lighter kick (as latest models suggest: Dessart et al. 2006), then not only would there be many more AIC MSP systems but they would also all have a greater possibility of residing in our Galactic target area and the population scaleheight would be lowered. They would also typically have smaller eccentricities.

Although we do not deal directly with low-mass X-ray binaries within this work, it is possible for us to compare the observed scale-heights of such systems – which suffer from less selection effects than pulsar observations – with our model MSP–MS scaleheight calculations. This is assuming that low-mass X-ray binaries are the progenitors of MSP–MS systems. Grimm, Gilfanov & Sunyaev (2002) found that Galactic field low-mass X-ray binaries have a scale-height of \( \approx 0.410 \) kpc. Interestingly enough, as shown in Table 4, our models overestimate this by almost a factor of 2. Such an outcome may be another implication that AIC NSs receive less momentum at birth than standard NSs formed from core-collapse SNe. However, it is not clear that MSPs that result from AIC NSs can be linked to an observable low-mass X-ray binaries phase (Hurley et al. 2009).

5.2 Electron capture supernovae

Another evolutionary scenario related to NS formation and velocity kicks is core-collapse electron capture SNe. This was discussed and modelled in Paper I and has also been accounted for in other population synthesis works (e.g. Ivanova et al. 2008). Briefly, core-collapse electron capture SNe are thought to arise when electrons are captured on to Mg atoms, depleting the electron force in an O–Ne–Mg stellar core of sufficient mass \( (1.4–2.5 \text{ M}_\odot, \text{ Nomoto 1984}) \) which is produced by initial progenitor masses in the range of \( 8–12 \text{ M}_\odot \) (although this mass range is model dependent; Podsiadlowski et al. 2004). The likelihood that a star born within the \( 8–12 \text{ M}_\odot \) limit will evolve to have an O–Ne–Mg core mass between 1.4 and \( 2.5 \text{ M}_\odot \) increases if the progenitor is able to interact with a companion and lose its outer hydrogen envelope, rather than evolve in an isolated environment (Podsiadlowski et al. 2004). Therefore, binary population synthesis is ideal for examining the likelihood and outcomes of such events. The resultant electron capture SN energy yield is low, sufficient to cause the explosion but not enough to impart any large velocity to the proto-NS (Kitaura, Janka & Hillebrandt 2006). Paper I found that the final MSP spin period and spin period derivative parameter space were altered when electron capture SNe were included. Less pulsar binary systems were disrupted, owing to the small momentum imparted during the SN, causing more MSPs to be produced. It is also reasonable to expect that including electron capture SNe in the \textsc{binkin} models, with SN kicks drawn from a distinct distribution with a smaller velocity dispersion than for standard NSs, will lead to a reduction in the pulsar scaleheights. This is a feature that will be fully explored in future models so that the impact of the electron capture SNe process on binary evolution outcomes and the resultant Galactic kinematics of pulsar populations can be quantified.

5.3 MSP–BH systems

In our models, we have assumed no SN velocity kick is given to BHs and have found that MSP–BH systems reside close to the Galactic plane. If BHs were instead to receive a SN kick selected from the same distribution as NSs, then it is clear that the scaleheights of populations including BHs would increase (Voss & Tauris 2003). However, it is not so obvious that the scaleheights would be similar to that of the equivalent NS populations (Pfahl et al. 2005). In particular, MSP–BH systems (and their progenitors) can be heavier on average than MSP–NS systems (and their progenitors) and the more massive systems will require a greater momentum to reach the same velocities as less massive systems. As such MSP–BHs, for example, could still have a significant difference in their resultant scaleheight to that of MSP–NSs even when both populations receive kicks from the same distribution. We would also expect the number of BH binary systems to decrease. Most likely, it would be the MSP–BH systems with a large orbital periods prior to BH formation (systems with initial orbital periods greater than \( \approx 100 \text{ d} \) which would be depleted. It is these systems that are not produced in the models of Pfahl et al. (2005) who assume SN kicks occur on nascent BHs. However, we must bear in mind that the final BH masses are calculated assuming that material ejected in the SN falls back on to the BH. There is less mass-loss associated with BH formation than for NSs and this means SN-induced binary disruption is less likely during BH formation (in the case of equivalent kick velocities).

We note that when discussing the MSP–BH population (or any of our model MSPs), we are defining a rapidly rotating NS to be an MSP based solely on its spin period. If instead we also include consideration of the magnetic field strength of these NSs, then the nomenclature may be misleading, especially if we are interested in observable MSPs. It turns out that all of the NSs in our model.
MSP–BH systems have magnetic fields residing on, or very close to, the assumed bottom magnetic field limit of $6 \times 10^7$ G (Paper I; Zhang & Kojima 2006). Previously (Paper I, Figs 9 and 12), we have assumed that any NS with a magnetic field less than this limit cannot accelerate the electrons in its atmosphere to produce the observed pulsar beam and as such is not observable as a pulsar. Therefore, if our assumptions regarding accretion on to NSs and how this translates to magnetic field decay are correct, then we have a lot of trouble producing observable MSP–BHs. Future observations of such systems will help greatly in constraining our evolutionary assumptions.

5.4 Initial distributions

In our models we have assumed a maximum birth height off the plane, $z_{\text{max}}$, of either 75 or 150 pc. Consistent with P90 and Sun & Han (2004), no significant variations of the MSP population scaleheights were found when varying this parameter. This suggests that the results are robust to changes in $z_{\text{max}}$ as long as a sensible choice is made. The majority of OB star formation has been shown by de Wit et al. (2005) to occur within $|z| \sim 200$ pc of the Galactic plane so choices within this range, such as for our models, would seem reasonable. In the future, it will be interesting to probe the effects of assuming a radial dependence in $z_{\text{max}}$ on the final pulsar population distributions. This may even be tied in with examining the effect of assuming bursts of star formation throughout the age of the Galaxy and accounting for Galactic arms when initiating the birth positions. This final point has previously been suggested as an important feature to incorporate into population synthesis models (Faucher-Giguere & Kaspi 2006).

We found that the Yusifov & Kucuk (2004) initial radial pulsar birth distribution gave the best fit of our models to observations. This distribution was based on observations of H\textsc{ii} regions within the Galaxy. However, it failed to reproduce the peak of the observable radial distribution – which the P90 initial radial distribution succeeded in reproducing. Yusifov & Kucuk (2004) recognized that their relation is approximate and suggested that a detailed analysis between models and observations of pulsar velocities and Population I stellar positions was required to develop a more realistic distribution. We are in a position to do this and as a result can suggest that the initial pulsar birth distribution of Yusifov & Kucuk (2004) perhaps be shifted towards smaller Galactic radii to peak at the inner H\textsc{ii} peak ($\sim 4.0–4.5$ kpc) depicted in fig. 3 of Paladini, Davies & DeZotti (2004).

5.5 Galactic model potentials

Even though our favoured model (Model C) utilizes the P90 form of the Galactic gravitational potential, we are in no way able to distinguish between this and the KG89 model as a more suitable representation of the Galactic potential. Both give similar pulsar population scaleheight results which are not surprising given their similarities as shown in Fig. 4. The form of the X08 potential is clearly distinct from the other two models, especially within the inner 1 kpc of the Galaxy (where X08 employ an extrapolation of their measurements), and leads to markedly increased scaleheights. On this basis, we do not favour the use of the X08 potential. However, we are not currently in a position to make strong conclusions in this area, especially when many previous pulsar, NS and X-ray binary population synthesis works (such as P90; Lorimer et al. 1993; Belczynski, Bulik & Rudak 2002a; Sun & Han 2004; Zuo, Li & Liu 2008) have used different Galactic gravitational potentials and their results compare well to observations. We note that Sun & Han (2004) comment that it is unclear whether the Milky Way has a peak in the gravitational potential at small Galactic radii (as present in the P90 and KG89 models).

A possibility in the future is to extend the Galactic gravitational potential analysis to consider Modified Newtonian Dynamics (MoND). Such an approach has already been taken by Wu et al. (2008), who compare MoND with cold dark matter models, and Zuo et al. (2008) who make use of MoND potentials to conduct population synthesis of X-ray binaries.

5.6 Close double compact systems and gamma-ray bursts

In this work, we have focused on pulsars and looked in detail at MSP systems. However, the models can also be extended to explore the formation of close double compact systems (NS–NS, BH–BH and NS–BH systems) in detail. The kinematics of these systems are of interest because of their link to gamma-ray bursts and, in particular, recent observations of the distances at which gamma-ray bursts appear to occur from their (assumed) host galaxy (Bloom, Kulkarni & Djorgovski 2002). Our combined \textsc{binpop} and \textsc{binkin} modules can provide model estimates for the projected distances from their host galaxy at which double compact systems coalesce and document the kinematic evolution of these systems in general. This will be the focus of a companion paper (Kiel, Hurley & Bailes 2009).

6 SUMMARY

We have examined in depth the Galactic dynamics and population characteristics (owing to stellar, binary and kinematic evolution) of pulsars.

Our main findings, reconfirming and updating many areas of pulsar evolutionary physics, can be summarized as follows (noting that overlap with previous work is detailed in Section 4):

(i) When using a peaked radial distribution for the birth locations of binaries, the population of pulsars that arises from these binaries also follows a peaked distribution where the location of the peak moves inwards in radius by as much as 0.5 kpc as the population evolves. Also, compared to the birth distribution, the initial shape is preserved inward of the peak but the distribution becomes more extended in the outer regions.

(ii) Starting with a uniform initial distribution of binaries cannot produce a final pulsar distribution that is peaked away from the centre of the Galaxy and therefore does not compare well to observations of pulsar locations which indicate a deficit of pulsars towards the Galactic centre.

(iii) The form of the Galactic potential does not produce significant differences in the final radial distribution of pulsars but can lead to noticeable differences in the calculated scaleheights of pulsars.

(iv) As the pulsar population ages, the peak of its velocity distribution moves to lower velocities. The velocity dispersion of this distribution (assuming a Maxwellian) almost halves over a period of 10 Gyr. The shape of the velocity distribution is significantly affected by the inclusion of binary evolution – this produces a more sharply peaked distribution.

(v) Similar to observations, we find that the majority of standard pulsars are isolated and that these dominate the statistics of the pulsar scaleheight calculations.

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(vi) Isolated pulsars have a greater scaleheight than binary pulsars except in cases where large velocity kicks are applied to the population resulting in many isolated pulsars being lost from the Galaxy and hence from the scaleheight calculations.

(vii) Isolated MSPs have greater scaleheights than binary MSPs (by as much as a factor of 2) however, limiting the region of the Galaxy considered (in terms of height off the plane) does reduce the difference in these scaleheights and brings them more in line with what observations suggest.

(viii) We find that 99 per cent of MSPs are in binary systems when we only consider SN disruption as a pathway for creating isolated MSPs. This does not agree with the observed MSP population. If we include a simple ablation model, we find instead that 64 per cent of MSPs are in binaries which adequately match the observed mix. Furthermore, accounting for ablation gives similar scaleheights for isolated and binary MSPs.

(ix) MSP systems with NS companions can receive large recoil velocities. There is a large scatter in the resulting peculiar motions of MSP–NS binaries, and it is possible for such systems to be found with low peculiar motion.

(x) The scaleheights of the MSP–MS and MSP–WD binary populations are very similar and follow similar radial distributions. These scaleheights are larger than that of the observed low-mass X-ray binary population in the Galaxy (often thought to be the precursors of MSP–MS binaries). However, many of the model MSPs in binary systems are formed from the AIC of a WD which, if given smaller kicks than for standard NSs at birth, would reduce the model scaleheights.

(xi) MSPs with WD companions are the most common of the binary MSPs. This is followed by MSP–MS, MSP–BH and MSP–NS binaries, respectively.

(xii) Restricting the model MSP population to only include MSPs with magnetic fields greater than $6 \times 10^7$ G drastically reduces the number of systems and changes the way that the population is distributed. This suggests that the underlying pulsar distribution of the Galaxy may differ greatly from the observed sample.

One future goal of pulsar astronomy is the detection of a pulsar orbiting a BH, and in terms of placing constraints upon general relativity MSPs in a close orbit around a BH would be an especially exciting observation. We find three distinct evolutionary pathways which result in the formation of MSP–BH systems. These pathways produce three distinct MSP–BH populations in terms of orbital period: those with periods of 10 d or less, those with periods of about 1000 d and those with periods of 10,000 d or greater. The short- and long-period populations are the most numerous and only the short-period systems are found further than 1 kpc from the Galactic plane. We find that owing to the amount of material accreted by the MSPs in our model MSP–BH binaries that the magnetic field decays below $6 \times 10^7$ G. This possibly suggests that we are overestimating the rate of accretion-induced magnetic field decay in our evolution model – the observation of an MSP–BH binary would confirm this possibility.

We emphasize to the reader that we are not presenting any of the models in this paper as a definitive representation of the true Galactic pulsar population. The uncertainty involved in the many parameters contained within BINSFX and BINSFX does not allow this. Moreover, because we do not consider selection effects in our model Galaxy, we cannot at this stage make definitive comparisons to observations as the possibility exists that the observed population may be biased in some manner. Lommen et al. (2006) suggest that observations of MSPs may preferentially detect binary MSPs because the isolated MSPs may be less luminous than their binary cousins. The intrinsic luminosity of pulsars is not something examined in this body of work. However, it will be discussed in detail in future work where selection effects are calculated within our upcoming BINSFX module (Kiel, Bailes & Hurley, in preparation). Supplementing our current pulsar population synthesis with selection effects will allow additional evaluation of the evolutionary codes and their scientific outcomes. It will also allow us to guide further surveys by selecting regions of the sky best suited for the specific pulsar survey and/or telescope of interest. Therefore, BINSFX will provide a powerful tool with which to constrain the theory and modelling of stellar, binary and Galactic kinematic evolution. Further constraints could be placed on binary evolution if population synthesis studies are extended to include additional stellar populations and their appropriate selection effects. For now, however, we are well on our way to producing a comprehensive treatment of pulsar population physics.

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erg s\(^{-1}\) – \(\Delta t\) \(\sim \) C/\(\Delta t\)\(^2\) for the threshold mass of a 0.02 M\(_\bigodot\).