ON THE THEORETICAL UNDERSTANDING
OF INCLUSIVE $\Lambda_b$ DECAYS

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1 Introduction

The difference between the measured lifetimes of the $\Lambda_b$ baryon and $B_d$ meson

$$\tau(\Lambda_b^0)/\tau(B^0) = 0.78 \pm 0.04$$ (1)

represents an intriguing problem of the present-day heavy quark physics. Indeed, the 20% difference contradicts the expectation that, at the scale of the $b$ quark mass, the spectator model should describe rather accurately the decays of hadrons $H_Q$ containing one heavy quark.

A calculation of the ratio $\tau(\Lambda_b)/\tau(B_d)$ can be attempted using a field theoretical approach developed for the analysis of the inclusive weak decays of the hadrons $H_Q$. The method is based on the expansion in powers of $m_Q^{-1}$, in the framework of the Wilson OPE. The widths are expressed in terms of hadronic matrix elements of high dimensional quark and gluon operators; since such matrix elements can be responsible of the large difference between $\tau(\Lambda_b)$ and $\tau(B_d)$, it is interesting to compute them by nonperturbative methods such as QCD sum rules.

The main aspects of the QCD analysis of the inclusive decays of hadrons $H_Q$ can be summarized considering the transition operator $\hat{T}(Q \rightarrow X_f \rightarrow Q)$:

$$\hat{T} = i \int d^4x \, T[\mathcal{L}_W(x)\mathcal{L}_W^\dagger(0)]$$ (2)

which describes an amplitude with the heavy quark $Q$ having the same momentum in the initial and final state. $\mathcal{L}_W$ is the effective weak Lagrangian governing the decay $Q \rightarrow X_f$. The inclusive width $H_Q \rightarrow X_f$ can be obtained from

$$\Gamma(H_Q \rightarrow X_f) = \frac{2}{2M_{H_Q}} <H_Q|\hat{T}|H_Q>$$ (3)

The large energy release in the heavy quark decay permits an expansion of $\hat{T}$ in terms of local operators $O_i$:

$$\hat{T} = \sum_i C_i O_i$$ (4)

with $O_i$ ordered according to the dimension, and the coefficients $C_i$ containing appropriate inverse powers of the heavy quark mass $m_Q$. The lowest dimension operator in (4) is $O_3 = \bar{Q}Q$. The next gauge and Lorentz invariant operator is the $D = 5$ chromomagnetic operator $O_4 = \bar{Q}\frac{2}{3}\sigma_{\mu\nu}G^{\mu\nu}Q$, whose hadronic matrix element

$$\mu_4^2(H_Q) = <H_Q|\bar{Q}\frac{2}{3}\sigma_{\mu\nu}G^{\mu\nu}Q|H_Q>$$ (5)

measures the coupling of the heavy quark spin to the spin of the light degrees of freedom in $H_Q$, and therefore is responsible of the mass splitting between hadrons belonging to the same $s_f$ multiplet ($s_f$ is the total angular momentum of the light degrees of freedom in $H_Q$). In the case of $b$-flavoured hadrons this mass difference has been measured, both for mesons $(M_B - M_\rho = 45.7 \pm 0.4$ MeV) and $\Sigma_b$ baryons $(M_{\Sigma_b} - M_{\Sigma_0} = 56 \pm 16$ MeV).

The matrix element of $\bar{Q}Q$ over $H_Q$ can be obtained using the following expansion stemming from the heavy quark equation of motion:

$$\bar{Q}Q = \bar{Q}\gamma^0 Q + \frac{O_3}{2m_Q^2} - \frac{O_4}{2m_Q} + O(m_Q^{-3})$$ (6)

$O_\pi$ is the kinetic energy operator $O_\pi = \bar{Q}(i\hat{D})^2Q$ whose matrix element

$$\mu_\pi^2(H_Q) = <H_Q|\bar{Q}(i\hat{D})^2Q|H_Q>$$ (7)

measures the average squared momentum of the heavy quark inside $H_Q$. 

At the order $O(m_Q^{-3})$ four-quark operators appear in (9):  
\[ \mathcal{O}_6^q = \bar{q} \Gamma q \bar{q} \Gamma Q \]  
(8) with $\Gamma$ an appropriate combination of Dirac and color matrices.

The resulting expression for the width $\Gamma(H_Q \rightarrow X_f)$ reads:

\[ \Gamma(H_Q \rightarrow X_f) = \Gamma_0^f \left[ A_0^f + \frac{A_1^f}{m_Q^2} + \frac{A_2^f}{m_Q^4} + \ldots \right]. \]  
(9)

$A_0^f$ and $\Gamma_0^f$ depend on the final state $X_f$; $A_1^f$ include perturbative short-distance coefficients and nonperturbative hadronic matrix elements incorporating the long range dynamics. The leading term in (9) corresponds to the partonic prediction $\Gamma_{\text{part}}(H_Q \rightarrow X_f) = \Gamma_0^f A_0^f$, with $A_0^f = 1 + c^2 \alpha_s/\pi + O(\alpha_s^2)$ and $\Gamma_0^f \propto m_Q^2$; differences among the widths of the hadrons $H_Q$ emerge at the next to leading order in $1/m_Q$, and are related to the different values of the matrix elements of the operators $\mathcal{O}_f$ of dimension larger than three. It is important to notice the absence of the first order term $m_Q^{-1}$ in (9).

The $D = 5$ operators $\mathcal{O}_G$ and $\mathcal{O}_\pi$ are SU(3) singlets; on the contrary, the $D = 6$ operators in (8) give different contributions when averaged over hadrons belonging to the same SU(3) light flavour multiplet, and therefore they are responsible of the different lifetimes of, e.g., $B^-$ and $B_s$, $\Lambda_b$ and $\Xi_b$. The spectator flavour dependence is related to the mechanisms of weak scattering and Pauli interference (both suppressed by the factor $m_Q$ with respect to the parton decay rate).

As for differences between mesons and baryons, they could already arise at $O(m_Q^{-2})$, due both to the chromomagnetic contribution and to the kinetic energy term in (8). In particular, the kinetic energy term is responsible of the difference for systems where the chromomagnetic contribution vanishes, namely in the case of $\Lambda_b$ and $\Xi_b$ having the light degrees of freedom in $S$- wave. However, the results of a calculation of $\mu_{\pi}^2$ for mesons and baryons support the conjecture that the kinetic energy operator has the same matrix element when computed on such hadronic systems. The approximate equality of the kinetic energy operator on $B_d$ and $\Lambda_b$ can also be inferred by using mass relations $\mu_{\pi}^2(\Lambda_b) - \mu_{\pi}^2(B_d) \simeq 0.002 \pm 0.024 \text{ GeV}^2$. Then, differences between meson and baryon lifetimes should occur at the $m_Q^{-3}$ level, thus involving the four-quark operators in (8). They can be classified as follows:

\[ \mathcal{O}_{V-A}^q = \bar{q} L \gamma_{\mu} q L \gamma_{\mu} q L \]

\[ \mathcal{O}_{S-P}^q = \bar{q} R q L \gamma_{\mu} q L \gamma_{\mu} q L \]

\[ T_{V-A}^q = \bar{q} L \gamma_{\mu} \frac{\lambda^a}{2} q L \bar{q} L \gamma_{\mu} \frac{\lambda^a}{2} q L \]

\[ T_{S-P}^q = \bar{q} R \frac{\lambda^a}{2} q L \bar{q} L \frac{\lambda^a}{2} q R \]  
(10)

$q_{R,L} = \frac{1 \pm \gamma_5}{2} q$, $\lambda_a$ Gell-Mann matrices).

For mesons, the matrix elements of the operators in (10) can be computed by vacuum saturation approximation:

\[ < B_q | \mathcal{O}_{V-A}^q | B_q >_{\text{VSA}} = \frac{f_{B_q}^2 M_B^2}{4} \]  
(11)

\[ < B_q | \mathcal{O}_{S-P}^q | B_q >_{\text{VSA}} = 0 \]  
(12)

etc. Such an approximation cannot be employed for baryons, where a direct calculation is required.

For $\Lambda_b$, a simplification can be obtained introducing:

\[ \tilde{\mathcal{O}}_{V-A}^q = \bar{Q}_L \gamma_{\mu} Q_L \gamma_{\mu} q_L^i \gamma_{\mu} q_L \]  
(13)

and

\[ \tilde{\mathcal{O}}_{S-P}^q = \bar{Q}_L q_R^i \bar{q}_L^i q_R \]  
(14)

($i$ and $j$ color indices): the $\Lambda_b$ matrix elements of the operators in (10) can be expressed in terms of $< \Lambda_b | \tilde{\mathcal{O}}_{V-A}^q | \Lambda_b >$ and $< \Lambda_b | \tilde{\mathcal{O}}_{S-P}^q | \Lambda_b >$, modulo $1/m_Q$ corrections contributing to subleading terms in the expression for the inclusive widths.

Parametrizing the matrix elements:

\[ < \tilde{\mathcal{O}}_{V-A}^q | \Lambda_b > = \frac{< \Lambda_b | \tilde{\mathcal{O}}_{V-A}^q | \Lambda_b >}{2 M_{\Lambda_b}} = f_{B_q}^2 M_B \frac{48}{r} \]  
(15)

\[ < \Lambda_b | \tilde{\mathcal{O}}_{S-P}^q | \Lambda_b > = - \tilde{B} < \Lambda_b | \tilde{\mathcal{O}}_{S-P}^q | \Lambda_b > \]  
(16)

one has that, for $f_{B} = 200$ MeV and $r = 1$, eq. (15) corresponds to $< \tilde{\mathcal{O}}_{V-A}^q | \Lambda_b > = 4.4 \times 10^{-3}$ GeV$^3$. Quark models predict $< \tilde{\mathcal{O}}_{V-A}^q | \Lambda_b > \simeq 0.75 - 2.5 \times 10^{-3}$ GeV$^3$, corresponding to $r \simeq 0.2 - 0.6$; larger values can be obtained using the mass splitting $\Sigma^*_b - \Sigma_b$ and $\Sigma^*_c - \Sigma_c$: $r \simeq 0.9 - 1.8$. In the valence quark approximation $\tilde{B} = 1$.

A value of $r$: $r \simeq 4 - 5$ would explain the difference between $\tau(\Lambda_b)$ and $\tau(B_d)$. A calculation by QCD sum rules, however, seems to exclude this possibility.
\section{\(\langle \hat{O}_{V-A}^q \rangle_{\Lambda_b}\) from QCD sum rules}

An estimate of \(\langle \hat{O}_{V-A}^q \rangle_{\Lambda_b}\) can be obtained by QCD sum rules, in HQET, analyzing the three-point correlator

\[
\Pi_{CD}(\omega, \omega') = (1 + \gamma^\mu) \Pi_i(\omega, \omega')
\]

\[
\approx i \int dxdy <0|T[J_C(x)\hat{O}_{V-A}^q(0)\bar{J}_D(y)]|0 > \times e^{i\omega x - i\omega' y}
\]

of the baryonic currents \(J(x)\) and \(\bar{J}(y)\) and of the operator \(\hat{O}_{V-A}^q\) in (13). The variables \(\omega, \omega'\) are related to the residual momentum of the currents: \(p^\mu = m_b v^\mu + \omega v^\mu\). Choosing \(J\) with a non-vanishing projection on the \(\Lambda_b\) state

\[
<0|J_C|\Lambda_b v) = f_{\Lambda_b} (\psi_v) C
\]

\((\psi_v\) is a spinor for a \(\Lambda_b\) of four-velocity \(v\)), the parameter \(f_{\Lambda_b}\) representing the coupling of \(J\) to \(\Lambda_b\), the matrix element \(\langle \hat{O}_{V-A}^q \rangle_{\Lambda_b}\) can be obtained by saturating the correlator (17) with baryonic states, and considering the double pole contribution in the variables \(\omega\) and \(\omega'\):

\[
\Pi^h(\omega, \omega') = \langle \hat{O}_{V-A}^q \rangle_{\Lambda_b} \frac{f_{\Lambda_b}^2}{(\Delta_{\Lambda_b} - \omega)(\Delta_{\Lambda_b} - \omega')}
\]

at the value \(\omega = \omega' = \Delta_{\Lambda_b}\). The mass parameter \(\Delta_{\Lambda_b}\) represents the binding energy of the light \(\Lambda_b\) degrees of freedom in the static color field generated by the \(b\)-quark: \(M_{\Lambda_b} = m_b + \Delta_{\Lambda_b}\); it must also be derived by QCD sum rules.

A possible interpolating field for \(\Lambda_b\) reads:

\[
J_C(x) = e^{ikT(T^i)(x)}(\Gamma) h_i(x) C(x)
\]

where \(T\) means transpose, \(i, j\) and \(k\) are color indices, and \(C\) is the Dirac index of the effective heavy quark field \(h_i(x)\): \(\tau\) is a light flavour matrix corresponding to zero isospin. In the \(m_b \to \infty\) limit, the light diquark in the \(\Lambda_b\) is in a relative \(0^+\) spin-parity state; therefore one can use in (21):

\[
\Gamma = C\gamma_5 (1 + b\gamma_\tau)
\]

\(C\) being the charge conjugation matrix and \(b\) a parameter. Arguments can be found in favour of the choice \(b = 1\) in [3].

The coupling \(f_{\Lambda_b}\) can be computed from:

\[
H_{CD}(\omega) = (1 + \gamma^\mu)_{CD} H(\omega)
\]

\[
\approx i \int dx <0|T[J_C(x)\bar{J}_D(0)]|0 > e^{i\omega x}
\]

In the Euclidean region the correlation functions (17) and (22) can be computed in QCD, in terms of a perturbative contribution and vacuum condensates:

\[
\Pi^{OPE}(\omega, \omega') = \int d\sigma d\sigma' \frac{\rho_1(\sigma, \sigma')}{(\sigma - \omega)(\sigma' - \omega')}
\]

\[
H^{OPE}(\omega) = \int d\sigma \frac{\rho_H(\sigma)}{(\sigma - \omega)},
\]

with the spectral functions given by:

\[
\rho_1(\sigma, \sigma') = \rho_1^{(pert)}(\sigma, \sigma') + \rho_1^{(D=3)}(\sigma, \sigma') < \bar{q}q >
\]

\[
+ \rho_1^{(D=4)}(\sigma, \sigma') < \bar{q}g \sigma Gq >
\]

\[
+ \rho_1^{(D=5)}(\sigma, \sigma') < \bar{q}g \sigma Gq >
\]

\[
+ \rho_1^{(D=6)}(\sigma, \sigma') < \bar{q}q >^2 + \ldots
\]

and a a similar expression for \(\rho_H(\sigma)\).

The various terms in \(\rho_1(\sigma, \sigma')\) and \(\rho_H(\sigma)\) can be found in [3].

A sum rule for \(\langle \hat{O}_{V-A}^q \rangle_{\Lambda_b}\) can be derived by equating the hadronic and the OPE representations of the correlator (17), and modeling the contribution of the higher resonances and of the continuum in \(\Pi^h\) as the QCD term outside the region \(0 \leq \omega \leq \omega_c, 0 \leq \omega' \leq \omega_c, \omega_c\) being an effective threshold. After a double Borel transform one gets:

\[
\frac{f_{\Lambda_b}^2}{2} (1 + b)^2 e^{-\frac{\Delta_{\Lambda_b}}{M}} \langle \hat{O}_{V-A}^q \rangle_{\Lambda_b}
\]

\[
= \int_{\omega_c}^{\infty} \int_{\omega_c}^{\omega_c} \frac{\omega}{\omega'} d\omega d\omega' e^{-\frac{\Delta_{\Lambda_b}}{M}} \rho_1(\sigma, \sigma')
\]

where \(E\) is a Borel parameter. The threshold \(\omega_c\) can be fixed in the QCD sum rule determination of \(f_{\Lambda_b}\) and \(\Delta_{\Lambda_b}\). \(\omega_c = 1.1 - 1.3\ GeV\); in correspondence one obtains \(f_{\Lambda_b} = (2.9 \pm 0.5) \times 10^{-2}\ GeV^3\) and \(\Delta_{\Lambda_b} = 0.9 \pm 0.1\ GeV\).

The resulting sum rule is depicted in fig.1. A stability window is observed for \(E > 0.2\ GeV\). In the duality region \(E \approx 0.2 - 0.3\ GeV\) (the same region considered in the QCD sum rule analysis of \(f_{\Lambda_b}\) and \(\mu_b^2(\Lambda_b)\)) we find:

\[
\langle \hat{O}_{V-A}^q \rangle_{\Lambda_b} \approx (0.4 - 1.20) \times 10^{-3}\ GeV^3
\]

which corresponds to \(r \approx 0.1 - 0.3\). This result is confirmed by an analysis based on the assumption of local quark-hadron duality, that amounts to calculate the matrix elements of \(\langle \hat{O}_{V-A}^q \rangle_{\Lambda_b}\) and \(f_{\Lambda_b}\).
by free quark states produced and annihilated by the baryonic currents in \{17\} and \{22\} and then averaging on a duality interval in $\omega, \omega'$. Considering, finally, the parameter $\hat{B}$ in eq.\{16\}, one gets $\hat{B} = 1$ since, in this computational scheme, only valence quark processes are taken into account.

3 Conclusions

Within the uncertainties of the method, QCD sum rules predict small values for the matrix elements $\langle \hat{O}_{V-A}^q \rangle_{\Lambda_b}$ and $\langle \hat{O}_{V-A}^q \rangle_{\Lambda_c}$, comparable with the outcome of constituent quark models. The conclusion is that the inclusion of $1/m_Q^4$ terms in the expression of the inclusive widths does not solve the puzzle represented by the difference between $\tau(\Lambda_b)$ and $\tau(B_d)$: using the formulae in \{28\} for the lifetime ratio, the value in \{28\} together with $\hat{B} = 1$ gives:

$$\tau(\Lambda_b)/\tau(B_d) \geq 0.94 \quad . \quad (29)$$

It seems unlikely that order $m_Q^4$ terms can solve the problem. We must conclude that, if the measurement of $\tau(\Lambda_b)$ and $\tau(B_d)$ will be confirmed, a reanalysis of the problem will be required. In particular, it has been proposed to consider the possibility of the failure of the assumption, made in the calculation of the lifetimes, of local quark-hadron duality in nonleptonic inclusive decays \{3\}. Meanwhile, it is interesting that new data are now available for other $b$-flavoured hadrons, e.g. $\Xi_b$, although with errors too large to perform a meaningful comparison with $\Lambda_b$. Such new information will be of great importance for the full understanding of the problem of the beauty hadron lifetimes.

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