Effects of Cosmological Constant on Clustering of Galaxies

Mir Hameeda, Sudhaker Upadhyay, Mir Faizal and Ahmed Farag Ali

1 Department of Physics, S.P. College, Srinagar, Kashmir, 190001, India & Inter University Center for Astronomy and Astrophysics, IUCAA, Pune-411007, India
2 Centre for Theoretical Studies, Indian Institute of Technology Kharagpur, Kharagpur-721302, India
3 Irving K. Barber School of Arts and Sciences, University of British Columbia - Okanagan, 3333 University Way, Kelowna, British Columbia V1V 1V7, Canada & Department of Physics and Astronomy, University of Lethbridge, Lethbridge, Alberta, T1K 3M4, Canada
4 Netherlands Institute for Advanced Study, Korte Spinhuissteeg 3, 1012 CG Amsterdam, Netherlands & Department of Physics, Faculty of Science, Benha University, Benha, 13518, Egypt

ABSTRACT

In this paper, we analyse the effect of the expansion of the universe on the clustering of galaxies. We evaluate the configurational integral for interacting system of galaxies in an expanding universe by including effects produced by the cosmological constant. The gravitational partition function is obtained using this configuration integral. Thermodynamic quantities, specifically, Helmholtz free energy, entropy, internal energy, pressure and chemical potential are also derived for this system. It is observed that they depend on the modified clustering parameter for this system of galaxies. It is also demonstrated that these thermodynamical quantities get corrected because of the cosmological constant.

Key words: Cosmology: theory—dark energy; Clusters: general—gravitation—fluctuation—large scale structure of universe—method; analytical

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1 OVERVIEW AND MOTIVATION

It has been established that the Universe has an acceleration in its expansion, and this is based on the observations from the Type Ia Supernovae (SNeIa) Riess et al. (2004). Thus, there seems to be some dark energy contribution to the total energy density of the Universe. Furthermore, the observations from the anisotropies in the cosmic microwave background radiation (CMBR) Spergel et al. (2003), and inferred matter power spectrum from large galaxy surveys Cole et al. (2005) can be used to make similar predictions. According to the cosmological constant Λ cold dark matter (ΛCDM) model, the baryons contribute only for ~4%, while the exotic cold dark matter (CDM) represents the bulk of the matter contributes ~25% and the cosmological constant Λ plays the role of the so called “dark energy” contributes ~70% of the universe Bahcall et al. (1999). It is expected that the general relativity might get corrected as its validity on the larger and smaller scales has not been verified Will (1993). Therefore, to explain the cosmic speed up and dark matter, it is possible to generalize the Hilbert-Einstein Lagrangian, which is linear in the Ricci scalar R. This is the basic idea behind the study of f(R) theories of gravity Capozziello et al. 2003; Carroll et al. 2004; Flanagan 2003; Allemandi et al. 2004; Nojiri & Odintsov 2007; Capozziello et al. 2005. It may be also noted that various models of dark energy have
been proposed to explain the late-time cosmic acceleration without the cosmological constant. For instance, these models are a non-canonical scalar field such as phantom Caldwell (2002), tachyon scalar field motivated by string theories Padmanabhan (2002), a fluid with a special equation of state called as Chaplygin gas Kamenshchik et al. (2001); Bento et al. (2002); Bilic et al. (2002, 2009) and a canonical scalar field, so-called quintessence Caldwell et al. (1998). In fact, some proposals of holographic dark energy have also been proposed Li (2004); Elizalde et al. (2005); Nojiri & Odintsov (2006). The ΛCDM model, in which dark energy is well-represented by Λ in Einstein’s gravity, and it is also supported by various cosmological observations. So, despite of several complications of baryonic astrophysics and lack of understanding behind the theoretical origin of the cosmological constant Λ Weinberg (1989), the ΛCDM model is considered as the standard cosmological model describing the Universe on large scales.

It is also possible to analyse the effect of dark energy in the form of cosmological constant directly on the partition function of galaxies. In this analysis, the galaxies can be approximated as point particles interacting through the Newton’s law, and the effect of dark energy can be introduced as an additional correction term in this partition function. In this approximation, we have a system of galaxies which can be approximated by a system of particles for our analysis, and thus formalism of statistical mechanics can be used for analysis this model Ahmad et al. (2002). Thus, it is possible to study the thermodynamics of interacting system of galaxies in the expanding universe from the using the formalism of statistical mechanics. The formalism of statistical mechanical has been used to analyse the observed peculiar velocity distribution function for a sample of galaxies within 50Mpc (H = 100) of the Local Group Raychaudhury & Saslaw (1996). In this study, a wide range of clustering properties for this sample of galaxies have been analysed. In this study, the effects of uncertainties in sampling on the estimated distribution function have also been studied. The peculiar velocity distribution function of galaxies has been used to analyse a system of galaxies with haloes Leong & Saslaw (2004). In this study, it was demonstrated that individual massive galaxies are usually surrounded by their own halos, and they are not embedded in common halos. This was done by comparing this study with the observed peculiar velocity distributions.

The spatial distribution function of galaxies at high redshift have also been analysed using the formalism of statistical mechanics Rahmani et al. (2009). It has been demonstrated that the redshifts of the galaxy spatial distribution function has the same form as predicted by gravitational quasi-equilibrium dynamics. This observation constrains the processes such as merging of galaxy. The probability that a galaxy cluster of a given shape exists has also been analysed using the formalism of statistical mechanics Yang & Saslaw (2012). This has been done using the observation that the distribution of galaxies is very close to quasi-equilibrium. This hold for both its linear and nonlinear regimes. The statistical mechanical formalism of cosmological many-body problem has also been used to analyse a system of two different kind of galaxies Ahmad et al. (2006). The general partition function for such a system has been obtained in the grand canonical ensemble. This has been used to obtain various thermodynamical quantities for such a system of galaxies. It has also been demonstrated that a softening parameter can be introduced in the partition function for galaxies, if the finite size of galaxies is taken into account Malik et al. (2009). Thus, it is well established that the formalism of statistical mechanics can be used for analysing the clustering of galaxies.

In this paper, we analyse the effect of the cosmological constant on the clustering of galaxies. This analysis is accomplished by deriving the gravitational partition function for galaxies in a universe with a cosmological constant. We utilize here the configuration integrals over a spherical volume. The exact equations of state for galaxies is also obtained, by computing Helmholzt free energy, entropy, internal energy, pressure and chemical potential, which depend on the corrected clustering parameter explicitly. The corrections to the clustering parameter are analysed for both point mass and non-point mass particles. We investigate the effect of the cosmological constant on the distribution function for galaxies. We analyse the effect of approximating a galaxy as a point mass and as an extended structure. Thus, we are able to analyse the effect of dark energy on the clustering of galaxies.

The organization of this paper is as follows. In section II, we elucidate the partition function for a system of galaxies with a cosmological constant. The various thermodynamic quantities get corrected, and such corrections are derived in section III. The details for general distribution functions are reported in section IV. In this last section we also summarize our results.

2 GRAVITATIONAL PARTITION FUNCTION

The a system of galaxies can be approximated as particle with pairwise interaction. It is assumed that the distribution is statistically homogeneous over large regions. The general partition function of a system of N such galaxies of mass m interacting gravitationally with a potential energy Φ, having momenta p, and average temperature T Ahmad et al. (2002);

\[
Z(T, V) = \frac{1}{N!} \int d^{3N} r_1 \ldots d^{3N} r_N \exp \left( - \frac{1}{2m} \sum_{i=1}^{N} \frac{p_i^2}{2m} + \Phi(r_1, r_2, r_3, \ldots, r_N) \right)^{T^{-1}}. 
\]

(1)

where N! takes the distinguish-ability of classical particles into account, and λ refers the normalization factor resulting from integration over momentum space.

Performing integration over momentum space yields,

\[
Z_N(T, V) = \frac{1}{N!} \left( \frac{2\pi m T}{\lambda^2} \right)^{N/2} Q_N(T, V),
\]

(2)

where the configurational integral, Q_N(T, V), is given by

\[
Q_N(T, V) = \int \ldots \int_{1 \leq i < j \leq N} \exp \left[ -\phi(r_{ij}) T^{-1} \right] d^{3N} r.
\]

(3)

In general, the gravitational potential energy, given as,

\[
\phi(r_{ij}) = \Phi(r_{ij}) = \Phi(r_1, r_2, \ldots, r_N),
\]

(4)
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is a function of the relative position vector \( r_{ij} = |r_i - r_j| \) and is the sum of the potential energies of all pairs. In a gravitational system, the potential energy \( \Phi(r_1, r_2, \ldots, r_N) \) is due to all pairs of particles composing the system.

Here, it is worth defining two particle function by

\[
 f_{ij} = e^{-\phi(r_{ij})/T} - 1, 
\]

which appears only if the interactions are present in the system, however, it disappears in absence of interactions.

It has been shown by Ahmad et al. (2002) that the configurational integral can be expressed as

\[
 Q_N(T, V) = \int \cdots \int \left[ (1 + f_{12})(1 + f_{13})(1 + f_{23}) \right. \\
 \left. \cdots (1 + f_{N-1,N}) \right] d^3 r_1 d^3 r_2 \cdots d^3 r_N . 
\]

For point masses, the Hamiltonian and, hence, the partition function diverge at \( r_{ij} = 0 \). This divergence has been removed by taking the extended nature of particles into account, via introducing a softening parameter which takes care of the finite size of each galaxy. Thus, by introducing the softening parameter, the Newtonian (interaction) potential energy between particles is given by

\[
 \Phi(r_{ij}) = -\frac{Gm^2}{(r_{ij}^2 + \epsilon^2)^{1/2}}, 
\]

where \( \epsilon \) represents the softening parameter.

In the cosmological constant \( \Lambda \) model, where the galaxies interact via Newtonian potential, the potential energy with the softening parameter, is given by Shtanov & Sahni (2010)

\[
 \Phi(r_{ij}) = -\frac{Gm^2}{(r_{ij}^2 + \epsilon^2)^{1/2}} - \frac{\Lambda r_{ij}^2}{6} . 
\]

Here, we do not introduce the softening parameter \( \epsilon \) in the second term of right hand side as corresponding Hamiltonian does not diverges.

Now, the expression of two particle function (5) for above potential energy reads,

\[
 f_{ij} = \exp \left\{ \frac{Gm^2}{(r_{ij}^2 + \epsilon^2)^{1/2}} + \frac{\Lambda r_{ij}^2}{6T} \right\} - 1 . 
\]

Assuming moderately dilute systems, the two particle function \( f_{ij} \) upon expansion gives

\[
 f_{ij} = \left[ \frac{Gm^2}{(r_{ij}^2 + \epsilon^2)^{1/2}} + \frac{\Lambda r_{ij}^2}{6T} \right] . 
\]

Evaluating the configuration integrals over a spherical volume of radius \( R_1 \), which utilize equations (3) and (6), gives

\[
 Q_1(T, V) = V . 
\]

Now, configuration integral \( Q_2(T, V) \) can be written as

\[
 Q_2(T, V) = 4\pi V \int_0^{R_1} \left[ 1 + \frac{Gm^2}{T(r^2 + \epsilon^2)^{1/2}} + \frac{\Lambda r^2}{6T} \right] r^2 dr . 
\]

Upon performing integration, this yields

\[
 Q_2(T, V) = V^2 \left[ 1 + \frac{\Lambda R_1^2}{10T} + \frac{Gm^2}{R_1 T} \left( \frac{1}{2} \sqrt{1 + \frac{\epsilon^2}{R_1^2}} \right) \right. \\
 \left. + \frac{1}{2} \frac{\epsilon^2}{R_1} \log \left( \frac{\epsilon/R_1}{1 + \sqrt{1 + \frac{\epsilon^2}{R_1^2}}} \right) \right] . 
\]

With the help of following definitions:

\[
 \alpha_1 \left( \frac{\epsilon}{R_1} \right) = \sqrt{1 + \frac{\epsilon^2}{R_1^2} + \frac{\epsilon^2}{R_1} \log \left( \frac{\epsilon/R_1}{1 + \sqrt{1 + \frac{\epsilon^2}{R_1^2}}} \right)} , \\
 \alpha_2 = \frac{\Lambda R_1^3}{15Gm^2} . 
\]

The above expression \( Q_2(T, V) \) can further be written in compact form as,

\[
 Q_2(T, V) = V^2 \left( 1 + \frac{3}{2} (\alpha_1 + \alpha_2) \left( \frac{Gm^2}{R_1 T} \right)^3 \right) . 
\]

Here we have utilized the scale transformations \( \rho \to \lambda^{-1} \rho \cdot T \to \lambda^{-1} T \) and \( R_1 \to \lambda R_1 \) also, to transform \( \frac{Gm^2}{R_1 T} \to \left( \frac{Gm^2}{R_1 T} \right)^3 \). Since \( R_1 \sim \rho^{-1/3} \approx (\bar{N}/V)^{-1/3} \), so we can write

\[
 \frac{3}{2} \left( \frac{Gm^2}{R_1 T} \right)^3 = \frac{3}{2} \left( \frac{Gm^2}{T} \right)^3 \rho : = x . 
\]

Thus, we can write equation (15) as:

\[
 Q_2(T, V) = V^2 (1 + \alpha x) , 
\]

where

\[
 \alpha \left( \frac{\epsilon}{R_1} \right) = \alpha_1 \left( \frac{\epsilon}{R_1} \right) + \alpha_2 . 
\]

For the point masses (i.e., \( \epsilon = 0 \)), \( \alpha \) reduces to

\[
 \alpha (\epsilon = 0) = 1 + \alpha_2 . 
\]

Following similar procedure, we obtain configurational integral for higher orders as:

\[
 Q_3(T, V) = V^3 (1 + \alpha x)^2 , 
\]

and

\[
 Q_4(T, V) = V^4 (1 + \alpha x)^3 . 
\]

Thus, for most general case, we have

\[
 Q_N(T, V) = V^N (1 + \alpha x)^{N-1} . 
\]

Hence, the gravitational partition function is obtained explicitly by substituting the value of \( Q_N(T, V) \) given in (22) to (2)

\[
 Z_N(T, V) = \frac{1}{N!} \left( \frac{2\pi m T}{\hbar^2} \right)^{3N/2} V^N (1 + \alpha x)^{N-1} . 
\]

Here, the effect of cosmological constant, embedded in \( \alpha \), can be seen in the expression of the gravitational partition function.
3 EQUATIONS OF STATE

We compute below the various thermodynamic quantities relevant for strongly interacting system of galaxies interacting through a Newtonian potential describing galaxies interacting with each other. It is well-known that the thermodynamic quantities can be easily calculated from the gravitational partition function. For example, Helmholtz free energy, defined, generally, by

\[ F = -T \ln Z_N(T, V) \]

is calculated as

\[ F = -T \ln \left( \frac{1}{N!} \left( \frac{2 \pi m T}{\lambda^2} \right)^{3N/2} V^N (1 + \alpha x)^{N-1} \right). \]  

(24)

Further simplification leads to

\[ F = N T \ln \left( \frac{N}{V} T^{-3/2} \right) - N T - (N - 1) T \ln (1 + \alpha x) - \frac{3}{2} N T \ln \left( \frac{2 \pi m}{\lambda^2} \right). \]  

(25)

Now, it is easy to compute entropy \( S \) for a given Helmholtz free energy with formula,

\[ S = - \left( \frac{\partial F}{\partial T} \right)_{N, V}. \]

Here entropy reads

\[ S = N \ln \left( \frac{V}{N} T^{3/2} \right) + (N - 1) \ln (1 + \alpha x) - \frac{3N}{2} \ln \left( \frac{2 \pi m}{\lambda^2} \right) + \ln N, \]  

(26)

For large \( N \) such that \( N - 1 \approx N \), this can further be simplified as

\[ S = N \ln \left( \frac{V}{N} T^{3/2} \right) - \ln (1 - \frac{\alpha x}{1 + \alpha x}) - 3 \frac{\alpha x}{1 + \alpha x} + S_0 - \ln N. \]  

(27)

where definition \( S_0 = \frac{1}{2} N + \frac{3}{2} N \ln \left( \frac{2 \pi m}{\lambda^2} \right) \) is utilized. Comparing this expression to its standard form \( \text{Ahmad et al. (2002)} \), the clustering parameter of galaxies in the expanding universe, \( \mathcal{B} \), is derived as by

\[ \mathcal{B} = \frac{\alpha x}{1 + \alpha x}. \]  

(28)

Evaluating the clustering parameter is worth because it plays a crucial role in finding various thermodynamic quantities. For the point masses, \( \epsilon = 0 \), the clustering parameter in modified potential becomes

\[ \mathcal{B}(\epsilon = 0) := B_0 = \frac{(1 + \alpha x) x}{1 + x(1 + \alpha x)}. \]  

(29)

Thus, due to cosmological constant modified potential, the clustering parameter for the point masses get following correction:

\[ B_0 = b \left( \frac{1 + \alpha x}{1 + b \alpha x} \right), \]  

(30)

where \( b = \frac{x}{1 + x} \). \( \text{Ahmad et al. (2002)} \) refers the original clustering parameter of Newtonian potential for point masses.

Employing expression \( (24) \) and \( (26) \), the internal energy, defined as \( U = F + TS \), for a system of galaxies is calculated by

\[ U = \frac{3}{2} N T (1 - 2 \mathcal{B}). \]  

(31)

It is evident from the above expression that the internal energy depends on the cosmological constant embedded in clustering parameter \( \mathcal{B} \).

The pressure and chemical potential, utilizing the standard notations and definitions \( P = -\left( \frac{\partial F}{\partial V} \right)_{N, T} \) and \( \mu = \left( \frac{\partial F}{\partial N} \right)_{V, T} \), respectively, are calculated by

\[ P = \frac{N T}{V} (1 - \mathcal{B}), \]  

(32)

\[ \mu = T \ln \left( \frac{N}{V} T^{-3/2} \right) + T \ln (1 - \mathcal{B}) - \frac{3}{2} T \ln \left( \frac{2 \pi m}{\lambda^2} \right) - \mathcal{B} T. \]  

(33)

Here, we observe that the pressure and chemical potential also get correction due to the cosmological constant as clustering parameter depends on cosmological constant explicitly.

4 GENERAL DISTRIBUTION FUNCTION

The definition of grand canonical partition function is,

\[ Z_G(T, V, z) = \sum_{N=0}^{\infty} z^N Z_N(V, T), \]

(34)

where \( z \) is the activity. The grand partition function for our gravitationally interacting system of galaxies is calculated by

\[ \ln Z_G = \frac{PV}{T} = N (1 - \mathcal{B}), \]  

(35)

where the expression \( (32) \) is utilized.

Now, the probability of finding \( N \) particles in volume \( V \) can be estimated by relation

\[ F(N) = \sum_{N=0}^{\infty} e^{\frac{N \epsilon}{\mathcal{B}}} \frac{1}{Z_G(T, V, z)} = \frac{e^{\frac{N \epsilon}{\mathcal{B}}} Z_N(V, T)}{Z_G(T, V, z)}. \]  

(36)

Here, with the help of \( (35) \), the distribution function for a system of point masses is computed, precisely, by

\[ F(N, \epsilon = 0) = N^N \left( \frac{1 + \mathcal{B}_0}{N (1 - \mathcal{B}_0)} \right)^{N - 1} \times \left( 1 + \frac{\mathcal{B}_0}{1 - \mathcal{B}_0} \right)^{-N} e^{-N \mathcal{B}_0 - \mathcal{B} (1 - \mathcal{B}_0)}. \]  

(37)

In the similar fashion, the distribution function for non-point mass particles is derived as

\[ F(N, \epsilon) = \frac{N^N}{N!} \left( 1 + \frac{N \mathcal{B}}{N (1 - \mathcal{B})} \right)^{N - 1} \times \left( 1 + \frac{\mathcal{B}}{1 - \mathcal{B}} \right)^{-N} e^{-N \mathcal{B} \mathcal{B} - \mathcal{B} (1 - \mathcal{B})}. \]  

(38)

Remarkably, the structure of resulting distribution function coincides exactly with the derived earlier \( \text{Ahmad et al. (2002)} \). The only difference here we get, the cosmological constant corrected clustering parameter \( \mathcal{B} \) in place of usual clustering parameter \( b \).
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Figure 1. Comparative study of distribution function $F(N)$ with and without dark energy corrections for $b = 0.3$, $N_0 = 10$. Here for point mass $\alpha_1 = 1$ with dark energy (i.e. $\alpha_2 = 1$) and without dark energy (i.e. $\alpha_2 = 0$) and for extended mass $\alpha_1 = 0.7$ with dark energy (i.e. $\alpha_2 = 1$) and without dark energy (i.e. $\alpha_2 = 0$).

5 DISCUSSION AND CONCLUSION

In this section, we draw a comparative analysis to emphasize the effect of dark energy corrections on the distribution function. Here, in figure 1, we consider both the cases of point mass in absence and presence of dark energy and the extended mass in absence and presence of dark energy. The point mass in absence of dark energy has the highest value peak $F(N)$, and the point mass with dark energy has the lowest value peak value of $F(N)$. The extended mass has a softening parameter, and so the peak values of extended mass for both these cases lies in between these two values. The peak value of the extended mass with dark energy is less than the peak value of extended mass in absence of dark energy. It is also observed that $F(N)$ for the point mass in absence of dark energy reduces faster than all the other cases. Thus, its value becomes lower than all the other cases, at later stages. This distribution can be used to compare the predicted values of $F(N)$, with the observed values and thus used to check the validity of this analysis.

In this paper, we have considered a strongly interacting system of galaxies in the expanding universe and derived, utilizing the configuration integrals over a spherical volume, the gravitational partition function for galaxies interacting with each other. The cosmological constant correction is evident in the expression obtained for gravitational partition function. We have computed various thermodynamical quantities, for example, Helmholtz free energy, entropy, internal energy, pressure and chemical potential, which depend on cosmological constant modified clustering parameter explicitly, to study the exact equations of state for galaxies. The cosmological constant modified clustering parameter is compared with the original clustering parameter obtained for Newtonian potential. Further, we have studied the general distribution function for such system. We would like to point out that unlike the Jacobson formalism Jacobson (1995), in this paper, the fundamental laws of gravity are not obtained using thermodynamics, but only the thermodynamical consequences of a system of galaxies are analysed.

The modification in the gravity action can affect the gravitational potential in the low energy limit and the modified potential reduces to the Newtonian one on the solar system scale as well. It has been seen in Capozziello et al. (2004); Milgrom (1983); Bekenstein (2004); Capozziello et al. (2006) that the modified gravitational potential could fit galaxy rotation curves without considering dark matter or dark energy. In this, this provides an opportunity to draw a formal analogy between the corrections due to the modified Newtonian potential and the dark energy models. In general, a relativistic gravity theory leads to a change to the Newton potential Schmidt (2004). In the post-Newtonian formalism, this could provide tests for the theory Will (1993); Capozziello & Troisi (2005); Capozziello et al. (2006); Allemandi et al. (2005). Verlinde’s derivation of laws of gravitation provides a new direction to understand gravity from the first principles. The entropic force interpretation gets relevance in various contexts Liu et al. (2010); Kislev & Timofeev (2010); Konoplya (2010); Banerjee & Majhi (2010); Nicolini (2010); Gao (2010); Myung & Kim (2010); Wei (2010); Easson et al. (2012); Wei et al. (2011). The Friedmann equations governing the dynamical evolution of the FRW universe from the viewpoint of entropic force together with the equipartition law of energy and the Unruh temperature is advocated Cai et al. (2010); Ling & Wu (2010). Corrections to Newton’s law of gravitation as well as modified Friedmann equations from the entropic force point of view are discussed in Sheykhi (2010); Sheykhi & Hendii (2011). Further entropic interpretation of gravity has also been used to study the modified Newton’s law Modesto & Randono (2010), the Newtonian gravity in loop quantum gravity Smolin (2010), the holographic dark energy Li & Wang (2010); Easson et al. (2011); Danielsson (2010) thermodynamics of black holes Tian & Wu (2010) and the extension to Coulomb force Wang (2010). It may be noted that it is possible to study the clustering of galaxies using this modified Newton’s law. In fact, Newton’s law also gets modified due to brane world effects, and the clustering parameter of galaxies has been studied using such a modified Newton’s law Hameed et al. (2016). Thus, it would be interesting to analyse the effect of modified Newton’s law obtained from entropic force on the clustering of galaxies.

It may be noted that the clustering of galaxies has also been studied using the modified Newtonian potential produced by $f(R)$ gravity De Martino et al. (2014); Capozziello et al. (2009). In fact these models can explain the dynamics of spiral and elliptical galaxies, even in absence of dark matter. This analysis was performed through the pressure profile of galaxy clusters. It was assumed that this is in hydrodynamic equilibrium within the potential well of the modified gravitational potential. Furthermore, it was demonstrated that this model is consistent with Planck data. We expect a similar effect to occur, if we perform this analysis by modifying the gravitational field by the cosmological constant. This is because the cosmological constant can be obtained from a suitable $f(R)$ gravity model Cruz-Dombriz & Dobado (2006). Now as it has been demonstrated that the $f(R)$ gravity is consistent with Planck data De Martino et al. (2014); Capozziello et al. (2009), we ex-
pect the model studied in this paper to also be consistent with the Planck data. This can be observed from the fact that both the modified Newtonian constant and the cosmological constant term will modify the gravitational potential energy term in the partition function, and hence, we expect on physical grounds that they will produce similar results. We would also like to comment that unlike the approach where the distribution of galaxies was studied using the standard Boltzmann-Vlasov equation, in our approach we analyses the thermodynamical properties of galaxies. Just like the other approaches, here, the galaxies are approximated as point particles, but calculating thermodynamic quantities for such systems makes it easier to make physical predication using this approach. We would also like to comment that as such a phase transition can also be studied for the model proposed in this paper, and the modified Newtonian potential by $f(R)$ gravity.

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