We consider a model for tensionless (null) super $p$-branes in the Hamiltonian approach and in the framework of a harmonic superspace. The obtained algebra of Lorentz-covariant, irreducible, first class constraints is such that the BRST charge corresponds to a first rank system.
1 Introduction

The tensionless (null) \( p \)-branes correspond to usual \( p \)-branes with their tension

\[ T = (2\pi\alpha')^{-(p+1)/2} \]

taken to be zero. This relationship between null \( p \)-branes and the tensionful ones may be regarded as a generalization of the massless-massive particles correspondence. On the other hand, the limit \( T \to 0 \) (because of \((\alpha')^{-1} \propto M_{\text{Plank}}^2\)) corresponds to the energetic scale \( E >> M_{\text{Plank}} \). In other words, the null \( p \)-brane is the high energy limit of the tensionful one. There exist also an interpretation of the null and free \( p \)-branes as theories, corresponding to different vacuum states of a \( p \)-brane, interacting with a scalar field background [1]. So, one can consider the possibility of tension generation for null \( p \)-branes (see [2] and references therein).

Another viewpoint on the connection between null and tensionful \( p \)-branes is that the null one may be interpreted as a "free" theory opposed to the tensionful "interacting" theory [3]. All this explains the interest in considering null \( p \)-branes and their supersymmetric extensions.

Models for tensionless \( p \)-branes with manifest supersymmetry are proposed in [4]. In [1] a twistor-like action is suggested, for null super-\( p \)-branes with \( N \)-extended global supersymmetry in four dimensional space-time. Then, in the framework of the Hamiltonian formalism, the initial algebra of first and second class constraints is converted into an algebra of first class effective constraints only. The obtained BRST charge corresponds to second rank theory. It is proven that there are no quantum anomalies when the so called ”generalized” \( \hat{q}\hat{p} \) operator ordering is applied. In the recent work [5], among other problems, the quantum constraint algebras of the usual and conformal tensionless spinning \( p \)-branes are considered.

In a previous paper [6], we announced for a null super \( p \)-brane model which possesses the following classical constraint algebra

\[
\begin{align*}
\{T_0(\sigma_1), T_0(\sigma_2)\} &= 0, \\
\{T_0(\sigma_1), T^A_\alpha(\sigma_2)\} &= 0, \\
\{T_j^A(\sigma_1), T_k^B(\sigma_2)\} &= [T_0(\sigma_1) + T_0(\sigma_2)]\partial_j\delta^p(\sigma_1 - \sigma_2), \\
\{T_j^A(\sigma_1), T^A_\alpha(\sigma_2)\} &= \delta^{AB}[s_j^B T^B_k(\sigma_1) + s_k^B T^A_j(\sigma_2)]\partial_j\delta^p(\sigma_1 - \sigma_2), \\
\{T^A_\alpha(\sigma_1), T^B_\beta(\sigma_2)\} &= -2i\delta^{AB}\hat{P}_{\alpha\beta} T_0(\sigma_1)\delta^p(\sigma_1 - \sigma_2), \\
\hat{P}_{\alpha\beta} &= P_\mu\sigma^\mu_{\alpha\beta}.
\end{align*}
\]

Here \( \sigma = (\sigma_1, ..., \sigma_p), (j, k = 1, ..., p), (A, B = 1, ..., N), \) where \( N \) is the number of the supersymmetries, \( \alpha, \beta \) are spinor indices and \( P_\mu \) is a Lorentz vector \((\mu = 0, 1, ..., D - 1)\). In [1] and below we do not write explicitly the dependence of the quantities on the time parameter \( \tau \).

In this letter, we consider the particular case of \( N = 1, D = 10 \) tensionless superbrane. We work in the Hamiltonian approach and in the framework of a harmonic superspace. Passing
to a system of Lorentz-covariant, irreducible first class constraints, we obtain a BRST charge as for a first rank theory.

2 Hamiltonian formulation

Let us begin with writing the initial Hamiltonian of the dynamical system under consideration

\[ H_0 = \int d^p \sigma [\mu^0 T_0 + \mu^j T_j + \mu^\alpha D_\alpha] \]

\( H_0 \) is a linear combination of the constraints \( T_0(\sigma), T_j(\sigma) \) and \( D_\alpha(\sigma) \). The latter are given by the expressions:

\[
\begin{align*}
T_0 &= p_\mu p_\nu \eta^{\mu\nu}, & \text{diag}(\eta_{\mu\nu}) &= (-, +, \ldots, +), \\
T_j &= p_\nu \partial_j x^\nu + p_{\theta\alpha} \partial_j \theta^\alpha, & \partial_j &= \partial/\partial \sigma^j, \\
D_\alpha &= -ip_{\theta\alpha} - (\dot{\theta})_\alpha.
\end{align*}
\]

Here \((x^\nu, \theta^\alpha)\) are the superspace coordinates, \(\theta^\alpha\) is a left Majorana-Weyl spinor \((\alpha = 1, \ldots, 16), p_\nu, p_{\theta\alpha}\) are the corresponding conjugated momenta, \(\dot{\theta} = p_\mu \sigma^\mu\), where \(\sigma^\mu\) are the 10-dimensional \(\sigma\)-matrices (for our conventions see the Appendix).

\( H_0 \) generalizes on the one hand the bosonic null p-brane Hamiltonian

\[ H^B = \int d^p \sigma (\mu^0 p^2 + \mu^j p_\nu \partial_j x^\nu), \]

and on the other - the \(N = 1\) Brink-Shwarz superparticle with Hamiltonian

\[ H^{BS} = \mu^0 p^2 + \mu^\alpha D_\alpha. \]

The constraints \([2]\) satisfy the following (equal \(\tau\)) Poisson bracket algebra

\[
\begin{align*}
\{T_0(\sigma_1), T_0(\sigma_2)\} &= 0, \\
\{T_0(\sigma_1), D_\alpha(\sigma_2)\} &= 0, \\
\{T_0(\sigma_1), T_j(\sigma_2)\} &= [T_0(\sigma_1) + T_0(\sigma_2)] \partial_j \delta^\mu(\sigma_1 - \sigma_2), \\
\{T_j(\sigma_1), T_k(\sigma_2)\} &= [\delta^\mu T_k(\sigma_1) + \delta^\mu T_j(\sigma_2)] \partial_\mu \delta^\nu(\sigma_1 - \sigma_2), \\
\{T_j(\sigma_1), D_\alpha(\sigma_2)\} &= D_\alpha(\sigma_1) \partial_j \delta^\nu(\sigma_1 - \sigma_2), \\
\{D_\alpha(\sigma_1), D_\beta(\sigma_2)\} &= 2i \delta_{\alpha\beta} \delta^\nu(\sigma_1 - \sigma_2).
\end{align*}
\]

From the condition that the constraints must be maintained in time, i.e. \([7]\)

\[ \{T_0, H_0\} \approx 0, \{T_j, H_0\} \approx 0, \{D_\alpha, H_0\} \approx 0, \]

it follows that in the Hamiltonian \(H_0\) one has to include the constraints

\[ T_\alpha = \dot{\theta}_\alpha \delta \]

instead of \(D_\alpha\). This is because the Hamiltonian has to be first class quantity, but \(D_\alpha\) are a mixture of first and second class constraints. \(T_\alpha\) has the following non-zero Poisson brackets

\[
\begin{align*}
\{T_j(\sigma_1), T_\alpha(\sigma_2)\} &= [T_\alpha(\sigma_1) + T_\alpha(\sigma_2)] \partial_j \delta^\nu(\sigma_1 - \sigma_2), \\
\{T_\alpha(\sigma_1), T_\beta(\sigma_2)\} &= 2i \delta_{\alpha\beta} T_0 \delta^\nu(\sigma_1 - \sigma_2).
\end{align*}
\]
In this form, our constraints are first class (their algebra coincides with the algebra \( \mathbb{H} \) for \( N = 1 \) and \( P_\mu = -p_\mu \)) and the Dirac consistency conditions \( \mathbb{D} \) (with \( D_\alpha \) replaced by \( T_\alpha \)) are satisfied identically. However, one now encounters a new problem. The constraints \( T_0, T_j \) and \( T_\alpha \) are not irreducible, i.e. they are functionally dependent:

\[
(\not \! D)^\alpha - D^\alpha T_0 = 0.
\]

It is known, that in this case after BRST-BFV quantization an infinite number of ghosts for ghosts appear, if one wants to preserve the manifest Lorentz invariance. The way out consists in the introduction of auxiliary variables, so that the mixture of first and second class constraints \( D^\alpha \) can be appropriately covariantly decomposed into first class constraints and second class ones. To this end, here we will use the auxiliary harmonic variables introduced in \( i \) and \( j \). These are \( u_\mu^a \) and \( v_\alpha^\pm \), where superscripts \( a = 1, \ldots, 8 \) and \( \pm \) transform under the 'internal' groups \( SO(8) \) and \( SO(1,1) \) respectively. The just introduced variables are constrained by the following orthogonality conditions

\[
u^a_\mu u_\mu^{b\nu} = C_{ab}, \quad u_\mu^+ u_\mu^{a\mu} = 0, \quad u_\mu^+ u_\mu^{-\mu} = -1,
\]

where

\[
u^\pm_\mu = v_\alpha^\pm \sigma_\mu^{\alpha \beta} v_\beta^\pm.
\]

\( C_{ab} \) is the invariant metric tensor in the relevant representation space of \( SO(8) \) and \( (u^\pm)^2 = 0 \) as a consequence of the Fierz identity for the 10-dimensional \( \sigma \)-matrices. We note that \( u_\mu^a \) and \( v_\alpha^\pm \) do not depend on \( \sigma \).

Now we have to ensure that our dynamical system does not depend on arbitrary rotations of the auxiliary variables \( (u_\mu^a, v_\alpha^\pm) \). It can be done by introduction of first class constraints, which generate these transformations

\[
I_{ab} = -(u_\nu^a p_{\nu^a}^b - u_\nu^b p_{\nu^b}^a + \frac{1}{2} v^+ a^{ab} p_{\nu^+}^a + \frac{1}{2} v^- a^{ab} p_{\nu^-}^a), \quad \sigma^{ab} = u_\mu^a u_{\nu^b} \sigma^{\mu \nu},
\]

\[
I_{-+} = \frac{1}{2} (v_\alpha^+ p_{\alpha^+}^a - v_{\alpha}^- p_{\alpha^-}^a),
\]

\[
I_{\pm a} = -(u_\nu^+ p_{\nu}^a + \frac{1}{2} v^+ a^{ab} p_{\nu}^a), \quad \sigma^\pm = u_\nu^\pm \sigma^\nu, \quad \sigma^a = u_\nu^\pm \sigma^\nu.
\]

In the above equalities, \( p_{\nu}^{a} \) and \( p_{\alpha}^{\pm a} \) are the momenta canonically conjugated to \( u_\nu^a \) and \( v_\alpha^\pm \).

The newly introduced constraints \( \mathbb{D} \) obey the following Poisson bracket algebra

\[
\{I_{ab}, I^{cd}\} = C_{bcd} I^{ad} - C_{bd} I^{ac} + C_{cd} I^{ab} - C_{bc} I^{ad}.
\]

This algebra is isomorphic to the \( SO(1,9) \) algebra: \( I^{ab} \) generate \( SO(8) \) rotations, \( I_{-+} \) is the generator of the subgroup \( SO(1,1) \) and \( I_{\pm a} \) generate the transformations from the coset \( SO(1,9)/SO(1,1) \times SO(8) \).
Now we are ready to separate $D^a$ into first and second class constraints in a Lorentz-covariant form. This separation is given by the equalities \[1\]:

\[
D^a = \frac{1}{p^+}[(\sigma^a v^+)\alpha D_a + (\not{p}\sigma^+\sigma^a v^-)\alpha G_a], \quad p^+ = p^\nu u_\nu^+,
\]

\[
D^a = (v^+\sigma^a \not{p})_\beta D_\beta, \quad G^a = \frac{1}{2}(v^-\sigma^a\sigma^+)_\beta D^\beta.
\]

Here $D^a$ are first class constraints and $G^a$ are second class ones:

\[
\{D^a(\vec{\sigma}_1), D^b(\vec{\sigma}_2)\} = -2iC^{ab}p^+T_0\delta^p(\vec{\sigma}_1 - \vec{\sigma}_2)
\]

\[
\{G^a(\vec{\sigma}_1), G^b(\vec{\sigma}_2)\} = iC^{ab}p^+\delta^p(\vec{\sigma}_1 - \vec{\sigma}_2).
\]

It is convenient to pass from second class constraints $G^a$ to first class constraints $\hat{G}^a$, without changing the actual degrees of freedom \[1\], \[1\]:

\[
G^a \rightarrow \hat{G}^a = G^a + (p^+)^{1/2}\Psi^a \Rightarrow \{\hat{G}^a(\vec{\sigma}_1), \hat{G}^b(\vec{\sigma}_2)\} = 0,
\]

where $\Psi^a(\vec{\sigma})$ are fermionic ghosts which abelianize our second class constraints as a consequence of the Poisson bracket relation

\[
\{\Psi^a(\vec{\sigma}_1), \Psi^b(\vec{\sigma}_2)\} = -iC^{ab}\delta^p(\vec{\sigma}_1 - \vec{\sigma}_2).
\]

It turns out, that the constraint algebra is much more simple, if we work not with $D^a$ and $\hat{G}^a$ but with $\hat{T}^a$ given by

\[
\hat{T}^a = (p^+)^{-1/2}[(\sigma^a v^+)\alpha D_a + (\not{p}\sigma^+\sigma^a v^-)\alpha \hat{G}_a]
\]

\[
= (p^+)^{1/2}D^a + (\not{p}\sigma^+\sigma^a v^-)\alpha \Psi^a.
\]

After the introduction of the auxiliary fermionic variables $\Psi^a$, we have to modify some of the constraints, to preserve their first class property. Namely $T_j, I^{ab}$ and $I^{-a}$ change as follows

\[
\hat{T}_j = T_j + \frac{i}{2}C^{ab}\Psi_a\partial_j\Psi_b,
\]

\[
\hat{I}^{ab} = I^{ab} + J^{ab}, \quad J^{ab} = \int dp\sigma j^{ab}(\vec{\sigma}), \quad j^{ab} = \frac{i}{4}(v^-\sigma^c\sigma^{ab}\sigma^d\sigma^+ v^-)\Psi^c\Psi^d,
\]

\[
\hat{I}^{-a} = I^{-a} + J^{-a}, \quad J^{-a} = \int dp\sigma j^{-a}(\vec{\sigma}), \quad j^{-a} = -(p^+)^{-1}j^{ab}p_b.
\]

As a consequence, we can write down the Hamiltonian for the considered model in the form:

\[
H = \int dp\sigma[\lambda^0 T_0(\vec{\sigma}) + \lambda^j \hat{T}_j(\vec{\sigma}) + \lambda^a \hat{T}_a(\vec{\sigma})] + \lambda_{ab}\hat{I}^{ab} + \lambda_{-+}I^{-+} + \lambda_{+-}I^{-+} + \lambda_{-a}I^{-a}.
\]

The constraints entering $H$ are all first class, irreducible and Lorentz-covariant. Their algebra reads (only the non-zero Poisson brackets are written):

\[
\{T_0(\vec{\sigma}_1), \hat{T}_j(\vec{\sigma}_2)\} = (T_0(\vec{\sigma}_1) + T_0(\vec{\sigma}_2))\partial_j\delta^p(\vec{\sigma}_1 - \vec{\sigma}_2),
\]

\[
\{\hat{T}_j(\vec{\sigma}_1), \hat{T}_k(\vec{\sigma}_2)\} = (\delta_j^l \hat{T}_l(\vec{\sigma}_1) + \delta_k^l \hat{T}_l(\vec{\sigma}_2))\partial_l\delta^p(\vec{\sigma}_1 - \vec{\sigma}_2),
\]
\{\hat{T}_j(\sigma_1), \hat{T}_a(\sigma_2)\} = (\hat{T}_a(\sigma_1) + \frac{1}{2} \hat{T}_a(\sigma_2)) \partial_j \delta^p(\sigma_1 - \sigma_2),
\{\hat{T}_\alpha(\sigma_1), \hat{T}_\beta(\sigma_2)\} = i\sigma^+_{\alpha\beta} T_0 \delta^p(\sigma_1 - \sigma_2),
\{I^{-+}, \hat{T}_a\} = \frac{1}{2} \hat{T}_a,
\{\hat{I}^{-a}, \hat{T}_a\} = (2p^+)^{-1}[p^a \hat{T}_a + (\sigma^+ \sigma_{ab} \nu^-)_\alpha \Psi_b T_0],
\{\hat{I}^{ab}, \hat{I}^{ad}\} = C^{bc} \hat{I}^{ad} - C^{ac} \hat{I}^{bd} + C^{ad} \hat{I}^{bc} - C^{bd} \hat{I}^{ac},
\{I^{-+}, I^{++}\} = I^{++},
\{I^{-+}, \hat{I}^{-a}\} = -\hat{I}^{-a},
\{\hat{I}^{ab}, \hat{I}^{+c}\} = C^{bc} I^{++} - C^{ac} \hat{I}^{+b},
\{I^{++}, \hat{I}^{-b}\} = C^{ab} I^{--} + \hat{I}^{ab},
\{\hat{I}^{-a}, \hat{I}^{-b}\} = -\int d^p \sigma (p^+)^{-2} j^{ab} T_0.

Having in mind the above algebra, one can construct the corresponding BRST charge \(\Omega\)
\[\Omega = \Omega^{\text{min}} + \pi_M P^M, \quad \{\Omega, \Omega\} = 0, \quad \Omega^* = \Omega,\] (6)

where \(M = 0, j, \alpha, ab, +, +, +, +, a, -a, \). \(\Omega^{\text{min}}\) in (3) can be written as
\[\Omega^{\text{min}} = \Omega^{\text{brane}} + \Omega^{\text{aux}},\]
\[\Omega^{\text{brane}} = \int d^p \sigma \left[ T_0 \eta^0 + \hat{T}_j \eta^j + \hat{T}_a \eta^a + P_0 \left( (\partial_j \eta^j) \eta^0 + (\partial_0 \eta^0) \eta^j \right) + \right.\]
\[+ P_k (\partial_j \eta^k) \eta^j + P_\alpha \left[ \eta^j \partial_\alpha \eta^\alpha - \frac{1}{2} \eta^\alpha \partial_j \eta^\alpha \right] - \frac{i}{2} P_\alpha \eta^\alpha \sigma^+_{\alpha\beta} \eta^\beta,\]
\[\Omega^{\text{aux}} = \hat{I}^{ab} \eta_{ab} + I^{-+} \eta_{++} + I^{++} \eta_{+a} + \hat{I}^{-a} \eta_{-a} + \]
\[+ (P^{ac} \eta^c_{ab} - P^{bc} \eta^b_{ac} + 2 P^{ab} \eta^0_{ac} + 2 P^{a0} \eta^0_{bc}) \eta_{ab} + \]
\[+ (P^{+a} \eta_{+a} - P^{-a} \eta_{-a}) \eta_{++} + (P^{-+} \eta^a_{-a} + P^{ab} \eta_{-b}) \eta_{+a} + \]
\[+ \frac{1}{2} \int d^p \sigma \left[ P_\alpha \eta^0 \eta_{-a} + (p^+)^{-1}[p^a P_\alpha - (\sigma^+ \sigma_{ab} \nu^-)_\alpha \Psi_b P_0] \eta^0 \eta_{-a} - \right.\]
\[\left. -(p^+)^{-2} j^{ab} P_0 \eta_{-b} \eta_{-a} \right].\]

These expressions for \(\Omega^{\text{brane}}\) and \(\Omega^{\text{aux}}\) show that we have found a set of constraints which ensure the first rank property of the model.
\(\Omega^{\text{min}}\) can be represented also in the form
\[\Omega^{\text{min}} = \int d^p \sigma \left[ (T_0 + \frac{1}{2} T_0^{gh}) \eta^0 + (\hat{T}_j + \frac{1}{2} T_j^{gh}) \eta^j + (\hat{T}_a + \frac{1}{2} T_a^{gh}) \eta^a \right] + \]
\[+ (\hat{I}^{ab} + \frac{1}{2} \hat{I}^{ab}_{gh}) \eta_{ab} + (I^{-+} + \frac{1}{2} I^{-+}_{gh}) \eta_{++} + (I^{++} + \frac{1}{2} I^{++}_{gh}) \eta_{+a} + (\hat{I}^{-a} + \frac{1}{2} I^{-a}_{gh}) \eta_{-a} + \]
\[+ \int d^p \sigma \partial_j \left( \frac{1}{2} P_k \eta^k \eta^j + \frac{1}{4} P_\alpha \eta^a \eta^j \right).\]

Here a super(sub)script \(gh\) is used for the ghost part of the total gauge generators
\[G^{\text{tot}} = \{\Omega, P\} = \{\Omega^{\text{min}}, P\} = G + G^{gh}.\]

We recall that the Poisson bracket algebras of \(G^{\text{tot}}\) and \(G\) coincide for first rank systems. The manifest expressions for \(G^{gh}\) are:
\[T_0^{gh} = 2 P_0 \partial_j \eta^j + (\partial_j P) \eta^j,\]
where \( ρ \)

and generally

Grassmann parity as follows:

Up to now, we introduced canonically conjugated ghosts \((η^M, P_M)\), \((\bar{η}_M, \bar{P}^M)\) and momenta \(π_M\) for the Lagrange multipliers \(λ^M\) in the Hamiltonian. They have Poisson brackets and Grassmann parity as follows (\(ε_M\) is the Grassmann parity of the corresponding constraint):

\[
\begin{align*}
\{η^M, P_N\} &= \delta^M_N, \quad ε(η^M) = ε(P_M) = ε_M + 1, \\
\{\bar{η}_M, \bar{P}^N\} &= -(-1)^{ε_Mε_N}\delta_M^N, \quad ε(\bar{η}_M) = ε(\bar{P}^M) = ε_M + 1, \\
\{λ^M, π_N\} &= \delta^N_M, \quad ε(λ^M) = ε(π_M) = ε_M.
\end{align*}
\]

The BRST-invariant Hamiltonian is

\[
H_\chi = H^{\text{min}} + \{\bar{χ}, Ω\} = \{\bar{χ}, Ω\},
\]

(7)

because from \(H_{\text{canonical}} = 0\) it follows \(H^{\text{min}} = 0\). In this formula \(\bar{χ}\) stands for the gauge fixing fermion (\(\bar{χ}^* = -\bar{χ}\)). We use the following representation for the latter

\[
\bar{χ} = χ^{\text{min}} + \bar{η}_M(χ^M + \frac{1}{2}ρ(\lambda^M)π^M), \quad χ^{\text{min}} = λ^M P_M,
\]

where \(ρ(\lambda^M)\) are scalar parameters and we have separated the \(π^M\)-dependence from \(χ^M\). If we adopt that \(χ^M\) does not depend on the ghosts \((η^M, P_M)\) and \((\bar{η}_M, \bar{P}^M)\), the Hamiltonian \(H_\chi\) from (7) takes the form

\[
\begin{align*}
H_\chi &= H^{\text{min}} + P_M\bar{P}^M - π_M(χ^M + \frac{1}{2}ρ(\lambda^M)π^M) + \\
&+ \bar{η}_M[\{χ^M, G_N\}η^N + \frac{1}{2}(-1)^{ε_N}\bar{P}Q\{χ^M, U^{Q}_{NP}\}η^Pη^N],
\end{align*}
\]

(8)

where

\[
H^{\text{min}}_χ = \{χ^{\text{min}}, Ω^{\text{min}}\},
\]

and generally \(\{χ^M, U^{Q}_{NP}\} \neq 0\) as far as the structure coefficients of the constraint algebra \(U^{M}_{NP}\) depend on the phase-space variables.
One can use the representation (8) for $H_\chi$ to obtain the corresponding BRST invariant Lagrangian

$$L_\chi = L + L_{GH} + L_{GF}.$$  

Here $L_{GH}$ stands for the ghost part and $L_{GF}$ - for the gauge fixing part of the Lagrangian. If one does not intend to pass to the Lagrangian formalism, one may restrict oneself to the minimal sector $(\Omega^{\min}, \chi^{\min}, H^{\min}_\chi)$. In particular, this means that Lagrange multipliers are not considered as dynamical variables anymore. With this particular gauge choice, $H^{\min}_\chi$ is a linear combination of the total constraints

$$H^{\min}_\chi = H^{\min}_{\text{brane}} + H^{\min}_{\text{aux}} =$$

$$= \int d^p \sigma \left[ \Lambda^0 T^0_{\text{tot}}(\sigma) + \Lambda^j T^j_{\text{tot}}(\sigma) + \Lambda^\alpha T^\alpha_{\text{tot}}(\sigma) \right] +$$

$$+ \Lambda_{ab} I_{ab}^+ - \Lambda_{-a} I^{- a} + \Lambda_{+a} I^{+ a} + \Lambda_{-a} I^{- a},$$

and we can treat here the Lagrange multipliers $\Lambda^0, ..., \Lambda_{-a}$ as constants. Of course, this does not fix the gauge completely.

### 3 Comments and conclusions

The introduced harmonic variables $u_{\mu}^a$, $v_\alpha^\pm$ helped us to construct $SO(1,1) \times SO(8)$ covariant quantities $(\sigma^a, p^+, \text{etc.})$ from $SO(1,9)$-covariant ones. We note here that this is an invertible operation. For any Lorentz vector $A_\mu$ we have

$$A_{\nu} = -u_{\nu}^+ A^- - u_{\nu}^- A^+ + u_{\nu}^a A_a,$$

where

$$A^a = u_{\nu}^a A_\nu, \quad A^\pm = u_{\nu}^\pm A_\nu.$$

For Lorentz spinors the reversibility is demonstrated in the equality (5) for example.

To ensure that the harmonics and their conjugate momenta are pure gauge degrees of freedom, we have to consider as physical observables only such functions on the phase space which do not carry any $SO(1,1) \times SO(8)$ indices. More precisely, these functions are defined by the following expansion

$$F(y, u, v; p_y, p_u, p_v) = \sum [u_{\alpha_1}^a \ldots u_{\alpha_k}^a p^a_{\nu_{k+1}} \ldots p^a_{\nu_{k+l}}]_{SO(8)\text{singlet}}$$

$$v^+_{\alpha_1} \ldots v^+_{\alpha_m} v^-_{\alpha_{m+1}} \ldots v^-_{\alpha_{m+n}} p^+_{\nu_1} \ldots p^+_{\nu_{m-\nu+r}} p^-_{\beta_1} \ldots p^-_{\beta_{m-\nu+r}} F^+_{\alpha_1 \ldots \alpha_{m+n} \beta_1 \ldots \beta_{m-\nu+r}} (y, p_y),$$

where $(y, p_y)$ are the non-harmonic phase space conjugated pairs.

In this letter we begin the investigation of a $p$-brane model with $N = 1$ supersymmetry in 10-dimensional flat space-time. Starting with a Hamiltonian which is a linear combination of first and mixed (first and second) class constraints, we succeed to obtain a new one, which is a linear combination of first class, irreducible and Lorentz-covariant constraints only. This is done with the help of the introduced auxiliary harmonic variables. Then we give manifest
expressions for the classical BRST charge, the corresponding total constraints and BRST-invariant Hamiltonian. It turns out, that in the given formulation our model is a first rank dynamical system. The problems of Lagrangian formulation, finding solutions of the classical equations of motion and quantization will be considered in the second part of the paper.

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**Appendix**

We briefly describe here our 10-dimensional conventions. Dirac $\gamma$-matrices obey

$$\Gamma_\mu \Gamma_\nu + \Gamma_\nu \Gamma_\mu = 2\eta_{\mu\nu}$$

and are taken in the representation

$$\Gamma^\mu = \begin{pmatrix} 0 & (\sigma^\mu)^{\dot{\beta}}_\alpha \\ (\bar{\sigma}^\mu)^{\dot{\alpha}}_\beta & 0 \end{pmatrix}. $$

$\Gamma^{11}$ and charge conjugation matrix $C_{10}$ are given by

$$\Gamma^{11} = \Gamma^0 \Gamma^1 \ldots \Gamma^9 = \begin{pmatrix} \delta^\alpha_\beta & 0 \\ 0 & -\delta^\dot{\beta}_{\dot{\alpha}} \end{pmatrix},$$

$$C_{10} = \begin{pmatrix} 0 & C^{\alpha\dot{\beta}} \\ (-C)^{\dot{\alpha}\beta} & 0 \end{pmatrix},$$

and the indices of right and left Majorana-Weyl fermions are raised as

$$\psi^\alpha = C^{\alpha\dot{\alpha}} \bar{\psi}_{\dot{\alpha}}, \quad \phi^{\dot{\alpha}} = (-C)^{\dot{\alpha}\beta} \phi^\beta.$$

We use $D = 10$ $\sigma$-matrices with undotted indices

$$(\sigma^\mu)^{\alpha\beta} = C^{\alpha\dot{\alpha}} (\bar{\sigma}^\mu)^{\dot{\beta}}_\alpha, \quad (\sigma^\mu)^{\alpha\beta} = (-C)^{\beta\dot{\beta}} (\sigma^\mu)^{\dot{\alpha}}_\beta,$$

and the notation

$$\sigma^{\mu_1 \ldots \mu_n} \equiv \sigma^{[\mu_1 \ldots \mu_n]}$$

for their antisymmetrized products.

From the corresponding properties of $D = 10$ $\gamma$-matrices, it follows:

$$(\sigma^\mu)^{\alpha\gamma} (\sigma^\nu)^{\gamma\beta} + (\sigma^\nu)^{\alpha\gamma} (\sigma^\mu)^{\gamma\beta} = -2\delta^\beta_\alpha \eta^{\mu\nu},$$

$$(\sigma_{\mu_1 \ldots \mu_{2s+1}})^{\alpha\beta} = (-1)^s (\sigma_{\mu_1 \ldots \mu_{2s+1}})^{\dot{\beta}\dot{\alpha}},$$

$$\sigma^{\mu_1 \ldots \mu_n} = \sigma^{\mu_1 \ldots \nu_n} + \sum_{k=1}^n (-1)^k \eta^{\mu_{k-1}\mu_k} \sigma^{\nu_{k-1}\nu_k \ldots \nu_n}.$$

The Fierz identity for the $\sigma$-matrices reads:

$$(\sigma^\mu)^{\alpha\beta} (\sigma^\mu)^{\gamma\delta} + (\sigma^\mu)^{\beta\gamma} (\sigma^\mu)^{\alpha\delta} + (\sigma^\mu)^{\gamma\alpha} (\sigma^\mu)^{\beta\delta} = 0.$$
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