Modeling of Downhole Weight on Bit Using Finite Element Method and Its Verification

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Abstract. During the drilling process of oil and gas well, drag and torque play a very important role in designing configuration of the well, diagnosing drilling states, and optimizing operations. Because of the drag between the drillstring and wellbore, the actual downhole weight on bit (DWOB) is unknown, especially for the horizontal well. In this study, a 3D finite element model was developed to calculate friction factor and DWOB. To validate the rationality of the developed finite element model, simulated data was compared with that of measured. The result indicates that the simulated data shows a good match with the measured data with average accuracy as high as 90%. It is concluded that the developed finite element model is reasonable and can be used to calculate and predict the friction factor and DWOB with high accuracy.

1. Introduction

Because of the friction caused by the contact of drillstring and wellbore, torque and drag exist and accumulate from bottom hole to surface [1]. It is evident that there is loss of torque and drag from surface to bottom hole for the friction between the drillstring and wellbore[2]. In general, surface torque is used to drive the drillstring and bit rotating around its axial and destruct rock[3]. While the drag is the incremental force needed for pulling or lowering the drillstring through the hole[4]. It is well known that excessive torque and drag could induce many troubles, such as tight hole conditions, sloughing hole, differential sticking, cuttings buildup caused by poor hole cleaning, and sliding wellbore friction[5, 6]. So, torque and drag have become a critical issue, which limit the drilling industry to go beyond a certain depth. Additionally, torque and drag are regard as important criteria for estimating drilling feasibility of directional wells, and reducing the occurrence of accident drill string failures[7, 8]. Meanwhile, the drill bit performance is often evaluated by the rate of penetration, which is affected by the DWOB[9]. Significant loss of drag means that it is difficult to load effective weight on bit. Generally, the surface weight on bit (SWOB) is regarded as a primary reference to adjust drilling operations. In fact, neither the accurate nor approximate value of DWOB is unknown[10].

To date, there are two dominant kinds of model for torque and drag calculating, including analytical analysis and finite element method (FEM). Researches have done a lot of studies on establishing and application of the analytical model[3, 4, 11, 12]. The main shortcoming of analytical analysis is that the stiffness of the drillstring is neglected and the drillstring is regard as a soft string[4, 12, 13]. In addition, the analytical model is built based on many assumptions. So, there is great difference between the calculated results and filed data, especially in complicate conditions. The rapid development of computing logic and the performance of computer make it possible to calculate torque...
and drag using FEM.

FEM has been used in drillstring analysis for many years. The basic principle of this method is that the whole drillstring is divided into small 3D beam elements. Bueno et al.\cite{14} established finite element model with a non-linear, 2D, quasi-static approach to estimate the contact forces caused by the drill string against the wellbore and the marine drilling riser system. The drillstring is modeled as non-deformed elastic beams. Moreover, contact points were assumed occur only at the tool-joints. Newman K R\cite{15} developed a 3D finite element model to analysis the blending of pipe inside wellbore. This model is also able to calculate the onset of buckling. Yang D et al.\cite{16} presented a 3D finite difference differential method for bottomhole assembly analysis under static loads. This model incorporates the contact response between drillstring and wellbore wall, the upper tangent point problem, stabilizer configurations, bent sub model and other considerations for numerical solutions. Ishak G et al.\cite{17} used explicit FEM to establish the static and dynamic interactions between the bit, the reamer and the formation of the bottomhole assembly while in the reaming operation. Wu pointed that finite element models can obtain more actual results than analytical models by considering complex boundary conditions and wellbore curvature \cite{18,19}.

In this study, a finite element model of drillstring was established to simulate the friction factor and DWOB. To verify the established finite element model, the data of DWOB calculated was compared with that of measured in field. The result indicates that the simulated data shows a good match with the measured in change trend with reasonable accuracy.

2. Modeling for Torque and Drag

2.1. Hamilton’s Principle

For each 3D beam element, there are two nodes at each side, and each node possesses 6 degrees of freedom, including three displacements and three rotations around. So, the discrete dynamic equation for an element can be expressed as Eq. (1).

\[
[M]^{e} \{\ddot{U}^{e}\} + [C]^{e} \{\dot{U}^{e}\} + [K]^{e} \{U^{e}\} = \{F\}^{e}
\]

where \{\ddot{U}^{e}\}, \{\dot{U}^{e}\}, \{U^{e}\} and \{F\}^{e} represent generalized displacement, velocity, acceleration, and force vectors acting on the element, respectively. \([M]^{e}\), \([C]^{e}\) and \([K]^{e}\) represent element mass, damping and stiffness matrix.

2.2. Transformation of Coordinate System

During the establishing of mathematic model, all mechanical analysis is based on element local-coordinate. While in horizontal sections, because of inclination and azimuth, the directions of X axis and Y axis for different nodes could be different. Fig.1 shows the difference between global coordinate system and local coordinate system.

![Figure 1. Local and global coordinate systems for horizontal Well](image-url)
To solve this issue, a coordinate transformation matrix is needed to convert the matrixes, such as mass matrix, stiffness matrix, damping matrix, force vectors, displacement vectors, velocity vectors and acceleration, from the local element coordinate system $xyz$ to the global coordinate system $XYZ$. Taking displacement matrix as an example, the transformation is expressed as Eq. (2).

$$
\begin{bmatrix}
U_x \\
U_y \\
U_z \\
\Phi_x \\
\Phi_y \\
\Phi_z
\end{bmatrix}
= 
\begin{bmatrix}
\cos \alpha & -\sin \alpha & 0 & u_x \\
\sin \alpha \cos \beta & \cos \alpha \cos \beta & -\sin \beta & u_y \\
\sin \alpha \sin \beta & \cos \alpha \sin \beta & \cos \beta & u_z \\
0 & \sin \alpha \cos \beta & \cos \alpha \cos \beta & \phi_x \\
0 & -\sin \alpha & 0 & \phi_y \\
\sin \alpha \sin \beta & \cos \alpha \sin \beta & \cos \beta & \phi_z
\end{bmatrix}
$$

where $\alpha$ and $\beta$ represent inclination and azimuth, $U_x$, $U_y$, $U_z$, $\Phi_x$, $\Phi_y$, and $\Phi_z$ are three translations and three rotations in global system, $u_x$, $u_y$, $u_z$, $\phi_x$, $\phi_y$ and $\phi_z$ are three translations and three rotations in local system.

### 2.3. Boundary Conditions

During finite element modeling, the main boundaries include rotary table, drill bit and stabilizers. At the rotary table, the displacement on $x$-axis and $y$-axis are constrained, and the drillstring can still move on axial direction. For different drilling states, the boundary conditions could be various, the detail as following:

**(i) Rotating drilling**

$$
U_x^i = 0, \quad U_y^i = 0, \quad T_z^i = T_{\text{surface}}
$$

where $T_{\text{surface}}$ is the torque on rotary table, $U_x^i$ and $U_y^i$ are the displacement on $x$-axis and $y$-axis of the first element, which nears the rotary table.

**(ii) Sliding drilling**

$$
U_x^i = 0, \quad U_y^i = 0, \quad T_z^i = 0
$$

At bottom hole, the radial displacement is constrained, because the size of bit is equal or slightly less than the hole size. While the axial displacement and rotation is released. For different drilling states, the boundary conditions could be various, the detail as following:

**(i) Rotating drilling**

$$
U_x^n = 0, \quad U_y^n = 0, \quad F_z^n = WOB_{\text{bottom}}, \quad T_z^n = \text{Torque}_{\text{bottom}}
$$

**(ii) Sliding drilling**

$$
U_x^n = 0, \quad U_y^n = 0, \quad F_z^n = 0, \quad T_z^n = 0
$$

In above equations, $n$ is the number of elements.

For the stabilizers, the radial displacement on $x$-axis and $y$-axis is constrained. While the axial displacement and rotation around drillstring axis is released.

$$
U_x^i = 0, \quad U_y^i = 0
$$

### 3. Computing Program and Verification

#### 3.1. Computing Program

Based on the logic order of the way that FEM has been introduced, a flowchart was proposed, as shown in Fig.2. C++ was chosen to be the programming language for this research, and Visual Studio 2012 was chosen as the programming platform. Considering the length of this study, the programming language was not described in detail.

#### 3.2. Verification of the FEM Model

A well in Rig Akita was selected to verify the finite element model. This well is a horizontal well, consisting of three sections. Down to 1705.2 m until Kick off point (KOP) is vertical section. From
1705.2 m to 2351.7 m until landing at heel point belongs to Build and turn section. And, from 2351.7 m to 3844 m until reaching the toe is horizontal section. Some of the data measured is shown in Table 1. The data which bit depth is less than hole depth was used to calculate friction. On the contrary, the data which bit depth is larger than hole depth was used to DWOB.

Figure 2. Flow chart of the finite element model for torque and drag analysis

Table 1. Real-time drilling data from 2505.3 m to 2505.99 m

| Bit Depth | Hole Depth | On Bottom ROP | Hook Load | Weight on Bit | Rotary RPM | Rotary Torque | Standpipe Pressure |
|-----------|------------|---------------|-----------|---------------|------------|---------------|--------------------|
| 2505.3    | 2505.65    | 0             | 60.9      | 1.2           | 61         | 4101.71       | 22087              |
| 2505.62   | 2505.65    | 0             | 58.8      | 7.2           | 61         | 5869.57       | 22611              |
| 2505.65   | 2505.65    | 0             | 55        | 6             | 61         | 5464.29       | 22886              |
| 2505.71   | 2505.71    | 5.74          | 54.7      | 6.6           | 61         | 5736.8        | 22886              |
| 2505.76   | 2505.76    | 5.74          | 54.2      | 6.9           | 61         | 5967.39       | 22972              |
Taking the data measured from 2500 m to 2800 m as an example, friction factors were calculated, as shown in Fig. 3. It can be observed that the friction factor are all among 0.2 and 0.25. These value are consistent with the cognition of friction factors.

Based on the calculated friction factors, the DWOB was calculated using the developed finite element model, as shown in Fig. 4. It indicates that the data calculated by the finite element model is coincided with that of tested. Therefore, the finite element model can be used to calculate and predict the friction factors and DWOB of the drilling.

![Figure 3. Friction factors from 2500 m to 2800 m](image1)

![Figure 4. Comparison of DWOB between the simulated and measured](image2)

In addition, it is found that there is different between the simulated and measured DWOB. To evaluate this different, the average accuracy of the finite element model results was calculated, as expressed in Eq. (3). It is noticed that the value of accuracy as high as 90 %. The results indicate that the finite element model can be applied to calculate DWOB with highly accuracy.

$$\text{Accuracy} = 1 - \frac{\text{DWOB}_\text{FEM} - \text{DWOB}_\text{Copilot}}{\text{SWOB} - \text{DWOB}_\text{Copilot}} \times 100% = 90\%$$ (3)

![Figure 5. Iteration procedures for friction factor calculation](image3)
The main disadvantage of finite element model is quite time-consuming, because the value of friction factor calculated is not stable, until 2000 times iterations. Taking depth at 2705.86 m for example, the calculating procedure is shown as Fig.5. It takes about 4 minutes to calculate one depth. Even using “Speedup Algorithm”, it still needs to calculate at least 15 times to get the friction factor for one depth point. This algorithm makes 60 minutes in total for one depth point.

4. Conclusions
(i) A 3D model for DWOB was developed based on the finite element method. This model can be used to calculate friction factor of the drillstring and DWOB for various drilling conditions, such as rotating drilling and sliding drilling.
(ii) The DWOB calculated using finite element model shows a good match with that of measured. The average accuracy of finite element model is up to 90%. The results indicate that the finite element model is proper and can be applied to calculate DWOB with high accuracy.
(iii) It takes about 60 minutes for finite element model to calculate the friction factor and DWOB on one depth point. According to the present situation, finite element model can be used for drag analysis, but cannot be used for real-time drilling monitoring, temporarily.

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