Matter in a Warped and Oscillating Background

Hael Collins†
Department of Physics
Carnegie Mellon University
Pittsburgh, PA 15213

Bob Holdom‡
Department of Physics
University of Toronto
Toronto, Ontario M5S 1A7, Canada

Abstract
We examine the role of matter in an oscillating background with a warped, compact extra dimension. This background is compatible with an $S^1/Z_2$ orbifold structure which allows chiral fermions to be included in the scenario. When the background oscillates rapidly, the leading coupling of these oscillations is to gauge fields rather than fermions. If the decay of these oscillations were to occur today, it could provide an alternative mechanism for generating the ultra high energy cosmic rays.

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† hael@physics.utoronto.ca
‡ bob.holdom@utoronto.ca
1. Introduction.

A remarkable property of actions that generalize the Einstein-Hilbert action for gravity is that they admit metrics with a periodic dependence on one or more of the coordinates. The simplest such example occurs for an action composed of a general set of curvature invariants, with up to four derivatives of the metric, and scalar field. In $4+1$ dimensions a metric periodic in one of the spatial dimensions provides a naturally compact space—by identifying the compactification length with the period—without any singularities or discontinuities in the metric. Moreover, these periodic solutions exist without the need for finely tuning any of the parameters in the action. The coefficients of the four-derivative terms determine an essentially unique compactification size.

Similarly, we can search for metrics that oscillate both in the fifth dimension and in time. Although these metrics do not describe the most general isotropic vacuum solutions for the action, they could have intriguing consequences for the early and the late evolution of the universe. In the very early universe, we shall see that the rapid decay of the oscillating background into gauge fields provides an example of how a universe with a Planck-scale time dependence can relax into one that can admit a realistic, slow evolution. Moreover, if some slowly relaxing, but rapid, oscillation persisted or arose recently, then it could contribute a flux of ultra high energy cosmic rays. To be successful, the theory would still require a mechanism to maintain these oscillations at a satisfactory rate of decay today.

In this article we shall explore the effect of including fermion and gauge fields in these oscillating backgrounds. Even for a purely static background, introducing chiral fermions requires that the compact extra dimension should have the topology of an $S^1/Z_2$ orbifold. The metric and the scalar field that supports the compact geometry must be respectively even and odd under this $Z_2$. When the metric additionally oscillates rapidly in time, the coupling of these fields with gravity leads to a potential decay channel for the oscillations. We shall show by expanding a fermion and an Abelian gauge field in Kaluza-Klein modes that to leading order the fermion zero mass mode does not couple to the oscillating component of the metric. The preferred channel for the relaxation of the metric is therefore into gauge fields.

In the following section we review the features of oscillating metrics in $4+1$ dimensions. Section 3 describes how to place the theory on an orbifold to allow for chiral fermions in a warped, static background. Section 4 examines the coupling of the time-dependent components of the metric with the Kaluza-Klein modes of a fermion and an Abelian gauge field placed in an oscillating, compact background. Finally, in section 5, we study the phenomenological signature in ultra high energy cosmic rays of a very small, but rapid, oscillation today.

2. Oscillating metrics.

At distances approaching the Planck length, the usual Einstein-Hilbert action should be supplemented by higher derivative curvature invariants. For example, an action with
up to four derivatives of the metric and a scalar field has the form\footnote{Our convention for the signature of the metric is \((- , + , + , + , + )\) while the Riemann curvature tensor is defined by \( R^{a}_{b c d} \equiv \partial_{a} \Gamma^{a}_{b c} - \partial_{b} \Gamma^{a}_{c d} + \Gamma^{e}_{b d} \Gamma^{a}_{c d} - \Gamma^{e}_{c d} \Gamma^{a}_{b d} \). Here the indices \( a, b, \ldots = 0, 1, 2, 3, y \) range over all coordinates while \( \mu, \nu \ldots = 0, 1, 2, 3 \) label the ordinary space-time coordinates.}

\[
S = M_{5}^{3} \int d^{4}x dy \sqrt{-g} \left[ -2\Lambda + R + a R^{2} + b R_{ab} R^{ab} + c R_{abcd} R^{abcd} \right] + M_{5}^{3} \int d^{4}x dy \sqrt{-g} \left[ -\frac{1}{2} \nabla_{a} \phi \nabla^{a} \phi + \Delta L_{\phi} \right] + \cdots \tag{2.1}
\]

\( M_{5} \) and \( \Lambda \) denote the Planck mass and cosmological constant respectively. This metric admits smooth, non-singular metrics that are compact in the fifth dimension,

\[
ds^{2} = g_{a b} dx^{a} dx^{b} = e^{A(y)} \eta_{\mu \nu} dx^{\mu} dx^{\nu} + dy^{2} \tag{2.2}
\]

where the exponent \( A(y) \) is a periodic function, when \( \Delta L_{\phi} \) is either an interaction, such as \( (\nabla_{a} \phi \nabla^{a} \phi)^{2} \) \footnote{Our convention for the signature of the metric is \((- , + , + , + , + )\) while the Riemann curvature tensor is defined by \( R^{a}_{b c d} \equiv \partial_{a} \Gamma^{a}_{b c} - \partial_{b} \Gamma^{a}_{c d} + \Gamma^{e}_{b d} \Gamma^{a}_{c d} - \Gamma^{e}_{c d} \Gamma^{a}_{b d} \). Here the indices \( a, b, \ldots = 0, 1, 2, 3, y \) range over all coordinates while \( \mu, \nu \ldots = 0, 1, 2, 3 \) label the ordinary space-time coordinates.}, or a Casimir term \footnote{Our convention for the signature of the metric is \((- , + , + , + , + )\) while the Riemann curvature tensor is defined by \( R^{a}_{b c d} \equiv \partial_{a} \Gamma^{a}_{b c} - \partial_{b} \Gamma^{a}_{c d} + \Gamma^{e}_{b d} \Gamma^{a}_{c d} - \Gamma^{e}_{c d} \Gamma^{a}_{b d} \). Here the indices \( a, b, \ldots = 0, 1, 2, 3, y \) range over all coordinates while \( \mu, \nu \ldots = 0, 1, 2, 3 \) label the ordinary space-time coordinates.} with differing values its components in the large ordinary \( x^{\mu} \) and the compact \( y \) directions. In either case, the existence of such solutions does not require finely tuning any of the parameters in the action. The effects of the \( R^{2} \) terms can be conveniently parameterized by

\[
\mu = 16 a + 5 b + 4 c, \quad \lambda = 5 a + b + \frac{1}{2} c \quad \text{and} \quad \nu = 3 a + b + c. \tag{2.3}
\]

\( \lambda \) and \( \nu \), in particular, represent the coefficients of a Gauss-Bonnet and a squared Weyl tensor respectively.

While the static metrics are adequate for studying a theory with a flat 3+1 dimensional long distance limit, models that are to include a realistic cosmology should evolve in time as well,

\[
ds^{2} = -e^{A(y)} dt^{2} + e^{A(y)} e^{B(t)} \delta_{i j} dx^{i} dx^{j} + e^{C(t)} dy^{2}; \tag{2.4}
\]

here we have still assumed an isotropy in the three large spatial dimensions but have allowed a different evolution in the compact dimension.

We shall principally consider a universe that oscillates rapidly so that the time dependence and the \( y \)-dependence, which fixes the size of the compact dimension, are on a similar footing. The fourth-order, two-variable differential equations that result from varying the action \( (2.1) \) are more difficult to solve than the static case \( (2.2) \), which can be solved numerically, but fortunately it is possible to build a solution order by order in a small amplitude expansion. To first order in \( \epsilon_{y}, \epsilon_{t} \ll 1 \), we find \footnote{Our convention for the signature of the metric is \((- , + , + , + , + )\) while the Riemann curvature tensor is defined by \( R^{a}_{b c d} \equiv \partial_{a} \Gamma^{a}_{b c} - \partial_{b} \Gamma^{a}_{c d} + \Gamma^{e}_{b d} \Gamma^{a}_{c d} - \Gamma^{e}_{c d} \Gamma^{a}_{b d} \). Here the indices \( a, b, \ldots = 0, 1, 2, 3, y \) range over all coordinates while \( \mu, \nu \ldots = 0, 1, 2, 3 \) label the ordinary space-time coordinates.}

\[
A(y) = \epsilon_{y} \cos(\omega_{y} y) + \cdots \]

\[
B(t) = \epsilon_{t} \cos(\omega_{t} t) + \cdots \quad C(t) = -3 \epsilon_{t} \cos(\omega_{t} t) + \cdots \tag{2.5}
\]

with

\[
\omega_{y} = \sqrt{-\frac{3}{\mu}} \quad \text{and} \quad \omega_{t} = \frac{1}{\sqrt{3 \mu - 16 \nu}} \tag{2.6}
\]
so that a periodic solution exists when $\mu < 0$ and $\nu < \frac{3}{16}\mu$—no fine-tuning is needed. In (2.6) we see explicitly that the higher derivative terms in the action determine the size of the extra dimension and the oscillation frequency. Proceeding to the next order in the small $\epsilon_y, \epsilon_t$ expansion [2] introduces some corrections to each of $A(y), B(t)$ and $C(t)$, which depend on both $t$ and $y$, and more importantly relates the small amplitudes to the size of the cosmological constant:

$$\Lambda = \frac{3}{4} \left( \omega^2_t \epsilon_t^2 + \omega^2_y \epsilon_y^2 \right).$$

The relative size of the $\epsilon_y$ and $\epsilon_t$ in this solution is undetermined. This relation allows us to state more precisely the regime in which the small amplitude expansion exists. When $\mu$ and $\nu$ have a natural size, $O(M^{-25})$, then a small amplitude translates to a small cosmological constant, $M_5 \Lambda^{-1/2} \gg 1$. Note however that the existence of periodic solutions—beyond the small amplitude regime—does not require a small cosmological constant as was shown numerically in [1].

In the large $3 + 1$ dimensions, once we have substituted the leading effects from the $y$-dependent scalar field, we discover that the density $\rho$ and the pressure $p$ for the non-compact spatial dimensions are

$$\rho = \frac{3}{4} \omega^2_t \epsilon_t^2 + \frac{\phi^2}{4} + \cdots \approx \frac{3}{2} \omega^2_t \epsilon_t^2 > 0$$

$$p = -\frac{3}{4} \omega^2_t \epsilon_t^2 + \frac{\phi^2}{4} + \cdots \approx 0$$

so that $\frac{3}{4} \omega^2_t \epsilon_t^2$ resembles an effective cosmological constant. The fundamental cosmological constant that appears in the action effects the warping of the extra dimension, as the pressure in that dimension indicates, $p_y = -2\Lambda + \frac{3}{2} \omega^2_t \epsilon_t^2$.

We find that the small amplitude expansion described above cannot be extended to the case of a de Sitter expansion $B(t) = kt + \cdots$ that coexists with the oscillations, if the rate of expansion is to be comparable to the leading oscillating terms in the metric, i.e. $k \sim O(M_5 \epsilon_t)$. In general, the dynamics that links the short scale oscillations with the large scale evolution of the universe is complicated and deserves further study.

If the amplitude of the oscillations starts with a value large compared to any expansion rate, the amplitude must eventually decrease sufficiently so as to allow a realistic cosmology. Given that the vacuum energy density in (2.8) is positive, a natural mechanism for its relaxation is through the decay of the rapid oscillations into energetic particles, analogous to the decay of the inflaton in inflation. In section 4, we shall study how fermions and gauge fields couple to the oscillating functions, $B(t)$ and $C(t)$, thus providing an explicit decay mechanism.

Note that the presence of the extra dimension is necessary for this decay. In $3 + 1$ dimensions the metric would not contain a $C(t)$ term and would therefore be conformally flat; then the oscillation could not decay into conformally coupled fields.

3. Chiral fermions.

In order to recover the standard model fields, the theory must contain chiral fermions. A difficulty arises for the usual Kaluza-Klein compactification with only one extra dimen-
sion since the Lorentz symmetry group $SO(4,1)$ has only one, non-chiral, spin-$\frac{1}{2}$ representation. Chiral spinor representations do arise when we break the full $SO(4,1)$ symmetry group, for example by placing the theory on an $S^1/Z_2$ orbifold in the fifth dimension [4].

We can similarly introduce an orbifold into the warped Kaluza-Klein picture with a static metric, (2.2). The field equations for the action (2.1) that determine $A(y)$ and $\phi(y)$ do not depend explicitly on $y$ so we are free to translate an extremum of $A(y)$ to $y = 0$. If the period of $A(y)$ is $y_c$ then we can compactify the space by restricting $y \in [-\frac{1}{2}y_c, \frac{1}{2}y_c]$ and identifying the endpoints. The solutions for $A(y)$ found numerically in [1] and analytically in a small amplitude expansion in [3] are manifestly even under $y \rightarrow -y$ so the background metric respects an $SO(3,1) \times Z_2$ symmetry.

This $Z_2$ invariance allows us to introduce an orbifold geometry in the extra dimension by identifying $y$ with $-y$. In order to define a theory consistently on this orbifold, the fields must be odd or even under this discrete $Z_2$. As we have seen, the background metric is even and this orbifold geometry will further constrain the allowed gravitational excitations of this background.

The scalar field, in contrast, must be odd, $\phi(-y) = -\phi(y)$. The reason is that the field equations for (2.1) relate $\phi'(y)$ to an expression that depends on $A(y)$ only through terms with even numbers of derivatives. Therefore, $\phi'(y)$ oscillates with the same period as $A(y)$ and is also even under $\phi'(y) = \phi'(-y)$. For the solutions found in [1] [2] [3], $\phi'(y)$ is everywhere positive so that after integrating we obtain a $\phi(y)$ that increases monotonically. Choosing the constant of integration so that $\phi(0) = 0$ produces a $\phi(y)$ that is odd. To accommodate the boundary values, we must further impose that $\phi(y)$ itself assumes values only in a compact space by identifying $\phi(y_c/2)$ and $\phi(-y_c/2)$. In figure 1, we show explicitly a example with this geometry. The form for $\phi(y)$ was found numerically for $\Delta \mathcal{L}_\phi = k(\nabla_a \phi \nabla^a \phi)^2$ but with the parameters chosen arbitrarily within the region of the $\{\Lambda, \mu, \lambda, k\}$-space with periodic solutions in $y$.

![Figure 1](image_url)

**Figure 1.** The behavior of $\phi(y)$ on the orbifold for $\Lambda = 1$, $\mu = 0.1$, $\lambda = 0$, and $\Delta \mathcal{L}_\phi = -\frac{1}{4}(\nabla_a \phi \nabla^a \phi)^2$. The initial condition is $A'(0) = 23.77364592$. Here $y \in [-0.6578, 0.6578]$ and $\phi \in [-0.744, 0.744]$. The endpoints of each dimension are identified so that both are compact.

In order to be compatible with this $Z_2$ orbifold structure, a separate scalar field should be included if we wish also to allow the rapid time oscillations. This requirement follows from the leading order behavior, $\dot{\phi}^2 = 3\omega_c^2 \epsilon^2$ [2]; upon integration we obtain a contribution that is even under the $Z_2$ symmetry which is incompatible with the symmetry of the scalar field that supports the compact extra dimension.
We now obtain chiral fermions through the standard construction \[4\] \[5\]. If we choose the following boundary conditions on a five dimensional fermion,
\[
\psi(x^\lambda, -y) = \gamma^5 \psi(x^\lambda, y),
\]
then expanding in a tower of Kaluza-Klein modes, we have a chiral zero mass mode,
\[
\psi^{(0)}_L(x^\lambda, y) = 0, \quad \psi^{(0)}_R(x^\lambda, y) = e^{-A(y)} \psi^{(0)}_R(x^\lambda),
\]
as well as a series of paired massive left and right modes \(\psi^{(n)}_{L,R}(x^\lambda, y)\) for \(n > 0\). Unlike the flat Kaluza-Klein expansion, the zero mass mode does depend on the fifth coordinate through the \(e^{-A(y)}\) factor.

4. Fermions and gauge fields in an oscillating background.

The presence of ordinary fermion or gauge fields affects the evolution of universe with rapid time oscillations. The interaction of such fields with the oscillating metric offers a route for the relaxation these oscillations. However, we shall see that the zero mass mode of the Kaluza-Klein expansion of a massless five dimensional fermion does not couple directly to the oscillating components of the metric, to leading order. A decay into gauge fields (or scalar fields) becomes the dominant channel for the relaxation of the metric.

Returning to a metric with a dependence both on time and the extra dimension (2.4), when \(B(t) \neq C(t)\) this metric is not conformally flat. This feature becomes more apparent if we define new coordinates
\[
\eta(t) \equiv \int^t e^{-\frac{1}{2}B(t')} dt' \quad \text{and} \quad r(y) \equiv \int^y e^{-\frac{1}{2}A(y')} dy',
\]
in terms of which the metric (2.4) becomes
\[
ds^2 = e^{A(r)} e^{B(\eta)} \left[ -d\eta^2 + \delta_{ij} dx^i dx^j + e^{C(\eta) - B(\eta)} dr^2 \right],
\]
where \(A(r), B(\eta)\) and \(C(\eta)\) are understood to be \(A(y(r)), B(t(\eta))\) and \(C(t(\eta))\).

We now introduce a fermion field \(\psi\) and an Abelian gauge field \(A_a\) through the action\[2\]
\[
S_{\psi,A} = \int d^4x dr \sqrt{-g} \left\{ e_A^a \bar{\psi} \Gamma^A(D_a - igA_a)\psi - \frac{1}{4} g^{ab} g^{cd} F_{ac} F_{bd} \right\},
\]
where \(F_{ab}\) is the gauge field strength. We work in the limit in which these fields do not significantly affect the background geometry. In this action, we have not assumed an

\[2\] Because of our choice for the metric’s signature, we have not included the usual factor of \(i\) in the fermion action. The \(\gamma\) matrices are defined by \(\Gamma^A = \{i\gamma^\mu, \gamma_5\}\), where \(\gamma^\mu\) and \(\gamma_5\) are the standard \(\gamma\) matrices, so that \(\{\Gamma^A, \Gamma^B\} = 2\eta^{AB}\) for \(\eta^{AB} = \text{diag}[-1, 1, 1, 1, 1]\). Refer to \[1\] or \[7\] for further details on fermions in a curved background.
orbifold structure in the extra dimension, but as in the previous section one can be readily introduced. The $e_A^a$ is a vierbein which connects the curved coordinates with a locally Lorentzian frame:

$$e_A^a e_B^b g_{ab} = \eta_{AB}. \quad (4.4)$$

The covariant derivative of a fermion,

$$D_a = \partial_a + \frac{i}{2} \omega_{aBC} \sigma^{BC}, \quad (4.5)$$

involves the spin-connection defined by

$$\omega_{aAB} = \frac{1}{2} e_A^b \left[ \partial_a e_B^b - \partial_b e_B^a \right] - \frac{1}{2} e_B^b \left[ \partial_a e_A^b - \partial_b e_A^a \right] - \frac{1}{2} e_A^c e_B^d [\partial_c e_D^d - \partial_d e_D^c] e_A^a \quad (4.6)$$

and $\sigma^{BC} = \frac{1}{4} [\Gamma^B, \Gamma^C]$. The contributions from the spin connection are canceled by introducing a rescaled fermion field defined by

$$\psi(x^\lambda, r) = e^{-A(r)} e^{-\frac{1}{4} B(\eta)} e^{-\frac{1}{4} C(\eta)} \bar{\Psi}(x^\lambda, r). \quad (4.7)$$

The fermion-gauge field action then becomes

$$S_{\psi, A} = \int d^4x dr \left\{ i \bar{\Psi} \gamma^\mu (\partial_\mu - ig A_\mu) \Psi + e^{\frac{1}{2}(B-C)} \bar{\Psi} \gamma^5 (\partial_r - ig A_r) \Psi \right\} + \int d^4x dr e^{\frac{1}{2}(A+C)} \left\{ -\frac{1}{4} \eta^{\mu\nu} \eta^{\lambda\rho} F_{\mu\lambda} F_{\nu\rho} - \frac{1}{4} e^{(B-C)} \eta^{\mu\nu} F_{\mu r} F_{\nu r} \right\}; \quad (4.8)$$

note that the first term no longer contains $A(r)$, $B(\eta)$ or $C(\eta)$. 

4.1. The Kaluza-Klein expansion.

To investigate the effect of the oscillating metric in the long distance limit, we expand the fermion and gauge fields in a tower of Kaluza-Klein modes. To simplify, we work in the axial gauge $A_r = 0$. The fermions are expanded separately in left- and right-handed fields,

$$\Psi_{L,R}(x^\lambda, r) \equiv \frac{1}{2} \left( 1 \mp \gamma_5 \right) \Psi = \sum_{n=0}^{\infty} \Psi_{L,R}^{(n)}(x^\lambda) f_{L,R}^{(n)}(r) \quad (4.9)$$

where $f_{L,R}^{(n)}(r)$ satisfy

$$\partial_r f_L^{(n)} = m_n f_R^{(n)} \quad \partial_r f_R^{(n)} = -m_n f_L^{(n)} \quad (4.10)$$

and obey the following orthogonality condition,

$$\int_{-r_c/2}^{r_c/2} dr f_{L}^{(m)*}(r) f_{L}^{(n)}(r) = \int_{-r_c/2}^{r_c/2} dr f_{R}^{(m)*}(r) f_{R}^{(n)}(r) = \delta^{mn}, \quad (4.11)$$

and
where \( r_c \) is the volume of the extra dimension. Analogously, we expand the gauge field

\[
A_{\mu}(x^\lambda, r) = \sum_{n=0}^{\infty} A^{(n)}_{\mu}(x^\lambda) h^{(n)}(r),
\]

(4.12)
defining the masses of the modes through

\[
\partial_r \left[ e^{\frac{1}{2} A} \partial_r h^{(n)} \right] = -M^2_n e^{\frac{1}{2} A} h^{(n)}
\]

(4.13)
and normalizing the states through

\[
\int_{-r_c/2}^{r_c/2} dr e^{\frac{1}{2} A(r)} h^{(m)}(r) h^{(n)}(r) = \delta^{mn}.
\]

(4.14)
The fermion-gauge interaction will induce couplings among the various Kaluza-Klein modes,

\[
G^{mnp}_{L,R} \equiv g \int_{-r_c/2}^{r_c/2} dr f^{(m)\ast}_{L,R}(r) f^{(n)}_{L,R}(r) h^{(p)}(r).
\]

(4.15)
The effective action that appears in four dimensions as a result of (4.8) is thus

\[
S_{\psi,A} = \int d^4x \left\{ \sum_{m,n} i \bar{\Psi}_L^{(m)} \gamma^\mu \left( \delta^{mn} \partial_{\mu} - i \sum_p G^{mnp}_{L} A^{(p)}_{\mu} \right) \Psi_L^{(n)} + \right.
\]

\[
+ \sum_{m,n} i \bar{\Psi}_R^{(m)} \gamma^\mu \left( \delta^{mn} \partial_{\mu} - i \sum_p G^{mnp}_{R} A^{(p)}_{\mu} \right) \Psi_R^{(n)}
\]

\[
+ e^{\frac{1}{2} (B-C)} \sum_n m_n \left( \bar{\Psi}_L^{(n)} \Psi_R^{(n)} + \bar{\Psi}_R^{(n)} \Psi_L^{(n)} \right) \}
\]

(4.16)
\[
+ \int d^4x e^{\frac{1}{2} C} \sum_n \left\{ -\frac{1}{4} \eta^{\mu\nu} \eta^{\lambda\rho} F_{\mu\lambda}^{(n)} F_{\nu\rho}^{(n)} - \frac{1}{2} e^{(B-C) M^2_n \eta} A^{(n)}_{\mu} A^{(n)}_{\nu} \right\}. \]

At low energies, well below the Planck scale, only the dynamics of the massless modes will be important in the effective theory. The \( r \)-dependent parts of the lowest-lying fermions are

\[
f^{(0)}_L = f^{(0)}_R = r_c^{-1/2}
\]
along with a factor of \( e^{-A(r)} \) from (4.7). The couplings among the massless modes simplify to

\[
g_0 \equiv G^{000}_L = G^{000}_R = \frac{g}{r_c} \int_{-r_c/2}^{r_c/2} dr h^{(0)}(r).
\]

(4.17)
The low energy effective action is thus

\[
S_{\text{eff}} = \int d^4x \left\{ i \bar{\Psi}^{(0)} \gamma^\mu \left( \partial_{\mu} - ig_0 A^{(0)}_{\mu} \right) \Psi^{(0)} - \frac{1}{4} e^{\frac{1}{2} C} F^{(0)}_{\mu\nu} F^{(0)\mu\nu} + \ldots \right\}
\]

(4.18)
In this effective action, the gravitational oscillations do not couple directly to the fermions, but rather to the gauge fields. In the small amplitude limit that we have assumed, the leading interaction is

$$\mathcal{L}_{\text{interaction}} = -\frac{1}{8} C(t) F_{\mu\nu}^{(0)} F^{(0)\mu\nu}. \quad (4.19)$$

This coupling offers a natural channel for the decay of the oscillating component of the metric. Similarly, any fundamental scalar fields in the theory would also allow the decay of the oscillating gravitational field. The kinetic energy term for the zero mass Kaluza-Klein mode of a massless five dimensional scalar field has a prefactor of $e^{B+\frac{1}{2}C}$.

If the fermion in (4.18) has a small realistic mass $m$, either through a fermion mass in the five dimensional action or if a massless fermion subsequently develops a dynamical mass, a mass term would produce a coupling between the fermion and the gravitational oscillations. However, this term would be suppressed by $m/M_5$ relative to (4.19).

5. Ultra high energy cosmic rays.

If some slowly decaying residual oscillations were to exist today, the gauge field products of this decay would provide a possible source for the ultra high energy cosmic rays. Here we shall only focus on demonstrating that a realistic cosmic ray spectrum can arise and establishing a rough limit on the rate of the decay and shall not attempt to develop a detailed model that produces this rate. The dominant signal would likely result from a decay of the oscillating background into a pair of gluons. An initial gluon of energy $E_0 \sim M_{Pl} \sim M_5$ will fragment into a high multiplicity jet of particles with a wide range of energies. For an initial photon pair the secondaries are only those produced as the photons scatter from the intergalactic radio background, and thus should not provide as strong a constraint as gluon-gluon production. Additionally, the presence of any gauge interactions beyond the standard model could provide other decay channels.

The observed flux of ultra-high energy cosmic rays, assumed here to be protons, can then set a bound on rate of production, or equivalently on the rate of decay of the effective vacuum energy density, $\rho_{\text{vac}}$. We estimate the flux of protons above some energy $E_>$ by

$$J_\gamma \approx \frac{1}{4\pi} \dot{\rho}_{\text{vac}} E_0^{-1} \ell N_\gamma. \quad (5.1)$$

$N_\gamma$ is the number of protons with energy $E > E_>$ in a jet produced by the initial gluon of energy $E_0 \sim M_{Pl}$. $\ell$ is the attenuation length of these protons due to scattering from the cosmic microwave background radiation [8]. When $E_\gamma \approx 10^{11}$ GeV then $\ell$ is approximately a few tens of Mpc [4]. This energy is just above the expected but not seen GZK cutoff [8], which is exceeded here by high energy protons originating within a distance $\ell$ of us.

We shall obtain an estimate of $N_\gamma$ from the perturbative analysis of multiparticle production in jets based on the modified leading log approximation (e.g. [10]). The

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3 In this approximation the main difference at leading order between a quark or gluon jet is that the multiplicity of particles in the gluon jet is enhanced by a factor of $\frac{9}{4}$ relative to the quark jet for the same initial jet energy. The mean particle energy of products will correspondingly be slightly lower in the gluon jet. In the following we have treated a gluon jet the same as a quark jet.
resulting “limiting spectrum” is known but since the initial gluon energy is so high, \( \tau \equiv \ln(E_0/\Lambda_{\text{QCD}}) \gg 1 \), we may simplify further with a Gaussian representation. We extract the following results from [10] where \( \xi \equiv \ln(E_0/E_0) \).

\[
\frac{dN}{d\xi} \propto \exp \left( -\frac{1}{2} \left( \xi - \overline{\xi} \right)^2 / \sigma^2 \right) \tag{5.2}
\]

\[
\sigma = \frac{\tau}{\sqrt{3z}} \left( 1 - \frac{3}{4z} \right) \quad \overline{\xi} = \tau \left( \frac{1}{2} + \sqrt{\frac{C}{\tau}} \right) \tag{5.3}
\]

Here \( z = \sqrt{16N_c \tau/b} \), \( C = a^2/(16N_c b) \), \( 3b = 11N_c - 2n_f \), and \( 3a = 11N_c + 2n_f/N_c^2 \). We also note that a Gaussian spectrum resembles the results from Monte Carlo simulations [11]. For the total multiplicity \( N = \int_0^\infty (dN/d\xi) \, d\xi \) we use the full “limiting spectrum” result,

\[
N = \Gamma(B) \left( \frac{z}{2} \right)^{1-B} I_{B+1}(z), \tag{5.4}
\]

where \( B = a/b \). This gives \( N \approx 7 \times 10^5 \). Assuming that 5% of the energy from each initial gluon emerges as protons, these results imply that \( N_\geq \approx 3000 \) for \( E_\geq = 10^{11} \text{ GeV} \) and \( n_f = 6 \). This value for \( N_\geq \) triples when the \( O(\tau^{-1/2}) \) corrections in (5.3) are absent. Another indication of the sensitivity of the result to the approximation is that no single value of \( n_f \) is actually correct over the range of energies involved, and \( N_\geq \) ranges from 1400 to 6000 as \( n_f \) ranges from 3 to 8. This last result also shows how \( N_\geq \) can be strongly affected by new physics beyond the standard model.

The observed integrated flux of cosmic rays with \( E > 10^{11} \text{ GeV} \) is approximately \( 2 \times 10^{-20} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \) [12], and so (5.1) and \( N_\geq \approx 3000 \) implies that

\[
\dot{\rho}_{\text{vac}} < 3 \times 10^{-53} \text{ g cm}^{-3} \text{ s}^{-1}. \tag{5.5}
\]

Thus the current vacuum energy density of the universe, assumed to be about two-thirds of the critical density, has a decay rate

\[
\frac{\dot{\rho}_{\text{vac}}}{\rho_{\text{vac}}} < 10^{-6} \frac{1}{t_{\text{universe}}}, \tag{5.6}
\]

where \( t_{\text{universe}} \approx 14 \text{ Gyr} \) is the age of the universe.

If this limit is saturated then we have a model for the ultra-high energy cosmic rays. The spectrum in (5.2) implies a hard energy spectrum \( dN/dE \sim 10^{-\alpha} \) with \( \alpha \approx 1.2 \) at the relevant energies. The observed flux spectrum \( dJ/dE \) is modified by the rapid decrease of the attenuation length by almost a factor of 100 between \( 4 \times 10^{10} < E < 10^{11} \text{ GeV} \) (the GZK cutoff). The frequently plotted \( E^3dJ/dE \) thus rises for \( E < 4 \times 10^{10} \text{ GeV} \), drops for \( 4 \times 10^{10} < E < 10^{11} \text{ GeV} \), and continues to rise for \( E > 10^{11} \text{ GeV} \). This behavior is a simple consequence of a hard initial spectrum of protons produced uniformly throughout space, and is quite consistent with the data above \( 10^{10} \text{ GeV} \). Below \( 10^{10} \text{ GeV} \) the observed flux rises much faster with decreasing energy, and some other source for cosmic rays must dominate.
This picture is similar to models involving decay of supermassive long-lived particles \[13\] \[14\] \[15\]. The primary difference is that the supermassive particles tend to congregate in the galactic halo, and this produces a galactic component to the signal in addition to a possible extragalactic one. But the galactic component would not produce a dip in the spectrum, and it would produce some amount of large scale anisotropy \[14\] \[15\]. In our picture the absence of a galactic component is natural, although there remains the question of just how spatially uniform the decay mechanism would be.

The production of cosmic gluon jets also ties in with another mechanism for producing air showers above the GZK cutoff, this time involving jets produced outside the \(\ell^3\) volume. This is because gluon jets contain neutrinos, and these energetic cosmic neutrinos have some probability for producing \(Z\)-bursts within the \(\ell^3\) volume, as they travel toward us \[10\]. In particular neutrinos with energy within \(\delta E/E_R = \Gamma_Z/M_Z \approx 3\%\) of the \(Z\)-resonance energy \(E_R = 4 (eV/m_\nu) \times 10^{12}\) GeV may annihilate with an enhanced cross section on the nonrelativistic relic antineutrinos (and vice versa) to produce the \(Z\). We estimate (with the same uncertainties as before) that the number of neutrinos in this energy band, produced per gluon jet, ranges from 300 to 1100 as \(E_R\) ranges from \(10^{13}\) to \(10^{11}\) GeV. The probability for such a neutrino to produce a \(Z\)-burst within the \(\ell^3\) volume is in the range 0.025\% to 1\%, depending on the relic neutrino clustering \[16\]. On the other hand there is an enhancement factor of about 100 for the neutrino flux relative to the direct proton flux since the former originates from the whole Hubble volume. Each \(Z\)-burst produces a couple of protons with typical energies \(E_p \approx E_R/30\), and thus this mechanism produces protons peaked in a fairly narrow energy range. It may even be of interest if this peak was somewhat below the GZK bound, where the protons-from-jets mechanism was deficient.

6. Conclusions.

A warped, compact background space-time with a compact scalar field, introduced to address the cosmological constant problem, does not present any obstructions to including chiral fermions. Since the compactness of the extra dimension resulted from a periodic solution to the field equations, rather than adding brane boundaries, the most appropriate approach is to give the universe an \(S^1/Z_2\) orbifold structure in this dimension.

A Kaluza-Klein expansion of massless fermion and gauge fields in a warped and oscillating background reveals that the zero mass modes of the fermion do not couple at leading order with the oscillating terms in the metric. Thus the gauge fields would most readily facilitate the decay of the oscillating gravitational field.

The decay of the oscillations in the metric should proceed rapidly in the early universe, at least until the amplitude is sufficiently small that the oscillatory effects are comparable to any de Sitter expansion or to effects from any matter and radiation present, in which case the simple picture developed in section 2 breaks down and the subsequent evolution is more complex. Yet it would be useful to determine when such oscillations could coexist with a familiar cosmology in the \(3 + 1\) dimensional effective theory since we have seen that a small, slowly relaxing oscillation today could provide a source for the observe ultra high energy cosmic rays.
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