Application of Fuzzy Controller Based on Chaos Optimal Design

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Abstract. It is difficult to tune parameters of fuzzy controller (FC), and control rules and membership functions of FC are hard to obtain optimization too. Therefore, the control results show strong overshoot and oscillation often. The authors propose a chaos optimal design method based on annealing strategy. According to random and ergodic features of chaos dynamics, the chaotic variables are applied to searching for parameters of FC, the optimal variables are transformed into chaotic variables by carrier-wave, the global search process is done using ergodic features of chaotic motion, and then an approximate global optimal solution is obtained. After that, the chaos local searching and optimization based on annealing strategy is cited, the parameters are optimized again within the limits of the approximate global optimal solution, which is realized by means of combination of global and partial chaos searching, which can converge quickly to global optimal value. Finally, simulations of third order and nonlinear system show that control result of the method is good.

1. Introduction

Fuzzy control has emerged as one of the most active and fruitful for research in the applications of fuzzy set theory, especially in the realm of industrial processes, which do not lend themselves to control by conventional methods because of a lack of quantitative data regarding the input-output relation. But fuzzy control also has its disadvantages such as hard to obtain optimal membership functions and control rules, especially when the processes are highly nonlinear and with high dimensions. The optimal parameters is acquired by trial-and-error methods using expert’s experiences[1], which takes a long time and is difficult to obtain optimal fuzzy systems. To seek an optimal FC, a number of exact methods have been proposed in literature, such as BP algorithm, gradient descent method, and so on[2,3], but they also have disadvantages of slow convergence and tend to become trapped in local minimum. Simulate annealing method has been widely applied to various optimization problems, too[4], but it requires subtle adjustment of parameters in the annealing schedule such as the size of the temperature steps, the temperature range, the number of re-starts and re-direction of the search, etc. It is not enough to design an optimal FC in detail with these methods alone.

In order to search the optimal parameters of FC, the authors propose a chaos optimal algorithm based on annealing strategy. Due to ergodic and random features of chaos dynamics, it can continually search all conditions according to itself rule within ranged space. First, the optimal variables are transformed into chaotic variables that are applied to global searching and optimization for parameters of FC, with an approximate global optimal solution obtained. After that, the chaos local searching and optimization based on annealing strategy is cited, the parameters are optimized again, which is realized
by means of combination of global and partial chaos searching, which can be expected to have higher ability of searching for global optimal solutions\cite{5}.

2. Rule of fuzzy control

The method of fuzzy inference in literature\cite{6} is adopted, three rules used for the fuzzy control are:

if \(E\) is NB)then\((u_F\) is NB)

if \(E\) is PB)then\((u_F\) is PB)

if \(E\) is AZ)then\((u_F\) is AZ)

Where \(E\) is the quantized value of error signal \(e(k)\), \(-1 \leq E \leq 1\). NB stands for negative big, PB for positive big and AZ for approximate zero. The membership functions for FC’s input \(E\) and output \(u_F\) use triangular membership functions. While the membership function for \(E\) are fixed, the membership functions for \(u_F\) may change according to the parameters (structure parameters of controller) \(x_1\), \(x_2\), they are shown in Fig 1:

Fig 1, Membership function

Fuzzy control algorithm is presented as follows:

(1) Case 1:

\[
0 < x_1 \leq x_2 < 1
\]

\[
u_F = \frac{E_{x_1}E[3x_2(2 - |E|) + E_x(3 - E^2)]}{3[2x_1(1 - E^2) + E_x(2|E| - E^2)]}
\]

(1)
Case 2:

Range ①:
\[ 0 < x_2 < x_1 < 1 \]
\[ u_F = \frac{E[3(1 - x_1^2) + 3x_1^2[E - x_1^2 E^2]]}{3(2x_1 + 2(1 - x_1)[E - x_1^2 E^2])} \]  

Range ②:
\[ E_d < |E| < 1 - E_d \]
\[ u_F = \frac{E[E_d[3x_1(2-E)+E_d(3-E^2)]-E_dE_d[3-E_d(1-E_1)]]}{3[E_2(1-E^2) + E_4[2|E-E^2| - E_0]]} \]

Range ③:
\[ 1 - E_d \leq |E| \leq 1 \]
\[ u_F = \frac{E[E_d[x_1(1+E^2)]-E_dE_d(3-E_d(1+E^2))]}{3[E_2(2+x_1E_2) - E_0E_0]} \]

Where,
\[ E_d = (x_1-x_2)/(1+x_1-x_2) \]
\[ E_1 = 1 - |E| \]
\[ E_2 = 1 + |E| \]
\[ E_3 = 1 - |E| \]
\[ E_4 = x_1 - x_2 \]
\[ E_5 = 1 - x_2 \]

3. Chaos optimization method

As discussed in the preceding section shows that the structure parameters \( x_1, x_2 \) of fuzzy controller are required to optimization. Supposing that the expected output is \( r_n \) and the network output is \( y_n \), the object function can be defined as follows:
\[ J = \int_0^t t\varepsilon(t)\,dt \]

Adoption Logistic mapping:
\[ q_{n+1} = \mu q_n(1 - q_n) \]
\[ n = 0, 1, 2, \ldots, N \]
\[ q_0 \in (0, 1) \]

Where,
\[ q = (q_1, q_2, \ldots, q_N) \]
are the number of chaotic variables, \( N \) is the frequency of chaos learning, \( \mu \) is control parameter. When \( \mu=4 \), the mapping is perfect mapping of \((0,1)\) interval and system gets into full chaos state, \( q_n \) are ergodic in \((0,1)\) interval and generate chaotic sequence. Since chaotic state is extreme sensitiveness to initial value, to fetch some initial values with tiny discrepancy to equation (6) can get a sequence of chaotic variables \( \{q_n\} \) with different traces. The chaotic variables are carried-wave and iterated respectively, the optimal current parameters (global suboptimization solutions) \( x_i^* \), \( i=1,2 \) are obtained. And then, the method of chaos partial searching based on annealing strategy is applied, which automatically reduces range of chaos searching and implement optimal method of combine global and partial searching, it can accelerate searching speed and the parameters are converged to global optimal solution.
As discussed in the preceding section, equations are presented as follows:

\[ x_{n+1} = \mu x_n (1 + x_n) \quad (\mu = 4, x_0 \in (0,1)) \]  

(7)

\[ x_{1,n} = x_i^* + z(t) x_n \]  

(8)

\[ x_{2,n} = x_i^* + z(t) x_n \]  

(9)

\[ z(t+1) = (1 - \alpha) z(t) \quad (0 < \alpha < 1) \]  

(10)

where, \( n=0,1,N \), \( z(t) \) is time parameter based on annealing strategy, \( \alpha \) is the decaying factor of \( z(t) \).

Constraint condition is as follows:

\[ 0 < x_1, x_2 < 1 \]

The optimization steps are shown as follows:

1) The algorithm initialization, \( n=0, t=0, \)

To fetch two different initial values random in \((0,1)\) interval and substitute in Logistic mapping of equation (7), respectively, two chaos variables \( x_{1,n}, x_{2,n} \) are gotten, according to system demand, each chaos variables \( x_{i,n} \) of \((0,1)\) interval are mapping to correspondent interval.

2) The chaos variables are substituted in the parameters \( x_1 \) and \( x_2 \) of equation (1) -(4), according to the equation (5) calculate the object function \( J \), let \( J^* = J(0) \), \( x_i^* = x_{i,0} \).

3) While \( n \geq N \), the algorithm proceeds next step, otherwise jump to step 5).

4) If \( J(N) < J^* \), then \( J^* = J(n) \), \( x_i^* = x_{i,n} \).

5) If \( J(N) \geq J^* \), then abandon \( x_{i,n} \), \( n=n+1 \), jump to step 2).

6) The global approximate optimal values \( x_i^* \) are obtained, \( t=t+1 \).

7) \( x_i^* \) are substituted in equation(8) and (9) to calculate global optimal values \( x_i^* \).

8) \( z(t) \) is calculated according to equation (10), if \( z(t) \leq Z \) (searching closing condition), the algorithm proceeds to the next step, otherwise jump step 6)

4. Application in the turbine speed control system and linear unipolar inverted pendulum

Mixed-flow turbine speed control system is a nonlinear time-varying system. Its rotation number \( y(s) \) on the input \( u(s) \) transfer function as follows:

\[ G(s) = \frac{e_y (1-T_y s)}{(1+T_y s)(1+e_{wy} T_y s)(e_y + T_y s)} \]  

(11)

where,

\[ T_1 = \frac{e_{hy} - e_{wy}}{T_W} ; \]

\[ e_n = e_g - e_{x_i} \]

\( e_y, e_r, e_h, e_{wy}, e_{qh} \) is the Turbine unit characteristic factor; \( e_g \) is self-regulating factor for the generator set; \( T_y, T_w, T_r \) are unit, diversion channel and the inertia time constant of relay device respectively. When \( T_r = 0.2 \cdot T_w = 0.85 \cdot T_y = 4.8 \cdot e_r = 0.2 \cdot e_h = 1.0 \cdot e_{wy} = 1.0 \cdot e_{qh} = 0.5[7] \) and the sampling period \( T = 0.1s \), using the discrete processing of ZOH(zero-order hold) devices on the controlled object, structure diagram of fuzzy control system is shown as Fig.2. The input minimum is \( u_{min} = 0 \) , and maximum is \( u_{max} = 15 \). Using chaotic optimization algorithm proposed in this paper to control. After Chaos coarse search and fine search optimization to quickly find the optimal value of fuzzy controller parameters, control results is shown as Fig.3. It can be seen from the figure, control system has advantages, such as stable, non-oscillatory, non-static error, less overshoot, fast response, and short adjustment time. It provides an effective way to address the global optimal design of fuzzy controller parameters in the industrial process control.
For another example. The mathematical model of linear unipolar inverted pendulum is as follows:

\[
\begin{align*}
(M + m) \ddot{x} + b \dot{x} - mL \dot{\theta} \cos \theta + mL \dot{\theta}^2 \sin \theta &= F \\
(I + mL^2) \ddot{\theta} - mgL \sin \theta &= mL \dot{\theta} \cos \theta
\end{align*}
\]  

(12)

Where,
- \( \theta \) is the Angle between the pendulum rod and the vertical upward direction (rad),
- \( \dot{\theta} \) is the angular velocity (rad/s),
- \( x \) is the displacement of the car (m),
- \( \dot{x} \) is the displacement velocity of the car (m/s),
- \( F \) is the force applied on the car (N),
- \( I \) is the inertia of the pendulum rod,
- \( L \) is the length from the axis of rotation of the pendulum rod to the center of mass of the rod,
- \( b \) is the friction coefficient of the car,
- \( m \) is the mass of the rod, and
- \( M \) is the mass of the car.

The mathematical model (12) of the monopole inverted pendulum system is taken as the simulation object. In the actual system, \( I = 0.0034 \text{kgm}^2 \), \( L = 0.25 \text{m} \), \( b = 0.1 \text{N} / \text{m} \), \( m = 0.109 \text{kg} \), \( M = 1.096 \text{kg} \), gravity acceleration \( g = 9.8 \text{m/s}^2 \), and the maximum and minimum values allowed to be input are \( u_{\text{max}} = 5 \text{V} \), \( u_{\text{min}} = -5 \text{V} \). The parameters of the fuzzy controller are designed by using the chaos optimization method studied above. Set the sampling period as \( 0.02 \text{s} \), the control period as \( 0.1 \text{s} \) and \( \lambda = 0.05 \). When the initial value of the system is \( \theta_0 = 20^\circ (0.34 \text{rad}) \), \( \dot{\theta}_0 = 0 \), \( x_0 = 0.1 \text{m} \), \( \dot{x}_0 = 0 \), the dynamic response curve is shown in Fig 4. When the system is stable and the pendulum rod is disturbed by \( 15^\circ (0.2617 \text{ rad}) \), its dynamic response curve is shown in Fig 5.
5. Conclusions
A chaotic optimization algorithm based on annealing strategy has been proposed in this paper. Design of fuzzy controller has the feature of global optimization. The simulation results show that the controller can improve response performance of the plant, which is efficient in searching for controller optimal parameters. It can be used to control the angle of inverted pendulum to get better effect. The control result has many advantages, such as small overshoot, short settling time, simple algorithm structure and easy to implement.

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