Simple Moment Analysis for a Kinetic Study of the Chromatographic Behavior of Spherical Particles and Silica Monoliths

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A simple procedure of moment analysis was proposed for a kinetic study of the rate processes in the columns packed with full-porous spherical particles and silica monoliths. Previous chromatographic data measured in reversed-phase HPLC systems using Mightysil and Chromolith columns were analyzed by a simple moment analysis. The surface of the packing materials is chemically modified with octadecyl alkyl ligands. A mixture of methanol and water (80/20, v/v) and alkylbenzene homologous series \( (C_nH_{2n+1}, n = 0 - 7) \) were used as the mobile-phase solvent and sample probes, respectively. More detailed information about the experimental conditions is provided in Supporting Information. The values of the intra-stationary phase diffusivity \( (D_i) \) and the surface diffusion coefficient \( (D_s) \), derived by the simple moment analysis, were almost the same as those by the conventional moment analysis. The simple moment analysis is effective for quantitative studies of mass transfer in chromatographic systems. The previous chromatographic data were also analyzed by assuming external porosity \( (\varepsilon_e) \) as typical values, i.e., 0.40 for spherical particles and 0.70 for silica monoliths. The resulting values of \( D_i \) and \( D_s \) were of the same order of magnitude as those derived by using \( \varepsilon_e \), experimentally measured. Even if \( \varepsilon_e \) is assumed to be typical values, the simple moment analysis is effective for preliminary studies of the mass-transfer kinetics in the columns.

Keywords: Moment analysis, first absolute moment, second central moment, rate theory, mass transfer kinetics, intra-stationary phase diffusivity, surface diffusion coefficient, spherical particles, silica monolith, column external porosity

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Introduction

High-performance liquid chromatography (HPLC) has been extensively used as one of the most powerful separation techniques. Various types of separation media have been used to achieve a faster separation with a high efficiency, such as silica monoliths and core-shell spherical particles.\(^1,2\) Compared with conventional full-porous spherical particles, the separation media have extremely different structural characteristics concerning the shapes and porosities. The chromatographic behavior of separation media has usually been analyzed from kinetic points of view by using ordinary rate equations,\(^3,4\) such as the van Deemter equation. However, it seems that the rate equations are not necessarily sufficient for a detailed and quantitative kinetic study of their chromatographic behavior because of the following three problems.

At first, the rate equations were not developed by considering the intrinsic structural characteristics of the separation media. Second, most of the rate equations neglect the existence of some mass-transfer processes in a column, \( i.e., \) external mass transfer and surface diffusion,\(^5,6\) which have an important influence on the column efficiency. Finally, the rate equations contain some fitting parameters, of which the physical definition is not necessarily clear. It is impossible to quantitatively derive kinetic information about chromatographic separations, such as diffusivities and mass transfer coefficients, by analyzing the flow rate dependence of HETP \( (H) \).

In spite of the drawbacks of the ordinary rate equations, they have been extensively used because their description is usually quite simple. For example, the van Deemter equation is written as follows.

\[
H = A + \frac{B}{u} + Cu
\]

where \( u \) is the flow velocity of the mobile phase. As is well known, \( A, B, \) and \( C \) are the coefficients of the \( A-, B-, \) and \( C \)-term, respectively. It is possible to divide the contributions of the \( A-, B-, \) and \( C \)-term to band broadening and to derive the values of \( A, B, \) and \( C \). However, it is impossible to quantitatively determine the values of the diffusivities and mass transfer coefficients from those of the coefficients and to analyze chromatographic behavior in detail, even if the correlation between \( H \) and \( u \) is experimentally measured as accurately as possible.

Moment theory based on the general rate model of chromatography is one of the strategies for representing chromatographic behavior. It has been used for deducing information about the retention equilibrium and the mass-transfer and reaction kinetics in the column from the first absolute \( (\mu_1A) \) and second central \( (\mu_2C) \) moment of elution peaks.\(^7,10\) In the moment theory, chromatographic processes in...
a column are described in detail by considering the contributions of plural rate processes to \( \mu_{2C} \). Some of them are not considered in the conventional rate equations. The physical meaning of all parameters included in the moment equations is clearly defined. A framework of the moment equations for various separation media, which have different shapes (spherical particle, cylindrical fiber, and flat plate) and different porosities (full-porous, partially-porous, and non-porous), has already been developed while appropriately considering their structural characteristics.\(^\text{10–15}\)

The moment theory is more effective than the rate theory for quantitatively and accurately analyzing the chromatographic behavior from kinetic points of view. However, it has not widely been used for analyzing the HPLC behavior. It seems to be one of the reasons that the moment analysis procedure of chromatographic data is mathematically more complicated.\(^\text{16–19}\) It is probably required to develop a convenient procedure for a moment analysis of the HPLC behavior by introducing some appropriate suppositions and simplifications.

In this study, at first, a simple procedure was considered for a moment analysis of the chromatographic behavior of two types of separation media, i.e., spherical particles and silica monoliths. Then, some previous chromatographic data were analyzed by applying the simple moment analysis procedure. The simple procedure seems to be effective for the spread of the moment analysis because it is easy for many researchers.

### Theory

**Moment equations for full-porous and non-porous separation media**

Moment equations of \( \mu_{1A} \) and \( \mu_{2C} \) for full-porous and non-porous separation media of various shapes are represented as follows.\(^\text{10,12,14}\)

\[
\mu_{1A} = \frac{Z}{u} \left[ 1 + \frac{(1 - \varepsilon_i)}{\varepsilon_i} (\varepsilon_i + (1 - \varepsilon_i) K_i) \right]
\]

\[
\mu_{2C} = \frac{Z}{u^2} \left[ \frac{2D_e}{u^2} \left[ 1 + \frac{(1 - \varepsilon_i)}{\varepsilon_i} (\varepsilon_i + (1 - \varepsilon_i) K_i) \right]^2 + \frac{2(1 - \varepsilon_i)}{\varepsilon_i} \left( 1 - \varepsilon_i - \frac{K_i^2}{k_a} + \frac{\xi^2}{\omega D_e} + \frac{\xi}{\theta K_i} \right) \right] (\varepsilon_i + (1 - \varepsilon_i) K_i)^2
\]

where \( Z \) is the column length, \( u \) is the interstitial velocity of the mobile phase, \( \varepsilon_i \) is the void fraction of a column (external porosity), \( \varepsilon_c \) is the porosity of separation media (internal porosity), \( K_i \) is the retention equilibrium constant, \( D_i \) is the axial dispersion coefficient, \( k_a \) is the rate constant of chemical interaction at the stationary phase surface, \( \xi \) is the external mass transfer coefficient, \( D_e \) is the effective intra-stationary phase diffusivity, \( \theta \) and \( \omega \) are respectively the numerical coefficients attached with \( k_i \) and \( D_c \), and \( \xi \) is the diffusion distance, which corresponds to the radius of spherical particles and cylindrical fibers or to the thickness of flat plates. The value of \( \varepsilon_i \) for full-porous and non-porous separation media is in the range \( 0 < \varepsilon_i < 1 \) and \( \varepsilon_i = 0 \), respectively. The value of \( K_i \) is equal to zero in the case of size exclusion chromatography. Because the influence of sample injection on \( \mu_{1A} \) and \( \mu_{2C} \) is usually negligibly small, they are neglected in Eqs. (2) and (3).

At first, \( \mu_{1A} \) is represented as Eq. (2) irrespective of the shape of the separation media. On the other hand, the second central moment equation depends on their shapes. The basic formula for \( \mu_{2C} \) is the same for all of the separation media. However, the values of \( \theta \) and \( \omega \) are systematically changed, i.e., 3 and 15 for spherical particles, 2 and 8 for cylindrical fibers, and 1 and 3 for flat plates. The values of \( \theta, \omega \), i.e., 3, 2, and 1, probably represent the dimension of mass transfer in the stationary phases. The ratios of \( \omega \) to \( \theta \) are also systematically changed, i.e., 5, 4, and 3. Giddings also reported the same values of \( \omega \) for the three different shapes of packing materials, i.e., uniform film, cylindrical rod, and spherical bead.\(^\text{3}\) The numerical values of \( \theta \) and \( \omega \) reflect the geometrical difference between the packing materials. The moment equations for a full-porous cylindrical rod can be used for analyzing chromatographic data measured by using silica monolithic columns.

### Simplification of moment analysis procedure

Equation (2) is the moment equation for \( \mu_{1A} \). Although it looks to be complicated, it is identical to the following equation.

\[
t_k = t_0 (1 + k)
\]

where \( t_k \) is the retention time, \( t_0 \) is the elution time of an inert tracer, and \( k \) is the retention factor. It was reported that \( t_k \) corresponded to \( \mu_{1A} \) with an error of less than 1%, even if an elution peak showed asymmetrical profile, e.g., the asymmetry factor at 10% peak height is equal to ca. 2.0.\(^\text{19}\) In Eq. (2), it is required to derive \( K_i \) from \( \mu_{1A} \) using the values of \( \varepsilon_i \) and \( e_c \). In contrast, Eq. (4) is quite simple and does not include \( e_i \) and \( e_c \). The retention behavior is represented by using only \( k \), which is equal to the ratio of \( (1 - e_i) K_i \) to \( e_i \). The total porosity \( e_i \) is equal to \( e_i + (1 - e_i) \). In addition, \( t_0 \) is equal to \( e_i Z/(e_i e_c) \).

It is suggested that a different moment analysis procedure without using \( K_i \) may be more convenient for many researchers. The moment analysis procedure concerning \( \mu_{2C} \) should also be simplified.

### Full-porous spherical particles

A detailed explanation is provided in Supporting Information for the simplification of moment analysis procedure of chromatographic behavior in the column packed with full-porous spherical particles. The value of HETP (\( H_{\text{total}} \)) is represented as follows.

\[
H_{\text{total}} = \frac{2 \varepsilon_i \gamma_1 D_n}{u_0} + 4 \gamma_1 R_p + \frac{2 R_n \gamma_1^2}{15(1 - e_i) D_n} u_0 + \frac{2 R_n \lambda^2}{3(1 - e_i) e_i} u_0^2 + \frac{2 \gamma_1^2}{(1 - e_i)(1 - e_i) k_a} u_0 (5)
\]

where \( u_0 \) is the superficial velocity of the mobile phase (= \( e_i u \)), \( R_p \) is the radius of spherical particles, \( \gamma_1 \) and \( \gamma_1^2 \) are the geometrical coefficients, and \( D_n \) is the molecular diffusivity. Equation (5) is simply written as follows by considering the flow rate dependence of \( H_{\text{total}} \).

\[
H_{\text{total}} = \frac{B^8}{u_0} + A^8 + C^2 u_0 + D^2 u_0^3 + E^2 u_0 (6)
\]

In the case of reversed-phase liquid chromatography (RPLC), the contribution of the chemical interaction (e.g., adsorption/desorption) kinetics, i.e., \( E^2 u_0 \), to \( H_{\text{total}} \) is usually assumed to be negligibly small because the reaction rate of physical adsorption is fast enough.\(^\text{8}\) Equation (6) is rearranged as follows.
\[ H_{\text{total}} - D^*u_0^2 = \frac{B^*}{u_0} + A^* + C^*u_0 \]  

(7)

Equation (7) represents the correlation of \((H_{\text{total}} - D^*u_0^2)\) with \(u_0\), and has the same formula with the van Deemter equation, i.e., Eq. (1). However, Eqs. (5) - (7) are essentially different from Eq. (1) in terms of the following two points. At first, the contribution of external mass transfer \((D^*\text{-term})\) to \(H_{\text{total}}\) is considered in Eq. (7). Second, the definition and physical meanings of the parameter \(C^*\) in Eq. (7) are clearly explained. As shown in Eqs. (5) - (7), \(D_i\) can quantitatively be calculated from the value of \(C^*\). The mechanism of mass transfer in the stationary phases can be analyzed in more detail by considering the contributions of pore diffusion and surface diffusion to \(D_i\). These are obvious differences between the van Deemter equation and Eqs. (5) - (7).

**Silica monolith (full-porous cylindrical fiber)**

Similar to the case of full-porous spherical particles, simplification of the moment analysis procedure of the chromatographic behavior in a column packed with silica monoliths is explained in detail in Supporting Information. The original moment equations for silica monolithic columns, i.e., Eqs. (2) and (3) \((\theta = 2, \omega = 8)\), were developed by assuming as a first approximation that the unit structure of the continuous porous packing material in monolithic columns consisted of a cylindrical silica skeleton of the stationary phase including meso- and micropores, which were surrounded by a through macropore spaces filled with the mobile-phase solvent.\(^\text{10}-\text{12},\text{14}\)

Similar to Eq. (5), Eq. (8) is derived in the same manner for silica monoliths.

\[
H_{\text{total}} = \frac{2\varepsilon_1\gamma \lambda D_x}{u_0} + 2\varepsilon_1\gamma \lambda^2 R_n + \frac{2R_n\lambda^2}{8(1-\varepsilon_1)D_x} u_0^2 \\
+ \frac{2R_n\lambda^2}{2(1-\varepsilon_1)\varepsilon} u_0^2 + \frac{2\nu_0}{(1-\varepsilon_1)(1-\varepsilon)}k_0 - u_0^2 
\]

(8)

where \(R_n\) is the radius of the cylindrical stationary phase silica skeleton and \(\gamma_1\) is the geometrical coefficient. Equation (8) is simply written as follows.

\[
H_{\text{total}} = \frac{B^*}{u_0} + A^* + C^*u_0 + D^*u_0^2 + E^*u_0^3 
\]

(9)

Equation (9) is rearranged as follows.

\[
H_{\text{total}} - D^*u_0^2 = \frac{B^*}{u_0} + A^* + C^*u_0 
\]

(10)

The contribution of \(E^*\)-term to \(H_{\text{total}}\) is similarly assumed to be negligibly small in the case of RPLC systems.\(^3\) Equation (10) represents the correlation of \((H_{\text{total}} - D^*u_0^2)\) with \(u_0\) and has the same formula with the van Deemter equation, i.e., Eq. (1).

**Results and Discussion**

*Full-porous spherical particles (Mightysil)*

Figures 1a and 1b illustrate the correlation between \(H\) and \(u_0\) for propylbenzene (PBz) and hexylbenzene (HBz) in the RPLC system using the Mightysil column, respectively. The solid line is the correlation line of open circles, which represent experimental data. Figure 1 also illustrates the flow rate dependence of the contributions of the \(A^*, B^*, C^*,\) and \(D^*\)-term to \(H_{\text{total}}\). The chromatographic behavior of the Mightysil column was also analyzed by the conventional moment analysis. The profiles of the straight and curved lines obtained by the conventional moment analysis are superposed with those in Fig. 1 (not shown). Both the simple and conventional moment analyses provided the same results. At first, as explained from Eq. (S17) to Eq. (S20) in Supporting Information, the contribution of the \(D^*\)-term to \(H_{\text{total}}\) was calculated as Eq. (5) on the basis of the \(k_i\) value, which was estimated by using the Wilson-Geankoplis equation. Then, the flow-rate dependence of the right-hand side in Eq. (7) was analyzed to obtain the coefficients, \(A^*, B^*,\) and \(C^*\). The values of the coefficients
were estimated by the least-squares method so that Eq. (7) represented the experimentally observed profile between $H_{base}$ and $u_0$ as well as possible. In addition, the value of $C^*$ was analyzed to study the mass-transfer kinetics in the stationary-phase particles, as explained in the following.

As indicated in Eqs. (5) – (7), the values of $D_ε$ can be calculated from the slope of the straight lines for the C*-term in Fig. 1. Table 1 lists the values of $D_ε$ determined by the simple moment analysis as $1.9 \times 10^{-10}$ and $2.3 \times 10^{-10}$ m$^2$/s$^{-1}$ for PBz and HBz, respectively. The same values of $D_ε$ were also obtained by the conventional moment analysis. The mechanism of mass transfer in the stationary phase can be analyzed in more detail by considering the contributions of pore diffusion and surface diffusion to $D_ε$. As listed in Table 1, almost the same values of $D_ε$ were obtained by both the simple and conventional moment analyses. Table 1 also lists the values of pore diffusivity ($D_p$), which were estimated by Eqs. (S36) - (S38). The value of the tortuosity factor of internal pores ($k^p$) was calculated as 8.4 for the Mightsyl column from the elution peak profiles of uracil.21 The comparison of $D_ε$ and $D_p$ indicates that the contribution of surface diffusion to the molecular migration of PBz and HBz in Mightsyl particles is ca. 5 – 8 times larger than that of pore diffusion. It is indicated that surface diffusion has a predominant role for intraparticle mass transfer.

A determination procedure of $D_ε$ and $D_p$ is explained in detail in Supporting Information. The accuracy of the resulting values of diffusivities and mass transfer coefficients determined in this study is affected by the uncertainties in the estimate of some kinetic parameters, i.e., $D_m$, $k_ε$, and $D_p$, because literature correlations were used for estimating their values. At first, as indicated in Eq. (S37), the value of $D_m$ was estimated by the Wilke-Chang equation.22 It is reported that the average error for estimating $D_m$ by various correlations is slightly less than 10%.23 The estimation error of $D_m$ affects the accuracy of the results in this study because the values of $k_ε$ and $D_p$ were calculated on the basis of the estimated value of $D_m$.

As explained by Eqs. (S17) – (S19) in Supporting Information, the $k_ε$ value was estimated by the Wilson-Geankoplis equation. The error of $k_ε$ made by the propagation of the uncertainty of $D_m$ should be smaller than about 6% because $D_m$ is estimated with an error of 10% or less and because $k_ε$ is proportional to $D_p^{0.75}$. Figures 1 and 2 indicate that the value of $D^*$-term increases with an increase in $u_0$ and that its contribution to $H_{base}$ is about 30% at maximum in the high flow rate range. It seems that the influence of the uncertainty of $k_ε$ on the error of the moment analysis is about 2% at maximum.

As described by Eq. (S36) in Supporting Information, the uncertainty of $D_p$ is proportional to that of $D_m$. However, the contribution of surface diffusion to intraparticulate mass transfer in Mightsyl particles is ca. 5 – 8 times larger than that of pore diffusion. This means that surface diffusion has a predominant role in the intraparticle diffusion as much as $C^*$ 83 - 89%. The influence of the error made in the estimation of $D_p$ on the determination of $D_ε$ is calculated to be ca. 1.1 - 1.7%. Similarly, as described later, the contribution of surface diffusion to intrastationary phase diffusion is larger than that of pore diffusion by a factor of ca. 2.5 - 6.0 for Chromolith column. The error of $D_ε$, made by propagation of the uncertainty of $D_m$, should be about 1.4 – 2.9% because about 71 – 86% of solute molecules migrate in silica-monolith by surface diffusion. The inaccuracy of $D_p$ does not seriously affect the accuracy of $D_ε$. In addition, the information about other parameters, e.g., $K_0$ and $k^2$, are necessary when the values of $k_ε$ and $D_p$ are estimated. The error in the estimations described above is responsible for the accuracy of the moment analysis. It seems that the overall error of the moment analysis is on the order of several percent.

As indicated in Eqs. (5) – (7), only $ε_ε$ is necessary as information about the porosities for the simple moment analysis when the $E^*$-term is neglected. Various methods have been proposed for measuring $ε_ε$ of HPLC columns packed with porous materials, e.g., Donan exclusion method,24 inverse size exclusion chromatography,25 and total pore blocking.26 Simple moment analysis of the chromatographic behavior of HPLC columns packed with spherical particles can accurately be conducted by using $ε_ε$ experimentally measured by the methods. On the other hand, it is reported that $ε_ε$ is typically around 0.40 - 0.41 in analytical columns packed with spherical particulate packing materials.9 Giddings also indicates that a fairly well-packed chromatographic column will typically have $ε_ε = 0.4 \pm 0.03$. Similar values of $ε_ε$ were experimentally measured in the range from ca. 0.35 to 0.43,25,32 It would be convenient that the simple moment analysis of chromatographic behavior of HPLC columns packed with spherical particles can be conducted by using the empirical $ε_ε$ values.

Figure S1a illustrates the correlation between $D_ε$ and $ε_ε$. The value of $ε_ε$ was hypothetically changed in the range from 0.35 to 0.45, which was wider than the previous observations.3,9 The $D_ε$ values were calculated at the different $ε_ε$ values by the simple moment analysis from the $H - u_0$ plots for PBz and HBz in Figs. 1a and 1b. When $ε_ε$ is assumed to be 0.40, the values of $D_ε$ are 1.6% and 2.6% larger than those of $D_ε$ at $ε_ε = 0.39$ for PBz and HBz, respectively. Although the $ε_ε$ value experimentally measured was 0.39 in this case, the $D_ε$ values were derived with an error of a few percent when $ε_ε$ was assumed to be 0.40. Figure S1a indicates that the relative error of $D_ε$ at $ε_ε$ between 0.35 and 0.45 against $D_ε$ at $ε_ε = 0.40$ is 7.1 - 9.2% and 12 - 16% for PBz and HBz, respectively. It would be impossible to describe the correlation between $D_ε$ and $ε_ε$ by using a simple equation. As indicated in Eq. (S14), all of the contributions of

| Column | Solute       | $k$    | $K_0$ | $D_ε$/m² s⁻¹ | $D_p$/m² s⁻¹ | $D_t$/m² s⁻¹ |
|--------|--------------|--------|-------|--------------|--------------|--------------|
| Mighty sil | Propylbenzene | 2.9 (2.9) | 54 (5.4) | $1.9 \times 10^{-10}$ | $3.2 \times 10^{-11}$ | $4.9 \times 10^{-11}$ |
|        | Hexylbenzene | 9.9 (9.9) | 18 (18)  | $2.3 \times 10^{-10}$ | $2.6 \times 10^{-11}$ | $1.8 \times 10^{-11}$ |
|        | Chromolith   | 1.2 (1.2) | 6.5 (6.5) | $1.8 \times 10^{-10}$ | $5.1 \times 10^{-11}$ | $3.9 \times 10^{-11}$ |

The values in parentheses are the retention and kinetic parameters derived by the conventional moment analysis.
$H_{\text{e}}^*,$ $H_{\text{e}}^\rho,$ $H_{\text{e}}^\phi,$ and $H_{\text{e}}^\nu$ to $H_{\text{med}}$ depend on the value of $e$, because they contain $e$. In addition, as described in Eqs. (S8) and (S12), $e$ is also included in both the functions for $\lambda$ and $v$.

As explained in Eqs. (S21) and (S22), the value of $D_e$ was calculated from that of $C^e$. Although the dependence of $D_e$ on $e$ is relatively complicated as explained above, the moment analysis is useful to clarify the correlation between $D_e$ and $e$, as illustrated in Figs. S1a and S2a.

Figure S1b similarly illustrates the correlation between $D_e$ and $e$. The value of $D_e$ was calculated by Eqs. (S34) and (S36) - (S38) from that of $D_e$, which was obtained by a simple moment analysis at different $e$ values. When $e$ is assumed to be 0.40, the values of $D_e$ are 4.1% and 5.0% larger than those of $D_e$ at $e$ = 0.39 (experimentally measured) for PBz and HBz, respectively. It seems that the $D_e$ values were derived with an error of several percent when $e$ was assumed to be 0.40. Figure S1b indicates that the relative error of $D_e$ at $e$ between 0.35 and 0.45 against $D_e$ at $e$ = 0.40 is ca. 17 – 23% and 21 – 31% for PBz and HBz, respectively. It is concluded from the results described above that preliminary information about the mass transfer in the stationary phase particles can be obtained with a moderate accuracy by assuming $e$ = 0.40. When accurate results of moment analysis are required, it is required to experimentally measure the value of $e$.

Full-porous cylindrical fiber (Chromolith)

Similar to Fig. 1, Fig. 2 illustrates the correlation between $H$ and $\nu_0$ for PBz and HBz in the RPLC system using the Chromolith column. Figure 2 also illustrates experimental data (open circles) and the contributions of the A*-, B*-, C*-, and D* term to $H_{\text{med}}$, which was analyzed by a simple moment analysis. The profiles of the straight and curved lines in Fig. 2 are superposed with those obtained by the conventional moment analysis (not shown). Both the simple and conventional moment analyses provided the same results for the chromatographic behavior of the Chromolith column.

As indicated in Eqs. (8) - (10), the values of $D_e$ can be calculated from the slope of the straight lines for the C*-term in Fig. 2. As listed in Table 1, the value of $D_e$ was calculated as 1.8 $\times$ 10$^{-10}$ and 2.8 $\times$ 10$^{-10}$ m$^2$s$^{-1}$ for PBz and HBz, respectively, by the simple moment analysis. On the other hand, $D_e$ was calculated as 1.7 $\times$ 10$^{-10}$ and 2.7 $\times$ 10$^{-10}$ m$^2$s$^{-1}$ for PBz and HBz by the conventional moment analysis. Almost the same values of $D_e$ were obtained by both the simple and conventional moment analyses.

Similar to the Mightysil column, it was tried to quantitatively analyze the mechanism of mass transfer in the stationary phase of the Chromolith column. The value of $D_e$ was calculated in the same manner. The hindrance parameter of internal pores ($K_i$) was estimated by Eq. (S38) around 0.80 for the three solutes in the RPLC systems using the Chromolith column. It was reported for the Chromolith column that the value of $K_i$ ranged from 5.5 to 6.7.31 In this study, an average value, i.e., 6.1, was used as $K_i$, which was a typical value for porous silica gel.3 As listed in Table 1, almost the same values of $D_e$ were obtained by both the simple and conventional moment analyses.

Table 1 also lists the values of $D_p$, which were estimated by Eqs. (S36) - (S38). The values of $D_p$ are several times larger than those of $D_e$, suggesting that surface diffusion plays an important role for the molecular migration in the stationary phase of the Chromolith column. The contribution of surface diffusion to intra-stationary phase diffusion of PBz and HBz is larger than that of pore diffusion by a factor of ca. 2.5 – 6.0.

As indicated in Eqs. (8) - (10), only $e$ is necessary for the simple moment analysis when the E*-term is neglected. It has been reported that $e$ is usually ranging from 0.65 to 0.75 in Chromolith columns.27,29,34,35 The values of $e$ were experimentally measured in the same range.18,36 It has been tried to analyze the chromatographic behavior of the Chromolith column by using the empirical $e$ values. Figure S2a illustrates the correlation between $D_e$ and $e$. The value of $e$ was hypothetically changed in the range of 0.65 to 0.75. The $D_e$ values were calculated at the different $e$ values by the simple moment analysis from the $H_\nu$ plots for PBz and HBz in Figs. 2a and 2b. When $e$ is assumed to be 0.70, the values of $D_e$ are 2.2% and 4.3% larger than those of $D_e$ at $e$ = 0.69 for PBz and HBz, respectively. Although the experimental value of $e$ was 0.69, the $D_e$ values were derived with an error of several percent when $e$ was assumed to be 0.70. Figure S2a indicates that the relative error of $D_e$ at $e$ between 0.65 and 0.75 against $D_e$ at $e$ = 0.70 is 9.7 – 15% and 17 – 29% for PBz and HBz, respectively.

Figure S2b illustrates the correlation between $D_e$ and $e$. The value of $D_e$ was calculated from that of $D_e$, which was obtained by the simple moment analysis at the different $e$ values. When $e$ is assumed to be 0.70, the values of $D_e$ are 1.3% and 2.4% larger than those of $D_e$ at $e$ = 0.69 (experimentally measured) for PBz and HBz, respectively. It seems that the $D_e$ values were derived with an error of a few percent when $e$ was assumed to be 0.70. Figure S2b indicates that the relative error of $D_e$ at $e$ between 0.65 and 0.75 against $D_e$ at $e$ = 0.70 is ca. 5.8 – 7.0% and 10 – 14% for PBz and HBz, respectively. It is only required to measure $e$, experimentally when more accurate results of
moment analysis are necessary.

Conclusions

It was tried to simplify the moment analysis procedure for a kinetic study on chromatographic behavior of spherical particles and silica monoliths. The simple procedure for a moment analysis of \( \mu \) was proposed without using \( K \) and \( \varepsilon \). However, similar to the conventional moment analysis, the contributions of axial molecular diffusion, Eddy diffusion, external mass transfer, intra-stationary phase diffusion, and reaction kinetics on the stationary phase surface to the variance of an elution peak could be quantitatively analyzed. Some chromatographic data measured by using the MightySil column (full-porous spherical particles) and the Chromolith column (silica monoliths) were analyzed by applying the simple moment analysis procedure. Both the simple and conventional moment analysis procedures provided almost the same values of \( D_a \) and \( D_s \). The simple moment analysis is effective for accurately estimating the mechanism of mass transfer in the stationary phases in detail with considering the contributions of pore diffusion and surface diffusion to \( D_s \). It was also tried to analyze the same chromatographic data by assuming \( \varepsilon_s = 0.40 \) for spherical particles and 0.70 for silica monoliths. Although the values of \( D_a \) and \( D_s \), change depending on the \( \varepsilon \) values, they were of the same order of magnitude with each other. Even if \( \varepsilon \) is hypothetically assumed to be typical values, the simple moment analysis is effective for a preliminary study of the mass-transfer kinetics in the columns. When more accurate results of moment analysis are required, the value of \( \varepsilon \) should experimentally be measured. In this study, the simple procedure of moment analysis was proposed for the analysis of chromatographic data measured by using HPLC columns packed with spherical particles and cylindrical rod fibers, such as silica monoliths. There is no necessary condition for a simple moment analysis, e.g., the types of solutes and stationary phases and the size and porosity of the packing materials, because it was theoretically developed on the basis of the general rate model of chromatography.

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Supporting Information

The simplification of moment analysis procedure is explained in detail. Supporting Information provides a detailed explanation about chromatographic experimental conditions and kinetic analysis of intraparticle mass transfer. Experimental results of the correlations between \( D_a \) and \( \varepsilon \) and that between \( D_s \) and \( \varepsilon_s \) are illustrated. This material is available free of charge on the Web at http://www.jsac.or.jp/analsci/.

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