Chiral Symmetry and Hyperfine Meson Splittings

Felipe J. Llanes-Estrada, Stephen R. Cotanch, Adam P. Szczepaniak, and Eric S. Swanson

Abstract. We briefly review theoretical calculations for the pseudoscalar-vector meson hyperfine splitting with no open flavor and report a many body field theoretical effort to assess the impact of chiral symmetry in the choice of effective potentials for relativistic quark models. Our calculations predict the missing $\eta_b$ meson to have mass near 9400 MeV. The radial excitation $\eta_c(2S)$ is in agreement with the measurements of the Belle and most recently Babar collaborations.

PACS. 11.30.Rd – 12.38.Lg – 12.39.Ki – 12.40Yx

 Shortly after the discovery of the $J/\psi$ it was understood that a rich spectroscopy of new mesons awaited classification. In this task the constituent quark model was a useful tool providing a simple periodic table where spectra and various radiative decays could be correlated with the help of a modest number of parameters. In this picture vector mesons are a $q\bar{q}$ pair, in an S or D wave, with spins parallel giving total angular momentum $J = 1$. Pseudoscalar mesons correspond to the $J = 0$ ground state with S-wave $q\bar{q}$ pairs spins antialigned. Ignoring the D-wave component, the only difference between both systems is the relative spin alignment. Any spectroscopic mass splitting can conveniently be incorporated in the quark model with a term, $A\sigma_1 \cdot \sigma_2$ that is reminiscent of the electron-nucleus spin-spin coupling, hence the name “hyperfine”. This was immediately noted by Appelquist et al. [1] who predicted a charmonium splitting, $\Delta M(J/\psi - \eta_c)$, of about 65 MeV. They extracted the amplitude $A$ by estimating the $J/\psi$ electron-positron width, $\Gamma_{e^+e^-}$, to be 4 keV. Using the currently accepted value of 5.3 keV, the splitting would be about 84 MeV, or about a factor of 2 smaller than the accepted experimental value of $3097 - 2980 \simeq 120$ MeV. The need for a confining potential [2] was soon understood and calculations (by Appelquist and Politzer, and independently Schnitzer [3]) including a confining strength yielded a larger splitting ($40 - 80$ MeV) than purely Coulombic potentials ($15 - 20$ MeV).

In retrospective we see that many of the early models utilized scalar confining potentials, which provided a good spin-orbit coupling and radial excitations, but underestimated the hyperfine splittings. In the early eighties, and with the $\eta_c$, experimental state now known, this splitting became a benchmark for new model calculations [4,5,6,7] which now also predicted the corresponding splitting in bottomonium. The variation in these predictions is summarized in Table I. Subsequently, further progress was achieved through improved, renormalized non-relativistic perturbative QCD calculations (NRQCD) [8,9] which described bottomonium as a non-relativistic system. However, the calculated radii of most $b\bar{b}$ states are too large indicating that a Coulombic description, where the relativistic splittings scale linearly with the quark mass, is not reliable and that strong interactions still induce important corrections at this scale [10]. Nevertheless, approximate ground state descriptions are feasible and useful for extracting $c$ and $b$ quark masses.

Non-perturbative lattice calculations with large error bars have also been performed [11,12] for bottomonium which yield about half, or less, the hyperfine splitting exhibited in charmonium. This again indicates the system is not fully Coulombic since the splitting is not proportional to the quark mass.

Extending this analysis accurately to the $\pi$-$\rho$ system is not currently feasible for either the perturbative or lattice approaches. Thus one still relies on constituent models where the hyperfine splitting has a $1/M^2$ dependence on the constituent quark mass [4]. This cannot describe the large $\pi$-$\rho$ splitting but not simultaneously the hadron scattering phase shifts [13]. On the other hand, we know that the pion’s mass is very low because of its Goldstone boson nature from spontaneous chiral symmetry breaking. Thus it is natural to seek a field-theoretical formulation of the

---

*On leave at University of Tuebingen, Inst. fuer Theoretische Physik, auf der Morgenstelle 14, D-72076 Tuebingen, Germany."
Table 1. Predictions for the splitting between vector and pseudoscalar $b\bar{b}$ mesons. CQM stands for Constituent Quark Model. Units are $MeV$.

| Date, Authors      | Model   | Splitting |
|--------------------|---------|-----------|
| 1983 Godfrey & Isgur| CQM     | 60        |
| 1983 McClary & Byars| CQM     | 101       |
| 1985 Igi & Ono     | CQM     | 60        |
| 1985 Igi & Ono     | CQM     | 90        |
| 1989 Song          | CQM     | 55        |
| 1994 Eichten & Quigg| CQM     | 141       |
| 1994 Eichten & Quigg| CQM     | 87/65/64  |
| 1994(98) Davies et al. | Lattice | 30-50     |
| 1998 Pineda & Yndurain| NRQCD   | 47(20)   |
| 2000 Lengyel et al.| CQM     | 46        |
| 2003 Ebert et al.  | CQM     | 60        |

A quark model which implements chiral symmetry consistently. Such an approach would predominantly attribute the hyperfine splitting in light mesons to chiral symmetry. This permits using a more moderate hyperfine potential to then describe the smaller splittings which are exhibited in light meson excited states and heavy mesons, both of which are not governed by chiral symmetry. Thus we consider the Hamiltonian (inspired in Coulomb gauge QCD)

\[ H_{eff} = T + V_C + V_T \]

\[ T = \int dx \bar{\Psi}(x)(-i\alpha \cdot \nabla + m_\beta)\Psi(x) \]

\[ V_C = -\frac{1}{2} \int dx dy \rho^a(x)\hat{V}(x-y)\rho^a(y) \]

\[ V_T = \frac{1}{2} \int dx dy J^a_i(x)J^a_j(y) \times \left( \delta_{ij} - \frac{\nabla_i \nabla_j}{\nabla^2} \right) \hat{U}(x-y) \]

Here $\rho^a = \bar{\Psi}^T a^a \Psi$ and $J^a_i = \bar{\Psi}^T a^a T^a_i \Psi$ are the quark color density and current, respectively. This Hamiltonian has been diagonalized previously \[14\] for $V_T = 0$ in the Bardeen-Cooper-Schrieffer (BCS) approximation for the vacuum. These earlier studies of the gap equation determined that the dynamical chiral symmetry breaking from only a longitudinal potential is relatively small and yields a low condensate $\langle \bar{\Psi} \Psi \rangle_0 \approx -(100 \, MeV)^2$. On the opposite limit, calculations for high quark masses using the Tamm-Dancoff (TDA) and Random Phase (RAP) approximations for both harmonic oscillator \[15\] and linear potentials \[16\] produce almost degenerate pseudoscalar and vector meson ground states. They are thus unable to describe the charmonium hyperfine splitting although the RPA can reproduce the $\pi-\rho$ splitting by sufficiently lowering the quark mass according to Thouless theorem. More recently a study \[18\] implementing chiral symmetry using $V_C = 0$ and a contact potential for $V_T$ to obtain a link with transverse one-gluon exchange, which is suppressed in our approach by the large gluon mass gap \[19\]. Because that model does not include radial excitations or confinement, we have generalized \[20\] the treatment by employing both a Coulomb instantaneous interaction and a transverse hyperfine potential. For the longitudinal Coulomb interaction we utilize a potential derived \[21\] from QCD through a BCS truncation of the gluon sector, represented in momentum space by

\[ \hat{V}(p) = C(p) = -\frac{8.07}{p^2} \log^{0.62} \left( \frac{m_g^2}{p^2} + 0.82 \right) \] for $p > m_g$

\[ \hat{V}(p) = -\frac{12.25 \, m_g^{1.93}}{p^{3.93}} \] for $p < m_g$.

This is numerically close to the standard Coulomb + linear potential. The transverse potential, due to non-explicit Lorentz covariance in Coulomb gauge QCD, can be different. Since this term has not been studied theoretically, we proceed phenomenologically and choose the same Coulomb tail as in Eq. \[4\]. It is then matched at low momentum to a Yukawa representing a massive gluon exchange which emerges from intermediate hybrid states in the Fock space truncation. Thus we take

\[ \hat{U}(p) = C(p) \] for $p > m_g$

\[ \hat{U}(p) = -\frac{C_h}{p^2 + m_g^2} \] for $p < m_g$.

The constant $C_h$ matches the potential continuously at the $m_g$ scale. Thus the only free potential parameter is $m_g$ which determines simultaneously the strength of the confining term and the logarithmic one-loop running of both $\hat{U}$ and $\hat{V}$. We adopt $m_g = 600 \, MeV$ and investigate alternative transverse potentials in a more detailed publication \[20\].

Calculating the resulting gap equation at zero quark mass we find a sizeable increase of the BCS quark condensate, to $-(178 \, MeV)^2$, which is now closer to the phenomenologically accepted values (this quantity is sensitive to the high energy behaviour of the potential as previously noted by Lagae \[22\]). In the chiral limit the calculated pion mass is effectively zero (numerically a fraction of an $MeV$) and the $\rho$ mass is about 780 $MeV$. For the vector mesons we include coupled $S$ and $D$ wave channels, since the Hamiltonian of Eq. \[10\] contains a tensor interaction.

Upon increasing the quark mass, the pion mass grows rapidly in the RPA whereas the $\rho$ mass only slowly increases yielding the hyperfine splitting plotted in Fig. \[1\] for various meson masses. This figure presents our preliminary results and reflects the success of this approach which incorporates chiral symmetry and is simultaneously applicable to a wide range of quark masses.

For the same model parameters, we also predict the mass of the missing $\eta_V$ state, a most important issue in hadronic spectroscopy \[23\]. We concur with NRQCD and lattice studies but predict a slightly larger splitting, (see below) of about 60 $MeV$. Subtracting this from the $T(9460)$ mass yields $\eta_V(9400)$. This decreasing hyperfine strength trend with increasing quark mass (see Fig. \[1\]) indicates that the potential is not yet scaleless. Note that in both PQCD and our approach (see Eq. \[4\] a hadron scale...
appears logarithmically in the coupling constant. Also for bottomonium there is a small difference between the RPA and TDA hyperfine splittings since the TDA $\eta_b$ mass is about 30 MeV lower than in the RPA. While insignificant when compared to the $\Upsilon(9460)$ mass, it should be accurately included when evaluating a small hyperfine splitting. Non-chiral preserving models, such as those based on Schrödinger’s equation, will thus underestimate the splitting by at least this 30 MeV. Although this is currently comparable to the quoted errors in both NRQCD and lattice calculations (20 – 30 MeV), it may become an issue in the future.

Finally, it is noteworthy that our approach naturally extends to radial excitations. For the $\psi(2S) - \eta_c(2S)$ splitting we obtain 56 MeV, in agreement with the BELLE result. Note added: the BABAR collaboration reports a possible detection of the $\eta_c(2S)$ corresponding to a hyperfine splitting of 55(4) MeV. The agreement is encouraging.

This work was supported by Spanish grants FPA 2000-0956, BFM 2002-01003 (F.L-E.) and the U. S. Department of Energy grants DE-FG02-97ER41048 (S.C.), DE-FG02-98ER40365 (A.S.) and DE-FG02-00ER41135, DE-AC05-84ER40150 (E.S.).

References

1. T. Appelquist et al., Phys. Rev. Lett. 34 (1975) 365.
2. E. Eichten et al., Phys. Rev. Lett. 34 (1975) 369.
3. H. J. Schnitzer, Phys. Rev. D 13 (1975) 74.
4. S. Godfrey, N. Isgur, Phys. Rev. D 32 (1985) 189.
5. R. McClary, N. Byars, Phys. Rev. D 28 (1983) 1692.
6. E. J. Eichten and C. Quigg, hep-ph/9402210.
7. K. Igi, S. Ono, Phys. Rev. D 33 (1986) 3349.
8. A. Pineda, F. J. Yndurain, Phys. Rev. D 61 (2000) 077505.
9. N. Brambilla, Y. Sumino, A. Vairo, Phys. Lett. B 513 (2001) 381.
10. H. Leutwyler, Phys. Lett. 98B (1981) 447.
11. C. T. H. Davies et al., hep-lat/9406117.
12. C. T. H. Davies et al., hep-lat/9802024.
13. P. Bicudo, J. E. Ribeiro, Z. Phys. C 38 (1988) 453.
14. S. L. Adler, A. C. Davis, Nucl. Phys. B 244 (1984) 460; A. Le Yaouanc et al., Phys. Rev. D 29 (1984) 1233; R. Alkofer, P.A. Amundsen, Nucl. Phys. B 306 (1988) 305; P. Bicudo and J. E. Ribeiro, Phys. Rev. D 42 (1990) 5; A. P. Szczepaniak, E. S. Swanson, Phys. Rev. D 55 (1997) 3987.
15. A. Le Yaouanc et al., Phys. Rev. D 31 (1985) 137.
16. P. Bicudo, J. E. Ribeiro, Phys. Rev. D 42 (1990) 1625.
17. F. J. Llanes-Estrada, S. R. Cotanch, Phys. Rev. Lett. 84 (2000) 1102; Nucl. Phys. A 697 (2002) 303.
18. A. P. Szczepaniak and E. S. Swanson, Phys. Rev. Lett. 87 (2001) 072001.
19. A. P. Szczepaniak, E. S. Swanson, C. R. Ji, and S. R. Cotanch, Phys. Rev. Lett. 76 (1996).
20. F. J. Llanes-Estrada, S. R. Cotanch, A. P. Szczepaniak, E. S. Swanson, manuscript in preparation.
21. A. P. Szczepaniak and E. S. Swanson, Phys. Rev. D 62 (2000) 094027.
22. J. F. Lagae, Phys. Rev. D 45 (1992) 317.
23. S. Godfrey, J. L. Rosner, hep-ph/0210399; S. Godfrey, hep-ph/0210400.
24. S.-K. Choi et al. (BELLE collaboration), Phys. Rev. Lett. 89 (2002) 102001; erratum-ibid. 89 (2002) 129901.
25. V. Lengyel et al., hep-ph/0007084.
26. D. Ebert, R. N. Faustov, V. O. Galkin, Phys. Rev. D 62 (2000) 034014; hep-ph/0304227; hep-ph/0308150.
27. B. Aubert et al., BABAR Collaboration, e-print hep-ex/0311038.