Modulation of harmonic emission spectra in relativistic laser interactions with overdense plasma

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Abstract. We have recently reported effects of plasma waves on high harmonic emission from laser-overdense plasma interactions [1] and found that the emission spectrum is modulated at the plasma frequency and identified circumstances favourable for modulation, in the plasma density profile, pulse shape and duration. Here we study the ultra-relativistic regime and have characterised the modulation observed in simulations. We find that the harmonic spectra show a behaviour distinct from the power law dependence $I_m \sim m^{-8/3}$ recently proposed by Baeva \textit{et al.} [2] on the basis of relativistic similarity theory.

1. Introduction

The generation of multiple harmonics of light incident on plasma targets is a topic of enduring interest, not only for its intrinsic importance as an index of the interaction physics but for its potential as a source of intense coherent pulses of radiation at short wavelengths [3-5]. With the availability of petawatt lasers, the pulse intensities cross what may be regarded as an ultra-relativistic (UR) threshold. One consequence is a greatly enhanced range of harmonics extending into the soft X-ray regime with wavelengths from 1 to 10 nm. Sources with this wavelengths hold diagnostic potential since the "water window" (from 2.3 nm to 4.4 nm) falls within the range.

Two recent papers have reported on aspects of harmonic generation in the UR regime. Dromey \textit{et al.} [3] used a petawatt laser and recorded a slow decay in intensity with harmonic order across the spectrum. They found conversion efficiencies in excess of $10^{-6}$ per harmonic. Baeva, Gordienko and Pukhov [2] (BGP) applied similarity theory to establish a power-law scaling of harmonic intensities $I_m \sim m^{-p}$ finding an index $p = 8/3$. These authors claim that the spectrum in the UR limit is \textit{universal} in that it is independent of the details of the interaction physics.

We report results from a set of particle-in-cell (PIC) simulations that show harmonic decay best fitted by values of $p$ across $5/4 \lesssim p \lesssim 10/3$. In contrast to the BGP model we contend that the harmonic spectrum may reflect details of the interaction physics, in particular the plasma
oscillations generated by bursts of relativistic electrons. These collective effects not only give rise to emission at the plasma frequency [5] but can explain the modulation observed in the spectrum [1]. The electron temperature was chosen to be 100 eV. The normalized quiver momentum $a_0 \sim 8.5\left(I_{20}\lambda_0^2\right)^{1/2}$ lay in the range 5-20 with densities between 20-140 $n_c$, with $n_c$ the critical density.

In [5] we showed that for $p$-polarized light in the moderately relativistic range ($a_0 \sim 0.5 - 2.0$) the spectrum extended to typically 50 harmonics with $P_m/P_1 > 10^{-6}$ where $P_m$ denotes the power radiated in the $m^{\text{th}}$ harmonic. The power radiated depends not only on the plasma density and the intensity and angle of incidence but critically on the density profile and on both the pulse length and shape. For $s$-polarized or for normally incident light, only odd harmonics are generated at intensities one or two orders of magnitude below $p$-polarized levels.

2. Numerical simulations

Fig. 1a shows harmonic power levels from a simulation at normal incidence with $a_0 = 10$, a value that broadly corresponds to that used in [3]. Since no information is available on the density in [3] we chose, an initial density $n_e/n_c = 40$. It is clear from Fig. 1a that the decay in power levels with harmonic number $m$ is well represented by $p = 8/3$ in agreement with the result from the BGP similarity analysis. At the same time we note that there is little or no evidence of any break in the spectrum predicted by the BGP analysis at a critical harmonic $m^{\ast} = \sqrt{8 \alpha \gamma_k^3}$ where $\alpha$ is the second derivative of the surface velocity and is of order 1; $\gamma_k$ is the maximum value of the relativistic factor associated with the motion of the plasma surface [2]. With this choice of parameters, $m^{\ast} \sim 100$.

![Figure 1: Harmonic spectrum generated from a laser pulse incident normally on a plasma target: $a_0 = 10, n_e/n_c = 40, \tau_p = 17$ fs and $\lambda_L = 1054$ nm, the line represents spectral decay $P_m \sim m^{-p}$ with a decay index $p = 8/3$. (b) relative intensity of harmonics scaled to $m = 238$, i.e. $P_m/P_{238} = (m/238)^{-p}$. The lines correspond to $p = 2.1$ (dashed), $p = 8/3$ (solid) and $p = 3.0$ (dotted).](image-url)

To allow ready comparison with the intensity scaling observed experimentally we represent data points from Fig. 1a in Fig. 1b in the form of an intensity scaling diagram corresponding to that presented in [3], with power levels normalized to their choice of $P_{238}$. The results in Fig. 1b show a range of scaling for $P_m \sim m^{-p}$ where $2.1 \lesssim p \lesssim 3$. In their scaling of power levels they found $2.2 \lesssim p \lesssim 2.7$. Allowing for the uncertainties affecting the parameters that govern harmonic generation, this comparison between the experiments and that from our simulations shows fair agreement. Note in passing that no overall comparison between PIC and observed spectra is possible since the measured spectral shape is governed by the absorption characteristics of the Al filter about different order carbon K-edges. Perhaps the most noteworthy difference is that whereas Dromey et al. find $p_{\text{max}} \sim 2.7$, our simulations show a $p_{\text{max}} \sim 3$, corresponding to the high harmonic end of the spectrum. Both sets of data support the BGP decay index $p = 8/3$.
for the experimental data it corresponds to their observed $p_{\text{max}}$ whereas it falls squarely in the middle of the range for the simulations. It is worth pointing out that although the index $p = 8/3$ is described as universal (independent of the plasma dynamics) Baeva et al. do allow [2] that over a restricted spectral range and for moderate intensities, an index $p = 10/3$ is required.

Results from previous PIC simulations [5], in the moderately relativistic range $a_0 \leq 2.0$ prompted us to question whether the concept of a universal decay index is in fact tenable. Broadly speaking, these results drew attention to plasma effects on the harmonic spectrum, effects that should be no less in evidence in the UR regime. If present, these findings would stand in sharp contradistinction to the BGP similarity model and its universal index.

Our earlier work [5] established that plasma line emission was present in the spectrum of harmonics. The source of plasma emission was identified as plasma waves excited in the supra-dense plasma by jets of Brunel-accelerated electrons generated when $p$-polarised light is incident obliquely on the plasma surface. The re-entrant electrons bunch to generate plasma oscillations within the surface layer that couple efficiently to the radiation field. We show in Fig. 2a a representation of electron density across a section of the plasma target illuminated by $p$-polarised light incident at 23 degrees to the normal to the target surface, in the form of an $(x, t)$ plot. This shows clear striations in electron density corresponding to plasma wave excitation. Contrast this with what happens in the case of normally incident light. Here there is no corresponding source of plasma wave excitation since now $E \cdot \nabla n = 0$. The principal effect on the plasma density in this case is attributed to the $v \times B$ electric field which gives rise to the density structures shown in Fig. 2b. Note that the much less pronounced striations at normal incidence appear both later in time and deeper into the surface plasma layer than is the case for $p$-polarised light.

Figs. 3ab show spectra with $n_e/n_c = 40$ and $a_0 = 10$ respectively, for light obliquely incident on the surface. The spectrum shows clear differences from that at normal incidence. It is apparent from Fig. 3a that the so-called universal decay represented by the coefficient $8/3$ which characterised the spectrum at normal incidence no longer describes the simulated spectrum. Moreover a comparison of the spectra for the two polarizations from Figs. 1a and 3a indicates that the relative levels of harmonic power are more than an order of magnitude more intense for obliquely incident light. To highlight the differences in the spectrum we show in Fig. 3b a diagram similar to that in Fig. 1b except for the normalization, which is now $P_{100}$ rather than $P_{238}$. Over the range of validity of BGP theory, i.e. up to about $m^* \sim 100$ we find that the spectrum decays as $p \sim 5/4$, i.e. significantly more slowly than predicted by the similarity theory index $8/3$. Immediately above
The spectral decay is locally described by the decay coefficient $p = 13/3$ and over the extended range by $p = 10/3$. Nowhere across the spectral range does $p = 8/3$ fit the spectrum. We infer that the distinctive modulation in the spectrum invalidates the concept of a universal decay index.

![Graph](image_url)

Figure 3: Harmonic spectrum generated from a laser pulse incident obliquely on a plasma target: (a) $a_0 = 10, n_e/n_c = 40, \tau_p = 17$ fs and $\lambda_L = 1054$ nm, $\theta = 23^\circ$, (b) relative intensity of harmonics scaled to $m = 100$, i.e. $P_m/P_{100} = (m/100)^{-p}$. The lines correspond to $p = 5/4$ (dotted), $p = 8/3$ (solid) and $p = 10/3$ (dashed), (c) $n_e/n_c = 80, a_0 = 20, \theta = 23^\circ, S = n_e/n_c a_0 = 4$. In this case the spectrum is best fitted by an index $p = 2.1$ (solid) and $p = 5/3$ (dotted).

We show in Fig. 3c the case $n_e/n_c = 80, a_0 = 20$ for which the BGP parameter $S$ defined as $S = n_e/n_c a_0$ is 4, as in Figs. 3ab. The harmonic spectra are quite distinct, emphasising that collective effects govern the interaction physics. There is a distinctive modulation, consistent with the effect found in [1] in a moderately relativistic regime. The structure evident from Figs. 3ab is even more pronounced in Fig. 3c, in which modulation effects dominate the spectrum. The decay index across the entire range is $p \sim 2.1$ though the range $m = 30 - 200$ is best fitted by $p \sim 5/3$.

3. Conclusion

We have shown from simulations that the high harmonics generated from the irradiation of overdense plasmas by laser light in the ultra-relativistic regime, decay in a more complex way than described by the “universal” decay index $p = 8/3$ [2]. This value appears to be truly universal only at normal incidence. At oblique incidence, plasma effects contribute to the spectrum overall both from emission at the plasma frequency and its harmonics and from a distinctive modulation of the harmonic spectrum at the plasma frequency. These effects serve to invalidate the concept of a universal decay index. The different values that apply across the spectral ranges are the outcome of an interplay between modulation effects and the natural harmonic decay. Over the restricted spectral range characterising the water-window, harmonic decay may be significantly weaker than would be the case were it governed by the spectral index found from similarity arguments.

References

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