Abstract

Gauge fixed domain wall fermions are investigated in the reduced model at small Yukawa couplings. We present chiral propagators at the waveguide boundaries using quenched numerical simulations and analytic methods. There is no evidence of mirror chiral modes at the waveguide boundaries.

Introduction

Recently it has been shown [1, 2] that nonperturbative gauge fixing [3] can be applied successfully to decouple the longitudinal gauge degrees of freedom from Abelian lattice chiral gauge theories in the limit of zero gauge coupling. After one gauge transforms a gauge-noninvariant lattice chiral gauge theory proposal like the Smit-Swift model or the domain wall waveguide model, one picks up the longitudinal gauge degrees of freedom (radially frozen scalars) explicitly in the action. The job of gauge fixing is to find a phase transition where the gauge symmetry would be restored with the scalars decoupled.

The gauge fixing approach deals with anomaly-free chiral gauge theories. It is manifestly local. The gauge fixing action involves a kinetic term for the longitudinal gauge degrees of freedom and allows for a renormalizable weak coupling expansion.

The so called reduced model is defined by taking the limit of zero gauge coupling, after gauge transforming the gauge-noninvariant theory. Hence it has coupling between fermions and the scalars. The spectrum of the reduced model derived from the gauge fixed theory should be devoid of the scalars and consist only of free fermions in the appropriate representation of the chiral group. This is precisely what was achieved with recent investigations of Smit-Swift [4] and domain wall models [5] and tuning counterterms posed no practical problems. In the reduced model there is no constraint on the fermion representation from anomalies since the anomaly vanishes trivially in this case, and one can still study the fermion spectra.

In this communication we extend our detailed study [2] of the U(1) lattice chiral gauge theory with domain wall fermions and gauge fixing in the reduced model limit. The reduced model has a Yukawa coupling $y$ which was put in by hand. Our previous study dealt with the situation at $y = 1$ and showed that the scalar fields were completely decoupled from the fermions and chiral modes were obtained only at the domain wall and at the anti-domain wall as in free domain wall fermions. This makes the gauge fixed domain wall model suitable for a U(1) chiral gauge theory at $y = 1$. On the other hand, at $y = 0$, fermion current considerations and also numerical simulations show that mirror chiral modes develop at the so-called waveguide boundaries making it unsuitable for the construction of a chiral gauge theory. Hence it is interesting to ask what happens at small nonzero values of Yukawa coupling. In general it is desirable to understand the properties of a model for all possible values of the coupling parameters. Here we investigate the gauge fixed domain wall waveguide model in the reduced limit for small nonzero values of the Yukawa coupling.
Gauge fixed Domain Wall Action

Kaplan’s free domain wall fermions on a $4 + 1$-dimensional $L^4L_s$ lattice ($0 \leq s \leq L_s - 1$, $s$ labeling the 5-th dimension) with periodic boundary conditions in the 5-th direction and the domain wall mass taken as

$$
m(s) = \begin{cases} 
- m_0, & 0 < s < L_s/2 \\
0, & s = 0, L_s/2 \\
m_0, & L_s/2 < s < L_s
\end{cases}
$$

possess a lefthanded (LH) chiral mode bound to the domain wall at $s = 0$ and a righthanded (RH) chiral mode bound to the anti-domain wall at $s = L_s/2$. For $m_0L_s \gg 1$, these modes have exponentially small overlap.

A 4-dimensional gauge field which is independent of $s$ is then coupled to fermions only for a restricted number of $s$-slices around say, the anti-domain wall with a view to coupling only to the RH mode at the anti-domain wall. The gauge field is thus confined within a waveguide,

$$
WG = (s : s_0 < s \leq s_1).
$$

The waveguide boundaries $s_0$ and $s_1$ should be reasonably far away from the domain wall and the anti-domain wall.

Obviously, the hopping terms from $s_0$ to $s_0 + 1$ and that from $s_1$ to $s_1 + 1$ would break the local gauge invariance of the action. This is taken care of by gauge transforming the action and thereby picking up the longitudinal gauge degrees of freedom or radially frozen scalar fields $\varphi$ at the waveguide boundary, leading to the action (lattice constant is taken to be unity throughout):

$$
S_F = \sum_{s \in WG} \overline{\psi} (D(U) - W(U) + m(s)) \psi + \sum_{s \notin s_0, s_1} \overline{\psi} (\varphi - w + m(s)) \psi \\
+ \sum_s \overline{\psi} \psi^s - \sum_{s \notin s_0, s_1} \left( \overline{\psi} P_L \psi^{s+1} + \overline{\psi}^{s+1} P_R \psi^s \right) \\
- y \left( \overline{\psi} \varphi P_L \psi^{s+1} + \overline{\psi}^{s+1} \varphi P_R \psi^s \right) - y \left( \overline{\psi} \varphi P_L \psi^{s+1} + \overline{\psi}^{s+1} \varphi P_R \psi^s \right)
$$

where we have suppressed all indices other than $s$. $\overline{\psi}$ and $\psi$ are the fermion fields, the projector $P_{L(R)}$ is $(1 \mp \gamma_5)/2$ and $y$ is the Yukawa coupling introduced by hand at the waveguide boundaries. $D(U)$ and $W(U)$ are respectively the gauge covariant Dirac operator and the Wilson term (with Wilson $r = 1$) in 4 space-time dimensions. $\varphi$ and $w$ are the 4-dimensional free Dirac and Wilson operators.

The gauge fixed pure gauge action for $U(1)$, where the ghosts are free and decoupled, is:

$$
S_B(U) = S_g(U) + S_{gf}(U) + S_{ct}(U)
$$

where, $S_g$ is the usual Wilson plaquette action and $S_{ct}$ is a counterterm. Previous studies show that only a gauge field mass counterterm is needed. $S_{gf}$ is the gauge fixing term which is not just a naive lattice transcription of the continuum covariant gauge fixing, it has in addition appropriate irrelevant terms. As a result, $S_{gf}$ has a unique absolute minimum at $U_{\mu x} = 1$, validating weak coupling perturbation theory around gauge coupling $g = 0$. For an explicit expression of $S_{gf}$, see [3].

Obviously, the action $S_B(U)$ is not gauge invariant. By giving it a gauge transformation the resulting action is $S_B(\varphi_{x}^\dagger U_{\mu x} \varphi_{x+\hat{\mu}})$. By restricting to the trivial orbit, $U_{\mu x} = 1$, we arrive at the reduced model action

$$
S_{reduced} = S_F(U = 1) + S_B(\varphi_{x}^\dagger \varphi_{x+\hat{\mu}})
$$

where $S_F(U = 1)$ is obtained quite easily from eq.[3] and

$$
S_B(\varphi_{x}^\dagger \varphi_{x+\hat{\mu}}) = -\kappa \sum_x \partial_x^\dagger \partial_x \varphi_x + \tilde{\kappa} \sum_x [\partial_x^\dagger \partial_x \varphi_x - B_x^2]
$$

now is a higher-derivative scalar field theory action and $B_x$ is given by,

$$
B_x = \sum_{\mu} \left( \frac{V_{x \mu - \hat{\mu}} + V_{\mu x}}{2} \right)^2 \text{ with } V_{\mu x} = \text{Im } \varphi_{x}^\dagger \varphi_{x+\hat{\mu}}.
$$

Perturbation theory around $g = 0$ translates in the reduced model to the same around $\tilde{\kappa} = \infty$.

In the following, we investigate the action at small nonzero $y$ by numerical methods. To follow up the numerical investigation, analytic studies will then be made again at small $y$ with slightly different boundary conditions.
showing a decreasing trend, indicating possible absence of mirror modes. Obviously the smaller the Yukawa coupling, the bigger the

mirror modes. The lower the Yukawa coupling, the sharper was the increasing trend of the chiral propagators with

for this lattice size, something that could signify a pole at zero momentum, thereby indicating possible presence of

wall action of Kaplan [4]).

diagram of the quenched reduced model, please see [7]. The (\(g\)gauge fields \(F_{MD}\) is the transition where gauge symmetry is restored). For a detailed discussion of the phase transition to a rotationally noninvariant broken phase (FMD), staying within the FM phase (in the full theory with dynamical
coupling parameters puts the theory quite close to the phase transition from a ferromagnetic broken phase (FM) to a rotationally noninvariant broken phase (FMD), staying within the FM phase (in the full theory with dynamical
gauge fields FM-FMD is the transition where gauge symmetry is restored). For a detailed discussion of the phase
determination by inverting the free fermionic action (obtained by putting \(\varphi = 1\) in the reduced fermionic action \(S_{F}(U = 1)\) with the respective values of the Yukawa couplings (the free fermionic action at \(y = 1\) is the free domain wall action of Kaplan [3]).

Figure 1: \(RR\) propagator at waveguide boundary \(s = 5\) and \(LL\) propagator at waveguide boundary \(s = 6\) at \(\kappa = 0.05\) and \(\tilde{\kappa} = 0.2\) (\(L_s = 22;\ a.p.b.c.\ in \(L_4\)), (a) \(y = 0.25\), (b) \(y = 0.50\), (c) \(y = 0.75\), (d) \(y = 1.0\).

Numerical results at small Yukawa coupling

At \(y = 0\), the domain wall and the anti-domain wall are detached from each other. In fact, in this case there is no coupling between the 4-dimensional worlds living on the \(s_0\)-th and the \((s_0 + 1)\)-th slices (and similarly for the \(s_1\)-th and the \((s_1 + 1)\)-th slices). From fermion current considerations, the LH chiral mode bound to the domain wall at \(s = 0\) would then necessitate the generation of a RH mode at \(s = s_0\) and also at \(s = s_1 + 1\). Similarly a LH mode, mirror to the RH chiral mode at the anti-domain wall at \(s = L_s/2\), would be generated at \(s = s_0 + 1\) and also at \(s = s_1\). Our numerical simulations at \(y = 0\) and similar studies in [3] clearly show that mirror chiral modes form at the waveguide boundaries. Generation of these mirror modes at the waveguide boundaries is independent of gauge fixing.

On the other hand, with gauge fixing at \(y = 1\) [4], the mirror chiral modes are certainly absent at the waveguide boundaries. In this case, the only chiral modes are at the domain wall and at the anti-domain wall and the spectrum is that of a free domain wall fermion.

To investigate the interesting question what happens at small \(y\), we looked for chiral modes at \(y = 0.75, 0.5, 0.25\), especially at the waveguide boundaries. Specifically we numerically evaluate the \(LL\) and the \(RR\) propagators in momentum space conjugate to the 4 space-time dimensions and in configuration space for the 5-th (flavor-like) dimension. This is done using quenched configurations of the scalar fields at \(\kappa = 0.05\) and \(\tilde{\kappa} = 0.2\). This choice of the coupling parameters puts the theory quite close to the phase transition from a ferromagnetic broken phase (FM) to a rotationally noninvariant broken phase (FMD), staying within the FM phase (in the full theory with dynamical
gauge fields FM-FMD is the transition where gauge symmetry is restored). For a detailed discussion of the phase
diagram of the quenched reduced model, please see [4]. The \((\kappa, \tilde{\kappa})\) point we have chosen for the present investigation is also where we had previously done all our work at \(y = 1\).

On a \(6^3 \times 16\) lattice with \(L_s = 22\) and \(m_0 = 0.5\), the chiral propagators at the waveguide boundaries showed an increasing trend as the fourth component of momentum \(p_4\) (with \(\vec{p} = 0\)) was decreased to the minimum value possible for this lattice size, something that could signify a pole at zero momentum, thereby indicating possible presence of mirror modes. The lower the Yukawa coupling, the sharper was the increasing trend of the chiral propagators with decreasing \(p_4\).

For a closer scrutiny we then took lattice sizes which were bigger in the 4-th direction to accommodate lower \(p_4\)-values. We found that for each \(y\) there is a big enough lattice size for which the chiral propagators ultimately start showing a decreasing trend, indicating possible absence of mirror modes. Obviously the smaller the Yukawa coupling, the bigger the \(L_4\) extension was required. Moreover, all the chiral propagators at these small Yukawa couplings matched exactly with the corresponding free chiral propagators. These free chiral propagators were numerically determined by inverting the free fermionic action (obtained by putting \(\varphi = 1\) in the reduced fermionic action \(S_F(U = 1)\) with the respective values of the Yukawa couplings (the free fermionic action at \(y = 1\) is the free domain wall action of Kaplan [3]).
Our results at the waveguide boundaries \( s = s_0 \equiv 5 \), and \( s = s_0 + 1 \equiv 6 \) are summarized in Fig.1 for the \( RR \) and the \( LL \) propagators respectively. In the figures, SF and FF respectively indicate data obtained by numerical simulation of the scalar-fermion reduced model, eq.(3), and by direct numerical inversion of the free fermion matrix (i.e. with \( \varphi = 1 \)) at the given Yukawa couplings. Error bars are smaller than the symbols. Dotted lines joining the data points are to guide the eye. Figure 1 also contains our previous data at \( y = 1 \) \([2]\) for comparison. From the figures, existence of poles at zero 4-momentum for these chiral propagators does not seem likely.

Similar numerical investigation is also carried out at the other waveguide boundary \( s_1, s_1 + 1 \). There too evidence for mirror modes is dim.

Perturbation Theory at \( y \neq 1 \)

To confirm the indication from our numerical simulation that, for small \( y \), the chiral propagators at the waveguide boundaries do not seem to have poles at zero momentum, it would be nice to calculate the propagators analytically. This can be done in perturbation theory in the coupling \( 1/\sqrt{2\kappa} \). This was also done for the fermion propagators to 1-loop in our previous investigation at \( y = 1 \) \([2]\).

The present case, however, is more complicated due to the presence of two more defects created at the two waveguide boundaries because of \( y \neq 1 \), in addition to the existing defects, namely, the domain wall at \( s = 0 \) and the anti-domain wall at \( s = L_s/2 \). The defects are the places across which translational invariance is not maintained and boundary conditions need to be imposed to match the propagators from two sides of each defect.

In this section we leave Kaplan boundary conditions and use Shamir boundary conditions \([3]\) because imposing boundary conditions in the Shamir case would be a lot less tedious and our conclusions would qualitatively be the same as in the Kaplan case.

The free Dirac operator in the Shamir case is defined on 4 space-time dimensions plus a finite 5-th dimension, \( 0 \leq s \leq L_s - 1 \) with rigid walls at \( s = 0 \) and \( s = L_s - 1 \). The domain wall mass is taken as \( -m_0 \) for all \( s \)-slices. Fermionic spectrum then consists of a LH chiral mode at \( s = 0 \) and a RH chiral mode at \( s = L_s/2 \). (For further discussion see \([3]\) and \([4]\).)

The waveguide in this case can be implemented by putting the same 4-dimensional gauge field over \( s \)-slices from \( s_0 + 1 \) to \( L_s - 1 \) with a view to gauging only the RH mode at \( s = L_s - 1 \). The hopping terms from \( s_0 \) to \( s_0 + 1 \) again break the local gauge invariance of the action. Longitudinal gauge degrees of freedom or radially frozen scalar fields \( \varphi \) enter the action explicitly after a gauge transformation. The reduced model is then obtained by imposing \( U_{ux} = 1 \):

\[
S_F^{(shamir)}(U = 1) = \sum_s \overline{\psi} \left( \theta - w - m_0 \right) \psi + \sum_s \overline{\psi} \psi - \sum_{s \neq s_0} \left( \overline{\psi} P_L \psi^{s+1} + \overline{\psi}^{s+1} P_R \psi^s \right) - y \left( \overline{\psi} \varphi^s \psi^{s+1} + \overline{\psi}^{s+1} \varphi \psi^s \right).
\]

The fermion propagators are obtained in momentum space for 4 space-time dimensions while staying in the coordinate space for the 5-th dimension. The procedure in the following is applicable to all values of the Yukawa coupling \( y \) including \( y = 0 \) and \( y = 1 \). Needless to say, it is also applicable to Kaplan boundary conditions.

In order to develop perturbation theory, in reduced model, we expand,

\[
\varphi_x = \exp(ib\theta_x) = 1 + ib\theta_x - \frac{1}{2}b^2\theta_x^2 + O(b^3), \quad b = \frac{1}{\sqrt{2\kappa}}.
\]

Thus the fermion action at the tree level is written as,

\[
S_F^{(0)} = \sum_{p,s,t} \overline{\psi}_p^s \left[ i\theta \delta_{s,t} + (M_y)_{st} P_L + \left( M_y^t \right)_{st} P_R \right] \psi_p^t
\]

where \( \overline{p}_\mu = \sin(p_\mu) \) and \( \overline{p} = \gamma_\mu \overline{p}_\mu \), and

\[
(M_y)_{st} = M_{st} + \bar{y}\delta_{s,s_0}\delta_{t,s_0+1}
\]

\[
(M_y^t)_{st} = M_{st}^t + \bar{y}\delta_{s,s_0+1}\delta_{t,s_0}
\]
where \( \bar{y} = 1 - y \). The \( M \) and \( M^\dagger \) are defined as,

\[
M_{st} = [1 - m_0 + \sum_\mu (1 - \cos p_\mu)] \delta_{s,t} - \delta_{s+1,t},
\]

\[
M^\dagger_{st} = [1 - m_0 + \sum_\mu (1 - \cos p_\mu)] \delta_{s,t} - \delta_{s-1,t}.
\]

(12)

The free fermion propagator can now formally be written as,

\[
\Delta(p) = \left[ (\not{p} + M_\bar{y} P_L + M^\dagger_\bar{y} P_R)^{-1} \right] = \left( -\not{p} + M^\dagger_\bar{y} \right) P_L G_L(p) + \left( -\not{p} + M_\bar{y} \right) P_R G_R(p)
\]

where,

\[
G_L(p) = \frac{1}{\sum_\mu p_\mu + \bar{y} M^\dagger_\mu}
\]

\[
G_R(p) = \frac{1}{\sum_\mu p_\mu + \bar{y} M_\mu}.
\]

(13)

General solution of \( G_L \) is obtained by writing (14) explicitly:

\[
\sum_s \left[ (\not{p} + MM^\dagger)_{ss'} + \bar{y} \left( M_{s,s_0+1} \delta_{s',s_0} + \delta_{s,s_0} M^\dagger_{s_0+1,s'} \right) + \bar{y}^2 \delta_{s,s_0} \delta_{s',s_0} \right] G_L(p)_{s',t} = \delta_{s,t}
\]

(16)

and similarly for \( G_R \). We show only the calculations for obtaining \( G_L \) and henceforth drop the subscript \( L \).

We now define two translationally invariant regions \( I \) and \( II \) and set appropriate notation for \( G \),

region \( I \) : \( 0 \leq s \leq s_0 \), \( G = G^{(1)} \),

region \( II \) : \( s_0 + 1 \leq s \leq L_s - 1 \), \( G = G^{(2)} \).

(17)

(18)

\( G^{(1)} \) and \( G^{(2)} \) are now forced to satisfy the translationally invariant equation (with \( B = 1 - m_0 + \sum_\mu (1 - \cos p_\mu) \)),

\[
[\not{p}^2 + 1 + B^2] G^{(1,2)}_{s,t} - B G^{(1,2)}_{s+1,t} - B G^{(1,2)}_{s-1,t} = \delta_{s,t}.
\]

(19)

The above equation (19) actually takes the system beyond the respective regions \( I \) and \( II \). This will be taken care of by boundary conditions later.

The solutions of eq. (19) are expressed as sum of homogeneous and inhomogeneous solutions,

\[
G^{(1)}(p)_{s,t} = g^{(1)}(t) e^{-\alpha s} + h^{(1)}(t) e^{\alpha s} + \frac{\cosh[\alpha(l - |s - t|)]}{2B \sinh(\alpha) \sinh(\alpha)}
\]

(20)

\[
G^{(2)}(p)_{s,t} = g^{(2)}(t) e^{-\alpha s} + h^{(2)}(t) e^{\alpha s} + \frac{\cosh[\alpha(l - |s - t|)]}{2B \sinh(\alpha) \sinh(\alpha)}
\]

(21)

where \( l = L_s/2 \), and

\[
\cosh(\alpha) = \frac{1}{2} \left( B + \frac{1 + \not{p}^2}{B} \right).
\]

(22)

The third terms in eqs. (20, 21) are the inhomogeneous solutions, which are the same in both the regions. Avoiding singularity in \( \alpha \) when \( B \) is zero restricts the allowed range of \( m_0 \) to \( 0 < m_0 < 1 \). In this work we have taken \( m_0 = 0.5 \).

The unknown functions \( g^{(1,2)}(t) \) and \( h^{(1,2)}(t) \) are determined from the following boundary conditions:

\[
G^{(1)}_{s_{0+1},t} - BG_{s_{0-1},t}^{(2)} = 0
\]

\[
G^{(2)}_{s_{0-1},t} - BG^{(2)}_{s_{0+1},t} = \delta_{s_{0+1},t}
\]

(23)
where $F = F - y(2 - y)$ and $F = p^2 + 1 + B^2$. Substituting $2B \sinh(\alpha) \sinh(\alpha) = X^{-1}$ and using the boundary conditions, eqs. (23), we arrive at,

$$A \cdot v(t) = X(t),$$

where, $v(t) = (g^{(1)} h^{(1)} g^{(2)} h^{(2)})$ is a 4-component vector, $X(t)$ is another 4-component vector and $A$ is a $4 \times 4$ matrix as given below,

$$A = \begin{pmatrix}
-\alpha & e^{\alpha} & 0 & e^{\alpha(L_s-1)} - Be^{-\alpha L_s} \\
0 & e^{-\alpha} & 0 & e^{\alpha(s_0+1)} - Be^{\alpha s_0+1} \\
F e^{-\alpha s_0} - Be^{-\alpha(s_0-1)} & (y-1)e^{-\alpha s_0} & 0 & (y-1)e^{\alpha(s_0+1)} \\
(y-1)e^{\alpha s_0} & e^{-\alpha(s_0-1)} - Be^{\alpha(s_0-1)} & 0 & \end{pmatrix}
$$

and

$$X(t) = \begin{pmatrix}
-X \cosh[\alpha(l - |1 - t|)] \\
BX \cosh[\alpha(l - |L_s - t|)] - X \cosh[\alpha(l - |L_s - 1 - t|)] \\
\delta_{s_0,t} + BX \cosh[\alpha(l - |s_0 - 1 - t|)] - FX \cosh[\alpha(l - |s_0 + 1 - t|)] + (1 - y)X \cosh[\alpha(l - |s_0 + 1 - t|)] \\
\delta_{s_0+1,t} + BX \cosh[\alpha(l - |s_0 + 2 - t|)] - FX \cosh[\alpha(l - |s_0 + 1 - t|)] + (1 - y)BX \cosh[\alpha(l - |s_0 - t|)]
\end{pmatrix}.$$

The solution to the eqs. (24) is very complicated in general, particularly for finite $L_s$. However, $g^{(1,2)}(t)$ and $h^{(1,2)}(t)$ can be obtained for finite $L_s$ by solving the above equations numerically for different $t$ values. This way we can easily construct the tree level fermion propagators at any given $s$-slice.

The solutions for $(G_R)_{s,t}$ and the resulting propagators are obtained in exactly the same way.

1-loop corrections to the tree level $LL$ and $RR$ propagators above can be easily found following [2] and as in $y = 1$ are found to be negligible. This is because the scalar fields are almost completely decoupled.

Using the analytic method described above, on a $512^3 \times 8192$ lattice with $L_s = 100$ and $m_0 = 0.5$, we plot in Fig. 2 tree level $RR$ propagators for $y = 0.25, 0.50, 0.75, 1.0$ at the waveguide boundary $s = s_0 = 49$. From the figure it is clearly seen that there are no poles in these propagators, ruling out any possibility of a mirror mode at the waveguide boundary at any nonzero $y$ (including arbitrarily small $y$).
Discussion

In our previous study [2] we investigated the U(1) gauge fixed domain wall waveguide model in the reduced limit at Yukawa coupling $y = 1$. Although the theory contained scalars explicitly in the action as a result of gauge noninvariance, we found no evidence of them. The fermionic spectrum was that of a free domain wall fermion.

Since the Yukawa interaction couples only the two slices across a waveguide boundary, there is a new defect introduced at a waveguide boundary when $y \neq 1$. Especially at $y = 0$ mirror chiral modes are developed at the waveguide boundaries. It is an interesting question to find out if these mirror modes persist to exist even for small values of the Yukawa coupling.

Our numerical investigations (at small $y$) of chiral propagators at the waveguide boundaries initially show a disturbing trend of increasing with decreasing momentum. On bigger lattices and smaller momenta, however, this trend does not continue and poles of the chiral propagators do not seem likely. In addition, the data agree quite well with free chiral propagators obtained by numerically inverting the fermion matrix after putting $\varphi = 1$ for all the $y$ values. Agreement with free propagators means that the scalar fields are decoupled.

To be able to go to much smaller momentum, we then perform an analytic calculation of the propagators at $y \neq 1$ in the perturbative limit $\tilde{\kappa} = \infty$. Although done with different boundary conditions than we started with, qualitative conclusions can at least be made and the analytic tree propagators certainly show a very good resemblance to the numerically calculated ones and confirm the view that there are no poles for these propagators at the waveguide boundaries.

Our conclusion would be that even for arbitrarily small or any nonzero Yukawa coupling, the fermion spectrum would be devoid of any ill-effects of the scalar fields. In fact the scalar fields are completely decoupled. This happens because the continuum limit is taken at the FM-FMD transition where the scalars do not scale, being far away from a ferromagnetic-paramagnetic transition. Obviously from a quenched numerical simulation alone, we cannot come to a firm conclusion. However, the 1-loop corrections to the perturbative tree level fermion propagators are negligibly small and fermion loops enter the calculation at least at the two loop level. Hence our conclusions about nonexistence of the mirror modes at the waveguide boundaries for small $y$ seem to be correct even for the situation with dynamical fermions as long as the phase diagram stays qualitatively the same with dynamical fermions. Once the continuum limit can be taken at the FM-FMD transition, even an arbitrarily small $y$ (which effectively mimics a scaling radial mode of the scalar fields) cannot produce the mirror modes and spoil the theory.

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