Abstract

The first version of a computer program \texttt{eett6f} for calculating cross sections of $e^+e^- \rightarrow 6$ fermions processes relevant for a $t\bar{t}$-pair production and decay at centre of mass energies typical for linear colliders is presented. \texttt{eett6f v. 1.0} allows for calculating both the total and differential cross sections at tree level of the Standard Model (SM). The program can be used as the Monte Carlo generator of unweighted events as well.

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2E-mail: kolodzie@us.edu.pl
PROGRAM SUMMARY

Title of program: eett6f

Version: 1.0 (April 2002)

Catalogue identifier: FJA0815b

Program obtainable from: CPC Program Library or on request by e-mail from the author

Licensing provisions: none

Computers: all

Operating systems: Unix/Linux

Programming language used: FORTRAN 90

Memory required to execute with typical data: 1 Mb

No. of bits in a word: 32

No. of bytes in distributed program, including test data, etc.: 400 Kb

Distribution format: ASCII

Keywords: $e^+e^- \rightarrow 6$ fermions, SM, lowest order six-fermion reactions, $t\bar{t}$ production and decay

Nature of physical problem
Description of $e^+e^- \rightarrow 6$ fermions processes relevant for a $t\bar{t}$-pair production and decay at centre of mass energies typical for linear colliders; all six fermion reactions containing a $b$ and $\bar{b}$ quarks and four other fermions of different flavours with a complete set of the Feynman diagrams in the lowest order of SM.

Method of solution
Matrix elements are calculated numerically with the helicity amplitude method. Constant widths of unstable particles are implemented by modifying mass parameters in corresponding propagators. The phase space integration is performed numerically utilizing a multi-channel Monte Carlo method.

Restrictions on complexity of the problem
Reactions containing fermions of the same flavour are not treated. No higher order effects are taken into account, except for assuming the fine structure constant and the strong coupling at appropriate scale and partial summation of the one particle irreducible loop corrections by introducing fixed widths of unstable particles.

Typical running time
The running time depends strongly on a desired precision of the result. The results of the appended test run have been obtained on a 800 MHz Pentium III processor with the use of Absoft FORTRAN 90 compiler in 490 seconds. In order to obtain a precision level below one per mille a few million calls to the integrand are required. This results in a typical running time of several hours. The running time becomes much shorter for approximated cross sections.
1 Introduction

Precise measurements of the top quark properties and interactions are planned at TESLA [1] and will most certainly belong to the research program of any future $e^+e^-$ collider [2]. The measurements should be confronted with theoretical predictions matching the same high precision level of a few per mille. It is obvious that in order to reach that high precision it is mandatory to include radiative corrections and, as the measurements of some top quark physical properties will be carried out at high energies, much above the $t\bar{t}$ threshold, it is crucial to know off-resonance background contributions to any specific 6 fermion decay channel and to estimate the effects related to the off-shellness of the $t\bar{t}$-pair.

In this article, a technical documentation of a numerical program, eett6f, is presented which allows for computer simulation of the 6 fermion reactions which are relevant for the top quark pair production and its decay into a specific 6 fermion final state

$$e^+(p_1) + e^-(p_2) \rightarrow b(p_3) + f_1(p_4) + f_1^*(p_5) + b(p_6) + f_2(p_7) + f_2^*(p_8),$$

(1)

where $f_1 = \nu_\mu, \nu_\tau, u, c$, $f_2 = \mu^-, \tau^-, d, s$, and $f_1', f_2'$ are the corresponding weak isospin partners of $f_1$, $f_2$, $f_1' = \mu^-, \tau^-, d, s$, $f_2' = \nu_\mu, \nu_\tau, u, c$, and the particle four momenta have been indicated in the parentheses. In the present version, v. 1.0, of the program, reactions (1) are treated in the lowest order of SM. Moreover, for the sake of simplicity, it is assumed that the actual values of $f_1$ and $f_2$ in (1) are different from each other, and that neither $f_1'$ nor $f_2'$ is an electron. The physics contents of the program is described in [3], where the details on the physical model, a method of calculation and physical results obtained with eett6f v1.0 have been discussed.

A similar analysis of the 6 fermion processes relevant for a $t\bar{t}$ production in $e^+e^-$ annihilation have been performed in [4], [5], where semileptonic channels of reaction (1) have been studied, and in [6], where purely hadronic channels of (1) have been analysed. Moreover, irreducible QCD background to top searches in semileptonic channels of (1) has been discussed in [7]. After this work has already been completed an extensive study of six fermion reactions, including (1) among others, in the massless fermion limit has appeared [8]. Using the same input parameters and separation cuts as in [8], the present program gives 5.8160(32) fb and 17.223(15) fb for the cross sections of $e^+e^- \rightarrow b\nu_\mu\mu^+b\tau^-\bar{\nu}_\tau$ and $e^+e^- \rightarrow b\bar{d}\mu^-\bar{\nu}_\mu$, respectively, at CMS energy of 500 GeV. These results are 2–3 standard deviations off the corresponding results of [8]. This difference can most probably be traced back to somewhat different implementation of the top quark width in the top quark propagator. In the present work, the top quark width is introduced both in the denominator and in the numerator of the top propagator while, in [8], it seems to be introduced in the denominator only. The invariant mass and angular distributions of the $b\bar{d}$ quark triple of $e^+e^- \rightarrow b\bar{d}\mu^-\bar{\nu}_\mu$ at $s^{1/2} = 500$ GeV shown in [8] are nicely reproduced by eett6f within accuracy of the plots.

The main advantage of the present program is that it allows for taking into account both the electroweak and QCD lowest order contributions. As light fermion masses are not neglected, the cross sections can be calculated without any kinematical cuts. Moreover, a number of options have been implemented in the program which make possible calculation of the cross
sections while switching on and off different subsets of the Feynman diagrams. It is possible to calculate cross sections in two different narrow width approximations discussed below, too.

Besides of a possibility of taking into account solely the electroweak contributions, or switching off the Higgs boson exchange diagrams, the program allows also for a simplified treatment of reaction \( \text{(4)} \) by utilizing a few different approximations: the double resonance approximation for W bosons

\[
e^+(p_1) + e^-(p_2) \rightarrow b(p_3) + W^+(p_{45}) + \bar{b}(p_6) + W^-(p_{78})
\]

\[
\rightarrow b(p_3) + f_1(p_4) + f_1^*(p_5) + \bar{b}(p_6) + f_2(p_7) + f_2^*(p_8),
\]

where only those 61 Feynman diagrams are taken into account which contribute to \( e^+e^- \rightarrow bW^+\bar{b}W^- \) and the W bosons are considered as being off-mass-shell and the double resonance approximation for a t- and \( \bar{t} \)-quark

\[
e^+(p_1) + e^-(p_2) \rightarrow t^*(p_{345}) + \bar{t}^*(p_{678})
\]

\[
\rightarrow b(p_3) + f_1(p_4) + f_1^*(p_5) + \bar{b}(p_6) + f_2(p_7) + f_2^*(p_8),
\]

with only two ‘signal’ diagrams contributing. The intermediate state momenta are \( p_{45} = p_4 + p_5, \) \( p_{78} = p_7 + p_8 \) in Eq. (2) and \( p_{345} = p_3 + p_4 + p_5, \) \( p_{678} = p_6 + p_7 + p_8 \) in Eq. (4).

The narrow width approximation for the W bosons (top quarks) is obtained by replacing the intermediate state W bosons (top quarks) in Eq. (2) (in Eq. (4)) with on mass shell W’s (top quarks) and by multiplying the cross section of the on mass shell reaction (2) ((4)) with the branching ratios corresponding to final state of reaction (1). The necessary partial decay width of the W and t are calculated according to the lowest order SM, and the fermion masses are neglected in the three body top decay with.

The necessary matrix elements of reactions (1), (2) and (4) are calculated with the helicity amplitude method described in [9] and [10] and phase space integrations are performed with the Monte Carlo (MC) method. More details on the multi-channel MC algorithm (see e.g. [11]), on which the MC integration and event generation is based, are given in the next section.

## 2 Monte Carlo integration and event generation

In order to improve the convergence of the MC integration the most relevant peaks of the matrix element squared related to the Breit-Wigner shape of the W, Z, Higgs and top quark resonances as well as to the exchange of a massless photon or gluon have to be mapped away. As it is not possible to find out a single parametrization of the \( n \)-dimensional phase space which would allow to cover the whole resonance structure of the integrand, it is necessary to utilize a multi-channel MC approach. In the multi-channel MC approach, the random numbers \( 0 \leq x_j \leq 1, \ j = 1, ..., n \), and hence four momenta of the final state particles, are generated according to one of \( N \) different probability densities \( f_i(x), \ i = 1, ..., N, \ x = (x_1, ..., x_n) \). The distribution \( f_i(x) \), which accounts for several different peaks of the integrand, is selected with probability \( a_i \). All the distributions \( f_i(x) \) are then combined in a single probability distribution \( f(x) \) which should cover possibly all the peaks of the integrand, or at least the most relevant ones

\[
f(x) = \sum_{i=1}^{N} a_i f_i(x), \quad \text{with} \quad \sum_{i=1}^{N} a_i = 1.
\]
The second condition of Eq. (3) guaranties that, if every distribution \( f_i(x) \) is normalized to unity, then the combined distribution \( f(x) \) is normalized to unity, too. Obviously, all the distributions \( f_i(x) \) and weights \( a_i, i = 1, ..., N \), are to be non-negative.

The basic phase space parametrizations which are used in the program are listed below. The 6 particle phase space of reaction (2) is parametrized in 3 different ways:

\[
d^{14}\text{Lips} = (2\pi)^{-14}dP_{S_2}(s, s_{345}, s_{678})dP_{S_2}(s_{345}, m_{21}^2, s_{45})dP_{S_2}(s_{678}, m_{06}^2, s_{78})
\]
\[
\times dP_{S_2}(s_{45}, m_{22}^2, m_{33}^2)dP_{S_2}(s_{78}, m_{21}^2, m_{33}^2)ds_{345}ds_{678}ds_{45}ds_{78},
\]

(7)

\[
d^{14}\text{Lips} = (2\pi)^{-14}dP_{S_2}(s, s_{34}, s_{5678})dP_{S_2}(s_{5678}, s_{678}, s_{78})dP_{S_2}(s_{34}, m_{21}^2, m_{33}^2)
\]
\[
\times dP_{S_2}(s_{56}, m_{55}^2, m_{66}^2)dP_{S_2}(s_{78}, m_{21}^2, m_{33}^2)ds_{34}ds_{5678}ds_{56}ds_{78},
\]

(8)

and

\[
d^{14}\text{Lips} = (2\pi)^{-14}dP_{S_2}(s, m_{22}^2, s_{45678})dP_{S_2}(s_{45678}, s_{45}, s_{678})dP_{S_2}(s_{678}, m_{06}^2, s_{78})
\]
\[
\times dP_{S_2}(s_{45}, m_{22}^2, m_{33}^2)dP_{S_2}(s_{78}, m_{21}^2, m_{33}^2)ds_{45678}ds_{45}ds_{678}ds_{78}.
\]

(9)

The 4 particle phase space of reaction (2) in the narrow W width approximation is parametrized in 2 different ways:

\[
d^{8}\text{Lips} = (2\pi)^{-8}dP_{S_2}(s, m_{21}^2, s_{456})dP_{S_2}(s_{456}, m_{02}^2, s_{56})dP_{S_2}(s_{56}, m_{02}^2, m_{66}^2)ds_{456}ds_{56}
\]

(10)

and

\[
d^{8}\text{Lips} = (2\pi)^{-8}dP_{S_2}(s, s_{34}, s_{56})dP_{S_2}(s_{34}, m_{21}^2, m_{33}^2)dP_{S_2}(s_{56}, m_{02}^2, m_{66}^2)ds_{34}ds_{56}.
\]

(11)

In Eqs. (7)–(10), \( s_{ijk...} = (p_i + p_j + p_k + ...)^2 \), \( i, j, k = 3, ..., 8 \), and \( dP_{S_2}(s, s', s'') \) is a two particle (subsystem) phase space element defined by

\[
dP_{S_2}(s, s', s'') = \delta^4(p - p' - p'') \frac{d^3p'}{2E'} \frac{d^3p''}{2E''} = \frac{|\vec{p}'|}{4\sqrt{s}} d\Omega',
\]

(12)

where \( \vec{p}' \) is the momentum and \( \Omega' \) is the solid angle of one of the particles (subsystems) in the relative centre of mass system, \( \vec{p}' + \vec{p}'' = 0 \). Making use of the rotational symmetry with respect to the \( e^+e^- \) beam axis reduces the dimension of the phase space elements to 13 in Eqs. (7)–(8) and to 7 in Eqs. (9) and (10).

Parametrizations (7)–(10) are used with different permutations of external particle momenta such that invariants \( s_{ijk...} \) possibly correspond to the virtuality of propagators of the gauge bosons, Higgs boson and/or top quarks. The invariants are then transformed to the interval \([0, 1]\) by performing mappings smoothing out peaks related to the propagators. All other integration variables of Eqs. (7)–(10), not related to peaks of the matrix element squared, are also transformed to the interval \([0, 1]\) with a simple mapping defined in the following way. Let \( y_j \in [c_j, b_j] \) be a variable not related to a peak in the matrix element squared and \( x_j \) be random variable uniformly distributed in the interval \([0, 1]\), then the relation

\[
y_j = (b_j - c_j) x_j + c_j
\]

(13)
defines the necessary mapping. After having all the integration variables transformed to the interval $[0, 1]$, any of phase space parametrizations (7–9) for reaction (1), or (10), (11) for reaction (2) in the narrow width approximation, for a given permutation of particle momenta, can be directly associated with the probability density $f_i(x)$, which is referred to as a kinematical channel. Altogether 59 (21) different channels are used in order to integrate the matrix element squared of reaction (1) (reaction (2) in the narrow width approximation).

The weights $a_i$ are calculated in the initial scanning run that starts with all the weights equal to each other. The weights $a_i$, $i = 1, ..., N$, for the actual run are calculated as ratios

$$a_i = \sigma_i / \sum_{j=1}^{N} \sigma_j,$$  \hspace{1cm} (14)

where $\sigma_j$ denotes the cross section obtained with the $i$-th kinematical channel in the initial scan.

Calculation of the cross section of reaction (4) in the narrow top width approximation does not require the multi-channel MC approach and can be performed with the single phase space parametrization (12).

### 3 Description of the program

**eett6f** is a package written in **FORTRAN 90**. It consists of 50 files including a makefile. They are stored in one working directory. The user should specify the physical input parameters in module **inprms.f** and select a number of options in the main program **csee6f.f**.

#### 3.1 Program input

The default values of the input parameters and options used in the program are those specified below.

#### 3.1.1 Physical parameters

The initial physical parameters to be specified in module **inprms.f** are the following.

Gauge boson masses and widths (GeV):

- $m_w = 80.419$ GeV, the W mass,
- $\Gamma_w = 2.12$ GeV, the W width,
- $m_z = 91.1882$ GeV, the Z mass,
- $\Gamma_z = 2.4952$ GeV, the Z width.

The electroweak (EW) mixing parameter $s_{\theta_2}$ is then calculated from

$$s_{\theta_2} = 1 - m_w^2 / m_z^2$$

utilizing $m_w^2 = m_w^2$, $m_z^2 = m_z^2$ in the fixed width scheme, and
\[ mw2 = mw**2 - i*mw*gamw, \quad mz2 = mz**2 - i*mz*gamz \] in the complex mass scheme.

\[ ralp0 = 137.03599976, \] an inverse of the fine structure constant in the Thomson limit,

\[ gmu=1.16639\times10^{-5} \text{ GeV}^{-2}, \] the Fermi coupling constant,

\[ alphas=0.1185, \] the strong coupling constant at \( mz \).

An inverse of the fine structure constant at \( mw \), \( ralpw \), is then calculated from

\[ ralpw=4.44288293815837/(2*sw2*gmumu**2). \]

\( mh=115.0 \text{ GeV} \), the Higgs boson mass,

\( gamh=0 \text{ GeV} \), the Higgs boson width.

If \( gamh = 0 \), then the Higgs width is calculated according to the lowest order of SM.

Fermion masses and widths:

\( me = 0.510998902 \text{ MeV} \), \( game = 0 \text{ MeV} \), for an electron,

\( mmu = 105.658357 \text{ MeV} \), \( gammu = 0 \text{ MeV} \), for a muon,

\( mtau=1.77703 \text{ GeV} \), \( gamtau = 0 \text{ GeV} \), for a lepton tau,

\( mu = 5 \text{ MeV} \), \( gamu = 0 \text{ MeV} \), for an up quark,

\( md=9 \text{ MeV} \), \( gamd = 0 \text{ MeV} \), for a down quark,

\( mc=1.3 \text{ GeV} \), \( gamc = 0 \text{ GeV} \), for a charm quark,

\( ms=150 \text{ MeV} \), \( gams = 0 \text{ MeV} \), for a strange quark,

\( mt=174.3 \text{ GeV} \), \( gamt = 1.5 \text{ GeV} \), for a top quark,

\( mb=4.4 \text{ GeV} \), \( gamb = 0 \text{ GeV} \), for a bottom quark.

\( ncol=3 \), the number of colours,

\( convc=0.389379292\times10^{12} \text{ fb GeV}^{-2} \), a conversion constant.

### 3.1.2 Main options

The following main options should be selected in the main program `csee6f.f`.

The number of different centre of mass (CMS) energies \( ne \)

\( ne = 1 \). **Recommended if the unweighted event are to be generated.**

The actual values of the CMS energies in the array `aecm` of size \( ne \):

\[ aecm=(/500.d0/). \]

The final state of \((\ddagger)\) by selecting a decay mode of the intermediate \( W^+, W^- \) bosons, \( iwp, iwm = 1 \) or 2, where 1 corresponds to a leptonic, and 2 to a hadronic decay mode, and by specifying the family index \( ifp, ifm = 1,2,3 \) of the fermion pair resulting from the \( W^+ \) or \( W^- \) decay. For example, the reaction \( e^+e^- \rightarrow b\nu_\mu\bar{\nu}_{\bar{\mu}}d\bar{u} \) corresponds to \( W^+ \) decaying leptonically and \( W^- \) decaying hadronically and the corresponding flags are:

\( iwp = 1 \)

\( ifp = 2 \)

\( iwm = 2 \)
ifm = 1. 
Whether or not to calculate the Born cross section utilizing the full 6 particle kinematics, 
iborn = 1 (yes) / else (no), with ncall0 calls to the integrand

iborn = 1
ncall0 = 20000. Recommended No of calls is a few millions.

Generate the unweighted events or not, imc = 1(yes)/else(no)?
imc = 0.
No standard event record is used. If imc = 1, then the final state particle momenta of the
accepted unweighted events are printed in the output.

Scan the Born cross section with nscan0 calls, iscan0 = 1(yes)/else(no), in order to find
the dominant kinematical channels, adjust integration weights and find out the maximum
value of the cross section 
iscan0 = 1. This option is strongly recommended.
nscan0 = 200. About 1 thousand is recommended.

Calculate an approximate cross section of (3), iwwa = 1 (yes) / else (no)?

iwwa = 1
ncallww = 20000. Recommended value is several hundred thousands.

Scan the Born cross section in the narrow W-width approximation with nscanww calls, iscanww
= 1(yes)/else(no)?
iscanww = 1. Recommended.
nscanww = 200. Recommended value is about 1 thousand.

Calculate an approximate cross section of (4), itopa = 1(yes)/else(no)?

itopa = 1
ncalla = 20000.
Include the Higgs boson exchange, ihiggs = 1(yes)/else(no)?

ihiggs = 1
Calculate the electroweak contributions only, iew = 1(yes)/else(electroweak + QCD
Feynman diagrams)?
i = 0.

Calculate the $t\bar{t}$ resonant diagrams only, itt = 1(yes)/else(all the diagrams)?
itt = 0.

Calculate the double W resonant diagrams only, iww = 1(yes)/else(all the diagrams)?
iww = 0.

Choose the scheme: ischeme = 1(complex mass scheme)/else(fixed width scheme)
ischeme = 1.
If ischeme=1, then should $\alpha_W$ be complex (iaplw=1) or real (ialpw=0)?
ialpw=1.
3.1.3 Auxiliary options

Impose cuts, icuts = 1(yes)/else(no)?

icuts = 0

If icuts = 1, then specify the kinematical cuts, e.g.:

c_{\text{lb}} = \cos(10\pi/180) – cosine of the charged lepton–beam angle,

c_{\text{qb}} = \cos(5\pi/180) – cosine of the quark–beam angle,

c_{\ell\ell} = 1 – cosine of the charged lepton–charged lepton angle,

c_{\ell q} = \cos(5\pi/180) – cosine of the charged lepton–quark angle,

e_{\text{cutl}} = 1 \text{ GeV} – minimum charged lepton energy,

e_{\text{cutq}} = 3 \text{ GeV} – minimum quark energy,

m_{qq} = 10 \text{ GeV} – minimum invariant mass of a quark pair,

m_{ll} = 0 \text{ GeV} – minimum invariant mass of a charged lepton pair,

Calculate distributions, idist = 1(yes)/0(no)?

idis=0.

If idis = 1, then specify parameters of the distributions:

\text{xmin} = (/ x_1, x_2, x_3, x_4/) – lower bounds,

\text{xmax} = (/ x_1, x_2, x_3, x_4/) – upper bounds,

\text{nbs} = (/ n_1, n_2, n_3, n_4/) – the corresponding numbers of bins in each distribution.

Constants \text{xli}, \text{xui} should be of type real(8) and ni of type integer. The number of desired distributions and the maximum number of bins, \text{nbmax} = \max\{n_1, n_2, n_3, n_4\} should be specified in module \text{distribs.f}.

The maximum of the fully differential cross section \text{crmax}, relevant only if iscan0 = 0 or iscanww = 0,

crmax=1000.

3.2 Routines of eett6f

The main program csee6f.f, each subroutine, function or module are located in a file named the same way as the routine itself, except for 3 functions: srr, src and scc located in a single file dotprod.f. The program consists of the following routines.

- **SUBROUTINE boost** – returns a four vector boosted to the Lorentz frame of velocity \( -\mathbf{v} \).
- **SUBROUTINE carlos** – the MC integration routine.
- **SUBROUTINE couplsmb** – returns the SM couplings.
• FUNCTION cross – calculates the cross section of \((1)\), approximated cross section of \((2 \text{–} 5)\) and the cross section of \((2)\) in the narrow \(W\)-width approximation.

• FUNCTION crosstopa – calculates the cross section of \((4)\) in the narrow top width approximation.

• The MAIN PROGRAM csee6f.

• MODULE distribs – contains parameters of distributions.

• MODULE drivec – contains driving flags and some kinematical variables.

• SUBROUTINE eebwbw – returns the squared matrix element averaged over initial spins and summed over final spins and colours of \(e^+e^- \to bW^+\bar{b}W^-\).

• SUBROUTINE eee – returns a contraction of a triple gauge boson coupling with three complex four vectors.

• SUBROUTINE eeee – returns a contraction of a quartic gauge boson coupling with four complex four vectors.

• SUBROUTINE eeff1 – returns the squared matrix element averaged over initial spins and summed over final spins and colours of \(e^+e^- \to f\bar{f}\).

• SUBROUTINE eeh – returns a contraction of the Higgs-gauge boson coupling with two complex four vectors.

• SUBROUTINE eett6f – returns the squared matrix element averaged over initial spins and summed over final spins and colours of reaction \((1)\) and approximations \((2 \text{–} 5)\).

• SUBROUTINES eev, eve and vee – return contractions of a triple gauge boson coupling with two polarization vectors leaving, respectively, the third, second and first Lorentz index uncontracted.

• SUBROUTINE fhfb – returns a set complex scalars: \(scal(\lambda_1, \lambda_2) = g_{S12}\bar{u}_1u_2\Delta_F\), where \(u_1, u_2\) are fermion spinors, \(\Delta_F\) is a scalar boson propagator and \(g_{S12}\) is a coupling of the scalar boson to fermion 1 and 2.

• SUBROUTINE fvfa – returns a set of four vectors \(\varepsilon^\mu(\lambda_1, \lambda_2) = \bar{u}_1(\lambda_1)\gamma_\nu(g_\nu^-(g_\nu^+(P_+ + g_\nu^+P_+)) \times u_2(\lambda_2)D_\nu^\mu\), where \(u_1, u_2\) are fermion spinors, \(P_\pm = (1 \pm \gamma_5)/2\), are chirality projectors, \(g_\nu^-(\pm)\) are the fermion-gauge boson couplings of definite chirality and \(D_\nu^\mu\) is the propagator of a gauge boson \(V\) in arbitrary linear gauge.

• SUBROUTINE fef – returns a set of matrix elements \(mat(\lambda_1, \lambda, \lambda_2) = \bar{u}_1(\lambda_1)\zeta(\lambda)(g_\nu^-(P_+ + g_\nu^+P_+)u_2(\lambda_2), where notation is the same as in \(fvfa)\).

• SUBROUTINE impcuts – imposes cuts on the kinematical variables of process \((1)\).

• MODULE inprms – contains initial input parameters.

• MODULE kincuts – contains kinematical cuts.

• SUBROUTINE kinee1212 – returns the four momenta, phase space normalization and flux factor for a \(2 \to 6\) process in the CMS. The phase space parametrization used is that of Eq. \((9)\).
• **SUBROUTINE kinee6f** – returns the four momenta, phase space normalization and flux factor for a $2 \to 6$ process in the CMS. The phase space parametrization used is that of Eq. (8).

• **SUBROUTINE kineeff1** – returns the four momenta, phase space normalization and flux factor for a $2 \to 2$ process in the CMS. The phase space is parametrized according to Eq. (12).

• **SUBROUTINE kineefff** – returns the four momenta, phase space normalization and flux factor for a $2 \to 4$ process in the CMS. The phase space is parametrized according to Eq. (11).

• **SUBROUTINE kineetbff** – returns the four momenta, phase space normalization and flux factor for a $2 \to 4$ process in the CMS. The phase space is parametrized according to Eq. (10).

• **SUBROUTINE kineett6f** – returns the four momenta, phase space normalization and flux factor for a $2 \to 6$ process in the CMS. The phase space parametrization used is that of Eq. (7).

• **SUBROUTINE kinff** – returns the final state four momenta and phase space normalization for a $2 \to 2$ process in the CMS.

• **FUNCTION lamsq** – the kinematic lambda function, $\lambda(\sqrt{x}, \sqrt{y}, \sqrt{z})$.

• **MODULE mathprms** – contains necessary arithmetical constants.

• **SUBROUTINE parfix** – returns parameters of a specific process (11) and the branching ratios for the narrow width approximations; initializes weights for the Monte Carlo integration.

• **MODULE parproc** – contains parameters for a specific process.

• **SUBROUTINE peu** – returns a set of generalized spinors: $u(\lambda, \lambda_1) = S_F \tilde{f}(\lambda)(g_V^{(-)} P_- + g_V^{(+)}) P_+) u_1(\lambda_1)$, where $S_F$ is a Feynman propagator of an internal fermion and the remaining notation is the same as in `fvfa`.

• **SUBROUTINE psnee1212** – returns a phase space normalization as the one of kinee1212 for a given set of external particle momenta.

• **SUBROUTINE psnee6f** – returns a phase space normalization as the one of kinee6f for a given set of external particle momenta.

• **SUBROUTINE psneeffff** – returns a phase space normalization as the one of kineeffff for a given set of external particle momenta.

• **SUBROUTINE psneetbff** – returns a phase space normalization as the one of kineetbff for a given set of external particle momenta.

• **SUBROUTINE psneett6f** – returns a phase space normalization as the one kineett6f for a given set of external particle momenta.

• **SUBROUTINE recpol** – returns real polarization vectors of a vector boson in the rectangular basis.
• FUNCTION scc (contained in dotprod.f) – returns the Minkowski dot product of two complex four vectors.

• SUBROUTINE spheric – returns spherical components $p_s$ of a four vector $p^\mu$, $p_s = (|p|, \cos \theta, \sin \theta, \cos \phi, \sin \phi)$, with $\theta$ and $\phi$ being a polar and azimuthal angles of momentum $p$.

• SUBROUTINE spinc – returns the contractions: $p^0 I - p \cdot \sigma$ and $p^0 I + p \cdot \sigma$, where $p^\mu = (p^0, p)$ is a complex four vector, $I$ is the $2 \times 2$ unit matrix and $\sigma$ are the Pauli matrices.

• SUBROUTINE spinornew – returns helicity spinors in the Weyl representation.

• SUBROUTINE spinr – returns the same contractions as in spinc for a real four vector $p^\mu$.

• FUNCTION src (contained in dotprod.f) – returns the Minkowski dot product of a real and a complex four vector.

• FUNCTION srr (contained in dotprod.f) – returns the Minkowski dot product of two real four vectors.

• SUBROUTINE uep – returns a set of generalized spinors: $u(\lambda_1, \lambda) = u_1(\lambda_1) \tilde{\varphi}(\lambda)(g_0^{(-)} P_+ + g_1^{(+)} P_-) S_F$, where notation is the same as in peu.

• SUBROUTINE wwidth – returns the partial width of the $W$-boson averaged over initial spins and summed over final spins, and colours for hadronic decay modes.

### 3.3 Run output

A sample of the listing of the test run output is given in Appendix A. It contains a specification of the process, information on the scheme choice, and values of the relevant physical parameters in the very beginning. Then, for each value of the CMS energy and for each cross section calculated, results of the initial scan and resulting weights for the actual run are printed, if $iscan = 1$. Sometimes the weights calculated according to Eq. (14) do not add up exactly to 1. Then, the last weight is changed slightly in order the second condition of Eq. (6) to be fulfilled. This change is supposed to be completely irrelevant numerically, but the message about it is printed. The final result for the total cross section is called Integral. Its value is printed together with the standard deviation and the actual number of calls used in the calculation. Finally, information on events acceptance efficiency is given, that means a fraction of accepted weight 1 events. If the event generation option is switched on, i.e. $imc = 1$, then the corresponding number of unwighted events will be printed out as collections of final state particle four momenta.

Whenever a maximum value of the cross section initially assigned in csee6f.f or found in the result of the initial scan is overflown by more than a factor 1.5, a corresponding message informing about it is printed. The integration is still valid, however, if the program is run as an event generator, it should be rerun. How to proceed in this situation is described in the next section. In the very end, all the calculated total cross sections together with the corresponding standard deviations are collected in the tabular form.
4 Use of the program

Up to now the program have been run only on Unix/Linux platforms. In order to run the program, the user should select a specific name of a FORTRAN 90/95 compiler, desired options and the name of the output file in the `makefile`. The output file is called `test` at present. The program can be then compiled, linked and run by executing a single command

```
make test
```

The results of the test run should reproduce those contained in file `testrun`.

The program can be run as the MC event generator of unweighted events by selecting

```
imc = 1
```

in `csee6f.f`. It is then recommended to run the program for a single CMS energy and to perform a scan with a relatively large number of calls, `nscan0` or `nscanw`. Attention should be paid to possible messages informing about updates of the maximum weight. In this case the program should be rerun, however, this time without the initial scan. The initial integration weights `aw0` or `awa` in `parfix.f` and the maximum weight `crmax` in `csee6f.f` should be updated according to the results of the prior scan. At present the efficiency of events acceptance is relatively low. However, in view of a relatively fast performance of the program, this should not be a serious limitation for the user.

5 Outlook

The following improvements of the program are envisaged in the near future. First of all, a new version is being prepared which will allow for calculations of reactions involving the same external fermion flavours. Another step will be an inclusion of the most relevant factorizable higher order effects: electroweak and QCD corrections to the top pair production, $e^+e^- \rightarrow t\bar{t}$, to the $t$ and $\bar{t}$ quark decays, as well as to the $W^+$ and $W^-$ decays.

References

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[2] T. Abe et al., American Linear Collider Working Group Collaboration, SLAC-R-570, Resource book for Snowmass 2001. K. Abe et al., hep-ph/0109166.

[3] K. Kołodziej, Eur. Phys. J. C23 (2002) 471, hep-ph/0110063.

[4] E. Accomando, A. Ballestrero, M. Pizzio, Nucl.Phys. B512 (1998) 19.
Appendix

A Listing of test run output

Approaching the process:
e+ e- -> b vm mu+ b d u

with the program ‘eett6f v. 1.0’ by

Karol Kolodziej
University of Silesia
Katowice, Poland
e-mail: kolodzie@us.edu.pl

Complex mass scheme
with sw2 = ( 0.222271237327414,-0.000778147496385)
and complex 1/alpha_w = (132.386398893114400, 3.953796222427256)

Gauge boson masses and widths: W, Gm_W, Z, Gm_Z
80.41900 2.12000 91.18820 2.49520

Higgs boson mass and width: H, Gm_H
0.11500E+03 0.49657E-02

1/alpha0 = 137.0359998 alphas = 0.1185000 G
mu = 0.116639E-04

t-quark mass and width:
174.30 1.50000

External fermion masses: e- b vm mu+ d u :
\[ E_{\text{cm}} = 500.000 \text{ GeV} \]

All cross sections in femtobarns (fb)

The Born cross section with full 6 particle kinematics:

| Kinematics | Weight | Integral | St. dev. | No of calls |
|------------|--------|----------|----------|-------------|
| 1          | 0.17241379E-01 | 0.77943026E+01 | 0.53027331E+00 | 200         |
| 2          | 0.17241379E-01 | 0.70433588E+00 | 0.14557680E+00 | 200         |
| 3          | 0.17241379E-01 | 0.46787757E+00 | 0.14942639E+00 | 200         |
| 4          | 0.17241379E-01 | 0.40042444E-01 | 0.17572985E-01 | 200         |
| 5          | 0.17241379E-01 | 0.46787757E+00 | 0.14942639E+00 | 200         |
| 6          | 0.17241379E-01 | 0.70179588E-02 | 0.50890185E-02 | 200         |
| 7          | 0.17241379E-01 | 0.18918195E-04 | 0.14788334E-04 | 200         |
| 8          | 0.17241379E-01 | 0.59135660E-02 | 0.26123591E-02 | 200         |
| 9          | 0.17241379E-01 | 0.65946999E-05 | 0.37561359E-05 | 200         |
| 10         | 0.17241379E-01 | 0.71059323E-02 | 0.32477486E-02 | 200         |
| 11         | 0.17241379E-01 | 0.12331601E+00 | 0.52550633E-01 | 200         |
| 12         | 0.17241379E-01 | 0.50746197E-01 | 0.85455398E-02 | 200         |
| 13         | 0.17241379E-01 | 0.52341207E-01 | 0.13094625E-01 | 200         |
| 14         | 0.17241379E-01 | 0.16697241E-02 | 0.91218523E-03 | 200         |
| 15         | 0.17241379E-01 | 0.21727387E-02 | 0.96316906E-03 | 200         |
| 16         | 0.17241379E-01 | 0.10069801E+00 | 0.44469111E-01 | 200         |
| 17         | 0.17241379E-01 | 0.55418681E-01 | 0.10461875E-01 | 200         |
| 18         | 0.17241379E-01 | 0.47690194E-03 | 0.25400283E-03 | 200         |
| 19         | 0.17241379E-01 | 0.12270501E-05 | 0.68571740E-06 | 200         |
| 20         | 0.17241379E-01 | 0.57639038E-02 | 0.20367189E-02 | 200         |
| 21         | 0.17241379E-01 | 0.10948697E-03 | 0.78871872E-04 | 200         |
| 22         | 0.17241379E-01 | 0.38725698E-03 | 0.32739976E-03 | 200         |
| 23         | 0.17241379E-01 | 0.31913662E-03 | 0.31771241E-03 | 200         |
| 24         | 0.17241379E-01 | 0.18057468E-04 | 0.15825867E-04 | 200         |
| 25         | 0.17241379E-01 | 0.75726379E-02 | 0.10358362E-02 | 200         |
| 26         | 0.17241379E-01 | 0.40524491E-01 | 0.40417673E-01 | 200         |
| 27         | 0.17241379E-01 | 0.48346637E-03 | 0.25434560E-03 | 200         |
| 28         | 0.17241379E-01 | 0.17689713E-05 | 0.10354945E-05 | 200         |
| 29         | 0.17241379E-01 | 0.24943550E-03 | 0.85703152E-04 | 200         |
| 30         | 0.17241379E-01 | 0.68130329E+01 | 0.55624830E+00 | 200         |
| 31         | 0.17241379E-01 | 0.80133941E+00 | 0.14035567E+00 | 200         |
| 32         | 0.17241379E-01 | 0.37030192E+00 | 0.13116529E+00 | 200         |
| 33         | 0.17241379E-01 | 0.27961633E-01 | 0.79658869E-02 | 200         |
| 34         | 0.17241379E-01 | 0.12381885E-02 | 0.65639060E-03 | 200         |
| 35         | 0.17241379E-01 | 0.38608372E-05 | 0.23584221E-05 | 200         |
| 36         | 0.17241379E-01 | 0.22584220E-03 | 0.16055018E-03 | 200         |
| 37         | 0.17241379E-01 | 0.10262033E-05 | 0.42675278E-06 | 200         |
38 0.17241379E-01 0.59892263E-04 0.17034981E-04 200
39 0.17241379E-01 0.11805922E-04 0.96111792E-05 200
40 0.17241379E-01 0.37482281E-02 0.13933930E-02 200
41 0.17241379E-01 0.69961565E-02 0.29473064E-02 200
42 0.17241379E-01 0.26317460E-03 0.96401737E-04 200
43 0.17241379E-01 0.14981430E-04 0.14211747E-04 200
44 0.17241379E-01 0.36580810E-02 0.31164792E-02 200
45 0.17241379E-01 0.65694693E-05 0.59639961E-05 200
46 0.17241379E-01 0.59283100E-01 0.19915949E-01 200
47 0.17241379E-01 0.34966532E-02 0.32846811E-02 200
48 0.17241379E-01 0.39467871E-01 0.24414577E-01 200
49 0.17241379E-01 0.33496371E-02 0.76289358E-03 200
50 0.17241379E-01 0.65454536E+00 0.11507542E+00 200
51 0.17241379E-01 0.11775709E-01 0.58766647E-02 200
52 0.17241379E-01 0.12389848E-01 0.28548786E-02 200
53 0.17241379E-01 0.19047822E-01 0.13103839E-01 200
54 0.17241379E-01 0.29502206E-01 0.16858165E-01 200
55 0.17241379E-01 0.62510122E+00 0.13584265E+00 200
56 0.17241379E-01 0.82405870E-02 0.22527034E-02 200
57 0.17241379E-01 0.55814360E-02 0.23231953E-02 200
58 0.17241379E-01 0.78977140E-02 0.36720050E-02 200

------------------------------------------------------------------------------

Born scanned with 11565 calls = 0.19039197E+02
The maximum value of the cross section= 0.27872147E+04
shifted to 0.41808220E+04

------------------------------------------------------------------------------

Weight 58 = 0.4148134056762879E-03 rescaled to 0.4148134056756181E-03

Calculating the Born cross section with the weights:
1 0.40938191E+00 0.40938191E+00
2 0.36993992E-01 0.44637590E+00
3 0.24574439E-01 0.47095034E+00
4 0.21031583E-02 0.47305350E+00
5 0.32439333E-02 0.47629743E+00
6 0.36860583E-03 0.47666604E+00
7 0.99364459E-06 0.47697763E+00
8 0.31059956E-03 0.47697763E+00
9 0.34637490E-06 0.47697798E+00
10 0.37322648E-03 0.47735120E+00
11 0.64769547E-02 0.48382816E+00
12 0.26653539E-02 0.48649351E+00
13 0.27491290E-02 0.48924264E+00
14 0.87699297E-04 0.48933034E+00
15 0.1141925E-03 0.48944446E+00
16 0.52898841E-02 0.49473344E+00
17 0.29107678E-02 0.49764421E+00
18 0.25048428E-04 0.49766926E+00
19 0.64448631E-07 0.49766932E+00
20 0.30273882E-03 0.49797206E+00
21 0.57506089E-05 0.49797781E+00
| Kinematics Weight | Integral | St.dev. | No of calls |
|-------------------|----------|---------|-------------|
| 22 0.20339986E-04 0.4979815E+00 |          |         |             |
| 23 0.16762084E-04 0.49801491E+00 |          |         |             |
| 24 0.94843646E-06 0.49801586E+00 |          |         |             |
| 25 0.39773936E-03 0.49841360E+00 |          |         |             |
| 26 0.21284769E-02 0.50054208E+00 |          |         |             |
| 27 0.25393213E-04 0.50056747E+00 |          |         |             |
| 28 0.92912076E-07 0.50056757E+00 |          |         |             |
| 29 0.13101157E-04 0.50058067E+00 |          |         |             |
| 30 0.35784246E+00 0.85842313E+00 |          |         |             |
| 31 0.42089308E-01 0.90051206E+00 |          |         |             |
| 32 0.19449451E-01 0.91996151E+00 |          |         |             |
| 33 0.14686352E-02 0.92143014E+00 |          |         |             |
| 34 0.65033652E-04 0.92149518E+00 |          |         |             |
| 35 0.20278362E-06 0.92149538E+00 |          |         |             |
| 36 0.11861960E-04 0.92150724E+00 |          |         |             |
| 37 0.53899508E-07 0.92150729E+00 |          |         |             |
| 38 0.31457348E-05 0.92151044E+00 |          |         |             |
| 39 0.62008511E-06 0.92151106E+00 |          |         |             |
| 40 0.19686903E-03 0.92170793E+00 |          |         |             |
| 41 0.36746070E-03 0.92207539E+00 |          |         |             |
| 42 0.13822779E-04 0.92208921E+00 |          |         |             |
| 43 0.78687301E-06 0.92209000E+00 |          |         |             |
| 44 0.19213421E-03 0.92228213E+00 |          |         |             |
| 45 0.34504971E-06 0.92228248E+00 |          |         |             |
| 46 0.31137396E-02 0.92539622E+00 |          |         |             |
| 47 0.18365550E-03 0.92557987E+00 |          |         |             |
| 48 0.20729799E-02 0.92765285E+00 |          |         |             |
| 49 0.17593374E-03 0.92782879E+00 |          |         |             |
| 50 0.34378833E-01 0.96220762E+00 |          |         |             |
| 51 0.61849819E-03 0.96282612E+00 |          |         |             |
| 52 0.65075477E-03 0.96347687E+00 |          |         |             |
| 53 0.10004530E-02 0.96447733E+00 |          |         |             |
| 54 0.15495510E-02 0.96602688E+00 |          |         |             |
| 55 0.32832332E-01 0.99885921E+00 |          |         |             |
| 56 0.43282221E-03 0.99929203E+00 |          |         |             |
| 57 0.29315502E-03 0.99958519E+00 |          |         |             |
| 58 0.41481343E-03 0.10000000E+01 |          |         |             |
| 59 0.00000000E+00 0.00000000E+00 |          |         |             |
| 60 0.00000000E+00 0.00000000E+00 |          |         |             |

Final result:
Integral = 0.201003E+02 Standard dev. = 0.27E+00 with 19980 calls

Efficiency of the events acceptance = 0.410E-02

The narrow W-width approximation, e+ e- -> b W+ b W- -> 6f:

Results of the initial scan:
Kinematics Weight Integral St.dev. No of calls
|   |   |   |   |   |
|---|---|---|---|---|
| 1 | 0.47619048E-01 | 0.23618560E+00 | 0.10375292E+00 | 200 |
| 2 | 0.47619048E-01 | 0.25574899E+00 | 0.11393103E+00 | 200 |
| 3 | 0.47619048E-01 | 0.90318927E-02 | 0.27636575E-02 | 200 |
| 4 | 0.47619048E-01 | 0.14178256E+00 | 0.63376575E-01 | 200 |
| 5 | 0.47619048E-01 | 0.29188135E-02 | 0.15778093E-02 | 200 |
| 6 | 0.47619048E-01 | 0.75109770E-02 | 0.28446236E-02 | 200 |
| 7 | 0.47619048E-01 | 0.87010486E-01 | 0.36285359E-01 | 200 |
| 8 | 0.47619048E-01 | 0.2008583E-01 | 0.10505322E-01 | 200 |
| 9 | 0.47619048E-01 | 0.25432033E+00 | 0.12331680E+00 | 200 |
|10 | 0.47619048E-01 | 0.14796519E+00 | 0.51432260E-01 | 200 |
|11 | 0.47619048E-01 | 0.14443649E+00 | 0.47481858E-01 | 200 |
|12 | 0.47619048E-01 | 0.13431004E+00 | 0.5781777E-01 | 200 |
|13 | 0.47619048E-01 | 0.62963498E-01 | 0.18297859E-01 | 200 |
|14 | 0.47619048E-01 | 0.14057286E+00 | 0.83384476E-01 | 200 |
|15 | 0.47619048E-01 | 0.16790199E+02 | 0.67749184E+00 | 200 |
|16 | 0.47619048E-01 | 0.61926860E-01 | 0.12581699E-01 | 200 |
|17 | 0.47619048E-01 | 0.1272293E+00 | 0.37044922E-01 | 200 |
|18 | 0.47619048E-01 | 0.15741819E+00 | 0.6466409E-01 | 200 |
|19 | 0.47619048E-01 | 0.1639987E+00 | 0.54156119E-01 | 200 |
|20 | 0.47619048E-01 | 0.21743058E+00 | 0.13237257E+00 | 200 |
|21 | 0.47619048E-01 | 0.13765610E+00 | 0.84254619E-01 | 200 |

Born scanned with 4200 calls = 0.19300621E+02
The maximum value of the cross section= 0.98788719E+03
shifted to 0.14818308E+04

Weight 21 = 0.7132210911535855E-02 rescaled to 0.7132210911536152E-02

Calculating the Born cross section with the weights:

|   |   |   |   |   |
|---|---|---|---|---|
| 1 | 0.12237203E-01 | 0.12237203E-01 |   |   |
| 2 | 0.13250817E-01 | 0.25488019E-01 |   |   |
| 3 | 0.46795867E-03 | 0.25955978E-01 |   |   |
| 4 | 0.73460104E-02 | 0.33301989E-01 |   |   |
| 5 | 0.15122900E-03 | 0.33453218E-01 |   |   |
| 6 | 0.38915728E-03 | 0.33842375E-01 |   |   |
| 7 | 0.45081704E-02 | 0.38350545E-01 |   |   |
| 8 | 0.10366808E-02 | 0.39387226E-01 |   |   |
| 9 | 0.13176795E-01 | 0.52564021E-01 |   |   |
|10 | 0.76663435E-02 | 0.60230365E-01 |   |   |
|11 | 0.74835153E-02 | 0.67713880E-01 |   |   |
|12 | 0.69588457E-02 | 0.74672726E-01 |   |   |
|13 | 0.32622525E-02 | 0.77934978E-01 |   |   |
|14 | 0.72833337E-02 | 0.85218312E-01 |   |   |
|15 | 0.86993051E+00 | 0.95514882E+00 |   |   |
|16 | 0.32085424E-02 | 0.95835737E+00 |   |   |
|17 | 0.65917015E-02 | 0.96494907E+00 |   |   |
|18 | 0.81561205E-02 | 0.97310519E+00 |   |   |
|19 | 0.84971292E-02 | 0.98160232E+00 |   |   |
|20 | 0.11265471E-01 | 0.99286779E+00 |   |   |
|21 | 0.71322109E-02 | 0.10000000E+01 |   |   |
The maximum weight updated from 0.14818308E+04 to 0.16809386E+04
The maximum weight updated from 0.16809386E+04 to 0.17190927E+04

Final result:
Integral = 0.206457E+02 Standard dev. = 0.18E+00 with 20000 calls

Efficiency of the events acceptance = 0.129E-01

Approximated cross section e+ e- \rightarrow t \bar{t} \rightarrow 6f:

Final result:
Integral = 0.193058E+02 Standard dev. = 0.66E-01 with 20000 calls

Born cross sections (fb):
ecm cs sd csta sda csbbww sdbbww
0.5000E+03 0.20100E+02 0.27E+00 0.19306E+02 0.66E-01 0.20646E+02 0.18E+00