High Dimensional Time Series Generators

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Abstract

Multidimensional time series are sequences of real valued vectors. They occur in different areas, for example handwritten characters, GPS tracking, and gestures of modern virtual reality motion controllers. Within these areas, a common task is to search for similar time series. Dynamic Time Warping (DTW) is a common distance function to compare two time series. The Edit Distance with Real Penalty (ERP) and the Dog Keeper Distance (DK) are two more distance functions on time series. Their behaviour has been analyzed on 1-dimensional time series. However, it is not easy to evaluate their behaviour in relation to growing dimensionality. For this reason we propose two new data synthesizers generating multidimensional time series. The first synthesizer extends the well known cylinder-bell-funnel (CBF) dataset to multidimensional time series. Here, each time series has an arbitrary type (cylinder, bell, or funnel) in each dimension, thus for \(d\)-dimensional time series there are \(3^d\) different classes. The second synthesizer (\textsc{RAM}) creates time series with ideas adapted from Brownian motions which is a common model of movement in physics. Finally, we evaluate the applicability of a 1-nearest neighbor classifier using DTW on datasets generated by our synthesizers.

1 Introduction

Multimedia retrieval is a common application which requires finding similar time series to a given query. This includes gesture recognition with modern virtual reality motion controllers, GPS tracking, speech recognition, and classification of handwritten letters. In these areas, time series of the same classes (e.g., same written character or same gestures) follow the same path in space, but have some temporal displacements. Tracking the GPS coordinates of two cars driving the same route from A to B is another example. We want these tracks (i.e. the trajectories of the time series) to be recognized as similar, although driving style, traffic lights, and traffic jams might result in temporal differences. Distance functions such as dynamic time warping (DTW) \cite{hubert1985}, edit distance with real penalties
(ERP) [6], and the dog-keeper distance (DK) [9] respect this semantic requirement. These time warping distance functions map pairs of time series representing similar trajectories to small distances and dissimilar time series to large distances. They basically follow the same idea by finding good alignments of the elements between the two elements (cf. Figure 1 for examples).

Of course, we are interested in fast algorithms for these time series distance functions. Unfortunately, time warping distance functions usually have quadratic runtime [4, 5]. There are some evaluations of different time warping distance functions on different data sets [8]. However, their performance has not been evaluated in relation to growing dimensionality.

**Computation time of distance function:** In order to evaluate the computation time of a distance function in relation to growing dimensionality, we need datasets with similar properties (e.g. size of dataset, length of time series, data distribution) but different dimensionality. Existing datasets of different dimensionality exist, but have different properties. Thus, datasets with similar properties but different dimensionalities are necessary to evaluate relations between the computation time and the dimensionality. To achieve similar properties on datasets with different dimensionality, we propose the usage of synthesized data.

**Tightness of lower bounds:** Nearest neighbor queries are also accelerated by pruning distance computations using cheap lower bounds [10, 15]. If a lower bound claims a large distance then there is no need to compute the exact but expensive distance value. The lower bound proposed by Keogh was extended to multidimensional time series in an unpublished paper [12] but there is no evaluation available regarding growing dimensionality. Again, having datasets with similar properties for different dimensionality is necessary for these evaluations. Those datasets could also be used to evaluate pruning strategies of index structures specialized to DTW [13] or metric index structures [2, 3, 7, 14] applicable for metric time series distances (e.g. DK [1] and ERP [6]).

**Accuracy of classifiers:** Nearest neighbor queries appear in supervised machine learning, for example 1-nearest neighbor classifiers. In order to compare properties of two classifiers using two distinct distance functions, labeled datasets are necessary. Since the comparison is meaningless when both classifier are perfectly accurate, we need to control the difficulty of the classification tasks. Thus, when designing dataset generators for the purpose of comparing the strength of two classifiers, we need to implement a parameter to control the difficulty.

**Time distortion of trajectories:** All of our proposed evaluation tasks consider time warping distance functions which yields the demand for time distorting dataset generators. Otherwise, evaluations would prefer implementations of distance functions which have an advantage on comparing perfectly aligned time series. The results of those evaluations could not be transferred to datasets with time distorted time series.

**Contribution:** To make such evaluation scenarios feasible, we propose two dataset generators for multidimensional, labeled, and time distorted time series. Both generators provide a
Figure 1: Examples of aligning two time series using DTW (left), DK (center), and ERP (right). Distances between states are marked with solid lines while the circled and squared time series are connected using dashed lines. The green blank circle is the gap element for ERP.

Figure 2: Examples for cylinder (left), bell (center), and funnel (right) time series resp..

2 Cylinder-Bell-Funnel

N. Saito proposed the well known 1-dimensional cylinder-bell-funnel dataset in his PhD thesis [16]. It is an artificial dataset consisting of three different classes of time series: cylinder, bell, and funnel.

For the time series synthesizer, let $\ell > 0$ be a fixed length of the time series and $\mathcal{N}$ be a standard normal distributed random variable. Furthermore, fixiate $a$ and $b$ uniformly distributed over $[\ell \cdot \frac{1}{8}, \ell \cdot \frac{2}{3}]$ and $[\ell \cdot \frac{6}{8}, \ell \cdot \frac{7}{8}]$, respectively and $\nu = 6 + \mathcal{N}$ per generated time series. Each time series has a prefix $\mathcal{P}$ of length $a$ and suffix $\mathcal{S}$ of length $\ell - b$ containing standard normal distributed random numbers ($\mathcal{N}, \ldots, \mathcal{N}$).

The middle parts of random cylinder ($\mathcal{C}$), bell ($\mathcal{B}$) and funnel ($\mathcal{F}$) time series are a plateau, a rising linear function, and a falling linear function, respectively. Their length is $b - a$, thus

$$\mathcal{C} := \mathcal{P} \times (\ldots, \nu + \mathcal{N}, \ldots) \times \mathcal{S} \quad (1)$$

$$\mathcal{B} := \mathcal{P} \times \left(\ldots, \nu \cdot \frac{i-a}{b-a} + \mathcal{N}, \ldots\right) \times \mathcal{S} \quad (2)$$

$$\mathcal{F} := \mathcal{P} \times \left(\ldots, \nu \cdot \frac{b-i}{b-a} + \mathcal{N}, \ldots\right) \times \mathcal{S} \quad (3)$$

where $\nu = 6 + \mathcal{N}$ is chosen once per time series (cf. Figure 2 for examples).

We canonically extend CBF to generate multidimensional time series by generating one of these types for each dimension with the same starting and ending positions $a$ and $b$ (cf. Figure 3 for a 2-dimensional example). Thus, the input parameters are the length of the time series and a vector $\mathbf{t}$ with values $c, b$, and $f$ claiming which type to synthesize per dimension. Given the dimensionality $n$, the multidimensional CBF generator produces a maximum number of $3^n$ different classes, which is the set of all combinations of $c, b$, and $f$. 

The rest of the paper is structured as follows. Section 2 presents the first dataset generator which is an extension of the well known cylinder-bell-funnel dataset [16]. Section 3 presents the RAM dataset generator which adapts ideas from Brownian motions. We evaluate the datasets using DTW in Section 4 in order to confirm that DTW yields an applicable 1-nearest neighbor classifier and that we can control its classification score.
Figure 3: An example for a 2-dimensional CBF time series (left). Its first dimension is a cylinder (center) and its second dimension is a bell (right).

3 Random Accelerated Motion Generator

The RAM generator produces classes by first generating base time series using impulse driven motions with random acceleration (cf. Section 3.1 for details). Representatives of the classes are generated by distorting the base time series in space and time.

3.1 Base Time Series Generator

The Brownian motion is a common model of random motion in physics, for example to model movement of molecules in gases. Basically, each next position of a molecule is obtained as a randomized position around the current position.

In order to achieve more curve like time series, we keep an impulse vector and add that to the current position to obtain the next position. In each step, we add a normal distributed random vector to the impulse vector. Hence we generate series by distorting the first derivative instead of the current position.

To model edges in the generated time series, we restrict the movement to a ball with a constant radius $R$. When the time series is about to leave the restricted area, we simply let it bounce off the sphere so the time series remains in the interior. Algorithm 1 provides the pseudocode for data generation and Figure 4 shows an example for a 2-dimensional time series. In the algorithm, uniformBall returns a uniform distributed vector from the interior of a union sphere and uniformSphere returns a uniform distributed vector on a union sphere.

Algorithm 1 Random Accelerated Motion Generator

1. Algorithm: rambase
2. Input: length $l$, dimensionality $n$, radius $r$
3. Output: time series $s$
4. let $s$ be an $n$-dimensional time series of length $l$
5. $v = (0, \cdots, 0)$
6. $\nu = normal(0, 1)$
7. $s_0 = uniformBall(r)$
8. for $i$ from 1 to $l - 1$
9. $v = v + uniformSphere(r)$
10. $s_i = s_{i-1} + v$
11. if $\|s_i\|_2 > r$
12. $// rescale p to stay within the ball with radius r$
13. $s_i = s_i \frac{r}{\|p\|_2}$
14. $v = reflect v on sphere at point p$
15. return $s$

3.2 Generating Representatives

We generate representatives of a class corresponding to a base time series by distorting the time series in space and time separately.

Distortion in Space Figure 5 shows examples from the character trajectories dataset [11].
Naturally, the representatives of a class do not match exactly. We try to imitate this property by adding noise to the first derivative of the time series. However, to prevent large divergence on long time series, we limit the maximum distance to the base time series by a distortion parameter.

Hence, to distort a given time series in space, we add a standard normal distributed random vector to the first derivative of each point analogously to the base time series generator. This also includes the first point of the time series by assuming that its predecessor is the null point \( \mathbf{0} = (0, \cdots, 0) \). Figure 6 shows two example time series and a copy for each with distorted derivatives.

Comparing Figure 5 and Figure 6 shows indeed that we could imitate the properties from the character trajectories dataset.

3.3 Dataset Generator

As mentioned in the beginning of this section, we use the base time series generator to generate the classes. The time distortion and space distortion
Algorithm 2 Time Distortion of a Time Series

1 Algorithm: timedistortion
2 Input: time series $s$ of length $L$
3 Output: time series $\tilde{s}$
4
5 // get arc length up to each point of the time series
6 $\ell_i = \sum_{j=1}^{i} \|s_j - s_{j-1}\|_2$
7 // get uniformly distributed values along the complete arc
8 $t = (0, \ell_{L-1})$
9 repeat $L-2$ times
10 $t = t \circ \text{uniform}([0, \ell_{L-1}])$
11 // interpolate between reparameterized points
12 $\tilde{s} = ()$
13 for $x$ in sort($t$)
14 // find correct index
15 $i = \min \{i \mid \ell_i \leq x < \ell_{i+1}\}$
16 // interpolation parameter
17 $u = \frac{x-\ell_i}{\ell_{i+1}-\ell_i}$
18 $\tilde{s} = \tilde{s} \circ ((1-u) \cdot s_i + u \cdot s_{i+1}) <++>$
19 return $\tilde{s}$

Figure 8: Two base time series and their distorted (blue) versions; length: 100; dimensionality: 2; radius: 75; distortion: 5 (left) and 25 (right).

algorithms generate the actual representatives of the classes.

Hence, to generate a dataset $D$ with $C$ classes each having $N$ representatives, first call rambase $C$ times to generate $T_{i,0}$ ($1 \leq i \leq C$). Then, for each $1 \leq i \leq C$, call timedistortion and spacedistortion $N$ times to generate $T_{i,j}$ ($1 \leq j \leq N$). The dataset consists of each $T_{i,j}$ for $1 \leq i \leq C$ and $1 \leq j \leq N$.

$C := \{T_{i,0} = \text{rambase}(L, n, r) \mid 1 \leq i \leq C\}$
$\mathcal{R}_i := \{T_{i,j} = \text{spacedistortion}(\text{timedistortion}(T_{i,0}), D)\}$

where $n$ is the desired dimensionality, $r$ is the radius of the bounding sphere, $L$ is the desired length of the time series, and $D$ the desired degree of distortion within each class. Figure 8 shows two examples of a base time series and their time and space distorted representatives.

4 Evaluation

As already emphasized in Section 1, we propose two dataset generators which produce datasets applicable for classification tasks. We claimed
that they provide a tuning parameter to control the difficulty classification tasks. In this section, we evaluate whether our proposed generators (from Section 2 and 3) satisfy our claims.

4.1 Cylinder-Bell-Funnel

The CBF dataset has no distortion parameter which makes it harder to evaluate the classification strength of time warping distance functions. However, we can influence the difficulty of the classification task by changing the number of representatives per class.

The heatmap in Figure 9 shows that for each dimensionality of the CBF dataset the classification score increases with growing class sizes. However, Figure 10 shows that there are a few cases where the classification score slightly decreases with growing class size. Hence, we can use the class size to roughly control the difficulty of the classification task with.

Figure 9 also shows that the classification strength decreases on higher dimensionality. We could observe that this behaviour does neither depend on the length of the time series nor on the number of classes.

We observe better classification scores using DTW than using the Euclidean distance. Hence, it appears that time warping distance functions (such as DTW) are necessary to achieve good classification scores on this dataset. Whether all time warping distance functions are applicable is out of scope.

4.2 Random Acceleration Motion

As mentioned in Section 3, RAM generates datasets of multidimensional labeled time series. Similar to the CBF datasets, Figure 11 shows that the classification score increases with growing class size. Also, this generator has a distortion parameter to control the noisiness of the time series. This parameter of the RAM synthesizer impacts the classification score as expected: The score decreases with increasing distortion.

Regarding the dimensionality, the RAM synthesizer seems to be complementary to the CBF synthesizer, since the classification scores increase with growing dimensionality (c.f. Figure 12).

Figure 11 furthermore shows that the DTW distance performs better than the Euclidean distance. Again, it appears time warping distance functions are better suited for solving classification tasks on these datasets.

5 Conclusion

We introduced two new dataset generators producing multidimensional labeled time series.
The datasets are applicable for classification tasks using time warping distance functions such as Dynamic Time Warping (DTW). Both generators provide parameters adjusting the difficulty of the classification task. Since the classification scores using DTW increase with growing dimensionality on the RAM datasets while decreasing on the CBF dataset, they seem to have some complementary properties. Thus, both synthesizers seem to be well suited for evaluating classifiers using time warping distance functions in relation to the dimensionality.

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Figure 11: Classification score using DTW (left) and Euclidean distance (right) for an example parameter set: radius 50, length 100, dimensionality: 3, number of classes 200.

Figure 12: Classification score using DTW (left) and Euclidean distance (right) for an example parameter set: radius 50, length 100, number of classes 200, distortion: 5.
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