New Glance at the Experimental Data for Low Lying Collective Excited States.

Vladimir P. Garistov
Institute for Nuclear Research and Nuclear Energy

December 8, 2018

Abstract

Recently the classification of low-lying excited $0^+$ states in even even deformed nuclei has been done. The available experimental data were represented as the energies parabolic distributed by number of monopole excitations. With other words each $0^+$ state now is determined as the collective state with the corresponding number of monopole type phonons $n$. In this short remark we discuss whether the experimental data for low-lying excited states possessing not equal to zero spins can also be described with parabolic distribution function depending on integer classification parameter and find any vindication of the connection between this integer parameter and the number of collective excitations building the corresponding state.

In our recent investigations of the yrast lines in even-even deformed nuclei we obtained that the energies of these lines can be described with great accuracy even if we use the simple rigid rotor model but if we consider yrast line be built with several number of crossing rigid rotor bands and if we make the bands heads be responsible for the behavior of the rotational bands. With other words we make the band head be responsible for the value of the moment of inertia of the nucleus staying in corresponding excited state.

We build the positive parity lines with crossing of several number of rotational $\beta$ - bands starting from different excited $0^+$ states that we consider as their heads. To understand the peculiarities of different excited $0^+$ states we analyzed a great amount of experimental data for low lying excited $0^+$ states in even-even nuclei.

We represent the available experimental data in the form of the energies of the $0^+$ excited states distributed by positive integer parameter and determine this classification parameter in the way giving us information about collective structure peculiarities of these states.

To specify the distribution function let us consider the monopole part of collective Hamiltonian for single level approach written in terms of boson creation and annihilation operators $R_+, R_-$ and $R_0$

$$H = \alpha R_+^0 R_+^0 + \beta R_+^0 R_0^0 + \frac{\beta \Omega^0}{2} R_0^0,$$

(1)
constructed with the pairs of fermion operators $a^\dagger$ and $a$.

$$R_j^+ = \frac{1}{2} \sum_m (-1)^{j-m} a_{jm}^\dagger a_{j-m}^\dagger,$$

$$R_j^- = \frac{1}{2} \sum_m (-1)^{j-m} a_{j-m} a_{jm},$$

$$R_j^0 = \frac{1}{4} \sum_m (a_{jm}^\dagger a_{j-m} - a_{j-m} a_{jm}^\dagger).$$

$$[R_{0}, R_{\pm}] = \pm R_{\pm}, \quad [R_{+}, R_{-}] = 2R_{0}$$

Further applying the Holstein-Primakoff transformation to the operators $R_+, R_-$ and $R_0$

$$R_+ = \sqrt{2\Omega - b^\dagger b} b; \quad R_- = b^\dagger \sqrt{2\Omega - b^\dagger b}; \quad R_0 = b^+ b - \Omega.$$

$$[b, b^\dagger] = 1, \quad [b, b] = [b^\dagger, b^\dagger] = 0.$$

the initial Hamiltonian (1) written in terms of pure bosons has the form:

$$H = Ab^\dagger b - Bb^\dagger b b^\dagger b.$$

$$A = \alpha(2\Omega + 1) - \beta\Omega, \quad B = \alpha - \beta.$$

$$|n\rangle = \frac{1}{\sqrt{n!}} (b^\dagger)^n |0\rangle, \text{where } b |0\rangle = 0$$

Thus the energy spectrum produced by Hamiltonian (2) is the parabolic function of the number of monopole bosons $n$

$$E_n = An - Bn^2 + C$$

This is the form we apply in our new representation of the experimental data of the low lying excited $0^+$ - states. Some of the distributions of the experimental energies of the excited $0^+$ states plotted

using (2) are shown in Figure 1.

This parabolic distribution (2) reproduces with a great accuracy experimental values of low lying $0^+$ excited states energies. Similarly, very nice agreement was obtained for all available experimental data of low lying $0^+$ excited states in a large region of the even-even nuclei. In Figure 2, we show the description of the positive parity yrast line experimental data with two crossing rotational $\beta$ bands. Along with the comparison with experiment there are shown the numbers of bosons $n$ for the bands heads.

Of course it is straightforward now to see whether the low lying excited states having different from zero spin can be also represented in the same form of the energies distributed by parabolic type function and can we connect the new classification parameter as a measure of collectivity determining each low lying state.
For this purpose let us shortly remind the Interacting Vector Boson Model (IVBM) developed some years ago by A. Georgieva, P. Raychev and R. Roussev [3].

IVBM is based on the introduction of two kinds of vector bosons (called p- and n-bosons), that "built up" the collective excitations in the nuclear system. The creation operators \( u^+_m(\alpha) \) of these bosons are assumed to be \( SO(3) \)-vectors and they transform according to two independent fundamental representations \((1, 0)\) of the group \( SU(3) \). The annihilation operators \( u_m(\alpha) = (u^+_m(\alpha))^\dagger \) transform according to the conjugate representations \((0, 1)\). These bosons form a "pseudospin" doublet of the group \( U(2) \) and differ in their "pseudospin" projection \( \alpha = \pm \frac{1}{2} \). The introduction of this additional degree of freedom leads to the extension of the \( SU(3) \) symmetry to \( U(6) \) so that the two kind of bosons \( u^+_m(\alpha = \pm \frac{1}{2}) \) transform according to the fundamental representation \([1]_6\) of the group \( U(6) \). The bilinear products of the creation and annihilation operators of the two vector bosons generate the noncompact symplectic group \( Sp(12, R) \):

\[
F^L_M(\alpha, \beta) = \sum_{k,m} C^{LM}_{1k1m} u^+_k(\alpha) u^+_m(\beta),
\]

\[
G^L_M(\alpha, \beta) = \sum_{k,m} C^{LM}_{1k1m} u_k(\alpha) u_m(\beta),
\]

\[
A^L_M(\alpha, \beta) = \sum_{k,m} C^{LM}_{1k1m} u^+_k(\alpha) u_m(\beta),
\]

where \( C^{LM}_{1k1m} \) are the usual Clebsh-Gordon coefficients and \( L \) and \( M \) define the transformational properties of (5) under rotations.

We consider \( Sp(12, R) \) to be the group of the dynamical symmetry of the model [3]. Hence the most general one- and two-body Hamiltonian can be expressed in terms of its generators. Using commutation relations between \( F^L_M(\alpha, \beta) \) and \( G^L_M(\alpha, \beta) \), the number of bosons preserving Hamiltonian can be expressed only in terms of operators \( A^L_M(\alpha, \beta) \):

\[
H = \sum_{\alpha, \beta} h_0(\alpha, \beta) A^0(\alpha, \beta) + \sum_{M, L} (-1)^M V^{L}(\alpha; \beta; \gamma; \delta) A^L_M(\alpha, \gamma) A^L_{-M}(\beta, \delta),
\]

where \( h_0(\alpha, \beta) \) and \( V^{L}(\alpha; \beta; \gamma; \delta) \) are phenomenological constants.

Being a noncompact group, the representations of \( Sp(12, R) \) are of infinite dimension, which makes it rather difficult to diagonalize the most general Hamiltonian. The operators \( A^L_M(\alpha, \beta) \) generate the maximal compact subgroup of \( Sp(12, R) \), namely the group \( U(6) \):

\[
Sp(12, R) \supset U(6)
\]

So the even and odd unitary irreducible representations /UIR/ of \( Sp(12, R) \) split into a countless number of symmetric UIR of \( U(6) \) of the type \([N, 0, 0, 0, 0, 0] = [N]_6\), where \( N = 0, 2, 4, ... \) for the even one and \( N = 1, 3, 5, ... \) for the odd
one \[3\]. Therefore the complete spectrum of the system can be calculated only through the diagonalization of the Hamiltonian in the subspaces of all the UIR of \( U(6) \), belonging to a given UIR of \( Sp(12, R) \).

Let us consider the rotational limit \[3\] of the model defined by the chain:

\[
U(6) \supset SU(3) \times U(2) \supset SO(3) \times U(1)
\]

\[
[N] (\lambda, \mu) \quad (N, T) \quad K \quad L \quad T_0
\]

where the labels below the subgroups are the quantum numbers \[\text{\textit{S}}\] corresponding to their irreducible representations. Their values are obtained by means of standard reduction rules and are given in \[3\]. In this limit the operators of the physical observables are the angular momentum operator

\[
L_M = -\sqrt{2} \sum_{M, \alpha} A_1^M(\alpha, \alpha)
\]

and the truncated ("Elliott") quadrupole operator

\[
Q_M = \sqrt{6} \sum_{M, \alpha} A_2^M(\alpha, \alpha),
\]

which define the algebra of \( SU(3) \).

The "pseudospin" and number of bosons operators:

\[
T_{+1} = \sqrt{\frac{3}{2}} A^0(p, n);
T_{-1} = -\sqrt{\frac{3}{2}} A^0(n, p);
T_0 = -\sqrt{\frac{3}{2}} [A^0(p, p) - A^0(n, n)];
N = -\sqrt{3}[A^0(p, p) + A^0(n, n)];
\]

define the algebra of \( U(2) \).

Since the reduction from \( U(6) \) to \( SO(3) \) is carried out by the mutually complementary groups \( SU(3) \) and \( U(2) \), their quantum numbers are related in the following way:

\[
T = \frac{\lambda}{2}; \quad N = 2\mu + \lambda
\]

Making use of the latter we can write the basis as

\[
| [N]; (\lambda, \mu = \frac{N}{2}); K, L, M; T_0 \rangle = | (N, T); K, L, M; T_0 \rangle
\]

The ground state of the system is:

\[
| 0 \rangle = | (0, 0); 0, 0, 0; 0 \rangle = | (N = 0, T = 0); K = 0, L = 0, M = 0; T_0 = 0 \rangle
\]

which is the vacuum state for the \( Sp(12, R) \) group.

Then the basis states \[3\] associated with the even irreducible representation of the \( Sp(12, R) \) can be constructed by the application of powers of raising
generators $F_M^L(\alpha, \beta)$ of the same group. The $SU(3)$ representations $(\lambda, \mu)$ are symmetric in respect to the sign of $T_0$.

Hence, in the framework of the discussed boson representation of the $Sp(12, R)$ algebra all possible irreducible representations of the group $SU(3)$ are determined uniquely through all possible sets of the eigenvalues of the Hermitian operators $N, T^2$, and $T_0$. The equivalent use of the $(\lambda, \mu)$ labels facilitates the final reduction to the $SO(3)$ representations, which define the angular momentum $L$ and its projection $M$. The multiplicity index $K$ appearing in this reduction is related to the projection of $L$ in the body fixed frame and is used with the parity to label the different bands in the energy spectra of the nuclei. The parity of the states is defined as $\pi = (-1)^T$. This allows us to describe both positive and negative bands.

The Hamiltonian, corresponding to this limit of IVBM is expressed in terms of the first and second order invariant operators of the different subgroups in the chain (7):

$$H = aN + \alpha_3 K_3 + \alpha_1 K_1 + \beta_3 \pi_3,$$

where $K_3$ are the quadratic invariant operators of the $U(n)$ - groups in (7), $\pi_3$ is the $SO(3)$ Casimir operator. As a result of the connections (9) the Casimir operators $K_3$ with eigenvalue $(\lambda^2 + \mu^2 + \lambda \mu + 3\lambda + 3\mu)$, is express in terms of the operators $N$ and $T$:

$$K_3 = 2Q_2 + \frac{3}{4}L^2 = \frac{1}{2}N^2 + N + T^2$$

After some transformations the Hamiltonian (12) takes the following form

$$H = aN + bN^2 + \alpha_3 T^2 + \beta_3 \pi_3 + \alpha_1 T_0^2,$$

and is obviously diagonal in the basis (10) labeled by the quantum numbers of the subgroups of chosen chain (7). Its eigenvalues are the energies of the basis states of the boson representations of $Sp(12, R)$:

$$E((N, T); KLM; T_0) = aN + bN^2 + \alpha_3 T(T + 1) + \beta_3 L(L + 1) + \alpha_1 T_0^2$$

Using the $(\lambda, \mu)$ labels facilitates and choosing for instance $(\lambda, 0)$ multiplet together with the reducing rules (9) after simple regrouping of the terms in (14) we can write the energy spectrum corresponding to this $(\lambda, 0)$ multiplet as:

$$E(\lambda) = A\lambda - B\lambda^2 + C$$

here $A$, $B$ and $C$ are the combinations of free model parameters of (14) $a$, $b$, $\alpha_3, \beta_3$ and $\alpha_1$.

Hence choosing any permitted by (9) $(\lambda, \mu)$ multiplet we again may classify the low lying excited states energies in even even nuclei applying the parabolic type distribution function and considering label $\lambda$ as a measure of collectivity of the corresponding excited states possessing different from 0 spins. In Figure
3. and Figure 4. are shown some examples for the classification (consider also \( n = \frac{1}{4} \)) of the energies of \( 2^+, 4^+, 6^+, \) and \( 8^+ \) excited states in \(^{162}\text{Dy}\) and also \( 2^+ \) states in \(^{240}\text{Pu}\) and \(^{250}\text{Cf}\) isotopes.

The experimental energies with great accuracy follow the parabolic distribution function (15) and similar agreement can be obtained for all spectra in even even nuclei. All experimental data are taken from [4].

We hope that this new interpretation of the experimental data for low lying collective excited states of even-even nuclei may be useful for divers aims in nuclear structure models. We also want to believe that it may be in help for experimentalists investigating low energies nuclear spectra especially when any ambiguous definition of the states spins exists.

References

[1] Vladimir P. Garistov "Phenomenological Description of the Yrast Lines", nucl-th/0201008, 2002

[2] T. Holstein, H. Primakoff, Phys. Rev. 58, (1940) 1098;
A.O. Barut, Phys. Rev. 139, (1965) 1433;
R. Marshalek Phys. Lett. B 97 (1980) 337;
C. C. Gerry, J. Phys. A 16, (1983) 11.

[3] Georgieva A., P. Raychev, R. Roussev, J. Phys. G: Nucl. Phys., 8, (1982), 1377-1389
Georgieva A., P. Raychev, R. Roussev, J. Phys. G: Nucl. Phys., 9, (1983), 521-534
V. P. Garistov, A. Georgieva, H. Ganev, Algebraic Methods in Nuclear Theory, collection of scientific papers edited by Anton N. Antonov, On Simultaneous Description of the Positive and Negative Bands in the Interacting Vector Boson Model, Sofia 2002

[4] Mitsuo Sacai Atomic Data and Nuclear Data Tables 31, 399-432 (1984);
Level Retrieval Parameters [http://iaeand.iaea.or.at/nudat/levform.html]
\[
\begin{align*}
\text{196 Pt} & \quad a = 0.409 \\
0^+ & \quad b = -0.01571
\end{align*}
\]

\[
\begin{align*}
\text{194 Pt} & \quad a = 0.487 \\
0^+ & \quad b = -0.019
\end{align*}
\]
\[ \begin{align*}
\text{\textit{162}} \text{Dy} & \quad 2^+ \\
\text{a} &= 0.61441 \\
b &= -0.03997 \\
c &= -0.15165
\end{align*} \]

\[ \begin{align*}
\text{\textit{162}} \text{Dy} & \quad 6^+ \\
a &= 0.78157 \\
b &= -0.05527 \\
c &= -0.14706
\end{align*} \]

\[ \begin{align*}
\text{\textit{162}} \text{Dy} & \quad 4^+ \\
a &= 0.78479 \\
b &= -0.08818 \\
c &= 0.35387
\end{align*} \]

\[ \begin{align*}
\text{\textit{162}} \text{Dy} & \quad 8^+ \\
a &= 0.7695 \\
b &= -0.0575 \\
c &= 0.2019
\end{align*} \]
\begin{align*}
\text{Pu}^{240} &: a=0.3984, b=-0.01647, c=-0.426 \\
\text{Cf}^{250} &: a=0.33527, b=-0.01497, c=-0.07086
\end{align*}