Supersymmetric Contributions to $B_s^0 \to K^+K^-$

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Abstract: Inspired by the existing calculation of $B \to \pi K$ decays in supersymmetry (SUSY), we evaluate the dominant SUSY contributions to $B_s^0 \to K^+K^-$. We show that the observables of this process can be significantly modified in the presence of SUSY. In particular, the branching ratio can be increased considerably compared to the prediction of the standard model (SM). The effect is even more dramatic for the CP-violating asymmetries $A_{\text{dir}}$ and $A_{\text{mix}}$. These asymmetries, expected to be small in the SM ($A_{\text{dir}}$ is predicted to take only positive values), change drastically with SUSY contributions. The measurement of these observables can therefore be used to detect the presence of physics beyond the SM, and put constraints on its parameters.

Keywords: $B$-Physics, Supersymmetry Phenomenology, CP violation.
1. Introduction

Current measurements of $B$ decays show hints of physics beyond the standard model (SM), in CP-violating asymmetries in penguin-dominated $\bar{b} \to \bar{s}q\bar{q}$ transitions ($q = u, d, s$) [1], in triple-product asymmetries in $B \to \phi K^*$ [2, 3], in the polarization measurements of $B \to \phi K^*$ [4, 5, 6] and $B \to \rho K^*$ [7, 8], and in $B \to \pi K$ decays (branching ratios and CP asymmetries) [9, 10, 11, 12, 13, 14, 15]. These discrepancies are (almost) all not yet statistically significant, being in the 1–2σ range. However, if these hints are taken together, the statistical significance increases. Furthermore, they are intriguing since they all point to new physics (NP) in $\bar{b} \to \bar{s}$ transitions. For this reason it is interesting to consider the effect of NP on $B$ decays dominated by the quark-level $\bar{b} \to \bar{s}$ process.

One such decay is $B_s^0 \to K^+K^-$. In the SM, its amplitude is given approximately by

$$A(B_s^0 \to K^+K^-) = -P' - T' \ .$$

Here the prime on the amplitude stands for a strangeness-changing decay. In the above, $P'$ and $T'$ are the gluonic penguin amplitude and the color-favored tree amplitude, respectively. These are estimated to obey the hierarchy $P' : T' \sim 1 : \bar{\lambda}$, where $\bar{\lambda} \sim 0.2$ [16]. There are other diagrams, but they are expected to be $O(\lambda^2)$, and have been neglected above.

The amplitude $P'$ is actually composed of three pieces, $P'_u$, $P'_c$ and $P'_t$, where the subscript refers to the internal quark in the loop:

$$P' = V_{ub}^* V_{us} P'_u + V_{cb}^* V_{cs} P'_c + V_{tb}^* V_{ts} P'_t$$

$$\approx V_{cb}^* V_{cs} (P'_c - P'_t) \ .$$

\[1.2\]
In writing the second line, we have used the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix to eliminate the $V_{ub}^* V_{ts}$ term, and we have dropped the $V_{ub}^* V_{us}$ term since $|V_{ub}^* V_{us}| \ll |V_{cb}^* V_{cs}|$.

Note: even though the term $V_{ub}^* V_{us} (P_u' - P_t')$ is at the level of other terms we have neglected, it can be retained by redefining the $T'$ amplitude:

$$T' \to T' + V_{ub}^* V_{us} (P_u' - P_t').$$  \hspace{1cm} (1.3)

In the rest of the paper we will adopt this redefinition. Thus, $T'$ has both a tree and a (small) penguin component.

At the quark level, $B_0^s \to K^+ K^-$ is described by $\bar{b} \to \bar{s} u u$. There are many potential NP contributions, which at the quark level take the form $\langle K^+ K^- | \bar{b} \Gamma_i s \bar{u} \Gamma_j u | B_0^s \rangle$, where the $\Gamma_{i,j}$ represent Lorentz structures, and color indices are suppressed. (We expect the size of all NP contributions to be at most of the order of $|P'|$.) This picture can be simplified by considering the strong phases. In Ref. [17], it was observed that the NP strong phases are negligible compared to that of the (dominant) SM contribution $P'$. (Note: each NP contribution can in principle have a different strong phase.) Briefly, the argument goes as follows. All strong phases are due to rescattering from intermediate states, with a suppression factor of about 10–20. In the SM, the $P'_c$ strong phase arises principally from the rescattering of the $\bar{b} \to \bar{c} c s$ tree diagram, $T'_c$. Since $T'_c$ is about 10–20 times bigger than $P'_c$, a strong phase of $O(1)$ is generated. By contrast, the NP strong phases can arise only from “self-rescattering,” i.e. rescattering from NP operators themselves. As a consequence, these phases are only about 5–10% as large as that of $P'$, and are therefore negligible. This leads to a great simplification: if one neglects the NP strong phases, one can combine all NP matrix elements into a single NP amplitude, with a single weak phase:

$$\sum c_{ij} \langle K^+ K^- | \bar{b} \Gamma_i s \bar{u} \Gamma_j u | B_0^s \rangle \equiv A_u e^{i \Phi_u},$$  \hspace{1cm} (1.4)

where the $c_{ij}$ are the coefficients of the operators and $\Phi_u$ is the effective NP weak phase.

Note that while this argument — that the NP strong phases are negligible — is quite general, there are still ways of evading this result. This can occur, for example, if certain NP amplitudes are larger than $|P'|$ and do not contribute to $B_0^s \to K^+ K^-$, but still contribute to the rescattering. This situation is perhaps unlikely, but the reader should be aware of these caveats.

Note also that the $T'$ strong phase is expected to be small. Thus, the relative strong phase between $T'$ and the NP is small compared to that of $P'$. Below, we will take this to be $(0 \pm 10)^\circ$.

In a previous article, three of us (DL, JM, JV) showed that one can measure the parameters $|A_u|$ and $\Phi_u$ by combining measurements of $B_0^s \to K^+ K^-$ and $B_0^d \to$
\[ \pi^+\pi^- \] In the present paper, we consider the generation of \( |A_u| \) and \( \Phi_u \) within a specific NP model: minimal supersymmetry (SUSY).

Naively, one would guess that all NP contributions to \( |A_u| \) and \( \Phi_u \) are suppressed

\[ \frac{M_2^2}{M_{NP}^2} \] where \( M_{NP} \sim 1 \text{ TeV} \), and are therefore small. However, there are SUSY contributions involving squark-gluino loops. Since these involve the strong coupling constant \( \alpha_s \), they are proportional to

\[ \frac{\alpha_s}{M_{NP}^2} \] and so can compete with the SM contributions which are of order \( \frac{\alpha}{M_W^2} \left[ \left( \frac{\alpha_s}{\alpha}\right) \left( \frac{M_W^2}{M_{NP}^2} \right) \sim 1 \right] \). Thus, there are large SUSY contributions to the NP parameters. Indeed, these are the dominant effects, and are the only ones which are considered below. As we will see, one can generate an \( |A_u| \) of the same order as \( |P'| \), so that the amplitude for \( B_s^0 \rightarrow K^+K^- \) can be written

\[ A(B_s^0 \rightarrow K^+K^-) = -P' - T' + A^re^{i\Phi_u}. \quad (1.5) \]

The effect of SUSY on the \( B_s^0 \rightarrow K^+K^- \) observables can therefore be sizeable, and we examine it here. We begin in Sec. 2 by establishing the SM predictions for the various observables in \( B_s^0 \rightarrow K^+K^- \). In Sec. 3, we evaluate the SUSY contributions to the NP parameters \( |A_u| \) and \( \Phi_u \). With this information, in Sec. 4 we calculate the combined effect of the SM and SUSY on the \( B_s^0 \rightarrow K^+K^- \) observables. We note that the presence of SUSY can dramatically change the values of these observables. Thus, their measurements can both establish the presence of NP and constrain the SUSY parameter space. We conclude in Sec. 5.

2. \( B_s^0 \rightarrow K^+K^- \): SM Results

We begin with general definitions of CP-violating asymmetries. For the decay \( B_s^0 \rightarrow f \), where \( f \) is a CP eigenstate, one can measure two such asymmetries. The direct CP asymmetry takes the form

\[ A_{dir} = \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2}, \quad (2.1) \]

where \( A \) is the amplitude for \( B_s^0 \rightarrow f \). \( \bar{A} \) is formed from \( A \) by changing the sign of the weak phases. The mixing-induced (indirect) CP asymmetry takes the form

\[ A_{mix} = -2 \frac{\text{Im} \left( e^{-i\phi_s} A^* \bar{A} \right)}{|A|^2 + |\bar{A}|^2}, \quad (2.2) \]

where \( \phi_s \) is the phase of \( B_s^0 \rightarrow B_s^0 \) mixing.

We now turn to specific expectations for \( B_s^0 \rightarrow K^+K^- \) within the SM. This process has three observables: the two CP asymmetries mentioned above, and the branching ratio. Without calculation, we can estimate the expected size of the CP asymmetries. For \( B_s^0 \rightarrow K^+K^- \), since the amplitude \( T' \) is subdominant, to leading
order this decay is described by a single amplitude, $V_{cb}^*V_{cs}(P_c' - P_t')$. As such, in the SM the direct CP asymmetry is expected to be small, of order $|T'/P'| \sim \lambda \sim 20\%$. Similarly, the mixing-induced CP asymmetry approximately measures $\phi_s$. Since $\phi_s$ is also expected to be very small (in the Wolfenstein parametrization $^{[19]}$, Im$V_{ts} \sim 5\%$), this asymmetry is expected to be correspondingly small.

In order to calculate the SM predictions for these three observables, we need the magnitudes and relative weak and strong phases of $P'$ and $T'$. The relative weak phase is $\gamma$, one of the three interior CP-violating angles of the unitarity triangle. This phase can be obtained from a fit to a variety of other measurements, some non-CP-violating. The latest analysis gives $\gamma = 61^{+7}_{-5}^\circ$ $^{[20]}$. Note that this error includes uncertainties in theoretical quantities. This value will be used in our analysis.

For the magnitudes and relative strong phase of $P'$ and $T'$, we can proceed in one of two ways. One approach is to use a particular theoretical framework to calculate these quantities (see, for instance, $^{[11, 21]}$). Alternatively, one can use measurements of $B^0_d \to \pi^+\pi^-$, along with flavor SU(3) symmetry, to obtain $P'$ and $T'$ $^{[12, 14, 22, 23]}$. In this paper, we adopt the latter approach.

Neglecting small contributions, the amplitude for the decay $B^0_d \to \pi^+\pi^-$ can be written

$$A(B^0_d \to \pi^+\pi^-) = -P - T . \quad (2.3)$$

As above, we can write

$$P = V_{ub}^*V_{ud}P_u + V_{cb}^*V_{cd}P_c + V_{tb}^*V_{td}P_t$$
$$= V_{ub}^*V_{ud}(P_u - P_t) + V_{cb}^*V_{cd}(P_c - P_t) . \quad (2.4)$$

The difference compared to $P'$ is that one cannot neglect the first term. On the other hand, we can absorb it into the definition of $T$:

$$T \to T + V_{ub}^*V_{ud}(P_u - P_t) . \quad (2.5)$$

Thus, $T$ is not a pure tree amplitude, but contains a penguin amplitude.

As with $B^0_s \to K^+K^-$, there are three measurements involving $B^0_d \to \pi^+\pi^-$: the two CP asymmetries and the branching ratio. These suffice to determine the magnitudes and relative strong phase of $P$ and $T$, given the knowledge of $\gamma$. Using flavor SU(3) symmetry, these can be related to the magnitudes and relative strong phase of $P'$ and $T'$ $^{[12, 14, 22, 23]}$:

$$\begin{align*}
\frac{T'}{T} &= \frac{V_{us}}{|V_{ud}|} \mathcal{R}_C , \\
\frac{P'/T'}{P/T} &= \frac{V_{cs}V_{ud}}{|V_{cd}V_{us}|} \xi . \quad (2.6)
\end{align*}$$

In the SU(3) limit we have $\mathcal{R}_C = 1$, $\xi = 1$ and $\theta' = \theta$, where $\theta'$ and $\theta$ are the relative strong phases of $P'$ and $T'$, and $P$ and $T$, respectively.
Due to U-spin breaking, $R_C$ gets both factorizable and non-factorizable contributions. The former have recently been calculated using QCD sum rules [24] and found to be sizeable:

$$R_C = 1.76^{+0.15}_{-0.17}. \quad (2.7)$$

It should be noticed, however, that factorizable corrections are absent in the double ratio $(P'/T')/(P/T)$. In our analysis, we use the central value of $R_C$. For the other quantities, we take $\xi = 1.0 \pm 0.2$ (which we vary), and $\theta' - \theta = 0^\circ$. Whenever we refer to the U-spin limit, we will mean $\xi = 1$ and $\theta' = \theta$, but always taking the value of Eq. (2.7) for $R_C$.

With the experimental measurements of $B^0_d \to \pi^+\pi^-$ and the theoretical values for the SU(3)-breaking parameters, we can obtain $P'$ and $T'$, which allow us to compute the SM expectations for the $B^0_s \to K^+K^-$ observables. The latest $B^0_d \to \pi^+\pi^-$ data is:

$$BR(B_d^0 \to \pi^+\pi^-) = \begin{cases} (5.5 \pm 0.5) \times 10^{-6} \text{ BaBar} [23] \\ (4.4 \pm 0.7) \times 10^{-6} \text{ Belle} [26] \\ (5.0 \pm 0.4) \times 10^{-6} \text{ Average} \end{cases}$$

$$A_{dir}(B_d^0 \to \pi^+\pi^-) = \begin{cases} -0.09 \pm 0.16 \text{ BaBar} [27] \\ -0.52 \pm 0.14 \text{ Belle} [28] \\ -0.33 \pm 0.11 \text{ Average} \end{cases}$$

$$A_{mix}(B_d^0 \to \pi^+\pi^-) = \begin{cases} 0.30 \pm 0.17 \text{ BaBar} [27] \\ 0.67 \pm 0.17 \text{ Belle} [29] \\ 0.49 \pm 0.12 \text{ Average} \end{cases}$$

Regarding $BR_{KK}^{SM}$, it is sometimes more useful to present the ratio of branching ratios of $B^0_s \to K^+K^-$ and $B^0_d \to \pi^+\pi^-$: $R_d^s \equiv BR(B_s^0 \to K^+K^-)/BR(B_d^0 \to \pi^+\pi^-)$ [24]. The SM $B^0_s \to K^+K^-$ predictions for all four quantities are shown in Table 1 (see also [14]). Obviously these values are correlated. Fig. 1 illustrates the main correlations between the observables, for different values of the SU(3) breaking parameter $\xi$.

Note that the CP asymmetries are allowed to take large values. This does not imply that our above argument about the expected smallness of these asymmetries is incorrect. Rather, it points to the largeness of the present experimental errors.

Despite the large regions, there is still room for NP. If any of the correlations is found to lie outside of the allowed regions, this is a signal of physics beyond the SM.

3. SUSY Contributions to $|A^u|$ and $\Phi_u$

In this section we evaluate the SUSY contributions to $|A^u|$ and $\Phi_u$. We adopt the following procedure:
Table 1: SM predictions for the branching ratio and mixing induced and direct CP-asymmetries. The impact of the uncertainty in the U-spin breaking parameter $\xi$ and CKM-angle $\gamma$ is shown.

| $\gamma = 61^\circ$ | $\xi = 1$ | $\xi = 1 \pm 0.2$ | $\gamma = (61^{+\frac{1}{2}})_{-\frac{1}{2}}^\circ$ |
|---------------------|------------|---------------------|---------------------------------|
| $BR_{KK}^{SM} \times 10^6$ | $(6.4, 42.6)$ | $(4.2, 61.9)$ | $(5.0, 60.7)$ |
| $R_d^{SM}$ | $(1.2, 9.3)$ | $(0.8, 13.5)$ | $(0.9, 13.2)$ |
| $A_{dir KK}^{SM}$ | $(0.15, 0.45)$ | $(0.12, 0.56)$ | $(0.08, 0.58)$ |
| $A_{mix KK}^{SM}$ | $(-0.32, -0.10)$ | $(-0.38, -0.09)$ | $(-0.34, 0.08)$ |

Figure 1: Correlations between the observables $A_{dir KK}^{SM} - BR_{KK}^{SM}$ and $A_{dir KK}^{SM} - A_{mix KK}^{SM}$, for $\gamma = 61^\circ$ and $\xi = 1$, $\xi = 0.9$ and $\xi = 1.1$.

1. We consider all operators generated at the heavy scale, taken to be $M_w$. We compute the SUSY contributions to the coefficients of these operators.

2. Using the renormalization group, we run the operator coefficients down to $m_b$. Operator mixing is included here.

3. We compute the matrix elements of the various operators at $m_b$. This allows us to calculate $|A_u|$ and $\Phi_u$.

We closely follow the approach of Grossman, Neubert and Kagan (GNK) [30]. One difference is that GNK are interested in isospin-violating effects ("trojan penguins"), while we consider both isospin-conserving and isospin-violating contributions to $|A_u|$ and $\Phi_u$. Another difference is that GNK calculate the NP contributions to $B \to \pi K$, while we concentrate on $B^0_\ell \to K^+ K^-$. Here the quark-level calculation is the same, and so our computation can be considered as a check.

We begin by listing all the operators which are generated by the new physics at
the heavy scale. The NP effective Hamiltonian is \[ H_{\text{eff}}^{\text{NP}} = \frac{G_F}{\sqrt{2}} \sum_{i,q=u,d} \left( c_i^q(\mu) O_i^q + \tilde{c}_i^q(\mu) \tilde{O}_i^q \right) + C_{8g}(\mu) Q_{8g} + \tilde{C}_{8g}(\mu) \tilde{Q}_{8g} \] \[ (3.1) \]

where
\[
O_1^q = (\bar{b}_s s_a)_{V-A}(\bar{q}_3 q_3)_{V+A} \quad \text{and} \quad O_2^q = (\bar{b}_s s_a)_{V-A}(\bar{q}_3 q_3)_{V-A} \\
O_3^q = (\bar{b}_s s_a)_{V-A}(\bar{q}_3 q_3)_{V-A} \\
O_4^q = (\bar{b}_s s_a)_{V-A}(\bar{q}_3 q_3)_{V-A} \\
Q_{8g} = (g_s/8\pi^2) m_b \bar{b} \sigma_{\mu\nu} (1 - \gamma_5) G^{\mu\nu} 
\]

In the above, \( \alpha \) and \( \beta \) are color indices, and the subscript \( V \pm A \) indicates that the Lorentz structure between quarks is \( \gamma\mu(1 \pm \gamma_5) \). Despite the fact that, at the quark level, \( B_s^0 \to K^+K^- \) is \( \bar{b} \to \bar{s}u\bar{u} \), \( d \)-quark operators must be included above since they mix with the \( u \)-quark operators upon renormalization to \( m_b \). Note that the above list includes the chromomagnetic operator \( Q_{8g} \). The operators \( \tilde{O}_i^q \) and \( \tilde{Q}_{8g} \) are obtained from \( O_i^q \) and \( Q_{8g} \) by chirality flipping.

The above list of operators includes new-physics contributions to electroweak-penguin operators. As we will see, these effects can be significant. This shows that, although the SM electroweak-penguin contributions to \( B_s^0 \to K^+K^- \) are negligible, the same does not hold for the NP.

We now must compute the NP contributions to the Wilson coefficients \( c_i^q(\mu) \) and \( \tilde{c}_i^q(\mu) \) at the heavy scale \( \mu = M_W \). As discussed above, the dominant contribution comes from QCD penguin and box diagrams with squark-gluino loops.

For our analysis, we follow Ref. \cite{30} and take the general minimal supersymmetric standard model at the electroweak scale without assuming any flavor models at high energies (e.g., at the scale of grand unification). Here, the SUSY flavor-changing neutral current problem is avoided by assuming that the down squark is decoupled from the strange and bottom squarks \cite{30}. That is, we write
\[
\tilde{d}_L = \tilde{d}_L^0 \\
\tilde{s}_L = \cos \theta_L \tilde{s}_L^0 - \sin \theta_L e^{-i\delta_L} \tilde{b}_L^0 \\
\tilde{b}_L = \sin \theta_L e^{i\delta_L} \tilde{s}_L^0 + \cos \theta_L \tilde{b}_L^0 . \]

(3.3)

In the above, the superscript ‘0’ indicates gauge eigenstates, and \( \delta_L \) is a new CP-violating phase. There are similar expressions for the right-handed squarks. The \( |\theta_{L,R}| \) are taken to be \( \leq 45^\circ \). Similarly, the up squark is assumed to be decoupled from the charm squark, and up-top squark mixing can also be ignored \cite{30}. With these approximations, the Wilson coefficients \( c_{5,6}^q(\mu) \) and \( c_{5,6}^q(\mu) \) vanish; the others are given in Appendix A.

Once we have calculated, for given values of the SUSY parameters, the Wilson coefficients at \( M_W \), the next step is to compute the renormalization-group running
of these, including operator mixing, down to $m_b$. The details of the computation are
given in Appendix B.

The final step in the program is to compute the hadronic matrix elements of
the operators in Eq. (3.2) for $B^0 \to K^+K^-$. These are calculated using the naive
factorization approach.

We define

$$A^Y_X \equiv \langle K^-|\langle \bar{b}u|X|B^0\rangle \langle K^+|\langle \bar{s}u|Y|0 \rangle \rangle ,$$

where $X$ and $Y$ refer to Lorentz structures. The pseudoscalar nature of the mesons implies that

$$A^{V+A}_{V+A} = A^{V-A}_{V-A} = -A^{V+A}_{V-A} = -A^{V-A}_{V+A} \equiv A$$

$$A^{S+P}_{S+P} = A^{S-P}_{S-P} = -A^{S-P}_{S+P} = -A^{S+P}_{S-P} \equiv S ,$$

which define the hadronic quantities $A$ and $S$. After Fierz rearranging and factor-
ization, the matrix elements of the operators read:

$$\langle O^u_1 \rangle = 2 \eta S , \quad \langle O^u_2 \rangle = 2 S ,$$

$$\langle O^u_3 \rangle = -\eta A , \quad \langle O^u_4 \rangle = -A , \quad \langle O^u_5 \rangle = \eta A ,$$

where $\eta = 1/N_C = 1/3$. The matrix elements of the operators $\tilde{O}^u_i$ are just $\langle \tilde{O}^u_i \rangle = -\langle O^u_i \rangle$, the minus sign coming from the change $A \to -A$ and $S \to -S$. Finally, we define

$$\chi \equiv -2S/A ; \quad \bar{c}_i^a \equiv c_i^a - \bar{c}_i^a$$

The NP amplitude can now be written as

$$\langle K^+K^-|H^{NP}_{\text{ eff}}|B^0_s \rangle = \frac{G_F}{\sqrt{2}} \left[ -\chi \left( \frac{1}{3} \bar{c}_1^u + \bar{c}_2^u \right) - \frac{1}{3} (\bar{c}_3^u - \bar{c}_6^u) - (\bar{c}_4^u - \bar{c}_5^u) \right]$$

$$-\lambda_i \frac{2\alpha_s}{3\pi} \tilde{C}_{8g} \left( 1 + \frac{\chi}{3} \right) A ,$$

where the coefficients $\bar{c}_i^a$ are evaluated at $m_b$. The hadronic quantities $\chi$ and $A$ can be calculated in terms of the meson masses, form factors and decay constants. Using
the following expressions for the factorized amplitudes,

$$\langle K^+|\bar{u}\gamma_\nu\gamma_5 s|0 \rangle = i\sqrt{2} f_{K \rho_\mu}$$

$$\langle K^+|\bar{u}\gamma_5 s|0 \rangle = - \frac{i\sqrt{2} f_K m_K^2}{m_u + m_s}$$

$$\langle K^-|\bar{u}\gamma_\mu u|B^0_s \rangle = \frac{m_B^2 - m_K^2 - q_\perp^2}{q_\perp^2} q_\perp^\mu F_{B \to K}$$

$$\langle K^-|\bar{u}u|B^0_s \rangle = \frac{1}{m_b} (m_B^2 - m_K^2) F_{B \to K} ,$$

with $q_\perp^\mu \equiv q_B^\mu - q_K^\mu = q_{K^-}^\mu \equiv p^\mu$, we find that

$$\chi = \frac{2m_K^2}{m_b(m_u + m_s)} \approx 1.18 ,$$

$$A = i\sqrt{2}(m_B^2 - m_K^2) f_K F_{B \to K} \simeq 1.37 \text{ GeV}^3 .$$
4. $B^0_s \to K^+K^-$: SM + SUSY

We are now ready to calculate the values of the various $B^0_s \to K^+K^-$ observables in the presence of SUSY. We begin with $BR_{kk}$ and $A_{dir}$. The parameters $P'$ and $T'$ are taken from the SM analysis (Sec. 2). The NP parameters $|A|$ and $\Phi$ are the modulus and argument of the amplitude in Eq. (3.8). Finally, we must address the question of the relative strong phase of $T'$ and $A^u$. If the factor $(P'_u - P'_t)$ were not present in $T'$ [Eq. (1.3)], we would say that the strong phase of $T'$ is the same as that of the NP, i.e. it is negligible and $\delta_{P'} - \delta_{NP} = 0$. However, $(P'_u - P'_t)$ is present. And since $P'_u$ can have a non-negligible strong phase due to rescattering from the $b \to s u \bar{u}$ tree diagram, the relative strong phase of $T'$ and $A^u$ can be nonzero. We take $\delta_{P'} - \delta_{NP} = (0 \pm 10)^0$. The quantities $BR_{kk}$ and $A_{dir}$ can now be obtained.

The mixing-induced CP asymmetry, $A_{mix}$, can also be affected by the presence of SUSY. However, in order to compute the allowed range, we must take into account the fact that this NP will also affect the $B^0_s \to \bar{B}^0_s$ mixing angle $\phi_s$. The SM predicts $\phi_s \approx 0$, because the combination of CKM matrix elements $(V_{ts}^* V_{tb})^2$ is real to a very good approximation [19]. On the other hand, in the SUSY scenario we consider, sizeable $\phi_s$ is possible. Barring the simultaneous existence of $LL$ and $RR$ mixing, we get the following expression for $\phi_s$:

$$\phi_s = \arg \left[ 1 + e^{-2i\delta_L} \frac{\sin^2 2\theta_L}{\lambda_t^2} \frac{\alpha_L^2}{\alpha_W^2} \frac{m_W^2}{m_{\tilde{g}}^2} \frac{1}{S_0(x_t)} \right. \times \left. \frac{11}{18} \left( G(x_{bLg}, x_{bLg}^c) + G(x_{sLg}, x_{sLg}^c) - 2G(x_{bLg}, x_{sLg}^c) \right) \right]

- \frac{2}{9} \left( F(x_{bLg}, x_{bLg}^c) + F(x_{sLg}, x_{sLg}^c) - 2F(x_{bLg}, x_{sLg}^c) \right). \tag{4.1}$$

Here $x_t \equiv m_t^2/m_W^2$ and $\lambda_t \equiv V_{tb} V_{ts}^*$. The loop functions $F$ and $G$, are given in Appendix A, and $S_0$ is

$$S_0(x) = \frac{x^4 - 12x^3 + 15x^2 - 4x + 6x^3 \ln x}{4(x - 1)^3}. \tag{4.2}$$

For the case of $RR$-mixing, we can use the same formula with $L \leftrightarrow R$. This allows us to compute $A_{mix}$ in the presence of SUSY.

The complete expressions for $BR_{kk}$, $A_{dir}$ and $A_{mix}$ depend on a number of unknown SUSY parameters. These are the gluino and squark masses, and the angles $\theta_{L,R}$ and $\delta_{L,R}$. (The relative strong phase $\delta_{P'} - \delta_{NP}$ has been discussed above.) Our aim here is to see how the space of allowed values for the $B^0_s \to K^+K^-$ observables is
definitions and values are [32]. For this purpose, we take the following ranges/values for the SUSY parameters. For the angles, we take $-\pi/4 \leq \theta_{L,R} \leq \pi/4$ and $-\pi \leq \delta_{L,R} \leq \pi$. For the masses we take $m_{\tilde{q}} = m_{\tilde{d}_{L,R}} = m_{\tilde{u}_{L,R}} = 250\text{ GeV}$, $250\text{ GeV} \leq m_{\tilde{u}_{L,R}} \leq 1000\text{ GeV}$, and $500\text{ GeV} \leq m_{\tilde{s}_{L,R}} \leq 1000\text{ GeV}$. We also take $m_{\tilde{q}_{R}} = m_{\tilde{q}_{L}}$.

Some further constraints are imposed on the set of input parameters. First, the same SUSY contributions to $B_s^0 \rightarrow K^+K^-$ will also affect $B \rightarrow \pi K$ decays. In particular, there will be effects on the quantities $R_\ast$ and $A_{CP}(\pi^+K^0)$ [30], whose definitions and values are [32]

$$R_\ast \equiv \frac{BR(B^+ \rightarrow \pi^+K^0) + BR(B^- \rightarrow \pi^-K^0)}{2[BR(B^+ \rightarrow \pi^0K^+) + BR(B^- \rightarrow \pi^0K^-)]} = 1.00 \pm 0.08 \quad (4.3)$$

$$A_{CP}(\pi^+K^0) \equiv \frac{BR(B^+ \rightarrow \pi^+K^0) - BR(B^- \rightarrow \pi^-K^0)}{BR(B^+ \rightarrow \pi^+K^0) + BR(B^- \rightarrow \pi^-K^0)} = -0.020 \pm 0.034 \quad (4.4)$$

In order to incorporate these two constraints we follow the approach in Ref. [30] where QCD-factorization is used, except for the strong phase related to $A_{CP}$ which we take as a free parameter. Second, there are bounds from $BR(B \rightarrow X_s + \gamma)$ and $\Delta m_s$ [33],

$$2.92 \times 10^{-4} < BR(B \rightarrow X_s + \gamma) < 4.12 \times 10^{-4} \quad (4.5)$$

$$\Delta m_s/\Delta m_s^{SM} > 0.9797 \quad (4.6)$$

The measured values (4.3)-(4.6) will therefore put additional constraints on the SUSY parameter space, which are taken into account in our analysis. In particular, the bounds (4.3) for $BR(B \rightarrow X_s + \gamma)$ have a strong effect on the angle $\delta_L$, which for $\theta_{L,R} = \pi/4$ is restricted to the ranges $0.86 < \delta_L < 1.35$ and $4.93 < \delta_L < 5.43$.

The NP amplitude $A^u$ is found to be small (even zero) for small $u$-squark masses ($m_{\tilde{u}} \sim 250\text{ GeV}$). However, for $u$- and $s$-squark masses close to 1 TeV and large $\tilde{s}-\tilde{b}$ mixing, $A^u$ can be as large as $3.3 \times 10^{-8}$ GeV (see figure 2). This number should be compared to the magnitude of the SM penguin amplitude $|P| \sim 3 \times 10^{-8}$ GeV. Since these values are similar, SUSY contributions can yield important deviations from SM predictions.

If we now allow for the variation of the SUSY parameters we find that $A_{mix}$ can take any possible value. Correspondingly, $A_{dir}$ can take any positive value, and negative values down to $-0.5$. A deviation from the SM range shown in Table 1 could be explained within SUSY.

Turning to the branching ratio, it can also receive a sizeable correction from the gluino contribution compared to the SM prediction. It can be almost 90% larger than the SM prediction in the U-spin limit. Even if one includes a large uncertainty of $\pm 20\%$ from the U-spin breaking parameter $\xi$, the supersymmetric prediction for

\[1\text{We take a wider range for } BR(B \rightarrow X_s + \gamma) \text{ to allow for various theoretical uncertainties.}\]
Figure 2: $\mathcal{A}^u$ versus the common mass $m_{\tilde{u}_L} = m_{\tilde{u}_R}$, for $m_{\tilde{d}_{L,R}} = m_{\tilde{b}_{L,R}} = m_{\tilde{g}} = 250$ GeV and several values of $m_{\tilde{s}_{L,R}}$, in the case of maximal $\tilde{s} - \tilde{b}$ mixing ($\theta_{R,L} = \pi/4$) and $\delta_L - \delta_R = \pi$.

Table 2: Allowed ranges of the $B_s^0 \to K^+K^-$ observables, including both SM + SUSY contributions.

| $BR_{K^+K^-}^{\text{susy}} \times 10^6$ | $R_d^{\text{susy}}$ | $A_{K^+K^-}^{\text{susy}}$ | $A_{\text{mix}_{K^+K^-}}^{\text{susy}}$ |
|--------------------------------------|-------------------|---------------------------|---------------------------------|
| (3.6, 79.1)                           | (0.7, 17.2)       | (-0.5, 1.0)               | (-1, 1)                         |

the $B_s^0 \to K^+K^-$ branching ratio can be up to 30% larger than that of the SM in the same regions of the SUSY parameter space. The same applies to the ratio $R_d^{\text{susy}}$.

Table 2 summarizes all of these results. We see that there is a wide range in the values of the observables which are not allowed by SM but that are easily accommodated by minimal SUSY. We stress that this result holds even in a situation of quite constrained parameter space, large hadronic uncertainties and a $\pm 20\%$ of SU(3) breaking in $\xi$.

5. Conclusions

At present, there are several hints of new physics (NP) in processes governed by $\bar{b} \to \bar{s}$ transitions. For this reason, it is useful to consider the effect of NP on $\bar{b} \to \bar{s}$ processes. One such decay is $B_s^0 \to K^+K^-$. There are many possible NP contributions to $B_s^0 \to K^+K^-$ decays. However, to a good approximation, all of these have strong phases which are small compared to those of the standard model (SM), and can therefore be neglected. In this limit, one can combine all NP contributions into a single term, parametrized by its magnitude $\mathcal{A}^u$ and weak phase $\Phi_u$. 

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In this paper, we have calculated the main supersymmetric (SUSY) contributions to $A_u$ and $\Phi_u$, assuming that $B_0^0 \to \pi^+\pi^-$ is unaffected. There are many SUSY effects. However, the principal ones come from squark-gluino loops, which involve strong couplings, and because $(\alpha_s/\alpha)(M_{\tilde{W}}^2/M_{\tilde{Q}}^2) \sim 1$, they are not suppressed compared to the SM ($M_{\tilde{N},p} \sim 1$ TeV). These are expected to be the dominant effects, and so we have included only these contributions. We have used naive factorization to compute the matrix elements and used data from $B \to \pi \pi$ decays to estimate the SM contribution.

In the presence of such SUSY contributions, the predictions of the SM for $B_s^0 \to K^+K^-$ decays can be significantly modified, particularly for $u$- and $s$-squark masses close to 1 TeV and large $\tilde{s}-\tilde{b}$ mixing. For example, we have found that the branching ratio can be increased. Even if one takes into account the large uncertainty due to the breaking of flavor SU(3) symmetry, the prediction of the SM + SUSY for the $B_s^0 \to K^+K^-$ branching ratio can be up to 30% larger than that of the SM alone.

The situation is even more dramatic for the CP-violating asymmetries $A_{dir}$ and $A_{mix}$. In the SM, these are predicted to be small, with $A_{dir}$ taking positive values only. On the other hand, in the presence of SUSY contributions, the range of $A_{mix}$ gets enlarged from $-1$ to 1, and $A_{dir}$ covers all the positive range, and also admits negative values forbidden to the SM.

We therefore conclude that the study of $B_s^0 \to K^+K^-$ decays is very useful with respect to new physics. The measurement of its observables can be used to detect the presence of NP. Furthermore, the precise values of these quantities can be used to constrain the NP parameter space. In particular, this holds true for the case of SUSY, which can significantly modify the SM predictions.

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Appendix A: Wilson Coefficients

Using the approximations discussed in the text, the non-vanishing Wilson coefficients (WC’s) at the heavy scale ($M_{\tilde{W}}$) are given by

$$c_1^q = \frac{\alpha_s^2 \sin 2\theta_W e^{i\delta_L}}{4\sqrt{2}G_F m_{\tilde{g}}^2} \left[ \frac{1}{18} F(x_{\tilde{b}_{L\tilde{g}},x_{\tilde{q}_{R\tilde{g}}}}) - \frac{5}{18} G(x_{\tilde{b}_{L\tilde{g}},x_{\tilde{q}_{R\tilde{g}}}}) + \frac{1}{2} A(x_{\tilde{b}_{L\tilde{g}}}) + \frac{2}{9} B(x_{\tilde{b}_{L\tilde{g}}}) \right]$$

$-(x_{\tilde{b}_{L\tilde{g}}} \to x_{\tilde{s}_{L\tilde{g}}})$
\[
c_2^q = \frac{\alpha_s^2 \sin 2\theta_L e^{i\delta_L}}{4\sqrt{2}G_F m_b^2} \left[ \frac{7}{6} F(x_{b_L\bar{g}}, x_{q_R\bar{g}}) + \frac{1}{6} G(x_{b_L\bar{g}}, x_{q_R\bar{g}}) - \frac{3}{2} A(x_{b_L\bar{g}}) - \frac{2}{3} B(x_{b_L\bar{g}}) \right] - (x_{b_L\bar{g}} \rightarrow x_{s_L\bar{g}}) \]
\[
c_3^q = \frac{\alpha_s^2 \sin 2\theta_L e^{i\delta_L}}{4\sqrt{2}G_F m_b^2} \left[ -\frac{5}{9} F(x_{b_L\bar{g}}, x_{q_L\bar{g}}) + \frac{1}{36} G(x_{b_L\bar{g}}, x_{q_L\bar{g}}) + \frac{1}{2} A(x_{b_L\bar{g}}) + \frac{2}{9} B(x_{b_L\bar{g}}) \right] - (x_{b_L\bar{g}} \rightarrow x_{s_L\bar{g}}) \]
\[
c_4^q = \frac{\alpha_s^2 \sin 2\theta_L e^{i\delta_L}}{4\sqrt{2}G_F m_b^2} \left[ \frac{1}{3} F(x_{b_L\bar{g}}, x_{q_L\bar{g}}) - \frac{7}{12} G(x_{b_L\bar{g}}, x_{q_L\bar{g}}) - \frac{3}{2} A(x_{b_L\bar{g}}) - \frac{2}{3} B(x_{b_L\bar{g}}) \right] - (x_{b_L\bar{g}} \rightarrow x_{s_L\bar{g}}), \tag{5.1} \]

where the functions \( F, G, A \) and \( B \) are
\[
F(x, y) = -\frac{x \ln x}{(x-y)(x-1)^2} - \frac{y \ln y}{(y-x)(y-1)^2} - \frac{1}{(x-1)(y-1)} \]
\[
G(x, y) = \frac{x^2 \ln x}{(x-y)(x-1)^2} + \frac{y^2 \ln y}{(y-x)(y-1)^2} + \frac{1}{(x-1)(y-1)} \]
\[
A(x) = \frac{1}{2(1-x)} + \frac{(1+2x) \ln x}{6(1-x)^2} \]
\[
B(x) = -\frac{11-7x+2x^2}{18(1-x)^3} - \frac{\ln x}{3(1-x)^4}, \tag{5.2} \]

and \( x_{q_i\bar{g}} \equiv m_{q_i}^2/m_b^2 \), where \( m_{q_i} (q = d, u) \) is the mass of the \( i^{th} \) quark mass eigenstate.

The expressions for the coefficients \( c_i^q \) are obtained from those in Eq. (5.1) via the exchange \( L \leftrightarrow R \). Note that there is a relative sign difference between our \( c_4^q \) and that given in Ref. [30]. Our computation of the WC’s agrees with Ref. [34].

Using the same approximations, the SUSY contribution to the WC of the chromomagnetic operator is given by
\[
\lambda \frac{2\alpha_s}{3\pi} C_{8g}^{\text{SUSY}} = \frac{8 \alpha_s^2 \sin 2\theta_L e^{i\delta_L}}{3 \cdot 4\sqrt{2}G_F m_b^2} \left[ f_{8 \text{SUSY}}^{\text{SUSY}}(x_{b_L\bar{g}}) - (b_L \leftrightarrow s_L) \right], \tag{5.3} \]

where the loop function is
\[
f_{8 \text{SUSY}}^{\text{SUSY}}(x) = \frac{-11 + 51 x - 21 x^2 - 19 x^3 + 6 x (-1 + 9 x) \log(x)}{72 (-1 + x)^4}. \tag{5.4} \]

**Appendix B: Renormalization-group evolution of the Wilson coefficients**

The QCD evolution of the Wilson coefficients (WC’s) is given by [35]
\[
\tilde{C}(\mu) = U_5(\mu, M_W) \overline{C}(M_W) \tag{5.5} \]
where $U_5(\mu, M_W)$ is the evolution matrix. Following GNK we work at leading order (LO)$^2$ neglecting electromagnetic corrections to the anomalous dimension matrix of the operators. The chromomagnetic operator $Q_{8g}$ is included. The leading logarithmic approximation depends only on the leading-order anomalous dimension matrix $\gamma^{(0)}$. We perform the running translating the basis to the $12 \Delta B = 1$ SM operators.

\[
H_{\text{eff}}^{SM} = \frac{G_F}{\sqrt{2}} \left\{ \lambda_u \sum_{i=1}^{2} C_i(\mu) Q_{i}^{u} - \lambda_t \left[ \sum_{i=3}^{10} C_i(\mu) Q_i + C_{7\gamma}(\mu) Q_{7\gamma} + C_{8g}(\mu) Q_{8g} \right] \right\}
\]

where $\lambda_i \equiv V^*_{ib} V_{is}$. In the above, $Q_1^{u}$ and $Q_2^{u}$ are tree operators, $Q_{3-6}$ are QCD penguin operators, and $Q_{7-10}$ are electroweak penguin operators:

\[
\begin{align*}
Q_1^{u} &= (\bar{b}_u u_\beta)_{V-A}(\bar{u}_\beta s_\alpha)_{V-A} \\
Q_2^{u} &= (\bar{b}_u u_\alpha)_{V-A}(\bar{u}_\alpha s_\beta)_{V-A} \\
Q_3 &= (\bar{b}_u s_\alpha)_{V-A} \sum_q (\bar{q}_\beta q_\beta)_{V-A} \\
Q_4 &= (\bar{b}_u s_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V-A} \\
Q_5 &= (\bar{b}_u s_\alpha)_{V-A} \sum_q (\bar{q}_\beta q_\beta)_{V+A} \\
Q_6 &= (\bar{b}_u s_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V+A} \\
Q_7 &= \frac{3}{2} (\bar{b}_u s_\alpha)_{V-A} \sum_q e_q (\bar{q}_\beta q_\beta)_{V-A} \\
Q_8 &= \frac{3}{2} (\bar{b}_u s_\beta)_{V-A} \sum_q e_q (\bar{q}_\beta q_\alpha)_{V-A} \\
Q_9 &= \frac{3}{2} (\bar{b}_u s_\alpha)_{V-A} \sum_q e_q (\bar{q}_\beta q_\beta)_{V-A} \\
Q_{10} &= \frac{3}{2} (\bar{b}_u s_\beta)_{V-A} \sum_q e_q (\bar{q}_\beta q_\alpha)_{V-A} \\
Q_{7\gamma} &= (e/8\pi^2)m_b \sigma_{\mu\nu}(1 - \gamma_5)F_{\mu\nu} s \\
Q_{8g} &= (g_s/8\pi^2)m_b \sigma_{\mu\nu}(1 - \gamma_5)G_{\mu\nu} s,
\end{align*}
\]

where $e_q$ is the electric charge of quark $q$.

In this basis the evolution matrix is $12 \times 12$, with the coefficients $C_i$ related to the NP $c_i^{u,d}$’s in Eq. (B.1) through

\[
\begin{align*}
c_1^{u} &= -\lambda_t (C_5 + C_7) & c_1^{d} &= -\lambda_t \left( C_5 - \frac{1}{2} C_7 \right) \\
c_2^{u} &= -\lambda_t (C_6 + C_8) & c_2^{d} &= -\lambda_t \left( C_6 - \frac{1}{2} C_8 \right) \\
c_3^{u} &= -\lambda_t (C_3 + C_9) & c_3^{d} &= -\lambda_t \left( C_3 - \frac{1}{2} C_9 \right) \\
c_4^{u} &= -\lambda_t (C_4 + C_{10}) & c_4^{d} &= -\lambda_t \left( C_4 - \frac{1}{2} C_{10} \right)
\end{align*}
\]

Note that the $c_{5,6}^{u,d}$ are zero at the $M_W$ scale in our case. We take them to be zero also at the $m_b$ scale, since the electroweak combination $c_{5,6}^{u}(m_b) - c_{5,6}^{d}(m_b)$ is at LO a function only of $c_{5,6}^{u,d}(M_W)$, and the QCD combination $(c_{5,6}^{u}(m_b) + 2c_{5,6}^{d}(m_b))/3$ is mostly dominated by the same combination at $M_W$, taking into account that all NP penguin WC’s are of similar size.

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