Flow Model Considering the Influence of Low Velocity Non-Darcy Flow and Reservoir Connectivity and its Application

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Abstract. During the development of low and ultra-low permeability sandstone reservoir, due to the starting pressure gradient of poor reservoir physical properties, it is difficult to establish effective displacement between oil and water wells, and high voidage replacement ratio and low formation pressure are common phenomena. So it is necessary to analyze the influential factor of high voidage replacement ratio of low and ultra-low permeability reservoir. A one source and one sink flow model considering low velocity non-Darcy flow and unconnected reservoirs was established by using the flow theory of unidirectional flow and radial flow, and the model was solved by numerical method. The results show that the unconnected layers of water well will lead to the increase of invalid water injection; the larger the starting pressure gradient is and the larger the volume proportion of unconnected layer is, the smaller the maximum annual liquid production rate is and the higher the cumulative voidage replacement ratio corresponding to maintaining the maximum liquid production rate is. It is theoretically proved that the existence of unconnected layers and starting pressure gradient are the factors leading to the existence of high voidage replacement ratio in low and ultra-low permeability reservoirs. The study provides a theoretical basis for the adjustment of high-efficiency water injection in reservoirs.

1. A Physical Model of One Source and One Sink Considering Low Velocity Non-Darcy Flow and Reservoir Connectivity

It is assumed that the reservoir is of low and ultra-low permeability, and the fluid flow in the reservoir conforms to the characteristics of low velocity non-Darcy flow. The connected reservoir of water injection well and oil production well is homogeneous and horizontal, and the fluid only flows from the injection well to the production well. The fluid flow in the connected layer conforms to the characteristics of plane unidirectional flow. There is a layer that is not connected with the surrounding oil well in the shooting interval of water injection well. The formation is in the shape of horizontal disc, homogeneous and equal thickness. The fluid flow in the unconnected layer conforms to the characteristics of plane radial flow. There is a layer that is not connected with the surrounding water wells in the shooting interval of oil production wells. The formation is in the shape of horizontal disc, homogeneous and equal thickness. The fluid flow in the unconnected layer conforms to the characteristics of plane radial flow. The physical model is shown in figure 1.
2. Mathematical Model

According to the mechanics of fluid flow through porous media, the flow equations of the four regions of the connected layer, the unconnected layers of the injection well and production well can be deduced as follows:

\[
\begin{align*}
\frac{dp_{w1}(t)}{dt} &= \frac{q_{w1}(t) - \bar{q}(t)}{V_1/2 \cdot \phi \cdot C_i} \\
\frac{dp_{w2}(t)}{dt} &= \frac{q_{w2}(t)}{V_2 \cdot \phi \cdot C_i} \\
\frac{dp_{o1}(t)}{dt} &= \frac{\bar{q}(t) - q_{o1}(t)}{V_1/2 \cdot \phi \cdot C_i} \\
\frac{dp_{o2}(t)}{dt} &= -\frac{q_{o2}(t)}{V_3 \cdot \phi \cdot C_i}
\end{align*}
\]

The cross flow of connected layer is:

\[
\bar{q}(t) = \frac{K_i W h_i}{\mu} \cdot \frac{p_{w1}(t) - p_{o1}(t) - \lambda_w \cdot L_1/2}{L_1/2}
\]

According to the theory of plane unidirectional flow, the injecting rate of connected layer in water injection well can be obtained:

\[
q_{w1}(t) = \frac{K_i W h_i}{\mu} \cdot \frac{p_{w1}(t) - p_{o1}(t) - \lambda_w \cdot L_1/2}{L_1/2}
\]

Liquid production of connecting layer in the production well can be obtained:

\[
q_{o1}(t) = \frac{K_i W h_i}{\mu} \cdot \frac{p_{o1}(t) - p_{w1} - \lambda_w \cdot L_1/2}{L_1/2}
\]

According to the theory of plane radial flow, the injecting rate in the unconnected layer of water injection well can be obtained:
Fluid production of unconnected layers in production wells can be obtained:

\[ q_{w2}(t) = \frac{2\pi K_s h_2}{\mu} \frac{p_{w2}(t) - \rho_{w2}(t) - \lambda_w L_2}{\ln(L_2/r_w)} \]  \hspace{1cm} (5)

The linear differential equation group can be obtained by arranging the system of linear equations (7):

\[
\begin{align*}
\frac{dp_{w1}(t)}{dt} & = \frac{4K_W h_1}{V_i\varphi \mu C_i} \left[ p_{w1}(t) - 2p_{o1}(t) + p_{w1}(t) \right] \\
\frac{dp_{w2}(t)}{dt} & = \frac{2\pi K_s h_2}{V_i\varphi \mu C_1 \ln(L_2/r_w)} \left[ p_{w2}(t) - \rho_{w2}(t) - \lambda_w L_2 \right] \\
\frac{dp_{o1}(t)}{dt} & = \frac{4K_W h_1}{L_i V_i \varphi \mu C_i} \left[ p_{o1}(t) - 2p_{o1}(t) + p_{w1}(t) \right] \\
\frac{dp_{o2}(t)}{dt} & = -\frac{2\pi K_s h_3}{V_i\varphi \mu C_i \ln(L_3/r_w)} \left[ p_{o2}(t) - \rho_{o2}(t) - \lambda_w L_3 \right]
\end{align*}
\]  \hspace{1cm} (7)

Using Euler or Runge Kutta numerical solution to solve the linear differential equation group (8), \( p_{w1}, p_{w2}, p_{o1} \) and \( p_{o2} \) can be obtained; then according to the formulas (3) ~ (6), the injecting rate \( q_{w1}, q_{w2} \) and the liquid production rate \( q_{o1}, q_{o2} \) can be obtained.

3. Application of the Model in the Analysis of High Voidage Replacement Ratio in Low and Ultra-low Permeability Reservoir

Assuming that the total volume of the connected layer and the unconnected layer in the reservoir is unchanged, change the volume ratio of the unconnected layer, and calculate the Injecting water rate and the pressure of the connected layer and the unconnected layer of water well at different time, and draw the curves as shown in Fig. 2 and Fig. 3. It can be seen from the figures that the pressure of the water well increases gradually with the continuous injecting water rate; after injection for a period of time, the pressure tends to be stable due to the continuous extraction of liquid in the connected layer, while the water rate in the unconnected layer has no place to leak out, and the pressure continues to rise, resulting in the pressure of the unconnected layer will be significantly higher than that of the connected layer. This shows that the existence of the unconnected layers of the water well will not only lead to some ineffective water injection due to the water absorption of the reservoir, but also may cause the water to overflow to the non-reservoir due to the serious pressure build-up.
According to the statistics of several groups of data, when the proportion of unconnected volume changes in a certain range under the same permeability and well spacing, there is a maximum value point in the relationship curve between the annual liquid velocity and the injection production ratio. For example, when the starting pressure gradient is 0.01mpa/m and the injection production well spacing is 300m, and the proportion of disconnected volume changes within the range of 0.1~0.6, there is a maximum value point in the relationship curve between the annual liquid production rate and the injection production ratio, as shown in Fig. 4.
According to the statistics of the maximum values of all liquid production velocities under the same permeability and the corresponding unconnected proportion and cumulative injection production ratio, the curve is drawn. It is found that the maximum annual liquid production velocity and unconnected proportion have a linear relationship with a negative slope, that is, the larger the unconnected proportion is, the smaller the maximum annual liquid production velocity is, as shown in Figure 5. It can be seen from the figure that the larger the unconnected proportion and the larger the starting pressure gradient, the smaller the maximum annual liquid production speed; while the larger the unconnected proportion and the larger the starting pressure gradient, the higher the cumulative injection production ratio corresponding to the maximum liquid production speed, as shown in Figure 6. Therefore, the existence of unconnected layers and start-up pressure gradient in low and ultra-low permeability oilfields are the factors leading to the existence of high voidage replacement ratio.
Figure 6. Relation curves between cumulative injection-production ratio and unconnected proportion corresponding to maximum annual liquid production rate

4. Conclusion
According to the characteristics of low and ultra-low permeability reservoirs, a physical model of one source and one sink considering low-speed non-Darcy flow and reservoir non-connectivity is established. The mathematical model is established by using the theory of plane unidirectional flow and plane radial flow. The calculation results of regional pressure and flow can be obtained by solving the equations with numerical method. It is proved theoretically that the existence of the unconnected layer at the end of the water well can not only lead to some ineffective water injection due to the water absorption of the reservoir, but also lead to the water overflow to the non reservoir due to the serious pressure build-up, so as to increase the ineffective water injection. The larger the proportion of unconnected layer volume and the larger the starting pressure gradient, the smaller the maximum annual liquid velocity is, and the higher the cumulative injection production ratio corresponding to maintaining the maximum liquid production velocity is. It shows that the existence of unconnected layers and starting pressure gradient in low and ultra-low permeability oilfields are the factors leading to the existence of high voidage replacement ratio.

5. References
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