Baryon Non-Conservation in Unified Theories, 
in the Light of Supersymmetry and Superstrings

Jogesh C. Pati

Department of Physics
University of Maryland
College Park, MD 20742

Abstract

The first part of this talk presents the general complexion of baryon and lepton number non-conservation that may arise in the context of quark-lepton unification, and emphasizes the importance of searching for both \( (B - L) \)-conserving proton decay modes—i.e. \( p \to \bar{\nu}K^+, \mu^+K^0 \), and \( e^+\pi^0 \) etc.—as well as \( (B - L) \)-violating transitions—i.e. \( p \to e^-\pi^+\pi^+ \), \( n - \bar{n} \)-oscillation and neutrinoless double beta decay.

The second part presents the status of grand unification with and without supersymmetry and spells out the characteristic proton decay modes, which if seen, will clearly show supersymmetry. The main theme of this talk, that follows next, pertains to two issues: (i) the need to remove the mismatch between MSSM and string-unifications; and especially (ii) the need to resolve naturally the problem of rapid proton decay, that generically arises in SUSY unification. Seeking for a natural solution to this second problem, it is noted that SUSY GUTS, including SUSY SO(10) and \( E_6 \), can at best accommodate proton-stability by a suitable choice of the Higgs-multiplets and discrete symmetries, but not really explain it, because they do not possess the desired symmetries to suppress both \( d = 4 \) and \( d = 5 \) proton-decay operators. By contrast, following a recent work, I argue that a class of string-solutions, possessing three families, does possess the desired symmetries, which naturally safeguard proton-stability from all potential dangers. They also permit neutrinos to have desired light masses. This shows that, believing in supersymmetry, superstring is needed just to understand why the proton is so stable. Some implications of the new symmetries, in particular the fact that they still lead to observable rates for proton-decay in the same context in which the mismatches between MSSM and string-unification are removed, are noted.

---

1Research supported by NSF grant No. PHY-9119745. Invited talk, presented at the Oak Ridge International Workshop on ”Baryon Instability”, March, 1996. Email: pati@umdhep.umd.edu
1 Introduction

Non-conservation of baryon and lepton numbers, in particular proton decaying into leptons, is one of the hallmarks of grand unification [1, 2, 3]. In this context, other forms of baryon and lepton non-conservation could be permitted as well – such as neutrinoless double beta decay and neutron-anti-neutron oscillation [4]. Unfortunately, experimental searches for any of these processes have not yet produced a positive result [5, 6, 7]. Nevertheless, it is known that some form of violation of baryon and/or lepton number must have occurred in the early universe to account for the observed excess of baryons over anti-baryons [8, 9], which is in fact crucial to the origin of life. In addition, as discussed below, violation of at least lepton number is strongly suggested by recent neutrino-oscillation experiments [10], which indicate non-vanishing but light masses for the neutrinos. From a purely theoretical viewpoint, to be presented in the following, it turns out that for a number of grand unified as well as superstring-derived models, observation of proton decay, into modes such as $\bar{\nu}_\mu K^+$ and/or $e^+ \pi^0$, should be within the reach of current and forthcoming experimental facilities.

It thus seems most encouraging and timely that SuperKamiokande, with the capability to improve the sensitivity of previous facilities - with respect to searches for both proton decay and neutrino-oscillations – by more than an order of magnitude, has just been turned on; and new facilities like SNO and ICARUS, as well as those designed to search for neutrino-less double beta decay and $n-\bar{n}$ oscillations with improved sensitivity are expected to be available in the near future.

On the theoretical front, the original motivation for a unity of the fundamental forces and that for questioning baryon and lepton-number conservation laws in the context of such unification ideas [1, 2, 3] have remained unaltered. But the perspective with regard to both issues has changed significantly over the last two-and-a-half decades, owing to the introduction of the ideas of supersymmetry [11] and superstrings [12]. In particular, supersymmetry, which seems to be an essential ingredient for higher unification (see discussion in Sec. 5) poses the problem of rapid proton decay. This is because, in accord with the standard model gauge symmetry $SU(2)_L \times U(1)_Y \times SU(3)_C$, a supersymmetric theory in general permits, in contrast to non-supersymmetric ones, dimension 4 and dimension 5 operators which violate baryon and lepton numbers [13]. Thus, unless these operators are suppressed to the extent needed (see discussions later), they pose
the danger of unacceptably rapid proton decay. It turns out that obtaining a natural solution to this problem gets even harder if one wishes to obtain at the same time non-vanishing but light masses for the neutrinos ($\leq$ few eV)\[10].

Bearing these issues in mind, I will first present here a brief summary of the status of non-supersymmetric and supersymmetric grand unification, and next a current perspective on baryon- and lepton-number conservation laws in the light of the ideas of supersymmetry and superstrings. In this latter part, following a recent work by me \[14\], I will also present a natural solution to the problem of rapid proton decay in the context of supersymmetry. It turns out that the solution in question needs certain symmetries which can not arise within conventional grand unification symmetries including $E_6$, but they do arise within superstring-derived three-family solutions. These symmetries play an essential role in safeguarding proton-stability from all potential dangers, to the extent desired, and simultaneously permit neutrinos to have light masses of a nature that is relevant to current experiments. This in turn provides a strong motivation for symmetries of string-origin. The extra symmetries in question lead to extra $Z'$—bosons, whose currents bear the hallmark of string theories. It turns out that there is an interesting correlation between the masses of the $Z'$—bosons and observability of proton decay.

In section 2, I present the need for $B - L$ violation and that for $SU(4)$-color. In section 3, general complexion of $(B,L)$ violations and the characteristic mass-scales associated with different processes are listed, and in Section 4, the main ideas as regards physics beyond the standard model are presented. Section 5 provides the current status of grand unification in the context of supersymmetry and raises the issue of compatibility between MSSM and string-unification. Certain attempts to achieve this compatibility are presented. In section 6, I present the problem of $d = 4$ and $d = 5$ proton-decay operators and propose a solution that naturally safeguards proton-stability from all potential dangers. Some concluding remarks are presented in Section 7.

2 B–L Violation and $SU(4)$-Color

As stated above, the observed excess of baryons over anti-baryons implies that some form of violation of $B$ and/or $L$ must have occurred in the early universe\[8, 9]. Such an excess could arise through $(B,L)$ violating processes which either conserve $B - L$, or violate it. Kuzmin, Rubakov and Shaposnikov pointed out, however, that any excess
generated through \((B - L)\)-conserving processes at very early moments of the universe (corresponding to temperatures \( \gg 1\text{TeV} \), i.e. \( t \ll 10^{-12}\text{sec} \)) is erased subsequently by purely electroweak effects\[14\]. At the same time, generating baryon-excess through electroweak effects alone does not seem to be adequate to account for the observed baryon-asymmetry\[15\]. These considerations suggest that baryogenesis must have its origin (at least in part) in processes which violate \( B - L \).

There is yet an independent motivation for violation of \( B - L \) which stems from considerations of neutrino masses. The reason is as follows. The simplest explanation for non-vanishing but light masses for the neutrinos\[10\] arises in the context of left-right symmetric gauge theories\[16\] and the so-called see-saw mechanism\[17\]. The minimal nonabelian version of a left-right symmetric gauge theory is provided by the symmetry\[1\]

\[
G_{224} = SU(2)_L \times SU(2)_R \times SU(4)^C
\]

which ensures (i) quantization of electric charge, (ii) quark-lepton unification (through \( SU(4) \)-color), as well as (iii) parity-conservation\[16\], at a basic level. Any such theory containing either \( SU(2)_L \times SU(2)_R \) or \( SU(4)^C \) necessarily implies the existence of right-handed neutrinos \((\nu^i_R)\), accompanying the left-hand ones \((\nu^i_L)\). The see-saw mechanism\[17\] assigns heavy Majorana masses \((M^i_R \gg 1\text{TeV})\) to the right-handed neutrinos, though not to the left-handed ones. This involves a breaking of left-right symmetry and thus parity spontaneously at a high scale\[16\]. Now, the Majorana masses for the \( \nu^i_R \)'s, in conjunction with the standard Dirac masses \( m^i_D \), naturally yield very light masses \((\ll m^i_D)\) for the known neutrinos:

\[
m(\nu^i_L) \sim (m^i_D)^2/M^i_R \quad (i = e, \mu, \tau)
\]

Noting that the Dirac mass \( m^i_D \) of the ith neutrino is expected to be comparable to the mass of the ith up-quark (barring QCD renormalization effects), it turns out that these masses for the \( \nu^i_L \)'s have just the right pattern to be relevant to the neutrino-oscillation experiments\[3, 18\] and to \( \nu_\tau \) being hot dark matter, with

\[
m(\nu^i_L) \sim (10^{-8})\text{eV, } 3 \times 10^{-3}\text{eV, } 1 - 10\text{eV} \quad (i = e, \mu, \tau)
\]

if \( M^i_R \sim 10^{12}\text{GeV} \), within a factor of 10 \[19\]. Heavy Majorana masses for \( \nu^i_R \)'s, however, needs spontaneous violation of lepton number L (with \( \Delta B = 0 \)) and therefore of \( B - L \).
at a heavy intermediate scale.

We thus see that both baryogenesis and neutrino masses suggest the need for microscopic violation of $B - L$. One can argue that spontaneous violation of $B - L$ becomes obligatory in theories in which it is gauged. This is because, in these theories, there is a massless spin-1 particle coupled to $B - L$. Such a particle would be inconsistent with the results of E"otvos-type experiments [20], unless it acquires mass spontaneously. Thereby, the associated charge, in this case $B - L$, must be violated spontaneously [1]. Now, the simplest symmetry that gauges $B - L$ is $SU(4)$-color, which unifies quarks and leptons by using the idea that lepton number is the fourth color [1]. In short, $(B - L)$-violation, is an integral feature of any theory containing $SU(4)$-color.

These considerations thus suggest that our very existence, requiring baryogenesis and therefore violation of $B - L$, bears the footprints of certain unification ideas - in particular that of quark-lepton unification through a symmetry-like $SU(4)$-color.

3 General Complexion of (B,L)-Violating Processes and Effective Mass Scales

The $(B, L)$ violating processes which conserve $B - L$ primarily involve only proton decaying into an anti-lepton plus mesons: e.g. $p \rightarrow e^+\pi^0, p \rightarrow \mu^+K^0, p \rightarrow \bar{\nu}K^+, p \rightarrow \bar{\nu}\pi^+$ etc. Once one permits violation of $B - L$, however, a whole new set of processes can in general occur. These include: (i) a nucleon decaying into a lepton plus mesons - i.e. $p \rightarrow e^-\pi^+\pi^+$ and $n \rightarrow e^-\pi^+$ etc., or proton decaying into a lepton + lepton + antilepton + mesons - e.g. $p \rightarrow e^-e^+\nu\pi^+$, (iii) Majorana masses for the neutrinos, (iv) neutrinoless double beta decay and (v) $n - \bar{n}$ oscillation.

Now, Majorana masses for the right handed neutrinos, that are needed for the see-saw mechanism [17], can arise by introducing the pair of Higgs multiplets $\Delta_L$ and $\Delta_R$ which transform as (3,1,10) and (1,3,10) of $G_{224}$ or equivalently a single Higgs multiplet 126 of $SO(10)$ [21], which contains $\Delta_L$ and $\bar{\Delta}_R$. (An alternative choice of Higgs multiplets will be presented later.) Parameters of the Higgs sector can be arranged such that the minimum of the potential induces a large VeV $< \Delta_R >> v_R >> 1$ TeV, while $< \Delta_L >\approx 0$ [22]. In the presence of the Yukawa coupling $h_M(\nu_R^T C^{-1} \nu_R \Delta_R + \nu_L^T C^{-1} \nu_L \Delta_L) + h_c$, such a VeV would induce a heavy Majorana mass for $\nu_R$. As mentioned before, this, in conjunction with the familiar Dirac mass, yields a very light Majorana mass for $\nu_L$. 

4
(see eq. (2)). The VeV of $\Delta_R$ would also break $G_{224}$ into the standard model symmetry $SU(2)_L \times U(1)_Y \times SU(3)'C$. In this way, $\langle \Delta_R \rangle$ breaks lepton number $L$ and $(B-L)$, each by two units. It also breaks parity and quark-lepton unification.

A specific set of diagrams which utilize $\langle \Delta_R \rangle \not= 0$ and/or Majorana masses for the neutrinos and thereby induce some of the (B-L) violating processes mentioned above are shown in figs. 1, 2, and 3. The amplitudes for these processes would, of course, depend upon the effective Yukawa, quartic and gauge couplings entering into the respective vertices, as well as on the masses of the intermediate particles - i.e. those of $\Delta_R$, the color-triplets and color-octets $\xi_3$ and $\xi_8$, as well as $W_R$ and $\nu_R$ - which enter into figs. 1, 2, and 3. Now, in minimal symmetry-breaking schemes of left-right symmetric grand unification models, such as those based on one- (or two-step)- breaking of SO(10), the masses of these intermediate particles typically turn out to be either superheavy $\sim 10^{15}$ GeV, or at least medium heavy $\sim 10^{12} GeV$\cite{23}. In this case, it is easy to verify that the rates of all these $(B, L)$-violating processes would be far too small to be observable. For example, even if effective Yukawa and quartic couplings are of order one, the amplitudes for $qqq \rightarrow l + (q\bar{q})$ (Fig. 2a) and $qqq \rightarrow q\bar{q}q$ (Fig. 3) would be of order $(1/M_{\text{eff}}^5) \leq (10^{-12} GeV)^5 \leq 10^{-60} GeV^{-5}$, where as one would need these amplitudes to be greater than or of order of $10^{-30} GeV^{-5}$ - i.e. $M_{\text{eff}} \leq 10^6 GeV$ (say), for the corresponding processes to have observable rates. Roughly, a similar conclusion can be drawn from a general operator analysis, based on construction of effective invariant operators\cite{24} and dimensional estimate. The results of such estimates for the effective mass-scales that would be necessary for the various $(B, L)$-violating processes to have observable rates are shown in Table I.

It needs to be said that while minimal symmetry-breaking schemes for SO(10) typically lead to effective mass-scales which are considerably larger than those shown in Table I and thus rates that are considerably smaller than what would be observable, there exist viable models of the Higgs system, some involving supersymmetry and thereby at least technically natural fine tuning, where relatively low effective mass-scales of the type shown in Table I and, therefore, observable rates for $n - \bar{n}$ oscillation\cite{25} and/or proton decaying into lepton plus mesons are obtained\cite{26}.

Thus, I believe that, from a broader theoretical perspective, and also because of the great significance of a positive result, if it should show, experimental searches for both
\begin{table}
\centering
\begin{tabular}{|l|l|l|}
\hline
Processes & Selection Rules & Eff. Mass Scale \\
\hline
I. & \( p \to e^+\pi^0, \bar{\nu}\pi^+ \) & \( \Delta B = \Delta L = -1 \) \\
\hline
& \( n \to e^+\pi^- \) & \( \Delta (B - L) = 0 \) \\
& \( p \to \bar{\nu}_\mu K^+, \mu^+K^0 \) & \( \sim 10^{15} \text{GeV} \) \\
\hline
II. & \( p \to e^-\pi^+\pi^+ \) & \( \Delta B = -\Delta L = -1 \) \\
\hline
& \( n \to e^-\pi^+ \) & \( \Delta (B - L) = -2 \) \\
& \( p \to e^-e^+\nu\pi^+ \) & \( \Delta (B + L) = 0 \) \\
& \( n \to e^-e^+\nu \) & \( \sim 10^{5} \text{GeV} \) \\
\hline
III. & \( p \to e^-\nu\pi^+\pi^+ \) & \( \Delta B = -\frac{2}{3} = -1 \) \\
\hline
& \( n \to e^-\nu\pi^+ \) & \( \Delta (B - L) = -4 \) \\
\hline
IV. & \( p \to e^+\nu\bar{\nu} \) & \( \Delta B = \frac{2}{3} = -1 \) \\
\hline
& \( \Delta (B - L) = +2 \) & \( \sim 10^{4.5} \text{GeV} \) \\
\hline
V. & \( n - \bar{n} \) & \( \Delta B = 2, \Delta L = 0 \) \\
\hline
VI. & \( nn \to ppe^-e^- \) & \( \Delta B = 0, \Delta L = 2 \) \\
\hline
\end{tabular}
\caption{Effective Mass-Scales based on operator analysis and dimensional estimates for the corresponding processes to have observable rates.}
\end{table}

(B - L)–conserving, as well as the (B - L)–violating processes shown in Table I, are strongly motivated.

To discuss these issues specifically in the context of grand unification and supersymmetry, I will first present briefly in the next section the motivations for certain theoretical ideas involving physics beyond the standard model, and then discuss grand unification with supersymmetry in the following section.

4 Going Beyond the Standard Model

The standard model of particle physics has brought a good deal of synthesis in our understanding of the basic forces of nature and has turned out to be brilliantly successful in terms of its agreement with experiments. Yet, as recognized for some time, it falls short as a fundamental theory because it introduces some 19 parameters. And it does not explain (i) the coexistence of the two kinds of matter: quarks and leptons; (ii) the coexistence of the electroweak and the QCD forces with their hierarchical strengths \( g_1 \ll g_2 \ll g_3 \), as observed at low energies; (iii) quantization of electric charge; (iv) family-replication; (v) inter and intrafamily mass-hierarchies; and (vi) the origin of diverse mass scales that span over more than 27 orders of magnitude from \( M_{\text{Planck}} \) to \( m_W \) to \( m_e \) to \( m_\nu \). There are in addition the two most basic questions: (vii) how does gravity fit into the whole scheme, especially in the context of a good quantum theory?, and (viii) why
is the cosmological constant so small or zero?

These issues constitute at present some of the major puzzles of particle physics and provide motivations for contemplating new physics beyond the standard model which should shed light on them. The ideas which have been proposed over the last two-and-a-half decades and which do show promise to resolve at least some of these puzzles include the following hypotheses:

1) **Grand Unification:** The hypothesis of grand unification \[1, 2, 3\] which proposes an underlying unity of the fundamental particles and their forces, appears attractive because it explains at once (i) the quantization of electric charge, (ii) the existence of quarks \(Q_e = -Q_p\), and (iii) the existence of the strong, the electromagnetic and the weak forces with \(g_3 \gg g_2 \gg g_1\) at low energies. These are among the puzzles listed above and grand unification resolves all three. By itself, it does not address, however, the remaining puzzles listed above, including the issues of family replication and origin of mass-hierarchies.

2) **Supersymmetry:** This is the symmetry that relates fermions to bosons \[11\]. As a local symmetry, it is attractive because it implies the existence of gravity. It has the additional virtue that it helps maintain a large hierarchy in mass-ratios such as \((m_\phi/M_U) \sim 10^{-14}\) and \((m_\phi/M_{p\ell}) \sim 10^{-17}\), without the need for fine tuning, provided, however, such ratios are put in by hand. Thus it provides a technical resolution of the gauge hierarchy problem, but by itself does not explain the origin of the large hierarchies.

3) **Preonic Substructures with Supersymmetry:** The idea \[27, 28\] that quarks, leptons and Higgs bosons are composites of a common set of constituents called “preons,” which possess supersymmetry, is still unconventional. On the negative side, the preonic approach needs a few unproven, though not implausible, dynamical assumptions as regards the preferred direction of symmetry breaking and saturation of the composite spectrum \[28\]. On the positive side, it has the advantage that it is far more economical in field-content and especially in parameters than the conventional grand unification models, because it has no elementary Higgs boson. Second, and most important, utilizing primarily the symmetries of the theory (rather than detailed dynamics) and the forbiddeness of SUSY-breaking, in the absence of gravity, the preonic approach provides simple explanations for the desired protection of composite quark-lepton masses and at the same time for the origins of family-replication, inter-family mass-hierarchy and
diverse mass scales [27]. It also provides several testable predictions. For this reason, I still keep an open mind about the preonic approach. To maintain a focus, however, I will assume in the rest of this talk that quarks, leptons and Higgs bosons are elementary.

(4) Superstrings: Last but not least, the idea of superstrings [12] proposes that the elementary entities are not truly pointlike but are extended stringlike objects with sizes $\sim (M_{\text{Planck}})^{-1} \sim 10^{-33}$ cm. These theories (which may ultimately be just one) appear to be most promising in providing a unified theory of all matter (spins 0, 1/2, 1, 3/2, 2, ...) and of all the forces of nature including gravity. Furthermore, by smoothing out singularities, they seem capable of yielding a well-behaved quantum theory of gravity.

In principle, assuming that quarks, leptons and Higgs bosons are elementary, a suitable superstring theory could also account for the origin of the three families and the Higgs bosons at the string unification scale, as well as explain all the parameters of the standard model. But in practice, this has not happened as yet. Some general stumbling blocks of string theories are associated with the problems of (i) a choice of the ground state (the vacuum) from among the many solutions and (ii) understanding supersymmetry breaking.

These provide in a nutshell motivations for physics beyond the standard model, which, as it turns out, has strong implications for non-conservation of baryon and lepton numbers. The ideas listed above are, of course, not mutually exclusive. In fact the superstring theories already comprise the idea of local supersymmetry and the central idea of grand unification. In the following, I first recall the status of conventional grand unification with supersymmetry and then discuss the issue of $(B, L)$-nonconservation in the context of these ideas.

5 Grand Unification and Supersymmetry

5.1 The Need for SUSY

It has been known for some time that the dedicated proton decay searches at the IMB and Kamiokande detectors [5] and more recently the precision measurements of the standard model coupling constants (in particular $\sin^2\theta_W$) at LEP [29] put severe constraints on the idea of grand unification. Owing to these constraints, the non-supersymmetric minimal $SU(5)$, and for similar reasons, the one-step breaking minimal non-supersymmetric $SO(10)$-model as well, are now excluded. [30] For example, minimal non-SUSY $SU(5)$
predicts: (i) $\Gamma(p \to e^+\pi^0)^{-1} < 6 \times 10^{31} yr$ and (ii) $\sin^2\theta_W(m_Z)|_{MS} = .214 \pm .004$, where as current experimental data show: (i) $\Gamma(p \to e^+\pi^0)_{\text{expt}}^{-1} > 6 \times 10^{32} yr$ and (ii) $\sin^2\theta_W(m_Z)_{\text{expt}}^{\text{LEP}} = .2313 \pm .0003$. The disagreement with respect to $\sin^2\theta_W$ is reflected most clearly by the fact that the three gauge couplings ($g_1, g_2$ and $g_3$), extrapolated from below, fail to meet by a fairly wide margin in the context of minimal non-supersymmetric $SU(5)$ (see Fig. 4).

But the situation changes dramatically if one assumes that the standard model is replaced by the minimal supersymmetric standard model (MSSM), above a threshold of about $1 TeV$. In this case, the three gauge couplings are found to meet at least approximately, provided $\alpha_3(m_Z)$ is not too low (see figs. 4 and 5 and discussions below). Their scale of meeting is given by

$$M_X \approx 2 \times 10^{16} \quad (\text{MSSM or SUSYSU}(5))$$

(4)

$M_X$ may be interpreted as the scale where a supersymmetric $GUT$ (like minimal SUSY $SU(5)$ or $SO(10)$) breaks spontaneously into the supersymmetric standard model gauge symmetry $SU(2)_L \times U(1) \times SU(3)^c$. Both because a straightforward meeting of the three gauge couplings (in accord with LEP data) is possible only provided SUSY is assumed, and also because SUSY provides at least a technical resolution of the gauge-hierarchy problem by preserving the input small ratio of $(m_W/M_X)$ in spite of quantum corrections, SUSY has emerged as an essential ingredient for higher unification.

With $M_X \sim 2 \times 10^{16} GeV$ and thus lepto-quark gauge boson masses $\sim 10^{16} GeV$, as opposed to $2 \times 10^{14} GeV$ for non-SUSY $SU(5)$, the dimension-6 gauge boson-mediated proton-decay amplitude of order $g^2/M_X^2$ would lead to proton lifetime of order $10^{37 \pm 1}$ years. This is too long to allow observable proton decay. For SUSY grand unification, there are, however, new contributions to proton-decay possibly from dimension 4 and necessarily from dimension 5 operators. These latter arise due to exchange of color-triplet (anti-triplet) Higgsinos, which lie in the $5(\bar{5})$ of $SU(5)$, or in the 10 of $SO(10)$. Since they are damped by just one power of the color-triplet Higgsino mass $m_{HC}$, these new contributions would lead to extra rapid proton decay, unless $M_{HC}$ is sufficiently heavy. For example, for SUSY $SU(5)$ (with low $\tan\beta \leq 2.5$), the experimental limit ($\geq 10^{32} yrs$) on $\Gamma(p \to \bar{\nu}_\mu K^+)^{-1}$ is met provided

$$m_{HC} \geq 2 \times 10^{16} GeV.$$
It is interesting that the requirement of coupling-unification for SUSY SU(5) puts an upper limit on $m_{HC}$ of about $2.4 \times 10^{16}$ GeV [33], which is barely compatible with the lower limit given above (eq.(5)).

### 5.2 SUSY GUT and Proton Decay Modes

Leaving out for a moment the issue of how to ensure naturally such a large mass for the triplet, while its doublet partner is light ($\leq 1 TeV$) (I will return to this issue in Sec. 6), if one takes the attitude that the $d = 4$ operators are forbidden by a discrete symmetry or R-parity (see discussions later), and that the parameters and/or the Higgs-spectrum and the couplings for SUSY SU(5) or SUSY SO(10) can be arranged so that the triplet is appropriately heavy [34], as noted above, proton decay would occur primarily through the $d = 5$ operators (rather than $d = 6$), with an observable rate $\sim (10^{32} - 10^{34} yrs)^{-1}$, which would be induced by the exchange of color-triplet Higgsinos. These bring a new complexion to proton decay modes.

Owing to symmetry of the bosonic components, the effective $d = 5$ operators of the form QQQL/M in the superpotential must involve at least two different families [35]. As a result, the non-vanishing operators relevant for proton decay, which arise effectively through exchange of color triplets, are of the form: (a) $(\phi_{u,t,c}\phi_{d}^{+}\phi_{s}\phi_{\nu\mu})$ and (b) $(\phi_{u}\phi_{di}\phi_{t,c}\phi_{\nu\mu})^{-}$. These give rise to $d = 5$ interactions which are quadratic in both fermion and boson operators. They need to be dressed by wino or gluino-exchange loops to yield effective four-fermion proton-decay interactions of the form $qqql$. Operators of class (a) lead to decay modes such as (see figs. 6)

$$p \rightarrow \bar{\nu}_{\mu}K^{+}, \quad n \rightarrow \bar{\nu}_{\mu}K^{0}$$

and also (see fig. 7)

$$p \rightarrow \bar{\nu}_{\mu}\pi^{+} \text{ and } n \rightarrow \bar{\nu}_{\mu}\pi^{0}, \text{ etc.}$$

which arise primarily through wino-exchange loops. Those of class (b) give rise, through both wino and gluino-exchange, to charged antilepton decay modes (see fig. 8):

$$p \rightarrow \mu^{+}K^{0}, \quad p \rightarrow \mu^{\pm}\pi^{0}, \text{ etc.}$$

Note that these do not include the canonical $p \rightarrow e^{\pm}\pi^{0}$-mode, which is induced, in SUSYSU(5) or minimal SUSYSO(10), primarily by the exchange of heavy gauge
bosons ($\sim 10^{16}\text{GeV}$) and thus strongly suppressed. Given the quark masses and the relevant mixing angles, it turns out that one typically expects the rate of $p \rightarrow \bar{\nu}_\mu K^+$-mode to be larger than that of both $p \rightarrow \bar{\nu}_\mu \pi^+$-mode, by about a factor of 2-10, and of $p \rightarrow \mu^+ K^0$-mode by as much as two to three orders of magnitude, and similarly for neutron-decay [32, 35, 33]. However, given the large top mass, which leads to large $\bar{t} - \bar{u}$ mixing through renormalization group corrections, it turns out that contribution from gluino-exchange can be quite important, especially for large tan $\beta \geq 40$ (such large tan $\beta$ is permitted for SUSY SO(10) though not for SUSY SU(5)). In this case, $p \rightarrow \mu^+ K^0$ can compete favorably and perhaps even dominate over the $p \rightarrow \bar{\nu}_\mu K^+$-mode [37]. In either case, we see that one characteristic signal of SUSY GUT is that strange particle decay modes – i.e. $p \rightarrow \bar{\nu}_\mu K^+$ and/or $p \rightarrow \mu^+ K^0$ – are at least prominent, and under some circumstances dominant [35]. There are regions in SUSY parameter space, pertaining to the masses of the SUSY particles, the mass of the color-triplets and tan $\beta$, for which the non-strange modes involving anti-leptons of the muon-family – i.e. $p \rightarrow \bar{\nu}_\mu \pi^+$ and possibly $p \rightarrow \mu^+ \pi^0$ – can be prominent or even dominant [38], but those involving antileptons of the first family – in particular the $p \rightarrow e^+ \pi^0$ mode – are strongly suppressed. These latter are, however, the dominant modes in non-SUSY GUTS, like minimal non-SUSY SU(5) and non-SUSY SO(10).

Thus observation of strange particle decay modes of the nucleon, like $\bar{\nu}K^+$ or $\mu^+ K^0$, as the dominant or at least prominent modes, would clearly be a strong signal in favor of supersymmetry. Furthermore, observation of certain non-strange decay modes of the proton like $\bar{\nu}_\mu \pi^+$ or $\mu^+ \pi^0$, as opposed to $e^+ \pi^0$, as prominent modes, should also be a strong hint for the dominance of $d = 5$ operators and thus for supersymmetry.

For the sake of completeness, it should be added, however, that there exist SUSY non-GUT models, for which even the $e^+ \pi^0$-mode can be prominent or even dominant. That is the case, for example, for the supersymmetric flipped $SU(5) \times U(1)'$-model [38], for which the mass of the relevant leptoquark gauge boson, determined by the point of meeting of only $\alpha_3$ and $\alpha_2$ (though not $\alpha_1'$), can be much lower than $10^{16}$ GeV. As a result the gauge-boson mediated $d = 6$-operator, which would lead to $e^+ \pi^0$ as the dominant mode, can be the primary source of proton decay and is likely to yield lifetimes of order $10^{32} - 10^{34}$ yrs. The $e^+ \pi^0$-mode can also arise as a prominent or dominant mode through effective $d = 4$ operators, which may be induced in string-derived solutions.
through higher dimensional operators \((d > 5)\) utilizing VEVs of appropriate fields (see discussions in Sec. 6). Thus, observation of \(e^+\pi^0\) as a prominent or dominant decay mode of the proton would certainly disfavor familiar SUSY GUTS, like \(SUSY SU(5)\) or \(SUSYSO(10)\), but it would be perfectly compatible with the dominance of \(induced d = 4\) operators, as mentioned above, or with the flipped \(SU(5)\) -model, and therefore with supersymmetry.

These points, as well as some related ones raised in the previous section, show that \textit{low-energy} studies of \textit{selection-rules} for \((B, L)\) non-conservation, pertaining to proton decay modes as well as \(n - \bar{n}\) oscillation, including a study of whether strange versus non-strange decay modes of the nucleon are prominent, can provide us with much information on possible new physics at very short distances, spanning from \((100\,\text{TeV})^{-1}\) to \((10^{17}\,\text{GeV})^{-1}\). I now return to the question of unification of couplings in MSSM.

5.3 MSSM-Unification and \(\alpha_3\)

Before entering into the question of doublet-triplet splitting and a natural suppression of the \(d = 4\) and \(d = 5\) proton decay operators, it would be useful to probe into the accuracy with which the three couplings meet and discuss the issue of a matching between MSSM and string-unifications. Minimal SUSY \(SU(5)\) predicts \(\sin^2\theta_W(m_Z)|_{\overline{MS}} = .2334 \pm .0036\), by using reasonable range of masses for the SUSY particles, and more importantly a value of \(\alpha_3(m_Z) = .12 \pm .01\) as an input\cite{29, 30, 31}, where the error bar in \(\alpha_3\) is more generous than that allowed by the present world average data (see below). Note that the predicted value of \(\sin^2\theta_W\) would agree with the observed one of \(\sin^2\theta_W(m_Z)_{\text{expt}} = .2313 \pm .0003\)\cite{29}, only provided \(\alpha_3(m_Z)\), is \textit{considerably higher than} .12, which is what is reflected clearly by Fig. 5b (taken from Ref. \cite{29}), if we demand a meeting of the three couplings.

This may be seen more succinctly by using the more accurately determined value of \(\sin^2\theta_W(m_Z) = .2313 \pm .0003\) as an input and thereby getting \(\alpha_3(m_Z)\) as an output\cite{29, 32}. If one ignores possible corrections from GUT-threshold and Planck-scale physics, it turns out that in this case one needs \(\alpha_3(m_Z) > .127\) to achieve coupling-unification within MSSM, assuming \(m_\tilde{q} \sim 1\,\text{TeV}\) and \(m_{1/2} < 500\,\text{GeV}\). Such high values of \(\alpha_3(m_Z) \geq .125\) (say) are, however, incompatible with its world-average value \cite{33},

\[
\alpha_3(m_Z) = .117 \pm .005
\]
which is based on high as well as low-energy determinations of \( \alpha_3 \). The former is based on LEP-data and the latter on the analysis involving \( J/\psi, \Upsilon \), deep inelastic scattering and lattice-calculations.

To summarize the situation with regard to coupling-unification, we see that MSSM, which may be embedded in SUSY SU(5) or SUSY SO(10), fares far better than non-SUSY GUT as regards achieving unification of the three gauge couplings. There does seem to be some discrepancy, however, between the predicted and the world-average values of \( \alpha_3 \), which, if genuine, would imply that the three couplings do not quite meet at a point in the context of MSSM. The discrepancy may be resolved through large corrections to the predicted \( \alpha_3 \) (as much as about \(-0.06\)) which may arise from GUT-threshold and Planck-scale physics. Alternatively, the discrepancy may have its origin in new physics, beyond MSSM, which may manifest at relatively low or intermediate scale. At this stage, not knowing precisely the GUT-threshold and Planck-scale corrections, it seems to me that one can not discard MSSM, nor can one accept it necessarily as the whole truth, representing the correct effective theory below the GUT-scale. It turns out that a resolution of this issue gets merged into a still bigger one pertaining to a matching between MSSM and string-unifications, which in turn has implications on the precise nature of (B,L)-nonconservation. I therefore discuss next the issue of this matching of unification from the two ends and the problem of low \( \alpha_3 \).

5.4 The Problem of Unification-Mismatch and Some Solutions

Achieving a complete unity of the fundamental forces together with an understanding of the origin of the three families and their hierarchical masses is among the major challenges still confronting particle physics. Conventional grand unification, despite all the merits noted in preceding sections, falls short in this regard in that owing to the arbitrariness in the Higgs sector, it does not even unify the Higgs exchange force, not to mention gravity. Superstring theory is the only theory we know that seems capable of removing these shortcomings. It thus seems imperative that the low energy data extrapolated to high energies be compatible with string unification.

It is, however, known \([40, 41]\) that while the three gauge couplings, extrapolated in the context of the minimal supersymmetric standard model (MSSM) meet, at least approximately \([32]\), provided \( \alpha_3(m_Z) \) is not too low (as discussed above), their scale of
meeting, \( M_X \approx 2 \times 10^{16} \, \text{GeV} \), is nearly 20 times smaller than the expected (one–loop level) string–unification scale \( \text{of} \, M_{st} \approx g_{st} \times (5.2 \times 10^{17} \, \text{GeV}) \approx 3.6 \times 10^{17} \, \text{GeV} \).

Babu and I recently noted that very likely there is a second mismatch concerning the value of the unified gauge coupling \( \alpha_X \) at \( M_X \) \cite{43}. Subject to the assumption of the MSSM spectrum, extrapolation of the low energy data yields a rather low value of \( \alpha_X \approx 0.04 \) \cite{12}, for which perturbative physics should work well near \( M_X \). On the other hand, it is known \cite{44} that non–perturbative physics ought to be important for a string theory near the string scale, in order that it may help choose the true vacuum and fix the moduli and the dilaton VEVs. The need to stabilize the dilaton in particular would suggest that the value of the unified coupling at \( M_{st} \) in four dimensions should be considerably larger than 0.04 \cite{45}. At the same time, \( \alpha_{st} \) should not be too large, because, if \( \alpha_{st} \gg 1 \), the corresponding theory should be equivalent by string duality \cite{16} to a certain weakly coupled theory that would still suffer from the dilaton runaway problem \cite{17}. Furthermore, \( \alpha_{st} \) at \( M_{st} \) should not probably be as large as even unity, or else, the one–loop string unification relations for the gauge couplings \cite{12} would cease to hold near \( M_{st} \) (e.g. in this case, the string threshold corrections for the gauge couplings should not be too large) and the observed (approximate) meeting of the three couplings would have to be viewed as an accident. \textit{In balance, therefore, the preceding discussions suggest that an intermediate value of the string coupling \( \alpha_{st} \sim .15 - .25 \) (say) at \( M_{st} \) in four dimensions, which might be large enough to stabilize the dilaton, but not so large as to disturb significantly the coupling unification relations, is perhaps the more desired value} \cite{43}. In short, the desired unification of the gauge couplings may well be semi-perturbative, rather than perturbative, in character. It is thus a challenge to find a suitable variant or alternative to MSSM which removes the mismatch not only with regard to the meeting point \( M_X \), but also with regard to the value of \( \alpha_X \), as mentioned above.

A third relevant issue noted in the last section is that the world average value of \( \alpha_3(m_Z) = 0.117 \pm 0.005 \) \cite{39} seems to be low compared to its value that is needed for MSSM unification. I note briefly a few alternative suggestions which have been proposed to address these issues, pertaining to removing the mismatch between MSSM and string-unifications.

**Matching Through String-Duality:** One suggestion in this regard is due to
Witten [48]. Using the equivalence of the strongly coupled heterotic $SO(32)$ and the $E_8 \times E_8$ superstring theories in $D = 10$, respectively to the weakly coupled $D = 10$ Type I and an $M$–theory, he observed that the 4-dimensional gauge coupling and $M_{st}$ can both be small, as suggested by MSSM extrapolation of the low energy data, without making the Newton’s constant unacceptably large. While this observation opens up a new perspective on string unification, its precise use to make $\alpha_{st} \approx 0.04$ at $M_{st}$ would seem to run into the dilaton runaway problem as in fact noted in Ref. [48].

**Matching Through SUSY GUT:** A second way in which the mismatch between $M_X$ and $M_{st}$ could be resolved is if superstrings yield an intact grand unification symmetry like $SU(5)$ or $SO(10)$ with the right spectrum – i.e., three chiral families and a suitable Higgs system including an adjoint Higgs at $M_{st}$ [49], and if this symmetry would break spontaneously at $M_X \approx (1/20$ to $1/50)M_{st}$ to the standard model symmetry. However, as yet, there is no realistic (or close-to realistic) string–derived GUT model [49]. Furthermore, for such solutions, there is the likely problem of doublet-triplet splitting and rapid proton decay (see discussions later).

**Matching Through Intermediate Scale Matter:** A third alternative is based on string–derived standard model–like gauge groups. It attributes the mismatch between $M_X$ and $M_{st}$ to the existence of new matter with intermediate scale masses ($\sim 10^9 – 10^{13} \text{ GeV}$), which may emerge from strings [50]. Such a resolution is in principle possible, but it would rely on the delicate balance between the shifts in the three couplings and on the existence of very heavy new matter which in practice cannot be directly tested by experiments. Also, within such alternatives, as well as those based on non–standard hypercharge normalization [51] and/or large string–scale threshold effects [52], $\alpha_X$ typically remains small ($\sim 0.04$), which is not compatible with the need for a larger $\alpha_X$, as suggested above.

**Matching Through ESSM – An Example of Semi-Perturbative Unification:** Babu and I recently noted [43] that all three issues raised above – i.e. (i) understanding fermion mass-hierarchy, (ii) finding a suitable alternative to MSSM which would be compatible with string-unification, and (iii) accommodating low $\alpha_3(m_Z)$ can have a common resolution through a certain variant of the MSSM spectrum, which was proposed some time ago [27]. The variant spectrum extends the MSSM spectrum by adding to it two light vector-like families $Q_{L,R} = (U,D,N,E)_{L,R}$ and $Q'_{L,R} = (U',D',N',E')_{L,R}$, two
Higgs singlets ($H_S$ and $H_λ$) and their SUSY partners, all as light as about 1 TeV and as heavy as about 100 TeV [53]. We refer to this variant as the Extended Supersymmetric Standard Model (ESSM). The combined sets ($Q_L|Q'_R$) and ($Q_R|Q'_L$) transform as $16$ and $\overline{16}$ of $SO(10)$ respectively. Barring addition of singlets, one can argue that ESSM is in fact the only extension of the MSSM, containing complete families of quarks and leptons, that is permitted by measurements of the oblique electroweak parameters and $N_ν$ on the one hand, and renormalization group analysis on the other hand [43]. While the derivation of such a spectrum in string theories, is not yet in hand, it is worth noting that the emergence of pairs of $27 + \overline{27}$ of $E_6$ or $\overline{16} + 16$ of $SO(10)$ in addition to chiral multiplets is rather generic in string theories [54, 55].

Babu and I performed a two-loop renormalization-group analysis for the running of the three gauge couplings for ESSM. In this analysis, we included the contributions of the Yukawa couplings of the two vector-like families with themselves and with the three chiral families. Such a pattern of Yukawa coupling, which leads to a see-saw mass-matrix for the 3 chiral and two vector-like families, is motivated by the inter-family mass-hierarchy [27, 56]. All the relevant (unknown) Yukawa couplings were assumed to have their fixed point values at the electroweak scale, so that the analysis is essentially parameter-free, except for the input gauge couplings and the variation in the ESSM-spectrum. Remarkably enough, we found that the three gauge couplings $α_{1,2,3}$ meet, even perfectly for many cases, for a fairly wide variation in the ESSM spectrum. A typical case of this meeting is shown in Fig. 9. The corresponding values of $α_X$, $M_X$ and $α_3(m_Z)$ |$_{MS}$, with vector-like quarks having masses $m_Q ≈ 1.5 - 2TeV$, are found to be [13]:

$$α_X ≈ .2 - .25, \quad M_X ≈ 10^{17}GeV, \quad α_3(m_Z)_{MS} ≈ .112 - .118$$ (10)

Raising $m_Q$ to $10 - 50 TeV$ would lower $α_X$ to about $.18 - .16$, and $M_X$ by about a factor of 2, leaving $α_3$ in the range shown above.

Thus we see that ESSM leads to coupling–unification, with an intermediate value of $α_X$, and a lower value of $α_3(m_Z)$ than that needed for MSSM unification, just as desired. The resulting $M_X ≈ 10^{17} GeV$, though higher than the MSSM value, is still lower than the one–loop string–unification scale of Ref. [12], which, for $α_X ≈ 0.2$, yields $M_{st} ≈ 6 \times 10^{17} GeV$. Considering the proximity of $M_X ≈ 10^{17} GeV$ to the expected string scale of $(5 − 8) \times 10^{17} GeV$, however, it would seem that contributions from the

16
infinite tower of heavy string-states, which have been neglected in the running of \( \alpha_i \)'s, quantum gravity and three and higher-loop effects \[57\] may well play an important role, especially for intermediate \( \alpha_X \approx .2 \), in bridging the relatively small gap between \( M_X \) and \( M_{st} \).

As a general comment, with an intermediate unified coupling \( \alpha_X \sim .2 \), the increased, though not overwhelming, importance of three and higher loop-effects, compared to the case of MSSM, cannot of course be avoided. Yet, for such a case, the three gauge couplings are still fairly weak \( (< .15, \text{say}) \) for most of the region of extrapolation – i.e. for \( Q < 10^{15.5} \text{ GeV (say)} \) (see Ref. \[13\]). As a result, perturbation theory is still fairly reliable, all the way up to \( M_X \), and the benefits of calculability are not lost for this case, in contrast to the case of a non-perturbative unification \[58\]. Thus, ESSM presents a good example of semi-perturbative unification, that is viable, and also seems desirable, if one wishes to stabilize the dilaton without losing the benefits of coupling-unification \[13\]. The main reason for devoting some discussion to these issues is that intermediate \( \alpha_X \sim .16 - .2 \) turns out to be crucial to ensure observable rate for proton decay in the same context in which rapid proton decay is prevented. This is what I discuss next.

## 6 The puzzle of proton–stability in Supersymmetry

In this section, I first outline the problem of the unsafe \( d = 4 \) as well as the color-triplet mediated and/or gravity-linked \( d = 5 \) proton decay operators, and then, following a recent paper by me \[14\], present a solution which suppresses these unsafe operators naturally, so as to ensure proton–stability, in accord with observation. The solution highlights the need for certain symmetries which cannot arise in conventional grand unification, but which do arise in string theories.

### 6.1 The Problem and Attempted Solutions in SUSY GUTS

As mentioned before, in accord with the standard model gauge symmetry \( SU(2)_L \times U(1)_Y \times SU(3)_C \), a supersymmetric theory in general permits, in contrast to non-supersymmetric ones, dimension 4 and dimension 5 operators which violate baryon and lepton numbers \[13\]. Using standard notations, the operators in question which may arise in the superpotential are as follows:

\[
W = [\eta_1 U\bar{D}\bar{D} + \eta_2 Q\bar{L}\bar{D} + \eta_3 L\bar{L}\bar{E}]
\]
\[ \lambda_1 QQQL + \lambda_2 UUDE + \lambda_3 LLH_2 H_2]/M. \] (11)

Here, generation, \(SU(2)_L\) and \(SU(3)_C\) indices are suppressed. \(M\) denotes a characteristic mass scale. The first two terms of \(d = 4\), jointly, as well as the \(d = 5\) terms of strengths \(\lambda_1\) and \(\lambda_2\), individually, induce \(\Delta(B-L) = 0\) proton decay with amplitudes \(\sim \eta_1 \eta_2/m_\tilde{q}^2\) and \((\lambda_{1,2}/M)(\delta)\) respectively, where \(\delta\) represents a loop-factor. Experimental limits on proton lifetime turns out to impose the constraints: \(\eta_1 \eta_2 \leq 10^{-24}\) and \((\lambda_{1,2}/M) \leq 10^{-25}\) GeV\(^{-1}\) [58]. Thus, even if \(M \sim M_{\text{string}} \sim 10^{18}\) GeV, we must have \(\lambda_{1,2} \leq 10^{-7}\), so that proton lifetime will be in accord with experimental limits.

Renormalizable, supersymmetric standard-like and \(SU(5)\) [60] models can be constructed so as to avoid, by choice, the \(d = 4\) operators (i.e. the \(\eta_{1,2,3}\)-terms) by imposing a discrete or a multiplicative \(R\)-parity symmetry: \(R \equiv (-1)^{3(B-L)}\), or more naturally, by gauging \(B - L\), as in \(G_{224} \equiv SU(2)_L \times SU(2)_R \times SU(4)_C\) or \(SO(10)\). Such resolutions, however, do not in general suffice if we permit higher dimensional operators and intermediate scale VEVs of fields which violate \((B-L)\) and \(R\)-parity (see below). Besides, \(B - L\) can not provide any protection against the \(d = 5\) operators given by the \(\lambda_1\) and \(\lambda_2\) - terms, which conserve \(B-L\). These operators are, however, expected to be present in any theory linked with gravity, e.g. a superstring theory, unless they are forbidden by some new symmetry.

As mentioned in Section 5, for SUSY grand unification models, there is the additional problem that the exchange of color-triplet Higgsinos which occur as partners of electroweak doublets (as in \(5 + \bar{5}\) of \(SU(5)\)) induce \(d = 5\) proton-decay operators [13]. Thus, allowing for suppression of \(\lambda_1\) and \(\lambda_2\) (by about \(10^{-8}\)) due to the smallness of the relevant Yukawa couplings, the color-triplets still need to be superheavy (\(\geq 10^{17}\) GeV) to ensure proton-stability [58], while their doublet partners must be light (\(\leq 1\) TeV). This is the generic problem of doublet-triplet splitting that faces all SUSY GUTS. Basically, four types of solutions to this problem have been proposed in the context of SUSY grand unification. They are as follows:

(i) The Case of Extreme Fine Tuning: In this case, utilizing cubic coupling in the superpotential of the form \(W = A \bar{5}_{H'} \cdot <24_H> \cdot 5_H + B \bar{5}_{H'} \cdot <1_H> \cdot .5_H\), one can assign masses to the triplets and the doublets in \(\bar{5}\) and \(5\) of \(SU(5)\) through the VEVs of both \(24_H\) (which is traceless) and \(1_H\). By arranging these two contributions to add for the triplets, but to almost cancel for the doublets, to one part in \(10^{14}\), one can keep the
doublets appropriately light, and the triplets superheavy \(50\). This case, however, needs extreme fine tuning.

(ii) The Missing Partner Mechanism \[61\]: In this case, by introducing suitable large-size Higgs multiplets, such as \(50_H + 50_{\bar{H}} + 75_H\), in addition to \(5_H + \overline{5}_H\) of \(SU(5)\), and allowing couplings of the form \(W = C 5_H \cdot \overline{50}_H \cdot <75_H> + D \overline{5}_H \cdot 50_H <75_H>\), one can give superheavy masses to the triplets (anti-triplets) in \(5(\overline{5})\) by pairing them with anti-triplets (triplets) in \(50(\overline{50})\). But there do not exist doublets in \(50(\overline{50})\) to pair up with the doublets in \(5(\overline{5})\), which therefore can remain light, provided a direct superheavy \(5 \cdot \overline{5}\) mass-term is prevented.

(iii) The Dimopoulos-Wilczek Mechanism \[62\]: Utilizing the fact that the VEV of \(45_H\) of \(SO(10)\) does not have to be traceless (unlike that of \(24_H\) of \(SU(5)\)), one can give mass to color-triplets and not to doublets in the 10 of \(SO(10)\), by arranging the VEV of \(45_H\) to be proportional to diag \((x,x,x,o,o)\), and introducing a coupling of the form \(\lambda 10_{H1} \cdot 45_H \cdot 10_{H2}\) in \(W\). Two 10’s are needed owing to the anti-symmetry of 45. Because of two 10’s, this coupling would leave two pairs of electroweak doublets massless. One must, however, make one of these pairs superheavy, by introducing a term like \(M 10_{H2} \cdot 10_{H2}\) in \(W\), so as not to spoil the successful prediction of \(\sin^2 \theta_W\) of SUSY GUT. In addition, one must also ensure that only \(10_{H1}\) but not \(10_{H2}\) couple to the light quarks and leptons, so as to prevent rapid proton decay. All of these can be achieved by imposing suitable discrete symmetries. There is, however, still some question as to whether the triplets can be sufficiently heavy \((\geq 10^{17}) GeV\) without conflicting with unification of the gauge couplings. One can avoid this issue altogether and ensure a strong suppression of color-triplet mediated proton decay, provided one introduces additional 45’s and 10 of \(SO(10)\) (see e.g. Babu and Barr, Ref. \[62\]).

(iv) The Case of Higgses as Pseudogoldstone Bosons \[63\]. A new line of approach, though not a complete model, has been proposed recently, in which suitable choice of discrete symmetries lead to accidental global symmetries of the Higgs-potential, which are broken explicitly by the Yukawa couplings of the model. The associated pseudogoldstone bosons are identified with the Higgs doublets. While this idea has some nice features, because it proposes to use only adjoint and fundamental representations for the Higgs scalars, the full consistenency of this idea in the context of a complete model in realizing electroweak-scale masses for the Higgs-doublets, and a desirable pattern of masses
for the fermions, remains to be shown. Furthermore, in this case, one needs to make heavy use of discrete symmetries to (a) ensure the accidental global symmetry of the Higgs-potential, (b) obtain a desired pattern of masses for the fermions, and (c) suppress undesirable flavor-changing neutral currents. Thus, the prospect of a natural origin of this class of models (i.e. of all the necessary discrete symmetries) from an underlying theory, like a string theory, is far from clear. Furthermore, the question of a natural suppression of the $d = 4$ -operators and of the gravity-linked $d = 5$ operators, of course, still remains open even in the context of this class of models.

In summary, solutions to the problem of doublet-triplet splitting needing either unnatural fine-tuning as in SUSY $SU(5)$ [60], or suitable choice of large number and/or large size Higgs multiplets and/or choice of discrete symmetries as in SUSY $SO(10)$ [62], missing-partner [61] and psuedogoldstone models [63], are technically feasible. They, however, do not seem to be compelling by any means because they have been invented for the sole purpose of suppressing proton-decay, without a deeper reason for their origin. Furthermore, such solutions are not easy to realize, and to date have not been realized, in string-derived grand unified theories [49].

These considerations show that, in the context of supersymmetry, the extraordinary stability of the proton is a major puzzle. The question in fact arises: Why does the proton have a lifetime exceeding $10^{39}$ sec, rather than the apparently natural value (for supersymmetry) of less than 1 sec? As such, the known longevity of the proton deserves a natural explanation. Rather than being merely accommodated, it ought to emerge as a compelling feature, owing to symmetries of the underlying theory, which should forbid, or adequately suppress, the unsafe $d = 4$ as well as $d = 5$ operators in Eq. (11). As discussed below, the task of finding such symmetries becomes even harder, if one wishes to assign non-vanishing light masses ($\lesssim$ few eV) to neutrinos. Following Ref. 14, I will present in this section, a class of solutions within supersymmetric theories, which (a) naturally ensure proton-stability, to the extent desired, and (b) simultaneously permit neutrinos to acquire light masses, of a nature that is relevant to current experiments [10, 18]. These solutions need either $I_{3R}$ and $B - L$ as separate gauge symmetries, as well as one extra abelian symmetry that lies beyond even $E_6$ [21]; or the weak hypercharge $Y (= I_{3R} + (B - L)/2)$ accompanied by two extra symmetries beyond those of $E_6$. The interesting point is that while the extra symmetries in question can not arise within conventional
grand unification models, including $E_6$, they do arise within a class of string-derived three generation solutions. This in turn provides a strong motivation for symmetries of string-origin. The extra symmetries lead to extra $Z'$-bosons, whose currents would bear the hallmark of string theories. It turns out that there is an interesting correlation between the masses of the $Z'$-bosons and observability of proton decay.

6.2 The need for symmetries beyond $SO(10)$ and $E_6$:

In what follows, I assume that operators (with $d \geq 4$), scaled by appropriate powers of Planck or string-scale mass, exist in the effective superpotential of any theory which is linked to gravity, like a superstring theory [12, 64], unless they are forbidden by the symmetries of the effective theory. For reasons discussed above, the class of theories – string-derived or not – which contains $B-L$, as in $G_{2311} \equiv SU(2)_L \times SU(3)^C \times U(1)_{I_{1R}} \times U(1)_{B-L}$, as a symmetry, the $d = 4$ operators in Eq.(11) are naturally forbidden. They can in general appear however through non-renormalizable operators if there exist VEVs of fields which violate $B-L$. This is where neutrino-masses become relevant. As discussed in Sec. 2, the familiar see-saw mechanism [17] that provides the simplest reason for known neutrinos $\nu^i_L$’s to be so light, assigns heavy Majorana masses $M^i_R$ to the right-handed neutrinos $\nu^i_R$, which in turn need spontaneous violation of $B-L$ at a heavy intermediate scale.

If $B-L$ is violated by the VEV of a field by two units, an effective $R$-parity would still survive [65], which would forbid the $d = 4$ operators. That is precisely the case for the multiplet 126 of $SO(10)$ or $(1,3,\overline{10})$ of $G_{224}$, which have commonly been used to give Majorana masses to $\nu_R$’s. Recent works show, however, that 126 and very likely $(1,3,\overline{10})$, as well, are hard – perhaps impossible – to obtain in string theories [66]. We, therefore, assume that this constraint holds. It will become clear, however, that as long as we demand safety from both $d = 4$ and $d = 5$ operators, our conclusion as regards the need for symmetries beyond $E_6$, would hold even if we give up this assumption.

Without 126 of Higgs, $\nu_R$’s can still acquire heavy Majorana masses utilizing product of VEVs of sneutrino-like fields $\overline{N}_R$ and $\overline{N}^i_L$, which belong to $16_H$ and $\overline{10}_H$ respectively. (as in Ref. [54]; see also [55].) In this case, an effective operator of the form $16 \cdot 16 \cdot \overline{10}_H \cdot \overline{10}_H / M$ in $W$, that is allowed by $SO(10)$, would induce a Majorana mass $(\sigma_R C^{-1} \sigma^L_R) (\langle N^i_L \rangle \langle N^i_L \rangle / M) + h c$ of magnitude $M_R \sim 10^{12.5}$ GeV, as desired, for
\[ \langle \tilde{N}_L \rangle \sim 10^{15.5} \text{GeV} \] and \( M \sim 10^{18} \text{GeV} \). However, consistent with \( SO(10) \) symmetry and therefore its subgroups, one can have an effective \( d = 5 \) operator in the superpotential \( 16^a \cdot 16^b \cdot 16^c \cdot 16_H/M \). This would induce the terms \( U_R D_R D_R \langle \tilde{N}_R \rangle/M \) and \( Q_L D \langle \tilde{N}_R \rangle/M \) in \( W \) (see Eq.(11)) with strengths \( \sim \langle \tilde{N}_R \rangle/M \sim 10^{15.5}/10^{18} \sim 10^{-2.5} \), which would lead to unacceptably short proton lifetime \( \sim 10^{-6} \) yrs. We thus see that, without having the 126 or \((1,3,\overline{10})\) of Higgs, \( B - L \) and therefore \( SO(10) \) does not suffice to suppress even the \( d = 4 \) - operators adequately while giving appropriate masses to neutrinos. As mentioned before, \( B - L \) does not of course prevent the \( d = 5, \lambda_1 \) and \( \lambda_2 \) - terms, regardless of the Higgs spectrum, because these terms conserve \( B - L \).

To cure the situation mentioned above, we need to utilize symmetries beyond those of \( SO(10) \). Consider first the presence of at least one extra \( U(1) \) beyond \( SO(10) \) of the type available in \( E_6 \), i.e. \( E_6 \to SO(10) \times U(1)_\psi \), under which 27 of \( E_6 \) branches into \((16_1 + 10_{-2} + 1_4)\), where 16 contains \((Q,L \mid U_R, D_R, E_R, \overline{E_R})\), with \( Q_\psi = +1 \); while 10 contains the two Higgs doublets \((H_1, H_2)^{(0,-2)}\) and a color-triplet and an anti-triplet \((H_3^{(-2/3,-2)} + H_3^{(2/3,-2)})\), where the superscripts denote \((B - L, Q_\psi)\). Assume that the symmetry in the observable sector just below the Planck scale is of the form:

\[
G_{\text{obs}} = [G_{\text{fc}} \subseteq SO(10)] \times \hat{U}(1)_\psi \times [U(1)'s]. \tag{12}
\]

It is instructive to first assume that \( \hat{U}(1)_\psi = U(1)_\psi \) of \( E_6 \) \footnote{18} and ignore all the other \( U(1)'s \). Ignoring the doublet-triplet splitting problem for a moment, we allow the flavor-color symmetry \( G_{\text{fc}} \) to be as big as \( SO(10) \). The properties of the operators in \( W \) given in Eq.(7), and the fields \( \overline{N}_R \), \((H_1, H_2)\) and the singlet \( \chi \subset 27 \), under the charges \( Y, I_{3R}, B - L, Q_\psi \) and \( Q_T \equiv Q_\psi - (B - L) \), are shown in Table 2. We see that the \( d = 4 \) operators \((\eta_i \text{-terms})\) carry nonvanishing \( B - L, Q_\psi \) and \( Q_T \), and are thus forbidden by each of these symmetries. Furthermore, note that when \( \overline{N}_R \subset 16 \) and \( \overline{N}_L \subset 16 \) acquire VEV, the charges \( I_{3R}, B - L \) as well as \( Q_\psi \) are broken, but \( Y \) and \( Q_T \) are preserved. Now \( Q_T \) would be violated by the VEVs of \((H_1, H_2) \sim 200 \text{ GeV} \) and of the singlets \( \chi^{(27)} \) and \( \chi^{(27)} \). Assume that \( \chi \) and \( \chi \) acquire VEVs \( \sim 1 \text{ TeV} \) through a radiative mechanism, utilizing Yukawa interactions, analogous to \((H_1, H_2)\). The \( d = 4 \) operators can be induced through nonrenormalizable terms of the type \[ 16 \cdot 16 \cdot 16 \cdot [\langle \overline{N}_R \subset 16 \rangle/M], [\langle 10 \rangle \langle 10 \rangle / M^2 \text{ or } \langle \overline{\chi} \subset 27 \rangle / M], \] where the effective couplings respect \( SO(10) \) and \( U(1)_\psi \). Thus we get \( \eta_i \leq (10^{15.5}/10^{18})(1 \text{ TeV} / 10^{18} \text{ GeV}) \sim 10^{-18} \), which is below the limit of \( \eta_1 \eta_2 \leq 10^{-24} \). Thus, \( B - L \) and \( Q_\psi \), arising within \( E_6 \), suffice
to control the $d = 4$ operators adequately, while permitting neutrinos to have desired masses.

Next consider the $LLH_2H_2$-term. While it violates $I_{3R}$, $B - L$ and $Q_\psi$, it is the only term that is allowed by $Q_T$. Such a term can arise through an effective interaction of the form $16 \cdot 16 \cdot (H_2 \subset 10)^2 \cdot (\langle N_R \subset 16 \rangle^2)/M^3$, and thus with a strength $\sim 10^{-5} \cdot (10^{18}\text{GeV})^{-1}$, which is far below the limits obtained from $\nu$-less double $\beta$-decay.

Although the two $d = 5$ operators $QQQL/M$ and $\overline{U} \overline{U} \overline{D} \overline{E}/M$ are forbidden by $Q_\psi$ and $Q_T$, the problem of these two operators still arises as follows. Even for a broken $E_6$-theory, possessing $U(1)_\psi$-symmetry, the color-triplets $H_3$ and $H'_3$ of 27 still exist in the spectrum. They are in fact needed to cancel the anomalies in $U(1)^3_\psi$ and $SU(3)^2 \times U(1)_\psi$ etc. They acquire masses of the form $M_3H_3H'_3 + hc$ through the VEV of singlet $\langle \chi \rangle$ which breaks $Q_\psi$ and $Q_T$ by four units. With such a mass term, the exchange of these triplets would induce $d = 5$ proton-decay operators, just as it does for SUSY $SU(5)$ and $SO(10)$. We are then back to facing either the problem of doublet-triplet splitting (i.e. why $M_3 \geq 10^{17}$ GeV) or that of rapid proton-decay (for $M_3 \sim 1$ TeV). In this sense, while the $E_6$-framework, with $U(1)_\psi$, can adequately control the $d = 4$ operators and give appropriate masses to the neutrinos (which $SO(10)$ cannot), it does not suffice to control the $d = 5$ operators, owing to the presence of color-triplets. As we discuss below, this is where string-derived solutions help in preserving the benefits of a $Q_\psi$-like charge, while naturally eliminating the dangerous color-triplets.

| Operators          | $I_{3R}$ | $B - L$ | $Y$   | $Q_\psi$ | $Q_T$ |
|--------------------|----------|---------|-------|----------|-------|
| $\overline{U} \overline{D} \overline{D}, QL\overline{D}$ | $1/2$    | $-1$    | $0$   | $3$      | $4$   |
| $LL\overline{E}$   | $1/2$    | $-1$    | $0$   | $3$      | $4$   |
| $QQQL/M$           | $0$      | $0$     | $0$   | $4$      | $4$   |
| $\overline{U} \overline{U} \overline{D} \overline{E}/M$ | $0$      | $0$     | $0$   | $4$      | $4$   |
| $LLH_2H_2/M$       | $1$      | $-2$    | $0$   | $-2$     | $0$   |
| $\overline{N}_R$   | $-1/2$   | $1$     | $0$   | $1$      | $0$   |
| $(H_1, H_2)$       | $(-1/2, 1/2)$ | $0$   | $(-1/2, 1/2)$ | $-2$ | $-2$ |
| $\chi$             | $0$      | $0$     | $0$   | $4$      | $4$   |

Table 2:
6.3 Doublet-Triplet Splitting In String Theories: A Preference 
For Non-GUT Symmetries:

While the problem of doublet-triplet splitting does not have a compelling solution within 
SUSY GUTS and has not been resolved within string-derived GUTS [19], it can be 
solved quite simply within string-derived standard-like [69, 70] or the \(G_{224}\)-models [54], 
because in these models, the electroweak doublets are naturally decoupled from the color-
triplets after string-compactification. As a result, invariably, the same set of boundary 
conditions (analogous to “Wilson lines”) which break \(SO(10)\) into a standard-like 
gauge symmetry such as \(G_{2311}\), either project out, by GSO projections, all color-triplets \(H_3\) and 
\(H'_3\) from the “massless”- spectrum [70], or yield some color-triplets with extra \(U(1)\) 
charges which make them harmless [69], because they can not have Yukawa couplings 
with quarks and leptons. In these models, the doublet triplet splitting problem is thus 
solved from the start, because the dangerous color - triplets simply do not appear in the 
massless spectrum [71].

At the same time, owing to constraints of string theories, the coupling unification 
relations hold [12] for the string-derived standard-like or \(G_{224}\)-models, just as they do 
for GUT-models. Furthermore, close to realistic models have been derived from string 
theories only in the context of such standard-like [69, 70], flipped \(SU(5) \times U(1)\) [38] 
and \(G_{224}\) models [54], but not yet for string-derived GUTS [19]. For these reasons, we 
will consider string-derived non-GUT models, as opposed to string-GUT-models, as the 
prototype of a future realistic string model, and use them as a guide to ensure (a) proton 
- stability and (b) light neutrino masses [72].

Now, if we wish to preserve the benefits of the charge \(Q_\psi\) (noted before), and still 
eliminate the color-triplets as mentioned above, there would appear to be a problem, 
because, without the color-triplets, the incomplete subset consisting of \(\{16_1+(2,2,1)_{-2}+ 
1_4\} \subset 27\) of \(E_6\) would lead to anomalies in \(U(1)_\psi^3, SU(3)^2 \times U(1)_\psi\) etc. This is where 
symmetries of string-origin come to the rescue.

6.4 Resolving the Puzzle of Proton-longevity through string-
symmetries

The problem of anomalies (noted above) is cured within string theories in a variety 
of ways. For instance, new states beyond those in the \(E_6\)-spectrum invariably appear in the
string-massless sector which contribute toward the cancellation of anomalies, and only certain combinations of generators become anomaly-free. To proceed further, we need to focus on some specific solutions. For this purpose, we choose to explore the class of string-derived three generation models, obtained in Refs. [69] and [70], which is as close to being realistic as any other such model that exists in the literature (see e.g. Refs. [54] and [38]). In particular, they seem capable of generating qualitatively the right texture for fermion mass-matrices and CKM mixings. We stress, however, that the essential feature of our solution [14], relying primarily on the existence of extra symmetries analogous to $U(1)\psi$, is likely to emerge in a much larger class of string-derived solutions.

After the application of all GSO projections, the gauge symmetry of the models developed in these references, at the string scale, is given by:

$$G_{st} = [SU(2)_L \times SU(3)_C \times U(1)_{3_R} \times U(1)_{B-L}] \times [G_M = \prod_{i=1}^{6} U(1)_i] \times G_H.$$  \hspace{1cm} (13)

Here, $U(1)_i$ denote six horizontal-symmetry charges which act non-trivially on the three families and distinguish between them. In the models of Refs. [69], [70], $G_H = SU(5)_H \times SU(3)_H \times U(1)^2_{H}$. There exists “hidden” matter which couples to $G_H$ and also to $U(1)_i$.

A partial list of the massless states for the solution derived in Ref. [69], together with the associated $U(1)_i$-charges, is given in Table 3. The table reveals the following features:

(i) There are three families of quarks and leptons (1, 2 and 3), each with 16 components, including $\nu_R$. Their quantum numbers under the symmetries belonging to SO(10) are standard and are thus not shown. Note that the $U(1)_i$ charges differ from one family to the other. There are also three families of hidden sector multiplets $V_i$, $\overline{V_i}$, $T_i$ and $\overline{T_i}$ which possess $U(1)_i$-charges.

(ii) The charge $Q_1$ has the same value ($\frac{1}{2}$) for all sixteen members of family 1, similarly $Q_2$ and $Q_3$ for families 2 and 3 respectively. In fact, barring a normalization difference of a factor of 2, the sum $Q_+ ≡ Q_1 + Q_2 + Q_3$ acts on the three families and on the three Higgs doublets $\overline{H_1}$, $\overline{H_2}$ and $\overline{H_3}$ in the same way as the $Q_\psi$ of $E_6$ introduced before. The analogy, however, stops there, because the solution has additional Higgs doublets (see table) and also because there is only one pair of color triplets $(D_{45}, \overline{D}_{45})$ instead of three. Furthermore, the pair $(D_{45}, \overline{D}_{45})$ is vector-like with opposite $Q_+$-charges, while $(H_3, H'_3)$, belonging to 27 of $E_6$, have the same $Q_\psi$-charge. In fact the pair $(D_{45}, \overline{D}_{45})$ can have an invariant mass conserving all $Q_\psi$-charges, but $(H_3, H'_3)$ can not.
(iii) It is easy to see that owing to different $U(1)_i$-charges, the color-triplets $D_{45}$ and $\overline{D}_{45}$ (in contrast to $H_3$ and $H_3^*$) can not have allowed Yukawa couplings to $(qq)$ and $(ql)$ - pairs. Thus, as mentioned before, they can not mediate proton decay.

(iv) Note that the solution yields altogether four pairs of electroweak Higgs doublets: $(h_1, h_2, h_3, h_{45})$ and $(\overline{h}_1, \overline{h}_2, \overline{h}_3, \overline{h}_{45})$. It has been shown \[69\] that only one pair – i.e. $\overline{h}_1$ or $\overline{h}_2$ and $h_{45}$ – remains light, while the others acquire superheavy or intermediate scale masses. Owing to differing $U(1)_i$-charges, the three families have Yukawa couplings with three distinct Higgs doublets. Since only one pair ($\overline{h}_1$ and $h_{45}$) remains light and acquires VEV, it turns out that families 1, 2 and 3 get identified with the $\tau$, $\mu$ and $e$-families respectively \[69\]. The mass-heirarchy and CKM mixings arise through higher dimensional operators, by utilizing VEVs of appropriate fields and hidden-sector condensates.

Including contributions from the entire massless spectrum, one obtains: $\text{Tr} U_1 = \text{Tr} U_2 = \text{Tr} U_3 = 24$ and $\text{Tr} U_4 = \text{Tr} U_5 = \text{Tr} U_6 = -12$. Thus, all six $U(1)_i$’s are anomalous. They give rise to five anomaly-free combinations and one anomalous one:

$$
\begin{align*}
U'_1 &= U_1 - U_2, \\
U'_2 &= U_4 - U_5, \\
U'_3 &= U_4 + U_5 - 2U_6, \\
\hat{U}_\psi &= U_1 + U_2 - 2U_3, \\
\hat{U}_\chi &= (U_1 + U_2 + U_3) + 2(U_4 + U_5 + U_6), \\
U_A &= 2(U_1 + U_2 + U_3) - (U_4 + U_5 + U_6).
\end{align*}
$$

(14)

One obtains $\text{Tr} Q_A = 180$. The anomalous $U_A$ is broken by the Dine-Seiberg-Witten (DSW) mechanism, in which the anomalous D-term generated by the VEV of the dilaton field is cancelled by the VEVs of some massless fields which break $U_A$, so that supersymmetry is preserved \[73\]. The solutions (i.e. the choice of fields with non-vanishing VEVs) to the corresponding F and D - flat conditions are, however, not unique. A few alternative possibilities have been considered in Ref. \[69\] (see also Refs. \[54\] and \[38\] for analogous considerations). Following our discussions in the previous subsection as regards non-availibility of 126 of $SO(10)$ or $(1,3,\overline{10})$ of $G_{224}$, I assume, for the sake of simplicity in estimating strengths of relevant operators, that $B - L$ is violated spontaneously at a scale $\sim 10^{15}$-$10^{16}$ GeV by one unit (rather than two) through the VEVs of elementary sneutrino-like fields $\tilde{N}_R \subset 16_H$ and $\tilde{N}_L^* \subset \overline{16}_H$ (as in Ref. \[54\]). Replacing VEVs of these elementary fields by those of products of fields including condensates,
as in Ref. [69], would only lead to further suppression of the relevant unsafe higher dimensional operators and go towards strengthening our argument [14] as regards certain symmetries being sufficient in preventing rapid proton-decay [74].

**A Longlived Proton:** We now reexamine the problem of proton-decay and neutrino-masses by assuming that in addition to $I_{3R}$ and $B-L$, or just $Y$, either $\hat{Q}_\psi \equiv Q_1 + Q_2 - 2Q_3$, or $\hat{Q}_\chi \equiv Q_1 + Q_2 + Q_3 + 2(Q_4 + Q_5 + Q_6)$ (see Eq. (10)), or both emerge as good symmetries near the string scale, which are broken by the VEVs of (i) sneutrino-like fields $\sim 10^{15} - 10^{16}$ GeV, (ii) electroweak doublets and singlets (denoted by $\phi$’s) $\sim 1$ TeV, and (iii) hidden-sector condensates. To ensure proton-stability, we need to assume that the hidden-sector condensate-scale is $\leq 10^{-2.5} M_{st}$. With the gauge coupling $\alpha_X$, at the unification-scale $M_X$, having nearly the MSSM value of .04 – .06, or even an intermediate value $\approx .16 -.2$ (say), as suggested in Ref. [43], this seems to be a safe assumption for most string models (see discussions later). The roles of the symmetries $Y$, $B-L$, $\hat{Q}_\psi$, $\hat{Q}_\chi$ and ($\hat{Q}_\chi + \hat{Q}_\psi$) in allowing or forbidding the relevant $(B,L)$ - violating operators, including the higher dimensional ones, which allow violations of these symmetries through appropriate VEVs, are shown in Table 4. Based on the entries in this table, the following points are worth noting:

(i) Inadequacy of the Pairs $(Y, B-L)$; $(Y, \hat{Q}_\psi)$; $(Y, \hat{Q}_\chi)$ and $(B-L, \hat{Q}_\chi)$: Table 4 shows that no single charge nor the pairs $(Y, B-L)$, $(Y, \hat{Q}_\psi)$, $(Y, \hat{Q}_\chi)$ and $(B-L, \hat{Q}_\chi)$ give adequate protection against all the unsafe operators. Let us next consider other pairs of charges.

(ii) Adequate Protection Through the Pair $(B-L$ and $\hat{Q}_\psi)$ or the Pair $(\hat{Q}_\chi$ and $\hat{Q}_\psi)$: Using Table 4, we observe that the pair $(B-L$ and $\hat{Q}_\psi)$, as well as the pair $(\hat{Q}_\chi$ and $\hat{Q}_\psi)$, forbid all unsafe operators, including those which may arise from higher dimensional ones, with or without hidden-sector condensates. In fact, members of the pairs mentioned above complement each other in the sense that when one member of a pair allows an unsafe operator, the other member of the same pair forbids it, and vice versa – a remarkable team effort. Note that the strengths of the $d=4$ and $d=5$ operators are controlled by the VEVs $\langle T_i / M \rangle^2$, $\langle \Phi / M \rangle^n$ and $\langle T_i T_j / M^2 \rangle^2$, which give more than necessary suppression, even if the condensate-scale is as large as about $10^{15}$ GeV (see estimates below).

(iii) $\hat{Q}_\psi$ removes Potential Danger From Triplets in The Heavy Tower As Well: Color
triplets in the heavy infinite tower of states with masses $M \sim M_{st} \sim 10^{18}$ GeV in general pose a potential danger for all string theories, including those for which they are projected out from the massless sector \[69\]. The exchange of these heavy triplets, if allowed, would induce $d = 5$ proton-decay operators with strengths $\sim \kappa/M$, where $\kappa$ is given by the product of two Yukawa couplings. Unless the Yukawa couplings are appropriately suppressed \[74\] so as to yield $\kappa \leq 10^{-7}$ \[59\], these operators would be unsafe. Note, however, that string-derived solutions possessing symmetries like $\hat{Q}_\psi$ are free from this type of danger. This is because, if $\hat{Q}_\psi$ emerges as a good symmetry near the string-scale, then the spectrum, the masses and the interactions of the color-triplets in the heavy tower would respect $\hat{Q}_\psi$. As a result, the exchange of such states can not induce $d = 5$ proton-decay operators, which violate $\hat{Q}_\psi$ (see Table 4).

In fact, for such solutions, the color-triplets in the heavy tower can appear only as vector-like pairs, with opposite $\hat{Q}_\psi$-charges (like those in 10 and $\overline{10}$ of $SO(10)$, belonging to 27 and $\overline{27}$ of $E_6$ respectively), so that they can acquire invariant masses of the type $M\{H_3\overline{H}_3 + H'_3\overline{H}'_3 + hc\}$, which conserve $\hat{Q}_\psi$. Such mass-terms cannot induce proton decay.

Thus we see that a symmetry like $\hat{Q}_\psi$ plays an essential role in safeguarding proton-stability from all angles \[14\]. Since $\hat{Q}_\psi$ distinguishes between the three families \[76\], it cannot, however, arise within single-family grand unification symmetries, including $E_6$. But it does arise within string-derived three-family solutions (as in Ref. \[69\]), which at once know the existence of all three families. In this sense, string theory plays a vital role in explaining naturally why the proton is so extraordinarily stable, in spite of supersymmetry, and why the neutrinos are so light.

**$Z'$-mass and proton decay rate:** If symmetries like $\hat{Q}_\psi$ and possibly $\hat{Q}_\chi$, in addition to $I_{3R}$ and $B-L$, emerge as good symmetries near the string scale, and break spontaneously so that only electric charge is conserved, there must exist at least one extra $Z'$-boson (possibly more), in addition to the (almost) standard $Z$ and a superheavy $Z_H$ (that acquires a mass through sneutrino-VEV) \[77\]. The extra $Z'$ boson(s) will be associated with symmetries like $\hat{Q}_T \equiv 2\hat{Q}_\psi - (B - L)$ and $\hat{Q}_\chi + \hat{Q}_\psi$, in addition to $Y$, that survive after sneutrino acquires a VEV. The $Z'$ bosons can acquire masses through the VEVs of electroweak doublets and singlets ($\phi$'s), as well as through the hidden-sector condensates like $\langle T_i T_j \rangle$, all of which break $\hat{Q}_T$ and $\hat{Q}_\chi + \hat{Q}_\psi$ (see Table 3). As mentioned before,
we expect the singlet $\phi$'s to acquire VEVs, at least radiatively (like the electroweak doublets), by utilizing their Yukawa couplings with the doublets, which at the string-scale is comparable to the top-Yukawa coupling. Since the $\phi$'s do not have electroweak gauge couplings, however, we would expect that their radiatively-generated VEV, collectively denoted by $v_0$, to be somewhat higher than those of the doublets ($v_{EW} \sim 200 \text{ GeV}$) -i.e., quite plausibly, $v_0 \sim 1 \text{ TeV}$. Thus, in the absence of hidden-sector condensates, we would expect $Z'$ to be light $\sim 1 \text{ TeV}$.

If the condensates like $\langle T_i T_j \rangle$ do form, however, they are likely to make $Z'$ much heavier than 1 TeV. Denoting the strength of $\langle T_i T_j \rangle$ by $\Lambda_c$, if $\Lambda_c \sim \Lambda_H$, where $\Lambda_H$ is the confinement-scale of the hidden-sector, we would expect $\Lambda_H$ and thus $Z'$ to be either superheavy $\sim 10^{15.5}-10^{16} \text{ GeV}$, or at least medium-heavy $\sim 10^8-10^{13} \text{ GeV}$ (see below).

The mass of the $Z'$-boson is correlated with the proton decay-rate. The heavier the $Z'$, the faster is the proton-decay. Looking at Table 4, we see that the strength of the effective $d = 4$ operators ($\overline{U} D \overline{D}$ etc.) is given by \((\langle N_R/M \rangle / (\langle T_i T_j \rangle / M^2)^2 \sim 10^{-2.5}(\Lambda_c/M)^4\), and that of the $d = 5$ operator ($QQQL/M$) is given by \((\langle T_i T_j \rangle / M^2)^2 \sim (\Lambda_c/M)^4\). The observed bound on the former ($\eta_{1.2} \leq 10^{-12}$) implies a rough upper limit of $(\Lambda_c/M)^4 \leq 10^{-9.5}$ and thus $\Lambda_c \leq 10^{15.5} \text{ GeV}$, while that on the latter (i.e. $\lambda_{1.2} \leq 10^{-7}$) implies that $\Lambda_c \leq 10^{16.2} \text{ GeV}$, where, for concreteness, we have set $M = 10^{18} \text{ GeV}$.

Thus, if $\Lambda_c \leq 1 \text{ TeV}$, $Z'$ would be light $\sim 1 \text{ TeV}$, and accessible to LHC and perhaps NLC. But, for this case, or even if $\Lambda_c$ is as heavy as $10^{15} \text{ GeV}$ (say), proton-decay would be too slow ($\tau_p \geq 10^{42} \text{ yrs.}$) to be observed. On the other hand, if $\Lambda_c \sim 10^{15.4} - 10^{15.6} \text{ GeV}$, the $Z'$-bosons would be inaccessible; but proton decay would be observable with a lifetime $\sim 10^{32}-10^{35}$ years [78]. To see if such a superheavy $\Lambda_c$ is feasible, let us recall the discussion in Sec. 5, where it was noted that an intermediate unified coupling $\alpha_X \approx 0.2$ at $M_X \sim 10^{17} \text{ GeV}$ (as opposed to the MSSM-value of $\alpha_X \approx 1/26$) is desirable to stabilize the dilaton and that such a value of $\alpha_X$ would be realized if there exists a vector-like pair of families having the quantum numbers of $16 + \overline{16}$ of $SO(10)$, in the $1 - 100 \text{ TeV}$-region [43]. With $\alpha_X \approx 0.16-0.18$ (say), and a hidden sector gauge symmetry like $SU(4)_H$ or $SU(5)_H$ [29], a confinement scale $\Lambda_H \sim \Lambda_c \sim 10^{15.5} - 10^{16} \text{ GeV}$ would in fact be expected. Thus, while rapid proton decay is prevented by string-derived symmetries of the type discussed here [14], observable rate for proton decay ($\tau_p \sim 10^{32}-10^{41} \text{ yrs.}$), which would be accessible to Superkamiokande and ICARUS, seems perfectly feasible and natural [79].
In summary, $\hat{Q}_\psi$ is a good example of the type of symmetry that can safeguard, in conjunction with $B - L$ or $\hat{Q}_\chi$, proton-stability from all angles, while permitting neutrinos to have desired masses. It even helps eliminate the potential danger from contribution of the color-triplets in the heavy tower of states. In this sense, $\hat{Q}_\psi$ plays a very desirable role. I do not, however, expect it to be the only choice. Rather, I expect other string-solutions to exist, which would yield symmetries like $\hat{Q}_\psi$, serving the same purpose [80]. At the same time, I feel that emergence of symmetries like $\hat{Q}_\psi$ is a very desirable constraint that should be built into the searches for realistic string-solutions.

To conclude this section, the following remark is in order. For the sake of argument, one might have considered an $SO(10)$-type SUSY grand unification by including 126 of Higgs to break $B - L$ and ignoring string-theory constraints [66]. One would thereby be able to forbid the $d = 4$ operators and give desired masses to the neutrinos [65]. But, as mentioned before, the problems of finding a compelling solution to the doublet-triplet splitting as well as to the gravity-linked $d = 5$ operators would still remain. This is true not just for SUSY $SO(10)$, but also for SUSY $E_6$, as well as for the recently proposed SUSY $SU(5) \times SU(5)$ - models [81]. By contrast, a string-derived non-GUT model, possessing a symmetry like $\hat{Q}_\psi$, in conjunction with $B - L$ or $\hat{Q}_\chi$, meets naturally all the constraints discussed above. This shows that string theory is not only needed for a unity of all forces, but also for ensuring natural consistency of SUSY-unification with two low-energy observations – proton stability and light masses for the neutrinos.

7 Summary and Concluding Remarks

Turning now to a summary of the first part of this talk,

- I noted in sections 1 and 2 that non-conservations of baryon and lepton numbers are implied on the one hand by ideas of higher unification [1, 2, 3], and on the other hand, by the need for baryogenesis [8, 9] and by neutrino-masses as well. The latter two in fact suggest some form of violation of $B - L$, which, very likely, includes a violation through heavy Majorana masses of the right-handed neutrinos.

- If $\Delta(B - L) = 0$ decay modes of the nucleon (i.e. $p \rightarrow \bar{\nu}K^+, \mu^+K^0, \bar{\nu}\pi^+, e^+\pi^0,$ etc.) turn out to be the only observed source of non-conservation of $B$ and $L$, as opposed to $\Delta(B - L) \neq 0$-transitions exhibited in Table 1 (i.e. $p \rightarrow e^-\pi^+\pi^+, n \leftrightarrow \bar{n}$ and
\(nn \rightarrow ppe^−e^−\) etc.), there would be no signal for new physics at about 100 TeV. That would of course conform with conventional wisdom, which is based on simple mechanisms for symmetry-breaking of SUSY GUTS and/or string-derived solutions, obtained to date. On the other hand, if the \(\Delta(B−L) \neq 0\) transitions such as \(p \rightarrow e^-\pi^+\pi^+\) or \(n−\bar{n}\) oscillation do show at some level, that would clearly point to new physics at low intermediate scales \(\sim 100\text{ TeV}\). This will be counter to conventional thinking, but just for that reason that may be quite revealing. I comment on this issue further in the following.

- Confining to the \(\Delta(B−L) = 0\) decay modes of the proton, assuming that they are discovered at SuperKamiokande and/or ICARUS, we will learn much from knowing which decay-channels are prominent or dominant. \textbf{First}, prominence of \(\bar{\nu}K^+\) and/or \(\mu^+K^0\)-mode would be a \textit{strong evidence} in favor of the dominance of \(d = 5\) over the \(d = 6\)-operators, and thereby in favor of supersymmetry, though not necessarily for SUSY GUTS. Prominence of the \(\mu^+K^0\)-mode would suggest large \(\tan\beta\)[37]. \textbf{Second}, prominence of \(\bar{\nu}_\mu\pi^+\) and/or \(\mu^+\pi^0\)-mode, together especially with \textit{non-observation} of the \(e^+\pi^0\)-mode would also provide the same signal. \textbf{Third}, prominence of the \(e^+\pi^0\)-mode would favor, in the context of supersymmetry, either the dominance of string-derived \(d = 4\) over \(d = 5\)-operators (see Sec. 6 and Ref. 14), or, for example, the flipped \(SU(5) \times U(1)′\)-model[38]. It would, of course, be compatible also with non-supersymmetric GUTS, whose unification-scales are raised, for example, through enriched Higgs-systems, so that the associated lifetimes exceed \(10^{33}\) yrs.

It is worth commenting on the issue of \(\Delta(B−L) = 0\) versus \(\Delta(B−L) \neq 0\) -transitions. Certain guidelines of simplicity, noted below, seem to suggest that the latter would be too slow to be observed. First, the straightforward meeting of the three gauge couplings, obtained in the context of either MSSM or ESSM, and the associated predictability of \(\sin^2\theta_W\) (see Sec. 5) would be lost, if one introduces low intermediate scale-physics (necessary for prominence of \(\Delta(B−L) \neq 0\) -transitions) [23, 24], and thereby somewhat arbitrary multi-stage running of the couplings. Second, neutrino masses that are relevant, especially for the MSW solution of the solar neutrino-puzzle and for \(\bar{\nu}_\tau\) being hot dark matter, suggest a superheavy, rather than a low intermediate-scale, Majorana mass for the right-handed neutrinos (see Secs. 2 and 6). Furthermore, in contrast to low intermediate-scales, such superheavy scales (e.g. \(< \bar{N}_R > \sim 10^{15.5}\)
GeV, see Sec. 6) do arise naturally in the context of string-solutions (see e.g. Ref. 53); and, they do not alter the simple running of three gauge couplings, except near the point of unification, which may be good anyway to remove the mismatch between MSSM and string-unifications (see Sec. 5). Thus, if one is permitted to possess a theoretical prejudice, based on grounds of simplicity, as narrated above, it would seem that low intermediate-scale physics of a nature that would lead to observable rates for \( \Delta(B - L) \neq 0 \)-modes is not likely to be realized, at least in the context of current level of thinking.

Nevertheless, I believe that it is essential to keep an open mind as regards the planning of experiments, precisely to find out if one’s prejudices are true after all. And, what if these prejudices turn out to be wrong? That has happened in the past. A case to the point is CP-violation. Imagine that CP-violation was not discovered in 1964 and that one did not know that it is needed to implement baryogenesis. As late as the early 70’s, judged purely from the point of view of theoretical models, based on just (u,d,s,c)-quarks, there was no compelling theoretical motivation for CP-violation. In the context of renormalizable gauge theories, one would have had to introduce extra Higgs-scalars, new gauge interactions, complex Yukawa couplings and/or a third family of quarks and leptons to implement CP-violation. Apriori, it would have appeared to be an unnecessary complication to introduce such extra matter and/or extra parameters. Judged from this point of view, CP-violation might have appeared unnatural or unlikely. Yet, CP-violation was discovered; so was its need to implement baryogenesis; and also the third family. It is, however, revealing to note in this context that even now we do not know the precise origin of CP-violation. There is some analogy of this case with that of \( (B - L) \)-violation. We do know from baryogenesis and neutrino masses that, very likely, \( B - L \) is violated (see Sec. 2). The issue that needs to be settled, notwithstanding the question of naturalness and simplicity of present theoretical models, is whether the rates for \( (B - L) \)-violating transitions would lie in an observable range. There is no other way to settle this question except to search for such transitions with the highest possible precision.

I, therefore, believe that experimental searches for both \( (B - L) \)-conserving–i.e. \( p \to \bar{\nu}K^+, \bar{\nu}\pi^+, \mu^+K^0, e^+\pi^0 \) etc. – as well as \( (B - L) \)-violating processes–i.e. \( p \to e^-\pi^+\pi^+, n \leftrightarrow \bar{n} \)-oscillation, neutrinoless double beta decay, etc.–are strongly motivated.
For this reason, I rejoice in the starting of the SuperKamiokande and look forward to the starting of ICARUS. I also greatly welcome proposal to set up searches for $n - \bar{n}$ oscillation here at OakRidge, which aim to probe into oscillation-periods of $10^{10} - 10^{11}$ sec.

The main purpose of this talk has been to address two issues:

(i) How to resolve the mismatch between MSSM and string-unification?, and especially,

(ii) How to prevent, naturally, rapid proton-decay in supersymmetry?

With regard to the first, I noted some alternative possibilities. Among these, the only one that has a chance of being directly tested at future high energy accelerators, including the LHC and the NLC, especially if the two vector-like families have masses below about 1.5 TeV, is the extended supersymmetric standard model (ESSM), which proposes a \textit{semi-perturbative unification} with intermediate unified coupling ($\alpha_X \sim .2$) in four dimensions \[43\].

With regard to the second issue, I have stressed that

- the extreme smallness of the strengths of the $d = 4$ (i.e. $\eta_i \leq 10^{-12}$) and the color-triplet mediated and/or gravity-linked $d = 5$ operators (i.e. $\lambda_{1,2}(M_{st}/M) < 10^{-7}$) deserves a natural explanation. The problem in this regard is somewhat analogous to that of understanding the smallness (or the vanishing) of the cosmological constant. Rather than being merely accommodated by a \textit{choice} of the Higgs multiplets and discrete symmetries, the small parameters associated with the $d = 4$ and the $d = 5$ operators should emerge as a \textit{compelling feature}, owing to \textit{symmetries} of the underlying theory, which would provide the desired protection against these unsafe operators.

- The symmetries in conventional SUSY GUTS including $SO(10)$, $E_6$ and $SU(5) \times SU(5)$ do not, however, suffice for the purpose–especially in the matter of suppressing naturally the $d = 5$ proton-decay operators. By contrast, I showed that a certain \textit{string-derived symmetry}, which cannot arise within SUSY GUTS as mentioned above, but which does arise within a class of \textit{three-generation string-solutions}, possessing non-GUT symmetries, suffices, in conjunction with $B - L$, to safeguard proton-stability from all potential dangers, including those which may arise from higher dimensional operators and the color-triplets in the heavy infinite tower of states \[14\]. At the same time, the symmetry in question permits neutrinos to acquire appropriate masses. We thus see
that just seeking for an understanding—as opposed to accommodating—proton-stability, in the context of supersymmetric unification, drives us to the conclusion that, at the fundamental level, the elementary particles must be string-like and not point-like. It seems remarkable that just a low-energy observation that proton is so stable, together with the demand of naturalness in understanding this feature, can provide us with such a deep insight.

- It is intriguing that one needs a family-dependent symmetry, like $\hat{Q}_\psi$, to achieve the desired protection against rapid proton decay, which cannot be realized for one- or two-family string-solutions, but which does emerge for a class of solutions possessing three families.

- A related remark: the necessity of such a family-dependent symmetry, which cannot arise within conventional GUT symmetries, as well as the lack of a compelling mechanism for doublet-triplet splitting in SUSY GUT theories (string-derived or not) suggest that the flavor-color gauge symmetry below the string-scale is very likely a non-GUT string-derived symmetry like $G_{2311}$ or $G_{224}$, or even flipped $SU(5) \times U(1)'$, rather than a GUT-symmetry like $SU(5)$, or $SO(10)$ or $E_6$. Recall that, owing to string-constraints, the benefits of coupling unification, quark-lepton unification and quantization of electric charge still hold for the former, just as they do for the latter.

- Last but not least, as discussed in Sec. 6, it is interesting that, while symmetries like $\hat{Q}_\psi$ provide the desired protection against rapid proton decay, observable rates for proton-decay (i.e. lifetimes of order $10^{32} - 10^{34}$ yrs) are nevertheless perfectly natural in the context of these solutions, provided, however, one assumes (at least for the class of solutions considered in Ref. [14]), a semi-perturbative unification with intermediate unified coupling ($\alpha_X \sim .18 - .2$) [43]. As discussed in Sec. 5, such intermediate coupling is suggested by ESSM, and it may well be needed to help stabilize the dilaton, while retaining the benefits of coupling-unification.

To conclude, the original motivations for a unity of the fundamental forces and that for questioning baryon and lepton-number conservations still persist. Supersymmetry and superstrings, while retaining these motivations, provide a new perspective with regard to both issues. As discussed here, several models of SUSY GUTS and superstring-derived models do in fact suggest that proton decay should occur at a rate that is accessible to ongoing searches. Observation of proton decay would strengthen
our belief in an underlying unity of all matter and of its forces. Determination of its dominant decay modes, would provide us with a wealth of knowledge regarding new physics at very short distances, spanning from $10^{-19}$ to $10^{-32}$ cm. The question now is an experimental one: Will proton decay be discovered at SuperKamiokande and/or ICARUS?

Acknowledgements

I wish to thank M. Dine, P. Langacker, R.N. Mohapatra, N. Polonsky, A. Raisin, F. Wilczek, E. Witten and especially K.S. Babu, K. Dienes, and A. Faraggi for most helpful discussions and communications on topics included in this talk. I wish to thank the organizers, especially Y. Kamyshkov and R.N. Mohapatra, for arranging a very fruitful meeting and for their hospitality. I also wish to thank Delores Kight for her care and efforts in typing this manuscript.

References

[1] J.C. Pati and Abdus Salam; Proc. 15th High Energy Conference, Batavia, reported by J.D. Bjorken, Vol. 2, p. 301 (1972); Phys. Rev. 8 (1973) 1240; Phys. Rev. Lett. 31 (1973) 661; Phys. Rev. D10 (1974) 275.

[2] H. Georgi and S.L. Glashow, Phys. Rev. Lett. 52 (1974) 438.

[3] H. Georgi, H. Quinn, and S. Weinberg, Phys. Rev. Lett. 33 (1974) 451.

[4] V. Kuzmin, JETP Lett. 12, 228 (1970); R.N. Mohapatra and R. E. Marshak, Phys. Rev. Lett. 44, 1316 (1980); S.L. Glashow in Recent Developments in Gauge Theories, Cargése, 1979. For a recent model, see R.N. Mohapatra, hep-ph/9604414, Proc. this Conference.

[5] For the status and prospect of SuperKamiokande, see Y. Totsuka, Nucl. Phys. B (Proc. Suppl.) 48, 547 (1996). For a review of the status of IMB, Kamiokande and SuperKamiokande searches for proton decay, see J. Stone, talk at this conference. For the prospect of ICARUS and the scope for detecting various $\Delta(B - L) = 0$ as well as $\Delta(B - L) \neq 0$ decay modes of the nucleon listed in Table 1, see in particular D. Cline, talk at this conference.
[6] For a review of the status of searches for $\nu$-less Double beta decay, including planned experiments, see H. Klapdor-Kleingrothaus, Proc. this Conference.

[7] For limits from the ILL experiment on $n - \bar{n}$ oscillation, see M. Baldo-Ceolin et al., Padova preprint DFPD 94/EP/13. For proposed Oak Ridge experiment, see Y. Kamyshkov, these Proceedings.

[8] A.D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. 5, 32 (1967).

[9] V. Kuzmin, JETP. Lett. 12, 335 (1970) (in Russian).

[10] For a review, see e.g. P. Langacker, invited talk, Erice School (1994), hep-ph/941339.

[11] Y.A. Gelfand and E.S. Likhtman, JETP Lett. 13 (1971) 323; J. Wess and B. Zumino, Nuc. Phys. B70 (1974) 139; Phys. Lett. 49B (1974) 52; D. Volkov and V.P. Akulov, JETP Lett. 16 (1972) 438.

[12] M. Green and J.H. Schwarz, Phys. Lett. 149B, 117 (1984); D.J. Gross, J.A. Harvey, E. Martinec and R. Rohm, Phys. Rev. Lett. 54, 502 (1985); P. Candelas, G.T. Horowitz, A. Strominger and E. Witten, Nucl. Phys. B 258, 46 (1985).

[13] S. Weinberg, Phys. Rev. D 26, 287 (1982); N. Sakai and T. Yanagida, Nucl. Phys. B197, 533 (1982).

[14] J.C. Pati, ”The Essential Role of String-Derived Symmetries in Ensuring Proton Stability and Light Neutrino Masses” hep-ph/9607446. Physics Letters, to appear. Some preliminary aspects of this work were alluded to at the conference. Its main results, reported here, developed after the conference.

[15] V. Kuzmin, Va. Rubakov and M. Shaposhnikov, Phys. Lett. 155B, 36 (1985). For a review, see A. Cohen, D. Kaplan and A. Nelson, Ann. Rev. Nucl. Part. Sc. 43, 27 (1993).

[16] J.C. Pati and A. Salam, Phys. Rev. D10, 275 (1974); R. N. Mohapatra and J.C. Pati, Phys. Rev. D11, 566, 2558 (1975); G. Senjanovic and R.N. Mohapatra, Phys. Rev. D12, 1502 (1975).
[17] M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity, ed. by D. Freedman and P. Van Nieuwenhuizen (North Holland 1979), p. 315; T. Yanagida, Proc. Workshop on “Unified Theories and Baryon Number of the Universe”, eds. O. Sawata and A. Sugamoto, KEK, Japan (1979); R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).

[18] This mass-pattern goes with the MSW-explanation of the solar neutrino puzzle (see S.P. Mikheyev and A. Yu. Smirnov, Nuov. Cim. 9 C, 17 (1986), and L. Wolfenstein, Phys. Rev. D 17, 2369 (1978)), and the indirect $\nu_e$-$\nu_\mu$ - oscillation - explanation of the LSND-experiment (see K.S. Babu, J.C. Pati and F. Wilczek, Phys. Lett. B 359, 351 (1995)).

[19] If there was no L-R symmetry and no $\nu_R$, $\nu_L$’s can still obtain Majorana masses through gravity-induced effects, using higher dimensional operators such as $\nu_L\nu_L\phi\phi/M$, where $\phi$ is the Higgs doublet, and $M$ is a characteristic mass-scale. With $<\phi> \sim 250 GeV$, and $M \sim 10^{18} GeV$, however, one obtains $m(\nu_L) \sim 10^{-7} eV$, which is too small to be relevant to the MSW solution for the solar neutrino puzzle.

[20] T.D. Lee and C.N. Yang, Phys. Rev. 98, 1501 (1955).

[21] For $SO(10)$: H. Georgi, in Particles and Fields (edited by C. Carlson), A.I.P. (1975); H. Fritzsch and P. Minkowski, Ann. Phys. (NY) 93, 193 (1975). For $E_6$: F. Gursey, P. Ramond and P. Sikivie, Phys. Lett. 60 B, 177 (1976).

[22] R.N. Mohapatra and G. Senjanovic (Ref. 17).

[23] See, e.g., J.C. Pati and A. Salam, Proc. First Workshop on Grand Unification, New Hampshire (April 1980), p. 115; J.C. Pati, A. Salam and U. Sarkar, Phys. Lett. 113B, 330 (1983); T. Rizzo and G. Senjanovic, Phys. Rev. D25, 235 (1982); F. del Aguila and L. Ibanez, Nucl. Phys. B 177, 60 (1981); D. Chang, R.N. Mohapatra, J.M. Gipson, R.E. Marshak and M. Parida, Phys. Rev. D 31, 1718 (1985).

[24] S. Weinberg, Phys. Rev. Lett. 43, 1566 (1979); F. Wilczek and A. Zee, Phys. Rev. Lett. 43, 1571 (1979).

[25] See R.N. Mohapatra, Proceedings this Conference and references therein.
[26] J.C. Pati, A. Salam and U. Sarkar (Ref. 23); J.C. Pati, Phys. Rev. D, Rap. Comm. 29, 1549 (1984).

[27] J.C. Pati, Phys. Lett. B 228, 228 (1989); J.C. Pati, M. Cvetic and H.S. Sharatchandra, Phys. Rev. Lett. 58, 851 (1987); K.S. Babu, J.C. Pati and H. Stremnitzer, Phys. Rev. Lett. 67, 1688 (1991); K.S. Babu, J.C. Pati and H. Stremnitzer, A hint from the inter-family mass-hierarchy: two vector-like families in the TeV range, Phys. Rev. D 51, 2451 (1995).

[28] For a recent review of the preonic approach, see J.C. Pati, "Towards a Unified Origin of Forces, Families and Mass Scales," hep-ph/9505227, Proc. of the 1994 Int’l. Conf. on B-Physics, held at Nagoya, Japan (Oct. 26-29, 1994), pages 164-181, Eds. A. Sanda and S. Suzuki, Publ. World Scientific.

[29] For a theoretical extraction of $\sin^2 \theta_W$ from LEP data, and discussion of coupling-unification, see P. Langacker and N. Polonsky, Phys. Rev. D 52, 3081 (1995). The value quoted corresponds to $m_H = 100\text{GeV}$ and $m_t = 180\text{GeV}$.

[30] For a review, see e.g., P. Langacker, talk at Gatlinburg Conference, June 94, hep-ph/9411247 and talk at this conference.

[31] P. Langacker and M. Luo, Phys. Rev. D 44 (1991) 817; U. Amaldi, W. de Boer and H. Furstenau, Phys. Lett. B 260 (1991); J. Ellis, S. Kelley and D. V. Nanopoulos, Phys. Lett. B 260 (1991) 131; F. Anselmo, L. Cifarelli, A. Peterman and A. Zichichi, Nuov. Cim. A 104 (1991) 1817. Earlier analysis containing the essential physics of the two-loop effects in SUSY GUTS may be found in W. Marciano and G. Senjanovic, Phys. Rev. D 25, 3092 (1982) and M. Einhorn and D.R.T. Jones, Nucl. Phys. B 196, 475 (1982).

[32] M. Bastero-Gil and J. Perez-Mercader, Nucl. Phys. B 450, 21 (1995); M. Bastero-Gil, private communications (to appear); T. Binoth and J.J. van der Bij, Z. Phys. C 58, 581 (1993); L. Clavelli and P. Coulter, hep-ph/9507261; J. Bagger, K. Matchev and D. Pierce, Phys. Lett. 348, 443 (1995); P. Chankowski, Z. Pluciennik and S. Pokorski, Nucl. Phys. B 439, 23 (1995); R. Barbieri, P. Ciafaloni and A. Strumia, Nucl. Phys. B 442, 461 (1995). These papers include exact one-loop (rather than step-function) threshold effects.
[33] J. Hisano, H. Murayama and T. Yanagida, Phys. Rev. D 49, 4966 (1994); Nucl. Phys. B 402, 46 (1993); J. Hisano, T. Moroi, K. Tobe and T. Yanagida, Tohoku University preprint (TU-470), hep-ph/9411298.

[34] See P. Nath, Talk at the conference.

[35] S. Dimopoulos, S. Raby and F. Wilczek, Phys. Lett. 112 B, 113 (1982).

[36] For earlier estimates on nucleon decay modes in SUSY GUT theories, and considerations of different regions of SUSY-parameter-space which lead to prominence of different decay modes, see S. Chada and M. Daniel, Phys. Lett. 137 B, 374 (1984); M. Belayev and M. I. Vyotsky, Phys. Lett. 127 B, 215 (1983); J. Milutnovic, P.B. Bal and G. Senjanovic, Phys. Lett. 140 B, 324 (1984); P. Nath, A.H. Chamseddine and R. Arnowitt, Phys. Rev. D 32, 2348 (1985) and K. Enqvist, A. Masiero and D.V. Nanopoulos, Phys. Lett. 156 B, 209 (1985). For recent more detailed analysis and estimates of rates for alternative nucleon decay modes, especially for SUSY SU(5), see Hisano et al. (Ref. 33), and for SUSY SO(10), see V. Lucas and S. Raby, hep-ph/9610293.

[37] For preliminary considerations of gluino-loops leading to possible prominence of \( p \to \mu^+K^0 \)-mode, see S. Chada, G.D. Coughlan, M. Daniel and G.G. Ross, Phys. Lett. 149 B, 477 (1984). For recent considerations showing prominence of this mode in supergravity models for the case of large \( \tan\beta \), see T. Goto, T. Nihei and J. Arafune, Phys. Rev. D 52, 505 (1995); and in particular K.S. Babu and S.M. Barr, Phys. Lett. B 381, 137 (1996).

[38] I. Antoniadis, J. Ellis, J. Hagelin and D.V. Nanopoulos, Phys. Lett. B231, 65(1989). For a review, see J.L. Lopez and D.V. Nanopoulos, hep-ph/9511266).

[39] Particle Data Group, L. Montanet et al., Phys. Rev. D 50, 1173 (1994).

[40] For an early review on this issue, see S. Weinberg, Summary talk, Proc. 26th Intl. Conf. on High Energy Physics, Dallas, Texas (1992).

[41] For a recent discussion, see K. Dienes, hep-th/9602045 (to appear in Phys. Rept.) and references therein.
[42] P. Ginsparg, Phys. Lett. B 197, 139 (1987); V.S. Kaplunovsky, Nucl. Phys. B 307, 145 (1988); Erratum: ibid. B 382, 436 (1992).

[43] J.C. Pati and K.S. Babu, "The Problems of Unification-Mismatch and Low $\alpha_3$: A Solution with Light Vector-Like Matter", hep-ph/9606213. Physics Lett., To appear.

[44] See e.g. M. Dine and N. Seiberg, Phys. Lett. 162 B, 299 (1985), and in Unified String Theories, Ed. by M. Green and D. Gross (World Scientific, 1986).

[45] If one were to use point particle field theory for an SU(N) gauge theory as a rough guide, one might expect perturbation theory to break down if the ratio of the 2-loop to the 1-loop amplitude given by $\alpha_N N/(2\pi)$ exceeds about 0.2, i.e., if $\alpha_N \geq (.3,.15)$ for $N = 3,6$ where $\alpha_N$ denotes the SU(N) gauge coupling $g^2 N/(4\pi)$. Assuming the MSSM spectrum and therefore $\alpha_X \simeq 0.04$, the question has however arisen, on grounds of necessity in this case, whether string-perturbation theory might break down even for such a weak coupling, see T. Banks and M. Dine, Phys. Rev. D 50, 7454 (1994). It seems fair to say that no convincing argument has arisen in favor of such a possibility.

[46] E. Witten, Nucl. Phys. B 433, 85 (1995); P. Horava and E. Witten, Nucl. Phys. B 460, 506 (1996); See J. Polchinski, hep-th/9511157, for a review and references therein.

[47] M. Dine and Y. Shirman, hep-th/9601175; M. Dine, hep-th/9508083.

[48] E. Witten, hep-th/9602070.

[49] See e.g. D. Lewellen, Nucl. Phys. B 337, 61 (1990); A. Font, L. Ibanez and F. Quevedo, Nucl. Phys. B 345, 389 (1990); S. Chaudhari, G. Hockney and J. Lykken, Nucl. Phys. B 456, 89 (1995) and hep-th/9510241; G. Aldazabal, A. Font, L. Ibanez and A. Uranga, Nucl. Phys. B 452, 3 (1995); ibid. B 465, 34 (1996); D. Finnell, Phys. Rev. D 53, 5781 (1996); A.A. Maslikov, I. Naunov and G.G. Volkov, Int. J. Mod. Phys. A 11, 1117 (1996); J. Erler, hep-th/9602032 and G. Cleaver, hep-th/9604183, and Z. Kakushadze and S.H. Tye, hep-th/9605221, and hep-th/9609027.
[50] K. Dienes and A. Faraggi, Nucl. Phys. **457**, 409 (1995); C. Bachas, C. Fabre and T. Yanagida, Phys. Lett. **B 370**, 49 (1996).

[51] J.A. Casas and C. Munoz, Phys. Lett. **B 214**, 543 (1988); L.E. Ibanez, Phys. Lett. **B 318**, 73 (1993); K. Dienes, A. Faraggi and J. March-Russell, [hep-th/9510223]; S. Chaudhuri, G. Hockney and J. Lykken, [hep-th/9510241].

[52] H. Nilles and S. Steiberger, [hep-th/9510009].

[53] In Ref. 42, a mass-scale of 1 TeV was used for the masses of the vector-like families. This was suggested by the SUSY-preon model (Ref. 27). With elementary quarks, this restriction is relaxed. This is because, following discussions in Refs. 27 and 43, it is easy to see that the masses of the known fermions restrict only the ratio of the VEVs of $H_S$ and $H_\lambda$ (see Ref. [43] for notations), where $< H_S >$ mixes chiral and vector-like families while $< H_\lambda >$ gives diagonal masses to the latter. I would in this note allow their masses to lie in the range of 1-100 TeV, which would still lead to intermediate unified coupling, as desired in Ref. [43]. Heavier masses of order 10-100 TeV are suggested by recent works on gauge-mediated SUSY-breaking, where vector-like quarks serve as messengers.

[54] I. Antoniadis, G.K. Leontaris and J. Rizos, Phys. Lett. **B 245**, 161 (1990).

[55] Z. Kakushadze and S.H. Tye (Ref. [49]).

[56] Such a see-saw pattern of mass-matrix arises naturally in the SUSY-preon model [27]. This is abstracted for elementary quarks [43] to preserve the explanation of inter-family mass-hierarchy. It remains to be seen whether such a pattern can be derived from a superstring theory for elementary quarks as well.

[57] An extension of the renormalization group analysis of Ref. [43] has recently been carried out by C. Kolda and J. March-Russell ([hep-ph/9609480]), by including 3-loop effects, which are found to preserve the essential features of the results of Ref. [43], for the case for which the extra matter is in $16 + \overline{16}$.

[58] See e.g. L. Maiani, G. Parisi and R. Petronzio, Nucl. Phys. **B 136**, 115 (1978).

[59] I. Hinchliff and T. Kaeding, Phys. Rev. **D 47**, 279 (1993).
[60] S. Dimopoulos and H. Georgi, Nucl. Phys. B 193, 150 (1981); N. Sakai, Z. Phys. C 11, 153 (1982).

[61] A. Masiero, D.V. Nanopoulos, K. Tamvakis and T. Yanagida, Phys. Lett. B 115, 380 (1982); B. Grinstein, Nucl. Phys. B 206, 387 (1982).

[62] S. Dimopoulos and F. Wilczek, in Proc. Erice Summer School (ed. by A. Zichichi), I.T.P. preprint NSF-ITP-82-07 (1981). For recent discussions on DW-mechanism, see K.S. Babu and S.M. Barr, Phys. Rev. D 48, 5354 (1993); ibid D 51, 2463 (1995).

[63] See, e.g., Z. Berezhiani, C. Csáki and L. Randall, Nucl. Phys. B 444, 61 (1995) and references therein.

[64] H. Kawai, D. Lewellen and S.H. Tye, Phys. Rev. Lett. 57, 1832 (1986) and Nucl. Phys. B 288, 1 (1987); I. Antoniadis, C. Bachas and C. Kounnas, Nucl. Phys. B 289, 87 (1987).

[65] R.N. Mohapatra, Phys. Rev. D 34, 3457 (1986), A. Font, L. Ibanez and F. Quevedo, Phys. Lett. B 288, 79 (1989); S.P. Martin, Phys. Rev. D 46, 2769 (1992).

[66] K.R. Dienes and J. March-Russell, hep-th/9604112; K.R. Dienes, hep-ph/9606467.

[67] For an alternative solution of $B-L$ violation, and of neutrino masses using hidden-sector condensates, see A. Faraggi and E. Halyo, Phys. Lett. B 307, 311 (1993).

[68] Our discussion in this section is similar in spirit, though not in content, to that of Weinberg in Ref. [13].

[69] A. Faraggi, Phys. Lett. B 278, 131 (1992); Nucl Phys. B 403, 101 (1993); Nucl. Phys. B 416, 63 (1994). The full massless spectrum is given in the first paper.

[70] A. Faraggi, Phys. Lett. B 274, 47 (1992).

[71] Although doublet and triplet coexist in the flipped $SU(5) \times U(1)_{\tau}$-model, the problem of doublet-triplet splitting is resolved in this model as well, owing to a natural missing-partner mechanism [38]. This too is a non-GUT-model like the $G_{224}$-model.
each of which possesses coupling unification at the level of strings, assuming that they originate from a string theory. One still needs to examine if the problems of $d = 4$ and gravity-linked $d = 5$ operators, which would seem to exist in the flipped $SU(5)$-model, can be resolved by utilizing extra symmetries of the type discussed in the text.

[72] If the problem of doublet-triplet splitting can be solved naturally for a string-derived GUT model, and if symmetries of a nature to be presented below can arise in these models, so as to prevent rapid proton decay, they would of course be just as acceptable as the string-derived non-GUT models.

[73] M. Dine, N. Seiberg and E. Witten, Nucl. Phys. B 289, 585 (1987).

[74] We remark that, while the vector-like $(16_H + \overline{16}_H)$ - pair or equivalently the $(B = (1, 2, \overline{7}) + \overline{B} = (1, 2, 4))$ - pair, does not appear in the specific solutions of Ref. [54], existence of such pairs is fairly generic in string theories (see e.g. Refs. [54] and [55]). It is important to note, however, that if sneutrino-like fields belonging to $16_H$ and $\overline{16}_H$ acquire large VEVs (as in Ref. [54]), one must ensure that strengths of operators like $16_i \cdot 16_H \cdot 10_H$ and $16_i \cdot \overline{16}_H \cdot 1_H$ in the superpotential are strongly suppressed, so that $\nu_L$-Higgsino mixing mass (which these operators would induce) remains below about 1 MeV. While such a suppression could happen through constraints of symmetries and the allowed pattern of VEVs, resolution of this problem turns out to be considerably easier in models like those of Ref. [69]. This is because, as mentioned in the text, $< \overline{N}_R >$ is effectively replaced in these models by VEVs of product of fields and a condensate, which naturally lead to sufficiently strong suppression of $\nu_L - \tilde{H}$ mixing (especially if the condensate-scale is $\leq 10^{11}$ GeV) through relevant higher dimensional operators. I thank K.S. Babu for drawing my attention to this potential problem.

[75] As far as we can see, Yukawa couplings of the color-triplets in the heavy tower need not all be suppressed to the same extent as those of the electroweak doublets.

[76] Note $\hat{Q}_\psi$ assigns the same charge to all sixteen members of a given family. In this sense, $\hat{Q}_\psi$ (though not $\hat{Q}_\lambda$) commutes with $SO(10)$. 

43
The possibility of extra $Z'$ bosons, with or without a string-origin, has been considered by several authors. See e.g. M. Cvetic and P. Langacker, [10] for a recent string-motivated analysis. The role of a string-derived symmetry (associated with an extra $Z'$) in preventing rapid proton-decay, has not however been noted in these previous works.

If $< \widetilde{N}_R > \neq 0$, the d=4 operators (see table 4) lead to proton decay rate $\propto (\eta_1 \eta_2)^2 \propto (\Lambda_c/M)^{16}$, which varies extremely rapidly with $\Lambda_c$. It is interesting that, despite this rapid variation, proton decay is observable for a plausible though narrow range of values of $\Lambda_c \approx 10^{15.5}$ GeV. If $< \widetilde{N}_R > = 0$, as in Ref. [14] (see also remarks in Ref. [19]), only the d=5 operators with strengths $\sim (< T_i T_j > /M^2)^2$ are relevant (see table 4). These will lead to proton decay rate $\propto (\Lambda_c/M)^8$, which would be observable if $\Lambda_c \sim 10^{16}$ GeV (see text). In contrast to the d = 5 operators, if the d = 4 operators ($\bar{U} \bar{D} \bar{D}$ and $Q \bar{L} \bar{D}$) induced as above dominate, they can yield (through $\bar{d}_R$-exchange) strange as well as non-strange particle decay modes of the nucleon. Thus, through these operators, the $e^+\pi^0$-mode can be prominent or even dominant.

In all these considerations, contribution of d=6 operators are neglected, because they are strongly suppressed if the relevant scale ($M_X$ or $M$) exceeds $10^{16}$ GeV.

For example, other pairs of string-derived symmetries such as $(\hat{F}, B - L)$, where $\hat{F} \equiv F_1 + F_2 - 2F_3$ and $F_i$ acts as the fermion number on the $i$th family (i.e. $F_i \equiv (3B + L)_i = (1, 1 \mid -1, -1)$ for $(q, l \mid l, l)_i$), may also play a role analogous to the pair $(\hat{Q}, B - L)$. As an additional possibility, it would be interesting to explore whether the recently proposed variants of the solutions in Refs. [18] and [38], which yield a leptophobic $Z'$ (see e.g. A. Faraggi, [14] and J. L. Lopez and D. V. Nanopoulos, [15]), can be developed to provide a consistent Higgs-mechanism and also prevent rapid proton decay, utilizing extra symmetries, as discussed here.

For alternative versions, see R. Barbieri, G. Dvali and A. Strumia, Phys. Lett. 333 B, 79 (1994); R. N. Mohapatra, [17] and S. M. Barr, [18].
The fundamental entities may of course well be an appropriate incarnation of superstrings, such as that that may occur in the M-theory or the D-branes. These may provide even newer symmetries (hopefully internal as well as space-time), beyond those exhibited by strings. Following the observation made here that superstrings provide symmetries beyond those in point-particle SUSY GUTS, which account naturally for the extreme smallness of the unsafe proton-decay operators, one is led to wonder whether such a departure from point-particle physics, brought forth by string or its ultimate incarnation, which would inevitably provide new symmetries, might be the source of resolutions of the many "naturalness" problems, including even that pertaining to the vanishing or the smallness of the cosmological constant, in a world where SUSY is broken.
| Family | States          | $Q_1$ | $Q_2$ | $Q_3$ | $Q_4$ | $Q_5$ | $Q_6$ | $Q_\psi$ | $Q_\chi$ |
|--------|----------------|-------|-------|-------|-------|-------|-------|----------|----------|
| 1      | $q_1$          | 1/2   | 0     | 0     | -1/2  | 0     | 0     | 1/2      | -1/2     |
|        | $L_1$          | 1/2   | 0     | 0     | 1/2   | 0     | 0     | 1/2      | 3/2      |
|        | $(\bar{U}, \bar{E})_1$ | 1/2   | 0     | 0     | 1/2   | 0     | 0     | 1/2      | 3/2      |
|        | $(\bar{D}, \bar{\nu芾})_1$ | 1/2   | 0     | 0     | -1/2  | 0     | 0     | 1/2      | -1/2     |
| 2      | $q_2$          | 0     | 1/2   | 0     | 0     | -1/2  | 0     | 1/2      | -1/2     |
|        | $L_2$          | 0     | 1/2   | 0     | 0     | 1/2   | 0     | 1/2      | 3/2      |
|        | $(\bar{U}, \bar{E})_2$ | 0     | 1/2   | 0     | 0     | 1/2   | 0     | 1/2      | 3/2      |
|        | $(\bar{D}, \bar{\nu芾})_2$ | 0     | 1/2   | 0     | 0     | -1/2  | 0     | 1/2      | -1/2     |
| 3      | $q_3$          | 0     | 0     | 1/2   | 0     | 0     | -1/2  | -1      | -1/2     |
|        | $L_3$          | 0     | 0     | 1/2   | 0     | 0     | 1/2   | -1      | 3/2      |
|        | $(\bar{U}, \bar{E})_3$ | 0     | 0     | 1/2   | 0     | 0     | 1/2   | -1      | 3/2      |
|        | $(\bar{D}, \bar{\nu芾})_3$ | 0     | 0     | 1/2   | 0     | 0     | -1/2  | -1      | -1/2     |

| Color Triplet | $D_{45} = (3, -2/3, 1_L, 0)$ | -1/2 | -1/2 | 0 | 0 | 0 | 0 | -1 | -1 |
|---------------|-------------------------------|------|------|---|---|---|---|----|----|
|               | $\bar{D}_{45} = (3^*, +2/3, 1_L, 0)$ | 1/2  | 1/2  | 0  | 0  | 0  | 0  | +1 | +1 |

| Higgs doublets | $h_1 = (1, 0, 2_L, 1/2)$ | -1   | 0    | 0  | 0  | 0  | 0  | -1 | -1 |
|                | $h_2 = (1, 0, 2_L, 1/2)$ | 0    | -1   | 0  | 0  | 0  | 0  | -1 | -1 |
|                | $h_3 = (1, 0, 2_L, 1/2)$ | 0    | 0    | -1 | 0  | 0  | 0  | +2 | -1 |
|                | $h_{45} = (1, 0, 2_L, 1/2)$ | 1/2  | 1/2  | 0  | 0  | 0  | 0  | 1  | 1  |

| Hidden Matter  | $V_1, \bar{V}_1$ | 0 | 1/2 | 1/2 | 1/2 | 0 | 0 | -1/2 | 2 |
|                | $T_1, \bar{T}_1$ | 0 | 1/2 | 1/2 | -1/2 | 0 | 0 | -1/2 | 0 |
|                | $V_2, \bar{V}_2$ | 1/2 | 0 | 1/2 | 0 | 1/2 | 0 | -1/2 | 2 |
|                | $T_2, \bar{T}_2$ | 1/2 | 0 | 1/2 | 0 | -1/2 | 0 | -1/2 | 0 |
|                | $V_3, \bar{V}_3$ | 1/2 | 1/2 | 0 | 0 | 0 | 1/2 | 1 | 2 |
|                | $T_3, \bar{T}_3$ | 1/2 | 1/2 | 0 | 0 | 0 | -1/2 | 1 | 0 |

Table 3: Partial List of Massless States from Ref. [69].

(i) The quark and lepton fields have the standard properties under $SU(3)^C \times U(1)_{B-L} \times SU(2)_L \times U(1)_{T_{1,2,3}}$, which are not shown, but those of color triplets and Higgses are shown. (ii) Here $\hat{Q}_L \equiv Q_1 + Q_2 - 2Q_3$ and $\hat{Q}_\chi = (Q_1 + Q_2 + Q_3) + 2(Q_4 + Q_5 + Q_6)$ (see Eq. (14)). (iii) The doublets $h_{1,2,3,45}$ are accompanied by four doublets $h_{1,2,3,45}$ with quantum numbers of conjugate representations, which are not shown. (iv) The $SO(10)$-singlets $\{\phi\}$ which possess $U(1)_\psi$-charges, and the fractionally charged states which become superheavy, or get confined, are not shown. In Ref. [69], since only $h_1$ and $h_{45}$ remain light, families 1, 2 and 3 get identified with the $\tau$, $\mu$ and $e$ - families respectively. Hidden matter $V_i, \bar{V}_i, T_i$ and $\bar{T}_i$ are $SO(10)$-singlets and transform as $(1, 3), (1, \bar{3}), (5, 1)$ and $(\bar{5}, 1)$, respectively, under $SU(5)_H \times SU(3)_H$. 

46
Table 4: The roles of $Y$, $B - L$, $\hat{Q}_\psi$, $\hat{Q}_\chi$ and $\hat{Q}_\chi + \hat{Q}_\psi$ in allowing or forbidding the relevant $(B, L)$ violating operators. Check mark ($\checkmark$) means “allowed” and cross (×) means “forbidden”. The mark † signifies that the corresponding operator is allowed if either two of the four fields are in family (1 or 2) and two are in family 3, with $i = 1$ and $j = 3$; or all four fields are in family (1 or 2) with $i = 1$ and $j = 2$. The mark (*) signifies that $(\hat{Q}_\chi + \hat{Q}_\psi)$ forbids $\hat{Q}_\psi$ in conjunction with $B - L$ or $\hat{Q}_\chi$, gives adequate protection against all unsafe operators. This establishes the necessity of string-derived symmetries like $\hat{Q}_\psi$ (which can not emerge from familiar GUTs including $E_6$) in ensuring proton-stability.

| Operators | Family Combinations | $Y$ | $B - L$ | $\hat{Q}_\psi$ | $\hat{Q}_\chi$ | $\hat{Q}_\chi + \hat{Q}_\psi$ | If Allowed |
|-----------|---------------------|-----|---------|----------------|----------------|-----------------------------|------------|
| $\hat{U} \hat{D} \hat{D}, \hat{Q} \hat{L} \hat{D}, LL \hat{E}$ | (a) All except (b) | $\checkmark$ | $\times$ | $\times$ | $\times$ | $\times$ | unsafe |
| | (b) 3 fields from 3 different families | $\checkmark$ | $\times$ | $\checkmark$ | $\times$ | $\times$ | unsafe |
| $(\hat{U} \hat{D} \hat{D} or \hat{Q} \hat{L} \hat{D})(\hat{N}_R/M)$ | All | $\checkmark$ | $\checkmark$ | $\times$ | $\checkmark$ | $\times$ | unsafe |
| $(\hat{U} \hat{D} \hat{D} or \hat{Q} \hat{L} \hat{D})(\hat{N}_R/M) \times ([\hat{h}_i/M]^2 or ("\phi"/M)^n)$ | All | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\times$ | $\times$ | safe |
| $(\hat{U} \hat{D} \hat{D} or \hat{Q} \hat{L} \hat{D})(\hat{N}_R/M) \times (T_i \hat{T}_j/M^2)^2$ | Some(†) | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | safe |
| $QQQL/M$ | All | $\checkmark$ | $\checkmark$ | $\times$ | $\checkmark$ | $\times$ | unsafe |
| $(QQQL/M)(\hat{N}_L/M)_{i=1,2}$ | e.g. (1, 2, 1, 3) | $\checkmark$ | $\times$ | $\checkmark$ | $\times$ | $\times$ | unsafe |
| $(QQQL/M)(\hat{N}_L/M)(\hat{N}_R/M)$ | All | $\checkmark$ | $\checkmark$ | $\times$ | $\checkmark$ | $\times$ | safe(?) |
| $(QQQL/M)(T_i \hat{T}_j/M^2)^2$ | Some(†) | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | safe |
| $UUDE/M$ | All | $\checkmark$ | $\checkmark$ | $\times$ | $\times$ | $\times$ | unsafe |
| $LLh_i \hat{h}_i/M$ | All | $\checkmark$ | $\times$ | $\times$ | $\times$ | $\times$ | unsafe |

Note that the pairs $(Y, B - L)$, $(Y, \hat{Q}_\psi)$, $(Y, \hat{Q}_\chi)$ and $(B - L, \hat{Q}_\chi)$ do not give adequate protection against the unsafe operators. But $\hat{Q}_\psi$, in conjunction with $B - L$ or $\hat{Q}_\chi$, gives adequate protection against all unsafe operators. This establishes the necessity of string-derived symmetries like $\hat{Q}_\psi$ (which can not emerge from familiar GUTs including $E_6$) in ensuring proton-stability.

$\langle \hat{N}_R \rangle \sim 10^{15.5}$ GeV, $\langle \phi/M \rangle^n \leq 10^{-9}$ and $M \sim M_{st} \sim 10^{18}$ GeV, and that hidden sector condensate-scale $\Lambda_c \leq 10^{15.5}$ GeV (see text). Note that the pairs $(Y, B - L)$, $(Y, \hat{Q}_\psi)$, $(Y, \hat{Q}_\chi)$ and $(B - L, \hat{Q}_\chi)$ do not give adequate protection against the unsafe operators. But $\hat{Q}_\psi$, in conjunction with $B - L$ or $\hat{Q}_\chi$, gives adequate protection against all unsafe operators. This establishes the necessity of string-derived symmetries like $\hat{Q}_\psi$ (which can not emerge from familiar GUTs including $E_6$) in ensuring proton-stability.
Fig. 6  Typical Diagrams for Proton Decay into $\bar{\nu}\kappa^+$ through $d=5$ operators

Fig. 7  Typical Diagrams for Proton Decay into $\bar{\nu}\pi^+$

Fig. 8  Typical Diagrams for Proton Decay into $\mu^+\kappa^0$
\( \alpha_1^{-1}(\mu) \)

\( \alpha_2^{-1}(\mu) \)

\( \alpha_3^{-1}(\mu) \)

MSSM

\( M_{\text{SUSY}} = M_Z \)
