Observation of many-body localization in a one-dimensional system with single-particle mobility edge

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In this work we experimentally study many-body localization (MBL) in a one-dimensional bichromatic quasiperiodic potential with a single-particle mobility edge (SPME) using ultracold atoms. We measure the time evolution of the density imbalance between even and odd lattice sites from an initial charge density wave, and analyze the corresponding relaxation exponents. We find clear signatures of MBL in this system when the corresponding noninteracting model is deep in the localized phase. We also critically compare and contrast our results with those from a tight-binding Aubry-André model, which does not exhibit an SPME.

Introduction.— In the past decade, it has been established that an isolated one-dimensional (1D) quantum system with strong quenched disorder can be localized, even if finite interactions are present [1–19]. Such a phenomenon, now known as many-body localization (MBL), represents a generic example of ergodicity breaking in isolated quantum systems. In particular, the eigenstate thermalization hypothesis (ETH) [20, 21] is strongly violated in such systems, leading to the inapplicability of textbook quantum statistical mechanics. Recently, experiments have found strong evidence for the existence of an MBL phase in 1D systems with random disorder [22–24] and in models with quasiperiodic disorder [25, 26] captured by the Aubry-André (AA) tight-binding lattice model [7]. One hallmark of the AA model is that the localization transition occurs sharply at a single disorder strength. As a result, across the transition, all single-particle eigenstates in the spectrum suddenly become exponentially localized without mobility edges.

In contrast, there are many other 1D models which exhibit a single-particle mobility edge (SPME) [27–35], i.e. a critical energy separating extended and localized eigenstates in the spectrum. As a result, a single-particle intermediate phase (SPIP) characterized by a coexistence of localized and extended eigenstates in the energy spectrum appears in the phase diagram (see Fig. 1). Experimental signatures of such an intermediate phase have been recently observed using ultracold atomic gases in a 1D quasiperiodic optical lattice described by a generalized Aubry-André (GAA) model including next-nearest neighbor tunneling [36, 37], as well as in a momentum-space lattice system with artificial gauge fields [38]. In the presence of interactions two natural questions arise: (i) Does an MBL phase exist in a model which in the limit of vanishing interactions exhibits an SPME? (ii) Does the SPIP survive finite interactions to become a many-body intermediate phase (MBIP)? This would suggest the existence of an intermediate phase, where extended and localized many-body states coexist in the energy spectrum. A system with delocalized non-ergodic many-body states is often referred to as non-ergodic metal phase [15, 16, 39]. Its existence is controversial and subject to current theoretical research. To date, there has not been any experimental investigation on these questions. Heuristically one can argue that an MBIP cannot exist because the extended states in the single-particle spectrum will delocalize the localized states as soon as interactions couple them [40]. Some recent work has indeed argued on theoretical grounds that no MBIP can exist in the thermodynamic limit in a system with random disorder [40, 41]. In contrast there have been extensive numerical simulations in the literature asserting the existence of an MBIP.

![Heuristic phase diagram of the generalized Aubry-André model](image)

**FIG. 1.** Heuristic phase diagram of the generalized Aubry-André model: The noninteracting GAA model exhibits three phases (single-particle extended, single-particle intermediate (SPIP), and single-particle localized), with the phase boundary denoted by A and B. Here Δ is the strength of the detuning lattice [Eq. (2)], while U is the strength of the Hubbard on-site interactions [Eq. (4)]. The situation with finite interactions is unknown in theory, although a full many-body localized (MBL) phase is believed to exist in the regime where the corresponding noninteracting system is single-particle localized. Below the single-particle localization transition point A interactions will lead to a thermal phase where the eigenstate thermalization hypothesis (ETH) holds. The existence of a many-body intermediate phase (MBIP, marked in gray) is highly debated.
in various different systems [9–17, 39, 42–46]. Given the
direct observation of the SPME in recent experi-
ments [37, 38], this issue takes on immediate experimen-
tal significance regarding the fate of this noninteracting
intermediate phase as interactions are added. Equally
important is the question of the existence of an MBL
transition in a system manifesting an SPME.

In this work, we address the questions raised above
by experimentally studying MBL in a model with an
SPME using fermionic ultracold atoms in optical lattices.
Moreover, we present a detailed comparison of the inter-
action effects on the thermalization dynamics in mod-
els with and without an SPME, i.e., the AA and GAA
model. In particular, we present experimental and nu-
merical exact diagonalization results for the relaxation
of an initial charge-density wave (CDW) [25] in the two
systems. As in the AA model [25], we observe full MBL
in the GAA model, but only in a regime where all single-
particle states in the corresponding noninteracting model
are localized. In addition, we find no discernible diffe-
rence in the relaxation dynamics of the two models for
all system parameters within the experimentally acces-
sible time scales. Hence, we conclude that the single-
particle extended states do not act as an efficient bath
for the localized states for the initial states and para-
ter regimes probed in this work. This apparent inability
of the extended states in inducing relaxation of the loca-
lized states and the observation of MBL itself in the GAA
model with SPME are the main results of our work.

Experiment.— Our experimental system consists of
a primary lattice with a wavelength of \( \lambda_p = 532 \text{ nm} \)
and two deep orthogonal lattices at a wavelength of
738 nm, which divide the atomic cloud into an array of
1D tubes with lattice spacing \( d = \lambda_p/2 \). A detuning
lattice (\( \lambda_d = 738 \text{ nm} \)) incommensurate with the primary
lattice introduces quasi-periodicity and enables the real-
ization of both the AA and the GAA model, depend-
ing on the primary lattice depth. In the noninteracting limit
such a system is described by the following continuum
Hamiltonian (incommensurate lattice model)

\[
\hat{H} = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + \frac{V_p}{2} \cos(2k_p x) + \frac{V_d}{2} \cos(2k_d x + \phi), \tag{1}
\]

where \( k_i = 2\pi/\lambda_i \) (\( i = p, d \)) is the wavevector of
the corresponding lattice, \( m \) is the mass of the atoms, \( V_i \)
(\( i = p, d \)) is the respective lattice depth, and \( \phi \) is the
relative phase between the primary and detuning lattice.
We will use the recoil energy of the primary lattice \( E_p^r = \hbar^2 k_p^2/(2m) \) with the reduced Planck constant \( \hbar \) as the
energy unit throughout this work.

In the tight-binding limit (i.e., when the primary lat-
tice potential \( V_p \) is deep) the continuum Hamiltonian in
Eq. (1) maps onto the 1D AA model,

\[
\hat{H}_{AA} = -J_0 \sum_{j,\sigma} \left( \hat{c}_{j+1,\sigma}^\dagger \hat{c}_{j,\sigma} + \text{h.c.} \right) + \Delta \sum_{j,\sigma} \cos(2\pi \alpha j + \phi) \hat{n}_{j,\sigma}, \tag{2}
\]

which describes our experiment sufficiently well at a primary
lattice depth \( V_p \gtrsim 8E_p^r \) [37]. In the above Hamilton-
ian, \( J_0 \) is the nearest-neighbor hopping energy, and \( \Delta \) is the strength of the detuning lattice. The operator \( \hat{c}_{j,\sigma}^\dagger \hat{c}_{j,\sigma} \) denotes the creation (annihilation) operator for
spin \( \sigma = \uparrow, \downarrow \) on lattice site \( j \), and \( \hat{n}_{j,\sigma} = \hat{c}_{j,\sigma}^\dagger \hat{c}_{j,\sigma} \) is the
the corresponding fermion number operator. The incommen-
surability \( \alpha = \lambda_p/\lambda_d \approx 532/738 \) is the ratio of primary
and detuning lattice wavelengths. The model defined by
Eq. (2), the AA model, is well-known to have a localiza-
tion transition at \( \Delta = 2J_0 \) when all energy eigenstates
convert from being extended to localized [7].

Beyond the tight-binding limit, corrections have to be
added to the AA model. These corrections can be derived
via a Wegner flow approach [36], leading to a GAA model
Hamiltonian \( \hat{H}_{GAA} = \hat{H}_{AA} + H' \), with

\[
\hat{H}' = J_1 \sum_{j,\sigma} \cos \left( 2\pi \alpha \left( j + \frac{1}{2} \right) + \phi \right) \left( \hat{c}_{j+1,\sigma}^\dagger \hat{c}_{j,\sigma} + \text{h.c.} \right) + \frac{J_2}{2} \cos\left( \right) \hat{n}_{j,\sigma}, \tag{3}
\]

For a detailed description of the parameters see [47].
Note that the GAA model of Eq. (3) is by definition non-
nearest-neighbor and therefore cannot be characterized
by a single dimensionless parameter \( \Delta/J_0 \) as in the AA
model. Thus, the self-duality symmetry of the nearest-
neighbor AA model is explicitly broken and an SPME
appears in the energy spectrum [36].

Experimentally, the GAA model is realized with a shal-
lower primary lattice with \( V_p = 4E_p^r \) [36, 37]. We employ
an atom cloud of about \( 5 \times 10^4 \) fermionic \(^{40}\text{K} \) atoms
at a temperature of \( 0.15(2)T_F \), where \( T_F \) is the Fermi
temperature in the dipole trap, and load it into the 3D
optical lattice. The gas consists of an equal spin mixture
of the states \( |\uparrow\rangle \equiv |m_F = -7/2 \rangle \) and \( |\downarrow\rangle \equiv |m_F = -9/2 \rangle \)
of the \( F = 9/2 \) ground state hyperfine manifold. On-
site interactions can be tuned via a magnetic Feshbach
resonance at 202.1 G, which is captured theoretically via
Fermi-Hubbard-type interactions

\[
\hat{H}_U = U \sum_j \hat{n}_{j,\uparrow} \hat{n}_{j,\downarrow}. \tag{4}
\]

Using a superlattice with wavelength \( 2\lambda_p \), an initial
CDW is created in the primary lattice, where only even
The experimental data is in reasonable agreement with exact diagonalization simulations with eight particles on 16 lattice sites, which were averaged for random initial spin configurations [47]. The offset is most likely caused by the harmonic trap present in the experiment [25].

We attribute the different behaviors of the imbalance dynamics of the AA model at different disorders to a many-body localized and many-body extended (i.e., ETH) phase [7, 25], above and below an interaction-dependent critical disorder strength respectively. Due to the remarkably similar dynamics in the GAA model, we infer that MBL exists in this model despite the presence of an SPME in the noninteracting limit. The data further shows that we have a many-body extended phase at weak detuning. Finally, we observe that for any detuning strength the imbalance time traces of the two models are indistinguishable within our resolution, both above and below the MBL transition. For strong detuning this behavior is expected since all single-particle states are localized and we infer that the interacting system is likely many-body localized.

Relaxation exponents.— To better quantify the relaxation dynamics, we fit the imbalance time traces using a power-law function $I \propto t^{-\xi}$ (see Fig. 2), and extract the resulting exponents $\xi$ as shown in Fig. 3. Note that a power-law description for a system with quasiperiodic potentials is not motivated by the standard Griffiths description which is presumably only applicable for randomly disordered systems [18, 54–56].
Nonetheless, we find our data to be well described by such power-laws. For a detailed discussion of the applicability of this picture see Ref. [52]. In the GAA model we observe that the exponents reach a value of 0.33(5) just above the single-particle localization transition point, for larger detuning lattice strengths the exponents decrease and finally converge to a constant positive plateau around $\Delta/J_0 \approx 3.0(2)$, which is significantly larger than the single-particle localization transition point $\Delta/J_0 \approx 2.6$ [36, 37]. Although $\xi = 0$ is expected in the MBL phase, we regard our system to be many-body localized in this regime and attribute the residual decay to the existence of external baths. Off-resonant photon scattering [51, 57] and couplings between different 1D tubes [50] give rise to a finite imbalance lifetime even in the many-body localized phase. Moreover, the experimental exponents are in reasonably good agreement with numerical simulations in a system with $L = 16$ sites [47]. This observation implies that MBL indeed occur in a system with an SPME, although only in the single-particle localized regime, as illustrated in Fig. 3.

As pointed out above, below the single-particle localization transition the imbalance decay is very fast, corresponding to a thermal phase. In contrast, for stronger detuning strengths we observe slow dynamics [52]. We now analyze this regime of interest in more detail. In the potential MBIP of the GAA model one could expect that the presence of extended states gives rise to a faster relaxation of the imbalance since the single-particle extended states may act as a bath for the coexistent localized states when coupled by interactions. In order to investigate this assumption, we compare the relaxation exponents of the GAA model and the AA model in Fig. 4. They turn out to be indistinguishable within the experimental uncertainties across all investigated detuning strengths. This fact provides an indication that the extended states in the noninteracting spectrum do not act as an effective bath thermalizing the whole system, at least within the time scales of our experiment. We also numerically investigate longer evolution times, where we find hints towards a faster relaxation in the intermediate regime in the GAA model, although this observation is not fully conclusive [47].

It has been proposed that an MBIP may also exist at large interactions due to symmetry-constrained dynamics [11]. We perform measurements at stronger interactions $U/J_0 = 4$ again for both models as shown in Fig. 4. The exponents at the same detuning strengths are overall larger at stronger interactions, accompanied by a shift of the critical disorder strength. Also for the case of strong interactions we find that the exponents are remarkably similar without any direct evidence for a potential intermediate phase. The fact that interacting AA and GAA models provide similar exponents with similar critical disorder strengths for MBL does not provide evidence for the existence of an MBIP, but neither contradicts it.

**Outlook.** — We have experimentally and numerically investigated the localization transition of the GAA model in the presence of interactions. We find that for
large enough detuning lattice strengths, the system likely reaches the many-body localized phase when all single-particle states in the noninteracting limit have been localized. Furthermore, we compare the experimental relaxation exponents in the AA model and the GAA model for multiple detuning and interaction strengths, and find that they are similar on short time scales, indicating that the coexistent extended states do not serve as an efficient bath within the experimentally accessible time scales for the initial states probed in this work. Therefore, our results do not rule out the existence of an MBIP, but do not provide any direct evidence. Our results suggest that the imbalance measurement alone may not be a reliable diagnostic to decisively detect the elusive nonergodic metal phase. In the future, it is worthwhile to extend the experimental measurements to much longer times in order to investigate the stability of MBL and reveal potential delocalization mechanisms introduced by the spin degree of freedom [58–63]. In addition, it is desirable to find a definitive experimental diagnostic for the possible many-body intermediate phase, which is currently lacking.

Acknowledgments.— We thank Ehud Altman for insightful discussions. We acknowledge financial support by the European Commission (UQUAM grant no. 319278, AQUS) and the Nanosystems Initiative Munich (NIM grant no. EXC4). Further, this work is supported at the University of Maryland by Laboratory for Physical Sciences and Microsoft.

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Supporting Materials

EXPERIMENTAL DETAILS

Data evaluation

To record the time traces as shown in Figs. 2 and S1 we take ten points which are evenly spaced on a logarithmic time scale either between 10 and 100τ in the GAA model or 10τ and 40τ in the AA model. One tunneling time in the AA model is τ ≃ 0.29 ms and in the GAA model τ ≃ 0.11 ms respectively. Each data point is averaged over six different detuning phases φ [see Eq. (1)] and error bars denote the standard error of the mean.

To determine the relaxation exponents ξ of the power-law I ∝ t−ξ, we fit a linear function to log(I) versus log(t). The error bars in Figs. 3, 4 and S2 denote the fit uncertainty of the slope of the linear function which directly corresponds to the relaxation exponent ξ.

Tube averaging and its impact

Our experiment is carried out in a three-dimensional optical lattice. The system is split into individual one-dimensional tubes along the x-direction via deep orthogonal lattices along the y- and z-direction with a depth of 40E_p each. The corresponding tuning rate J_x along these axes is reduced by a factor J_x/J_0 = 6 × 10^{-4} in the GAA model and J_x/J_0 = 2 × 10^{-3} in the AA model. Due to the Gaussian-shaped intensity distribution of the laser beams (beam waist 150 μm), inner and outer tubes have slightly different values of V_p and V_d. Given the measured in-situ cloud size (FWHM) of 42 μm in the x-y-plane, the lattice depth in the outermost tube is 14 % smaller than that in the central tube. Correspondingly, the ratio of the tight-binding parameters Δ/J_0 is about 30 % smaller there. In our detection sequence, the bandmapping procedure [48, 49] practically averages over all 1D systems such that our measured imbalance reveals the average dynamics of tubes with different lattice depths, weighted by the respective atom numbers. As demonstrated in Ref. [37], tube averaging only has a very small impact on the imbalance evolution. The reason is that in the outer wings, which are mostly affected by the Gaussian laser beams, the average occupation is smaller and the averaged imbalance, as a local probe of the system, is rather insensitive to those low-density regions.

On the other hand, due to the lower lattice depth in the outer wings, the high-energy extended states see a smaller detuning strength Δ/J_0 and thus do not yet localize, while the center of the system is already in the fully localized phase. This effect therefore leads to an overall broadening of the single-particle intermediate regime as compared to the central tube only. Consequently, there may be MBL in the single-particle intermediate regime, which is broadened due to the harmonic trap in our system as compared to the SPIP illustrated in Fig. 3. This can enlarge the SPIP up to Δ/J_0 ≃ 3.5 [37]. Resolving this intriguing question, however, requires a better experimental resolution, in particular for the determination of the MBL transition point, and precise knowledge of the boundaries of the single-particle intermediate phase since they strongly depend on the experimental parameters and initial density distribution [37].

Model parameters

In this paper, we investigated two lattice models which are valid in different regimes. The AA model is the tight-binding approximation of the continuum Hamiltonian in Eq. (1) and implemented in the experiment by a deep (V_p = 8E_p) primary lattice such that next-nearest-neighbor hopping can be neglected. Relevant parameters in this model are the nearest-neighbor tunneling amplitude J_0 and the detuning strength Δ. In the GAA model, the tight-binding description is no longer valid and corrections have to be added to the terms of the AA model, which lead to the appearance of an SPME. Up to first order, these are the correction to the nearest-neighbor tunneling amplitude in the primary lattice due to the detuning lattice J_1, the next-nearest-neighbor hopping amplitude in the primary lattice J_2 and a correction to the detuning strength and thus to the on-site potential Δ'. We employ two methods to calculate these parameters, an analytical calculation based on the first band Wannier functions of the primary lattice and the numerical Wegner flow approach.

The tight-binding parameters of the AA-model as well as the next-nearest neighbor tunneling amplitude can be computed analytically via the unperturbed Wannier functions w_j of the primary lattice at site j:

\[ J_0 = -\langle w_0 | \hat{H}_0 | w_1 \rangle \equiv -\int_{-∞}^{∞} dx \, w_0^*(x) \hat{H}_0 w_1(x), \]

\[ Δ = \frac{V_d α^2}{2 E_p} \langle w_0 | \cos(2αk_p x) | w_0 \rangle, \]  

\[ J_2 = -\langle w_0 | \hat{H}_0 | w_2 \rangle, \]  

where \( \hat{H}_0 = -\frac{ℏ^2}{2m} \frac{d^2}{dx^2} + \frac{1}{2} V(x) \). Note that the parameter J_2 is independent of the detuning strength V_d. When the experimental lattice depths V_p and V_d are known, the parameters in Eq. (S01) can be computed directly.

The remaining parameters J_1 and Δ', however, require the Wannier functions of the detuned primary lattice and cannot be computed in that manner. Instead, the Wegner flow method [36] is required to generate the full set of parameters. Starting from the continuum model [Eq.(1)] with lattice depth V_p and V_d the GAA Hamiltonian in
The relevant model parameters used in the experiment. While in the AA model, higher order corrections are negligible, they have to be accounted for in the GAA model. Note the the sign of $J_1$ and $J_2$ is opposite to $J_0$ due to our convention in Eqs. (2) and (3).

Additionally, we would like to specify how the connection between the GAA and AA model is established. In the experiment we choose the model via the primary lattice depth $V_p$ and set the desired detuning strength via the depth of the incommensurate lattice $V_d$. In the simulation we fixed the primary lattice strength to be $V_p = 4E_p^\ast$, and choose various different values for the detuning strength $V_d$. For each $(V_p, V_d)$ pair, we first generate the corresponding GAA model parameters using the Wegner flow method [36] (see Table S2). These parameters then enable us to simulate the temporal evolution of the density imbalance in the GAA model. In order to obtain the corresponding AA model results, we remove the $H'$ term in Eq. (3) from the GAA model Hamiltonian, and calculate the dynamics accordingly. The conversion from $V_d$ to $J/d$ is thus independent of the model. This is unlike the experiment where due to the different primary lattice depths $J_0$ has a different value and thus $V_d$ has to change in order to get the same detuning in units of $J/d$. This circumstance is visualized in Table S1 and Fig. S2.

### Time traces and exponents for $U/J_0 = 4$

In the main text we focused on the case of weak interactions ($U/J_0 = 1$) and found that the imbalance cannot resolve a difference in the relaxation dynamics of the models, induced by a potential many-body intermediate phase. For completeness we show the data for stronger interactions ($U/J_0 = 4$) here, in particular the corresponding time traces (Fig. S1) and relaxation exponents (Fig. S2). We basically observe the same behavior as for weak interactions, namely, an indistinguishability of the imbalance time traces accompanied by the same relaxation exponents within our experimental resolution.

Moreover, we observe, as expected from previous studies on the AA model [25, 52], an interaction-dependent transition point from the extended to the localized phase. The critical detuning is presumably the same in the AA and the GAA model and extracted to be $\Delta/J_0 = 4.0(4)$ and thus significantly larger than for $U/J_0 = 1$. Our experimental results are also in good agreement with the exact diagonalization simulations in a system with $L = 16$ sites. The experimental and numerical exponents in Fig. S2 deviate at large detuning strengths due to residual decay mechanisms in the experiment. As mentioned in the main text these are off-resonant photon scattering [51, 57] and coupling between neighboring 1D
tubes [50].

FIG. S2. Power-law exponents: Measured relaxation exponents as a function of the detuning strength for the AA model and the GAA model at $U/J_0 = 4$. The error bars denote the uncertainty of the fit. The rectangles are the numerically extracted exponents and the uncertainty is represented by the shaded region. The gray region indicates the regime of the single-particle intermediate phase (SPIP) which is much narrower in the AA model.

NUMERICAL SIMULATIONS

In this section of the supplementary material we present details of our numerical simulations in a system with up to $L = 16$ sites. In particular, because we are dealing with an interacting system, it is inconvenient to work with the continuum model in Eq. (1). Instead, all simulations are based on lattice models, including both the AA model in Eq. (2) and the GAA model in Eq. (3).

The quench dynamics of an initial CDW state

The temporal evolution of the density imbalance studied in our experiment can be simulated efficiently in a system with $L \leq 16$ sites. For $L > 16$, the finite size calculation becomes prohibitively difficult in the presence of interactions because of the exponential increase in the Hilbert space size. Moreover, the system size has to be a multiple of four in order to account for the charge-density wave initial state and an equal spin mixture. As a result, we choose to work with $L = 8, 12,$ and 16 only. We take open boundary conditions and fix $\alpha = 532/738$ in the AA and GAA model, in accordance with the experiment. All other parameters in the GAA model are generated by the Wegner flow method from the continuum model in Eq. (1) for each pair of $V_p$ and $V_d$. The initial CDW state is chosen to have zero magnetization and quarter-filling ($L/4$ up spin and $L/4$ down spin fermions). These spins are randomly distributed throughout all even sites, and no doublons are allowed in the initial states. The resulting Hilbert space dimension is 784 for $L = 8$, 48400 for $L = 12$, and 3312400 for $L = 16$. Each density imbalance result is obtained as an average over 8 random initial state realizations and 10 random phases $\phi$. Due to the large Hilbert space dimension for $L = 16$, such a calculation is most efficiently carried out using the kernel polynomial method (KPM) [64].

Figure S3 shows exemplary time traces of the density imbalance $I$ for three typical values of $V_d$, from which we can extract a power-law fit and obtain the corresponding exponents $\xi$, which are used extensively in this work. We can further perform an approximate finite-size scaling analysis in the interacting system ($U/J_0 = 4$). Specifically, we first calculate the time traces of $I$ in a system of $L = 8, 12,$ and 16 sites, and then extrapolate the results to $L = \infty$ by plotting the exponent $\xi$ at a given $V_d$ as a linear function of $1/L$, as shown in Fig. S4. The intercept on the vertical axis yields the extrapolated exponent, which we denote as $\xi_\infty$. Such a result is shown as the $L = \infty$ curve in Fig. S5. From a comparison of the data points and the linear extrapolation function we can see that this analysis tends to overestimate $\xi_\infty$ for smaller $V_d$, but works better when the system is more localized.
FIG. S4. **Finite-size scaling of the interacting exponents:** The interacting exponent $\xi$ in the $L \to \infty$ limit is estimated crudely by extrapolating the corresponding exponent in a system with $L = 8$, $12$, and $16$, respectively.

FIG. S5. **Estimate of the localization transition point:** In this plot we again have $U = 4J_0$, and $V_d = 4E_p^r$. The results for $L = 8$, $12$, and $16$ are included in the plot, together with a dataset ($L = \infty$) obtained by finite-size scaling. We also reproduce the experimental exponent $\xi$ from Fig. S2.

**Numerical results at longer times**

It is helpful to go beyond the current experimental results by numerically calculating the quench dynamics at much longer times (although in a small system). The key question we want to answer is whether there is a qualitative difference between the dynamics at short ($< 100\tau$) and long ($> 100\tau$) time scales. This question is motivated by the possibility that the coupling between extended and localized states might be small such that differences in the dynamics only become visible at longer times.

FIG. S6. **Numerical results for the density imbalance in the GAA and AA models in the long time limit:** The interaction strength is $U/J_0 = 4$. The AA model is shown in green, the GAA model in blue and the brightness translates to the corresponding detuning strength. The finite value in the lower traces is due to finite-size errors.

We carry out numerical simulations to explore the relaxation dynamics at longer time scales between 100 and 500$\tau$, a regime that cannot be reached in the present experiment due to residual external baths. Hence, although these numerical simulations are carried out in a much smaller system ($L = 16$), they provide an important complementary perspective for our experimental results. Figure S6 shows the computed imbalance time traces for three different detuning strengths. The three curves are chosen such that the corresponding noninteracting system is in the extended, intermediate, and localized regime, respectively [36]. One can clearly identify a thermal regime ($V_d = 0.2E_p^r$) which is characterized by a fast initial decay and a small stationary imbalance, which we mostly attribute to finite-size effects in the simulations. Contrarily, the MBL regime ($V_d = 0.5E_p^r$) is characterized by a large and almost non-decaying imbalance. Finally, the third trace ($V_d = 0.55E_p^r$) taken below the MBL transition exhibits slow dynamics [52] that can be consistently fit to a power-law description $I \propto t^{-\xi}$ between 100$\tau$ and 500$\tau$, which provides a good opportunity for us to explore potential differences between short and long-term dynamics. Specifically, we can extract the relaxation exponent $\xi$ within this time scale, and check if there is an appreciable difference between the AA and GAA model. The results are shown in Fig. S7.

The results in Fig. S7 suggest that for strong detuning lattices ($V_d \gtrsim 0.7E_p^r$) the relaxation exponents $\xi$ extracted from both models are very similar, and decrease towards zero, suggesting the existence of an MBL phase at large $V_d$, which is consistent with our experimental results that were obtained at shorter time scales. Within
the single-particle intermediate phase \((0.5E_p^r < V_d < 0.65E_p^r)\), however, the exponents of the GAA model are slightly larger than those of the AA model, indicating that the single-particle extended states might possibly contribute to the relaxation of the system at this longer time scale. However, due to finite-size limitations of this calculation, these results are not fully conclusive.

**FIG. S7.** Numerical relaxation exponents \(\xi\): The exponents are extracted from power-law fits between \(100\tau\) and \(500\tau\). The system size is \(L = 16\), the primary lattice depth is \(4E_p^r\), and \(U/J_0 = 4\). Error bars denote the uncertainty of the fit.