Longitudinal fluctuations of rotors of disk knife refiners

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Abstract. The subject of the study is the longitudinal fluctuations of the rotors of disk knife refiners. The dynamic and mathematical models of rotors of disk mills are developed. As a result of the research, a methodology for calculating the longitudinal fluctuations of rotors is proposed and tested. The error in determining the lower frequencies of free fluctuations of the mill rotors does not exceed 9%. The lowest frequencies of free longitudinal fluctuations of the rotors of existing mills are in the range of 85-165 Hz, and the second and subsequent frequencies of free fluctuations are in the frequency range above 1 kHz. When designing and operating disk knife refiners, it is necessary that the minimum inter-knife gap between the rotor and stator does not exceed the amplitude of fluctuations of the mill disk. The developed calculation procedure can be used in other industries, for example, mining and metallurgy.

1. Introduction

Knife refiners are the main technological equipment for refining fibrous materials in the pulp and paper industry [1-4]. The inter-knife gap in these machines is a fraction of a millimeter [5,6]. When designing and operating knife refiners, it is necessary that the minimum inter-knife gap between the rotor and stator does not exceed the amplitude of disk fluctuations in the longitudinal direction [7]. Therefore, it is necessary to study the longitudinal fluctuations of the mill rotors and to develop a method of fluctuation calculation. The purpose of the work is to study the longitudinal fluctuation of the mill rotor.

2. Methods and materials

2.1. Dynamic rotor model

The following assumptions are made: the study is conducted in a linear formulation; the dispersion of fluctuations in the rotor design is not taken into account; the mill rotor shaft is modeled as a uniform rod; disk deformations, the effect of radial bearings and couplings are not taken into account. It is assumed that the centers of mass of the cross sections of the rod lie on the same axis, and the sections themselves remain flat, their transverse deformations are not taken into account; the effects of bending and torsional fluctuations are not taken into account. The main factor in the excitation of fluctuations is considered to be the forceful effect of the milled semi-finished product.

The dynamic model is represented as a homogeneous rod with a cross-sectional area $S$, elastic modulus $E$, mass $m$, and equivalent length $l$ (figure 1). The mass of the disk $m_d$ is taken as a concentrated
A dissipative force, proportional to the vibration speed of the disk, acts on the disk. The rotor rotates at a constant frequency $\omega$. The total stiffness of the thrust bearing and its mounting are denoted by $C_y$. The mechanical and hydrodynamic effect of the milled material on the disk is modeled in the form of axial force $F(t)$, consisting of constant, periodic and random components [8-10].

2.2. The mathematical model of the rotor

Let us select the element $dy$ of the rod (figure 2) and consider its equilibrium in the direction of the Y axis. If the rod has constant characteristics $ES = const$, $\rho S = const$ along the length, then the homogeneous equation describing its free fluctuations takes the following form:

$$\frac{\partial^2 u}{\partial y^2} - \frac{1}{a^2} \frac{\partial^2 u}{\partial t^2} = 0,$$

(1)

where $a$ is velocity of longitudinal waves in the rod.

We divide the variables by introducing a time factor harmoniously changing with time

$$u(y, t) = v(y) \cos \omega t.$$

(2)

With the joint solution of (1) and (2), we will obtain

$$\ddot{v} + \frac{\omega^2}{a^2} v = 0.$$

(3)

We will represent the solution (3) in the form

$$v(y) = A \cos \frac{\omega}{a} y + B \sin \frac{\omega}{a} y.$$

(4)

Boundary conditions for the study of the model:

if $y=0$
\[ ES \frac{\partial u}{\partial y} - m_g \frac{\partial^2 u}{\partial t^2} - F(t) = 0. \]  
(5)

if \( y = l \)

\[ ES \frac{\partial u}{\partial y} + C_y u = 0. \]  
(6)

3. Results and discussion

Substitution of (2) and (4) into the boundary conditions (5) and (6) gives a system of linear homogeneous equations. Given only the first harmonic of the axial force \( F(t) = F_0 \cos \omega t \) and equating the main determinant of the system of equations to zero, we find the frequency equation, the real roots of which at \( \omega = \omega_0 \) are the frequencies of the longitudinal fluctuations of the rotor. The lowest is the frequency of free longitudinal fluctuations

\[
\omega_{b1} = \sqrt{\left( \frac{C_y}{(m_g + m_g)} \right)^2 + \left( \frac{C_s}{m_g} \right)^2} \sqrt{\frac{\omega_0^2}{\omega_{m}^2} + \frac{\omega_0^2}{\omega_{m}^2}},
\]

where \( \omega_0 = \left[ C_y / (m_g + m_g) \right]^{1/2} \) is the lowest partial frequency of free fluctuations of an absolutely rigid rotor in the longitudinal direction, \( \omega_{m} = \left( C_s / m_g \right)^{1/2} \) is lower partial frequency of free fluctuations of the disk at the sealed end of the rod in the longitudinal direction, \( C_s = \frac{ES}{l} \) is longitudinal stiffness.

The lower frequencies of the free longitudinal fluctuations of the rotors of existing mills are presented in Table 1.

| Mill brand | The lowest frequency of free fluctuations, Hz |
|------------|---------------------------------------------|
|            | Calculation | Experiment | Error, % |
| MD-31      | 98          | 92         | 7        |
| MD-5Sh1A   | 86          | 80         | 8        |
| MD-4Sh6    | 94          | 86         | 9        |
| MD-23      | 127         | 136        | 7        |

The lowest frequencies of free longitudinal fluctuations of the rotors of existing mills are in the range of 85-165 Hz, and the second and subsequent frequencies of free fluctuations are in the frequency range above 1 kHz. When developing methods and means of vibration protection of rotors in the longitudinal direction, it is necessary to take into account only their lowest frequency of free fluctuations. The error in determining this frequency does not exceed 9%. 

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The dispersion of rotor vibrations occurs in the bearings, shaft material, and the medium in which the rotor rotates. It is impossible to analytically account for all types of dispersion. The introduction of a member that generalizes all types of dispersion of vibrations is known in [11–13]. The parameters of inelastic resistances in this case are determined experimentally. The amplitude of the displacements of the rotor disk in the longitudinal direction \( y = 0 \) in the low- and mid-frequency region, taking into account the dissipative forces in the system

\[
S_a = S_{ct} \left[ \left( 1 + \frac{C_Y}{C_c} \right) \right] / \left\{ 1 - \frac{\omega^2}{\omega_e^2} = \frac{\omega^2}{(\omega_e)^2} \right\}^2 + \frac{\omega^2}{(\omega_0)^2} \chi_{pn}^2 \right\}^{1/2},
\]

where \( S_{ct} = F_1/C_y \) - static displacement of an absolutely rigid rotor in the longitudinal direction, \( \chi_{pn} \) - dynamic stress coefficient at resonance in the longitudinal direction.

The amplitude-frequency characteristic of the rotor in the longitudinal direction with reference to the mill MD-31 is presented in figure 3.

\[\text{Figure 3. The amplitude-frequency characteristic of the rotor in the longitudinal direction in relation to the mill MD-31 if } F_1 = 10^3 \text{ H: o - excluding inelastic resistances; x - taking into account non-elastic resistances.}\]

The longitudinal fluctuations of the mill rotor are excited not only at the set frequencies [14], but, as shown by experimental studies, at the reverse frequencies and their harmonics. The source of these excitations is the imperfection of the structural elements of mills, in particular the end runout of the disk [15].

When designing and operating knife refiners, it is necessary that the condition \( A > S_a \) is fulfilled, where \( A \) is the minimum inter-knife gap between the rotor and the mill stator. If this condition is not met, boundary friction occurs between the rotor and stator, which leads to a decrease in the reliability of the knife headset.

4. Conclusion
A method for calculating the longitudinal fluctuations of the mill rotor has been developed and tested. The error in determining the frequencies of free bending fluctuations of the mill rotors does not exceed 9%.

The lowest frequencies of free fluctuations of the rotors of existing mills are in the range of 85-165 Hz, and the second and subsequent frequencies of free fluctuations are in the frequency range above 1 kHz.
When designing and operating knife refiners, it is necessary that the minimum inter-knife gap between the rotor and stator does not exceed the amplitude of fluctuations of the mill disk.

The developed calculation procedure can be used in other industries, for example, mining and metallurgy.

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