Observation of SQUID-Like Behavior in Fiber Laser with Intra-Cavity Epsilon-Near-Zero Effect

Jiaye Wu, Xuanyi Liu, Boris A. Malomed, Kuan-Chang Chang, Minghe Zhao, Kang Qi, Yanhua Sha, Ze Tao Xie, Marco Clementi, Camille-Sophie Brès, Shengdong Zhang, Hongyan Fu,* and Qian Li*

Establishing relations between fundamental effects in far-flung areas of physics is a subject of great interest in the current research. Realization of a novel photonic system akin to the radio-frequency superconducting quantum interference device (RF-SQUID), in a fiber laser cavity with epsilon-near-zero (ENZ) nanolayers as intra-cavity components is reported here. Emulating the RF-SQUID scheme, the photonic counterpart of the supercurrent, represented by the optical wave, circulates in the cavity, passing through effective optical potential barriers. Different ENZ wavelengths translate into distinct spectral outputs through the variation of cavity resonances, emulating the situation with a frequency-varying tank circuit in the RF-SQUID. Due to the presence of the ENZ element, the optical potential barrier is far lower for selected frequency components, granting them advantage in the gain-resource competition. The findings reported in this work provide a deeper insight into the ultrafast ENZ photonics, revealing a new path toward the design of nanophotonic on-chip devices with various operational functions, and offer a new approach to study superconducting and quantum-mechanical systems.

1. Introduction

The past two decades have seen the rapid development of epsilon-near-zero (ENZ) linear and nonlinear photonics.[1–4] Combining ultrahigh intensity and ultrashort duration of pulses with the near-zero permittivity or refractive index and large optical nonlinearities,[5–8] unconventional light–matter interaction and pulse shaping,[9–12] frequency translation,[13,14] generation of the second-, third-, and higher harmonics,[15–20] as well as the terahertz generation,[21] have been demonstrated, giving rise to nanophotonic applications, such as optical switching,[22,23] electro-optical modulation,[24] photonic memory,[25] etc.

The currently available studies of ENZ in optics are typically conducted in conservative settings (rather than in laser cavities), with light passing the sample a finite number of times, namely, once (transmission), twice (reflection, disregarding internal reflections), or several times (in multilayers).[17,18,21] In such cases, ENZ media can be regarded as electromagnetic (EM) ideal fluids,[26] relating ENZ photonics to fluid dynamics. However, ENZ materials can exhibit an altogether different intra-cavity phenomenology in laser cavities, involving dissipative effects, an unlimited number of passes (roundtrips), and light–matter interactions.

A completely different area of physics is based on the concept and applications of superconductivity (SC).[27–29] A spectacular recent achievement in this field is the discovery of SC at 250 K in lanthanum hydride under high pressure.[29] Much earlier, a very important ramification of studies of the SC dynamics was initiated by the prediction of the Josephson effect in junctions formed by two bulk SCs separated by a narrow layer of a dielectric material.[30,31] The physics of Josephson junctions (JJs) has itself grown into a vast field of experimental and theoretical
In particular, an SC loop interrupted by one or two weak links, in the form of JJs, is the basis of "superconducting quantum interference devices" (SQUIDs), which provide the most sensitive tool for measuring weak magnetic fields (for instance, in the studies of biomagnetism). The weak link represents an effective potential barrier for the charge carriers in the superconductors: tunneling across this barrier results in quantum interference at the output of the SQUID. We further remark that a (small) imaginary part of this complex potential could be used to describe any excess ohmic loss inside the system. If the loop contains a single weak-link JJ, it is called a radio-frequency (RF) SQUID.

Of course, the nature of the Josephson effect is purely quantum-mechanical. However, it is a macroscopic quantum phenomenon, hence its observation (in particular, in the RF-SQUID configuration) and the relevant models (based on the commonly known classical sine-Gordon equation for the phase SQUID configuration) and the relevant models (based on the studies of biomagnetism) seem as quasi-classical dynamical regimes. This fact suggests a possibility to look for similar phenomenology in classical-wave settings. In this vein, in photonics it is intuitively attractive to consider ENZ and conventional optical media as ones emulating SCs and dielectric materials, respectively. Specifically, in the former case the complex refractive index—which results in slowing down, bending, and attenuation of light—represents the counterpart of the complex electric impedance of the SQUID operating in the RF regime. In particular, the imaginary part of the effective index represents the photonic counterpart of the electric inductance in the RF-SQUID scheme. The fiber-laser–SQUID similarity may be further extended for media combining symmetrically placed gain and loss elements, in the framework of parity-time symmetric optical and electric systems.

In superconductivity and optics alike, the essence of the SQUID-like behavior is that, when a certain physical parameter related to the SC-dielectric junction (or, in the case of its optical counterpart, the ENZ-non-ENZ junction) changes, the oscillation frequency of the tank circuit (TC, or in terms of the optical counterpart) shifts. Inspired by this principle, in this work we hypothesize a mode-locking wavelength shift due to the quasi-SC role of the ENZ material, which is represented by its complex refractive index. We propose a phenomenological model of intra-cavity mode reselection, which assumes that, in the ENZ setup, certain frequency components pass a far lower optical potential barrier, thus acquiring an advantage in the gain-resource competition. The overall operation scheme resembles the one implemented in the RF-SQUID by means of the frequency variation in the TC. In this context, it is relevant to stress that, while SQUID is a tool for the precise detection of weak magnetic fields, the fiber-laser scheme, experimentally elaborated in this work, offers a similarly operating system for the identification of ENZ wavelength, \( \lambda_{\text{ENZ}} \).

Thus, we design a corresponding experimental setup and demonstrate the novel phenomenon similar to the quantum-mechanical operation of the SQUID, namely, RF-SQUID-like behavior in a fiber laser with intra-cavity ENZ effects controlled by a variable resonance frequency. When the EM wave passes the optical potential barrier induced by the ENZ material, different ENZ wavelengths \( \lambda_{\text{ENZ}} \) lead to distinct spectral outputs via wavelength reselection and spectrum redistribution; while for the setting without the ENZ element a CW output is obtained, instead of the mode-locking (ML) regime, due to the polarization-evolution mismatch and strong absorption. This fact enables us to identify \( \lambda_{\text{ENZ}} \) directly within the cavity, by means of comparison of different intra-cavity specimens providing the ENZ effect. The results reported below provide deeper insight into ultrafast ENZ photonics and unveil a pathway toward the design of novel nanophotonic on-chip optical devices with various functionalities. The results also suggest new possibilities for the study of the SQUID-like behavior at optical frequencies, several orders of magnitude beyond the operation speed of a traditional RF-SQUID.

2. The RF-SQUID and SQUID-Like Behavior

A typical electric RF-SQUID consists of an SC ring interrupted by a single JJ, coupled to an oscillatory TC as shown in Figure 1. The JJ acts as an SC-dielectric structure, and the TC can be constructed as a simple inductor–capacitor (LC) circuit, whose fundamental resonance frequency is \( \omega_0 = (LC)^{-1/2} \). When the RF-SQUID operates in the dispersive mode detecting a magnetic field, the induced current \( I \) does not exceed the critical value \( I_c \). Then, the variation of the oscillation frequency in the tank circuit, \( \Delta \omega_0 \), can be detected by measuring the respective voltage (see Figure 1), and a change in the magnetic field changes the inductance of JJ and leads to a change in the resonant frequency of the TC. With \( I > I_c \) (for a particular flux), the RF-SQUID operates in the resistive mode, and \( I \) decays exponentially until the JJ recovers its superconducting state. The TC must supply energy to the SQUID, which reduces the voltage across the TC, but does not necessarily change its oscillation frequency. In this mode, if only a small magnetic field is present, \( I_c \) would not be essentially exceeded.

Thus, the RF-SQUID is built of two main components, the JJ serving as the sensor of the magnetic field, and TC that translates the detected field into a change of the oscillation frequency. It is worth mentioning that the RF-SQUID, with its intrinsic inductance, can operate even without a TC, but in this work we are considering the basic setup with a TC. In optics, the refractive index may be regarded as representing the "resistance" of the medium. Having near-zero permittivity and refractive index, ENZ materials realize ultra-low optical potential barriers, exerting low resistance onto the propagating optical wave. Therefore, this photonic setup resembles the SQUID scheme, with the ENZ element embedded in the laser cavity being a counterpart of the JJ, cf. in Figure 1. It is mentioned in Section 1 that the intra-cavity
behavior of the ENZ element is different from that previously reported for extra-cavity setups. The basic reason for the difference is that, in the former case, light keeps traversing the ENZ in the course of roundtrips, until it is emitted out of the cavity. As a result, the effect of the wavelength/frequency preference accumulates. This wavelength-selecting feature of the optical cavity provides emulation of the TC function in the electric RF-SQUID, as sketched in Figure 1. Resembling the RF-SQUID, this photonic counterpart of TC should also provide two operation modes. Namely, in the mode-locking regime the variation of \( \lambda_{\text{ENZ}} \) entails a change in the cavity's resonant (mode-locking) frequency, as represented by \( \lambda_{\text{ML}} \). On the other hand, in the mismatched regime the variation of \( \lambda_{\text{ENZ}} \) keeps producing the CW output with an unchanged wavelength. A comparison table between the RF-SQUID and its optical counterpart is shown in the Supporting Information.

3. The Phenomenological Picture: Mode Reselection under the Action of the Optical-Potential-Barrier

To realize the wavelength (frequency) shift within the cavity following the SC-dielectric analogue, that is, in terms of the complex and rapidly varying index in the ENZ region, a mode-reselection-induced wavelength shift scheme should be utilized instead of the adiabatic\(^{11}\) and self-phase-modulation-induced “time refraction”\(^{14}\) that is, a frequency shift between two segments of an optical medium separated by a boundary moving in time.\(^{40}\) Therefore, a weak-power nonlinear-polarization-evolution (NPE) cavity is considered, in which the added ENZ element should not greatly change the original mode-locking mechanism, rather acting as a mode reselector. Here, we propose a phenomenological picture based on the optical-potential-barrier concept to realize the mode reselection.

To demonstrate this more intuitively, we use the ellipsometry data from a 2 cm \( \times \) 2 cm 300-nm thick ENZ indium tin oxide (ITO) nanolayer sample, which is fabricated on a pure silica substrate with the free-carrier concentration of \( 8.273 \times 10^{20} \) cm\(^{-3} \) and mobility 22.19 cm\(^2\) (Vs), exhibiting the ENZ point at \( \lambda_{\text{ENZ}} = 1538 \) nm where the real part of the permittivity vanishes (see Section S2, Supporting Information). The complex permittivity and the corresponding complex refractive index of the sample are measured by the variable-angle spectroscopic ellipsometer (VASE), with the measured and modeled index curves shown in Figure 2a,b.

The complex permittivity curves of the ENZ ITO feature a trend similar to that in the basic Drude model. Therefore, to simplify the description of the proposed phenomenological model, we use the Drude model to illustrate the frequency dependence of the complex permittivity

\[
\varepsilon = \varepsilon_r + i\varepsilon_i = \varepsilon_{\infty} - \frac{\alpha^2}{\omega^2 + \gamma^2} + i \frac{\alpha^2 \gamma^2}{(\omega^2 + \gamma^2)^2} \omega \tag{1}
\]
where $\varepsilon_{\infty}$ is the high-frequency limit of the permittivity, the plasma frequency is $\omega_p = N e^2/(\varepsilon_{\infty} m^*)$, and the damping rate is $\gamma = e/(\mu m^*)$. Here, $N$ is the density of free carriers with effective mass $m^*$ and mobility $\mu$, but when the frequency is related to the wavelength as usual, $\omega = 2\pi c/\lambda$. In Equation (1), the complex permittivity can be converted to a complex refractive index, namely,

$$
n_n = n + ik = \frac{\sqrt{\epsilon_n^2 + \epsilon_t^2 + \epsilon_i}}{2} + i \sqrt{n^2 - \epsilon_t} \tag{2}
$$

Due to the existence of $\epsilon_t \neq 0$, Equation (2) always yields $n > 0$. The dependence of $n$ and $k$ on the wavelength, corresponding to the measurements and models, shown in Figure 2a-b, respectively. Note that the numerical VASE model (solid lines), which takes into account all effects, agrees well with the predictions (dashed lines), and demonstrates the same trend as the simple Drude model (the dashed lines).

The laser cavity with the population inversion, gain, and loss is a non-Hermitian optical system.$^{[41]}$ In such a setting, the distribution of the refractive index may be treated as an effective optical potential.$^{[42]}$ Following its quantum-mechanical counterpart, well-known in the context of the parity-time symmetry.$^{[43]}$ The respective potential barrier affects the speed of light in the medium. In quantum mechanics, the wave function confined by potential barriers can form a trapped state. In optics, similarly, nanophotonic structures with periodic modulation of the refractive index can trap localized EM modes.$^{[44,45]}$

As seen from Figure 2a, the height of the optical potential barrier, determined by the real part $\nu$ of the refractive index, drops significantly at $\nu > \lambda_{\text{ENZ}}$. The distribution of $n$ is not even across the spectrum, therefore the EM waves with different values of $\nu$ experience different “resistance”, with the longer-wavelength components having to pass lower potential barriers. This bias results in a wavelength preference, giving the longer-wavelength components an advantage (e.g., higher velocity and a shorter evolution). Therefore, the modelocking is lost if a pure glass substrate, or an ITO sample whose ENZ out of the laser operating range, is inserted in the cavity. Also, in the latter case, the extinction factor $k$ is relatively large, as seen in Figure 2b, hence the strong absorption further destroys the ML regime at all wavelengths. Thus, the proposed phenomenological picture strongly suggests that the role of ENZ ITO is that its highly dispersive real and imaginary parts of the permittivity act as a mode re-selector in the cavity while preserving the NPE condition.

In Figure 2c, the experimental values of the influence factors from Equation (3) are $a_1 = 0.51$ and $a_2 = 0.19$. The theoretically predicted optimal regime coincides with the experimental data within the cavity’s operation band, the respective value of $a_1$ being 12.5% larger than the experimental one, which qualitatively verifies the phenomenological consideration. Once the dominant wavelength is selected by the cavity, sidebands form as they would in the cavity without the ENZ element, thus finalizing the spectrum redistribution. The build-up of the spectrum is illustrated in Figure 2e.

When the ENZ condition is no longer satisfied within the considered band, the benefit of low $n$ vanishes, which results in phase accumulation and creates mismatch in the polarization evolution. Therefore, the mode locking is lost if a pure glass substrate, or an ITO sample whose ENZ out of the laser operating range, is inserted in the cavity. Also, in the latter case, the extinction factor $k$ is relatively large, as seen in Figure 2b, hence the strong absorption further destroys the ML regime at all wavelengths. Thus, the proposed phenomenological picture strongly suggests that the role of ENZ ITO is that its highly dispersive real and imaginary parts of the permittivity act as a mode re-selector in the cavity while preserving the NPE condition.

where $P(\lambda)$ is the effective potential height for wavelength $\lambda$, and, in our phenomenological model, $\lambda$ corresponding to the lowest $P$ in the spectrum is the advantageous wavelength for the mode locking. In Equation (3), $a_1$, $a_2$, and $a_3$ are weight coefficients that represent the influence of each factor. They are subject, by definition, to condition $a_1 + a_2 + a_3 = 1$. The first, second, and third terms in Equation (3) are contributions to the overall potential height from $n$, $k$, and the cavity’s structure. In particular, the presence of the cavity term $n_t$ in Equation (3) indicates that the combined potential barrier is most passable not exactly at $\lambda = \lambda_{\text{ENZ}}$, but at the wavelength determined by the interplay of the ENZ element and the cavity. Therefore, the actual wavelength corresponding to the ML regime may be either $\lambda_{\text{ML}} > \lambda_{\text{ENZ}}$ or $\lambda_{\text{ML}} < \lambda_{\text{ENZ}}$. This value should be found from the minimum condition, $dP/d\lambda = 0$. The usual RF-SQUID also works in the regime realizing a minimum of the JJ potential barrier.$^{[39]}$

In Figure 2d, the experimental values of the influence factors from Equation (3) are $a_1 = 0.51$ and $a_2 = 0.19$. The theoretically predicted optimal regime coincides with the experimental data within the cavity’s operation band, the respective value of $a_1$ being 12.5% larger than the experimental one, which qualitatively verifies the phenomenological consideration. Once the dominant wavelength is selected by the cavity, sidebands form as they would in the cavity without the ENZ element, thus finalizing the spectrum redistribution. The build-up of the spectrum is illustrated in Figure 2e.
4. The Wavelength Reselection and Spectrum Redistribution

Here, we realize the proposed photonic SQUID-like setup in the experiment, which makes it necessary to vary the optical wavelength/frequency in the resonant laser cavity. If the ENZ sample is absent, the cavity operates in its ML state at wavelength $\lambda_{ML}$ (corresponding to the original resonant frequency, $\omega_0$). If the sample’s ENZ wavelength is present in the laser operation range, a shift in $\lambda_{ML}$ should be observed. Similar to the action of the real SQUID, which detects the external magnetic field and converts it into the electric signal, the photonic emulator “detects” the presence of the ENZ and translates it into a shift of the laser emission, as shown in Figure 1. In this section, we use the same sample mentioned in the phenomenological model, whose ENZ characteristics are shown in Figure 3a.

To realize the ENZ internal feedback, we designed the experimental setup displayed in Figure 3. The laser configuration consists of different fiber pieces, fiber pigtailed optical components, and free-space optics for the ML of the NPE. To incorporate the sample in the free space of the laser cavity, we use the $\sigma$-shaped design of the fiber laser, as shown in Figure 3c. A branch of the contour is realized at PBS1, where one of the beams is directed to the ENZ sample perpendicularly, and the transmitted light is reflected by the highly reflective silver mirror, passing through the ENZ nanolayer reversely, getting back into the cavity. This design allows the removal of ENZ ITO sample at any time, and the cavity reverting back to the conventional mode-locked operation without any change and recalibration of the optical path.

Without the ENZ ITO sample, the loop cavity stably yields a 57 MHz train of 15 nm wide (full width at half maximum) 250-fs ultrashort pulses centered at 1571 nm (see Supporting Information). When the ENZ ITO sample with $\lambda_{ENZ} = 1538$ nm is inserted, the center wavelength shifts to 1553 nm, as shown in Figure 3b. The shift is $\approx 120\%$ of the spectral width of the pulse, while the pulse’s spectral and temporal shapes remain almost unchanged. Thus, the stably reproducible spectrum is redistributed around the new central wavelength (see Supporting Information). This effect persists when different wavelengths of the light source ($\lambda_{ML}$) are used, see Supporting Information.

Note that, by adjusting the waveplates (WPs), the central wavelength of the pulse can also be shifted. However, in this
Figure 4. The comparison between the results for different ENZ ITO samples. a) The real and imaginary parts of the permittivity for $\lambda_{ENZ} = 1538$ and 1274 nm. b) The output spectra shown on the logarithmic scales for these two samples and, in addition, for the setup without the ENZ ITO sample (the long-dashed line). c) The relation between values of $\lambda_{ENZ}$ and the corresponding $\lambda_{ML}$, as obtained from measurements performed with twelve different ITO samples, having $\lambda_{ENZ} = 1274.15$, 1434.00, 1451.99, 1474.05, 1532.84, 1537.71, 1566.95, 1567.49, 1578.83, 1642.48, 1657.92, and 1668.29 nm. The first two samples (with $\lambda_{ENZ} = 1274.15$ and 1434.00 nm) are out of the laser’s operation range, producing CW outputs, which have no corresponding $\lambda_{ML}$.

5. Further Assessment of the Intra-Cavity ENZ Effect

Although thin-film ENZ elements are used in many optical setups,[14,22] the role of ENZ in this work is altogether different. As many factors could potentially influence and/or contribute to the observed phenomenon, we analyze them as follows, noting the ability of ITO to switch its characteristics by means of saturable absorption.[46–48] In our experiments, the nonlinear saturable absorption is also observed (see Supporting Information), which may contribute constructively to the hybrid mode locking. However, if the optical-switching effect of the ITO film indeed plays the main role, the laser is expected to achieve the ML state regardless of $\lambda_{ENZ}$. The nonlinear Kerr effect, on the other hand, is trivial in this context, due to the weak intensity in the fiber laser cavity. Additionally, using the two-temperature model,[6,7] the calculation of the intensity-induced ENZ wavelength shift shows that it is smaller than 0.1 nm, which may be neglected.

To corroborate the validity of the operation regime outlined above and rule out essential effects produced by factors unrelated to the ENZ effect, in this section we switch to the sample with $\lambda_{ENZ} = 1274$ nm, placed far outside the operation band of the cavity. The comparison of the complex permittivity of the two samples measured by VASE is presented in Figure 4a. It is seen that the sample with $\lambda_{ENZ} = 1274$ nm features negative real permittivity and a relatively high intrinsic loss (the imaginary part) within the cavity’s operation band, while the sample with $\lambda_{ENZ} = 1538$ nm exhibits ENZ and lower loss in the considered band.

Output spectra of the cavity with the two different ITO samples are displayed in Figure 4b. One can see that the output of the sample with $\lambda_{ENZ} = 1274$ nm has the CW form, without any signature of the ML regime. In this case, no phase-locked spectrum can be observed, no matter how much pump energy is applied. On the contrary to the previous situation, only high-intensity CW
appears at wavelength 1532 nm, and the laser fails to reach a new steady state for spectral reconstruction and phase-locking. This can be explained by the fact that the ITO’s switching effect is not the cause of the appearance of the spectrum shown in Figure 3b, and, within the considered operation band, the sample with $\lambda_{\text{ENZ}} = 1274 \text{ nm}$ exhibits no ENZ effect (i.e., the refractive index in this range is no longer close to zero). With no adjustment of the optical path, the output may only take the CW form, and even the saturable absorption alone cannot realize the WP-adjustment-free hybrid mode-locking. This behavior further demonstrates that the wavelength reselection and spectrum redistribution are caused by the unique ENZ effects.

Additionally, to evaluate the impact of $\lambda_{\text{ENZ}}$ on the actual value of $\omega_{\text{ML}}$, the experiment was reproduced with a set of twelve pieces of ITO samples with different values of $\lambda_{\text{ENZ}}$ inserted in the cavity (see Supporting Information). The preparation of the sample is technically complex and time-consuming, because tuning $\lambda_{\text{ENZ}}$ from 1274 nm across the C-band to 1668 nm is a challenging procedure. The results are displayed in Figure 4c, which shows a trend for longer $\lambda_{\text{ENZ}}$ to have a greater effect on the wavelength reselection, especially for $\lambda_{\text{ENZ}} > (\lambda_{\text{ML}})_0 = 1571 \text{ nm}$. It is worthy to note that the established value of $\lambda_{\text{ML}}$, which is not necessarily located between $\lambda_{\text{ENZ}}$ and original $\lambda_{\text{ML}}$, which is explained by the ML mapping in Figure 4c. In line with these considerations, for wavelengths $\lambda_{\text{ENZ}}$ which are too short (namely, 1274 and 1434 nm), lying outside of the operation range of the cavity, a CW output is obtained.

The relation between $\lambda_{\text{ENZ}}$ and $\lambda_{\text{ML}}$ in Figure 4c indicates a “detection capability” of the ENZ-based setup, based on the wavelength reselection and spectrum redistribution. On the other hand, similar to the resistive mode in the RF-SQUID, with $I > I_c$ and decaying $I$, here the CW output persists at $\omega_{\text{ENZ}} > \omega_{\text{gain,min}}$, when the laser cavity cannot operate in the ML regime. The relation of $\lambda_{\text{ENZ}}$ to $\lambda_{\text{ML}}$, shown in Figure 4c, unveils a novel method to directly identify an initially unknown $\lambda_{\text{ENZ}}$ by putting it into the cavity and comparing the results with those obtained for a known $\lambda_{\text{ENZ}}$, without referring to expensive and non-ubiquitous ellipsometry measurements. This method can be utilized for designing new instruments for optical measurements.

Conclusively, considered from the perspective of the system, rather than the ENZ material itself, there are fundamental differences between the observed SQUID-like behavior and previous extra-cavity frequency-shift experiments (see, e.g., refs. [13, 14]), namely, i) the number of wavelength components inside the cavity is more diverse than in extra-cavity settings, with the former arrangement exhibiting the mode selection, competition for gain resources, and traveling through ENZ TCQ as the additional selection condition in multiple roundtrips; ii) the temporal overlap of pulses in a pulse train in the laser cavity is not guaranteed to initiate a pump-probe relation in the time-refraction frequency shift; iii) due to the weak power level, the size of the calculated Kerr-induced refractive-index change is only 1.8% at the ENZ wavelength (from 0.63 to 0.6413), and the corresponding permittivity changes from 0 to 0.0226, which is insignificant. Thus, the single crucial factor is, indeed, the rapidly changing complex permittivity, which shapes the unique dispersion and absorption curves of ITO in the ENZ region.

If the SQUID-like operation in the NPE laser cavity is considered as a whole, the following inferences can be made. For an NPE laser without ENZ, the polarization components form an artificial saturable absorber. The optical modes launched by the CW source propagate, while their polarizations rotate and evolve. Those with the highest transmission for the given polarization will resonate and lase. In such a system, the aforementioned exclusive dispersion and absorption curves of ENZ in the ENZ region act as the mode-reselector, maintained by the original NPE mode-locking regime. These inferences confirm the SQUID-like behavior predicted by the underlying assumption.

Additionally, it is worth mentioning that the response of a SQUID is periodic in the magnetic field. As the field is increased, the TC frequency varies periodically between a maximum and minimum value. Our proposed ENZ optical system is limited by the operation range of the laser (which in turn, is limited by the gain bandwidth of the EDF), therefore, just like SQUID, it has upper and lower limits of the frequency. The periodicity is not observed in our experiments, due to the fact that the ranges of $\lambda_{\text{ENZ}}$ and the laser operation range have similar limited scales. Finally, to give a clear picture of the development of the intra-cavity

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**Table 1. Comparison of intra-cavity-ENZ-based laser systems.**

| Ref. | Operation | ENZ component | $\lambda_{\text{ENZ}}$ [nm] | $\lambda_{\text{output}}$ [nm] | Pulse width | Repetition rate | $P_{\text{avg}}$ / $P_{\text{pump}}$ | Features |
|------|-----------|---------------|-----------------------------|-----------------------------|-------------|----------------|-----------------|----------|
| This work | Mode-locked | None | None | 1571 | 250 fs | 57 MHz | 10.47 mW / 130 mW | $\lambda_{\text{ML}}$ can be tuned by WPs. CW output at NPE mismatch. |
| Hybrid mode-locked | ITO thin film | 1451.99–1668.29 | 1551.88–1557.54 | 11.82 mW / 240 mW | Sample exhibits large modulation depth and sub-ps response time. |
| [22] | Mode-locked by transient bleach | ITO nanocrystal | ≈1300–1600 | ≈1560 | ≈593 fs | 16.62 MHz | 0.26 mW / 12 mW | Demonstration of ENZ-based Q-switched lasers outside C-band. |
| [46] | Q-switched by saturable absorption | ITO thin film | ≈2000 | 1862.5 | 526–882 ns | 241–113 kHz | 97.2 mW / 700 mW | Resolution ENZ to ML can be tuned by $\lambda_{\text{ENZ}}$. |
| [47] | Q-switched by saturable absorption | ITO nanocolumns array | 1293.5 | 1064.63 | 579.6–1060 ns | 126.1–67.4 kHz | 152 mW / 1.85 W | Demonstration of ENZ-based Q-switched lasers outside C-band. |
| [48] | ITO with nanostructures | N.A. | 2062.8 | 2.42 $\mu$s | 20.53 kHz | 312 mW / 2.56 W | None None 1571 250fs 57MHz 10.47mW/130mW | Features |
ENZ studies, we further summarize a comparison of intra-cavity-ENZ-based laser systems in Table 1. As can be concluded from the table, our SQUID-like ENZ laser setup provides wavelength tunability by switching ENZ samples, as well as a shorter pulse width and a higher repetition rate; while the Q-switched systems generally exhibit higher efficiencies than the mode-locked ones.

6. Conclusions

In this work, we have proposed and phenomenologically substantiated the novel RF-SQUID-like operation regime in the fiber laser with the intra-cavity ENZ element. The predicted behavior has been realized in the experimental setup. With \( \lambda_{\text{ENZ}} \) taken in the range of the cavity’s operation band, different values of \( \lambda_{\text{ENZ}} \) yield differently-shifted mode-locked pulses, created by the wavelength reselection and spectrum redistribution in the setup. On the other hand, only the CW output occurs in the absence of the ENZ element under the action of the added-phase-induced polarization-evolution mismatch. The balance between the (small) real and imaginary parts of the effective complex optical potential barrier, and contributions to it from intrinsic characteristics of the laser cavity, determine the observed wavelength reselection and spectrum redistribution scenarios. This approach offers a new method for identifying \( \lambda_{\text{ENZ}} \), without using high-precision instruments. The results of this work offer a deeper insight into ENZ photonics, as well as additional tools for the investigation of superconductivity by means of photonic analogues.

Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

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Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

The data that support the findings of this study are available in the supplementary material of this article.

Keywords

epsilon-near-zero, fiber laser, indium tin oxide, mode-locking, nonlinear polarization evolution

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