Coupled-channel effects in the $pp \rightarrow ppK^+K^-$ reaction

A. Dzyuba$^a$, M. Büscher$^b$, M. Hartmann$^b$, I. Keshelashvili$^{b,c,1}$, V. Koptev$^a$, Y. Maeda$^d$, A. Sibirtsev$^{b,e}$, H. Ströher$^b$, C. Wilkin$^f,\ast$

$^a$High Energy Physics Department, Petersburg Nuclear Physics Institute, 188350 Gatchina, Russia
$^b$Institut für Kernphysik, Forschungszentrum Jülich, 52425 Jülich, Germany
$^c$High Energy Physics Institute, Tbilisi State University, 0186 Tbilisi, Georgia
$^d$Research Center for Nuclear Physics, Osaka University, Ibaraki, Osaka 567-0047, Japan
$^e$Helmholtz-Institut für Strahlen- und Kernphysik (Theorie), Universität Bonn, D-53115 Bonn, Germany
$^f$Physics and Astronomy Department, UCL, London, WC1E 6BT, UK

Abstract

The cross sections for the $pp \rightarrow ppK^+K^-$ reaction were measured at three beam energies 2.65, 2.70, and 2.83 GeV at the COSY-ANKE facility. The shape of the $K^+K^-$ spectrum at low invariant masses largely reflects the importance of $KK$ final state interactions. It is shown that these data can be understood in terms of an elastic $K^+K^-$ rescattering plus a contribution coming from the production of a $K^0\bar{K}^0$ pair followed by a charge-exchange rescattering. Though the data are not yet sufficient to establish the size of the cusp at the $K^0\bar{K}^0$ threshold, the low mass behaviour suggests that isospin-zero production is dominant.

Key words: Final state interactions, cusp phenomena.
PACS: 13.60.Le, 14.40.Aq, 13.75.Lb

The COSY-ANKE collaboration has recently published data on the differential and total cross sections for the $pp \rightarrow ppK^+K^-$ reaction at three beam

1 Current address: Department of Physics and Astronomy, University of Basel, CH-4056 Basel, Switzerland
* Corresponding author.
Email address: cw@hep.ucl.ac.uk (C. Wilkin).
energies $T_p = 2.65$, 2.70, and 2.83 GeV, which correspond to excess energies of $\varepsilon = 50.6$, 66.6, and 108.0 MeV, respectively [1]. The $K^+K^-$ invariant mass $M_{\text{inv}}(KK)$ spectra show a strong signal for the production and decay of the $\phi$ meson. This sits upon an apparently non-resonant background. However, the distributions in the $Kp$ and $Kpp$ invariant masses prove that this background is strongly distorted by a $K^-p$ final state interaction (fsi) [2]. After taking this fsi into account, as well as the one between the outgoing protons, most of the distributions are well described by Monte Carlo simulations.

![Figure 1](image_url)

Fig. 1. Differential cross section for the $pp \rightarrow ppK^+K^-$ reaction at 2.65 GeV as a function of the $K^+K^-$ invariant mass compared to Monte Carlo simulations of $\phi$ (dotted) and non-$\phi$ (dashed) contributions and their sum (solid histogram).

Nevertheless, as seen in the $K^+K^-$ invariant mass spectrum at 2.65 GeV shown in Fig. 1, the simulation underestimates the experimental points for $M_{\text{inv}}(KK) < 995$ MeV/c$^2$. Of itself, this could be dismissed as a fluctuation, though it is important to realise that the ANKE spectrometer has a very good acceptance in this region [3]. Furthermore, a similar phenomenon was observed for the same mass region in the 2.70 and 2.83 GeV ANKE data, as well as in those obtained by the DISTO collaboration at a slightly higher energy [4]. Moreover, an analogous effect was also noted for the $pn \rightarrow dK^+K^-$ reaction, where the experimental systematics are rather different [5].

The simulation of the $pp \rightarrow ppK^+K^-$ spectrum of Ref. [1], shown in Fig. 1 for the 2.65 GeV data, includes only the final state interactions in the $K^-p$ and $pp$ systems. To investigate the low $K^+K^-$ mass region in finer detail, we have taken these results and divided them by the simulation. Although the resulting error bars are rather large, the ratios at all three energies are mutually consistent and Fig. 2 shows the weighted averages of the points at the three energies.

An enhancement at low $K^+K^-$ masses is, of course, to be expected from an
Fig. 2. Ratio of the $K^+K^-$ invariant mass spectra from the $pp \rightarrow ppK^+K^-$ reaction to the simulation presented in Ref. [1]. The experimental points correspond to the weighted average of data taken at 2.65, 2.70, and 2.83 GeV. The solid curve is the result of a best fit of Eq. (1) to these data, with the parameters being given in Table 1. The dot-dashed curve is the best fit when the elastic rescattering is neglected and the dashed when the charge-exchange term is omitted.

Fig. 3 illustrates the three types of contribution that are to be considered. In all cases the large blob represents the $pp \rightarrow ppK\bar{K}$ production distorted by the final state interactions in the $pp$ and $\bar{K}p$ systems, as described in the

Fig. 3. Diagrammatic representation of (i) direct production of $K^+K^-$ pairs, where the large blob includes the $fsi$ effects in the $pp$ and $K^-p$ systems considered in Ref. [1]. The elastic $K^+K^-$ $fsi$ is illustrated in (ii) and the production of virtual $K^0\bar{K}^0$ pairs followed by a charge exchange $fsi$ in (iii).

Although the diagrams of Fig. 3 are easy to visualise in the charge basis, the evaluation is somewhat simpler in the isospin basis because the $K\bar{K}$ scattering lengths are normally quoted in this way. Let $B_0$ and $B_1$ be the bare $pp \rightarrow ppK\bar{K}$ amplitudes for producing $s$-wave $K\bar{K}$ pairs in isospin-0 and 1 states, respectively. These amplitudes, which already include the $fsi$ in the $K^-p$ and $pp$ channels [1], are then distorted through a $fsi$ corresponding to elastic scattering. This leads to enhancement factors of the form $1/(1 - i k A_I)$, where $k$ is the momentum in the $K^+K^-$ system and $A_I$ is the $s$-wave scattering length in each of the two isospin channels. The charge-exchange $fsi$ of Fig. 3 (iii) depends upon the $K^0\bar{K}^0 \rightarrow K^+K^-$ scattering length, which is proportional to the difference between $A_0$ and $A_1$, and on the momentum $q$ in the $K^0\bar{K}^0$ system. In total therefore, the enhancement factor has a momentum dependence of the form

$$F = \left| \frac{B_1/(B_1 + B_0)}{(1 - i \frac{1}{2} q[A_1 - A_0]) (1 - i k A_1)} + \frac{B_0/(B_1 + B_0)}{(1 - i \frac{1}{2} q[A_0 - A_1]) (1 - i k A_0)} \right|^2,$$  \hspace{1cm} (1)

where the $\frac{1}{2}$ are isospin factors. In this way we have extended the $K^-p$ and
The $pp$ fsi factorisation hypothesis of Ref. [1] to include also the $K\bar{K}$ system.

The cusp structure arises because $q$ changes from being purely real above the $K^0\bar{K}^0$ threshold to purely imaginary below this point. The strength of the effect clearly depends upon $A_0 - A_1$, but the shape of the signal also depends upon the interference with the direct $K^+K^-$ production amplitude.

The $K\bar{K}$ scattering lengths are significant due to the presence of the $(a_0, f_0)$ scalar resonances [9], but there is a large uncertainty in their numerical values, which reflects the uncertainty in the positions and widths of these states. A useful summary of the different estimates before 2004 is to be found in Ref. [10]. It is generally agreed that the isospin $I = 1$ scattering length has a small real part [11][12][13][14] and we take $A_1 = (0.1 + i0.7)$ fm. Values of the isoscalar scattering length can be extracted from many fits to data [14][15][16][17] but that deduced by the BES collaboration, $A_0 = (−0.45 + i1.63)$ fm [18] seems to be the most reliable.

There is an even bigger uncertainty in the (complex) ratio of the production amplitudes,

$$B_1/B_0 = C e^{i\phi_C},$$

which is completely unknown a priori. We have therefore fitted the points shown in Fig. 2 with Eq. (1) so as to determine the values of the unknown magnitude $C$ and phase $\phi_C$ within this approach. This has been done separately for the cases where (i) all the terms in Eq. (1) are retained, (ii) for purely elastic fsi, when the $q[A_1 - A_0]$ terms are neglected, and (iii) purely charge-exchange fsi, when the $kA_1$ terms are discarded. The corresponding results are presented in Table 1 and the associated curves in Fig. 2.

Table 1
The fit results for the magnitude and phase of the ratio of the $I = 1$ and $I = 0$ amplitudes of Eq. (2). The data of Fig. 2 are fitted using the ansatz of Eq. (1) with (i) both elastic and charge-exchange fsi, (ii) elastic fsi alone, and (iii) purely charge-exchange fsi. The corresponding curves are shown together with the experimental points in Fig. 2.

| Fit par. | el.+c.e. | el. alone | c.e. alone |
|----------|---------|-----------|------------|
| $C$      | $0.62^{+0.16}_{-0.11}$ | $0.88^{+0.05}_{-0.21}$ | $0.56^{+0.07}_{-0.21}$ |
| $\phi_C$ (deg) | $-81^{+36}_{-25}$ | $159^{+31}_{-21}$ | $-131^{+8}_{-20}$ |
| $\chi^2/ndf$ | 1.2 | 1.2 | 2.5 |
Although all three sets of curves in Fig. 2 reproduce the data in an acceptable way, it has to be stressed that the two final state interactions must be bought as a single package. The effects of neglecting either the elastic or charge-exchange \textit{fsi} have only been considered in order to indicate the separate influences of the two types of contribution.

The full fit of Fig. 2 demonstrates a cusp effect at the $K^0\bar{K}^0$ threshold but only as a sharp discontinuity in the slope of the cross section ratio. This is qualitatively similar to the case where the elastic rescattering is neglected. On the other hand, the scenario where only elastic rescattering is retained gives (statistically) an equally good description of the data. This shows a smooth increase down to the $K^+K^-$ threshold and no anomalous behaviour at the $K^0\bar{K}^0$ threshold. The nature of the solutions also differs in that the full one requires mainly $I = 0$ production, whereas the contributions from the two isospins would be rather similar if the channel coupling were disregarded.

The $KK$ elastic and charge-exchange \textit{fsi} both enhance the cross section at low masses and, since this region represents a larger fraction of the total spectrum at low excess energies, it is clear that these will also affect the energy dependence of the total production cross section [19]. This is indeed the case, as shown by Fig. 4, where the extra contributions from the \textit{fsi} allow the low and high energy data to be described simultaneously.

![Graph](image)

**Fig. 4.** Total cross section for the $pp \rightarrow ppK^+K^-$ reaction as a function of the excess energy. The experimental data are taken from Refs. [1] (closed circles), [4] (open circle), [2] (closed squares), [20] (open triangle), and [21] (open square). The dot–dashed curve is that of four-body phase space normalised on the 108 MeV point. The dashed curve corresponds to the fit which includes final state interactions between the $K^-$ and the protons and between the two protons themselves, as described in Ref. [1]. The further inclusion of the \textit{fsi} between the kaon pair leads to the solid curve.
The $K\bar{K}$ final state interaction approach used here does not rely on knowing the basic production mechanism. The analysis of the $pp \to ppK^+K^-$ data given in Ref. [1] suggests that the main terms driving the reaction might be linked to $Y^*$ excitation and decay. However, even if one assumed that the major contribution to the cross section came from a combination of $a_0$ and $f_0$ production, where these resonances are described by Flatté shapes [22], this would lead to a similar structure to that of the present work, though only in the vicinity of the $K\bar{K}$ thresholds. Thus the observation of cusps or smooth enhancements at low $K^+K^-$ invariant mass should not be taken as evidence that the underlying production mechanism is necessarily driven by the formation of these scalar resonances.

On the other hand, if $(a_0, f_0)$ production were indeed dominant, then one could put the kaon mass difference directly into the Flatté descriptions of these resonances [23,24]. However, after fitting the $pp \to ppK^+K^-$ data in terms of $a_0$ and $f_0$ production amplitudes, such a procedure would give results that differed little from ours in the near-threshold region, provided that the corresponding values of the scattering lengths were used.

On the basis of the parameters quoted in Table 1, it is seen that the best fit is achieved with a production of $I = 0$ $K\bar{K}$ pairs in the near-threshold region that is about three times stronger than for $I = 1$. This sensitivity originates mainly from the very different $I = 0/I = 1$ scattering lengths, which is a general feature of the various analyses [10,11,12,13,14,15,16,17,18]. This suggests that the production of $I = 0$ pairs is dominant in the $pp \to ppK^+K^-$ reaction, independent of the exact values of the scattering lengths.

In summary, on the basis of the existing knowledge of the low energy $K\bar{K}$ interaction, we would expect there to be a cusp-like structure in the $K^+K^-$ invariant mass spectrum from the $pp \to ppK^+K^-$ reaction. The details, however, are unclear because of the uncertainties in the relative amplitudes for $I = 0$ or $I = 1$ production of $KK$ pairs, as well as in the $K\bar{K}$ scattering lengths. As seen in Fig. 2, the data themselves would be consistent with either a cusp or simply a strong but smooth low mass enhancement.

Although the energy dependence of the total cross section is better reproduced when the $K\bar{K}$ rescattering is included, this is mainly a reflection of it enhancing the cross section at low $K^+K^-$ masses. A similar improvement is found if the charge-exchange $f_{si}$ is neglected in the fitting process. To establish the actual nature of the behaviour in the cusp region, better data are needed and it is hoped that this might be achieved by working at lower excess energy [25].
on this subject with Professors D.V. Bugg, A. Gal, and B.S. Zou. Support
from other members of the ANKE collaboration is also much appreciated.

References

[1] Y. Maeda et al., Phys. Rev. C 77 (2008) 015204.
[2] P. Winter et al., Phys. Lett. B 635 (2006) 23.
[3] S. Barsov et al., Nucl. Instrum. Methods A 462 (2001) 364.
[4] F. Balestra et al., Phys. Rev. C 63 (2001) 024004.
[5] Y. Maeda et al., Phys. Rev. Lett. 97 (2006) 142301; Y. Maeda et al. (in
preparation).
[6] T.H. Tan, Phys. Rev. Lett. 23 (1969) 395.
[7] R.H. Dalitz and S.F. Tuan, Ann. Phys. (N.Y.) 10 (1960) 307.
[8] R.H. Dalitz, Revs. Mod. Phys. 33 (1962) 471; R.H. Dalitz in Strangeness in
Nuclei, ed. St. Kistryn and O.W.B. Schult (World Scientific, Singapore, 1993)
p. 203.
[9] W.-M. Yao et al., J. Phys. G 33 (2006) 1, but see also the minireview in
S. Eidelman et al., Phys. Lett. B 592 (2004) 1.
[10] V. Baru et al., Phys. Lett. B 586 (2004) 53.
[11] D.V. Bugg, V.V. Anisovich, A. Sarantsev, and B.S. Zou, Phys. Rev. D 50 (1994)
4412.
[12] S. Teige et al., Phys. Rev. D 59 (1999) 012001.
[13] M.N. Achasov and V.V. Gubin, Phys. Rev. D 68 (2003) 014006.
[14] A. Antonelli, [arXiv:hep-ex/0209069]
[15] M.N. Achasov et al., Phys. Lett. B 485 (2000) 349.
[16] R.R. Akhmetshin et al., Phys. Lett. B 462 (1999) 380.
[17] R. Kamiński and L. Leśniak, Phys. Rev. C 51 (1995) 2264.
[18] M. Ablikim et al., Phys. Lett. B 607 (2005) 243.
[19] W. Oelert et al., Int. J. Mod. Phys. A 22 (2007) 502.
[20] M. Wolke, PhD thesis, University of Münster (1997).
[21] C. Quentmeier et al., Phys. Lett. B 515 (2001) 276.
[22] S. Flatté, Phys. Lett. B 63 (1976) 224.
[23] C. Hanhart, B. Kubis, J.R. Pelaez, Phys. Rev. D 76 (2007) 074028; idem arXiv:0712.0473 [nucl-th].

[24] D.V. Bugg, arXiv: 0802.0934 [hep-ph], arXiv:0804.3450 [hep-ph].

[25] M. Hartmann et al., COSY proposal 191, available from www.fz-juelich.de/ikp/anke/en/proposals.shtml.