Fine-tuning the Color-Glass Condensate with the nuclear configurational entropy

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Abstract – The dipole-nucleus forward scattering amplitude rules the onset of the gluon anomalous dimension, in the Color-Glass Condensate regime. In this model, the onset of quantum regime is here derived as a critical stable point in the nuclear configurational entropy, matching the fitted experimental data in the literature with accuracy of ∼ 1%. It corroborates the informational entropy paradigm in high-energy nuclear physics.

Introduction. – In the specific case of high-energy nuclear physics, the informational entropy setup has been brought out into a multiplicity of scopes in hadronic processes, in the lattice QCD setup, and in the AdS/QCD correspondence as well [1–4]. In this case, the stability of mesons [1] and the dominance of glueball states [2] were derived, in the context of the configurational entropy. The informational entropy has its roots in the Shannon work of the communication theory, measuring the shape complexity of spatially localized configurations, as the expected value of the information contained in each message, in the communication theory. The concept of the informational entropy was refreshed by means of the relative configurational entropy [5–8], that up to now has relied on the energy density that represents the studied systems. Both denominations, configurational and informational entropy, are currently used throughout the literature and hereupon we also utilize both namings. The informational entropy counts on the Fourier transform of square-integrable, bounded, position-dependent functions. Such rich concept computes the inherent shape complexity related to configurations of physical systems that are spatially localized. Here we propose the informational entropy as a quite natural tool for the study of cross-sections in nuclear physics. In fact, the cross-sections are also physically adequate to define the configurational entropy, since they are also spatially localized, square-integrable, position-dependent bounded functions. There are close parallels between the mathematical expressions for the thermodynamical entropy and the informational entropy. As the informational entropy has been calculated for energy densities, the choice of the total reaction cross-section can be, therefore, also appropriate for this purpose. In fact, the amount of information needed to define the detailed quantum states of the system, given its macroscopic description, can be thus implemented.

For configurations encoded by the measure of their cross-section in nuclear physics, information entropy can be thought of as being a measure of the weights associated with different modes, in the momentum space, that comprehends the nuclear physical system to be studied. The higher the informational entropy, the more the information necessary to portray the shape. Cross-sections in the Color-Glass Condensate (CGC) are here analysed, with respect to the critical points of the associated informational entropy.

The configurational entropy and KKT model. – In the last decades, the mechanism of hadron interactions at high-energy regime within Quantum Chromodynamics (QCD) has been drawing the attention of particle physicists. The dependence of the total hadronic cross-section on the energy permits to relate such aspect to the reaction mechanism based on QCD, in particular, a suggestion of the appropriate role of the partons in the high-energy reactions with hadrons. QCD at high energies can be described as a many-body theory of partons which are weakly coupled albeit non-perturbative due to the large number of partons.

The so-called Color-Glass Condensate is the effective theory describing high-energy scattering in QCD. It provides an insight into outstanding conceptual issues in
QCD at asymptotically high energies. For instance, in the case of Deep Inelastic Scattering (DIS) of a lepton scattering off a hadronic target, it provides the useful environment to study the particle production at high energies. During the collision one can represent the hadron as a collection of constituents, partons, which are nearly on-shell excitations carrying some fraction of the total hadron longitudinal momentum. This variable is equal to the empirical Bjorken variable $x_{Bj}$. The inclusive cross-sections in DIS can be expressed in terms of the Lorentz invariants, such as $x$ which corresponds, at lowest order in the perturbation theory, to the longitudinal momentum fraction carried by a parton in the hadron, the virtual photon four-momentum squared $q^2 = -Q^2 < 0$ exchanged between the electron and the hadron, and the center-of-mass energy squared $s$. The CGC has been proposed as a new state of matter, characterized by gluon saturation and by a typical momentum (saturation) scale, $Q_s$, growing with the energy. The Color-Glass Condensate determines the critical interface that separates the linear and the saturation regimes of QCD dynamics [9–12]. Reference [9] considered a sea of gluons in gold ions that seem to other ions as a gluon wall, constituting the Color-Glass Condensate itself, comprising gluons of high density. The higher the energy, the higher the momentum states that are occupied by the gluons, and concomitantly the weaker the coupling among the gluons [9–11]. Hence, the gluon density saturates, corresponding to a multiparticle Bose condensate state. Reference [3] thoroughly paves the road for this interpretation in the informational entropy setup.

In the QCD framework, the total hadronic cross-section has been shown to be correctly described by the minijet model, with the inclusion of a window in the $p_T$-spectrum associated to the saturation physics [10]. This model was also successfully used for the dipole-nucleus forward scattering amplitude, which described the onset of the gluon anomalous dimension in the Color-Glass Condensate regime [9]. Partons emulate localized-energy systems, being spatially coherent field configurations [5]. The configurational entropy is here proposed to identify both the onset of instability of a spatially bound configuration as well as the approach towards the optimal state corresponding to the minimum in the configurational entropy. This setup has been widely verified in different contexts, with great experimental, phenomenological and observational accuracy (see [1–8] and references therein).

The starting point for defining the configurational entropy, generally for $p$ spatial dimensions, is the Fourier transform of the energy density, given by [13]:

$$\rho(k) = \frac{1}{(2\pi)^p} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \rho(r) e^{ik \cdot r} d^p r.$$  

(1)

The configurational entropy is defined from the modal fraction, that is constructed upon of the Fourier transform of the energy density associated with a physical system. Then we define the nuclear modal fraction which reads

$$f(k) = \frac{|\rho(k)|^2}{\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} |\rho(k)|^2 d^p k}.$$  

(2)

The nuclear configurational entropy is, then, defined as [5,6]

$$S_c[f] = -\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(k) \log f(k) d^p k.$$  

(3)

Critical points of the configurational entropy imply that the system has informational entropy that is critical with respect to the maximal entropy $f_{\text{max}}(k)$, corresponding to more dominant states [1,3]. It has been comprehensively scrutinised and applied in a variety of models that range from higher-spin mesons and glueballs stability [1,2]—in the holographic nuclear physics panorama—to Bose-Einstein condensates of long-wavelength gravitons [3].

On the other hand, the investigation of deep inelastic scattering of leptons on protons and/or nuclei shows that during the interaction the gluons with small $x$, a longitudinal momentum fraction carried by a parton in the hadron, can be produced. The data, obtained in different experiments such as DESY collider HERA, can be successfully interpreted by the phenomenological descriptions based on the gluon saturation concept [14–16]. It provides an idea that at high-energy regimes the nuclei can be considered as systems of gluons with small $x$ value that are strongly correlated. For such dense systems, the theory has been called Color-Glass Condensate based on QCD and has been developed in [11]. At the Relativistic Heavy-Ion Collider (RHIC) one can observe the window for the gluon distributions at the energy of a center-of-mass 200 GeV. For instance, in the deuteron-induced reaction of gold, hadrons, which are detected on the beam direction, are produced mainly by quark-gluon interactions. In such a collision, it is possible to investigate the small-$x$ components of the gold nuclei wave function. Such fact leads to the dependence on the number of nucleons and the connection with the rapidity $y$ of the measured particles by $Q_s^2 \sim A^{1/3} e^{\lambda y}$. The value of $\lambda \sim 0.2–0.3$ is obtained by fitting the HERA data. It should be worth noting that at RHIC energies the saturation scale for gold nuclei is expected to be $\sim 2$ GeV [11].

In order to compute the informational entropy associated to the Color-Glass Condensate, let us start by calculating the spatial Fourier transform of the total hadronic cross-sections defined in the minijet saturation model [10]. Instead of the energy density, we here use the cross-sections as a more suitable quantity to describe nuclear systems, being also spatially localized, square integrable, functions. We will analyze the informational entropy of cross-sections in the KKT model. Analogously to the definitions (1), (2), the nuclear configurational entropy is hereon defined by means of the Fourier transform of the

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cross-section,
\[
\sigma(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sigma(r) e^{ik \cdot r} dr.
\]
(4)

It yields the modal fraction:
\[
f_\sigma(k) = \frac{\left| \sigma(k) \right|^2}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sigma(k) \right|^2 dk}.
\]
(5)

The configurational entropy is, then, defined as [5]
\[
S_c[f_\sigma] = -\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_\sigma(k) \log f_\sigma(k) dk.
\]
(6)

Critical points of the nuclear configurational entropy imply that the system has informational entropy that is critical with respect to the maximal entropy \( f_{\text{max}}(k) \), corresponding to more dominant states [1,3,4].

This model is based upon the so-called saturation window, that consists of a gap lying between the non-perturbative and the perturbative regimes of QCD. In the minijet model [10] the total hadronic cross-section can be decomposed as follows:
\[
\sigma_{\text{total}} = \int_{0}^{Q_{\text{QCD}}} d\sigma + \int_{Q_{s}^2}^{Q_s^2} d\sigma + \int_{Q_s^2}^{Q_s^2} d\sigma.
\]
(7)

where \( d\sigma = \frac{d\sigma}{dp_\perp^2} dp_\perp^2 \), and \( \sigma_0 \) characterizes the non-perturbative component [14]. The last split component, \( \sigma_{\text{QCD}} \), was calculated using the perturbative QCD with respect to a low transverse momenta cut-off. The cross-section \( \sigma_{\text{sat}} \) stands for the saturated component. Such component contains data about interactions below the saturation scale \( (Q_s) \) where the parameter \( \sqrt{s} \) fixes the center-of-mass energy. It is determined by the solution of the non-linear evolution equation associated to Color-Glass Condensate physics [15] and given by \( Q_s^2(y) = \left( \frac{\alpha_s}{\pi} \right) \lambda \), where \( x \) is the Bjorken variable, with \( Q_0^2 = 0.3 \) GeV\(^2\) and \( x_0 = 0.3 \times 10^{-4} \) fixed by the initial condition [9,10].

The saturation exponent \( \lambda \) has been estimated on the basis of different approaches for the QCD dynamics, being \( \approx 0.3 \) [16], agreeing with the HERA data [17].

In the above-mentioned model [10], the saturated component is given by the following equation \( \sigma_{\text{sat}} \) [18]:
\[
\sigma_{\text{sat}} = \int \sigma_{\text{dip}}(x,r) |\Psi_p(r)|^2 d^2 r,
\]
(8)

where \( r \) is the dipole radius and the proton wave function \( \Psi_p \) is chosen as
\[
|\Psi_p(r)|^2 = \frac{1}{2\pi S_p} e^{-r^2/2S_p^2}
\]
(9)

with \( S_p = 0.74 \) fm and the dipole-proton cross-section [9]
\[
\sigma_{\text{dip}}(x,r) = 2\pi R_p^2 \left\{ 1 - \exp \left[ -\frac{1}{4} (rQ_s)^2 \right] \right\},
\]
(10)

where \( R_p = 0.9 \) fm and \( y \) is the total rapidity interval\(^1\). The dipole scattering amplitude should be given by the impact parameter \( (b) \) dependent solution of a non-linear evolution equation, such as the Balitsky-Kovchegov equation [15]. In the KKT model [9], the expression for the quark dipole-target scattering amplitude regards the relation \( Q_s = \pm \frac{2}{3} Q_s [9] \), and the anomalous dimension reads
\[
\gamma(y,r^2) = \frac{1}{2} \left( 1 + \frac{\xi(y,r^2)}{7\zeta(3)c} + \frac{\xi(y,r^2)}{2\zeta(3)c} \right),
\]
(11)

with \( c \) a free parameter that describes the onset of the quantum regime, which was fixed to be \( c = 4 \) in the literature [9,10], and
\[
\xi(y,r^2) = -4 \log \frac{|rQ_{s0}|}{\lambda(y-y_0)}.
\]
(12)

The saturation scale is expressed as \( Q_s^2(y) = A^2 \sqrt{\rho} x^{-\lambda} \), where the parameters \( A = 0.6 \) GeV and \( \lambda = 0.3 \) are fixed by DIS data [22]. In the theory of the Color-Glass Condensate, the parameter \( \lambda \) changes with the energy, being a function of the variable \( y = \log(1/x) \). In eq. (12) the initial saturation scale is defined by \( Q_{s0}^2 = Q_s^2(y_0) \) with \( y_0 = 0.6 \) being the lowest value of rapidity at which the low-\( x \) quantum evolution effects are essential. This parametrization can describe the dAu RHIC data [9,10].

In general, free parameters of this model are \( y_0 \), which sets the initial value of \( y \), at which the quantum evolution sets in and the parameter \( c \) in (11), which describes the onset of quantum regime, both fitted to RHIC dAu data reported by the BRAHMS Collaboration [23]. The infrared cutoff \( \mu = 1 \) GeV and the momentum scale range 0–1 GeV are fixed by lower-energy data.

Now eqs. (4)–(6) that define the nuclear configurational entropy are used to compute the cross-section saturated

\(^1\)It is worth mentioning that the wave function in eq. (8) can be further generalized in the context of cyclic defects [19–21].
component (8). After awkward computations, the results are presented in fig. 1: The informational entropy computed by eq. (6) is shown in fig. 1, for different values of the Bjorken variable $x$. Small $x$ values are employed, according to refs. [9,10].

The minima in the curves that depict the informational entropy in fig. 1 occur at $c = 4.038$, for $x = 10^{-7}$; at $c = 4.041$, for $x = 10^{-5}$; and at $c = 4.050$, for $x = 10^{-4}$. The fitted parameter $c$ establishes the onset of the quantum regime, and was assumed to be $c = 4$, in refs. [9,10], to derive the results therein presented.

**Conclusions.** – Here we showed that such is a natural choice provided by the analysis of the minima of the informational entropy, associated to the onset of the gluon anomalous dimension, in the Color-Glass Condensate regime. For the Bjorken variable $x = 10^{-7}$, the error is $\approx 0.94\%$, whereas for $x = 10^{-4}$ the error is $\approx 1.23\%$. This is the supremum among the error for any curve for a fixed Bjorken variable. Indeed, dense systems of gluons of small $x$ represent the Color-Glass Condensate based on QCD [11], and the Bjorken variable $x_0 = 0.3 \times 10^{-4}$ was fixed by the initial conditions. Hence, all values for the Bjorken variable must be adopted as being smaller than this initial condition. A direction to be further studied encompasses quantum mechanics fluctuations, regarding eqs. (8), (9). The wave function used in those equations can be explored in the context of the topological defects proposed, e.g., in refs. [24–26], in the configurational entropy setup. Moreover, a modified KKT model [9] uses a modification of the KKT model assuming that the saturation momentum scale is given by $y_0 = 4.6$ and the onset of the quantum regime can be taken as a different value [10,27,28].

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