Statistical analysis of 2D patterns and its application to astrometry

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Abstract. A general statistical procedure for analysis of finite 2D patterns, inspired by analysis of heavy-ion data, is developed. The method is verified in the study of publicly available data obtained by the Gaia-ESA mission. We prove that the procedure can be sensitive to the limits of accuracy of measurement, but it can also clearly identify the real physical effects on the large background of random distributions. As an example, the method confirms presence of binary and ternary star systems in the studied data. At the same time the possibility of statistical detection of gravitational microlensing effect is discussed.

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1. Introduction

The motivation of the present study was to modify and generalize the known method for analyzing anisotropic flow in relativistic nuclear collisions [1, 2], which is effectively applied in the current experiments [3] (and citations therein). The method is based on the use of the Fourier expansion of azimuthal distributions of produced particles and allows us to obtain important information on mechanism of nuclear collisions. However, mathematical frame of this method is more general and corresponding formalism can be after minor modifications used even for quite different kind of analysis. Our present idea is focused primarily on astrometry. Recently, some similarity between spiral structures in galactic patterns and heavy ion collisions has been discussed in [4]. However, our approach is different. The astrometric data are decomposed into the set of limited, finite patterns whose parameters are statistically analyzed. The input data are taken from the Gaia survey [5, 6].

The detailed description of Fourier analysis and its application to astrometry is given in the extended version of this report [7]. In the same paper we have described also a complementary statistical method for analysis of the patterns of stars, which deals with angular distances and is important for identification of binary and ternary star systems. The results obtained by application of both methods are presented and discussed in the next section. Sec. 2.1 is devoted to the discussion on gravitational microlensing effect and conditions of its observation.

2. Application to astrometry

The simulations described in [7] have been compared with the data of the recent Gaia catalogue DR1 [6]. We present the results of our analysis from the regions marked in Fig.1.

First, we checked the Fourier analysis, the results are shown in Fig.2. The upper part corresponds to a dense field C and very similar results can be obtained over other regions at the galactic plane. This result can be compared with the Monte-Carlo calculation in [7]. From the
Figure 1: Analyzed regions in the GAIA survey, where $\delta$ is angular radius of the events, $M$ is their multiplicity and $N_e$ is number of events. Analysis is done only for events $2 \leq M \leq 80$.

| Area | $l \times b$ | $\delta$ [deg] | $\langle M \rangle$ | $N_e$ |
|------|-------------|----------------|-------------------|------|
| C    | $\langle 140, 180 \rangle \times \langle -10, 10 \rangle$ | 0.005 | 2.60 | 6962671 |
| N    | $\langle -180, 180 \rangle \times \langle 60, 80 \rangle$ | 0.02 | 3.26 | 1437067 |
| S    | $\langle -180, 180 \rangle \times \langle -80, -60 \rangle$ | 0.02 | 3.30 | 1448647 |

Figure 2: Dependence of Fourier coefficients $\langle v^2_1 \rangle$, $\langle v^2_2 \rangle$, $\langle v^2_3 \rangle$ on multiplicity $M$ for the events in the area C (upper panels) and N&S (lower panels). The green line is a linear fit of data taken in $\langle v^2_2 \rangle$ panel.

The negative slope of the line one can estimate the radius of the empty spots. The result $M \langle v^2_1 \rangle > 1$ for the sparse field N&S suggests the presence of clustering.

To analyze obtained results in more detail, we applied the complementary method on angular distances. The results are presented in the upper part of Fig. 3 and can be compared with the corresponding Monte-Carlo simulation in [7]. In fact we observe a 2D representation of the effect reported in [8] (Sec. 4.4.1., Fig. 17), which is due to reduced resolution of two sources at angular distances $d \lesssim 3''$. In the same figure we observe also a peak at small separation $d \lesssim 1''$. In the
Figure 3: Distributions of angular distances in the region C for $G \leq 15$ mag (lower panels) and for all $G$ (upper panels). The left and central panels are 3D plots, where the unit of $x, y$ is 1″. The right panels are the ratios of measured distribution of relative distances $\hat{d}_{ij}$ to the interpolation of corresponding random Monte-Carlo data.

... cited paper such peak is observed only in sparse field and is interpreted as a presence of binary stars. A simulation described in the same paper suggests that reduced efficiency at $d \lesssim 3″$ correlates with fainter magnitudes $G$ in the field. This can be in agreement with the lower part of Fig.3, where the sources with $G > 15$ mag are excluded and as a result the resolution dip is reduced.

Similar analysis for region N&S is demonstrated in Fig.4. Obviously, in the sparse field the resolution is improved. Apart of the pronounced peak at $d \lesssim 1″$, we observe a clear excess of pairs separated by $d \lesssim 8″$. The whole excess, giving one broader peak for $G \leq 15$ mag can represent binaries.

The next results are related to the ternary star systems. We compare random Monte-Carlo simulation in [7] with the corresponding data from the field N&S in Fig.5. Again, we observe an excess in the region of small ternary distances, roughly $\hat{d}_\alpha \lesssim 0.1$ or equivalently $d_\alpha \lesssim 15″$. The excess is broader and less dependent on $G$ than for binaries. It can represent the bounded ternary star systems.

2.1. Gravitational microlensing

The principle of the gravitational microlensing effect is explained in [9]. For a small angular distance $\beta$ between two stars the light beams from the more distant one ($S_2$) and passing the gravitational field of the star on the way ($S_1$), can reach the observer $O$ by two ways. The
angles seen by the observer are

\[ \theta_{12} = \frac{\beta \pm \sqrt{4\alpha_0^2 + \beta^2}}{2} \; ; \quad \alpha_0 = \sqrt{\frac{4\kappa M}{c^2} \frac{D_{da}}{D_d D_s}} \]  

(1)

where \( \kappa \) is gravitational constant, \( M \) is the mass of the star \( S_1 \) and \( D_d, D_s, D_{ds} \) are distances \( OS_1, OS_2, S_1S_2 \). For \( \beta = 0 \) the Einstein ring with angular radius \( \theta = \alpha_0 \) is created. Apparently

\[ |\theta_2| \leq \alpha_0 \leq \theta_1. \]  

(2)

For observation of the effect it is important that angular separation \( \beta \) of the pair is close to \( \alpha_0 \). And for \( \alpha_0 \) estimate the distance \( D_d \) is critical. For example \( D_d \approx 10 - 10^2 \) l.y., \( D_s \approx 2D \)
Figure 6: The Monte-Carlo simulation (upper panels) of distribution \( P(\hat{d}_{ijk}, \kappa) \) (left), the alignment function \( \langle \kappa \rangle \) (centre) and integrated distribution \( P(\kappa) \) (right). The lower panels display on two scales the comparison of the \( \langle \kappa \rangle \) from the data region N&S (blue) with the Monte-Carlo simulation (red).

and \( M \approx M_S \) give roughly \( \alpha_0 \approx 20 \) mas. Such separation is probably beyond present Gaia resolution. In fact the minimum separation we have registered in any of the regions C,N,S is 59 mas.

There can be the following signature of the gravitational microlensing effect. The light sources \( S_2(\theta_2), S_1, S_2(\theta_1) \) as seen by the observer, should have a small separation \( d_{ijk} \) (or \( \hat{d}_{ijk} \)) and should be aligned in a line, or make a narrow triangle within the errors of measurement. So, as a measure of the alignment we define the parameter \( \kappa \):

\[
\kappa = \frac{1}{3} \left( \cos^2 \eta_1 + \cos^2 \eta_2 + \cos^2 \eta_3 \right),
\]

where \( \eta_i \) are angles in the observed triangle. For \( \kappa = 1 \) there is maximum alignment (e.g. \( \eta_1, \eta_2, \eta_3 = 0, 0, \pi \)) and for \( \eta_1 = \eta_2 = \eta_3 = \pi/3 \) we have minimum \( \kappa = 1/4 \). In the upper panels of Fig.6 we have shown results of the related Monte-Carlo simulation. In the lower part we have shown its comparison with the data from the field N&S (note rescaled variable \( \hat{d}_{ijk} \) instead of \( \hat{d}_{ijk} \)). It is important, that the shape of (normalized) Monte-Carlo distribution \( P(\hat{d}_{ijk}, \kappa) \) does not depend on the multiplicity of events \( \geq 3 \). The same holds for the distribution \( P(\kappa) \) and the dependence \( \langle \kappa \rangle \) on \( \hat{d}_{ijk} \). One can observe a perfect agreement with the data for \( \hat{d}_{ijk} \gtrsim 5'' \). On contrary, for the smaller distances among the three sources there is a clear excess of the alignment. Can there be a connection between this effect and the gravitational microlensing - image splitting? Or, more probably, is the excess only another form of the distortion of measurement at small separations? One should add that the conditions for observation of the
aligned triplet described above can be rather strict. For $\beta < \alpha_0$ the sources $S_2(\theta_2), S_2(\theta_1)$ are getting strongly magnified, so the source $S_1$ may not be resolved. On the other hand, brightness of $S_2(\theta_2)$ falls rapidly for $\beta > \alpha_0$ [9].

3. Summary and conclusion

We have proposed the statistical method for analysis of the finite 2D patterns. Each pattern (event) involves random points located inside the disc with fixed radius. We have demonstrated that the method can identify tiny deviations from purely random distributions, for example a tendency to clustering or anti-clustering.

The method has been applied to the analysis of astrometric data obtained by the Gaia mission. In this case the events are sets of the stars observed inside discs of small fixed angular radius in the galactic reference frame, which cover chosen region of the sky. In parallel the statistics of random events (star patterns) is generated by the Monte-Carlo code. Comparison of relevant distributions obtained from the data and Monte-Carlo events proves the presence of binary and ternary star systems in the Gaia survey.

A special attention has been paid to the discussion on possibility of detection of the gravitational microlensing and image splitting effect. Our present conclusion is that the statistical method suggested above can be useful for detection of this effect, however further effort is needed to extract reliable results from the data.

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