Probing Higgs $C P$ properties at the CEPC in the $e^+e^- \rightarrow ZH \rightarrow l^+l^-H$ using optimal variables

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Received: 2 August 2022 / Accepted: 15 October 2022 / Published online: 1 November 2022
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Abstract In the Circular Electron Positron Collider (CEPC), a measurement of the Higgs $C P$ mixing through $e^+e^- \rightarrow ZH \rightarrow l^+l^- (e^+e^-/\mu^+\mu^-)H(\rightarrow b\bar{b}/c\bar{c}/gg)$ process is presented, with 5.6 ab$^{-1}$ $e^+e^-$ collision data at the center-of-mass energy of 240 GeV. In this study, the $C P$-violating parameter $\tilde{c}_{Z\gamma}$ is constrained between the region of $-0.30$ and $0.27$ and $\tilde{c}_{ZZ}$ between $-0.06$ and $0.06$ at 68% confidence level. This study demonstrates the great potential of probing Higgs $C P$ properties at the CEPC.

1 Introduction

The historic discovery of Higgs boson with a mass around 125 GeV in 2012 by the ATLAS and CMS collaborations at the Large Hadron Collider (LHC) [1,2] completed the Standard Model (SM). This particle provides a new portal to search for new physics beyond the SM (BSM). The Higgs boson is predicted to be a scalar particle ($J^P = 0^{++}$) under the SM of particle physics. As a result, any observation of charge-parity violation ($CPV$) in Higgs would be a sign of physics BSM and could account for the explanation of the observed baryon asymmetry of the universe.

At present, the Higgs boson $CP$-mixing measurements are performed at hadron colliders. The hypothesis of spin-1 or spin-2 Higgs has been excluded by ATLAS and CMS at 99% confidence level (CL) with $\sqrt{s} = 7$ and 8 TeV, 25 fb$^{-1}$ data [3]. Studies of the $CP$ properties of the Higgs boson interactions with gauge bosons have been performed by the ATLAS [4–6] and CMS [7–9] experiments, and the results show no deviations from the SM predictions. ATLAS and CMS also finished analyses of $Ht\bar{t}$ coupling, which provides an alternative and independent avenue for $CP$ testing in the Higgs sector because it is particularly sensitive to deviations from the SM coupling [10,11]; Their results show exclusion of the pure $CP$-odd structure of the top quark Yukawa ($t\tilde{t}H$) coupling at 3.9$\sigma$ (3.2$\sigma$) and the fractional contribution of the $CP$-odd component is measured to be $f^{Ht\bar{t}}_{CP} = 0.00 \pm 0.33$.

However, small anomalous contributions were not excluded. At the HL-LHC [12], the $CP$-odd $VVH$ couplings are introduced and the expected results constrain the $CP$-odd parameters $\tilde{c}_{Z\gamma}$ between $-0.22$ and $0.22$ and the $\tilde{c}_{ZZ}$ between $-0.33$ and $0.33$ at the 68% confidence level.

In terms of probing the $CP$-odd Higgs couplings, a lepton collider operating as a Higgs factory has great advantages, as it is free of the QCD background and has tunable and precisely defined initial energies. Several future lepton collider, including the International Linear Collider (ILC) [13], the $e^+e^-$ Future Circular Collider (FCC-ee) [14], the Compact Linear Collider (CLIC) [15], and the Circular Electron-Positron Collider (CEPC) [16], have been proposed with the capability of precise measurement of Higgs boson parameters. There are plenty of approaches to the Higgs CP measurements at the future lepton colliders by using differ-
ent probes, like the VH process, the $t\bar{t}H$ process, the VBF process and the $H \rightarrow \tau^+\tau^-$ process [17]. For example, the VBF process, an ongoing study of the $Z$-fusion process at 1.4 TeV CLIC and 1 TeV ILC are mentioned in Ref [18]. For the past CP-odd VWH couplings studies, at the ILC [19], the CP-mixing angle of $\tau$ pair from Higgs boson decays can reach an accuracy of 4.3° with 2 $ab^{-1}$ of polarized data at 250 GeV [18,20] and the anomalous couplings $b_Z$ and $\bar{b}_Z$ with the new Lorentz structures can reach $<1\%$ [21]. The best 95% CL limits at the loose working point (90% b-tagging efficiency) on CP-violating couplings $\tilde{c}_{HW}$ and $\tilde{c}_{HB}$ are $[-7.0 \times 10^{-3}, 7.0 \times 10^{-3}]$ and $[-3.0 \times 10^{-2}, 3.0 \times 10^{-2}]$, respectively at the 3 TeV energy stage of CLIC [22] via $e^+e^- \rightarrow v\bar{v}H$ process. The CP violation parameter $\tilde{g}$ can reach the limit of $[-0.04, 0.01]$ at the CEPC [23] which is similar to this letter, but the current study obtain a better result after using full simulation and the technique of optimal observable. As for the past CP study at the future linear $e^+e^-$ colliders (including ILC at 500 GeV and CLIC at 380 GeV) [24], their results show the potential of measuring CP violation in the top-quark sector at future $e^+e^-$ collider.

The CEPC will operate at a center-of-mass energy of $\sqrt{s} \sim 240$ GeV which is close to the maximum of the Higgs boson production cross-section through the $e^+e^- \rightarrow ZH$ process. Over one million Higgs bosons with an integrated luminosity of 5.6 $ab^{-1}$ will be produced. In comparison to the LHC, the cleaner environment of the CEPC allows significantly better exclusive measurements of Higgs boson decay channels. So, in the future, more precise Higgs-gauge boson coupling studies can be performed, such as this Higgs CP measurement through $e^+e^- \rightarrow ZH \rightarrow (e^+e^-/\mu^+\mu^-)H \rightarrow (b\bar{b}/c\bar{c}/gg)$ process [25].

This letter is organized as follows: in Sect. 2, we introduces the theory framework for the analysis of Higgs CP-mixing. The MC samples and event selections are described in Sects. 3 and 4 respectively. Section 5 describes the strategy for analysis and interprets the results of this study. The conclusions are presented in Sect. 7.

2 Theory framework

To parametrize BSM effects in a general way, we assume that the new physics sector is characterized by a scale $\Lambda$, which is significantly higher than the electroweak scale, and the SM is supplemented with 59 independent dimension-6 operators [26]. This Lagrangian can be schematically cast as:

$$L_{\text{eff}} = L_{\text{SM}}^{(4)} + \frac{1}{\Lambda^2} \sum_{k=1}^{59} \alpha_k O_k,$$

where the $\alpha_k$ is the coupling of operator $O_k$. Apart from the SM tree contributions, we only consider effects of order $1/\Lambda^2$ on the decay amplitude. In the broken-symmetry phase, the effective Lagrangian Eq. (1) generates the terms [27] [28]:

$$L_{\text{eff}} \supset c_{ZZ}^{(1)} HZ \mu Z^\mu + c_{ZZ}^{(2)} HZ \mu \nu Z^\mu \nu + c_{Z\bar{Z}} HZ \mu \nu \bar{Z}^\mu \nu \bar{Z}^\nu + c_{AZ} HZ \mu \nu A^\mu \nu + c_{A\bar{Z}} HZ \mu \nu A^\mu \nu \bar{Z}^\nu + HZ \mu \nu \bar{Z}^\nu (c_{V} + c_{AY}) \bar{Z} + Z_{\mu} \bar{Z}^\nu (g_{V} - g_{AY}) \bar{Z} - \text{gen.} Q_{\mu} A_{\mu} \bar{Y} \bar{Y} \mu \bar{Y},$$

which includes the relevant tree-level SM terms. The Higgs-gauge couplings of Eq. (2) are given by:

$$
\begin{align*}
    c_{ZZ}^{(1)} &= m_Z^2 \left( \sqrt{2} G_F \right)^{1/2} \left( 1 + \tilde{\alpha}_{ZZ}^{(1)} \right), \\
    c_{ZZ}^{(2)} &= \left( \sqrt{2} G_F \right)^{1/2} \tilde{\alpha}_{ZZ}, \\
    c_{Z\bar{Z}} &= \sqrt{2} G_F \tilde{\alpha}_{Z\bar{Z}}, \\
    c_{AZ} &= \left( \sqrt{2} G_F \right)^{1/2} \tilde{\alpha}_{AZ}, \quad \text{(3)} \\
    c_{A\bar{Z}} &= \sqrt{2} G_F \tilde{\alpha}_{A\bar{Z}},
\end{align*}
$$

where the $\tilde{\alpha}_{ZZ}$ and $\tilde{\alpha}_{Z\bar{Z}}$ are CP-violation parameters.

The differential cross-section for $e^+e^- \rightarrow ZH \rightarrow l^+l^-H(\rightarrow b\bar{b}/c\bar{c}/gg)$ is given by:

$$
\frac{d\sigma}{d\cos \theta_1 d\cos \theta_2 d\psi} = \frac{1}{m_H^2} N_\sigma \left( q^2 \right) J \left( q^2, \theta_1, \theta_2, \phi \right),
$$

where the definitions of three angles are shown in Appendix A. $N_\sigma \left( q^2 \right)$ is the normalization factor and it can be written in terms of the dimensionless parameters $r$ and $s$ as:

$$
N_\sigma \left( q^2 \right) = \frac{1}{210 (2\pi)^3} \frac{1}{\sqrt{r}} \frac{\sqrt{\lambda(1,s,r)}}{\sqrt{s}},
$$

where $\lambda$ is the käännönen functions and, the constant dimensionless parameters are given by the following:

$$
\begin{align*}
    s &= \frac{q_{th}^2}{m_Z^2} \approx 3.68, \\
    r &= \frac{m_Z^2}{m_H^2} \approx 0.53, \\
    \gamma Z &= \frac{\Gamma_Z}{m_H} \approx 0.020.
\end{align*}
$$

$J$ can be expressed by:

$$
J \left( q^2, \theta_1, \theta_2, \phi \right) = J_1 \left( 1 + \cos^2 \theta_1 \cos^2 \theta_2 + \cos^2 \theta_1 + \cos^2 \theta_2 \right) + J_2 \sin^2 \theta_1 \sin^2 \theta_2 + J_3 \cos \theta_1 \cos \theta_2 + (J_4 \sin \theta_1 \sin \theta_2 + J_5 \sin \theta_1 \sin \theta_2) \sin \phi + J_6 \sin \theta_1 \sin \theta_2 + J_7 \sin \theta_1 \sin \theta_2 \cos \phi + J_8 \sin \theta_1 \sin \theta_2 \sin 2\phi.
$$
where the explicit form of the $J_i$ in terms of the EFT coefficients and Standard Model parameters was computed by the authors of [27] and for convenience is given in Appendix B.

Among all the BSM variables, only $\hat{a}_{AZ}$ and $\hat{a}_{ZZ}$, shown in Eq. (B2), contribute to the CP-odd. So those are the CP-violating parameters that we need to study.

In addition to simplify the analysis, we only constrain the CP-violating parameters with the assumption that all other parameters are zero. Three of the $J_i$ functions shown in Eq. (B1), namely $J_4$, $J_5$, $J_8$, are CP-odd and vanish in the SM at tree level, whereas the remaining six functions are CP-even. As a result, the differential cross-section can be represented as follows:

\[
\frac{d\sigma}{d\cos\theta_1 d\cos\theta_2 d\phi} = N \times \left(J_{\text{even}}(\theta_1, \theta_2, \phi) + \hat{a}_{AZ} \times J_{\text{odd}_1}(\theta_1, \theta_2, \phi) + \hat{a}_{ZZ} \times J_{\text{odd}_2}(\theta_1, \theta_2, \phi)\right),
\]

where $J_{\text{odd}_1}$ and $J_{\text{odd}_2}$ are part of $J_{\text{odd}}$.

Two optimal variables combining the information from $\{\theta_1, \theta_2, \phi\}$ can be defined as following:

\[
\omega_1 = \frac{J_{\text{odd}_1}(\theta_1, \theta_2, \phi)}{J_{\text{even}}(\theta_1, \theta_2, \phi)},
\]

\[
\omega_2 = \frac{J_{\text{odd}_2}(\theta_1, \theta_2, \phi)}{J_{\text{even}}(\theta_1, \theta_2, \phi)},
\]

where $\omega_1$ and $\omega_2$ combine the information from 3-dimension phase space and can be used to measure $\hat{a}_{AZ}$ and $\hat{a}_{ZZ}$, respectively.

It is an optimal use of the complete information from the decay as it is more convenient and efficient than using the full multi-dimensional distribution, as shown in [29]. Therefore, we can use the one-dimensional distribution of the single variable $\omega$ to substitute the multi-dimensional distribution without any loss of information. Furthermore, it can help us improve the result compared to the one in Ref. [23] which only uses the $\phi$ information.

The parametric curves with different $\hat{a}_{AZ}$ values are shown in Fig. 1 and with different $\hat{a}_{ZZ}$ values in Fig. 2.

3 Monte Carlo samples

The SM Higgs and background samples, generated with Whizard 1.95 [30] and fully simulated with the CEPC baseline [16] detector design, are used to calculate the selection efficiencies and study background. The details of the event generation can be found at Ref. [31]. Moreover, CP-mixing Higgs samples are generated according to Eq. (1). The study assumes that $\sqrt{s}$ is 240 GeV. The mass of the Higgs boson is set to be 125 GeV. All the generations are normalized to the expected yields with an integrated luminosity of 5.6 ab$^{-1}$.

4 Event selection

The signal sample is the process of $e^+e^- \rightarrow ZH \rightarrow l^+l^-H (\rightarrow b\bar{b}/c\bar{c}/gg)$, containing two jets and two leptons with opposite charges in the final state. Only irreducible backgrounds are considered in the study, mainly $e^+\mu^- \rightarrow ZZ \rightarrow l^+l^-q\bar{q}$ because that the reducible background has very little contribution according to a previous study [32].

Each event must contain two isolated tracks with opposite charges, reconstructed as $e^+e^-$ or $\mu^+\mu^-$. The energy of each isolated lepton candidate must be above 20 GeV. The polar angle of the muon pair system is required to be in the range of $|\cos \theta_{\mu^+\mu^-}| < 0.81$. The invariant mass of the muon pair must be within the $Z$ mass window, which is defined as from 77.5 GeV to 104.5 GeV. The invariant mass of the muon pair’s recoil system, denoted as $M_{\text{recoil}}^{\mu^+\mu^-}$, can provide a clear signature of the $\mu\mu H$ events. The definition of $M_{\text{recoil}}^{\mu^+\mu^-}$ is:

\[
M_{\text{recoil}}^{\mu^+\mu^-} = \sqrt{\left(\sqrt{s} - E_{\mu^+\mu^-}\right)^2 - p_{\mu^+\mu^-}^2} = \sqrt{s} - 2E_{\mu^+\mu^-}\sqrt{s} + m_{\mu^+\mu^-}^2,
\]

in which $\sqrt{s}$ = 240 GeV, and $E_{\mu^+\mu^-}$ and $m_{\mu^+\mu^-}$ stand for the energy and mass of the muons, respectively. A Higgs mass window is defined by requiring $M_{\text{recoil}}^{\mu^+\mu^-}$ between 124 GeV and 140 GeV.

The remaining particles in the event are used to reconstruct exactly two jets with a polar angle $\theta_{\text{jet}}$ in the range of $|\cos \theta_{\text{jet}}| < 0.96$, using ee-kt algorithm [33]. The invariant mass of the pair of jets is required to be between 100 GeV and 150 GeV to reject the background.

Compared to the analysis of the $ZH \rightarrow \mu^+\mu^-H$ decay, the analysis of the $ZH \rightarrow e^+e^-H$ decay suffers from large background. A cut based event selection is performed for the $ZH \rightarrow e^+e^-H$ process. The electron-positron pair is required to have its invariant mass in the range of 85–95 GeV and the polar angle of each electron is required to satisfy $|\cos \phi_e| < 0.95$. The other selection criteria are same as $\mu^+\mu^-H$ analysis. It should be noticed that the effect of $Z$-fusion in $e^+e^-H$ process is neglected in this study since its cross section is rather small.

The expected signal and background yields during the event selections are summarized in Table 1 for $\mu^+\mu^-H$ and $e^+e^-H$ analysis, respectively.
5 Fitting strategy and result

After the event selections, the correlations among $(\theta_1, \theta_2, \phi)$ and the variables for selection, such as $\cos \theta_{l^{+}l^{-}}$, Mass$_{l^{+}l^{-}}$, $M_{\text{recoil}}$, $\cos \theta_{\text{jet}}$, Mass$_{jj}$, are carefully investigated and the impacts on CP study are negligible.

5.1 $\mu^+\mu^-H$ results obtained by $\omega$-fitting

The CP-violating parameters $\hat{\alpha}_{A\bar{Z}}$ and $\hat{\alpha}_{Z\bar{Z}}$ can be measured through the optimal variable $\omega_1$ and $\omega_2$. The estimation of $\hat{\alpha}_{A\bar{Z}}$ and $\hat{\alpha}_{Z\bar{Z}}$ uses a maximum-likelihood fit which could be constructed as:

$$L(\tilde{\alpha} | \alpha) = \prod_{\text{data}} f(x_j | \tilde{\alpha}),$$

where $\tilde{\alpha}$ are CP-violating parameters ($\hat{\alpha}_{A\bar{Z}}$ and $\hat{\alpha}_{Z\bar{Z}}$) to be estimated, and $x$ is the dataset. For each $\tilde{\alpha}$ hypothesis, the profile of a negative log-likelihood (NLL) is calculated. The best-estimated $\tilde{\alpha}$, as well as its central confidence interval at a 68% (95%) confidence level (CL), can be determined at $\Delta \text{NLL} = \text{NLL} - \text{NLL}_{\text{min}} = 0.5$ (1.96).

The main sensitive variable for the CP test in this analysis is the optimal variable $\omega$ (stand for $\omega_1$ and $\omega_2$), which combines all 3 kinematic variables of $\theta_1$, $\theta_2$, and $\phi$, and the function used to determine the $\tilde{\alpha}$-value can be defined as:

$$f^{\tilde{\alpha}}(\omega) = N_{\text{sig}} * f^{\tilde{\alpha}}_{\text{sig}}(\omega) + N_{\text{bkg}} * f^{\tilde{\alpha}}_{\text{bkg}}(\omega),$$

where $f^{\tilde{\alpha}}_{\text{sig}}(\omega)$ and $f^{\tilde{\alpha}}_{\text{bkg}}(\omega)$ are probability density functions (PDFs) of the signal and background, and $N_{\text{sig}}$ and $N_{\text{bkg}}$ are the yields of them, respectively.

For the modeling of signal, PDFs are generated according to different $\tilde{\alpha}$ hypotheses, and background PDF is fixed to the MC simulation. The $M_{\text{recoil}}$ distribution is essential to discriminate signal over background. The background, dominated by the $e^+e^- \rightarrow ZZ \rightarrow \mu^+\mu^-q\bar{q}$, is modeled by a second-order polynomial, while the signal is modeled by the Crystal Ball function [34]. Figure 3 shows the fit result of $\omega$ and $M_{\text{recoil}}$.
Fig. 2 Parametric curves with different $\tilde{\alpha}_{Z\tilde{Z}}$ values. The optimal variable $\omega_2$ is defined in the text. (The value of $\omega_2$ here is multiplied by 1000 for numerical convenience.)

Table 1 Event yields of cut flow. Signal events are $ZH \rightarrow l^+l^-H$, $H \rightarrow b\bar{b}/c\bar{c}/gg$ combined. Background is the $e^+e^-/\mu^+\mu^-/\text{jet pair}$ process.

|                      | Signal | Background |
|----------------------|--------|------------|
| $ZH \rightarrow \mu^+\mu^-H(\rightarrow b\bar{b}/c\bar{c}/gg)$ channel | $2.62 \times 10^4$ | $1.25 \times 10^6$ |
| Lepton pair selection | $1.59 \times 10^4$ | $9.91 \times 10^3$ |
| All selection        | $1.48 \times 10^4$ | $5.60 \times 10^3$ |
| $ZH \rightarrow e^+e^-H(\rightarrow b\bar{b}/c\bar{c}/gg)$ channel | $2.72 \times 10^4$ | $1.77 \times 10^6$ |
| Lepton pair selection | $8.76 \times 10^3$ | $8.77 \times 10^3$ |
| All selection        | $7.15 \times 10^3$ | $4.59 \times 10^3$ |

The individual fitting with each single CPV parameter has been performed by assuming $\tilde{\alpha}_{A\tilde{Z}}$ is a free parameter with $\tilde{\alpha}_{Z\tilde{Z}} = 0$ and vice versa. The expected and observed $\Delta NLL$ curves are shown in Fig. 4 as a quadratic function of $\tilde{\alpha}$. It corresponds to the SM prediction that $\tilde{\alpha}$ equals to zero, and the results of $\tilde{\alpha}_{Z\tilde{Z}}$ and $\tilde{\alpha}_{A\tilde{Z}}$ with confidence interval at 68% (95%) represent the sensitivity to a $CP$-odd Higgs, which are shown in Table 2.

5.2 $\mu^+\mu^-H$ results obtained by $\phi$-fitting

Because the $\phi$ contains the most information among the three kinematic variables, it is straightforward and feasible to fit to $\phi$.

Similar to the Eq. (13), the function used to obtain the $\tilde{\alpha}$ is as following:

$$f(\phi) = N_{\text{sig}} \ast f_{\text{sig}}(\phi) + N_{\text{bkg}} \ast f_{\text{bkg}}(\phi).$$

where the definitions of $f_{\text{sig}}(\phi)$, $f_{\text{bkg}}(\phi)$, $N_{\text{sig}}$ and $N_{\text{bkg}}$ are the same as those used to fit to $\omega$ above.

After fitting with a single $CPV$ parameter, the fit results are shown in Fig. 5, the $\Delta NLL$ curves in Fig. 6.

The comparison with the results of $\omega$-fitting is shown in Table 2. It can be seen that the results of the $\phi$-fitting is slightly worse than those of the $\omega$-fitting as expected, since fewer kinematic variables, i.e., less information, are used.
5.3 Combined results obtained by $\omega$-fitting

By using the same process shown in Sect. 5.1, the $e^+e^-H$ results with $\omega$-fitting can be easily obtained. Neglecting the migration between $e^+e^-H$ and $\mu^+\mu^-H$, the combined likelihood is equal to the multiplication of the likelihood of $e^+e^-H$ and $\mu^+\mu^-H$ channels.

According to the latest note [35], CEPC Higgs operation can be upgraded to 20 ab$^{-1}$. With this increasing luminosity, the updated numerical results after applying the conversion which is described in Sect. 6 are shown in Table 4. This result is about two times better than that of 5.6 ab$^{-1}$.

6 Discussion

Because studies of different $CP$-odd $VVH$ coupling use different symbols or methods, this makes it difficult to visually
used is given by:

\[ \Delta L = \frac{1}{\Delta} \text{fit to } \phi \]

For HL-LHC [12], the CPV Lagrangian which HL-LHC used is given by:

\[ \mathcal{L}_{CPV} = \frac{v}{\sqrt{2}} \left[ \tilde{c}_{Y \phi} A_{\mu \nu} \tilde{A}^{\mu \nu} + \tilde{c}_{Z \phi} \frac{v^2 \tilde{g}_1^2 + \tilde{g}_2^2}{2} Z_{\mu \nu} \tilde{Z}^{\mu \nu} + \tilde{c}_{Z Z} \left( \frac{\tilde{g}_1^2 + \tilde{g}_2^2}{2} \right) Z_{\mu \nu} \tilde{Z}^{\mu \nu} + \tilde{c}_{WW} \frac{v^2}{2} W_{\mu \nu} \tilde{W}^{\mu \nu} \right] , \]  

(15)

where \( g_1 \) and \( g_2 \) are the \( U(1)_Y \) and \( SU(2)_L \) gauge coupling constants.

Comparing Eqs. (15) and (2), the connections of the \( CP \)-odd related parameters between the two equations are obvious,

\[ \left( \sqrt{2} G_F \right)^{1/2} \tilde{A}_{Z \phi} H Z_{\mu \nu} \tilde{Z}^{\mu \nu} = \frac{H}{v} \tilde{c}_{Z \phi} \frac{v^2 \tilde{g}_1^2 + \tilde{g}_2^2}{4} Z_{\mu \nu} \tilde{Z}^{\mu \nu} , \]

\[ \left( \sqrt{2} G_F \right)^{1/2} \tilde{A}_{Z Z} H Z_{\mu \nu} \tilde{Z}^{\mu \nu} = \frac{H}{v} \tilde{c}_{Z Z} \frac{v^2 \tilde{g}_1^2 + \tilde{g}_2^2}{2} Z_{\mu \nu} \tilde{Z}^{\mu \nu} , \]

(16)

where \( g_1 \) and \( g_2 \) equal to 0.358 and 0.648, respectively.\( v = (\sqrt{2} G_F)^{-1/2} = 2 M_W / g \), and \( e \) is the EM coupling which equals to 0.313.

Converted with the Eq. (16), all the results are summarized in Table 3. The \( 1 \sigma \) bounds on \( \tilde{c}_{Z \phi} \) and \( \tilde{c}_{Z Z} \) are \([-0.30, 0.27]\) and \([-0.06, 0.06]\) respectively. These results are significantly better than HL-LHC on the \( \tilde{c}_{Z \phi} \) and \( \tilde{c}_{Z Z} \) due to a cleaner environment and higher luminosity in CEPC.

We use the \( CP \)-violation variables \( \tilde{c}_{Z \phi} \) and \( \tilde{c}_{Z Z} \) which are used by HL-LHC as a benchmark. After similar conversions which are shown in the Appendix C, we can get transformed results of other analysis.

For the previous CEPC \( CP \) study [23], the \( CP \) violation parameter \( \tilde{g} \) can reach the limit of \([-0.04, 0.01]\), it means the \( 1 \sigma \) bounds on \( \tilde{c}_{Z Z} \) is \([-0.20, 0.05]\) after converting with the Eq. (C2). Our study achieves a better result than this previous study because that the technique of optimal observable makes use of the complete kinematics information.

Also, converted with the Eq. (C5), the \( 2 \sigma \) bounds on \( \tilde{c}_{Z \phi} \) and \( \tilde{c}_{Z Z} \) are \([-0.36, 0.36]\) and \([-0.24, 0.24]\) respectively in CLIC study [22]. Assuming that their constraints are in the interference term, it can be approximated that the \( 1 \sigma \) bounds on \( \tilde{c}_{Z \phi} \) and \( \tilde{c}_{Z Z} \) are \([-0.18, 0.18]\) and \([-0.12, 0.12]\). This

\[ \Delta NLL \text{ curve (fit to } \tilde{A}_{Z \phi} \text{ with } \tilde{c}_{Z Z} = 0) \]

\[ \Delta NLL \text{ curve (fit to } \tilde{A}_{Z \phi} \text{ with } \tilde{c}_{Z Z} = 0) \]
result is comparable to us as expected and the comparison is shown in Table 3.

7 Conclusion

In summary, the Higgs CP is studied by analyzing the \( e^+e^- \rightarrow ZH \rightarrow (\mu^+\mu^-/e^+e^-)H \rightarrow (b\bar{b}/c\bar{c}/gg) \) process in a 5.6 ab\(^{-1}\) \( e^+e^- \) collision sample with \( \sqrt{s} = 240 \) GeV at the CEPC. The simplest CP mixing model and two optimal variables improving results compared to just using the angular distribution between the H and Z decay planes. The CP-violating parameter \( \tilde{c}_{Z\gamma} \) is determined to be greater (less) than 0.30 (−0.27) and \( \tilde{c}_{ZZ} \) greater (less) than 0.06 (−0.06) at 95% confidence level. Considering possibly the increasing luminosity such as 20 ab\(^{-1}\), the sensitivities to new physics could be further improved. The CP-violating parameter \( \tilde{c}_{Z\gamma} \) is determined to be greater (less) than 0.16 (−0.14) and \( \tilde{c}_{ZZ} \) greater (less) than 0.03 (−0.03) at 68% confidence level. In the \( \tilde{c}_{ZZ} \) part, there is an order of magnitude improvement over the HL-LHC.

Acknowledgements This study is supported by National Natural Science Foundation of China (NSFC) under Grant No. 12035008 and No.12075271, the Innovation fundation of IHEP under Grant No. E2545AU210.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors’ comment: Anyone who need the raw data can contact the corresponding author.]

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Funded by SCOAP3. SCOAP3 supports the goals of the International Year of Basic Sciences for Sustainable Development.

Appendix A

Here we describe the angle conventions used in our results. Using the conventions for the axes giving in Fig. 7. We choose the direction of the \( Z \) direction to be defined by the momentum of the on-shell \( Z \) boson in the \( e^+e^- \) state rest frame. The \( \theta_1 \) is the angle between the momentum of \( e^- \) and the \( z \) axis. The angle \( \theta_2^- \) is the angle between the direction of flight of the \( e^- \) and the \( z \) axis in the \( e^+e^- \) rest frame. To best exploit the crossing symmetry of the two processes, one should describe the reaction using the angle \( \theta_2^+ \) measured from the \( z \) axis to

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**Table 3**: Summary of 1σ bounds on \( \tilde{c}_{Z\gamma} \) and \( \tilde{c}_{ZZ} \) from various analyses considered in our study, HL-LHC analysis, and CLIC analysis. The optimal variables improve results compared to just using the angular distribution between the H and Z decay planes. The simplest CP mixing model and two optimal variables improving results compared to just using the angular distribution between the H and Z decay planes. The simplest CP mixing model and two optimal variables improving results compared to just using the angular distribution between the H and Z decay planes.

| Collider | \( pp \) | \( e^+e^- \) | \( e^+e^- \) |
|----------|----------|-----------------|-----------------|
| \( E (\text{GeV}) \) | 14,000 | 3000 | 240 |
| \( \mathcal{L} (\text{fb}^{-1}) \) | 3000 | 5000 | 5600 |
| \( \tilde{c}_{Z\gamma} (1\sigma) \) | [−0.22, 0.22] | [−0.18, 0.18] | [−0.30, 0.27] |
| \( \tilde{c}_{ZZ} (1\sigma) \) | [−0.33, 0.33] | [−0.12, 0.12] | [−0.06, 0.06] |

**Table 4**: Summary of 1σ and 2σ bounds on CP-violating parameters \( \tilde{c}_{Z\gamma} \), \( \tilde{c}_{ZZ} \) from various analyses considered in our study with different channels by fitting to \( \omega \) with 5.6 ab\(^{-1}\) and 20 ab\(^{-1}\). The bounds are given at the CEPC. The simplest CP mixing model and two optimal variables improving results compared to just using the angular distribution between the H and Z decay planes. The simplest CP mixing model and two optimal variables improving results compared to just using the angular distribution between the H and Z decay planes. The simplest CP mixing model and two optimal variables improving results compared to just using the angular distribution between the H and Z decay planes.

| Channel | \( \tilde{c}_{Z\gamma} \) | \( \tilde{c}_{ZZ} \) | \( \tilde{c}_{Z\gamma} \) | \( \tilde{c}_{ZZ} \) |
|---------|-----------------|-----------------|-----------------|-----------------|
| \( \mu^+\mu^- H \) channel | [−0.36, 0.33] | [−0.08, 0.07] | [−0.19, 0.17] | [−0.04, 0.04] |
| 68% CL(1\sigma) | [−0.70, 0.67] | [−0.15, 0.15] | [−0.37, 0.35] | [−0.08, 0.08] |
| 95% CL(2\sigma) | [−0.51, 0.47] | [−0.11, 0.11] | [−0.28, 0.24] | [−0.06, 0.06] |
| \( e^+e^- \) channel | [−1.00, 0.95] | [−0.21, 0.21] | [−0.53, 0.49] | [−0.11, 0.11] |
| Combined results | [−0.30, 0.27] | [−0.06, 0.06] | [−0.16, 0.14] | [−0.03, 0.03] |
| 68% CL(1\sigma) | [−0.58, 0.55] | [−0.12, 0.12] | [−0.31, 0.28] | [−0.06, 0.06] |
the direction of flight of the $e^+$. Our Eq. 4 are therefore written in terms of the angle:

$$\theta_2^+ \equiv \theta_2 = \pi - \theta_2^-,$$  

(A1)

Also $\phi$ is the angle between the normal of the planes defined by the $z$ direction and the momenta of $\ell^-$ and $e^-$. It is measured positively from the $\ell^+\ell^-$ plane to the $e^+e^-$ plane.

Appendix B

For completeness, here we list the various $J_i$ coefficients computed first in [27]. These coefficients are conveniently expressed in terms of components of the matrix element as

$$J_1 = 2 r s \left( g_3^A + g_3^L \right) \left( |H_{1,V}|^2 + |H_{1,A}|^2 \right),$$

$$J_2 = \kappa \left( g_3^A + g_3^L \right) \left[ \kappa \left( |H_{1,V}|^2 + |H_{1,A}|^2 \right) + \lambda \Re \left( H_{1,V} H_{2,V}^* + H_{1,A} H_{2,A}^* \right) \right],$$

$$J_3 = 32 r s g_3 g_V \Re \left( H_{1,V} H_{2,V}^* \right),$$

$$J_4 = 4 \sqrt{r s} \left( g_3^2 + g_V^2 \right) \Re \left( H_{1,V} H_{2,A}^* + H_{1,A} H_{2,V}^* \right),$$

$$J_5 = \frac{1}{2} \sqrt{r s} \left( g_3^2 + g_V^2 \right) \left[ 2 \kappa \left( |H_{1,V}|^2 + |H_{1,A}|^2 \right) + \lambda \Re \left( H_{1,V} H_{2,V}^* + H_{1,A} H_{2,A}^* \right) \right],$$

$$J_6 = 4 \sqrt{r s} g_3 g_V \Re \left[ 4 \kappa \Re \left( H_{1,V} H_{2,A}^* \right) \right. + \lambda \Re \left( H_{1,V} H_{2,V}^* + H_{1,A} H_{2,A}^* \right) \left. \right],$$

$$J_7 = \frac{1}{2} \sqrt{r s} \left( g_3^2 + g_V^2 \right) \left( |H_{1,V}|^2 + |H_{1,A}|^2 \right),$$

$$J_8 = 2 r s \sqrt{\kappa} \left( g_3^2 + g_V^2 \right) \Re \left( H_{1,V} H_{2,V}^* + H_{1,A} H_{2,A}^* \right),$$

$$J_9 = 2 r s \left( g_3^2 + g_V^2 \right) \left| H_{1,V} \right|^2 + \left| H_{1,A} \right|^2 \right)^2.$$  

(B1)

The expressions for $H_{i,V/A}$ at $\mathcal{O}(1/\Lambda^2)$ are:

$$H_{1,V} = - \frac{2 m_H \left( \sqrt{2} G_F \right)^{1/2}}{r - s} \frac{g_V}{s} \left( 1 + \hat{\alpha}_{ZZ} \hat{\alpha}_{ZZ}^* - \frac{\kappa Q_{i,g_{em}}(r - s)}{s g_V} \hat{\alpha}_{AZ} \right),$$

$$H_{1,A} = \frac{2 m_H \left( \sqrt{2} G_F \right)^{1/2}}{r - s} \frac{g_A}{s} \left( 1 + \hat{\alpha}_{ZZ} \hat{\alpha}_{ZZ}^* - \frac{\kappa}{r} \hat{\alpha}_{AZ} \right),$$

$$H_{2,V} = - \frac{2 m_H \left( \sqrt{2} G_F \right)^{1/2}}{r - s} \frac{g_V}{s} \left( 2 \hat{\alpha}_{ZZ} - \frac{Q_{i,g_{em}}(r - s)}{s g_V} \hat{\alpha}_{AZ} \right),$$

$$H_{2,A} = \frac{4 m_H \left( \sqrt{2} G_F \right)^{1/2}}{r - s} \frac{g_A}{s} \hat{\alpha}_{ZZ},$$

$$H_{3,V} = - \frac{2 m_H \left( \sqrt{2} G_F \right)^{1/2}}{r - s} \frac{g_V}{s} \left( 2 \hat{\alpha}_{ZZ} + \frac{Q_{i,g_{em}}(r - s)}{s g_V} \hat{\alpha}_{AZ} \right),$$

$$H_{3,A} = \frac{4 m_H \left( \sqrt{2} G_F \right)^{1/2}}{r - s} \frac{g_A}{s} \hat{\alpha}_{ZZ}.$$  

(B2)

Appendix C

As mentioned in the Sect. 6. Some conversion are useful to compare with other study.

In the previous CEPC CP study [23], the CP violation parameter $g$ can reach the limit of $[0.04, 0.01]$ by using the CPV Lagrangian as following:

$$\mathcal{L}_{HZZ} = - \frac{1}{4} g_1 Z_{\mu\nu} Z^{\mu\nu} h - g_2 Z_{\mu} \partial_{\mu} Z^{\mu} h$$

$$+ g_3 Z_{\mu} Z^{\mu} h - \frac{1}{4} \bar{g} Z_{\mu\nu} \bar{Z}^{\mu\nu} h,$$

(C1)

in this function, $g_0 = e M_Z / \left( c_\mu s_\mu \right)$ is the $HZZ$ coupling in the Standard Model. Taking the convenient of [36], $g_3$ is a small number in units of $g_0$, while $g_1, g_2, \bar{g}$ are small numbers in units of $e^2 / \left( g_0 s^2 \right)$, so that the interaction is consistent with the dimension of mass.

In order to compare with this results, we can use the equations here:

$$\frac{H}{\mu} \bar{Z}_{\mu} g_1^2 + g_2^2 4 Z_{\mu\nu} \bar{Z}^{\mu\nu} \nu = \frac{1}{4} \bar{g} Z_{\mu\nu} \bar{Z}^{\mu\nu} h,$$

(C2)

while $\bar{g}$ is the number in units of $e^2 / \left( g_0 s^2 \right)$. So we can easily reach the limit of $\bar{g}$ to $-0.013 \sim 0.013$ at 68% confidence level with 5.6 ab$^{-1}$ in our study, this is a better result than the previous CEPC CP result [23].

In CLIC study [22], the best 95% C.L. limits on CP-violating couplings $\bar{c}_{HW}$ and $\bar{c}_{HB}$ are $[-7.0 \times 10^{-3}, 7.0 \times 10^{-3}]$ and $[-3.0 \times 10^{-2}, 3.0 \times 10^{-2}]$, respectively.

The SM EFT Lagrangian containing the Wilson coefficients in the SILH basis of dimension-6 CP-violating operators can be defined in terms of the mass eigenstates after
electroweak symmetry breaking (Higgs boson, W, Z, photon, etc.) as follows:

\[
\mathcal{L}_{\text{CPV}} = \frac{1}{4} \tilde{g}_{hzz} C^{\mu \nu} \tilde{G}^{\mu \nu} h - \frac{1}{2} \tilde{g}_{hyy} F_{\mu \nu} \tilde{F}^{\mu \nu} h - \frac{1}{4} \tilde{g}_{hzz} Z_{\mu \nu} \tilde{Z}^{\mu \nu} h - \frac{1}{2} \tilde{g}_{hyz} \mu \nu \tilde{h}_{\mu \nu},
\]

\[\text{(C3)}\]

where \(W_{\mu \nu}, Z_{\mu \nu}\) and \(F_{\mu \nu}\) are the field strength tensors of W-boson, Z-boson and photon, respectively.

The relations between Lagrangian parameters in the mass basis \[\text{[22]}\] and in the gauge basis \(\text{(Eq.(C3))}\) are as following:

\[\tilde{g}_{hzz} = \frac{2g}{cWmW} \left[ \tilde{c}_{HHB} \tilde{s}_{WW}^2 - 4\tilde{c}_p \tilde{s}_{WW}^2 + \tilde{s}_{WW}^2 \tilde{c}_{HW} \right],\]

\[\tilde{g}_{hyz} = \frac{gsW}{cWmW} \left[ \tilde{c}_{HW} - \tilde{c}_{HB} + 8\tilde{c}_y \tilde{s}_{WW}^2 \right].\]

\[\text{(C4)}\]

Comparing Eqs. (15) and (C3), the connections of the \(CP\)-odd related parameters between the two equations are obvious:

\[\frac{1}{4} \tilde{g}_{hzz} Z_{\mu \nu} \tilde{Z}^{\mu \nu} h = \frac{H}{v} \tilde{c}_{ZZ} \frac{g_1^2 + g_2^2}{4} Z_{\mu \nu} \tilde{Z}^{\mu \nu},\]

\[\frac{1}{2} \tilde{g}_{hyz} \mu \nu \tilde{h}_{\mu \nu} = \frac{H}{v} \tilde{c}_{Z} \frac{g_1^2 + g_2^2}{2} Z_{\mu \nu} \tilde{A}^{\mu \nu}.\]

\[\text{(C5)}\]

Converted with the Eq. (C5), the 2\(\sigma\) bounds on \(\tilde{c}_{ZZ}\) and \(\tilde{c}_{Z}\) are \([-0.36, 0.36]\) and \([-0.24, 0.24]\) respectively in CLIC study.

For other more study, we can also compare the Lagrangian they used in a similar way to get the comparable results.

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