Dynamic Response of Ising System to a Pulsed Field

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The dynamical response to a pulsed magnetic field has been studied here both using Monte Carlo simulation and by solving numerically the meanfield dynamical equation of motion for the Ising model. The ratio $R_p$ of the response magnetisation half-width to the width of the external field pulse has been observed to diverge and pulse susceptibility $\chi_p$ (ratio of the response magnetisation peak height and the pulse height) gives a peak near the order-disorder transition temperature $T_c$ (for the unperturbed system). The Monte Carlo results for Ising system on square lattice show that $R_p$ diverges at $T_c$, with the exponent $\nu z \approx 2.0$, while $\chi_p$ shows a peak at $T_c^\ast$, which is a function of the field pulse width $\delta t$. A finite size (in time) scaling analysis shows that $T_c^\ast = T_c + C(\delta t)^{-1/2}$, with $x = \nu z \approx 2.0$. The meanfield results show that both the divergence of $R$ and the peak in $\chi_p$ occur at the meanfield transition temperature, while the peak height in $\chi_p \sim (\delta t)^y$, $y \approx 1$ for small values of $\delta t$. These results also compare well with an approximate analytical solution of the meanfield equation of motion.

PACS number(s): 05.50.+q

I. INTRODUCTION

The dynamic response of the Ising systems has recently been studied extensively employing computer simulations [1]. In particular, the study of dynamical response of Ising systems to oscillating magnetic field [2,3] has led to many intriguing dynamic phenomena, like dynamic hysteresis and the fluctuation induced dynamic symmetry breaking transitions in (low e.g., one, two or three dimensional) Ising system in presence of an oscillating field. Acharyya and Chakrabarti also noted [3,4] some anomalous behaviour in the growth of pulse susceptibility, in Ising systems under pulsed magnetic fields of finite durations.

Usually, when a cooperatively interacting thermodynamic system in equilibrium is perturbed (with the perturbation having a step function like variation with time), then the relaxation of the system (to the equilibrium appropriate to the perturbed state) is observed to follow the common Debye type form with a single relaxation time. The standard (Debye) form for any response function (say magnetisation of Ising system) $m(t)$ is

$$m(t) \sim m(\infty) + A \exp(-t/\tau),$$

where $\tau$ is the relaxation time, $m(\infty)$ denotes the new equilibrium value, $A$ is a constant. As the critical temperature is approached $\tau$ shows a critical slowing down; $\tau$ diverges at the critical temperature $T_c$:

$$\tau \sim \xi^z \sim (T - T_c)^{-\nu z},$$

where $\xi$ is the correlation length, $z$ is the dynamic exponent and $\nu$ is the correlation length exponent [1].

Here, we have investigated in details the response of pure Ising systems to pulsed magnetic fields of finite duration, using Monte Carlo simulations for two dimensional Ising systems, and solving numerically the meanfield equation of motion. We have studied the response behaviour for 'positive' pulses, where the pulsed field is in the direction of the spontaneous magnetisation in the ordered phase. In the disordered phase, of course, this notion is immaterial. One can also study the effect of 'negative' pulses on the spontaneous order, where the field direction is opposite to spontaneous magnetisation. Although many intriguing features of the domain growth etc are expected for such 'negative' pulse problem, we confine here to the study of 'positive' pulses only. We have measured the ratio $R_p$ of the response magnetisation (pulse) half-width ($\Delta t$) to that ($\delta t$) of the pulsed external field, and the ratio $\chi_p$ of the response magnetisation peak height ($m_p$) to the field pulse height ($h_p$) giving the pulse susceptibility. The temperature variation of these two quantities for various pulse width durations and heights of the external field, have been investigated.

We find that for weak pulses, while the width-ratio $R_p$ diverges at the order-disorder transition point $T_c$ of the unperturbed Ising system, the pulse susceptibility $\chi_p$ does not diverge and for low dimensions, e.g., in two dimension studied here, a smeared peak in $\chi_p$ occurs at an effective $T_c^\ast$, which approaches $T_c$ as the field pulse width $\delta t$ increases ($\chi_p$ diverges at $T_c$ as $\delta t \to \infty$). A meanfield analysis for the absence of divergence of $\chi_p$ for finite $\delta t$ has been developed. Also, a finite time scaling analysis, similar to Fisher’s finite size (length) scaling [5], has been developed and compared with the observation of the effective transition temperature $T_c^\ast$ with the external field pulse width $\delta t$.

We have organised this paper as follows: In section II, the model and the simulation techniques have been described. In section III, the results are given. The paper ends with concluding remarks in section IV.
II. THE MODEL AND SIMULATION

A ferromagnetically interacting (nearest neighbour) Ising system in presence of a time dependent magnetic field can be described by the Hamiltonian

$$H = - \sum_{ij} s_i^z s_j^z - h(t) \sum_i s_i^z,$$

(2.1)

where \(s_i^z\)'s are spin variable having their value \(\pm 1\) and \(h(t)\) is the time varying longitudinal magnetic field. Here, we have considered the time variation of \(h(t)\) as follows:

$$h(t) = h_p \quad \text{for} \quad t_0 < t < t_0 + \delta t,$$

(2.2)

$$= 0 \quad \text{elsewhere},$$

where \(h_p\) is the amplitude of the field and \(\delta t\) is the duration or the active period of the external field.

In our simulation, we have considered a 500\(\times\)500 square lattice in two dimension. At each site of the lattice there is a spin variable \(s_i^+ = (\pm 1)\). We update the lattice by stepping sequentially over it following the Glauber single spin flip dynamics. One such full scan over the lattice is unit time step (Monte Carlo step or MCS) here. First, we allowed the system to reach the equilibrium (at any temperature \(T\)) and only after that the magnetic field \(h(t)\) has been switched on (\(t_0\) is thus much larger than the relaxation time of the system). One can also use random updating sequences. However, it takes more time (MCS) to stabilise the system. We expect all the features of the response studied here remains qualitatively unchanged for random updating sequences, and we give the results here for sequential updating only. We have measured the maximum height (above the equilibrium value) \(m_p\) and half-width \(\Delta t\) of the response magnetisation. Here, as mentioned before, the following two quantities have been defined to characterise the response of the system:

- pulse width ratio \(R_p = \frac{\Delta t}{\delta t}\)

and

- pulse susceptibility \(\chi_p = \frac{m_p}{h_p}\).

It may be noted that \(\chi_p\) reduces to normal static susceptibility as \(h_p \to 0\) and \(\delta t \to \infty\). At each temperature for fixed \(h_p\) and \(\delta t\), the numerical values of \(R_p\) and \(\chi_p\) are obtained averaging over 20 random Monte Carlo realisations (initial seed). We have studied the temperature variation of these two quantities. The results are given and discussed in the next section. The observation for finite peak in \(\chi_p\) at an effective transition temperature \(T_c^e\) (which converges to \(T_c\) as \(\delta t \to \infty\)) is analysed in view of finite time scaling behaviour, as mentioned before.

To study the similar response in the case where the fluctuations are absent, we have considered the following meanfield dynamical equation of motion for the kinetic Ising system:

$$\tau_0 \frac{dm}{dt} = -m + \tanh \left( \frac{m + h(t)}{T} \right).$$

(2.3)

Here \(\tau_0\) is the microscopic relaxation time, \(m\) is the average magnetisation (in the meanfield approximation), \(h(t)\) is the time varying pulse field having the same time variation described in equation (2.1) and \(T\) denotes the temperature. We have solved numerically the above equations using fourth order Runge-Kutta method. We have evaluated the pulse width ratio \(R_p\) and pulse susceptibility \(\chi_p\) at various temperatures for fixed pulse width \(\delta t\) and height \(h_p\). The numerical results are given in the next section, where we have compared the results for finite peak height in \(\chi_p\) (at \(T_c = 1\)), with an approximate analytic estimate of \(\chi_p\) in such cases.

III. RESULTS

A. Monte Carlo studies

As mentioned before, our results here are for two dimensional Ising system on square lattice of size 500\(\times\)500. We applied an external field of amplitude \(h_p\) for a duration \(\delta t\) after bringing the system into a steady state. For this, the typical number of Monte Carlo steps required for this size of the lattice chosen here, is observed to be of the order of \(10^6\). The response magnetisation has amplitude \(m_p\) (measured from the equilibrium value) and a half-width \(\Delta t\). Fig. 1 shows a typical time variation of magnetic field \(h(t)\) and the corresponding response magnetisation \(m(t)\). The dynamical response is characterised by two quantities; the width-ratio \(R_p\) and the pulse-susceptibility \(\chi_p\). The temperature variation of these two quantities has been studied. Fig. 2 shows the temperature variations of \(R_p\) and \(\chi_p\) for fixed values of \(h_p(= 0.5)\) and for three values of \(\delta t\) (= 5, 10, 25 MCS).

Since \(m_p\) is bounded from above, a large \(h_p\) would saturate \(m_p\) and hence \(\chi_p\) becomes small (due to the saturation). Also, for extremely small \(h_p\), it becomes difficult to identify \(m_p\) from the noise, and hence the estimate of \(\chi_p\) becomes erroneous. We found \(h_p \cong 0.5\) to be well within the above optimal range. From the figure it is clear that \(R_p\) has a sharp divergence at \(T_c (\cong 2.30)\), somewhat larger than \(T_c\), the Onsager transition temperature, due to the small size of the system) almost irrespective of the values of the pulse width \(\delta t\). But \(\chi_p\) shows peak at different points (significantly above \(T_c \cong 2.27\)) depending upon the values of the field pulse width \(\delta t\). As \(\delta t\) value increases, it is observed that the peak shifts towards \(T_c\) from above (and also the peak height grows).

Let us try to understand why the width-ratio \(R_p\) diverges at \(T_c\), while height-ratio or pulse susceptibility \(\chi_p\) shows peak at some higher value \(T_c^e(\delta t)\) depending upon
the value of $\delta t$: $T_c^e(\delta t) \rightarrow T_c$ as $\delta t \rightarrow \infty$. The pulse-like perturbation probes the response of the system at finite frequencies. Consequently, the $\chi_p(m_p/h_p)$ cannot diverge as the response magnetisation height $m_p$ is not an equilibrium value corresponding to the pulse height $h_p$; rather the $m_p$ results are bounded by the time window of width $\delta t$. On the other hand, the response will take its own relaxation time to come to its equilibrium value (irrespective of the value of $\delta t$), when the field is switched off (at $t_0 + \delta t$). This leads to the divergence of $\Delta t$, due to critical slowing down, as $T$ approaches $T_c$.

The sharp divergence of width-ratio $R_p$ is identified as the consequence of critical slowing down and the point of divergence is the critical temperature $T_c$ for the ferro-param transition. In fact, since the relaxation after the withdrawal of the pulse is unrestricted by the pulse-width, we can assume that $\Delta t \sim \tau \sim |T - T_c|^{-\nu z}$. We therefore plot in Fig. 3, $R_p^{-1/\nu z}$ versus $T$ and find a straight line plot with $\nu z \cong 2.0$ in two dimensional case. This compares well with the previous estimates of the value of $\nu z$ [6].

As the growth of the height of magnetisation response (and its maximum value $m_p$) is very much bounded by the time window $\delta t$ of the applied field pulse, the anomalous behaviour of $\chi_p$ (having finite peak at a shifted temperature $T_c^e (\delta t)$) may be considered to be due to the finite size (in time) effect. Similar to the finite size (in length) scaling theory of Fisher [5], where a finite size system shows effective (non-singular or non-divergent) pseudo-critical behaviour at $T_c^e (L)$ when the correlation length $\xi$ becomes of the order of the system size $L$, we suggest a finite time $(\delta t)$ scaling behaviour here for $\chi_p$: If the relaxation time $\tau \sim \xi^z \sim |T - T_c|^{-\nu z}$, where $\nu$ is the correlation length exponent and $z$ is the dynamic exponent, then $\chi_p$ would show peak at the temperature $T_c^e$ here when $\tau(T_c^e) \sim \delta t$ or $T_c^e (\delta t) - T_c |^{-\nu z} \sim \delta t$, or

$$T_c^e (\delta t) \sim T_c + C(\delta t)^{-1/\nu z},$$

where $C$ is some constant. In fact, Fig. 4 shows that the effective peak position $T_c^e$ indeed approaches $T_c$ as $\delta t \rightarrow \infty$. The inset shows the plot of $T_c^e$ with $(\delta t)^{-1/\nu z}$, which gives a straight line for $x = \nu z \cong 2.0$. This again suggests $\nu z \cong 2.0$ and also the extrapolated value of $T_c$ becomes about 2.29, which compares well with the Onsager value, comparable to the previous estimate.

**B. Meanfield results**

We have solved the meanfield equation for response magnetisation $[2,3]$ using fourth order Runge-Kutta method. Fig. 1 shows the typical variation of response magnetisation and field. Here also we have measured the width-ratio and the pulse susceptibility and studied the temperature variation of these two quantities. Fig. 5 shows the temperature variation of $R_p$ and $\chi_p$ for different values of $\delta t$. $R_p$ diverges and $\chi_p$ peaks at the same order-disorder transition point ($T_c = 1$ here). We have also studied the variation of the maximum value of $\chi_p$ ($\chi_p^{\max}$ at $T = T_c$) with respect to the duration of the pulsed field. It may also be mentioned that the peak height was found to increase with increasing pulse width $\delta t$, and $\chi_p$ diverges as $\delta t \rightarrow \infty$; $\chi_p^{\max} \sim (\delta t)^y; y \cong 1.0$

Fig. 6 shows the variation of $\chi_p^{\max}$ with $\delta t$.

In order to comprehend these observations, we solve the equation $[2,3]$ in a linearised limit (large $T$ and small $h_p$; specifically $T > 1$, $h_p \rightarrow 0$). In such limit, the equation of motion becomes

$$\tau_0 \frac{dn}{dt} = -em + h(t)/T;\ x = (T - 1)/T. \quad (3.2)$$

One can solve the above equation using a form $m(t) = m_0 e^{-t/\tau}$ which gives

$$\tau_0 \frac{dm_0}{dt} e^{-t/\tau} - \frac{\tau_0}{\tau} m_0 e^{-t/\tau} = -em_0 e^{-t/\tau} + \frac{h(t)}{T}, \quad (3.3)$$

giving $\frac{\tau}{\tau_0} = e^{-1}$, and

$$\tau_0 \frac{dm_0}{dt} e^{-t/\tau} = h(t)/T. \quad (3.4)$$

Integrating the last equation for $h(t) = h_p$ for a finite time width $\delta t$ and $h(t) = 0$ elsewhere, one gets $m_p \sim h_p \delta t/(T_c \tau_0)$ at $T = T_c = 1$ (when $\tau \rightarrow \infty$). This gives

$$\chi_p^{\max} = \chi_p(T_c) = m_p(T_c)/h_p \sim \delta t/(T_c \tau_0) \sim \delta t/\tau_0. \quad (3.5)$$

We have checked the above linear relationship of $\chi_p^{\max}$ with $\delta t$ for extremely small values of pulse field amplitude $h_p$ (see Fig. 6).

**IV. SUMMARY**

We have studied the Glauber (order parameter nonconserving) dynamics of an Ising system under a time varying external magnetic field, when the field is applied as a pulse of finite time width, after the system reaches the equilibrium. The time variation of the response magnetisation is studied as a function of pulse width $\delta t$, height $h_p$ and temperature $T$ of the system. We have measured specifically $R_p = \Delta t/\delta t$ and $\chi_p = m_p/h_p$, where $\Delta t$ is the time width of the response magnetisation and $m_p$ is the maximum height of the response magnetisation above its equilibrium value.

Our computer simulation results for square lattice showed $R_p \sim |T_c - T|^{-\nu z}$ with $T_c \cong 2.30$, the Onsager value, and $\chi_p$ has a peak $\chi_p^{\max}$ at $T_c^e > T_c$, such that a finite size (in time) scaling behaviour is observed: $T_c^e = T_c + C(\delta t)^{-1/\nu z}$, with $\nu z \cong 2.0$. The numerical solutions of the mean field equation $[2,3]$ showed $R_p \sim 1/(T - 1)$ and the pulse susceptibility peak value $\chi_p^{\max} \sim \delta t$, occuring at $T = T_c = 1$. Theoretical analysis for the finite size (in time) scaling behaviour (of $T_c^e$ in the Monte Carlo case) and for the peak $\chi_p^{\max}(\delta t)$ (in the mean field case) are also given.
ACKNOWLEDGEMENTS

MA acknowledges JNCASR for financial support and SERC, IISc Bangalore for computational facilities.

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Figure Captions

Fig.1. Time variation of magnetic field \( h(t) \) and the response magnetisation \( m(t) \) in the Monte Carlo case. For \( h_p = 0.5 \) and \( \delta t = 50 \).

Fig.2. Temperature variations of (a) \( R_p \) and (b) \( \chi_p \) for different values of \( \delta t \) in the Monte Carlo case. The symbol circle is for \( \delta t = 5 \); the square is for \( \delta t = 10 \) and the cross is for \( \delta t = 25 \).

Fig.3. Variation of \( R_p^{-1/\nu_z} \) against \( T \), with \( \nu_z = 2 \).

Fig.4 Variation of \( T_c^\circ \) with respect to \( 1/\delta t \) in the Monte Carlo case. Inset shows the variation of \( T_c^\circ \) with respect to \( (\delta t)^{-1/x} \); \( x = 2 \).

Fig.5 Temperature variation of (a) \( R_p \) and (b) \( \chi_p \) for different values of \( \delta t \) in the mean field case. The symbol triangle is for \( \delta t = 8 \); the plus is for \( \delta t = 16 \) and the cross is for \( \delta t = 32 \).

Fig.6. Variation of \( \chi_p^{\max} \) with respect to \( \delta t \) in the mean field case.
Fig. 3

$P_p(1/Nz)$ vs. $T$

- $P_p(1/Nz)$ ranges from 0.1 to 0.7.
- $T$ ranges from 2 to 3.
Fig. 4
Fig. 6

\[ h_p = 0.001 \]