Reliability Analysis of Arch Dam Based on Optimized Complex Method: a Case Study of Shapai Arch Dam

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Abstract. The research on crack development and reliability index is very important for roller compacted concrete (RCC) arch dam. There are many methods for studying reliability of a structure, such as JC method or finite element method. But it is inevitable to take derivatives of complex functions in this method. However, the complex method can avoid this problem. Therefore, the optimized complex method is applied to the study of the distribution law of the reliability index of Shapai high RCC arch dam. And the possibility of dam cracking and failure are further analysed based on the calculate results. The research results provide the basis for the safety of Shapai arch dam and also provide reference for other similar arch dams.

1. Introduction

The RCC dam adopts low slump concrete and roller technology for full section construction, which has the advantages of short construction period and low cost. But due to the temperature control requirements, the induced joint or peripheral joint is often set in the RCC arch dam body, which makes the dam structure relatively complex and the structure safety problem of the arch dam more prominent. Especially when studying the possibility of dam cracking and the reliability of the dam structure, the traditional methods such as arch-beam load-distributed method often can't solve the problem.

Engineering reliability refers to the ability of engineering structure to meet the expected functions of safety, applicability and durability under the specified time and conditions. Influenced by material performance, production quality, the change of temperature, humidity and load, the factors that affect the reliability of the structure are uncertain and random. Generally, the reliability index of the structure can only be measured by probability. Since the 1940s, American A M Floydenthal introduced the concept of statistical mathematics into the study of reliability theory, many professionals have analysed and expanded reliability. Such as Chen Gang [1] et al. proposed a reliability analysis method based on artificial neural network. Wang Dong [2] et al. proposed structural reliability analysis by Monte-Carlo based on conditional expectation variance reduction and antithetic variable sampling. Huang Liandi [3] et al. proposed a reliability method for arch dam based on BP network and particle swarm optimization algorithm. Chen Hongjie [4] et al. used the gradient optimization method to calculate the reliability index of each element of the dam and obtained the evolution rule of the reliability index of Dayang arch dam. Jang Y Z [5] et al. introduced the improved response surface method (RSM) to obtain the failure probability of a single uncertainty, and introduced the PNET method to modify the overall reliability of multiple uncertainties. Wang Xiaolong [6] et al. proposed an analysis method of non-probabilistic reliability of anti-sliding stability of arch dam considering fluid-
structure interaction. These methods respectively explore the reliability of arch dam from different perspectives and put forward relevant viewpoints. But for the arch dam, the three dimensional shell structure function and its derivative forms are very complex. So it is necessary to introduce a new method to study the reliability of arch dam. At present, due to the advantage of complex method, which does not require the derivative operation of functional functions, many scholars have applied it in various practical fields. For example, Chen Han [7] et al. applied the complex method to the trajectory planning of time optimal manipulator. Li Zhenyu [8] applied the complex method to the optimal design of the spillway section. Zhang Shibo [9] et al. applied the complex method to the reconstruction of traffic engineering accidents. Li Shouyi [10] et al. applied the complex method to the optimization design of arch dam shape, which achieved good results. This paper introduces the optimized complex method into the reliability analysis of Shapai RCC arch dam. The distribution law of the reliability index is studied. The possibility of dam cracking and failure and its causes according to the results is further analysed. The research results provide the basis for the safety of Shapai arch dam and also provide reference for other similar arch dams.

2. Basic idea of complex method

The complex shape is a polyhedron composed of \( n+1 \leq k \leq 2n \) vertices in the feasible region of an \( n \)-dimensional design space. The basic idea of the complex method is derived from the simplex method of the unconstrained optimization algorithm. Its iterative process is: select \( k \) vertices as the vertices of the initial complex shape within the feasible region of the design variables, and then compare size of the objective function values, where the point with the largest objective function value is a bad point, and the center (centroid) of the other points outside the bad point is used as the mapping center to find the mapping point of the bad point. If the mapping point is better than the bad point, replace it and a new complex shape is formed, and iteratively repeated so that the complex shape continues to approach the best advantage until the accuracy is satisfied.

Specific iterative steps of the complex method:

1. Generate the first vertex of the initial complex shape:
   - Deterministic method: artificially select a vertex in the feasible region.
   - Random method: use random number \( r_i^{(1)} \) to generate randomly:
   \[
   x_i^{(1)} = x_i^0 + r_i^{(1)} (x_i^u - x_i^0), \quad i = 1, 2, \ldots, n
   \]  
   Check its feasibility. If it is not satisfied, we should regenerate random numbers and select points again. If there are more than 10 random variables, the limit state equation is highly non-linear, and it is difficult to find the first vertex by using the above two methods, so the improved method in this paper is used.

2. Optimization method: Substitute the randomly generated vertices into the limit state equation \( g \) (it is generally not equal to zero), and first take \( |g| \) as the objective function and use the complex method for minimal optimization to meet the equation, and the first feasible vertex can be found successfully.

3. Randomly generate the remaining \( 2n-1 \) vertices of the initial complex shape:
   \[
   x_i^{(j)} = x_i^0 + r_i^{(j)} (x_i^u - x_i^0), \quad i = 1, 2, \ldots, n; \quad j = 2, 3, \ldots, n
   \]  
   Check its feasibility. Assuming that there are \( s \) points that satisfy the constraint, first find the center of the point set formed by the \( s \) points:
   \[
   \hat{x}^{(s)} = \frac{1}{s} \sum_{j=1}^{s} x^{(j)}
   \]  
   If the \( s+1 \) vertex is an infeasible point then the line between \( x^{(s+1)} \) and \( \hat{x}^{(s)} \) is reduced by half to center \( \hat{x}^{(s)} \). And check the feasibility of the new point \( x^{(s+1)} \) again. If the point is still not a feasible point, then the distance is reduced by half along the original line to \( \hat{x}^{(s)} \). Repeat this way, if it doesn't work yet, then replace \( x^{(s+1)} \) with another point to try; or you can use the optimization method in step 1 to find a new feasible point. And the rest points can be found in the same manner until all the \( 2n \) vertices of the initial complex shape can become feasible points.
(4) Form a complex shape and find the worst and best points. Form a complex shape of one-dimensional vertices, calculate the function value $\beta(x^{(j)})$ of each vertex, $j = 1, 2, ..., 2n$. Before the calculation, the correlative non-normally distributed random variables need to be changed into uncorrelated normally distributed random variables. Then compare the function values of each vertex to find the worst point $x^{(h)}$ and the best point $x^{(l)}$, the formula is as follows:

$$\beta(x^{(h)}) = \max_{1 \leq j \leq 2n} \beta(x^{(j)}), 1 \leq j \leq 2n; \beta(x^{(l)}) = \max_{1 \leq j \leq 2n} \beta(x^{(j)}), 1 \leq j \leq 2n$$

(4) turn to step (6).

(5) Find the mapping point $x^{(a)}$ and calculate the center points $\bar{x}^{(a)}$ of the remaining vertices without the worst point $x^{(h)}$:

$$\bar{x}^{(a)} = \frac{1}{2n-1} \sum_{j=1}^{2n} x^{(j)}$$

(5) Checking the feasibility:

i. If $\bar{x}^{(a)}$ is an infeasible point (the feasible region is a nonconvex feasible region at this time), then in the hypercube with the starting point $x^{(l)}$ and the end point $\bar{x}^{(a)}$, reuse the random numbers to generate the vertices of the new complex shape, that is $x_i^l = x_i^{(l)}, x_i^u = x_i^o, i = 1, 2, ..., n$, and return to step (1).

ii. If $\bar{x}^{(a)}$ is a feasible point, then select a mapping coefficient $\alpha (\alpha \geq 1, and \alpha = 1.3$ in this method), and the worst point $x^{(h)}$ is mapped by $\alpha$ times through point $\bar{x}^{(a)}$, which is the mapping point $x^{(a)}$:

$$x^{(a)} = \bar{x}^{(a)} + \alpha (\bar{x}^{(a)} - x^{(h)})$$

(6) And check its feasibility. If $x^{(a)}$ is the infeasible point, then reduce $\alpha$ by half until $x^{(a)}$ becomes the feasible point.

(6) Compare the objective function value of the worst point in the mapping point region:

1. If $\beta(x^{(a)}) < \beta(x^{(h)})$, replace $x^{(h)}$ with $x^{(a)}$. Form a new complex shape and return to step (3).

2. If $\beta(x^{(a)}) > \beta(x^{(h)})$, reduce $\alpha$ by half, then calculate $x^{(a)}$ and $\beta(x^{(a)})$, and compare the new $\beta(x^{(a)})$ and $\beta(x^{(h)})$ again.

Repeat the process until $\alpha$ is less than a predetermined value $\zeta$ ($\zeta = 10^{-5}$ in this method). If the objective function is still no improvement, change the direction of mapping, find out the next bad points $x^{(sh)}$ of all vertices in the complex shape, the formula is as follows:

$$\beta(x^{(sh)}) = \max_{1 \leq j \leq 2n \ j \neq h} \beta(x^{(j)})$$

(7) And return to step (4) with the next bad point instead of the worst point.

(7) Stop searching. If the function values of the vertices of the complex shape satisfy the following convergence criteria:

$$\left[ \frac{1}{2n} \sum_{j=1}^{2n} \left( \beta(x^{(l)}) - \beta(x^{(j)}) \right) \right] ^{\frac{1}{2}} < \varepsilon$$

(8) then stop searching, where $\varepsilon$ is a predetermined small positive number, this method is $\varepsilon = 10^{-7}$. At this time, the optimal solution or the design check point is $x^* = x^{(l)}$, also the corresponding reliable index is $\beta(x^*) = \beta(x^{(l)})$, otherwise return to step (4).

(8) The judgment optimal solution. When stopping the search after meeting the convergence criterion, the optimal solution can be obtained. Then the new data file with the optimal solution as the first vertex can also be generated automatically, also we can be returned to the first step according whether is required, and the new complex shape can be reconstituted randomly within the whole feasible region for the optimization of the new round. Repeat this way until the objective function is no longer improved, and is judged to be the optimal solution, then the calculation is terminated. This helps to find the global optimal solution.
3. General engineering situation of Shapai arch dam

Shapai Hydropower Station is located on the Caopo River in Wenchuan County, Aba Tibetan and Qiang Autonomous Prefecture, Sichuan Province. The hub consists of RCC arch dams, flood discharge tunnels, diversion power generation tunnels, and hydropower plant. The normal water storage level of the reservoir is 1866.0m, the total storage is 18 million m$^3$, and the total installed capacity of the power station is 36,000kW. The shape of the RCC arch dam is a three-center circular single-curve arch dam, with a maximum dam height of 132 m, a dam crest thickness of 9.5 m, and a dam bottom thickness of 28.0 m. The overhang degree of the upstream dam surface is 0.12, the downstream dam surface is a polyline, the arc height ratio is 2.13, the thickness height ratio is 0.238, the maximum center angle is 92.48°, and the total concrete volume of the dam is about 393,000 m$^3$.

As shown in figure 1, Shapai arch dam is provided with two induced joints and two transverse joints to limit the development of cracks and improve the stress distribution of the dam body. In order to understand the influence of induced joints on cracking of RCC arch dam, the reliability analysis of Shapai concrete arch dam is conducted.

![Figure 1. Schematic diagram of induced joint scheme of Shapai arch dam](image)

4. Reliability analysis of dam structure

4.1. Random design variable

According to the actual engineering parameters and the selection criteria of related random variables, in this paper the statistical characteristics of each random variable cited are shown in table 1.

| Variable                      | Distribution type   | Mean value | Standard deviation |
|-------------------------------|---------------------|------------|--------------------|
| Upstream Water Load $H$/m    | Normal distribution | 1.0        | 0.16               |
| Tensile Strength $f_{ct}$/MPa | Normal distribution | 2.0        | 0.3                |
| Compressive Strength $f_c$/MPa | Normal distribution | 20.0       | 3.0                |
| Fracture Toughness $K_{IC}$/kN/cm$^{3/2}$ | WEIBULL distribution | 0.96 | 0.1                |

4.2. The establishment of functions

According to the suggestion of professor Chen Zuping [10], the generalized double-shear strength criterion is selected as the point failure criterion for arch dam, and the point failure functions of Shapai arch dam is established as follows:

Generalized compression, namely $\sigma_2 \leq \frac{\sigma_1 + \sigma_3}{1 + \alpha}$,

$$z = f_{ct} - \sigma_1 + \frac{\alpha}{2} (\sigma_2 + \sigma_3)$$  \hspace{1cm} (9)
Generalized stretch, namely \( \sigma_2 \geq \frac{\sigma_1 + \alpha \sigma_3}{1 + \alpha} \),
\[ z = f_{ct} - \frac{1}{2}(\sigma_2 + \sigma_3) + \alpha \sigma_3 \] (10)

Here, \( \alpha = \frac{f_{ct}}{f_{cc}} \)
\( f_{ct}, f_{cc} \)—— equivalent tensile strength and equivalent compressive strength.
\( \sigma_1, \sigma_2, \sigma_3 \)—— the first, second, and third principal stresses, and the tensile stress is positive.
These are all functions of upstream water load.

In this paper, a three-dimensional finite element method was used to calculate 288 nodes on the upstream and downstream dam surfaces, and the stress \( \sigma_1, \sigma_2, \sigma_3 \) were obtained under the action of 14 different water levels in the upstream. Then, these 14 groups of stress and water levels were taken as samples and trained by BP network, and the relationship between the stress component and the upstream water level was obtained, and the relationship is \( \sigma_i = f^i(H), i=1,2,\ldots,288 \).

Due to the particularity of Shapai RCC arch dam (having induced joints), the strength parameters of each part are different, so the dam body and induced joints have different functions, so the reliability analysis of dam body and induced joints should be carried out respectively:

(1) Dam body
\[ f_{ct} = f_t, \]
\[ f_{cc} = f_c \] (11) (12)

Here, \( f_c, f_t \) are the compressive and tensile strength of concrete body.

(2) Induced joints
\[ f_{ct} = \frac{\varphi K_{IC}}{\lambda \pi (a+ry)} \] (13)
\[ \lambda = \left[ \frac{2b}{\pi (a+ry)} \tan \left( \frac{\pi (a+ry)}{2b} \right) \right]^{\frac{1}{2}} \] (14)
\[ \varphi = \int_0^{\frac{\pi}{2}} \left( \sin^2 \alpha + \frac{a^2}{c^2} \cos^2 \alpha \right)^{\frac{1}{2}} d\alpha \] (15)
\[ \gamma_y = \frac{1}{\pi} \left( \frac{K_{IC}}{f_t} \right)^2 \] (16)

Here, \( K_{IC} \)—— fracture toughness of concrete
\( f_t \)—— concrete tensile strength
\( \lambda_y \)—— related parameters of concrete softening zone
\( a, b, c \)—— dimensional parameters related to induced joints arrangement

4.3. Calculation and analysis of reliability index
In this paper, the reliability index of 288 nodes on upstream and downstream Shapai RCC arch dam surface is analysed and calculated by the above complex method. The reliability index of each node is calculated, and the maximum and minimum values are listed in table 2. The distribution of the reliability index of upstream and downstream dam surface is shown in figure 2 and figure 3.

Figure 2. Reliability index contour line of upstream dam surface.
Figure 3. Reliability index contour line of downstream dam surface.
Table 2. Maximum and minimum values of dam surface and occurrence positions of Shapai arch dam.

|                | Upstream | Downstream |
|----------------|----------|------------|
|                | Dam body | Dam body   |
|                | Induced joint | Induced joint |
|                | transverse joint | Transverse joint |
| β max          | 9.98     | 9.13       |
| Position       | 1780m    | 1840m      |
|                | Right arch crown | Middle of left half arch |
| β min          | 3.12     | 2.66       |
| Position       | 1790m    | 1770m      |
|                | Right arch abutment | Arch crown |

Combined with the above figure, the reliability calculation results are analysed as follows:

(1) Downstream dam surface. The reliability of downstream dam is lower than upstream dam. The maximum point of reliability is at the elevation of 1840 in the middle of the left half arch, $\beta=9.13$. The minimum point is at 1750 elevation in the $2\#$ induced joint, $\beta=0.53$. The reason why the reliability index here is so small is that this place is at the turning point of connection between arch abutments. Also the induced joint causes the stress concentration at this place. The reliability indexes of the induced joints on the downstream dam surface are generally low. The reliability indexes of the joints of the $2\#$ induced joints from 1750m to 1780m are less than 0.65, and the failure probability is greater than 25.87%. This shows that the possibility of $2\#$ induced joint cracking is quite large. The reliability index of the $3\#$ induced joints from 1750m to 1830m elevation is less than 2.0, and the failure probability is greater than 22.8%, and the maximum is up to 24%. It is indicated that the $3\#$ induced joint is more likely to form a penetrating crack from the 1750m to 1830m, and the possibility of cracking is greater than that of other parts, and the possibility of $2\#$ induced joint cracking is close behind. This is mainly because the induced joint in the downstream of the dam is in the tensile stress zone, and the strength of the induced joint is significantly weakened compared with the strength of the concrete body, so the possibility of damage to the induced joint is great. In addition, the reliability index around the transverse joint of the downstream dam surface has also increased, which is largely due to the fact that the transverse joint releases a large amount of stress during the construction period, so the stress level has decreased a lot.

(2) Upstream dam surface. The reliability index of the arch crown part is large, indicating that the reliability is relatively high, and the maximum value of its reliability index is at 1790 elevation, reaching 10.2; while the reliability index of the arch abutment is small, indicating that the reliability is relatively low. This is mainly because the upstream arch abutment is controlled by tension, and the arch crown portion is controlled by pressure, also the compressive strength of concrete is far greater than the tensile strength. The distribution of reliability indexes of the left and right half arch is roughly symmetrical. The smallest $\beta$ of the left half arch appears at the 1790 elevation arch abutment, and its value is 3.12, which is mainly due to the rapid change of the shape of the arch abutment, which causes a certain degree of stress concentration under the torsion of the arch beam. Due to the small principal stress at the 1790 elevation induced joint, there is a large reliability index $\beta$. The smallest $\beta$ of the right half arch appears at the right arch abutment of 1750 elevation, meanwhile there is the junction of the induced joint. And its reliability is poor because of the abrupt change of shape at the connection of arch abutment, also the induced joint causes strong stress concentration.
5. Conclusions
The research on crack development and reliability index is very important for the safety of high RCC arch dam. In this paper, the optimized complex method is applied to study the distribution law of reliability index of Shapai high RCC arch dam. And the possibility of the dam cracking and failure and its causes are further analysed according to the results. The following results are obtained through the research:

(1) The reliability of the downstream dam is lower than upstream, and the reliability of the left arch dam is higher than right. Meanwhile, because the downstream induced joint is in the tensile stress zone, and the strength of the concrete is weakened by the split joint, the reliability index of the induced joint is generally reduced. Consequently, the 3# induced joint is most likely to crack, and followed by 2# induced joint.

(2) The reliability of the arch crown is higher than that of the arch abutment, which is consistent with the distribution of tensile stress and compressive stress. At 1750 elevation of the right arch, the stress concentration is caused by induced joint intersection, and the reliability index is relatively small.

(3) By using the optimized complex method, the first point that is more suitable to the needs can be found effectively, which greatly improves the operation efficiency. Meanwhile, the application of the complex method in the reliability analysis of the arch dam has effectively analysed the reliability index of upstream and downstream dam and the failure probability of each point of dam surface. The research results can be used for reference in practical projects.

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