Natural Electroweak Symmetry Breaking from Scale Invariant Higgs Mechanism

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\textbf{Abstract}

We construct a minimal viable extension of the standard model (SM) with classical scale symmetry. Its scalar sector contains a complex singlet in addition to the SM Higgs doublet. The scale-invariant and CP-symmetric Higgs potential generates radiative electroweak symmetry breaking à la Coleman-Weinberg, and gives a natural solution to the hierarchy problem, free from fine-tuning. Besides the 125 GeV SM-like Higgs particle, it predicts a new CP-even Higgs (serving as the pseudo-Nambu-Goldstone boson of scale symmetry breaking) and a CP-odd scalar singlet (providing the dark matter candidate) at weak scale. We systematically analyze experimental constraints from direct LHC Higgs searches and electroweak precision tests, as well as theoretical bounds from unitarity, triviality and vacuum stability. We demonstrate the viable parameter space, and discuss implications for new Higgs and dark matter (DM) searches at the upcoming LHC runs and for the on-going direct detections of DM.

\textbf{Keywords:} Scale Symmetry, Radiative Higgs Mechanism, Naturalness, LHC, Dark Matter

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\section{1. Introduction}

The LHC discovery of a 125 GeV Higgs-like particle\textsuperscript{1}[1,2] seems to provide the last missing piece of the standard model (SM) of particle physics, but the SM apparently fails to accommodate dark matter (DM) and neutrino masses. Higgs mechanism\textsuperscript{3} is the cornerstone of the SM, which hypothesizes a single spin-0 Higgs doublet to realize the spontaneous electroweak symmetry breaking (EWSB) and gives rise to a physical remnant — the Higgs boson. This generates\textsuperscript{4} the observed masses for spin-1 weak bosons and all three families of spin-$\frac{1}{2}$ SM fermions via gauge and Yukawa interactions of the Higgs boson. However, the Higgs boson could not fix its own mass and an \textit{ad hoc} negative mass term is input by hand at the weak scale. As such, it is customary to think that the Higgs mass will be unstabilized against the Planck scale by quantum corrections unless large fine-tuned cancellation of the associated quadratical divergences is imposed\textsuperscript{5}. Historically, seeking resolutions to this naturalness problem has been the major driving force behind numerous “beyond SM” extensions on the market, ranging from supersymmetry and compositeness to large or small extra dimensions, despite none of them has been seen so far at the LHC.

The naturalness theorem\textsuperscript{6} asserts that the absence of large corrections can only be maintained through certain symmetry which protects the Higgs mass term. This means that the symmetry must increase when the Higgs mass approaches zero. It is important to note that the Higgs mass is the unique dimensionful parameter in the SM Lagrangian, and only causes soft breaking of the scale symmetry\textsuperscript{7}. Such a scale symmetry will also be explicitly broken by the trace anomaly with dimension-4 operators at quantum level. But this only leads to logarithmic running of coupling constants and cannot generate quadratical divergence in the dimension-2 Higgs mass term\textsuperscript{8}. Hence, the SM itself could be technically natural up to high scales\textsuperscript{9} and free from fine-tuning in the Higgs mass renormalization due to the softly broken classical scale invariance\textsuperscript{10}.\textsuperscript{9}

It is even more tempting to restore the full scale symmetry for the SM Lagrangian by setting a vanishing Higgs mass. This justifies the use of a scale-invariant regularization method for loop corrections, which automatically

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\textsuperscript{1}After the SM is extended with singlet right-handed neutrinos, their dimension-3 heavy Majorana mass-term provides another soft breaking of scale invariance. Our present construction will naturally generate this Majorana mass term via spontaneous symmetry breaking.
\textsuperscript{2}The SM Higgs sector with a 125 GeV Higgs boson is free from triviality bound, but suffers a vacuum stability bound at the scale $\mu = 10^{12}$ GeV\textsuperscript{8}. We will analyze both triviality and vacuum stability bounds for the present model.
ensures the absence of quadratical divergence in the Higgs mass renormalization. (The simplest regulator respecting classical scale symmetry is the dimensional regularization \([\mathcal{I}]\).) Thus, such a scale-invariant SM Lagrangian or its scale-symmetric extensions will stabilize the weak scale up to a high ultraviolet (UV) cutoff \(\Lambda_{\text{UV}}\) provided \(\mathcal{I}\): (1) no intermediate scales\(^3\) would mix with the weak scale; (2) no Landau poles (or instabilities) appear in the running couplings (or Higgs potential) over the energies up to \(\Lambda_{\text{UV}}\).

With such a fully scale-invariant SM Lagrangian, one can radiatively generate nonzero Higgs mass and spontaneous EWSB via Coleman-Weinberg mechanism \([10]\). In consequence, the weak scale is nicely induced at quantum level via dimensional transmutation. This further reduces one more free-parameter from the conventional SM. But, unfortunately such a minimal version has its Higgs potential unbounded from below at one-loop given the experimentally observed masses of top quark and weak gauge bosons. In addition, the radiatively induced Higgs mass is too low to even survive the LEP-II Higgs search bound \(M_h > 114.4\text{ GeV} (95\% \text{ C.L.})\) \([13]\). Hence, the SM Higgs sector has to be properly extended and some interesting attempts appeared in recent years \([14, 15]\).

In this work, we construct the minimal viable extension of the SM with classical scale symmetry. Its Higgs sector contains a Higgs doublet and a complex gauge-singlet scalar. The Higgs potential is scale-invariant, as well as CP-conserving. The model predicts two CP-even Higgs boson and one CP-odd scalar at weak scale. Among the two CP-even states, one provides the observed 125 GeV Higgs boson and another serves as a pseudo-Nambu-Goldstone boson from scale symmetry breaking. The CP-odd scalar is a potential dark matter candidate. We will demonstrate that including the complex singlet scalar not only helps to lift the radiative mass of the Higgs boson to coincide with the current LHC Higgs data \([11, 12]\), but also nicely generate the Majorana mass term for right-handed neutrinos from scale-invariant Yukawa interaction. We systematically analyze experimental and theoretical constraints on the parameter space of our model. These include experimental bounds from the direct LHC Higgs measurements and the indirect electroweak precision tests, as well as the theoretical constraints from unitarity, triviality and vacuum stability. Finally, we note that our approach also differs from the previous studies \([14, 15]\) (à la Coleman-Weinberg) invoking extra scalars or certain hidden gauge groups. Those extended gauge groups include the \(U(1)_X\) (sometimes \(U(1)_B-L\)) or the left-right gauge group, or the vector dark \(SU(2)_L\), or certain strongly interacting hidden sector. An extensive analysis of a complex singlet scalar with the global \(U(1)\) (or \(Z_2\)) symmetry and maximal CP-violation was given in \([15]\), which differs from our CP-symmetric and scale-invariant Higgs sector (without extra global or local symmetry). Our model also differs from \([16]\) which considered two real scalar singlets with an extra \(Z_2\) to ensure stability of the \(Z_2\)-odd singlet as DM. In contrast, our model builds the imaginary component of the complex singlet as DM and its stability is automatically protected by CP invariance; we further include right-handed neutrinos for light neutrino mass-generations via TeV scale seesaw.

This paper is organized as follows. Sec.2 sets up the model construction for our classically scale-invariant Higgs potential. Then, we present the full one-loop corrections, identify the physical states, and derive their mass spectrum and couplings. In Sec.3 we study both experimental and theoretical constraints on the parameter space of the model. Sec.3.5 presents our results and discusses the physical implications. Finally, we conclude in Sec.4.

2. Model Structure and Radiative Electroweak Symmetry Breaking

In this section, we construct the minimal viable extension of the SM with classical scale-invariance. It only contains an extra gauge-singlet complex scalar \(S\) in addition to the conventional Higgs doublet \(H\). Our extended Higgs sector is CP invariant (similar to the SM) and respects the classical scale symmetry. This will naturally induce radiative EWSB and predict two new scalar states in addition to the observed 125 GeV light Higgs boson. This minimal construction maximally preserves all the original SM symmetries, and further incorporates three right-handed neutrinos for mass-generation of light neutrinos via TeV scale seesaw.

2.1. The Model Structure

In our construction, the extended Higgs sector consists of the SM Higgs doublet \(H\) and a complex singlet scalar \(S\), so its Lagrangian is,

\[ \mathcal{L}_S = (D^\mu H)^\dagger D_\mu H + \partial^\mu S^\dagger \partial_\mu S - V^{(0)}(H, S), \]  

\(^3\)Our present model will extend the scale-invariant SM Lagrangian with a complex Higgs singlet and three right handed neutrinos at TeV scale. Hence, it is a technically natural effective field theory (EFT), all the way up to its UV cutoff (above which a more complete theory arises and is assumed to properly retain classical scale symmetry). We do not concern detailed Planck-scale dynamics \([12]\), given the lack of a full theory of quantum gravity. This EFT is also free from little hierarchy problem because it invokes no extra heavy state at intermediate scales. We thank Nima Arkani-Hamed for discussing this point.

2
where the Higgs doublet $H$ is expressed in component form, 

$$H = \left( \frac{1}{\sqrt{2}}(v_\phi + \eta^0 + i\pi^0) \right),$$  
(2.2)

and $D^\mu$ is the covariant derivative under SM gauge group. In (2.2), $\phi$ is the SM-like Higgs field, with the vacuum expectation value (VEV), $v_\phi \approx 246$ GeV, to be determined from radiative EWSB. The gauge-singlet scalar field $S$ has the following component form, 

$$S = \frac{1}{\sqrt{2}}(v_\eta + \eta^0 + i\chi^0),$$  
(2.3)

where $\eta$ has $J^P = 0^+$. Thus, under either C or CP operation it transforms as, $S \rightarrow S^*$. This means that $\eta$ and $\chi$ belong to the CP-even and CP-odd fields, respectively.

Then, we can write down the most general scale-invariant and CP-symmetric Higgs potential with the Higgs doublet $H$ and complex singlet $S$, 

$$V^{(0)}(H,S) = \frac{\lambda_1}{6} (H^\dagger H)^2 + \frac{\lambda_2}{6} |S|^4 + \lambda_3 (H^\dagger H)|S|^2 + \frac{\lambda_4}{2} (H^* H)(S^2 + S^*2) + \frac{\lambda_5}{12} (S^2 + S^*2)^2 + \frac{\lambda_6}{12} (S^4 + S^*4),$$  
(2.4)

which contains six dimensionless real coupling constants $\{\lambda_i\}$. Here the cubic couplings and mass terms are forbidden by the scale-invariance. In the above potential, the general mixing between Higgs doublet and singlet is represented by the third and fourth terms via the quartic couplings $\lambda_3$ and $\lambda_4$. For practical analysis, we find it convenient to introduce the following coupling combinations:

$$\lambda_\phi \equiv \lambda_1, \quad \lambda_\eta \equiv \lambda_2 + \lambda_5 + \lambda_6, \quad \lambda_\xi \equiv \lambda_3 - \lambda_5 + \lambda_6, \quad \lambda_{\eta\xi} \equiv \frac{1}{2} \lambda_2 - \lambda_6, \quad \lambda_{\phi\xi} \equiv \lambda_3 + \lambda_4, \quad \lambda_{\phi\eta} \equiv \lambda_1 - \lambda_4.$$  
(2.5)

Thus, we infer the quartic scalar interactions in terms of component fields,

$$V^{(0)} = \frac{1}{24} \left[ \lambda_\phi \phi^4 + \lambda_\eta \eta^4 + \lambda_\xi \chi^4 + \lambda_{\phi\xi} \left( \phi^0 \pi^0 + 2\pi^+ \pi^- \right)^2 + \frac{1}{4} \left( \lambda_{\phi\xi}^2 \phi^2 \eta^2 + \lambda_{\phi\eta}^2 \phi^2 \chi^2 + \lambda_{\eta\xi}^2 \eta^2 \chi^2 \right) \right] + \frac{1}{12} \left( \lambda_{\phi\xi} \phi^2 + 3\lambda_{\phi\eta} \eta^2 + 3\lambda_{\eta\xi} \chi^2 \right) \left( \phi^0 \pi^0 + 2\pi^+ \pi^- \right)^2,$$

(2.6)

In terms of these variables, the tree-level potential (2.4) or (2.6) is bounded from below under the conditions,

$$\lambda_\phi > 0, \quad \lambda_\eta > 0, \quad \lambda_\xi > 0, \quad (\lambda_{\phi\eta}^2) < \frac{1}{3} \lambda_\phi \lambda_\eta, \quad (\lambda_{\phi\xi}^2) < \frac{1}{3} \lambda_\phi \lambda_\xi,$$

(2.7a)

and

$$\lambda_{\eta\xi} > -\frac{1}{3} \sqrt{\lambda_\phi \lambda_\eta \lambda_\xi}, \quad \lambda_{\phi\eta} \lambda_{\eta\xi} - 3\lambda_{\phi\xi} \lambda_{\eta\xi} > -\frac{1}{3} \sqrt{[\lambda_{\phi\eta} \lambda_{\eta\xi} - 9 (\lambda_{\phi\xi})^2] [\lambda_{\phi\eta} \lambda_{\xi} - 9 (\lambda_{\phi\xi})^2]},$$

(2.7b)

Finally, we include three right-handed Majorana neutrinos, which will account for the observed light neutrino masses via seesaw mechanism [17]. In our construction, we conjecture that the pure singlet sector (including singlet scalar $S$ and singlet neutrino $N_j$) always conserves CP. This requires the Yukawa interactions between $S$ and $N_j$ to be CP symmetric. Thus, we can write down the gauge- and scale-invariant Yukawa interactions for neutrino sector, 

$$\mathcal{L}_\nu = -\left( Y_{ij}^\nu \overline{L}_i \tilde{H} N_j + h.c. \right) - \frac{1}{2} Y_{ij}^N (S + S^*) \overline{N}_i N_j,$$

(2.8)

where $\tilde{H} = i \sigma_2 H^*$, and $N_j = N_j^\nu$ is a 4-component Majorana spinor denoting the singlet (right-handed) neutrinos. Our construction builds the singlet neutrino $N_1$ as a Majorana spinor starting from the symmetric phase, and $N_j$ will acquire Majorana mass after spontaneous scale symmetry breaking. In the above, $[Y_{ij}^\nu]$ denotes Yukawa couplings of the Higgs doublet $H$ with left-handed lepton doublet $L_i$, and singlet neutrino $N_j$, while $Y^N$ represents the Yukawa couplings between the singlet Higgs $S$ and singlet Majorana neutrinos $N_j$. It is straightforward to verify CP invariance of the above $S$-$N_j$ Yukawa interactions since $(S + S^*)$ and $\overline{N}_i N_j$ respect CP symmetry, respectively. Besides, since the operator $(S + S^*) \overline{N}_i N_j$ equals its own Hermitian conjugate, the Yukawa couplings $Y_{ij}^N$ are real. In the practical analysis, we will always choose $Y^N$ in the diagonal basis, and for simplicity we set $Y^N$ to be degenerate, $Y^N = y_N I_{3x3}$. We note that because our gauge-singlet sector conserves CP, the CP-odd scalar $\chi \propto \text{Im}(S)$ has vanishing Yukawa coupling with the singlet neutrinos $N_j$ in Eq. (2.8). This is a key feature of our model which

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*The coupling normalizations in (2.4) and (2.5) have been chosen such that the associated Feynman rules of the scalar quartic interactions take the simple form of $\pm \lambda_i$. The Lanczos Library provides a comprehensive list of theorems and proofs about the Lanczos algorithm. In (2.8), the linear dependence between the parameters $\lambda_i$ and the Yukawa couplings $Y_{ij}^\nu$ is essential, and the tensor product of the parameters $\lambda_{\phi\eta}$ and the Yukawa matrix $Y_{ij}^\nu$ is a common application in quantum field theory. The matrix $\overline{N}_i N_j$ represents the Majorana mass term for the singlet neutrinos. In (2.8), the parameter $y_N$ is related to the neutrino mass eigenvalue and the CP-violating phases. The matrix $I_{3x3}$ is the identity matrix of size $3 \times 3$. The Lanczos algorithm is a powerful tool for approximating the eigenvalues and eigenvectors of large sparse Hermitian matrices, which is fundamental in numerical linear algebra and quantum physics.*
ensures that the pseudo-scalar $\chi$ always appears in pair via CP-invariant Higgs potential (2.6) and is thus stable. Hence, the $\chi$ boson provides a natural dark matter candidate.

We note that the Yukawa interactions (2.8) will generate seesaw masses for light neutrinos,

$$m_\nu = m_D M_N^{-1} m_D^T,$$

where $m_D = Y^\nu v_\nu / \sqrt{2}$ and $M_N = \sqrt{2} Y^N v_\eta$. For our construction, we will set the singlet scalar VEV $v_\nu = O(\text{TeV})$. Inputting the scale of light neutrino masses from oscillation data $m_\nu = O(0.01 \text{eV})$ and taking $Y^N = O(1)$, we find, $m_D = O(m_\nu)$ with $m_\nu$ the electron mass. Thus, $Y^\nu = O(m_\nu/v_\nu)$ is around the size of the electron Yukawa coupling of the SM. Hence, in the following effective potential analysis we can safely ignore the tiny Dirac Yukawa coupling $Y^\nu$, and only retain the Majorana Yukawa term in (2.8).

2.2. Mass Eigenvalues at Tree-Level

For the present study, we will determine the physical vacuum and Higgs mass-eigenvalues from minimizing the full scalar potential up to one-loop,

$$V(H, S) = V^{(0)}(H, S) + V^{(1)}(H, S),$$

(2.10)

where $V^{(1)}(H, S)$ is the one-loop contribution from all relevant fields running in the loop. Such a minimization with multiple scalars is technically complicated in general. Following the approach of Gildener and Weinberg [18], we first minimize the tree-level potential (2.4), and keep in mind that the potential couplings become running at quantum level and depend on the renormalization scale $\mu$. Thus, the minimization of the tree-level potential is performed at a particular scale $\mu = \Lambda$, and gives a “flat” direction among the scalar VEVs. Further including one-loop corrections will lift this flat direction and generate the true physical vacuum (corresponding to the spontaneous breaking of classical scale invariance).

Starting from the tree-level scalar potential (2.4), we analyze its minimization with respect to the Higgs fields $H$ and $S$ at the scale $\mu = \Lambda$, and derive the conditions,

$$\frac{\lambda}{\lambda^0} = -3 \lambda^0_{\phi}(\Lambda) \lambda_{\eta}(\Lambda) = -\lambda_{\phi}(\Lambda) = -\lambda_{\eta}(\Lambda).$$

(2.11)

This defines the flat direction of the potential, and further implies [cf. (2.7a)], $\lambda_{\phi}(\Lambda) > 0$, $\lambda_{\eta}(\Lambda) > 0$, and

$$\lambda^0_{\phi}(\Lambda) = -\frac{1}{4} \sqrt{\lambda_{\phi}(\Lambda) \lambda_{\eta}(\Lambda)}, \quad \text{or,} \quad \lambda_{\eta}(\Lambda) = \frac{9 \lambda^0_{\phi}(\Lambda)^2}{\lambda_{\phi}(\Lambda)}.$$  

(2.12)

Then, we can compute the tree-level mass spectrum from the scalar potential (2.4). Expanding the Higgs fields in terms of their components (2.2)-(2.3), and using the definitions (2.5), we deduce the quadratic Higgs mass-terms,

$$V^{(0)}_{\text{mass}} = \frac{1}{2} \left( \begin{array}{c} \phi \\ \eta \end{array} \right) \left( \begin{array}{cc} \lambda_{\phi} v_\phi^2 + \lambda_{m_{\phi}} v_\phi^2 + 2 \lambda_{\eta} v_\phi v_\eta \\ 2 \lambda_{m_{\phi}} v_\phi v_\eta \end{array} \right) \left( \begin{array}{c} \phi \\ \eta \end{array} \right)$$

(2.13)

$$+ \frac{1}{4} \left( \lambda_{m_{\phi}} v_\phi^2 + \lambda_{m_{\eta}} v_\eta^2 \right) \chi^2 + \frac{1}{12} \left( \lambda_{\phi} v_\phi^2 + 3 \lambda_{m_{\phi}} v_\phi^2 + 2 \lambda_{\eta} v_\phi v_\eta + 2 \pi^{-} \right).$$

The mass terms of the CP-even components ($\phi, \eta$) form a $2 \times 2$ matrix, and can be diagonalized by an orthogonal rotation,

$$\left( \begin{array}{c} \phi \\ \eta \end{array} \right) = \mathbb{O} \left( \begin{array}{c} h \\ \sigma \end{array} \right),$$

(2.14)

where ($h, \sigma$) are the CP-even mass-eigenstates. The rotation matrix $\mathbb{O}$ is defined with mixing angle $\omega$, and can be determined as follows,

$$\mathbb{O} = \left( \begin{array}{cc} \cos \omega & \sin \omega \\ -\sin \omega & \cos \omega \end{array} \right), \quad \cot 2\omega \equiv \frac{1}{4 \lambda_{m_{\phi}}} \left[ \lambda_{\eta} - \lambda_{m_{\phi}} v_\phi v_\eta - \left( \lambda_{\phi} - \lambda_{m_{\phi}} \right) v_\phi v_\eta \right].$$

(2.15)

Accordingly, we derive the tree-level mass-eigenvalues for all scalar states after mass-diagonalization,

$$M^2_h = \frac{1}{2} \left( \lambda_{\phi} v_\phi^2 + \lambda_{m_{\phi}} v_\phi^2 - 2 \lambda_{\eta} v_\phi v_\eta \tan \omega \right), \quad M^2_\chi = \frac{1}{2} \left( \lambda_{\phi} v_\phi^2 + \lambda_{m_{\phi}} v_\phi^2 + 2 \lambda_{\eta} v_\phi v_\eta \cot \omega \right),$$

$$M^2_h = \frac{1}{2} \left( \lambda_{m_{\phi}} v_\phi^2 + \lambda_{m_{\eta}} v_\eta^2 \right), \quad M^2_\phi = M^2_\eta = \frac{1}{2} \left( \lambda_{\phi} v_\phi^2 + 3 \lambda_{m_{\phi}} v_\phi^2 \right).$$

(2.16)
Besides, from Eq. (2.8) and taking $Y^N = y_N I \bar{3}$ for simplicity, we infer a tree-level mass formula for right-handed neutrinos,

$$M_N = \sqrt{2} y_N v_\eta.$$  

(2.17)

Implementing the minimum condition (2.11) at scale $\mu = \Lambda$, we obtain, $\cot \omega = v_\eta/v_\phi$ for the mixing angle, and reexpress the tree-level masses,

$$M_h^2 = \frac{v_\phi^2}{3} \left[ \lambda_\phi(\Lambda) - 3 \lambda_m^*(\Lambda) \right], \quad M_\chi^2 = \frac{v_\phi^2}{6 \lambda_m^*(\Lambda)} \left[ 3 \lambda_m^*(\Lambda) \lambda_m^*(\Lambda) - \lambda_\phi(\Lambda) \lambda_{\eta \chi}(\Lambda) \right],$$

$$M_\sigma^2 = M_\phi^2 = M_\chi^2 = 0, \quad M_N = y_N v_\phi \sqrt{\frac{2 \lambda_\phi(\Lambda)}{-3 \lambda_m^*(\Lambda)}}.$$  

(2.18)

As expected, we find three massless would-be Nambu-Goldstone bosons $\{\pi^+, \pi^0\}$ eaten by $\{W^\pm, Z^0\}$, and another massless CP-even state $\sigma$ which is the Nambu-Goldstone boson of spontaneously broken classic scale symmetry. As will be shown below, the $\sigma$ particle will acquire its radiative mass along the flat direction à la Coleman-Weinberg [10], and thus becomes a pseudon-Nambu-Goldstone boson at quantum level. Hence, we have only two massive scalar bosons at tree-level, the CP-even state $h$ and the CP-odd component $\chi$.

We note that the pseudo-scalar $\chi$ is protected by CP symmetry, namely, because of the CP conservation associated with the Higgs potential (2.4) and singlet Yukawa sector (involving $S$ and $N$), the $\chi$ boson always appears in pair and thus provides a stable dark matter candidate. In the following analysis, we will identify the CP-even Higgs boson $h$ with the 125 GeV new state discovered at the LHC [1][2]. In Sec. 3.5, we will find that the other way of identifying $\sigma$ boson with the 125 GeV state is excluded by the current data.

2.3. Radiative EWSB from One-Loop Effective Potential

So far we have been working on the tree-level Higgs potential (2.4), where the flat direction (2.11) does not pick up any true physical vacuum for the EWSB. Therefore, it is important to compute the one-loop potential $V^{(1)}$. This will become dominant along the flat direction (2.11), and thus produce realistic radiative EWSB à la Coleman-Weinberg. According to E. Gildener and S. Weinberg [18], we cast the one-loop effective potential into the general form at the renormalization scale $\mu = \Lambda$,

$$V^{(1)}(\varphi) = A \varphi^4 + B \varphi^4 \log \frac{\varphi^2}{\Lambda^2},$$

(2.19)

where $\varphi$ is the “radial” combination of the Higgs fields at $\mu = \Lambda$,

$$\varphi^2 = \phi^2(\Lambda) + \eta^2(\Lambda) = \frac{\phi^2(\Lambda)}{\sin^2 \omega}.$$  

(2.20)

Since the one-loop potential (2.19) is computed at $\mu = \Lambda$ and along the flat direction (2.11), the relation $\cot \omega = v_\eta/v_\phi = \eta(\Lambda)/\phi(\Lambda)$ holds, as inferred below (2.17). From this we can deduce the second equality of (2.20). The coefficients $A$ and $B$ are dimensionless loop-generated constants, under the $\overline{MS}$ scheme [18][15].

$$A = \frac{1}{64 \pi^2 v_\phi^4} \left\{ \text{Tr} \left[ M_\chi^2 \left( \frac{3}{2} + \log \frac{M_\chi^2}{v_\phi^2} \right) \right] + 3 \text{Tr} \left[ M_\sigma^2 \left( -\frac{5}{6} + \log \frac{M_\sigma^2}{v_\phi^2} \right) \right] - 4 \text{Tr} \left[ M_\phi^2 \left( -1 + \log \frac{M_\phi^2}{v_\phi^2} \right) \right] \right\},$$

(2.21a)

$$B = \frac{1}{64 \pi^2 v_\phi^4} \left( \text{Tr} M_\sigma^2 + 3 \text{Tr} M_\chi^2 - 4 \text{Tr} M_\phi^2 \right),$$

(2.21b)

where the traces are taken over all internal degrees of freedom, and $M_{V,S,F}$ represent involved tree-level masses of vectors, scalars and fermions evaluated at $\mu = \Lambda$. This scale may be determined from minimizing the one-loop potential (2.19), $dV^{(1)}(\varphi)/d\varphi|_{\varphi=v_\phi} = 0$, yielding

$$\Lambda = v_\phi \exp \left[ \frac{A}{2B} + \frac{1}{4} \right].$$  

(2.22)

We note that it is possible to implement an inverse identification of the CP-even states, such that $h$ becomes a tree-level massless state (and thus the pseudo-Nambu-Goldstone boson of scale symmetry breaking), whereas $\sigma$ acquires nonzero mass at tree-level. But, as we will find in Sec. 3.5, this setup is excluded by the theoretical and experimental constraints.
Moreover, the one-loop potential \( V^{(1)} \) will generate a mass term for the \( \sigma \) pseudo-Nambu-Goldstone boson along the flat direction. With (2.22), we compute this loop-induced \( \sigma \) mass,

\[
M_{\sigma}^2 = \frac{d^2V^{(1)}(\phi)}{d\phi^2} \bigg|_{\phi = \Phi} = 8v_\phi^2B.
\]  

(2.23)

For the present model, we consider the relevant tree-level masses, \( M_t, M_s, M_{W}, M_{Z}, M_{t}, \) and \( M_{\sigma}, \) which include the masses of scalars and right-handed neutrinos in (2.18), as well as the heavy SM fields of top quark and \( (W, Z) \) vector bosons. With the aid of (2.11) and (2.20), we can write down the one-loop potential (2.19) in terms of \( \phi \) and its VEV, \( v_\phi \approx 246 \text{ GeV}, \)

\[
V^{(1)}(\phi) = \Lambda' \phi^4 + B' \phi^6 \log \frac{\phi^2}{\Lambda^2},
\]  

(2.24)

with the coefficients under \( \overline{\text{MS}} \) scheme,

\[
\begin{align*}
\Lambda' &= \frac{1}{64\pi^2 v^4_\phi} \left( M_h^4 \left( -\frac{3}{2} + \log \frac{M_h^2}{v_\phi^2} \right) + M_s^4 \left( -\frac{3}{2} + \log \frac{M_s^2}{v_\phi^2} \right) + 6M_W^4 \left( -\frac{5}{6} + \log \frac{M_W^2}{v_\phi^2} \right) + 3M_Z^4 \left( -\frac{5}{6} + \log \frac{M_Z^2}{v_\phi^2} \right) - 12M_t^4 \left( -1 + \log \frac{M_t^2}{v_\phi^2} \right) - 6M_N^4 \left( -1 + \log \frac{M_N^2}{v_\phi^2} \right) \right), \\
B' &= \frac{1}{64\pi^2 v^4_\phi} \left( M_h^4 + M_s^4 + 6M_W^4 + 3M_Z^4 - 12M_t^4 - 6M_N^4 \right).
\end{align*}
\]  

(2.25a, 2.25b)

In the coefficients above, we note that top quark carries a color factor \( N_c = 3, \) while the three singlet neutrinos \( \{N_j\} \) have an extra factor of \( 1/2 \) due to their Majorana nature. We can readily verify that the coefficients (2.25) are related to the original definition (2.21) via,

\[
\begin{align*}
\Lambda &= \sin^4 \omega \left( \Lambda' + B' \log \sin^2 \omega \right), \\
B &= B' \sin^4 \omega.
\end{align*}
\]  

(2.26)

With (2.22) and (2.23), we further deduce,

\[
\begin{align*}
M_{\phi}^2 &= 8Bv_\phi^2 \sin^2 \omega, \\
\Lambda &= v_\phi \exp \left[ \frac{\Lambda'}{2B'} + \frac{1}{4} \right],
\end{align*}
\]  

(2.27a, 2.27b)

where \( \Lambda \) is the renormalization scale at which the flat direction conditions of (2.11) hold. From (2.27a), the positivity condition of squared-mass \( M_{\phi}^2 \) requires \( B' > 0, \) which takes the form

\[
M_N^4 - 6M_N^2 > 12M_t^4 - 6M_W^4 - 3M_Z^4 - M_h^4.
\]  

(2.28)

As anticipated, given the current data of mass measurements on the right-hand-side of (2.28), this condition cannot be fulfilled by the SM particle content alone. Hence, the Higgs sector of classically scale-invariant SM has to be properly extended for realizing the radiative EWSB. Finally, with (2.27b) we can simplify the one-loop effective potential (2.24) by eliminating the coefficient \( \Lambda' \),

\[
V^{(1)}(\phi) = B' \phi^6 \left[ \log \frac{\phi^2}{v_\phi^2} - \frac{1}{2} \right].
\]  

(2.29)

We see that the one-loop potential (2.29) is bounded from below for large values of \( \phi \), provided \( B' > 0 \) which is ensured by the positivity condition via (2.27a) and realized in (2.28).

Before concluding this section, let us summarize the present model. Aside from the three exactly massless would-be Goldstone bosons \( (\pi^+, \pi^0) \) eaten by \( (W^+, Z^0) \), the scalar particle spectrum consists of the CP-even state \( h \) and CP-odd state \( \chi \), with nonzero tree-level masses, and an additional CP-even scalar \( \sigma \), which is a pseudo-Nambu-Goldstone boson of scale symmetry breaking, with radiatively induced mass. Furthermore, we have three singlet Majorana neutrinos \( N_j \), with masses generated from tree-level Yukawa interactions with singlet scalar \( S \). The Higgs potential (2.24) includes six scalar-couplings, along with two VEVs \( (v_\phi, v_\eta) \). As explained, we can utilize the minimization condition (2.11) to express \( v_\phi \) in terms of \( v_\phi \approx 246 \text{ GeV}, \) and eliminate one of the coupling parameters in the potential (say, \( \lambda_g \)) according to the dimensional transmutation at scale \( \Lambda \). Furthermore, identifying the mass-eigenstate \( h \) with the LHC Higgs discovery \( M_h = 125 \text{ GeV}, \) we can eliminate one more coupling \( \lambda_g \) [as shown...
in (2.15). With these, we find that the present model contains five input parameters in total, which, without losing
generality, may be chosen as, \{\lambda_m, \lambda_m', \lambda_x, \lambda_y, \chi\}. Given the defined mixing angle in (2.15), and the tree-level
masses of \chi and singlet neutrinos \eta N in (2.18), we can express the five independent inputs in terms of the more
physically transparent parameter set\footnote{Alternatively, it is possible to choose \{\sin \omega, M_{\chi}, M_{\eta}, \lambda_x, \lambda_y\} as the inputs. But we find that the two sets of inputs are physically equivalent for describing the parameter space.}:
\[
\{ \sin \omega, M_\chi, M_\eta, \lambda_x, \lambda_y \}. \tag{2.30}
\]
The other couplings are non-independent and can be expressed as functions of them,
\[
\lambda_\phi = \frac{3M_h^2}{v^2} \cos^2 \omega, \quad \lambda_m^+ = -\frac{M_h^2}{v^2} \sin^2 \omega, \quad \lambda_y = \frac{3M_h^2}{v^2} \sin^2 \omega \tan^2 \omega,
\]
\[
\lambda_{yx} = \left(\frac{2M_h^2}{v^2} - \lambda_m\right) \tan^2 \omega, \quad y_N = \frac{M_N \tan \omega}{\sqrt{2}v_\phi}. \tag{2.31}
\]
In the following section, we will systematically analyze the theoretical and experimental constraints on the allowed
parameter space of this model.

3. Experimental and Theoretical Constraints on the Parameter Space

In this section, we study various experimental and theoretical constraints on the viable parameter space. From
experimental side, we will analyze the direct Higgs measurements at the LHC and Tevatron, and the indirect
electroweak precision tests. For theoretical constraints, we will derive limits from the perturbative unitarity, triviality and vacuum stability. Finally, we present the combined numerical results and discuss their physical implications.

3.1. Constraints from Direct Higgs Searches of the LHC

As mentioned earlier, we will identify the CP-even Higgs boson \( h \) with the 125 GeV new state discovered by the
LHC. Given the mixing between CP-even components of the Higgs doublet \( H \) and singlet \( S \) in (2.15), we note that
\( h \) couplings with other SM fields are suppressed by a factor of \( \cos \omega \), relative to the corresponding SM values. We will perform a global fit of our model with the LHC Higgs measurements, and derive the favored range of the mixing angle \( \omega \).

To preform the global fit with LHC Higgs data, we start from a model-independent effective Lagrangian formulation,
where deviations of the associated couplings from their SM values are taken as free parameters to be determined
by data. For the current analysis, our effective Lagrangian includes Higgs couplings to the vector bosons, and heavy
fermions (with top quark integrated out). Thus, we can generally write down this effective Lagrangian,
\[
\mathcal{L}_{\text{eff}} = \left(1 + \delta_{h}\right) C_{hWW}^{\text{SM}} h W^+ W^- + \left(1 + \delta_{h}\right) C_{hZZ}^{\text{SM}} h Z^+ Z^- - \left(1 + \delta_{h}\right) C_{h\bar{b}b}^{\text{SM}} h \bar{b} b - \left(1 + \delta_{h}\right) C_{h\tau\tau}^{\text{SM}} h \tau \bar{\tau},
\]
where the coefficients \( C_{hXY}^{\text{SM}} \) denote the SM Higgs couplings to the fields \( XY \), and potential deviations are parametrized by the corresponding \( \{\delta_{h}\} \) which vanish in the pure SM.

For the present model, we find that the \( h \) couplings in (3.1) deviate from their SM values by the common suppression factor \( \cos \omega \), i.e., \( \delta_\omega = \cos \omega - 1 = -\frac{1}{\omega^2} + O(\omega^4) < 0 \). With the LHC Higgs data, we can constrain the value of mixing angle \( \omega \) since it is the only model parameter entering this analysis. Also, the decay channel \( h \to \sigma \sigma \) would be kinematically accessible for \( M_{\sigma} > 2 M_{\sigma} \). But, we find that the cubic \( h-\sigma-\sigma \) coupling vanishes along the flat direction (2.11) up to one-loop due to the nature of \( \sigma \) being pseudo-Goldstone of scale symmetry breaking. Thus, the decay mode \( h \to \sigma \sigma \) is absent.

Using the latest Higgs measurements from Lepton-Photon-2013 [2][19], we perform a global fit of the mixing
parameter \( \omega \) via effective Lagrangian (3.1), by minimizing the \( \delta \chi^2 \) function,
\[
\delta \chi^2 = \sum_{ij} (\hat{\rho}_{ij} - \hat{\rho}_{ij}^{\exp}) (\sigma^2)^{-1} (\hat{\rho}_{ij} - \hat{\rho}_{ij}^{\exp}), \tag{3.2}
\]
where \( \hat{\rho}_{ij} = [\sigma \times \text{Br}]/[\sigma \times \text{Br}]_{\text{SM}} \) denotes the Higgs signal strength for each given channel, \( j = \gamma \gamma, WW^*, ZZ^*, \bar{b}b, \tau \bar{\tau} \), at ATLAS, CMS and Tevatron. The error matrix is defined as, \( (\sigma^2)^{-1} \) \( j_i = \sigma_j \rho_{ij} \sigma_j \), where \( \sigma_j \) gives the corresponding error and \( \rho_{ij} \) denotes the correlation matrix. We present our findings in Table 1.
| sin ω | 68% C.L. | 95% C.L. | Best Fit | δσ^2_/d.o.f. |
|-------|----------|----------|----------|---------------|
| CMS   | (0.14, 0.45) | (0, 0.55) | 0.33     | 0.36          |
| All Data | (0, 0.26) | (0, 0.37) | –        | 0.85          |

Table 1: Global fit of Higgs mixing parameter sin ω with the LHC and Tevatron data from Lepton-Photon-2013 [2][10].

Table[1] shows that the CMS data alone prefers a nonzero Higgs mixing at 68% C.L. and a best fit of |sin ω| = 0.33, while including all data from ATLAS/CMS and Tevatron puts a tighter limit on the allowed range of mixing angle ω, still consistent with the SM case with zero mixing. We note that the signal strengths measured by the current CMS data are lower than the SM predictions in h → γγ, WW, ZZ channels [2]. This is more consistent with the prediction from universal cos ω suppression in our present model, so our fit mildly prefers ω ≠ 0 at 68% C.L. (although still consistent with ω = 0 at 95% C.L.). Table[1] also shows that the CMS data give a better fitting quality, as expected. On the other hand, ATLAS gives somewhat enhanced signal strengths in h → γγ, ZZ channels [2]. Thus, the combined fit (including all data) allows less room for the cos ω suppression, and puts a tighter upper limit on |sin ω|. This combined fit is consistent with ω = 0 at 68% C.L., but with a poorer fitting quality (due to the mild discrepancies between the current CMS and ATLAS data mentioned above).

3.2. Constraints from Indirect Electroweak Precision Tests

The present model contains two CP-even Higgs bosons in mass-eigenstates, h and σ. The 125 GeV SM-like Higgs boson h has suppressed couplings relative to the SM values by a factor of cos ω, whereas the couplings of σ scalar are proportional to the factor sin ω [cf. Eq. (2.15)]. Thus, it is important to analyze the oblique corrections via S and T parameters [20]. (It is easy to check that the contributions to other oblique parameters are subleading as compared to (S, T), and are negligible for the current analysis.) With electroweak precision tests [21], we can thus place indirect constraints on the parameter space.

Analytical expressions of the oblique corrections from an arbitrary number of Higgs doublet and singlets were given in [22]. For our Higgs sector, we have the following results,

\[ \Delta S = \frac{\sin^2 \omega}{24\pi} \left[ \log R_{\sigma h} + \hat{G}(M_{\sigma}^2, M_{h}^2) - \hat{G}(M_{\sigma}^2, M_{\sigma}^2) \right], \]  

(3.3a)

\[ \Delta T = \frac{3\sin^2 \omega}{16\sin^2 \theta_w M_W^2} \left[ M_Z^2 \left( \log R_{Z\sigma} - \log R_{Zh} \right) - M_W^2 \left( \log R_{W\sigma} - \log R_{Wh} \right) \right], \]  

(3.3b)

where

\[ R_{ij} \equiv M_i^2/M_j^2, \]  

(3.4a)

\[ \hat{G}_{ij} \equiv -\frac{7}{4} + 9R_{ij} - 2R_{ij}^2 + \left( 12 - 4R_{ij} + R_{ij}^2 \right) \hat{F}_{ij} + \left( -10 + 18R_{ij} - 6R_{ij}^2 + R_{ij}^3 + \frac{9 + R_{ij}}{1 - R_{ij}} \right) \log R_{ij}, \]  

(3.4b)

\[ \hat{F}_{ij} \equiv \begin{cases} \frac{1}{2} \log \left( \frac{R_{ij}(R_{ij} - 4)}{R_{ij} - 2} \right) R_{ij} & (R_{ij} > 4), \\ \frac{1}{2} \sqrt{(4 - R_{ij})/R_{ij}}, & (R_{ij} \leq 4). \end{cases} \]  

(3.4c)

In the formulas (3.3), θ_w denotes the weak mixing angle, and M_σ is the loop-induced mass of σ scalar [cf. Eq. (2.27a)]. Given the experimental values of (M_σ, M_3) and the LHC data of M_h = 125 GeV, we find that the oblique parameters (3.3) are functions of (sin ω, M_h), where the radiative scalar mass M_σ still depends on (M_3, M_h), as shown in Eqs. (2.27a) and (2.25b). Thus, imposing the electroweak precision data [21] and inputting M_h, we can use oblique corrections (3.3) to derive constraints on the parameter space of (sin ω, M_h) or (M_3, M_h), as will be presented in Sec. 3.3.

3.3. Constraints from Perturbative Unitarity

The SM has two essential features — the perturbative renormalizability and unitarity. We require the same for the present extension. It is trivial to say that the full S-matrix would be unitary because computing an exact S-matrix cannot be practically done. Therefore, it is important to study the perturbative unitarity [23][24] of a given model, which will ensure us to trust the theory predictions based on tree-level and one-loop analyses.
For perturbative unitarity analysis, we are concerned with the high energy behaviors of scattering amplitudes involving longitudinal weak gauge bosons for the in/out states. In the high energy regime, such scattering amplitudes can be given by the corresponding Goldstone boson scattering amplitudes according to the equivalence theorem [2].

Thus, we can perform a systematical coupled-channel unitarity analysis for the scalar sector of our model. Our Higgs sector contains three would-be Nambu-Goldstone bosons ($\pi^\pm, \pi^0$) eaten by ($W^\pm, Z^0$), as well as three neutral physical states ($h, \sigma, \chi$).

It is convenient to impose the unitarity condition on the partial wave amplitudes of the scattering processes. The $s$-wave amplitude is inferred for a given $S$-matrix element $T$,

$$a_0 = \frac{1}{2\pi} \int_0^\pi T \, d\cos \theta,$$  \hspace{1cm} (3.5)

and its unitarity condition is given by $|\text{Re} \, a_0| < \frac{1}{2}$ for the properly normalized in/out states. For the coupled-channel analysis in the scalar sector, we consider a set of electrically neutral in/out states,

$$\left( \begin{array}{c} \pi^+\pi^- \, \chi, \, \pi^0\pi^0, \, \phi^0\phi^0, \, \phi^0\eta, \, \phi^0\eta', \, \phi^0\eta^0, \, \phi^0\eta^0, \, \phi^0\eta, \, \phi^0\eta', \, \phi^0\eta^0 \end{array} \right).$$  \hspace{1cm} (3.6)

In the unitarity analysis, we mainly concern the high energy scattering where the masses of in/out states are negligible. Thus, for convenience we can work in their weak-eigenbasis, before the mass-diagonalization of ($\phi, \eta$) fields,

$$\left( \begin{array}{c} \pi^+\pi^- \, \chi, \, \pi^0\pi^0, \, \phi^0\phi^0, \, \phi^0\eta, \, \phi^0\eta', \, \phi^0\eta^0, \, \phi^0\eta^0, \, \phi^0\eta, \, \phi^0\eta', \, \phi^0\eta^0 \end{array} \right).$$  \hspace{1cm} (3.7)

There are many tree-level diagrams contributing to the scattering processes with the above in/out states. They can be classified into three categories: diagrams with quartic contact interactions, diagrams with scalar-exchanges, and diagrams with gauge boson exchanges. For the high energy scattering regime, the scalar-exchange contributions are suppressed by $E^{-3}$ relative to the contact interactions. On the other hand, the diagrams with gauge boson exchanges have momentum-dependent vertices, which may compensate for the propagator-induced suppression and give $O(E^3)$ contributions. But they are always proportional to the squared electroweak gauge couplings which are subdominant compared to the pure quartic scalar couplings in the contact interactions. Hence, similar to the SM case [24], it suffices to consider the leading contact interaction diagrams for the current unitarity analysis.

Using the weak-eigenbasis (3.7) as in/out states, we compute all possible two-body scattering processes with the contact interactions (2.6), and derive the following matrix amplitudes,

$$T = \begin{pmatrix} T_1 & 0 \\ 0 & T_2 \end{pmatrix}, \quad T_1 = \begin{pmatrix} \frac{2}{3} \lambda_0 & \frac{1}{3} \lambda_0 & \frac{1}{3} \lambda_0 & \frac{1}{3} \lambda_0 & \frac{1}{3} \lambda_0 & \frac{1}{3} \lambda_0 \\ \frac{1}{3} \lambda_0 & \frac{2}{3} \lambda_0 & \frac{1}{3} \lambda_0 & \frac{1}{3} \lambda_0 & \frac{1}{3} \lambda_0 & \frac{1}{3} \lambda_0 \\ \frac{1}{3} \lambda_0 & \frac{1}{3} \lambda_0 & \frac{2}{3} \lambda_0 & \frac{1}{3} \lambda_0 & \frac{1}{3} \lambda_0 & \frac{1}{3} \lambda_0 \\ \frac{1}{3} \lambda_0 & \frac{1}{3} \lambda_0 & \frac{1}{3} \lambda_0 & \frac{2}{3} \lambda_0 & \frac{1}{3} \lambda_0 & \frac{1}{3} \lambda_0 \\ \frac{1}{3} \lambda_0 & \frac{1}{3} \lambda_0 & \frac{1}{3} \lambda_0 & \frac{1}{3} \lambda_0 & \frac{2}{3} \lambda_0 & \frac{1}{3} \lambda_0 \\ \frac{1}{3} \lambda_0 & \frac{1}{3} \lambda_0 & \frac{1}{3} \lambda_0 & \frac{1}{3} \lambda_0 & \frac{1}{3} \lambda_0 & \frac{2}{3} \lambda_0 \end{pmatrix},$$  \hspace{1cm} (3.8)

which is a coupled 5 x 5 non-diagonal sub-block $T_1$. Thus, substituting (3.8) into (3.5), we derive the $s$-wave amplitude, $a_0 = T / (16\sigma)$. After diagonalization, we deduce the eigenvalues,

$$a_0[\text{diagonal}] = -\frac{1}{16\sigma} \text{diag} \left( \frac{1}{2} \lambda_0, \frac{1}{2} \lambda_0, x_1, x_2, x_3, \frac{1}{2} \lambda_0, x_1, x_2, x_3, \frac{1}{2} \lambda_0, x_1, x_2, x_3 \right).$$  \hspace{1cm} (3.9)

where $(x_1, x_2, x_3)$ are given by the roots of the following cubic equation,

$$4x^3 - 2\left(2\lambda_0 + \lambda_0 + x_3 \right)x^2 + \left[2\lambda_0\lambda_0 + 2\lambda_0\lambda_0 - 4\lambda_0\lambda_0^2 + 4\lambda_0\lambda_0 - 4\lambda_0\lambda_0^2 + A_{\text{diagonal}} \right] x + \left[4\lambda_0^2 + 2\lambda_0\lambda_0 - 4\lambda_0\lambda_0 + 4\lambda_0\lambda_0^2 - 4\lambda_0\lambda_0^2 + A_{\text{diagonal}} \right] = 0.$$  \hspace{1cm} (3.10)

In the region of small Higgs mixing angle $\omega \ll 1$, we find simple solutions for $(x_1, x_2, x_3)$ with

$$x_{1,2} = -\frac{1}{4} \lambda_0 \pm \sqrt{16\lambda_0^2 + 4\lambda_0^2 + \lambda_0^2}, \quad x_3 = 0.$$  \hspace{1cm} (3.11)

For our numerical analysis in Sec.3.5, we will use the exact solutions of (3.10).

In the above coupled channel analysis, the eigenvalues in (3.9) are all functions of our input parameters (2.30). Thus, imposing the unitarity condition $|\text{Re}a_0| < \frac{1}{2}$, we can derive constraints on the parameter space, which will be presented in Sec.3.5.
3.4. Constraints from Triviality and Vacuum Stability

In this subsection, we analyze both triviality and stability bounds for the present model. The renormalization group (RG) equations for SM gauge couplings \((g', g, g_s)\) and top Yukawa coupling \(y_t\) are given by [26],

\[
\begin{align*}
\frac{dg'}{dt} & = (4\pi)^2 \left( \frac{1}{4} + \frac{11}{2} g'^3 \right), \\
\frac{dg}{dt} & = (4\pi)^2 \left( -\frac{19}{2} g^3 \right), \\
\frac{dg_s}{dt} & = (4\pi)^2 \left( -7g_s^3 \right),
\end{align*}
\]

where \(t = \log \mu\). In addition, the Yukawa coupling \(y_N\) of right-handed neutrinos is defined in Eq. (2.8), and its RG equation reads,

\[
\frac{dy_N}{dt} = (4\pi)^2 \left( +9y_N^3 \right).
\]

Thus, we can derive the RG evolutions for \((g', g, g_s, y_t, y_N)\). The initial conditions for \((g', g, g_s)\) are defined at \(\mu = M_Z\), while for \((y_t, y_N)\), we define, \(y_t(M) = \sqrt{2M/\nu_d}\) and \(y_N(M_N) = M_N/(\sqrt{2} \nu_y)\).

It is also straightforward to compute the divergent parts of one-loop corrections to the scalar quartic couplings in (2.5). These include the vertex corrections and wavefunction renormalizations. Thus, we derive their RG equations as follows,

\[
\begin{align*}
\frac{d\lambda_\phi}{dt} & = (4\pi)^2 \left[ 4\lambda_\phi^2 + 3\lambda_m^2 + 3\lambda_m^2 \left( 4y_t^2 - g^2 - 3g^2 \right) + \frac{9}{4} \left( 2g^4 + \left( g^2 + g'^2 \right)^2 - 16y_t^4 \right) \right], \\
\frac{d\lambda_\eta}{dt} & = (4\pi)^2 \left[ 3\lambda_\eta^2 + 12(\lambda_m)^2 + 3\lambda_m^2 + 24\lambda_m^2 \nu_y^2 - 288y_N^2 \right], \\
\frac{d\lambda_y}{dt} & = (4\pi)^2 \left[ 3\lambda_y^2 + 12(\lambda_m)^2 + 3\lambda_m^2 \right], \\
\frac{d\lambda_m}{dt} & = (4\pi)^2 \left[ 4\lambda_m^2 + 2\lambda_m \lambda_m^2 + \lambda_m \lambda_m^2 + \lambda_m \lambda_i^2 + \frac{1}{2} \lambda_m^2 \left( 4y_t^2 + 2y_N^2 \right) - g^2 - 3g^2 \right], \\
\frac{d\lambda_i}{dt} & = (4\pi)^2 \left[ 4\lambda_i^2 + 2\lambda_i \lambda_i^2 + \lambda_i \lambda_i^2 + \lambda_i \lambda_i^2 + \frac{1}{2} \lambda_i^2 \left( 4y_t^2 - g^2 - 3g^2 \right) \right], \\
\frac{d\lambda_\eta}{dt} & = (4\pi)^2 \left[ 4\lambda_\eta^2 + 4\lambda_\eta \lambda_\eta^2 + \lambda_\eta \lambda_\eta + \lambda_\eta \lambda_\eta \right].
\end{align*}
\]

The right-hand-sides of (3.14) depend on all inputs of the model parameters (2.30). Since the \(\lambda_i\)’s are defined at a particular renormalization scale \(\mu = \Lambda\) where the tree-level flat direction conditions hold, we will define the initial conditions of RG equations at \(\Lambda\), where \(\Lambda\) is determined in terms of physical masses \((M_t, M_N)\) via (2.27) and (2.25).

As shown in (3.14), the scalar self-couplings \(\{\lambda_i\}\) have positive contributions to their beta functions and thus tend to make them nonasymptotically free, whereas the Yukawa couplings \((y_t, y_N)\) can give negative corrections via box diagrams. When \(\lambda_i\)’s dominate the beta functions, these quartic scalar couplings will encounter Landau poles during the RG running. Requiring \(\lambda_i\) not to diverge will thus impose constraints (the triviality bounds) on the scalar masses\(^7\) for a given UV cutoff \(\Lambda_{UV}\). For practical numerical analysis, we will set a condition for all scalar couplings, \(\max |\lambda_i(\Lambda)| < (4\pi)^2\), for \(\mu < \Lambda_{UV}\). As we have explicitly checked, raising this upper limit from \((4\pi)^2\) up to infinity does not produce any visible numerical difference. Similar feature was also noted for the SM triviality analysis [28].

Then, we turn to the vacuum stability of the Higgs potential. To ensure the stability of physical vacuum requires that the Higgs potential is bounded from below. For the leading order, we employ the bounded-from-below conditions (2.7a-2.7b) for tree-level Higgs potential (2.6), with couplings improved by one-loop RG running (3.14) to ensure vacuum stability at high scales. This will constrain the parameter space for each given cutoff scale \(\Lambda_{UV}\). Besides, the one-loop Higgs potential (2.29) is stabilized under the condition (2.28). For the present analysis we study the conditions for a stable physical vacuum. As an alternative, it may be possible that the vacuum is merely meta-stable [27, 28], which would be worth of a future study.

3.5. Viable Parameter Space: Combined Numerical Analysis

In this subsection, we systematically present numerical analysis of the viable parameter space by imposing the experimental and theoretical constraints studied in Sec 3.1-3.4.

\(^7\) The pure SM with a 125 GeV Higgs boson is free from triviality up to Planck scale, since the SM triviality bound only requires \(M_6 \lesssim 180\) GeV [27].
Figure 1: Experimental and theoretical constraints on the parameter space of $|\sin \omega| - M_\chi$ in plots (a)-(b), and of $M_\nu - M_\chi$ in plots (c)-(d). In each plot, the red region is excluded by the electroweak precision tests at 95% C.L., while the green region is excluded (95% C.L.) by the global fit of direct Higgs measurements at the LHC and Tevatron (as partially overlapped by the red contour). In plots (c)-(d), the shaded black region is forbidden due to $|\sin \omega| \leq 1$. The scattered blue-dots are simulated in each plot to represent the parameter space, allowed by the triviality, stability and perturbative unitarity bounds. We set two benchmark values of right-handed neutrino masses, $M_\nu = 0.5, 1$ TeV, for the plots (a)(c) and (b)(d), respectively.
Inspecting the five independent input parameters of (2.30), we will analyze the viable parameter space in the two-dimensional plane of $|\sin \omega, M_N|$ or $\{M_\sigma, M_\chi\}$ by scanning the allowed ranges of scalar couplings $\{\lambda_{1,2}, \lambda_{m,n}\}$ for each given sample mass $M_N$ of right-handed neutrinos.

For the experimental constraints in Sec. 3.1-3.2, we note that the LHC Higgs measurements of $h (125 \text{ GeV})$ only depend the Higgs mixing angle $|\sin \omega|$, while the electroweak oblique corrections (3.3) are functions of $|\sin \omega, M_N|$, or, equivalently, $|\sin \omega, M_\sigma, M_\chi|$ via (2.27a). For the numerical analysis, we will set two benchmark values of the right-handed neutrino mass, $M_N = 0.5, 1 \text{ TeV}$. In Fig. 1(a)-(b), we first present the experimental constraints in the shaded regions. The red region in each plot is excluded by the precision tests at 95% C.L. via oblique corrections (3.3a)-(3.3b). The vertical green band displays the excluded parameter region on $|\sin \omega|$ by the global fit of direct Higgs measurements of the LHC and Tevatron at 95% C.L. (Table I), which is partly overlapped by the red contour of precision bound. In Fig. 1(c)-(d), we impose the same experimental constraints in the plane of $M_\sigma - M_\chi$.

For the theoretical constraints in Sec. 3.3-3.4, they depend on all input parameters of (2.30). For Fig. 1(a)-(b), we simulate 900 random points in $|\sin \omega - M_N|$ plane for the remaining two input couplings $\{\lambda_1, \lambda_m\}$ under the unitarity bound, the triviality bound, and stability conditions. These scattered points in Fig. 1 are statistically represented by the small blue-dots. Our unitarity constraints are derived from tree-level partial wave analysis, while the triviality and vacuum stability bounds invoke loop corrections and RG runnings up to the UV cutoff scale $\Lambda_{\text{UV}}$. In the numerical simulations, we scan over the range of $\Lambda_{\text{UV}} \geq 10^5 \text{ GeV}$ for RG evolutions. Note that the positivity condition (2.28) also sets a generic lower bound on $M_N$ for a given input of $M_N$. We find, $M_N > 0.79 (1.57) \text{ TeV}$ for $M_N = 0.5 (1.0) \text{ TeV}$, as indicated in Fig. 1. It is evident that the full analysis favors relatively small mixing between the Higgs doublet and singlet with $|\sin \omega| \lesssim 0.3$. In parallel, for Fig. 1(c)-(d), we simulate 900 random points in $M_\sigma - M_\chi$ plane for input couplings $\{\lambda_1, \lambda_m\}$ under the same theoretical constraints as in plots (a)-(b), shown by the small blue-dots. We see that for each given $M_\chi$, the mass of $\chi$ receives a lower bound, while for a given $M_\sigma$, the $\sigma$ mass acquires an upper bound. For instance, taking $M_\chi \lesssim 2 \text{ TeV}$ will impose an upper limit $M_\sigma \lesssim 400 (300) \text{ GeV}$ for $M_\chi = 0.5 (1.0) \text{ TeV}$. Our parameter space predicts $M_\sigma$ to be significantly lighter than $M_\chi$. This is expected, since the mass of pseudo-Nambu-Goldstone boson $\sigma$ is radiatively generated.

As a final remark, we note that the definition of the Higgs mixing angle $\omega$ flips sign if we identify the 125 GeV state as the pseudo-Nambu-Goldstone boson $\sigma$ of the scale symmetry breaking, where the small mixing angle $\omega$ will correspond to a small singlet VEV $v_\eta$. In this case, the theoretical bounds heavily constrain the small mixing region, while the large mixing range remains excluded by experimental bounds. After detailed numerical scan of parameter space, we conclude that this inverse-identification scenario is ruled out by the combined theoretical and experimental constraints of Sec. 3. Besides, we find that replacing the complex singlet $S$ by a real singlet scalar is also excluded by these constraints. Thus, the present model gives the minimal viable construction.

4. Conclusions and Discussions

The recent LHC Higgs discovery [1][2] opens up a new era for particle physics, pressuring us to seek natural Higgs mechanism and explore the associated new physics (including the dark matter candidate).

In this work, we constructed the minimal viable extension of the SM with classical scale symmetry. It realizes radiative electroweak symmetry breaking (EWSB) à la Coleman-Weinberg and gives a natural solution to the hierarchy problem. In addition to a SM-like light Higgs boson $h$ of mass 125 GeV, it predicts two new states at weak scale: one CP-even Higgs $\sigma$ serving as the pseudo-Nambu-Goldstone boson of scale symmetry breaking, and a CP-odd scalar singlet $\chi$ providing the dark matter (DM) candidate. Furthermore, the model nicely accommodates three right-handed Majorana neutrinos $\nu_{1,2,3}$ and generates light neutrino masses via TeV scale seesaw.

In Sec. 2.1-2.2, we presented the model, whose Higgs sector contains the SM Higgs doublet plus a new complex singlet. We formulated the scale-invariant and CP-symmetric Higgs potential (2.4) at tree-level, and determined the flat direction (2.11), as well as realizing the TeV scale seesaw of light neutrino masses (2.9) via Yukawa interaction (2.8). In Sec. 2.3, we systematically studied the radiative EWSB à la Coleman-Weinberg, and derived complete Higgs mass-spectrum in (2.16) and (2.27a).

In Sec. 3.1, we first analyzed experimental constraints from global fit of the direct Higgs measurements at the LHC and Tevatron (cf. Table I). Then, in Sec. 3.2, we derived oblique corrections in (3.3) and analyzed the corresponding electroweak precision tests. In Sec. 3.3-3.4, we studied theoretical constraints from unitarity, triviality and vacuum stability for this model. Combining both the experimental and theoretical constraints, we analyzed the viable parameter space in Sec. 3.5 which are presented in Fig. 1(a)-(d).

Finally, we discuss signals of the predicted new $\sigma$ and $\chi$ bosons at the upcoming runs of the LHC (14 TeV). The
σ\) boson has couplings to \(W W/ZZ\) and quarks/leptons under the suppression\(^{[1]}\) of \(\sin \omega\). Hence, it can be produced at the LHC via gluon fusions \(gg \to \sigma\) \(\to ZZ, WW\) as its major detection channels, where \(ZZ\) and \(WW\) decay leptonically. It also has an interesting decay mode, \(\sigma \to hh\), for \(M_\sigma > 250\) GeV. On the other hand, the DM candidate \(\chi\) can be produced in pair at the LHC, giving raise to missing energy. Since \(\chi\) only appears in the Higgs potential\(^{[1]}\) and couples to \(h\) and \(\sigma\) besides its self-coupling, we expect its major LHC production channel comes from the associate production, \(pp \to Z\chi' \to Z\chi\chi\) and \(pp \to W\chi' \to W\chi\chi\), with \(Z \to \ell^+\ell^-\) and \(W \to \ell v (\ell = e, \mu)\).

We also note that our model predicts a relatively heavy scalar DM particle \(\chi\) with mass of \(O(\text{TeV})\). The positivity condition\(^{[1,28]}\) generally sets a lower bound on \(M_\chi\), and requires, \(M_\chi > 0.79 (1.57)\) TeV for \(M_\chi = 0.5 (1.0)\) TeV. We verify its viability as a cold DM by computing the thermal relic density. Since \(M_\chi\) is heavier than all other particles in the model, it is reasonable to consider that all particles are in thermal equilibrium around the time when \(\chi\) freezes out. There is considerable rate for a pair of \(\chi\) annihilating into other two-body final states, which arise from contact interactions and exchanges of scalars. Thus, given the measured DM relic density \(\Omega_{DM}\), we can derive constraints on the parameter space\(^{[1,29]}\). As for the DM direct detection, the relevant couplings concern the \(\chi^{-}\chi^{-h}\) or \(\chi^{-}\chi^{-\sigma}\) vertices with \(h\) or \(\sigma\) interacting with the SM fermions. The corresponding effective contact interactions of this DM with nucleons can be tested by direct detection experiments, such as XENON100\(^{[10]}\), and CDMS-II & EDELWEISS\(^{[31]}\). So far, the XENON100 detection gives the best bound on spin-independent cross sections of TeV scale DM\(^{[30]}\). A systematical DM analysis for our present model is beyond the current scope and will be given elsewhere.

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\footnote{This suppression is due to the Higgs mixing \(^{[7]}\) for \((h, \sigma)\). So, the production and decay signals of \(\sigma\) are similar to that of the heavier Higgs state \(H^0\) from new physics models with extended two-Higgs-doublet sector\(^{[29]}\).}
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