Composite quarks and leptons in higher space-time dimensions

M. Chaichian\textsuperscript{a}, J.L. Chkareuli\textsuperscript{b} and A. Kobakhidze\textsuperscript{a,b}

\textsuperscript{a}High Energy Physics Division, Department of Physics, University of Helsinki and Helsinki Institute of Physics, FIN-00014 Helsinki, Finland

\textsuperscript{b}Andronikashvili Institute of Physics, Georgian Academy of Sciences, 380077 Tbilisi, Georgia

Abstract

A new approach towards the composite structure of quarks and leptons in the context of the higher dimensional unified theories is proposed. Owing to the certain strong dynamics, much like an ordinary QCD, every possible vectorlike set of composites appears in higher dimensional bulk space-time, however, through a proper Sherk-Schwarz compactification only chiral multiplets of composite quarks and leptons survive as the massless states in four dimensions. In this scenario restrictions related with the 't Hooft's anomaly matching condition are turned out to be avoided and, as a result, the composite models look rather simple and economic. We demonstrate our approach by an explicit construction of model of preons and their composites unified in the supersymmetric $SU(5)$ GUT in five space-time dimensions. The model predicts exactly three families of the composite quarks and leptons being the triplets of the chiral horizontal symmetry $SU(3)_h$ which automatically appears in the composite spectrum when going to ordinary four dimensions.
1 Introduction

The observed replication of quark-lepton families and hierarchy of their masses and mixings are one of the major puzzles of modern particle physics. In this respect it is conceivable to think that quark and lepton spectroscopy finds its ultimate explanation in terms of the subfermions (preons) and their interactions in analogy with an explanation of hadronic spectroscopy in the framework of the quark model. However, a direct realization of this program seems to meet serious difficulties. Among the problems appeared the basic one is, of course, that related with the dynamics responsible for a production of the composite quarks and leptons whose masses \( m_f \) are turned out to be in fact much less than a compositeness scale \( \Lambda_C \) which must be located at least in a few TeV region to conform with observations [1]. Indeed, if, as usual, one considers underline preon theory to be QCD-like, then one inevitably comes to the vectorlike bound state spectra where most naturally \( m_f \sim \Lambda_C \). To overcome this difficulty one has to require the presence of some chiral symmetry which being respected by the strong preon dynamics makes quark and lepton bound states to be massless. As ’t Hooft first argued [2], such a chiral symmetry to be preserved in the spectrum of massless composite fermions must yield the same chiral anomalies as those appearing in the underline preon theory. However, this anomaly matching condition is turned out to be too restrictive to drive at the physically interesting self-consistent models. As a result, most of existing models [3, 4, 5] are rather complex and controversial and often contain too many exotic composite states apart from the ordinary quarks and leptons.

Supersymmetric preon models [4, 5] follow to somewhat different pattern of the anomaly matching condition since in this case the physical composites, quarks and leptons, appear as both the three-fermion (”baryons”) and scalar-fermion (”mesons”) bound states. More interestingly, these models may provide a new dynamical alternative for obtaining light composite fermions, which emerge as (quasi)Goldstone fermions [4] when the starting global symmetry \( G \) of the superpotential is spontaneously broken down to some lesser symmetry \( H \). In the recent years, there was renewal of interest in supersymmetric preon models [3] based on a powerful technique developed within the strongly interacting \( N = 1 \) supersymmetric gauge theories [9]. However, despite these attractive features of supersymmetric theories the supersymmetric composite models generally suffer from the same drawbacks as the more traditional non-supersymmetric ones.

In this Letter we suggest a new approach towards the composite structure of quarks and leptons proposing a presence of extra space-time dimensions at the small distances comparable or a bit larger than a radius of compositeness \( R_C \sim 1/\Lambda_C \). It is well known that the compactification of extra space-time dimensions (depending on the detail of dimensional reduction) was happened quite successful to get realistic four dimensional models where supersymmetry [7, 8, 9], gauge symmetry [10, 11] and certain discrete symmetries such as \( P \)
and CP are broken in an intrinsically geometric way. Following to this line of arguments we find that owing to a certain Scherk-Schwarz compactification [7] the composite quarks and leptons are turned out to be massless in four dimensions, while all unwanted states (residing in the bulk) are massive. In this way the restrictions related with an original 't Hooft anomaly matching condition can be avoided. Thereby, the physical composite models look rather simple and economic as we will show shortly by a few examples of the elementary preons and their composite unified in the SU(5) SUSY GUT initially appearing in five space-time dimensions (5D).

2 Supersymmetry in 5D and Scherk-Schwarz compactification

Before turning to the construction of composite models let us recall some aspects of the \( N = 1 \) 5D supersymmetry and Scherk-Schwarz compactification which are relevant for our subsequent discussion. Consider in 5D the \( N = 1 \) supersymmetric gauge theory with a local symmetry group \( G \) under which the matter fields transform according to one of its irreducible representation \( R \). The \( N = 1 \) supersymmetric Yang-Mills supermultiplet \( \mathcal{V} = (A^M, \lambda^i, \Sigma, X^a) \) in 5D contains a vector field \( A^M = A^{M\alpha}T^\alpha \), a real scalar field \( \Sigma = \Sigma^\alpha T^\alpha \), two gauginos \( \lambda^i = \lambda^{i\alpha}T^\alpha \), which form a doublet under the \( R \)-symmetry group \( SU(2)_R \), and auxiliary fields \( X^a = X^{a\alpha}T^\alpha \) being a triplet of \( SU(2)_R \)-triplet indices, respectively; \( \alpha \) runs over the \( G \) group index values and \( T^\alpha \) are generators of \( G \) algebra. These fields are combined into the \( N = 1 \) 4D vector supermultiplet \( \mathcal{V} = (A^m, \lambda^1, X^3) \) and a chiral supermultiplet \( \Phi = (\Sigma + iA^4, \lambda^2, X^1 + iX^2) \). The matter fields are collected in the hypermultiplet \( \mathcal{H} = (h^i, \Psi, F^i) \) which contains the scalar fields \( h^i \) being a doublet of \( SU(2)_R \), Dirac fermion \( \Psi = (\psi_1, \psi_2^+)^T \) being the \( SU(2)_R \) singlet, and also the \( SU(2)_R \)-doublet of auxiliary fields \( F^i \). All those fields form two \( N = 1 \) 4D chiral multiplets, \( H = (h^i, \psi_1, F^1) \) and \( H^c = (h^2, \psi_2, F^2) \) transforming according to the representations \( R \) and anti-\( R \) of gauge group \( G \), respectively. The 5D supersymmetric and \( G \)-symmetric action then can be then written as (see, e.g. [3]):

\[
S = \int d^5x \int d^4\theta \left[ H^{c} e^{V} H^{c+} + H^{+} e^{V} H \right] + \\
\int d^5x \int d^2\theta \left[ H^{c} \left( \partial_4 - \frac{1}{\sqrt{2}} \Phi \right) H + h.c. \right]. \tag{1}
\]

The above theory (1) is in fact vectorlike and, hence, anomaly-free.

Now let us compactify the extra fifth dimension \( x^4 \) on a circle of radius \( R_C \). Aside from the trivial (periodic) boundary conditions under the \( 2\pi R_C \) translation of extra dimension
one can impose to the 5D fields the following non-trivial \((U(1)\text{-twisted})\) ones:

\[
H(x^m, x^4 + 2\pi R_C, \theta) = \exp(i2\pi q_H)H(x^m, x^4, e^{i\pi(q_H+q_{H^c})\theta}),
\]
\[
H^c(x^m, x^4 + 2\pi R_C, \theta) = \exp(i2\pi q_{H^c})H(x^m, x^4, e^{i\pi(q_H+q_{H^c})\theta}),
\]
\[
V(x^m, x^4 + 2\pi R_C, \theta, \bar{\theta}) = V(x^m, x^4, e^{i\pi(q_H+q_{H^c})\theta}, e^{-i\pi(q_H+q_{H^c})\bar{\theta}}),
\]
\[
\Phi(x^m, x^4 + 2\pi R_C, \theta) = \Phi(x^m, x^4, e^{i\pi(q_H+q_{H^c})\theta}),
\]

where \(q_H\) and \(q_{H^c}\) are the \(R\) charges of the superfields \(H\) and \(H^c\), respectively. Due to the periodicity conditions (2) the component fields are Fourier expanded as:

\[
h^1(x^m, x^4) = \sum_{n=-\infty}^{\infty} e^{ix^4(n+q_H)/R_C} h^{1(n)}(x^m),
\]
\[
h^2(x^m, x^4) = \sum_{n=-\infty}^{\infty} e^{ix^4(n+q_{H^c})/R_C} h^{2(n)}(x^m),
\]
\[
\psi_1(x^m, x^4) = \sum_{n=-\infty}^{\infty} e^{ix^4\left(n+\frac{q_H-q_{H^c}}{2}\right)/R_C} \psi^{1(n)}(x^m),
\]
\[
\psi_2(x^m, x^4) = \sum_{n=-\infty}^{\infty} e^{ix^4\left(n-\frac{q_H-q_{H^c}}{2}\right)/R_C} \psi^{2(n)}(x^m),
\]
\[
\lambda^1(x^m, x^4) = \sum_{n=-\infty}^{\infty} e^{ix^4\left(n-\frac{q_H+q_{H^c}}{2}\right)/R_C} \lambda^{1(n)}(x^m),
\]
\[
\lambda^2(x^m, x^4) = \sum_{n=-\infty}^{\infty} e^{ix^4\left(n+\frac{q_H+q_{H^c}}{2}\right)/R_C} \lambda^{2(n)}(x^m),
\]
\[
A^m(x^m, x^4) = \sum_{n=-\infty}^{\infty} e^{ix^4 \frac{n}{R_C}} A^{m(n)}(x^m),
\]
\[
\left(\Sigma + iA^4\right)(x^m, x^4) = \sum_{n=-\infty}^{\infty} e^{ix^4 \frac{n}{R_C}} \left(\Sigma + iA^4\right)^{(n)}(x^m).
\]

Let us note now that all the fields with the non-trivial \(R\)-charges are necessarily turned out to be massive when reducing the theory from 5D to 4D. Particularly, zero modes of all the fermionic fields in (3) and those of the scalars \(h^1\) and \(h^2\) have a masses \(\frac{q}{R_C}\), where \(q\) are the corresponding \(R\) charges, while the zero modes of the gauge fields \(A^m\) and adjoint scalar \((\Sigma + iA^4)\) are massless. Nevertheless, the latter picks up the mass of the order of \(\sim \frac{1}{R_C}\), radiatively since the supersymmetry is broken by the above Scherk-Schwarz compactification, so that in general only gauge fields \(A^m\) are left to be massless. However, if the \(R\) charges of the superfields \(H\) and \(H^c\) are equal \((q_H = q_{H^c})\) then, as one can quickly confirm from (3), the zero modes of \(\psi_1\) and \(\psi_2\) happen to be massless as well. Note also that for the composite operators containing the above superfields the \(R\) charge assignment, and thus the spectrum of the massless zero modes, can be rather different. This is a key point we will use below in the construction of composite models of quarks and leptons.
3 One-generation composite model

Let us consider $N = 1$ supersymmetric $G \otimes SU(N)_{HC}$ gauge theory in 5D, where $G$ is a gauged part of some hyperflavor symmetry $G_{HF}$, which includes all the observed symmetries (color $SU(3)_C$ and electroweak $SU(2)_W \otimes U(1)_Y$, or grand unified symmetry $SU(5)$ etc), and $SU(N)_{HC}$, which describes hypercolor interactions responsible for the formation of hypercolorless bound states from preons. We assume that the preons and anti-preons are resided in the 5D hypermultiplets $P = (P, P^c)$ and transform under hypercolor gauge group $SU(N)_{HC}$ as its fundamental ($P \sim N$) and anti-fundamental ($P^c \sim \overline{N}$) representations, respectively. The preons should carry also the quantum numbers related with the hyperflavor symmetry group $G_{HF}$. The hypercolor gauge group $SU(N)_{HC}$ has to be asymptotically free as is in the case of an ordinary QCD. Otherwise, the theory will not be well defined as an interacting quantum field theory (because of the Landau pole appeared) and can be consistently treated only as a low energy limit of some other theory. Thus, the asymptotic freedom of the $SU(N)_{HC}$ restricts a number of the allowed hyperflavors $N_{HF}$ to be

$$\frac{N_{HF}}{2} \leq N \quad (4)$$

Now let us take $G$, the gauged part of a total hyperflavor symmetry $G_{HF}$, to be the minimal grand unified group, i.e. $G \equiv SU(5)$, so that the preons transform under the $SU(5) \otimes SU(N)_{HC}$ as:

$$P_{(5)} \sim (5, N),$$

$$P^c_{(\overline{5})} \sim (\overline{5}, \overline{N}),$$

$$P_{(s)i} \sim (1, N),$$

$$P^c_{(s)i} \sim (1, \overline{N}), \quad (5)$$

where $i = 1, ..., N_g$. Therefore, the total number of flavors is $N_{HF} = 5 + N_g$. The $SU(5)$ singlet preons (anti-preons) $P_{(s)i}$ ($P^c_{(s)i}$) in (5) are actually necessary in order to produce the entire set of composite quark and leptons transforming as $\overline{5} + 10$ representations of $SU(5)$. We call them “generation” preons. Thus, the preons carry all “basic” quantum numbers presently observed in quark-lepton phenomenology at low energies, such as three colors, two weak isospin components (being unified within the $SU(5)$) and the generation numbers as well.

Within the framework described above the minimal possible hypercolor group is $SU(3)_{HC}$ which admits a single ($N_g = 1$) “generation” preon and, thus, in total only six hyperflavors of preons, $N_{HF} = 6$. This hypercolor interaction is assumed to be responsible for the formation of hypercolorless “baryons”

$$\overline{D}_1 \sim P_{(5)} P_{(5)} P_{(5)} \sim 10, \quad D_1 \sim P^c_{(\overline{5})} P^c_{(\overline{5})} P^c_{(\overline{5})} \sim 10,$$
\[ D_2 \sim P_{(5)} P_{(s)} \sim 10, \quad \overline{D}_2 \sim P^c_{(5)} P^c_{(s)} \sim \overline{10} \] (6)

and “mesons”

\[ \overline{Q} \sim P^c_{(5)} P_{(s)} \sim 5, \quad Q \sim P_{(5)} P^c_{(s)} \sim 5, \]
\[ M \sim P^c_{(5)} P_{(s)} \sim 24 + 1, \quad S \sim P^c_{(s)} P_{(s)} \sim 1 \] (7)

at a compositeness scale \( \Lambda_C \) (antisymmetrized products in (6) are meant). All these bound states come out in vectorlike \( SU(5) \) representations and they are in fact the \( N = 1 \) 4D superfields.

As in the previous section, compactifying the extra dimensions on a circle of radius \( R_C \) (and assuming that \( R_C > 1/\Lambda_C \)) we impose Scherk-Schwarz boundary conditions to the preonic superfields (3) of type

\[ P_{(5)}(x^m, x^4 + 2\pi R_C, \theta) = e^{i2\pi q_5} P_{(5)}(x^m, x^4, e^{i\pi(q_5 + q_\tau)}\theta), \]
\[ P_{(5)}(x^m, x^4 + 2\pi R_C, \theta) = e^{i2\pi q_5} P_{(5)}(x^m, x^4, e^{i\pi(q_5 + q_\tau)}\theta), \]
\[ P_{(s)}(x^m, x^4 + 2\pi R_C, \theta) = e^{i2\pi q_5} P_{(s)}(x^m, x^4, e^{i\pi(q_5 + q_\tau)}\theta), \]
\[ P^c_{(s)}(x^m, x^4 + 2\pi R_C, \theta) = e^{i2\pi q_5} P^c_{(s)}(x^m, x^4, e^{i\pi(q_5 + q_\tau)}\theta), \] (8)

where

\[ q_5 + q_\tau = q_s + q_\tau. \] (9)

The vector supermultiplets and the adjoint superfields are periodic as in (2). Expanding the 5D preonic fields as in (3) one can see that all fermionic preons are massive in 4D, thus low energy preonic theory can be treated as a consistent quantum theory since the gauge anomalies are absent. Obviously, supersymmetry is broken by the above boundary conditions (8). Specifying the boundary conditions for the preonic fields one can easily obtain \( R \)-charges for the composite states (3) and (7) as well:

\[ \overline{D}_1 \sim 3q_5, \quad D_1 \sim 3q_5, \quad D_2 \sim 2q_5 + q_s, \quad \overline{D}_2 \sim 2q_5 + q_\tau, \]
\[ \overline{Q} \sim q_\tau + q_s, \quad Q \sim q_5 + q_\tau, \quad M \sim q_\tau + q_5, \quad S \sim q_\tau + q_s. \] (10)

Since the \( R \)-charges (10) for the composite states differ from those of preons (8), one can expect different spectrum of composite zero modes. Particularly, we are looking for such an assignment of preonic \( R \)-charges (8) which lead to massless composite fermions in 4D in \((5 + 10)\) representation of \( SU(5) \) that are nothing but composite quarks and leptons. It is evident from (3) and (7) that we should identify the fermionic components of \( \overline{Q} \) superfield with an anti-quintet of \( SU(5) \) where down-type anti-quark and lepton doublet are resided. The \( SU(5) \) decuplet where quark doublet, up-type antiquark and charged anti-lepton are resided can be identified with fermionic components of either \( D_1 \) or \( D_2 \) superfields.
fermionic zero modes of $Q$ and $D_1$ will be massless if the $R$-charges (8) along with the equation (9) satisfy also the following equations:

$$q_5 + q_s = \frac{q_5 + q_5}{2}$$

(11)

$$3q_5 = \frac{q_5 + q_5}{2}.$$  

(12)

Solving the equations (9, 11, 12) one has to remember that due to periodicity $R$-charges $q$ are defined up to an arbitrary integer number, $q = q + k$, $k \in \mathbb{Z}$. To ensure that only a desired set of fermionic zero modes are massless in 4D we restrict general $U(1)$-twisted boundary conditions to some discrete $Z_K$ ones. It is easy to verify then that any $K \neq 2, 3, 4, 6, 9, 12$ will provide the desired solutions of (9, 11, 12):

$$q_5 = \frac{5}{K}, q_5 = \frac{1}{K}, q_s = \frac{2}{K}, \text{ and } q_5 = \frac{4}{K}.$$  

(13)

The minimal choice is $Z_5$-twisted boundary conditions with $R$-charges $q_5 = 0, q_5 = \frac{1}{5}, q_s = \frac{2}{5}$, and $q_5 = -\frac{1}{5}$, so that only one generation of composite quarks and leptons are massless in 4D at low energies. All extra composite states are massive with masses of the order of the order of $1/R_C$.

If one identifies the quark-lepton decuplet of $SU(5)$ with fermionic components of $D_2$ superfield then one has to determine the $R$-charges from the equations (9, 11) and the equation

$$2q_5 + q_s = \frac{q_5 + q_5}{2},$$

(14)

instead of (12), appears. Any $Z_K$-twisted boundary conditions with $K \neq 2, 3, 4, 6, 9, 12$ and

$$q_5 = -\frac{2}{K}, q_5 = -\frac{4}{K}, q_s = \frac{1}{K}, \text{ and } q_5 = -\frac{7}{K}.$$  

(15)

will lead to the desired solutions. The minimal possibility is again $Z_5$ but now with the following $R$-charges: $q_5 = -\frac{2}{5}, q_5 = \frac{1}{5}, q_s = \frac{1}{5}$, and $q_5 = -\frac{7}{5}$. It looks quite intriguing that just composite quarks and leptons (without any extra states) unified within the $SU(5)$ gauge theory emerge at low energies in 4D from a simple and economic preon dynamics discussed above.

### 4 Three-generation composite model

One can easily extend the above model with one generation of composite quarks and leptons to the case of three composite generations by simply copying the above structure thrice, thus resulting in a model with hypercolor group $SU(N)_1 \otimes SU(N)_2 \otimes SU(N)_3$. However,
a more interesting way is based on treating the $SU(5)$-singlet preons in (5) as the carriers of quantum numbers associated with quark-lepton generations. Thus we will take three “generation” preons (anti-preons) $P_{(s)}$, $P^c_{(s)}$ ($i = 1, 2, 3$) and the global $SU(3)_P$ symmetry of 5D preonic Lagrangian will be interpreted as a “horizontal” hyperflavor symmetry $SU(3)_h$ for quark-lepton families (see below). Therefore, we will also require this symmetry to be survied upon the Scherk-Schwarz compactification, that is to say, the $R$-charges for all three “generation” preons are the same. Now altogether there are $N_{HF} = 8$ hyperflavors of preons and thus, due to the asymptotic freedom constraint, the minimal hypercolor group is $SU(4)_{HC}$. While, following to arguments used in the previous section such a composite model leading to three quark-lepton generations can easily be constructed, it seems to be more interesting to take the $SU(5)_{HC}$ as the hypercolor group. Apart from the possibility to treat all the massles composites in the same way as the pure baryonic composites, this case may be of a special interest as the case suggesting some starting extra hypercolor-hyperflavour symmetry ($HC \leftrightarrow HF$) in the 5D. The composite “baryons” and “mesons” are then:

$$
\begin{align*}
\overline{D}_1 & \sim P_{(5)}^c P_{(5)} P_{(5)} P_{(s)} P_{(s)} \sim (\overline{10}, 3), \\
D_2 & \sim P_{(5)} P_{(5)} P_{(5)} P_{(s)} P_{(s)} \sim (10, 1), \\
\overline{Q} & \sim P_{(5)} P_{(5)} P_{(5)} P_{(5)} P_{(s)} \sim (\overline{3}, 3), \\
S & \sim P_{(5)} P_{(5)} P_{(5)} P_{(5)} P_{(s)} \sim (1, 1), \\
D_1 & \sim P_{(5)}^c P_{(5)}^c P_{(5)}^c P_{(s)}^c P_{(s)}^c \sim (10, 3), \\
\overline{D}_2 & \sim P_{(5)}^c P_{(5)}^c P_{(5)}^c P_{(s)}^c P_{(s)}^c \sim (\overline{10}, 1), \\
Q & \sim P_{(5)}^c P_{(5)}^c P_{(5)}^c P_{(5)}^c P_{(s)}^c P_{(s)}^c \sim (5, 3), \\
\overline{S} & \sim P_{(5)}^c P_{(5)}^c P_{(5)}^c P_{(5)}^c P_{(s)}^c P_{(s)}^c \sim (1, 1) \\
\end{align*}$$

and

$$
\begin{align*}
\overline{Q}' & \sim P_{(5)}^c P_{(s)} \sim (\overline{5}, 3), \\
M & \sim P_{(5)}^c P_{(5)} \sim (24 + 1, 1), \\
Q' & \sim P_{(5)} P_{(s)} \sim (5, 3), \\
I & \sim P_{(5)} P_{(s)} \sim (1, 8 + 1),
\end{align*}
$$

respectively, transforming under $SU(5) \otimes SU(3)_h$ as indicated in brackets (anti-symmetrization of all the $SU(5)$ and $SU(3)_h$ indices are meant in (16)). One can see that the $SU(5)$ decuplets (anti-decuplets) in (13), being the triplets (anti-triplets) and singlets of the global family symmetry $SU(3)_h$, are pure baryonic composites. As to the $SU(5)$ anti-quintets (quintets), being triplets (anti-triplet) of the $SU(3)_h$, they appear as both baryonic (16) and mesonic (17) composites. Also some other states, singlets and adjoints of $SU(5)$ and $SU(3)_h$, appear in the composite spectrum (16,17). Now, as soon as the fermionic zero modes proposed for the $D_1$ supermultiplet in (16) are massless, one has to ensure that the zero modes of fermionic components of the baryonic anti-quintet $\overline{Q}$ in (16) or mesonic anti-quintet $\overline{Q}'$ in (17) (but not of the both) are also massless in order the low energy composite model to be anomaly-free, thus giving an unique assignments of the massless composites to the representation ($\overline{5} + 10, 3$) of the $SU(5) \otimes SU(3)_h$. Remarkably, one can come to this
basic consequence even if starts with an arbitrary number of the generation preons $P_{(s)}^i$ ($P_{(s)}^i$ ($i = 1, 2, ..., N_g$). Since, according to the above construction $\mathcal{D}_1$, a number of the composite $SU(5)$ decuplets is given by the $N_g(N_g - 1)/2$, while the composite anti-quintets by the number $N_g$ by itself (whether they are the baryonic or meson composites), one is unavoidably come to the $SU(5)$ anomaly cancellation condition of type

$$\frac{N_g(N_g - 1)}{2} = N_g$$

(18)

from which immediately follows that $N_g = 3$. Thus the above model actually predicts three full generations of composite quarks and leptons being the triplets of the chiral global family symmetry $SU(3)_h$ automatically appeared in the composite spectrum.

Proceeding as in the previous section, one can easily determine the desired $R$-charges. If we identify the composite quarks and leptons with fermionic zero modes of the baryonic composites $D_1$ and $\overline{Q}$ in (16) then preonic $R$-charges along with the equation (9) must satisfy the following equations:

$$3q_5 + 2q_s = \frac{q_5 + q_5}{2}$$

(19)

$$4q_5 + q_s = \frac{q_5 + q_5}{2}$$

(20)

The desired solutions is provided by $Z_6$-twisted boundary conditions (which is the minimal one) with $R$-charges defined as:

$$q_5 = q_5 = \frac{1}{6}, \quad q_s = -\frac{1}{6} \text{ and } q_s = \frac{1}{6}.$$  

(21)

In the case when the composite $SU(5)$ decuplets are identified with fermionic zero modes of the baryonic composite $D_1$ $\mathcal{D}_1$, while the composite anti-quintet with the mesonic composite $\overline{Q}$ $\mathcal{D}_1$ one should replace the equation (20) by the equation (11). It is easy to verify that the minimal solution will be once again provided by $Z_6$-twisted boundary conditions but now with the following $R$-charges:

$$q_5 = q_5 = \frac{1}{6}, \quad q_s = \frac{1}{3} \text{ and } q_s = 0.$$  

(22)

Remarkably, only three generations of composite quarks and leptons emerge as a massless states, while all other composites are massive, thus decoupling from the low-energy particle spectrum.
5 Discussion and conclusion

Some questions concerning the dynamics of the composite models discussed above must be further elaborated. The major ones are: How the SU(5) and subsequently the electroweak symmetries are broken? How the masses for composite quarks and leptons are generated? Can one naturally explain the hierarchies of masses and mixings of composite quarks and leptons? Here we will briefly outline some possible scenarios one can think about.

In fact, one can use the SU(5)-adjoint superfield Φ to break SU(5) symmetry down to the SU(3)C ⊗ SU(2)W ⊗ U(1)Y Standard Model gauge group. In the supersymmetric uncompactified limit there are degenerate flat vacuum directions for the scalar component of Φ. Among these vacua one can certainly find the SU(5)-breaking and SU(3)C ⊗ SU(2)W ⊗ U(1)Y-invariant one. In such a vacuum the preons will acquire SU(5) non-invariant masses but this does not affect their subsequent dynamics resulting in formation of composite states. The degeneracy of vacuum states of course are lifted when one takes into account supersymmetry breaking effects due to the Scherk-Schwarz compactification. Alternatively, one can break SU(5)-symmetry through the condensation of the scalar components of composite mesonic superfield M(7,17). Similarly, to break SU(2)W ⊗ U(1)Y electroweak symmetry one can use the doublet (anti-doublet) components of composite quintets (anti-quintets). Since, the supersymmetry is broken, one inevitably faces with gauge hierarchy problem which can be resolved by fine-tuning as in the usual non-supersymmetric GUTs. Alternatively, one can think that the solution to the gauge hierarchy problem appears due to the strong renormalization of the electroweak Higgs mass which is driven to an infrared stable fixed-point of the order of electroweak scale, while being of the order of GUT scale at higher energies [14]. Relatively large extra dimensions play crucial role in this scenario by inducing fast (power-law) evolution of gauge and Yukawa couplings.

The same mechanism could explain the observed hierarchies of quark-lepton masses and mixings along the lines discussed in [15]. These scenarios can be actually operative in the case of composite quarks and leptons as well. However, following to a more traditional way, one can think that the hierarchy of quark-lepton masses and mixings are related with spontaneous breaking of the global chiral SU(3)h horizontal symmetry appeared in our model together the three quark-lepton generations predicted. That is, as one can presently think, the main benefit of the above consideration. Actually, the chiral horizontal symmetry SU(3)h is known [16] to work successfully both in quark and lepton sector and can readily be extended to the composite quarks and leptons as well. It would be interesting to gauge this symmetry within the preon model. However, a direct gauging of the chiral horizontal symmetry typically leads to the SU(3)h triangle anomalies in the effective 4D theory. One way to overcome this problem is to introduce some extra massless states which properly cancel these anomalies in a traditional way. Another, and perhaps more interesting possibility, is to cancel 4D anomalies
by Callan-Harvey anomaly inflow mechanism [17] assuming a presence of 4D hypersurface (3-brane) in 5D bulk space-time where the composite quarks and leptons are localized.

From purely phenomenological point of view it is certainly interesting to study whether the compositeness scale, as well as the compactification one, can be lowered down to the energies accessible for the high energy colliders. Of course, these and related issues deserve more careful investigation.

Various extensions of the simple models presented here are also interesting to study. One can consider different gauge groups and more extra dimensions as well. Particularly, one can study the possibility to unify the $SU(5)$ symmetry with the gauged horizontal $SU(3)_h$ and/or hypercolor $SU(N)_{HC}$ symmetries within a single gauge group (for earlier attempts see, e.g. [18]). It is certainly interesting to investigate the dynamical emergence of gauge symmetries themselves with the composite gauge bosons within the approach undertaken in this paper. And finally, from more fundamental point of view it could be encouraging to study string theories where the string excitations are identified with preons rather than the physical quarks and leptons (for earlier discussion, see [19]).

To conclude, we have proposed a new approach towards the quark and lepton compositeness within the higher dimensional unified theories where owing to proper Scherk-Schwarz compactification the composite quarks and leptons are turned out to be massless in four dimensions, while all unwanted states (residing in the bulk) are massive. The prototype models discussed here are rather simple and economic, so we think this approach will help to construct the largely realistic composite models of quarks and leptons in a not distant future.

**Acknowledgements**

This work was supported by the Academy of Finland under the Project 163394. One of us (JLC) would like to acknowledge a warm hospitality during his visit to High Energy Physics Division, Department of Physics, University of Helsinki where part of this work was done.
References

[1] Particle Data Group, *Eur. Phys. J.* **C15** (2000) 1.

[2] G. ’t Hooft, *Recent Developments In Gauge Theories*, Proceedings, Nato Advanced Study Institute, New York (Plenum, New York, 1980).

[3] For a review and references of earlier works see e.g. L. Lyons, *Prog. Part. Nucl. Phys.* **10** (1983) 227.

[4] For earlier supersymmetric composite models see the review: R. R. Volkas, G.C. Joshi, *Phys. Rep.* **159** (1988) 303 and references therein.

[5] M.J. Strassler, *Phys. Lett.* **B376** (1996) 119; A.E. Nelson, M.J. Strassler, *Phys. Rev.* **D56** (1997) 4226; M.A. Luty, R.N. Mohapatra, *Phys. Lett.* **B396** (1997) 161.

[6] N. Seiberg, *Phys. Rev.* **D49** (1994) 6857; *Nucl. Phys.* **B435** (1995) 129; For a review, see K. Intriligator, N. Seiberg, *Nucl. Phys. Proc. Suppl.* **45BC** (1996) 1.

[7] J. Scherk, J. H. Schwarz, *Phys. Lett.* **B82** (1979) 60.

[8] I. Antoniadis, *Phys. Lett.* **B246** (1990) 377; A. Pomarol, M. Quirós, *Phys. Lett.* **B438** (1998) 255; I. Antoniadis, S. Dimopoulos, A. Pomarol, M. Quirós, *Nucl. Phys.* **B544** (1999) 503; R. Barbieri, L.J. Hall, Y. Nomura, *Phys. Rev.* **D63** (2001) 105007.

[9] G. Dvali, M. Shifman, *Nucl. Phys.* **B504** (1997) 127; M. Chaichian, A.B. Kobakhidze, M. Tsulaia, *Phys. Lett.* **B505** (2001) 222.

[10] Y. Kawamura, *Prog. Theor. Phys.* **103** (2000) 613; *Prog. Theor. Phys.* **105** (2001) 691; G. Altarelli, F. Feruglio, *Phys. Lett.* **B511** (2001) 257; A.B. Kobakhidze, *Phys. Lett.* **B514** (2001) 131; hep-ph/0108049; L. Hall, Y. Nomura, hep-ph/0103125; Z. Berezhiani, I. Gogoladze, A. Kobakhidze, hep-ph/0104288 ; T. Kawamoto, Y. Kawamura, hep-ph/0106163; A. Hebecker, J. March-Russell, hep-ph/0107004; R. Barbieri, L.J. Hall, Y. Nomura, hep-ph/0107039; J.A. Bagger, F. Feruglio, F. Zwirner, hep-th/0107128; T. Li, hep-th/0107136; hep-ph/0108120; L.J. Hall, H. Murayama, Y. Nomura, hep-th/0107245; N. Maru, hep-ph/0108002; T. Asaka, W. Buchmuller, L. Covi, hep-ph/0108021; L. Hall, Y. Nomura, T. Okui, D. Smith, hep-ph/0108071.

[11] C. Csaki, G.D. Kribs, J. Terning, hep-ph/0107266; H.-C. Cheng, K.T. Matchev, J. Wang, hep-ph/0107268.

[12] R.N. Mohapatra, A. Pèrez-Lorenzana, *Phys. Lett.* **B468** (1999) 195; D. Chang, R.N. Mohapatra, hep-ph /0103432.
[13] N. Arkani-Hamed, T. Gregoire, J. Wacker, hep-th/0101233; D. Marti, A. Pomarol, hep-th/0106256.

[14] M. Chaichian, A.B. Kobakhidze, Phys. Lett. B478 (2000) 299; A.B. Kobakhidze, hep-th/0012191.

[15] K.R. Dienes, E. Dudas, T. Gherghetta, Phys. Lett. B436 (1998) 55; Nucl. Phys. B537 (1999) 47; S.A. Abel, S.F. King, Phys. Rev. D59 (1999) 095010; A.B. Kobakhidze, Phys. Atom. Nucl. 64 (2001) 941; M. Bando, T. Kobayashi, T. Noguchi, K. Yoshioka, Phys. Lett. B480 (2000) 187.

[16] J.L. Chkareuli, JETP Lett. 32 (1980) 671; Z.G. Berezhiani, J.L. Chkareuli, Yad. Fiz. 37 (1983) 1043; F. Wilczek, preprint NSF-ITP-83-08 (1983); Z.G. Berezhiani, Phys. Lett. B129 (1983) 99; J.C. Wu, Phys. Rev. D36 (1987) 1514; Z. Berezhiani, Phys. Lett. B417 (1998) 287; J.L. Chkareuli, C.D. Froggatt, H.B. Nielsen, in preparation.

[17] C.G. Callan, J.A. Harvey, Nucl. Phys. B250 (1985) 427.

[18] J.L. Chkareuli, JETP Lett. 32 (1980) 671, ibid. 36 (1983) 493; I. Montvay, Phys. Lett. 95B (1980) 227; J.E. Kim, H.S. Song, Phys. Rev. D23 (1981) 2102; M. Chaichian, Yu.N. Kolmakov, N.F. Nelipa, preprint HU-TFT-82-15 (1982); Z. Phys. C43 (1989) 381.

[19] J. C. Pati, M. Cvetic, H.S. Sharatchandra, Phys.Rev.Lett. 58 (1987) 851.