Charm partner of the exotic $X(5568)$ state and its properties

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The mass, decay constant and width of a hypothetical charmed partner $X_c$ of the newly observed exotic $X_b(5568)$ state are calculated using a technique of QCD sum rule method. The $X_c = [su][cd]$ state with $J^P = 0^+$ is described employing two types of the diquark-antidiquark interpolating currents. The evaluation of the mass $m_{X_c}$ and decay constant $f_{X_c}$ is carried out utilizing the two-point sum rule method by including vacuum condensates up to eight dimensions. The widths of the decay channels $X_c \rightarrow D_{s0}^\mp \pi^\mp$ and $X_c \rightarrow D^0 K^0$ are also found. To this end, the strong couplings $g_{X_c D_s \pi}$ and $g_{X_c DK}$ are computed by means of QCD sum rules on the light-cone and soft-meson approximation.

I. INTRODUCTION

The D0 Collaboration recently reported the observation of a narrow structure $X_b(5568)$ in the decay process $X_b(5568) \rightarrow B_b^0 \pi^\pm$, $B_b^0 \rightarrow J/\psi \phi$, $J/\psi \rightarrow \mu^+ \mu^-$, $\phi \rightarrow K^0 \bar{K}^0$ based on $p\bar{p}$ collision data at $\sqrt{s} = 1.96$ TeV collected at the Fermilab Tevatron collider [1]. The new state $X_b(5568)$ is considered to posses quantum numbers $J^{PC} = 0^{++}$. The D0 Collaboration provides the value $m_{X_b} = 5567.8 \pm 2.9$ (stat)$^{+3.0}_{-1.9}$ (syst) MeV for its mass, and estimates $\Gamma = 21.9 \pm 6.4$ (stat)$^{+5.0}_{-2.5}$ (syst) MeV for its decay width. As it was emphasized in Ref. [1] this is the first observation of a hadronic state with four different quark flavors. Thus, $X_b$ state is composed of $b$, $s$, $u$, $d$ quarks.

Suggestions concerning the possible quark-antiquark organization of $X_b$ were made already in Ref. [1]. Thus, within the diquark-antidiquark model, the $X_b$ state with positive charge, i.e. the particle $X_b^+$ may be described as $[b\bar{u}] [d\bar{s}]$ or $[s\bar{u}] [\bar{b}\bar{d}]$ bound states, whereas $X_b^0$ may have the structures $[b\bar{d}] [\bar{s}\bar{u}]$ or $[s\bar{d}] [\bar{b}\bar{u}]$. Alternatively, $X_b$ may be considered as a molecule composed of $B$ and $K$-mesons.

This is a valuable discovery, because the charmonium-like resonances that populate $XYZ$ family of ”traditional” exotic states, contain $c\bar{c}$ charm quark-antiquark pair and hence, the number of the quark flavors in these particles does not exceed three. Properties of known exotic states extracted from experimental data and theoretical calculations can be found in, for instance, review papers [2–9] and references therein.

The newly observed state $X_b(5568)$ has immediately attracted interests of physicists and stimulated theoretical studies of $X_b$ in the context of different approaches [10–18]. Thus, in Refs. [10, 11] we have calculated the mass, decay constant and width of the $X_b(5568)$ state within the diquark-antidiquark picture $X_b = [su][bd]$ considering the exotic state with positive charge. Our predictions for the mass $m_{X_b}$, and for the width of its decay $\Gamma(X_b^+ \rightarrow B_b^0 \pi^\mp)$ are in agreement with the experimental data. It is worth noting that in the context of the diquark model some parameters of $X_b$ were also analyzed in Refs. [12, 13]. In these works the authors use various versions for the diquark-antidiquark type interpolating currents with different Lorentz structures. It is remarkable, that the obtained values for $m_{X_b}$ are in agreement with each other and also consistent with experimental data of D0 Collaboration. The molecule picture for $X_b$ was realized in Ref. [18], where the $X_b(5568)$ state was taken as the $B\bar{K}$ bound state. The questions of quark-antiquark organization of this particle and its partners were addressed in Ref. [17].

In the present work we are going to continue our investigation of the new family of the four-quark exotic states by considering the hypothetical charmed partner of the $X_b(5568)$ state, which is composed of the $c$, $s$, $u$, $d$ quarks. We assume that this state bears the same quantum numbers as its counterpart, i.e. $J^{PC} = 0^{++}$. We also accept that it has the internal structure $X_c = [su][\bar{c}\bar{d}]$ in the diquark-antidiquark model. Thus, the partner state $X_c$ is a neutral particle. Our aim is to determine the parameters of the state $X_c$, i.e. to find its mass, decay constant and widths of the strong $X_c \rightarrow D_{s0}^\mp \pi^\mp$ and $X_c \rightarrow D^0 K^0$ decays. For these purposes, we apply methods presented in a rather detailed form in Refs. [10, 11, 19].

This work is structured in the following way. In Section III we introduce the interpolating currents employed in QCD sum rule calculations. Here we find the mass and decay constant of $X_c$ using the two-point QCD sum rule approach. The widths of the strong decays $X_c \rightarrow D_{s0}^\mp \pi^\mp$ and $X_c \rightarrow D^0 K^0$ are subject of Sect. III. Explicit expression for the spectral density required in computation of the mass and decay constant of the exotic $X_c$ is moved to Appendix A.
II. THE MASS AND DECAY CONSTANT OF $X_c$  

As it has been noted above, we use the two-point QCD sum rule approach in order to compute mass and decay constant of the $X_c$ state. To this end, we consider the two-point correlation function given as

$$\Pi(p) = i \int d^4x e^{ipx} \langle 0|J_{1(2)}^X(x) J_{1(2)}^X(0)|0\rangle,$$  

(1)

where $J_{1(2)}^X(x)$ are the interpolating currents with required quantum numbers. We consider $X_c$ state as a particle with the quantum numbers $J^{PC} = 0^{++}$. Then in the diquark-antidiquark model the current $J_1^X(x)$ is given by the following expression

$$J_1^X(x) = \varepsilon^{ijk} \varepsilon^{imn} [\bar{s}^i(x) C\gamma_\mu u^k(x)] \left[\bar{c}^m(x) \gamma_\mu C\bar{d}^n(x)\right].$$  

(2)

Alternatively, one may introduce the interpolating current

$$J_2^X(x) = \varepsilon^{ijk} \varepsilon^{imn} [\bar{s}^i(x) C\gamma_\mu u^k(x)] \left[\bar{c}^m(x) \gamma_\mu d^n(x)\right].$$  

(3)

In Eqs. (2) and (3) $i$, $j$, $k$, $m$, $n$ are color indexes and $C$ is the charge conjugation matrix.

Let us note that the current $J_1^X(x)$ has been employed throughout in Refs. [10, 11] for exploration the exotic $X_b(5568)$ state. The sum rules derived in these works, after trivial replacements of corresponding parameters, can easily be applied to analyze the $X_c$ state. Therefore, in what follows we concentrate on the current $J_2^X(x)$ omitting, in what follows, the subscript in its definition.

The representation of the function $\Pi(p)$ in terms of the physical quantities does not depend on the form of the interpolating current and is the same for both $J_{1(2)}^X(x)$

$$\Pi^{\text{Phys}}(p) = \frac{\langle 0|J_{1(2)}^X(x)|X_c(p)\rangle}{m^2_{X_c} - p^2} + \ldots$$  

(4)

where $m_{X_c}$ is the mass of the $X_c$ state, and dots stand for contributions of the higher resonances and continuum states. We define the decay constant $f_{X_c}$ through the matrix element

$$\langle 0|J_{1(2)}^X|X_c(p)\rangle = f_{X_c} m_{X_c}. $$  

(5)

Then, for the correlation function we obtain

$$\Pi^{\text{Phys}}(p) = \frac{m_{X_c}^2 f_{X_c}^2}{m^2_{X_c} - p^2} + \ldots$$  

(6)

The Borel transformation applied to Eq. (6) yields

$$B_p \Pi^{\text{Phys}}(p) = m_{X_c}^2 f_{X_c}^2 e^{-m_{X_c}^2 / M^2} + \ldots$$  

(7)

The theoretical expression for the same function, $\Pi^{\text{QCD}}(p)$, has to be determined employing the quark-gluon degrees of freedom. Contracting the quark fields we find for the correlation function $\Pi^{\text{QCD}}(p)$:

$$\Pi^{\text{QCD}}(p) = i \int d^4x e^{ipx} \langle 0|J_{1(2)}^X(x) J_{1(2)}^X(0)|0\rangle.$$  

(8)

where $S_q^i(x)$ and $S_{\bar{q}}^i(x)$ are the light ($q \equiv u, d$ or $s$) and $c$-quark propagators, respectively. In Eq. (8) we introduce the notation

$$\bar{S}_q^i(x) = CS_q^{ij}C(x).$$

In the $x$-space the light quark propagator $S_q^i(x)$ has the form

$$S_q^i(x) = i \delta_{ij} \frac{\not{x}}{2\pi^2 x^4} - \delta_{ij} \frac{m_q}{4\pi^2 x^2} - \delta_{ij} \frac{\langle 7q \rangle}{12} + i \delta_{ij} \frac{\not{x} m_q}{48 \pi^2 x^2} - \delta_{ij} \frac{x^2}{192 \pi^2} G_{7qGq} + i \delta_{ij} \frac{x^2 \not{x} m_q}{152 \pi^2} G_{7qGq}.$$  

(9)

For the $c$-quark propagator $S_{\bar{q}}^i(x)$ we employ the expression from Ref. [20]

$$S_{\bar{q}}^i(x) = i \int d^4k \frac{e^{-ikx}}{(2\pi)^4} \left[ \delta_{ij} \left( \frac{k^2 + m_c^2}{k^2} \right) - \frac{g G_{ij}^{\alpha \beta} \delta_{ij} m_c}{4 (k^2 - m_c^2)} \right].$$  

(10)

In Eqs. (9) and (10)

$$G_{ij}^{\alpha \beta} \equiv G_{ij} G_{\alpha \beta}, \quad a = 1, 2 \ldots 8,$$

where $i, j$ are color indexes, and $t^a = \lambda^a / 2$ with $\lambda^a$ being the standard Gell-Mann matrices. The first term in Eq. (10) is the perturbative propagator of a massive quark, the next two terms are nonperturbative gluon corrections. In the nonperturbative terms the gluon field strength tensor $G_{ij}^{\alpha \beta}$ is fixed at $x = 0$.

The correlation function $\Pi^{\text{QCD}}(p^2)$ is given by a simple dispersion integral

$$\Pi^{\text{QCD}}(p^2) = \frac{\rho^{\text{QCD}}(s)}{s - p^2} + \ldots,$$  

(11)

where $\rho^{\text{QCD}}(s)$ is the corresponding spectral density. It can be computed using mathematical methods described in Refs. [10, 11]. Therefore, here we omit details of calculations and provide explicit expressions for both $\rho^{\text{QCD}}(s)$ in Appendix A.
Applying the Borel transformation on the variable $p^2$ to the invariant amplitude $\Pi^{QCD}(p^2)$, equating the obtained expression $\mathcal{B}_p\Pi^{\text{Phys}}(p)$, and subtracting the continuum contribution, we finally obtain the required sum rule. Thus, the mass of the $X_c$ state can be evaluated from the sum rule

$$m^2_{X_c} = \int_{(m_c+m_{X_c})^2}^{s_0} ds \rho^{QCD}(s)e^{-s/M^2}.$$  \hspace{1cm} (12)

whereas for the decay constant $f_{X_c}$ we employ the formula

$$f_{X_c}^2 m_{X_c}^2 e^{-m_{X_c}^2/M^2} = \int_{(m_c+m_{X_c})^2}^{s_0} ds \rho^{QCD}(s)e^{-s/M^2}.$$ \hspace{1cm} (13)

The last two expressions are the sum rules needed to evaluate the $X_c$ state’s mass and decay constant, respectively. For numerical computation we need values of the quark, gluon and mixed condensates. Additionally, QCD sum rules contain $c$ and $s$ quark masses. The values of used parameters are moved to Table I.

Sum rule calculations imply fixing regions for the parameters $s_0$ and $M^2$, where they can be varied. For $s_0$ we employ

$$7.56 \text{ GeV}^2 \leq s_0 \leq 8.12 \text{ GeV}^2.$$ \hspace{1cm} (14)

We find the range $2 \text{ GeV}^2 < M^2 < 4 \text{ GeV}^2$ as a reliable region for varying the Borel parameter. Here the effects of the higher resonances and continuum states, and contributions of the higher dimensional condensates satisfy well known requirements of QCD sum rule calculations. It is not difficult to see that, in these intervals, the dependences of the mass and decay constant on $M^2$ and $s_0$ are very weak, and we expect that the sum rules give the firm predictions (see Figs. 1 and 2). We estimate errors of the numerical computations by varying the parameters $M^2$ and $s_0$ within the accepted ranges, as well as taking into account uncertainties coming from other input parameters.

For the mass and decay constant of the $X_c$ state we find:

$$m_{X_c} = (2590 \pm 60) \text{ MeV},$$

$$f_{X_c} = (0.20 \pm 0.03) \cdot 10^{-2} \text{ GeV}^4,$$ \hspace{1cm} (15)

when using the interpolating current $J^{X_c}_1$, and

$$m_{X_c} = (2634 \pm 62) \text{ MeV},$$

$$f_{X_c} = (0.11 \pm 0.02) \cdot 10^{-2} \text{ GeV}^4$$ \hspace{1cm} (16)

in the case of $J^{X_c}_2$. As is seen, for the mass of the $X_c$ state the different interpolating currents lead to predictions, which are very close to each other. The result for the mass of the $X_c$ state obtained in Ref. [14]

$$m_{X_c} = (2.55 \pm 0.09) \text{ GeV},$$ \hspace{1cm} (17)

within the errors is in agreement with our predictions.

| Parameters        | Values                           |
|-------------------|----------------------------------|
| $m_c$             | $(1.275 \pm 0.025) \text{ GeV}$  |
| $m_s$             | $(95 \pm 5) \text{ MeV}$        |
| $\langle \overline{q}q \rangle$ | $(-0.24 \pm 0.01)^3 \text{ GeV}^3$ |
| $\langle s\bar{s} \rangle$ | $0.8 \langle \overline{q}q \rangle$ |
| $\langle \overline{q}qG_q \rangle$ | $(0.012 \pm 0.004) \text{ GeV}^4$ |
| $m_0^2$           | $(0.8 \pm 0.1) \text{ GeV}^2$    |
| $\langle \overline{q}q \rangle$ | $m_0^2 \langle \overline{q}q \rangle$ |

TABLE I: Input parameters used in calculations.

FIG. 1: The mass $m_{X_c}$ as a function of the Borel parameter $M^2$ for different values of $s_0$. In calculations the current $J^{X_c}_2$ is used.

III. THE STRONG DECAYS OF THE $X_c$ STATE

Predictions for the mass of the $X_c$ state obtained in the previous section allow us to continue our exploration by considering its possible decay channels and to calculate

FIG. 2: The decay constant $f_{X_c}$ vs Borel parameter $M^2$ for $J^{X_c}_2$. The values of the parameter $s_0$ are shown in the figure.
their decay widths. From the quark content and assigned quantum numbers, it is easy to conclude that the $X_c$ state can decay into $D^-_s(sar{c}) + \pi^+(ud)$ or $D^0(uar{c}) + K^0(sar{d})$. In other words, $X_c \rightarrow D^-_s \pi^+$ and $X_c \rightarrow D^0 K^0$ transitions are kinematically allowed decay channels of the $X_c$ state. Our aim in this section is to find widths of these decays. To this end, we calculate the strong couplings $g_{X_c,D_s\pi}$ and $g_{X_c,D_K}$ using the method of QCD sum rule on the lightcone in conjunction with the soft-meson approximation [19].

We start our analysis from the decay $X_c \rightarrow D^-_s \pi^+$. In order to calculate the required strong coupling $g_{X_c,D_s\pi}$ we consider the correlation function

$$\Pi(p, q) = i \int d^4x e^{ipx} \pi(q) | J^{D_s}(x) J^{X_c}(0) \rangle \langle 0 |.$$  (18)

Here the interpolating current $J^{X_c}(x)$ is given by Eq. (3), whereas for $D_s$ we use

$$J^{D_s}(x) = \bar{c}(i \gamma_5 s^I(x).$$  (19)

It is not difficult to find $\Pi(p, q)$ in terms of the physical degrees of freedom:

$$\Pi^{\text{Phys}}(p, q) = \left( \frac{0| J^{D_s} | D_s(p) \rangle \langle D_s(p) \pi(q) | X_c(p') \rangle}{p^2 - m_{D_s}^2} \right)\times \frac{\langle X_c(p') | J^{X_c} | 0 \rangle}{p'^2 - m_{X_c}^2} + \ldots,$$  (20)

where by dots we denote contributions of the higher resonances and continuum states. Here $p$, $q$, and $p' = p + q$, are the momenta of $D_s$, $\pi$, and $X_c$ states, respectively. In order to finish computation of the correlation function we introduce the matrix elements

$$\langle 0 | J^{D_s} | D_s(p) \rangle = \frac{f_{D_s} m_{D_s}^2}{m_c + m_s},$$

$$\langle X_c(p') | J^{X_c} | 0 \rangle = f_{X_c} m_{X_c},$$

$$\langle D_s(p) \pi(q) | X_c(p') \rangle = g_{X_c,D_s\pi} p \cdot p',$$  (21)

where $f_{D_s}$ and $m_{D_s}$ are the decay constant and mass of the $D_s$ state, whereas $f_{X_c}$ and $m_{X_c}$ are the same parameters of the $D_s$ meson.

We calculate $\Pi^{\text{Phys}}(p, q)$ in the soft-meson limit $q = 0$, and after some manipulations described in Refs. [11, 19], for the Borel transformation of the correlation function find

$$\Pi^{\text{Phys}}(M^2) = \frac{f_{D_s} f_{X_c} m_{X_c} m_{D_s}^2 g_{X_c,D_s\pi}}{(m_c + m_s) m^2} \times \frac{1}{M^2} e^{-m^2/M^2},$$  (22)

where $m^2 = (m_{X_c}^2 + m_{D_s}^2)/2$.

To proceed, we have to calculate $\Pi^{\text{QCD}}(p, q)$ in terms of the quark-gluon degrees of freedom and find QCD side of the sum rule. Contractions of $s$ and $c$-quark fields in Eq. (18) yield

$$\Pi^{\text{QCD}}(p, q) = \int d^4x e^{ipx} e^{-iqx} \left[ \gamma_5 S_{\pi}^{ij}(x) \gamma_5 \right] \times S_m^{\pi l}(x) \langle \pi(q) | \pi^0(0) \rangle d_m^0(0) | 0 \rangle,$$  (23)

where $\alpha$ and $\beta$ are the spinor indexes.

Skipping technical details, which can be found in Refs. [11, 19], we provide final expression for the spectral density, which is given as a sum of the perturbative and nonperturbative components

$$\rho_{\text{coup}}(s) = \rho_{\text{pert}}(s) + \rho_{\text{n-pert}}(s).$$  (24)

where

$$\rho_{\text{pert}}(s) = \frac{f_\pi \mu_\pi}{16 \pi^2 s} \sqrt{s(4m_\pi^2 - 4m_\pi^2)} (s + 2m_c m_s - 2m_\pi^2),$$  (25)

and

$$\rho_{\text{n-pert}}(s) = \frac{f_\pi \mu_\pi}{72} \left\{ 6(\bar{q}q) - 2m_c \delta(s - m_\pi^2) + s m_\pi \delta^{(1)}(s - m_\pi^2) + \left( 6(m_c - m_\pi) \delta^{(1)}(s - m_\pi^2) - 3s(m_c - m_\pi) \delta^{(2)}(s - m_\pi^2) - 2s_m \delta^{(3)}(s - m_\pi^2) \right) \right\}.$$  (26)

In Eq. (26) $\delta^{(n)}(s - m_\pi^2) = (d/ds)^n \delta(s - m_\pi^2)$ that appear when extracting the imaginary part of the pole terms.

As is seen, in the soft limit the spectral density depends only the parameters $f_\pi$ and $\mu_\pi$ through the pion’s local matrix element

$$\langle 0 | \bar{c}(0) i \gamma_5 u(0) | \pi(q) \rangle = f_\pi \mu_\pi,$$  (27)

where

$$\mu_\pi = \frac{m_\pi^2}{m_u + m_d} = -\frac{2(\bar{q}q)}{f_\pi^2}. $$  (28)

Continuum subtraction performed in the standard way leads to the final sum rule for evaluating of the strong coupling

$$g_{X_c,D_s\pi} = \frac{1}{f_{D_s} f_{X_c} m_{X_c} m_{D_s}^2} \left( 1 - M^2 \frac{d}{dM^2} \right) M^2 \int_{m_c}^{s_0} ds \rho_{\text{QCD}}(s).$$  (29)

The width of the decay $X_c \rightarrow D^-_s \pi^+$ can be found applying the standard methods and is given in Ref. [11]:

$$\Gamma \left( X_c \rightarrow D^-_s \pi^+ \right) = \frac{g_{X_c,D_s\pi} m_{D_s}^2}{24\pi} \lambda(m_{X_c}, m_{D_s}, m_\pi) \times \left[ 1 + \frac{\lambda^2(m_{X_c}, m_{D_s}, m_\pi)}{m_{D_s}^2} \right],$$  (30)
where
\[ \lambda(a, b, c) = \frac{\sqrt{a^4 + b^4 + c^4 - 2(a^2b^2 + a^2c^2 + b^2c^2)}}{2a}. \]

Equations (29) and (30) are final expressions that will be used for numerical analysis of the decay channel \( X_c \rightarrow D_s^- \pi^+ \).

The investigation of the transition \( X_c \rightarrow D^0 K^0 \) can be carried out in the same manner as for the decay \( X_c \rightarrow D_s^- \pi^+ \). One needs only to replace the parameters of the particles in accordance with the prescription \( \pi \rightarrow K \), and \( D_s \rightarrow D \). Nevertheless, below we write down some key expressions.

Thus, the analysis of the vertex \( X_cDK \), which is necessary to derive the sum rule for the coupling \( g_{X_cDK} \), is founded on the correlation function
\[
\Pi_K(p, q) = i \int d^4xe^{ipx}(K(q)|T\{J^D(x)X_c^+(0)\}|0),
\]
where for \( D^0 \) meson we employ the interpolating current
\[
J^D(x) = \bar{c}(x)i\gamma_5u(x).
\]

In the soft-meson limit \( q = 0 \), the Borel transformation of the correlation function \( \Pi_{K}^{\text{Phys}}(p, q) \) is given by
\[
\Pi_{K}^{\text{Phys}}(M^2) = \frac{f_Df_{X_c}m_{X_c}m_{DK}^2g_{X_cDK}m^2}{(m_c + m_u)} \times \frac{1}{M^2}e^{-m^2/M^2}.
\]

In the formula above \( m^2 = (m_{X_c}^2 + m_{DK}^2)/2 \), and \( m_D \) and \( f_D \) are the mass and decay constant of D meson, respectively.

In terms of the quark-gluon degrees of freedom the same function is determined by means of the formula
\[
\Pi_K^{\text{QCD}}(p, q) = \int d^4xe^{ipx}e^{ik_c}e^{imn}\left[\gamma_5\tilde{S}_d^{ij}(x)\gamma_5\right] \times \tilde{S}_c^{m}(x)\gamma_5\tilde{S}_d^{m}(x)\gamma_5\left(K(q)|\bar{c}(0)u(0)d(0)\gamma_5c(0)\right).
\]

Its imaginary part gives us the spectral density \( \rho_{\text{comp}}^{\text{QCD}}(s) \), which now depends on the K meson local matrix element
\[
\langle 0|\bar{c}(0)i\gamma_5s(0)|K(q)\rangle = \frac{f_Km_K^2}{m_s + m_d},
\]
with \( m_K \) and \( f_K \) being the mass and decay constant of the K meson. The remaining analysis is the same as for the \( X_c \rightarrow D_s^- \pi^+ \) decay: after evident changes in the relevant final expressions, they can be utilized for studying the \( X_c \rightarrow DK \) transition, as well.

The QCD sum rules derived above contain, as input parameters, the masses and decay constants of the \( D_s, D, \pi \) and \( K \) mesons. They are collected in Table II. It is worth noting that for the decay constants \( f_D \) and \( f_{D_s} \) we use the lattice result from Ref. [21].

| Parameters | Values |
|------------|--------|
| \( m_{D_s} \) | (1968.30 ± 0.10) MeV |
| \( f_{D_s} \) | (260.1 ± 10.8) MeV |
| \( m_D \) | (1864.84 ± 0.05) MeV |
| \( f_D \) | (218.9 ± 11.3) MeV |
| \( m_K \) | 497.61 MeV |
| \( f_K \) | 156 MeV |
| \( m_s \) | 139.57 MeV |
| \( f_s \) | 131 MeV |

TABLE II: Input parameters used in the coupling calculations.

| Parameters | Values |
|------------|--------|
| \( g_{X_cDK} \) | (0.51 ± 0.10) GeV⁻¹ |
| \( \Gamma_{D_s} \) | (8.0 ± 2.0) MeV |
| \( g_{X_cDK} \) | (1.57 ± 0.34) GeV⁻¹ |
| \( \Gamma_{DK} \) | (55.4 ± 14.0) MeV |

TABLE III: The sum rule predictions for the strong couplings and corresponding decay widths.

The results of the numerical calculations of the strong couplings and decay widths are shown in Table II. We find that the transition \( X_c \rightarrow DK \) may be viewed as the dominant decay channel of the \( X_c \) state. The total width of this particle computed by taking into account the explored decay channels equals to
\[
\Gamma_{X_c}^1 \simeq (63.4 ± 14.2) \text{ MeV},
\]
and
\[
\Gamma_{X_c}^2 \simeq (53.7 ± 11.6) \text{ MeV},
\]
for the first and second interpolating currents, respectively. It is seen that results obtained for the total width of the \( X_c \) state using various interpolating currents, within errors, are compatible with each other, nevertheless the difference between the central values are sizeable. The experimental exploration of the \( X_c \) state, and its observation may extend our knowledge about the nature and internal structure of the new exotic states.

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Appendix: A

In this appendix we have collected the results of our calculations of the spectral density

$$\rho_{\text{QCD}}(s) = \rho_{\text{pert}}(s) + \sum_{k=3}^{8} \rho_k(s), \quad (A.1)$$

used for evaluation of the $X_c$ meson mass $m_{X_c}$ and its decay constant $f_{X_c}$ from the QCD sum rule. In Eq. (A.1) by $\rho_k(s)$ we denote the nonperturbative contributions to $\rho_{\text{QCD}}(s)$. In calculations we have neglected the masses of the $u$ and $d$ quarks and taken into account terms $\sim m_s$. The explicit expressions for $\rho_{\text{pert}}(s)$ and $\rho_k(s)$ in the case of the current $J_{1c}^q(x)$ are presented below as integrals over the Feynman parameter $z$. Note that in $\rho_k(s)$, we keep only the term containing the gluonic contribution.

\begin{align*}
\rho_{\text{pert}}(s) &= \frac{1}{6144\pi^6} \int_0^a \frac{dz \langle z \rangle^4}{(z-1)^3} \left[ m_c^2 + s(z-1) \right]^3 \left[ m_c^2 + 3s(z-1) \right], \\
\rho_3(s) &= \frac{1}{64\pi^3} \int_0^a \frac{dz \langle z \rangle^2}{(z-1)^2} \left[ m_c^2 + s(z-1) \right] \left\{ \langle d\bar{d} \rangle m_c \left[ m_c^2 + s(z-1) \right] + m_s \langle (\bar{s}s) - 2(\bar{u}u) \rangle \left[ m_c^2 + 2s(z-1) \right] (z-1) \right\}, \\
\rho_4(s) &= \frac{1}{9216\pi^4} \langle \alpha_s G^2 \rangle \int_0^a \frac{dz \langle z \rangle^2}{(z-1)^3} \left\{ 2m_c^4 \left[ z(7z-15) + 9 \right] + 3m_c^2 s(z-1) \left[ z(13z-30) + 18 \right] + 12s^2(z-1)^3(2z-3) \right\}, \\
\rho_5(s) &= \frac{m_c^6}{192\pi^4} \int_0^a \frac{dz}{(z-1)} \left[ 3m_c \langle d\bar{d} \rangle \left[ m_c^2 + s(z-1) \right] + m_s (z-1) \langle (\bar{s}s) - 3(\bar{u}u) \rangle \left[ 2m_c^2 + 3s(z-1) \right] \right], \\
\rho_6(s) &= \frac{g^2}{1296\pi^4} \int_0^a dz \langle (\bar{u}u)^2 + (\bar{d}d)^2 + (\bar{s}s)^2 \rangle \left[ 2m_c^2 + 3s(z-1) \right], \\
\rho_7(s) &= \frac{1}{576\pi^2} \langle \alpha_s G^2 \rangle \int_0^a dz \left\{ 4m_c \langle d\bar{d} \rangle + m_s \langle (\bar{u}u) (4z+2) - 3(\bar{s}s) \rangle \right\}, \\
\rho_8(s) &= -\frac{11}{36864\pi^2} \langle \alpha_s G^2 \rangle \int_0^a dz \langle z \rangle,
\end{align*}

(A.2)

where $a = (s - m_c^2)/s$.

[1] V. M. Abazov et al. [D0 Collaboration], Observation of a new $B^0_s \pi^\pm$ state, [arXiv:1602.07588] [hep-ex].
[2] E. S. Swanson, The New heavy mesons: A Status report, Phys. Rept. 429, 243 (2006).
[3] E. Klempt and A. Zaitsev, Glueballs, Hybrids, Multi-quarks. Experimental facts versus QCD inspired concepts, Phys. Rept. 454, 1 (2007).
[4] S. Godfrey and S. L. Olsen, The Exotic XYZ Charmonium-like Mesons, Ann. Rev. Nucl. Part. Sci. 58, 51 (2008).
[5] M. B. Voloshin, Charmonium, Prog. Part. Nucl. Phys. 61, 455 (2008).
[6] M. Nielsen, F. S. Navarra, and S. H. Lee, Phys. Rep. 497, 41 (2010).
[7] R. Faccini, A. Pilloni and A. D. Polosa, Exotic Heavy Quarkonium Spectroscopy: A Mini-review, Mod. Phys. Lett. A 27, 1230025 (2012).
[8] A. Esposito, A. L. Guerrieri, F. Piccinini, A. Pilloni and A. D. Polosa, Four-Quark Hadrons: an Updated Review, Int. J. Mod. Phys. A 30, 1530002 (2014).
[9] H.-X. Chen, W. Chen, X. Liu, and S.-L. Zhu, Arxiv: 1601.02092 [hep-ph], 2016.
[10] S. S. Agaev, K. Azizi and H. Sundu, Mass and decay constant of the newly observed exotic $X(5568)$ state, Phys. Rev. D, to be published, [arXiv:1602.08642] [hep-ph].
[11] S. S. Agaev, K. Azizi and H. Sundu, Width of the exotic $X_0(5568)$ state through its strong decay to $B^0_s \pi^\pm$, [arXiv:1603.00290] [hep-ph].
[12] Z. G. Wang, Analysis of the $X(5568)$ as scalar tetraquark state in the diquark-antidiquark model with QCD sum rules, [arXiv:1602.08711] [hep-ph].
[13] W. Wang and R. Zhu, Can $X(5568)$ be a tetraquark state?, [arXiv:1602.08806] [hep-ph].
[14] W. Chen, H. X. Chen, X. Liu, T. G. Steele and S. L. Zhu, Investigation of the $X(5568)$ as a fully open-flavor $s\bar{d}$ tetraquark state, [arXiv:1602.08916] [hep-ph].
[15] C. M. Zanetti, M. Nielsen and K. P. Khemchandani, A QCD sum rule study for a charged bottom-strange scalar meson, [arXiv:1602.09041] [hep-ph].
[16] C. J. Xiao and D. Y. Chen, Possible $B^{(*)}\bar{K}$ hadronic molecule state, [arXiv:1603.00228] [hep-ph].
[17] Y. R. Liu, X. Liu and S. L. Zhu, $X(5568)$ and its partner states, [arXiv:1603.01131] [hep-ph].
[18] X. H. Liu and G. Li, Could the observation of $X(5568)$ be resulted by the near threshold rescattering effects?, [arXiv:1603.00708] [hep-ph].
[19] S. S. Agaev, K. Azizi and H. Sundu, Strong $Z_c^+(3900) \rightarrow J/\psi \pi^+; \eta_c \rho^+$ decays in QCD, Phys. Rev. D 93, 074002 (2016).
[20] L. J. Reinders, H. Rubinstein and S. Yazaki, Hadron Properties from QCD Sum Rules, Phys. Rept. 127, 1 (1985).
[21] A. Bazavov et al. [Fermilab Lattice and MILC Collaborations], B- and D-meson decay constants from three-flavor lattice QCD, Phys. Rev. D 85, 114506 (2012).