Correlated Non-Linear Phase Noise in Multi-Subcarrier Systems

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Abstract: We explore correlated nonlinear phase noise (NLPN) in multi-subcarrier systems. We derive an analytical model for predicting the covariance between the NLPN affecting different subcarriers, and offer a simple algorithm which uses the correlations for improved NLPN mitigation.

OCIS codes: (060.2330) Fiber optics communications; (060.4080) Modulation.

1. Introduction

Reduction of symbol rate, i.e. subcarrier multiplexing (SCM), is a powerful technique that allows optimizing the system with respect to fiber Non-Linear Interference Noise (NLIN) [1], and reducing certain aspects of transceiver complexity [2]. Yet, although the overall NLIN power reduces when optimizing the number of subcarriers, a greater portion of it is due to Non-Linear Phase Noise (NLPN) [3]. In addition, the use of Probabilistic Shaping (PS), which makes the modulation more ‘Gaussian-like’ [4], is expected to further increase the amount of generated NLPN [5].

It has been shown that the relatively long correlation times that characterize NLPN can be used in order to effectively mitigate it by means of adaptive equalization [6]. However, SCM reduces the correlation length (in terms of number of symbols), making the use of adaptive equalization more challenging. A remedy to this situation is to take advantage of the fact that all subcarriers are jointly processed by the same receiver, and hence correlations between them can be taken advantage of.

In this paper we demonstrate the existence of strong correlations between the NLPN of different subcarriers in SCM systems. We derive an analytical expression for the correlation and validate it with numerical simulations. We report of an initial attempt to exploit these correlations by means of an RLS-based joint processing algorithm, which is already producing encouraging results. It is expected that customized algorithms aiming at exploiting the correlations between subcarriers will significantly improve system performance.

2. Estimating NLPN correlations

The phase-noise \( \phi_n^{(j)} \) affecting the \( n \)-th symbol of the \( j \)-th subcarrier in the channel of interest (COI) as a result of a single interfering channel (IC), is given by [6]

\[
\phi_n^{(j)} = \sum_{k,m} X_{k,m}^{(j)} b_{n-k}^* b_{n-m},
\]

where \( b_n \) are the symbols of the IC, and \( X_{k,m}^{(j)} \) are the interaction coefficients (which are independent of the signal power or properties). Since the subcarriers are spectrally close to one another they are affected by the IC in a similar way and hence the coefficients \( X_{k,m}^{(j)} \) are similar for different subcarriers, producing a correlation between subcarriers’ NLPN. It can be shown that the covariance between the NLPN of the \( i \)-th and \( j \)-th subcarriers is given by

\[
\text{Cov}(i,j) = \langle \phi_i^{(i)} \rangle \langle \phi_j^{(j)} \rangle - \langle \phi_i^{(i)} \rangle \langle \phi_j^{(j)} \rangle = P^2 \left[ S_1^{(i,j)} + (M - 2) S_2^{(i,j)} \right],
\]

where \( P \) is the launch power of the IC, \( M \) is the normalized kurtosis of the IC’s constellation [7], and

\[
S_1^{(i,j)} = \sum_{k,m} X_{k,m}^{(i)} X_{k,m}^{(j)*}, \quad S_2^{(i,j)} = \sum_{k} X_{k,k}^{(i)} X_{k,k}^{(j)*}.
\]
The values of $S_1^{(i,j)}$ and $S_2^{(i,j)}$ can be found analytically, using a similar derivation to that performed in [6]. Once these quantities are evaluated, the covariance matrix characterizing the NLPN of all subcarriers in the WDM channel can be easily expressed, for every symbol constellation and launch power.

A split-step Fourier simulation was set up to validate the analytical derivation. The transmitter generates 21 pol-muxed WDM channels on a 50-GHz DWDM grid. Each channel is then divided into $N_s$ subcarriers, each with symbol rate $32/N_s$ GBaud and separated by 50/$N_s$ GHz. As a COI, we picked the central channel in the WDM spectrum, as shown in Fig. 1a. Transmitted modulation formats are (unshaped) 64-QAM, and PS (shaped) 64-QAM, shaped with a Maxwell-Boltzmann distribution to give a constellation entropy of 5 bit/symb in each polarization. The kurtosis of the unshaped and shaped constellations was found analytically, using a similar derivation to that performed in [6]. Once these covariance matrices characterize the NLPN of all subcarriers in the WDM channel can be easily expressed, for every symbol constellation and launch power.

Results, comparing simulations (markers) with analytical predictions (solid lines), are shown in Fig. 1b and 1c in the exemplary case of 8 subcarriers. Fig. 1b shows the correlation between the lowest-frequency subcarrier of the COI and the other subcarriers, after 10 spans. It can be seen that the correlation decreases with frequency, but is always above 0.3. Shaping mildly increases NLPN correlation with respect to unshaped 64-QAM. Fig. 1c shows the correlation between the first and second subcarriers (top two curves) and the first and last subcarriers (two bottom curves) as a function of the link length. In both cases, the correlation reduces with the number of spans, where the reduction is faster in the case of the larger frequency separation. Although the analytic expression slightly overestimates the actual correlation at short distances, the overall agreement between theory and simulations is self-evident.

3. Joint mitigation of NLPN

The main importance of the correlation between the NLPN affecting different subcarriers is the ability to employ joint phase estimation algorithms to improve performance. As an initial demonstration of this principle, we used a simple joint phase estimation algorithm, described schematically in Fig. 2a. Each of the subcarriers is individually detected, and a standard decision-directed RLS equalizer is used to estimate the NLPN. The outputs of the $N_s$ equalizers are then averaged, providing a single estimation of the NLPN, which is less noisy than those of the individual equalizers. The averaged phase estimation is then removed from each subcarrier’s signal, providing a cleaner estimate of the
transmitted data.

The joint equalization procedure was applied to an exemplary scenario of a $5 \times 100$km link, carrying unshaped 64-QAM constellation and using 11 WDM channels. The EDFAs had a noise figure of 5dB, and all other parameters are as described in the previous section. The RLS equalizers’ forgetting factor was optimized independently for each configuration, so as to provide optimal performance. Figure 2b shows the Q-factor of the equalized signal, as a function of the launch power and the number of subcarriers. The differences between the un-equalized cases (solid curves) originate from the reduction in NLIN power at lower symbol rates. Figure 2c shows the peak Q-factor gain (i.e. the difference between the peak of the unequalized case to that of the equalized cases), using both individual equalization (blue bars) and joint equalization (red bars). It is evident that the benefit of individual equalization decreases as the number of subcarriers increases. This is because the temporal correlation length of the NLPN decreases, which makes the equalization more difficult. In contrast, the benefit from joint processing increases with the number of subcarriers.

4. Conclusion

We have demonstrated the existence of correlations between the NLPN of different subcarriers in a multi-subcarrier system. The correlations were found to be fairly strong and an accurate analytic approach to finding them has been presented. We have shown a simple equalization scheme for benefiting from these correlations, and argue that much larger benefits should be achievable in equalizers specifically designed to target this feature.

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