Using genetic algorithm to determine the optimal order quantities for multi-item multi-period under warehouse capacity constraints in kitchenware manufacturing

D Saraswati, D K Sari and V Johan

Industrial Engineering Department, Trisakti University, Jakarta, Indonesia

E-mail: docki_saraswati@trisakti.ac.id

Abstract. The study was conducted on a manufacturer that produced various kinds of kitchenware with kitchen sink as the main product. There were four types of steel sheets selected as the raw materials of the kitchen sink. The problem was the manufacturer wanted to determine how much steel sheets to order from a single supplier to meet the production requirements in a way to minimize the total inventory cost. In this case, the economic order quantity (EOQ) model was developed using all-unit discount as the price of steel sheets and the warehouse capacity was limited. Genetic algorithm (GA) was used to find the minimum of the total inventory cost as a sum of purchasing cost, ordering cost, holding cost and penalty cost.

1. Introduction
Companies are often faced with two problems of inventory management. A policy requires inventory in large quantities, and as the consequences, they will require a large space in a warehouse to stock the materials. On the other hand, inventories in small amounts are at risk of running out, which causes back-order or lost-sales. The best known inventory model is the classic economic order quantity (EOQ). The application of EOQ is widely used in a real environment. The extension of this model is relaxing some of its assumptions [1]. Matsuyama [2] described the dependence of the purchase price on ordering quantity with a certain function in order to maximize the one day’s average profit. Mendoza and Ventura [3] developed EOQ model with transportation cost in which all unit discount was included to find the optimal inventory policy. Taleizadeh and Pentico [4] developed the EOQ model with all unit discounts and partial backordering.

Genetic algorithm (GA) is a search procedure to identify optimal or near optimal solutions from a population. Stockton and Quinn [5] proposed GA to solve economic lot size in deterministic inventory model. Using GA, the other group of researchers, Ongkunaruk et al. [6] developed a multi-item replenishment as a group from a single supplier with shipment constraint, budget constraint and transportation constraint to minimize the total expected cost per unit time. An algorithm for multi-item replenishment from a single supplier was presented by Goyal [7] with 15 items in order to determine the optimum ordering frequency.

In this study, the inventory policy was applied in the environment with multi-item multi period in a kitchenware manufacturer that produced household appliances with steel sheets as the main raw materials. Another factor to consider was the limited capacity of the warehouse and the discounts rate for ordering raw materials in a certain amount. Following that, this paper presented the genetic
algorithm as an approach to model the problem and determined how much to order based on minimizing the total inventory cost.

2. Problem definition
This study was done in a kitchenware manufacturer with kitchen sink as the main products (Figure 1). The raw material of kitchen sink is steel sheets. During the planning horizon, the manufacturer produced 79 items for 18 product family of kitchen sink. The planning horizon in this research was four periods. Based on ABC classification there were only four types of steel sheets in class A, namely steel sheets; S/S 201: 0.3 mm x 1220 mm (item 1), S/S 430: 0.37 mm x 660 mm (item 2), S/S 201: 0.5 mm x 1220 mm (item 3) and S/S 430: 0.4 mm x 1220 mm (item 4). The demand for the four types of steel sheets is based on the planned order release in four periods (Table 1). Ordering cost is the cost of preparing an order from the supplier. Demand and ordering cost for each item are shown in Table 1. Holding cost consists of cost of capital, storage and handling cost, taxes, insurance, obsolescence, pilferage and deterioration.

![Figure 1. Types of kitchen sinks.](image)

The objective of this study was to determine the optimal order quantities in order to minimize the total inventory cost including ordering, holding and purchasing. The notations used in the inventory modeling are the same as Pasandideh et al. [8]. The notations are as follows:

- \( B_i \) : batch size of item \( i (i = 1,2,3,4) \)
- \( V_{i,t} \) : number of orders of item \( i \) in period \( t (t = 1,2,3,4) \)
- \( D_{i,t} \) : demand of item \( i \) in period \( t \) (kg)
- \( H_i \) : holding cost of item \( i \) per kg per period (Rp/kg/period)
- \( A_i \) : ordering cost of item \( i \) per order (Rp/order)
- \( P_i \) : purchase cost of item \( i \) per unit (Rp/unit)
- \( q_{i,k} \) : upper bound quantity of the \( k \)-th price break of item \( i \) (kg)
- \( P_{i,k} \) : purchase cost of item \( i \) in period \( t \) determined by the discount schedule of \( Q_{i,t} \) (Rp/kg)
- \( m_{i,k} \) : discount rate of purchasing item \( i \) at price break \( k (0<m_{i,k}<1) \)
- \( Q_{i,t} \) : ordering quantity of item \( i \) in period \( t \) (kg)
- \( X_{i,t} \) : beginning inventory of item \( i \) in period \( t = 0 \), beginning inventory for all items = 0 (kg)
- \( TC \) : total inventory cost for all items in all period. (Rp.)
- \( S \) : available warehouse capacity (kg)
- \( S_i \) : storage space needed per unit of item \( i \) (kg/unit)
- \( U_{i,t,k} \) : binary variable, equals to 1 if item \( i \) is purchased at price break \( k \) in period \( t \), and 0 otherwise.
- \( W_{i,t} \) : binary variable, equals to 1 if an order of item \( i \) is placed in period \( t \), and 0 otherwise.
Table 1. Demand and ordering cost for 4 items in 4 periods.

|    | $D_{i,t}$ (kg) | 1     | 2     | 3     | 4     |
|----|----------------|-------|-------|-------|-------|
| 1  | 8576.79        | 17737.64 | 12016.15 | 9065.65 |
| 2  | 3658.45        | 8160.12  | 6140.44  | 2740.93  |
| 3  | 5564.63        | 17039.91 | 9252.2   | 19322.71 |
| 4  | 7189.96        | 5055.39  | 2821.97  | 5815.29  |

|    | $A_i$ (Rp/order) | 1     | 2     | 3     | 4     |
|----|------------------|-------|-------|-------|-------|
| 1  | 926,000          | 935,000 | 875,000 | 888,000 |

Table 2 shows the discount rate for the unit price of 4 types of stainless sheets. The price includes shipping costs from supplier located in Surabaya to Jakarta. The supplier’s quantity discount applies all-unit discount (AUD) method. The unit of holding cost is defined as 25% of the unit price. Stainless steel is sold by suppliers in the form of batch. Each batch weighs 5 kg. Therefore, the purchase of stainless steel should be a multiplication of 5.

Table 2. Quantity discount for 4 items.

| Quantity (kg) | 1 | 2 | 3 | 4 |
|---------------|---|---|---|---|
| 0 ≤ $Q_i$ < 5000 | 26,900 | 25,800 | 22,575 | 21,450 |
| 5000 ≤ $Q_i$ < 10000 | 26,400 | 25,300 | 22,075 | 20,950 |
| $Q_i$ ≥ 10000 | 25,900 | 24,800 | 21,575 | 20,450 |

Each pallet for stainless steel roll cannot exceed 3500 kg because of the company’s forklift capacity, while the warehouse capacity is enough for 15 pallets. Therefore, the available capacity for holding the 4 types of stainless steel rolls are 15 times 3500 kg equals to 52,500 kg.

3. Problem formulation

The beginning inventory of an item $i$ in period $t$, $X_{i,t}$, ($i = 1, 2, 3, 4$ and $t = 1, 2, 3, 4$) is as follows (Figure 2):

$$X_{i,t} = X_{i,t-1} + Q_{i,t-1} + D_{i,t-1}$$

(1)

Since $Q_{i,t} + X_{i,t} ≥ D_{i,t}$, therefore $B_{i,t} × V_{i,t} + X_{i,t} ≥ D_{i,t}$.

The purchasing cost per unit with all-units quantity discounts (AUD) and 3 price break points is defined as follows:

$$P_i = \begin{cases} P_{i,1} & 0 ≤ Q_{i,t} < q_{i,2} \\ P_{i,2} & q_{i,2} ≤ Q_{i,t} < q_{i,3} \\ P_{i,3} & q_{i,3} ≤ Q_{i,t} \end{cases}$$

(3)

The mathematical model for finding the minimum total cost:

$$Min TC = \sum_{i=1}^{4} \sum_{t=1}^{4} A_i + \sum_{i=1}^{4} \sum_{t=1}^{3} \left( \frac{X_{i,t} + X_{i,t+1}}{2} H_i \right) + \sum_{i=1}^{4} \sum_{t=1}^{3} \sum_{k=1}^{3} U_{i,t,k} (P_{i,k} × Q_{i,t})$$

(4)

Subject to

$$X_{i,t} = X_{i,t-1} + Q_{i,t-1} - D_{i,t-1}$$

(5)

$$X_{i,1} = 0$$

(6)

$$\sum_{i=1}^{4} S_i X_{i,t} ≤ S$$

(7)
where $Q_{t,i} = B_{i,t} \times V_{i,t}$ (8)

\[
\sum_{k=1}^{K} U_{i,t,k} = 1; U_{i,t,k} \in \{0,1\}
\]

(9)

Equation (4) explains the minimum total cost as the sum of total ordering cost, total inventory holding cost, and total purchasing cost. Equation (5) describes the inventory at period $t$ is the inventory of item $i$ at period $t-1$ plus ordering quantity of item $i$ at period $t-1$ minus demand of item $i$ at period $t-1$. Inventory of item $i$ in period $t$ is $X_{i,t}$ as illustrated in Figure 2. Equation (6) assumes there is no inventory at the beginning of period 1 for all items $X_{1,1} = X_{2,1} = X_{3,1} = X_{4,1} = 0$. Equation (7) requires the space for item $i$ at period $t$ should be less or equal to the warehouse capacity. Equation (8) calculates the batch size times the number of ordering of item $i$ at period $t$ equal to ordering quantity for item $i$ at period $t$. Equation (9) is a binary variable. The value equals 1 if item $i$ is purchased at price break $k$ in period $t$ and 0 otherwise.

Warehouse capacity

Chromosomes that do not meet the specified constraints will get penalty. It happens if the storage space used exceeds the warehouse capacity. The penalty value (PV) is derived from the sum of squared space used to store the 4 items in the same period minus the available warehouse capacity. If the total inventory of four items in each period is less than the capacity of the warehouse, there is no penalty or penalty value = 0.

\[
\text{if } \sum_{i=1}^{4} X_{i,t} + Q_{i,t} \geq S \ ; \ PV = \left\{ \sum_{i=1}^{4} \left( X_{i,t} + Q_{i,t} \right) - S \right\}^2 \text{ and if } \sum_{i=1}^{4} X_{i,t} \leq S \ ; \ PV = 0
\]

(10)

Total penalty value (TPV) = \left\{ \sum_{i=1}^{4} \left( X_{i,t} + Q_{i,t} \right) - S \right\}^2

(11)

4. Genetic algorithms

GA begins with an initial set of solutions while the optimal ordering quantity for each item from a single supplier in each period is called a population. Each ordering quantity in the population is called a chromosome. Each chromosome is composed set of gens. The chromosome forms a matrix ($m \times n$) with $m$ rows for items and $n$ columns for periods [8]. Figure 3 shows the matrix of gene for $i$-th row and the $j$-th column. This matrix indicates the ordering quantity of item $i$ in period $t$. 
There are 20 chromosomes in the population and each chromosome forms a $4 \times 4$ matrix. All genes are generated by the random number generator in the form of a $4 \times 80$ matrix with a range of 500-3500 (Figure 4). Range 500-3500 is used since the minimum order quantity fixed by the supplier equals to 500 kg and the maximum weight that can be transported by the company’s material handling is 3500 kg.

The generated random number becomes the initial population. Since the supplier has determined the weight of each batch is 5 kg, then the initial population was adjusted to a multiple of 5 kg per batch. For example in cell $(1,C1)$ the value of 528.31 became 530. In this case, there were 20 chromosomes in the population with 80 genes. Each gene represented the batch size of each item in each period ($B_{i,t}$). As shown at Table 3, after obtaining the value of $B_{i,t}$ for 20 chromosomes and calculating the ordering quantity ($Q_{i,t}$), the inventory began ($X_{i,t}$).

**GA parameters**

Since there were 20 chromosomes in population, the population size (pop-size) was 20. The crossover probability ($P_c$) = 0.7 indicated that as many as 0.7 of the total population would be crossover. The mutation probability ($P_m$) = 0.05 was set for only one gene mutated per chromosome. The mutation probability in this study was 0.05 obtained from 1 divided by 20. The number of generations (GEN) was not determined because iteration was complete when the average fitness generation declined.
Table 3. Batch size, order quantities and beginning inventory.

| Chromosome | \( B_{i,t} \) | \( Q_{i,t} \) | \( X_{i,t} \) |
|------------|-------------|-------------|-------------|
| 1          | 530        | 1205        | 3185        |
| 2          | 3250       | 1725        | 2925        |
|            | 3100       | 855         | 1090        |
| 4          | 3355       | 1020        | 2225        |
| 5          | 1540       | 3000        | 1365        |
| 6          | 2345       | 1450        | 1835        |
| 7          | 730        | 2345        | 1130        |
| 8          | 3375       | 695         | 1550        |
| 20         | 2355       | 12245       | 1595        |
|            | 771        | 2345        | 3455        |
|            | 781        | 2345        | 1130        |
|            | 797        | 2345        | 2985        |
|            | 808        | 2055        | 3360        |

Chromosomes

The fitness value of each chromosome was determined by evaluating the suitable objective function, in which the objective function was the summation of purchase cost, ordering cost and holding cost.

\[
\text{Total ordering cost (TOC)} = \sum_{i=1}^{4} \sum_{t=1}^{4} A_i W_{i,t} \quad (12)
\]

\[
\text{Total holding cost (THC)} = \sum_{i=1}^{4} \sum_{t=1}^{4} \left( \frac{X_{i,t} + X_{i,t+1} + Q_{i,t}}{2} \right) H_i \quad (13)
\]

\[
\text{Total purchasing cost (TPC)} = \sum_{i=1}^{4} \sum_{t=1}^{4} \sum_{k=1}^{3} U_{i,t,k} (P_k \times Q_{i,t}) \quad (14)
\]

\[
\text{Total inventory cost (TC)} = \text{TOC} + \text{THC} + \text{TPC}
\]

The total cost or objective function of 20 chromosomes in the first generation was IDR3,450,434,433.

Fitness and chromosome selection

The purpose of this study was to obtain inventory planning with minimum total cost. The selection process was done by making the chromosome with a small objective function had a high selected probability. This selection used fitness value equal to 1/objective function. As an example, the fitness value of the first chromosome in the first generation was 0.00000000029. Next, the fitness probability was calculated for each chromosome with the formula = fitness value /total fitness value. For example, the probability of the fitness value for chromosome 1 was 0.0502.

Selection of chromosomes was carried out using a strategy roulette wheel, where the selection was done randomly. In the selection process, it was necessary to calculate the cumulative probability. Roulette wheel was rotated according to the number of chromosomes that was as much as 20 times to generate random numbers \( R \) with a range of 0-1. In each round, a chromosome was selected for the new population. If \( R<C[1] \) then chromosome 1 would be selected as a parent. Otherwise, the \( i \)-th chromosome should be selected as the parent for \( C[i-1]-R<C[i] \). For example, in the generation of random numbers in the first round was obtained a number of 0.7978. As shown in Table 4, the first random number \( R[1] \) was larger than \( C[15] \) but smaller than \( C[16] \) then chromosome 16 should be selected as the chromosome in the new population.
Table 4. Chromosome selection.

| Chromosome | Objective function | Fitness Value | Fitness proportion (p) | Cumulative p | Random number | Selected chromosome |
|------------|--------------------|---------------|------------------------|-------------|--------------|---------------------|
| 1          | 0.0000000000      | 0.0502        | 0.0502                 | 0.7978      | 16           |                     |
| 2          | 0.0000000000      | 0.0503        | 0.1005                 | 0.5345      | 11           |                     |
| 3          | 0.0000000000      | 0.0494        | 0.1499                 | 0.2258      | 5            |                     |
| 4          | 0.0000000000      | 0.0502        | 0.0502                 | 0.7978      | 16           |                     |
| 5          | 0.0000000000      | 0.0503        | 0.1005                 | 0.5345      | 11           |                     |
| 6          | 0.0000000000      | 0.0494        | 0.1499                 | 0.2258      | 5            |                     |
| 7          | 0.0000000000      | 0.0502        | 0.0502                 | 0.7978      | 16           |                     |
| 8          | 0.0000000000      | 0.0503        | 0.1005                 | 0.5345      | 11           |                     |
| 9          | 0.0000000000      | 0.0494        | 0.1499                 | 0.2258      | 5            |                     |
| Total      | 0.0000000000      | 0.0502        | 0.0502                 | 0.7978      | 16           |                     |

Crossover

Crossover operations were carried out in two stages: selection and operation. In the selection process, individuals were randomly selected for crossing with a probability of 0.7 \((P_c = 0.7)\). If the generated random number was smaller than the specified \(P_c\) value, crossover was performed. Crossover would not be accomplished if the random number was greater than the value of \(P_c\) and the value of the parent became the same. The selected chromosomes would be paired with each other. If the number was odd, then one of the chromosomes was not selected for crossover process. The Chromosome was \(4 \times 4\) matrix, and crossover operation was executed by selecting the number of rows and columns randomly (Figure 5).

Mutation

The number of chromosomes that performed the gene mutation process was determined by the mutation probability parameter \((P_m)\). The number of gen had the value of 16 because the matrix shape was \(4 \times 4\) with 16 cells. Total genes = number of genes in chromosome x number of chromosomes in the population = \(16 \times 20 = 320\) genes. The cell was mutated because it had a random number smaller than the mutation probability \((P_m)\). In the first generation, there were 21 genes mutated.

![Figure 5. Crossover operations.](image-url)
New generation

Iteration continues to produce a new generation. That is, the results of the first generation will become the initial population for the second generation, and so on. Iteration will be terminated if the stopping criteria have been fulfilled by obtaining the best chromosome with a minimum objective function value of the whole generations. As shown in Table 5, the best value of objective function is given by chromosome #7 with the total cost of IDR 3,374,322,432.

Table 5. Ordering quantity \((Q_{ij})\).

| Raw materials                      | Item \((i)\) | \(j=1\) | \(j=2\) | \(j=3\) | \(j=4\) |
|-----------------------------------|--------------|---------|---------|---------|---------|
| SS 201 0.30 mm x 1220 mm x coil (kg) | 1            | 8700    | 17955   | 13400   | 7350    |
| SS 201 0.50 mm x 1220 mm x coil (kg) | 2            | 4235    | 7810    | 7620    | 1080    |
| SS 430 0.37 mm x 660 mm x coil (kg)  | 3            | 7080    | 16060   | 8780    | 19390   |
| SS 430 0.40 mm x 1220 mm x coil (kg) | 4            | 8320    | 5220    | 2810    | 4550    |

5. Conclusion

This article addressed the inventory problem for multi-item multi period by considering the warehouse capacity. Moreover, this inventory model was implemented in the kitchenware manufacturer whose ordering of the raw materials was from a single supplier. Stainless steel is the main raw materials for kitchenware products. This article used metaheuristic approach, genetic algorithm (GA), to determine the ordering lot size of stainless steel based on minimizing total inventory cost. Moreover, this work can be explored for different value of GA’s parameters such as population size or probability of crossover \((P_c)\) or probability of mutation \((P_m)\).

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