The “Supercritical Pile” Model of GRB: Thresholds, Polarization, Time Lags

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Abstract. The essence of the “Supercritical Pile” model is a process for converting the energy stored in the relativistic protons of a Relativistic Blast Wave (RBW) of Lorentz factor \( \Gamma \) into electron – positron pairs of similar Lorentz factor, while at the same time emitting most of the GRB luminosity at an energy \( E_p^\prime \approx 1 \text{ MeV} \). This is achieved by scattering the synchrotron radiation emitted by the RBW in an upstream located “mirror” and then re-intercepting it by the RBW. The repeated scatterings of radiation between the RBW and the “mirror”, along with the threshold of the pair production reaction \( p \gamma \rightarrow e^+ e^- \), lead to a maximum in the GRB luminosity at an energy \( E_p^\prime \approx 1 \text{ MeV} \), independent of the value of \( \Gamma \). Furthermore, the same threshold implies that the prompt \( \gamma \) ray emission is only possible for \( \Gamma \) larger than a minimum value, thereby providing a “natural” account for the termination of this stage of the GRB as the RBW slows down. Within this model the \( \gamma \) ray (\( E \approx 100 \text{ keV} – 1 \text{ MeV} \)) emission process is due to Inverse Compton scattering and it is thus expected to be highly polarized if viewed at angles \( \theta \approx \Gamma \) to the RBW’s direction of motion. Finally, the model also predicts lags in the light curves of the lower energy photons with respect to those of higher energy; these are of purely kinematic origin and of magnitude \( \Delta t \approx 10^2 \text{ s} \), in agreement with observation.

INTRODUCTION

The discovery of GRB afterglows by BeppoSAX and the ensuing determination of their redshifts [1], ushered a new era in GRB physics and in our understanding of their time development. While the issue of their distance and energetics was settled by these observations, a number of issues concerning the physics of the RBWs that give rise to the GRB phenomenon still remain open as novel issues have been raised with the advent of observations and accumulation of more GRB of known redshifts [3]. Among the older issues that still remain open is that of the narrow distribution of the GRB peak energy \( E_p \) [2], in view of its sensitive dependence on the RBW Lorentz factor \( E_p \propto \Gamma^4 \) within the synchrotron model of GRB. Another such issue is that of conversion of the energy stored in relativistic protons in the RBW into electrons. The presence of protons was necessary in the early RBW models [4] for transporting and releasing the energy carried off by the GRB RBW to the distances demanded by observation (\( R \approx 10^{16} \text{ cm} \)). While more recent MHD models [5, 6] are immune from this requirement (though they still need a mechanism for dissipating the magnetic energy), sweeping and accumulating the ISM matter by these MHD flows will store a similar amount of energy into relativistic protons as do more conventional models, still demanding a mechanism for converting this energy
to radiation. Models generally resolve this issue by assuming the equipartition of the total energy density between protons and relativistic electrons of arbitrary distributions, as demanded by the need to account for their observed spectral characteristics.

In our view, the most compelling argument in favor our model is that it can answer in a rather straightforward way both the issue of the the limited range in $E_p$ and that of conversion the energy stored into relativistic protons to radiation. At the same time, it has additional implications which seem to be in agreement with the mounting GRB phenomenology; these will be discussed in the following sections.

“SUPERCRITICAL PILE”: THE THRESHOLDS

We assume the presence of a population of relativistic protons of form $n(\gamma_p) = n_0 \gamma_p^{\beta}$ on the frame co-moving with the RBW, i.e. moving with Lorentz Factor (LF) $\Gamma$ with respect to the observer (and also at zero angle to his/her line of sight; see [7] for details). We consider synchrotron photons from $e^- e^+$ pairs of energy $\gamma_e$, $\varepsilon_s = b \gamma_e^2$ ($b$ is the value measured in units of the electron mass $m_e c^2$) reflecting off an upstream “mirror” and being re-intercepted by the RBW. Their energy will now be $\varepsilon_s^f = \Gamma^2 b \gamma_e^2$. The threshold for $e^- e^+$ pair production of these photons with a proton of Lorentz factor $\gamma_p$, $\gamma_e b \Gamma^2 \gamma_e^3 = 2$. Assuming that the proton and the electrons are drawn from the relativistic thermal post-shock particle population, $\gamma_e = \gamma_p \Gamma$, leading to the kinematic threshold condition

$$b \Gamma_{th}^5 > 2 \text{ or } \Gamma_{th} > \left(\frac{2}{b}\right)^{1/5}$$

(1)

Assuming equipartition for the magnetic field for a GRB with total energy $E = 10^{52} E_{52}$ erg, restricted to an angle $\theta \leq 1-\Gamma$ leads to $\Gamma_{th} > 90 (E_{52} = R_{16})^{1/12}$.

For this reaction network to be self-sustained, at least one of the reflected synchrotron photons must pair produce with the protons on the RBW (after its reflection by the “mirror”) to replace the electron that produced it. Therefore the plasma optical depth $\tau$ to the pair producing reaction $p \gamma ! pe^- e^+$ must be at least as large as the inverse of the number of synchrotron photons produced by a given electron. An electron of energy $\gamma$ produces $N_\gamma$ of $\gamma = b \gamma^2 = 1 = b \gamma$ photons, yielding $\tau = n_{com} \sigma_{p\gamma} \Delta_{com} > 1 = N_\gamma$, where $n_{com} \Delta_{com}$ are the comoving density and width of the RBW. Considering that the column density is a Lorentz invariant $n_{com} \Delta_{com} = nR$ and taking into account the kinematic threshold relation (Eq. 1) the dynamic threshold condition reads

$$\sigma_{p\gamma} n_{com} = \sigma_{p\gamma} R n \gamma \gamma > b \Gamma \text{ or } \sigma_{p\gamma} R n \Gamma^4 > 2$$

(2)

This latter condition (and the physics behind it) are akin to those of those of a “supercritical” nuclear pile, hence the nomenclature of this model. For the typical values of $n$ and $R$ used in association with GRBs, i.e. $n = 1 n_0 \text{ cm}^{-3}$ and $R = 10^{16} R_{16} \text{ cm}$ and considering that $\sigma_{p\gamma} = 5 \times 10^{-27} \text{ cm}^2$, the criticality condition yields $\Gamma > 375 (n_0 R_{16})^{1/4}$ (Eq. 2 is slightly different from that given in [7]; we would like to thank P. Mészáros for this correction which does not affect the results otherwise).
Assuming the width of the reflecting “mirror” to be thinner than the width of the RBW, blast waves with column densities higher than that implied by Eq. (2) will release the energy stored in relativistic protons explosively on times scales comparable to the RBW light crossing time scale; otherwise, the duration of the burst will be comparable to the time it takes the RBW to cross the width of the “mirror”. In this case, prominent emission is halted until the proton column has been built up significantly to conform to the dynamic threshold. For RBW with \( \Gamma \) between those of Eqs. (2) and (1) and assuming the presence of a “mirror” \( \gamma \) ray emission continues as long as its LF is greater than that implied by the kinematic threshold (Eq. 1). Eventually, when the value of the LF drops below this value, \( \gamma \) ray emission stops and the RBW enters the stage of afterglow.

**THE GRB SPECTRA**

Consider a RBW of Lorentz factor \( \Gamma \). Because of the relativistic focusing of emitted radiation, we need only consider a section of the blast wave of opening half angle \( \theta = 1 - \Gamma \). The shocked electrons of the ambient medium (and pairs from the \( p \gamma \rightarrow e^+ e^- \) process) produce, as discussed above, synchrotron photons of energy \( \varepsilon_s \sim b \Gamma^2 \). These, upon their scattering by the “mirror” and re-interception by the RBW, are boosted to energy \( \varepsilon = \varepsilon_s \Gamma^2 = b \Gamma^4 \) (in the RBW frame). These photons will then be scattered by the following electron populations: (a) By electrons of \( \gamma' \), originally contained in the RBW and/or cooled since the explosion. (b) By the hot (\( \gamma' \)), recently shocked electrons to produce inverse Compton (IC) radiation at energies correspondingly \( \varepsilon_1' \sim b \Gamma^4 \) and \( \varepsilon_2' \sim b \Gamma^6 \) at the RBW frame. (the SSC process will also yield photons at \( \varepsilon_{ssc} \sim b \Gamma^4 \), however it turns out that this is not as important and it is ignored here).

At the lab frame, the energies of these three components, i.e. \( \varepsilon_s, \varepsilon_1, \varepsilon_2 \) will be higher by roughly a factor \( \Gamma \), i.e. they will be respectively at energies \( b \Gamma^3 \), \( b \Gamma^5 \) and \( b \Gamma^7 \). Assuming that the process operates near its kinematic threshold, \( b \Gamma^5 \sim 2 \), at the lab frame these components will occur at energies \( \varepsilon_s \sim \Gamma^2 \), \( \varepsilon_1 \sim 1 \text{ MeV} \) and \( \varepsilon_2 \sim 10 \text{ GeV} \) (\( \Gamma = 100 \))^2. This model therefore, produces “naturally” a component in the \( vF_v \) spectral distribution which peaks in the correct energy range. It also predicts the existence of two additional components at an energies \( m_e c^2 \Gamma^2 \) and \( m_e c^2 = \Gamma^2 \). The high energy emission has been observed from several GRBs [8].

**OTHER ISSUES**

The simplicity by which this model deals with several outstanding issues of GRB suggest that one should attempt to test its viability by addressing additional GRB systematics and properties.

A. **Time Lags.** It has been observed that, in general, the light curves of soft photons lag with respect to those of the harder ones. A systematic study of these lags in a sample of GRBs with known redshift has determined that these lags range between 0.01 – 0.1 sec, with their magnitude in inverse correlation to the GRB peak luminosity [9]. The model we presented above can produce lags of the order of magnitude observed: According
to the basic premise of our model the emission at $E \approx 500$ keV is due to bulk IC scattering of synchrotron photons, by the “cold” electrons of the RBW, after being reflected the “mirror”. The eventual energy of these photons depends on the angle of their direction after scattering at the “mirror” with respect to the velocity vector of the RBW (this being highest for a “head-on” collision). Because the distance between the “mirror” and the RBW is of order $R=\Gamma^2$, one can easily estimate that the path difference between the soft and hard photons are of the same order of magnitude i.e. $\Delta L \sim R=\Gamma^2$. Therefore the corresponding time lags (assuming the observer to be along the direction of the velocity of the RBW) should be $\Delta t \sim \Delta L/c=\Gamma^2 \times 10^{25} R_{16}=\Gamma^2$ sec, in agreement with observations.

B. Polarization. The recent results of high (80% 20%) polarization of GRB 021211 [10] has raised the interest of the community in this particular aspect of GRBs. While models employing synchrotron radiation can produce at best polarization $\lesssim 70\%$ (for totally uniform field geometry), models producing the 100 - 1000 keV radiation by the inverse Compton process can potentially produce polarization approaching 100% for particular orientation of the observer [11, 12, 13]. This is a purely geometric effect: Thomson scattering of unpolarized radiation to an angle $\theta = 90^\circ$ with respect to the incident direction erases all electric field orientations but that perpendicular to the plane defined by the incident and scattering directions leading to 100% polarization in this direction. Since in the lab frame this direction corresponds to an angle $\theta = 1=\Gamma$, the polarization should rise from 0 to 100% in going from $\theta = 0$ to $\theta = 1=\Gamma$ and drop again for larger angles. However, in our model we scatter not unpolarized radiation but the synchrotron radiation from the RBW, which is itself polarized. This leads to a non-zero polarization even for angles close to $\theta = 0$ thus enhancing the probability that we observe a high polarization signal.

C. General Considerations We would like to point out that the arguments presented above have completely ignored the possibility of particle acceleration at the RBW of GRBs (a fundamental requirement of most models). We have dealt with the conversion in pairs of the energy stored only in the thermal population of protons. A non-thermal component will ease the thresholds of Eqs. (1, 2) and allow high energy emission long after the end of the prompt GRB phase, as it appears to be the case in some GRBs [8].

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