Chimera-like States in Structured Heterogeneous Networks

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Chimera-like states are manifested through the coexistence of synchronous and asynchronous dynamics and have been observed in various systems. To analyze the role of network topology in giving rise to chimera-like states we study a heterogeneous network model comprising two group of nodes, of high and low degrees of connectivity. The architecture facilitates the analysis of the system, which separates into a densely-connected coherent group of nodes, perturbed by their sparsely-connected drifting neighbors. It describes a synchronous behavior of the densely-connected group and scaling properties of the induced perturbations.

Nonlinear interactions of coupled oscillators, such as the Kuramoto model with sinusoidal couplings, give rise to fascinating collective behaviors$^{1-3}$, one of which is the emergence of chimera states in identical and symmetric networks, namely, the coexistence of stable synchronous and fluctuating asynchronous dynamical patterns$^4$. Studies have shown that the chimera-like states also occur in complex networks, which exhibit disparate behaviors for nodes of different connectivities, for instance higher-degree nodes are more prone to synchronization than lower-degree nodes in scale free networks$^5$. However, the rich structure of complex networks makes it difficult to analyze the dynamics and gain insight into the collective dynamics and its properties. To facilitate the analysis and better understand the role of network heterogeneity, we examine a structured heterogeneous network model comprising only two interacting groups of nodes, one of which is densely connected while the other is sparsely connected. As in other complex networks this model exhibits a chimera state-like behavior whereby high-degree nodes form a synchronized domain while low-degree nodes remain unsynchronized. The main benefit of this topology is that it is amenable to analysis and provides insight that is difficult to obtain otherwise; we derive a self-consistent mean field theory for these networks, which explains quantitatively their behavior, especially the synchronized dynamics of the densely connected component under perturbation induced by the sparsely connected nodes.

I. INTRODUCTION

The dynamics of coupled oscillators has been widely studied to understand the synchronization behaviors in complex systems$^{1-3}$. Among the fascinating phenomena observed in these systems the chimera states, which exhibit a coexistence of synchronous and asynchronous domains, have attracted particular interest recently$^4$. It came as a surprise that the symmetry-breaking chimera states were observed in symmetric and homogeneous media$^5$. This phenomena is also reminiscent of the coexistence of equilibrium-like domains in non-equilibrium systems$^6$. The analysis of chimera states is generally difficult, due to the heterogeneous interactions between a large number of variables, and appropriate approximations are required. One of the approximations is based on mapping the system onto a low dimensional manifold$^7$, which facilitates a simplification that leads to tractable solutions. Such approaches are either based on the continuous self-consistent mean field theory or rely on the assumption of Ott-Antonsen reduction to simplify the problem$^8$. Carefully chosen initial conditions are also necessary to produce chimera states in these symmetric systems$^9$. In addition to the theoretical discovery, chimera states have also been observed and studied in different experiments$^{10-13}$. While it is fascinating to understand the emergence of chimera states as symmetry-broken states in very large homogeneous symmetric networks, real systems are typically heterogeneous and of finite size$^{14,15}$. To explore synchronization behavior in real systems, one has to take into account their complex and heterogeneous network structure$^{16,17}$. Two recent examples are the modelling of neural activities in the resting-state functional networks$^{18}$ and the modular neural networks of C. Elephants$^{19}$, both of which consider irregular network topologies. Studies of networks comprising non-identical constituents with heterogeneous natural frequencies or coupling strengths, and irregular topologies have shown similar phenomena where synchronized and unsynchronized subpopulations coexist$^{17,18,20}$. Some of the features ex-
hibited are different from those of symmetric systems, e.g., in the case of all-to-all coupled oscillators with scale-free distributed coupling strengths, the phase locked subpopulations are the oscillators with small coupling strength\textsuperscript{24,26}. Another study that does not rely on the system’s symmetry\textsuperscript{22} investigated sinusoidally coupled oscillators in Erdös-Rényi and scale-free networks, which exhibit chimera-like state behavior from arbitrary initial conditions.

An interesting observation from studies in scale free networks is that nodes of high degrees are more likely to synchronize than nodes of low degrees\textsuperscript{2}. This is an example where heterogeneity in network topology results in disparate behaviors across the system.\textsuperscript{21} Also in models with chaotic oscillators, it was shown that hubs synchronize while low-degree nodes do not.\textsuperscript{32-35} It is proposed in a theoretical study of neural culture that neurons with more inputs, i.e., with high in-degrees, are the leaders of the burst activities.\textsuperscript{36} In general, the strong interactions that the highly-connected oscillators have with their neighbors distinguish them from the nonsynchronizing low-degree nodes.\textsuperscript{3} However, the complex structure of heterogeneous networks, such as scale free or small world networks is not amenable to analysis due to the combined complexity of the network topology and intractability of the large scale-dynamical model. In this work, we investigate a simple heterogeneous network architecture which supports an easy-to-reach chimera-like state but is amenable to analysis. It exposes the synchronized equilibrium-like behavior of the densely-connected components and facilitate the derivation of perturbation originated by the sparsely-connected components; finite-size scaling of these perturbation and the corresponding order parameter is also be derived, shedding light on the interplay between the two types of nodes.

II. THE MODEL

We consider a heterogeneous network with degree distribution of two peaks, i.e., the network comprises two classes of nodes, one of which has extensive connectivity scaled with system size $N$ while the nodes in the other has finite connectivity. Denote the two classes as the dense group $D$ and the sparse group $S$ respectively. By assigning a large number of intra-group connections between constituents of the dense group $D$, each node inside $D$ can receive large number of coordinated signals from within the group, dominating the fluctuations of other neighboring nodes, potentially leading to coherent intra-group dynamics. Conversely, the nodes in the sparse group can be easily perturbed by fluctuation induced by any of its neighbors due to their limited connectivity.

Specifically, to simplify the construction, each node $i \in D$ is connected to $d_D(\propto N)$ other nodes randomly chosen from the same class $D$, and each node $j \in S$ is connected to $d_S(\sim O(1))$ other nodes randomly chosen from the whole system. Let $n_D$ and $n_S$ denote the number of nodes in the dense and sparse groups, respectively. For simplicity, we consider the special case where node $i \in D$ is connected to all the rest members of $D$ so that the dense group forms a complete subgraph, and both groups include the same number of nodes, i.e., $d_D = n_D - 1$ and $n_D = n_S = N/2$. Fig. 1 is an instance of the proposed network structure with $N = 40$ and $d_S = 4$. We would like to remark that the chimera-like state discussed below is not restricted to the special case of the dense group being a complete subgraph and $n_D = n_S$, but is a more general phenomena in the networks of a similar construction to that described above. Nevertheless, the simplification makes the analysis easier.

The governing dynamics we consider is the Kuramoto model with phase lag in the chimera state\textsuperscript{37,38}:

$$
\dot{\theta}_i = \omega + \frac{K}{n_D} \sum_{j \in \partial_i} \sin(\theta_j - \theta_i - \alpha) \\
= \omega + \frac{K}{n_D} \sum_{j \in \partial_i \cap D} \sin(\theta_j - \theta_i - \alpha) \\
+ \frac{K}{n_D} \sum_{j \in \partial_i \cap S} \sin(\theta_j - \theta_i - \alpha),
$$

where $\theta_i$ is the phase of node $i$, $\omega$, $K$ and $\alpha$ are the natural frequency, coupling strength and the phase lag respectively, and $\partial_i$ stands for the set of nodes adjacent to node $i$ where we have isolated the contribution from the dense and sparse members. Any node $i \in D$ experiences the global mean field induced by the whole dense group $r_D e^{i\psi_D} := 1/n_D \sum_{j \in D} e^{i\theta_j}$ and the local field of its sparse neighbors $\rho_i e^{i\varphi_i} := \sum_{j \in \partial_i \cap S} e^{i\theta_j}$, both terms are of order 1.

$$
\dot{\theta}_i = \omega + \frac{K}{n_D} \sin \alpha + K r_D \sin(\psi_D - \theta_i - \alpha) \\
+ \frac{K}{n_D} \rho_i \sin(\varphi_i - \theta_i - \alpha).
$$

By construction, the dense group itself forms a $d_D$ regular subgraph, which can reach perfect phase synchronization

![Figure 1. An instance of the heterogeneous network composed of a densely connected (black nodes) and sparsely connected (white nodes), where $N = 40$, $d_S = 4$.](image)
if the contribution from the sparse neighbors are negligi-
ble, i.e., \( \theta_i = \Omega + \text{const}. \) This will hold true in the thermodynamic
limit \( N \rightarrow \infty \); in which case the densely-connected group \( D \) on its own exhibits an equilibrium-like behav-
ior within the global non-equilibrium system. In finite
systems, the effect of sparse neighbors can be regarded as
a finite size perturbation with strength of order \( 1/N \) on the
coherent dynamics of the densely-connected members
\( \theta_i(t) = \Omega + \epsilon_i(t) \).

The argument cannot be applied to the sparse group.
Firstly, we notice that for node \( j \in S \), \( |\theta_j| \leq \omega +
Kd_j/\epsilon \Omega \ll \Omega \), thus it does not follow the coherent dy-
namics of the densely-connected group and global syn-
chronization of the whole system is not attainable. Sec-
ondly, the irregularity of the sparse subgraph and pres-
ence of phase lag interfere with phase synchronization.
Finally, sparse variables are easily disturbed by noisy sig-
als from their neighbors due to their finite connectivity.
We define the global order parameter of the sparse group
\( r_S := |1/NS \sum_{i \in S} e^{i\theta_i}| \) measuring their incoherence.

III. ANALYSIS AND RESULTS

Without loss of generality, let \( \omega = 0 \) in the follow-
ing discussion. In Fig. 2 we demonstrate the dynamics
of a network with \( N = 200 \) and \( d_S = 6 \) governed by
Eq. (1). The dense variables dynamics is coherent with
a high phase velocity exhibiting minor deviations from
each other, revealed by the small deviation of \( r_D(t) \) from
1, shown in Fig. 2(b). On the other hand, the sparse
variables are drifting slowly and incoherently. The ob-
served partial synchronization phenomenon constitutes a
chimera-like state. Interestingly, the chimera-like state
is easily obtained in the proposed topology by starting
with arbitrary initial conditions, as shown in Fig. 2. This
is similar to what has been observed previously. We
remark that the chimera-like states observed here are
different from the classical symmetry-breaking chimera
states due to the lack of symmetry between the
dense and sparse groups.

In the following, we analyse the deviation of the
densely-connected variables from perfect synchronization
due to the interaction with the sparsely-connected
system. The method used is similar to the self-consistent
mean field theory commonly used in the analysis of the
Kuramoto model with inhomogeneous natural frequen-
cies, in which the system is divided into two subpop-
ulations to be determined, a phase-locked group and a
drifting group, depending on the natural frequency with
reference to the mean field order parameter. Assuming
the system is in the stationary state, the two subpop-
ulations are described by certain probability densities to be
solved self-consistently to obtain the order parameter. A
similar method with a space-dependent order parameter
was used to analyze chimera states in finite dimensional
spaces.

In the current study, the system has homogeneous
natural frequencies but highly inhomogeneous network
topologies, and the phase-locked group and drifting group
can be identified by the network connectivities. Instead
of solving the invariant density for the whole system, we
utilize the property of time scale separation in the sys-
tem and isolate the synchronized group for considera-
tion. One advantage of the chosen network topology is
that the node in the dense group receives overwhelmingly
more inputs from the same group than from the sparse
group, allowing one to expand the deviation from per-
flect synchronization in orders of \( 1/N \). We then express

![Figure 2](image1.png)

**Figure 2.** (Color online) (a) Dynamical process of 5 densely
(top) and 5 sparsely connected variables (bottom) in a net-
work of size \( N = 200 \), where the sparse degree is \( d_S = 6 \),
coefficient \( K = 10 \), and phase \( \alpha = 0.2 \). In this case,
\( \Omega = -K(n_D - 1)/n_D \sin(\alpha) \approx 9.7 \). (b) Evolution of the global
order parameters for the densely (top) and sparsely
connected variables (bottom). Inset: Power spectrum of \( r_D(t) \), which
has its main peak at \( \omega = 2\Omega \).

![Figure 3](image2.png)

**Figure 3.** (Color online) Evolution of the global order param-
eters for the dense (top) and sparse (bottom) groups, starting
from different initial conditions. Blue: the initial phases of
all variables are drawn from the uniform distribution ranging
from 0 to \( \pi \). Green: the initial phases of all variables are
drawn from the uniform distribution ranging from 0 to \( 2\pi \),
while that of sparse variables share the same phase value.
the dynamics of the dense group as small deviations from perfect synchronization and solve for the small deviation self-consistently.

Expanding the phase of a dense group variable \(i\) with respect to small perturbation \(\varepsilon_i\) such that \(\theta_i(t) = \Omega t + \varepsilon_i(t)\), and defining \(g(\varepsilon) = 1 - r_D(t)\), \(f(\varepsilon) = \psi_D(t) - \Omega t\) one obtains

\[
r_D e^{i \psi_D} = e^{i \Omega t} \frac{1}{n_D} \sum_{i \in D} e^{i \varepsilon_i},
\]

\[
e^{i \Omega t} \left[ 1 + \frac{i}{n_D} \sum_{i \in D} \varepsilon_i - \frac{1}{2 n_D} \sum_{i \in D} \varepsilon_i^2 + O(\varepsilon^3) \right]
\]

and from results in leading order in \(\varepsilon^2\) (See Appendix A),

\[
g(\varepsilon) \approx \frac{1}{2 n_D} \sum_{i \in D} \varepsilon_i^2 - \frac{1}{2 n_D^2} \left( \sum_{i \in D} \varepsilon_i \right)^2,
\]

\[
f(\varepsilon) \approx \frac{1}{n_D} \sum_{i \in D} \varepsilon_i.
\]

Assuming \(\varepsilon_i\), and subsequently \(g(\varepsilon)\) and \(f(\varepsilon)\) are small, then Eq. (2) can be expressed as

\[
\dot{\varepsilon}_i = K \sin \alpha + K (1 - g(\varepsilon)) \sin(f(\varepsilon) - \varepsilon_i - \alpha)
\]

\[
+ \frac{K}{n_D} \rho_i \sin(\varphi_i - \Omega t - \varepsilon_i - \alpha)
\]

\[
\approx (K \cos \alpha) [f(\varepsilon) - \varepsilon_i] - [K \partial_{\varepsilon_i} g(\varepsilon)]_{\varepsilon_i=0} \sin \alpha \varepsilon_i
\]

\[
+ \frac{K}{n_D} \rho_i \sin(\varphi_i - \Omega t - \alpha)
\]

\[
- \frac{K}{n_D} \rho_i \cos(\varphi_i - \Omega t - \alpha) \varepsilon_i,
\]

where only first order terms in \(\varepsilon_i\) are retained. Furthermore, in the large size limit \(1/n_D\) is considered to be small and the last term of Eq. (6) can be ignored since it contains the product of two small quantities, \(\varepsilon_i\) and \(1/n_D\). Assuming the fluctuations \(\varepsilon_i\) are uncorrelated across sites, \(f(\varepsilon) = 1/n_D \sum_{i \in D} \varepsilon_i\) smooths out the fluctuations of individual variables and becomes smaller compared to any single fluctuations \(\varepsilon_i\); it is therefore omitted in the following. Additionally, to leading order we have

\[
\partial_{\varepsilon_i} g(\varepsilon)_{\varepsilon_i=0} = -1/n_D^2 \sum_{j \neq i} \varepsilon_j,
\]

which vanishes for large \(n_D\), such that the corresponding term in Eq. (6) can be omitted for fixed \(\alpha < \pi/2\), leading to

\[
\dot{\varepsilon}_i = - (K \cos \alpha) \varepsilon_i + \frac{K}{n_D} \rho_i \sin(\varphi_i - \Omega t - \alpha).
\]

By definition of the local mean field of \(\rho_i\) and \(\varphi_i\), the last term of Eq. (7) can be expressed as

\[
\frac{K}{n_D} \rho_i \sin(\varphi_i - \Omega t - \alpha) = \frac{K}{n_D} \sum_{j \in \Omega_i \cap S} \sin(\theta_j - \Omega t - \alpha).
\]

Since the phase velocity \(|\dot{\theta}_j| \leq K d_S/n_D\) of the sparsely-connected variables \(j\) is much smaller than the oscillation of \(\Omega\), as shown in Fig. 2(a), the two time scales within the \(\sin\) function can be separated, allowing one to view \(\rho_i\) and \(\varphi_i\) as quench variables within a time frame of a few periods. As \(\Omega = -K(n_D - 1)/n_D \sin(\alpha) \approx -K \sin(\alpha)\) for large \(n_D\). Eq. (7) can be integrated to provide

\[
\varepsilon_i(t) = c_t e^{-(K \cos \alpha) t} + \frac{1}{n_D} \rho_i \sin(\varphi_i - \Omega t - 2\alpha).
\]

This solution Eq. (8) has a decay term of rate \(K \cos(\alpha)\) and a stationary oscillatory term of frequency \(\Omega = -K \sin(\alpha)\). The stationary part of \(\varepsilon_i\) scales as \(1/n_D\), which is consistent with the derivation that treats both \(\varepsilon_i\) and \(1/n_D\) as small quantities at the same time. We can therefore deduce that the dense group converges faster to the steady solution for smaller \(\alpha\) values, where \(\alpha = \pi/2\) signals the onset of instability of the chimera-like state, as shown in Fig. 4.

The deviation \(\varepsilon_i(t)\) has an oscillatory behavior of frequency \(\Omega\). From Eq. (4), we expect that \(r_D(t) = 1 - g(\varepsilon)\) oscillates primarily at a frequency \(2\Omega\), which is verified by the power spectrum of \(r_D(t)\) from the numerical experiment in the inset of Fig. 2(b).

In the steady state, substituting the solution of \(\varepsilon_i(t) = \rho_i/n_D \sin(\varphi_i - \Omega t - 2\alpha)\) into Eq. (1), one obtains an expression for \(g(\varepsilon)\), and consequently for \(r_D\). It is observed that the distribution of sparsely-connected variables phases is close to uniform, as illustrated in Fig. 4. Thus we compute the time average \(r_D\) by averaging over the phase angles of the sparse variables, where phases are assumed to be independently and uniformly distributed in \([0, 2\pi]\), i.e., \(P(\theta_j) = 1/2\pi, \forall j \in S\). It is sufficient to consider a single instance of \(\varepsilon_i(t)\)

\[
\begin{align*}
\varepsilon_i &= \frac{\rho_i}{n_D} \sin(\varphi_i),
\end{align*}
\]

Since \(\mathbb{E}[\sin(\theta_j)] = 0, \mathbb{E}[\sin^2(\theta_j)] = 1/2, \mathbb{E}[\sin^4(\theta_j)] = 3/8\), it is straightforward to obtain the mean and variance of
The obtained solution of $r_D$ in Eq. (11) is independent of $\alpha$. Nevertheless, we remark that the derivation breaks down when $\alpha$ is close to $\pi/2$, in particular for $\alpha \sim \tan^{-1}(n_D^2)$, where the term $[K \partial_x g(\varepsilon)]_{\varepsilon=0} \sin \alpha \varepsilon_i$ starts to dominate $(K \cos \alpha) \varepsilon_i$ in Eq. (9). We can see this effect in Fig. 4 where for a wide range of $\alpha$ values away from $\pi/2$, the deviation to perfect synchronization in the stationary state is independent of $\alpha$. When $\alpha$ gets closer to $\pi/2$, the deviation starts to increase; the onset of this deviation occurs later for larger systems, as expected.

IV. DISCUSSION AND CONCLUSION

We study a simple heterogeneous network architecture that supports easy-to-reach chimera-like states and is amenable to analysis. The network comprises two interconnected components of densely connected and sparsely connected variables. The heterogeneity separates the time scales of the dynamics of the two components, leading to disparate behaviors of the two groups. We showed that the densely connected group maintains a synchronized and coherent dynamics, while the dynamics of sparse group variables is incoherent. Furthermore, we derived a self-consistent mean field theory for these networks, by viewing the contribution of the sparse variables as $1/n_D$-scale perturbations to the dense group’s dynamics. Using this framework we obtained expressions for the
dense group order parameter that exhibits phase synchronization, and subsequently its dependence on phase lag parameter, network size and network connectivity. The model facilitates the analysis of large systems comprising interacting oscillatory variables to provide new insight and understanding of their complex dynamical behaviors.

In general, the densely connected nodes in the complex network may not form a regular subgraph. One possible generalization of our model is to consider randomly removal of connections in the dense group, and study the resulting erosion of synchronization in the framework developed in related work. In more complex heterogeneous network topologies such as scale free networks, although our analysis is not directly applicable, we suggest that similar mechanism accounts for the disparate synchronization behaviors for the high-degree nodes and other nodes, i.e., the high-degree nodes interact with each other strongly and in a combinatorial sense they have higher chance to arrange themselves to a synchronized cluster, while the incoherent perturbations from the low-degree nodes are less significant, as preliminarily illustrated in numerical simulations. We envisage that our model and similar variants could be employed to explore the properties of other heterogeneous network architectures.

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Appendix A: The Expressions of $g(\varepsilon)$ and $f(\varepsilon)$

In this section we derive the expressions of $g(\varepsilon)$ and $f(\varepsilon)$ up to leading order of $\varepsilon^2$. From Eq. (3), we see that $r_D = 1 - g(\varepsilon)$ is the magnitude of the complex vector $\frac{1}{n_D} \sum_{i \in D} e^{i \varepsilon_i} \approx 1 - \frac{1}{2 n_D} \sum_{i \in D} \varepsilon_i^2 + \frac{i}{n_D} \sum_{i \in D} \varepsilon_i$; then

$$r_D \approx \left( 1 - \frac{1}{2 n_D} \sum_{i \in D} \varepsilon_i^2 \right)^2 + \left( \frac{1}{n_D} \sum_{i \in D} \varepsilon_i \right)^2 \approx \left( 1 - \frac{1}{n_D} \sum_{i \in D} \varepsilon_i^2 \right)^2 + \frac{1}{n_D} \sum_{i \in D} \varepsilon_i \approx 1 - \frac{1}{2 n_D} \sum_{i \in D} \varepsilon_i^2 + \frac{1}{2 n_D} \left( \sum_{i \in D} \varepsilon_i \right)^2,$$

where we have used the approximation $\sqrt{1 + x} \approx 1 + x/2$ for small $x$. Therefore, we obtain the expression

$$g(\varepsilon) = \frac{1}{2 n_D} \sum_{i \in D} \varepsilon_i^2 - \frac{1}{2 n_D^2} \left( \sum_{i \in D} \varepsilon_i \right)^2.$$

Notice that $f(\varepsilon)$ is the phase of the complex vector $\frac{1}{n_D} \sum_{i \in D} e^{i \varepsilon_i}$; after some manipulation we find that

$$f(\varepsilon) = \tan^{-1} \left( \frac{1}{1 - \frac{1}{2 n_D} \sum_{i \in D} \varepsilon_i^2} \right) \approx \frac{1}{n_D} \sum_{i \in D} \varepsilon_i^2 + \frac{1}{2 n_D} \sum_{i \in D} \varepsilon_i \sum_{i \in D} \varepsilon_i^2.$$

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