Two-photon detuning and decoherence in cavity electromagnetically induced transparency for quantized fields

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The interaction of a quantized field with three-level atoms in Λ configuration inside a two-mode cavity is analyzed in the small noise approximation. The atoms are in a two-photon detuning with respect to the carriers of the field. We calculate the stationary quadrature noise spectrum of the field outside the cavity in the case where the input probe field is a squeezed state and the input pump field is a coherent state. The mean value of the field is unaltered in all the analysis: the atoms shows electromagnetically induced transparency (EIT). The effect of the atoms’ base level decoherence in the cavity output field is also studied. It is found that the output field is very sensitive to two-photon detuning.

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I. INTRODUCTION

It was recently shown that a probe field, after interacting with atoms that show electromagnetically induced transparency (EIT), does not necessarily preserve the noise properties of an initially squeezed state. This was first shown for the stationary output field from a two-mode cavity filled with atoms in Λ configuration (see Fig. 1) in a fully quantized model. In addition to the expected reduction of squeezing for frequencies in resonance with the normal mode splitting of the atom-cavity system, an interchange of noise properties between the squeezed state probe and coherent state pump field was found for frequencies for which the mean value of the field is unaltered (well inside the EIT transparency window and the cavity bandwidth) [1]. In the case of an initially squeezed state propagating in an EIT medium, a similar result holds; in this case the interchange of the noise properties between probe and pump oscillates with the propagating distance [2]. In both cases the transfer of noise properties is maximum for equal strengths of the probe and pump fields. These results mean that, although the mean value of the field is unaltered, the quantum state is indeed altered by the interaction with EIT media. Except for the expected absorption due to the linewidth of the excited level, the transfer of noise properties between probe and pump oscillates with respect to the carriers of the field. We calculate the stationary quadrature noise spectrum of the field outside the cavity in the case where the input probe field is a vacuum state and the probe mode is not driven. In Sec. III we analyze the cavity output field for the case where the incoming probe field is a vacuum squeezed state and the probe mode is not driven. In Sec. IV we analyze the case where the probe mode is driven. In Sec. V we study the influence of decoherence due to dephasing in the atom base levels. Finally we present our conclusions in Sec. VI.

II. CAVERY OUTPUT FIELD EQUATIONS

FIG. 1: The atoms are in Λ configuration, Γ₁ (Γ₂) represent the radiative decay constant to state |1⟩ (|2⟩). The probe (pump) field has a detuning δ₂ (δ₁) with respect to the corresponding dipolar transition. Γ₁₂ accounts for decoherence due to dephasing in the atom base levels.

Consider the case of N three-level atoms in Λ configuration inside a cavity that sustains two modes of the electromagnetic field. The annihilation field operators for each mode are denoted by $a_1$ and $a_2$. Each mode $a_i$ has a detuning $\delta_i$ with respect to the transitions between level $|i\rangle$ and $|0\rangle$, $i = 1, 2$. We will work with a two-photon detuning, $\delta_1 = \delta_2 = \delta$. The atomic system equations are given by [1]

$\dot{\rho}_{11} = -i [H, \rho_{11}] + \Gamma_1 \rho_{11} - \Gamma_1 \rho_{12} + \Gamma_1 \rho_{21}$

$\dot{\rho}_{22} = -i [H, \rho_{22}] + \Gamma_2 \rho_{22} - \Gamma_2 \rho_{12} + \Gamma_2 \rho_{21}$

$\dot{\rho}_{12} = -i [H, \rho_{12}] - \Gamma_1 \rho_{12} + \Gamma_2 \rho_{21}$

$\dot{\rho}_{21} = -i [H, \rho_{21}] - \Gamma_2 \rho_{21} + \Gamma_1 \rho_{12}$

where $H = H_{atom} + H_{cavity}$, with $H_{atom}$ being the atomic Hamiltonian and $H_{cavity}$ the cavity Hamiltonian.
\[
\begin{align*}
\frac{d}{dt} \hat{W}_1 & = \frac{1}{3}(-2\Gamma_1 - \Gamma_2)(1 + \hat{W}_1 + \hat{W}_2) - 2i g_1 \Sigma_0 \hat{a}_1 \\
& \quad + 2i g_1 \hat{a}_1^\dagger \Sigma_{10} - i g_2 \hat{\Sigma}_{02} \hat{a}_2 \\
& \quad + i g_2 \hat{a}_2^\dagger \hat{\Sigma}_{20} + \hat{F}_{W_1}, \\
\frac{d}{dt} \hat{W}_2 & = \frac{1}{3}(-\Gamma_1 - 2\Gamma_2)(1 + \hat{W}_1 + \hat{W}_2) - i g_1 \Sigma_0 \hat{a}_1 \\
& \quad + 2i g_2 \hat{a}_1^\dagger \hat{\Sigma}_{10} - 2i g_2 \hat{\Sigma}_{02} \hat{a}_2 \\
& \quad + 2i g_2 \hat{a}_2^\dagger \hat{\Sigma}_{20} + \hat{F}_{W_2}, \\
\frac{d}{dt} \hat{\Sigma}_{10} & = (-\frac{\Gamma_1 + \Gamma_2}{2} + i\delta) \hat{\Sigma}_{10} + i g_1 \hat{W}_1 \hat{a}_1 \\
& \quad - i g_2 \Sigma_{12} \hat{a}_2 + \hat{F}_{10}, \\
\frac{d}{dt} \hat{\Sigma}_{20} & = (-\frac{\Gamma_1 + \Gamma_2}{2} + i\delta) \hat{\Sigma}_{20} + i g_2 \hat{W}_2 \hat{\Sigma}_{21} \\
& \quad - i g_1 \Sigma_{21} \hat{a}_1 + \hat{F}_{20}, \\
\frac{d}{dt} \hat{\Sigma}_{21} & = -\Gamma_{12} \hat{\Sigma}_{21} - i g_1 \hat{a}_1^\dagger \hat{\Sigma}_{10} + i g_2 \Sigma_{02} \hat{a}_2, \\
\end{align*}
\]

where \( \hat{\Sigma}_{ij} = \sum_{k=1}^{N} \hat{a}_j^k \) are the collective operators that represent the sum of individual atomic operators \( \hat{a}_j^k = |i\rangle\langle j| \) associated with the \( k \)th atom.

Each intracavity mode interacts with its own collection of modes in the outside field. This means that either they have different polarizations or their difference in frequency is large. Using input-output theory \( \text{[4]} \) to relate the inside field with the outside field we obtain the following equations for the intracavity modes \( \text{[1]} \):

\[
\begin{align*}
\frac{d}{dt} \hat{a}_1(t) & = -i g_1 \hat{\Sigma}_{10}(t) - \frac{\gamma_1}{2} \hat{a}_1(t) + \sqrt{\gamma_1} \hat{a}_{\text{in}}(t), \\
\frac{d}{dt} \hat{a}_2(t) & = -i g_2 \hat{\Sigma}_{20}(t) - \frac{\gamma_2}{2} \hat{a}_2(t) + \sqrt{\gamma_2} \hat{a}_{\text{in}}(t),
\end{align*}
\]

where \( \gamma_i \) is the decay rate of cavity mode \( i = 1, 2 \). The operators \( \hat{a}_{\text{in}}(t) = -1/\sqrt{2\pi} \int_{-\infty}^{\infty} d\omega e^{-i\omega(t-t_0)} \hat{b}_i(t_0, \omega) \) represent the field entering the cavity. The operator \( \hat{b}_i(t_0, \omega) \) represents the outside mode associated with cavity mode \( i \) at the initial time \( t_0 \) and frequency \( \omega \). We will call the outside field associated with the modes labeled by index \( i = 1 (i = 2) \) the pump (probe) field. The incoming field is given by \( \hat{a}_{\text{out}}(t) = 1/\sqrt{2\pi} \int_{-\infty}^{\infty} d\omega e^{-i\omega(t-t_0)} \hat{b}_i(t_1, \omega) \).

The operators \( \hat{b}_i(t_1, \omega) \) represent the outside mode associated with the cavity mode \( i \) at time \( t_1 > t_0 \) and frequency \( \omega \). All the operators are in a reference frame rotating with the corresponding cavity frequencies. The laboratory frame notation can be obtained by the transformation \( \Sigma_{ij} = \Sigma_{0j} \exp(-i\delta_j - \omega_j) \). Due to the rotating frame, the spectrum frequency \( \omega \) will represent the detuning from the cavity frequency. The incoming and outgoing fields are related by

\[
\begin{align*}
\hat{a}_{1\text{in}}(t) + \hat{a}_{1\text{out}}(t) & = \sqrt{\gamma_1} \hat{a}_1(t), \\
\hat{a}_{2\text{in}}(t) + \hat{a}_{2\text{out}}(t) & = \sqrt{\gamma_2} \hat{a}_2(t).
\end{align*}
\]

The Langevin fluctuation operators \( \hat{F} \)'s are assumed to be delta correlated, with zero mean:

\[
\langle \hat{F}_x \rangle = 0,
\]

\[
\langle \hat{F}_x(t)\hat{F}_y(t') \rangle = D_{xy}\delta(t-t'),
\]

where \( x \) and \( y \) label the fluctuation operators.

The atom diffusion coefficients \( D_{xy} \) can be obtained using the generalized Einstein relations \( \text{[3]} \). The nonzero diffusion coefficients are given in \( \text{[1]} \).

We will consider the following initial conditions for the incoming field. For frequencies different from the intracavity frequencies, each mode outside the cavity is, for the probe field, a \( \theta = 0 \) quadrature vacuum squeezed state and, for the pump field, a vacuum state. When the frequency is equal to the probe intracavity frequency, the initial condition is, for the probe field, a \( \theta = 0 \) squeezed state (mean value different from zero) and for the pump field a coherent state.

Defining the field quadrature \( \theta \) for the field \( i \) and frequency \( \omega \) as

\[
\hat{Y}_i(\omega, t) = \hat{b}_i(\omega, t) \exp(i\theta) + \hat{b}_i^\dagger(\omega, t) \exp(-i\theta),
\]

we have that, for the given initial conditions, the \( \theta = 0 \) quadrature noise operator for the probe field is,

\[
\Delta Y_{2\theta}(\omega, t = t_0) = \langle (\hat{Y}_1(\omega, t_0) - \langle \hat{Y}_1(\omega, t_0) \rangle)^2 \rangle = e^{-2r} \cos^2 \theta + e^{2r} \sin^2 \theta,
\]

and \( \Delta Y_{1\theta}(\omega, t = t_0) = 1 \) for the pump field, where \( r > 0 \) measures the maximum level of squeezing of the \( \theta = 0 \) quadrature.

That means that the outside modes are initially in a vacuum, which can be squeezed, except for the resonant modes with the cavity which can be in a squeezed state but with field mean value different from zero. We will chose that value in such a way that inside the cavity we have \( \langle \hat{a}_i \rangle = \alpha_i \).

The system of Eqs. \( \text{[1]}, \text{[2]} \) and \( \text{[3]} \) is transformed to c-number equations and solved for the stationary case in the small noise approximation. The method is described in detail in \( \text{[8]} \) for two-level systems. A similar system (the resonance case without decoherence) is discussed in \( \text{[1]} \). We will suppose \( \Gamma_1 = \Gamma_2 = \Gamma \) and \( \gamma_1 = \gamma_2 = \gamma \). We will concentrate on the case \( \delta > 0 \) and \( \omega > 0 \), the negative case being symmetric.

III. PROBE FIELD IS A SQUEEZED VACUUM

In this section the probe mode of the cavity is not driven, its mean value being \( \langle \hat{a}_2 \rangle = 0 \). The pump mode is driven and has \( \langle \hat{a}_1 \rangle = \alpha \).
A. Numerical results

The results for the output field, although analytical, are to big to give here. In order to gain insight into the behavior of the cavity output field we will plot it for different detunings.

In Fig. 2 we plotted the $\theta = 0$ quadrature of the field 2 for different detunings. When $\delta = 0$ we obtain a reduction of the incoming squeezing for some observation frequencies $\omega$. We will call the peak of these frequencies $\omega_{\delta=0}$. This reduction of squeezing is due to the normal mode splitting of the atom-cavity system. When $\delta$ increases, we observe that the one peak shown for the case $\delta = 0$ develops into a double peak (see the plot for $\delta = 2\Gamma$ and $\delta = 4\Gamma$). One of the peaks is for an observation frequency $\omega < \omega_{\delta=0}$ and with a height larger than the other peak for $\omega > \omega_{\delta=0}$. For $\delta$ even greater, the position of the peak for $\omega < \omega_{\delta=0}$ approaches $\omega = 0$ and the maximum of the peaks no longer shows noise squeezing. This can be seen in Fig. 3(a). For the case $\delta = 100\Gamma$ the excess noise is approximately the same as the initial condition for the $\theta = \pi/2$ quadrature. In Fig. 3(b) we plotted the $\theta = \pi/2$ quadrature. In this plot we can see a reduction of noise for the same frequencies where we have an increase of noise for the $\theta = 0$ quadrature. For the case where $\delta = 100\Gamma$ the $\theta = \pi/2$ quadrature of the output field shows squeezing, the numerical value is approximately the same as the initial condition for the $\theta = 0$ quadrature. This result, together with the excess noise shown by the output field for the $\theta = 0$ quadrature, may lead us to suspect that for that frequency we have a $\pi/2$ rotation of the initial condition. For frequencies $\delta < 100\Gamma$ we have a similar behavior but the rotation is not complete.

The maximum position for the peak for $\omega < \omega_{\delta=0}$ increases with the number of atoms, $N$, and diminishes with increasing two-photon detuning $\delta$.

In Fig. 2 we show what happens with the peaks for $\omega > \omega_{\delta=0}$. We can see that, as $\delta$ increases, the observation frequency for these peaks also increases and its height diminishes. The frequency observation of these peaks also increases with $N$.

B. Analytical results

In order to find analytical expressions for the extremal position, we Taylor expand the field in $\gamma$. Comparing the term of order $O(\gamma)$ with the term of order $O(\gamma^2)$, we conclude that the expansion is valid for $\gamma \ll \Gamma$ and $\omega \geq \sqrt{g^2N}$. This expansion allows us to find the position of the extremum for $\omega > \omega_{\delta=0}$. For that extremum, the peak is located for

$$\omega_{\max} = \frac{1}{2} \left( \delta + \sqrt{4\Omega^2 + 4Ng^2 } \right).$$  \hfill (7)

In the case of $N$ two-level atoms inside a cavity, it is known that the absorption spectrum has a peak for frequency $\frac{1}{2} \left( \delta + \sqrt{4\Omega^2 + 4Ng^2 + \delta^2} \right)$. This frequency corresponds to the vacuum Rabi splitting for $N$ atoms inside the cavity. As we expect that for large detuning of the probe field the three-level atoms behave similarly to two-level atoms, we can interpret the frequency $\omega_{\max}$ as the usual absorption peak for two-level atoms inside a cavity, modified by the presence of an external field (the pump field).

In order to obtain analytical results for the position and value of the peak for $\omega < \omega_{\delta=0}$, we use the numerically observed fact that the position of this peak increase with $N$ and decrease with increasing $\delta$. We write then $\delta = \lambda N$ and take the limit $N \rightarrow \infty$. We obtain

$$\Delta Y_{\theta_2}(\omega) = \frac{1}{M} \left\{ e^{-2r} \left( \cos(\theta) \left( \delta_c^2 (4\omega_c^2 + 1) - 4 \right) - 4 \sin(\theta)\delta_c \right)^2 
+ e^{2r} \left( 4 \cos(\theta)\delta_c + \sin(\theta) \left( \delta_c^2 (4\omega_c^2 + 1) - 4 \right) \right)^2 \right\},$$

where

$$M = 16 \left( 4\omega_c^2 + 1 \right)^2 \delta_c^4 + 8 \left( 1 - 4\omega_c^2 \right) \delta_c^2,$$

$$\omega = \omega_\gamma \gamma, \ \delta_c = \delta C$$

and the cooperativity parameter $C = g^2N/\gamma$. When $\theta = 0$ the maximum is located at

$$\omega_{\max}/\gamma = \pm \sqrt{\frac{4 - \delta_c^2}{2\delta_c}},$$

if $\delta_c \leq 2$ and $\omega_{\max} = 0$ if $\delta_c > 2$. To see the validity of the former limit for a given $\delta$, we study the expansion of the output field in the parameter $1/N$. Comparing the order 0 with the order 1 for $\omega_{\max}$, it can be seen that the limit is valid when $\gamma \ll \omega_{\max}$ and $\delta^2 \gg 4\Gamma$. 

![Graph of $\omega/\Gamma$ versus $\Delta Y_{2\theta_2}$](image-url)
The value of the $\theta$ quadrature for this maximum is

$$\Delta Y_{2\theta_2}(\omega_{\text{max}}) = \cos^2(\theta)e^{2r} + \sin^2(\theta)e^{-2r}.$$ 

The previous equation means that in the limit $N \to \infty$ the effect of two-photon detuning in a cavity-atom system is to rotate by $\pi/2$ the initial condition for the frequency $\omega_{<\text{max}}$.

We interpret the peak at frequency $\omega_{\text{max}}$ in the following way. For each interaction of the field with the atoms, a small rotation of the maximum squeezed quadrature happens. This rotation is explained by the phase gained by the field induced by the two-photon detuning $10$. The velocity of this rotation depends on the two-photon detuning and spectrum frequency. The frequency positions $\omega_{\text{max}}$ correspond to the frequencies where the buildup of the rotation is maximal.

In summary, the spectrum at $\omega_{\text{max}}$ is explained by the energy splitting due to the cavity-atom system (this splitting increases with atom number and detuning). The spectrum at $\omega_{<\text{max}}$ is explained by the phase gained by the field due to two-photon detuning.

IV. PROBE FIELD IS A SQUEEZED STATE

In this section, the incoming squeezed and coherent states which are resonant with the respective probe and pump modes of the cavity, are such that $\langle a_2 \rangle = \langle a_1 \rangle = \alpha$.

A. Numerical Results

In Figs. 3 and 4 we plot the $\theta = 0$ and $\theta = \pi/2$ quadratures of the output field. When $\omega/\Gamma > 1/4$ the extrema found have a qualitatively similar behavior to that studied in the previous section, namely, the extremum found for $\delta = 0$ around $\omega/\Gamma \approx 5$ divides into two as $\delta$ increases. The location of one of these extrema increase with $\delta$ (not shown in the plot) and the other decreases. We will focus on the case of spectrum frequency $\omega < \omega_{\delta=0}$. Unlike in the squeezed vacuum case, we can see from the figures that the pump field also has an increase of noise. For $\delta \approx 1$ the maximum noise is given by approximately $\epsilon^4/4$ instead of $\epsilon^4$ as in the vacuum squeezed case. In the next section we will obtain an analytical expression, valid for some parameters, for the positions and values of these extrema. For $\omega/\Gamma < 1/4$ we observe the interchange of squeezing described in 11 for $\delta = 0$. It can be seen that, for the parameters in the figures, this interchange of squeezing apparently does not depend on the
we conclude that the expansion is valid for $\gamma \omega \geq \delta$, two-photon detuning $\delta$. The probe field is in a broadband vacuum squeezed state except for the resonance mode with the cavity. The pump field is in a coherent state. For $\omega$ in the vicinity of zero, it can be observed how the transfer of squeezing between the probe and pump fields does not depend on $\delta$. For $\omega/\Gamma > 1$, a maximum, whose frequency decreases with increasing $\delta$, can be seen in the pump and probe output fields. Parameters: $g = -0.005\Gamma$, $\Omega_1 = \Omega_2 = \Gamma$, $\gamma = 0.06\Gamma$, $N = 1000000$, $r = 2$.

two-photon detuning $\delta$. We found that this is true until $\delta$ is so large that the excess noise peak enters the domain of the frequencies where we had the interchange of squeezing. In the next section we will better characterize this behavior.

B. Analytical Results

In order to find analytical expressions for the extremal positions, we Taylor expand the field in $\gamma$. Comparing the term of order $O(\gamma)$ with the term of order $O(\gamma^2)$, we conclude that the expansion is valid for $\gamma \ll \Gamma$ and $\omega \geq \sqrt{g^2N}$. This expansion allows us to find the position of the extremum for $\omega > \omega_\delta = 0$. For that extremum, the peaks are located at

$$\omega_{\text{max}} = \frac{1}{2} \left( \delta + \sqrt{8\Omega^2 + 4Ng^2 + \delta^2} \right).$$

In order to obtain analytical results for the positions and values of the peaks for $\omega < \omega_\delta = 0$, we use the numerically observed fact that the positions of these peaks increase with $N$ and decrease with increasing $\delta$. We write then $\delta = \lambda N$ and take the limit $N \to \infty$. For the $\theta = 0$ quadrature we obtain (the expression for all quadratures is too large to write here)

$$\Delta Y_{2\theta=0}(\omega) = \frac{1}{R} \left\{ \begin{array}{l}
4e^{2r} (4\omega_\gamma^2 + 1) \delta_c^2 + 4 (4\omega_\gamma^2 + 1) \delta_c^2 + 4 + \\
e^{-2r} \left( (4\omega_\gamma^2 + 1)^3 \delta_c^4 - 32 (4\omega_\gamma^4 + \omega_\gamma^2) \delta_c^2 + 64\omega_\gamma^2 \right) \end{array} \right\},$$

where

$$R = (1 + 4\omega_\gamma^2) M.$$  

To see the validity of the former limit for a given $\delta$, we study the expansion of the output field in the parameter $1/N$. Comparing the order 0 with the order 1 for $\omega_{\text{max}}$, it can be seen that the limit is valid when $\gamma \ll \Gamma$, $\delta^2 \gg 4C\Gamma$, and $C\gamma \gg \Omega^2$.

When the term proportional to $e^{2r}$ is much larger than the other two, the spectral frequency for the extremum is given by Eq. (10). Substituting this extremum in the expression for the quadratures of the field, $\Delta Y_{1\theta}(\omega_{\text{max}})$, we obtain

$$\Delta Y_{2\theta}(\omega_{\text{max}}) = \Delta Y_{1\theta+\pi/2}(\omega_{\text{max}}) = \frac{1}{4} \left( (e^{-r} + e^r)^2 + (-e^{-2r} + e^{2r}) \cos(\theta) \sin(\theta) \delta_c \right).$$  

(10)
The previous equation, valid for $\delta \leq 2$, summarizes the most interesting behavior of the interaction of a broadband probe squeezed state with a pump coherent state in cavity EIT. The characteristics of the stationary output field, for spectral frequency $\omega_{<\text{max}}$, are as follows. (i) The probe maximum squeezing quadrature is rotated, from the $\theta = 0$ quadrature, corresponding to the initial condition, to $\theta = 3\pi/4$. (ii) The probe field is no longer a minimal uncertainty state. (iii) The pump field is also squeezed. In fact, the quadrature spectrum of the pump field is equal to the quadrature spectrum of the probe field rotated by $\pi/2$. We explain these results as a combination of quadrature noise rotation due to two-photon detuning, as explained in Sec. III, and interchange of noise properties between pump and probe, as explained in [1].

It is known [1] that the frequency where the exchange of squeezing happens is $\omega_{sq} = \gamma g a / (\sqrt{2} \sqrt{g^2 N / (\Gamma + 2g^2 \alpha^2)})$. We can guess then that, as long as $\delta$ is such that $\omega_{max} \gg \omega_{sq}$, the interchange of squeezing for frequencies inside the cavity linewidth is unaffected by the two-photon detuning. We could not find an analytical probe of this hypothesis but we checked it numerically for the parameters in the figures.

We explain now the transfer of squeezing, between probe and pump fields, that takes place in the spectral frequency $\omega_{sq}$. In order to understand this phenomenon, we recall what happens in the case of Gaussian states propagating in EIT media [2]. For this case, there is a transfer of squeezing between the pump and probe field as a function of spectral frequency and propagation distance. When the spectral frequency is the same as that of the driven fields, nothing happens. This the reason why, in the cavity, although the field interacts several times with the atoms before leaving the cavity, nothing happens for $\omega = 0$. When the spectral frequency is detuned with respect to the driven frequency, transfer of squeezing starts to happen. What we obtain for the cavity output field is the transfer of squeezing that builds up during the several times the field interacts with the medium as it bounces between the cavity mirrors. As the spectral frequency increases, the transfer of squeezing increases, but at the same time, due to the frequency width of the cavity $\gamma$, the mode associated with this frequency interacts less with the atoms. The frequency where the maximum of the transfer of squeezing happens, $\omega_{sq}$, is of an equilibrium between both processes. For small detuning, the distance scale of the rotation of the maximum squeezed quadrature is larger than the scale of the transfer of squeezing. This explains why small detuning does not affect the buildup of the transfer of squeezing.

As noted before, the effect of the quantum field-atom interaction, for the output field spectrum at frequency $\omega_{<\text{max}}$, given by Eq. (10), can be viewed as the combination of rotation of the quadrature of maximum squeezing with interchange of noise properties between probe and pump fields.

V. DECOHERENCE OF ATOM BASE LEVELS

A. Probe is a squeezed vacuum

When the probe is a squeezed vacuum, the mean value of the output field is the same as the mean value of the input field and the mean values of the dipole operators are zero. So there is no fluorescence. The decoherence between the base levels of the atoms ($\Gamma_{12} \neq 0$) affects only the statistical properties of the incoming field, namely, degrading the maximum squeezed quadrature. In order to see this we start discussing the case where the spectral frequency is the same as the two-photon detuning ($\omega = 0$). Substituting $\alpha_2 = 0$ but allowing $\Gamma_{12} \neq 0$ in the system equations, we found the following expression for the $\theta = 0$ quadrature:

$$\Delta Y_{2\theta_2=0}(\omega = 0) = \frac{1}{B^2} \left( 16e^{2r} C^2 \delta^2 \Gamma_{12}^4 + e^{-2r} \left( \Omega^4 - 4C^2 \Gamma_{12}^2 + 2 \Omega^2 \Gamma_{12} + (\Gamma^2 + \delta^2) \Gamma_{12}^2 \right) + \frac{1}{B} 8 \Omega \Gamma_{12} (\Omega^2 + \Gamma_{12}) \right),$$

where

$$B = \delta^2 \Gamma_{12}^2 + (\Omega^2 + 2 \Omega \Gamma_{12} + \Gamma_{12})^2.$$

The $\pi/2$ quadrature, $\Delta Y_{2\theta_2=\pi/2}$, has a similar expression but with $r$ substituted by $-r$.

Due to the term $e^{2r}$, we identify the first term of Eq. (11) with a $\pi/2$ rotation of part of the noise of the $\pi/2$ quadrature initial condition. It is known that the field emitted from a medium has a phase of $\pi/2$ with respect to the incoming field. The fact that part of the $\pi/2$ quadrature incoming noise is rotated to $\theta = 0$ noise is already known from the fluorescence of two-level atoms [11]. The novelty here is that there is no fluorescence (the mean value of the field is zero) but there is still transformation of noise from the $\pi/2$ quadrature to the $\theta = 0$ quadrature. This transformation of noise is proportional to the two-photon detuning, being zero when $\delta = 0$. The other two terms of Eq. (11) are different from zero when $\delta = 0$; we interpret them as degradation of squeezing due to the interaction of the squeezed state in media subject to decoherence.

For $\omega > \gamma$ we could not find numerically any significant difference from the plots of Sec. III due to decoherence in the base levels.

B. Probe is a squeezed state

Here we analyze the influence of the atom base level decoherence ($\Gamma_{12} \neq 0$) in the cavity output field. We compare our result with the cavity output field studied in Sec. IV.
We could not find analytical solutions; we carry out our study numerically. The parameters not specified in this section are the same used in Sec. IV.

For \( \omega > \gamma \) we could not find any significant difference from the plots in Sec. IV due to decoherence in the base levels. The biggest values of \( \Gamma_{12} \) we use are approximately the ones that turn the system equations unstable. For the parameters in the plots, this value is around \( \Gamma_{12} = \Gamma/210 \). This means that the special characteristics found in the previous section for spectrum frequency \( \omega_{<\text{max}} \), namely quadrature rotation and noise interchange between probe and pump, are not sensitive to base level decoherence, as long as \( \omega_{<\text{max}} > \gamma \).

For \( \omega < \gamma \) we found that the transfer of squeezing between the probe and pump is destroyed for very small \( \Gamma_{12} \). This is shown in Fig. 7 (compare with Figs. 5 and 8 for \( \omega < \gamma \)). As can be seen in the figure, for \( \Gamma_{12} \) of the order of \( \Gamma_{12} = 0.0005 \Gamma \), the pump quadrature [Fig. 7(b)] does not show any squeezing. We explain this in the following way. As \( \alpha_2 = \alpha_1 \), the effect of decoherence is to break the dark state and some fluorescence comes from the atom (the emitted field is different from zero [12]).

The cavity forces the atoms to emit photons inside its linewidth. This fluorescence destroys the noise properties of the squeezed state for frequencies inside the cavity linewidth (\( \omega < \gamma \)).

The two-photon detuning also has a huge effect in destroying the transfer of squeezing between the probe and pump fields. We show this in Fig. 8. We use a value of the decoherence rate (\( \Gamma_{12} = \Gamma/100000 \)) for which the transfer of squeezing is almost unaffected for \( \delta = 0 \). We plot then the probe and pump \( \theta = 0 \) noise spectra for \( \delta = 0, 2 \Gamma, 5 \Gamma \). Remember that for \( \Gamma_{12} = 0 \) the cases \( \delta = 2 \Gamma, 5 \Gamma \) would be similar to the \( \delta = 0 \) case. It can be observed in the figure how, unlike in the case \( \Gamma_{12} = 0 \), the noise strongly depends on \( \delta \), and increases for all frequencies if \( \delta \) increases.

VI. SUMMARY AND CONCLUSIONS

We have studied the output field of a two-mode cavity sustaining two-photon detuning (\( \delta_1 = \delta_2 = \delta \); see Fig. 1) with \( N \) atoms in \( \Lambda \) configuration. The atoms show EIT, and the mean value of the field is practically unaltered by the interaction of the cavity+atom system. Nevertheless, there is a coherent alteration of the incoming noise properties in the output field. This alteration depends strongly on the two-photon detuning and the mean value of the modes inside the cavity. Decoherence tends to destroy the quantum properties of the state (i.e., the squeezing) for frequencies inside the cavity linewidth. The particular characteristics of the output field are as follows.

In the case of an incoming coherent pump and a vacuum squeezed probe, the output pump field remains coherent but the output probe field suffers large alterations with respect to the incoming broadband vacuum squeezed state. In addition to a frequency [see Eq. (7)] where absorption of squeezing is expected due to normal splitting of the cavity+atom system, we found that, for a spectral frequency which decreases with increasing \( \delta \) [see Eq. (1)], the quadratures of the output probe field shows a partial rotation of the incoming quadrature probe field. In some cases, \( \delta^2 \gg 4 \Omega^2 \), this rotation can be complete and the incoming \( \theta = 0 \) quadrature squeezed field is transformed into a \( \theta = \pi/2 \) quadrature squeezed field. The effect of decoherence is, in addition to degrading the incoming vacuum squeezed state, to rotate part of the incoming \( \theta = \pi/2 \) quadrature noise to the \( \theta = 0 \) quadrature for frequencies inside the cavity linewidth.

In the case where the cavity probe mode is also pumped and both modes inside the cavity have the same strength, both modes suffer large alteration with respect to the incoming quantum state. In the case of \( \omega > \gamma \), the output field has a maximum which decreases with increasing \( \delta \). Nevertheless we no longer have a rotation of the quadrature of the incoming field as in the previous case. When \( \delta^2 \gg 4 \Gamma^2 \) and \( C \gamma \gg \Omega^2 \), the characteristics of the stationary output field, for spectral frequency \( \omega_{<\text{max}} \), are that (i) the probe maximum squeezing quadrature is ro-
FIG. 8: Output probe (a) and pump (b) field fluctuations for the $\theta = 0$ quadrature in cavity EIT as a function of two-photon detuning $\delta$ when there is decoherence in the atom base levels. Unlike the case $\Gamma_{12} = 0$, here it can be seen how the output field strongly depends on two photon detuning. Parameters: $g = -0.005\Gamma$, $\Omega_1 = \Omega_2 = \Gamma$, $\gamma = .06\Gamma$, $N = 1000000$. $\Gamma_{12} = \Gamma / 10000$

tated, from the $\theta = 0$ quadrature, corresponding to the initial condition, to $\theta = 3\pi/4$; (ii) the probe field is not longer a minimal uncertainty state; and (iii) the pump field is also squeezed. In fact, the quadrature spectrum of the pump field is equal to the quadrature spectrum of the probe field rotated by $\pi/2$. We explain these results as a combination of quadrature noise rotation due to two-photon detuning and interchange of noise properties between pump and probe.

When $\omega < \gamma$ the transfer of fluctuation between probe and pump is unaltered by the two-photon detuning as long as $\omega_{\text{max}} > \gamma$. Nevertheless, the combination of decoherence and two-photon detuning has a large effect in the field fluctuation for frequencies inside the cavity linewidth, destroying the squeezed state for small $\Gamma_{12}$.

Our results shows, that interaction of quantum fields in cavity EIT is a much richer phenomeno than believed, and that the quantum nature of both fields, probe and pump, should be taken into account when dealing with cavity EIT.

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