Chiral-Odd Generalized Parton Distributions from
Exclusive $\pi^o$ Electroproduction

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Exclusive $\pi^o$ electroproduction is suggested for extracting both the tensor charge and
the transverse anomalous magnetic moment from experimental data. A connection be-
tween partonic degrees of freedom, given in terms of Generalized Parton Distributions,
and Regge phenomenology is discussed. Calculations are performed using a physically
motivated parametrization that is valid at values of the skewness, $\zeta \neq 0$. Our method
makes use of information from the nucleon form factor data, from deep inelastic scat-
tering parton distribution functions, and from lattice results on the Mellin moments
of generalized parton distributions. It provides, therefore, a step towards a model
independent extraction of generalized distributions from the data, alternative to other
mathematical ansatze available in the literature.

1 Introduction

Exclusive $\pi^o$ electroproduction from nucleons and nuclei allows one to ex-
tract the chiral odd Generalized Parton Distributions (GPDs), the tensor charge, and other quantities related to
transversity, from experimental data [2]. In this reaction only the C-parity odd combinations
quantum numbers of the t-channel exchanges are, in fact, selected. In a description in terms
of partonic degrees of freedom, i.e. based on the handbag diagram at leading order, the
helicity structure for this C-odd process relates to the quark helicity flip, or chiral odd
generalized parton distributions whose integrals were shown to be related to the tensor
charge $\delta q$, and to the transverse anomalous magnetic moment, $\kappa^T_q$. This differs markedly
from Deeply Virtual Compton Scattering (DVCS), and from both vector meson and charged
$\pi$ electroproduction, where the axial charge can enter the amplitudes.

In this talk, we start from writing the cross section and spin asymmetries for $ep \to e'\pi^op$ using the helicity amplitudes formalism. We then study the sensitivity of the various
components of the cross section, and of the target transverse asymmetry, $A_{UT}$, to both $\delta q$
and $\kappa^T_q$, in kinematical ranges where experimental data are currently being analysed. The
Compton Form Factor, $H$, containing the chiral even GPD, $H$ was already extracted with
high accuracy from the data, using cross section and beam spin asymmetries measurements.
Within our suggested scenario, the presently analysed $\pi^o$ electroproduction data, as well as
future measurements, will allow one to obtain information on $H_T$, and $E_T$, containing the
chiral-odd GPDs, $H_T$ and $E_T = 2H_T + E_T$. We subsequently discuss the structure of our
GPDs parametrization in relation with the constraints obtained from the data.

*U.S. Department of Energy grants no. DE-FG02-01ER4120 (S.A. and S.L), and no. DE-FG02-
92ER40702 (G.R.G.)
2 Dependence of $\pi^0$ Electroproduction Observables on Transversity Moments

The differential cross section for pion electroproduction off an unpolarized target is

$$\frac{d^4\sigma}{d\Omega dt} = \Gamma \left\{ \frac{d\sigma_T}{dt} + \epsilon_L \frac{d\sigma_L}{dt} + \epsilon \cos 2\phi \frac{d\sigma_{TT}}{dt} + \sqrt{2\epsilon_L(\epsilon + 1)} \cos \phi \frac{d\sigma_{LT}}{dt} \right\}. \quad (1)$$

where $\epsilon$ is the photon polarization parameter, and for longitudinal polarization alone, $\epsilon_L = (Q^2/v^2)\epsilon$; $\Gamma$ is a kinematical factor including the Mott cross section, $\sigma_M = 4\alpha^2 e^4 \cos^2(\theta/2)/Q^4$.

The different contributions in Eq. (1) are written in terms of helicity amplitudes, for instance (we use the notation: $f_{\Lambda,\Lambda';0,0'}$):

$$\frac{d\sigma_T}{dt} = N \left( |f_{1,+;0,+}|^2 + |f_{1,+;0,-}|^2 + |f_{1,-;0,+}|^2 + |f_{1,-;0,-}|^2 \right)$$

$$= N \left( |f_1|^2 + |f_2|^2 + |f_3|^2 + |f_4|^2 \right) \quad (2)$$

with $N = [M(s - M^2)]^{-1} G$, where we used the Hand convention, multiplied by a geometrical factor $G = 1/\sqrt{8\pi}$. In addition to the unpolarized observables listed above, a number of observables directly connected to transversity can be written (see e.g. [3]). Here we show the transversely polarized target asymmetry,

$$A_{UT} = 2\Im m(f_3^* f_1 - f_4^* f_2) \left( \frac{d\sigma_T}{dt} \right). \quad (3)$$

It is important to realize that the relations between observables and helicity amplitudes is general, independent of any particular model. In [2] we, in fact, used the formalism above in both a Regge model, as well as using GPDs. The connection with the parton model and the transversity distribution is uncovered through the GPD decomposition of the helicity amplitudes. In the factorization scenario the amplitudes for exclusive $\pi^0$ electroproduction, $f_{\Lambda,\Lambda';0,0'}$, can be decomposed into a “hard part”, $g_{\Lambda,\Lambda';0,0'}$ and a “soft part”, $A_{\Lambda',\Lambda,\Lambda'}$ (where $\Lambda$ ($\Lambda'$) are the initial (final) proton helicities, and $\lambda$ ($\lambda'$) are the initial (final) quark helicities) through the products of $\gamma^* + g \rightarrow \pi^0 + q$ amplitudes, $g_{\Lambda,\Lambda';0,0'}$, with the matching quark-hadron helicity structures that, in turn, contain the GPDs, in the form

$$f_{\Lambda,\Lambda';0,0'} = \sum_{\Lambda',\lambda} g_{\Lambda,\lambda;0,0'} A_{\Lambda',\lambda';\Lambda,\lambda}. \quad (4)$$

The $A_{\Lambda',\lambda';\Lambda,\lambda}$ structures are functions of $x_{Bj}, t$ and $Q^2$; they are analogous to the Compton Form Factors in DVCS. They implicitly contain an integration over unobserved quark momenta. Because only one of the $g$ transverse photon functions is non-zero [2], the relation to the quark-hadron amplitudes is quite simple Note that because of the pion chirality, $0^-$, the quark must flip helicity at the pion vertex,

$$\frac{d\sigma_T}{dt} = \left\{ 2 |A_{++,-}|^2 g_1^2 + |A_{--,+}|^2 g_2^2 + |A_{++,-}|^2 g_3^2 \right\} \quad (5a)$$

where we wrote explicitly the $Q^2$-dependence of the $g$ functions from the pion vertex, $g_i(s,t,Q^2) = \tilde{g}_i(s,t) F_i(Q^2)$, $i = 1, 5$, for each amplitude. These depend on whether the $\pi^0$ is produced within an interaction with a vector or an axial vector meson (see Ref. [2] for DIS 2008.
details). Similar expressions containing bilinear products of the imaginary and real parts of the quark-hadron amplitudes, \( A \), can be written for the other contributions.

A formal proof of factorization was given only in the case of longitudinally polarized virtual photons producing longitudinally polarized vector mesons [4]. Endpoint contributions are surmised to be larger in electroproduction of transversely polarized vector mesons, and therefore prevent factorization. Notwithstanding current theoretical approaches, many measurements conducted through the years, display larger transverse contributions than expected [5]. In this paper we suggest as an alternative avenue a QCD based model, that predicts different \( Q^2 \) behaviors for meson production via natural and unnatural parity exchanges.

At leading order, using the notations of Ref.[6], one can write the helicity amplitudes in terms of linear combinations of Meson Production Form Factors (MPFFs). The MPFFs defining \( \pi^0 \) production are

\[
F \equiv F^{p \to \pi^0} = \frac{1}{\sqrt{2}} \left[ \frac{2}{3} F^u + \frac{1}{3} F^d \right]
\]

with \( F = H_T, E_T, \tilde{H}_T, \tilde{E}_T \), and the corresponding \( F^q \) defined in terms of the chiral odd-GPDs, \( F = F_T \) as:

\[
F^q(\zeta, t) = i\pi \left[ F^q(\zeta, \zeta, t) - F^{\bar{q}}(\zeta, \zeta, t) \right] + P \int_{-1+\zeta}^{1} dX \left( \frac{1}{X-\zeta} + \frac{1}{X} \right) F^q(X, \zeta, t).
\]

3 A Bottom-Up Parametrization of Generalized Parton Distributions

We performed calculations using a phenomenologically constrained model from the parametrization of Refs.[7]. We summarize the AHLT model for the unpolarized GPD. The parameterization’s form is:

\[
H(X, \zeta, t) = G(X, \zeta, t) R(X, \zeta, t),
\]

where \( R(X, \zeta, t) \) is a Regge motivated term that describes the low \( X \) and \( t \) behaviors, while the contribution of \( G(X, \zeta, t) \), obtained using a spectator model, is centered at intermediate/large values of \( X \):

\[
G(X, \zeta, t) = C \frac{X}{1-X} \int d^2 k_\perp \frac{\phi(k^2, \lambda)}{D(X, k_\perp)} \frac{\phi(k'^2, \lambda)}{D(X, k'_\perp)}.
\]

Here \( k \) and \( k' \) are the initial and final quark momenta respectively; explicit expressions are given in [7]. The \( \zeta = 0 \) behavior is constrained by enforcing both the forward limit: \( H^q(X, 0, 0) = q_{val}(X) \), where \( q_{val}(X) \) is the valence quarks distribution, and the following relations:

\[
\int_0^1 dX H^q(X, \zeta, t) = F_1^q(t), \quad \int_0^1 dX E^q(X, \zeta, t) = F_2^q(t),
\]

which define the connection with the quark’s contribution to the nucleon form factors. Notice the AHLT parametrization does not make use of a “profile function” for the parton distributions, but the forward limit, \( H(X, 0, 0) \equiv q(X) \), is enforced non trivially. This affords us
the flexibility that is necessary to model the behavior at $\zeta, t \neq 0$. $\zeta$-dependent constraints are given by the higher moments of GPDs. The $n = 1, 2, 3$ moments of the NS combinations: $H^{u-d} = H^u - H^d$, and $E^{u-d} = E^u - E^d$ are available from lattice QCD [8], $n = 1$ corresponding to the nucleon form factors. In a recent analysis a parametrization was devised that takes into account all of the above constraints. The parametrization gives an excellent description of recent Jefferson Lab data in the valence region.

The connection to the transversity GPDs is carried out similarly to Refs. [9] for the forward case by setting:

$$H^q_T(X, \zeta, t) = \delta q H^q_{val}(X, \zeta, t)$$

$$E^q_T \equiv 2H_T + E_T = \kappa^q_T H^q_T(X, \zeta, t)$$

where $\delta q$ is the tensor charge, and $\kappa^q_T$ is the tensor anomalous moment introduced, and connected to the transverse component of the total angular momentum in [10]. Notice that our unpolarized GPD model can be adequately extended to describe $H_T$ since it was developed in the valence region, and transversity involves valence quarks only.

In Fig. 1 we show the sensitivity of $A_{UT}$ to to the values of the $u$-quark and $d$-quark tensor charges. The values in the figure were taken by varying up to 20% the values of the tensor charge extracted from the global analysis of Ref. [9], i.e. $\delta u = 0.48$ and $\delta d = -0.62$, and fixing the transverse anomalous magnetic moment values to $\kappa^u_T = 0.6$ and $\kappa^d_T = 0.3$. This is the main result of this contribution: it summarizes our proposed method for a practical extraction of the tensor charge from $\pi^0$ electroproduction experiments. Therefore our model can be used to constrain the range of values allowed by the data.

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