Experimental demonstration of indefinite causal order induced quantum heat extraction

Huan Cao\textsuperscript{1,2}, Ning-ning Wang\textsuperscript{1,2}, Zhih-Ahn Jia\textsuperscript{1,2}, Chao Zhang\textsuperscript{1,2}, Yu Guo\textsuperscript{1,2}, Bi-Heng Liu\textsuperscript{1,2}, Yun-Feng Huang\textsuperscript{1,2}, Chuan-Feng Li\textsuperscript{1,2} and Guang-Can Guo\textsuperscript{1,2}

\textsuperscript{1}CAS Key Laboratory of Quantum Information, University of Science and Technology of China, Hefei, 230026, China
\textsuperscript{2}CAS Center For Excellence in Quantum Information and Quantum Physics, Hefei, 230026, China

In the classical world, physical events always happen in a fixed causal order. However, it was recently revealed that quantum mechanics allows events to occur with indefinite causal order (ICO). In this study, we use an optical quantum switch to experimentally investigate the application of ICO in thermodynamic tasks. Specifically, we demonstrate that when a working system interacts with two thermal reservoirs in an ICO, non-classical heat transfer can be observed, even through they share the same temperature. Using such a process, we simulate an ICO refrigeration cycle and investigate its properties. We also show that by passing through the ICO channel multiple times, one can extract more heat per cycle and thus obtain a higher refrigeration efficiency. Our results provide inspirations for further improving the efficiency of quantum thermodynamic tasks and shed new light on the development of a new class of thermodynamic resource theories without presumed causal order.

It is a deeply rooted concept that in a physical theory, there is a well-defined pre-existing classical causal structure for which physical events happen in a definite causal order. However, from the Bell-Kochen-Specker theorem \textsuperscript{1,2}, we known that quantum mechanics is incompatible with the viewpoint that physical observables have pre-existing values independent of the measurement context. Inspired by this, many recent studies on causal structure of quantum mechanics have shown that if we assume the causal relation to obey the laws of quantum mechanics, it is possible for two events to occur with a superposed causal order. Thus, there is no pre-existing causal relation \textsuperscript{3,4}. The quantum causal structure becomes especially crucial when quantum physics and general relativity become relevant \textsuperscript{5,6}. A typical example is the quantum spacetime causal structure in the study of quantum gravity \textsuperscript{10,11}. For instance, when a massive object is in a superposition of two or more distinct spatial positions, the states of the gravitational field produced by the object will also be in a superposition, implying that the spacetime causal order that originates from the spacetime geometry will also be in a superposition.

In addition to the fundamental properties of indefinite causal order (ICO) and causal structure, the applications of ICO as an operational resource in quantum information and computation also attract considerable research interests \textsuperscript{15,16}. It provides remarkable enhancements ranging from channel discrimination \textsuperscript{17}, communication and computation complexity \textsuperscript{18–20} to quantum metrology \textsuperscript{21,22}, quantum information transmission \textsuperscript{23,24} etc. Recently, some of these predictions have also been studied in the experiment, in which optical quantum switch is utilized to simulate the ICO process \textsuperscript{26–31}.

In thermodynamics, entropy in closed systems always tends to increase definitely. An interesting question is what will happen when apply ICO in thermodynamic tasks. One of such example is the recent discovery of ICO-based quantum refrigeration \textsuperscript{22}. There are several different ways for quantum refrigeration: the standard one is powered by energy injected by a time dependent driving force \textsuperscript{33,34}; the Maxwell demon can also be used to steer the heat by means of a feedback control loop \textsuperscript{35,36}; while another method is to use invasive quantum measurements as a resource to power the refrigeration \textsuperscript{37}. All the above refrigeration protocols works in a world with pre-existing causal structures. The ICO-based protocol provides a good supplement for which no pre-existing causal relation is assumed \textsuperscript{32}. This sheds new light on both of the quantum causal structure and quantum thermodynamics.

In this paper, we experimentally demonstrate the advantages of ICO in performing quantum thermodynamic tasks \textsuperscript{22,32,38,39}. In particular, by using an optical quantum simulator, we demonstrate the ICO induced heat extraction and investigate its feasibility to construct a quantum refrigerator. We also show that by interacting with reservoirs in an ICO multiple times, one can extract more heat from the reservoirs. The high accuracy achieved in our experiment is expected to lead to more operational protocols and thus contribute to a broader research into the use of ICO as a resource.

\textit{Protocol outline}.— Before detailing the experimental realization of the ICO-driven quantum heat extraction, let us first recall how the protocol works. Consider a system with Hamiltonian $\mathcal{H}$ and energy eigenstates $|n\rangle$ for energy level $E_n$. After thermocontact with a thermal reservoir with inverse temperature $\beta$, the resulting equilibrium state of the system is always $T = e^{-\beta\mathcal{H}/Z} = \sum_n e^{-\beta E_n} Z |n\rangle \langle n|$ regardless of the initial system state $\rho$, where $Z = \text{Tr}(e^{-\beta\mathcal{H}}) = \sum_n e^{-\beta E_n}$ is the partition function. This thermodynamics operation can be char-
acterized by a quantum channel, namely, a completely positive trace preserving (CPTP) map $T: \mathcal{L}(\mathcal{H}) \rightarrow \mathcal{L}(\mathcal{H})$ for which $T(\rho) = T$ for all density operators $\rho$. We denote the Kraus operators for $T$ as $K_i$. The Kraus decomposition is thus $T(\rho) = \sum_i K_i \rho K_i^\dagger$, where the Kraus operators $\{K_i\}$ satisfy $\sum_i K_i^\dagger K_i = I$.

Now consider the situation where the system in state $\rho$ undergoes thermocontact sequentially with two identical thermal reservoirs with the same temperature. If we assume the definite causal order, then the process is given either by $T^1 \circ T^2(\rho)$ or $T^2 \circ T^1(\rho)$, or potentially a classical probabilistic mixture of these two processes, and the same thermal state $T$ is obtained. However, when in a world with ICO, the two events ‘ thermocontact with thermal reservoir 1 firstly’ and ‘ thermocontact with thermal reservoir 2 firstly’ can occur in a superposed causal order. This will yield an interesting result—the resulting state is different from $T$. Such an operation can be optically simulated using the quantum switch [17, 20, 40]. The action of ICO is achieved by routing photons through two channels with the visiting order being tailored by the control qubit [20, 41], when control qubit $|\phi_c\rangle = |1\rangle$ or $|\phi_c\rangle = |0\rangle$, the operations $T^2 \circ T^1$ or $T^1 \circ T^2$ is carried out respectively. We denote the corresponding channel as $S^T$. In terms of Kraus operators, we have $S^T(\rho_c \otimes \rho) = \sum_{ij} M_{ij} (\rho_c \otimes \rho) M_{ij}^\dagger$ and

$$M_{ij} = |0\rangle \langle 0|_c K_i^1 K_j^2 + |1\rangle \langle 1|_c K_j^3 K_i^1$$  \hspace{1cm} (1)

where $K_i^1$ ($K_j^2$) represents the $i$-th ($j$-th) Kraus operator of the thermalizing channels $T^1$ ($T^2$).

Considering the most simple non-trivial case, a two-level system, the ground state is $|0\rangle$ (the excited state is $|1\rangle$) with energy $E_0 = 0$ ($E_1 = \Omega$), thus, the Hamiltonian for the system is $H = \Omega |1\rangle \langle 1|$. The thermal state at a given temperature is $\rho = \text{diag}(1, e^{-\beta \Omega}) / Z$, where $Z = 1 + e^{-\beta \Omega}$. In the following we set $\Omega = 1$ for simplicity. If the ancillary control qubit is initialized as $|\phi_c\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, the resultant output state undergoing ICO with two identical thermalizing channels will be

$$S(\rho_c \otimes \rho) = \sum_{ij} M_{ij} (\rho_c \otimes \rho) M_{ij}^\dagger$$  \hspace{1cm} (2)

$$= \frac{1}{2} \left[ |0\rangle \langle 0|_c + |1\rangle \langle 1|_c \right] \otimes T + |0\rangle \langle 1|_c + |1\rangle \langle 0|_c \otimes T \rho T \right]$$

Where the control qubit is $\rho_c = |\phi_c\rangle \langle \phi_c|$. Note that the ICO evolution inside the quantum switch entangles the control qubit with the resultant system state; if the control qubit is projected into $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$, the system state will collapse into

$$\text{Tr}_c [|\pm\rangle \langle \pm|_c S(\rho_c \otimes \rho)] = \frac{1}{2} (T \pm T \rho T)$$  \hspace{1cm} (3)

with the corresponding probability $p_\pm = \frac{1}{2} \text{Tr}[T \pm T \rho T]$. We note that the temperature of output system state could be different from the thermal state $T$, with the temperature depending on the measurement outcome in $|\pm\rangle_c$ basis and initial system state $\rho$. This intriguing phenomenon suggests that the ICO can either extract heat from or dump heat into the reservoir. Hence ICO is potentially useful for realizing thermodynamic tasks, such as refrigerating or cooling the input system.

**Experimental implementation.**— We build a high-performance tabletop photonic quantum switch to realize the ICO process. A spontaneous parametric down conversion (SPDC) produces heralded single photons at 780 nm. The heralded photons, which act as working substances, are then fed into a Mach-Zehnder interferometer, as shown in Fig. 1. Here we use photonic polarization to simulate the two energy levels of the working system, where horizontal (vertical) polarization state $H$ ($V$) represents the ground (excited) state. The population of the excited state is related to the temperature of the working system. A system state at an arbitrary temperature is prepared by randomly rotating the polarization of the photons into $H$ or $V$ with a probability proportional to its temperature. Energy detection can be realized by measurements in Pauli $\sigma_z$ basis; thus, it is possible to detect the temperature. To realize the ICO quantum channel (see eq. [2]), the beam splitter 1 (BS1) introduces two spatial modes as the control qubit. The polarization qubit undergoes the causal order $T^1 \circ T^2$ in one spatial mode, while $T^2 \circ T^1$ in the other one. BS2 then coherently combines these spatial modes and projects the control qubit onto $|\pm\rangle$. The ICO is valid when the spatial modes inside quantum switch admit the interferometer. A phase-locking system is adopted to ensure the stability.

---

**Figure 1.** Schematic of the experimental apparatus. The quantum switch contains two identical thermalizing channels (with pink planes underneath) in an indefinite causal order. One of the causal order is presented by a bluish optical path, the other is presented by a red optical path. The BS2 concludes the superposition. BS: beam splitter, PBS: polarization beam splitter, HWP: half wave plate, IF: interferometer filter.
of the path interferometer, with an average interferometric visibility of more than 99.7% for four hours, which sufficiently guarantees the ICO performance (See Section II in the Supplementary Material for details). The action of the thermalizing channel can be simulated using a probabilistic mixture of two fully amplitude damping (AD) channels \[ \rho_k = \sum_{i=0}^{3} \rho K_i \rho K_i^\dagger, \] where the \{K_0, K_1, K_2, K_3\} constitute a channel damping to ground (excited) state, and \[ \bar{p} = \{p_0, p_1, p_2, p_3\} \] denote the probability vectors, which depend on the temperature of the thermalizing channel. The AD channel is realized using polarization M-Z interferometer-like setup with half-wave plates (HWPs) and polarization beam splitters (PBSs), as shown in Fig. 1. We have performed the process tomography of the thermalizing channel at several different temperatures. The average process fidelity exceeded 99.9%, verifying the credible simulation of the channel (see section II in Supplementary Material for details).

**Results of the ICO process.**— We first verify the non-classical heat transfer driven by the ICO. We implement numbers of experiments by traversing the temperatures of thermalizing channel which belongs to the reservoir in the following discussion. The system state is initialized into the thermal state with the same temperature \(1/\beta_C\) as the reservoir, \(\rho = T\). Fig. 2 (a) shows the measured energy change \(\Delta E\) of the system qubit (as the working substance) after passing through the ICO channel. The horizontal coordinate \(E_C\) = \(e^{-\beta_C}/(1 + e^{-\beta_C})\) ranges from 0 to 0.5, corresponding to the temperatures from zero to infinity (here we use the energy of the thermal state of the working system to represent the reservoir’s temperature for simplicity). By extrapolating our experimental data (red and blue dots in Fig. 2 (a)) to theoretical prediction (red and blue lines in Fig. 2 (a)), we find that the working substance can extract the heat flow from the reservoir when the control qubit is measured in \(|-\rangle\), even though they initially share the same temperature. This intriguing phenomenon is applicable in arbitrary temperature case with the exceptions of zero and infinity. The heat transfer decreases when the temperature increases, whereas the successful probability \(p_+ = \text{Tr} \{p^{-} \langle S (\rho_k \otimes \rho) \rangle \} \) increases (inset in Fig. 2 (a)). The weighted energy change \(\Delta \tilde{E}_{+} = p_+ \Delta E_{+}\) is also presented (orange and green dots and lines 2 (a)). For comparison, we also measure the energy transfer when the control qubit is measured in computational basis \{\{0\}, \{1\}\}, by exemplifying the reservoir’s temperature to be \(E_C = 0.25\) (Fig. 2(b)). Such a case yields the classical outcome in which the output working substance has the same temperature as the reservoir, and no heat extraction could achieve.

Obviously, the non-classical heat transfer driven by ICO can be used for thermodynamic tasks. For example, when the working substance appears at the heating branch, we can interact it with an external hot reservoir to release the extracted heat, thus refrigerating the reservoir: when the working substance appears at the cooling branch, we can send it back to the reservoir to erase the unwanted heat exchange. For the opposite purpose (i.e., to construct a heat engine or cool the working substance), one can instead discard the heating branch and reserve the cooling branch. An interesting question is whether the working substance can become colder or hotter after passing through the ICO channel multiple times.
Results of multi-pass ICO.— Our second result is to investigate this multi-pass strategy. We start with the working substance at the same temperature as the reservoirs. As an example, we still adopt the initial temperature to be $E_C = 0.25$, as shown by the first point in Fig.3(a). At each step, a single run of quantum switch is carried out. Hence one initial state will generate a two-branch outcome (indicated by the arrows in Fig.3(a)). Here we only consider the case in which the working substance becomes colder, and then send it into the next step as initial state. In the experiment, since photons will be annihilated after being measured in each step, we use the measurement results (classical information) to determine the state preparation in the next step to simulate this iterative process (as the loop depicted in Fig.1). The experimental results for the 10-steps ICO are summarized in Fig.3(a), in which the iteration process is indicated by the arrows. We observe that when the multi-pass ($N \geq 2$) ICO is implemented, a colder working substance in the unwanted branch could jump into a higher temperature compared to in the single-step process ($N = 1$). This means that the multi-pass ICO may release the restriction for external reservoir for heat dumping. Interestingly, Fig.3(a) shows that the working substance will quickly saturates to a specific temperature, which means the output working substance remains unchanged in its input state when the control qubit is measured to be $|+\rangle$. We theoretically calculate this steady-state solution for all temperatures of the reservoir and also experimentally sample five points $E_C = \{0.05, 0.15, 0.25, 0.35, 0.45\}$, finding that the working substance always tends toward this steady-state solution after several iterations (see section I of Supplementary Material for details).

Construction of a quantum refrigerator—In the following we consider the refrigeration task driven by ICO. We construct an operational cycle to realize a refrigerator. The schematic diagram of a single cycle of quantum refrigerator is shown in Fig.4, of which the complete process is described as follows. In the first stroke (i), the working substance is initialized by classically interacting with the cold reservoir (Preparing a colder working substance required additional work cost, thus is excluded in our discuss). Then interacts with the two cold reservoirs superposed in ICO. In the second stroke (ii), the control qubit is measured. If the control qubit is collapsed into $|−\rangle$, the working system successfully extracts heat from the cold reservoir, followed by proceeding to next stroke. If the control qubit collapses into $|+\rangle$, two alternative strategies are available as listed above: (a) the working system classically contacts the cold reservoir to recover its initialized state, thereby undoing the unwanted heat change, and a new cycle is implemented. (termed classical strategy); and (b) the ICO quantum switch is repeatedly passed through until the desired outcome is obtained. (termed multi-pass strategy). In the third stroke (iii), The working substance makes classical thermal contact with external hot reservoir for heat release, followed by a classical thermal contact with cold reservoir once again for initialization; subsequently, a new cycle is started.

To evaluate the performance of the quantum refrigerator, we introduce the coefficient of performance, which is calculated by dividing the heat change from the cold reservoir by the work cost of measurement $E_C$. To calculate the heat flow, we adopt the assumption that all the thermalizations are isochoric, which assures that the internal energy change of working system refers to the heat exchange with reservoirs. The heat flow out of cold reservoirs per cycle is related to the temperature of particle entering ($β_-$) and leaving the external hot reservoir ($β_H$) (depicted by the particles on the centreline).
to be measured and the result is stored in a register. A following operation is determined by the information in the register, as shown by the loop in Fig. 1. Since we assume the control qubit is degenerate in energy, the measurement itself does not cost energy. Rather, the energy cost comes from resetting the register to its initial state in order to proceed to the next cycle, which refers to Landauer’s erasure [45]. The work cost is given by $\Delta W = \frac{1}{S} \Delta S$, where $S = (p_+ \ln p_+ + p_- \ln p_-)$ is the Shannon entropy of the register and $\beta_R$ is the inverse temperature of the resetting reservoir. Therefore the coefficient of quantum refrigerator is given by

$$\eta = \frac{\Delta E}{\bar{n} \Delta W}$$

(4)

where $\bar{n} = \frac{1}{p_+}$ is the average number of measurements consumed to realize one cycle of the quantum refrigerator.

For the classical strategy, the coefficient of quantum refrigerator can be directly calculated by Eq. (4). When it comes to multi-pass strategy, it potentially contains many steps per cycle. The heat flow extraction or work cost in each step is not identical. However, we can rationally approximate the coefficient for multi-pass strategy by assuming that the quantum refrigerator working at the steady-state point (see section I of Supplementary Material for details). That is because single cycle is much likely to undergo multiple steps and evolve into equilibrium, especially for a low temperature of cold reservoir in practice, where the probability of measurement outcome $|\rangle$ approaches to zero with the temperature decreasing. We comparatively present the coefficients of quantum refrigerator under both the classical and multi-pass strategy in Fig. 3(d), in which the solid lines show the theoretical prediction and dots show the experimental results for several sample temperatures $E_C = \{0.05, 0.15, 0.25, 0.35, 0.45\}$. The results show that the multi-pass strategy is always superior to classical strategy. The results also provide a deep insights into how to improve the efficiency of quantum thermodynamic tasks by replacing irreversible isochoric thermalizations with a reversible process to the extent possible.

We provide the strategies of quantum refrigerator that involve both quantum and classical operation. Despite a measurement inside strokes, both the outcomes are all taken into account, hence the refrigerator cycle does not depend on postselection to gain its advantages. In addition, instead of feedback control loop in which the control qubit is measured, the quantum refrigerator can also be realized without projection but rather consumption of purity of control qubit [52]. In the present work, we have demonstrated ICO-driven nonclassical heat transfer between system and reservoirs even though they share the same initial temperature, for which heat change via direct thermalization would be inaccessible. Our findings are beneficial for advancing a new class of resource theories in quantum thermodynamics based ICO. Especially in the case where the control qubit is degenerate in energy, which means it can not be used for direct thermalization, the ICO provide an effective way to utilize its free energy. Our results confirm that the ICO can be a useful resource in quantum thermodynamics, which provide a new paradigm of work extraction alternative to other non-classical features [46, 47]. Therefore we expect that our work will advance further investigations on the exotic properties of indefinite causal order, as well as its superiority in quantum tasks.

We thank Yong-Xiang Zheng, Xue Li, Xiao Liu for beneficial discussions. This work was supported by National Natural Science Foundation of China (11734015, 62075208, 11774335, 11874345, 11821404, 11904357), Anhui Initiative in Quantum Information Technologies (AHY070000, AHY020100, AHY060300), National Key Research and Development Program of China (2017YFA0303400, 2016YFA0301300, 2016YFA0301700), Key Research Program of Frontier Sciences, CAS (QYZDY-SSW-SLH003), Science Foundation of the CAS (ZDRW-XH-2019-1), the Fundamental Research Funds for the Central Universities, Science and Technological Fund of Anhui Province for Outstanding Youth (2008085J02).

Noted——Recently, we become aware of a related work by Nie et al. which experimentally study quantum thermodynamics driven by ICO on nuclear spins using the nuclear magnetic resonance system [48].

---

[1] J. S. Bell, Physics Physique Fizika 1, 195 (1964).
[2] S. Kochen and E. P. Specker, in The logico-algebraic approach to quantum mechanics (Springer, 1975), pp. 293–328.
[3] O. Oreshkov, F. Costa, and Č. Brukner, Nature communications 3, 1 (2012).
[4] Č. Brukner, Nature Physics 10, 259 (2014).
[5] D. Oriti, Approaches to quantum gravity: Toward a new understanding of space, time and matter (Cambridge University Press, 2009).
[6] S. Hosenfelder, Experimental search for quantum gravity (Springer, 2017).
[7] C. Marletto and V. Vedral, Physical review letters 119, 240402 (2017).
[8] S. Bose, A. Mazumdar, G. W. Morley, H. Ulbricht, M. Tóroš, M. Paternostro, A. A. Geraci, P. F. Barker, M. Kim, and G. Milburn, Physical review letters 119, 240401 (2017).
[9] A. Peres and D. R. Terno, Reviews of Modern Physics 76, 93 (2004).
[10] M. Christodoulou and C. Rovelli, Physics Letters B 792, 64 (2019).
[11] B. S. DeWitt, Physical Review 160, 1113 (1967).
[12] C. Rovelli, Physical Review D 42, 2638 (1990).
[13] R. Gambini, R. A. Porto, and J. Pullin, New Journal of Physics 6, 45 (2004).
[14] L. Hardy, in Quantum reality, relativistic causality, and closing the epistemic circle (Springer, 2009), pp. 379–401.
[15] M. M. Taddei, R. V. Nery, and L. Aolita, Physical Review Research 1, 033174 (2019).
[16] D. Jia, F. Costa, et al., Physical Review A 100, 052319 (2019).
[17] G. Chiribella, Physical Review A 86, 040301 (2012).
[18] P. A. Guérin, A. Feix, M. Araújo, and Č. Brukner, Physical review letters 117, 100502 (2016).
[19] A. Feix, M. Araújo, and Č. Brukner, Physical Review A 92, 052326 (2015).
[20] M. Araújo, F. Costa, and Č. Brukner, Physical review letters 113, 250402 (2014).
[21] X. Zhao, Y. Yang, and G. Chiribella, Physical Review Research 2, 033174 (2019).
[22] C. Mukhopadhyay, M. K. Gupta, and A. K. Pati, arXiv preprint arXiv:1812.06655 (2018).
[23] D. Ebler, S. Salek, and G. Chiribella, Physical review letters 120, 120502 (2018).
[24] S. Salek, D. Ebler, and G. Chiribella, arXiv preprint arXiv:1809.06655 (2018).
[25] G. Chiribella, M. Banik, S. S. Bhattacharya, T. Guha, M. Alimuddin, A. Roy, S. Saha, S. Agrawal, and G. Kar, arXiv preprint arXiv:1810.10457 (2018).
[26] L. M. Procopio, A. Mochanaki, M. Araújo, F. Costa, I. A. Calafell, E. G. Dowel, D. R. Hamel, L. A. Rozema, Č. Brukner, and P. Walther, Nature communications 6, 1 (2015).
[27] G. Rubino, L. A. Rozema, A. Feix, M. Araújo, J. M. Zeuner, L. M. Procopio, Č. Brukner, and P. Walther, Science advances 3, e1602589 (2017).
[28] K. Goswami, C. Giarmatzi, M. Kewming, F. Costa, C. Branciard, J. Romero, and A. White, Physical review letters 121, 090503 (2018).
[29] Y. Guo, X.-M. Hu, Z.-B. Hou, H. Cao, J.-M. Cui, B.-H. Liu, Y.-F. Huang, C.-F. Li, G.-C. Guo, and G. Chiribella, Physical Review Letters 124, 030502 (2020).
[30] K. Goswami, Y. Cao, G. Paz-Silva, J. Romero, and A. White, Physical Review Research 2, 033292 (2020).
[31] K. Wei, N. Tischler, S.-R. Zhao, Y.-H. Li, J. M. Arrazola, Y. Liu, W. Zhang, H. Li, L. You, Z. Wang, et al., Physical review letters 122, 120504 (2019).
[32] D. Felce and V. Vedral, Physical Review Letters 125, 070603 (2020).
[33] M. Campisi, J. Pekola, and R. Fazio, New Journal of Physics 17, 035012 (2015).
[34] M. Campisi and R. Fazio, Journal of Physics A: Mathematical and Theoretical 49, 345002 (2016).
[35] K. Maruyama, F. Nori, and V. Vedral, Reviews of Modern Physics 81, 1 (2009).
[36] C. Elouard, D. Herrera-Martí, B. Huard, and A. Auffeves, Physical Review Letters 118, 260603 (2017).
[37] L. Buffoni, A. Solfanelli, P. Verrucchi, A. Cucoli, and M. Campisi, Physical review letters 122, 070603 (2019).
[38] T. Guha, M. Alimuddin, and P. Parashar, arXiv preprint arXiv:2003.01464 (2020).
[39] S. Markes and L. Hardy, in Journal of Physics-Conference Series (2011), vol. 306, p. 012043.
[40] G. Chiribella, G. M. D’Ariano, P. Perinotti, and B. Valiron, Physical Review A 88, 022318 (2013).
[41] K. A. Fisher, R. Prevedel, R. Kaltenbaek, and K. J. Resch, New Journal of Physics 14, 033016 (2012).
[42] H. Lu, C. Liu, D.-S. Wang, L.-K. Chen, Z.-D. Li, X.-C. Yao, L. Li, N.-L. Liu, C.-Z. Peng, B. C. Sanders, et al., Physical Review A 95, 042310 (2017).
[43] L. Mancino, M. Sbroscia, I. Gianani, E. Roccia, and M. Barbieri, Physical Review Letters 118, 130502 (2017).
[44] K. Abdelkhalek, Y. Nakata, and D. Reeb, arXiv preprint arXiv:1609.06981 (2016).
[45] M. Perarnau-Llobet, K. V. Hovhannisyan, M. Huber, P. Skrzypczyk, N. Brunner, and A. Acín, Phys. Rev. (2015).
[46] G. Francica, J. Goold, F. Plastina, and M. Paternostro, npj Quantum Information 3, 1 (2017).
[47] X. Nie, X. Zhu, C. Xi, X. Long, Z. Lin, Y. Tian, C. Qiu, X. Yang, Y. Dong, J. Li, et al., arXiv preprint arXiv:2011.12580 (2020).