Instrumenting an SMT Solver to Solve Hybrid Network Reachability Problems

Abstract—PDDL+ planning has its semantics rooted in hybrid automata (HA) and recent work has shown that it can be modeled as a network of HAs. Addressing the complexity of nonlinear PDDL+ planning as HAs requires both space and time efficient reasoning. Unfortunately, existing solvers either do not address nonlinear dynamics or do not natively support networks of automata.

We present a new algorithm, called HNSolve, which guides the variable selection of the dReal Satisfiability Modulo Theories (SMT) solver while reasoning about network encodings of nonlinear PDDL+ planning as HAs. HNSolve tightly integrates with dReal by solving a discrete abstraction of the HA network. HNSolve finds composite runs on the HA network that ignore continuous variables, but respect mode jumps and synchronization labels. HNSolve admissibly detects dead-ends in the discrete abstraction, and posts conflict clauses that prune the SMT solver’s search. We evaluate the benefits of our HNSolve algorithm on PDDL+ benchmark problems and demonstrate its performance with respect to prior work.

I. INTRODUCTION

Recent planners ([7] and [6]) for PDDL+ [19] represent actions, processes, events, and state variables as a network of synchronized hybrid automata (HA), but there are no suitable algorithms for reasoning about nonlinear change in a network of automata. We address nonlinear PDDL+ problems by adapting the dReal Satisfiability Modulo Theories (SMT) solver [10], [22], which has been previously shown to address nonlinear PDDL+ as a single hybrid automaton. It is well-known that reasoning about the explicit parallel composition of a network of automata as a single automaton is usually a poor choice because it grows exponentially in the size of the individual synchronized automata [2], [11], [14]. We base our HA network SMT encoding upon prior work [14] that represents each automaton independently and adds synchronization constraints. We show that a direct network encoding results in better scalability in the number of encoding steps than a single automaton encoding if it is also matched with an appropriate search strategy.

We extend and reinterpret the approach taken by Bryce et al. [10] within our proposed HNSolve algorithm. They guide dReal to systematically select variable assignments that correspond to discrete-feasible runs of a hybrid automaton (i.e., ignoring continuous variables). In this manner, they force dReal to perform a heuristic depth-first search that considers all runs through the hybrid automaton. This led to increased scalability because dReal did not consider variable assignments that correspond to discrete-infeasible runs. A similar technique is employed by the dReach [22] and BACH [12] algorithms, but these encode a different SMT instance for each discrete run and do not benefit from learned conflict clauses.

HNSolve, like the work of Bryce et al. [10], guides dReal variable selection to construct discrete-feasible runs, but differs in two main aspects. First, HNSolve constructs composite runs for a network of HAs and not a single HA. Each composite run represents a feasible sequence of synchronized transitions within each automaton. HNSolve also performs a heuristic depth-first search over the possible composite runs. The second difference is that HNSolve also learns conflict clauses that it adds to the SMT encoding. While searching the space of discrete-feasible composite runs, HNSolve may be unable to find a run that extends the current partial run. dReal’s current variable assignment encodes a partial run prefix, and the conflict clause blames the current variable assignment encoding the prefix. We find that adding conflict clauses is a critical new aspect of our approach, and demonstrate that it improves solver scalability as the number of steps in the encodings increase. Developing problem-specific SMT solver algorithms was also recently explored in the context of program analysis [25]. We hope to advance the understanding of how problem-specific, and arguably invasive, modifications to SMT solvers can bring about improved performance.

We evaluate the HNSolve algorithm on several PDDL+ planning benchmarks encoded as networks of hybrid automata, as developed in prior work [7]. We compare flat, precompiled encodings of the networks to our direct encoding of the networks. We show that our encoding outperforms encodings based upon explicit parallel composition and is competitive with the state of the art in PDDL+. We also demonstrate the advantages of using the HNSolve algorithm to improve performance.

II. HYBRID AUTOMATA BACKGROUND

We discuss how to represent HA networks in the $\mathcal{L}_F$ language and their semantics, as follows.

First-order Theories of the Reals: The $\mathcal{L}_F$ language represents the first-order signature over the reals with the set $\mathcal{F}$ of computable real functions.

Definition 1 ($\mathcal{L}_F$-Formulas). $\mathcal{L}_F$-formulas are first-order formulas over real numbers, whose signature allows an arbi-
The syntax is:
\[ t := c \mid x \mid f(t(x)); \]
\[ \varphi := t(x) > 0 \mid t(x) \geq 0 \mid \varphi \land \varphi \mid \varphi \lor \varphi \mid \exists x_1 \varphi \mid \forall x_i \varphi. \]

A function is Type 2 computable if it can be algorithmically evaluated up to an arbitrary numerical accuracy. All common continuous real functions are Type 2 computable.

**Networks of Hybrid Automata**: We solve HA network reachability problems by expressing the network in \( L_{\mathbb{R}, x} \) and then unrolling it over \( k \) steps and associating a goal region \( (x_i^0) \). The goal is also an \( L_{\mathbb{R}, x} \) formula of the form \( \text{mode}_k \land \varphi(x_k^0) \) where \( \text{mode}_k \) defines the HA modes to reach and \( \varphi(x_k^0) \) defines the variable values to reach.

**Definition 2 (\( L_{\mathbb{R}, x} \)-Representations of Hybrid Automata)**. A hybrid automaton in \( L_{\mathbb{R}, x} \)-representation is a tuple
\[ H = (X, Q, \text{flow}, \text{inv}, \text{jump}, \text{init}, L) \]
where
- \( X \subseteq \mathbb{R}^n \) for some \( n \in \mathbb{N} \),
- \( Q = \{q_1, \ldots, q_m\} \) is a finite set of modes,
- \( \text{flow} = \{\text{flow}_k(x^i, x^i, t) : q \in Q\} \) is the set of ODEs describing the flow for each mode,
- \( \text{inv} = \{\text{inv}_{q}(x) : q \in Q\} \) is a set of invariants for each mode,
- \( \text{jump} = \{\text{jump}_{q 
rightarrow q'}(x^i, x'^i) : q, q' \in Q\} \), where each element is a transition from mode \( q \) to \( q' \) using the synchronization label set \( \ell \). Each formula \( \text{jump}_{q 
rightarrow q'}(x^i, x'^i) \) is of the form \( \varphi(x^i) \land \psi(x^i, x'^i) \), where \( \varphi(x^i) \) is a conjunction specifying the guard, \( \psi(x^i, x'^i) \) is a conjunction specifying the discrete update,
- \( \text{init} = \{\text{init}_q(x) : q \in Q\} \) is the set of initial states, and
- \( L \) is a finite set of synchronized event labels.

Gao et al. [22] describe how to unroll this encoding for a single automaton. The important aspects of the unrolling are to time stamp each of the important continuous variables to denote their value at the start \( (x_i^0) \) and end \( (x_i^1) \) of the mode at step \( i \), as well as the time \( t_i \) spent in step \( i \).

**Definition 3. (HA Runs)** Each run \( \tau \) on \( H \) is a series of states
\[ (t_0, q_0, s(x_0^0), s(x_0^1)), \]
\[ (t_1, q_1, s(x_1^0), s(x_1^1)), \]
\[ \ldots, \]
\[ (t_k, q_k, s(x_k^0), s(x_k^1)) \]
where \( t_i \) is the time spent in step \( i \), \( q_i \) is a mode, and \( s(x_i^0) \) and \( s(x_i^1) \) are respective valuations on \( x_i^0 \) and \( x_i^1 \) upon entering or leaving \( q_i \).

The init set defines what constitutes a legal initial mode \( q_0 \) and state valuation \( s(x_0^0) \). The flow and inv sets define legal pairs of state valuations \( s(x_i^0) \) and \( s(x_i^1) \) and occupancy times \( t_i \). The jump set defines legal pairs of modes \( q_i \) and \( q_{i+1} \), state valuations \( s(x_i^1) \) and \( s(x_{i+1}^0) \), and occupancy times \( t_i \).
values $\vec{x}$ represented by the box, each constraint $f(\vec{x}) \geq -\delta$ is satisfied.

**Hybrid Network Solver:** HNSolve wraps the SAT solver and ICP solver by suggesting variable assignments and adding conflict clauses. Where dReal would normally make decisions without realizing it is solving a hybrid network reachability problem, HNSolve organizes its search around potentially viable runs on the network. To see this, consider a situation where dReal has made several variable assignments and decided that $q_0$ and $q_k$ are the initial and final modes of a run satisfying the goal($x^*_k$). For the sake of example, the first automaton $H_1$ does not include jumps that connect $q_0$ to $q_k$. A simple shortest path algorithm can recognize that there is no path from $q_0$ to $q_k$, but dReal will continue to make assignments unnecessarily and eventually backtrack.

HNSolve solves a discrete abstraction [1], [1] of the hybrid reachability problem and coordinates the SAT solver in making assignments corresponding to its solution. Its approach is to find a sequence of mode vectors and synchronization labels of the form

$q_0, L_0, q_1, L_1, \ldots, L_k, q_k$

that are consistent with the jump transitions and synchronize. HNSolve then extracts the literal assignments corresponding to this sequence and guides the SAT solver to realize this run. If the SAT or ICP solvers discover an inconsistency, then HNSolve rebuilds the run from the point to which the SAT solver backtracks.

HNSolve interfaces with the SAT solver through three main methods: getTrail(), assertLit() and assertClause(). The getTrail() method returns the SAT solver’s assignment stack, including all premises, decision variable assignments, and inferred variable assignments. The assertLit() and assertClause() methods assert a literal or clause assignment, respectively, and return unsat, $\delta$-sat, consistent, or backtrack. We note that there is no need for a corresponding retractLit() method because backtracking is handled by the SAT solver as part of the calls to assertLit() and assertClause().

HNSolve, Algorithm [1] first computes the cost (described below) to reach each mode in each automaton in line [2] and then encodes the problem in $L_B$, in line [3]. The algorithm works in two phases. In lines [5] - [14], the solver generates a run suffix $P$. Each run is represented by a set of literals of the form $\{mode_1 = q_0, mode_2 = q_0, \ldots, sync_l, \ldots, mode_k = q_k, mode_2 = q_k, \ldots\}$, defining the modes of each automaton at each step and the synchronization labels of each transition. If the current run represented by the SAT solver’s trail cannot be extended, then genRun() returns a “nil” run. Lines [8] - [12] create a conflict clause from the decision variables on the SAT solver’s trail (i.e., negate the corresponding literals) and assert the new clause. The assertClause() method returns unsat when the SAT solver determines the new clause causes unsatisfiability; otherwise, the SAT solver will backtrack as part of assertClause() and we will find a new run on line [6].

Upon finding a non-nil run, lines [15] - [26] assert each literal in the run from the beginning of the run to the end (i.e., following a run forward to one of the goal modes of each automaton). After successfully asserting each literal on the run, HNSolve asserts the nil literal, which signals the SAT solver to complete any remaining assignments. If the run leads to a $\delta$-sat solution, then the algorithm returns; otherwise, if asserting a literal is not consistent (i.e., returns backtrack or unsat), then the solver attempts to find a new run on line [6].

The genRun() method (Algorithm [2]) finds a run on the network that is consistent with the current SAT solver assignment $T$. It uses depth-first search (line 3 and Algorithm [5]) to find a search stack $S$. The search stack $S$ includes a transition for each automaton for each step 0 to $k-1$. From $S$, the genRun method extracts the literals needed to encode the run (lines 7-14). The literals include mode choices (line 10) and synchronization labels (lines 11-13).

The depth-first search (Algorithm [5]) generates the search stack $S$ corresponding to a run on the network. It selects an initial mode of $H_1$ at depth 0, an initial mode of $H_2$ at depth 1, and so on. It selects a jump from the initial mode of $H_1$ (chosen at depth 0) at depth $|N|$, a jump for $H_2$ at depth $|N| + 1$, and so on. Thus, the first $N$ levels of the stack correspond to initial modes, the second $|N|$ levels to the zeroth step, and similarly for later steps. Lines 8-18 generate the

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**Algorithm 1: HNSolve algorithm.**

HNSolve ($N$, $G$, $k$, $M$)

input: A network of automata $N$, a reachability property $G$, and step and delay bound $k$ and $M$.

cost ← getRunCosts($N$);
encodeSMT($N$, $G$, $k$, $M$);

while true do

repeat

$P$ ← genRun($N$, getTrail(), cost, $k$);

if $P = nil$ then

$C$ ← $\bigvee_{l \in \text{decisions(getTrail())}} \neg l$;

res ← assertClause($C$);

if res = unsat then

return unsat

end

end

until $|P| \neq nil$;

for $l$ from 0 upto $|P|$ do

if $l < |P|$ then

res ← assertLit($P[l]$);

else

res ← assertLit(nil);

end

if res = $\delta$-sat then

return $\delta$-sat;

else if res $\neq$ consistent then

break;

end

end

end
successors, line 19 filters the successors (described below),
and lines 20-27 conduct the recursive step of the search.

Algorithm 4 removes successors that are either inconsistent
with the SAT solver’s current assignment (lines 8-9) or do not
synchronize with the “sibling” jumps previously chosen for the
current step (lines 13-20). The synchronization check involves
three cases. The first and second case are for when either the
current automaton or the sibling will persist its mode (i.e.,
not synchronize). We require that for a mode \( q_i \) to persist
in automaton \( H_j \), the automaton must not be compelled to
synchronize with its siblings at the current time step. More
formally, the mode persistence is allowed if the public labels
\( L_j \cap \ell_i \) for each chosen sibling jump \( q_j \xrightarrow{\ell_i} q'_j \) do not intersect
with the labels \( L_i \). The third case checks that two jumps agree
on publicly communicated labels. The jumps in \( succ' \) are those
that are possible in the discrete sense, but may not be possible
if we were to consider their continuous variables in the guards,
updates, or mode invariants.

After filtering the possible successors, the depth-first search
will sort the jumps by increasing cost, where cost is defined
by the successor mode’s value in \( cost \). The cost of each mode
is defined by the minimum number of steps from an initial
mode:

\[
\text{cost}(q) = \begin{cases} 
0 & : \text{init}_{q}(\vec{x}) \in \text{init}_i; \\
\min_{\text{jump}_{q_i \xrightarrow{\ell_i} q'_j} \in \text{jump}_i} \text{cost}(q') + 1 & : \text{otherwise} 
\end{cases}
\]

In the next section, we detail the \( L_{eff} \) encoding that we use
to express the network HA reachability problem. We follow
with a section describing how we experiment with HNSolve
by omitting it entirely (i.e., use dReal alone), omitting lines 6-
12 of Algorithm 2 to avoid learning conflict clauses, or using
it in its entirety. We note that omitting conflict clauses results
in an algorithm similar to that described by Bryce et al. [10],
aside from our generalization to a network of automata.

IV. NETWORK ENCODING

We encode the parallel composition of a network of hybrid
automata implicitly, as follows. We encode the mode at step
\( i \) of each automaton with literals of the form \( \text{mode}_i^1 = q_1, \ldots, \text{mode}_i^m = q_m \). We constrain the possible composable
jumps with synchronized jump constraints, and noops. With
these constraints, we avoid pre-computing all possible \( O(2^{\text{jump}_i}) \) parallel jumps per step. Instead, we encode
\( O(|\text{jump}|m) \) synchronized jump constraints and \( O(|Q|m) \)
noops. To determine which jumps must synchronize, we introduce
literals for each label and constrain their values with the
appropriate jumps. Noop (stutter) clauses encode cases where
an automaton does not synchronize any of its transitions, and
its mode persists.

We define the implicit parallel composition for a \( k \)-step
M-delay reachability problem as the conjunction of clauses
describing each individual automaton, and the goal:
Algorithm 4: Filter algorithm.

Filter(succ, i, N, S, step)

input: A set of transitions succ, an index of the current automaton i, a network of automata N, a search stack S, and the current time step step.

1. succ' ← \{
2. siblings ← \{
3. for j = 0 \ldots |S||\%|N| do
4. s ← siblings \cup S.get(|S| - (i - j));
5. end
6. for (qi, ℓ) \rightarrow q' \in succ do
7. if T.contains(mode\_step, qi, \% \perp) or \exists ℓ. T.contains(sync\_step, ℓ) then continue;
8. end
9. for (qi, ℓ) \rightarrow q' \in siblings do
10. if ℓ_1 \cap ℓ \cap ℓ_i = \emptyset \text{ or } ℓ_i = \emptyset \text{ and } ℓ_1 \cap ℓ \cap ℓ_i = \emptyset \text{ or } ℓ \cap ℓ_1 \cap ℓ_i \neq ℓ_i \cap ℓ_1 \cap ℓ_j \text{ then break;}
11. end
12. if syncs then succ' ← succ' \cup (qi, ℓ) \rightarrow q'
13. end
14. return succ';

\begin{align*}
\exists X \bar{x}^0 \exists X \bar{x}^1 \ldots \exists X \bar{x}^n \exists X \bar{x}^k

&\exists [0,M] t_0 \ldots \exists [0,M] t_n, \\
&\left( \bigwedge_{j=1}^{n} \text{autom}(H_j, k) \right) \land \text{goal}(\bar{x}^k) \land \left( \bigwedge_{i=0}^{k-1} \bigvee_{\ell \in L_j \cup \ldots \cup L_n} \text{sync}^i \right)
\end{align*}

The \bar{x}^0 and \bar{x}^i variables denote the values of continuous variables at the start and end of step i. The \bar{t} variables denote the duration of step i. The encoding ensures that each automaton behaves appropriately, the goal is satisfied, and at least one non-noop transition occurs in each step.

The \text{autom}(H_j, k) clauses define the behavior of each automaton \(H_j\) as:

\begin{align*}
\text{init}_j(\bar{x}_0^j) \land \bigwedge_{i=0}^{k} \text{maintain}_j(i) \land \bigwedge_{i=0}^{k-1} \left( \bigvee_{q \in Q_j} \text{noop}(q, i) \right) \lor \bigvee_{q \in Q_j} \text{trans}_j(\text{jump}_{q \rightarrow q'}(\bar{x}_i^j, \bar{x}_i^{j+1}), i)
\end{align*}

which constrain the initial state, the continuous change in each mode at each step, and the transitions between steps.

The \text{init}_j(\bar{x}_0^j) clause constrains the initial values of the variables and the initial mode. It defines:

\begin{align*}
\bigvee_{q \in Q_j} \text{init}_q(\bar{x}_0^j) \land (\text{mode}_0^i = q)
\end{align*}

to constrain the assignments to \(\bar{x}_0^j\) and the initial mode.

The \text{maintain}_j(i) clause defines how the flows and invariants of the automaton \(H_j\) govern continuous change:

\begin{align*}
\text{flow}_j(\bar{x}_i^j, \bar{x}_i^{j+1}, t_i) \land \\
\bigvee_{[0,t_i] \cap \bar{t}} \forall \bar{x}_i \left( \text{flow}_j(\bar{x}_0^j, \bar{x}_i, t) \rightarrow \text{inv}_j(\bar{x}_i) \right)
\end{align*}

where we note that the nested universal quantifiers ensure that the invariant holds for the entire time the mode is occupied. The nested quantifiers are a unique aspect of our encoding that enables us to reason about nonlinear change [23].

The \text{flow}_j(\bar{x}_i^j, \bar{x}_i^{j+1}, t_i) clause defines

\begin{align*}
\bigwedge_{q \in Q_j} (\text{mode}_i^j = q) \rightarrow \text{flow}_q(\bar{x}_i^j, \bar{x}_i^{j+1}, t_i)
\end{align*}

The \text{inv}_j(\bar{x}_i) clause enables the invariants of the active modes by defining:

\begin{align*}
\bigwedge_{q \in Q_j} (\text{mode}_i^j = q) \rightarrow \text{inv}_q(\bar{x}_i)
\end{align*}

Noop clauses\(\text{noop}(q, i)\) model asynchronous behavior where the automated does not synchronize, and define:

\begin{align*}
\left( \bigwedge_{\ell \in L_j} \neg \text{sync}^i \right) \land (\text{mode}_i^j = q) \land (\text{mode}_{i+1}^j = q)
\end{align*}

Jump transition clauses\(\text{trans}_j(\text{jump}_{q \rightarrow q'}(\bar{x}_i, \bar{x}_i^{j+1}), \ell)\) define how each jump must synchronize and constrain the variables and modes:

\begin{align*}
\left( \bigwedge_{\ell \in L_j} \text{sync}^i \land \bigwedge_{\ell \notin L_j} \neg \text{sync}^i \right) \land \text{jump}_{q \rightarrow q'}(\bar{x}_i^j, \bar{x}_i^{j+1}) \land \\
(\text{mode}_i^j = q) \land (\text{mode}_{i+1}^j = q')
\end{align*}
V. EMPIRICAL EVALUATION

Our evaluation studies the effectiveness of our network encoding and HNSolve algorithm. Specifically, we compare three configurations of our solver on several hybrid automaton encodings of PDDL+ problems. The configurations include an unmodified dReach/dReal solver, the addition of the HNSolve algorithm without clause learning, and HNSolve with clause learning. We evaluate the configurations on single automaton encodings [10], and networks of automata encodings based on that of Bogomolov et al. [7]. With the network encodings, we either take the parallel composition and encode a single automaton, or encode the network as described in the previous section.

We note that the encodings used in prior work differ in whether they include a “lock” for the actions. Bryce et al. [10] hand-encode a single automaton for each problem that ensures no two actions can occur at the same time. In a network of automata, where each action is represented by its own automaton, Bogomolov et al. [7] ensure that no two actions occur, start, or end simultaneously by introducing a lock automaton. The network of automata models each action so that it must acquire and release the lock when it occurs (atomic actions) or starts/ends (durative actions). This causes the network encoding to require twice the number of encoding steps than the single automaton encoding used by Bryce et al. [10]. We notice that the two step lock is only necessary when \(\epsilon\)-separation of the actions. Since Bryce et al. [10] do not model \(\epsilon\)-separation we can match their required number of encoding steps with the network of automata by using a single lock transition that synchronizes with each action.

We also compare HNSolve with dReal and existing planners, including SpaceEx [3]-[5], [20], CoLin [15], and UP-Murphi [17]. We reproduce previously published results [7] for the SpaceEx, CoLin, and UPMurphi approaches, but report runtimes for HNSolve and dReal from the same machine, a 2.6 GHz Intel Core i7 and 8GB RAM.

Our approach inherits some of the limitations of using SpaceEx with the Bogomolov et al. [7] network representation of PDDL+. The encoding does not respect the “must” semantics of PDDL+ where processes and events must occur when enabled. However, this limitation is not realized in our chosen benchmarks because any use of a process or event is advantageous to the plan. We also note that dReal (and HNSolve as a result) find \(\delta\)-satisfiable solutions to the \(L_{\mathbb{R}^2}\) encoding. Owing to the undecidable nature of nonlinear hybrid systems, dReal cannot guarantee that a \(\delta\)-satisfiable solution, which bounds the values of the continuous variables, contains a realizable plan. Defining an appropriately small value for \(\delta\) minimizes this concern. We also note that dReal in itself is not a full planner. We report results for the minimum step length required to find a plan. A number of strategies for exploring different step lengths in parallel or in sequence have been studied in SAT based planning and can be applied here. We note that these considerations must be incorporated when comparing the results for our approach with that of the other planners.

**Domains:** We use the Generator and Car domains from the literature [7] and the Dribble domain [10]. We compare on linear and nonlinear versions of Generator and Car, but only

| k | Dom | Inst | \(f \) | \(f+H \) | \(f+H+L\) | \(C \) | \(C+H \) | \(C+H+L\) | \(N \) | \(N+H \) | \(N+H+L\) |
|---|-----|------|-----|-------|--------|-----|------|--------|-----|------|--------|
| 3 | Gen | 1    | 0.22 | 0.16  | 0.17   | 1.52 | 0.11  | 0.12   | 0.30 | 0.13  | 0.13   |
| 7 | Gen | 1    | -    | 1.66  | 1.15   | -   | 3.45  | 3.41   | 1.39 | 0.75  | 0.77   |
| 11 | Gen | 2    | -    | -     | -      | 735.00 | 738.10 | 20.69 | 3.62 | 3.62  |
| 15 | Gen | 3    | -    | -     | -      | -   | -    | -      | -   | 12.23 | 12.77  |
| 19 | Gen | 4    | -    | -     | -      | -   | -    | -      | -   | 37.77 | 36.05  |
| 23 | Gen | 5    | -    | -     | -      | -   | -    | -      | -   | 100.21 | 92.26  |
| 27 | Gen | 6    | -    | -     | -      | -   | -    | -      | -   | 511.00 | 360.63 |
| 31 | Gen | 7    | -    | -     | -      | -   | -    | -      | -   | 561.06 | 482.03 |

**TABLE I**

Runtime (s) on linear instances. "-" indicates a timeout.
a nonlinear version of Dribble.

The Car domain includes only atomic actions and processes. The actions are to start or stop the Car, and accelerate or decelerate. The moving process models one-dimensional kinematics (distance as a function of velocity and velocity as a function of acceleration) and the wind-resistance process models the drag effect upon velocity. Additional actions for acceleration or deceleration increase the branching factor of the problem. As the problems scale, each instance includes actions to accelerate and decelerate by 1,...,k units. The linear and nonlinear versions of the domain differ in whether they include the nonlinear wind-resistance process.

The Generator domain includes two durative actions: generate, and refuel. The generate action has a duration of 1000 time units and consumes fuel at a linear rate. Its at-end effect satisfies the goal. Its overall condition requires that the fuel level is non-negative. The instances scale in the number of tanks required to refuel the Generator so that its overall condition is satisfied. The refuel actions increase the fuel level in the Generator continuously, by a linear rate (in the linear version) or a nonlinear rate (in the nonlinear version). For example, the refuel action defines the effects linearly as

\[
\text{(increase \ (fuel \ ?g) \ (* \ #t \ 2))}
\]

or nonlinearly

\[
\begin{align*}
\text{(increase \ (ptime \ ?t) \ (* \ #t \ 1))} \\
\text{(increase \ (fuel \ ?g))} \\
\quad \text{(*/ \ #t \ (* \ 0.1 \ (* \ (ptime \ ?t) \ (ptime \ ?t))))}
\end{align*}
\]

The Dribble domain involves a process that effects the position \(x\) of a ball. The position changes continuously based upon the ball velocity \(v\). The velocity changes continuously due to gravity \((-g)\) and drag \((-0.1v^2)\). The available actions are dribble \(f\) which decrease velocity by \(f \in \{0,1,2,4\}\). The dribble actions have the precondition that velocity is zero (i.e., the ball is at the top of its arc). The bounce event assigns velocity to \(-0.9v\) and has the condition that the ball position \(x\) is zero. The initial state places the ball at \(x = 1\) with velocity \(v = 0\) and the goal is to reach \(1.5 \leq x \leq 3.0\). The problem, while it does not scale, can be solved for plan lengths greater than one. We find plans (using two-step locking) for steps \(k = 8,12,16,...,40\), which correspond respectively to 2,3,4,...,10 dribble actions interleaved with the same number of bounce events.

**Results:** Tables I and II list runtime results for dReal and HNSolve on the respective linear and nonlinear instances. The columns list the number of encoding steps \(k\), domain,
instance, and run times in seconds for each encoding and solver configuration. The first three columns of results are denoted by “F” for a hand-coded flat encoding based upon the instances studied by Bryce et al. [10]. The second three columns of results are denoted by “C” for the automatically generated parallel composition of the network encoding into a single automaton. The last three columns denoted by “N” are the instances encoded with the network encoding. Within each group of columns, we denote by “F”, “C”, and “N” the results that do not use the new HNSolve layer. The columns with “+H” denote results for HNSolve without clause learning, and those with “+H+L”, for HNSolve with clause learning. Entries with “-” indicate a timeout of 20 minutes was reached.

The results in the Generator domain are listed in the tables with the number of encoding steps $k$ for the two-step lock encoding used in the C and N columns, and the steps in parentheses for the F encodings. The F encodings model the generate action with three steps, and each refuel action with two steps. The C and N encodings model each action with four steps, but is able to achieve the goal before releasing the final lock. Thus, each C and N instance uses one generate action (3 steps), and a number of refuel actions equal to the instance number (4 steps each). The linear results show that all solver configurations have difficulty scaling on the F and C encodings, as reported by [10]. The critical factor is that the encoding grows very large with the size of the instances (see Figure 1). Despite the poor scalability due to the size of the encoding, HNSolve (C+H, C+H+L) can provide some modest improvement over dReal (C). The network encoding N performs significantly better because it uses a tighter encoding. We also see the same trends as for the F and C encodings when comparing the different solver options; HNSolve (N+H, N+H+L) outperforms dReal (N) considerably and clause learning (+L) has a large impact.

The Car1 instances use the one-step lock and the Car2 instances use the two-step lock encoding. The F+H results on Car1 are most similar to the results reported by [10]. Our results are somewhat different because they are based upon dReal3, where the prior work results were collected on dReal2. The major difference between these versions of dReal are its use of the IBEX interval constraint solver (dReal3), and the Realpaver solver (dReal2). We see that the Car1 instances heavily favor the F encodings, and the use of dReal over HNSolve. In all cases, we see an improvement over dReal (F, C, N) by using HNSolve (+H). We see less of an improvement, and sometimes worse performance when using clause learning (+L). This may be due to the overhead associated with storing the clauses or a reshaping of the search space that leads to more backtracking.

The Car2 instances favor the N encoding and HNSolve with clause learning (N+H+L). It appears that the difference is that the relatively shorter encoding lengths in Car1 do not impact the encoding size as in Car2. In Car2 where the encoding length is double that of Car1, the HNSolve is needed to explore the search space. This result is largely consistent with the trend demonstrated in the Generator domain, where HNSolve performs best as the number of encoding steps increases.

Lastly, the Dribble domain highlights how both HNSolve and the network encoding have a positive impact upon performance. The F+H configuration and encoding is closest to that reported by Bryce et al. [10], and illustrates an improvement over previously published results that we attribute to a difference in dReal version.

Table III compares HNSolve (N+H+L) with the results reported by [10] (denoted as dReal2, and similar to F+H) and the other planners on linear instances of the Generator and Car domains. As above, each instance scales the respective number of tanks to fill the Generator (where each tank is required) and levels of acceleration/deceleration. We see that HNSolve scales much better than dReal on the Generator domain.

VI. RELATED WORK

While PDDL+ [19] has been an accepted language for planning with continuous change for nearly a decade, very few planners have been able to handle its expressivity. Planners either assume that all continuous change is linear [7], [15], [16], [26] or handle nonlinear change by discretization [17]. LP-SAT [26] is very similar in spirit to our work because it uses a SAT solver to solve Boolean constraints and an LP solver to solve continuous (linear) constraints. The nature of the encodings is somewhat different in that our encoding makes use of the hybrid system semantics of PDDL+ in a very direct fashion. LP-SAT more closely resembles classical planning as SAT encodings. Unlike our work, LP-SAT does not incorporate heuristics.

More recent work [9], [13] has extended the LP-SAT approach by adapting its encoding for use in contemporary SMT solvers, including dReal and Z3 [24]. Unlike our work on HNSolve, these works focus solely on planning and not model checking hybrid systems. The advantage of focussing on planning encodings is that it is easier to implement the “must” semantics of PDDL+ and adapt existing techniques for SAT-based planning. Nevertheless, not all problems are best phrased as PDDL+, and approaches for reasoning about hybrid systems are necessary.
Bogomolov et al. [7] and Della Penna et al. [17], like our work, make use of the planning as model checking paradigm. Unlike our work, Bogomolov et al. [7] encode a network of linear hybrid automata and we can handle nonlinear automata. Bogomolov et al. [7] use the SpaceEx model checker [20], which performs a symbolic search over the hybrid automata.

Coles et al. [16] and [15] approach PDDL+ from the perspective of heuristic state space search. Coles et al. [16] exploit piecewise linear representations of continuous change to derive powerful pruning conditions for forward heuristic search.

VII. CONCLUSION

We have described a new specialization of the dReal SMT solver called HNSolve and an associated network of hybrid automata encoding. The combination of HNSolve and the network encoding helps find PDDL+ plans as the number of encoding steps increases, especially with the use of a two-step lock encoding. We have shown that the approach scales up reasoning about PDDL+ planning and that it is competitive with the state of the art.

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