Generalized second law and phantom cosmology

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Abstract

The accretion of phantom fields by black holes within a thermodynamic context is addressed. For a fluid violating the dominant energy condition, the case of a phantom fluid, the Euler and Gibbs relations permit two different possibilities for the entropy and temperature: a situation in which the entropy is negative and the temperature is positive or vice versa. In the former case, if the generalized second law (GSL) is valid, then the accretion process is not allowed whereas in the latter, there is a critical black hole mass below which the accretion process occurs. In a universe dominated by a phantom field, the critical mass drops quite rapidly with the cosmic expansion and black holes are only slightly affected by accretion. All black holes disappear near the big rip, as suggested by previous investigations, if the GSL is violated.

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1. Introduction

Data on type Ia supernova distances and other complementary cosmological observations revealed that presently the expansion of the universe is in an accelerated phase [23] and [27]. The observed acceleration requires the existence of a new component, termed dark energy, which dominates presently over all other forms of energy and is characterized by a negative pressure. The dark energy is frequently modeled as a homogeneous scalar field $\phi$ with a suitable potential $V(\phi)$. According to the value of the equation of the state parameter $w$, defined by the ratio between the pressure and the energy density ($w = P/\varepsilon$), three different cases can be distinguished: ‘quintessence’, when $w$ is in the range $-1 < w < -1/3$ and the kinetic term $\dot{\phi}^2/2$ is positive; ‘cosmological constant’, the particular case when $w = -1$ and only the potential term $V(\phi)$ contributes to both the pressure and the energy density of the field; and finally, fields with a negative kinetic term leading to values of $w < -1$, dubbed ‘phantom’ fields. These unusual fields appear in some string [12] and super gravity theories [21] and
have also some weird properties: they violate the dominant energy condition \((P + \epsilon < 0)\), the energy density increases with the cosmic time and a universe dominated by a phantom field has a future singularity, the ‘big rip’ [7]. However, quantum effects may eventually drive the universe out of the future singularity [22]. A further difficulty with phantom fields concerns quantum instabilities of the vacuum. Processes involving the graviton-mediated decay of vacuum into two ghost-quanta and two photons have been studied by [8], who have shown that the divergent nature of the phase space can only be avoided by imposing a Lorentz noninvariant momentum cutoff, which cannot guarantee the masslessness of the graviton. In spite of these theoretical difficulties and of the fact that most cosmological data are presently in favor of a cosmological constant as the driving expansion acceleration mechanism, lower values of the equation of state parameter, e.g., \(w < -1\) cannot be completely excluded [1, 7, 11]. In particular the analysis by [28], based on supernova data, assuming a flat universe and adopting a prior based on the two degree field (2dF) redshift survey constraint on the total matter density parameter, leads to the conclusion that the equation of state parameter lies in the range \(-1.48 < w < -0.72\) at the 95% confidence level.

Since an accelerated expansion driven by a phantom field remains a possibility, further consequences related to the presence of such a field in the universe deserve more detailed investigations, in particular those related to applications of the generalized second law [5] or the entropy bound [6]. A phantom field may completely modify the evolution of black holes (BHs), since studies performed by [2, 3] and confirmed later by [14] suggest that all BHs lose mass and disappear completely near the singularity. In this paper, the effects of accretion of a phantom fluid by BHs are revisited in the light of the generalized second law (GSL) and the holographic bound to the entropy, both supposed here to hold in a universe dominated by a phantom field.

2. The evolution of black holes embedded in a phantom field

2.1. The generalized second law

Some studies performed recently on phantom cosmologies and on possible violations either of the GSL or the entropy bound, are often based on different interpretations of the GSL, thus leading to disparate conclusions. Therefore, it seems necessary first to clarify some aspects related to the entropy and energy conservation in an expanding universe.

In relativistic cosmology, in the absence of entropy sources such as bulk viscosity or particle production, the entropy is a conserved quantity within a comoving unit volume. The total entropy within a volume delimited by a specific horizon (event, apparent or causal) is an ill-defined quantity. It grows in an open universe but it can decrease in a closed universe which has already attained the collapsing phase. This does not mean that the second law of thermodynamics is satisfied in the former case but violated in the latter, since in both examples the evolution is adiabatic, e.g., no entropy sources are present inside the considered volume. Similar considerations can be made for the energy inside a comoving unit volume. If the cosmic expansion is adiabatic, then from the first law

\[
\frac{d(\epsilon a^3)}{dt} + P \frac{da^3}{dt} = 0. 
\]  

(1)

In a laboratory, an expanding volume loses energy adiabatically to the external world at the rate given by equation (1) and the sum of the energies inside the considered volume and the outside world (supposed to be delimitated by adiabatic walls) remains constant. In an expanding, homogeneous and unbound universe, all comoving regions are alike in content and each of them may be regarded as a closed system having no external world to which the lost energy
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\(- (P d a^3)\) can be transferred, since all regions experience identical losses. The usual idea of an expanding volume performing work on its surroundings cannot apply in this case because the expansion of the space itself is responsible for the energy losses [16].

If BHs are present, they can either accrete energy or emit particles via the Hawking mechanism, thus producing entropy. In this case, it has been conjectured in [5] that the second law holds only for the sum of the black hole and matter-radiation entropies. This conjecture should be understood as follows. Consider a thermodynamic system \(\Sigma\) consisting of ordinary matter with entropy \(S_{init}^{m+r}\) and black holes with entropy \(S_{init}^{bh}\), representing the sum of the horizon areas inside the system \(\Sigma\). The initial total entropy of \(\Sigma\) is

\[
S_{init}^{\Sigma} = S_{init}^{m+r} + S_{init}^{bh}.
\] (2)

If the initial equilibrium is disrupted because different processes occur, for instance, matter can be accreted by black holes or collapse to form new black holes, the entropy of \(\Sigma\) will be changed. When the new equilibrium state is established, the total final entropy of the system will be \(S_{final}^{\Sigma}\) and the GSL states that

\[
S_{final}^{\Sigma} \geq S_{init}^{\Sigma}.
\] (3)

In phantom cosmologies, since \(P + \varepsilon < 0\), the Euler relation \(Ts = P + \varepsilon\) allows two alternatives: either the entropy density \(s\) is negative, being the temperature \(T\) positive or the entropy density is positive but in this case the temperature associated with the phantom fluid is negative. These two possibilities follow also from the Gibbons relation, assuming that the energy density is a state function of the temperature and the integrability condition. In statistical mechanics, the entropy of a system is a measure of the (logarithm) number of available states, thus a positively defined quantity. Other definitions as the Shannon entropy, which measures the uncertainty of a discrete random variable in the information theory are also non-negative. However, since the consequences of both possibilities (\(S < 0\) and \(S > 0\)) have been examined in the literature, here the same procedure will be adopted. The first possibility leads to the pair of equations

\[
s = \kappa \left( 1 + w \right) T \frac{1}{\varepsilon} \quad \text{and} \quad \varepsilon = \kappa T \left( \frac{T}{(1 + w)} \right)^{\frac{1}{2}}.
\] (4)

where \(\kappa\) is a constant. This branch of solutions was adopted, among others, by [19] and [17] in their analyses of the entropy evolution in a world model dominated by a phantom field. Note that since the energy density as a function of the scale factor varies as \(\varepsilon \propto a^{-3(1+w)}\), the entropy per comoving unit volume is constant, e.g., \(sa^3 = \text{constant}\). In [17], the variation of the total entropy was defined by the sum of the phantom fluid entropy inside the event horizon plus the area entropy of the event horizon itself as suggested by [10]. This approach requires an additional hypothesis concerning the temperature characterizing the fluid, usually assumed to be equal to the horizon temperature given by the Gibbons–Hawking relation [13]. Under these conditions, the total entropy is always zero and the GSL is verified [17], including the de Sitter case [24]. The assumption of thermal equilibrium with the horizon requires that \(|R_H/R_H| > (R_H/c)\) or, in other words, the time scale in which the event horizon varies should be larger than the radiation crossing time, a condition satisfied only for cosmologies with small departures from the de Sitter model. Moreover, if we require further that the characteristic wavelength of the field quanta be smaller than the horizon radius, then necessarily the fluid temperature must be higher than the Gibbons–Hawking temperature.

The second solution is expressed by the pair [14]

\[
s = \kappa \left[ \frac{T}{(1 + w)} \right]^{\frac{1}{2}} \quad \text{and} \quad \varepsilon = \kappa \left[ \frac{T}{(1 + w)} \right]^{\frac{T}{(1+w)}}
\] (5)
where now the temperature is negative. In spite of not being common in physics, this concept is not meaningless. Experiments on the nuclear spin relaxation of a LiF crystal, after exposition in a magnetic field, indicate that the spin state is properly described by a negative spin temperature, since the system loses internal energy as it gains entropy [25]. Different authors have addressed to the thermodynamic properties of systems at negative temperatures (see, for instance, [26]). The basic requirement for the existence of a negative temperature is that the entropy density should not be a monotonically increasing function of the energy density. This is exactly the situation resulting from equations (5), since \( \frac{ds}{d\varepsilon} \propto \varepsilon^{-w/(1+w)} < 0 \) for a phantom field and such a slope is proportional to \( T^{-1} \). It is worth mentioning that negative temperatures are not colder than absolute zero but instead are hotter than infinite temperatures. As a consequence, if a system with a negative temperature interacts with another system having a positive temperature, the energy will always flow from the former to the latter.

2.2. Accreting a phantom field

In light of the above considerations, a BH (having \( T > 0 \), and therefore acting as a source) embedded in a fluid with a negative temperature will absorb energy. The problem of a scalar field in the presence of a BH was already considered by different authors. References [4] and [18] discussed fields with positive kinetic energy by solving the wave equation in a Schwarzschild background and concluding that the accretion rate depends only on the kinetic term (see also [9, 15]). A different approach was adopted by [2], who considered stationary solutions for the accretion of a fluid onto a Schwarzschild BH, generalizing the early results by [20] and concluding that the accretion rate is proportional to the quantity \((P + \varepsilon)\), which is equal to twice the kinetic energy of a homogeneous scalar field and negative for a phantom field. These results are a simple consequence of the first law: the amount of energy \( dE \) crossing the BH horizon in a time interval \( dt \) is

\[
dE = 4\pi r_g^2 (P + \varepsilon) c \, dt,
\]

where \( r_g = 2G M_{bh}/c^2 \) is the gravitational radius. The absorbed energy produces a variation of the BH entropy equal to

\[
dS_{bh} = \frac{8\pi GM \, dM}{hc} \, dt.
\]

Since \( T = dE/dS \), using the above equations and the well-known relation for the BH temperature, e.g., \( kT = \hbar c/(8\pi GM) \), one obtains

\[
\frac{dM}{dt} = \frac{16\pi (GM_{bh})^2}{c^5} \dot{\phi}^2
\]

which coincides with rates derived by other methods mentioned above and gives consistency to the thermodynamic approach.

Consider now a comoving volume \( V \propto a^3 \) containing a phantom fluid and a black hole of mass \( M_{bh} \) (the generalization for the case including \( N \) black holes is immediate and trivial). Assuming further that distortions in the spacetime inside the cavity, due to the presence of the black hole, can be neglected in a first approximation, the total entropy can be written as

\[
S = \frac{4\pi GM_{bh}^2}{hc} + \kappa \varepsilon \frac{\pi c^3}{a^2} V
\]

where the first term represents the black hole entropy and the second, the phantom fluid entropy inside the comoving volume \( V \). Due to the accretion process, in a short time interval the BH mass varies by an amount \( \Delta M_{bh} \) and the energy density of the phantom field varies by
an amount $\Delta \epsilon$. Under these conditions, using equation (9), the total entropy variation in the cavity is

$$\Delta S = \frac{8\pi GM_{bh}}{\hbar c} \Delta M_{bh} + \frac{\kappa}{(1+w)} e^{-\frac{\epsilon}{\kappa}} \Delta \epsilon V.$$ \hspace{1cm} (10)

For a phantom fluid modeled by a scalar field, only the kinetic term contributes to the accretion as discussed above, thus energy conservation inside the cavity implies that

$$c^2 \Delta M_{bh} = -\frac{1}{2} \Delta \phi^2 V = -\frac{1}{2} (1+w) \Delta \epsilon V.$$ \hspace{1cm} (11)

The above equation says that if $w > -1$ (quintessence fluid) a negative variation of the field energy density implies a positive variation in the black hole mass whereas if $w < -1$ (phantom field), a decrease in the field energy density implies also a decrease in the black hole mass since the kinetic term is now negative. Using the above result, equation (10) can be recast as

$$\Delta S = \left[ \frac{8\pi GM_{bh}}{\hbar c} - \frac{2\kappa c^2}{(1+w)^2} e^{-\frac{\epsilon}{\kappa}} \right] \Delta M_{bh}.$$ \hspace{1cm} (12)

Since in the accretion process $\Delta M_{bh} < 0$, in order to satisfy the GSL it is required that

$$M_{bh, \text{crit}} \leq \frac{\kappa \hbar c^3}{8\pi G (1+w)^2} e^{-\frac{\epsilon}{\kappa}}.$$ \hspace{1cm} (13)

This relation implies that there is a critical mass above which the BH cannot accrete the phantom fluid, otherwise the GSL is violated. The existence of a critical mass can be easily understood. As the BH accretes phantom energy its entropy decreases. Thus, in order to have an increase in the total entropy, the field entropy must increase and compensate the loss in the BH entropy. This is possible only if the phantom fluid has a negative temperature, since in this case a decrease in the energy density increases the entropy. However, negative variations in the BH entropy are proportional to the BH mass and, above a certain value, they cannot be counterbalanced by increasing the field entropy. Such a situation would be completely different had we adopted the solution in which the phantom fluid has a positive temperature. In this case, using the same reasoning as before leads to the condition for the validity of the GSL

$$\frac{8\pi GM_{bh}}{\hbar c} - \frac{2\kappa c^2}{(1+w)^2} e^{-\frac{\epsilon}{\kappa}} \leq 0.$$ \hspace{1cm} (14)

This condition cannot be satisfied since the second term on the left-hand side of the above equation is always positive when $w < -1$. Thus, the negative entropy solution for the phantom fluid has more drastic consequences since it implies that the accretion process is not possible unless the GSL is violated.

A more quantitative analysis of the fate of BHs embedded in a phantom field requires knowledge of its entropy, which remains indeterminate since the constant $\kappa$ is unknown. However, an upper bound can be derived from the holographic principle which states that

$$\frac{4\pi}{3} R^3 \kappa \epsilon^{\frac{1}{2}} \leq \frac{\pi}{l_p^2} R^2$$ \hspace{1cm} (15)

where $R$ is the event horizon radius and $l_p$ is the Planck length. Using the time solutions for the energy density and the event horizon radius in a phantom-dominated universe (see, for instance, [14]), equation (15) can be recast as

$$\left[ \frac{(t_0 - t_0)}{(t_0 - t)} \right]^{\frac{1}{2}} \leq \frac{3|1+3w|}{8 \sqrt{2\lambda z} c l_p^2 z_0}.$$ \hspace{1cm} (16)
where $t_*$ is the big rip time, $t_0$ is the present age of the universe and $s_0 = \kappa e^{\frac{t}{t_0}}$ is the present phantom entropy density. If $w < -1$, the left-hand side of equation (16) goes to zero as the universe expands and approaches the singularity, whereas it is equal to the unity at the present time. Thus, the holographic bound is satisfied if

$$s_0 \leq \frac{3|1+3w|}{8} \sqrt{\Omega_\Lambda H_0} c_l^2 \Omega_{1/L}^{1/2}$$

or, numerically

$$s_0 \simeq 9.37 \times 10^{36} |1+3w| \text{ cm}^{-3}.$$  \hspace{1cm} (18)

Using entropy conservation within a comoving volume and the entropy bound given by equation (17) into equation (13) a more generous upper limit can be derived, e.g.,

$$M_{bh, crit} \leq \frac{2GM_p^2}{3(1+w)^2 \Omega_\Lambda H_0^2} \left( \frac{(t_* - t)}{t_* - t_0} \right)^{\frac{2}{1+w}}$$

where $M_P$ is the Planck mass.

In order to perform some numerical estimates, let us suppose $w = -3/2$, the same value adopted by [7] in their calculations, implying $(t_* - t_0) \simeq 22.7$ Gyr. Presently, the critical mass is about $3.6 \times 10^{33} M_\odot$, a quite huge value, allowing all BHs in the universe to accrete the phantom field. However, for the adopted value of the equation of state parameter, the critical mass decreases as $(t_* - t)^6$ and is about $10^9 M_\odot$ at 85.2 Myr before the big rip. Thus, a BH having presently such a mass will still accrete the phantom field beyond that time only if its mass has substantially decreased; otherwise the limit imposed by the GSL avoids further accretion. The mass evolution of the BH due to accretion of the phantom fluid is given by

$$M_{bh}(t) = \frac{M_{bh,0}}{1 + \frac{8w}{3(1+w)} \left( \frac{1}{(t_* - t)} - \frac{1}{(t_* - t_0)} \right)}$$

where $t_0$ is the Planck time scale. The above equation shows that only near the singularity is the mass of the BH substantially altered and that the evolution in these late phases is independent of the initial BH mass $M_{bh,0}$. A simple calculation indicates that a BH with the present mass of $10^9 M_\odot$ will suffer a relative mass loss, up to 85.2 Myr before the singularity, of about $10^{-11}$, a quite insignificant amount. This means that, just after reaching the critical mass limit, the accretion process stops. Estimates for lower mass BHs lead to similar results.

3. Conclusions

For phantom fields, the violation of the dominant energy condition leads to two alternatives: either the entropy density is negative, being the temperature positive or the entropy density is positive but in this case the associated temperature is negative. In the latter case, if the GSL is supposed to be valid, then there is a critical BH mass above which the accretion process is not allowed. However, the accretion process is significant only near the singularity and, as a consequence BHs reach the critical mass value before the big rip, having lost a negligible amount of mass. If the negative entropy solution is adopted, the GSL forbids the accretion process. In both cases, only a violation of the GSL is consistent with the scenario devised by [2].
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