Quantum anti-Zeno effect

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Prevention of a quantum system’s time evolution by repetitive, frequent measurements of the system’s state has been called the quantum Zeno effect (or paradox). Here we investigate theoretically and numerically the effect of repeated measurements on the quantum dynamics of the multilevel systems that exhibit the quantum localization of the classical chaos. The analysis is based on the wave function and Schrödinger equation, without introduction of the density matrix. We show how the quantum Zeno effect in simple few-level systems can be recovered and understood by formal modeling the measurement effect on the dynamics by randomizing the phases of the measured states. Further the similar analysis is extended to investigate of the dynamics of multilevel systems driven by an intense external force and affected by frequent measurement. We show that frequent measurements of such quantum systems results in the delocalization of the quantum suppression of the classical chaos. This result is the opposite of the quantum Zeno effect. The phenomenon of delocalization of the quantum suppression and restoration of the classical-like time evolution of these quasiclassical systems, owing to repetitive frequent measurements, can therefore be called the ‘quantum anti-Zeno effect’. From this analysis we furthermore conclude that frequently or continuously observable quasiclassical systems evolve basically in a classical manner.

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Dynamics of a quantum system, which it is not being observed, can be described by the Schrödinger equation. In the von Neumann axiomatics of quantum mechanics it is postulated that any measurement gives rise to an abrupt change of the state of the system under consideration and projects it onto an eigenstate of the measured observable. The measurement process follows irreversible dynamics, e.g. due to coupling with the multitude of vacuum modes if spontaneous radiation is registered, and causes the disappearance of coherence of the system’s state: to the decay of the off-diagonal matrix elements of the density matrix or randomization of the phases of the wave function’s amplitudes.

It is known that a quantum system undergoes relatively slow (Gaussian, quadratic or cosine type but not exponential) evolution at an early period after preparation or measurement [1]. Therefore, the repetitive frequent observation of the quantum system can inhibit the decay of unstable [2] system and suppress dynamics of the driven by an external field [3, 4] system. This phenomenon, namely the inhibition or even prevention of the time evolution of the system from an eigenstate of observable into a superposition of eigenstates by repeated frequent measurement, is called the quantum Zeno effect (paradox) or the quantum watched pot [2-5]. Usually derivation and investigation of the quantum Zeno effect is based on the von Neumann’s postulate of projection or reduction of the wave-packet in the measurement process. However, the outcome of the variation of the quantum Zeno effect in a three-level system, originally proposed by Cook [3] and experimentally realized by Itano et al. [4] has been explained by Frerichs and Schenzle [6] on the basis of the standard three-level Bloch equations for the density matrix in the rotating-wave approximation with the spontaneous relaxation. Thus, the quantum Zeno effect can be derived either from the ad hoc collapse hypothesis [2-4] or formulated in terms of irreversible quantum dynamics without additional assumptions, i.e. as the dynamical quantum Zeno effect [6,7]. Moreover, the postulate of the ‘collapse of the wave function’ models the actual measurement process only roughly [6].

Aharonov and Vardi [8] showed that frequent measurements can not only stop the quantum dynamics but it also can induce time evolution of the observable system. They used the von Neumann projection postulate and predicted an evolution of the system along a presumed trajectory due to a sequence of measurements performed on states that slightly change from measurement to measurement. Altenmüller and Schenzle [9] have demonstrated that such a phenomenon can be realized replacing the collapse hypothesis by an irreversible physical interaction.

It should be noted that most of the systems analyzed in the papers mentioned above consist only of the few (usually two or three) quantum states and are purely quantum. Consequently, it is of interest to investigate the influence of the repeated frequent measurement on the evolution of the multilevel quasiclassical systems, the classical counterparts of which exhibit chaos. It has been established [10-12] that chaotic dynamics of such systems, e.g. dynamics of nonlinear systems strongly driven by a periodic external field, is suppressed by the quantum interference effect and it gives rise to the quantum localization of the classical dynamics in the energy space of the system. Thus, the quantum localization phenomenon strongly limits the quantum motion. As it was stated above, the repeated frequent measurement or continuous observation of the quantum system can inhibit its dynamics as well. Therefore, it is natural to expect that frequent measurement of the suppressed system will result in the additional freezing of the system’s state.

In connection with this question we should refer to the papers where the influence of small external noise, environment and measurement induced effects on the quantum chaos is analyzed (see [13-20] and references therein). The
general conclusion of such investigations is that noise, interaction with the environment and measurement induce the decoherence, irreversibility and delocalization. However, the direct link between measurements of the suppressed chaotic systems and the quantum Zeno effect, to the best of our knowledge, have not yet been analyzed. We can only refer to papers [21] where some preliminary relation between the quantum Zeno effect and the influence of repeated measurement on the dynamics of the localized quantum system is presented.

The purpose of this paper is to investigate theoretically and numerically the influence of dense measurement on the evolution of the multilevel quasiclassical systems.

The analysis of the measurement effect on the dynamics of the quantum systems is usually performed with the aid the density matrix formalism. However, the investigation of the quantum dynamics of the multilevel systems affected by repeated measurements is very difficult analytically and much time consuming in numerical calculations. The analysis based on the wave function and Schrödinger equation is considerably easier tractable and more evident. So, first we will show how the quantum Zeno effect in a few-level-system can be described in terms of the wave function and Schrödinger equation without introducing of the density matrix and how the measurements can be incorporated into the equations of motion.

Further we will use the same method for the analysis of the dynamics of the multilevel system affected by repeated frequent measurement. We reveal that repetitive measurement of the multilevel systems with quantum suppression of classical chaos results in the delocalization of the states superposition and restoration of the chaotic dynamics. Since this effect is reverse to the quantum Zeno effect we call this phenomenon the 'quantum anti-Zeno effect'.

II. DYNAMICS OF TWO-LEVEL SYSTEM

Let’s consider the simplest quantum dynamical process and the influence of frequent measurements on the outcome of the dynamics. Time evolution of the amplitudes $a_1(t)$ and $a_2(t)$ of the two-state wave function

$$\Psi = a_1(t)\Psi_1 + a_2(t)\Psi_2$$

(2.1)

of the system in the resonance field (in the rotating wave approximation) or of the spin-half system in a constant magnetic field can be represented as

$$a_1(t) = a_1(0)\cos\frac{1}{2}\Omega t + ia_2(0)\sin\frac{1}{2}\Omega t$$

$$a_2(t) = ia_1(0)\sin\frac{1}{2}\Omega t + a_2(0)\cos\frac{1}{2}\Omega t,$$

(2.2)

where $\Omega$ is the Rabi frequency. We introduce a matrix $A$ representing time evolution during the time interval $\tau$ (between time moments $t = k\tau$ and $t = (k+1)\tau$ with integer $k$) and rewrite Eq. (2.2) in the mapping form

$$\begin{pmatrix} a_1(k+1) \\ a_2(k+1) \end{pmatrix} = A \begin{pmatrix} a_1(k) \\ a_2(k) \end{pmatrix}$$

(2.3)

where the evolution matrix $A$ is given by

$$A = \begin{pmatrix} \cos\varphi & i\sin\varphi \\ i\sin\varphi & \cos\varphi \end{pmatrix}, \quad \varphi = \frac{1}{2}\Omega\tau.$$  

(2.4)

Evidently, the evolution of the amplitudes from time $t = 0$ to $t = T = n\tau$ can be expressed as

$$\begin{pmatrix} a_1(n) \\ a_2(n) \end{pmatrix} = A^n \begin{pmatrix} a_1(0) \\ a_2(0) \end{pmatrix}.$$  

(2.5)
One can calculate matrix $A^n$ by the method of diagonalization of the matrix $A$. The result naturally is

$$A^n = \begin{pmatrix} \cos n\varphi & i\sin n\varphi \\ i\sin n\varphi & \cos n\varphi \end{pmatrix}. \quad (2.6)$$

Note that $n\varphi = \frac{1}{2}\Omega T$.

Equations (2.2)–(2.6) represent time evolution of the system without the intermediate measurements in the time interval $0 \div t$. If at $t = 0$ the system was in the state $\Psi_1$, i.e. $a_1(0) = 1$ and $a_2(0) = 0$, and if $\Omega T = \pi$ then at the time moment $t = T$ we would certainly find the system in the state $\Psi_2$, i.e. it would be $|a_1(T)|^2 = 0$ and $|a_2(T)|^2 = 1$, a certain (with the probability 1) transition between the states. Note, that such quantum dynamics without the time moment $t$ time moment probabilities $p_i$ i.e. the phases $\alpha$ of iterations equals zero. This results in the equation for the probabilities

$$a_1(k) = |a_1(k)| e^{i\alpha_1(k)}, \quad a_2(k) = |a_2(k)| e^{i\alpha_2(k)}, \quad (2.7)$$
i.e. the phases $\alpha_1(k)$ and $\alpha_2(k)$ after every act of the measurement are random. Randomization of the phases after the measurement act can also be confirmed by the analysis of the definite measurement process, e.g. in the V-shape tree-level configuration with the spontaneous transition to the ground state [3-7]. Every measurement of the system’s state results in the mutually uncorrelated phases $\alpha_1(k)$ and $\alpha_2(k)$. After the full measurement of the system’s state these phases are uncorrelated with the phases of the amplitudes before the measurement too. That is why, according to the measurement postulate the outcome of the measurement does not depend on the phases of the amplitudes in the expansion of the wave function through the eigenfunctions of the measured observation. This will result in the absence of the influence of the interference terms in the expressions derived below on the transition probabilities between the eigenstates.

Now we derive equations for the transition probabilities between the states in the case of evolution with intermediate measurements. From equations (2.3) and (2.4) we have

$$|a_1(k+1)|^2 = |a_1(k)|^2 \cos^2 \varphi + |a_2(k)|^2 \sin^2 \varphi + |a_1(k)a_2(k)| \sin[\alpha_1(k) - \alpha_2(k)] \sin 2\varphi,$$

$$|a_2(k+1)|^2 = |a_1(k)|^2 \sin^2 \varphi + |a_2(k)|^2 \cos^2 \varphi - |a_1(k)a_2(k)| \sin[\alpha_1(k) - \alpha_2(k)] \sin 2\varphi. \quad (2.8)$$

After the measurement in the time moment $t = k\tau$ the phase difference $\alpha_1(k) - \alpha_2(k)$, according to the above statement, is random and the contribution of the last term in expressions (2.8) on the average for the large number of iterations equals zero. This results in the equation for the probabilities

$$\begin{pmatrix} p_1(k+1) \\ p_2(k+1) \end{pmatrix} = M \begin{pmatrix} p_1(k) \\ p_2(k) \end{pmatrix}, \quad (2.9)$$

where

$$M = \begin{pmatrix} \cos^2 \varphi & \sin^2 \varphi \\ \sin^2 \varphi & \cos^2 \varphi \end{pmatrix}. \quad (2.10)$$

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is the evolution matrix for the probabilities. The full evolution from the initial time \( t = 0 \) until \( t = T \) with the \((n - 1)\) equidistant intermediate measurement is described by the equation

\[
\begin{pmatrix}
p_1(n) \\
p_2(n)
\end{pmatrix} = M^n \begin{pmatrix}
p_1(0) \\
p_2(0)
\end{pmatrix}.
\] (2.11)

The result of the calculation of the matrix \( M^n \) by the method of diagonalization of the matrix \( M \) is

\[
M^n = \frac{1}{2} \begin{pmatrix}
1 + \cos^n 2\varphi & 1 - \cos^n 2\varphi \\
1 - \cos^n 2\varphi & 1 + \cos^n 2\varphi
\end{pmatrix}.
\] (2.12)

From Eqs. (2.11) and (2.12) we recover the quantum Zeno effect obtained by the density matrix technique [3-6]: if initially the system is in the state \( \Psi_1 \), than the result of the evolution until the time moment \( T = n\tau = \pi/\Omega \) (after the \( \pi \)-pulse) with the \((n - 1)\) intermediate measurement will be characterized by the probabilities \( p_1(T) \) and \( p_2(T) \) for finding the system in the states \( \Psi_1 \) and \( \Psi_2 \) respectively:

\[
p_1(T) = \frac{1}{2}(1 + \cos^n 2\varphi) \approx \frac{1}{2}(1 + e^{-\pi^2/4n}) \approx 1 - \frac{\pi^2}{4n} \to 1,
\]

\[
p_2(T) = \frac{1}{2}(1 - \cos^n 2\varphi) \approx \frac{1}{2}(1 - e^{-\pi^2/4n}) \approx \frac{\pi^2}{4n} \to 0, \quad n \to \infty.
\] (2.13)

We see that results of equations (2.11)-(2.13) represent the inhibition of the quantum dynamics by measurements and coincide with those obtained by the density matrix technique [3-6]. This also confirms correctness of the proposition that the act of the measurement can be represented as randomization of the amplitudes’ phases. Further we will use this proposition and the same method for the analysis of the repeated measurement influence for the quantum dynamics of multilevel systems which classical counterparts exhibit chaos. We restrict ourselves to the strongly driven by a periodic force systems with one degree of freedom. The investigation is also based on the mapping equations of motion for such systems.

**III. QUANTUM MAPS FOR MULTILEVEL SYSTEMS**

In general the classical equations of motion are nonintegrable and the Schrödinger equation for strongly driven systems can not be solved analytically. However, mapping forms of the classical and quantum equations of motion greatly facilitates the investigation of stochasticity and quantum–classical correspondence for the chaotic dynamics. From the standpoint of an understanding of the manifestation of the measurements for the dynamics of the multilevel systems the region of large quantum numbers is of greatest interest. Here we can use the quasiclassical approximation and convenient variables are the angle \( \theta \) and the action \( I \). Transition from classical to the quantum (quasiclassical) description can be undertaken replacing \( I \) by the operator \( \hat{I} = -i\hbar \frac{\partial}{\partial \theta} \) [22, 23]. (We use the system of units with \( \hbar = 1 \)). One of the simplest systems in which the dynamical chaos and its quantum localization can be observed is a system with one degree of freedom described by the unperturbed Hamiltonian \( H_0(I) \) and driven by periodic kicks. The full Hamiltonian \( H \) of the driven system can be represented as

\[
H(I, \theta, t) = H_0(I) + k \cos \theta \sum_j \delta(t - j\tau)
\] (3.1)

where \( \tau \) and \( k \) are the period and strength of the perturbation, respectively.
The intrinsic frequency of the unperturbed system is $\Omega = \frac{dH_0}{dI}$. In particular, for a linear oscillator $H_0 = \Omega I$.

For $H_0 = I^2/2$ we have widely investigated rotator which results to the so-called standard map [12, 24], while the Hamiltonian (3.1) with $H_0 = \omega I_0 (I_0 + I)^{1/2}$ and $k = 2\pi bF/\omega^{3/2}$ (where $b \simeq 0.411$) models the highly excited atom in a monochromatic field of the strength $F$ and frequency $\omega$ [23,25-27].

Integration of the classical equations of motion for the Hamiltonian (3.1) over the perturbation period $\tau$ leads to the classical map for the action and angle

$$I_{j+1} = I_j + k \sin \theta,$$

$$\theta_{j+1} = \theta_j + \tau \Omega (I_{j+1}). \tag{3.2}$$

In the case of rotator the unperturbed frequency is $\Omega (I_{j+1}) = I_{j+1}$ and the map (3.2) coincides with the investigated in great detail standard map [12,22,24].

For the derivation of the quantum equations of motion we expand the state function $\psi(\theta,t)$ of the system through the quasiclassical eigenfunctions, $\varphi_n(\theta) = e^{in\theta}/\sqrt{(2\pi)}$, of the Hamiltonian $H_0$,

$$\psi(\theta,t) = (2\pi)^{-1/2} \sum_n a_n(t)^i n e^{-in\theta}. \tag{3.3}$$

Here the phase factor $i^{-n}$ is introduced for the maximal simplification of the quantum map. Integrating the Schrödinger equation over the period $\tau$, we obtain the following maps for the amplitudes before the appropriate kicks [21, 23]

$$a_m(t_{j+1}) = e^{-iH_0(m)\tau} \sum_n a_n(t_j) J_{m-n}(k), \quad t_j = j\tau \tag{3.4}$$

where $J_m(k)$ is the Bessel function.

The form (3.4) of the map for the quantum dynamics is rather common: similar maps can be derived for the monochromatic perturbations as well, e.g. for an atom in a microwave field [23, 27]. A particular case of map (3.4), corresponding to the model of a quantum rotator $H = I^2/2$, has been comprehensively investigated with the aim of determining the relationship between classical and quantum chaos [12,22,24]. It has been shown that under the onset of dynamical chaos at $K \equiv \tau k > K_c = 0.9816$, motion with respect to $I$ is not bounded and it is of a diffusion nature in the classical case, while in the quantum description diffusion with respect to $m$ is bounded, i.e. the diffusion ceases after some time and the state of the system localizes exponentially. The exponential localization length $\lambda$ of the quantum state is usually defined as follows:

$$\lim_{N \to \infty} |a_m(N\tau)|^2 \sim \exp\left(-\frac{2|m-m_0|}{\lambda}\right) \tag{3.5}$$

where $m_0$ is the initial action. It has been shown in papers [10-12] that for a quantum rotator the localization length is $\lambda \simeq k^2/2$. The effect of quantum limitation of dynamic chaos is extremely interesting and important. It reveals itself for many quantum systems which classical counterparts exhibit chaos. Note, that for the rotator the exact quantum description coincides with the quasiclassical one.

Classical dynamics of the system described by map (3.2) in the case of global distinct stochasticity is diffusion-like with the diffusion coefficient in the $I$ space

$$B(I) = \langle (\Delta I)^2/2\tau \rangle = k^2/4\tau. \tag{3.6}$$
From equations (3.4) we obtain the transitions probabilities $P_{n,m}$ between $n$ and $m$ states during the period $\tau$:

$$P_{n,m} = J_{m-n}^2(k). \quad (3.7)$$

Using the expression $\sum_n n^2 J_n^2(k) = k^2/2$ and approximation of the uncorrelated transitions we can formally evaluate the local quantum diffusion coefficient in the $n$ space [21,25,26]

$$B(n) = \frac{1}{2\tau} \sum_m (m - n)^2 J_{m-n}^2(k) = \frac{k^2}{4\tau} \quad (3.8)$$

Therefore, the expression for the local quantum diffusion coefficient coincides with the classical equation (3.6).

However, it turns out that such a quantum diffusion takes place only for some finite time $t \leq t^* \simeq \tau k^2/2$ [28] after which an essential decrease of the diffusion rate is observed. Such a behavior of quantum systems in the region of strong classical chaos is called "the quantum suppression of classical chaos" [10,11]. This phenomenon turns out to be typical for models (3.1) with nonlinear Hamiltonians $H_0(I)$ and for other quantum systems. Thus, the diffusion coefficient (3.8) derived in the approximation of uncorrelated transitions (3.7) does not describe the true quantum dynamics in the energy space.

The quantum interference effect is essential for such dynamics and it results in the quantum evolution being quantitatively different from the classical motion. Quantum equations of motion, i.e. the Schrödinger equation and the maps for the amplitudes, are linear equations with respect to the wave function and probability amplitudes. Therefore, the quantum interference effect manifests itself even for quantum dynamics of the systems, the classical counterparts of which are described by nonlinear equations and chaotic dynamics of them exhibit a dynamical chaos. On the other hand, quantum equations of motion are very complex as well. Thus, the Schrödinger equation is a partial differential equation with the coordinate and time dependent coefficients, while the system of equations for the amplitudes is the infinite system of equations. Moreover, for the nonlinear Hamiltonian $H_0(m)$ the phases' increments, $H_0(m)\tau$, during the free motion between two kicks while reduced to the basic interval $[0, 2\pi]$ are the pseudorandom quantities as a function of the state’s quantum number $m$. This causes a very complicated and irregular quantum dynamics of the classically chaotic systems. We observe not only very large and apparently irregular fluctuations of probabilities of the states’ occupation but also almost irregular fluctuations in time of the momentum dispersion (see curves (a) in the figures 1 and 2).

However, the quantum dynamics of such driven by the external periodic force systems is coherent and the evolution of the amplitudes $a_m(t_{j+1})$ in time is linear: they are defined by the linear map (3.4) with the time independent coefficients. The nonlinearity of the Hamiltonian $H_0(I)$, being the reason of the classical chaos, causes the pseudorandom nature of the increments of the phases, $H_0(m)\tau$, as a function of the state’s number $m$ (but constant in time). These increments of the phases remain the same for each kick. So, the dynamics of the amplitudes $a_m(t_{j+1}) = |a_m(t_{j+1})| e^{i\alpha_m(t_{j+1})}$ and of their phases, $\alpha_m(t_{j+1})$, is strongly deterministic and non-chaotic but very complicated and apparently irregular. For instance, the phases $\alpha_m(t_{j+1})$ are phases of the complex amplitudes, $a_m(t_{j+1})$, which are linear combinations (3.4) of the complex amplitudes, $a_n(t_j)$, before the preceding kick with the pseudorandom coefficients, $e^{iH_0(m)\tau}J_{m-n}(k)$. Nevertheless, the iterative equation (3.4) is a linear transformation with coefficients regular in time. That is why, we observe for such dynamics the quasiperiodic reversible in the time evolution [12] with the quantum localization of the pseudochaotic motion.
In paper [23] it has been demonstrated that this peculiarity of the pseudochaotic quantum dynamics is indeed due to the pseudorandom nature of the phases, $H_0(m)\tau$, in Eq. (3.4) as a function of the eigenstate’s quantum number $m$ (but not of the kick’s number $j$). Replacing the multipliers exp $[-iH_0(m)\tau]$ in Eq. (3.4) by the expressions $\exp[-i2\pi g_m]$, where $g_m$ is a sequence of random numbers that are uniformly distributed in the interval $[0, 1]$, we observe the quantum localization as well [23]. The essential point here is the independence of the phases $H_0(m)\tau$ or $2\pi g_m$ on the step of iteration $j$ or time $t$. This is the main core of difference from the randomness of the phases due to the measurements under consideration in the next Section.

IV. INFLUENCE OF REPETITIVE MEASUREMENT ON THE QUANTUM DYNAMICS

Each measurement of the system’s state projects the state into one of the energy state $\varphi_m$ with the definite $m$. Therefore, if we make a measurement of the system after the kick $j$ but before the next ($j+1$) kick we will find the system in the states $\varphi_m$ with the appropriate probabilities $p_m(j) = |a_m(t_j)|^2$.

In principle, such a measurement can be performed in the same way as in the experiment of Itano et al. [4], i.e. by the short-impulse laser excitation of the system from state $\varphi_m$ to some higher state followed by the irreversible return of the system to the same state $\varphi_m$ with registration of the state’s population by photon counting. After the measurement of the state’s $\varphi_m$ population, the probability of finding the system in the state $\varphi_m$ coincides with that before the measurement. However there is no interference between the state’s $\varphi_m$ amplitude $\tilde{a}_m(t_j)$ after the measurement and amplitudes of other states, $a_n(t_j)$, i.e. the cross terms containing the amplitude $\tilde{a}_m(t_j)$ in the expressions for probabilities vanish. But interference between the unmeasured states remains and the cross terms containing the amplitudes of the unmeasured states do not vanish.

In the calculations of the system’s dynamics the influence of the measurements can be taken into account in the same way as in the Section II, i.e. through randomization of phases of the amplitudes after the measurement of the appropriate state’s populations. The phases of amplitudes after the measurement are completely random and uncorrelated with the phases before the measurement, after another measurements and with the phases of other measured or unmeasured states. Therefore, after the full measurement of the system after the kick $j$, all phases of the amplitudes $a_m(t_j)$ are random. So, this full measurement of the system’s state influences on the further dynamics of the system through the randomization of the phases of amplitudes (see Section II for analogy). This fact can be expressed by replacement in Eqs. (3.4) of the amplitudes $a_m(t_{j+1})$ by the amplitudes $e^{i\beta_m(t_{j+1})}a_m(t_{j+1})$ with the random phases $\beta_m(t_{j+1})$. The essential point here is that the phases $\beta_m(t_{j+1})$ are different, uncorrelated for the different measurements, i.e. for different time moments of the measurement $t_{j+1}$. This is the principal difference of the random phases $\beta_m(t_{j+1})$ from the phases $H_0(m)\tau$ in Eqs. (3.4) which are pseudorandom variables as functions of the eigenstate’s quantum number $m$ (but not of the time moment $t_{j+1}$).

In such a way, introducing the appropriate random phases we can analyze the influence on the system’s dynamics of the full measurements of the system’s state performed after every kick, after every $N$ kicks or of the measurements of the population probabilities just of some states, e.g. only of the initial state. Note that there is no need to measure more frequently than after every kick because the results of the subsequent measurements before the next kick repeat the results of the preceding measurements (after the last kick).

Instead of representing the detailed quantum dynamics expressed as the evolution of all amplitudes in the expansion
of the wave function (3.3) we can represent only dynamics of the momentum dispersion

$$\langle (m_j - m_0)^2 \rangle = \sum_m (m - m_0)^2 |a_m(t_j)|^2$$

(4.1)

where $m_0$ is the initial momentum (quantum number). Such a representation of the dynamics is simpler, more picturesque and more comfortable for comparison with the classical dynamics.

In figures 1 and 2 we show the results of numerical analysis of the influence of measurements of the system’s state on the quantum dynamics of the rotator and of the system with random distribution of energy levels, i.e. for random phases $H_0(m)\tau$ in Eqs. (3.4) as a function of the eigenstate’s quantum number $m$. We see that quantum diffusion-like dynamics of the systems without measurements, represented by curves (a), after sufficiently short time $t^* \approx \tau k^2/2$ (of the order of 50$\tau$ in our case) ceases and the monotonic increase of the momentum dispersion $\langle (m - m_0)^2 \rangle \approx 2Bt$ for time $t \ll t^*$ turns for the time $t \gg t^*$ into the stationer (on the average for same time interval $\Delta t \geq t^*$) distribution with the momentum dispersion $\langle (m_{st} - m_0)^2 \rangle \approx \lambda^2/2 \approx k^4/8$. This is a demonstration of the effect of quantum suppression of the classical chaos.

In the case of measurement of the only initial, $\varphi_{500}$, state’s population after every kick (which technically is achieved by introduction of the random phase $\beta_{500}(t_{j+1})$ after every kick $j$) we observe monotonic, though slow, increase of the momentum dispersion for very long time, until $t \sim 600\tau$ in our case (curves (b) in figures 1 and 2). After such time the population of the initial state on the average becomes very small and measurements of this state’s population almost does not influence on the systems dynamics.

The dynamics with measurements of all states every 200 kicks represented by curves (c) is a staircase-like: fast increase of the momentum dispersion after the immediate measurement turns into the quantum suppression of the diffusion-like motion for $\Delta t \geq t^*$ until the next measurement destroys the quantum interference and induces the succeeding diffusion-like motion.

The quantum dynamics of the kicked rotator or some other system with measurements of all states’ populations after every kick as represented by the curves (d) is essentially classical-like: the momentum dispersion increases linearly in time with the classical diffusion coefficient (3.6) for all time of the calculation.

Theoretically such differences of dynamics can be understood from the iterative equations for the momentum dispersion:

$$\langle (m_{j+1} - m_0)^2 \rangle = \sum_m (m - m_0)^2 |a_m(t_{j+1})|^2,$$

(4.2)

where

$$|a_m(t_{j+1})|^2 = \sum_{n,n'} J_{m-n}(k) J_{m-n'}(k) a_n(t_j) a_{n'}^*(t_j).$$

(4.3)

Substitution of Eq. (4.3) into Eq. (4.2) yields

$$\langle (m_{j+1} - m_0)^2 \rangle = \sum_{m,n} (m - m_0)^2 J_{m-n}^2(k) |a_n(t_j)|^2 + 2 \sum_{m,n,n'<n} (m - m_0)^2 J_{m-n}(k) J_{m-n'}(k) Re[a_n(t_j) a_{n'}^*(t_j)].$$

(4.4)

For the random phase differences of the amplitudes $a_n(t_j)$ and $a_{n'}^*(t_j)$ with $n' \neq n$, which is a case after the measurement of the system’s state, the second term of Eq. (4.4) on the average equals zero (see Section II for
clarification). Then from Eq. (4.4) we have

\[
\langle (m_{j+1} - m_0)^2 \rangle = \sum_n |a_n(t_j)|^2 \sum_m \langle m - m_0 \rangle^2 J_{m-n}^2(k) = \sum_m |a_m(t_j)|^2 \left( m^2 - m_0^2 + \frac{k^2}{2} \right) = \langle (m_j - m_0)^2 \rangle + \frac{k^2}{2}.
\]

In the derivation of Eq. (4.5) we have used the summation expressions

\[
\sum_m m J_{m-n}^2(k) = 0 \quad \text{and} \quad \sum_m m^2 J_{m-n}^2(k) = n^2 + \frac{k^2}{2}.
\]

Therefore, according to Eq. (4.5) for the uncorrelated phases of the amplitudes \(a_n(t_j)\) and \(a_n^*(t_j)\) with \(n' \neq n\) the dispersion of the momentum as a result of every kick increases on the average in the magnitude \(k^2/2\), the same as for the classical dynamics. For dynamics of isolated quantum systems without measurements or unpredictable interaction with the environment the second term of Eq. (4.4), due to the quantum interference between the amplitudes of different states arisen from the same initial states’ superposition, compensate (on the average for sufficiently large time interval \(\Delta t \geq t^*\)) the first term of Eq.(4.4) and so the quantum suppression of dynamics may be observed.

Similar analysis can be used for the investigation of the influence of the measurements on quantum dynamics of another quantum systems with quantum localization of the classical chaos as well.

As it has already been stated above the influence of the repetitive measurement on quantum dynamics is closely related with the affect of the unpredictable interaction between the system and the environment. It should be noticed that in general for the analysis of the measurement effect and to facilitate the comparison between quantum and classical dynamics of chaotic systems it is convenient to employ the Wigner representation, \(\rho_W(x,p,t)\), of the density matrix \([19, 29]\). The Wigner function of the system with the Hamiltonian of the form \(H = p^2/2m + V(x,t)\) evolves according to equation

\[
\frac{\partial \rho_W}{\partial t} = \{H, \rho_W\}_M = \{H, \rho_W\} + \sum_{n \geq 1} \frac{\hbar^{2n} (-1)^n}{2^{2n}(2n+1)!} \frac{\partial^{2n+1} V}{\partial x^{2n+1}} \frac{\partial^{2n+1} \rho_W}{\partial p^{2n+1}},
\]

where by \(\{\ldots\}_M\) and \(\{\ldots\}\) are denoted the Moyal and the Poisson brackets, respectively. The terms in Eq. (4.6) containing Planck’s constant and higher derivatives give the quantum corrections to the classical dynamics generated by the Poisson brackets. In the region of regular dynamics one can neglect the quantum corrections for very long time if the characteristic actions of the system are large. For classically chaotic motion the exponential instabilities lead to the development of the fine structure of the Wigner function and exponential growth of its derivatives. As a result, the quantum corrections become significant after relatively short time even for macroscopic bodies \([19, 28]\). The extremely small additional diffusion-like terms in Eq. (4.6), which reproduce the effect of interaction with the environment or frequent measurement, prohibits development of the Wigner function’s fine structure and removes barriers posed by classical chaos for the correspondence principle \([19, 29]\).

V. CONCLUSIONS

From the above analysis we can conclude that the influence of the repetitive measurement on the dynamics of the quasiclassical multilevel systems with the quantum suppression of the classical chaos is opposite to that for the few-level quantum system. The repetitive measurement of the multilevel systems results in delocalization of the states superposition and acceleration of the chaotic dynamics. In the limit of the frequent full measurement of
the system’s state the quantum dynamics of such systems approaches the classical motion which is opposite to the quantum Zeno effect: inhibition or even prevention of time evolution of the system from an eigenstate of observable into a superposition of eigenstates by repeated frequent measurement. Therefore, we can call this phenomenon the ‘quantum anti-Zeno effect’.

It should be noted that the same effect can be derived without the ad hoc collapse hypothesis but from the quantum theory of irreversible processes, in analogy with the method used in the papers [6, 9]. Even the simplest detector follows irreversible dynamics due to the coupling to the multitude of vacuum modes which results in the randomization of the quantum amplitudes’ phases, decay of the off-diagonal matrix elements of the density matrix and to smoothing of the fine structure of the Wigner distribution function, i.e. just what we need to obtain the classical equations of motion.

So, the quantum-classical correspondence problem caused by the chaotic dynamics is closely related with the old problem of measurement in quantum mechanics. In the case of frequent measurement or unpredictable interaction with the environment the quantum dynamics of the quasiclassical systems approaches the classical-like motion.

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1. L. A. Khalfin, Zh. Eksp. Teor. Fiz. 33, 1371 (1958) [Sov. Phys. JETP 6, 1503 (1958)]; J. Swinger, Ann. Phys. 9, 169 (1960); L. Fonda, G. C. Ghirardi, and A. Rimini, Rep. Prog. Phys. 41, 587 (1978); G.-C. Cho, H. Kasari, and Y. Yamaguchi, Prog. Theor. Phys. 90, 803 (1993).

2. B. Misra and E. C. G. Sudarshan, J. Math. Phys. 18, 756 (1976); C. B. Chiu, E. C. G. Sudarshan, and B. Misra, Phys. Rev. D 16, 520 (1977).

3. R. J. Cook, Phys. Scr. T21, 49 (1988).

4. W. M. Itano, D. J. Heinzen, J. J. Bollinger, and D. J. Wineland, Phys. Rev. A 41, 2295 (1990).

5. E. Joos, Phys. Rev. D 29, 1626 (1984); T. Petrosky, S. Tasaki, and I. Prigogine, Phys. Lett. A 151, 109 (1990); P. Knight, Nature 344, 493 (1990); A. Schenzle, Contemp. Phys. 37 303 (1996).

6. V. Frerichs and A. Schenzle, Phys. Rev. A 44, 1962 (1991).

7. S. Pascazio and M. Namiki, Phys. Rev. A 50, 4582 (1994); H. Nakazato, M. Namiki, S. Pascazio, and H. Rauch, Phys. Lett. A217, 203 (1996).

8. Y. Aharonov and M. Vardi, Phys. Rev. D 21, 2235 (1980).

9. T. P. Altenmüller and A. Schenzle, Phys. Rev. A 48, 70 (1993).
10. G. Casati, B. V. Chirikov, J. Ford, and F. M. Izrailev, in Stochastic Behavior in Classical and Quantum Hamiltonian Systems, edited by G. Casati and J. Ford Lecture Notes in Physics Vol. 93 (Springer-Verlag, Berlin, 1979), p. 334.

11. G. Casati, B. V. Chirikov, D. L. Shepelyansky, and I. Guarneri, Phys. Rep. 154, 77 (1987).

12. F. M. Izrailev, Phys. Rep. 196, 299 (1990).

13. E. Ott, T. M. Antonsen, Jr., and J. D. Hanson, Phys. Rev. Lett. 53, 2187 (1984).

14. S. Adachi, M. Toda, and K. Ikeda, J. Phys. A: Math. Gen. 22, 3291 (1989).

15. T. Dittrich and R. Graham, Ann. Phys. 200, 363 (1990).

16. R. Blümel, A. Buchleiter, R. Graham, L. Sirko, U. Smilansky, and H. Walther, Phys. Rev. A 44, 4521 (1991).

17. P. Goetsch and R. Graham, Phys. Rev. E 50, 5242 (1994).

18. D. Cohen and S. Fishman, Phys. Rev. Lett. 67, 1945 (1991).

19. W. H. Zurek, Phys. Today 44, 36 (1991); W. H. Zurek and J. P. Paz, Phys. Rev. Lett. 72, 2508 (1994).

20. K. Shiokawa and B. L. Hu, Phys. Rev. E 52, 2497 (1995).

21. B. Kaulakys, in Quantum Communications and Measurement, eds V. P. Belavkin, O. Hirota, and R. L. Hudson (Plenum Press, 1995), p. 193, quant-ph/9503018; V. Gontis and B. Kaulakys, J. Tech. Phys. 38, 223 (1997).

22. G. M. Zaslavskii, Stochastic Behavior of Dynamical Systems (Nauka, Moscow, 1984; Harwood, New York, 1985).

23. V. G. Gontis and B. P. Kaulakys, Liet. Fiz. Rink. 28, 671 (1988) [Sov. Phys.-Collec. 28(6), 1 (1988)].

24. A. J. Lichtenberg and M. A. Lieberman, Regular and Stochastic Motion (Springer-Verlag, New York, 1983).

25. V. Gontis and B. Kaulakys, J. Phys. B: At. Mol. Opt. Phys. 20, 5051 (1987).

26. B. Kaulakys, V. Gontis, G. Hermann, and A. Scharmann, Phys. Lett. A 159, 261 (1991).

27. V. Gontis and B. Kaulakys, Lithuanian J. Phys. 31, 75 (1991).

28. G. Casati and B. Chirikov, "The legacy of chaos in quantum mechanics", in Quantum chaos: between order and disorder, Ed. G. Casati and B. V. Chirikov (Cambridge University, 1994), p.3.

29. B. Kaulakys, Lithuanian J. Phys. 36, 343 (1996); B. Kaulakys, quant-ph/9610041.
Fig. 1 Dependence of the dimensionless momentum dispersion, $\langle (m - m_0)^2 \rangle$, as defined by Eq. (4.2) for the quantum rotator with $m_0 = 500$, $\tau = 1$ and $k = 10$ on the discrete dimensionless time $j$ for the dynamics according to Eq. (3.4): (a) without the intermediate measurements, (b) with measurements of the initial state, $\varphi_{500}$, after every kick, (c) with measurements of all states every 200 kicks and (d) with measurements of all states after every kick.

Fig. 2. Same as in Fig. 1 but for the system with random distribution of energy levels, i.e. for random phases $H_0(m)\tau$ in Eqs. (3.4) defined as $2\pi g_m$ where $g_m$ is a sequence of random numbers that are uniformly distributed in the interval $[0, 1]$. 
