Manifestation of additional dimensions of space-time in mesoscopic systems

A.F. Andreev

P.L. Kapitza Institute for Physical Problems, Russian Academy of Sciences, Kosygin str., 2, 119334 Moscow, Russia

Abstract

We note that the existence of physical states which are coherent superpositions of states with even and odd numbers of fermions means the existence, together with \( x, y, z, t \), of additional spinor dimensions of space-time. A system with variable number of electrons is described in which such superpositions are realized. Experiments with mesoscopic condensed matter systems are suggested which generalize the experiment of Nakamura et al. and may provide direct observation of such superpositions and, thereby, justify the reality of a superspace with additional spinor dimensions introduced in quantum field theory to account for supersymmetry. The nature of additional dimensions of space-time is elucidated for nonrelativistic systems.

1 Introduction

In 70-th the notion of superspace is introduced in quantum field theory to make supersymmetry possible. In addition to the usual dimensions (coordinates) \( x, y, z, t \), spinor fermionic (anticommuting) dimensions of space-time are introduced. If supersymmetry is indeed discovered in the next generation of experiments at Fermilab and CERN within a decade or two (see the review paper [1]), this will amount to the discovery of the new dimensions.

There is another general problem in which the existence of additional spinor coordinates of space-time plays an important role. In 1952, Wick, Wightman, and Wigner [2] show that the coherent superpositions of states with even and odd numbers of fermions are incompatible with the Lorenz invariance and introduce the superselection rule, according to which such superpositions are physically impossible. In actuality (as is pointed out in [3, 4]), the superselection rule is the alternative to the existence of additional spinor coordinates.

The point is that the vector \( |\text{odd}\rangle \) of any state with an odd number of fermions, being spinor of the odd rank, is multiplied by \((-1)\) upon the rotations \( O(2\pi) \) of the coordinate system through an angle of \( 2\pi \) about any axis and upon the double time reversal \( R^2 \). The vectors \( |\text{even}\rangle \) of the states with an even
number of fermions do not change under the $O(2\pi)$ and $R^2$ transformations. For this reason, the existence of a coherent superposition

$$|\ast\rangle = u|\text{even}\rangle + v|\text{odd}\rangle$$

where $u$ and $v$ are nonzero complex numbers, $|u|^2 + |v|^2 = 1$, implies the existence of a strange state $|\ast\rangle$ which changes physically under the $O(2\pi)$ and $R^2$ transformations, because the corresponding change of the state vector does not amount to the appearance of a common phase factor.

If $x, y, z, t$ completely characterize the space-time, then the $O(2\pi)$ and $R^2$ transformations coincide with the identical transformation, which can change nothing physically. The superselection rule is necessary in this case. If, in addition to $x, y, z, t$, the spinor coordinates exist, $O(2\pi)$ and $R^2$ are physically real transformations changing the sign of the additional coordinates. In this case, the superselection rule is not necessary. Thus, the proof of the possible existence of states $|\ast\rangle$ corresponding to the coherent superpositions of states with even and odd numbers of fermions is simultaneously the proof of the reality of a superspace with additional spinor dimensions.

Such proof is presented below. Namely, we show that states $|\ast\rangle$ can be realized in a simple system with variable number of electrons which form, together with an environment, an isolated common system with a fixed number of electrons. In the case considered, the interaction of the system with the environment can be described as an external field acting on the system. This field has the spinor character; i.e., it changes sign under the $O(2\pi)$ and $R^2$ transformations. All eigenstates of the Hamiltonian of the system are coherent superpositions $|\ast\rangle$ of the states with even and odd numbers of electrons, so at temperatures well below the characteristic energy difference, one (ground) of the states is automatically realized.

Irrespective of whether supersymmetry really exists or not, the existence of additional dimensions of space-time results in the existence of corresponding universal degrees of freedom for systems of fermions. Below, the nature of these degrees of freedom is elucidated for nonrelativistic systems. We show that they correspond to the continuous change of the number of fermions in the system. In principle, there are two ways to change the average number $\langle N \rangle$ of fermions continuously between two neighboring integral values $N$ and $N + 1$. The states with nonintegral $\langle N \rangle$ can be either impure states corresponding to incoherent mixtures of states $N$ and $N + 1$, or pure states $|\ast\rangle$ considered above. The first possibility would be the only one if the superselection rule is valid. We show that the change of $\langle N \rangle$ in a system from $N$ to $N + 1$ through coherent superpositions $|\ast\rangle$ can be (and should be) interpreted as “the motion” of the system “as a whole” along the additional fermionic dimensions of space-time. Since the coherent change of $\langle N \rangle$ is accompanied by the change of the spin of the system, the considered universal degrees of freedom include also the spin degrees of freedom.

We consider condensed matter systems such as metallic nanoparticles influenced by gate potentials at extremely low temperatures. The nanoparticles
behave like mesoscopic quantum dots (MQD), i.e., all ordinary spatial degrees of freedom of the particles are completely frozen out. Near some critical values of the gate potentials, the particles possess electronic degrees of freedom which are not still frozen and which are described adequate in terms of the additional space-time coordinates.

We suggest experiments with MQDs which generalize the experiment of Nakamura et al. [5] on the observation of quantum coherence between the states with different (but even in both cases) numbers of electrons. The implementation of these experiments will directly demonstrate the coherence between the states with even and odd numbers of electrons and, thereby, justify the reality of the new dimensions.

2 Realization of states $|\ast\rangle$

We are going to demonstrate that the superselection rule is not selfconsistent. We show that states $|\ast\rangle$ can be naturally realized in a simple realistic system with variable number of electrons because this system is governed by the Hamiltonian whose eigenstates are all coherent superpositions of the states with even and odd numbers of electrons. The idea (see [6]) is as follows.

The number of electrons is a conserved quantity which is analogous, in this respect, to the momentum. Physical systems with Hamiltonians whose eigenstates are all coherent superpositions of the states with different momenta are well known. A particle in an external potential field depending on the particle coordinate is the simplest example. In actuality, this particle is part of an isolated system consisting of the particle and a certain massive body, the interaction with which can be described as an external field acting on the particle. As known, this is justified only if certain conditions are fulfilled. First, the states of a massive body must adiabatically adjust to the changes in the particle coordinate in order to prevent excitation of the body degrees of freedom. Second, all measurements must be made only with the particle. No direct influence upon a massive body is possible.

Let us consider single electron with fixed spin projection (strong external magnetic field is applied along $z$-axis) which can be localized in one of two quantum dots (I and II) and can tunnel from one dot to another. We are going to consider everything inside the dot I as system with variable number of electrons we are investigating (analogous to the particle in the above example) and everything inside the dot II as environment (analogous to a massive body).

This can be done by the following two steps. First, we consider our initial system (single electron) as a part of a larger system with variable number of electrons consisting of both quantum dots whose Hilbert space $S$ (see Fig.1) corresponds to the general superposition $|\rangle$ of all states $|n, m\rangle$ with the numbers of electrons $n$ in the dot I and $m$ in the dot II running through values 0, 1 independently. Let us introduce in $S$ the operators $a, a^+$ and $b, b^+$ of annihilation and creation of electrons in the dot I and the dot II, respectively, so $n = a^+ a$ and $m = b^+ b$. 
The Hilbert space \( s \) of our initial system (the straight line \( AB \) in Fig.1) corresponds to the general superposition \(|⟩_i\) of states \(|0,1⟩\) and \(|1,0⟩\). The characteristic feature of the space \( s \) is that the quantum number \( m \) of the environment is determined unambiguously by the quantum number \( n \) of the system:

\[
m \approx 1 - n. \tag{2}\]

Here the sign \( \approx \) means, as in the book of Dirac \[7\], that the corresponding equality is the following auxiliary condition on the state vector

\[
(m + n - 1) |⟩_i = 0, \tag{3}\]

but not an operator identity in the whole space \( S \). It is this property that allows the interaction of the system with the environment to be treated as an external field acting on the system. This is a fermionic analogy to the adiabatic condition in the above example.

The second step is to project the Hilbert space \( s \) of the system into the Hilbert space \( s_I \) (the straight line \( OB \) in Fig.1) of the dot 1 considered as a separate system. The corresponding state vectors \(|⟩_I\) are superpositions of states \(|0⟩ \equiv |0,0⟩\) and \(|1⟩ \equiv |1,0⟩\). The projection is the result of the transformation \(|⟩ \to U |⟩\) of state vectors in the space \( S \) with

\[
U = n + \sigma(1 - n)b, \tag{4}\]

where \( \sigma = \pm 1 \). We have

\[
U |0,1⟩ = \sigma |0,0⟩ = \sigma |0⟩; \quad U |1,0⟩ = |1,0⟩ = |1⟩. \tag{5}\]

The operator \( U \) is not unitary in \( S \):

\[
U^*U = n + (1 - n)m. \tag{6}\]
But in $s$, it satisfies the condition

$$U^+U \approx 1$$

because

$$n + (1 - n)m \approx n + (1 - n)^2 = n + (1 - n) = 1.$$  

The condition (7) gives the possibility to introduce the transformed Hermitian operator

$$F_T = UFU^+$$

acting in $s_I$ for each Hermitian operator $F$ acting in $s$ in such a way that the matrix elements do not change. In fact, let

$$|1\rangle_I = U|1\rangle_i, \quad |2\rangle_I = U|2\rangle_i$$

be two states in $s_I$ corresponding to two arbitrary states $|1\rangle_i$ and $|2\rangle_i$ in $s$. According to (10), (9), and (7) we have

$$\langle 2|F_T|1\rangle_I = \langle 2|U^+UFU^+U|1\rangle_i = \langle 2|F|1\rangle_i.$$

The Hamiltonian of single electron can be represented in $s$ as

$$H = en + Em + Vab^+ + V^*ba^+,$$

where $e$ and $E$ are energies when electron is localized in the dot I and the dot II, respectively, $V$ is the tunneling amplitude.

After simple calculations, we find quantities transformed to $s_I$:

$$UnU^+ = n,$$

$$UmU^+ = nm + (1 - n)(1 - m) \approx 1 - n,$$

$$Uab^+U^+ = -\sigma a(1 - m) \approx -\sigma a,$$

where the condition $m \approx 0$ in $s_I$ is used.

Finally, the Hamiltonian of the system interacting with the environment is

$$H_I = UHU^+ \approx en + E(1 - n) + \eta a + \eta^* a^+,$$

where $\eta = -\sigma V$.

We note that $H_I$ does not contain operators $b$ and $b^+$ of the environment. The interaction of the system with the environment is described as an external field $\eta$ acting on the system. The field $\eta$, as well as the operators $a$ and $a^+$, and other spinor quantities, change sign under the $O(2\pi)$ and $R^2$ transformations so that, for a given field value, Hamiltonian (16) is not invariant about these transformations. Due to the presence of terms linear in electron operators, all
eigenstates of the Hamiltonian are coherent superpositions of the states with even and odd numbers of electrons. Actually, (16) is the Hamiltonian of the two level system

$$|*\rangle = u |0\rangle + v |1\rangle.$$  

(17)

## 3 Additional spinor dimensions

Thus, to describe the above system correctly, we have to introduce additional spinor dimensions of space-time. Assuming that the corresponding coordinates are the nonrelativistic limit of the coordinates considered in the field theory, we introduce (as in [3], [4]) a Pauli spinor $\theta_\alpha$ where $\alpha = 1, 2$ is a spin index. Additional coordinates $\theta_\alpha$ are Grassmann coordinates satisfying anticommutation relations

$$\{\theta_\alpha, \theta_\beta\} = 0.$$  

(18)

Whatever the actual superspace structure is in the relativistic case, this simplest possibility is quite general in the nonrelativistic limit.

In the case considered above due to the presence of the strong magnetic field along $z$-axis, the system (the dot I) should be treated as “moving” along $\theta \equiv \theta_1$. Quantum mechanics with anticommuting coordinates is well known (see for example [8]). The “wave function” $\Psi$ is an analytical function of $\theta$. Due to the condition $\theta^2 = 0$, the most general $\Psi(\theta)$ is

$$\Psi(\theta) = uI + v\theta,$$  

(19)

where $I$ is the unit of Grassmann algebra. By identifying $I = |0\rangle$ and $\theta = |1\rangle$ we see that states $|*\rangle$ in (17) are identical to (19). The physical change of states $|*\rangle$ under the $O(2\pi)$ and $R^2$ transformations is connected with the spinor nature of the physical coordinate $\theta$.

Operators $a^+$ and $a$ in the Hamiltonian (16) play the role of canonical coordinate $a^+ = \theta$ and momentum $a = \partial/\partial \theta$ corresponding to the additional dimension of space-time.

## 4 Mesoscopic quantum dots

To elucidate the physical meaning of the degrees of freedom corresponding to the additional dimensions, it is very helpful to consider ground states of a mesoscopic quantum dot at different values of the gate potential or at different values of the electron chemical potential.

We consider a metal particle with a large but finite number $N$ of electrons, connected by tunnel junctions to macroscopic leads and (or) to other particles of the same type, and being influenced by a gate potential. Let us suppose that the temperature and all tunneling amplitudes are much smaller than the energy difference $\delta \epsilon \sim E_F/N$ between the first excited and ground states of the particle at a given number of electrons, $E_F \sim 10^4 K$ is Fermi energy. Under these
conditions the metal particle behaves as a mesoscopic quantum dot (MQD), i.e., it is a quasi-closed system in which all ordinary degrees of freedom associated with spatial motion of electrons are completely frozen. Inasmuch as even at lowest experimentally possible temperature $T \sim 1\text{mK}$ the number of electrons can not be larger than $10^7$, we have to deal with metallic nanoparticles of the type obtained by Ralph et al.\cite{9}.

Thus, at a given $N$ the metal particle is in its ground state $|N\rangle$ with the energy $E_N$. Minimization of $E_N = E_N(U)$ with respect to $N$ at a given gate potential $U$ determines the ground state value of $N = N(U)$. We see that in MQD, a change in the number of electrons accompanying a change of the gate potential occurs as a result of first-order phase transitions between phases characterized by different integral values of $N$ (see Fig.2). The jumps of the number of electrons occur at critical values of the gate potential.

It is important here to take into account the so-called parity effect, i.e. the fact that in Fermi systems, the ground state energy $E_N$ calculated with an accuracy appropriate for a mesoscopic system contains in an explicit form the number of particles in the combination $(-1)^N$. In order to deal only with quasi-continuous functions at large $N$, we should introduce two different functions $E_N^o$ and $E_N^e$, individually for odd and even values of $N$, extrapolated to the same value of $N$. The difference $P = E_N^o - E_N^e$, which is usually positive, can be considered as a quantitative characteristics of the parity effect. Due to the parity effect, the steps corresponding to even $N$ are longer than the steps corresponding to odd $N$. (see Fig.2). With increasing $P$ the length of even steps increases, while the length of odd steps decreases until the odd steps disappear at all. The jumps of the number of electrons at critical values of the gate potential become equal to 2. The phase transitions between states with neighboring even numbers of electrons take place at large $P$. A clear example of a system characterized by large $P$ is the so-called single-Cooper-pair box (see \cite{5}), i.e. a superconducting MQD in which all electrons form Cooper-pairs and the energy of an additional single electron (the superconducting energy gap) is
very large.

To classify all possible ground states of a normal MQD at different values of the gate potential, let us put \( N = N_e + n \) where the quasicontinuous number \( N_e \approx N (N >> 1!) \) runs through all even numbers, while \( 0 \leq n \leq 2 \). There are three states for each \( N_e \) with \( n = 0, 1, 2 \), respectively, the combinations \((N_e, n = 2)\) and \((N_e + 2, n = 0)\) being identical.

Fig. 2 shows the dependence \( n = n(U) \) at a given \( N_e \). There are two critical values of the gate potential, \( U_{c1} = U_{c1}(N_e) \) and \( U_{c2} = U_{c2}(N_e) \), corresponding to the conditions

\[
E_0(N_e, U_{c1}) = E_1(N_e, U_{c1}),
\]

(20)

and

\[
E_1(N_e, U_{c2}) = E_2(N_e, U_{c2}),
\]

(21)

respectively, where \( E_n(N_e, U) \equiv E_{N_e+n}(U) \).

As the parity effect increases (with changing \( N_e \)), the quantities \( U_{c1} \) and \( U_{c2} \) approach one another, so that for certain \( N_e = N_{ec} \) (or \( P = P_c \)) determined by the equation

\[
U_{c1}(N_{ec}) = U_{c2}(N_{ec}),
\]

(22)
a triple point may exist where all three states have the same energy. We note that the triple point has actually been observed experimentally [10].

For an even larger \( P \) (\( P > P_c \)) there is only one critical value, \( U_c(N_e) \), where

\[
E_0(N_e, U_c) = E_2(N_e, U_c).
\]

(23)

The jump of \( N \) at \( U = U_c \) is equal to 2.

At critical values of the gate potential, the ground state is degenerate. According to the quantum superposition principle (no superselection rule!), there are infinite sets of ground states of the form

\[
|\ast\rangle = u |0\rangle + v |1\rangle,
\]

(24)

and

\[
|\ast\rangle = u |1\rangle + v |2\rangle,
\]

(25)

at \( U = U_{c1} \) and \( U = U_{c2} \) for \( P < P_c \), and at \( U = U_c \) for \( P > P_c \), respectively, where \( |n\rangle \equiv |N - N_e\rangle \).

The ground states of the form (24), (25), and (26) are characterized by nonintegral averaged numbers of electrons \( < \hat{N} > = N_e + |v|^2, N_e + 1 + |v|^2 \), and \( N_e + 2|v|^2 \), respectively. One can say that these states correspond to “phase coexistence” regions (vertical segments in Fig. 2) of first-order phase transitions taking place at \( U = U_{c1}, U_{c2}, \) and \( U_c \).
The “phase coexistence” of all three phases with \( n = 0, 1, 2 \),

\[
|t\rangle = (1 - w_1 - w_2)^{1/2} |0\rangle + w_1^{1/2} e^{i\phi_1} |1\rangle + w_2^{1/2} e^{i\phi_2} |2\rangle
\]  

(27)
takes place at the triple point \( U = U_{c1} = U_{c2} \). These ground states are characterized by two physical (superconducting!, see \([3, 4]\)) phases, \( \phi_1 \) and \( \phi_2 \), and \( w_1 > 0, w_2 > 0, w_1 + w_2 < 1 \).

A more general consideration is the following. Near critical values \( U_{c1}, U_{c2} \), and \( U_c \) of the gate potential the MQD has some number of states which are close in energy to the ground state. All other states have a much higher energy and can be neglected in low-frequency dynamics. In this sense one can say that the MQD possesses degrees of freedom active at low temperatures and low frequencies (much below \( \delta \epsilon \)). Active degrees of freedom are characterized by the Hilbert spaces (24), (25), (26), or (27) near \( U_{c1}, U_{c2}, U_c \), or near the triple point, respectively.

To take spin into account, let us suppose that the state \( |0\rangle \) and the state \( |2\rangle \) are spin-singlets, and the state with \( n = 1 \) has a total spin \( 1/2 \). Otherwise these states are “incompletely frozen” with respect to the Bose degrees of freedom. Then in the most general case (realized near the triple point), four states, \( |0\rangle \), \( |1, \alpha\rangle \), and \( |2\rangle \) where \( \alpha = 1, 2 \) is a spin index, are close in energy. The Hilbert space of active degrees of freedom corresponds to the general superposition

\[
|g\rangle = c_0 |0\rangle + \sum_\alpha c_{1, \alpha} |1, \alpha\rangle + c_2 |2\rangle,
\]  

(28)
where \( c_0 \), \( c_{1, \alpha} \), and \( c_2 \) are complex numbers. Active degrees of freedom near \( U_{c1}, U_{c2}, \) and \( U_c \), far from the triple point are described by (28) with \( c_2 = 0 \), \( c_0 = 0 \), and \( c_{1, \alpha} = 0 \), respectively.

Let us show that the active degrees of freedom described by (28) correspond to “motion” of the MQD along the spinor dimensions \( \theta_\alpha \) of superspace. Due to the anticommutation relations (18), the most general wave function \( \Psi(\theta_\alpha) \) of the MQD is

\[
\Psi(\theta_\alpha) = c_0 I + \sum_\alpha c_{1, \alpha} \theta_\alpha + c_2 \theta_1 \theta_2.
\]  

(29)
By identifying \( I = |0\rangle \), \( \theta_\alpha = |1, \alpha\rangle \), and \( \theta_1 \theta_2 = |2\rangle \), we see that the Hilbert space (28) is identical to (29).

The Hamiltonian of the MQD is expressed in terms of the coordinates \( \theta_\alpha \equiv a_\alpha^+ \) and momenta \( \partial/\partial \theta_\alpha = a_\alpha \) operators satisfying the canonical relations for Fermi operators. The operators \( a_\alpha \) and \( a_\alpha^+ \) represent universal collective characteristics of any system of fermions under conditions such that the Bose degrees of freedom are completely frozen. In exactly the same manner, the conventional coordinate and momentum operators describe the dynamics of a system with respect to a collective Bose degree of freedom under conditions such that all other degrees of freedom are frozen.

The MQD is characterized by the following gauge invariant quantities

\[
n = a_\alpha^+ a_\alpha, \quad S = (1/2) a_\alpha^+ \sigma_{\alpha \beta} a_\beta,
\]  

(30)
where $\sigma_{\alpha\beta}$ are the Pauli matrices. The operator $n$ corresponds to the parameter $n$ introduced above. The operator $S$ is the operator of the total spin of the MQD. The operators $a_\alpha$ and $a_\alpha^+$ are therefore a generalization of the spin operators to the case where, together with $x, y, z, t$, there exist additional dimensions described by $\theta_\alpha$.

5 Experiments

The states (24), (26), and generally (28) are the stationary states of the MQD at critical values of the gate potential when the interaction of the MQD with the environment or with other MQDs is neglected. Manipulating by the gate potentials as functions of time in a system of two (or more) MQDs one can realize conditions when even infinitesimal tunneling amplitudes cause observable tunneling transitions of electrons between different MQDs. This happens at the moments when the energies of different states of the total system coincide, the states (24), (26), and (28) playing the role of asymptotic states. The coherence of these states can be demonstrated by observations of interference phenomena (see [5] and below).

It is important to note that due to the relatively large dimensions of MQDs one can measure the charge of a MQD without essential disturbance of neighbouring MQDs. This can be done (as in the experiment of Nakamura et al. [5]) using a probe electrode connected to the MQD under study through a tunneling contact or (as in the experiment of Aassime et al. [11]) using a probe electrometer based on a single-electron transistor.

Let us consider the simplest experimental situation. The role of quantum dots (the dots I and II) in the discussion of Section II can be played by two MQDs. The gate potentials should be close to corresponding critical values $U_{c1}$ for both MQDs. The sum of the parameters $n$ (determined by (30)) should be equal to one. Under these conditions the tunneling of the additional (with reference to the state with both $n$ equal to zero) single electron between MQD I (considered as a system) and MQD II (considered as environment) is the only active degree of freedom. All other degrees of freedom correspond to much higher energies and can be neglected.

Thus, to prove experimentally the existence of the new dimensions of space-time, one has to demonstrate the coherence of the superpositions (17) for the two level system (MQD I) governed by the Hamiltonian (16). The corresponding time-dependent Schrödinger equations are

$$i\dot{u} = \eta v, \quad i\dot{v} = e(t)v + \eta^*u,$$

(31)

where the energy origin is chosen so that $E = 0$. We suppose that the gate potential of the MQD I can be varied to make the electron energy $e$ depending on time: $e = e(t)$.

The system considered by Nakamura et al. [5] to demonstrate the coherence between states with different, but even in both cases, numbers of electrons is also equivalent to a two level system described by the equations (31). In
the case considered by Nakamura et al. the role of the system (MQD I), the environment (MQD II), and the spinor field \( \eta \) are played by a single-Cooper-pair box, a macroscopic superconductor (Cooper-pair reservoir), and by the Josephson coupling constant, respectively.

Below we consider two experiments. The first experiment is the experiment of Nakamura et al. performed with our two-level system \( \text{(31)} \). Before the initial time \( t = 0 \), the two-level system has been in the ground state with the gate potential of the MQD I such that \( e \gg |\eta| \). Accordingly, \( u = 1 \) and \( v = 0 \). At \( t = 0 \), the gate potential rapidly changes to a value for which \( e = 0 \). Then, the potential remains constant for a time \( \Delta t \), after which it rapidly regains its initial value. On the time interval between \( t = 0 \) and \( t = \Delta t \), the system obeys Eqs.\( \text{(31)} \) with \( e = 0 \) and initial conditions \( u(t = 0) = 1 \) and \( v(t = 0) = 0 \). Then \( u(t) = \cos |\eta|t \) and \( v(t) = -i(\eta^*/|\eta|) \sin |\eta|t \). At \( t = \Delta t + 0 \), one measures the average charge of the system

\[
|v(\Delta t)|^2 = (1/2)(1 - \cos 2|\eta|\Delta t)
\]  

(32)
as a function of pulse duration \( \Delta t \). The observed oscillations would indicate that the system coherently oscillates between the states with \( n = 0 \) and \( n = 1 \) on the time interval \((0, \Delta t)\). As pointed out in Section III, the system considered is an oscillator moving along the fermionic coordinate \( \theta_1 \).

The Nakamura-type experiment considered above can be modified by passing from the single-pulse to two-pulse technique. As above, let the two-level system be at \( t < 0 \) in the ground state \( u = 1 \) and \( v = 0 \), \( e \gg |\eta| \). The amplitude of the first gate-potential rectangular pulse is the same as above (i.e., corresponds to \( e = 0 \)), but its duration is fixed at \( t_1 = \pi/4|\eta| \). Immediately after the pulse at \( t = t_1 + 0 \), the system is in the state with \( u = v = \sqrt{1/2} \). In the interval between \( t = t_1 \) and \( t = t_1 + \Delta t \), the potential is equal to its initial value corresponding to \( e \gg |\eta| \). Under this condition, the tunneling interaction of the system with environment can be ignored and it behaves as a closed system in its pure state characterized by

\[
u(t) = -i(\eta^*/|\eta|) \sqrt{1/2} \exp i\varphi(t),
\]  

(33)
with the relative phase of the ground \((n = 0)\) and excited \((n = 1)\) states linearly depending on time: \( \varphi(t) = -e(t - t_1) \).

The second gate-potential pulse with parameters of the first pulse is switched on at time \( t_1 + \Delta t \). Using Eqs.\( \text{(31)} \), one can see that, after completion of the second pulse at time \( 2t_1 + \Delta t \) \((\Delta t \ll t_1 \ \text{because} \ e \gg |\eta|)\), the average charge of the system is

\[
|v|^2 = 1/2(1 + \cos e\Delta t).
\]  

(34)
The observation of oscillations \( \text{(34)} \) as a function of time delay \( \Delta t \) between the pulses would demonstrate that the state \( \text{(33)} \) is realized. This would be direct experimental proof of the reality of a superspace with additional spinor dimensions.

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