Abstract

We analyze a scenario where the right-handed neutrinos make part of a strongly coupled conformal field theory and acquire an anomalous dimension $\gamma < 1$ at a large scale $\Lambda$. Their Yukawa couplings to the Higgs become irrelevant at the fixed point and they are suppressed at low scales giving rise naturally to a small (sub-meV) Dirac neutrino mass which breaks the conformal invariance. We derive an upper bound on $\gamma$ from loop-induced flavor changing neutral currents. Neutrino Yukawa couplings can be sizable at electroweak scales and therefore the invisible decay of the Higgs in the neutrino channel can be comparable to the $c\bar{c}$ and $\tau\bar{\tau}$ modes and predict interesting Higgs phenomenology. If lepton number is violated in the conformal theory an irrelevant Majorana mass operator for right-handed neutrinos appears for $\gamma > 1/2$ giving rise to an inverse see-saw mechanism. In this case light sterile neutrinos do appear and neutrino oscillation experiments are able to probe our model.
It is by now a well-established experimental fact that neutrinos have nonzero masses. On the other hand, the nature and absolute scale of these masses are far from clear. The most important theoretical implication from this experimental evidence is that there have to exist right-handed neutrino fields, as direct (Majorana) mass terms for left-handed neutrinos are forbidden by the electroweak gauge symmetry of the Standard Model (SM). It is then possible to write down Yukawa couplings of the neutrinos to the Higgs field, thereby generating Dirac mass terms ($m^D_\nu$) after electroweak symmetry breaking (EWSB) in the standard way. On top of that one can allow for Majorana mass terms for the right-handed neutrinos as the latter are totally sterile with respect to the SM. The most successful model so far consists of assigning to the right-handed neutrino a huge Majorana mass ($m^M_\nu$), possibly generated by the breaking of a Grand Unified Theory (GUT) at a scale of the order $10^{16}$ GeV. Sub-eV neutrino masses are then achieved via the so-called see-saw mechanism \(^\text{[1]}\) by noting that the light eigenvalue of this system is given approximately by

$$m_\nu \simeq \frac{(m^D_\nu)^2}{m^M_\nu}.$$  \(^\text{(1)}\)

This is a very neat way to explain the smallness of the neutrino masses without resorting to unnaturally small Yukawa couplings. The see-saw mechanism then implies that neutrinos are Majorana particles, a fact that can be tested experimentally by observing neutrinoless double beta ($0\nu\beta\beta$) decay of certain nuclei. The only drawback of this otherwise beautiful and simple mechanism is that it involves either physics at an energy scale inaccessible to present and future high energy colliders or tuning the Yukawa couplings to extremely small values. It was further suggested in Refs. \(^\text{[2, 3]}\) that small Yukawa couplings can be naturally achieved by assuming that physics right below the GUT scale is governed by a strongly coupled infrared (IR) fixed point that results in positive anomalous dimensions for the matter fields such that Yukawa couplings become irrelevant operators. Conformal symmetry is broken at some intermediate scale $M_W \ll M_{\text{int}} \ll M_{\text{GUT}}$ and Yukawa hierarchies can be generated in a natural way \(^\text{[1]}\).

In this letter we would like to report on an alternative and natural way to obtain small neutrino masses. We will assume that only the right-handed neutrinos $N_R$ make part of a conformal theory with a fixed point at the scale $\Lambda$: in the language of Ref. \(^\text{[8]}\) right-handed neutrinos are unparticles with large anomalous dimension that couple to SM fields through irrelevant operators. Moreover if lepton number is not conserved in the conformal theory irrelevant Majorana mass terms for right-handed neutrinos are present, together with dimension-five couplings between the leptons and Higgs. This can yield interesting modifications of our mechanism. Conformal symmetry breaking will be induced

\(^\text{[1]}\) Certain supersymmetric models also allow for a suppression of flavor dependence of the soft masses \(^\text{[3, 4, 5, 6]}\), a mechanism that is sometimes referred to as conformal sequestering. Furthermore models with dynamical symmetry breaking involving quasi-conformal behavior and large anomalous dimensions have also been proposed in Ref. \(^\text{[7]}\).
by the electroweak breaking at the scale of the Dirac neutrino mass while all the neutrino phenomenology [as e.g. $\mu \to e\gamma$ and other flavor changing rare processes, or $h \to \nu\bar{\nu}$] will take place at scales where the right-handed neutrinos keep their unparticle nature giving rise to interesting and new phenomena.

Our four-dimensional (unparticle-like) approach should have a five-dimensional counterpart where the conformal invariance is broken by a mass gap [9]. It is essentially different to higher-dimensional theories where right-handed neutrinos propagate in a five-dimensional space and conformal invariance is broken by an IR brane [10].

If the theory is strongly coupled the field $N_R$ may acquire a large anomalous dimension $\gamma$ and its propagator in two-component spinor notation is given by [9]

$$\Delta(p, \gamma) = -iB_\gamma \bar{\sigma}^\mu p_\mu (-p^2 - i\epsilon)^{-1+\gamma}, \quad B_\gamma = \frac{\Gamma(1 - \gamma)}{(4\pi)^2 \Gamma(1 + \gamma)},$$

where the particle limit is reached for $\gamma = 0$ and $B_0 = 1$. For $\gamma > 0$, the renormalizable operator with the Standard Model fields $\bar{\ell}LH \bar{N}_R$ (where $H$ is the Standard Model Higgs doublet and $\ell$ the leptonic doublet) becomes irrelevant.

We will now assume that the UV theory conserves lepton number and come back to the effect of Majorana mass terms at the end of the paper. The effective Lagrangian at the scale $\Lambda$ is then given by

$$\mathcal{L}(\Lambda) = \Lambda^{-\gamma} \bar{\ell}LH \bar{N}_R + h.c.$$  \hspace{1cm} (3)

where we are fixing the Yukawa coupling at the $\Lambda$ scale as $h_\nu(\Lambda) = 1$. The fact that the Yukawa coupling in Eq. (3) is sequestered by the conformal dynamics for scales $\mu < \Lambda$ can be made explicit by redefining $\bar{N}_R$ in terms of fields $\bar{\nu}_R$ with canonical dimension as

$$\bar{N}_R = B_\gamma^{1/2} \mu^\gamma \bar{\nu}_R.$$  \hspace{1cm} (4)

Therefore for scales $\mu < \Lambda$ one can write the effective Lagrangian

$$\mathcal{L}(\mu) = B_\gamma^{1/2} \left( \frac{\mu}{\Lambda} \right)^\gamma \bar{\ell}LH \bar{\nu}_R + h.c.$$  \hspace{1cm} (5)

The coefficient $B_\gamma$ varies very little over the values of $\gamma$ considered here. Although it diverges as $(1 - \gamma)^{-1}$ for $\gamma$ close to one, it does so with a small coefficient and one has only a mild variation $B_\gamma \approx 0.09 - 0.16$ for $\gamma$ in the range 0.5 -- 0.95.

When the Higgs field acquires a vacuum expectation value, $\langle H \rangle = v/\sqrt{2}$, the resulting Dirac mass term represents a tiny relevant perturbation to the conformal sector that will eventually drive it away from the fixed point. We thus face the intriguing possibility that electroweak breaking itself is responsible for the breaking of the conformal symmetry and the generation of neutrino masses. To see at which scale this happens notice that

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$^2$Other (different) aspects of this possibility have been considered in Ref. [12].
conformal dynamics will still be governing the right-handed neutrino sector until the mass becomes of the order of the renormalization group scale. This happens when

$$\mu_c = B_{\gamma}^{1/2} \frac{v}{\sqrt{2}} \left( \frac{\mu_c}{\Lambda} \right)^{\gamma},$$

and hence the physical Dirac mass will be

$$m_{\nu}^D = \mu_c = v \left( \frac{B_{\gamma}}{2} \right)^{\frac{1}{2(1-\gamma)}} \left( \frac{v}{\Lambda} \right)^{\frac{1}{1-\gamma}}.$$  

(7)

For a given neutrino mass Eq. (7) gives a relation between the scale \( \Lambda \) and the anomalous dimension \( \gamma \) of the right handed neutrino. A plot of \( \gamma \) as a function of \( \Lambda \) for various values of \( m_{\nu}^D \) is given in Fig. 1. It is very interesting to observe that as \( \Lambda \) is lowered from the GUT to the TeV scale, the anomalous dimension roughly varies from \( \gamma = 1/2 \) [i.e. the critical value at which a Majorana mass term becomes an irrelevant perturbation, see Eq. (17)] to \( \gamma = 1 \) [the value at which the propagator becomes UV sensitive.]

![Figure 1: The anomalous dimension \( \gamma \) of the right-handed neutrino as a function of the scale \( \Lambda \). The two lines correspond to neutrino masses of \( m_{\nu}^D = 0.01 \text{ eV} \) (dashed blue line) and 1 eV (dash-dotted black line).](image)

Next consider the case of three generations. The best experimental values for mass squared differences are \( m_2^2 - m_1^2 \simeq 7.6 \times 10^{-5} \text{ eV}^2 \) \([13]\) and \( |m_3^2 - m_2^2| \simeq 2.7 \times 10^{-3} \text{ eV}^2 \) \([14]\). In the case of a regular hierarchy, \( m_3 > m_2 > m_1 \), this implies the bound on the hierarchy

$$\frac{m_3}{m_2} \lesssim 6,$$

(8)

while \( m_2/m_1 \) can be any number greater than one. Conversely, for the case of an inverted hierarchy, \( m_2 > m_1 > m_3 \), one has \( m_2/m_1 \approx 1 \) while \( m_2/m_3 \) is unconstrained from above. Let us now introduce three right-handed neutrinos. Several possibilities can be proposed to reproduce the little neutrino hierarchy.
• The first possibility is having all the three right-handed neutrinos with identical anomalous dimension. In either of the two hierarchy schemes, the ratio of the two heavier masses is close to unity, and the small splitting will be accounted for by SM corrections to the Yukawa couplings. The same holds true if the lightest neutrino mass is of the same order.

• A much lighter state can also be naturally achieved. Let us work in the regular hierarchy scheme for definiteness and assign a larger anomalous dimension $\gamma_1$ to one of the right-handed neutrinos while keeping the other two equal, e.g. $\gamma_2 = \gamma_3 = \gamma$. At the scale $\mu_c$ the two heavy neutrinos decouple and the running mass for the third neutrino equals

$$m_1(\mu_c) = \epsilon \mu_c \ll \mu_c, \quad \epsilon = \left( \frac{\mu_c}{\Lambda} \right)^{\gamma_1 - \gamma} \ll 1.$$ (9)

However below $\mu_c$ the strongly coupled sector will flow away from the fixed point. Without making further assumptions on that sector we do not know which value $m_1$ will flow to in the IR.

• Assuming that the right-handed neutrino sector becomes weakly coupled at the scale $\mu_c$, the physical mass $m_1$ will be given by Eq. (9) to good approximation.

• On the other hand, assuming that the flow below $\mu_c$ is governed by a different IR fixed point with an anomalous dimension $\gamma'_1$ for the remaining neutrino, we compute

$$\mu'_c = m_1 = \epsilon^{1-\gamma_1} \mu_c \ll m_2, m_3,$$ (10)

leading to a further suppression of $m_1$ below $\mu_c$. For this to be efficient $\epsilon$ needs not even be particularly small. Instead of generating it from a difference in the $\gamma$’s it can just as well originate from a moderate Yukawa hierarchy at the high scale, e.g. $h_2 = h_3 \sim 1$ while $h_1 \equiv \epsilon \sim 0.1$.

• Finally the last (obvious) possibility is that different neutrinos $\nu_i$ belong to different conformal theories at the scale $\Lambda_i$ and develop different anomalous dimensions $\gamma_i$ which can then describe different masses as in Fig. [1]

Given that the Yukawa couplings and hence the Dirac masses grow with energy one should be worried about possible flavor changing neutral currents (FCNC). There are strong experimental bounds on decay channels such as $\mu \to e\gamma$ and $\mu \to 3e$. We will see how these relate to a bound on the anomalous dimension $\gamma$. Focusing on $\mu \to e\gamma$, one needs to evaluate the diagrams in the left panel of Fig. [2] The amplitudes actually go smoothly to zero when the Dirac masses are turned off, so we will calculate their effects perturbatively. The leading contribution comes from two mass insertions. All diagrams are IR and UV finite for $\gamma < 1$, and the amplitude can be parametrized as $^{3}$

$^{3}$Under the assumption that all external momenta are small compared to $M_W$. 

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Figure 2: Diagrams contributing to $\mu \to e\gamma$ (left panel) and the branching ratio of that reaction as a function of the scale $\Lambda$ (right panel). The two lines correspond to $m_3 = 0.05$ eV (dashed blue line) and 1 eV (dash-dotted black line).

\[
\mathcal{A} \propto \sum_i U_{ei} U_{\mu i}^* \frac{\pi \gamma B_{\gamma} (h_i v \Lambda^{-\gamma})^2}{\sin(\pi \gamma) M_W^{2-2\gamma}} = \frac{\pi \gamma}{\sin(\pi \gamma)} \sum_i U_{ei} U_{\mu i}^* \left( \frac{m_i}{M_W} \right)^{2-2\gamma}.
\]

where we can see that the $\gamma$ dependence is a typical unparticle effect since the right-handed unparticle propagates along the internal lines of the diagram. Normalizing this to the main channel $\mu \to e\nu\nu$ we get for the branching ratio

\[
B(\mu \to e\gamma) = \frac{3}{32 \pi} \left| \frac{\pi \gamma}{\sin(\pi \gamma)} \sum_i U_{ei} U_{\mu i}^* \left( \frac{m_i}{M_W} \right)^{2-2\gamma} \right|^2.
\]

The result for the SM with massive Dirac neutrinos [15], recovered in the limit $\gamma \to 0$, is known to be many orders of magnitudes below the experimental bound $B(\mu \to e\gamma) < 1.2 \times 10^{-11}$ [16]. Due to its exponential dependence on the anomalous dimension $B(\mu \to e\gamma)$ can nevertheless reach this bound if $\gamma$ becomes close enough to one. Using the best fit values for $U$ [17], the existing bound implies that $\gamma \lesssim 0.86$ for the case of a regular hierarchy with $m_1 \ll m_2, m_3$ [16]. Future experiments [18] aim to improve the sensitivity to $\sim 10^{-14}$ which would push down the sensitivity to $\gamma \sim 0.81$. These bounds will be moving closer to $\gamma = 1$ when the neutrinos become more degenerate. In Fig. 2 we plot the branching ratio as a function of the scale $\Lambda$. We expect similar bounds to hold from the $\mu \to 3e$ channel as well as from $\mu \to e$ conversion [19].

Another very interesting effect that arises is that at a given scale $\mu$ the neutrino Yukawa coupling is given by

\[
h_\nu(\mu) = B^{1/2}_\gamma \left( \frac{\mu}{\Lambda} \right)^\gamma,
\]

and it can be sizable at the LHC scales which are sensitive to the electroweak scale. In the left panel of Fig. 3 the Yukawa coupling is plotted as a function of $\Lambda$ for the same values

\footnote{This does not change much for an inverted hierarchy as long as $\theta_{13}$ is not too close to $\pi/2$.}
of the neutrino masses as in Fig. 1 and \( \mu = v \). From this one can see that for a rather low cutoff e.g. \( \Lambda \sim 10 \) TeV, corresponding to \( \gamma \sim 0.8 - 0.9 \), the neutrino Yukawa can be of the same order as the \( \tau \) or \( c \) couplings, i.e. at the percent level. In such an extreme case one can even hope to have a sizable fraction of Higgs decaying to neutrinos at the LHC. In fact one can easily compute the width \( \Gamma(h \rightarrow \nu \bar{\nu}) \) as the imaginary part of the one-loop correction to the Higgs inverse propagator with a neutrino-loop internal line and using the unparticle right-handed neutrino propagator given in Eq. (2). The result is given by

\[
\Gamma(h \rightarrow \nu \bar{\nu}) = h_\nu^2(m_H) \frac{m_H}{16\pi} \frac{2}{\Gamma(1-\gamma)\Gamma(3+\gamma)}
\]

(14)

where \( m_H \) is the Higgs pole mass and \( h_\nu(m_H) \) is the neutrino Yukawa coupling defined in Eq. (13) at the scale \( \mu = m_H \). In the particle limit \( \gamma \rightarrow 0 \) the last factor in (14) goes to one and one recovers the Standard Model expression

\[
\Gamma_{SM}(h \rightarrow \nu \bar{\nu}) = h_{\nu \sm}^2 \frac{m_H}{16\pi}
\]

(15)

By comparison of (14) and (15) one can see that the main difference between both expressions is the ”conformal running” of the neutrino Yukawa coupling. In the right panel of Fig. 3 we plot the branching ratio with respect to the dominant decay mode \( h \rightarrow b \bar{b} \)

\[
B(h \rightarrow \nu \bar{\nu}) = \frac{\sum_i \Gamma(h \rightarrow \nu_i \bar{\nu}_i)}{\sum_i \Gamma(h \rightarrow \nu_i \bar{\nu}_i) + \Gamma(h \rightarrow b \bar{b})}
\]

(16)

for the value of the Higgs mass \( m_H = 130 \) GeV and corresponding to three neutrino flavors (quasi) degenerate in mass. One can see that for \( \Lambda \lesssim 10 \) TeV the branching ratio corresponding to the three neutrino channel is comparable (or dominant) to the branching ratio into \( c \bar{c} \) and \( \tau \bar{\tau} \), which might have implications for light Higgs searches. More details will be given elsewhere [19].
Let us finally comment on the possibility that the theory above the scale $\Lambda$ violates lepton number. For values of $\gamma > 1/2$ the right handed Majorana mass operator

$$\mathcal{L}^M = \frac{1}{2} \Lambda^{1-2\gamma} \bar{N}_R N_R + h.c.,$$

is an irrelevant perturbation and does not lead to a breakdown of the conformal symmetry. It can also be immediately verified that at the scale $m^D_\nu$ the right-handed neutrino Majorana mass $m^M_{\nu_R}(\mu) = B_\gamma (\mu/\Lambda)^{2\gamma} \Lambda$ is parametrically suppressed with respect to the Dirac mass at the scale of conformal breaking

$$m^M_{\nu_R} = \Lambda \left( \frac{B_\gamma v^2}{2\Lambda^2} \right)^{\gamma} = m^D_\nu \left( \frac{m^D_\nu}{\Lambda} \right)^{2\gamma-1}. \quad (18)$$

However the fact that lepton number is violated also allows for further higher dimension operators, the most important one being

$$\mathcal{L}^L = c \Lambda^{-1} (H \ell_L)^2. \quad (19)$$

If this operator is generated from integrating out heavy right-handed neutrinos at $\Lambda$ the constant $c$ will be of $\mathcal{O}(1)$. This is of course the standard see-saw mechanism. On the other hand if the right-handed neutrinos are conformal (as we are assuming in this paper) we cannot integrate them out. Nevertheless due to the presence of the irrelevant lepton-number violating operator $\mathcal{L}^M$ loop corrections will still generate $\mathcal{L}^L$. For instance, the diagram in Fig. 4 gives

$$c(\gamma) = \frac{\lambda (B_\gamma)^2}{16\pi^2(2\gamma - 1)} \left[ 1 - \left( \frac{v}{\Lambda} \right)^{4\gamma-2} \right], \quad (20)$$

where $\lambda$ is the Higgs quartic coupling and one Majorana-mass insertion was used. Similar contributions will be generated by box diagrams containing electroweak gauge bosons in internal lines. Once electroweak symmetry is broken a left-handed neutrino mass

$$m^M_{\nu_L} = c \frac{v^2}{\Lambda}, \quad (21)$$

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5We assume here, for simplicity, that the dimension of the mass operator is twice that of the neutrino field, as it is the case for chiral fields in superconformal theories [11].
is generated. For $\mu < v$ this mass will run much less compared to the strongly running Dirac and right-handed Majorana masses and hence the dominant neutrino mass will be $m_{\nu_L}^M$. The neutrino mass matrix at the scale of conformal breaking

$$\mathcal{M}_\nu = \begin{pmatrix} m_{\nu_L}^M & m_{\nu}^D \\ m_{\nu}^D & m_{\nu_R}^M \end{pmatrix}$$

(22)

has entries given by Eqs. (7), (18), and (21) respectively. Due to the hierarchy $m_{\nu_L}^M \gg m_{\nu}^D \gg m_{\nu_R}^M$ we now have an inverted see-saw mechanism, with an extremely light and almost completely sterile right-handed neutrino. The light mass eigenvalue $m_{\nu_L}^M$ and the mixing angle $\alpha$ are given by

$$m_{\nu_R}^M = m_{\nu_L}^M \alpha^2, \quad \alpha = \left( \frac{B^{1/2}}{\sqrt{2}c} \right)^{1/\gamma} \left( \frac{m_{\nu_L}^M}{v} \right)^{\frac{2\gamma - 1}{1 - \gamma}}.$$ 

(23)

Figure 5: Contour plots for fixed values of the mass of the heaviest –mainly $\nu_L$– (solid) and the lightest –mainly $\nu_R$– (dashed) mass eigenstates. The labels indicate the values of the corresponding mass eigenvalues $m$ as $\log_{10}(m/eV)$. The region below the thick (blue) line corresponds to mixing angles $\alpha > 0.1$.

The light and heavy eigenvalues are displayed in Fig. 5. For $\gamma \simeq 1/2$ the mixing becomes of $\mathcal{O}(1)$ and the two eigenvalues are similar. Such a scenario would lead to modifications in the predictions for neutrino masses and mixings as new sterile neutrinos participate in the oscillations. Although mixing schemes with sterile neutrinos have been

\textsuperscript{6}A similar scenario has previously been proposed in theories with extra dimensions [10].
proposed to explain the LSND anomaly [21], global fits including recent data produce poor results [22]. We therefore impose a rough cutoff of $\alpha < 0.1$ above which (the region below the thick line in Fig. 5) we consider our model slightly disfavoured. It is thus interesting to notice that neutrino oscillation experiments are able to probe our model even for large values of $\Lambda$, as required in the lepton number violating case [7]. Needless to say in this case Majorana neutrino masses are sensitive to neutrino-less double beta-decay experiments [23]. Finally, when lepton number is violated in the conformal sector, since the only $\gamma$-dependence for left-handed masses comes from the prefactor $c(\gamma)$, it is natural to expect near-degenerate neutrino masses even for sizable differences in their corresponding anomalous dimensions.

To conclude a conformally invariant right-handed neutrino sector represents a natural way to obtain sub-eV neutrino masses if the anomalous dimensions lie in the interval $1/2 < \gamma < 1$. Electroweak symmetry breaking triggers conformal breaking, which finally occurs at the neutrino Dirac mass scale. The unusually strong energy dependence of the right-handed neutrino field induces a series of interesting phenomena which could be detected at future experiments. Our model also predicts lepton flavor violating reactions such as $\mu \rightarrow e\gamma$ at a much larger rate than in the Standard Model, and even opens up the possibility to experimentally determine the anomalous dimensions of the right-handed neutrinos in forthcoming experiments. Finally for rather low scales $\Lambda \sim 10$ TeV the neutrino Yukawa couplings can be comparable with those for charm and tau at the weak scale and induce sizable (invisible) Higgs decay into the $\nu\bar{\nu}$ channel. If the conformal theory violates lepton number small Majorana masses can be generated without heavy states. Light sterile neutrinos then appear and neutrino oscillation experiments are able to probe our model and put a lower bound on the anomalous dimension of the right-handed neutrino as $\gamma \gtrsim 0.6$.

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We should point out here that it seems impossible to expect a signal from Higgs decay for such large values of $\Lambda$, as the Yukawa coupling at the weak scale will still be largely suppressed. As for the $\mu \rightarrow e\gamma$ process the diagrams in Fig. 2 contain an internal left-handed neutrino line and now there are no unparticles propagating in internal lines. The result should then correspond to the usual one in the Standard Model calculation (i.e. that in Eq. (11) for $\gamma \rightarrow 0$) which is, for realistic neutrino masses, far away from the experimental bounds.
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