Proton-and neutron-rich breakups near zero binding energy

B Mukeru
Department of Physics, University of South Africa, P O Box 392, Pretoria 0003, South Africa
E-mail: mukerb1@unisa.ac.za

Keywords: elastic scattering, breakup cross section, proton- and neutron-rich nuclei, open quantum systems, binding energy

Abstract
An analysis of the elastic scattering and breakup cross sections in the $^{17}F/^{17}O + ^{208}Pb$ reactions in the zero binding energy limit, is presented. The main motivation was to investigate whether finite reaction observables can be obtained for a neutron-rich system in this limit, where it can be regarded as an open quantum system. It is first found that as the binding energy tends to zero, the ground-state wave function of the $^{17}O$ ($^{16}O+n$) system falls asymptotically as $\sim 1/r^\ell_0$ (where $\ell_0 = 2$, is the ground-state orbital angular momentum). For both projectiles, finite elastic scattering and breakup cross sections are obtained in the zero binding energy limit, where they also become insensitive to the variation of the binding energy. For the $^{17}F$ projectile, this is due to the core-proton Coulomb barrier, whereas for the $^{17}O$ projectile, it is due to the $\sim 1/r^6$ asymptotic behavior of the ground-state wave function in this limit. In conclusion, finite reaction observables in the breakup of an open quantum system can be obtained for $\ell_0 \geq 2$.

1. Introduction

The physics of halos and other loosely-bound systems has attracted an extensive attention over the past few decades (see for instance [1–7], for some of the latest developments in this field). Given their weak ground-state binding energy, halo nuclei are mainly characterized by the extension of the matter density to the peripheral region (well outside the core nucleus radius), and a ground-state that is strongly coupled to the continuum, particularly for a neutron-halo system where there is no core-neutron Coulomb barrier. Therefore, given a strong correlation between the low binding energy and the extension of the density matter, if the ground-state binding energy tends to zero, the density matter can be expected to extend to infinity, leading to an open quantum system (for example, see [8–11], for more discussion on open quantum systems). In this case, the ground-state wave function may no longer fulfills the natural condition of square-integrability, and a closed gap between the ground-state and the continuum. The bound-state can then be, to some extent, regarded as embedded in the continuum, probably 'leading' to the well-known bound-state in the continuum (BIC) phenomenon. Born out of quantum mechanical curiosity [12], this phenomenon has become a topic of great interest, with practical applications in various fields, as exemplified by [13–32]. With the current state-of-art experimental equipment, extremely loosely-bound systems are no longer out of sight as evidenced by [33]. In some studies, the spectroscopy of newly observed systems is determined through the analysis of the spectroscopic factor as function of the ground-state binding energy, where this energy ranges from 0 to around 1 MeV (see for example [3]). This makes the study of the breakup process in the zero binding energy limit of utmost importance.

Due to the Coulomb barrier between two charged fragments, the proton-halo ground-state matter density is not expected to extend to infinity, thus preventing this system from reaching the state of an open quantum system, as shown for example in [34], for the $^8B \rightarrow ^7Be + p$ system. In this reference, it is obtained that the ground-state wave function converges to a finite function as the binding energy approaches zero, where it remains square-integrable. This agrees with [35], where a general asymptotic expression of the proton-halo wave function is derived in the zero binding energy limit. Studying the breakup process in the zero binding energy limit [34], the breakup cross section is reported to remain finite and become independent of the binding energy, similar to the ground-state wave function. The continuum-continuum couplings effect, which is otherwise
known to increase as the binding energy decreases, is also found to be independent of the ground-state binding energy in this limit. Notice that apart from a dissociation of the projectile into its constituent fragments, the breakup process is regarded as an excitation (transition) of the projectile from the bound-state to the continuum. It could be interesting to investigate whether a finite breakup cross section in the zero binding energy limit is associated with the valence nucleon being embedded in the continuum. Although this could be a formidable task, it might shed some light on whether a BIC can be associated with a finite breakup cross section in the zero binding energy limit.

Although the conclusion reached in [34], can be expected to extend to any system with two charged fragments, it is not the case for neutron-rich systems. For a neutron-halo system, where there is no Coulomb barrier, the extension of the matter density to infinity in the zero binding energy limit, means that finite reaction observables may not be obtained. For example, for an s-wave ground-state, the wave function falls asymptotically as \( C \exp(-k_b r) \), where \( C \) is the normalization coefficient and \( k_b \) the ground-state wave number. This implies that in the zero binding energy limit \((k_b \to 0)\), the wave function will approach \( C \). However, for a non-s-wave ground-state, the asymptotic expression of the wave function obtained in [35], suggests that it remains square-integrable even for \( k_b \to 0 \). In this case, finite reaction observables may be expected even for a neutron-halo system in the zero binding energy limit.

In this paper, I study the \(^{17}\text{F} + ^{208}\text{Pb}\) and \(^{17}\text{O} + ^{209}\text{Pb}\) breakup reactions, where the projectiles are modeled as \(^{17}\text{F} \to ^{16}\text{O} + \text{p} \) and \(^{17}\text{O} \to ^{16}\text{O} + \text{n} \). Their ground-states are identified as d-wave (\(1d_{5/2}\)) state, with ground-state binding energies \(S_p = -0.601 \text{ MeV} \) \((^{16}\text{O} + \text{p})\) and \(S_n = -4.144 \text{ MeV} \) \((^{16}\text{O} + \text{n})\), respectively [36]. The two main objectives of this study are: (1) using the expression of the non-s-wave ground-state wave function in the asymptotic region from [35], to investigate whether finite elastic scattering and breakup cross sections can be obtained for a neutron-rich system in the zero binding energy limit, and (2) to extend the conclusion of [34] to other proton-halo systems. This study will also serve to further emphasize the peripheral nature of the breakup process (which is well documented as exemplified by [37–41]), and the crucial importance of the ground-state orbital angular momentum in the breakup of open quantum systems. To this end, the same procedure as in [34] is adopted, where apart from the experimental ground-state binding energies, four values, arbitrary chosen in the interval \([-0.01 \text{ keV}: S_{p,n}\)] are considered. For practical reason, the value \(-0.01 \text{ keV}\) which obtained by scaling \(S_p\) and \(S_n\), by about \(6 \times 10^4\) and \(4 \times 10^4\) respectively, can be regarded as approaching zero compared to the experimental values. Among other factors, the choice of these reactions is primarily motivated by the non-zero orbital angular momentum in the ground-states of the projectile nuclei. The elastic scattering and breakup cross sections are obtained through a numerical solution of the coupled differential equations, derived from the Continuum-Discretized Coupled-Channel (CDCC) formalism [42, 43]. This formalism is uniquely designed to handle breakup reactions induced by weakly-bound projectiles, due to its accurate inclusion of the continuum-continuum couplings in the coupling matrix elements. The detail of the numerical calculations is given in section 2, the results are presented and discussed in section 3, and the conclusions are reported in section 4.

### 2. Numerical calculations

The detail of the CDCC formalism (which is not repeated in the present work), can be obtained in [42, 43]. This section outlines the description of the projectiles and the numerical parameters used to generate the projectiles’ ground-state and continuum wave functions as well as the ones used to solve the CDCC coupled differential equations (CDCC model space).

#### 2.1. Projectile description

As already indicated, the projectiles \(^{17}\text{F}\) and \(^{17}\text{O}\), are modeled as \(^{17}\text{F} \to ^{16}\text{O} + \text{p} \) and \(^{17}\text{O} \to ^{16}\text{O} + \text{n} \), where the nucleon (proton/neutron) is bound to the \(^{16}\text{O}\) core nucleus by \(S_p = -0.601 \text{ MeV}\), and \(S_n = -4.144 \text{ MeV}\) binding energies, respectively [36]. The ground-states are identified by \(\alpha_0 \equiv (\ell_0, s, I, J_0^\pi) = (2, \frac{1}{2}^+, 0, \frac{5}{2}^+)\) quantum numbers, where \(\ell_0 = 2\), is the orbital angular momentum, \(s = \frac{1}{2}\), the nucleon’s spin, \(I = 0\), the spin of the core nucleus, and \(J_0^\pi = \frac{5}{2}^+\) (\(\pi\) is the parity), the total angular momentum \((J_0 = \ell_0 + s)\). Both systems exhibit first-excited bound states, with binding energies \(\varepsilon_{ex} = -0.106 \text{ MeV} \) \((^{17}\text{F})\), and \(\varepsilon_{ex} = -3.273 \text{ MeV} \) \((^{17}\text{O})\) [44], located in the \(2\ell_0+\) state. Their continuum contains resonances in the \(d_{5/2}\) state, with energy \(\varepsilon_{res} = 4.4 \text{ MeV} \) \((^{17}\text{F})\), and \(\varepsilon_{res} = 0.941 \text{ MeV} \) \((^{17}\text{O})\) [44]. The single-particle wave function \(\psi_0^{\ell_0 s I}(k, r)\) \([\alpha \equiv (\epsilon, s, J)]\), \(m_j\) is the z-projection of \(J\) that describes the relative motion of the core nucleus and the valence nucleon, is
\[
\Psi^{\text{ins}}_b(k, r) = \frac{\phi^j_b(k, r)}{r} [Y_J(\Omega_r) \otimes X_l_{\text{in}}],
\]

where \(\phi^j_b(k, r)\) is the radial wave function (with \(k\), the wave number, and it is related to the energy \(\varepsilon\) in the continuum through \(k = \sqrt{2\mu_{\text{core}}/\varepsilon}\), where \(\mu_{\text{core}}\) is the core-nucleon reduced mass), \(r\), the core-nucleon radial coordinate, \(Y_J(\Omega_r)\), the usual spherical Harmonics, with \(\Omega_r = (\theta_r, \varphi_r)\), the solid angle in the direction of the vector \(r\), expressed in spherical coordinates. The wave function \(\Psi^{\text{ins}}_b(k, r)\) is an eigenfunction of the following internal Hamiltonian

\[
H_p = -\frac{\hbar^2}{2\mu_{\text{core}}} \nabla^2 + V_{\text{coul}}(r),
\]

where \(\nabla^2\) is the usual nabla operator, and \(V_{\text{coul}}(r)\), the core-nucleon interacting potential, which is present in the present case is given by

\[
V_{\text{coul}}(r) = \left[ V_0 + \frac{V_0\ell' \cdot s}{2} \right] f(r, R_0, a) + V_C(r),
\]

with \(V_0\) and \(V_0\ell'\cdot s\) being the depths of the central and spin–orbit coupling terms of the nuclear component, \(R_0\), the radius, \(a\), the diffuseness, and \(V_C(r)\), the Coulomb component (which is considered here to be a Coulomb pointsphere potential), where \(V_C(r) = 0\), for \(^{16}\text{O} + n\) system. In this equation, the function \(f(r, R_0, a)\) is defined as

\[
f(r, R_0, a) = [1 + \exp(r - R_0)/a]^{-1}.
\]

In the asymptotic region \((r \gg R_0)\) where nuclear forces are nonexistent, the radial bound-state wave function \(\phi^j_b(k_b, r)\) is normalized according to

\[
\phi^j_b(k_b, r) \xrightarrow{r \to \infty} C_{\ell_b} \exp[k_b r],
\]

where \(C_{\ell_b}\) is the single-particle asymptotic normalization coefficient (ANC), \(W_{0\ell_0\ell_0\ell_0\ell_0}\), the Whittaker function [45], and \(\eta_b = \kappa_0/k_b\) the Sommerfeld parameter, with \(\kappa_0 = Z_c Z_v e^2 \mu_{\text{core}}/\hbar^2\) and \(k_b = \sqrt{2\mu_{\text{core}} S_p / \hbar^2}\). In the zero binding energy limit \((S_p \to 0, \eta_b \to \infty)\), the bound-state wave function in equation (5), becomes [35]

\[
\phi^j_b(k_b, r) \xrightarrow{r \to \infty} C_{\ell_b} \left(\frac{\kappa_0 r}{2\eta_b}\right)^{1/4} \exp[-\eta_b \ln(\eta_b) - 2\sqrt{2\kappa_0} r].
\]

From this equation, one can deduce that in the \(S_p \to 0\) limit,

\[
\int_0^\infty |\phi^j_b(k_b, r)|^2 dr < \infty,
\]

which shows that in this limit, the proton-halo system has a square-integrable wave function, as also observed in \([34]\), for the \(^7\text{Be} + p\) system. For the \(^{14}\text{O} + n\), where \(\eta_b = 0\), in the asymptotic region, one has

\[
\phi^j_b(k_b, r) \xrightarrow{r \to \infty} C_{\ell_b}(-i)^\ell_b k_b r h^j_{\ell_b}(ik_b r),
\]

where \(h^j_{\ell_b}(ik_b r)\), is the spherical Hankel function [45]. In the zero binding energy limit, one obtains [35]

\[
\phi^j_b(k_b, r) \xrightarrow{r \to \infty} C_{\ell_b} \left(\frac{2\ell_b - 1}{(k_b r)^{\ell_b}}\right)!,
\]

where \(C_{\ell_b} \propto k_b^{\ell_b}\) [35]. With this proportionality, the wave function falls asymptotically as \(\sim 1/r^{\ell_b}\). Consequently, the wave function also satisfies equation (7), depending on the value of \(\ell_b\), in particular for \(\ell_b \geq 2\). This reveals the crucial importance of the orbital angular momentum \(\ell_b\) for the neutron-rich system as \(S_p \to 0\). For an s-wave bound-state \((\ell_b = 0)\), equation (8) reduces to

\[
\phi^j_b(k_b, r) \xrightarrow{r \to \infty} C_{\ell_b} \exp(-k_b r),
\]

such that \(\lim_{S_p \to 0} \phi^j_b(k_b, r) \to C_{\ell_b} \delta\), in which case the wave function does not satisfy equation (7). Owing to the peripheral nature of the breakup process, one can expect a finite breakup cross section for the \(^{14}\text{O} + n\) as \(S_p \to 0\), due to the ground-state orbital angular momentum \(\ell_b\). This assertion is what the present study intends to highlight.

In the CDCC formalism, the projectile-target wave function is expanded on the projectile bound and discretized bin states. To this end, once the relative momentum \(k\) has been truncated by \(k_{\text{max}}\) and the interval \([0: k_{\text{max}}]\) sliced into \(N_b\) bins of width \(\Delta k_i = k_i - k_{i-1}\), the pure scattering wave functions are transformed into square-integrable bin wave functions, using for example the binning technique as follows [46].
The reduced radii \( r_i \) are converted to absolute values \( R_i \), as \( R_i = r_i (A_i^{1/3} + A_j^{1/3}) \), for the core-target, and \( R_i = r_i A_i^{1/3} \), for nucleon-target. \( A_i \) and \( A_j \) are core and target atomic masses, respectively. The reduced Coulomb radius is \( r_j = 1.30 \) fm. The spin-orbit term of the real \( n + ^{208}\text{Pb} \) potential was also included, with parameters \( V_{so} = 3.241 \text{MeV} \cdot \text{fm}^2, \lambda_{so} = 1.076 \) fm, and \( \alpha_{so} = 0.59 \) fm.

### Table 1. Potential parameters of the nucleon-target and core-target optical potentials used in the CDCC calculations.

|  | \( V_0 \) (MeV) | \( r_0 \) (fm) | \( a_0 \) (fm) | \( W_r \) (MeV) | \( r_\ell \) (fm) | \( a_\ell \) (fm) | \( R_\ell \) (fm) | \( a_\ell \) (fm) |
|---|---|---|---|---|---|---|---|---|
| \( p + ^{208}\text{Pb} \) | 59.100 | 1.244 | 0.646 | 0.520 | 1.244 | 0.646 | 8.41 | 1.246 | 0.615 |
| \( n + ^{208}\text{Pb} \) | 11.626 | 1.235 | 0.647 | 12.409 | 1.235 | 0.647 | 0.53 | 1.248 | 0.510 |
| \( ^{16}\text{O} + ^{208}\text{Pb} \) | 130.00 | 1.150 | 0.600 | 20.000 | 1.310 | 0.490 | \( \cdots \) | \( \cdots \) | \( \cdots \) |

### Table 2. Parameters used in the numerical solution of the CDCC coupled differential equations.

| \( \ell_{\text{max}} \) \( (\ell') \) | \( \lambda_{\text{max}} \) (\( \ell' \)) | \( k_{\text{max}} \) (fm\(^{-1}\)) | \( r_{\text{max}} \) (fm) | \( \Delta r \) (fm) | \( I_{\text{max}} \) (\( \ell' \)) | \( R_{\text{max}} \) (fm) | \( \Delta R \) (fm) |
|---|---|---|---|---|---|---|---|
| 7 | 6 | 1.0 | 200 | 0.1 | 10000 | 1000 | 0.05 |

\[
\phi_i^{\ell}(k_i, r) = \sqrt{\frac{2}{\pi W_{i0}}} \int_{k_{i-1}}^{k_i} g_{i0}(k) \phi_i^{\ell}(k, r) dk, \tag{11}
\]

where \( \phi_i^{\ell}(k_i, r) \), are square-integrable bin wave functions, \( g_{i0}(k) \), is some weight function, \( W_{i0} = \int_{k_{i-1}}^{k_i} g_{i0}(k) dk \), the normalization coefficient, with \( i = 1, 2, \ldots, N_p \), and \( \alpha_{i} \equiv (i, \ell', s, j) \). The substitution of the discretized projectile-target wave function into the Schrödinger equation, yields a finite set of coupled differential equations, which are solved numerically.

### 2.2. Numerical parameters

To numerically solve the two-body Schrödinger equation and obtain the projectiles’ different states, I used the following parameters of the \( V_\ell(r) \) potential, \( V_0 = -56.7 \text{ MeV}, V_5 = 25.14 \text{ MeV} \cdot \text{fm}^2 (^{16}\text{O} + p) \), \( V_6 = 24.01 \text{ MeV} \cdot \text{fm}^2 (^{16}\text{O} + n) \), \( R_0 = 3.023 \) fm, and \( a = 0.6415 \) fm, taken from [44]. This potential reproduces the ground and first-excited bound-states as well as the resonance energies. The same parameters were also used for the non-resonant continuum states. The depth \( V_0 \) was adjusted to obtained the other ground-state binding energies considered, i.e., \( S_{p,n} = -10 \text{ keV}, -10 \text{ keV}, -0.1 \text{ keV}, \) and \(-0.01 \text{ keV} \). The different core-target and nucleon-target optical potential parameters are listed in table 1. The parameters of the neutron-target potential were obtained from the global parametrization of [47], whereas the ones of the proton-target potential were taken from [48]. The latter were adopted as they provide a better fit of the \( ^{17}\text{F} + ^{208}\text{Pb} \) elastic scattering experimental data. The CDCC model space parameters used in the numerical solution of the coupled differential equations, are summarized in table 2. In this table, \( \ell_{\text{max}} \), is the maximum core-nucleon orbital angular momentum, \( \lambda_{\text{max}} \), the maximum order in the potential multipole expansion, \( k_{\text{max}} \), the maximum relative momentum, \( r_{\text{max}} \), the maximum matching radius for bin integration, \( \Delta r \), the integration step size associated with \( r_{\text{max}} \), \( R_{\text{max}} \), the maximum matching radius in the numerical integration of the coupled differential equations, \( \Delta R \), the integration step size associated with \( R_{\text{max}} \) and \( I_{\text{max}} \), the maximum orbital angular momentum of the projectile-target relative center-of-mass motion. The interval \( [0: k_{\text{max}}] \) was discretized into momentum bins of widths, \( \Delta k = 0.05 \text{ fm}^{-1} \), for \( s- \) and \( p- \) states, \( \Delta k = 0.10 \text{ fm}^{-1} \), for \( f- \) and \( d- \) states, \( \Delta k = 0.15 \text{ fm}^{-1} \) for \( g- \) states, and \( \Delta k = 0.20 \text{ fm}^{-1} \), for higher partial waves. Finer bins were considered in the resonant states. These parameters were selected in accordance with the convergence requirements, in particular for \( S_{p,n} \lesssim -10 \text{ keV} \), where a larger \( I_{\text{max}} \) was required. The numerical calculations were performed using Fresco [49].

### 3. Results and discussion

#### 3.1. Projectile structure

In figure 1, the ground-state wave functions are shown for both \( ^{16}\text{O} + p \) [panel (a)] and \( ^{16}\text{O} + n \) [panel (b)] systems, considering the different ground-state binding energies. From panel (a), one sees that the \( ^{16}\text{O} + p \) ground-state wave function falls asymptotically to zero in accordance with equation (6). Also, for \( S_p \lesssim -10 \text{ keV} \), the wave function converges to a finite function as it becomes insensitive to the variation of \( S_p \), and it is more extended to the asymptotic region compared to \( S_p \geq -100 \text{ keV} \). Given the convergence of the ground-state wave function, finite breakup cross sections can be anticipated for \( S_p \lesssim -10 \text{ keV} \).
The function is directly proportional to the dipole electric response function, indicating the crucial role of this function in the response function. According to the zero binding energy limit, the Coulomb breakup cross section possibly increases with $S_n$, where $S_n$ is the ground-state separation energies. The scattering wave functions were calculated at a continuum energy $\varepsilon = 4.144$ MeV (which corresponds to the $^{16}$O+n experimental ground-state separation energy), in the $f_{1/2}$ partial-wave.

In order to assess the implication of a large extension of the ground-state wave function to the peripheral region, I analyze the function $I(r) = \phi^b_{\ell}(k_b, r) \phi^j_{\ell}(k, r)$, which reflects the radial behavior of the dipole electric response function. According to the first-order perturbation theory \[50, 51\], the Coulomb breakup cross section is directly proportional to the dipole electric response function, indicating the crucial role of this function in the Coulomb breakup process. The function $I(r)$ is plotted in panel (c) for $^{16}$O+p and panel (d) for $^{16}$O+n. The scattering wave functions $\phi^j_{\ell}(k, r)$ were calculated at a continuum energy $\varepsilon = 4.144$ MeV (which corresponds to the $^{16}$O+n experimental ground-state binding energy), in the $f_{1/2}$ partial-wave. In panel (c), as one could expect, the main contribution to this function stretches well outside the core radius ($r \gg 10$ fm) (the root-mean-square radius of the $^{16}$O+p system calculated for $S_p = -601$ keV, is $\sqrt{\langle r^2 \rangle} = 3.70$ fm), and quickly decays to

![Figure 1](image1.png)

![Figure 2](image2.png)
zero for \( r \geq 20 \text{ fm} \), regardless the binding energy. On the other hand, in panel (d), the effect of the large extension of the \(^{16}\text{O} + n\) ground-state wave function is remarkably clear. It is noticed that while the function \( I(r) \) falls rapidly to zero for \( S_n \geq -100 \text{ keV} \), similar to panel (c), for \( S_n \leq -10 \text{ keV} \), it becomes oscillatory in the asymptotic region, due to the slow decay of the ground-state wave function in this energy region. If one were to consider pure scattering wave functions, there would be a convergence issue in the breakup calculations in the zero binding energy limit. However, in the CDCC formalism, the pure scattering wave functions are transformed into square-integrable bin wave functions as shown by equation (11). To display how this transformation affects the function \( I(r) \) in figure 1(d), the function \( F(r) = \phi^b_{\ell}(k_\theta, r)\phi^b_{\ell,j}(k_\rho, r) \) is plotted in figure 3. To obtain the bin wave functions \( \phi^b_{\ell,j}(k_\rho, r) \), the momentum \( k_\rho \) was truncated by \( k_{\text{max}} = 0.4 \text{ fm}^{-1} \), and the interval \([0: k_{\text{max}}]\) sliced into 10 equally-spaced bins. Observing this figure, it follows that the oscillatory pattern in figure 1(c) has been washed away, such that the integral \( \int_0^{\infty} I(r) \, dr \) has no convergence issues. This highlights the uniqueness of the CDCC method to handle breakup calculations in the zero binding energy limit.

3.2. Elastic scattering and breakup cross sections

The elastic scattering cross sections, which were calculated at \( E_{\text{lab}} = 170 \text{ MeV} \) for both projectiles, are plotted in figure 4(a) (for \(^{17}\text{F} + ^{208}\text{Pb} \) reaction) and in figure 4(b) (for \(^{17}\text{O} + ^{208}\text{Pb} \) reaction). Inspecting panel (a), one sees that the elastic scattering cross section is not sensitive to the variation of the ground-state wave function, despite the fact that for \( S_p \leq -10 \text{ keV} \), the ground-state wave function is relatively more extended to the asymptotic

---

Figure 3. Plots of the function \( I(r) = \phi^b_{\ell}(k_\theta, r)\phi^b_{\ell,j}(k_\rho, r) \), for three different \( S_n \) energies. The bin wave functions were calculated for \( k_{\text{max}} = 0.4 \text{ fm}^{-1} \), with the interval \([0: k_{\text{max}}]\) sliced into 10 equally-spaced bins.

Figure 4. Elastic scattering cross sections of the \(^{17}\text{F} + ^{208}\text{Pb} \) [panel (a)] and \(^{17}\text{O} + ^{208}\text{Pb} \) [panel (b)] reactions as functions of the centre-of-mass (c.m.) angle \( \theta \) for different ground-state separation energies. The experimental data for the \(^{17}\text{F} + ^{208}\text{Pb} \) reaction were taken from [52].
region compared to the case of \( S_p \geq -100 \text{ keV} \), as seen in figure 1(a). A good agreement with the experimental data from \([52]\) is also observed. It follows that the ground-state binding energy below the experimental value has no effect on the \( {}^{17}\text{F} + {}^{208}\text{Pb} \) elastic scattering cross section, such that any energy in this range provides a better fit of the experimental data. In other words, the ground-state binding energy that provides a better fit of the experimental data from \([52]\) were compared with the experimental data, a similar conclusion would have been reached.

Considering the \( {}^{17}\text{O} + {}^{208}\text{Pb} \) reaction in panel (b), it follows that the scattering cross section decreases as the binding energy decreases from \( S_n = -4144 \text{ keV} \) to \(-10 \text{ keV} \), while it also becomes insensitive to the variation of this energy for \( S_n \leq -10 \text{ keV} \). This lack of sensitivity can be attributed to the fact that the ground-state wave function falls asymptotically as \( \sim 1/r^{6/5} \) in the \( S_n \to 0 \) limit, leading to the convergence of the radial integral of the elastic scattering matrix elements. In conclusion, a finite elastic scattering cross section for a neutron-rich projectile is obtained in the zero binding energy limit, for \( \ell_p = 2 \). This result is quite important for a better understanding of the breakup dynamics of an open quantum system. Another observation in figure 4(b), is the persistence of the Coulomb-nuclear interference peak (CNIP) around 30°, similar to panel (a), which is not completely suppressed even as \( S_n \to 0 \). For the \( {}^{11}\text{Be} + {}^{208}\text{Pb} \) reaction which also involves a neutron-halo projectile, the CNIP is found to be completely suppressed at an incident well above the Coulomb barrier \([53]\).

Among other factors, the persistence of the CNIP in this case can be mainly attributed to the centrifugal barrier in the ground-state of the \( {}^{16}\text{O} + \text{n} \) system, which is absent in the \( {}^{10}\text{Be} + \text{n} \) system (see for instance \([7]\) for more discussion). The angular distributions breakup cross sections for both \( {}^{17}\text{F} + {}^{208}\text{Pb} \) and \( {}^{17}\text{O} + {}^{208}\text{Pb} \) reactions, obtained when all the different couplings are included in the coupling matrix elements, are shown in figure 5. Starting with \( {}^{17}\text{F} + {}^{208}\text{Pb} \) reaction [panel (a)], one observes, unlike in the case of elastic scattering, a substantial increase of the breakup cross section from \( S_p = -601 \text{ keV} \) to \( S_p = -10 \text{ keV} \). This is attributed to a relatively large extension of the ground-state wave function as reported in figure 1(a). As anticipated from figure 1, finite breakup cross sections are obtained for \( S_p \leq -10 \text{ keV} \), where they are also insensitive to the variation of \( S_n \), in agreement with conclusion drawn in \([34]\). A further look at this figure, one sees that the theoretical calculations overestimate the experimental data (obtained from \([52]\)) at lower angles. This is also the case in \([48, 52]\), where the same reaction is analyzed. It could be that more breakup dynamics such as projectile and target excitations, among others, are needed in order to successfully describe the data. Turning to the \( {}^{17}\text{O} + {}^{208}\text{Pb} \) reaction [panel (b)], a substantial increase of the breakup cross section from \( S_n = -4144 \text{ keV} \) to \( S_n = -10 \text{ keV} \) is noticed. Finite breakup cross sections are as well obtained for \( S_n \leq -10 \text{ keV} \), where they also become insensitive to the variation of \( S_n \). This is ascribed to the fact that the ground-state wave function falls asymptotically as \( \sim 1/r^{6/5} \). In conclusion, a finite breakup cross section of a neutron-rich projectile in the zero binding energy limit is obtained \( \ell_p = 2 \). This amounts to saying that finite reaction observables for an open quantum system can be obtained, provided the ground-state orbital angular momentum is non-zero. This highlights the crucial importance of the ground-state orbital angular momentum in the zero binding energy limit for an open quantum system.
To ensure that the convergence of the breakup cross section in the zero binding energy is exclusively related to the convergence of the ground-state wave function, I show in Figure 6, the breakup cross sections in the absence of couplings among continuum states (continuum-continuum couplings), i.e., only couplings to and from the ground-state are taken into account. These couplings are known to have a large effect on the breakup cross section. Observing this figure, it can be noticed that the breakup cross sections for both systems remain finite for $S_{p,n} \leq -10$ keV, which points to a negligible effect of the continuum-continuum couplings.

4. Conclusions

In this paper, I have studied the breakup of proton- and neutron-rich systems in the zero binding energy limit, considering $^{17}\text{F}$ and $^{17}\text{O}$ nuclei, which are modeled as $^{17}\text{F} \rightarrow ^{16}\text{O} + p$ and $^{17}\text{F} \rightarrow ^{16}\text{O} + n$. The main objective was to investigate whether finite elastic scattering and breakup cross sections can be obtained for a neutron-rich system in the zero binding energy limit. To this end, the experimental binding energy values were artificially reduced down to $S_{p,n} = -0.01$ keV. For practical reason, this value was then considered to be in the zero energy limit compared to $S_p = -0.601$ MeV and $S_n = -4.144$ MeV. It is first shown that the $^{16}\text{O} + p$ system has a square-integrable ground-state wave function in the zero binding energy limit. In the same limit, the $^{16}\text{O} + n$ ground-state wave function falls asymptotically as $\sim 1/r^{\ell_b}$ (where $\ell_b = 2$, is the orbital angular momentum in the ground-state). Therefore, the square-integrability of the neutron-rich ground-state wave function becomes strong in the zero binding energy limit. In this binding energy limit, finite $^{17}\text{F} + ^{208}\text{Pb}$ and $^{17}\text{O} + ^{208}\text{Pb}$ elastic scattering and breakup cross sections are obtained, where they become insensitive to the variation of the binding energy. For the $^{17}\text{F} + ^{208}\text{Pb}$ reaction, this is due to the Coulomb barrier in the $^{16}\text{O} + p$ system, whereas for the $^{17}\text{O} + ^{208}\text{Pb}$ reaction, it is due to the $\sim 1/r^{\ell_b}$ asymptotic behavior of the $^{16}\text{O} + n$ ground-state wave function.

In conclusion, the results obtained in [34], can be generalized to other proton-rich systems. Finite reaction observables in the breakup of an open quantum system can be obtained, provided $\ell_b \geq 2$. It follows that for $s$-wave neutron-halos such as $^{11}\text{Be}$, $^{15,19}\text{C}$ where $\ell_b = 0$, finite reaction observables in the $S_{p,n} \rightarrow 0$ limit may not be expected, in particular for the elastic scattering cross section. These results provide an important step into the breakup dynamics of an open quantum system, and highlight the crucial role of the orbital angular momentum.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).
ORCID iDs

B Mukeru  
https://orcid.org/0000-0002-9646-594X

References

[1] Watanabe S et al 2014 Phys. Rev. C 89 044610
[2] Crawford H L et al 2019 Phys. Rev. Lett. 122 052501
[3] Kobayashi N et al 2014 Phys. Rev. Lett. 112 242501
[4] Hamamoto I 2017 Phys. Rev. C 95 044325
[5] Cao X N, Liu Q and Guo J Y 2019 Phys. Rev. C 99 014309
[6] Ahn D S et al 2019 Phys. Rev. Lett. 123 212501
[7] Mukeru B 2021 Int. Journ. Mod. Phys. E 30 2150006
[8] Breuer H P and Petruccione F 2002 The Theory of Open Quantum Systems (Oxford: Oxford University Press)
[9] Okolowicz J, Ploszajczak M and Rotter I 2003 Phys. Rep. 374 271
[10] Dobaczewska J et al 2007 Prog. Part. and Nucl. Phys. 59 632
[11] Michel N, Nazarewicz W, Okolowicz J and Ploszajczak M 2010 J. Phys. G: Nucl. Part. Phys. 37 064042 (12pp)
[12] Neumann V J and Wigner E 1929 Über merkwürdige diskrete Eigenwerte Phys. Z. 30 465
[13] Dreisow F et al 2009 Opt. Lett. 34 2405
[14] Gentry C M and Popović M A 2014 Opt. Lett. 39 4136
[15] Ha S T et al 2018 Nat. Nanotechnol 13 1042
[16] Kodigala A et al 2017 Nature (London) 541 196
[17] Foley J M and Phillips J D 2015 Opt. Lett. 40 2637
[18] Romano S et al 2018 Materials 11 326
[19] Foley J M, Young S M and Phillips J D 2014 Phys. Rev. B 89 165111
[20] Maxim V G, Antonov A A and Kivshar Y S 2020 Phys. Rev. Lett. 125 093903
[21] Zhen B, Hsu C W, Lu L, Stone A D and Soljacic M 2014 Phys. Rev. Lett. 113 257401
[22] Liang Y et al 2020 Nano Lett. 20 6351
[23] Wei H C et al 2016 Nat. Rev. Mat. 1 9 16048
[24] Molina M I, Miroshnichenko A E and Kivshar Y S 2012 Phys. Rev. Lett. 108 070401
[25] Karl N et al 2019 Appl. Phys. Lett. 115 141103
[26] Bykov D, Bezus E A and Doskolovich L L 2019 Phys. Rev. A 99 063805
[27] Koshelev K, Bogdanov A and Kivshar Y 2019 Sci. Bull. 64 856
[28] Sadrizadeh Z et al 2019 Phys. Rev. B 100 115303
[29] Minkov M, Williamson I A D, Xiao M and Fan S 2018 Phys. Rev. Lett. 121 263901
[30] Koshelev K et al 2018 Phys. Rev. Lett. 121 193903
[31] Ordonez G and Na K 2006 Phys. Rev. A 73 022113
[32] Overvig A, Yu N and Alù A 2021 Phys. Rev. Lett. 126 073901
[33] Cook K J et al 2020 Phys. Rev. Lett. 124 212503
[34] Mukeru B 2021 Chin. Phys. C 45 054107
[35] Okolowicz J, Michel N, Nazarewicz W and Ploszajczak M 2012 Phys. Rev. C 85 064320
[36] Wang M et al 2017 Chin. Phys. C 41 030003
[37] Capel P and Nunes F M 2007 Phys. Rev. C 75 054609
[38] Capel P, Phillips D R and Hammer H W 2018 Phys. Rev. C 98 034610
[39] Yang J and Capel P 2018 Phys. Rev. C 98 054602
[40] Capel P and Rollet Y 2017 Phys. Rev. C 96 015801
[41] Moschini L, Yang J and Capel P 2019 Phys. Rev. C 100 044615
[42] Iseri Y et al 1986 Prog. Theor. Phys. Suppl. 89 84
[43] Austern N et al 1987 Phys. Rev. 154 125
[44] Sparenberg M, Baye D and Imanishi B 2000 Phys. Rev. C 61 054610
[45] Abramowitz M and Stegun I 1970 Handbook of Mathematical Functions (New York, NY: Dover)
[46] Tostevin J A, Nunes F M and Thompson I J 2001 Phys. Rev. C 63 024617
[47] Koning A J and Belaroch J P 2003 Nucl. Phys. A 713 231
[48] Kucuk Y and Moro A M 2012 Phys. Rev. C 86 034601
[49] Thompson J I 1988 Comput. Phys. Rep. 7 167
[50] Bertulani C A and Baur G 1988 Phys. Rep. 163 299
[51] Alder K et al 1956 Rev. Mod. Phys 28 432
[52] Liang J E et al 2009 Phys. Lett. B 681 22
[53] Duan F F et al 2020 Phys. Lett. B 811 135942