COMMENTS ON LORENTZ AND CPT VIOLATION

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This contribution to the CPT’13 meeting briefly introduces Lorentz and CPT violation and outlines two recent developments in the field.

1. Introduction

The idea that small observable violations of Lorentz symmetry could provide experimental access to Planck-scale effects continues to draw attention across several subfields of physics. In the three years since the previous meeting in this series, considerable progress has been made on both experimental and theoretical fronts. This contribution to the CPT’13 proceedings contains a brief introduction, followed by comments on two topics of recent interest: nonminimal fermion couplings and Riemann-Finsler geometry.

2. Basics

A satisfactory theoretical description of Lorentz violation must incorporate coordinate independence, realism, and generality. A powerful approach uses effective field theory, starting with General Relativity coupled to the Standard Model and adding to the Lagrange density all observer-invariant terms that contain Lorentz-violating operators combined with controlling coefficients. This yields the comprehensive realistic effective field theory for Lorentz violation called the Standard-Model Extension (SME).

The SME also describes general CPT violation, which in the context of realistic effective field theory is accompanied by Lorentz violation. The full SME contains operators of arbitrary mass dimension $d$, while the minimal SME restricts attention to operators of renormalizable dimension $d \leq 4$.

Observable signals of Lorentz violation are governed by the SME coefficients. Experiments typically search for particle interactions with background coefficient values, which can produce effects dependent on the par-
particle velocity, spin, flavor, and couplings. Many investigations of this type have been performed, achieving impressive sensitivities that in some cases exceed expectations for suppressed Planck-scale effects. If the SME coefficients are produced by spontaneous Lorentz violation, as is necessary when gravity is based on Riemann geometry, then they are dynamical quantities that must incorporate massless Nambu-Goldstone modes. These modes have numerous interpretations, including serving as an alternative origin for the photon in Einstein-Maxwell theory and the graviton in General Relativity, or representing new spin-dependent or spin-independent forces, among other possibilities. Massive modes can also appear.

3. Nonminimal fermion sector
In the nonminimal sector of the SME, the number of Lorentz-violating operators grows rapidly with the mass dimension \( d \). Systematicallycataloguing and characterizing the possibilities is therefore indispensable in the search for Lorentz violation. In the CPT’10 proceedings, I outlined some features appearing in the treatment of quadratic operators of arbitrary \( d \) in the photon Lagrange density. In the intervening three-year period, investigations of the quadratic fermion sector for arbitrary \( d \) have also been performed. The Lagrange density for propagation and mixing of any number of fermions has been developed and applied to describe general Lorentz violation in the neutrino sector. More recently, quadratic operators of arbitrary \( d \) have been studied for a massive Dirac fermion.

Many nonminimal operators generate effects that are in principle observable, and each such operator generates a distinct experimental signal. For quadratic operators, which characterize particle propagation and phase-space features of particle decays, the Lorentz-violating effects can include direction dependence (anisotropy), wave-packet deformation (dispersion), and mode splitting (birefringence). In the neutrino sector, for example, some operators control flavor-dependent effects in neutrino and antineutrino mixing, producing novel energy and direction dependences that involve both Dirac- and Majorana-type couplings. Others govern species-independent effects, which can differ for neutrinos and antineutrinos and can produce propagation times varying with energy and direction, in some cases exceeding that of light. A few operators produce ‘countershaded’ effects that cannot be detected via oscillations or propagation but change interaction properties in processes such as beta decay.

Analogous effects appear in the description of a massive Dirac fermion in the presence of Lorentz violation, for which the exact dispersion relation
for arbitrary $d$ is known in closed and compact form.\textsuperscript{13} For example, the fermion group velocity is anisotropic and dispersive, while the fermion spin exhibits a Larmor-like precession caused by birefringent operators. Using field redefinitions to investigate observability reveals that many operators of dimension $d$ produce no effects or are physically indistinguishable from others of dimensions $d$ or $d \pm 1$. Nonetheless, the number of observable coefficients grows as the cube of $d$. To date, almost all the nonminimal coefficient space for fermions is experimentally untouched. This offers an open arena for further exploration with a significant potential for discovery.

4. Riemann-Finsler geometry

The surprising ‘no-go’ result that the conventional Riemann geometry of General Relativity and its extension to Riemann-Cartan geometry are both incompatible with explicit Lorentz violation raises the questions of whether an alternative geometry is involved and, if so, whether a corresponding gravitational theory exists. The obstruction to explicit Lorentz violation, which disappears for the spontaneous case, can be traced to the generic incompatibility of the Bianchi identities with the external prescription of coefficients for Lorentz violation. It is therefore reasonable to conjecture that a natural geometrical setting would include metric distances depending locally on the coefficients in addition to the Riemann metric.\textsuperscript{2}

Support for this conjecture has recently emerged with the discovery that a fermion experiencing explicit Lorentz violation tracks a geodesic in a pseudo-Riemann-Finsler geometry rather than a conventional geodesic in pseudo-Riemann spacetime.\textsuperscript{16} Riemann-Finsler geometry is a well-established mathematical field with numerous physical applications (see, e.g., Ref. 17), such as the famous Zermelo navigation problem of obtaining the minimum-time path for a ship in the presence of ocean currents. A large class of Riemann-Finsler geometries is determined by the geodesic motion of particles in the SME.\textsuperscript{16,18} Among these are the canonical Randers geometry, which is related to the 1-form SME coefficient $a_{\mu}$, and numerous novel geometries of simplicity comparable to the Randers case. One example of the latter is $b$ space, a calculable Riemann-Finsler geometry that also is based on a 1-form and has Finsler structure complementary to that of Randers space. Physically, this geometry underlies the geodesic motion of a fermion in curved spacetime in the presence of chiral CPT-odd Lorentz violation. All the SME-inspired geometries exhibit mathematically interesting features connected to physical properties. For instance, when the SME coefficients are covariantly constant, the trajectories are conventional Rie-
mann geodesics and special Riemann-Finsler geometries known as Berwald spaces result. Many open challenges remain in this area, ranging from more technical questions such as resolving singularities or classifying geometries to physical issues such as generalizing Zermelo navigation or uncovering implications for the SME. The prospects appear promising for further insights to emerge from this geometrical approach to Lorentz violation.

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