Analytical study of temperature layer fields of pectin-containing raw material under controlled heat exposure

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Abstract. Shelf life and quality of raw materials depend on optimal conditions for long-term storage. The optimal storage time, is the longest period, at which the loss of quality and nutritional value of raw materials do not exceed the allowable limit, depends on the temperature and humidity conditions. Throughout the entire storage period, the constant temperature should be maintained as long as possible, since its fluctuations even within +5°C can seriously affect the storage duration. On the territory of the Russian Federation, the temperature of outdoor air changes during the day from about 5 to 15°C, reaching 30°C in some regions. The seasonal temperature also varies significantly. Optimal conditions for storing pectin-containing raw materials are temperature limits of 0 − 5°C. One of the modes of storage of raw materials is storage in a refrigerated state, when the temperature of the raw materials and the surrounding space is lowered to −10°C. Such conditions can be achieved by equipping storage facilities with artificial cooling. Using thermophysical methods, it is possible to objectively assess the optimal storage conditions for raw materials and predict storage periods.

1. Statement of the problem and discussion of the results
To achieve optimal conditions for storing pectin-containing raw materials in a metal silo of a cylindrical shape, it is necessary to maintain the temperature range (from \( t_1 = t_{\text{min}} \) to \( t_2 = t_{\text{max}} \)). The silo design can have heat-insulated bases or a heat-insulated lateral surface. The respiration of raw materials causes heating, which potentially decreases the storage time. To find a solution the boundary-value heat transfer problem can be formulated as follows [1–4]: There is a heat equation for an infinite cylinder, an unlimited plate, a finite rod with a thermally insulated lateral surface, and for a small bulk.

\[
\frac{\partial t(\xi, \tau)}{\partial \tau} = a \left( \frac{\partial^2 t(\xi, \tau)}{\partial \xi^2} + \frac{\nu - 1}{\xi} \frac{\partial t(\xi, \tau)}{\partial \xi} \right) + \frac{q}{c} \quad (0 < \xi < \xi_{\nu}; \xi_1 = h; \xi_2 = R; \tau > 0).
\]  

(1)

Under the initial condition, we obtain

\[
\tau(\xi, 0) = t_1 = t_{\text{min}} = \text{const}
\]  

(2)
Under the symmetry condition, it is
\[
\frac{\partial t(0, \tau)}{\partial \xi} = 0
\]  
(3)
and under the first row boundary condition
\[
t(\xi, \tau) = \begin{cases} 
  t_1 = t_{\min} & \text{for } 0 < \tau < \tau_1 \\
  t_2 = t_{\max} & \text{for } \tau_1 < \tau < \tau_2
\end{cases}
\]  
(4)
\[
q = q_1 \pm q_2 e^{-k\tau}
\]  
(5)

Here we use
- \( t(\xi, \tau) \) – temperature;
- \( t_{\min} = \) const – the minimum temperature on the surface of the layer;
- \( t_{\max} = \) const – the maximum allowable temperature on the surface of the layer, this may be the temperature of the medium;
- \( \tau \) – time; \([0, \tau_1]\) – operating time of the cooling device;
- \([\tau_1, \tau_2]\) – time of spontaneous heating of the material (cooling device shutdown time);
- \( \xi \) – current coordinate;
- \( h \) – the height of the layer of oilseeds in a metal silo;
- \( R \) – silo circular radius;
- \( \nu \) – shape parameter (\( \nu = 1 \) – plate, rod, \( \nu = 2 \) – cylinder);
- \( a \) – thermal diffusivity coefficient;
- \( c \) – specific heat of the raw material layer;
- \( q = q_1 \pm q_2 e^{-k\tau} \) – experimentally established dependences of the specific heat of respiration of raw flax [5] (lower sign) and sunflower, soy [24] (upper sign).
- \( k = \) const \( > 0 \) – empirical coefficient (attenuation coefficient).

The formulation of the boundary value problem (1)–(5) for storing granular grass meal with a specific heat of respiration of raw materials \( q = 0 \) is given in [6].

An analytical solution to the problem (1)–(5) by the methods of mathematical physics is obtained in the following form:

\[
T_\nu(X, F_0) = 1 - \frac{F_{0_1}}{F_{0_2}} - \sum_{m=1}^{\infty} 4 \sin(x F_0)(P_{\nu_1} \sin \beta + P_{\nu_2} \cos \beta) \frac{\nu \lambda m \phi_\nu}{\mu'} + \sum_{n=1}^{\infty} 2A_{\nu'} \frac{\mu''_{\nu'} \mu_{\nu'}}{\mu''_{\nu'} F_{0_2} - \mu''_{\nu'} F_{0_2} - 1} \left[ P_{\nu_1} \frac{F_{0_2}}{\mu''_{\nu'} (F_{0_2} = 1)} \right] \times e^{\mu_{\nu}^2 F_{0} + B_{\nu'}};
\]

\[
x = \frac{\pi m}{F_{0_2}}; \quad \beta = 2x(F_0 - \frac{F_{0_1}}{2});
\]

\[
P_{11} = \sinh \sqrt{x} \sin \sqrt{x} \cosh(\sqrt{x} X) \cos(\sqrt{x} X) - \cosh \sqrt{x} \cos \sqrt{x} \sinh(\sqrt{x} X) \sin(\sqrt{x} X);
\]
\[ P_{12} = \cosh \sqrt{x} \cos \sqrt{x} \cosh(\sqrt{x}X) \cos(\sqrt{x}X) + \sinh \sqrt{x} \sin \sqrt{x} \sinh(\sqrt{x}X) \sin(\sqrt{x}X); \]

\[ \phi_1 = \cosh(2\sqrt{x}) + \cos(2\sqrt{x}); \]

\[ \mu_{n_1} = \frac{\pi}{2}(2n - 1); \quad n \in \mathbb{N}; \quad (6) \]

\[ A_{n_1} = (-1)^n \cos(\mu_{n_1}X) \]

\[ A_{n_2} = \frac{J_0(\mu_{n_2}X)}{J_1(\mu_{n_2})}; \]

\[ B_1 = \frac{P_{0_1}}{2} \left[ 1 - X^2 \pm \frac{2P_{0_2}}{P_dP_{0_1}}(1 - \frac{\cos(\sqrt{P_d}X)}{\cos \sqrt{P_d}})e^{-P_dF_0} \right]; \]

\[ P_{21} = \text{bei}\sqrt{2x}\text{ber}(\sqrt{2x}X) - \text{ber}\sqrt{2x}\text{bei}(\sqrt{2x}X); \]

\[ P_{22} = \text{ber}\sqrt{2x}\text{ber}(\sqrt{2x}X) + \text{bei}\sqrt{2x}\text{bei}(\sqrt{2x}X); \]

\[ \phi_2 = \text{ber}^2\sqrt{2x} + \text{bei}^2\sqrt{2x}; \]

\[ \mu_{n_2} - \text{successive positive roots of the characteristic equation} \]

\[ J_0(\mu) = 0; \quad (7) \]

\[ B_2 = \frac{P_{0_1}}{4} \left[ 1 - X^2 \pm \frac{4P_{0_2}}{P_dP_{0_1}}(1 - \frac{J_0(\sqrt{P_d}X)}{J_1}\sqrt{P_d})e^{-P_dF_0} \right]; \]

- \( F_0 = \frac{\alpha_0}{h}; \quad F_{0_1} = \frac{\alpha_0}{h}; \quad F_{0_2} = \frac{\alpha_0}{h}; \) – the number of Fourier;
- \( X = \frac{\xi}{h}, \quad X = \frac{\xi}{R} \) – dimensionless coordinates (\( \xi = z \) for the plate, \( \xi = r \) for the cylinder);
- \( P_d = \frac{kh^2}{\alpha_0} \) – the Predvoditelev number;
- \( P_{0_1} = \frac{\pi h^2}{\alpha_0\Delta t}, \quad P_{0_2} = \frac{\pi h^2}{\alpha_0\Delta t} \) – the Pomerantsev number;
- \( \Delta t = t_2 - t_1, \quad t_1 < t_2; \quad J_0(X), \quad J_1(X) \) – Bessel functions of the first kind of zeroth and first order, respectively;
- \( \text{ber}X, \quad \text{bei}X \) – Thomson functions;
- \( \gamma \) – the density of absolutely dry material.
The solutions averaged over the volume (6) have the form:

$$\bar{T}_v(F_0) = \frac{\bar{T}(\tau) - t_1}{\Delta t} = 1 - \frac{F_{01}}{F_{02}} - \sum_{m=1}^{\infty} \frac{2^\nu \Phi^\nu \sin(xF_0)}{m\pi \sqrt{2x\phi_\nu}}$$

$$-\sum_{m=1}^{\infty} \frac{2^\nu(e^{\mu^2 F_0} - e^{\mu^2 F_0})}{\mu^2_n(e^{\mu^2 F_0} - 1)} e^{-\mu^2 F_0} + \overline{B}_v;$$

where

$$\Phi_1 = \sinh(2\sqrt{x}) \cos\left(\frac{\pi}{4} - \beta\right) + \sin(2\sqrt{x}) \sin\left(\frac{\pi}{4} - \beta\right);$$

$$\Phi_2 = \sum_{p=0}^{\infty} \frac{(-1)^p p(\sqrt{2})^{2p-1}}{p^2} \left[ \text{ber}\sqrt{2x} \sin(\beta - \frac{\pi}{2}) - \text{bei}\sqrt{2x} \cos(\beta - \frac{\pi}{2}) \right];$$

$$\overline{B}_1 = P_{01} \left[ \frac{1}{3} \pm \frac{P_{02}}{P_d P_{01}} \left( 1 - \tan\sqrt{T_d} \right) e^{-P_d F_0} \right];$$

$$\overline{B}_2 = P_{01} \left[ \frac{1}{4} \pm \frac{4P_{02}}{P_d P_{01}} \left( 1 - \frac{2}{\sqrt{T_d}} \right) e^{-P_d F_0} \right].$$

Knowing the average temperature $\bar{T}(F_0)$, one can find the cooling rate $\frac{\partial\bar{T}(F_0)}{\partial F_0}$ and the specific energy consumption, which are necessary to maintain the storage mode under study:

$$Q_\nu = c_\gamma (\bar{T}(\tau) - t_1)$$

For sufficiently large values of the Fourier numbers for large storage duration of raw materials, or high values of thermal inertia of the material, or sufficiently small height of the bulk of the raw material, from (8) we can obtain simplified formulas for calculating the average temperature of the stored material.

Hence, with $m = 1$ and $F_0 \gg F_{01}$; $F_0 \gg F_{02}$ for $v = 1$ (similarly for $v = 2$) we have:

$$\bar{T}_1(\tau) = t_1 + (t_2 - t_1) \left[ 1 - \frac{\tau_1}{\tau_2} \right] \left( 1 - \frac{\tau_1}{\tau_2} \right) \left( 1 - \frac{\tau_1}{\tau_2} \right) + \frac{P_{01}}{3}$$

(10)

Taking into account that the last term in the second bracket of formula (11) is much less than the unity, we obtain

$$\bar{T}_1(\tau) = t_1 + (1 - \frac{t_2}{t_1})(t_2 - t_1) + \frac{P_{01}}{3}$$

(11)

It is easy to show that $t_1 < \bar{T}_1(\tau) < t_2$, always corresponds to the physics of the process.

More accurate than formulas (10) and (11), but also quite elementary, are the following relationships for calculating temperature fields:

$$t_1(z, \tau) = t_1 + (t_2 - t_1) \left[ 1 - \frac{\tau_1}{\tau_2} \right] \left( 1 - \frac{\tau_1}{\tau_2} \right) \left( 1 - \frac{\tau_1}{\tau_2} \right) + \frac{P_{01}}{3}$$

(12)
where \( B_{m1} = \frac{2 \sin(xF_{01})}{\pi m} \)

\( B_{n1} = (-1)^n \frac{2(1 - e^{\mu_2^2 (F_{01} - F_{02})})}{\mu_n (1 - e^{-\mu_2^2 F_{02}})} \).

From the average values of the temperature of a raw material layer

\[
\overline{t_1}(\tau) = t_1 + (t_2 - t_1)\left[1 - \frac{\tau_1}{\tau_2}\right] - \sum_{m=1}^{\infty} B_{m1} \cos \beta - \sum_{n=1}^{\infty} B_{n1} e^{-\mu_n^2 F_0} + \frac{P_{01}}{3}]
\]

where

\[
\overline{B}_{m1} = \sqrt{\frac{2}{\pi}} B_{m1}
\]

\[
\overline{B}_{n1} = \frac{2(1 - e^{\mu_2^2 (F_{01} - F_{02})})}{\mu_n (1 - e^{-\mu_2^2 F_{02}})}
\]

In addition to the determining of temperature fields, the analytical solutions [5, 7-9] obtained allow to solve the inverse problem of finding the time necessary to reach the desired temperature at any point in the bulk or the average temperature of the bulk layer at the chosen values of \( \tau_1 \) and \( \tau_2 \).

By setting \( \tau_1 \), it is possible to determine the interval \([\tau_1, \tau_2]\), when the bulk temperature reaches its upper limit. The upper limit is the optimal ratio of periods \([0, \tau_1]\) and \([0, \tau_2]\), which enables to minimize energy consumption for maintaining the storage mode.

Thus, a boundary problem proposed and solved describes the temperature field of a layer of pectin-containing raw materials under controlled temperature influences.

It is known that the respiration rate of fruit and vegetables rise with increasing temperature. The temperature range 0 – 40°C, which is applied in practice, obeys the Arrhenius law and is characterized by function

\[
F = e^{\frac{E}{RT}}
\]

where \( E \) is the activation energy, which is assumed to be sufficiently large, \( T \) is the absolute temperature; \( R \) is the universal gas constant. The dependence of the specific heat of respiration for pectin-containing raw materials with different humidity during storage up to 10–15 days on temperatures up to +25°C can be represented as

\[
q = q_0^{k_1 \tau}
\]

The same pattern was observed during isothermal and adiabatic storage of pectin-containing raw materials. The coefficients \( q_0 \) and \( k_1 \) in equation (14) are the functions of source moisture of the raw material, \( q_0 \) is the specific heat of respiration of the raw material at 0°C.

Because of respiration of pectin-containing raw materials in the bulk layer under the influence of biological processes of carbon dioxide, moisture and excess heat and its poor thermal conductivity, the temperature of the lower layers of the raw materials is usually higher than the temperature of the outer (upper) layers [5]. Studies [3] show that with a sufficient degree
of accuracy, it is possible to adopt the exponential law of reducing the respiration rate of raw materials with a layer height, namely:

\[ q_2 = q_0 e^{-k_2z} \]

where \( z \) is the coordinate, \( q_m \) is the specific heat of respiration of the raw material at 0°C for \( z = 0 \).

Therefore, the total dependence of the specific heat of respiration of seeds on time \( \tau \) and the \( z \) coordinate is expressed by the formula

\[ q = q_0 e^{k_1 \tau - k_2z} \quad (15) \]

Raw material bulk accepted as a homogeneous and isotropic medium can be stored in a cylindrical silo. The lateral surface of the silo is heat-insulated, but the temperatures of the base and upper surface vary arbitrarily from time to time. It is required solve the heterogeneous heat equation for an unlimited plate, where the height of the product bulk is less than the diameter of the silo. Hence, the heat transfer problem can be formulated as follows [6]

\[ \frac{\partial t(z, \tau)}{\partial \tau} = a \frac{\partial^2 t(z, \tau)}{\partial z^2} + \frac{q_0}{c} e^{k_1 \tau - k_2z} \quad (0 < z < h; \tau > 0), \]

under the initial condition:

\[ t(z, 0) = t_0 = \text{const}, \quad (16) \]

and the boundary conditions:

\[ t(0, \tau) = f_1(\tau), \quad (17) \]
\[ t(h, \tau) = f_2(\tau), \quad (18) \]

where \( a \) is the thermal diffusivity, \( h \) is the height of the embankment, \( t_0 \) is the temperature of the product at the beginning of the storage process, \( s \) is the specific heat of the product embankment, \( f_1(\tau) \) and \( f_2(\tau) \) are the specified limited and continuous functions. By substituting \[20\]

\[ t(z, \tau) = v(z, \tau) + \frac{q_0}{c_s(k_1 - ak_2^2)} e^{k_1 \tau - k_2z} \]

where \( v(z, \tau) \) is a new sought function, we reduce equation (15) to a homogeneous one, eliminating a heat source,

\[ \frac{\partial t(z, \tau)}{\partial \tau} = a \frac{\partial^2 t(z, \tau)}{\partial z^2} \]

but adding new initial and boundary conditions \[22\]:

\[ v(z, 0) = t_0 - \frac{q_0}{c_s(k_1 - ak_2^2)} e^{-k_2z} = \phi(z) \]
\[ v(0, \tau) = f_1(\tau) - \frac{q_0}{c_s(k_1 - ak_2^2)} e^{k_1 \tau} = \phi_1(\tau) \]
\[ v(h, \tau) = f_2(\tau) - \frac{q_0}{c_s(k_1 - ak_2^2)} e^{k_1 \tau - k_2h} = \phi_2(\tau) \]
By the integral finite sine transform methods, we obtained an analytical solution of the boundary-value problem recorded for the distribution of the temperature field in the bulk of products.

For the special case of constant temperatures of the base of the bulk at $t(0, \tau) = t_1 = \text{const}$, $t(h, \tau) = t_2 = \text{const}$, and taking into account the known relations \[22\], we obtained the solution in the form \[23\]:

$$t(z, \tau) = \frac{q_0}{c(k_1 - ak_2^2)} e^{k_1 - k_2 z + t_1 + (t_2 - t_1) \frac{z}{h}} + \left(\frac{2}{\pi}\right) \sum_{n=1}^{\infty} \frac{t_0 - t_1 - (-1)^n(t_0 - t_2)}{n\pi} \sin \left(\frac{n\pi z}{h}\right) e^{-an^2\frac{z^2}{h}}$$

$$- \frac{2q_0}{c(k_1 - ak_2^2)} \sum_{n=1}^{\infty} \left[1 - (-1)^n e^{-k_2 h} \sin \left(\frac{n\pi z}{h}\right) \right] \left[ e^{-\frac{a h^2 n^2 z^2}{(k_2 h)^2 + (n\pi)^2}} + \frac{k_1 \tau - e^{-an^2\frac{z^2}{h}}}{k_1 h^2 + (n\pi)^2} \right]$$

The dependence obtained was used to model the storage process in the bulk. We used the data on the storage of pectin-containing raw materials in a warehouse (batch No. 1). Due to the fact that the difference in humidity across the layers during the laying of the batch ($x = 0$) fluctuated slightly ($\pm 0.2 \%$), the value $k_2$ equals zero. The moisture value of the pectin-containing raw materials in the layer at the initial instant of time is $U(Z, 0) = 8.4 \%$, the temperature value is $t_0 = 16^\circ C$, $k_1 = 0.8 \cdot 10^{-7} c - 1$, $q_0 = \frac{0.01ZW}{kg}$. This corresponds to estimates of the release heat of raw materials with such humidity and temperature, according to the work data.

The calculation results for the bulk with a height of 3 m are given in the table 1

| $F_0, 10^3$ | $X = 0.17$ | $X = 0.2$ | $X = 0.83$ |
|-------------|------------|------------|------------|
|             | $t_{\text{exponential}}$ | $T_{\text{estimated}}$ | $t_{\text{exponential}}$ | $T_{\text{estimated}}$ | $t_{\text{exponential}}$ | $T_{\text{estimated}}$ |
| 1           | 20.6       | --         | 20.1       | --         | 19.2       | --         |
| 3           | 21.4       | 20         | 21.2       | 13         | 20.8       | --         |
| 6           | 21.9       | --         | 21.5       | --         | 21.0       | --         |
| 9           | 22.3       | --         | 21.8       | --         | 21.3       | --         |
| 13          | 22.6       | 21         | 22.0       | 20         | 21.5       | --         |
| 17          | 23.0       | --         | 22.2       | --         | 21.7       | --         |
| 0           | 23.2       | 31         | 22.3       | 29         | 21.9       | --         |
| 5           | 23.4       | --         | 22.4       | --         | 21.9       | --         |
| 0           | 23.5       | --         | 22.5       | --         | 21.9       | --         |
Figure 1. Estimated Values.

Figure 2. Experimental values.
Here is $F_0 = \frac{a \tau}{h^2}$ the Fourier number, and $X = \frac{z}{h}$ is the dimensionless coordinate.

The feasibility of the model proposed is obvious in real-life process at low heights of the bulk of pectin-containing raw materials and the Fourier number $F_0 < 0.02$. In other cases, there are discrepancies between the calculated and experimental values, which can be explained by a number of reasons. On the one hand, the function of the heat source does not take into account the change in humidity during the storage, which is amounted from 0.6 to 1.0 % as a storage result. In addition, the accelerating effect of temperature on the heat release intensity is not taken into account in the model. The temperature difference at the beginning and end of storage reached 15°C for the middle layer. We can conclude that it is necessary to move on to the models, which consider both joint heat and moisture transfer, and non-linearity in temperature. The mathematical models proposed allow to predict and control the temperature fields of product bulk and thereby affect their quality and storage duration.

2. Conclusions

The analytical solution obtained can be applied to engineering calculations of pectin-containing raw material storage with low humidity conditions under controlled temperature influences. The solution considering the specific heat of respiration of the raw material enables, to determine the rate of cooling or heating, the lowest specific energy consumption to maintain the required mode, as well a to choose the optimal ratio of the duration of the periods of operation and shutdown of the cooling device.

References

[1] Trisvyatsky L A, Lesik B V and Kudrina V N 1991 Storage and technology of agricultural products
[2] Hasselberg K and Herppich W B 2005 Ozontes Wasserzur Qualitätssicherungbei-Waschmohren Landtechnik 6 350–351
[3] Lagaron J M 2005 Improving packaged food quality and safety. Part 2: Nanocomposites Food Additives and Contaminants (22) 10 994–998
[4] 2008 Auf die richtige Auswahl des Gefrierverfahrenskommtesan Lebensmittel-technik 10 26–27
[5] Callens A 2008 Frostenmit Stickstoff Lebensmitteltechnik. 1-2 48–49
[6] Alekseev G V, Voronenko B A, Verboloz E I , Romanchikov S A and Loza A A 2017 Ball under action of periodic point load Advances in Engineering Research 133 36–41
[7] Voronenko B A 2013 An analytical study of the drying process of wheat germ by infrared radiation Scientific journal NRU ITMO. Series "Processes and Food Production Equipment" 2
[8] Lykov A V and Mikhailov Yu A 1963 Theory of heat and mass transfer Gosenergoizdat
[9] Lykov A V and Mikhailov Yu A 1952 Theory of thermal conductivity Gosenergoizdat
[10] Moldovanov D V and Titova N E 2018 Energy and resource saving as non-alternative components of modern production Colloquium-journal (1) 8 (19) 12–15.