Couplings of heavy mesons with soft pions in QCD

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QCD sum rules are used to calculate the couplings of heavy-light quark pseudoscalar and vector mesons \( D, D^* \) and \( B, B^* \) with soft pions, both for finite and infinitely heavy quark mass. The couplings are also computed in the framework of a QCD relativistic potential model; in this approach the relativistic corrections due to the light quark are relevant.

1. INTRODUCTION

The strong coupling constant \( g_{D^* D \pi} \), defined by the matrix element

\[
\langle \pi^+ (q) | D^0(q_2) | D^{*+}(q_1, \epsilon) \rangle = g_{D^* D \pi} (\epsilon \cdot q)
\]

(1)
governs the decay \( D^{*+} \to D^0 \pi^+ \); from the experimental branching ratio \( BR(D^{*+} \to D^0 \pi^+) = 68.1 \pm 1.0 \pm 1.3 \% \) and from the measurement \( \Gamma(D^{*+}) < 131 \) KeV \( \) the upper limit \( g_{D^* D \pi} \leq 20.6 \) can be derived.

The analogous coupling constant in the \( B \) channel, \( g_{B^* B \pi} \), describes the contribution of the \( B^* \) pole to the form factor \( F^{B \to \pi}_{1}(q^2) \) of the semileptonic decay \( B \to \pi \ell \nu \):

\[
F_{1}^{B \to \pi}(q_{\text{max}}^2) \simeq \frac{f_{B^*}}{4} \frac{g_{B^* B \pi}}{E_\pi + \delta_B},
\]

(2)

where \( f_{B^*} \) is the \( B^* \) leptonic decay constant and \( \delta_B = m_{B^*} - m_B \). If the pole of \( B^* \) dominates the form factor \( F_{1}^{B \to \pi} \) not only at zero recoil but for all accessible values of the transferred momentum \( q \), the coupling \( g_{B^* B \pi} \), together with \( f_{B^*} \), completely determines \( F_{1}^{B \to \pi} \), with interesting phenomenological consequences for the measurement of \( V_{ub} \) using this decay.

In the limit of infinitely heavy quark \( b \), \( g_{B^* B \pi} \) scales according to the relation \( \| \):

\[
g_{B^* B \pi} = \frac{2m_{B^*}}{f_\pi} g,
\]

(3)

where \( g \) is a low energy parameter independent of the heavy scale; it acts as a coupling constant in the effective Lagrangian constructed using heavy quark and chiral symmetries to describe the interaction of heavy-light quark \( 0^- \) and \( 1^- \) states with the octet of Nambu-Goldstone pseudoscalars \( \| \). The upper limit \( g \leq 0.7 \) can be derived from the experimental bound on \( g_{D^* D \pi} \); in ref.\( \| \) the estimate \( g \approx 0.4 \) has been given using heavy-quark chiral symmetry relations and data on the semileptonic decay \( D \to K \ell \nu \).

The couplings \( g_{D^* D \pi} \), \( g_{B^* B \pi} \), and \( g \) have been computed in the framework of nonrelativistic constituent quark models \( \| \); in particular, within this approach the value \( g \approx 1 \) is obtained \( \| \), which exceeds the above mentioned experimental bound.

\( g_{D^* D \pi} \) and \( g \) are also available from QCD sum rules \( \| \) and, recently, from light-cone sum rules \( \| \). In ref.\( \| \) we have reconsidered the calculation in \( \| \), introducing a number of improvements both in the theoretical and in the hadronic part of the QCD sum rule, as discussed below.

2. COUPLINGS FROM QCD SUM RULES

In order to calculate, for example, the off-shell process \( B^{*-}(q_1) \to B^0(q_2) + \pi^-(q) \) we have considered the time-ordered product of two quark currents interpolating the \( B \) and \( B^* \) mesons, between the vacuum and the pion state:

\[
i \int dx < \pi^-(q)|T(V_\mu(x) J_5(0)|0 > e^{-iqx} = A q_\mu + B P_\mu
\]

(4)
where $V_\mu = \bar{u}_{\gamma} b$, $j_\mu = \bar{b}_i \gamma_\mu d$ and $P = q_1 + q_2$; $A$, $B$ are scalar functions of $q_1^2$, $q_2^2$, $q^2$. In the soft pion limit ($q \to 0$) and for large Euclidean momenta $q_1$ and $q_2$ both the invariant functions (we have considered $A$) can be computed in QCD by the operator product expansion. In our calculation we have taken into account the contributions proportional to the quark condensate, the terms proportional to the mixed quark-gluon condensate (which are missing in [9,10]) and the terms proportional to the matrix element $< \pi(q)\bar{D}_\mu g\sigma G\gamma_5 d(0) >= -if_\pi m_\pi^2 q_\mu$ (with $m_\pi^2 = 0.2$ GeV$^2$ [9]). The following relations have been used:

$$
< \pi(q)\bar{D}_\mu g\sigma G\gamma_5 d(0) > = \frac{m_\pi^2 - \bar{q} q}{f_\pi} q_\mu
$$

$$
< \pi(q)\bar{D}_\mu g\sigma G\gamma_5 d(0) > = \frac{i m_\pi^2 - \bar{q} q}{4f_\pi} q_\mu.
$$

(5)

On the other hand, a dispersive representation can be given for $A$:

$$
A(0,q_1^2,q_2^2) = \frac{1}{\pi^2} \int ds \frac{\rho(s,s')}{(s-q_1^2)(s'-q_2^2)}
$$

with the spectral function $\rho$ expressed in terms of hadronic states. In the region $m_B^2 \leq s, s' \leq s_0$ (with $s_0$ a small threshold) $\rho(s,s')$ gets contribution from the $B$ and $B^*$ poles, only. Above the threshold other contributions must be taken into account (in addition to the hadronic continuum) which are not suppressed by the borelization procedure since, in the soft pion limit, a single Borel transform in the variable $q_1^2 = q_2^2$ must be performed; these resonance-continuum contributions are called "parasitic terms". By studying models for such terms, we have observed that they generally contribute to the Borel transformed sum rule for $g_{B^\ast B \pi}$ as follows:

$$
\frac{\gamma}{M^2} + d_0 + d_1 e^{-\delta/M^2} = e^{\delta/M^2} f^{QCD}(M^2)
$$

(7)

where $M$ is the Borel parameter, $\delta = s_0 - m_B^2$, $\gamma = f_B f_{B^*} m_B^2 g_{B^* B \pi}(3m_B^2 + m_B^2)/4 m_B m_{B^*}$ and $\Omega = (m_B^2 + m_{B^*}^2 - 2m_\pi^2)/2$; $f^{QCD}$ is the Borel transformed QCD side of the sum rule (the expression can be found in [12]).

The terms $d_0$ and $d_1$ parametrize the parasitic contributions: they do not depend on $M$ and therefore can be removed performing derivatives in $M^2$.

The numerical analysis has been carried out using the parameters: $m_c = 1.35$ GeV, $m_b = 4.6$ GeV, the standard values of the condensates and the effective thresholds $s_0 = 6 - 8$ GeV$^2$ in the case of the $c$ quark and $s_0 = 32 - 36$ GeV$^2$ in the case of the $b$ quark. The result is:

$$
\begin{align*}
\tilde{f}_B f_{B^*} g_{B^* B \pi} &= 0.56 \pm 0.12 \text{ GeV}^2 \\
f_D f_{D^*} g_{D^* D \pi} &= 0.34 \pm 0.08 \text{ GeV}^2 \\
\tilde{F}^2 g &= 0.035 \pm 0.008 \text{ GeV}^3;
\end{align*}
$$

(8)

in the last equation the result concerning the limit $m_B \to \infty$ is reported, with $\tilde{F}$ given by $\tilde{F} = f_B \sqrt{m_B}$ in this limit.

The results in (8) agree with the light-cone sum rules calculation in (1).

The prediction for $g_{D^* D \pi}$, $g_{B^* B \pi}$, and $g$ can be given once the values of the leptonic constants $f_D$ etc. are put in (8). Using the leptonic constants obtained by two-point function QCD sum rules without radiative corrections (notice that the calculation for the strong couplings has been carried out at the order $\mathcal{O}(\alpha_s) = 0$) we get: $g_{D^* D \pi} = 9 \pm 2$, $g_{B^* B \pi} = 20 \pm 4$ and $g = 0.39 \pm 0.16$. On the other hand, if radiative corrections at the order $\mathcal{O}(\alpha_s)$ are included in the calculation of the leptonic constants we obtain: $g_{D^* D \pi} = 7 \pm 2$, $g_{B^* B \pi} = 15 \pm 4$, $g = 0.21 \pm 0.06$. Within the uncertainties, there is agreement for $g_{D^* D \pi}$ and $g_{B^* B \pi}$; the difference between the values for $g$ reflects the important role of radiative corrections in the determination of $f_B$ in the limit $m_B \to \infty$.

Using the central values for $g_{D^* D \pi}$ and $g_{B^* B \pi}$ we predict: $\Gamma(D^+ \to \pi^0) = 10 - 17 \text{ keV}$ and $F_1^{B \to \pi}(0) = 0.30 - 0.40$ (in the hypothesis of dominance of the $B^*$ pole for this form factor).

3. COUPLINGS FROM A QCD RELATIVISTIC POTENTIAL MODEL

The value $g \simeq 0.2 - 0.4$ obtained by QCD sum rules must be compared with the outcome of constituent quark models: $g \simeq 1$ [3]. Since in these models the light quark is treated as a nonrelativistic particle, one could wonder about the role
of relativistic corrections.

Relativistic effects can be taken into account in the framework of the potential model described in [14]; in this model the quark kinematics is relativistic, and the interquark interaction is described by the Richardson potential, linear at large distances and coulombic (with QCD corrections) at short distances.

Within this approach the following equation has been derived for $g$ [15] (in the limit $m_b \to \infty$):

$$g = \frac{1}{4m_B} \int_0^{\infty} dk |\tilde{u}_B(k)|^2 \times 
\times \frac{E_q + m_q}{E_q} \left[ 1 - \frac{k^2}{3(E_q + m_q)^2} \right],$$

(9)

where $k$ is the relative quark-antiquark momentum inside the meson, $\tilde{u}_B(k)$ is the $B$ and $B^*$ wave function (we neglect the spin splitting between $B$ and $B^*$ which is of the order $1/m_b$), $E_q = \sqrt{k^2 + m_q^2}$ ($q$ is the light quark); the wave function normalization is: $\int_0^{\infty} dk |\tilde{u}_B(k)|^2 = 2m_B$.

In the non relativistic limit $E_q \approx m_q \gg k$ one obtains: $g = 1$. This is the result of constituent quark models; notice that, in this limit, eq.(9) is similar to the expression for $g$ derived by Kamal and Xu in [8] in the framework of the Bauer-Stech-Wirbel model.

On the other hand, if one considers the limit $m_q \to 0$ (our fit of the meson masses fixes the light quark mass to the value $m_q = 38$ MeV) the equation for $g$ gives the result: $g = \frac{1}{4}$, in agreement with QCD sum rules. This allows us to conclude that the reduction of the value of $g$ from the result of non relativistic constituent quark models finds an explanation (as observed also in [10]) in the effects of the relativistic motion of the light quark.

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