SPONTANEOUS CURRENT-LAYER FRAGMENTATION AND CASCADING RECONNECTION IN SOLAR FLARES. I. MODEL AND ANALYSIS

MIROSLAV BÁRTA1,2, JÖRG BÜCHNER1, MARIAN KARLICKÝ2, AND JAN SKÁLA2,3

1 Max Planck Institute for Solar System Research, D-37191 Katlenburg-Lindau, Germany; barta@mps.mpg.de
2 Astronomical Institute of the Academy of Sciences of the Czech Republic, CZ-25165 Ondřejov, Czech Republic
3 Faculty of Science, University of J. E. Purkinje, CZ-40096 Ústí nad Labem, Czech Republic

Received 2010 November 17; accepted 2011 May 24; published 2011 July 26

ABSTRACT

Magnetic reconnection is commonly considered to be a mechanism of solar (eruptive) flares. A deeper study of this scenario reveals, however, a number of open issues. Among them is the fundamental question of how the magnetic energy is transferred from large, accumulation scales to plasma scales where its actual dissipation takes place. In order to investigate this transfer over a broad range of scales, we address this question by means of a high-resolution MHD simulation. The simulation results indicate that the magnetic-energy transfer to small scales is realized via a cascade of consecutively smaller and smaller flux ropes (plasmoids), analogous to the vortex-tube cascade in (incompressible) fluid dynamics. Both tearing and (driven) “fragmenting coalescence” processes are equally important for the consecutive fragmentation of the magnetic field (and associated current density) into smaller elements. At the later stages, a dynamic balance between tearing and coalescence processes reveals a steady (power-law) scaling typical of cascading processes. It is shown that cascading reconnection also addresses other open issues in solar-flare research, such as the duality between the regular large-scale picture of (eruptive) flares and the observed signatures of fragmented (chaotic) energy release, as well as the huge number of accelerated particles. Indeed, spontaneous current-layer fragmentation and the formation of multiple channelized dissipative/acceleration regions embedded in the current layer appear to be intrinsic to the cascading process. The multiple small-scale current sheets may also facilitate the acceleration of a large number of particles. The structure, distribution, and dynamics of the embedded potential acceleration regions in a current layer fragmented by cascading reconnection are studied and discussed.

Key words: acceleration of particles – magnetic reconnection – magnetohydrodynamics (MHD) – Sun: flares – turbulence

Online-only material: color figures

1. INTRODUCTION

It is generally conjectured that solar flares represent a dissipative part of the release of the magnetic energy accumulated in active regions of the Sun. The “standard” CSHKP model (see, e.g., Shibata & Tanuma 2001; Mandrini 2010; Magara et al. 1996, and references therein) agrees well with the observed large-scale dynamics of eruptive events. In this model, flares are initiated by the eruption of a flux rope (in many cases observed as a filament) via, for example, a kink or torus instability (e.g., Kliem & Török 2006; Török & Kliem 2005; Williams et al. 2005; Kliem et al. 2010), evolving later into a coronal mass ejection (CME). The latter is trailed by a large-scale current layer behind the ejecta (Lin & Forbes 2000). In this trailing current layer, reconnection is supposed to give rise to various observed phenomena, such as hot soft X-ray/EUV flare loops rooted in He chromospheric ribbons, hard X-ray (HXR) sources in loop tops and footpoints, and radio emissions of various types. Some authors (e.g., Ko et al. 2003; Lin et al. 2007) argue that the bright thin ray-like structures sometimes observed behind CMEs may represent a manifestation of the density increase connected with this current sheet (CS).

However, closer analysis of the classic CSHKP model revealed some of its open issues. One is that the timescales of reconnection in such a thick flare CS appeared to be much longer than the typical flare duration. In other words, the reconnection rate in such configurations has been found to be insufficient for the rapid energy release observed in flares. Later, it was found that the dissipation necessary for reconnection in the practically collisionless solar corona is an essentially plasma-kinetic process (see, e.g., Büchner 2006), which takes place at very small spatial scales. Hence, the question arises of how sufficiently thin CSs can build up within the global-scale, thick CME-trailing current layer: The actual physical mechanism that provides the energy transfer from the global scales, at which the energy is accumulated, to the much smaller scales, at which the plasma-kinetic dissipation takes place, is an open issue.

Addressing these questions, Shibata & Tanuma (2001) suggested the concept of cascading (or fractal, as they call it) reconnection. According to their scenario, a cascade of nonlinear tearing instabilities occurs in the continually stretched current layer formed behind a CME. Multiple magnetic islands (helical flux ropes in three dimensions), also called plasmoids, are formed, interleaved with thin CSs. Due to increasing separation of the plasmoids in the continually vertically extending trailing part of the CME, the interleaved CSs are subjected to further filamentation until the threshold for secondary tearing instability is reached. This process continues further; third and higher levels of tearing instabilities arise until the widths of the CSs reach the kinetic scale.

This scenario has recently been supported by the analytical theory of plasmoid instability by Loureiro et al. (2007). They show that high–Lundquist number systems, with high enough CS length-to-width ratios, are not subject to slow Sweet–Parker reconnection, but they are inherently unstable to the formation of plasmoids on very short timescales. Samtaney et al. (2009),
Bhattacharjee et al. (2009), and Huang & Bhattacharjee (2010) confirmed predictions of this analytical theory by numerical simulations with high Lundquist numbers. Ni et al. (2010) generalize the model by including the presence of shear flows around the CS. Uzdensky et al. (2010) further relate the theory of plasmoid instability to the concept of fractal reconnection suggested by Shibata & Tanuma (2001). Shepherd & Cassak (2010) and Huang et al. (2010) study the plasmoid instability numerically at smaller scales and investigate its relation to Hall reconnection. They found various regimes of parameters where different types of reconnection prevail.

Eventually, however, kinetic scales are reached where dissipation and particle acceleration take place, most likely via the kinetic coalescence of micro-plasmoids and, possibly, their shrinkage (Drake et al. 2005; Karlický & Bártá 2007; Karlický et al. 2010).

In addition to the issue of energy transport, there are also other questions that remain open in the CSHKP model. One is its apparent inability to explain the acceleration of the number of particles in its single diffusion region around the X-line that would correspond to the fluxes inferred from HXR observations in the thick-target model (Fletcher 2005; Krucker et al. 2008, and references therein). Furthermore, HXR and radio (e.g., decimetric spikes; see Karlický et al. 1996, 2000; Bártá & Karlický 2001) observations indicate that particle acceleration takes place via multiple concurrent small-scale events distributed chaotically in the flare volume, rather than by a simple compact acceleration process hosted by a single diffusion region. Such observations are usually referred to as signatures of fragmented/chaotic energy release in flares.

Because of these difficulties, an alternative concept based on so-called self-organized criticality (SOC) has been proposed (Aschwanden 2002; Vlahos 2007). This model is based on the idea of multiple small-scale CSs embedded in the chaotic (braided) magnetic fields that are formed as a consequence of random motions at the system boundary (photosphere). Multiple CSs can host multiple reconnection sites, which provides a natural explanation of the observed signatures of fragmented energy release. At the same time, they provide a larger total volume of diffusion regions, perhaps sufficient to also account for the observed particle fluxes quantitatively. Organized large-scale structures (i.e., coherent structures like flare-loop arcades) should be, in the case of SOC-based models, achieved by the so-called avalanche principle: a small-scale energy-release event can trigger similar events in its vicinity provided the system is in a marginally stable state (due to the continuous pumping of energy and entropy through the boundary). Nevertheless, it is difficult to achieve such coherent large-scale structures as are usually observed in solar flares in the frame of SOC-based models.

Thus, solar flares appear to be enigmatic phenomena exhibiting duality between the regular, well-organized dynamics of flares observed at large scales and the signatures of fragmented/chaotic energy release seen in observations related to flare-accelerated particles. While the coherent global eruption (flare) picture seems to be in agreement with the CSHKP scenario, the observed fragmented-energy-release signatures favor the SOC-based class of models.

In the present paper, we suggest that cascading reconnection can address these three pressing questions (i.e., energy transport across the scales, accelerated-particle fluxes, and the organized/chaotic picture duality) as being closely related to each other. In our view, energy is transferred from large to small scales by the cascade of fragmentation of originally large-scale magnetic structures to smaller elements. We identify two elementary processes of this fragmentation (see Section 3). In the course of these processes, the initial current layer also fragments into multiple small-scale, short-lived CSs. These CSs are hierarchically embedded within the thick current layer in a qualitatively self-similar manner. In this sense, the cascading fragmentation resembles the SOC models, but now the chaotic distribution of small-scale currents results from the internal instabilities of the global current layer. The fragmented current layer represents a modification of the standard CSHKP model, and thus it keeps coherent the large-scale picture of solar flares. At the same time, it addresses the observed signatures of fragmented energy release and the question of efficient particle acceleration. We believe that cascading reconnection in solar coronal current layers can thus address the three main problems mentioned above en bloc, and it reconciles the two concepts of the standard CSHKP and SOC-based models, hitherto seen as antagonistic.

The paper is organized as follows. First, we describe the model used in our investigations. Then we present results of our high-resolution MHD simulation of cascading reconnection in an extended, global, eruption-generated current layer. We identify the processes that lead to the fragmentation of magnetic and current structures into smaller elements. Then we analyze the resulting scaling law of the energy cascade. We describe the structure, distribution, and dynamics of small dissipation regions embedded in an initially thick current layer. Finally, we discuss the implications that cascading reconnection has for the theory of solar flares.

2. MODEL

Generally speaking, a solar flare involves three kinds of processes that take place in different scale domains; see Figure 1. At the largest scales, magnetic-field energy is accumulated. During this stage, flux rope (filament) is formed and its magnetic energy increases. Eventually, it loses its stability and is ejected. This process already represents (ideal) release of the magnetic energy at large scales. Subsequently, a current layer is formed.

![Figure 1. Large-scale magnetic reconnection from the point of view of the theory of dynamical systems. Schematic view of the cascade of energy transfer from large to small scales and the window of scales resolved in our simulations. The lower abscissa shows the wavenumber k; corresponding characteristic scales $\lambda = 2\pi/k$ are shown on the upper abscissa. The numeric values on the scale axis correspond to scaling based on typical coronal parameters (for details on scaling, see Section 2).](image)
and stretched behind the ejected flux rope. Energy transfer from the large scales, at which the magnetic energy has been accumulated, to the small dissipation scale occupies the intermediate range of scales. The dissipation itself takes place at the smallest, kinetic scales.

In this paper, we aim to study energy transfer from large to small scales by means of numerical simulation. Despite the high spatial resolution, our simulation is still within the MHD regime. Also, we do not address the very process of energy accumulation, that is, flux-rope formation and energization, or its instability and subsequent current-layer formation. Instead, we assume a relatively thick and extended current layer to be already formed at the initial state of our study. In order to cover a large range of scales, we limit ourselves to two-dimensional geometry, while allowing for all three components of velocity and magnetic field (commonly referred to as a 2.5D model). This is a reasonable assumption since observations show that the typical length of flare arcades along the polarity-inversion line (PIL) is much larger than the dimension across the PIL.

In the range of scales that we are interested in, the evolution of magnetized plasma can be adequately described by a set of compressible resistive one-fluid MHD equations (e.g., Priest 1984):

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0 \\
\rho \frac{\partial u}{\partial t} + \rho (u \cdot \nabla) u = -\nabla p + j \times B + \rho \mathbf{g} \\
\frac{\partial B}{\partial t} = \nabla \times (u \times B) - \nabla \times (\eta j) \\
\frac{\partial U}{\partial t} + \nabla \cdot S = \rho u \cdot \mathbf{g}.
\]

The set of Equations (1) is solved by means of the finite volume method (FVM). For numerical solution, it is first rewritten in its conservative form. The (local) state of the magnetofluid is then represented by the vector of basic variables \( \Psi \equiv (\rho, \rho u, B, U) \), where \( \rho, u, B \), and \( U \) are the plasma density, plasma velocity, magnetic field strength, and total energy density, respectively. The energy flux \( S \) and auxiliary variables, plasma pressure \( p \) and current density \( j \), are defined by the formulae

\[ \nabla \times B = \mu_0 j \]

\[ U = \frac{p}{\gamma - 1} + \frac{1}{2} \rho u^2 + \frac{B^2}{2 \mu_0} \]

\[ S = \left( U + \rho + \frac{B^2}{2 \mu_0} \right) u - \frac{u \cdot B}{\mu_0} B + \frac{\eta}{\mu_0} j \times B, \]

and \( \mathbf{g} \) is the gravitational acceleration at the photospheric level. Microphysical (kinetic) effects enter into the large-scale dynamics by means of transport coefficients, here via a (generalized) resistivity \( \eta \). In general, the role of non-ideal terms in the generalized Ohm’s law increases as the current density becomes more concentrated via CS filamentation. To quantify this intensification, we use the current-carrier drift velocity \( v_D = |j|/(en_c) \) as the threshold for non-ideal effects to take place. Such behavior is suggested by theoretical considerations and confirmed by kinetic (Vlasov and PIC codes) numerical experiments (Büchner & Elkina 2005, 2006; Karlický & Bártá 2008). In particular, we assume the following law for (generalized) resistivity (see also Kliem et al. 2000):

\[
\eta(r, t) = \begin{cases}
0 & |v_D| \leq u_{cr} \\
\left( \frac{1}{C_{\text{fr}}, (r, t)} - v_{cr} \right) \frac{v_D}{v_0} & |v_D| > u_{cr}
\end{cases}
\]
The partial differential Equations (1) are of a mixed hyperbolic–parabolic type. We utilized the time-splitting approach for their solution. First, the hyperbolic (conservative) part is solved using a second-order FVM leapfrog scheme. In a second step, the magnetic-diffusivity term is solved by means of a (semi-implicit) alternating-direction implicit (ADI) scheme (Chung 2002).

We initially solve the MHD system of Equations (1) in a two-dimensional simulation box on a global (coarse) Cartesian grid. The horizontal and vertical dimensions of the calculation box are 800 and 6400 grid cells, respectively. Using a mirroring boundary condition at \( x = 0 \) in the symmetric CS (see below), we obtain a doubled box with an effective grid of \( 1600 \times 6400 \) cells (see also Báta et al. 2008a for details). We use the following reference frame. The \( z \)-axis corresponds to the vertical direction, while the \( y \)-axis corresponds to the invariant (i.e., \( \partial / \partial y = 0 \)) direction along the PIL. The \( x \)-axis is perpendicular to the current layer and centered at the initial current maximum. The simulation is thus performed in the \( xz \)-plane, while the \( xy \)-plane corresponds to the solar photosphere; the PIL is located at \( x = 0 \), \( z = 0 \) (see Figure 5 in Báta et al. 2008a).

The simulation is performed in dimensionless variables. They are obtained by the following normalization. The spatial coordinates \( x, y, \) and \( z \) are expressed in units of the CS half-width \( L_A \) at the photospheric level (\( z = 0 \)). Time is normalized to the Alfvén-wave transit time \( \tau_A = L_A / V_{A,0} \) through the CS, \( V_{A,0} = B_0 / \sqrt{\mu_0 \rho_0} \) is the asymptotic value at \( x \to \infty \) and \( z = 0 \) of the Alfvén speed at \( t = 0 \). Equation (2) for anomalous resistivity in the dimensionless variables then reads \( \eta = C (|j| / \rho - v_{cr}) \) for \( |j| / \rho > v_{cr} \), where \( C \) and \( v_{cr} \) are now dimensionless parameters. We used \( C = 0.003 \) and \( v_{cr} = 15.0 \) in our simulation. The choice of the threshold \( v_{cr} \) is not arbitrary as it is closely related to the numerical resolution reached by the code; see the discussion in Section 4. The parameter \( C \) was adjusted to reach peak anomalous resistivities on the order of \( 10^6 \) times higher than the Spitzer resistivity in the solar corona; similar values for resistivity based on nonlinear wave–particle interaction are indicated by Vlasov simulations (Büchner & Elksina 2006).

If not specified otherwise, all quantities in the paper are expressed in this system of dimensionless units. In order to apply our results to actual solar flares, appropriate scaling of dimensionless variables, however, has to be performed. The gravitational stratification included in our model introduces a natural length scale. Assuming an ambient coronal temperature of \( T = 2 \) MK, the corresponding scale height for a fully ionized hydrogen plasma is \( L_G = 120 \) Mm. The value used in our simulation is \( L_G = 200 L_A \), hence \( L_A = 600 \) km. For this scaling the flare-arcade loop top is \( \approx 10,000 \) km high, which corresponds well to observed values. The initial CS width \( 2L_A = 1200 \) km is roughly in accordance with the fact that the CS was formed by the stretching of the magnetic field in the trail of an ejected flux rope/outflow, which itself has typical transverse dimensions \( \approx 5000 \) km (Vršnak et al. 2009). It also corresponds (in order of magnitude) to estimates made from observations of thin layers trailing behind CMEs, which are sometimes interpreted as signatures of CSs (Ko et al. 2003; Lin et al. 2007). For the ambient magnetic field in the vicinity of the current layer, we assume \( B_{z,0} = 40 \) G (see, e.g., the discussion in Kliem et al. 2000).

The initial state has been chosen to be in the form of a vertical generalized Harris-type CS with the magnetic field \( B = \nabla \times A + \hat{e}_z B_z \) slightly decreasing with height \( z \) (Bárta et al. 2010):

\[
A(x, y, z; t = 0) = - B_0 \ln \left( e^{y z / L_A} + e^{-y z / L_A} \right) \hat{e}_y
\]

\[
B_z(x, y, z; t = 0) = B_{z,0}
\]

\[
\rho(x, y, z; t = 0) = \rho_0 \exp \left( - \frac{z}{L_G} \right).
\]

In the following, we will refer to \( B_z \) and \( A \) as the principal components and \( B_z \) as the guide field. The characteristic width of the initial CS varies with \( z \) as

\[
w_{CS}(z) = \frac{d \cdot z^2 + z + z_0}{z + z_0}
\]

and \( B_{z,0}, \rho_0, d, \) and \( z_0 \) are appropriately chosen constants:

\[B_{z,0} = 0.2, \rho_0 = 1.0, B_0 = \sqrt{B_{z,0}^2 + B_z^2} = 1.0, d = 0.003, \text{ and } z_0 = 20.0.\]

The initial state given by Equation (3) corresponds to a stratified atmosphere in the presence of gravity (which is consistent with Equations (1)). The divergence of the magnetic field lines toward the upper corona is in agreement with the expansion of the coronal field. It also favors upward motion of secondary plasmoids formed in the course of CS tearing (Bárta et al. 2008b). This leads to further filamentation of the CSs that develop between the plasmoids (Shibata & Tanuma 2001).

The current density and magnetic field at the initial state are extended in the second step fulfilling \( \nabla \cdot B = 0 \). In order to satisfy the MHD boundary conditions, symmetric \( (q(-z) = q(z)) \) relations are
used for $\rho \, B_x$, $B_z$, $U$, and anti-symmetric ($q(-z) = -q(z)$) for $B_z$ at the bottom boundary, while velocities are set to zero there ($u = 0$). This ensures that the principal magnetic-field component is vertical at the bottom boundary and that the total magnetic flux passing through that boundary does not change on the rather short timescales of the eruption, as enforced by the presence of a dense solar photosphere (Bárta et al. 2008a). Mirroring boundary conditions (symmetric in $\rho$, $u_x$, $B_x$, $B_z$, and $U$ and anti-symmetric in $u_y$ and $B_y$) are used for the left side of the boundary at $x = 0$ (i.e., the center of the CS). We use these symmetries to construct (mirror) the left half of the full effective box.

The asymptotic plasma beta parameter at $x \to \infty$ and $z = 0$ is $\beta = 0.1$ and the ratio of specific heats is $\gamma = 5/3$ (adiabatic response).

The coarse mesh sizes are $\Delta x = \Delta z = 0.045$. Thus, with the reference frame established above, the entire box corresponds to $(-36, 36) \times (0, 288)$ in the $xz$-plane. The simulation was performed over 400 normalized Alfvén times. To save disk space, only the most interesting interval $t = 300-400$ has been recorded with a step of 0.5 $\tau_A$.

Note that the initial state described by Equation (3) is not an exact MHD equilibrium. Nevertheless, the resulting field variations are much weaker than those introduced by reconnection.

At the very beginning, in order to trigger reconnection, the system is perturbed by enhanced resistivity localized in a small region surrounding the line $x = 0$, $z = 30$ in the invariant direction $y$ for a short time $0 < t < 10$ (see also Magara et al. 1996). This short perturbation sets up a localized inflow, which somewhat compresses the current layer around the selected point. It should mimic the effect of various irregularities that can be expected during CS stretching in actual solar eruptions (see also Riley et al. 2007). Later, the resistivity is switched on only if the threshold according to Equation (2) is exceeded. As the threshold for anomalous resistivity cannot be reached on a coarse grid (the threshold for mesh refinement is reached earlier than the threshold for anomalous resistivity onset), the condition in Equation (2) is actually checked only at the smallest resolved scale; for the large-scale dynamics, we take $\eta = 0$.

3. ANALYSIS OF RESULTS

We used the above described numerical code in order to study which mechanisms are involved in the transfer of free magnetic energy from large to small scales. Thanks to the adaptive mesh, we were able to cover scales from $4.5 \times 10^{-3} \, L_A$ to $\approx 300 \, L_A$ (the larger dimension of the simulation box), that is, over almost five orders of magnitude.

The early sytem evolution can be briefly described as follows. After the localized initial resistivity pulse, a flow pattern sets up that leads to CS stretching (in the $z$-direction) and compression (in the $x$-direction). Eventually, the condition in Equation (2) for anomalous resistivity is reached at the smallest resolved scale and the first tearing occurs. The dynamics of the plasmoids formed by the tearing process lead to further stretching of the CSs interleaved with the mutually separating plasmoids. This leads to further generation of tearing. Later, after $t \approx 300$, the smallest magnetic structures still consistent with the resolution start appearing. Here, we present an analysis of this more developed stage of cascade. Results are shown in Figures 4 and 5. Figure 4 shows the state of the magnetic field and current density at $t = 316$. For better orientation, auxiliary lines are added indicating the locations where $B_x = 0$ and $B_z = 0$. Their intersections show the positions of O-type and X-type “nulls,” the points where only the guide field remains finite. Areas indicated by blue lines are consecutively zoomed (from left to right panels). The leftmost panel shows the entire simulation box and the rightmost corresponds to a zoom at the limit of the AMR-refined resolution. The figure shows how parts of the current layer are stressed and thinned between separating magnetic islands/plasmoids formed by tearing instabilities. The current-layer filamentation is most pronounced if one compares the zoomed views of the same selected area at the initial state (Figure 3, right panel) and the system state at $t = 316$ (Figure 4, third panel). During the dynamic evolution, the even more thinly stretched current layers become, after some time, unstable to the next level of tearing and even smaller plasmoids are formed. The zoomed figures show that cascading reconnection has formed plasmoids at the smallest resolved scales. The $x$-sizes of the largest and smallest resolved plasmoids in Figure 4 range from $\approx 10$ down to $\approx 0.01$; the $z$-sizes are from $\approx 0.2$ to $\approx 70$.
Plasmoids formed by tearing instability are not only subject to separation but can also approach each other. As a result, the magnetic flux piles up and transverse (i.e., horizontal, perpendicular to the original current layer) CSs are formed between pairs of plasmoids approaching each other. Earlier simulations with lower effective resolution treated plasmoid merging as a coalescence process without any internal structure of the small-scale (sub-grid) CS between the magnetic islands since their thickness was not resolved (Tajima et al. 1987; Kliem et al. 2000; Báta et al. 2008b). If resolved, however, the transverse CS does not just dissipate. Instead, it is subject to tearing instability in the direction perpendicular to the primary current layer. This is shown in Figure 5. The most detailed resolution (rightmost panel) clearly reveals the formation of the O-point at \( x = 0L_A, z = 12.79L_A \) and two adjacent X-points at \( x = 0.37L_A, z = 12.78L_A \) and \( x = -0.37L_A, z = 12.78L_A \). We call this process “fragmenting coalescence” in order to emphasize that even smaller structures are formed during the merging of two plasmoids. Thus, both the tearing and (fragmenting) coalescence processes contribute to the fragmentation of the original thick and smooth current layer.

### 3.1. Fragmentation of CS: Scaling

In order to study the scaling properties of the continued fragmentation of the magnetic structures associated with the current layer, we performed both a one-dimensional Fourier and a wavelet analysis of the magnetic field along the vertical axis \( \{x = 0, y = 0, z \in (0, 288)\} \). We use the \( B_z \) component for this study, since \( B_z = 0 \) there due to the boundary condition. The results are shown in Figure 6. The upper panel shows the magnetic field and current density in the subset of the entire computational domain \( (z \in (0, 200); \text{note the rotated view}) \) where the current layer is fragmented. Panel (b) shows the profile of \( B_z \) along the current-layer axis, and panels (c) and (d) the Fourier and wavelet analyses of this profile. The Fourier power spectrum exhibits power-law scaling with spectral index \( s = -2.14 \) over the rather wide range of scales 300 km–10,000 km. This clearly indicates the cascading nature of the continued fragmentation.

The energy-transfer cascade ends at \( \approx 300 \text{ km} \) in Figure 6. This is closely related to the dissipation threshold \( v_{ci} \) that was chosen to be \( v_{ci} = 15.0 \) in our simulation. By selecting this value, we shifted the dissipation-scale domain into the window of resolved scales; see the discussion in Section 4.

### 3.2. Fragmentation of CS: Diffusion Regions

The CSs are filamented down to the resolution limit of our simulations (in reality, to kinetic scales). The smallest current-density structures contain dissipative/acceleration regions. In the following, we will study the structure, distribution, and
dynamics of these non-ideal regions embedded in the global current layer.

Cascading reconnection and consequent fragmentation of the current layer may also have significant impact on particle acceleration in solar flares. Instead of the single diffusion region assumed in the “classic” picture of solar reconnection, cascading fragmentation causes the formation of large numbers of thin non-ideal channels. The structuring of non-ideal regions in our simulation is depicted in Figure 7. The left panel shows two areas of dissipation around $x = 0, z = 36$ and $x = 0, z = 44$. A closer look (right panel, note the large zoom), however, reveals that the bottom dissipation region is structured and it is in fact formed by two regions of finite magnetic diffusivity that are associated with two X-points at $x = 0, z = 35.40$ and $x = 0, z = 35.97$ interleaved with a (micro) plasmoid. The multiple dissipative regions embedded in the global current layer are favorable for efficient (and possibly multi-step) particle acceleration. At the same time, they provide a natural explanation of fragmented energy release as has been inferred from HXR and radio observations (Aschwanden 2002; Karlický et al. 2000). Since they are embedded in the large-scale current layer, the “classic” well-organized global picture of eruptions is simultaneously preserved.

Figure 7 shows that the X-points formed in the thinned CSs between magnetic islands are connected with the thin channels of magnetic diffusivity. Hence it is appropriate to study the distribution and dynamics of these non-ideal regions by means of tracking the X-points associated with them. We present such analysis in Figure 8.

Figure 7. Filamentation and splitting of diffusive regions at $t = 328$. Detailed treatment shows how the magnetic diffusivity is concentrated into multiple thin channels. The blue-line-bounded areas indicate the diffusion regions where the generalized anomalous resistivity (Equation (2)) is finite. Field lines are dashed to be better distinguished from the resistive region boundaries. The right panel displays a detailed view of the rectangular box in the left. Red and green lines are as in Figure 4.

(A color version of this figure is available in the online journal.)
(panels (b) and (d)) and plasmoid merging (c). As can be seen from panels (b) and (d), X-points can become magnetically connected to (the right X-point in panel (d)) or disconnected from (panel (b)) the bottom boundary during their lifetimes. Note also the splitting (and subsequent merging) of the X-point at \( x = 0, z = 12.8 \) into an X–O–X configuration between \( t \approx 360 \) and \( t \approx 380 \) in panel (a). This process maps the tearing in the transverse (horizontal) CS formed between the interacting plasmoid and the loop arcade (see also Figure 5). Note that Figure 8 can be compared with Figure 5 in Samtaney et al. (2009). The main difference is just the presence of the off-plane X-points formed by the fragmentation of the CS between coalescing plasmoids in our simulation.

4. DISCUSSION

Reconnection in the trailing current layer behind an ejected flux rope (filament) is a key feature of the “standard” CSHKP scenario of solar flares. A large amount of free magnetic energy is accumulated around this rather thick (relative to plasma kinetic scales) and very long layer. The thickness of this layer was estimated from the observed brightening (Ko et al. 2007; Li et al. 2007) and also based on a typical transverse dimension of a filament (Vršnak et al. 2009). In both ways one obtains the order of magnitude of \( \approx 1000 \) km. On the other hand, collisionless reconnection requires dissipation at very small scales, in thin CSs with typical width on the order of \( \approx 10 \) m in the solar corona (Büchner 2007). The fundamental question arises of how the accumulated energy is transferred from large to small scales or, in other words, what the mechanisms of direct energy cascade in magnetic reconnection are. We addressed this question using high-resolution AMR simulation covering a broad range of scales to investigate the MHD dynamics of an expanding current layer in the solar corona.

4.1. Mechanisms of Direct Energy Cascade

Our simulations reveal the importance of continued fragmentation of the current layer due to the interaction of two basic processes: the tearing instability of stretched CSs and the fragmenting coalescence of flux ropes/plasmoids formed by the tearing and subsequently forced to merge by the tension of the ambient magnetic field. After ejection of the primary flux rope (i.e., the filament/CME), a trailing current layer is formed behind it, which becomes long and thins out. As has been pointed out through theoretical analysis by Loureiro et al. (2007), current layers with high enough length-to-width ratios become unstable to fast plasmoid instability. Moreover, any irregularity in the plasma inflow that stretches the sheet facilitates the tearing (Lazarian & Vishniac 1999).

Plasmoids that form are subjected to the tension of the ambient magnetic field, which causes them to move (Bárta et al. 2008b). The motion can lead to their increasing separation. A secondary current layer then formed between them becomes, in turn, stretched and a secondary tearing instability can take place. This simulation result, illustrated in Figure 4, fully confirms the scenario suggested by Shibata & Tanuma (2001), developed further by Loureiro et al. (2007) and Uzdensky et al. (2010) into the analytical theory of chain plasmoid instability. The results are also in qualitative agreement with the simulations of plasmoid instability by Samtaney et al. (2009) and Bhattacharjee et al. (2009), which were performed, however, with constant resistivity.

In addition to that, our simulation has shown that the converging motion of plasmoids leads to a magnetic-flux pileup between mutually approaching plasmoids. Consequently, secondary (oppositely directed) CSs are formed perpendicular to the original current layer. While previous studies found only unstructured current density pileups between merging magnetic islands, our enhanced (by AMR) resolution reveals secondary tearing-mode instabilities that take place in the direction transverse to that of the primary CS (see Figure 5). This process represents a new mechanism of fragmentation and changes our view of coalescence instability, which has been hitherto commonly considered to be a simple merging process of two plasmoids, contributing to the inverse energy cascade only. Note that this behavior is different from that seen for plasmoids at the dissipation scales.
in PIC simulations (Drake et al. 2005; Karlický & Bártá 2007), where plasmoids merge without subsequent tearing.

One can suppose that with even higher spatial resolution one would see more subsequent tearing-mode instabilities alternating with the fragmenting coalescence of the resulting magnetic islands/flux ropes. As a result, third- and higher-order CSs could form.

To sum up, the results of our simulation support the idea that both the tearing (Shibata & Tanuma 2001; Loureiro et al. 2007; Uzdensky et al. 2010) and “fragmenting coalescence” processes lead to the formation of consecutively smaller magnetic structures (plasmoids/flux ropes) and associated current filaments. Subsequent stretching and compression cause filamentation of the current. This situation is schematically depicted in Figure 9, which can be seen as a generalization of the scheme in Figure 6 in Shibata & Tanuma (2001). One can expect that this cascade will continue down to the scales at which the magnetic energy is finally dissipated. Note that the physics and the corresponding scaling laws may change at intermediate (but still relatively small) scales when additional contributions to the generalized Ohm’s law become significant (e.g., a Hall term; see recent simulations by Shepherd & Cassak 2010 and Huang et al. 2010).

4.2. Impact on Reconnection Efficiency

Reconnection in the CS between merging plasmoids is fast, since it is driven by ambient-field magnetic tension that naturally pushes the flux ropes together. Thus, even shorter timescales can be reached by this process than by tearing cascade in the stretched CS (Shibata & Tanuma 2001). Yet another point makes the overall reconnection process more efficient. Many magnetic-flux elements, except those ejected outward to the escaping CME, reconnect several times: first, during the primary tearing and plasmoid formation, and then again during plasmoid coalescence. Since coalescence leads to a follow-up tearing instability (in the transverse direction), the remaining magnetic flux is subjected to another act of magnetic reconnection. This process resembles the recurrent separator reconnection simulated by Parnell et al. (2008).

4.3. Relation to Turbulence Onset

To some extent the initial situation of global, smooth, and relatively thick sheets is similar to that of turbulence onset in a sheared flow in (incompressible) fluid dynamics (FD), as schematically shown in Figure 10. Usually, the typical length scale of shear flows (the counterpart of the width of current layers) is much larger than the dissipative (molecular) scale. The mechanism of energy transfer from large to small scales in classical FD is mediated by a cascade of vortex tubes. Large-scale vortices formed by shear flows can mutually interact, giving rise to increased velocity shear at the smaller scales in the space between them. Each small shear flow element formed by this process can be, itself, subjected to this fragmentation. Based on our simulation results, we suggest a similar scenario for current-layer fragmentation. The role of the vortex tubes in FD is, in MHD, taken over by flux ropes/plasmoids.

By analogy with the onset of turbulence in sheared flows, one could expect that a dynamical balance would arise between fragmentation and coalescence processes in a later more developed stage. This should manifest in a power-law scaling rule. Using AMR, we reached a rather broad (five orders of magnitude) range of scales. This allowed us to perform a one-dimensional scaling analysis of the magnetic-field structures formed along the current layer for the first time. The scaling rule found does indeed exhibit a power-law distribution with index $s = -2.14$ (Figure 6). Since our resolution still does not allow us to make this scaling analysis exclusively within the small selected subdomain around the CS center where one could expect isotropic “turbulence” (we lack a sufficient range of scales for that), it is difficult to compare the spectral index found over the whole (clearly anisotropic) simulation domain with the values expected from the theory of fully developed isotropic turbulence.

The obtained power-law distribution is also in qualitative agreement with the concept of fractal reconnection of Shibata & Tanuma (2001) and with the hierarchical analytical model of plasmoid instability (Loureiro et al. 2007) as described by Uzdensky et al. (2010). They use distribution functions for plasmoid width and contained flux in order to characterize the statistical properties of the plasmoid hierarchy rather than power spectra. We plan to perform similar analysis of our results in a future study in order to also compare the results quantitatively.

In order to obtain as broad a scale range as possible in the plane where reconnection occurs, we performed these simulations using the 2.5D approach. The question arises as to what extent a full three-dimensional treatment would change the resulting picture. In the FD cascade, vortices are deformed and their cross sections change along their main...
axis, even in the topological sense. The object defined as a single vortex tube in one place can be split into two in another location. One can expect a similar behavior of plasmoids/flux tubes in MHD. There, they could be subject to kink and similar instabilities with \( k_x > 0 \). Such processes would naturally lead to modulation of the reconnection rate along the PIL. Observations indicating such effects have already been presented (McKenzie & Savage 2009). To some extent, the expected behavior can also be obtained for kink instabilities of tiny current channels at the dissipative scale studied by three-dimensional PIC simulations (Zhu & Winglee 1996; Karlický & Báta 2008).

Nevertheless, Edmondson et al. (2010) found no evidence of such in their three-dimensional AMR MHD simulation, which used, however, a different setup. The sizes of the plasmoids formed under three-dimensional perturbation in the direction that corresponds to the invariant y-axis in our 2.5D case have been found to be very short, preventing kink-like instabilities from developing. On the other hand, the resulting plasmoid lengths might depend on the initial guide field (Edmondson et al. 2010; Dahlburg et al. 2005).

To sum up, the answer to this question can be found only via full three-dimensional simulations with initial setups similar to the one we used here. Therefore, we plan to extend our current 2.5D simulations with very high in-plane resolution and moderately resolved structuring in the third dimension.

### 4.4. Fragmented Energy Release and Particle Acceleration

Cascading fragmentation of the current layer is closely related to another puzzling question of current solar-flare research, the apparent contradiction between observed regular large-scale dynamics and signatures of fragmented energy release in (eruptive) flares. This duality is reflected by two classes of flare models: the “classic” CSHKP scenario based on magnetic reconnection in a single global flare CS and the class of SOC models based on the avalanche of small-scale reconnection events in multiple CSs formed as a consequence of either chaotic (Aschwanden 2002; Vlahos 2007) or regular but still complex boundary motions causing, for example, magnetic braiding (Wilmot-Smith et al. 2010).

The model of cascading reconnection has the potential to provide a unified view of these seemingly very different (see the discussion in the next paragraph) approaches. From the global point of view, it coincides with the classic CSHKP model keeping the regular picture of the process at large scales. At the same time, due to the internal current-layer fragmentation, the tearing/coalescence cascade forms multiple small-scale CSs and potential diffusive regions. As a consequence, fragmented energy release (e.g., by particle acceleration) can take place in these tiny regions. To some extent, this finding can be seen as a follow-up to the “bursty” reconnection regime found by Kliem et al. (2000). Those authors show that intermittent signals (X-ray, radio) can be related to chaotic pulses of the (resistive) electric field in the dissipation regions around X-points. The pulsed regime is a consequence of nonlinear interplay between the governing MHD equations and the anomalous resistivity model. In our view, however, these pulses result from the sub-grid physics unresolved in earlier simulations. Essentially, what has been seen as a single dissipative region around a single X-point in the coarse-grid models is in fact a ("fractal-like") set of non-ideal areas around multiple X-points (see Figure 7) that are interleaved with very-small-scale mutually interacting plasmoids. This view based on high-resolution simulation is perhaps more closely aligned with the term “fragmented energy release,” which assumes that energy dissipation is accomplished via many concurrent small-scale events appearing in multiple sites distributed in space. In this context, it is interesting to note how surprisingly well the phenomenological resistivity model (Equation (2)) used by Kliem et al. (2000) mimics the sub-grid scale physics, as it is able to qualitatively reproduce the temporal behavior of the resistive electric field even without resolving the actual processes that are responsible for it. Also note that a possible role for tearing and coalescence in the fragmentation of the energy release in solar flares has already been mentioned by Kliem (1990).

We would like to emphasize that there is a fundamental difference between the fragmented energy release by cascading reconnection and SOC models. It is rooted in the fact that in cascading reconnection the complexity/chaoticity is due to intrinsic current-layer dynamics (i.e., due to spontaneous fragmentation), while it is introduced in SOC models through (external) boundary conditions (chaotic boundary motions). In fact, these two concepts are opposed to some extent. While in SOC-based models the global flare picture is built as an avalanche of many small-scale events (bottom→top process in the scale hierarchy), in cascading reconnection small-scale structures are formed as a consequence of the internal dynamics of large-scale CSs (a top→bottom process).

Fragmented energy release is closely related to the problem of the number of particles accelerated in solar flares. The single diffusion region assumed in the CSHKP model provides far too small a volume for accelerating such strong fluxes of particles as are inferred from HXR observations. This argument has been used in favor of SOC-based models as they provide energetic-particle spectra and time-profile distributions as observed and explain large energetic particle fluxes.

We suggest that, however, the inclusion of cascading reconnection into the CSHKP model provides even more capabilities than SOC models. It could explain both the distribution and the number of accelerated particles based on a physical consideration of the many small-scale CSs that can host tiny diffusive channels that can all act as acceleration regions (see Figure 7).

Here it is appropriate to make one technical comment: in an MHD simulation with the resistivity model described by Equation (2) or its dimensionless version, the size and number of diffusive regions are controlled mostly by the threshold \( v_{cr} \) for the onset of (anomalous) diffusivity. The higher the chosen \( v_{cr} \) is, the thinner the CSs can become, and the smaller and more numerous the embedded diffusion regions are. Since one has to resolve these diffusive regions in the simulation, one has to choose the threshold \( v_{cr} \) low enough to be able to resolve the dissipation regions appropriately. In ideally resolved simulations, covering all scales down to the real physical dissipation length, the chosen critical velocity \( v_{cr} \) could be on the order of the physically relevant value, the electron thermal speed \( v_{th} \) (Büchner 2007). In dimensionless units, this corresponds to

\[
v_{cr} = L_A/d \sqrt{m_e/m_i} \sqrt{\beta/2},
\]

where \( m_i \) and \( m_e \) are the proton and electron masses, respectively. For a simulation with technically limited spatial resolution, one has to choose a (much) smaller value of \( v_{cr} \) in order to resolve the smallest possible CSs, before dissipation sets in, with a reasonable number of grid points. Since the resolution in our current AMR simulation is higher than in earlier models, we could choose a more realistic value of \( v_{cr} = 15.0 \) (while older simulations used \( v_{cr} = 3.0 \); see, e.g., Kliem et al. 2000; Báta et al. 2008a). This allowed us to track more fragmented, smaller reconnection regions.
If we extrapolate this trend, we can expect that with even higher resolution one would find even more and tinier diffusive regions. They would be grouped hierarchically (self-similarly), occupying a sub-space of the global current layer. This kind of distribution is indicated in the wavelet spectra (white islands in Figure 6(d)), and also by the positions and motion of the associated X-points in Figure 8. The latter shows a structured grouping of “null points” and their various lifetimes.

5. CONCLUSIONS

Our simulation has shown that cascading reconnection due to the formation and fragmenting coalescence of plasmoids/flux ropes is a viable physical model of fragmented magnetic energy release in large-scale systems, such as solar flares. Cascading reconnection addresses three key problems in current solar-flare research at once: the gap between energy accumulation and dissipation scales, the duality between the regular global-scale dynamics and the fragmented energy-release signatures simultaneously observed in solar flares, and the issue of particle acceleration. All these problems arising from observations appear to be closely related via cascading reconnection.

In order to further evaluate the relevance of cascading reconnection for actual solar flares, it is desirable, however, to identify and predict model-specific observables and to search for them in observed data. Possible specific signatures and their comparison with observations are presented in Bártá et al. (2011).

This research was performed under the support of the European Commission through the SOLAIRE Network (MTRN-CT-2006-035484) and grant P209/10/1680 of the Grant Agency of the Czech Republic, grant 30030701 of the Grant Agency of the Czech Academy of Science, and research project AV0Z10030501 of the Astronomical Institute of the Czech Academy of Science. The authors thank Dr. Antonius Otto for inspirational discussions and the anonymous referee for valuable comments that helped to improve the quality of the paper.

REFERENCES

Aschwanden, M. J. 2002, Space Sci. Rev., 101, 1
Bártá, M., Büchner, J., & Karlický, M. 2010, Adv. Space Res., 45, 10
Bártá, M., Büchner, J., Karlický, M., & Kotrč, P. 2011, ApJ, 730, 47
Bártá, M., & Karlický, M. 2001, A&A, 379, 1045
Bártá, M., Karlický, M., & Žemlička, R. 2008a, Sol. Phys., 253, 173
Bártá, M., Vršnak, B., & Karlický, M. 2008b, A&A, 477, 649
Berger, M. J., & Oliker, J. 1984, J. Comput. Phys., 53, 484
Bhattacharjee, A., Huang, Y., Yang, H., & Rogers, B. 2009, Phys. Plasmas, 16, 112102
Büchner, J. 2006, Space Sci. Rev., 124, 345
Büchner, J. 2007, Plasma Phys. Control. Fusion, 49, 325
Büchner, J., & Eltina, N. 2005, Space Sci. Rev., 121, 237
Büchner, J., & Eltina, N. 2006, Phys. Plasmas, 13, 082304.1
Chung, T. J. 2002, Computational Fluid Dynamics (Cambridge: Cambridge Univ. Press)
Dahlburg, R. B., Klimchuk, J. A., & Antiochos, S. K. 2005, ApJ, 622, 1191
Drake, J. F., Shay, M. A., Thonhaui, W., & Swisdak, M. 2005, Phys. Rev. Lett., 94, 095001.1
Dreher, J., & Grauer, R. 2005, Parallel Comput., 31, 913
Edmondson, J. K., Antiochos, S. K., DeVore, C. R., & Zurbuchen, T. H. 2010, ApJ, 718, 72
Fletcher, L. 2005, Space Sci. Rev., 121, 141
Huang, Y., & Bhattacharjee, A. 2010, Phys. Plasmas, 17, 062104
Huang, Y.-M., Bhattacharjee, A., & Sullivan, B. P. 2010, arXiv:1010.5284
Karlický, M., & Bártá, M. 2007, A&A, 464, 735
Karlický, M., & Bártá, M. 2008, Sol. Phys., 247, 335
Karlický, M., Bártá, M., & Rybáek, J. 2010, A&A, 514, A28
Karlický, M., Jiřička, K., & Sobotka, M. 2000, Sol. Phys., 195, 165
Karlický, M., Sobotka, M., & Jiřička, K. 1996, Sol. Phys., 168, 375
Kliem, B. 1990, Astron. Nachr., 311, 399
Kliem, B., Karlický, M., & Benz, A. O. 2000, A&A, 360, 715
Kliem, B., Linton, M. G., Török, T., & Karlický, M. 2010, Sol. Phys., 266, 91
Kliem, B., & Török, T. 2006, Phys. Rev. Lett., 96, 255002
Ko, Y., Raymond, J. C., Lin, J., Lawrence, G., Li, J., & Fludra, A. 2003, ApJ, 594, 1068
Krucker, S., et al. 2008, A&ARv, 16, 155
Lazarian, A., & Vishniac, E. T. 1999, ApJ, 517, 700
Lin, J., & Forbes, T. G. 2000, J. Geophys. Res., 105, 2375
Loureiro, N. F., Schekochihin, A. A., & Cowley, S. C. 2007, Phys. Plasmas, 14, 100703
Magara, T., Mineshige, S., Yokoyama, T., & Shibata, K. 1996, ApJ, 466, 1054
Mandrini, C. H. 2010, in IAUP Symp. 264, Magnetic Energy Release: Flares and Coronal Mass Ejections, ed. A. G. Kosovichev, A. H. Andrei, & J.-P. Roelot (Cambridge: Cambridge Univ. Press), 257
McKenzie, D. E., & Savage, S. L. 2009, ApJ, 697, 1569
Ni, L., Germaschewski, K., Huang, Y., Sullivan, B. P., Yang, H., & Bhattacharjee, A. 2010, Phys. Plasmas, 17, 052109
Parnell, C. E., Haynes, A. L., & Galsgaard, K. 2008, ApJ, 675, 1656
Priest, E. R. 1984, Solar Magnetohydrodynamics (Geophysics and Astrophysics Monographs; Dordrecht: Reidel)
Riley, P., Lionello, R., Mikic, Z., Linker, J., Clark, E., Lin, J., & Ko, Y. 2007, ApJ, 655, 591
Sanctaney, R., Loureiro, N. F., Uzdensky, D. A., Schekochihin, A. A., & Cowley, S. C. 2009, Phys. Rev. Lett., 103, 105004
Shepherd, L. S., & Cassak, P. A. 2010, Phys. Rev. Lett., 105, 015004
Shibata, K., & Tanuma, S. 2001, Earth Planets Space, 53, 473
Tajima, T., Sakai, J., Nakajima, H., Kosugi, T., Brunel, F., & Kundu, M. R. 1987, ApJ, 321, 1031
Török, T., & Kliem, B. 2005, ApJ, 630, L97
Uzdensky, D. A., Loureiro, N. F., & Schekochihin, A. A. 2010, Phys. Rev. Lett., 105, 235002
van der Holst, B., & Keppens, R. 2007, J. Comput. Phys., 226, 925
Vlahos, L. 2007, in Magnetic Complexity, Fragmentation, Particle Acceleration, and Radio Emission from the Sun, ed. K.-L. Klein & A. L. MacKinnon (Lecture Notes in Physics, Vol. 725; Berlin: Springer), 15
Vršnak, B., et al. 2009, A&A, 499, 905
Williams, D. R., Török, T., Demoulin, P., van Driel-Gesztelyi, L., & Kliem, B. 2005, ApJ, 628, L163
Williams-Smith, A. L., Pontin, D. I., & Hornig, G. 2010, A&A, 516, A5
Zhu, Z., & Winglee, R. M. 1996, J. Geophys. Res., 101, 4885