Evidence for the flavor singlet axial anomaly related effects in $\phi$ meson electromagnetic production at large momentum transfers

N.I. Kochelev$^{1,2,3}$ and V. Vento$^1$

$^1$ Departament de Física Teòrica and Institut de Física Corpuscular, Universitat de València-CSIC, E-46100 Burjassot (Valencia), Spain
$^2$ BLTP, Joint Institute for Nuclear Research, Dubna, Moscow region, 141980 Russia
$^3$ Institute of Physics and Technology, Almaty, 480082, Kazakhstan

Abstract

The gluonic contributions to the conventional PCAC formulas due to flavor singlet axial anomaly have been instrumental in explaining the mass of the $\eta'$ and providing a plausible explanation for solving the spin crisis. We show that they also play an important role in the description of photo- and electroproduction of vector mesons at low energy and high momentum transfers. We calculate the contributions of this type to $\phi$ meson electromagnetic production in a model, which contains also a soft pomeron, and find agreement with recent CLAS data.

Pacs: 12.40.Nn, 13.60.Le,13.88.+e
Keywords: axial anomaly, vector mesons, gluon, large momentum transfer.

kochelev@thsun1.jinr.ru
vicente.vento@uv.es
1 Introduction

The CLAS data on $\phi$ meson photo- and electroproduction \cite{1, 2} at large momentum transfers are widely under discussion \cite{3, 4, 5}. The main interest lies in unveiling the structure of the nucleon, in particular its strange quark content \cite{6, 7}. The relation between the structure of the nucleon, the so called proton spin problem, and the gluonic contributions related to the flavor singlet axial anomaly, hereafter abbreviated by FSAA, have been understood for some time \cite{6}. The latter also furnishes an explanation for OZI rule violations, describing why decays of heavy resonances that involve disconnected quark graphs, i.e. graphs that can only be connected via gluon lines, are suppressed. Thus it is to be expected that the FSAA mechanism be manifest in those channels of vector meson electromagnetic production where gluonic degrees of freedom are important.

The FSAA arises as a consequence of the complicated vacuum structure of QCD. The breaking of the $U_A(1)$ symmetry in the theory requires a complex mechanism, associated with the periodicity of the QCD potential as a function of the topological charge, producing a massless pole in the correlator of two anomalous currents \cite{8}. The pole is not associated to any physical particle because the currents are gauge dependent. It was first introduced by Veneziano \cite{9} to explain the large $\eta'$ mass in the framework developed by Witten \cite{10}.

The role of the FSAA in understanding the structure of the nucleon has been pointed out by Shore and Veneziano \cite{11}. We consider its contribution to $\phi$ meson elastic electromagnetic production off the nucleon, which is a suitable scenario to investigate the OZI rule. In $\phi$ meson production the contribution of the valence quark exchange is much smaller than in $\rho$ and $\omega$ meson production, therefore this reaction gives the opportunity to investigate the role of the gluonic exchanges in vector meson production.

2 The dynamics associated with the FSAA mechanism

The $U_A(1)$ symmetry of QCD is anomalous. The anomaly is reflected in the divergence of the current by the appearance of a term proportional to the topological charge density operator
\[ Q(x) = \frac{\alpha_s}{8\pi} G^a_{\mu\nu}(x) G^a_{\mu\nu}(x). \]  

The axial singlet Ward identities are anomalous and the mechanism which provides the large mass to the \( \eta' \) is the non vanishing of the zero momentum correlator of two topological charges. As a consequence the anomalous gluonic current

\[ K_{\mu} = \frac{\alpha_s}{4\pi} \epsilon_{\mu\nu\alpha\beta} A^a_{\nu}(\partial_{\alpha} A^a_{\beta} + \frac{g_s}{3} f_{abc} A^b_{\alpha} A^c_{\beta}) \]  

has a massless pole

\[ i \int d^4 x e^{ikx} < 0|TK_{\mu}(x)K_{\nu}(0)|0 >_{k \to 0} \to \frac{g_{\mu\nu}}{k^2} \lambda^4 \]  

which is related to the topological susceptibility of the QCD vacuum

\[ \chi(0) = -\lambda^4 = i \int d^4 x < 0|TQ(x)Q(0)|0 >. \]  

The matrix elements between the \( Q \)-state and the pseudoscalar isosinglet quark states

\[ \phi^i_5 = \bar{q}_i \gamma 5 q_i, \]  

where \( i = u, d, s \), are non zero. The \( Q \)-operator allows transitions between states with different quark flavors and provides the mechanism for the observed large OZI violation in \( \eta - \eta' \) system.

The diagonalization of the propagator matrix for the operator \( Q \) and quark nonet states \( \pi^0, \eta^8 \) and \( \eta' \) leads to the following gauge invariant propagator

\[ < GG > = -A, \]  

where \( G \) is the linear combination of \( Q \) and \( \phi^i_5 \) operators. For the physical mesons the propagator is given by

\[ < \eta^\alpha \eta^\beta > = \frac{-\delta^{\alpha\beta}}{k^2 - m_{\eta^\alpha}^2}, \]  

where \( \eta^\alpha = \eta', \eta, \pi^0 \).

\( A \) in \( (6) \) is determined from the topological susceptibility by the equation

\[ \chi(0) = -A(1 - A \sum_q \frac{1}{m_q < \bar{q}q >})^{-1}. \]
We approximate $A$ by the susceptibility of the Yang Mills theory, $\chi(0)|_{YM}$, and take for the latter the value given by the Veneziano-Witten formula formula\(^{1}\) \[10, 9\]

$$\chi(0)|_{YM} = -\frac{f^2}{6}(m^2_{\eta'} + m^2_\eta - 2m^2_K) \approx (180 \text{MeV})^4.$$ \hspace{1cm} (9)

We should stress that the $G$-propagator (6) does not depend on the transfer momentum $k^2$, therefore the effective interaction induced by the $G$-exchange is a point-like interaction. Below we shall show that this property is responsible for the large $G$-pole contribution to the electromagnetic $\phi$ meson production at large momentum transfers.

The contribution of the $G$-pole to the physical amplitudes leads to modifications on the predictions by the OZI rule. An example where this mechanism is at work, of interest here, is the generalized $U(1)$ Golberger-Treiman relation for isosinglet axial-vector nucleon form factor at zero momentum transfer \[9\]

$$2M_NG^0_A = F g_{N\pi} + 2N_f A g_{GNN} = F_{m} g_{\eta_0 N N},$$ \hspace{1cm} (10)

where $F \approx \sqrt{2N_f f_\pi}$, $f_\pi = 93$ MeV. The $G$-nucleon coupling constant $g_{GNN}$ has been defined by the interaction

$$L_{QNN} = ig_{GNN} G\bar{N} \gamma_5 N.$$ \hspace{1cm} (11)

One can interpret the formula (10) as establishing a relation between the coupling constant of the physical $\eta'$ meson and that of the nonphysical $\eta_0$ meson in OZIQCD. The $G$-pole contribution in (10) just describes the OZI violating piece of the coupling due to intermediate gluonic states (see Fig.1).

The contribution of the $G$-pole leads to a significant reduction of the flavor singlet axial charge from naive OZI expectation of $G^0_A \approx G^0_A \approx 0.6$ and provides an explanation of the famous spin crisis (see reviews \[3, 4\]). The contribution should be negative and its effect in (10) is to make $G_0^A$ reach its experimental value of $\approx 0.3$. This allows us to estimate the value of $G$-nucleon coupling constant, for $N_f = 3$, as

---

\(^{1}\)The lattice calculations of the topological susceptibility \[13\] get $\chi(0) = -(175 \pm 5$ MeV$)^4$, confirming this value.
Figure 1: The contributions to the $\eta' - NN$ interaction a) due to OZI conserving piece and b) the gluonic $G$-pole contribution.

$$g_{GNN} \approx -\frac{0.3M_N}{N_f A} \approx -89.35\text{GeV}^{-3}. \quad (12)$$

The $G$-pole contribution to the $\eta'(\eta) \rightarrow \gamma\gamma$ decay has been discussed by Shore [12]. Defining the interaction vertex as

$$<\gamma\gamma|\eta'> = -i g_{\gamma'\gamma\gamma}\epsilon_{\mu\nu\alpha\beta} k_1^\mu k_2^\nu \epsilon^\alpha(k_1) \epsilon^\beta(k_2), \quad (13)$$

a modified formula for the effective coupling of the $\eta'$ meson with photons has been obtained,

$$F g_{\gamma'\gamma\gamma} + 2N_f Ag_{G\gamma\gamma} = \frac{4}{\pi} \alpha_{em}. \quad (14)$$

One can understand this relation in a similar way as for the effective $\eta'$-nucleon coupling. There are two ways for the coupling to take place: i) an OZI preserving one; ii) another which incorporates OZI violating terms determined via the $G$-pole (Fig.2).

Let us consider the $G$-pole contribution to the $\eta'\gamma\phi$ coupling [13]. The interaction of the $\phi$ meson with the $s$ quarks is assumed to be photon-like

$$L_{\phi ss} = C_\phi \bar{s} \gamma_\mu s \phi_\mu. \quad (15)$$

Due to this vector meson-photon analogy the generalization of (14) to the case of $\eta' \rightarrow \gamma\phi$ decay is straightforward (Fig.3)

$^2$ Similar considerations can be perform for $\rho$ and $\omega$ mesons.
Figure 2: The contributions to the $\eta' - \gamma\gamma$ decay: a) OZI conserving part and b) $G$-pole contribution.

\[
F g_{\eta'\gamma\phi} + 2N_f A g_{G\gamma\phi} = -C_\phi \sqrt{\frac{\alpha_{em}}{\pi^3}}. \tag{16}
\]

The value of $g_{\eta'\gamma\phi}$ can be extracted from experimental width $\Gamma_{\phi \to \eta'\gamma} = 2.99 \times 10^{-4}$ MeV \cite{16}. Using the calculated decay width

\[
\Gamma_{\phi \to \eta'\gamma} = \frac{(M_\phi^2 - M_{\eta'}^2)^3}{96\pi M_\phi^3} |g_{\eta'\gamma\phi}|^2, \tag{17}
\]

we obtain

\[
|g_{\eta'\gamma\phi}| = 0.233 \text{GeV}^{-1}. \tag{18}
\]

Unfortunately the $\phi$-meson coupling with the strange quarks is not well known. In order to estimate it we use three scenarios. The naive quark model (NQM) calculation provides us with

\[
C_\phi = g_{\phi KK} \approx \frac{g_{K^+K^-} + g_{K_LK_S}}{2} = 4.53, \tag{19}
\]

where the values of the $\phi - KK$ coupling from \cite{14} was used. The vector meson dominance (VMD) model at the quark level gives

\[
C_\phi = \frac{g_\phi}{3} \approx 4.37, \tag{20}
\]

where $g_\phi = 13.1$ is determined from electronic width of $\phi$ meson. The last estimate comes from the Nambu-Jona-Lasinio (NJL) model \cite{15}

\[
C_\phi = 5.33. \tag{21}
\]
Figure 3: The contributions to the $\eta' - \gamma \phi$ vertex a) OZI conserving part and b) $G$-pole contribution.

Our procedure does not determine the sign of the some of coupling constants. However in the large $N_c$ limit of the theory, the sign of the first term in (16), should be the same as that of the OZIQCD term in the right hand side of the equation. Therefore we can write,

$$|g_{G\gamma\phi}| = \frac{|F g_{\eta'\gamma\phi}| - |C_{\phi}\sqrt{\alpha_{em}}/\pi^2|}{2N_f A}$$

which gives $|g_{G\gamma\phi}^{NQM}| = 2.88 \text{ GeV}^{-4}$, $|g_{G\gamma\phi}^{VDM}| = 3.08 \text{ GeV}^{-4}$ and $|g_{G\gamma\phi}^{NJL}| = 1.91 \text{ GeV}^{-4}$.

3 $G$-pole contribution to the $\phi$ meson photo- and electroproduction

It is very well known that at large energy and small momentum transfer the main contribution to the elastic electromagnetic production of the vector mesons comes from the pomeron exchange (see [17] and references therein). At low energies the pomeron still gives the main contribution to the $\phi$ production cross section since OZIQCD is valid for reactions at small momentum transfers [3, 4, 5] (Fig.4a). We use the Donnachie-Landshoff (DL) model [18] to describe the soft pomeron contribution to the elastic $\phi$ meson production differential cross section. This model is based on two gluon exchange contributions to quark-quark scattering [19].
In the DL model the contribution of the pomeron to the elastic cross section is given by

$$\frac{d\sigma_P}{dt} = \frac{81m_0^3\beta^2_0/\alpha'_s e^+e^-}{\pi\alpha_{em}} \frac{F(t)^2}{(2\mu_0^2 + Q^2 + m_0^2 - t)^2(Q^2 + m_0^2 - t)^2} \left( \frac{S}{S_0} \right)^{2\alpha_P(t)^{-2}},$$

(23)

where

$$F(t) = \frac{4M_K^2 - 2.8t}{(4M_K^2 - t)(1 - t/0.7)^2}$$

(24)

is the electromagnetic nucleon form factor, and $\beta_0 = 2$ GeV$^{-1}$, $\beta_s = 1.5$ GeV$^{-1}$, $\mu_0 = 1.2$ GeV, $S_0 = 1/\alpha'_P$ and pomeron trajectory is $\alpha_P(t) = \alpha_P(0) + \alpha'_P t$, with $\alpha_P(0) = 1.08$ and $\alpha'_P = 0.25$ GeV$^{-1}$, $S = W^2$. The contribution of the DL pomeron to the cross section for the $\phi$ meson photo- and electroproduction for CLAS kinematics is presented in Figs. 5 and 6 by dashed lines.

For small $-t \leq 1$ GeV$^2$ the pomeron contribution describes the data rather well, but for large $-t \geq 1$ GeV$^2$, the deviation from the experiment is large. Furthermore, the electroproduction data show some evidence for a dip near $-t \approx 1$ GeV$^2$, which is impossible to obtain within the DL model. There are some attempts to describe large $t$ CLAS photoproduction data by the $u-$channel contributions [3, 4]. But in this case one should expect a large suppression of this contributions in $\phi$-meson electroproduction due to the strong $Q^2$ dependence of the form factors. This suppression is not seen in the CLAS data (Fig.6).

The main idea behind our approach to the large $-t$ behavior of the cross

---

Figure 4: The contributions of the a) pomeron b) G-pole exchange to the electromagnetic $\phi$-meson production.
The contributions of the pomeron (dashed line) and G-pole exchange (dot-dashed line) with NQM coupling to the φ-meson photoproduction at $W = 2.76$ GeV. Solid lines are the total contribution for the different $φ−s$ quark couplings. The data are from the CLAS collaboration [1].

section is that the G-pole exchange mechanism will soften the decrease of the cross section from the pomeron. The reason is very simple, the G propagator \( (6) \) does not have any $t$ dependence \([6]\). From the analysis of the nucleon spin dependent structure function $g_1(x, Q^2)$ we know that the G-pole contribution, which can be considered as the gluon contribution to structure function, has only weak (logarithmic) $Q^2$ dependence \([4]\). Therefore the G-pole exchange should give the flattening at large $-t$ of the cross section in both photo- and electroproduction. The absolute value of the G-pole contribution (Fig.4b) strongly depends on its couplings with nucleon, photon and φ-meson and is given by \([4]\)

$$\frac{dσ_G}{dt} = -\frac{A^2g_{Gγφ}^2g_{GN}^2t(t-M_φ^2)^2}{64\pi((S+Q^2+M_N^2)^2-4SM_N^2)}F_1(t)^2,$$

\(^3\)We neglect the possible weak $Q^2$ dependence of the G-pole contribution to the matrix element of the reaction.
Figure 6: The contribution of the pomeron and G-pole exchange to the \( \phi \)-meson electroproduction for \( < Q^2 > = 1.45 \) GeV\(^2\) and \( W = 2.3 \) GeV. The data are from the CLAS collaboration \[2\]. The notation is the same as in Fig.5.

where \( F_1(t) \) is the flavor singlet axial form factor of the nucleon which we take of the form (see \[20\])

\[
F_1(t) = \frac{1}{(1 - t/M_{f_1}^2)^2}. \tag{26}
\]

In \[20\] \( M_{f_1} = 1.285 \) GeV is the mass of the flavor singlet \( f_1 \) meson.

It should be noted that the G-pole exchange induces a nucleon spin-flip, which produces a factor \( t \) in Eq.(25), and leads to an additional enhancement of the G contribution at large \( -t \), as compared with pomeron contribution, which is nonspin-flip \[23\]. Moreover the energy dependence of the G contribution corresponds to a fix pole with zero Regge slope. Therefore the large \( t \) Regge suppression to vector meson production given by \( (S/M_N^2)^{2\alpha_R^t} \) with slope \( \alpha_R' \approx 0.9 \) GeV\(^{-2}\) for the usual Regge trajectories, e.g. \( \pi^0 \) and \( \eta \), is absent for the G-pole exchange.
The result of the calculation of the $G$ contribution with the NQM $\phi$—strange quark coupling is presented in Figs. 5 and 6 by the dot-dashed line. In them the sum of the pomeron and $G$ contributions within all models for the couplings are shown by the solid lines. It is evident that $G$ contribution determines the behavior of the cross section at large $-t$. The agreement with the data can be, of course, improved by the inclusion of another meson exchanges, for example $\eta$, $\pi^0$ and $f_1$, but already our simple model based only on pomeron and $G$-pole contributions reproduces the main features of the CLAS data.

4 Conclusion

We have shown that gluonic degrees of freedom play a very important role in the electromagnetic $\phi$-meson production at small energy and large $-t$. At small $-t$ the cross-section is described rather well by the Donnachie-Landshoff soft pomeron model. At large $-t$, both in the photo- and electro-production cross sections, an extremely interesting phenomena related to the complex topological structure of QCD vacuum, takes place. We have shown that at large momentum transfer, the point-like interaction induced by the axial anomaly gives the dominant contribution. It should be mentioned that a similar phenomena should appear also in $\rho$, $\omega$ and $J/\Psi$ production at low energy and large momentum transfer, although it might be masked by other mechanisms [21]. We expect, though, that the sum of pomeron and $G$-pole contributions to $J/\Psi$ photoproduction might be dominant near threshold, because here the minimal momentum transfer is large, $-t_{\text{min}} = 2.2 \text{ GeV}^2$. This mechanism might explain the unusual flat energy dependence of the $J/\Psi$ cross section observed at Cornell [24] for $E_\gamma \leq 12 \text{ GeV}$.

The $G$ exchange is an unnatural parity exchange. Therefore it can be separated from the usual hard gluonic exchange at large $-t$ [24] by looking at the angular distribution of the $\phi$ decay. This procedure was suggested in [23] to disantangle the anomalous unnatural parity exchange $f_1$ and the natural parity hard pomeron contribution in vector meson photoproduction at large energies. In this connection we should point that CLAS photoproduction data [1] show a big change of the $K$-mesons angular distribution at large $-t$, which might be a signal for a large $G$-pole exchange contribution.

From our point of view vector meson electroproduction at large momentum transfer opens the new opportunity to investigate the complex structure
of the QCD vacuum.

Acknowledgements

We are grateful to A.E.Dorokhov, S.B. Gerasimov, N.N.Achasov and V.L.Yudichev for useful discussion. We thank R. Schumacher for providing us with the CLAS data. One of us (N.I.K) is grateful to the University of Valencia for the warm hospitality. This work was partially supported by DGICYT PB97-1227, RFBR-01-02-16431 and INTAS-2000-366 grants.

References

[1] CLAS Collaboration, E.Anciant et al., Phys. Rev. Lett. 85 (2000) 4682.
[2] CLAS Collaboration, K.Lukashin et al., hep-ex/0101030.
[3] J.-M.Laget, Phys. Lett. B489 (2000) 313.
[4] Y.Oh, A.I.Titov and T.-S.H.Lee, nucl-th/0004053.
    Y.Oh, A.I.Titov, S.N.Yang, T.Morii, Nucl. Phys. A684 (2001) 354.
[5] Q.Zhao, B.Saghai and J.S.Al-Khalili, nucl-th/0102027.
[6] M.Anselmino, A.Efremov and E.Leader, Phys. Rep. 261 (1995) 1.
[7] A.E.Dorokhov,N.I.Kochelev and A.Yu.Zubov, Int. J. of Mod.Phys. A8 (1993) 603.
[8] D.I.Dyakonov and M.I.Eides, Sov. Phys. JETP. 54 (1981) 2.
[9] G.Veneziano, Nucl. Phys. B159 (1979) 213.
[10] E.Witten, Nucl. Phys.B156 (1979) 269.
[11] G.Veneziano, Mod. Phys. Lett. A4 (1989) 1605;
    G.M.Shore and G.Veneziano, Nucl. Phys.B381 (1992) 23.
[12] G.M.Shore, Nucl. Phys. B569 (2000) 107 and hep-ph/9908273.
[13] B.Alles, M.D’Elia and A. Di Giacomo, Nucl.Phys. B494 (1997) 281.
[14] M.N.Achasov et. al., hep-ex/0009036

[15] M.K.Volkov, Sov.J. Part.Nucl. 17 (1986) 282;
D.Ebert, M.K.Volkov and V.L.Yudichev, J. Phys. G25 (1999) 2025.

[16] Particle Data Group, C. Caso et al., Eur. Phys. J. C 3, 1 (1998).

[17] A.Donnachie and P.V.Landshoff, Phys. Lett. B478 (2000) 146.

[18] A.Donnachie and P.V.Landshoff, Phys. Lett. B185 (1987) 403

[19] F.E.Low, Phys.Rev. D12 (1975) 163;
S.Nussinov, Phys.Rev.Lett.34 (1975) 1275;
P.V.Landshoff and O.Nachtmann, Z. Phys. C35 (1987) 405.

[20] N. I. Kochelev, D.-P. Min, Y. Oh, V. Vento, and A. V. Vinnikov, Phys. Rev. D 61, (2000) 094008.

[21] N.I. Kochelev and V. Vento, in preparation.

[22] J.-M. Laget and R.Mendez-Galain, Nucl. Phys. A581 (1995) 397.

[23] Y. Oh, N. I. Kochelev, D.-P. Min, V. Vento, and A. V. Vinnikov, Phys. Rev. D 62, (2000) 017504.

[24] B. Gittelman et al. Phys. Rev. Lett. 35 (1975) 1616.