TIME (A-)SYMMETRY IN A RECOLLAPSING QUANTUM UNIVERSE

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Abstract: It is argued that Hawking’s ‘greatest mistake’ may not have been a mistake at all. According to the canonical quantum theory of gravity for Friedmann type universes, any time arrows of general nature can only be correlated with that of the expansion. For recollapsing universes this seems to be facilitated in part by quantum effects close to their maximum size. Because of the resulting thermodynamical symmetry between expansion and (formal) collapse, black holes must formally become ‘white’ during the collapse phase (while physically only expansion of the universe and black holes can be observed). It is conjectured that the quantum universe remains completely singularity-free in this way (except for the homogeneous singularity) if an appropriate boundary condition for the wave function is able to exclude past singularities (as is often assumed).

1 Conditioned entropy in quantum cosmology

Invariance under reparametrizations of time may be considered as a specific consequence of Mach’s principle (which requires the absence of any preferred or ‘absolute’ time parameter). In quantum theory this leads to a time-independent Schrödinger equation (Hamiltonian constraint), since any parametrization of physical time (or ‘clocks’) would require the concept of a trajectory in configuration space. For example, in canonical quantum gravity the wave function of the universe is dynamically described by the ‘stationary’ Wheeler-DeWitt equation $H\Psi_{\text{universe}} = 0$ in superspace (the configuration space of geometry and matter). The conventional time dependence has then to be replaced by the resulting quantum correlations between all dynamical variables of the universe including those describing physical clocks, in particular the spatial metric (see Page and Wootters, 1983). However, this procedure leaves open the problem of how to formulate the asymmetry in time which is manifest in most observed phenomena.
For example, entropy as the thermodynamical measure of time asymmetry is defined in quantum theory as a functional of the density matrix $\rho$,

$$S = -k \text{Trace}\{\hat{P}\rho \ln(\hat{P}\rho)\}.$$  \hspace{1cm} (1)

This definition requires an appropriate ‘relevance concept’ or ‘generalized coarse graining’ which is represented by a ‘Zwanzig projection’ $\hat{P}$ (an idempotent operator on the space of density matrices – cf. Zeh, 1992). Well known examples of relevance concepts are Boltzmann’s neglect of particle correlations, or the neglect of all long-range correlations (quantum and classical) in the form of replacing the density matrix by a direct product $\hat{P}_{\text{local}}\rho := \prod_i \rho_{\Delta V_i}$ of density matrices $\rho_{\Delta V_i}$ for separate volume elements $\Delta V_i$, each of them obtained from $\rho$ by tracing out the rest of the world (the ‘environment’). The latter procedure gives rise to the usually presumed local concept of an entropy density. Under an appropriate Zwanzig projection, the density matrix in (1) may even represent a pure (‘real’) state, $\rho = |\psi><\psi|$, which should, however, depend on some time variable in order to allow the entropy to grow.

Physical entropy, in contrast to the entropy of information, is objectively defined as a function of macroscopic variables (such as those characterizing density, volume, shape, position or temperature), regardless of whether they are known. Therefore, the wave function $\psi$ to be used in (1) cannot be identified with $\Psi_{\text{universe}}$ (which is a superposition of macroscopically different states), but must instead represent some ‘relative state’ (conditioned wave function) for the microscopic degrees of freedom with respect to ‘given’ macroscopic variables of the universe (including clocks). In the framework of a global quantum description, this state is understood as the ‘present collapse component’ (or as ‘our Everett branch’) that has resulted indeterministically from all measurements or measurement-like processes of the past. While the unitary part of von Neumann’s dynamical description of measurements leads to a superposition of macroscopic ‘pointer positions’, its components can be considered as dynamically decoupled from one another once they have decohered. Measurements and decoherence represent the quantum mechanical aspect of time asymmetry (Joos and Zeh, 1985; Gell-Mann and Hartle, contributions to this conference) that also has to be derived from the structure of the Wheeler-DeWitt wave function.

A procedure for deriving the approximate concept of a time-dependent wave function $\psi(t)$ from the Wheeler-DeWitt equation has been proposed by means of the WKB approximation (geometric optics) valid for part of the dynamical variables of the universe. These variables may be those describing the spatial geometry (Banks, 1985), those forming the ‘mini superspace’ of all monopole amplitudes on a Friedmann sphere (Halliwell and Hawking, 1985), or all macroscopic variables which define an appropriate ‘midi superspace’. For example, Halliwell and Hawking assumed that the wave function of the universe can approximately be written as a sum of the form

$$\Psi_{\text{universe}} \approx \sum_r e^{iS_r(\alpha, \Phi)} \psi_r(\alpha, \Phi; \{x_n\}),$$  \hspace{1cm} (2)

where $\alpha = \ln a$ is the logarithm of the expansion parameter, $\Phi$ is the monopole amplitude of a massive scalar field which represents matter in this model, while the variables $x_n$
(with $nJ > J0$) represent all multipole amplitudes of order $n$. The exponents $S_r(\alpha, \Phi)$ are Hamilton-Jacobi functions with appropriate boundary conditions, while the relative states $\psi_r$ are assumed to depend only weakly on $\alpha$ and $\Phi$. If the corresponding orbits of geometric optics in mini superspace are parametrized in the form $\alpha(t_r), \Phi(t_r)$, one may approximately derive from the Wheeler-DeWitt equation a Schrödinger type evolution

$$i\frac{\partial}{\partial t_r}\psi_r(t_r, \{x_n\}) = H_x\psi_r(t_r, \{x_n\})$$

for the ‘relative states’ $\psi_r(t_r, \{x_n\}) := \psi_r(\alpha(t_r), \Phi(t_r), \{x_n\})$. It may apply within the limits of geometric optics along most parts of the trajectories on each WKB sheet $S_r(\alpha, \Phi)$, but one must keep in mind that this dynamical approximation does not define the states $\psi_r(t_r, \{x_n\})$ from which the entropy is to be calculated.

In order to be able to describe the dynamics of the observed quantum world, equation (3) must contain the description of the above-mentioned measurements and measurement-like interactions in von Neumann’s unitary form

$$\psi_r \propto \left(\sum_k c_k \psi_k^S\right) \psi_0^A \rightarrow t_r \sum_k c_k \psi_k^S \psi_k^A,$$

valid in the direction of ‘increasing time’. For proper measurements the ‘pointer positions’ $\psi_k^A$ of the ‘apparatus’ $A$ must decohere through further ‘measurements’ by the environment, and thus lead to newly separated world branches, each one with its own corresponding ‘conditioned (physical) entropy’. The formal entropy corresponding to the ensemble of different values of $k$ would instead have to be interpreted as describing ‘lacking knowledge’.

This required asymmetry with respect to the direction of the orbit parameter $t_r$ means that (3) may be meaningfully integrated, starting from the wave function representing the present state of the observed world, only into the ‘future’ direction of $t_r$ (where it has to describe the entangled superposition of all outcomes of future measurements). In the ‘backward’ direction of time this calculation does not reproduce the correct quantum state, since the unitary predecessors of the non-observed components would be missing. This is particularly important if the trajectories are continued backwards into the inflationary era, or even into the Planck era where different trajectories in mini superspace (and in the case of recollapsing universes even both of their ‘ends’) have to interfere with one another in order to form the complete boundary condition for the total Wheeler-DeWitt wave function (the ‘intrinsic’ initial condition).

Entropy is expected to grow in the same direction of time as that describing measurements. Any such asymmetry requires a very special cosmic initial condition; the existence of measurement-like processes in the quantum world requires essentially a non-entangled initial state (Zeh, 1992). Since the unitary dynamics (3) was derived as an approximation from the Wheeler-DeWitt equation, its initial condition for $\psi_r(t_r)$, too, must be derived from $\Psi_{universe}$. There are no free boundary conditions for trajectories or their relative states.

In order to obtain an appropriate asymmetry of the Wheeler-DeWitt wave function, it will be assumed in accordance with current models of the quantum universe that the
Wheeler-DeWitt Hamiltonian for the gauge-free multipoles on the Friedmann sphere is of the form

\[ 2e^{3\alpha} H = \frac{\partial^2}{\partial \alpha^2} - \frac{\partial^2}{\partial \Phi^2} - \sum_n \frac{\partial^2}{\partial x_n^2} + V(\alpha, \Phi, \{x_n\}), \quad (5) \]

with a potential \( V \) that becomes ‘simple’ (e.g. constant) in the limit \( J \alpha \rightarrow -\infty \). In his talk, Julian Barbour gave an example for how complicated the effective potential in configuration space becomes instead once the particle concept has emerged from the general quantum state of the fundamental fields. The hyperbolic nature of (5) defines an initial value problem with respect to \( \alpha \) which then also allows one to choose a ‘simple’ (or symmetric) initial condition (SIC) for \( \Psi_{universe} \) in the limit of small \( a \). Its qualitative aspects may be illustrated by a WKB approximation with respect to \( \alpha \) (Conradi and Zeh, 1991; Conradi, 1992)

\[ \Psi_{universe}(\alpha, \Phi, \{x_k\}) \rightarrow \frac{1}{(-V)^{1/4}} \exp \left[ \int_{-\infty}^{\alpha} \sqrt{-V(\alpha', \Phi, \{x_k\})} d\alpha' \right] \rightarrow \Psi(\alpha) \quad (6) \]

for \( \alpha \rightarrow -\infty \). The explicit form of the ‘initial’ wave function resulting from the no-boundary condition (Hartle and Hawking, 1983) is less obvious, but need not be different from (6).

If the initial simplicity of the relative states \( \psi_r \) of (2) can be derived from this or some similar simple structure of the total wave function close to the singularity, this means that ‘early times’ (in the thermodynamical sense) must correspond to small values of \( a \). However, classical trajectories in the mini superspace spanned by \( a \) and \( \Phi \) return to small values of \( a \) for closed universes with cosmological constant \( \Lambda \leq 0 \) (even though they are clearly not symmetric in the generic case – see Fig. 1).* How, then, can one distinguish between the Big Bang and the Big Crunch? Or is that distinction really required for the definition of an arrow of time in quantum gravity?

**Fig. 1:** Asymmetric classical trajectory in mini superspace. (After Hawking and Wu, 1985 – see also Laflamme, this conference.) \( a \) is plotted upwards, \( \Phi \) from left to right.

* The ‘no-boundary’ condition, defined as a boundary condition for the Wheeler-DeWitt wave function, is sometimes also used for deriving special ‘initial’ conditions for trajectories at one of their ends, which are then classically continued through all of their history (cf. Laflamme and Shellard, 1987). The required classical conditions at small values of \( a \) are thereby often in violent conflict with the uncertainty relations. However, such a selection of trajectories is neither compatible with the usual probability interpretation of quantum mechanics, nor with the structure of the Wheeler-DeWitt wave function derived from its boundary condition. By no means should these trajectories be used to calculate ‘corrections’ to the wave function from which they were obtained as approximate and limited concepts. (Classically, the exceptional condition of a ‘bounce’ at small values of \( a \), sometimes derived in this way, would describe the middle of a universe’s history, not its beginning or end.)
Dotted curve corresponds to $V = -a^4 + m^2 a^6 \Phi^2 = 0$. In more than two-dimensional mini superspace, the trajectories need not intersect themselves. If the corresponding wave packets (Fig. 2) do not even overlap thereby, this would in reduced dimensions be described as their decoherence from one another.

The contributions of Murray Gell-Mann, Jim Hartle and Larry Schulman to this conference indicate that it is not, provided the considered universe is very young compared to its total lifetime. A symmetric (double-ended) low entropy condition for an assumed $\Psi_{universe}(t)$ would be allowed even if the latter obeyed a unitary time dependence (although it would then represent a very strong constraint). In quantum gravity, however, where there is no general time parameter $t$, one has to conclude that a ‘simple’ condition for $\psi_r(t_r)$ can either be derived from the boundary condition for $\Psi_{universe}$ at both ends of a turning quasitrajectory in mini superspace, or at none. (Any asymmetric selection criteria for trajectories or their relative states – for example by means of a time-directed probability interpretation – would introduce an absolute direction of time ‘by hand’.) In the second case, the asymmetry of the world would have to be explained as a ‘great accident’ occurring at one end. In the first case, all ‘statistical’ arrows of time must reverse their direction together with the expansion of the universe. Integrating (3) in the asymmetric sense of (4) beyond the cosmic turning point would precisely correspond to presupposing the quantum mechanical arrow to keep its direction.

If the concept of trajectories through mini superspace were applicable at all for this purpose (cf. however Sect. 2), the derivation of thermodynamically asymmetric universes would require the existence of two extremely different regions at small values of $a$, together with a proof that almost all trajectories compatible with the structure of the correct Wheeler-DeWitt wave function have one of their ends in each of them. This would not only seem to be in conflict with the sensitivity of the trajectories to their initial conditions, but also with the statistical interpretation of entropy. In correct quantum description, the broad ‘initial wave packet’ which represents the whole assumed low entropy region would, if exactly propagated through superspace according to the Wheeler-DeWitt equation and reflected from the repulsive curvature potential at large $a$, have to reproduce the complementary ‘initial’ wave packet that represents the high entropy region at the boundary of small $a$ without thereby interfering with the low entropy region. The condition of reflection (integrability for $a \rightarrow \infty$) restricts the otherwise complete freedom of choosing the intrinsic initial values (corresponding to the hyperbolic nature of the Wheeler-DeWitt equation) by a factor of 1/2. Regardless of all open problems of dynamical consistency, no properties of the Wheeler-DeWitt Hamiltonian or in the no-boundary condition seem to indicate the existence of two that much contrasting regions for small values of $a$.

If the arrow of time is instead correlated with the expansion, the derived dynamics (3) for $\psi_r$ has always to be applied in the direction of growing values of $a$. In particular, considering the inflation of the early universe as ‘causing’ a low entropy state at one end of the trajectory only would be equivalent to presuming an arrow of causality in a certain direction of it (instead of deriving this asymmetry as claimed).

Notice that in quantum gravity there is no problem of consistency of the lifetime of the recollapsing universe with its supposedly much longer Poincaré cycles (that is, with the mean time intervals between two statistical fluctuations of cosmic size), as it would
arise from the mentioned double-ended boundary conditions under deterministic (such as unitary) dynamics. The exact dynamics $H\Psi_{\text{universe}} = 0$, understood as an intrinsic initial value problem in the variable $\alpha$, constitutes a well-defined one-ended condition, while the reversal of the arrows of time described by the time-dependence $\psi_r(t_r)$ is facilitated by the required corrections to the derived unitary dynamics. These corrections have to describe decoherence and inverse branchings on the return leg.

I am thus trying to convince Stephen Hawking that he did not make a mistake before he changed his mind about the arrow of time! Even in classical general relativity, the asymmetry of individual trajectories in mini superspace (pointed out by Don Page, 1985) would not be sufficient for drawing conclusions on much stronger thermodynamical asymmetries.

2 Reversal of the expansion of the universe in quantum gravity

Within the canonical quantum theory of gravity it appears therefore hardly possible for the arrow of time to maintain its direction when the universe starts recollapsing. However, the above picture of wave functions approximately evolving along separate WKB orbits in mini superspace is not a sufficient representation of the dynamics described by the Wheeler-DeWitt equation – not even far outside the Planck region. As will be shown, the approximation of geometric optics does not justify the continuation of classical trajectories through the whole history of a universe. For example, a trajectory chosen to be compatible with the WKB approximation of the wave function at one end, and found to be incompatible with it at the other one, would not indicate an asymmetric arrow of time along this trajectory, but simply demonstrate that the concept of trajectories must have broken down in between.

Wave mechanically, trajectories have to be replaced by narrow wave packets which separately solve the wave equation. The exact dynamics for $\Psi_0(\alpha, \Phi)$ in mini superspace (now replacing the approximation $e^{iS(\alpha, \Phi)}$) is described by

$$2e^{3\alpha}H\Psi_0(\alpha, \Phi) = \frac{\partial^2 \Psi_0}{\partial \alpha^2} - \frac{\partial^2 \Psi_0}{\partial \Phi^2} + [ - e^{4\alpha} + m^2 e^{6\alpha} \Phi^2 ] \Psi_0(\alpha, \Phi) = 0. \quad (7)$$

The $\alpha$-dependent oscillator potential for $\Phi$ suggests the ansatz

$$\Psi_0(\alpha, \Phi) = \sum_n c_n(\alpha) \Theta_n \left( \sqrt{m e^{3\alpha}} \Phi \right), \quad (8)$$

where the functions $\Theta_n$ are the oscillator eigenfunctions. In the adiabatic approximation, the coefficients $c_n(\alpha)$ decouple dynamically,

$$\frac{d^2 c_n(\alpha)}{d\alpha^2} + [ - e^{4\alpha} + (2n + 1) m e^{3\alpha} ] c_n(\alpha) = 0. \quad (9)$$

In this case, coherent oscillator wave packets exhibit the least possible dispersion, and may therefore be expected to resemble the trajectories of geometric optics best.

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As demonstrated by Kiefer (1988), the usual (here ‘final’ with respect to the intrinsic wave dynamics) condition of square integrability for \( \alpha \to +\infty \) leads to the classically expected reflection of quasitrajectories from the repulsive curvature-induced potential \(-e^{4\alpha}\). (Without such a condition, wave packets would not return at all.) For example, a further WKB approximation to (9), together with Langer’s pasting to the exponentially decreasing WKB solutions at the classical turning point, leads to

\[
c_n(\alpha) \propto \cos[\phi_n(\alpha) + n\Delta\phi] + \cos[\phi_n(\alpha) - n\Delta\phi + \delta_n]
= \text{‘expanding universe’ + ‘collapsing universe’},
\]

where the \( \phi_n \)'s are monotonic functions of \( \alpha \) (approximately proportional to \( n \)) while \( \delta_n = (\pi/4)m^2(2n + 1)^2 \) is the ‘scattering’ phase shift enforced by the ‘final’ (large \( a \)) condition. The two cosines correspond to the expanding and recollapsing parts of the histories of classical universes in mini superspace (in a merely relative sense, of course). \( \Delta\phi \) is the phase of the classical \( \Phi \)-oscillation at the point of maximum \( \alpha \) (describing the asymmetry of the trajectory). If the constants of integration at our disposal from (9), which determine the size and phase of the coefficients \( c_n \), are now chosen to form coherent states from the first cosine on the rhs of Eq. (10), these phase relations are then completely changed by the large phase shift differences \( \delta_n - \delta_{n-1} \propto n \) resulting from the second cosine. While the term representing the expanding universe (Fig. 2) nicely resembles the corresponding part of a classical trajectory (Fig. 1), the reflected wave is smeared out over the whole allowed region (Fig. 3). This spreading must also be described by the corresponding Klein-Gordon current. From a sharp \((n\text{-independent})\) potential barrier in ‘time’ \( \alpha \), the wave packets would instead be reflected without any dispersion.

This dispersion of the wave packet will become even more important for more macroscopic universes (higher mean oscillator quantum numbers \( \bar{n} \)), since the phase shift differences are proportional to \( n \). The result depicted by Fig. 3 may therefore be expected to represent a generic property of Friedmann type quantum universes. Quasiclassical trajectories must then never be continued beyond the turning point in order to end in a well defined region of high entropy. The wave mechanical continuation leads instead to a superposition of many recollapsing universes (each of which cannot be intrinsically distinguished from an expanding one). Cosmological quantum effects of gravity thus seem to be essential not only at the Planck scale! The phase relations of the resulting superpositions of quasitrajectories on the return leg in mini superspace are however destroyed by decoherence.
– now ‘irreversibly’ acting in the opposite direction of the trajectory (with increasing $a$ again) because of the (formally) final condition at the (formal) Big Crunch. (The phase shifts $\delta_n$ could as well have been put into the first cosine with a negative sign, since there is no absolute direction of probabilistic ‘scattering’ from one wave packet into the other. One has to be careful to avoid any notion of absolute time.) A related result has independently been obtained by Kiefer (1992b). This is further evidence that the unitary dynamics (3) cannot be continued along trajectories beyond the turning point at maximum $a$.

Although wave packets solving the Wheeler-DeWitt equation in mini superspace can thus be defined to be intrinsically asymmetric, they are physically determined (as Everett branches) by their decoherence from one another. Wave packets in the complete configuration space (which never decohere, since they do not possess an environment) are not to describe the whole ‘quantum world’, but merely the (limited) causal connections which give rise to the latter’s ‘classical appearance’.

3 Black-and-white holes

A formal reversal of the arrow of time (in particular if facilitated through quantum effects near the turning point of the universal expansion) must drastically affect the internal structure of black holes (Zeh, 1992). For comparison, consider black holes which would form during the expansion of a time-asymmetric universe, and which are massive enough to survive the turning point (cf. Penrose’s diagram in Fig. 4). If the arrow of time is now formally reversed along a (quasi)trajectory through mini or midi superspace in order to form a quasiclassical time-symmetric universe, black holes cannot continue ‘losing hair’ any further by radiating their higher multipoles away (by means of retarded radiation) when the universe starts recollapsing. They must instead grow hair by means of the now coherently incoming (advanced) radiation that has to drive the matter apart again.

Fig. 4: Time-asymmetric classical universe with a homogeneous Big Bang only (Penrose, 1981).

The reversal of all arrows of time has of course to include the replacement of time-directed ‘causality’ by what would formally represent a ‘conspiracy’. A mere reversal of the expansion would not by itself be able to ‘cause’ a reversal of the thermodynamical or radiation arrows without simultaneous reversal of the time-direction of this causation. The (fork-like) causal structure (see Zeh, 1992) must hence be contained in the dynamical structure of the universal wave function that results from the intrinsic initial condition by means of the Wheeler-DeWitt equation. Black holes must therefore formally disappear as ‘white holes’ during the recollapse phase of the universe.

This surprising fate of black holes thus seems to become important only in the very distant future (long after horizons and singularities may be expected to have formed in their interiors). However, our simultaneity with a black hole is not well defined because of the time translation invariance of the Schwarzschild metric. Fig. 5 shows a spherical black hole in Kruskal-type coordinates (a modified Oppenheimer-Snyder scenario) after translation of the Schwarzschild time coordinate $t$ such that the turning point of the universal expansion is now at $t = 0$ (hence also at the corresponding Kruskal time coordinate
The resulting ‘black-and-white hole’ must then also exhibit a thermodynamically symmetric appearance, although it need not be symmetric in non-conserved microscopic or macroscopic properties (‘hair’). If past horizons and singularities can in fact be excluded by an appropriate initial condition at the Big Bang (as it is claimed for the Weyl tensor hypothesis), the same conclusion must hold in quantum gravity also for future horizons and singularities. One may therefore conjecture a completely singularity-free quantum world (i.e., a wave function vanishing at all singularities).

**Fig. 5:** ‘Black-and-white hole’ originating from a thermodynamically active (i.e., non-pathological) collapsing spherical matter distribution, with the Kruskal time coordinate $v = 0$ chosen to coincide with the time of maximum size of the universe. If the quantum effects studied in Sect. 2 are essential, this classical picture is not meaningful itself in the region of ‘quantum behaviour’ around $v = 0$. Only a probabilistic connection can then exist between its upper and lower parts.

Fig. 6 shows the same situation as Fig. 5 from our perspective of a young universe (after a back-translation of the Schwarzschild time coordinate such that $t_{today} = 0$). From this perspective, the time coordinate $t = t_{turn}$ appears to be very ‘close’ to where one would expect the future horizon to form. The ‘strange’ thermodynamical and quantum effects now also appear to occur close to the horizon, thereby preventing it from forming.

**Fig. 6:** Same black-and-white hole as in Fig. 5 considered from our perspective of a young universe.

This reversal of the gravitational collapse cannot be observed from a safe distance, although it could be experienced by suicidal methods within relatively short proper times if a black hole were available in our neighborhood. If the black-and-white hole is massive enough, this kind of ‘quantum suicide’ must be quite different from the classically expected one by means of tidal forces. In a classical picture, travelling through a black-and-white hole may reduce the proper distance between the Big Bang and the Big Crunch considerably, but unfortunately we could not survive as information and memory gaining systems. This consideration should at least demonstrate that the classical (Kruskal-Szekeres) continuation of the Schwarzschild metric beyond the horizon is absolutely doubtful for thermodynamical and quantum mechanical reasons!

Before Stephen Hawking changed his mind about the time arrow in a recollapsing universe, he had conjectured (Hawking, 1985) that the arrow is reversed inside the horizon of a black hole, since “it would seem just like the whole universe was collapsing around one” (cf. also Zeh, 1983). This consequence would however not describe the situation in a thermodynamically time-symmetric universe.

Penrose’s black holes, hanging like stalactites from the ‘ceiling’ (the Big Crunch) in Fig. 4, must now also become symmetric, as shown in Fig. 7. Black-and-white holes in equilibrium with thermal radiation (as studied by Hawking, 1976) would instead consist of thermal radiation at both ends. They would possess no ‘hair’ at all, neither to lose nor to grow. The classically disconnected upper and lower halves of Fig. 7 should rather be
interpreted as two of the many Everett branches of the quantum universe, each of them representing an expanding quasiclassical world.

**Fig. 7:** Time-symmetric, singularity-free universe with black-and-white holes together with (small) black or white holes.

The absence of singularities from this quantum universe thus appears to be a combined thermodynamical and quantum effect. However, one may equivalently interpret the result as demonstrating that in quantum cosmology the thermodynamical arrow is a consequence of the absence of inhomogeneous singularities – a generalization (or symmetrization) of Penrose’s Weyl tensor condition.

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Hawking: Your symmetric initial condition for the wave function is wrong!
Zeh: Do you mean that it does not agree with the no-boundary condition?
Hawking: Yes.
Zeh: It was not meant to agree with it, although we found it to be very similar to the explicit wave functions you gave in the literature for certain regions of mini superspace. This is however not essential for my argument. It requires only that the multipole wave functions $\psi_r$ become appropriately ‘simple’ (low-entropic and factorizing) for small values of $a$ (as you too seem to assume, although only at that ‘end’ of the trajectory where you start your computation).

Barbour: Did I understand you correctly to say that the criteria Kiefer used to obtain his solution was of the kind I call Schrödinger type, namely that there should be no blowing up of the wave function anywhere in the configuration space?
Zeh: Yes – if by blowing-up solutions you mean the exponentially increasing ones. Otherwise you would not be able to describe reflection (turning trajectories) by means of wave packets. I think this assumption corresponding to the usual normalizability is natural (or ‘naive’ according to Karel Kuchař) if the expansion parameter $a$ is considered as a dynamical quantum variable (as it should in canonical quantum gravity).

Barbour: Could it be that worries about the turning point are an artifact of the extreme simplicity of the model? Consider in contrast a two-dimensional oscillator in a wave packet corresponding to high angular momentum!
Zeh: The described quantum effects at the turning point are due to the specific Friedmann potential with an oscillator constant for $\Phi$ exponentially increasing with $\alpha$. They do not seem to disappear if added degrees of freedom possess similarly ‘normal’ potentials (e.g. polynomials multiplied by positive powers of $a$). This seems to be the case in Friedmann-type models.

Kuchař: Did you study decoherence between $\psi$’s corresponding to one $S$, or also the decoherence corresponding to different $S$’s?
Zeh: I expect decoherence to become effective (with increasing $\alpha$) between different trajectories in mini superspace (cf. Kiefer, 1987), between macroscopically different branches of the multipole wave functions $\psi_r$ along every trajectory, and between different WKB sheets corresponding to different $S$’s except at very small and large $a$ (cf. Halliwell, 1989; Kiefer, 1992a). Otherwise equation (3) would not be valid as an independent approximation on different sheets.
Griffiths: In applying ordinary quantum mechanics to a closed system, I do not know how to make sense out of the ‘wave function of the closed system’. I need the unitary transformations that take me from one time to another. Is there any analogy of this in quantum gravity? For if not, it is hard to see how quantum gravity can be used to produce a sensible description of something like the world we live in.

Zeh: Your question seems to apply to quantum gravity in general. I think that it is sufficient for the wave function of the universe to contain correlations between all physical variables – including those describing clocks. In classical theory these correlations would be essentially unique, since they would be represented by the trajectories in the complete configuration space which remain after eliminating any (physically meaningless) time parameter. In quantum theory there are no trajectories that could be parametrized. These quantum correlations must of course obey ‘intrinsic’ dynamical laws as they are described by the Wheeler-DeWitt equation. From them one tries to recover the time-dependent Schrödinger equation (which has to describe the ‘observed world’) as an approximation when spacetime (the history of spatial geometry) is recovered as a quasiclassical concept.

Lloyd: Could you clarify how black holes would grow hair in the contraction phase? Is it through interference between incoming radiation and the Hawking radiation?

Zeh: Only the advanced radiation is essential, since black holes can form by losing hair even if Hawking radiation is negligible. This is a pure symmetry consideration. A final condition which is thermodynamically and quantum mechanically (although not in its details) the mirror image in time of an initial condition that leads to black holes must consequently lead to their time-reversed phenomena. If Hawking radiation is essential (as for small mass), the black hole may disappear before \( t(a_{\text{max}}) \) is reached, but again before an horizon forms.

Hawking: The no-boundary condition can only be interpreted by means of semiclassical concepts such as the saddle point method.

Zeh: I would prefer to understand such a fundamental conclusion as the arrow of time in terms of an exact (even though incomplete) description. In particular, your opposite conclusion about the arrow of time seems to be introduced by the direction of computation (along the assumed trajectories) by using approximations, similar to how it is often erroneously argued in the theory of chaos by using ‘growing errors’ in the calculation for explaining the increase of ‘real’ physical entropy!

– Did I understand you correctly during your talk that you – at the time when you made what you call your ‘mistake’ – also expected black holes to re-expand during the recollapse of the universe?

Hawking: Yes. I did not understand black holes sufficiently until I changed my mind.