A Simulation Method for Assembly Process of Branch Cables Based on the Minimal Energy Principle

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Abstract. To solve the simulation problem of branch cables in virtual assembly process, a cable simulation method based on the minimal energy principle is proposed. Firstly, to describe the postures and deformations of the branch cables, a discrete Cosserat rod model is established. Next, according to the description of the minimal energy principle, each system is in the most stable state when its energy takes the minimum, so the postures of the cables could be solved by calculating the extremum of energy optimization model. After that, on the circumstance of ignoring low-speed movement of the cables, the energy could be obtained by the summation of the tensile, bending and torsional deformations potential energy. Then, the Levenberg-Marquardt algorithm is applied to solve the energy optimization model, and the postures of cables is calculated through Hermite interpolation. Finally, the verification example was established to simulate the deformations of the branch cables, which verifying the high calculating speed of algorithm and the authenticity of simulation.

1. Introduction

1.1. Background
Cables play an important role in the signal and energy transmission in electronic equipment, and their layouts will directly affect the working stability of the devices. To economize the labor and time cost of cable design and debug process, the assembly process simulation could be applied to discover the design defects and to verify the design results in advance, leading to a suitable guide of the actual process. This simulation studies the deformed results of cables when they are moving, and its core lies in the establishment of cable physical model and the theory to solve simulation.

1.2. Literature reviews
In the course of the assembly process, cables often occur tensile, bending and torsional deformations. To take the simulated accuracy and calculating speed into consideration, the physical model of cables should fully describe the deformations, and an appropriate theory should be applied to meet the needs of interactive display.

Terzopoulos et al[1] proposed a dynamic spline model, which treated the centreline as a D-NURBS curve, and solved the cable’s model by the lagrangian functions. Wakamatsu et al[2] expressed the cables by using the parametric equations, and simulated the drag operations accurately by the finite element method. Loock et al[3] established a mass spring model which used torsional springs to describe the anti-bending characteristic of the cable, brought a fast simulation of bending deformations. Selle et al[4] added the torsional springs on the basis of the mass springs model, which were used to
express the anti-torsional characteristic of the model, and simulated multiple hairs successfully. Pai[5] applied the Cosserat rod model, and used different coordinate systems of the thin rod to express the bending and torsional deformations, then simulated the deformed results of surgical sutures.

In the respect of modelling of branch cables, Bergou et al[6] established a centreline-angle coupled model of the rope, and got good deformations results of tree branches by solving the kinetic equations. Zhao et al[7] treated the plants as triangular mesh, then used finite element method to solve the deformations of complex plants.

For the theory used to solve the simulation, Wang et al[8] built the kinetic equations based on the Kirchhoff rod theory[9], and is solved the model numerically by the differential quadrature method. Spillmann et al[10] considered the speed and angular velocity of the rope, and used the semi-implicit Euler method with respect to time to update the speed and position, then completed the dynamic simulation of the rope.

The simulation theory above not only provided high precision, but also brought slower speed in calculating, and spent too much time in solving the nonlinear mechanical equations. The minimal energy principle is an energy optimization method, which states that every system has a tendency to develop to the minimum energy state, and the system takes the most stable state when its own the extremum of energy. In this way, it’s obvious that the postures of the branch cables could be solved by calculating the extremum of energy, that’s what we studied in the article below.

2. Method

2.1. Cable modelling

The cables are expressed by the Cosserat rod model[5], which contains multiple coordinate systems lying on the cables. Then, the postures are described by each origin point of the coordinate systems, while the deformations are described by its axes. To avoid the complex partial differential and integral problems, the cables are discretized into a series of end-to-end cable segments, and each coordinate system lies on the middle of the segment.

![Figure 1. The coordinate systems lying on the discrete cable.](image)

In figure 1 above, the cable is discretized into N segments. By using the Rodrigues’ rotation formula and the quaternion \( q = (q_1, q_2, q_3, q_4)^T \) and, each coordinate system \( d_i \) can be described as:

\[
d_i = R_i \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
2(q_1^2 + q_2^2) - 1 & 2(q_2q_3 - q_1q_4) & 2(q_3q_4 + q_1q_2) \\
2(q_2q_3 + q_1q_4) & 2(q_1^2 + q_3^2) - 1 & 2(q_3q_4 - q_1q_2) \\
2(q_4q_2 - q_1q_3) & 2(q_3q_4 + q_1q_2) & 2(q_1^2 + q_3^2) - 1
\end{pmatrix}
\]

(1)

According to Cosserat theory, the bending and torsion vectors \( \omega \) are used to describe the strains of the rod, which are refer to the angle of bending and torsional deformations along each axis, and its value depend on the derivative of each \( d \) to the arc coordinate system \( s \) of the cable.

\[
\omega = \begin{pmatrix}
\omega_1 \\
\omega_2 \\
\omega_3
\end{pmatrix} = \frac{1}{2} \sum_{i=1}^{3} (d^k \times \frac{d}{ds} d^k)
\]

(2)
Then the postures and deformations can be expressed by \( P = [p_1, ..., p_{N+1}]^T \) and \( Q = [q_1, ..., q_N]^T \).

At this time, the position of each segment is given by the points on both sides, and the deformations between adjacent segments is described by the bending and torsional vector.

### 2.2. Potential energy

After the cable been treated as discrete segments, some presumption should be listed to simplify the solution steps. It’s easy to learn that the tensile deformations have just a small impact on the total length, leading to the smaller influence on the bending and torsional deformations. Vice versa, the latter don’t have much impact on the former. Then, we can assume that the tensile deformations only exist in each segment, while the bending and torsional deformations occur between adjacent segments.

Since we have got the relationships between the moment \( M \) and the vector \( \omega \) \(^9\), the stiffness matrix \( K \) is introduced to describe the ability of cable to resist deformations, and the moment are:

\[
 M = K \omega = \begin{bmatrix} k_b & k_b & k_b \\ k_b & k_b & k_b \\ k_b & k_b & k_b \\ k_b & k_b & k_b \\ k_b & k_b & k_b \\ k_b & k_b & k_b \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \frac{\pi r^4}{4} \begin{bmatrix} E \\ E \\ 2G \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}
\]

(3)

Where the parameter \( k_b \) and \( k_t \) refer to anti-bending stiffness and anti-torsional stiffness, related to the radius \( r \), the Young’s modulus \( E \) and the shear modulus \( G \).

Known that the potential energy is equivalent to the work done by the external force, the bending and torsional potential energy could be expressed by the integration of the moment to the angle:

\[
 E_{bi} = \sum_{i=1}^{N} \int_0^{\Delta i} M_i d\phi = \sum_{i=1}^{N} \int_0^{\Delta i} k_b \omega \cdot l_i d\omega + \int_0^{\Delta i} k_t \omega \cdot l_i d\omega = \frac{l_i}{2} (k_b \omega_{i,1}^2 + k_b \omega_{i,2}^2 + k_t \omega_{i,3}^2)
\]

(4)

For the tensile potential energy, the deformations include the directional part and the length part, then the square of strain should be written as the dot product of the gap vector.

\[
 E_{si} = \int_0^{l_i} F_3 dx = \int_0^{l_i} k_s \epsilon \cdot l_i d\epsilon = \frac{k_s l_i}{2} \left( \frac{p_{i+1} - p_i}{l_c} \right) \left( \frac{p_{i+1} - p_i}{l_c} - d_i \right)
\]

(5)

Where the parameter \( k_s = E_s \pi r^2 \) refers to tensile strength, and the parameter \( l_c = L / N \) is the length of each discrete segment.

Above all, the bending and torsional deformations occur in \( p_i \) can be calculated by \( d_i \) and \( d_{i+1} \), while the tensile deformations in each segment can be calculated by \( p_i \) and \( p_{i+1} \). For all \( N \) parts of cable segments, when we ignore the low-speed moving, the total energy \( E \) is:

\[
 E = E_p = \sum_{i=1}^{N} E_{si} + \sum_{i=1}^{N-1} E_{bi}
\]

(6)

Now the postures of the single cable can be calculated by obtaining the minimum energy value.

### 2.3. The operations on branch cables

We can learn that the postures of each discrete cable segment depend on the quaternion and points. So the core to handle the branch cables lies in the operation on the branch point and the branch segment.

All the occasions about the branch cables can be divided into the simplest situation shown in figure 2, which contains the ramose part \( B_2 \), \( B_3 \) and the main part \( B_1 \). The three parts are intersect at the common segment \( C \) between \( p_s \) and \( p_{s+1} \), and part \( C \) belongs to all three parts simultaneously.
After separating the branch cables to three parts, it’s obvious that there are two parts of the bending and torsional potential energy at $p_e$, and the radius of the part $B_1$ should be bigger than the other two.

While computing the total energy of the branch cables, we can also calculate the energy of each part separately, and the sum of them calculates the tensile potential energy of the part $C$ for three times, then the total energy can be easily described as:

$$E_{\text{total}} = E(B_1) + E(B_2) + E(B_3) - 2E'(C)$$ (7)

After calculating the positions and quaternions, two vectors are needed for interpolation. The start vector $V_1 = d_c$ follows the direction of the common part, which is shown in the figure 3. To keep the curve smoothly, the another vector $V_3 = (d_{i-1} + d_i)/2$ should be the average of $d_{i-1}$ and $d_i$. When it comes to the endpoints of the cables, the vector should be $V = d$ owing to the direction of the connector of cables.

Based on the points and the vectors, the curve $C_3$ is described by the Hermite interpolation:

$$p(u) = (2u^3 - 3u^2 + 1)p_e + (-2u^3 + 3u^2)p_i + (u^3 - 2u^2 + u)V_1 + (u^3 - u^2)V_2$$ (8)

Where the position $p(u)$ represents $p_e$ while $u = 0$, and represents $p_i$ while $u = 1$.

3. Solution

3.1. Optimization model

According to the minimal energy principle, the cables take stable state while its energy acquire the minimum value.

Since the deformations are controlled by boundary conditions that represent the constraints of the optimization model, the first restriction is about the endpoints and tail-ends of cables, and it’s believed that every end of cables should follow the position and the direction of its connectors.

$$C_{\text{end}} = \begin{bmatrix} p_{e, \text{p}} \\ q_{e, \text{p}} \end{bmatrix} - \begin{bmatrix} p_{\text{handle}} \\ q_{\text{handle}} \end{bmatrix} = 0$$ (9)
Secondly, considering the limitation of quaternion, its modulus should also meet:

\[ C_q = q_i^T q_i - 1 = 0 \quad i = 1, 2, \ldots, N \]  

(10)

When it comes to the iterative parameters, we get rid of the terminal point and terminal direction, and the parameters \( [p_2, \ldots, p_N]^T \) and \( [q_2, \ldots, q_{N-1}]^T \) are used in iteration, then the constraint \( C_{end} \) is ignored. To avoid the problem of local optimization, we transform the single-objective optimization model into a multi-objective optimization model, which is shown as:

\[ \min E = \sum E_p + \sum (k_q C_q)^2 = \frac{1}{2} \sum F^2 \]  

(11)

Where the parameter \( k_q = 5000 \) refers to the coefficient of penalty term.

3.2. Algorithm and result

To solve the least square optimization, the Levenberg-Marquardt algorithm is used, which unites the least square method and the trust region method at the same time. Each round of calculation starts from the initial point, after assuming a reliable displacement, then in the area with the current point as the centre and the displacement as the radius, and the real iterative increment can be solved by finding the optimal point of the approximate function of the objective function.

\[ x = \arg\min \frac{1}{2} \sum F^2 = \frac{1}{2} \arg\min f^T f \]
\[ x_{k+1} = x_k - (J_k^T J_k + \lambda_k I)^{-1} J_k^T f_k \]  

(12)

Where the parameter \( x_k \) refers to \( [p_2, \ldots, p_N]^T \) and \( [q_2, \ldots, q_{N-1}]^T \) by the \( k \) time, and the parameter \( J_k \) is the jacobian matrix of objective function \( f'_k \) to iterative parameters \( x_k \). And the parameter \( \lambda_k \) is introduced to turn the matrix into an invertible matrix, and we set it as \( \lambda_k = |J_k^T f_k| \).

In each round of iteration, the present postures are used as iterative parameters to solve the new postures, then update the postures by iterating. After repeating those steps for times, the simulation of cable assembly process could be completed. The figure 4 shows one of the state of branch cables during the simulation, it’s obvious that this research has got good results.

![Figure 4. The postures of branch cables during the simulation.](image-url)
4. Conclusion

In this paper, we introduced a method based on the minimal energy principle to simulate branch cables, and proposed the way to disperse and interpolate the branch cables, then used the Levenberg-Marquardt algorithm to solve the energy optimization model. Based on this method, we can get the postures of the branch cables in the simulation, which is used for the assembly process simulation.

The minimal energy principle can be well applied to the simulation, and the calculating speed is greatly improved while maintaining the accuracy.

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