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Two-Boson Exchange Physics: A Brief Review

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Abstract. Current status of the two-boson exchange contributions to elastic electron-proton scattering, both for parity conserving and parity-violating, is briefly reviewed. How the discrepancy in the extraction of elastic nucleon form factors between unpolarized Rosenbluth and polarization transfer experiments can be understood, in large part, by the two-photon exchange corrections is discussed. We also illustrate how the measurement of the ratio between positron-proton and electron-proton scattering can be used to differentiate different models of two-photon exchange. For the parity-violating electron-proton scattering, the interest is on how the two-boson exchange (TBE), $\gamma Z$-exchange in particular, could affect the extraction of the long-sought strangeness form factors. Various calculations all indicate that the magnitudes of effect of TBE on the extraction of strangeness form factors is small, though can be large percentage-wise in certain kinematics.

Keywords Two-boson exchange · proton electromagnetic form factors · proton strange form factors

1 Introduction

As the only stable hadron, the study of the structure of the proton provides a unique and excellent testing ground of QCD. In the one-photon exchange (OPE) approximation, it is well-known that the proton’s electric ($G_E$) and magnetic ($G_M$) form factors (FFs) can be extracted from the reduced differential cross section of the electron-proton ($ep$) elastic scattering as one has

$$\sigma_R(Q^2, \epsilon) \equiv \frac{d\sigma}{d\Omega_{lab}} \varpi_\text{Mott} = G^2_M + \frac{\epsilon}{\tau} G^2_E,$$

where $\tau = Q^2/4M^2$, $\epsilon^{-1} = 1 + 2(1 + \tau)\tan^2\theta/2$, $Q^2 = -q^2$ is the momentum transfer squared, $\theta$ the laboratory scattering angle, $0 \leq \epsilon \leq 1$, and $\sigma_\text{Mott}$ is the Mott cross section. For fixed $Q^2$, varying angle $\theta$, i.e. $\epsilon$, and adjusting incoming electron energy as needed to plot $\sigma_R$ versus $\epsilon$ will give the FFs, a method often called the Rosenbluth, or longitudinal-transverse (LT), separation technique. Experiments employing such a technique over the last half century all indicate that $R = \mu_pG_E/G_M \sim 1$ for $Q^2 < 6 \text{GeV}^2$ as often quoted in textbooks.

On the other hand, it has been shown [1] that, again in the OPE approximation, the ratio $R$ can be accessed in $ep$ scattering with longitudinally polarized electron by measuring the polarizations of the recoiled proton parallel $P_l$ and perpendicular $P_t$ to the proton momentum in the scattering plane,

$$\frac{G_E}{G_M} = -\sqrt{\frac{\tau(1+\epsilon)}{2\epsilon}} \frac{P_t}{P_l}$$

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Polarization transfer (PT) experiment of this kind is only possible recently at JLab. It came as a big surprise that these PT experiments yield values of $R$ markedly different from 1 in the same range of $Q^2$ \[2\]. It prompts intensive efforts, both experimentally and theoretically, to understand this apparent discrepancy as the form factors, which describe the spatial distribution of the charge and magnetization inside the proton, are among the most fundamental observables in hadron physics.

On the experimental side, the first step is to verify this discrepancy. A new global analysis of the world’s cross section data was carried out \[3\]. It found that the great majority cross sections were consistent and the disagreement with polarization transfer measurements remains. A set of extremely high precision measurements of $R$ was soon performed using a modified Rosenbluth technique \[4\], with the detection of recoil proton to minimize the systematic uncertainties, and confirmed the discrepancy.

On the theoretical side, the immediate step taken was to carefully reexamine the radiative corrections which were known to be as large as 30% of the uncorrected cross section in certain kinematics. Of various radiative corrections, only proton-vertex and two-photon exchange (TPE) corrections contained $\epsilon$ dependence. The proton-vertex corrections had been investigated thoroughly by Maximon and Tjon and \[2\] found to be negligible. Realistic evaluations of the TPE corrections are hence called for to see whether they are able to resolve this discrepancy. Both hadronic and partonic model calculations have found that TPE effects can account for more than half of the discrepancy which we will briefly review in Sec. 2.

Because TPE effects were found to play an important role in the parity-conserving electron-proton scattering, it was natural to speculate that $\gamma Z$-exchange could also play a non-negligible role in the parity-violating $ep$ elastic scattering which has been intensively studied in order to extract the proton strangeness form factors. Theoretical efforts in this regard will be briefly reviewed in Sec. 3. Finally, Sec. 4 will be devoted to summary and discussions on the outlook of the future research on TBE effects, in particular the TPE effects in the parity-conserving electron-proton scattering.

### 2 Two-photon exchange in elastic electron-proton scattering

In this section, we discuss (1) the attempts to evaluate the TPE contributions to $ep$ scattering, focusing mostly on two types of model calculations, one hadronic and the other partonic, (2) possible phenomenological parametrization of the TPE effect, and (3) use of $e^+p/e^-p$ ratio and normal asymmetries as constraints on models.

#### 2.1 Model calculations of TPE

In the well-known work of Mo and Tsai \[6\], the radiative corrections were treated in the soft-photon approximation. In the case of the box and cross-box diagrams depicted in Fig. 1, it amounts to evaluating the integrand within the integration over the four-momentum of the internal by taking $k = 0$ and $k = q$. After the discrepancy between LT and PT experiments in extracting $R$ is confirmed, Tjon and his collaborator \[7\] immediately set out to treat the TPE effects rigorously, in a hadronic model including finite size of the proton but only the elastic nucleon intermediate states. They made use of the package FeynCalc \[8\] and LoopTools \[9\] which enable them to carry out integrals over four-momentum $k$ with numerator in the integrand containing power of $k$ up to 4. Their results for $R$, corrected with TPE effects, are shown in the left panel of Fig. 2 and indicates that TPE corrections can account for more than half of the discrepancy. Later efforts to include higher resonances like $\Delta(1232)$ etc. improve
the agreement with the data quantitatively, but not qualitatively. We refer the readers to a recent review [10] for details on these developments.

The partonic calculation of the TPE corrections was reported in [11]. The generalized parton distributions which appear in hard exclusive processes were used to evaluate the handbag diagram where the incident electron scatters from quarks in the proton. The results of this partonic calculation are shown on the right panel of Fig. 2 which are qualitatively similar to the hadronic model calculations.

![Figure 2](image_url)

**Fig. 2** TPE corrections to $\mu_y G_E/G_M$. The PT data are from [2] while the data obtained using LT separation method are from [3, 12]. Left panel: results of the hadronic model calculation of [7], where the figure is taken. Right panel: results of the partonic calculation of [11], where the figure is taken.

### 2.2 Phenomenological parametrization of two-photon exchange effects

Including the TPE effects, $\sigma_R$ of Eq. (1) takes the following general form [13]:

$$\sigma_R(Q^2, \varepsilon) = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2) + F(Q^2, \varepsilon),$$  

(3)

where $F(Q^2, \varepsilon)$ is a real function describing the effect of $1\gamma \otimes 2\gamma$ interference. Charge conjugation and crossing symmetry require that $F$ should, at a fixed value of $Q^2$, satisfy the following constraint [14], $F(Q^2, y) = -F(Q^2, -y)$, where $y = \sqrt{(1 - \varepsilon)/(1 + \varepsilon)}$. Two parametrizations were considered in [15],

$$I : \sigma_R = G_M^2(Q^2) \left(1 + \frac{\varepsilon}{\tau} R^2 \right) + A_1(Q^2)y + A_2(Q^2)y^3,$$

(4)

$$II : \sigma_R = G_M^2(Q^2) \left(1 + \frac{\varepsilon}{\tau} R^2 \right) + B_1(Q^2)y + B_2(Q^2)y(\ln|y|)^2.$$  

(5)

Eq. (4) (parametrization I) is chosen if $F(Q^2, y)$ is assumed to be analytic to allow a Taylor expansion around $y = 0$. It vanishes at $\varepsilon = 1$ ($y = 0$) and stays finite at $\varepsilon = 0$ ($y = 1$), a feature agrees with the results of the model calculations of [7, 11]. Eq. (5) (parametrization II) is allowed if $F(Q^2, y)$ is required to be only smooth, but not analytic in $0 \leq y \leq 1$. The logarithmic or double-logarithmic functions are considered since such types of functions often appear in the loop diagrams [11]. With functions $A_1(Q^2)$ and $B_1(Q^2)$ assumed to be proportional to $G_D(Q^2) = 1/(1 + Q^2/0.71)^2$ and $R$ taken from experiments. It was found that data for $\sigma_R$ can be fitted with both fits I and II nicely [15].
2.3 $e^+p/e^-p$ ratio and normal asymmetries

The amplitudes for positron-proton ($e^+p$) and electron-proton scatterings can be written as $T^{(\pm)} = \pm T_1 + T_2\gamma$, where $(\pm)$ correspond to the charge of positron and electron, and $T_1, T_2, T_2\gamma$ denote the scattering amplitudes with $1\gamma$ and $2\gamma$ exchanged, respectively. We then have

$$R^{(\pm)} \equiv \frac{\sigma(e^+p)}{\sigma(e^-p)} \simeq 1 + Re \left( \frac{T_2\gamma}{T_1} \right),$$

i.e., measurements of the ratio of $e^+p$ and $e^-p$ cross sections provide a direct probe of the real part of the TPE amplitude. Thus measurement of $R^{(\pm)}$ presents a stringent test on any model of TPE. For example, even though both parametrizations I and II of Eqs. (4) and (5) describe well the experimental data for $R^{(\pm)}$ of comparable quality, their predictions for $R^{(\pm)}$ are very different as shown in Fig. 3. However, a recent measurement of $R^{(\pm)}$ from Novosibirsk at $Q^2 = 1.6$ GeV$^2$, $\epsilon = 0.4$, gives $R^{(\pm)} = 1.056 \pm 0.011$ clearly rules out parametrization II. Several experiments are being performed or planned at JLab and OLYMPUS to measure $R^{(\pm)}$ for $0.5 < Q^2 < 2.2$ GeV$^2$ with $\epsilon > 0.2$. We refer the readers to Ref. 10 for more details.

The imaginary part of $T_{2\gamma}$ can be accessed from measurement of the target normal asymmetry $A_N \equiv (\sigma^+ - \sigma^-)/(\sigma^+ + \sigma^-)$, where $\sigma^{(\pm)}$ is the cross section for unpolarized electrons scattering from a proton target with spin parallel (antiparallel) to the direction normal to the scattering plane. To order $\alpha_{em}$, the target normal asymmetry is given by $A_N = 2Im(T_{2\gamma}/T_{1\gamma})$. No data on $A_N$ have been collected yet. The imaginary part of $T_{2\gamma}$ can also be accessed by measuring the beam normal asymmetry for electrons polarized normal to the scattering plane from unpolarized protons. Discussions about the experimental and theoretical status of beam normal asymmetries can be found in 17.

3 Two-boson exchange in parity-violating elastic electron-proton scattering

Strangeness content in the proton is one of the most intriguing questions in hadron structure study. Indications on the contribution of strange quarks to the nucleon properties came from neutrino, electron deep inelastic scatterings and pion-nucleon sigma term. Other observables were later suggested, including double polarizations in photo- and electroproduction of $\phi$ meson 18, and asymmetry $A_{PV}$ in scattering of longitudinally polarized electrons from unpolarized protons.

Parity-violating asymmetry $A_{PV} \equiv (\sigma_R - \sigma_L)/(\sigma_R + \sigma_L)$, where $\sigma_R(L)$ is the cross section with a right-handed (left-handed) electron, arises from the interference of weak and electromagnetic amplitudes. Weak neutral current elastic scattering is mediated by the $Z$-exchange and measures form factors which are sensitive to a different linear combination of the three light quark distributions. When combined with proton and neutron electromagnetic form factors and with the use of charge symmetry, the strange electric and magnetic form factors, $G_E^s$ and $G_M^s$, can then be determined 19. Specifically, within the one-boson exchange approximation, $A_2$ in $A_{PV} = A_1 + A_2 + A_3$ is given as

$$A_2 = a G_E^s G_E^s + \frac{\tau G_M^s G_M^s}{\sigma(G_E^s)^2 + \tau(G_M^s)^2} = a G_E^s \frac{\beta G_M^s}{\sigma_R(Q^2, \epsilon)},$$

Fig. 3 Predictions for the ratio $R^{(\pm)}$ of $e^+p$ and $e^-p$ cross sections with parametrizations I (left panel) and II (right panel) of Eqs. (4) and (5), respectively, as compared with the recent data of 16.

\[ \text{Fig. 3 Predictions for the ratio } R^{(\pm)} \text{ of } e^+p \text{ and } e^-p \text{ cross sections with parametrizations I (left panel) and II (right panel) of Eqs. (4) and (5), respectively, as compared with the recent data of 16.} \]
where \( \beta = \tau G_E^p/\epsilon G_E^p \), \( a = G_F Q^2/4\pi\alpha\sqrt{2} \) with \( \alpha \) the fine structure constant, while \( A_1 \) and \( A_3 \) depend only on nucleon electromagnetic form factors, Weinberg angle \( \theta_W \), and \( G_A^Z \), which can be determined from other measurements. This is a clean technique to access the charge and magnetization distributions of the strange quark within nucleons and four experimental programs SAMPLE, HAPPEX, A4, and G0 [20] had been undertaken to measure this important quantity, which is small and ranges from 0.1 to 100 ppm. It is hence imperative to reduce theoretical uncertainty on the radiative corrections in order to arrive at a more reliable interpretation of experiments.

Leading order radiative corrections to \( A_{PV} \), including the box diagrams in Fig. 4 and other diagrams, have been extensively studied by Marciano and Sirlin (MS) [21] and widely used in the global analyses. Among those corrections, the interference between \( \gamma Z \) exchange (\( \gamma Z E \)) of Fig. 4 with OPE diagram, was evaluated within the zero momentum transfer approximation, i.e., \( Q^2 = 0 \). As we have seen in Sec. 2, it is important to evaluate loop diagrams rigorously and accurately. This has been done in [22, 23], and by Tjon and his collaborators [24] for both the interference between \( \gamma Z E \) and OPE diagrams, as well as that between TPE and \( Z \)-exchange diagrams. Both groups employed the hadronic model developed in [7] and included nucleon and the \( \Delta \) in the intermediate states.

Partonic calculations on the interference between TPE and \( Z \)-exchange, and that between \( \gamma Z E \) with OPE have been carried in [23] and [20], respectively.

4 Summary and Outlook

We have briefly reviewed the advances made in the last decade to understand the discrepancy in the values of \( \mu_p G_E/G_M \) of the proton as extracted from the traditional Rosenbluth separation of elastic cross sections and the new polarization transfer measurements. Model calculations, both hadronic and partonic, strongly indicate that the two-photon exchange corrections can at least account for more than half of the discrepancy observed. However, we are still not yet in the stage of quantifying the two-photon exchange effects as the theoretical models still suffer from uncertainties such as the on-shell approximation for the vertices in hadronic calculations and the extrapolations beyond the region of validity in partonic calculations. Considerable challenge remains on the theoretical side to obtain a reliable estimate of two-photon exchange effects.

Just as in the case of nucleon-nucleon interaction, phenomenological parametrization of the two-photon exchange corrections could be useful. One of the attempts made in this direction is discussed to illustrate its potential usefulness. \( e^+p/e^-p \) ratio is known to probe the real part of the TPE amplitude and can serve as a stringent test on TPE models. We show that indeed a recent measurement of \( e^+p/e^-p \) ratio does rule out one of the proposed phenomenological parametrizations of the two-photon exchange corrections. More high precision experiments as planned in JLab, OLYMPUS, etc. will all be very helpful to further quantify the TPE effects.

At the end of the day, the final goal of this game is to include TPE contributions in the analyses of experimental observables and subsequently extract the form factors which are the aims where we begin. Such an analysis has been attempted in [27] and more will definitely come.

TPE corrections have been found or speculated to play an important role in many other observables like hyperfine splitting in hydrogen, pion form factors, \( R_{EM} \) and \( R_{SM} \) ratios in \( N \rightarrow \Delta \) excitation, as well as in reactions like electron-nucleus scatterings, deep-inelastic scatterings etc. We refer the interested readers to the recent excellent reviews [17] and [10] for details.

Recent studies on the two-boson exchange corrections in the parity-violating elastic electron-proton scattering, where the proton strangeness form factors have been extracted, are also discussed. It is found
that the modification incurred in going beyond the MS approximation is indeed significant for some data and the TBE corrections to $A_{PV}$ depends strongly on $Q^2$ and $\epsilon$. However, because the effects from nucleon and $\Delta$ intermediate states tend to cancel out each other, the total effects are not large. It remains to be seen what role of other higher resonances might play. This will be a daunting task in hadronic model calculation since it will introduce many not well-determined coupling constants and cut-off parameters. Recently, a dispersion calculation of $\gamma^Z E$ corrections to $Q_{weak}$ has been attempted\[28\]. Many studies along this line have ensued\[29, 30\]. It will be very helpful and welcome if the dispersion relation approach could also be attempted for the TPE corrections.

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