Fermi liquid to Luttinger liquid transition at the edge of a two-dimensional electron gas

M. Hilke\textsuperscript{b}, D.C. Tsui, M. Grayson\textsuperscript{a}, L.N. Pfeiffer\textsuperscript{a} and K.W. West\textsuperscript{a}

\textsuperscript{a} Dept. of Elect. Eng., Princeton University, Princeton, New Jersey, 08544
\textsuperscript{b} Walter Schottky Institut, Technische Universit"at München, Garching, 85748, Germany

(Received March 28, 2001; PACS numbers: 73.40.In, 71.10.Pm, 72.20.My, 73.21.Fs, 73.40.Lq)

We present experimental results on the tunneling into the edge of a two-dimensional electron gas (2DEG) obtained with a GaAs/AlGaAs cleaved edge overgrown structure in a strong perpendicular magnetic field. While the 2DEG exhibits typical fractional quantum Hall features of a very high mobility sample, we observe the onset of a non-linear current-voltage characteristic in the vicinity of \( \nu = 1 \). For filling factor \( \nu < 1 \) the system is consistent with a non-Fermi liquid behavior, such as a Luttinger liquid, whereas for \( \nu > 1 \) we observe an Ohmic tunneling resistance between the edge and a three dimensional contact, typical for a Fermi liquid. Hence, at the edge, there is a transition from a Luttinger liquid to a Fermi liquid. Finally, we show that the Luttinger liquid exponent at a given filling factor is not universal but depends on sample parameters.

In one dimension and in the presence of interactions, a metal can have a Fermi surface in agreement with Luttinger’s theorem \cite{1}. However, fermionic quasi-particles are no longer possible and the elementary excitations are replaced by bosonic charge and spin fluctuations dispersing with different velocities. Hence, this one-dimensional metal is no longer a Fermi-liquid but a Luttinger liquid (LL) \cite{2}. Models describing one-dimensional interacting Fermions were first considered by Tomonaga and Luttinger \cite{3}.

In a pioneering work on the fractional quantum Hall (FQH) states, Wen \cite{4} showed that the edge modes can be represented as chiral LLs. The chirality is due to the presence of a magnetic field, which forces the edge states to propagate in one direction. A unique feature of the chiral LL is the absence of back-scattering, i.e., no localization can occur. A key theoretical result is the existence of power-law correlation functions, which lead to the vanishing of the momentum distribution function at \( k_F \) following a power-law, i.e., \( n(k) \sim |k - k_F|^\alpha \), where \( \alpha \) is related to the interaction strength. As a consequence, the tunneling current-voltage (I-V) characteristics follows \( I \sim V^\alpha \). For the particular case, where the filling factor \( \nu = 1/3 \), Wen predicted that \( \alpha = 3 \), hence the tunneling current should vanish like \( I \sim V^3 \) when tunneling from a Fermi liquid into a Luttinger liquid. This is very different from the Fermi liquid-to-Fermi liquid tunneling which would be Ohmic.

Following the predictions of Wen \cite{4} and others \cite{5}, several experimental attempts were made in order to observe this power-law dependence. The first experiments considered a gate induced constriction to tunnel between two FQHE liquids \cite{6}. Unfortunately, in some cases the results were consistent with a power-law \cite{6} but not in others \cite{7}. This was largely attributed to the smoothness of the potential barrier causing the possible reconstruction of the edge and an energy dependent tunneling barrier. Chang et al. \cite{8} avoided this problem by growing a sharp tunneling barrier on the cleaved edge of a two-dimensional electron gas (2DEG). They obtained a good power-law over more than a decade in voltage to obtain a tunneling exponent \( (\alpha \sim 2.7 \text{ at } \nu = 1/3) \) close to Wen’s prediction.

When moving away from the primary fraction \( \nu = 1/3 \) to \( \nu = p/(2np \pm 1) \) (where \( p \) and \( n \) are positive integers), the edge cannot be described anymore by a single LL edge mode but requires several additional modes, the number and nature of which depends strongly on the particular fraction and, moreover, the disorder becomes important because of possible inter-mode scattering. The overall structure of these states is reviewed in ref. \cite{9}. As a consequence, the recent experimental result from Grayson et al. \cite{10} came as a surprise, because instead of observing a plateau-like structure between \( \nu^{-1} = 2 \) and \( 3 \), as expected from both the composite fermion theory \cite{11,12,13} and a disordered edge in the hierarchical model \cite{14}, they observed a linear dependence of the exponent on the inverse filling factor, \( \alpha \sim \nu^{-1} \). Recent theories have attempted to account for this behavior using different approaches \cite{14} and are currently under debate. Subsequent experimental work, indicated a weak plateau feature at \( \alpha \approx 2.7 \), suggesting a stable single edge mode, but for \( \nu^{-1} = 3 \) or 4.5, depending on the sample \cite{13}. At low filling factors, a tunneling resonance consistent with a Luttinger liquid behavior was observed recently \cite{15}.

In this article, we probe for the first time the edge of the 2DEG over a large range in filling factors from \( \nu = 1/3 \) to \( \nu = B = 0 \), hence also higher Landau levels. In order to achieve this we start with a very high mobility 2DEG \((\mu > 10^8 \text{cm}^2/\text{Vs})\) confined in a symmetrically doped GaAs/AlGaAs quantum well. The quantum well is then placed in the molecular beam epitaxy (MBE) chamber and cleaved for a subsequent growth along the (110) direction. First an atomically sharp barrier of (Al\textsubscript{p}Ga\textsubscript{1-p})\textsubscript{As} is grown and then a 5000 \AA n-doped GaAs layer \cite{16}. In order to probe a large voltage range,
we fixed the height of the barrier at a high value, i.e., 200meV by using \( x = 20\% \) as barrier material. The low temperature (30mK) and zero field tunneling resistance can be varied by using different barrier widths (20-120Å).

For the thinnest barrier (20 Å) the tunneling resistance is even smaller than the 2D resistance, whereas for the thickest 120 Å barrier the tunneling resistance is 200kΩ. Therefore, most of the results in this work are based on the 60 Å and 120 Å barriers as they cover our \( B \)-field range of interest, including the low field regime. Thanks to the high tunneling resistance we were able for the first time to measure the 3D Shubnikov-de Haas oscillations directly on the edge because the 2D does not short the edge. This enables us to extract the effective 3D Fermi energy directly. In Fig. 1, we have plotted the magnetoresistance of the edge, measured with an AC resistance bridge and obtain a 3D Fermi energy of 33 meV \((n_{3D} = 4.5 \times 10^{11} cm^{-3})\). This can be compared to the Fermi energy of the 2DEG, which is 13 meV for a 2DEG density of \( n = 2.2 \times 10^{11} cm^{-2} \) in a 300 Å quantum well (6 meV). Hence, the Fermi energy mismatch is 20 meV. The value of the mismatch is important when evaluating the internal electrical field building up at the interface due to the mobile carriers. This bend bending could affect the density at the edge of the 2DEG compared to the density of the bulk 2DEG.

![Image](image_url)

**FIG. 1.** Magnetoresistance measured directly on the edge as a function of the normalized inverse magnetic field at 30mK. The contacts used were 1 and 8 as represented in the sketch of fig. 2.

The magnetoresistance trace of the 2DEG system is shown in Fig. 2 and exhibits typical features between Landau levels characteristic of a very high mobility 2DEG. In the same figure, the tunneling resistance across the barrier, between the 2DEG and the edge, is plotted as a function of the magnetic field for different voltages. The overall magnetic field dependence is dominated by an exponential increase of the resistance as a function of \( B \). Two effects contribute to this increase in resistance. Indeed, for a perfect barrier the momentum conservation parallel to the barrier is suppressed by a quantizing magnetic field. However, scattering along the barrier can reduce this effect. Second, the penetration length orthogonal to the barrier is exponentially suppressed by the square of the magnetic length.

The most important feature in this figure is that below 6.5 T the tunneling resistance is independent on the voltage, whereas above 6.5 T the resistance is strongly voltage dependent, i.e., the resistance is not Ohmic. When using a narrower barrier, such as 60 Å, we obtain the same \( B \)-field for the onset of non-linearity, but at a much smaller tunneling resistance. Hence, the onset of non-linearity only depends on \( B \) and not on the value of the tunneling resistance. This overall behavior is very suggestive to a transition from a Fermi liquid (Ohmicity) to a LL, where \( B_C \approx 6.5 \) T would be the critical field of the transition. This \( B \)-field corresponds to a filling factor of \( \nu = 1.3 \). In order to investigate this behaviour further we now analyze the non-linearity in more detail and compare it with Wen’s theoretical prediction for a chiral LL.

![Image](image_url)

**FIG. 2.** The magnetoresistance, \( R_{4-2,5-6} \), of the 2DEG as a function of the magnetic field is plotted in dotted lines. The tunneling resistance, \( R_{6-8,6-8} \), is plotted for voltages of \( 2,1,0.5,0.16,0.06 \) mV applied across the barrier. \( R_{4-2,5-6} \) was obtained by measuring the voltage between contacts 5 and 6 and applying an AC (\(< 5Hz\)) current (10nA) between contacts 4 and 2. For \( R_{6-8,6-8} \) we applied a negative DC bias on contact 8 corresponding to the edge. All traces were obtained at 30 mK.

We start by plotting in Fig. 3 the I-V characteristics across the barrier for different \( B \)-fields at 30 mK. The I-V’s are obtained by measuring the DC traces, unlike refs. [8][10], where an AC voltage was applied. For clarity we have only plotted the IV traces, where a positive bias was applied to the edge, i.e., electrons tunnel out of the 2DEG. The negative bias data is identical up to a voltage of about 2 mV, above which strong asymmetries arise. For a more detailed discussion of these asymmetries beyond 2 mV the reader is referred to [13]. When limiting our range of interest from 0.2 to 2 mV, two regimes can be identified: for \( B \leq B_C \), where \( B_C \approx 6.5 \) T, the I-V’s are linear, but for \( B > B_C \) the I-V’s follow a power-law larger than 1. To extract the power-law exponent, \( \alpha \), we have performed a least square fit including all data in that range. The fits are shown in dotted lines along with
the data in solid lines. The quality of the fit is very good over this voltage range, but below 0.2 mV there are deviations from the power-law, which can be attributed to finite temperature effects and to limitations in our experimental sensitivity. Above 2 mV we also have deviations from a power-law, which are probably not related to the LL behavior, because similar deviations are also seen at B=0 and are not symmetric with respect to the sign of the bias. For the data above 6.5 T the shapes of the IV’s are very similar to the ones in refs. [8], which were attributed to the LL behavior.

When further increasing B, the power-law exponent α increases gradually and is plotted in Fig. 4 for two different barriers 120 Å and 60 Å. Below 6.5 T, α is essentially constant. The crossing of the linear extrapolation between the points above 7 T (dotted line) and α = 1 is at 6.7 T, which is very close to the onset of nonlinearity BC = 6.5 T of Fig. 2. Interestingly, the small difference in resistance at different voltages between 4.5 and 6.5 T in Fig. 2 is reflected in Fig. 4, by a value of α slightly above 1, for the same B-field range.

![Figure 3](image3.png)

**FIG. 3.** Current voltage characteristics on a logarithmic scale at B=4, 4.5, 5, ..., 14, 15.3 T. The thick line corresponds to B=6.5 T. The dotted lines are the linear log-log fits to the data.

For reference, we have also plotted the results for the 120 Å barrier width in Fig. 4. Although, the two barriers (60 and 120 Å) have very different tunneling resistances, the overall dependence of α is very similar. Because the tunneling resistance of the 60 Å barrier is much smaller we can extract the exponent α to a larger B. The experimental limit is given by our current noise level (10-100 fA). For fields below 4 T the tunneling resistance of the 60 Å barrier becomes comparable to the two-terminal resistance of the 2D, implying that the barrier resistance can no longer be extracted. In contrast, the 120 Å barrier has a tunneling resistance much larger than the two-terminal resistance even below 4 T (at B=0 the tunneling resistance is still 200 kΩ). Therefore, using the 120 Å barrier we can also cover all the low field behavior. The error bars shown in Fig. 4 are given by the mean square deviation from a power-law. The extraction of the exponent is very robust when comparing positive to negative bias. Indeed, when extracting the exponents from the negative bias IV’s (not shown) the exponents fall within less than 10%.

![Figure 4](image4.png)

**FIG. 4.** Exponent α extracted from the power-law part of the I-V curves as a function of B and filling factor ν. The circular dots are for a 120Å wide barrier and the square dots for a 60Å wide barrier.

Thus, for B > BC the I-V’s follow a power-law indicative of a Luttinger liquid, whereas below BC we have a standard Fermi-liquid, hence BC represents the transition between a Fermi-liquid and a Luttinger-liquid. Moreover, this demonstrates that below BC the rich structure of the fractional quantum Hall system is dominated at the edge by a standard Fermi-liquid mode. In our samples this transition occurs at a filling factor of νC ≃ 1.3. This is very different to the value obtained in refs. [9], where the extrapolation to α = 1 would yield ν ≃ 0.73. Further, we obtain in our case a slope of α ≃ 2.0ν−1 − 0.55 whereas in ref. [10] they obtained α ≃ 1.16ν−1 − 0.58. Hence, experimentally, for a fixed 2DEG filling factor the value of the exponent is dependent on sample parameters, although the onset of nonlinearity does not depend on the barrier width nor on the magnitude of the tunneling resistance, when all other parameters are the same (like for our samples with different barrier widths 60Å and 120Å). Hence, the value of the exponent for a given filling factor is different to the chiral LL predictions by Wen [4].

A possible explanation could be a shift in the local density distribution close to the edge. Indeed, our tunneling edge is at the interface between two differently doped semiconductors, which leads to the band bending within the characteristic depletion length [11]. In the simplest case of 2 adjacent 3D n-doped semiconductors, the local electron density of the side with the lowest Fermi energy is enhanced at the interface. For a 2D-3D interface this
is not necessarily the case anymore. Indeed, Levitov and co-workers [12] used a Thomas-Fermi model, but without including quantum confinement effects, nor a magnetic field, to calculate the density distribution of the 2DEG close to the edge for a similar structure. They found that the 2D edge density could be about 25% smaller than in the bulk, when the 3D has a 20 meV higher Fermi energy than the 2D like in our case. They further calculated that the 2D edge density increases continuously with increasing 3D density. However, when comparing two samples with the same 2D Fermi energy of 17 meV but two different 3D Fermi energies (33 and 62 meV), we found a very similar onset of non-linearity for both samples at \( \nu \simeq 0.65 \). This suggests that a more involved theoretical treatment is necessary in order to account for a possible edge density redistribution. In particular, it may be crucial to include the effects of a large \( B \)-field as it is possible that the average density at the edge is dependent on \( B \).

Assuming, however, that we do have a different 2D filling factor at the edge than in the bulk and that the effective \( \nu_{\text{edge}} = 1 \) for the edge occurs at 6.7 T because the \( \nu = 1 \) state is expected to behave like a standard Fermi-liquid [11], we would obtain an edge density 24% smaller than the bulk density. For the samples used by Grayson et al. [10], the intercept where \( \alpha = 1 \) would occur at \( \nu_{\text{edge}} = 1 \) if the density at the edge of the 2DEG is assumed to be 40% larger than in the bulk. Rescaling our data by -24% (\( \nu_{\text{edge}} = 0.76 \times \nu \)) and Grayson’s data by +40% we find in both cases that \( (\alpha - 1) \sim 1.6(\nu_{\text{edge}}^{-1} - 1) \). This is very intriguing, but it is not clear whether this is generic or not. This shift in edge vs. bulk 2D density, could also explain the behavior in ref. [15], where they observe a weak plateau behavior for lower filling factors than expected.

In the following we compare our results with existing theories. Most theoretical results fall in two main classes. In one group, calculations are consistent with \( (\alpha - 1) = 2(\nu_{\text{edge}}^{-1} - 1) \) and in the other group the calculations are consistent with \( (\alpha - 1) = (\nu_{\text{edge}}^{-1} - 1) \) [14]. However, none of the experimental curves fall clearly on one of these theoretical dependencies, even if we assume a shift in the edge versus bulk 2D density. We believe that a more detailed analysis of the edge distribution is needed in order to resolve this issue.

In conclusion, we have observed a Luttinger liquid - to - Fermi liquid transition by tunneling into the edge of a 2DEG system. For high filling factors, our results indicate that a Fermi-liquid outer edge mode dominates the edge physics at higher filling factors. We further showed that the Luttinger liquid exponent is not universal in relation to the bulk two-dimensional filling factor but that it can depend on other sample parameters. This dependence, however, could be due to a shift in the 2D edge density.

We would like to acknowledge Albert Chang, Leonya Levitov and Shivaji Sondhi, for helpful discussions. This work was supported in part by the National Science Foundation.

\[ \text{References:} \]

[1] J.M. Luttinger, Phys. Rev. 119, 1153 (1960).
[2] F.D.M. Haldane, J. Phys. C 14, 2585 (1981).
[3] J.M. Tomonaga, Prog. Theor. Phys. 5, 544 (1950) and J.M. Luttinger, J. Math. Phys. 4, 1154 (1963).
[4] X.G. Wen Phys. Rev. Lett. 64, 12206 (1990); Phys. Rev. B 41, 12838 (1990); Phys. Rev. B 44, 5708 (1991).
[5] C.L. Kane and M.P.A. Fisher, Phys. Rev. B 46, 2768 (1992); Phys. Rev. B 46, 15233 (1992).
[6] F.P. Milliken, C.P. Umbach and R.A. Webb, Sol. State Comm. 97, 309 (1996); I.J. Maasila and V.J. Goldman, Phys. Rev. B 55, 4081 (1997); Phys. Rev. B 57, R4273 (1998); P.J. Turley et al., Physica B 249-251, 410 (1998); D.C. Glattli (EP2DS) Physica E 1999.
[7] M. Ando, A. Endo, S. Katsumoto, and Y. Iye, Physica B 249-251, 426 (1998).
[8] A.M. Chang, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. 77, 2538 (1996).
[9] X.G. Wen, Adv. Phys. 44, 405 (1995).
[10] M. Grayson, D.C. Tsui, L.N. Pfeiffer, K.W. West, and A.M. Chang Phys. Rev. Lett. 80, 1062 (1998).
[11] A.V. Shytov, L.S. Levitov and B.I. Halperin Phys. Rev. Lett. 80, 141 (1998).
[12] L.S. Levitov, A.V. Shytov and B.I. Halperin, cond-mat/0005016.
[13] C.L. Kane, M.P.A. Fisher and Polchinski Phys. Rev. Lett. 72, 4129 (1994).
[14] S. Conti and G. Vignale, J. Phys. - Cond. Mat., 10, L779 (1998); J.H. Han and D.J. Thouless, Phys. Rev. B 55, 1926 (1997); J. H. Han, Phys. Rev. B 56, 15806 (1997); U. Zülicke and A.H. MacDonald, Phys. Rev. B 60, 1836 (1999); A.M.M. Pruisken, B. Skoric and M.A. Baranov, Phys. Rev. B 60, 16838 (1999); A. Alekseev, V. Cheianov, A.P. Dmitriev, V. Yu Kachorovskii, cond-mat/9904074; A. Lopez and E. Fradkin, Phys. Rev. B 59, 15323 (1999); D.V. Khveshchenko, Sol. St. Commun. 111, 501 (1999); D.-H. Lee and X.-G. Wen, cond-mat/9809160.
[15] A.M. Chang, M.K. Wu, C.C. Chi, L.N. Pfeiffer and K.W. West, Phys. Rev. Lett. 86, 143 (2001).
[16] M. Grayson, D.C. Tsui, L.N. Pfeiffer, K.W. West and A.M. Chang, Phys. Rev. Lett. 86, 2645 (2001).
[17] L.N. Pfeiffer, K.W. West, H.L. Stormer, J.P. Eisenstein, K.W. Baldwin, D. Gershoni and J. Spector, Appl. Phys. Lett. 56, 1697 (1990).
[18] M. Hikle et al. in preparation (2001).
[19] M. Grayson, C. Kurdak, D.C. Tsui, S. Parhizkar, S. Lyon and M. Shayegan, Sol. St. Electronics 40, 233 (1996).