Abstract We discuss M(atrix) theory compactification on $T^6$. This theory is described by the large $N$ limit of the world volume theory, of $N$ Kaluza-Klein monopoles in eleven dimensions. We discuss the BPS states, and their arrangement in $E_6$ multiplets. We then propose the formulation of the world volume theory of KK monopoles in eleven dimensions that decouples from the bulk. This is given by a large $N_1$ m(atrix) theory with eight supercharges, corresponding to the quantum mechanics theory of $N_1$ zero-branes inside the Type IIA Kaluza-Klein monopole. Various limits of the construction are considered.
1 Introduction

It has been realized in the last couple of years that all the superstring theories in ten
dimension are only some limits of a different theory, called M-theory [1, 2], that lives in eleven
dimensions. The superstring theories, as their name suggests, were described by the interaction
of strings, and are the only known consistent quantum theories that include gravity. When taking
the strong coupling limit of the Type IIA string theory to obtain M-theory, the strings became
membranes. It might be that M-theory can be described as a theory of interacting membranes, however such a description is not known. Instead another description
was proposed [3], that M-theory in the infinite momentum frame (IMF) is the large \( N \),
strong coupling limit of \( SU(N) \) quantum mechanics, whose Lagrangian coincides with that
of \( N \) zero-brane at short distances. This is known as M(atrix) theory. The M(atrix) theory
description of M-theory has by now passed many tests. It has the correct graviton graviton
scattering [3, 4, 5] and the correct interaction between the extended objects [3] of M-theory
with [13], and without [6] \( \beta \beta \) [14, 15, 16] eleven dimensional momentum transfer. Taking
the limit that corresponds to weakly coupled Type IIA theory gives the correct description
of string interactions [14, 13, 16].

M(atrix) theory compactification on \( T^d \) have recently been extensively studied. The low
energy description of M(atrix) theory compactified on \( T^d \) are given by a super Yang-Mills
theory in \((d+1)\) dimensions with sixteen supercharges, compactified on a dual torus \( \tilde{T}^d \) [3, 17].
For \( d \leq 3 \) these theories are renormalizable and hence do not require extra information to
be defined in the ultraviolet. For \( d \geq 4 \) the theories are not renormalizable and hence ill
defined as they stand.

For all \( d \) the behavior at strong coupling is of interest, as it is related in many beautiful
ways to known stringy phenomena [18, 19, 20, 21, 22]. For \( d \geq 4 \) this is also related to the
ultraviolet behavior.

Clearly for \( T^d \) with \( d > 3 \) one has to add information how to treat the ultraviolet regime.
An obvious possibility is to regulate the theories using string theory, by mapping the SYM
theories to some brane [23] configurations in string theory. One however may suspect that
this will necessarily introduce bulk effects (and in particular quantum gravity effects). For
example the theory on the branes includes excitation of open strings stuck to the brane, in
general two open strings can meet, become a closed string and leave the brane. This process
is the coupling of the bulk to the brane theory. If this coupling is not suppressed then it seems
unlikely we can describe the theory on the brane by itself. This is only a problem for \( d > 3 \).
For \( d = 4, 5 \) the ultraviolet behavior has been suggested to be governed by the \((2,0)\) theory
in six dimensions and by the theory of coincident NS-five branes at zero string coupling
respectively [24, 23]. In these theories it was argued that the bulk decouples. In particular
the theory of the NS-five branes at zero coupling, which describes the compactification of
the M(atrix) theory on \( T^5 \), was argued to be connected to the theory of non-critical strings
living in \( 5+1 \) dimensions [26, 23], which do not include gravity.

In this paper we discuss the compactification of M(atrix) theory on \( T^6 \). We propose
that this is described by the theory of \( N \) coinciding Kaluza-Klein (KK) monopoles in eleven
dimensions, in a limit where the theory on the KK monopoles decouples from the bulk. The
analog of the non-critical strings of the NS-five brane theory are now membranes inside the world volume of the KK monopole. Similar to the case of the string theories and M-theory, we do not propose a description in terms of interacting membranes. Instead we propose a description (in the IMF) of that theory as a m(atrix) theory.

In section (2) we review the compactification on $T^d$ with $d \leq 5$. In section (3) we discuss the compactification on $T^6$ and the representation of the U-duality group. In section (4) we propose a m(atrix) theory description of the theory of KK monopoles in eleven dimensions. Taking certain compactification limits we get known descriptions of the theory supporting our proposal. We also make a clear map between compactification of M(atrix) theory on $T^6$ and the compactification of the proposed theory. We end with some speculations.

While this paper was finalized \cite{27, 28, 29} appeared, with some overlap of ideas and results.

2 Compactification: The General Picture

The super-Yang-Mills prescription for compactification \cite{3, 17} breaks down if the number of compactified direction is larger than three. This is because these theories are non-renormalizable, and thus ill defined. One may hope to some how add some extra degrees of freedom to regulate the theory in the UV, using string theory. The idea is to match to the super Yang-Mills description at low energy a configuration of branes in string theory. From the knowledge of string dualities and behavior at large string coupling many observations can be made on the strong coupling limit of those theories. If these theories need extra information in the UV, than one has to take various limits of parameters in order to decouple the bulk and still leave a non trivial theory on the brane. As every thing is embedded in string theory the limit is a well defined theory (hopefully non trivial). Even before we worry about the decoupling from the bulk, the comparison to brane configuration in string theory gives some information on these theories.

First let us set-up some notation. Compactification of the M(atrix) theory are defined by the parameters: the Planck length $l_p$, the radii of the compact directions $L_i$, and $R$ which is the radius of the compact eleventh direction. $R$ determines the coupling of the uncompactified $(0 + 1$ dimensional) theory,

\begin{align}
\frac{g_{ym}^2}{g_s} &= g_s l_s^{-3} = \frac{R^3}{l_p^3} \\
g_s &= R/l_s, \quad l_s^2 = \frac{l_p^3}{R}.
\end{align}

The M(atrix) theory compactification is defined as the large $N$ $SU(N)$, super-Yang-Mills (SYM) theory, with sixteen supercharges, in $d + 1$ dimensions on a dual torus with radius ($\Sigma$), and coupling ($g_{ym}$) given by (we drop factors of $2\pi$),

\begin{align}
\Sigma_i &= \frac{l_s^2}{L_i} = \frac{l_p^3}{RL_i}.
\end{align}
\[ g_{ym}^2 = g_s l_s^{-3} \prod_i \left( \frac{l_s^2}{L_i} \right) = \frac{R^3}{l_6^3} \prod_i \left( \frac{l_s^3}{RL_i} \right). \] (4)

As the super-Yang-Mills is just the theory of coincident branes in string theory these expressions can basically be derived from standard T-duality. The string coupling of the system of coincident branes is then

\[ G_s = g_s \prod_i \frac{l_s}{L_i}. \] (5)

2.1 Examples

We now briefly review the known result of compactification up to \( T^5 \), stressing the qualitative understanding coming from the relations to brane configurations in string theory.

2.1.1 \( T^2 \)

We will start with M(atrix) theory on \( T^2 \). The Yang-Mills coupling is given by \( g_{ym}^2 = \frac{R}{L_1 L_2} \).

We model this by a theory of coincident D-2-branes in a string theory with coupling \( G_s \) and string length \( l_s \) obeying

\[ g_{ym}^2 = G_s l_s^{-1}. \] (6)

We compare some energy scales in the Yang-Mills theory and in the string theory. Take for example the non-threshold bound state of a D2-brane and a D0-brane in string theory, and compare that in the appropriate approximation to the energy of a magnetic flux in the Yang-Mills theory. The energy of the bound state in the string theory is \([30, 23]\)

\[ E_s = \left( \frac{\Sigma_1 \Sigma_2}{G_s l_s^3} \right)^2 + \left( G_s l_s^{-2} \right)^{1/2}. \] (7)

The energy in the YM theory is then the first term in the expansion of equation (7) above the D2 brane mass. One gets that the extra energy above the D2-brane energy is

\[ \frac{l_s}{G_s \Sigma_1 \Sigma_2}. \] (8)

The energy of one flux of magnetic field in the YM theory is

\[ E_{ym} = (g_{ym}^2 \Sigma_1 \Sigma_2)^{-1}. \] (9)

Comparing equation (8) to equation (9) gives equation (6).

We now look at what happens when the YM coupling becomes large. We think of the limit as a limit in M-theory. This limit corresponds to fixing the eleven dimensional Planck length and taking the string coupling to be large, thus the string length is also very small. This is the same as large \( G_s \). A collection of D2-branes at weak string coupling lives in ten dimensions that is why the YM theory has seven scalars representing the transverse excitations. But at strong string coupling the eleventh dimension opens up and the two-branes are free to move in a fully rotational invariant way in eleven dimensions so one expects at strong coupling
which in this case is the infra-red a fixed point with \( SO(8) \) symmetry \[18, 16\]. The length of the new eleventh dimension is

\[
R_{11} = G_s l_s = g^2_{ym} l^2_s = \frac{l^3_p}{L_1 L_2}. \tag{10}
\]

2.1.2 \( T^4 \)

One models the M(atrix) theory on \( T^4 \) with a theory of D4-branes in string theory with string coupling \( G_s \). This was analyzed in \[21\]. Similar consideration as before gives \( g^2_{ym} = G_s l_s \). Now we are interested in the ultraviolet behavior which is like looking at the strong coupling behavior (as the YM coupling has dimensions of length). In the Type IIA string as the string coupling becomes strong the eleventh dimension opens up. Unlike the case of the D2-brane, the D4-brane in string theory is wrapped around the eleventh direction and is an M5-brane. The radius of the circle is just

\[
R_{11} = G_s l_s = g^2_{ym} = \frac{l^6_p}{RL_1 L_2 L_3 L_4}. \tag{11}
\]

Thus the ultraviolet theory is the theory of coincident M5-branes wrapped on \( T^5 \) \[24\] with radii \( \Sigma_i \) given by equation (4), and \( R_{11} \) given by equation (11).

2.1.3 \( T^5 \)

The theory here is the theory of D5-branes. The YM coupling and the string coupling are

\[
g^2_{ym} = \frac{l^9_p}{R^2 L_1 L_2 L_3 L_4 L_5}, \tag{12}
\]

\[
G_s = \frac{l^6_p}{RL_1 L_2 L_3 L_4 L_5}. \tag{13}
\]

The ultraviolet behavior is the strong coupling behavior. Now \( g^2_{ym} = G_s l_s^2 \), using S-duality of the string theory this is a theory of NS-five branes in string theory with

\[
g^2_{NS} = G_s^{-1} (G_s^{1/2} l_s)^2 = l_s^2. \tag{14}
\]

Thus in the ultraviolet the string coupling goes to zero but leaving an interacting theory on the NS-five branes \[25\]. It was argued in \[24, 25\] that this theory contains non-critical string with finite tension, and thus M(atrix) theory on \( T^5 \), may be described by a non-critical string theory.

3 Matrix theory on \( T^6 \)

We model the theory by coinciding D6-branes in a string theory with coupling \( G_s \). The YM coupling is

\[
g^2_{ym} = \frac{l^{12}_p}{L_1 L_2 L_3 L_4 L_5 L_6 R^3}. \tag{15}
\]
But now one has the relation
\[ g_{ym}^2 = G_s l_s^3 = L_p^3 \]
where \( L_p \) is the eleventh dimensional Planck length defined in the new string theory as
\[ L_p = G_s^{1/3} l_s. \]

Now let us explore the ultraviolet region. Again this is just the strong coupling region and the eleventh dimension opens up to size
\[ R_{11} = G_s l_s = \frac{l_p^9}{L_1 L_2 L_3 L_4 L_5 L_6 R^2} \]
The other sides are
\[ \Sigma_i = \frac{l_p^3}{R L_i} \]
The M-theory interpretation of the D6-brane is as of a KK monopole of eleven dimension [36], so M(atrix) theory on \( T^6 \) is described by the large \( N \) limit of \( N \) KK monopoles in eleven dimensions.

Let us give a short description of some of the properties of the KK monopole. The multi-KK monopole is a gravitational background, in \( n + 4 \) dimensions, of the form \( R^n \times N_4 \) where \( N_4 \) is the Euclidean multi-Taub-NUT metric [31]. It is given by,
\[
\begin{align*}
    ds^2 &= -dt^2 + \sum_{i=1}^{n-1} dx_i^2 + V^{-1} (dx_7 + \vec{w} d\vec{r})^2 + V d\vec{r}^2, \\
    V &= 1 + \sum_{l=1}^{N} \frac{4A}{|\vec{F} - \vec{r}_l|} \\
    \vec{\nabla} \times \vec{w} &= \nabla V.
\end{align*}
\]
For this metric to be non-singular \( x_7 \) must have periodicity of \( 16\pi A \). \( \vec{r}_l \) denotes the location of the centers of the different KK monopoles. We will refer to the compact direction \( x_7 \) as the NUT direction.

The world volume theory of the KK monopole in eleven dimensions is described by a vector multiplet with three scalars [32, 33]. When two KK monopoles coincide there is an enhanced gauged symmetry coming from extra massless states. The configuration of KK monopoles includes a sphere of area proportional to the distance between the centers of the KK monopoles. This is because the physical radius of the NUT direction goes to zero at \( \vec{r} = \vec{r}_l \). A two brane wrapped around the sphere has zero mass in the limit in which the KK monopoles coincide, giving the extra massless states.

Let us now recover the compactification on \( T^5 \) by taking one of the sizes, say \( L_6 \rightarrow \infty \). In this case \( \Sigma_6 \rightarrow 0 \). Let us now take this direction as the coupling direction, the theory

\footnote{For some recent discussion see [32, 34, 35].}
is well approximated by the theory of KK monopoles in ten dimensional Type IIA string theory with coupling proportional to $\Sigma_6$. The new string coupling becomes
\[
G_{s}^{\text{new}} = (\Sigma_{6} L_{p}^{-1})^{3/2} = \left( \frac{L_{1} L_{2} L_{3} L_{4} L_{5}}{L_{6}^{3} L_{p}} \right)^{1/2}. \tag{23}
\]
and the new string length is
\[
(l_{s}^{\text{new}})^{2} = (G_{s}^{\text{new}})^{-2/3} L_{p}^{2} = \frac{l_{p}^{9}}{L_{1} L_{2} L_{3} L_{4} L_{5} R^{2}}. \tag{24}
\]
The KK monopole has five dimensional compact world volume directions with sizes $\Sigma_{i}$, $i = 1 \cdots 5$, and the Taub-NUT direction is just $R_{11}$ from equation (18). This direction however is also very small in the limit $L_{6} \to \infty$, so we T-dualize along this direction. We end up with a theory on NS-five brane in type IIB, with string length $l_{s}^{\text{new}}$, the NS-five branes are wrapped on the five-torus of sides $\Sigma_{i}$, there is an extra sixth direction which is compact given by T-dualizing $R_{11}$
\[
L_{6}^{\text{new}} = \frac{(l_{s}^{\text{new}})^{2}}{R_{11}} = L_{6}, \tag{25}
\]
and there is a new string coupling (because of the T-duality) given by
\[
G_{s}^{\text{new}}(IIB) = G_{s}^{\text{new}} \frac{l_{s}^{\text{new}}}{R_{11}} = \frac{L_{1} L_{2} L_{3} L_{4} L_{5} R}{l_{6}^{3}}. \tag{26}
\]
The YM coupling on the NS-five branes is
\[
g_{NS}^{2} = G_{s}^{\text{new}}(IIB) (L_{s}^{\text{new}})^{2} = \frac{l_{p}^{3}}{R} = l_{s}^{2}. \tag{27}
\]
Taking $L_{6} \to \infty$ we get exactly the result in the $T^{5}$ case. Of course one does not have to take that limit, and then one gets the theory of NS-five branes with one transverse compact direction.

### 3.1 BPS states and U-duality

In this section we describe the BPS states and show they fall into the U-duality group representations. An analysis of the moduli space of M(atrix) theory compactification was done in [37].

For M-theory on $T^{6}$ the U-duality group is $E_{6}$. What are the BPS states? Let us look at configurations of branes that can wrap around some dimensions occupied by the KK monopole in eleven dimensions and preserve some of the original supersymmetries of M-theory. Let us for convenience choose to work in Type IIA language, with The KK monopole becoming the D6-brane. There are two different possibilities. First there are the bound states of the D6-brane and another object, which preserves half of the original supersymmetries, they form non-threshold bound states (from the point of view of the IIA theory), with mass
\[
M^{2} = M_{6}^{2} + M_{D}^{2}, \tag{28}
\]
where $M_D$ is the mass of the lighter object (we drop factors of $2\pi$) and

$$M_6 = \frac{\Sigma_1 \cdots \Sigma_6}{G_s l_s^7} = \frac{V_6}{G_s l_s^7}. \quad (29)$$

In the YM theory they will correspond to states with energy

$$E = \frac{M_p^2}{2M_6}. \quad (30)$$

The configurations are

1. An elementary string parallel to the D6-brane forms a non-threshold bound state with it, there are 6 such states. For the elementary string $M_{\text{string}} = \frac{\Sigma}{l_s}$, this gives for the Yang-Mills (YM) energy

$$E = \frac{\Sigma^2 G_s l_s^3}{2V_6} = \frac{\Sigma^2 g_{ym}^2}{2V_6}. \quad (31)$$

These can be represented in the YM theory as electric fluxes.

2. A four brane forms a non-threshold bound state, there are 15 of those. Now $M_4 = \frac{\Sigma_1 \cdots \Sigma_4}{G_s l_s^5}$, this gives a YM energy of

$$E = \frac{(\Sigma_1 \cdots \Sigma_4)^2}{2V_6 G_s l_s^5} = \frac{V_6}{2g_{ym}^2 (\Sigma_5 \Sigma_6)^2}. \quad (32)$$

These can be represented in the YM theory as magnetic fluxes.

3. A KK monopole of IIA string wrapped around the six dimensions. There are 6 such states. If the Taub-Nut direction is $\Sigma_6$ then

$$M_{kk} = \frac{\Sigma_1 \cdots \Sigma_6 \Sigma_6}{G_s l_s^8}. \quad (33)$$

This gives a YM energy of

$$E = \frac{(\Sigma_1 \cdots \Sigma_5 \Sigma_6)^2}{2G_3 l_s^9 V_6} = \frac{V_6 \Sigma_6^2}{2g_{ym}^6}. \quad (33)$$

These are not easily represented in the YM theory. This is the same problem as representing the transverse five-brane in M(atrix) theory (see also below).

These states transform under $SL(6, \mathbb{Z})$ as the 6, 15, and 6 respectively. Together these states gives the 27 of $E_6$. Let us see how they transform into one another. Start with the state of the D6-brane with world volume in directions $(1, 2, 3, 4, 5, 6)$, bounded to a D4-brane with world volume directions $(1, 2, 3, 4)$. If we perform T-duality on directions $(2, 3, 4)$, then S-duality and then T-duality again on directions $(2, 3, 4)$ we end up with the bound states of
a D6-brane and a fundamental string stretched in the 1 direction. Similar transformation maps the bound state of the D6-brane and the D4-brane to the bound state of the D6-brane and the KK monopole of ten dimensions. So to get the $E_6$ one needs a transformation of the form $(T^3ST^3)$, to be present in the regulated form of the world sheet theory\footnote{This kind of symmetry was used in \cite{20,11} to describe the NS-five brane in M(atrix) theory, in a very similar set-up.}. This is just the uplifting of the S-duality of the $3 + 1$ dimensional YM theory (notice we always turn the D6-brane to a D3-brane by T-duality, then S-dual and then T-dual again. This always gives us back a D6-brane). A possible manifestation of this symmetry in an “improved” SYM was recently considered in \cite{27}.

Second there are the bound states at threshold that preserve one quarter of the original supersymmetries. The mass of the bound states is then just

$$M = M_6 + M_D$$

So the YM energy is just $M_D$. The states are

1. A D2-brane wrapped on any two of the six dimensions is a threshold bound state with the D6-brane, there are 15 such states. Now, $M_2 = \Sigma_1 \Sigma_2 / G_s l_s^3$, so the YM energy is

$$E = \frac{\Sigma_1 \Sigma_2}{G_s l_s^3} = \frac{\Sigma_1 \Sigma_2}{g_{ym}^2}.$$ (35)

These can be described by instanton configurations in the YM, as can be seen from the coupling of the D6-brane world volume to background RR charges \footnote{This kind of symmetry was used in \cite{20,11} to describe the NS-five brane in M(atrix) theory, in a very similar set-up.}. Notice also that the membrane living inside the D6 brane worldvolume has a tension, using equation (16),

$$T_2 \sim g_{ym}^2 = L_p^{-3}.$$ (36)

2. The NS-brane gives another 6 threshold bound states. Now $M_{NS} = \Sigma_1 \Sigma_2 / G_s l_s^6$, this then gives a YM energy of

$$E = \frac{V_6}{g_{ym}^2 \Sigma_6}.$$ (37)

These are not easily represented as a background. This is the same problem as for the KK monopoles above.

3. A graviton moving in one of the six directions gives 6 states which are threshold bound states with the D6-brane. The YM energy is then $E = 1 / \Sigma_1$. These are just the KK modes in six dimensions.

These states form a $\bar{27}$ of $E_6$. Again they transform to each other by $SL(6, Z)$ and a $T^3ST^3$ transformation. The brane of the $27$ and $\bar{27}$ are mapped into each other by electric magnetic duality in ten dimensions (Hodge duality).
4 m(atrix) Theory of KK Monopoles

In the previous section we made the observation that M(atrix) theory on $T^6$ is the large $N$ limit of coincident KK monopoles in eleven dimensions. The low energy theory of the KK monopoles in eleven dimensions is known and is described by a vector multiplet in seven dimensions. However this can not be the whole story as it is ill defined. We are faced with the prospect of understanding the theory of KK monopoles in eleven dimensions, and finding the limit of parameters where the bulk decouples.

We start by noting an interesting analogy. In the case of $T^5$ it has been argued that the theory includes non-critical strings, and may actually be formulated as a non-critical string theory [25]. For some possible constructions see [39, 40]. This is also formulated as the theory of coinciding NS-five branes. The compactification of M(atrix) theory on $T^6$ can be thought as the strong coupling limit of coinciding D6 branes. If we compactify a transverse dimension to $N$ coinciding NS-branes, with vanishing string coupling, a series of S-dualities and T-dualities brings us to the situation of strongly coupled D6 branes. The string like excitation of the NS-five brane theory are formally mapped to membrane excitations. All this sounds much like the relation between ten dimensional string theories and M-theory. In that case it is not clear that M-theory can be formulated as the theory of membranes. However one candidate for the description of M-theory (in the IMF) is the BFSS M(atrix) theory. As we will see the analogy is quite complete.

We propose a definition of the theory of coincident KK monopoles in eleven dimensions (in the IMF) as the large $N$ quantum mechanics of zero-branes with some extra matter restricted to the Coulomb branch. This theory will be called m(atrix) theory as compared with M(atrix) theory.

Imagine in the original M(atrix) model of BFSS we wish to add some p-brane. From the point of view of the zero-branes this is some background, for instance a four-brane is described by turning on a background with $\varepsilon^{ijkl} X_i X_j X_k X_l \neq 0$ [20]. Alternatively, if the p-brane zero-brane system preserves a quarter of the supersymmetries, then one can model this by adding more matter content that exactly reproduces the same effect. For example in the case of a D4-brane this is done by adding hypermultiplets [41]. So we end up with a quantum mechanics theory, with eight supercharges, describing the effect of the p-brane on the zero-brane. If one restricts to the branch where the zero-branes are inside the p-brane (in the D4-brane it is the Higgs branch), this describes the theory on the brane as a m(atrix) theory (the theory should be in a limit where the Higgs branch and the Coulomb branch are disconnected). In the large $N$, and strong coupling limit, the eleventh dimension will open up and the D4-brane will become M5-brane. This was one of the approaches taken, to describe the $(2,0)$ theory as a m(atrix) theory [39]. A similar suggestion was made in [12].

We wish to describe the KK monopole of eleven dimensions. We should start with a brane in ten dimensions with the following properties. First it should become the KK monopole of eleven dimensions at strong Type IIA string coupling. Second a configuration with extra zero-branes leaves a quarter of the supersymmetries unbroken. These two criteria are satisfied by the KK monopole in ten dimensions (the D6-brane does not satisfy the second criterion).
We propose that there is a theory that lives on the world volume of the KK monopole in eleven dimensions, and decouples from the bulk. This theory is the restriction to the Coulomb branch, of the large $N_1$ limit of gauge quantum mechanics, with eight supercharges, and some matter. Its Lagrangian coincides with the Lagrangian describing the interaction between $N_1$ zero-branes inside the KK monopole of ten dimensions.

The motion of the zero branes inside the KK monopole corresponds to the Coulomb branch of the QM theory while the motion outside the KK monopole corresponds to the Higgs branch of the QM theory. We wish to decouple the theory on the KK monopole from the ten-dimensional bulk. For this we need to decouple the Coulomb branch from the Higgs branch. Since the mass of the hypermultiplets on the Coulomb branch is proportional to the gauge coupling, in the strong gauge coupling limit the Higgs branch decouples from the Coulomb branch.

The KK monopole in ten-dimensions has one special direction. The metric is of the form $R^6 \times N_4$ where $N_4$ stands for the four dimensional Euclidean Taub-NUT metric. One of the four directions in $N_4$ is a circle (the NUT direction). It is convenient to perform T-duality along this circle. The resulting configuration consists of a collection of NS-five branes and D-strings in Type IIB theory. We choose the orientation of the branes as follows

\begin{align}
\text{NS – fivebrane} & \{0, 1, 2, 3, 4, 5\} \\
\text{D1 – brane} & \{0, 6\}
\end{align}

(38) (39)

The decoupling of the Coulomb branch corresponds to the strong coupling limit, which is like flowing to the infra-red. Thus we only need the corresponding zero modes. The zero modes of this theory can be deduced using the same reasoning as in [46, 47].

If we want to describe $m$ KK monopoles in eleven dimensions we need to take $m$ NS-five branes on the circle $x_6$. There will be $N_1$ D1-branes wrapped around the $x_6$ direction. These in turn can break into segments stretched between each pair of adjacent NS five branes. The theory we are interested in is the theory on the D1 branes in the large $N_1$ limit. This theory contains 8 supercharges and is 0+1 dimensional. The R-symmetry for this theory is $SO(5) \times SO(3)$ (or rather its extension to include spinor representations) as will be explained below.

The matter content consists of a gauge group $U(N_1)^m$ with $m$ hypermultiplets transforming in the bi-fundamental representations of each of the adjacent factors in the gauge group. The $k$-th multiplet is transforming under the $(N_1, \bar{N}_1)$ representation (with its complex conjugate) of $U(N_1)_k \times U(N_1)_{k+1}$. There is cyclic symmetry such that the $m$-th $U(N_1)$ group is coupled with a bi-fundamental representation to the first $U(N_1)$ group. The freedom in adding an arbitrary constant phase to the position of the branes on the circle leads to the decoupling of the $U(1)$ above. Similarly by translation invariance along the (1,2,3,4,5) directions we get only $m - 1$ moduli for the $U(1)$ factors of the gauge group. This sets the sum over all these moduli to be zero. A possible non-zero parameter can be introduced for the sum of these moduli to be non-zero along the lines discussed in section 4 of [14]. For $m = 1$ the gauge group is $U(N_1)$ with matter in the adjoint representation. A mass term for the adjoint can be introduced as for the $m \neq 1$ case. We will shortly see that this mass term is essential to reproduce the correct result.
The presence of these branes break the ten-dimensional Lorentz symmetry into two groups $SO(1,5) \times SO(3)$, where the first group acts on $(0,1,2,3,4,5)$ and the second group acts on $(7,8,9)$. These groups are identified as R-symmetries of the corresponding 0 + 1 dimensional theory. There are 4 parameters for a given NS five brane which describe its position in the transverse space $(6,7,8,9)$. The $(7,8,9)$ position of the NS brane, which transform as a vector of the $SO(3)$ group, parametrizes the FI couplings of the $U(1)$ fields in the gauge group. A $(7,8,9)$ distance between each two adjacent NS five branes is proportional to the FI coupling of the $U(1)$ field in between these two branes. The $x_6$ positions of the NS branes give the 0 + 1 dimensional gauge couplings. Denote the $x_6$ position of the NS five branes by $t_i$, $i = 1, \ldots, m$. Then the gauge coupling of the $i$-th gauge group is $g_i^2 = \frac{g_s^2}{t_i^2|t_i - t_{i+1}|}$.

We can give an alternative description in the zero brane – KK monopoles system. The $(7,8,9)$ positions of the KK monopoles are the FI couplings of the $U(1)$ factors in the gauge group. In addition the QM gauge couplings correspond to periods of the two form along the cycles given by the multi Taub-NUT solutions.

This theory was used to describe zero-brane near orbifolds, and M(atrix) theory in the presence of ALE singularity in [43, 44, 45]. There the analysis was focused on the Higgs branch, while here we discuss the Coulomb branch. In the analysis of [43, 44] they looked at the quantum mechanics on an ALE space, the metric on the Higgs branch is then the ALE metric. This is different than the metric of the Taub-NUT (which we might expect to get if the quantum mechanics we are discussing, is the one describing the effect of the KK monopole in ten dimensions on zero branes). Both metric have the same form but the defining function is $V = 1 + \frac{4}{r}$ in the Taub-NUT case and $V = \frac{1}{r}$ in the ALE case. If however $A \to \infty$ we can drop the 1. This is equivalent from our original point of view to taking a KK monopole with a large circumference of $x_6$ which after T-duality reduces to the case where the D1 brane is on a very small circle i.e. we have a theory in (0 + 1) dimensions. This is however the limit that we are looking at, for the decoupling of the Coulomb branch from the Higgs branch.

\[3\]

4.1 Dynamics

The description of the moduli space of vacua is as follows. The following discussion should be taken in the context of the Born-Oppenheimer approximation which serves as a pictorial description of the physics. The ends of the D1 branes can move inside the NS branes. The position of the D1 is given by 5 scalars in the $(1,2,3,4,5)$ directions which parametrize the Coulomb branch. A motion into the Higgs branch can be done by going over to the Origin of the Coulomb branch. A set of $m$ D1 branes can reconnect along the circle, each between two adjacent NS branes, and form a closed D1 brane which is wrapped around the $x_6$ direction. At this point the D1 brane can move far from the system of NS branes along the $(7,8,9)$ positions. This is a transition to the Higgs branch where the $(7,8,9)$ distance parametrizes the expectation value for the hypermultiplets. The transition from the Coulomb to the Higgs branch describes the interaction of the KK monopoles with the bulk.

\[3\] The vanishing of that term (or rather the limit $A \to \infty$) was connected in [28, 29] to the decoupling from the gravity fields in the bulk.
Another transition possible from the QM point of view is to turn on a FI term for one of the $U(1)$ fields. This corresponds to moving the corresponding NS brane away from the other NS branes along the $(7,8,9)$ directions. For this to happen all the D1 segments from both sides of the NS brane should reconnect in such a way that no segments are left attached to the NS brane. In field theory this means that the $U(N_1) \times U(N_1)$ theory connected to this NS brane is Higgsed by giving expectation value to the bi-fundamental hypermultiplet localized on this NS brane. The value of the FI term sets the expectation value for this hypermultiplet. The resulting gauge group is $U(N_1)$ which is the diagonal embedding of these two gauge groups. It is coupled to the other bi-fundamentals which are located at the two ends of the D1 branes on neighboring NS branes. We are left with a system of $m - 1$ NS branes coupled to segments of $N_1$ D1 branes with a matter content just as for $m - 1$ KK monopoles and $N_1$ D0 branes. For the KK monopole – D0 system this mechanism is just a decoupling of one KK monopole from the other $m - 1$ KK monopoles by moving it along the $(7,8,9)$ directions.

It should be emphasized that the two transitions described above correspond to effects which happen in the bulk of space-time and does not correspond to the description of the theory on the KK monopoles. This is since these transitions are to or in the Higgs branch. A necessary condition for having a Coulomb branch is to set the FI parameters to zero and thus to the same $(7,8,9)$ positions for the NS branes.

When all gauge couplings of the QM theory are infinite and there are no FI terms the QM theory has a $SU(m)$ global symmetry. This symmetry is visible in the brane construction by setting the positions of the NS branes along the $(6,7,8,9)$ directions to coincide. When the gauge couplings are finite this symmetry is broken explicitly to $U(1)^{m-1}$. For this reason such a symmetry is not visible in the Lagrangian formulation of the theory since it describes the theory around the weakly coupled region. Only the diagonal part is visible. In the IR limit, the couplings are sent to infinity and thus we expect this symmetry to be visible in the spectrum (unless all states are singlets of this symmetry). From the point of view of the KK monopole theory this symmetry is expected. It just corresponds to the enhanced gauge symmetry on the world volume of the KK monopoles when $m$ of them coincide. For the eleven dimensional KK monopoles 3 moduli are needed to be tuned per one KK monopole (the FI parameters), while for the ten-dimensional theory an additional scalar is needed to be tuned (the gauge coupling).

Let us look at the case $m = 1$. The QM theory contains a $U(N_1)$ gauge theory coupled to hypermultiplets in the adjoint representation. There is a mass term for the adjoint hypermultiplet coming from non-trivial boundary conditions on the D1 branes ending on the NS brane. When the mass of the adjoint is zero the number of supersymmetries is enhanced from 8 to 16. The QM theory then becomes just the BFSS proposal. At this point the QM gauge theory will not be able to identify if the eleven dimensional theory contains a KK monopole or not. Thus the theories for $m = 0$ (BFSS) and $m = 1$ may appear the same. However, a mass term for the adjoint field can be introduced only in the presence of the NS brane. Only then there can be non-trivial boundary conditions on the D1 branes which will allow for the non-zero mass. In the limit in which the mass term is very large the adjoint field decouples and we are left with a supersymmetric YM theory with gauge group $U(N_1)$.
and 8 supercharges. So our proposal now includes another parameter in the gauge theory. Clearly for our proposal to be true this parameter can not be zero.

If this parameter is not infinite there are two problems. First, it gives another parameter that does not seem to exist in the theory we are after. Second, in order to have total decoupling from the bulk, the D1 brane segments should never be able to close, even at one point of the moduli space. If the mass parameter is zero, the segments can join at the origin of the Coulomb branch, and thus there is no real total decoupling. If the mass parameter is non zero, there is no such point. However as we are dealing with a 1 + 1 dimensional field theory the moduli space is only in the sense of a Born-Oppenheimer approximation. To ensure decoupling we should then take the mass parameter to infinity. For the case \( m = 1 \) this leaves us with a large \( N_1 U(N_1) \) QM gauge theory with eight supercharges.

The large \( N \) limit of \( U(N) \) QM gauge theory with 16 supercharges can be thought of as describing a membrane in eleven dimensions in the lightcone frame \[48\]. A large \( N U(N) \) QM gauge theory with 8 supercharges appears to describe a membrane in seven dimensions in the light cone frame (for a general review see \[49\]). This fits nicely with our result, as the theory in 6 + 1 dimensions should include a membrane.

Going back to the \( m \neq 1 \) case. The bi-fundamental hypermultiplets receive their mass from moduli on the Coulomb branch. Due to the special properties on the \( x_6 \) circle, there are only \( m - 1 \) such mass moduli. A restriction comes from the sum of these masses to be zero. The above discussion for the case \( m = 1 \) then suggests that an additional mass parameter needs to be introduced using the special boundary conditions on the D1 branes. This mass parameter should probably be taken to infinity.

### 4.2 Some Limits of m(atrix) Theory

Let us consider some limits of this construction. This will illuminate some relations to other results, and strengthen our argument. If the original KK monopole has \( d \) compact directions then the theory will be that of a D\((d+1)\)-brane in the presence of NS-five branes. The D\((d+1)\) brane is oriented along the \( x_6 \) direction and along \( d \) more directions that are parallel to the NS-five brane. In the \( x_6 \) direction the D\((d+1)\) brane can break into segments stretched in between two adjacent NS branes. The corresponding \( d + 1 \) dimensional theory which describes the system of KK monopoles with \( d \) compact directions has 8 supercharges and contains the same matter content as described in the previous subsection.

#### 4.2.1 The Non-Critical String Limit

The m(atrix) theory we have just described can be thought of as describing the theory that lives on the D6 branes in the IMF at strong Type IIA coupling (actually infinite as the NUT direction is infinitely large). If one of the directions of the D6-brane is compact and vanishingly small this can be T-dualized to give the theory on D5 branes at infinite coupling. By S-duality this gives the theory on Type IIB NS-five branes at zero string coupling. This theory was argued to be non trivial and to contain non-critical strings \[25\]. In our construction this means that one of the NS-five brane directions is vanishingly small and we can T-dualize it to get a system of NS-five brane in Type IIA and a D2 brane along
$x_6$ and let's say $x_1$. In the infinite gauge coupling limit this becomes a $(1 + 1)$ dimensional theory.

Our proposal is that the Coulomb branch of this large $N_1$ theory describes the theory of non-critical strings in $(5 + 1)$ dimensions, in the IMF. This is the analogous construction to that of Type IIA strings of [13]. In this case these strings are closed so they are mapped to closed non-critical strings. Let us look at the case $m = 1$. This will be a $U(N_1)$ gauge theory in $1 + 1$ dimensions. Classically the moduli space is $(R^4)^{N_1}/S_{N_1}$. If this would be exactly correct one will reproduce the spectrum of the Green-Schwartz superstring in six dimensions, which is not Lorentz invariant [14]. However unlike the case of [13] here we have only eight supercharges, one expects the moduli space to be corrected. A possible description for Type IIA NS-five branes at zero coupling was given in [40] and also discussed in [39].

There is a possible mirror symmetry argument to relate the present construction to the construction of [40, 39]. The theory on the Higgs branch which was studied in [40] has a brane description in terms of D2 branes stretched between D4 branes on the circle. In the strong Type IIA coupling this system turns into membranes stretched between five branes in M-theory. On the other hand the system of D2 branes stretched between NS branes in Type IIA discussed above turns into the same system in the strong Type IIA coupling. The M-theory system cannot distinguish from which system it has descended. This translates into mirror symmetry between the two theories where the Higgs branch of one theory is mapped to the Coulomb branch of the other theory and vice versa. Clearly many details need to be worked out.

4.2.2 The Weakly Coupled D6 Limit

Let us take the NUT direction to be very small. The KK monopole in eleven dimensions can then be thought of as a D6 brane in weakly coupled Type IIA boosted to infinite momentum along one of its world volume directions. This should be described by open superstring theory with some Dirichlet directions.

Now the set up is a collection of D1 branes wrapped around a large $x_6$ direction and NS-five branes which span directions $(1, 2, 3, 4, 5)$. In general the D1 branes will break in between the NS-five branes. The condition that we restrict to the Coulomb branch is the condition that the D1 brane can not move away from the NS-five branes. This can happen if some of the D1 brane segments join to form a circle, and the D1 brane can then leave.

The analysis of the boundary condition on the D1 brane segments is the same as in [46]. On the Coulomb branch the D1 brane can move inside the NS-five brane, but can not leave the NS-five brane, thus the boundary condition for the Coulomb branch are Dirichlet boundary conditions in directions $(7, 8, 9)$ and Neumann in directions $(1, 2, 3, 4, 5)$. This is just what one expects for an open string on a D6 brane in the IMF. Each D1 brane segment has two labels which denote on which NS-five brane it ends. They transform under the global hidden symmetries described earlier. They correspond to the Chan-Paton factors of the open string. One should construct the “long strings” in the large $N$ limit, similar to

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4For a recent analysis of the Coulomb branch of some 1+1 dimensional field theories see [50].

5A. H. would like to thank discussions on this issue with Jacques Distler.
If the D1 branes join and leave the NS-five brane (the Higgs branch), far along the flat direction they will be approximately described by just the world sheet theory of the D1 branes, which has the right interactions to describe the closed Type IIA strings [15].

### 4.2.3 Connection to the (2,0) Theory

If the KK monopole in eleven dimensions had two compact world volume directions that are small and its NUT direction was also small than our theory should reduce to the theory of Type IIA D4 branes in the IMF. In our construction we see that we have a theory of $N$ D3 branes oriented along directions $(1, 2, 6)$ and NS-five branes along directions $(1, 2, 3, 4, 5)$. This is the theory that was originally studied in [16]. The theory of Type IIA D4 branes in the IMF is then described by the Coulomb branch of the large $N$ limit of the theory on the D3 branes. However, this is related by S-duality (mirror symmetry in the 2 + 1 dimensional infra-red theory) to the Higgs branch of the large $N$ limit of D3 branes oriented the same way together with D5 branes replacing the NS five branes. The Higgs branch of this theory is the same as $N$ D1 branes oriented orthogonal to D3 branes.

Now take the proposed theory to describe the M5 brane in the IMF [39]. It consists of the theory of large $N$ D0 branes and D4 branes. Suppose one of the directions of the M5 brane, say $x^1$, which is not the IMF direction, is compact and small. We can take this direction to be a string coupling direction. This reduces to the description of weakly coupled Type IIA D4 branes in the IMF. We can T-dualize along this compact $x^1$ direction to get a system of which consists of $N$ D1 branes orthogonal to D3 branes. This is the system which was discussed in the last paragraph. This correspondence needs a further investigation. It suggests an interesting connection, along the lines of mirror symmetry, between the two approaches for defining the $(0, 2)$ theory using the Higgs branch and the KK monopole theory using the Coulomb branch.

### 4.3 Back to M(atrix) theory on $T^6$

When compactifying the BFSS M(atrix) theory on a torus we are led to theories that live on branes. As was stressed in the introduction, one is looking for a theory that lives on the brane, is well defined and decouples from the bulk. We have given in the previous section a proposal for a m(atrix) description of the theory that lives on the KK monopole of eleven dimensions, and decouples from the bulk, in the infinite momentum frame. Now one needs to connect the variables of the KK theory to those of M(atrix) theory on $T^6$. The parameters of the theory living on the KK monopole in the limit we have considered are the six lengths of the world volume, and the tension of the membrane that lives inside. The analogous data of M-theory are the ten lengths and the tension of the membrane which is $\sim l_p^{-3}$. In our m(atrix) model the length scale should then be $l_g \sim T^{-1/3}_{\text{membrane}}$. Bellow this scale the theory is reasonably described by 6+1 dimensional SYM. Above this scale the SYM description breaks down, and one has to use a different theory, the m(atrix) model. The mapping between the variables of the M(atrix) theory on $T^6$ and the theory of compactified KK monopole is as

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6This theory was analyzed in [17].
follows. The NUT direction is infinite. This corresponds to the decoupling of the Coulomb branch from the Higgs branch, and thus from the bulk. The sides of the KK monopole are (as in section (3))

\[ \Sigma_i = \frac{l_p^3}{R L_i}, \]  

(40)
and the tension of the membrane is (from section (3)):

\[ T_{\text{membrane}}^{-1} = L_p^3 = g_{ym}^2 = \frac{l_p^{12}}{L_1 L_2 L_3 L_4 L_5 L_6 R^3} = l_g^3 \]  

(41)

This maps the variables of the M(atrix) compactification to the description given in section (4).

There are problems with compactifying more than three directions in our proposal. This is because one ends up with a large \( N_1 \) gauge theory in more than four dimensions which are not renormalizable. We hope our proposal is a first step in the right direction.

5 Discussion

In this paper we have analyzed the compactification of M(atrix) theory on \( T^6 \). We were led to explore the theory living on the KK monopole of eleven dimensions. We suggested a non-perturbative description of the theory living on the KK monopole that decouples from the bulk. It is given by a m(atrix) description of a large \( N \), strong coupling limit, \( SU(N) \) quantum mechanics with eight supercharges and appropriate matter content. The analogy with the relationship between string theory and the M(atrix) description of M-theory was stressed. As is the case with M(atrix) theory, the description of other branes (membrane, four-brane, etc), inside the seven dimensional world volume theory, can arise as some background. Their existence will be related to the central charges of the supersymmetry algebra \([6]\). It will also be interesting to do a similar construction for KK monopoles of the Heterotic string. This presumably will give the analogs of the Heterotic M(atrix) model.

Compactifications of the theory were discussed, but it is not yet possible to get the full description of M(atrix) theory on \( T^6 \). This model however as it has a lot of similarity to the BFSS model and its relation to the string theory may be useful in better understanding M(atrix) theory. Of course this theory as a theory which presumably does not contain gravity is of interest in its own right.

Let us end with some speculation. If one wants to describe certain theories in the IMF it seems possible to do that by taking the large \( N \), strong coupling limit of some supersymmetric \( SU(N) \) quantum mechanics. This was given in [38] for the M5 brane and in this paper for the KK monopole of eleven dimensions. These results in a theory in \( 5 + 1 \) and \( 6 + 1 \) dimensions, respectively. In order to end with a theory in \( 10 + 1 \) dimensions (in the IMF) one needs to start with a nine-brane in Type IIA theory\footnote{Perhaps this is the nine-brane recently discussed in \[32\].}. Then take the strong coupling limit to decouple from some unknown bulk, and of course the large \( N \) limit. Maybe this
is a possible interpretation of the BFSS model. The limits we need to take in the (5 + 1) and (6 + 1) dimensional theories, in order to decouple the bulk, are the exact same limits we need to take in the BFSS model in order to get a unitary Lorentz invariant theory.

This may suggest (as has been suggested by others) that M-theory can be embedded in a higher dimensional theory.

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