Towards Relativistic Atomic Physics and Post-Minkowskian Gravitational Waves

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Abstract

A review is given of the formulation of relativistic atomic theory, in which there is an explicit realization of the Poincare' generators, both in the inertial and in the non-inertial rest-frame instant form of dynamics in Minkowski space-time. This implies the need to solve the problem of the relativistic center of mass of an isolated system and to describe the transitions from different conventions for clock synchronization, namely for the identifications of instantaneous 3-spaces, as gauge transformations. These problems, stemming from the Lorentz signature of space-time, are a source of non-locality, which induces a spatial non-separability in relativistic quantum mechanics, with implications for relativistic entanglement.

Then the classical system of charged particles plus the electro-magnetic field is studied in the framework of ADM canonical tetrad gravity in asymptotically Minkowskian space-times admitting the ADM Poincare' group at spatial infinity, which allows to get the general relativistic extension of the non-inertial rest frames of special relativity. The use of the York canonical basis allows to disentangle the tidal degrees of freedom of the gravitational field from the inertial ones. The York time is the inertial gauge variable describing the general relativistic remnant of the gauge freedom in clock synchronization. However now each solution of Einstein’s equations dynamically determines a preferred notion of instantaneous 3-spaces. The linearization of this canonical formulation in the weak field approximation will allow to find Hamiltonian Post-Minkowskian gravitational waves with an asymptotic background and without Post-Newtonian expansion in non-harmonic 3-orthogonal gauges.

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Due to the developments in atom interferometry proposals are appearing for using them as detectors of gravitational waves. While this is an exciting challenge at the experimental level, it raises theoretical problems concerning the framework to be used to put together atomic physics, special relativity and gravity. All experiments on the Earth are using a non-relativistic formulation of quantum mechanics, with special and general relativistic corrections when extended to space near Earth (see GPS, the experiment on the Space Station, geodesy satellites), and a post-Newtonian description of gravity seen as an external potential to be added to the Schrödinger equation. Atoms and photons are assumed to move on geodesics which strictly speaking do not exist in the non-relativistic context (see for instance Ref.[1]). As a consequence it would be desirable to have a relativistic framework where all the actors of the game are consistently defined at least at the classical level with a subsequent introduction of quantization for matter (no accepted formulation of quantum gravity exists).

Regarding atoms, usually they are described as quantum particles with some structure in a sector of quantum field theory with a fixed number of particles. Moreover instead of treating them as relativistic bound states, they are approximated with non-relativistic quantum particles. However, non-relativistic quantum mechanics and the theory of entanglement, with the associated foundational problems connected to quantum non-locality, are formulated in Galilei space-time. In it both Newtonian time and the Euclidean 3-space, with the associated notion of spatial distance, are absolute and Maxwell equations do not exist. The 'photons' in the discussions about atom interferometry, entanglement and teleportation are only states with two polarizations in a two-dimensional Hilbert space: their carrier cannot be a ray of light in the eikonal approximation moving along a null geodesic. The existing inclusion of electro-magnetism at the order $1/c$ made by atomic physics destroys the Galilei group and does not allow a consistent definition of the Poincaré one, namely a consistent special relativistic formulation of atomic physics in Minkowski space-time.

Therefore the first problem is to find a classical description of charged particles and of the electro-magnetic field in Minkowski space-time. Then the theory has to be reformulated in Einstein’s general relativity in asymptotically Minkowskian space-times so to have a geometrical description of gravity with a smooth limit in the weak field approximation to Minkowski space-time (Post-Minkowskian gravity) and then to Galilei space-time with the Post-Newtonian expansions.

Once this classical framework is under control, one must understand how to include the quantum aspects of atomic physics. The first step is to develop a consistent description of relativistic quantum mechanics in Minkowski space-time, in which atoms and photons are simulated by massive and massless particles with some spin structure respectively. Moreover there must be a well defined non-relativistic limit for massive particles. Then a phenomenological inclusion of gravity could be done by having the special relativistic quasi-classical (mean value of position operators in a scheme like the one used in Ehrenfest theorem) worldlines of atoms and photon replaced with Post-Minkowskian time-like and null geodesics and with suitable Post-Newtonian potentials describing the mutual gravitational interactions of particles.

Here I review the status of the construction of this framework and its open problems. Since we are still far away from a complete control, it is difficult to say which could be the implications at the experimental level at this stage.
The main problem in going from Galilei space-time to Minkowski one and then to Einstein’s ones (or at least at their weak field approximation) is the absence of an intrinsic notion of \textit{instantaneous 3-space} where to give the Cauchy data at least for Maxwell equations, so to have predictability for future times. Already in special relativity, where the space-time is absolute, there is no notion of simultaneity, of instantaneous 3-spaces and of spatial distance. The light postulates say that the two-way (or round-trip; only one clock is involved) velocity of light is a) isotropic and b) constant (a standard constant $c$ replaces the standard of length in existing relativistic metrology). The one-way velocity of light between two observers depends on how their clocks are synchronized (in general is not isotropic and point-dependent). Usually one uses Einstein’s convention for clock synchronization: an inertial observer A send a ray of light at $x^0_i$ towards the (in general accelerated) observer B; the ray is reflected towards A at a point P of B world-line and then reabsorbed by A at $x^0_f$; by convention P is synchronous with the mid-point between emission and absorption on A’s world-line, i.e. $x^0_P = x^0_i + \frac{1}{2} (x^0_f - x^0_i)$. This convention selects the Euclidean instantaneous 3-spaces $x^0 = ct = \text{const.}$ of the inertial frames centered on A. Only in this case the one-way velocity of light between A and B coincides with the two-way one, $c$. However if the observer A is accelerated the convention breaks down, because if only the world-line of the accelerated observer A (the 1+3 point of view) is given, then the only way for defining instantaneous 3-spaces is to identify them with the Euclidean tangent planes orthogonal to the 4-velocity of the observer (the local rest frames). But these planes (they are tangent spaces not 3-spaces!) will intersect each other at a distance from A’s world-line of the order of the acceleration lengths of A, so that all the either linearly or rotationally accelerated frames, centered on accelerated observers, based either on Fermi coordinates or on rotating ones will develop \textit{coordinate singularities}. Therefore their instantaneous 3-spaces cannot be used for a well-posed Cauchy problem for Maxwell equations. See Refs[2, 3, 4] for more details and a rich bibliography on these topics.

In general relativity, where the space-time becomes dynamical and the equivalence principle forbids the existence of global inertial frames, the problem of \textit{clock synchronization} becomes unavoidable and fundamental. Already near the Earth there is the ACES mission of ESA [5], programmed for 2013, which will make possible a measurement of the gravitational redshift of the Earth from the two-way link among a microwave clock (PHARAO) on the Space Station and similar clocks on the ground: the proposed microwave link should make possible the control of effects on the scale of 5 picoseconds. This will be a test of post-Newtonian gravity in the framework of Einstein’s geometrical view of gravitation: the redshift is a measure of the $1/c^2$ deviation of post-Newtonian null geodesics from Minkowski ones.

As a consequence we need a fully relativistic formulation of the classical background of atomic physics, considered as an effective theory of positive-energy scalar (or spinning) particles with mutual Coulomb interaction plus the transverse electro-magnetic field in the radiation gauge, valid for energies below the threshold of pair production. To build the Poincare’ generators of such an isolated system we have to face the problem of clock synchronization (determination of an instantaneous 3-space), the problem that the Poincare’ boosts, differently from the Galilei ones, are interaction-dependent and the old problem of the relativistic extension of the Newtonian center of mass. Moreover we need a formulation in which the change of the clock synchronization convention does not alter the physical results, namely it must be formulated as a gauge transformation of the theory [3]. Clearly,
this theory must put inertial and non-inertial frames of Minkowski space-time on the same level: this will allow the transition to general relativity, where global inertial frames are absent and the instantaneous 3-spaces are dynamical [6], since each solution of Einstein’s equations dynamically selects a preferred clock synchronization convention.

Parametrized Minkowski theories [7], [3], [2], with the associated inertial and non-inertial rest-frame instant form of dynamics, solve these problems and, together with the results of Refs.[8, 9, 10], allow to get a formulation of relativistic atomic physics [11], [12], [4], both in inertial and non-inertial frames of Minkowski space-time. To formulate this theory without these problems and, together with the results of the 3+1 point of view [3], in which we assign: a) the world-line of an arbitrary time-like observer; b) an admissible 3+1 splitting of Minkowski space-time, namely a nice foliation with space-like instantaneous 3-spaces (i.e. a clock synchronization convention).

This allows to define a global non-inertial frame centered on the observer and to use observer-dependent Lorentz-scalar radar 4-coordinates \( \sigma^A = (\tau; \sigma^r) \), where \( \tau \) is a monotonically increasing function of the proper time of the observer and \( \sigma^r \) are curvilinear 3-coordinates on the 3-space \( \Sigma_\tau \) having the observer as origin. If \( x^\mu \mapsto \sigma^A(x) \) is the coordinate transformation from the inertial Cartesian 4-coordinates \( x^\mu \) to radar coordinates, its inverse \( \sigma^A \mapsto x^\mu \) defines the embedding functions \( z^\mu(\tau, \sigma^r) \) describing the 3-spaces \( \Sigma_\tau \) as embedded 3-manifold into Minkowski space-time. The induced 4-metric on \( \Sigma_\tau \) is the following functional of the embedding \( g_{AB}(\tau, \sigma^r) = [z^\mu_A \eta_{\mu\nu} z^\nu_B](\tau, \sigma^r) \), where \( z^\mu_A = \partial z^\mu / \partial \sigma^A \) and \( \eta_{\mu\nu} = \epsilon (+ - - -) \) is the flat metric (\( \epsilon = \pm 1 \) according to either the particle physics \( \epsilon = 1 \) or the general relativity \( \epsilon = -1 \) convention). While the 4-vectors \( z^\mu(\tau, \sigma^r) \) are tangent to \( \Sigma_\tau \), so that the unit normal \( \mu^\mu(\tau, \sigma^r) \) is proportional to \( \epsilon^{\alpha\beta\gamma} [z^\alpha_1 z^\beta_2 z^\gamma_3](\tau, \sigma^r) \), we have \( z^\mu_\tau(\tau, \sigma^r) = [N^{\mu} + N^r z^r_\mu](\tau, \sigma^r) \) (\( N(\tau, \sigma^r) = \epsilon [z^\mu_\tau l^\mu](\tau, \sigma^r) \) and \( N_r(\tau, \sigma^r) = -\epsilon g_{rr}(\tau, \sigma^r) \) are the lapse and shift functions).

The foliation is nice and admissible if it satisfies the conditions:
1) \( N(\tau, \sigma^r) > 0 \) in every point of \( \Sigma_\tau \) (the 3-spaces never intersect);
2) \( \epsilon g_{rr}(\tau, \sigma^r) > 0 \), so to avoid the coordinate singularity of the rotating disk, and with the positive-definite 3-metric \( h_{rs}(\tau, \sigma^u) = -\epsilon g_{rs}(\tau, \sigma^u) \) having three positive eigenvalues (these are the Møller conditions [2, 4]);
3) all the 3-spaces \( \Sigma_\tau \) must tend to the same space-like hyper-plane at spatial infinity (so that there are always asymptotic inertial observers to be identified with the fixed stars).

These conditions imply that global rigid rotations are forbidden in relativistic theories [2].

The 4-metric \( g_{AB}(\tau, \bar{\sigma}) \) on \( \Sigma_\tau \) has the components \( \epsilon g_{rr} = N^2 - N_r N^r, -\epsilon g_{rr} = N_r = h_{rs} N^s, h_{rs} = -\epsilon g_{rs} = \sum_{a=1}^3 e(\alpha_r) e(\alpha_s) = \gamma^{1/3} \sum_{a=1}^3 e^2 \sum_{\alpha=1}^3 \gamma_{\alpha a} R_{\alpha}(\theta^i) V_{a0}(\theta^0) \), where \( e(\alpha_r) e(\alpha_s) \) are cotriads on \( \Sigma_\tau \), \( \gamma(r, \sigma^r) = \text{det} h_{rs}(\tau, \sigma^r) \) is the 3-volume element on \( \Sigma_\tau \), \( \lambda_a(\tau, \sigma^r) = [\gamma^{1/3} e^{\sum_{\alpha=1}^3 \gamma_{\alpha a} R_{\alpha}(\theta^i) V_{a0}(\theta^0)}(\tau, \sigma^r) \) are the positive eigenvalues of the 3-metric (\( \gamma_{\alpha a} \) are suitable numerical constants) and \( V(\theta^i(\tau, \sigma^r)) \) are diagonalizing rotation matrices depending on three Euler angles. The components \( g_{AB} \) or the quantities \( N, N_r, \gamma, R_{\alpha}, \theta^i \) play the role of the inertial potentials generating the relativistic apparent forces in the non-inertial frame. It can be shown [4] that the Newtonian inertial potentials are hidden in the functions \( N, N_r \) and \( \theta^i \).

In parametrized Minkowski theories one considers any isolated system (particles, strings, fields, fluids) admitting a Lagrangian description, because it allows, through the coupling
to an external gravitational field, the determination of the matter energy-momentum tensor and of the ten conserved Poincare’ generators $P^\mu$ and $J^{\mu\nu}$ (assumed finite) of every configuration of the system. Then one replaces the external gravitational 4-metric in the coupled Lagrangian with the 4-metric $g_{AB}(\tau, \sigma^r)$ of an admissible 3+1 splitting of Minkowski space-time and the matter fields with new ones knowing the instantaneous 3-spaces $\Sigma_r$. For instance a Klein-Gordon field $\tilde{\phi}(x)$ will be replaced with $\phi(\tau, \sigma^r) = \tilde{\phi}(z(\tau, \sigma^r))$; the same for every other field. Instead for a relativistic particle with world-line $x^\mu(\tau)$ we must make a choice of its energy sign: then it will be described by 3-coordinates $\eta^r(\tau)$ defined by the intersection of the world-line with $\Sigma_r$: $x^\mu(\tau) = z^\mu(\tau, \eta^r(\tau))$. In this way we get a Lagrangian depending on the given matter and on the embedding $z^\mu(\tau, \sigma^r)$, which is invariant under frame-preserving diffeomorphisms. As a consequence, there are four first-class constraints (an analogue of the super-Hamiltonian and super-momentum constraints of canonical gravity) implying that the embeddings $z^\mu(\tau, \sigma^r)$ are gauge variables, so that all the admissible non-inertial frames are gauge equivalent, namely physics does not depend on the clock synchronization convention and on the choice of the 3-coordinates $\sigma^r$: only the appearances of phenomena change by changing the notion of instantaneous 3-space.

The inertial rest-frame instant form of dynamics for isolated systems is obtained by choosing the 3+1 splitting corresponding to the intrinsic inertial rest frame of the isolated system centered on an inertial observer: the instantaneous 3-spaces, named Wigner 3-space due to the fact that the 3-vectors inside them are Wigner spin-1 3-vectors [3, 7], are orthogonal to the conserved 4-momentum $P^\mu$ of the configuration. In Ref.[4] there is the extension to admissible non-inertial rest frames, where $P^\mu$ is orthogonal to the asymptotic space-like hyper-planes to which the instantaneous 3-spaces tend at spatial infinity. This non-inertial family of 3+1 splittings is the only one admitted by the asymptotically Minkowskian space-times covered by canonical gravity formulation discussed at the end of the paper.

In the inertial rest frames there are only three notions of collective variables, which can be built by using only the Poincare’ generators (they are non-local quantities knowing the whole $\Sigma_r$) [8]: the canonical non-covariant Newton-Wigner center of mass (or center of spin), the non-canonical covariant Fokker-Pryce center of inertia and the non-canonical non-covariant Møller center of energy. All of them tend to the Newtonian center of mass in the non-relativistic limit. See Ref.[3] for the Møller non-covariance world-tube around the Fokker-Pryce 4-vector identified by these collective variables. As shown in Refs.[8, 9, 10] these three variables can be expressed as known functions of the rest time $\tau$, of the canonically conjugate Jacobi data (frozen Cauchy data) $\vec{z} = M c \vec{x}_{NW}(0)$ ($\vec{x}_{NW}(\tau)$ is the standard Newton-Wigner 3-position) and $\vec{h} = P/\sqrt{\vec{Q}^2}$ of the invariant mass $M c = \sqrt{\epsilon P^2}$ of the system and of its rest spin $\vec{S}$. It is convenient to center the inertial rest frame on the Fokker-Pryce inertial observer. As a consequence, every isolated system (i.e. a closed universe) can be visualized as a decoupled non-covariant collective (non-local) pseudo-particle described by the frozen Jacobi data $\vec{z}$, $\vec{h}$ carrying a pole-dipole structure, namely the invariant mass and the rest spin of the system, and with an associated external realization of the Poincare’ group. The universal breaking of Lorentz covariance is connected to this decoupled non-local collective variable and is irrelevant because all the dynamics of the isolated system leaves inside the Wigner 3-spaces and is Wigner-covariant. In each Wigner 3-space $\Sigma_r$ there is a unfaithful internal realization of the Poincare’ algebra, whose generators are built by using the energy-momentum tensor of the isolated system. While the internal energy and angular momentum
are $M c$ and $\vec{S}$ respectively, the internal 3-momentum vanishes: it is the rest frame condition. Also the internal Lorentz boost (whose expression in presence of interactions is given for the first time) vanishes: this condition identifies the covariant non-canonical Fokker-Pryce center of inertia as the natural inertial observer origin of the 3-coordinates $\sigma^r$ in each Wigner 3-space. As a consequence [11] there are three pairs of second class (interaction-dependent) constraints eliminating the internal 3-center of mass and its conjugate momentum inside the Wigner 3-spaces [12]: this avoids a double counting of the collective variables and allows to re-express the dynamics only in terms of internal Wigner-covariant relative variables. In the case of relativistic particles the reconstruction of their world-lines requires a complex interaction-dependent procedure delineated in Ref.[10]. See Ref.[11] for the comparison with the other formulations of relativistic mechanics developed for the study of the problem of relativistic bound states.

In Refs.[9] and [11], there is the formulation of relativistic atomic physics in the inertial rest frames (extended to the non-inertial ones in Ref.[4]). This was possible because we considered Grassmann-valued electric charges for the particles ($Q_i^2 = 0, Q_i Q_j = Q_j Q_i \neq 0$ for $i \neq j$), which give rise to a two-level charge structure after quantization. This allows
a) to make an ultraviolet regularization of Coulomb self-energies;
b) to make an infrared regularization eliminating the photon emission;
c) to express the Lienard-Wiechert potentials only in terms of the 3-coordinates $\eta^r_i(\tau)$ and the conjugate 3-momenta $\kappa^r_i(\tau)$ in a way independent from the used (retarded, advanced,..) Green function.

All this amounts to reformulate the dynamics of the one-photon exchange Feynman diagram as a Cauchy problem with well defined classical potentials. Moreover there is a canonical transformation [11] sending the above system in a transverse radiation field (in- or out-fields) decoupled, in the global rest frame, from Coulomb-dressed particles with a mutual interaction described by the sum of the Coulomb potential plus the Darwin potential. Therefore for the first time we are able to obtain results, previously derived from instantaneous approximations to the Bethe-Salpeter equation for the description of relativistic bound states (see the bibliography of Ref.[9]), starting from the classical theory. Moreover, for the first time, at least at the classical level, we have been able to avoid the Haag theorem according to which the interaction picture does not exist in QFT.

Therefore now we have an acceptable special relativistic description of massive particles and of the electro-magnetic field in the radiation gauge. We are now developing the rest-frame description of massless particles with helicity [13] (see also Appendix C of Ref.[14]) to simulate "photons" as an eikonal geometrical optic approximation of rays of light. Also a quasi-classical description of a relativistic two-level atom is under study [15]. These ill be the building blocks needed to try to give a relativistic description of the classical background of an atom interferometer.

Before introducing gravity let us consider the problem of quantizing the inertial and non-inertial rest frame instant form of dynamics in special relativity. In refs.[16] there is the quantization of positive-energy free scalar and spinning particles in a family of non-inertial rest frames where the instantaneous 3-spaces are space-like hyper-planes with differentially rotating coordinates. We take the point of view not to quantize the inertial effects (the appearances of phenomena): the embedding $z^\mu(\tau, \sigma^r)$ remains a c-number and we get results compatible with atomic spectra. Instead the quantization of fields in non-inertial frames is
an open problem due to the no-go theorem of Ref. [17] showing the existence of obstructions to the unitary evolution of a massive quantum Klein-Gordon field between two space-like surfaces of Minkowski space-time. Its solution, i.e. the identification of all the 3+1 splittings allowing unitary evolution, will be a prerequisite to any attempt to quantize canonical gravity taking into account the equivalence principle (global inertial frames do not exist!).

Let us make a digression on relativistic entanglement. The formulation of entanglement in non-relativistic quantum mechanics is based on the assumption (the zeroth postulate) that a composite system with two (or more) subsystems is described by a Hilbert space which is the tensor product of the Hilbert spaces of the subsystems: $H = H_1 \otimes H_2$. When interactions between the subsystems are present, one makes a unitary transformation to $H = H_1 \otimes H_2 = H_{com} \otimes H_{rel}$, where the Hilbert spaces $H_{com}$ and $H_{rel}$ describe the decoupled Newtonian center of mass of the two subsystems and their relative motion respectively. Then $H_{com}$ can be replaced with a frozen Hamilton-Jacobi Hilbert space $H_{com,HJ}$, containing only non-evolving Jacobi data for the center of mass, by a unitary transformation.

Due to the non-locality of the canonical non-covariant (Newton-Wigner) center of mass, described by the frozen Jacobi data $\vec{z}$ and $\vec{h}$, the previous type of unitary equivalence breaks down in relativistic quantum mechanics and therefore in relativistic quantum atomic physics. As shown in Ref. [14], the only consistent relativistic quantization is based on the Hilbert space $H = H_{com,HJ} \otimes H_{rel}$, which has the correct non-relativistic limit. The frozen nature of the center of mass avoids the violation of relativistic causality implied by the Hegerfeldt theorem [18] about the instantaneous spreading of the wave packets.

The breaking of the zeroth postulate at the relativistic level, $H \neq H_1 \otimes H_2$, is due to the fact that, already in the non-interacting case, in the tensor product of two quantum Klein-Gordon fields, $\phi_1(x_1)$ and $\phi_2(x_2)$, most of the states correspond to configurations in Minkowski space-time in which one particle may be present in the absolute future of the other particle. This is due to the fact that the two times $x_1^0$ and $x_2^0$ are totally uncorrelated, or in other words there is no notion of instantaneous 3-space (clock synchronization convention). Also the scalar products in the two formulations are completely different as shown in Ref. [19]. In S-matrix theory this problem is eliminated by avoiding the interpolating states at finite (the problem of the Haag theorem) and going the the asymptotic (in the times $x_i^0$) limit of the free in- and out- states. However in atomic physics we need interpolating states, and not S-matrix, to describe a laser beam resonating in a cavity and intersected by a beam of atoms!

The non validity of the zeroth postulate for composite systems implies that Einstein’s notion of separability is not valid since in $H = H_{com,HJ} \otimes H_{rel}$ the composite system must be described by means of relative variables in a Wigner 3-space (this is a type of weak form of relationism different from the formulations connected to the Mach principle). As a consequence every component of an isolated relativistic quantum system is entangled with every other component, even if it is not causally connected. These kinematical non-locality and kinematical spatial non-separability introduced by special relativity reduce the relevance of quantum non-locality in the study of the foundational problems of quantum mechanics [20] which have to be rephrased in terms of relative variables. Moreover, the control of Poincare’ kinematics will force to reformulate the experiments connected with Bell inequalities and teleportation in terms of isolated systems containing: a) the observers with their measuring apparatus (Alice and Bob as macroscopic quasi-classical objects); b) the particles of the
protocol (but now the ray of light, the "photons" carrying the polarization, move along null geodesics); c) the environment (macroscopic either quantum or quasi-classical object).

Finally let us consider the reformulation of the previous results in the canonical formulation of metric and tetrad gravity given in Refs.[21, 22], which is the final version of previous works in Refs.[23]. In Einstein’s general relativity space-time is dynamical, gravity is described by the 4-metric tensor and the equivalence principle says that global inertial frames do not exist. To get its Hamiltonian formulation we must use the same 3+1 formalism previously introduced for parametrized Minkowski theories and replace the Einstein-Hilbert action with the ADM one. However now the basic dynamical variable is the 4-metric $g_{AB}(\tau, \sigma^r)$ and not the embedding $z^\mu(\tau, \sigma^r)$ ($z_A^\mu$ are now the transformation coefficients for tensors from world 4-coordinates $x^\mu$ to adapted radar 4-coordinates). Since tetrad gravity is more natural for the coupling to the fermions, the 4-metric is decomposed in terms of cotetrads, $g_{AB} = E_A^\alpha(\tau, \sigma^r) E_B^{(\alpha)} ((\alpha)$ are flat indices), and the ADM action, now a functional of the 16 fields $E_A^\alpha(\tau, \sigma^r)$, is taken as the action for ADM tetrad gravity. This leads to an interpretation of gravity based on a congruence of time-like observers endowed with orthonormal tetrads: in each point of space-time the time-like axis is the unit 4-velocity of the observer, while the spatial axes are a (gauge) convention for observer’s gyroscopes.

This canonical formulation holds in a special class of globally hyperbolic, asymptotically Minkowskian at spatial infinity, topologically trivial space-times with boundary conditions killing super-translations so that the asymptotic symmetries are reduced to the ADM Poincaré’ group as shown in Refs.[23]. The ADM energy turns out to be the Hamiltonian of ADM tetrad gravity.

The absence of super-translations implies that the non-inertial rest frames are the only family of 3+1 splittings admitted by these asymptotically Minkowskian space-times. Therefore the instantaneous 3-spaces are asymptotically orthogonal to the ADM 4-momentum, are non-inertial rest frames of the 3-universe and admit asymptotic inertial observers (the fixed stars). As a consequence the 3-universe contained in an instantaneous 3-space can be described as a decoupled non-covariant non-observable pseudo-particle carrying a pole-dipole structure, whose mass and spin are those identifying the configuration of the ”gravitational field plus matter” isolated system present in the 3-universe.

Moreover, if we switch down the Newton constant, we get the description of the matter present in these space-times in the non-inertial rest frames of Minkowski space-time (deparametrization of general relativity) with the ADM Poincaré’ group collapsing in the Poincaré’ group of particle physics.

The kinematical Poincaré’ group connecting inertial frames in special relativity and its enlargement to the group of frame-preserving diffeomorphisms required for the treatment of non-inertial frames in parametrized Minkowski theories are now replaced by the full spatio-temporal diffeomorphism group enlarged with the O(3,1) gauge group of the Newman-Penrose approach (the extra gauge freedom acting on the tetrads in the tangent space of each point of space-time and reducing from 16 to 10 the number of independent fields like in metric gravity). The relativity principle of special relativity is replaced with the principle of general covariance (invariance in form of physical laws).

Since in general relativity the whole chrono-geometrical structure of space-time, described by the 4-metric and the associated line element, is dynamical, it turns out that every solution of Einstein equations dynamically selects its preferred instantaneous 3-spaces (modulo
coordinate transformations) \[6\]. As a consequence also the clock synchronization convention acquires a dynamical character. The gravitational field, i.e. the 4-metric, is not only the potential of the gravitational interaction but it also teaches relativistic causality to the other fields (it says to each massless particle which are the allowed trajectories in each point). This geometrical property is lost when the 4-metric is split in a background plus a perturbation (like in quantum field theory and string theory for being able to define a Fock space), since the chrono-geometrical structure is frozen to the one of the background. In Refs.\[21, 22\] such a splitting is never done, since there is an asymptotic Minkowskian background.

Let us remark that in the ADM Hamiltonian formulation of general relativity in the York canonical basis of Ref.[21] we use the same decomposition of the 4-metric $g_{AB}$ in terms of the quantities $N, N_r, \gamma, R_{\bar{a}}, \theta^i$, like in the non-inertial frames of special relativity. However now we have that:

a) the quantities $R_{\bar{a}}(\tau,\sigma^r)$, $\bar{a} = 1, 2$, become the physical tidal degrees of freedom of the gravitational field (the polarizations of the gravitational waves in the linearized theory);

b) the lapse and shift functions $N$ and $N_r$ and the angles $\theta^i$ (determining the 3-coordinates $\sigma^r$ on $\Sigma_\tau$) remain inertial gauge variables, namely they play the role of inertial potentials like in special relativity;

c) the 3-volume element $\gamma(\tau,\sigma^r)$ is determined by the super-Hamiltonian constraint (the Lichnerowicz equation) in terms of the other variables;

d) there is an extra inertial potential (replacing $\gamma$ of special relativity) describing the non-dynamical part of the freedom in choosing the clock synchronization convention (once this gauge freedom is fixed the final shape of the instantaneous 3-space is dynamically determined; it is the remnant of the special relativistic gauge freedom), i.e. the trace $K(\tau,\sigma^r)$ of the extrinsic curvature (also named the York time) of the non-Euclidean 3-space $\Sigma_\tau$, which is a functional of $g_{AB}$ in special relativity and has no Newtonian counterpart; in the York canonical basis it is the only gauge variable described by a momentum (a reflex of the Lorentz signature of space-time) and it gives rise to a negative kinetic term in the weak ADM energy vanishing only in the gauges $\gamma(\tau,\sigma^r) = 0$;

e) finally the extra $O(3,1)$ gauge freedom for the tetrads (the gauge freedom for each observer to choose three gyroscopes as spatial axes and to choose the law for their transport along the world-line) can be fixed by choosing tetrads adapted to the 3+1 splitting (the time-like tetrad is the unit normal to the 3-space) with the so-called Schwinger time gauges.

In Ref.[22] we study the coupling of $N$ charged scalar particles plus the electro-magnetic field to ADM tetrad gravity in this class of asymptotically Minkowskian space-times without super-translations. To regularize the self-energies both the electric charge and the sign of the energy of the particles are Grassmann-valued. The introduction of the non-covariant radiation gauge allows to reformulate the theory in terms of transverse electro-magnetic fields and to extract the generalization of the Coulomb interaction among the particles in the Riemannian instantaneous 3-spaces of global non-inertial frames.

After the reformulation of the whole system in the York canonical basis, we give the restriction of the Hamilton equations and of the constraints to the family of non-harmonic 3-orthogonal Schwinger time gauges, in which the instantaneous Riemannian 3-spaces have a non-fixed trace $^3K$ of the extrinsic curvature but a diagonal 3-metric.

Starting from the results obtained in the family of non-harmonic 3-orthogonal Schwinger gauges, it will be possible to define a linearization of ADM canonical tetrad gravity plus
matter in the weak field approximation, to obtain a formulation of Hamiltonian Post-Minkowskian gravity (without Post-Newtonian expansions) with non-flat Riemannian 3-spaces and asymptotic Minkowski background: i.e. with a decomposition of the 4-metric tending to the asymptotic Minkowski metric at spatial infinity, $g_{AB} = \eta_{AB} + h_{AB} \rightarrow \eta_{AB}$ (the small perturbation $h_{AB}$ vanishes at spatial infinity). We will show that a consequence of this approach is the possibility of describing part (or maybe all) dark matter as a relativistic inertial effect determined by the gauge variable $^3K(\tau, \sigma^r)$ (not existing in Newtonian gravity, where the Euclidean 3-space is an absolute notion): the rotation curves of galaxies would then experimentally determine a preferred choice of the instantaneous 3-spaces.

It is at this stage that it will be possible to see how to try to simulate the classical background of atom interferometry in presence of Post-Newtonian gravity as it is done in Ref.[1].

Finally, if we will replace the matter with a perfect fluid (for instance dust), this will allow us to try to see whether the York canonical basis can help in developing the back-reaction [24] approach to dark energy, according to which dark energy is a byproduct of the non-linearities of general relativity when one considers mean values on large scales.
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