The Nambu-Goto and Polyakov Strings via the Relativistic Particles

Davoud Kamani

Faculty of Physics, Amirkabir University of Technology (Tehran Polytechnic)
P.O.Box: 15875-4413, Tehran, Iran
e-mail: kamani@aut.ac.ir

Abstract

We assume the bosonic string is a composite object of the relativistic particles. The behavior of the relativistic particles in a curve enables us to obtain the Nambu-Goto and the Polyakov actions of the bosonic string. We observe that the particles of these strings move with non-constant speeds along them.

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1 Introduction

There have been some attempts to understand the string bits [1]-[13]. The origin of the idea of the string bits can be traced to the earliest days of the dual models [14, 15]. Recently, the string bits have been discussed in the pp-wave background [1, 2, 3, 4, 5]. The idea of the string bits also appeared in the 'tHooft’s works [6]. There are string bit models which are based on supersymmetric quantum mechanics [7]. Some other pictures for strings in terms of the bits language, corresponding to the quantum gravity, have been studied [8, 9, 10]. For more investigation on the string bits also see the Ref. [16, 17, 18, 19].

Classically, the Nambu-Goto and Polyakov strings are equivalent. We are motivated to find difference of them, classically, in terms of their bits. That is, classical behavior of a bit of the Nambu-Goto string is different from the behavior of a bit of the Polyakov string. We consider relativistic particles as the string bits.

We shall give a picture for the bosonic strings in terms of the relativistic point particles. Although the actions of a string are analogue of the actions of a relativistic point particle, however, apparently there is no relation between these particles and strings. We connect the actions of a particle to the Nambu-Goto and Polyakov actions of a string. This connection enables us to study the motion of a string bit along the string. We observe that the string particles are not at rest relative to the string. The motion of a bit of the Nambu-Goto string is different from the motion of a bit of the Polyakov string. As physical results, these motions determine the string mass density and, put an upper bound on the string length scale. Some quantum considerations simplify the classical speed formulas, and reveal non-uniform distribution of the particles on the string.

In all string-bit models when the number of bits goes to infinity the usual string is recovered. Similarly, in our model, string is a continuum of infinite relativistic particles.

This paper is organized as follows. In section 2, the action of a moving particle in the string will be studied. In section 3, the actions of the Nambu-Goto string and the Polyakov string through their particles will be obtained. In section 4, the motion of the particles of the Nambu-Goto string and the Polyakov string, relative to these strings, will be analyzed. In section 5, by canonical quantization, the particle speeds will be simplified.
2 Action of a relativistic point particle in the string

2.1 Relativistic point particle

Consider a relativistic point particle in the $D$-dimensional curved spacetime (e.g., see [20]). Let the coordinates of this spacetime be $\{X^\mu | \mu = 0, 1, ..., D - 1\}$. The curve $X^\mu(t)$, with the parameter “$t$” along it, describes the world-line of the particle. The Nambu-Goto form of the action of the particle is given by

$$S_{NG}^{(pp)} = -m \int dt \left( -G_{\mu\nu}(X) \frac{dX^\mu}{dt} \frac{dX^\nu}{dt} \right)^{1/2},$$

(1)

where $G_{\mu\nu}(X)$ is the metric of the curved spacetime with one negative eigenvalue, corresponding to the time direction, and $D - 1$ positive eigenvalues, associated to the space directions, and $m$ is the particle mass. In fact, this action shows the invariant proper time along the particle world-line.

Introducing an independent degree of freedom, i.e., world-line metric $h(t)$, leads to the Polyakov form of the particle action

$$S_P^{(pp)} = \frac{1}{2} \int dt \left( h^{-1}(t)G_{\mu\nu}(X) \frac{dX^\mu}{dt} \frac{dX^\nu}{dt} - h(t)m^2 \right).$$

(2)

The action (2) works for both massive and massless particles, while the action (1) only is appropriate for the massive particles. Both of them have world-line reparametrization invariance. Note that under the reparametrization $t \rightarrow t'(t)$ the world-line metric $h(t)$ transforms to $h'(t')$ such that

$$h'(t') dt' = h(t) dt.$$

(3)

Each of the actions (1) and (2) has its own advantages. However, at the classical level they are equivalent. That is, use the equation of motion of $h(t)$,

$$h^2(t) = \frac{1}{m^2} G_{\mu\nu}(X) \frac{dX^\mu}{dt} \frac{dX^\nu}{dt},$$

(4)

to eliminate $h(t)$ from (2), the action $S_P^{(pp)}$ reduces to the action $S_{NG}^{(pp)}$. From now on we call the particles with the actions (1) and (2) as the Nambu-Goto particle and the Polyakov particle, respectively.

2.2 Relativistic point particle in the string

The string shape is a mathematical curve $C$. We assume the string is a composite object of the relativistic particles, distributed along the curve $C$. Now we obtain the action of a point
particle which moves along the curve $C$. Each particle on the string curve is distinguished by its position, \textit{i.e.} the worldsheet coordinate $\sigma$. All particles on the string curve have the common parameter $\tau$ on their world-lines, where $\tau$ is the time coordinate of the string worldsheet. This enables us to choose the parameter $\tau$ on the particle world-line. The various world-lines are distinguished by various values of $\sigma$. Thus, the string worldsheet is constructed by infinite number of the particle world-lines. In this picture, the particle “$p$” has a worldsheet-line in the string worldsheet, \textit{i.e.} $\sigma_p(\tau)$, and a world-line in the spacetime, \textit{i.e.} $X^\mu(\sigma, \tau)$, where the coordinates $X^\mu(\sigma, \tau)$ describe the string worldsheet in the spacetime.

For an arbitrary world-line action of particle is given by (1) or (2). For a particle in the string we should replace the variable $\frac{dX^\mu}{dt}$ by $\frac{dX^\mu}{d\tau}$,

\[
\frac{d}{d\tau} X^\mu(\sigma_p(\tau), \tau) = \partial_\tau X^\mu + \dot{\sigma}_p \partial_\sigma X^\mu,
\]

\[
\partial_\tau X^\mu(\sigma_p(\tau), \tau')|_{\tau' = \tau} = \frac{d}{d\sigma_p(\tau)} X^\mu(\sigma_p(\tau), \tau),
\]

\[
\dot{\sigma}_p = \frac{d\sigma_p(\tau)}{d\tau}.
\]

Thus, the motion of the particle in the spacetime is described by the motion of the string which comes from the term $\partial_\tau X^\mu$, and the motion of the particle relative to the string which is given by the term $\dot{\sigma}_p \partial_\sigma X^\mu$.

According to these, the actions of a particle (moving in the string) are as in the following

\[
S_{NG}^{(pp)} = -m \int d\tau \left( -H^{ab} G_{\mu\nu}(X) \partial_a X^\mu \partial_b X^\nu \right)^{1/2},
\]

for the Nambu-Goto particle, and

\[
S_P^{(pp)} = \frac{1}{2} \int d\tau \left( h^{-1}(\tau) H^{ab} G_{\mu\nu}(X) \partial_a X^\mu \partial_b X^\nu - h(\tau) m^2 \right),
\]

for the Polyakov particle. The indices “$a$” and “$b$” run over $\tau$ and $\sigma$, and the symmetric matrix $H^{ab}$ has the definition

\[
H^{ab} = \begin{pmatrix} 1 & \dot{\sigma}_p \\ \dot{\sigma}_p & \ddot{\sigma}_p \end{pmatrix}.
\]

The actions (6) and (7) under the world-line reparametrization $\tau \longrightarrow \tau'(\tau)$ with $h'(\tau')d\tau' = h(\tau)d\tau$, are invariant. Furthermore, by the equation of motion of $h(\tau)$, \textit{i.e.},

\[
h^2(\tau) = -\frac{1}{m^2} H^{ab} G_{\mu\nu}(X) \partial_a X^\mu \partial_b X^\nu,
\]

these actions classically are equivalent.
3 String actions from the relativistic particles

Apart from the square root, the form of the action (6) is far from the Nambu-Goto action of string. Similarly, although for the massless particle the action (7) has the form similar to the Polyakov action of string, however, we cannot identify $h^{-1}(\tau)H^{ab}$ with the intrinsic metric of the string worldsheet, i.e. $g^{ab}$. This is due to the fact that $\det H^{ab} = 0$, and hence $H^{ab}$ is not invertible, while $g^{ab}$ is an invertible matrix with the inverse $g_{ab}$.

In fact, as we shall do, it is possible to obtain the Nambu-Goto string and the Polyakov string from the actions (6) and (7), respectively.

3.1 The Nambu-Goto string

Consider particles in an interval $(\sigma - \frac{d\sigma}{2}, \sigma + \frac{d\sigma}{2})$ of the string, which is located at the position $\sigma$. Assume $d\sigma$ contains a large number of the particles. This enables us to define a mass density for the string. Let $\rho$ be the mass density of the string, i.e. $\rho = dm/d\sigma$. Therefore, from the action (6) (after integration over $\sigma$) we obtain

$$S_{NG} = -\int d\sigma \int d\tau \rho (-H^{ab}h_{ab})^{1/2},$$

where $h_{ab}$ is the pull-back of the spacetime metric $G_{\mu\nu}(X)$ on the string worldsheet

$$h_{ab} = G_{\mu\nu}(X)\partial_a X^\mu \partial_b X^\nu.$$

The equation (10) describes the action of infinite number of the relativistic particles in a curve. This system may be any one-dimensional physical object. Demanding it be a special object imposes special behavior on the particles. We want it be a Nambu-Goto string. This leads to the following behavior for each string bit which is given by the equation

$$H^{ab}h_{ab} = \frac{1}{(2\pi\alpha'\rho)^2} \det h_{ab}. $$

(12)

This implies the Nambu-Goto action for the string

$$S_{NG} = -\frac{1}{2\pi\alpha'} \int d\sigma \int d\tau \sqrt{-\det h_{ab}}.$$

(13)

In fact, if the relativistic particles along the string curve $C$ form the Nambu-Goto string, then the matrix $H^{ab}$ and the induced metric $h_{ab}$ have to relate to each other through the equation (12). In other words, the equation (12) means that when a relativistic particle is inside the Nambu-Goto string curve $C$, it should obey the obligatory equation (12). However,
if this relativistic particle is a bit of the Polyakov string, it should obey another obligatory equation (see Eq.(14)).

According to the matrix (8), the covariance of the equation (12) implies \( \dot{\sigma} \neq 0 \). Thus, for a given spacetime and worldsheet, i.e. known \( G_{\mu\nu}(X) \) and \( X^\mu(\sigma, \tau) \), the equation (12) gives \( \dot{\sigma} \) in terms of \( \sigma \) and \( \tau \). That is, the motion of a particle inside its string is influenced by the background metric and the geometrical shape of the string. In other words, the particle feels an acting potential through a complex combination of the background geometry and the string shape. In the next section the equation (12) will be analyzed.

### 3.2 The Polyakov string

The reparametrization invariance can be used to make the gauge choice \( h(\tau) = l^2 \), where \( l \) is a characteristic length of the theory. With this gauge the mass term of (7) becomes constant, and hence we ignore it. Again this system may be any one-dimensional physical object. Requesting the Lagrangian of (7) be the Lagrangian density of the Polyakov string leads to the special behavior of the relativistic particles which is described by the equation

\[
H^{ab}h_{ab} = -\frac{l^2}{2\pi\alpha'} \sqrt{g}g^{ab}h_{ab},
\]

(14)

where \( g \equiv -\det g_{ab} \). This equation is analogue of the equation (12). According to this, the action (7) gives the following Lagrangian for a Polyakov particle in the curve \( C \),

\[
\mathcal{L}[X(\sigma, \tau), g_{ab}(\sigma, \tau)] = -\frac{1}{4\pi\alpha'} \sqrt{g}g^{ab}(\sigma, \tau)G_{\mu\nu}(X)\partial_aX^\mu(\sigma, \tau)\partial_bX^\nu(\sigma, \tau).
\]

(15)

Consider an interval \((-\Delta\sigma_p/2, \Delta\sigma_p/2)\) of the string which its center is located at \( \sigma_p \). Thus, its Lagrangian is proportional to the factor \( \Delta\sigma_p \). The total Lagrangian of the string is given by the summation of the Lagrangians of the intervals, i.e. \( \sum_p \mathcal{L}[X(\sigma_p, \tau), g_{ab}(\sigma_p, \tau)] \Delta\sigma_p \). When the infinitesimal length \( \Delta\sigma_p \) goes to zero, the summation changes to the integral, and hence we obtain the Polyakov action for the string

\[
S_P = -\frac{1}{4\pi\alpha'} \int d\sigma \int d\tau \sqrt{g}g^{ab}G_{\mu\nu}(X)\partial_aX^\mu\partial_bX^\nu.
\]

(16)

Note that the Polyakov action of the string gives the equation

\[
h_{ab} = \frac{1}{2}g_{ab}(g^{cd}h_{cd}).
\]

(17)

This is the equation of motion of the metric \( g_{ab} \). Contracting it by \( H^{ab} \) and using the equation (14) we obtain

\[
H^{ab}g_{ab} = -\frac{l^2}{\pi\alpha'} \sqrt{g}.
\]

(18)

This equation is equivalent to (14), which will be used.
3.3 The resulted structure for the bosonic string

The motion of a particle relative to the string curve, is given by $V^\mu = \dot{\sigma} \partial_\sigma X^\mu$. Since the variable $\dot{\sigma}$ appears in our calculations, we call $\dot{\sigma}$ as speed. For the rest particle in the string curve, *i.e.* $\dot{\sigma} = 0$, the only nonzero element of the matrix $H^{ab}$ is $H^{00} = 1$. Thus, having covariance, *i.e.* appearance of the indices “$a$” and “$b$” in the actions (6) and (7) and hence in the string actions (13) and (16), implies the motion of the particle along the string curve. Therefore, we obtain the following statement: “If the fundamental string is a collection of infinite relativistic particles, they should move along the string curve. Motion of each of them depends on the background metric and geometrical shape of the string. In addition, we shall see that the distribution of the particles along the string is not uniform.”

4 Analysis of the particle behavior in the string

For simplicity we refer to $\dot{\sigma}$ as the particle speed. In fact, the string bits form a chain of point-like constituents in which, from the point of view of the observer on the string, they enjoy a Galilei invariant dynamics. Thus, we can use $\dot{\sigma} = d\sigma/d\tau$ as speed.

4.1 Upper and lower bounds of the particle speed

Before obtaining the explicit form of the speed $\dot{\sigma}_p$, it is useful to find upper and lower bounds of it. For simplicity drop the index “$p$” from the particle coordinate $\sigma_p$. For a Nambu-Goto particle the action (6) implies the inequality $H^{ab} h_{ab} < 0$, which is

$$h_{11}\dot{\sigma}^2 + 2h_{01}\dot{\sigma} + h_{00} < 0.$$  \hspace{1cm} (19)

This inequality also holds for the Polyakov particles, which can be seen from the equation (9).

For discussing about the inequality (19) we separate the cases $h_{11} > 0$ and $h_{11} < 0$. For those areas of the string worldsheet with $h_{11} > 0$, the range of the particle speed is given by

$$v_- < \dot{\sigma} < v_+,$$  \hspace{1cm} (20)

where $v_+$ and $v_-$ are as in the following

$$v_{\pm} = \frac{-h_{01} \pm \sqrt{-\det h_{ab}}}{h_{11}}.$$  \hspace{1cm} (21)

In the case in which we receive the simple Lorentz gauge, *i.e.*, the flat induced metric $h_{00} = -1$, $h_{01} = 0$ and $h_{11} = 1$, we have $v_\pm = \pm 1$ and hence $-1 < \dot{\sigma} < 1$.  

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For those areas of the worldsheet which have $h_{11} < 0$, when at least one of the conditions
\begin{align}
\dot{\sigma} &> v'_+,
\dot{\sigma} &< v'_-, \tag{22}
\end{align}
holds, the inequality (19) is satisfied. The speeds $v'_\pm$ are defined by
\begin{equation}
 v'_\pm = \frac{h_{01} \pm \sqrt{-\det h_{ab}}}{|h_{11}|}. \tag{24}
\end{equation}

The actual particle motion, relative to the string, is given by the spacetime vector $V^\mu = \dot{\sigma} \partial_\sigma X^\mu$. Since $\partial_\sigma X^\mu$ for both closed and open strings is a known function of $\sigma$ and $\tau$, it is sufficient to obtain explicit form of the speed $\dot{\sigma}$.

### 4.2 Speed of a particle of the Nambu-Goto string

The particles of the Nambu-Goto string have the speed equation (12). This equation gives the following speeds
\begin{equation}
 \dot{\sigma}_\pm = \frac{1}{h_{11}} \left[ -h_{01} \pm \sqrt{\left(\frac{h_{11}}{(2\pi\alpha')^2} - 1\right) \det h_{ab}} \right]. \tag{25}
\end{equation}

We observe that the background metric, string shape and the mass density of the string determine the speeds of the particles.

Since there is $\det h_{ab} < 0$ (which can be seen from (13)), the square root leads to the condition $h_{11} \leq (2\pi\alpha')^2$. For negative values of $h_{11}$ this inequality always holds. For positive $h_{11}$ we have
\begin{equation}
 \rho(\sigma, \tau) \geq \frac{1}{2\pi\alpha'} \sqrt{h_{11}(\sigma, \tau)}. \tag{26}
\end{equation}

We shall observe that, due to the quantum effects, only the equality holds.

### 4.3 Speed of a particle of the Polyakov string

Each of the equations (14) or (18) can be used for investigating the particle speed of the Polyakov string. In fact, through the equation (17) they are equivalent. For analogy with the Nambu-Goto string, we consider the equation (14). This equation leads to the following speeds for the particles of the Polyakov string
\begin{equation}
 \dot{\sigma}_\pm = \frac{1}{h_{11}} \left[ -h_{01} \pm \left( -\det h_{ab} - \frac{t^2 h_{11}}{2\pi\alpha'} \sqrt{g} g^{ab} h_{ab} \right)^{1/2} \right]. \tag{27}
\end{equation}
Compare the speeds (25) and (27). We note that the behavior of the particles of the Nambu-Goto string is different from the behavior of the particles of the Polyakov string. There are two possible speeds for the particles. Therefore, there are two kinds of the particles: particles with the speed $\dot{\sigma}_+$ and particles with the speed $\dot{\sigma}_-$. Since the direction of the motion of the particles along the string should be the same, each string contains one kind of these particles. Thus, there are two kinds of the Nambu-Goto string and two kinds of the Polyakov string.

The condition (19) and the equation (14) imply that $g^{ab}h_{ab}$ is positive. Also we have $\det h_{ab} < 0$. According to these, the expression under the square root of (27) for $h_{11} \leq 0$ always is positive. For the case $h_{11} > 0$ we obtain the condition

$$\frac{l^2}{2\pi\alpha'} \leq \frac{-\det h_{ab}}{h_{11}(\sqrt{g^{cd}h_{cd}})}.$$  

This inequality should hold for any $\sigma$ and $\tau$. This implies that the quantity $\frac{l^2}{2\pi\alpha'}$ is equal or less than the minimum value of the right-hand-side of (28), i.e.,

$$\frac{l^2}{2\pi\alpha'} \leq \left(\frac{-\det h_{ab}}{h_{11}(\sqrt{g^{cd}h_{cd}})}\right)_{\text{min}}.$$  

This determines the upper bound of the length scale “$l$”.

For both open string and closed string we know the explicit form of the function $X^\mu(\sigma, \tau)$. From the gauge choice of $g_{ab}$ we can also use $g_{ab} = \eta_{ab}$. Thus, for a given background geometry, i.e. known $G_{\mu\nu}(X)$, the right-hand-sides of the equations (25) and (27) are known functions of $\sigma$ and $\tau$. In other words, (25) and (27) define differential equations for the function $\sigma(\tau)$.

## 5 Some quantum considerations

### 5.1 The Nambu-Goto case

For the given coordinates $(\sigma, \tau)$, the action (10) defines the Lagrangian of a particle inside the string curve. This Lagrangian defines the following canonical momentum, which is conjugate to the particle coordinate $X^\mu(\sigma, \tau)$,

$$\Pi^{(1)}_\mu(\sigma, \tau) = \frac{\rho}{\sqrt{-H_{ab}h_{ab}}} G_{\mu\nu}(\partial_\tau X^\nu + \dot{\sigma}\partial_\sigma X^\nu).$$  

Therefore, equal time canonical quantization gives

$$[X^\mu(\sigma, \tau), \Pi^{(1)}_\nu(\sigma', \tau)] = i\eta^{\mu\nu}\delta(\sigma - \sigma').$$  

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In the same way, the action (13) also defines the canonical momentum

$$\Pi^{(2)}(\sigma, \tau) = \frac{1}{2\pi\alpha'} \frac{1}{\sqrt{-\det h_{ab}}} G_{\mu\nu}(h_{11} \partial_{\tau} X^\nu - h_{01} \partial_{\sigma} X^\nu).$$

(32)

This is conjugate to the coordinate of a point of the string, i.e. $X^\mu(\sigma, \tau)$. This leads to the following quantization

$$[X^\mu(\sigma, \tau), \Pi^{(2)}_{\nu}(\sigma', \tau)] = i\eta_{\mu\nu} \delta(\sigma - \sigma').$$

(33)

Since both points refer to the same particle position, we should have $\Pi^{\mu}_{(1)} = \Pi^{\mu}_{(2)}$, and hence

$$[h_{11} - (2\pi\alpha' \rho)^2] \partial_{\tau} X^\mu = [h_{01} + (2\pi\alpha' \rho)^2 \dot{\sigma}] \partial_{\sigma} X^\mu.$$

(34)

This implies the equations

$$\rho(\sigma, \tau) = \frac{1}{2\pi\alpha'} \sqrt{h_{11}(\sigma, \tau)},$$

$$\dot{\sigma} = -\frac{h_{01}}{h_{11}}.$$

(35)

These are consistent with the equation (25). However, the particle speed, extracted from the quantum considerations, is simpler than the classical case. Furthermore, we observe that the mass density $\rho$ is not constant. In other words, on a time slice, the matter distribution on the string is inhomogeneous. Note that some string-bit-models reveal homogeneity and some others give inhomogeneity for distribution of bits along the string (e.g. see Ref. [8, 9, 10] and references therein).

### 5.2 The Polyakov case

Again equality of the canonical momenta, extracted from the Lagrangian of a particle in the string (i.e. from the left-hand-side of (14)) and Lagrangian of a point part of the string (i.e. from the right-hand-side of (14)) leads to the equations

$$\frac{l^2}{2\pi\alpha'} \sqrt{g} g^{00} = -1,$$

$$\dot{\sigma} = g^{01} g^{00}.$$

(36)

Combining this speed with one of the speed equations, e.g. (18), implies the condition

$$g_{00} + 2g_{01} g^{01}_{g^{00}} + g_{11} \left(\frac{g^{01}_{g^{00}}}{g^{00}}\right)^2 + \frac{l^2}{\pi\alpha'} \sqrt{g} = 0.$$

(37)

That is, equality of the particle speed from the classical and quantum point of views gives the condition (37).
6 Conclusions

We assumed a bit-structure for the bosonic string. We considered each bit as a relativistic point particle. We obtained the Nambu-Goto form and the Polyakov form of action of a bit which moves along the string. From these particle actions we acquired the Nambu-Goto string and the Polyakov string.

We achieved the following structure for the fundamental string: “If a string is made of infinite relativistic particles, they are not fixed in the string. They move with non-constant speeds along the string. The motion of each of them completely depends on the background geometry and string shape. In addition, distribution of the particles along the string is not uniform”.

Some upper and lower bounds determine the range of the particle speed $\dot{\sigma}$. Explicit forms of the speed indicate that the speed depends on the position of the particle on the string worldsheet. This dependence comes via the induced metric of the worldsheet. Therefore, each of the speed equations provides a differential equation for the function $\sigma(\tau)$. The reality of the particle speed, inside the Polyakov string, determines an upper bound for the characteristic length scale of the theory.

We observed that the particles of the Nambu-Goto string have non-uniform distribution in the string. In addition, we saw that canonical quantization gives another speed for the particle in the string. For a Nambu-Goto particle this speed is consistent with the classical case. For a Polyakov particle the consistency of the speeds puts a condition on the elements of the intrinsic metric of the worldsheet.

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