On mass limits for leptoquarks from
$K^0_L \to e^\mp \mu^\pm$, $B^0 \to e^\mp \tau^\pm$ decays

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Abstract

The contributions of the scalar leptoquark doublets into widths of the $K^0_L \to e^\mp \mu^\pm$, $B^0 \to e^\mp \tau^\pm$ decays are calculated in MQLS model with Higgs mechanism of the quark-lepton mass splitting. The resulted mass limits for the scalar leptoquarks are investigated in comparison with those for vector and chiral gauge leptoquarks. It is shown that the scalar leptoquark mass limits are essentially weaker (about 200 GeV) than those (a few hundreds TeV) for gauge leptoquarks. The search for such scalar leptoquark doublets at LHC and the search for the leptonic decays $B^0 \to l_i^+ l_j^-$ are of interest.

The search for a new physics beyond the Standard Model (SM) is now one of the aims of the high energy physics. One of the possible variants of such new physics can be the variant induced by the possible four color symmetry [1] between quarks and leptons. The four color symmetry can be unified with the SM by the gauge group

$$G_{new} = G_c \times SU_L(2) \times U_R(1)$$

(1)

where as the four color group $G_c$ can be the vectorlike group [1, 2, 3]

$$G_c = SU_V(4)$$

(2)

the general group of the chiral four color symmetry

$$G_c = SU_L(4) \times SU_R(4)$$

(3)

or one of the special groups

$$G_c = SU_L(4) \times SU_R(3), \ G_c = SU_L(3) \times SU_R(4)$$

(4)

of the left or right four color symmetry. According to these groups the four color symmetry predicts the vector gauge leptoquarks the left and right gauge leptoquarks and the left (or right) gauge leptoquarks respectively. The most stringent mass limits the vector leptoquarks are resulted from $K^0_L \to e^\mp \mu^\pm$ decay and are known to be of order of $10^3$ TeV [4, 5, 6]. The four color symmetry allows also the existence of scalar leptoquarks and such particles have been phenomenologically introduced in ref. [7] and have been discussed in a number of papers. The experimental lower mass limits for the scalar leptoquarks from their direct search depend on additional assumptions and are about 250 GeV or slightly less [8].

It should be noted that in the case of Higgs mechanism of the quark-lepton mass splitting the four color symmetry predicts [2, 3, 9] the scalar leptoquarks

$$s_{\alpha \alpha}^{(\pm)} = \begin{pmatrix} s_{1 \alpha}^{(\pm)} \\ s_{2 \alpha}^{(\pm)} \end{pmatrix}.$$
with doublet structure under electroweak group $SU(2)_L$. Due to their Higgs origin these scalar leptoquark doublets interact with the fermions with coupling constants which are proportional to the ratios $m_f/\eta$ of the fermion masses $m_f$ to the SM VEV $\eta$. As a result these coupling constants are small for the ordinary $u-, d-, s-$ quarks ( $m_u/\eta \sim m_d/\eta \sim 10^{-5}$, $m_s/\eta \sim 10^{-3}$ ) but they are more significant for $c-, b-$ quarks and, especially, for $t$-quark ( $m_c/\eta \sim m_b/\eta \sim 10^{-2}$, $m_t/\eta \sim 0.7$ ).

The analysis of the contributions of these scalar leptoquark doublets into radiative corrections $S-, T-, U-$ parameters showed [10, 11] that these scalar leptoquarks can be relatively light, with masses below 1 TeV. It is interesting now to know what are the mass limits for these scalar leptoquarks which are resulted from $K^0\rightarrow e^+\mu^-$ decay and from other decays of such type.

In this talk I discuss the mass limits for the scalar leptoquark doublets which are resulted from $K^0\rightarrow e^+\mu^-$, $B^0\rightarrow e^+\tau^-$ decays in comparison with the corresponding mass limits for the vector leptoquarks in frame of MQLS - model and with those for the chiral gauge leptoquarks.

The interaction of the gauge and scalar leptoquarks with down fermions can be described in the model independent way by the lagrangians

$$L_{Vdl} = (d_{p\alpha} [(g^L_d)_{p\alpha} \gamma^\mu P_L + (g^R_d)_{p\alpha} \gamma^\mu P_R] l) V_{\alpha\mu} + h.c., \tag{6}$$

$$L_{Sdl} = (d_{p\alpha} [(h^L_m)_{p\alpha} P_L + (h^R_m)_{p\alpha} P_R] l) S_{\alpha\mu} + h.c., \tag{7}$$

where $(g^L_d)_{p\alpha}$ and $(h^L_m)_{p\alpha}$ are the phenomenological coupling constants, $p, i = 1, 2, 3,...$ are the quark and lepton generation indexes, the indexes $k, m$ numerate the gauge and scalar leptoquarks, $\alpha = 1, 2, 3$ is the $SU(3)$ color index and $P_{L,R} = (1 \pm \gamma_5)/2$ are the left and right operators of fermions.

The MQLS - model ( with $V^1_{\alpha\mu} \equiv V_{\alpha\mu}$, $V_{\alpha\mu}$ is the vector leptoquark of the model ) gives for the phenomenological coupling constants in (6), (7) the expressions

$$(g^L_d)_{p\alpha} = \frac{g_4}{\sqrt{2}} (K^L_{2\alpha})_{pi}, \tag{8}$$

$$(h^L_m)_{p\alpha} = (h^L_{2m})_{p\alpha} = h^L_{pi} c_m^{(\pi)}, \tag{9}$$

where $g_4 = g_{st}(M_c)$ is the $SU_V(4)$ gauge coupling constant related to the strong coupling constant at the mass scale $M_c$ of the $SU_V(4)$ symmetry breaking and

$$h^L_{pi} = -\sqrt{3}/2 \frac{1}{\sin \beta} \left[ m_{dp} (K^L_{2\alpha})_{pi} - (K^R_{2\alpha})_{pi} m_{li} \right], \tag{10}$$

$\eta$ is the SM VEV, $\beta$ is the two Higgs doublet mixing angle of the model, $m_{dp}, m_{li}$ are the quark and lepton masses and $c_m^{(\pm)}$ are the scalar leptoquark mixing parameters entering in superpositions

$$S^{(\pm)} = \sum_{m=0}^{3} c^{(\pm)} S_m, \quad S^{(-)} = \sum_{m=0}^{3} c^{(-)} S_m \tag{11}$$

of three physical scalar leptoquarks $S_1$, $S_2$, $S_3$ with electric charge 2/3 and a small admixture of the Goldstone mode $S_0$ (in general case the scalar leptoquarks $S^{(+)}_{2a}$ and $S^{(-)}_{2a}$ with electric charge 2/3 are mixed and can be written as superpositions [11] ). The matrices $K^L_{a\alpha}, a = 1, 2$ describe the (down for $a = 2$) fermion mixing in the leptoquark currents and in general case they can be nondiagonal.

In particular case of the two leptoquark mixing the superpositions [11] can be approximately written as

$$S^{(+)}_2 = cS_1 + sS_2, \quad S^{(-)}_2 = -sS_1 + cS_2 \tag{12}$$
where $c = \cos \theta$, $s = \sin \theta$, $\theta$ is the scalar leptoquark mixing angle.

Omitting the details of calculations we present the final expressions for the widths of the $K_L^0 \to e\mu$, $B^0 \to e\tau$ decays with account of the contributions of the vector leptoquark $V_{\alpha\mu}$ and of the scalar leptoquarks $S_1, S_2$ with mixing (12) in MQLS - model for $K_L^2 = K_R^2 = I$

$$
\Gamma(K_L^0 \to e\mu) = \frac{m_{K^0} f_{K^0}^2}{64\pi} \left(1 - \frac{m_\mu^2}{m_{K^0}^2}\right)^2 \left\{ \frac{4\pi\alpha_{em}}{m_V^2}(-\bar{m}_{K^0} + m_\mu/2) - \frac{h_1 h_2}{2} \left(m_\mu \left(\frac{1}{m_S^2}\right)_L - \bar{m}_{K^0} c_\alpha \left(\frac{1}{m_{S_1}^2} - \frac{1}{m_{S_2}^2}\right)\right)^2 + L \leftrightarrow R \right\},
$$

$$
\Gamma(B^0 \to e\tau) = \frac{m_{B^0} f_{B^0}^2}{32\pi} \left(1 - \frac{m_\tau^2}{m_{B^0}^2}\right)^2 \left\{ \frac{4\pi\alpha_{em}}{m_V^2}(-\bar{m}_{B^0} + m_\tau/2) - \frac{h_1 h_3}{2} \left(m_\tau \left(\frac{1}{m_S^2}\right)_L - \bar{m}_{B^0} c_\alpha \left(\frac{1}{m_{S_1}^2} - \frac{1}{m_{S_2}^2}\right)\right)^2 + L \leftrightarrow R \right\},
$$

where

$$
h_p = -\sqrt{3/2} \frac{1}{\eta\sin \beta} (m_d - m_\mu),
$$

$$
\left(\frac{1}{m_S^2}\right)_L = \frac{s^2}{m_{S_1}^2} + \frac{c^2}{m_{S_2}^2}, \quad \left(\frac{1}{m_S^2}\right)_R = \frac{c^2}{m_{S_1}^2} + \frac{s^2}{m_{S_2}^2}.
$$

$$
\bar{m}_{K^0} = m_{K^0}^2/(m_s + m_d), \quad \bar{m}_{B^0} = m_{B^0}^2/(m_b + m_d),
$$

the form factor $f_{K^0}$ is defined as

$$
\langle 0 | \bar{s} \gamma^\mu \gamma^5 d | K^0(p) \rangle = i f_{K^0} p^\mu, \quad \langle 0 | \bar{s} \gamma^5 d | K^0(p) \rangle = - i \bar{m}_{K^0} f_{K^0},
$$

$p_\mu$ is 4-momentum of the decaying meson (the same definition has been used for $f_{B^0}$) and the relations $K^0 = (\bar{s}d)$, $K^0_L = (\bar{s}d - \bar{d}s)/\sqrt{2}$, $B^0 = (bd)$ have been taken into account.

In formulae (13), (14) we imply that

$$
\Gamma(K_L^0 \to e\mu) = \Gamma(K_L^0 \to e^-\mu^+) + \Gamma(K_L^0 \to e^+\mu^-) = 2\Gamma(K_L^0 \to e^-\mu^+),
$$

$$
\Gamma(B^0 \to e\tau) = \Gamma(B^0 \to e^-\tau^+) + \Gamma(B^0 \to e^+\tau^-) = 2\Gamma(B^0 \to e^-\tau^+).
$$

It is interesting to note that the contributions of the scalar leptoquarks in (13), (14) can have the opposite sign relatively to vector leptoquark contribution, so that the destructive interference of these contributions can take place in the decays under consideration. The magnitude of the scalar leptoquark contributions depends on the scalar leptoquark masses on the masses of quarks and leptons and on the scalar leptoquark mixing angle in (12).

The interaction of the chiral gauge leptoquarks with fermions can be obtained from (6) by believing

$$
(g_L^{L})_{pi} = \frac{g_L}{\sqrt{2}}(K_L^L)_{pi}, \quad (g_R^{L})_{pi} = 0,
$$

$$
(g_L^{R})_{pi} = 0, \quad (g_R^{R})_{pi} = \frac{g_R}{\sqrt{2}}(K_R^R)_{pi}
$$

and takes the form

$$
L_{N^d} = \frac{g_L}{\sqrt{2}}(d_{PC}((K_L^L)_{pi}\gamma^\mu P_L)_L)V_{\alpha\mu}^L + \frac{g_R}{\sqrt{2}}(d_{PC}((K_R^R)_{pi}\gamma^\mu P_R)_L)V_{\alpha\mu}^R + h.c.,
$$

3
where \( g_L^4 \), \( g_R^4 \) are the gauge coupling constants of the group (3) or (4) which are related to strong coupling constant by the equation

\[
g_L^4 g_R^4 / \sqrt{(g_L^4)^2 + (g_R^4)^2} = g_{st}. \tag{21}
\]

As a result of calculations we have obtained the widths of the \( K_L^0 \to e\mu, B^0 \to e\tau \) decays with account of the contributions of the chiral gauge leptoquarks \( V^L, V^R \) in the form

\[
\Gamma(K_L^0 \to e\mu) = \frac{m_{K^0} f_{K^0}^2 m_e^2}{64\pi} \left(1 - \frac{m_e^2}{m_{K^0}^2}\right)^2 \left[\frac{(g_4^L)^4}{4m_{V^L}^2} |\kappa_{21}^L|^2 + L \leftrightarrow R\right], \tag{22}
\]

\[
\Gamma(B^0 \to e\tau) = \frac{m_{B^0} f_{B^0}^2 m_\tau^2}{32\pi} \left(1 - \frac{m_\tau^2}{m_{B^0}^2}\right)^2 \times \left[\frac{(g_4^L)^4}{4m_{V^L}^2} |\kappa_{31}^L|^2 + L \leftrightarrow R\right], \tag{23}
\]

where the parameters

\[
\kappa_{ij}^{L,R} = (K_2^L,R)_{2i}(K_2^L,R)_{1j} - (K_2^L,R)_{1i}(K_2^L,R)_{2j}, \tag{24}
\]

\[
k_{ij}^{L,R} = (K_2^L,R)_{3i}(K_2^L,R)_{1j} \tag{25}
\]

account the effects of the fermion mixing in the leptoquark currents.

We have numerically analysed the widths (13), (14), (22), (23) in dependence on the leptoquark masses with varying the values of the quark masses in the scalar leptoquark coupling constants and in (17) . The gauge coupling constants we relate to \( g_{st}(M_c) \) at \( M_c = 1000 \text{GeV} \), in the case of chiral gauge leptoquarks we assume also that \( g_4^L = g_4^R (= \sqrt{2}g_{st}) \). For simplicity we assume \( K_2^L = K_2^R = I \) and for definiteness we believe \( \sin \beta = 0.7 \). We use the values of the form factors \( f_{K^0} = 160 \text{MeV}, f_{B^0} = 170 \text{MeV} \) and the life times \cite{8} of \( K_L^0 \) and \( B^0 \) mesons.

![Figure 1: Branching ratio of \( K_L^0 \to e\mu \) decay in dependence on the mass \( m_{S^L} \) of the chiral scalar leptoquark \( S^L \) for 1)\( m_d = 11 \text{MeV}, m_s = 175 \text{MeV} \), 2)\( m_d = 14 \text{MeV}, m_s = 230 \text{MeV} \), 3)\( m_d = 22 \text{MeV}, m_s = 350 \text{MeV} \).](image)
Fig. 1 shows the branching ratio of $K^0_L \to e\mu$ decay in dependence on the mass $m_{SL}$ of the chiral scalar leptoquark $S^L$. The horizontal dashed line shows the experimental upper limit $Br(K^0_L \to e\mu) < 4.7 \times 10^{-12}$ [8]. The curves 1, 2 correspond to quark masses

\begin{align*}
m_d &= 11\,MeV, \ m_s = 175\,MeV, \quad (26) \\
m_d &= 14\,MeV, \ m_s = 230\,MeV \quad (27)
\end{align*}

which are taken at the mass scale $\mu = 1\,GeV$ and (approximately) at $\mu = 750\,MeV$ respectively. Strictly speaking one should use the quark masses at the mass of $K^0_L$-meson but this is a nonperturbative region and we cannot calculate them. Nevertheless one can hope that the quark masses at $m_{K^0_L}$ are larger that those at $750\,MeV$ and, hence the contribution of the scalar leptoquark is also larger. To illustrate how such contribution could affect the branching ratio of $K^0_L \to e\mu$ decay we present also the curve 3 for the quark masses

$$m_d = 22\,MeV, \ m_s = 350\,MeV$$

which are chosen twice as large as those at $\mu = 1\,GeV$. As seen in all three cases the lower mass limits for the chiral scalar leptoquark are small, below the mass limits from the direct search for scalar leptoquarks. The contributions of the leptoquarks of the scalar or pseudoscalar types are slightly larger but the corresponding mass limits are still below the direct scalar leptoquark mass limits.

Figure 2: Branching ratio of $K^0_L \to e\mu$ decay in dependence on the mass $m_V$ of the vector leptoquark $V$ for the quark masses $m_d = 11\,MeV, \ m_s = 175\,MeV$ (curve 1), $m_d = 14\,MeV, \ m_s = 230\,MeV$ (curve 2), $m_d = 22\,MeV, \ m_s = 350\,MeV$ (curve 3) for the case a) with neglect of the scalar leptoquark contribution and for the case b) with account of the pseudoscalar leptoquark contribution and on the chiral gauge leptoquark mass $m_{VL}$ (curve 4a).

The branching ratios of $K^0_L \to e\mu$ decay as the functions of the masses $m_V, m_{VL}$ of the vector and chiral gauge leptoquarks $V, V^L$ are shown in Fig.2. The curves 1, 2, 3 correspond to the quark masses $m_d = 11\,MeV, \ m_s = 175\,MeV$ (case a) when only the vector leptoquark $V$ is taken into account with neglecting the scalar leptoquarks and for the case b) with account of both the vector leptoquark $V$ and the pseudoscalar leptoquark $S^P$ with $m_{SP} = 250\,GeV$. For the case 1a) the lower vector leptoquark mass limit is of order of $m_V > 1100\,TeV$. 

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As seen, in these cases (due to the destructive interference of the pseudoscalar and vector leptoquark contributions and due to the mass dependence in (17)) the lower mass limit for the vector leptoquark can be decreased to about \(m_V > 800 \text{TeV}\) or slightly below in dependence on the quark masses. The curve 4a) corresponds to case when only the chiral gauge leptoquark \(V^L\) is taken into account. The lower mass limit for the chiral gauge leptoquark in this case is of order of \(m_{V^L} > 260 \text{TeV}\). One can hope that the account of the scalar leptoquarks can result in further decreasing the mass limit for the chiral gauge leptoquark.

As concerns the \(B^0 \rightarrow e\tau\) decay the current experimental limit on its branching ratio \(Br(B^0 \rightarrow e\tau) < 5.3 \cdot 10^{-4} \) [8] gives the relatively weak limits on the leptoquark masses. For example the lower mass limits from \(B^0 \rightarrow e\tau\) decay for the scalar leptoquarks are only of order of a few GeV, i.e. they lie essentially below the mass limit from the direct search for scalar leptoquarks. The mass limits from \(B^0 \rightarrow e\tau\) decay for the gauge leptoquarks are also weaker than those from \(K^0_L \rightarrow e\mu\) decay nevertheless they are of interest as the new independent on \(K^0_L \rightarrow e\mu\) decay mass limits.

\[\text{Figure 3: Branching ratio of } B^0 \rightarrow e\tau \text{ decay in dependence on the mass } m_V \text{ of the vector leptoquark } V \text{ (curve 1) and on the chiral gauge leptoquark mass } m_{V^L} \text{ (curve 2).}\]

The Fig.3 shows the branching ratios of \(B^0 \rightarrow e\tau\) decay as the functions of the masses \(m_V, m_{V^L}\) of the vector and chiral gauge leptoquarks \(V, V^L\). The curve 1 corresponds to the case when only the vector leptoquark \(V\) is taken into account with neglecting the scalar leptoquarks. In this case the lower mass limit for the vector leptoquark is of order of \(m_V > 3.4 \text{TeV}\). The curve 2 corresponds to the case when only the chiral gauge leptoquark \(V^L\) is taken into account. The lower mass limit for the chiral gauge leptoquark in this case is of order of \(m_{V^L} > 1.6 \text{TeV}\). Because the contributions into \(B^0 \rightarrow e\tau\) decay from the scalar leptoquarks with masses allowed by their direct search are small the interference of the scalar and gauge leptoquark contributions in the \(B^0 \rightarrow e\tau\) decay is negligible.

In conclusion one can say that the mass limits for the scalar leptoquark doublets from the current experimental bounds on the branching ratios of \(K^0_L \rightarrow e\mu\) and \(B^0 \rightarrow e\tau\) decays (in contrast to the corresponding mass limits for the gauge leptoquarks) occur to be small, of order of or below the mass limits from the direct search for scalar leptoquarks. The search for these scalar leptoquarks at LHC as well the further search for the \(B^0\) leptonic decays of \(B^0 \rightarrow l_i^+ l_j^-\) type are of interest.

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