A few years after the discovery of superconductivity in CuO$_2$-systems, a new oxide-superconductor has been found: Sr$_2$RuO$_4$. This system has actually the same layered perovskite structure as La$_2$CuO$_4$, but behaves otherwise very differently. In its stoichiometric composition Sr$_2$RuO$_4$ is metallic, displays Fermi liquid behavior and becomes superconducting at the rather low transition temperature $T_c \approx 1.5$K. The electronic properties of Sr$_2$RuO$_4$ are determined by the three $4d_{t_2g}$ orbitals ($d_{xz}, d_{yz}, d_{xy}$) of the Ru$^{4+}$-ions which form three bands that cross the Fermi level [3]. As a result we have two electron-like and one hole-like Fermi surface [4]. There are clear indications that the superconducting state is unconventional. The transition temperature is highly sensitive to the non-magnetic impurities [5] and the NQR-experiments do not show any Hebel-Slichter peak in $1/T_1$. Shortly after the discovery of Sr$_2$RuO$_4$ it has been suggested that this superconductor might form odd-parity (spin-triplet) Cooper pairs in contrast to the even-parity states belonging to the two-dimensional $d_{xy}$-plane. The odd-parity state which is compatible with spin fluctuation feedback mechanism analogous to the A-phase in $^3$He and the presence of ferromagnetism in related compounds such as SrRuO$_3$ [3].

Meanwhile a number of experiments point indeed towards odd-parity pairing. In $\mu$SR-experiments the enhancement of the zero-field relaxation rate in the superconducting state indicates the presence of intrinsic magnetism [9]. This occurs in connection with broken time-reversal symmetry of the superconducting state [10]. We will give in the next section a brief argument on the reason why this result suggests odd-parity pairing. Even more compelling evidence is provided by the $^{17}$O NMR Knight shift data which show that the spin susceptibility is not affected by the superconducting state for magnetic fields parallel to the RuO$_2$-plane. The odd-parity state which is compatible with all the present data has the form of $d(k) = \tilde{z}(k_x \pm ik_y)$. In this talk we will discuss the symmetry aspects of the superconducting state, first ignoring the multi-band structure of this compound. In the second part, however, we will analyze the influence of the different orbitals and give a brief outline of the idea of orbital dependent superconductivity. Finally, we will examine weak coupling theories based on ferromagnetic spin fluctuations which could stabilize the most promising candidate, $d(k) = \tilde{z}(k_x \pm ik_y)$.

II. SYMMETRY OF THE SUPERCONDUCTING STATE

The layered perovskite structure of Sr$_2$RuO$_4$ corresponds to the tetragonal point group $D_{4h}$. The possible Cooper pairing states can be classified according to the irreducible representations of $D_{4h}$ which include four one-dimensional and one two-dimensional representation for both even and odd parity [10]. This determines the orbital symmetry of the even-parity (spin-singlet) states completely. In the case of odd-parity pairing, however, the Cooper pairs have a spin degree of freedom whose orientation is completely determined, if spin-orbit coupling is taken into account, i.e. the spin and the crystal lattice orientation are not independent. In Table I we list both even- and odd-parity states using simple basis functions on a cylindrical Fermi surface, i.e. expressed in momenta $k$ with $|k| = k_F$. The pair wavefunction can be expressed in the standard notation by a scalar function $\psi(k)$ for even and by a vector function $d(k)$ for odd parity pairing states [10].

It is rather unlikely to find pairing among electrons on different RuO$_2$-layers as they are well separated. As a consequence, even-parity states belonging to the two-dimensional $E_u$-representation can be ruled out, because they would require interlayer pairing and would actually by symmetry not have any pairing amplitude within the plane. All other representations possess intra-layer pairing states. In particular, the two-dimensional representation $E_u$ contains the basis states $\{z_{k_x}, z_{k_y}\}$. In Table II we list the three possible combinations of these two $E_u$-basis states. All three states break a symmetry in addition to the U(1)-gauge symmetry. The a-phase has broken time reversal symmetry,
while the b- and c-phase lower the crystal field symmetry. Within a weak-coupling approach the a-phase is the most stable state as we will see below, since its gap has no nodes.

Let us now compare these states with some experiments mentioned above. The only state which breaks time reversal symmetry is the a-phase in Table II, \(d(k) = \hat{z}(k_x \pm ik_y)\). Other possible time reversal symmetry breaking states would involve the complex combinations of two states belonging to different representations. In general, this would lead to double phase transitions and only below the second transition the state with broken time reversal symmetry would appear. This is in clear contradiction with the experimental \(\mu\)SR-data which show only one phase transition, the onset of superconductivity, coincident with the appearance of intrinsic magnetism, the sign for the violation of time reversal symmetry \[14\]. Thus, this experiment is a strong indication for the odd-parity pairing state \(\hat{z}(k_x \pm ik_y)\) which is the two-dimensional analog to the \(\lambda\)-phase in superfluid \(^3\)He.

This interpretation is supported by measurements of the spin susceptibility in the superconducting state using \(^{17}\)O-Knight shift \[11\]. For odd-parity states the uniform susceptibility has the form

\[
\chi_{\mu\nu} = \chi_0 \left[ \delta_{\mu\nu} - \langle (1 - Y(k,T))\text{Re}d_k^{\dagger}(k)d_k(k) \rangle_{FS} \right]
\]

where \(Y(k,T)\) is the angle-dependent Yosida function, \(\chi_0\) is the Pauli spin susceptibility and \(\langle . . . \rangle_{FS}\) denotes the average over the Fermi surface \[13\]. For the state \(\hat{z}(k_x \pm ik_y)\) we obtain,

\[
\chi_{\mu\nu}(T) = \chi_0 \delta_{\mu\nu} \left\{ \frac{\langle Y(k,T) \rangle_{FS}}{1} \right\}_{H \parallel \hat{z}} \quad \text{H \perp \hat{z}}
\]

The experiment can only be performed for the field in the basal plane where it agrees perfectly with the expected result \[11\].

Based on these experimental findings we can derive the corresponding Ginzburg-Landau theory which is based on a two-dimensional order parameter \(\eta = (\eta_x, \eta_y)\) so that \(d(k) = \hat{z}(k \cdot \eta)\). The free energy has the following general form,

\[
F = \int d^3r \left[ a(T - T_c)|\eta|^2 + b_1|\eta|^4 + \frac{b_2}{2} (\eta_x^2\eta_y^2 + \eta_x^2\eta_y^2) + b_3|\eta_x|^2|\eta_y|^2 + K_1(|D_x\eta_x|^2 + |D_y\eta_y|^2) + K_2(|D_x\eta_x|^2 + |D_y\eta_y|^2) + \right. \\
+ \left. K_3(D_x\eta_x)^*(D_y\eta_y) + K_4(D_y\eta_y)^*(D_x\eta_x) + \text{c.c.} \right] + K_5(|D_z\eta_x|^2 + |D_z\eta_y|^2) + \frac{(\nabla \times \mathbf{A})^2}{8\pi}, \quad (3)
\]

where \(a, b_i\) and \(K_i\) are real coefficients and \(\mathbf{D} = \nabla - i2e\mathbf{A}/\hbar c\) is the gauge-invariant gradient \[10\]. The choice \(b_2 > 0, b_3 < b_2\) stabilizes the a-phase with \(\eta = \eta_0(1, \pm i)\). Based on this free energy Agterberg showed that the vortex lattice has square lattice form for fields along the \(z\)-axis \[14\]. The orientation of the lattice depends on the coefficients of the free energy. Recent neutron scattering experiments show indeed a square vortex lattice with the main axis orientation parallel to that of the crystal lattice \[18\].

III. ORBITAL DEPENDENT SUPERCONDUCTIVITY

As we mentioned above the Fermi liquid state is formed by the bands belonging to the three \(t_{2g}\)-orbitals, \(d_{yz}, d_{zx}\) and \(d_{xy}\). By symmetry the \(d_{xy}\)-band is distinct from the two bands belonging to the other two orbitals. It can be shown that the pair scattering between the two types of bands is weak due to the special character of the orbitals \[14\] \[15\]. Therefore we may assume that the superconductivity is associated with one of the two subsystems while the other is only participating via the pair component induced by the weak pair scattering, which represents a form of proximity effect in the momentum space. We call this phenomena “orbital dependent superconductivity” (ODS) \[14\].

The superconducting state \(\hat{z}(k_x \pm ik_y)\) opens a gap over the whole Fermi surface. As a consequence of ODS, however, this gap is very different in magnitude for the Fermi surfaces with intrinsic and induced superconductivity. This aspect appears, in particular, in thermodynamic quantities such as the specific heat which seems to preserve a large amount of low-energy density of states (DOS) down to rather low temperatures \[14\]. Theses states close to the Fermi level are associated with the orbitals which are not intrinsically superconducting. Only at rather low temperature the gap induced on these bands is expected to become visible in thermodynamic quantities. Experimentally this virtual residual DOS has been found to be of the order of 40 - 50 % of the normal state DOS.

The recent analysis of London penetration depth and coherence length by Riseman and coworkers led to the further strong evidence for ODS identifying \(d_{xy}\) as the orbital relevant for superconductivity \[18\]. This yields the value for the expected residual DOS of about 43 % derived from the effective mass experiments in the de Haas-van Alphen
measurements, which is very consistent with the specific heat data \[1\]. In addition Agterberg has shown that the \( d_{xy} \)-orbital would lead to coefficients of the Ginzburg-Landau theory in the proper range to account for the orientation of the vortex lattice \[17\].

IV. SPIN FLUCTUATIONS AND THE SYMMETRY OF THE SUPERCONDUCTING STATE

In a recent NMR experiment Imai and coworkers extracted the uniform spin susceptibility for each of the orbitals separately from their \( ^{17}\)O-Knight shift data taken in the normal state over wide temperature range \[13\]. The susceptibility associated with the \( d_{xy} \)-orbital is considerably larger than the contributions of the other two bands. Moreover, it is significantly increasing with lowering temperature, while the other two bands have a more or less temperature-independent susceptibility. This suggests that the tendency towards ferromagnetism is stronger for the \( d_{xy} \)-orbital than the others. In view of the idea of orbital dependent superconductivity we can simplify the following discussion by restricting ourselves to a single band belonging to the relevant orbital (\( d_{xy} \)) assuming also ferromagnetic spin fluctuations as the dominant mechanism for pairing. Before going into the details of the symmetry analysis of the resulting pairing state, we would like to give here an argument on why ferromagnetic spin fluctuations are important in \( \text{Sr}_2\text{RuO}_4 \). There is a series of ferromagnetic compounds related to \( \text{Sr}_2\text{RuO}_4 \), the Ruddlesden-Popper series \( \text{Sr}_{n+1}\text{Ru}_n\text{O}_{3n+1} \), which are multi-layer compounds with \( n \) as the number of \( \text{RuO}_2 \)-planes per unit cell. The infinite-layer (3D) \( \text{SrRuO}_3 \) is a ferromagnet with \( T_C \approx 165\text{K} \) whereby the band structure calculations give a good understanding within the Stoner theory \[12\]. For \( n = 3 \) one finds \( T_C \approx 148\text{K} \) and for \( n = 2 \) \( T_C \approx 104\text{K} \) (although the ferromagnetism in this latter case is controversial \[21,22\]). This demonstrates the tendency that with decreasing layer number \( n \) \( T_C \) is reduced and vanishes finally. Hence, we may consider \( n \) as the parameter controlling a quantum phase transition between a ferromagnetic and a magnetically disordered phase. In the schematic phase diagram depicted in Figure 1 we see that the single-layer compound \( \text{Sr}_2\text{RuO}_4 \) would lie very near to the quantum critical point so that ferromagnetic spin fluctuations play an important role. We may argue that the moderately enhanced Wilson ratio \( R_W \approx 1.4 \) would contradict this conclusion. Note, however, that the two-dimensionality plays an important role in reducing \( R_W \) as pointed out recently by Julian and coworkers \[23\]. In two dimensions it should diverge as \( R_W \propto (1 - a)^{-1/2} \) as the quantum phase transition is approached (controlling parameter \( a \to 1 \)) which is weaker than in three dimensions where \( R_W \propto -\ln(1 - a)/(1 - a) \). Therefore the comparison with other (three-dimensional) nearly ferromagnetic systems could be misleading. Moreover there are other mechanisms renormalizing the mass.

A. Phenomenological model

Now we discuss a single band model for Cooper pairing assuming that the (relevant) \( d_{xy} \)-band has cylindrical symmetry. The Hamiltonian has the form,

\[
\mathcal{H} = \sum_{k,s} \epsilon_k c^\dagger_{ks} c_{ks} + \frac{1}{2} \sum_{k,k',s_1,s_2,s_3,s_4} V_{k,k';s_1s_2s_3s_4} c^\dagger_{ks_1} c^\dagger_{ks_2} c_{k's_3} c_{k's_4},
\]

(4)

where \( \epsilon_k \) denotes the electron band energy measured from the Fermi energy and \( c^\dagger_{ks} \) and \( c_{ks} \) are the Fermion creation and annihilation operators. The effective pairing interaction is mediated by the spin fluctuations (paramagnon exchange) which we describe here by the static susceptibility \( \chi_{\mu\nu}(q) \),

\[
V_{k,k';s_1s_2s_3s_4} = -\frac{r^2}{4} \sum_{\mu,\nu} \chi_{\mu\nu}(k - k') a^\mu_{s_1s_4} a^\nu_{s_2s_3}
\]

(5)

where \( I \) is an interaction constant. We ignore the dynamical part of the spin fluctuations and adopt a weak coupling approach where the interaction is finite in a certain range around the Fermi energy. The corresponding cutoff frequency \( \omega_c \) limiting the attractive region is not so easy to define, but is sometimes brought into connection with the largest paramagnon frequency, \( \gamma q_c (1 - a)/\chi_0 \) with a cutoff wave vector around \( 2k_F \) and \( \gamma \) a phenomenological parameter \[24\]. For the static susceptibility we use for simplicity the small-\( q \) approximation \( \chi_{\mu\nu}(q) = \chi_0 \delta_{\mu\nu}/(1 - a + cq^2) \) where \( c \) is the spin stiffness \( (q^2 = q_x^2 + q_y^2) \) \[23\]. The factor \( (1 - a)^{-1} \) describes the Stoner enhancement. This approximation is certainly an oversimplification of the real situation and a more realistic form based on band structure data may be found in Ref. \[13\]. This simplification would, however, not invalidate our further discussion, since we will concentrate on aspects related to symmetry of the states by comparing the classified states in Table I and II. The momentum structure should affect all states essentially in the same way.
The standard BCS paring meanfield scheme leads to the self-consistence equation given by

$$d^\alpha(k) = \frac{I^2}{4N} \sum_{k',\beta} \left[ \sum_\mu \chi_{\mu\nu}(k-k') \delta_{\alpha\beta} - \chi_{\alpha\beta}(k-k') - \chi_{\beta\alpha}(k-k') \right] d^\beta(k') \tanh \left( \frac{E_{k'}}{2k_BT} \right)$$

with $E_k = [\epsilon_k + |d(k)|^2]^{1/2}$ and excluding non-unitary pairing states, i.e. we impose the condition $d^* \times d = 0$.

B. Spin rotation symmetric case

In the absence of spin-orbit coupling $\chi_{\mu\nu}(q) = \chi(q) \delta_{\mu\nu}$ as given above. Assuming a small cutoff energy we can expand $\chi_k(k-k')$ in Legendre polynomials $P_l(\theta)$ where $\theta$ denotes the angle between $k$ and $k'$ for momenta lying on the Fermi surface. The lowest and only relevant component is $P_{l=1}(\theta) = \cos(\theta)/\sqrt{\pi}$ which yields the self-consistence equation

$$d^\alpha(k) = \frac{g}{N} \sum_{k'} \frac{k \cdot k'}{k^2} d^\alpha(k') \tanh \left( \frac{E_{k'}}{2k_BT} \right)$$

with the coupling constant $g \approx (I^2\chi_0/(8kF))/\sqrt{c(1-c)}$. The superconducting transition temperature is obtained from the linearized self-consistence equation,

$$k_BT_c = 1.14\omega_c \exp(-1/N(0)g)$$

where $N(0)$ is the DOS of the $d_{xy}$-band at the Fermi level. In this simplified calculation we have, however, ignored the effect of the spin fluctuation on the renormalization of the quasiparticles forming the Cooper pairs. This leads to an additional renormalizing factor in the exponent of Eq. (6), $\exp((1+\lambda)/N(0)g)$ where $\lambda$ is approximately proportional to $g$ for the contribution due to the spin fluctuations [24,13]. Note, however, that the multi-band structure makes this analysis somewhat more complicated than in the pure single-band case [13].

The transition temperature $T_c$ given in Eq. (7) is the same for any state of the form $d(k) = \sum_{\alpha,\mu} d_{\alpha\mu} \hat{n}_\alpha k_\mu$ where $\hat{n}_\alpha$ is the unit vector in $\alpha$-direction in the basis for the $d$-vectors. In order to find the states which are actually stable in the self-consistence equation we have to go beyond the linearized form. Already the Ginzburg-Landau theory which is easily derived from Eq. (7) using $d_{\alpha\mu}$ as order parameters, is sufficient.

$$F = f_0 \left\{ \sum_{\alpha,\mu} \ln \left( \frac{T}{T_c} \right) |d_{\alpha\mu}|^2 + \frac{b}{16} \sum_{\alpha,\beta,\mu,\nu} |d_{\alpha\mu}|^2 |d_{\beta\nu}|^2 + d_{\alpha\mu}^* d_{\alpha\nu} d_{\beta\mu}^* d_{\beta\nu} + d_{\alpha\mu}^* d_{\alpha\nu} d_{\beta\mu}^* d_{\beta\nu} \right\}$$

with $b = 7\zeta(3)/(8\pi k_BT_c)^2$. By examination we find that states which have gaps $|d(k)|^2$ without nodes are more stable than those with nodes. All states belonging to the one-dimensional representations, $A_1u, A_2u, B_1u, B_2u$ and the $\alpha$-phase of the $E_u$-representation, as listed in Table I and II, have nodeless gaps. Within this free energy analysis they are degenerate. Thus the spin symmetric weak-coupling theory results in six pairing states of the same condensation energy, a property specific to two dimension. In three dimensions there is only one (non-degenerate) state of this kind, the BW-state, which forms the B-phase of superfluid $^3$He [13].

We will now investigate further mechanisms which would favor one state over the others. One such mechanism is based on the feedback effects of the superconducting state on the spin fluctuations which is well known to stabilize the A-phase in $^3$He. Let us give here a simplified approach to this mechanism which, however, contains all the essential physics, following Leggett [13]. The presence of superconductivity modifies the spin susceptibility entering the pairing interaction.

$$\chi_{\mu\nu}(q) = \chi_N(q) \delta_{\mu\nu} + \delta\chi_{\mu\nu}(q)$$

$$= \chi_N(q) \delta_{\mu\nu} + f(q) \delta\chi_{0\mu\nu}.$$  

We introduce the form factor $f(q)$ for the correction of the (uniform) susceptibility [13,26] which is given by Eq. (1),

$$\delta\chi_{0\mu\nu} = \chi_0 2b (\text{Red}_\mu^* (k) d_\nu (k))_{FS} + O(|d|^4)$$

where we have expanded $\delta\chi_{0\mu\nu}$ in terms of $d$ close to $T = T_c$. Inserting this expression as a correction to the interaction into the self-consistence equation we can easily derive an additional fourth-order term to the Ginzburg-Landau free energy in Eq. (7) of the form,
\[
\delta F = \kappa \sum_{\alpha, \beta, \mu, \nu} (\text{Re} \, d_{\alpha \mu}^* \delta_{\beta \mu}) \{ \delta_{\alpha \beta} \sum_{\alpha'} d_{\alpha' \nu}^* d_{\alpha' \nu} - 2(\text{Re} \, d_{\alpha \nu}^* d_{\beta \nu}) \},
\]

where \( \kappa \) is a positive constant \[13,20\]. Inserting the different nodeless states we find that \( \tilde{z}(k_x \pm ik_y) \) is energetically favored over all the other states, in close analogy to the A-phase of \(^3\)He.

C. The effect of spin-orbit coupling

The spin fluctuation feedback mechanism stabilized the a-phase in the fourth order term of the free energy expansion. In second order there is still complete degeneracy among all the states due to the spin rotation symmetry. We now discuss the effect of spin-orbit coupling which lifts this degeneracy. With spin-orbit coupling the spin susceptibility entering into the pairing interaction has reduced symmetry in the sense that it needs to be a scalar only under simultaneous rotations of both spin and orbital part and not anymore under their separate transformation. On the lowest level of expansion equivalent to the term (proportional to the first Legendre polynomial) given in Eq.(3) we find the general form invariant under cylindrical symmetry,

\[
\chi_{\mu \nu} (\mathbf{k} - \mathbf{k}') = \chi^{(1)}_{\mu \nu} \mathbf{k} \cdot \mathbf{k}' + \chi^{(2)}_{\mu \nu} k_\mu k'_\nu
\]

which we approximate by essentially three phenomenological parameters, \( \chi^{(1)}_{x,y} = g_1, \chi^{(1)}_{z} = g_2 \) and \( \chi^{(2)}_{x,y} = -\chi^{(2)}_{y,x} = g_3 \). In the linearized gap equation this susceptibility leads to

\[
v d^0 (\mathbf{k}) = \int \frac{d^2 k'}{\pi k_F^2} \sum_{\beta} \left[ (2g_1 + g_2 - 2\chi^{(1)}_{\alpha \beta}) k' \cdot \delta_{\alpha \beta} - g_3 (k_\alpha k'_\beta - k_\beta k'_\alpha) \right] d^3 (\mathbf{k}')
\]

with \( k_F T_c = 1.14 \omega_c \exp(-4/I^2 N(0)v) \) and \( v \) is an eigenvalue in this equation. In Table III we give a list of the eigenvalues and eigenstates. The states separate into three subsets of doubly degenerate eigenvalues (transition temperature) according to their total angular momentum \( J_z \). The stable state is decided by the choice of the parameters \( g_{1,3} \) (\( g_3 \sim |g_1 - g_2| \)). Therefore, if the spin fluctuations are enhanced for spin orientations in the basal plane the state \( \tilde{z}(k_x \pm ik_y) \) can reach the highest transition temperature. Indeed recent experiments by Mukuda et al. show that the ferromagnetic spin fluctuations are apparently stronger in-plane than out-of-plane \[27\]. Whether this is the case for the \( d_{xy} \)-orbital remains to be investigated.

V. CONCLUSIONS

The interpretation of various experiments led to the conclusion that the most likely superconducting state of \( \text{Sr}_2\text{RuO}_4 \) is given by the time reversal symmetry breaking pairing state of the form \( \mathbf{d}(\mathbf{k}) = \tilde{z}(k_x \pm ik_y) \) reminding of the A-phase of superfluid \(^3\)He. This state is compatible with all currently available experimental data. Also recent investigations on Josephson contacts between \( \text{Sr}_2\text{RuO}_4 \) and \( \text{Pb} \) should be included here \[28\], although their interpretation is not completed yet \[29,30\]. Furthermore, there is good reason for the assumption that superconductivity can be mainly associated with the \( d_{xy} \)-band, while the other two orbitals participate only passively via proximity-induced pairing. This could, in particular, account for the large apparent residual DOS in the superconducting state. It seems rather unlikely that a non-unitary state as proposed earlier to explain the large residual DOS \[17\] is realized here, since such a state is very difficult to stabilize unlike the unitary state \( \tilde{z}(k_x \pm ik_y) \) \[35\]. We have shown that a spin fluctuation based mechanism favors the state \( \tilde{z}(k_x \pm ik_y) \). There are basically two stabilizing mechanisms: (1) the spin fluctuation feedback analogous to \(^3\)He and (2) the spin-orbit coupling effects enhancing the ferromagnetic spin fluctuation with spin orientations parallel to \( \text{RuO}_2 \)-plane.

In conclusion, we would like to emphasize that \( \text{Sr}_2\text{RuO}_4 \) is probably the first odd-parity superconductor emerging out of a strongly correlated electron system which is a clear Fermi liquid. In the case of the heavy Fermion superconductors \( \text{UPt}_3 \) and \( \text{UBE}_{13} \), two other candidates for odd-parity pairing, the situation is considerably more complicated. Most experimental data for \( \text{Sr}_2\text{RuO}_4 \) show that the properties of the superconductor can be investigated in a controlled and clean way. Therefore, \( \text{Sr}_2\text{RuO}_4 \) may in future become the textbook example for the study and presentation of unconventional superconductivity. The fact that the superconducting order parameter has two components, yields also a large space for unusual phenomena \[10,22\]. These include the formation of domains separated by domain walls, non-axial vortices or various collective modes, to mention only a few.
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| $\Gamma$ | $\psi(k)$ | $\Gamma$ | $d(k)$ |
|---------|-----------|---------|-------|
| $A_{1g}$ | 1 | $A_{1u}$ | $\hat{x}k_x + \hat{y}k_y$ |
| $A_{2g}$ | $k_xk_y(k_x^2 - k_y^2)$ | $A_{2u}$ | $\hat{x}k_y - \hat{y}k_x$ |
| $B_{1g}$ | $k_x^2 - k_y^2$ | $B_{1u}$ | $\hat{x}k_x - \hat{y}k_y$ |
| $B_{2g}$ | $k_xk_y$ | $B_{2u}$ | $\hat{x}k_y + \hat{y}k_x$ |
| $E_u$ | - | $E_u$ | $\{\hat{z}k_x, \hat{z}k_y\}$ |

TABLE I. List of possible pairing states for the tetragonal point group $D_{4h}$ with even and odd parity.
TABLE II. The three phases of the $E_u$-representation with basis \{\hat{z}k_x, \hat{z}k_y\}.

| Phase | \(d(k)\) |
|-------|------------------|
| a-phase | \(\hat{z}(k_x \pm ik_y)\) |
| b-phase | \(\hat{z}(k_x \pm k_y)\) |
| c-phase | \(\hat{z}k_x, \hat{z}k_y\) |

TABLE III. The eigenvalues and eigenstates of the gap equation Eq. (14) with their total angular momentum \(J_z\).

| \(d(k)\) | \(J_z\) | \(v\) |
|---------|---------|------|
| \(\hat{x}k_x + \hat{y}k_y\) | 0 | \(2(g_2 - g_3)\) |
| \(\hat{x}k_y - \hat{y}k_x\) | 0 | \(2(g_2 - g_3)\) |
| \(\hat{x}k_x - \hat{y}k_y\) | \(\pm 2\) | \(2(g_2 + g_3)\) |
| \(\hat{x}k_y + \hat{y}k_x\) | \(\pm 2\) | \(2(g_2 + g_3)\) |
| \{\(\hat{z}k_x, \hat{z}k_y\)\} | \(\pm 1\) | \(2g_1\) |

FIG. 1. Schematic phase diagram of the ferromagnetic and superconducting members of the Ruddlesen-Popper series. The number of layers is the parameter which determines the quantum phase transition between the two phases. (FM: ferromagnetic; SC: superconducting).