We study nonperturbative features of QCD using the dual Ginzburg-Landau theory. The color confinement is realized through the dual Higgs mechanism, which is brought by QCD-monopole condensation. We investigate the infrared screening effect on the color confinement due to the light-quark pair creation. By solving the Schwinger-Dyson equation, we find that the dynamical chiral-symmetry breaking is largely brought by the confining force.

1. Dual Higgs Mechanism for Color Confinement

We study color confinement and dynamical effects of light quarks in the dual Ginzburg-Landau (DGL) theory. Because of the asymptotic freedom, QCD in the infrared region exhibits the nonperturbative features like the color confinement and the dynamical chiral-symmetry breaking (DχSB). The color confinement is characterized by the vanishing of the color dielectric constant and squeezing of the color electric flux, so that it has been regarded as the dual version of the superconductor using the duality of gauge theories. In this analogy, the confinement is brought by the dual Meissner effect originated from QCD-monopole condensation, which corresponds to Cooper-pair condensation in the superconductivity. As for the appearance of monopoles in QCD, 't Hooft proposed an interesting idea of the abelian gauge fixing, which is defined by the diagonalization of a suitable gauge dependent variable. In this gauge, QCD is reduced into an abelian gauge theory with magnetic monopoles, which appear from the hedgehog-like configuration corresponding to the nontrivial homotopy class on the nonabelian manifold, \( \pi_2(\text{SU}(N_c)/\text{U}(1)^{N_c-1}) = \mathbb{Z}^{N_c-1} \).

We compare the QCD vacuum with the superconductor in terms of the abelian gauge fixing. In the superconductor, there are two kinds of degrees of freedom, the gauge field (photon) and the matter field corresponding to the electron and the metallic lattice, which provide the Cooper pair. On the other hand, there is only the gauge field in the pure gauge QCD, and therefore it seems difficult to find the analogous point between these two systems. However, in the abelian gauge, the diagonal part and the off-diagonal part of gluons play different roles. While the diagonal part behaves as the gauge field, the off-diagonal part behaves as the charged matter and provides QCD-monopoles, whose condensation leads to the dual Higgs
mechanism, mass generation of the dual gauge field. Thus, QCD can be regarded as a similar system to the dual superconductor in the abelian gauge. Recent studies using the lattice gauge theory have reported many evidences on the abelian dominance scheme and monopole condensation for the color confinement.

2. Dual Ginzburg-Landau Theory and Quark Confinement Potential

The DGL theory is considered as an infrared effective theory of QCD in the abelian gauge. Its Lagrangian in the Zwanziger form is described by the diagonal gluon \( \vec{A}_\mu \) and the dual gauge field \( \vec{B}_\mu \),

\[
\mathcal{L}_{\text{DGL}} = -\frac{1}{n^2} [n \cdot (\partial \wedge \vec{A})]^{\nu} [n^{\ast} \cdot (\partial \wedge \vec{B})]_{\nu} - \frac{1}{2n^2} [n \cdot (\partial \wedge \vec{A})]^2 - \frac{1}{2n^2} [n \cdot (\partial \wedge \vec{B})]^2 + \sum_{\alpha=1}^{3} \left[ \left| (i \partial_{\mu} - g \vec{\epsilon}_{\alpha} \cdot \vec{B}_{\mu}) \chi_{\alpha} \right|^2 - \lambda (|\chi_{\alpha}|^2 - v^2)^2 \right] \tag{1}
\]

apart from the quark sector. (The notations are the same as those in Refs.1 and 2.) The self-interaction of the QCD-monopole field \( \chi_{\alpha} \) is introduced phenomenologically like the Ginzburg-Landau theory in the superconductivity. There is the dual gauge symmetry corresponding to the local phase invariance of the QCD-monopole field \( \chi_{\alpha} \) as well as the residual gauge symmetry embedded in SU(3)$_c$. When QCD-monopoles are condensed, the dual Higgs mechanism occurs accompanying mass generation of the dual gauge field \( \vec{B}_{\mu} \) and the spontaneous breaking of the dual gauge symmetry. On the other hand, the residual gauge symmetry is never broken in this process.

In this framework, the Dirac condition \( eg = 4\pi \) for the dual gauge coupling constant \( g \) is naturally derived in the same way as in the Grand Unified Theory.\[3,1\] In view of the renormalization group, the DGL theory is not asymptotically free in terms of \( g \) similar to the scalar QED. Hence, asymptotic freedom is expected for the gauge coupling constant \( e \) owing to the Dirac condition. Thus, the DGL theory qualitatively shows asymptotic freedom in terms of \( e \)\[3,1\] which seems a desirable feature for an effective theory of QCD.

First, we investigate the \( Q-Q \) system in the quenched level using the DGL theory. The \( Q-Q \) static potential includes the Yukawa and linear part,

\[
V(r) = -\frac{\bar{Q}^2}{4\pi} \cdot \frac{e^{-m_Br}}{r} + kr, \quad k = \frac{\bar{Q}^2 m_B^2}{8\pi} \ln\left(\frac{m_B^2 + m_\chi^2}{m_B^2}\right), \tag{2}
\]

where \( \bar{Q} \) denotes the color electric charge of the color source. Here, \( m_B \) is the mass of the dual gauge field \( \vec{B}_{\mu} \), whose inverse corresponds to the cylindrical radius of the flux tube. The expression of the string tension \( k \) is quite similar to the energy per unit length of the Abrikosov vortex in the type-II superconductor.\[3\]

3. Quark Pair Creation, Infrared Screening Effect and \( D\chi_{\text{SB}} \)
Next, we consider the dynamical effect of light quarks, which should be taken into account for the study of \(D\chi_{\text{SB}}\). A long hadron string can be cut through the light \(q\bar{q}\) pair creation, which is estimated by using the Schwinger formula. This provides the screening effect on the long-range part of the confinement potential, as is observed in the lattice QCD with light dynamical quarks. Taking account of such an infrared screening effect, we introduce the corresponding infrared cutoff \(\varepsilon\) to the gluon propagator as

\[
D_{\mu\nu}^{sc} = -\frac{1}{k^2} \left\{ g_{\mu\nu} + (\alpha_c - 1) \frac{k_\mu k_\nu}{k^2} \right\} - \frac{1}{k^2} \frac{m_B^2}{k^2 - m_B^2} \cdot \frac{\varepsilon^\lambda \mu\beta \varepsilon_{\lambda\nu\gamma\delta} n^\alpha n^\gamma k^\beta k^\delta}{(n \cdot k)^2 + \varepsilon^2}
\]

without breaking the residual gauge symmetry. Here, we have introduced the infrared cutoff to the non-local factor \(\frac{1}{(n \cdot k)^2}\), which provides the strong and long-range correlation as the origin of the confinement potential. Using this gluon propagator, we obtain a compact formula for the screened quark potential

\[
V_{\text{linear}}^{sc}(r) = k \cdot \frac{1 - e^{-\varepsilon r}}{\varepsilon}
\]

apart from the Yukawa part. This screened potential certainly exhibits the saturation for the longer distance than \(\varepsilon^{-1}\).

Finally, we investigate \(D\chi_{\text{SB}}\) in the DGL theory. We use the Schwinger Dyson (SD) equation with the gluon propagator including the nonperturbative effects on the confinement and the infrared screening. We find that QCD-monopole condensation largely contributes to the dynamical generation of the quark mass. As the physical interpretation, the dual Higgs mechanism leads to the confining force (a strong and long-range attractive force) between the \(q\bar{q}\) pair with opposite color charges, which promotes \(q\bar{q}\) pair condensation similarly in the Nambu-Jona-Lasinio model.

In conclusion, we have studied nonperturbative features of QCD using the dual Ginzburg-Landau theory. The confinement potential has been reproduced through the dual Higgs mechanism, which is brought by QCD-monopole condensation. We have investigated the infrared screening effect on the linear confinement potential due to the light-quark pair creation, and obtained a compact formula for the screened potential. \(D\chi_{\text{SB}}\) have been also studied in terms of the SD equation. We have found that \(D\chi_{\text{SB}}\) is largely brought by the confining force between the light \(q\bar{q}\) pair.

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