ON STABILITY OF CLASSES OF SOLUTIONS TO PARTIAL
DIFFERENTIAL RELATIONS CONSTRUCTED BY
QUASICONVEX FUNCTIONS AND NULL LAGRANGIANS
WITH RESPECT TO PRECOMPACT FAMILIES IN $C_{\text{loc}}$

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Abstract. We prove theorems on stability of classes of solutions to partial
differential relations constructed by quasiconvex functions and null Lagrangians
with respect to precompact families in $C_{\text{loc}}$.

Let $G$ be the class of $W^{1,k}_{\text{loc}}$-solutions $u: V \to \mathbb{R}^m$ (defined on domains $V \subset \mathbb{R}^n$) to the equation
\[
F(u'(x)) = G(u'(x)) \quad \text{a.e. } x \in V,
\]
where $F: \mathbb{R}^{m \times n} \to \mathbb{R}$ is a nonnegative quasiconvex function and $G: \mathbb{R}^{m \times n} \to \mathbb{R}$ is
a null Lagrangian. Here $u'(x)$ denotes the Jacobi matrix of $u$ at $x \in V$.

Let $F$ be the class of mappings $v \in W^{1,k}_{\text{loc}}(V; \mathbb{R}^n)$ (defined on domains $V \subset \mathbb{R}^n$) for which there exists a finite measurable function $K: V \to [1, +\infty)$, finite almost everywhere, such that
\[
F(v'(x)) \leq KG(v'(x)) \quad \text{a.e. } V.
\]
Then for $v: V \to \mathbb{R}^m$ of the class $\mathfrak{F}$ and for a.e. $x \in V$ we can define
\[
K(x, v) = \begin{cases} 
\frac{F(v'(x))}{G(v'(x))} & \text{if } G(v'(x)) > 0; \\
1 & \text{if } F(v'(x)) = 0.
\end{cases}
\]

The class $\mathfrak{G}$ has some stability property if any mapping $v \in \mathfrak{F}$ for which the
function $K(x, v)$ is close to 1 also close to some mapping $u \in \mathfrak{G}$.

In [17], the author has obtained some results on stability of $\mathfrak{G}$ in the case when the
discrepancy between $K(x, v)$ and 1 is measured in the norm of $L^\infty(V)$. In this case $v$
belongs to the classes $\mathfrak{G}(K) := \{ v: V \to \mathbb{R}^m, v \in \mathfrak{F}, \text{ess sup}_{x \in V} K(x, V) < K \}$,
$K \geq 1$. Note that the class $\mathfrak{G}(K)$ consists of $W^{1,k}_{\text{loc}}$-solutions $v: V \to \mathbb{R}^m$ (defined
on domains $V \subset \mathbb{R}^n$) of the inequality
\[
F(v'(x)) \leq KG(v'(x)) \quad \text{a.e. } V.
\]
The aim of the present paper is to prove that a mapping $v \in \mathfrak{G}(K)$ is close to
some $u \in \mathfrak{G}$ in the case when the function $K(x, v)$ is close to 1 only some integral
sense.

Our results are analogues of N. A. Kudryavtseva and Yu. G. Reshetnyak’s re-
results [36] on stability of M"obius transformations with respect to precompact (in
$C_{\text{loc}}$) families of mappings with bounded distortion. A mapping $v \in W^{1,n}_{\text{loc}}(V; \mathbb{R}^n)$

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of an open set $V \subset \mathbb{R}^n$ is an (orientation-preserving) mapping with $K$-bounded distortion, $K \geq 1$, if $v$ satisfies the distortion inequality
\begin{equation}
|v'(x)|^n \leq K \det v'(x) \quad \text{a.e. } V,
\end{equation}
where $|v'(x)|$ is the operator norm of the matrix $v'(x)$. If, in addition, $v$ is topological, then $v$ is $K$-quasisymmetric. The distortion inequality is the particular case of (2) with the following functions $F(v'(x)) = |v'(x)|^n$ and $G(v'(x)) = \det v'(x)$. The theory of quasisymmetric mappings and mappings with bounded distortion is the key part of modern geometric analysis which has many diverse applications, for example, see monographs [2, 8, 6, 22, 23, 24, 26, 30, 34, 39, 40, 43, 44, 45, 46, 47, 48, 52, 53] and the bibliography therein. In this monograph the results on stability of M"obius transformations are playing an important role. Other examples of classes of mappings which can be described as solutions of (1) with some function $F$ and $G$ can be found in [7, 9, 10, 11, 12, 28, 29, 30, 31, 49, 50]. The author has obtained some results on other properties of mappings of classes $\Theta(K)$ and $\mathfrak{g}$ in [14, 15, 16, 17, 18, 19, 20, 21].

1. Notation and Terminology

Let $A$ be a set in $\mathbb{R}^n$. The topological boundary of $A$ is denoted by $\partial A$. The diameter of $A$ is defined as $\text{diam } A := \sup \{ |x - y| : x, y \in A \}$. The outer Lebesgue measure of $A$ is denoted by $|A|$.

The set $\mathbb{R}^{m \times n} := \{ \zeta = (\zeta_{\mu})_{\mu=1,\ldots,m} : \zeta_{\mu} \in \mathbb{R}^n, \mu = 1, \ldots, m, \nu = 1, \ldots, n \}$ consists of all real $(m \times n)$-matrices. We identify a matrix $\zeta = (\zeta_{\mu})_{\mu=1,\ldots,m} \in \mathbb{R}^{m \times n}$ with the linear mapping $(\zeta_1, \ldots, \zeta_m) : \mathbb{R}^n \to \mathbb{R}^m$, where $\zeta_{\mu}(x) := \sum_{\nu=1}^{n} \zeta_{\mu \nu} x_{\nu}$, $\mu = 1, \ldots, m$, $x = (x_1, \ldots, x_n) \in \mathbb{R}^n$. The operator norm in $\mathbb{R}^{m \times n}$ is defined as $|\zeta| := \sup \{|\zeta(x)| : x \in \mathbb{R}^n, |x| < 1 \}$. The number of $k$-tuples of ordered indices in $\Gamma^k_n := \{ I = (i_1, \ldots, i_k) : 1 \leq i_1 < \cdots < i_k \leq n, i_\nu \in \{1, \ldots, n\}, \nu = 1, \ldots, k \}$ equals the binomial coefficient $\binom{n}{k} := \frac{n!}{k!(n-k)!}$. Given $x \in \mathbb{R}^n$ and $I \in \Gamma^k_n$, we put $x_I := (x_{i_1}, \ldots, x_{i_k}) \in \mathbb{R}^k$. If $I \in \Gamma^k_n$ and $J \in \Gamma^k_m$, then $\det_J I \zeta := \det \begin{pmatrix} \zeta_{i_1 j_1} & \cdots & \zeta_{i_1 j_k} \\ \vdots & \ddots & \vdots \\ \zeta_{i_k j_1} & \cdots & \zeta_{i_k j_k} \end{pmatrix}$ is the $k \times k$-minor of the matrix $\zeta \in \mathbb{R}^{m \times n}$.

The Jacobi matrix of $u = (u_1, \ldots, u_m) : U \subset \mathbb{R}^n \to \mathbb{R}^m$ at a point $x \in U$ is the matrix $u'(x) := \left( \frac{\partial u_\mu}{\partial x_\nu}(x) \right)_{\mu=1,\ldots,m, \nu=1,\ldots,n}$. If $I \in \Gamma^k_n$ and $J \in \Gamma^k_m$ then $\frac{\partial u_\mu}{\partial x_{i_1}}(x) = \frac{\partial (u_{i_1}, \ldots, u_{i_k})}{\partial x_{i_1}}(x) := \det_J I u'(x)$ and $\frac{\partial u_\mu}{\partial x_{i_1}}(x) := \left( \frac{\partial u_\mu}{\partial x_{i_1}}(x), \ldots, \frac{\partial u_\mu}{\partial x_{i_1}}(x) \right), \mu = 1, \ldots, m$.

Let $\mathcal{V}$ be a real vector space equipped with a norm $| \cdot |$. We say that a function $\Phi : \mathcal{V} \to \mathbb{R}$ is positively homogeneous of degree $p \in \mathbb{R}$ if $\Phi(t x) = t^p \Phi(x)$ for all $t > 0$ and $x \in \mathcal{V} \setminus \{0\}$.

Following Ch. B. Morrey [11], we say that a continuous function $F : \mathbb{R}^{m \times n} \to \mathbb{R}$ is quasiconvex, if
\begin{equation}
|B(0,1)| F(\zeta) \leq \int_{B(0,1)} F(\zeta + \varphi'(x)) \, dx
\end{equation}
for all $\varphi \in C^\infty_c(B(0,1); \mathbb{R}^m)$ and $\zeta \in \mathbb{R}^{m \times n}$. Let $p \geq 1$. Following M. A. Sychev [51], we say that a quasiconvex function $F$ is strictly $p$-quasiconvex if, for $\zeta \in \mathbb{R}^{m \times n}$ and $\varepsilon, C > 0$, there is $\delta = \delta(\zeta, \varepsilon, C) > 0$ such that, for each mapping $\varphi \in C^\infty(B(0,1); \mathbb{R}^m)$ satisfying $\| \varphi' \|_{L^p(B(0,1); \mathbb{R}^{m \times n})} \leq C|B(0,1)|^{1/p}$, the condition
\[ \int_{B(0,1)} F(\zeta + \varphi'(x)) \, dx \leq |B(0,1)|(F(\zeta) + \delta) \] implies \[ \{x \in B(0,1) : |\varphi'(x)| \geq \varepsilon \} \leq \varepsilon |B(0,1)|. \]

Observe that in the mathematical literature the term strictly quasiconvexity is also used for another property (which is close but nonequivalent to ours) consisting in the fact that the strict inequality in the definition of quasiconvexity is valid for nonzero mappings \( \varphi \) (for example, see [27]). In this article we use the term in the sense of M. A. Sychev’s definition [51]. In the case \( p > 1 \) the notion of strictly \( p \)-quasiconvexity for functions \( F \) of this article is equivalent to the notion of strictly closed \( p \)-quasiconvexity from J. Kristensen’s article [33] which is defined in terms of the theory of gradient Young measures (see [33, Proposition 3.4]). Observe that we can replace the ball \( B(0,1) \) in the definitions of quasiconvexity and strictly \( p \)-quasiconvexity by an arbitrary bounded domain \( U \) with \( |\partial U| = 0 \) (for example, see [42]). A function \( G : \mathbb{R}^{m \times n} \to \mathbb{R} \) is a null Lagrangian if both functions \( G \) and \( -G \) are quasiconvex. The term “null Lagrangian” appeared due to \( \partial U \). The only the affine combinations of minors (called quasi-affine functions) are null Lagrangians [13, 37] (also see [3, 4, 5, 26, 41, 12]); i.e.

\[
G(\zeta) = \gamma_0 + \sum_{k=1}^{\min\{m,n\}} \sum_{J \in \Gamma^l_k \, l \in \Gamma^r_k} \gamma_{Jl} \det J \zeta, \quad \zeta \in \mathbb{R}^{m \times n},
\]

for some \( \gamma_0, \gamma_{Jl} \in \mathbb{R} \).

Let \( C_{\text{loc}}(V; \mathbb{R}^m) \) be the space \( C(V; \mathbb{R}^m) \) furnished with the topology of locally uniform convergence.

2. Statement of the Main Results

Let \( n, m, k \in \mathbb{N} \) such that \( 2 \leq k \leq \min\{n, m\} \). We need the following hypothesis on continuous functions \( F : \mathbb{R}^{m \times n} \to \mathbb{R} \) and \( G : \mathbb{R}^{m \times n} \to \mathbb{R} \) (see [17]):

(H1) \( F \) is quasiconvex;

(H1') \( F \) is strictly \( k \)-quasiconvex;

(H2) \( G \) is a null Lagrangian;

(H3) \( F \) and \( G \) are positively homogeneous of degree \( k \);

(H4) \( \sup \{ K \geq 0 : F(\zeta) \geq KG(\zeta), \ \zeta \in \mathbb{R}^{m \times n} \} = 1; \)

(H5) \( c_F := \inf \{ F(\zeta) : \zeta \in \mathbb{R}^{m \times n}, |\zeta| = 1 \} > 0. \)

By (H3), the representation (5) for the null Lagrangian \( G \) consists only of \( (k \times k) \)-minors; i.e.,

\[
G(\zeta) = \sum_{J \in \Gamma^l_k \, l \in \Gamma^r_k} \gamma_{Jl} \det J \zeta, \quad \zeta \in \mathbb{R}^{m \times n}.
\]

It follows from (H4) that \( \mathcal{S} = \mathcal{S}(1). \)

The following theorems are the main results of the present paper.

**Theorem 2.1.** Suppose \( F \) and \( G \) satisfy (H1)–(H5). Let \( V \) be a bounded domain in \( \mathbb{R}^n \), \( K \geq 1 \), and let \( \mathcal{S} \subset \mathcal{S}(K) \cap C(V; \mathbb{R}^m) \) such that \( \mathcal{S} \) is precompact in \( C_{\text{loc}}(V; \mathbb{R}^m) \). Then for a compact subset \( U \subset V \) there exists a function \( \alpha(\varepsilon) = \alpha_{S,V}(\varepsilon), 0 \leq \varepsilon \leq \varepsilon_0, \lim_{\varepsilon \to 0} \alpha(\varepsilon) = \alpha(0) = 0, \) such that, for every mapping \( v \in \mathcal{S} \) with \( \|K(\cdot,v) - I\|_{L^1(V)} < \varepsilon_0 \), there is a mapping \( u : V \to \mathbb{R}^m \) in the class \( \mathcal{S} \)
such that
\[
\|v - u\|_{C(U; \mathbb{R}^m)} \leq \alpha(\|K(\cdot, v) - 1\|_{L^1(V)}).
\]

**Theorem 2.2.** Suppose \(F\) and \(G\) satisfy (H1') and (H2)--(H5). Let \(V\) be a bounded domain in \(\mathbb{R}^n\), \(K \geq 1\), and let \(S \subset \mathcal{G}(K) \cap C(V; \mathbb{R}^m)\) such that \(S\) is precompact in \(C_{\text{loc}}(V; \mathbb{R}^m)\). Then for a compact subset \(U \subset V\) there exists a function \(\beta(\cdot) \leq 0\), \(0 \leq \varepsilon < \varepsilon_0\), \(\lim_{\varepsilon \to 0} \beta(\varepsilon) = \beta(0) = 0\), such that, for every mapping \(v \in S\) with \(\|K(\cdot, v) - 1\|_{L^1(V)} < \varepsilon_0\), there is a mapping \(u: V \to \mathbb{R}^m\) in the class \(\mathcal{G}\) such that
\[
\|v - u\|_{C(U; \mathbb{R}^m)} + \|v' - u'\|_{L^k(U; \mathbb{R}^{m \times n})} \leq \beta(\|K(\cdot, v) - 1\|_{L^1(V)}).
\]

3. Proof of Theorem 2.2

To prove Theorem 2.2 we need the following auxiliary lemma from [17].

**Lemma 3.1 ([17] Lemma 1).** Let \(F\) and \(G\) satisfy (H2)--(H5). Let \(K \geq 1\), \(V \subset \mathbb{R}^n\) be a domain, and \(S = \{v: V \to \mathbb{R}^m\} \subset \mathcal{G}(K)\). Suppose that \(S\) is uniformly bounded in \(C_{\text{loc}}(V; \mathbb{R}^m)\) and \(S\) is uniformly bounded in \(W^{1,k}_{\text{loc}}(V; \mathbb{R}^m)\).

Let us prove Theorem 2.1. Proceeding by way of contradiction, assume that there are a compact subset \(U \subset V\), a number \(\varepsilon > 0\), and a sequence \((v_l) \in S\) with \(\|K(\cdot, v_l) - 1\|_{L^1(V)} \leq 1/l\) such that the inequality
\[
\|v_l - u\|_{C(U; \mathbb{R}^m)} > \varepsilon
\]
holds for all mappings \(u: V \to \mathbb{R}^m\) of the class \(\mathcal{G}\). Since \(S\) is precompact in \(C_{\text{loc}}(V; \mathbb{R}^m)\) and \(\|K(\cdot, v_l) - 1\|_{L^1(V)} \to 0\) as \(l \to \infty\), from the sequence \((v_l)\) we can extract subsequence (we denote it by \((v_l)\) again) such that it converges locally uniformly in \(V\) to some mapping \(v: V \to \mathbb{R}^m\) and
\[
K(\cdot, v_l) \to 1 \quad \text{a.e. in } V
\]
as \(l \to \infty\). Since \(S \subset \mathcal{G}(K)\), from Lemma 3.1 we obtain that the sequence \((v_l)\) is uniformly bounded in \(W^{1,k}_{\text{loc}}(V; \mathbb{R}^m)\). It follows from the general properties of the Sobolev spaces that \(v \in W^{1,k}_{\text{loc}}(V; \mathbb{R}^m)\) (for example, see [13] Chapter I, Theorem 1.1]). We have
\[
F(v_l'(x)) \leq K(x, v_l)G(v_l'(x)) \quad \text{a.e. in } V.
\]
and
\[
K(x, v_l) \leq K \quad \text{a.e. in } V.
\]
Multiply both sides of (11) by an arbitrary nonnegative function \(\eta \in C_0^\infty(V)\) and integrate over \(V\). Eventually, we obtain \(\int_V \eta F(v_l') \leq \int_V \eta K(\cdot, v_l)G(v_l')\). Passing to the limit in the last inequality over \(l\) and using the theorem on weak semicontinuity of the functionals of calculus of variations [11] Theorem II.4], the theorem on weak continuity of minors [13] Chapter II, Lemma 4.9], (10), and (12), we obtain
\[
\int_V \eta F(v') \leq \liminf_{l \to \infty} \int_V \eta F(v_l') \leq \limsup_{l \to \infty} \int_V \eta F(v_l') \leq \limsup_{l \to \infty} \int_V \eta K(\cdot, v_l)G(v_l') \leq \int_V \eta G(v').
\]
By the arbitrariness of \( \eta \), the last inequality means validity of (2) for \( v \) with \( K = 1 \). It follows that \( v \in \mathfrak{S} \). The sequence \( (v_l) \) converges locally uniformly in \( V \) to \( v \). This contradicts the assumption (9). Theorem 2.1 is proven.

4. Proof of Theorem 2.2

To prove Theorem 2.2, we need the following auxiliary proposition from [17].

**Proposition 4.1** ([17] Proposition 1). Let \( p > 1 \), and suppose that \( F: \mathbb{R}^{m \times n} \rightarrow \mathbb{R} \) is a strictly \( p \)-quasiconvex function satisfying \( c|\zeta|^p \leq F(\zeta) \leq C(|\zeta|^p + 1) \), \( \zeta \in \mathbb{R}^{m \times n} \), with some constants \( 0 < c < C < \infty \). Let \( V \subset \mathbb{R}^n \) be a bounded domain with Lipschitz boundary, and let \( (v_l)_{l \in \mathbb{N}} \), \( v_l \in W^{1,p}(V; \mathbb{R}^m) \), be a sequence of mappings such that \( v_l \rightharpoonup v \) in \( L^1(V; \mathbb{R}^m) \) for some mapping \( v \in W^{1,p}(V; \mathbb{R}^m) \). Suppose that \( \int_V \eta F(v_l') \rightarrow \int_V \eta F(v') < \infty \). Then \( v_l \rightharpoonup v \) in \( W^{1,p}(V; \mathbb{R}^m) \).

Let us prove Theorem 2.2. Assume that there is no function with necessary properties. Then for some number \( \varepsilon > 0 \) and some compact subset \( U \subset V \) and every \( l \in \mathbb{N} \) there exists a mapping \( v_l \in \mathfrak{S} \) with \( \| F(\cdot, v_l) - 1 \|_{L^1(V)} \leq 1/l \) such that the inequality

\[
\| v_l - u \|_{C(U; \mathbb{R}^m)} + \| v_l' - u' \|_{L^k(U; \mathbb{R}^{m \times n})} > \varepsilon
\]

holds for each mapping \( u \in \mathfrak{S} \). Arguing as the proof of Theorem 2.1, we obtain that for the sequence \( (v_l) \) there is a subsequence (denote it again by \( (v_l) \)) converging locally uniformly in \( V \) to some mapping \( v: V \rightarrow \mathbb{R}^m \) from the class \( \mathfrak{S} \) and satisfying (13) for any nonnegative function \( \eta \in C^\infty_0(V) \). We have that \( v \) satisfies (1). Combining (13) with (1), we have

\[
\liminf_{l \to \infty} \int_V \eta F(v_l') = \limsup_{l \to \infty} \int_V \eta F(v_l') = \int_V \eta F(v').
\]

It means that there is a subsequence (denoted again by \( (v_l) \)) such that \( \int_V \eta F(v_l') \rightarrow \int_V \eta F(v') \). Observe that this subsequence depends on the chosen function \( \eta \). Taking an appropriate collection of \( \eta \) and using Proposition ???, we find that there is a subsequence (for which we preserve the notation \( (v_l) \)) such that \( \| v_l' - v' \|_{L^k(U; \mathbb{R}^{m \times n})} \to 0 \).

Using the locally uniform convergence of \( (v_l) \) to \( v \in \mathfrak{S} \), we arrive at a contradiction with (14). Theorem 2.2 is proven.

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