Matter and Radiation in Strong Magnetic Fields of Neutron Stars

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Abstract. Neutron stars are found to possess magnetic fields ranging from $10^8$ G to $10^{15}$ G, much larger than achievable in terrestrial laboratories. Understanding the properties of matter and radiative transfer in strong magnetic fields is essential for the proper interpretation of various observations of magnetic neutron stars, including radio pulsars and magnetars. This paper reviews the atomic/molecular physics and condensed matter physics in strong magnetic fields, as well as recent works on modeling radiation from magnetized neutron star atmospheres/surface layers.

1. Introduction
The study of neutron stars (NSs) entails many different areas of physics, often applied under extreme conditions. In this paper, we focus on the superstrong magnetic fields of NSs.

The first question one would ask is: From a physics point of view, how strong a magnetic field is considered “strong”? We know that an electron in a uniform magnetic field $B$ gyrates around the field line at the frequency $\omega_{ce} = eB/(me^c)$. In quantum mechanics, this transverse motion is quantized into Landau levels, with the cyclotron energy (the Landau level spacing) given by $\hbar\omega_{ce} = h\omega_{ce} = 11.58 B_{12}$ keV, where $B_{12} = B/(10^{12}$ G). The natural (atomic) unit for the field strength, $B_0$, is set by $h\omega_{ce} = e^3/a_0$ (where $a_0$ is the Bohr radius), i.e.,

$$B_0 = m_e^2 e^3 c/\hbar^3 = 2.35 \times 10^9 \text{ G}. \quad (1)$$

Thus when studying the property of matter, strong magnetic fields means $B \gg B_0$. Another critical physics strength (the “QED field”), $B_Q$, is set by $h\omega_{ce} = m_e c^2$, or

$$B_Q = B_0/\alpha^2 = 4.4 \times 10^{13} \text{ G}. \quad (2)$$

We shall see that effects of QED become important when $B \gg B_Q$.

Magnetic fields of such magnitude are indeed superstrong by terrestrial standard, but they are commonly found on the surfaces of NSs (see Reisenegger et al. 2005 for a review). To date, our knowledge of NS magnetic fields has been based on indirect inferences to a large extent. NSs have many different observational manifestations: (i) The most commonly observed NSs are radio pulsars, of which more than 1600 are known today. For radio pulsars, one measures the period $P$ and period derivative $\dot{P}$ by timing the arrivals of radio pulses, and assuming that pulsar spindown is due to magnetic dipole radiation, i.e., $I\dot{\Omega} = -(2/3c^3)\Omega^3(BR^3)^2$, one then gets an estimate of the NS surface field $B$. For most radio pulsars, the result is $B \sim 10^{11-13}$ G. For a
smaller population of older, millisecond pulsars, one finds $B \sim 10^{8-9}$ G. Such field reduction is thought to be due to accretion that cycled a dead “ordinary” pulsar into a millisecond pulsar, but the detail of how such recycling works is not clear at present. (ii) Accreting X-ray pulsars in X-ray binary systems emit X-rays because of ongoing accretion from a binary companion. For many systems, one use the measured spin period together with theoretical ideas of spin equilibrium (i.e., spinup due to accretion of matter is balanced by magnetic braking – loss of stellar angular momentum via magnetic fields) to deduce $B \sim 10^{12}$ G. For a few X-ray pulsars, electron cyclotron features were seen in the X-ray spectra, from which one can deduce $B \sim 10^{12-13}$ G. In the last few years, half dozen or so accreting millisecond pulsars, with $B \sim 10^{9}$ G, have been discovered — these are thought to be the recycling process “caught in the act”. (iii) Finally, we have magnetars, NSs endowed with magnetic fields stronger than $10^{14}$ G. Observations in the last 5-8 years have revealed two related classes of young NSs, called anomalous X-ray pulsars (AXPs) and Soft Gamma Repeaters (SGRs) (see Wood & Thompson 2005). AXPs and SGRs share many common properties, e.g., they have $P = 5-12$ s, and the measured $\dot{P}$ implies a dipole field $\gtrsim 10^{14}$ G; their quiescent X-ray luminosity $L_x \sim 10^{34-36}$ erg/s is much larger than the spindown luminosity $\dot{I} \Omega$ (thus, unlike radio pulsars, the energy cannot be powered by rotation); some are associated with supernova remnants or young star clusters; they show occasional bursts and flares, including the spectacular giant flares from three SGRs (the March 5 1979 flare of SGR 0525-66 with energy $E > \sim 6 \times 10^{44}$ erg, the August 27 1998 flare from SGR 1900+14 with $E \gtrsim 2 \times 10^{44}$ erg, and the December 27 2004 flare from SGR 1806-20 with $E \sim 4 \times 10^{46}$ erg). A proper interpretation of these observations requires detailed understanding of various physics in strong magnetic fields.

2. Thermal Radiation and Neutron Star Atmospheres/Surfaces

The study of thermal, surface emission from isolated NSs can potentially provide invaluable information on the physical properties and evolution of NS (equation of state at super-nuclear densities, superfluidity, cooling history, magnetic field, surface composition, different NS populations, etc. See, e.g., Yakovlev & Pethick 2004 for review). In the last few years, considerable observational resources (e.g. Chandra and XMM-Newton) have been devoted to such study (e.g. Kaspi et al. 2005). For example, the spectra of a number of radio pulsars (e.g., PSR B1055-52, B0656+14, Geminga and Vela) have been observed to possess thermal components that can be attributed to emission from NS surfaces and/or heated polar caps. Phase-resolved spectroscopic observations are becoming possible, revealing the surface magnetic field geometry and emission radius of the pulsar (e.g., De Luca et al. 2005). Chandra has also uncovered a number of compact sources in supernova remnants with spectra consistent with thermal emission from NSs, and useful constraints on NS cooling physics have been obtained (e.g., Yakovlev & Pethick 2004). Surface X-ray emission has also been detected from a number of SGRs and AXPs. Fits to the quiescent magnetar spectra with blackbody or with crude atmosphere models indicate that the thermal X-rays can be attributed to magnetar surface emission at temperatures of $(3-7) \times 10^6$ K (see, e.g., Tiengo et al. 2005). One of the intriguing puzzles is the absence of spectral features (such as ion cyclotron line around 1 keV for typical magnetar field strengths) in the observed thermal spectra. Clearly, detailed observational and theoretical studies of surface emission can potentially reveal much about the physical conditions and the nature of magnetars.

Of particular interest are the seven isolated, radio-quiet NSs (so-called “dim isolated NSs”; see Haberl 2005). These NSs share the common property that their spectra appear to be entirely thermal, indicating that the emission arises directly from the NS atmospheres, uncontaminated by magnetospheric processes. Thus they offer the best hope for inferring the precise values of the temperature, surface gravity, gravitational redshift and magnetic field strength. The true nature of these sources, however, is unclear at present: they could be young cooling NSs, or NSs kept
hot by accretion from the ISM, or magnetars and their descendants. Given their interest, these isolated NSs have been intensively studied by deep Chandra and XMM-Newton observations. While the brightest of these, RX J1856.5-3754, has a featureless spectrum remarkably well described by a blackbody, absorption lines/features at $E \simeq 0.2$–$2$ keV have been detected in four other sources. The identifications of these features, however, remain uncertain, with suggestions ranging from cyclotron lines to atomic transitions of H, He or mid-Z atoms in a strong magnetic field (see Ho & Lai 2004). Another puzzle concerns the optical emission: For four sources, optical counterparts have been identified, but the optical flux is larger (by a factor of 4-10) than the extrapolation from the black-body fit to the X-ray spectrum.

The thermal emission is mediated by the outermost layer of the NS. In order to properly interpret the current and future observations, it is crucial to have a detailed understanding of the physical properties of the matter and radiation in strong magnetic fields ($B \sim 10^{11}$–$10^{16}$ G) and to calculate the emergent thermal radiation spectra from the NSs.

Some basics about NS atmospheres are in order. Because of the strong gravity, the NS atmosphere is highly compressed, with scale height $0.1$–$10$ cm and density $\sim 0.1$–$10^3$ g/cm$^3$. Thus we are dealing with a highly nonideal gas and effects like pressure ionization are important. The physical properties of the atmosphere, such as the chemical composition, equation of state, and especially the radiative opacities, directly determine the characteristics of the thermal emission. While the surface composition of the NS is unknown, a great simplification arises due to the efficient gravitational separation of light and heavy elements. A pure H atmosphere is expected even if a small amount of fallback/accretion occurs after NS formation. A He atmosphere results if H is completely burnt up, and a heavy-element (e.g., C, O, or Fe) atmosphere may be possible if no fallback/accretion occurs. The strong magnetic field makes the atmospheric plasma anisotropic and birefringent. If the surface temperature is not too high, atoms and molecules may form in the atmosphere. Moreover, if the magnetic field is sufficiently strong, the NS envelope may transform into a condensed phase with very little gas above it. A superstrong magnetic field will also make some QED effects (e.g., vacuum polarization) important in calculating the surface radiation spectrum.

NS atmosphere models with zero magnetic field have been extensively studied in recent years, using the opacity and equation of state data from the OPAL project (e.g., Gansicke et al. 2002). These models may be applicable to weakly magnetized ($B \lesssim 10^8$ G) NSs. So far most studies of magnetic NS atmospheres have focused on hydrogen and moderate field strengths of $B \sim 10^{12}$–$10^{13}$ G (e.g., Zavlin & Pavlov 2002). These models take into account the transport of different photon modes through a mostly ionized medium. The opacities adopted in the models include free-free transitions and electron scattering. However, since a strong magnetic field greatly increases the binding energies of bound species (e.g., the ground-state binding energy of H atom is 160 eV at $10^{12}$ G and 540 eV at $10^{14}$ G; see Lai 2001), atoms, molecules, and other bound states may have appreciable abundances in the atmosphere (e.g., Lai & Salpeter 1997; Potekhin et al. 2004). Thus these magnetic atmosphere models are expected to be valid only for relatively high temperatures ($T \gtrsim$ a few $\times 10^6$ K) where hydrogen is almost completely ionized. As the magnetic field increases, we expect these models to break down at even higher temperatures as bound atoms, molecules and condensate become increasingly important. Models of magnetic iron atmospheres (with $B \sim 10^{12}$ G) were studied by Rajagopal et al. (1997). Because of the complexity in the atomic physics and radiative transport, these Fe models are necessarily crude.

In the last few years, our group at Cornell has carried out systematic studies of NS atmospheres, with particular focus on radiative transfer in the superstrong ($B \gtrsim 10^{14}$ G) magnetic field regime (Ho & Lai 2001,2003,2004; Lai & Ho 2002,2003; see also Zane et al. 2001). Our current models treat ionized H and He atmospheres, and include full angle-dependent transport of the photon polarization modes and ion cyclotron resonance in the opacity. We have explored the novel physical effect of vacuum polarization on radiative transfer in strong
magnetic fields. The theory of quantum electrodynamics (QED) predicts a nontrivial dielectric property of the vacuum in magnetic fields; this significantly affects photon propagation in the atmospheric plasma. Our study showed that vacuum polarization can dramatically modify the spectra (including the high-energy tail and cyclotron line strength) and polarization signals of magnetic NSs. We have also studied the equation of state, polarizibility tensor (which determines the photon polarization modes) and opacities of magnetic, partially ionized H plasmas, and have constructed the first NS atmosphere models that include self-consistent treatment of the thermodynamics and opacities of bound H atoms in strong magnetic fields (Ho et al. 2003; Potekhin et al. 2004, 2005). In addition, we have studied emission property of condensed NS surface (van Adelsberg et al. 2005).

3. Matter in Strong Magnetic Fields: Brief Overview

For \( b \equiv B/B_0 \gg 1 \), the electron cyclotron energy \( \hbar \omega_{ce} \) is much larger than the typical Coulomb energy, so that the properties of atoms, molecules and condensed matter are qualitatively changed by the magnetic field. In such a strong field regime, the usual perturbative treatment of the magnetic effects (e.g., Zeeman splitting of atomic energy levels) does not apply. Instead, the Coulomb forces act as a perturbation to the magnetic forces, and the electrons in an atom/molecule settle into the ground Landau level. Because of the extreme confinement of the electron (the cyclotron radius of the electron, \( R \equiv h/(eB) \), is much less than the Bohr radius \( a_0 \)), in the transverse direction (perpendicular to the field), the Coulomb force becomes much more effective in binding the electrons along the magnetic field direction. The atom attains a cylindrical structure. Moreover, it is possible for these elongated atoms to form molecular chains by covalent bonding along the field direction. Interactions between the linear chains can then lead to the formation of three-dimensional condensates.

We now discuss some basic properties of different bound states in strong magnetic fields (see Lai 2001 for a review).

(i) Atom: For \( b \gg 1 \), the H atom is elongated and squeezed, with the transverse size (perpendicular to \( B \)) \( \sim \hat{R} = a_0/b^{1/2} \ll a_0 \) and the longitudinal size \( \sim a_0/(\ln b) \). Thus the ground-state binding energy \( |E| \sim 0.16(\ln b)^2(\text{au}) \) (where 1 au = 27.2 eV; the factor 0.16 is an approximate number based on numerical calculations). Thus \( |E| = 160,540 \) eV at \( B = 10^{12}, 10^{14} \) G respectively. In the ground state, the guiding center of the electron’s gyromotion coincides with the proton. The excited states of the atom can be obtained by displacing the guiding center away from the proton; this corresponds to \( \hat{R} \rightarrow R_s = (2s+1)^{1/2} \hat{R} \) (where \( s = 0, 1, 2, \cdots \)). Thus \( E_s \sim -0.16 \{ \ln[b/(2s+1)] \}^2(\text{au}) \).

This simple picture of the H energy levels is modified when we consider the effect of finite proton mass: Even for a “stationary” H atom, the energy \( E_s \) is changed to \( E_s + s\hbar \omega_{cp} \), where \( \hbar \omega_{cp} = 6.3 B_{12} \) eV is the proton cyclotron energy. Thus the extra energy \( s\hbar \omega_{cp} \) (which can be thought of as a “recoil” term) becomes increasingly important with increasing \( B \). Moreover, the effect of center-of-mass motion is nontrivial: When the atom moves perpendicular to the magnetic field, a strong electric field is induced in its rest frame and can significantly change the atomic structure (the “motional Stark effect”); indeed, the mobility of the neutral atom across the magnetic field is limited. As a result, the dependence of the atomic energy on the transverse momentum is complicated. This effect leads to a large shift and broadening of the energy levels and significant modification to the ionization equilibrium.

We can imagine constructing a multi-electron atom (with \( Z \) electrons) by placing electrons at the lowest available energy levels of a hydrogenic ion. The lowest levels to be filled are the tightly bound states with \( \nu = 0 \). When \( a_0/Z \gg \sqrt{2Z-1}R \), i.e., \( b \gg Z^3 \), all electrons settle into the tightly bound levels with \( s = 0, 1, 2, \cdots, Z-1 \). Reliable values for the energy of a multi-electron atom for \( b \gg 1 \) can be calculated using the Hartree-Fock method or density functional theory, which takes into account the electron-electron direct and exchange interactions in a
self-consistent manner.

(ii) **Molecules and Chains:** In a strong magnetic field, the mechanism of forming molecules is quite different from the zero-field case. The spins of the electrons in the atoms are aligned antiparallel to the magnetic field, and thus two atoms in their ground states do not bind together according to the exclusion principle. Instead, one H atom has to be excited to the $s = 1$ state before combining (by covalent bond) with another atom in the $s = 0$ state. Since the “activation energy” for exciting an electron in the H atom from $s = 0$ to $(s + 1)$ is small, the resulting H$_2$ molecule is stable. Moreover, in strong magnetic fields, stable H$_3$, H$_4$ etc. can be formed in the similar manner. The dissociation energy of the molecule is much greater than the $B = 0$ value: e.g., for H$_2$, it is 40,350 eV at $10^{12}$, $10^{14}$ G respectively. A highly magnetized molecule exhibits excitation levels much different from a $B = 0$ molecule.

(iii) **Condensed Matter:** As more atoms are added to a molecule, the energy per atom in a H$_n$ molecule saturates, becoming independent of $n$; this occurs at $n \gtrsim [b/(ln b)^2]^{1/5}$ ($\sim 3 - 5$ for field strengths of interest). We then have a 1D metal. To obtain the basic scaling relation, we can consider a uniform cylinder model in which a 1D ion lattice is embedded in an electron Fermi sea. The radius of the cylinder and the ion spacing are of order $a \sim Z^{1/5}b^{-2/5}$ and the energy per “atom” is $E \sim -Z^{9/5}b^{2/5}$ (where we have restored the ion charge number $Z$). By placing parallel chains together (with spacing $\sim a$) we form a 3D condensed matter (e.g., in a body-centered tetragonal lattice). The energy per unit cell is again of order $E \sim -Z^{9/5}b^{2/5}$. The radius of the cell is $R \sim Z^{1/5}b^{2/5}$, corresponding to the zero-pressure density $\sim 10^5AZ^{3/5}B_{12}^{6/5}$ g cm$^{-3}$ (where $A$ is the mass number of the ion).

Although the simple uniform electron gas model and its Thomas-Fermi type extensions give a reasonable estimate for the binding energy for the condensed state, they are not adequate for determining the cohesive property of the condensed matter. In principle, a three-dimensional electronic band structure calculation is needed to solve this problem. So far the only attempt to this problem has been the preliminary calculations by Jones (1986) for a few elements and several values of field strengths using density functional theory. The binding energies of 1D chain for some elements have been obtained using Hartree-Fock method (Neuhauser et al 1987; Lai et al 1992; Lai 2001). Density functional theory has also been used to calculate the structure of linear chains in strong magnetic fields (Jones 1986; Medin & Lai 2006). Numerical calculations carried out so far indicate that for $B_{12} = 1 - 10$, linear chains are unbound for large atomic numbers $Z \gtrsim 6$ (Neuhauser et al 1987; Medin & Lai 2006). In particular, the Fe chain is unbound relative to the Fe atom; this is contrary to what some early calculations have indicated. Therefore, the chain-chain interaction must play a crucial role in determining whether the three dimensional zero-pressure Fe condensed matter is bound or not. The main difference between Fe and H is that for the Fe atom at $B_{12} \sim 1$, many electrons are populated in the $\nu \neq 1$ states, whereas for the H atom, as long as $b \gg 1$, the electron always settles down in the $\nu = 0$ tightly bound state. Therefore, the covalent bonding mechanism for forming molecules is not effective for Fe at $B_{12} \sim 1$. However, for a sufficiently large $B$, when $a_0/Z \gg \sqrt{2Z + 1}R$, or $B_{12} \gg 100(Z/26)^3$, we expect the Fe chain to be bound in a manner similar to the H chain or He chain (Medin & Lai 2006).

(iv) **Phase Diagram:** Having understood the different bound states of H (for example), the next question is: What is the physical condition of the H surface layer of a NS as a function of $B$ and $T$? What is the phase diagram? Clearly, we are dealing with a highly magnetized, dense ($0.1-10^3$ g cm$^{-3}$) and partially ionized plasma; further complications arise from the strong coupling between the center-of-mass motion and the internal structure of atoms/molecules. Calculations indicate that there are two possible regimes: (1) Under “normal” conditions ($B \lesssim 10^{14}$ G and $T \gtrsim 10^6$ K), the surface layer (photosphere) is gaseous and nondegenerate, with a mixture of p, e, H atoms, and H$_2$, etc. (depending on $B, T, \rho$; (2) Under more “extreme” conditions ($T \gg 10^{14}$ G and/or $T < 10^6$ K), there is a phase transition from the gaseous phase
to the condensed metallic phase; as $B$ increases, the vapor density (above the metal) decreases, and we then have a situation in which the surface consists of condensed metallic H from which radiation directly comes out. The precise boundary between the two regimes (or the precise critical temperature) is currently uncertain (Lai & Salpeter 1997; Lai 2001; van Adelsberg et al. 2005).

4. Effect of Vacuum Polarization on Radiation from Magnetars and Dim Isolated Neutron Stars

The magnetized plasma of a NS atmosphere is birefringent. A X-ray photon, with energy $E \ll E_{Be} = \hbar eB/(mc) = 1.16B_{14}$ MeV [the electron cyclotron energy; $B_{14} = B/(10^{14} \text{G})$], propagating in such a plasma can be in one of the two polarization modes: The ordinary mode (O-mode) has its electric field $\mathbf{E}$ oriented along the $\mathbf{B}$-$\mathbf{k}$ plane ($\mathbf{k}$ is along direction of propagation), while the extraordinary mode (X-mode) has its $\mathbf{E}$ perpendicular to the $\mathbf{B}$-$\mathbf{k}$ plane. Since charge particles cannot move freely across the magnetic field, the X-mode photon opacity (e.g., due to free-free absorption or electron scattering) is suppressed compared to the zero-field value, $\kappa_X \sim (E/E_{Be})^2\kappa_{(B=0)}$, while the O-mode opacity is largely unchanged, $\kappa_O \sim \kappa_{(B=0)}$ (e.g., Meszaros 1992). The standard treatment of radiative transfer in magnetized NS atmospheres involves solving the transfer equations for the intensities of the two photon modes.

Vacuum polarization can change this picture in an essential way (see Fig. 1). In the presence of a strong magnetic field, vacuum itself becomes birefringent due to virtual $e^+e^-$ pairs. Thus in a magnetized NS atmosphere, both the plasma and vacuum polarization contribute to the dielectric tensor of the medium (e.g., Gnedin et al. 1978; Meszaros & Ventura 1979). The vacuum polarization contribution is of order $10^{-4}(B/B_Q)^2f(B)$ (where $B_Q = m_e^2c^3/e\hbar = 4.414 \times 10^{13}$ G, and $f \sim 1$ is a slowly varying function of $B$), and is quite small unless $B \gg B_Q$. However, even for “modest” field strengths, vacuum polarization can have a dramatic effect through a “vacuum resonance” phenomenon. This resonance arises when the effects of vacuum polarization and plasma on the polarization of the photon modes “compensate” each other. For a photon of energy $E$ (in keV), the vacuum resonance occurs at the density $\rho_V \sim 0.964Y_e^{-1}B_{14}^2E^{2f-2} g \text{ cm}^{-3}$, where $Y_e$ is the electron fraction (Lai & Ho 2002). Note that $\rho_V$ lies in the range of the typical densities of a NS atmosphere. For $\rho \gtrsim \rho_V$ (where the plasma effect dominates the dielectric tensor) and $\rho \lesssim \rho_V$ (where vacuum polarization dominates), the photon modes are almost linearly polarized — they are the usual O-mode and X-mode described above; at $\rho = \rho_V$, however, both modes become circularly polarized as a result of the “cancellation” of the plasma and vacuum polarization effects (Fig. 1). When a photon propagates outward in the NS atmosphere, its polarization state will evolve adiabatically if the plasma density variation is sufficiently gentle. Thus the photon can convert from one mode into another as it traverses the vacuum resonance. The conversion probability $P_{conv}$ depends mainly on $E$ and atmosphere density gradient; for a typical atmosphere density scale height ($\sim 1$ cm), adiabatic mode conversion requires $E \gtrsim 1-2$ keV (Lai & Ho 2003a). Because the O-mode and X-mode have vastly different opacities, the vacuum polarization-induced mode conversion can significantly affect radiative transfer in magnetar atmospheres (see Fig. 2). In particular, the effect tends to deplete the high-energy tail of the thermal spectrum (making it closer to blackbody) and reduce the width of the ion cyclotron line or other spectral lines (Ho & Lai 2003,2004; Lai & Ho 2003a). It is tempting to suggest that the absence of lines in the observed thermal spectra of several AXPs (e.g., Morii et al. 2003; Patel et al. 2003; Tiengo et al. 2005) is a consequence of the vacuum polarization effect at work in these systems.

Although we believe our previous works have captured the qualitative effects of vacuum polarization, significant work is needed to quantify these and to incorporate them into realistic NS atmosphere models:

As mentioned above, all previous studies (including our own) of magnetic NS atmospheres
Figure 1. A diagram illustrating mode conversion due to vacuum resonance. The two curves show the ellipticity of the photon modes as a function of plasma density (for a given external B field, photon energy and direction of propagation). At densities away from the resonance density $\rho_V$, the two modes are almost linearly polarized (with polarization ellipses orthogonal to each other); at $\rho = \rho_V$, both modes are circularly polarized. In the adiabatic limit, an O-mode (X-mode) photon from the high-density region will convert to the X-mode (O-mode) as it traverses the vacuum resonance density $\rho_V$, with its polarization ellipse rotated by $90^\circ$.

Figure 2. Right panel: A diagram illustrating how vacuum polarization-induced mode conversion affects radiative transfer in a magnetar atmosphere: When the vacuum polarization effect is turned off, the X-mode photosphere (where optical depth $\approx 1$) lies deeper than the O-mode; with the vacuum polarization effect included, the X-mode photon effectively decouples from the atmosphere at the vacuum resonance layer, which lies at a lower density than the original X-mode photosphere. Thus vacuum resonance can dramatically affect the emergent radiation spectra.

rely on solving the transfer equations for the specific intensities of the two photon modes. These equations cannot properly handle the vacuum-induced mode conversion phenomenon described above. This is because mode conversion intrinsically involves the interference between the modes. In particular, photons with energies 0.3-2 keV (this is the energy range in which the bulk of the radiation comes out and spectral lines are expected for $B \sim 10^{14}$ G) are only partially converted across the vacuum resonance (i.e., the adiabatic condition is neither strongly satisfied nor violated). Such partial-conversion case is impossible to treat with transfer equations based on normal modes. Our previous calculations (Ho & Lai 2003, 2004) considered only two extreme limits ("complete conversion" and "complete no conversion"); we found that the atmosphere spectra calculated in the two limits differ appreciably (particularly in the spectral line width). The other problem with the modal description of radiative transport is that it is valid only in the limit of large Faraday depolarization (Gnedin & Pavlov 1974), which is not always satisfied.
near the vacuum resonance, especially for superstrong magnetic fields (Lai & Ho 2003a). Also, in the presence of dissipation, the two photon modes can collapse near the resonance (i.e., they become identical; Soffel et al. 1983; Lai & Ho 2003a), making the modal description meaningless.

Clearly, to account for the vacuum resonance effect in a quantitative manner, one must go beyond the modal description of the radiation field by formulating and solving the transfer equation in terms of the photon intensity matrix (or Stokes parameters). The general transfer equations for the photon intensity matrix in a magnetized, birefringent plasma-vacuum medium has been derived by Lai & Ho (2003a). These equations are expected to have oscillatory terms because of wave interference, and proper averaging will be necessary.

We note that a proper treatment of the vacuum resonance effect is important not only for magnetars, but also crucial for interpreting the emission from dim isolated NSs, whose nature remains uncertain (see §1). Indeed, the recent detections of absorption features around 0.2–1 keV in several such NSs most likely imply surface magnetic fields $B \sim 7 \times 10^{13}$ G (While the line identifications are not entirely certain, these absorption features likely result from the combination of ion cyclotron resonance and atomic transitions; see Ho & Lai 2004). For such magnetic field strengths, the absorption lines lie in the “partial mode conversion” regime (see above) and therefore a quantitative treatment of the vacuum resonance effect is crucial for the interpretation of the lines. We also note that even for “ordinary” NSs (with $B \sim 10^{12}$–$10^{13}$ G), vacuum resonance has a profound effect on the polarization signals of the surface emission; this may provide a direct probe of strong-field QED in the regime inaccessible at terrestrial laboratories (Lai & Ho 2003b). Such polarization signals will be of interest for future X-ray polarimetry detectors/missions.

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