An experiment to distinguish between diffusive and specular surfaces for thermal radiation in cryogenic gravitational-wave detectors

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In cryogenic gravitational-wave detectors, one of the most important issues is the fast cooling of their mirrors and keeping them cool during operation to reduce thermal noise. For this purpose, the correct estimation of thermal-radiation heat transfer through the pipe-shaped radiation shield is vital to reduce the heat load on the mirrors. However, the amount of radiation heat transfer strongly depends on whether the surfaces reflect radiation rays diffusely or specularly. Here, we propose an original experiment to distinguish between diffusive and specular surfaces. This experiment has clearly shown that the examined diamond-like carbon-coated surface is specular. This result emphasizes the importance of suppressing the specular reflection of radiation in the pipe-shaped shield.

1. Introduction

For the detection of gravitational waves, i.e., space-time ripples predicted by Einstein’s general theory of relativity, several gravitational-wave detectors have been operated or are under development. KAGRA [1] (the Large-scale Cryogenic Gravitational wave Telescope (LCGT), interferometer under construction in Japan) and ET (the Einstein Telescope, under development in Europe) [2] have two key advantages over other detectors: they will be located at underground sites with small seismic vibration, and they are equipped with cooled mirrors (220 mm diameter at 20 K in the case of KAGRA [3,4]) to reduce thermal noise [5] in the mirror and its suspension [6]. Although the mirror must be surrounded by a radiation shield to be cooled, a hole in the shield with a size comparable to that of the mirror is necessary to allow the laser beam to pass through. The mirror will absorb a fraction of the thermal radiation entering from the hole and will thus be heated. This situation is problematic because it increases the cooling time of the mirror [7] and the target temperature of the mirror cannot be achieved. To reduce the undesirable thermal radiation, a pipe-shaped radiation shield, called a duct shield, will be used to decrease thermal radiation. The duct shield is designed to minimize the thermal radiation [8–13] entering the cryostat, which is estimated using the ray-tracing calculations described in Sect. 2.
For the ray-tracing calculation, two types of limiting surfaces exist [14]. One type reflects the incoming radiation to all directions uniformly (diffusive surface). The other type reflects rays according to the reflection law; i.e., the reflected ray forms the same angle with the surface normal as the incident ray and lies within the plane defined by the incident ray and the surface normal (specular surface). The amount of radiation heat transfer through the pipe strongly depends on whether the surfaces reflect radiation rays diffusely or specularly. Figure 1 shows that radiation undergoes a smaller number of reflections in a specular pipe than in a diffusive pipe. Figure 2 shows the calculation result of the thermal-radiation power passing through a pipe with $L/d = 50$ ($L$ and $d$ are the length and diameter of the pipe, respectively). Unless the reflectivity of the pipe is extremely high, the power passing through the diffusive pipe is reduced by approximately $10^{-4}$, which is the portion directly transferred without being reflected by the pipe. Depending on the reflectivity of the pipe, the power passing through a specular pipe is several orders of magnitude larger than that through a diffusive pipe. In cryogenic gravitational-wave detectors, such large amounts of radiation power reaching the mirror are problematic because the additional heat input entering through the duct shields hinders the cooling of the mirror. To reduce this heat input, donut-shaped plates called baffles [9–13] will be inserted.

Consequently, it is important to examine whether the surfaces involved are diffusive or specular. Experiments have shown that the inner surfaces of the examined metal pipes were specular for thermal radiation [8,11,13]. Here, we have conducted a completely new type of experiment to distinguish between diffusive and specular surfaces for further evidence of the specular reflection.
2. Theoretical analysis

This study considers heat transfer by thermal radiation from a sphere (inner sphere) to a surrounding sphere (outer sphere), positioned as shown in Fig. 3. For simplicity, we neglect emission of thermal radiation from the outer sphere itself (i.e., the temperature of the outer sphere is assumed to be 0 K), and consider only the thermal radiation emitted from the inner sphere and its reflections by the spheres. For diffusive surfaces, the amount of heat transfer does not depend strongly on the position of the inner sphere, in contrast to the case of specular surfaces. In the following, the intuitive explanation of heat transfer is first described. Next, the calculations of the exact amount of radiation heat transfer from the inner sphere to the outer sphere are presented.

- Diffusive surfaces

When the inner sphere is positioned at the center of the outer sphere, the equation

\[ Q_{\text{diff, center}}(T) = \frac{A_1 \sigma T^4}{\epsilon + \frac{A_1}{A_2} \left( \frac{1}{\epsilon} - 1 \right)} \]  

(1)

can be obtained (e.g., Ref. [15]). Here, \( A_i \) is the surface area, where the index \( i = 1, 2 \) represents the inner sphere and the outer sphere, respectively. \( T \) is the temperature of the inner sphere, and \( \sigma \) is the Stefan–Boltzmann constant. We assume that both spheres have the same emissivity \( \epsilon \). Irrespective of the position of the inner sphere inside the outer sphere, rays from the inner sphere are reflected mainly by the outer sphere owing to the larger surface area of the outer sphere than that of the inner sphere. Therefore, the heat transfer does not depend strongly on the position of the inner sphere inside the outer sphere.

- Specular surfaces

When the inner sphere is at the center of the outer sphere, all the rays from the inner sphere are reflected alternately by the inner and outer spheres. This heat transfer is equivalent to the case where the two bodies are infinite parallel planes (diffusive and specular infinite parallel planes have the same amount of heat transfer because all the rays are reflected alternately by the two planes):

\[ Q_{\text{spec, center}}(T) = \frac{\frac{A_1 \sigma T^4}{\epsilon - 1}}{\frac{1}{\epsilon} - 1} \]  

(2)

(e.g., Ref. [15]). If the inner sphere is shifted from the center, rays are reflected by the outer sphere more times; namely, more heat can be absorbed by the outer sphere. Thus, the radiation can conduct more heat from the inner sphere to the outer sphere and Eq. (2) represents the minimum heat transfer in the specular calculation.

The exact radiation heat transfer from the inner sphere to the outer sphere can be calculated by tracing rays with the Monte Carlo method. In the calculation, rays (total power \( \epsilon A_1 \sigma T^4 \)) were emitted from uniformly distributed positions on the inner sphere, according to Lambert’s cosine law, and were then repeatedly reflected by the two spheres. If the surface was diffusive, the ray was reflected in a
random direction with a probability proportional to $\cos \theta$ ($\theta$ is the angle between the surface normal and the reflected ray). If the surface was specular, the ray was reflected according to the reflection law. When the ray arrived at the spheres, a portion $\epsilon$ of its power was absorbed and the remaining portion $1 - \epsilon$ was reflected (the reflectivity is equal to $1 - \epsilon$). This calculation for the direction and the power of the ray was repeated until the power of each ray became sufficiently smaller than 1% of its initial power. The number of traced rays was increased until the standard deviation of the heat-transfer values became smaller than 1%. Here, the number of traced rays was 10,000. The ray-tracing calculation yields heat transfer in the form of $C_p(\epsilon(T))A_1\sigma T^4$ using the factor $C_p$, which depends on the position of the inner sphere inside the outer sphere. Here, it is important to note that the emissivity of the outer sphere is calculated as a function of the temperature of the inner sphere. The emissivity depends strongly on the wavelength of the thermal radiation and thus on the temperature of the radiator (the inner sphere), as described in Sect. 5.

3. Experimental methods

The experimental setup is shown in Fig. 4. In the interior of the outer sphere (220 mm in diameter, made of aluminum A1070), the inner sphere (30 mm in diameter, fabricated from oxygen-free copper) was suspended by a nylon wire. The outer sphere was composed of two hemisphere-shaped vacuum chambers, as shown in Fig. 5. The inner sphere and the inner surface of the outer sphere were buffed under condition #400 of JIS (Japanese Industry Standard); namely, the surfaces were polished by whetstones $\sim 30 \mu$m in diameter. The buffed surfaces were coated with a diamond-like carbon (DLC) coating with a thickness of 1 $\mu$m. Aluminum tape was used to cover the hole (except the nylon wire) to prevent thermal radiation of 300 K from reaching the outer sphere through the pipe.

At the beginning of the experiment, the inside of the outer sphere was kept at a pressure smaller

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3 Materials to absorb stray light or thermal radiation are necessary in cryogenic interferometric gravitational-wave detectors. Because these detectors require high vacuum, the materials used in KAGRA must be vacuum-compatible. From this point of view, the DLC coating is a promising candidate [7,16].

4 The diffusive or specular nature of this pipe was not investigated because the aim of this experiment was to examine the spheres. Alternatively, the hole in the aluminum tape was made as small as possible to reduce thermal radiation through the pipe, calculated in Sect. 4.1.
Fig. 4. Experimental setup. The inner sphere is suspended inside the outer sphere (vacuum chamber) by a nylon wire. The thick dotted lines represent the DLC coating.

Fig. 5. (a) Photograph of the inner sphere suspended inside the top half of the outer sphere. (b) Photograph of the bottom half of the outer sphere.

than $10^{-3}$ Pa to prevent heat conduction by gas molecules. This vacuum was also necessary to keep the sphere surfaces free of water molecules, which can change the emissivity of the surfaces. The outer sphere was immersed in liquid nitrogen. The temperatures of the spheres were monitored with thermometers (DT-670-CU, calibrated, Lake Shore Cryotronics, Inc.) attached on the inner sphere (thermometer 1) and the outer sphere (thermometer 2). The heat transferred from the inner sphere to the outer sphere was derived from the temperature change of the inner sphere:

$$Q = -MC(T)\frac{dT}{dt},$$

where $M$ is the mass of the inner sphere ($M = 127$ g), $C$ is the specific heat of copper (e.g., Ref. [17]), $T$ is the temperature of the inner sphere, and $t$ is the time. The temperature of the outer sphere was kept at the boiling point of liquid nitrogen, 77 K.
For the measurement, the inner sphere was first suspended at the center of the outer sphere. Next, the inner sphere was shifted up by 5 cm inside the outer sphere to examine the dependence of the heat transfer on the position of the inner sphere.

4. Error analysis

4.1. Extra heat

The amount of extra heat on the inner sphere is plotted in Fig. 6. The thermal conduction by the nylon wire and the thermometer wires was estimated as follows. The thermal conduction between two points with temperatures $T_x$ and $T_y$ connected by $N$ wires with thermal conductivity $\kappa$, length $\ell$, and cross-sectional area $S_{\text{wire}}$ is calculated as:

$$Q = \frac{NS_{\text{wire}}}{\ell} \int_{T_x}^{T_y} \kappa(T')dT'.$$

(4)

Here, because the nylon wire and the thermometer wires were attached to the inner sphere and the flange (which was at room temperature), we considered $T_x$ to be the temperature of the inner sphere and $T_y = 300$ K. The nylon wire had a cross section of $1 \text{ mm} \times 0.1 \text{ mm}$ and a length of 530 mm. Thermometer 1 had four phosphor bronze wires with 0.127 mm diameter and 914 mm length. The values of the thermal conductivity of nylon and phosphor bronze from Refs. [18] and [19], respectively, were used.

The thermal radiation entering through the pipe that is absorbed by the inner sphere is

$$Q = \sigma T_r^4 S_{\text{hole}} \frac{\Omega}{2\pi} = 4 \times 10^{-4} \text{ W},$$

(5)

where the room temperature is assumed to be $T_r = 300$ K and the emissivity of the pipe and absorptivity of the inner sphere are assumed to be unity for a pessimistic estimation. $S_{\text{hole}}$ is the surface area of the hole ($5 \text{ mm} \times 5 \text{ mm}$) on the aluminum tape and $\Omega$ is the solid angle of the inner sphere when viewed from the hole.

4.2. Errors related to the inner sphere

We evaluated the uncertainty of the position of the inner sphere inside the outer sphere as 1 cm. Because the length of the nylon wire was greater than 500 mm, a 1% tilt of the entire experimental setup caused a 0.5 cm shift of the inner sphere in the horizontal direction. The thermal contraction of the nylon wire was estimated to be 0.5 cm, which caused a shift of the inner sphere in the vertical direction, according to the thermal expansion coefficient of nylon [18].

The ratio of the surface area covered by thermometer 1 to the total surface area of the inner sphere was approximately 10%.

4.3. Thermal radiation emitted from the outer sphere

As described in Sect. 2, thermal radiation emitted from the inner sphere (temperature $T$) transfers heat $C_p(\epsilon(T))A_1\sigma T^4$ to the outer sphere. Similarly, the outer sphere (temperature $T_n = 77$ K) transfers heat $C_p'(\epsilon(T_n))A_2\sigma T_n^4$ to the inner sphere. The net heat transfer from the inner sphere to the outer sphere is calculated as:

$$Q(T, T_n) = C_p(\epsilon(T))A_1\sigma T^4 - C_p'(\epsilon(T_n))A_2\sigma T_n^4$$

(6)

$$= A_1\sigma \left[ C_p(\epsilon(T))T^4 - C_p'(\epsilon(T_n))T_n^4 \right].$$

(7)
Equation (7) is obtained considering that no heat is transferred when both spheres have the same temperature. Because $\epsilon(T) > \epsilon(T_n)$, under the condition that $T > T_n$ (the coating looks thinner for longer wavelengths, as discussed in Sect. 5), the second term can be neglected when $T^4 \gg T_n^4$. When $T = 150\, \text{K}$, $T_n^4 / T^4 \sim 0.07$.

5. Results and discussion

The results of the experiment are shown in Fig. 7. The heat transfer shown in Fig. 8 was calculated from the cooling speed using Eq. (3). Each data point has an error bar equal to 10% of the heat transfer value, caused by the fact that thermometer 1 covers approximately 10% of the surface area of the inner sphere. Because the calculation results for the diffusive surfaces were independent of the shift (0, 1, 5 cm) within a calculation uncertainty of 1%, only one curve is shown as “Calculation: diffusive”.

Above 150 K, the experimental results are consistent with those of the calculation that assumes specular surfaces. In the experiment, the heat transfer with the shifted inner sphere was larger than that with the centered inner sphere. This fact cannot be explained by the diffusive calculation. Considering the explanation by the specular calculation, the experimental results for the case of the centered inner sphere can be compared with the calculated results for the case where the shift is 1 cm because of the position uncertainty of the inner sphere described in Sect. 4.

Below 150 K, the experimental results become smaller than the calculated values. Possible causes for this disagreement are the extra heat, which amounts to 10% of the heat transfer of interest, and thermal radiation emitted from the outer sphere itself, which cannot be neglected below 150 K, as described in Sect. 4.

The fact that the experimental results are consistent with the specular-surface calculation indicates that the surfaces can be regarded as flat planes at the examined radiation wavelength. The wavelength of blackbody radiation is approximately $10\, \mu\text{m}$ at $300\, \text{K}$ and longer at lower temperatures. On the other hand, the surface roughness of the spheres was of the order of $10\, \mu\text{m}$. Consequently, a surface with roughness smaller than that wavelength can be approximated as a flat plane.
Fig. 7. Experimental results of the temperature of the inner sphere. The outer sphere was maintained at 77 K.

Fig. 8. The amount of heat transfer (per surface area of the inner sphere) obtained from the cooling of the inner sphere. The calculation results for diffusive surfaces and specular surfaces are also shown.

By fitting the calculation to the experimental results, we obtained the emissivity of DLC for thermal radiation emitted from the inner sphere with temperature $T$:

$$\epsilon = 0.3 \times \left( \frac{T}{300 \text{ K}} \right).$$  \hspace{1cm} (8)

This equation can be explained as follows: the emissivity is equal to the absorptivity at the radiation wavelength $\lambda$, which is inversely proportional to the radiator temperature $T$, according to Wien’s displacement law. When the coating has a constant absorption coefficient, the absorptivity is proportional to the coating thickness and inversely proportional to $\lambda$. Namely, the coating appears thinner at longer wavelengths. Thus, the absorptivity is proportional to $T$.

It is important to note that this experiment can distinguish the diffusive or specular nature of the outer sphere, but not of the inner sphere. In our calculations, we considered the case of a diffusive
inner sphere and a specular outer sphere and the case of a specular inner sphere and a diffusive outer sphere. The result depended only on whether the outer sphere was diffusive or specular and was independent of the inner sphere within a calculation accuracy of 1%.

6. Conclusion
We have conducted a novel experiment to distinguish between diffusive and specular surfaces. Although it is theoretically clear that this type of experiment can distinguish between diffusive and specular surfaces, we have verified this method experimentally for the first time. This experiment has demonstrated that the examined DLC-coated surfaces are specular. Although several studies have already shown the specular reflection of thermal radiation [8,11,13], this experiment provides further evidence of the specular reflection of thermal radiation from a novel viewpoint. Therefore, this experiment emphasizes the importance of the baffles inserted in the duct shield in the cryogenic gravitational-wave detectors to suppress this specular reflection of radiation.

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