Pomeron exchange and $t$-dependence of the scattering amplitude

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Abstract

Constraints on the $t$-dependence of the hadronic scattering amplitude at asymptotic energies are derived by considering the exchange of the Pomeron, as a Regge pole, between off-shell gluons. Covariant reggeization ensures pure spin $\alpha$ exchange, where $\alpha$ is the Regge trajectory of the Pomeron. The structure of the amplitude, as a function of $t$, has been derived without a specific choice for the partonic wave functions of the hadrons. New terms appear, with respect to the standard approach, and allow to describe non trivial properties of the diffraction cone in agreement with experimental data, as shown in a specific example.

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1 Introduction

Field theoretical descriptions of hadronic two-body processes at large $s$ and small $|t|$, based on the idea that strong interactions are mediated by colored gluons exchanged between color singlet states, have a long history. The identification of the Pomeron with two, or more, gluon exchanges [1], gave rise to the BFKL equation [2, 3] whose asymptotic solution is the so-called ”Lipatov Pomeron”. The Born approximation to the Pomeron, used also recently in many calculations of high energy processes, developed in a conformal field theory, at least in the leading-log approximation.

At large momentum transfer squared, the perturbative approach will give sensible results for the observables of any hadronic exclusive process. Non perturbative effects can however be important at small $|t|$, surely they are present in the Born approximation, and corrections to the Pomeron pole in perturbative QCD are larger than expected in the next-to-leading approximation [4]. It could be interesting, at this stage, to consider a phenomenological model where, first, the Pomeron is exchanged as a Regge pole between off-shell gluons and, then, off-shell gluons couple to a color singlet, e.g. a $q\bar{q}$ state. A more sophisticated model, where quark and many-gluon components of the Pomeron are considered, would be more realistic, but far more involved. The quark component, however, can be taken into account, in the simplest case, and this will be done in the following.

Covariant reggeization [5, 6, 7] provide us with a differential technique that generates reggeized scattering amplitudes free from kinematical singularities and satisfying automatically factorization. The most important properties of this approach were its direct relation with Toller poles [8] and the possibility to include easily spins of the particles coupled to the Pomeron ensuring, for the latter, pure spin $\alpha$ exchange where $\alpha$ is its Regge trajectory [6, 9]. It could seem, at first sight, that this method does not have any
predictive power since the couplings depend on a large number (five for the
gluonic component) of unknown functions of $t$. As will be seen later, con-
straints on the coupling of two off-shell gluons to "white" hadrons, yielding
even $C$ exchange, simplify noticeably this picture \[2, 10, 11\].

Unless a specific choice of the partonic wave functions for the interacting
hadrons and of the Pomeron couplings to off-shell gluons is made, only for-
mal properties of the scattering amplitude can be derived with the aforemen-
tioned technique. It turns out, however, that these properties are interesting
enough to justify the general treatment of Section 2, where conditions on the
scattering amplitude, frequently required on phenomenological grounds, are
derived with a limited number of assumptions. In Section 3, a model will be
considered for the Pomeron as a double Regge pole and formal constraints
on the $t$ dependence of the amplitude will be derived. In this step a possi-
ble treatment of the non-perturbative region is proposed based on a suitable
regularization procedure. In Section 4, the new amplitude is compared with
the standard Regge formalism by considering a particular example, proton-
antiproton scattering at high energy. Generalizations to $\gamma$ induced processes,
like photoproduction, will be also touched upon. Concluding remarks appear
in Section 5.

2 Covariant reggeization

Consider first Fig. 1, the bare Pomeron in our approach. $\gamma, \delta, \nu$ and $\rho$ are
gluons and the zigzag line the Pomeron Regge pole. Let $Q = k’ + r/2, P =
k - r/2, \Delta = r$ and $\Delta^2 = t < 0$. If $(P \cdot Q)$ is large, the contribution for this
graph can be written in the form \[3, 7\]

$$\mathcal{M}_g = \frac{2^\alpha \Gamma(\alpha + 3/2)}{\sqrt{\pi} \Gamma(\alpha + 1)} \xi^+ M_g$$

(1)
where
\[
M_g = C_{\nu\rho}^+ C_{\gamma\delta}^+ \frac{\partial}{\partial P_{\alpha_1}} \frac{\partial}{\partial P_{\alpha_2}} \frac{\partial}{\partial Q_{\beta_1}} \frac{\partial}{\partial Q_{\beta_2}} (P \cdot Q)^\alpha
\]
and \(\xi_+ = \exp(-i\pi\alpha/2)/\sin(\pi\alpha/2)\) is the signature; \(\alpha\) is the Pomeron trajectory \((\alpha = \alpha(t))\). Expressions (1, 2) ensure pure spin \(\alpha\) exchange but, since "external" particles are off-shell gluons, the usual mass shell conditions cannot be used after all the derivatives have been done. Hence, if the reduced Regge coupling \(C_{\nu\rho}^+\) is written in the general form
\[
C_{\nu\rho}^+ = g_1 P_{\alpha_1} P_{\alpha_2} P_{\nu} P_{\rho} + g_2 P_{\alpha_1} P_{\alpha_2} g_{\nu\rho} +
\]
\[
+ g_3 g_{\alpha_2 \nu} P_{\alpha_1} P_{\rho} + g_4 g_{\alpha_1 \rho} P_{\alpha_2} P_{\nu} + g_5 g_{\alpha_1 \rho} g_{\alpha_2 \nu}
\]
the coefficients \(g_i\) depend a priori on all the invariants \(t, k^2\) and \((k - r)^2\): \(g_i \equiv g_i(t, k^2, (k - r)^2)\). An expression, analogous to (3) holds for the upper vertex \(C_{\gamma\delta}^+\).

A first simplification, if the Pomeron is coupled to hadrons, \(a\) and \(b\) with masses \(m_a\) and \(m_b\), as in Fig. 2, comes from Ward identities. The gluon-particle amplitude, call it \(f_{\nu\rho}\), vanishes at the lowest order when saturated with \(k^\nu\) or \((k - r)^\rho\) \([2, 12]\) and, neglecting non leading terms of the form \(r^\nu f_{\nu\rho}\), only the terms with coefficients \(g_2\) and \(g_5\) in (3) give a non zero contribution. Notice that all the terms, having origin from the derivatives in eq. (4), are leading; at the end they will give rise to contributions going as \((P \cdot Q)^\alpha\). The term proportional to \(g_2\), that is usually neglected, represents a new and interesting feature of the scattering amplitude.

Consider first, for the sake of simplicity, the scattering hadrons as \(q\bar{q}\) bound states; extrapolation to real mesons or baryons will not change the main result of the model. The formal evaluation of the vertices, for example the lower one shown in Fig. 3, can then be done as follows. Momentum-space techniques \([13]\) can be used by choosing, in the \(s \to \infty\) limit, a reference frame where the large components of the momenta of the incoming and outgoing particles are along the \(z\)-axis. Hence, setting \(\omega = \sqrt{s}/2\), from Fig. 2 one
Figure 1: Scattering of off-shell gluons $g^* + g^* \rightarrow g^* + g^*$ with Pomeron exchange.

has

\[ p_a = (p_{a+}, p_{a-}, \vec{p}_{a\perp}) = (2\omega, m_a^2/(2\omega), 0), \quad p_b = (m_b^2/(2\omega), 2\omega, 0) \]
and
\[ r = (r_+, r_-, \vec{r}_\perp) \]
with
\[ r_\pm = \frac{\pm t (p_{a\pm} + p_{b\pm})}{p_{a+} p_{b-} - p_{a-} p_{b+}} \sim \pm O(t/\omega) \]

In the following, the masses and $r_+, r_-$ will be neglected with respect to $\omega$.

In Fig. 3, that represents one of the three possible diagrams to be evaluated, the particle with momentum $l_1$ is a quark and $l_2$ an antiquark with $l_1 - l_2 = p$.

\footnote{For the four vector $a$, the infinite momentum variables are defined as $a_\pm = a^0 \pm a^3$ and $\vec{a}_\perp = (a^1, a^2)$}
The leading contribution of the lower part of this graph will have a tensor structure of the form $l_1^\sigma \cdot l_2^\tau$, or $l_1^{\nu} \cdot l_2^{\rho}$ when the gluon propagators in Feynman gauge are taken into account. With the position $l_1 = p/2 + z$, $l_2 = -p/2 + z$ the integration over $d^4k\; d^4z = (1/4)dk_+ \; dk_- \; d\vec{k}_\perp \; dz_+ \; dz_- \; d\vec{z}_\perp$ can be formally performed.

Whatever the form of the vertex $V$ and the functions $g_2$ and $g_5$ could be, the integrals over $k_+$ and $z_+$, at the lower vertex, can be done by closing over the respective poles. The same procedure applies to the integrals over $k'_-$ and $z'_-$ at the upper vertex. Keeping always the leading terms in $\omega$, the integrations over the transverse momenta and $z_-$ can be performed implicitly; they involve in fact the unknown functions $g_2$, $g_5$ and $V$. Only the integrals over the "large" components of $k$ and $k'$, $k_-$ and $k'_+$, remain undone. While $|k_-|$ and $|k'_+|$ are large, they satisfy the constraint $k_-, k'_+ \ll \omega$ and, for simplicity sake, are neglected with respect to $\omega$. A more rigorous approach would not change sensibly the final form of the amplitude whose structure is determined from the model chosen for the Pomeron propagator.
Let now $X$ be a second rank tensor constructed from the four-momenta $k, k'$ and $r$. When the aforesaid integrations have been performed, the following correspondences can be established

$$g_2 g_{\nu\rho} X^{\nu\rho} \to V_2^{(b)}(t), \quad g_5 g_{\alpha_1 \nu} g_{\alpha_2 \rho} X^{\nu\rho} \to V_5^{(b)}(t) X^{--} \quad (4)$$

$$g_2 g_{\gamma\delta} X^{\gamma\delta} \to V_2^{(a)}(t), \quad g_5 g_{\beta_1 \gamma} g_{\beta_2 \delta} X^{\gamma\delta} \to V_5^{(a)}(t) X^{++} \quad (5)$$

where $V_{2,5}^{(a,b)}(t)$ are unknown functions of $t$, unless a specific choice for the wave functions and $g_2, g_5$ is made.

For the scattering of identical particles, $a \equiv b$, one gets, from the $M_g$ part of the propagator,

$$\alpha^2 (\alpha - 1)^2 \left[ \frac{1}{\omega^2} V_2^2(t)(P \cdot Q)^{\alpha} + \frac{1}{4} V_2(t) V_5(t)(k_-^2 + k'_+ + k_+^2)(P \cdot Q)^{\alpha-2} + \frac{\omega^2}{4} V_5^2(t)(P \cdot Q)^{\alpha-2} \right] \quad (6)$$

where

$$P \cdot Q \simeq \frac{k_- k'_+}{2}. \quad (7)$$

is the large variable. As far as the practical evaluation of the integrals in (4,5) is concerned, while the term with $g_5$ is standard, the calculation of the term with $g_2$ will be far from trivial and gives rise, in perturbative QCD, to singularities both in the infrared and in the ultraviolet regions. The latter singularities can be avoided if the function $g_2$ helps to make the integrals convergent. The integrals over $k_-$ and $k'_+$ factorize and are both of the form

$$\left( \frac{1}{2} \right)^{\alpha-m} \int \frac{1}{\omega/\rho} \int d\omega \int d\omega \, k_-^{\alpha+n-m}$$

where $n, m$ are integers and the scale $\rho, \rho > 1$, has been chosen the same at both vertices for simplicity sake. The integration of eq. (6), with the position $\rho^2 = s_0/4$, gives

$$\frac{\alpha^2 (\alpha - 1)^2}{2^{\alpha-1}} \left( \frac{\omega^2}{\rho^2} \right)^{\alpha} \left[ \frac{V_2(t)}{\rho (\alpha + 1)} + \frac{\rho V_5(t)}{\alpha - 1} \right]^2 = \rho^2$$
\[ \frac{\alpha^2}{2^{\alpha-1}(\alpha + 1)^2} \left( \frac{s}{s_0} \right)^\alpha \left[ \frac{2(\alpha-1)}{\sqrt{s_0}} V_2(t) + \frac{\sqrt{s_0}(\alpha+1)}{2} V_5(t) \right]^2. \]  

(8)

Hence the model requires the presence of a large scale \( s_0 \).

By defining
\[ h(\alpha) = \frac{\alpha^2 \Gamma(\alpha + 3/2)}{(\alpha + 1)\Gamma(\alpha + 2)}, \]
and collecting in \( V_2 \) and \( V_5 \) unknown constants the result, for the gluonic Pomeron contribution to the amplitude, is
\[ A_g = \frac{-h(\alpha)}{\sin(\pi\alpha/2)} \left( \frac{\text{is}}{s_0} \right)^\alpha [(\alpha-1)V_2(t) + (\alpha+1)V_5(t)]^2. \]  

(10)

Equation (10) can be easily generalized to the case \( a \neq b \). It is sufficient to substitute the term, within squared brackets in (10), with the product:
\[ [(\alpha-1)V_2^{(a)}(t) + (\alpha+1)V_5^{(a)}(t)] \cdot [(\alpha-1)V_2^{(b)}(t) + (\alpha+1)V_5^{(b)}(t)]. \]

In addition to well established properties of the amplitude, like Regge behaviour, a new feature appears. If a linear trajectory is adopted for the Pomeron, a term vanishing with \( t \) is present in (10) since \( V_2(t) \), as shown below, can be made regular at \( t = 0 \). This term can have an important rôle in the calculation of the forward slope and in the dip region. At any rate, it shows that the \( t \)-dependence can be more involved than commonly believed. Corrections to eq. (10) will give rise to other contributions that can be summarized as follows.

- Non-leading contributions will appear in the amplitude because of the neglected terms in eq. (3).
- Since the integrals over \( k_- \) and \( k'_+ \) comprise a region where these variables are not large, and hence do not correspond to the exchange of a Regge pole, the contribution of this region must be subtracted from eq. (10). The most important term will appear from the integration
of the coefficient of $V_5(t)$ in eq. (3), since it is multiplied by $\omega^2$. The contribution to the amplitude will be proportional to $sV_5^2(t)$.

• More important will be the contribution due to the presence of quarks in the Pomeron. The coupling of the Pomeron to a quark in the hadron has the form [6, 7]

$$C_\alpha^+ C_\beta^+ \frac{\partial}{\partial P_\beta} \frac{\partial}{\partial Q_\alpha} (P \cdot Q)^\alpha$$

where

$$C_\alpha^+ (\frac{1}{2}, \frac{1}{2}, J) = (f_1 Q_\alpha + f_2 \gamma_\alpha)$$

and an analogous expression for $C_\beta^+$. In the eikonal approximation, $Q_\alpha$ and $\gamma_\alpha$ give the same contribution and, for the amplitude, one obtains

$$A_q = \frac{\alpha(\alpha + 1) \Gamma(\alpha + 3/2)}{\sin(\pi\alpha/2) \Gamma(\alpha + 1)} \left( -\frac{is}{4} \right)^\alpha V_5^2(t). \quad (11)$$

with a $t$-dependence different from the one found in eq. (10).

• Non-leading trajectories will give the same contribution, with the appropriate change in the trajectory function $\alpha(t)$, as in eq. (11).

3 A dipole Pomeron model

The amplitude in eq. (10) is an asymptotic estimate and, in order to preserve unitarity, it is preferable to keep the Pomeron intercept at one and adopt a dipole Pomeron model [14] to account for rising cross sections. It is well possible to consider instead a supercritical Pomeron with an intercept slightly higher than one [15] but, as will be clear later, the formalism of Section 2 is particularly suited for a model where the Pomeron is a double pole in the J-plane. Eq. (10) distinguishes clearly the functional dependence on $t$ and
on $\alpha(t)$. Since the procedure to obtain a dipole Pomeron amounts to derive eq. (10) with respect to $\alpha(t)$, the answer is unique.

Only the gluonic component of the dipole Pomeron will be considered in the following and, setting

$$W \equiv W(\alpha, t) = (\alpha - 1)V_2(t) + (\alpha + 1)V_3(t),$$

from eq. (10) the imaginary and real part of the amplitude assume the form:

$$\Im A^{(d)}_g = \left( \frac{s}{s_0} \right)^\alpha W \left[ \ln \left( \frac{s}{s_0} \right) W + 2 \frac{dW}{d\alpha} + \frac{dh}{d\alpha} W \right],$$

and

$$\Re A^{(d)}_g = - \left( \frac{s}{s_0} \right)^\alpha W \left[ \ln \left( \frac{s}{s_0} \right) \cot \left( \frac{\pi \alpha}{2} \right) - \frac{\pi}{2 \sin^2(\pi \alpha/2)} \right] +$$

$$+ \cot \left( \frac{\pi \alpha}{2} \right) \left[ \frac{dh}{d\alpha} W + 2h \frac{dW}{d\alpha} \right].$$

The trajectory can be chosen in the form $\alpha(t) = 1 + \alpha' t$, with the conventional value for $\alpha'$, $\alpha' = 0.25 \text{ GeV}^{-2}$. As a consequence of the derivation, with respect to $\alpha(t)$, factorization of residues is lost in eqs. (13) and (14). The factorization breaking term is proportional to $d \ln W/d\alpha$.

The total cross section

$$\sigma_T = \frac{\Im A^{(d)}(s, 0)}{s},$$

the differential cross section

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2} \left| A^{(d)}(s, t) \right|^2,$$

\[A\text{ possible dependence of } g_1 \text{ and } g_2 \text{ on } \alpha, \text{ as would happen for example in a dual model, does not change the final conclusions.}\]
the forward slope and the ratio between the real and imaginary part of
\( A^{(d)}(s,t) \) can be derived from the total amplitude \( A^{(d)} \) that will be the sum
of three terms

\[
A^{(d)} = A_g^{(d)} + A_q^{(d)} + A_{n.l.},
\]

where \( A_{n.l.} \) represents the contribution of non-leading trajectories, as simple
poles, and \( A_q^{(d)} \) is the derivative of eq. (11) with respect to the Pomeron
trajectory.

Before trying to obtain more concrete informations from this model, it is
important to spend a word on the integrals implied in the correspondences
(4, 5). At large \(|t|\), perturbative QCD is expected to give the correct answer
and the integration over the gluon propagator, that in this regime has the
form \( D(k_\perp^2) \simeq 1/k_\perp^2 \), does not give rise to problematic results. However,
for small \(|t|\), divergencies will arise in the derivatives of \( V_2(t) \) and \( V_5(t) \);
for example, in the Born approximation, two-gluon exchange results in a
diverging slope at \( t = 0 \) [11]. First and second derivatives of the amplitude
are related to the slope and curvature parameters, both have been measured
experimentally in \( p-\bar{p} \) scattering [16]. The correct, but unknown, behaviour
of the gluon propagator in the infrared must restore the physical properties
of the scattering amplitude.

It is possible to avoid the need of a precise representation for the non
perturbative gluon propagators since a superconvergence relation exists for its
discontinuity in Landau gauge [17, 18]. Let \( \sigma(k^2) \) be the discontinuity along
the positive real \( k^2 \)-axis of \( D(k^2) \), the structure function of the transverse
gauge propagator, \( \pi \sigma(k^2) = ImD(k^2 + i0) \). Then, if \( N_f < 10 \) where \( N_f \)
is the number of flavours, the superconvergence relation

\[
\int_0^\infty d\lambda^2 \sigma(\lambda^2) = 0
\]

follows from the dispersion relation

\[
D(k^2) = -\int_0^\infty d\lambda^2 \frac{\sigma(\lambda^2)}{k^2 - \lambda^2 + i\epsilon}.
\]
For large $|t|$, $\sigma(\lambda^2) \sim \delta(\lambda^2)$ reproduces the usual Feynman rule for the gauge propagator. Hence it is possible to add, to every troublesome integrals in eqs. (1, 2) a term of the form

$$R(t) \int \int \sigma(x)\sigma(y) \, dx \, dy = 0$$

and regularize the divergence at $t = 0$. This procedure does not introduce free parameters since the value of all integrals is fixed in the perturbative region and, according to [11], confinement effects become sensible only at quite small $k^2$, for example $|\vec{k}_\perp| \sim 2m_\pi$. In this narrow $t$ interval, linear, or quadratic, extrapolations should be possible with suitable matching conditions, at least from a phenomenological point of view [19]. It is assumed, in the following that this regularization takes into account all nonperturbative effects.

### 4 Comparison with the standard approach

Consider now proton-antiproton elastic scattering as a typical process that, at very high energy, can be described with the help of eqs. (12), (13) and (14). The limitation to high energies is due to the neglect in the following of nonleading terms: mesonic trajectories and corrections to the Pomeron contribution.

With the conventional Pomeron trajectory, $\alpha_P = 1 + \alpha^' \, t$ with $\alpha^' = 0.25 \, GeV^{-2}$, eq. (13) gives

$$W = \frac{1}{4} t (V_2 + V_5) + 2V_5,$$

while

$$\frac{dW}{d\alpha} = V_2 + V_5,$$

where $V_2$ and $V_5$ are unknown functions of $t$. Since

$$h(\alpha)|_{t=0} = \frac{3\sqrt{\pi}}{16}, \quad \left. \frac{d\ln h}{d\alpha} \right|_{t=0} = \frac{8}{3} - 2 \ln 2,$$
the total cross section can be written as
\[
\sigma_T = \frac{3\sqrt{\pi}}{4s_0} V_5^2(0) \left[ \ln \left( \frac{s}{s_0} \right) + \frac{V_2(0)}{V_5(0)} + \frac{11}{3} - 2 \ln 2 \right].
\] (18)

The differential cross section (in \(mb^2\)) has the form
\[
\frac{d\sigma}{dt} = \frac{1}{16\pi s^2} \left( \frac{s}{s_0} \right)^{2\alpha} \frac{h^2 W^4}{\sin^2(\pi \alpha/2)} \times \left[ \ln \left( \frac{s}{s_0} \right) + \frac{d \ln(h W^2)}{d\alpha} - \frac{\pi}{2} \cot \left( \frac{\pi \alpha}{2} \right) \right]^2 + \frac{\pi^2}{4}. \] (19)

and, from eq. (9), it is easy to obtain
\[
\frac{d \ln h}{d\alpha} = \frac{2}{\alpha(\alpha + 1)} - \psi(1 + \alpha) + \psi(\alpha + 3/2),
\]
where \(\psi(z)\) is the logarithmic derivative of the gamma function.

The slope in the forward direction, in this approach, is a cumbersome expression that presents, however, many interesting features. As will be shown later, there is in fact the possibility to overcome discrepancies, present in the standard approach, appearing when the experimental slope is fitted at different energies. In order to have a compact form for this observable, it is convenient to introduce the following notation:
\[
f(s) = \ln \left( \frac{s}{s_0} \right) + \frac{8}{3} - 2 \ln 2,
\]
and \(V_5'(0)/V_5(0) = v_5',\ V_2(0)/V_5(0) = v_2,\ V_2'(0)/V_5(0) = v_2'.\) With these definitions, the forward slope is
\[
b \equiv \frac{d}{dt} \left( \ln \frac{d\sigma}{dt} \right) \bigg|_{t=0} = 2\alpha' f(s) + \frac{v_2 + 1}{2} + 4v_5' + 2 \left[ v_2' - v_5' v_2 - \frac{1}{8}(v_2 + 1)^2 + \frac{\alpha'}{6} \left( \frac{7\pi^2}{2} - \frac{89}{3} \right) \right] \frac{f(s) + v_2 + 1}{(f(s) + v_2 + 1)^2 + \pi^2/4}. \] (20)
In order to see how problems in fitting experimental data can be removed in a specific example, let us consider $p - \bar{p}$ elastic scattering at the Tevatron [20]. Measurements of the elastic slope, near $t = 0$, give $b = 15.35 \pm 0.18 \text{ GeV}^{-2}$, at $\sqrt{s_1} = 546 \text{ GeV}$, and $b = 16.98 \pm 0.24 \text{ GeV}^{-2}$ at $\sqrt{s_2} = 1800 \text{ GeV}$. Assuming an $s$-dependence of the slope $b = b_0 + 2\alpha' \ln(s/s_0)$, the data at these energies, where only the Pomeron probably contributes, give

\[ v_5' \]

Figure 4: Allowed region in the $(v_5', v_2')$ plane. Continuous lines are determined from the slope, with errors, at $\sqrt{s} = 1800 \text{ GeV}$, dashed lines refer to $\sqrt{s} = 546 \text{ GeV}$ and dotted lines to $\sqrt{s} = 62.5 \text{ GeV}$. Data are from [20, 22].
\( \alpha' = 0.34 \pm 0.07 \text{GeV}^{-2} \) to be compared with a lower value when the data at the CERN-ISR \cite{21,22} are included in the fit. From 546 GeV to 1800 GeV, the total cross section increases by 18.8 \( \pm \) 2.5 \( \text{mb} \), according to \cite{20}. Then eq. (18) implies that \( V_s^2(0)/s_0 = 5.9 \pm 0.8 \text{mb} \) while the measured values of the total cross section at these energies give

\[
v_2 - \ln s_0 = -7.0 \pm 1.6.
\]

By keeping the central value for \( (v_2 - \ln s_0) \) and fixing the value of \( s_0 \), for example \( s_0 = 9 \text{GeV}^2 \), it is possible to find the allowed region, in the plane \((v'_2, v'_5)\), determined from the experimental slopes, and their errors, at the energies \( \sqrt{s}_1 \) and \( \sqrt{s}_2 \). In this calculation, \( \alpha' \) is always 0.25 \( \text{GeV}^{-2} \). Figure 4 shows the region where \( b \), in eq. (21), satisfies the experimental bounds given by \cite{20}. If \( s_0 \) is increased, above 9 GeV\(^2\), the parallelogram representing the allowed region moves down and to the right, in the \((v'_5, v'_2)\) plane, preserving its form. In this figure, also the boundary determined by the ISR data \cite{22} at \( \sqrt{s} = 62.5 \text{GeV} \) is shown. This can be considered as an extreme example, since I am not aware of other parametrizations, within the Regge framework, that succeed in reproducing the total cross sections, measured by \cite{20}, at both energies. Usually the cross section at 1800 GeV is underestimated.

The term proportional to \( t \), in the vertex, has now an important rôle and makes the determination of the slope somewhat independent on the actual value of the total or the differential cross section in the forward direction. It is plausible that, for \( t \) different from zero, this term could help in explaining non trivial properties of the forward cone. In order to substantiate this belief, it is interesting to consider the real and imaginary part of the amplitude, eqs. (13) and (14). There are both theoretical \cite{23} and phenomenological \cite{24,25} reasons for the presence of a zero in the real part of the even signature amplitude near \( t = 0 \). Looking at eq. (14), this requirement can be written

\[3\text{Remember that, in this model, } s_0 > 4 \text{GeV}^2 \text{ (see Section 2).}\]
For example, the flattening of the slope for the production of $J/\psi$ can be explained in this approach without requiring a drastic change in the slope of the Pomeron trajectory. According to the generalization proposed after eq. (10), the differential cross section for the photoproduction of the $J/\psi$ is obtained from eq. (19) with the substitution

$$W \rightarrow \sqrt{UW}$$

where the new vertex $U(t)$ has the form given in eq. (16), but refers to the vertex $\gamma$-Pomeron-$J/\psi$. An attempt along this line has been considered in ref. [26]. In the case of electroproduction, this vertex will be a function of $t$ and $Q^2$, where $q^2 = -Q^2$ is the square of the fourmomentum of the off-shell photon. Data for the diffractive production of vector mesons have been published by H1 [27] and ZEUS [28] Collaborations. The scarcity of
experimental data and their large errors, for the cases of interest here, where only the Pomeron is exchanged, does not allow an analysis similar to the one performed for the $p - \bar{p}$ case. It is plausible, however, that the greater flexibility reached in this model will help in accounting for the variation of the slope with energy.

5 Concluding remarks

The proposed approach to the hadronic scattering at asymptotic energies regards the Pomeron propagator as a Regge dipole, while its coupling to the quarks in the hadrons reflects the Pomeron structure in terms of gluons and quarks. Covariant reggeization determines the general form of the scattering amplitude for the interaction of two off-shell gluons when the Pomeron is exchanged. Since the internal colour structure of the hadrons has not been specified, only general properties of the amplitude for the hadronic process, imposed by the method of reggeization, can be derived.

From the operative point of view, the main difference with the standard approach consists in the appearance of a new term in the amplitude, that, if a linear trajectory is adopted for the Pomeron, vanishes linearly in the forward direction. The importance of this term in the description of experimental data, especially when the slope in the forward direction is considered, is shown in the specific example of $\bar{p} - p$ scattering. The Pomeron trajectory can be fixed, once for all, in the description of the forward slopes at different energies. In the dipole Pomeron model [14], adopted in this paper, the increase of the total cross section can be obtained with a unity intercept for the Pomeron trajectory, while its slope coincides with the conventional value of 0.25 GeV$^{-2}$. Corrections due to the presence of quarks in the Pomeron and non leading contributions have been explicitly evaluated in Section 2. The case of photon induced processes is also briefly discussed.
It would be tempting to parametrize the functions appearing in the vertex (see eq. (16)) in terms of exponentials, and/or ratios of polynomials, and obtain an amplitude where the number of free parameters is comparable to other parametrizations. From a phenomenological point of view, this approach could be adequate. However, in my opinion, the correct way to increase the predictability of the model must start from the explicit calculation of the vertex with different forms of the hadron wave functions, available in the literature. The comparison of the result with the experimental data requires, in addition, the knowledge of the Pomeron couplings to off-shell gluons. A hint for the latter couplings could come from the (skewed) parton distribution functions in the Pomeron.

The limits of the model are set from the chosen framework: Regge exchange and momentum space technique are the main ingredients of this calculation. Both are supposed to describe correctly the scattering amplitude only in the small \(|t|\) region. The choice of a different form for the Regge trajectories \([29]\) could provide a smooth interpolation between soft and hard behaviour of the scattering amplitude, from an exponential decrease to a power law in \(t\) \([29]\). It would be quite interesting to study, within the present model, the effect of a Pomeron trajectory, with a two-pion square root threshold, on the differential cross section \([30]\). This calculation, that could also explain the presence of kinks in the differential cross section well before the dip, will be considered elsewhere.

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