QCD Phase Diagram: Phase Transition, Critical Point and Fluctuations

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Abstract

A summary of discussions on selected topics related to QCD phase diagram, phase transition, critical point, fluctuations and correlations at the Quark Matter 2009 conference are presented.

1. Introduction

Understanding the properties of the nuclear matter when subjected to extremes of temperature and density is one of the goals of the relativistic heavy-ion collision program. The knowledge of the properties manifested in these collisions should have some connection to our understanding of the early universe and how hadrons acquire their masses. QCD suggests we should at least expect two types of transitions for nuclear matter in limits of high temperature \((T)\) and densities: deconfinement transition and chiral phase transition. Theoretically the deconfinement measure is the order parameter, Polyakov loop \((L(T) \sim \lim_{r \to \infty} \exp[-V(r)/T])\), where \(V(r)\) is the potential between a static quark-antiquark pair separated by a distance \(r\). The measure of chiral transition is the order parameter, the chiral condensate \(\langle \bar{\psi}\psi \rangle(T)\). For a confined phase for \(T < T_c\) (critical temperature for transition) we have \(L(T) \approx 0\) and \(\langle \bar{\psi}\psi \rangle(T) \neq 0\), and for a deconfined phase for \(T > T_c\) with \(L(T) \neq 0\) and \(\langle \bar{\psi}\psi \rangle(T) \approx 0\). In this proceedings we summarize the theoretical and experimental understanding of the QCD phase diagram as was discussed at the QM2009.

2. Lattice QCD results

Lattice calculations simulate a quantum statistical ensemble in thermal equilibrium at fixed \(T\) with partition function \(Z = \text{Tr} e^{-H/T}\), where \(H\) is the QCD Hamiltonian. The lattice spacing \(a\) and \(N_t\) sites in imaginary time are related to \(T\) as \(aN_t = 1/T\). Usually \(a\) is varied to change \(T\) keeping \(N_t\) fixed. An approach based on “\(T\) integral method” was presented, where \(N_t\) is varied and \(a\) fixed to change \(T\) \cite{1}. There are few aspects to be noted while interpreting lattice results. To connect with reality, lattice results needs to be presented as extrapolations to continuum limit. The calculations are now done on lines of constant physics, along which the variations in observables can be attributed to changes in temperature and not also to changes in the Hamiltonian. Choice of proper actions (see recent work \cite{2}), spatial volume and setting of quark masses are important for interpreting the results. Results are also dependent on the choice of number of quark flavours. The current view is, computation with 2 light quarks and a heavy strange quark (2+1) are close to a realistic picture. At the conference, several studies from Lattice QCD were presented. These provided insights on the order of phase transition, transition temperature \((T_c)\), Equation-Of-State (EOS) and QCD critical point (QCP), some of these are discussed below.
2.1. Order of phase transition

QCD calculations on lattice at high temperature and $\mu_B = 0$ MeV has established the quark-hadron transition to be a cross-over \cite{3}. Figure 1 shows the lattice chiral susceptibility $\chi(N_s, N_t) = \frac{\partial^2}{(\partial m^2_{ud})(T/V)} \log Z$, where $m_{ud}$ is the mass of the light u,d quarks, $N_s$ is the spatial extension, $N_t$ euclidean time extension, and $V$ the system volume. The susceptibility plotted as a function of $6/g^2$ ($g$ is the gauge coupling and $T$ grows with $6/g^2$) shows a pronounced peak around the $T_c$. The peak and width are independent of volume (varied by a factor 8) thereby establishing the transition to be an analytic cross-over \cite{3}. For a first-order phase transition the height of the susceptibility peak should have been $\propto V$ and the width of the peak $\propto 1/V$, while for a second-order transition the singular behaviour should have been $\propto V^\alpha$, $\alpha$ is a critical exponent. A lattice result discussing the nature of phase transition at finite $T$ and baryon density \cite{4} was presented. It was based on idea that the chemical potential ($\mu^*_q$) which gives a minimum of the effective potential does not increase when it passes through the mixed phase. Whereas for a cross-over transition it shows a monotonic increase with density when $T > T_c$. The results are shown in the right panel of Fig. 1. This calculation which uses P4-improved staggered quark action, in canonical ensemble with $m_\pi = 700$ MeV and number of quark flavour of 2, results in a first order phase transition for $T/T_c < 0.83$ and $\mu_q/T > 2.3$ \cite{4}.

2.2. Transition Temperature

The point of sharpest change in temperature dependence of the chiral susceptibility ($\chi_{\bar{\psi}\psi}$), the strange quark number susceptibility ($\chi_s$) and the renormalized Polyakov-loop ($L$) are used to estimate the QCD transition temperature in lattice calculations. The recent results on chiral and deconfinement phase transition temperatures were presented by two groups (HotQCD/RBC \cite{5} and Budapest/Wuppertal \cite{6}). Using the observable $\chi_{\bar{\psi}\psi}/T^2$ the HotQCD/RBC and Budapest/Wuppertal groups get the chiral phase transition temperatures as 192 (4)(7) MeV and 152(3)(3) MeV respectively. Using the observable $L$ the deconfinement transition temperature remained the same for HotQCD/RBC and is 170(4)(3) MeV from Budapest/Wuppertal group. The two groups differ in the transition temperature for both chiral and deconfinement transitions. The possible sources of differences in the two approaches could be ambiguity in locating $T_c$ for a cross-over, physical observable used to set the scale, preferred renormalization of chiral susceptibilities and choice of actions. It was suggested at the conference to study the QCD thermodynamics with a theoretically firmly established Wilson type fermion discretization. Such a large difference in the value of $T_c$ has serious consequences for heavy-ion phenomenological studies. As energy density ($\epsilon$) $\sim T^4$, the lack of accurate determination of $T_c$ leads to about 60% uncertainty in $\epsilon$ at $T_c$. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Left: Susceptibility for the light quarks for $N_t=6$ as a function of $6/g^2$, where $g$ is the gauge coupling ($T$ grows with $6/g^2$) \cite{3}. Right: Derivative of logarithm of partition function in canonical ensemble formalism as a function of the quark number density \cite{4}.}
\end{figure}
2.3. Equation of State

Figure 2 (left) shows the variation of $\epsilon/T^4$ vs. $T$. As $\epsilon/T^4 \propto$ effective degrees of freedom (dof) of the system ($g_{\text{eff}}$), the trend shows a change in $g_{\text{eff}}$ below and above $T_c$. This is interpreted as a transition from a state with hadronic dof to a state with quark-gluon dof. At high $T$ the $\epsilon/T^4$ is about 10-15% lower compared to Stefan-Boltzman (SB) limit indicating that the matter still has strong interaction effects. The deviations from ideal gas behaviour could be understood in terms of effective thermal masses of quarks and gluons, and a consequence of this sizable interaction at high $T$ could lead to the existence of coloured resonance states. It was shown that at very high $T \sim 10^5 - 10^7$ MeV the lattice calculations approach the SB limit and has good agreement with calculations from perturbation theory [9]. The functional relation between pressure ($p$) and $\epsilon$ is the EOS. The velocity of sound in the medium ($c_s^2$) is $dp/d\epsilon$ at fixed entropy. $p/\epsilon$ is plotted as a function of $\epsilon^{1/2}$ computed from Lattice in Fig. 2 (right) [8]. This is an important ingredient to heavy-ion phenomenology calculations, as the $c_s$ decides the rate of cooling of the high-$T$ system and provides input on the system composition at different evolution times. For the dependence of thermodynamics of QCD plasma on number of colors see [10].

3. Experimental results on Charge Correlations

The STAR experiment at RHIC presented preliminary results on charged hadron azimuthal correlations based on 3-particle correlation technique [11]. The results from Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV at midrapidity for $0.15 < p_T < 2$ GeV/c between same charged and opposite charged hadrons with respect to reaction plane (plays the role of third particle) are shown in Fig. 3. The observable, $\langle \cos(\phi_a + \phi_p - 2\phi_{RP}) \rangle$ represented the difference between azimuthal correlations projected onto the direction of the angular momentum vector and correlations projected onto the collision event plane. The difference between the same charge and opposite charge correlations could not be explained by models such as HIJING and UrQMD and by incorporating realistic values for the elliptic flow in such simulations. The signal seems to be consistent with the predictions for the existence of metastable domains in QCD vacuum leading to local Parity violation in the vicinity of deconfinement transition expected to be achieved in heavy-ion collisions [12].

For such phenomena where the massless quarks can change their chirality due to interactions with gluon fields, there could be separation of positive charges from negative charges along the direction of angular momentum of the collision as a result of large magnetic fields reached in the collisions (especially in non-central collisions). Deconfinement allows for the possibility of quarks traveling over distances greater than nucleonic scales and chiral symmetry restoration is essential, because a chiral condensate will tend to erase any asymmetry between the number of right- and left-handed fermions. The observable presented is parity-even [11], making it susceptible to physical processes not related to parity violation effects. Thus understanding of the
physical background is crucial. The signal was also observed to be not restricted in the low $p_T$ region as naively might be expected for parity-violation effects.

4. QCD Critical Point

4.1. Theory Calculations

Several QCD based models predict the existence of an end point at high $\mu_B$ for the first order phase transition in the QCD phase diagram [13]. However the exact location depends on the model assumptions used. A summary of these model results can be found in Ref. [14]. One such calculation based on linear $\sigma$ model coupled to two flavors of identical mass quarks was presented at the conference [15]. This work extends the previous investigations by including thermal fluctuations of the meson and fermion fields. It is observed that when all other parameters of the model are fixed, the calculation allows for existence of zero, one or two critical points in phase diagram depending on the value of the vacuum pion mass. A typical result from this calculation for varying mass of pion is shown in Fig. 4. Given the ambiguity in predictions of QCP in models, studies on lattice was expected to provide reliable estimates. However lattice calculations at finite $\mu_B$ have important issues to be addressed. Typically, for any lattice computation one needs to evaluate the expectation value of an observable $X$, $\langle X(m,\mu) \rangle = \int DU \exp(-S_G) X(m,\mu) \text{Det} M(m,\mu)$, where $M$ is the Dirac matrix in $x$, colour, spin, flavour space for sea quarks of mass $m_s$, $S_G$ is the gluonic action, and the observable $X$ may contain fermion propagators of mass $m_v$. The Det $M$ for non zero $\mu$ is not positive definite, hence numerical methods of evaluation of the expectation values is difficult, this is commonly refered to as the sign problem. There are several ways suggested to overcome this issue. (i) Reweighting the partition function in the vicinity of transition temperature and $\mu = 0$ [16], (ii) Taylor expansion of thermodynamic observables in $\mu/T$ about $\mu = 0$ [17] and (iii) Choosing the chemical potential to be imaginary will make the fermionic determinant positive [18]. The first two methodologies yield an existence of QCP (as shown in Fig. 3), whereas the third procedure gives a QCP only when the first co-efficient in the Taylor expansion of generic quark mass on the chiral critical surface ($m_c$) as a function of $\mu/T$ ($\mu/m_c(0) = 1 + \sum_{k=1} c_k \left( \frac{\mu}{T} \right)^{2k}$) is positive. The calculations seem to support a negative value of $c_1$ indicating an absence of QCP\footnote{1 Developments after QM2009 have shown that with decreasing lattice spacing, $c_1$ becomes positive and the expected picture of the phase diagram seems to be recovered (O. Philipsen at CPOD2009).}. This result was intensely discussed at the conference with

![Figure 3: Azimuthal charge correlations in data compared to simulation results for 200 GeV Au+Au [13]. Blue symbols mark opposite-charge correlations, and red are same-charge. Markers connected by solid lines represent the data.](image)
suggestion to do the calculation with larger spatial volume, check the stability of the results for different values of $N_t$ and for a realistic 2+1 flavour in continuum limit. A new calculation on lattice using the Canonical ensemble was presented at the conference showing the existence of an end point for $T_E \sim 160$ MeV and $\mu_E \sim 600$ MeV [20]. Further improvements are expected when calculations will use a realistic pion mass, instead of $m_\pi = 700$ MeV used in the presented work.

4.2. Signatures of QCD Critical Point and Experimental Results

The characteristic signature of QCP is large fluctuations in event-by-event conserved quantities like net-charge, net-baryon number and net-strangeness [21]. The variance of these distributions ($\langle (\delta N)^2 \rangle$) are proportional to square of the correlation length ($\xi$). The finite size and finite time effects attained in high energy heavy-ion collisions, limits the value of the $\xi$ achieved in the collisions. Model calculations suggest it could be small (2-3 fm) [22], thereby making it extremely challenging to measure in the experiments. Motivated by the fact that non-Gaussian features in above observables increase if the system freezes-out closer to QCP, it has been suggested to measure higher moments (non-zero skewness and kurtosis indicates non-Gaussianity) of net-charge or net-baryon number distributions. Further it has been shown that higher moments ($\langle (\delta N)^2 \rangle \sim \xi^{4.5}$ and $\langle (\delta N)^4 \rangle \sim \xi^7$) have stronger dependence on $\xi$ compared to variance and hence have higher sensitivity [23]. Another important reason to look for higher moments comes from lattice calculations of the quadratic and quartic net-charge, net-baryon and net-strangeness (these denoted by $N_X$) fluctuations (evaluated at vanishing chemical potential), $\chi^2_X = \frac{1}{VT} \langle N_X^2 \rangle$ and $\chi^4_X = \frac{1}{VT} \langle (N_X^4) - 3 \langle N_X^2 \rangle^2 \rangle$ [24]. $\chi^2_X$ show a rapid rise in the transition region whereas the $\chi^4_X$ show a maximum at $T_c$. This maximum is most pronounced for the baryon number fluctuations and would diverge at QCP due to long-range correlations [19]. So studying higher moments allows for a connection between QCD calculations on lattice and experimental data. Experimentally it is difficult to measure all the produced baryons. This issue was addressed theoretically, where it was shown that net-proton number fluctuations would faithfully reflect the singularity of the charge and baryon number susceptibility due to the iso-spin blindness of $\sigma$ field [23]. Hence looking for non-monotonic variation of higher moments of net-proton distribution as a function of $\sqrt{s_{NN}}$ (or $T - \mu_B$) is sufficient to locate the QCP. The first results on moments of net-proton
distribution at RHIC energies was presented at the conference and shown in Fig. 5 [25]. The evolution of the moments from peripheral to central collisions follow the expectations (dashed lines) from central limit theorem (equations shown in Fig. 5). This study at $\mu_B < 30$ MeV provides an understanding and a formulation for the physics background for the observable expected to be sensitive to long range fluctuations as expected for QCP.

Several other interesting signatures of QCP and associated results from SPS were discussed at the conference, one of them was based on the idea that the presence of a critical point deforms the isoentropic trajectories in the $T - \mu_B$ phase plane. The critical point serves as an attractor of the hydrodynamical trajectories. This later feature was debated at the conference. If such a scenario exists, it would lead to an experimental signal of drop in $\bar{p}/p$ ($\sim e^{-2\mu_B/T}$) vs. $p_T$ (in intermediate region) [27]. The suggestion was put to test with the existing data, by looking at the slope of $\bar{p}/p$ vs. $p_T$ at available $\sqrt{s_{NN}}$ at SPS and RHIC energies. As shown in the Fig. 5 (middle panel) there is no large drop in slope is observed for intermediate $p_T$ range. Another novel idea discussed was based on study of critical dynamics around QCP with relativisic dissipative hydrodynamics [28]. Using the idea of coupling the density fluctuations to thermal energy it was shown that sound modes around QCP will be suppressed. This will then lead to disappearance of mach-cone like signals observed at top RHIC energies [29]. NA49 experiment presented multiplicity and mean $p_T$ fluctuation results as a function of beam energy and ion size [26]. Within the experimental acceptance no strong non-monotonic dependence of fluctuations was observed for Pb+Pb collisions as a function of $\sqrt{s_{NN}}$ (or $\mu_B$). However the system size dependence (using the central C+C, Si+Si and Pb+Pb data) seems to indicate a non-monotonic behaviour around $\mu_B \sim 250$ MeV and $T \sim 178$ MeV. This was shown to be consistent with a theory calculation including a QCP and with $\xi \sim 6$ fm and folding in the experimental acceptance (shown as solid line in Fig. 5 (right)). This study further underscores the need for a more detailed investigation in the upcoming critical point search programs at RHIC and SPS.

5. Phase Diagram

The current understanding of the QCD phase diagram as discussed at the conference is depicted in Fig. 6. From the QCD calculations on lattice it is now established theoretically that the quark-hadron transition at $\mu_B = 0$ MeV is a cross-over [3]. The critical temperature for a quark-hadron phase transition lies within a range of 170-190 MeV (vertical band in Fig. 6) [3, 6].
Most calculations on lattice also indicate the existence of QCD critical point for $\mu_B > 160$ MeV $^{[16, 19, 20]}$. The exact location is not yet known unambiguously. Two such predictions computed on lattice are shown in Fig. 6 for a $T_c$ of 176 MeV $^{[16, 19]}$. High energy heavy-ion collision experiments have seen distinct signatures which suggest that the relevant degrees of freedom at top RHIC $^{[30]}$ and SPS energies $^{[31]}$ in the initial stages of the collisions are quark and gluons. Specifically the direct photon data is used to show that the initial temperature ($T_{\text{init}}$, Fig. 6) reached at RHIC and SPS is greater than $T_c$ predicted by lattice $^{[32]}$. Further the understanding of suppression in high $p_T$ hadron production in heavy-ion collisions relative to $p+p$ collisions at RHIC requires a medium energy density > 1 GeV/fm$^3$ (critical energy density from lattice for a phase transition) $^{[30]}$. The experiments have also measured the temperature at which the inelastic collisions ceases (Chemical freeze-out) and elastic collisions ceases (Kinetic freeze-out) $^{[33]}$. These temperatures (as shown in Fig. 6) are extracted from the measured particle ratios and transverse momentum distributions using model calculations which assume the system is in chemical and thermal equilibrium. It is interesting to note that the difference between the two freeze-out temperatures becomes smaller at high $\mu_B$ (estimated at chemical freeze-out). New experimental programs at RHIC $^{[34]}$, SPS $^{[35]}$, FAIR $^{[36]}$ and NICA facilities have been designed to search for the QCD critical point in coming years. Whereas the experimental program at LHC (probing the region of $\mu_B \sim 0$ MeV of the phase diagram) will provide an unique opportunity to understand the properties of matter governed by quark-gluon degrees of freedom at unprecedented high initial temperatures (higher plasma life time) achieved in the Pb+Pb collisions at 5.5 TeV $^{[37]}$. A novel theoretical proposal was made at the conference on the existence of a quarkyonic phase around $\mu_B$ values corresponding to AGS energies $^{[38]}$. This is in addi-
tion to confined and de-confined phases. The matter in such a phase is expected to have energy density and pressure that of a gas of quarks, and yet be confined. Baryon-Baryon correlations to look for nucleation of baryon rich bubbles surrounded by baryon free regions was discussed as a signature of such a phase [39].

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