Coherent States for Generalized Laguerre Functions

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Abstract

We explicitly construct a Hamiltonian whose exact eigenfunctions are the generalized Laguerre functions. Moreover, we present the related raising and lowering operators. We investigate the corresponding coherent states by adopting the Gazeau-Klauder approach, where resolution of unity and overlapping properties are examined. Coherent states are found to be similar to those found for a particle trapped in a Pöschl-Teller potential of the trigonometric type. Some comparisons with Barut-Girardello and Klauder-Perelomov methods are noticed.
1 Introduction

Coherent states were first investigated by Schrödinger in 1926 [1], where he introduced harmonic oscillator coherent states. Coherent states are mathematical tools which provide a close connection between classical and quantum formalisms. In fact, there appeared many applications of them [2, 3, 4, 5] and recently they were used to study orbital magnetism of two-dimensional electrons [6] and noncommutative magnetism [8].

In 1971, Barut and Girardello [9] proposed a method for constructing coherent states. They defined them as eigenstates of the lowering operator of the system. A generalization of their approach was suggested by Gazeau and Klauder [10]. Klauder and Perelomov, separately, proposed another definition [11, 12], which is actually known as the Klauder-Perelomov approach. In the latter coherent states are defined as the states generated by the action of the elements of the related dynamical symmetry group on the Hilbert space whose basis vectors are some special functions.

On the other hand, special functions are often investigated by using the factorization method. This method consists of constructing raising and lowering operators which generate orthogonal bases in terms of special functions [13, 14].

Motivated by the recent developments concerning special functions [13, 14], we consider a Hilbert space whose elements are generalized Laguerre functions. By constructing raising and lowering operators acting on these states one can obtain an explicit realization of the Hamiltonian which is defined to be diagonal in this Hilbert space. We deal with the coherent states of this system in terms of the Gazeau-Klauder method. The states turn out to be similar to those found for a particle trapped in a Pöschl-Teller potential of the trigonometric type. Moreover, we compare the obtained Gazeau-Klauder coherent states to the Barut-Girardello and Klauder-Perelomov ones.

In section 2 we review some properties of generalized Laguerre functions. In terms of the differential equation and the recurrence relations satisfied by these functions we construct operators which are acting as raising and lowering operators on them. Then, a Hamiltonian is defined such that generalized Laguerre functions are its eigenfunctions. In section 3 we construct coherent states of this system following the approach of Barut-Girardello. Section 4 is devoted to building coherent states in terms of the method given by Gazeau-Klauder. In section 5, we first give an explicit realization of the dynamical symmetry group $su(1,1)$. Then, we use elements of this group to generate coherent states from the Klauder-Perelomov definition. We present conclusions and some proposals in the final section.

2 Hamiltonian formalisms

We start by reviewing some properties related to the associated Laguerre polynomials $L_n^\alpha(x)$ [17], which will be used. By definition $L_n^\alpha(x)$ are

$$L_n^\alpha(x) = \frac{1}{n!} e^{x-x_0} \frac{d^n}{dx^n} (e^{-x} x^{n+\alpha}) = \sum_{m=0}^{n} (-1)^m \binom{n+\alpha}{n-m} x^m, \quad (1)$$
where \( \binom{p}{n} = \frac{p(p-1)\cdots(p-n+1)}{n!}, \binom{p}{0} = 1 \) and \( L_n^0(x) = L_n(x) \). The generating function corresponding to associated Laguerre polynomials is
\[
\sum_{n=0}^{\infty} L_n^\alpha(x)x^n = \frac{e^{\frac{x}{1-z}}}{(1-z)^{\alpha+1}}.
\] (2)

Note that \( L_n^\alpha(x) \) are orthogonal with respect to the following weight function
\[
\rho(x) = x^\alpha e^{-x}, \quad \alpha > -1,
\] (3)
satisfy the differential equation
\[
\left[ x \frac{d^2}{dx^2} + (\alpha - x + 1) \frac{d}{dx} + n \right] L_n^\alpha(x) = 0,
\] (4)
and the recurrence relations
\[
(n+1)L_{n+1}^\alpha(x) - (2n + \alpha + 1 - x)L_n^\alpha(x) + (n + \alpha)L_{n-1}^\alpha(x) = 0,
\]
\[
x \frac{d}{dx} L_n^\alpha(x) = n L_n^\alpha(x) - (n + \alpha)L_{n-1}^\alpha(x).
\] (5)

In terms of \( L_n^\alpha(x) \) one can define the generalized Laguerre functions as [16]
\[
\psi_n^\alpha(x) = \sqrt{\frac{n!\alpha+1}{\Gamma(n+\alpha+1)}} L_n^\alpha(x), \quad \alpha > -1,
\] (6)
where \( \Gamma(n+\alpha+1) \) is the Gamma function
\[
\Gamma(n+\alpha+1) = \int_0^\infty e^{-t} t^{n+\alpha} dt,
\] (7)
with \( \Gamma(n) = (n-1)! \). The generalized Laguerre functions \( \psi_n^\alpha(x) \) can be shown to obey the orthonormality condition
\[
\int_0^\infty \psi_n^\alpha(x) \psi_{n'}^\alpha(x)x^{-1} dx = \delta_{nn'}.
\] (8)

Using the differential equation (4) and the recurrence relations (5) of associated Laguerre polynomials, it is easy to derive the differential equation
\[
\left[ x \frac{d^2}{dx^2} + \frac{1}{4}(2\alpha + 2 - x + \frac{1-\alpha^2}{x}) + n \right] \psi_n^\alpha(x) = 0,
\] (9)
and the recurrence relation
\[
\sqrt{(n+\alpha+1)(n+1)} \psi_{n+1}^\alpha(x) + \sqrt{(n+\alpha)n} \psi_{n-1}^\alpha(x) - (2n + \alpha + 1 - x) \psi_n^\alpha(x) = 0,
\] (10)
of the generalized Laguerre functions.
By exploring the above formulas, we can define the raising operator $A^+$ and the lowering operator $A^-$ for the generalized Laguerre functions:

$$
A^+ = -x \frac{d}{dx} - \frac{1}{2}(2n + \alpha + 1 - x), \\
A^- = x \frac{d}{dx} - \frac{1}{2}(2n + \alpha + 1 - x).
$$

They act on $\psi_n^\alpha(x)$ as follows

$$
A^+ \psi_n^\alpha(x) = -\sqrt{(n+1)(n+\alpha+1)} \psi_{n+1}^\alpha(x), \\
A^- \psi_n^\alpha(x) = -\sqrt{n(n+\alpha)} \psi_{n-1}^\alpha(x).
$$

(12)

(\psi_n^\alpha(x)) can be written in terms of $A^+$ and $\psi_0^\alpha(x)$:

$$
\psi_n^\alpha(x) = \frac{1}{\sqrt{n!(\alpha+1)_n}} (A^+)^n \psi_0^\alpha(x), \\
\psi_0^\alpha(x) = \frac{1}{\Gamma(\alpha+1)} x^{\frac{\alpha}{2}} e^{-\frac{x}{2}},
$$

(13)

where the shifted factorial is $(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)} = a(a+1)...(a+n-1)$. Observe that the following relation are satisfied

$$
A^+A^- \psi_n^\alpha(x) = n(n+\alpha) \psi_n^\alpha(x), \\
A^-A^+ \psi_n^\alpha(x) = (n+1)(n+\alpha+1) \psi_n^\alpha(x).
$$

(14)

One can define a Hamiltonian $H$ in terms of raising and lowering operators \[11\] in such a way that

$$
H = A^+A^-,
$$

(15)

where the generalized Laguerre functions \[6\] satisfy the eigenvalue equations

$$
H \psi_n^\alpha(x) = e_n \psi_n^\alpha(x),
$$

(16)

with

$$
e_n = n(n+\alpha), \quad n = 0, 1, 2...
$$

(17)

Note that the obtained spectrum \[17\] is similar to that found for a particle trapped in a Pöschl-Teller potential of the trigonometric type. We remind the reader that the coherent states for this system are constructed by using the Gazeau-Klauder method \[18\], as well as other methods \[19, 20\].

3 Barut-Girardello coherent states

According to Barut-Girardello definition, coherent states are defined to be the eigenvalues of lowering operator:

$$
A^- |z, \alpha > = z |z, \alpha >, \quad \text{where } z \in \mathbb{C}. \quad |z, \alpha > \text{ can be written in terms of the generalized Laguerre functions } \psi_n^\alpha(x) \text{ as}
$$

$$
|z, \alpha > = N(z)^{-1} \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{n!(\alpha+1)_n}} |\psi_n^\alpha >, \quad \text{(19)}
$$
where $N(z)$ is the normalization factor
\[ N(z) = \frac{\sqrt{\Gamma(\alpha + 1)I_\alpha(2|z|)}}{|z|^\frac{\alpha}{2}}. \] (20)

The Barut-Girardello coherent states (19) become
\[ |z,\alpha> = |z|^\frac{\alpha}{2} \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{n!\Gamma(n + \alpha + 1)}}|\psi_n^\alpha>. \] (21)

After some computations, we simplify the last equation to
\[ |z,\alpha> = \sqrt{x - \alpha \Gamma(\alpha + 1)I_\alpha(2|z|)}e^{z J_\alpha(2\sqrt{xz})}|\psi_0^\alpha>, \] (22)

where $J_\alpha(2\sqrt{xz})$ is the Bessel function
\[ J_\alpha(2\sqrt{xz}) = (xz)^\frac{\alpha}{2} \sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n!\Gamma(n + \alpha + 1)}}(xz)^n. \] (23)

We can see that the overlapping of two coherent states does not vanish
\[ <z_1,\alpha|z_2,\alpha> = \frac{I_\alpha(2\sqrt{z_1z_2})}{\sqrt{I_\alpha(2|z_1|)I_\alpha(2|z_2|)}}. \] (24)

By the choice of the measure:
\[ d\mu(z,\alpha) = \frac{2}{\pi}K_\alpha(2|z|)I_\alpha(2|z|)d^2z, \] (25)

one can show that the resolution of unity is satisfied
\[ \int |z,\alpha><z,\alpha|d\mu(z,\alpha) = 1. \] (26)

As a consequence, for any state $|\Psi> = \sum_{n=0}^{\infty} c_n|\psi_n^\alpha>$ in the Hilbert space, one can construct the analytic function
\[ f(z) = \frac{N(z)}{\sqrt{\Gamma(\alpha + 1)}} <z,\alpha|\Psi> = \sum_{n=0}^{\infty} \frac{c_n}{\sqrt{n!\Gamma(n + \alpha + 1)}}z^n. \] (27)

Therefore, the state $|\Psi>$ can be expressed in terms of the Barut-Girardello coherent states (22) in such a way that
\[ |\Psi> = \int d\mu(z,\alpha) \frac{|z|^\frac{\alpha}{2}}{\sqrt{I_\alpha(2|z|)}} f(z)|z,\alpha>, \] (28)

and we have
\[ <\Psi|\Psi> = \int d\mu(z,\alpha) \frac{|z|^\alpha}{I_\alpha(2|z|)}|f(z)|^2 < \infty. \] (29)

Barut-Girardello coherent states also have been considered by Brif [21] in the framework of the Lie algebra $su(1,1)$. He constructed coherent states as eigenstates of the lowering operator of $su(1,1)$. We should also mention reference [22], where the Barut-Girardello coherent states are constructed.
4 Gazeau-Klauder coherent states

Consider a Hamiltonian $H$ with discrete spectrum $(e_n)$ which is bounded below and has been adjusted so that $H \geq 0$. Moreover, assume that the eigenvalues are nondegenerate and arranged in increasing order. With these assumptions Gazeau and Klauder suggested a method where the coherent states are characterized by a real two-parameter set $\{|J,\gamma>, \quad J \geq 0, -\infty < \gamma < +\infty\}$, such that

$$|J,\gamma> = N(J)^{-1} \sum_{n=0}^{\infty} \frac{J_n}{\sqrt{\rho_n}} e^{-ie_n\gamma} |\psi_n^\alpha >. \quad (30)$$

The positive constants $\rho_n$ are defined by

$$\rho_n = e_1 e_2 ... e_n, \quad (31)$$

and the normalization factor $N(J)$

$$N(J)^2 = \sum_{n=0}^{\infty} \frac{J_n}{\rho_n}. \quad (32)$$

We would like to apply this approach to derive the corresponding coherent states for the generalized Laguerre functions (6). For this purpose, we start by noting that the energy spectrum (17) is arranged in the strictly increasing order:

$$0 = e_0 < e_1 < e_2 ... < e_n < .... \quad (33)$$

Therefore, by inserting (17) into (31), we find

$$\rho_n = n!(\alpha + 1)_n, \quad (34)$$

as well as

$$N(J)^2 = \frac{\Gamma(\alpha + 1)}{J_2} I_{\alpha}(2\sqrt{J}), \quad (35)$$

where $I_{\alpha}(2\sqrt{J})$ are the modified Bessel functions

$$I_{\alpha}(2\sqrt{J}) = \sum_{n=0}^{\infty} \frac{J_n^{\alpha+\frac{1}{2}}}{n!\Gamma(n + \alpha + 1)}. \quad (36)$$

In Gazeau-Klauder approach, one needs to specify the radius of convergence [10], which plays an important role in investigating the resolution of unity. It is defined by

$$R = \lim_{n \to \infty} \sqrt[2]{\rho_n}. \quad (37)$$

Then from (34), we observe that

$$R = \lim_{n \to \infty} \sup n!(\alpha + 1)_n = \infty. \quad (38)$$
In the present approach, the positive constants $\rho_n$ are assumed to arise as the moments of a probability distribution

$$\rho_n = n!(\alpha + 1)_n = \int_0^{R=\infty} J^n \rho(J)dJ, \quad (39)$$

which leads to the following expression for $\rho(J)$

$$\rho(J) = \frac{2}{\Gamma(\alpha + 1)} J^{\frac{\alpha}{2}} K_{\alpha}(2\sqrt{J}). \quad (40)$$

$K_{\alpha}(2\sqrt{J})$ are the $\alpha-$order modified Bessel functions of the second kind

$$K_{\alpha}(2\sqrt{J}) = \frac{\pi}{2 \sin(\pi \alpha)} (I_{-\alpha}(2\sqrt{J}) - I_{\alpha}(2\sqrt{J})). \quad (41)$$

Actually, (30) becomes

$$|J, \gamma >= J^{\frac{\alpha}{2}} I_{\alpha}(2\sqrt{J}) \psi_0 \sum_{n=0}^{\infty} \frac{J^{\frac{\alpha}{2}}}{\Gamma(n + \alpha + 1)} e^{-in(\alpha+\gamma)} |L_n^\alpha >. \quad (42)$$

To complete the construction of coherent states $|J, \gamma >$, we need to check some requirements. For this, we consider the relation

$$\int |J, \gamma > < J, \gamma | d\mu(J, \gamma) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} d\gamma \left[ \int_0^{\infty} k(J) |J, \gamma > < J, \gamma | dJ \right], \quad (43)$$

where $k(J)$ is defined by [10]

$$k(J) = \begin{cases} \frac{N(J)^2 \rho(J)}{\rho(J) \equiv 0,} & 0 \leq J < R, \\ \rho(J) \equiv 0, & J > R. \end{cases} \quad (44)$$

According to our data, $k(J)$ is nothing but

$$k(J) = 2I_{\alpha}(2\sqrt{J}) K_{\alpha}(2\sqrt{J}) \equiv k_{\alpha}(J). \quad (45)$$

Now it is easy to observe that the resolution of unity is satisfied

$$\int |J, \gamma > < J, \gamma | d\mu(J, \gamma) = 1, \quad (46)$$

where $\gamma \in [-\pi, \pi]$. The temporal stability is immediate

$$e^{-iHt} |J, \gamma > = |J, \gamma + t >. \quad (47)$$

The action of identity is given by

$$< J, \gamma | H | J, \gamma > = N(J)^{-2} \sum_{n=0}^{\infty} \frac{n(n + \alpha)}{n!(\alpha + 1)_n} J^n. \quad (48)$$
Using (35) and making some calculations, we find

\[ \langle J, \gamma | H | J, \gamma \rangle = J. \]  

(49)

The overlapping is

\[ \langle J', \gamma' | J, \gamma \rangle = \frac{2}{N(J)N(J')} \sum_{n=0}^{\infty} \frac{(JJ')^{\frac{n}{2}}}{n!(\alpha + 1)^n} e^{-i\alpha(\gamma - \gamma')} \].

(50)

It is clear from the last equation that two different coherent states are not orthogonal to each other. If we made a restriction such that \( \gamma = \gamma' \), then the overlapping becomes

\[ \langle J', \gamma' | J, \gamma \rangle = \frac{I_{\alpha}(2\sqrt{JJ'})}{\sqrt{I_{\alpha}(2\sqrt{J})I_{\alpha}(2\sqrt{J'})}}. \]

(51)

We conclude that the obtained coherent states (42) are similar to those found for particle moving in the Pöschl-Teller potential [18] by adopting Gazeau-Klauder approach. Given this similarity, one can translate directly the physical interpretation of Pöschl-Teller coherent states to generalized Laguerre ones. We remind our readers that the authors of [18] have studied the spatial and temporal features of the mentioned coherent states.

We are going to discuss the weighting distribution \( |c_n|^2 \) corresponding to our coherent states (42), which can be written in terms of the \( J \) parameter

\[ |c_n|^2 = \frac{J^n}{N(J)^2 \rho_n}. \]

(52)

Furthermore, there is a parameter called Mandel parameter \( Q \) which plays an important role, since it can determine the nature of the weighting distribution \( |c_n|^2 \) as we will show. It is defined by [23, 24]

\[ Q = \frac{(\Delta n)^2}{\langle n \rangle} - 1, \]

(53)

where \( \langle n \rangle \) are the mean values

\[ \langle n \rangle = \sum_{n=0}^{\infty} \frac{n J^2}{N(J)^2 \rho_n}, \]

(54)

and the spread is

\[ \Delta n = [\langle n^2 \rangle - \langle n \rangle^2]^\frac{1}{2}. \]

(55)

Note that, \( Q = 0 \) yields \( (\Delta n)^2 = \langle n \rangle \). Thus, for \( Q = 0 \) the weighting distribution becomes to be Poissonian

\[ |c_n|^2 = \frac{1}{n!} \langle n \rangle^n e^{-\langle n \rangle}. \]

(56)

Otherwise, it is super-Poissonian or sub-Poissonian for \( Q \) strictly positive or negative. To determine explicitly the nature of \( |c_n|^2 \) in our case, we need to evaluate (54) and (55). A direct calculation leads to

\[ \langle n \rangle = \frac{\sqrt{J} I_{\alpha+1}(2\sqrt{J})}{I_{\alpha}(2\sqrt{J})}, \]

(57)
and
\[ \Delta n = \left[ < n > - < n >^2 + J \frac{I_{a+2}(2\sqrt{J})}{I_a(2\sqrt{J})} \right]^\frac{1}{2}. \] (58)

Then (53) becomes
\[ Q = \sqrt{J} \left[ \frac{I_{a+2}(2\sqrt{J})I_a(2\sqrt{J}) - (I_{a+1}(2\sqrt{J}))^2}{I_{a+1}(2\sqrt{J})I_a(2\sqrt{J})} \right]. \] (59)

Since \( (I_{a+1}(2\sqrt{J}))^2 \geq I_{a+2}(2\sqrt{J})I_a(2\sqrt{J}) \), we realize immediately that we have a negative \( Q \), which implies that the weighting distribution (52) is sub-Poissonian. The Poissonian case can be recovered when \( J \) is large (for more details see [18]).

We close this section by noting that the obtained overlapping property for Gazeau-Klauder approach at \( \gamma = \gamma' \) (51) is similar to that derived from Barut-Girardello definition (24) although, the coherent states (22) and (42) are not the same. However, by choosing \( \gamma = 0 \) and \( J \) as a complex parameter, we can reproduce the Barut-Girardello coherent states (22) from the Gazeau-Klauder ones (42).

5 Klauder-Perelomov coherent states

Following the Klauder-Perelomov definition, coherent states of a given system can be constructed in terms of its dynamical symmetry group. Thus, we need first to determine the appropriate dynamical symmetry group of the system described by the generalized Laguerre functions (6). Starting from (14), one can show that the following commutation relation is satisfied
\[ [A^-, A^+] \psi_n^\alpha(x) = (2n + \alpha + 1) \psi_n^\alpha(x). \] (60)

Let us introduce the operator \( A_3 \) defined to satisfy
\[ A_3 \psi_n^\alpha(x) = \frac{1}{2} (2n + \alpha + 1) \psi_n^\alpha(x). \] (61)

Now the operators \( A^-, A^+ \) and \( A_3 \) generate the \( su(1,1) \) Lie algebra
\[ [A^-, A^+] = 2A_3, \quad [A_3, A^+] = A^+, \quad [A_3, A^-] = -A^- \] (62)

The corresponding Casimir operator is
\[ C = A_3^2 - \frac{1}{2} (A^+ A^- + A^- A^+). \] (63)

The value of \( C \) in the Hilbert space of the generalized Laguerre functions (6) is
\[ \frac{1}{4} (\alpha + 1)(\alpha - 1). \] (64)

This means that the unitary irreducible representations of \( su(1,1) \) are determined by the \( \alpha \)-parameter.
The Klauder-Perelomov definition of coherent states consists of applying the operator $e^{\xi A^+}$ on the ground state $\psi_0^\alpha$, such that

$$|\xi, \alpha> = e^{\xi A^+ - \xi A^-} |\psi_0^\alpha>,$$  \hspace{1cm} (65)

which leads to

$$|z, \alpha> = (1 - |z|^2)^{\alpha+1} e^{z A^+} |\psi_0^\alpha>,$$  \hspace{1cm} (66)

where $z = \frac{\xi}{|\xi|} \tanh |\xi|$ is a complex number satisfying the condition $|z| < 1$. The last equation can be reorganized as follows

$$|z, \alpha> = (1 - |z|^2)^{\alpha+1} \sum_{n=0}^{\infty} \sqrt{\frac{(\alpha + 1)n}{n!}} z^n |\psi_n^\alpha>.$$  \hspace{1cm} (67)

Finally, we find that

$$|z, \alpha> = (\frac{1 - |z|^2}{1 - z})^{\alpha+1} e^{z A^+} |\psi_0^\alpha>.$$  \hspace{1cm} (68)

The overlapping property is

$$<z_1, \alpha|z_2, \alpha> = [(1 - |z_1|^2)(1 - |z_2|^2)]^{\alpha+1} (1 - z_1 \bar{z}_2)^{-\alpha-1},$$  \hspace{1cm} (69)

which shows that the $su(1, 1)$ coherent states are normalized, but are not orthogonal to each other. The resolution of unity can be obtained with an appropriate choice of the measure. By choosing it as

$$d\mu(z, \alpha) = \frac{\alpha}{\pi} \frac{d^2 z}{(1 - |z|^2)^2},$$  \hspace{1cm} (70)

we get

$$\int |z, \alpha> <z, \alpha| d\mu(z, \alpha) = 1.$$  \hspace{1cm} (71)

As noted for Barut-Girardello coherent states, for any state $|\Psi> = \sum_{n=0}^{\infty} c_n |\psi_n^\alpha>$ in the Hilbert space, one can construct an analytic function

$$f(z) = (1 - |z|^2)^{-\frac{\alpha+1}{2}} <z, \alpha|\Psi> = \sum_{n=0}^{\infty} c_n \sqrt{\frac{(\alpha + 1)n}{n!}} (\bar{z})^n.$$  \hspace{1cm} (72)

In terms of the $su(1, 1)$ coherent states \((68)\) $|\Psi>$ can be written as

$$|\Psi> = \int d\mu(z, \alpha)(1 - |z|^2)^{\frac{\alpha+1}{2}} f(z)|z, \alpha>,$$  \hspace{1cm} (73)

leading to

$$<\Psi|\Psi> = \int d\mu(z, \alpha)(1 - |z|^2)^{\alpha+1} |f(z)|^2 < \infty.$$  \hspace{1cm} (74)

Note that the coherent states derived in the third approach \((68)\) are completely different from the previous ones, namely the Barut-Girardello \((22)\) and the Gazeau-Klauder \((42)\) coherent states. Moreover, their overlapping property is also different from the others \((24)-(30)\).

We close this section by noting that Klauder-Perelomov coherent states have been studied by Trivinov in the context of more physical problem of the "singular oscillator" described in terms of the Hamiltonian $p^2 + w^2(t)x^2 + g/x^2$ \([25]\).
6 Conclusions

Generalized Laguerre functions are considered as a basis of a Hilbert space. Raising, lowering and Hamiltonian operators are constructed. The corresponding coherent states are investigated by using three different methods. The resolution of unity and the overlapping properties have been considered in each case. We found that although one can recover the Barut-Girardello coherent states \( \gamma = 0 \) and \( J \) is a complex parameter, they differ from the coherent states obtained in terms of the Klauder-Perelomov approach.

Obviously, other special functions can be studied by the same method presented here. Moreover, one could examine the states minimizing the Robertson-Schrödinger uncertainty relation (intelligent states) in terms of the generalized Laguerre functions.

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