Non-leptonic two-body weak decays of $\Lambda_c(2286)$

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(Dated: July 29, 2018)

Abstract

We study the non-leptonic two-body weak decays of $\Lambda_c^+(2286) \rightarrow B_n M$ with $B_n$ ($M$) representing as the baryon (meson) states. Based on the $SU(3)$ flavor symmetry, we can describe most of the data reexamined by the BESIII Collaboration with higher precisions. However, our result of $\mathcal{B}(\Lambda_c^+ \rightarrow p\pi^0) = (5.6 \pm 1.5) \times 10^{-4}$ is larger than the current experimental limit of $3 \times 10^{-4}$ (90% C.L.) by BESIII. In addition, we find that $\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^+ K^0) = (8.0 \pm 1.6) \times 10^{-4}$, $\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^+ \eta') = (1.0^{+1.6}_{-0.8}) \times 10^{-2}$, and $\mathcal{B}(\Lambda_c^+ \rightarrow p\eta') = (12.2^{+14.3}_{-8.7}) \times 10^{-4}$, which are accessible to the BESIII experiments.
I. INTRODUCTION

Recently, the BESIII Collaboration has reanalyzed the two-body weak decays of $\Lambda^+_c(2286)$ with the final states to be the combinations of baryon ($B_n$) and pseudoscalar meson ($M$) particles, where $\Lambda^+_c \equiv \Lambda^+_c(2286)$ along with $\Xi^+_c(2470)$ belongs to the lowest-lying antitriplet charmed baryon ($B_c$) state. In particular, the decay branching ratios of $\Lambda^+_c \to p\bar{K}^0, \Lambda\pi^+, \Sigma^+\pi^0$ and $\Sigma^0\pi^+$ have been measured at the level of $10^{-2}$ with high precisions [1]. In addition, the Cabibbo-suppressed $\Lambda^+_c \to p\eta$ decay has been observed for the first time [2]. According to the measurements of the two-body $\Lambda^+_c \to B_nM$ decays since 2016 [1], there have been 4 measured branching fractions listed in PDG [3], given as

\[
B(\Lambda^+_c \to p\bar{K}^0) = (3.16 \pm 0.16)\%,
\]
\[
B(\Lambda^+_c \to \Lambda\pi^+) = (1.30 \pm 0.07)\%,
\]
\[
B(\Lambda^+_c \to \Sigma^+\pi^0) = (1.24 \pm 0.10)\%,
\]
\[
B(\Lambda^+_c \to \Sigma^0\pi^+) = (1.29 \pm 0.07)\%,
\]

(1)

together with the new data [2], given by

\[
B(\Lambda^+_c \to p\eta) = (1.24 \pm 0.28 \pm 0.10) \times 10^{-3},
\]
\[
B(\Lambda^+_c \to p\pi^0) < 3 \times 10^{-4} \text{ (90\% C.L.)}.
\]

(2)

Note that the limit of $B(\Lambda^+_c \to p\pi^0)$ in Eq. (2) comes from the original data of $B(\Lambda^+_c \to p\pi^0) = (7.95 \pm 13.61) \times 10^{-5}$ [4] by BESIII, while the $\Lambda^+_c \to \Sigma^+K^0, p\eta'$ and $\Sigma^+\eta'$ decays, along with the neutron modes, have not been seen yet. It is interesting to see if these current data can be understood.

Theoretically, the factorization approach is demonstrated to well explain the $B$ and $b$-baryon decays [5–7], such that it is also applied to the two-body $\Lambda^+_c \to B_nM$ decays [8], of which the amplitudes are derived as the combination of the two computable matrix elements for the $\Lambda^+_c \to B_n$ transition and the meson ($M$) production. However, the factorization approach does not work for most of the two-body $\Lambda^+_c \to B_nM$ ones. For example, the decays of $\Lambda^+_c \to \Sigma^+\pi^0$ and $\Xi^0K^+$ are forbidden in the factorization approach [9], but their branching ratios turn out to be measured. As a result, several theoretical attempts to improve the factorization by taking into account the nonfactorizable effects have been made [10–14]. In contrast with the QCD-based models, the $SU(3)$ symmetry approach is independent of the
detailed dynamics, which has been widely used in the $B$ meson \[15-17\], $b$-baryon \[18, 19\] and $\Lambda_c^+$ ($\Xi_c$) \[9, 20–23\] decays. With this advantage, the two-body $\Lambda_c^+ \to B_n M$ decays can be related by the $SU(3)$ parameters, which receive possible non-perturbative and non-factorizable contributions \[9, 20–24\], despite of the unknown sources. The minimum $\chi^2$ fit with the p-value estimation \[3\] can statistically test if the $SU(3)$ flavor symmetry agrees with the data. Being determined from the fitting also, the $SU(3)$ parameters are taken to predict the not-yet-measured modes for the future experimental tests. However, the global fit was once unachievable without the sufficient data and the use of the symmetry for $\Lambda_c^+ \to B_n M$. Clearly, the reexamination with the global fit to match the currently more accurate data is needed. Note that, to study the $\Lambda_c^+ \to B_n \eta^{(')}$ decays, the singlet state of $\eta_1$ should be included \[16, 17\]. In this report, we will extract the $SU(3)$ parameters in the global fit, and predict the branching fractions to be compared with the future BESIII experimental measurements.

II. FORMALISM

From Fig. II there are four types of diagrams for the non-leptonic charm quark decays, where Figs. IIa–IIc with the $W$-boson emissions directly connected to quark pairs are the so-called tree-level processes, while Fig. IIi with the $W$-boson in the loop corresponds to the penguin-level ones. In Fig. IIe, the $c \to du\bar{s}$ transition that proceeds through $|V_{cd}V_{us}| \simeq \sin^2 \theta_c$ is the doubly Cabibbo-suppressed one, with $\theta_c$ being the well-known Cabibbo angle. Meanwhile, the $c \to uqq'\bar{s}\bar{s}$ transitions in Fig. IIi have the higher-order contributions from the quark loops, with the effective Wilson coefficients \[25\] calculated to be smaller than the tree-level ones by one order of magnitude. As a result, the decay processes in Figs. II: and
are both excluded in the present study. Accordingly, the effective Hamiltonian for the 
$c \to su\bar{d}$ and $c \to uq\bar{q}$ transitions with $q = (d, s)$ in Figs. 1 and 2, respectively, is given by

\[ \mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ V_{cs} V_{ud} [c_1 O_1 + c_2 O_2] + \sum_{q=d,s} V_{cq} V_{uq} [c_1 O_1^q + c_2 O_2^q] \right\}, \]

with the current-current operators $O_{1,2}^{(q)}$, written as

\[ O_1 = (\bar{u}d)_{V-A}(\bar{s}c)_{V-A}, \quad O_2 = (\bar{s}d)_{V-A}(\bar{u}c)_{V-A}, \]
\[ O_1^q = (\bar{u}q)_{V-A}(\bar{q}c)_{V-A}, \quad O_2^q = (\bar{q}q)_{V-A}(\bar{u}c)_{V-A}, \]

where $G_F$ is the Fermi constant, $V_{ij}$ are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, and $(\bar{q}_1 q_2)_{V-A}$ stands for $\bar{q}_1 \gamma_\mu (1-\gamma_5) q_2$. The operators $O_{1,2}$ and $O_{1,2}^q$ in Eq. (3) lead to the so-called Cabibbo-allowed and Cabibbo-suppressed decay modes due to the factor of $|\langle V_{cq} V_{uq} \rangle| = \sin \theta_c$. The Wilson coefficients $c_{1,2}$ in Eq. (3) are scale-dependent. In the NDR scheme \cite{25, 27}, one has that $(c_1, c_2) = (1.27, -0.51)$ at the scale $\mu = 1 \text{ GeV}$. Note that one is able to recombine $V_{cs} V_{ud} [c_1 O_1 + c_2 O_2]$ and $\sum_{q=d,s} V_{cq} V_{uq} [c_1 O_1^q + c_2 O_2^q]$ in Eq. (3) into $V_{cs} V_{ud} [c_+ O_+ + c_- O_-]$ and $V_{cd} V_{ud} [c_+ \hat{O}_+ + c_- \hat{O}_-]$ with $\hat{O}_\pm \equiv O_\pm^q - O_\pm^{\ast q}$, respectively, where $c_\pm = c_1 \pm c_2$, $O_{\pm}^{(q)} = (O_1^{(q)} \pm O_2^{(q)})/2$ and $V_{cs} V_{us} = -V_{cd} V_{ud}$.

For the four-quark operator $(\bar{q}^i q_k)(\bar{q}^j c)$ from the effective Hamiltonian in Eq. (4), $(\bar{q}^i q_k)(\bar{q}^j c)$ that belongs to the $SU(3)$ triplet of $q_i = (u, d, s)$ can be decomposed as the irreducible forms of $3 \times 3 \times 3 = 3 + 3' + 6 + \bar{T}_5$, which are in terms of the $SU(3)$ flavor symmetry with the Lorentz-Dirac structures being disregarded. Consequently, the Cabibbo-allowed operators $O_-$ and $O_+$ fall into 6 and $\bar{T}_5$, respectively, instead of $3 + 3'$ that actually appear in the penguin operators. Therefore, in the $SU(3)$ picture the Cabibbo-allowed operators $O_-$ and $O_+$ are presented as \cite{20, 21}

\[ O_6 = \frac{1}{2} [ (\bar{u}d)(\bar{s}c) - (\bar{s}d)(\bar{u}c) ], \]
\[ O_{\bar{T}_5} = \frac{1}{2} [ (\bar{u}d)(\bar{s}c) + (\bar{s}d)(\bar{u}c) ], \]

which are formed as the tensor notations of $H(6)^{ij}$ and $H(\bar{T}_5)^{ijk}$, respectively. Note that the Cabibbo-suppressed operators $\hat{O}_-$ and $\hat{O}_+$ have similar irreducible forms, leading to their own $\hat{H}(6)^{ij}$ and $\hat{H}(\bar{T}_5)^{ijk}$ \cite{20, 21}. As a result, the effective Hamiltonian in Eq. (3) under the $SU(3)$ representation becomes

\[ \mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ V_{cs} V_{ud} [c_- H(6) + c_+ H(\bar{T}_5)] + V_{cd} V_{ud} [c_- \hat{H}(6) + c_+ \hat{H}(\bar{T}_5)] \right\}, \]
where the non-zero entries are

\[

t_{22}^2(6) = 2, \quad t_{13}^2(1\bar{5}) = t_{31}^2(1\bar{5}) = 1,
\]

\[
\hat{t}_{22}^3(6) = \hat{t}_{32}^3(6) = -2,
\]

\[
\hat{t}_{12}^2(1\bar{5}) = \hat{t}_{21}^2(1\bar{5}) = -\hat{t}_{13}^3(1\bar{5}) = -\hat{t}_{31}^3(1\bar{5}) = 1.
\]

(7)

To proceed, we take the amplitudes of $B_c \rightarrow B_n M$ under the $SU(3)$ representations. First, the $B_c$ state acts as $\bar{3}$ under the $SU(3)$ flavor symmetry, written as

\[
(B_c)^i = (\Xi_c^0, -\Xi_c^+, \Lambda_c^+),
\]

(8)

by which one defines $T_{ij} = \epsilon_{ijk} (B_c)^k$. Second, $B_n$ is the baryon octet, given by

\[
(B_n)^j_i = \begin{pmatrix}
\frac{1}{\sqrt{6}} \Lambda + \frac{1}{\sqrt{2}} \Sigma^0 & \Sigma^+ & p \\
\Sigma^- & \frac{1}{\sqrt{6}} \Lambda - \frac{1}{\sqrt{2}} \Sigma^0 & n \\
\Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}} \Lambda
\end{pmatrix}
\]

(9)

To include the octet ($\pi, K, \eta_8$) and singlet $\eta_1$, $M$ is presented as the nonet, given by

\[
(M)^i_j = \begin{pmatrix}
\frac{1}{\sqrt{2}} (\pi^0 + \cos \phi \eta - \sin \phi \eta') & \pi^- & K^- \\
\pi^+ & \frac{1}{\sqrt{2}} (\pi^0 - \cos \phi \eta - \sin \phi \eta') & \bar{K}^0 \\
K^+ & K^0 & -\sin \phi \eta + \cos \phi \eta'
\end{pmatrix}
\]

(10)

where $(\eta, \eta')$ are the mixtures of $(\eta_1, \eta_8)$, decomposed as $\eta_1 = \sqrt{2/3} \eta_q + \sqrt{1/3} \eta_s$ and $\eta_8 = \sqrt{1/3} \eta_q - \sqrt{2/3} \eta_s$ with $\eta_q = \sqrt{1/2} (u \bar{u} + d \bar{d})$ and $\eta_s = s \bar{s}$. Explicitly, the $\eta - \eta'$ mixing matrix is given by

\[
\begin{pmatrix}
\eta \\
\eta'
\end{pmatrix} = \begin{pmatrix}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi
\end{pmatrix} \begin{pmatrix}
\eta_q \\
\eta_s
\end{pmatrix},
\]

(11)

with the mixing angle $\phi = (39.3 \pm 1.0)^\circ$. Subsequently, the amplitude of $B_c \rightarrow B_n M$ is derived as

\[
A(B_c \rightarrow B_n M) = \langle B_n M | H_{\text{eff}} | B_c \rangle = \frac{G_F}{\sqrt{2}} [V_{cs} V_{ud} T(B_c \rightarrow B_n M) + V_{cd} V_{ud} \hat{T}(B_c \rightarrow B_n M)],
\]

(12)

where $T(B_c \rightarrow B_n M) = T(\mathcal{O}_{1\bar{5}}) + T(\mathcal{O}_6)$ are given by

\[
T(\mathcal{O}_{1\bar{5}}) = a H^i_{jk}(1\bar{5})(B_c)^j(M)^i + b H^i_{jk}(1\bar{5})(B_c)^j(M)^i,
\]

\[
T(\mathcal{O}_6) = \cdots
\]
\[ + c H_{jk}^i (15) (B_n)_i^j (M)_j^k (B_c)_k^i + d H_{jk}^i (15) (M)_j^k (B_n)_i^j (B_c)_k^i \\
+ h_1 H_{jk}^i (15) (B_n)_i^j (M)_j^k (B_c)_k^j, \]

\[ T(O_6) = e H_{ij}^i (6) T_{ik}^i (B_n)_i^k (M)_j^j + f H_{ij}^i (6) T_{ik}^i (M)_j^k (B_n)_i^j \\
+ g H_{ij}^i (6) (B_n)_i^k (M)_j^j T_{kl}^k + h_2 H_{ij}^i (6) T_{ik}^i (B_n)_i^j (M)_i^k, \]

(13)

with \((c_+, c_-)\) absorbed in the \(SU(3)\) parameters of \((a, b, c, d, h_1)\) and \((e, f, g, h_2)\), respectively, while \(\hat{T}(B_c \to B_n M)\) is given by replacing \(H(6, 15)\) in \(T(B_c \to B_n M)\) with \(\hat{H}(6, 15)\), respectively. Since the amplitudes are derived from the effective Hamiltonian in Eq. (3), where the \(c \to su \bar{d}\) and \(c \to uq \bar{q}\) transitions are the tree-level processes, \(T(B_c \to B_n M)\) and \(\hat{T}(B_c \to B_n M)\) are named as the tree-level \((T)\) amplitudes. In Eq. (13), the expansions of \(T(B_c \to B_n M)\) and \(\hat{T}(B_c \to B_n M)\) are shown in Table I. Note that, although we

| Decay modes | \(T(O_{15})\) | \(T(O_6)\) |
|-------------|----------------|----------------|
| \(T(\Lambda_c^+ \to p\bar{K}^0)\) | \(a + c\) | \(-2e\) |
| \(T(\Lambda_c^+ \to \Lambda\pi^+)\) | \(\sqrt{\frac{2}{3}} (a + b - 2c)\) | \(-\sqrt{\frac{8}{3}} (e + f + g)\) |
| \(T(\Lambda_c^+ \to \Sigma^+\pi^0)\) | \(-\sqrt{\frac{2}{3}} (a - b)\) | \(\sqrt{2} (e - f - g)\) |
| \(T(\Lambda_c^+ \to \Sigma^0\pi^+)\) | \(\sqrt{\frac{2}{3}} (a - b)\) | \(-\sqrt{2} (e - f - g)\) |
| \(T(\Lambda_c^+ \to \Xi^0 K^+)\) | \(b + d\) | \(-2f\) |
| \(\hat{T}(\Lambda_c^+ \to p\pi^0)\) | \(-\sqrt{\frac{2}{3}} (b + c)\) | \(\sqrt{2} (f + g)\) |
| \(\hat{T}(\Lambda_c^+ \to \Lambda K^+)\) | \(\sqrt{\frac{3}{4}} (-a + 2b + 2c + 3d)\) | \(\sqrt{3} (e - 2f + g)\) |
| \(\hat{T}(\Lambda_c^+ \to \Sigma^0 K^+)\) | \(-\sqrt{\frac{2}{3}} (a + d)\) | \(\sqrt{2} (e - 2f + g)\) |
| \(\hat{T}(\Lambda_c^+ \to \Sigma^+ K^0)\) | \((a + d)\) | \(2 (e - 2f + g)\) |
| \(T(\Lambda_c^+ \to \Sigma^+ \eta)\) | \(-d \sin \phi + \sqrt{\frac{2}{3}} (a + b) \cos \phi + h_1 (\sqrt{2} \cos \phi - \sin \phi)\) | \(-\sqrt{2} [e + (f - g)] \cos \phi\) |
| \(T(\Lambda_c^+ \to \Sigma^+ \eta')\) | \(d \cos \phi + \sqrt{\frac{2}{3}} (a + b) \sin \phi + h_1 (\cos \phi + \sqrt{2} \sin \phi)\) | \(-\sqrt{2} [e + (f - g)] \sin \phi\) |
| \(\hat{T}(\Lambda_c^+ \to p\eta)\) | \(-(a + c + d) \sin \phi + \sqrt{\frac{2}{3}} (b - c) \cos \phi + h_1 (\sqrt{2} \cos \phi - \sin \phi)\) | \(-\sqrt{2} [\sqrt{2} e \sin \phi - (f - g) \cos \phi]\ |
| \(\hat{T}(\Lambda_c^+ \to p\eta')\) | \((a + c + d) \sin \phi + \sqrt{\frac{2}{3}} (b - c) \cos \phi + h_1 (\cos \phi + \sqrt{2} \sin \phi)\) | \(\sqrt{2} [\sqrt{2} e \cos \phi + (f - g) \sin \phi]\) |

TABLE I. The tree-level amplitudes for the \(\Lambda_c^+ \to B_n M\) decays.
follow the approach in Ref. [21], the $h_{1,2}$ terms are newly added for the singlet $\eta_1$. Due to $c_-/c_+ \simeq 2.4$, the contribution of $O_6(\hat{O}_6)$ to the decay branching ratio can be 5.5 times larger than that of $O_{15}(\hat{O}_{15})$, such that $O_{15}(\hat{O}_{15})$ is negligible. However, we will examine if the reduction is reasonable, in case the interferences between $O_6(\hat{O}_6)$ and $O_{15}(\hat{O}_{15})$ can be sizable. Subsequently, we only keep the SU(3) parameters $e$, $f$, $g$ and $h_2$ from $O_6$ to simplify the amplitudes. Since $e$, $f$, $g$ and $h_2$ are complex numbers, we have 7 real independent parameters to be determined by the data, given by

$$e, fe^{i\delta_f}, ge^{i\delta_g}, h_2 e^{i\delta_{h_2}},$$

(14)

where $e$ is set to be real, while an overall phase can be removed without losing generality.

To calculate the decay widths, we use [3]:

$$\Gamma(\Lambda_c \to B_n M) = \frac{|\vec{p}_{cm}|}{8\pi m_{\Lambda_c}^2} |A(\Lambda_c \to B_n M)|^2,$$

(15)

where $|\vec{p}_{cm}| = \sqrt{[(m_{\Lambda_c}^2 - (m_{B_n} + m_M)^2)] [(m_{\Lambda_c}^2 - (m_{B_n} - m_M)^2)]/(2m_{\Lambda_c})}$, with the integrated-over variables of the phase spaces in the two-body decays.

III. NUMERICAL RESULTS AND DISCUSSIONS

For the numerical analysis, we use the minimum $\chi^2$ fit to find the SU(3) parameters in Eq. (14). The theoretical inputs for the CKM matrix elements are given by [3]

$$(V_{cs}, V_{ud}, V_{us}, V_{cd}) = (1 - \lambda^2/2, 1 - \lambda^2/2, \lambda, -\lambda),$$

(16)

with $\lambda = 0.225$ in the Wolfenstein parameterization. There are 9 branching ratios of $\Lambda_c \to B_n M$, which are the data inputs, given in the last column of Table I. The equation of the $\chi^2$ fit is given by

$$\chi^2 = \sum_{i=1}^{9} \left( \frac{B_{th}^i - B_{ex}^i}{\sigma_{ex}^i} \right)^2,$$

(17)

where $B_{th}^i$ and $B_{ex}^i$ stand for the branching ratios from the theoretical SU(3) amplitudes in Table I and experimental data inputs in Table I, with $\sigma_{ex}^i$ as the $1\sigma$ experimental errors, while $i = 1, 2, ..., 9$ denote the 9 observed decay modes involved in the global fit, respectively. Consequently, we obtain

$$(e, f, g, h_2) = (0.257 \pm 0.006, 0.121 \pm 0.015, 0.092 \pm 0.021, 0.111 \pm 0.081) \text{GeV}^3,$$

$$(\delta_f, \delta_g, \delta_{h_2}) = (79.0 \pm 6.8, 35.2 \pm 8.8, 102.4 \pm 29.8)^\circ,$$

$$\chi^2/d.o.f = 2.4,$$

(18)
where \( d.o.f \) stands for the degrees of freedom. The statistical p-value to be smaller than 0.05 will show the inconsistency between the theory and data \cite{3}, which is equivalent to \( \chi^2/d.o.f > 3 \) here. In our case, the value of \( \chi^2/d.o.f = 2.4 \) indicates a tolerable result to accommodate the current data of \( B(\Lambda_c \to B_n M) \) under the \( SU(3) \) flavor symmetry, where the contributions of \( O_{15}(\hat{O}_{15}) \) and the broken effects of \( SU(3) \) are both neglected. Explicitly, it is found that the largest contributions to \( \chi^2 \) are from \( B(\Lambda_c^+ \to \Lambda K^+, \Sigma^0 K^+) \), whereas the individual \( \chi^2 \) values from the other seven data show no apparent violation of \( SU(3) \) or the sextet dominating assumption. The value of \( \chi^2/d.o.f. \) being as large as 2.4 could suggest that the decays of \( \Lambda_c^+ \to (\Lambda K^+, \Sigma^0 K^+) \) should be reexamined by BESIII with more precisions. Due to the lack of sufficient data, it leaves the room for more precise examinations by the future experimental measurements on the \( SU(3) \) flavor symmetry with or without \( O_{15}(\hat{O}_{15}) \). With the parameters in Eq. (18) we can reproduce the branching ratios of the measured two-body \( \Lambda_c \) decays as shown in Table II where the results based on the heavy quark effective theory (HQET) \cite{24}, Sharma and Verma (SV) in Ref. \cite{23}, pole model (PM) \cite{11} and current algebra (CA) \cite{11} are also listed. In our fit, the \( SU(3) \) amplitudes in Eq. (18) have considerable imaginary parts, being included in \( \delta_{f,g,h_2} \). Nonetheless, the studies in Refs. \cite{12, 24} depend on real ones. For a test, we turn off \( \delta_{f,g,h_2} \), which causes an unsatisfactory fit to the data with \( \chi^2/d.o.f \approx 14 \gg 2.4 \) in Eq. (18), suggesting that the imaginary parts are necessary to fit the nine data well. It is similar that, in the \( D \to MM \) decays, the imaginary parts with the \( SU(3) \) flavor symmetry are also considerable, which correspond to the strong phases calculated from the on-shell quark loops in the next-leading-order QCD models \cite{29}. We hence conclude that the \( \Lambda_c^+ \to B_n M \) decays are like the \( D \to MM \) ones, where the phases are in accordance with the higher order contributions in the QCD models, which have not been well developed yet.

From Table II by keeping both \( O_6 \) and \( O_{15} \), we obtain that

\[
\begin{align*}
A(\Lambda_c^+ \to \Sigma^0 \pi^+) &= - A(\Lambda_c^+ \to \Sigma^+ \pi^0), \\
A(\Lambda_c^+ \to \Sigma^+ K^0) &= \sqrt{2} A(\Lambda_c^+ \to \Sigma^0 K^+) ,
\end{align*}
\]

where \( \Lambda_c^+ \to \Sigma^0 \pi^+ \) being identical to \( \Lambda_c^+ \to \Sigma^+ \pi^0 \) represents the conservation of the isospin \( (SU(2)) \) symmetry. By neglecting \( O_{15} \), the relations in Eq. (19) can be extended to

\[
\begin{align*}
A(\Lambda_c^+ \to \Sigma^0 \pi^+) &= - A(\Lambda_c^+ \to \Sigma^+ \pi^0), \\
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\end{align*}
\]
\[
\sqrt{6}A(\Lambda_c^+ \to \Lambda\pi^+) + \sqrt{2}A(\Lambda_c^+ \to \Sigma^0\pi^+) = 2A(\Lambda_c^+ \to pK^0),
\]
\[
\sqrt{6}A(\Lambda_c^+ \to \Lambda\pi^+) - \sqrt{2}A(\Lambda_c^+ \to \Sigma^0\pi^+) = \frac{2\sqrt{2}}{\lambda}A(\Lambda_c^+ \to p\pi^),
\]
resulting in
\[
B(\Lambda_c^+ \to \Sigma^0\pi^+) = B(\Lambda_c^+ \to \Sigma^+\pi^0),
\]
\[
B(\Lambda_c^+ \to \Sigma^+K^0) = 2B(\Lambda_c^+ \to \Sigma^0K^+),
\]
\[
B(\Lambda_c^+ \to p\pi^0) \simeq \frac{\lambda^2}{2}[3B(\Lambda_c^+ \to \Lambda\pi^+) + B(\Lambda_c^+ \to \Sigma^0\pi^+) - B(\Lambda_c^+ \to pK^0)].
\]

With the inputs of the \(SU(3)\) parameters in Eq. (18), we show \(B(\Lambda_c^+ \to \Sigma^+K^0) = (8.0 \pm 1.6) \times 10^{-4}\) and \(B(\Lambda_c^+ \to \Sigma^0K^+) = (4.0 \pm 0.8) \times 10^{-4}\) to agree with the second relation in Eq. (21), which can be used to test the assumption of the dominant \(O_6\) contributions in comparison with the future measurements. We remark that the factorization approach predicts \(B(\Lambda_c^+ \to\)

| Branching ratios | Our results HQET [24] | SV [23] | PM [11] | CA [11] | Data [1–3] |
|------------------|-----------------------|--------|--------|--------|------------|
| \(10^2B(\Lambda_c^+ \to pK^0)\) | 3.3 ± 0.2 | 1.23 | 2.67 ± 0.74 | 1.20 | 3.46 | 3.16 ± 0.16 |
| \(10^2B(\Lambda_c^+ \to \Lambda\pi^+)\) | 1.3 ± 0.2 | 1.17 | — | 0.84 | 1.39 | 1.30 ± 0.07 |
| \(10^2B(\Lambda_c^+ \to \Sigma^+\pi^0)\) | 1.3 ± 0.2 | 0.69 | — | 0.68 | 1.67 | 1.24 ± 0.10 |
| \(10^2B(\Lambda_c^+ \to \Sigma^0\pi^+)\) | 1.3 ± 0.2 | 0.69 | 0.87 ± 0.20 | 0.68 | 1.67 | 1.29 ± 0.07 |
| \(10^2B(\Lambda_c^+ \to \Sigma^0K^+)\) | 0.5 ± 0.1 | 0.07 | — | — | — | 0.50 ± 0.12 |
| \(10^4B(\Lambda_c^+ \to p\pi^0)\) | 5.6 ± 1.5 | — | 2 | — | — | — |
| \(10^4B(\Lambda_c^+ \to \Lambda\pi^+)\) | 4.6 ± 0.9 | — | 14 | — | — | 6.1 ± 1.2 |
| \(10^4B(\Lambda_c^+ \to \Sigma^0\pi^+)\) | 4.0 ± 0.8 | — | 4 | — | — | 5.2 ± 0.8 |
| \(10^4B(\Lambda_c^+ \to \Sigma^0K^+)\) | 8.0 ± 1.6 | — | 9 | — | — | — |
| \(10^2B(\Lambda_c^+ \to \Sigma^+\eta)\) | 0.7 ± 0.4 | 0.25 | 0.50 ± 0.17 | — | — | 0.70 ± 0.23 |
| \(10^2B(\Lambda_c^+ \to \Sigma^+\eta')\) | 1.0\(^{+1.6}_{-0.8}\) | 0.08 | 0.20 ± 0.08 | — | — | — |
| \(10^4B(\Lambda_c^+ \to p\eta)\) | 12.4 ± 4.1 | — | 21 | — | — | 12.4 ± 3.0 |
| \(10^4B(\Lambda_c^+ \to p\eta')\) | 12.2\(^{+14.3}_{-8.7}\) | — | 4 | — | — | — |
\(\Sigma^0 \pi^+, \Sigma^0 K^+ \simeq 0\), which contradicts the relations from the \(SU(3)\) symmetry. This is due to the fact that, when the decay proceeds with the \(\Lambda^+_c \rightarrow \Sigma^0\) transition, together with the recoiled meson \(\pi^+\) or \(K^+\), the \(c \rightarrow s\) transition currents transform \(\Lambda^+_c \rightarrow \Lambda = (ud - du)s\), which is unable to correlate to \(\Sigma^0 = (ud + du)s\) \([30]\), leading to \(B = 0\).

In Eq. (21), the simple estimation based on the data inputs gives that \(B(\Lambda^+_c \rightarrow p\pi^0) = (5.1 \pm 0.7) \times 10^{-4}\), which agrees with our numerical fitting result of \(B(\Lambda^+_c \rightarrow p\pi^0) = (5.6 \pm 1.5) \times 10^{-4}\), but is larger than the experimental upper bound of \(3 \times 10^{-4}\) \((90\%\text{C.L.})\) in Eq. (2) by BESIII. To check if there is a discrepancy here, we have taken the original data of \(B(\Lambda^+_c \rightarrow p\pi^0) = (7.95 \pm 13.61) \times 10^{-5}\) \([4]\) by BESIII as the input. In this case, we get \(\chi^2/d.o.f = 4.7\), which is two times larger than the value in Eq. (18), showing that the fitting cannot accommodate the present data of \(B(\Lambda^+_c \rightarrow p\pi^0)\). Apart from the \(SU(3)\) flavor symmetry, we estimate that \(B(\Lambda^+_c \rightarrow p\pi^0) \simeq 5 \times 10^{-4}\) in the approach of the factorization, which is also larger than the experimental upper bound. It is clear that a dedicated search for this mode with a more precise measurement should be done. An improved sensitivity to measure \(\Lambda^+_c \rightarrow p\pi^0\) will clarify if the currently unmovable discrepancy exists or not.

It is also interesting to see that \(B(\Lambda^+_c \rightarrow \Sigma^+\eta')\) and \(B(\Lambda^+_c \rightarrow p\eta')\) fitted to be \((1.0^{+1.6}_{-0.8}) \times 10^{-2}\) and \((12.2^{+14.3}_{-8.7}) \times 10^{-4}\) are as large as their \(\eta\) counterparts, respectively, while \(B(\Lambda_b \rightarrow \Lambda\eta) \simeq B(\Lambda_b \rightarrow \Lambda\eta')\) \([31]\). We note that there is a similar term in Ref. \([32]\) as the \(h_{1,2}\) terms, which relates \(\Lambda^+_c \rightarrow \Sigma^+\eta'\) to \(\Xi_c^0 \rightarrow \Sigma^0\eta'\). In contrast, the theoretical approach in Ref. \([23]\) is based on the \(SU(3)\) flavor symmetry also, but without the \(h_{1,2}\) terms to include the singlet \(\eta_1\), such that it leads to \(B(\Lambda^+_c \rightarrow \Sigma^+(p)\eta') < B(\Lambda^+_c \rightarrow \Sigma^+(p)\eta)\) \([23]\). Finally, we remark that, with the \(SU(3)\) symmetry, we can extend our study to the two-body \(\Xi_b^{\pm,0}\) decays, which are also accessible to the current experiments. Since the two-body \(\Lambda^+_c \rightarrow B_n V\) with \(V\) the vector meson and three-body \(\Lambda^+_c\) decays are observed, which require the interpretations, the approach of the \(SU(3)\) symmetry can be useful.

**IV. CONCLUSIONS**

We have studied the two-body \(\Lambda^+_c \rightarrow B_n M\) decays, which have been recently reanalyzed or newly measured by BESIII. With the \(SU(3)\) flavor symmetry, we can describe the data except that for \(\Lambda^+_c \rightarrow p\pi^0\). We have found that \(B(\Lambda^+_c \rightarrow p\pi^0) = (5.6 \pm 1.5) \times 10^{-4}\), which is almost \(2\sigma\) above the experimental upper bound of \(3 \times 10^{-4}\). We hope that the
future experimental measurement of $\mathcal{B}(\Lambda_c^+ \to p\pi^0)$ can resolve this discrepancy. Unlike the previous results, we have predicted that $\mathcal{B}(\Lambda_c^+ \to \Sigma^+\eta') = (1.0^{+1.6}_{-0.8}) \times 10^{-2}$ and $\mathcal{B}(\Lambda_c^+ \to p\eta') = (12.2^{+14.3}_{-8.7}) \times 10^{-4}$ which are as large as their $\eta$ counterparts, due to the newly added $h_{1,2}$ terms with the singlet $\eta_1$ in the $SU(3)$ flavor symmetry. With the $SU(3)$ symmetry, one is able to study $\Lambda_c^+ \to B_nV$ and the three-body $B_c$ decays, which have been observed but barely interpreted. Moreover, the extensions to study the $\Xi_b^{+,0}$ decays are possible, which are also accessible to the current experiments.

ACKNOWLEDGMENTS

We would like to thank X.G. He for useful discussions. This work was supported in part by National Center for Theoretical Sciences, MoST (MoST-104-2112-M-007-003-MY3), and National Science Foundation of China (11675030).

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