Algebraic decay of non-adiabatic spin transfer torque in magnetic domain walls with Rashba spin orbit interaction

D. Wang\textsuperscript{1,\ast} and Yan Zhou\textsuperscript{2,†}

\textsuperscript{1}College of Engineering Physics, Shenzhen Technology University, Guangdong 518118, P. R. China
\textsuperscript{2}School of Science and Engineering, The Chinese University of Hong Kong, Shenzhen, Guangdong 518172, P. R. China

(Dated: October 25, 2019)

Abstract

Spin transfer torque in a two dimensional electron gas system without space inversion symmetry was theoretically investigated by solving the Pauli-Schrödinger equation for the itinerant electrons inside magnetic domain walls. Due to the presence of the Rashba spin orbit coupling induced by the broken inversion symmetry, the non-adiabaticity, which is defined to measure the relative importance of the non-adiabatic, field-like torque to the adiabatic, damping-like torque, exhibits an inverse power law decay as the domain wall width is increased. This algebraic decay is much slower than the exponential decay observed for systems without the Rashba spin orbit coupling, and may find applications in innovative design of spintronic devices utilising magnetic topological textures such as magnetic domain walls and skyrmions.
From ancient times, the conventional way to manipulate a ferromagnet’s magnetic state is through application of an external magnetic field. The situation changed drastically in the last two decades, following the innovative proposition by Berger\textsuperscript{1} and Slonczewski\textsuperscript{2} of using high density electric current to exert a torque, dubbed the spin transfer torque (STT), on local magnetization. After more than twenty years of intensive quest for unconventional methods for magnetization manipulation with high energy efficiency and operation speed, we now have many alternatives at our disposal, such as ultrashort laser pulses\textsuperscript{3}, electric field\textsuperscript{4} and even magneto-elastic waves\textsuperscript{5}. Even with so many competitors on the arena for the manipulation of magnetization, electric current based methods, including the original STT, and the later developments of Rashba spin orbit torque (RSOT)\textsuperscript{6–10} and spin Hall effect\textsuperscript{11} in systems with spin orbit interaction, attract more attention due to their easy implementation and compatibility with current semiconductor technology.

Investigations\textsuperscript{12–14} following Berger and Slonczewski’s seminal works showed that the STT should be a sum of two terms, one damping-like torque which is already given in their original papers, and the other an additional field-like torque,

\[ \tau = \alpha \hat{j} \cdot \nabla \hat{M} + \beta \hat{M} \times (\hat{j} \cdot \nabla \hat{M}), \]

for a continuous distribution of magnetization characterized by the normalized magnetization vector, \( \hat{M} = \mathbf{M}/M \), where \( \mathbf{M} \) is the magnetization vector and \( M \) the saturation magnetization. \( \hat{j} \) is a unit vector pointing to the direction of the electric current flowing in a ferromagnet. \( \alpha \) and \( \beta \) are decomposition coefficients. As shown by a subsequent model quantum mechanical investigation on STT inside a magnetic domain wall (DW)\textsuperscript{14}, the damping-like torque is given by the continuous rotation of the itinerant electron spin towards the local magnetization, while the field-like torque is attributable to the misalignment between the electron spin and the local magnetization inside the rigid DW. With this microscopic interpretation, the damping-like torque (first term in Eq. (1)) and the field-like torque (second term in Eq. (1)) are usually called the adiabatic torque and the non-adiabatic torque, respectively.

For applications and understanding of the origin of the STT, an important feature of the non-adiabaticity of the STT, \( \beta/\alpha \), is that it decays exponentially as the DW width is increased\textsuperscript{14}. This feature of the non-adiabatic STT renders the utilization of it to drive DW motion in materials with sizable DW width difficult. However, we would like to emphasize that this exponential decay is obtained without considering the spin orbit interaction. As we will show below, by including the spin orbit interaction, the exponential decay is reduced to an algebraic one, thus facilitating the utilization of the non-adiabatic STT in a wider
range of material systems with spin orbit interaction.

We consider a special form of spin orbit interaction in solids, the Rashba spin orbit interaction\textsuperscript{15} in a two dimensional (2D) electron system without spatial inversion symmetry. In a 2D electron gas, there could be electric field built up along the inversion symmetry breaking direction. In the rest frame of a moving electron, this static electric field is transformed into a magnetic field, and can influence the spin dynamics of moving electrons\textsuperscript{16}. Hence the Hamiltonian for itinerant electrons has the form\textsuperscript{7–9}

\begin{equation}
H = \frac{\mathbf{p}^2}{2m_e} + \mu_B \mathbf{\sigma} \cdot \mathbf{M} + \frac{\alpha_R}{\hbar} \mathbf{\sigma} \cdot (\mathbf{p} \times \hat{z}),
\end{equation}

where $m_e$ is the electron mass, $\mu_B$ the Bohr magneton, and $\hbar$ the reduced Planck’s constant. $\mathbf{p} = -i\hbar \nabla$ is the momentum operator. $\alpha_R$ is the Rashba constant, which characterizes the broken inversion symmetry\textsuperscript{15}. $\mathbf{\sigma} = \hat{x}\sigma_x + \hat{y}\sigma_y + \hat{z}\sigma_z$ is the vector Pauli matrix, which is also the electron spin operator if a multiplicative constant is ignored, with $\sigma_x$, $\sigma_y$ and $\sigma_z$ being the Pauli matrices. It is obvious from Eq. (2) that the effective Rashba field is perpendicular to the symmetry breaking direction, which is $\hat{z}$ in the current case. The Rashba spin orbit interaction term can be absorbed into the kinetic energy term, forming a covariant derivative operator\textsuperscript{17}. The Hamiltonian (2) describes the energy of conduction electrons in a solid, interacting through the $s$-$d$ exchange interaction with the localized electrons. For the purpose of illustrating the effect of spin orbit interaction on the STT, we only consider the spin dynamics of itinerant electrons dictated by the Hamiltonian (2), while the local magnetic moments are assumed to be static, as described by the magnetization texture $\mathbf{M}$. As our model Hamiltonian does not include the Coulomb interaction between

FIG. 1. Spatial derivative of the equilibrium itinerant magnetization distribution ($\mathbf{m}'$) inside the DW region, with the DW width $\lambda k_F = 1$ (a) and $\lambda k_F = 70$ (b). The spatial derivative of the underlying $s$-$d$ exchange field ($\mathbf{M}'$) is also displayed. The misalignment between $\mathbf{M}'$ and $\mathbf{m}'$ is mainly caused by the Rashba interaction.
FIG. 2. Numerically calculated STT, \( \tau \), for \( \lambda k_F = 1 \) (a) and \( \lambda k_F = 70 \) (b). The corresponding decomposition coefficients \( \alpha \) and \( \beta \) are displayed in the insets. The observable oscillation for \( \lambda k_F = 1 \) is due to the quantum confinement effect induced by the presence of the short DW. As the DW width is increased, the quantum oscillation is smoothed out, as shown in (b).

electrons explicitly, there is no physical exchange interaction between electron spins. We use the magnetization texture \( M \) to simulate the exchange interaction between conduction electrons.

The magnetization texture considered is a Néel DW described by the unit magnetization vector \( \hat{M} = \chi \hat{x} \text{sech}(x/\lambda) - q \hat{z} \tanh(x/\lambda) \), which is the renowned Walker profile\(^{18}\). \( \lambda = \sqrt{A/K} \) is the DW width that is determined by the material specific exchange and anisotropy constants \( A \) and \( K \). \( \hat{x} \) and \( \hat{z} \) are unit vectors pointing along the \( x \) and \( z \) directions, respectively. The charge \( q \) and the chirality \( \chi \rangle^{19} \) of the DW are topological numbers to quantify its topological characteristics. We assume then the current is flowing along the \( x \) direction, and the electrons are moving in the 2D \( xy \) plane. The eigenvalue problem corresponding to the Pauli-Schrödinger equation with the Hamiltonian (2) and the Walker magnetization profile as given above is difficult to solve analytically. We adopt a scattering matrix method to numerically solve the eigenvalue problem\(^{14,20,21}\). The physical picture behind such a scattering method is simple: We inject plane waves which are the solutions to a uniform magnetization distribution from both \( \pm \infty \), then let them evolve according to the Hamiltonian (2) and match the evolved waves at the DW center, by requiring the continuity of wave functions and their first order derivatives. With the wave functions thus obtained, we employ a semiclassical approach with relaxation time approximation\(^{22}\) to calculate the steady state STT. Further details and particulars can be found in Ref. 23.

For the convenience of numerical calculation and a direct comparison between different quantities, we convert the local exchange field \( M \) and the Rashba coupling constant \( \alpha_R \)
FIG. 3. Non-adiabaticity $\beta(0)/\alpha(0)$ as a function of the DW width $\lambda$, with individual coefficients $\alpha(0)$ and $\beta(0)$ shown in the inset. Obviously, the decay of the non-adiabaticity is not simply exponential. The solid line in the main panel is the result fitted according to Eq. 5, while those in the inset are merely guides to the eye.

into effective wave numbers through relations $\hbar^2 k_B^2/2m_e = \mu_B M$ and $\hbar^2 k_R^2/2m_e = \alpha_R$, and measure them in terms of the Fermi wave number $k_F$ for the electron gas with only the first term in the Hamiltonian [2]. In our following numerical results, we use the values $k_B/k_F = 0.4$ and $k_R/k_F = 0.1$ unless stated otherwise. The numerically obtained spatial derivative of the equilibrium itinerant magnetization distribution $m' = d\mathbf{m}/dx$ is given in Fig. 1, together with the background local magnetization’s spatial derivative, for two typical DW width values, $\lambda k_F = 1$ and $\lambda k_F = 70$. As the transition from the non-adiabatic to adiabatic behaviour is defined by the critical DW width $\lambda c = k_F^2/k_B^2 = 6.25$ for $k_B/k_F = 0.4$, the value we used to introduce the exchange interaction for itinerant magnetization, $\lambda k_F = 1$ corresponds to the extreme non-adiabatic situation and $\lambda k_F = 70$ the adiabatic one. In the extreme non-adiabatic limit, the spatial variation of the itinerant magnetization is distributed over the whole simulated region and the misalignment between $\mathbf{m}'$ and $\mathbf{M}'$ is not negligible. In the adiabatic limit, however, only the spatial variation is confined to the DW center region and the misalignment between $\mathbf{m}'$ and $\mathbf{M}'$ is still there. The oscillation of the itinerant magnetization observable for short DWs is attributable to the quantum confinement effect induced by the presence of the local magnetization profile $\mathbf{M}$. It actually decays away very quickly: For $\lambda k_F = 3$ (not shown here), the quantum oscillation is already not discernable. The misalignment between $\mathbf{m}'$ and $\mathbf{M}'$ can be traced back to the finite Rashba coupling present in our calculation. As was discussed in Ref. 23, the definite parity for the itinerant magnetization components arises due to the parity-time, or particle-hole, symmetry of the Hamiltonian [2]. It is interesting to note that the same symmetry was also observed for magnons inside DWs[23].
A qualitatively similar behaviour is observed for the STT, as shown in Fig. 2. In the extreme non-adiabatic limit, $\lambda k_F = 1$, both the adiabatic and non-adiabatic components of the STT contribute, manifested by the finite value of both $\alpha$ and $\beta$. As we consider only the dynamics of the itinerant magnetization $\mathbf{m}$, the actual decomposition of the STT is

$$\tau = \alpha \hat{m}' + \beta \hat{m} \times \hat{m}'\text{,}$$

using the unit vector along the direction of the derivative of the itinerant magnetization vector $\hat{m}' = \mathbf{m}'/m'$ instead of the derivative of the local magnetization vector. The magnitude of $\alpha$ and $\beta$ is comparable to each other, signifying the significant contribution of the non-adiabatic STT. As the DW width is increased, both $\alpha$ and $\beta$ decrease in magnitude, with the relative importance of the non-adiabatic component decreasing, too. As the non-adiabatic STT acts as an effective field-like torque in the Landau-Lifshitz-Gilbert equation\textsuperscript{25} describing the magnetization dynamics phenomenologically, the relative importance of the non-adiabatic STT warrants more investigation.

To measure the relative importance of the non-adiabatic STT, we follow Ref. \textsuperscript{14} to define the non-adiabaticity of the STT as the ratio between the non-adiabatic and adiabatic coefficients at the DW center, $\beta(0)/\alpha(0)$, where the spatial variation of the local magnetization is maximized. The non-adiabaticity thus defined is plotted in Fig. 3 along with the individual coefficients as a function of the DW width. Surprisingly, the behaviour of the non-adiabaticity in the presence of the Rashba spin orbit coupling shows a much slower decay in the DW width, as compared to the case without the Rashba coupling. Although the non-adiabaticity in Fig. 3 decays away as a whole, its decay is not exponential anymore, but resembling more like a power law decay, which is also in contrast to the previously obtained oscillatory behaviour through analytical treatment of a semiclassical kinetic equation\textsuperscript{26} or the unusual linear increase scaling extracted from density functional theory calculation\textsuperscript{27}.

The behaviour of the STT non-adiabaticity can be understood qualitatively by a perturbation analysis of the Pauli-Schrödinger equation. Using a unitary transformation, we can transform the Hamiltonian\textsuperscript{22} into a different form in the spinor space. The rotation angle is related to the magnetization angle $\theta$ and the electron’s transverse momentum $k_y$.\textsuperscript{23} The Hamiltonian thus obtained has both position dependent and independent parts. In the adiabatic ($\lambda \to \infty$) and weak Rashba coupling ($\alpha_R \to 0$) limit, the position dependent part can be treated as a perturbation to the position independent Hamiltonian. To the lowest order, the equilibrium itinerant magnetization $\mathbf{m}$ is everywhere parallel to the local magnetization $\mathbf{M}$, $\mathbf{m} \propto \mathbf{M}$. Correspondingly, their spatial derivatives are also parallel to
FIG. 4. Dependence of the non-adiabaticity on the Rashba coupling strength for $\lambda k_F = 10$, with $\alpha(0)$ and $\beta(0)$ shown in the inset. The solid line in the main panel is fitted to a parabola, while those in the inset are only guides to the eye.

each other, $\mathbf{m}' \propto \mathbf{M}' = (\hat{x}\cos \theta - \hat{z}\sin \theta)\theta'$. Recalling the definition of the magnetization angle $\theta$, we can immediately calculate its spatial derivative, $\lambda \theta' = q\chi \text{sech}(x/\lambda)$, which is proportional to the product of the charge and the chirality. The phenomenological decomposition of the STT (3) then have the form $\boldsymbol{\tau} = \alpha q\chi (\hat{x}\cos \theta - \hat{z}\sin \theta) + \hat{y}\beta q\chi$. The lowest order perturbation result for the STT gives $\boldsymbol{\tau} \propto \mathbf{m}'$, where only the adiabatic component contributes. According to this expression, the adiabatic coefficient at the DW center $\alpha(0)$ should scale inversely proportional to the DW width $\lambda$, $\alpha(0) \propto 1/\lambda$. Including the first order correction to wave functions, a finite non-adiabatic STT at the DW center emerges, which is proportional to

$$q\chi \frac{a}{\lambda^2} + k_R \frac{b + ce^{-\gamma \lambda}}{\lambda}, \quad (4)$$

where $a$, $b$, $c$ and $\gamma$ are constants determined by the effective exchange splitting $k_B$ and possibly the Rashba coupling strength $k_R$. The exponential and inverse power law terms are brought about by the terms with non-zero and zero momentum transfers in the momentum space effective potential\textsuperscript{23,28}. Using this result for the coefficient $q\chi \beta(0)$, the non-adiabaticity has the form

$$\frac{\beta(0)}{\alpha(0)} = \frac{a}{\lambda} + q\chi k_R (b + ce^{-\gamma \lambda}). \quad (5)$$

The corresponding fit to the expression (5) is displayed in Fig. 3. The agreement of the fit to the numerical result is satisfactory.

The first order perturbative result can only be used to understand the DW width dependence of the non-adiabaticity of the STT in the adiabatic limit. In fact, there could be higher order terms contributing to the multiplicative coefficients $a$, $b$ and $c$ of the exponential and power functions of $\lambda$. Especially for short DWs, the non-adiabacity in Fig. 4 shows...
FIG. 5. Decomposition coefficients $\alpha$ and $\beta$ for different combinations of the DW charge $q$ and chirality $\chi$ with $\lambda k_F = 10$ (a, b) and $\lambda k_F = 70$ (c, d). It is evident that both $\alpha$ and $\beta$ are almost completely determined only by the product $q\chi$, rather than their separate values.

significant nonlinear dependence on the Rashba coupling constant, although the nonlinear behaviour of both individual coefficients exhibit mild impact of higher order terms, cf. insets of Fig. 4.

Finally, we would like to check the influence of underlying DW’s topology on the STT. As can be seen from Eq. (5) for the non-adiabatic STT, in addition to a term independent of the topological features of the DW, the term proportional to the Rashba interaction depends on the product of the DW’s charge and chirality. Although this behaviour is in stark contrast to that of the non-adiabatic RSOT which is solely determined by the product of charge and chirality, it is actually borne out by the numerical results, as shown in Fig. 5. For short DWs, the Rashba contribution to $\beta$ is smaller compared to the contribution to the STT arising from the spatial variation of the magnetization, and the non-adiabatic STT is dominated by the spatial variation of the magnetization. In the adiabatic limit, the Rashba contribution becomes important, so the non-adiabatic coefficient shows the characteristic $q\chi$ behaviour (Fig. 5(d)), although there is still some
deviation from the perfect \( q \chi \) behaviour due to the contribution from the spatial variation of the magnetization.

To conclude, we have investigated the DW width scaling of STT in magnetic DWs with a sizable Rashba spin orbit interaction, which is originated from the broken inversion symmetry at ferromagnet/heavy metal interfaces. In the conventional case where only the spatial variation is responsible for the emergence of the STT, an exponential decay for the non-adiabaticity, which is used to measure the relative importance of the non-adiabatic to the adiabatic torques, was observed. In contrast, in DWs with non-vanishing Rashba spin orbit coupling, the decay of the non-adiabaticity is algebraic, much slower than an exponential behaviour. Due to the presence of the finite Rashba spin orbit coupling, the non-adiabatic STT torque exhibits unusual dependence on the topology of the underlying DW, especially in the adiabatic limit where the Rashba contribution is dominant. This topological feature of the non-adiabatic STT is absent in systems without the Rashba spin orbit interaction.

**ACKNOWLEDGEMENTS**

We would like to express our gratitude to Prof. Jiang Xiao for valuable comments and discussions, especially for bringing us to the topic of STT in magnetic DWs with Rashba spin orbit interaction and sharing his code on STT simulation. Y. Z. acknowledges the support by the President’s Fund of CUHKSZ, Longgang Key Laboratory of Applied Spintronics, National Natural Science Foundation of China (Grant No. 11974298, 61961136006), and Shenzhen Fundamental Research Fund (Grant No. JCYJ20170410171958839).

---

* wangdaowei@sztu.edu.cn
† zhouyan@cuhk.edu.cn

1. L. Berger, J. Appl. Phys. 49, 2156 (1978); Phys. Rev. B 54, 9353 (1996).
2. J. Slonczewski, J. Magn. Magn. Mater. 159, L1 (1996).
3. A. Kirilyuk, A. V. Kimel, and Th. Rasing, Rev. Mod. Phys. 82, 2731 (2010).
4. H. Ohno, D. Chiba, F. Matsukura, T. Omiya, E. Abe, T. Dietl, Y. Ohno, and K. Ohtani, Nature (London) 408, 944 (2000).
5. L. Dreher, M. Weiler, M. Pernpeintner, H. Huebl, R. Gross, M. S. Brandt, and S. T. B. Goennenwein, Phys. Rev. B 86, 134415 (2012); Erratum: *ibid.* 98, 099901 (2018).
K. Obata and G. Tatara, Phys. Rev. B 77, 214429 (2008).

A. Manchon and S. Zhang, Phys. Rev. B 78, 212405 (2008); *ibid.* 79, 094422 (2009).

I. Garate and A. H. MacDonald, Phys. Rev. B 80, 134403 (2009).

A. Matos-Abiague and R. L. Rodríguez-Suárez, Phys. Rev. B 80, 094424 (2009).

X. Wang and A. Manchon, Phys. Rev. Lett. 108, 117201 (2012).

J. E. Hirsch, Phys. Rev. Lett. 83, 1834 (1999); S. F. Zhang, Phys. Rev. Lett. 85, 393 (2000); J. Sinova, D. Culcer, Q. Niu, N. A. Sinitsyn, T. Jungwirth, and A. H. MacDonald, Phys. Rev. Lett. 92, 126603 (2004).

Z. Li and S. Zhang, Phys. Rev. Lett. 93, 127204 (2004).

A. Thiaville, Y. Nakatani, J. Miltat, and Y. Suzuki, Europhys. Lett. 69, 990 (2005).

J. Xiao, A. Zangwill and M. D. Stiles, Phys. Rev. B 73, 054428 (2006).

Yu. A. Bychkov and E. I. Rashba, JETP Lett. 39, 78 (1984).

P. Gambardella and I. M. Miron, Philos. Trans. R. Soc. London, Ser. A 369, 3175 (2011).

K.-W. Kim, H.-W. Lee, K.-J. Lee, and M. D. Stiles, Phys. Rev. Lett. 111, 216601 (2013).

N. L. Schryer and L. R. Walker, J. Appl. Phys. 45, 5406 (1974).

H.-B. Braun, Adv. Phys. 61, 1 (2012).

K. Xia, M. Zwierzycki, M. Talanana, P. J. Kelly, and G. E. W. Bauer, Phys. Rev. B 73, 064420 (2006).

M. Zwierzycki, P. A. Khomyakov, A. A. Starikov, K. Xia, M. Talanana, P. X. Xu, V. M. Karpan, I. Marushchenko, I. Turek, G. E. W. Bauer, G. Brocks, and P. J. Kelly, Phys. Stat. Sol. B 245, 623 (2008).

N. W. Ashcroft and N. D. Mermin, *Solid State Physics*, World Publishing Corporation, Beijing, 2004.

D. Wang and Y. Zhou, J. Magn. Magn. Mater. 493, 165694 (2020).

D. Wang, Y. Zhou, Z.-X. Li, Y. Nie, X.-G. Wang, and G.-H. Guo, IEEE Trans. Magn. 53, 1300110 (2017).

L. D. Landau, E. M. Lifshitz, and L. P. Pitaevski, *Statistical Physics*, 3rd ed. (Pergamon, Oxford), Part 2, 1980; T. L. Gilbert, IEEE Trans. Magn. 40, 3443 (2004).

S. Bohlens and D. Pfannkuche, Phys. Rev. Lett. 105, 177201 (2010).

Z. Yuan and P. J. Kelly, Phys. Rev. B. 93, 224415 (2016).

V. K. Dugaev, J. Barnaś, A. Lusakowski, and L. A. Turski, Phys. Rev. B 65, 224419 (2002).