Active learning of phase and group velocities

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Abstract. I report on my observations during an active-learning activity on the topic of phase and group velocities with in-service physics teachers and a small group of high school students. The activity was designed to be as active as possible, and to guide the participants to the discovery (or reminder) that phase velocity is not the only velocity relevant to wave propagation. I observed that both groups, even after having been exposed to waves with dispersion, had difficulties using any velocity but phase velocity in their predictions and their reasoning. In the article, I describe the activity, and the observations. I discuss the possible reasons for the participants’ difficulties and suggest approaches that might improve the active-learning outcome.

1. Introduction
I first encountered the topic of phase and group velocities when I was a physics student. And, it was relevant to me only because the teacher said that while some waves travel at phase velocity greater than the speed of light, this does not violate special relativity, because the group velocity remains smaller than the speed of light and thus information cannot travel faster than the speed of light. So I wondered, what happens to the wavefront? In my mind the wavefront, being the first crest, must travel with phase velocity, and it can certainly carry information. How to reconcile this with what was said above? In my student experience, no further time was devoted to this topic. Even after studying Fourier analysis, the topic never came up again. Based on this experience, I believe that many physics teachers graduate without fully understanding this phenomenon. I find this problematic, because information transfer in continuous media mostly happens via waves, be it sound, vibration, earthquake, tsunami, electromagnetic waves of radio and mobile devices, … virtually every transfer of information not done by a physical carrier is done by waves. Understanding the concepts behind information transfer in waves, I believe, is very important, especially for physics teachers. It is also important for physicists, but physicists are often satisfied with a formalistic answer. For a physicist, it might suffice to go through the derivation of group or front velocities, and as long as the mathematics is correct, they will trust the result. On the other hand, physics teachers are mostly faced with students whose formal knowledge is not advanced enough, not trusted enough, or who simply do not feel that formalism is a ‘real’ answer, but who may have heard of faster-than-light phase velocities and are intrigued.

Jumping ahead in time, I achieved a position where I had the opportunity to ask in-service physics teachers how they perceived this phenomenon. I wanted to do this in an active learning environment using formative tasks to gain insight into their understanding. The topic of teaching dispersion with experiments is discussed well in [1] (and references therein), where authors propose an experiment with electromagnetic waves in coaxial cables. In their opinion, the experiment is suitable for advanced
undergraduate labs. A similar mechanical proposal was given in [2], where the author described an active learning setup, tested on a small group of students who were interviewed while completing the various tasks. A further literature search revealed reports that address the propagation of a wave in a dispersive medium, but do not focus on the different velocities [3–5]. Some approaches use acoustics [8], some use water waves [9], some use electromagnetic waves [10, 11], and some suggest simulations [12]. I found no reports on attempts to produce an active learning sequence on the topic. I reproduced the setup in [2], developed a modified set of tasks, and tested the activity on a larger group of in-service physics teachers (N=25) at a professional development event, and a group of high school students (N=6), who participated in a physics-themed weekend event and chose to participate in this activity. The activity was not designed with any specific research goal in mind. I wanted to see how efficient the active learning tasks were and whether participants would learn the intended content. However, the problems the participants encountered during the activity offered a great deal of insight into their reasoning. I recorded my observations after the activity and tried to find explanations for the problems. The p-prims framework of diSessa [7] proved a good starting point. However, the conceptual resources that could be identified in their reasoning were not exactly p-prims. They were chunks of formal knowledge such as the law of motion or the law of conservation of energy. As such, they were more sophisticated than p-prims, yet not necessarily developed enough to qualify as concepts. Therefore, I call them simply ideas. I report on which ideas I identified, and on the impact that I believe they have on the learning process. Based on the identified ideas, I suggest modifications to the presented activity to increase its teaching efficiency.

2. The activity and the observations
I present the tasks and the responses together, because each activity gives insight into a specific way of thinking that is best described together with the activity instead of a separate section about observations. The activity was the same for both groups, so I only describe it once. I report mainly on the observations with teachers, since the student group was much smaller and there were almost no differences between their approaches. My analysis focuses on the ideas that the teachers appear to have used and how they impact the learning process and the difficulties that the teachers encountered.

2.1. Wave velocity of a regular wave
We filmed in advance a propagating nondispersive wave, and a dispersive one. From the movies we extracted frames at regular time intervals and aligned them vertically to produce figures 1 and 2. Participants were first presented with the nondispersive wave (figure 1). They were asked to find two or more methods to determine its velocity, such as:

(a) Determining the frequency, \( \nu \), from the time stamps, the wavelength, \( \lambda \), from the pictures, and using equation \( c = \lambda \cdot \nu \) to determine the wave velocity \( c \). Surprisingly, this was the most commonly suggested (and applied) method, even though it is the most complicated. I call this the “velocity from equation” idea.

(b) Following the motion of a crest. From the pictures, they determined the distance travelled by a crest, \( s \), and from the time stamps, the elapsed time, \( t \). They used equation \( c = s/t \) to determine the wave velocity, \( c \). I call this the “velocity from the crest” idea.

(c) Only very few suggested following the motion of the front of the wave instead of the crest. I call this the “common-sense velocity” idea, because it seems to invoke the usual, common-sense use of velocity: a means to determine the time it takes for something (in this case the front of the disturbance) to travel between two points.

In both groups, the two predominant methods were (a) and (b). When discussing with their peers why they did not think of (c), they responded mostly that they considered this method to be equivalent to method (b). To me this shows how deeply rooted the perception is that any wave is nondispersive. Methods (a) and (b) give the phase velocity, while method (c) gives the group velocity. In this report, we will use these labels for ease of understanding, even though in the activity, the naming of the velocities comes much later. We also do not differentiate between group and front/signal velocity. This
is because it is beyond the scope of the activity, which is to familiarize the participants with the fact that the phase velocity is not the only reasonable velocity of a wave.

2.2. Wave velocity of a dispersive wave
In the next activity, participants were presented with a different series of pictures, this time of a dispersive wave (figure 2). The wave was produced with a Klein-Gordon string made of sticks and adhesive tape [2]. They were asked to determine the wave velocity in all the three ways discussed above. Methods (a) and (b) gave results consistent with each other, while method (c) gave a smaller value. After this was apparent, they were asked what to do with the fact that the three methods do not give matching results. I expected that they would answer that there are obviously two relevant velocities. I checked that they had studied phase and group velocities when they were university students, and expected them to remember enough about the topic to be familiar with the phenomenon. Instead, they appeared very confused and unsure of what to do. After some deliberation, they chose the phase velocity to be the wave velocity. When asked what to do with group velocity, they responded that the discrepancy is probably due to experimental uncertainty, and that maybe some sort of average between the two velocities should be calculated. From this reply, I learned that: (i) The task gives an opportunity to give importance to experimental uncertainties. In the activity, we completely neglected them, because I have not anticipated that the participants would find them relevant, but if we had estimated them, it would be clear that the discrepancy between phase and group velocities cannot be explained by experimental uncertainties, which is also clear from lines (B) and (C) in figure 2. (ii) The idea that the wave velocity is the result of the relation \( c = \lambda \cdot \nu \), or the velocity of the crest, is very deeply rooted. I expected them to use the ‘common-sense velocity’ idea, for the motion of the first displacement of the medium. Instead, they chose the velocity of the crest as the velocity of the wave.

To address the possibility that participants might think that the wave velocity is by definition the phase velocity, in their next task they were asked to predict how long it would take for the wave to travel a given distance \( L \). I believe that this task is independent of the definition of wave velocity. I expected them to use the motion of the front in figure 2 (indicated by line C) to extrapolate the time. Again, surprisingly, they used the phase velocity, even though it is obvious from figure 2 that the crest vanishes

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**Figure 1.** A sequence of photographs at almost equal time intervals representing the propagation of a regular (nondispersive) wave. The inverse of the slopes of lines B (dotted lines) represents the phase velocity. The inverse of the slope of line C (solid line) represents the group velocity. It is apparent that they match well.
as it reaches the front and does not overtake the front. We even previously discussed that a blind person would know that a wave has reached them by feeling the first displacement of the medium.

![Figure 2](image)

**Figure 2.** A sequence of photographs at equal time intervals representing the propagation of a dispersive wave. The inverse of the slopes of lines B (dotted lines) represent the phase velocity. The inverse of the slope of line C (solid line) represents the group velocity. The difference between them is obvious.

I believe this is due to another idea: the idea that “the front is a crest”. This idea was never explicitly stated, but I believe it can be inferred from their reasoning. Given the vast experience teachers have with propagating pulses, it is not hard to believe that this idea is part of their experiential base. It explains the difficulties and offers ideas on how to address them. The idea is correct for nondispersive waves. If the disturbance was created as a crest, it will continue to travel as a crest. Since the crest travels with phase velocity and the front is a crest, the front should travel with phase velocity. In dispersive waves, however, the ‘front is a crest’ idea is no longer correct. The front of the disturbance alternates between crest and trough. At this point something should happen in the activity that would challenge the ‘front is a crest’ idea. It appears that this was achieved in [2] where the students observed a real-life dispersive wave and noticed that “there was something strange about this pulse” (reference [2], page 6). Since we were unaware of this possible idea beforehand, we instead attempted to activate the ‘common-sense velocity’ idea and expected the challenge to arise from the fact that the ‘velocity from equation’ idea and the ‘common-sense velocity’ idea do not give the same predictions. However, the observations above suggest that when dealing with waves, the ‘velocity from equation’ idea is much stronger than the ‘common-sense velocity’ idea so the latter was almost never activated, and the participants had no reason to distrust their initial idea.

The fact that the ‘front is a crest’ idea was not challenged could also explain why they attempted to explain the discrepancy between the phase and group velocities with experimental error. It might be that even if the ‘common-sense velocity’ idea had been activated, without noticing that the front is not a
crest, they would see no reason to expect that the front should travel with anything other than phase velocity.

We discussed the situation, and I suggested graphically representing the velocities derived from (b) and (c) in figures 1 and 2. These are the lines B (for phase velocity derived from (b)) and C (for group velocity derived from (c)). The inverses of the slopes of lines B and C represent the respective velocities. From the lines, it is clear that in figure 2, crests and troughs travel faster than the front, while in figure 1, they all travel at the same velocity.

I proceeded to define the phase and group velocities relating phase velocity to the velocity derived from (b), and group velocity to the velocity derived from (c). Participants were asked to identify which is the velocity given by the equation \( c = \lambda \cdot v \) and they correctly identified it to be phase velocity by comparing the results from (a), (b) and (c) in the case of the dispersive wave.

With these two activities, I wanted to give teachers some experience in the field of dispersive waves. I expected that the concrete example they analysed would provide an anchor for their previous theoretical knowledge, which would then be recalled more easily. However, it appears that simply presenting them with experience was not enough. Based on the observations, I believe that participants’ knowledge of waves is deeply anchored in nondispersive waves, and to provide a different anchor more scaffolding is needed. To improve learning, I believe that three activities should be added: First, a real model of the Klein-Gordon string should be used, a video of its motion should be made beforehand and the video, in slow motion if necessary, should be shown before the pictures in figure 2 are discussed. Second, some ‘compare and contrast’ type of tasks should be added to alert the participants to the differences and similarities between the two wave propagations. Third, a peer discussion about the implications of the acquired knowledge should be added. This discussion would offer the possibility to clarify some participants’ ideas. Unfortunately, we have not been able to implement these improvements by the time of the writing of this report.

2.3. Other interesting observations

Activities on the Fourier decomposition of waves were added to the session, because I have noticed that some teachers have difficulty with the formal tool of decomposing a finite pulse into a series, or spectrum, of infinite purely sinusoidal waves, hereafter called wavelets. I have found that the PhET simulation Fourier – making waves [6] can be very useful to address this topic. The simulation enables the user to choose the amplitudes of a base frequency and eight higher harmonics to compose a wave of a particular shape. The challenge posed to our participants was to produce a finite wave. The expected outcome was that they would realize that choosing more than one frequency can shape the wave so that it becomes finite. They successfully completed the task. The important learning goal was that infinite wavelets, pure cosines, can produce a finite superposition.

However, one teacher expressed that they have difficulty imagining that wavelets would be infinite, if the pulse is finite. Their picture was that the wavelets have a beginning (front) at the same time and place as the beginning of the pulse. From my discussion with the teacher, I believe this is because the teacher might consider a wavelet to be a physical entity, an actual infinite wave in a medium. The difficulties seem to stem from two points: One, a real medium cannot be infinite, and two, a pulse that was just created could not have produced an infinite wave immediately after its creation. In this picture, wavelets require time to travel along the medium. This might include the picture that they travel with their phase velocity. To address this, I suggest stressing the following points: (I) The only real wave shape on the medium is the one that is being decomposed. The wavelets are mathematical tools. They do not actually appear on a real medium, only their superposition does, which is finite. (II) If a wavelet had a front, it would be a truncated sinusoid, just like the wave in the activity, graphically represented in figure 2. Note (again) that the front does not travel with phase velocity, but with a smaller (group) velocity.

Comparison between the group of in-service teachers and the interested group of high school students revealed no significant differences. The way in which high-school students approached the topic and the problems they encountered were very similar to those of the teachers.
3. Conclusions
The activity provided participants with a comparison between a nondispersive and a dispersive wave. It was carried out with the aim of providing participants with some real-life experience with dispersive waves, which would form an anchor for their theoretical knowledge. I expected that being given the experience they would quickly achieve the important learning goals. It turned out that just providing the experience did not suffice. However, from their thinking I was able to identify a few ideas that seem to be crucial in their reasoning.

I have identified two specific cases where they have problems, and both can, in my opinion, be explained by the combination of two ideas: ‘velocity from equation’ and ‘front is a crest’. In the situation when they have to determine the time it will take for the wave to reach the other end, ‘velocity from equations’ seems to be the first idea that is activated, as evidenced by the fact that for the great majority this was the first method of determining the velocity. If then the ‘front is a crest’ idea is activated, they have no reason to challenge their idea that the phase velocity will give the right answer. In the case of the Fourier transform, the teacher with whom I discussed the situation in more depth seemed to have the idea that all the wavelets have to start at some point and that their front propagates with phase velocity. This directly suggests that the ‘front is a crest’ idea has been activated.

The reasoning of the teachers appears to be remarkably similar to my reasoning in student years, as described in the introduction. This reinforces my assumption that many teachers graduate without fully understanding the topic, and, therefore, encounter similar problems as I did. This report offers a tentative explanation for these problems, at the core of which is the ‘front is a crest’ idea.

There were no significant differences observed between the groups of teachers and the motivated group of high-school students. I believe that for both groups nondispersive waves are basically a new situation. For high-school students because they have never encountered them before, and for the teachers because they is poorly covered at university level and they never encountered them since. I feel confident concluding that in both these groups, the ‘velocity from equation’ idea is the preferred idea that is activated in relation to waves, even to a point where it is difficult to activate the ‘common-sense velocity’ idea. Usually, replacing a naïve idea with a more formally structured one is a good thing. However, in this activity we wanted to rely on both ideas to arrive at the conclusion that there are two relevant velocities, but the naïve idea was very difficult to activate. However, from my observations described above, addressing the ‘front is a crest’ idea might be even more beneficial than trying to activate the ‘common-sense velocity’ idea.

Participants showed no problem following the explanation of the teacher, but they performed poorly on the tasks designed to let them actively acquire the knowledge for themselves. If my explanation is correct, then additional ‘compare and contrast’ tasks should be added to the activity to alert the participants to the differences and maybe address the ‘front is a crest’ idea. Such activities have been performed in [2] and might be the reason why the students in [2] performed better at identifying the two velocities. Some discussion of the implication of the new knowledge should also be added to make the participants reflect on the meaning and importance of the knowledge and to perhaps clarify some of their ideas. Whether these additions really improve learning remains to be studied.

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