Partial $K$–way Negativities of Pure Four qubit Entangled States

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It has been shown by Versraete et. al [F. Versraete, J. Dehaene, B. De Moor, and H. Verschelde, Phys. Rev. A65, 052112 (2002)] that by Stochastic local operations and classical communication (SLOCC), a pure state of four qubits can be transformed to a state belonging to one of a set of nine families of states. By using selective partial transposition, we construct partial $K$–way negativities to measure the genuine $4$–partite, tripartite, and bi-partite entanglement of single copy states belonging to the nine families of four qubit states. Partial $K$–way negativities are polynomial functions of local invariants characterizing each family of states, as such, entanglement monotones.

Detection and measurement of multipartite entanglement is an important open question. Different parts of an N-partite composite system may be entangled to each other in distinctly different ways. In particular, four qubit ($ABCD$) states may have $4$–partite, tri-partite, and bi-partite entanglement. The bipartite entanglement, may in turn be present for a given pair of qubits or for more than one qubit pairs. Similarly, possible candidates for tripartite entanglement are subsystems, $ABC, ABD, ACD,$ and $BCD$. A four qubit state may have four-partite entanglement generated by three qubit coherence or tripartite entanglement due to two qubit quantum correlations. Negativity $^{1}$ based on Peres Horodecki PPT separability criterion $^{2, 3}$ has been shown to be an entanglement monotone $^{2, 4, 5}$. We proposed a characterization of three qubit states $^{6, 7}$ based on global negativities and partial $K$–way negativities ($2 \leq K \leq 3$). The $K$–way partial transpose with respect to a subsystem of an N-partite composite system is constructed by partial transposition subject to specific constraints on transposed matrix elements for each value of $K$ ($2 \leq K \leq N$). The $K$–way partial transpose may also be constructed for a given set of $K$ subsystems. The $K$–way negativity ($2 \leq K \leq N$), defined as the negativity of $K$–way partial transpose, quantifies the $K$-way coherences of the composite system. The underlying idea of selective transposition to construct a $K$–way partial transpose with respect to a subsystem, presented for the first time in ref. $^{8}$ and then applied in ref. $^{9}$, shifts the focus from $K$-subsystems to $K$–way coherences of the composite system. By $K$–way coherences, we mean the quantum correlations responsible for GHZ state like entanglement of a $K$-partite system. For an N-partite entangled state, the partial $K$-way negativity is the contribution of a $K$–way partial transposes ($2 \leq K \leq N$) to the global negativity.

In this article, we present analytical expressions and numerical calculations of partial $K$–way negativities for single copy pure four qubit states. Entanglement being a nonlocal property of the composite system, it cannot be increased by local operations and classical communication (LOCC). Versraete et al. $^{10}$ have shown that a pure state of four qubits can be transformed, by Stochastic local operations and classical communication (SLOCC), to a state belonging to one of a set of nine families of states, corresponding to nine distinct ways of entangling four qubits. We use the global and $K$–way partial transposes, to construct measures of genuine $4$–partite, tri-partite, and bi-partite entanglement for all the nine families of states. It has been pointed out by Versraete et al. $^{10}$, that the polynomial functions of local invariants characterizing each family are entanglement monotones. The calculated global and partial $K$–way negativities are polynomial functions of local invariants for each family of states, as such entanglement monotones.

The nine families of states are grouped in to two classes, with (i) class I containing states for which partial three-way negativity is zero and (ii) class II states characterised by nonzero partial three-way negativity.

The necessary definitions to construct global and $K$–way partial transpose are given in section I, and partial $K$-way negativities defined in section II. Calculation of pairwise and three qubit entanglement for specific groups of qubits is discussed in section III. Comments on notation and classification of nine families of states are included in section IV. In section V, entanglement of a general four qubit state is analyzed and the entanglement measures of states belonging to class I are shown to be related to those for the general four qubit state. Monogamy inequalities for the states in Class I are also presented. Section VI presents the states with non zero partial three-way negativities. The results are summarized in section VII.

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I. THE GLOBAL AND K-WAY PARTIAL TRANSPOSE OF 4-QUBIT STATE

We consider qubits one, two, three and four located at labs $A$, $B$, $C$ and $D$, respectively, constituting a composite system $ABCD$ in state $\hat{\rho}$. The Hilbert space, $C^4$, associated with the quantum system is spanned by basis vectors of the form $|i_1i_2i_3i_4\rangle$, where $i_m = 0$ or $1$, for $m = 1$ to $4$. To simplify the notation we denote the vector $|i_1i_2i_3i_4\rangle$ by $\prod_{m=1}^{4} i_m$ and write a general four qubit pure state as

$$\rho = \sum_{i_1-i_4, j_1-j_4} \prod_{m=1}^{4} i_m |i_m\rangle\langle i_m| \prod_{m=1}^{4} j_m |j_m\rangle\langle j_m|.$$  \hspace{1cm} (1)

The partial transpose of $\hat{\rho}$ with respect to qubit $p$ is defined as

$$\hat{\rho}_p^T = \sum_{i_1-i_4, j_1-j_4} \prod_{m=1}^{4} i_m |i_m\rangle\langle i_m| \prod_{m=1}^{4} j_m \prod_{m=1,m\neq p}^{4} j_m |j_m\rangle\langle j_m|.$$  \hspace{1cm} (2)

The partial transpose $\hat{\rho}_G^T$ of a state having free entanglement is non positive. The Global negativity defined as

$$N_p^G = \frac{1}{d_p-1} (\|\hat{\rho}_G^T\|_1 - 1),$$  \hspace{1cm} (3)

measures the entanglement of subsystem $p$ with it’s complement. Here $\|\hat{\rho}\|_1$ is the trace norm of $\hat{\rho}$. The factor $1.0/(d_p - 1)$ ensures that the maximum value of negativity is one. Global negativity vanishes on PPT states.

We label a given matrix element $\prod_{m=1}^{4} i_m |i_m\rangle\langle i_m| \prod_{m=1}^{4} j_m$ by a number $K = \sum_{m=1}^{4} (1 - \delta_{i_m,j_m})$, where $\delta_{i_m,j_m} = 1$ for $i_m = j_m$, and $\delta_{i_m,j_m} = 0$ for $i_m \neq j_m$. The $K$-way partial transpose ($K > 2$) of $\rho$ with respect to subsystem $p$ is obtained by selective transposition such that

$$\prod_{m=1}^{4} i_m \hat{\rho}_K^T |i_m\rangle\langle i_m| \prod_{m=1}^{4} j_m = \prod_{m=1}^{4} i_m \hat{\rho} |i_m\rangle\langle i_m| \prod_{m=1}^{4} j_m,$$

if $\sum_{m=1}^{4} (1 - \delta_{i_m,j_m}) = K$, and $\delta_{i_p,j_p} = 0$ \hspace{1cm} (4)

and

$$\prod_{m=1}^{4} i_m \hat{\rho}_K^T |i_m\rangle\langle i_m| \prod_{m=1}^{4} j_m = \prod_{m=1}^{4} i_m \hat{\rho} |i_m\rangle\langle i_m| \prod_{m=1}^{4} j_m,$$

if $\sum_{m=1}^{4} (1 - \delta_{i_m,j_m}) \neq K.$ \hspace{1cm} (5)

while

$$\prod_{m=1}^{4} i_m \hat{\rho}_2^T |i_m\rangle\langle i_m| \prod_{m=1}^{4} j_m = \prod_{m=1}^{4} i_m \hat{\rho} |i_m\rangle\langle i_m| \prod_{m=1}^{4} j_m,$$

if $\sum_{m=1}^{4} (1 - \delta_{i_m,j_m}) = 1$ or $2$, and $\delta_{i_p,j_p} = 0$ \hspace{1cm} (6)

and

$$\prod_{m=1}^{4} i_m \hat{\rho}_2^T |i_m\rangle\langle i_m| \prod_{m=1}^{4} j_m = \prod_{m=1}^{4} i_m \hat{\rho} |i_m\rangle\langle i_m| \prod_{m=1}^{4} j_m,$$

if $\sum_{m=1}^{4} (1 - \delta_{i_m,j_m}) \neq 1$ or $2.$ \hspace{1cm} (7)
Recalling that a single qubit matrix element of spin flip operator \( \hat{\sigma}^m_x \) is given by \( \langle i_m | \hat{\sigma}^m_x | j_m \rangle = (1 - \delta_{i_m,j_m}) \), \( \hat{\rho}^T_F \) is obtained by restricting the transposition to the group of matrix elements of \( \hat{\rho} \) in which the number of spins being flipped is \( K \). In case matrix \( \rho \) is a real matrix, the definition of \( \hat{\rho}^T_F \) used here gives the same results as that used in refs.[6, 8]. However, if state operator is represented by a complex Hermitian matrix, the contribution of \( \hat{\rho}^T_F \), as defined by Eqs. (6) and (7), to global negativity is different. The correction ensures that, \( \hat{\rho}^T_F = \hat{\rho}^T_F \), in case the state \( \hat{\rho} \) has only pairwise entanglement. The \( K \)-way negativity calculated from \( K \)-way partial transpose of matrix \( \rho \) with respect to subsystem \( p \), is defined as \( N^K_p = \frac{1}{d_p - 1} \left( \left\| \hat{\rho}^T_F \right\|_1 - 1 \right) \). Using the definition of trace norm and the fact that \( tr(\hat{\rho}^T_F) = 1 \), we get \( N^K_p = \frac{2}{d_p - 1} \sum_i |\lambda^K_i|, \lambda^K_i \) being the negative eigenvalues of matrix \( \hat{\rho}^T_F \). The negativity \( N^K_p \) \((K = 2 \text{ to } 4)\), depends on \( K \)-way coherences and is a measure of all possible types of entanglement attributed to \( K \)-way coherences. By \( K \)-way coherences, we mean the quantum correlations responsible for genuine \( K \)-partite entanglement of the composite system.

II. PARTIAL K-WAY NEGATIVITIES

Global negativity with respect to a subsystem \( p \) can be written as a sum of partial \( K \)-way negativities. Using \( Tr(\hat{\rho}^T_F) = 1 \), the negativity of \( \hat{\rho}^T_F \) is given by

\[
N^K_p = -\frac{2}{d_p - 1} \sum_i \langle \Psi^G_i | \hat{\rho}^T_F | \Psi^G_i \rangle = -\frac{2}{d_p - 1} \sum_i \lambda^G_i, \tag{8}
\]

where \( \lambda^G_i \) and \( |\Psi^G_i\rangle \) are, respectively, the negative eigenvalues and eigenvectors of \( \hat{\rho}^T_F \). It is straight forward to verify that

\[
\hat{\rho}^T_F = \sum_{K=2}^{4} \hat{\rho}^T_F - 2\hat{\rho}. \tag{9}
\]

For a pure state with qubit \( p \) separable, the expansion of Eq. (9), leads to the equality

\[
\left\| \hat{\rho}^T_F \right\|_1 = \sum_{K=2}^{4} \left( \sum_i \left| \langle \Psi^G_i | \hat{\rho}^T_F | \Psi^G_i \rangle \right| - 2 \sum_i |\langle \Psi^G_i | \hat{\rho} | \Psi^G_i \rangle| \right) = \sum_{K=2}^{4} tr(\hat{\rho}^T_F) - 2 tr(\hat{\rho}) = 1, \tag{10}
\]

where \( \Psi^G_i \) are eigen functions of a positive global partial transpose \( \hat{\rho}^T_F \). On the other hand, when qubit \( p \) is entangled that is \( N^K_p > 0 \), by substituting Eq. (10) in Eq. (9), we get

\[
-2 \sum_i \lambda^G_i = -2 \sum_{K=1}^{4} \sum_i \langle \Psi^G_i | \hat{\rho}^T_F | \Psi^G_i \rangle + 4 \sum_i \langle \Psi^G_i | \hat{\rho} | \Psi^G_i \rangle. \tag{11}
\]

Defining partial \( K \)-way negativity \( E^K_p \) \((K = 2 \text{ to } 4)\) as

\[
E^K_p = -\frac{2}{d_p - 1} \sum_i \langle \Psi^G_i | \hat{\rho}^T_F | \Psi^G_i \rangle, \tag{12}
\]

\[
E^0_p = -\frac{2}{d_p - 1} \sum_i \langle \Psi^G_i | \hat{\rho} | \Psi^G_i \rangle, \tag{13}
\]

we may split the global negativity for qubit \( p \) as

\[
N^K_p = \sum_{K=2}^{4} (E^K_p - E^0_p) + E^0_p. \tag{14}
\]
The eigen vectors of global partial transpose and $K$–way partial transpose are not the same except when the global partial transpose is equal to $K$–way partial transpose. Given that $\hat{\rho}^T_K |\Psi^K\rangle = \beta^K_\nu |\Psi^K\rangle$, and representing by $\beta^K_\nu$ and $\beta^K_\nu^+$, the negative and positive eigenvalues of $\hat{\rho}^T_K$, we may rewrite the partial $K$–way negativity as

$$E'_K = \frac{2}{d_p - 1} \left( \sum_{\mu} \sum_i |\beta^K_\mu^-| \langle \Psi^G_i^- | \Psi^K_\mu^- \rangle^2 - \sum_{\mu} \sum_i |\beta^K_\mu^+| \langle \Psi^G_i^- | \Psi^K_\mu^+ \rangle^2 \right). \quad (15)$$

It follows from Eq. (15) that $E'_K > 0$, if and only if one or more eigenvalues of $\hat{\rho}^T_K$ are negative and eigen functions of $\hat{\rho}^T_K$ have finite overlap with eigenfunctions of $\hat{\rho}^T_K$ corresponding to negative eigenvalues of $\hat{\rho}^T_K$. We notice that, in the limiting case, with all the matrix elements having $\sum_{m=1}^{4} (1 - \delta_{im,jm}) = K$, and $\delta_{ip,jp} = 0$ satisfying

$$\left\langle \prod_{m=1}^{4} i_m | \hat{\rho} | \prod_{m=1}^{4} j_m \right\rangle = \left\langle j_p \prod_{m=1,m\neq p}^{4} i_m | \hat{\rho} | i_p \prod_{m=1,m\neq p}^{4} j_m \right\rangle, \quad (16)$$

we obtain $\hat{\rho}^T_K = \hat{\rho}$ and $K$–way coherences, if present cannot be detected by $K$–way partial transpose. Only positive quantities $(E'_K - E''_K)$ ($K > 1$) are used to measure the $K$–way coherences of the system. A positive $K$–way partial transpose of a pure state represents another pure state $|\Psi'\rangle$ of the system, having larger overlap with a given $|\Psi^G\rangle$ than $|\Psi\rangle$, leading to $E'_K - E''_K \leq 0$. This result follows from the observation that for a pure separable state $|\Phi^A \otimes \Phi^B\rangle$, with partial transpose

$$\hat{\rho}^T_G = \langle (\Phi^A)^* \otimes \Phi^B \rangle \langle (\Phi^A)^* \otimes \Phi^B \rangle,$$

the overlaps satisfy

$$\left| \langle (\Phi^A)^* \otimes \Phi^B | \Phi^A \otimes \Phi^B \rangle \right|^2 - \left| \langle (\Phi^A)^* \otimes \Phi^B | (\Phi^A)^* \otimes \Phi^B \rangle \right|^2 \leq 0, \quad (17)$$

where equality holds, only, when all probability amplitudes in the expansion of $\Phi^A$ are real. For two and three qubit states in canonical form $E''_K = 0$, as such, the partial negativity $E'_K$ ($K = 2, 3$) turns out to be the measure of $K$–way entanglement. The necessary condition for a 4–qubit pure state not to have genuine 4–partite entanglement is that at least one of the global negativities $N^p_G$ is zero, where $p$ is one of the subsystems or one part of a bipartite split of the composite system.

### III. How is the Pairwise and Three Qubit Entanglement Distributed in a Four Qubit State?

It is common practice to trace out subsystem $AD$ to obtain the entanglement of pair $BC$. State reduction is an irreversible local operation and it is believed that the entanglement of the pair $BC$ in the reduced system is either the same or less than that in the composite system $\hat{\rho}$. One can, however, obtain a measure of 2–way coherences involving a given pair of subsystems by using a 2–way partial transpose constructed from the state operator $\hat{\rho}$ by restricting the transposed matrix elements to those for which the state of the third and fourth qubit does not change. For instance, $\hat{\rho}^{TA-AB}_2$ is obtained from the matrix $\rho$ by applying the condition

$$\langle i_1 i_2 i_3 i_4 | \hat{\rho}^{TA-AB}_2 | j_1 j_2 i_3 i_4 \rangle = \langle j_1 i_2 i_3 i_4 | \hat{\rho} | i_1 j_2 i_3 i_4 \rangle;$$

if $\sum_{m=1}^{4} (1 - \delta_{im,jm}) = 1$ or 2, and $\sum_{m=3}^{4} (1 - \delta_{im,jm}) = 0$, \quad (18)

and for all other matrix elements

$$\langle i_1 i_2 i_3 i_4 | \hat{\rho}^{TA-AB}_2 | j_1 j_2 j_3 i_4 \rangle = \langle i_1 i_2 i_3 i_4 | \hat{\rho} | j_1 j_2 j_3 i_4 \rangle.$$

(19)
The negativity $N_{2}^{A-AB} = \frac{1}{\sigma_{-1}} \left( \| \rho_{2}^{A-AB} \|_{1} - 1 \right)$ measures the 2-way coherences involving the pair of subsystems $AB$. It is easily shown that for a four qubit system
\begin{equation}
\rho_{2}^{A} = \rho_{2}^{A-AB} + \rho_{2}^{A-AC} + \rho_{2}^{A-AD} - 2\tilde{\rho}.
\end{equation}
Substituting Eq. (20) in the definition of $E_{2}^{A}$ that is
\begin{equation}
E_{2}^{A} = -2 \sum_{i} \langle \Psi_{i}^{G-} | (\rho_{2}^{A-AB} + \rho_{2}^{A-AC} + \rho_{2}^{A-AD} - 2\tilde{\rho}) | \Psi_{i}^{G-} \rangle,
\end{equation}
we get the relation
\begin{equation}
E_{2}^{A} - E_{0}^{A} = (E_{2}^{A-AB} - E_{0}^{A}) + (E_{2}^{A-AC} - E_{0}^{A}) + (E_{2}^{A-AD} - E_{0}^{A}),
\end{equation}
where $E_{2}^{A-AB}$, $E_{2}^{A-AC}$ and $E_{2}^{A-AD}$ are contributions of $\rho_{2}^{A-AB}$, $\rho_{2}^{A-AC}$, and $\rho_{2}^{A-AD}$ to $E_{2}^{A}$.
We can also construct the partially transposed matrices $\rho_{3}^{A-ABC}$, $\rho_{3}^{A-ABD}$, and $\rho_{3}^{A-ACD}$ such that
\begin{equation}
\rho_{3}^{A} = \rho_{3}^{A-ABC} + \rho_{3}^{A-ABD} + \rho_{3}^{A-ACD} - 2\tilde{\rho}.
\end{equation}
Here $\rho_{3}^{A-ABC}$ is constructed subject to the conditions
\begin{equation}
\langle i_{1}i_{2}i_{3}i_{4} | \rho_{3}^{A-ABC} | j_{1}j_{2}j_{3}j_{4} \rangle = \langle i_{1}i_{2}i_{3}i_{4} | \tilde{\rho} | j_{1}j_{2}j_{3}j_{4} \rangle; \quad \text{if} \quad \sum_{m=1}^{3} (1 - \delta_{i_{m},j_{m}}) = 3,
\end{equation}
\begin{equation}
\langle i_{1}i_{2}i_{3}i_{4} | \rho_{3}^{A-ABD} | j_{1}j_{2}j_{3}j_{4} \rangle = \langle i_{1}i_{2}i_{3}i_{4} | \tilde{\rho} | j_{1}j_{2}j_{3}j_{4} \rangle; \quad \text{for all other matrix elements.}
\end{equation}
Analogous restrictions are applied to construct $\rho_{3}^{A-ABD}$, $\rho_{3}^{A-ACD}$ and $\rho_{3}^{AB-BCD}$ etc. Using Eqs. (12) and (23), we obtain
\begin{equation}
E_{3}^{A} - E_{0}^{A} = -2 \sum_{i} \langle \Psi_{i}^{G-} | (\rho_{3}^{A-ABC} + \rho_{3}^{A-ABD} + \rho_{3}^{A-ACD} - 3\tilde{\rho}) | \Psi_{i}^{G-} \rangle,
\end{equation}
\begin{equation}
= (E_{3}^{A-ABC} - E_{0}^{A}) + (E_{3}^{A-ABD} - E_{0}^{A}) + (E_{3}^{A-ACD} - E_{0}^{A}),
\end{equation}
where
\begin{equation}
E_{3}^{A-ABC} = -2 \sum_{i} \langle \Psi_{i}^{G-} | (\rho_{3}^{A-ABC}) | \Psi_{i}^{G-} \rangle,
\end{equation}
\begin{equation}
E_{3}^{A-ABD} = -2 \sum_{i} \langle \Psi_{i}^{G-} | (\rho_{3}^{A-ABD}) | \Psi_{i}^{G-} \rangle,
\end{equation}
\begin{equation}
E_{3}^{A-ACD} = -2 \sum_{i} \langle \Psi_{i}^{G-} | (\rho_{3}^{A-ACD}) | \Psi_{i}^{G-} \rangle.
\end{equation}

IV. ENTANGLEMENT OF FOUR QUBITS CANONICAL STATES

A composite system state $\tilde{\rho}$, obtained from an N-partite state $\tilde{\rho}$ through local unitary operations, differs from the former in being characterized by a different set of partial $K$-way negativities. A canonical state $\tilde{\rho}_{c}$ obtained from $\tilde{\rho}$ through entanglement conserving local unitary operations is a state written in terms of the minimum number of local basis product states $|11\rangle$. The coefficients in a canonical form are local invariants. The partial $K$-way negativities $E_{K}^{p}$ (defined in Eq. (12)), when calculated for a canonical state are functions of local invariants having unique values, as such qualify to be entanglement monotones for all the states lying on the orbit. For a canonical state $\tilde{\rho}_{c}$, $E_{K}^{p} - E_{0}^{p}$, measures the $K$-way entanglement of subsystem $p$ with its complement.
Veerstraete et al. (11), have shown that a pure state of four qubits can be transformed by Stochastic local operations and classical communication (SLOCC) to a state belonging to one of a set of nine families of states. In the next two sections, we focus on the calculation of partial $K$-way negativities for states representing the nine families of states. The analytical and numerical results are a pointer to the similarities and principle differences between the different sets of four qubit states. In the case of two parameter states, partial $K$-way negativity contours are plotted.
TABLE I: List of coefficients to represent $G_{abcd}$, $L_{abc2}$, $L_{a2b2}$, and $L_{a203\oplus 1}$ by a general state $|\Psi\rangle$. The Schmidt coefficients $\mu_0$, $\mu_1$, and squared global negativity $\left(N_G^A\right)^2$ are also listed.

| $|\Psi\rangle$ | $\alpha$ | $\beta$ | $\gamma$ | $\delta$ | $A$ | $B$ | $C$ | $D$ | $\mu_0$ | $\mu_1$ | $\left(N_G^A\right)^2$ |
|----------------|----------|----------|----------|----------|------|------|------|------|--------|--------|----------------------|
| $G_{abcd}$     | $a \mu b$ | $a \mu b$ | $b \mu c$ | $b \mu c$ | $a \mu d$ | $a \mu d$ | $b \mu c$ | $b \mu c$ | $\frac{1}{2}$   | $\frac{1}{2}$ | $1$                  |
| $L_{abc2}$     | $a \mu b$ | $a \mu b$ | $a \mu b$ | $c$       | $c$       | $d$       | $b$       | $c$       | $0$        | $\frac{1}{2}$   | $\frac{1}{2}$ - $d^4$ |
| $L_{a2b2}$     | $a$      | $b$      | $b$      | $b$      | $a$       | $0$       | $0$       | $0$       | $1 - |a|^2$ | $|a|^2$ | $4\left(|a|^2 - |a|^4\right)$ |
| $L_{a203\oplus 1}$ | $a$      | $b$      | $a$      | $b$      | $a$       | $0$       | $0$       | $0$       | $1 - |a|^2$ | $|a|^2$ | $4\left(|a|^2 - |a|^4\right)$ |

to identify the range of parameter values for which a particular $K$–way entanglement mode is dominant. We use the notation of ref. [10] to represent different families of four qubit states without going into the detailed meaning of the symbols used. The nine families of states are grouped in to two main classes on the basis of partial $K$–way entanglements associated with each family of states. Class I includes all the states having zero partial $3$–way entanglements. The most general states, having non zero partial $K$–way entanglements for $K = 2$, 3, and 4, belong to class II, which also includes three other states characterized by $E_3 \neq 0$. The states having genuine $K$–partite entanglement of a single type, that is four partite, tripartite, or bipartite are obtained for particular parameter values.

V. CLASS I - $E_3 = 0$

The families of states, $G_{abcd}$, $L_{abc2}$, $L_{a2b2}$, and $L_{a203\oplus 1}$ of ref. [10] belong in class I. A common feature of these states is a positive three way partial transpose, independent of the qubit with respect to which partial transpose is constructed. In the normal form, the states do not have three qubit correlations. All the states have genuine four–partite entanglement, which is destroyed when a measurement is made on the state of a single qubit. In addition the states may have four-partite entanglement due to pairwise entanglement. In this case, the mixed state $\rho_{ABC} = T_D \left(\tilde{\rho}\right)$ may, in turn, have W-like tripartite entanglement if two qubit coherences for qubit pairs $AB$, $AC$, and $BC$ are simultaneously nonzero. The reduced two qubit mixed states have pairwise entanglement.

Consider a general four qubit state $\hat{\rho} = |\Psi\rangle \langle \Psi|$ where

$$|\Psi\rangle = \alpha |0000\rangle + \beta |0111\rangle + \gamma |0101\rangle + \delta |0110\rangle + A |1111\rangle + B |1000\rangle + C |1010\rangle + D |0101\rangle,$$

with qubits 1, 2, 3, and 4, held by parties A, B, C, and D respectively. With proper choice of coefficients, as listed in Table I, the state $|\Psi\rangle$ represents all possible states belonging to class I.

By writing the state $|\Psi\rangle$ in Schmidt form for qubit $A$, the global negativity of partial transpose with respect to qubit $A$ is found to be $N_G^A = 2\sqrt{\mu_0\mu_1}$, where

$$\mu_0 = \left(|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2\right), \quad \text{and} \quad \mu_1 = \left(|A|^2 + |B|^2 + |C|^2 + |D|^2\right).$$

The Schmidt coefficients $\mu_0$, $\mu_1$, and squared global negativity $\left(N_G^A\right)^2$ are also listed in Table I. The eigenvector of partially transposed operator $\tilde{\rho}_{G}^{TA}$ corresponding to negative eigen value $\lambda^- = -\sqrt{\mu_0\mu_1}$ reads as

$$|\Psi_G^{A^-}\rangle = \frac{1}{\sqrt{2}} \left(\frac{\alpha}{\sqrt{\mu_0}} |1000\rangle + \frac{\beta}{\sqrt{\mu_0}} |1011\rangle + \frac{\gamma}{\sqrt{\mu_0}} |1101\rangle + \frac{\delta}{\sqrt{\mu_0}} |1110\rangle \right)$$

$$- \frac{1}{\sqrt{2}} \left(\frac{A}{\sqrt{\mu_1}} |0111\rangle + \frac{B}{\sqrt{\mu_1}} |0100\rangle + \frac{C}{\sqrt{\mu_1}} |0010\rangle + \frac{D}{\sqrt{\mu_1}} |0001\rangle \right).$$

Next, we construct $\rho_1^{TA}$ and $\rho_2^{TA}$ by applying the constraints given in Eqs. (14) and (17). It is straightforward to obtain

$$E_4^A = -2 \langle \Psi_G^{A^-} | \tilde{\rho}_1^{TA} | \Psi_G^{A^-}\rangle$$

$$= \frac{4}{N_G^A} \left(|A|^2 |\alpha|^2 + |B|^2 |\beta|^2 + |C|^2 |\gamma|^2 + |D|^2 |\delta|^2\right),$$

where $\tilde{\rho}_1^{TA}$ and $\tilde{\rho}_2^{TA}$.
where the qubits ABC (measurement operators $M$) measurement is made on the state of a particular qubit. Consider a projective measurement of qubit $K$ leading to the monogamy inequalities $P$ occurs with probability $N$.

Distinct features of states belonging to families $G_{abcd}$, $L_{abc}$, $L_{a2b2}$, and $L_{a2b3e1}$ are discussed below.

$$E_2^A = -2 \langle \Psi_G^A \mid \hat{\rho}_G^A \mid \Psi_G^A \rangle = N_G^A - E_4^A, \quad E_0^A = 0,$$

(33)

giving

$$N_G^A E_4^A = 4 \left( |A|^2 |\alpha|^2 + |B|^2 |\beta|^2 + |C|^2 |\chi|^2 + |D|^2 |\delta|^2 \right),$$

(34)

$$N_G^A E_2^A = 4 |A|^2 \left( |\beta|^2 + |\delta|^2 + |\chi|^2 \right) + 4 |B|^2 \left( |\delta|^2 + |\alpha|^2 + |\chi|^2 \right)$$

$$+ 4 |C|^2 \left( |\beta|^2 + |\alpha|^2 + |\delta|^2 \right) + 4 |D|^2 \left( |\alpha|^2 + |\beta|^2 + |\chi|^2 \right).$$

(35)

### A. Monogamy inequalities for four qubit states

We notice that for four qubit state $|\Psi\rangle$ of Eq. (29),

$$\left( N_G^A \right)^2 = N_G^A E_4^A + N_G^A E_2^A,$$

(36)

leading to the monogamy inequalities

$$N_G^A E_4^A \leq \left( N_G^A \right)^2, \quad N_G^A E_2^A \leq \left( N_G^A \right)^2.$$  

(37)

The partial $K$–way negativities calculated for the state $|\Psi\rangle$ represent the amount of entanglement lost when a measurement is made on the state of a particular qubit. Consider a projective measurement of qubit $D$, using measurement operators $M_0 = |0\rangle \langle 0|$ and $M_1 = |1\rangle \langle 1|$. The resulting three qubit state is a pure state decomposition (PSD) written as

$$\rho_{PSD}^{ABC} = P_0 |\Phi_0\rangle \langle \Phi_0| + P_1 |\Phi_1\rangle \langle \Phi_1|,$$

(38)

where the qubits $ABC$ are found in state

$$|\Phi_0\rangle = \frac{1}{\sqrt{P_0}} \left( \alpha |000\rangle + \delta |011\rangle + B |110\rangle + C |101\rangle \right),$$

(39)

with probability $P_0 = |B|^2 + |C|^2 + |\alpha|^2 + |\delta|^2$, and the three qubit state

$$|\Phi_1\rangle = \frac{1}{\sqrt{P_1}} \left( \beta |001\rangle + \chi |010\rangle + A |111\rangle + D |100\rangle \right),$$

(40)

occurs with probability $P_1 = |A|^2 + |D|^2 + |\beta|^2 + |\chi|^2$. Entanglement lost on measuring the state of qubit $D$ is $N_G^D E_D^4 + N_G^D E_D^2$. Defining the global negativity of state $\rho_{PSD}^{ABC}$ as

$$[N_{PSDG}^A(\rho^{ABC})] = [N_G^A(P_0 |\Phi_0\rangle \langle \Phi_0|)] + [N_G^A(P_1 |\Phi_1\rangle \langle \Phi_1|)],$$

(41)

we obtain

$$[N_{PSDG}^A(\rho^{ABC})] = 4 \left( B^2 + C^2 \right) \left( \alpha^2 + \delta^2 \right) + 4 \left( A^2 + D^2 \right) \left( \beta^2 + \chi^2 \right)$$

$$= [N_G^A]^2 - N_G^A E_D^4 - N_G^A E_D^2 - AD$$

$$= N_G^A E_D^2 - N_G^A E_D^2 - AD.$$

(42)

Distinct features of states belonging to families $G_{abcd}$, $L_{abc}$, $L_{a2b2}$, and $L_{a2b3e1}$ are discussed below.
FIG. 1: Contour plots displaying $N^A_G E_4^A$, and $N^A_G E_2^A$ for the states $G_{abcd}$ as a function of parameters $a$ and $b$.

B. Set of states $G_{abcd}$

The set of states $G_{abcd}$

$$G_{abcd} = \frac{a + d}{2} (|0000⟩ + |1111⟩) + \frac{a - d}{2} (|1100⟩ + |0011⟩) + \frac{b + c}{2} (|1010⟩ + |0101⟩) + \frac{b - c}{2} (|0110⟩ + |1001⟩), \quad (43)$$

are characterized by $N^A_G = 1$ and $E^A_3 = 0$. This result is consistent with the observation that the 3-tangle of the mixed states obtained by tracing out a single qubit of $G_{abcd}$ state is always equal to zero. Substituting $\alpha = A = \frac{a + d}{2}$, $\beta = B = \frac{a - d}{2}$, $\chi = C = \frac{b + c}{2}$, and $\delta = D = \frac{b - c}{2}$ in Eqs. (32) and (33) we get

$$E^A_4 = \frac{1}{4} \left( |a + d|^4 + |a - d|^4 + |b + c|^4 + |b - c|^4 \right), \quad (44)$$

and $E^A_2 = 1 - E^A_4$. Using the equality

$$\hat{\rho}^A_2 = \hat{\rho}^{T_2 - AB}_2 + \hat{\rho}^{T_2 - AC}_2 + \hat{\rho}^{T_2 - AD}_2 - 2\hat{\rho}, \quad (45)$$

we further split $E^A_2$ as

$$E^A_2 = -2 \langle \Psi^A_G | \hat{\rho}^{T_2 - AB}_2 | \Psi^A_G \rangle - 2 \langle \Psi^A_G | \hat{\rho}^{T_2 - AC}_2 | \Psi^A_G \rangle - 2 \langle \Psi^A_G | \hat{\rho}^{T_2 - AD}_2 | \Psi^A_G \rangle + 4 \langle \Psi^A_G | \hat{\rho} | \Psi^A_G \rangle, \quad (46)$$

$$-2 \langle \Psi^A_G | \hat{\rho}^{T_2 - AD}_2 | \Psi^A_G \rangle + 4 \langle \Psi^A_G | \hat{\rho} | \Psi^A_G \rangle, \quad (47)$$
obtaining

\[ E_{2}^{A-AB} = \frac{1}{2} \left( |a + d|^{2} |a - d|^{2} + |b + c|^{2} |b - c|^{2} \right), \]

\[ E_{2}^{A-AC} = \frac{1}{2} \left( |a + d|^{2} |b + c|^{2} + |a - d|^{2} |b - c|^{2} \right), \]

\[ E_{2}^{A-AD} = \frac{1}{2} \left( |a + d|^{2} |b - c|^{2} + |a - d|^{2} |b + c|^{2} \right). \] (48)

The states have genuine 4-partite entanglement but no genuine tripartite entanglement. Fig. (1) displays \( N_{G} E_{2}^{A} \), and \( N_{G} E_{2}^{A} \) for the states \( G_{abcd} \) for the special case where coefficients \( a, b, \) and \( d = c = \sqrt{(1 - a^{2} - b^{2})/2} \) are real coefficients.

1. **Entanglement lost on measuring the state of qubit D in state \( G_{abcd} \)**

On tracing over qubit \( D \), we get the three qubit state

\[ \rho^{ABC}_{PSD} = Tr_{D}(G_{abcd}) = \frac{1}{2} \left[ |W_{0}\rangle \langle W_{0}| + |W_{1}\rangle \langle W_{1}| \right], \] (49)

which is a mixture of normalized W-like states

\[ |W_{0}\rangle = 2 \left[ \frac{a + d}{2} |000\rangle + \frac{a - d}{2} |110\rangle + \frac{b + c}{2} |101\rangle + \frac{b - c}{2} |011\rangle \right], \] (50)

and

\[ |W_{1}\rangle = 2 \left[ \frac{a + d}{2} |111\rangle + \frac{a - d}{2} |001\rangle + \frac{b + c}{2} |010\rangle + \frac{b - c}{2} |100\rangle \right]. \] (51)

The loss of qubit \( D \) results in total loss of four-partite entanglement of the state \( G_{abcd} \). Besides that the two-way coherences involving qubit \( D \) are also annihilated. Recalling that in the state \( G_{abcd} \) all qubits have equal amount of \( K \)-way coherences, the mixed state \( \rho^{ABC}_{PSD} \) has

\[ \left[ N_{4}^{A}(\rho^{ABC}) \right]^{2} \leq \left( N_{G}^{A} E_{2}^{A-AB} + N_{G}^{A} E_{2}^{A-AC} \right) \] (52)

where

\[ N_{G}^{A} E_{2}^{A-AB} + N_{G}^{A} E_{2}^{A-AC} = \left( |a|^{2} + |d|^{2} \right) \left( |b|^{2} + |c|^{2} \right) \] (53)

On the other hand if party D measures the state of qubit \( D \) and communicates classically to parties \( A, B, \) and \( C \), W-like entangled states become available to parties \( A, B, \) and \( C \). Qubit \( D \) is found in state \( |0\rangle \), with probability \( \frac{1}{2} \), leaving the qubits \( ABC \) in normalized state \( |W_{0}\rangle \), whereas the result \( |1\rangle \) for the state of qubit \( D \) collapses the four qubit state to three qubit state \( |W_{1}\rangle \). Defining \( \rho_{W_{0}} = |W_{0}\rangle \langle W_{0}| \) and \( \rho_{W_{1}} = |W_{1}\rangle \langle W_{1}| \), it is found that

\[ \left[ N_{G}^{B} \left( \frac{1}{2} \rho_{W_{0}} \right) \right]^{2} + \left[ N_{G}^{B} \left( \frac{1}{2} \rho_{W_{1}} \right) \right]^{2} = \left( |a|^{2} + |d|^{2} \right) \left( |b|^{2} + |c|^{2} \right), \] (54)

as such

\[ \left[ N_{PSDC}^{B} \left( \rho^{BCD} \right) \right]^{2} = N_{G}^{A} E_{2}^{A-AB} + N_{G}^{A} E_{2}^{A-AC}. \] (55)

Entanglement loss due to loss of qubit \( D \) is caused by loss of information about the state of qubit \( D \). On further state reduction, two qubit entangled states are obtained from the states \( |W_{0}\rangle \), and \( |W_{1}\rangle \).

C. **Set of states \( L_{abc} \)**

For the set of normalized states

\[ L_{abc} = \frac{a + b}{2} (|0000\rangle + |1111\rangle) + \frac{a - b}{2} (|1100\rangle + |0011\rangle) + c (|1010\rangle + |0101\rangle) + d |0110\rangle, \] (56)
with $d^2 = 1 - 2|c|^2 - |b|^2 - |a|^2$ we get $(N_G^A)^2 = 1 - d^4$. The product of global negativity and 4-way negativity is found to be

$$N_G^A E_4^A = 4 \left( \frac{|a + b|^4}{2} + \frac{|a - b|^4}{2} + |c|^4 \right), \quad (57)$$

where as the product

$$N_G^A E_2^A = N_G^A (E_2^{AB} + E_2^{AC} + E_2^{AD}), \quad (58)$$

with

$$N_G^A E_2^{AB} = \frac{1}{2} |a + b|^2 |a - b|^2 + |c|^2 d^2, \quad (59)$$

$$N_G^A E_2^{AC} = 2 |a + b|^2 |c|^2 + |a - b|^2 d^2, \quad (60)$$

and

$$N_G^A E_2^{AD} = 2 |a - b|^2 |c|^2 + |a + b|^2 d^2. \quad (61)$$

### D. Sets of states $L_{a2b2}$ and $L_{a203b1}$

The general form for the family of states $L_{a2b2}$ is

$$L_{a2b2} = a((0000) + (1111)) + b((0101) + (1010)) + c((0110) + (0011)), \quad (62)$$

where

$$c^2 = 1 - 2|a|^2 - 2|b|^2 \leq 0 \leq |b| \leq \frac{1}{\sqrt{2}}, \quad 0 \leq |a| \leq \frac{1}{\sqrt{2}}. \quad (63)$$

The calculated squared Global negativity is given by $(N_G^A)^2 = 1 - 4c^4$ and $N_G^A E_4^A = 4 \left( |a|^4 + |b|^4 \right)$. The pairwise partial negativities read as

$$N_G^A E_2^{AB} = N_G^A E_2^{AD} = 4c^2 \left( |a|^2 + |b|^2 \right), \quad N_G^A E_2^{AC} = 8 |a|^2 |b|^2. \quad (64)$$

The states

$$L_{a203b1} = a((0000) + (1111)) + b((0101) + (0110) + (0011)) \quad (65)$$

have

$$(N_G^A)^2 = 4 \left( a^4 + 3a^2b^2 \right), \quad N_G^A E_4^A = 4a^4, \quad N_G^A E_2^A = 12 |a|^2 |b|^2, \quad (66)$$

with

$$N_G^A E_2^{AB} = N_G^A E_2^{AC} = N_G^A E_2^{AD} = 4 |a|^2 |b|^2. \quad (67)$$

and $trD \left( |L_{a203b1}\rangle \langle L_{a203b1}| \right)$ displays W-like entanglement. That means we can extract pairwise entanglement from the normal form.

### VI. CLASS II - $E_3 \neq 0$

The sets of states $L_{ab3}$ and $L_{a4}$, having 4-partite, tripartite and bi-partite entanglement are grouped in class II, along with the states $L_{0b3}$, $L_{073}$, and $L_{0b1}0331$ having $E_3 \neq 0$ while $E_4 = 0$. 


The states in the family $L_{ab3}$ have bipartite, tripartite as well as 4-partite entanglement. The normalized two parameter state has the general form

$$L_{ab3} = a \left( |0000\rangle + |1111\rangle \right) + \frac{a + b}{2} \left( |0101\rangle + |1010\rangle \right) + \frac{a - b}{2} \left( |0110\rangle + |1001\rangle \right) + \frac{ic}{\sqrt{2}} \left( |0001\rangle + |0010\rangle + |0111\rangle + |1011\rangle \right),$$

where $0 \leq |a| \leq \frac{1}{\sqrt{3}}$, $0 \leq |b| \leq 1$, and $c = \sqrt{(1 - b^2 - 3a^2)/2}$. Figs. 2 and 3 display the contour plots of global negativity and partial two, three and four way negativities calculated numerically from state operator partially transposed with respect to qubit $A$. For the two parameter states $L_{ab3}$, maximum value of $N_G^A E_4^A = 0.5$, occurs for $a = \frac{1}{\sqrt{3}}$, $b = 0$, $c = 0$ that is for the state

$$\Psi = \frac{1}{\sqrt{3}} \left( |0000\rangle + |1111\rangle \right) + \frac{1}{2\sqrt{3}} \left( |0101\rangle + |1010\rangle \right)$$

$$+ \frac{1}{2\sqrt{3}} \left( |0110\rangle + |1001\rangle \right).$$

Single parameter states
FIG. 3: Contour plots displaying $N_G^A(E_A^4 - E_0^A)$ and $N_G^A(E_2^4 - E_0^A)$ for the states $L_{ab}$, as a function of parameters $a$ and $b$.

\[
L_{a4} = a \left( |0000\rangle + |0101\rangle + |1010\rangle + |1111\rangle \right) \sqrt{1 - 4a^2/3} \left( i |0001\rangle + |0110\rangle - i |1011\rangle \right),
\]  

(69)

are also characterized by partial two, three and four way negativities such that

\[
(N_G^2)^2 = \frac{8}{9} \left( a^2 - 2a^4 + 1 \right), \quad N_G^A E_4^A = 8a^4,
\]

(70)

\[
N_G^A E_3^A = \frac{4}{9} \left( 4a^2 - 32a^4 + 1 \right), \quad N_G^A E_2^A = \frac{4}{9} \left( 10a^4 - 2a^2 + 1 \right).
\]

(71)

The global negativity and partial negativities as a function of parameter $0 \leq a \leq \frac{1}{2}$ are displayed in Fig. (4) for qubit $A$ and in Fig. (5) for qubit $D$. We notice that the state is in Schmidt like form for qubit $A$ with $E_0^A = 0$.

B. The states $L_{05^{03}_3}$, $L_{07^{01}_3}$ and $L_{03^{11}_3}^{03_{11}}$

A common feature of the states $L_{05^{03}_3}$, $L_{07^{01}_3}$ and $L_{03^{11}_3}^{03_{11}}_0$ is $E_4 = 0$. The state

\[
L_{05^{03}_3} = \frac{1}{2} \left( |0000\rangle + |0101\rangle + |1000\rangle + |1110\rangle \right)
\]

(72)
FIG. 4: Global and partial negativities for qubit $A$ versus parameter $a$ for the states $L_a$.

has only genuine tripartite and bi-partite entanglement, while $(N_A^G)^2 = (N_B^D)^2 = 0.75$. The tripartite entanglement for qubits $ABC$, $ABD$, and $ACD$ is found to be equal that is

$$N_A^G E_{3}^{A-ABC} = N_A^G E_{3}^{A-ABD} = N_A^G E_{3}^{D-ACD} = 0.25,$$

while $N_D^G E_{2}^{D-BD} = 0.25$. For the mixed state

$$\rho_{PSD} = Tr_D(|L_{0\oplus 3}\rangle \langle L_{0\oplus 3}|) = \frac{3}{4} |T_0\rangle \langle T_0| + \frac{1}{4} (|010\rangle \langle 010|),$$

where

$$T_0 = \frac{1}{\sqrt{3}}(|000\rangle + |100\rangle + |111\rangle),$$

the relation

$$\left[ N_A^G \left( \frac{3}{4} |T_0\rangle \langle T_0| \right) \right]^2 + \left[ N_A^G \left( \frac{1}{4} (|010\rangle \langle 010|) \right) \right]^2 = N_A^G E_{3}^{A-ABC},$$

holds. Three tangle of the mixed state is also found to be 0.25 if qubit $B$, $C$ or $D$ is traced out and zero if qubit $A$ is traced out. Qubit $A$ has genuine tripartite entanglement in state

$$L_{0\oplus 3} = \frac{1}{2} (|000\rangle + |101\rangle + |110\rangle + |111\rangle),$$

giving $(N_A^G)^2 = N_A^G E_{3}^{A} = 0.75$, and $N_A^G E_{3}^{A-ABC} = N_A^G E_{3}^{A-ABD} = N_A^G E_{3}^{A-ACD} = 0.25$. The partial negativities for qubit $D$ are

$$(N_D^G)^2 = 1.0, \quad N_D^G E_{3}^{D} = 0.5, \quad N_D^G E_{2}^{D} = 0.5,$$

$$N_D^G E_{3}^{D-ABD} = N_D^G E_{3}^{D-ACD} = 0.25, \quad N_D^G E_{2}^{D-BD} = N_D^G E_{2}^{D-CD} = 0.25.$$
FIG. 5: Global and partial negativities for qubit $D$ versus parameter $a$ for the states $L_{a4}$.

The state $Tr_D(|L_{0a03}\rangle \langle L_{0a03}|)$, has GHZ like correlations between the qubits $ABC$, whereas, $Tr_A(|L_{0a03}\rangle \langle L_{0a03}|)$, has pairwise residual entanglement.

The product of separable qubit $A$ and three qubit GHZ state constitutes the state

$$L_{0a010\bar{a}1} = \frac{1}{\sqrt{2}}(|0000\rangle + |0111\rangle),$$

(79)

with $(N_A)^2 = 0$, $(N_G)^2 = N_G^p E^p_3 = 1.0$ for $p = B$ or $C$ or $D$.

VII. CONCLUSIONS

To summarise, the nine families of four qubit states obtained by Versraete et al. [10] can be grouped in two distinct classes. Four sets of states, $G_{abcd}$, $L_{abc2}$, $L_{a2b2}$, and $L_{a20a1}$, have non zero partial 4-way and 2-way negativities, while 3-way partial negativities are zero in the normal form. The families of states $L_{a03}$ and $L_{a4}$ are distinctly different from the sets of states in the first category in that the states have bi, tri as well as 4-partite entanglement in the normal form. The states $L_{0a03}$ and $L_{0a03}$ characterized by $E_4 = 0$, $E_3 \neq 0$, $E_2 \neq 0$, along with the state $L_{0a010\bar{a}1}$, having $E_4 = 0$, $E_3 \neq 0$, $E_2 = 0$, are also included in Class II. Partial four way negativity is a measure of genuine 4-partite entanglement of the state. Three-way partial negativity determines the probabilistic entanglement that becomes available to the three parties after the fourth party measures the state of the qubit it holds. Two-way negativity measures the pairwise entanglement.

The coefficients in a normal form being local invariants, the partial $k$-way negativities are proper entanglement measures satisfying the conditions of normalization, convexity and monotonicity. Whereas, the global negativity with respect to a given qubit $p$ gives information about the amount of multipartite entanglement that is lost on the loss of qubit $p$, the partial $K$-way negativities give detailed information about the nature and distribution of quantum correlations lost due to the loss of a qubit. The partial $K$-way negativities are meaningful polynomials of local invariants. Local unitary rotations on the state in normal form may enhance a given set of partial $K$-way negativities at the cost of others. For a state that is not in normal form, the partial $K$-way negativities measure the coherences present in the composite system. The monogamy relations obtained, naturally, for the 4-partite, tripartite and bipartite entanglement of a given qubit, provide further insight into entanglement distribution in four qubit states. We believe that quantifying the multipartite quantum correlations through partial $K$-way negativities will facilitate
the construction and implementation of quantum information processing protocols.

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