The Sunyaev-Zel’dovich Effect and the Value of $\Omega_o$

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Abstract. We consider the Sunyaev-Zel’dovich (SZ) effect as a probe of $\Omega_o$: Using a self-consistent modeling of X-ray clusters, we examine the dependence of both the mean Compton $y$ parameter and the SZ source counts on $\Omega_o$. These quantities increase with decreasing $\Omega_o$ due to the earlier epoch of structure formation in low-density cosmogonies; the results depend only on the quantity of gas heated to the virial temperature of collapsed objects and are independent of the spatial distribution of the gas in the potential wells. Specifically, we compare two models which reproduce the present-day abundance of clusters - a biased, critical universe and an unbiased, open model with $\Omega_o = 0.3$. We find that the mean $y$ parameter approaches the current FIRAS limit of $y < 2.5 \times 10^{-5}$ for the open model, demonstrating the importance of improving this limit on spectral distortions, and that the SZ source counts and corresponding redshift distribution differ significantly between the two cosmogonies; millimeter surveys covering a large area should thus provide interesting constraints on the density parameter of the Universe and on the evolution of the heated gas fraction in virialized objects.

Key words: cosmology

1. Introduction

One of the fundamental differences between high- and low-density cosmogonies concerns the rate of structure formation. If we measure the density of the Universe by the dimensionless parameter $\Omega_o \equiv 8\pi G \rho/3H_o^2$, where $H_o$ is the current value of the Hubble constant, then the linear growth factor, $D_s(z)$, which governs the rate of structure formation, tends to a constant for redshifts $1 + z < 1/\Omega_o$. In other words, the objects and structure observed today existed relatively unchanged out to redshifts of order $1/\Omega_o - 1$. This has been illustrated and proven in a formal way by Oukbir and Blanchard (1992,1995): The redshift distribution of clusters of a given mass is almost independent of the primordial power spectrum, but depends sensitively on the cosmic density, with a tail extending to large redshifts for low $\Omega_o$. This provides us with a probe of $\Omega_o$.

Because of their rarity, which is naturally interpreted in gaussian theories of structure formation as the result of their being density perturbations on the tail of the probability distribution, galaxy clusters are particularly sensitive to differences in the linear growth rate. The discovery of significant numbers of clusters at redshifts greater than 1 would be a strong argument for a low-density universe. The problem is finding, if they exist, such high redshift clusters. Optical identifications of clusters become quite difficult at large redshifts due to the large number of projected foreground and background galaxies at the same magnitudes as the cluster galaxies; moreover, our lack of understanding of galaxy evolution and its possible dependence on environment raise doubts on the interpretation, in the present context, of optically detected, high redshift clusters. Identifications based on the X-ray emission of the intrachannel medium (ICM), heated to the virial temperature of the gravitational potential well, fare better in this regard, but the X-ray luminosity, and thus the detectability of a cluster, strongly depends on the core radius of the gas distribution; the $n^2$ dependence of the free-free emissivity guarantees that the core region of the cluster dominates the total luminosity. The problem is that the physics determining this core radius is not well understood.

The same hot gas responsible for the X-ray emission also produces a spectral distortion of the cosmic microwave background (CMB) known as the Sunyaev-Zel’dovich (SZ) effect (Sunyaev & Zel’dovich 1972). Inverse Compton scattering of the photons off the hot electrons in the ICM causes the diffusion of low energy photons from the Rayleigh-Jeans region into the Wien tail of the (originally unperturbed blackbody) spectrum of the CMB. This results in a truly unique spectral signature – at wavelengths longward of 1.34 mm, the cluster appears as a decrement in the mean sky brightness, while at shorter wavelengths one observes the cluster as a source of emission in excess of the mean sky brightness. Quantitatively, one expresses the change in sky brightness relative to the mean CMB intensity, which we will refer to as the surface brightness of the source, although it is negative at low frequencies, as

$$i_\nu = y(\theta)j_\nu(x),$$

where $j_\nu$ describes the frequency dependence, and the Compton $y$ parameter, which determines the magnitude of the distortion, is given by an integral along the line-of-sight through the cluster:
\[
y \equiv \int \frac{dT}{m_e c^2} n_e \sigma_T.
\] (2)

In this latter expression, \( T \) is the temperature of the ICM (strictly speaking, of the electrons), \( m_e \) is the electron rest mass, \( n_e \) is the electron density and \( \sigma_T = 6.65 \times 10^{-25} \text{cm}^2 \) is the Thompson cross section. Note that \( y \) is a function of angular position \( \theta \) on the cluster image. The spectral function \( j_\nu \) may be written in terms of the dimensionless frequency \( x \equiv h_\nu T_o/kT_o \), where \( h_\nu \) is Planck’s constant and \( T_o \) is the current temperature of the CMB – 2.726K (Mather et al. 1994) – as

\[
j_\nu(x) = 2\left(\frac{KT_o}{(h_\nu c)^2}\right)^3 \frac{x e^x}{(e^x - 1)^2} \left[ \frac{\tanh(x/2)}{x} - 4 \right].
\] (3)

The flux of a cluster, which is measured in mJy (= \( 10^{-26} \text{ergs/s/cm}^2/\text{Hz} \)), is the integral of the surface brightness \( i_o(\theta) \) over the solid angle subtended by the cluster:

\[
S_o(x, M, z) = j_\nu(x) D_o^{-2}(z) \int d\nu \frac{kT(M, z)}{m_e c^2} n_e(M, z) \sigma_T;
\] (4)

the integral is over the cluster volume and the angular distance \( D_o(z) = 2\eta H_o^{-1}\Omega_o z + (\Omega_o - 2)(\sqrt{1 + \Omega_o z} - 1)/\Omega_o(1+z)^3 \).

We see from this expression that the integrated effect of a cluster, i.e., its flux, depends only on the quantity of gas at temperature \( T \), which we will take to be the virial temperature, and not on the spatial distribution of the gas; as we have seen, this is in contrast to the X-ray flux.

The other property of importance in the present context is the distance independence of \( y \): for a given set of cluster properties, the value of \( y \) remains independent of redshift. This implies that the mean distortion, averaged over the full sky, equally weights the contributions from clusters at low and high redshifts. Because the cluster population extends back to larger redshifts in an open universe, we expect the mean \( y \) to be bigger than for a critical universe (Cavaliere et al. 1991; Markevitch et al. 1991). This is the characteristic which makes the SZ effect a potent probe of \( \Omega \).

The situation is similar for the source counts: The fact that one can observe clusters at high \( z \) means that the counts will be larger for a low-density universe (Korolov et al. 1986; Markevitch et al. 1994). As an instructive exercise, we may compare the behavior of the SZ and X-ray fluxes with simple scaling laws: \( f_S \sim M/T D_o^2(z) \); while for the X-ray flux we have \( f_X \sim M H_o^2 D_o^3(z) \). We have written the baryonic mass of the cluster as \( M \), its temperature as \( T \) and its gas density as \( n \). The dependence of the X-ray flux on gas density, and therefore on the spatial distribution of the ICM, is manifest by the explicit appearance of \( n \) in the expression for \( f_X \). Using \( T \sim M^{2/3} (1+z) \) (see below), we find that \( f_S/f_X \sim (1+z)^{4/3}/n \sim (1+z)^{1.3} \), the last equality following for self-similar scaling, \( n \propto (1+z)^{1.3} \). In comparison with X-ray observations, one probes deeper in redshift with the SZ effect and, at the same time, avoids a sensitivity to the core radius of the gas distribution.

2. Mean Spectral Distortion and Source Counts

For our calculations we adopt the Press-Schechter (1974) mass function:

\[
\frac{dn}{d\ln M} d\ln M = \sqrt{2\pi} \frac{\hat{\rho}}{M} \nu(M, z) \left( -\frac{d\ln \sigma}{d\ln M} \right) e^{-\nu^2/2} d\ln M,
\] (5)

in which \( \hat{\rho} \) is the comoving, or present-day, mass density of the Universe. The quantity \( \nu = \delta D_o(z)/\sigma(M) \) gives the height of the over-dense regions collapsing at redshift \( z \), with a linear density contrast of \( \delta \), relative to the amplitude of the density perturbations at that epoch, \( \sigma(M)/D_o(z) \), where \( \sigma(M) \) is the amplitude of the density perturbations today. The linear overdensity corresponding to a just virialized object, \( \delta \), depends weakly on redshift if \( \Omega_o \neq 1 \) and equals 1.68 if \( \Omega_o = 1 \) (Oukbir & Blanchard 1995).

This formula demonstrates quantitatively the origin of the particular sensitivity of clusters to the linear growth rate. Clusters typically represent perturbations of several \( \nu \) today, and thus rapidly disappear as the average perturbation amplitude, \( \sigma(M)/D_o(z) \), decreases towards higher \( z \).

As per the discussion in the Introduction, the mean Compton distortion depends only on the temperature distribution of the baryons, not on their spatial distribution:

\[
<y> = \int \sigma_T c dt \int dT \frac{dT}{dT} \frac{kT}{m_e c^2}
\]

\[
f_B \int \frac{dz}{dz} \sigma_T (1+z)^3 \int d\ln M \frac{d\ln M}{d\ln M} \frac{\chi M kT(M, z)}{m_p m_e c^2},
\] (6)

where, in the first line, \( \bar{n}_e \) is the mean electron density. In the second line, \( m_p \) is the proton mass, \( \chi \) is the number of electrons per baryon, and the extra factor of \((1+z)^3\) changes the comoving density in eq. 1 to a proper density. For a primordial gas composition, \( \chi = 0.88 \).

Detailed X-ray imaging of clusters, such as Coma, indicate baryon fractions \( f_B \) as large as \( 0.05h^{-3/2} \) (\( h \equiv H_o/100 \text{km/s/Mpc} \)) (Briel et al. 1992; Mushotsky 1993). This poses a problem for flat cosmologies (White et al. 1993a) if one accepts the baryon density dictated by primordial nucleosynthesis – \( \Omega_B h^2 = 0.012 \) (Walker et al. 1991) – for one expects \( f_B = \Omega_B/\Omega_o \). In the open model we have chosen, with \( \Omega_o = 0.3 \) and \( h = 0.5 \), presented below, the observed baryon fraction of clusters is consistent with the theory of primordial nucleosynthesis. To isolate the effects of \( \Omega_o \), we choose the same value of \( f_B = 0.2 \) for the critical model, even though this represents a violation of primordial nucleosynthesis theory. One may easily scale the results presented in our figures to any desired baryon fraction by multiplying \( <y> \) by \( f_B/0.20 \) and by sliding the counts at a given \( S_o \) to \( f_B/0.20 S_o \), everything else being equal.

Straightforward scaling arguments tell us that the temperature of a virialized object \( T \sim M/R \), and that the virial radius \( R \sim M^{1/3} \Delta^{-1/3} \Omega_o^{-1/3} (1+z)^{-1} \). The spherical collapse model permits the calculation of the (non-linear) over-density, \( \Delta(z) \), at the time of virialization and of the other numbers needed to find the coefficients of these scaling relations. The result for a critical universe is in good agreement with hydrodynamical simulations (Evrard 1990), which give \( T(M, z) = T_{15} M_1^{2/3} (1+z) \) with \( T_{15} = 6.4 h^{2/3} \text{keV} \); it is convenient to express the mass \( M \) of a cluster in terms of \( M_{15} \equiv M/10^{15} M_o \). Taking this as a normalization, we henceforth adopt a temperature–mass relation of the form: \( T(M, z) = T_{15} (z) M_1^{2/3} (1+z) \) with \( T_{15}(z) = (6.4 \text{keV}) h^{2/3} \Omega_o^{1/3} (\Delta(z)/178)^{1/3} \) (\( \Delta = 178 \) in a critical universe, independent of \( z \)).

To perform the integral in eq. 1, we must also choose the form and normalization of the power spectrum, \( \sigma(M) \), at the present epoch. In this paper, we use a power–law form for
the power spectrum $- \sigma(M) = M^{-\alpha}$ - which translates into $P(k) \propto k^\alpha$, with $\alpha = (n + 3)/6$, in Fourier space. One usually expresses the normalization of the power spectrum in terms of the amplitude of the density perturbations in spheres of radius $8h^{-1}$ Mpc, or $\sigma_8$. Following Oukbir & Blanchard (1995), we determine $n$ and $\sigma_8$ by transforming the PS mass function with the temperature–mass relation and fitting to the observed temperature distribution at the present epoch as determined by Edge et al. (1990) and Henry & Arnaud (1991). For a critical universe with $h = 0.5$, we find $n = -1.85$ and $\sigma_8 = 0.6$ (Henry & Arnaud 1991; Blanchard & Silk 1992; Bartlett & Silk 1993; White et al. 1993b; Oukbir & Blanchard 1995). Note that a cold dark matter model with $h = 0.5$ has $n = -1$ on cluster scales and thus does not provide a good fit to the shape of the temperature distribution function (in addition, when the model is normalized to the COBE fluctuation amplitude, $\sigma_8$ is $\sim 1$, i.e. much too large for $\Omega_0 = 1$).

Because the mean density determines the mass contained in spheres of radius $8h^{-1}$ Mpc, $\sigma_8$ will depend on $\Omega_0$ if one is trying to fit a given cluster temperature distribution. For concreteness, we choose an unbiased low-density model; fixing $\sigma_8 = 1$ while adjusting $n$ and $\Omega_0$, Oukbir & Blanchard (1995) find that $n = -1.42$ and $\Omega_0 = 0.32$ provide the best fit to the temperature distribution function. Here again we assume $h = 0.5$. This will serve as our example of a low-density universe.

We prefer this fitting to the temperature function over one using the X-ray luminosity function because the temperature–mass relation is essentially based on energetics, while the core radius strongly influences the X-ray luminosity of a cluster. Since each model is now normalized to the present-day cluster abundance, a comparison of their predictions permits us to directly examine the influence of $\Omega_0$ on SZ observations.

We compare the mean CMB spectral distortion in the two models in figure 1, where we show the mean $y$ value integrated out to redshift $z$ as a function of $z$. The various curves for each value of $\Omega_0$ correspond to different lower bounds on the mass integral in eq. (6). With the cutoff mass set to zero, the integrated $< y >$ in an Einstein–de-Sitter universe reaches a value of $\sim 3 \times 10^{-6}$, while in the low-density universe it reaches an asymptotic value at large redshift of more than $5 \times 10^{-5}$! - well above the present FIRAS limit of $y < 2.5 \times 10^{-5}$ (Mather et al. 1994). However, one should take into account that the gas in low temperature halos cools within a Hubble time, thereby providing a low mass cutoff to the integral; this mass is typically of the order of $10^{12} M_\odot$, almost independent of redshift (Blanchard et al. 1992). One should also take into account Compton cooling, which becomes efficient at redshifts greater than $\sim 5$. Nevertheless, in a low-density universe at redshift 5, the difference between a cutoff mass of $10^{12} M_\odot$, or even one as large as $10^{13}$, and a zero mass cutoff is quite small; and the predicted $< y >$ reaches $10^{-5}$ - just below the current limit. We conclude that the expected mean Compton distortion for a low-density Universe remains just below the current limits from FIRAS and that an effort to reduce the limits on $< y >$ would provide an interesting constraint on low density models (or a detection!). In a critical universe, on the other hand, $< y >$ should be about one order of magnitude below the present limit; with an $f_B$ as predicted by primordial nucleosynthesis, this becomes even lower, by a factor of $\sim 3$.

Next consider the number of SZ sources on the sky brighter than a given threshold $S_\nu$ (expressed in mJy). We simply integrate the PS mass function over all objects with a flux density brighter than $S_\nu$:

$$\frac{dN}{dV}(S_\nu) = \int dz \frac{dV}{dz} \int dM d\nu \frac{dM dn}{dM}.$$  

(7)
Fig. 2. The SZ source counts (right) and redshift distribution for several values of $S_\nu$ (left). The calculation is for a wavelength of 0.75 mm and an $h = 0.5$.

In this equation, $M_{\text{min}}$ is the mass corresponding to the threshold, $S_\nu$, as determined by relation (8):

$$S_\nu = (8 \text{ mJy} h^{8/3}) f_\nu(x) f_B \Omega_o^{1/3} M_{15}^{5/3} \Delta(z)^{1/3} (1+z) D^{-2}(z),$$

where $f_\nu(x)$ and $D(z)$ are the dimensionless parts of eq. (3) and the angular distance, respectively (in both cases, without the factor 2). This procedure gives the number of objects per solid angle on the sky as a function of the total flux density of the objects - in other words, it assumes that the sources are unresolved in the experimental beam.

We present the calculated source counts at 0.75 mm for the two cosmogonies in figure 2. The redshift distribution for each model is also shown, in the adjacent panel. In a low–density universe one predicts a larger number of sources at a given threshold and a broader distribution in redshift, extending out to higher redshifts, than would be the case in a critical universe. Thus, we see the effect of the geometry on the linear growth rate: in a low–density universe, clusters exist out to larger redshifts and, hence, appear in greater numbers in the source counts. In a manner similar to X-ray selected clusters, the redshift distribution of SZ clusters offers a unique way to determine the mean density of the universe - the shape of this distribution provides a robust indicator of the mean density.

The advantage of SZ cluster catalogs, over X-ray selected samples, is that the detection criteria are insensitive to the gas core radius and its evolution.

3. Discussion

The Sunyaev-Zel’dovich effect is a natural complement to optical and X-ray studies of galaxy clusters, and, in contrast, suffers neither from projection effects nor from a sensitivity to the spatial distribution of the hot, intracluster gas. It simply probes the quantity of gas heated to the virial temperature of collapsed objects (we consider here only unresolved sources). In this light, the SZ effect serves as the more robust probe of the evolution of this gas fraction and of the value of $\Omega_o$, the latter determining the epoch of structure formation: In a low–density universe, structure must exist back to a redshift $1 + z \sim 1/\Omega_o$, for the linear growth of fluctuations stops at lower redshifts due to the more rapid expansion of space. A low–density universe thus provides a longer baseline over which to integrate the cluster SZ effect and should produce both a larger $<y>$ and greater number counts, with a larger mean redshift, than a critical universe. This is what we see in figures 1 and 2.

As demonstrated in figure 1, the mean spectral distortion of the CMB depends surprisingly little on the lowest masses one includes in the calculation, at least if one agrees that masses as low as, say, $10^{13} M_\odot$ can safely be included in the computation. Then, the $<y>$ contributed by structure back to a redshift 5 in our low–density model reaches $\sim 10^{-5}$; beyond a redshift of $\sim 5$, Compton cooling dominates and should effectively eliminate the hot gas in most structures. This is only a factor of 2–3 below the current FIRAS limit; lower cosmic densities would predict distortions even closer to the FIRAS limit. This dependence of $<y>$ on $\Omega_o$ has already been remarked on by Cavaliere et al (1991) and Markevitch et al (1991). Further effort to increase the sensitivity to Compton distortions would provide useful constraints on low–density models, or, perhaps, the more interesting possibility of a detection. Motivation for pushing down limits on $<y>$ also comes from consideration of the physical state of a uniform intergalactic medium: One could probe the quantity of gas in the medium (Bartlett et al. 1995).

As first noted by Korolev et al (1986), cluster source counts also prove to be a useful probe of $\Omega_o$. The counts expected in
our two fiducial models are shown in figure 2. A SZ selected cluster catalog would be the best choice for studying the evolution of the gas fraction in clusters and $\Omega_\Lambda$. Of particular importance is the difference in the number of detected sources, at a given threshold, between low and high density cosmologies and the fact that the detected clusters will extend out to much larger redshifts in the low density case - a smoking gun for a low value of $\Omega_\Lambda$ (which is to say that if such high-redshift clusters exist, this would be a very strong indication for a low-density universe, although if they do not exist, one might still argue that $\Omega_\Lambda$ is low, but that the hot gas fraction drops rapidly with look-back time). A study of this kind requires mapping a significant portion of the sky at millimeter and sub-millimeter wavelengths. This would be possible with a satellite similar to the SAMBA/COBRA mission proposed to the European Space Agency; such a mission could cover most of the sky with a sensitivity to SZ point sources on the order of $\sim 100$ mJy. From figure 2, we see that this is a sensitivity perfectly matched for detailed studies of the evolution of clusters and $\Omega_\Lambda$.

We emphasize what we feel to be an important aspect of the present work: that in comparing a high and a low–density model, to distinguish the influence of $\Omega_\Lambda$, we have insisted that the power spectra in each case reproduce the present-day abundance of galaxy clusters as measured by the temperature distribution function.

We have not considered the fluctuations induced in the CMB by the SZ effect in this paper, but leave this to a future work (Barbosa et al. 1996). This issue has been addressed by numerous authors using a variety of approaches (Shaeffer & Silk 1988; Cole & Kaiser 1989; Bond & Meyers 1991; Markevitch et al. 1992; Bartlett & Silk 1994; Colafrancesco & Vittorio 1994). In contrast to the mean CMB spectral distortion and the SZ source counts, the induced temperature fluctuations depend on the spatial distribution of the gas within clusters and on the distribution of the clusters themselves.

Finally, we remark that flat, low-density models, with a non-zero cosmological constant, would produce distortions and counts somewhere in between the predictions presented here - linear growth of density perturbations in such models turns off at lower redshifts than in the equivalent open model, but still higher than for the $\Omega_\Lambda = 1$ case.

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