A motion compensation method for airborne SAR imagery

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Abstract: A novel motion compensation method, which is based on the approximate total least squares (ATLS) algorithm, for wide-beamwidth SAR systems is proposed in this paper. The method is suitable for situations where both the estimated phase errors and the geometry of SAR imagery are corrupted by noise. The precondition that the noise belongs to a certain distribution model is not necessary, which makes it more robust for many kinds of scene content. As a consequence, higher accuracy of the estimated motion error and better focused image are achieved. Simulated and real data imaging demonstrate the validity of the proposed method.

Keywords: airborne synthetic aperture radar (SAR), approximate total least squares (ATLS), motion errors, phase gradient autofocus (PGA)

Classification: Microwave and millimeter wave devices, circuits, and systems

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1 Introduction

Motion compensation is an effective technique to obtain high azimuth resolution in synthetic aperture radar (SAR) systems [1]. Motion errors are usually compensated by the flight information provided by inertial navigation system (INS) and global positioning system (GPS) mounted on-board the aircraft. Alternatively, motion errors beyond the measurement capabilities could be directly estimated from the raw data by means of motion compensation techniques [2, 3].

Up to now, some databased motion compensation approaches have been proposed, in which autofocus technique is employed. The phase gradient autofocus (PGA) algorithm is one of the most popular autofocus algorithms [4]. Improvements in the PGA algorithm are presented in previous literature [5, 6, 7]. These improvements either cannot provide an accurate estimation in low signal-to-noise ratio (SNR) cases, or the calculation of weights, which are related to the signal-to-clutter of the received signal, leads to unavoidable errors and high computational complexity.

The objective of this paper is to present a novel motion compensation method, which utilizes the approximate total least squares (ATLS) algorithm as the estimation kernel. The conventional estimation algorithms suppose the geometry of SAR imagery, is precisely known. Contrarily, the ATLS algorithm is optimal for the case when perturbations exist both in the estimated phase errors and in SAR imaging geometry, which makes it a robust PGA-based algorithm for motion errors estimation from the raw data. It avoids the calculation of weights as well. This method is evaluated by both simulated and real SAR data tests, and is found to be an efficient approach for motion error estimation.

The rest of this paper is organized as follows. Firstly, the motion error estimation model for airborne SAR is introduced. Then the proposed motion compensation method and the experimental settings and results are described in detail. Finally, conclusions are driven.

2 Motion error estimation model

The transverse of airborne SAR motion error geometry is shown in Fig. 1. Supposing the horizontal and vertical component of line of sight (LOS) deviations are $y(t)$ and $h(t)$ respectively, the difference between the actual and nominal position at point $P$ in the cross-track direction could be calculated by

$$\Delta R = R_1 - R_0 \approx -y(t) \sin \theta_{tk} + h(t) \cos \theta_{tk}$$  \hspace{1cm} (1)

where $\theta_{tk}$ is the incidence angle of the $k$th range bin, and it is given by

$$\theta_{tk} = \arccos \frac{H}{r_0 + k\Delta r}$$  \hspace{1cm} (2)

where $H$ represents the height of the platform above the horizon. $r_0$ is the range to the zeroth sample, and $\Delta r$ is the range bin size. Hence, the phase errors due to the LOS deviations could be written as

$$\Delta \Phi = \frac{4\pi}{\lambda} \Delta R = \frac{4\pi}{\lambda} [-y(t) \sin \theta_{tk} + h(t) \cos \theta_{tk}]$$  \hspace{1cm} (3)
where \( \lambda \) is the wavelength of the system. For ground targets, different incidence angles result in different phase errors. Several strips in the cross-track direction could be chosen to estimate their azimuth phase errors using the standard stripmap PGA. Then the motion errors at azimuth time \( t \) can be calculated by substituting the phase errors into equation (3). For each chosen strip, the incidence angle of midpoints is used to calculate the motion errors.

The phase error matrix could be obtained after getting the phase errors by PGA.

\[
b = \left[ \Delta \Phi_{t_1} \quad \Delta \Phi_{t_2} \quad \ldots \quad \Delta \Phi_{t_k} \right]^T.
\]

Substituting equation (4) into equation (3) yields a linear equation

\[
b = Ax
\]

where

\[
A = \frac{4\pi}{\lambda} \begin{bmatrix}
-\sin \theta_{t_1} & \cos \theta_{t_1} \\
-\sin \theta_{t_2} & \cos \theta_{t_2} \\
\vdots & \vdots \\
-\sin \theta_{t_k} & \cos \theta_{t_k}
\end{bmatrix}
\]

\[
x = \begin{bmatrix} y(t) \\ h(t) \end{bmatrix}.
\]

\( A \) is the incidence angle matrix, and \( x \) represents the motion error vector.

There are two error sources in the motion error estimation model. One comes from the inaccurate geometry of SAR imagery. The other is the estimation error of phase errors obtained by the standard stripmap PGA. All of these factors will have a notable influence on the estimation accuracy of the motion error.

![Fig. 1. Geometry of airborne SAR motion.](image)

3 Motion error estimation

3.1 ATLS estimation of motion errors

The relationship between the estimated phase errors and the motion errors is built through the geometry of SAR imagery as shown in section 2. The ATLS algorithm is introduced to compensate the noise presenting in both the phase error matrix \( b \) and the incidence angle matrix \( A \) in the aim of getting the motion errors accurately, and the noise could be arbitrary. After getting the phase error gradients with the standard stripmap PGA, (5) can be rewritten as
\[ \dot{b} + e = (A + E)\dot{x} \]  

where \( \dot{b} \) represents the phase error gradient matrix, and \( \dot{x} \) is the motion error gradient matrix. \( e \) and \( E \) are the perturbations existing in \( \dot{b} \) and \( A \), respectively. Take several linear transformations into (8) [8], and it yields

\[
\begin{bmatrix}
[A : -\dot{b}] \\
[E : -e]
\end{bmatrix}
\begin{bmatrix}
\dot{x} \\
1
\end{bmatrix} = 0.
\]

(9)

Here let \( D = [E : -e] \) be the noise matrix, and \( S = [A : -\dot{b}] \) be the augmented matrix.

The total least squares (TLS) algorithm is a natural generalization of the least squares algorithm when the data in both \( A \) and \( b \) is perturbed. It has the ability of compensating the noise in \( A \) and \( b \) systematically. The TLS solution could be derived from

\[
\min ||D||_F^2 \ \text{subject to} \ \dot{b} + e = (A + E)\dot{x}
\]

(10)

where \( ||\cdot||_F^2 \) denotes the Frobenius norm of a matrix. The classical TLS problem aims to find the minimal (in the Frobenius norm sense) corrections of \( S \) on the given matrix \( S \). If \( S \) has a full rank, an analytic expression of the TLS problem exists, which is similar to the least squares solution

\[
\dot{x}_{TLS} = (A^H A - \sigma_{min}^2 I)^{-1} A^H \dot{b}
\]

(11)

where \( \sigma_{min} \) is the smallest singular value of \( S \). Suppose the singular value decomposition (SVD) of \( S \) is

\[
S = U\Sigma V^H
\]

(12)

\[
\Sigma = \begin{bmatrix}
\Sigma_p & 0 \\
0 & 0
\end{bmatrix}, \quad \Sigma_p = \text{diag}(\sigma_1, \sigma_2, \cdots, \sigma_p), \quad \sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_p > 0
\]

(13)

where \( \Sigma \) is a diagonal matrix. The non-zero diagonal elements \( \sigma_1, \sigma_2, \cdots, \sigma_p \) of \( \Sigma \), assumed to be positive and arranged in decreasing order, are the singular values of \( S \). \( U \) and \( V \) are matrices with orthonormal columns, named left singular vectors and right singular vectors respectively. The singular vectors associated with small singular values tend to oscillate significantly. An approximation is added to the TLS problem to reduce the negative influence of the factors on the analytic solution (11), which is done by taking a truncation on the small singular values of \( S \). By doing this on \( S \), it is possible to produce a more accurate TLS solution, which is the ATLS solution. Substituting the approximation into (12) yields

\[
\hat{S}_r = U\hat{\Sigma}_r V^H
\]

(14)

\[
\hat{\Sigma} = \begin{bmatrix}
\Sigma_r & 0 \\
0 & 0
\end{bmatrix}, \quad \Sigma_r = \text{diag}(\sigma_1, \sigma_2, \cdots, \sigma_r), \quad \sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r > 0
\]

(15)

where \( r \) is the truncated parameter. By substituting (14) into (11), the ATLS algebraic solutions of motion error gradients could be derived

\[
\dot{x}_{ATLS} = (A^H A - \sigma_r^2 I)^{-1} A^H \dot{b}
\]

(16)
where \( \sigma_r \) is the \( r \)th singular value of \( S \). The motion errors in the LOS direction are then obtained by integrating (16). The ATLS algorithm is a numerical linear algebra tool to find a more accurate solution to the airborne SAR motion error estimation.

### 3.2 Threshold determination

The choice of the truncated parameter \( r \) has a significant influence on the final results of the estimated motion errors. With different choices of \( r \), various solutions \( \hat{x}_{ATLS} \) are obtained. After the truncation, the singular values of the matrix \( S \), indexed from \( r + 1 \) to the end, are set to zero, and the corresponding singular vectors are disregarded. Then the ATLS solution \( \hat{x}_{ATLS} \) does not contain any high frequency components. The residual matrix is given by

\[
\|S - \hat{S}\|_F = \sqrt{\sigma_{r+1}^2 + \sigma_{r+2}^2 + \cdots + \sigma_{p+1}^2} \tag{17}
\]

which indicates that the residual matrix decreases with the increasing of \( r \) and would represent the truncated error. The value of (17) is set as the threshold of the ATLS algorithm. \( r \) is updated to the optimal value with each iteration. The optimal value is obtained when the SAR image is well focused.

### 3.3 ATLS framework

The detailed processing procedures of the proposed method are given as follows

Step 1: After range compression and range cell migration correction, a standard stripmap PGA algorithm is applied to get the phase error gradient.

Step 2: Based on the motion error estimation model, the augmented matrix \( S \) could be obtained. Calculate the singular values of \( S \), and the truncated parameter is chosen to get the ATLS algebraic solutions of motion errors gradient by (16).

Step 3: Then integrate the estimated motion error gradient, and calculate the phase errors in each range bin through (5). Phase correction could be imposed by complex multiplication as in PGA.

Step 4: The estimation and correction process is repeated iteratively. The algorithm is driven toward convergence as the image becomes more focused.

### 4 Experimental results

This section gives some results of motion compensation experiments adopting real-measured SAR data to show the effectiveness of the proposed method. The data is collected by a strip-mode unmanned aerial vehicle (UAV) SAR system, which is built by the Institute of Electronics, Chinese Academy of Sciences. The system parameters are listed in Table I, which indicates that the swath is wide enough for generating range-dependent phase errors.

#### 4.1 Simulation experiment

A simulation experiment is provided to demonstrate the effectiveness of the proposed method. A known range-dependent phase error is appended to a well focused image, generating a blurred image, and then removed by the proposed method. Fig. 2 shows the results of the estimated phase errors from a selected rural image, and indicates that the estimated phase errors are keep a tight pace with the
appended ones. The results of simulation experiment with airborne SAR image are shown in Fig. 3 with size of $1024 \times 1024$ pixels. By applying the ATLS algorithm, the appended phase errors are estimated and removed accurately, and the restored image is nearly indistinguishable from the original.

### Table 1. Airborne SAR system parameters

| Parameter                | Value       |
|--------------------------|-------------|
| Frequency band           | Ku          |
| Bandwidth                | 600 MHz     |
| Height                   | 6999 m      |
| Pulse repetition frequency| 755 Hz     |
| Swath                    | 8000 m      |
| PRF-velocity ratio       | 16          |
| Range resolution         | 0.25 m      |
| Azimuth resolution       | 0.25 m      |

(a) Estimated phase errors in near range  

(b) Estimated phase errors in far range  

Fig. 2. Estimated phase errors.

(a) Original focused image  
(b) Blurred image in near  
(c) Restored image in near  

(d) Original focused image in far  
(e) Blurred image in far  
(f) Restored image in far  

Fig. 3. The simulation experiments.
4.2 Real data validation

Real SAR experiments are adopted to show the effectiveness of the proposed method. Fig. 4 illustrates the vertical component of motion errors estimated with the least squares (LS), weighted least squares (WLS) [7], and ATLS algorithm respectively as shown with solid lines. The dashed line is the height variations of aircraft recorded by the INS i.e. the vertical component of motion error, which could reveal the trend of height variations. Obviously, the ATLS algorithm makes the most accurate estimation.

Fig. 5 presents the results of whole scene with range-dependent phase errors corrected by the standard PGA algorithm and Fig. 6 presents the proposed method. The size of the image is 2048 × 27790 pixels. The dominant points used for PGA are chosen arbitrarily, and the far range points are taken as an example. Fig. 5 shows that the far range is better focused than the near range because PGA cannot cope with range-dependent phase errors. The size of small image is 1024 × 1024 pixels. In Fig. 6, the motion error is estimated and compensated accurately by the ATLS algorithm, so the whole scene is well focused from near range to far range.

![Comparison of different methods with INS data.](image1)

![The wide beamwidth SAR image by PGA.](image2)
5 Conclusion

In this paper, a novel motion compensation method for wide-beamwidth SAR systems is presented. The method adopts the approximate total least squares (ATLS) algorithm as the estimation kernel to get motion errors in the cross-track direction from the SAR raw data. The proposed method could treat the noise in both the estimated phase errors and the geometry of SAR imagery systematically, and does not need to model the noise to be a certain pattern. Both simulation and real data tests indicate the robust and efficiency of this method.

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