Unparticle physics in top pair signals at the LHC and ILC

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received 14 February 2008; accepted in final form 11 August 2008
published online 17 September 2008

PACS 14.80.-j – Other particles (including hypothetical)
PACS 12.90.+b – Miscellaneous theoretical ideas and models
PACS 12.38.Qk – Experimental tests

Abstract – We study the effects of unparticle physics in the pair productions of top quarks at the LHC and ILC. By considering vector, tensor and scalar unparticle operators, as appropriate, we compute the total cross-sections for pair production processes depending on the scale dimension $d_U$. We find that the existence of unparticles would lead to measurable enhancements on the SM predictions at the LHC. In the case of ILC this may become two orders of magnitude larger than that of SM, for smaller values of $d_U$, a very striking signal for unparticles.

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Introduction. – Scale invariance is broken in the quantum field theory by the masses of particles. Recently, Georgi [1,2] has introduced a new scheme based on the existence of a non-trivial scale-invariant sector at a very large scale $M_U$. The fields of this sector and Standard Model (SM) fields can interact via the exchange of a connector stuff. Below this mass scale, non-renormalizable operators are suppressed by power of $1/M_U$. Renormalization effects of the scale-invariant Banks and Zaks (BZ) sector [3] induce dimensional transmutation at the scale $\Lambda_U$. Below the scale $\Lambda_U$, BZ operators transform to so-called unparticle operators with scale dimension $d_U$. So far there have been many studies exploring various aspects of unparticles, including also supersymmetric extensions and colored versions [4].

Top quark physics is a very interesting and active field of research [5,6]. One of the most distinctive features of the top quark which makes it so interesting is its large mass. Due to this large mass, the top quark is considered as an ideal tool for probing new physics beyond SM. The CERN Large Hadron Collider (LHC) will be, in a sense, a top quark factory with about $10^7$ top pair signals per year. This large statistics will also enable us to determine the top quark properties very accurately, at the LHC. On the other hand, although the cross-section for $t\bar{t}$ production at the International Linear Collider (ILC) [7] is about three orders less than the LHC, the very clean environment of the ILC experiments, as well as the polarization of the initial beams make it an attractive platform for further and complementary investigation of top quarks.

In this work we exploit implications of unparticle physics in the pair productions of top quarks, and show how the interactions between unparticles and SM fields can be probed at the LHC with $\sqrt{s} = 14$ TeV, and at ILC with $\sqrt{s} = 0.5$ TeV by obtaining the modifications on SM predictions for the $t\bar{t}$ production cross-sections. A work which addresses some effects of unparticle physics on top quark pair production primarily at Tevatron has recently appeared while this work was still in progress [8]. In this reference, what they have done concerning the LHC regime was the display of $tt$ production cross-section for vector unparticle as a function of $d_U$, as well as two other plots displaying the $3\sigma$ reaches in the interaction scale $\Lambda$. Instead, in this letter we present all the cross-sections as functions of the scale dimension $d_U$ for all processes considered, and obtain the effects of unparticle physics on these production cross-sections by computing the contributions of all types of unparticles — vector, scalar and tensor unparticle separately, one at a time. Another feature of our work worth emphasizing is that we analyze the top pair production cross-sections by deriving all the expressions analytically. Thus we think that these two works, one being primarily on the Tevatron [8], and ours being exclusively on the LHC and ILC, together form a complete set concerning the top pair production in unparticle physics.

Using the scale invariance, the following propagators for unparticles with different Lorentz structures can be:

\begin{align*}
\frac{1}{\sqrt{-g}} \int \frac{d^4 x d^4 y}{\sqrt{-g(x) g(y)}} \epsilon^\mu_{\alpha} \epsilon^\nu_{\beta} \langle \bar{U}(0) U(x) | \bar{U}(y) U(0) \rangle &= \int \frac{d^4 x d^4 y}{\sqrt{-g(x) g(y)}} \epsilon^\rho_{\alpha} \epsilon^\rho_{\beta} \langle \bar{U}(0) U(x) | \bar{U}(y) U(0) \rangle \\
\frac{1}{\sqrt{-g}} \int \frac{d^4 x d^4 y}{\sqrt{-g(x) g(y)}} \epsilon^\rho_{\alpha} \epsilon^\rho_{\beta} \langle \bar{U}(0) U(x) | \bar{U}(y) U(0) \rangle &= \int \frac{d^4 x d^4 y}{\sqrt{-g(x) g(y)}} \epsilon^\rho_{\alpha} \epsilon^\rho_{\beta} \langle \bar{U}(0) U(x) | \bar{U}(y) U(0) \rangle \\
\frac{1}{\sqrt{-g}} \int \frac{d^4 x d^4 y}{\sqrt{-g(x) g(y)}} \epsilon^\rho_{\alpha} \epsilon^\rho_{\beta} \langle \bar{U}(0) U(x) | \bar{U}(y) U(0) \rangle &= \int \frac{d^4 x d^4 y}{\sqrt{-g(x) g(y)}} \epsilon^\rho_{\alpha} \epsilon^\rho_{\beta} \langle \bar{U}(0) U(x) | \bar{U}(y) U(0) \rangle
\end{align*}
obtained [1,9]:

Scalar: $\Delta_S = \frac{i A_{d\bar{d}}}{2 \sin(\pi d_{d\bar{d}})} (-q^2) d_{u}^{2} - 2$, 

Vector: $\Delta_V = \frac{i A_{d\bar{d}}}{2 \sin(\pi d_{d\bar{d}})} (-q^2) d_{u}^{2} - 2 \pi_{\mu\nu}$, 

Tensor: $\Delta_T = \frac{i A_{d\bar{d}}}{2 \sin(\pi d_{d\bar{d}})} (-q^2) d_{u}^{2} - 2 T_{\mu\nu,\rho\sigma}$, 

where

$$\pi_{\mu\nu}(q) = g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2},$$

$$T_{\mu\nu,\rho\sigma}(q) = \frac{1}{2} \left( \pi_{\mu\sigma}(q) \pi_{\nu\rho}(q) + \pi_{\mu\rho}(q) \pi_{\nu\sigma}(q) \right) - \frac{2}{3} \pi^{\mu\nu}(q) \pi^{\rho\sigma}(q) \right) \right)$$

and $A_{d\bar{d}} = \frac{16 \pi^2}{(2 \pi)^2} \frac{\Gamma(d_{d\bar{d}} + 1)}{\Gamma(d_{d\bar{d}} - 1)} (\frac{d_{d\bar{d}}}{2})$. 

The SM gauge-invariant effective interactions of scalar, vector and tensor unparticles with the SM fields are given by [9,10]

$$\lambda_0 \frac{1}{A_{d\bar{d}}} \bar{f} f O_{U\bar{d}} - \lambda_0 \frac{1}{A_{d\bar{d}}} \bar{f} f O_{U\bar{d}}, - \lambda_1 \frac{1}{A_{d\bar{d}}} \bar{f} f \gamma_\mu f O_{U\bar{d}} - \lambda_1 \frac{1}{A_{d\bar{d}}} \bar{f} f \gamma_\mu f O_{U\bar{d}},$$

where $\lambda_i (i = 0, 1, 2)$ are dimensionless effective couplings labeling scalar, vector and tensor unparticle operators, respectively. $c_\alpha$, $c_\sigma$ represent vector and axial vector couplings of vector unparticle, respectively. $D_\mu$ is the covariant derivative, $f$ are SM fermions, and $G_{\alpha\beta}$ are the gluon field strengths.

In the second section, we give the analytical expressions of differential cross-sections for $t\bar{t}$ (and $t\bar{t}$) productions in $pp$ collisions. In the third section, the corresponding expressions for $e^+e^-$ collisions are given. In the fourth section, we present the results of our numerical analysis, for LHC and ILC regimes, and finally discuss our conclusions.

$t\bar{t}$ (and $t\bar{t}$) production in $pp$ collisions with unparticles. – In QCD $t\bar{t}$ production originates either from quark-antiquark annihilation or gluon fusion. Below we give the explicit analytical expressions of the leading-order (LO) differential cross-sections for pair productions with the contributions of vector, tensor and scalar unparticles. We have assumed that unparticles are colorless.

The color- and spin-averaged partonic differential cross-sections for $t\bar{t}$ productions via quark-antiquark annihilations are given by:

i) Vector unparticle

$$\frac{d\sigma}{dt}(q\bar{q} \rightarrow t\bar{t}) = \frac{A_V^2}{8 \pi s^2 (s \bar{t})^{2 - 2d_{u}}} \left( c_{u}^2 + c_{d}^2 \right) (m^8 + 2m^6 + (s + \bar{t})^4 + 2(m^2 + (s + \bar{t})^2)^2 + 4c_{u}^2 c_{d}^2 m^2 + 2(s + \bar{t})^4 + 2m^2) + \frac{d\sigma_{qq}}{dt},$$

where $A_V = \frac{\lambda_2^2 A_{d\bar{d}}}{2 \pi d_{d\bar{d}} (s \bar{t})^{2 - 2d_{u}}}$, $\lambda_2$ is the strong-coupling constant, $m$ denotes top quark mass, $A$ refers to scale $A_{d\bar{d}}$ up to which effective theories are valid and $\frac{d\sigma_{qq}}{dt}$ is the SM part:

$$\frac{d\sigma_{qq}}{dt} = \frac{4 \pi a_s^2}{9 \pi^2} \left[ 2m^2 - 4(m^2 - (s + \bar{t})^2 + \frac{d\sigma_{qq}}{dt} \right].$$

ii) Tensor unparticle

$$\frac{d\sigma}{dt}(q\bar{q} \rightarrow t\bar{t}) = \frac{A_T^2}{8 \pi s^2 (s \bar{t})^{2 - 2d_{u}}} \left( c_{u}^2 + c_{d}^2 \right) (m^8 + 2m^6 + 4(s + \bar{t})^4 + 4(s + \bar{t})^2 + 4m^2 + 4(s + \bar{t})^2)^2 + 4c_{u}^2 c_{d}^2 m^2 + 2(s + \bar{t})^4 + 2m^2 + 3(s + \bar{t})^4 + 2(s + \bar{t})^2 + 2m^2),$$

$$\frac{d\sigma_{qq}}{dt} = \frac{4 \pi a_s^2}{9 \pi^2} \left[ 2m^2 - 4(m^2 - (s + \bar{t})^2 + \frac{d\sigma_{qq}}{dt} \right].$$

iii) Scalar unparticle

$$\frac{d\sigma}{dt}(q\bar{q} \rightarrow t\bar{t}) = \frac{A_S^2}{4 \pi s^2 (s \bar{t})^{2 - 2d_{u}}} \left[ \frac{d\sigma_{qq}}{dt} + \frac{d\sigma_{qq}}{dt} \right],$$

where $A_S = \frac{\lambda_3^2 A_{d\bar{d}}}{3 \pi d_{d\bar{d}} (s \bar{t})^{2 - 2d_{u}}}$. Note that there is no interference with the SM part in this case.

In unparticle physics there are also Flavor-Violating (FV) processes $q\bar{q} \rightarrow t\bar{t}$ via $t$-channel exchange of unparticles. Here we have two cases. In the case of $q = q'$, the $s$-channel SM reaction is to be included. But the case of $q \neq q'$, obviously does not have an SM counterpart at tree level due to the non-existence of FCNC vertices in SM. Differential cross-sections in the FV case are given below.

a) The case $q = q'$ (q, q’ = u, c):

$$\frac{d\sigma_{FV}^{uu}}{dt}(q\bar{q} \rightarrow t\bar{t}) = \frac{A_V^2}{16 \pi s^2 (s \bar{t})^{2 - 2d_{u}}} \left( c_{u}^2 + c_{d}^2 \right) (m^8 + 2m^6 + \frac{d\sigma_{qq}}{dt} \right).$$

b) The case $q = q'$ (q, q’ = d, s):

$$\frac{d\sigma_{FV}^{dd}}{dt}(q\bar{q} \rightarrow t\bar{t}) = \frac{A_T^2}{16 \pi s^2 (s \bar{t})^{2 - 2d_{u}}} \left( c_{u}^2 + c_{d}^2 \right) (m^8 + \frac{d\sigma_{qq}}{dt}).$$
where and $\tilde{c}_v, \tilde{c}_a$ are FF vector and axial vector couplings of vector unparticle, respectively.

$$\frac{d\sigma_{FV}^{\gamma q}}{dt}(q\bar{q} \rightarrow t\bar{t}) = \frac{A F}{18\pi s^2(-t)^{3/2 - 2\alpha_u}} \left[ 8m^{16} - 48\bar{m}m^{14} + 3\bar{t}(32\bar{m} + 43\bar{t})m^{12} - 2\bar{t}^2(26\bar{m} + 10\bar{t})m^{10} + 3\bar{t}^2(128\bar{m} + 41\bar{t}s + 85\bar{t}^2s - 12\bar{t}^3(12\bar{m}^2 + 13\bar{t}s + 19\bar{t}^2) + 2\bar{t}^2m^6 + \bar{t}^3(576\bar{m}^3 + 2202\bar{m}s + 1212\bar{t}^2s + 143\bar{t}^3)s^3 - 18\bar{t}^4(64\bar{m}^3 + 82\bar{t}s^3 + 28\bar{t}^2s^3 + 3\bar{t}^3)s^2 + 9\bar{t}^5(32\bar{m}^4 + 64\bar{t}s^4 + 42\bar{t}^2s^4 + 10\bar{t}^3s + 1\bar{t}^4) \right] - \frac{8\alpha_A A_T}{27\bar{s}^3(-t)^{3/2 - 2\alpha_u}} \left[ 2m^{10} + (2\bar{s} - 5\bar{t})m^8 - 4\bar{s}m^6 + 2\bar{t}(6\bar{m}^2 + 10\bar{t}s + 5\bar{t}^2)s^4 - 2\bar{t}^2(12\bar{m}^2 + 18\bar{t}s + 5\bar{t}^2)s^2 + 3\bar{t}^2(s + \bar{t})^2(4s + 1\bar{t}) + \frac{\alpha_0^{D\bar{q}}}{dt}, (8) \right] \right] + \frac{4\alpha_A A_S}{9\bar{s}^3(-t)^{3/2 - 2\alpha_u}} (\bar{s}m^2 + (m^2 - \bar{t})^2) + \frac{\alpha_0^{D\bar{q}}}{dt}. (9) \right]$$

b) The case $q \neq q'$ ($q, q' = u, c$).

The corresponding differential cross-sections for vector, tensor and scalar unparticles would be the terms involving $A_{TF}^V$ only in eq. (7), $A_{TF}^T$ in eq. (8) and $A_{TF}^S$ in eq. (9), respectively, which are obtained by disregarding the SM parts and interferences between the unparticle and SM sectors.

After completing quark-antiquark annihilations, we now analyze $tt$ productions originating from gluon fusion. In this case, to produce $tt$ we have four Feynman diagrams in two cases of tensor and scalar unparticle exchanges, as the vector unparticle do not couple to gluons. One out of four is the $s$-channel exchange of unparticle and the other three are SM ($s, t$ and $u$ channel) diagrams. The diagram mediated by unparticles does not interfere with that of the SM model $s$-channel mediated by gluon. Hence, the differential cross-sections for the process $gg \rightarrow tt$ have the following forms:

i) Tensor unparticle

$$\frac{d\sigma}{dt}(gg \rightarrow tt) = \frac{3A_T^2}{16\pi s^2} \left\{ \frac{1}{t - \bar{m}} \left[ 2m^6 + 2(2\bar{s} + 5\bar{t})m^4 - 2\bar{t}m^2 + \bar{t}^2\bar{s} + 4\bar{t}^2s \right] - \frac{1}{\bar{u} - \bar{m}} \left[ 2m^6 - 2(2\bar{s} + 5\bar{t})m^4 + 2\bar{t}m^2 + \bar{t}^2\bar{s} + 4\bar{t}^2s \right] \right\} + \frac{\alpha_0^{g\bar{q}}}{dt}.$$

where $\frac{d\sigma_{gg}}{dt}$ is the SM part of the differential cross-section of the gluon fusion in $tt$ production, and is given by

$$\frac{d\sigma_{gg}}{dt} = \frac{\pi\alpha_s^2}{s^2} \left\{ \frac{3}{4s^2} (m^2 - \bar{t})(m^2 - \bar{u}) \right\}$$

$$\frac{1}{2(t - \bar{m})^2} (3m^2 - \bar{s}m^2 + \bar{t}(\bar{s} + \bar{t})) - \frac{1}{2(\bar{u} - \bar{m})^2} (3m^2 - \bar{s}m^2 + \bar{u}(\bar{s} + \bar{u}))$$

$$\frac{3}{8\bar{s}(t - m^2)} (3m^4 - 2(\bar{s} + \bar{t})m^2 + \bar{t}\bar{s} + \bar{t}^2) - \frac{3}{8\bar{s}(\bar{u} - m^2)} (3m^4 - 2(\bar{s} + \bar{u})m^2 + \bar{u}\bar{s} + \bar{u}^2)$$

$$\frac{1}{24(\bar{t} - m^2)(\bar{u} - m^2)} (m^2(2m^2 - \bar{t} + \bar{u})).$$

ii) Scalar unparticle

$$\frac{d\sigma}{dt}(gg \rightarrow tt) = \frac{3A_S^2}{256\pi s^2} \left\{ \frac{1}{t - \bar{m}} m(2m^4 + (5\bar{s} - 4\bar{t})m^2 - 2\bar{t}^2 + \bar{s}^2 - \bar{t}) + \frac{\alpha_0^{g\bar{q}}}{dt} \right\}$$

where $A_S = \frac{2\lambda^2\alpha_{ud}}{m(\alpha_{ud})^2\sqrt{s} + \alpha_{ud}}.$

Finally, the cross-sections for the processes $qq' \rightarrow tt$ ($q, q' = u, c$) for the $tt$ case in unparticle physics are:

i) Vector unparticle

$$\frac{d\sigma}{dt}(qq' \rightarrow tt) = \frac{A_V^2}{16\pi s^2} \left\{ \frac{1}{(-t)^{3/2 - 2\alpha_u}} \left[ \frac{(\bar{t}^2 + \bar{t})}{t - \bar{m}} m(2m^6 - 8\bar{t}m^4 + 8\bar{t}^2m^2 - \bar{t}^2\bar{s} + 3\bar{t}^3) + \frac{1}{6(-t)^{3/2 - 2\alpha_u} \bar{u}^2} \left[ (\bar{c}_v^2 + \bar{c}_a^2)(3m^6 - (\bar{t} + \bar{u})(3m^6 - 6m^6\bar{t} + 4\bar{t}\bar{u}(\bar{t} + \bar{u})) + m^4(2\bar{s} + 3\bar{t} + 2\bar{u}) + 2\bar{c}_v^2\bar{c}_a^2) \right] \right. \right\} + \frac{\alpha_0^{g\bar{q}}}{dt}.$$

where

$$\frac{d\sigma}{dt} = \frac{\pi\alpha_s^2}{s^2} \left\{ \frac{3}{4s^2} (m^2 - \bar{t})(m^2 - \bar{u}) \right\}$$

$$- \frac{1}{2(t - \bar{m})^2} (3m^2 - \bar{s}m^2 + \bar{t}(\bar{s} + \bar{t})) - \frac{1}{2(\bar{u} - \bar{m})^2} (3m^2 - \bar{s}m^2 + \bar{u}(\bar{s} + \bar{u}))$$

$$- \frac{3}{8\bar{s}(t - m^2)} (3m^4 - 2(\bar{s} + \bar{t})m^2 + \bar{t}\bar{s} + \bar{t}^2) - \frac{3}{8\bar{s}(\bar{u} - m^2)} (3m^4 - 2(\bar{s} + \bar{u})m^2 + \bar{u}\bar{s} + \bar{u}^2)$$

$$- \frac{1}{24(\bar{t} - m^2)(\bar{u} - m^2)} (m^2(2m^2 - \bar{t} + \bar{u})).$$
In this section we present the differential cross-sections
\[
\frac{d\sigma}{dt}(q'q \to tt) = \frac{A_T^2}{18s^2} \left\{ \left[ \frac{1}{(t-\ell)^2-4\delta\ell} \frac{1}{4\pi^2s^2} \left( 8m_{16} - 48m_{14} \right) + \frac{3(32s^2 + 43\delta\ell)m_{12} - 2\delta\ell(264s + 107\ell)m_{10} + 3\ell^2(128\delta^2) + 141t\delta s + 85\delta l^2 \right) m_{18} - 12\delta^3(124s^2 + 134\delta s + 192\delta l^2) + 102^3(576s^3 + 2202\delta s^3 + 1212t\delta s^3 + 143\delta^2) m_{14} + 18\delta^4(64s^3 + 82\delta s^3 + 28\delta^2 s^3 + \ell^2) m_{12} + 94^3(324s^4 + 64t^2 s^3 + 10^3 t^3 s^2 + t^4 l^2) \right] + [\ell \leftrightarrow \ell'] \right\} 
\]
\[
+ \frac{1}{(t-\ell)^2-4\delta\ell (-\ell)^2-4\delta\ell} \left[ 12m_{16} + 26m_{14}(t + \ell) + m_{12}(-28\delta^2 + 6\ell^2 - 28\delta^2 - 9\ell^2 \ell^2 + 4\ell^2) + 17\ell^2 + 4\ell^2 \right) m_{10}(20\ell^3 - 27\ell^2 \ell - 27\ell^3 \ell^2 + 20\ell^5) + 4m\ell^2(36\ell^2 + 149\ell^2 \ell + 149\ell^2 \ell + 36\ell^3) + 3m^2 \ell^2 \ell^2 (52\ell^3 + 231\ell^2 \ell + 231\ell^3 \ell + 52\ell^5) + m^2 (-6\ell^2 + 136\ell^2 \ell + 384\ell^2 \ell^2 + 136\ell^3 \ell - 64\ell^4) + m^4 \ell^2 (36\ell^4 + 45\ell^2 \ell^2 + 124\ell^2 \ell^2 + 45\ell^2 \ell^2 + 36\ell^4) \right). \tag{14}
\]
\[
\frac{d\sigma}{dt}(q'q \to tt) = \frac{A_T^2}{4\pi^2s^2} \left\{ \left[ \frac{1}{(t-\ell)^2-4\delta\ell} \frac{1}{4\pi^2s^2} \left( 2m^4 + 2m^2 \ell + \ell - \ell \ell \right) \right] + \frac{1}{6(t-\ell)^2-4\delta\ell (-\ell)^2-4\delta\ell} \left( m^4 + m^2 \ell^2 - 2\ell^4 \right) \right) + [\ell \leftrightarrow \ell'] \right\}. \tag{15}
\]
We are finished with the construction of the analytical expressions for the partonic differential cross-sections for the \( tt \) and \( t \bar{t} \) productions in \( pp \) collisions. Next we will derive the corresponding expressions for the process \( e^+ e^- \to tt \) in the next section.

### \( tt \) production in \( e^+ e^- \) collisions with unparticles

- In this section we present the differential cross-sections for the processes \( e^+ e^- \to tt \) which occur via the s-channel exchanges of unparticles, and of usual electroweak bosons, \( \gamma \) and \( Z \), exchanges in the SM case.

i) Vector unparticle

\[
\frac{d\sigma}{dt}(e^+ e^- \to tt) = \frac{3A_T^2}{8\pi s^2(s)^4-2\delta\ell} \left[ c_s^2(2m^4 - 4(s + \ell)^2 \ell^2 + (s + \ell)^2 + \ell^2) + c_s^2(2m^4 - 4t^2 \ell^2 \ell + (s + \ell)^2 + \ell^2) \right] + \frac{3A_T c_s \cos(\delta dt\pi)}{s_s^2(s)^2-2\delta\ell} \left[ c_s^2(2m^4 - 4(s + \ell)^2 \ell^2 + (s + \ell)^2 + \ell^2) \right] + \frac{3A_T c_s \cos(\delta dt\pi)}{s_s^2(s)^2-2\delta\ell} \left[ c_s^2(2m^4 - 4(s + \ell)^2 \ell^2 + (s + \ell)^2 + \ell^2) \right] - 2\ell u - s\ell (t + \ell) + \left( [v_c v_c (\ell - \ell) s^2 + a_c a_c c_s^2 (8m^6 - 2(5s + 4(\ell + \ell))) + 2(s^2 + 3s(\ell + \ell) + (\ell + \ell) m^2 - s(2\ell u + s(\ell + \ell)))] + 2c_s c_c (a_c v_c s^2 (m^4 + s^2 - 2\ell u + s(\ell + \ell)) \right) + v_c v_c (2m^4 + 2s^2) + \frac{d\sigma^0}{dt}. \tag{16}
\]

where \( s_W = \sin \theta_W \) and \( c_W = \cos \theta_W \), with \( \theta_W \) being Weinberg angle and \( d\sigma^0/df \) is the SM part given by

\[
\frac{d\sigma^0}{dt} = \frac{6\pi a_s^2}{s^2} \left[ \frac{(2m^4 + 2s^2 - 2\ell u - s(\ell + \ell)] + \frac{1}{4\pi^2s^2(s)^4-2\delta\ell} \left[ 4a_c v_c s_c v_s [s + 2\ell^2 - 3m^2 + 4(\ell^2 + s^2) + 32\ell^2 \ell + 2\ell^2 - 4m^2 \ell - 4m^2 s_c^2 (a_c^2 + v_c^2)] + s_s^2 \ell^2 \ell^2 [s - m^2 - 2\ell u - s(\ell + \ell)] \right]. \tag{17}
\]

In eqs. (16) and (17) \( v_c \), \( a_c \) and \( v_t \), \( a_t \) are the vector and axial vector couplings of \( Z \) to electron and top quark, respectively. Note that the vector unparticle contribution interferes with those of the photon and \( Z \) boson of the SM.

ii) Tensor unparticle

\[
\frac{d\sigma}{dt}(e^+ e^- \to t\bar{t}) = \frac{3A_T^2}{2\pi s^2(s)^4-2\delta\ell} \left[ 32m^8 - 32(s + 4\ell) m^6 + 2(5s^2 + 64t^2 s + 96\delta t^2) m^4 - 4(3s^3 + 13s^2 t^2 + 40t^2 s + 32t^3) m^2 + 4s^4 + 4s^3 t^2 + 4s^2 t^2 \ell^2 + 8s^3 t^2 \right] - 6A_T c_s \frac{\cos(\delta dt\pi)}{s_s^2(s)^2-2\delta\ell} \left[ 8m^6 - 4(\ell + \ell) m^4 + 2(s^2 + 8s t^2 + 12t^2) m^2 - (s + 2t^2) \right] + \frac{3\alpha A_T [s - M_Z^2 \cos(\delta dt\pi) + \Gamma_Z M_Z \sin(\delta dt\pi)]}{2c_s^2 s_W s_W^2(s)^2-2\delta\ell} \left[ v_c v_c (\ell - \ell) s^2 + 3(\ell - \ell)^2 + v_c v_c (\ell - \ell) u^2 \right] + \frac{d\sigma^0}{dt}. \tag{18}
\]

Here we again have the interference terms with the corresponding SM contributions as in the vector unparticle case.

iii) Scalar unparticle

\[
\frac{d\sigma}{dt}(e^+ e^- \to t\bar{t}) = \frac{3A_T^2}{4\pi^2s^2(s)^4-2\delta\ell} \left[ \frac{s + 2m^2}{\ell^2} + \frac{d\sigma^0}{dt}. \tag{19}
\]

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It is seen from eq. (19) that the scalar unparticle contribution does not interfere with those of the neutral electroweak bosons of the SM.

Now we are ready to turn our attention to the analysis of the total cross-sections at the LHC and ILC energies.

**Numerical analysis, discussions and conclusions.**

- In fig. 1, we present the total cross-section for the process $q\bar{q} \rightarrow t\bar{t}$ by setting $\Lambda = 1\,\text{TeV}$, $\lambda_0 = \lambda_1 = \lambda_2 = 1$ and $c_v = c_a = 1$ at LHC regime with the CM energies of $14\,\text{TeV}$ for the flavor-conserving case. From this figure we see that the main contribution comes from the vector unparticle, and it enhances the SM result substantially for the values of $d_{tt}$, $1 < d_{tt} < 1.5$. The tensor contribution, however, is tiny as expected and insensitive to the variations of the scale dimension. Hence, the contribution of the tensor unparticle does not appear in fig. 1. Here we should note that for vector unparticle exchange the high-energy behavior of the amplitude scales as $(\hat{s}/\Lambda_v^2)^{d_{tt}-1}$, while it is further suppressed by $(\hat{s}/\Lambda_v^2)$ in the case of tensor unparticle. The scalar contribution is quite large and comparable with that of vector unparticle. It has been shown that there are constraints imposed on the parameters of unparticle physics stemming from some physical processes like $\mu$ decay [11]. That is, if the coupling strength of the vector unparticle, $c_v$ and $c_a$, for the FV case are chosen to be of the same magnitude as in the FC case, then $d_{tt}$ must be bounded from below, namely $d_{tt} > 2$ for vector unparticle and $d_{tt} > 3$ for the tensor unparticle, without any constraint on $d_{tt}$ for scalar unparticle. The results are depicted in fig. 2 for vector and scalar contributions, and in fig. 3 for tensor contribution. These contributions are almost insensitive to the variation of $d_{tt}$ beyond certain critical values and is about $40\,\text{pb}$, most of which comes from $s$-channel SM diagram. The results for the $q\bar{q} \rightarrow t\bar{t}$ are very small and thus we have not depicted them in this work. In fig. 4 we plot the dependence of the total cross-section for the process $gg \rightarrow t\bar{t}$ at $\Lambda = 1\,\text{TeV}$, $\lambda_0 = \lambda_2 = 1$ at LHC energies. Notice that this is a FC process. As we have already mentioned in the previous section, the vector unparticle does not give any contribution for this channel. In this case the main contribution comes from the scalar unparticle; the enhancement due to this contribution is quite large especially for $d_{tt} < 1.5$ and becomes slightly
negative for $\delta_{tt} > 1.6$. The result for $\delta_{tt} = 1.9$ is $343.8 \, \text{pb}$ as the SM value at LO is $448 \, \text{pb}$. The contribution of tensor unparticle varies from $440 \, \text{pb}$ for $\delta_{tt} = 1.1$ to $448.9 \, \text{pb}$ for $\delta_{tt} = 1.9$. In fig. 5, we plot the cross-sections for the $qq \rightarrow tt$ ($q = u, c$) processes for the scalar and vector unparticle contributions. For the range of $2 < \delta_{tt} < 3$, the total cross-sections for $tt$ production take values from $4.9 \, \text{pb}$ to $0.01 \, \text{pb}$ for vector unparticle exchange, and from $0.37 \, \text{pb}$ to $0.0014 \, \text{pb}$ for scalar unparticle exchange, as shown in this figure. The result for tensor unparticle exchange varies between $0.11$ and $0.00005 \, \text{pb}$ for the range of $3 < \delta_{tt} < 4$ and thus we have not depicted it. We have used the CTEQ5 parton distributions [12] in our computation for LHC. Turning to the case of $tt$ production at the ILC, we see from fig. 6 that virtual effects of vector and scalar unparticles are substantial specifically for the range of $1 < \delta_{tt} < 1.5$. As the SM value is about $0.76 \, \text{pb}$, these contributions are about $116 \, \text{pb}$ and $71 \, \text{pb}$, respectively for $\delta_{tt} = 1.1$. The contribution of the tensor unparticle is very small, however. It changes between $0.774$ and $0.762 \, \text{pb}$ for the range of $1 < \delta_{tt} < 2$.

In summary we have explored the phenomenology of unparticle physics with the top quark pair productions in $pp$ and $e^+e^-$ collisions. Depending on the relevant ranges of the scale dimension $d_W$ we have found significant enhancements as compared to the SM predictions for the total cross-sections. For the case of like-sign top pair production at LHC, our numerical result agrees with that of [8], in the case of vector unparticle. Our results for ILC are rather striking as compared to the LHC results because, for smaller values of $\delta_{tt}$, the enhancements of the cross-sections exceed SM predictions by two orders of magnitude, while the corresponding enhancements for the LHC case are only about a factor of three at the most.

Effects of unparticle physics can also be investigated on single productions of top quarks via FV processes which have no counterparts in the SM at leading order. These processes have been investigated for different types of colliders, namely, lepton-hadron, and linear $e^+e^-$ colliders, as well as LHC in ref. [13]. This study reveals, as we have expected, that there could be observable effects of unparticles, with rather striking signals for certain values of $\delta_{tt}$, which may provide further insight on unparticle physics.

REFERENCES

[1] GEORGI H., Phys. Rev. Lett., 98 (2007) 221601 [arXiv: hep-ph/0703260].
[2] GEORGI H., Phys. Lett. B, 650 (2007) 275 [arXiv: 0704.2457 [hep-ph]].
[3] BANKS T. and ZAKS A., Nucl. Phys. B, 196 (1982) 189.
[4] CHEUNG K., KEUNG W. Y. and YUAN T. C., Phys. Rev. Lett., 99 (2007) 051803 [arXiv:0704.2588 [hep-ph]]; LUO M. and ZHU G., arXiv:0704.3532 [hep-ph]; CHEN C. H. and GENG C. Q., arXiv:0705.0876 [hep-ph]; DING G. J. and YAN M. L., Phys. Rev. D, 76 (2007) 075005 [arXiv:0705.0794 [hep-ph]]; LIANG Y., Phys. Rev. D, 76 (2007) 056006 [arXiv:0705.0837 [hep-ph]]; ALIEV T. M., CORNELL A. S. and GAUR N., Phys. Lett. B, 657 (2007) 77 [arXiv:0705.1326 [hep-ph]]; BHATTACHARYYA G., CHOU DHURY D. and GHOSH D. K., arXiv:0708.2835 [hep-ph] and references therein.
[5] BENEKE M. et al., arXiv:hep-ph/0003033 and references therein.
[6] WAGNER W., Rep. Prog. Phys., 68 (2005) 2409 [arXiv:hep-ph/0507207] and references therein.
[7] LHC/LC STUDY GROUP (WEIGLEIN G. et al.), Phys. Rep., 426 (2006) 47 [arXiv:hep-ph/0410364] and references therein; ECFA/DESY LC PHYSICS WORKING GROUP (AGUILAR-SAavedra J. A. et al.), arXiv:hep-ph/ 0106315.
[8] CHOU DHURY D. and GHOSH D. K., arXiv:0707.2074 [hep-ph].
[9] CHEUNG K., KEUNG W. Y. and YUAN T. C., Phys. Rev. D, 76 (2007) 055003 [arXiv:0706.3155 [hep-ph]].
[10] CHEN S. L. and HE X. G., arXiv:0705.3946 [hep-ph].
[11] CHOU DHURY D., GHOSH D. K. and MAMTA, arXiv:0703.3637 [hep-ph].
[12] CTEQ COLLABORATION (LAI H. L. et al.), Eur. Phys. J. C, 12 (2000) 375 [arXiv:hep-ph/9903282].
[13] ALAN A. T., PAK N. K. and SENOL A., arXiv:0710.4239 [hep-ph].