**Abstract**

In this paper, we study the Joule-Thomson expansion for RN-AdS black holes immersed in perfect fluid dark matter. Firstly, the negative cosmological constant could be interpreted as thermodynamic pressure and its conjugate quantity with the volume gave us more physical insights into the black hole. We derive the thermodynamic definitions and study the critical behaviour of this black hole. Secondly, the explicit expression of Joule-Thomson coefficient is obtained from the basic formulas of enthalpy and temperature. Then, we obtain the isenthalpic curve in $T - P$ graph and demonstrate the cooling-heating region by the inversion curve. At last, we derive the ratio of minimum inversion temperature to critical temperature and the inversion curves in terms of charge $Q$ and parameter $\lambda$. 

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I. INTRODUCTION

Since Bekenstein and Hawking’s first study, black holes as thermodynamic systems have been an interesting research field in the theory of gravity [1–5]. The properties of AdS black hole thermodynamics have been studied since the early work of Hawking-Page phase transition [5]. Another notable thing is that the dark energy which corresponds to the cosmological constant is introduced to the first law of black hole thermodynamics. Particularly, the negative cosmological constant could be interpreted as thermodynamic pressure and treated as a thermodynamic variable [6, 7]. And its conjugate quantity could be regarded as volume of black holes once the thermodynamic laws of black hole mechanics were generalized [8]. Based on this idea, the charged AdS black hole thermodynamic properties and plenty of other studies were successfully investigated [9–24].

In the extended phase space (including $P$ and $V$ terms in the black hole thermodynamics), phase transition of charged AdS black holes is remarkably coincident with van der Waals liquid-gas phase transition [18, 25, 26]. An interesting aspect of van der Waals system is Joule-Thomson expansion which indicates that the heating and cooling zone emerge through the throttling process. Since the phase structure and the critical behaviour of AdS black holes are similar to van der Waals system, the Joule-Thomson expansion of AdS black holes was firstly investigated by Ökcü and Aydıner [27]. In the extended thermodynamics, one identifies the enthalpy $H$ with the mass of the black hole [8]. Joule-Thomson expansion is characterized by the invariance of enthalpy, so the throttling process is also an isenthalpic process. The slope of the isenthalpy curves equals the Joule-Thomson coefficient $\mu$ which determines the final change of temperature in this system. One can use the sign of the Joule-Thomson coefficient to determine the heating and cooling zone. Subsequently, there were many studies on Joule-Thomson expansion in various black holes [28–49].

The standard model of cosmology suggests that our universe is compiled of dark matter, dark energy and baryonic matter, and the existence of dark matter and dark energy has been proven by several experiments and observations [50]. There exists a greater velocity than expected which means that more mass is required and it is thought to be provided by dark matter [51, 52]. As one of the dark matter candidates, the perfect fluid dark matter (PFDM) has been considered [53, 54]. While astrophysical observations show that there exists a supermassive black hole surrounded by dark matter halo [55, 56] and many black hole
solutions within dark matter have been proposed. Particularly, spherically symmetric black hole solutions surrounded by PFDM have been obtained \[54\ 57\ 59\]. Plenty of properties about this kind of black holes were discussed in \[60\ 71\]. Here we intend to study the Joule-Thomson expansion of RN-AdS black hole immersed in PFDM.

This paper is organized as follows. In Section (II), we investigate the thermodynamic properties of RN-AdS black holes immersed in PFDM. In Section (III), we discuss the Joule-Thomson expansion of this kind of black holes, which includes the Joule-Thomson coefficient, the inversion curves and the isenthalpic curves. Furthermore, we compare the critical temperature and the minimum of inversion temperature, and the influence of the PFDM parameter \(\lambda\) and charge \(Q\) on inversion curves is discussed. Finally, we discuss our result in Section (IV).

II. RN-ADS BLACK HOLE IMMERSED IN PFDM

The minimum coupling of dark matter field with gravity, electromagnetic field and cosmological constant is described by the action \[54\ 59\ 72\],

\[
S = \int \sqrt{-g} \left( L_{\text{DM}} + \frac{F_{\mu\nu}F^{\mu\nu}}{4} - \frac{\Lambda}{8\pi G} + \frac{R}{16\pi G} \right) d^4x,
\]

where \(G\) is gravitational constant, \(\Lambda\) is cosmological constant, \(F_{\mu\nu}\) is electromagnetic tensor and \(L_{\text{DM}}\) is the Lagrangian which is related to the density of PFDM.

The spacetime metric of static and spherically symmetric RN-AdS black holes immersed in PFDM is defined as \[60\ 72\]

\[
ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2d\Omega^2,
\]

where \(d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2\) and \(f(r)\) is given by

\[
f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3} r^2 + \frac{\lambda}{r} \ln \left( \frac{r}{|\lambda|} \right),
\]

with \(Q\) being charge of the black hole and \(\lambda\) related to the dark matter density and pressure. This metric reduces to the RN-AdS black hole when \(\lambda = 0\). For the given spacetime metric in the case of \(\lambda \neq 0\) the stress energy-momentum tensor of the dark matter distribution is that of an anisotropic perfect fluid reads as

\[
T^\mu_\nu = \text{diag}(-\rho, p_r, p_\theta, p_\phi),
\]
where density, radial and tangential pressures are given by

\[
\rho = -p_r = \frac{\lambda}{8\pi r^3}, \quad p_\theta = p_\phi = \frac{\lambda}{16\pi r^3}.
\] (5)

Note that for a PFDM distribution we shall restrict ourselves to the case \( \lambda > 0 \) which gives positive energy density.

The cosmological constant \( \Lambda \) is often parameterized by the AdS radius \( l \) according to

\[
\Lambda = -\frac{(d - 1)(d - 2)}{2l^2}.
\] (6)

And the negative cosmological constant is related to thermodynamic pressure \( P \),

\[
P = -\frac{\Lambda}{8\pi}.
\] (7)

The entropy of the black hole at the horizon is given by

\[
S = \pi r_+^2,
\] (8)

The event horizon \( r_+ \) is the solution of \( f(r_+) = 0 \), which indicates the black hole mass \( M \) can be expressed in event horizon \( r_+ \) as

\[
M = \frac{r_+}{2} + \frac{4}{3} \pi Pr_+^3 + \frac{Q^2}{2r_+} + \frac{1}{2} \lambda \ln \left( \frac{r_+}{\lambda} \right).
\] (9)

The first law of thermodynamics can be expressed as

\[
dM = TdS + \Phi dQ + VdP + Ad\lambda.
\] (10)

where the thermodynamic volume \( V \) of black hole, the electrostatic potential \( \Phi \) and the conjugate quantity \( A \) are given by

\[
V = \frac{4}{3} \pi r_+^3, \quad \Phi = \frac{Q}{r_+}, \quad A = \frac{1}{2} \ln \left( \frac{r_+}{\lambda} \right).
\] (11)

The Hawking temperature \( T \) is obtained as

\[
T = \left( \frac{\partial M}{\partial S} \right)_{Q,P,\lambda} = \frac{\lambda}{4\pi r_+^2} + 2Pr_+ + \frac{1}{4\pi r_+} - \frac{Q^2}{4\pi r_+^3}.
\] (12)

The Hawking temperature \( T \) versus event horizon \( r_+ \) and entropy \( S \) are shown in FIG. (1) for different values of \( Q \) with \( \lambda = 1 \) and \( P = 0.075 \). The Hawking temperature is zero corresponding to the extreme black hole. Since the slope of the \( T - S \) graph is related to
specific heat capacity, it’s positive or negative values determine the stability of the system with respect to fluctuations. Hence the FIG. (1) shows that there exists a critical point which indicates the phase transition [42]. In this case, we think that the asymptotic AdS black hole in a charge fixed canonical ensemble shows a first-order phase transition similar to that of a van der Waals fluids.

\[ Q = 0.5 \]
\[ Q = 0.6 \]
\[ Q = 0.7 \]
\[ Q = 0.8 \]

FIG. 1: The temperature \( T \) of RN-AdS black hole immersed in PFDM versus \( r_+ \) and \( S \) for different values of \( Q \), where \( \lambda = 1 \) and \( P = 0.075 \).

Next we will study the critical behaviour of this black hole. From Eq. (12), we get the expression of pressure,

\[ P(r_+, T) = \frac{T}{2r_+} - \frac{1}{8\pi r_+^2} + \frac{Q^2}{8\pi r_+^4} - \frac{\lambda}{8\pi r_+^3}. \]  

(13)

At the critical point, we have

\[ \frac{\partial P}{\partial r_+} \bigg|_{r=r_c} = \frac{\partial^2 P}{\partial r_+^2} \bigg|_{r=r_c} = 0, \]  

(14)

\[ \frac{\partial T}{\partial r_+} \bigg|_{r=r_c} = \frac{\partial^2 T}{\partial r_+^2} \bigg|_{r=r_c} = 0, \]  

(15)

so the critical point \( r_c \) is obtained as

\[ r_c = \frac{1}{2} \sqrt{9\lambda^2 + 24Q^2} - \frac{3\lambda}{2}. \]  

(16)

By using the Eqs. (12), (13) and (16), we can determine the critical temperature \( T_c \) and critical pressure \( P_c \),

\[ T_c = \frac{16Q^2 - 3\lambda \left( \sqrt{9\lambda^2 + 24Q^2} - 3\lambda \right)}{\pi \left( \sqrt{9\lambda^2 + 24Q^2} - 3\lambda \right)^3}, \]  

(17)

\[ P_c = \frac{\lambda \left( 3\lambda - \sqrt{9\lambda^2 + 24Q^2} \right) + 6Q^2}{\pi \left( \sqrt{9\lambda^2 + 24Q^2} - 3\lambda \right)^4}. \]  

(18)
It is clear that the critical variables depend on the charge $Q$ and parameters $\lambda$. Here we focus on the critical temperature because it will be useful to analyze the Joule-Thomson expansion of the black hole in the next section. One can plot the critical point $r_c$ and the critical temperature $T_c$ with varying $Q$ and $\lambda$ in FIG. 2. The critical temperature decreases while charge $Q$ increases. If the temperature is less than the critical temperature $T_c$, the black hole can undergo a first-order phase transition between the small black hole and the large black hole, which is similar to the van der Waals fluids. Furthermore, one can go back to the RN-AdS case as in [27] by taking the limit $\lambda \rightarrow 0$. Thus, the influence of PFDM on the critical behaviours of black hole is obvious.

\[ \lambda = 0 \quad \lambda = 0.1 \quad \lambda = 0.5 \quad \lambda = 1 \quad \lambda = 2 \]

\[ 0.0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad Q \]

\[ 0.0 \quad 0.5 \quad 1.0 \quad 1.5 \quad 2.0 \quad T_c \]

**FIG. 2:** Critical radius $r_c$ and critical temperature $T_c$ versus $Q$ of RN-AdS black hole immersed in PFDM.

We derive the thermodynamic definitions of RN-AdS black holes immersed in PFDM, and we study the critical behaviour of this black hole. In the next section, we investigate the Joule-Thomson expansion of RN-AdS black holes immersed in PFDM.

### III. Joulle-Thomson Expansion

The instability of the black holes appears in the process of Joule-Thomson expansion. And we introduce Joule-Thomson coefficient $\mu$ to determines the cooling and heating phases of the isenthalpic expansion.

The specific heat at constant pressure for the black holes is obtained according to the first law of thermodynamics,

\[ C_p = T \left( \frac{\partial S}{\partial T} \right)_{P,Q,\lambda} \] \hspace{1cm} (19)

Regarding parameters $Q, P, \lambda$ as constant, and substituting the Eqs. (11) and (12) into (19),
we have the isobaric heat capacity of this black hole,
\[
C_p = \frac{2\pi r_+^2 [r_+ (\lambda + 8\pi Pr_+^3 + r_+) - Q^2]}{r_+ (-2\lambda + 8\pi Pr_+^3 - r_+) + 3Q^2}.
\]

(20)

For the Joule-Thomson expansion of the van der Waals fluids, the fluids passes through a porous plug from one side to the other with pressure declining during throttling process. Therefore, we apply this corresponding concept to black hole thermodynamics and the enthalpy \( M \) of the black hole keeps constant throughout this process. Besides, the partial differential of temperature versus pressure of the black hole is defined as Joule-Thomson coefficient \( \mu \),
\[
\mu = \left( \frac{\partial T}{\partial P} \right)_M = \frac{1}{C_p} \left[ T \left( \frac{\partial V}{\partial T} \right)_P - V \right].
\]

(21)

By substituting Eqs (12), (20) and (11) into (21), we obtain
\[
\mu = \frac{2r_+ [r_+ (5\lambda + 16\pi Pr_+^3 + 4r_+) - 6Q^2]}{3 [r_+ (\lambda + 8\pi Pr_+^3 + r_+) - Q^2]}.
\]

(22)

The Joule-Thomson coefficient \( \mu \) versus the horizon \( r_+ \) is shown in FIG. (3) where the parameter \( \lambda = 0, 0.5, 1 \) and 2, pressure \( P = 0.075 \) and charges \( Q = 0.7 \). There exist both a divergence point and a zero point at each different value of \( \lambda \) and the influence of PDFM on JT coefficient is apparent. The divergence point here reveals the information of Hawking temperature and corresponds to the extremal black hole, and it is clear that the divergence point of the blue curve in FIG. (3) is consistent with the zero point of Hawking temperature in FIG. (1) in the case of \( Q = 0.7 \).

![FIG. 3: Joule-Thomson coefficient \( \mu \) of RN-AdS black hole immersed in PDFM for \( Q = 0.7 \), \( P = 0.075 \) and \( \lambda = 0, 0.5, 1 \) and 2.](image)

Therefore, we are only interested in the right part of FIG. (3) where the temperature is above zero. Besides, the Joule-Thomson coefficient of RN-AdS black hole immersed in
PFDM can be both positive and negative, which means that there remain both cooling and heating stages. For \( \mu > 0 \), the temperature of the black hole decreases in other words the black hole is cooling down when pressure goes down in the process of throttling. On the contrary, \( \mu < 0 \) means that the black hole heats up after the throttling process. So there are inversion points at which the cooling-heating transition occurring when \( \mu = 0 \), and the temperature of the black hole at \( \mu = 0 \) is called the inversion temperature.

One can obtain the inversion temperature \( T_i = V \frac{\partial T}{\partial V} \) by \( \mu = 0 \). While using the Eqs. (11) and (12), inversion temperature \( T_i \) in terms of inversion pressure \( P_i \) is expressed as

\[
T_i = V \left( \frac{\partial T}{\partial V} \right)_P = \frac{2P_i r_+}{3} + \frac{Q^2}{4\pi r_+^3} - \frac{\lambda}{6\pi r_+^2} - \frac{1}{12\pi r_+}.
\]

(23)

Bringing this equation into Eq. (13) at \( P = P_i \), we derive

\[
P_i = \frac{3Q^2}{8\pi r_+^4} - \frac{5\lambda}{16\pi r_+^3} - \frac{1}{4\pi r_+^2},
\]

(24)

\[
T_i = \frac{Q^2}{2\pi r_+^3} - \frac{3\lambda}{8\pi r_+^2} - \frac{1}{12\pi r_+^2} - \frac{1}{6\pi r_+}.
\]

(25)

From above results, the inversion curves \( T_i - P_i \) in FIG. 4 with different \( Q \) and \( \lambda \) is plotted, which is similar to the case of charged RN AdS black hole [27]. At low pressure, the inversion temperature \( T_i \) decreases with the increase of charge \( Q \) and increases with the increase of \( \lambda \). This phenomenon is just the opposite of high pressure situation. In addition, there are some numerical effects of PFDM on the inversion temperature, but no giant differences from charged AdS black hole. It can be observed that the inversion temperature still increases monotonically with the growth of inversion pressure, and the inversion curves are not closed which is different from the situation of the van der Waals fluids.

FIG. 4: Inversion curves of RN-AdS black hole immersed in PFDM.
The Joule-Thomson expansion occurs in the isenthalpic process. And for a black hole, enthalpy is mass $M$. The isenthalpic curves can be obtained from Eqs. (9) and (13) in FIG. (5) where inversion curves and isenthalpic curves are both presented.

![Graphs of isenthalpic curves with varying $M$ values](image)

(a) $\lambda = 1$, $Q = 0.7$ and $M = 2, 2.25, 2.5, 2.75, 3.$

(b) $\lambda = 1$, $Q = 0.8$ and $M = 2, 2.25, 2.5, 2.75, 3.$

(c) $\lambda = 1$, $Q = 0.7$, and $M = 2, 2.25, 2.5, 2.75, 3.$

(d) $\lambda = 0.1$, $Q = 0.8$ and $M = 2, 2.25, 2.5, 2.75, 3.$

FIG. 5: The isenthalpic curves of RN-AdS black hole immersed in PFDM. From bottom to top, the isenthalpic curves correspond to increasing values of $M$.

The intersections of the black inversion curves and the blue isenthalpic curves indicating the cooling-heating transition coincide with the extreme points of isenthalpic curves. And the isenthalpic curves have positive slope above the inversion curves indicating that the cooling occurs. And the isenthalpic curves have negative slope under the inversion curves indicating that the heating occurs. We can conclude that the region above this inversion curve is cooling zone while the one below is heating zone. The temperature rises when the pressure goes down in the heating zone. In contrast, the temperature is lower with reducing pressure in the cooling zone. Moreover, comparing the four graphs the temperature and pressure decrease with larger charge $Q$ or smaller $\lambda$, and PFDM statistically have an influence on the isenthalpic curves.
Furthermore, one can obtain the minimum inversion temperature $T_{i\text{min}}$ corresponding to $P_i = 0$ as

$$T_{i\text{min}} = \frac{4 \left[ \lambda \left( 5\lambda - \sqrt{25\lambda^2 + 96Q^2} \right) + 16Q^2 \right]}{\pi \left( \sqrt{25\lambda^2 + 96Q^2} - 5\lambda \right)^3}. \tag{26}$$

The ratio of inversion temperature to critical temperature $T_{i\text{min}}/T_c$ is obtained according to Eqs. (26) and (17),

$$\frac{T_{i\text{min}}}{T_c} = \frac{4 \left( \sqrt{9\lambda^2 + 24Q^2} - 3\lambda \right)^3 \left[ \lambda \left( 5\lambda - \sqrt{25\lambda^2 + 96Q^2} \right) + 16Q^2 \right]}{\left( \sqrt{25\lambda^2 + 96Q^2} - 5\lambda \right)^3 \left[ 16Q^2 - 3\lambda \left( \sqrt{9\lambda^2 + 24Q^2} - 3\lambda \right) \right]^3}. \tag{27}$$

It has been shown in [27] that the ratio for the RN-AdS black holes equals $1/2$, but in the case of PFDM this value is corrected. In fact, the ratio grows with increasing $Q/\lambda$, and the ratio versus $Q/\lambda$ is shown in FIG. (6). When $Q/\lambda \to 0$, in other words $Q \to 0$ or $\lambda \to \infty$, the ratio has a leading term which is $25/54$,

$$\frac{T_{i\text{min}}}{T_c} = \frac{25}{54} + \frac{4Q^2}{27\lambda^2} + O\left[ (Q/\lambda)^4 \right], \tag{28}$$

and it is close to the situation of Schwarzschild black hole within PFDM.

For $Q/\lambda \to \infty$,

$$\frac{T_{i\text{min}}}{T_c} = \frac{1}{2} - \frac{\lambda^2}{128Q^2} + O\left[ (\lambda/Q)^3 \right]. \tag{29}$$

Furthermore, by taking the limit $\lambda \to 0$, the ratio is approaching $1/2$ corresponding to the RN-AdS case [27].

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig6.png}
\caption{The ratio $T_{i\text{min}}/T_c$ of RN-AdS black hole immersed in PFDM.}
\end{figure}
IV. CONCLUSION

In this paper, we discuss the Joule-Thomson expansion for RN-AdS black holes immersed in PFDM. In the extended phase space, the cosmological constant is identified with the pressure and its conjugate quantity as the thermodynamic volume. Since the black hole mass is interpreted as enthalpy, an isenthalpic process can be applied to calculate the temperature change during the Joule-Thomson expansion. By computing the Joule-Thomson coefficient \( \mu \), the divergent point coincides with the extremal black hole while the zero point is the inversion point that determines the transition from heating/cooling phases. And we calculate the inversion curves in the \( T - P \) plane as well as the corresponding isenthalpic curves. And we can use inversion curve distinguish the cooling and heating regions for different values of \( \lambda \) and \( Q \). Our calculations show that the effect of PFDM density parameter \( \lambda \) on Joule-Thomson coefficient and the inversion temperature is obvious. In addition, one can easily go back to the RN-AdS case by taking the limit \( \lambda \to 0 \). We also discuss the isenthalpic curves with different values of \( M \) during the throttling process.

Furthermore, we derive the ratio of minimum inversion temperature to critical temperature and the inversion curves in terms of charge \( Q \) and parameter \( \lambda \). It has been shown in [27] that the ratio for the charged RN-AdS black holes is equal to 1/2, but in the case of PFDM the ratio depends on the charge \( Q \) and parameter \( \lambda \). In fact, the ratio of \( T_{i}^{\text{min}}/T_{c} \) depends on \( Q/\lambda \), which is shown in FIG. [6]. The ratio is about 25/54 for small \( Q/\lambda \), and it is corresponding to the situation of RN-AdS black holes for \( \lambda = 0 \).

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