Molecular Ensemble Based Remote Quantum Storage for Charge Qubit via Quasi-Dark State

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(Dated: May 6, 2009)

We propose a quantum storage scheme independent of the current time-control schemes, and study a “quantum data bus” (transmission line resonator) in a hybrid system consisting of a circuit QED system integrated with a cold molecular ensemble. Here, an effective interaction between charge qubit and molecule is mediated by the off-resonate field in the data bus. Correspondingly, the charge state can be mapped into the collective quasi-spin state of the molecular ensemble via the standard dark state based adiabatic manipulation.

PACS numbers: 03.67.Lx, 33.80.Ps, 85.25.Cp

I. INTRODUCTION

Quantum storage is crucial to quantum information processing [1, 2, 3]. Its importance rests with two issues: mapping the qubits with shorter coherence time into a system with relatively longer coherence time [4, 5, 6]; and storing the quantum states of the physical system sufficiently operated into the states of the system manipulated feasibly [7]. Superconducting qubits, such as the charge qubits, flux qubits and phase qubits, are hopeful candidates for the basic blocks in the architecture of quantum computer [8] since they exhibit feasibility to be well manipulated and well integrated [9, 10, 11, 12, 13, 14]. Currently the decoherence of the superconducting qubits is relatively fast so that no enough logical operations can be performed for a practical quantum computing. Thus people expect that the superconducting qubits can be stored in some system for which the decoherence time is long enough so that the stored qubits can be operated within an enough time.

To overcome the disadvantage of superconducting qubits a hybrid quantum processor has been suggested interfac ing the solid state system and the molecular ensemble [15, 16], in which the ensemble of cold polar molecules was used as a long-lived quantum memory storing the information of superconducting quantum qubits by collective spin state via microwave Raman processes in a high-Q stripline cavity [16]. Further more, a holographic quantum computing has been demonstrated using this protocol [17]. These conceptual designs used the real photons as the data bus, and the corresponding schemes depend on an exact time control to switch photon-qubit and photon-molecular couplings effectively on and off. Based on the above considerations, to avoid the difficulty in exactly controlling evolution time, we study the data bus role of quantum transmission line. By adiabatically eliminating the variables of the transmission line as quantum data bus, we obtain dark state for the effective coupling between molecular ensemble and the charge qubit. And then adiabatically manipulating the control field, the superconducting charge qubit can be mapped into the collective quasi-spin state of the molecular ensemble, and a reversing operation in a latter time will regenerate the charge qubit state. During the adiabatic passage the transmission line is virtual excited and no real photons in the transmission line are exchanged between the charge qubit and the molecular ensemble. In this sense the effective dark state of the charge qubit and the molecular ensemble computing system plays crucial role.

The paper is organized as follows. In Sec. II we describe the circuit QED setup hybridized with embedded molecular ensemble. In Sec. III, we demonstrate how the off-resonate coplanar cavity field induces the effective coupling of charge qubit to collective excitation of molecular ensemble and the existence of the quasi-dark state at parameter resonance. In Sec. IV, we show that using the effective dark state, quantum storage of the charge qubit state can be implemented through an adiabatic manipulation.

II. MODELING THE HYBRID SYSTEM IN TERMS OF COLLECTIVE EXCITATIONS

We consider a hybrid system shown in Fig. 1 that consists of a cold molecular ensemble, a Cooper-pair box (charge qubit) and the stripline cavity formed by a transmission line resonator [18, 17]. The molecular ensemble consists of \( N \) identical and noninteracting A-type molecules, each with a ground state \( |b\rangle \), an excited state \( |a\rangle \) and a metastable lower excited state \( |c\rangle \) (see in Fig. 1(b)). At the antinodes of quantum electromagnetic field in stripline cavity, the polar molecules with large electric-dipole moment achieve the strong coupling, similarly as that in the circuit QED system [18]. We place the Cooper-pair box and the molecular ensemble in two antinodes at a distance. In some proper conditions [19], the Cooper-pair box can be reduced to a two-level quantum system (charge qubit) with Jaynes-Cummings type coupling to the single mode cavity field. The Hamilto-
FIG. 1: Schematic diagram of a hybrid system where the cold molecular ensemble is coupled to a circuit QED system consisting of a Cooper-pair box and a stripline cavity in (a). The Cooper-pair box (as a two level system (b)) and cold molecules consisting of a Cooper-pair box and a stripline cavity in (a). The molecular ensemble is coupled to a circuit QED system as in Ref. [7] and invoke the bosonic operators

\[ A = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |b\rangle_{jj} \langle a|, \quad C = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |b\rangle_{jj} \langle c| \] (3)

to describe the collective excitation of the ensemble. To this end we rewrite the above Hamiltonian \( H = H_0 + H_1 \) in terms of the free part

\[ H_0 = \frac{\omega_a}{2} \sigma_z + \omega_a^\dagger a + \omega_c A^\dagger A + \omega_c C^\dagger C \] (4)

and the interaction part

\[ H_1 = g (a \sigma_+ + h.c.) + \zeta (a A^\dagger + h.c.) + \Omega (e^{-i\omega f t} A^\dagger C + h.c.), \] (5)

where \( \zeta = \xi \sqrt{N} \). In the large \( N \) limit with the low excitation condition that there are only a few atoms occupying the level \( |a\rangle \) or \( |c\rangle \) [20], the operator \( A \) \( (C) \) behaves as a boson which satisfies the bosonic commutation relation

\[ [A, A^\dagger] = 1, \quad [C, C^\dagger] = 1. \] (6)

In the same limit, we also have

\[ [A, C] = 0, \quad [A, C^\dagger] \rightarrow 0, \] (7)

which means that two boson modes described by \( A \) and \( C \) are independent with each other.

III. INDUCED EFFECTIVE COUPLING OF CHARGE QUBIT TO COLLECTIVE EXCITATIONS OF MOLECULAR ENSEMBLE

We emphasize again that, the current hybrid system based quantum storage protocol requires an exact time control for couplings of charge qubit to photons and photons to molecular ensemble. Namely, one can switch on (off) the couplings exactly in an arbitrarily given instance. Such a time control process includes two steps: a) couple the photon to the charge qubit firstly; b) then switch off the previous coupling and switch on the coupling of the photon to the molecular ensemble again. Due to the “time-energy uncertainty”, this kind of time control protocol would inevitably result in experimental errors. It is much appreciated that we can avoid these errors. With these considerations, we detune stripline cavity field slightly from both the charge qubit and the molecular ensemble. Thus the effective interaction between the charge qubit and the molecular ensemble will be induced by adiabatically eliminating photon degrees of freedom.

Initially, we prepare the microwave field in the vacuum state, then the interaction between the charge qubit and the molecular ensemble is induced by the virtual excitation of the transmission line field, with no exchange of the real photons between the charge qubit (molecular ensemble) and the field. The process can be schematically

\[ H_1 = \frac{\omega_a}{2} \sigma_z + \omega_a^\dagger a + g(a \sigma_+ + h.c.), \] (1)

where \( a \) \( (a^\dagger) \) is the quantum cavity field annihilation and (creation) operator, \( \sigma_- \) \( (\sigma_+) \) the lowering (raising) operator, and \( g \) the strength of the interaction between the stripline cavity field and the Cooper-pair box. The parameters \( \omega_a \) and \( \omega_c \) denote the frequency of charge qubit and the quantum cavity field respectively. Here we have assumed \( \hbar = 1 \) for convenience in this paper. Initially we can prepare the polar molecule in the ground state \( |b\rangle \), which is coupled to the excited state \( |a\rangle \) by the quantum electromagnetic field in the stripline cavity. While the coupling of the ground state \( |b\rangle \) to the metastable excited state \( |c\rangle \) is realized through the classical control field with the Rabi frequency \( \Omega \). Thus the Hamiltonian for the coupled molecular ensemble-stripline cavity can be written as

\[ H_2 = \sum_{j=1}^{N} (\omega_a |a\rangle_{jj} \langle a| + \omega_c |c\rangle_{jj} \langle c|) + \omega_a^\dagger a + \sum_{j=1}^{N} \left( \xi a \langle a|_{jj} (b| + \Omega e^{-i\omega f t} |a\rangle_{jj} \langle c| + h.c.) \right). \] (2)

where \( \xi \) is the interaction strength of the quantum cavity field to the molecular ensemble, \( \omega_a \) and \( \omega_c \), the molecule’s energy level spacings with respect to the excited state and the metastable excited state respectively, and \( \omega_f \) the frequency of the control field.

Here we adopt the quasi-spin wave approach developed
shown in Fig. 2. Here both the qubit and the molecular ensemble interact simultaneously with the continuum of the cavity field whose spectral structure is illustrated in the top of Fig. 2. We will show as follows that, under some conditions related to the large detuning the virtual photon transition happens between the qubit (molecular ensemble) and the transmission line cavity. The photon-assisted transitions can not happen in practice, but the effective interaction between qubit and the molecular ensemble is induced, which is similar to the conventional contact interaction. Therefore, our present protocol does not need an exact time control for the coupling of the photon qubits to the charge qubit (to the molecular ensemble). Next we study in details the virtual photon exchange process by deriving the effective Hamiltonian through a canonical transformation. In the rotating frame in terms of a time dependent unitary operator $U = \exp(i\omega t C^\dagger C)$, we get a time independent Hamiltonian

$$H^R = U^\dagger H U - iU^\dagger \partial_t U.$$  (8)

When we adopt the frequency-matching condition, $\omega_2 = \omega_c + \omega_f$, i.e., resonantly coupling the metastable lower excited state $|c\rangle$ to the excited state $|a\rangle$, the time-independent Hamiltonian can be decomposed into two parts, $H^R = H_0 + H_1$, including the “free” part Hamiltonian

$$H_0 = \omega_0 \sigma_z + \omega_2 a + \omega_3 a^\dagger a C^\dagger C + \Omega(A^\dagger C + h.c.)$$  (9)

and the interaction part Hamiltonian

$$H_1 = g(a\sigma_+ + h.c.) + \zeta(a A^\dagger + h.c.),$$  (10)

which describes the coupling of stripline cavity field to both the charge qubit and the molecular ensemble.

Using Fröhlich’s transformation, we can obtain the effective Hamiltonian

$$H_{\text{eff}} = e^{-S} H^R e^S \approx H_0 + \frac{1}{2} [H_1, S],$$  (11)

where $S$ is an anti-Hermitian operator satisfying

$$H_1 + [H_0, S] = 0.$$  (12)

The corresponding anti-Hermitian operator $S$ for Fröhlich transformation adopts the following form

$$S = (\eta_1 a\sigma_+ + \eta_2 a A^\dagger + \eta_3 a C^\dagger) - h.c.,$$  (13)

where the coefficients $\eta_1, \eta_2, \text{ and } \eta_3$ will be determined by Eq. (12)

$$\begin{align*}
\eta_1 &= \frac{g}{\omega_2 - \omega_0}, \\
\eta_2 &= \frac{\zeta (\omega - \omega_a)}{(\omega - \omega_a)^2 - \Omega^2}, \\
\eta_3 &= \frac{\zeta \Omega}{(\omega - \omega_a)^2 - \Omega^2}. 
\end{align*}$$  (14)

In order to obtain the effective coupling between the charge qubit and the molecular ensemble we consider the lightly off resonant case and take appropriate experimental conditions so that the coefficients $\eta_1, \eta_2$ and $\eta_3$ are all as small as enough, i.e., $|\eta_1|, |\eta_2|, |\eta_3| \ll 1$, which are also requisite by the usual second order perturbation theory. We assume that the microwave cavity field is initially in the vacuum state, i.e., $\langle a^\dagger a \rangle = 0$. Averaged over the vacuum state of the photon field, in the two photon resonance case, the effective Hamiltonian (11) in the interaction picture reads,

$$H_{\text{int}} = \Delta B^\dagger B + (g_m\sigma - B^\dagger + \Omega d B^\dagger D + h.c.),$$  (15)

where the bosonic operators $B$ and $D$ are defined

$$B = \alpha A + \beta C, \quad D = \beta A - \alpha C,$$  (16)

with some time-dependent coefficients

$$\alpha = \frac{(\eta_2 g + \eta_1 \zeta)}{\sqrt{(\eta_2 g)^2 + (\eta_2 g + \eta_1 \zeta)^2}}.$$  (17)

$$\beta = \frac{(\eta_3 g)}{\sqrt{(\eta_2 g)^2 + (\eta_2 g + \eta_1 \zeta)^2}}.$$  (18)

The effective Hamiltonian (15) shows that the photon degrees of freedom is effectively eliminated and it mediates an effective interaction between the charge qubit and the molecular ensemble. The two photon resonance condition is

$$\omega_B - \omega'_g = \omega_B - \omega_D = \Delta$$  (19)

with $\omega_B$ and $\omega_D$ being the effective energy level spacings with respect to the states of $B$-excitation and $D$-excitation respectively, $\omega'_g$ the effective frequency of the charge qubit after the Fröhlich transform. The complicated coefficients in $H_{\text{int}}$ are listed as follows

$$g_m = -\frac{1}{2} \sqrt{(\eta_3 g)^2 + (\eta_2 g + \eta_1 \zeta)^2},$$  (20)
\[ \omega' = \omega_0 - \eta_1 g, \]  
\[ \omega_B = \omega_a - \eta_2 \zeta + 2 \left( \Omega - \frac{1}{2} \eta_3 \zeta \right) g, \]  
\[ \omega_D = \omega_a - \eta_2 \zeta - 2 \left( \Omega - \frac{1}{2} \eta_3 \zeta \right) g, \]

\[ \Omega_d = -\eta_2 \zeta + \left( \Omega - \frac{1}{2} \eta_3 \zeta \right) (b^2 - a^2). \]

In order to find invariant subspaces of \( H_{int} \), we define the dark state operator

\[ F = \sigma_+ \cos \theta - D \sin \theta \]  

with \( \theta(t) \) satisfying \( \tan \theta(t) = g_m / \Omega_d(t) \). It mixes the states of the qubit excitation and the collective molecular excitations. We introduce the composite vacuum state \( |0\rangle = |b\rangle \otimes |g\rangle \), where \( |b\rangle \) is the ground state with all \( N \) molecules staying in the same single particle ground state \( |b\rangle \). Evidently an eigenstate of \( H_{int} \) with vanishing eigenvalue is constructed as

\[ |\Psi\rangle = F^\dagger |0\rangle. \]  

We notice that \( |\Psi\rangle \) is not an ideal dark state, actually it is a superposition state of the \( A \)-excitation concerning the excited state which will decay during the usual manipulation process. In this sense, we name it quasi-dark state.

To construct other eigenstates of the interaction Hamiltonian, we introduce the operators as follows

\[ Q_\pm = \sqrt{\Theta \pm \Delta} B \pm \sqrt{\Theta \pm \Delta} (\sigma_+ \sin \theta + D \cos \theta), \]

where \( \Theta = \sqrt{\Delta^2 + 4(g_m^2 + \Omega_d^2)} \). Using these operators, we can construct the eigenstates \( |\Phi_{\pm}\rangle = Q^\dagger_{\pm} |0\rangle \), with the corresponding eigenvalues \( E_{\pm} = \pm \sqrt{\Theta \pm \Delta}(g_m^2 + \Omega_d^2)/\Theta \pm \Delta). \]

**IV. QUASI-DARK STATE BASED ADIABATIC MANIPULATION FOR QUANTUM STORAGE**

In this section, we will show how to store the quantum information of the charge qubit in the molecular ensemble by using the above quasi-dark state. In this quantum storage scheme, adiabatically modulating the control field, the charge qubit state can be mapped into the collective excitation state of the molecular ensemble in appropriate experimental conditions.

Now we consider whether the charge qubit state can be mapped into the molecular ensemble effectively. We assume that the Rabi frequency of the control field is much smaller than the frequency detuning between the cavity field and the \( A \)-excitation of molecule, i.e., \( |\Omega| \ll |\omega - \omega_a| \). Then the values of \( \alpha \) and \( \beta \) are approximated as \( \alpha \approx 1 \) and \( \beta \approx 0 \) from the Eqs. \([17]\) and \([18]\). In this case, there exist other eigenstates with non-zero eigenvalues besides the quasi-dark state. To avoid the system to transit to other eigenstates and keep itself within the quasi dark state, the manipulation of the control field should satisfy the adiabatic condition

\[ \frac{|\Phi_{\pm}| \partial_t |\Psi\rangle}{E_{\pm} - \Omega} = \frac{g_m(\theta + |\Delta|)}{\sqrt{\Theta - |\Delta|}(g_m + \Omega_d^2)} |\Omega| \ll 1 \]  

according the quantum adiabatic theorem \([22, 23] \). From the Eqs. \([20, 21]\) of the coefficients, with the adiabatic decrease of the control field, the mixing angle will increase from 0 to \( \pi/2 \).

Because the composite vacuum state \( |g\rangle \otimes |b\rangle \) is an eigenstate of the interaction Hamiltonian also, if the mixing angle \( \theta \) changes from 0 to \( \pi/2 \), an arbitrary superposition of charge qubit state will undergo the following evolution through the quasi-dark state,

\[ \gamma |g\rangle + \delta |c\rangle \otimes |b\rangle \rightarrow |g\rangle \otimes (\gamma |b\rangle + \delta (|c\rangle + \beta |a\rangle)), \]

where \( |a\rangle = A^\dagger |b\rangle \), \( |c\rangle = C^\dagger |b\rangle \). This means that we can transfer the information of the charge qubit state into the collective excitation state of the molecular ensemble without any real photon exchange. This storage process can be schematically shown in Fig. 3. Here, rotating the mixing angle \( \theta \) from 0 to \( \pi/2 \) by changing the strength of the control field adiabatically, the information of the charge qubit state is stored in the molecular ensemble. And the reverse manipulation will make the molecular ensemble release the information to the charge qubit again.

**FIG. 3: Mapping the charge qubit into the molecular ensemble by virtual photon transition.**

We have approximately considered the approach of mapping the charge qubit into the molecular ensemble. To realize the storage of a charge qubit, next we will consider some experimental data and show how the coefficients change with respect of the control field in details. From the references \([15, 18]\), we choose the experimental data: \( g = 20 \text{MHz}, \) \( \zeta = 20 \text{MHz}, \omega = 6.044 \text{GHz} \) and \( \omega_a = 5.844 \text{GHz} \). The effective coupling \( g_m = 2.01 \text{MHz} \) which almost does not depend on the control field. The mixing angle \( \theta \) changes from \( \pi/2 \) to 0 by adiabatically modulating the control field with the Rabi frequency \( \Omega \).
varying from 0 to 30MHz, (see, e.g., Fig. 4). The frequency of charge qubit needs to be modulated (see, e.g., Fig. 5) with the variance of the control field to satisfy the resonance condition (19). The change range is about 5MHz, which is very small comparing to the detuning between the cavity and the charge qubit with the values of $198 \rightarrow 202$MHz. The appearance of the excited state $|a\rangle$ will induce noise during the storage process. The magnitude of coefficient $\beta \sin \theta$ for the excited state decides the degree of the noise. Just as shown in Fig. 6, the value of the coefficient is very small with the slowly increase of the control field and the influence of the noise can be omitted. All these results indicate that the charge qubit can be mapped into the collective excitation of the molecular ensemble in current experimental condition effectively without any real photon exchange.

![FIG. 4: Variance of the mixing angle $\theta$ by adiabatic modulation of the control field with the Rabi frequency $\Omega$ changing from 0 to 30MHz.](image1)

![FIG. 5: Modulation of the frequency of the charge qubit with the change of the control field. Here the variance of the qubit frequency is denoted by the frequency difference between charge qubit and the $A$-excitation of the molecular ensemble and the unit is GHz.](image2)

V. CONCLUSIONS

In summary, we have explored and studied a quantum storage scheme for charge qubit, which is based on the virtual photon exchange between charge qubit and the collective excitations of polarized molecules. Our investigation concerns a hybrid system consisting of a superconducting circuit QED system and an ensemble of polarized molecules. Through the adiabatic elimination using the Fr"ohlich's transformation, we explicitly derive an effective coupling between the qubit and the molecular ensemble. It shows that the effective dark state can be dynamically produced as an entanglement of qubit plus molecular ensemble, which means that the qubit state can directly be mapped into the state of molecular ensemble without middle time control. The corresponding numerical results show the feasibility of this experimental scheme.

The work is supported by National Natural Science Foundation of China under Grant Nos. 10547101 and 10604002, the National Fundamental Research Program of China under Grant No. 2006CB921200.
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