A PSO Approach for Optimum Design of Multivariable PID Controller for nonlinear systems

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Abstract—The aim of this research is to design a PID Controller using particle swarm optimization (PSO) algorithm for multiple-input multiple output (MIMO) Takagi-Sugeno fuzzy model. The conventional gain tuning of PID controller (such as Ziegler-Nichols (ZN) method) usually produces a big overshoot, and therefore modern heuristics approach such as PSO are employed to enhance the capability of traditional techniques. However, due to the computational efficiency, only PSO will be used in this paper. The results show the advantage of the PID tuning using PSO-based optimization approach.

I. INTRODUCTION

PID control, which is usually known as a classical output feedback control for SISO systems, has been widely used in the industrial world [1] and [2]. The tuning methods of PID control are adjusting the proportional, the integral and the derivative gains to make an output of a controlled system track a target value properly. Several researchers focus on multiple-input-multiple output MIMO control systems. Because many industrial processes are MIMO systems which need MIMO control techniques to improve performance, though they are naturally more difficult than SISO systems. As we know, MIMO PID controller design has developed over a number of years. Luyben (1986) proposed a simple tuning method for decentralized PID controllers in MIMO system from single-loop relay tests [3]. Yusof and Omatu (1993) presented a multivariable self-tuning PID controller based on estimation strategies [4]. Wang et al. (1997) proposed a tuning method for fully cross-coupled multivariable PID controller from decentralized relay feedback test to find the critical oscillation frequency of the system by first designing the diagonal elements of multivariable PID controller independent of off-diagonal ones [5]. Recently, the computational intelligence has proposed particle swarm optimization (PSO) [6,7] as opened paths to a new generation of advanced process control. The PSO algorithm, proposed by Kennedy and Eberhart [6] in 1995, was an evolution computation technology based on population intelligent methods. In comparison with genetic algorithm, PSO is simple, easy to realize and has very deep intelligent background. It is not only suitable for scientific research, but also suitable for engineering applications in particular. Thus, PSO received widely attentions from evolution computation field and other fields. Now the PSO has become a hotspot of research. Various objective functions based on error performance criterion are used to evaluate the performance of PSO algorithms.

In this paper, a scheduling PID tuning parameters using particle swarm optimization strategy for MIMO nonlinear systems. This paper has been organized as follows: In section 2, a brief review of the TS fuzzy model formulation is given. Estimation method of recursive weighted least-squares (RWLS) in section 3. In section 4, PID control systems of multivariable processes. Finally, some conclusions are made in section 5.

II. TAKAGI-SUGENO FUZZY MODEL OF A MIMO PROCESS

Generally, modeling process consists to obtain a parametric model with the same dynamic behavior of the real process. In this section, we are interested to the problem of the MIMO process identification[8]. We consider a MIMO system with \( n_1 \) inputs and \( n_0 \) outputs. The MISO models are a input-output NARX (Non linear Auto Regressive with eXogenous input) defined by:

\[
y_i(k+1) = f_i(x_l(k)) \quad l = 1, 2, ..., n_0
\]

(1)

With the regression vector represented by:

\[
x_l(k) = \{[y_1(k)]^{n_{y1}}, [y_2(k)]^{n_{y2}}, ..., [y_{n_0}(k)]^{n_{y0}}, [u_1(k)]^{n_{u1}}, [u_2(k)]^{n_{u2}}, ..., [u_{n_1}(k)]^{n_{ud}}\}
\]

(2)

\( n_y \) and \( n_u \) define the number of delayed outputs and inputs respectively. \( n_d \) is the number of pure delays. \( n_y \) is a \( n_0 \times n_0 \) matrix and \( n_u \times n_d \) are \( n_0 \times n_i \) matrices. \( f_l \) are unknown nonlinear functions. MISO models are estimated independently [9], so to simplify the notation, the output index \( l \) is omitted and we will be interested only in the multi-input, mono-output case. The Takagi-Sugeno MISO rules are estimated from the system input-output data[15]. The base rule contains \( r \) rules of the following form:

\[
R_r: \text{If } y_l(k) \text{ is } A_{r1} \text{ and if } u_{nu}(k-n+1) \text{ is } A_{rn} \text{ then }
\]

\[
y_l(k+1) = \sum_{j=1}^{n} a_{rj} y_l(k-j+1) + \sum_{j=1}^{n} b_{ij} u_l(k-j+1) + \sum_{l=1}^{nu} \sum_{j=1}^{n} b_{ij} u_l(k+j-1) + c_i \quad i=1,2,...,r
\]

(3)
III. Estimation method of recursive weighted least-squares (RWLS)

For nonlinear systems the online adaptation is necessary to obtain a model able to continue the system in its evolution. The system described by relation (4) can also be rewritten as:

\[ y_i(k) = \theta_i^T \varphi_i(k - 1) \]  

(4)

with \( \theta \) being a system parameter vector and \( \varphi \) a regression vector. It should be noted that the system (5) is in general nonlinear but it is linear with respect to its unknown parameter vectors. Based on parameterizations \([4]\), the identification algorithm giving estimates \( \hat{\theta}(k) \) of \( \theta(k) \) can be obtained using the RWLS.

We define:

\[ \varphi_i(k - 1) = [\mu_i y_i(k - 1) \ldots \mu_i y_i(k - n)]^T \]

(5)

\[ \theta_i = [a_{i1} \ldots a_{in} b_{i1} \ldots b_{in} c_i] \]

(6)

\[ \varphi_i(k - 1) = \left[ \varphi_{i1} (k - 1) \varphi_{i2} (k - 1) \ldots \varphi_{ir} (k - 1) \right]^T \]

(7)

\[ \theta_i(k) = \theta_i(k - 1) + L_i(k)[y_i(k) - \varphi_i(k)\theta_i(k - 1)] \]

(8)

\[ L_i(k) = \frac{P(k - 1) + \varphi_i(k)P(k - 1)\varphi_i^T}{1/\mu_i + \varphi_i(k)P(k - 1)\varphi_i^T} \]

(9)

\[ P(k) = P_i(k - 1 - L_i(k)\varphi_i(k)P_i(k - 1) \]

(10)

for \( k = 1, \ldots, N \), \( P(k - 1) \) is a covariance matrix and \( L(k) \) referred to the estimator gain vector. A common choice of initial value is to take \( \theta_i(0) = 0 \) and \( P_i(0) = \alpha I \) where \( \alpha \) is a large number.

IV. PID control systems of multivariable processes

Consider a multivariable PID control structure as shown in Fig. 1, where:

Desired output vector: \( Y_d = [y_{d1}, y_{d2}, \ldots, y_{dn}]^T \).

Actual output vector: \( Y = [y_1, y_2, \ldots, y_n]^T \).

Error vector: \( E = Y_d - Y = [y_{d1} - y_1, y_{d2} - y_2, \ldots, y_{dn} - y_n] = [e_1, e_2, \ldots, e_n]^T \).

Control input vector: \( U = [u_1, u_2, \ldots, u_n]^T \).

\[ n \times n \] multivariable processes:

\[
H(z) = \begin{bmatrix}
h_{11}(z) & \ldots & h_{1n}(z) \\
\vdots & \ddots & \vdots \\
h_{n1}(z) & \ldots & h_{nn}(z)
\end{bmatrix}
\]

(11)

\[ n \times n \] multivariable PID controller:

\[ C(z) = \begin{bmatrix}
c_{11}(z) & \ldots & c_{1n}(z) \\
\vdots & \ddots & \vdots \\
c_{n1}(z) & \ldots & c_{nn}(z)
\end{bmatrix}
\]

(12)

The form of \( k_{ij}(z) \), for \( i, j \in \mathbb{N} \) and \( n = \{1, 2, \ldots, n\} \), is given by:

\[ C(z) = k_{pij}(1 + \frac{z}{T_{zi}(z-1)} + \frac{(z-1)}{T_{di}z}) \]

(13)

where \( kp \) is the proportional gain, \( Ti \) is the integral time constant, and \(Td\) is the derivative time constant. It can be also rewritten (9) as:

\[
C(z) = k_{pij} + \frac{k_{ij}z}{(z-1)} + \frac{k_{di}(z-1)}{z}
\]

(14)

where \( k_i = kp/Ti \) is the integral gain and \( kd = kp * Td \) is the derivative gain. For convenience, let \( K = [k_{pij}; k_{ij} ; k_{di}]^T \) represent the gains vector of \( i^{th} \) row and \( j^{th} \) column[12]. In the design of a PID controller, the performance criterion or objective function is first defined based on our desired specifications and constraints under input testing signal. Typical output specifications in the time domain are peak overshooting, rise time, settling time, and steady-state error, to name a few. Three kinds of performance criteria usually considered in the control design are integral of the Absolute Error (IAE), integral of Square Error (ISE) and integral of Time weighted Square Error (ITSE) which are given as:

\[
IAE = |e_1(k) + e_2(k) + \ldots + e_n(k)|
\]

(15)

\[
ISE = \sum e_1(k)^2 + \sum e_2(k)^2 + \ldots + \sum e_n(k)^2
\]

(16)

\[
ITSE = \sum k*e_1(k)^2 + \sum k*e_2(k)^2 + \ldots + \sum k*e_n(k)^2
\]

(17)

Therefore, for the PSO-based PID tuning, these performance indexes (Eqs. (17)-(19)) will be used as the objective function. In other word, the objective in the PSO-based optimization is to seek a set of PID parameters such that the feedback control system has minimum performance index.

A. Tuning of PID using Z-N method

The first method of Z-N tuning is based on the open-loop step response of the system. The open-loop systems Shaped response is characterized by the parameters, namely the process time constant \( T \) and \( L \). These parameters are used to determine the controllers tuning parameters (see TABLE.1). The second method of Z-N tuning is closed-loop tuning method that requires the determination of the ultimate gain and ultimate period. The method can be interpreted as a technique of positioning one point on the Nyquist curve [13].
achieved by adjusting the controller gain (Ku) till the system undergoes sustained oscillations (at the ultimate gain or critical gain), whilst maintaining the integral time constant ( Ti ) at infinity and the derivative time constant (Td) at zero (see TABLE II).

| Controller | kp | Td=kd/kp | Ti=kp/ki |
|------------|----|----------|----------|
| P          | 0.5ku | - | 0 |
| PI         | 0.45ki | 1.5kp/Pu | 0 |
| PID        | 0.6ku | 2kp/Pu | kp/Pu/8 |

B. Implementation of PSO-Based PID Tuning

1) Particle swarm optimization (PSO): Particle swarm optimization was introduced by Kennedy and Eberhart by simulating social behavior of birds flocks in (199[10]. The PSO algorithm has been successfully applied to solve various optimization problems [14]. The PSO works by having a group of m particles. Each particle can be considered as a candidate solution to an optimization problem and it can be represented by a point or a position vector Xi = [X1, ..., Xid] in a d dimensional search space which keeps on moving toward new points in the search space with the addition of a velocity vector Vij = [V1, ..., Vi], to further facilitate the search procedure. The initial positions and velocities of particles are random from a normal population in the interval [0, 1]. All particles move in the search space to optimize an objective function f. Each member of the group gets a score after evaluation on objective function f. The score is regarded as a fitness value. The member with the highest score is called global best. Each particle memorizes its previous best positions. During the search process all particles move toward the areas of potential solutions by utilizing the cognitive and social learning components. The process is repeated until any prescribed stopping criterion is reached. After any iteration, all particles update their positions and velocities to achieve better fitness values according to the following:

\[ V_{pd}^{t+1} = \omega V_{pd}^t + c_1 r_1 (pbest_i - X_{pd}^t) \]

\[ + c_2 r_2 (gbest - X_{id}), \]

\[ X_{pd}^{t+1} = X_{pd}^t + V_{pd}^{t+1}, \]

where:

- t is the current iteration number,
- pbest is pbest of particle i,
- gbest is gbest of the group,
- r1, r2 are two random numbers in the interval [0, 1],
- c1, c2 are positive constants and ω is the inertia weight, is a parameter used to control the impact of the previous velocities on the current velocity. It influences the tradeoff between the global and local exploitation abilities of the particles. Weight is updated as:

\[ \omega = \omega_{max} - (\omega_{max} - \omega_{min}) \frac{iter}{iter_{max}} \]

where ωmin, ωmax, iter, and itermax are minimum, maximum values of ω, the current iteration number and pre-specified maximum number of iteration cycles, respectively.

2) Proposed PSO-PID Controller: This paper presents a PSO-PID controller for searching the optimal controllers parameters of MIMO nonlinear system, kp, ki, and kd, with the PSO algorithm. Each individual Kp contains 3*m members kp, ki, and kd, Its dimension is n^3*m. The searching procedures of the proposed PSO-PID controller were shown as below [12]. Optimal design for both conventional PID controllers can be fulfilled using PSO technique. Based on the PSO technique, the PID controller can be tuned to some parameters values that minimize those fitness functions given in (15), (16) and (17). The algorithmic steps for the PSO is as follows:

- **Step 1:** Select the number of particles, generations, tuning accelerating coefficients c1 and c2 and random numbers r1, and r2 to start the optimal solution searching.
- **Step 2:** Initialize the particle position and velocity.
- **Step 3:** Select the particles individual best value for each generation.
- **Step 4:** Select the particles global best value, particle near the target among all the particles, is obtained by comparing all the individual best values.
- **Step 5:** Update particle individual best pbest, global best gbest, in the velocity equation (18) and obtain the new velocity.
- **Step 6:** Update the new velocity value in Eq. (19) and obtain the position of the particle.
- **Step 7:** Find the optimal solution with a minimum (IAE, ISE, ITSE) from the updated new velocity and position values.

V. SIMULATION RESULTS

This section presents a simulation example to show an application of the proposed control algorithm and its satisfactory performance. The MIMO nonlinear system is characterized by the equation (16), (16)[17].

\[ y_1(k) = \frac{a_1 y_1(k-1) y_2(k-1)}{1 + a_2 y_1(k-1) + a_3 y_2(k-1)} + a_4 u_1(k-2) + a_5 u_1(k-1) + a_6 u_2(k-1) \]

\[ y_2(k) = \frac{b_1 y_2(k-1) \sin (y_2(k-2))}{1 + b_2 y_1(k-1) + b_3 y_2(k-1)} + b_4 u_2(k-2) + b_5 u_2(k-1) + b_6 u_1(k-1) \]

The system parameters are: a1 = 0.7, a2 = 1, a3 = 1, a4 = 0.3, a5 = 1, a6 = 0.2, b1 = 0.5, b2 = 1, b3 = 1, b4 = 0.5, b5 = 1 and db = 0.2 which is used as a test for control techniques introduced in this paper, to demonstrate the effectiveness of the proposed algorithms. Here y1 and y2 are the outputs, u1 and u2 are the inputs which is uniformly bounded in the region [−2, 2].

We choose [y1(k-1), y1(k-2), u1(k-1), u1(k-2), u2(k-1), u2(k-2)] and [y2(k-1), y2(k-2), u2(k-1), u2(k-2)] as inputs variables, and the number of fuzzy rules is four. The setup applied in this work was the following: the population
size was 20, the stopping criterion was 30 generations, 
\( \omega_{min} = 0.5 \), \( \omega_{max} = 0.9 \), and \( c_1 = c_2 = 2 \).

![Fig. 2. System response (y1)](image)

![Fig. 3. System response (y2)](image)

### TABLE III
**Optimized PID Parameters (y1)**

| Tuning Method | kp       | ki       | kd       |
|---------------|----------|----------|----------|
| Z-N PID       | 32.3431  | 3.2943   | 4.4829   |
| PSO-PID1 (ISE)| 26.7236  | 46.2192  | 30.136   |
| PSO-PID1 (IAE)| 8.7263   | 46.2192  | 33.136   |
| PSO-PID1 (ITSE)| 5.3487  | 15.7988  | 45.4369  |

### TABLE IV
**Optimized PID Parameters (y2)**

| Tuning Method | kp       | ki       | kd       |
|---------------|----------|----------|----------|
| Z-N PID       | 38.0743  | 4.4619   | 5.5734   |
| PSO-PID1 (ISE)| 35.7236  | 16.2192  | 9.136    |
| PSO-PID2 (IAE)| 35.5246  | 17.2514  | 9.7983   |
| PSO-PID3 (ITSE)| 45.3487  | 15.7988  | 41.4369  |

In the conventionally Z-N tuned PID controller, the systems response produces high overshoot, but a better performance obtained with the implementation of PSO-based PID controller tuning. In the PSO-based PID controllers (PSO-PID), different performance index gives different results. These are shown in TABLE. III and TABLE. IV. Comparative results for the PID controllers are given below in TABLE. V and TABLE. VI where the step response performance is evaluated based on the overshoot, settling time and Rise Time. The corresponding plot for the step responses are shown in Fig. 2 and Fig.3.

### VI. CONCLUSIONS

This paper presents a design method for determining the PID controller parameters using the PSO method for MIMO nonlinear systems. The proposed method integrates the PSO algorithm with performance criterions into a PSO-PID controller. The comparison between PSO-based PID (PSO-PID) performance and the ZN-PID is presented. The results show the advantage of the PID tuning using PSO-based optimization approach.

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