A novel method and system for calibrating the spring constant of atomic force microscope cantilever based on electromagnetic actuation

Yanling Tian, Chongkai Zhou, Fujun Wang, a) Jinyi Zhang, Zhiyong Guo and Dawei Zhang

Key Laboratory of Mechanism Theory and Equipment Design of Ministry of Education,
School of Mechanical Engineering, Tianjin University, Tianjin 300354, China

It is crucial to calibrate atomic force microscope (AFM) cantilevers for the development and further applications of AFM in precision engineering such as nanonewton force measurement. This paper presents a novel approach to calibrate the spring constant of AFM cantilever based on electromagnetic actuation and null position measurement. According to the method, a calibration system was designed. In order to optimize the static and dynamic characteristics of the calibration system, the analytical models for the electromagnetic force and the suspension mechanism stiffness have been developed. Finite Element Analysis (FEA) has been utilized to further investigate the precision of analytical modeling. The null position measurement method was utilized to monitor the deformation of the flexible beam, and then the deformation was compensated by the electromagnetic force. Experiments were carried out based on the developed prototype, and the results show that the electromagnetic force conversion rate is 40.08 μN/mA. Finally, a typical AFM cantilever was calibrated and the spring constant is (30.83 ± 0.24) N/m. The uncertainty of the proposed null position measurement method is better than 0.78%, which verifies the effectiveness and feasibility of the calibration method and system.

I. INTRODUCTION

With the rapid development of micro/nano manufacturing, more and more attention has been paid to the micro/nano newton force measurement.1-3 Atomic force microscope (AFM) has been becoming more and more important for the measurement of the modulus of polymers,4 the strength of chemical bonds,5 and the intermolecular interaction force between single molecule.6 The interaction force between the tip and the sample surface can be measured through the deflection of probe cantilever and the Hook’s law.7 Thus, the calibration of the spring constant of cantilever has become one of the major concerns, especially in the quantitative measurement of micro-scale force.8 There are variety kinds of probes utilized with different shapes and functions. Unfortunately, the spring

a) Author to whom correspondence should be addressed. Electronic mail: wangfujun@tju.edu.cn
The constants of AFM cantilevers are generally provided with a tolerance varying in a wide range by the manufacturers. The main reasons for which are given as follows: firstly, the thickness of cantilever usually varies from 0.5 μm to 5 μm. Considering the present manufacturing conditions and cost for AFM cantilevers, the thickness uncertainty of cantilevers is in the range from 10% to 25% of the desired value. Secondly, the defects and deficiencies of geometry and material in different cantilevers are commonly not taken into account by the manufacturers. Furthermore, the material properties such as Young’s modulus can be seriously influenced by surface effects, leading to additional errors for the spring constant.\(^9\) As a result, it is necessary to study the calibrating method of AFM cantilever.

Recently, several methods have been proposed to calibrate the spring constant.\(^10\) According to the calibration principle, these can be grouped into three general categories: dimensional analysis, dynamic approach and static method.\(^11,13\) In dimensional analysis, the most famous method is Sader method.\(^14,15\) The spring constant is calculated according to the cantilever material properties and geometrical size.\(^16\) The measured thickness and inaccurate Young’s modulus are the main error sources for these kinds of methods, and the errors range from 10% to 25%.\(^17\) In dynamic approach, the spring constant is commonly obtained through the resonant response of cantilever. The most common dynamic methods include the Cleveland method,\(^18\) and thermal noise method.\(^19\) For the Cleveland method, it is difficult to add a small mass on the same position of cantilever routinely, which is easy to damage the cantilever.\(^20\) Some commercial AFM systems are equipped with thermal noise method to calibrate cantilevers, because it is convenient to implement and get the calibration results. The dynamic approaches are simple and do less damage to the cantilever, but they are not suitable for all kinds of cantilevers. By dynamic method low spring constant cantilevers can be calibrated easily and accurately, while it is difficult to calibrate the higher ones. In addition, the spring constant of the cantilevers are obtained through measurement of the cantilever vibrations, during which the environmental vibrations inevitably caused some errors.\(^21\) In static method, a micronewton force is loaded on the top of the cantilever, and the subsequent deflection of the cantilever is measured. Based on the Hooke’s law, the spring constant can be calculated immediately. The micronewton force can be obtained from a pre-calibrated cantilever, which is then used to calibrate the unknown probe cantilever, and the method is called the reference cantilever method.\(^22\) Nowadays, the International System of Units (SI) traceability in nanoforce metrology has been paid more and more attention by researchers, who usually use balance mechanical structure approach.\(^23\) A novel nanopositioning system to calibrate the cantilever is designed based on an electromagnetic force compensated
balance. In consequence, the nanonewton force calibrator (NFC) gets an accurate result with the standard error less than 1%. A further improved development technique based on electromagnetic force compensated and a laser interferometer for the displacement measurement is proposed to reduce the largest uncertainty. However, the measurement devices are expensive or not easy to install. The deflection of the cantilever is not measured immediately, which brings some errors to the calibration results and further reduces the accuracy of the calibration system.

In order to overcome the above-mentioned shortcomings, a novel cantilever calibration method and system have been designed, which includes an electromagnetic actuation, a suspension mechanism, an x/y/z 3-Degree of freedom (3-DOF) high precision positioning stage and two capacitive sensors. In order to avoid the influence of the bending flexible beam to the calibrated result, the null position measurement method is proposed. Accordingly, the concrete operating procedure for the spring constant calibration is introduced. Finally, a commercial AFM cantilever spring constant is calibrated, and the uncertainties of the proposed system are evaluated.

II. THE NOVEL CALIBRATION SYSTEM AND THE NULL MEASUREMENT METHOD

For calibration cantilevers using electromagnetic actuation, an SI-traceable force can be generated with the null measurement method. The electromagnetic force is introduced by a coil solenoid and permanent magnet. The material of the permanent magnet is Nd$_2$Fe$_{14}$B. The permanent magnet has a constant strength of $1.20 \pm 0.01$ T along the axial direction according to the datasheet from the manufacturer.

The suspension mechanism installed the permanent magnet is driven to the undeformed position by the electromagnetic force, which realizes the null measurement method. With the electromagnetic force and the measured bending displacement of the AFM cantilever, the spring constant can be obtained.

The self-developed spring constant calibration system is shown in Fig. 1. To facilitate the adjustment of AFM probe position, three manual coarse positioning platforms are arranged orthogonally in x/y/z axis. A 3-dimensional (3D) printed probe holder is mounted on the vertical coarse positioning stage to fix the AFM cantilever. The material of the 3D printed probe holder is Polyamide 66, which has good toughness and high strength. An x/y/z 3-DOF high precision positioning stage is assembled on the two horizontal coarse positioning platforms to realize micro/nano movement of the Null Driving System (NDS).
The NDS is an important part of the proposed calibration system, which mainly includes the electromagnetic system and the capacitive sensors. As shown in Fig. 2, the electromagnetic system is composed of the upper plate, the suspension mechanism, permanent magnet, connecting ring, coil solenoid and fixing platform. The coil solenoid and permanent magnet organize the electromagnetic device to generate the electromagnetic force. The detailed information of the suspension mechanism includes the flexible beam, the rigid plate and the support ring. The permanent magnet is adhered on the bottom surface of the central circular rigid plate.

In the calibration process, the AFM probe tip contacts with the effective area of the rigid platform, which is the circular area with a diameter of 2 mm and the same with the permanent magnet diameter. Correspondingly, the capacitive sensor measures the deformation in the sensing area, and the deformation of the suspension mechanism occurs on the flexible beam. All the components of the electromagnetic system are screwed on the fixing platform, and mounted on the 3-DOF high precision positioning stage together with the capacitive sensor.
Based on the developed NDS, the null measurement method is proposed to calibrate the spring constant of AFM cantilever. The NDS and a capacitive sensor are installed together upon the precision positioning stage. The whole measurement process can be divided into three steps. At the beginning, the AFM cantilever is driven down to the center of the rigid plate until approximately touching it. Then the high precision positioning stage actuates the NDS to approach the AFM probe tip gradually. The capacitive sensor is so sensitive to the changing position of the platform that the distance measurement $z_0$ is marked, with the AFM probe tip touching the center, which is shown in Fig. 3(a). After that, the high precision positioning stage further raises NDS and the capacitive sensor with micrometers $z_1$. Because the interaction force between the rigid plate and AFM probe tip, the free part of the flexible beam is raised less than the fixed end. Simultaneous, AFM probe tip is bent upwards. Figure 3(b) illustrates that the capacitive sensor measures the displacement value $z_2$ of the cantilever.

![Schematic drawing of AFM cantilever calibration progress](image)

Fig. 3. The schematic drawing of AFM cantilever calibration progress. (a) the fixed suspension is driven upwards and the AFM probe tip is touched to the rigid platform; (b) the fixed suspension is driven upwards with a displacement $z_1$; (c) the electromagnetic force actuates the rigid platform to the original position $z_0$.

For the most important part, the electromagnetic actuator gets a linear increasing input current, and generates a stable electromagnetic force between the coil solenoid and the permanent magnet. And then, the permanent magnet and platform are driven up, with upward force transmitting to the rigid plate. The input current will increase until the measurement distance decreases from $z_2$ to $z_1$, and the final current $i$ will be recorded by the dSPACE (digital Signal Processing And Control Engineering) DS1103 R&D control board in time. As indicated in Fig. 3(c), the displacement of the precision positioning stage $z_1$ turns into the bending displacement of AFM probe ultimately. If the electromagnetic force $F_{em}$ is calibrated to the linear input current $i$, the corresponding force acting on AFM probe can be obtained.
As a result, the spring constant $k_c$ of the cantilever could be calculated by

$$ k_c = \frac{F_{em}}{z_1} \quad \text{(1)} $$

So another key work is to determine the conversion rate between force and current.

III. Characteristic Analysis

A. The electromagnetic device

The related geometrical parameters of the electromagnetic device are shown in Fig. 4, where the height, inner and outer radius of the coil solenoid are $l_c$, $a_1$ and $a_2$, respectively. The diameter and height of the permanent magnet are $d_m$ and $l_m$, respectively. The electromagnetic force $F_{em}$ along $z$-axis can be calculated by (2).

\[
F_{em} = B_{rem} V \frac{dH(z/a_1)}{d(z/a_1)} \quad \text{(2)}
\]

where $H \left( \frac{z}{a_1} \right) = \chi n F(\alpha, \beta) \left[ \frac{F(\alpha, \beta + \frac{z}{a_1}) + F(\alpha, \beta - \frac{z}{a_1})}{2F(\alpha, \beta)} \right] \quad \text{(3)}$

where $F(\alpha, \beta)$ is the Fabry factor

\[
F(\alpha, \beta) = \beta \ln \left( \frac{\alpha + (\alpha^2 + \beta^2)^{1/2}}{1 + (\beta^2)^{1/2}} \right) \quad \text{(4)}
\]

Besides the geometrical size, there are also some other electromagnetic parameters: the residual magnetism $B_{rem}$ and volume $V$ of the permanent magnet, the number turns of the coil solenoid $n$, the packing factor of the coil solenoid $\chi$ and the input current $i$. Two other factors $\alpha = a_2/a_1$ and $\beta = l_c/2a_1$ are introduced to better describe the coil.
geometry, where, $H$ is the strength of magnet field, and $z$ is the distance from the permanent magnet center to the coil solenoid center.\(^{28}\)

The electromagnetic forces at various positions under different current conditions are analyzed with (2) using the physical parameters defined in TABLE I. The calculating results are shown in Fig. 5 and it can be seen when the permanent magnet is positioned near the end of the coil solenoid, the maximum magnetic field gradient $dH/dz$ occurs at the end of the coil solenoid and the electromagnetic force reaches the maximum.

### TABLE I Parameters of the electromagnetic actuation

| Parameters                  | Variables | Value         |
|-----------------------------|-----------|---------------|
| Remanence of the magnet     | $B_{rm}$  | 1.20 $\pm$ 0.01 T |
| Length of the coil          | $l_c$     | 10.00 $\pm$ 0.02 mm |
| Inside radius of the coil   | $a_1$     | 2.50 $\pm$ 0.02 mm |
| Outside radius of the coil  | $a_2$     | 5.00 $\pm$ 0.02 mm |
| Turns of the coil           | $n$       | 210           |
| Length of the magnet        | $l_m$     | 2.00 $\pm$ 0.02 mm |
| Diameter of the magnet      | $d_m$     | 2.00 $\pm$ 0.02 mm |

Fig. 5. The calculating force on the magnet at different positions  
Fig. 6. Electromagnetic force varying with current by calculations

When the permanent magnet is positioned at the end of the coil solenoid, the impact of the changing position along $z$-axis to the electromagnetic force is minimum.\(^{29}\) According to the calculation results, Fig. 6 shows the corresponding electromagnetic force varies with the input current. The fitting curve shows the relationship between electromagnetic force and the input current. Hence, the analysis results can be used to optimize the system.
B. The flexible suspension mechanism

The flexible suspension mechanism plays a significant role in the electromagnetic actuator system in terms of supporting the permanent magnet and connecting the main area of the rigid plate. Because the suspension mechanism needs to generate bending deformation in calibrating process, the copper/beryllium (Cu/Be) alloy with high elasticity is chosen as the material. In order to optimize the property of the suspension mechanism, the stiffness matrix model and Finite Element Analysis (FEA) are implemented.

![Model of the suspension mechanism](image)

(a) (b)

Fig. 7. The model of the suspension mechanism: (a) the flexible beam; (b) the leaf-type flexible hinge

As is shown in Fig. 7(a), the working part of the suspension mechanism is a flexible beam, which generates approximately all the deformation. The flexible beam is fixed by the support ring. Considering the loading force is applied on the effective part of rigid plate, it is obvious that the beam is bent along z-axis direction. But the torsion along y-axis is not restrained. In order to constraint the undesirable movement, the flexible beam is fabricated much thinner than other parts, which is efficient to improve the property. So the flexible beam can be regarded as a leaf-spring flexible hinge and the stiffness matrix analysis is a quite convenient and accurate way to analyze this kind of flexible hinge.\(^{30,31}\) The global coordinate of the suspension mechanism is shown in Fig. 7(a), and the model of traditional leaf-type flexible hinges is displayed in Fig. 7(b), of which the length, width and thickness are \(l, w\) and \(t\), respectively. It is located at end of the flexible hinge, and the loading force is applied on the midpoint of the other end.
It is assumed that the relationship between force and deformation is linear. When forces and moments are applied on a certain point around the axis, the infinitesimal translation and rotational displacements of the center point are formulated by (5) and (6).

\[ \delta = CF \quad \text{\^MERGEFORMAT (5)} \]
\[ F = KX \quad \text{\^MERGEFORMAT (6)} \]

where \( F \) and \( X \) are the set of translational and rotational displacements, respectively. Further, the force and moment, translational and rotational displacement will not be distinguished. The matrix \( C \) and \( K \) are inverse matrices of each other, and the former is called the compliance matrix and the other is called the stiffness matrix.

The compliance matrix of the leaf-spring hinge in the local coordinate is given as following:

\[
\begin{bmatrix}
\delta_x \\
\delta_y \\
\delta_z \\
\theta_x \\
\theta_y \\
\theta_z \\
\end{bmatrix} =
\begin{bmatrix}
\frac{4l^3}{Etw} + \frac{l}{Gtw} & 0 & 0 & 0 & 0 & -\frac{6l^2}{Etw} \\
0 & \frac{l}{Etw} & 0 & 0 & 0 & 0 \\
0 & 0 & 4\frac{l^3}{Et^w} + \frac{l}{Gtw} & \frac{6l^2}{Et^w} & 0 & 0 \\
0 & 0 & \frac{6l^2}{Et^w} & \frac{12l}{Et^w} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{l}{Gk_0l^3w} & 0 \\
-\frac{6l^2}{Etw} & 0 & 0 & 0 & 0 & \frac{12l}{Et^w} \\
\end{bmatrix}
\begin{bmatrix}
F_x \\
F_y \\
F_z \\
M_x \\
M_y \\
M_z \\
\end{bmatrix}
\quad \text{\^MERGEFORMAT (7)}
\]

where \( E \) and \( G \) are the Young’s modulus and the shear modulus of the material, respectively. \( k_0 \) is a geometric parameter, \( kw/bt \) and \( xi = -l_x, y_1 =0, l_e=4 \) mm. \( F_n \) and \( \delta_n \) are the force and translational displacement in \( n \)-axis, respectively, and \( M_n \) and \( \theta_n \) are the moment and rotational displacement around the \( n \)-axis, respectively.

The stiffness matrix of the flexible beam is the inverse of compliance matrix, so the stiffness matrix of the flexible beam device can be obtained as

\[ K = C^{-1} \quad \text{\^MERGEFORMAT (8)} \]

For the suspension mechanism, the reference point \( o \) is the center of the flexible beam, and coordinate of point \( o_1 \) is \((-4,0,0)\). The loaded force along \( z \)-axis acts at the central point of the suspension mechanism, and the vector force to the central can be defined. According to (7) and (8), the calculation stiffness of the suspension mechanism is 31.14 N/m.
In the FEA process of the suspension mechanism, the geometrical model is established in the Unigraphics NX 8.0 software, and then imported into the ANSYS Workbench software to generate mesh and further simulate the static and dynamic property. The material is cooper alloy with Young’s module 128 GPa, Poisson’s ratio 0.34 and density 8300 kg/m$^3$. The length, width and thickness of the flexible beam are 4.80 ± 0.02 mm, 1.00 ± 0.02 mm and 0.050 ± 0.001 mm, respectively. The thickness of the other parts of the suspension mechanism is 0.150 ± 0.005 mm.

The flexible beam connects two parts: the rigid plate and the outer located circular part. The inner and outer diameters of the support ring are 16 mm and 26 mm, respectively. The diameter of central circular rigid plate is 6 mm.

![Image](image1.png)

**Fig. 8.** The simulation analysis of the suspension mechanism: (a) The mechanical stress distribution with 10 μN; (b) The deformation of the central platform; (c) 1st modal: linear motion along z-axis at 109.92 Hz; (d) 2nd modal: rotation about x-axis at 452.17 Hz; (e) 3rd modal: rotation about y-axis at 1369.8 Hz.

According to the theoretical calculation, the z-axis stiffness of the suspension mechanism is 31.14 N/m, which matches well with the FEA simulation results of 27.54 N/m. As shown in Fig. 8(a), when an initial deformation of 0.005 mm is imposed in the center rigid plate, the stress mainly distributes in the flexible beam. The maximal stress is 0.104 MPa, which is far lower than the yield strength of the material and occurs at the root of the flexible beam.

The maximum elastic deformation of the central rigid plate is approximately 0.34 nm, as shown in Fig. 8(b), which illustrates the central platform can be regarded as a rigid component. The first three model shapes are shown in Fig. 8(c), (d), and (e), which reveals the first modal shape is the translation along z-axis direction with the natural frequency of 109.92 Hz.
IV. EXPERIMENTS

The experiment setup is shown in Fig. 9. Three manual coarse mobile platforms (WN115TM25M, winner optical instruments, China) were arranged in the x-axis y-axis and z-axis to realize a large stroke of 25 mm × 25 mm × 25 mm with the minimum adjustment amount of 10 μm. The x/y/z 3-DOF high precision positioning stage (P-517.3CD, PI, Germany) was utilized to realize precise movement for the NDS. The AFM probe was attached on a 3D printed holder, which was designed with optimized shape and high stiffness. A home-made direct current circuit was used to provide the current for the coil, which received the command signals from the I/O interface of a dSPACE DS1103 R&D control board. The deformation of the suspension mechanism was measured in real time by a capacitive sensor (Lion Precision, CPL290, USA) and another capacitive sensor was measured the precision positioning stage to make sure the accurate driving displacement. In order to reduce the errors induced by the environmental vibration, the whole system was located on a Newport RS-4000 optical table.

A. The measurement of the coil current

A home-made direct current circuit was utilized to input current to the coil of the electromagnetic actuation. The schematic diagram of the circuit is shown in Fig. 10, where, \( V_i \) and \( V_m \) were the control signal and the measured voltage of the precision resistor, respectively. The coil was connected to a precision resistor in series, so the current of the coil was the same with the resistor, which could be calculated by Ohm’s law. Generally, a larger current would heat the circuit device, especially the precision resistor, leading to the decrease of the current resolution and further the electromagnetic force resolution, so the input current should be less than 50 mA.
B. The calibration of the electromagnetic force

The electromagnetic actuation was developed with the same electromagnetic parameters as the simulation. The schematic diagram of the electromagnetic force is shown in Fig. 11 (a). A manual platform (WN05VM13, winner optical instruments, China) was utilized to realize the contact of the loadcell and probe suspension. A loadcell (F329UB00A0, NOVATECH, UK) was applied to calibrate the electromagnetic force. Its stiffness is 1300 N/m based on the information from the manufacturer. A capacitive sensor (Lion Precision, CPL290, USA) was used to measure the deformation of the suspension mechanism, shown in Fig. 11 (b). For the same current, the relationship between the loadcell measured force $F_{\text{loadcell}}$ and the actual electromagnetic force $F_{\text{em}}$ was defined by

$$F_{\text{em}} = F_{\text{loadcell}} + K_{\text{flex}} F_{\text{loadcell}}^2 + K_{\text{loadcell}}^2$$  \(\ast\) MERGEFORMAT (9)

where $K_{\text{flex}}$ and $K_{\text{loadcell}}$ are the stiffness of the flexible beam and the loadcell, respectively.

After the loadcell contacting with the suspension mechanism, different currents were input to the coil, simultaneously. The corresponding electromagnetic forces were calculated by (9) and the related data were listed in TABLE II. The loadcell was removed from the system and the deformations of the suspension mechanism were
measured with different electromagnetic forces, as shown in TABLE II. Figure 12 shows the relationship between electromagnetic force and input current and Fig.13 shows linear relationship between electromagnetic force and the displacement of rigid plate. The experimental stiffness of the flexible beam is 27.17 N/m, which agrees well with the theoretical and the simulation results.

### TABLE II. Experimental results

| Parameters                        | Values     |
|-----------------------------------|------------|
| Current $i$ (mA)                  | 5 10 15 20 25 30 35 40 45 50 |
| Deformation of the suspension mechanism $z$ (μm) | 7.35 14.77 22.61 29.87 37.45 45.08 52.28 59.01 66.24 74.13 |
| Measured force of loadcell $F$ (μN) | 187 363 559 756 973 1182 1378 1569 1776 1973 |

Fig. 12. The linear relationship between electromagnetic force and input current

Fig. 13. The linear relationship of the electromagnetic force and the displacement of the rigid plate

**C. The calibration of AFM cantilever**

The type of cantilever studied in this paper is the tapping model probe (Tap300Al, Budget Sensors). The nominal dimensions provided by the manufacturer for the length ($l$), width ($w$) and thickness ($t$) are 125 μm, 30 μm and 4 μm, respectively. The nominal spring constant is 40 N/m, but it can actually vary from 20 N/m to 75 N/m. The AFM probe was installed to a 3D printed holder. The holder approached to the rigid plate by the $x$/$y$/$z$ manual coarse positioning platform. Meanwhile, the CCD camera monitored whether the cantilever tip touched the effective area of rigid plate. The capacitive sensor (CS) measured the displacements of the suspension mechanism and the NDS, respectively. When the probe tip contacted the effective area of rigid plate, the CS got the original position $z_0$ of the suspension mechanism. The precision position stage actuated the NDS along $z$-axis direction from 0 μm to 1.05 μm.
with 0.21 μm step, which was defined as \( z_1 \) and shown in Fig. 14. With the NDS going up, the flexible beam bent from 0 μm \((z_0)\) to -0.51 μm \((z_2)\), shown in Fig. 15. Meanwhile, the tip of AFM cantilever was upward, because of the interactive force between cantilever tip and rigid plate. Hence, the displacement of cantilever tip was 0.54 μm. After that, a linear increasing current was input to the coil and the electromagnetic force actuated the rigid plate upwards, and then the flexible beam was back to the original position \( z_0 \). The increasing current recorded the final value 0.81 mA, shown in Fig. 16. According to the experimental results, the total bending displacement of AFM cantilever was 1.05 μm.

In order to verify the accuracy of calibration system, we have made 100 times experiments, and divided the data into 10 groups by the calibration data. The data of each group are the average value of 10 times, which is listed in the TABLE III. The average input current is 0.81 mA, and it is known that the \( k_{F,i} \) is 40.08 μN/mA, so the
The electromagnetic force is 32.46 μN. The average displacement of AFM cantilever is 1.05 μm. According to (1), the spring constant of the AFM cantilever is 30.83 N/m.

| Parameters                  | Values     | Average |
|-----------------------------|------------|---------|
| Number of groups            | 1 2 3 4 5 6 7 8 9 10 |
| Current i (mA)              | 0.81 0.80 0.80 0.81 0.80 0.83 0.82 0.82 0.81 0.80 0.81 |
| Electromagnetic force $F_{em}$ (μN) | 32.46 32.06 32.06 32.46 32.06 33.27 32.87 32.87 32.46 32.06 |
| Deformation of the cantilever $z_1$ (μm) | 1.05 1.04 1.05 1.08 1.04 1.08 1.06 1.04 1.05 1.04 1.05 |
| Spring constant (N/m)       | 30.92 30.83 30.54 30.06 30.83 30.80 31.01 31.60 30.92 30.83 30.83 |

The same probe was calibrated by a commercial AFM, JPK NanoWizard 3 NanoScience AFM system using the thermal noise method. The corresponding calibration stiffness is 29.89 N/m. From the Euler-Bernoulli beam theory, the normal spring constant of the rectangular cantilever with a uniform cross section can be obtained by $k_{dimensional} = Ec/l^3/(4l^3)$, where $E$ is the Young’s modulus of the cantilever material and $l$, $b$, $t$ are the length, width and thickness of the cantilever, respectively. The calibration results with the proposed method, dimensional method and thermal noise method are shown in TABLE IV. The nominal spring constant is 40 N/m, quoted by the manufacturer.

| Cantilevers | Spring constants obtained from various methods (N/m) |
|-------------|-----------------------------------------------------|
| Tap300Al    | Proposed method 30.83 | Dimensional method 27.03 | Thermal noise method 29.89 |

D. The accuracy analysis of the proposed method

In the paper, a novel method is presented to calibrate AFM cantilevers based on electromagnetic force and precision displacement measurement. We have investigated the calibration accuracy by theoretical and experiment analysis.

For the theoretical analysis, the electromagnetic force is determined by (2) to (4). Based on the law of error propagation, the standard deviation of the electromagnetic force can be defined as

$$\sigma_{F_{em}} = \sqrt{\left( \frac{\partial F_{em}}{\partial B} \right)^2 \sigma_B^2 + \left( \frac{\partial F_{em}}{\partial a_1} \right)^2 \sigma_{a_1}^2 + \left( \frac{\partial F_{em}}{\partial a_2} \right)^2 \sigma_{a_2}^2 + \left( \frac{\partial F_{em}}{\partial l} \right)^2 \sigma_l^2 + \left( \frac{\partial F_{em}}{\partial t} \right)^2 \sigma_t^2}$$

\* MERGEFORMAT \(^{(10)}\)
where $\sigma_B$, $\sigma_{a_1}$, $\sigma_{a_2}$, $\sigma_{l}$, and $\sigma_i$ are the standard deviations of $B$, $a_1$, $a_2$, $l$, and $i$. The error of the input current is 0.01 mA and TABLE I shows the errors of other parameters. The standard deviation of the electromagnetic force $\sigma_{F_{\text{me}}}$ is calculated to 0.85 $\mu$N (2.61%). The error of measured displacement is 30 nm (0.3%) from the commercial data sheet.

For the experimental analysis, according to the experimental results, we can get the error of the electromagnetic force $\delta_{F_{\text{me}}}$ by

$$
\delta_{F_{\text{me}}} = \sqrt{\frac{1}{10} \sum_{j=1}^{10} \delta_j^2 + e_i^2}
$$

\* MERGEFORMAT (11)

where $\delta_j$ is limit error of the measured electromagnetic force, which can be calculated by the data in the TABLE III; $e_i$ is the uncertain systematic error, which is influenced by the error of input current $i$. According to the analysis above, the error of current $i$ is 0.01 mA and the ratio $k_{F-i}$ is 40.08 $\mu$N/mA. Hence, $e_i$ can be calculated to 0.40 $\mu$N. The error of electromagnetic force $\delta_{F_{\text{me}}}$ is obtained to 0.84 $\mu$N (2.59%). The displacement of the cantilever is measured by a commercial capacitive sensor. The error of the capacitive sensor is 30 nm (0.30%) according to its commercial data sheet.

Figure 17 shows the variation of the mean values of the spring constant. The error bars represent standard of deviation of 10 groups measurement results.

![Graph showing the variation of the mean values of the spring constant](image)

Fig. 17. The mean values of the spring constant

For the accuracy analysis of the suspension mechanism. According to (7) and (8), the standard deviation $\sigma_s$ of the suspension mechanism can be obtained by
\[
\sigma_s = \sqrt{\left(\frac{\partial K}{\partial l}\right)^2 \sigma_l^2 + \left(\frac{\partial K}{\partial w}\right)^2 \sigma_w^2 + \left(\frac{\partial K}{\partial t}\right)^2 \sigma_t^2}
\]

\(^{\text{MERGEFORMAT}}\) (12)

where \(\sigma_l, \sigma_w, \sigma_t\) are the standard deviation of length \(l\), width \(w\) and thickness \(t\) of the suspension mechanism. The calculation result of the suspension mechanism \(\sigma_s\) is 0.41 N/m (1.5\%). The most influential parameter is the thickness of the suspension mechanism.

**E. The uncertainty estimation for spring constant measurement**

The uncertainty of the calibration result of the mentioned cantilever is estimated. Based on the estimation from type A evaluation, the relative uncertainty of the repeatability, \(u(\bar{k})/\bar{k}\), can be defined by

\[
\frac{u(\bar{k})}{\bar{k}} = \frac{s}{\bar{k}\sqrt{n}}
\]

\(^{\text{MERGEFORMAT}}\) (13)

where \(s\), \(\bar{k}\) and \(n\) are the standard deviation, the mean value of the spring constant and the number of the measurements using to calculate the mean value for the cantilever. According to TABLE III, the relative standard deviation \((s/\bar{k})\) is 1.3\% and \(n\) is 10. Hence, the relative uncertainty is calculated to be 1.3\%/\(\sqrt{10}\) = 0.41\%.

The uncertainty is focusing on two main factors, force and displacement, owing to the uncertainty from the designed instrument and the environment. Firstly, we estimated the uncertainty \(u(F_{\text{em}})\) of the force \(F_{\text{em}}\) by

\[
u(F_{\text{em}}) = \left| \frac{\partial k}{\partial F_{\text{em}}} \right| \sigma_{F_{\text{em}}}
\]

\(^{\text{MERGEFORMAT}}\) (14)

where the mean standard deviation of measured force \(\sigma_{F_{\text{em}}} = \frac{\sigma_{F_{\text{em}}}}{\sqrt{10}}\), and standard deviation of measured force

\[
\sigma_{F_{\text{em}}} = \sqrt{\frac{1}{10-1} \sum_{i=1}^{10} (F_{\text{em}} - \bar{F}_{\text{em}})^2} \quad \bar{F}_{\text{em}} \text{ is the average value of the electromagnetic force. According to TABLE III, the relative uncertainty } u(F_{\text{em}}) / F_{\text{em}} \text{ of the force is calculated to be 0.40\%.}
\]

Secondly, the displacement \(z_1\) was measured by the capacitive sensor. The uncertainty for the displacement measurement \(u(z_1)\) was mainly determined by two factors: the measurement system and the tip-sample contact displacement. For the uncertainty \(u_i(z_1) / z_1\) of the measurement system, it can be estimated from type B evaluation. A capacitive feedback sensor was integrated to the PI stage to measure the displacement. The PI stage was
controlled with a range of 10 μm. The maximum difference from the real value is 30 nm from the data sheet. For the conservative estimation, the uncertainty of the capacitive sensor is 30 nm over a range of 10 μm. The relative uncertainty \( u(z_i)/z_i \) could be conservatively estimated to be 0.30% (30 nm/10 μm). However, as to the tip-sample contact displacement, when probe tip contacts the surface of rigid plate, the probe tip is pushed into the surface and the depth \( \delta_{tip} \) should be analyzed. The Hertz analysis can be used to solve the problem and it can be calculated by\textsuperscript{35}

\[
\delta_{tip} = \left( \frac{9F_t^2}{16RE^2} \right)^{1/3}
\]

\* MERGEFORMAT (15)

where \( F_t \) is the resistance force from rigid plate to probe tip, \( E^* \) is equivalent Young’s modulus and \( 1/E^* = (1-v^2_{tip})/E_{tip} + (1-v^2_{platform})/E_{platform} \), \( v_{platform} \) and \( E_{tip}, E_{platform} \) are the Young’s modulus and Poisson’s ratios of bearing platform material and probe tip, respectively. \( R \) is the radius of probe tip. During the calibration progress, the average force \( F_t \) is equal to 32.46 μN, and the values of \( R, E_{platform}, v_{platform}, E_{tip} \) and \( v_{tip} \) are 10 nm, 128 GPa, 0.34, 190 GPa, 0.278, respectively. The weight of the probe can be ignored. From (15), the depth \( \delta_{tip} \) is calculated as 0.20 nm.

Comparing to the radius of probe tip (10 nm) and the displacement of \( z_i \) (1.05 μm), the relative uncertainty of tip-sample contact displacement \( u_{tip}(k_c)/k_c \) is negligible.

The uncertainty from the orientation of the cantilever with respect to the balance axis can be expressed by\textsuperscript{36}

\[
\frac{k_c}{k_a} = \frac{1}{\cos \theta (\cos \theta - \mu \sin \theta)}
\]

\* MERGEFORMAT (16)

where \( k_a \), the stiffness at \( \theta \), is determined by the ratio of normal force \( F_n \) to the normal displacement \( z \). \( \mu \) is the coefficient of the static friction of the rigid plate. \( k_a \) is the stiffness measured at 0°. The misalignment \( \theta \) in Fig. 3 is bounded to ±1° and assuming the coefficient of the friction \( \mu \) is 0.22. According to (16), the relative uncertainties of the stiffness, \( u_{a}(kc)/k_c \), is 0.42%.

The measured spring constant \( (k_a) \) is the combined stiffness of two spring connected in series: the cantilever \( (k_c) \) and the probe holder \( (k_h) \). Hence, the uncertainty for the calibrated spring constant due to the load frame can be given by the following:

\[
\frac{1}{k_c} = \frac{1}{k_a} - \frac{1}{k_h}
\]

\* MERGEFORMAT (17)

The stiffness of the probe holder \( (k_h) \) is calculated to be 20000 N/m with the limits of ±500 N/m from the designed parameters. The relative uncertainty due to the probe holder stiffness, \( u_{h}(kc)/k_c \), is derived by
\[
\frac{u_c(k_c)}{k_c} = \frac{1}{\sqrt{3}} \left| \frac{k_m - k_n}{k_m} \right| \delta k_h = \frac{1}{\sqrt{3}} \left| \frac{k_m - k_n}{k_m} \right| \delta k_h \quad \text{(18)}
\]

where \( \delta k_h \) is the deviation of the probe holder stiffness, \( k_m = 30.83 \) N/m, \( k_n = 20000 \) N/m and \( \delta k_h = 500 \) N/m. Thus, the uncertainty of the probe holder \( u_h(k_c)/k_c \) is calculated to be 0.003%.

The temperature was 21 ± 0.1 °C during the tests in the constant temperature and humidity laboratory. The tiny change of the temperature does not affect the calibration results of the cantilever. Hence, the uncertainty of the temperature changes \( u_T(k_c)/k_c \) can be negligible.

We summarized the combined uncertainties with the calibrated spring constant of cantilever in TABLE V. The relative combined uncertainty \( u_c(k_c) \) is obtained by the root-squared sum of all the relative uncertainties. Thus, the calculated \( u_c(k_c) \) is 0.78% and the spring constant with uncertainty is \( 30.83 \pm 0.24 \) N/m.

### TABLE V Uncertainty budget for the spring constant.

| Uncertainty component | Uncertainty source | Relative uncertainty value (%) |
|-----------------------|--------------------|--------------------------------|
| \( u_t(\bar{k})/\bar{k} \) | Repeatability | 0.41 |
| \( u(F_{en})/F_{en} \) | Electromagnetic force | 0.40 |
| \( u(z_i)/z_i \) | Displacement | 0.30 |
| \( u_{tip}(z)/k_c \) | Tip-sample contact | negligible |
| \( u_{tip}(z)/k_c \) | displacements | negligible |
| \( u_0(kc)/k_c \) | Orientation of the cantilever | 0.42 |
| \( u_0(kc)/k_c \) | Probe holder stiffness | 0.003 |
| \( u_T(kc)/k_c \) | Temperature | negligible |
| \( u_c(kc)/k_c \) | Relative combined uncertainty | 0.78 |
| Spring constant with uncertainty (N/m) | | 30.83 ± 0.24 |

### V. DISCUSSION

This paper presents a novel method and system to calibrate the spring constant of AFM cantilever based on the electromagnetic actuator and the designed suspension mechanism. The stiffness of investigated flexible beam is 27.17 N/m, which is determined by the structure and shape of the suspension mechanism. The developed system is suit to calibrate high spring constant cantilevers. For the lower spring constant cantilevers, the stiffness of the system should be decreased by designing new symmetrical parallel flexible beams, circular flexible suspension or other kinds of compliant mechanisms with a lower stiffness, which we will study systematically in future work.
VI. CONCLUSION

A novel method and calibration system used to measure the spring constant of AFM cantilevers have been developed. The calibration system mainly includes the electromagnetic actuation and the suspension mechanism. Analytical modeling has been carried out to facilitate the design and characteristic analysis. The system prototype has been established and experimental tests have been implemented. According to the experimental results, it is known that the electromagnetic force conversion rate is 40.08 μN/mA. The spring constant of a commercial AFM cantilever has been calibrated and the value is (30.83 ± 0.24) N/m. The uncertainty of the method using null position measurement techniques is better than 0.78%, indicating the feasibility of the developed method and system in calibrating the spring constant of AFM cantilever.

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REFERENCES

1. F. Wang, C. Liang, Y. Tian, X. Zhao, D. Zhang, IEEE/ASME Transactions on Mechatronics, 21, 1262, (2016).
2. E. Enikov, L. Minkov, and S. Clark, IEEE Transactions on Industrial Electronics, 52, 1005, (2005).
3. C. Liang, F. Wang, Y. Tian, X. Zhao, D. Zhang, Review of Scientific Instruments, 88, 115101, (2017).
4. C. Clifford, M. Seah, Applied Surface Science, 25, 1915, (2005).
5. M. Grandbois, M. Beyer, M. Rief, H. Clausen-schaumann, H. Gaub Science, 283, 1727, (1999).
6. B. Heymann, H. Grubmüller, Physical Review Letters, 26, 6126, (2000).
7. G. Matei, E. Thoreson, J. Pratt, D. Newham, N. Burnham, Review of Scientific Instruments, 77, 2527, (2006).
8. S. Kennedy, D. Cole, R. Clark Review of Scientific Instruments, 80, 125103, (2009).
9. A. Labuda, M. Kocun, M. Lysy, T. Walsh, J. Meinhold, T. Proksch, W. Meinhold, C. Anderson, R. Proksch, Review of Scientific Instruments, 87, 073705, (2016).
10. Y. Song, S. Wu, L. Xu, X. Fu Sensors, 15, 5865, (2015).
11. C. Gibson, D. Smith, C. Roberts Nanotechnology, 16, 234, (2005).
12. J. Riet, A. Katan, C. Rankl, S. Stahl, A. Buul Ultramicroscopy, 111, 1659, (2011).
13. R. Gates, M. Reitsma Review of Scientific Instruments, 78, 086101, (2007).
14. J. Sader, L. White, Journal of Applied Physics, 74, 1, (1993).
15. J. Sader Journal of Applied Physics, 84, 64, (1998).
16 J. Neumeister, W. Ducker Review of Scientific Instruments, 65, 2527, (1994).
17 S. Kim, J. Boyd Measurement Science & Technology, 17, 2343, (2006).
18 J. Cleveland, S. Manne, D. Bocek, P. Hansma Review of Scientific Instruments, 64, 403, (1993).
19 J. Hutter, J. Bechhoefer Review of Scientific Instruments, 64, 1868, (1993).
20 S. Gao, U. Brand Measurement Science & Technology, 25, 044014, (2014).
21 N. Burnham, X. Chen, C. Hodges, G. Matei, E. Thoreson, C. Roberts, M. Davies, S. Tendler Nanotechnology, 14, 1, (2002).
22 C. Gibson, G. Watson, S. Myhra Nanotechnology, 7, 259, (1996).
23 P. Cumpson, J. Hedley Nanotechnology, 14, 1279, (2003).
24 C. Diethold, M. Kühnel, F. Hilbrunner, T. Fröhlich, E. Manske Measurement, 51, 343, (2014).
25 M. Kim, J. Choi, J. Kim, Y. Park Measurement Science & Technology, 11, 3351, (2007).
26 C. Diethold, M. Kühnel, T. Ivanov, Ivo W. Rangelow, T. Fröhlich IMEKO-International Measurement Federation Secretariat, (2015).
27 S. Moore, M. Coskun, T. Alan, A. Neild, S. Moheimani Journal of Microelectromechanical Systems, 24, 1092, (2015)
28 J. Tian, Y. Tian, Z. Guo, F. Wang, D. Zhang, X. Liu, B. Shirinzadeh International Journal of Precision Engineering & Manufacturing, 17, 1415, (2016).
29 W. Robertson, B. Cazzolato, A. Zander IEEE Transactions on Magnetics, 48, 2479, (2012).
30 Z. Guo, Y. Tian, J. Tian, X. Liu, F. Wang, D. Zhang Microsystem Technologies, 23, 2285, (2017).
31 C. Liang, F. Wang, Y. Tian, X. Zhao, D. Zhang, Robotics and Computer-Integrated Manufacturing, 44, 208, (2017).
32 H. Pham, I. Chen Precision engineering, 29, 467, (2005).
33 E. Wang, C. Liang, Y. Tian and X. Zhao, IEEE/ASME Transactions on Mechatronics, 20, 2205, (2015).
34 F. Wang, C. Liang, Y. Tian, IEEE Transactions on Industrial Electronics, 21, 1055, (2016).
35 K. Johnson Cambridge: Cambridge Univ. Press, (1985).
36 M. Kim, J. Choi, Y. Park, J. Kim Metrologia, 43, 389, (2006).
37 M. Kim, J. Pratt, U. Brand, C. Jones Metrologia, 49, 70, (2012).