An Introduction to Dynamical
Electroweak Symmetry Breaking

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In these lectures, I present an introduction to the theory and phenomenology of
dynamical electroweak symmetry breaking.

1 Lecture 1: The Dynamics of Electroweak Symmetry Breaking

1.1 What’s Wrong with the Standard Model?

In the standard higgs model, one introduces a fundamental scalar doublet:

\[ \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \]

with potential:

\[ V(\phi) = \lambda \left( \phi^+ \phi - \frac{v^2}{2} \right)^2. \]

While this theory is simple and renormalizable, it has a number of shortcomings. First, while the theory can be constructed to accommodate the breaking of electroweak symmetry, it provides no explanation for it – one simply assumes that the potential is of the form in eqn. 2. In addition, in the absence of supersymmetry, quantum corrections to the Higgs mass are naturally of order the largest scale in the theory

\[ m_H^2 \propto \Lambda^2, \]

leading to the hierarchy and naturalness problems. Finally, the \( \beta \) function for the self-coupling \( \lambda \)

\[ \Rightarrow \beta = \frac{3\lambda^2}{2\pi^2} > 0, \]

leading to a “Landau pole” and triviality.
The hierarchy/naturalness and triviality problems can be nicely summarized in terms of the Wilson renormalization group. Define the theory with a fixed UV-cutoff:

\[ \mathcal{L}_\Lambda = D^\mu \phi^\dagger D_\mu \phi + m^2(\Lambda)\phi^\dagger \phi + \frac{\lambda(\Lambda)}{4}(\phi^\dagger \phi)^2 + \kappa(\Lambda) \frac{\Lambda^2}{36\Lambda^2} (\phi^\dagger \phi)^3 + \ldots \]  

(5)

Here \( \kappa \) is the coefficient of a representative irrelevant operator, of dimension greater than four. Next, integrate out states with \( \Lambda' < k < \Lambda \), and construct a new Lagrangian with the same low-energy Green’s functions:

\[ \mathcal{L}_\Lambda \Rightarrow \mathcal{L}_{\Lambda'} \]

\[ m^2(\Lambda) \rightarrow m^2(\Lambda') \]

\[ \lambda(\Lambda) \rightarrow \lambda(\Lambda') \]

\[ \kappa(\Lambda) \rightarrow \kappa(\Lambda') \]  

(6)

The low-energy behavior of the theory is then nicely summarized in terms of the evolution of couplings in the infrared a three-dimensional representation of this flow in the infinite-dimensional space of couplings shown in Figure 1.

From Figure 1, we see that as we scale to the infrared the coefficients of irrelevant operators, such as \( \kappa \), tend zero; i.e. the flows are attracted to the

\^{a}For convenience, we ignore the corrections due to the weak gauge interactions. In perturbation theory, at least, the presence of these interactions does not qualitatively change the features of the Higgs sector.
finite dimensional subspace spanned (in perturbation theory) by operators of
dimension four or less; this is the modern understanding of renormalizability.
On the other hand, the coefficient of the only relevant operator (of dimension
2), \( m^2 \), tends to infinity. This leads to the naturalness & hierarchy problem. Since we want \( m^2 \propto v^2 \) at low energies we must adjust the value of \( m^2(\Lambda) \) to
a precision of
\[
\frac{\Delta m^2(\Lambda)}{m^2(\Lambda)} \propto \frac{v^2}{\Lambda^2}.
\]
Finally, the coefficient of the only marginal operator \( \lambda \) tends, because of the
positive \( \beta \) function, 0. If we try to take the continuum limit, \( \Lambda \to +\infty \), the
theory becomes free or trivial. This last statement implies that, in and of itself, the
standard one-doublet higgs model is incomplete.

The analysis we have presented is based on perturbation theory and is valid in the domain of attraction of the “Gaussian fixed point” (\( \lambda = 0 \)). In
principle, however, the Wilson approach can be used non-perturbatively and
take into account the presence of nontrivial fixed points or large anomalous
dimensions. In a conventional Higgs theory, neither of these effects is thought to occur — these issues will, however, be relevant in theories of dynamical
electroweak symmetry breaking.

1.2 Solving the Naturalness/Hierarchy Problems

There are only two ways of dealing with any hierarchy, political or otherwise: we can either stabilize or eliminate it.

The conservative approach of stabilizing the hierarchy can be implemented
by introducing a symmetry which protects the scalar masses. One approach is
supersymmetry. In this case each scalar is associated with a fermionic superpartner
and the chiral symmetry of the superpartners of the scalar higgs
protects the mass from receiving corrections of \( \mathcal{O}(\Lambda^2) \). In practice this occurs
because of a cancelation between loop-diagrams involving scalars and fermions,
for example
\[
\delta m^2_H \propto \log \Lambda^2.
\]

An alternative approach to stabilizing the hierarchy is to use the “composite higgs” approach of Georgi and Kaplan. In these models, the higgs is a
Goldstone boson whose mass is protected by a (spontaneously broken) chiral
symmetry. In these models electroweak symmetry breaking is due to “vacuum
(mis)-alignment.”
Models of dynamical electroweak symmetry are based on the radical approach of eliminating the hierarchy. Here electroweak symmetry breaking is due to chiral symmetry breaking in a gauge theory with massless fermions. We will concentrate on this approach in what follows.

1.3 Technicolor: A Dynamical Electroweak Symmetry Breaking

The simplest theory of dynamical electroweak symmetry breaking is technicolor. Consider an $SU(N_{TC})$ gauge theory with fermions in the fundamental representation of the gauge group

$$\Psi_L = \left( \begin{array}{c} U \\ D \end{array} \right)_L \ U_R, D_R \quad \quad (9)$$

The fermion kinetic energy terms for this theory are

$$\mathcal{L} = \bar{U}_L i \gamma^\mu U_L + \bar{U}_R i \gamma^\mu U_R + \bar{D}_L i \gamma^\mu D_L + \bar{D}_R i \gamma^\mu D_R , \quad \quad (10)$$

and, like QCD in $m_u, m_d \to 0$ limit, have a chiral $SU(2)_L \times SU(2)_R$ symmetry.

As in QCD, exchange of technigluons in the spin zero, isospin zero channel is attractive

$$\bar{U}D \rightarrow \langle \bar{U}U \rangle = \langle \bar{D}D \rangle \neq 0, \quad \quad (11)$$

causing the formation of a condensate which dynamically breaks $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$. These broken chiral symmetries imply the existence of three massless Goldstone bosons, the analogs of the pions in QCD.

Now consider gauging $SU(2)_W \times U(1)_Y$ with the left-handed fermions transforming as weak doublets and the right-handed ones as weak singlets (in this one-doublet model we will take the left-handed technifermions to have hypercharge zero and the right-handed up- and down-technifermions to have hypercharge $\pm 1/2$). The spontaneous breaking of the chiral symmetry breaks the weak-interactions down to electromagnetism. The would-be Goldstone bosons become

$$\pi^\pm, \pi^0 \rightarrow W^\pm_L, Z_L, \quad \quad (12)$$

the longitudinal components of the $W$ and $Z$ which acquire a mass

$$M_W = \frac{g_{FTC}^2}{2}. \quad \quad (13)$$
Here \( F_{TC} \) is the analog of \( f_\pi \) in QCD. In order to obtain the experimentally observed masses, we must have that \( F_{TC} \approx 250 \text{GeV} \) and hence this model is essentially QCD scaled up by a factor of

\[
\frac{F_{TC}}{f_\pi} \approx 2500. \tag{14}
\]

While I have described only the simplest model above, it is straightforward to generalize to other cases. Any strongly interacting gauge theory with a chiral symmetry breaking pattern \( G \to H \), in which \( G \supset SU(2)_W \times U(1)_Y \) and breaks to a subgroup \( H \supset U(1)_{em} \) (with \( SU(2)_W \times U(1)_Y \not\subset H \)) will break the weak interactions down to electromagnetism. In order to correspond to experimental results, however, we must also require that \( H \) contain “custodial” \( SU(2)_C \) which insures that the \( F \)-constant associated with the \( W^\pm \) and \( Z \) are equal and therefore that the relation

\[
\rho = \frac{M_W}{M_Z} \sin \theta_W = 1 \tag{15}
\]

is satisfied at tree-level. If the chiral symmetry is larger than \( SU(2)_L \times SU(2)_R \), theories of this sort will contain additional (pseudo-)Goldstone bosons which are not “eaten” by the \( W \) and \( Z \). For simplicity, in the remainder of this lecture, we will discuss the phenomenology of the one-doublet model.

1.4 The Phenomenology of Dynamical Electroweak Symmetry Breaking

Of the particles that we have observed to date, the only ones directly related to the electroweak symmetry breaking sector are the longitudinal gauge-bosons. Therefore, we expect that the most direct signatures for electroweak symmetry breaking to come from the scattering of longitudinally gauge bosons. At high-energies, we may use the equivalence theorem to reduce the problem of longitudinal gauge boson \((W_L)\) scattering to the corresponding (and generally simpler) problem of the scattering of the Goldstone bosons \((\pi)\) that would be present in the absence of the weak gauge interactions.

In order to correctly describe the weak interactions, the symmetry breaking sector must have an (at least approximate) custodial symmetry and the most general effective theory describing the behavior of the Goldstone bosons is an

\[\text{Except, possibly, for the third generation. See the discussion of topcolor in lecture 3.}\]
effective chiral lagrangian^{10} with an \( SU(2)_L \times SU(2)_R \rightarrow SU(2)_V \) symmetry breaking pattern. This effective lagrangian is most easily written in terms of a field

\[
\Sigma = \exp(i\pi^a \sigma^a / F_{TC}) ,
\]

where the \( \pi^a \) are the Goldstone boson fields, the \( \sigma^a \) are the Pauli matrices, and where the field \( \Sigma \) which transforms as

\[
\Sigma \rightarrow L \Sigma R^\dagger
\]

under \( SU(2)_L \times SU(2)_R \).

The interactions can then be ordered in a power-series in momenta. Allowing for custodial \( SU(2) \) violation, the lowest-order terms in the effective theory are

\[
\frac{F_{TC}^2}{4} \text{Tr} [D^\mu \Sigma D^\mu \Sigma] + \frac{F_{TC}^2}{2} \left( \frac{1}{\rho} - 1 \right) \text{Tr} T_3 \Sigma^\dagger D^\mu \Sigma]^2
\]

where

\[
D^\mu \Sigma = \partial^\mu \Sigma + ig W^\mu \Sigma - i \Sigma g' B^\mu ,
\]

and the gauge-boson kinetic terms

\[
- \frac{1}{2} \text{Tr} [W^{\mu\nu} W_{\mu\nu}] - \frac{1}{2} \text{Tr} [B^{\mu\nu} B_{\mu\nu}] .
\]

In unitary gauge, \( \Sigma = 1 \) and the lowest-order terms in eqn. \ref{eqn:lowest_order_terms} give rise to the \( W \) and \( Z \) masses

\[
g^2 F_{TC}^2 \frac{W^{-\mu} W^\mu}{4 \rho\cos^2 \theta} ,
\]

\[
g^2 F_{TC}^2 \frac{Z^\mu Z_\mu}{8 \rho\cos^2 \theta} .
\]

So far, the description we have constructed is valid in any theory of electroweak symmetry breaking. The interactions in eqn. \ref{eqn:lowest_order_terms} result in universal low-energy^{1} theorems

\[
\mathcal{M}[W^+_L W^-_L \rightarrow W^+_L W^-_L] = \frac{\sin^2 \theta}{2} ,
\]

\[
\mathcal{M}[W^+_L W^-_L \rightarrow Z_L Z_L] = \frac{\sin^2 \theta}{2} (4 - \frac{3}{\rho}) ,
\]

\[
\mathcal{M}[Z_L Z_L \rightarrow Z_L Z_L] = 0 .
\]

These amplitudes increase with energy and, at some point, this growth must stop. What dynamics cuts off growth in these amplitudes? In general, there are three possibilities:
• New particles

• The born approximation fails $\rightarrow$ strong interactions

• both.

In the case of QCD-like technicolor, we take our inspiration from the familiar strong interactions. The data for $\pi\pi$ scattering in QCD in the $I = J = 1$ channel is shown in Figure 2. After correcting for the finite pion mass, we see that the scattering amplitude follows the low-energy prediction near threshold, but at higher energies the amplitude is dominated by the $\rho$-meson whose appearance (1) enhances the scattering cross-section and (2) cuts-off the growth of the scattering amplitude at higher energies. In a QCD-like technicolor theory, then, we expect the appearance of a vector meson whose mass we estimate by scaling by $F_{TC}/f_{\pi} \approx 2500$. That is,

$$M_{\rho_{TC}} \approx 2 \text{ TeV} \sqrt{\frac{3}{N_{TC}}},$$

where we have included large-$N_{TC}$ scaling to estimate the effect of $N_{TC} \neq 3$.

The most direct experimental signature of dynamical electroweak symmetry breaking is to look for these “technivector mesons.” At the LHC, gauge
In particular, the $M_{T}$ transverse mass ($M_{T}$) mixing signal from the gauge boson scattering signal discussed above. The vector-meson $\rho$ mixing in QCD:

$$
\begin{align*}
\text{keptonic cuts} & \quad \text{jet cuts} \\
|p(t)| & < 3.5 \quad |p(j_{\text{tag}})| > 0.8 \text{ TeV} \\
p_{T}(l) & > 40 \text{ GeV} \quad 3.0 < |p(j_{\text{tag}})| < 5.0 \\
p_{T}(\rho) & > 50 \text{ GeV} \quad p_{T}(j_{\text{tag}}) > 40 \text{ GeV} \\
|p_{T}(l)| & > 0.5 M_{T} \quad p_{T}(j_{\text{tag}}) > 60 \text{ GeV} \\
M_{T} & > 500 \text{ GeV} \quad |p(j_{\text{tag}})| < 3.0 
\end{align*}
$$

Note that, in addition to high-$p_{T}$ gauge bosons, one expects forward “tag” jets (with a typical transverse momentum of order $M_{W}$) from the quarks which radiate the initial gauge bosons. The signal expected is shown in Figure 3 for $M_{\text{PTC}} = 1.0 \text{ TeV, 2.5 TeV}$. Note the scale: events per 50 GeV bin of transverse mass ($M_{T}$) per 100 fb$^{-1}$!

A complementary signal is provided through the technicolor analog of “vector-meson dominance.” In particular, the $W$ and $Z$ can mix with the technirho mesons in a manner exactly analogous to $\gamma-\rho$ mixing in QCD:

$$
\begin{align*}
\begin{tikzpicture}
\node (q) at (0,0) [draw] {q};
\node (p) at (1,0) [draw] {p};
\node (l) at (2,0) [draw] {l};
\draw (q) -- (p);
\draw (p) -- (l);
\end{tikzpicture}
\end{align*}
$$

Note that this process does not have a very forward jet and is distinguishable from the gauge boson scattering signal discussed above. The vector-meson mixing signal at the LHC is shown in Figure 3 for $M_{\text{PTC}} = 1.0 \text{ TeV, 2.5 TeV}$.

Figure 3: Gauge boson scattering signal plus background (grey) and background (black) for $W^{\pm}Z$ production at LHC for technirho masses of 1.0 TeV and 2.5 TeV. Signal selection requirements shown in table above.

boson scattering occurs through the following process,
Figure 4: Vector meson mixing signal plus background (grey) and background (black) for $W^\pm Z$ production at LHC for technirho masses of (a) 1.0 TeV and (b) 2.5 TeV.

A dynamical electroweak symmetry breaking sector will also have effect two gauge-boson production at a high-energy $e^+e^-$ collider such as the NLC. For example, if gauge-boson re-scattering is dominated by a technirho meson, it can be parameterized in terms of a $ZWW$ form-factor

$$F_T = \exp \left[ \frac{1}{\pi} \int_0^\infty ds' \delta(s', M_\rho, \Gamma_\rho) \left\{ \frac{1}{s' - s - i\epsilon} - \frac{1}{s'} \right\} \right],$$

(28)

where

$$\delta(s) = \frac{1}{96\pi v^2} s + \frac{3\pi}{8} \left[ \tanh \left( \frac{s - M_\rho^2}{M_\rho^2 \Gamma_\rho} \right) + 1 \right].$$

(29)

This two gauge-boson production mechanism interferes with continuum production, and by an accurate measurement of the decay products is is possible to reconstruct the real and imaginary parts of the form-factor $F_T$. The expected accuracy of a 500 GeV NLC with 80 fb$^{-1}$ is shown in Figure 5.

1.5 Low-Energy Phenomenology

Even though the most direct signals of a dynamical electroweak symmetry breaking sector require (partonic) energies of order 1 TeV, there are also effects which may show up at lower energies as well. While the $O(p^2)$ terms in the

$^6$If the technicolor theory satisfies a “KSRF” relation this re-scattering” effect is exactly equivalent to the vector-meson mixing effect discussed above.
Figure 5: ZWW form-factor measurement at a 500 GeV NLC with $80 fb^{-1}$. Predictions are shown for the standard model, and for technicolor for various technirho masses.

effective lagrangian are universal, terms of higher order are model-dependent. At energies below $M_{\rho TC}$, there are corrections to 3-pt functions:

\[ -ig \frac{b_L}{16\pi^2} \text{Tr} W^{\mu\nu} D_\mu \Sigma D_\nu \Sigma^\dagger, \]

(31)

and

\[ -ig' \frac{b_R}{16\pi^2} \text{Tr} B^{\mu\nu} D_\mu \Sigma^\dagger D_\nu \Sigma, \]

(32)

as well as corrections to the 2-pt functions:

\[ + gg' \frac{l_0}{16\pi^2} \text{Tr} \Sigma B^{\mu\nu} \Sigma^\dagger W_{\mu\nu}. \]

(34)

In these expressions, the $l$s are normalized to be $O(1)$. 
The corrections to the 3-point functions are typical, following Hagiwara, et al.

given above,

\[
\frac{i}{e \cot \theta} \mathcal{L}_{WWZ} = g_1 (W^\dagger_{\mu} W^\mu Z^\nu - W^\dagger_{\mu} Z^\nu W^\mu) \\
+ \kappa_Z W^\dagger_{\mu} W^\mu Z^\nu + \frac{\lambda_Z}{M_W} W^\dagger_{\mu} W^\mu Z^\nu, \tag{35}
\]

and

\[
\frac{i}{e} \mathcal{L}_{WW\gamma} = (W^\dagger_{\mu} W^\mu A^\nu - W^\dagger_{\mu} A^\nu W^\mu) \\
+ \kappa_\gamma W^\dagger_{\mu} W^\mu F^\mu \nu + \frac{\lambda_\gamma}{M_W} W^\dagger_{\mu} W^\mu F^\mu \nu. \tag{36}
\]

Re-expressing these coefficients in terms of the parameters in $\mathcal{L}_{\rho_4}$ given above, we find

\[
g_1 \approx \frac{\alpha_i i_4}{4 \pi \sin^2 \theta} = \mathcal{O}(10^{-2} - 10^{-3}), \quad \kappa_Z \approx \mathcal{O}(10^{-2} - 10^{-3}), \quad \kappa_\gamma \approx \mathcal{O}(10^{-2} - 10^{-3}),
\]

and $\lambda_{Z, \gamma}$ from $\mathcal{L}_{\rho_5}$ implying that

\[
\lambda_{Z, \gamma} = \mathcal{O}(10^{-4} - 10^{-5}). \tag{38}
\]

The best current limits coming from Tevatron experiments are shown in Figure 6. Unfortunately, they do not reach the level of sensitivity required. The situation is somewhat improved at the LHC, as shown in Figure 7, or at a 500 or 1500 GeV NLC, as shown in Figure 8.
Figure 7: Experimental reach of LHC to probe anomalous gauge-boson vertices given an integrated luminosity of 100 fb\(^{-1}\).

Figure 8: Experimental reach of a 500 GeV (solid) or 1500 GeV (dashed) NLC to probe anomalous gauge-boson vertices, assuming 80 fb\(^{-1}\) or 190 fb\(^{-1}\) respectively.
Figure 9: 95% confidence level bounds on the oblique parameters $S$ and $T$ for $\alpha_S = 0.115$ (solid) and 0.124 (dashed).

The corrections to the 2-pt functions give rise to contributions to the "oblique parameters" $S$

$$S \equiv 16\pi \left[ \Pi_{33}^\prime(0) - \Pi_{3Q}^\prime(0) \right]$$

$$= -\pi l_{10} \approx 4\pi \left( \frac{F_{\rho TC}^2}{M_{\rho TC}^2} - \frac{F_{A TC}^2}{M_{A TC}^2} \right) N_D , \quad (39)$$

and $T$

$$\alpha T \equiv \frac{g^2}{\cos^2 \theta M_Z^2} \left[ \Pi_{11}(0) - \Pi_{33}(0) \right] = \rho - 1 . \quad (40)$$

Current experimental constraints imply the bounds shown in Figure 9 at 95% confidence level for different values of $\alpha_S$. Scaling from QCD, we expect a contribution to $S$ of order

$$S \approx 0.28 N_D (N_{TC}/3) , \quad (41)$$

for an $SU(N_{TC})$ technicolor theory with $N_D$ technidoublets. From these we see that, with the possible exception of $N_D = 1$ and $N_{TC} = 2$ or 3, QCD-like technicolor is in conflict with precision weak measurements.

Dynamical Electroweak Symmetry Breaking provides a natural and attractive mechanism for producing the $W$ and $Z$ masses. Generically models of this type predict strong $WW$-Scattering, signals of which may be observable at the LHC. While the simplest QCD-like models serve as a useful starting
point, they are excluded (except, perhaps, for an $SU(2)_{TC}$ model with one doublet) since they would give rise to unacceptably large contributions to the $S$ parameter. In the next lecture we will discuss the additional interactions and features required in (more) realistic models to give rise to the masses to the ordinary fermions.

2 Lecture 2: Flavor Symmetry Breaking and ETC

2.1 Fermion Masses & ETC Interactions

In order to give rise to masses for the ordinary quarks and leptons, we must introduce interactions which connect the chiral-symmetries of technifermions to those of the ordinary fermions. The most popular choice is to introduce new broken gauge interactions, called extended technicolor interactions (ETC), which couple technifermions to ordinary fermions. At energies low compared to the ETC gauge-boson mass, $M_{ETC}$, these effects can be treated as local four-fermion interactions

\[ g^2_{ETC} M_{ETC}^{-2} (\bar{U} \psi U_R) (\bar{q} q_L). \]  

After technicolor chiral-symmetry breaking, such an interaction gives rise to a mass for an ordinary fermion

\[ m_q \approx \frac{g^2_{ETC}}{M_{ETC}^2} \langle \bar{U} U \rangle_{ETC}, \]  

where $\langle \bar{U} U \rangle_{ETC}$ is the value of the technifermion condensate evaluated at the ETC scale (of order $M_{ETC}$). The condensate renormalized at the ETC scale in eqn. 43 can be related to the condensate renormalized at the technicolor scale as follows

\[ \langle \bar{U} U \rangle_{ETC} = \langle \bar{U} U \rangle_{TC} \exp \left( \int_{\Lambda_{TC}}^{M_{ETC}} d\mu \frac{d}{d\mu} \gamma_m(\mu) \right), \]  

where $\gamma_m(\mu)$ is the anomalous dimension of the fermion mass operator and $\Lambda_{TC}$ is the analog of $\Lambda_{QCD}$ for the technicolor interactions.

For QCD-like technicolor (or any theory which is "precociously" asymptotically free), $\gamma_m$ is small over in the range between $\Lambda_{TC}$ and $M_{ETC}$ and using dimensional analysis we find

\[ \langle \bar{U} U \rangle_{ETC} \approx \langle \bar{U} U \rangle_{TC} \approx 4\pi F_{TC}^3. \]
In this case eqn. 43 implies that
\[
\frac{M_{\text{ETC}}}{g_{\text{ETC}}} \approx 40 \text{ TeV} \left( \frac{F_{\text{TC}}}{250 \text{ GeV}} \right)^{\frac{3}{2}} \left( \frac{100 \text{ MeV}}{m_q} \right)^{\frac{1}{2}}.
\]  
(46)

In order to orient our thinking, it is instructive to consider a simple “toy” extended technicolor model. The model is based on an $SU(N_{\text{ETC}})$ gauge group, with technicolor as an extension of flavor. In this case $N_{\text{ETC}} = N_{\text{TC}} + N_F$, and we add the (anomaly-free) set of fermions
\[
Q_L = (N_{\text{ETC}}, 3, 2)_{1/6}, \quad L_L = (N_{\text{ETC}}, 1, 2)_{-1/2},
\]
\[
U_R = (N_{\text{ETC}}, 3, 1)_{2/3}, \quad E_R = (N_{\text{ETC}}, 1, 1)_{-1},
\]
\[
D_R = (N_{\text{ETC}}, 3, 1)_{-1/3}, \quad N_R = (N_{\text{ETC}}, 1, 1)_0,
\]
where we display their quantum numbers under $SU(N_{\text{ETC}}) \times SU(3)_C \times SU(2)_W \times U(1)_Y$. We break the ETC group down to technicolor in three stages
\[
SU(N_{\text{TC}} + 3) \quad \Lambda_1 \quad \downarrow \quad m_1 \approx \frac{4\pi F^3}{\Lambda_1^2}
\]
\[
SU(N_{\text{TC}} + 2) \quad \Lambda_2 \quad \downarrow \quad m_2 \approx \frac{4\pi F^3}{\Lambda_2^2}
\]
\[
SU(N_{\text{TC}} + 1) \quad \Lambda_3 \quad \downarrow \quad m_3 \approx \frac{4\pi F^3}{\Lambda_3^2}
\]
resulting in three isospin-symmetric families of degenerate quarks and leptons, with $m_1 < m_2 < m_3$. Note that the heaviest family is related to the lightest ETC scale!

Before continuing our general discussion, it is worth noting a couple of points. First, in this example the ETC gauge-boson do not carry color or weak charge
\[
[G_{ETC}, SU(3)_C] = [G_{ETC}, SU(2)_W] = 0.
\]  
(47)
Furthermore, in this model there is one technifermion for each type of ordinary fermion: that is, this is a “one-family” technicolor model. Since there are eight left- and right-handed technifermions, the chiral symmetry of the technicolor theory is (in the limit of zero qcd and weak couplings) $SU(8)_L \times SU(8)_R \to SU(8)_V$. Such a theory would lead to $8^2 - 1 = 63$ (pseudo-)Goldstone bosons. Three of these Goldstone bosons are unphysical — the corresponding degrees of freedom become the longitudinal components of the $W^\pm$ and $Z$ by the Higgs mechanism. The remaining 60 must obtain a mass, and the condition in eqn.
will be modified in a realistic model. We will return to the issue of pseudo-Goldstone bosons below.

The most important feature of this or any ETC-model is that a successful extended technicolor model will provide a dynamical theory of flavor! As in the toy model described above and as explicitly shown in eqn. above, the masses of the ordinary fermions are related to the masses and couplings of the ETC gauge-bosons. A successful and complete ETC theory would predict these quantities, and hence the ordinary fermion masses.

Needless to say, constructing such a theory is very difficult. No complete & successful theory has been proposed. Examining our toy model, we immediately see a number of shortcomings of this model that will have to be addressed in a more realistic theory:

• What breaks ETC?
• Do we require a separate scale for each family?
• How do we obtain quark mixing angles?
• \( T_3 = \pm \frac{1}{2} \) fermions have equal masses, hence the \( u_R \) & \( d_R \) must be in different representations of ETC.
• What about right-handed technineutrinos and \( m_\nu \)?

2.2 Flavor-Changing Neutral-Currents

Perhaps the single biggest obstacle to constructing a realistic ETC model (or any dynamical theory of flavor) is the potential for flavor-changing neutral currents. Quark mixing implies transitions between different generations: \( q \to \Psi \to q' \), where \( q \) and \( q' \) are quarks of the same charge from different generations and \( \Psi \) is a technifermion. Consider the commutator of two ETC gauge currents:

\[
[\bar{q}\gamma\Psi, \bar{\Psi}\gamma q'] \supset \bar{q}\gamma q'.
\]

Hence we expect gauge bosons which couple to flavor-changing neutral currents. In fact, this argument is slightly too slick: the same is true of charged-current weak interactions as well! However in that case the gauge interactions, \( SU(2)_W \) respect a global \( (SU(5) \times U(1))^5 \) chiral symmetry leading to the usual GIM mechanism.

Unfortunately, ETC interactions cannot respect GIM (exactly); they must distinguish between the various generations in order to give rise the masses of the different generations. Therefore, flavor-changing neutral-current interactions are (at least at some level) unavoidable.
The most severe constraints come from possible $|\Delta S| = 2$ interactions which give rise to contributions to the $K_L-K_S$ mass difference. In particular, we would expect that in order to produce Cabbibo-mixing the same interactions which give rise to the $s$-quark mass could give rise to the flavor-changing interaction

$$\mathcal{L}_{|\Delta S|=2} = \frac{g_{ETC}^2}{M_{ETC}^2} \theta_{sd}^2 \bar{s}L^\mu d\bar{\tau}'\mu d + h.c.,$$

where $\theta_{sd}$ is of order the Cabbibo angle. Such an interaction contributes to the kaon mass splitting

$$(\Delta M_{K}^2)_{ETC} = \frac{g_{ETC}^2}{M_{ETC}^2} \theta_{sd}^2 \langle \bar{K}^0|\bar{s}L^\mu d\bar{\tau}'\mu d|K^0 \rangle + c.c.$$ (50)

Using vacuum insertion approximation we find

$$\langle \Delta M_{K}^2 \rangle_{ETC} \simeq \frac{g_{ETC}^2}{2M_{ETC}^2} \Re(\theta_{sd}^2) \frac{f_K^2}{N_D^{3/2}} M_{ETC}.$$ (51)

Experimentally we know that $\Delta M_K < 3.5 \times 10^{-12}$ MeV and hence that

$$\frac{M_{ETC}}{g_{ETC} \sqrt{\Re(\theta_{sd}^2)}} > 600 \text{ TeV}$$ (52)

Using eqn. 43 we find that

$$m_{q,\ell} \simeq \frac{g_{ETC}^2}{M_{ETC}^2} \langle \bar{T}T \rangle_{ETC} < \frac{0.5 \text{ MeV}}{N_D^{3/2} \theta_{sd}^2}$$ (53)

showing that it will be difficult to get $s$-quark mass right, let alone the $c$-quark!

2.3 Pseudo-Goldstone Bosons

As illustrated by our toy model above, a “realistic” ETC theory may require a technicolor theory with a chiral symmetry structure bigger than the $SU(2)_L \times SU(2)_R$ discussed in detail in the previous lecture. The prototypical model of this sort is the one-family model incorporated in our toy model. As discussed there the theory has $SU(8)_L \times SU(8)_R \to SU(8)_V$ ⇒ chiral symmetry breaking structure resulting in 63 Goldstone bosons, 60 of which are physical. The quantum numbers of the 60 remaining Goldstone bosons are shown in table 2.3. Clearly, these objects cannot be massless in a realistic theory!

In fact, the ordinary gauge interactions break the full $SU(8)_L \times SU(8)_R$ chiral symmetry explicitly. The largest effects are due to QCD and the color
SU(3)$_C$ | SU(2)$_V$ | Particle
--- | --- | ---
1 | 1 | $P^{0\mu}$, $\omega_T$
1 | 3 | $P^{0,\pm}_T$, $P^{0,\pm}_T$
3 | 1 | $P^{0\mu}_3$, $P^{0\mu}_T$
3 | 3 | $P^{0,\pm}_3$, $P^{0,\pm}_T$
8 | 1 | $P^{0(LR)}_8$, $P^{0\mu}_T$
8 | 3 | $P^{0,\pm}_8$, $P^{0,\pm}_T$

Table 1: Quantum numbers of the 60 physical Goldstone bosons (and the corresponding vector mesons) in a one-family technicolor model. Note that the mesons that transform as a 3's of QCD are complex fields.

Octets and triplets mesons get masses of order 200 — 300 GeV, in analogy to the electromagnetic mass splitting $m_{\pi^+} - m_{\pi^0}$ in QCD. Unfortunately, the others are massless to $O(\alpha)$!

Luckily, the ETC interactions (which we introduced in order to give masses to the ordinary fermions) are capable of explicitly breaking the unwanted chiral symmetries and producing masses for these mesons. This is because in addition to coupling technifermions to ordinary fermions, there will also be ETC interactions which couple technifermions to themselves. Using Dashen’s formula, we can estimate that such an interaction can give rise to an effect of order

$$F_{TC}^2 M_{\pi T}^2 \propto \frac{g_{ETC}^2}{M_{ETC}^2} \langle (T T)^2 \rangle_{ETC}.$$  (54)

In the vacuum insertion approximation for a theory with small $\gamma_m$, we may rewrite the above formula using eqn. 43 and find that

$$M_{\pi T} \simeq 55 \text{ GeV} \sqrt{\frac{m_f}{1 \text{ GeV}}} \sqrt{\frac{230 \text{ GeV}}{F_{TC}}}.$$  (55)

It is unclear that this large enough.

In addition, there is a particularly troubling chiral symmetry in the one-family model. The $SU(8)$-current $\bar{Q} \gamma_{\mu} \gamma_5 Q - 3 \bar{L} \gamma_{\mu} \gamma_5 L$ is spontaneously broken and has a color anomaly. Therefore we have a potentially dangerous weak scale axion! An ETC-interaction of the form

$$\frac{g_{ETC}^2}{M_{ETC}^2} (\bar{Q}_L \gamma^\mu L_L) (\bar{Q}_R \gamma^\mu Q_R),$$  (56)
is required to give to an axion mass, therefore we must embed $SU(3)_C$ in ETC.

Finally, before moving on I would like to note that there is an implicit assumption in the analysis of gauge-boson scattering presented in the last lecture. We have assumed that elastic scattering dominates. In the presence of many pseudo-Goldsone bosons, $WW$ scattering could instead be dominated by inelastic scattering. This effect has been illustrated in an $O(N)$-Higgs model with many pseudo-Goldstone Bosons, solved in large-$N$ limit. Instead of the expected resonance structure at high energies, the scattering can be small and structureless at all energies.

2.4 ETC etc

There are other model-building constraints on a realistic TC/ETC theory. For completeness, I list them here:

- ETC should be asymptotically free.
- There can be no gauge anomalies.
- Neutrino masses, if nonzero, must be small.
- There should be no extra massless, or light, gauge bosons.
- Weak CP-violation, without strong CP-violation.
- Isospin-violation in fermion masses without large $\Delta \rho$.
- Accomodate a large $m_t$.
- Small corrections to $Z \rightarrow b\bar{b}$ and $b \rightarrow s\gamma$.

Clearly, building a fully realistic ETC model will be quite difficult! However, as I have emphasized before, this is because an ETC theory must provide a complete dynamical explanation of flavor. In the remainder of this lecture, I will concentrate on possible solutions to the flavor-changing neutral-current problem(s). As I will discuss in detail in the last lecture, I believe the outstanding obstacle in ETC or any theory of flavor is providing an explanation for the top-quark mass, i.e. dealing with the last three issues listed above.
2.5 Technicolor with a Scalar

At this point, it would be easy to believe that it is impossible to construct a model of dynamical electroweak symmetry breaking. Fortunately, there is at least an existence proof of such a theory: technicolor with a scalar.\footnote{Such a theory is also the effective low-energy model for a “strong-ETC” theory in which the ETC interactions themselves participate in electroweak symmetry breaking.}\footnote{\textsuperscript{20}} Admittedly, while electroweak symmetry breaking has a dynamical origin in this theory, the introduction of a scalar reintroduces the hierarchy and naturalness problems we had originally set out to solve.

In the simplest model one starts with a one doublet technicolor theory, and couples the chiral-symmetries of technifermions to ordinary fermions through scalar exchange:

\begin{equation}
\langle \bar{T}T \rangle
\end{equation}

The phenomenology of this model has been studied in detail,\footnote{\textsuperscript{33}} and the allowed region is shown in Figure \textbf{10}.

2.6 Walking Technicolor and the Gap Equation

Up to now we have assumed that technicolor is, like QCD, precociously asymptotically free and $\gamma_m(\mu)$ is small for $A_{TC} < \mu < M_{ETC}$. However, as discussed above it is difficult to construct an ETC theory of this sort without producing...
dangerously large flavor-changing neutral currents. On the other hand, if $\beta_{TC}$ is small, $\alpha_{TC}$ can remain large above the scale $\Lambda_{TC}$ — i.e. the technicolor coupling would “walk” instead of run. In this same range of momenta, $\gamma_m$ may be large and, since

$$\langle TT \rangle_{ETC} = \langle TT \rangle_{TC} \exp \left( \int_{\Lambda_{TC}}^{M_{ETC}} \frac{d\mu}{\mu} \gamma_m(\mu) \right)$$

(58)

this could enhance $\langle TT \rangle_{ETC}$ and fermion masses.

In order to proceed further, however, we need to understand how large $\gamma_m$ can be and how walking affects the technicolor $\chi$-symmetry breaking dynamics. These questions cannot be addressed in perturbation theory. Instead, what is conventionally done is to use a nonperturbative approximation for $\gamma_m$ and chiral-symmetry breaking dynamics based on the “rainbow” approximation to Schwinger-Dyson equation for shown in Figure 11. Here we write the full, nonperturbative, fermion propagator in momentum space as

$$iS^{-1}(p) = Z(p) / (p - \Sigma(p))$$

(59)

The linearized form of the gap equation in Landau gauge (in which $Z(p) \equiv 1$ in the rainbow approximation) is

$$\Sigma(p) = 3C_2(R) \int \frac{d^4k}{(2\pi)^4} \frac{\alpha_{TC}((k - p)^2)}{(k - p)^2} \frac{\Sigma(k)}{k^2}.$$  

(60)

Being separable, this integral equation can be converted to a differential equation which has the approximate (WKB) solutions

$$\Sigma(p) \propto p^{-\gamma_m(\mu)} \cdot p^{\gamma_m(\mu) - 2}$$

(61)

where the anomalous dimension of the fermion mass operator is

$$\gamma_m(\mu) = 1 - \sqrt{1 - \frac{\alpha_{TC}(\mu)}{\alpha_C}} ; \quad \alpha_C \equiv \frac{\pi}{3C_2(R)}.$$  

(62)
One can give a physical interpretation of these two solutions. Using the operator product expansion, we find

$$\lim_{p \to \infty} \Sigma(p) \propto m(p) + \frac{\langle TP_p \rangle}{p^2},$$

and hence the first solution corresponds to a “hard mass” or explicit chiral symmetry breaking, while the second solution corresponds to a “soft mass” or spontaneous chiral symmetry breaking. If we let $m_0$ be the explicit mass of a fermion, dynamical symmetry breaking occurs only if

$$\lim_{m_0 \to 0} \Sigma(p) \neq 0.$$  \hspace{1cm} (64)

A careful analysis of the gap equation, or equivalently the appropriate effective potential, implies that this happens only if $\alpha_{TC}$ reaches a critical value of chiral symmetry breaking, $\alpha_C$ defined in eqn. 62. Furthermore, the chiral symmetry breaking scale $\Lambda_{TC}$ is defined by the scale at which

$$\alpha_{TC}(\Lambda_{TC}) = \alpha_C$$  \hspace{1cm} (65)

and hence, at least in the rainbow approximation, at which

$$\gamma_m(\Lambda_{TC}) = 1.$$ \hspace{1cm} (66)

In the rainbow approximation, then, chiral symmetry breaking occurs when the “hard” and “soft” masses scale the same way. It is believed that even beyond the rainbow approximation $\gamma_m = 1$ at the critical coupling.

### 2.7 Implications of Walking: Fermion and PGB Masses, $S$

If $\beta(\alpha_{TC}) \simeq 0$ all the way from $\Lambda_{TC}$ to $M_{ETC}$, then $\Rightarrow \gamma_m(\mu) \simeq 1$ in this range. In this case, eqn. 43 becomes

$$m_{q,l} = \frac{g^2_{ETC}}{M_{ETC}^2} \times \left( \langle TT \rangle_{ETC} \cong \langle TT \rangle_{TC} \frac{M_{ETC}}{\Lambda_{TC}} \right).$$ \hspace{1cm} (67)

We have previously estimated that flavor-changing neutral current requirements imply that the ETC scale associated with the second generation must be greater than of order 100 to 1000 TeV. In the case of walking the enhancement of the technifermion condensate implies that

$$m_{q,l} \simeq \frac{50 - 500 \text{ MeV}}{N_D^{3/2} \theta_{sd}},$$ \hspace{1cm} (68)
arguably enough to accommodate the strange and charm quarks.

While this is very encouraging, two caveats should be kept in mind. First, the estimates given are for limit of “extreme walking”, i.e. assuming that the technicolor coupling walks all the way from the technicolor scale $\Lambda_{TC}$ to the relevant ETC scale $M_{ETC}$. To produce a more complete analysis, ETC-exchange must be incorporated into the gap-equation technology in order to estimate ordinary fermion masses. Studies of this sort are encouraging, it appears possible to accommodate the first and second generation masses without necessarily having dangerously large flavor-changing neutral currents. The second issue, however, is what about the third generation quarks, the top and bottom? As we will see in the next lecture, because of the large top-quark mass, further refinements or modifications will be necessary to produce a viable theory of dynamical electroweak symmetry breaking.

In addition to modifying our estimate of the relationship between the ETC-scale and ordinary fermion masses, walking also influences the size of pseudo-Goldstone boson masses. In the case of walking, Dashen’s formula for the size of pseudo-Goldstone boson masses in the presence of chiral symmetry breaking from ETC interactions, eqn. 54, reads:

$$F_{TC}^2 M_{\pi T}^2 \propto \frac{g_{ETC}^2}{M_{ETC}^2} \langle (\bar{T}T)^2 \rangle_{ETC}$$

$$\approx \frac{g_{ETC}^2}{M_{ETC}^2} \langle (\bar{T}T)_{ETC} \rangle^2$$

$$\simeq \frac{g_{ETC}^2 M_{ETC}^2}{M_{ETC}^2 \Lambda_{TC}^2} \langle (\bar{T}T)_{TC} \rangle^2 ,$$

where, consistent with the rainbow approximation, we have used the vacuum-insertion to estimate the strong matrix element. Therefore we find

$$M_{\pi T} \simeq g_{ETC} \left( \frac{4\pi F_{TC}^2}{\Lambda_{TC}} \right)$$

$$\simeq g_{ETC} \left( \frac{750 \text{ GeV}}{N_D} \left( \frac{1 \text{ TeV}}{\Lambda_{TC}} \right) \right) ,$$

i.e. walking also enhances the size of pseudo-Goldstone boson masses!

Finally, what about $S$? As emphasized by Lane, the assumptions of previous estimate of $S$ included that:

- techni-isospin is a good symmetry, and
- Technicolor is QCD-like, i.e.
1. Weinberg’s sum rules are valid,
2. the spectral functions saturated by lowest-resonances,
3. that the masses/couplings of resonances can be scaled from QCD.

A “realistic” walking technicolor theory would be very unlike QCD:
- Walking ⇒ different behavior of spectral functions.
- Many flavors/PGBs and non-fundamental representations makes scaling from QCD suspect.

For this reason the analysis given previously does not apply, and a walking theory could be phenomenologically acceptable. Unfortunately, technicolor being a strongly-coupled theory, it is not possible to give a compelling argument that the value of $S$ in a walking technicolor theory is definitely acceptable.

3 Lecture 3: Top in Models of Dynamical Symmetry Breaking

3.1 The ETC of $m_t$

Because of its large mass, the top quark poses a particular problem in models of dynamical electroweak symmetry breaking. Consider an ETC interaction (c.f. eqn. 43) giving rise to the top quark mass

$$\Psi_L \rightarrow \frac{g_{ETC}^2}{M_{ETC}} (\bar{U}_L U_R) (\bar{L}_R Q_L),$$

(71)

yielding

$$m_t \approx \frac{g_{ETC}^2}{M_{ETC}^2} \langle \bar{U}U \rangle_{ETC}.$$  

(72)

In conventional technicolor, using

$$\langle \bar{U}U \rangle_{ETC} \approx \langle \bar{U}U \rangle_{TC} \approx 4\pi F_{TC}^3$$

(73)

we find

$$\frac{M_{ETC}}{g_{ETC}} \approx 1 \text{TeV} \left( \frac{F_{TC}}{250 \text{GeV}} \right)^{\frac{2}{3}} \left( \frac{175 \text{GeV}}{m_t} \right)^{\frac{1}{3}}.$$  

(74)

That is, the scale of top-quark ETC-dynamics is very low. Since $M_{ETC} \simeq \Lambda_{TC}$ and

$$\langle \bar{U}U \rangle_{ETC} = \langle \bar{U}U \rangle_{TC} \exp \left( \int_{\Lambda_{TC}}^{M_{ETC}} \frac{d\mu}{\mu} \frac{\gamma_m(\mu)}{\mu} \right),$$

(75)

we see that walking can’t alter this conclusion. As we will see in the next few sections, a low ETC scale for the top quark is very problematic.
3.2 ETC Effects on \( Z \to b\bar{b} \)

For ETC models of the sort discussed in the last lecture, in which the ETC gauge-bosons do not carry weak charge, the gauge-boson responsible for the top-quark mass couples to the current

\[
\xi(\bar{\Psi}_i^\alpha L \gamma^\mu Q_i^L) + \xi^{-1}(U_R^\alpha \gamma^\mu t_R),
\]

(or h.c.) where \( \alpha \) is the technicolor index and and the contracted \( i \) are weak-indices. The part of the exchange-interaction coupling left- and right-handed fermions leads to the top-quark mass.

Additional interactions arise from the same dynamics, including

\[
- \frac{g_{ETC}^2}{M_{ETC}^2}(U_R^\alpha \gamma^\mu t_R) (U_R^\alpha \gamma^\mu t_R)
\]

and

\[
- \frac{g_{ETC}^2}{M_{ETC}^2} (\bar{Q}_L^i \gamma^\mu \Psi_L^i) (\bar{\Psi}_L^i \gamma^\mu Q_L^i).
\]

The last interaction involves \( b_L \) and the technifermions. After a Fierz transformation, the left-handed operator becomes the product of weak triplet currents

\[
- \frac{1}{2} \frac{g_{ETC}^2}{M_{ETC}^2} (\bar{Q}_L^i \gamma^\mu \tau_{ij} Q_L^j) (\bar{\Psi}_L^k \gamma^\mu \tau_{kl} \Psi_L^l),
\]

where the \( \tau \) are the Pauli matrices, plus terms involving weak singlet currents (which will not concern us here).

The exchange of this ETC gauge-bosons produces a correction of the coupling of the \( Z \) to \( b\bar{b} \)

\[
\text{Diagram}
\]

The size of this effect can be calculated by comparing it to the technifermion weak vacuum-polarization diagram

\[
\frac{\mu}{Z} \text{U.D.} \rightarrow \pi^{\mu\nu}_{ij} = \left(q^2 g^{\mu\nu} - q^\mu q^\nu\right) \delta_{ij} \pi(q^2),
\]

25
which, by the Higgs mechanism yields

\[ \pi(q^2) = \frac{e^2 v^2}{4 \sin^2 \theta} \frac{1}{q^2}. \]  

(82)

Therefore, exchange of the ETC gauge-boson responsible for the top-quark mass leads to a low-energy effect which can be summarized by the operator

\[ -\frac{e}{2 \sin \theta \cos \theta} \frac{g_{ETC}^2 v^2}{M_{ETC}^2} \xi^2 (Q_L Z r_3 Q_L). \]  

(83)

Hence this effect results in a change in the \( Zb\bar{b} \) coupling

\[ \delta g_L = +\frac{1}{4 \sin \theta \cos \theta} \xi^2 \frac{g_{ETC}^2 v^2}{M_{ETC}^2}, \]  

(84)

which, using the relation in eqn. 74, results in

\[ \frac{\delta \Gamma_b}{\Gamma_b} \approx 2 \frac{g_{ETC}^2 v^2}{g_L^2 + g_R^2} \approx -6.5\% \cdot \xi^2 \cdot \left( \frac{m_t}{175 \text{ GeV}} \right). \]  

(85)

It is convenient to form the ratio \( R_b = \Gamma_b / \Gamma_h \), where \( \Gamma_b \) and \( \Gamma_h \) are the width of the \( Z \) boson to \( b \)-quarks and to all hadrons, respectively, since this ratio is largely independent of the “oblique” corrections \( S \) and \( T \). The shift in eqn. 85 results in a shift in \( R_b \) of approximately

\[ \frac{\delta R_b}{R_b} \approx \frac{\delta \Gamma_b}{\Gamma_b} (1 - R_b) \approx -5.1\% \cdot \xi^2 \cdot \left( \frac{m_t}{175 \text{ GeV}} \right). \]  

(86)

Recent LEP results on \( R_b \) are shown in Figure 12. As we see, the current experimental value of \( R_b \) is about 1.8\( \sigma \) above the standard model prediction, while a shift of -5.1\( \% \) would (given the current experimental plus systematic experimental error \( \sigma \) one \( \sigma \) corresponds to a shift of 0.7\( \% \)) lower \( R_b \) by approximately 7\( \sigma \)! Clearly, conventional ETC generation of the top-quark mass is ruled out.

It should be noted, however, that there are nonconventional ETC models in which \( R_b \) may not be a problem. The analysis leading to the result given above assumes that (see eqn. 76) the ETC gauge-boson responsible for the top-quark mass does not carry weak-SU(2) charge. It is possible to construct models \( \delta \) where this is not the case. Schematically, the group-theoretic structure of such a model would be as follows
Figure 12: Contours in the $R_b$-$R_c$ plane from LEP data, corresponding to 68% and 95% confidence levels assuming Gaussian systematic errors. The Standard Model prediction for $m_t=175\pm 6$ GeV is also shown. The arrow points in the direction of increasing values of $m_t$.

$$ETC \times SU(2)_{light}$$

\[ \downarrow f \]

$$TC \times SU(2)_{heavy} \times SU(2)_{light}$$

\[ \downarrow u \]

$$TC \times SU(2)_{weak}$$

where ETC is extended technicolor, $SU(2)_{light}$ is (essentially) weak-$SU(2)$ on the light fermions, $SU(2)_{heavy}$ (originally embedded in the ETC group) is weak-$SU(2)$ for the heavy fermions, and $SU(2)_{light} \times SU(2)_{heavy}$ break to their diagonal subgroup (the conventional weak-interactions, $SU(2)_{weak}$) at scale $u$.

In this case a weak-doublet, technicolored ETC boson coupling to

$$\xi \bar{Q}_L \gamma^\mu U_L + \frac{1}{\xi} \bar{t}_R \gamma^\mu \Psi_R ,$$

(87)

is responsible for producing $m_t$. A calculation analogous to the one above yields a correction

$$Z_{uwc} \rightarrow \delta g_L = -\frac{1}{4} \frac{e}{\sin \theta \cos \theta} \frac{\xi^2 g^2_{ETC} v^2}{M^2_{ETC}}$$

(88)
of the opposite sign. In fact, the situation is slightly more complicated: there is also an extra $Z$-boson which also contributes. The total contribution is found[4] to be

$$\frac{\delta R_b}{R_b} \approx +5.1\% \cdot \xi^2 \cdot \left( \frac{m_t}{175\text{GeV}} \right) \left( 1 - \frac{\sin^2 \alpha f^2}{\xi^2} \right)$$  \hspace{1cm} (89)$$

where $\tan \alpha = g'/g$ is the ratio of the $SU(2)_{\text{light}}$ and $SU(2)_{\text{heavy}}$ coupling constants. The overall contribution to $R_b$ is very model-dependent, but could be within the experimentally allowed window.

### 3.3 Isospin Violation: $\Delta \rho$

#### „Direct” Contributions

ETC-interactions must violate weak-isospin in order to give rise to the mass splitting between the top and bottom quarks. This could induce dangerous $\Delta I = 2$ technifermion operators[4]

$$\mathcal{L} = \frac{g_{\text{ETC}}^2}{M_{\text{ETC}}^2} (\bar{\Psi}_R \gamma_\mu \tau_3 \Psi_R)^2.$$  \hspace{1cm} (90)$$

We can estimate the contribution of these operators to $\Delta \rho$ using the vacuum-insertion approximation

$$\Delta \rho \approx \frac{2g_{\text{ETC}}^2 N_D F_{\text{ETC}}^4}{v^2}$$  \hspace{1cm} (91)$$

which yields

$$\Delta \rho \approx 12\% \cdot \left( \frac{\sqrt{N_D F_{\text{ETC}}}}{250\text{GeV}} \right)^2 \cdot \left( \frac{1\text{TeV}}{M_{\text{ETC}}/g_{\text{ETC}}} \right)^2.$$  \hspace{1cm} (92)$$

If we require that $\Delta \rho \leq 0.4\%$, we find

$$M_{\text{ETC}} > 5.5\text{ TeV} \cdot \left( \frac{\sqrt{N_D F_{\text{ETC}}}}{250\text{GeV}} \right)^2,$$  \hspace{1cm} (93)$$

i.e. $M_{\text{ETC}}$ must be greater than required for $m_t \simeq 175\text{ GeV}$.

There is another possibility. It is possible that $N_D F_{\text{ETC}}^2 \ll (250\text{ GeV})^2$, if the sector responsible for the top-quark mass does not give rise to the bulk of EWSB. In this scenario, the constraint is

$$F_{\text{ETC}} < \frac{105\text{ GeV}}{N_D^{1/2}} \cdot \left( \frac{M_{\text{ETC}}/g_{\text{ETC}}}{1\text{TeV}} \right)^{1/2}.$$  \hspace{1cm} (94)$$
Figure 13: Momentum dependent dynamical masses of the technifermions responsible for the t- and b-quark masses, based on a gap-equation analysis.

However, this modification would enhance the effect of ETC-exchange in \( Z \rightarrow b\bar{b} \).

"Indirect" Contributions to \( \Delta \rho \)

Isospin violation in the ordinary fermion masses suggests the existence of isospin violation in the technifermion dynamical masses. Indeed, an analysis of the gap equation shows that if the \( t \)- and \( b \)-quarks get masses from technifermions in the same technidoublet the dynamical masses of the corresponding technifermions are as shown in Figure 13. At a scale of order \( M_{ETC} \) the technifermions and ordinary fermions are unified into a single gauge group, so it is not surprising that their masses are approximately equal at that scale. Below the ETC scale, the technifermion dynamical mass runs (because of the technicolor interactions), while the ordinary fermion masses do not. As shown in Figure 13, therefore, we expect that \( \Sigma_U(0) - \Sigma_D(0) \approx m_t - m_b \).

We can estimate the contribution of this effect to \( \Delta \rho \)

\[
\Sigma_U^0 \propto \frac{N_D d (\Sigma_U^0(0) - \Sigma_D^0(0))^2}{16\pi^2 v^2},
\]

where \( N_D \) is the number of technidoublets and \( d=\)dimension of TC representation. If we require \( \Delta \rho \leq 0.4\% \), this yields

\[
N_D d \left( \frac{\Delta \Sigma(0)}{175 \text{GeV}} \right)^2 \leq 2.7.
\]

This is perhaps possible if \( N_D = 1 \) and \( d = 2 \) (i.e. \( N_{TC} = 2 \)), but is generally problematic.
3.4 Evading the Unavoidable

The problems outlined in the last two sections, namely potentially dangerous ETC corrections to the branching ratio of $Z \rightarrow b\bar{b}$ and to the $\rho$ parameter, rule out the possibility of generating the top-quark mass using conventional extended technicolor interactions. A close analysis of these problems, however, suggests a framework for constructing an acceptable model: arrange for the $t$- and $b$-quarks to get the majority of their masses from interactions other than technicolor. If this is the case, the top- and bottom-quark masses can run substantially below the ETC scale as shown in Figure 14, allowing for

$$\Delta \Sigma(0) \simeq m_t(M_{ETC}) - m_b(M_{ETC}) \ll m_t.$$  \hspace{1cm} (97)

Since the technicolor/ETC interactions would only be responsible for a portion of the top-quark mass in this type of model, the problems outlined in the previous two sections are no longer relevant. In order to produce a substantial running of the third-generation quark masses, the third-generation fermions must have an additional strong-interaction not shared by the first two generations of fermions or (at least in an isospin-violating way) by the technifermions.

3.5 An Aside: Top-Condensate Models

Before constructing a model of the sort proposed in last section, we should pause to consider another possibility. Having entertained the notion that the top-quark mass may come from a strong interaction felt (at least primarily) by the third generation, one should ask if there is any longer a need for technicolor! After all, any interaction that gives rise to a quark mass must break the weak interactions. Furthermore, since $m_t \simeq M_W, M_Z$, the top-quark is
much heavier than other fermions it must be more strongly coupled to
symmetry-breaking sector. Perhaps all 45 of electroweak-symmetry breaking is
due to a condensate of top-quarks, \( \langle \bar{t}t \rangle \neq 0 \).

Consider a spontaneously broken/strong gauge-interaction, e.g. top-color:

\[
SU(3)_{tc} \times SU(3) \xrightarrow{M} SU(3)_{QCD},
\]

where \( SU(3)_{tc} \) is a new, strong, top-color interaction coupling to the third-
generation quarks and the other \( SU(3) \) is a weak, color interaction coupling to
the first two generations. At scales below \( M \), one has the ordinary QCD plus
interactions which couple primarily to the third generation quarks and can be
summarized by an operator of the form

\[
\mathcal{L} \supset -\frac{4\pi\kappa}{M^2} \left( \bar{t}_\lambda \gamma_\mu \frac{\lambda^a}{2} Q \right)^2,
\]

where \( \kappa \approx \frac{g_{tc}^2}{4\pi} \) is related to the top-color coupling constant. Consider
what happens as, for fixed \( M \), we vary \( \kappa \). For small \( \kappa \), the interactions are
perturbative and there is no chiral symmetry breaking. For large \( \kappa \), since the
new interactions are attractive in the spin-zero, isospin-zero channel, we expect
chiral symmetry breaking with \( \langle \bar{t}t \rangle \propto M^3 \). If the transition between these two
regimes is \textit{continuous}, as it is in the bubble 46 or mean-field approximation, we
expect that the condensate will behave as shown in Figure 15.

In order to produce a realistic model of electroweak symmetry breaking
based on these considerations, one must introduce extra interactions to split
the top- and bottom-quark masses. A careful analysis then shows that it is
not possible to achieve a phenomenologically acceptable theory unless 45 the
scale \( M \gg v \). Since the weak scale \( v \) is fixed, this implies that the condensate
\( \langle \bar{t}t \rangle \ll M^3 \), and the top-color coupling \( \kappa \) must be finely tuned

\[
\frac{\Delta \kappa}{\kappa_c} \equiv \frac{\kappa - \kappa_c}{\kappa_c} \propto \frac{\langle \bar{t}t \rangle}{M^3}.
\]
In this region, one has simply reproduced the standard-model with the Higgs-boson \( \phi \) produced dynamically as a \( t_R Q_L \) bound state!

3.6 Topcolor-Assisted Technicolor (TC2)

Recently, Chris Hill has proposed a theory which combines technicolor and top-condensation. Features of this type of model include

- Strong Technicolor dynamics at 1 TeV which dynamically generates most of electroweak symmetry breaking;
- Extended Technicolor dynamics at scales much higher than 1 TeV which generate the light quark and lepton masses, and small contributions to the third generation masses \( (m_{t,b,\tau}) \) of order 1 GeV;
- Strong Topcolor dynamics also at a scale of order 1 TeV which generates \( \langle \bar{t} t \rangle \neq 0, m_t \sim 175 \text{ GeV} \);
- Topcolor does not form \( \langle \bar{b} b \rangle \), and therefore there must be isospin violation. This may be acceptable because...
- Topcolor contributes a small amount to EWSB (with an “F-constant” \( f_t \sim 60 \text{ GeV} \));
- Extra pseudo-Goldstone bosons (“Top-pions”) which get mass from ETC interactions which allow for mixing of third generation to first two.

Hill’s Simplest TC2 Scheme

The simplest scheme which realizes these features has the following structure:

\[
G_{TC} \times SU(2)_{EW} \times \\
SU(3)_{tc} \times SU(3) \times U(1)_H \times U(1)_L \\
(g_3^c > g_3) \quad \downarrow \quad M \approx 1 \text{ TeV} \quad (g_1^H > g_1^T) \\
G_{TC} \times SU(3)_C \times SU(2)_{EW} \times U(1)_Y \\
\downarrow \quad \Lambda_{TC} \approx 1 \text{ TeV} \\
SU(3)_C \times U(1)_{EM}
\]

Here \( U(1)_H \) and \( U(1)_L \) are \( U(1) \) gauge groups coupled to the (standard model) hypercharges of the third-generation and first-two generation fermions respectively. Below \( M \), this leads to the effective interactions:

\[
- \frac{4\pi\kappa_{tc}}{M^2} \left[ \bar{\psi} \gamma^\mu \frac{\lambda^a}{2} \psi \right]^2,
\]

(101)
from top-color exchange and the isospin-violating interactions

\[ - \frac{4\pi\kappa_1}{M^2} \left[ \frac{1}{3}\bar{\psi}_L\gamma_\mu\psi_L + \frac{4}{3}\bar{t}_R\gamma_\mu t_R - \frac{2}{3}\bar{b}_R\gamma_\mu b_R \right]^2, \]  

(102)

from exchange of the “heavy-hypercharge” \((Z')\) gauge boson.

Since the interactions in eqn. (102) are attractive in the \(\bar{t}t\) channel, but repulsive in the \(\bar{b}b\) channel, the couplings \(\kappa_{tc}\) and \(\kappa_1\) can be chosen to produce \(\langle \bar{t}t \rangle \neq 0\) and a large \(m_t\), but not \(\langle \bar{b}b \rangle = 0\). In the Nambu-Jona-Lasinio approximation, we require

\[ \kappa^t = \kappa_{tc} + \frac{1}{3}\kappa_1 > \kappa_c \left( = \frac{3\pi}{8} \right)_{NJL} > \kappa^b = \kappa_{tc} - \frac{1}{6}\kappa_1. \]  

(103)

3.7 \(\Delta \rho\) in TC2

Direct Contributions

Couplings of the (potentially strong) \(U(1)_H\) group are isospin violating, at least in it’s couplings to the third generation. Isospin violating couplings to technifermions could be very dangerous, as shown above. For example, in the one-family technicolor model, if \(U(1)_H\) charges proportional to \(Y\):

\[ \Delta \rho^T \approx 152\% \kappa_1 \left( \frac{1 \text{ TeV}}{M} \right)^2. \]  

(104)

If \(M \approx 1\) TeV, we must have \(\kappa_1 \ll 1\). From eqn. (103) above, this implies a fine tuning of \(\kappa_{tc}\). In order to avoid this problem, one must construct a model in which the \(U(1)_H\) couplings to technifermions are isospin symmetric – “Natural TC2”.

Indirect/Direct Contribution

Since there are additional (strong) interactions felt by the third-generation of quarks, there are new “two-loop” contributions to \(\Delta \rho\):

\[ \Rightarrow \Delta \rho^{tc} \approx 0.53\% \left( \frac{\kappa_{tc}}{\kappa_c} \right) \left( \frac{1 \text{ TeV}}{M} \right)^2 \left( \frac{f_t}{64 \text{ GeV}} \right)^4. \]  

(106)

From this we find that \(M \gtrsim 1.4\) TeV.
Electroweak Constraints on Natural TC2

If the \( U(1)_H \) couplings to technifermions are isospin-symmetric, electroweak phenomenology is specified by \( M_{Z'}^2, \tan \phi = g_L^T / g_H^H \), and the charges \( Y_H \) of ordinary fermions. To get a feeling for the size of constraints on these models from electroweak phenomenology, consider a “baseline” model: \( Y_H = Y \). While this may be unrealistic, it is flavor universal. In this case the third generation picked out by it’s couplings to \( SU(3)_H \).

Constraints (arising from \( Z-Z' \) mixing as well as \( Z' \) exchange) from all precision electroweak data are shown in Figure 16. We see that, even in light of current LEP data, natural TC2 with a \( Z' \) mass of order 1-2 TeV is allowed.

Where have we come from, where are we going?

In these lectures I have tried to provide an introduction to modern theories of dynamical electroweak symmetry breaking. We have come a long way, and it is worth reviewing the logical progression that has brought us here:

- The search for a natural and dynamical explanation for electroweak symmetry breaking implies we should explore technicolor and related models (lecture 1);
- Accommodating/explaining \( u, d, s, c \) masses in such theories without large flavor-changing neutral-currents leads us to consider “walking” technicolor (lecture 2);
- Accommodating the \( b \) and, especially, the \( t \) mass without large corrections to \( \Delta\Gamma_b \) and \( T \) leads us to consider top-color assisted technicolor related model(s) (lecture 3).
Despite the progress that has been made, no complete and consistent model exists. As I have emphasized, model building difficult because

- Technicolor is a non-decoupling theory, the natural dynamical scale must be of order 1 TeV. Therefore, there are always potentially large low-energy effects;
- Technicolor theories are strongly-coupled and we have no reliable calculational methods. (QCD-like theories are already excluded.);
- Extended technicolor theories must provide a dynamicalexplanation of flavor.

Ultimately, these problems are not likely to be solved without experimental direction.

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