Industrial control system (ICS) attacks are usually targeted attacks that use the ICS entry approach to get a foothold within a system and move laterally throughout the organization. In recent decades, powerful attacks such as Stuxnet, Duqu, Flame, and Havex have served as wake-up calls for industrial units. All organizations are faced with the rise of security challenges in technological innovations. This paper aims to develop aggregation operators that can be used to address the decision-making problems based on a spherical fuzzy rough environment. Meanwhile, some interesting properties of idempotence, boundedness, and monotonicity for the proposed operators are analyzed. Moreover, we use this newly constructed framework to select ICS security suppliers and validate its acceptability. Furthermore, a different test has been performed based on a new operator to strengthen the suggested approach. Additionally, comparative analysis based on the novel extended TOPSIS method is presented to demonstrate the superiority of the proposed technique. The results show that the conventional approach has a larger area for information representation, better adaptability to the evaluation environment, and higher reliability of the evaluation results.

1. Introduction

Many governments are launching initiatives to encourage the implementation of electronic and manufacturing innovations, including Germany’s industry 4.0 systems, the United States’ reindustrialization, and China’s “Made in China 2025” strategy to advance next-generation information technology. All of these have motivated the continuous development of industrial control systems (ICS). All of these have prompted the development of industrial control systems (ICSs) to proceed. Information technology, while adding new development strength to ICS, it also introduces new security flaws. ICS plays a crucial role in the national economy and people’s livelihood of crucial national infrastructure. Furthermore, in recent years, a number of notable ICS attacks, such as Stuxnet, Duqu, Flame, and Havex, have raised the concern for authorities and industry sectors. As a result, many manufacturers are working to keep an increasingly open ICS safe. And evaluating ICS security suppliers and selecting the best one are a vital part of management decisions. A typical multi attribute decision-making (MADM) problem is selecting an ICS security supplier and selecting the best one are a vital part of management decisions. A typical multi attribute decision-making (MADM) problem is selecting an ICS security supplier and selecting the best one are a vital part of management decisions. A typical multi attribute decision-making (MADM) problem is selecting an ICS security supplier and selecting the best one are a vital part of management decisions. A typical multi attribute decision-making (MADM) problem is selecting an ICS security supplier and selecting the best one are a vital part of management decisions.
described q-spherical fuzzy sets and discussed their method to spherical fuzzy TOPSIS and presented an illustrative example for Order of Preference by Similarity to Ideal Solution (OPE-SIS) method. Kutlu [32] extended the TOPSIS technique to spherical fuzzy sets and implemented the aforementioned concept as a practical tool with the help of fuzzy measures. They have been efficient when dealing with complex fuzzy evaluation problems. Abdullah et al. [31] implemented the cubic fuzzy soft set (CFS) and addressed its related concepts. Khan et al. [29] presented generalized PFSS and their application in DM systems. Further, Ashraf et al. [30] introduced the idea of cubic PFSS. Abdullah et al. [31] implemented the cubic FFS approach to address the MADM problem and show it with a numerical example of a petroleum circulation center evaluation problem to demonstrate the usage and application of the proposed ranking technique. It should observe that both IFSs and FFS still have some limitations, although they have been efficient when dealing with complex fuzzy information in some practical applications. The novel idea of spherical fuzzy sets (SFSs) was introduced by Ashraf et al. [8], an advanced tool of FSS, IFS, and PFSS. They investigated the fundamental characteristics of SFSs and compared them with those of PFSSs. Graphically, the spaces of spherical and picture membership grades are investigated. They have been efficient when dealing with complex fuzzy evaluation problems. Abdullah et al. [31] implemented the cubic fuzzy soft set (CFS) and addressed its related concepts. Khan et al. [29] presented generalized PFSS and their application in DM systems. Further, Ashraf et al. [30] introduced the idea of cubic PFSS. Abdullah et al. [31] implemented the cubic FFS approach to address the MADM problem and show it with a numerical example of a petroleum circulation center evaluation problem to demonstrate the usage and application of the proposed ranking technique. It should observe that both IFSs and FFS still have some limitations, although they have been efficient when dealing with complex fuzzy information in some practical applications. The novel idea of spherical fuzzy sets (SFSs) was introduced by Ashraf et al. [8], an advanced tool of FSS, IFS, and PFSS. They investigated the fundamental characteristics of SFSs and compared them with those of PFSSs. Graphically, the spaces of spherical and picture membership grades are investigated. They implemented the aforementioned concept as a practical application to demonstrate the problem of evaluating a food circulation center. Kutlu [32] extended the TOPSIS technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method to spherical fuzzy TOPSIS and presented an illustrative example in the MADM problem. Kahraman [33] described q-spherical fuzzy sets and discussed their application in the MADM problem. Zeng [34] discussed T-spherical fuzzy Einstein interactive aggregation operators and their application to selecting photovoltaic cells. Emergency decision making based on SFSs to address the uncertainty in COVID-19 situation is discussed in [6, 7].

The rough set (RS) theory, proposed by Professor Pawlak in 1982, is an essential mathematical tool for dealing with ambiguous, inconsistent, and incomplete data and information [35]. The fuzzy rough set (FRS) can be combined with RSs to manage information with continuous attributes and investigate data inconsistencies. The FRS model has shown to be highly effective in many application areas because it is a powerful tool for analyzing inconsistent and ambiguous information. FRS set theory is a rough set theory extension that handles continuous numerical attributes [36]. The significance of the FRS theory may be observed in a variety of applications. Pan et al. [37] established the additive consistent fuzzy preference relation to improve the rough set model of the fuzzy preference relation. Li et al. [38] suggested a practical FRS approach for robust feature selection. Feng et al. [39] reduced multi-granulation using uncertainty measures based on FRSs, eliminating the negative and positive regions. Sun et al. [40] used a constructive technique to provide three multi-granulation FRSs over two universes. In this study by Liu [41], in the framework of interval-valued fuzzy and fuzzy probabilistic approximation space models, a decision-theoretic RS model was investigated. Zang used axiomatic and constructive techniques to integrate rough set theory and interval-valued fuzzy set theory and [42] presented a novel paradigm based on FRSs with extended interval values. Zhang et al. [43] offered FR based feature selection based on information entropy to minimize heterogeneous data. By combining granular variable precision FRSs and general fuzzy relations, Wang and Hu [44] provided a random set of fuzzy relationships. Vluymans et al. [45] introduced a new type of classifier for unbalanced multi-instance data based on FRS theory. Shaheen et al. [46] described the application of generalized hesitant fuzzy rough sets (GHFRS) in risk analysis. Khan et al. [47] addressed the use of a probabilistic hesitant FRS in a decision support system. Tang et al. [48] proposed the decision-theoretic rough set model with q-rung orthopair fuzzy information, as well as its application in evaluating stock investments. Liang et al. [49] suggested q-Rung orthopair fuzzy sets on decision-theoretic rough sets for three-way decisions under group DM. Zhang et al. [50] proposed group DM using incomplete q-rung orthopair fuzzy preference relations. For MADM, Hussain et al. [51] presented a covering-based q-rung orthopair fuzzy rough set model hybrid with the TOPSIS approach. In practice, these extensions of the q-rung orthopair fuzzy rough set successfully handle DMs’ evaluation values in MAGDM problems. Some decision-making techniques are discussed in [53–58].

In this research, motivated by the above discussion, we plan to introduce a new concept of spherical fuzzy rough set (SFRSs). Also, the hybrid technique of aggregation operators and expanded TOPSIS procedure under spherical fuzzy rough setting is presented, in order to benefit of the advantages of the TOPSIS method and SFSs. As a consequence,
the SFS is the generalized structure of fuzzy set like IF set and PyF set. Thus SFRS is capable of handling more uncertainty than FS, IF set, PyF set, and rough set. Therefore, in this paper, a novel improved TOPSIS-based method and novel algebraic norm-based aggregation operators are established to address with such circumstances of unknown weight information of both DMs and criteria weights and to solve the MAGDM problem after computing all the weights. In order to solve the decision-making problems (DMPs), choosing the ideal opinion, which is better connected to each order to solve the decision-making problems (DMPs), the MAGDM problem after computing all the weights. In this paper, a novel improved TOPSIS-based method and novel algebraic norm-based aggregation operators are established to introduce to find out the criteria weights under SFR information used in this paper. The entropy measure is introduced to find out the criteria weights under SFR information used in this paper. The challenges in the industrial control systems security and sustainability are addressed by using proposed advance methodology.

In summary, the main contributions of the present study are as follows:

(1) Novel idea of spherical fuzzy rough aggregation operators namely spherical fuzzy rough weighted averaging, spherical fuzzy rough ordered weighted averaging and spherical fuzzy rough hybrid weighted averaging operators is introduced and their basic operational laws are investigated.

(2) A case study of industrial control service provider selection is also presented to demonstrate the applicability of the established operators.

(3) To validate the findings, different test on aggregation operators are implemented.

(4) Finally, comparisons with the spherical fuzzy rough TOPSIS method are made to interpret the outcomes. The ranking of the obtained results is presented graphically.

2. Preliminaries

In this section, we will put forward some basic literature concerning IFS, PFS, SFS, relation and rough sets, which will be helpful for subsequent sections.

Definition 1. Let \( S = \{s, \xi_G(s), \psi_G(s) \} | s \in \mathcal{S} \),

where \( \xi_G: \mathcal{S} \rightarrow [0, 1] \) and \( \psi_G: \mathcal{S} \rightarrow [0, 1] \) denotes the MemD and NMemD of an object \( s \) to the set \( \mathcal{S} \) such that \( 0 \leq \xi_G(s) + \psi_G(s) \leq 1 \). For an alternative \( s \in \mathcal{S} \), \( \pi_G(s) = 1 - (\xi_G(s) + \psi_G(s)) \) is known as the degree of hesitancy.

Definition 2. Assume a universal set \( \mathcal{S} \). A PFS \( G \) on a set \( \mathcal{S} \) is of the form as follows:

\[ G = \{ (s, \xi_G(s), \eta_G(s), \psi_G(s)) | s \in \mathcal{S} \} \]  

where \( \xi_G: \mathcal{S} \rightarrow [0, 1], \eta_G: \mathcal{S} \rightarrow [0, 1] \) and \( \psi_G: \mathcal{S} \rightarrow [0, 1] \) denotes the positive membership (PM), neutral membership (NeuM) and negative membership (NM) of an object \( s \) in the set \( \mathcal{G} \) such that \( 0 \leq \xi_G(s) + \eta_G(s) + \psi_G(s) \leq 1 \). For an alternative \( s \in \mathcal{G} \), \( \pi_G(s) = 1 - (\xi_G(s) + \eta_G(s) + \psi_G(s)) \) is known as the degree of hesitancy.
The set $g^\ast(s)$ is said to be a successor neighborhood of an element $s$ with respect to $g$. The pair $(\mathcal{S},g)$ represents the approximation space with respect to $g^\ast$. Consider an approximation space $(\mathcal{S},g)$ and $b$ be any subset of $\mathcal{S}$, then lower and upper approximation of $b$ is represented and defined as follows:

\begin{align}
\tilde{g}(b) &= \{s \in \mathcal{S} | g^\ast(s) \subseteq b\}, \\
\tilde{g}(b) &= \{s \in \mathcal{S} | g^\ast(s) \cap b \neq \emptyset\}.
\end{align}

Therefore, the pair $(\tilde{g}(b), \tilde{g}(b))$ is called rough set. Then the mapping $g(\mathcal{S}), g(b) : \mathcal{P}(\mathcal{S}) \rightarrow \mathcal{P}(\mathcal{S})$ denotes the approximation operators.

### 3. Spherical Fuzzy Rough Sets

SFS is the most significant generalization of IFS, PyFS and PFS which provide more space for experts for assigning values to PM, NeuM and NM. Here we will present the hybrid notion of SFS and rough sets to get the novel concept of SF rough set. The new score and accuracy mapping are defined for the developed model and studied their desirable properties of the presented model with detail.

**Definition 7.** Suppose a universal set $\mathcal{S}$ and let $g \in \mathbb{SFS}(\mathcal{S} \times \mathcal{S})$ be SF relation. Then

(i) $g$ is reflexive if $\xi_g(s,s) = 1$, $\eta_g(s,s) = 0$, and $\psi_g(s,s) = 0$, $\forall s \in \mathcal{S}$;

(ii) $g$ is symmetric if $\forall (s,r) \in \mathcal{S} \times \mathcal{S}$,

\[
\xi_g(r,s) = \xi_g(s,r), \quad \eta_g(r,s) = \eta_g(s,r), \quad \text{and} \quad \psi_g(r,s) = \psi_g(s,r);
\]

(iii) $g$ is transitive if $\forall (s,t) \in \mathcal{S} \times \mathcal{S}$,

\[
\xi_g(r,t) \geq \bigvee_{x \in \mathcal{S}} [\xi_g(r,s) \wedge \xi_g(s,t)], \quad \eta_g(r,t) = \bigwedge_{x \in \mathcal{S}} [\eta_g(r,s) \vee \eta_g(s,t)], \quad \text{and} \quad \psi_g(r,t) = \bigvee_{x \in \mathcal{S}} [\psi_g(r,s) \vee \psi_g(s,t)].
\]

Alternatively, the relation $g$, is transitive if it holds the following:

For all $s, r \in \mathcal{S}$ and $\lambda_1, \lambda_2, \lambda_3 \in [0, 1]$

(a) $\forall t \in \mathcal{S}$, $\xi_g(r,s) \geq \lambda_1$, and $\xi_g(s,t) \geq \lambda_1 \Longrightarrow \xi_g(r,t) \geq \lambda_1$,

(b) $\forall t \in \mathcal{S}$, $\eta_g(r,s) \leq \lambda_2$, and $\eta_g(s,t) \leq \lambda_2 \Longrightarrow \eta_g(r,t) \leq \lambda_2$,

(c) $\forall t \in \mathcal{S}$, $\psi_g(r,s) \leq \lambda_3$, and $\psi_g(s,t) \leq \lambda_3 \Longrightarrow \psi_g(r,t) \leq \lambda_3$.

**Definition 8.** Suppose a universal set $\mathcal{S}$ and let $g \in \mathbb{SFS}(\mathcal{S} \times \mathcal{S})$ be any SF relation. The pair $(\mathcal{S},g)$ denotes a SF approximation space. Let $b$ be any subset of $\mathbb{SFS}(\mathcal{S})$ i.e. $b \subseteq \mathcal{F}(\mathcal{S})$. Then based on SF approximation space $(\mathcal{S},g)$, the upper and lower approximations of $b$ are denoted by $\tilde{g}(b)$ and $g(b)$ and are given as follows:

\[
\tilde{g}(b) = \{s \in \mathcal{S} \mid (s, \xi_g(b)(s), \eta_g(b)(s), \psi_g(b)(s)) \in g(b)\},
\]

\[
g(b) = \{s \in \mathcal{S} \mid (s, \xi_g(b)(s), \eta_g(b)(s), \psi_g(b)(s)) \in \tilde{g}(b)\},
\]

where

\[
\xi_g(b)(s) = \bigvee_{t \in \mathcal{S}} [\xi_g(t,s) \wedge \xi_g(t,t)],
\]

\[
\eta_g(b)(s) = \bigwedge_{t \in \mathcal{S}} [\eta_g(t,s) \vee \eta_g(t,t)],
\]

\[
\psi_g(b)(s) = \bigvee_{t \in \mathcal{S}} [\psi_g(t,s) \vee \psi_g(t,t)].
\]

Example 1. Consider a fixed set $\mathcal{S} = \{s_1, s_2, s_3, s_4, s_5\}$. Let $(\mathcal{S}, g)$ be an SF approximation space and let $g \in \mathbb{SFS}(\mathcal{S} \times \mathcal{S})$ be a SF relation which is given in Table 1.

Consider the professional experts presented the optimum decision normal object in the form of SFS, that is,

\[
b = \{\langle s_1, 0.8, 0.1, 0.4 \rangle, \langle s_2, 0.6, 0.3, 0.5 \rangle, \langle s_3, 0.9, 0.2, 0.3 \rangle, \langle s_4, 0.7, 0.3, 0.4 \rangle, \langle s_5, 0.4, 0.3, 0.5 \rangle\}.
\]
\begin{align*}
\xi_{\mathcal{G}(b)}(s_1) &= \bigvee_{t \in S} \left[ \xi_{\mathcal{G}}(s, t) \vee \xi_t(t) \right], \\
&= (0.6 \lor 0.8) \lor (0.7 \lor 0.6) \lor (0.6 \lor 0.9) \\
&= (0.7 \lor 0.7) \lor (0.6 \lor 0.4) = 0.9.
\end{align*}

\begin{align*}
\eta_{\mathcal{G}(b)}(s_1) &= \wedge_{t \in S} \left[ \eta_{\mathcal{G}}(s, t) \land \eta_t(t) \right] \\
&= (0.3 \land 0.1 \land (0.6 \land 0.3) \land (0.5 \land 0.2) \\
&\land (0.3 \land 0.3) \land (0.3 \land 0.3) = 0.1.
\end{align*}

\begin{align*}
\psi_{\mathcal{G}(b)}(s_1) &= \wedge_{t \in S} \left[ \psi_{\mathcal{G}}(s, t) \land \psi_t(t) \right] \\
&= (0.1 \land 0.4) \land (0.2 \land 0.5) \land (0.5 \land 0.3) \land (0.6 \land 0.4) \\
&\land (0.1 \land 0.5) = 0.1.
\end{align*}

(9)

\begin{align*}
(\xi_2, \eta_2, \psi_2) &= (0.9, 0.1, 0.2), (\xi_3, \eta_3, \psi_3) = (0.9, 0.1, 0.2), (\xi_4, \eta_4, \psi_4) = (0.9, 0.1, 0.2), \\
(\xi_5, \eta_5, \psi_5) = (0.9, 0.1, 0.3), \\
(\xi_6, \eta_6, \psi_6) = (0.4, 0.1, 0.7), \\
(\xi_7, \eta_7, \psi_7) = (0.4, 0.1, 0.5), (\xi_8, \eta_8, \psi_8) = (0.3, 0.1, 0.6), (\xi_9, \eta_9, s_0) = (0.4, 0.1, 0.6), (\xi_{10}, \eta_{10}, \psi_{10}) = (0.2, 0.1, 0.6), \\
\mathcal{G}(b) &= \{s_0, 0.9, 0.1, 0.1\}, (s_2, 0.9, 0.1, 0.2), (s_3, 0.9, 0.1, 0.2), (s_4, 0.9, 0.1, 0.2), (s_5, 0.9, 0.1, 0.3), \\
g(b) &= \{s_1, 0.4, 0.1, 0.7\}, (s_2, 0.4, 0.1, 0.5), (s_3, 0.3, 0.1, 0.6), (s_4, 0.4, 0.1, 0.6), (s_5, 0.2, 0.1, 0.6), \\
g(b) &= (\overline{g(b)}, \mathcal{G}(b)) = \begin{cases} s_1, (0.4, 0.1, 0.7), (0.9, 0.1, 0.1), s_2, (0.4, 0.1, 0.5), (0.9, 0.1, 0.2), s_3, (0.3, 0.1, 0.6), (0.9, 0.1, 0.2), \\
s_4, (0.4, 0.1, 0.6), (s_0, 0.9, 0.1, 0.2), s_5, (0.2, 0.1, 0.6), (0.9, 0.1, 0.3) \end{cases}.
\end{align*}

(10)

Definition 9. Assume two SFRNs \( g(b_1) = (g(b_1), \mathcal{G}(b_1)) \) and \( g(b_2) = (g(b_2), \mathcal{G}(b_2)) \). Then some basic operations on them are given as follows:

(i) \( g(b_1) \cup g(b_2) = (g(b_1) \cup g(b_2), \mathcal{G}(b_1) \cup \mathcal{G}(b_2)) \);

(ii) \( g(b_1) \cap g(b_2) = (g(b_1) \cap g(b_2), \mathcal{G}(b_1) \cap \mathcal{G}(b_2)) \);

(iii) \( g(b_1) \oplus g(b_2) = (g(b_1) \oplus g(b_2), \mathcal{G}(b_1) \oplus \mathcal{G}(b_2)) \);

(iv) \( g(b_1) \ominus g(b_2) = (g(b_1) \ominus g(b_2), \mathcal{G}(b_1) \ominus \mathcal{G}(b_2)) \);

(v) \( g(b_1) \subseteq g(b_2) = (g(b_1) \subseteq g(b_2), \mathcal{G}(b_1) \subseteq \mathcal{G}(b_2)) \);

(vi) \( \lambda g(b_1) = (\lambda g(b_1), \lambda \mathcal{G}(b_1)) \) for \( \lambda \geq 1 \);

(vii) \( g(b_1) \lambda = ((g(b_1)) \lambda, (\mathcal{G}(b_1)) \lambda) \) for \( \lambda \geq 1 \);

(viii) \( g(b_1) \gamma = (g(b_1) \gamma, \mathcal{G}(b_1) \gamma) \) where \( g(b_1) \gamma \) and \( \mathcal{G}(b_1) \gamma \) denote the complement of SFR rough approximation operators \( g(b_1) \) and \( \mathcal{G}(b_1) \), i.e., \( \mathcal{G}(b_1) \gamma = (\psi, \eta, \xi) \) and \( \mathcal{G}(b_1) \gamma = (\psi, \eta, \xi) \).

(ix) \( g(b_1) = g(b_2) \mathcal{G}(b_2) = g(b_2) \mathcal{G}(b_2) \mathcal{G}(b_2) = (\mathcal{G}(b_1) \mathcal{G}(b_2) \mathcal{G}(b_3) \mathcal{G}(b_4)) \).

Definition 10. Let the SFRN \( g(b) = (g(b), \mathcal{G}(b)) = ((\xi, \eta, \psi), (\xi, \eta, \psi)) \). Then the score function defined as:

\[ S(g(b)) = \frac{1}{6} (4 + \xi + \bar{\psi} - \eta - \bar{\eta} - \bar{\psi}) \], \( S(g(b)) \in [0, 1] \).

(11)
The accuracy of the SFRN $g(b) = (g(b), \overline{g}(b)) = ((\xi, \eta, \psi), (\xi, \eta, \overline{\psi}))$ for $g(b)$ is defined as:

$$h(g(b)) = \frac{1}{2}(\xi + \overline{\xi} - \psi - \overline{\psi}), \quad s(g(b)) \in [-1, 1]. \quad (12)$$

Consider two SFRNs $g(b_1) = (g(b_1), \overline{g}(b_1))$ and $g(b_2) = (g(b_2), \overline{g}(b_2))$, then

- (i) If $\overline{S}(g(b_1)) > \overline{S}(g(b_2))$, then $g(b_1) > R(b_2)$,
- (ii) If $\overline{S}(g(b_1)) < \overline{S}(g(b_2))$, then $g(b_1) < R(b_2)$,
- (iii) If $\overline{S}(g(b_1)) = \overline{S}(g(b_2))$, then $g(b_1) = g(b_2)$.

Proposition 1. Assume for any two SFRNs $g(b_1) = (g(b_1), \overline{g}(b_1))$ and $g(b_2) = (g(b_2), \overline{g}(b_2))$ with respect to $\overline{S}$F approximation space $(\overline{S}, \overline{g})$. Then the following properties are SFRNs.

- (i) $-g(b_1) = b_1$, where $-g(b_1)$ is the complement of $g(b_1)$;
- (ii) $g(b_1) \cup g(b_2) = g(b_1) \cup g(b_1)$ and $g(b_1) \cap g(b_2) = g(b_1) \cap g(b_1)$;
- (iii) $-g(b_1) \cup g(b_2) = (\sim g(b_1)) \cap (\sim g(b_2))$;
- (iv) $-g(b_1) \cap g(b_2) = (\sim g(b_1)) \cup (\sim g(b_2))$;
- (v) If $b_1 \leq b_2$, then $g(b_1) \leq g(b_2)$;
- (vi) If $b_1 \geq b_2$, then $g(b_1) \geq g(b_2)$;
- (vii) $g(b_1) \cap g(b_2) \leq g(b_1) \cap g(b_2)$.

4. Spherical Fuzzy Rough Averaging Aggregation Operators

Aggregation operators have the ability to aggregate the collective information of several professional experts into a single value. Here we will put forward the hybrid study of SFRS and averaging aggregation operators to get SFR averaging aggregation operators and presented their desirable properties.

4.1. Spherical Fuzzy Rough Weighted Averaging Aggregation Operators. This subsection is devoted for the study of SFRWA aggregation operators and presented the fundamental properties of SFRWA operators.

Definition 11. Let $g(b_i) = (g(b_i), \overline{g}(b_i)) \ (i = 1, 2, \ldots, n)$ be the collection of SFRNs. Assume $u = (u_1, u_2, \ldots, u_n)^T$ be the weight vectors with $\sum_{i=1}^{n} u_i = 1$ and $0 \leq u_i \leq 1$. Then the SFRWA aggregation operators are given as follows:

$$SFRWA(g(b_1), g(b_2), \ldots, g(b_n)) = (\sum_{i=1}^{n} u_i g(b_i), \sum_{i=1}^{n} u_i \overline{g}(b_i)). \quad (13)$$

The aggregation result for the above definition is described in Theorem 1.

Theorem 1. Consider $g(b_i) = (g(b_i), \overline{g}(b_i)) \ (i = 1, 2, \ldots, n)$ be the collections of SFRNs. Consider the weight vectors $u = (u_1, u_2, \ldots, u_n)^T$ with $\sum_{i=1}^{n} u_i = 1$ and $0 \leq u_i \leq 1$. Then the aggregation result for SFRWA operator is given as follows:

Proof. To get the required proof, we will use mathematical induction.

By using the operation law, we have

$$g(b_1) \oplus g(b_2) = (g(b_1) \oplus g(b_2), \overline{g}(b_1) \oplus \overline{g}(b_2))$$

$$\lambda g(b_1) = (\lambda g(b_1), \lambda \overline{g}(b_1))$$

$$\lambda g(b_1) = \left\{\left(1 - \frac{2}{\xi_1}\right)^{\lambda} \eta_1, \frac{1}{\xi_1}, \frac{1}{\psi_1} \right\}, \left(\sqrt{1 - \frac{2}{\xi_1} \frac{1}{\xi_1}}, \eta_1, \frac{1}{\psi_1} \right)$$

$$\lambda g(b_1) = \left\{\left(1 - \frac{2}{\xi_1}\right)^{\lambda} \eta_1, \frac{1}{\xi_1}, \frac{1}{\psi_1} \right\}, \left(\sqrt{1 - \frac{2}{\xi_1} \frac{1}{\xi_1}}, \eta_1, \frac{1}{\psi_1} \right)$$

$$\lambda g(b_1) = \left\{\left(1 - \frac{2}{\xi_1}\right)^{\lambda} \eta_1, \frac{1}{\xi_1}, \frac{1}{\psi_1} \right\}, \left(\sqrt{1 - \frac{2}{\xi_1} \frac{1}{\xi_1}}, \eta_1, \frac{1}{\psi_1} \right)$$
Suppose \( n = 2 \), then

\[
\text{SFRWA}(g(b_1), g(b_2)) = [\phi_{i=1}^n u_i g(b_i), \phi_{i=1}^n u_i \bar{g}(b_i)]
\]

\[
= \left\{ \left( \prod_{i=1}^n \left( 1 - \frac{2}{\xi_i} \right)^{u_i} \prod_{i=1}^n \eta_i \prod_{i=1}^n \psi_i \right), \left( \prod_{i=1}^n \left( 1 - \frac{2}{\xi_i} \right)^{u_i} \prod_{i=1}^n \eta_i \prod_{i=1}^n \psi_i \right) \right\}.
\]

(16)

Thus, the result holds for \( n = 2 \).

Now suppose that result is true for \( n = k \)

\[
\text{SFRWA}(g(b_1), g(b_2), \ldots, g(b_k)) = \left\{ \left( \prod_{i=1}^k \left( 1 - \frac{2}{\xi_i} \right)^{u_i} \prod_{i=1}^k \eta_i \prod_{i=1}^k \psi_i \right), \left( \prod_{i=1}^k \left( 1 - \frac{2}{\xi_i} \right)^{u_i} \prod_{i=1}^k \eta_i \prod_{i=1}^k \psi_i \right) \right\}.
\]

(17)

Let us show that the result holds for \( n = k + 1 \), thus we have

\[
\text{SFRWA}(g(b_1), g(b_2), \ldots, g(b_k), g(b_{k+1})) = \left\{ \left( \prod_{i=1}^{k+1} \left( 1 - \frac{2}{\xi_i} \right)^{u_i} \prod_{i=1}^{k+1} \eta_i \prod_{i=1}^{k+1} \psi_i \right), \left( \prod_{i=1}^{k+1} \left( 1 - \frac{2}{\xi_i} \right)^{u_i} \prod_{i=1}^{k+1} \eta_i \prod_{i=1}^{k+1} \psi_i \right) \right\}.
\]

(18)

This shows that the result is true for \( n = k + 1 \). Therefore, the result is hold for all \( n \geq 1 \).

Based on Theorem 1, \( g(b) \) and \( \bar{g}(b) \) are SFRNs. Therefore, \( \phi_{i=1}^n u_i g(b_i) \) and \( \phi_{i=1}^n u_i \bar{g}(b_i) \) are also SFRNs. Thus, it is clear that SFRWA (\( g(b_1), \ldots, g(b_n) \)) is also a SFRN based on SF approximation space \( (\mathcal{D}, g) \).

Some fundamental and desirable properties of SFRWA operator are presented in Theorem 2.

**Theorem 2.** Let \( g(b_i) = (g(b_i), \bar{g}(b_i)) (i = 1, 2, \ldots, n) \) be the collections of SFRNs with \( i = (i_1, i_2, \ldots, i_n)^T \) with \( \sum_{i=1}^n u_i = 1 \) and \( 0 \leq u_i \leq 1 \). Then SFRWA operator satisfies the following properties.

(i) **Idempotency:** if \( g(b_i) = g(\mathcal{D}) \) for all \( i = 1, 2, \ldots, n \) where \( g(\mathcal{D}) = (g(\mathcal{D}), \bar{g}(\mathcal{D})) = ((\tau, \xi), (\tau, \xi, \bar{\tau}), (\tau, \tau, \bar{\tau})) \), then SFRWA \((g(b_1), g(b_2), \ldots, g(b_n)) = g(\mathcal{D})\).

(ii) **Monotonicity:** consider that \( \mathcal{D}_1 \leq \mathcal{D}_2 \) and \( \mathcal{D}_2 \leq \mathcal{D}_3 \), then SFRWA \((g(b_1), g(b_2), \ldots, g(b_n)) \leq \mathcal{D}_3\).

(iii) **Boundedness:** let \( (g(b))^- = (\min \, g(b_i), \max \bar{g}(b_i)) \) and \( (g(b))^+ = (\min, g(b_i), \max \bar{g}(b_i)) \), then \( (g(b))^- \leq \text{SFRWA}(g(b_1), g(b_2), \ldots, g(b_n)) \leq (g(b))^+ \).

**Proof**

(i) **Idempotency:** as \( g(b_i) = g(\mathcal{D}) \) (for all \( i = 1, 2, \ldots, n \), where \( g(\mathcal{D}) = (g(\mathcal{D}), \bar{g}(\mathcal{D})) = ((\tau, \xi, \bar{\tau}), (\tau, \tau, \bar{\tau})) \))
For all \( i, (b_i) = g(\mathcal{L}) = (g(\mathcal{L}), \overline{g}(\mathcal{L})) = ((r_i, s_i, t_i), (\tau, \varsigma, \iota)) \). Therefore,

\[
((r_i, s_i, t_i), (\tau, \varsigma, \iota)) \overset{g, \overline{g}}{\longrightarrow} \left( \left( \prod_{i=1}^{k} (1 - \frac{2}{\tau_i})^{u_i} \right)^{1}, \left( \prod_{i=1}^{k} (1 - \frac{2}{\varsigma_i})^{u_i} \right)^{1} \right) \left( \prod_{i=1}^{k} (1 - \frac{2}{\iota_i})^{u_i} \right)^{1}
\]

\[
= \left( \left( \prod_{i=1}^{k} (1 - \frac{2}{\tau_i})^{u_i} \right)^{1}, \left( \prod_{i=1}^{k} (1 - \frac{2}{\varsigma_i})^{u_i} \right)^{1} \right) \left( \prod_{i=1}^{k} (1 - \frac{2}{\iota_i})^{u_i} \right)^{1}
\]

Hence,

\[
SFRWA(g(b_1), \ldots, g(b_n)) = g(\mathcal{L}).
\]

(ii) Monotonicity: since \( g(\mathcal{L}_i) = (g(\mathcal{L}_i), \overline{g}(\mathcal{L}_i)) = ((r_{i\tau}, s_{i\varsigma}, t_{i\iota}), (\tau, \varsigma, \iota)) \) and \( g(b_i) = (g(b_i), \overline{g}(b_i)) \) for \( i = 1, 2, \ldots, n \),

\[
g(\mathcal{L}_i) \leq g(b_i) \quad \text{and} \quad \overline{g}(\mathcal{L}_i) \leq \overline{g}(b_i).
\]

Therefore,

\[
SFRWA(g(\mathcal{L}_1), g(\mathcal{L}_2), \ldots, g(\mathcal{L}_n)) \leq SFRWA(g(b_1), g(b_2), \ldots, g(b_n)).
\]

(iii) Boundedness: proof is easy and follow from (i) and (ii).

4.2. Spherical Fuzzy Rough Ordered Weighted Averaging Aggregation Operators. This subsection is devoted for the study of SPROWA aggregation operators, which weigh the ordered position of the argument. Then we have presented the fundamental properties of SFROWA operators.

**Definition 12.** Let \( g(b_i) = (g(b_i), \overline{g}(b_i)) \) \( (i = 1, 2, \ldots, n) \) be the collection of SFRNs. Assume \( u = (u_1, u_2, \ldots, u_n)^T \) be the weight vectors with \( \sum_{i=1}^{n} u_i = 1 \) and \( 0 \leq u_i \leq 1 \). Then the SFROWA aggregation operators are given as follows:
The aggregation result for the above definition is described in Theorem 3.

\[
\text{SFRWA}(g(b_1), g(b_2), \ldots, g(b_n)) = \left( \mathbf{a}_{\nu}^{\text{SFRWA}}(b_1), \mathbf{a}_{\nu}^{\text{SFRWA}}(b_2), \mathbf{a}_{\nu}^{\text{SFRWA}}(b_3), \ldots, \mathbf{a}_{\nu}^{\text{SFRWA}}(b_n) \right)
\]

\[
\text{SFROWA}(g(b_1), g(b_2), \ldots, g(b_n)) = \left( \mathbf{a}_{\nu}^{\text{SFROWA}}(b_1), \mathbf{a}_{\nu}^{\text{SFROWA}}(b_2), \mathbf{a}_{\nu}^{\text{SFROWA}}(b_3), \ldots, \mathbf{a}_{\nu}^{\text{SFROWA}}(b_n) \right)
\]

where \( g(b_i) = (g_1(b_i), \bar{g}_1(b_i)) \) denotes the largest value of permutation from the collection \( g(b_i) \).

**Proof.** Proof is easy and followed from Theorem 1.

Some fundamental and desirable properties of SFRWOA operator are presented in Theorem 4.

**Theorem 4.** Let \( g(b_1) = (g(b_1), \bar{g}(b_1)) \) \((i = 1, 2, \ldots, n)\) be the collections of SFRNs with \( u = (u_1, u_2, \ldots, u_n)^T \) with \( \sum_{i=1}^{n} u_i = 1 \) and \( 0 \leq u_i \leq 1 \). Then SFROWA operator satisfies the following properties.

(i) **Idempotency:** \( g(b) = g(g(b)) \) for all \( b = (b_1, b_2, \ldots, b_n) \), \( g(b) = (g(b), \bar{g}(b)) = ((\xi, \zeta, \xi), (\gamma, \delta, \gamma)) \).

Then SFROWA \((g(b_1), g(b_2), \ldots, g(b_n)) = g(b)\).

(ii) **Monotonicity:** consider that \( g(b) = (g(b), \bar{g}(b)) \) \((i = 1, 2, \ldots, n)\) be another family of SFRVs with \( g(b_i) \leq g(b_i) \) and \( \bar{g}(b_i) \leq \bar{g}(b_i) \), then

\[
\text{SFROWA}(g(b_1), g(b_2), \ldots, g(b_n)) \leq \text{SFROWA}(g(b_1), g(b_2), \ldots, g(b_n)).
\]

(iii) **Boundedness:** \((g(b))^{-1} = (\min g(b), \bar{g}(b))\) and \((g(b))^+ = (\max g(b), \bar{g}(b))\), then \((g(b))^{-1} \leq \text{SFROWA}(g(b_1), g(b_2), \ldots, g(b_n)) \leq (g(b))^+\).

**Proof.** Proofs directly followed from Theorem 2.

\[
\text{SFRHA}(g(b_1), g(b_2), \ldots, g(b_n)) = \left( \mathbf{a}_{\nu}^{\text{SFRHA}}(b_1), \mathbf{a}_{\nu}^{\text{SFRHA}}(b_2), \mathbf{a}_{\nu}^{\text{SFRHA}}(b_3), \ldots, \mathbf{a}_{\nu}^{\text{SFRHA}}(b_n) \right)
\]

\[
\text{SFRHA}(g(b_1), g(b_2), \ldots, g(b_n)) = \left( \mathbf{a}_{\nu}^{\text{SFRHA}}(b_1), \mathbf{a}_{\nu}^{\text{SFRHA}}(b_2), \mathbf{a}_{\nu}^{\text{SFRHA}}(b_3), \ldots, \mathbf{a}_{\nu}^{\text{SFRHA}}(b_n) \right)
\]

\[
\text{SFRWA}(g(b_1), g(b_2), \ldots, g(b_n)) = \left( \mathbf{a}_{\nu}^{\text{SFRWA}}(b_1), \mathbf{a}_{\nu}^{\text{SFRWA}}(b_2), \mathbf{a}_{\nu}^{\text{SFRWA}}(b_3), \ldots, \mathbf{a}_{\nu}^{\text{SFRWA}}(b_n) \right)
\]

\[
\text{SFROWA}(g(b_1), g(b_2), \ldots, g(b_n)) = \left( \mathbf{a}_{\nu}^{\text{SFROWA}}(b_1), \mathbf{a}_{\nu}^{\text{SFROWA}}(b_2), \mathbf{a}_{\nu}^{\text{SFROWA}}(b_3), \ldots, \mathbf{a}_{\nu}^{\text{SFROWA}}(b_n) \right)
\]

\[
\text{SFRHA}(g(b_1), g(b_2), \ldots, g(b_n)) = \left( \mathbf{a}_{\nu}^{\text{SFRHA}}(b_1), \mathbf{a}_{\nu}^{\text{SFRHA}}(b_2), \mathbf{a}_{\nu}^{\text{SFRHA}}(b_3), \ldots, \mathbf{a}_{\nu}^{\text{SFRHA}}(b_n) \right)
\]
Theorem 6. Let \( g(b_i) = (g(b_i), \overline{g}(b_i)) \) be the collections of SFRNs with weight vector \( v = (v_1, v_2, \ldots, v_n)^T \) such that \( \sum_{i=1}^{n} v_i = 1 \) and \( 0 \leq v_i \leq 1 \). Consider \( u = (u_1, u_2, \ldots, u_n)^T \) be the associated weight vectors with \( \sum_{i=1}^{n} u_i = 1 \) and \( 0 \leq u_i \leq 1 \). Then SFRHA operator satisfies the following properties.

(i) Idempotency: if \( g(b_i) = g(L) \) for all \( i = 1, 2, \ldots, n \) where \( g(L) = (g(L), \overline{g}(L)) = ((r, s, \ell, \tau), (\sigma, \delta, \pi)) \), then \( SFRHA(g(b_1), g(b_2), \ldots, g(b_n)) = g(L) \).

(ii) Monotonicity: consider that \( g(L_i) = (g(L_i), \overline{g}(L_i)) \) for \( i = 1, 2, \ldots, n \) be another family of SFRVs with \( g(L_i) \leq g(b_i) \) and \( \overline{g}(L_i) \leq \overline{g}(b_i) \), then \( SFRHA(g(L_1), g(L_2), \ldots, g(L_n)) \leq SFRHA(g(b_1), g(b_2), \ldots, g(b_n)) \).

(iii) Boundedness: let \( (g(b))^+ = (\min g(b_i), \min \overline{g}(b_i)) \) and \( (g(b))^+ = (\max g(b_i), \max \overline{g}(b_i)) \), then \( (g(b))^+ \leq SFRHA(g(b_1), g(b_2), \ldots, g(b_n)) \leq (g(b))^+ \).

Proof.\footnote{Proofs directly followed from Theorem 2.}\label{proof6}

\section{5. Decision Support Algorithm}

Here, we have developed a framework for addressing uncertainty in decision making (DM) under spherical fuzzy rough information. Consider a DM problem with \( \{a_1, a_2, \ldots, a_n\} \) be a set of alternative and \( \{\tau_1, \tau_2, \ldots, \tau_h\} \) be a set of attributes with \( (\delta_1, \delta_2, \ldots, \delta_h) \) be the weight vector, such that each \( \delta_i \in [0,1], \sum_{i=1}^{h} \delta_i = 1 \). To test the reliability of kth alternative \( a_k \) under the rth attribute \( \tau_r \), let a set of decision makers (DMs) \( \{D_1, D_2, \ldots, D_j\} \) and \( (\eta_1, \eta_2, \ldots, \eta_j) \) be DMs weights such that \( \eta_j \in [0,1], \sum_{j=1}^{j} \eta_j = 1 \). The expert evaluation matrix is described as follows:

\[ M = \begin{bmatrix}
(g(b_{11}), \overline{g}(b_{11})) & (g(b_{12}), \overline{g}(b_{12})) & \cdots & (g(b_{1h}), \overline{g}(b_{1h})) \\
(g(b_{21}), \overline{g}(b_{21})) & (g(b_{22}), \overline{g}(b_{22})) & \cdots & (g(b_{2h}), \overline{g}(b_{2h})) \\
(\vdots & \vdots & \ddots & \vdots) \\
(g(b_{n1}), \overline{g}(b_{n1})) & (g(b_{n2}), \overline{g}(b_{n2})) & \cdots & (g(b_{nh}), \overline{g}(b_{nh}))
\end{bmatrix} \tag{33}
\]

where the pair \((g(b), \overline{g}(b))\) = \((\xi_{g(b)}, \eta_{\overline{g}(b)}, \psi_{g(b)}, \overline{\psi}_{\overline{g}(b)})\) such that \(0 \leq (\xi_{g(b)})^2 + (\eta_{\overline{g}(b)})^2 + (\psi_{g(b)})^2 + (\overline{\psi}_{\overline{g}(b)})^2 \leq 1\) and \(0 \leq (\xi_{\overline{g}(b)})^2 + (\eta_{g(b)})^2 + (\psi_{\overline{g}(b)})^2 + (\overline{\psi}_{g(b)})^2 \leq 1\).

Step 1: construct the experts evaluation matrices \( M^j \) as follows:

\[ M^j = \begin{bmatrix}
(g(b_{11}), \overline{g}(b_{11})) & (g(b_{12}), \overline{g}(b_{12})) & \cdots & (g(b_{1h}), \overline{g}(b_{1h})) \\
(g(b_{21}), \overline{g}(b_{21})) & (g(b_{22}), \overline{g}(b_{22})) & \cdots & (g(b_{2h}), \overline{g}(b_{2h})) \\
(\vdots & \vdots & \ddots & \vdots) \\
(g(b_{n1}), \overline{g}(b_{n1})) & (g(b_{n2}), \overline{g}(b_{n2})) & \cdots & (g(b_{nh}), \overline{g}(b_{nh}))
\end{bmatrix} \tag{34}
\]

where \( j \) represents the number of considered decision makers/experts.

Step 2: evaluate normalized experts’ matrices \( M^j \) that is

\[ M^j = \begin{bmatrix}
(g(b), \overline{g}(b)), & \text{for benefit attributes}, \\
(g(b)^p, \overline{g}(b)^p), & \text{for cost attributes}.
\end{bmatrix} \tag{35}
\]
5.2. Real Life Case Study. An ICS is a large class of automation systems used in manufacturing and industrial facilities to provide monitoring and control functions. Since more industrial control systems are connected to information technology (IT) networks and more IT technologies are implemented to industrial control systems to improve production efficiency, network security has emerged as a significant challenge. An ICS actually is the aggregate of a variety of system which is a vital part of national infrastructure such as electric power, transportation, energy, aviation and aerospace, and so on. To protect these systems from attack while maintaining normal operations, ICS-aware control is essential. More than 80% of vital infrastructure relevant to people’s livelihoods must depend on ICS to operate automatically. With the advancement of technology, ICS is increasingly facing more new threats like as viruses, Trojans, and hackers. The scale level of ICS security manufacturers is currently irregular. Many ICS security manufacturers also provide security systems that protect individual parts or groups of components, such as the industrial firewall, industrial gate, and so on. The difference between ICS and traditional information security is that ICS needs more necessary security rather than the impregnability of a single point of protection. To ensure comprehensive safety protection for ICS, an integrated safety system is required that can run throughout the entire manufacturing life cycle. Industrial control network security analysis and decision-making is an effective approach for resolving the problem because it can predict risks and help to make decisions before the fault of the industrial control network system. Therefore, it is essential for industrial enterprises to evaluate the entire capability of ICS security providers from various perspectives in order to select the best supplier finally. A company must now select one of four ICS security suppliers $\lambda_i$ ($i = 1, 2, 3, 4$) for better coordination. In addition, the invited decision makers are divided into three expert panels whose weighting vector is $\{0.19, 0.35, 0.46\}$ decided to evaluate the comprehensive capabilities of these four suppliers from the following four aspects:

1. $\mathfrak{A}_1$ is the safety prevention and control capacity.
2. $\mathfrak{A}_2$ is the product fit
3. $\mathfrak{A}_3$ is the abnormal response time
4. $\mathfrak{A}_4$ is future development capability of the enterprise

Expert Information = $\{(E)^1, (E)^2, (E)^3\}$, where each expert panel is required to provide unified evaluation results in the form of q-rung orthopair fuzzy rough values with unknown expert and criteria weight information.

Step 1. The expert evaluation matrices under spherical fuzzy rough values are enclosed in Tables 2–4:

Step 2. The normalized experts’ matrices $\mathcal{A}^{ij}$ are enclosed in Tables 5–7:

Step 3. The collective performance of the experts using spherical fuzzy rough weighted averaging aggregation operator is enclosed in Table 8:

Step 4. In this step, we calculate the aggregated preference values of each alternative in the revised expert ideal matrix using the proposed list of aggregation operators as follows:

Case 1. Using SFRWA aggregation operator: The aggregated preference values of each alternative using SFRWA aggregation operator are enclosed in Table 9:

Case 2. Using SFROWA aggregation operator: The aggregated preference values of each alternative using SFROWA aggregation operator is enclosed in Table 10:

Case 3. Using SFRHWA aggregation operator: The aggregated preference values of each alternative using SFRHWA aggregation operator is evaluated as follows:
### Table 2: Expert-1 information.

| $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ |
|-------------|-------------|-------------|-------------|
| $((0.84, 0.34, 0.40), (0.50, 0.80, 0.30))$ | $((0.43, 0.39, 0.78), (0.46, 0.23, 0.78))$ | $((0.67, 0.50, 0.30), (0.82, 0.12, 0.43))$ | $((0.31, 0.21, 0.71), (0.55, 0.21, 0.04))$ |

### Table 3: Expert-2 information.

| $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ |
|-------------|-------------|-------------|-------------|
| $((0.61, 0.15, 0.53), (0.59, 0.37, 0.40))$ | $((0.16, 0.35, 0.62), (0.45, 0.43, 0.61))$ | $((0.61, 0.35, 0.47), (0.42, 0.33, 0.49))$ | $((0.55, 0.17, 0.54), (0.29, 0.39, 0.83))$ |

### Table 4: Expert-3 information.

| $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ |
|-------------|-------------|-------------|-------------|
| $((0.85, 0.25, 0.15), (0.67, 0.50, 0.30))$ | $((0.14, 0.23, 0.88), (0.42, 0.33, 0.49))$ | $((0.78, 0.38, 0.18), (0.51, 0.48, 0.24))$ | $((0.29, 0.39, 0.83), (0.34, 0.21, 0.61))$ |

### Table 5: Normalized Expert-1 information.

| $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ |
|-------------|-------------|-------------|-------------|
| $((0.84, 0.34, 0.40), (0.50, 0.80, 0.30))$ | $((0.78, 0.39, 0.43), (0.51, 0.23, 0.46))$ | $((0.67, 0.50, 0.30), (0.82, 0.12, 0.43))$ | $((0.71, 0.21, 0.31), (0.63, 0.21, 0.55))$ |

### Table 6: Normalized Expert-2 information.

| $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ |
|-------------|-------------|-------------|-------------|
| $((0.61, 0.15, 0.53), (0.59, 0.37, 0.40))$ | $((0.62, 0.35, 0.16), (0.61, 0.43, 0.45))$ | $((0.61, 0.35, 0.47), (0.42, 0.33, 0.49))$ | $((0.74, 0.17, 0.55), (0.83, 0.39, 0.29))$ |
For SFRHWA operator, firstly the weighted collected matrix is enclosed in Table 11:

Respected score value is evaluated in Table 12:

Ordered weighted metric by using score value is enclosed in Table 13:

The aggregated preference values of each alternative using SFRHWA aggregation operator is evaluated in Table 14:

Step 5. Score of aggregated overall preference values of each alternative is enclosed in Table 15:

Step 6. Rank of the alternatives $\lambda_k (k = 1, 2, \ldots, 4)$ is enclosed in Table 15:

From the above computational process, we concluded that alternative $\lambda_3$ is the best among others, and therefore it is highly recommended.

The ranking results of the alternative using proposed aggregation operators are shown in Figure 1.

5.3. Reliability and Validity Test. In practice, selecting the perfect option from the group’s decision matrices is a challenging task. The approach for analyzing the reliability and validity of DM systems was developed by Wang and Triantaphyllou [52]. The testing procedure is as follows.

Test Step 1: the appropriate and effective MAGDM technique is to present the appropriate choice without modification and without changing the comparable position of each decision criterion by substituting the normalized element for the worse element of the alternative.

Test Step 2: the transitive property must be satisfied using an efficient and appropriate MAGDM approach.

Test Step 3: when a MAGDM problem is reduced to a minor one. A combined alternative rating should be similar to the original rating of the un-decomposed problem. To rank the alternative, we utilize the same
methods adopted in the MAGDM problem on minor issues.

The MAGDM problem was scaled down to achieve the best results, and the same suggested DM technique was applied. If the same procedure is applied to a small problem, the result will be the same as the MAGDM problem.

5.4. Validity Test (Proposed Numerical Illustration). The collected expert information in proposed case study is enclosed in Table 8:

Test Step 1: we replace the worst component of the alternative by presenting the effective choice without a modification and also without changing the

| Proposed operators | Score values of alternatives | Ranking |
|--------------------|------------------------------|---------|
| SFRWA              | 0.6878                       | 0.8063  | 0.8080 | 0.6779 | $\lambda_3 > \lambda_2 > \lambda_1 > \lambda_4$ |
| SFROWA             | 0.6827                       | 0.8020  | 0.8045 | 0.6759 | $\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4$ |
| SFRHWA             | 0.2915                       | 0.3777  | 0.3785 | 0.2892 | $\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4$ |
comparable position of each decision criterion. Table 16 contains the most recent expert decision matrix compilation.

Case 1: using SFRWA aggregation operator: the aggregated preference values of each alternative in the updated collected expert’s decision matrix using SFRWA aggregation operator are enclosed in Table 17:

Case 2: using SFROWA aggregation operator: the aggregated preference values of each alternative in the updated collected expert’s decision matrix using SFROWA aggregation operator are enclosed in Table 18:

![Graphical representation of alternatives using EWA, EOWA and EHWA operators.](image)

Figure 1: Graphical representation of alternatives using EWA, EOWA and EHWA operators.

Rank of the alternatives $\lambda_k$ ($k = 1, 2, \ldots, 4$) is enclosed in Table 19:

When we use Test Step 1, we get the same result $\lambda_3$, which is also the case when we use our established method.

The ranking results of the alternative in the validity test under proposed aggregation operators are shown in Figure 2.

Test Step 2 and 3: Steps 2 and 3 of the validity tests is now being tested to indicate that the presented method is reliable and valid. To accomplish this, we first divided the MAGDM problem into three smaller sub-problems, such as $[\lambda_1, \lambda_2, \lambda_3]$, $[\lambda_2, \lambda_1, \lambda_4]$, and $[\lambda_3, \lambda_1, \lambda_4]$. We now implement our presented DM approach to the smaller problems that have been converted and give us the ranking of alternatives as: $\lambda_1 > \lambda_3 > \lambda$, $\lambda_2 > \lambda_1 > \lambda_4$ and $\lambda_1 > \lambda_2 > \lambda_4$ respectively. We analyzed that $\lambda_1 > \lambda_2 > \lambda_4 > \lambda_4$ is the same as the standard decision-making approach results when assigning a detailed ranking.

6. TOPSIS Methodology

We present a strategy for dealing with uncertainty in DM under the presence of SFR information. Consider a DM problem with $[2_1, 2_2, \ldots, 2_n]$ be a set of alternatives and $[\tau_1, \tau_2, \ldots, \tau_n]$ be a set of attributes with $(\delta_{1t}, \delta_{2t}, \ldots, \delta_{nt})$ be the weight vector, such that each $\delta_t \in [0, 1]$, $\sum_{t=1}^{n} \delta_t = 1$. To test the reliability of $k$th alternative $\lambda_k$ under the $t$th attribute $\tau_t$, let a set of decision makers (DMs) $\{D_1, D_2, \ldots, D_n\}$ and $(\eta_1, \eta_2, \ldots, \eta_j)$ be DMs weights such that $\eta_j \in [0, 1]$, $\sum_{j=1}^{n} \eta_j = 1$. The expert evaluation matrix is described as follows:

$$M = \left[ g^i(b) \right]_{g \times h} = \left[ \begin{array}{cccc} (g(b_{11}), \overline{g}(b_{11})) & (g(b_{12}), \overline{g}(b_{12})) & \cdots & (g(b_{1h}), \overline{g}(b_{1h})) \\ (g(b_{21}), \overline{g}(b_{21})) & (g(b_{22}), \overline{g}(b_{22})) & \cdots & (g(b_{2h}), \overline{g}(b_{2h})) \\ \vdots & \vdots & \ddots & \vdots \\ (g(b_{g1}), \overline{g}(b_{g1})) & (g(b_{g2}), \overline{g}(b_{g2})) & \cdots & (g(b_{gh}), \overline{g}(b_{gh})) \end{array} \right]$$

(37)

where the pair $(g(b), \overline{g}(b)) = \{(\xi_{g(b)}(s), \eta_{g(b)}(s), \psi_{g(b)}(s)), (\overline{g}(b), \xi_{g(b)}(s), \eta_{g(b)}(s), \psi_{g(b)})\}$ such that $0 \leq (\xi_{g(b)}(s))^2 + (\eta_{g(b)}(s))^2 + (\psi_{g(b)}(s))^2 \leq 1$ and $0 \leq (\overline{g}(b))^2 + (\xi_{g(b)}(s))^2 + (\eta_{g(b)}(s))^2 + (\psi_{g(b)}(s))^2 \leq 1$.

Step 1: construct the experts evaluation matrices $M_i$.

$$M_i = \left[ \begin{array}{cccc} (g^i(b_{11}), \overline{g}^i(b_{11})) & (g^i(b_{12}), \overline{g}^i(b_{12})) & \cdots & (g^i(b_{1h}), \overline{g}^i(b_{1h})) \\ (g^i(b_{21}), \overline{g}^i(b_{21})) & (g^i(b_{22}), \overline{g}^i(b_{22})) & \cdots & (g^i(b_{2h}), \overline{g}^i(b_{2h})) \\ \vdots & \vdots & \ddots & \vdots \\ (g^i(b_{g1}), \overline{g}^i(b_{g1})) & (g^i(b_{g2}), \overline{g}^i(b_{g2})) & \cdots & (g^i(b_{gh}), \overline{g}^i(b_{gh})) \end{array} \right]$$

(38)
Table 16: Updated Expert’s collective matrix.

|  | $\mathcal{A}_1$ | $\mathcal{A}_2$ | $\mathcal{A}_3$ | $\mathcal{A}_4$ |
|---|---|---|---|---|
| $\lambda_1$ | $((0.78, 0.22, 0.31), (0.59, 0.52, 0.35))$ | $((0.20, 0.31, 0.78), (0.44, 0.32, 0.54))$ | $((0.69, 0.40, 0.29), (0.63, 0.27, 0.34))$ | $((0.37, 0.23, 0.76), (0.37, 0.26, 0.71))$ |
| $\lambda_2$ | $((0.80, 0.07, 0.27), (0.74, 0.26, 0.27))$ | $((0.34, 0.24, 0.67), (0.17, 0.22, 0.82))$ | $((0.81, 0.16, 0.22), (0.86, 0.11, 0.13))$ | $((0.09, 0.18, 0.91), (0.34, 0.29, 0.72))$ |
| $\lambda_3$ | $((0.22, 0.12, 0.81), (0.13, 0.33, 0.77))$ | $((0.89, 0.16, 0.09), (0.63, 0.37, 0.35))$ | $((0.22, 0.28, 0.74), (0.19, 0.15, 0.83))$ | $((0.67, 0.15, 0.39), (0.91, 0.11, 0.14))$ |
| $\lambda_4$ | $((0.27, 0.26, 0.70), (0.34, 0.32, 0.73))$ | $((0.53, 0.32, 0.39), (0.76, 0.42, 0.33))$ | $((0.43, 0.42, 0.61), (0.22, 0.32, 0.76))$ | $((0.57, 0.25, 0.42), (0.75, 0.20, 0.33))$ |

Table 17: Aggregated information using SFRWA.

| $\lambda_1$ | 
|---|---|---|---|---|
| $((0.5644, 0.2808, 0.5172), (0.5101, 0.3164, 0.4867))$ |
| $\lambda_2$ | 
| $((0.6151, 0.1582, 0.4798), (0.6302, 0.2109, 0.4175))$ |
| $\lambda_3$ | 
| $((0.6835, 0.1689, 0.3495), (0.7074, 0.2020, 0.3733))$ |
| $\lambda_4$ | 
| $((0.4882, 0.3034, 0.4943), (0.6385, 0.2977, 0.4649))$ |

Table 18: Aggregated information using SFROWA.

| $\lambda_1$ | 
|---|---|---|---|---|
| $((0.5465, 0.2793, 0.5392), (0.5422, 0.3439, 0.4644))$ |
| $\lambda_2$ | 
| $((0.6165, 0.1549, 0.4779), (0.6176, 0.2159, 0.4321))$ |
| $\lambda_3$ | 
| $((0.7474, 0.1663, 0.2724), (0.5809, 0.2315, 0.3918))$ |
| $\lambda_4$ | 
| $((0.4862, 0.3064, 0.4928), (0.6391, 0.3067, 0.4649))$ |

Table 19: Ranking of alternative.

| Proposed operators | Score values of alternatives | Ranking |
|---|---|---|---|
| SFRWA-test | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4$ |
| SFROWA-test | $0.5789$ | $0.6631$ | $0.7162$ | $0.5944$ | $\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4$ |

Figure 2: Graphical representation of alternatives using test EWA and EOWA operators.
where \( j \) represents the number of considered decision makers/experts.

Step 2: evaluate normalized experts’ matrices \( \mathcal{N}^j \) that is
\[
\mathcal{N}^j = \begin{cases} 
\left( g(b), \mathcal{G}(v) \right), & \text{for benefit attributes}, \\
\left( g(b)^*, \mathcal{G}(v)^* \right), & \text{for cost attributes}.
\end{cases}
\]

(39)

SFRWA\( (g(b_1), g(b_2), \ldots, g(b_n)) = \frac{\{\mu^r_{i=1} u_i, g(b_i), \mu^n_{i=1} u_i g(b_i)\}}{1 - \prod_{i=1}^{n} (1 - \xi_i^u)} \prod_{i=1}^{n} \eta_i^u \prod_{i=1}^{n} \psi_i^r}. \]

(40)

Step 4: determine the positive and negative ideal solutions \( I^+ \) and \( I^- \) for DM’s by using normalized decision matrix are defined as follows:
\[
I^+ = \left\{ N^{(j)}: \max_i \left[ SC\left(N^{(j)}_{ij}\right) \right], \ (j = 1, 2, \ldots, n), \right\}
\]
\[
I^- = \left\{ N^{(j)}: \min_i \left[ SC\left(N^{(j)}_{ij}\right) \right], \ (j = 1, 2, \ldots, n). \right\}
\]

(41)

\[
DI\ S^{(c)}_{N} = \frac{1}{4n} \left( \sum_{j=1}^{n} \left( \left( \mu_{DM}^{(N^+)} - \mu_{P^{+}IS^0}^{(N^+)} \right)^q \right) \right)
\]

\[
\left( \left( \mu_{DM}^{(N^-)} - \mu_{P^{-}IS^0}^{(N^-)} \right)^q \right) \right)^{1/q}
\]

(42)

and for each \( i = 1, 2, \ldots, m. \)

Step 6: the closeness indices denoted by \( (C^* I_i) \) are calculated as follows:
\[
C^* I_i = \frac{DIS^{(c)}_{N}}{DIS^{(c)}_{N} + DIS^{(c)}_{P^-}} \]

(43)

In ascending order, rank the measured \( (C^* I_i) \) values; the alternative with the greatest value is our best choice.

### 6.1. Numerical Illustration

Step 1: the expert evaluation matrices under spherical fuzzy rough values are enclosed in Tables 2–4 which is given in real life case study.

Step 2: the normalized experts’ matrices \( \mathcal{N}^j \) is enclosed in Tables 5–7 which is given in real life case study.

Step 3: compute the collective performance of the experts using spherical fuzzy rough weighted averaging aggregation operator.

Step 4: by using equation (14) the WGDM from \( DM \ (N^+)^{0}) \) to \( P^+ IS^0 \) and \( N^+ IS^0 \), are computed as follows:

Step 5: by using equation (14) the WGDM from \( DM \ (N^+)^{0}) \) to \( P^+ IS^0 \) and \( N^+ IS^0 \), are computed as follows:

\[
\begin{array}{cccc}
0.3327 & 0.2680 & 0.0302 & 0.3501 \\
0.2256 & 0.3442 & 0.3490 & 0.0651
\end{array}
\]

Step 6: the closeness indices denoted by \( (C^* I_i) \) are calculated in Table 22.

Ranking results using SFR-TOPSIS methodology is shown in Figure 3:
7. Conclusion

The application of the Internet in industry has broken down the conventional industry's heavily regulated industrial environment, allowing for intelligent manufacturing. Utilizing information technologies to improve conventional industrial organizations has been an essential trend in industrial enterprise development. Simultaneously, it implies that industrial firms would face new safety challenges. Since enterprise is more closely linked to the national economy and people's livelihoods, the authorities and the public focus primarily on ICS safety. In this paper, a new approach of spherical fuzzy rough aggregation operators has been developed to successfully resolve the issue of ICS security supplier selection. To demonstrate the applicability and rationality of the suggested approach, different tests based on the aforementioned operators have been performed. According to the experimental findings, using the proposed technique to evaluate the ICS supplier is reasonable and acceptable. An addition, the comparison analysis performed by the developed models with the SFR-TOPSIS technique validates the capability, superiority, and reliability of the presented techniques.

In the future, we will aim to use the developed approach in more domains, as well as investigate advanced theories and algorithms in diverse fields.

### Data Availability

The data used in the manuscript are hypothetical and can be used by anyone by just citing this article.

### Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.
Authors’ Contributions

All authors have contributed equally to this article.

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