Research Article

Image Processing Based on Fuzzy Mathematics Theory

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In order to further improve the problems of poor rationality and weak antinoise ability of existing image processing algorithms and technical algorithms, an image processing research method based on fuzzy mathematical theory is proposed. First, aiming at the ill-posed problem of the PFCM algorithm, the neutrality and rejection degree are used to construct a regular term and embed the algorithm objective function to enhance the correlation between the attribute parameters of the fuzzy set of the sample graph, so as to solve the ill-posed problem of the PFCM algorithm. Secondly, in view of the same noise sensitivity problem of PFCM algorithm as a traditional fuzzy clustering algorithm, combined with the robust ideas of FCM_S1 and FCM_S2 algorithms, the objective function of robust segmentation algorithm for graph fuzzy clustering (RPFCM_s) is constructed. The misclassification rate of the clustering algorithm proposed in this study in image segmentation is reduced by 38%–76%, and the misclassification rate of the corresponding segmentation result of the ATPFCA algorithm is reduced by 5%–77%. Therefore, the algorithm not only improves the effective segmentation efficiency of the fuzzy mathematical theory algorithm for the processing of uneven grayscale images but also enhances the anti-noise robustness of the algorithm.

1. Introduction

Since it was put forward in 1965, fuzzy mathematics theory has played a key role in studying and dealing with fuzzy phenomena. The basic concept of fuzzy mathematics theory is a fuzzy set. In recent years, scholars have deepened their research on fuzzy mathematics theory, and fuzzy mathematics theory has been widely used in many industries and fields. Specifically, fuzzy mathematics theory accurately defines fuzzy phenomena through the in-depth analysis and research method application of fuzzy phenomena. In fact, there are many fuzzy phenomena in our work and life. The help of fuzzy mathematics theory can help us analyze and solve these problems. Computer image processing technology is to better recognize and calculate the image with the help of computer technology. The purpose of the integration of the two is to further improve the accuracy and accuracy of image processing [1]. Figure 1 shows an image fusion method based on fuzzy mathematics theory.

2. Literature Review

Image segmentation is an important pre-processing step of pattern recognition, machine vision, image understanding, and analysis. After decades of research, it still receives wide attention from many scholars [2]. There are currently many methods for image segmentation, which can be broadly divided into four types: threshold-based segmentation methods, edge detection-based segmentation methods, region-based segmentation methods, and theory-based segmentation methods. In recent 10 years, with the proposal of new theories in various disciplines, image segmentation algorithms based on these theories are also emerging, and the theoretical system of image segmentation algorithms is being further developed. The image segmentation algorithm based on edge detection will focus on the edge part of the image. The boundaries between different homogeneous regions in the image have unique properties, and the gray value will jump along the direction perpendicular to the edge. Using this characteristic, the jumping pixel points in
the image to be segmented can be detected, and the set of these points is the boundary to distinguish different homogeneous targets and backgrounds. The method of edge detection is to use the difference operator of gray mutation in the pixel neighborhood and the change of the first or second derivative of the edge to judge whether the sample points are boundary pixels and finally realize image segmentation. The theoretical system of image segmentation algorithms is being further developed. Classical edge detection operators include the Roberts operator, Prewitt operator, Sobel operator, Laplacian operator, Canny operator, and so on. Since this kind of algorithm is based on the principle of gray value jump in pixel neighborhood, this kind of algorithm is more sensitive to noise, especially the second derivative. Therefore, for noisy images, false edges may appear, resulting in unreasonable boundary contour. If the noise resistance of the algorithm is improved, the actual edge area will be weakened, resulting in edge missing detection. In both cases, the detected edge will deviate from the actual edge, resulting in unreasonable segmentation results [3–5].

For the image itself, the pixels in the same region have the same or similar features, the feature distribution in different regions is inconsistent, and the pixel features between different regions are abrupt and uncertain. At the same time, the influence of some external factors on the drawing process also affects the drawing, such as angle of view, illumination, shadow, spatial position, and so on, the target area and background area have a certain similarity, and the boundary between the target and background is uncertain. These pseudo similarity and uncertainty can be called fuzziness, which makes image segmentation become a typical ill-structured problem, resulting in that most clustering segmentation methods are only suitable for one or several types of image segmentation [6]. In order to obtain more satisfactory segmentation results and make full use of the uncertainty and fuzziness of image pixels, fuzzy clustering was once pushed to the forefront of the research of image clustering and segmentation algorithm, the use of dimension clustering technology to address image segmentation is a new trend and is also one of the hot topics in the image segmentation industry.

Based on this research, this study proposes image processing research based on fuzzy mathematical theory. By closely correlating neutrality and rejection degree, the correlation between various attribute parameters is improved, and the misclassification and misclassification of the original graph fuzzy clustering algorithm due to ill-posed problems are solved. According to the characteristics of the proposed algorithm’s objective function, the convergence of the algorithm is deduced and proved by the idea of divide and conquer, and the convergence of the algorithm’s objective function is verified by using standard grayscale images and noise interference images. The test results show that the improved algorithm RPFCM has faster convergence performance than FCM, IFCM, FC-PFS and PFCM, and other algorithms; the RPFCM algorithm has better segmentation performance than the existing PFCM algorithm.

3. 3D Model Construction Based on Image Processing

In recent decades, the problem of restoring structure 3D models from multiple two-dimensional plane images has been widely and deeply studied. In particular, the 3D scene structure can be restored and reconstructed (structure from motion, SFM) during camera movement, so that feature points can be detected from uncalibrated images or image sequences, feature point matching can be performed, and the 3D structure model of the scene can be established. The basic idea is shown in Figure 2.

In machine learning and image processing, data is usually represented as vectors. The complexity of many machine learning algorithms is exponentially related to the dimensional nature of the data. The larger the dimension, the greater the complexity of data processing and the more time it takes. In order to reduce time consumption and avoid the waste of machine resources, data dimensionality must be reduced. Dimensionality reduction of data means the loss of data, but the actual data are often related. On this basis, the loss of data can be reduced during dimensionality reduction. Basic Component Analysis (PCA) is a widely used method of reducing the size of data based on solid data. The main idea of PCA is to reduce the n-dimensional nature of the data to k-scale (k < n). In this way, the purpose of data reduction and feature extraction is realized [7–9]. The specific process of the PCA algorithm is as follows: suppose there is a data set \( S \), including \( m \) samples, and the dimension of each sample is \( n \), that is:

\[
S = (x_1, x_2, x_3, x_4, \ldots, x_m),
\]

\[
x_j = (x_{1j}, x_{2j}, x_{3j}, x_{4j}, \ldots, x_{mj}).
\]

The data set \( S \) is represented in the form of a matrix, and then each row of the matrix is a sample, and each column is a dimension, that is:

![Image Processing Diagram](Image A)

**Figure 1:** An image fusion method based on fuzzy mathematics theory.
Zero the average value of each row of $S$, that is, subtract the average value of this row:

$$\bar{x}_i = x_i - \bar{x},$$

$$\bar{x} = \frac{1}{m} \sum_{i=1}^{m} x_i.$$  \hspace{1cm} (3)

The matrix consisting of $\bar{x}_i$ is recorded as:

$$\bar{S} \in \mathbb{R}^{m \times n}.$$  \hspace{1cm} (4)

Then calculate the covariance matrix $C$ of matrix $\bar{S}$:

$$C = \frac{1}{m} \bar{S}^T \bar{S}$$

$$= \frac{1}{m} \sum_{i=1}^{m} x_i^T \bar{x}_i,$$  \hspace{1cm} (5)

where $C \in \mathbb{R}^{m \times n}$ calculate the eigenvalues of the covariance matrix $C$ and the corresponding eigenvectors, place the eigenvectors as a matrix from top to bottom according to their respective eigenvalues, and take the first $k$-line to form a $P$-matrix. PCA matrix. Then, after reducing the size, the matrix is:

$$\bar{z} = \bar{z}p.$$  \hspace{1cm} (6)

In summary, PCA, by its very nature, takes the direction of greatest variability and “correlates” the data in each orthogonal direction, i.e., independent of the different orthogonal directions, and can be used well for image compression according to PCA characteristics.

In order to apply the PCA algorithm to 3D model reconstruction and process images faster and better, it is necessary to deform the PCA algorithm or segment large-resolution images [10]. From the singular value decomposition, the PCA algorithm is a variant of the singular value decomposition algorithm (SVD). The singular value decomposition algorithm can decompose any matrix. The formula is as follows:

$$A = U \sum V^T.$$  \hspace{1cm} (7)

$A$ is an $m \times n$ matrix, $U$ is an $m \times m$ matrix, $V$ is an $n \times n$ matrix, and $\sum$ is an $m \times n$ matrix. Where $u$ and $V$ are the left singular vector and right singular vector of $A$, respectively, and $\sum$ is the singular value of $A$. Multiply both ends of the equal sign of formula (7) by $U^T$ to obtain:

$$U^T A = U^T U \sum V^T.$$  \hspace{1cm} (8)

According to the above requirements and steps of algorithm time, we use the PCA algorithm to compress the concrete test block. The resolution of the image of the test block is $768 \times 1024$. The flow of compression processing is as follows.

Step 1. Decompose the image into $n$ sub-images, and the size of each sub-image is $K$. With the different value of $K$, the number $n$ of sub-images changes accordingly. In the compression experiment, the $K$ value is arranged from small to large, and the values are $K = 2$, $K = 3$ and $K = 4$, respectively. The number of corresponding sub-images $n$ is: $N = 196608, N = 87495, N = 49216$

Step 2. Calculate the average value of $N$ sub-images:

$$\text{IMG} = \frac{\text{IMG}_1 + \text{IMG}_2 + \cdots + \text{IMG}_N}{N}.$$  \hspace{1cm} (9)

The above formula represents the average of $N$ sub-images, then subtracts the average of the images from these $N$ sub-images and takes $N$ new sub-images.

Step 3. Calculate the covariance matrix $R$ of the new sub-image, which is defined as:

$$R(i, j, t, l) = \frac{1}{N} \sum_{S=1}^{N} \text{IMG}_S(i, j) \text{IMG}_S(t, l).$$  \hspace{1cm} (10)

In the above formula, $i, j, t, l$, respectively, represent the positions of elements in the image.

Step 4. Calculate the eigenvalue and eigenvector of $R$, and calculate the inner product and maximum eigenvector of each sub-image to obtain the compressed image [11, 12].

In order to further verify that the image compression will not affect the image modeling effect, it is necessary to scale the model. And it is compared with the conversion results of the original picture model. After the dense point cloud reconstruction, the reconstructed model is scaled according to the actual size of the shear wall structure to obtain the transformation relationship. After reconstruction, verify the size of the model through the pre-calibration points on the shear wall. In order to further prove the feasibility and applicability of the algorithm in practical engineering, this chapter selects a rainbow bridge as an engineering example for modeling. The original Rainbow Bridge image modeling time is 118398.108 s, about 32 hours, 53 minutes, and 18 seconds. The modeling time of the picture compressed by the...
PCA algorithm is 41057.931 s, about 11 hours, 23 minutes, and 17 seconds. In order to make the comparison result more intuitive, this example only intercepts the picture of the rainbow bridge model and does not add the reconstructed useless information to the comparison. It can be seen from the time comparison that this method is still applicable when the target is a large structure, which can greatly reduce the modeling time [13, 14].

4. Graph Fuzzy Clustering and Robust Segmentation Algorithm

4.1. Regularized Graph Fuzzy Clustering Algorithm. The objective function of the graph fuzzy clustering (PFCM) algorithm can be described as:

$$\min f_m(\mu, \eta, \xi, V) = \sum_{k=1}^{c} \sum_{i=1}^{n} \left( \frac{\mu_{ik}}{1 - \eta_{ik} - \xi_{ik}} \right)^{m} \|x_i - v_k\|^2 + \sum_{k=1}^{c} \sum_{i=1}^{n} \eta_{ik} \left( \log \eta_{ik} + \xi_{ik} \right).$$  (11)

The expression of the optimization objective function corresponding to the improved graph fuzzy clustering algorithm is:

$$\min f_m(\mu, \eta, \xi, V) = \sum_{k=1}^{c} \sum_{i=1}^{n} \left( \frac{\mu_{ik}}{1 - \eta_{ik} - \xi_{ik}} \right)^{m} \|x_i - v_k\|^2 + \sum_{k=1}^{c} \sum_{i=1}^{n} \eta_{ik}^{\alpha} \xi_{ik} + \eta_{ik} \xi_{ik}^{\beta}. $$  (12)

The constraints are as follows:

$$\begin{align*}
0 & \leq \mu_{ik}, \xi_{ik}, \eta_{ik} \leq 1, 0 \leq \mu_{ik} + \xi_{ik} + \eta_{ik} \leq 1, & i = 1, 2, \ldots, n, k = 1, 2, \ldots, c, \\
\sum_{k=1}^{c} \frac{\mu_{ik}}{1 - \eta_{ik} - \xi_{ik}} &= 1, & i = 1, 2, \ldots, n, \\
\sum_{k=1}^{c} (\eta_{ik} - \frac{1}{c} \xi_{ik}) &= 1, & i = 1, 2, \ldots, n,
\end{align*}$$  (13)

where $n$ represents the number of clustering samples; $c$ represents the number of cluster categories; $\mu_{ik}, \eta_{ik}, \xi_{ik}$ represent the membership degree, neutrality degree, and rejection degree of the $i$th sample belonging to class $k$, respectively; $v_k$ represents the clustering center of class $k$; $m$ represents the fuzzy index.

The objective function of unconstrained optimization is obtained by the Lagrange multiplier method, and its expression is:

$$L = \sum_{k=1}^{c} \sum_{i=1}^{n} \left( \frac{\mu_{ik}}{1 - \eta_{ik} - \xi_{ik}} \right)^{m} \|x_i - v_k\|^2 + \sum_{k=1}^{c} \sum_{i=1}^{n} \eta_{ik}^{\alpha} \xi_{ik} + \eta_{ik} \xi_{ik}^{\beta} + \sum_{i=1}^{n} k_i \left( 1 - \sum_{k=1}^{c} \frac{\mu_{ik}}{1 - \eta_{ik} - \xi_{ik}} \right)^{m} + \sum_{i=1}^{n} \beta_i \left( 1 - \sum_{k=1}^{c} (\eta_{ik} + \frac{1}{c} \xi_{ik}) \right).$$  (14)

The expression of the rejection index can be obtained by using the Yager complement operator (combined with constraints) as follows:

$$\xi_{ik} = 1 - (\mu_{ik} + \eta_{ik}) - (1 - (\mu_{ik} + \eta_{ik})^{m})^{(1/\alpha)}.$$  (15)
4.2. Experimental Simulation and Result Analysis. In order to verify the effectiveness and anti-noise of the improved regularized graph fuzzy clustering robust segmentation algorithm, the segmentation tests are carried out on different images and images with different intensity types of noise. In order to arrange the layout neatly, images of different sizes are displayed as the same size, and the layout of other chapters is also the same. The test running environment is Matlab 2014a. Set the test parameter fuzzy index \( m = 2.0 \), iteration termination error \( \varepsilon = 1 \times 10^{-4} \), and maximum iteration times \( t_{\text{max}} = 10^3 \).

4.2.1. Segmentation Test and Result Analysis of Noise-Free Image. Three gray images of brain CT image, coin image, and water cup image with the size of 256 \( \times \) 256 are selected. FCM algorithm, IFCM algorithm, FC-PFS algorithm, PFCM algorithm, PRFCM algorithm, and PFCM_s algorithm are used for the segmentation test, respectively. The corresponding error rates as a result of the segmentation of the various algorithms are shown in Table 1.

From the above results, the segmentation results obtained by the FCM algorithm, IFCM algorithm, FC-PFS algorithm, and RPFCM algorithm cannot store image details, but only a rough outline of the image can be obtained, but the PFCM algorithm can store image details but image sound contains noisy dots. From the incorrect classification of the results of the segmentation of the six algorithms shown in Table 1, it can be seen that the level of incorrect classification of the PRFCM_s algorithm is lower than that of the other five algorithms. The results of brain CT and water cup map were particularly obvious, and the misclassification rate decreased by 0.42%–25.66%; For the more complex water cup graph, the misclassification rate of the PRFCM_s algorithm is also reduced. Therefore, in contrast, the regularized graph fuzzy clustering robust segmentation algorithm has certain segmentation advantages [15, 16].

4.2.2. Segmentation Test and Result Analysis of Gaussian Noise Interference Image. The 256 \( \times \) 256 brain CT images, gear images, and photographer images were selected as the raw image data for the anti-Gaussian noise performance test of the RPFCM_s algorithm. Using the FCM algorithm, the IFCM algorithm, the FC-PFS algorithm, the PFCM, and the PFCM algorithm, add the Gaussian noise with an average value of 0, the variance 57, 81, 81 (normalized variance of 0.05, 0.1, and 0.1) on the three images. The RPFCM_s algorithms are used for segmentation experiments, and the values of the peak signal-to-noise ratios corresponding to the segmentation results obtained by different algorithms are shown in Table 2.

Out of the segmentation results obtained by the six segmentation algorithms, the segmentation results obtained by the FCM algorithm, IFCM algorithm, FC-PFS algorithm, PFCM algorithm, and RPFCM algorithm all contain more noisy points, and the details are blurred; Contains the results of segmentation obtained by the RPFCM_s algorithm. The number of noise points is relatively rare, and the segmentation effect is better than the other five algorithms. The peak signal and noise ratios of the anti-Gaussian noise shown in Table 2 show that the FCM, IFCM, FC-PFS, PFCM, and RPFCM algorithms are not significantly different as a result of segmentation, but the peak signal-to-noise ratio is a segmentation of the RPFCM_s algorithm. agrees. The value of the noise ratio is much larger than the first five algorithms. The improved algorithm of RPFCM_s shows that it is strong against Gaussian noise interference [17].

4.2.3. Segmental Testing and Analysis of the Results of Salt and Pepper Noise Interference. The RPFCM_s algorithm selected a 256 \( \times \) 256 square shape, a gear shape, and a pepper shape as the initial drawing data for the salt and pepper anti-noise performance test. The three images were added with a salt and pepper noise of 0.2 probability, and the FCM algorithm, IFCM algorithm, FC-PFS algorithm, PFCM algorithm, RPFCM, and RPFCM_s algorithms were used for segmentation testing. The values of the ratios obtained as a result of the segmentation obtained by the various algorithms are shown in Table 3.

From the results of the above segmentation, it can be seen that the segmentation results of the three images obtained by the RPFCM_s algorithm are more detailed and complete, with only a small amount of noise, while the segmentation results obtained by the other five algorithms are blurred, and a lot of noise. In addition, the peak signal-to-noise ratios of the six algorithms shown in Table 3 are significantly higher than the values of the corresponding peak-to-noise ratios obtained by the RPFCM_s enhanced algorithm. from the peak signal-to-noise ratio corresponding to the other five algorithms. Therefore, the RPFCM_s algorithm has a strong ability to resist salt and pepper noise [18].

4.2.4. Segmentation Test and Result Analysis of Mixed Noise Interference Image. The cell image, pepper image, and gear image with the size of 256 \( \times \) 256 were selected as RPFCM_s algorithm anti mixed noise performance test of the original image data. Add Gaussian noise with the mean value of 0, the variance of 0.05, and salt and pepper noise with a probability of 0.1 to the cell diagram and pepper diagram, add Gaussian noise with the mean value of 0, the variance of 0.1, and salt and pepper noise with the probability of 0.1 to the gear diagram, and use FCM algorithm, IFCM algorithm, fc-pfs algorithm, PFMC algorithm, rpfc and rpfc_m algorithm for segmentation test. The peak signal-to-noise ratio corresponding to the segmentation results obtained by different algorithms is shown in Table 4.

From the results obtained by the six segmentation algorithms, the segmentation results obtained by the FCM algorithm, the IFCM algorithm, the FC-PFS algorithm, the PFCM algorithm, and the RPFCM algorithm all contain a large number of noise points. Reduced and more detailed. Compared to the other five algorithms, the RPFCM_s algorithm has better segmentation performance. In addition, the peak values of the signal-to-noise ratios corresponding to the segmentation results of each algorithm shown in Table 4 can be seen. The RPFCM_s algorithm is much larger than
the other five algorithms, which effectively demonstrates that RPFCM_s has a strong resistance to mixed noise interference. Add neighborhood means information to traditional FCM algorithm, IFCM algorithm, FC-PFS algorithm, and PFCM algorithm, respectively, to obtain FCM_s algorithm, IFCM_s algorithm, FC-PFS_s, and FC-PFS_s algorithm is combined with the idea of fuzzy c-means robust clustering algorithm to obtain FC-PFS_s algorithm and FC-PFS_s algorithm. Test the robustness of different gray images and compare them with the PRFCM_s algorithm [19].

| Noisy image | FCM algorithm | IFCM algorithm | FC-PFS algorithm | PFCM algorithm | PRFCM algorithm | PFCM_S algorithm |
|-------------|---------------|----------------|------------------|----------------|-----------------|------------------|
| Brain CT image | 13.57 | 15.85 | 13.57 | 2.76 | 13.57 | 2.34 |
| Coin image | 5.01 | 6.54 | 5.01 | 5.33 | 5.01 | 3.71 |
| Cup image | 18.61 | 26.31 | 18.61 | 12.46 | 18.61 | 1.64 |

4.2.5. Test and Analysis of Robust Segmentation of Image Disturbed by Gaussian Noise. A basketball court remote sensing image, a river remote sensing image, and a 256 × 256 brain CT image were selected, with a mean value of 0 and a variance of 44, 81, and 81 (normalized variance 0.03, 0.1, and 0.1, respectively) of Gaussian noise. The results of the segmentation experiment are shown in Table 5.

From the above results, the IFCM_s algorithm has the worst segmentation result and serious noise point pollution. Among them, the segmentation result of brain CT image shows that the IFCM_s algorithm blurs the details; FCM_s algorithm, FC-PFS_s algorithm, and PFCM_s algorithm have the same segmentation results; and the segmentation results obtained by the RPFCM_s algorithm are much better than the IFCM_s algorithm and better than the other three algorithms. From the values of the peak signal-to-noise ratios corresponding to the five segmentation algorithms shown in Table 5, the values of the peak signal-to-noise ratios corresponding to the segmentation results of the IFCM_s algorithm are the lowest; As a result of the segmentation of the FC-PFS_s algorithm, the values of the corresponding peak signal and noise ratios are compared. The FCM_s algorithm is slightly larger; As a result of the segmentation of the PFCM_s algorithm, the values of the corresponding peak signal-noise ratios are those of the FCM_s algorithm and the FC-PFS_s algorithm; As a result of the segmentation of the RPFCM_s algorithm, the value of the corresponding peak signal-to-noise ratio is relatively high, indicating that the improved RPFCM_s algorithm performs better noise interference [20].

4.2.6. Test and Analysis of Robust Segmentation of Salt and Pepper Noise Interference Image. Three images of the Chinese character Shu map, river remote sensing map, and photographer map with the size of 256 × 256 are selected to segment salt and pepper noise with the additional probability of 0.1, 0.15, and 0.15, respectively. The results are shown in Table 6.

From the peak values of the signal-to-noise ratio corresponding to the six segmentation algorithms shown in Table 6, it can be seen that the peak signal-to-noise ratio of the RPFCM_s enhanced algorithm is much higher than that of the IFCM_s and PFCM_s algorithms, FCM_s and FC-PFS_s, which further shows that PRFCM_s algorithm has strong anti-salt and pepper noise interference ability.
4.2.7. Test and Analysis of Robust Image Segmentation with Mixed Noise Interference. Three grayscale images of a river remote sensing map, a Chinese character and a block map with the size of 256 \times 256 are selected. Add 0 averages for remote sensing maps and Chinese characters, Gaussian noise with 57 variances (normalized variance 0.05), salt and pepper noise with a probability of 0.1, and add an average of 0 to the variable block map. 72 (0.08 normalized variance) Gaussian noise and salt and pepper noise with a probability of 0.1, the results are shown in Table 7.

For remote sensing image and Chinese character space image, IFCM_s algorithm has the worst segmentation results, and there is little difference among FCM_s algorithm, FC-PFS_s algorithm, PFCM_s algorithm and improved algorithm PRFCM_s. For the block diagram, the segmentation results of the five algorithms are equivalent. As a result of the segmentation shown in Table 7, the peak signal-to-noise ratio of the corresponding PPFCM_s algorithm shows that the peak signal-to-noise ratio is higher than the other four algorithms. The improved algorithm has a relatively strong ability to counteract mixed noise interference. To test the integrity of the enhanced RPFCM algorithm, select a barrel map (size 215 \times 300) and a photographer map (size 256 \times 256) and add Gaussian noise, salt and pepper noise, Gaussian and salt and pepper mixed noise, respectively. The PRFCM algorithm is used for clustering and segmentation test. In order to better observe the convergence curve of the algorithm, the parameters $\alpha$ and $\omega$ are 0.38 and 2 respectively. The results are shown in Figure 3.

As shown in Figure 3(a)–3(d), the clustering objective function obtained by the regularized graph fuzzy clustering algorithm for segmenting the original image and the image interfered by Gaussian, salt and pepper and mixed noise decreases monotonically and tends to be stable with the change of iteration times $T$. Indicating that the clustering model is reasonable and the algorithm is convergent.

In order to study the convergence speed of PRFCM algorithm, FCM, IFCM, FC-PFS, PFCM, PRFCM, and PRFCM_s algorithms are used to test the convergence of rice grain map (size 256 \times 256), photographer map (size 256 \times 256), river remote sensing map (size 306 \times 342) and wheel map (size 232 \times 205). The number of convergence iterations and time cost measured by the segmentation results of each algorithm are shown in Table 8.

It can be seen from the test results in Table 8 that the number of convergence iterations decreases in the order of FCM, IFCM, FC-PFS, PFCM, PRFCM, and PRFCM_s. At the same time, the number of convergence iterations of PRFCM and PRFCM_s algorithm proposed in this study is more significantly reduced than PFCM algorithm. Although FCM algorithm converges more slowly than PRFCM algorithm, the time cost of FCM algorithm is significantly less than PRFCM algorithm because of its low computational complexity. From the test results, the PRFCM algorithm proposed in this study has certain potential advantages over FCM, IFCM, FC-PFS, and PFCM algorithms. The robust algorithm PRFCM_s of PRFCM has very obvious advantages over other algorithms in terms of convergence iteration times and time overhead.

For three images (all 256 \times 256 in size), salt and pepper noise with probability of 0.3, 0.1 and 0.2 is added to them respectively. FCM, FC-PFS, TPFCA, FCM_s, FLICM, FC-PFS_s and the algorithm ATPFCA proposed in this chapter are used for segmentation test. The test results are shown in Table 9.

Experimental results show that the total stubbornness-based graphic blur clustering (TPFCA) algorithm can effectively segment different types of images with uneven gray distribution, resulting in a high error rate (maximum signal-to-noise ratio), traditional dimmer C-Medium cluster and graphical dimmer cluster algorithms decreased from approximately 30% to 55% (increased by approximately 19%–33%). The introduction of total divergence improves the effective segmentation of uneven images to a certain extent. Compared with traditional fuzzy c-means clustering, graphic fuzzy clustering, robust fuzzy c-means clustering, fuzzy local information c-means clustering and robust graphic fuzzy clustering, the adaptive robust graphic fuzzy clustering (ATPFCA) algorithm based on total divergence has a stronger ability to segment gray uneven images. The misclassification rate (peak signal-to-noise ratio) of the segmentation result is reduced by about 38%–76% (increased by about 21%–67%) compared with the above clustering algorithm. Compared with the TPFCA algorithm, the effective segmentation degree of uneven images is further improved. For noisy images, the error rate (peak signal-to-noise ratio) of the ATPFCA algorithm corresponding to the segmentation results is reduced by about 5%–77%.

| Noisy image                  | FCM_s  | IFCM_s | FC-PFS_s | PFCM_s | PRFCM_s |
|-----------------------------|--------|--------|----------|--------|---------|
| Remote sensing map of basketball court | 17.7437 | 7.2047 | 17.7843  | 17.0811 | 18.2856 |
| River remote sensing map     | 14.5630 | 7.3170 | 14.6632  | 14.1162 | 14.6283 |
| Brain CT                     | 15.6738 | 10.7040| 14.6663  | 14.8166 | 16.3224 |

| Noisy image                  | FCM_s  | IFCM_s | FC-PFS_s | PFCM_s | PFCM_s algorithm |
|-----------------------------|--------|--------|----------|--------|------------------|
| Han Zishu                   | 13.0804| 9.0026 | 13.0005  | 11.8381| 13.7060          |
| River remote sensing map     | 16.7610| 8.2306 | 17.8418  | 14.8110| 18.5745          |
| Photographer                 | 11.8502| 8.7006 | 13.0820  | 11.7160| 13.2305          |

Table 5: Peak signal-to-noise ratio of segmentation results of different algorithms (unit: dB).

Table 6: Peak signal-to-noise ratio of segmentation results of different algorithms (unit: dB).
increased by about 6%–230%, especially up to 230% relative to FCM) compared with FCM, FC-PFS, TPFCA, FCM_s, FLICM, literature algorithm and FC-PFS_s. The embedding of adaptive pixel spatial neighborhood information not only improves the effective segmentation degree of the algorithm for gray uneven images but also enhances the anti-noise

### Table 7: Mixed noise interference block image and segmentation results.

| Noisy image                | FCM_s | IFCM_s | FC-PFS_s | PFCM_s | PFCM_s algorithm |
|----------------------------|-------|--------|----------|--------|-----------------|
| Han Zishu                  | 14.1110 | 7.1681 | 14.1701 | 15.0066 |
| River remote sensing map   | 23.0870 | 8.8004 | 23.1885 | 24.1303 |
| Photographer               | 18.3008 | 18.7328 | 19.4553 | 20.11034 |

### Table 8: Convergence iteration times of different algorithms.

| Noisy image     | FCM algorithm | IFCM algorithm | FC-PFS algorithm | PFCM algorithm | PRFCM algorithm | PFCM_s algorithm |
|-----------------|---------------|----------------|------------------|----------------|-----------------|------------------|
| Rice grain      | 8             | 10             | 12               | 8              | 7               | 2                |
| Photographer    | 6             | 12             | 14               | 7              | 5               | 3                |
| River remote sensing map | 8          | 13             | 19               | 8              | 10              | 7                |

Figure 3: Clustering segmentation test curve of barrel diagram and its noise interference diagram. (a) No noise interference. (b) Gaussian noise interference. (c) Salt and pepper noise interference. (d) Mixed noise interference.
robustness of the algorithm. The embedding of adaptive pixel spatial neighborhood information not only improves the effective segmentation degree of the algorithm for gray uneven images but also enhances the anti-noise robustness of the algorithm.

5. Conclusion

This study modifies and improves the existing graphic fuzzy clustering algorithm. First, aiming at the ill-posed problem of the PFCM algorithm, this study uses neutrality and rejection to construct regular terms and embed the objective function of the algorithm to enhance the correlation between the attribute parameters of fuzzy sets of sample graphics, so as to solve the ill-posed problem of PFCM algorithm. Second, aiming at the noise sensitivity of the PFCM algorithm, which is the same as the traditional fuzzy clustering algorithm, combined with the robust idea of FCM_s1 and FCM_s2 algorithms, the objective function of a reliable segmentation algorithm for a graphical blur cluster (RPFCM_s) was developed. Experiments have shown that the RPFCM_s algorithm achieves better segmentation results and effectively suppresses noise interference in images through several standard images, Gaussian noise, salt and pepper noise, and mixed noise segmentation.

Table 9: Error rate and peak signal-to-noise ratio of salt and pepper noise image segmentation results.

| Noisy image             | Evaluating indicator | FCM   | IFCM_s | TPFCA | FCM_s2 | FLICM | FC-PFS_s | ATPFCA |
|-------------------------|----------------------|-------|--------|-------|--------|-------|----------|--------|
| Synthetic diagram       | Misclassification rate (%) | 38.06 | 38.01  | 38.07 | 34.27  | 35.30 | 34.31    | 23.26  |
|                         | Signal-to-noise ratio (dB) | 9.1328| 9.1461 | 9.1327| 16.3671| 14.7200| 16.4256  | 30.2343|
| Airport remote sensing map | Misclassification rate (%) | 10.67 | 10.23  | 11.60 | 6.30   | 6.76  | 6.12     | 4.15   |
|                         | Signal-to-noise ratio (dB) | 9.1776| 9.3444 | 8.8514| 10.8285| 10.5206| 10.8607  | 12.6045|
| Industrial remote sensing map | Misclassification rate (%) | 30.18 | 28.41  | 25.10 | 20.73  | 21.37 | 21.16    | 10.80  |
|                         | Signal-to-noise ratio (dB) | 10.458| 10.5523| 10.7501|14.6770 |14.3500 |14.6004  |17.5355 |

(1) Aiming at the ill-posed problem in the existing graph fuzzy clustering algorithm, this study improves the correlation between various attribute parameters by closely associating the neutrality and rejection degree, and solves the phenomenon of misclassification and misclassification in the original graph fuzzy clustering algorithm due to the ill-posed problem. According to the nature of the objective function of the proposed algorithm, the unity of the algorithm was developed and proved with the idea of division and victory, and the unity of the objective function of the algorithm was confirmed using standard gray images and noise interference images.

(2) Experimental results under the same hardware conditions show that the improved RPFCM algorithm performs faster merging performance than other algorithms such as FCM, IFCM, FC-PFS, and PFCM. The RPFCM algorithm shows better segmentation performance than existing PFCM algorithms.

(3) Like other fuzzy clustering algorithms, the PRFCM algorithm still has the defect of being sensitive to noise. Therefore, the pixel gray spatial neighborhood information is integrated into the objective function of the clustering segmentation algorithm to obtain the PRFCM_s algorithm. From the test results, it can be seen that the algorithm has a stronger noise suppression ability. Because the neighborhood information weight factor in the PRFCM_s algorithm is an artificial parameter, which is not conducive to its further promotion, the artificial parameter needs to be optimized in the next step to facilitate the successful promotion and application of the algorithm.

Image-based 3D reconstruction technology is a mathematical process and calculation technology that uses 2D projection to restore 3D information about an object. Among them, the vision-based three-dimensional reconstruction technology uses a camera as a sensor to obtain two-dimensional images, and comprehensively uses image processing, visual computing and other technologies to reconstruct the three-dimensional information of objects with computer programs to complete the scene reproduction of the real environment, so that humans can better perceive external information. At present, due to the continuous improvement of 3D reconstruction algorithms, the increasingly automated modeling process, the lighter labor intensity, and the reduction of equipment costs, 3D reconstruction based on computer vision is suitable for reconstruction of any scene.

Data Availability

The labeled data set used to support the findings of this study is available from the author upon request.

Conflicts of Interest

The author declares that there are no conflicts of interest.
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