Canonical brackets from continuous symmetries: 
Abelian 2-form gauge theory

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Abstract: We derive the canonical (anti-)commutation relations amongst the creation and annihilation operators of the various basic fields, present in the four (3 + 1) - dimensional (4D) free Abelian 2-from gauge theory, with the help of continuous symmetry transformations within the framework of Becchi-Rouet-Stora-Tyutin (BRST) formalism. We show that all the six continuous symmetries of the theory lead to the exactly the same non-vanishing (anti-)commutator amongst the creation and annihilation operators of the normal mode expansion of the basic fields of the theory.

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1. Introduction

Symmetry principles play a crucial role as far as the theoretical description of the fundamental interactions of nature is concerned [1]. Three out of four fundamental interactions, present in nature (i.e. electromagnetic, weak and strong), are governed by the local continuous symmetries known as gauge symmetries. The notion of local gauge invariance generates the interaction term for these theories which are endowed with first-class constraints in the language of Dirac’s prescription for the classification scheme [2].

In recent past, the 2-form \( B^{(2)} = \frac{1}{2!} (dx^\mu \wedge dx^\nu) B_{\mu\nu} \) Abelian gauge field \( B_{\mu\nu} \) and corresponding gauge theories have been studied thoroughly in different contexts [3-5]. Its study is important because of its relevance in the context of (super)string theories [6,7]. As far as the canonical quantization of such kind of gauge theories are concerned, Becchi-Rouet-Stora-Tyutin (BRST) formalism is one of the most intuitive approaches. The BRST quantization of such 2-form gauge theories has already been carried out in different contexts [8,9]. It has also been shown that the 4D free Abelian 2-form gauge theory provides a field theoretic model for Hodge theory where all the de Rham cohomological operators of the differential geometry find their physical realizations in terms of the symmetry transformations (and their corresponding generators) of the theory [10]. The quantization of the above Abelian 2-form gauge theory has been a topic of research interest. The most familiar quantization scheme is the usual canonical formalism.

In the context of the canonical method of quantization, three steps are followed. In the first step, we use the spin-statistics theorem in order to distinguish the nature of the fields (i.e. bosonic/fermionic) of a given theory. In the second step, we calculate the canonical (graded) Poisson brackets between the field variables and their corresponding conjugate momenta and promote them up to the level of (anti-)commutators. In the third and final step, we express the field variables and corresponding conjugate momenta in terms of normal mode expansion of the basic fields, that include the creation and annihilation operators. The relevant physical quantities (e.g. conserved charges, Hamiltonian, etc.) are expressed in terms of creation and annihilation operators where the concept of normal ordering is required.

The main motivation of our present investigation is to derive the canonical (anti-)commutation relations amongst the creation and annihilation operators with the help of continuous symmetry transformations (and their corresponding generators) in the context of 4D free Abelian 2-form gauge theory. Towards this goal, although, we have taken the help of spin-statistic theorem and the concept of normal ordering but we have not exploited the
concept of (graded) Poisson brackets. Instead, in the place of latter, we have taken the help of continuous symmetry transformations (and their corresponding generators) to obtain the canonical brackets amongst the creation and annihilation operators. It is worthwhile to mention that we have already calculated the canonical brackets amongst the creation and annihilation operators with the help of symmetry transformations in the context of 2D free and 2D interacting Abelian 1-form gauge theory with Dirac fields [11].

Our paper is organized as follows. In the Sec. 2, we discuss the symmetries of the 4D free Abelian 2-form gauge theory. Our Sec. 3 contains the conserved charges, that have been derived with the help of Noether’s theorem, and the normal mode expansion of the basic fields in terms of the creation and annihilation operators. We have explicitly derived the (anti-)commutation relations amongst the creation and annihilation operators with the help of symmetry principle in Sec. 4 of our manuscript. In Sec. 5, we have calculated, for the sake of completeness, the same (anti-)commutation relations with the help of conventional Lagrangian formalism. Finally, we have made some concluding remarks in Sec. 6.

2. Lagrangian formalism: symmetries

We begin with the Becchi-Rouet-Stora-Tyutin (BRST) invariant Lagrangian density of a 4D free Abelian 2-form gauge theory\(^1\)(see, e.g., [12])

\[
\mathcal{L} = \frac{1}{2} \left( \partial^\nu B_{\kappa\mu} - \partial_\kappa B^{\nu\mu} - \partial^\nu \phi_1 \right) - \partial_\mu \bar{\beta} \partial^\mu \beta \\
- \frac{1}{2} \left( \frac{1}{2} \varepsilon_{\mu\nu\kappa\lambda} \partial^\nu B^{\kappa\lambda} - \partial_\mu \phi_2 \right) \frac{1}{2} \varepsilon^{\mu\nu\sigma\tau} \partial_\kappa B_{\sigma\tau} - \partial^\nu \phi_2 \\
+ \left( \partial_\mu \bar{C}_\nu - \partial_\nu \bar{C}_\mu \right) (\partial^\nu C^\mu) - \frac{1}{2} (\partial \cdot \bar{C}) (\partial \cdot C),
\]

(1)

where \(B_{\mu\nu}\) is antisymmetric \((B_{\mu\nu} = -B_{\nu\mu})\) tensor gauge field. \(\phi_1\) and \(\phi_2\) are massless scalar fields, \((\bar{C}_\mu)C_\mu\) are the fermionic (anti-)ghost fields (with \(C_\mu^2 = C_\mu^2 = 0, C_\mu C_\nu + C_\nu C_\mu = 0\)) and \((\bar{\beta})\beta\) are the bosonic (anti-)ghost fields.

The above Lagrangian density remains quasi-invariant under the following on-shell nilpotent \((s^a_0 s^a_0 = 0)\) and absolutely anticommuting \((s^a s^b + s^b s^a = 0)\)

\(^1\)We choose the flat metric \((\eta_{\mu\nu})\) with the signatures \((+1, -1, -1, -1)\) so that \(A \cdot B = \eta_{\mu\nu} A^\mu B^\nu = A_0 B_0 - A_i B_i\) is the dot product between two non-null vectors \(A_\mu\) and \(B_\mu\) where the Greek indices \(\mu, \nu, \eta, \ldots = 0, 1, 2, 3\) and Latin indices \(i, j, k, \ldots = 1, 2, 3\). The choice of 4D totally antisymmetric Levi-Civita tensor \((\varepsilon_{\mu\nu\rho\kappa})\) is such that \(\varepsilon_{0123} = +1 = -\varepsilon_{0123}, \varepsilon_{\mu\nu\rho\kappa} \varepsilon^{\mu\nu\rho\kappa} = -4!, \varepsilon_{\mu\nu\rho\kappa} \varepsilon^{\mu\nu\rho\kappa} = -3! \delta^\mu_0 \delta^\nu_0 \delta^\rho_0 \delta^\kappa_0\), etc. The component \(\varepsilon_{0ijk} = \epsilon_{ijk}\) is the 3D Levi-Civita tensor.
The (anti-)co-BRST symmetry transformations can be given as:

\[ s_{ab}B_{\mu \nu} = (\partial_\mu \bar{C}_\nu - \partial_\nu \bar{C}_\mu), \quad s_{ab}C_\mu = -\partial_\mu \bar{\beta}, \quad s_{ab}\bar{C}_\mu = -(\partial^\nu B_{\nu \mu} - \partial_\mu \phi_1), \]
\[ s_{ab}\phi_1 = \frac{1}{2} (\partial \cdot \bar{C}), \quad s_{ab}\beta = -\frac{1}{2} (\partial \cdot C), \quad s_{ab}\bar{\beta} = 0, \quad s_{ab}\phi_2 = 0, \quad (2) \]

\[ s_bB_{\mu \nu} = (\partial_\mu C_\nu - \partial_\nu C_\mu), \quad s_bC_\mu = \partial_\mu \beta, \quad s_b\bar{C}_\mu = (\partial^\nu B_{\nu \mu} - \partial_\mu \phi_1), \]
\[ s_b\phi_1 = \frac{1}{2} (\partial \cdot C), \quad s_b\bar{\beta} = -\frac{1}{2} (\partial \cdot \bar{C}), \quad s_b\beta = 0, \quad s_b\phi_2 = 0. \quad (3) \]

It can be checked that under the above (anti-)BRST symmetry transformations the kinetic term remains invariant. Furthermore, we have following on-shell nilpotent (anti-)co-BRST \((s^2_{(a)b} = 0)\) symmetry transformations under which the gauge fixing term of the Lagrangian density \((1)\) remains invariant. The (anti-)co-BRST symmetry transformations can be given as:

\[ s_{ad}B_{\mu \nu} = \varepsilon_{\mu \nu \kappa \xi} \partial^\kappa C^\xi, \quad s_{ad} \bar{C}_\mu = -\left(\frac{1}{2} \varepsilon_{\mu \nu \kappa \sigma} \partial^\nu \partial^\kappa B^{\nu \sigma} - \partial_\mu \phi_2\right), \]
\[ s_{ad} C_\mu = \partial_\mu \beta, \quad s_{ad} \phi_2 = \frac{1}{2} (\partial \cdot C), \quad s_{ad} \bar{\beta} = -\frac{1}{2} (\partial \cdot \bar{C}), \quad s_{ad} (\bar{\beta}, \phi_1, \partial^\mu B_{\mu \nu}) = 0, \quad (4) \]

\[ s_d B_{\mu \nu} = \varepsilon_{\mu \nu \kappa \xi} \partial^\kappa \bar{C}^\xi, \quad s_d C_\mu = \left(\frac{1}{2} \varepsilon_{\mu \nu \kappa \sigma} \partial^\nu \partial^\kappa B^{\nu \sigma} - \partial_\mu \phi_2\right), \]
\[ s_d \bar{C}_\mu = -\partial_\mu \bar{\beta}, \quad s_d \phi_2 = \frac{1}{2} (\partial \cdot \bar{C}), \quad s_d \beta = -\frac{1}{2} (\partial \cdot C), \quad s_d (\bar{\beta}, \phi_1, \partial^\mu B_{\mu \nu}) = 0, \quad (5) \]

These transformations leave the Lagrangian density \((1)\) quasi-invariant. The (anti-)BRST and (anti-)co-BRST symmetry transformations obey the two key properties (i) the nilpotency of order two \((i.e. \ s^2_{(a)b} = 0, \ s^2_{(a)d} = 0)\), and (ii) the absolute anticommutativity \((i.e. \ s_b s_{ab} + s_{ab} s_b = 0, \ s_d s_{ad} + s_{ad} s_d = 0)\). The former property \((i.e. \ nilpotency)\) shows the fermionic nature of (anti-)BRST as well as (anti-)co-BRST symmetries and the latter property \((i.e. \ absolute \ anticommutativity)\) shows that the BRST and anti-BRST symmetries are independent of one another (also true in the case of co-BRST and anti-co-BRST symmetry transformations).

The anticommutator of the above two symmetries \((i.e. \ \{s_b, s_d\} = s_\omega = -\{s_{ab}, s_{ad}\})\) lead to a another symmetry, named as bosonic symmetry:

\[ s_\omega B_{\mu \nu} = \varepsilon_{\mu \nu \rho \kappa} \partial^\rho (\partial_\sigma B^{\sigma \kappa}) + \frac{1}{2} \varepsilon_{\nu \xi \sigma \eta} \partial_\mu (\partial^\xi B^{\nu \sigma \eta}) - \frac{1}{2} \varepsilon_{\mu \xi \sigma \eta} \partial_\nu (\partial^\xi B^{\nu \sigma \eta}), \]
\[ s_\omega C_\mu = -\frac{1}{2} \partial_\mu (\partial \cdot C), \quad s_\omega \bar{C}_\mu = \frac{1}{2} \partial_\mu (\partial \cdot \bar{C}), \quad s_\omega (\beta, \bar{\beta}, \phi_1, \phi_2) = 0. \quad (6) \]
Under above symmetry transformation the Lagrangian density (1) goes to a total spacetime derivative, therefore it is symmetry of the theory. Moreover, the ghost sector of the present theory also possess a continuous symmetry called the ghost symmetry \((s_g)\) and can be realized as

\[
s_g C_\mu = +\Sigma C_\mu, \quad s_g \bar{C}_\mu = -\Sigma \bar{C}_\mu, \quad s_g \beta = +2\Sigma \beta, \quad s_g \bar{\beta} = -2\Sigma \bar{\beta},
\]

where \(\Sigma\) is a continuous global scale parameter. Thus the 4D free Abelian 2-form gauge theory is endowed with, in totality, six continuous symmetries.

3. Conserved charges and normal mode expansions

We have seen (cf. Sec. 2) the 4D free Abelian 2-form gauge theory endowed with, in totality, six continuous symmetry transformations. According to the Noether’s theorem these continuous symmetries lead to the derivation of six conserved currents \((J^r_\mu, r = b, ab, d, ad, g, \omega)\) corresponding to each symmetry transformations. The zeroth component \((J^0_\mu)\) of these conserved currents lead to the following conserved charges (i.e. \(Q^r = \int d^3x \, J^r_\mu\))

\[
Q_b = \int d^3x \left[ (\partial^0 \bar{C}^i - \partial^i \bar{C}^0)(\partial_i \beta) + \frac{1}{2} (\partial^0 \beta)(\partial \cdot \bar{C}) - \frac{1}{2} (\partial_i B^0 - \partial^0 \phi_1)(\partial \cdot C) + H^{0ij}(\partial_i C_j) - \epsilon_{ijk}(\partial^i \phi_2)(\partial^j \bar{C}^k) + (\partial^0 C^i - \partial^i \bar{C}^0)(\partial^0 B_{0i} + \partial^i B_{ji} - \partial_i \phi_1) \right],
\]

\[
Q_{ab} = \int d^3x \left[ (\partial^0 C^i - \partial^i \bar{C}^0)(\partial_i \bar{\beta}) + \frac{1}{2} (\partial^0 \bar{\beta})(\partial \cdot C) - \frac{1}{2} (\partial_i B^0 - \partial^0 \phi_1)(\partial \cdot \bar{C}) + H^{0ij}(\partial_i \bar{C}_j) - \epsilon_{ijk}(\partial^i \phi_2)(\partial^j \bar{C}^k) + (\partial^0 \bar{C}^i - \partial^i C^0)(\partial^0 B_{0i} + \partial^i B_{ji} - \partial_i \phi_1) \right],
\]

\[
Q_d = \int d^3x \left[ \epsilon_{ijk} (\partial^i \bar{C}^k)(\partial_0 B^{0j} + \partial_j B^{0i} - \partial^i \phi_1) + \frac{1}{2} (\partial \cdot C)(\partial_0 \bar{\beta}) + \frac{1}{2} \left( - \epsilon_{ijk} \partial_0 B^{jk} + 2 \epsilon_{ijk} \partial^i B^{0k} \right)(\partial^0 \bar{C}^0 - \partial^0 \bar{C}^0) + \frac{1}{4} \epsilon_{ijk} (\partial^j B^{ik})(\partial \cdot \bar{C}) - \frac{1}{2} (\partial \cdot C)(\partial^i \phi_2) - (\partial_i \bar{\beta})(\partial^0 C^i - \partial^i \bar{C}^0) + (\partial_i \phi_2)(\partial^0 \bar{C}^i - \partial^i \bar{C}^0) \right],
\]

5
\[ Q_{ad} = \int d^3x \left[ \varepsilon_{ijk} (\partial^j C^k)(\partial_0 B^{0i} + \partial_i B^{0i} - \partial^i \phi_1) + \frac{1}{2} (\partial \cdot \bar{C})(\partial_\beta) \right. \\
\left. + \frac{1}{2} \left(-\varepsilon_{ijk} \partial_0 B^{0k} + 2 \varepsilon_{ijk} \partial^j B^{0k}\right)(\partial^i C^0 - \partial^0 C^i) \right. \\
\left. + \frac{1}{4} \varepsilon_{ijk} (\partial^j B^{jk})(\partial \cdot C) - \frac{1}{2} (\partial \cdot C)(\partial^0 \phi_2) \right. \\
\left. - (\partial_\beta)(\partial^0 \bar{C} - \partial^i \bar{C}^0) + (\partial_\beta \phi_2)(\partial^0 C^i - \partial^i C^0) \right], \quad (11) \]

\[ Q_g = \int d^3x \left[ (\partial^0 C^i - \partial^i C^0)\bar{C}_i + (\partial^0 \bar{C}^i - \partial^i \bar{C}^0)C_i - \frac{1}{2} (\partial \cdot C)\bar{C}_0 \right. \\
\left. - \frac{1}{2}(\partial \cdot \bar{C})C_0 - 2\beta(\partial^0 \bar{\beta}) + 2\bar{\beta}(\partial^0 \beta) \right], \quad (12) \]

\[ Q_\omega = \int d^3x \left[ \frac{1}{2} \partial_i (\partial \cdot C)(\partial^i \bar{C}^0 - \partial^i \bar{C}^0) + \frac{1}{2} \partial_i (\partial \cdot C)(\partial^0 \bar{C}^i - \partial^i \bar{C}^0) \right. \\
\left. - \varepsilon_{ijk} \partial^j (\partial_0 B^{0k} + \partial_k B^{0k})(\partial^i \phi_1) + \varepsilon_{jlm} H^{0ij} \partial_i (\partial^0 B^{0m}) \right. \\
\left. - \frac{1}{2} \varepsilon_{ijk} \partial_0 (\partial^0 B^{jk})(\partial_l \phi_1) - \frac{1}{2} \varepsilon_{ijk} \partial_i (\partial^j B^{0k})(\partial_0 B^{0l} + \partial_m B^{ml}) \right. \\
\left. - \frac{1}{2} \varepsilon_{ijk} (\partial_0 B^{jk})\partial^0 (\partial_0 B^{0i} + \partial_i B^{0i}) + \frac{1}{2} \varepsilon_{ijk} (\partial^0 B^{0k})\partial_0 (\partial_i B^{0l}) \right. \\
\left. + \varepsilon_{ijk} (\partial^0 B^{0k})\partial^0 (\partial_0 B^{0i} + \partial_i B^{0i}) - \frac{1}{2} \varepsilon_{jlm} H^{0ij} \partial_i (\partial^0 B^{lm}) \right. \\
\left. - \frac{1}{2} \varepsilon_{ijk} H^{0ij} \partial^k (\partial_l B^{00}) + \frac{1}{2} \varepsilon_{ijk} H^{0ij} \partial_0 (\partial_0 B^{0k} + \partial_l B^{0k}) \right. \\
\left. + \varepsilon_{ijk} \partial^j (\partial_0 B^{0k} + \partial_k B^{0k}) (\partial_0 B^{0i} + \partial_m B^{mi}) + (\partial_\omega \phi_2)(\partial^0 \bar{C} \cdot H^{0ij}) \right], \quad (13) \]

these charges are turn out to be the generators of the corresponding symmetry transformations.

The Euler-Lagrange equations of motion derived from the Lagrangian density (1) can be given as

\[ \Box B_{\mu \nu} = 0, \quad \Box \phi_1 = \Box \phi_2 = 0, \quad \Box \beta = \Box \bar{\beta} = 0, \]

\[ \Box C_\mu = \frac{3}{2} \partial_\mu (\partial \cdot C), \quad \Box \bar{C}_\mu = \frac{3}{2} \partial_\mu (\partial \cdot \bar{C}), \quad (14) \]

we choose the gauge condition such that \((\partial \cdot C) = (\partial \cdot \bar{C}) = 0\). With these gauge conditions the last two equations in (14) reduce to the following form:

\[ \Box C_\mu = 0, \quad \Box \bar{C}_\mu = 0. \quad (15) \]
The normal mode expansions of the basic fields, that are present in our theory, in terms of the creation and annihilation operators can be given as

\[
B_{\mu\nu}(x) = \int \frac{d^3x}{(2\pi)^3 \cdot 2k_0} \left( b_{\mu\nu}(\vec{k}) e^{+i k \cdot x} + b_{\mu\nu}^+(\vec{k}) e^{-i k \cdot x} \right),
\]

\[
C_\mu(x) = \int \frac{d^3x}{(2\pi)^3 \cdot 2k_0} \left( c_\mu(\vec{k}) e^{+i k \cdot x} + c_\mu^+(\vec{k}) e^{-i k \cdot x} \right),
\]

\[
\bar{C}_\mu(x) = \int \frac{d^3x}{(2\pi)^3 \cdot 2k_0} \left( \bar{c}_\mu(\vec{k}) e^{+i k \cdot x} + \bar{c}_\mu^+(\vec{k}) e^{-i k \cdot x} \right),
\]

\[
\beta(x) = \int \frac{d^3x}{(2\pi)^3 \cdot 2k_0} \left( \tilde{b}(\vec{k}) e^{+i k \cdot x} + \tilde{b}^+(\vec{k}) e^{-i k \cdot x} \right),
\]

\[
\bar{\beta}(x) = \int \frac{d^3x}{(2\pi)^3 \cdot 2k_0} \left( \tilde{c}(\vec{k}) e^{+i k \cdot x} + \tilde{c}^+(\vec{k}) e^{-i k \cdot x} \right),
\]

\[
\phi_1(x) = \int \frac{d^3x}{(2\pi)^3 \cdot 2k_0} \left( f_1(\vec{k}) e^{+i k \cdot x} + f_1^+(\vec{k}) e^{-i k \cdot x} \right),
\]

\[
\phi_2(x) = \int \frac{d^3x}{(2\pi)^3 \cdot 2k_0} \left( f_2(\vec{k}) e^{+i k \cdot x} + f_2^+(\vec{k}) e^{-i k \cdot x} \right),
\]

where \(k_\mu(= k_0, k_i)\) is the momentum 4-vector and \(b_{\mu\nu}, c_\mu, \bar{c}_\mu, b, \tilde{b}, f_1, f_2\) are the annihilation operators and \(b_{\mu\nu}^+, c_\mu^+, \bar{c}_\mu^+, b^+, \tilde{b}^+, f_1^+, f_2^+\) are the creation operators of the basic fields \(B_{\mu\nu}, C_\mu, \bar{C}_\mu, \beta, \bar{\beta}, \phi_1, \phi_2\) respectively.

It is interesting to note that the gauge conditions (i.e. \((\partial \cdot C) = (\partial \cdot \bar{C}) = 0\)) imply the following relationships

\[
k^\mu c_\mu(\vec{k}) = k^\mu c_\mu^+(\vec{k}) = k^\mu \bar{c}_\mu(\vec{k}) = k^\mu \bar{c}_\mu^+(\vec{k}) = 0,
\]

along with \(k^2 = k^\mu k_\mu = 0\).

4. Canonical brackets from symmetry principle

The conserved charges, (cf. (8) - (13)), are turn out to be the generator of the continuous symmetry transformations as follows [13,14]

\[
s_r \Phi = \pm i \left[ \Phi, Q_r \right]_{\pm}, \quad r = b, ab, d, ad, \omega, g,
\]

where \(\Phi\) is any generic field of the theory and \(Q_r\) are the conserved charges. The (+)− signs, as the subscripts on the square bracket, correspond to the (anti-)commutators for the generic field \(\Phi\) being (fermionic)bosonic in nature.
The explanation for the $(+)-$ signs, in front of the square bracket on the r.h.s. (i.e. $\pm i \left[ \Phi, Q_r \right]_\pm$) is given below:

(i) negative sign is to be taken into account only for the $s_r = s_b$, $s_{ab}$, $s_d$, $s_{ad}$ (e.g. $s_b B_{\mu \nu} = - i \left[ B_{\mu \nu}, Q_b \right]$, $s_d C_\mu = - i \left[ C_\mu, Q_d \right]$, etc.), and
(ii) for $s_r = s_g$, $s_\omega$ the negative sign is to be taken into account only for the bosonic fields and the positive sign is to be chosen for the fermionic fields (e.g. $s_g B_{\mu \nu} = - i \left[ B_{\mu \nu}, Q_g \right]$, $s_\omega C_\mu = + i \left[ C_\mu, Q_\omega \right]$ etc.).

Let us take an example in order to illustrate our formalism:

\[
s_d B_{\mu \nu} = \varepsilon_{\mu \nu \kappa} \partial^\kappa \tilde{C}^\alpha = - i \left[ B_{\mu \nu}, Q_d \right] \implies \]
\[
s_d B_{\nu \kappa} = \varepsilon_{\nu \kappa \eta} \partial^\eta \tilde{C}^\alpha = - i \left[ B_{\nu \kappa}, Q_d \right],
\]
\[
s_d B_{ij} = \varepsilon_{ij k} \left( \partial^k \tilde{C}^\alpha - \partial^k \tilde{C}^0 \right) = - i \left[ B_{ij}, Q_d \right],
\]

(19)

where $Q_d$ is co-BRST charge and it turns out to be the generator of the continuous symmetry transformations $s_d$. Let us choose $t = 0$ for the sake of simplicity in all the computations. Firstly, let us calculate the commutation relation corresponding to the $B_{0i}$ component. The l.h.s. of the second equation of (19) can be expressed in terms of creation and annihilation operators as follows

\[
\varepsilon_{ijk} \left( \partial^j \tilde{C}^k \right) = i \int \frac{d^3 k}{\sqrt{(2\pi)^3 \cdot 2k_0}} \varepsilon_{ijk} k^j \left( \tilde{c}^k (\vec{k}) e^{-ik \cdot \vec{x}} - (\tilde{c}^k)\dagger (\vec{k}) e^{+ik \cdot \vec{x}} \right)
\]

\[
= \frac{i}{2} \int \frac{d^3 k}{\sqrt{(2\pi)^3 \cdot 2k_0}} \varepsilon_{ijk} k^j \left[ \left( \tilde{c}^k (\vec{k}) e^{-ik \cdot \vec{x}} - (\tilde{c}^k)\dagger (\vec{k}) e^{+ik \cdot \vec{x}} \right)
+ \left( \tilde{c}^k (\vec{k}) e^{-ik \cdot \vec{x}} - (\tilde{c}^k)\dagger (\vec{k}) e^{+ik \cdot \vec{x}} \right) \right].
\]

(20)

The reason for breaking it into two similar terms will be clear later when we compare the exponentials. In the second term of the above equation, changing $\vec{k} \rightarrow -\vec{k}$ and rearranging the terms, we obtain

\[
\varepsilon_{ijk} \left( \partial^j \tilde{C}^k \right) = \frac{i}{2} \int \frac{d^3 k}{\sqrt{(2\pi)^3 \cdot 2k_0}} \varepsilon_{ijk} k^j \left[ \left( \tilde{c}^k (\vec{k}) + (\tilde{c}^k)\dagger (-\vec{k}) \right) e^{-ik \cdot \vec{x}}
- \left( \tilde{c}^k (-\vec{k}) + (\tilde{c}^k)\dagger (\vec{k}) \right) e^{+ik \cdot \vec{x}} \right].
\]

(21)

Now the r.h.s. of equation (19), we re-express $B_{0i}(\vec{x})$ in terms of creation and annihilation operators as follows (at $t = 0$):

\[
-i [B_{0i}(\vec{x}), Q_d] = -i \left[ \int \frac{d^3 k}{\sqrt{(2\pi)^3 \cdot 2k_0}} \left( b_{0i}(\vec{k}) e^{-ik \cdot \vec{x}} + b_{0i}^\dagger (\vec{k}) e^{+ik \cdot \vec{x}} \right), Q_d \right]
\]

\[
= -i \int \frac{d^3 k}{\sqrt{(2\pi)^3 \cdot 2k_0}} \left( [b_{0i}(\vec{k}), Q_d] e^{-ik \cdot \vec{x}} + [b_{0i}^\dagger (\vec{k}), Q_d] e^{+ik \cdot \vec{x}} \right).
\]

(22)
Comparing the exponentials from the r.h.s. of the equations (21) and (22), we obtain the following relationship

\[
[b_0(\vec{k}), Q_d] = -\frac{1}{2} \varepsilon_{ijk} k^j \left( \tilde{c}^k(\vec{k}) + (\tilde{c}^k)^\dagger(\vec{-k}) \right)
\equiv -\int \frac{d^3p}{2} \varepsilon_{ijk} p^i \left( \tilde{c}^k(\vec{p}) + (\tilde{c}^k)^\dagger(\vec{-p}) \right) \delta^{(3)}(\vec{k} - \vec{p}), \quad (23)
\]

\[
[b_0^\dagger(\vec{k}), Q_d] = \frac{1}{2} \varepsilon_{ijk} k^j \left( \tilde{c}^k(-\vec{k}) + (\tilde{c}^k)^\dagger(\vec{k}) \right)
\equiv \int \frac{d^3p}{2} \varepsilon_{ijk} p^i \left( \tilde{c}^k(-\vec{p}) + (\tilde{c}^k)^\dagger(\vec{p}) \right) \delta^{(3)}(\vec{k} - \vec{p}). \quad (24)
\]

The relevant part of \(Q_d\) (that have non-vanishing (anti-)commutation relations with \(B_0(\vec{x})\)) can be given as (cf. (10))

\[
Q_d \approx \int d^3y \varepsilon_{ijk} (\hat{\partial}^j \tilde{c}^k)(\hat{\partial} B^0). \quad (25)
\]

The above equation for \(Q_d\) can be re-expressed in terms of the normal mode expansions of the basic fields (cf. (16))

\[
Q_d \approx \int \frac{d^3y}{(2\pi)^3 \cdot \sqrt{2p_0 \cdot 2q_0}} \varepsilon_{ijk} (-p^j q_0) \left( \tilde{c}^k(\vec{p}) b^0(\vec{q}) e^{-i (\vec{p} + \vec{q}) \cdot \vec{y}} - \tilde{c}^k(\vec{p}) (b^0)^\dagger(\vec{q}) e^{-i (\vec{p} - \vec{q}) \cdot \vec{y}} - (\tilde{c}^k)^\dagger(\vec{p}) b^0(\vec{q}) e^{+i (\vec{p} - \vec{q}) \cdot \vec{y}} + (\tilde{c}^k)^\dagger(\vec{p}) (b^0)^\dagger(\vec{q}) e^{+i (\vec{p} + \vec{q}) \cdot \vec{y}} \right). \quad (26)
\]

Integrating the above equation with respect to \(d^3y\) and \(d^3q\)

\[
Q_d \approx -\frac{1}{2} \int d^3p \varepsilon_{ijk} p^j \left( \tilde{c}^k(\vec{p}) b^0(-\vec{p}) + (\tilde{c}^k)^\dagger(\vec{p}) (b^0)^\dagger(-\vec{p}) - (\tilde{c}^k)^\dagger(\vec{p}) b^0(\vec{p}) - \tilde{c}^k(\vec{p}) (b^0)^\dagger(\vec{p}) \right). \quad (27)
\]

In the first two terms of above equation changing \(\vec{p} \rightarrow -\vec{p}\) and collecting the coefficient of \(b^0(\vec{p})\) and \((b^0)^\dagger(\vec{p})\), we obtain

\[
Q_d \approx \frac{1}{2} \int d^3p \varepsilon_{ijk} p^j \left[ \left( (\tilde{c}^k)^\dagger(\vec{p}) + \tilde{c}^k(-\vec{p}) \right) b^0(\vec{p}) + \left( (\tilde{c}^k)^\dagger(-\vec{p}) + \tilde{c}^k(\vec{p}) \right) (b^0)^\dagger(\vec{p}) \right]. \quad (28)
\]

\[\text{We have used the following definition of Dirac } \delta\text{-function}
\int d^3x e^{\pm i(\vec{p} - \vec{q}) \cdot x} = (2\pi)^3 \delta^{(3)}(\vec{p} - \vec{q}), \quad \int d^3q f(\vec{q}) \delta^{(3)}(\vec{p} - \vec{q}) = f(\vec{p}).\]
Now substituting this value of $Q_d$ in expression (23), we have following expression that contain the two existing commutators

\[
[b_{bi} (\vec{k}), Q_d] = \frac{1}{2} \int d^3p \ (\varepsilon_{ijk} p^i) \left( ((\tilde{c}^k)^\dagger (\vec{p}) + \tilde{c}^k (\vec{p})) [b_{bi} (\vec{k}), b^{0l}(\vec{p})] \\
+ (\tilde{c}^k(\vec{p}) + (\tilde{c}^k)^\dagger (\vec{p})) [b_{bi} (\vec{k}), (b^{0l})^\dagger(\vec{p})] \right). \tag{29}
\]

In fact, we have eight commutators out of which four commutators, amongst the (anti-)ghost fields with the rest of the bosonic fields, are zero because the (anti-)ghost fields are decoupled from the rest of the bosonic fields present in the theory. Now comparing the r.h.s. of (23) and (29), we obtain following commutators

\[
[b_{bi} (\vec{k}), b^{0l}(\vec{p})] = 0, \quad [b_{bi} (\vec{k}), (b^{0l})^\dagger(\vec{p})] = - \delta_i^l \delta^{(3)}(\vec{k} - \vec{p}). \tag{30}
\]

It is straightforward to check that, if we substitute the value of $Q_d$ (cf. (28)) in the equation (24), in stead of equation (23), we get the exactly same set of canonical commutation relations (cf. (30)).

Secondly, let us calculate the commutation relations corresponding to the $B_{ij}$ component (cf. (19))

\[
s_d B_{ij} = \varepsilon_{ijl} (\partial^0 \tilde{c}^l - \partial^l \tilde{c}^0) = -i [B_{ij}, Q_d]. \tag{31}
\]

The l.h.s. of above equation can be expanded in the terms of creation and annihilation operators in the following manner

\[
\varepsilon_{ijl}(\partial^0 \tilde{c}^l - \partial^l \tilde{c}^0) = i \int \frac{d^3k}{(2\pi)^3 \cdot 2k_0} \varepsilon_{ijl} \left[ (k^0 \tilde{c}^l(\vec{k}) - k^l \tilde{c}^0(\vec{k})) e^{-ik \cdot \vec{x}} \\
- (k^0 (\tilde{c}^l)^\dagger(\vec{k}) - k^l (\tilde{c}^0)^\dagger(\vec{k})) e^{+ik \cdot \vec{x}} \right] \\
= \frac{i}{2} \int \frac{d^3k}{(2\pi)^3 \cdot 2k_0} \varepsilon_{ijl} \left[ (k^0 \tilde{c}^l(\vec{k}) - k^l \tilde{c}^0(\vec{k})) e^{-ik \cdot \vec{x}} \\
- k^l \tilde{c}^0(\vec{k}) - k^l (\tilde{c}^0)^\dagger(\vec{k}) e^{-ik \cdot \vec{x}} - (k^0 (\tilde{c}^l)^\dagger(\vec{k}) - k^0 \tilde{c}^l(\vec{k} - \vec{k})) e^{-ik \cdot \vec{x}} \\
- k^l \tilde{c}^0(\vec{k}) - k^l (\tilde{c}^0)^\dagger(\vec{k}) e^{+ik \cdot \vec{x}} \right]. \tag{32}
\]

In the r.h.s. of equation (31), we re-express $B_{ij}(\vec{x})$ in terms of creation and annihilation operators as follows

\[
-i [B_{ij}(\vec{x}), Q_d] = -i \left[ \int \frac{d^3k}{(2\pi)^3 \cdot 2k_0} \left( b_{ij}(\vec{k}) e^{-ik \cdot \vec{x}} + b_{ij}^\dagger(\vec{k}) e^{+ik \cdot \vec{x}} \right), Q_d \right] \\
= -i \int \frac{d^3k}{(2\pi)^3 \cdot 2k_0} \left( [b_{ij}(\vec{k}), Q_d] e^{-ik \cdot \vec{x}} + [b_{ij}^\dagger(\vec{k}), Q_d] e^{+ik \cdot \vec{x}} \right). \tag{33}
\]
Now comparing the exponentials from the equations (32) and (33), we get

$$\left[ b_{ij}(\vec{k}), Q_d \right] = -\frac{1}{2} \varepsilon_{ijl} \left( k^0 \tilde{c}^l(\vec{k}) - k^0(\tilde{c}^l)^\dagger(-\vec{k}) - k^l \tilde{c}^0(\vec{k}) - k^l(\tilde{c}^0)^\dagger(-\vec{k}) \right)$$

$$= -\frac{1}{2} \int d^3p \varepsilon_{ijl} \delta^{(3)}(\vec{k} - \vec{p}) \left( p^0 \tilde{c}^l(\vec{p}) - p^0(\tilde{c}^l)^\dagger(-\vec{p}) - p^l \tilde{c}^0(\vec{p}) - p^l(\tilde{c}^0)^\dagger(-\vec{p}) \right),$$

(34)

$$\left[ b_{ij}^\dagger(\vec{k}), Q_d \right] = \frac{1}{2} \varepsilon_{ijl} \left( k^0(\tilde{c}^l)^\dagger(\vec{k}) - k^0(\tilde{c}^l)^\dagger(-\vec{k}) - k^l \tilde{c}^0(\vec{k}) - k^l \tilde{c}^0(-\vec{k}) \right)$$

$$= \frac{1}{2} \int d^3p \varepsilon_{ijl} \delta^{(3)}(\vec{k} - \vec{p}) \left( p^0(\tilde{c}^l)^\dagger(\vec{p}) - p^0(\tilde{c}^l)^\dagger(-\vec{p}) - p^l(\tilde{c}^0)^\dagger(\vec{p}) - p^l(\tilde{c}^0)^\dagger(-\vec{p}) \right).$$

(35)

The relevant part of $Q_d$ (that have non-vanishing commutation relations with $B_{ij}(\vec{x})$) can be given as (cf. (10))

$$Q_d \approx -\frac{1}{2} \int d^3y \varepsilon_{lmn}(\partial^0 B^{mn})(\partial^l \tilde{C} - \partial^l \tilde{C}^l),$$

(36)

re-expressing $Q_d$ in terms of normal mode expansions of basic fields (cf. (16)):

$$Q_d \approx -\frac{1}{2} \int \frac{d^3y \ d^3p \ d^3q}{(2\pi)^3 \cdot \sqrt{2p_0 \cdot 2q_0}} \varepsilon_{lmn} p^0 \left[ \left( q^0 b^{mn}(\vec{p}) \tilde{c}^l(\vec{q}) - q^l b^{mn}(\vec{p}) \tilde{c}^0(\vec{q}) \right) e^{-i(\vec{p} - \vec{q}) \cdot \vec{y}} - \right.$$

$$- \left. \left( q^0 b^{mn}(\vec{p}) \tilde{c}^l(\vec{q}) - q^l b^{mn}(\vec{p}) \tilde{c}^0(\vec{q}) \right) e^{-i(\vec{p} - \vec{q}) \cdot \vec{y}} - \left( q^0 (b^{mn})^\dagger(\vec{p}) \tilde{c}^l(\vec{q}) - q^l (b^{mn})^\dagger(\vec{p}) \tilde{c}^0(\vec{q}) \right) e^{+i(\vec{p} + \vec{q}) \cdot \vec{y}} \right].$$

(37)

Integrating out $d^3y$ and $d^3q$ from the above equation, we obtain

$$Q_d \approx -\frac{1}{2} \int \frac{d^3p}{2} \varepsilon_{lmn} \left[ p^0 b^{mn}(\vec{p}) \left( \tilde{c}^l(\vec{p}) - (\tilde{c}^l)^\dagger(\vec{p}) \right) - p^0 (b^{mn})^\dagger(\vec{p}) \left( \tilde{c}^l(\vec{p}) - (\tilde{c}^l)^\dagger(\vec{p}) \right) + p^l b^{mn}(\vec{p}) \left( \tilde{c}^0(\vec{p}) + (\tilde{c}^0)^\dagger(\vec{p}) \right) + p^l (b^{mn})^\dagger(\vec{p}) \left( \tilde{c}^0(\vec{p}) + (\tilde{c}^0)^\dagger(\vec{p}) \right) \right].$$

(38)
Now substituting this value of $Q_d$ in the l.h.s. of expression (34), we have following expression having four commutators

$$
\left[ b_{ij}(\vec{k}), Q_d \right] = -\frac{1}{2} \int \frac{d^3p}{2} \varepsilon_{lmn} \left( p^0 [b_{ij}(\vec{k}), b^{mn}(\vec{p})] (\bar{c}^l(-\bar{p}) - (\bar{c}^l)^{\dagger}(-\bar{p})) 
- p^0 [b_{ij}(\vec{k}), (b^{mn})^{\dagger}(\bar{p})] (\bar{c}^l(\bar{p}) - (\bar{c}^l)^{\dagger}(\bar{p})) 
+ p^l [b_{ij}(\vec{k}), b^{mn}(\vec{p})] (\bar{c}^0(-\bar{p}) + (\bar{c}^0)^{\dagger}(\bar{p})) 
+ p^l [b_{ij}(\vec{k}), (b^{mn})^{\dagger}(\bar{p})] (\bar{c}^0(\bar{p}) + (\bar{c}^0)^{\dagger}(\bar{p})) \right),
$$

(39)

In fact there exist, in totality, sixteen commutators out of which eight commutators (that involve the (anti-)ghost fields) are zero because of the fact that the (anti-)ghost fields are decoupled from the rest of the bosonic fields present in the theory. Hence, the (anti-)commutators of the (anti-)ghosts fields with the rest of all the bosonic fields are zero. Now rearranging the various terms, we have following expression

$$
\left[ b_{ij}(\vec{k}), Q_d \right] = -\frac{1}{2} \int \frac{d^3p}{2} \varepsilon_{lmn} \left( [b_{ij}(\vec{k}), b^{mn}(\vec{p})] (p^0 \bar{c}^l(-\bar{p}) - p^0 (\bar{c}^l)^{\dagger}(\bar{p})) 
+ p^l \bar{c}^0(-\bar{p}) + p^l (\bar{c}^0)^{\dagger}(\bar{p}) 
- [b_{ij}(\vec{k}), (b^{mn})^{\dagger}(\bar{p})] (p^0 \bar{c}^l(\bar{p}) - p^0 (\bar{c}^l)^{\dagger}(\bar{p}) - p^l \bar{c}^0(\bar{p})) 
- p^l (\bar{c}^0)^{\dagger}(-\bar{p}) \right). 
$$

(40)

Comparing the r.h.s. of (34) and (40), we get the following commutation relations amongst the creation and annihilation operators

$$
[b_{ij}(\vec{k}), t^{mn}(\vec{p})] = 0, \quad [b_{ij}(\vec{k}), (b^{mn})^{\dagger}(\bar{p})] = -(\delta_i^m \delta_j^n - \delta_i^n \delta_j^m) \delta^{(3)}(\vec{k} - \vec{p}).
$$

(41)

It is worthwhile to mention that, instead of substituting the value of $Q_d$ (cf. (38)) into equation (34), if we substitute its value into equation (35), we get exactly same set of canonical commutation relations as in (41).

Similar exercise can also be done with the other basic fields of the present theory. It is straightforward to check that, with the help of symmetry principle, we get the same set of (anti-)commutation relations amongst the creation and annihilation operators as obtained from the conventional Lagrangian formalism (cf. (49) below). It is worthwhile to mention that all the six continuous symmetries of the present 4D free Abelian 2-form gauge theory lead to the exactly same set of (anti-)commutation relations amongst the creation and annihilation operators (cf. (49) below).
5. Canonical brackets from Lagrangian formalism

It is evident to calculate the canonical conjugate momenta form the Lagrangian (1) and are listed below:

\[
\Pi_{(\phi_1)} = \dot{\phi}_1 + \partial_t B^{0i}, \quad \Pi_{(\phi_2)} = -\dot{\phi}_2 + \frac{1}{2} \varepsilon^{ijk} \partial_i B_{jk}, \quad \Pi_{(\beta)} = -\dot{\beta},
\]

\[
\Pi_{(\bar{\beta})} = -\dot{\bar{\beta}}, \quad \Pi_{(C)}^0 = \frac{1}{2} (\partial \cdot \vec{C}), \quad \Pi_{(C)}^i = - (\partial^i \vec{C} - \partial \vec{C}^i),
\]

\[
\Pi_{(\bar{C})} = (\partial^0 C^i - \partial_i C^0), \quad \Pi_{(C)}^{ij} = \frac{1}{2} H^{0ij} - \frac{1}{2} \varepsilon^{ijk} (\partial_k \phi_2),
\]

\[
\Pi_{(\bar{C})}^0 = -\frac{1}{2} (\partial \cdot C), \quad \Pi_{(C)}^{0i} = \frac{1}{2} (\partial_0 B^{0i} + \partial_j B^{ji} - \partial_i \phi_1),
\]

(42)

here we have adopted the left derivative prescription for the fermionic fields.

Therefore, the canonical (anti-)commutation relations amongst the basic fields of the theory and corresponding canonically conjugate momenta can be given as (at equal time \(t\))

\[
[\phi_1(\vec{x}, t), \dot{\phi}_1(\vec{y}, t)] = i \delta^{(3)}(\vec{x} - \vec{y}), \quad [\phi_2(\vec{x}, t), \dot{\phi}_2(\vec{y}, t)] = -i \delta^{(3)}(\vec{x} - \vec{y}),
\]

\[
[\beta(\vec{x}, t), \dot{\beta}(\vec{y}, t)] = -i \delta^{(3)}(\vec{x} - \vec{y}), \quad [\bar{\beta}(\vec{x}, t), \dot{\bar{\beta}}(\vec{y}, t)] = -i \delta^{(3)}(\vec{x} - \vec{y}),
\]

\[
\{C_0(\vec{x}, t), \dot{\bar{C}}^0(\vec{y}, t)\} = 2i \delta^{(3)}(\vec{x} - \vec{y}),
\]

\[
\{C_0(\vec{x}, t), \dot{\bar{C}}^j(\vec{y}, t)\} = -i \delta^j_i \delta^{(3)}(\vec{x} - \vec{y}),
\]

\[
\{\bar{C}_0(\vec{x}, t), \dot{\bar{C}}^0(\vec{y}, t)\} = -2i \delta^{(3)}(\vec{x} - \vec{y}),
\]

\[
\{\bar{C}_0(\vec{x}, t), \dot{\bar{C}}^j(\vec{y}, t)\} = i \delta^j_i \delta^{(3)}(\vec{x} - \vec{y}),
\]

\[
[B_{0b}(\vec{x}, t), \dot{\bar{B}}^b(\vec{y}, t)] = i \delta^b_i \delta^{(3)}(\vec{x} - \vec{y}),
\]

\[
[B_{0j}(\vec{x}, t), \dot{\bar{B}}^b(\vec{y}, t)] = i (\delta^b_i - \delta^b_j) \delta^{(3)}(\vec{x} - \vec{y}).
\]

(43)

All the rest of (anti-)commutators are zero. Now our aim is to calculate the (anti-)commutation relations amongst the creation and annihilation operators of the theory. In order to simplify our computations, we re-express the normal mode expansions of the basic fields (cf. (16)) according to [13]

\[
B_{\mu\nu}(\vec{x}, t) = \int d^3x \left( f^*_k(x) b_{\mu\nu}(k) + f_k(x) b^*_k(\mu\nu) \right),
\]

\[
C_\mu(\vec{x}, t) = \int d^3x \left( f^*_k(x) c_\mu(k) + f_k(x) c^*_\mu(k) \right),
\]

\[
\bar{C}_\mu(\vec{x}, t) = \int d^3x \left( f^*_k(x) \bar{c}_\mu(k) + f_k(x) \bar{c}^*_\mu(k) \right),
\]

\[
\beta(\vec{x}, t) = \int d^3x \left( f^*_k(x) b(k) + f_k(x) b^*_k(k) \right),
\]

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\[
\tilde{\beta}(\vec{x}, t) = \int d^3x \left( f^*_k(x) \bar{b}(k) + f_k(x) \bar{b}^*(k) \right),
\]
\[
\phi_1(\vec{x}, t) = \int d^3x \left( f^*_k(x) f_1(k) + f_k(x) f^*_1(k) \right),
\]
\[
\phi_2(\vec{x}, t) = \int d^3x \left( f^*_k(x) f_2(k) + f_k(x) f^*_2(k) \right),
\]
(44)

where \(f_k(x)\) and \(f^*_k(x)\) are defined as
\[
f_k(x) = \frac{e^{-ik\cdot x}}{\sqrt{(2\pi)^3 2k_0}}, \quad f^*_k(x) = \frac{e^{ik\cdot x}}{\sqrt{(2\pi)^3 2k_0}}. \quad (45)
\]

These functions (i.e. \(f_k(x)\) and \(f^*_k(x)\)) form an orthonormal set. It is self-evident from the following conditions
\[
\int d^3x f^*_k(x) i \overrightarrow{\partial_0} f_{k'}(x) = \delta^{(3)}(\vec{k} - \vec{k'}),
\]
\[
\int d^3x f^*_k(x) i \overrightarrow{\partial_0} f^*_k(x) = 0, \quad \int dx f_k(x) i \overrightarrow{\partial_0} f_k(x) = 0. \quad (46)
\]

In the above equation, the following standard definition of operator \(\overrightarrow{\partial_0}\) has been taken into account
\[
A \overrightarrow{\partial_0} B = A(\partial_0 B) - (\partial_0 A) B. \quad (47)
\]

Using the above relationships, we express the creation and annihilation operators in terms of the basic fields and the orthonormal functions (i.e. \(f_k(x)\) and \(f^*_k(x)\)) as follows:
\[
b_{\mu}(k) = \int d^3x B_{\mu\nu}(x) i \overrightarrow{\partial_0} f_k(x), \quad b^\dagger_{\mu}(k) = \int d^3x f^*_k(x) i \overrightarrow{\partial_0} B_{\mu\nu}(x),
\]
\[
c_{\mu}(k) = \int d^3x C_{\mu}(x) i \overrightarrow{\partial_0} f_k(x), \quad c^\dagger_{\mu}(k) = \int d^3x f^*_k(x) i \overrightarrow{\partial_0} C_{\mu}(x),
\]
\[
c_{\bar{\mu}}(k) = \int d^3x \bar{C}_{\bar{\mu}}(x) i \overrightarrow{\partial_0} f_k(x), \quad c^\dagger_{\bar{\mu}}(k) = \int d^3x f^*_k(x) i \overrightarrow{\partial_0} \bar{C}_{\bar{\mu}}(x),
\]
\[
b(k) = \int d^3x \beta(x) i \overrightarrow{\partial_0} f_k(x), \quad b^\dagger(k) = \int d^3x f^*_k(x) i \overrightarrow{\partial_0} \beta(x),
\]
\[
\bar{b}(k) = \int d^3x \bar{\beta}(x) i \overrightarrow{\partial_0} f_k(x), \quad \bar{b}^\dagger(k) = \int d^3x f^*_k(x) i \overrightarrow{\partial_0} \bar{\beta}(x),
\]
\[
f_1(k) = \int d^3x \phi_1(x) i \overrightarrow{\partial_0} f_k(x), \quad f^\dagger_1(k) = \int d^3x f^*_k(x) i \overrightarrow{\partial_0} \phi_1(x),
\]
\[
f_2(k) = \int d^3x \phi_2(x) i \overrightarrow{\partial_0} f_k(x), \quad f^\dagger_2(k) = \int d^3x f^*_k(x) i \overrightarrow{\partial_0} \phi_2(x). \quad (48)
\]
Now it is straightforward to check, using above relationships (48), that we have the following non-vanishing (anti-)commutation relations amongst the creation and annihilation operators of the 4D free Abelian 2-form gauge theory:

\[
\begin{align*}
[b(\vec{k}), b^\dagger(\vec{k}')] &= \delta^{(3)}(\vec{k} - \vec{k}'), \\
[f_1(\vec{k}), f_1^\dagger(\vec{k}')] &= -\delta^{(3)}(\vec{k} - \vec{k}'), \\
\{c_0(\vec{k}), (\bar{c}^0)\dagger(\vec{k}')\} &= 2\delta^{(3)}(\vec{k} - \vec{k}'), \\
\{c_i(\vec{k}), (\bar{c}^i)\dagger(\vec{k}')\} &= \delta^i_j \delta^{(3)}(\vec{k} - \vec{k}'), \\
[b_i(\vec{k}), (b^0)_i\dagger(\vec{k}')] &= -\delta^i_j \delta^{(3)}(\vec{k} - \vec{k}'), \\
[b_{ij}(\vec{k}), (b^{mn})_i\dagger(\vec{k}')] &= -\left(\delta^m_i \delta^n_j - \delta^n_i \delta^m_j\right)\delta^{(3)}(\vec{k} - \vec{k}').
\end{align*}
\]

(49)

All the rest of the (anti-)commutators amongst the creation and annihilation operators are zero.

6. Conclusion

We have derived all the (anti-)commutation relations amongst the creation and annihilation operators of 4D free Abelian 2-form gauge theory with the help of symmetry principle. The key feature of our present investigation is that while deriving the (anti-)commutation relations amongst the creation and annihilation operators, although we have taken the help of spin-statistics theorem and normal ordering but we have not used the concept of (graded) Poisson brackets. Instead of latter, we have taken the help of continuous symmetry transformations present in the theory. All the six continuous symmetries (and their corresponding generators) present in the theory lead to the same set of (anti-)commutation relations amongst the creation and annihilation operators. This is a unique feature of our present investigation.

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