Analysis of adhesive elastic contact between a silica glass lens and silicone rubber using the JKR theory

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Abstract Contact between a silica glass lens and silicone rubber is experimentally investigated by simultaneously measuring displacement, force and contact radius. The relationship between these three parameters is derived using elastic theory. The discrepancy between the theoretical relationship and the experimental results is observed to increase as the deformation of the silicone rubber increases. Under smaller deformation conditions, the elastic theory shows good agreement with the experimental results, although infinite stress on the edge of the contact area is predicted in the theory, and time dependence and adhesion hysteresis are observed in all experiments. It is suggested that time dependence and adhesion hysteresis in contact are not induced by the deformation of the bulk of the silicone rubber, but are induced by surface effects. The result suggests that the applicability limit of the elastic theory must be carefully considered in the JKR analysis of point contact for polymers.

1. Introduction

Adhesion phenomena play a significant role in the engineering fields of contact mechanics and tribology, and are essential for general production methods that rely on adhesive forces. Understanding the adhesive elastic contact that takes place in adhesion phenomena has important applications in device design, e.g. for grip-and-release devices [1]. To describe adhesive elastic contact, Johnson, Kendall and Roberts proposed the JKR model [2]. In the JKR model, an elastic parabolic punch is compressed against a flat rigid surface, or conversely, a hard parabolic punch is compressed against a flat elastic substrate (i.e. the point contact model). This model is the simplest model for adhesive elastic contact, and assumes elastic deformation while taking into account the work of adhesion between materials. Since the JKR model assumes the linear elasticity of a material (the static state) and the equilibrium state of total energy, it is difficult to guarantee a good approximation for a practical contact system of polymeric materials which does not insure perfectly elasticity. However, the JKR model is sometimes used when considering the contact between polymeric materials [3]. Barthel [4] has reviewed many experimental studies and discussed the effects of the non-elasticity on the contact, including non-linearity, time dependence, etc. However, this has been dealt with based on the equilibrium state of total energy in the contact model. Although the polymer has non-elasticity, it must exhibit linear elasticity in a smaller deformation condition. In the present study, the theoretical consideration of the non-equilibrium state is discussed using the JKR model. The contact between a silica glass lens and silicone rubber is investigated experimentally, where the displacement, the force, and the contact radius of the contact model are measured.
simultaneously. By analyzing our experimental results while varying the amount of deformation in the silicone rubber, the applicability limit of the elastic theory is investigated.

2. Theoretical considerations

Figure 1 shows a schematic illustration of the point contact model describing contact between a rigid parabolic punch and a semi-infinite elastic body. This model is axisymmetric, and we assume linear elasticity, small deformation, and no friction between the two bodies.

The theory of point contact can be expressed using the JKR theory. However, it is difficult to realize the equilibrium state of total energy in experiments, as attaining a perfectly static state during an experimental process is nearly impossible. Therefore, we must extend the JKR theory to the non-equilibrium state. In Figure 1, the force \( F \) between the rigid parabolic punch and the semi-infinite elastic body is balanced using the relation \( F = P_H - P_B \) when the displacement \( \delta \) is given. Here \( P_H \) is the compressive force due to the Hertz stress distribution and \( P_B \) is the adhesive force due to the Boussinesq stress distribution. Note that infinite stress is predicted on the edge of the contact area by the JKR stress distribution [5]. Then, the force can be expressed as

\[
F = \frac{2Ea}{1-v^2} \left( \delta - \frac{a^2}{3R} \right).
\]  

The above expression shows the relationship between the force \( F \), the contact radius \( a \) and the displacement \( \delta \). Using equation (1), the Young’s modulus can be experimentally evaluated by measuring these three parameters, and the linear elasticity can be discussed based on experimentally obtained values of the Young’s modulus.

3. Experimental procedure

In the experiments, the force \( F \), the contact radius \( a \), and the displacement \( \delta \) must be measured, and these values are used in equation (1). To accomplish this we constructed an experimental system as schematically illustrated in Figure 2. In the experiment, the displacement \( \delta \) was controlled by a vertical motorized stage, while the force \( F \) and the contact radius \( a \) were measured simultaneously by digital
balance and microscope, respectively. The experiment was set up on the clean bench of a vibration isolation table, and the room temperature was set to 20 °C with 35% humidity. A silicone rubber sheet with thickness 10 mm was used as the semi-infinite elastic body, and a silica glass lens with radius of curvature 207.6 mm and diameter 40 mm was used as the rigid parabolic punch. The experiment was performed by controlling the displacement of the lens against the silicone rubber using the motorized stage. The displacement was controlled in steps of 1 µm, and the waiting duration between steps was set to 60 seconds to account for the relaxation time of the silicone rubber. First, we set the displacement to 0 µm when the two bodies are in contact. Second, for the loading process, we varied the motorized stage from 0 µm to the maximum loading displacement. Third, for the unloading process, we varied the motorized stage until the lens detached from the silicone rubber. The combined loading and unloading processes were considered as one cycle. The maximum loading displacements were chosen to be 30, 60 and 120 µm on each cycle for confirming the elasticity. The force and the contact radius were measured just before the displacement of the lens changed.

![Figure 2. Schematic illustration of the experimental system.](image)

**Figure 2.** Schematic illustration of the experimental system. (a) Lens: Silica glass lens with radius of curvature $R=0.2076$ m and diameter 40 mm, (b) Motorized stage: Motorized vertical movement with resolution 1 µm, (c) Polymer: Silicone rubber sheet with thickness 10 mm, (d) Digital balance: Strain gauge type with resolution 0.01 g, (e) Microscope: Resolution 1600×1200 pixel.

4. Results and discussion

Figures 3 and 4 show the measured force and contact radius for each cycle as functions of the displacement, for maximum displacements of 30, 60, and 120 µm. The loading and unloading processes in Figures 3 and 4 refer to increasing and decreasing of the displacement, respectively (i.e. compressing and decompressing the lens to the rubber). Regions for which the contact radius is constant are observed, and are indicated in Figure 4 as i, ii, and iii. These regions correspond to regions i, ii, and iii in Figure 3 of each cycle. In the constant-radius regions, the deformed equation from equation (1) can be used,

\[ F = \frac{2Ea_{\text{const}}}{1-\nu^2} \left( \delta - \frac{a_{\text{const}}^2}{3R} \right), \]

\[ (2) \]
where $a_{\text{const.}}$ is the constant value of contact radius in regions i, ii, or iii. Equation (2) indicates that the force $F$ is a linear function of the displacement $\delta$. Furthermore, the slopes of regions i, ii, and iii in Figure 3 can be approximated linearly. Using equation (2) and the approximated slope, the Young’s modulus $E$ is evaluated as

$$E = \frac{(1 - \nu^2)a}{2a_{\text{const.}}}$$

where $\alpha$ is the approximated slope of regions i, ii, and iii in Figure 3. The approximated slope $\alpha$, the constant value of contact radius $a_{\text{const.}}$, and the evaluated Young’s modulus $E$ in each cycle are calculated in Table 1.

![Figure 3](image1.png)

**Figure 3.** In the loading process, an increase of compressive force is observed. In the unloading process, the compressive force is decreased until the maximum adhesive forces, and then the lens is detached from the silicone rubber. The slopes of regions i, ii, and iii can be approximated linearly.

![Figure 4](image2.png)

**Figure 4.** In the loading process, an increase in the contact radius is observed. In the unloading process, the contact radius is constant in regions i, ii, and iii of each cycle at first, and then progressive decreasing of the contact radius is observed.

|   | $\alpha$ (N/m) | $a_{\text{const.}}$ (mm) | $E$ (Pa) |
|---|---------------|-----------------|--------|
| i | 4580.3        | 3.18            | 0.539  |
| ii| 5432.1        | 3.93            | 0.518  |
| iii| 6281.8       | 4.99            | 0.471  |

Table 1. Evaluation results.

From Table 1, a decrease in the evaluated Young’s modulus is observed as the maximum loading displacement is increased. This tendency exhibits typical strain-stress characteristics of polymers in large deformation, which means that the linear elasticity is not satisfied. Therefore, there is the limit of the linear elasticity in the experimental contact system.

The displacement was controlled in 1 $\mu$m step-movements with a waiting duration of 60 seconds between steps. The maximum strain of vertical deformation was chosen under 1.2% and the contact radius was observed to be smaller than 2.4% of the lens radius $R=0.2076$ m. Although the experimental conditions appear to satisfy the assumptions of the theory, a decrease in the Young’s modulus is observed as shown in Table 1. It is considered that this tendency is attributed to the
increase of the effects from the infinite stress distribution on the edge of the contact area by the Boussinesq. However, it is clear that the Young’s modulus should have a constant value in limit of small deformations. Hence, it can be concluded that the Young’s modulus of the silicone rubber is 0.539 MPa in the linear elasticity regime.

Figure 5. The calculated results of the expected force using the estimated $E=0.539$ MPa with equation (1) as a function of the displacement (a) and the contact radius (b). The experimental results from each cycle are also plotted.

5. Conclusions
Using the experimentally determined Young’s modulus $E=0.539$ MPa, the expected force of the linear elastic state can be calculated using equation (1) with the experimentally measured values of displacement $\delta$ and contact radius $a$. The calculated results of the expected force and the experimental results for all cycles are plotted in Figure 5 as a function of the displacement (a) and the contact radius (b). As shown in Figure 5 (a) and (b), the cycle of maximum displacement $120 \mu m$ shows a larger discrepancy than the cycle of maximum displacement $30 \mu m$. This discrepancy seems to increase as the displacement increases. However, the expected force for the cycle with maximum displacement $30 \mu m$ shows good agreement with the experimental results. This result suggests that the applicability limit of the elastic theory must be carefully considered in the JKR approximation of point contact using polymeric materials. Furthermore, in all experiments, time dependence is observed in the contact radius and the force, for a given displacement. Adhesion hysteresis [6], which is the difference between the loading path and the unloading path, is observed in Figure 5 (a). This means that the relationships among parameters show the elasticity, even when the time dependence and adhesion hysteresis are observed in case of extremely small deformation contacting processes. This suggests that the time dependence and the adhesion hysteresis in the contact is not induced by the deformation of the bulk of the silicone rubber, but would be induced by other effects such as a surface effect.

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