A Contact Model for the Effect of Particle Coating on Improving the Flowability of Cohesive Powders

Renwei Mei, Hong Shang
Department of Aerospace Engineering, Mechanics & Engineering Science*

James F. Klausner, and Elizabeth Kallman
Department of Mechanical Engineering, University of Florida*

Abstract

Coating or blending of fine particles on the surface of primary powder particles is often applied to improve and control the flow behavior of the powder. The effect of coating primary powder particles with fine particles on the cohesion force between primary powder particles is quantitatively examined. The JKR theory is extended to include the effect of particle coating on the force-displacement relationship due to surface energy and elastic deformation. It is shown that the cohesion force between two primary particles in the presence of a fine coating particle is directly proportional to the size ratio of the coating particle to the host powder particle and results in a drastic reduction in the cohesion force. Through discrete element simulation of powder flow, which uses the force-displacement relationship based on the extended JKR theory, the improved flowability is demonstrated. Competing effects of the coating and shear induced migration on the macroscopic behavior of the powder flow are discussed. The effect of coating on improving the flowability is also experimentally demonstrated by comparing the measured angles of repose at a static condition and the flow rates of the gravity driven flow through a funnel for powders with and without particle coating.

1. Introduction

Fine powders, 30 μm or less, are of significant industrial importance. Unfortunately, these materials are the most difficult to process and handle of all powders. Fine powders are characterized by cohesive behavior. Inter-particle surface forces, such as Van der Waals force, capillary force, electrostatic force and the like, are relatively strong. They play an important role in particle agglomeration which results in poor flow behavior. Many efforts have been made to improve the flowability using techniques such as aeration, vibration, and coating or blending with nanometer size particles [1, 2, 3, 4, 5]; and there is an increasing use of the coating of fine particles (or flow conditioners) on the primary (or host) powder particles to improve the flowability of the otherwise cohesive powders. Experimental investigations on the effects of particle coating have been reported [6, 7]. By measuring the rheological parameters of bulk powder structure, represented by tensile strengths and plastic deformation coefficients of bulk powders, Kono et al. [7, 8] studied the working mechanism of flow conditioners (anti-caking agents, glidants or lubricants) under small deformation. Hollenbach et al. [9] demonstrated that the bulk properties of treated powder as a solid are strongly influenced by interactions between the host powder and the conditioner particles. However, a basic understanding of the underlying improved flowability mechanism on the microscopic level is lacking [5]. Quantitative modeling of powder flows in the presence of coating particles is scarce.

It has long been recognized that the attractive force between adjacent particles is responsible for the cohesive behavior. A better understanding of the flow of the cohesive powders would most likely come with a better understanding of the inter-particle forces. Visser [5] pointed out that the dominant interactive force between fine particles is the Van der Waals force of attraction. Other attractive forces, such as capillary and electrostatic forces, may also operate; but in general, they are smaller than the omnipresent Van der Waals force.
For most of dry processes, the capillary force is negligible. The electrostatic force may be controlled by humidity.

In quantitatively discussing the effect of particle coating on reducing cohesion between the host particles, the Van der Waals force is difficult to use directly since it is singular upon the contact of two surfaces. Specifying an initial separation distance may be appropriate in some cases but the result will be very sensitive to the initial separation distance (c.f. Visser [5]). It is also difficult to implement in the simulation, modeling, or any quantitative estimation of powder flow behavior. From the viewpoint of modeling cohesive powders, the following issues must be addressed:

i. How do coating particles change the microscopic behavior of the powder?
ii. How does the size of the coating particles affect cohesion and the flowability of the powder?
iii. How does the coating of fine particles affect the rheological behavior of the powder flow when the powder is in motion?

In this paper, the effects of fine particle coating on the surface of host powder particles is quantitatively examined. The first two issues will be addressed by extending the JKR theory [10] for the force-displacement relationship between two primary host particles in the presence of a coating particle and accounting for both the elastic deformation and the attractive Van der Waals force. Surface energy is used to represent the effect of Van der Waals force in order to model the force-displacement relationship at contact. Except for cases involving high pressure, high density compaction or high velocity impact, the deformation on the surface of the powder upon contact may be considered to be within the elastic range. The extended JKR theory shows that the cohesion force between two primary powder particles in the presence of a fine coating particle is directly proportional to the size ratio of the coating particle to the host powder particle resulting in a drastic reduction in the cohesion force. The third issue raised above is more involved. It is addressed by presenting the results obtained from a discrete element simulation of powder flows between a moving and stationary plate to illustrate the effect of coating on the velocity and concentration profiles. The model and simulation results indicate that the exact number of coating particles has only minor effect on the flowability once the cohesion force is reduced by one or more coating particles. For coated particles, the angle of repose is noticeably smaller than that for uncoated ones. For a gravity driven powder flow through a funnel, the treated powder flows easily while blockage occurs frequently with the untreated powders. Test results also show the angle of repose and flow rate are not sensitive to the exact number of coating particles on the host particle surface resulting from two different coating methods.

2. Reduction of the cohesion force and stiffness in the force-displacement relationship via particle coating

Consider the deformation of two primary powder particles of size $R_1$ and $R_2$ under a load $P$ as shown in Fig. 1. In the absence of a fine particle, the JKR theory [10] gives

$$P = \frac{4E'^*a^3}{3R'^*} - 2\sqrt{2\pi\Gamma E'^*a^3},$$  \hspace{1cm} (1)

where $a = \frac{a^2}{R'^*}$, $R'^* = \frac{R_1 + R_2}{2}$. In (1) & (2), the respective first terms on the right hand side (RHS) result from the elastic deformation and the energy of cohesion of both surfaces, $a$ is the radius of the contact area, $E$ and $\nu$ are Young’s modulus and Poisson ratio of the materials. For identical spheres, $R'^* = \frac{R}{2}$. In (1) & (2), the respective first terms on the right hand side (RHS) result from the elastic deformation while the respective second terms result from the surface force. Due to the attractive surface force, the maximum tensile force $(-P_c)$ which is required to separate the particles at contact is given by

$$-P_c = \frac{3}{2} \pi \Gamma R'^*. \hspace{1cm} (5)$$

![Fig. 1 Deformation of two primary particles under a load P.](image)
At the point of separation, the displacement is given by

$$\alpha = -\frac{3}{4} \left( \pi^2 R^* / E^* \right)^{1/3}. \quad (6)$$

The maximum separation force $-P_c$ can also be interpreted as the adhesion force between two primary powder particles. If the effect of the surface energy is small, the second terms on the RHS of eqns. (1-2) may be neglected, and the force-displacement relationship for Hertzian contact results,

$$P = \frac{4}{3} E^* R^*^{1/2} a^{3/2} \quad \text{for } \Gamma = 0. \quad (7)$$

For a fine particle of radius $R_t = R_t / R_1 < R_1$ in contact with the primary powder particles, $R^*$ is approximately equal to $R_1$, as easily seen from (3).

Effect of one coating particle in the contact area

Consideration is now given to the load-displacement relationship between two identical spheres in the presence of a very fine coating sphere of radius $R_1 < R_1$ as shown in Fig. 2. Assuming that the load is small so that the Hertzian contact theory can be applied locally at each contact, the force-displacement relationship at each contact is given by (1). In general, the material of the coating particle is different from that of the powder so that each material possesses different surface energy. The effective surface energy at the contact of dissimilar materials is given by

$$\Delta\gamma = \gamma_1 + \gamma_2 - \gamma_{12}. \quad (8)$$

where $\gamma_1$ and $\gamma_2$ are the intrinsic surface energies of solids 1 and 2 and $\gamma_{12}$ is the energy of the interface in contact. For same materials, $\gamma_{12} = 0$ so that $\Delta\gamma = 2\gamma = \Gamma$ and is sometimes referred to as the cohesive energy. Since there are two contacts on the coating particle, the total displacement due to the relative motion of the host particles

$$-a = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 - 2R_1 - 2R_t}, \quad (9)$$

contributes only half to the surface deformation at contact, where $(x_i, y_i, z_i)$ are the coordinates at the center of the $i$th host particle. Hence,

$$\frac{1}{2} \alpha \sim \frac{a^2}{R_t} - \sqrt{2\pi \Delta\gamma E^* a^3}. \quad (10)$$

The load-contact area relationship at the contact is given by

$$P \sim \frac{4E^* a^3}{3R_t} - 2\sqrt{2\pi \Delta\gamma E^* a^3}. \quad (11)$$

In the above,

$$E_i^{-1} = \frac{1 - \nu_i^2}{E_i}$$

is the inverse of the effective Young's modulus of the primary particles in the presence of fine coating particles. The cohesion force, which must be overcome in order to separate these two primary particles, in the presence of a fine coating particle, is thus

$$-P_c \sim \frac{3}{2} \pi\Delta\gamma R_t. \quad (12)$$

At the point of separation, the displacement is

$$\alpha \sim \frac{3}{2} \left[ \frac{\pi^2 \Delta\gamma^2 R_t^{1/3}}{E_i^{1/2}} \right]^{1/3}. \quad (13)$$

Comparing (13) with (5), it is easily concluded that for a comparable surface energy, $\Delta\gamma/E = O(1)$, the cohesion force in the presence of a fine coating particle is much smaller than that for the untreated powders,

$$-P_c \ll -P_c \quad \text{for } \frac{R_t}{R_1} \ll 1. \quad (15)$$

For a typical case of $R_t < 100 \text{ nm}$ and $R_t > 5 \mu m$, it gives $2R_t/R_1 < 0.04$ which implies an order of magnitude drop in the cohesion force. Due to the drastic reduction in the cohesion force, $-P_c \ll -P_c$, it is much less likely for the powders to adhere upon collision during flow.

The reduction in $P_c$ also implies that the repulsive force due to elastic deformation now dominates over the surface force because of a much smaller value of $R_t$ in (10). Neglecting the effect of $\Delta\gamma$ in (10), a leading order approximation for the force-displacement relationship is obtained

$$P \sim \frac{4E^* a^3}{3R_t} \sim \frac{4E^* a^3}{3R_t} \left[ \frac{aR_t}{2} \right]^{3/2} = 2\pi \frac{1}{3} E^* R^*^{1/2} a^{3/2} \quad (16)$$

which is valid except in a very small neighborhood near $a=0$. Near $a=0$, the absolute error in the $P-a$ relationship given by (8) is still small due to a much reduced cohesion force $-P_c$. It is interesting to note that while the cohesion force is reduced by a factor of $R_t/2R_t$, the stiffness is only reduced by a factor of $2(R_t/R_1)^{1/2}$ by comparing (7) with (8). In essence, the force-displacement relationship for the primary powder particles in the presence of coating...
particle behaves like that of cohesionless granular particles with a reduced stiffness (associated with the elastic deformation) for a given relative particle displacement $a$.

The importance of eqn. (15) is that it quantitatively relates the reduction in the cohesion force between two primary powder particles to the size ratio of the coating particle to the host powder particle. This quantitative relationship is not possible if the attractive surface force is described using the Van der Waals force expression due to the singular nature of the force at contact. The asymptotic relationship given by (16) clearly shows how the size of the coating particle, $R_r$, quantitatively changes the microscopic response of the powder particles to variations in the relative displacement.

Effect of multiple coating particles in the contact area

In reality, there may be more than one coating particle in the vicinity of the contact point between two host particles as shown in Fig. 3. Thus it is necessary to consider the effect of neighboring coating particles on the $P-a$ relationship. Assuming that all coating particles adhere to host particles, it can be estimated that the average number of coating particles on the surface of each host powder particles is

$$n = x \frac{\rho_l}{\rho_r} \left( \frac{R_l}{R_f} \right)^3$$

(17)

where $x$ is the mass fraction of the coating particles, and $\rho$ denotes density. Since the surface area of a spherical powder particle available for coating particle is $4\pi R_l^2$, the number of coating particles per unit surface area is $n/4\pi R_l^2$. It is easily seen that the average center-to-center distance, $L$, among the coating particles on each powder particle surface is

$$L = \left( \frac{4\pi R_l^2}{m} \right)^{1/2}$$

(16)

However, in the contact region the other host powder particle also has, on average, an equal number of coating particles on its surface. Effectively, the area concentration of the coating particle at contact is double the amount given by (17) so that

$$\frac{L}{R_f} = \left[ \frac{2\pi \rho_l R_l}{x \rho_r R_f} \right]^{1/2}$$

(18)

For a typical case with $x=0.01$, $\rho_l \sim \rho_r$, $R_l=5 \mu m$, and $R_f=0.1 \mu m$, the above gives $L/R_f \sim 3.5$.

For a coating particle centered at $(r, z)=0$ as shown in Fig. 3a, the surface-to-surface distance between the host particles at $r$ before the deformation is

$$\delta(r) \sim 2R_r + r^2/R_1$$

(20)

At $r \sim L$, eqns. (19) give

$$\delta(L) \sim 2R_r \sim \pi \frac{\rho_l}{\rho_r} \left( \frac{R_l}{R_f} \right)^3$$

(21)

For the typical case under consideration ($x=0.01$, $R_l/R_1=0.02$), one obtains $(\delta \sim 2R_1)/R_f \sim 0.25$. This implies that the central coating particle in Fig. 3a would have deformed significantly before the neighboring coating particles can even contact with the other primary particle. Thus it is unlikely that the neighboring coating particles have an effect on the $P-a$ relationship between two primary particles in this configuration.

However, for the configuration shown in Fig. 3b, it is possible to have 3 or even more coating particles undergoing simultaneous deformation between two host powder particles. As a first approximation, it is assumed that the mutual interaction caused by the neighboring coating particles on the global deformation of host particles be neglected so that each contact can be treated independently. Since the coating particles are close to $(r, z)=0$, the deformation may be treated as if each coating particle is at $(r, z)=0$. Thus the linear superposition gives the following approximate $P-a$ relationship,

$$P \sim m \left( \frac{4\pi \gamma E^* d^3}{3R_f} - 2\sqrt{2\pi \gamma E^* d^3} \right)$$

(22)

$$a/2 \sim d^2/2 \sqrt{2\pi \gamma E^*}$$

(23)

where $m$ is the number of fine particles in contact. Due to multiple contacts, the cohesion force $(-P_{cm})$ is now given by

$$-P_{cm} = m \frac{3}{2} \pi \gamma R_f$$

(24)

Since the configuration in Fig. 3b is more stable than that in Fig. 3a in a nearly static condition and it takes three points to support a surface, it is conceivable that $m$ may take a value of 3 when the particle collision velocity is low and the coating is
At a high particle collision velocity, the particle collision time is shorter so that it is conceivable that the deformation may center only on one coating particle. Although the value of $m$ is not known in general and a probabilistic approach for choosing $m$ may be better to describe the effects of multiple coating particles, it is noted that a reduction in the cohesion force by a factor of $\frac{R_d}{2mR_f}$ is still quite significant even for $m=3$.

Fig. 4 shows the dimensionless force-displacement curve between two primary powder particles for three cases: i) no coating particle; ii) one coating particle ($m=1$); and iii) three coating particles ($m=3$). The scales $-P_c$ and $|\alpha|$ are given by (5-6). The size ratio is $R_d/R_1=0.02$ and $\Delta \gamma = \Gamma$. It is seen that the tensile portion of the $P-\alpha$ curve is drastically reduced in the presence of coating for both $m=1$ and $3$. The slope of the curve, which is a measure of the stiffness, is also reduced for $m=1$ and 3. It is important to recognize that once the cohesion force becomes insignificant the powder behaves essentially as cohesionless granular particles, and the macroscopic behavior of the granular flow is in general insensitive to the exact stiffness. Thus, there should be only small differences in using either $m=1, 2$ or 3 in the discrete element simulation which allows the effect of fine particle coating on powder flow to be studied. As long as $m$ is not very large the dynamic behavior of the powder flow should not be very sensitive to the exact concentration of coating particles. This hypothesis will be explored next.

3. Discrete element simulation for powders in a Couette flow

The numerical method used in this work is described in detail in Walton & Braun [10] and Walton [11]. The motions of a large number of individual particles between a plate moving with a constant velocity $V$ and a stationary plate of height $H$ are tracked (Fig. 5). A square array of hemispheres are placed on both plates to simulate the roughness of the wall or the effect of a moving layer of the same powder. The interactions among powder particles and between the particles with the wall are described by the above extended JKR model for the force-displacement relationship. Periodic boundary conditions are imposed on the mean flow ($x$) direction and the spanwise ($z$) direction. After the steady state is reached, the time-area averaged velocity and concentration profiles between the plates are analyzed. To illustrate the effects of coating on the macroscopic behavior of the powder, two average volume concentrations ($\phi_0$) of the powders are considered. They are $\phi_0=0.42$ and 0.53. The rest of the parameters are kept the same and are given as follows. The velocity of the moving plate is $V=3.0$ m/s. $\Gamma=0.2$ N/m is used for untreated powder and $\Delta \gamma =0.2$ N/m is used for the treated powder. The radii of particles are $R_1=10 \text{ \mu m}$ and $R_d=0.1 \text{ \mu m}$ with $E_1=6 \times 10^{10}$ N/m$^2$, $\nu_1=0.25$, $\rho_1=2.6 \times 10^3$ kg/m$^3$, $E_f=6 \times 10^{10}$ N/m$^2$, and $\nu_f=0.25$. The friction coefficient (which is used in determining the maximum tangential contact force) is 0.4. A large value of $H$ ($=60R_1$) is chosen so that the actual location of the stationary plate has little effect on the velocity profile of untreated powder. $L=W=18R_1$ are used.

Fig. 6a compares the mean velocity profiles, $u = V_1/V$, of the powder for treated ($m=1 & 3$) and untreated powders at a mean particle concentration $\phi_0=0.53$. The $y$-coordinate is normalized by $H$ as $y^* = y/H$. Only a thin layer of the untreated powder
moves under the shearing of the moving plate. The rest of the untreated powder remains nearly stationary, like a solid due to strong cohesion. For the treated powders \((m=1 \& 3)\), the particles in the middle region move as a plug with relative ease. The difference in the velocity profiles between \(m=1\) and \(m=3\) for the treated powders does exist; but the difference is much smaller than between \(m=3\) and the untreated powder. \textbf{Fig. 6b} compares the concentration profiles \(\phi(y/H)\) for the treated and untreated powders under the same conditions. The treated powders \((m=1 \& 3)\) have more uniform distributions than the untreated powder. The particle concentration \(\phi\) is around 0.56 in the lower shear region and is around 0.5 near \(\eta=1\). For the untreated powder, the particle concentration in the region \(0.7<\eta<1\) where \(\frac{\partial u}{\partial y}\) is finite is noticeably lower than that in the low shear region where \(\phi\approx 0.58\) for \(0.0<\eta<0.7\). Apparently, particles migrate from regions of high shear to regions of low shear. In the high shear region, the random fluctuating velocity component (or the kinetic energy) of the particle is also large and particles have larger mean free path than in the lower shear region. Since a larger mean free path implies a lower concentration, the lower shear region always has higher concentration than the high shear region if sufficiently long time is given to allow particles to adjust. However, this shear induced migration of particles is not so effective at \(\phi_0=0.53\) for the treated particles as seen from \textbf{Fig. 6b}. Thus, coating at this high concentration range seems to reduce the extent of shear induced migration. Only small differences in the velocity and concentration of the treated powders between \(m=1\) and \(m=3\) are observed. This is consistent with the observation that the macroscopic behavior is insensitive to the actual number of coating particles. \textbf{Figs. 7a-b} compares the mean velocity and concentration profiles of the powder for treated \((m=1 \& 3)\).
and untreated powders at a lower average particle concentration ($\phi_0 = 0.42$). While there is indeed very little difference in the velocity and concentration profiles for the treated powders between $m=1$ and $m=3$, the difference between untreated powder and the treated powder ($m=3$) is small as well, in contrast to the case with $\phi_0 = 0.53$. Apparently, the coating has little effect on the steady state average particle velocity and concentration profiles. This result appears surprising since $\phi_0 = 0.42$ cannot be considered as a truly low concentration case. It is interesting to note, however, that the concentration profiles for all three cases are highly non-uniform. The shear induced migration causes the powder to move to the low shear region ($0 < \eta < 0.5$). In the low shear region, the particle concentration is around 0.56 which is close to that of the treated powder in the case of $\phi_0 = 0.53$. Comparing Fig. 6b with Fig. 7b, it is seen that for $\phi_0 = 0.42$, the shearing at $\eta = 1$ drives powder to the region of the lower shear and packs the powder to a concentration of around $\phi \sim 0.56$. The rest of the powder is then distributed continuously in the high shear region. Using a linear variation to roughly approximate the concentration $\phi$ at $0.5 < \eta < 1$, the average concentration in the flow field is $\phi_{av} = 0.56 \times 0.5 + \frac{1}{2} (0.56 + 0) \times 0.5 = 0.42$. Thus, for $\phi_0 = 0.42$, the dominant factor in determining the macroscopic behavior of the powder is the shear induced migration, whether the powder is coated or not. After a significant portion of the powder migrates to the lower shear region, the local concentration in the high shear region is much lower than the mean concentration and collisions occur less frequently so that the effect of fine particle coating is not as significant. Yet, small differences in the particle concentration profiles between $m=3$ and the untreated powder can be noticed for $0.5 < \eta < 1$.

4. Experimental quantification of the effects of particle coating on powders

While the analysis on the force-displacement relationship provides insight into the mechanism of flowability improvement and the discrete element simulation demonstrates qualitatively how particle coating modifies the macroscopic velocity profile of the powder, the results are nevertheless limited to spherical particles. In reality, powder particles may be non-spherical and may have straight edges and flatter surfaces. The potential contact area can be larger than that of spherical particles. This implies a further increase in the cohesion force among untreated powder particles and a reduction in the flowability in comparison with spherical powder particles. To assess the effect of particle coating on the flowability of real powders, the static angle of repose and the gravity driven flow rate of powder through a funnel are measured for: i) silicon carbide powder coated with fine silica particles using a magnetic driven impact impingement process; ii) silicon carbide powder simply blended with fine silica particles; and iii) untreated silicon carbide powder. The untreated powder has a mean diameter of 10 $\mu$m and the coating particle has a mean diameter of 100 nm. Fig. 8 shows the surface morphologies of these two treated powders, obtained from Scanning Electronic Microscope photographs. It is noted that simple blending results in significantly less fine particles on the surface of the powder.

To measure the angle of repose, a pile of powder is prepared, illuminated with a light source, and projected on a screen. The image of the pile is

![ Typical Scanning Electronic Microscope photographs of a) blended and b) coated powders. The untreated powder has almost smooth surface under the magnification of $10^5$. ](image)
recorded on a film negative from which the angle of repose is measured. The average value is obtained from 4 samples for each case. Table 1 compares the measured angle of repose for three cases. The angles of repose of the two treated powders are clearly lower than that of the untreated powder. Since the host powders are the same in these three cases, the reduction in the angle of repose is the direct consequence of the reduced cohesion force due to the presence of fine coating particles.

Table 1 also compares the mean and standard deviation of the flow rate (gram/sec) through a funnel obtained over 100 measurements for each case. The funnel is fabricated from Plexiglas. The cone angle is 10° and the vertical section has a length of 60 mm and discharge diameter of 5 mm. For the untreated powder, the flow is frequently blocked and the measurement is not meaningful. The treated powder flows easily. There is little difference in the measured flow rate for powders treated with blending and with coating. Although there are many more fine particles on the surface of host particles generated by the coating process than by simply blending (see Fig. 8), there is little difference in the angle of repose and flow rate with a nominal 1% mass fraction of the fine particles. This is consistent with the observation that the exact number of coating particles on the host particle surface has only minor effect on the flowability once the cohesion force is reduced by one or more coating particles.

5. Summary

The extended JKR model for the force-displacement relationship shows that the reduction in the cohesion force between two primary particles is proportional to the size ratio of the coating particle to the host powder particle. The drastic reduction in the cohesion force results in an improved flowability. For Couette flow at high average concentration, when the shear induced migration is not dominant, the treated powder exhibits a fluid-like behavior while the untreated powder shows a solid-like behavior under otherwise identical shearing conditions. When the shear induced migration dominates the mean flow behavior of the powder, the effect of coating is not obvious. The macroscopic behavior of the powder flow is not sensitive to the exact number of coating particles on the surface of host particles. Measurements of angle of repose and flow rate in a funnel support the results from the analysis and simulation.

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Nomenclature

| a | particle contact radius [m] |
| E | Young's modulus [N/m²] |
| H | height of the Couette flow cell in simulation [m] |
| L | average distance between coating particles [m] |
| m | number of coating particles at contact [-] |
| P | loading on particle [N] |
| R | particle radius [m] |
| V | velocity of powder [m/s] |
| x | mass fraction of coating particles [-] |

Greek Symbols:

| a | displacement [m] |
| \phi | volume concentration of solids [-] |
| \Gamma, \gamma | surface energy [N/m] |
| \tau | intrinsic surface energy [N/m] |
| \nu | Poisson ratio [-] |
| \rho | material density [kg/m³] |

Subscript:

| c | cohesive force |
| f | in the presence of fine particles |
| m | multiple coating particles at a contact |
| s | separation |
| x | flow direction |
| 1,2 | material 1 and 2 |
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Renwei Mei
Dr. Renwei Mei is an Associate Professor in the Department of Aerospace Engineering, Mechanics & Engineering Science at the University of Florida. He received his B.S. in 1982 at Zhejiang University, China, and his Ph.D. in 1990 at University of Illinois at Urbana-Champaign. He joined the University of Florida as a faculty in 1990. His research interests include particulate multiphase flows and boiling heat transfer.

James F. Klausner
Dr. James Klausner is an Associate Professor in the Department of Mechanical Engineering at the University of Florida. In 1984 he received his B.S. degree in Marine Engineering from the United State Merchant Marine Academy and his M.S. and Ph.D. in Mechanical Engineering from the University of Illinois at Urbana-Champaign in 1986 and 1989, respectively. He joined the University of Florida in 1989 and has been engaged in research in multiphase flow and boiling heat transfer, photocatalytic oxidation, energy storage, and particulate mechanics.

Hong Shang
Hong Shang received her B.S. and M.S. in Fluid Machinery Engineering in June 1988 and 1991, respectively, at Xi'an Jiaotong University, China. Currently she is working towards her Ph.D in powder flow mechanics at the University of Florida.

Elizabeth Kallman
Elizabeth Kallman received her B.S. in Mechanical Engineering in 1995 at the University of Florida. She is currently pursuing her Ph.D degree at UC Berkeley.