Novel Collective Excitation in Spin Textured Edges of Quantum Hall Systems.

M. Franco and L.Brey

Instituto de Ciencia de Materiales (CSIC), Cantoblanco, 28049 Madrid, Spain

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We study the electric and magnetic properties of the edge of a two-dimensional electron gas in presence of a magnetic field and at filling factor unity. The existence of a spin textured edge is proved as a function of the Zeeman coupling and of the smoothness of the confining potential. We also calculate the low energy excitation of the spin textured phase. We obtain that in addition to the classical edge magnetoplasmons at small wavevectors, there is an almost dispersionless excitation, with a finite gap of energy at zero wavevector. This excitation is associated with the broken symmetry of the spin textured edge phase.

There is great interest in understanding the properties of the edge states of a two-dimensional electron gas (2DEG) in presence of a strong magnetic field $B$ in the quantum Hall effect (QHE) regime. The edge states are important because they can control the magnetotransport of the 2DEG in a broad class of mesoscopic and macroscopic systems. Also, under normal conditions the only gapless excitations in the QHE regime are edge excitations.

Because of the screening properties of the 2DEG, the structure of the edge states changes when the smoothness of the edge confining potential, $V_0(x)$, varies. For studying the edge states properties, we consider a semi-infinite $x-y$ plane, with a straight edge parallel to the $y$ direction. In the case of sharp confinement potentials the electron density falls from the bulk value to zero in a distance of the order of the magnetic length $\ell=\sqrt{\hbar c/eB}$. For a sufficiently smooth confining potential, it has been proposed theoretically that the edge separates into incompressible and compressible regions. This picture appears to be in reasonable agreement with recent experiments. For intermediate smooth confining potentials, due to the exchange interactions it has been suggested that stripes of charge density corresponding to filling factor $\nu=1$ can be stabilized at the edge of the 2DEG. This edge reconstruction can explain transport experiments in quantum dots and spin textures at the edges of quantum Hall systems have been predicted.

The low-energy collective excitations localized at the edge of the system have dispersion relations that depend strongly on the type of reconstruction at the edge. For a sharp edge, a magnetoplasmon with a dispersion of the form $q \ln q$ is expected. Here $q \parallel \vec{y}$ is the wavevector of the excitation. For smooth compressible edges, in addition to the magnetoplasmon, the existence of new branches of acoustic excitations have been proposed and measured. For the striped phase, the collective excitations consist of a set of $q \ln q$ branches and a number of acoustic modes equal to the number of stripes present at the edge of the 2DEG.

In this work we study the electrical and magneto-magnetic properties of the edge states of the 2DEG at $\nu=1$ as a function of the Zeeman coupling, $\vec{g} = g\mu_B B$, and of the smoothness of the confining potential. In particular we are interested in the existence and properties of a spin textured edge. Edge textures are configurations of the spin density that possess a topological charge density at the edge of the system. The spin field, $\mathbf{n}(r)$, of the spin texture has the form

$$n_x + in_y = \sqrt{1-f^2(x)} e^{i(G_x y + \theta)} , \quad n_z = f(x) , \quad (1)$$

here $G_x$ is the wavevector of the spin texture and $\theta$ is an arbitrary phase. In the polarized bulk we have $f(x) = -1$. In the QHE regime the topological charge density coincides with the real charge density.

The charge density associated with the texture only depends on $x$ and is proportional to $G_s df/dx$. The system can develop a spin texture in order to modulate the charge density profile in the $x$ direction and therefore to screen the edge confinement. The lowering of the confinement energy competes with the cost in exchange and Zeeman energies. Therefore the spin textured edge occurs only for smooth enough confinement potential and for small enough Zeeman coupling.

The two main results of this work are the following:

1) We obtain the phase diagram of the system as a function of the smoothness of the confining potential and of the Zeeman energy. By performing a full unrestricted Hartree-Fock (HF) calculation we obtain the range of parameters where a spin textured edge can be expected. Given a confining potential, we obtain that the maximum value of $\tilde{g}$ where the spin texture exists is considerable smaller than the obtained in reference [10]. This discrepancy occurs because reference [10] only considers the competition of the spin textured state with the striped phase. However, we obtain that making $V_0$ smoother, the sharp edge becomes unstable to a smooth charge modulation in both the $x$ and $y$ directions before it becomes unstable to the stripe phase.
ii) We have studied the dispersion relation of the collective excitations in a spin textured edge phase. We obtain that in addition to magnetoplasmon like excitations and the bulk spin density waves, there exists a low energy excitation associated with the broken symmetry of this phase. This mode is almost dispersionless, and it has a finite energy at zero \( q \). The existence of this gap is due to the finite width of the spin texture in the \( \hat{x} \) direction. We turn now to the details of our calculations and results.

**Microscopic Hamiltonian and HF approximation.-** We are interested in properties of the edge states of the \( \nu=1 \) quantum Hall state. In this regime we assume that the electron-electron interaction and Zeeman energies are much smaller than the Landau level splitting, and we therefore restrict the orbital Hilbert space to the lowest Landau level. Since the confining potential only depends on the \( x \) coordinate, it is convenient to work in the Landau gauge, \( A = Bx\hat{y} \). The Hamiltonian of this system has the form (Through this work we take \( \ell \) as the unit of length and \( e^2/\ell e \) as the unit of energy):

\[
H = \sum_{k,\alpha} \left( V_0(k) + \frac{\hat{q}}{2} \right) c_{k,\alpha}^\dagger c_{k,\alpha} + \frac{1}{2L_xL_y} \sum_{k,k',q,\alpha,\beta} v(p)e^{-p^2/2}e^{ip_y(k-k'+p_y)} c_{k,\alpha}^\dagger c_{k',\beta}^\dagger c_{k'+p_y,\beta} c_{k'+p_y,\beta}.
\]

Here \( \alpha, \beta = \pm \) (up)-(down) are spin indices, \( L_x \) and \( L_y \) are the sample dimensions, \( v(p) \) is the Fourier transform of the Coulomb interaction, and \( c_{k,\alpha} \) creates an electron with spatial wavefunction \( \psi_k(r) = \frac{1}{\sqrt{L_xL_y}} e^{iky} e^{-(x-k)^2/2} \), and spin \( \alpha/2 \).

In order to change the smoothness of the edge continuously we take \( V_0 \) as the potential created by a distribution of positive charge which falls linearly from its bulk value, \( 1/2\pi, \) to zero, over a region of width \( W \). In this way the edge is smoother in direct proportion to \( W \). This form of \( V_0 \) has been used in previous works. For small values of \( W \) the edge is sharp. Chamon et al [7], obtained that in the HF approximation the stripe phase is stable with respect the sharp edge for values of \( W \) larger than \( W_s=9.0 \). Working also in the HF approximation Karlhede et al [10] obtained a critical value of \( W_s=6.8 \) for the existence of a textured edge at \( \tilde{g}=0 \). These two calculations do not allow the charge density to vary along the edge. In this work we allow the system to modulate the charge in both directions \( \hat{x} \) and \( \hat{y} \).

In order to solve the Hamiltonian Eq.1 we make the HF pairing of the second-quantized operators, allowing for the possibility of different broken translational symmetry and spin order in the ground state. To characterize the different solutions, it is very convenient to introduce the operators

\[
\rho_{\alpha,\beta}(q) = \frac{2\pi}{L_xL_y} \sum_k e^{-iq_x(x-k_y)/2} c_{k,\alpha}^\dagger c_{k+q_y,\beta}^\dagger (3)
\]

which are easily related to the charge \( n(q) \) and spin \( S(q) \) density operators. By solving selfconsistently the Hartree-Fock equations we obtain the expectation values of the energy and of the different density operators.

**Phase Diagram.-** The different solutions of the electric and magnetic edge structure can be characterized by the expectation values of the products \( c_{k,\alpha}^\dagger c_{k,\beta} \) by the expectation values of the operators \( \rho_{\alpha,\beta}(q) \). In this work we find the following type of solutions: (see Fig 1):

i) Spin polarized compact edge (SPCE). In this state \(<c_{k,\alpha}^\dagger c_{k,\beta}> = \delta_{k,\alpha} \delta_{k,\beta} \delta_{\alpha,-}\), and there is a maximum wavevector such that all states with smaller momentum are occupied. This solution is the sharpest edge possible, and it is the ground state for small values of \( W \).

ii) Spin polarized charge density wave (SPCDW). In this state only the majority spin electronic states are occupied, i.e. \(<\rho_{\alpha,\beta}(q)> \propto \delta_{\alpha,\beta} \delta_{\alpha,-}\). In this class of solutions, the system modulates the charge along the \( \hat{x} \) direction in order to screen the edge potential. In the QHE regime the system only can modulate smoothly the charge along the \( \hat{x} \) direction by modulating also the charge along the \( \hat{y} \) direction. At \( \tilde{g} \rightarrow \infty \), the SPCDW state has lower energy than the SPCE state for \( W > W_{CDW} \approx 7 \), while the stripe phase has lower energy than the SPCE at \( W_s = 9.0 \). We find that the SPCDW state always has lower energy than the stripe phase: the stripe phase is not a stable solution of the system.

iii) Spin textured edge (STE). In this class of solutions, \(<c_{k,\alpha}^\dagger c_{k,\beta}> = \delta_{k,\alpha} \delta_{k,\beta} \delta_{\alpha,-}\) but all the \(<\rho_{\alpha,-\alpha}(q)> \) can be different from zero. In the calculation we obtain that the operators \( \rho_{\alpha,-\alpha}(q) \) are different from zero only for one wavevector of the form \( q = (0,G_s) \). Minimizing the energy with respect ito \( G_s \) we obtain microscopically the periodicity of the spin texture. We do not obtain higher harmonics of the spin texture because, in order to get a constant charge density along the edge, only one wavevector of the spin texture is possible.

Since \(<\rho_{\alpha,-\alpha}(0,G_s)> \) is different from zero, the STE phase breaks the translational invariance along the edge and the spin rotational symmetry about the magnetic field. However the STE is invariant under a symmetry composed of a translation along the edge and a spin rotation. The states related with this symmetry correspond to the different values of \( \theta \) in Eq.1.

For \( \tilde{g} = 0 \), the STE has lower energy than the SPCE for \( W > W_s = 6.7 \). This value of \( W_s \) increases with \( \tilde{g} \), and for \( \tilde{g} > \tilde{g}_c \approx 0.005 \) the system prefers to screen the edge potential by forming a SPCDW state rather than by creating a STE. This value of \( \tilde{g}_c \) is about ten times smaller than the obtained by Karlhede et al [10]. This discrepancy occurs because in reference [10] the STE is assumed to compete only with the striped phase, and not
with the SPCDW state.

It is important to note that the width of the charge and spin modulation in the \( \hat{x} \) direction can be much larger than \( G_s \). For example for \( \tilde{\gamma} = 0 \) and \( W = 8 \), \( G_s \approx 0.85 \) and the width of the modulation is of the order of 4 magnetic lengths.

iv) Spin textured and charge density wave state. This is a fully broken symmetry ground state where the expectation value of all \( \langle \rho_{\alpha,\beta}(q) \rangle \) are not zero. This state is a mixture of charge density waves and spin textures and it is reached from both the STE and SPCDW states by making the edge confinement smoother. This phase corresponds to the shadow region in Fig.1.

**Collective excitations.** We now study the collective excitations of the spin textured edge. As commented above, in the STE phase the order parameters different from zero are: \( \langle \rho_{+,-}(0, G_s) \rangle \) and \( \langle \rho_{\alpha,\alpha}(q_z, 0) \rangle \). With these order parameters the wavefunction of the STE is a Slater determinant of the form \( |STE> = \prod_i d_i^\dagger(0)|G_s> \)

\( d_k = u_k c_{-,k} + v_k c_{+,k + G_s} \), where \( u_k \) and \( v_k \) are obtained from the expectation values of the \( \rho_{\alpha,\beta} \) operators and verify the relation \( u_k^2 + v_k^2 = 1 \). The set of eigenstates of the Hartree-Fock Hamiltonian orthogonal to the \( d \) states have the form: \( b_k^i = -v_k c_{-,k} + u_k c_{+,k + G_s}^i \).

Using the \( d \) and \( b \) states, the low energy collective excitations of the system can be characterized by a quantum number \( q \) and correspond to linear combinations of electron-hole pairs of the form \( d_{k+q}^i d_q \) and \( b_{k+q}^i d_k^\dagger \). From the form of the \( d \) and \( b \) operators the excitations have the general expression

\[ |\psi_q^i> = \sum_{k,\alpha,\alpha'} a_{k,\alpha,\alpha'}^i(q) c_{k+q+G_s,\alpha}^\dagger c_{k,\alpha',\alpha'}^i |STE> \quad (4) \]

where \( g_+ = 0 \) and \( g_+ = G_s \), and the coefficients \( a_{k,\alpha,\alpha'}^i \) are obtained by minimizing the energy of the excitations, \( h\omega_i(q) = |<\psi_i^q | H |\psi_q^i> | <\psi_i^q | \psi_i^q> \).

Due to the existence of the spin texture, the collective excitations are a mixture of spin and charge density excitations. Also note that because the STE is invariant under a translation along the edge plus a spin rotation \( \tilde{\gamma} \), any electron spin flip is accompanied by a change of the electron wavevector in \( \pm G_s \).

In Fig. 2 we plot the lowest energy collective excitations for the \( \tilde{\gamma} = 0 \) and \( W = 8 \) case. The spectrum consists basically of two parts: i) a continuum of excitations starting from an energy \( \tilde{\gamma} + 4\pi \rho_s (q - G_s)^2 \), where \( \rho_s \) is the spin stiffness of the 2DEG at \( \nu = 1 \), and ii) a set of discrete branches around \( q = 0 \). The former excitations are localized in the bulk part of the system and they are the well known bulk spin density waves. \( 23 \) The dispersion relation of the spin density waves starts at \( q = -G_s \), because in the STE phase the electron hole pairs involving a majority spin flip have the form (see Eq. 4) \( c_{k+G_s,\alpha}^\dagger c_{k,\alpha} \).

On the other hand the low energy excitations starting at \( q = 0 \) are localized at the edge of the system and they correspond to edge excitations of the STE phase.

We describe now the character of the edge excitations of the STE phase. At small wavevectors, all but one of the low energy excitations are gapless at \( q = 0 \), and have a dispersion of the form \( q \ln q \). The analysis of the coefficients \( a_{k,\alpha,\alpha'}^i \) of these gapless excitations, reveals that they are localized at the edge of the system in a region of thickness \( q \). These excitations correspond to the classical edge magnetoplasmons \( 33 \) of the system. The difference with the edge magnetoplasmon of the spin polarized compact edge is that in the STE the spin and charge excitations are mixed.

As mentioned above, in addition to the magnetoplasmon, there is a low energy excitation which is practically dispersionless at small wavevectors and which has a finite gap at \( q = 0 \). This excitation anticrosses with the edge magnetoplasmon, see inset of Fig.2. It is localized at the edge of the system but with a thickness equal to the spatial width of the charge modulation in the \( \hat{x} \) direction. This thickness is much bigger than the wavevector of the excitation, and therefore this mode is almost dispersionless. In Fig. 2, we plot the lowest energy collective excitations of the system. As commented above, in the STE phase there exists a low energy excitation which is practically dispersionless at small wavevectors and which has a finite gap at \( q = 0 \). This excitation anticrosses with the edge magnetoplasmon, see inset of Fig. 2. It is localized at the edge of the system but with a thickness equal to the spatial width of the charge modulation in the \( \hat{x} \) direction. This thickness is much bigger than the wavevector of the excitation, and therefore this mode is almost dispersionless.
should be a probe of the existence of the STE phase. It could be possible to detect the existence of this mode by time-resolved magnetotransport experiments or by measuring the transmission of electromagnetic waves.

In summary, we have studied the electronic and magnetic structure of the edge of a 2DEG in the $\nu=1$ QHE regime. We have obtained the phase diagram of the system as a function of the Zeeman coupling and the smoothness of the confinement potential. We obtain the range of parameters where a spin textured edge phase is expected. We have also studied the collective excitations of this phase. We have found the existence of a low energy gapfull collective excitation associated with the broken symmetry of the spin textured phase.

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FIG. 1.
Phase diagram, as a function of $g= g\mu_B B$ and $W$, of the edge of a 2DEG at $\nu = 1$. The shadow region corresponds to the spin textured and charge density wave phase. $\tilde{g}$ is in units of $e^2/\ell \epsilon$ and $W$ in units of $\ell$.

FIG. 2.
Low energy collective excitations of the spin textured edge phase. The results correspond to $\tilde{g} = g\mu_B B=0$ and $W=8$. The energy is in units of $e^2/\ell \epsilon$ and the wavevector $q$ in units of $\ell^{-1}$.