Muon conversion to electron in nuclei within the BLMSSM

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(Dated: October 9, 2018)

Abstract

In a supersymmetric extension of the standard model with local gauged baryon and lepton numbers (BLMSSM), there are new sources for lepton flavor violation, because the right-handed neutrinos, new gauginos and Higgs are introduced. We investigate muon conversion to electron in nuclei within the BLMSSM in detail. The numerical results indicate that the $\mu \to e$ conversion rates in nuclei within the BLMSSM can reach the experimental upper bound, which may be detected in the future experiments.

PACS numbers: 12.60.-i, 14.40.-n, 12.38.Qk

Keywords: BLMSSM, conversion, experimental bounds

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I. INTRODUCTION

The observations of neutrino oscillations \cite{1-3} imply that neutrinos have tiny masses and are mixed \cite{4-7}, which have demonstrated that lepton flavor in neutrino sector is not conserved. Nevertheless, in the Standard Model (SM) with three tiny massive neutrinos, the expected rates for the charged lepton flavor violating (LFV) processes are very tiny \cite{8-11}. Thus, Lepton-flavor violation is a window of new physics beyond the SM. Among the various candidates for new physics that produce potentially observable effects in LFV processes, one of the most appealing model is supersymmetric (SUSY) extension of the SM. Here, we can use the neutrino oscillation experimental data to restrain the input parameters in the new models. A neutral Higgs with mass $m_{h^0} = 125.1$ GeV reported by ATLAS \cite{12} and CMS \cite{13, 14} gives a strict constraint on relevant parameter space of the model.

The present sensitivities of the $\mu \rightarrow e$ conversion rates in different nuclei \cite{15-17} are collected here,

$$
\begin{align*}
\text{CR}(\mu \rightarrow e : \text{Au}) &< 7 \times 10^{-13}, \\
\text{CR}(\mu \rightarrow e : \text{Ti}) &< 4.3 \times 10^{-12}, \\
\text{CR}(\mu \rightarrow e : \text{Pb}) &< 4.6 \times 10^{-11}. 
\end{align*}
$$

These processes have close relation with $l_i \rightarrow l_i \gamma$. In the work \cite{18}, the $\mu \rightarrow e$ conversion was studied in $\mu \nu$SSM. For the models beyond SM, one can violate R parity with the non-conservation of baryon number ($B$) or lepton number ($L$) \cite{19-22}. A minimal supersymmetric extension of the SM with local gauged $B$ and $L$ (BLMSSM) was first proposed by the author \cite{23, 24}. The local gauged $B$ is used to explain the matter-antimatter asymmetry in the universe. Right-handed neutrinos in BLMSSM lead to three tiny neutrino masses through the See-Saw mechanism and can account for the neutrino oscillation experiments. So lepton number ($L$) is expected to be broken spontaneously around TeV scale.

In BLMSSM, the lightest CP-even Higgs mass and the decays $h^0 \rightarrow \gamma \gamma$, $h^0 \rightarrow ZZ(WW)$ were studied in the work \cite{25}. The neutron and lepton electric dipole moments (EDMs) were researched in the CP-violating BLMSSM \cite{26, 27}. In BLMSSM, there are also other works \cite{28, 30}. In this work, we analyze the processes on muon conversion to electron in nuclei within the BLMSSM. Compared with MSSM, there are new sources to enlarge the processes via loop contributions. The new scores are produced from: 1. the coupling of new
neutralino(lepton neutralino)-slepton-lepton; 2. the right-handed neutrinos mixing with left-handed neutrinos; 3. the sneutrino sector is extended, whose mass squared matrix is \(6 \times 6\). In some parameter space of BLMSSM, large corrections to the processes are obtained, and they can easily exceed their experiment upper bounds. Therefore, to enhance the processes on muon conversion to electron in nuclei is possible, and they may be measured in the near future.

After this introduction, we briefly summarize the main ingredients of the BLMSSM, and show the needed mass matrices and couplings in section II. In section III, the processes \(\mu \rightarrow e + q\bar{q}\) are studied in the BLMSSM. The input parameters and numerical analysis are shown in section IV, and our conclusion is given in section V. Some functions are collected in the Appendix.

II. BLMSSM

BLMSSM is the supersymmetric extension of the SM with local gauged \(B\) and \(L\), whose local gauge group is \(SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_B \otimes U(1)_L\). The exotic leptons \(\hat{L}_4, \hat{E}^c_4, \hat{N}^c_4, \hat{L}^c_5, \hat{E}_5\) and \(\hat{N}_5\) are introduced to cancel \(L\) anomaly. As well as, the exotic quarks \(\hat{Q}_4, \hat{U}^c_4, \hat{D}^c_4, \hat{Q}^c_5, \hat{U}_5\) and \(\hat{D}_5\) are introduced to cancel \(B\) anomaly. To break lepton number and baryon number spontaneously, the Higgs superfields \(\hat{\Phi}_L, \hat{\varphi}_L\) and \(\hat{\Phi}_B, \hat{\varphi}_B\) are introduced, respectively. The exotic quarks obtain masses from nonzero vacuum expectation values(VEVs) of \(\hat{\Phi}_B\) and \(\hat{\varphi}_B\). While, exotic leptons get masses from VEVs of \(H_u\) and \(H_d\). \(H_u\) and \(\hat{\varphi}_L\) give masses to light neutrinos through See-Saw mechanism. In the BLMSSM, the superfields \(\hat{X}\) and \(\hat{X}'\) are introduced to make the heavy exotic quarks unstable. The above mentioned exotic lepton, quark and Higgs superfields are shown in table 1.

The superpotential of BLMSSM is

\[
\mathcal{W}_{BLMSSM} = \mathcal{W}_{MSSM} + \mathcal{W}_B + \mathcal{W}_L + \mathcal{W}_X ,
\]

\[
\mathcal{W}_B = \lambda_Q \hat{Q}_4 \hat{Q}_5^c \hat{\Phi}_B + \lambda_U \hat{U}^c_4 \hat{U}_5 \hat{\varphi}_B + \lambda_D \hat{D}^c_4 \hat{D}_5 \hat{\varphi}_B + \mu_B \hat{\Phi}_B \hat{\varphi}_B ,
\]

\[
+ Y_{\nu_4} \hat{Q}_4 \hat{H}_u \hat{U}_5^c + Y_{\nu_4} \hat{Q}_4 \hat{H}_d \hat{D}_5^c + Y_{\nu_5} \hat{Q}_5^c \hat{H}_d \hat{U}_5 + Y_{\nu_5} \hat{Q}_5^c \hat{H}_u \hat{D}_5 ,
\]

\[
\mathcal{W}_L = Y_{\nu_4} \hat{L}_4 \hat{H}_d \hat{E}_4^c + Y_{\nu_4} \hat{L}_4 \hat{H}_u \hat{N}_4^c + Y_{\nu_5} \hat{L}_5^c \hat{H}_d \hat{E}_5 + Y_{\nu_5} \hat{L}_5^c \hat{H}_u \hat{N}_5 ,
\]

\[
+ Y_{\nu_5} \hat{L}_5 \hat{H}_u \hat{N}_5^c + \lambda_{\nu \nu} \hat{N}^c \hat{N}^c \hat{\varphi}_L + \mu_L \hat{\Phi}_L \hat{\varphi}_L ,
\]

\[
\mathcal{W}_X = \lambda_1 \hat{Q}_5 \hat{X} + \lambda_2 \hat{U}^c_5 \hat{U}_5 \hat{X}' + \lambda_3 \hat{D}^c_5 \hat{D}_5 \hat{X}' + \mu_X \hat{X} \hat{X}' .
\]
where $W_{MSSM}$ is the superpotential of the MSSM. To save space in the text, the soft breaking terms $\mathcal{L}_{\text{soft}}$ of the BLMSSM is not shown here.

In this model, we introduce the superfields $\tilde{N}_c$ to produce tiny masses of three light neutrinos. The mass matrix of neutrinos in the basis $(\psi_{\nu L}^I, \psi_{\nu R}^I)$ is expressed as

$$Z_{\nu}^\top \begin{pmatrix} 0 & \frac{v_\nu}{\sqrt{2}} (Y_{\nu})^{IJ} \\ \frac{v_\nu}{\sqrt{2}} (Y_{\nu}^T)^{IJ} & \frac{v_\nu}{\sqrt{2}} (\lambda_{N_{c}})^{IJ} \end{pmatrix} Z_{\nu} = \text{diag}(m_{\nu^{\alpha}}), \alpha = 1 \cdots 6, I, J = 1, 2, 3,$$

$$\psi_{\nu L}^I = Z_{N_{\nu}}^{I\alpha} k_{N_{\alpha}}^0, \quad \psi_{\nu R}^I = Z_{N_{\nu}}^{(I+3)\alpha} k_{N_{\alpha}}^0, \quad \chi_{N_{\alpha}}^0 = \begin{pmatrix} k_{N_{\alpha}}^0 \\ \lambda_{N_{\alpha}}^0 \end{pmatrix} . \quad (3)$$

Here, $\chi_{N_{\alpha}}^0$ represent the mass eigenstates of neutrino fields mixed by left-handed and right-handed superfields.
handed neutrinos.

The new gaugino $\lambda_L$ mixes with the superpartners of the $SU(2)_L$ singlets $\Phi_L$ and $\varphi_L$, then they produce three lepton neutralinos

$$\mathcal{L}_{\chi^0_L} = \frac{1}{2}(i\lambda_L, \psi_{\Phi_L}, \psi_{\varphi_L}) \begin{pmatrix} 2M_L & 2v_L g_L & -2\bar{v}_L g_L \\ 2v_L g_L & 0 & -\mu_L \\ -2\bar{v}_L g_L & -\mu_L & 0 \end{pmatrix} \begin{pmatrix} i\lambda_L \\ \psi_{\Phi_L} \\ \psi_{\varphi_L} \end{pmatrix} + \text{h.c.} \tag{4}$$

One can use $Z_L$ to diagonalize the mass matrix in Eq.(4) and obtain three lepton neutralino masses in the end.

From Eqs. (2) and the soft breaking terms $\mathcal{L}_{soft}$ of the BLMSSM, the mass squared matrix of sneutrino gets corrections and reads as

$$\begin{pmatrix} (M^2_{\tilde{\nu}})_{LL} & (M^2_{\tilde{\nu}})_{LR} \\ (M^2_{\tilde{\nu}})_{LR}^\dagger & (M^2_{\tilde{\nu}})_{RR} \end{pmatrix}. \tag{5}$$

$(M^2_{\tilde{\nu}})_{LL}$, $(M^2_{\tilde{\nu}})_{LR}$ and $(M^2_{\tilde{\nu}})_{RR}$ are shown here

$$(M^2_{\tilde{\nu}})_{LL} = \frac{(g_1^2 - g_2^2)(v_1^2 - v_2^2)}{8} \delta_{IJ} + g_1^2(v_1^2 - v_2^2)\delta_{IJ} + m_{\tilde{\nu}_I}^2 \delta_{IJ} + (m^2_{\tilde{\nu}})_{IJ},$$

$$(M^2_{\tilde{\nu}})_{LR} = \frac{\mu^* v_u}{\sqrt{2}}(Y_{\nu})_{IJ} - \frac{v_u}{\sqrt{2}}(A^\nu_{IJ} + \frac{v_d}{\sqrt{2}}(A^l_{IJ}),$$

$$(M^2_{\tilde{\nu}})_{RR} = \frac{g_2^2(v_1^2 - v_2^2)}{4} \delta_{IJ} - g_1^2(v_1^2 - v_2^2)\delta_{IJ} + m_{\tilde{\nu}_I}^2 \delta_{IJ} + (m^2_{\tilde{\nu}})_{IJ}. \tag{6}$$

The unitary matrix $Z_L$ is used to rotate sneutrino mass squared matrix to mass eigenstates.

Because of the introduction of right handed neutrinos, in BLMSSM the mass squared matrix of sneutrino is $6 \times 6$. In the base $\tilde{\nu}^T = (\tilde{\nu}, \tilde{\nu}^c)$, the concrete forms for the sneutrino mass squared matrix $\mathcal{M}_{\tilde{\nu}}$ are shown here

$$\mathcal{M}_{\tilde{\nu}}^2(\tilde{\nu}_I^c, \tilde{\nu}_J) = \frac{g_1^2 + g_2^2}{8} \delta_{IJ} + g_1^2(\overline{\nu}_I^c - \overline{\nu}_J^c)\delta_{IJ} + \frac{\mu}{2} \overline{(Y^\dagger Y_{\nu})_{IJ}} + (m^2_{\tilde{\nu}})_{IJ},$$

$$\mathcal{M}_{\tilde{\nu}}^2(\tilde{\nu}_I^c, \tilde{\nu}_J^c) = -\frac{g_1^2}{8} \delta_{IJ} + \frac{\mu}{2} \overline{(Y^\dagger Y_{\nu})_{IJ}} + 2g_1^2 \lambda_{\nu^c}^\dagger \lambda_{\nu^c} + (m^2_{\tilde{\nu}})_{IJ},$$

$$\mathcal{M}_{\tilde{\nu}}^2(\tilde{\nu}_I^c, \tilde{\nu}_J^c) = \frac{\mu \overline{Y_{\nu}}}{\sqrt{2}} \overline{(Y_{\nu})_{IJ}} + \frac{\overline{v_u}}{\sqrt{2}} (A_{\nu^c})_{IJ} \lambda_{\nu^c} + (m^2_{\tilde{\nu}})_{IJ} \tag{7}$$

The superfields $\tilde{\nu}^c$ in BLMSSM lead to the corrections for some couplings existed in MSSM. We give out the corrected couplings such as: W-lepton-neutrino and Z-neutrino-
neutrino

\[ \mathcal{L}_{W/L\nu} = -\frac{e}{\sqrt{2} s_W} W^\mu_\nu \sum_{l=1}^3 \sum_{\alpha=1}^6 Z_{N_{\nu l}}^{j l \alpha} \chi_{N_\alpha}^0 \gamma^\mu P_L e^l, \]

\[ \mathcal{L}_{Z/\nu} = -\frac{e}{2 s_W c_W} Z^\mu_\nu \sum_{l=1}^3 \sum_{\alpha, \beta=1}^6 Z_{N_{\nu l}}^{j l \alpha} Z_{N_{\nu l}}^{j l \beta} \chi_{N_\alpha}^0 \gamma^\mu P_L \chi_{N_\beta}^0, \]

(8)

where \( P_L = \frac{1 - \gamma^5}{2} \) and \( P_R = \frac{1 + \gamma^5}{2} \). We use the abbreviation \( s_W = \sin \theta_W, c_W = \cos \theta_W \), and \( \theta_W \) is the Weinberg angle.

Some other adapted couplings are collected here: chargino-lepton-sneutrino, Z-sneutrino-sneutrino and charged Higgs-lepton-neutrino

\[ \mathcal{L}_{\chi^\pm L\nu} = -\frac{3}{2} \sum_{l=1}^3 \sum_{i=1}^6 \sum_{j=1}^2 \bar{\chi}^0_j (Y^I_l Z^j \nu P_L + \frac{e}{s_W} Z^j \nu P_L) e^l \tilde{\nu}^i + h.c. \]

(9)

\[ \mathcal{L}_{\chi^0 Z\nu} = \frac{e}{2 s_W c_W} Z^\mu_\nu \sum_{l=1}^3 \sum_{i,j=1}^6 Z^i \nu Z^i \nu P_L \tilde{\chi}^0_j \gamma^\mu \tilde{\nu}^j. \]

(10)

\[ \mathcal{L}_{H^\pm L\nu} = \sum_{l=1}^3 \sum_{\alpha=1}^6 \bar{H}^\pm e^l \left( Y^I_l \cos \beta Z^l_{N_{\nu \alpha}} P_L - Y^I_l \sin \beta Z^l_{N_{\nu \alpha}} P_R \right) \chi_{N_\alpha}^0 \]

\[ - \sum_{l=1}^3 \sum_{\alpha=1}^6 H^\pm e^l \left( Y^I_l \sin \beta Z^l_{N_{\nu \alpha}} P_L + Y^I_l \sin \beta Z^l_{N_{\nu \alpha}} P_R \right) \chi_{N_\alpha}^0 + h.c. \]

(11)

In BLMSSM, there are new couplings that are deduced from the interactions of gauge and matter multiplets \( ig\sqrt{2} T^a_{ij} (\lambda^a \phi_j A^*_i - \bar{\lambda}^a \phi_i A^*_j) \). After calculation, the lepton-slepton-lepton neutralino couplings are obtained

\[ \mathcal{L}_{l\chi^0_i L} = \sum_{i=1}^3 \sum_{j=1}^6 \sqrt{2} g_L \chi^0_j \left( Z^i_{N_{\nu l}} Z^j_{N_L} P_L - Z^i_{N_{\nu l}} Z^j_{N_L} P_R \right) l^l \bar{L}^+_i + h.c. \]

(12)

III. \( \mu \rightarrow e + q\bar{q} \) IN THE BLMSSM

In this section, the LFV processes \( \mu \rightarrow e + q\bar{q} \) are studied in the BLMSSM. Both penguin-type diagrams and box-type diagrams have contributions to the effective Lagrangian. For convenience, the penguin and box diagrams are analyzed in the generic form, which can simplify the work.
FIG. 1: The penguin diagrams for $\mu \to e + q\bar{q}$, with $F$ representing Dirac(Majorana) particles.

A. the penguin diagrams

When the external leptons are all on shell, we can generally obtain the $\gamma$-penguin contributions in the following form

$$T_{\gamma-p} = -\bar{e} (p_1) \left[ q^2 \gamma_\mu \left( C_1^L P_L + C_1^R P_R \right) + m_\mu i \sigma_\mu q' \left( C_2^L P_L + C_2^R P_R \right) \right] \mu (p)$$

$$\times \frac{e^2}{q^2} \bar{q} (p_2) \gamma^\mu q (p_3)$$

(13)

The relevant Feynman diagrams are shown in Fig.1. The final Wilson coefficients $C_1^L, C_1^R, C_2^L$ and $C_2^R$ are obtained from the sum of these diagrams’ amplitudes.

The contributions from the virtual neutral fermion diagram in the top-left of Fig.1 are denoted by $C_\alpha^{L,R}(n)$, $\alpha = 1, 2$. We give out the deduced results in the following form,

$$C_1^L (n) = \sum_{F=\chi^0, \chi^0_L, \nu} \sum_{S=L, L, H^\pm} \frac{1}{6m_{FW}^2} H^{SF\bar{e}}_R H^{S*\mu F}_L I_1(x_F, x_S) ,$$

$$C_2^L (n) = \sum_{F=\chi^0, \chi^0_L, \nu} \sum_{S=L, L, H^\pm} \frac{m_F}{m_{W}} H^{SF\bar{e}}_L H^{S*\mu F}_L \left[ I_2(x_F, x_S) - I_3(x_F, x_S) \right] ,$$

$$C_\alpha^R (n) = C_\alpha^L (n) \bigg|_{L \leftrightarrow R}, \quad \alpha = 1, 2.$$  

(14)

with $x = m^2/m_{FW}^2$ and $m$ representing the mass for the corresponding particle. $H^{SF\bar{e}}_{L,R}$ and $H^{S*\mu F}_{L,R}$ are the corresponding couplings of the left(right)-hand parts in the Lagrangian. The one-loop functions $I_i(x_1, x_2), i = 1 \ldots 4$ are collected in Appendix.
The diagram in top-right of Fig.1 represents the virtual charged Fermion diagram and its contribution is

\[
C^L_1(c) = \sum_{F=x^\pm} \sum_{S=\bar{\nu}} \frac{1}{6m_W^2} H^S_{R,F} H^S_{F,L} \left[ I_3(x_F, x_S) - 2I_4(x_F, x_S) - I_1(x_F, x_S) \right],
\]

\[
C^L_2(c) = \sum_{F=x^\pm} \sum_{S=\bar{\nu}} \frac{m_F}{m_\mu m_W^2} H^S_{R,F} H^S_{F,L} \left[ I_3(x_F, x_S) - I_4(x_F, x_S) - I_1(x_F, x_S) \right],
\]

\[
C^R_\alpha(c) = C^L_\alpha(c) \big|_{L \to R}, \quad \alpha = 1, 2.
\]

On account of the mixing of three light neutrinos and three heavy neutrinos, the virtual \( W \) diagrams in the bottom of Fig.1 give corrections to the charged LFV process \( \mu \to e + q\bar{q} \). We show the coefficients \( C^L,R_\alpha(W)(\alpha = 1, 2) \)

\[
C^L_1(W) = \sum_{F=\nu} \frac{-1}{2m_W^2} H^W_{L,F} H^W_{L,F} \left[ I_2(x_F, x_W) + I_1(x_F, x_W) \right],
\]

\[
C^L_2(W) = \sum_{F=\nu} \frac{1}{m_W} H^W_{L,F} H^W_{L,F} (1 + \frac{m_\mu}{m_\mu}) \left[ 2I_2(x_F, x_W) - \frac{1}{3} I_1(x_F, x_W) \right],
\]

\[
C^R_\alpha(W) = 0, \quad \alpha = 1, 2.
\]

The sum of the total coefficients in Eqs. (14) (15) (16) are

\[
C^L,R_\alpha = C^L,R_\alpha(n) + C^L,R_\alpha(c) + C^L,R(W), \quad \alpha = 1, 2.
\]

The contributions from \( Z \)-penguin diagrams are depicted by the Fig.1, similar as \( \gamma \)-penguin diagrams,

\[
T_{Z-p} = \frac{e^2}{m_Z} \varepsilon(p_1) \gamma_\mu (N_L P_L + N_R P_R) \mu(p) \bar{q}(p_2) \gamma^\mu \left( H^Z_{L} \bar{e} F P_L \right.
\]

\[
+ H^Z_{R} \bar{e} F P_R \big) \delta(p_3),
\]

\[
N_{L,R} = N_{L,R}(S) + N_{L,R}(W).
\]

The concrete forms of the effective couplings \( N_L(S), N_R(S), N_L(W) \) and \( N_R(W) \) read as

\[
N_L(S) = \frac{1}{2e^2} \sum_{F=x^\pm, S=\bar{\nu}} \sum_{S=L, \bar{L}, H} \left[ \frac{2e^2}{m_W^2} H^S_{R,F} H^S_{F,L} G_1(x_F, x_{S_1}, x_{S_2}) \right.
\]

\[
+ \left. H^S_{R,F} H^S_{F,L} G_2(x_F, x_{S_1}, x_{S_2}) \right] + \sum_{F=x^\pm} \sum_{S=L} \left[ H^S_{R,F} H^S_{F,L} G_2(x_F, x_{S_1}, x_{S_2}) \right].
\]

\[
N_R(S) = N_L(S) \big|_{L \to R},
\]
FIG. 2: The box diagrams for $\mu \to e + q\bar{q}$, with $F$ representing Dirac(Majorana) particles.

\[
N_L(W) = \frac{c_W}{\epsilon s_W} \sum_{F=\nu} H_L^{WF} e^2 H_L^{W*} \mu F [G_3(x_F, x_W) + 2(x_i + x_j)[I_1(x_F, x_W) - I_2(x_F, x_W)]]
\]
\[
+ \frac{1}{e^2} \sum_{F_1, F_2=\nu} H_L^{WF_1} e^2 H_L^{W*} \mu F_1 H_L^{Z*} F_1 F_2 (\frac{3}{32\pi^2} - G_2(x_W, x_{F_1}, x_{F_2})
\]
\[+ x_j \left[ 1 G_4(x_W, x_{F_1}, x_{F_2}) + G_5(x_W, x_{F_1}, x_{F_2}) \right],
\]
\[
N_R(W) = \frac{c_W}{\epsilon s_W} \sum_{F=\nu} H_L^{WF} e^2 H_L^{W*} \mu F [2\sqrt{x_i x_j}[I_1(x_F, x_W) - I_2(x_F, x_W)]]
\]
\[
+ \frac{1}{e^2} \sum_{F_1, F_2=\nu} H_L^{WF_1} e^2 H_L^{W*} \mu F_1 H_L^{Z*} F_1 F_2 \sqrt{x_i x_j} (2G_1(x_W, x_{F_1}, x_{F_2})
\]
\[\frac{1}{3} G_4(x_W, x_{F_1}, x_{F_2}) - 2G_5(x_W, x_{F_1}, x_{F_2}) \]  
(19)

The concrete expressions for the functions $G_i$ ($i = 1, ..., 7$) are collected are in appendix.

B. The box-type diagrams

The box-type diagrams drawn in Fig.2 can be written as
\[ T_{\text{box}} = e^2 \sum_{q=u,d} \bar{q} \gamma_\alpha q \gamma_\alpha \left( B_q^L P_L + B_q^R P_R \right) \mu , \quad (20) \]

\[ B_q^L = B_q^{(n)} L_R + B_q^{(c)} L_R + B_q^{(W)} L_R (q = u, d) . \quad (21) \]

\( B_q(n) \) represent the contributions from the virtual neutral Fermion diagrams in the first line of Fig. 2.

\[ B_q^L(n) = \frac{4}{\sum_{i,j=1} B_q^L(n)} \sum_{i,j=1,k,l=1} \left\{ \frac{1}{8e^2 m_W^2} G_6 \left( x_{\chi_i^0}, x_{\chi_j^0}, x_{\nu_k}, x_{\bar{\nu}_l} \right) \left[ \left( H_R^{\tilde{l}_k \chi_i^0} \right)^* H_R^{\tilde{l}_k \chi_i^0} H_R \bar{\chi}_i \gamma_5 \left( H_R^{\tilde{l}_k \chi_i^0} \right)^* \right] \right. \]

\[ - \left( H_R^{\tilde{l}_k \chi_i^0} \right)^* H_R^{\tilde{l}_k \chi_i^0} \left( H_R^{\tilde{l}_k \chi_i^0} \right)^* H_R \bar{\chi}_i \gamma_5 \left( H_R^{\tilde{l}_k \chi_i^0} \right)^* \right] \}

\[ B_q^R(n) = B_q^L(n) \bigg|_{L \leftrightarrow R}, \quad (22) \]

The virtual charged Fermion in the middle line of Fig. 2 give contributions denoted by \( B_q(c) \).

\[ B_q^L(c) = \frac{2}{\sum_{i,j=1,k,l=1} B_q^L(n)} \sum_{i,j=1,k,l=1} \left\{ \frac{1}{8e^2 m_W^2} G_6 \left( x_{\chi_i^0}, x_{\chi_j^0}, x_{\nu_k}, x_{\bar{\nu}_l} \right) \left[ \left( H_R^{\tilde{l}_k \chi_i^0} \right)^* H_R^{\tilde{l}_k \chi_i^0} H_R \bar{\chi}_i \gamma_5 \left( H_R^{\tilde{l}_k \chi_i^0} \right)^* \right] \right. \]

\[ - \left( H_R^{\tilde{l}_k \chi_i^0} \right)^* H_R^{\tilde{l}_k \chi_i^0} \left( H_R^{\tilde{l}_k \chi_i^0} \right)^* H_R \bar{\chi}_i \gamma_5 \left( H_R^{\tilde{l}_k \chi_i^0} \right)^* \right] \}

\[ B_q^R(c) = B_q^L(c) \bigg|_{L \leftrightarrow R} . \quad (23) \]

The virtual \( W \) produces corrections through the diagrams in the last line of Fig. 2.

\[ B_q^L(W) = -\frac{1}{2e^2} \frac{\partial}{\partial x_W} G_2 \left( x_W, x_{\nu}, x_{\bar{\nu}} \right) \left( H_R^{W\mu \nu} H_R^{W^* \nu \bar{\nu}} H_R^{W^* \mu \bar{\nu}} H_R^{W^* \bar{\mu} \bar{\nu}} \right), \]

\[ B_u^L(W) = 2 \frac{\partial}{\partial x_W} \left( H_R^{W^* \mu \nu} H_R^{W^* \nu \bar{\nu}} H_R^{W^* \mu \bar{\nu}} H_R^{W^* \bar{\mu} \bar{\nu}} \right), \]

\[ B_d^R(W) = B_u^R(W) = 0. \quad (24) \]
C. $\mu - e$ conversion rate

Once we know the effective Lagrangian relevant to this process at the quark level, we can calculate the conversion rate

\[ CR = 4\alpha \frac{Z_{\text{eff}}^4}{Z} \left| F \left( q^2 \right) \right|^2 m_\mu^5 \left[ |Z (A_1^L - A_2^R) - (2Z + N) \bar{D}_u^L - (Z + 2N) \bar{D}_d^L| \right]^2 \]

\[ + |Z (A_1^R - A_2^L) - (2Z + N) \bar{D}_u^R - (Z + 2N) \bar{D}_d^R|^2 \frac{1}{\Gamma_{\text{capt}}}, \quad (25) \]

\[ \bar{D}_q^L = D_q^L + \frac{Z_q^L + z_q^R}{2} \frac{F_L}{m_Z^2 s_W^2 c_W^2}, \]

\[ \bar{D}_q^R = \bar{D}_q^L \bigg|_{L\leftrightarrow R}, \quad (q = u, d). \quad (26) \]

with $Z$ and $N$ representing the proton and neutron numbers in a nucleus. $Z_{\text{eff}}$ is an effective atomic charge determined in refs [31, 32]. $F \left( q^2 \right)$ is the nuclear form factor and $\Gamma_{\text{capt}}$ denotes the total muon capture rate, while $\alpha$ is the fine structure constant.

IV. NUMERICAL RESULTS

In this section, we discuss the numerical results, and consider the experimental constraints from the lightest neutral CP-even Higgs mass $m_{h_0} \simeq 125.1$ GeV [12–14] and the neutrino experiment data. In this model, the LFV processes $l_j \rightarrow l_i \gamma$ and $l_j \rightarrow 3l_i$ are studied in our previous work [30], and their constraints are also taken into account. In this work, we use the parameters [28, 29]

\[ L_4 = \frac{3}{2}, \quad \mu = 0.5 \text{GeV}, \quad (A_l)_{ii} = -2000 \text{GeV}, \quad (A_l')_{ii} = A'_L = 300 \text{GeV}, \]

\[ (A_{Nc})_{ii} = (A_N)_{ii} = -500 \text{GeV}, \quad (A_u)_{ii} = (A_d)_{ii} = (A'_d)_{ii} = (m_{Nc})_{ii} = 1000 \text{GeV}, \]

\[ (A'_u)_{ii} = 800 \text{GeV}, \quad (m_Q)_{ii} = (m_Q')_{ii} = (m_D)_{ii} = 2 \times 10^3 \text{GeV}, \quad \lambda_{Nc} = 1. \quad (27) \]

The Yukawa couplings of neutrinos $(Y_\nu)^{IJ}$, $(I, J = 1, 2, 3)$ are at the order of $10^{-8} \sim 10^{-6}$, whose effects to this processes are tiny and can be ignored savely. To simplify the numerical discussion, we use the following relations

\[ (A_l)_{ii} = A_L, \quad (A_{Nc})_{ii} = (A_N)_{ii} = AN, \quad (A_l')_{ii} = A'_L, \]

\[ (m_{Nc})_{ii} = M_{Nc}^2, \quad (m_Q)_{ii} = (m_Q')_{ii} = s_m^2, \quad \text{for} \ i, j = 1, 2, 3. \]

\[ (m_{L}^2)_{ij} = (m_{R}^2)_{ij} = M_{LJ}, \quad \text{for} \ i, j = 1, 2, 3 \text{ and } i \neq j. \quad (28) \]
If we do not specially declare, the non-diagonal elements of the used parameters should be zero.

A. $\mu \rightarrow e$ conversion rate in nuclei Au

The experimental upper bound for the $\mu \rightarrow e$ conversion rate in nuclei Au is around $7.0 \times 10^{-13}$. The parameters $m_2 = 1000\text{GeV}, M_{L_f} = 10^4 \text{GeV}^2$ are supposed in the calculation of this process. The parameter $m_1$ is related to the mass matrix of the neutralino, which means the contributions from neutralino-slepton can be influenced by the parameter $m_1$. For $S_m^2 = 13 \text{TeV}^2$, $\tan\beta = 5.0$, $\tan\beta_L = 2.0$, and $g_L = \frac{1}{6}$, we plot the results versus $m_1$ with $V_{Lt} = 3000 \text{GeV}$ and 6000 GeV in Fig.3. We can see that the results decrease quickly with the increase of $m_1$. As $V_{Lt} = 6000 \text{GeV}$, the results are slightly smaller than the corresponding results with $V_{Lt} = 3000 \text{GeV}$. This implies that $m_1$ is a sensitive parameter and has a strong effect on muon conversion to electron in nuclei. Compared with $m_1$, the effect from $V_{Lt}$ is very small.

![Graph](image)

**FIG. 3**: With $m_2 = 1000\text{GeV}, M_{L_f} = 10^4 \text{GeV}^2, S_m^2 = 13 \text{TeV}^2$, $\tan\beta = 5.0$, $\tan\beta_L = 2.0$, and $g_L = \frac{1}{6}$, $\mu \rightarrow e$ conversion rate in nuclei Au versus $m_1$ with $V_{Lt} = 3000 \text{GeV}$ (solid line) and 6000 GeV (dotted line).

$\tan\beta$ is related to $v_u$ and $v_d$, and appears in almost all mass matrices of particles contributing to the $\mu \rightarrow e$ processes. With $m_1 = 500 \text{GeV}, V_{Lt} = 3000 \text{GeV}$, $\tan\beta_L = 2.0$, and $g_L = \frac{1}{6}$, Fig.4 shows the variation of the $\mu \rightarrow e$ conversion rate in nuclei Au with the parameter $\tan\beta$ and $S_m^2$. It indicates that the results change significantly with $\tan\beta$. When $\tan\beta$ is in the region $(0 \sim 6)$, the results decrease significantly, but in the range of $\tan\beta > 6$, we find that the results increase sharply. Only when the value of $\tan\beta$ is about 6, the results
of $\mu \to e$ conversion rate in nuclei Au are close and not higher than the experimental upper bound.

![Graph](image)

**FIG. 4:** With $m_1 = 500 \text{GeV}$, $m_2 = 1000 \text{GeV}$, $M_{L_f} = 10^4 \text{GeV}^2$, $V_{Lt} = 3000 \text{GeV}$, $\tan \beta_L = 2.0$, and $g_L = \frac{1}{6}$, the contribution to $\mu \to e$ conversion rate in nuclei Au versus $\tan \beta$ with $S_m^2 = 12 \text{TeV}^2$ (solid line) and $16 \text{TeV}^2$ (dotted line).

The parameters $g_L$, $\tan \beta_L$ and $V_{Lt}$ all present in the mass squared matrices of sleptons, sneutrinos and lepton neutralinos. Therefore, these three parameters affect the results through slepton-neutrino, sneutrinos-chargino and slepton-lepton neutralino contributions.

As $m_1 = 1000 \text{GeV}$, $\tan \beta = 5.5$, $\tan \beta_L = 2$, $S_m^2 = 16 \text{TeV}^2$, $g_L$ versus $V_{Lt}$ are scanned in Fig.5. We find that the allowed scope of $V_{Lt}$ shrinks and the value of $V_{Lt}$ decreases with the enlarging $g_L$. Therefore, the value of $g_L$ should not be too large. Generally, we take $0.05 \leq g_L \leq 0.3$ and $V_{Lt} \sim 3 \text{TeV}$ in our numerical calculations.

As the parameters $m_1 = 1000 \text{GeV}$, $\tan \beta = 6.0$, $S_m^2 = 12 \text{TeV}^2$ and $V_{Lt} = 3000 \text{GeV}$, we plot the allowed results with $\tan \beta_L$ versus $g_L$ in Fig.6. When $g_L < 0.43$, the parameter $\tan \beta_L$ can vary in the region of $(0 \sim 4)$. It implies that $g_L$ is a sensitive parameter to the numerical results and the value of $g_L$ should not be larger than 0.43.

**B. $\mu \to e$ conversion rate in nuclei Ti**

In a similar way, the $\mu \to e$ conversion rate in nuclei Ti is numerically studied and its experimental upper bound is around $4.3 \times 10^{-12}$. In this subsection, we use the parameters $\tan \beta = 2.0$, $\tan \beta_L = 2.0$, $g_L = \frac{1}{6}$ and $V_{Lt} = 3000 \text{GeV}$. The parameter $m_2$ presents in the mass matrixes of neutralino and chargino. This parameter affects the numerical results through the neutralino-slepton and chargino-sneutrino contributions. With $S_m^2 = \ldots$
FIG. 5: For $\mu \rightarrow e$ conversion rate in nuclei Au, the allowed parameter space in the plane of $g_L$ versus $V_{Lt}$ with $m_1 = 1000$ GeV, $m_2 = 1000$ GeV, $\tan \beta = 5.5$, $S_m^2 = 16$ TeV$^2$, $M_{Lf} = 10^4$ GeV$^2$ and $\tan \beta_L = 2$.

FIG. 6: For $\mu \rightarrow e$ conversion rate in nuclei Au, the allowed parameter space in the plane of $\tan \beta_L$ versus $g_L$ with $m_1 = 1000$ GeV, $m_2 = 1000$ GeV, $\tan \beta = 6$, $S_m^2 = 12$ TeV$^2$, $M_{Lf} = 10^4$ GeV$^2$ and $V_{Lt} = 3000$ GeV.

10 TeV$^2$, $M_{Lf} = 10^4$ GeV$^2$, we plot the results versus $m_2$ with $m_1 = 1000$ GeV and 2000 GeV by the solid and dotted lines in Fig.7. We can see that the results decrease with the increase of $m_2$. The results of dotted line are slightly larger than solid line and all the results are in the region $(1.4 \times 10^{-12} \sim 6.4 \times 10^{-12})$. This implies that $m_2$ should have impact on the results to some extent.

$M_{Lf}$ are the non-diagonal elements of $\tilde{m}_L^2$ and $\tilde{m}_R^2$ in the slepton mass matrix. For $m_1 = 1000$ GeV and $S_m^2 = 12$ TeV$^2$, we study the $\mu \rightarrow e$ conversion rate in nuclei Ti versus $M_{Lf}$ with $m_2 = 1000$ GeV (solid line) and $m_2 = 2000$ GeV (dotted line) in Fig.8. As
FIG. 7: With $\tan \beta = 2.0$, $S_m^2 = 10 \text{TeV}^2$, $M_{Lf} = 10^4 \text{GeV}^2$, $\tan \beta_L = 2.0$, $g_L = \frac{1}{6}$ and $V_{Lt} = 3000 \text{GeV}$, $\mu \to e$ conversion rate in nuclei Ti versus $m_2$ is plotted for $m_1 = 2000 \text{GeV}$ (solid line) and 1000 GeV (dotted line).

$M_{Lf} = 0$, the conversion ratio for $\mu \to e$ is almost zero, but the results increase sharply with $M_{Lf} > 0$. We deduce that non-zero $M_{Lf}$ is a sensitive parameter and has a strong effect on muon conversion to electron in nuclei.

FIG. 8: With $m_1 = 1000 \text{GeV}$, $\tan \beta = 2.0$, $S_m^2 = 12 \text{TeV}^2$, $\tan \beta_L = 2.0$, $g_L = \frac{1}{6}$ and $V_{Lt} = 3000 \text{GeV}$, $\mu \to e$ conversion rate in nuclei Ti versus $M_{Lf}$ with $m_2 = 1000 \text{GeV}$ (solid line) and 2000 GeV(dotted line).

C. $\mu \to e$ conversion rate in nuclei Pb

The experimental upper bound of $\mu \to e$ conversion rate in nuclei Pb is around $4.6 \times 10^{-11}$. In this subsection, we use the parameters $m_2 = 1000 \text{GeV}$, $\tan \beta_L = 2.0$, $g_L = \frac{1}{6}$ and $M_{Lf} = 10^4 \text{GeV}^2$. $S_m$ are the diagonal elements of $m_L^2$ and $m_R^2$ in the slepton mass matrix,
which can affect slepton-neutralino and slepton-lepton neutralino contributions in the $\mu \rightarrow e$ process.

With $m_1 = 1000$ GeV, $V_{Lt} = 3000$ GeV, we plot the conversion ratio for $\mu \rightarrow e$ in nuclei Pb versus $S_m$ with $\tan \beta = 2.0$ (solid line) and $\tan \beta = 3.0$ (dotted line) in Fig.9. These two lines decrease quickly with $S_m$ enlarging from 1400 GeV to 3000 GeV, which indicates that $S_m$ is a very sensitive parameter to the numerical results. When $S_m > 3000$ GeV, the results decrease slowly and the conversion ratios are around $10^{-12} \sim 10^{-13}$.

![Figure 9](image_url)

FIG. 9: With $m_1 = 1000$ GeV, $m_2 = 1000$ GeV, $M_{Lf} = 10^4$ GeV$^2$, $\tan \beta_L = 2.0$, $g_L = \frac{1}{6}$ and $V_{Lt} = 3000$ GeV, the contribution to $\mu \rightarrow e$ conversion rate in nuclei Pb versus $S_m$ with $\tan \beta = 2$ (solid line) and 3 (dotted line).

We focus on $V_{Lt}$ which is a special parameter in BLMSSM, and with $m_1 = 500$ GeV, $\tan \beta = 13$, we plot the conversion ratio for $\mu \rightarrow e$ in nuclei Pb versus $V_{Lt}$ with $S_m^2 = 5$ TeV$^2$ (solid line) and $S_m^2 = 6$ TeV$^2$ (dotted line) in Fig. 10. Overall, the results of dotted line are about $0.5 \times 10^{-11} \sim 1.2 \times 10^{-11}$ larger than the solid line. In the range of $V_{Lt} = (0 \sim 10000$ GeV), the two lines decrease quickly with the enlarging $V_{Lt}$. We can see $S_m$ and $V_{Lt}$ are sensitive parameters to the numerical results.

V. DISCUSSION AND CONCLUSION

In the framework of the BLMSSM model, we study the LFV processes $\mu \rightarrow e + q\bar{q}$. In the processes, we consider some new parameters and contributions, such as the newly introduced parameters $g_L$, $\tan \beta_L$ and $V_{Lt}$. Combined with the numerical results discussed in the Section IV, different parameters have different effects on the processes. The parameter $g_L$ presents in the mass squared matrices of sleptons, sneutrinos and lepton neutralinos. Numerical
FIG. 10: With \( m_1 = 500 \text{ GeV} \), \( m_2 = 1000 \text{ GeV} \), \( \tan \beta = 13 \), \( M_{L_f} = 10^4 \text{ GeV}^2 \), \( \tan \beta_L = 2.0 \), and \( g_L = \frac{1}{6} \), \( \mu \rightarrow e \) conversion rate in nuclei Pb versus \( V_{Lt} \) with \( S_m^2 = 5 \text{ TeV}^2 \) (solid line) and \( S_m^2 = 6 \text{ TeV}^2 \) (dotted line).

analysis shows that \( g_L \) has obvious influence on the results, the value of \( g_L \) should not be too large. As sensitive parameters, \( S_m \) and \( M_{L_f} \) are respectively diagonal and non-diagonal elements of matrixes for \( m_L \) and \( m_R \). Both \( S_m \) and \( M_{L_f} \) have significant impacts on the results. \( \tan \beta \) is related to \( v_u \) and \( v_d \), and appears in almost all mass matrices of particles contributing to the \( \mu \rightarrow e \) processes. The value of \( \tan \beta \) is critical to these processes. With the improvement of experimental accuracy, we believe that there will be some discoveries for \( \mu \) to \( e \) conversion in the near future.

VI. ACKNOWLEDGMENTS

Supported by National Natural Science Foundation of China (No. 11535002 and No.11575052), Physics laboratory center of Hebei GEO University(Hebei experimental teaching demonstration center).

VII. APPENDIX

In this section, we give out the corresponding one loop functions. \( G_2(x_1, x_2, x_3) \) and \( G_3(x_1, x_2, x_3) \) have infinite term, and to obtain finite results we use \( \overline{MS} \) subtraction and \( DR \) scheme.
\[ I_1(x_1, x_2) = \frac{1}{96\pi^2} \left[ \frac{11 + 6\ln x_2}{(x_2 - x_1)^2} - \frac{15x_2 + 18x_2\ln x_2}{(x_2 - x_1)^3} + \frac{6x_2^2 + 18x_2^2\ln x_2}{(x_2 - x_1)^4} \right], \]
\[ \quad \quad \quad + \frac{6x_1^2\ln x_1 - 6x_1^2\ln x_2}{(x_2 - x_1)^4}. \] (29)

\[ I_2(x_1, x_2) = \frac{1}{32\pi^2} \left[ \frac{3 + 2\ln x_2}{(x_2 - x_1)} - \frac{2x_2 + 4x_2\ln x_2}{(x_2 - x_1)^2} - \frac{2x_2^2}{(x_2 - x_1)^3} + \frac{2x_2^2\ln x_2}{(x_2 - x_1)^3} \right], \] (30)

\[ I_3(x_1, x_2) = \frac{1}{16\pi^2} \left[ \frac{1 + \ln x_2}{(x_2 - x_1)} + \frac{x_1\ln x_1 - x_2\ln x_2}{(x_2 - x_1)^2} \right]. \] (31)

\[ I_4(x_1, x_2) = \frac{1}{16\pi^2} \left[ - \frac{1 + \ln x_1}{(x_2 - x_1)} - \frac{x_1\ln x_1 - x_2\ln x_2}{(x_2 - x_1)^2} \right]. \] (32)

\[ G_1(x_1, x_2, x_3) = \frac{1}{16\pi^2} \left[ \frac{x_1\ln x_1}{(x_2 - x_1)(x_1 - x_3)} + \frac{x_2\ln x_2}{(x_2 - x_1)(x_2 - x_3)} + \frac{x_3\ln x_3}{(x_3 - x_1)(x_3 - x_2)} \right], \] (33)

\[ G_2(x_1, x_2, x_3) = \frac{1}{16\pi^2} \left[ - (\Delta + 1 + \ln x_\mu) + \frac{x_1^2\ln x_1}{(x_1 - x_2)(x_1 - x_3)} + \frac{x_2^2\ln x_2}{(x_2 - x_1)(x_2 - x_3)} + \frac{x_3^2\ln x_3}{(x_3 - x_1)(x_3 - x_2)} \right]. \] (34)

\[ G_3(x_1, x_2) = -\frac{1}{16\pi^2} \left( (\Delta + \ln x_\mu + 1) + \frac{x_2^2\ln x_2 - x_1^2\ln x_1}{(x_2 - x_1)^2} + \frac{x_2 + 2x_2\ln x_2}{x_1 - x_2} - \frac{1}{2} \right), \]

\[ G_4(x_1, x_2, x_3) = \frac{1}{32\pi^2} \left[ \frac{2x_1^3[3x_1(x_1 - x_2 - x_3) + x_2^2 + x_2x_3 + x_3^2]\ln x_1}{(x_1 - x_3)^3(x_1 - x_2)^3} - \frac{2(3x_1^2 - 3x_1x_2 + x_2^2)x_2\ln x_2}{(x_1 - x_2)^3(x_2 - x_3)} + \frac{2(3x_1^2 - 3x_1x_3 + x_3^2)x_3\ln x_3}{(x_1 - x_3)^3(x_2 - x_3)} \right. \]
\[ - \left. \frac{x_1[5x_1^2 - 3x_1(x_2 + x_3 + x_2x_3)]}{(x_1 - x_2)^2(x_1 - x_3)^2} \right] \right], \]

\[ G_5(x_1, x_2, x_3) = \frac{1}{16\pi^2} \left[ \frac{x_1^2(2x_1 - x_2 - x_3)\ln x_1}{(x_1 - x_2)^2(x_1 - x_3)^2} + \frac{x_2(x_2 - 2x_1)\ln x_2}{(x_1 - x_2)^2(x_2 - x_3)} \right. \]
\[ - \left. \frac{x_3(2x_1 - x_3)\ln x_3}{(x_1 - x_2)^2(x_1 - x_3)^2} + \frac{x_3^2(2x_1 - x_3)\ln x_3}{(x_1 - x_2)(x_1 - x_3)^2(x_2 - x_3)^2} \right]. \] (35)

\[ G_6(x_1, x_2, x_3, x_4) = \frac{1}{16\pi^2} \left[ \frac{x_1^2\ln x_1}{(x_1 - x_2)(x_1 - x_3)(x_1 - x_4)} + \frac{x_2^2\ln x_2}{(x_2 - x_1)(x_2 - x_3)(x_2 - x_4)} \right. \]
\[ + \left. \frac{x_3^2\ln x_3}{(x_3 - x_1)(x_3 - x_2)(x_3 - x_4)} + \frac{x_4^2\ln x_4}{(x_4 - x_1)(x_4 - x_2)(x_4 - x_3)(x_4 - x_2)} \right]. \] (36)

\[ G_7(x_1, x_2, x_3, x_4) = \frac{1}{16\pi^2} \left[ \frac{x_1\ln x_1}{x_2\ln x_2} + \frac{x_2\ln x_2}{x_3\ln x_3} + \frac{x_3\ln x_3}{x_4\ln x_4} + \frac{x_4\ln x_4}{(x_4 - x_1)(x_4 - x_2)(x_4 - x_3)} \right]. \]
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