Study on the influence of beam transverse position on the cavity bunch length measurement

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Abstract. Monopole modes in the resonant cavities are wildly used to obtain the beam current and the bunch length, while dipole modes are used to measure the beam transverse position. It is generally recognized that the monopole modes are independent of the beam transverse offset. In this paper, the influence of beam transverse offset on the bunch length measurement using monopole modes is analyzed. The simulation results show that the relative error of the bunch length measurement is less than 1 % when the beam offset is within 1 mm.

1. Introduction
Bunch root mean square (rms) length is one of the important beam parameters. As an effective way to measure the rms bunch length, resonate cavities are wildly used for the past few years [1, 2, 3, 4]. The measurement can be achieved by extracting the monopole modes because it contains the information about the rms bunch length. In previous studies, it is assumed the bunch passes through the beam drift tube on the axis of the cavity, and the bunch length is measured without regard to beam transverse offset, since the monopole mode is deemed to be independent of the beam transverse position. But in reality, the measurement needs at least two monopole modes, and the beam position have effect on the two modes. Therefore, the errors must be considered. In this paper, by taking the cavity bunch length monitor in the National Synchrotron Radiation Laboratory infrared free-electron laser facility (FELiChEM) [5], the influence of beam transverse offset on the bunch length measurement is analyzed.

2. Theoretical basis
2.1. Measurement method
When a bunch passes through the cavity, many monopole modes such as TM_{010} mode and TM_{020} are excited. The output voltage of a monopole mode can be written as [6, 7]

\[ V = \frac{1}{2} \omega q \sqrt{\frac{Z(R/Q_0)}{Q_{ext}}} \exp\left(-\frac{\omega^2 \sigma^2}{2}\right), \]

where \( \omega \) is the mode working frequency, \( q \) is the bunch charge, \( Z \) is the impedance of the detector, \( (R/Q_0) \) is the normalized shunt impedance, \( Q_{ext} \) is the external quality factor of the mode and \( \sigma \) is the rms bunch length. We need at least two monopole modes in different frequencies, and the
The bunch charge $q$ and the bunch length $\sigma$ can be obtained by solving the simultaneous equations

$$\begin{align*}
V_1 &= \frac{1}{2} \omega_1 \sqrt{\frac{Z(R/Q_0)}{Q_{ext1}}} q \exp\left(-\frac{\omega_1^2 \sigma^2}{2}\right) \\
V_2 &= \frac{1}{2} \omega_2 \sqrt{\frac{Z(R/Q_0)}{Q_{ext2}}} q \exp\left(-\frac{\omega_2^2 \sigma^2}{2}\right),
\end{align*}$$

(2)

It follows that the bunch length can be described as:

$$\sigma = \sqrt{2 \ln[k_2 V_1/(k_1 V_2)]/(\omega_2^2 - \omega_1^2)}. \quad (3)$$

Taking the output voltage of the two monopole modes, $V_1$ and $V_2$, into Eq. (3), the bunch length can be obtained.

### 2.2. Cavity monitor

According to the requirements of FELiChEM, the cavity bunch length monitor has been designed, as shown in Fig. 1.

![Figure 1. The cavity bunch length monitor.](image)

This monitor is composed of two cavities. The TM$_{010}$ mode in the first cavity and the TM$_{020}$ mode in the second cavity are used to measure the bunch length. The two modes resonate at 0.9515 GHz and 6.1847 GHz, respectively. The signal coupler consists of two coaxial probes with axial symmetry.

### 2.3. Influence of beam transverse position

From Eq. (1), it can be seen that the output voltage is in direct proportion to $(R/Q_0)$ of the mode. $(R/Q_0)$ can be described as [6]:

$$\frac{R}{Q_0} = \frac{\left| \int E ds \right|^2}{\omega U}, \quad (4)$$

where $U$ is the stored energy of the mode in the cavity, and the numerator indicates integration of the electric field of the mode along the beam orbit. $(R/Q_0)$ is a function of beam offset. Monopole modes are axially symmetric and have a field maximum on the cavity axis. The electric field intensity distributions of TM$_{010}$ mode and TM$_{020}$ mode along the cavity diameter are shown in Fig. 2.

Compared with moving on axis, the beam passes the cavity with a small offset leads to the electric field intensity of the two modes decrease slightly on the beam orbit, which means the...
output voltages decline. Although the amplitude of the signal of the TM010 mode is insensible to the beam offset, it is still changing with the position.

Meanwhile, when a bunch passes off axis of the cavity, a series of dipole modes such as TM_{110} are excited. The dipole modes have effect on the monopole modes. For example, Fig. 3 shows that the wide-band TM_{110} mode disturb the TM_{010} mode in frequency domain, which leads to the increase of TM_{010} mode amplitude. This interference is still exist even through the TM_{110} mode is narrow-band.

According to Eq. (3), any small changes of $V_1/V_2$ will bring about great impact on the measuring results. Therefore, the influence of beam transverse offset on the bunch length measurement must be taken into account.

3. Simulation results

To study the influence of beam transverse offset on bunch length measurement, the simulations with the bunch passing through the cavity monitor at different transverse positions are performed. We evaluate the output signal level with the 3D electromagnetic field simulation program CST, setting a bunch charge of 1 nC. Figure 4 is the waveform in time domain and frequency domain from the second cavity with beam charge 1nC and bunch length 2ps in CST.
The output voltages in the time domain are difficult to be acquired because of signal attenuation and frequency mixing, so we have the amplitude in frequency domain instead of it.

![Graph showing output signal in time domain and frequency domain.](image)

**Figure 4.** The output signal in time domain and frequency domain.

3.1. Influence on monopole mode
The bunches of 2 ps and 5 ps are used in the simulations. The output voltage from two coaxial probes of the first cavity in different beam offsets are shown in Fig. 5.

It can be seen that the larger the beam offset value is, the more greatly the output voltage changes. The change directions of the output amplitude from the two ports are inverse, because the electric field of dipole modes is equal and opposite at the two axisymmetric coaxial probes. In this case, monopole mode attenuation is weaker than interference from the dipole modes.

The output voltage from two coaxial probes of the second cavity in different beam offsets are shown in Fig. 6.

It can be seen that the output amplitude decreases with increasing beam offsets. From Fig. 2, it can be seen that the electric field intensity of the TM$_{020}$ mode drops faster than that of the TM$_{010}$ mode near the axis. Therefore, the monopole mode attenuation is stronger than the interference from the dipole modes. The dipole modes only lead to difference between the two output signals.

3.2. Influence on bunch length measurements
According to Eq. (3), the bunch length measurement results depend on $V_1/V_2$. Simulation results of bunch length in different beam offsets are shown in Table 1.

It can be seen that the error of the rms bunch length measurements increases with the increase of beam offset, especially for shorter bunch measurements. For the FELiChEM facility, the rms bunch length ranges from 2 ps to 5 ps. Under normal operation, the beam transverse offset can be controlled within 1 mm, so the influence of beam transverse position on bunch length measurements is less than 1 %. The errors can be ignored.
Figure 5. The output voltage from two coaxial probes of the first cavity.

Figure 6. The output voltage from two coaxial probes of the second cavity.

Table 1. Simulation results of bunch length in different beam offsets.

| Set Value of the Bunch Length (ps) | Beam Offset (mm) | Simulation Result of the Bunch Length (ps) | Relative Error (%) | Set Value of the Bunch Length (ps) | Beam Offset (mm) | Simulation Result of the Bunch Length (ps) | Relative Error (%) |
|-----------------------------------|------------------|------------------------------------------|-------------------|-----------------------------------|------------------|------------------------------------------|-------------------|
| 2                                 | 0                | 1.99903                                   | 0.049             | 5                                 | 0                | 5.00046                                   | 0.009             |
| 2                                 | 0.25             | 1.99457                                   | 0.271             | 5                                 | 0.25             | 4.99716                                   | 0.057             |
| 2                                 | 0.50             | 2.00692                                   | 0.346             | 5                                 | 0.50             | 5.00261                                   | 0.052             |
| 2                                 | 0.75             | 2.01034                                   | 0.517             | 5                                 | 0.75             | 5.00406                                   | 0.081             |
| 2                                 | 1.00             | 2.01721                                   | 0.861             | 5                                 | 1.00             | 5.00963                                   | 0.193             |
| 2                                 | 2.00             | 2.13032                                   | 6.516             | 5                                 | 2.00             | 5.05084                                   | 1.017             |
| 2                                 | 3.00             | 2.30553                                   | 15.276            | 5                                 | 3.00             | 5.13123                                   | 2.625             |
| 2                                 | 4.00             | 2.53220                                   | 26.610            | 5                                 | 4.00             | 5.23755                                   | 4.751             |
| 2                                 | 5.00             | 2.87085                                   | 43.542            | 5                                 | 5.00             | 5.40865                                   | 8.173             |

4. Summary
In this paper, the influence of beam transverse offset on the cavity bunch length monitor is studied by theory and simulation. The main sources of error are attenuation of the monopole mode interference from the dipole modes. For different cavities and different modes, the two kinds of errors above lead to very different results. Due to the alignment system, beam transverse offset of most free-electron laser facilities can be controlled within 1 mm, so the influence of beam offset is very small, and it can be ignored.

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