New Population Synthesis Techniques in the Analysis of Interacting Binaries

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Abstract. Novel approaches to understanding the observed properties of interacting binaries containing compact accretors such as neutron stars and white dwarfs are examined. Explaining the evolution of these systems is a computationally challenging problem because the vector space of initial conditions that describes the progenitor binaries is wide-ranging. There are large variations in the chemical abundance (e.g., metallicity), binary mass correlations, and assumed input physics. In this paper we compare two very different strategies to synthesize a specific subset of the currently observed population of compact binaries. Both involve the pre-computing a large grid of representative models. In the first case, the grid of initial conditions is densely packed thereby allowing us to identify the spectrum of initial conditions and the most probable evolutionary channels leading to the formation of the observed binaries. In the second, the grid is accurately interpolated to provide us with the ensemble properties of the currently observed population of interacting binaries (e.g., Cataclysmic Variables). As an example of the utility of the first approach, we have taken advantage of the multicore processing power of the fast, new stellar evolution code known as MESA to compute an extensive grid of binary evolution tracks for low- and intermediate-mass X-ray binaries. The grid is about two orders of magnitude larger than any previous computation of X-ray binary evolution and includes more than 40,000 models. It comprises 60 initial donor masses over the range of 1 to 4 M⊙ and, for each of these, 700 initial orbital periods over the range of 10 to 250 hours were chosen. Using a ‘traceback’ analysis, we show how the extremely massive neutron star (1.97 M⊙) in the binary pulsar PSR J1614-2230 is likely to have evolved. We find that the initial donor stars which produce the closest relatives to PSR J1614-2230 are likely to have had a mass of between approximately 3.4 to 3.8 M⊙. Nonetheless, we conclude that it is difficult to form high-mass neutron stars unless they are born with masses larger than the 1.4 M⊙ canonical value.

1. Introduction
Interacting binaries are stellar systems composed of two stars in mutual orbit that are undergoing (or have undergone) mass transfer or systemic mass loss. They are generally very rich in physics, especially with respect to the study of radiative hydrodynamics, nuclear physics, and high-energy physics. An important sub-class of these systems is comprised of interacting binaries containing compact accretors (i.e., white dwarfs [WDs], neutron stars [NSs], and black holes [BHs]). If the separation between the stars is sufficiently small, the companion star (donor) can become very tidally and rotationally distorted. If the separation is decreased, it will eventually overflow its Roche lobe (corresponding to the critical gravitational equipotential surface) and matter will generally flow towards...
the compact accretor (see figure 1). The matter that is stripped from the donor has too much angular momentum to be directly captured by the accretor and thus an accretion disk forms around it (see Hilditch [1] for more details).

There are several sub-populations of interacting binaries containing compact objects that are delineated based on the nature of the accretor (see, e.g., Eggleton [2] and Warner [3]). If the accretor is a white dwarf, then the binary is known as a Cataclysmic Variable (CV). If it is a neutron star, then the binary is referred to as an X-ray binary and this terminology is further refined depending on whether the donor is a low-mass or intermediate-mass star. While it is true that we generally have good models to describe the properties of these systems, it is important to understand their ensemble properties. In particular we would like to understand how these systems formed and subsequently evolved thereby producing the relative numbers of the various classes of interacting binaries that we actually observe at this epoch in the Galaxy’s history. Only when we can synthesize all of the classes of interacting binaries in the correct numbers and with the correct properties can we say that we truly have a robust, unified picture for their formation and evolution.

In this paper I discuss two distinctly different approaches to addressing this issue. The first approach takes advantage of the tremendous increases in computational speed; it is now possible to calculate extensive grids of evolutionary models for a multidimensional parameter space of initial conditions. Although this approach can be viewed as somewhat of a brute force method, it does have its advantages. For example, if the grid is sufficiently-well resolved, the models can produce new and interesting types of evolution that require further investigation (e.g., ultracompact accreting millisecond pulsars). Moreover, it may be possible to trace the progenitor evolution of astrophysically important systems such as the binary millisecond pulsar PSR J1640-2230. For the second approach, population synthesis techniques are used to generate a Galactic-disk population of CVs (see, e.g., Howell, Nelson and Rappaport [4], and references therein). In this case a grid of evolutionary models is pre-computed and then interpolated based on the actual initial properties of individual primordial binaries that comprise the ensemble. The strengths and weaknesses of both of these approaches are also discussed.

2. Methodology
In order to analyze the evolution of interacting binaries containing compact objects, the detailed structure and response of the donor star to mass loss must be calculated. The analysis of stellar structure and evolution was originally carried out using analytic and semi-analytic methods (see, e.g., Chandrasekhar [5] and Schwarzschild [6]). However, the advent of computers greatly facilitated the solution of the highly coupled and non-linear partial differential equations describing their evolution. For the calculations presented in this paper, the evolution and response of the donor to mass-loss is calculated in tandem with the orbital dynamics and the quasi-hydrostatic rules that govern the mass-loss rate. They are briefly described in the two following sections.
2.1. Stellar Structure and Evolution of the Donor

Aside from changes in the chemical composition, the structure and evolution of a spherically symmetric (one-dimensional) donor star is governed by four coupled partial differential equations. Adopting a Lagrangian viewpoint, we take the independent variables to be $t$, the elapsed time, and $m_r$, the mass contained within a sphere of radius $r$. These equations ensure mass continuity, energy and momentum conservation, and account for energy transport by either radiation, conduction and/or convection. They can be expressed as:

\[
\begin{align*}
\frac{\partial P}{\partial m_i} &= \left( \frac{G}{4\pi} \right) \frac{m_i}{r^2} - \frac{1}{4\pi r^2} \left( \frac{\partial^2 r}{\partial t^2} \right) m_i, \\
\frac{\partial r}{\partial m_i} &= \frac{1}{4\pi r^2 \rho}, \\
\frac{\partial L}{\partial m_i} &= \epsilon_{\text{nuc}} - \epsilon_{\nu} - T \left( \frac{\partial S}{\partial t} \right) m_i, \\
\frac{\partial T}{\partial m_i} &= \left( \frac{3}{64\pi^2 ac} \right) \frac{L_k}{r^7 T^7} \quad \text{[Radiative/Conductive]} \\
\frac{\partial T}{\partial m_i} &= \left( \frac{T}{P} \right) \left( \frac{\partial P}{\partial m_i} \right) \nabla_{\text{ad}} \quad \text{[Convective]},
\end{align*}
\]

where $P$ is the pressure, $T$ is the temperature, $L$ is the luminosity, $S$ is the specific entropy, $\rho$ is the density, $\epsilon$ is the specific energy generation/degradation (nuclear energy generation and neutrino losses, respectively), $\kappa$ is the radiative/conductive opacity, and $\nabla_{\text{ad}}$ is the adiabatic temperature gradient.

In order to solve these equations, the appropriate boundary conditions and physical constraints must be applied. Specifically, we enforce $L=0$ and $r=0$ at the centre of the star, and match an atmosphere at the surface that indirectly gives us $T$ and $P$ there. The input physics serves to provide us with the equations of constraint. The set of constraints can be written as:

\[
\begin{align*}
P &= P(\rho, T, X_i) \\
\epsilon_{\text{nuc}} &= \epsilon_{\text{nuc}}(\rho, T, X_i) \\
\epsilon_{\nu} &= \epsilon_{\nu}(\rho, T, X_i) \\
\kappa &= \kappa(\rho, T, X_i).
\end{align*}
\]

The equations represent the equation of state, the nuclear energy generation, neutrino losses, and the radiative/conductive opacities, respectively. Note that $X_i$ is the abundance fraction (by mass) of the $i$th chemical constituent. The temporal evolution of $X_i$ is computed using an auxiliary set of differential equations.

The method of solution that is most widely used, yielding both high accuracy and good computational speed, is the Henyey Method (described by Kippenhahn, Weigert and Hofmeister [7]). The algorithm used to solve the differential equations falls into the category of ‘relaxation methods’. The method requires an initial guess for the values of the physical variables at the shell interfaces (i.e., a ‘block start’) at a particular instant in time. Once a set of approximate solutions has been applied to the difference equations, a more exact solution is computed. This iterative process continues until the desired accuracy is achieved. Since a temporal sequence of evolutionary models is being computed, the properties of the model computed at $t_0$ are then used as an approximation for the interior properties of the next model in the sequence ($t_0 + \delta t$). The physical structure of the new model is allowed to relax to the prescribed accuracy.
2.2. Binary Dynamics

For the interacting binaries that we are considering, the separation is so small that the binary system circularizes and the donor synchronizes on very short timescales. Thus, to a first approximation, we can treat the orbit as Keplerian and assume that the donor (of mass $M_2$) is completely constrained by Roche geometry. The latter constraint implies that there is an approximate analytic relationship between the orbital separation $A$ and the spherical-volume-equivalent Roche lobe radius $R_L$.

Furthermore, if the donor star is filling its Roche lobe, then we can assume that its radius $R_2$ is equal to $R_L$. This yields a relation between the intrinsic binary properties ($A$, $M_1$, and $M_2$) and the macro-properties of the donor star being modeled ($R_2$ and $M_2$):

$$
\frac{R_2}{A} = \frac{R_L}{A} = \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1 + q^{1/3})}, \quad q = \frac{M_2}{M_1}, \quad M_T = M_1 + M_2.
$$

(3)

Mass loss from the donor is typically driven by the expansion of the donor on its nuclear timescale or the shrinking of the orbit as a result of orbital angular momentum dissipation. Assuming a circular orbit, we can write the orbital angular momentum ($J_{\text{orb}}$) and its time derivative as:

$$
J_{\text{orb}} = M_1 M_2^2 \sqrt{\frac{G}{M_1 + M_2}} \Rightarrow 2 \left( \frac{J_{\text{orb}}}{J} \right) = 2 \left( \frac{M_2}{M_1} \right) + 2 \left( \frac{M_1}{M_2} \right) - \left( \frac{M_T}{M_1} \right) + \left( \frac{A}{A} \right).
$$

(4)

Since the contributions to the orbital angular momentum dissipation can be written as

$$
\left( \frac{\dot{J}_{\text{orb}}}{J} \right) = \left( \frac{\dot{J}_{\text{GR}}}{J} \right) + \left( \frac{\dot{J}_{\text{MB}}}{J} \right) + \left( \frac{\dot{J}_{\text{M}}}{{\delta M}_T} \right), \quad \left( \frac{\dot{J}_{\text{M}}}{{\delta M}_T} \right) = \alpha (1 - \beta) \left( \frac{M_T}{M_1} \right) \left( \frac{M_2}{M_1} \right),
$$

(5)

$$
\left( \frac{\dot{J}_{\text{GR}}}{J} \right) = -1.3 \times 10^{-8} \left( \frac{M_T}{M_1} \right)^{\eta_5} \left( \frac{M_1}{M_1} \right) \left( \frac{M_2}{M_1} \right) \left( \frac{P_{\text{orb}}}{1 \text{ hr}} \right)^{-3/5} \text{ yr}^{-1}, \text{ and}
$$

$$
\left( \frac{\dot{J}_{\text{MB}}}{J} \right) = -7.1 \times 10^{-6} \left( \frac{M_T}{M_1} \right)^{\eta_5} \left( \frac{M_1}{M_1} \right)^{-1} \left( \frac{R}{R_0} \right)^{\gamma} \left( \frac{P_{\text{orb}}}{1 \text{ hr}} \right)^{-3/5} \text{ yr}^{-1},
$$

where $P_{\text{orb}}$ is the orbital period, we have a mathematically well-posed algorithm to calculate the evolution of the binary system once the response of the donor to mass-loss has been computed. Note that the subscripts GR and MB correspond to orbital angular momentum losses due to gravitational radiation (as predicted by General Relativity) and magnetic braking, respectively. The magnetic braking formula is a parameterization of the Verbunt-Zwaan law as described by Rappaport, Verbunt and Joss [8]. The parameter $\gamma$ is normally assigned a value between 2 and 4. The third contribution to angular momentum losses can be ascribed to systemic mass-loss. For example, if the accretor is a neutron star and if the mass transfer rate exceeds the Eddington limit, the excess mass may be lost from the binary system in a wind that carries away orbital angular momentum ($\delta J_{\delta M_1}$). The parameter $\alpha$ is a measure of the specific angular momentum that is carried away by the mass (e.g., normally a ‘fast Jeans’ mode’ corresponding to the specific angular momentum of the accretor), and $\beta$ is the fraction of the mass that is transferred from the donor that is permanently captured by the accretor.

In a typical evolutionary sequence, the structure of the donor is calculated first and then its radius relative to the Roche lobe radius is compared thereby allowing the mass-loss rate to be calculated for a given $\delta r$. As long as the timesteps are sufficiently small compared to the relevant physical timescales of the donor (e.g., its Kelvin time), the evolution of the orbital separation can be calculated explicitly.
3. First Approach: Dense Grid of Initial Conditions

One strategy that can be adopted to understand the overall behavior of a particular class of interacting binaries is to create a set of initial conditions that is sufficiently well-resolved that most of the salient details can be determined from the evolutionary calculations. This approach was adopted by Lin et al. [9] who chose to explore the evolution of low-mass x-ray binaries (LMXBs) and intermediate-mass x-ray binaries (IMXBs) that contain NS accretor. We computed a grid of 42,000 evolutionary tracks each of which contained thousands of models corresponding to specific ages of the donor star. We chose 60 different initial donor masses in the range of 1 to 4 M⊙, and for each mass, 700 evenly (logarithmically) spaced initial orbital periods in the range of 10 – 250 hours were selected.

The initial conditions were used as an input into the newly developed stellar evolution program known as MESA star [10] which simultaneously solves equations (1) and (2). A brief description of the MESA integrator and other numerical details are given in the next section. An application of this approach to the astrophysically important binary millisecond pulsar PSR J1614-2230 will be described in section 3.3.

3.1. Numerical Method (MESA)

The core integrator which was used to compute the evolution and response of the donor to mass-loss was developed by Paxton et al. [10] as part of an open-source suite of numerical simulation modules known as MESA (Modules for Experiments in Stellar Astrophysics). MESA star was used to solve the equations of stellar structure and evolution using adaptive mesh techniques and advanced timestep controls. The code finds its origins in the work of Eggleton [11] who developed the EZ code and made it publicly available. It has been extensively modified through the incorporation of superior input physics (such as the equation of state) and it also employs sophisticated software engineering tools that can take full advantage of cutting-edge HPC architectures. In particular the MESA modules are ‘thread-safe’ thus allowing applications to use multicore processors (i.e., it supports the shared-memory model based on OpenMP). The MESA codebase is always being updated as the physics and numerical algorithms are being improved. The suite of packages can be downloaded from the project webserver at http://mesa.sourceforge.net/. A complete description of the code (and relevant links) are also provided at the site.

The code that drives the binary evolution is very similar to that described in the paper by Madhusudhan et al. [12]. One of the big differences concerns the treatment of mass transfer near the Eddington limit. For sub-Eddington rates the capture fracture was taken to be 90% (i.e., β=0.9). For super-Eddington rates, the mass accretion by the neutron star is simply set equal to the rate corresponding to the Eddington limit. The evolution proceeds until either an age of 10 Gyr is reached or until mass transfer becomes dynamically unstable. For all the cases shown in the next section, the initial mass of the neutron star was taken to be the ‘canonical’ mass of 1.4 M⊙.

3.2. Results

The evolution of L/IMXBs can best be understood by examining how their observables (orbital period and mass of the donor) change with time. The various subclasses of evolution are clearly evident in figure 2. A set of representative tracks are labelled as CV, UC, and G. In each figure the red box denotes the region occupied by systems at the onset of their evolution (i.e., the set of initial conditions). The CV tracks are very analogous to the evolution that is expected for Cataclysmic Variables (interacting binaries with WD accretors). The systems start out having largely unevolved (H-rich) donors and evolve towards shorter \( P_{\text{orb}} \) until magnetic braking ceases resulting in an orbital period gap between approximately 2 and 3 hours (mass transfer ceases temporarily). As the donor’s mass decreases, the system reaches a minimum orbital period (about 70 minutes) at which point the donor is becoming degenerate and expanding (leading to a larger \( P_{\text{orb}} \)). The donor continues to lose mass as it becomes a brown dwarf. This type of evolution has been discussed extensively by Rappaport, Verbunt, and Joss [8], and Howell, Nelson, and Rappaport [4].
If the donors are able to evolve on a short enough nuclear timescale, it is possible for them to become sub-giants and/or red giants before too much mass is stripped away. In this case (tracks denoted by G in figure 2) $P_{\text{orb}}$ increases as mass is stripped away from the giant. This occurs because the radius of red giants is strongly dependent on the masses of their He-rich degenerate cores for $M_2 < 2.2 \, M_\odot$ (as opposed to their own mass) and because the orbital period is constrained by the radius of the red giant (and its mass). Eventually so little mass is contained in the envelope of the giant (i.e., outside its He-rich core) that it collapses leaving behind a cooling helium WD in a detached binary system. Since the accretor is a neutron star that has been spun up by accretion torques (thus making it a pulsar), the system is properly referred to as a binary millisecond pulsar (see Podsiadlowski, Rappaport, and Pfahl [13], and Nelson, Dubeau, and MacCannell [14], and references therein).

The dividing line between these two extremely diverse sets of behaviors is known as the bifurcation (Pylyser and Savonije [15]). The bifurcation is extremely sensitive to the assumed input physics (for example, magnetic braking) because it depends sensitively on the relative magnitudes of the mass-loss timescale and the nuclear timescale. For systems whose initial conditions place them extremely close to the bifurcation, yet below it, it is possible for them to evolve to extremely short orbital periods ($P_{\text{orb}}$ as small 6 minutes) before their orbital period starts increasing again. These evolutionary tracks are shown as the ultracompact (UC) cases in figure 2. The reason why these systems can reach such short orbital periods can be explained by the fact that the donor stars had almost formed a helium-degenerate core before mass loss precluded any further growth. Thus the donors in the systems are very helium rich and consequently much smaller than their hydrogen-rich counterparts. Given the constraint of Roche geometry, this guarantees that their orbital periods will be extremely short (see Fedorova and Ergma [16], and Nelson and Rappaport [17] for further details).

A pictorial presentation of all 42,000 systems is shown in the upper-left panel of figure 3. Each panel is a pixel map that is divided into 1300 X 1200 cells. As each of the tracks passes through a pixel, the cumulative time that is spent by all of the systems evolving through that particular pixel is recorded. The cumulative time that the ensemble of systems spends in each pixel varies from $10^{10}$ yr.
(red pixels) to $10^3$ years (violet pixels). Thus the $M_2-P_{\text{orb}}$ plane in figure 3 can be viewed as a (logarithmic) relative probability density plot. The other three panels correspond to the types of evolution discussed above. One of the benefits of this type of analysis is that we can clearly see the subset of initial conditions that leads to the formation of each type of system.

![Figure 3. Relative color probability plots for the ensemble of models.](image)

Figure 3. Relative color probability plots for the ensemble of models. The initial grid of models (i.e., initial conditions) is visible in the upper right portion of each panel, outlined by a red box. All 42,000 systems are superposed in the upper-left panel of the figure. Specific subsets are plotted in the other three panels. See Lin et al. [9] for more details.

Other potentially observable properties of the binary system (such as the mass-transfer rate) can also be plotted in order to gain insight into the evolution of these systems. Figure 4 shows the interrelationship of the orbital period, mass of the donor and the mass-transfer rate. Note that the mass-transfer rates of the ultracompact become very high just as they reach their minimum periods.

3.3. Application to PSR J1644-2230

Demorest et al. [18] recently made a very precise measurement of the mass of the neutron star (1.97 ± 0.04$M_\odot$) in the binary millisecond pulsar J1644-2230. This is the highest mass ever measured for a neutron star and has extremely important consequences for our understanding of high-energy physics. There has been considerable debate as to the equation of state that prevails inside neutron stars. Some models invoke exotic forms of matter such as strange quarks and kaon condensates (soft equations of state) to construct neutron star models. These models do not allow for the existence of high-mass neutron stars ($\geq 2 M_\odot$ or larger). Clearly this new determination of the mass rules out the softer equations of state (see figure 5).
One of the questions that can be addressed by computing a dense grid of models concerns the initial conditions that lead to the formation of this binary. Could this system have formed from an L/IMXB? According to our results (Lin et al. [9]), the closest relatives of J1614-2230 had an initial donor mass of 3.4 - 3.8 M\(_{\odot}\) (assuming the natal mass of the neutron star to be 1.4 M\(_{\odot}\)). But we found that it is difficult to form neutron stars as massive as 1.97 M\(_{\odot}\) unless their initial masses are somewhat higher (1.6 ± 0.1 M\(_{\odot}\)). In this case, the initial donor mass and \(P_{\text{orb}}\) would likely have been \(\equiv 4.25\) M\(_{\odot}\) and \(\equiv 49\) hours, respectively.

**Figure 4.** Color probability plots showing the dependence on the mass-transfer rate (\(\dot{M}\)).

**Figure 5.** Mass-radius diagram for several equations of state used to compute the structure of neutron stars. The black curves correspond to “normal-matter” equations of state while the green curves (labelled SQM) correspond to strange-quark equations of state. Regions excluded by general relativity (GR), causality, and rotational constraints are also indicated. The figure is reproduced from the article by Lattimer and Prakash [19].
4. Second Approach: Population Synthesis using Grid Interpolation

Population synthesis techniques are used in an attempt to reproduce all of the various types of stars and multiple systems that would be detected at any epoch in the galaxy’s history. The details associated with this method as it pertains to interacting binaries can be found in de Kool [20], Politano [21], and Howell, Nelson, and Rappaport [4] (and references). Based on empirical probability rules, a set of primordial binaries is created. The separation and masses of the two components are completely specified. For the case of Cataclysmic Variables, the individual evolutions of each system then have to be computed. In the past this difficulty limited the number of primordial binaries that could be evolved. Moreover, many important (but low probability) phases could be accidentally excluded.

The large number of dimensions of parameter space often forced workers in the field to make simplifications such as ignoring the effects of nuclear evolution on the donor star (e.g., Nelson, MacCannell, and Dubeau [22]). However another strategy was recently employed by Goliasch and Nelson [23] wherein a grid of carefully constructed evolutionary tracks is pre-computed (starting from Zero-Age Cataclysmic Variables [ZACVs]) and then interpolated in order to obtain the synthesized population at the current epoch (i.e., Present-Day Cataclysmic Variables [PDCVs]). The advantage of this method is that if the grid is well constructed, the only computational cost comes from the Monte Carlo calculation to obtain the set of primordial binaries and then from the multidimensional interpolation of the evolutionary grid.

4.1. Population Synthesis

A Monte Carlo technique was used to seed the Galaxy with the observed numbers and types of stars (see Goliasch and Nelson [23] for detailed information). The parameters that were adopted are listed in table 1. We started by creating $10^8$ primordial binaries. The primary masses (corresponding to the component that evolves to form the WD accretor) were chosen according to the Miller and Scalo [24] initial mass function (IMF). The donor mass was then chosen according to a specific correlation function (generally had a weak dependence on $M_1$). The separation was chosen based on either a uniform distribution in log $P_{\text{orb}}$ or log $A$. The birthrate function was assumed to be either constant (0.6 stars/yr with $M > 0.8 M_\odot$) or exponentially decaying for an assumed galactic (disk) age of 10 Gyrs.

| Parameters                  | Adopted Values                           |
|-----------------------------|------------------------------------------|
| Primary mass $M_1$ (0.8 - 10 $M_\odot$) | Miller & Scalo IMF                       |
| Initial mass ratio correlation ($M_2$) | Uncorrelated OR $q^{1/4}$ OR $q^1$        |
| Primordial separation ($A$)   | Uniform in Log $P_{\text{orb}}$ OR Log $A$ |
| Common Envelope efficiency ($\alpha_{\text{CE}}$) | 1/3 OR 1                                 |
| Birthrate Function (BRF)      | Constant OR $\propto e^{t/t}$ ($t=3.5\text{Gyr}$) |
| Age of the Galaxy            | 10$^{10}$ years                          |

The most problematic parameter to ascertain is the so-called common envelope efficiency factor which is a measure of the energetic efficiency for the removal of the envelope of the primary during the common envelope phase ($\alpha_{\text{CE}}\Delta E_{\text{binding}}/\Delta E_{\text{orbital}}$). The ‘standard model’ assumed a mass correlation probability proportional to $q^{1/4}$, $\alpha_{\text{CE}} = 1$, and a constant birthrate function. A 1-D Henyey code was then used to calculate a full evolutionary grid of about 300 models (the rudiments of the code are described in section 2.1). The grid was then interpolated for each set of initial binary parameters ($M_1$, $M_2$, $A$) and a probability density function was computed for all present-day observable parameters of the (ensemble) population.
4.2. The Present-Day Population of Cataclysmic Variables

Figure 6 shows the computed probability density for the intrinsic population of PDCVs in the orbital period-mass transfer rate plane. There are 1000 pixels along the abscissa and 1000 pixels along the ordinate and the colors represent the relative logarithmic probability of finding a CV with those properties. It should be noted that the highest probability density has arbitrarily been normalized to a value of 8.0 on the logarithmic probability scale. Figure 7 attempts to crudely take into account observational selection effects by assuming that they are dominated by the accretion brightness (which is proportional to the mass-transfer rate). In this context, the importance of the systems with very high mass-transfer rates can be easily seen.

Also apparent in both figures is a ‘period gap’ that occurs for orbital periods of between approximately 2 to 3 hours. According to the interrupted magnetic braking paradigm that has been implemented in this model, when the donor’s interior becomes fully convective, magnetic braking is greatly attenuated. Since this angular momentum dissipation mechanism is largely driving mass transfer, mass transfer ceases (the donor shrinks beneath its Roche lobe because it is thermally bloated) until gravitational radiation losses can drive the donor back into Roche-lobe contact at a \( P_{\text{orb}} \) of about 2 hours. An orbital-period minimum can also be seen for a \( P_{\text{orb}} \) of about 70 minutes. The value of the minimum period is smeared out because of the presence of both He and CO white dwarf accretors, and because of the nuclear evolution of the donor before it became a ZACV. Although this effect was never previously investigated, Goliasch and Nelson [23] show that it is not highly significant. In fact, the population synthesis shows that the volume of the phase space of initial conditions is so small that it is very unlikely that nature can produce the ultracompact AM CVn systems via this channel. They are more likely produced either as a result of the formation of a double degenerate or a helium-star progenitor system.

Figure 8 shows a histogram of the presently observed CVs in the galaxy. It should be noted, however, that the observations are subject to many serious selection effects. Nonetheless, the salient features of the distribution are very evident: there is a period gap between roughly 2 and 3 hours, and there is a minimum orbital period at about 80 minutes. Note that the observed period gap is...
approximately reproduced in the folded data based on the population synthesis results for the standard model (see figure 9). Moreover, the relative number of long-period CVs (< 12 hours) is a reasonable facsimile of what is observed based on figure 8. Note that very few AM CVn systems are synthesized. Also note that there is an enhancement in the number of systems near the minimum period as is observed (the “period spike”) but that it tends to be smeared out by the phenomena noted above.

Figure 8. Orbital-period histogram showing the currently observed population of CVs that was generated using the Ritter Cataclysmic Binaries Catalog (version 7.10). The label AM CVn denotes the existence of several short-period AM Canum Venaticorum systems that typically have helium-rich donors.

Figure 9. Folded orbital-period histogram of PDCVs for the standard model. The black curve corresponds to the actual (intrinsic) number of systems that are found in a particular $P_{\text{orb}}$ bin. The green curve corresponds to the “observed” population that was weighted by $M_{\text{dot}}$.

5. Conclusion
Two different strategies for analyzing a subset of the present-day population of compact binaries have been compared. Both require that grids of evolutionary tracks be computed. The computations for both approaches were massively scalar and required about 400000 and 250000 core-hr, respectively, on MS2 (RQCHP). The main advantage of the second approach is that the primordial population of binaries can be easily recomputed and thus the sensitivity of the synthesized results to various physical assumptions affecting the formation process (such as $\alpha_{\text{CE}}$) can be reasonably quantified. The disadvantage is that the grid needs to be carefully constructed so that difficult regions in initial condition space (e.g., dynamical instabilities) are properly covered. The first approach provides greater physical insight into how the more complicated evolutionary channels can lead to the formation of specific binary systems. As was shown for the case of PSR J1644-2230, the grid can also be used to identify the spectrum of initial conditions corresponding to the most probable evolutionary channels leading to the formation of the observed system (i.e., traceback analysis).

Some of the issues that are currently being addressed include: (1) the inclusion of the effects of X-ray irradiation on the donors in L/IMXBs; (2) an examination of the effects of disk instabilities in those same systems; and, (3) an expansion of the previous population synthesis which takes into account the chemical evolution of stars in the disk and halo of our galaxy (i.e., the inclusion of low-metallicity Population II stars).
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