Spectral flow in vortex dynamics of d-wave superconductors

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(June 18, 1997)

We calculate Ohmic and Hall flux flow conductivities in the mixed state of d-wave superconductors taking into account the spectral flow along the anomalous branch of the states bound to vortex cores. Two parameters governing the strength of the spectral flow at low temperatures are found. They are determined by the ratios of the width of the localized levels 1/τ and 1) the interlevel spacing in the core, and 2) ∆²/E_F. These parameters determine three regimes of vortex dynamics. In the moderately clean limit the contribution of the spectral flow to the conductivities coincides with that for s-superconductors. In the superclean regime in low magnetic fields both Ohmic and Hall conductivities acquire τ-independent universal values. The contribution of the spectral flow in the superclean regime is suppressed in higher fields. We also discuss the case of higher temperatures.

I. INTRODUCTION

The problem of vortex motion in superconductors/superfluids and flux-flow Hall effect attracted attention of researchers for years. The renewed interest to this problem is due to recent developments in the physics of the Hall effect in high-temperature superconductors and in the investigations of the superfluid ³He. A microscopic theory of mutual friction (friction between the superfluid and normal components in presence of quantized vortices) was developed long time ago. This theory (see also Refs. 2 3) enables us to calculate the forces on a vortex (or, equivalently, Ohmic and Hall conductivities) as functions of microscopic and external parameters. The temperature dependence of the forces on a vortex predicted theoretically is in good agreement with the experimental data for ³He-B. The theory also proposes an explanation for the sign reversal of the Hall angle as an intrinsic property of vortex motion in conventional and high-T_c superconductors. This theory applies to the regime of free flux flow when pinning is not important. In superconductors this region can be reached experimentally either by preparation of high purity untwinned samples or by using high density driving currents.

However, all the previous calculations were performed for the isotropic s-pairing. Since this theory is used for analysis of the data for high-T_c materials it is necessary to understand whether its arguments are sensitive to the symmetry of the pairing state and the presence of gap nodes.

The total non-dissipative force on a vortex can be subdivided into the following three contributions of different physical origin (see Ref. 3 and references therein):

\[ \mathbf{F} = \rho \mathbf{v}_L \times (\mathbf{v}_L - \mathbf{v}_s) + \rho_n \mathbf{v}_s \times (\mathbf{v}_s - \mathbf{v}_n) + C \mathbf{v}_s \times (\mathbf{v}_n - \mathbf{v}_L). \]  

where \( \rho, \rho_n \) are the densities of the fluid and the normal component, respectively, \( \mathbf{v} \) is the vector of the circulation of the superfluid velocity around the vortex directed along the vortex axis, \( \kappa = \pi \hbar / m \) for a 1-quantum vortex. \( \mathbf{v}_n \) and \( \mathbf{v}_s \) are the velocities of the normal and superfluid components far from the vortex core and \( \mathbf{v}_L \) is the velocity of the vortex (the vortex lattice). The first term in (1) is the Magnus force describing the transfer of momentum from the superfluid vacuum to the vortex. The second one is the Iordanskii force due to asymmetric scattering of excitations by the vortex core. These first two terms could be rewritten as \( \mathbf{v} \times [\rho_s (\mathbf{v}_s - \mathbf{v}_L) + \rho_n (\mathbf{v}_n - \mathbf{v}_L)] \) and thought about as contributions of the superfluid and normal components.

In this article we concentrate on the last term in (1). This term describes the transfer of momentum between the heat bath and the bound states in the vortex core specific for Fermi superfluids which is due to the spectral flow (SF) along the anomalous branch of bound states.

For s-wave superconductors the coefficient \( C \) in front of this term depends on the ratio of the interlevel distance \( \Delta \) in the core \( \omega_0 \sim \Delta^2 / E_F \) to the inverse relaxation time \( \tau \) for the bound states. When the width of the levels \( 1/\tau \) is greater than the interlevel spacing \( \omega_0 \), the spectrum is continuous and the SF is allowed. In this limit \( C = C_0 \), which is the density of the fluid in the normal state \( (C_0 = m k_F^2 / 2 \pi \) in 2D, \( C_0 = m k_F^2 / 3 \pi^2 \) in 3D). The difference \( \rho - C_0 \sim \rho (\Delta / E_F)^2 \) is small and \( \mathbf{v}_L \) dependence of the total force (1) is weak. In the opposite limit \( \omega_0 \tau \gg 1 \) the spectrum is discrete and the SF is suppressed, \( C \to 0 \).

Another explanation of the dependence of the strength of the spectral flow on \( \tau \) was proposed by Stone. He
showed that quasiclassical bound states inside the vortex core rotate with the angular velocity $\omega_0$ and this rotation suppresses the spectral flow. However, relaxation effectively prevents the rotation if the relaxation time $\tau$ is less than the period of rotation $2\pi/\omega_0$. Therefore, the SF is suppressed in the limit of large $\omega_0\tau$ and allowed in the opposite limit. In the regime of free flux flow the only contribution to the dissipative force is also due to the bound states. The dissipative force is small in both limits and is maximal at $\omega_0\tau \sim 1$.

At finite $T \sim T_c$ a contribution of the continuous spectrum of states above the gap to $C$ is also essential. Since the interlevel distance above the gap is given by the cyclotron frequency $\omega_c = e|H/m^*c| \sim \omega_0(H/H_{c2})$, this contribution of the continuous spectrum is suppressed in the limit $\omega_c\tau \gg 1$.

In this article we address the question of applicability of the theory to the d-wave case, namely, to $d_{x^2-y^2}$-pairing state, which is believed to represent the order parameter in high-$T_c$ materials, though the analysis for an arbitrary pairing state with points of gap nodes in 2D (or lines in 3D) would be similar. The structure of the states bound to vortex cores in d-wave superconductors is different from that in s-wave materials. In particular, low-energy states are extended in directions of gap nodes. In addition, at energies below the maximum value of the gap $\Delta_0$ bound states coexist with the states in the continuous spectrum. So, one can expect new features in the contribution of these low-lying levels to vortex dynamics.

The interlevel spacing in cores of d-wave vortices is much less than in s-superconductors: $\omega_c \ll \epsilon_0 \ll \Delta_0^2/E_F$ if $H \ll H_{c2}$. Comparison of $\epsilon_0$ to the width $1/\tau$ of the levels suggests the dimensionless parameter $\epsilon_0\tau$ governing the strength of the SF.

On the other hand, the angular velocity of rotation of Andreev bound states $\omega_0(\theta)$ is angle-dependent in d-wave superconductors (it follows the angle dependence of the gap; see Section 2). Its maximal value $\omega_0^{\text{max}}$ on the order of $\Delta_0^2/E_F$ ($\Delta_0$ is the maximal value of the gap) is much greater than $\epsilon_0$. So, this approach proposes a different scale for $1/\tau$ as a boundary between the regimes of extreme and suppressed SF. In addition, near the gap nodes the interlevel distance $\omega_0(\theta)$ is very small (see Section 2), i.e., the density of states is large. So the contribution of this region needs special attention.

One of our purposes is to understand, which parameter governs the strength of the SF in the d-wave case. In particular, this question is important for analysis of experimental data: one should locate the boundary between the regimes of extreme and suppressed SF. In addition, it is interesting to estimate the force on a vortex and Ohmic and Hall conductivities in various limiting cases to compare the results to experiments and previous theories. In particular, we look for results different from those for s-superconductors. We found new regimes of vortex dynamics in the superclean limit in superconductors with gap nodes (d-wave), different from those in isotropic (s-wave) superconductors. The SF force depends on both parameters discussed above, $\epsilon_0\tau$ and $\Delta_0^2\tau/E_F$ (as well as on $\omega_c\tau$ at finite $T$).

It turns out that it is convenient to explore this problem using the kinetic equation approach developed recently by Stone for s-wave vortices. We apply it to the d-wave case and find the solution of the equation. To use the kinetic equation (KE) one should first investigate the spectrum of the bound states (Section 2). Then, in Section 3, we solve the KE and discuss its implications on the force in various limiting cases in Section 4. We consider the 2D case (applicable directly to thin films and pancake vortices). Generalization to 3D is straightforward (see discussion).

In Section 5 we explore the KE in terms of the density matrix in the basis of exact eigenstates of Bogolyubov-de Gennes (BdG) hamiltonian, and evaluate parameters governing the strength of the SF in a different way. Finally, we summarize and discuss our results in Section 6.

II. SPECTRUM OF THE BOUND STATES

In this Section we describe states inside and near vortices in d-wave superconductors. We consider a material with a circular particle-like Fermi surface (generalization to the case of a hole-like Fermi surface or coexisting hole- and particle-like parts of a Fermi surface is straightforward). We assume that the coherence length $\xi_0$ in the superconducting state is much greater than the inverse Fermi momentum $k_F^{-1}$ (or, equivalently, $\Delta_0 \ll E_F$). Even in high-$T_c$ materials, where $E_F/\Delta_0$ is not very large, it is still of the order of 7–10. Therefore we can use the quasiclassical approach and solve the BdG equations in Andreev approximation. We also assume that the magnetic field $H \ll H_{c2}$ so that vortices are far apart (and, of course, $H > H_{c1}$ so that the sample is in the mixed state).

In $d_{x^2-y^2}$-wave state the distribution of the order parameter in the vicinity of a vortex is given by

$$\Delta(r, \theta) = \Delta_0(r)e^{i\varphi}(\hat{a}_a^2 - \hat{b}_b^2)$$

where $\hat{a}$ and $\hat{b}$ are the crystal axes, $r$, $\varphi$ are polar coordinates in the position space. We put $\hat{a} = \hat{x}$, $\hat{b} = \hat{y}$. The absolute value of the gap $\Delta_0(r)$ equals zero at the center of the core and saturates at the bulk value $\Delta_0$ far from the core, at distances $r \gg \xi_0$. In the bulk $\Delta(\theta) = \Delta_0 \cos(2\theta)$, where the angle $\theta$ is defined by $k = k_F(\cos\theta; \sin\theta)$, and there are 4 gap nodes (in 2D case). Close to a gap node $\theta_0$ the order parameter is given by $\Delta(\theta) = \Delta'(\theta - \theta_0)$.

We consider clean superconductors ($\Delta(\theta)\tau(\theta) \gg 1$) so that the motion of quasiparticles inside and near the core is ballistic. In the quasiclassical approach the states are
described by the direction of the momentum $\theta$ and the spatial position of the line, on which the state is situated. This spatial position can be described by the impact parameter $b$ or, equivalently, by the angular momentum $l = k_F b$. In d-wave vortices bound states could extend far outside vortex cores. The characteristic extension of these states in the direction of their momentum is of the order of the angle-dependent coherence length $\xi(\theta) = v_F/|\Delta(\theta)|$ where $|\Delta(\theta)|$ is the angle-dependent d-wave superconducting order parameter.

Below the gap there is one chiral branch of bound states. Due to a special symmetry of (1D) BdG Hamiltonian in Andreev approximation for a state situated at a diameter $(l = 0)$ the eigenenergy of the bound state is exactly zero and the eigenfunction is

$$\Psi(x) \propto \left( \frac{1}{i} \right) \exp \left( -\frac{1}{v_F} \int_0^x dy \Delta(y, \theta) \right)$$

with the extension of the order of $\xi(\theta)$. In Ref. [13] the spectrum of states with impact parameters smaller than the vortex core size was calculated perturbatively, the correction to the energy being

$$E = -b \frac{2 \int dx \frac{\Delta(x, \theta)}{x} \exp \left( -\frac{2}{v_F} \int_0^x dy \Delta(y, \theta) \right)}{\int dx \exp \left( -\frac{2}{v_F} \int_0^x dy \Delta(y, \theta) \right)}.$$ (5)

For small values of the impact parameter the logarithmically divergent integral in the numerator of the previous expression should be cut off from below at the core size $\xi_0$, that is,

$$E(l, \theta) = -\omega_0(\theta, 0) l,$$ (6)

where the interlevel distance is

$$\omega_0(\theta, 0) = \Omega_0(\theta - \theta_0)^2 \ln(1/|\theta - \theta_0|)$$ (7)

in a vicinity of a gap node $\theta_0$, $\Omega_0 = 2\Delta^2/\nu_F k_F$. Far from the nodes $\omega_0(\theta)$ is of the order of $\Omega_0$. However, for impact parameters larger than the core size the lower cutoff is given by the impact parameter itself. Therefore, to the logarithmic accuracy one finds that

$$E(l, \theta) \sim -\Omega_0(\theta - \theta_0)^2 \ln \left( \frac{\xi(\theta)}{\max(\xi_0, l/k_F)} \right) l,$$ (8)

At large values of the impact parameter $|b| \gg \xi(\theta)$ the energy of the bound state saturates at the gap value $\Delta(\theta)$, the asymptotic behavior being given by

$$E(l, \theta) \sim -|\Delta(\theta)| \text{sign}(l) + \text{const} \frac{E_F}{l}.$$ (9)

where $\text{const}$ is of the order of unity. Saturation of the spectrum occurs at $l \sim E_F/|\Delta(\theta)| \sim k_F \xi(\theta)$ where spatial variations of the order parameter along the quasiclassical trajectory are slow.

In (charged) superconductors the effect of magnetic field on motion of quasiparticles leads to an additional term in the dispersion relation $\delta E = -\omega_c l$. This term is important for excitations with momenta close to gap nodes, where $\omega_0(\theta, 0)$ is small.

Close to the nodes $(k = \pm \hat{a}, \pm \hat{b})$ the extension $\xi(\theta)$ of the wave function of a localized state is very large and can exceed the intervortex distance $R_v$, which establishes the characteristic distance of the changes in the order parameter and superfluid velocity in the vortex lattice. For angles $\theta$ close to a gap node $\theta_0$ such that $|\theta - \theta_0| < \theta_H$, the wave $\omega_0(\theta)$ becomes comparable to the cyclotron frequency $\omega_c$ of the external field, which also contributes to $\omega_0(\theta)$. We model this region by putting $\omega_0(\theta) \equiv \omega_0(\theta_H) \sim \omega_c$ for these angles close to gap nodes. The only characteristic of the spectrum in this region, which affects the value of the force, is the interlevel distance $\epsilon_0$ (see below).

In addition to the just described branch of bound states there exist branches of the states with energies above the gap $\Delta(\theta)$, that is, in the continuous spectrum. For a one-quantum vortex there is one chiral branch in the continuous spectrum, which crosses all energy levels once. The interlevel spacing in the continuous spectrum is $\omega_0 = \omega_c$.

For later analysis we need to know the angle dependence of the interlevel spacing

$$\omega_0(\theta, E) = -\left( \frac{\partial E}{\partial l} \right)_\theta$$ (10)

for different energies $E$. For very small energies $|E| \ll \Delta(\theta)H$ this dependence is described by Eq. (7) above with the just discussed condition of saturation near nodes. For higher energies $\Delta(\theta)H \ll |E| < \Delta_0$ the spectrum near the gap nodes is “continuous”:

$$\omega_0(\theta, E) = \omega_c, \quad |\theta - \theta_0| < \theta_E$$ (11)

(where $\theta_E$ is defined by $|\Delta(\theta_E)| = |E|, \theta_E < \pi/4$; $\theta_E = |E|/\Delta'$ for $|E| < \Delta_0$). On approaching this region from outside $\omega_0(\theta, E)$ decreases. To investigate the behavior of $\omega_0(\theta, E)$ as we approach this vicinity from outside we use the expression (9) and find the relationship between $l$ and $\theta$ at given $E$ as $|\theta - \theta_E|$. We find that $l \propto (\theta - \theta_E)^{-1} \rightarrow \infty$ and $\omega_0 \sim \Omega_0(\theta - \theta_0)^2$. However, for $l > k_F R_v$ the states are sensitive to the velocity fields of other vortices in the lattice, and therefore $\omega_0(\theta, E)$ saturates at a finite value. This value is on the order of $\omega_c$, which also contributes to $\partial E/\partial l$. So, the behavior of $\omega_0(\theta, E)$ as $\theta$ approaches $\theta_E$ from outside is quantitatively similar to the behavior of $\omega_0(\theta, 0)$ as $\theta \rightarrow 0$.

As $|E| \rightarrow \Delta_0$ the region of the continuous spectrum grows and at $|E| > \Delta_0$ the interlevel distance $\omega_0(\theta, E) = \omega_c$ for all $\theta$. The difference in the behavior
of $\omega_0(\theta, E)$ and, in particular, coexistence of the regions of continuous and discrete spectrum at the same energy $E$, determines difference in the solutions of the KE at different energies (and therefore different temperatures).

We can use the quasiclassical approximation as long as the temperature is larger than the interlevel spacing. The maximal value of $\omega_0(\theta, E)$ is on the order of $\Omega_0$, i.e., we need $T \gg \Omega_0$.

Now, as we know the structure of the spectrum we can proceed to consideration of the dynamics of the distribution function.

III. KINETIC EQUATION FOR THE DISTRIBUTION FUNCTION

We consider the problem in the frame of the vortex lattice, so that $v_L = 0$. To calculate the effect of the SF (linear in the velocity of the normal component with respect to the vortex lattice $v_n - v_L$) we put the external superflow $v_s = 0$ but keep the velocity of the normal component $v_n$ finite. To the first order, the SF force is a linear function of $v_n$, possessing $D_4$-symmetry of the pairing state. Therefore, the force on a vortex is determined by two parameters:

$$F_{SF} = d_{SF}^\perp C_0 \kappa \times v_n + d_{SF}^\parallel C_0 \kappa v_n. \quad (12)$$

In particular, the longitudinal and transverse components of the force do not depend on the direction of $v_n$. For simplicity we choose $v_n = v_n \hat{x}$. The dynamics of the distribution function (or the density matrix) can be described by the KE. Since $\theta$ and $l$ are canonically conjugate, this equation reads (cf. Ref. [4]):

$$\frac{\partial n(l, \theta)}{\partial \tau} = -\frac{\partial E}{\partial \theta} \frac{\partial n}{\partial \theta} + \frac{\partial E}{\partial l} \frac{\partial n}{\partial l} = -\frac{n - n^{eq}}{\tau}, \quad (13)$$

where $n^{eq} = n_F(E - v_n k) = n_F(E(l, \theta) - v_n k_F \cos \theta)$ is the distribution function in equilibrium with the heat bath (the normal component), $n_F$ is the Fermi distribution function. The treatment of the collisions in the relaxation time approximation is, of course, not accurate. Nevertheless, we believe that it qualitatively correctly describes the physics of the system. The relaxation time $\tau$ can be angle and energy-dependent, and we can easily incorporate this dependence into our results of this Section. However, for investigation of limiting cases we put $\tau(\theta) = \text{const.}$

We study the stationary case and put $\partial n = 0$ in Eq. (13). If we change the variables $l, \theta$ to $E, \theta$, the equation takes the form

$$\frac{\partial n(E, \theta)}{\partial \theta} = \frac{n(E, \theta) - n^{eq}(E, \theta)}{\omega_0(E, \theta) \tau}, \quad (14)$$

The equation (14) can be easily solved:

$$n(E, \theta) = C(E) \exp \left( \int_0^\theta \frac{d\varphi}{\omega_0(E, \varphi) \tau} \right) - \int_0^\theta \frac{d\chi}{\omega_0(E, \chi) \tau} n_F(E - v_n k_F \cos \chi) \exp \left( \int_\chi^\theta \frac{d\varphi}{\omega_0(E, \varphi) \tau} \right). \quad (15)$$

The energy-dependent constant $C(E)$ should be determined from the condition of $2\pi$-periodicity of $n(E, \theta)$ in $\theta$.

Since we know the distribution function we can substitute it into the expression for the SF force (the force on vortex from the heat bath, which in the case of a crystal is the lattice with the impurities). Since the momentum of the excitations is given by $\vec{k}$,

$$P = \frac{1}{2} \int \frac{d\vartheta dl}{2\pi} k_F (\cos \theta; \sin \theta)(n - n_0) \quad (16)$$

where $n_0(l, \theta) = \Theta(l)$ stands for normalization purposes and ensures that $P = 0$ in equilibrium at $T = 0$; $\Theta(l) = 0$ for $l < 0$, $\Theta(l) = 1$ for $l > 0$; the prefactor $1/2$ compensates for the double counting of particles and holes. The contribution of the heat bath to the force is [cf. (13)]

$$F_{SF} = \frac{1}{2} \int \frac{d\vartheta dl}{2\pi} k_F (\cos \theta; \sin \theta) \frac{n^{eq} - n}{\tau}. \quad (17)$$

To take into account all the branches we should sum up the expressions (16) for all of them. However, non-chiral branches never contribute to the force.

IV. REACTIVE AND DISSIPATIVE FORCES IN VARIOUS LIMITS

In principle one can substitute the solution (13) of the KE into the expression for the force (16), and if the temperature dependence of the gap and the relaxation time is known, one obtains the temperature dependence of the SF force. However, it is interesting to investigate the behavior of the force in different physical limits analytically.

We are looking for an expression for the force to the first order in $v_n$, so we expand the KE to this order. Namely, we characterize the deviation of the distribution function from its equilibrium value at $v_n = 0$ by a function $\nu$:

$$n(E, \theta) = n_F(E) - \frac{\partial n_F}{\partial E} v_n k_F \nu(E, \theta). \quad (17)$$

The stationary KE takes the form

$$\frac{\partial \nu}{\partial \theta} = \frac{\nu - \nu^{eq}}{\omega_0(E, \theta) \tau} \quad (18)$$

where $\nu^{eq} = \cos \theta$. This equation has a unique solution, which is an odd function of $\theta$: $\nu(\theta + \pi) = -\nu(\theta)$. The quasiclassical equations of motion
\[ \dot{\theta} = \frac{\partial E}{\partial t}, \quad \dot{l} = -\frac{\partial E}{\partial \theta} \]  

(19)
describe the motion along a curve of fixed energy in \((\theta, l)\) plane, and the KE \(18\) keeps track of variations of the distribution function \(\nu\) along this curve in the stationary case. The time of motion along this trajectory is given by

\[ t(\theta) = -\int_{\theta_0}^{\theta} \frac{d\varphi}{\omega_0(E, \varphi)} \]  

(20)
and the KE describes the relaxation of \(\nu\) towards its equilibrium \((t\text{-dependent})\) value:

\[ \frac{\partial \nu}{\partial t} = -\frac{\nu - \nu^{eq}}{\tau}. \]  

(21)
The solution of this equation depends on the function \(\nu^{eq}(t)\) and the relaxation time \(\tau\).

The total time of motion along the closed trajectory

\[ t_0(E) = \int_{0}^{2\pi} \frac{d\varphi}{\omega_0(E, \varphi)} \]
determines the structure of the energy levels. The quantization of the quasiclassical motion \(13\) shows that in a vicinity of energy \(E\) the spectrum is equidistant with the interlevel distance

\[ \epsilon_0(E) = \frac{2\pi}{t_0(E)}. \]  

(22)
For small \(|E| \ll \Delta \theta_H\) the interlevel spacing equals \(\epsilon_0 = \epsilon_0(E = 0)\) and is given by Eq.\(13\).

The solution \(15\) can be simplified in various limits. The behavior of the force is different in three regimes: moderately clean with \(\omega_0^0 \ll 1\), “very clean” with \(\epsilon_0 \ll 1 \ll \Omega_0 \tau\) and “extremely clean” when \(\epsilon_0 \ll 1\). Usually the term “super clean” is used for the regime with \(\Omega_0 \tau \gg 1\). In the mixed state of d-wave materials vortex dynamics is different in two subregions. Therefore we use the terms “very clean” and “extremely clean” for these two subregions. Note that \(\epsilon_0 \ll 1\) depends on external field \(H\) and temperature, while \(\Omega_0 \tau\) is determined by temperature only. In subsections A, B, and C below we consider these three regimes at low temperatures \(T \ll T_c\) and \(\Omega_0 \tau \gg 1\). At corresponding energies \(|E| \ll \Delta \theta_H\) the quasiclassical interlevel spacing is given by \(\epsilon_0(\theta, 0)\). The case of higher \(T\) is discussed in subsection D.

A. Moderately clean limit

In the moderately clean case the parameter \(\omega_0 \tau\) is small for all values of \(\theta\): \(\omega_0^0 \ll 1\), and \(\nu^{eq}\) is a slow function of \(t\) (on the scale of \(\tau\)). Therefore, the solution \(\nu\) of the KE everywhere is very close to \(\nu^{eq}\). Simple calculation shows that the value of the SF force in this limit coincides with that in the s-wave case. The contribution of the bound states is given by:

\[ F_{SF}^{\text{bound}} \approx -\frac{1}{2} \int \frac{d\theta d\varphi}{2\pi} k_F(\cos \theta; \sin \theta) \frac{\partial n^{eq}}{\partial \theta} \omega_0(\theta, 0) \]

\[ = 2\kappa C_0 v_0 \int \frac{dE d\theta}{2\pi} \left( -\frac{\partial F}{\partial E} \right) (\cos \theta; \sin \theta) \sin \theta \]

\[ = 2\kappa C_0 v_0 \int \frac{d\varphi}{2\pi} (\cos \theta; \sin \theta) \sin \theta \times [n_F(-|\Delta(\theta)|) - n_F(|\Delta(\theta)|)]. \]  

(23)
In the continuous spectrum only the chiral branch contributes:

\[ F_{SF}^{\text{cont}} = 4\kappa C_0 v_0 \int \frac{d\theta}{2\pi} (\cos \theta; \sin \theta) \sin \theta \times [1 - n_F(-|\Delta(\theta)|)]. \]

(24)
So, the total force is

\[ F_{SF} = C_0 \kappa \times \nu_n, \quad d_{SF} \approx 1. \]  

(25)
The first correction (in small \(\tau\)) gives the dissipative force:

\[ \delta d_{SF} = \tau \langle \omega_0(\theta, 0) \rangle, \]  

(26)
where the angle brackets denote averaging over angles \(\theta\).

In this limit the SF force almost exactly cancels the Magnus force, therefore the total non-dissipative force (and the Hall conductivity) is given by the force, proportional to the difference \(\rho - C_0\) (see Introduction) and a correction to \(25\):

\[ \delta d_{SF} = -\tau^2 \langle \omega_0(\theta, 0) \rangle. \]  

(27)
So, the total dissipative and non-dissipative forces are of the same order as in an s-wave superconductor with the same value of the gap \(\Delta_0\).

The result \(25\) is independent of the structure of the vortex core. However, the corrections \(26, 27\) depend on this structure.

B. Very clean limit

In this limit \(\epsilon_0 \ll 1 \ll \omega_0^0\) the value of \(\omega_0 \tau\) is small near the nodes but it is large far from the nodes. \(t(\theta)\) is almost constant far from the nodes, but it varies rapidly (on \(\tau\) time scale) in their \(\theta\)-vicinities where \(\theta_\tau = (\Omega_0 \tau)^{-1}\). In an internode region \(\theta_0 + a < \theta < 0 + \pi/2 - a\) (where \(\theta_\tau \ll a \ll 1\) and \(\theta_0 = 0, \pi/2, \pi, 3\pi/2\) is a gap node) the point in \((\theta, l)\)-plane, describing the quasiclassical motion \(13\), spends an amount of time small compared to the relaxation time \(\tau\). It follows from the KE that \(\nu\) is almost constant in such a region (variations of \(\nu\) are much less than unity; we assume, and this assumption is justified by the form of the solution below, that
everywhere on on $\theta$-circle $\nu$ is of the order of 1 or less). On the contrary, as the point in $(\theta, t)$-plane approaches a node its motion gets slower and $\nu$ adjusts to a local value of $\nu^{eq}$ (the positive $t$-direction is the negative $t$-direction, therefore the jump in the value of $\nu$ occurs on the right-hand side of a node, see below). In a small $\theta$-vicinity of a gap node $\nu^{eq}$ is almost constant and the solution of the KE (21) can be represented as $\nu = A + B e^{-\nu^{eq}/\tau}$ with constant $A$ and $B$. In the very clean limit, the change in the exponent across a gap node $t_0/4\pi = \pi/2c_0\tau$ is very large, and matching the solutions in two different regions leads to the conclusion that in the internode region $[\theta_0, \theta_0+\pi/2]$ one has $\nu = \nu^{eq}(\theta_0+\pi/2)$, and $\nu$ jumps between two constant values near the nodes. In terms of $\nu$ the force is given by

$$F_{SF} = 2\kappa C_0 v_n \int \frac{dEd\theta}{2\pi} \left( \frac{\partial n_F}{\partial E} \right) \frac{\nu^{eq} - \nu}{\omega_0 \tau}. \tag{28}$$

Using the KE we can substitute the last fraction in the rhs by $\partial \nu / \partial \theta$. This angle derivative of $\nu$ has sharp peaks near the nodes, and simple integration shows that in the very clean regime

$$d^\parallel_{SF} = d^\perp_{SF} = \frac{2}{\pi}. \tag{29}$$

C. Extremely clean limit

In the extremely clean limit $\tau$ is very large, namely, $\epsilon_0 \tau \gg 1$. It means that the period of a quasiclassical trajectory $t_0$ is much less than $\tau$. Therefore, $\nu$ does not have enough time to change its value during the motion, i.e., $\nu = \text{const}$. Since the solution of the KE (21) should be an odd function of $\theta$, this constant is in fact 0. So, to the first order the total force is given by

$$F_{SF} = 2\kappa C_0 v_n \int \frac{dEd\theta}{2\pi} \left( \frac{\partial n_F}{\partial E} \right) \frac{\nu^{eq}}{\omega_0 \tau} (\cos \theta; \sin \theta).$$

At low $T \ll T_c \theta_H$ the leading contribution to the force in this limit is dissipative:

$$d^\parallel_{SF} = \frac{1}{\epsilon_0 \tau}, \quad d^\perp_{SF} = \frac{\pi}{4(\epsilon_0 \tau)^2}. \tag{30}$$

$\epsilon_0$ is the only characteristic of the spectrum, which enters the expression for the force in this limit.

D. Higher temperatures

Moderately clean limit. At higher temperatures $T > T_c \theta_H$ one should account for energy dependence of $\omega_0(\theta, E)$ at characteristic $|E| \sim T$. In the moderately clean regime $\omega_0^{max} \tau$ is still very small (this parameter depends on $T$ only through $\Delta_0$ and $\tau$) and the force is given by Eq. (23). To obtain corrections one should substitute $\omega_0(\theta, E)$ in Eqs. (21), (22) by $\omega_0(\theta, E)$ and average over energies. At low $T \ll T_c$, these corrections remain of the same order of magnitude as at $T \ll T_c \theta_H$. As $T$ approaches $T_c$ they decrease down to $\omega_0 \tau$ and $-\omega_0 \tau^2$, respectively.

Superclean limit. In the superclean regime $\omega_0^{max} \tau \gg 1$ the situation is more complicated. The time of quasiclassical motion along the closed trajectory at a given energy $E \gg \Delta / \theta_H$ has two contributions. The time of motion outside $\theta_E$-vicinities of gap nodes is in the order of $2\pi / \epsilon_0$ (cf. Section 2), while the motion inside these vicinities takes time on order of $\theta_E / \omega_c \gg 2\pi / \epsilon_0$.

Low fields. If $\epsilon_0 \tau \ll 1$ then the sample is in the very clean limit (subsection B) at all temperatures, i.e., the system (19) spends a large amount of time (in units of $\tau$) near the nodes and a small amount of time between the nodes. Therefore, the solution of the KE is of the same form as in the subsection B. In $\theta_E$ vicinities of the nodes $\nu \equiv \nu^{eq}$. Using the KE (21) it is easy to show that on the interval $[\theta_0, \theta_0+\pi/2 - \theta_E]$ the solution is given by $\nu = \nu^{eq}(\theta_0+\pi/2 - \theta_E)$ to the accuracy $o(1)$. The transition between these two types of behavior of the solution occurs in an intermediate region of the width $\theta_c \ll 1$.

There are two contributions to the integral over $\theta$ in (28): one from $\theta_E$-vicinities of the nodes, and the other from the boundaries of these regions where $\nu$ jumps to its equilibrium value in the internode region. The integration shows that (to the accuracy $o(1)$)

$$d^\parallel_{SF} = \frac{4}{\pi} \int_{0}^{\Delta_0} dE \left( \frac{\partial n_F}{\partial E} \right) \left( 1 - \frac{E}{\Delta_0} \right), \tag{31}$$

$$d^\perp_{SF} = \frac{4}{\pi} \int_{0}^{\Delta_0} dE \left( \frac{\partial n_F}{\partial E} \right) \left[ \arcsin \frac{E}{\Delta_0} + \sqrt{1 - \frac{E^2}{\Delta_0^2}} \right], \tag{32}$$

The last line of Eq. (32) represents the contribution of energies $|E| > \Delta_0$ and is exactly the same as in the $s$-wave case (cf. Ref. 5).

At very low temperatures $T \ll \Delta_0$ these expressions give (29). We see that the SF force is almost $T$-independent in this region of temperatures. In the opposite limit $T \gg \Delta_0(T)$ (which corresponds to $|T_c - T| \ll T_c$) we have $d^\parallel_{SF} = 0$ and $d^\perp_{SF} = 1$ regardless of the value of $\omega_0^{max} \tau$: in this region the SF force exactly compensates for the Magnus force, and the vortex does not experience any force in the limit of the normal state (note, however, that our approach is not applicable to a very narrow vicinity of $T_c$ where $\Delta_0 / \tau \ll 1$).

Higher fields. In the other case $\epsilon_0 \tau \gg 1$ the time of periodic motion is small at sufficiently low temperatures $T \ll T_c \omega_c \tau$. In this limit the situation is similar to that of subsection C, and spectral flow is suppressed. To obtain an expression for the force one should
average over energies with $\epsilon_0$ substituted by $\epsilon_0(E) = \pi \omega_c / 4 \theta_E$. Evaluation shows that:

\[
d_{SF}^\parallel \sim \frac{T}{T_c} \frac{1}{\omega_c \tau}, \quad d_{SF}^\perp \sim \left( \frac{T}{T_c \omega_c \tau} \right)^2.
\]  

(33)

At low $T \ll T_c$ one can calculate prefactors in these expressions:

\[
d_{SF}^\parallel = \frac{8 \ln 2}{\pi} \frac{T}{\Delta \omega_c \tau}, \quad d_{SF}^\perp = \frac{4\pi}{3} \left( \frac{T}{\Delta \omega_c \tau} \right)^2.
\]

If $\omega_c \tau \gg 1$ the temperature in this extremely clean regime can reach the values $T \sim T_c$. At these temperatures the contribution of the states above the gap $\Delta_0$ to $d_{SF}^\parallel, d_{SF}^\perp$ is of the same order as the contribution of the states below the gap. At $T \sim T_c$ expressions correspond to conductivities of an s-wave superconductor at $T \sim T_c$ or a normal metal in the limit $\omega_c \tau \gg 1$.

If $\omega_c \tau \ll 1 \ll \epsilon_0 \tau$ another regime is possible: at higher temperatures $T, \omega_c \tau \ll T \approx T_c$ the quasiclassical motion near the nodes becomes slow (on $\tau$ time scale), the system is in the very clean regime, and the force is described by Eqs. (31), (32).

V. ON THE DENSITY MATRIX

The parameter $\epsilon_0 \tau$ is determined by the interlevel distance, while the relation of $\Omega_0 \tau$ to the structure of the spectrum is unclear. In this Section we consider the general KE for the density matrix in terms of exact (not quasiclassical) eigenstates of the Hamiltonian. We demonstrate a solution of this equation and evaluate characteristic parameters determining the strength of the SF. We discuss the origin of the parameters $\Omega_0 \tau$ and $\epsilon_0 \tau$.

The KE for the density matrix $\hat{\rho}$ is

\[
\frac{\partial \hat{\rho}}{\partial t} = i[\hat{\rho}, \hat{H}] - \frac{\hat{\rho} - \hat{\rho}^q}{\tau}.
\]  

(34)

In the stationary case

\[
i \rho_{mn}(E_n - E_m) = \frac{\rho_{mn} - \rho_{mn}^q}{\tau}
\]

and therefore

\[
\left( \frac{\hat{\rho} - \hat{\rho}^q}{\tau} \right)_{mn} = \frac{\rho_{mn}^q}{1 + i\tau(E_m - E_n)} i(E_n - E_m).
\]

(35)

Here $n, m$ enumerate the eigenstates of $\hat{H}$. In two extreme cases we have:

\[
i(E_n - E_m) \rho_{mn}^q, \quad \text{if} \quad |E_m - E_n| \tau \ll 1, \quad (36)
\]

\[-\rho_{mn}^q / \tau, \quad \text{if} \quad |E_m - E_n| \tau \gg 1. \quad (37)
\]

The SF force on the vortex is

\[
F_{SF} = \frac{1}{2} \sum_{m,n} P_{mn} \left( \frac{\hat{\rho}^q - \hat{\rho}}{\tau} \right)_{mn}
\]

(38)

where $P_{mn}$ are matrix elements of the operator of linear momentum.

At low energies $E = -\omega_0(\theta)l$ and quantization leads to a hermitian operator $\hat{H} = i\omega_0(\theta)\partial_\theta + i\partial_\theta \omega_0(\theta)$ as a Hamiltonian. The eigenfunctions of $\hat{H}$ are

\[
f_N(\theta) = \frac{\epsilon_0}{\omega_0(\theta)} \exp \left( -iE \int^0_\theta d\varphi \right)
\]

(39)

(cf. Refs. [2, 17] where a non-hermitian operator $i\omega_0(\theta)$ was used). $E_N = N\epsilon_0$ are determined by periodicity of the wave functions. Matrix elements of the momentum operator $P_{nm}$ [which also determines the density matrix $\hat{\rho}^q = n_F(\hat{H} - v_n \hat{P})$] depend only on the difference $N = m - n$. In terms of $P^+ = P_x + i P_y$, the force is given by

\[
F^+_N = \frac{v_n}{4} \sum_N (P^+_N)^2 \frac{iN}{1 + iN\epsilon_0 \tau}
\]

(40)

at low $T \ll T_c \sqrt{H/H_c}$ (one can split summations over $n$ and $N = m - n$ in [23] provided that $N \gg \Omega_0$). To evaluate the force at higher temperatures one should substitute $\epsilon_0$ by $\epsilon_0(E \sim T) \sim \omega_c T_c / T$. Matrix elements of $P^+$ are

\[
P_N^+ = k_F \sum_0^{2\pi} \frac{dx}{2\pi} e^{-iN x} e^{i\theta(x)}
\]

where $\theta(x = -\epsilon_0 t)$ is defined by Eq. (20), and are non-zero only for $N = 4k + 1$. Analysis shows that for small $N$ (such that $N \sim 1$) one has $P_N^+ = 2\sqrt{2} (-1)^k k_F / \pi N$. $P_N^+$ deviates from this expression at $N \sim \Omega_0 / \epsilon_0 \sim \theta_H^{-1}$ (these $N$ are on the order of the maximal value of $d\theta / dx$ where it slightly increases, and $P_N^+$ rapidly decreases for $|N| \gg \theta_H^{-1}$). So, this dependence determines two energy scales: the interlevel distance $\epsilon_0$ and $\epsilon_0(\Omega_0 / \epsilon_0) = \Omega_0$. Analyzing Eq. (40) one can single out the same three regimes as in the previous Section.

In the moderately clean regime one can neglect the second term in the denominator in (40). Two regions of $N$ contribute to (only reactive) force: the region of small $|N|$, which gives a contribution $2/\pi$ to $d_{SF}^\parallel$, and the region $N \sim \Omega_0 / \epsilon_0$ contributing $1 - 2/\pi$ to $d_{SF}^\perp$.

In the very clean regime $\epsilon_0 \tau \ll 1 \ll \Omega_0 \tau$ the second term in the denominator of Eq. (40) suppresses the contribution of the region $N \sim \Omega_0 / \epsilon_0$ as this term is on the order of $\Omega_0 \tau \gg 1$ in this region. On the other hand, $N \sim (\epsilon_0 \tau)^{-1}$ contribute to the dissipative force in this regime, the contribution to $d_{SF}^\parallel$ being $2/\pi$. Finally, in the extremely clean regime $\epsilon_0 \tau \gg 1$ the denominator in (40) suppresses all the terms, and to the first order the force is $F_{SF} = P^q / \tau$. Calculation of the equilibrium
The value of the linear momentum (at given \( v_n \)) leads to the result (40).

In the s-wave case \( P^0_\tau \neq 0 \) only for \( N = 1 \), and Eq.(40) gives the same result as in Ref. 3

Thus, we found that the reactive force is due to transitions between the levels separated by distances a) on the order of interlevel distance, and b) on the order of maximal value of \( \omega_0(\theta) \). However at finite relaxation rate \( 1/\tau \) only the terms with energy difference \( N\epsilon_0 < 1/\tau \) in [cf. Eq.(40)] are effective. The dissipative force, naturally, comes from transitions over c) the distance 1/\( \tau \). Contributions b) and c) compete with each other: the former is suppressed if \( \Omega_0\tau \gg 1 \), and the latter in the opposite limit. In the extremely clean limit all the transitions are suppressed.

VI. SUMMARY AND DISCUSSION

Our analysis shows that like in s-superconductors, in d-materials the SF along anomalous branches of the spectrum of fermions bound to vortex cores produces a contribution to the force on a moving vortex. In the regime of extreme SF (fast relaxation in the core) the value of the non-dissipative force is the same as in the s-wave case. This result could be anticipated: in fact, consideration of the action for a superfluid shows that the SF force is proportional to the number of the states below the Fermi level \( (C_0/m) \) regardless of the pairing symmetry.

In the superclean regime coexistence of bound states with states in the continuous spectrum leads to new features in vortex dynamics. As the relaxation time grows at constant \( T \) and \( H \) (the system gets cleaner), the spectral flow contribution to the reactive force is gradually suppressed. The motion of a vortex with respect to the normal component (crystall lattice) produces a perturbation of the Bogolyubov-de Gennes hamiltonian \( \hat{v}\hat{P} \) leading to transitions between energy levels. Transitions between the levels \( n \) and \( m \) do not lead to a net loss of momentum of quasiparticles if the time derivative \( E_n - E_m \) of the phase of an off-diagonal element of the density matrix \( \rho_{mn} \) is much larger than the relaxation rate 1/\( \tau \). Such transitions effectively contribute to the force only for \( |E_n - E_m|\tau < 1 \). In s-wave vortices the operator of linear momentum leads to transitions only between the neighboring bound levels with energy spacing \( \omega_0 \), and the parameter \( \epsilon_0\tau \) determines the total force on a vortex (cf. Ref. 3). In the d-wave case the perturbation of the hamiltonian effectively leads to transitions between levels with energy differences in the range \( \epsilon_0; \Omega_0 \). So, features of vortex dynamics depend on two parameters, \( \Omega_0\tau \) and \( \epsilon_0\tau \). The reactive force is completely suppressed only in the extremely clean limit.

The dissipative contribution is small in the limits of very large (extremely clean limit) and very small (moderately clean limit) \( \tau \) and it acquires its maximal value in the intermediate (very clean) region. In this limit dissipative and reactive contributions are equal and independent on \( \tau \) and the details of the spectrum.

In connection with this discussion of different symmetries of the pairing state let us mention that our calculations are not sensitive to the sign of the order parameter. Only the anisotropy of the absolute value of the gap counts. So, the results are equally applicable to d-wave and extremely anisotropic s-wave superconductors (with gap nodes).

Let us discuss some conditions and restrictions we used in our calculations. We have mentioned above that simple quasiclassical description of Andreev bound states is valid in the clean limit \( \Delta(\theta)\tau \gg 1 \). In the moderately clean limit the main contribution to the force is due to the states away from the nodes. Therefore, the contribution of vicinities of the nodes is inessential, and the condition \( \Delta_0\tau \gg 1 \) is sufficient. In the superclean limit the main contribution comes from vicinities of the gap nodes of the width \( \max(\theta,\theta_E) \). The condition \( \Delta(\theta)\tau \gg 1 \) can be translated to \( E_F\tau \gg 1 \), which is even weaker than \( \Delta_0\tau \gg 1 \).

In our calculations we omitted the logarithmic factors [cf. Eqs.(5),(6)]. In fact these factors are unimportant in the moderately clean limit, where mostly the regions away from the nodes contribute, and in these regions the logarithms are absent (see Section 2). In the superclean limit, where the contribution of the states close to the nodes is essential, the logarithmic factors could only affect the boundaries between the different regimes. It follows from the derivation in Sections 4,5 that they could not change the results in the very clean regime. As for the extremely clean limit, the logarithms enter the result only through \( \epsilon_0 \).

We studied 2D case so far. To account for the third dimension (with \( H \parallel \hat{z} \)) we can integrate over all modes of motion along the third axis \( \hat{z} \). z-dependence of the spectrum of the bound states is slow (within an order of magnitude), therefore our results would not change qualitatively in 3D.

Using our results one can investigate temperature, magnetic field and purity dependences of transverse \( \sigma_H \) and longitudinal \( \sigma_O \) conductivities, which are given by (cf. (3) and Ref. 24): \[ \sigma_O = \frac{|e|e}{mH} \rho d S_F \] \[ \sigma_H = \frac{ec}{mH} \rho (1 - d^\perp S_F). \] \[ \sigma_O \approx \sigma_H \approx \sigma \] and the Hall angle is very small. The superconductor is always in this regime \( (\Delta_0\tau / E_F \ll 1 \) or
\[ T_c - T \ll T_c \] just below \( T_c \), and this region could extend down to some finite or zero \([\Delta_0^2(T = 0) \tau(T = 0)/E_F \ll 1]\) temperature. In the latter case the superclean limit is unreachable.

In the very clean limit the conductivities are given by Eqs.\((11), (12)\) and Eqs.\((21), (22)\). This regime is a sub-region of the superclean one with an additional condition that \( c_0\tau \ll \max(1; \sqrt{\frac{H_c^2}{HT/T_c}}) \). If in addition \( T \ll T_c \) then \( \sigma_0 \) and \( \sigma_H \) are \( T \) and \( \tau \) independent and \( \propto H^{-1} \) [cf. Eq.\((29)\)]. Experimentally the superclean regime in high-\( T_c \) compounds was reached, e.g., in Ref.\((23)\).

In all the three regimes the conductivities depend on \( T \) directly and through \( \Delta_0 \). In all the three regimes the conductivities in the moderately clean regime is similar to the clean regime, while in the superclean regime it may be important.

In conclusion, we presented an analysis of the contribution of the quasiparticles bound to vortex cores to vortex dynamics in all regimes though it can be described by motion of quasiclassical energy levels only in the moderately clean limit.

The behavior of Hall and dissipative (Ohmic) conductivities in the moderately clean regime is similar to that in s-wave superconductors, while in the superclean regime new features were found.

As this article was in preparation, I learned about a recent work by N.Kopnin and G.Volovik \cite{24} devoted to the same subject. We note that our results for finite temperatures \( T > T_c \sqrt{H/H_c^2} \) differ from those of Ref.\((24)\).

**ACKNOWLEDGMENTS**

It is my pleasure to thank N.Kopnin, S.Simon, M.Stone, and G.Volovik for very useful discussions of the issues considered in this article and E.Fradkin for stimulating questions. I am very grateful to G.Volovik communications with whom helped me to correct a mistake in the consideration of the very clean limit. The work was supported by NSF through the grants DMR 89-20538COOP, DMR-94-24511, and DMR-91-20000.

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22 By averaging over energies we mean calculation of the integral \( \int dE(-\partial E n_F) \ldots \)

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