I briefly review the formalism for treating the leading-twist two-gluon states appearing in processes which involve $\eta$ and $\eta'$ mesons. The constraints on the size of the lowest order Gegenbauer coefficients of the two-gluon distribution amplitude are obtained from the fit to the $\eta$ and $\eta'$ transition form factor data. The results are applied to $\chi \rightarrow \eta \eta'$ decays and deeply virtual electroproduction of $\eta$ and $\eta'$ mesons.

1. Introduction

The description of the hard exclusive processes involving light mesons is based on the factorization of the short- and long-distance dynamics and the application of the perturbative QCD. The former is represented by the process-dependent and perturbatively calculable elementary hard-scattering amplitude, in which meson is replaced by its (valent) Fock states, while the latter is described by the process-independent meson distribution amplitude (DA), which encodes the soft physics. The lowest Fock state of flavour-nonsinglet mesons consists of quark and antiquark, while for flavour-singlet meson the two-gluon state appears additionally. In this paper I give a status report on work of Ref. [2], and discuss the proper treatment and the importance of these two-gluon states.

On the basis of recent results on $\eta$ and $\eta'$ mixing, we adopt the following representation of $\eta$ and $\eta'$ in an octet-singlet basis:

$$|\eta\rangle = \cos \theta_8 |\tilde{\eta}_8\rangle - \sin \theta_1 |\tilde{\eta}_1\rangle,$$

$$|\eta'\rangle = \sin \theta_8 |\tilde{\eta}_8\rangle + \cos \theta_1 |\tilde{\eta}_1\rangle.$$
\[ |\eta'| = \sin \theta_8 \langle \tilde{\eta}_8 | + \cos \theta_1 \langle \tilde{\eta}_1 |, \quad (1) \]

where the pure octet (\( \tilde{\eta}_8 \)) and singlet (\( \tilde{\eta}_1 \)) states are given by
\[ \langle \tilde{\eta}_1 | = \frac{f_1}{2\sqrt{6}} \phi_1(x)(u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3} + \phi_g(x)|gg\rangle, \]
\[ \langle \tilde{\eta}_8 | = \frac{f_8}{2\sqrt{6}} \phi_8(x)(u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}, \quad (2) \]

and higher Fock states are neglected. In this representation, the mixing dependence is solely embedded in the \( \theta_8 \) and \( \theta_1 \) angles, while in more general approach different distribution amplitudes \( \phi_8^P \) and \( \phi_1^P \) could be assumed for \( P = \eta, \eta' \). The numerical values \( f_8 = 1.26 f_\pi, f_1 = 1.17 f_\pi, \theta_8 = -21.2^\circ, \) and \( \theta_1 = -9.2^\circ \) are used in this work. Alternatively, one could use the recently suggested quark-flavour basis, but the analysis of DA evolution is more straightforward in the above given octet-singlet basis.

2. Two gluon distribution amplitude and the transition form factor for the flavour-singlet meson

Employing for this section more transparent notation \( \phi_q \equiv \phi_1 \), the DA evolution equation for \( \tilde{\eta}_1 \) takes the matrix form
\[ \mu_F^2 \frac{\partial}{\partial \mu_F^2} \begin{pmatrix} \phi_q(x, \mu_F^2) \\ \phi_g(x, \mu_F^2) \end{pmatrix} = V(x, u, \alpha_S(\mu_F^2)) \otimes \begin{pmatrix} \phi_q(u, \mu_F^2) \\ \phi_g(u, \mu_F^2) \end{pmatrix}, \quad (3) \]

where \( \otimes \) denotes the usual convolution symbol. The kernel \( V \) is 2x2 matrix with a well defined expansion in \( \alpha_S \). The evolution of the flavour-singlet pseudoscalar meson distribution amplitude (DA) has been investigated in a number of papers \(^5,^6,^7\). Most of the results \(^5,^6\) are in agreement up to differences in conventions. On the other hand, the consistent set of conventions has to be used in calculation of both the hard-scattering and the distribution amplitude, and these are not easy to extract from the literature.

Following the recent analysis of the pion transition form factor \(^8\), we have performed a detailed next-to-leading order (NLO) analysis of the \( \tilde{\eta}_1 \) transition form factor taking into account both hard-scattering and perturbatively calculable DA part, and this enabled us to fix and test the convention we are using, and to make a connection with other conventions.

The hard-scattering amplitude one obtains by evaluating the \( \gamma^* + \gamma \rightarrow q\bar{q} \) and \( \gamma^* + \gamma \rightarrow gg \) amplitudes which we denote by \( T_{q\bar{q}}(u, Q^2) \) and \( T_{gg}(u, Q^2) \), respectively. Owing to the fact that final state quarks and gluons are taken to be massless and onshell, \( T_{q\bar{q}} \) and \( T_{gg} \) contain collinear singularities, which
have to be factorized out in order to obtain the finite quantities $T_{H,qar{q}}$ and $T_{H,gg}$:

$$T(u,Q^2) = \left( T_{H,qar{q}}(x,Q^2,\mu_F^2) \ T_{H,gg}(x,Q^2,\mu_F^2) \right) \otimes Z^{-1}(x,u,\mu_F^2). \quad (4)$$

On the other hand, the unrenormalized quark and gluon distribution amplitudes $\phi_q(u)$ and $\phi_g(u)$ are defined in terms of $\langle 0 | \tilde{\Phi}(-z) \gamma^+ \gamma_5 \Omega \tilde{\Phi}(z) | \eta \rangle$ and $\langle 0 | G^{\alpha\beta}(-z) \Omega \tilde{G}_{\alpha\beta}(z) | \eta \rangle$, respectively. The renormalization introduces the mixing of these quark and gluon composite operators and

$$\phi(u) = \left( \begin{array}{c} \phi_q(u) \\ \phi_g(u) \end{array} \right) = Z(u,x,\mu_F^2) \otimes \left( \begin{array}{c} \phi_q(x,\mu_F^2) \\ \phi_g(x,\mu_F^2) \end{array} \right). \quad (5)$$

We note that $Z$ represents a 2x2 matrix. Perturbation theory cannot be used for a direct evaluation of $\Phi(u)$, but replacing $|\eta\rangle$ by $|\bar{q}q\rangle$ or $|gg\rangle$ enables us to obtain the perturbatively calculable DA part and to determine matrix $Z$. Finally, the $\eta_1$ transition form factor is given by

$$F_{\gamma^*\eta_1}(Q^2) = \frac{f_1}{2\sqrt{6}} T(u,Q^2) \otimes \phi(u) = \frac{f_1}{2\sqrt{6}} T(x,Q^2,\mu_F^2) \otimes \phi(u,\mu_F^2). \quad (6)$$

Hence, the singularities, which appear in the calculation of the hard-scattering (4) and the DA part (5) should cancel, and we have used this requirement to check the consistency of our calculation.

By differentiating (5) with respect to $\mu_F^2$ one obtains the DA evolution equation (3) with evolution potential $V$ expressed in terms of $Z$ $V = -Z^{-1} \otimes (\mu_F^2 \partial / \partial \mu_F^2) Z$. The solutions of the leading-order (LO) evolution equation are given by

$$\phi_q(x,\mu_F^2) = 6x(1-x) \left[ 1 + \sum_{n=2}^{\infty} \frac{\beta_n}{\alpha_s(\mu_0^2)} C_{n-10}^{5/2}(2x-1) \right]$$

$$\phi_g(x,\mu_F^2) = x^2(1-x)^2 \sum_{n=2}^{\infty} \frac{\beta_n}{\alpha_s(\mu_0^2)} C_{n-10}^{5/2}(2x-1), \quad (7)$$

where

$$B_n^q(\mu_F^2) = B_n^+(\mu_0^2) \left( \frac{\alpha_s(\mu_0^2)}{\alpha_s(\mu_F^2)} \right)^{\gamma_n^{n_0}} + B_n^- \left( \mu_0^2 \right) \left( \frac{\alpha_s(\mu_0^2)}{\alpha_s(\mu_F^2)} \right)^{\gamma_n^{n_0}}$$

$$B_n^{q}(\mu_F^2) = \rho_n^+ B_n^+(\mu_0^2) \left( \frac{\alpha_s(\mu_0^2)}{\alpha_s(\mu_F^2)} \right)^{\gamma_n^{n_0}} + B_n^- \left( \mu_0^2 \right) \left( \frac{\alpha_s(\mu_0^2)}{\alpha_s(\mu_F^2)} \right)^{\gamma_n^{n_0}}. \quad (8)$$
The coefficients $B_n^\pm(\mu_0^2)$, i.e., $B_n^\pm(g)(\mu_0^2)$, represent nonperturbative input at scale $\mu_0^2$, while $\gamma_n^\pm = 1/2 \left( \gamma_n^{qq} + \gamma_n^{gg} \right) \pm \sqrt{\left( \gamma_n^{qq} - \gamma_n^{gg} \right)^2 + 4 \gamma_n^{qg+gq}}$ and

$$\rho_n^+ = 6 \frac{\gamma_n^{qq}}{\gamma_n^{q}-\gamma_n^{gg}}, \quad \rho_n^- = \frac{1}{6} \frac{\gamma_n^{gg}}{\gamma_n^{n}-\gamma_n^{qq}}$$

are defined in terms of anomalous dimensions $\gamma_n^{qq} = \gamma_n^{(0)q}, \gamma_n^{gg}$.

The DA $\phi_q$ is normalized to 1, but, since $\int_0^1 dx \phi_g(x, \mu_2^2 F) = 0$, there is no such natural way to normalize $\phi_g$. It is important to emphasize that any change of the normalization of the gluon DA is accompanied by the corresponding change in the hard-scattering part. Namely, for $\phi_g \to \sigma \phi_g$, the projection of $gg$ state on the $\bar{\eta}_1$ state, which can be derived from the definition of the gluon distribution amplitude, gets modified by factor $1/\sigma$.

By inspecting Eqs. (7-9), it is easy to see that the change of the normalization of $\phi_g$ can be translated into the change of the off-diagonal anomalous dimensions

$$\gamma_n^{qq} \to \sigma \gamma_n^{qq}, \quad \gamma_n^{gg} \to \sigma \gamma_n^{gg}$$

and of the coefficient $B_n^-(\mu_0^2) \to \sigma B_n^-(\mu_0^2)$.

3. Applications

Using the mixing scheme defined in Eq. (1), we have obtained the NLO leading-twist prediction for the $\eta$ and $\eta'$ transition form factors. For the treatment of $\phi_q(x, \mu_2^2 F)$, we use the well-known LO result for the flavour-nonsinglet meson distribution amplitude. We truncate the Gegenbauer
series at $n = 2$, and fit our results to the experimental data\textsuperscript{9}. The fits are carried through with $\mu_R = Q/\sqrt{2}$ and $\mu_F = Q$, with $\alpha_S$ evaluated from the two-loop expression with $n_f = 4$ and $\Lambda_{\overline{MS}}^{(4)} = 305$ MeV. For $Q^2 \geq 2$ GeV$^2$ and $\mu_0 = 1$ GeV, the results of the fits read

$$
B_2^g(\mu_0^2) = -0.04 \pm 0.04 \quad B_1^2(\mu_0^2) = -0.08 \pm 0.04 \quad B_2^g(\mu_0^2) = 9 \pm 12. \quad (13)
$$

The existing experimental data and their quality allow us to obtain not more than a constraint on the value of $B_2^g$. As expected, we have observed a strong correlation between $B_1^2$ and $B_2^g$. The quality of the fit is shown in Fig. 1.

Figure 1. $\eta$ (below) and $\eta'$ (above) transition form factors. The shaded area corresponds to the in Eq. (13) given range for $B_1^2(\mu_0^2)$ and $B_2^g(\mu_0^2)$.

As a first application of the above given results, we have analyzed the $\chi_{c0} \to \eta\eta, \eta'\eta'$ decays. Following previous work\textsuperscript{10}, we take $\chi_{c0}$ as a non-relativistic $c\bar{c}$ bound state, and obtain the decay amplitudes for $c\bar{c} \to (q\bar{q})(q\bar{q})$ and $c\bar{c} \to (gg)(gg)$. The ratio $\Gamma_{\eta'\eta'}/\Gamma_{\eta\eta}$ is suitable for investigating the sensitivity to the gluon contributions. Despite possibly large value of $B_2^g$, we have observed only modest dependence on the variations of $B_1$ and $B_2$ in the allowed range (13) (up to 20 % difference).

Finally, I report on our result for the two gluon contribution to the deeply virtual electroproduction of $\eta, \eta'$ mesons. In this case, the subprocess amplitudes $q+\gamma^*_L \to (q\bar{q})(q\bar{q})+q$ and $q+\gamma^*_L \to (gg)+q$ have to be evaluated. Our result for the former amplitude $H_{P_{\gamma}^{(0)}}$ is in agreement with the results from the literature\textsuperscript{11,12}. In terms of subprocess Mandelstam variables ($\hat{s}, \hat{u}, \hat{t} = t$),
the latter amplitude reads

\[ H_{0+,0+}^{P(g)} = \frac{4\pi\alpha_s}{Q} \frac{C_F}{N_C} \frac{1}{2\sqrt{\mu_f}} \frac{Q\sqrt{-u\bar{u}}}{Q^2 + s} \int_0^1 d\tau \frac{\phi_g(\tau)}{\tau(1-\tau)} \frac{t}{\bar{u}\bar{s}} \left( \frac{1}{\tau} - \frac{1}{1-\tau} \right). \tag{14} \]

In deeply virtual electroproduction of mesons (DVEM), the limit \( t \to 0 \) has to be considered. For \( t = 0 \) and \( \hat{s} + \hat{u} = -Q^2 \), our result (14) gives \( H_{0+,0+}^{P(g)} = 0 \). We conclude that the two-gluon contribution is suppressed.

4. Conclusions

We have performed a detailed analysis of the proper inclusion of the two gluon states in the (octet-singlet scheme based) description of the hard exclusive processes involving \( \eta \) and \( \eta' \) mesons. Normalization and conventions have been fixed and discrepancies found in the literature have been resolved. From the fit of the \( \eta, \eta' \) transition form factors to experimental data, we have obtained the range for the \( B_1^2, B_2^g \) and \( B_8^g \) coefficients of the \( \phi_1, \phi_g \) and \( \phi_8 \) DAs, respectively. Expected strong correlation between \( B_1^2 \) and \( B_2^g \) has been observed. The results have been applied to \( \chi_c^0 \to \eta\eta(\eta'\eta') \) decay, where only a moderate dependence on \( B_2^g \) has been found. For DVEM, the \( \gamma^* q \to (gg)q \) subprocess has been found to be suppressed for small momentum transfer \( t \). We conclude that for considered processes the theoretical and experimental uncertainties do not allow further restriction of the parameter range since only a modest dependence on the value of \( B_2^g \) has been observed. This contrasts the findings for \( \eta'g^*g^* \) vertex\(^{13} \).

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