Superstring-Inspired $E_6$ Unification,  
Shadow Theta-Particles and Cosmology

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Abstract

We construct a new cosmological model considering the superstring-inspired $E_6$ unification in the 4-dimensional space at the early stage of the Universe. We develop a concept of parallel existence in Nature of the ordinary and shadow worlds with different cosmological evolutions.
1 Introduction

We have developed a concept of parallel existence of the ordinary (O) and shadow (Sh) worlds assuming the superstring-inspired $E_6$ unification in the 4-dimensional space [1]. The 'heterotic' superstring theory $E_8 \times E_8'$ was suggested as a more realistic model for unification of all fundamental gauge interactions with gravity [2]. This ten-dimensional theory can undergo spontaneous compactification. The integration over six compactified dimensions of the $E_8$ superstring theory leads to the effective theory with the $E_6$ unification in the four-dimensional space. Superstring theory has led to the speculation that there may exist another form of matter - "shadow matter" - in the Universe, which only interacts with ordinary matter via gravity, or gravitational-strength interactions [3]. The shadow world, in contrast to the mirror world [4], can be described by another group of symmetry (or by a chain of groups of symmetry), which is different from the ordinary world symmetry group.

Our model is based on the following assumptions:

• Grand Unified Theory (GUT) is inspired by Superstring theory [2], which predicts the existence of the $E_6$ unification in the 4-dimensional space at the energy scale $\sim 10^{18}$ GeV.
• The Shadow world is responsible for the dark energy (DE) and dark matter (DM).
• We assume that $E_6$ unification had a place in the O- and M-worlds at the early stage of our Universe. This means that at the very high energy scale $\sim 10^{18}$ GeV there exist mirror world (MW) and the group of symmetry $E_6 \times E_6'$ [1].
• We have adopted for the O-world the breaking $E_6 \rightarrow SO(10) \times U(1)$, while for the Sh-world we have considered the breaking $E_6' \rightarrow SU(6)' \times SU(2)'$ with aim to have in the Sh-world an extra $SU(2)'_\theta$ group at low energies. Here ordinary (shadow) world is described by non-primed (primed) symbols.

The study [1] is a development of the ideas considered previously in Refs. [5]. However, in present investigation we give a new interpretation of the possible accelerating expansion of the Universe using a cosmological quintessence model with superstring-inspired $E_6$ unification.

2 $E_6$ unification breakdown in the ordinary and shadow worlds

Developing the ideas of Refs. [1], we have considered the existence of theta-particles in the shadow world. These "theta-particles" were introduced by Okun in Refs. [6], who suggested that there exists in Nature the symmetry group

$$SU(3)_C \times SU(2)_L \times SU(2)_\theta \times U(1)_Y,$$

which, in contrast to the Standard Model (SM) group, has an additional non-Abelian $SU(2)_\theta$ group whose gauge fields are neutral massless vector particles – 'thetons'. These
'thetons' have a macroscopic confinement radius $1/\Lambda_\theta$. In Refs. [1] we have assumed that such a group of symmetry exists in the shadow world at low energies and having $\Lambda_\theta \sim 10^{-3}$ eV provides a tiny cosmological constant.

According to the assumptions of Refs. [1], in the ordinary world, from the SM-scale up to the $E_6$ unification scale, there exists the following chain of symmetry groups:

$$SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow [SU(3)_C \times SU(2)_L \times U(1)_Y]_{\text{SUSY}}$$

$$\rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X \times U(1)_Z \rightarrow SU(4)_C \times SU(2)_L \times SU(2)_R \times U(1)_Z$$

$$\rightarrow SO(10) \times U(1)_Z \rightarrow E_6.$$ (2)

But the following chain of symmetry groups exists in the shadow world:

$$SU(3)'_C \times SU(2)'_L \times SU(2)'_R \times U(1)'_Y \rightarrow [SU(3)'_C \times SU(2)'_L \times SU(2)'_R \times U(1)'_Y]_{\text{SUSY}}$$

$$\rightarrow SU(3)'_C \times SU(2)'_L \times SU(2)'_R \times U(1)'_X \times U(1)'_Z \rightarrow SU(4)'_C \times SU(2)'_L \times SU(2)'_R \times U(1)'_Z$$

$$\rightarrow SU(6)' \times SU(2)'_R \rightarrow E'_6.$$ (3)

3  New shadow gauge group $SU(2)'_\theta$

The reason for our choice of the $SU(2)'_\theta$ group was to obtain a new scale in the shadow world at extremely low energies [1], [5]. By comparison with the content of the 27-plet of $E_6$, having 16 fermions (see [1]), we have considered theta-quarks as $\theta$–doublets and shadow leptons as $\theta$–singlets. The scalars $\phi'_\theta$ also can be considered as doublets of $SU(2)'_\theta$.

In Refs. [1] we have considered the running coupling constant $\alpha^{-1}_2(\mu)$ for high energies $\mu > M'_t$ (where $M'_t$ is the top-quark mass in the Sh-world), assuming the existence of three generations of theta-quarks and two doublets of scalar fields $\phi'_\theta$. Of course, near the scale $\Lambda'_\theta$ only theta-quarks of the first generation contribute, and it is easy to obtain the value $\Lambda'_\theta \sim 10^{-3}$ eV. Theta-quarks of the first generation are stable, due to the conservation of the theta-charge [6]. We also have considered a complex scalar field $\varphi_\theta = (1, 1, 1, 0)$, which is a singlet under the symmetry group $G' = SU(3)'_C \times SU(2)'_L \times SU(2)'_R \times U(1)'_Y$.

4  Cosmological Constant, DE and DM

From the point of view of particle physics the cosmological constant naturally arises as an energy density of the vacuum. For the present epoch, the Hubble parameter $H$ is given by the following value [7]: $H = 1.5 \times 10^{-42}$ GeV, and the critical density of the Universe is

$$\rho_c = 3H^2/8\pi G = (2.5 \times 10^{-12} \text{ GeV})^4.$$ (4)

For the ratios of densities $\Omega_X = \rho_x/\rho_c$, cosmological measurements give the following density ratios of the total Universe [7]: $\Omega_0 = \Omega_r + \Omega_m + \Omega_\Lambda = 1$. Here $\Omega_r$ is a relativistic
(radiation) density ratio, and $\Omega_\Lambda = \Omega_{DE}$. The measurements give: $\Omega_{DE} \sim 75\%$ - for the mysterious dark energy, $\Omega_M \approx \Omega_B + \Omega_{DM} \sim 25\%$, $\Omega_B \approx 4\%$ - for (visible) baryons, $\Omega_{DM} \approx 21\%$ - for dark matter. Here we propose that a plausible candidate for DM is a shadow world with its shadow quarks, leptons, bosons and super-partners, and the shadow baryons are dominant: $\Omega_{DM} \approx \Omega_{B'}$. Then we see that $\Omega_{B'} \approx 5\Omega_B$, what means that the shadow baryon density is larger than the ordinary baryon density.

We can calculate the dark energy density using the results given by [7]:

$$\rho_{DE} \approx 0.75 \rho_c \approx (2.3 \times 10^{-3} \text{ eV})^4. \quad (5)$$

The $\Lambda CDM$-cosmological model predicts that the cosmological constant $\Lambda$ is equal to $\Lambda = \rho_{\text{vac}} = \rho_{DE}$. Given by Eq. (5), the cosmological constant $\Lambda$ is extremely small:

$$\Lambda \approx (2.3 \times 10^{-3} \text{ eV})^4. \quad (6)$$

The evolution of the Universe is described by an equation of state:

$$w = \frac{p}{\rho}, \quad (7)$$

where $w$ in general is assumed to be constant, but may be time-dependent. If the cosmological constant describes the dark energy (DE), then $\rho_{DE} = -p_{DE}$, and $w = w_{DE} = -1$. Recent cosmological observations [7] give the following value of $w$: $w = -1.05 \pm 0.13$ (statistical) $\pm 0.09$ (systematic).

5 Quintessence model of DE, DM and matter

The main idea of our present investigation is to show the absence of the SM and SM’ contributions to the cosmological constant $\Lambda$. We relate the value (6) only with the $SU(2)_\theta'$ gauge group’s contribution.

We assume that there exists an axial $U(1)_A$ global symmetry in our theory with currents having $SU(2)'_g$, $SU(3)_c$ and $SU(3)'_c$ anomalies, which are spontaneously broken at the scale $f$ by a singlet complex scalar field $\varphi$, with a VEV $\langle \varphi \rangle = f$. As a result, we have three Nambu-Goldstone (NG) bosons:

$$\varphi^{(i)} = (f + \sigma^{(i)}) \exp(i\alpha^{(i)}/f), \quad i = 0, 1, 2, \quad (8)$$

where index $i = 0, 1, 2$ corresponds to $SU(2)'_g$, $SU(3)'_c$ and $SU(3)_c$ gauge groups, respectively. The boson $\alpha^{(i)}$ (imaginary part of the singlet scalar field $\varphi^{(i)}$) is an axion and could be identified with a massless Nambu-Goldstone (NG) boson if the $U(1)_A$ symmetry is not spontaneously broken. However, the spontaneous breaking of the global $U(1)_A$ by corresponding instantons inverts $\alpha^{(i)}$ into the pseudo Nambu-Goldstone (PNG) bosons.
The singlet complex scalar field $\varphi^{(i)}$ reproduces a Peccei-Quinn (PQ) model [8]. Near the vacuum, a PNG mode $\alpha^{(i)}$ emerges the following PQ axion potential:

$$V_{PQ}(\alpha^{(i)}) \approx \left(\Lambda^{(i)}\right)^4 \left(1 - \cos(\alpha^{(i)}/f)\right).$$

(9)

This axion potential exhibits minima at

$$\cos(\alpha^{(i)}/f) = 1, \quad \text{i.e.} \quad (\alpha^{(i)})_{\text{min}} = 2\pi nf, \quad n = 0, 1, ...$$

(10)

For small fields $\alpha^{(i)}$ we expand the effective potential (9) near the minimum:

$$V_{PQ} \approx \frac{(\Lambda^{(i)})^4}{2f^2}(\alpha^{(i)})^2 + ... = \frac{1}{2}m_i^2(\alpha^{(i)})^2 + ..., \quad (11)$$

and hence the PNG axion mass squared is given by:

$$m_i^2 \sim \frac{(\Lambda^{(i)})^4}{f^2}. \quad (12)$$

Let us assume that at the cosmological epoch when $U(1)_A$ was spontaneously broken, the value of the axion fields $\alpha^{(i)}$ was deviated from zero and initial values were: $\alpha^{(i)}_m \sim f$. The scale $f \sim 10^{18}$ GeV (near the $E_6$ unification scale) makes it natural that the $U(1)_A$ symmetry was broken before inflation.

Now quintessence is described by the PNG scalar field $\alpha \equiv \alpha^{(i)}$ minimally coupled to gravity and leading to the late time inflation. The action for quintessence is given by

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} (\nabla \alpha)^2 - V(\alpha)\right], \quad (13)$$

where $V(\alpha)$ is the potential of the field. In the flat Friedmann spacetime the action (13) leads to the following field and Einstein equations:

$$\ddot{\alpha} + 3H \dot{\alpha} + \frac{dV}{d\alpha} = 0. \quad (14)$$

$$H^2 = \frac{8\pi G}{3} \left[\frac{1}{2} (\dot{\alpha})^2 + V(\alpha)\right], \quad (15)$$

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{3} \left[\frac{1}{2} (\dot{\alpha})^2 - V(\alpha)\right]. \quad (16)$$

The equation of state is given by

$$w^{(i)} = \frac{(\dot{\alpha})^2 - 2V(\alpha^{(i)})}{(\dot{\alpha})^2 + 2V(\alpha^{(i)})}. \quad (17)$$
6 Quintessence model for $SU(2)_{\theta}'$ gauge group

This is a case for $i = 0$. Solving Eq. (14) for $\alpha^{(0)}$ we can introduce an axion $\alpha_{\theta}$ by the notation $\alpha^{(0)} = \alpha_{\theta}$. The equation of motion (EOM) of the classical field $\alpha_{\theta}$ is:

$$\frac{d^2 \alpha_{\theta}}{dt^2} + 3H \frac{d\alpha_{\theta}}{dt} + V' (\alpha_{\theta}) = 0,$$

where according to Eq. (9),

$$V (\alpha_{\theta}) = (\Lambda_{\theta}')^4 (1 - \cos(\alpha_{\theta}/f)),$$

$$V' (\alpha_{\theta}) = (\Lambda_{\theta}')^4 \frac{\sin(\alpha_{\theta}/f)}{f}.$$  \hfill (19)

If now $\sin(\alpha_{\theta}/f) = 0$, but $\cos(\alpha_{\theta}/f) = -1$, then $\dot{\alpha}_{\theta} = 0$, and

$$V (\alpha_{\theta}) = 2(\Lambda_{\theta}')^4.$$

In this case $\alpha_{\theta} = \text{const}$, $\rho_{\theta} = \text{const}$, $w = w_{\theta} = -1$, what means that axion $\alpha^{(0)} = \alpha_{\theta}$ gives the contribution to DE with DE density equal to the value (20):

$$\rho_{\text{DE}} = 2(\Lambda_{\theta}')^4.$$

Using the result (3), we obtain the following estimate of the $SU(2)_{\theta}'$ group’s gauge scale:

$$\Lambda_{\theta}' \simeq 2.0 \times 10^{-3} \text{ eV}.$$ \hfill (22)

If $\Lambda_{\theta}' \sim 10^{-3} \text{ eV}$ and $f \sim 10^{18} \text{ GeV}$, then from Eq. (12) we obtain the estimate of the theta-axion mass: $m_0 \sim \Lambda_{\theta}'^2 / f \sim 10^{-42} \text{ GeV}.$

7 Quintessence model for $SU(3)_c$ and $SU(3)_{\theta}'$ gauge groups

In the case $i = 1, 2$ we have $\Lambda^{(2)} = \Lambda_{QCD} \sim 0.3 \text{ GeV}$ and $\Lambda^{(1)} = \Lambda_{QCD}^{(1)} \approx 1.5 \Lambda_{QCD}$ (see [1]), which are much larger than $\Lambda_{\theta}'$. We can consider $\alpha^{(1,2)} \neq 0$ if $V' = dV / d\alpha^{(1,2)} \neq 0$ in Eq. (14).

Assuming that at present epoch we have $(\dot{\alpha}_{(1,2)})^2 = 2V (\alpha^{(1,2)})$, we obtain from Eq. (17): $w^{(1,2)} = w_m = 0$, what describes the matter contributions to $\rho_{(QCD)}$ and $\rho_{(QCD)}'$ with the result:

$$\frac{\rho_{\text{DM}}}{\rho_B} \simeq \left( \frac{\Lambda_{QCD}'}{\Lambda_{QCD}} \right)^4 = (1.5)^4 \simeq 5.$$  \hfill (23)
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