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Anisotropic Lp Poisson Disk Sampling for NPR Image with Adaptively Shaped Pieces

Tao WANG†, Zhongying HU†, Nonmembers, and Kiichi URAHAMA(✉), Member

SUMMARY A non-photorealistic rendering technique is presented for generating images such as stippling images and paper mosaic images with various shapes of paper pieces. Paper pieces are spatially arranged by using an anisotropic Lp poisson disk sampling. The shape of paper pieces is adaptively varied by changing the value of p. We demonstrate with experiments that edges and details in an input image are preserved by the pieces according to the anisotropy of their shape.

key words: non-photorealistic rendering, poisson disk sampling, anisotropic distance, stippling images, paper mosaics

1. Introduction

Stippling is a non-photorealistic halftoning technique where round-shaped points are scattered for reproducing grayscale tones in input images. Several methods have been proposed for placing and producing points in synthetic stipple drawings. For example, stippling with round dots [1] such as in Fig. 1 (a) is the popular technique. Feng et al. [2] extended the shape of points to ellipse as in Fig. 1 (b), however their method is not halftoning. A method [3] of arranging line segments uses only short line segments of fixed length as in Fig. 1 (c). As is observed in these examples, the shape of stipple is fixed in the previous methods or some shapes of stipple are mixed by hands. In NPR paper mosaics [4], [5], the shape of paper pieces is usually unified to round disks, ellipses, or squares with slightly jaggy peripheries. However, in hand-made paper mosaics, various shapes of pieces are mixed and pasted along the direction of edges as are examples shown in Fig. 1 (d), (e).

In this paper, to emphasize the visual recognition of grayscale variation and to show clear contour and the detail of objects, we present a method for generating stippling images with adaptively shaped points by using anisotropic Lp poisson disk sampling (PDS) with grayscale ordered scanning of pixels. Furthermore, for generating paper mosaic images with variously shaped pieces, a method has been presented by using PDS with Lp distance of which the value of p is adaptively changed according to the gradient intensity at each pixel.

2. Grayscale ordering Poisson Disk Sampling

We denote the size (number of pixels) of input image as n.

Then we set the ordering of pixel scan by their grayscale. The procedure is as follows:
1. Set the radius ri for each pixel.
2. Add uniform random number ni of the range [0,1] to the gray-level di of each pixel to get fi = di + ni.
3. Attach the number k(= 1, 2, ..., n) to every pixel in the increasing order of fi.
4. Paste a point at the pixel of k = 1.
5. Increase k by one. If k = n then finish, else go to step 6.
6. If there is no point in the circle of radius ri around the center pixel of number k, then paste a new point at that pixel and go to step 5, else skip that pixel and go to step 5.

The final step 6 in this procedure requires the distance between two pixels in the input image. In the usual isotropic PDS, the distance between pixels i and j is \( \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \) where \((x_i, y_i)\) is the coordinate of pixel i [6].

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The authors are with Kyushu University, Fukuoka-shi, 815-8540 Japan.

a) E-mail: urahama@design.kyusyu-u.ac.jp
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Fig. 1 Stippling images by previous methods and paper mosaics.

Fig. 2 Stippling images by isotropic PDS.
3. Anisotropic $L_p$ Distance

In the poisson disk sampling result of Fig. 2, the disk is a circle. This is due to using the Euclidean distance between pixels as above. In this paper, for changing the shape of disks adaptively, we adopt anisotropic $L_p$ distance:

$$\sqrt[\alpha]{(|x_i - x_j|^p + |y_i - y_j|^p + \alpha * |d_i - d_j|^p)^{1/p}}$$

where $(x_i, y_i)$ is the $(x, y)$ coordinate of pixel $i$ and $d_i$ is its luminance. This is a distance in 3-dimensional space where $d_i$ is on the third coordinate $Z$. An image forms a 2-dimensional manifold, i.e. a surface in this 3-dimensional space. The coefficient $\alpha$ is a parameter controlling anisotropy. If $\alpha = 0$, Eq. (1) reduces to the usual isotropic $L_p$ distance, particularly when $\alpha = 0$, $p = 2$ it becomes the Euclidean distance. The anisotropic $L_p$ disk of radius $r$ with the center at pixel $j$ is given by the set of pixels $i$ of which coordinate $(x_i, y_i)$ satisfies

$$\sqrt[\alpha]{(|x_i - x_j|^p + |y_i - y_j|^p + \alpha * |d_i - d_j|^p)^{1/p}} \leq r$$

i.e. the inner region in a $L_p$ circles. The popular isotropic $L_p$ disks with $\alpha = 0$ are shown below in Fig. 3 for $p = 1, 2, \infty$. Sides of the square in Fig. 3 (a) are inclined by 45 degrees, hence it is suitable to be pasted around slanted edges, while the sides of Fig. 3 (c) are horizontal or vertical, hence it is well fitted to horizontal or vertical edges in input images. If we set $\alpha > 0$, these disks are distorted along the gradient of luminance. The anisotropic $L_p$ disks with different gradient of input images are as follows in Table 1.

4. Anisotropic $L_p$ PDS Stippling

To emphasize our visual recognition of grayscale variation, we distort the shape of points adaptively by extending the above isotropic PDS to an anisotropic PDS as $\sqrt[\alpha]{(|x_i - x_j|^p + |y_i - y_j|^p + \alpha * |d_i - d_j|^p)^{1/p}}$ with $\alpha > 0$, $p = 2$. In the anisotropic PDS, the radius is set as

$$r_i = (a + b * d_i / 255)[1 + \alpha * (g_x^2 + g_y^2)]^{1/4}$$

where $g_x$ and $g_y$ are the horizontal and vertical gradients at pixel $i$. We calculate them by using the Sobel filter. The second coefficient in the right hand side of this expression of the radius is attached for equalizing the area of every disk.

Thus, in this paper, we propose the anisotropic PDS with grayscale ordering of pixels. After arranging points by this anisotropic PDS, we paste disks at every point. These disks are also extended to spatially anisotropic shapes. When drawing anisotropic disks, we set the radius as $r_i = (a + b * d_i / 255)[1 + \alpha * (g_x^2 + g_y^2)]^{1/4}$ where $d_{min}$ denotes the minimum grayscale of pixels in the image. Different in the shape of disks between the isotropic PDS and the

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\begin{array}{|c|c|c|}
\hline
\text{gradiention} & p = 1 & p = 2 & p = \infty \\
\hline
\text{anisotropic } & \text{disks with different gradient.} & \text{disks with different gradient.} & \text{disks with different gradient.} \\
\hline\end{array}
\]

![Examples of isotropic $L_p$ disks.](image)

![Stippling image of isotropic PDS and that of anisotropic PDS.](image)

![Stippling image of a butterfly.](image)
anisotropic one is shown in Fig. 4(b) and Fig. 4(c). We set \( \alpha = 0.8 \) for these stippling images.

In Fig. 5(a), the image of butterfly contains thin lines which are difficult to reproduce by stippling of round dots. The stippling image by the isotropic PDS with grayscale ordering of pixels is shown in Fig. 5(b). However lines in Fig. 5(b) are thicker than Fig. 5(a) and some small spots are noisy. Figure 5(c) is the stippling image by the anisotropic PDS with grayscale ordering of pixels. We set \( \alpha = 0.1 \). The width of lines are remained as thin as in Fig. 5(a) owing to extremely thinned disks placed in them. Spots are also clearly seen in Fig. 5(c). Noticeably the small head of the butterfly is clearly preserved while it is merged with the wing limbs in Fig. 5(b). This example reveals the high capability of the proposed method for preservation of fine structures in input images. The next example is the image of peppers in Fig. 6(a). The stippling image by the isotropic PDS with grayscale ordering of pixels is shown in Fig. 6(b) and Fig. 6(c) is the stippling image by the anisotropic PDS with grayscale ordering of pixels. We set \( \alpha = 0.1 \). Owing to some ellipses skewed along the gradients of grayscale in Fig. 6(a), the 3D shapes of peppers are enhanced particularly at their peripheries.

5. Adaptive Anisotropic \( L_p \) PDS

As was mentioned in Sect. 2 about the directionality of disks in Table 1, they are desirable to be pasted according to the direction of luminance gradient at each pixel. Hence we vary the value of \( p \) at each pixel \( i \) as follows:

If \( |g_{xi}| < \epsilon \) and \( |g_{yi}| < \epsilon \), \( p_i = 2 \)
else if \( \max(|g_{xi}|, |g_{yi}|) / \min(|g_{xi}|, |g_{yi}|) > \varphi \), \( p_i = \infty \)
else \( p_i = 1 \).

Because the area of paper pieces varies with the value of \( p \). To make the areas even, we normalize the radius of \( p = 1 \) disk to \( \pi r/2 \) and the radius of \( p = \infty \) to \( \pi r/4 \) with respect to the radius \( r \) of \( p = 2 \) as the standard size. The procedure of anisotropic \( L_p \) poisson disk sampling is shown as follows:

1. Set the standard radius \( r \) of disks.
2. Calculate the gradient \( g_{xi}, g_{yi} \) at each pixel.
3. Attach the number \( k(=1, \ldots, n) \) to every pixel in the
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(a) input      (b) blurred luminance

(c) isotropic PDS  (d) anisotropic PDS

(e) isotropic disks of radius $2r_i/3$  (f) isotropic disks of radius $3r_i/4$

(g) anisotropic disks of radius $2r_i/3$  (h) anisotropic disks of radius $3r_i/4$

Fig. 8  Paper mosaic images of peppers.

increasing order of $|g_{xi}| + |g_{yi}|$.

4. Calculate the value of $p_i$ at each pixel on the basis of $g_{xi}$, $g_{yi}$.

5. Calculate the radius $r_i$ adjusted according to $p_i$.

6. Place a point at the pixel of $k = 1$.

7. Increase $k$ by one. If $k = n$, then finish, else go to step 8.

8. If there is no point in the circle of radius $r_i$ around the center pixel of number $k$, then place a new point at that pixel and go to step 7, else skip that pixel and go to step 7.

If we paste each disk in its own size of radius $r$, then disks are heavily overlapped mutually, hence each disk cannot discriminated from each other, so we weaken their overlapping. We allow their slight overlapping according to the hand-made examples in Fig. 1. We paste anisotropic $L_p$ disks of radius $2r_i/3$ or $3r_i/4$ at the places obtained by the PDS. In addition this shortening of disk radius, we add random fluctuation to the color of disks for serving visual differentiation of each disk, that is, we add random noise to input color images and paste that color of center pixels to each disk.

We set $r = 20$, $\epsilon = 1$, $\varphi = 3$ in the experiments below. The example of input image is shown in Fig. 7 (a) of which PDS points in Fig. 7 (c) is the isotropic PDS ($\alpha = 0$) and Fig. 7 (d) is the anisotropic PDS ($\alpha = 0.5$). Pasted isotropic disks are shown in Fig. 7 (e), (f) and anisotropic disks in Fig. 7 (g), (h). Last example of input image is shown in Fig. 8 (a) of which PDS points are shown in Fig. 8 (c) ($\alpha = 0$), and Fig. 8 (d) ($\alpha = 0.5$). The pasted disks are shown in Fig. 8 (e), (f) and Fig. 8 (g), (h).

6. Conclusion

We have presented an anisotropic PDS technique for generating NPR images such as stippling images and paper mosaic images with adaptively shaped disks and verified its superiority to the conventional isotropic PDS. Our procedure preserves edges and enhances our visual perception of three dimensional geometry of objects in input images.

Extension of this method to the multiple PDS in stippling images such as white and black disks on gray backgrounds, and adding irregularity to the peripherals of paper pieces is under study.

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