Resonant parameters of the $Y(4220)$

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Abstract

The vector charmoniumlike state $Y(4220)$ was reported recently in the cross sections of $e^+e^- \rightarrow \pi^+\pi^- h_c$, $\omega \chi_{c0}$, $\pi^+\pi^- J/\psi$, and $D^0 D^{*-}\pi^+ + c.c.$ measured by the BESIII experiment. A combined fit is performed to the cross sections of these four final states to measure the resonant parameters of the $Y(4220)$. We determine a mass $M = (4219.6 \pm 3.3 \pm 5.1) \text{ MeV}/c^2$ and a total width $\Gamma = (56.0 \pm 3.6 \pm 6.9) \text{ MeV}$ for the $Y(4220)$, where the first uncertainties are statistical and the second ones systematic. We determine the lower limit of its leptonic decay width of around 30 eV, which can be compared with the theoretical expectations of different models. We also estimate its partial decay width to $\pi\pi J/\psi$ in different scenarios. These information is essential for the understanding of the nature of this state.

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I. INTRODUCTION

During the last decade, many new states with hidden charm-quark pair were discovered \cite{1}. The number of observed states in experiments is more than that of the predicted charmonium states in potential models for the mass above the $D \bar{D}$ threshold \cite{2} and there are also charged states which are obviously not charmonium states. These states, such as the $X(3872)$ \cite{3}, the $Y(4260)$ \cite{4}, and the $Z_c(3900)$ \cite{5,6}, are referred to as charmoniumlike states or $XYZ$ particles \cite{1}.

Among these new charmoniumlike states, there are many vector states with quantum numbers $J^{PC} = 1^{--}$ that are usually called $Y$ states, like the $Y(4260)$ \cite{4}, the $Y(4360)$ \cite{7}, and the $Y(4660)$ \cite{8}. The $Y$-states show strong coupling to hidden-charm final states in contrast to the vector charmonium states in the same energy region ($\psi(4040)$, $\psi(4160)$, $\psi(4415)$) which couples dominantly to open-charm meson pairs. These $Y$ states are good candidates for new types of exotic particles and stimulated many theoretical interpretations, including tetraquarks, molecules, hybrids, or hadrocharmonia \cite{1}.

These $Y$ states were observed at $B$ factories with limited statistics since they are produced from initial state radiation processes with data collected at around 10.6 GeV in the bottomonium energy region \cite{4,7,8}. The high precision cross section measurements and the study of these states in different final states in direct $e^+e^-$ annihilation in the charmonium energy region from the BESIII experiment supply new insight into their properties.

In 2013, BESIII reported the cross section measurement of $e^+e^- \rightarrow \pi^+\pi^- h_c$ at 13 center-of-mass (c.m.) energies from 3.9 to 4.2 GeV and found the magnitude of the cross sections is at the same order as that of $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ but with a different line shape. Although no quantitative results were given, the resonant structure at around 4.22 GeV/$c^2$ is obvious \cite{9}. A combined fit to the BESIII data together with the CLEO-c measurement at 4.17 GeV \cite{10} results in a resonant structure, $Y(4220)$, with a mass of $(4216 \pm 18)$ MeV/$c^2$ and a width of $(39 \pm 32)$ MeV \cite{11}, different from any of the known $Y$ and excited $\psi$ states in this mass region \cite{12}.

In 2014, BESIII reported the cross section measurement of $e^+e^- \rightarrow \omega_{\chi_{c0}}$ at 9 c.m. energies from 4.21 to 4.42 GeV. By assuming the $\omega_{\chi_{c0}}$ signals come from a single resonance, BESIII reported a resonant structure with the mass and width of $(4230 \pm 8 \pm 6)$ MeV/$c^2$ and $(38 \pm 12 \pm 2)$ MeV, respectively, and the statistical significance is more than $9\sigma$ \cite{13}. This structure is in good agreement with the $Y(4220)$ observed in $e^+e^- \rightarrow \pi^+\pi^- h_c$ \cite{11}, and combined fits assuming the structures at 4.22 GeV/$c^2$ are the same have been tried by the authors of Refs. \cite{14,15}.

BESIII updated the measurements with higher energy data up to 4.6 GeV included, in both $e^+e^- \rightarrow \pi^+\pi^- h_c$ \cite{16} and $\omega_{\chi_{c0}}$ \cite{17} processes. In addition, more data points are added even at low energy, although with low integrated luminosity, to further constrain the line shape in $e^+e^- \rightarrow \pi^+\pi^- h_c$ \cite{16} process. While the structure in $\omega_{\chi_{c0}}$ mode was affected only slightly with the new measurements at high energies \cite{17}, in the $e^+e^- \rightarrow \pi^+\pi^- h_c$ mode, the $Y(4220)$ was observed with improved significance together with a new structure, the $Y(4390)$. The resonant parameters are $M = (4218.4 \pm 4.0 \pm 0.9)$ MeV/$c^2$ and $\Gamma = (66.0 \pm 9.0 \pm 0.4)$ MeV for the $Y(4220)$, and $M = (4391.6 \pm 6.3 \pm 1.0)$ MeV/$c^2$ and $\Gamma = (139.5 \pm 16.1 \pm 0.6)$ MeV for the $Y(4390)$ \cite{16}. The updated cross sections of $e^+e^- \rightarrow \omega_{\chi_{c0}}$ and $\pi^+\pi^- h_c$ are shown in Fig. \ref{fig:1}, where the measurements at energy points both with integrated luminosities larger than 40 pb$^{-1}$ (referred to as ‘$XYZ$ data sample’ hereafter) and with integrated luminosities
smaller than 20 pb$^{-1}$ (referred to as ‘R-scan data sample’ hereafter) are presented.

The process $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ at c.m. energies up to 5.0 GeV was first studied by the BABAR experiment, where the $Y(4260)$ was observed [4]. Belle measured the cross sections of $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ at c.m. energies between 3.8 and 5.0 GeV and reported that $Y(4260)$ alone cannot describe the line shape satisfactorily [18]. Improved measurements with both BABAR [19] and Belle [5] full data samples confirmed the existence of non-$Y(4260)$ component in $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ but the line shape was parametrized with different models. Recently, BESIII reported a precise measurement of $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ cross sections at c.m. energies from 3.77 to 4.60 GeV (as shown in Fig. 1) using data samples with an integrated luminosity of 9 fb$^{-1}$ [20]. While the nature of the events at around 4 GeV is still ambiguous, the dominant resonant structure, the so called $Y(4260)$, was found to have a mass of $4222.0 \pm 3.1 \pm 1.4$ MeV/c$^2$ and a width of $(44.1 \pm 4.3 \pm 2.0)$ MeV, in good agreement with the $Y(4220)$ observed in $e^+e^- \rightarrow \pi^+\pi^-h_c$ [16]. In addition, a new resonance with a mass of around 4.32 GeV/c$^2$ is needed to describe the high precision data.

BESIII also reported a measurement of the $e^+e^- \rightarrow D^0D^{*-}\pi^+ + c.c.$ cross sections at c.m. energies from 4.05 to 4.60 GeV with the same data samples [21], which is a significant improvement over the previous measurement at Belle [22]. Two resonant structures in good agreement with the $Y(4220)$ and $Y(4390)$ observed in $\pi^+\pi^-h_c$ [16] are identified over a smoothly increasing non-resonant term which can be parametrized with a three-body phase

FIG. 1: The measured cross sections of $e^+e^- \rightarrow \omega c_0, \pi^+\pi^-h_c, \pi^+\pi^-J/\psi$, and $D^0D^{*-}\pi^+ + c.c.$ by the BESIII experiment. The dots are from the XYZ data sample and the triangles are from the R-scan data sample. The error bars are the sum in quadrature of the statistical and uncommon systematic errors. Here, for each process the correlated systematic uncertainties (13.3%, 14.8%, 5.8%, and 4.6% for $\omega c_0, \pi^+\pi^-h_c, \pi^+\pi^-J/\psi$, and $D^0D^{*-}\pi^+ + c.c.$, respectively) are not shown.
space amplitude. The cross sections of $D^0D^*-\pi^+ + c.c.$ are also shown in Fig. 1.

An obvious feature in the above four channels from the BESIII measurements is that there is a common structure at around 4.22 GeV/$c^2$, i.e., the $Y(4220)$. As such a state is not observed in other open charm final states [23], these four final states are probably the dominant decay modes of the $Y(4220)$. By applying constraints to the resonant parameters in a simultaneous fit to the cross sections of these four processes, we may obtain the best knowledge on the $Y(4220)$, including resonant parameters (mass, width, coupling to lepton pair, and decay branching fractions), and thus a better understanding of its nature [1], especially whether it is an exotic state, such as a tetraquark state in the diquark-antidiquark model [24], a vector molecular state of $DD_1(2420)$ [25], a mixture of two hadrocharmonium states [26], an $\omega\chi_{cJ}$ molecule [27, 28], or a charmonium-hybrid state [29].

II. THE DATA AND THE FIT FORMALISM

We use the measured cross sections of $e^+e^-\to\omega\chi_{c0}$, $\pi^+\pi^-h_c$, $\pi^+\pi^-J/\psi$, and $D^0D^*-\pi^+ + c.c.$ processes [14, 17, 20, 21] by BESIII experiment only to measure the parameters of the resonances presented. The data are shown in Fig. 1 where the dots with error bars are from the XYZ data sample and the triangles with error bars are from the R-scan data sample. Here, the error bars are the sum in quadrature of the statistical and uncorrelated systematic errors, and the correlated systematic uncertainties common to the energy points in each process are removed since they have no effect on the fitted resonant parameters.

We parametrize the cross section with the coherent sum of a few amplitudes, either resonance represented by a Breit-Wigner (BW) function or non-resonant production term parametrized with a phase space term. The BW function used in this article is

$$BW(s) = \sqrt{\frac{12\pi|e^+e^-B_f^2|}{s-M^2+iMT}} \sqrt{\frac{PS_n(s)}{PS_n(M)}},$$

where $M$ is the mass of the resonance; $\Gamma$ and $\Gamma_{e^+e^-}$ are the total width and partial width to $e^+e^-$, respectively; $B_f$ is the branching fraction of the resonance decays into final state $f$; and $PS_n$ is the $n-$body decay phase space factor which increases smoothly from the mass threshold with the $\sqrt{s}$ [12].

In fitting to the data shown in Fig. 1, we assume the observed structures at around 4.22 GeV/$c^2$ in all reactions and structures at around 4.39 GeV/$c^2$ in $e^+e^-\to\pi^+\pi^-h_c$ and $D^0D^*-\pi^+ + c.c.$ are due to the same resonant states, i.e., we assume the cross sections are due to the $Y(4220)$ only for $\omega\chi_{c0}$, the $Y(4220)$ and $Y(4390)$ for $\pi^+\pi^-h_c$, the $Y(4008)$, $Y(4220)$ and $Y(4320)$ for $\pi^+\pi^-J/\psi$, and the $Y(4220)$ and $Y(4390)$ for $D^0D^*-\pi^+ + c.c.$, that is,

$$\sigma_{\omega\chi_{c0}}(s) = |BW_1(s)|^2,$$
$$\sigma_{\pi^+\pi^-h_c}(s) = |BW_1(s) + BW_3(s) e^{i\phi_1}|^2,$$
$$\sigma_{\pi^+\pi^-J/\psi}(s) = |BW_0(s) + BW_1(s) e^{i\phi_2} + BW_2(s) e^{i\phi_3}|^2,$$
$$\sigma_{D^0D^*-\pi^+ + c.c.}(s) = |\sqrt{PS_3(s)} + BW_1(s) e^{i\phi_4} + BW_3(s) e^{i\phi_5}|^2,$$

where $BW_0$, $BW_1$, $BW_2$, and $BW_3$ represent the $Y(4008)$, $Y(4220)$, $Y(4320)$, and $Y(4390)$, respectively, and $\phi$ is the relative phase between the amplitudes.
We do a combined fit using a least squares method with MINUIT package in the CERN Program Library [30]. The $\chi^2$ function is constructed as

$$\chi^2 = \sum_{j=1}^{4} \sum_{i=1}^{n} \frac{(\sigma_{ij}^{\text{data}} - \sigma_{ij}^{\text{fit}})^2}{\delta^2_{ij}},$$

where $\sigma_{ij}^{\text{data}}$ and $\sigma_{ij}^{\text{fit}}$ are the measured and fitted cross sections of the $i$th energy point in the $j$th mode, $\delta_{ij}$ is the corresponding total error with common systematic errors removed. The sum is performed over all the measured cross section points from the above mentioned four modes. The $\chi^2$ is minimized to obtain the best estimation of the resonant parameters.

III. FIT RESULTS

We fit BESIII data on $e^+e^- \rightarrow \omega\chi_{c0}$, $\pi^+\pi^-h_c$, $\pi^+\pi^-J/\psi$, and $D^0D^*-\pi^+ + c.c.$ cross sections simultaneously. Two solutions, four solutions, and four solutions with the same minimum values of $\chi^2$ are found with the two, three, and three amplitudes interfering with each other for $e^+e^- \rightarrow \pi^+\pi^-h_c$, $\pi^+\pi^-J/\psi$, and $D^0D^*-\pi^+ + c.c.$, respectively. The masses and the widths of the resonances are identical but the partial widths to $e^+e^-$ and relative phases are different in different solutions for each process. There are no multiple solutions for $e^+e^- \rightarrow \omega\chi_{c0}$ since only one amplitude is used.

Figure 2 shows the fit results with a goodness-of-the-fit of $\chi^2/ndf = 241/273 = 0.9$, where the solid curves show the projections from the best fit, the dashed curves show the fitted resonance components from different solutions, and the corresponding mass, width, and the product of the branching fraction to specific mode and the $e^+e^-$ partial width for each resonance are listed in Table I.

From the fit we obtain $M = (4219.6 \pm 3.3)\,\text{MeV}/c^2$ and $\Gamma = (56.0 \pm 3.6)\,\text{MeV}$ for the $Y(4220)$ where the errors are combined statistical and uncorrelated systematic uncertainties. We can find that the resonant parameters are significantly different from those of the $Y(4260)$ determined from low statistics experiments BABAR [19] and Belle [5], although they are obviously the same resonant structure.

IV. SYSTEMATIC ERRORS

The systematic uncertainties in the resonant parameters in the combined fit to the cross sections of $e^+e^- \rightarrow \omega\chi_{c0}$, $\pi^+\pi^-h_c$, $\pi^+\pi^-J/\psi$, and $D^0D^*-\pi^+ + c.c.$ are mainly from the absolute c.m. energy measurement, the c.m. energy spread, cross section measurements, parametrization of the resonances and background shape.

The systematic uncertainties on the mass and width from the absolute c.m. energy measurement and the c.m. energy spread are taken from the original BESIII publications [16, 17, 20, 21], where we take the largest values conservatively when they are different in different modes. The uncertainty from the cross section measurement in each mode is divided into two categories, correlated and uncorrelated systematic uncertainties. For the uncorrelated systematic uncertainties, they are added with the statistical errors in quadrature, as shown...
and is 13.3%, 14.8%, 5.8%, and 4.6% for $\omega_{\chi}$.

The correlated uncertainty from the cross section measurement in each mode in Figs. 1 and 2, i.e., the fit errors have covered uncorrelated systematic uncertainties in the cross sections. The correlated uncertainty from the cross section measurement in each mode is common for all data points [16, 17, 20, 21], which only affects the $B \times \Gamma_{e^+e^-}$ measurement and is 13.3%, 14.8%, 5.8%, and 4.6% for $\omega_{\chi}$, $\pi^+\pi^- h_c$, $\pi^+\pi^- J/\psi$, and $D^0 D^{*-}\pi^+ + c.c.$, respectively.

Instead of using a constant total width, we assume a mass dependent width to estimate the uncertainty due to signal parametrization. To model the $\pi^+\pi^- J/\psi$ cross section near 4 GeV, an exponential function as used in Ref. [20] is taken instead of using the $Y(4008)$ resonance. We consider the systematic bias introduced by possible additional resonances in the processes under study. The fit scenarios include adding an additional phase space term for $\omega_{\chi}$; using three resonances, the $Y(4220)$, $Y(4320)$ and $Y(4390)$, to fit $\pi^+\pi^- h_c$, $D^0 D^{*-}\pi^+ + c.c.$, or both of them. The shifts of the masses and widths are taken as systematic uncertainties.

The overall systematic uncertainties are obtained by adding all the sources of systematic uncertainties in quadrature assuming they are independent, which are 16.7 MeV/$c^2$ and 31.6 MeV for the mass and width of the $Y(4008)$, respectively; 5.1 MeV/$c^2$ and 6.9 MeV for the mass and width of the $Y(4220)$, respectively; 20.9 MeV/$c^2$ and 23.1 MeV for the mass...
TABLE I: The resonant parameters from the combined fit to $e^+e^- \rightarrow \omega\chi_{c0}$, $\pi^+\pi^- h_c$, $\pi^+\pi^- J/\psi$, and $D^0D^{*-}\pi^+ + c.c$. Here $M$, $\Gamma$, and $(B_i \times \Gamma e^{+e^-})_j$ are the mass (in MeV/$c^2$), total width (in MeV), and the product of the branching fraction to specific final state and the $e^+e^-$ partial width (in eV), respectively, where $i$ presents a final state and $j$ indicates a resonance added in the fit in different processes. The fitted mass and width for each resonance are shown in the upper table separately. All the errors are statistical from fit only. $\phi$ is the relative phase (in rad).

|        | Y(4008)     | Y(4220)     | Y(4320)     | Y(4390)     |
|--------|-------------|-------------|-------------|-------------|
| $M$    | 3846.3 ± 45.5 | 4219.6 ± 3.3 | 4333.2 ± 19.9 | 4391.5 ± 6.3 |
| $\Gamma$ | 345.6 ± 58.2 | 56.0 ± 3.6  | 104.3 ± 44.9 | 153.2 ± 11.4 |

|        | Solution I  | Solution II | Solution III | Solution IV |
|--------|-------------|-------------|--------------|-------------|
| $(B_\omega\chi_{c0} \times \Gamma e^{+e^-})_{Y(4220)}$ | 3.4 ± 0.4    |             |             |             |
| $(B_{\pi^+\pi^- h_c} \times \Gamma e^{+e^-})_{Y(4220)}$ | 4.0 ± 1.1    | 4.0 ± 1.1   |             |             |
| $(B_{\pi^+\pi^- h_c} \times \Gamma e^{+e^-})_{Y(4390)}$ | 11.7 ± 2.4   | 11.7 ± 2.5  |             |             |
| $\phi_1$ | 3.1 ± 0.4   | −3.2 ± 0.4  |             |             |
| $(B_{\pi^+\pi^- J/\psi} \times \Gamma e^{+e^-})_{Y(4008)}$ | 5.5 ± 0.3    | 6.6 ± 0.7   | 6.9 ± 0.7   | 8.3 ± 0.7   |
| $(B_{\pi^+\pi^- J/\psi} \times \Gamma e^{+e^-})_{Y(4220)}$ | 2.5 ± 0.2    | 3.5 ± 0.7   | 10.5 ± 1.1  | 15.1 ± 1.3  |
| $\phi_2$ | 0.1 ± 0.1   | 0.8 ± 0.3   | −1.8 ± 0.2  | −1.0 ± 0.1  |
| $(B_{\pi^+\pi^- J/\psi} \times \Gamma e^{+e^-})_{Y(4320)}$ | 0.7 ± 0.2    | 13.3 ± 3.8  | 1.0 ± 0.5   | 19.4 ± 3.2  |
| $\phi_3$ | 2.2 ± 0.2   | −2.0 ± 0.2  | 1.4 ± 0.6   | −2.7 ± 0.1  |
| $(B_{D^0D^{*-}\pi^+ + c.c.} \times \Gamma e^{+e^-})_{Y(4220)}$ | 5.3 ± 0.6    | 43.3 ± 3.2  | 6.9 ± 0.8   | 56.7 ± 4.2  |
| $\phi_4$ | 2.2 ± 0.1   | −2.2 ± 0.1  | −2.7 ± 0.1  | −0.8 ± 0.1  |
| $(B_{D^0D^{*-}\pi^+ + c.c.} \times \Gamma e^{+e^-})_{Y(4390)}$ | 39.7 ± 4.3   | 61.6 ± 6.6  | 265.5 ± 16.6 | 412.0 ± 26.0 |
| $\phi_5$ | 1.9 ± 0.1   | 1.5 ± 0.2   | 4.7 ± 0.1   | 4.2 ± 0.1   |

and width of the $Y(4320)$, respectively; and 20.8 MeV/$c^2$ and 16.4 MeV for the mass and width of the $Y(4390)$, respectively.

V. SUMMARY AND DISCUSSIONS

From a combined fit to the $e^+e^- \rightarrow \omega\chi_{c0}$, $\pi^+\pi^- h_c$, $\pi^+\pi^- J/\psi$, and $D^0D^{*-}\pi^+ + c.c$. cross sections measured by BESIII, we determine the mass of the $Y(4220)$ as $(4219.6 ± 3.3 ± 5.1)$ MeV/$c^2$ and the width of $(56.0 ± 3.6 ± 6.9)$ MeV, and the relative production rates in these four decay modes.

The leptonic decay width for a vector state is an important quantity for discriminating various theoretical interpretations of its nature. The magnitude of the leptonic decay width determines how the strong decay widths sum up to the total width. Smaller leptonic decay width means that the strong decay widths will be relatively enhanced and vice versa. As the $Y(4220)$ is the dominant component in $e^+e^- \rightarrow \pi^+\pi^- J/\psi$, we assume the theoretical interpretations of $Y(4260)$ may apply also for the $Y(4220)$.

The recent estimate of Lattice QCD (LQCD) for the leptonic decay width of the $Y(4220)$ is about 40 eV [31] as a feature of the hybrid scenario; the predicted upper limit of the
Y(4220) leptonic decay width is about 500 eV if the Y(4220) is a hadronic molecule dominated by $DD_1(2420)$ \cite{32}; the leptonic decay width is only about 23 eV for the $\omega\chi_{c0}$ molecule interpretation \cite{33} where no contributions from the open charm decay channel are included in the analysis.

By considering the isospin symmetric modes of the measured channels, we can estimate the lower limit on the leptonic partial width of the $Y(4220)$ decays. For an isospin-zero charmoniumlike state, we expect

$$B_{\pi\pi h_c} = \frac{3}{2} \times B_{\pi^+\pi^- h_c},$$

$$B_{\pi\pi J/\psi} = \frac{3}{2} \times B_{\pi^+\pi^- J/\psi},$$

$$B_{D\bar{D}^*\pi} = 3 \times B_{D^0\bar{D}^{*+}\pi^+ + c.c.}.$$

So we have

$$\Gamma_{e^+e^-} = \sum_i B_i \times \Gamma_{e^+e^-}$$

$$= B_{\omega\chi_{c0}} \times \Gamma_{e^+e^-} + B_{\pi\pi h_c} \times \Gamma_{e^+e^-} + B_{\pi\pi J/\psi} \times \Gamma_{e^+e^-} + B_{D\bar{D}^*\pi} \times \Gamma_{e^+e^-} + ...$$

By inserting the numbers from Table II, considering the solutions with the smallest $B \times \Gamma_{e^+e^-}$, we obtain

$$\Gamma_{e^+e^-} = (3.4 \pm 0.4 \pm 1.8) + \frac{3}{2} \times (4.0 \pm 1.1 \pm 3.2) + \frac{3}{2} \times (2.5 \pm 0.2 \pm 0.8)$$

$$+ 3 \times (5.3 \pm 0.6 \pm 1.5) + ...$$

$$= (29.1 \pm 2.5 \pm 7.0) + ... \text{ eV}$$

$$> (29.1 \pm 2.5 \pm 7.0) \text{ eV},$$

where the first errors are from fit and the second errors are the systematic errors with the uncertainties from different fit scenarios discussed above included. That is, the lowest value of the $\Gamma_{e^+e^-}$ of the $Y(4220)$ is around 30 eV. This lower limit on the leptonic partial width of the $Y(4220)$ is close to the prediction from LQCD for a hybrid vector charmonium state \cite{31}.

On the other hand, if we take the Solutions with the largest $B \times \Gamma_{e^+e^-}$ in Table II, we obtain $\Gamma_{e^+e^-} = (202 \pm 13 \pm 23) + ... \text{ eV}$. This means that the leptonic partial width of the $Y(4220)$ can be as large as 200 eV or even higher based on current information, to be compared with the predicted upper limit of 500 eV from the molecular scenario \cite{32}. The other combinations of the Solutions result in $\Gamma_{e^+e^-}$ values between $(30+...) \text{ and } (200+...) \text{ eV}$.

If we assume these modes saturate the $Y(4220)$ decays, we determine the $Y(4220)$ decay branching fractions to the above four modes. For the most interesting mode, $Y(4220) \rightarrow \pi\pi J/\psi$, we obtain $B_{\pi\pi J/\psi} = (12.9 \pm 1.3 \pm 3.9)\%$ (or a partial width of $(7.2 \pm 0.8 \pm 2.2) \text{ MeV}$) for the case of smallest $B \times \Gamma_{e^+e^-}$; and $B_{\pi\pi J/\psi} = (11.2 \pm 1.1 \pm 1.9)\%$ (or a partial width of $(6.3 \pm 0.7 \pm 1.1) \text{ MeV}$) for the case of highest $B \times \Gamma_{e^+e^-}$. In these two particular cases, the branching faction of the $Y(4220) \rightarrow \pi\pi J/\psi$ is very big. We may also calculate the $B_{\pi\pi J/\psi}$ in the most extreme case, i.e., taking the smallest $B \times \Gamma_{e^+e^-}$ for $\pi\pi J/\psi$ and largest $B \times \Gamma_{e^+e^-}$ for other modes, we find $B_{\pi\pi J/\psi} = (2.1 \pm 0.3 \pm 0.7)\%$ (or a partial width of $(1.2 \pm 0.2 \pm 0.4) \text{ MeV}$).

However, the assumption that the $\omega\chi_{c0}$, $\pi^+\pi^- h_c$, $\pi^+\pi^- J/\psi$, and $D^0 D^{*+}\pi^+ + c.c.$ modes saturate the $Y(4220)$ decays may not be true. Being well above the thresholds of many final
states with $\eta_c$, such as $\pi\eta_c$, $\omega\eta_c$, and $\phi\eta_c$, and final states like $\eta h_c$, $\pi\psi(2S)$, and $K\bar{K}J/\psi$, $Y(4220)$ may decay into such final states with substantial rates. In addition, the decays into open charm final states other than $D\bar{D}\pi$ such as $D\bar{D}$, $D\bar{D}^* + c.c.$, $D^*\bar{D}^*$, $D_s^+D_s^-$, $D_s^+D_s^- + c.c.$ are also possible, although the charmed mesons are in relative P-wave. The $Y(4220)$ is very close to the $D^*_sD^*$ threshold, the possible coupling to this mode should also be investigated. Further information on these final states will be important for a deeper understanding of the nature of the $Y(4220)$.

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