A robust $H_{\infty}$-based steering assistance system for the wheeled tractor

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Abstract
Agricultural machine automatic navigation poses great challenge to the precise agricultural technology system nowadays. To this end, this paper proposes a novel steering assistance system (SAS) to assist drivers in the path-tracking. First, the driver steering model is investigated through the driver simulator tests. Combining the wheeled tractor kinematics model, a driver-vehicle model is developed. Then, a polytopic linear parameter-varying (LPV) system is adopted to describe the uncertainties, including time-varying driver model parameters and velocity, in the model, based on which an output-feedback robust controller is developed to ensure robust stability within the polytope space. Moreover, a regional pole placement method is adopted to improve the transient performance of the system. Finally, driver-in-the-loop and field tests conducted to value the controller. The results show the effectiveness of the proposed method to improve the path-tracking performance for the agricultural machine navigation, while reducing the physical and mental workload of drivers. This control method is expected to be a paradigm for the precise navigation system of the agricultural machinery.

Keywords
Agricultural machinery, automatic navigation, an output-feedback robust controller, driver-vehicle model, path tracking

Introduction
Agricultural machine automatic navigation technology is an important technology to realize the precise agriculture.$^{1,2}$ It has been used in the farming, fertilization, spraying,
harvesting and other agricultural production process. There exits two main concerns which would have an effect on the automatic navigation technology. One is the acquisition of the multiple-information. The environment perception technique can be an effective method which can use onboard sensors to collect the environment information and agricultural machine states such as position, heading angle and speed. Mature technology for precise environment perception can depend on the GNSS (Global Navigation Satellite System), such as GPS (Global Positioning System), GLONASS, GALILEO and BDS (Bei Dou Navigation Satellite System). So far, the most common application in the agricultural machine automatic navigation system is RTK-GPS (real-time kinematic GPS) with the cm-level accuracy. O’Connor first applies RTK-GPS to the navigation system of the tractor, which can minimize the tracking error below 2.5 cm for the straight line running. Unal develops a GPS-guided autonomous robot for the detection of soil and plant test processes. In addition, the rise of the machine vision technique is also providing a feasible scheme for the environment detection. Hiremath et al. propose a vision-based farm navigation system for the robots in the field work. Reference designs a machine version algorithm to apply to the small vehicles in the orchard.

The second problem is the accuracy of path-tracking on the basis of precise environment perception. Many studies have been conducted to investigate the path-tracking controller, including model-based and model-independent methods. Bai et al. study the influencing factors of tracking accuracy during the curve-line running and presents a self-tuning model control method for the agricultural machine navigation, which can adjust the control input dynamically. The test results indicates that the curve-line tracking error can be less than 18.57 cm. Reference designs a path-tracking system for the tractor and trailer system based on the nonlinear model predictive control algorithm and conducts the field tests to verify the effectiveness of the proposed control method.

It should be noted that abovementioned methods are depend on the agricultural machinery modeling. Model-independent control method can also be an alternative strategy. PID control and fuzzy control are widely used in the model-independent control. Norremark et al. introduce the PID controller into the wheel speed control of weeding machines. The path-tracking error are 1.6 cm and 2.2 cm during running with a speed of 0.31 m/s and 0.52 m/s. As for the fuzzy control, Kannan et al. present a multi-functional wheeled tractor for automatic ploughing, sowing, and soil moisture sensing. The fuzzy controller is used to change the speed of driving wheels and control the steering angle.

However, the model-independent method is difficult to design the proper controller parameters, which determine the control performance for the system. Moreover, the parameters self-tuning method also requires a large number of experimental resources. Therefore, the design of the controller in this work is based on the wheeled tractor two-wheels kinematics model. Nevertheless, as described in, the kinematics model uses the linear approximation method with assuming that a small heading angle and a constant velocity. These assumptions would cause the weak stability of the control system. The robustness of the system cannot be guaranteed during the varying speeds. Considering that the existing agricultural machines basically rely on the manual operation, we mainly study the driver steering assistance system (SAS) to assist drivers in realizing the precise path-tracking.
With regard to the time-varying parameters in the model, the robust controller have also been used to handle the system uncertainties. Reference designs the active front steering system to maintain the vehicle stability while considering the uncertainty of cornering stiffness. The driver steering angle input is regarded as an external disturbance in. A robust gain-scheduled H∞ control method is proposed to realize the integrated control of active steering system and direct yaw-moment control system for electric vehicles. The tests results show that the proposed method can effectively ensure the vehicle lateral stability under the effect of the external disturbances. Reference develops a Takagi-Sugeno (T-S) fuzzy robust controller to enhance the vehicle handling performance. The extreme steering manipulation demonstrates the superiority of the proposed control method. Furthermore, the driver steering model is investigated and integrated into the vehicle model. A driver-vehicle model is established, based on which a robust controller is designed to assist the drivers in tracking the desired path for handover scenarios. To achieve good cooperation between drivers and steering assistance system, the driver activity is considered in the reference. Then a Takagi-Sugeno (T-S) fuzzy robust controller is designed to design the lane-keep assistance system. In this work, the driver model parameters and vehicle velocity are regarded as the time-varying parameters. The polytopic methodology with finite vertices is used to describe the system uncertainties. Then a linear parameter-varying (LPV) model is reconstructed. LPV method are widely used to handle the model uncertainties and facilitate the controller design.

Assuming that agricultural machines are endowed with the ability to obtain the surrounding environment information and their own states. This paper aims to extend the previous path-tracking work for the agricultural machine automatic navigation. The main contributions of this paper are as follows. First, the driver preview model is used to describe the diver steering maneuver during tracking the desired path. The driver model parameters are identified through the driver tests, based on which a driver-vehicle model is established. Then, an output-feedback robust controller with regional pole placement is presented to design the steering assistance system and improve the path-tracking performance of the agricultural machines. Finally, the simulator and field tests are conducted. The results show the effectiveness of the proposed controller to enhance the path-tracking performance and improve the comfort of the drivers.

The remainder of this paper is organized as follows. “Modeling” develops a driver-wheeled tractor kinematics model. In “Controller design”, a robust H∞ output-feedback controller with regional pole placement is designed. The simulator experiments and field tests are conducted to verify the effectiveness of the proposed control method in “Driver simulator tests”. The conclusions are presented in “Conclusion”.

**Modeling**

The wheeled tractor motion control is dependent on the agricultural machinery modeling. Here, we adopt a wheeled tractor two-wheels kinematics model, which takes take into account the lateral and yaw motion of the wheeled tractor. Then, a single-point preview model is applied to investigate the drivers’ steering manipulation behavior during the path-tracking.
Vehicle kinematic model

A wheeled tractor kinematics model as shown in Figure 1 is applied to the motion control of the vehicle. It can guarantee a feasible path with satisfying kinematic geometric constraints in real driving conditions. We define a system with the state vector. Then wheeled tractor steering kinematic model can be described by:

\[
\dot{\chi} = f(\chi, u) = \begin{bmatrix} \cos \varphi \\ \sin \varphi \\ \tan \delta / l \end{bmatrix} v, \tag{1}
\]

where \( y \) is the lateral position of the agriculture vehicle CG. \( \varphi \) represents the yaw angle between the \( x \)-axis and a reference line on the vehicle inertial frame XOY. \( l \) and \( v \) are the wheel base and vehicle velocity respectively. \( \delta \) represents the front wheel steering angle control input, which consists of the driver’s steering angle input \( \delta_d \) and the SAS’s steering angle input \( \delta_c \).

Driver-vehicle model

In this section, a sing-point preview model, as shown in Figure 2, is introduced to represent the driver’s steering manipulation \( \delta_d \) during the vehicle’s motion in the small-curve path, which can be represented as:

\[
\delta_d = \frac{G}{aT_l s^2 + T_l s + 1} Y_z, \tag{2}
\]

where \( G \) is the steering gain of the driver. \( T_l \) is the driver’s delay time. \( a \) represents a constant value. \( Y_z \) is the deviation of the lateral position between the reference path and the driver’s preview point, which can be given by: \( Y_z = Y_d - (Y + v T_c \varphi) \). \( T_c \) and \( Y_d \) are the preview

![Figure 1](image-url). Wheeled tractor kinematics model.
time of the driver and the desired tracking path, respectively. Considering the identification of the driver parameters, the driver model (2) is discretized first as follows:

$$\delta_d((\eta + 1)\Delta) = \theta_1 \delta_d(\eta \ast \Delta) + \theta_2 \delta_d((\eta - 1)\Delta) + \rho_1 Y_d((\eta - 1)\Delta) + \rho_2 Y((\eta - 1)\Delta) + \rho_3 \phi((\eta - 1)\Delta),$$  \hspace{1cm} (3)

With $\theta_1 = 2 - \frac{\Delta}{aT_l}$, $\theta_2 = \frac{\Delta}{aT_l} - \frac{\Delta^2}{a^2T_l^2} - 1$, $\rho_1 = \rho_2 = \frac{G\Delta^2}{aT_l^2}$, $\rho_3 = \nu T_c G\frac{\Delta^2}{aT_l^2}$

where $\Delta$ is the sampling time of the driver simulator. The parameters $a$, $T_l$ and $G$ are identified through the tests. The driver simulator is shown in Figure 3. The wheeled tractor is established by the Carsim full-vehicle model. The driver can operate the steering

**Figure 2.** Single-preview driver model.

**Figure 3.** Driver simulator tests.
wheel based on the desired path. Then the steering angle can be obtained through the angle sensor and transmit to the Carsim model. The Carsim is embedded into the PXI and can exchange information via the CAN bus. The recursive least squares (RLS) method is adopted to identify the parameters.

Then, combing equations (1) and (2), the diver-in-the-loop model is established based on the identification results. The steering input of the SAS is donated by $\delta_c$.

With assuming that a small yaw angle and constant velocity, the model can be represented as:

$$\dot{x} = Ax + B_1u + B_2w,$$

where $x = [\varphi, Y, \delta_d, \dot{\delta}_d, \int_0^t(Y - Y_t)dt]$, $u = \delta_c$, $w = Y_d$.

$$A = \begin{bmatrix} 0 & 0 & v/l & 0 & 0 \\ v & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0 \\ -vT_c & -1 & 0 & 0 & 0 \end{bmatrix}, B_u = \begin{bmatrix} v/l \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, B_u = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

With $a_{41} = -\frac{GT_c}{aT_i}, a_{42} = -\frac{G}{aT_i}, a_{43} = -\frac{1}{aT_i}, a_{44} = -\frac{1}{aT_i}$.

where $\dot{\delta}_d$ is driver steering angle rate. the vehicle states $\varphi, Y, \delta_d, \dot{\delta}_d, \int_0^t(Y - Y_t)dt$ can be obtained through onboard sensors. These states are treated as the measured states and represented by: $y = C_1x$, with $C_1 = I$. During the controller design, we mainly consider the vehicle path-tracking performance and driver comfort. Thus, the performance output can be defined as $z = [\delta_d, \dot{\delta}_d, Y_z]$ and can be expressed as: $z = C_2x + Dw$.

$$C_2 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0 \\ -vT_c & -1 & 0 & 0 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

### Controller design

The diver-vehicle model can be rewritten as:

$$\begin{cases} \dot{x} = Ax + B_1u + B_2w \\ y = C_1x \\ z = C_2x + Dw \end{cases},$$

A robust output-feedback controller is designed to assist the driver. Considering that the driver parameters can change for different driving conditions, simultaneously the velocity demand is different, a polytopic linear parameter-varying (LPV) method is adopted to rewrite the model (5) and represent the uncertainties set $[T_i, G, v]$ with eight vertices. $G_{\text{min}}, T_{\text{dmin}}$ and $v_{\text{min}}$ represent the minimum values of the driver steering gain $G_h$, delay time $T_d$ and vehicle velocity $v$, respectively, while $G_{\text{max}}, T_{\text{dmax}}$ and $v_{\text{max}}$ are the maximum values for these parameters. Through introducing the method in reference,27
the rectangular region constructed by parameter set \([1/T_i, 1/T_i^2]\) can be simplified as a parallelogram region. Defining two vertices \(R_1\) and \(R_2\) with:

\[
R_1 = \left( \frac{1}{T_{i\min}}, \frac{7T_i^2_{\max} + 2T_i_{\min}T_i_{\max} - T_i^2_{\min}}{8T_i^2_{\min}T_i^2_{\max}} + \frac{(T_i_{\max} - T_i_{\min})^2}{8T_i^2_{\min}T_i^2_{\max}} N(t) \right)
\]

\[
R_2 = \left( \frac{1}{T_{i\max}}, \frac{7T_i^2_{\min} + 2T_i_{\min}T_i_{\max} - T_i^2_{\max}}{8T_i^2_{\min}T_i^2_{\max}} + \frac{(T_i_{\max} - T_i_{\min})^2}{8T_i^2_{\min}T_i^2_{\max}} N(t) \right)
\]

The point \(\ell\) in the parallelogram region can be represented by:

\[
\ell = \omega_1 R_1 + \omega_2 R_2, \text{ with } \omega_1 + \omega_2 = 1
\]

where \(\omega_1 = \frac{T_{i\max} - T_i}{T_{i\max} - T_{i\min}}\), \(\omega_2 = \frac{T_i - T_{i\min}}{T_{i\max} - T_{i\min}}\).

Then the parameter set \([1/T_i, 1/T_i^2]\) can be represented by only two vertices.

The uncertainties of the driver model parameters can be expressed with a rectangular domain as shown in Figure 4. Then, the model (5) can be furthermore represented as a polytopic model:

\[
\begin{align*}
\dot{x} &= \Theta_i(A_ix + B^1_iu + B^2_iw) \\
y &= C^1_ix \\
z &= \Theta_i(C^2_ix + Dw)
\end{align*}
\]  

\(\begin{bmatrix}
i = 1, 2 \ldots 8, \end{bmatrix}\)  

\(\text{(6)}\)

Figure 4. Polytope for the time-varying parameters.
where

\[ \sum_{i=1}^{8} \Theta_i = 1, \Theta_1 = \omega_1 \cdot \frac{G - G_{\min}}{G_{\max} - G_{\min}} \cdot \frac{\nu - \nu_{\min}}{\nu_{\max} - \nu_{\min}}, \Theta_2 = \omega_1 \cdot \frac{G_{\max} - G}{G_{\max} - G_{\min}} \cdot \frac{\nu - \nu_{\min}}{\nu_{\max} - \nu_{\min}}, \]

\[ \Theta_3 = \omega_1 \cdot \frac{G - G_{\min}}{G_{\max} - G_{\min}} \cdot \frac{\nu_{\max} - \nu}{\nu_{\max} - \nu_{\min}}, \Theta_4 = \omega_1 \cdot \frac{G_{\max} - G}{G_{\max} - G_{\min}} \cdot \frac{\nu - \nu_{\min}}{\nu_{\max} - \nu_{\min}}, \]

\[ \Theta_5 = \omega_2 \cdot \frac{G - G_{\min}}{G_{\max} - G_{\min}} \cdot \frac{\nu - \nu_{\min}}{\nu_{\max} - \nu_{\min}}, \Theta_6 = \omega_2 \cdot \frac{G_{\max} - G}{G_{\max} - G_{\min}} \cdot \frac{\nu - \nu_{\min}}{\nu_{\max} - \nu_{\min}}, \]

\[ \Theta_7 = \omega_2 \cdot \frac{G - G_{\min}}{G_{\max} - G_{\min}} \cdot \frac{\nu_{\max} - \nu}{\nu_{\max} - \nu_{\min}}, \Theta_8 = \omega_2 \cdot \frac{G_{\max} - G}{G_{\max} - G_{\min}} \cdot \frac{\nu - \nu_{\min}}{\nu_{\max} - \nu_{\min}}. \]

The parameter matrices in model (6) are given by:

\[ A_1 = A_i(T_{i,\min}, G_{i,\min}, \nu_{\min}), A_2 = A_i(T_{i,\min}, G_{i,\min}, \nu_{\max}), A_3 = A_i(T_{i,\min}, G_{i,\max}, \nu_{\min}), \]
\[ A_4 = A_i(T_{i,\max}, G_{i,\max}, \nu_{\min}), A_5 = A_i(T_{i,\max}, G_{i,\min}, \nu_{\min}), A_6 = A_i(T_{i,\max}, G_{i,\min}, \nu_{\max}), \]
\[ A_7 = A_i(T_{i,\max}, G_{i,\max}, \nu_{\min}), A_8 = A_i(T_{i,\max}, G_{i,\max}, \nu_{\max}), B_1^i = B_3^i = B_4^i = B_7^i = B_8^i (v_{\min}), \]
\[ B_2^i = B_4^i = B_6^i = B_8^i (v_{\max}), \]
\[ C_1^i = C^i(T_{i,\min}, G_{i,\min}, \nu_{\min}), C_2^i = C^i(T_{i,\min}, G_{i,\min}, \nu_{\max}), C_3^i = C^i(T_{i,\min}, G_{i,\max}, \nu_{\min}), \]
\[ C_4^i = C^i(T_{i,\max}, G_{i,\max}, \nu_{\min}), C_5^i = C^i(T_{i,\max}, G_{i,\max}, \nu_{\max}), C_6^i = C^i(T_{i,\max}, G_{i,\max}, \nu_{\max}). \]

In this section, an output-feedback robust controller is developed to assist the driver, so that the close-loop system can be robust to the time-varying model parameters. The output-feedback control law can be written as:

\[
\begin{align*}
\dot{\zeta} &= \hat{A}_{\zeta} + \hat{B}y, \\
u &= \hat{C}_{\zeta} + \hat{D}y
\end{align*}
\]

where \(\zeta\) are the states of the controller. \(\hat{A} \in \mathbb{R}^{5 \times 5}, \hat{B} \in \mathbb{R}^{5 \times 5}, \hat{C} \in \mathbb{R}^{1 \times 5}\) and \(\hat{D} \in \mathbb{R}^{1 \times 5}\) are the undetermined controller matrices. Combining equations (6) and (7), the close-loop system can be expressed as:

\[
\begin{align*}
\dot{\vartheta} &= A_{\Delta} \vartheta + B_{\Delta} w, \\
z &= C_{\Delta} \vartheta + D_{\Delta} w
\end{align*}
\]

where \(\vartheta = \begin{bmatrix} x \\ \zeta \end{bmatrix}, A_{\Delta} = \begin{bmatrix} A_i + B_i^j \hat{D}C_1 & B_i^j \hat{C} \\ \hat{B}C_1 \end{bmatrix}, B_{\Delta} = \begin{bmatrix} B_i^2 \\ 0 \end{bmatrix}, C_{\Delta} = [C_i^2 0].\) The reference path \(Y_p\) is treated as the external disturbance for the close-loop system. The design of the controller aims to attenuate the disturbance on the system performance. The H\(\infty\) performance of the system can be represented as:

\[
\|z\|_2 \leq \gamma \|w\|_2.
\]

where \(\gamma\) can represent the degree of the interference suppression. Through minimizing \(\gamma\), the disturbance can be mitigated. Then the controller matrices \(\hat{A}, \hat{B}, \hat{C}\) and \(\hat{D}\) are solved to
guarantee the performance while satisfying the asymptotically stable. The Lemma 1 is introduced.

**Lemma 1:** The close-up system is stable and the $H_\infty$ performance can be guaranteed if and only if a positive matrix $\Psi$ that can satisfy the following inequality:

$$
\begin{bmatrix}
\text{dua}\{A_\Delta^T\Psi\} & \Psi B_\Delta & C_\Delta^T \\
\ast & -\gamma I & D^T \\
\ast & \ast & -\gamma I
\end{bmatrix} < 0,
$$

(10)

where $\text{dua}(\Phi) = \Phi^T + \Phi$. $\ast$ is the symmetric matrix. The matrix $\Psi$ and its inverse matrix are subject to the blocks as follows:

$$
\Psi = \begin{bmatrix}
\tilde{Y} & \tilde{N} \\
\tilde{N}^T & \tilde{W}
\end{bmatrix},
\Psi^{-1} = \begin{bmatrix}
\tilde{X} & \tilde{M} \\
\tilde{M}^T & \tilde{Z}
\end{bmatrix},
$$

(11)

Then we can obtain

$$
\Psi \begin{bmatrix}
\tilde{X} & I \\
\tilde{M}^T & 0
\end{bmatrix} = \begin{bmatrix}
I & \tilde{Y} \\
0 & \tilde{N}^T
\end{bmatrix}
$$

By defining:

$$
F_1 = \begin{bmatrix}
\tilde{X} & I \\
\tilde{M}^T & 0
\end{bmatrix},
F_2 = \begin{bmatrix}
I & \tilde{Y} \\
0 & \tilde{N}^T
\end{bmatrix}
$$

(12)

Through multiplying by a congruence transformation to (10) with $\text{diag} \{F_1^T, I, I\}$. We can get the following equivalent matrix equation (13).

$$
\begin{bmatrix}
\text{dua}\{AX + B_1\tilde{C}\} & \tilde{A}^T + (A + B_1\tilde{D}\tilde{C}_1) & B_2 & \left(C_2\tilde{X}\right)^T \\
\ast & \text{dua}\{YA + \tilde{B}\tilde{C}_1\} & \tilde{Y}B_2 & C_2^T \\
\ast & \ast & -\gamma I & D^T \\
\ast & \ast & \ast & -\gamma I
\end{bmatrix} < 0,
$$

(13)

where

$$
\tilde{A} = \tilde{Y}(A + B_1\tilde{D}\tilde{C}_1)\tilde{X} + \tilde{N}\tilde{B}\tilde{C}_1\tilde{X} + \tilde{Y}B_u\tilde{C}\tilde{M}^T + \tilde{N}\tilde{A}\tilde{M}^T, \tilde{B} = \tilde{Y}, B_1\tilde{D} + \tilde{N}\tilde{B}, \\
\tilde{C} = \tilde{D}\tilde{C}_1\tilde{X} + \tilde{C}\tilde{M}^T, \tilde{D} = \tilde{D}.
$$

In addition, the inequality (14) can be included as a constraint for the controller design:

$$
\begin{bmatrix}
\tilde{X} & I \\
I & \tilde{Y}
\end{bmatrix} > 0,
$$

(14)

Through the Schur complement, it can be included that $\tilde{Y} > 0$ and $I - \tilde{X}\tilde{Y} > 0$. The matrixes $\tilde{M}$ and $\tilde{N}$ can be obtained first by the singular value decomposition:
Then the controller parameter matrices can be calculated by:

\[
\begin{align*}
\hat{D} &= \tilde{D} \\
\hat{C} &= (\tilde{C} - \hat{D}C_1\tilde{X})(\tilde{M}^T)^{-1} \\
\hat{B} &= \tilde{N}^{-1}(\tilde{B} - \tilde{Y}B_1\hat{D}) \\
\hat{A} &= \tilde{N}^{-1}\left[\hat{A} - \tilde{\gamma}(A' + B_1\hat{D}C_1)\tilde{X}\right](\tilde{M}^T)^{-1} \\
-\hat{B}C_1\tilde{X}(\tilde{M}^T)^{-1} - \tilde{N}^{-1}\tilde{Y}B_1\hat{C}
\end{align*}
\]

Furthermore, to guarantee the transient performance of the controller, a pole placement method is adopted in this work to assign the poles of the system on the designed region. Due to the disturbance and uncertainties of the model, the precise pole placement is not feasible for the controller design. Thus, as described in,\textsuperscript{20} a regional pole placement method is more effective. Here, the disk region $D(-\rho, \Re)$ is selected as the LMI region, which represents a circle with a center point $(-\rho, 0)$ and radius $\Re$. Through assigning the poles on the desired left half plane of axis, the transient performance can be guaranteed. The Lemma 2 is introduced here to complete the pole placement.

**Lemma 2:** For a matrix $A$, the eigenvalues are contained in the region $D(-\rho, \Re)$ if satisfying the following equation:

\[
\begin{bmatrix}
-\Re & \rho \Psi + \Psi A \\
* & -\Re \Psi
\end{bmatrix} < 0, \tag{16}
\]

Then multiplying a congruence transformation to the equation (16) with $\text{diag}\{F_1^T, F_1^T\}$. The LMI condition can be obtained as follows:

\[
\begin{bmatrix}
-\Re \tilde{X} & -\Re I & A\tilde{X} + B_1\tilde{C} + \rho \tilde{X} & A + B_1\hat{D}C_1 + \rho I \\
* & -\Re \tilde{Y} & \hat{A} + \rho I & \hat{Y}A + \hat{B}C_1 + \rho \tilde{Y} \\
* & * & -\Re \tilde{X} & -\Re I \\
* & * & * & -\Re \tilde{Y}
\end{bmatrix} < 0, \tag{17}
\]

According to the Lemma 1 and Lemma 2, a robust output-feedback controller is proposed for the polytopic model (10).
**Theorem 1**: For the close-loop system (8), the $H_\infty$ performance and transient stability can be guaranteed if and only if the equations (13) and (17) hold at each vertex, where $i = 1, 2, \ldots, 8$.

\[
\begin{bmatrix}
\text{dua}\left\{\hat{A}_i \hat{X} + B_1^i \hat{C}\right\} & \hat{A}^T + (A_i + B_1^i \hat{D}C_1) & B_i^2 (C_i^2 \hat{X})^T \\
* & \text{dua}\left\{Y A + \hat{B}C_1\right\} & \hat{Y} B_i^2 (C_i^2)^T \\
* & * & -\gamma I \quad \hat{D}^T \\
* & * & * & -\gamma I
\end{bmatrix} < 0, \quad (18)
\]

\[
\begin{bmatrix}
-\Re \hat{X} & -\Re I & A_i \hat{X} + B_1^i \hat{C} + \rho \hat{X} & A_i + B_1^i \hat{D}C_1 + \rho I \\
* & -\Re Y & \hat{A} + \rho I & \hat{Y} A_i + \hat{B}C_1 + \rho \hat{Y} \\
* & * & -\Re \hat{X} & -\Re I \\
* & * & * & -\Re \hat{Y}
\end{bmatrix} < 0, \quad (19)
\]
The controller parameter matrices are:

\[
\begin{align*}
\hat{D}_i &= \tilde{D}_i \\
\hat{C}_i &= \left(\tilde{C}_i - \hat{D}_i C_i \tilde{X}_i\right)\left(\tilde{M}_i \tilde{X}_i\right)^{-1} \\
\hat{B}_i &= \tilde{N}_i^{-1}\left(\tilde{B}_i - \tilde{Y}_i B_i D_i\right) \\
\hat{A}_i &= \tilde{N}_i^{-1}\left[\tilde{A}_i - \tilde{Y}_i (A_i + B_i \hat{D}_i C_i \tilde{X}_i)\right]\left(\tilde{M}_i \tilde{X}_i\right)^{-1} \\
-\hat{B}_i &C_i \tilde{X}_i \left(\tilde{M}_i \tilde{X}_i\right)^{-1} - \tilde{N}_i^{-1} \tilde{Y}_i B_i \hat{C}_i
\end{align*}
\]

where \(\tilde{M}_i \tilde{N}_i^T = I - \tilde{X}_i \tilde{Y}_i\). The controller parameter matrices are given by:

\[
\hat{A} = \sum_{i=1}^{8} \Theta_i \hat{A}_i, \hat{B} = \sum_{i=1}^{8} \Theta_i \hat{B}_i, \hat{C} = \sum_{i=1}^{8} \Theta_i \hat{C}_i, \hat{D} = \sum_{i=1}^{8} \Theta_i \hat{D}_i.
\]

The performance index \(\gamma\) can be minimized by the LMI toolbox.

**Driver simulator tests**

In this section, the driver simulator tests are conducted to verify the effectiveness of the control method based on a full vehicle model. During the tests, the driver can operate the steering wheel according to the desired path. Then the steering angle information are transmitted to the Dspace, based on which the robust controller was used to calculate the assistance steering angle. Finally, the total steering angle can send to the vehicle model which is established the Carsim software. The real test platform is given in Figure 2. Figure 5 is the experimental set-up, where the steering motor is based on the PID controller. The master controller MicroAutobox is used to compile the proposed control method. The NI/Labview is adopted to show the animations. The control diagram is shown in Figure 6. A PID control is also set as a comparison test to verify the superiority of the proposed controller. Moreover, the field test is conducted to verify the proposed control method.

![Figure 6. Control diagram of the steering assistance system.](image-url)
Before the tests, the driver model parameter are required to identify first A skilled driver was trained through the driver simulator. A small-curve path is used to implement the identification tests. The identification results of the driver is shown in Figure 7. It can be seen that the RLS method is effective to identify the driver parameters.

Then, the performance indices are defined as: $J_1 = \int \| \delta_d \|^2$ and $J_2 = \int \| \dot{\delta}_d \|^2$, which are used to describe the driver physical workload and mental workload, respectively. The

![Figure 7. Driver identification results.](image)

![Figure 8. Poles position of the system.](image)
poles of the system with and without the controller are given in Figure 8 respectively, in which the x-axis and y-axis represent the real axis and imaginary axis of the plane. The disk plane in the tests is set with $D(-10,8)$. It can be seen that all poles can be in the desired position, thereby improving the transient performance of the system on the basis of stability. The two different scenarios are set to verify the control method.

Figure 9. Reference path of the scenario 1.

Figure 10. Lateral deviation for scenario 1.
Scenario 1 test

In this section, a single-lane-change manoeuvre is set to verify the control performance during a machine vehicle runs from straight-line to the curve-line. The reference path and path-tracking error are shown in Figure 9 and Figure 10, respectively. It is clear that the path-tracking performance of the machine vehicle can be improved with both two controllers. However, the proposed robust controller can be better than PID control. It can be seen that the absolute value of maximum deviation between the machine vehicle lateral position and desired path would be 10 cm approximately without the proposed control method at 5~10 s. It is because the driver may not keep a preview well on the desired path during the lane-change from the straight-line to the curve-line. In case introducing the proposed controller into the system, the absolute value of maximum deviation can be reduced to 2 cm, while 6 cm for PID control. It shows that the path-tracking performance of the robust control excels the PID control. This is because the driver model through the identification can match the real driver characteristics, based on which the robust control method can provide a proper steering angle to assist the wheeled tractor in the path-tracking. This is beneficial to prompt the precise agricultural system. PID algorithm is not a model-based control method. It would be

| Indices          | Physical load | Mental load |
|------------------|---------------|-------------|
| Without controller | 2.56          | 3.5         |
| With controller  | 2.3           | 2.4         |

Table 1. Physical and mental workload of drivers for scenario 1.

![Reference path](image)

Figure II. Reference path of the scenario 2.
difficult to adjust control parameters based on the engineering experience and achieve a better control performance. In addition, the quantified driver performance indices including physical workload $J_1 = \int \| \delta_d \|^2$ and mental workload $J_2 = \int \| \dot{\delta}_d \|^2$ are given in Table 1 with and without the proposed control method. Here, the driver steering angle $\delta_d$ and steering angle rate $\dot{\delta}_d$ are collected from the simulator tests and used to calculate the workloads. It is obviously that two workloads of drivers can be reduced with the proposed control method, in which the physical load can be reduced by 26.86% and mental load is reduced by 4.2%. This is due to the role of the steering assistance system.

**Scenario 2 test**

Considering the complex condition in the farm work for the agricultural machines, a snake-like line is set to further test the control method. The reference path and path-tracking error are shown in Figure 11 and Figure 12, respectively. As can be seen, the lateral error obviously increase without the control compared to the single-lane-change scenario, while this can be reduced by applying the control. The proposed robust control has a better performance to track the reference path compared to the PID control.

### Table 2. Physical and mental workload of drivers for scenario 2.

| Indices             | Physical load | Mental load |
|---------------------|---------------|-------------|
| Without controller  | 9.6           | 15.4        |
| With controller     | 7.9           | 11.5        |
The absolute maximum value of the later deviation can be reduced by 75% with the robust control, while 33.3% with the PID control. Additionally, the driver performance indices during the agricultural machine’s running are given in Table 2. As can be seen, the driver workload can also be reduced under the effect of the steering assistance system, in which the physical load $J_1 = \int \| \delta_d \|^2$ is reduced by 37.6% and $J_2 = \int \| \dot{\delta}_d \|^2$ is reduced by 31.3%.

**Figure 13.** Set-up of the wheeled tractor.

**Figure 14.** Lateral deviation with and without the controller.
Field test

In this section, a wheeled tractor as shown in Figure 13 is used to verify the proposed controller. The GPS is used to obtain the reference path. Industrial Personal Computer (IPC) is in the charge of controlling the motor. The proposed control method is embedded into the Dspace. A straight-line scenario is set to be tracked by a driver. The tracking performance without and with the proposed controller is shown in Figure 14. It is clear that the maximum deviation without the controller can be up to 1 m during path-tracking. It is unfavorable for precise agriculture. As can be seen from the results, the deviation can be reduced significantly with the proposed controller. This can also demonstrate that the proposed controller is effective to improve the path-tracking performance.

Conclusion

In this paper, a steering assistance system for the wheeled tractors is designed to assist drivers in path-tracking. To facilitate the controller design, a driver-machine vehicle model is established first. Considering the uncertainties of driver parameters and vehicle velocity in the model, a linear parameter-varying (LPV) method is employed to reconstruct the model, in which the uncertainties are handled by the polytopic methodology with finite vertices. Then an output-feedback robust control method is proposed to improve the path-tracking accuracy and simultaneously reduce the workloads of drivers. The system transient performance can also be guaranteed through the regional poles assignment. The driver simulator testes with different scenarios are implemented to verify the effectiveness. The results show that the proposed control method can reduce the lateral deviation for path-tracking while ensuring the comfort of drivers. A comparative test with the PID control demonstrates the superiority of the proposed control method. The field tests also validate that the proposed controller can improve the path-tracking performance.

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