Solving large scale unit dilemma in electricity system by applying commutative law

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Abstract. The conventional system, pooling resources with large centralized power plant interconnected as a network, provides a lot of advantages compared to the isolated one including optimizing efficiency and reliability. However, such a large plant need a huge capital. In addition, more problems emerged to hinder the construction of big power plant as well as its associated transmission lines. By applying commutative law of math, ab = ba, for all a, b ∈ R, the problem associated with conventional system as depicted above, can be reduced. The idea of having small unit but many power plants, namely “Listrik Kerakyatan,” abbreviated as LK provides both social and environmental benefits that could be capitalized by using proper assumption. This study compares the cost and benefit of LK to those of conventional system, using simulation method to prove that LK offers alternative solutions to answer many problems associated with the large system. Commutative Law of Algebra can be used as a simple mathematical model to analyze whether the LK system as an eco-friendly distributed generation can be applied to solve various problems associated with a large scale conventional system. The result of simulation shows that LK provides more value if its plants operate in less than 11 hours as peaker power plant or load follow power plant to improve load curve balance of the power system. The result of simulation indicates that the investment cost of LK plant should be optimized in order to minimize the plant investment cost. This study indicates that the benefit of economies of scale principle does not always apply to every condition, particularly if the portion of intangible cost and benefit is relatively high.

1. Introduction
Many power system in the world still applies conventional system, pooling resources with large centralized power plant interconnected as a network. From this network, power then delivers to the customers using transmission and distribution lines. Interconnected system provides a lot of advantages compared to the isolated one including intermingling power, variety of loads and access to a variety of power resources. Until recently, we are still relying on conventional system, which aimed at optimizing efficiency and reliability. However, such a large plant need a huge capital that should be borrowed from financial institution with complicated lender guidelines. In addition, more problems emerged to hinder the construction of big power plant as well as its associated transmission lines. Those problems include land acquisition, right of way permit, environmental as well as social obligation. The above challenges have resulted in the slippage of many big power plants and transmission lines projects in Indonesia. The fact described above becomes the prognosis of this paper:
The overall cost due to the problems risen in constructing large power plant under the conventional system may offset its benefit coming from its economies of scale and interconnection system.

Mathematical approach has potential benefit for problem solving if we could use it beyond the routine acquisition of isolated techniques and procedures [1]–[4]. Furthermore, mathematical can be considered as a social process [5] as reported by some studies. For instance, Carraher et al. [6] studied the relations between mathematic and cultural activities in Brazil and Cobb [7] studied the sociocultural perspectives on mathematical development.

Under the micro scientific view, electricity concept cannot be separated from mathematical theory, from complex number to analyze electric circuit to Fourier and Laplace Transform to analyze magnetic field and wave propagation [8], [9]. However, for the macro view in the real world, there is very rare studies that use mathematical approach to solve the real world problem, such as a large power system dilemma, a focal issue in this study. This paper will show that such problem can be analyzed and solved by applying simple mathematical principle, known as commutative law of math, \( ab = ba \), for all \( a, b \) [10]. Commutative algebra has been known for a long time [11] and some author call it as commutative property [12].

By using commutative law, the problem associated with conventional system as depicted above, can be reduced as follows. Such principle can be illustrated as an algebraic expression that \( 1000 \times 1 = 1 \times 1000 \), which means that “to build one large unit plant of 1000 MW equals to building 1000 small units of 1 MW. The idea of having small unit but many power plants in this paper is relevant to the recent initiative of Sekolah Tinggi Teknik PLN, Jakarta, namely “Listrik Kerakyatan” abbreviated as LK. This initiative offers an option to avoid problems that challenge conventional system by adopting the concept of small scale distributed generation, which plants are built scattered closed to the customer side. Distributed generation provides benefits including voltage improvement and stability and could reduce power losses [13]. For example, Arunachalam et.al. reported the implementation of Decentralization Distributed Generation in India [14] and [15] reported that distributed generation may increase energy security and reduce carbon pollution. In addition, LK creates opportunity for local people to own and manage electricity their selves using available and easy renewable energy resources.

This study conduct cost and benefit analysis to compare the tangible and intangible value of large scale centralized conventional system with those of Listrik Kerakyatan by using simulation.

### 2. The conceptual framework of LK

![Figure 1. Conventional system vs distributed generation](image)

As illustrated in Figure 2, it takes around five years serial process to build a large scale power plants such as for 1000 MW Coal Fired Plant, much longer than building a small LK unit, which takes less than six months and can be constructed simultaneously as many numbers we want.
3. The benefit of LK

The small size of LK provides opportunity for local business players to own small scale power plant or waste to energy plant because the investment cost per package of LK is relatively low, less than one billion rupiah for the unit size of 30 kW to 50 kW. Such small size plant also needs only small area (200-1000 m²) and flexible to be located anywhere around school, housing, market, mall, hotel, and office. For remote areas that have no electricity, LK can be managed locally as a hybrid system. Technically, LK will reduce the transmission losses and the problem of excessive transformer overload and drop voltage. From the energy balance aspect, LK offers the additional renewable energy capacity to the system without any transmission lines construction.

On top of the above advantages, LK will promote to maximize the use of local material and manpower especially for biomass power plant. LK may create energy security because very little dependency to the import material and expertise.

If LK can be risen up to the national scale, the government target to have 100 percent electrification ratio and the target of national energy policy to have 23 percent renewable energy by 2025 can be achieved in less than three years. Unlike, the giant project that needs foreign funding, LK can be funded by local people equity and national bank with special interest for small and medium business scheme.

For the big city, the biomass plant of LK offers the option to apply zero waste concept without any truck go to landfill. For the remote areas that are still lack of electricity and cannot be reached by the large size project, LK offers the option to have their own power plant to manage and serve their own community using hybrid of Solar and Biomass plant.

4. Power plant type of LK

LK is a hybrid power plants that can be mixed of Solar PV, Biogas, Biomass, or Wind as main sources. If we want to improve reliability, battery backup or diesel engine may be added based on the requirement. Micro hydro may be also included if the resource is available near the area. STT PLN has conducted trial for LK model at two locations, LK 1 at Pondok Kopi district East Jakarta and LK2 in STT PLN Campus Duri Kosambi West Jakarta. The hybrid for trial consists of solar PV and biomass plant using municipal waste.

5. From waste to energy

Municipal waste can be converted to thermal and electrical energy by using dual processes, digestion and briquetting.
5.1. Anaerobic digesting
The food waste and any degradable organic waste is putting into the digester and convert into methan gas and leachate through Anaerobic digestation process. This is the first stage of zero waste process that can eliminate the bad smell of waste. This process can eliminate the waste odor and produce leachate that can be used as an alternative enzyme required for “peuyeumisasi” (composting process for the rest of mixed waste). This process also produce methan gas that can be used for cooking or generate electricity.

5.2. Bricketting peuyeumisasi
After digesting process, there are still more than 50 percent of waste remaining, not taking into account about 30%-40% of usable waste such as plastic bottle, can, and other valuable material. This remaining waste can be converted into bricket through the process similar to composting, called “peuyeumisasi.”. This process will dry the waste and reduce the waste volume and eliminate the waste odor. It takes 10 days to proceed peuyeumisasi, that will be converted further into bricket or pellet form using shredder machine.

Fresh domestic or municipal waste typically has low caloric value around 1000 kcal/kg waste. The process of peuyeumisasi may increase the caloric value to the range of 2400 kcal/kg to 3000 kcal/kg. The gasifier unit is used to converted waste “pellet” or waste bricket into syngas as fuel to run the generator set engine.

5.3. The drawback of LK
There are some disadvantages of LK or distributed generation as a consequence of a huge number and scattered plans that should be supported by a proper control and monitoring system, which means additional capital for control and information technology support facilities. The second drawback is the lack of supporting regulation or incentive for small scale, people owned generation, kind of small scale IPP. Although LK provides a lot of benefit for clean environment and additional income for local people, there is no legal base to provide compensation for such social and environmental benefit. The minister decree for Renewable Energy Feed in Tariff does not count such intangible benefit for the nation and this rule is applied for high level IPP investor, not suitable for local people. Another drawback of LK is the lack of skilled labour, particularly for renewable energy power plants. Moreover, there is additional cost related to improve reliability and stability of LK by putting battery for stabilizer, which price is still very expensive.

6. Pilot projects
STT PLN has undertaken pilot projects of LK in Duri Kosambi Campus (LK2) and in public space at Pondok Kopi Village (LK 1), aimed at processing one ton of municipal waste using dual process of digesting and bricketing. The waste volume in each location is 1 (one) ton of mixed waste per day and the PV total capacity of the two locations is 13 kW. The actual expenses spent for LK 1 and LK 2 will be used for further calculation in this paper. The capital expenditure (capex) and operational expenditure (opex) of the two locations are shown below:

- Building and facilities : IDR 198,353,540
- Anaerobic digestion facilities : IDR 56,965,000
- Bricketting organic waste facilities (Peuyeumisasi) : IDR 482,250,000
- Gasification and genset facilities : IDR 152,483,100
- PV and installation facilities : IDR 246,613,400
- Total Capex of LK2 at STT PLN Campus : IDR 1,136,715.040; Eq. US$: 87,500
- The annual opex of LK1 Pondok Kopi : IDR 148,800.000; Eq. US$: 11,500
- The annual opex of LK2 at STT PLN Campus : IDR 74,400.000; Eq. US$: 5,700
7. Products and revenue projection from LK1 and LK2

After a year trial at two locations has resulted in various products that may potentially make money including waste treatment or tipping fee, valuable waste material (e.g. bottle, can, wood, metal), brick for fuel, organic fertilizer, and income from selling electrical energy, as shown in Table 1. In addition, LK will provide social benefit as more local people involved in LK related business. LK will also render environmental benefits because LK applies a zero waste concept that can reduce huge number of waste transportation from communities to landfill area. More over, LK will save fossil fuel as natural resources and contribute to the global green environment program.

7.1. Perspective Revenue of LK

LK will be sustained if it is treated as a proper business model that should have secured revenue that are potentially can be coming from the following products and services:

a) Waste treatment or tipping fee, which due should be paid by local communities or by government or combination of both ways. For the purpose of simulation in this paper, the tipping fee is IDR 200,000 per ton of waste, which number is realistic because it is much lower than the current provincial waste treatment spending in Jakarta (IDR 325,000/ton). The local government may get budget saving and environmental bonus as LK will make less and less truck transporting waste to the Landfill area.

b) Income from selling usable and valuable material (bottle, can, cartoon, wood, etc), which this kind of business has been done by ordinary people for decades. Based on observation during one year trial, the potential quantity of this material is 40% of total waste. We pick up a moderate number of IDR 1000 per ton of material, which number is the average actual value between the minimum IDR 600 and IDR 3600.

c) Organic fertilizer as by product of anaerobic digesting process for digestable waste or composting process of organic waste. We pick a number IDR 5000 per litre from the market price of similar product.

d) Although energy sales can be a good potential income, but there is no supporting regulation available that provide suitable Feed in Tariff (FIT) for people owned small scale renewable in Indonesia. Currently, the existing rule is applied for big IPP (independent power producer) investors and the FIT is very low, only less than 10 cents US$, same for every kind of renewable energy. For the purpose of simulation, this paper will use the FIT for Java System, which is IDR 867,- equal to 7 cents US$.

e) Energy from biogas (200 kg degradable waste) is around 6 kWh per day @ IDR 867 /kWh

f) Energy from biomass (480 kg organic waste x 0.75 kWh) is around 435 kWh per day @ IDR 867/kWh

| Table 1. LK 1 and LK 2 revenue estimation from 2 ton of waste |
|-------------------------------------------------------------|
| Revenue Projections of LK 1 & LK 2                          | Total per period | Number/year | Unit     | Total per year |
| Estimated Tipping Fee: 2 ton @ IDR 200,000/ton              | IDR 400,000      | 365         | day      | IDR 146,000,000 |
| Valuable waste @ IDR 1000 per kg x 400 kg                  | IDR 400,000      | 365         | day      | IDR 146,000,000 |
| Organic fertilizer (@IDR 5000 x 2000 liter/month)          | IDR 10,000,000   | 12          | month    | IDR 120,000,000 |
| Biogas energy from degradable waste (6kWh/day) @ IDR 867/kWh| IDR 4,482        | 365         | day      | IDR 1,635,930   |
| Biomass energy from bricket gasifier plant (435 kWh/day @ IDR 867/kWh) | IDR 537,840      | 365         | day      | IDR 196,311,600 |
| PV Solar 13 kWp @ 4hr x IDR 867/kWh                        | IDR 60,690       | 365         | day      | IDR 22,151,850   |
7.2. Specific cost and revenue of LK

Based on the result from pilot project of LK1 and LK2, we can calculate the specific cost per item of work as shown in Table 2. The calculation has considered some additional equipment required to improve the production and quality.

| Item                                | Unit cost (IDR) | Unit         | Eq.US$ |
|-------------------------------------|-----------------|--------------|--------|
| Civil work                          | 800,000         | IDR/m²       | 62 US$/m² |
| Biogas unit                         | 615,225         | IDR/kg       | 47 US$/kg |
| Bricketing unit without gasifier    | 219,650,000     | IDR/ton      | 16896 US$ |
| Gasifier unit                       | 10,516,076      | IDR/kW       | 809 US$/kW |
| PV unit                             | 21,800,303      | IDR/kWp      | 1,677 US$/kWp |

**Table 2. Specific cost of LK**

| Operational Cost                     |                   |              |
|--------------------------------------|-------------------|--------------|
| Supervisor                           | 60,000,000        | IDR/year     | 4,615 US$/year |
| Operator                             | 38,400,000        | IDR/year     | 2,954 /year    |
| Operation and maintenance material   | 15,000,000        | IDR/year     |              |
| Over head                            | 10,000,000        | IDR/unit office | 769 US$/unit |

Revenue is estimated based on available data and market price of similar product and service as follow:

a) Valuable waste (bottle, can, paper, etc) estimate volume is 40% of the total waste and picking the smallest market price of similar products (IDR 500 /kg). If the volume of waste is 1 ton, it may generate revenue: 40% x 1000 x 500 = IDR 200,000/day

b) Tipping fee is assumed IDR 200,000/ton, much lower compare to the existing cost in Jakarta (IDR 325,000 per ton)

c) Organic fertilizer IDR 1,000 /litre, much lower than market price of similar product.

d) Electric energy (kWh) is calculated using minister decree: For Jawa/Bali, the production cost (BPP) is IDR 867 /kWh. Energy from biogas organic 2 kWh per ton waste, and for bricket may produce: 480kg/ton x 0.75 kWh/kg = 216 kWh/ton

e) If PV installed, additional revenue will be get from PLN: 4 hour time Capacity kWp).

| Table 3. Specific revenue of LK per ton per day |
|-----------------------------------------------|---------|---------|
| Revenue item                                  | Per Day (IDR) | Per Year (IDR) | Eq. US$/year* |
| Valuable waste IDIR per ton/day               | 200,000  | 73,000,000 | 5615          |
| Tipping fee /ton/day                          | 100,000  | 36,500,000 | 2,808         |
| Organic fertilizer /liter                     | 1,000    | 365,000   | 28            |
| kWh digester/ton                              | 2        | 639,480   | 49            |
| kWh Briket/ton                                | 216      | 69,063,840| 5,313         |
| Income from waste/ton/year                    | 181,486,760| 181,486,760| 13,961        |
| Additional rev from PV per year               | 3,836,880| 295       |
| Total income for 1 ton waste                  | 191,078,960| 14,698   |

7.3. Specific cost per ton of waste

Specific cost per ton of waste is helpful to get a rough estimation of LK project investment as a preliminary figure before moving to further feasibility study. One of LK criteria depicts that the unit LK should be small enough in order to limit the investment requirement and local people affordability. Therefore, we propose the largest LK package capacity is 10 ton per day. Table 4 shows ten typical cost of LK from the lowest one ton to the largest 10 tons.
Table 4. Typical specific cost per ton of waste

| Volume (ton/day) | Specific cost (USD) | Capex (USD) | Opex (USD) | kWh/day | Capacity (kW) | Op Hr/day |
|-----------------|---------------------|-------------|------------|---------|--------------|-----------|
| 1               | 31,000              | 31,000      | 7,185      | 144     | 25           | 6         |
| 2               | 31,000              | 62,000      | 10,585     | 288     | 45           | 6         |
| 3               | 24,800              | 74,360      | 14,435     | 432     | 75           | 6         |
| 4               | 24,800              | 99,150      | 18,285     | 576     | 29           | 6         |
| 5               | 24,800              | 123,940     | 22,870     | 720     | 120          | 6         |
| 6               | 24,800              | 146,730     | 27,750     | 864     | 150          | 6         |
| 7               | 24,800              | 173,520     | 30,860     | 1,008   | 175          | 6         |
| 8               | 24,800              | 198,310     | 36,927     | 1,152   | 200          | 6         |
| 9               | 24,800              | 223,100     | 41,515     | 1,296   | 225          | 6         |
| 10              | 22,320              | 223,156     | 47,546     | 1,440   | 250          | 6         |

8. LK packages and financial simulation

One of LK criteria depicts that the unit LK should be small enough in order to limit the investment requirement and local people affordability. Therefore, we propose six categories of small size of LK capacity from the smallest 1 (one) ton per day to the largest LK package of 10 ton per day, the size of which is based on waste volume in each location. Table 5 shows the simulation result of financial indicators for each category including payback period, Net Present Value (NPV), Internal Rate of Return, Electricity Product and Capacity.

\[
NPV = -C_0 + \sum_{i=1}^{T} \frac{C_i}{(1 + r)^i}
\]  

Where IRR is the value of i if NPV = 0, Energy is electrical energy (E), which is power (P) or LK capacity multiplies by time E (kWh) = P (kW) x h (hours).

Table 5. NPV, IRR, and payback period of five module packages (10 years basis)

| Pack, Modul | Volume (ton/day) | Payback Period | Net Present Value (USD) | Internal Rate of Return* | Energy kWh/day | Capacity (kW)** |
|-------------|-----------------|----------------|-------------------------|--------------------------|----------------|-----------------|
| I           | 1               | 5 years        | 8.352                   | 5.2%                     | 144            | 25              |
| II          | 2               | 4 years        | 38.088                  | 11.9%                    | 288            | 50              |
| III         | 3               | 3 years        | 83.919                  | 21%                      | 432            | 75              |
| IV          | 4               | 3 years        | 117.325                 | 22%                      | 576            | 100             |
| V           | 5               | 3 years        | 146.558                 | 22%                      | 720            | 120             |
| VI          | 10              | 3 years        | 307.673                 | 26%                      | 1,440          | 250             |

9. Intangible benefit

In addition to the real revenue as written above, LK provides intangible benefit in the form of social benefit and environmental benefit. Social benefit can be earned from people opportunity to get income and wellbeing improvement under the zero waste milieu. Environmental benefit can be valued from fossil fuel cost saving and carbon reduction [16]–[18]. For the purpose of simulation in this paper, the social benefit that will be taken into consideration is potential additional income tax, both from LK as company and from thousands of individuals who get new jobs as LK employees.

9.1. Cost benefit comparison between conventional system and LK system
Comparing to LK distributed generation system, the conventional system (CS) has two main merits. First, CS is cheaper in price per unit capacity due to the principle of economies of scale [19]. Second, CS has much better reliability than LK because of interconnection system advantage. However, CS has to incur at least three additional indirect costs. First, CS plants construction time (around 5 years) is much longer than those of LK plants (6 months). Second, the high cost of transmission lines construction, around USD 114/kW to USD227/kW [20]. Third, large scale fossil fuel power plant construction time is much longer than LK and even worse, most of the big size power plants construction are facing many problems that cause delay or project slippage. Delay means cost due to the loss of opportunity to get income from energy services.

On the other hand, LK offers three benefits that may answers problems of CS system. First, land acquisition problem is not a big issue in LK since LK capacity is small and the plants can be scattered wherever the land available. Second, LK construction time is less than a year, much shorter than CS that takes around five years. Third, LK power plants does not require a high cost transmission lines because it can be connected directly to the low voltage or medium voltage distribution lines. In addition to the three advantages described above, LK provides social and environmental benefits. Among many social benefits, for simple simulation in this paper, we will take the social benefit derived from the increasing start up businesses and job opportunities for local people. This intangible benefit is capitalized in term of additional tax revenue, both from company and individual taxes. The environmental benefit is derived from the less CO2 pollution that will be capitalized using equivalent carbon reduction (CDM) formula. Despite its benefits, there are two disadvantages that may need additional costs related to LK, higher per unit capital expenditure and cost for control and monitoring a huge numbers and scattered LK power plants. The cost-benefit comparison between LK and CS can be represented in simple mathematical expression namely Delta Value as follows:

\[
\Delta V_{LK-CS} = \Delta B_{LK-CS} - \Delta C_{LK-CS} = (B_{LK} - B_{CS}) - (C_{LK} - C_{CS}) \tag{2}
\]

Where, \( \Delta V_{LK-CS} \) is a monetary value of LK (USD), benefit if the value is positive and cost if the value is negative. \( B_{LK} \) is a total benefits of LK consist of \( B_{LK1} \) (Social benefit) and \( B_{LK2} \) (Environmental benefit). \( B_{CS} \) is a total benefits of CS consist of \( B_{CS1} \) (Benefit of economies of scale) and \( B_{CS2} \) (Benefit of reliability). \( C_{LK} \) is additional cost for control and monitoring cost of LK. \( C_{CS} \) is a total costs of CS consist of \( C_{CS1} \) (transmission lines cost) and \( C_{CS2} \) (construction delay cost).

9.2. LK benefit – cost simulation

The purpose of this simulation in this paper, we will use the following numbers: Total LK capacity \( P \) = 1000 MW, which number is taken from the largest unit of power plant in Indonesia. Production time is annual based, \( h = 8760 \) hours. Period of simulation: 10 (ten) year basis. Feed in tariff, \( f = 0.01 \) USD/kWh, the price of which is taken from the current government regulated tariff for electrical energy.

As depicted previously, LK provides opportunity for local business players to own small scale power plant or waste to energy plant. It means that LK will give social benefit as a lot job opportunity to people to run thousands of LK plants around the country. The method that will be used to quantify such social benefit in this paper will be derived from the potential additional tax income for the nation, both from company taxes and individual income taxes. For simplification, other social benefit of LK including more comfortable neighbourhood, increasing buying power, reducing unemployment rate, and energy security improvement are not be included in this calculation. For simulation purpose, we will take the moderate package of 5 ton waste per day, which employs around 12 workers with regional minimum wage per year is IDR 40,000,000 or equivalent USD 3,000
Refer to Table 5, the package of 5 ton of waste per day is estimated to have Net Present Value of USD 146,558. The country will have potential income from added value and corporate income tax at least 20% or USD29,312. Estimated capacity of LK with 5 ton waste per day is 120 kW, so the total potential corporate income tax from 1000 MW LK is 1000,000/120 x USD29,312 = USD 244,263.410.

The individual tax potential is calculated from the income of 12 workers who work in 5 ton package waste (120 kW capacity). The total income in this LK is 12 x USD3000 = USD36,000 and the potential tax (15%) from 1000 MW LK is 15% x 1,000,000/120 x USD 36,000 = USD 300,000,000.

Every kWh of electrical energy is similar to 0.00012 Ton coal equivalent and if the price of coal is USD40/ton, then any kWh used of renewable LK plant will save: 0.00012 x USD40 = USD 0.0048. Since the package of 5 ton waste may produces 720 kWh/day (Table 5), the total environmental benefit resulted from 1000 MW LK plants will be: 1,000,000/120 x 720 x 365 x USD 0.0048 = USD 10,512,000.

A huge number of plants under distributed generation system, require rigorous control and monitoring that can be expressed as additional cost. For the purpose of this simulation, we will pick the moderate value of this cost as 5% of Capital Expenditure (Capex). For the package of 5 ton waste, the capex is USD123,940 and the total cost of 1,000 MW LK will be: 1,000,000/120 x 5% x USD123,940 = USD 51,641,667.

The benefit of the conventional system is derived from the lower investment cost per unit of large power plant and the better system reliability compare to that of LK. We will use Coal Power Plant to represent the large CS power plant and the average capital cost of this kind in Indonesia is USD 1,218/kW (Wahid, 2016). While the capital cost for LK plant is calculated from the typical cost of 5 ton waste package LK (Table 4) with 20 hours daily operation. Capex for 5 ton is USD 123,940 and the capacity is 720 kWh/20 h = 36 kW. The cost per kW of LK= 123,940/36 = 3,443 USD/kW. So, the total cost of 1000 MW LK plants will be: 1000,000 x USD (3,443 – 1,218) = USD 2,224,777,778.

Conventional system has better reliability since it has a lot of redundant power plants that interconnected one into another. Reliability can be represented by the delta reliability (less duration of interruption compare to the LK system), in hours per year. For the purpose of this calculation we assume that the maximum of shortage in LK plants is 10 days or 240 hours a year. So, we can pick three conditions of delta; 72 hours, 120 hours, days, and 240 hours outages per year. For instance, if the delta is 120 hours, the benefit will be: 1000,000 kW x 120 hours x 0,01 USD/kWh = USD 1,200,000.

Transmission line (T/L) cost is taken from Andrade, Juan et. Al (2016), which stipulated that the Grid connection cost for Generating Technology is within the range of 114 USD/kW for the small plant and 227USD/kW for the large plant that far from load. So, we can pick the number of 227USD/kW as transmission lines cost for this simulation. If the total capacity of plants is 1000 MW, the T/L cost will be: 1000,000 x USD 227 = USD 227,000,000.

In the electricity industry, revenue is coming from energy sales in USD/kWh. It other words, any single hour delay means the potential loss of income is similar to the capacity of unit in kW times the tariff. For example: if capacity is 1000 MW or 1,000,000 kW and average tariff in Indonesia 0,07 USD, then the potential loss is USD 70,000 per hour or USD 1,680,000 per day of delay. This value will be used to calculate the total delay cost for several conditions. For simulation purpose, we will pick 3 (three) different conditions: one month delay, six month delay, and one year delay. For example, if the project slippage is one month, the cost will be: 30 x USD 1,680,000 = USD 50,400,000.

10. Result
The overall simulation of Cost-Benefit calculation is using the equation (2) and simulated for nine different cases by combining three different numbers for reliability and project delay. Case 1: 72 hours outage; 1 month slippage, Case 2: 72 hours outage; 6 month slippage, Case 3: 72 hours outage; 12 month slippage, Case 4: 120 hours outage; 1 month slippage, Case 5:
120 hours outage; 6 month slippage, Case 6: 120 hours outage; 12 month slippage, Case 7: 240 hours outage; 1 month slippage, Case 8: 240 hours outage; 6 month slippage, Case 9: 120 hours outage; 12 month slippage. The overall simulation result of Cost-Benefit calculation is put in the Table 6.

| Cases | B_{LK1} | B_{LK2} | B_{CSI} | B_{CSI} | C_{LK} | C_{CSI} | C_{CS2} | ΔV_{LK-CS} |
|-------|---------|---------|---------|---------|--------|---------|---------|-----------|
| 1     | 290,417 | 10,512  | 503.389 | 720     | 51,642 | 227,000 | 50,400  | 22,579    |
| 2     | 290,417 | 10,512  | 503.389 | 720     | 51,642 | 227,000 | 302,400 | 274,579   |
| 3     | 290,417 | 10,512  | 503.389 | 1,200   | 51,642 | 227,000 | 604,800 | 576,979   |
| 4     | 290,417 | 10,512  | 503.389 | 1,200   | 51,642 | 227,000 | 50,400  | 22,099    |
| 5     | 290,417 | 10,512  | 503.389 | 1,200   | 51,642 | 227,000 | 302,400 | 274,099   |
| 6     | 290,417 | 10,512  | 503.389 | 2,400   | 51,642 | 227,000 | 604,800 | 576,499   |
| 7     | 290,417 | 10,512  | 503.389 | 2,400   | 51,642 | 227,000 | 50,400  | 20,899    |
| 8     | 290,417 | 10,512  | 503.389 | 2,400   | 51,642 | 227,000 | 302,400 | 272,899   |
| 9     | 290,417 | 10,512  | 503.389 | 2,400   | 51,642 | 227,000 | 604,800 | 575,299   |

Table 6 can be used as a template to calculate and compare Listrik Kerakyatan system to Conventional system by simulating different input variable that match the local condition. From table 6, we can observe that the most dominant variable is the economies of scale $B_{CS}$ that mainly depends on specific cost per kW of LK power plant. Result in Table 6 shows that in all cases, $ΔV_{LK-CS}$ positives. It means that LK system may offer more benefit compare to the conventional system if LK operates only 6 hours per day. In this case, LK does not operate as a base load but as a peaker or load follower plant, which runs during the peak load condition.

However, the case will be different if the 5 ton LK power plants operates longer than 6 hours. For instance if LK operates for 11 hours a day, leads to the capacity of 65 kW (720 kWh/11 h). In this case the cost per kW of LK becomes 1,894 USD/kW and the result shows $ΔV_{LK-CS}$ negative for case 1, case 4, and case 7 as shown in Table 7.

| Cases | BLK1 | BLK2 | BCS1 | BCS2 | CLK | CCS1 | CCS2 | $ΔV_{LK-CS}$ |
|-------|------|------|------|------|-----|------|------|-------------|
| 1     | 290,417,179 | 10,512,000 | 675,527,778 | 720,000 | 51,641,667 | 227,000,000 | 50,400,000 | -149,560,265 |
| 2     | 290,417,179 | 10,512,000 | 675,527,778 | 720,000 | 51,641,667 | 227,000,000 | 302,400,000 | 102,439,735 |
| 3     | 290,417,179 | 10,512,000 | 675,527,778 | 720,000 | 51,641,667 | 227,000,000 | 604,800,000 | -404,839,735 |
| 4     | 290,417,179 | 10,512,000 | 675,527,778 | 1,200,000 | 51,641,667 | 227,000,000 | 50,400,000 | -150,040,265 |
| 5     | 290,417,179 | 10,512,000 | 675,527,778 | 1,200,000 | 51,641,667 | 227,000,000 | 302,400,000 | 101,959,735 |
| 6     | 290,417,179 | 10,512,000 | 675,527,778 | 1,200,000 | 51,641,667 | 227,000,000 | 604,800,000 | -404,359,735 |
| 7     | 290,417,179 | 10,512,000 | 675,527,778 | 2,400,000 | 51,641,667 | 227,000,000 | 50,400,000 | -151,240,265 |
| 8     | 290,417,179 | 10,512,000 | 675,527,778 | 2,400,000 | 51,641,667 | 227,000,000 | 302,400,000 | 100,759,735 |
| 9     | 290,417,179 | 10,512,000 | 675,527,778 | 2,400,000 | 51,641,667 | 227,000,000 | 604,800,000 | 403,159,735 |

### 11. Conclusion
Commutative Law of Algebra can be used as a simple mathematical model to analyse whether the Listrik Kerakyatan model as an eco-friendly distributed generation can be applied to solve various problems associated with a large scale Conventional System. The result of simulation shows that LK provides more value if its plants operate in less than 11 hours as peaker power plant or load follower power plant to improve load curve balance of the power system. The result of simulation indicates that the investment cost of LK plant should be optimized in order to minimize the plant investment cost. This study indicates that the benefit of economies of scale principle does not always apply to every conditions, particularly if the portion of intangible cost and benefit is relatively high.
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