The calculation method of the length of contact of car tires with the road surface

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Abstract. In the tasks of modeling the motion stability, controllability and braking dynamics of cars and buses, information is required on the car tires contact length with the road surface. The car tires contact length is usually calculated from geometrical considerations (using the Hedekel formula). In this case, the experimental data of this value turn out to be less than the calculated ones due to the deformation features of the car tires. To account for this reduction in the contact patch length, the authors developed a general method for determining the correction coefficient. Previously, they developed a similar method for low-profile radial passenger car tires. Now it is valid for all passenger and truck tires. This will allow calculating the car tires contact length more correctly.

1. Introduction
In modeling of the properties of movement stability, controllability and braking dynamics of cars and buses, one has to deal with modeling the contact patch length [1-15]. The scheme of the relationship of these performance properties with the contact patch length is shown in figure 1.

Correct calculation of its value determines, among other things, the accuracy of the calculation of the parameters of the sideslip and oscillations of operated wheels.

The contact patch length determines the results of calculating the parameters of the slip and oscillations phenomena of the operated wheels. There are two methods to determine contact patch length. The first method is experimental [1, 16-24]. These articles present the comparison of theoretical and practical research of the contact patch between tire and road in static conditions without applying lateral forces. The authors determined and analyzed the geometric parameters of the contact patch such as shape, sizes, and area depending on the load applied over the tire and the air pressure in the tire.

Also, the influence of radial load and the air pressure against the surface of the contact patch was determined and analyzed. It can be concluded that the experimental method of determining the contact patch length is complex and time-consuming.

The second method is calculation. This way is easier than the experimental one. The contact patch length is calculated by the Hedekel formula [3, 25, 26]. At the same time, the real values of the contact patch length turn out to be less than the calculated ones due to the deformation features of car tires.
The purpose of this work is to develop a general method for determining the correction coefficient $K_h$ for the car tires. This correction coefficient $K_h$ will help calculate the car tires contact patch length more correctly.

**Figure 1.** The scheme of the relationship of the vehicle active safety properties with the contact patch length.

2. Research methodology

The contact patch length $l_c$ is usually calculated from geometric considerations, using the scheme of figure 2, by a simple relationship, which is commonly called the Hedekel formula [26].

In figure 2, $P_z$ is normal wheel load in N; $\Delta z$ is radial tire deflection, mm; $R_0$ is free wheel radius in mm; $l_c$ is contact patch length in mm; $b_c$ is width patch length in mm.

According the Hedekel formula:

$$l_{c0} = 2\sqrt{R_0^2 - (R_0 - \Delta z)^2} = 2\sqrt{\Delta z(2R_0 - \Delta z)}.$$  (1)
In formula (1), $l_{c0}$ is calculated contact patch length according the Hedekel formula in mm; $R_0$ is free wheel radius in mm; $\Delta z$ is radial tire deflection in mm.

$$\Delta z = \frac{P_z}{C_{tz}}. \quad (2)$$

In formula (2), $P_z$ is normal wheel load in N; $C_{tz}$ is radial rigidity of the tire in N/mm.

To calculate the radial rigidity of car tires with the participation of the author E V Balakina, universal dependencies [15] were previously obtained for calculations shown in table 1.

In Table 1, $C_{tz}$ is tire radial rigidity in N/mm; $B_t$ is profile width in mm; $D_t$ is tire outer diameter in mm; $P_{z\text{max}}$ is maximum normal load on the tire according to the passport in N; $\frac{P_z}{P_{z\text{max}}}$ is the ratio of workload to the maximum; $\frac{P}{P_{\text{max}}}$ is the ratio of working pressure to maximum.

However, the validity of the Hedekel formula obtained from geometrical considerations requires additional research.

For this purpose, the authors developed a method. Its essence is that the contact patch lengths are measured experimentally for a large number of car tires; then they are calculated using the Hedekel formula. Comparison results were analyzed and a correction coefficient was determined, by which the calculated contact patch lengths should be multiplied to obtain refined results that are close to the experimental ones. This makes it possible to accurately determine the car tires contact patch length by calculation without using time-consuming experiments.

The experimental contact patch length is somewhat less than the calculated one. This is due to the peculiarities of tread deformation in the areas above the contact patch. The value of this difference is determined by the tire design. Therefore, the secondary calculation of the contact patch length is based on the specified reduction. This can be done as follows:

$$l_c = K_h \cdot l_{c0}. \quad (3)$$

In formula (3), $K_h$ is a coefficient of the contact patch length reduction ($K_h = l_{ce}/l_{c0}$).
Table 1. Recommended universal constraints for calculating the radial rigidity of car tires

| Tire type         | Recommended universal relationship for the calculation of radial rigidity | Maximum relative error, % | Average relative error, % |
|-------------------|--------------------------------------------------------------------------|---------------------------|----------------------------|
| Passenger radial  | $C_{tz} = 166.1429 \cdot \prod_{i=1}^{4} Y_{pi}$                         | 13                        | 4.0                        |
|                   | $Y_{p1} = -0.15771 + 0.0067 B_{z}$                                      |                           |                            |
|                   | $Y_{p2} = 0.62391 + 0.0006172 D_{z}$                                     |                           |                            |
|                   | $Y_{p3} = 0.4053341 + 0.000118836 P_{z_{max}}$                          |                           |                            |
|                   | $Y_{p4} = 0.3185 + 0.849 \frac{P}{P_{max}}$                            |                           |                            |
| Passenger diagonal| $C_{tz} = 201.6842 \cdot \prod_{i=1}^{4} Y_{pi}$                        | 18                        | 6.8                        |
|                   | $Y_{p1} = -2.10538 + 0.01747 B_{z}$                                     |                           |                            |
|                   | $Y_{p2} = 0.74784 + 0.00040 D_{z}$                                      |                           |                            |
|                   | $Y_{p3} = 0.0631225 + 0.00018329 P_{z_{max}}$                          |                           |                            |
|                   | $Y_{p4} = 0.25407 + 0.96191 \frac{P}{P_{max}}$                         |                           |                            |
| Truck diagonal    | $C_{tz} = 785.95 \cdot \prod_{i=1}^{5} Y_{pi}$                         | 17                        | 5.4                        |
|                   | $Y_{p1} = 0.68765 + 0.0010287 B_{z}$                                     |                           |                            |
|                   | $Y_{p2} = 1 + 0.000004 D_{z}$                                           |                           |                            |
|                   | $Y_{p3} = 0.7721584 + 0.00000785 P_{z_{max}}$                          |                           |                            |
|                   | $Y_{p4} = 0.46727 + 0.6995645 \frac{P}{P_{z_{max}}}$                   |                           |                            |
|                   | $Y_{p5} = 0.8821875 + 0.1422837 \frac{P}{P_{z_{max}}}$                 |                           |                            |
| Truck radial      | $C_{tz} = 839.436 \cdot \prod_{i=1}^{2} Y_{pi}$                        | 10                        | 3.5                        |
|                   | $Y_{p1} = 0.284905 + 0.0000267347 P_{z_{max}}$                          |                           |                            |
|                   | $Y_{p2} = 0.2854095 + 0.776038 \frac{P}{P_{z_{max}}}$                  |                           |                            |
Thus, the contact patch length of the tire with the road can be calculated from the final dependence (4):
\[
l_c = 2K_h\sqrt{\Delta z(2R_0 - \Delta z)}.
\] (4)

FSUE «NAMI» measured the values of the radial rigidity $C_{rz}$ and the contact patch length $C_{lz}$ of passenger and truck tires of different models and structures (47 each) for given normal wheel loads $P_z$. A photograph of the experimental installation is shown in figure 3.

![Figure 3](image)

**Figure 3.** Photograph of the experimental stand to determine the characteristics of tires.

According to the experimental data, we calculated tire deformation $\Delta z$ by the formula (2). To obtain the correction coefficient $K_h$, the experimental values $l_{ce}$ are divided into the corresponding calculated ones $l_{c0}$. The results are shown in table 2 and figure 4.

![Figure 4](image)

**Figure 4.** The results of the calculation of the correction coefficient using the Hedekel formula.
Table 2. Calculation results

| Tire model | Tire type | $P_{c}$, N | $C_{l_{c}}$, m/m | $z_{c}$, mm | $R_{h_{l}}$, mm | $F_{c}$, m/m² | $b_{c}$, mm | $l_{c}$, mm | $l_{c}$, mm | $k_{h}$ |
|------------|-----------|-------------|------------------|--------------|----------------|----------------|-------------|-----------|-----------|-------|
| 1          | passenger | 3236        | 158              | 26           | 299            | 19800          | 125         | 173       | 243.9016195 | 0.7093 |
| 2          | passenger | 3236        | 177              | 25.5         | 299            | 19800          | 125         | 182       | 241.6505742 | 0.75315 |
| 3          | passenger | 1863        | 126              | 15.5         | 310            | 1080           | 100         | 109       | 193.594938 | 0.56303 |
| 4          | passenger | 4707        | 186              | 33           | 325            | 26100          | 153         | 186       | 285.3839519 | 0.65175 |
| 5          | passenger | 4707        | 196              | 31.5         | 325            | 24400          | 154         | 175       | 279.1612437 | 0.62688 |
| 6          | passenger | 6227        | 322              | 23.6         | 342.5          | 18320          | 116         | 183       | 249.8722874 | 0.73237 |
| 7          | passenger | 7061        | 342              | 24.5         | 342.5          | 18640          | 117         | 184       | 254.419458 | 0.72322 |
| 8          | truck     | 27810       | 910              | 33.3         | 480.5          | 41030          | 207         | 225       | 351.524736 | 0.64007 |
| 9          | truck     | 30410       | 970              | 35.3         | 480.5          | 43350          | 208         | 232       | 361.5367754 | 0.64171 |
| 10         | truck     | 27810       | 865              | 32.7         | 481.5          | 41200          | 228         | 220       | 348.8312486 | 0.63068 |
| 11         | passenger | 30410       | 1010            | 33.6         | 481.5          | 41880          | 210         | 206       | 353.4280125 | 0.58286 |
| 12         | passenger | 3432        | 165              | 25           | 308            | 19300          | 123         | 194       | 243.1049156 | 0.79801 |
| 13         | passenger | 3432        | 176              | 24           | 308            | 21300          | 123         | 196       | 238.3946308 | 0.82217 |
| 14         | passenger | 3432        | 173              | 24.5         | 307.5          | 20600          | 124         | 195       | 240.5597639 | 0.81061 |
| 15         | truck     | 27810       | 246              | 23.5         | 330.5          | 21800          | 129         | 191       | 244.7958333 | 0.78024 |
| 16         | truck     | 27810       | 263              | 21.5         | 330.5          | 20400          | 130         | 177       | 234.5143919 | 0.75475 |
| 17         | truck     | 24517       | 760              | 36.3         | 572            | 61510          | 249         | 292       | 401.0461819 | 0.7281 |

Note: Calculation results.
From figure 4 it follows that $K_h \approx 0.7$ for all tires, except low-profile radial passenger tires. Thus, for almost all tires $l_c \approx 1.4\sqrt{\Delta z(2R_0 - \Delta z)}$. It has been previously established [25] that for passenger radial low-profile tires $l_c \approx 1.2\sqrt{\Delta z(2R_0 - \Delta z)}$.

3. Conclusion
The authors developed a general universal method for determining the value of the car tires contact length with the road surface. This method is valid for radial, diagonal, truck and passenger tires, including low profile tires. The application of the method will allow calculating the car tires contact length with the road more correctly and improving the accuracy of modeling the movement stability and controllability of cars.

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