The transition to turbulence in the boundary flow of superfluid $^4$He is investigated using a vortex-free vibrating wire. At high wire vibration velocities, we found that stable alternating flow around the wire enters a turbulent phase triggered by free vortex rings. Numerical simulations of vortex dynamics demonstrate that vortex rings can attach to the surface of an oscillating obstacle and expand unstably due to the boundary flow of the superfluid, forming turbulence. Experimental investigations indicate that the turbulent phase continues even after stopping the injection of vortex rings, which is also confirmed by the simulations.

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Quantum turbulence in superfluid $^4$He consists of quantized vortices that have a simple structure: a normal fluid core and a quantized circulation [1]. Since this structure prevents the core from becoming a superfluid, vortices in a superfluid adopt the form of a ring or a line attached to boundaries. A vortex ring propagates in a superfluid at a self-induced velocity and eventually attaches to a boundary. If boundary flow is applied, it is likely that it will cause a vortex line attached to a boundary to expand [2], forming turbulence. However, this simple transition to turbulence triggered by vortex rings has not been observed experimentally since remnant vortices always remain attached to boundaries.

Quantized vortices may nucleate during transition to the superfluid state [3], and counterflow occurring during cooling can force vortices to expand, causing the number of vortices to multiply [4]. Free vortices that are not attached to boundaries dissipate by mutual friction or by reconnection of vortices [4, 5], while vortices remain attached between boundaries [6]. Therefore, an oscillating obstacle such as a sphere [7], grid [8], or wire [9, 10] can always generate turbulence in superfluid $^4$He around an oscillating velocity of 50 mm/s, much lower than the Landau velocity of about 60 m/s at which the superfluidity collapses [1]. It appears to be difficult to establish a vortex-free state on the surface of an obstacle in superfluid $^4$He, in which the critical velocity for the transition to turbulence is expected to be very high. A vortex-free state has only been achieved in the vicinity of a moving ion, with vortex nucleation occurring at velocities exceeding 10 m/s [11].

In a recent study [12], we reported that slow filling of a superfluid and using a filter of vortices can prevent vortices from attaching to boundaries, which prevents the generation of turbulence by oscillation, even above a velocity of 1 m/s. This method enables the formation of an oscillating obstacle whose surface is effectively free of remnant vortices. In the present paper, we report experimental and numerical investigations of the responses of a vortex-free oscillating obstacle sprinkled with vortex rings. In the experiments, we successfully observed a transition to turbulence triggered by vortex rings using a vortex-free vibrating wire. Numerical simulations based on a vortex filament model revealed a new process, namely, the turbulence is generated by vortex rings propagating to the oscillating obstacle, where the oscillations cause them to grow and then sustain them.

The experimental setup consisted of two vibrating wires, similar to those described in a previous paper [12]. These 3-µm NbTi superconducting wires, were formed into semicircles, the two arms of which were attached to columns mounted on a copper plate, as shown in the inset in Fig. 1. The wires labeled A and B in Fig. 1 were located in a small chamber with a 0.1-mm-diameter pinhole. Wire A was oriented so that it pointed towards the center of wire B, perpendicular to its semicircle face. The distance between the arms was 1.6 mm for wire A and 1.8 mm for wire B. The heights of the columns could be varied so as to adjust the heights of the wires. The distance between the apexes of the wire loops was 1.8 mm. The wires and the chamber were located in a cell...
in a magnetic field of 25 mT. A heat exchanger made of sintered silver powder was mounted in the bottom of the cell for cooling the helium. A RuO$_2$ thermometer was mounted on the cell wall. The responses of the vibrating wires were measured in vacuum at 30 mK. The resonance frequencies were 1590 Hz for wire A and 1030 Hz for wire B. In the present paper, we report experimental results obtained at 30 mK.

In the experiment, the cell was filled with superfluid $^4$He cooled below 100 mK over 48 h. This careful filling prevented the formation of remanent vortices. The pinhole of the chamber also had the effect of filtering vortices [12]. As Fig. 1(A) shows, vibrating wire A was unable to generate turbulence in the superfluid. To test the stability of the superfluid, we applied mechanical vibrations to the cryostat and warmed the superfluid to 1 K. However, even under these conditions the vibration of the wire was too stable to collapse, indicating that no turbulence was generated. In contrast, vibrating wire B generated turbulence, as Fig. 1(B) shows. It appears that vortices remain attached to wire B. Thus, we were able to produce both a vortex–free wire and a vortex–attached wire in a chamber.

Turbulence in a superfluid generated by an oscillating obstacle is accompanied by free vortex rings [13, 14]. Therefore, many vortex rings are expected to be generated in the chamber by the vibration of wire B, and these vortex rings can propagate through the surrounding fluid and attach to the surface of the wires. However, after the vibration of wire B was stopped, we found that the velocity of wire A exhibited the same dependence as that shown in Fig. 1(A), indicating that there was no transition to turbulence. The vortices generated by wire B seem to be unable to form remanent vortices attached to wire A. A previous study [12] reported that vortices that nucleate during a superfluid transition form bridges between an obstacle and the surrounding wall to become remanent vortices. If these bridge vortices are forced to oscillate largely by the vibration of the obstacle, free vortex rings can be generated by reconnection of the vortices, forming turbulence [15]. Free vortex rings in turbulence are expected to be too small to form bridges between a wire and the surrounding wall. Vortices attaching to the surface of wire A might vanish if there is no boundary flow. This condition is suitable for studying the transition to turbulence triggered by free vortex rings. Wire A can be used to detect the transition to turbulence, and wire B can be used to generate free vortex rings.

After stopping the vibration of the generator (wire B), we increased the velocity of the detector (wire A) above 1 m/s, as shown in Fig. 2(a). Maintaining the driving force of the detector at 1 nN, we increased the velocity of the generator so as to generate turbulence. At the moment of transition to turbulence, the detector’s velocity suddenly dropped to a low value of around 60 mm/s.

This result indicates that free vortex rings were generated by the generator, propagating to the detector and interfering with its vibration. We again stopped the vibration of the generator, but the detector’s velocity still remained low, indicating that the detector had entered a turbulent phase. This is a surprising result because free vortex rings not only disturb the wire vibration but also trigger a turbulent phase. During the transition to turbulence, free vortex rings may attach to the surface of the detector, expanding due to boundary flow and forming turbulence. Even after the generator ceases to generate free vortex rings, vortices attached to the detector’s surface may still develop by the flow and form turbulence. As the driving force was reduced, the velocity of the detector remained low until the flow around it entered a laminar phase, as shown in Fig. 2(b). While the generator was not vibrating, we further increased the driving force of the detector and found that the detector’s velocity traces the initial values shown in Fig. 2(a), indicating that there was no turbulence. Vortices nucleating in the turbulent phase were unable to affect the condition of the detector.

It has been reported that free vortex rings can affect the vibration of a wire in superfluids [13, 14]. In superfluid $^4$He, free vortex rings disturb the vibration of a wire, decreasing the critical velocity of turbulence [16]. This result implies that vortex rings might trigger a transition to turbulence. However, the wire could generate turbulence independently, indicating that remanent vortices were attached to the wire from the beginning. Under this condition, it is difficult to understand how vortex rings cause turbulence, since the remanent vortices would also affect the transition to turbulence. In the present study, however, the absence of remanent vortices clarifies this difficulty. In $^3$He–B, instead of disturbing the vibration
of a wire, vortex rings rather increase it \[13\], since vortices can shield the wire from thermal excitations in the superfluid. This shielding effect may limit observations of dissipation.

These observations were verified by performing numerical simulations. In modeling the experimental situation, an oscillating sphere was considered rather than a vibrating wire; this is because it is easier to calculate the case of a sphere, while it still reflects the essence of the physics that we are interested in. The problems we desire to solve numerically are what occurs when vortex rings coming from the generator collide with an oscillating sphere, and whether after collision these vortices form turbulence or not. The vortex filament model is used in the simulation \[17, 18\]. The superfluid velocity \( \mathbf{v}_{\text{vor}} \) generated by the vortices is calculated using Biot-Savart integration. An additional velocity field \( \mathbf{v}_b \) is imposed in order to satisfy the boundary condition on the surface of the sphere for a superfluid \( (\mathbf{v}_s - \mathbf{v}_p) \cdot \mathbf{n} = 0 \), where \( \mathbf{v}_s = \mathbf{v}_{\text{vor}} + \mathbf{v}_b \) is the total superfluid velocity, \( \mathbf{v}_s \) is the sphere velocity, and \( \mathbf{n} \) is the unit normal vector to the surface of the sphere. The additional velocity \( \mathbf{v}_b \) takes the form

\[
\mathbf{v}_b = \nabla \Phi_b + \nabla \Phi_u, \tag{1}
\]

where \( \Phi_b \) is the velocity potential that cancels the normal velocity component to the sphere made by the vortices \[17\] and

\[
\Phi_u = -\frac{1}{2} \left( \frac{R}{r} \right)^3 \mathbf{v}_p \cdot \mathbf{r}, \tag{2}
\]

is the scalar velocity potential made by the moving sphere. Here \( R \) denotes the radius of the sphere, and \( \mathbf{r} \) is the position from the center of the sphere. The reconnection process is assumed to occur when a vortex becomes close to another vortex or a spherical boundary within the computational resolution. The parameters used in the calculation were chosen to match those of the experiments, such as a sphere radius of 3 \( \mu \)m and an oscillation frequency of 1590 Hz. The velocity of the oscillation is chosen to be 137 mm/s which is much higher than the observed critical velocity, 50 mm/s close to the velocity, and 30 mm/s much lower than the velocity. The experiments were performed at a very low temperature of 30 mK, where the normal fluid component is negligible. Thus, we performed the numerical simulation at the zero temperature limit to neglect mutual friction. Vortex rings all having a radius of 0.74 \( \mu \)m \[19\] were injected every 0.05 ms vertically from the bottom \[20\] (Fig. 3(a)).

The numerical simulation reveals a new explanation for the transition from vortex rings to turbulence. The incoming vortex rings collide successively with the sphere. The vortices are then split into lots of segments of vortex loops that attach to the sphere. Smaller vortex loops than the sphere radius will be split into smaller loops by reconnection during running on its surface, eventually vanishing. In contrast, large loops still remain attached to the sphere. The ends of a large vortex loop are dragged by a sphere motion; however, the other parts of the loop can move with each self-induced velocity rather independently. Consequently, the sphere motion stretches a vortex line, up to a length of the oscillation amplitude (Fig. 3(b)). Some of the large vortices that have grown sufficiently detach from the sphere. Since they have a low self-induced velocity, they are unable to escape before the sphere returns. These vortices are struck by the sphere many times, so that vortex segments become attached to the sphere. These processes are repeated with the sphere oscillating. The remaining vortex segments start to grow, and finally the sphere becomes covered with a vortex tangle (Fig. 3(c)). The volume that the created vortex tangle occupies is smaller in the simulation for an oscillation velocity of 50 mm/s than for that in the case of 137 mm/s; this is due to the different oscillation amplitudes of these two simulations, since the sphere can stretch vortices only up to the oscillation amplitude (Figs. 3(c) and (f)). For 30 mm/s, in contrast, though the vortices that had collided with the sphere grew slightly by the oscillation, they reconnect immediately to themselves, flying away from the sphere. Therefore, vortices will disappear soon after stopping the injection of vortex rings. The roughness of the sphere surface should be concerned with respect to the stretch of a vortex loop. The roughness could affect only the motion of loop ends attached to the surface, thus not changing much the present scenario.

As mentioned in the experimental section, the vortex tangle in the detector survives even after the vortex ring supply from the generator has been stopped. To confirm
The computational box (25 mm × 25 mm) is covered with a tangle, the vortices that exit the superfluid. The time development for this case is very similar to that for the previous case. Collisions between large vortices and the sphere leave many vortex segments attached to the sphere’s surface. These segments behave as seeds for turbulence, and start to grow. As a result, the vortex tangle is sustained by the sphere even after the supply of vortex rings has stopped, which is consistent with the observations.

The numerical simulations of vortex dynamics demonstrate that vortex rings can attach to the surface of an oscillating obstacle and expand due to fast boundary flow to form turbulence. Thus the combination of experimental and numerical investigations reveals new insights into the phenomenon of quantum turbulence.

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In the experiment, we observed a delay between the generation of vortex rings by a generator to the turbulent transition of a detector. We estimated the typical size of vortex rings to be about 0.7 μm from this delay time.

[20] The frequency of the vortex ring emission from the generator should be higher than the vibration frequency of the generator of an order of 1 kHz. See Ref. 15.

FIG. 4: (color online) Time evolution of vortex lines for an oscillating velocity of 137 mm/s with continuous vortex–rings injection (red solid line, 1), for 137 mm/s with the injection of eight vortex–rings (green dashed line, 2), for 50 mm/s with continuous vortex–ring injection (blue dotted line, 3), and for 50 mm/s with the injection of eight vortex–rings (pink dotted line, 4). The arrows indicate cusps on the solid line (see text).