New N=1 Superconformal Field Theories
and their Supergravity Description

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Abstract

In this note we construct a new class of superconformal field theories as mass deformed $N = 4$ super Yang-Mills theories. We will argue that these theories correspond to the fixed points which were recently found [1] studying the deformations of the dual IIB string theory on $AdS_5 \times S^5$.

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1 Introduction

The investigation of superconformal field theories has already a long history. One appealing feature of superconformal models is that they often exhibit a strong-weak coupling duality symmetry (S-duality). In a remarkable recent development it became clear that in a large class of superconformal field theories a new type of duality symmetry arises, namely they can be equivalently described by supergravity in anti-de-Sitter (AdS) spaces [2]. In particular, there is a correspondence between four-dimensional superconformal field theories and supergravity on $AdS_5 \times M^5$, where $M^5$ is a certain five-dimensional Einstein space. In the simplest case, $M^5$ is given by $S^5$, and the corresponding superconformal field theory is just $N = 4$ super Yang-Mills with $SU(n)$ gauge symmetry. This is nothing else as the superconformal theory which lives on the world volume of $n$ parallel D3 branes. Another well studied example for $M^5$ is the coset space $T^{1,1}$ which leads to a $N = 1$ superconformal gauge theory, which is the celebrated superconformal theory of D3 branes probing a conifold singularity [3].

The prescription [4, 5, 6, 7] of the holographic map allows for several non-trivial checks of the conjectured AdS/CFT correspondence. For example, the central charge of the conformal field theory is inversely proportional to the volume of $M^5$. This check works very nicely for the correspondence between the coset space $T^{1,1}$ and the superconformal field theory from D3 branes at the conifold singularity. Recently, deformations of the usual IIB string theory on $AdS_5 \times S^5$ has been studied [8, 9, 1] by analyzing critical points of $N = 8$ gauged supergravity. In [1] a new fixed point was found, leaving $N = 2$ unbroken in the bulk (corresponding to $N = 1$ on the brane). The information about this fixed point that has been extracted from the supergravity description are the global symmetries and the ratio of the central charges of the undeformed UV theory and the interacting IR theory: $c_{IR}/c_{UV} = 27/32$. The aim of this letter is to show that this new fixed point corresponds to a particular mass deformation of the $N = 4$ super-Yang-Mills theory. We will show that our new fixed point field theories obtained by mass deforming the $N = 4$ theory indeed reproduce the global symmetry as well as the ratio $c_{IR}/c_{UV}$ from the supergravity side.

In section 2 we will first briefly review the method [10] of deforming a supersymmetric field theory with a marginal operator to obtain a new class of superconformal models. Then we recall the existence of a family of $N = 1$ superconformal theories as mass deformed $N = 2$ theories. Deforming
the finite $N = 2$ $SU(n)$ theory with $2n$ flavors by a mass for the adjoint chiral multiplet leaves an $N = 1$ theory with a quartic superpotential. By the method of [10] it can be established that this quartic superpotential is a marginal deformation of the IR physics. The superconformal theory along the fixed line parametrized by the marginal operator is precisely the superconformal theory of $n$ D3 branes probing a conifold.

Then we will argue that the same arguments will basically establish that deforming the $N = 4$ SUSY gauge theory by a mass term to one of the adjoint chiral multiplets will lead to a one parameter family of $N = 1$ superconformal theories. They can be expressed as $N = 1$ theories with two massless adjoints $A$ and $B$ deformed by a quartic superpotential $W \sim (AB)^2$. This has to be contrasted with the mass deformation of the $N = 4$ theories by a mass for a full hypermultiplet studied e.g. in [11]. While latter one leaves an $N = 2$ theory, our deformation leaves only $N = 1$ unbroken.

In section 3 we will turn to the dual supergravity description provided by the supersymmetric fixed point found in [1] from the deformation of the $AdS_5 \times S^5$ supergravity. While the field theory considerations presented up to this point are actually valid for an arbitrary gauge group, only the $SU(n)$ theories will be realized on D3 branes probes in IIB$^*$.

### 2 The new conformal theories

Let us first briefly recall the construction of [10] to establish the existence of a family of new $N = 1$ superconformal theories by mass deforming a given superconformal theory. The mass deformation causes a flow from the original theory in the UV to the deformed theory in the IR. Specifically, start with a theory at a fixed point with a marginal operator provided by the superpotential

$$W = gX\phi\phi',$$  \hspace{1cm} (1)

and add the following mass term to the superpotential

$$W_{\text{mass}} = mX^2.$$  \hspace{1cm} (2)

$^*$Allowing for orientifolds, the $SO$ and $Sp$ examples are also accessible.
Via its equation of motion the heavy field can be integrated out, and one obtains the new, non-renormalizable superpotential

$$W_{\text{new}} = -\frac{g^2}{2m}(\phi\phi')^2.$$  \hspace{1cm} (3)

As shown in [10] this is again a marginal operator.

### 2.1 The flow from finite $N = 2$ theories to superconformal $N = 1$ theories

In this section we briefly recall how this method establishes the existence and S-duality of a family of $N = 1$ superconformal theories as mass deformed finite $N = 2$ theories. In the simplest case the $N = 2$ theory is $SU(n)$ gauge theory with $n$ fundamental hypermultiplets, whose $\beta$-function is zero. In $N = 1$ language the superpotential has the form

$$W = gQ\bar{Q}X,$$  \hspace{1cm} (4)

where $X$ is an adjoint scalar multiplet and the $2n$ fundamental fields $Q$ and $\bar{Q}$ originate from the hypermultiplets. Now giving mass to $X$ via $W_{\text{mass}} = mX^2$ breaks the supersymmetry to $N = 1$ with, after integrating out $X$, the marginal operator

$$W_{\text{new}} = h(Q\bar{Q})^2, \quad h = \frac{g^2}{2m}.$$  \hspace{1cm} (5)

As discussed in [10] there is a second way to flow to the curve parametrized by this operator. They consider supersymmetric quantum chromodynamics (SQCMD), that is $SU(n)$ gauge theory with $2n$ flavors, a singlet meson field $N$ and:

$$W = \lambda NQ\bar{Q} + \frac{m_0}{2}N^2.$$  

Again there is the marginal operator $\ast$ $(Q\bar{Q})^2$, since $\beta_{\text{gauge}} \propto (n - n\gamma_Q) \propto \beta_{\lambda}$ and hence vanishing of the $\beta$ functions only imposes one constraint on the

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$\ast$ Keeping track of the index structure the second term in the superpotential should really read $N_r^a N_s^a - \frac{1}{n} N_r^a N_s^a$ for $n$ colors. Similarly the $(Q\bar{Q})^2$ operator will read $(Q_r^a Q_s^a)(Q_r^b \bar{Q}_s^b) - \frac{1}{n} (Q_r^a Q_s^a)(Q_r^b \bar{Q}_s^b)$. Here and in the rest of the paper we will use the compact and sloppy notation $(Q\bar{Q})^2$. 

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two couplings. Integrating out $N$ from the superpotential, we generate the marginal operator with a coupling $\lambda/2m_0$.

One interesting point is that the S-duality of the finite $N = 2$ theory, $g \leftrightarrow \frac{1}{g}$, translates directly into a $N = 1$ S-duality, $h \leftrightarrow \frac{1}{h}$, on the fixed line parametrized by $h$. The special point where this operator is turned off corresponds to $N = 1$ with no superpotential. In SQCMD this can be achieved at $m_0 = \infty$, in $N = 2$ by $g = 0$. At this point the global symmetry is enhanced. In SQCMD there is another special point with this enhanced symmetry, $m_0 = 0$. Here we have Seiberg’s dual $SU(n)$ with $2n$ flavors and $W = Nq\tilde{q}$. In the $N = 2$ language this corresponds to the free magnetic theory at $g = \infty$. Therefore the $N = 2$ S-duality means from the $N = 1$ point of view that the theory is selfdual under Seiberg duality.

This type of $N = 1$ gauge theory with gauge group $SU(n) \times SU(n)$, bifundamental chiral matter fields and with quartic superpotential precisely appears as the superconformal theory living on $n$ D3 branes probing a conifold, respectively as the dual supergravity on $AdS_5 \times T^{1,1}$ ($T^{1,1} = (SU(2) \times SU(2))/U(1)$). In a T-dual brane picture [13, 14] à la Hanany-Witten [15], the mass deformations corresponds to the rotation of one of the two NS branes by a certain angle. On the other hand, the flow from $N = 2$ to $N = 1$ corresponds in the supergravity context to deforming the $N = 2$ orbifold space $S^5/Z_2$ by a blow up to the coset space $T^{1,1}$.

2.2 The flow from $N = 4$ theories to superconformal $N = 1$ theories

By the same reasoning as in the finite $N = 2$ case we can also study the $N = 4$ theory. This is really just a special case of a finite $N = 2$ theory. The matter content is just one adjoint hypermultiplet. So the analysis from above applies to this case as well. This has however some interesting implications. So let us spell out this “result” that is implicit in the analysis of [10].

From the $N = 1$ point of view, the $N = 4$ theory provides us three adjoint chiral fields $A$, $B$ and $X$. The superpotential is just the cubic expression $W = g f^{abc} A_a B_b X_c$. Now we add the mass term for the chiral field $X$. As a result we get that any $N = 1$ theory with two adjoint matter fields $A$ and $B$ allows for a marginal deformation by adding the quartic superpotential

$$W = \frac{g^2}{2m} f^{abc} f^{dec} A_a B_b A_d B_c. \quad (6)$$
Not all values of this marginal coupling are distinct. There exists an S-duality inherited from the $N = 4$ theory, mapping strong coupling to weak coupling. This is difficult to see from the field theory point of view, but it is a direct consequence of type IIB S-duality and the AdS/CFT correspondence once the dual supergravity description is established. As in the case of the finite $N = 2$ theory this S-duality seems to imply a selfduality of the 2 adjoint theories under Seiberg duality. Let us add a few comments about this model:

(i) The self-duality under Seiberg duality is not in apparent conflict with the models with two adjoints discussed by [16] where always an ADE-type of superpotential is present.

(ii) In contrast to the previous case we cannot reach the marginal operator eq.(6) from an SQCMD description with four singlet meson fields. Four massive singlets would yield

$$W = (\text{tr}AB)^2$$

which can only be identified with (6) if the product of two $f^{abc}$ symbols can be written as a product of $\delta^{ab}$ symbols. This is however only possible for $SU(2)$.

(iii) This deformation is not the same as the deformation of the $N = 4$ theory by a mass term for a full hypermultiplet as e.g. studied in [11]. Latter one leaves an $N = 2$ SUSY unbroken and the mass deformation is given by one complex parameter. In our case it is just one real mass parameter.

(iv) As in the previous case of deforming $N = 2$ models there will be again a description in terms of deformed brane configurations, now in terms of a brane box. In contrast to the realization of the mass deformed $N = 4$ theory on the interval studied by [12] where one gives a complex mass to a hypermultiplet leaving $N = 2$ unbroken, the brane box naturally give the possibility to incorporate a real mass for a chiral multiplet (breaking down to $N = 1$) by a very similar mechanism. A more detailed description of this duality will be given in [17].

*6 We are grateful to M. Strassler for correcting us on this point.
3 The dual supergravity description

Following the ideas of [2] one would expect these conformal field theories to have a dual supergravity description. Since the field theory arises as a mass deformation of $N = 4$ SYM, the dual supergravity description should be a deformation of the usual IIB string theory on $AdS_5 \times S^5$. In [8, 9, 1] such deformations were studied by analyzing critical points of $N = 8$ gauged supergravity. In [1] a new fixed point was found, leaving $N = 2$ unbroken in the bulk (corresponding to $N = 1$ on the brane). We will argue that this deformation indeed corresponds to the dual of the superconformal field theories we were studying in this paper.

As a first piece of evidence for our identification of the conformal $N = 1$ theory obtained by mass deforming the $N = 4$ theory with the SUGRA solution of [1] let us compare the global symmetries. According to [1] the subgroup of $SO(5)$ unbroken by the solution is $SU(2) \times U(1)$. The $SU(2)$ in the field theory rotates the 2 adjoints into each other. The $ABAB$ superpotential is invariant. The $U(1)$ is the $U(1)_R$ symmetry of the $N = 1$ theory under which $A$ and $B$ both have charge $1/2$.

A more quantitative test is to compare the ratio of the central charges $c$ of the undeformed UV and the deformed IR theory. The UV central charge will be given just by the free field contributions. The central charge of the IR conformal theory can be calculated from the anomaly of the R-charge, since they sit in the same supermultiplet. This calculation can be found in great detail e.g. in [7].

On the supergravity side the ratio can be calculated by comparing the volume of the undeformed SUGRA solution with that of the deformed. The prediction is $27/32$. Let us show that this value is reproduced by our proposed dual field theory.

As in [7] we calculate the central charge $c$ and the axial charge $a$ computing [18] the correlators among the energy momentum tensor $T$ and the R-current $R$:

$$\partial_\mu < TTR > \sim a - c$$

(8)

and, with the same proportionality factor,

$$\frac{9}{16} \partial_\mu < RRR > \sim 5a - 3c.$$  

(9)
First consider the UV theory. The UV theory is the unbroken $N = 4$ theory. It has $c = 1/4 \cdot (N_c^2 - 1)$. To see this use the above relations. We have $(N_c^2 - 1)$ gauginos with $r = 1$ and $3 \cdot (N_c^2 - 1)$ matter fermions with $r = -1/3$ (the superpotential has to have $r = 2$, so the scalars have $r = 2/3$ and the fermions $r = -1/3$).

$\partial_\mu < TTR >$ is given by the sum of all $r$-charges,

$$\partial_\mu < TTR > = (N_c^2 - 1) \cdot \left[ 1 \right] + 3 \cdot (N_c^2 - 1) \cdot \left[ -\frac{1}{3} \right] = 0$$  \hspace{1cm} (10)

hence $a - c = 0$ or $a = c$. Moreover

$$\frac{9}{16} \partial_\mu < RRR > = \frac{9}{16} \left\{ (N_c^2 - 1) \cdot \left[ 1 \right]^3 + 3 \cdot (N_c^2 - 1) \cdot \left[ -\frac{1}{3} \right]^3 \right\}$$

$$= \frac{1}{2} \cdot (N_c^2 - 1)$$  \hspace{1cm} (11)

hence $5a - 3c = \frac{1}{2} \cdot (N_c^2 - 1)$ and with $a = c$ we get

$$c_{UV} = \frac{1}{4} \cdot (N_c^2 - 1).$$  \hspace{1cm} (12)

In the IR we see the mass deformed $N = 1$ theory, so $W = ABX + X^2$ produces $W = (AB)^2$, and we are left with $(N_c^2 - 1)$ gauginos with $r$-charge $r = 1$ plus $2 \cdot (N_c^2 - 1)$ matter fermions $A, B$ with $r$-charge $r = -1/2$. Therefore

$$\partial_\mu < TTR > = (N_c^2 - 1) \cdot \left[ 1 \right] + 2 \cdot (N_c^2 - 1) \cdot \left[ -\frac{1}{2} \right] = 0,$$  \hspace{1cm} (13)

and hence still $a = c$. In addition we have

$$\frac{9}{16} \partial_\mu < RRR > = \frac{9}{16} \left\{ (N_c^2 - 1) \cdot \left[ 1 \right]^3 + 2 \cdot (N_c^2 - 1) \cdot \left[ -\frac{1}{2} \right]^3 \right\}$$

$$= \frac{27}{64} \cdot (N_c^2 - 1),$$  \hspace{1cm} (14)

and hence $5a - 3c = 2c = \frac{27}{64} \cdot (N_c^2 - 1)$ or

$$c_{IR} = \frac{27}{128} \cdot (N_c^2 - 1).$$  \hspace{1cm} (15)
Comparing with above we find \( \frac{c_{IR}}{c_{UV}} = \frac{27}{32} \) as predicted by supergravity! It would be interesting to compare also the chiral spectrum of the superconformal field theory with the spectrum of the scalar Laplacian of the deformed \( \mathbb{S}^5 \) manifold.

Note that the numerical value \( \frac{27}{32} \) is precisely the same as the one obtained in the related setup of the conifold as viewed as a mass deformation of the \( Z_2 \) orbifold [7, 12]. This lead [1] to the speculation that these two theories are indeed related. Here we see that they are quite distinct. The reason for the matching of the numerical values is just due to the mechanism by which we deform: a finite theory with a cubic superpotential (the only choice in a finite theory) is deformed by a mass term, giving rise to quartic superpotential while killing 1/3 of the fields. The superpotential uniquely fixes the \( r \)-charge which in turn determines the central charge.

Having identified the deformation of [1] leading to an \( N = 1 \) superconformal field theory in the dual language, one might hope that we can understand the deformations leading to the \( N = 0 \) theories in a similar spirit.

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