Quintessence and the Relic Density of Neutralinos

Pierre Salati\textsuperscript{a,b} \textsuperscript{*}

\textit{a) Laboratoire de Physique Théorique LAPTH, B.P. 110, F-74941 Annecy-le-Vieux Cedex, France.}
\textit{b) Université de Savoie, B.P. 1104, F-73011 Chambéry Cedex, France.}

July 17, 2003 – LAPTH–951/02

The archetypical model for the recently discovered dark energy component of the universe is based on the existence of a scalar field whose dynamical evolution comes down today to a non–vanishing cosmological constant. In the past – before big–bang nucleosynthesis for that matter – that scalar field could have gone through a period of kination during which the universe has expanded at a much higher pace than what is currently postulated in the standard radiation dominated cosmology.

I examine here the consequences of such a period of kination on the relic abundance of neutralinos and find that the latter could be much higher – by three orders of magnitude – than what is estimated in the canonical derivation. I shortly discuss the implications of this scenario for the dark matter candidates and their astrophysical signatures.

I. INTRODUCTION.

The recent WMAP observations of the Cosmic Microwave Background (CMB) anisotropies \cite{1}, combined either with the determination of the relation between the distance of luminosity and the redshift of supernovae SNeIa \cite{2}, or with the large scale structure (LSS) information from galaxy and cluster surveys \cite{3}, give independent evidence for a cosmological average matter density of $\Omega_m = 0.27 \pm 0.04$ \cite{1}. This value may be compared to a baryon density of $\Omega_b = 0.044 \pm 0.004$ as indicated by nucleosynthesis \cite{4} and the relative heights of the first acoustic peaks in the CMB data. A significant fraction of the matter in the universe is dark and non–baryonic. The cosmological observations also point towards a flat universe with $\Omega_{\text{tot}} = 1.02 \pm 0.02$ and strongly favour the existence of a cosmological constant which contributes a fraction $\Omega_\Lambda = 0.73 \pm 0.04$ to the closure density. The pressure–to–density ratio $w$ of that fluid is negative with a value of $w = -1$ in the case of an exact cosmological constant. That $\Omega_\Lambda$ component is called dark energy as opposed to the $\Omega_m$ dark matter contribution.

The nature of the astronomical dark matter is still unresolved insofar. The favorite candidate for the non–baryonic component is a weakly–interacting massive particle (WIMP). The so–called neutralino naturally arises in the framework of supersymmetric theories. Large efforts have been devoted in the past decade to pin down these evading species. New experimental techniques have been devised to look for the direct and indirect astrophysical signatures of the presence of neutralinos in our Milky–Way \cite{5} as well as in extra–galactic systems \cite{6}. The uncertainty on the theoretical estimates of the various signals has been considerably reduced. As an illustration, the energy spectrum of secondary spallation antiprotons – the natural background to a putative neutralino–induced antiproton extra radiation – is now well under control \cite{7}. Another example of the level of sophistication which the theoretical investigations have reached is provided by the calculations of the neutralino relic abundance $\Omega_\chi$. The observation that this relic density – depending on the numerous parameters of the model – falls in the ballpark of the measured value for $\Omega_m$ has been a crucial argument in favor of supersymmetric particles as a viable option to non–baryonic dark matter.

A large number of processes – typically $\sim 2000$ – are now taken into account and the corresponding diagrams are automatically generated and calculated with the help of numerical codes such as micrOMEGAs \cite{8}. Co–annihilations are taken into account and the thermal averaging $\langle \sigma v \rangle$ of the product of the velocity by the cross section is performed.

Surprisingly enough, calculations of $\Omega_\chi$ are based on the assumption that the universe is dominated by radiation when neutralinos decouple from the primordial plasma and reach their relic density. This hypothesis is presumably correct as soon as primordial nucleosynthesis (BBN) sets in at a time of $\sim 1$ second. We have however little information on the earlier pre–BBN period, a crucial stage during which neutralinos freeze out. If the expansion of the universe is modified with respect to a pure radiation–dominated behaviour, the quenching of these species could be drastically

\textsuperscript{*}E–mail: salati@lapp.in2p3.fr
modified. An increase in the expansion rate $H$ accelerates the decoupling of neutralinos and translates into larger values for the relic density $\Omega_\chi$.

Exploring the effects of a modified expansion history of the universe onto the relic abundance of neutralinos is no longer a mere academic exercise. Such an analysis has become mandatory inasmuch as a new and unexpected component – the dark energy $\Omega_\Lambda$ – has been discovered. The potential interplay between that component and its matter counterpart $\Omega_m$ is worth being explored and may have unexpected consequences. The archetypal model for the cosmological dark energy is the so–called quintessence [9,10] and relies on the existence of a neutral scalar field $\Phi$ with Lagrangian density

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - V(\Phi). \quad (1)$$

Should the field $\Phi$ be homogeneous and the metric be flat, the energy density may be expressed as

$$\rho_\Phi \equiv T^0_0 = \frac{\dot{\Phi}^2}{2} + V(\Phi), \quad (2)$$

whereas the pressure obtains from $T_{ij} \equiv -g_{ij}P$ so that

$$P_\Phi = \frac{\dot{\Phi}^2}{2} - V(\Phi). \quad (3)$$

If the kinetic term $\dot{\Phi}^2/2$ is negligible with respect to the contribution of the potential $V(\Phi)$, a pure cosmological constant with $w_\Phi = P_\Phi/\rho_\Phi = -1$ is recovered since $\rho_\Phi = -P_\Phi = V(\Phi)$. As indicated by cosmological observations, this is the case today. But the field $\Phi$ has been continuously rolling down. Should the kinetic term $\dot{\Phi}^2/2$ have dominated over the potential $V(\Phi)$ in the early universe, a period of kination – i.e., domination by the kinetic energy of the field $\Phi$ – would have ensued with drastic effects on the expansion rate of the universe [11].

In section II, we briefly recall why a pure cosmological constant should be disregarded and replaced by a dynamical dark energy component in the form of a scalar field, the so–called quintessence whose salient features are presented. The existence of tracking solutions provides a natural solution to the problem of initial conditions. We also pay some attention to the difficulty of generating a kination–dominated expansion in the early universe together with a cosmological constant today [10]. We show that this difficulty may be circumvented depending on the potential $V(\Phi)$ that drives the evolution of the scalar field and we propose examples where quintessence boosts the expansion rate in the past while it still accounts for $\Omega_\Lambda$ today. Following a suggestion by [12], we investigate in section III the effects of kination on the thermal decoupling of neutralinos and derive an approximate relation between their relic abundance $\Omega_\chi$ and their annihilation cross section in the presence of kination. Section IV is devoted to a discussion of the consequences of this scenario on the astrophysical signatures of neutralino dark matter.

## II. KINATION AND THE EXPANSION RATE OF THE UNIVERSE.

Two difficulties arise with a pure cosmological constant. The coincidence or fine–tuning problem lies in the fact that the vacuum energy $\rho^0_\Lambda$ comes into play only today and is therefore of order the closure density $\rho^0_\Lambda$, an exceedingly small value with respect to the typical Planck energy scale $M^4_{\text{Planck}}$ set by particle physics

$$\frac{\rho^0_\Lambda}{M^4_{\text{Planck}}} \sim \frac{4 \text{ keV cm}^{-3}}{\{1.22 \times 10^{19} \text{ GeV}\}^4} \sim 10^{-123}. \quad (4)$$

The other issue is related to the initial conditions in which a pure cosmological constant has to be prepared. At the Planck time, the corresponding vacuum energy density $\rho^i_\Lambda = \rho^0_\Lambda$ needs to be exceedingly fine–tuned with respect to the radiation density

$$\frac{\rho^i_\Lambda}{\rho^0_{\text{rad}}} \sim \left(\frac{T_0}{M_{\text{Planck}}}\right)^4 \sim 10^{-125}. \quad (5)$$

Because a pure cosmological constant does not vary in time, similar values are obtained in the previous relations. This has led to some confusion between the fine–tuning and the initial condition problems. Quintessence actually solves only the latter difficulty while the fine–tuning of $\rho^0_\Lambda$ with respect to $M^4_{\text{Planck}}$ is forced by direct observation.
The idea of quintessence is based on the existence of a scalar field $\Phi$ that rolls down its potential $V$ according to the homogeneous Klein–Gordon equation

$$\ddot{\Phi} + 3H \dot{\Phi} + \frac{\partial V}{\partial \Phi} = 0. \quad (6)$$

Should $\Phi$ satisfy to that relation, its energy–momentum tensor would be immediately conserved as may be inferred from the identity

$$D_\mu T^{\mu \alpha} = (\partial_\alpha \varphi) \cdot \left\{ D_\mu (\partial_\mu \varphi) + \frac{dV}{d\varphi} \right\}. \quad (7)$$

That energy–momentum tensor obtains from the Lagrangian density (1) and may be written as

$$T_{\mu \nu} = \partial_\mu \varphi \partial_\nu \varphi - g_{\mu \nu} L. \quad (8)$$

Its conservation translates into the adiabatic expansion of quintessence as

$$\frac{d}{dt} (\rho \Phi a^3) = -P_\Phi \frac{da^3}{dt}, \quad (9)$$

where the dark energy density $\rho_\Phi$ and pressure $P_\Phi$ have been defined in relations (2) and (3). The expansion rate $H = \dot{a}/a$ is increased by the presence of quintessence

$$H^2 = \frac{8 \pi G}{3} \left\{ \rho_B + \rho_\Phi \right\}, \quad (10)$$

where the energy density of the background $\rho_B = \rho_{\text{rad}} + \rho_m$ is dominated by radiation in the past and by matter since equality. Notice that at fixed equation–of–state $w = P/\rho$, the energy density scales as

$$\rho \propto a^{-3 (1 + w)}. \quad (11)$$

As discussed in [10], we are looking for tracking solutions of (6) for which $w_B \geq w_\Phi \simeq \text{constant}$. This allows a natural and smooth decrease of the quintessence energy density $\rho_\Phi$ that remains subdominant until recently when it takes over from the background $\rho_B$. This scenario is realized only if $|V'/V|$ decreases with $V$ as time goes on. This translates into the condition

$$\Gamma = \frac{V''V}{V'^2} > 1, \quad (12)$$

where the prime denotes the derivative with respect to the field $\Phi$. As long as the background dominates – for $\Omega_\Phi = \rho_\Phi/\left(\rho_\Phi + \rho_B\right) \ll 1$ – the parameter $\Gamma$ is related to the equation–of–state $w_\Phi$ of the tracking solution through

$$w_\Phi = \frac{w_B - 2 \left(\Gamma - 1\right)}{1 + 2 \left(\Gamma - 1\right)}. \quad (13)$$

Inverse power law potentials $V \propto \Phi^{-\alpha}$ with $\alpha > 0$ naturally drive quintessence since $\Gamma - 1 = 1/\alpha > 0$. On the contrary, exponential potentials $V \propto \exp \left(-\Phi/M\right)$ for which $\Gamma = 1$ and $w_\Phi = w_B$ should be disregarded.

As shown in [10], the tracking solutions are attractors towards which the field $\Phi$ relaxes for a large range of values of the initial energy density $\rho_\Phi^0$. Quintessence solves therefore the initial condition problem to a large extent – actually to the extent that $\rho_\Phi^0$ is comprised between $\rho_\Lambda^0$ and $\rho_B^0$. The Klein–Gordon equation (6) may be written as

$$\frac{|V'|}{V} = 3 \sqrt{\frac{\kappa}{\Omega_\Phi}} \sqrt{1 + w_\Phi} \left\{ 1 + \frac{1}{6} \frac{d \ln x}{d \ln a} \right\}, \quad (14)$$

where $\kappa = 8 \pi G / 3$ and

$$x = \frac{\dot{\Phi}^2/2}{V} = \frac{1 + w_\Phi}{1 - w_\Phi}. \quad (15)$$
Along the tracking solution, \( w_\Phi \) and \( x \) are constant. When quintessence pops above the background – when \( \rho_\Phi \sim \rho_B \) – relation (14) translates into
\[
\frac{\Phi}{M_{\text{Planck}}} \simeq \frac{\alpha}{\sqrt{24\pi}} \sqrt{\frac{\Omega_\Phi}{1 + w_\Phi} \sim O(1)},
\]
in the case of an inverse power law potential. The field \( \Phi \) is naturally of order the Planck mass \( M_{\text{Planck}} \) when quintessence takes over from the background. The actual coincidence – forced by direct observation – lies in the fact that quintessence becomes dominant today. As its energy density – and therefore the potential \( V \) – are of order the closure density \( \rho_C \), this implies a significant amount of fine–tuning as
\[
V(\Phi_0) = \mu^4 \left\{ \frac{M_{\text{Planck}}}{\Phi_0} \right\}^\alpha \simeq \mu^4 \sim \rho_C^0 \simeq 10^{-47} \text{GeV}^4,
\]
where \( \Phi_0 \sim M_{\text{Planck}} \) is the present value of the scalar field. We conclude at this stage that quintessence does not solve the coincidence or fine–tuning problem [13]. This point cannot be too strongly emphasized.

As also sketched in [10], the entire scenario of quintessence would be in jeopardy should the initial energy density \( \rho_\Phi^i \) exceed \( \rho_B^i \). If so, the initial position from which the scalar field starts to roll lies too high on the potential. The field \( \Phi \) falls precipitously. As its kinetic energy becomes dominant, a kination stage ensues during which the potential \( V \) plays no role. The Hubble friction – see the Klein–Gordon equation (6) – slows down the fall and the field gets frozen at the value \( \Phi_F \) such that
\[
\frac{\Phi_F}{M_{\text{Planck}}} \simeq \sqrt{\frac{3}{4\pi}} \left\{ 1 + \frac{1}{2} \ln \Omega_i \right\},
\]
where \( \Omega_i = \rho_\Phi^i/\rho_B^i \). If, in the early universe, quintessence drives a period of kination with \( \Omega_i \gg 1 \), the field gets frozen at \( \Phi_F \sim 10 - 100 M_{\text{Planck}} \), a value that exceeds by far what is required by the conventional quintessence scheme where \( \Phi_0 \sim M_{\text{Planck}} \). The field has overshot the tracking solution and its energy density has become so small – the potential \( V \) decreases with increasing \( \Phi \) – that it cannot account for \( \rho_\Lambda \). A period of early kination seems therefore incompatible with a near constant dark energy density that becomes dominant today.

The problem raised by oversooting lies in the incompatibility between the conditions \( \Phi_F \gg M_{\text{Planck}} \) and \( \Phi_0 \sim M_{\text{Planck}} \). However the latter has been derived in the framework of inverse power law potentials. We anticipate that the oversooting difficulty could be circumvented with other potentials. As an illustration, we consider the toy–model where
\[
V(\Phi) = \mu^4 \exp \left\{ \frac{M}{\Phi} \right\},
\]
with \( M \sim O(M_{\text{Planck}}) \). In order for dark energy to overcome the background only today, we still need – as before – a considerable amount of fine–tuning as \( \mu^4 \sim \rho_C^0 \simeq 10^{-47} \text{GeV}^4 \). That potential has nevertheless the correct behaviour with \( \Gamma = V'^2/V^2 = 1 + 2 \Phi/M > 1 \) and an associated equation–of–state
\[
w_\Phi = - \left\{ 1 + \frac{M}{4\Phi} \right\}^{-1},
\]
on the tracking solution. The initial condition problem is solved to the extent that \( \rho_\Phi^i \) is now larger than \( \rho_B^i \) and consequently drives a period of kination. In that case, the field rapidly freezes at \( \Phi_F \gg M_{\text{Planck}} \sim M \) – irrespective of its precise initial value – and the potential reaches an asymptotic value of
\[
V(\Phi_F \gg M) \simeq \mu^4 = \rho_\Lambda^0.
\]
This example is remarkable as early kination is now mandatory in order to get today a cosmological constant with \( w_\Phi \simeq -1 \).
In our counter–example to the overshooting no go theorem, the potential (19) has a sharp decrease for small values of the field and then varies smoothly for $\Phi \gg M_{\text{Planck}}$. A dynamical realization of that idea – proposed in [14] – is based on

$$V\{\Phi_1, \Phi_2\} = M^{\alpha+4} \{\Phi_1 \Phi_2\}^{-\alpha/2},$$

where two scalar fields come into play. If one of the fields starts not too far from the tracking solution whereas the other one is dropped from an elevated position on the potential, we still get kination in the past whereas a cosmological constant is recovered today. This behaviour occurs for a wide range of initial conditions on $\Phi_1$ and $\Phi_2$. Other examples are presented in [14] with the same trend and we conclude at this stage that a period of early dominant kination could perfectly occur at the time of neutralino decoupling without precluding a significant contribution today from a near constant dark energy density.

In the standard big bang model, the early universe is only filled with a gas of ultra–relativistic particles that behaves like a radiation of photons. If the kinetic energy $\dot{\Phi}^2/2$ of some additional scalar field component dominates both the potential $V(\Phi)$ and the radiation energy density $\rho_{\text{rad}}$, a period of kination sets in. The overall energy density decreases consequently like

$$\rho_{\text{tot}} \simeq \rho_\Phi \simeq \frac{\dot{\Phi}^2}{2} \propto a^{-6},$$

with respect to the scale factor $a$. This amounts to say that the derivative $\partial V/\partial \Phi$ is negligible with respect to the damping term $3H\dot{\Phi}$ in the Klein–Gordon equation (6) as the field tumbles down the potential. The energy density of radiation is related to the temperature $T$ of the primordial plasma through

$$\rho_{\text{rad}} = g_{\text{eff}}(T) \frac{\pi^2}{15} T^4.$$

The effective number $g_{\text{eff}}$ of degrees of freedom allows to express $\rho_{\text{rad}}$ in units of the corresponding photon density $\rho_\gamma$. The evolution of the scale factor $a$ with respect to the temperature $T$ follows from the requirement that the entropy of the radiation is conserved so that

$$S_\gamma \propto 4 \frac{\pi^2}{45} h_{\text{eff}}(T) T^3 a^3,$$

remains constant as $T$ drops. The effective number $h_{\text{eff}}$ of entropic degrees of freedom is very close to $g_{\text{eff}}$ since

$$h_{\text{eff}} \simeq g_{\text{eff}} \simeq \sum_B \frac{g_B}{2} + \sum_F \frac{7g_F}{16}.$$

The sum is carried over the bosonic $g_B$ and fermionic $g_F$ spin states that correspond to those species that are massless at temperature $T$. We infer that at any given time, the scale factor $a$ and the temperature $T$ are related through

$$a/a_0 = \left( \frac{h_{\text{eff}}(T_0)}{h_{\text{eff}}(T)} \right)^{1/3} \frac{T_0}{T},$$

where $a_0$ is the scale factor at some temperature of reference $T_0$. During the decoupling of neutralinos, $g_{\text{eff}}$ and $h_{\text{eff}}$ do not change drastically so that $a$ varies approximately like $T^{-1}$ at that time. This will prove to be helpful when we derive an approximate formula for the relic density $\Omega_\chi$ in the next section. Numerical results are nevertheless based on equation (27).

We parametrize the contribution of quintessence to the overall energy density through

$$\eta_\Phi = \frac{\rho_\Phi}{\rho_\gamma^0}.$$

The ratio $\eta_\Phi$ of the quintessence–to–photon energy densities is defined at temperature $T_0$. The latter has been set equal to 1 MeV. Because quintessence should not upset the conventional BBN scenario, we expect in principle $\eta_\Phi \leq 1$. Note however that a non–vanishing value for $\eta_\Phi$ may still alter the pre–BBN era because quintessence may be the dominant form of energy at that time if kination holds. Its energy density varies actually like

$$\frac{\rho_\Phi}{\rho_\gamma^0} = \eta_\Phi \left( \frac{h_{\text{eff}}(T)}{h_{\text{eff}}(T_0)} \right)^2 \left( \frac{T}{T_0} \right)^6,$$
whereas for radiation, the temperature dependence is smoother with
\[
\frac{\rho_{\text{rad}}}{\rho_0} = g_{\text{eff}}(T) \left( \frac{T}{T_0} \right)^4.
\] (30)

Going back in time, \(\rho_\Phi\) rises more steeply than \(\rho_{\text{rad}}\). Even for small values of \(\eta_\Phi\), quintessence dominates over the radiation at early times. Up to a numerical constant, we conclude that the expansion rate \(H\) is increased by a factor of \(\sqrt{\eta_\Phi} \ (T/T_0)\) with respect to the conventional radiation dominated cosmology. The next section is devoted to the consequences of such a change in \(H\) on the decoupling of neutralinos.

### III. THE FREEZE–OUT OF NEUTRALINOS.

At early times – as long as the temperature \(T\) exceeds their mass \(m_\chi\) – neutralinos are in chemical equilibrium. They steadily annihilate into lighter species while the reverse process is concomitantly active. The annihilation–production reaction
\[
\chi + \overline{\chi} \rightleftharpoons f - f, \ W^+ W^-, \ Z^0 Z^0, \ H H \ldots
\] (31)

reaches its thermodynamical equilibrium. This implies a neutralino density
\[
n_\chi^0 = g_\chi T^3 e^{-y} \left( \frac{y\, \eta_\chi^2}{2\, \pi} \right)^{3/2},
\] (32)

that only depends on the mass–to–temperature ratio \(y = m_\chi/T\) as well as on the number \(g_\chi\) of spin states. As soon as \(T\) drops below \(m_\chi\), as the result of the overall adiabatic expansion, the neutralino population becomes non–relativistic and the annihilations take over the thermal productions. Neutralinos are severely depleted until their density \(n_\chi^0\) is so low that they fail to annihilate with each other. Reaction (31) stops to be in equilibrium. The resulting freeze–out occurs typically for values of the mass–to–temperature of \(y_\Phi \sim 20\). Because the probability for a neutralino to encounter a partner has become less than unity per typical expansion time \(H^{-1}\), the density \(n_\chi\) remains subsequently constant per covolume – per volume that expands with the expanding universe. The relic abundance \(\Omega_\chi\) readily obtains from the value of \(n_\chi^0\) at freeze–out.

Assuming that there are as many species \(\chi\) than antiparticles \(\overline{\chi}\) – which is an obvious statement if neutralinos are Majorana fermions as is the case for instance in the supersymmetric extensions of the standard model – the density \(n_\chi\) evolves according to the basic equation
\[
\frac{dn_\chi}{dt} = -3H \, n_\chi - <\sigma_{\text{an}} v > \, n_\chi^2 + <\sigma_{\text{an}} v > \, n_\chi^0 n_\chi^2.
\] (33)

The first expression in the right–hand side refers to the dilution resulting from the expansion of the universe. The second term accounts for the neutralino annihilations. The last expression – for which detailed balance has been assumed – describes the back–creations of \(\chi \overline{\chi}\) pairs from lighter species. The neutralino density is given by \(n_\chi^0\) as long as a thermodynamical equilibrium is reached for reaction (31). In the non–relativistic regime at stake here, it is given by relation (32). In terms of the codensity \(f_\chi = n_\chi/T^3\), the evolution equation simplifies into
\[
\frac{df_\chi}{dt} + \{<\sigma_{\text{an}} v > n_\chi\} \, f_\chi = <\sigma_{\text{an}} v > T^3 f_\chi^0 n_\chi^2.
\] (34)

As discussed in the previous section, the temperature \(T\) varies roughly as the inverse of the scale factor \(a\). The codensity \(f_\chi\) corresponds therefore to the number of particles inside a typical volume \(\propto a^3\) that follows the expansion of the universe. In order to solve equation (34), two typical time scales may be defined. To commence, as long as reaction (31) has reached its equilibrium, the time derivative \(df_\chi/dt\) may be neglected and we recover \(f_\chi = f_\chi^0\) as we should. The characteristic time scale with which the neutralino codensity \(f_\chi\) relaxes towards its kinetic – and thermodynamical – equilibrium value \(f_\chi^0\) is set by the annihilation rate
\[
\tau_{\text{rel}}^{-1} = <\sigma_{\text{an}} v > n_\chi.
\] (35)

Then, the time scale of the variations of the equilibrium \(f_\chi^0\) itself may be expressed as
\[
\tau_{\text{eq}}^{-1} = -\frac{df_\chi^0}{dt} \log \left\{f_\chi^{0.2} T^3 \right\} \approx 2H \left\{\frac{m_\chi}{T} \right\},
\] (36)
in the non–relativistic regime. As is clear in Fig. 1, the neutralino freeze–out proceeds in two stages.

(i) At high temperature, as long as $\tau_{\text{rel}} \ll \tau_{\text{eq}}$, $f_\chi$ has plenty of time to relax towards the equilibrium value $f_\chi^0$ that evolves at a much slower pace during this stage. As a consequence, the annihilation reaction (31) reaches thermodynamical equilibrium. A very good approximation for the neutralino density is provided by $f_\chi^0$ as featured in Fig. 1 where the various neutralino density curves all merge for $y \lesssim 10$ whatever the value of the kination parameter $\eta_\phi$. As illustrated by Fig. 2 and 3, relaxation becomes progressively less efficient as long as the temperature decreases. The freeze–out takes place at the very moment when $\tau_{\text{rel}}$ crosses $\tau_{\text{eq}}$.

(ii) From that moment on – below the freeze–out temperature $T_F$ – the relaxation time $\tau_{\text{rel}}$ well exceeds $\tau_{\text{eq}}$. Whilst $f_\chi^0$ drops down and vanishes, $f_\chi$ still decreases a little bit and reaches an asymptotic value of $f_\chi^{\text{asy}}$ that leads directly to the relic abundance $\Omega_\chi h^2$.

To illustrate our discussion, we have considered a generic bino–like species with mass $m_\chi = 250$ GeV and $\tan \beta = 50$. The thermally averaged annihilation cross section has been approximated by

$$< \sigma v > \approx \tilde{a} + \tilde{b} x ,$$

where the parameter $x$ stands for the temperature–to–mass ratio $T/m_\chi$. We have chosen the values $\tilde{a} = 0.1$ pb and $\tilde{b} = 21$ pb for the annihilation cross section. Before freeze–out, the neutralino density $n_\chi$ is set equal to $n_\chi^0$. As soon as decoupling has taken place, we integrate the Boltzmann equation (33) in terms of the codensity $F_\chi = f_\chi / \kappa$ where the coefficient $\kappa(T) = h_{\text{eff}}(T) / h_{\text{eff}}(T_0)$ accounts for the reheating of the radiation as its massive species annihilate and are converted into lighter populations. As is clear from equation (27), the codensity $F_\chi$ evolves exactly as $a^{-3}$. For convenience, we have plotted $f_\chi$ instead of $F_\chi$ as a function of the $m_\chi / T$ ratio in Fig. 1 for three different values of $\eta_\phi$. In Figs. 2 and 3, the age of the universe $t_U$ as well as the typical time scales $\tau_{\text{rel}}$ and $\tau_{\text{eq}}$ are also featured as a function of the mass–to–temperature ratio $m_\chi / T$. The kination parameter $\eta_\phi$ has been respectively set equal to 0 and 1. When the scalar field dominates over the radiation – see Fig. 3 – the expansion of the universe is accelerated with much smaller values for $t_U$. The evolution of $\tau_{\text{rel}}$ with $y$ is not affected by kination. On the contrary, $\tau_{\text{eq}}$ is much smaller than in the conventional radiation dominated cosmology. Freeze–out is therefore reached at a higher temperature with $y_F = 13.5$ for $\eta_\phi = 1$ instead of $y_F = 22.7$ when $\eta_\phi = 0$. The decoupling temperature increases
FIG. 2. The age of the universe $t_U$ as well as the typical time scales $\tau_{\text{rel}}$ and $\tau_{\text{eq}}$ are featured as a function of the mass–to–temperature ratio $y = m_\chi/T$. The freeze–out occurs at $y_F = 22.7$ when $\tau_{\text{rel}}$ overcomes $\tau_{\text{eq}}$. The standard radiation dominated cosmology is assumed here with $\eta_\Phi = 0$ so that $t_U$ evolves like $y^2$.

FIG. 3. Same as in Fig. 2 with a kination parameter of $\eta_\Phi = 1$. The scalar field dominates over the radiation and the expansion is accelerated with respect to the conventional situation. This implies smaller values for the age $t_U$ that evolves now like $y^3$. The freeze–out point is therefore reached earlier with $y_F = 13.5$, hence a larger neutralino relic density.

by a factor of $\sim 2$. This mild variation nevertheless implies a strong rise in the neutralino relic density as a result of
the strong dependence of \( n_0^\chi \) on \( y \) – see relation (32). Fig. 1 features a growth of the asymptotic codensity \( f_\text{asy}^\chi \) by a factor of \( \sim 350 \) when \( \eta_\Phi \) is varied from 0 to 0.01. When \( \eta_\Phi \) is set equal to 1, that increase reaches a factor of \( \sim 2900 \).

The previous example illustrates the large increase of the neutralino relic abundance when kination takes over radiation in the pre–BBN period. The effect of \( \eta_\Phi \) on \( \Omega_\chi h^2 \) may also be understood in the framework of the simple approximation which we derive now. Integrating the cross section \( \sigma_\text{an} V \) over the thermal distribution of neutralinos is beyond the scope of this work. We will simply assume here that relation (37) applies. This is certainly correct as long as the annihilation does not proceed through a s–channel resonance for which the presence of a pole implies an unusual behaviour of the cross section and we will postpone this problem to a later analysis. The freeze–out occurs at temperature \( T_F \) when \( \tau_{\text{rel}} = \tau_{\text{eq}} \). This condition translates into

\[
\left\{ \tilde{a} + \tilde{b} x_F \right\} T_F^3 f_F = 2 y_F H (T_F) .
\]  

(38)

At temperature \( T \), the expansion rate \( H \) is increased with respect to the conventional situation by a factor of \( \sqrt{1 + \alpha x^2} \) where \( x = T/m_\chi \). The parameter \( \alpha \) describes the relative contribution of quintessence to the overall energy density as compared to the radiation

\[
\alpha = \frac{\eta_\Phi}{g_{\text{eff}}(T)} \left( \frac{m_\chi}{T_0} \right)^2 \kappa^2 .
\]

(39)

The evolution of \( f_\chi \) after decoupling is followed by integrating relation (34) from \( x = x_F \) down to \( x = 0 \) while neglecting the right–hand side term. This leads to a decrease of the codensity \( f_\chi \) just after freeze–out from \( f_F \) down to its asymptotic value \( f_\text{asy}^\chi \) with

\[
f_\text{asy}^\chi = \frac{f_F}{\mu} .
\]

(40)

The decrease factor \( \mu \) depends on \( x_F \) and on the parameter \( \alpha \) which we evaluate at the decoupling temperature \( T_F \). This assumption is reasonable because most of the evolution of \( f_\chi \) takes place immediately after freeze–out as featured in Fig. 1. We readily obtain

\[
\mu(x_F, \alpha) - 1 = \frac{2}{x_F} \sqrt{1 + \alpha x_F^2} \left\{ \frac{\tilde{a} A + \tilde{b} x_F B}{\tilde{a} + \tilde{b} x_F} \right\} ,
\]

(41)

where the factors \( A \) and \( B \) depend on the combination \( u = \sqrt{\alpha} x_F \) through the relations

\[
A(u) = \frac{\ln \left\{ u + \sqrt{1 + u^2} \right\}}{u}
\]

(42)

and

\[
B(u) = \frac{\sqrt{1 + u^2} - 1}{u^2} .
\]

(43)

In the conventional scenario for which \( \alpha = 0 \), the previous expressions simplify into \( A(0) = 1 \) and \( B(0) = 1/2 \). The freeze–out codensity may be derived from the condition (38)

\[
f_F = \sqrt{\frac{32 \pi^4}{45}} \left\{ g_{\text{eff}}(T_F) \right\}^{1/2} \sqrt{1 + \alpha x_F^2} \left\{ \frac{m_\chi M_P}{x_F} \left( \tilde{a} + \tilde{b} x_F \right) \right\}^{-1} ,
\]

(44)

where \( M_P \) denotes the Planck mass. The asymptotic codensity \( f_\text{asy}^\chi \) is defined by relation (40) and we infer for the present epoch a neutralino relic density of

\[
\rho_\chi = \frac{4}{11} \frac{1}{\kappa(T_F)} T_F^3 f_\text{asy}^\chi m_\chi .
\]

(45)

The factor \((4/11)(1/\kappa)\) accounts for the reheating of the photon background that occurs in the period extending from the neutralino freeze–out until now. Most of the relevant dark matter candidates have a mass \( m_\chi \) in the range between 100 GeV and 10 TeV. Notice also that the freeze–out temperature–to–mass ratio \( x_F \sim 0.1 \) – 0.2 is not very
sensitive to the quintessence parameter $\eta_\Phi$. We can safely take $g_{\text{eff}}(T_F) \simeq h_{\text{eff}}(T_F) \simeq 45$ for a numerical estimate of the previous expression in order to get
\[
\rho_\chi \simeq 0.69 \text{ keV cm}^{-3} \frac{\sqrt{1 + \alpha x_F^2}}{\mu x_F^2} \left\{ \frac{3 \times 10^{-27} \text{ cm}^3 \text{s}^{-1}}{< \sigma_{\text{ann}} v >} \right\}. \tag{46}
\]
That result is to be compared to the closure density
\[
\rho_C^0 = 1.879 \times 10^{-29} \text{ g cm}^{-3} \simeq 10.6 \text{ keV cm}^{-3} \hbar^2, \tag{47}
\]
hence a neutralino relic abundance that may be expressed as
\[
\Omega_\chi h^2 \simeq 6.6 \times 10^{-2} \frac{\sqrt{1 + \alpha x_F^2}}{\mu x_F^2} \left\{ \frac{3 \times 10^{-27} \text{ cm}^3 \text{s}^{-1}}{< \sigma_{\text{ann}} v >} \right\}, \tag{48}
\]
where the annihilation cross section is taken at decoupling and may differ from its present value if $\tilde{b}$ well exceeds $\tilde{a}$. The approximation (48) tends to overestimate the relic abundance but is still acceptable. In our generic example illustrated in the previous figures, we have numerically derived a value of $\Omega_\chi h^2 = 0.145$ for $\eta_\Phi = 0$ and of $\Omega_\chi h^2 = 440$ for $\eta_\Phi = 1$ whereas expression (48) respectively gives $\Omega_\chi h^2 = 0.16$ and 465.

\section*{IV. DISCUSSION AND PROSPECTS.}

We have shown that the neutralino relic abundance increases if a period of kination takes place during the freeze-out of the species. We derive now an estimate of the corresponding boost factor as a function of $\eta_\Phi$ and $m_\chi$. If $\tilde{a}$ dominates over $\tilde{b} x_F$ in the expression of the annihilation cross section, we may even simplify further relation (48) in order to get
\[
\Omega_\chi h^2 \sim \frac{2 \times 10^{-27} \text{ cm}^3 \text{s}^{-1}}{\tilde{a}} \tag{49}
\]
in the conventional radiation dominated cosmology. We have taken a value of $y_F \sim 20$ for the mass-to-decoupling temperature ratio in that case. On the contrary, if quintessence is the dominant form of energy with a large value for $\eta_\Phi$, the neutralino fossil abundance becomes
\[
\Omega_\chi h^2 \sim \sqrt{\eta_\Phi m_{100}} \left\{ \frac{1.3 \times 10^{-23} \text{ cm}^3 \text{s}^{-1}}{\tilde{a}} \right\}, \tag{50}
\]
where $u = \sqrt{\alpha x_F}$ has already been defined. With a value of $y_F \sim 10$ – see the previous section – this implies
\[
u \simeq 1.5 \times 10^4 \sqrt{\eta_\Phi m_{100}}, \tag{51}
\]
so that the logarithm yields a contribution $\sim 10$. The parameter $m_{100}$ denotes the neutralino mass in units of 100 GeV. A crude estimate of the relic abundance in this regime ensues
\[
\Omega_\chi h^2 \sim \sqrt{\eta_\Phi m_{100}} \left\{ \frac{1 - 2 \times 10^{-24} \text{ cm}^3 \text{s}^{-1}}{\tilde{a}} \right\} \tag{52}
\]
We derive a boost factor of $\sim 10^3 \sqrt{\eta_\Phi m_{100}}$ with respect to the conventional cosmology. If now $\tilde{b} x_F$ is the leading term as regards the annihilation cross section, we find that the relic abundance which is normally given by
\[
\Omega_\chi h^2 \sim \left\{ \frac{10^{-25} \text{ cm}^3 \text{s}^{-1}}{b} \right\}, \tag{53}
\]
is increased to
\[
\Omega_\chi h^2 \sim \sqrt{\eta_\Phi m_{100}} \left\{ \frac{1.2 \times 10^{-22} \text{ cm}^3 \text{s}^{-1}}{b} \right\}. \tag{54}
\]
in the presence of quintessence. The boost factor is still of the order of $\sim 10^3 \sqrt{\eta_\Phi} m_{100}$. In our fiducial illustration, we actually obtained an increase of the neutralino relic abundance by a factor of 3,000 with $m_{100} = 2.5$ and $\eta_\Phi = 1$ in good agreement with the benchmark value which has been derived here.

The increase of $\Omega_\chi h^2$ with $\eta_\Phi$ has interesting consequences and brings up new perspectives as regards neutralino dark matter. To commence, the various avatars of the minimal or non-minimal supersymmetric extensions of the standard model start to be constrained — should R parity be conserved — by the accelerator data on the one hand and by the requirement that the neutralino relic abundance should not overclose the universe or even exceed the observed value of $\Omega_m h^2 = 0.135 \pm 0.009$ [15]. If a period of kination takes place in the pre–BBN period, the various SUSY configurations in the $\Omega_\chi h^2$, $m_\chi$ plane that are so far allowed are shifted upwards with the consequence of becoming forbidden. Exploring in greater detail this question is a worthwhile project.

We already anticipate that configurations with a very small relic density — for instance those for which poles dominate in the annihilation mechanism — would become cosmologically attractive if $\eta_\Phi$ is large enough. The difference with the conventional cosmology lies in the significant enhancement of the annihilation cross section of neutralino dark matter candidates. At fixed $\Omega_\chi h^2$, notice that $\sigma_{\text{ann}} v$ increases precisely by the same factor of $\sim 10^3 \sqrt{\eta_\Phi} m_{100}$ which we have derived above. This means a general enhancement of the various indirect signatures for supersymmetric dark matter. If neutralinos dominate the mass budget of the Milky Way halo, they should still annihilate today and produce gamma–rays, antiprotons and positrons which may be detected through the corresponding distortions in the various energy spectra. As a matter of fact, the recent HEAT experiment has confirmed [16] an excess around 8 GeV in the positron spectrum of cosmic rays. A large boost factor of $\sim 10^3$ – $10^4$ in most of the supersymmetric parameter space is needed to explain that excess in terms of a homogeneous distribution of annihilating galactic neutralinos. A certain degree of clumpiness is actually expected in most of the numerical simulations but even in the extreme case of [17], it does not exceed a few hundreds. A period of kination in the early universe could provide an alternate explanation for that boost factor.

Another potential consequence of quintessence is the rehabilitation of a fourth generation heavy neutrino in the realm of the dark matter candidates. In the conventional cosmology, a 100 GeV stable neutrino provides today a contribution of $\sim 10^{-3}$ to the closure density. Once again, kination at the time of decoupling would enhance that relic density and make that species cosmologically relevant.

Notice finally that scenarios with extra–dimensions have the same effect as kination. The expansion rate is also increased and may even evolve as $T^4$ — to be compared to a $T^3$ behaviour in our case and to a $T^2$ dependence in the conventional radiation dominated universe. Implications of such scenarios on neutralino dark matter should be investigated. In the case of a low reheating temperature at the end of inflation, neutralinos are not thermally produced. Depending on the details of the scenario — decay of an inflaton field [18] or on the contrary decay of moduli fields [19] — the relic abundance is decreased or increased.

A key ingredient of our study is the contribution $\eta_\Phi$ of the quintessential scalar field to the overall energy density at the onset of BBN. A detailed analysis of the light element yields in the presence of kination [20] is mandatory at that stage in order to explore a promising scenario or to derive constraints on $\eta_\Phi$. The existence of dark energy opens up an exciting line of research and the study of its implications on the astronomical dark matter problem will certainly bring surprising results.

ACKNOWLEDGEMENTS

I would like to thank M. Joyce for having pointed out to me this problem in suggesting that a period of kination could potentially affect the freeze–out of neutralinos and modify their relic density.

[1] G. Hinshaw et al., astro–ph/0302217;
C. L. Bennett et al., astro–ph/0302207;
D. N. Spergel et al., astro–ph/0302209.
[2] S. Perlmutter et al. [Supernova Cosmology Project Collaboration], Astrophys. J. 517, 565 (1999).
[3] W. J. Percival et al., astro–ph/0105252, submitted to MNRAS.
[4] S. Burles and D. Tytler, Astrophys. J. 499, 699 (1998) and Astrophys. J. 507, 732 (1998).
[5] A. Bouquet, P. Salati and J. Silk, Phys. Rev. D 40, 3168 (1989);
H. U. Bengtsson, P. Salati and J. Silk, Nucl. Phys. B 346, 129 (1990);
L. Bergstrom, P. Ullio and J. Buckley, Astropart. Phys. 9, 137 (1998).
[6] E. A. Baltz, C. Briot, P. Salati, R. Tailliet and J. Silk, Phys. Rev. D 61, 023514 (2000).
[7] A. Bottino, F. Donato, N. Fornengo and P. Salati, Phys. Rev. D 58, 123503 (1998);
F. Donato, D. Maurin, P. Salati, A. Barrau, G. Boudoul and R. Tailliet, Astrophys. J. 536, 172 (2001).
[8] micrOMEGAs: A program for calculating the relic density in the MSSM, G. Belanger, F. Boudjema, A. Poulov and
A. Semenov, hep-ph/0112278, Comput. Phys. Commun. 149 (2002) 103–120.
[9] R. R. Caldwell, R. Dave and P. J. Steinhardt, Phys. Rev. Lett. 80, 1582 (1998);
E. J. Copeland, A. R. Liddle and D. Wands, Phys. Rev. D 57, 4686 (1998);
A. R. Liddle and R. J. Scherrer, Phys. Rev. D 59, 023509 (1999);
P. J. Peebles and A. Vilenkin, Phys. Rev. D 59, 063505 (1999);
P. Binétruy, Phys. Rev. D 60, 063502 (1999);
P. Brax and J. Martin, Phys. Lett. B 468, 40 (1999);
A. Riazuelo and J. Uzan, Phys. Rev. D 62, 083506 (2000).
[10] P. J. Steinhardt, L. Wang and I. Zlatev, Phys. Rev. D 59, 123504 (1999).
[11] M. Joyce, Phys. Rev. D 55, 1875 (1997);
M. Joyce and T. Prokopec, Phys. Rev. D 57, 6022 (1998);
P. G. Ferreira and M. Joyce, Phys. Rev. D 58, 023503 (1998);
M. Joyce and T. Prokopec, JHEP 0010, 30 (2000).
[12] M. Joyce, private communication. The potential importance of a period of kination on the freeze–out of neutralinos has
been mentioned in M. Joyce’s thèse d’habilitation that was defended in december 2001.
[13] E. Copeland, talk given at the 18th IAP Astrophysics Colloquium : On the Nature of Dark Energy, held in Paris, France,
July 1–5, 2002, eds. P. Brax, J. Martin and J.P. Uzan, Frontier Group (ISBN: 2 914601-13.1) (2002) p. 101–109.
[14] F. Rosati, astro-ph/0302159.
[15] J. Ellis, T. Falk, G. Ganis and K. A. Olive, Phys. Rev. D 62, 075010 (2000);
J. Ellis, K. A. Olive and Y. Santosso, Phys. Lett. B 539, 107 (2002).
[16] S. Coutu et al., Proceedings of ICRC 2001;
S. W. Barwick et al. [HEAT Collaboration], Astrophys. J. 482, L191 (1997);
S. W. Barwick et al. [HEAT Collaboration], Phys. Rev. Lett. 75, 390 (1995).
[17] C. Calcaneo–Roldan and B. Moore, Phys. Rev. D 62, 123005 (2000).
[18] G. F. Giudice, E. W. Kolb and A. Riotto, Phys. Rev. D 64, 023508 (2001).
[19] S. Khalil, C. Munoz and E. Torrente–Lujan, hep-ph/0202139.
[20] R. Bean, S. H. Hansen and A. Melchiorri, Phys. Rev. D 64, 103508 (2001).