SUPERSYMMETRIC SUM RULES
IN MINIMAL SUPERSTRING UNIFICATION

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ABSTRACT

The Minimal superstring unification, assuming orbifold compactification, provides interesting and rather detailed implications on physics at low energy. The interesting feature of this model is that the masses of the spectrum are related since all of them are functions of only two parameters: the goldstino angle $\theta$ and the gravitino mass $m_{3/2}$. This fact will help us in studying the modification of the supersymmetric magnetic moments sum rules which are very sensitive to the supersymmetry breaking. We write these rules in case of exact supersymmetry in a form close to the supersymmetric mass relations, namely $\sum_J (-1)^{2J}(2J)A_J = 0$, where $A_J$ is the anomalous magnetic moment of the spin $J$ particle. We show that the anomalous magnetic moments of the W-boson and the gauginos can be written as functions of $\theta$ and $m_{3/2}$. Then we obtain a modified version of the supersymmetric magnetic moment sum rule in the context of the minimal superstring unification.
1 Introduction

The anomalous magnetic transitions among members of a vector or higher spin supermultiplet are related by model independent sum rules [1]. These rules reduce to \( g_{1/2} = 2 \) for chiral multiplets and to \( g_{1/2} = 2 + 2h \), \( g_1 = 2 + h \) for vector multiplets, where \( g_j \) is the gyromagnetic ratio of a given spin \( j \) particle as defined by \( \mu_j = \frac{e}{2M} g_j J \) and \( h \) is a real number characterizing the magnetic transition between the spin-0 and spin-1 states.

The relevant question that has been addressed in many places, e.g. [2] and [3], is what is the impact of the breaking of supersymmetry on these rules. Clearly, the problem of modifying these sum rules is a problem of SUSY breaking since the anomalous magnetic moment depends on the supersymmetric spectrum which is determined in terms of the SUSY soft breaking terms. In the Minimal Supersymmetric Standard Model (MSSM) with supersymmetry broken explicitly but softly by a universal mass \( \tilde{m} \) for all scalar particles, the total contribution of the anomalous magnetic moments of the W-boson \( \Delta K_{WW} \), the anomalous magnetic moments of the charginos \( a_{\omega_1} \) and \( a_{\omega_2} \) and the magnetic transition between the spin-1 and spin-0 states in a vector multiplet \( \Delta K_{WH} \) has been considered in [2]. However, in this case the sum rules

\[
\Delta K_{WW} = a_{\omega_1} = a_{\omega_2} = \Delta K_{WH}
\]

results to be badly broken without any interesting functional relation among the four quantities. In Ref. [3], we pursued the same strategy in the case of spontaneous breaking of the global N=1 supersymmetry. The hope was that the spontaneous nature of supersymmetry breaking could guarantee the survival of some interesting relations among the various transition magnetic moments. We considered a SUSY spontaneous breaking realized a’la Fayet-Iliopoulos in the realization of Ref. [4]. In that model the following mass splitting is obtained:

\[
\begin{align*}
\Delta m_{\text{gauge}}^2 &= \mu^2, \\
\Delta m_{\text{matter}}^2 &= \frac{1}{4} \mu^2
\end{align*}
\]

where \( \Delta m_{\text{gauge}} \) and \( \Delta m_{\text{matter}} \) denote the mass differences between the fermionic and bosonic components in the vector and scalar multiplet respectively, and \( \mu \) is proportional
to the v.e.v of the Fayet-Iliopoulos term. Our result showed that the sum rules (4) are again badly broken without any surviving clear pattern.

The investigation of this problem in the full realistic case of spontaneous broken N=1 supergravity is the aim of this paper. The most popular way to break SUSY is to assume that the flatness of moduli and dilaton directions of the effective potential are lifted by non-perturbative dynamics and that SUSY breaking arises from the non-vanishing VEV’s of the F-term of modulus $T$ and/or dilaton $S$ supermultiplets. The soft terms become, in general, functions of the the gravitino mass $m_{3/2}$ and the goldstino angle $\theta$. We have shown that the scheme of minimal superstring unification provides interesting and rather detailed implications on physics to be tested in LEPII. In this paper we study the modification of the supersymmetric sum rules in the minimal superstring unification.

The outline of the paper is as follows. In Section 2 we present a brief review of the minimal superstring unification, and we determine the masses of the supersymmetric spectrum which will be needed for calculating the anomalous magnetic moment. In Section 3, we calculate the anomalous magnetic moment of the spin-1 gauge boson and spin-1/2 partner, respectively showing that they are given in terms of only two parameters: the goldstino angle $\theta$ and the gravitino mass $m_{3/2}$. Then we discuss the modification of the supersymmetric sum rules in the context of the minimal superstring unification. In Section 4, we present our conclusions.

2 The Minimal Superstring Unification Model

In this section, we give a brief review of the construction of the soft SUSY breaking terms in the minimal superstring unification models, where the orbifold compactification with large threshold correction is assumed. The soft breaking terms have the form:

The scalar masses are

$$m_i^2 = m_{3/2}^2 (1 + n_i \cos^2 \theta)$$

where $n_i$ are the modular weights which are given by

$$n_{Q_L} = n_{D_R} = -1, \quad n_{u_R} = -2, \quad n_{L_L} = n_{E_R} = -3, \quad n_{H_1} = -2, \quad n_{H_2} = -3$$
as was explained in [8]. The gaugino masses are

\[ M_1 = \sqrt{3}m_3/2 (\sin \theta - 0.02 \cos \theta) \]  
\[ M_2 = \sqrt{3}m_3/2 (\sin \theta + 0.06 \cos \theta) \]  
\[ M_3 = \sqrt{3}m_3/2 (\sin \theta + 0.12 \cos \theta) \]

The trilinear coupling is

\[ A_t = -m_3/2 (\sqrt{3} \sin \theta - 3 \cos \theta) \]

The \( A_t \) term is the only term relevant to the radiative symmetry breaking since we assume that the only top-Yukawa coupling is nonvanishing: Finally, the bilinear coupling is

\[ B = m_3/2 (-1 - \sqrt{3} \sin \theta + 2 \cos \theta) \]

Given the boundary conditions in equations (4) to (9) at the compactification scale \( M_S = 3.6 \times 10^{17} \) GeV, we have to determine the evolution of the couplings according to their renormalization group equation (RGE) to finally compute the mass spectrum of the SUSY particles at the weak scale. For a detailed discussion of these points see Ref. [7].

At the weak scale we find

\[ A_t = m_{3/2} \left[ (3.817 \cos \theta + 5.739 \sin \theta) - r(3.446 \cos \theta + 2.495 \sin \theta) \right], \]  
\[ B = m_{3/2} \left[ (-1 + 2.057 \cos \theta - 0.64137 \sin \theta) - r(1.723 \cos \theta + 1.24767 \sin \theta) \right], \]  
\[ m_{H_1}^2 = m_{3/2}^2 \left( 1 - 1.85 \cos^2 \theta + 0.182 \cos \theta \sin \theta + 1.67 \sin^2 \theta \right), \]

and

\[ m_{H_2}^2 = m_{3/2}^2 \left( 1 - 3.14 \cos^2 \theta + 0.18 \cos \theta \sin \theta + 1.67 \sin^2 \theta \right) \]
\[ + m_{3/2}^2 r \left(-1.5 - 3.11 \cos^2 \theta - 11.58 \cos \theta \sin \theta - 16.44 \sin^2 \theta \right) \]
\[ + m_{3/2}^2 r^2 \left( 5.94 \cos^2 \theta + 8.6 \cos \theta \sin \theta + 3.22 \sin^2 \theta \right). \]

where

\[ r = \frac{Y_t(0)}{Y_t(0) + 5 \times 10^{-4}} \]

and \( Y_t = \frac{\lambda^2}{(4\pi)^2} \) and \( Y_t(0) \) is the t-Yukawa coupling at \( M_S \).

The electroweak symmetry breaking requires the following conditions among the renormalized \( \mu_1^2, \mu_2^2 \) and \( \mu_3^2 \) quantities:
\[ \mu_1^2 + \mu_2^2 > 2\mu_3^2, \quad |\mu_3|^4 > \mu_1^2\mu_2^2, \]  

(14)

and

\[ \mu^2 = \frac{m_{H_1}^2 - m_{H_2}^2}{\tan^2 \beta - 1} - \frac{M_Z^2}{2}, \quad \sin 2\beta = \frac{-2B\mu}{m_{H_1}^2 + m_{H_2}^2 + 2\mu^2} \]  

(15)

where \( \tan \beta = \langle H_2^0 \rangle / \langle H_1^0 \rangle \). Using equations (10)-(15) we find that \( \mu \) and \( \tan \beta \) are given in terms of the goldstino angle and the gravitino mass. Fig.1 shows \( \tan \beta \) as a function of \( \theta \) for different values of \( m_{3/2} \). Thus all the low energy quantities can be determined in terms of only the gravitino mass \( \theta \) and the gravitino angle \( m_{3/2} \). As we have explained in [8] a further constraint on the parameter space is entailed by the demand of colour and electric charge conservations. In particular, the latter conservation yields the most powerful constraint [9] and we find that \( \theta \) is limited approximately by \( \theta \in [0.98, 2] \) rad, and \( m_{3/2} \) is larger than 55 GeV.

As we will see in the next section the mass of the squarks, sleptons and charginos are needed to calculate the anomalous magnetic moment of the W-boson and the gauginos \( \omega_1 \) and \( \omega_2 \). Here we will show explicitly the dependence of these masses on the gravitino mass \( m_{3/2} \) and the goldstino angle \( \theta \). It is well known that the complete expressions for the first two generations squark and slepton mass parameters are given by

\[ m_j^2 = m^2 + \sum_{j=1}^3 f_j M_j^2 + Y_j \tilde{\alpha}_1 S(t) + (T_j^3 - Q_j \sin^2 \theta_W) M_Z^2 \cos^2 \beta \]  

(16)

where the sum is over the gauge groups \( U(1) \), \( SU(2) \) and \( SU(3) \) and

\[ f_j = \frac{c_j(f)}{b_j} [1 - \frac{1}{(1 - \frac{\alpha_{\text{string}} b_j t}{2\pi})^2}], \]

with \( b_j = (11, 1, -3) \) for \( U(1) \), \( SU(2) \) and \( SU(3) \) respectively and \( c_j \) is \( \frac{N^2-1}{N} \) (0) for the fundamental (singlet) representation of \( SU(N) \) and \( Y^2 \) for \( U(1)_Y \). From equation (16) we find that the masses of the superpartners are functions of \( \theta \) and \( m_{3/2} \). For example the mass of the superpartner of the up quark, \( m_\tilde{u}^2 \), is given by

\[ m_\tilde{u}^2 = m_{3/2}^2 \left( 1 - 0.7 \cos^2 \theta + 5.53 \cos \theta \sin \theta + 23.829 \sin^2 \theta \right) \]

\[ - m_{3/2} r \left( 0.5 + 1.037 \cos^2 \theta + 3.86 \cos \theta \sin \theta + 5.48 \sin^2 \theta \right) \]

\[ + m_{3/2}^2 r^2 \left( 1.979 \cos^2 \theta + 2.866 \cos \theta \sin \theta + 1.037 \sin^2 \theta \right) \]

\[ + M_Z^2 \cos 2\beta (1/2 - 2/3 \sin^2 \theta_W). \]  

(17)
Similar formulae for the mass of the superpartner of the down quark $m_{\tilde{d}}^2$, the mass of selectron $m_{\tilde{e}}^2$ and the mass of sneutrino $m_{\tilde{\nu}}^2$ can be obtained.

The squark mass spectrum of the third generation is more complicated for two reasons:

1. the effects of the third generation Yukawa couplings need not be negligible.
2. there can be substantial mixing between the left and the right top squark fields so that they are not mass eigenstates.

Let us keep the top Yukawa coupling only and neglect the others. Therefore, the relations for the mass of $\tilde{b}$, $\tilde{\tau}$ will be as for the first two generations. The masses of the stop are given by:

$$m_{1,2}^2 = \frac{1}{2}(m_{LL}^2 + m_{RR}^2 \pm ((m_{LL}^2 - m_{RR}^2)^2 + 4m_{RL}^4)^{1/2})$$  \hspace{1cm} (18)

where $m_{LL}$, $m_{RR}$ and $m_{RL}$ are as defined in Ref. [10]. From equation (18) we can easily see that the mass of the stop quarks are also functions of only $m_{3/2}$ and $\theta$. Fig. 2 shows the relation between light stop and the $m_{3/2}$ and $\theta$.

Finally, we are also interested in the mass of the charginos $\omega_1$ and $\omega_2$. They are given by

$$M_{2,1}^2 = 1/2(M_2^2 + \mu^2 + 2M_W^2$$

$$\pm \sqrt{(M_2^2 - \mu^2)^2 + 4M_W^4 \cos^2 2\beta + 4M_W^2(2M_2^2 + \mu^2 + 2M_2\mu \sin 2\beta)}.$$ \hspace{1cm} (19)

It is also clear that $M_{2,1}^2$ are functions of $\theta$ and $m_{3/2}$. Fig.3 shows the mass of the lightest chargino as a function of the goldstino angle $\theta$ for different values of the gravitino mass $m_{3/2}$.

3 Supersymmetric Sum Rules In the Minimal Superstring Unification

In supersymmetric theories it was shown that the existing sum rule holds at any order in perturbation theory implying that the anomalous magnetic moment of spin 1/2 particle in chiral supermultiplet is identically zero (i.e. $g_{1/2} = 2$), and

$$\Delta K_{WW} = a_{\omega_1} = a_{\omega_2} = \Delta K_{WH}$$ \hspace{1cm} (20)

for the vector multiplet. These rules have been verified in case of massles ordinary fermions [11] and for massive fermions [12] and [2], where we have $m_b = 0$ with fixed ratio.
\[
\left(\frac{m_W}{m_t}\right)^2 = \alpha, \\
\Delta K_{WW}^{q\bar{q}l} = a_{W_1}^{q\bar{q}l} = a_{W_2}^{q\bar{q}l} = \Delta K_{WH}^{q\bar{q}l} = \frac{-g^2}{32\pi^2}G(\alpha)
\]

with
\[
G(\alpha) = \frac{2}{\alpha^2}[3\alpha + (3 - 2\alpha)\ln(1 - \alpha)].
\]

This function has limit \(\alpha \to 0\) \(G(\alpha) = 1\) and so reproduces the case of negligible \(m_W\). Using the value of the top quark mass \(m_t = 174\), we find that \(\alpha = 0.211389\) and
\[
\Delta K_{WW}^{q\bar{q}l} = a_{W_1}^{q\bar{q}l} = a_{W_2}^{q\bar{q}l} = \Delta K_{WH}^{q\bar{q}l} = \frac{-g^2}{16\pi^2} \times 0.495082.
\]

The work of Refs. [2] and [3] showed that these sum rules are very sensitive to supersymmetry breaking, and even in a very simplified version of a theory with supersymmetry is broken by a universal mass \(\tilde{m}\) for all the scalar, the sum rules (21) is badly broken.

The breaking of supersymmetry in the minimal superstring unification is a consistent way and we have determined explicitly the allowed region in the parameter space where all the experimental and theoretical constraints are satisfied [8]. The interesting feature of this model is that the masses of the supersymmetric particles, as we explained in the previous section, are related since all of them are functions of \(\theta\) and \(m_{3/2}\). This arises again the hope of modifying these sum rules.

We know that, in supersymmetric theories, strong mass relations hold
\[
STr M^2 = \sum_J (-1)^{(2J)}(2J + 1)M_J = 0
\]
and in the case of spontaneously broken local supersymmetry these relations get modified to
\[
STr M^2 = \sum_J (-1)^{(2J)}(2J + 1)M_J = 2(N - 1)m_{3/2} - 2R_i^i F_i F^j
\]
with
\[
R_j^i = [\log det(G_k^i)]_j^i, \quad F_i = exp(-G/2)(G^{-1})_j^i G_j
\]
where \(G\) is the Kähler potential, and \(G^i\) is the derivative of \(G\). In Ref. [3] the F-term of the dilaton field \(S\) and modulus field \(T\), which are the only fields contributing in breaking supersymmetry, are parameterized as follow
\[
(G_{0S}^S)^{1/2}F_0^S = \sqrt{3}m_{3/2}\sin\theta
\]
\[ (G_{0T}^T)^{1/2} F_0^T = \sqrt{3} m_{3/2} \cos \theta \] (28)

We have shown that in this scheme of supersymmetry breaking all the supersymmetric masses are functions of \( \theta \) and \( m_{3/2} \). Then in the minimal superstring unification the relation (25) takes the form

\[ STr M^2 = \sum_J (-1)^{(2J)} (2J + 1) M_J = f(m_{3/2}, \theta) \] (29)

and in case of pure dilaton \( i.e. \theta = \pi/2 \) it becomes

\[ STr M^2 = \sum_J (-1)^{(2J)} (2J + 1) M_J = 2(N - 1) m_{3/2}. \] (30)

We find that the supersymmetric magnetic moment sum rules can be written in a form close to the relations in Eq. (24), namely

\[ \sum_J (-1)^{(2J)} (2J) A_J = 0 \] (31)

where \( A_J \) is the anomalous magnetic moment of the spin \( J \) particle. This simple relation gives the supersymmetric sum rules given in Ref. [1] in both cases of chiral and vector supermultiplet. We can see this easily: in case of chiral supermultiplet this relation leads to \( A_{1/2} = 0 \) and in the case of vector multiplet it leads to

\[ 2 \Delta K_{WW} - a_{\omega_1} - a_{\omega_2} = 0 \]

and since in exact supersymmetry \( a_{\omega_1} = a_{\omega_2} \) we find

\[ \Delta K_{WW} = a_{\omega_1} = a_{\omega_2} \]

which is given in Eq. (31).

The question now is how will this relation be modified in case supersymmetry is broken in the context of the minimal superstring unification and in particular in case of the pure dilaton? Can we get a relation similar to the modified mass relation in Eq. (30)? To answer these questions, we will calculate the total contribution of the anomalous magnetic moment of the W-boson \( \Delta K_{WW} \), and the anomalous magnetic moment of the charginos \( a_{\omega_1} \) and \( a_{\omega_2} \). We will concentrate on the case of vector supermultiplet, since in the case of chiral supermultiplet, as we explained in [3], the anomalous magnetic moment of the fermionic member is different from zero and depends only on the mass of the superpartner,
i.e. in the minimal superstring unification it depends only on the \( \theta \) and \( m_{3/2} \). The result of the one loop contributions to the anomalous magnetic moment of the W-boson, \( \Delta K_{WW} \) is given by:

For first two quark generations,

\[
\Delta K_{WW} = -\frac{g^2}{16\pi^2}.
\]

while for the third quark generation,

\[
\Delta K_{WW} = -0.95347 \times \frac{g^2}{16\pi^2}.
\]

Then the quark contribution to the anomalous magnetic moment of W-boson is given by

\[
\Delta K_{WW}(q) = -1.4767 \times \frac{g^2}{16\pi^2} \quad (32)
\]

The squark contribution to the anomalous magnetic moment of W-boson is given by

\[
\Delta K_{WW} = \frac{g^2}{16\pi^2} \int_0^1 dx \frac{(x^3 - x^2)(\bar{b} - \bar{a} - 1 + 2x)}{bx + \bar{a}(1 - x) - x(1 - x)} + \frac{2g^2}{16\pi^2} \int_0^1 dx \frac{(x^3 - x^2)(\delta a - \bar{b} - 1 + 2x)}{\bar{a}x + b(1 - x) - x(1 - x)} \quad (33)
\]

for one generation. The first two squark generations are identically the same, since \( \bar{a} = \left(\frac{m_{\tilde{u}}}{m_W}\right)^2 \) and \( \bar{b} = \left(\frac{m_{\tilde{d}}}{m_W}\right)^2 \), while the third squark generation contribution is different because \( \bar{a} \) is given by \( \bar{a} = \left(\frac{m_{\tilde{t}}}{m_W}\right)^2 \). As we explained in the previous section \( m_2^\alpha \) and \( m_2^\beta \) are functions of \( m_{3/2} \) and \( \theta \) and therefore \( \Delta K_{WW} \) is also a function of \( m_{3/2} \) and \( \theta \).

The lepton contribution to \( \Delta K_{WW} \) is given by

\[
\Delta K_{WW}(l) = -0.5 \times \frac{g^2}{16\pi^2}. \quad (34)
\]

The slepton contribution to \( \Delta K_{WW} \) is given by

\[
\Delta K_{WW}(\tilde{l}) = 3 \times \frac{g^2}{16\pi^2} \int_0^1 dx \frac{(x^3 - x^2)(\bar{b} - \bar{a} - 1 + 2x)}{bx + \bar{a}(1 - x) - x(1 - x)}, \quad (35)
\]

where \( \bar{a} = \left(\frac{m_{\tilde{l}}}{m_W}\right)^2 \) and \( \bar{b} = \left(\frac{m_{\tilde{e}}}{m_W}\right)^2 \). It is clear that \( \Delta K_{WW}(\tilde{l}) \) is also a function of \( \theta \) and \( m_{3/2} \). In Fig.4 we plot the total contribution of quarks, squarks, leptons and sleptons to the anomalous magnetic moment of the W-boson, \( \Delta K_{WW}(q, \tilde{q}, l, \tilde{l}) \), as a function of the gravitino mass \( m_{3/2} \) and for different values of the goldstino angle \( \theta \).
The anomalous magnetic moment of the gauginos $a_{\omega i}$ are given by:

$$a_{\omega i} = \frac{g^2}{16\pi^2} \left( \frac{g^2}{16\pi^2} \left( 4 \int_0^1 dx \frac{x^2(x - 1)}{ax - x(1 - x)} - 2 \int_0^1 dx \frac{x^2(x - 1)}{\tilde{a}(1 - x) - x(1 - x)} \right) \right. - 2 \left. \int_0^1 dx \frac{\tilde{b}x^2(1 - x)}{bx - x(1 - x)} + 4 \int_0^1 dx \frac{\tilde{b}x^2(1 - x)}{b(1 - x) - x(1 - x)} \right)$$

(36)

for the first two generations of the quark and squark, and $\tilde{a} = (\frac{m_\tilde{u}}{m_\tilde{u}})^2$, $\tilde{b} = (\frac{m_\tilde{d}}{m_\tilde{d}})^2$. While for the third generation it is given by

$$a_{\omega i} = \frac{g^2}{16\pi^2} \left( 2 \int_0^1 dx \frac{x^2(x - 1)}{ax - x(1 - x)} - \int_0^1 dx \frac{x^2(x - 1)}{\tilde{a}(1 - x) - x(1 - x)} \right)$$

$$- \int_0^1 dx \frac{\tilde{b}x^2(1 - x)}{bx + a(1 - x) - x(1 - x)} + 2 \int_0^1 dx \frac{\tilde{b}x^2(1 - x)}{ax + b(1 - x) - x(1 - x)}$$

(37)

where $\tilde{a} = (\frac{m_\tilde{d}}{m_\tilde{d}})^2$, $\tilde{b} = (\frac{m_\tilde{d}}{m_\tilde{d}})^2$ and $a = (\frac{m_\tilde{u}}{m_\tilde{u}})^2$. These quantities are given in terms of $\theta$ and $m_{3/2}$, and therefore $a_{\omega i}$ is a function of $\theta$ and $m_{3/2}$. Finally, the lepton contribution is given by

$$a_{\omega i} = \frac{g^2}{16\pi^2} \left( -3 \int_0^1 dx \frac{x^2(x - 1)}{\tilde{a}(1 - x) - x(1 - x)} - \int_0^1 dx \frac{x^2\tilde{b}(1 - x)}{bx - x(1 - x)} \right)$$

(38)

Similar formulae for $a_{\omega_2}$ can be obtained by replacing $m_\tilde{a}$ by $m_\tilde{d}$ and vice versa, also by changing the value of the electric charge of $u$-quark to the negative value of the electric charge of the $d$-quark and vice versa.

We are interested in finding the modification of the supersymmetric sum rules due to the breaking of supersymmetry. It is clear that it is difficult to deduce a relation among the $\Delta K_{WW}$ and $a_{\omega_i}$ from the above results. So we will try to use the above results to find the functions that depend on the two parameters which $\Delta K_{WW}$ and $a_{\omega_i}$ depend on, namely, $\theta$ and $m_{3/2}$ and fit all the above results of $\Delta K_{WW}$ and $a_{\omega_i}$.

Using the Statistics Nonlinear Fit package in Mathematica we have obtained the functions that fit the results of $\Delta K_{WW}$, $a_{\omega_1}$ and $a_{\omega_2}$ as follows:

$$\Delta K_{WW} = \frac{g^2}{16\pi^2} \left( -0.495082 - 0.02(m_{3/2}/m_W)^2 \cos \theta - 0.2(m_{3/2}/m_W)^2 \sin \theta \right)$$

(39)

$$a_{\omega_1} = \frac{g^2}{16\pi^2} \left( -0.495082 + 0.12(m_{3/2}/m_W)^2 \cos \theta + 0.28(m_{3/2}/m_W)^2 \sin \theta \right)$$

(40)
\[ a_{\omega_2} = \frac{g^2}{16\pi^2} \left( -0.495082 - 0.08 \left( \frac{m_{3/2}}{m_W} \right)^2 \cos \theta + 0.03 \left( \frac{m_{3/2}}{m_W} \right)^2 \sin \theta \right). \]  

Equations (39)-(41) show explicitly that \( \Delta K_{WW}, a_{\omega_1} \) and \( a_{\omega_2} \) are functions of \( \theta \) and \( m_{3/2} \). Also in the limiting case of exact supersymmetry (i.e. \( m_{3/2} = 0 \)) we obtain the values of \( \Delta K_{WW}, a_{\omega_1} \) and \( a_{\omega_2} \) in equation (23). So that in case of breaking supersymmetry in the context of minimal superstring unification we have

\[
\sum (-1)^{\left(\begin{array}{c} 2J \end{array}\right)} A_J = \frac{g^2}{16\pi^2} \left[ -0.08 \left( \frac{m_{3/2}}{m_W} \right)^2 \cos \theta - 0.7 \left( \frac{m_{3/2}}{m_W} \right)^2 \sin \theta \right].
\]

This modified rule gives a relation between the anomalous magnetic moment among members of a vector supermultiplet in case of breaking supersymmetry in the context of the minimal superstring unification. It is also clear that in the case of pure dilaton, where \( \theta = \pi/2 \), these modified rules reduce to

\[
\sum (-1)^{\left(\begin{array}{c} 2J \end{array}\right)} A_J = \frac{g^2}{16\pi^2} \left[ -0.07 \left( \frac{m_{3/2}}{m_W} \right)^2 \right] - 0.64 a_{\omega_1} + 0.7 a_{\omega_2} = -1.16 \times \frac{g^2}{16\pi^2}
\]

This relation between the anomalous magnetic moment of different particles within the same supermultiplet could be a possible way of testing SUSY theories.

## 4 Conclusion

We studied the modification of supersymmetric sum rules in the minimal superstring unification. We determined the mass spectrum of squarks, sleptons and the charginos to calculate the anomalous magnetic moment of the W-boson and the charginos. The interesting feature of the minimal superstring unification is that all the spectrum is determined in terms of two parameters.

We obtained supersymmetric magnetic moment sum rules that relate the anomalous magnetic moment of the members of a vector supermultiplet, and they reduce to the SUSY
sum rules defined in [1] in the case of exact supersymmetry.

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**Figure Captions**

**Fig. 1** The values of $\tan \beta$ as a function of the goldstino angle.

**Fig. 2** The lightest chargino mass in the region of interest for LEPII searches, as a function of the goldstino angle. The horizontal lines correspond to the visibility at LEPII. While the vertical line corresponds to the pure dilaton breaking.

**Fig. 3** The lightest stop mass as a function of the goldstino angle versus.

**Fig. 4** The total contribution of quarks, squarks, leptons, and sleptons to the anomalous magnetic moment of W-boson as a function of the goldstino angle.
\[ \tan(\beta) \]

goldstino angle \([\text{rad.}]\)
the anomalous magnetic moment of the W boson

the gravitino mass [GeV]