Experimental quantum “Guess my Number” protocol using multiphoton entanglement

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We present an experimental demonstration of a modified version of the entanglement-assisted “Guess my Number” protocol for the reduction of communication complexity among three separated parties. The results of experimental measurements imply that the separated parties can compute a function of distributed inputs by exchanging less classical information than by using any classical strategy. And the results also demonstrate the advantages of entanglement-enhanced communication, which is very close to quantum communication. The advantages are based on the properties of Greenberger-Horne-Zeilinger states.

One of the most challenging applications of quantum mechanics for information processing is the reduction of the amount of communication needed to compute a function of a number of inputs distributed between distant parties 1, 2, 3, 4, 5, 6, 7, 8. Compared to the classical scenario, the quantum scenario with the assistance of quantum entanglement has significant advantages; i.e., less classical information is needed from one place to another than the classically required amount of communication, or the quantum scenario can reduce the communication complexity. A particularly attractive and stimulating way of showing the quantum advantages in a multiparty scenario was proposed by Steane and van Dam as a method for always winning the television contest “Guess my Number” (GMN) 9. Steane and van Dam stressed that “A laboratory demonstration of entanglement-enhanced communication would be … a landmark in quantum physics and quantum information science” 9.

Previous experiments have demonstrated the reduction of two-party communication complexity by using two-photon nonmaximally entangled states 6 and the reduction of N-party communication complexity using a single qubit 8. However, the GMN game can provide a more direct and effective demonstration of the reduction of multiparty communication complexity. Though the high detection efficiencies required for the original GMN game 6 have prevented further progress, recently the study of modified versions of the GMN game requiring lower detection efficiencies has prompted 9.

In this paper we demonstrate the reduction of multiparty communication complexity by using genuine multiqubit entanglement. In the experiment, a three-party quantum GMN protocol using multiphoton entanglement is demonstrated to implement a significant reduction of the communication complexity between three separated parties. The protocol is based on a further modification of the GMN game which preserves all the essential features of the original game. The quantum advantages are based on the properties of a polarization-entangled Greenberger-Horne-Zeilinger (GHZ) state 10, 11.

The modified GMN game is as follows: a team of three contestants, Alice, Bob, and Charlie, plays against a TV program’s host. Before the game starts, the contestants decide on a common strategy. Each contestant is then isolated in a separate booth. They can take anything they want with them into the booths, except clocks or devices which would allow any two of them to share a temporal reference 9. At random moments, the host gives the contestant j (j = A, B, C) a randomly chosen number n_j = 0, 1/2, 1, or 3/2 of apples, so that the sum of all three, n = n_A + n_B + n_C, is an integer number. The team’s task is to ascertain whether n is even or odd. Each contestant gives the host a bit value b_j and the team’s answer to the question “what is the parity of n?” is the sum modulo two of these three bits, b = (b_A + b_B + b_C) mod 2. “b = 0” means that they think n is an even number, and “1” means that they think n is an odd number. The contestants are permitted to refuse to answer (i.e., not to give the host a bit) in any round. The host is permitted to give apples to some of the contestants only. Valid rounds are those in which the host has given apples to all three contestants and all of them have given a bit to the host. The host must guarantee that valid rounds are equally distributed among the 32 possible variations of apples.

Considering the above rules, since the contestants can refuse to give bits to the host, the first possible classical strategy for them is to use fixed local instructions like “give the bit b_j = 0 upon receiving n_j = 0 apples, give b_j = 1 upon receiving n_j = 1 apples, and give nothing in other cases.” Of course, the host can recognize this strategy easily. If the host does not prevent their strategy, as a result the team definitely wins the game with a probability of 100%. However, in order to guarantee that valid rounds are equally distributed among the possible vari-
ations of apples, the host will insist on those variations that the contestants are refusing to give bits, so that the contestants cannot take advantage of this possibility.

Another possible classical strategy is to use a secret sequence of local instructions like “give $b_j = 0$ upon receiving $n_j = 0$ apples, give $b_j = 1$ upon receiving $n_j = 1$ apples, and give nothing in other cases, in rounds numbered two, seven and so on; give . . . in rounds numbered one, six and so on; etc.” In other words, the team could make a common table before the game, which displays the corresponding instructions for every round, by randomly selecting only two or three of the four numbers of apples to give bits so that every possible variation appears in the table with the same frequency. This strategy allows the team to win every round if all the contestants agree on which round they are partaking. However, the contestants cannot take advantage of this strategy because none of them knows which valid round he (she) currently finds himself (herself) in, since the host is permitted to give apples to some of the contestants only, and also because the contestants do not share temporal references, since clocks and timing devices are forbidden.

The best classical strategies (i.e., those allowed by classical physics) are previously decided local instructions like “give $b_j = 1$ on receiving $n_j = 0$ or 1/2 apples, and give $b_j = 0$ on receiving $n_j = 1$ or 3/2 apples.” A careful examination reveals that this strategy, which allows the team to win the game with probability of 3/4, is indeed optimal.

Oppositely, the contestants can always win the game by using the following entanglement-assisted protocol.

(1) Each contestant receives a photon belonging to a three-photon entanglement system initially prepared in the GHZ states

$$|\text{GHZ}\rangle = \frac{1}{\sqrt{2}}(|HHH\rangle + |VVV\rangle),$$  

(1)

where $|H\rangle$ and $|V\rangle$ represent horizontal and vertical polarization, respectively.

(2) Each contestant $j$ applies to his photon the rotation

$$R(n_j) = |H\rangle\langle H| + e^{in_j\pi/2}|V\rangle\langle V|,$$

(2)

where $n_j$ is his number of apples.

(3) Each contestant then measures the polarization of his photon in the \{|+,|-\} basis, where $|+\rangle = 1/\sqrt{2}(|H\rangle + |V\rangle)$ and $|−\rangle = 1/\sqrt{2}(|H\rangle − |V\rangle)$.

(4) Sometimes, due to the inefficiency of the detectors, contestant $j$ does not detect his photon. In these cases, he will not give $b_j$ to the host. Note that the inefficiencies keep the contestants from using the detections as a method to have common references in time. When all contestants give the host a bit, their sum modulo 2 is the correct answer, due to the following property of state $|\text{GHZ}\rangle$:

for any $n_A + n_B + n_C$ integer (where $n_j = 0, 1/2, 1, or 3/2$), the result of applying a rotation to each photon is

$$R(n_A) \otimes R(n_B) \otimes R(n_C)|\text{GHZ}\rangle$$

$$= \begin{cases} |\text{GHZ}\rangle & \text{if } n_A + n_B + n_C \text{ is even} \\ |\text{GHZ}^\perp\rangle & \text{if } n_A + n_B + n_C \text{ is odd}, \end{cases}$$

(3)

where $|\text{GHZ}\rangle$ and $|\text{GHZ}^\perp\rangle$ can be reliably distinguished by local measurements in the \{|+,|-\} basis. This can be checked by rewriting the states in that basis:

$$|\text{GHZ}\rangle = \frac{1}{2}(|+++\rangle + |−−−\rangle),$$

(4)

$$|\text{GHZ}^\perp\rangle = \frac{1}{2}(|−++\rangle + |−−−\rangle).$$

(5)

Therefore, the contestants can always win the GMN game with the assistance of quantum entanglement, while they can only win the game with a probability of 3/4 without quantum entanglement. Further analysis indicates that simulating the quantum advantage would require two bits of communication between the contestants. Therefore, the GMN game demonstrates that quantum entanglement can be used to reduce the communication complexity.

We have implemented the three-party quantum GMN protocol using three-photon entanglement. The experimental setup is shown in Fig. 11.

The first step is preparing three-photon GHZ states using the techniques similar to those in previous experiments 12, 13, 14, 15. Ultraviolet pump pulses pass through a $\beta$-barium borate (BBO) crystal twice to produce two pairs of polarization-entangled photons. The states of both pairs after passing through the two additional HWP’s in modes 2 and 4 are

$$|\psi\rangle_{12} = |\psi\rangle_{34} = (1/\sqrt{2})(|HH\rangle + |VV\rangle).$$

(6)

By adjusting the position of a delay mirror, the two photons in modes 2 and 4 simultaneously arrive at the polarizing beam splitter (PBS), which transmits horizontally polarized photons while reflects vertically polarized photons. The PBS is used as a parity check: detectors D2 and D4 fire only when the inputs of the PBS are both $H$ photons or both $V$ photons. Then we can produce the following GHZ state

$$|\Psi\rangle_{1234} = (1/\sqrt{2})(|HHHH\rangle + |VVVV\rangle).$$

(7)

To confirm that the state is in a coherent superposition, we measure the coincident count rates of $+++$ and $+++−$ as a function of the delay mirror’s position to observe interference. After optimal fitting we find the maximum visibility at zero delay is approximately 86%, which indicates that the four photons are indeed in a coherent superposition. When the photon of mode 4
is projected to 45°, the state of remaining three photons will be a three-photon GHZ state

$$|\Psi⟩_{123} = (1/\sqrt{2})(|HHH⟩ + |VVV⟩).$$

(8)

Then we use this three-photon GHZ state to demonstrate the three-party quantum GMN game.

The next step is that the host starts the game and distributes the apples. During the game, at random moments, the host gives contestant $j$ ($j = A, B, C$) a number $n_j$ of apples. In the experiment, the three photons of modes 1, 2, and 3 are distributed to the three contestants and the contestant $j$ then performs the rotation $R(n_j)$ according to the corresponding number of apples, which is implemented using a suitably chosen combina-

![FIG. 1: Experimental setup for the three-party GMN game using three-photon entanglement. Two pairs of polarization-entangled photons are produced by spontaneous parametric down-conversion. A mode-locked Ti:sapphire femtosecond laser, with pulse duration of 200 fs and repetition rate of 76 MHz at 788 nm central wavelength, generates ultraviolet (UV) pulses with 394 nm wavelength, with an average pump power of about 550 mW, after a frequency-doubled process in a $LiB_2O_3$ crystal. The UV pulses pass through a 2 mm thick BBO crystal twice to generate approximately 18 000 forward pairs and 14 000 backward pairs of entangled photons with full width at half maximum (FWHM) = 2.8 nm interference filters at 788 nm. One photon in each pair is overlapped at the PBS temporally and spatially. Three photons of modes 1, 2, and 3 are distributed to the three separated parties A, B and C respectively, while the photon polarization of mode 4 is fixed at 45° as a trigger for the four-fold coincidence. The three parties use $R(n_A)$, $R(n_B)$ and $R(n_C)$, which are implemented using the combination of half-wave plate (HWP) and quarter-wave plate (QWP), to accomplish the rotations required for the quantum GMN protocol respectively. Finally, three polarizers (POL) are used to implement the projective measurements of the linear polarization of photons.

![FIG. 2: Measurement results of the quantum GMN experiment for the 32 possible variations of apples. The square dots (corresponding to the right y axis) represent the number of played rounds. Each variation has been played with approximately the same frequency, in order to guarantee the game’s fairness. The histogram (corresponding to the left y axis) shows the experimental probabilities of winning. The dashed line (1.0) and the solid line (0.75) represent the theoretical maximum quantum and classical probabilities of winning the GMN game, respectively. The experimental probabilities of winning the game are significantly higher than the maximum classical probabilities.

\begin{align*}
R(0) &= |H⟩⟨H| + |V⟩⟨V| \quad \text{nothing} \\
R(1/2) &= |H⟩⟨H| + i|V⟩⟨V| \quad \text{a QWP at 0°} \\
R(1) &= |H⟩⟨H| − |V⟩⟨V| \quad \text{a HWP at 0°} \\
R(3/2) &= |H⟩⟨H| − i|V⟩⟨V| \quad \text{HWP+QWP at 0°},
\end{align*}

(9)

where half an apple is equivalent to a QWP at 0°, while one apple is equivalent to a HWP at 0°. When a fourfold coincidence event is detected it implies that all the three contestants have received the apples and given a bit to the host, respectively, which is a valid round in the game.

During the game the 32 possible variations of apples should be the same frequency. In the experiment we produced the 32 cases through the different combinations of $R(n_j)$. Each case was repeated for enough rounds to reduce the statistical fluctuations and the total round numbers of each case were almost identical. In each case, we measured 8 kinds of coincidences (i.e., $+++$, $++, +−−−$, $++$, $−−−$) and the measurement time of each coincidence was 30 min.

We recorded the 8 numbers of counts $N_{+++}$, ..., $N_−−−$ during 4 h. According to Eq. (8), the number of rounds in which the players answer “even” is

$$N_{\text{even}} = N_{+++} + N_{+−−} + N_{−++} + N_{−−+}$$

(10)

and the number of rounds in which the players answer “odd” is

$$N_{\text{odd}} = N_{−++} + N_{+++} + N_{+−−} + N_{−−−}.$$  

(11)

Therefore, the experimental probability of winning the GMN game is $N_{\text{even}}/(N_{\text{even}} + N_{\text{odd}})$ when $n$ is even and
$N_{\text{odd}}/(N_{\text{even}} + N_{\text{odd}})$ when $n$ is odd. The experimental results are shown in Fig. 2. In all of the 32 variations of apples the experimental probability of winning is higher than the best classical value of 0.75. The number of played rounds of each possible is about 1000. In each variation of apples, the probability of winning is about 0.85 and differs from the best classical value by about 9 standard deviations. On the other hand, the total number of rounds in which the answer was correct is 28768, while the number of rounds in which the answer was incorrect is 5032. Therefore, the mean value of the experimental probability of winning is $P_Q = 0.851 \pm 0.002$, which differs from the classical result with more than 52 standard deviations. This clearly illustrates the advantage of the entangled-assisted strategy. The imperfection of $P_Q$ (the theoretical quantum prediction is $P_Q = 1$) is mainly due to the visibility limitation of multiphoton entanglement and a slight drift of the interference position in the experiment. During the experiment, the laser system cannot be stabilized for enough long time. Therefore the best interference position cannot always be fixed at the same point for enough time.

In conclusion, we have performed an experimental implementation of a quantum protocol which outperforms the best classical strategy for winning a modified version of the GMN game preserving all the essential features of the original one. Our results demonstrate the advantages of entanglement-assisted communication and confirm one of the most challenging predictions of quantum information processing and quantum computation [17, 18, 19]. The experimental triumph in the GMN game shows that entanglement is a physical resource that can be used to reduce the classical communication cost of some distributed computations in a multiparty scenario. The entanglement-assisted reduction of classical communication complexity has a number of potential applications in computer networks, very large scale integrated (VLSI) circuits, and data structures [20], and deserves further research.

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