Two-Loop Matching Onto Dimension Eight Operators in the Higgs-Glue Sector

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This letter presents results for the two-loop matching coefficients for the dimension eight operators that contribute to Higgs production via gluon fusion. The coefficients can be used to calculate the first correction to the infinite top mass limit to Higgs production with large transverse momentum at two loops. To date such processes have been studied at two loop order only in the leading term in the top mass expansion. These corrections become enhanced in processes with large final state invariant mass, typical of multijet processes.

One significant objective of the Large Hadron Collider (LHC) is to produce and study the Higgs particle. To fully understand this sector of the standard model, it is important to have precise theoretical control over the observables associated with the Higgs in a hadronic production environment. The dominant mechanism for Higgs production is gluon fusion through a top-quark loop (for review see [1]). Given that the process starts at one loop, it could easily be enhanced by new physics. Thus an accurate prediction for the cross section and other associated observables is of vital importance, and much work has been done with this in mind [2, 3, 4, 5]. On the other hand, the fact that the process starts at one loop also means that radiative corrections, which are known to be large, are more difficult to calculate. However, the calculations can be greatly simplified by working in an effective theory where the top quark has been integrated out. Doing so effectively reduces the order of the calculation by one loop at the cost of introducing errors that are suppressed by inverse powers of the top mass. To date, the focus has been on the lowest mass dimension six operator generated,

\[ F_{\text{eff}}^6 = C H F_{\mu \nu} F^{\mu \nu} / v. \] (1)

The two loop result [6, 7] for the coefficient \( C \) is given by

\[ C = \frac{g^2}{48\pi^2} + \frac{g^4}{4\pi^4} \left( \frac{5}{192} C_A - \frac{1}{64} C_F \right) \] (2)

For \( m_h < 2m_t \), this leading order (in inverse powers of \( m_t \)) contribution does an excellent job of approximating the inclusive Higgs production rate, with errors on the order of a few percent for a light Higgs. On the other hand, observables requiring a large transverse momentum for the Higgs, like production in association with jets or its transverse momentum spectrum, will be more susceptible to larger power corrections. Such processes have been calculated to two loops in the infinite top mass limit, that is, only including the dimension six operator Eq. (1) [6, 7, 8]. The full \( m_t \) dependence for Higgs plus jet observables is presently only known at one loop [10, 11, 12, 13]. To extend these observables to two loops is of considerable difficulty, however, the calculation is simplified by working in the effective theory. The mass corrections to these results can be included by first calculating the matching coefficients to the set of dimension eight operators, and then using those results to calculate the cross section. The goal of this paper is to present the aforementioned Wilson coefficients. In a forthcoming paper the results for Higgs plus jet cross sections

\[ R = \sigma_{\text{inc}} / \sigma_{p_t>p_t^0} \approx R_{\text{SM}} \] (3)

is approximately model independent if all the new masses are sufficiently large that the effective field theory is well behaved. The corrections to this statement arise from the dimension eight operators. That is

\[ \delta = 1 - R/R_{\text{SM}} \propto C_8 C_6 \] (4)

where \( C_8 \) corresponds to some linear combinations of the Wilson coefficients introduced below. Thus, if \( \delta \) is measured and found to be non-zero, then whether or not one can conclude there must be light new particles in the spectrum can only be determined once one determines if the contribution from the dimension eight operators is sufficiently small. This calculation will be taken up in a future paper.

THE OPERATOR BASIS

Below (Eqs. (5) to (8)) is the list of all possible operators with mass dimension eight coupling gluons to the Higgs, consistent with requirements of Lorentz and color gauge invariance. After using integration by parts to remove any derivatives on the Higgs field, the Bianchi identity was used to remove any remaining relations, thus giving a linearly independent basis. A minimal basis con-
FIG. 1: Representative diagrams for the Higgs-two-gluon Vertex necessary for fixing operators with two gluon Feynman rules.

consists of four operators, and is given by

\begin{align*}
O_a &= \frac{HD_\alpha F^\alpha_{\mu\nu} D^\alpha F^{\alpha\mu\nu}}{m_t^3} \\
O_b &= \frac{HF^{\alpha\mu\nu} D_\delta F^{\beta\delta\mu}}{m_t^3} \\
O_c &= \frac{HD^\alpha F^{\alpha\mu\nu} D_\delta F^{\beta\delta\nu}}{m_t^3} \\
O_d &= \frac{H F^{\alpha\mu\nu} F^{\delta\nu}_{\beta\mu} F^{\alpha\sigma}_{\mu\sigma} f^{abc}}{m_t^3}
\end{align*}

(5) (6) (7) (8)

Three of the four operators couple to two gluons and the Higgs, while the fourth is only involved in process involving three or more gluons. Its color factor only includes the antisymmetric color structure constants. Note that the basis includes two operators \(O_b\) and \(O_c\) that can be traded for operators involving quark bilinears using the equations of motion. However, calculating off-shell will allow us to utilize the Low Energy Theorem, giving a computationally simpler way to calculate the matching coefficients, as discussed below. One can then use the equations of motion to simplify the basis after matching.

METHODS

Canonical matching involves calculating in the full and effective theory and then taking the difference to find the matching coefficients. Alternatively, one can extract the matching coefficient by asymptotically expanding the integrals around hard loop momenta which are taken to be of order \(m_t\) and ignoring other regions which would cancel in the matching. Calculating in this way reduces the amount of work involved.

Matching onto the basis requires asymptotically expanding double boxes in the large top mass limit. The Low Energy Theorem (LET) for the Higgs (see \cite{16} for an overview) allows one to reduce the complexity of the calculation. In its basic form, the Low Energy Theorem states that the amplitude for the process \(X \rightarrow Y + H\) can be related to the process \(X \rightarrow Y\) as

\[ \lim_{p_h \to 0} M(X \rightarrow Y + H) = \sum_i \lambda_i m_{q_i} \frac{d}{dm_{q_i}} M(X \rightarrow Y) \]

where \(p_h\) is the four momentum of the Higgs boson, and \(m_{q_i}\) and \(\lambda_i\) are the masses and couplings of the particles coupling to the Higgs. Diagrammatically this is shown in Figure 3.

To use the LET in the matching, one first calculates off shell the corrections to the Higgs–two-gluon vertex and match onto three of the operators (This vertex has been investigated in the onshell limit for total Higgs production\cite{17}), typical diagrams are found in Figure 1. Then one calculates the top-quark contribution to the three-gluon vertex in QCD. Relating this quantity to the Higgs-three-gluon vertex in the limit of vanishing Higgs four-momentum, one can fix the fourth operator’s matching coefficient. The low energy limit itself is off-shell, hence the inclusion of operators that vanish by the equations of motion. The Low Energy Theorem approach eliminates the need to calculate the 135 two-loop box diagrams for the Higgs–three-gluon effective vertex. Instead one need only calculate 57 two-loop triangle diagrams.

The hard contribution of the integrals is obtained by Taylor expanding in the external momenta. The resulting expansion leaves one with a sum of bubble diagrams which are much simpler to evaluate. An efficient method to accomplish the expansion is to first reduce all integrals to scalar integrals and then perform the Taylor expansion following Tarasov \cite{18, 19}. In this method, one
takes an integral of the form

$$I(s) = \int \prod_{i=1}^{L} d^{d}k_{i} \frac{1}{\prod_{j=1}^{n}(k_{j}^{2} - m_{j}^{2})^{\nu_{j}}} e^{i \sum_{i=1}^{L} k_{i} \cdot a_{i}}$$  \quad (9)$$

where $\bar{k}_{j}$ and $m_{j}$ are the momentum and mass associated with the jth propagator. For each loop momentum, we have introduced an auxiliary vector $a_{i}$ and the exponential factor $e^{i \sum_{j=1}^{L} k_{j} \cdot a_{j}}$. Differentiating with respect to the auxiliary vectors allows one to produce any numerator in the loop momenta in the integral from the scalar integral. But before differentiating to produce the desired tensor integrals, one passes to the $\alpha$ representation, so the above integral has the form

$$I(s, m) = \frac{\Gamma(\sum_{i}^{n} \nu_{i})}{\prod_{i} \Gamma(\nu_{i})} \int_{0}^{\infty} d\alpha_{i} \prod_{i}^{n} \alpha_{i}^{\nu_{i}-1} (D(\alpha))^{-d/2}$$

$$\times \exp\left(\frac{Q(s, \alpha)}{D(\alpha)} - i \sum_{i}^{n} \alpha_{i} m_{i}^{2}\right)$$ \quad (10)

where $s$ are the kinematic invariants formed from the external momenta and auxiliary vectors. $D(\alpha)$ and $Q(s, \alpha)$ are polynomials in $\alpha$ and $s$, uniquely determined by the topology of the diagram. As noted above, differentiating the integral with respect to the auxiliary vectors generates the desired numerator. In the $\alpha$-representation the differentiation generates a polynomial in the external momenta whose coefficients are proportional to scalar integrals having the same form as $I$, but with shifted spacetime dimension and powers of propagators. This is a simple consequence of the fact that $Q$ is polynomial in $\alpha$ and the kinematic invariants. The shift in spacetime dimension accounts for each derivative bringing down an inverse power of $D(\alpha)$. After differentiation, one sets the auxiliary vectors to zero.

Having reduced the diagram to scalar integrals, the Taylor expansion can be performed similarly. The Taylor expansion in the $\alpha$ representation is equivalent to writing out the Taylor series in the $\exp\left(\frac{Q(s, \alpha)}{D(\alpha)}\right)$ factor, and distributing through the $\alpha$ integrations over the terms. Again this results in further shifts in spacetime dimensions and powers of propagators.

After the reduction to scalar integrals and Taylor expansion, one is left with bubble integrals with propagators of arbitrary powers, and shifted spacetime dimensions. For the case of the two-loop calculations in the large mass expansion, all these integrals were of the form

$$\int \frac{d^{d}k_{1} d^{d}k_{2}}{(k_{1}^{2} - m_{2}^{2})^{\nu_{1}}((k_{1} + k_{2})^{2})^{\nu_{2}}(k_{2}^{2} - m_{2}^{2})^{\nu_{3}}}$$ \quad (11)

where a simple analytic result is known for all $\nu_{i}$ and spacetime dimension $d$. To insure gauge invariance of the final results, all effective action vertices were computed in the background field gauge, and renormalized with the $\overline{\text{MS}}$ scheme at the scale $2m_{i}$.

All diagrams were generated with FeynArts\textsuperscript{20} and then analyzed within Mathematica.

**CALCULATIONAL CHECKS**

We have checked that we reproduce the known matching results for the dimension six operator to two loops. This check works independently of the choice of external states, so we reproduce the matching to the dimension six operator in both processes computed.

From the calculation we can extract the anomalous dimensions of the operator basis and compare it to known results, thus providing another non-trivial check on the calculation. In the method of regions, each region develops infrared and ultraviolet divergences, but only the sum over the regions contains the divergences (both UV and IR) of the full theory\textsuperscript{12}. Thus UV divergences of the soft regions must cancel with IR divergences of the hard region. With knowledge of the UV divergences of the effective theory (contained in the anomalous dimensions of the effective operators), and the one loop matching, one can predict the IR divergences of the hard region. The anomalous dimensions of the effective operators have been computed before\textsuperscript{21,22} (the operators considered there were pure QCD operators, but have the same QCD renormalization properties since the higgs field is a color singlet). Thus it becomes a simple matter to check that the coefficients of the logarithms in the asymptotic expansion are the one loop matching coefficients times the renormalization factor needed to substract the UV divergences of the effective theory. Thus schematically if we have in a calculation for the hard region (HR)

$$HR = A_{uv} * (\epsilon_{uv}^{-1} + \text{Log}(\frac{\Lambda^{2}}{\mu^{2}})) +$$

$$B_{IR} * (\epsilon_{IR}^{-1} + \text{Log}(\frac{\Lambda^{2}}{\mu^{2}})) + \text{Finite},$$

the effective field theory (EFT) then has

$$EFT = C_{uv} * (\epsilon_{uv}^{-1} + \text{Log}(\frac{p^{2}}{\mu^{2}})) +$$

$$D_{IR} * (\epsilon_{IR}^{-1} + \text{Log}(\frac{p^{2}}{\mu^{2}})) + \text{Finite}.$$  

Where $\Lambda$ is the hard scale, and $p$ is the effective theory scale. The two are reproducing the full theory when $B_{IR} = -C_{UV}$. The only UV logarithms of the hard region correspond to the top quark mass renormalization. The two types of logarithms are easily distinguished by the associated group theory factors due to the differing representations of quarks and gluons.
RESULTS

The effective lagrangian resulting from integrating out the top to this mass order is:

\[ L_{\text{eff}} = C_1 \frac{H F_{\mu \nu} F^{\mu \nu}}{m_t^4} + C_2 \frac{H D_{\alpha} F_{\mu \nu} D_{\alpha} F^{\mu \nu}}{m_t^4} + \]
\[ + C_3 \frac{H F_{\alpha \beta} F^{\alpha \beta}}{m_t^4} + C_4 \frac{H D_{\alpha} F_{\mu \nu} D_{\alpha} F^{\mu \nu}}{m_t^4} + C_5 \frac{H F_{\alpha} D_{\alpha} D_{\alpha}}{m_t^4} \]

Where

\[ C_1 = \frac{g_2^2}{16\pi^2} + \frac{g_1^2}{4\pi^2} \left( \frac{5}{192} C_A - \frac{1}{64} C_F \right) \]
\[ C_2 = -\frac{7g_2^2}{2880\pi^2} - \frac{g_1^2}{64\pi^2} \left( \frac{129}{34560} C_A - \frac{7}{1920} C_F \right) \log \left( \frac{\pi \epsilon m_t^2}{\mu^2} \right) \]
\[ C_3 = \frac{g_3^2}{180\pi^2} + \frac{g_1^2}{6\pi^2} \left( \frac{49}{9600} C_A + \frac{37}{5760} C_F \right) \]
\[ + \frac{1}{320} \left( \frac{C_A - 4 C_F}{4 C_F} \right) \log \left( \frac{\pi \epsilon m_t^2}{\mu^2} \right) \]
\[ C_4 = \frac{g_2^2}{2240\pi^2} + \frac{g_1^2}{2\pi^2} \left( \frac{101}{601200} C_A + \frac{1}{3240} C_F \right) \]
\[ + \frac{1}{17280} \left( 29 C_A - 9 C_F \right) \log \left( \frac{\pi \epsilon m_t^2}{\mu^2} \right) \]
\[ C_5 = \frac{g_2^2}{80\pi^2} + \frac{g_1^2}{\pi^2} \left( \frac{1169}{518400} C_A + \frac{73}{51840} C_F \right) \]
\[ + \frac{1}{17280} \left( 56 C_A - 81 C_F \right) \log \left( \frac{\pi \epsilon m_t^2}{\mu^2} \right) \]

\[ \lambda = \frac{m_t}{\mu} \] is the yukawa coupling to the top quark. One takes \( m_t \) in what ever scheme one renormalizes the hard region. Then one runs the operators to the low scale using a scheme that is consistent with the scheme used in the matching. For simplicity the coefficients are listed in the MS scheme.

CONCLUSION

We have presented the order \( \alpha_s^2(m_t/m_t)^3 \) Lagrangian coupling the Higgs directly to gluons. This basis will prove useful in understanding the higher order QCD corrections to Higgs production in association with jets, where the range of validity of the standard effective field theory begins to break down due to large final state invariant masses. In a forth coming paper the basis will be used to examine the ghn induced Higgs production with a large transverse momentum observable at the LHC.

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