1 Introduction

The concept of deformation quantization of a symmetric manifold $M$ has been defined by Bayen, Flato, Fronsdal, Lichnerovich, and Sternheimer in [1]. Deformation quantization means a formal $\ast$-product

$$f_1 \ast f_2 = f_1 \cdot f_2 + \sum_{k=1}^{\infty} C_k(f_1, f_2)t^k, \quad f_1, f_2 \in C^\infty(M)$$

with some additional properties, where $C_k : C^\infty(M) \times C^\infty(M) \to C^\infty(M)$ are bidifferential operators.

In the special case of the unit disc in $\mathbb{C}$ with $SU_{1,1}$-invariant symplectic structure, a formal $\ast$-product and explicit formulae for $C_k$, $k \in \mathbb{N}$, are derivable by a method of Berezin [4, 2].

Our intention is to replace the ordinary disc with its q-analogue. We are going to produce $U_qSU_{1,1}$-invariant formal deformation of our quantum disc and to obtain an explicit formula for $C_k$, $k \in \mathbb{N}$, using a q-analogue of the Berezin method [10].

Our work is closely related to the paper of Klimek and Lesniewski [5] on two-parameter deformation of the unit disc. The explicit formulae for $C_k$ we provide below work as a natural complement to the results of this paper.

2 Covariant symbols of linear operators

Everywhere in the sequel the field of complex numbers $\mathbb{C}$ is assumed as a ground field. Let also $q \in (0, 1)$.

Consider the well known algebra $Pol(\mathbb{C})_q$ with two generators $z, z^*$ and a single commutation relation $z^*z = q^2zz^* + 1 - q^2$. Our intention is to produce a formal $\ast$-product

$$f_1 \ast f_2 = f_1 \cdot f_2 + \sum_{k=1}^{\infty} C_k(f_1, f_2)t^k, \quad f_1, f_2 \in Pol(\mathbb{C})_q, \quad (2.1)$$
(with some remarkable properties) to be given by explicit formulae for bilinear operators $C_k : \text{Pol}(\mathbb{C})_q \times \text{Pol}(\mathbb{C})_q \to \text{Pol}(\mathbb{C})_q$.

We describe in this section the method of producing this $*$-product whose idea is due to F. Berezin.

It was explained in [5] that the vector space $D(\mathbb{U})'_q$ of formal series $\sum_{j,k=0}^{\infty} a_{jk} z^j z^k$ with complex coefficients is a $q$-analogue of the space of distributions in the unit disc $\mathbb{U} = \{ z \in \mathbb{C} | |z| < 1 \}$. Equip this space of formal series with the topology of coefficientwise convergence. Since $\{ z^j z^k \}_{j,k \in \mathbb{Z}_+}$ constitute a basis in the vector space $\text{Pol}(\mathbb{C})_q$, $\text{Pol}(\mathbb{C})_q$ admits an embedding into $D(\mathbb{U})'_q$ as a dense linear subvariety.

Consider the unital subalgebra $\mathbb{C}[z]_q \subset \text{Pol}(\mathbb{C})_q$ generated by $z \in \text{Pol}(\mathbb{C})_q$. Let $\alpha > 0$. We follow [5] in equipping the vector space $\mathbb{C}[z]_q$ with the scalar product $(z^j, z^k)_{\alpha} = \delta_{j,k} (q^{2k}; q^2)^{\alpha}_k$, $j, k \in \mathbb{Z}_+$, where $(a; q^2)_k = (1-a)(1-q^2a) \cdots (1-q^{2(k-1)})a$.

Let $L^2_{\alpha}(d\nu_\alpha)_q$ be the a completion of $\mathbb{C}[z]_q$ with respect to the norm $\| \psi \|_{\alpha} = (\psi, \psi)_{\alpha}^{1/2}$. It was demonstrated in [5] that the Hilbert space $L^2_{\alpha}(d\nu_\alpha)_q$ is a $q$-analogue of the weighted Bergman space. Let $\widehat{z}$ be a linear operator of multiplication by $z$:

$$\widehat{z} : L^2_{\alpha}(d\nu_\alpha)_q \to L^2_{\alpha}(d\nu_\alpha)_q; \quad \widehat{z} : \psi(z) \mapsto z \cdot \psi(z),$$

and denote by $\widehat{z}^*$ the adjoint operator in $L^2_{\alpha}(d\nu_\alpha)_q$ to $\widehat{z}$. The definition of the scalar product in $L^2_{\alpha}(d\nu_\alpha)_q$ implies that the operators $\widehat{z}$, $\widehat{z}^*$ are bounded. Equip the space $\mathcal{L}_\alpha$ of bounded linear operators in $L^2_{\alpha}(d\nu_\alpha)_q$ with the weakest topology in which all the linear functionals

$$l_{\psi_1, \psi_2} : \mathcal{L}_\alpha \to \mathbb{C}, \quad l_{\psi_1, \psi_2} : A \mapsto (A \psi_1, \psi_2)_\alpha, \quad \psi_1, \psi_2 \in \mathbb{C}[z]_q$$

are continuous. The following proposition is a straightforward consequence of the definitions (see the proof in [5]).

**Proposition 2.1** Given any bounded linear operator $\widehat{f}$ in the Hilbert space $L^2_{\alpha}(d\nu_\alpha)_q$, there exists a unique formal series $f = \sum_{j,k=0}^{\infty} a_{jk} z^j z^k \in D(\mathbb{U})'_q$ such that $\widehat{f} = \sum_{j,k=0}^{\infty} a_{jk} \widehat{z}^j \widehat{z}^k$.

Thus we get an injective linear map $\mathcal{L}_\alpha \to D(\mathbb{U})'_q$, $\widehat{f} \mapsto f$. The distribution $f$ is called a covariant symbol of the linear operator $\widehat{f}$.

**Remark 2.2.** For an arbitrary $f \in \text{Pol}(\mathbb{C})_q$, there exists a unique operator $\widehat{f} \in \mathcal{L}_\alpha$ with the covariant symbol $f$. Specifically, for $f = \sum_{j,k=0}^{N(f)} a_{jk} z^j z^k$, one has $\widehat{f} = \sum_{j,k=0}^{N(f)} a_{jk} \widehat{z}^j \widehat{z}^k$.

We follow F. Berezin in producing the $*$-product of covariant symbols using the ordinary product of the associated linear operators.
Let $f_1, f_2 \in \operatorname{Pol}(\mathbb{C})_q$ and $\hat{f}_1, \hat{f}_2 \in \mathcal{L}_u$ be the operators whose covariant symbols are $f_1, f_2$. Under the notation $t = q^4\alpha$, let $m_t(f_1, f_2)$ stand for the covariant symbol of the product $\hat{f}_1 \cdot \hat{f}_2$ of the linear maps $\hat{f}_1, \hat{f}_2$. Evidently, we have constructed a bilinear map $m_t : \operatorname{Pol}(\mathbb{C})_q \times \operatorname{Pol}(\mathbb{C})_q \to D(\mathbb{U})'_q$.

The $*$-product $f_1 \ast f_2$ of $f_1, f_2 \in \operatorname{Pol}(\mathbb{C})_q$ is to be introduced by replacement of the one-parameter family of distributions $m_t(f_1, f_2)$, $t \in (0, 1)$, with its asymptotic expansion as $t \to 0$.

### 3 $*$-Product

The term 'order one differential calculus over the algebra $\operatorname{Pol}(\mathbb{C})_q$,' stand for a $\operatorname{Pol}(\mathbb{C})_q$-bimodule $\Omega^1(\mathbb{C})_q$ equipped with a linear map $d : \operatorname{Pol}(\mathbb{C})_q \to \Omega^1(\mathbb{C})_q$ such that

i) $d$ satisfies the Leibniz rule $d(f_1 f_2) = df_1 \cdot f_2 + f_1 \cdot df_2$ for any $f_1, f_2 \in \operatorname{Pol}(\mathbb{C})_q$,

ii) $\Omega^1(\mathbb{C})_q$ is a linear span of $f_1 \cdot df_2 \cdot f_3$, $f_1, f_2, f_3 \in \operatorname{Pol}(\mathbb{C})_q$ (see [1]).

One can find in [1] a construction of that kind of order one differential calculus for a wide class of prehomogeneous vector spaces $V$. In the case $V = \mathbb{C}$ we deal with this calculus is well known; it can be described in terms of the following commutation relations:

$$z \cdot dz = q^{-2}dz \cdot z, \quad z^* dz^* = q^2 dz^* z^*, \quad z^* dz = q^2 dz \cdot z^*, \quad z \cdot d z^* = q^{-2} dz^* z.$$

The partial derivatives $\frac{\partial^{(r)}}{\partial z}$, $\frac{\partial^{(r)}}{\partial z^*}$, $\frac{\partial^{(l)}}{\partial z}$, $\frac{\partial^{(l)}}{\partial z^*}$ are linear operators in $\operatorname{Pol}(\mathbb{C})_q$ given by

$$df = \frac{\partial^{(r)}}{\partial z} dz + \frac{\partial^{(r)}}{\partial z^*} dz^* = dz \frac{\partial^{(l)}}{\partial z} + dz^* \frac{\partial^{(l)}}{\partial z^*},$$

with $f \in \operatorname{Pol}(\mathbb{C})_q$.

Let $\square : \operatorname{Pol}(\mathbb{C})_q^\otimes 2 \to \operatorname{Pol}(\mathbb{C})_q^\otimes 2$, $m_0 : \operatorname{Pol}(\mathbb{C})_q^\otimes 2 \to \operatorname{Pol}(\mathbb{C})_q$ be linear operators given by

$$\square(f_1 \otimes f_2) = \left( \frac{\partial^{(r)}}{\partial z} f_1 \otimes 1 \right) \cdot q^{-2}(1 - (1 + q^{-2}) z^* \otimes z + q^{-2} z^2 \otimes z^2) \cdot \left( 1 \otimes \frac{\partial^{(l)}}{\partial z} f_2 \right),$$

$$m_0(f_1 \otimes f_2) = f_1 f_2, \text{ with } f_1, f_2 \in \operatorname{Pol}(\mathbb{C})_q.$$

**Theorem 3.1** For all $f_1, f_2 \in \operatorname{Pol}(\mathbb{C})_q$, the following asymptotic expansion in $D(\mathbb{U})'_q$ is valid:

$$m_t(f_1, f_2) \sim_0 f_1 \ast f_2,$$

with

$$f_1 \ast f_2 = f_1 \cdot f_2 + \sum_{k=1}^{\infty} C_k(f_1, f_2) t^k \in \operatorname{Pol}(\mathbb{C})_q[[t]], \quad (3.1)$$

$$C_k(f_1, f_2) = m_0 \left( \left( p_k \left( \square \right) - p_{k-1} \left( \square \right) \right) (f_1 \otimes f_2) \right), \quad (3.2)$$

and $p_k(x)$, $x \in \mathbb{Z}_+$, are polynomials given by

$$p_k(x) = \sum_{j=0}^{k} \frac{(q^{-2k}; q^2)_j}{(q^2; q^2)_j} q^{2j} \prod_{i=0}^{j-1} (1 - q^2((1 - q^2)^2 x + 1 + q^2) + q^{4i+2}). \quad (3.3)$$
This statement is to be proved in the next section, using the results of [10] on a q-analogue of the Berezin transform [12]. We are grateful to H. T. Koelink who attracted our attention to the fact that the polynomials \( p_k(x) \) differ from the polynomials of Al-Salam – Chihara [7] only by normalizing multiples and a linear change of the variable \( x \).

4 A q-analogue of the Berezin transform

Remind the notation \( t = q^{4\alpha} \), with \( q \in (0, 1) \), \( \alpha > 0 \).

Consider the linear map \( \text{Pol}(\mathbb{C})_q \rightarrow L\alpha \) which sends a polynomial \( \hat{f} = \sum_{jk} b_{jk} z^j \hat{z}^k \) to the linear operator \( \hat{f} = \sum_{jk} b_{jk} z^j \hat{z}^k \). The polynomial \( \hat{f} \) will be called a contravariant symbol of the linear operator \( \hat{f} \).

Note that our definitions of covariant and contravariant symbols agree with the conventional ones, as one can observe from [10] (specifically, see proposition 6.6 and lemma 7.2 of that work).

The term ‘q-transform of Berezin’ will be stand for the linear operator \( B_{q,t} : \text{Pol}(\mathbb{C})_q \rightarrow (D(U))'_q \), \( B_{q,t} : f \mapsto \hat{f} \), which sends the contravariant symbols of linear operators \( \hat{f} = \sum_{jk} b_{jk} z^j \hat{z}^k \) to their covariant symbols.

**Remark 4.1.** It is easy to extend the operators \( B_{q,t} \) onto the entire ‘space of bounded functions in the quantum disc’ via a non-standard approach to their construction (see [10]).

[10], proposition 5.5 imply

**Proposition 4.1** Given arbitrary \( \hat{f} \in \text{Pol}(\mathbb{C})_q \), the following asymptotic expansion in the topological vector space \( D(U)_q' \) is valid:

\[
B_{q,t} \hat{f} \sim \sum_{k=1}^{\infty} ((p_k(\Box) \hat{f} - p_{k-1}(\Box) \hat{f}) t^k,
\]

with \( \Box \) being a q-analogue of the Laplace-Beltrami operator

\[
\Box f = \frac{(1 - zz^*)^2}{z^* z} \frac{\partial^{(l)} \partial^{(l)} f}{\partial z^* \partial z} = q^2 \frac{\partial^{(r)} \partial^{(r)} f}{\partial z^*} (1 - zz^*)^2,
\]

with \( f \in \text{Pol}(\mathbb{C})_q \) and \( p_k, k \in \mathbb{Z}_+ \), being polynomials given by (3.3).

It follows from the definition of the bilinear maps \( m_t \), \( t \in (0, 1) \), that for all \( i, j, k, l \in \mathbb{Z}_+ \), \( f_1, f_2 \in \text{Pol}(\mathbb{C})_q \),

\[
m_t(z^i f_1, f_2) = z^i m_t(f_1, f_2),
\]

\[
m_t(f_1, f_2 z^k) = m_t(f_1, f_2) z^k,
\]

\[
m_t(z^j, z^k) = B_{q,t}(z^j z^k).
\]
Hence for all \(i, j, k, l \in \mathbb{Z}_+\) one has
\[
m_t((z^i z^*)^j, (z^k z^*)^l) = z^i B_{q,t}(z^j z^k) z^l.
\] (4.1)

We are about to deduce theorem 3.1 from (4.1) and proposition 4.1. In fact, one can easily demonstrate as in [10, proposition 8.3] that
\[
\square (f_2(z) \cdot f_1(z)) = q^2 \frac{\partial^{(r)} f_2(z^*)}{\partial z^*} (1 - z z^*)^2 \frac{\partial^{(l)} f_1(z)}{\partial z} =
\]
\[
= \frac{\partial^{(r)} f_2(z^*)}{\partial z^*} q^{-2} (1 - (1 + q^{-2}) z z^* + q^{-2} z^2 z^2) \frac{\partial^{(l)} f_1(z)}{\partial z}
\]
for arbitrary polynomials \(f_1(z), f_2(z^*)\). What remains is to compare this expression for \(\square\) with the definition of \(\tilde{\square}\) and apply the fact that \(\{z^i z^* j\}_{i, j \in \mathbb{Z}_+}\) constitute a basis in the vector space \(\text{Pol}(\mathbb{C})_q\).

5 A formal associativity

**Proposition 5.1** The multiplication in \(\text{Pol}(\mathbb{C})_q[[t]]\) given by the bilinear map
\[
m : \text{Pol}(\mathbb{C})_q[[t]] \times \text{Pol}(\mathbb{C})_q[[t]] \rightarrow \text{Pol}(\mathbb{C})_q[[t]],
\]
\[
m : \sum_{j=0}^{\infty} a_j t^j \times \sum_{k=0}^{\infty} b_k t^k \mapsto \sum_{i=0}^{\infty} \left( \sum_{j+k=i} a_j b_k \right) t^i,
\] (5.1)
with \(\{a_j\}_{j \in \mathbb{Z}_+}, \{b_k\}_{k \in \mathbb{Z}_+} \in \text{Pol}(\mathbb{C})_q\), is associative.

**Proof.** Introduce the algebra \(\text{End}_{\mathbb{C}}(\mathbb{C}[z]_q)\) of all linear operators in the vector space \(\mathbb{C}[z]_q\), and the algebra \(\text{End}_{\mathbb{C}}(\mathbb{C}[z]_q)[[t]]\) of formal series with coefficients in \(\text{End}_{\mathbb{C}}(\mathbb{C}[z]_q)\). To prove our statement, it suffices to establish an isomorphism of the algebra \(\text{Pol}(\mathbb{C})_q[[t]]\) equipped with the multiplication \(m\) and a subalgebra of \(\text{End}_{\mathbb{C}}(\mathbb{C}[z]_q)[[t]]\) given the standard multiplication. Let \(\mathcal{I} : \text{Pol}(\mathbb{C})_q \rightarrow \text{End}_{\mathbb{C}}(\mathbb{C}[z]_q)[[t]]\) be such a linear operator that for all \(j, k, m \in \mathbb{Z}_+\)

\[
\mathcal{I}(z^j z^m) : z^m \mapsto \begin{cases} (q^{2m}; q^{-2})_k z^{m-k+j} & , \quad k \leq m \\ (t q^{2m}; q^{-2})_k & , \quad k > m \\ 0 & \end{cases}.
\]

(More precisely, one should replace the rational function \(1/(t q^{2m}; q^{-2})_k\) of an indeterminate \(t\) with its Taylor expansion.)

The following lemma follows from the construction of [10, section 7].

---

1The outward sum clearly converges in the topological vector space \(\text{Pol}(\mathbb{C})_q[[t]]\).
Lemma 5.2  The linear map
\[ Q : \text{Pol}(\mathbb{C})[[t]] \rightarrow \text{End}_\mathbb{C}(\mathbb{C}[z])[[t]], \]
\[ Q : \sum_{j=0}^\infty f_j t^j \mapsto \sum_{j=0}^\infty I(f_j) t^j, \]
is injective, and for all \( \psi_1, \psi_2 \in \text{Pol}(\mathbb{C})[[t]] \) one has \( Qm(\psi_1, \psi_2) = (Q\psi_1) \cdot (Q\psi_2) \).

Lemma 5.2 implies the associativity of the multiplication \( m \) in \( \text{Pol}(\mathbb{C})[[t]] \). Thus, proposition 5.1 is proved. ■

Define a linear operator \( \ast \) in \( \text{Pol}(\mathbb{C})[[t]] \) by
\[ \left( \sum_{j=0}^\infty f_j t^j \right)^\ast = \sum_{j=0}^\infty f_j^\ast t^j, \quad \{ f_j \}_{j \in \mathbb{Z}^+} \subset \text{Pol}(\mathbb{C}). \]

Proposition 5.3 \( \ast \) is an involution in \( \text{Pol}(\mathbb{C})[[t]] \) equipped by \( m \) as a multiplication:
\[ m(\psi_1, \psi_2)^\ast = m(\psi_2^\ast, \psi_1^\ast), \quad \psi_1, \psi_2 \in \text{Pol}(\mathbb{C})[[t]]. \]

Proof. For all \( f_1, f_2 \in \text{Pol}(\mathbb{C}) \) one has
\[ (m_0(f_1 \otimes f_2))^\ast = m_0(f_2^\ast \otimes f_1^\ast), \]
with \( \Box^{21} = c_0 \Box c_0 \), and \( c_0 \) being the flip of tensor multiples. What remains is to observe that the coefficients of \( p_n(x) \), \( n \in \mathbb{Z}^+ \), are real, and to apply (5.1), (3.1), (3.2). ■

6  \( U_q\mathfrak{su}_{1,1} \)-invariance

Remind some well known results on the quantum group \( SU_{1,1} \) and the quantum disc (see, for example, [3, 9]).

The quantum universal enveloping algebra \( U_q\mathfrak{sl}_2 \) is a Hopf algebra over \( \mathbb{C} \) determined by the generators \( K, K^{-1}, E, F \), and the relations
\[ KK^{-1} = K^{-1}K = 1, \quad K^{\pm 1}E = q^{\pm 2}EK^{\pm 1}, \quad K^{\pm 1}F = q^{\mp 2}FK^{\pm 1}, \quad EF - FE = (K - K^{-1})(q - q^{-1}). \]
Comultiplication \( \Delta : U_q\mathfrak{sl}_2 \rightarrow U_q\mathfrak{sl}_2 \otimes U_q\mathfrak{sl}_2 \), counit \( \varepsilon : U_q\mathfrak{sl}_2 \rightarrow \mathbb{C} \) and antipode \( S : U_q\mathfrak{sl}_2 \rightarrow U_q\mathfrak{sl}_2 \) are given by
\[ \Delta(K^{\pm 1}) = K^{\pm 1} \otimes K^{\pm 1}, \quad \Delta(E) = E \otimes 1 + K \otimes E, \quad \Delta(F) = F \otimes K^{-1} + 1 \otimes F, \quad \varepsilon(E) = \varepsilon(F) = \varepsilon(K^{\pm 1} - 1) = 0, \]

\[^2\text{The convergence of the series } \sum_{j=0}^\infty I(f_j) t^j \text{ in the space } \text{End}_\mathbb{C}(\mathbb{C}[z])[[t]] \text{ is obvious.}\]
Hence it is also true for $F_m$ that the linear maps polynomials $U$ property has already been mentioned. So we need only to consider (the quantum disc considered there).

The following relations determine a structure of $U_q\mathfrak{sl}_2$-module algebra on $\mathbb{C}[z]_q$:

$$K^{\pm 1} z = q^{\pm 2} z, \quad F z = q^{1/2} z, \quad E z = -q^{1/2} z^2.$$ (6.1)

Equip $U_q\mathfrak{sl}_2$ with an involution:

$$E^* = -K F, \quad F^* = -EK^{-1}, \quad (K^{\pm 1})^* = K^{\mp 1},$$

and let $U_q\mathfrak{su}_{1,1}$ stand for the Hopf $*$-algebra produced this way. An involutive algebra $F$ is said to be $U_q\mathfrak{su}_{1,1}$-module algebra if it is $U_q\mathfrak{sl}_2$-module algebra, and the involutions in $F$ and $U_q\mathfrak{su}_{1,1}$ agree as follows:

$$(\xi f)^* = (S(\xi))^* f^*, \quad \xi \in U_q\mathfrak{su}_{1,1}, \ f \in F.$$ (6.2)

(6.1) determines a structure of $U_q\mathfrak{su}_{1,1}$-module algebra in $\text{Pol}(\mathbb{C})_q$. Thus, each of the vector spaces $\text{Pol}(\mathbb{C})_q$, $\text{Pol}(\mathbb{C})_q[[t]]$ is equipped with a structure of $U_q\mathfrak{su}_{1,1}$-module.

**Proposition 6.1** $\text{Pol}(\mathbb{C})_q[[t]]$ with the multiplication defined above and the involution $*$ is a $U_q\mathfrak{su}_{1,1}$-module algebra.

**Proof.** Since $\text{Pol}(\mathbb{C})_q$ is a $U_q\mathfrak{su}_{1,1}$-module algebra, (6.2) is valid for $F = \text{Pol}(\mathbb{C})_q$. Hence it is also true for $F = \text{Pol}(\mathbb{C})_q[[t]]$. What remains is to prove that $\text{Pol}(\mathbb{C})_q[[t]]$ is a $U_q\mathfrak{sl}_2$-module algebra. For that, by a virtue of (6.1), (6.2), it suffices to demonstrate that the linear maps $m_0$ and $\Box$ are morphisms of $U_q\mathfrak{sl}_2$-modules. As for $m_0$, this property has already been mentioned. So we need only to consider $\Box$. Given any polynomials $f_1(z^*), f_2(z)$, it follows from $\Box(f_1(z^*)f_2(z)) = \sum_{jk} b_{jk} z^{*j} z^k, b_{jk} \in \mathbb{C}$, that

$$\Box((f_1(z^*) \otimes f_2(z)) = \sum_{jk} b_{jk} z^{*j} \otimes z^k,$$ and

$$\Box((g_1(z)f_1(z^*) \otimes f_2(z)g_2(z^*)) = (g_1(z) \otimes 1)\Box(f_1(z^*) \otimes f_2(z))(1 \otimes g_2(z^*)).$$

Thus, it suffices to prove that $\Box$ is a morphism $U_q\mathfrak{sl}_2$-modules. This latter result is obtained in [8]. (It is a consequence of $U_q\mathfrak{su}_{1,1}$-invariance of the differential calculus in the quantum disc considered there).

**Remark 6.2.** The works [8, 9] deal with the $U_q\mathfrak{su}_{1,1}$-module algebra $D(\mathbb{U})_q$ of 'finite functions in the quantum disc’. (The space $D(\mathbb{U})_q$ mentioned in this work is dual to $D(\mathbb{U})_q$). The relations (3.1) – (3.3) determine a formal deformation of $D(\mathbb{U})_q$ in the class of $U_q\mathfrak{su}_{1,1}$-module algebras, that is, it allows one to equip $D(\mathbb{U})_q[[t]]$ with a structure of $U_q\mathfrak{su}_{1,1}$-module algebra over the ring $\mathbb{C}[[t]]$. 

7
7 Concluding notes

We have demonstrated that the method of Berezin allows one to produce a formal deformation for a q-analogue of the unit disc. In [11], q-analogues for arbitrary bounded symmetric domains were constructed. We hope in that essentially more general setting, the method of Berezin will help remarkable results to be obtained.

References

[1] F. Bayen, M. Flato, C. Fronsdal, A. Lichnerovich, D. Sternheimer, Deformation theory and quantization, Annals of Phys., 111 (1978), part I: 61 – 110, part II: 111 – 151.

[2] F. A. Berezin, General concept of quantization, Commun. Math. Phys., 40 (1975), No 2, 153 – 174.

[3] V. Chari, A. Pressley. A Guide to Quantum Groups, Cambridge Univ. Press, 1995.

[4] M. Cahen, S. Gutt, J. Rawnsley, Quantization on Kähler manifolds III, Letters in Mathematical Physics, 30 (1994), 291 – 305.

[5] S. Klimek, A. Lesniewski, A two-parameter quantum deformation of the unit disc, J. Funct. Anal. 115, (1993), 1 – 23.

[6] A. Klimyk, K. Schmüdgen, Quantum Groups and Their Representations, Springer, Berlin-N.-Y.-Heidelberg, 1997.

[7] H. T. Koelink, Yet another basic analogue of Graf’s addition formula, J. Comput. and Appl. Math., 68 (1996), 209 – 220.

[8] D. Shklyarov, S. Sinel’shchikov, L. Vaksman. On function theory in quantum disc: integral representations, E-print: math.QA/9808015.

[9] D. Shklyarov, S. Sinel’shchikov, L. Vaksman. On function theory in quantum disc: covariance, E-print: math.QA/9808037.

[10] D. Shklyarov, S. Sinel’shchikov, L. Vaksman. On function theory in quantum disc: a q-analogue of the Berezin transform, E-print: math.QA/9809018.

[11] S. Sinel’shchikov, L. Vaksman. On q-analogues of bounded symmetric domains and Dolbeault complexes, Mathematical Physics, Analysis and Geometry, 1, (1998), 75 – 100; E-print 1997, q-alg/9703003.

[12] A. Unterberger, H. Upmeier, The Berezin transform and invariant differential operators, Comm. Math. Phys., 164 (1994), 563 – 598.