Coupled analysis for active control and energy harvesting from flow-induced vibration

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Abstract. The aerospace community has long studied active flutter control. Recently, a novel idea, which is energy harvesting from wing vibration under the assumption that a certain magnitude of vibration is allowed, has been attracting attention. The balancing problem between piezoelectric energy harvesting and active control is an active research topic. In our previous work, we developed a coupled analysis system for active control of piezoelectric–structure–fluid interaction problems. In the present study, we integrate a piezoelectric energy harvester model. With numerical examples, we considered the limit cycle oscillation (LCO) of a cantilevered beam in an axial flow. Then, we showed that LCO suppression was achieved while the energy scavenged by harvesters was at least as much as the energy consumed by the active control.

Keywords: Fluid–structure interaction, Piezoelectric material, Energy harvesting, Active control, Partitioned iterative coupled analysis

1. Introduction

Recently, for applications such as environmental research and telecommunications, unmanned airframes have been designed to allow high-altitude and long-endurance flights, which require very efficient aerodynamic and structural designs. Because of the absence of pilots, many design constraints are relaxed. Many researchers have been investigating high-aspect-ratio slender and highly flexible wings with a low structural weight [1]. Such wings may undergo large deformation, which results in geometrical nonlinearity and possible nonlinear flow. High-aspect-ratio wings often exhibit limit cycle oscillation (LCO) [1, 2, 3], which is a self-sustained oscillation with a limited amplitude that remains constant over time and is caused by nonlinearities in structural dynamics and aerodynamics. Although destructive vibration must be suppressed, if a certain magnitude of vibration is allowed under the assumption that it will not damage the wing or degrade aircraft performance, this vibration can serve as a source of energy.
Anton and Inman developed a miniature unmanned aerial vehicle equipped with piezoelectric materials on a wing spar to harvest energy from wing vibration [4]. As an extension of this study, a multifunctional wing spar comprising an energy harvester, a thin-film battery, an actuator, and a sensor was experimentally and numerically investigated [5, 6]. This multifunctional wing spar makes it feasible to perform active control and energy harvesting concurrently. Because the suppression of wing vibration decreases the energy scavenged by harvesters, a balancing problem between active control and harvesting is naturally engendered. The balance between harvesting energy from LCO and its active control was investigated by Tsushima et al. [7]. In their study, the fluid force was calculated by a simple mathematical model based on the potential flow theory. Because LCO is caused by very complicated nonlinear behavior, a detailed fluid–structure interaction (FSI) model with large-scale degrees of freedom is appropriate for accurately capturing LCO. In the present study, we developed a high-fidelity coupled analysis for numerical evaluation of the balancing problem. For harvesters, actuators, and sensors, we focused on piezoelectric materials. Because the piezoelectric effect transforms electrical energy into mechanical energy and vice versa, piezoelectric materials are widely used for harvesters, actuators, and sensors. The problem we tackled is a complex multi-physics problem with coupling among FSI, piezoelectric energy harvesting, and active control by piezoelectric actuators and sensors.

Several existing studies have addressed high-fidelity FSI simulations with piezoelectric energy harvesters. Akaydin et al. proposed a partitioned staggered scheme composed of a computational fluid dynamics solver (FLUENT) and a simple piezo-beam solver [8]. The structural response of the piezoelectric beam was modeled with a single degree of freedom. They neglected any contribution of higher modes. Unlike partitioned schemes, Ravi and Zilian employed a monolithic scheme that solves sub-systems simultaneously [9]. They applied the finite element method to fluid analysis, structural analysis for host structures, and electromechanical analysis for piezoelectric materials. Regarding FSI simulation with active control, in contrast, few studies have been reported. In our previous work, we developed a fluid–structure–piezoelectricity coupled analysis considering active control systems to simulate the active control of FSI-induced vibration by piezoelectric sensors and actuators [10]. The study was based on the partitioned iterative method. The advantage of partitioned approaches is that they allow the use of existing sub-solvers, which makes it easy to develop and manage partitioned method-based analysis systems. In the partitioned iterative method, sub-systems are executed both separately and iteratively.

In the present study, we considered five solvers. The first is a solver for flow analysis. The second is a solver for updating the fluid mesh, which is required because we employ an arbitrary Lagrangian–Eulerian (ALE) method. The third is a solver for monolithic structure–electrostatic interaction (SEI) analysis, which models the behavior of host structures and piezoelectric materials. The fourth is a solver for circuit analysis to model the piezoelectric energy harvesters. The fifth is a solver for the calculation of control inputs from control outputs. The purpose of the present study was to combine the five solvers based on the partitioned iterative scheme.

The rest of this paper is organized as follows. In Section 2, the formulations of the problem with coupling among host structures, piezoelectric materials, and surrounding fluid are given. Then, we describe a partitioned iterative scheme-based coupled system to combine the first, second, and third solvers. In Section 3, models for the piezoelectric energy harvester and active control system are explained. In addition, we explain how to calculate the energy gen-
enerated by harvesters and the energy consumed by the active control system. Then, the fourth and fifth solvers are incorporated into the coupled system described in Section 2. In Section 4, we show numerical examples to demonstrate the validity of the analysis. Finally, concluding remarks are given in Section 5.

2. Fluid–structure–electrostatic interaction analysis

The problem we considered in this study included host structures, piezoelectric materials, and the surrounding fluid. Because piezoelectricity is a coupled phenomenon between structural dynamics and electrostatics, a fluid–SEI (FSEI) analysis method is necessary. In this section, we describe the formulation of the FSEI analysis method. Although, we also need to consider circuits and control laws for our target problem, this section does not account for the modeling of piezoelectric energy harvesters and active control systems, which is addressed in Section 3.

2.1. Governing equations

Under the assumption of incompressible viscous flow, fluid dynamics are governed by the Navier–Stokes equation in an ALE frame of reference,

\[ \rho^F \left( \frac{\partial \mathbf{v}^F}{\partial t} \right)_X + (\mathbf{v}^F - \mathbf{v}^F) \cdot \nabla \mathbf{v}^F - \nabla \cdot \mathbf{\sigma}^F = \rho^F \mathbf{b}^F, \]  

with the following continuity equation:

\[ \nabla \cdot \mathbf{v}^F = 0, \]  

where \( \rho^F \) is the density of the fluid, \( \mathbf{v}^F \) is the velocity vector for the fluid, \( \mathbf{v}^F \) is the mesh velocity vector, \( \mathbf{\sigma}^F \) is the stress tensor for the fluid, \( \mathbf{b}^F \) is the body force vector applied to the fluid, and \( \frac{\partial \mathbf{v}^F}{\partial t} \) represents the referential time derivative of the solution in the spatial configuration. A Newtonian fluid is assumed and \( \mathbf{\sigma}^F \) is defined as follows:

\[ \mathbf{\sigma}^F = -p^F \mathbf{I} + \mu \left( \nabla \mathbf{v}^F + \nabla \mathbf{v}^F^T \right), \]  

where \( p^F \) is the pressure of the fluid, \( \mathbf{I} \) is the second order unit tensor, and \( \mu \) is the viscosity of the fluid. The stress field \( \mathbf{\sigma}^F \) is subjected to the following traction condition:

\[ \mathbf{\sigma}^F \mathbf{n}^F = \mathbf{h}^F, \]  

where \( \mathbf{n}^F \) is the outward normal vector and \( \mathbf{h}^F \) is the prescribed traction.

Piezoelectricity is a combination of mechanical and electrical phenomena. The mechanical behavior is governed by the following Cauchy momentum equation:

\[ \rho_0^S \frac{\partial^2 \mathbf{u}^S}{\partial t^2} - \nabla_X \cdot (\mathbf{FS}) = \rho_0^S \mathbf{b}_0^S, \]  

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where \( \rho^S_0 \) is the structural density, \( u^S \) is the displacement vector, \( b^F_0 \) is the body force vector, \( F \) is the deformation gradient, \( S \) is the second Piola–Kirchhoff stress, and \( \frac{D}{Dt} \) represents the material derivative. The nabla operator \( \nabla_X \) refers to the initial configuration. To consider the geometrical nonlinearity, the total Lagrange formulation is employed. Therefore, the governing equation is described with respect to the initial configuration. The value for the initial configuration is denoted by the subscript 0. The stress field is subjected to the following traction condition:

\[
FSn^S_0 = h^S_0, \tag{6}
\]

where \( n^S_0 \) is the outward normal vector and \( h^S_0 \) is the prescribed traction force.

Because the electric equilibrium is satisfied instantly, the electrical dynamics can be assumed to be quasi-static. The electrical behavior is governed by the following form of Gauss’s law:

\[
\nabla_X \cdot D = r_0, \tag{7}
\]

where \( D \) is the electric displacement and \( r_0 \) is the electric body charge. The electric displacement is subjected to the following traction condition:

\[
Dn^S_0 = h^E_0, \tag{8}
\]

where \( h^E_0 \) is the prescribed surface charge.

The constitutive equations are modeled by the following relation:

\[
S = C e - \varepsilon E, \tag{9}
\]
\[
D = \varepsilon^T e + \varepsilon E. \tag{10}
\]

where \( e \) is the Green–Lagrange strain, \( E \) is the electric field vector, \( C \) is the fourth order elastic tensor in a constant strain field, \( \varepsilon \) is the third order piezoelectric tensor, and \( \varepsilon \) is the dielectric tensor in a constant strain field. The piezoelectric tensor determines the degree of the coupling effect. Therefore, when \( \varepsilon = 0 \) holds, the mechanical response is governed purely by elasticity. By properly setting \( \varepsilon \), both piezoelectric materials and St. Venant–Kirchhoff elastic materials, which are used for modeling host structures, can be described. \( e \) and \( E \) are written as follows:

\[
e = \frac{1}{2} (F^T F - I), \tag{11}
\]
\[
E = -\nabla_X \phi, \tag{12}
\]

where \( \phi \) is the voltage field.

The equilibrium of force and geometrical compatibility at the FSI interface is expressed as follows:
\[
\sigma^F n^F + \sigma^S n^S = 0 , \tag{13}
\]
\[
\mathbf{v}^F = \frac{D u^S}{Dt} , \tag{14}
\]
where \( \sigma^S \) is the Cauchy stress tensor for the structure.

### 2.2. Partitioned iterative method

Our coupled analysis system is based on the partitioned iterative method, where multiple sub-analysis systems are executed sequentially and iteratively. For FSEI analysis, we use three sub-analysis systems, namely one analysis system for updating the fluid mesh, one for fluids, and one for piezoelectric materials and host structures. For the flow analysis, which solves Eqs. (1-4), the velocity field and pressure field are solved simultaneously. To avoid instabilities, the streamline upwind/Petrov–Galerkin [11] and pressure-stabilizing/Petrov–Galerkin [12] methods are employed. For the analysis of piezoelectricity, which solves Eqs. (5-12), a monolithic SEI analysis is performed, following the approach in many previous studies [13, 14]. As mentioned in the previous subsection, the SEI analysis can handle pure elasticity by setting \( e = 0 \). Therefore, the voltage field \( \phi \) for piezoelectric materials and the displacement field \( \mathbf{u}^S \) for piezoelectric materials and host structures are solved simultaneously by the monolithic SEI analysis. For the spatial discretization, the finite element method is employed. Regarding the time integration, the Newmark-\( \beta \) method and the backward Euler method are employed for the structural and fluid governing equations, respectively.

For the fluid mesh update, pseudo-elastic smoothing [15] is employed, as done in many other studies that used FSI analysis [10, 16]. In this method, the mesh deformation in the fluid domain is virtually governed by linear elastic equations. Given the displacement at the FSI interface, the updated fluid mesh coordinates can be obtained by solving the equations. Then, the fluid mesh velocity \( \mathbf{v}^F \) is calculated. Because the representation of the FSI interface in the fluid domain and that in the structural domain are always consistent, Eq. (14) is satisfied.

The partitioned iterative method adopted here is based on the Dirichlet–Neumann approach [17], which is briefly described below. First, the mesh update \( \mathcal{M} \) is performed as follows:

\[
\mathbf{x}^M = \mathcal{M}(\mathbf{u}^S) . \tag{15}
\]

The input is a guesstimate of the displacement \( \mathbf{u}^S \). The output is the fluid mesh coordinate \( \mathbf{x}^M \). After the mesh update, the mesh velocity \( \dot{\mathbf{x}}^F \), which is needed in the flow analysis, can be calculated. Second, the flow analysis \( \mathcal{F} \) is performed as follows:

\[
(\mathbf{v}^F, p^F) = \mathcal{F}(\mathbf{x}^M) . \tag{16}
\]

Because the displacement of the FSI interface is given, the velocity at the FSI interface is known. The velocity at the FSI interface is imposed as a Dirichlet boundary condition in the flow analysis. After the flow analysis, the fluid force on the FSI interface can be calculated using the equilibrium of force [Eq. (13)]. Third, the SEI analysis \( \mathcal{S} \) is performed as follows:
The fluid force is imposed as a Neumann boundary condition in the SEI analysis. After the SEI analysis, the voltage field $\phi$ is also obtained, although it is not written explicitly in Eq. (17). Convergence is checked by comparing the input of Eq. (15) and the output of Eq. (17).

From Eqs. (15-17), the FSEI problem is equivalent to the following nonlinear equation:

$$\mathbf{u}^s = \mathcal{S}\mathcal{E}\left(\mathcal{F}\left(\mathcal{M}\left(\mathbf{u}^s\right)\right)\right).$$

To solve Eq. (18), the Broyden method, a quasi-Newton method, is employed.

### 3. Modeling of active control system and energy harvester

#### 3.1. Modeling of active control system

To impose electrical conditions, electrodes are attached to a piezoelectric material. In the present study, we assume that the piezoelectric material is sandwiched between two electrodes as shown in Fig. 1. In this figure, the black lines AB and CD represent the electrodes. It should be noted the electrodes are assumed to be very thin and are therefore not modeled structurally [18]. Instead, the electrodes are just boundaries at the edges of the piezoelectric material, where electrical boundary conditions for sensors or actuators are imposed. As shown in Fig. 1, Electrode 2 is grounded. Therefore, 0 V is imposed on Electrode 2. On the edges that do not have electrodes (AC, BD), a zero surface charge condition is imposed. The electrical boundary condition for Electrode 1 changes depending on whether the device is operating as a sensor or an actuator.

![Figure 1: Piezoelectric material sandwiched by two electrodes](image)

In the piezoelectric sensor model, a zero surface charge condition is imposed on Electrode 1. After the SEI analysis [Eq. (17)], the distribution of the voltage on the electrode is obtained. Then, the voltage measured by the sensor, denoted by $V_{\text{sensor}}$, is assumed to be the spatially averaged voltage on the electrode [19, 20]. In the piezoelectric actuator model, conversely, an applied voltage, denoted by $V_{\text{actuator}}$, is applied to Electrode 1. $V_{\text{actuator}}$ is determined from $V_{\text{sensor}}$ via a control law. Because $V_{\text{sensor}}$ is explicitly determined if the voltage field $\phi$ is given, the following equation holds:

$$V_{\text{actuator}} = C(\phi),$$

where $C$ denotes the control law.

For the active control system, we assume that the energy consumption by actuators is dominant. Its calculation is explained below. The present study employed self-powered active
vibration control [21]. In this scheme, a piezoelectric material as an actuator is placed in a circuit with a resistor and a power supply as shown in Fig. 2.

Current that flows into Electrode 1 (see Fig. 2) is taken to be positive. The voltage of the power supply is written as

\[ U = V_{\text{actuator}} + \frac{dQ_{\text{actuator}}}{dt} R, \]  

where \( Q_{\text{actuator}} \) is the charge stored on Electrode 1. As mentioned above, \( V_{\text{actuator}} \) is applied to Electrode 1 as a Dirichlet boundary condition. Therefore, \( Q_{\text{actuator}} \) is obtained in the post-processing. After the SEI analysis, the surface charge that corresponds to the Dirichlet boundary condition is calculated. Then, \( Q_{\text{actuator}} \) can be calculated by spatially integrating the surface charge on Electrode 1. The energy consumed by the actuator from time \( t_1 \) to time \( t_2 \) is written as \( \int_{t_1}^{t_2} U \cdot \frac{dQ_{\text{actuator}}}{dt} dt. \)

3.2. Modeling of energy harvesting

The model for the piezoelectric energy harvester consisted of a closed circuit with a piezoelectric material and a resistor. Then, the energy generated by the energy harvester was modeled as the energy consumed by the resistor [22, 23]. Figure 3 shows a schematic diagram of this model.

The two electrodes are connected by a resistor with resistance \( R \). The following circuit equation holds:
where $V_{\text{harvester}}$ is the voltage on Electrode 1. Similar to the modeling of the piezoelectric sensor, the spatial average of the distributed voltage on Electrode 1 is equal to $V_{\text{harvester}}$. $Q_{\text{harvester}}$ is the charge stored on Electrode 1. Current that flows into Electrode 1 (see Fig. 3) is taken to be positive. The distribution of the charge on Electrode 1 is denoted as $q_{\text{harvester}}$, which at time $t$ is calculated as:

$$q_{\text{harvester}} = \frac{1}{S} \int_0^t \frac{dQ_{\text{harvester}}}{dt} dt = \frac{1}{S} \int_0^t \frac{V_{\text{harvester}}}{R} dt,$$

where $S$ is the area of Electrode 1 and Eq. (21) is used. Because $V_{\text{harvester}}$ is explicitly determined if the voltage field $\phi$ is given, the following equation holds:

$$q_{\text{harvester}} = R(\phi),$$

where $R$ denotes the circuit analysis.

The energy generated by the piezoelectric energy harvester from time $t_1$ to time $t_2$ can be written as $\int_{t_1}^{t_2} R \cdot \left( \frac{dQ_{\text{harvester}}}{dt} \right)^2 dt$.

### 3.3. Integration of models for active control system and energy harvesting

We incorporated the models for the active control system and energy harvesting into the analysis described in Section 2.2. The following nonlinear equation is solved:

$$(u^*, \phi) = S E \left( \mathcal{T}(u^*, \phi) \right).$$

In the function $\mathcal{T}$, the calculation of control laws, the circuit analysis, and the combination of the mesh update and the fluid analysis are performed in parallel without interference. As mentioned above, the combination of the mesh update and the fluid analysis can be represented as $(v^p, p^p) = F(M(u^p))$, the calculation of control laws can be represented as Eq. (19), and the circuit analysis can be represented as Eq. (23). $V_{\text{harvester}}$ is applied to Electrode 1 of an actuator as a Dirichlet boundary condition in the SEI analysis. $q_{\text{harvester}}$ is applied to Electrode 1 of the harvester as a Neumann boundary condition in the SEI analysis. Similar to Eq. (18), Eq. (24) is solved using the Broyden method. Unlike in the previous section, a guessestimate of the voltage field $\phi$ is required in addition to that of the displacement field $u^s$. Figure 4 shows the flowchart of the proposed analysis for our target multi-physics problem.

In the flowchart, the variables with the subscript $k$ represent those at the $k$-th iteration. To distinguish the guessestimates of the displacement and voltage from the solution of the SEI analysis, we use $\tilde{u}^s$ and $\tilde{\phi}$. As shown in Fig. 4, there are two types of residual, namely $r^1$ and $r^2$. If both residuals are sufficiently small, the iteration at the current time step is finished. The convergence is checked as follows:
where \( \varepsilon^1 \) and \( \varepsilon^2 \) are the convergence criteria for the displacement and voltage, respectively. In the present study, they were both set to \( 1.0 \times 10^{-4} \). The update of the displacement and voltage is based on the Broyden method.

\[
\left[ \frac{r^1_{(k)}}{r^1_{(t)}} \right] \leq \varepsilon^1, \left[ \frac{r^2_{(k)}}{r^2_{(t)}} \right] \leq \varepsilon^2, \tag{25}
\]

Figure 4: Flowchart of coupled analysis for the multi-physics problem with coupling among FSI, piezoelectric energy harvesting, and active control composed of piezoelectric actuators and sensors

4. Numerical examples

An FSI problem from the study of Tang et al. [24], who presented numerical simulations of the LCO of a cantilevered beam in axial flow, is considered here. After presenting the problem settings, we show some numerical results. First, under the assumption that neither the piezoelectric energy harvester nor the active control system is driven, we reproduced the LCO shown by Tang et al. [24]. Second, we modeled the case where the energy harvester is driven. Third, we modeled the case where the active control system is driven. Finally, a con-
current operation case was modeled.

4.1. Problem setting

Figure 5 shows the problem geometry and boundary conditions for the flow analysis. A cantilevered beam was placed in a uniform flow. The root of the cantilever beam, the black region in Fig. 5, was fixed. The red region represents a flexible plate. The length of the fixed part \( L_0 \) was 0.0058 m and that of the flexible part \( L \) was 0.58 m. The thickness of the beam \( h \) was 0.0005 m. The mass ratio \( \mu_M \) is defined as follows:

\[
\mu_M = \frac{\rho_r L}{\rho_0^2 h^2}.
\]  

(Tang et al. reported that LCO with the first vibration mode occurs at \( L_0/L = 0.01 \) and \( \mu_M = 0.01 \) \cite{24}.

Figure 6 shows the beam around the root and an enlarged view of a pair of piezoelectric layers. The red and gray regions are the piezoelectric materials. The blue region is the host structure. The polarization in the top (red) and bottom (gray) portions is in the \(-y\) and \(+y\) direction, respectively. There are 10 pairs of piezoelectric layers in the
The flexible structure with piezoelectric materials was discretized by the solid-shell elements proposed by Klinkel et al. [14], which allow the use of high-aspect-ratio elements. The width of the flexible part was divided into 150 sections and the height was divided into 2 sections. The number of nodes and elements in the structural mesh was 453 and 300, respectively. The fluid domain was discretized by linear triangular elements. The number of nodes and elements in the fluid mesh was 35,423 and 67,634, respectively.

The material properties are shown in Table 1. To satisfy the condition $\mu_M = 0.01$, the density was set to be very small here.

The time step was 0.005 s. The Newmark-$\beta$ parameters $\beta$ and $\gamma$ were 0.3025 and 0.6, respectively. This parameter set provides numerical damping, especially for high-frequency modes. In the study of Tang et al. [24], Kelvin–Voigt-type structural damping was employed. Here, numerical damping served the same function.

### Table 1: Material properties

| Property                      | Piezoelectric material | Host structure | Fluid |
|-------------------------------|------------------------|----------------|-------|
| Young’s modulus (N/m²)        | $6.7 \times 10^{10}$   | $6.7 \times 10^{10}$ | -     |
| Poisson’s ratio               | 0.31                   | 0.31           | -     |
| Density (kg/m³)               | $2.84 \times 10^3$     | $2.84 \times 10^3$ | $2.45 \times 10^{-2}$ |
| Viscosity (kg/m · s)          | -                      | -              | $1.84 \times 10^{-5}$ |
| Piezoelectric coefficients    |                        |                |       |
| (C/m²)                        | $\varepsilon_{13} = \varepsilon_{23}$ | $-9.3$         | -     |
|                              | $\varepsilon_{51} = \varepsilon_{62}$ | 14.6           | -     |
|                              | $\varepsilon_{33}$     | 20.3           | -     |
| Permittivity (C/m² · N)       | $15.3 \times 10^{-9}$  | -              | -     |

### 4.2. Simulation of LCO under open-circuit condition

![Figure 7: Time history of vertical tip displacement](image-url)
In the LCO simulation, all piezoelectric materials were assumed to operate under the open-circuit condition, that is, without piezoelectric energy harvesting or active control. The essential boundary condition was set as 0 V on line BE (Fig. 6). In addition, zero surface charge was applied to lines AD, DE, EF, CF, AB, and BC as the natural boundary conditions. The inlet flow velocity was 50 m/s. Figure 7 shows the time history of the vertical tip displacement. It can be seen that the vibration increased over time. After about 17 s, the increase rate of the vibration decreased. Then, after about 25 s, the vibration reached a steady state, with an amplitude of about 0.03 m and a frequency of about 3 Hz. As shown, LCO was reproduced in the simulation.

Figure 8 shows snapshots of the velocity norm distribution around the beam at 25.15 s to 25.45 s. As shown, the first mode is dominant, which is consistent with the study of Tang et al. [24].

4.3. Simulation of energy harvesting from LCO

In this simulation, energy harvesting became active at 26 s with the open-circuit condition (Section 4.2). Only 1 of the 10 pairs of piezoelectric layers was used as the energy harvester, as shown in Fig. 9. The resistance $R$ was set to 50,000 $\Omega$. Figures 10 and 11 show the results
for Pair 1 driven as a harvester after 26 s. Figure 10 shows the time history of the electric current, which flows in accordance with the change of the charge on the electrodes induced by the deformation of the piezoelectric materials. Figure 11 shows the time history of the vertical tip displacement. After the energy harvester turned on, the amplitude increased slightly and LCO was maintained.

![Figure 10: Time history of electric current with Pair 1 driven as a harvester after 26 s](image1)

![Figure 11: Time history of vertical tip displacement with Pair 1 driven as a harvester after 26 s](image2)

The relation between the reduction of the amplitude and the generated energy for various pairs used as harvesters is shown in Fig. 12. In this figure, the reduction of the amplitude in the open-circuit condition is plotted as “Damping,” and “Generated energy” indicates the total generated energy during 5 s (from 35 s to 40 s). Although energy harvesting has a shunt damping effect, the amplitude increased when Pair 1, 2, 9, or 10 was driven as a harvester. When the first vibration mode is dominant, a large piezoelectric effect occurs at the root because the magnitude of the piezoelectric effect is mainly proportional to the strain $e_{11}$. 

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However, this large effect is not generated when the pair closer to the root is used as the harvester. Turning on energy harvesting triggers a transition of the state of the surrounding flow, and the energy pumped into the structure from the fluid is changed.

![Graph showing the relation between damping and generated energy.](image)

**Figure 12: Relation between damping and generated energy**

### 4.4. Simulation of active control of LCO

In this simulation, active control turned on at 26 s with the open-circuit condition (Section 4.2). Only 1 of the 10 pairs of piezoelectric layers was used as the active control system, as shown in Fig. 13. The resistance $R$ was set to 200 Ω [21]. The top and bottom portions were used as an actuator and sensor, respectively.

![Diagram of the pair of piezoelectric layers used as an active control system.](image)

**Figure 13: Pair of piezoelectric layers used as an active control system**

The applied voltage $V_{\text{actuator}}$ was determined by the following direct velocity feedback control [10, 19, 25]:

$$V_{\text{actuator}} = G \frac{dv_{\text{sensor}}}{dt},$$

where $G$ is the feedback gain and is a negative value. This control method enhances the damping of the system. Especially, when a sensor and an actuator are at the same location
(the so-called collocation condition), this control method enhances the damping for all vibration modes [26]. As shown in Fig. 13, a sensor and an actuator are placed at almost the same location in the present study.

![Figure 14: Time history of measured voltage with Pair 3 driven as an active control system after 26 s](image)

![Figure 15: Time history of applied voltage with Pair 3 driven as an active control system after 26 s](image)

Figures 14, 15, 16, and 17 show the results with $G = -0.4$. Figures 14 and 15 show the time histories of the measured voltage and the applied voltage, respectively, with Pair 3 driven as an active control system after 26 s. Figure 16 shows the time history of the vertical tip displacement. After turning on the active control, the vibration is gradually suppressed until it reaches a steady state. The relation between the reduction of the amplitude and the consumed energy for various pairs used as harvesters is shown in Fig. 17. In this figure, the definition of “Damping” is the same as in Fig. 12, and “Generated energy” indicates the total consumed energy during 5 s (from 35 s to 40 s). Although the direct velocity feedback control increases the damping of the system, the amplitude increased when Pair 1, 9, or 10 was driven as an ac-
tive control system. Both the energy harvesting described in the previous subsection and the direct velocity feedback control enhanced the damping locally. Therefore, it is reasonable that the same tendency with respect to “Damping” is observed in Figs. 12 and 17. In Fig. 17, the consumed energy is negative for all cases. This means that self-powered active control is achieved.

Figure 16: Time history of vertical tip displacement with Pair 3 driven as an active control system after 26 s

Figure 17: Relation between damping and consumed energy

4.5. Simulation of harvesting energy from LCO and its active control

In this simulation, active control and energy harvesting were concurrently turned on at 26 s
with the open-circuit condition (Section 4.2). Pairs 3 and 4 were used as the active control system with direct velocity feedback control. Pairs 7, 8, 9, and 10 were used as energy harvesters. Other pairs were operated in an open-circuit condition. For active control, the resistance $R$ was set to 200 $\Omega$. For energy harvesting, $R$ was set to 50,000 $\Omega$. Six values of feedback gain $G$ were tested: $G = -1.2, -1.0, -0.8, -0.6, -0.4$, and $-0.2$. Figure 18 shows the relation among the damping, the energy generated by the harvesters, and the energy consumed by the active control system. As the feedback gain increased, the contribution of the active control increased, and the damping became larger. The energy consumed by active control also increased. When the vibration was diminished, the degree of deformation became smaller, which led to a smaller piezoelectric effect. Therefore, the generated energy decreased as the damping increased. For all cases, we can see that the generated energy exceeded the consumed energy. Especially, in the case with $G = -1.2$, about 50% of the damping was achieved with the generated energy covering the consumed energy.

![Figure 18: Relation among damping, energy generated by harvesters, and energy consumed by active control systems](image)

5. Conclusion

The motivation of the present study is to numerically investigate the balancing problem between piezoelectric energy harvesting from LCO and the active control of LCO by piezoelectric sensors and actuators. In our previous work, we developed a partitioned iterative method-based coupled analysis system for active control of piezoelectric–structure–fluid interaction problems. In the present study, we integrate a piezoelectric energy harvester model. The proposed analysis system comprises five sub-analysis systems: the fluid mesh update, the fluid analysis, the SEI analysis, the circuit analysis, and the control law. We used numerical examples to show the simulation of harvesting energy from LCO, that of the active control,
and that of the concurrent operation case. Then, we confirmed that a large fraction of the suppression is achieved while the energy scavenged by the harvesters exceeds the energy consumed by active control. Therefore, we conclude that the concurrent operation is promising and worth further investigation.

In future work, we will examine the feasibility of active flutter control with energy harvesting for unmanned air vehicles with multifunctional wing spars. This investigation will require three-dimensional analysis because the LCO of the wing is not the LCO of the axial flow used in the present study. In addition, we need to consider a higher mass ratio.

Acknowledgement

This work was supported by the Japan Society for the Promotion of Science, KAKENHI Grant Number JP19H01098 (Grant-in-Aid for Scientific Research A).

References

[1] F. Afonso, J. Vale, É. Oliveira, F. Lau, A. Suleman: A review on non-linear aeroelasticity of high aspect-ratio wings, Prog. Aerosp. Sci., 89 (2017), 40–57.

[2] E. Jonsson, C. Riso, C. A. Lupp, C. E. S. Cesnik, J. R. Martins, B. I. Epureanu: Flutter and post-flutter constraints in aircraft design optimization, Prog. Aerosp. Sci., 109 (2019), 100537.

[3] J. Xiang, Y. Yan, D. Li: Recent advance in nonlinear aeroelastic analysis and control of the aircraft, Chinese J. Aeronaut., 27 (2014), 12–22.

[4] S. Anton, D. Inman: Vibration energy harvesting for unmanned aerial vehicles, in Active and Passive Smart Structures and Integrated Systems 2008. International Society for Optics and Photonics, San Diego, 2008.

[5] S. Anton, A. Erturk, D. Inman: Multifunctional unmanned aerial vehicle wing spar for low-power generation and storage, J. Aircr., 49 (2012), 292–301.

[6] Y. Wang, D. Inman: Simultaneous energy harvesting and gust alleviation for a multifunctional composite wing spar using reduced energy control via piezoceramics, J. Compos. Mater., 47 (2012), 125–146.

[7] N. Tsushima, W. Su: Flutter suppression for highly flexible wings using passive and active piezoelectric effects, Aerosp. Sci. Technol., 65 (2017), 78–89.

[8] H. D. Akaydin, N. Elvin, Y. Andreopoulos: Energy harvesting from highly unsteady fluid flows using piezoelectric materials, J. Intell. Mater. Syst. Struct., 21:13 (2010), 1263–1278.

[9] S. Ravi, A. Zilian: Simultaneous finite element analysis of circuit-integrated piezoelectric energy harvesting from fluid-structure interaction, Mech. Syst. Signal Pr., 114 (2019), 259–274.

[10] S. Kaneko, G. Hong, N. Mitsume, T. Yamada, S. Yoshimura: Numerical study of active control by piezoelectric materials for fluid–structure interaction problems, J. Sound Vib., 24 (2018), 23–35.
A. N. Brooks, T. J. Hughes: Streamline upwind/Petrov–Galerkin formulations for convection dominated flows with particular emphasis on the incompressible Navier–Stokes equations, *Comput. Methods Appl. Mech. Eng.*, 32:1 (1982), 199–259.

T.E. Tezduyar: Stabilized finite element formulations for incompressible flow computations, *Adv. Appl. Mech.*, 28 (1992), 1–44.

H. Allik, T. J. Hughes: Finite element method for piezoelectric vibration, *Int. J. Numer. Meth. Engng.*, 2 (1970), 151–157.

S. Klinkel, W. Wagner: A geometrically non-linear piezoelectric solid shell element based on a mixed multi-field variational formulation, *Int. J. Numer. Meth. Engng.*, 65 (2006), 349–382.

K. Stein, T. E. Tezduyar, R. Benney: Mesh moving techniques for fluid-structure interactions with large displacements, *J. Appl. Mech.*, 70:1 (2003), 58–63.

T. Sawada, T. Hisada: Fluid–structure interaction analysis of the two-dimensional flag-in-wind problem by an interface-tracking ALE finite element method, *Comput. Fluids*, 36:1 (2007), 136–146.

P. Causin, J. F. Gerbeau, F. Nobile: Added-mass effect in the design of partitioned algorithms for fluid–structure problems, *Comput. Methods Appl. Mech. Eng.*, 194 (2005), 4506–4527.

S. Ravi, A. Zilian: Monolithic modeling and finite element analysis of piezoelectric energy harvesters, *Acta. Mech.*, 228 (2017), 2251–2267.

S. Narayanan, V. Balamurugan: Finite element modelling of piezolaminated smart structures for active vibration control with distributed sensors and actuators, *J. Sound Vib.*, 262 (2003), 529–562.

T. Nishigaki, Y. Odawara, M. Endo: Vibration sensing and control of a flexible beam using piezoelectric films, *Trans. Jpn. Soc. Mech. Eng.*, 63 (1997), 3728–3734.

K. Nakano, M. Ohori, A. Tagaya: Feasibility study on self-powered active vibration control using a piezoelectric actuator, in *Active and Passive Smart Structures and Integrated Systems 2010. International Society for Optics and Photonics*, San Diego, 2010.

H. Sodano, G. Park, D. Inman: Estimation of electric charge output for piezoelectric energy harvesting, *Strain*, 40:2 (2004), 49–58.

C. De Marqui, A. Erturk, D. J. Inman: An electromechanical finite element model for piezoelectric energy harvester plates, *J. Sound Vib.*, 327 (2009), 9–25.

L. Tang, M. Paidoussis, J. Jiang: Cantilevered flexible plates in axial flow: energy transfer and the concept of flutter-mill, *J. Sound Vib.*, 326 (2009), 263–276.

Y. H. Lim, V. V. Varadan, V. K. Varadan: Closed loop finite element modeling of active structural damping in the time domain, *Smart Mater. Struct.*, 8 (1999), 390–400.

M. J. Balas: Direct velocity feedback control of large space structures, *J. Guid. Control*, 2 (1979), 252–253.