Study of Mesh Stiffness of Spur Gear Tooth by Considering Pitting Defect under Dynamic Load Conditions

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Abstract. Gear drives are one of the most essential mechanism of power transmission systems in numerous industrial applications such as automobiles, aerospace and wind turbines, etc. As the speed of gear transmission increases, the study of the dynamic behavior of the gears are more important in the gear design. Gear mesh stiffness plays an eminent role in gear dynamics and it varies in the existence of gear fault such as pitting, spalling and crack. The dynamic performance of the gears are affected by module, contact ratio, pressure angle and transmission error, etc. In order to understand the dynamic properties of spur gear system, it is necessary to calculate the mesh stiffness of the gear tooth pair effectively. In this paper, a comparative study has been carried out using FEM between healthy gear and fault gear by considering pitting defect. Gear mesh stiffness of healthy and fault gear has been compared under dynamic load distribution in gear tooth contacts.

1. Introduction

Dynamic analysis of gears has become a major research interest in high speed rotating power transmission. The gear tooth stiffness and mesh stiffness of a gear pair are showing significant contribution to evaluate dynamic characteristics of gears [1]. Due to high service load, severe operating conditions or fatigue, pits may develop in gears. Pitting is one of the common fatigue failure mode in gear teeth instigated by a small part of material breach out from the surfaces of gear teeth after a definite number of meshing cycles [2]. The dynamic performance of gears fluctuates, when pitting appears on the gear tooth surfaces, whereas gear mesh stiffness reduces [3]. In general, the gear tooth damage level can be estimated based on the variation in the stiffness of gear tooth [4, 5]. In this area several studies has been interpreted to analyze stiffness of the gear tooth with and/or without defects on the tooth flank surface. A novel analytical method was developed to study the effect of spalling defect on the dynamic performance of a spur gear pair, [6]. The correlation of gear tooth damage and the fall in mesh stiffness is more significant while simulating the dynamic response of the gear pair [7]. Zhiguo Wan et al. [8] observed, vibration signal fluctuates as mesh stiffness vary during one complete mesh period. Thus, the time varying mesh stiffness is one of the primary excitation sources of vibration. Many authors proposed a man-made pit in different shapes on the tooth surface approaching the pitch line area in order to simulate the pitting defect [9, 10]. In Ref. [11], the author produced many circular pits artificially in three levels to represent the slight, moderate and severe pitting damages on their experimental test gears as shown in figure. 1. In this present work, the severe pitting damages were modeled using circular pits as highlighted in Ref. [11]. The main focus of this
study is to determine the pitted gear tooth mesh stiffness under dynamic load conditions in a complete mesh cycle. In this scenario, the dynamic load was applied at several points of tooth contact of healthy and pitted gear during the gear tooth mesh period. The gear tooth mesh stiffness was estimated using finite element analysis results. At last, the effects of dynamic load on healthy and pitted gear tooth were analyzed and the percentage of decrease in mesh stiffness was evaluated for the defected gear tooth pair.

2. Tooth Pitting CAD Modeling
In the current investigation, we considered same physical parameters for both the driver and driven gear, which are listed in table 1. The gear quality has been taken as per DIN 3962-1 is DIN8 [12]. According to the Ref. [11], the gear tooth pits are designed in circular shape in order to enhance this work.

![Slight pitting](image1) ![Moderate pitting](image2) ![Severe pitting](image3)

**Figure 1.** Manmade circular pitting damage [11].

The healthy and fault gear are modeled using Solidworks software as shown in figure.2. In faulty gear, all the pits are designed with 2 mm diameter and 0.8 mm depth.

| S.No. | Parameters                                      | Driver and Driven Gear |
|-------|-------------------------------------------------|------------------------|
| 1     | Number of teeth                                 | 11                     |
| 2     | Module (mm)                                     | 4.5                    |
| 3     | Pressure angle                                  | 20°                    |
| 4     | Tip diameter (mm)                               | 63.2                   |
| 5     | Pitch circle diameter (mm)                      | 49.5                   |
| 6     | Base circle diameter (mm)                       | 46.5                   |
| 7     | Root circle diameter (mm)                       | 42.9                   |
| 8     | Center distance (mm)                            | 49.5                   |
| 9     | Circular arc thickness on pitch circle (mm)     | 9.279                  |
| 10    | Addendum Modification                           | 3.038                  |
| 11    | Backlash                                        | 0.15                   |
| 12    | Speed (rpm)                                     | 333                    |
| 13    | Transmission ratio (i)                          | 1:1                    |
In [13], the gear tooth involute profile was generated from base circle and continued as a straight line up to the root circle. In addition to this the gear tooth fillet curve was ignored in order to replicate the single tooth as a cantilever beam as shown in figure 3. (a). We also implemented this model for healthy as well as faulty gear tooth design. X.Liang et. al. [14], has extended this model by adding a single circular pit center on pitch line as shown in figure- 3.(b). The position and size of a single pit was represented by three variables: the distance between the tooth root and the center of the circular pit (u), the circular pit radius (r), and the pitting depth (δ). The details of unsymmetrical distribution of severe pitting damage along the tooth width are given below.

2.1 Severe Pitting Model: The method adopted in [13] for multiple pit model was considered in this study. In this study, multiple pits were created on the gear tooth surface with 2 mm diameter and 0.8 mm depth in order to facilitate severe pitting. The circular pits were modeled above the pitch line by maintaining distance between pit circle center and tooth root. Thus, each circular pit was arranged in a straight line along the face width by (u, r, δ): 5.8 mm, 1 mm and 0.8 mm respectively. Similarly, the circular pits on the next row can be stated by (8.3 mm, 1 mm, 0.9 mm). In our work the severe pitting model, the multiple pits are created with 38 circles on one gear pair, among that 19 pits equally distributed on each of the mating teeth. Therefore, all 38 pit circles are settled on the addendum of the gear tooth.

3. Dynamic Load, Mesh Stiffness and FEM simulations

3.1. Dynamic Load

In general, gear systems are designed by using static analysis. The deviation in profile, circular pitch, and tooth alignment will impart to non-conjugate action. At low speeds, these manufacturing errors produce concealed dynamic loads, but at higher speeds they become more conspicuous. There are several factors which affect their dynamic performance. The complications related to the tooth profile error are utmost important which has been clearly summarized by Buckingham [15]:
"Errors on gear-tooth profiles, caused by elastic deformation under load or by inaccuracies of production, or both, act to change the relative velocities of the mating members. This varying velocity of the rotating members results in a varying load cycle on the teeth of the gears; the amount of this load variation depends largely upon the extent of the effective masses of the revolving gears, the extent of the effective errors and the speed of the gears."

The dynamic loads acting on the tooth have been calculated using Buckingham dynamic load relations as follows [15].

\[ F_d = F_t + \sqrt{(f_a (2f_2 - f_a))} \]  
\[ f_a = \frac{f_1 f_2}{f_1 + f_2} \]  
For 20° Pressure angle \[ f_1 = 0.00120 \left( \frac{1}{R_1} + \frac{1}{R_2} \right) m V^2 \]  
\[ f_2 = \left( \frac{e}{d} + 1 \right) \]  
\[ d = \left( \frac{F_t}{L} \right) \left[ \frac{E_1 Z_1 + E_2 Z_2}{E_1 Z_1 E_2 Z_2} \right] \]  
\[ Z = \frac{y}{(0.242 + 7.25y)} \]  
\[ F_t = F \sin \alpha \]  
\[ F = \frac{P}{V} \]  

4. FEM analysis of healthy and fault gear

In this study, the three-dimensional model of spur gear with involute profile is employed in both the healthy and faulty spur gear models. For this analysis, the driver and driven gear tooth were assumed as rigid body and have same material properties which are listed in the table.2. The perfect and defective gear is designed using SOLID185 element in ANSYS as shown in figure.4.

Table.2 Material properties

| Material                  | Steel-Steel |
|---------------------------|-------------|
| Young’s Modulus (N/mm²)   | 2.1 × 10⁵   |
| Poisson’s ratio            | 0.3         |

In this analysis, dynamic load is calculated using Buckingham’s equations (1), (2), (3), (4), (5) and (6) which is listed in the table.3. The dynamic load is applied at more than several points of contact when the gear pair rotates from the point of engagement to disengagement of the single gear tooth pair [18].
Figure 4. FEM model.

Table 3 Dynamic load value.

| Torque (Nm) | Speed (rpm) | Pitch Line Velocity (m/s) | Effective Mass | $f_1$ (N) | $f_2$ (N) | $f_a$ (N) | $F_r$ (N) | $F_d$ (N) |
|-------------|-------------|---------------------------|----------------|-----------|-----------|-----------|-----------|-----------|
| 213.8       | 333         | 0.86                      | 10.35          | 13.822    | 16151.6   | 1273.24   | 10830     | 14925     |

The calculated dynamic load is uniformly distributed in each node at all the points of contact along the direction of face width.

4.1. Mesh Stiffness calculation

Mesh stiffness can be defined as combined overall tooth stiffness of the tooth in contact. The mesh stiffness $k_m$ of single tooth contact pair can be written as [16, 17].

$$k_m = \frac{1}{\frac{1}{k_{g1}} + \frac{1}{k_{g2}}}$$  \hspace{1cm} (9)

where $k_{g1}$ is the driver gear tooth stiffness, $k_{g2}$ is the driven gear tooth stiffness.

$$k = \frac{F_d}{\delta}$$  \hspace{1cm} (10)

where $k$ is the tooth stiffness, $F_d$ is the normal force or dynamic load applied to the tooth profile along the line of action and $\delta$ is the deflection of tooth in the direction of applied load $F_d$.

The deflection of the tooth can be calculated as:

$$\delta = \delta_x \sin \alpha + \delta_y \cos \alpha$$  \hspace{1cm} (11)

where $\delta_x$ and $\delta_y$ are the displacement of the contact element of the tooth in $x$ and $y$ direction and $\alpha$ is the pressure angle as shown in figure 6.
Figure 5. Gear Pair arrangement [18].

Healthy Gear.

Fault Gear.

Figure 6. Deformation at 0, 5, 10 and 15 degree angle of rotation of a gear pair.
Figure 7. Deformation at 20 and 25 degree angle of rotation of a gear pair.

5. Result and Discussion

The healthy and fault gear are analyzed under dynamic conditions using FEM from the point of engagement to disengagement. The displacements of gear tooth pair without and with pitting are evaluated at 22 contact points. However, the few displacements of healthy and fault gear tooth pair at 0, 5, 10, 15, 20, 25 degree angle of rotation are represented in Figure 6 and 7. The evaluated deflections from these analyses are used to calculate the mesh stiffness of single gear tooth pair without and with pitting. The time varying mesh stiffness are plotted in Figure 8. The maximum observed in the healthy and fault single gear tooth pair are $9.564 \times 10^7$ N/mm and $7.581 \times 10^7$ N/mm respectively.

Figure 8. Mesh stiffness comparison of healthy and fault gear pair.

The percentage of mesh stiffness reduction in single gear tooth pair with pitting is witnessed about 24.99% as shown in the stable 4.
Table 4. Percentage of mesh stiffness reduction.

| Percentage of reduction (%) | Gear tooth pair without pitting | Gear tooth pair without pitting |
|-----------------------------|---------------------------------|---------------------------------|
| Average Mesh Stiffness $\times 10^7$ (N/mm) | 7.2949 | 5.4713 |
| 24.99 | |

6. Conclusion

The finite element model of the single gear tooth pair without and with pitting are analyzed to investigate the changes in mesh stiffness. The artificial circular pits are considered to mimic the pitting defect as mentioned in ref. [13]. Based on quantifying the percentage of reduction in mesh stiffness, it is required to reduce the dynamic load effect in order to improve the mesh stiffness of the gear tooth pair. This study reveals the strong awareness on the importance of mesh stiffness evaluation during dynamic mesh period. So that, the calculated mesh stiffness values can be used in dynamic analysis of gear systems.

Nomenclature,

- $F_d$ – Dynamic Load (N)
- $P$ – Power to be transmitted (kW)
- $f_a$ – Accelerating force (N)
- $E$ – Modulus of elasticity (N/mm$^2$)
- $f_1$ – Force required to accelerate the connecting masses (N)
- $m$ – Effective mass
- $f_2$ – Limiting acceleration load (N)
- $y$ – Form factor
- $F_t$ – Applied tangential load (N)
- $Z$ – Elasticity factor of shaft
- $e$ – Measured error on pair of mating teeth (mm)
- $F$ – Force along the line of action (N)
- $d$ – Total elastic deformation of mating teeth under applied Load (mm)
- $V$ – Pitch line velocity (m/s)
- $L$ – Face width of gears (mm)

7. References

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