Basic Tools for Studies on the Formation and Disruption of Star Clusters: the Luminosity Function

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Abstract. The luminosity function (LF) of young star clusters is often approximated by a power law function. For clusters in a wide range of galactic environments this has resulted in fit indices near $-2$, but on average slightly steeper. A fundamental property of the $-2$ power law function is that the luminosity of the brightest object ($L_{\text{max}}$) scales linearly with the total number of clusters, which is close to what is observed. This suggests that the formation of Young Massive Clusters (YMCs) is a result of the size of the sample, i.e. when the SFR is high it is statistically more likely to form YMCs, but no particular physical conditions are required. In this contribution we provide evidence that the LF of young clusters is not a $-2$ power law, but instead is curved, showing a systematic decrease of the (logarithmic) slope from roughly $-1.8$ at low luminosities to roughly $-2.8$ at high luminosities. The empirical LFs can be reproduced by model LFs using an underlying cluster IMF with a Schechter type truncation around $M_* \approx 2 \times 10^5 M_\odot$. This value of $M_*$ can not be universal since YMCs well in excess of this $M_*$ are known in merging galaxies and merger remnants. Therefore, forming super massive clusters ($\gtrsim 10^6 M_\odot$) probably requires conditions different from those in (quiescent) spiral galaxies and hence is not only the result of a size-of-sample effect. From the vertical offset a cluster formation efficiency of $\sim 10\%$ is derived. We find indications for this efficiency to be higher when the SFR is higher.

1 Introduction: the luminosity function of young star clusters

It has been well established that the luminosity function (LF) of young star clusters is profoundly different from that of old globular clusters (GCs). Where the latter is peaked (when presented in logarithmic luminosity bins or in magnitude bins), the former rises down to the detection limit. When approximating the LF by a power law function of the form

$$\frac{dN}{dL} \propto L^{-\alpha}$$

many studies find $\alpha \approx 2$ for clusters in widely varying environments (for overviews see de Grijs et al. 2003; Whitmore 2003; Larsen 2006). However, the values found for $\alpha$ are often slightly higher (i.e the LF is steeper, see e.g. Dolphin & Kennicutt 2002; Elmegreen et al. 2002; Larsen 2002) and the range is rather large ($1.8 \lesssim \alpha \lesssim 2.8$) to be explained by statistical fluctuations or photometric uncertainties.

The LF is often related to the cluster initial mass function (CIMF). However, the relation between the LF and the CIMF is not one-to-one, since the LF consists of clusters of different ages and evolutionary fading causes clusters of the same mass, but with different ages, to contribute at different luminosities in the LF. The empirically determined CIMF seems also well described by a power law, with an index of $-2$ (e.g.
Elmegreen & Efremov 1997; Zhang & Fall 1999; McCrady & Graham 2007), with the spread between different studies much smaller than that found for the LF.

Although the LFs of the individual cluster systems are most of the time within a few $\sigma$ compatible with a $-2$ power law, a comparison between the different results shows that the deviations from the $-2$ power law are not randomly scattered, but instead show some systematic variations, in the sense that the LF is

1. **steeper at higher luminosities** (Whitmore et al. 1999; Benedict et al. 2002; Larsen 2002; Mengel et al. 2005; Gieles et al. 2006a,b; Hwang & Lee 2008);

2. **steeper in redder filters** (e.g. Dolphin & Kennicutt 2002; Elmegreen et al. 2002; Gieles et al. 2006a,b; Haas et al. 2008; Cantiello et al. 2009).

The steepening with increasing luminosity is illustrated in Fig. 1 where we show results of power law fits to LFs of clusters in different galaxies taken from literature. The horizontal bars indicate the fit range used and the vertical bars give the uncertainties in the power law index. From this a decrease of $-\alpha$ (i.e. steepening) with increasing luminosity is seen. This suggests that the CIMF is not a universal continuous power law. If it was a continuous power law with the same index at all ages and in all galaxies, all LFs should be power laws with the same index. Age dependent extinction or bursts in the formation rate would not cause a difference between the CIMF and the LF. An addition of identical power laws always results in the same power law. Luminosity dependent extinction could cause a deviation from a $-2$ power law. This will be addressed in Section 3.

The fact that the LF gets steeper at the bright-end is an indication that the CIMF is truncated at some upper mass $M_{up}$. When an abrupt truncation of the CIMF is assumed, it is possible to roughly estimate the index of the bright-end of the LF. Assume that the CIMF is fully populated, i.e. the mass of the most massive cluster actually formed, $M_{max}$, is equal to $M_{up}$. Then assume a constant formation history of clusters: $dN_{up}/d\tau =$constant. The luminosities of these clusters, $L_{up}$, are age dependent. The light-to-mass ratio, or the flux of a cluster of constant mass, scales roughly with age as $\tau^{-\zeta_\lambda}$, with $0.6 \lesssim \zeta_\lambda \lesssim 1$ depending on the wavelength $\lambda$, such that $\partial L_{up}/\partial \tau \propto \tau^{-\zeta_\lambda-1} \propto L_{up}^{1+1/\zeta_\lambda}$. The same arguments hold for the luminosities of the 2nd, 3rd, etc. most massive clusters, such that the bright-end of the LF is

$$\frac{dN}{dL}(\text{bright}) \propto \frac{dN}{d\tau} \frac{\partial \tau}{\partial L} \propto L^{-1-1/\zeta_\lambda}.$$  

(2)

For the $V$-band $\zeta_V = 0.7$ (Boutloukos & Lamers 2003; Gieles et al. 2007), such that for the $V$-band LF the index at the bright-end is $-2.5$. For the $U(I)$-band $\zeta_{U(I)} \simeq 1(0.6)$ and then equation (3) predicts a logarithmic slope of $-2(-2.7)$, i.e. shallower(steeper) at blue(red) wavelengths. The faint-end of the LF should still be a power law with index $-2$ at all wavelengths or flatter if mass dependent disruption is important. This double power law shape for the LF was found by Gieles et al. (2006b) and Whitmore et al. (1999) for the LFs of clusters in M51 and the Antennae, respectively.

In Section 2 we will model the LF in more detail and show that also this double power law shape is not the best approximation of the LF.
2 Modelling the luminosity function

In order to understand the luminosity function (LF) of star clusters we create semi-analytic cluster population models. For this the distribution function of initial masses ($M_i$) with age needs to be defined first. We define the probability of forming a cluster with an initial mass between $M_i$ and $M_i + dM_i$ at a time between $t$ and $t + dt$, as $dN/(dM_i dt)$. For a Schechter (1976) type CIMF this then gives

$$\frac{dN}{dM_i dt} = A M_i^{-2} \exp(-M_i/M_*),$$

(4)

where $A$ is a constant that scales with the cluster formation rate (CFR) and $M_*$ is the mass where the exponential drop occurs. Note that we fix the power law index in the Schechter CIMF to $-2$.

An equivalent distribution function for the present day masses ($M$) of clusters, $dN/(dM dt)$, can be found from multiplying equation (4) by $|\partial M_i/\partial M|$ which describes the relation between the initial and the present cluster mass to taken into account cluster dissolution. This was done by Fall & Zhang (2001) and Jordán et al. (2007) to model the mass function of globular clusters using a constant mass loss rate. Lamers et al. (2005) and Gieles (2009) apply a mass dependent mass loss rate to model age and mass distributions of cluster populations using a power law and a Schechter CIMF, respectively. We refer the aforementioned studies and Larsen (2009) for full descriptions of the formulae.

An equivalent distribution function of luminosity ($L$) and $t$ can be acquired by multiplying $dN/(dM dt)$ by the age dependent mass-to-light ratio ($\Upsilon$) (see also Fall 2006; Larsen 2009), such that
Figure 2. **Top panel:** Model for the LF based on a constant cluster formation rate, a Schechter cluster initial mass function with $M_* = 2 \times 10^5 \, M_\odot$ and a moderate cluster disruption time (full line). The dotted line shows a power law with index $-2$ for comparison. **Bottom panel:** the relative contribution of different age groups. The logarithmic age ranges are indicated.

\[ \frac{dN}{dL dt} = \frac{dN}{dM dt} \Psi(t). \]  
(5)

Since $\Psi(t)$ needs to be taken from SSP models (we adopt the Bruzual & Charlot (2003) models for solar metallicity and a Salpeter stellar IMF) from here on the results need to be solved numerically.

The luminosity function (LF) is then found by integrating $dN/(dL dt)$ over all ages

\[ \frac{dN}{dL} = \int \frac{dN}{dL dt} dt. \]  
(6)

The logarithmic slope at each $L$, which is a probe of the shape of the LF, is found from

\[ -\alpha(L) = \frac{d \log (dN/dL)}{d \log L}. \]  
(7)

An example of a model LF based on a Schechter CIMF and mass dependent dissolution is shown in the top panel of Fig. 2. The bottom panel shows how clusters of different ages contribute to the LF. A feature of the truncated CIMF is that at the bright-end mainly young clusters contribute to the LF. This feature was indeed found from age determinations of the brightest clusters in a sample of spiral galaxies (Larsen 2009).

3 Comparison to the cluster populations of M51, M74 and M101

Here the model of the previous section is compared to the LFs of star clusters in 3 spiral galaxies: M51 (NGC 5194), M74 (NGC 628) and M101 (NGC 5457). We use the data from Haas et al. (2008); Hwang & Lee (2008) (M51), Barmby et al. (2006) (M101) and Gieles et al. (2010, in prep) (M74).
First we take the model from Fig. 2 and apply a vertical offset such that it matches the empirical LFs. This directly gives us the cluster formation rate (CFR) which enters in the model through the variable $A$ in equation (4). A comparison to the global SFR in these galaxies (Table 1) shows that the cluster formation efficiency ($\Gamma \equiv \text{CFR}/\text{SFR}$, Bastian 2008) is roughly 10%, with a tendency of $\Gamma$ to increase with increasing SFR. Note that this fraction could be affected by disruption if clusters are dissolving but are still picked up and therefore part of the LF.

Then we break the LFs in different luminosity segments and determine the logarithmic slopes in these intervals using a maximum likelihood estimate. The results are shown in Fig. 3. As can be seen the steepening of the LF with increasing $L$ as shown in Fig. 1 is also found within the LF of individual cluster populations (left panel). In the right panel we show that the filter dependent logarithmic slopes for the M51 LF are also in good agreement with the model.

So far we have ignored the effect of extinction on the individual clusters. If luminous objects are more extincted, then the LF should be steeper in the bluer filter. Since the opposite is found, it seems that extinction does not play a vital role in shaping the LF.

**Table 1.** Comparison of the CFR derived from the LF of Fig. 2 to the SFR for the cluster populations considered here.

| Galaxy | SFR [$M_\odot$ yr$^{-1}$] | CFR [$M_\odot$ yr$^{-1}$] | $\Gamma$ |
|--------|---------------------------|-----------------------------|--------|
| M74    | $\sim$1                   | 0.05                        | 0.05   |
| M101   | $\sim$3                   | 0.25                        | 0.08   |
| M51    | $\sim$5                   | 0.90                        | 0.18   |
4 Summary

We show that the LF of young star clusters is not a universal $-2$ power law, but instead shows evidence for curvature, in the sense that the logarithmic slope ($-\alpha$) decreases towards brighter luminosities. This observational trend can be reproduced by a model based on a Schechter type cluster initial mass function with $M_\star \approx 2 \times 10^5 \, M_\odot$. A property of this model is that $-\alpha(L)$ is smaller (steeper) in redder filters at the same $L$, which is what we find for the rich cluster population of the interacting galaxy M51.

Acknowledgments. MG thanks Pauline Barmby for kindly providing the luminosities of the M101 clusters and the organisers of the meeting Galaxy Wars: Stellar Populations and Star Formation in Interacting Galaxies for an enjoyable conference.

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