Universality of $q_T$ resummation for electroweak boson production

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Abstract. We perform a global analysis of transverse momentum distributions in Drell-Yan pair and $Z$ boson production in order to investigate universality of nonperturbative contributions to the Collins-Soper-Sterman resummed form factor. Our fit made in an improved nonperturbative model suggests that the nonperturbative contributions follow universal nearly-linear dependence on the logarithm of the heavy boson invariant mass $Q$, which closely agrees with an estimate from the infrared renormalon analysis.

Transverse momentum distributions of heavy Drell-Yan lepton pairs, $W$, or $Z$ bosons produced in hadron-hadron collisions present an interesting example of factorization for multi-scale observables. If the transverse momentum $q_T$ of the electroweak boson is much smaller than its invariant mass $Q$, $d\sigma/dq_T$ at an $n$-th order of perturbation theory includes large contributions of the type $\alpha_s^n \ln^m(q_T^2/Q^2)/q_T^2$ ($m = 0, 1 \ldots 2n - 1$), which must be summed through all orders of $\alpha_s$ to reliably predict the cross section [1]. Such resummation is realized in the Collins-Soper-Sterman (CSS) formalism [2], which describes soft and collinear QCD radiation in a wide range of energies by introducing a resummed form factor $\tilde{W}(b)$ in impact parameter ($b$) space.

While the short-distance contributions ($b \lesssim 1 \text{ GeV}^{-1}$) to the CSS form factor $\tilde{W}(b)$ can be calculated in perturbative QCD, long-distance nonperturbative contributions from $b > 1 \text{ GeV}^{-1}$ are not yet fully computable, even though their basic form can be deduced from the infrared renormalon analysis [3]. The factorization theorem behind the CSS formalism predicts that the nonperturbative contributions are universal in unpolarized Drell-Yan-like and semi-inclusive DIS processes. Consequently the function $\mathcal{F}_{NP}(b, Q)$ that describes the nonperturbative terms can be constrained in a global fit to the hadronic $q_T$ data, just as the $k_T$-integrated parton densities are constrained with the help of inclusive scattering data. $\mathcal{F}_{NP}(b, Q)$ must be known precisely in order to successfully measure the $W$ boson mass, because uncertainties in $\mathcal{F}_{NP}(b, Q)$ may affect the measured value of $M_W$ at the level comparable to the targeted accuracy of the measurement, $\delta M_W \approx 30 \text{ MeV}$ at the Tevatron and $15 \text{ MeV}$ at the LHC. It is therefore interesting to investigate if $\mathcal{F}_{NP}(b, Q)$ found in the $q_T$ fit is consistent with the universality hypothesis.

1 Talk given by P. Nadolsky at the XIII International Workshop on Deep Inelastic Scattering (DIS 2005, April 27-May 1, 2005, Madison, WI, U.S.A.).
and whether its preferred form is compatible with the renormalon analysis.

These issues were explored recently in Ref. [4], where a global analysis of $q_T$ data from fixed-target Drell-Yan pair production and Tevatron and whether its preferred form is compatible with the renormalon analysis. Although $\mathcal{F}_{NP}(b, Q)$ primarily parametrizes the “power-suppressed” terms, i.e., terms proportional to positive powers of $b$, its form found in the fit is correlated with the assumed behavior of the leading-power terms (logarithmic in $b$ terms) at $b < 2 \text{ GeV}^{-1}$. The exact behavior of $\tilde{W}(b)$ at $b > 2 \text{ GeV}^{-1}$ is of reduced importance, as $\tilde{W}(b)$ is strongly suppressed at such $b$. For these reasons, we closely followed the procedure of the previous global $q_T$ analysis [5], while paying close attention to the model of the leading-power terms at perturbative and moderately nonperturbative transverse distances, $b < 2 \text{ GeV}^{-1}$.

The large-$b$ contributions were introduced by using the $b_*$ model [2], as

$$\tilde{W}(b) = \tilde{W}_{\text{pert}}(b_*) e^{-\mathcal{F}_{NP}(b, Q)}.$$  

Here $\tilde{W}_{\text{pert}}(b_*)$ is the perturbative part of $\tilde{W}(b)$, i.e., its leading-power part evaluated at a finite order of $\alpha_s$. $\tilde{W}_{\text{pert}}(b_*)$ depends on the variable $b_* \equiv b/(1 + b^2/b_{\text{max}}^2)^{1/2}$ and serves as an approximation for all leading-power terms. Its shape is varied at all $b$ by adjusting a single parameter $b_{\text{max}}$. The $b_*$ model with a relatively low $b_{\text{max}} = 0.5 \text{ GeV}^{-1}$ was a choice of the previous $q_T$ fits [5, 6]. However, it is natural to consider $b_{\text{max}}$ above $1 \text{ GeV}^{-1}$ in order to avoid ad hoc modifications of $\tilde{W}_{\text{pert}}(b)$ in the $b$ region where perturbation theory is still applicable. In Ref. [4], we proposed a modification in the $b_*$ model that allowed us to increase $b_{\text{max}}$ at least up to $\approx 3 \text{ GeV}^{-1}$, while preserving correct resummation of the large logarithms at small $b$ and numerical stability of the Fourier-Bessel transform. If a very large $b_{\text{max}}$ comparable to $1/\Lambda_{\text{QCD}}$ is taken, $\tilde{W}_{\text{LP}}(b)$ essentially coincides with $\tilde{W}_{\text{pert}}(b)$, extrapolated to large $b$ by using the known, although not always reliable, dependence of $\tilde{W}_{\text{pert}}(b)$ on $\ln b$. Hence, the new prescription can be also used to test viability of extrapolation of $\tilde{W}_{\text{pert}}(b)$ to large $b$, reminiscent of similar extrapolations introduced in the alternative models [7, 8].

Following the renormalon analysis and Ref. [5], we assumed a Gaussian form of the nonperturbative function, $\mathcal{F}_{NP}(b, Q) \equiv \tilde{a}(Q) b^2$, with

$$a(Q) \equiv a_1 + a_2 \ln [Q/(3.2 \text{ GeV})] + a_3 \ln [100 x_1 x_2].$$  

(2)

The dependence of $\mathcal{F}_{NP}$ on $\ln Q$ is a consequence of renormalization-group invariance of the soft-gluon radiation. The coefficient $a_2$ of the $\ln Q$ term has been related to the vacuum average of the Wilson loop operator and evaluated within lattice QCD as $0.19^{+0.12}_{-0.09}$ GeV$^2$ [9]. To see if the universal Gaussian behavior is consistent with the data, we first examined the values of $a(Q)$ that are independently preferred by each bin of $Q$ in 5 examined experimental data sets. Fig. 1(a) shows the best-fit values of $a(Q)$ obtained in independent fits to the data in each bin of $Q$ for $b_{\text{max}} = 1.5 \text{ GeV}^{-1}$. The best-fit $a(Q)$ follow a nearly linear dependence on $\ln Q$, and the slope $a_2 \equiv da(Q)/d(\ln Q)$ is close to the renormalon analysis expectation of $0.19 \text{ GeV}^2$ [9]. Such nearly linear behavior of $a(Q)$ is observed in the entire range $b_{\text{max}} = 1 - 2 \text{ GeV}^{-1}$, and it less pronounced at $b_{\text{max}}$ outside of the interval 1-2 GeV$^{-1}$. Since the best-fit $a(Q)$ in each $Q$ bin are essentially
Figure 1. (a) The best-fit values of $a(Q)$ obtained in independent scans of $\chi^2$ for the contributing experiments. The vertical error bars correspond to the increase of $\chi^2$ by unity above its minimum in each $Q$ bin. The slope of the line is equal to the central-value prediction from the renormalon analysis [9].

(b) The best-fit $\chi^2$ and coefficients $a_1$, $a_2$, and $a_3$ in $F_{NP}(b, Q)$ for different values of $b_{\text{max}}$. The size of the symbols approximately corresponds to 1\(\sigma\) errors for the shown parameters.

independent, we conclude that the data support the universality of $F_{NP}$, when $b_{\text{max}}$ lies in the range $1 - 2$ GeV\(^{-1}\). In addition, each experimental data set individually prefers a nearly quadratic dependence on $b$, $F_{NP} = a(Q)b^2 - \beta$, with $|\beta| < 0.5$ in all experiments.

Next, we performed a simultaneous fit of our model to all the data. Fig. 1(b) shows the dependence of the best-fit $\chi^2$, $a_1$, $a_2$, and $a_3$ on $b_{\text{max}}$. As $b_{\text{max}}$ is increased above 0.5 GeV\(^{-1}\) assumed in the studies [5, 6], $\chi^2$ rapidly decreases, becomes relatively flat at $b_{\text{max}} = 1 - 2$ GeV\(^{-1}\), and grows again at $b_{\text{max}} > 2$ GeV\(^{-1}\). The global minimum of $\chi^2$ is reached at $b_{\text{max}} \approx 1.5$ GeV\(^{-1}\), where all data sets are described equally well, without major tensions among the five experiments. The magnitudes of $a_1$, $a_2$, and $a_3$ are reduced when $b_{\text{max}}$ increases from 0.5 to 1.5 GeV\(^{-1}\). In the whole range $1 \leq b_{\text{max}} \leq 2$ GeV\(^{-1}\), $a_2$ agrees with the renormalon analysis estimate. The coefficient $a_3$, which parametrizes deviations from the linear $\ln Q$ dependence, is considerably smaller ($< 0.05$) than both $a_1$ and $a_2$ ($\sim 0.2$). This behavior supports the conjecture in [7] that $a_3$ is small if the exact form of $\tilde{W}_{\text{perl}}(b)$ is maximally preserved.

The preference for the values of $b_{\text{max}}$ between 1 and 2 GeV\(^{-1}\) indicates, first, that the data do favor the extension of the $b$ range where all leading-power terms are ap-
proximated by their finite-order expression $\tilde{W}_{\text{pert}}(b)$. In $Z$ boson production, this region extends up to $3 - 4 \text{ GeV}^{-1}$ as a consequence of the strong suppression of the large-$b$ tail by the Sudakov exponent. The fit to the $Z$ data is actually independent of $b_{\text{max}}$ within the experimental uncertainties for $b_{\text{max}} > 1 \text{ GeV}^{-1}$. In the low-$Q$ Drell-Yan process, the continuation of $b\tilde{W}_{\text{pert}}(b)$ far beyond $b \approx 1 \text{ GeV}^{-1}$ is disfavored because of large higher-order corrections to $b\tilde{W}_{\text{pert}}(b)$ at $b$ around $1.5 \text{ GeV}^{-1}$. To summarize, the extrapolation of $\tilde{W}_{\text{pert}}(b)$ to $b > 1.5 \text{ GeV}^{-1}$ is disfavored by the low-$Q$ data sets, if a purely Gaussian form of $F_{\text{NP}}$ is assumed. The Gaussian approximation is adequate, on the other hand, in the $b_*$ model with $b_{\text{max}}$ in the range $1 - 2 \text{ GeV}^{-1}$.

In $Z$ boson production, our best-fit $a(M_Z) = 0.85 \pm 0.10 \text{ GeV}^2$ agrees with $0.8 \text{ GeV}^2$ found in the extrapolation-based models [7, 8], and it is about a third of $2.7 \text{ GeV}^2$ predicted by the BLNY parametrization. In the low-$Q$ Drell-Yan case, our $a(Q) = 0.2 - 0.4 \text{ GeV}^2$ is close to the average $\langle a \rangle = 0.19 - 0.28 \text{ GeV}^2$ in four $Q$ bins of the E288 and E605 data found in the model [7]. To describe the low-$Q$ data, Ref. [7] allowed a large discontinuity in the first derivative of $\tilde{W}(b)$ at $b$ equal to the separation parameter $b_{\text{max}}^{QZ} = 0.3 - 0.5 \text{ GeV}^{-1}$, where switching from the exact $\tilde{W}_{\text{pert}}(b)$ to its extrapolated form occurs. In the revised $b_*$ model, such discontinuity does not happen, and $\tilde{W}_{\text{LP}}(b)$ is closer to the exact $\tilde{W}_{\text{pert}}(b)$ in a wider $b$ range than in Ref. [7].

The best-fit parameters in $F_{\text{NP}}$ found in the new model are quoted in Ref. [4]. The global fit places stricter constraints on $F_{\text{NP}}$ at $Q = M_Z$ than the Tevatron Run-1 $Z$ data alone. Theoretical uncertainties from a variety of sources may be substantial in the low-$Q$ Drell-Yan process, which is indicated, in particular, by the dependence of the agreement with the low-$Q$ data on an arbitrary factorization scale $C_3$ in $\tilde{W}_{\text{pert}}(b)$. The low-$Q$ uncertainties do not substantially affect predictions at the electroweak scale. The $\mathcal{O}(\alpha_s^2)$ corrections and scale dependence are smaller in $W$ and $Z$ production, and, in addition, the term $\alpha_s^2 \ln Q$, which arises from the soft factor $\mathcal{I}(b,Q)$ and dominates $F_{\text{NP}}$ at $Q = M_Z$, shows little variation with $C_3$. Consequently, the revised $b_*$ model with $b_{\text{max}} \approx 1.5 \text{ GeV}^{-1}$ increases our confidence in the transverse momentum resummation at electroweak scales by exposing the soft-gluon origin and universality of the dominant nonperturbative contributions at collider energies.

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