Stability of Multiquark Systems

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Abstract. We give a brief review of developments in the field of exotic hadrons formed of more than three quarks and/or antiquarks. In particular we discuss the stability of multiquark systems containing heavy flavours. We show that the gluon exchange model and the chiral constituent quark model based Goldstone boson (pseudoscalar meson) exchange give entirely different results.

INTRODUCTION

Here we consider systems formed of more than three quarks and/or antiquarks ($q^m \bar{q}^n$ with $m + n > 3$). These are a category of exotic hadrons. They are "exotic" with respect to "ordinary" hadrons which are either mesons ($q \bar{q}$) or baryons ($q^3$) systems. The existence or nonexistence of stable exotics against strong decays is crucial for understanding some aspects of the strong interactions. If discovered, their properties could be an important test for the validity of various quark models. Most theoretical and experimental effort has been devoted so far to systems described by the colour state [222]$_C$. These are the tetraquarks $q^2 \bar{q}^2$ [1], the pentaquarks $q^4 \bar{q}$ [2,3] and the hexaquarks $q^6$ [1]. They have a baryonic number $B = 0$, 1 and 2 respectively. Multibaryon systems with $B$ up to 9 have also been studied within the SU(3) Skyrme model [4].

The most celebrated example of exotics is the H-particle (H- for hexaquark) predicted in the context of the MIT bag model more than 20 years ago [1]. It is a dibaryon with the flavor content of two $\Lambda$ baryons, i.e. it contains light quarks only. Its existence remains controversial (see below).

From theoretical general arguments one expects an increase in stability of multiquark systems if they contain heavy flavours $Q = c$ or $b$. A recent review of the experimental efforts to search for the H-particle and charmed-strange pentaquarks can be found in [5]. In summary, no evidence for the production of a deeply bound...

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H-particle has been observed, the production cross section being one order of magnitude than the theoretical estimates. By deeply bound state we understand a compact object, bound by about 80 MeV, as in Jaffe’s picture [1]. However, a molecular type structure, like that of the deuteron, is not excluded and there are some suggestive signals to be confirmed. Within the confidence level of the analyzed experiments, no convincing evidence for the production of pentaquarks with the flavour content \( uuds\bar{c} \) and \( udds\bar{c} \) has been observed either. However the existence of pentaquarks is not ruled out. The analysis done so far can provide a good starting point for future search in high statistics charm experiments at CERN [6] or Fermilab [7].

If experimentally discovered, the properties of multiquark systems would help to put constraints on phenomenological interquark forces. Indeed the theoretical predictions are model dependent. Here we are concerned with constituent quark models which simulate the low-energy limit of QCD and discuss theoretical predictions for compact objects. We compare results from constituent quark models where the spin-dependent term of the quark-quark interaction is described by the chromomagnetic part of the one gluon exchange (OGE) interaction [8] with results we obtained from a chiral model where the quarks interact via Goldstone boson exchange (GBE) [9,10], i.e. pseudoscalar mesons. In the latter model the hyperfine splitting in hadrons is due to the short-range part of the Goldstone boson exchange interaction between quarks, instead of the OGE interaction of conventional models. The GBE interaction is flavor dependent and its main merit is that it reproduces the correct ordering of positive and negative parity states in all parts of the considered spectrum. Moreover, the GBE interaction induces a strong short-range repulsion in the \( \Lambda-\Lambda \) system, which suggests that a deeply bound H-particle should not exist [11], in agreement with the high-sensitivity experiments at Brookhaven [12].

In the stability problem we are interested in the quantity

\[
\Delta E = E(q^m\bar{q}^n) - E_T
\]

where \( E(q^m\bar{q}^n) \) represents the multiquark energy and \( E_T \) is the lowest threshold energy for dissociation into two hadrons: two mesons for tetraquarks, a baryon + a meson for pentaquarks and two baryons for hexaquarks. In the right-hand side of (1) both terms are calculated within the same model. A negative \( \Delta E \) suggests the possibility of a stable compact multiquark system.

We first give a brief summary of the situation for tetraquarks, pentaquarks and hexaquarks. Next we outline the main characteristics of the GBE model, which is more recent and less known than the OGE model. In the last section we compare the results which we obtained in the frame of the GBE model with those from the literature, based on the OGE model.
THE TETRAQUARKS

The light tetraquarks are related to the study of meson-meson scattering as e.g. \( \pi\pi, \pi\eta, \pi\eta', \pi K, \) etc. and to the identification of scalar mesons, i.e. mesons with quantum numbers \( J^{PC} = 0^{++} \), having masses and decay properties which do not fit into a \( q\bar{q} \) bound state. One expects a non-\( q\bar{q} \) scalar component to play an important role in the mass range below 1800 MeV. The well observed isovector \( a_0(980) \) and the isoscalar \( f_0(980) \) mesons are interpreted as being \( q^2\bar{q}^2 \) states [1] or in a more realistic version as \( KK \) molecules [13] (see also [14]).

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A recent reanalysis of the \( \pi\pi \) scattering [17] reduces the interval of the \( \sigma \) mass to 400-800 MeV so that its central value returns to 600 MeV. The scalars \( a_0(980) \), the \( f_0(980) \), the \( \sigma(600) \) and the \( \kappa(900) \) (found in the analysis of \( \pi K \) scattering) are possible members of a scalar nonet (see e.g. [18]) and satisfy the Gell-Mann-Okubo mass formula [19].

The heavy tetraquarks have been studied in a variety of models and experimental search of double charmed tetraquarks (\( cc\bar{u}\bar{d} \)) are planned at CERN [6]. In the following let us denote by \( q \) a light quark \( u, d \) or \( s \) and by \( Q \) a heavy one \( c, b \) or \( t \). Theoretical work has focussed on tetraquarks of type \( QQq\bar{q} \) or equivalently \( QQq\bar{q} \) (see e.g. [20]). Note that one can also have \( QQq\bar{q} \) systems. These have two distinct thresholds \( QQ + q\bar{q} \) and \( Qq + Qq \). The latter is the same as for \( QQqq \) and it can be shown [21] that \( m_{QQ} + m_{q\bar{q}} \leq 2m_{Qq} \) which means that \( QQ + q\bar{q} \) is the lower threshold. Then assuming that the mass of \( QQq\bar{q} \) is the same as that of \( QQq\bar{q} \), the latter has more chance to be bound. Also, it is more convenient for variational studies, where an upper bound is more conclusive about stability. Moreover \( QQq\bar{q} \) has no meson-antimeson annihilation channels as \( QQq\bar{q} \) does have. As an example, detailed arguments in favour of the stability of \( cc\bar{u}\bar{d} \) as compared to \( cc\bar{u}\bar{d} \) are given in [6].

**TABLE 1.** Estimate of the size \( R \sim (\alpha_s(m_Q) m_Q)^{-1} \) of a \( QQ \) pair

| \( QQ \) | \( m_Q(\text{GeV}) \) | \( \alpha_s(m_Q) \) | \( (\text{fm}) \) |
|-------|----------------|---------------|------|
| \( cc \) | 1.5            | 0.44          | 0.29 |
| \( bb \) | 5.0            | 0.28          | 0.14 |
| \( tt \) | 175.0          | 0.13          | 9.10^{-3} |

The stability of \( QQq\bar{q} \) relies on the fact that \( QQ \) brings a small kinetic energy into the system and forms a tightly bound pair of size \( (\alpha_s(m_Q) m_Q)^{-1} \) (see Table 1). Then two heavy quarks act as an almost point-like heavy color antitriplet source. If \( Q \) is heavy enough, as it is the case of \( t \), the short range Coulomb attraction plays an important role in the formation of the tetraquark system and leads to...
The claim in Ref. [22] is that $c$ and $b$ are not heavy enough to enter such a mechanism. The alternative is the existence of weakly bound two heavy meson systems due to a potential determined at long distance by one pion exchange and calculable in chiral perturbation theory. One pion exchange meson-meson interactions have also been discussed in Refs. [23] and [24].

Lattice gauge calculations became also recently available [25] and they may help in shedding more light into the intermeson potential and to isolate contributions of various mechanisms [26].

As mentioned above here we discuss results for stability looking at the quantity (1). We compare value of $\Delta E$ obtained in the literature from the OGE model [27] with those obtained in [28] from the GBE model. Ref. [28] considers only the most favourable configuration which is $\bar{3} \bar{3} S = 1, I = 0$. This means that $QQ$ is in a $\bar{3}$ color state and $\bar{q}q$ in a $3$ color state. The mixing of the $6\bar{6}$ is neglected because one expects that this plays a negligible role in deeply bound heavy systems. Then the Pauli principle requires $S_{12} = 1$ for $QQ$ and $S_{34} = 0, I_{34} = 0$ for $\bar{q}q$, if the relative angular momenta are zero for both subsystems. This gives a state of total spin $S = 1$ and isospin $I = 0$. In the channel of light quarks having $S_{\text{light}} = S_{34} = 0, I_{\text{light}} = I_{34} = 0$, as above, the lattice gauge calculations [25] produce a strong short-range attraction for $b\bar{b}q\bar{q}$, which is consistent with constituent quark model calculations [20,27,28].

THE HEXAQUARKS

We shall discuss hexaquarks before pentaquarks because they have been proposed first in historical order [1].

In the light sector the well known example is the H-particle which has been extensively studied in the literature. A recent and comprehensive review can be found in Ref. [29]. From the time it was proposed by Jaffe lots of theoretical studies have been performed within a variety of models as the bag model, the Skyrme model, constituent quark models, lattice calculations, QCD sum rules, etc. The results spread over a wide range of predictions depending on the model parameters and the approximations involved. In each model there are predictions for a bound state or for an unstable state. In the flavor singlet $uuddss$ system with $J^P = 0^+, I = 0$ the GBE model induces a strong repulsion of $847$ MeV above the $\Lambda\Lambda$ threshold [11]. This implies that the H-particle should not exist as a compact object, in contrast to Jaffe’s picture. A molecular type structure, as that of the deuteron, is not excluded however.

In the heavy sector attention has been focused on hexaquarks of type $uuddsQ$. These systems are like the H-particle where one of the $s$ quarks has been replaced by a heavy one. Then when $Q = c$ the particle is denoted by $H_c$ and when $Q = b$ the particle is denoted by $H_b$. In the context of a diquark model [30] the charmed hexaquark is found to be unstable but the bottom hexaquark is found to be stable by about 10 MeV. Calculations based on a chromomagnetic interaction give both
$H_c(I=0,J=3)$ and $H_b(I=0,J=2$ or $3)$ stable by 7.7 MeV up to 13.8 MeV [31]. In the GBE model both $H_c$ and $H_b$ turn out to be unstable, with about the same amount of repulsion above the respective thresholds, as for the H-particle [32]. This means that the heavy flavor has no effect on the stability in these cases.

THE PENTÄQUARKS

The pentaquarks $P_{cs}^0 = uuds\bar{c}$ and $P_{cs}^− = udds\bar{c}$ have been proposed as stable systems against strong decays nearly simultaneously in Refs. [2,3], about ten years after Jaffe’s proposal for the H-particle. The more realistic calculations [33] which take into account the SU(3)-flavor breaking, etc. also lead to stability. In the OGE model the stable pentaquarks have negative parity (i.e. the parity of the antiquark) and require strangeness.

In the GBE model the best candidates to stability are not necessarily strange and have positive parity [34]. To understand these differences let us make the simplifying assumption that the heavy antiquark has an infinite mass. Then we need to care only about the light quarks, which we assume identical. The wave function of the light subsystem, containing radial, spin, flavor and color parts must be antisymmetric. The color part has necessarily the symmetry $[211]_C$ in order to form a color singlet together with the antiquark. Let us consider the spin $S = 0$ state of $q^4$. There are two ways to construct a totally antisymmetric state:

1) assume that the orbital part is symmetric i.e. has symmetry $[4]_O$. Then the flavor-spin part must have the symmetry $[31]_{FS}$. The inner product rules of the permutation group [35] require that the flavor part must be $[211]_F$. In the FS coupling this state reads

$$|1\rangle = ([4]_O[211]_C[211]_{OC} ; [211]_F[22]_S[31]_{FS}) \quad (2)$$

This state has $L = 0$, thus positive parity. Together with the antiquark this leads to a negative parity pentaquark. Obviously its flavor part $[211]_F$ requires strangeness. In the GBE model these pentaquarks are unbound [36]. In the CS coupling the state (2) has the same form but with C and F indices interchanged. It is the most favourable state in the OGE model [2,3], because it has the lowest allowed symmetry in the $CS$ space.

2) assume that the flavor-spin part is symmetric. Then the Pauli principle requires that the orbital part should have the symmetry $[31]_O$. The spin state is $[22]_S$, as before, so that inner product rules require the symmetry $[22]_F$ for the flavor part in order to get $[4]_{FS}$. In the FS coupling this state reads

$$|2\rangle = ([31]_O[211]_C[14]_{OC} ; [22]_F[22]_S[4]_{FS}) \quad (3)$$

The lowest angular momentum associated to $[31]_O$ is $L = 1$ so that this state has negative parity and together with the antiquark gives a positive parity pentaquark. The $[22]_F$ symmetry indicates that strangeness is not required. The state $[4]_{FS}$ is
the most favourable in the GBE model because it has the lowest symmetry in the $FS$ space and, as will be shown in the next section, the GBE hyperfine interaction has a flavor-spin operator which, of course, takes the lowest expectation value for the most symmetric $FS$ state.

The extension to the heavy flavor sectors of the Skyrmion approach \[37\] allowed to calculate the spectra of the lowest lying pentaquarks containing charm and bottom antiquarks \[38\]. Interestingly, the conclusions are similar to those of the GBE model: 1) the lowest pentaquarks have positive parity for any flavor content and 2) strangeness is not necessary in order to gain stability.

Finally, binding due to the long range one pion exchange has also been considered \[39\] and leads to a molecular type structure.

## THE GBE MODEL

Here we refer to the GBE model as originally proposed by Glozman and Riska \[9\]. Its present status can be found in Ref. \[40\]. Besides the pseudoscalar meson exchange, both the vector and scalar meson exchanges are now incorporated in the model. The calculations presented here are based on the nonrelativistic version of Ref. \[10\].

The origin of the model lies in the spontaneous breaking of chiral symmetry in QCD which implies the existence of constituent quarks with a dynamical mass and Goldstone bosons (pseudoscalar mesons). Accordingly, it is assumed that the underlying dynamics in the low energy regime is due to Goldstone boson exchange between constituent quarks. In a nonrelativistic reduction for the quark spinors the quark meson vertex is proportional to $\vec{\sigma} \cdot \vec{q} \lambda^F$, with $\lambda^F$ the Gell-Mann matrices, $\vec{\sigma}$ the Pauli matrices, and $\vec{q}$ the momentum of the meson. This generates a meson exchange interaction which is spin and flavor dependent. In the coordinate space the corresponding interaction potential contains two terms. One is the Yukawa potential tail and the other is a contact $\delta$-interaction. When regularized, this generates the short range part of the quark-quark interaction. It is this short range part which dominates over the Yukawa part in the description of baryon spectra and leads to a correct order of positive and negative parity states both in nonstrange and strange baryons.

The dominant interaction is reinforced by the short-range part of the vector meson exchange (two correlated pions) \[41\].

The model is supported by the independent phenomenological analysis of the $L = 1$ baryons \[42\], by the $1/N_c$ expansion studies of the $L = 1$ nonstrange baryons \[43\] and by lattice studies \[44\].

### A schematic GBE model

In a schematic model the dominant GBE interaction takes the form
TABLE 2. Results for $\Delta E$ for charmed tetraquarks, pentaquarks and hexaquarks. Each case corresponds to the most favourable $I, J^P$ state.

| System | Parity | $\Delta E$ for OGE$^a$ | $\Delta E$ for GBE$^b$ |
|--------|--------|------------------------|------------------------|
| uucc   | $+$    | 19 MeV (I=0,J=1) [27]  | $-$ 185 MeV (I=0,J=1) [28]. |
| uuds\bar{c} | $-$   | $-$51 MeV (I=1/2,J=1/2) [33] | 488 MeV (I=1/2,J=1/2) [36] |
| uu\bar{d}c | $+$    | unbound                | $-$75.6 MeV (I=0,J=1/2) [34] |
| uu\bar{d}sd | $+$    | $-$7.7 MeV (I=0,J=3) [31] | 625 MeV (I=0,J=0 or 1) [32] |

$^a$ In all cases a nonrelativistic Hamiltonian is used. It contains a linear confinement and a chromomagnetic spin-spin interaction.

$^b$ We use the nonrelativistic version of Ref. [10]

$$V_\chi = -C_\chi \sum_{i < j} \lambda_i^F \cdot \lambda_j^F \vec{\sigma}_i \cdot \vec{\sigma}_j$$

with $C_\chi \cong 30$ MeV, determined from the $\Delta$-N splitting [9]. It is useful to give an estimate of $\Delta E$, based on the interaction (4). As an example, let us consider pentaquarks containing heavy flavor, for which equation (1) becomes

$$\Delta E = E(q^4Q) - E(q^3) - E(q\bar{Q})$$

First we suppose that the confinement energy roughly cancels out in (5). Next, as in the previous section, we suppose that $m_Q \to \infty$. As a consequence, the quark-antiquark interaction can be neglected in the expectation value of (4) both for the pentaquark and the emitted heavy meson. Using the Casimir operator technique one finds that the contribution of $V_\chi$ to $E(q^3)$ is $-14 C_\chi$. Now we have to distinguish the two cases introduced in the previous section.

**Negative parity pentaquarks**. In this case, in a harmonic oscillator basis, the contribution of the kinetic energy to $\Delta E$ is $3/4 \hbar \omega$. This difference is exactly the kinetic energy associated to the extra degree of freedom in the pentaquark, corresponding to the relative motion between the $q^3$ and $q\bar{Q}$ subsystems. Using again the Casimir operator technique one finds that the state (2) leads to $\langle V_\chi \rangle = -16C_\chi$. The separation energy becomes

$$\Delta E = \frac{3}{4} \hbar \omega - (16 - 14) C_\chi = 128 \text{ MeV}. \quad (6)$$

where the numerical value results from taking $\hbar \omega = 250$ MeV and $C_\chi = 30$ MeV [9]. The fact that $\Delta E$ is positive indicates that the GBE interaction leads to unbound negative parity pentaquarks of a compact type. This is confirmed by the more precise estimates [36] where $\Delta E$ obtained from a variational method is several hundred MeV for all strange or nonstrange pentaquarks under consideration, containing $c$ or $b$ antiquarks.

**Positive parity pentaquarks**. In this case the state (3) suggests that there is a unit of orbital excitation in the pentaquark due to the symmetry state $[31]_O$ of the $q^4$ subsystem. This leads to $\Delta K.E. = 5/4 \hbar \omega$. But at the same time the contribution of the spin-flavor interaction becomes more attractive than for negative
parity pentaquarks, giving $\langle V_\chi \rangle = -28C_\chi$ due to the higher symmetry $[4]_{FS}$ present in (3). Then one has

$$\Delta E = 5/4 \hbar \omega - (28 - 14) C_\chi = -100 \text{ MeV.} \quad (7)$$

This proves that the attraction due to $V_\chi$ overcomes the excess in the kinetic energy due to the orbital excitation. This cannot happen for the OGE interaction which has a spin-color structure, thus is flavor-blind, and does not distinguish between the $[31]_{FS}$ and the $[4]_{FS}$ states. For this reason the positive parity pentaquarks are expected to be even more unbound than the negative parity ones. In particular the $uudd\bar{c}$ pentaquark will be unbound in any OGE model (see Table 2) inasmuch as the OGE interaction predicts unbound negative parity pentaquarks of the same flavor [33].

DISCUSSION

The main objective of this talk is to compare results for stability obtained from two constituent quark models: OGE and GBE. We illustrate the discussion with results for charmed multiquark systems as shown in Table 2. From this table and the results reported above for the H-particle one important conclusion can be drawn: when the GBE interaction stabilizes a system, the OGE interaction destabilizes it and vice-versa.

When the quark $b$ is used instead of $c$ the results are not so strikingly different but still show large differences in the predictions of the two models.

The challenging question of the existence of exotics remains unanswered so far. It is worthwhile to perform more elaborate calculations, based for example, on the resonating group method, in order to better understand the role played by various mechanisms and the dynamics of exotics. The experimental search would be of great help in putting constraints on the various effective quark-quark interactions.

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