DOMINANT CONTRIBUTION IN PION PRODUCTION
SINGLE-SPIN ASYMMETRIES

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Abstract

Working with a completely collinear twist-3 factorized cross-section formula, we identify two largely dominant partonic sub-processes, which contribute to the single-spin asymmetries in semi-inclusive pion production, in the region of large $p_T$ and medium–large $x_F$.

1 Introduction

During the past years, different models have been developed in an attempt to explain the mechanism behind the single-spin asymmetries observed experimentally in high-energy hadronic interactions. The approach based on the study of the hadronic cross-section contribution given by the twist-3 components in the operator product expansion of parton matrix elements turns out to be particularly interesting: taking into account such terms provides a consistent model. However, at the same time the complexity of the calculational framework unfortunately increases, since twist-3 contribution are characterized by the presence of an additional gauge-field term, which in turn implies an extra gluon in the sub-processes, see for example [1, 2].

Restricting our analysis therefore to a particular class of processes (pion production in proton–proton collisions), our principal aim is to identify which, if any, among all possible partonic sub-processes provide the dominant contributions to the asymmetry and to understand the origin of the suppression of the other terms. We can thus list a set of criteria (which we call “selection rules”) summarizing these mechanisms. To simplify our analysis, we shall extract a totally collinear cross-section formula, in the axial gauge and in the limit of $x_F \rightarrow 1$, valid for large $p_T$.

2 The model

We shall now go into detail, first by providing an expression for the twist-3 contribution to the cross-section through the study of the pole behavior of the Bjorken variables, and then by analyzing the causes of the suppression of many other sub-processes.

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2.1 The poles

Working in axial gauge, thus setting $A^+ = 0$, allows us to write the twist-3 contribution to the cross-section in the following way:

$$d\sigma^{(\tau=3)} \simeq \text{Tr} \left\{ \Phi_A^\alpha(x_1, x_2) S^\beta(x_1, x_2) \right\} g_{\perp\alpha\beta},$$

(1)

where $\Phi_A^\alpha(x_1, x_2)$ is the multi-parton matrix element and the index $\alpha$ is completely transverse, due to gauge choice. Moreover, in the axial gauge, the relation between $\Phi_A^\alpha(x_1, x_2)$ and $\Phi_F^\alpha(x_1, x_2)$ assumes a very simple form (see [3], Eq. 7.3.30):

$$(x_2 - x_1) \Phi_A^\alpha(x_1, x_2) = -i \Phi_F^\alpha(x_1, x_2),$$

(2)

demonstrating that if $\Phi_F^\alpha(x_1, x_2)$ is different from zero for $x_1 = x_2$, then $\Phi_A^\alpha(x_1, x_2)$ must have a pole.

The analysis of the hard part is also crucial for the pole structure; there are two different possibilities for the extra gluon, generated at twist-3, to interact significantly: with the on-shell fragmenting parton (the so-called final-state interactions, FSI) and with the on-shell parton coming from the unpolarized nucleon (initial-state interactions, ISI); the important feature of these interactions is the presence of an extra internal propagator, whose Dirac structure has the form

$$\cdots \frac{k^\nu}{2(P \cdot k)} \left( \frac{2k_\alpha - (x_2 - x_1)\gamma_\alpha P}{x_2 - x_1 - i\varepsilon} \right) \cdots,$$

(3)

where $k^\mu$ is the four-momentum of the on-shell parton and $P^\mu$ is the four-momentum of the polarized hadron.

By also taking into account the pole behavior originating in the multi-parton matrix element, it is possible to separate the trace over the Dirac indices into two traces, each one with a different pole structure: the first, known as the single-pole contribution, where the $(x_2 - x_1)$ term in the numerator cancels the pole contribution of the matrix element, and the other, called the double-pole contribution, where no such cancelation occurs. In order to maintain the cross-section a real quantity, we are forced to take the imaginary part of these poles, remembering that

$$\text{Im} \left( \frac{1}{(x_2 - x_1 \pm i\varepsilon)} \right) = \mp i\pi \delta(x_2 - x_1),$$

(4)

and

$$\text{Im} \left( \frac{1}{(x_2 - x_1 \pm i\varepsilon)^2} \right) = \mp i\pi \delta'(x_2 - x_1).$$

(5)

Using these relations and integrating the derivative of the delta function by parts, we obtain the following expression for the twist-3 contribution to the cross-section:

$$d\sigma^{(\tau=3)} = \int dx \, dx' \, \frac{dz}{z^3} \, \varepsilon_T^\mu S_\perp \left\{ \frac{dG_F(x, x)}{dx} H_{DP}(x, x', z) + G_F(x, x) H_{SP}(x, x', z) \right\} f(x') \, D(z),$$

(6)

where we have omitted the color factors and the sum over flavor indices; $\varepsilon_T^\mu$ is the antisymmetric tensor in the transverse directions, $G_F(x, x)$ is the multi-parton distribution function evaluated at the pole (owing to the delta functions), $f(x')$ is the unpolarized quark density and $H$ represents the hard-scattering partonic cross-sections, with $DP$ and $SP$ standing respectively for double pole and single pole.
2.2 “Selection Rules”

Given such an expression for the cross-section at twist three, we list here the set of principles we have adopted to identify the possibly dominant contributions:

- first, we expect DP contributions to be much more relevant than SP ones, owing to the presence of the derivative of the multiparton density function, which endows the asymmetry with a behavior in $x$ roughly as $A_N \sim \frac{1}{(1-x)}$ (for $x_F$ approaching unity, the Bjorken $x$ of the incoming parton also approaches unity), thus enhancing the contribution of such terms for growing $x_F$;

- for $x_F \to 1$ and $|T| \ll |U| \ll |S|$, we expect the $t$-channel diagrams to be dominant; for the same reason, remembering the power suppression of the hard parts given in Eq. 3, we expect FSI to give a greater contribution than ISI;

- we neglected the contributions given by polarized gluons and by sea quarks since these may reasonably be expected to be small.

In order to test our model and the selection rules described above, we have evaluated the single-spin asymmetries for the reaction $p^\uparrow p \to \pi^0 + X$ for the STAR kinematical range ($\sqrt{S} = 200$ GeV and $1.3$ GeV/$c < P_{hT} < 2.8$ GeV/$c$, see for example [4]). Restricting our analysis to the contribution given only by the $t$-channel diagram involved in the process, in Fig. 1a we present a comparison between the data points and the resulting prediction given by our model; we note that there is good agreement with data for values of $x_F$ greater than 0.4 – 0.5.

![Figure 1](image.png)

**Figure 1.** (a) The theoretical curve represents the prediction for the SSA in $\pi^0$ production evaluated at $P_{hT} = 2.3$ GeV/$c$, compared to STAR data points. (b) Here we plot the same curve as in Fig. 1a, compared to the FSI DP term in a quark–gluon (here labeled $qg$) sub-process and the FSI DP in a quark–quark subprocess.

In Fig. 1b we also plot the total asymmetry, but together with the contribution given by the two major sub-processes we have identified, i.e. the $t$-channel FSI DP terms. Comparing these curves, we can see how the two sub-processes mentioned provide almost entirely the value of the asymmetry in the kinematical range of $x_F > 0.4$; for lower values of this variable, we expect all the neglected contribution to become more important.
3 Conclusions

To summarize then:

- we have obtained an expression providing predictions for the single-spin asymmetries for pion production consistent with data, in a completely collinear framework, without appealing to any collinear expansion;

- using such an expression and a simple set of criteria, we have also been able to identify two largely dominant subprocesses, which are almost entirely responsible for the asymmetries in the $x_F \to 1$ limit.

References

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