A Mutual Inductance Measurement Method for the Wireless Power Transfer System

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Abstract. The characteristic of the Magnetic Resonance Wireless Power Transfer (MR-WPT) system is related on the coupling coefficients between coils, so it is very important to accurately measure mutual inductance. The method of measuring mutual inductance using the primary current and the secondary voltage, when the secondary circuit is opened, is commonly used and most reliable. However, a standard sine wave generator and a linear power amplifier are needed for measuring the mutual inductance. Due to the high frequency inverter of WPT system may be has distortion, it is not recommended to be as the sine wave generator for measuring the mutual inductance. The proposed method demonstrate that, although the output current waveform of the high frequency inverter and the voltage waveform on the secondary coil are highly distorted in PWM (Pulse Width Modulation) control mode, the base component’s amplitudes of these waveforms are investigated by FFT (Fast Fourier Transform), and then, the mutual inductance is calculated by using these values. According to the above contents, the experimental platform is built, and then the experiments on the different duty cycle ratio and the different frequencies, that is, including the different quantities of its high-frequency harmonics, has been done. In addition, the same operation as above is repeated on the different distances, and the results are compared with the theoretical calculation values. Consequently, the results show that the method is very close to the theoretical value, and it is very stable in whole distance range.

1. Introduction

In recent years, more and more portable products have been used in people's life, which has caused the demand for wireless power transfer. Additionally, the realization of wireless power transfer system has been promoted as the rapid development of switching elements. Wireless power transfer can directly transmit without connecting to the power plug, hence there is no bad contact, may cause the electric fire, and the process that someone go to plug the connector is not needed, thus WPT has a merit for achieving the unmanned process. Also, there is no need for any connections, so it is very feasible in the special applications such as electrical energy transfer to the water, human body and disaster areas. Magnetic coupling resonance method is most commonly used, and the research using this method is also the most active. It has the advantage that the transmission distance is long and has little effect on the organism. There are two common methods for measuring mutual inductance, namely, secondary open circuit voltage measurement method and coupling two coil series connection inductance measurement method. The mutual inductance could be calculated by using the integral calculation,
and according to the geometric topology and alignment state of the coils, the calculation formula is different. Here, only the circular spiral coil will be considered, where this coil model is the most commonly used. The method for calculating the mutual inductance between two parallel spiral coils was proposed in the literature [1]. In the literature [2], the different topologies of the two-coil magnetic resonant wireless power transfer systems were analyzed, and it is pointed out that the series-series topology was the best selection, therefore in this paper only series topology is considered. The optimal transmission condition equations in the magnetic resonance WPT, is suggested in the literatures [3-5], in all of which the coupling coefficient or mutual inductance is included. From this point of view, the mutual inductance between the coils is also critical, in addition to the quality factor and the load characteristic. The mutual inductance measurement method, which is utilizing the primary current and secondary open-circuit voltage, was used in the literature [6], but there is no description about of the measurement process in detail. Generally speaking, own high-frequency inverter of WPT system cannot directly be used for measurement of mutual inductance, rather than only the use of standard sine wave oscillator and linear power amplifier. Therefore, this method only can be used for the offline measurement of mutual inductance or/and the evaluation of mutual inductance, and not suitable for practical WPT system. The other method using series connection for measuring the mutual inductance, is concerned to the measuring the inductances, which are obtained by two different series connection of two coils (Two different connection types are respectively the joint of the dotted terminal of one coil and the dotted terminal of another coil, and the joint of the dotted terminal of one coil and another terminal of another coil). This method may only be suitable for the low-frequency circuits, and there is no matter when the interval between two coils is relatively close. The impact of the terminal-terminal connection cable cannot be ignored when the distance is far away, moreover, two times measuring and changing the terminal connections is demanded. Similarly as the reason of above mentioned, only may be used for measuring, is not appropriate for the practical application. The method depicted in here is to use FFT operation instead of the standard function generator and the linear power amplifier, and own designed high-frequency inverter itself can be directly used for measuring the mutual inductance. Hence, it can be suitable for the on-line measurements in case of the distance changes between two coils, and thus more practical. This method is especially aimed at WPT systems. The paper is described as follows: In Section 2, the general principle of series-series topology two-coil WPT system and the critical role of mutual inductance are described, and then the calculation formula based on the integral operation and the common measurement method of mutual inductance are reviewed. In Section 3, it is observed, the output current waveform when our own designed high-frequency inverter is operate on PWM control mode, and the problems encountered when these waveforms will be employed to measure the mutual inductance is demonstrated. And then, the solutions that our own designed high-frequency inverter will be applied to the practical WPT system by using FFT, are put forward. In Section 4, the example experiment platform is set up, including the coil design process and then the experiments are done and the experimental results are analyzed. Finally, the whole content is concluded in Section 5.

2. Mutual inductance calculation method and measurement method

2.1 Series-series topology WPT system

The series-series topology WPT equivalent circuits may be depicted as in Figure 1.

![Figure 1](image-url)
When the primary and the secondary circuit are serial topologies, based on the mutual inductance circuit theory, the following equation can be provided by applying Kirchhoff’s voltage law

\[
\begin{align*}
U_S &= Z_1 I_1 - j\omega M I_2 \\
0 &= j\omega M I_1 - I_2 Z_2
\end{align*}
\]

(1)

Where, \(Z_1\) and \(Z_2\) are the primary and the secondary impedances, respectively, and expressed as follows.

\[
\begin{align*}
Z_1 &= R_1 + j\omega L_1 + \frac{1}{j\omega C_1} \\
Z_2 &= R_2 + j\omega L_2 + \frac{1}{j\omega C_2}
\end{align*}
\]

(2)

Wireless power transfer systems frequently is in variable situations, such as the transmission distance changes or/and the load impedance changes. In order to improve the performance of WPT system (the performances mentioned here includes the following norms, namely available transmission distance, maximum transmission power, transmission efficiency, environmental interference and system stability, etc. If there is no special notes, at most cases only implies to transmission power and transmission efficiency), it is necessary to accurately measure the mutual inductance and the load impedance. In literature [7], the maximum output power condition is given, and the equation is as follows.

\[
k_{\text{critical}} = \left(\frac{Q_1 Q_2}{2}\right)^{1/2}
\]

(3)

Where, \(Q_1\) and \(Q_2\) are the quality coefficients of two coils, respectively. If the driving frequency is \(\omega\), the inductances of the primary and the secondary coils are \(L_1\) and \(L_2\), the primary and the secondary resistors are \(R_1\) and \(R_2\) respectively, then the Equation (3) is transformed into the Equation (4) by using the relationship between the coupling coefficient and the mutual inductance \((k=M(L_1 L_2)^{1/2})\).

\[
M_{\text{critical}} = \left(\frac{1}{L_1 L_2}\right)^{1/2} \left(\frac{1}{Q_1 Q_2}\right)^{1/2} = \left(\frac{1}{R_1 R_2}\right)^{1/2} / \omega
\]

(4)

2.2 The method of the Mutual Inductance Calculation

The following is a brief description of the mutual inductance calculation method proposed in the literature [1]. As shown in Figure 2, the radius of the ring_1 is \(r_1\), the radius of the ring_2 is \(r_2\), the distance between the planes where the two rings are placed is \(c\), and the literal offset between two rings is \(d\). The mutual inductance between two non-coaxial single-turn parallel circular rings is calculated according to Equation (5) [1].

\[
m = 2\mu_0 \frac{\phi}{\pi} \int_0^{1} \frac{1 - d \cos \phi}{k \sqrt{r_1^2 + r_2^2}} \Psi(k) \, d\phi
\]

(5)
Where, \( m \) is the mutual inductance, \( k^2 = 4\alpha V \left( (1 + \alpha V)^2 + \beta^2 \right)^{-1} \), \( \alpha = r_2 r_1^{-1} \), \( \beta = c r_1^{-1} \),

\[
V = \left( 1 + d^2 r_2^{-2} - 2d r_2^{-1} \cos \phi \right)^{1/2}, \quad \Psi(k) = \left( 1 - k^2 / 2 \right) K(k) - E(k),
\]

\[
K(k) = \int_{0}^{\pi/2} \left( 1 - k^2 \sin^2 \theta \right)^{-1/2} d\theta,
\]

\[
E(k) = \int_{0}^{\pi/2} \left( 1 - k^2 \sin^2 \theta \right)^{1/2} d\theta.
\]

\( K(k) \) and \( E(k) \) is the first type elliptic integral and the second type elliptic integral, respectively.

The planar circular spiral coil is regarded as the superposition set of the several monolithic coaxial rings with the different radii on the same plane [8], and likewise, the mutual inductance between two spiral coils may be regarded as the superposition set of the several pairs of the single-turn coil mutual inductances. According to the Equation (5), the equation for calculating the mutual inductance between two spiral coils is shown in Equation (6).

\[
M = \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} M \left(R_i, R_j, c, d\right)
\]

(6)

\( N_s \) is the number of turns of the spiral coils, \( R_i \) and \( R_j \) is the corresponding radii of \( i \)-th and \( j \)-th turn of the spiral coil, respectively.

2.3 The Method for Measuring the Mutual Inductance

The one general measuring method of the mutual inductances is to use the values of the primary current and the voltage on the secondary coil measured in case of the secondary circuit opening, and then the mutual inductance is obtain by the Equation (7).

According to the principle of electromagnetic induction, the secondary open circuit voltage depends on primary current and mutual inductance.

\[
M = \frac{U_2}{(\omega I_1)}
\]

(7)

In Equation (7), \( \omega \) is the frequency of the alternative driving power sources, and \( I_1, U_2 \) are the primary current and the secondary voltage when the secondary circuit is opened respectively. Another measuring method of mutual inductance is to measure the inductances in two cases of forward and backward connections, and the following Equation (8) can be used to calculate the mutual inductance. The inductance when the two coils are connected by forward is equal to the sum value that the twice times of the mutual inductance is added to the sum value of two coil’s inductances. Vice versa, the inductance when the two coils are connected by backward is equal to the value that the twice times of the mutual inductance is subtracted from the sum value of two coil’s inductances. It is intended that the difference between the two inductance values is naturally equal to four times of the mutual inductance.

\[
M = \frac{L_{1st} - L_{2nd}}{4}
\]

(8)

In Equation (8), \( L_{1st} \) and \( L_{2nd} \) is the inductance values measured in case of the forward connection and reverse, respectively.

3. Mutual Inductance Measurement Using FFT

When the WPT system is applied, the mutual inductance between the coils is easy to be changed due to the various reasons, and this phenomenon is inevitable. The mutual inductances between the coils are the most important parameters in the WPT system, therefore, without accurately determining of the mutual inductance, the most optimal state cannot be achieved in the WPT system. Aimed to this problem, in spite of, the output current waveform of the WPT’s own high-frequency power supply may be distorted, directly using of this power supply has the advantages of the on-line measurement of mutual inductance.

H-Bridge power switching driver mode is a common driving method in the high-power wireless power transfer system. In order to avoid the faulty of burning out the power switching elements due to direct conduction of the upper and lower switching elements, the dead zone part must be added when the on/off state of the power switching elements is alternative. The duty cycle control means that the
time ratio between the switch on and off, so it contains the high frequency harmonic components that cannot be ignored in the driving waveform. There are the stray inductances on H bridge PCB (Printed Circuit Board) and the output capacitance of the power switching elements, to form a closed loop (in the literature [9]), as a result, become appeared the high resonant frequency, and this high frequency wave affect to the output current waveform. Although the series connection circuit of the resonance inductance and the resonance capacitance becomes a filter circuit, it also contains different degrees of high frequency harmonics on different duty cycle ratios. In the Figure 3, the primary current and the secondary voltage waveforms measured on the WPT experimental platform presented in Section 4 is showed, when the secondary circuit is opened. It is difficult to directly find out the amplitude of the base wave from the measured waveform, so the mutual inductance cannot be deduced directly from the Equation (7).

4. Experiment
The planar spiral coil has the merits of the relatively balanced magnetic field, the simple structure and the thin coil thickness, thus this structure is often used in the magnetic resonance WPT systems. The coil used in the experiment is also a planar spiral structure, the geometrical parameters are as follows; the diameter of the minimum circle (most inner) is 14 cm, the diameter of the maximum circle (most outside) is 30 cm, the number of turns is 8 and the distance between the circles is 1 cm. The inductance and the capacitance values measured by the network Analyzer Keysight E5061B are listed on Table 1.
Table 1. Parameters of TX and RX circuit

| Parameter                     | Driving Frequency (KHz) | Primary Coil Inductance L₁ (uH) | Secondary Coil Inductance L₂ (uH) | Primary Resonance Conductor C₁ (nF) |
|-------------------------------|-------------------------|---------------------------------|-----------------------------------|-------------------------------------|
| Value                         | 551                     | 17.491                          | 17.448                            | 4.77                                |
| 80                            |                         | 17.491                          | 17.448                            | 225                                 |

Figure 4 is the sketch diagram of the WPT experimental platform circuit. AC is a city power supply, the adjustable rectifier is able to adjust the DC voltage (adjustable range DC 25V-200V), Cᵢ is the input filter capacitor. T₁, T₂, T₃ and T₄ is the power switching components of H bridge, including a reverse diode inside of the switch components itself, the resistance Rᵢ is for the protection of the over current and reserved to measure the current. C₁ and L₁ respectively is the resonant capacitor and resonant inductance of the primary circuit, its self-resonant frequency is equal to the driving frequency of the drive power switching components. Drive circuit is the driving circuit that includes four isolated power modules and turns the drive signal output from DSP28335 into the available level for the gate of the power switch components. DSP28335 is a digital signal processor for controlling the PWM and DC input voltage.

Figure 4. Sketch Diagram of the WPT experimental platform

Figure 5 is a photograph of the experimental platform. The Current Transformer in the photograph is the Keysight 1147B, which is used to measure the output current of the primary coil. The oscilloscope in the photograph is Keysight InfiniiVision MSOX2024A, used to measure the voltage on the secondary coil. The mutual inductance results of calculating with the Equation (6) and using the proposed method are shown in Figure 6. The red line in the figure is the variation curve of the mutual inductance to the distance between the coils, that is calculated with by Equation (6) using MATLAB.

Figure 5. Photograph of Experimental Platform
meanwhile the black line and the blue line are the curves of using the proposed method, specifically, the blue '-' line is the mutual inductances in case of when the dead zone time is 16.7% and the frequency is 551KHz, the black '-' line is the mutual inductances in case of when the dead time zone 5% and the frequency is 551KHz, the black '-' line is the mutual inductance in case of when the dead time zone is 16.7% and the frequency is 80KHz, and the black '-' line is the mutual inductance in case of when the dead time zone is 5% and the frequency is 80KHz.

As known from Figure 6, the mutual inductance-distance variation curve obtained under the different driving conditions is very close to the theoretical calculation value, accordingly, it demonstrates that the proposed method is available for the practical WPT systems.

The reasons for the gap between the experimental results and the theoretical calculations are as follows; Firstly, the spiral coils used in these experiments is composed with not only a flat spiral part, but also two straight wire parts, secondly, the manual manufacturing may be a bit of error in the space. And then in the process of the experiment, a small alignment error of the pair of coils may be happened. Finally, despite of the secondary coil is opened, but strictly speaking, the inductor itself has the stray capacitance, forming a closed circuit in some frequencies.

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