Loops, Cutoffs and Anomalous Gauge Boson Couplings

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Abstract

We discuss several issues regarding analyses which use loop calculations to put constraints on anomalous trilinear gauge boson couplings (TGC’s). Many such analyses give far too stringent bounds. This is independent of questions of gauge invariance, contrary to the recent claims of de Rújula et. al., since the lagrangians used in these calculations are gauge invariant, but the $SU(2)_L \times U(1)_Y$ symmetry is nonlinearly realized. The real source of the problem is the incorrect use of cutoffs – the cutoff dependence of a loop integral does not necessarily reflect the true dependence on the heavy physics scale $M$. If done carefully, one finds that the constraints on anomalous TGC’s are much weaker. We also compare effective lagrangians in which $SU(2)_L \times U(1)_Y$ is realized linearly and nonlinearly, and discuss the role of custodial $SU(2)$ in each formulation.

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Although the standard model of the weak and electromagnetic interactions has been extremely successful in explaining all experimental results to date, the gauge structure of the theory has not yet been tested. This task will be accomplished in the coming years in such experiments as LEP200 and the TeVatron, where the three-gauge-boson self couplings will be directly measured. Of course, it is hoped that new physics will be observed. With this in mind, these experiments will search for, among other things, anomalous trilinear gauge boson couplings (TGC’s) not found in the standard model.

If one wants to parametrize new physics such as anomalous TGC’s, the easiest way to do so is to use an effective lagrangian. In order to define the low-energy effective lagrangian which parametrizes the new physics, it is necessary only to specify the particle content and the symmetries of the low-energy theory. In dealing with anomalous TGC’s, one basically has three choices:

1. Linearly Realized $SU(2)_L \times U(1)_Y$: Here, symmetry breaking is accomplished through the Higgs mechanism, and the low-energy theory includes the standard model Higgs doublet.

2. Nonlinearly Realized $SU(2)_L \times U(1)_Y$: In these effective lagrangians, the symmetry breaking mechanism is unspecified – the low-energy theory contains only those pseudo-Nambu-Goldstone bosons which are eaten to give mass to the $W$’s and $Z$’s. These are also known as “chiral lagrangians”.

3. Only $U(1)_{em}$ Gauge Invariance: In this case, the low-energy effective lagrangian is required only to obey electromagnetic gauge invariance. Massive $W$’s and $Z$’s and their interactions are simply put in by hand.

At this point we would like to make a comment regarding the first choice above. It is, in fact, a very strong assumption to assume that, even in the presence of new physics which gives rise to anomalous TGC’s, $SU(2)_L \times U(1)_Y$ is still broken to $U(1)_{em}$ via a single Higgs doublet. After all, given that there is new physics, the method of symmetry breaking might be quite different from that of the standard model. Furthermore, anomalous TGC’s often involve the longitudinal components of the gauge bosons, which are intimately connected to the symmetry breaking mechanism. Most conclusions based upon linearly realized $SU(2)_L \times U(1)_Y$ could be altered if one changes, for example, the Higgs content. Of course, this does not imply that it is incorrect to use an effective lagrangian based on linearly realized $SU(2)_L \times U(1)_Y$. However, it is more conservative to use nonlinearly realized $SU(2)_L \times U(1)_Y$. We will briefly return later to a comparison of the two formulations.

Up to now, most analyses which concern themselves with anomalous TGC’s use the third option – the effective lagrangian obeys only electromagnetic gauge invariance. That
Here, we have written only a subset of the possible terms, including explicit masses for the $W$ and $Z$, and several triple gauge boson vertices [1]. In the above, $V^\mu$ represents either the photon or the $Z$, $W^\mu$ is the $W^-$ field, $\mathcal{D}_\mu$ is the electromagnetic covariant derivative, and $W_{\mu\nu} = \mathcal{D}_\mu W_\nu - \mathcal{D}_\nu W_\mu$ (and similarly for $V_{\mu\nu}$). The $g_4^Z$ term is a CP violating TGC called (for obscure reasons) the anapole coupling, which we will use in our examples below.

Now, given that these anomalous three-gauge-boson vertices will not be measured for several years, it is reasonable to ask whether limits can be placed on them using current data. Obviously the only constraints can come through contributions to loop-induced processes, and a number of papers have considered this possibility [2]. Let us illustrate a typical such calculation.

Consider the CP violating anapole coupling $g_4^Z$ defined in Eq. (1) above. This will contribute at one loop to the $W$- and $Z$-masses. However, because it is CP violating, there will be a nonzero result only if this anomalous TGC appears at both vertices in the loop diagrams, as in Fig. 2. Now, the anapole coupling is nonrenormalizable, and hence the diagrams diverge. The standard way to regularize the divergent loop integrals is to simply insert a cutoff. If this is done, then one finds (keeping only the highest divergence)

$$\delta\pi_{WW} (q^2) = -\frac{(g_4^Z)^2}{6\pi^2} \frac{\Lambda^6}{M_W^2 M_Z^2}$$
$$\delta\pi_{ZZ} (q^2) = -\frac{(g_4^Z)^2}{144\pi^2} \frac{\Lambda^4}{M_W^4} q^2. \tag{2}$$

Since the values of $\delta\pi_{WW}$ and $\delta\pi_{ZZ}$ at $q^2 = 0$ are unequal, there will be a nonzero contribution to the deviation of the $\rho$-parameter from unity, often parametrized using the $T$-parameter of Peskin and Takeuchi [3]. Taking $\Lambda$ to be the scale of new physics, say 1 TeV, and using the present limit of $|T| < 0.8$ [4], one obtains a very stringent constraint on $g_4^Z$:

$$g_4^Z < 3.5 \times 10^{-4} \left(\frac{1 \text{ TeV}}{\Lambda}\right)^3. \tag{3}$$

Intuitively, this result seems suspect. After all, the anomalous coupling contributes only at one loop, and the quantity which is used to constrain $g_4^Z$, the $\rho$-parameter, has
only been measured to a precision of a couple of percent or so. It’s not as if this limit comes from an extremely well-measured process such as $\mu \rightarrow e\gamma$, for example.

In fact, there is a recent paper by de Rújula et. al. [5], in which they claim that this constraint is a considerable overestimate (and similarly for calculations which predict large measurable effects in loop-induced processes involving anomalous TGC’s). The reason, they say, is that the lagrangian used (Eq. (1)) is not $SU(2)_L \times U(1)_Y$ gauge invariant.

This claim is only partly correct. It is true that the result in Eq. (3) is an overestimate, for reasons we will explain later. However, the reason has nothing whatsoever to do with gauge invariance. The point is that the lagrangian in Eq. (1), which obeys only electromagnetic gauge invariance, is equivalent, term by term, to a chiral lagrangian in which $SU(2)_L \times U(1)_Y$ is present but nonlinearly realized [6].

Briefly, the proof goes as follows. To construct a lagrangian which contains a nonlinear realization of $SU(2)_L \times U(1)_Y$, broken to $U(1)_{em}$, one introduces the matrix-valued scalar field $\xi(x) \equiv \exp[iX_a\phi^a/f]$, in which the $\phi^a$ are the Nambu-Goldstone bosons, and the $X_a$ are the broken generators. (The aficionados may remark that, as we have written it, $\xi(x)$ respects custodial $SU(2)$, since there is a common decay constant $f$ for each of the $\phi^a$’s. However, we could equally well have ignored this symmetry by writing $f_a$ – the proof is independent of the existence of a custodial $SU(2)$.) With $\xi(x)$, one can define a nonlinear “covariant derivative”

$$D_{\mu}(\xi) \equiv \xi^\dagger\partial_{\mu}\xi - i\xi^\dagger W_\mu\xi,$$

in which $W_{\mu} = g_2W_{\mu}^a T_a + g_1B_{\mu} Y$. In terms of $D_{\mu}(\xi)$, one constructs the three fields

$$e A_{\mu} \equiv i \text{tr}[QD_{\mu}(\xi)],$$
$$\sqrt{g_1^2 + g_2^2} Z_{\mu} \equiv 2i \text{tr}[(T_3 - Y)D_{\mu}(\xi)],$$
$$g_2 W_{\mu}^\pm \equiv i\sqrt{2} \text{tr}[T_{\pm} D_{\mu}(\xi)].$$

The significance of these three quantities, $A_{\mu}$, $Z_{\mu}$ and $W_{\mu}$ is that, under arbitrary $SU(2)_L \times U(1)_Y$ transformations, they transform purely electromagnetically. Thus, any lagrangian which is constructed using these quantities and which is required to obey electromagnetic gauge invariance will automatically obey the full $SU(2)_L \times U(1)_Y$ symmetry. In unitary gauge, these fields reduce to the standard photon, $W$ and $Z$ fields:

$$A_{\mu} \leftrightarrow A_{\mu}, \quad Z_{\mu} \leftrightarrow Z_{\mu}, \quad W_{\mu}^\pm \leftrightarrow W_{\mu}^\pm .$$

With this construction one sees explicitly that $SU(2)_L \times U(1)_Y$ gauge invariance is automatic for any lagrangian which obeys $U(1)_{em}$ gauge invariance. Therefore, the claim that
Eq. (1) is not gauge invariant is simply a red herring.

The real reason that Eq. (3), and other results like it, are incorrect is due to the improper use of cutoffs [7]. This can be seen as follows. Suppose we knew what the full theory was at the new physics scale, $M$, which is much larger than $m$, the scale of the light physics. Now let us calculate the effect of the heavy physics on a light particle mass such as $M_W$, for example. The contribution one obtains after integrating out the heavy particles has the following form:

$$
\delta \mu^2 (m, M) = a M^2 + b m^2 + c \frac{m^4}{M^2} + \cdots
$$

(7)

The dots represent terms which are of higher order in the small mass ratio $m^2/M^2$, and the coefficients $a$, $b$, $c$, ... may depend at most logarithmically on this ratio. Note that there is no term of the form $M^4/m^2$. There is an old paper of Weinberg [8] which explains that such terms are not allowed since only logarithmic infrared divergences are allowed at zero temperature in four spacetime dimensions.

Now suppose we split this calculation up into a “high-energy” piece and a “low-energy” piece by choosing a cutoff $\Lambda$ such that $M \gg \Lambda \gg m$. In this case, the two contributions can have the form

$$
\delta \mu^2_{\text{he}} (m, \Lambda, M) = a' M^2 + b' \Lambda^2 + c' \frac{\Lambda^4}{m^2} + \cdots
$$

$$
\delta \mu^2_{\text{le}} (m, \Lambda, M) = b'' \Lambda^2 + c'' \frac{\Lambda^4}{m^2} + \cdots
$$

(8)

Obviously, this is just a reorganization of the full calculation (Eq. (7)), so we must have

$$
\delta \mu^2 (m, M) = \delta \mu^2_{\text{he}} (m, \Lambda, M) + \delta \mu^2_{\text{le}} (m, \Lambda, M).
$$

(9)

Furthermore, the full result is independent of the cutoff $\Lambda$, which implies that

$$
a = a' , \quad b' = -b'' , \quad c' = -c'' , \quad \cdots
$$

(10)

In other words, all quadratic and higher dependence on $\Lambda$ in the low-energy piece of the calculation is simply cancelled by counterterms coming from the high-energy piece of the calculation! Note also that the coefficient $b''$ is unrelated to the coefficient $a$. That is, a calculation of the quadratic divergence in the low-energy theory does not tell you how the full calculation depends on $M^2$. 

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The point is that there is no physical significance to terms containing the cutoff \( \Lambda \). Another way of saying this, perhaps more to the point, is that *cutoffs do not accurately track the true heavy mass dependence of the full theory.*

There is one exception to this statement – the case of a logarithmic divergence. Suppose that, in the full theory, there were a term of the form

\[
\delta \mu^2 \sim \frac{d^2}{m^2} \log \left( \frac{M^2}{m^2} \right). \tag{11}
\]

If one used a cutoff, the high-energy and low-energy contributions would be

\[
\delta \mu_{he}^2 \sim \frac{d' m^2}{\Lambda^2} \log \left( \frac{M^2}{\Lambda^2} \right),
\]

\[
\delta \mu_{le}^2 \sim \frac{d'' m^2}{m^2} \log \left( \frac{\Lambda^2}{m^2} \right). \tag{12}
\]

Clearly, cancellation of the \( \Lambda \)-dependence requires \( a = a' = a'' \). This is the only case in which the low-energy cutoff dependence accurately reflects the true dependence on the heavy mass \( M \).

Before returning to the example of the anapole, let us explicitly demonstrate in a toy model the fact that the cutoff dependence found in a low-energy loop calculation is in general unrelated to the true heavy mass dependence. Consider a renormalizable model with only two scalars: \( \psi \), which has mass \( M \), and \( \phi \), of mass \( m \). The potential describing the interactions of these two scalars can be split up into two pieces, depending on whether the potential is odd or even under separate reflections of the fields. We have

\[
U = U_+ + U_-,
\]

with

\[
U_+ = \frac{1}{2} m^2 \phi^2 + \frac{1}{2} M^2 \psi^2 + \lambda \phi^4 + \lambda' \psi^4 + \lambda'' \phi^2 \psi^2,
\]

\[
U_- = \frac{1}{3} h \psi^3 + \frac{1}{3} g \psi \phi^3. \tag{13}
\]

Note that, in principle, other terms could have been included in \( U_- \), but we assume that these are the only terms which appear in the lagrangian at some scale, \( \mu_0 \).

Now consider integrating out the field \( \psi \), so that the potential is a function of \( \phi \) only, \( V(\phi) \), and consider just the \( V_\cdot \) piece of the potential. At tree and one-loop levels, the lowest order terms which appear are (see Fig. 2)

\[
\text{tree level: } V_-(\phi) = \frac{hg^3}{81M^6} \phi^9,
\]

\[
\text{one-loop level: } V_-(\phi) = \frac{hg}{92\pi^2} \psi^3 \left[ 1 + 2 \log \left( \frac{M^2}{\mu_0^2} \right) \right]. \tag{14}
\]
What is important to note here is that the dependence of the $\phi^3$ term on the heavy mass $M$ is logarithmic.

Now suppose that the heavy physics, characterized by scale $M$, were unknown. In this case one assumes the most general form for $V_-$,

$$V_-(\phi) = aM\phi^3 + \frac{b}{M}\phi^5 + \frac{c}{M^3}\phi^7 + \frac{d}{M^5}\phi^9 + ...$$  \hspace{1cm} (15)$$

In order to make the connection with the anapole calculation we showed you earlier, consider the contribution of the $d$-term to the $a$-term. In other words, $aM\phi^3$ plays the role of the $\rho$-parameter, while $(d/M^5)\phi^9$ acts like an anomalous $WWV$ coupling. Here one finds a contribution at 3 loops (Fig. 3),

$$\delta a \sim \left(\frac{\Lambda^2}{16\pi^2M^2}\right)^3 d.$$ \hspace{1cm} (16)$$

The upshot is that one finds that the cutoff dependence of the $\phi^3$ term goes like $\Lambda^6$. However, we have already determined the true dependence on the heavy mass to be logarithmic. This demonstrates explicitly that cutoffs do not accurately track the true dependence of the full theory on the heavy mass scale. This also demonstrates that the issue of gauge invariance is indeed a red herring, since it is clear that same type of problems arise in models which contain only scalars.

Let us now return to our original example of the anapole contribution to the $W$- and $Z$-masses. Given the problems with cutoffs, how can one extract physically meaningful bounds on the anapole coupling, given that there is a nonzero contribution to the $\rho$-parameter? The easiest way to do this (but by no means the only way) is not to use cutoffs at all to regularize the divergent integrals. Instead, one uses dimensional regularization, supplemented by the decoupling subtraction renormalization scheme.

Using dimensional regularization, the divergent pieces of the diagrams in Fig. 1 are

$$\delta\pi_{WW} \left( q^2 \right) |_{q^2=0} = -\frac{\left(g_T^2\right)^2 3M_W^2}{4\pi^2} \left[ 1 + \frac{M_Z^2}{M_W^2} - \frac{M_Z^4}{M_W^4} \right] \frac{2}{\epsilon},$$

$$\delta\pi_{ZZ} \left( q^2 \right) |_{q^2=0} = 0,$$ \hspace{1cm} (17)$$

where $\epsilon = n - 4$ in $n$ spacetime dimensions. The key point now is the following. In the most general effective lagrangian, there will be a term contributing directly to the $T$-parameter
The contribution of Eq. (17) to $\Delta \rho$ renormalizes this direct contribution:

$$\alpha T(\mu^2) = \alpha T(\mu'^2) - \frac{3}{8\pi^2} \left(g_4^Z(\mu'^2)\right)^2 \left[1 + \frac{M_Z^2}{M_W^2} - \frac{M_W^4}{M_Z^4}\right] \ln \left(\frac{\mu'^2}{\mu^2}\right).$$  \hspace{1cm} (18)

This shows how the two operators in the effective lagrangian, $T$ and $g_4^Z$, mix as the lagrangian is renormalized and evolved down from scale $\mu'$ to scale $\mu$ (in the absence of thresholds). This is a point which seems to have been overlooked in many of the analyses which deal with anomalous TGC’s. Even in an effective (nonrenormalizable) lagrangian, the parameters must be renormalized. In general, this requires an infinite number of counterterms, but this is of no consequence since the effective lagrangian already contains an infinite number of terms. Note also that there is, in general, a contribution which depends quadratically on the heavy mass scale $M$. This is contained in the initial condition $T(M^2)$, which is, however, incalculable if one does not know the underlying theory.

If one assumes no accidental cancellation between the two terms on the right hand side of Eq. (18), one can now use the experimental limit on $|T|$ to constrain the anapole coupling:

$$g_4^Z < 0.24 \hspace{1cm} (@ 1 \text{ TeV}).$$  \hspace{1cm} (19)

This is 3 orders of magnitude weaker than the bound found using cutoffs (Eq. (3))!

Before concluding, we would like to return to a subject we briefly touched upon at the beginning, namely the comparison of conclusions based upon effective lagrangians with linearly and nonlinearly realized $SU(2)_L \times U(1)_Y$. One of the terms in Eq. (1) is the electric quadrupole moment of the $W$,

$$i\frac{\lambda_V}{M^2} W^\dagger_{\lambda \mu} W^\mu_{\nu} V^{\nu \lambda}. \hspace{1cm} (20)$$

In the literature, one often sees statements to the effect that $\lambda_\gamma = \lambda_Z$ (modulo cot $\theta_W$). The reasons given vary – occasionally gauge invariance or custodial $SU(2)$ symmetry are invoked, and sometimes this relation is required in order to avoid too large contributions to well-measured quantities which are in agreement with the standard model. The fact is, none of these reasons is valid – there is no reason, in general, to require $\lambda_\gamma = \lambda_Z$.

There are basically two sources of confusion. First of all, if one calculates the contribution to $\Delta \rho$ using a cutoff, one finds

$$\Delta \rho \sim (\lambda_\gamma - \lambda_Z) \frac{\Lambda^4}{M_W^4}. \hspace{1cm} (21)$$
This has led some authors to require $\lambda_\gamma = \lambda_Z$ in order to avoid large contributions to $\Delta \rho$ for large values of the cutoff. However, as we have argued above, this cutoff behaviour is not physically meaningful – this type of term is cancelled by a counterterm coming from the high-energy theory, and no conclusions as to the relative size of $\lambda_\gamma$ and $\lambda_Z$ should be drawn.

A more important, and subtle, source of confusion is that the relation $\lambda_\gamma = \lambda_Z$ is true, to a good approximation, if one uses an effective lagrangian with a linearly realized $SU(2)_L \times U(1)_Y$ gauge symmetry. If there is only one Higgs doublet, then it necessarily follows that

$$W_{3\mu} = Z_\mu \cos \theta_W + A_\mu \sin \theta_W. \quad (22)$$

Since the standard model Higgs sector possesses an approximate custodial $SU(2)$ symmetry, one is led quite naturally in this context to $\lambda_\gamma = \lambda_Z$. Even if one were to add more Higgs doublets, for example, this relation would continue to be approximately true.

On the other hand, and this is where the subtlety arises, if one realizes the symmetry nonlinearly, then it does not necessarily follow that $\lambda_\gamma = \lambda_Z$. Although the symmetry breaking sector continues to possess an approximate custodial $SU(2)$ symmetry, the relation among $W_{3\mu}$, $Z_\mu$, and $A_\mu$ need not be that found in Eq. (22) above [9]. In general, there is no reason, neither gauge invariance nor custodial $SU(2)$, for $\lambda_\gamma$ and $\lambda_Z$ to be related.

This is the point of this discussion. In general, an effective lagrangian has an infinite number of terms, the coefficients of which are arbitrary and independent. It is possible to relate some of these coefficients by imposing certain symmetries, or to construct the effective lagrangian in a special way. However, the lagrangian thus obtained is less general. This is the case for an effective lagrangian with $SU(2)_L \times U(1)_Y$ realized linearly. The assumption of the breaking of the symmetry via the Higgs mechanism results in certain relationships among the parameters of the low-energy effective lagrangian. These relationships are not present if one makes no assumption about the mechanism of symmetry breaking, i.e. if one realizes the symmetry nonlinearly. Again, this is not to say that the linearly realized effective lagrangian is incorrect; however, it is more constrained than an effective lagrangian with nonlinearly realized $SU(2)_L \times U(1)_Y$.

To conclude,

1. Bounds on anomalous trilinear gauge boson couplings which are obtained from their contributions to loop diagrams are often significantly overestimated (and similarly for predictions of large effects in loop-induced processes).

2. This is unrelated to any questions of gauge invariance. The fact is, any lagrangian which obeys Lorentz invariance and electromagnetic gauge invariance is equivalent,
term by term, to a lagrangian in which the $SU(2)_L \times U(1)_Y$ symmetry is present, with the breaking $SU(2)_L \times U(1)_Y \to U(1)_{em}$ nonlinearly realized.

3. The real source of the problem is the incorrect use of cutoffs in estimating the effect of heavy masses in the loop diagrams. In general, the cutoff behaviour does not properly track the true dependence on the heavy mass scale $M$. (The one exception is the case of a logarithmic divergence.)

4. A more straightforward way to do the calculation is not to use cutoffs at all to regularize the divergent integrals. Instead one uses dimensional regularization, supplemented by the decoupling subtraction renormalization scheme, to calculate the effect of anomalous triple-gauge-boson couplings in loops.

5. In general, there is no reason to have relationships such as $\lambda_\gamma = \lambda_Z$ among the parameters of the low-energy effective lagrangian – neither gauge invariance nor custodial $SU(2)$ symmetry requires this. Such relations arise naturally when one uses an effective lagrangian in which the breaking $SU(2)_L \times U(1)_Y \to U(1)_{em}$ is linearly realized. However, this is a strong assumption – it is more conservative to use the nonlinearly realized version of the effective lagrangian in calculations involving anomalous trilinear gauge boson couplings.

Acknowledgments

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[9] For an explicit example, see the appendix of Ref. 7.
Figures

Figure 1. Contribution of the CP violating anapole TGC (blob) to $W$- and $Z$-boson propagators.

Figure 2. Diagrams which result in $\phi^3$ and $\phi^9$ couplings once the heavy field $\psi$ is integrated out. The $\psi$ field is denoted by lines in bold type, while the fine (external) lines represent the $\phi$ field.

Figure 3. 3-loop contribution of the $\phi^9$ coupling to the $\phi^3$ coupling.