On the scaling law of ramp structures in scalar turbulence

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\textbf{ABSTRACT}

Ramp structures widely exist in scalar turbulence, such as temperature, water vapor, and carbon dioxide (CO\textsubscript{2}), which refer to the phenomenon that the physical quantity increases slowly with time and then suddenly drops. ramp structures lead to large gradients on a small scale and result in intermittency and anisotropy of turbulent flows. in this paper, wavelet analysis is used to analyze observed data from the beijing 325-m meteorological tower to extract ramp structures in temperature, water vapor, and CO\textsubscript{2} signals. ramp structures in CO\textsubscript{2} signals are different from those in temperature and water vapor in terms of the averaged temporal scale and normalized amplitude, and the ramp duration almost equals the cliff duration, which means ramp structures in CO\textsubscript{2} signals are not easy to generate and different physical mechanisms may exist. in addition, both the ascending and descending part of ramp structures are linearly fitted. it is found that a scaling law exists between the slope and duration in the ascending part in the three scalar signals. the corresponding power exponents are slightly different. furthermore, the same rule exists in the descending part of ramp structures, which indicates that self-similarity may be a universal law in scalar turbulence. moreover, the maxima of selected ramp structures show the same pattern, i.e. there are ramp structures in the maximum sequence, which proves that small-scale ramp structures are superimposed on large-scale ramp structures.

1. Introduction

There are many phenomena and processes related to one or more scalar fields advected by turbulent flows, such as the transport of scalars by turbulence, which is one of the central issues of fundamental physics of turbulence and is important in engineering, oceanographic and atmospheric applications (Gotoh and Watanabe 2012). The concentration of such a scalar exhibits a complex evolving structure over a broad range of space and time scales (Shraiman and Siggia 2000). Overall, these scalars can be classified as either passive or active, depending on whether they feed back to the velocity field (Ostilla-Monico et al. 2015). Obviously, temperature is an active scalar and CO\textsubscript{2} a passive scalar. These scalar signals often show ramp-cliff structures at some scales (Warhaft 2000; Mazzitelli and Lanotte 2012). These ramp-cliff patterns are common features of scalar turbulence, and have been observed in a variety of turbulent shear flows in both stably and unstably stratified conditions (Wroblewski et al. 2007). A typical ramp-cliff structure in scalar turbulence signals is characterized by a gradual ascending part, the ramp, followed by a steep descending part, the cliff. However, the order is reversed for cliff-ramp structures, with a steep rise and a more gradual drop. In this paper, we call both ‘ramp structures’. These structures are quasi-singular and play a key role in the scalar statistics: they dominate larger and larger scalar fluctuations, leading to strong small-scale intermittency (Kraichnan 1994; Celani et al. 1999).

Many effective methods have been developed to extract ramp structures. The so-called pseudo-wavelet, considering a visually identified pattern in the turbulence signal as the specified wavelet function, can extract ramp structures and estimate their durations (Qiu, Kyaw Tha Paw, and Shaw 1995). However, combining conventional spectral and wavelet analysis, spawlet analysis (Petenko 2001) can yield information about the duration and periodicity of ramp structures.
Furthermore, to avoid any predefined characteristics and parameters, a background noise testing method has been developed to study various structures, including ramp structures, in the stable boundary layer (Kang, Belušić, and Smith-Miles 2013; 2015).

By using these methods, some results have shown that ramp structures are multi-scale in temperature and wind speed signals (Belušić and Mahrt 2012). However, it was unclear as to how ramp structures with different scales influence structure function analysis. So, Shapland et al. (2012a; 2012b) investigated the structure function of two-scale ramp structures. Also, considering the multi-scale characteristic of ramp structures, Song et al. (2014) found that a scaling law exists in wind speed signals, which indicates that ramp structures in wind speed signals are self-similar.

In this paper, we concentrate on ramp structures in the scalar turbulence signals and their self-similarity. The paper is organized as follows: In section 2 we introduce the dataset and the methods used to extract ramp structures. Then, in section 3, the results are presented, showing how we find scaling laws in scalar turbulence signals. In section 4 we draw conclusions about the self-similarity and outline future research plans, including applications of the scaling laws.

2. Data and methods

2.1 Data

The data are from eddy covariance systems mounted at seven levels (8 m, 16 m, 47 m, 80 m, 140 m, 220 m, and 280 m) on the 325-m Beijing meteorological tower. The system consists of two parts: one is for measuring wind speed and temperature; and the other is for water vapor and CO₂. Actual instantaneous values of three-component velocity and temperature of urban turbulence are measured by three-dimensional sonic anemometer-thermometers (Wind Master, Gill, USA). Water vapor and CO₂ are sampled by LI-7500 open-path gas analyzers (Li-Cor, Inc., USA) (Liu et al. 2016). The meteorological tower is in north-central Beijing (39.967°N, 116.367°E) and was set up in 1978 by the Institute of Atmospheric Physics, Chinese Academy of Sciences. The underlying surface is a typical complex urban area consisting of trees, rivers, roads, and buildings of various heights (Song et al. 2014). The dataset (10 Hz) used in this paper is from the 47-m level of the tower, from 0000 LST 1 July 2015 to 2400 LST 11 September 2015. Rainy days are excluded, and we select the data from 1000 to 1500 LST on clean or cloudless days to ensure turbulence is fully developed.

2.2 Methods

2.2.1 Linear regression of the ascending and descending parts

A ramp structure consists of two parts, i.e. the ascending part and descending part. In this paper we refer to the ascending part as the ‘ramp’ and the descending part as the ‘cliff’. Also, we consider the difference between the maximum and minimum in the ramp part as the amplitude of the ramp structure. Then, the amplitude divided by the average of the ramp part is considered as the normalized amplitude. First, we carry out linear fitting of the ascending part and descending part, separately, to obtain the corresponding slope. It is clear that the slope of the rising part is positive and the slope of the descending part is negative. Figure 1 gives an example of how to obtain the slopes and durations of ramp structures by using linear regression.

However, the scalar sequences are so complex that it is not easy to distinguish and pick out ramp structures from noisy signals. So, in the next section, we introduce the discrete wavelet transform to solve the problem.

2.2.2 Extracting ramp structures by the discrete wavelet transform

The scalar signals are decomposed into frameworks and details through the discrete wavelet transform at a

![Ramp structure and its decomposition](image-url)

**Figure 1.** Plot of a typical ramp structure (black solid line). By means of discrete wavelet analysis, it is decomposed into four parts. The green part is the extracted framework at the last scale. The left of the dotted line is the ramp and the right the cliff. The red solid lines are the results of linear regression. The slope of the ramp part is 0.016 and the slope of cliff part −0.102. The amplitude of the ramp structure is 0.7°C and the normalized amplitude is 0.03.
certain scale. By scaling and shifting, the mother wavelet is transformed into orthogonal basis functions in the corresponding space at the first scale. The orthogonal basis functions approximate the original sequence and form a new series called the 'framework'. This approximates the original sequence at the first level. The difference between the framework and the original sequence is named the 'detail'. The next operation involves letting the framework be the original sequence, and the mother wavelet is transformed into basis functions again at the second scale or level. Hence, one obtains a new framework and detail. The procedure is repeated until the sequence is decomposed at the last scale or level. Figure 1 shows an example of obtaining the framework of ramp structures through wavelet decomposition.

In this paper, we carry out discrete wavelet decomposition (mother wavelet: db3) at three scales to obtain the main framework. As mentioned above, ramp structures are quasi-singular. Therefore, the wavelet coefficients at turning points of ramp structures are larger than the neighbors, according to which we select ramp structures and think of these points as the ends of ramp parts. It should be noted that, due to the extreme complexity of Table 1. Averaged characteristics of parameters of ramp structures.

| Scalar Turbulence | Total Duration | Ramp Duration | Cliff Duration | Averaged Amplitude | Normalized Amplitude |
|-------------------|----------------|---------------|----------------|--------------------|----------------------|
| Temperature       | 72 s           | 56 s          | 16 s           | 1.1°C              | 0.031                |
| CO₂               | 36 s           | 19 s          | 17 s           | 0.125 mmol m⁻³     | 0.008                |
| H₂O               | 55 s           | 42 s          | 13 s           | 26.1 mmol m⁻³      | 0.037                |

Figure 2. (a) Multi-scale ramp structures in temperature with duration ranging from 5 s to 200 s. (b, c) Power laws in ramp patterns, for (b) the ascending part, i.e. the ramp, and (c) the descending part, i.e. the cliff. The small inset figures are log-log plots. The corresponding power exponents are −0.92 and −0.81, respectively, and the determination coefficients of linear regression marked in the top-right section in the log-log plots are 0.81 and 0.86, which shows a clean linear relationship between ln(k) (or ln(−k)) and ln(t).
turbulence signals, the procedure above cannot guarantee that all extracted structures are ramp-cliff in pattern. So, in the end we abandon some collected series where there are no obvious descending cliffs or turning points by manual intervention. The abandoned structures take up less than 5% of the extracted structures, which has little influence on the following analysis.

3. Results and discussion

For temperature signals, we select 663 ramp structures with different durations ranging from several seconds to hundreds of seconds, which can be seen in Figure 2 (a). Likewise, 613 ramp structures are extracted in water vapor signals and 456 in CO₂ signals. Ramp patterns in temperature have a duration of 72 s on average, which is the characteristic temporal scale — the most important feature for ramp structures. Also, the total duration in temperature is larger than water vapor (55 s) and CO₂ (36 s). Other details, such as ramp duration and cliff duration, can be seen in Table 1. It seems that, for scalar turbulence, ramp structures in temperature and water vapor have something in common compared to CO₂ signals. The normalized amplitude for CO₂ is 0.008, which is smaller than that for temperature (0.031) and water vapor (0.037). The characteristic scale of ramp structures in CO₂ signals is also smaller than that in temperature and water vapor, and the

![Figure 3](attachment:image.png)

Figure 3. Scaling laws found in (a, b) CO₂ and (c, d) water vapor signals for the (a, c) ramp parts and (b, d) cliff parts. The small inset figures are log-log plots, i.e. $x$ is $\ln(k)$ or $\ln(-k)$ and $y$ is $\ln(t)$. The power exponents of the rising parts and descending parts are $-0.71$ and $-0.69$ in (a) and (b), respectively, for CO₂. The scaling exponents are in (c) and (d) for water vapor are $-0.83$ and $-0.72$, respectively. The coefficients of determination for the linear regression marked in the top-right section in the log-log plots are greater than 0.75, which indicates that the power law is clear between $k$ or $-k$ and $t$. 
ramp scale is almost equal to the cliff scale, which means ramp structures in CO\textsubscript{2} signals are difficult to generate. It is possible that there are different physical mechanisms for ramp structures in CO\textsubscript{2} signals.

Based on selected signals in temperature, we calculate the slopes and durations of ramp and cliff parts. It is found that a power law exists between the slope and duration in these two parts. The power exponents are $-0.92$ and $-0.81$, respectively, i.e. $t = k^{0.92}$ for the ramp and $t = |k|^{-0.81}$ for the cliff, where $t$ is the duration time and $k$ the slope. The result can be seen in Figure 2(b) and (c). The former is for ramp parts and the latter for cliff parts. In these figures, the $x$-coordinate is $\ln(k)$ or $\ln(-k)$, to clearly show the results, which is a semi-log plot. The two small figures are log-log plots, i.e. the $x$-coordinate is $\ln(k)$ or $\ln(-k)$ and the $y$-coordinate is $\ln(t)$. Similar power laws can also be found in the water vapor and CO\textsubscript{2} signals. These results are presented in Figure 3. The $x$-coordinate is also $\ln(k)$ or $\ln(-k)$. Figure 3(a) and (b) are for CO\textsubscript{2}, and Figure 3(c) and (d) are for H\textsubscript{2}O, with Figure 3(a) and (c) showing the ramp parts and Figure 3(b) and (d) the cliff parts. The power exponents of the ascending and descending parts are $-0.71$ and $-0.69$, respectively, for CO\textsubscript{2}. Furthermore, the scaling exponents are $-0.83$ and $-0.72$ for H\textsubscript{2}O. The power law exponents of scalars are slightly different, which indicates that self-similarity exists in these three scalar signals and may be a universal law in scalar turbulence.

Ramp-cliff patterns can be observed not only in scalar turbulence signals but also in their maximum sequence. This indicates that ramp structures are self-similar and the small-scale ramp structures are superimposed on the large-scale ramp structures. Figure 4(a) represents the maximum sequence of temperature in chronological order, which shows ramp-like structures in the red solid line. Figure 4(b), (c) and (d) show some examples of self-similarity in temperature signals. The red parts are small-scale ramp structures, and several of them are found in the ascending part. In particular, Figure 4(c) shows a cliff-ramp pattern where the cliff is the ascending part and the ramp the descending part. However, in its ramp part there is another cliff-ramp structure.

4. Conclusions and future work

In this paper, based on observed temperature, water vapor, and CO\textsubscript{2} data from the Beijing 325-m meteorological tower, we find that power laws exist in scalar turbulence signals,
which indicates that these signals are self-similar. The large-scale ramp structure contains small-scale structures. Plus, different physical mechanisms for ramp structures in CO₂ signals may exist. Ramp-like structures can also be found in fine-particle (PM₃₅) series and wind power series. Ramp patterns in wind power series are called ramp events. Although the physical mechanism and the reason why ramp structures exist in these signals are unknown, we believe that for scalar turbulence signals, wind speed series and PM₃₅ series, self-similarity is the common characteristic from the mathematical perspective. Future research will focus on finding power laws in PM₃₅ series and wind power signals, in the hope that the power law can modify and improve the prediction results of numerical models regarding ramp structures or ramp events.

**Disclosure statement**

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