A Unique Settling Pattern for Two Circular Particles Having Different Densities

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Abstract. In this work the sedimentation of two circular particles having different densities in a vertical channel has been numerically studied using the lattice Boltzmann method (LBM). The case of a single particle settling in a narrow channel has been simulated to validate the computational code. The relationship between the Reynolds number and the particle-to-fluid density ratio was presented. For the case of two particles, a unique pattern of motion, represented by the process of “drafting-kissing-oscillating-separating”, has been revealed. The vortex structures along with the particle displacements are shown to illustrate this pattern.

1. Introduction
The behaviour of systems involving the motion of particles immersed in fluids exists in a wide range of phenomena of interest to both scientists and engineers. The complexity of particle suspensions is primarily due to the hydrodynamic interactions between particles when the Reynolds number is non-negligible. The comprehension of these interactions is of great interest from both fundamental and practical points of view. For instance, the well-known phenomenon of “drafting-kissing-tumbling” (DKT) that was first observed by Fortes et al. [1], was also numerically studied by Feng et al. [2]. Recently, Wang et al. [3] used the lattice Boltzmann Method (LBM) to investigate the DKT phenomenon of two non-identical particles. They demonstrated that the effect of the diameter difference on the DKT process was significant. Similar work has also been conducted by Nie et al. [4], who studied the influence of inter-particle distance on the DKT process. Nie et al. [5] reported the grouping behaviours of multiple particles settling along their line-of-centers in a narrow channel. They showed that the settling particles separated into several groups resulting from the particle-particle interaction, with each group settling at the same velocity. Furthermore, their work demonstrated that this type of grouping behavior strongly depended on the number of particles and the Reynolds number. [6] More recently, Nie et al. [6] studied the settling of two circular particles in a narrow channel and revealed some new features of the settling behaviour of particles.

The two-particle sedimentation system is simple but rich in dynamics and worthy of extensive examination. In general, the magnitude of the interaction between the particles is governed by several variables, among which the density difference is a significant factor characterizing the difference of inertia between the particles. However, to the best of the authors’ knowledge, the interactions between a heavy particle and a light one in fluids are not well understood. A considerable attention should be paid to this issue because it provides valuable insights into the hydrodynamic interactions between unequal particles in finite-Reynolds-number-regime. In view of this, the present work aims to investigate the settling behaviour of two particles with different densities in a vertical channel via the
lattice Boltzmann method. Particular attention is paid to the flow pattern resulted from the hydrodynamic interactions between these particles.

2. Numerical method
The motion of fluid is solved through the LBM in this work. The discrete lattice Boltzmann equations of a single-relaxation-time model are \[ f_i(x + e_i, \Delta t, t + \Delta t) - f_i(x, t) = -\frac{1}{\tau} \left[ f_i(x, t) - f_i^{(eq)}(x, t) \right] \] (1)

where \( f_i(x, t) \) is the distribution function for the microscopic velocity \( e_i \) in the \( i \)th direction, \( f_i^{(eq)}(x, t) \) is the equilibrium distribution function, \( \Delta t \) is the time step of the simulation, \( \tau \) is the relaxation time which is related to the fluid viscosity, \( c_s \) is the speed of sound, and \( w_i \) denotes weights related to the lattice model. The fluid density \( \rho \) and velocity \( u \) are determined by the distribution function \[ \rho = \sum_i f_i, \quad \rho u = \sum_i f_i e_i \] (2)

For the two-dimensional D2Q9 lattice model used here, the discrete velocity vectors are
\[
e_i = \begin{cases} (0,0) & \text{for } i = 0, \\ (\pm 1,0)c, (0,\pm 1)c, & \text{for } i = 1 \text{ to } 4, \\ (\pm 1,\pm 1)c, & \text{for } i = 5 \text{ to } 8, \end{cases}
\] (3)

where \( c = \Delta x/\Delta t \) and \( \Delta x \) is the lattice spacing. The speed of sound has the relation \( c_s^2 = c^2/3 \). Following Qian et al. [7], the equilibrium distribution function is chosen as
\[
f_i^{(eq)}(x, t) = w_i \rho_f \left[ 1 + \frac{3e_i \cdot u}{c^2} - \frac{9{(e_i \cdot u)}^2}{2c^4} - \frac{3u^2}{2c^2} \right] \] (4)

where \( w_i \) are set to be \( w_0 = 4/9, w_{1-4} = 1/9, \) and \( w_{5-8} = 1/36 \). By performing a Chapman–Enskog expansion, the macroscopic mass and momentum equations in the low Mach number limit can be recovered. The kinematic viscosity of the fluid is determined using the equation \( \nu = c_s^2 (0.5) \Delta t \).

In the LBM, a special treatment for a moving boundary is usually needed to ensure the no-slip boundary condition on the surface of a particle. In this work, an improved bounce-back scheme proposed by Lallemand & Luo [8] is used.

The purpose of this work is to reveal a unique pattern of particle motion for two particles with different densities. Fig. 1 shows the schematic diagram of the present problem. Two circular particles, each of diameter \( d \), are released from rest in a vertical channel of width \( L \) and height \( H \). The particles are initially symmetrical with respect to the channel centerline and the distance between their centers is \( S \). The densities of the two particles are denoted \( \rho_s \) (for the light particle) and \( \rho_s' \) (for the heavy particle). In the simulations the parameters are fixed as \( \rho_s = 1.5, S = 2d \) and \( L = 5d \).
3. Numerical results

The present computational code has been validated in our previous work. However, additional computations have been carried out to add further credibility to the present results. The settling of one single particle in a 5d channel has been simulated for the particle-to-fluid density ratio ($\frac{s}{\rho_f}$) ranging from 1.002 to 3.5. The relationship between $Re$ and $\gamma = \frac{s}{\rho_f} - 1$ is illustrated in Fig. 2. It is clearly seen that a linear relation ($Re \sim \gamma$) holds for low Reynolds numbers ($Re < 10$). The reason for this is that the drag on the particle linearly increases with its settling velocity when the flow is dominated by viscous effects at low $Re$. In contrast, when the Reynolds number becomes large the nonlinear effects are significant and the relation between the drag and the settling velocity becomes quadratic. This results in the relationship of $Re \sim \gamma^{0.5}$ for high Reynolds numbers ($Re \gg 10$). These observations are in good agreement with the previous computational results [9].

Figure 1. Schematic diagram of the present problem

Figure 2. Dependence of the Reynolds number ($Re$) on the value of $\gamma = \frac{s}{\rho_f} - 1$
It has been shown that a periodic state may be reached for two particles with different densities [6]. Fig. 2 shows the time evolution of the horizontal displacements for both particles. The particle displacements are normalized in this way: \( X_1' = \frac{X_1}{d} \) and \( X_2' = \frac{X_2}{d} \). Because the velocity scale is chosen as \( U_0 \), the time is then normalized through \( t' = \frac{tU_0}{d} \). In this particular case the parameters are as follows: \( s_1 = 1.5 \), \( s_2 = 1.585 \) and \( \varepsilon = 0.236 \). The Reynolds number, based on the maximum settling velocity of the two particles, is about 8.33.

It is clearly shown that the two particles are periodically oscillating in the channel when they are settling. In other words, the two particles are moving in the same averaged speed. Particularly, Fig. 2 reveals that the light particle is on the opposite side of the channel to the heavy one. In addition, the amplitude of \( X_2' \) is seen to be larger than that of \( X_1' \), which suggests that the oscillation is stronger for the heavy particle.

\[
\begin{align*}
\text{(a)} & \quad \text{(b)}
\end{align*}
\]

**Figure 3.** Time history of horizontal displacement for (a) the light particle \( (X_1') \) and (b) the heavy particle \( (X_2') \), respectively.

Fig. 3 reveals a unique pattern of motion for two particles with different densities. In order to illustrate this point, the evolution of \( X_2' \) during one period is shown in Fig. 4. Due to the existence of the light particle, the motion of the heavy one is significantly affected, which is known as the hydrodynamic interaction. It is the hydrodynamic interaction that gives rise to the occurrence of the unique pattern of particle motion which is characterized by the process of “drafting-kissing-oscillating-separating”, as shown in Fig. 4. The letters (‘A’~ ‘J’) shown in Fig. 4 denote the different stages of this pattern. The instantaneous flow field (vortex contour) is presented in Fig. 5, which corresponds to the time represented by each letter.

\[
\begin{align*}
\text{drafting} & \quad \text{kissing} & \quad \text{oscillating} & \quad \text{separating}
\end{align*}
\]

**Figure 4.** Enlargement of the heavy particle’s horizontal displacement in one period.

As shown in Fig. 5(a), the light particle is located at the wake of the heavy one, resulting in the fact that the light particle experiences a smaller drag and settling with a larger velocity. As a result, the light particle will come close to the heavy one until they nearly touch, as seen in Fig. 5(b). This
process is called “drafting”. As shown in Fig. 5(c), the two particles will contact with each for a time before they separate, which is known as the process of “kissing”. It is worth remarking that the particles do not come to actual contact due to the lubrication force between the particles. When the Reynolds number increases, the inertia of particles becomes strong enough and they tend to collide. Eventually the interparticle repulsion keeps the two particles apart. Then they are seen to oscillate in the channel due to the effect of inertia, as shown in Fig. 5(d) and (e). Owing to the fact that the two particles have different densities, the process of “tumbling” does not occur, unlike the cases reported in the reference. Instead, the heavy particle is seen to leave the light one, as shown in Fig. 5(f). This process is denoted as “separating” in this work. However, once the light particle is pulled into the wake of the heavy one, the process of “separating” stops and the “drafting” begins. Then the pattern is repeated. However, a significant difference takes place this time, as shown in Fig. 5(h). In comparison with Fig. 5(c), it is seen that the two particles exchange their positions.

![Figure 5](image)

**Figure 5.** Instantaneous vorticity contour at different times which correspond to the small circles labelled in Figure 3.

4. Conclusion
The lattice Boltzmann method is used to simulate the sedimentation of two circular particles in a narrow channel. The two particles have different densities. The method is firstly validated through the case of the settling of a single particle. A unique pattern, which is characterized by the motion of “drafting-kissing-oscillating-separating”, has been revealed in this work. Unlike the phenomenon of DKT reported in previous literature, the process of “tumbling” does not occur because the light particle cannot go to the bottom of the heavy particle. It has also been revealed that the heavy particle may be pushed to the left or right side of the channel during the process of “kissing” due to the interparticle repulsion.

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