Isospin Breaking in the Nucleon Isovector Axial Charge
from QCD Sum Rules

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Abstract

The isospin breaking in the nucleon isovector axial charge, $g_A^3$, are calculated within the external field QCD sum-rule approach. The isospin violations arising from the difference in up and down current quark masses and in up and down quark condensates are included; electromagnetic effects are not considered. We find $\delta g_A^3 / g_A^3 \approx (0.5 - 1.0) \times 10^{-2}$, where $\delta g_A^3 = (g_A^3)_p + (g_A^3)_n$ and $g_A^3 = [(g_A^3)_p - (g_A^3)_n]/2$. Using the Goldberger-Treiman relation, we also obtain an estimate of the isospin breaking in the pion-nucleon coupling constant, $(g_{pp\pi} - g_{nn\pi}) / g_{NN\pi} \approx (2 - 7) \times 10^{-3}$.
The nucleon isovector axial charge (or the nucleon axial vector coupling constant) is defined through the nucleon matrix element of the isovector axial current at zero momentum transfer

\[ \langle N| \bar{u} \gamma_\mu \gamma_5 u - \bar{d} \gamma_\mu \gamma_5 d |N \rangle = g_A^3 \overline{U}(p) \gamma_\mu \gamma_5 U(p) , \]  

(1)

where \( U(p) \) denotes the nucleon spinor. Assuming isospin symmetry, one finds \( (g_A^3)_p = -(g_A^3)_n \), and the value of \( (g_A^3)_p = 1.2573 \pm 0.0028 \), extracted from the neutron beta decay, has been quoted in the literature [1]. In nature, the isospin symmetry is broken by the current quark mass difference as well as the electromagnetic interaction, and thus \( (g_A^3)_p \neq -(g_A^3)_n \).

Previous studies of the nucleon isovector axial charge in the framework of external field QCD sum-rule method have been made by various authors [2,3]. However, to our best knowledge, the isospin breaking effects have been ignored in these studies. The goal of this Letter is to examine the difference between \( (g_A^3)_p \) and \( (g_A^3)_n \) using the external field QCD sum-rule approach, which has been used in studying various nucleon matrix elements of bilinear quark operators [2–12]. The isospin violation is reflected in \( m_u \neq m_d \) and the isospin breaking in the vacuum condensates. Electromagnetic effects will not be included.

Let us start from the correlation function of the nucleon interpolating field in the presence of a constant external isovector axial vector field \( Z^\mu \)

\[ \Pi_Z(q) \equiv i \int d^4xe^{iq \cdot x} \langle 0 | T[\eta_N(x)\overline{\eta}_N(0)] | 0 \rangle_Z , \]  

(2)

where \( \eta_N \) is the nucleon interpolating field introduced in Ref. [13]

\[ \eta_p(x) = \epsilon_{abc} \left[ u_a^T(x)C \gamma_\mu u_b(x) \right] \gamma_5 \gamma^\mu d_c(x) , \]  

(3)

\[ \eta_n(x) = \epsilon_{abc} \left[ d_a^T(x)C \gamma_\mu d_b(x) \right] \gamma_5 \gamma^\mu u_c(x) , \]  

(4)

where \( u_a(x) \) and \( d_c(x) \) stand for the up and down quark fields, \( a, b \) and \( c \) are the color indices, and \( C = -C^T \) is the charge conjugation operator. The subscript \( Z \) in Eq. (2) denotes that we are evaluating the correlation function in the presence of the external isovector axial vector field \( Z_\mu \); the correlator Eq. (2) should be calculated with an additional term
\[ \Delta L \equiv -Z_\mu \left[ \pi \gamma^\mu \gamma^5 u - \bar{d} \gamma^\mu \gamma^5 d \right] , \]  

added to the usual QCD Lagrangian. The up and down quark fields then satisfy the modified equations of motion:

\[ (i\slashed{D} - m_u - Z \gamma^5)u(x) = 0 , \]  
\[ (i\slashed{D} - m_d + Z \gamma^5)d(x) = 0 . \]  

To first order in the external field, the correlation function can be written as

\[ \Pi_Z(q) = \Pi_0(q) + Z \lambda \Pi^\lambda(q) , \]  

where \( \Pi_0(q) \) is the correlation function in the absence of the external field which gives rise to the usual mass sum rules. Here we are interested in the linear response to the external field given by \( \Pi^\lambda(q) \). The QCD sum rules for \( \Pi^\lambda(q) \) differ from those for \( \Pi_0(q) \). The phenomenological representation for \( \Pi^\lambda(q) \) at the hadron level contains a double pole at the nucleon mass whose residue contains the matrix element of interest. In addition there are single pole terms which arise from the transition matrix element between the ground state nucleon and excited states. These later contributions are not exponentially damped after Borel transformation relative to the double pole term and should be retained in a consistent analysis of the sum rules. On the theoretical side of the sum rules expressed in terms of an operator product expansion (OPE) the external field contributes in two different ways—by directly coupling to the quark fields in the nucleon current and by polarizing the QCD vacuum.

The linear response of the correlation function, \( \Pi^\lambda(q) \), has three distinct invariant structures [2,3]:

\[ \Pi^\lambda = \Pi_1(q^2)q^\lambda \gamma^5 + \Pi_2(q^2)\gamma^\lambda \gamma^5 + \Pi_3(q^2)i\sigma_{\rho\sigma}q^\rho \gamma^5 . \]  

So, one may derive three QCD sum rules from the three invariant functions, \( \Pi_1(q^2) \), \( \Pi_2(q^2) \), and \( \Pi_3(q^2) \), respectively. In principle, the predictions based on these sum rules should be the
same. In practice, however, one has to truncate the OPE and use a simple phenomenological ansatz for the spectral density; thus these sum rules usually have different merits. In particular, some sum rules work better than the others. This pattern has been seen in various external field sum rules \[2\]–\[12\]. As discussed extensively in Ref. \[3\], this may be attributed to the different asymptotic behavior of various sum rules. The phenomenological side of the external field sum rules contains single pole terms arising from the transition between the ground state and the excited states, whose contribution is not suppressed relative to the double pole term and thus contaminates the double pole contribution. The degree of this contamination may vary from one sum rule to another. The sum rule with smaller single pole contribution works better. We refer the reader to Refs. \[2,3,10–12\] for more discussions on this point. As pointed out in Refs. \[2,3\], the sum rule obtained from $\Pi_1(q^2)$ is the most stable one for the problem under study. As such, we shall focus on this stable sum rule and disregard the sum rules based on $\Pi_2(q^2)$ and $\Pi_3(q^2)$.

It is straightforward to obtain the external field sum rules following the techniques given in the literature. To include the isospin violation effects, we retain the terms linear in current quark masses and isospin breaking in the condensates. Here we truncate the OPE at the same level as in the previous studies. The OPE result for $\Pi_1(q^2)$ in the proton case is given by

$$
\Pi_1(q^2) = -\frac{1}{16\pi^4} q^2 \ln(-q^2) - \frac{1}{16\pi^2} \left< \frac{\alpha_s}{\pi} G^2 \right>_0 \frac{1}{q^2} + \frac{1}{3\pi^2} \frac{\kappa}{q^2} + \frac{4}{3\pi^2} m_u \left< \Sigma u \right>_0 \frac{1}{q^2} \left( q^2 \right)
$$

where $\left< \hat{O} \right>_0 \equiv \left< 0 | \hat{O} | 0 \right>$, and $\chi$ and $\kappa$ denote the linear response of condensates to the external field

$$
\left< 0 | \Sigma \gamma_\mu \gamma_5 u - \bar{d} \gamma_\mu \gamma_5 d | 0 \right>_Z = 2 Z \chi , \\
\left< 0 | \Sigma \tilde{G}^\mu_\nu \gamma_\nu u - \bar{d} \tilde{G}^\mu_\nu \gamma_\nu d | 0 \right>_Z = 2 Z \kappa ,
$$

with $\tilde{G}^\mu_\nu = \frac{1}{2} \epsilon^\mu_\nu\lambda_\rho G^\lambda\rho$. Here we have omitted all the polynomials in $q^2$ which vanish under
the Borel transformation, and neglected the responses of the corresponding isoscalar current to the external isovector field. The analogous result for the neutron is

\[
\Pi_1(q^2) = \frac{1}{16\pi^2} q^2 \ln(-q^2) + \frac{1}{16\pi^2} \left( \frac{\alpha_s}{\pi} G^2 \right)_0 \frac{1}{q^2} - \frac{1}{3\pi^2} \frac{\kappa}{q^2} - \frac{4}{3\pi^2} m_d \langle \bar{d}d \rangle_0 \frac{1}{q^2}
\]

\[
- \frac{20}{9} \langle \bar{d}d \rangle_0 \frac{1}{q^4} + \frac{2}{9} m_d \langle \bar{d}d \rangle_0 \chi \frac{1}{q^4} + \frac{2}{3} m_u \langle \bar{u}u \rangle_0 \chi \frac{1}{q^4}.
\]

The resulting QCD sum rules can be written as

\[
M^4 E_1 L^{-4/9} + \frac{b}{4} L^{-4/9} + \frac{4}{3} \tilde{\kappa} L^{-68/81} + \frac{16}{3} \left( 1 - \frac{\gamma}{2} - \frac{\delta m}{2\hat{m}} \right) \hat{m} a L^{-4/9}
\]

\[
+ \frac{20}{9} (1 - \gamma) a^2 L^{-4/9} - \frac{2}{9} \left( 1 - \frac{\gamma}{2} - \frac{\delta m}{2\hat{m}} \right) \hat{m} a \tilde{\chi} \frac{1}{M^2} L^{-4/9}
\]

\[
- \frac{2}{3} \left( 1 + \frac{\gamma}{2} + \frac{\delta m}{2\hat{m}} \right) \hat{m} a \tilde{\chi} \frac{1}{M^2} L^{-4/9} = \chi^2 \left[ \frac{(g_A^2)^n}{M^2} + A_p \right] e^{-M_p^2/M^2},
\]

for the proton case and

\[
M^4 E_1 L^{-4/9} + \frac{b}{4} L^{-4/9} + \frac{4}{3} \tilde{\kappa} L^{-68/81} + \frac{16}{3} \left( 1 + \frac{\gamma}{2} + \frac{\delta m}{2\hat{m}} \right) \hat{m} a L^{-4/9}
\]

\[
+ \frac{20}{9} (1 + \gamma) a^2 L^{-4/9} - \frac{2}{9} \left( 1 + \frac{\gamma}{2} + \frac{\delta m}{2\hat{m}} \right) \hat{m} a \tilde{\chi} \frac{1}{M^2} L^{-4/9}
\]

\[
- \frac{2}{3} \left( 1 - \frac{\gamma}{2} - \frac{\delta m}{2\hat{m}} \right) \hat{m} a \tilde{\chi} \frac{1}{M^2} L^{-4/9} = -\tilde{\chi}^2 \left[ \frac{(g_A^2)^n}{M^2} + A_n \right] e^{-M_n^2/M^2},
\]

for the neutron case, where we only keep the terms up to first order in isospin violation and have defined \( a \equiv -(2\pi)^2 \langle \bar{u}u \rangle_0 + \langle \bar{d}d \rangle_0 / 2, b \equiv (2\pi)^2 \langle \alpha_s G^2 \rangle_0, \hat{m} \equiv (m_u + m_d)/2, \delta m = m_d - m_u, \gamma \equiv \langle \bar{d}d \rangle_0 / \langle \bar{u}u \rangle_0 - 1, \tilde{\kappa} \equiv -(2\pi)^2 \kappa, \tilde{\chi} \equiv -(2\pi)^2 \chi, \) and \( \tilde{\chi}^2 \equiv 32\pi^4 \chi^4, \) with \( \langle 0 | \eta_{p(n)} | p(n) \rangle = \lambda_{p(n)} v_{p(n)} \) and \( \pi_{p(n)} v_{p(n)} = 2M_{p(n)}. \) Here \( A_p \) and \( A_n \) are the phenomenological parameters that represent the sum over the contributions from all off-diagonal transitions between the nucleon and excited states, and \( E_1 \equiv 1 - e^{-s_0/M^2} \left( \frac{s_0}{M^2} + 1 \right), \) which accounts for the sum of the contributions involving excited states only, where \( s_0 \) is an effective continuum threshold. We have also taken into account the anomalous dimension of the various operators through the factor \( L \equiv \ln(M^2/\Lambda_{QCD}^2) / \ln(\mu^2/\Lambda_{QCD}^2). \) We take the renormalization scale \( \mu \)
and the QCD scale parameter $\Lambda_{\text{QCD}}$ to be 500 MeV and 150 MeV \[13\], respectively. It is easy to see that the sum rules give rise to $(g_A^3)_p = -(g_A^3)_n$ when isospin violation is switched off.

To analyze the above sum rules and extract the quantities of interest, we adopt the numerical optimization procedures used in Refs. \[10\] \[12\]. The sum rules are sampled in the fiducial region of Borel $M^2$, where the contributions from the high-dimensional condensates remain small and the continuum contribution is controllable. We choose $0.8 \leq M^2 \leq 1.4 \text{GeV}^2$ which has been identified as the fiducial region for the nucleon mass sum rules \[4\]. Here we adopt these boundaries as the maximal limits of applicability of the external field sum rules. The sum-rule predictions are obtained by minimizing the logarithmic measure $\delta(M^2) = \ln[\text{maximum}\{\text{LHS,RHS}\}/\text{minimum}\{\text{LHS,RHS}\}]$ averaged over 150 points evenly spaced within the fiducial region of $M^2$, where LHS and RHS denote the left- and right-hand sides of the sum rules, respectively. Note that the coupling strengths $\lambda_{p(n)}^2$ also appear in the external field sum rules. Here we use the experimental values for the nucleon masses and extract $\lambda_{p(n)}^2$ from the nucleon mass sum rules given in Ref. \[14\] [Eqs. (17) and (21)], using the same optimization procedure as described above. Since we neglect the electromagnetic interactions, we correct the neutron and proton masses such that $M_n - M_p = 2.06\text{MeV}$, where the central value for the electromagnetic contribution, $(M_n - M_p)_{\text{el}} = -0.76\text{MeV} \[13\]$, has been used. We then extract $(g_A^3)_{p(n)}$, $A_{p(n)}$, and $(s_0)_{p(n)}$ from the sum rules Eqs. \[14\] \[15\].

For vacuum condensates, we use $a = 0.55\text{GeV}^3 (m_u + m_d \simeq 11.8\text{MeV})$ and $b = 0.5\text{GeV}^4 \[4\]$. The parameters $\chi$ and $\kappa$ have been estimated previously. Here we just quote the values, $\chi = -2f_\pi^2 \[2,3\]$ and $\kappa \simeq -0.2 \chi \[16\]$, with $f_\pi = 93\text{MeV}$. The quark mass difference $\delta m$ has been determined by Gasser and Leutwyler, $\delta m/(m_u + m_d) = 0.28 \pm 0.03 \[15\]$. The value of $\gamma$ has been estimated previously in various approaches \[17\] \[25\]. All of them, with the exception of Refs. \[24\] \[25\], indicate an interval of $-0.01 \leq \gamma \leq -0.006$. For these $\gamma$ values, we obtain

$$\delta g_A^3 \simeq (0.5 - 1.0) \times 10^{-2},$$

(16)
FIG. 1. The left-hand side (solid) and right-hand side (dashed) of Eq. (14) as functions of Borel $M^2$, with $\gamma = -0.008$ and the optimized values for $(g_A^3)_{p}$, $A_p$ and $(s_0)_p$.

where $\delta g_A^3 = (g_A^3)_{p} + (g_A^3)_{n}$ and $g_A^3 = [(g_A^3)_{p} - (g_A^3)_{n}] / 2$. With smaller magnitude for $\gamma$, we get smaller values for $\delta g_A^3$. To see how well the sum rules work, we plot the left- and right-hand sides of the sum rule Eq. (14) as functions of $M^2$ in Fig. 1, with $\gamma = -0.008$. One can see that the two sides have a good overlap. This is typical for other values of $\gamma$ and for the neutron case.

As emphasized above as well as in the literature, the contribution from the transition between the ground state nucleon and the excited states is not suppressed relative to the double pole term of interest. This contribution is included through a single constant parameter, $A_{p(n)}$. This, as pointed out in Ref. [26], is an approximation. In principle, $A_{p(n)}$ should also be dependent on $M^2$. The impact of approximating $A_{p(n)}$ as a constant on the extracted quantities is expected to be small [26]. Moreover, we have treated the continuum threshold $s_{p(n)}^0$ as a free parameter to be extracted from the sum rules. This should partially account for the $M^2$ dependence of $A_{p(n)}$.

It is also worth pointing out that unlike the mass there are no experimental values for the
couplings, $\lambda_{p(n)}^2$. One usually evaluates these parameters from the mass sum rules by fixing the mass at the experimental value. This means that the uncertainties associated with $\lambda_{p(n)}^2$ will give rise to additional uncertainties in the determination of the nucleon matrix elements of various current, besides the uncertainties in the external field sum rules themselves. This is a general drawback of external field sum-rule approach and/or QCD sum-rule calculations based on three point functions. Here we have not considered the uncertainties associated with $\lambda_{p(n)}^2$.

From our calculation of the isospin breaking in the nucleon isovector axial charge, we may estimate the isospin splitting in the pion-nucleon coupling constants by invoking the Goldberger-Treiman relation. The pion nucleon couplings are defined through the interactions

$$L_{pp\pi} = g_{pp\pi} \pi \gamma_5 \pi_0 P , \quad L_{nn\pi} = -g_{nn\pi} \pi \gamma_5 \pi_0 N .$$

(17)

Note that both $g_{pp\pi}$ and $g_{nn\pi}$ are positive in this notation. The Goldberger-Treiman relation then states

$$(g_A^3)_p = \frac{g_{pp\pi} f_\pi}{M_p} , \quad (g_A^3)_n = -\frac{g_{nn\pi} f_\pi}{M_n} .$$

(18)

Using our results for $(g_A^3)_p$ and $(g_A^3)_n$, we find

$$\frac{g_{pp\pi} - g_{nn\pi}}{g_{NN\pi}} \approx (2 - 7) \times 10^{-3} ,$$

(19)

where $g_{NN\pi} = (g_{pp\pi} + g_{nn\pi})/2$. Since the Goldberger-Treiman relation only holds approximately and there may be corrections to this relation which are originated from the isospin breaking effects, our estimate here is only qualitative. Nevertheless, the estimate given by Eq. (19) is qualitatively compatible with the recent result obtained by Henley and Meissener [27] from the QCD sum rules based on three point function, though the magnitude is somewhat smaller than that given in Ref. [27]. Our estimate here is also consistent with those found in various models [28–31]. On the other hand, Ref. [32] gives a result which has an opposite sign.
In summary, we have calculated the isospin breaking in the nucleon isovector axial charge within the external field QCD sum-rule method. We included the isospin breaking effects due to the difference in current quark mass difference and in quark condensates, and neglected the electromagnetic effects. We found a small isospin violation in the nucleon isovector axial charge, $\delta g_A^3/g_A^3 \approx (0.5 - 1.0) \times 10^{-2}$. This, upon using the Goldberger-Treiman relation, leads to an estimate of the isospin breaking in the pion-nucleon coupling constant, $(g_{pp\pi^0} - g_{nn\pi^0})/g_{NN\pi^0} \approx (2 - 7) \times 10^{-3}$, which is qualitatively consistent with previous studies.

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