Adaptive Backstepping Control for a Class of Non-Triangular Structure Nonlinear Systems

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ABSTRACT In this paper, an adaptive control scheme is proposed based on backstepping approach for a class of second-order nonlinear systems with non-triangular uncertainties. The triangular form of the control system is destroyed by such uncertainties and the traditional backstepping can not be directly used in the controller design. Compared with the existing results, nonlinear terms and unknown parameters appear in every channel. Thus the linear type of relation between the state vector $x$ and $z$ is difficult to maintain. To overcome such difficulty, a new backstepping-based adaptive control scheme is proposed. A significant of our new scheme is to introduce the feedback gain term $l_i z_i x_i^2$ in virtual control signals. Under the proposed control scheme, not only the non-triangular uncertainties can be restrained effectively but also the estimation of parameter has been realized successfully. Finally simulation studies are used to verify the effectiveness of the proposed scheme.

INDEX TERMS Backstepping, adaptive control, nonlinear system, uncertainties, non-triangular structure.

I. INTRODUCTION

Backstepping technique [1] as a promising approach has been widely used in the controller design and stability analysis for nonlinear systems which are required with strictly triangular structure. However, the system structure of triangular nonlinear systems may be affected by all sorts of uncertainties including modeling errors, unknown parameters, delay, disturbance, unknown failures, input and state delay and so on [2]–[12]. As we all know, uncertainties caused by modeling errors, nonlinear parameters and external disturbance can not be avoid and are often represented as an unknown nonlinear function of system states, inputs and time-variable $t$. To guarantee the system performance, such uncertainties must be fully considered in controller design. But unfortunately, the triangular structure of the control system will be destroyed by such uncertainties when we take it into full account. Such a problem usually exists in practical systems, for example the servo system driven by twin motors shown in [14]. As a result, backstepping approach will no longer be applicable even if for systems whose main body has triangular structure, for example, $\dot{x}_i = x_{i+1} + \Delta_i$ where $\Delta_i$ is non-triangular structure uncertainties [13], [14]. Then how to extend the traditional backstepping technique on above non-triangular structure systems seems very important.

In this paper, we address such a problem for a class of non-triangular structure nonlinear systems. In the system model, we use unknown nonlinear functions $\Delta_i$ appearing in every channel to represent non-triangular structural terms which include all unmeasurable uncertainties. To obtain single term about $\|x\|_2$, the uncertainties term caused by $\Delta_i$ was enlarged by using fundamental inequality in every design step. Then all non-triangular uncertainties $\delta_i \|x\|_2$ will be left to the last step and deal with together. Feedback gain term $l_i z_i x_i^2$ introduced in virtual signal $\alpha_i$ is exactly to reduce the non-triangular uncertainties $\Delta_i$. The main contributions of this proposed control laws for nonlinear systems with non-triangular structure uncertainties can be summarized as follows: (1) The control problem is investigated for second-order system with unknown non-triangular uncertainties. An adaptive control law has been proposed by backstepping approach; (2) Unlike the existing results of semi global stability, the proposed control scheme only needs $z_i^2(0) + z_i^2(0) \leq \rho$; (3) Compared with the existing results in [13], [14], the existence of nonlinear functions $\phi_i(\cdot)$ and unknown parameter $\theta$ make the controller design more...
difficult. Not only the non-triangular uncertainties can be restrained effectively but also the estimation of parameter has been realized successfully under the proposed control scheme.

The paper is organized as follows: Section II describes controlled system model with non-triangular uncertainties. Section III presents the designed adaptive controller and analysis of the closed-loop system. The main result is also given in this section. Simulation results is given in Section IV to verify the effectiveness of the proposed control scheme. Finally, the paper is concluded in Section V.

II. PROBLEM STATEMENT

In practice, many control systems can be written as a second-order model, for example, [15], [16] the tank gun control system (1), the missile system (2) and joint system of pneumatic muscle (3), etc. Following these three systems will be given simply.

- The mathematical model of gun control system is given below:
\[
\begin{align*}
\dot{q} &= -\frac{R_a}{L}i_q - \frac{K_p}{L}w_g + \frac{K}{L}u_q \\
\dot{w}_g &= \frac{K_i}{J}i_q - \frac{1}{J}\dot{q}(t)
\end{align*}
\]

where \( L \) is the inductance of motor, \( R_a \) is armature resistance, and \( J \) presents total load inertia; \( K_p \) and \( K_i \) are respectively the motor torque coefficient and back EMF(electromotive force) coefficient of motor; \( i \) is reduction ratio, \( \omega_g \) is angular velocity of gun. \( \dot{q}(t) \) is uncertainties.

- The missile system can be shown as
\[
\begin{align*}
\dot{\alpha} &= \frac{\mu QS}{mV} \cos \alpha C_{ao}[\alpha, \delta, M_{ab}] + q \\
\dot{\delta} &= \frac{\mu QS d}{I} C_m[\alpha, \delta, M_{ab}]
\end{align*}
\]

where \( \alpha \) is the angle of attack and \( q \) is the rate of pitch. \( \delta \) is the rudder angle which is the input signal of the missile system. \( m \) is the mass of missile and \( V \) is the flight velocity. \( Q, S, d \) denote the dynamic pressure, characteristic area, characteristic length, respectively. \( M_{ab} \) is mach number, \( I \) is the moment of inertia and \( \mu = 180/\pi \). \( C_{ao}[\alpha, \delta, M_{ab}], C_m[\alpha, \delta, M_{ab}] \) denote aerodynamic force.

- The system model of pneumatic muscle is
\[
T(t) = J\ddot{\theta}(t) + b_\theta \dot{\theta}(t) = F_1(t)d_1 - F_2(t)d_2 + \vartheta(t)
\]

where \( J \) represents the moment of inertia for the mechanism of PMAs, \( b_\theta \) represents the coefficient of damping, \( \vartheta(t) \) indicates an unknown term such as external disturbances and unmodeled dynamics for the mechanism of PMAs. \( F_1 \) and \( F_2 \) are two pulling forces of PMAs.

Remark 1: Based on the above analysis, many practical systems can be written as a second-order model. At the same time, uncertainties including unknown modeling errors, external disturbance and so on can not be avoided in the establishment of the mathematical model. Especially when the non-triangular uncertainties exist, the traditional backstepping will be invalided. So the research on adaptive control for second-order systems with non-triangular uncertainties is reasonable and meaningful.

Because of second-order systems being common in practice, To study the control scheme design for nonlinear systems with non-triangular uncertainties. To illustrating our design ideas, the following second system with unknown parameters is considered
\[
\begin{align*}
\dot{x}_1 &= x_2 + f_1(x_1) + \varphi_1^T(x_1)\theta + \Delta_1(x_1, x_2) \\
\dot{x}_2 &= bu + f_2(x_1, x_2) + \varphi_2^T(x_1, x_2)\theta + \Delta_2(x_1, x_2) \\
y &= x_1
\end{align*}
\]

where \( x = (x_1, x_2)^T \) are system states, \( y \in R \) is output, \( u \) is input. \( \varphi_i \in R^p(i = 1, 2), f_i \in R \) are known and sufficient smooth functions, \( \theta \in R^n \) and \( b \in R \) are unknown constant parameters. \( \Delta_i(x_1, x_2) \in R, (i = 1, 2) \) is unknown nonlinear functions and represent all terms can not be modeled or linearly parameterized.

Assumption 1: There exist positive constants \( \delta_i \) and \( d_i \) such that
\[
|\Delta_i| \leq \delta_i \parallel (x_1, x_2) \parallel_2 + d_i
\]

Remark 2: Note that existing results on adaptive control of nonlinear systems by using backstepping techniques normally require the unknown modeling error term \( \Delta_i \) to satisfy triangular conditions. Namely, to unknown modeling errors existing in every state equation certain strong requirements such as \( |\Delta_i| \leq \alpha_i(x_1, \ldots, x_i) \) where \( \alpha_i(x_1, \ldots, x_i) \) is a known function and \( |\Delta_i| \leq D_i \) where \( D_i \) is an unknown constant are imposed. These requirements indicate that the bounding functions of \( \Delta_i \) must satisfy a semi-strict feedback form, or triangular structure of system states. Such requirements are no longer needed under the proposed adaptive control scheme in this paper.

Remark 3: Such a problems had been studied by authors and some results were given in [13] and [14]. Although above triangular structure requirements on \( \Delta_i \) have been removed successfully, the strong requirements on the magnitudes or strength of these unknown nonlinear modeling errors are difficult to satisfy in practice. In addition, results shown in [13] and [14] need that nonlinear functions \( f_i(x_1, \ldots, x_i), \varphi_i(x_1, \ldots, x_i)(i = 1, \ldots, n - 1) \) and unknown parameter \( \theta \) can not to appear in system model due to the linear type of relation between the state vector and its transformed vector obtained by coordinate change will be damaged. Such requirements are no longer need in this paper.

Assumption 2: \( b \neq 0 \) and \( \text{sign}(b) \) is known.

Assumption 3: Reference signal \( y_0(t) \) and its derivative are known and bounded.

III. DESIGN OF ADAPTIVE CONTROLLERS

Our control purpose is to design adaptive control scheme by backstepping techniques to guarantee the semi-globally...
uniformly stability of closed loop system. To carry out the
design of control law and adaptive update laws, the following
change of coordinates are introduced.
\[
\begin{align*}
    z_1 &= x_1 - y_r \\
    z_2 &= x_2 - \alpha_1 - y_r^{(1)}
\end{align*}
\]
(6)
where variable \(z_1\) is tracking error. \(\alpha_1\) is the virtual control in step 1.

Step 1: From (5) and (6) the derivative of \(z_1\) can be rewritten as
\[
\dot{z}_1 = \dot{x}_1 - \dot{y}_r
\]
\[
= x_2 + f_1(x_1) + \psi^T_1(x_1)\theta + \Delta_1(x_1, x_2) - y_r^{(1)}
\]
\[
= z_2 + \alpha_1 + f_1(x_1) + \psi^T_1 \theta + \Delta_1(x_1, x_2)
\]
(7)
where \(\alpha_1\) is the virtual control. We consider the following
Lyapunov function
\[
V_1 = \frac{1}{2} z_1^2 + \frac{1}{2} \dot{\theta}^T \Gamma^{-1} \dot{\theta}
\]
(8)
where \(\dot{\theta} = \theta - \hat{\theta}\) and \(\hat{\theta}\) is an estimate of unknown parameters \(\theta\). \(\Gamma\) is a positive definite matrix. Virtual control \(\alpha_1\) can be chosen as
\[
\alpha_1 = -k_1 z_1 - f_1(x_1) - \psi^T_1 \dot{\theta} - \frac{1}{4 \varepsilon_{1,1}} z_1 - \frac{1}{4 \varepsilon_{1,2}} z_1 - l_1 z_1 x_1^2
\]
(9)
where \(k_1 > 0, \varepsilon_{1,i} > 0, l_1 > 0\) are design parameters. Then the derivative of \(V_1\) is
\[
\dot{V}_1 = z_1 \dot{z}_1 - \dot{\theta}^T \Gamma^{-1} \dot{\theta}
\]
\[
= z_1 z_2 - k_1 z_1^2 - l_1 z_1^2 x_1^2 - \sum_{i=1}^{2} \frac{1}{4 \varepsilon_{1,i}} z_i^2 + z_1 \Delta_1
\]
\[
- \dot{\theta}^T \Gamma^{-1} (\dot{\theta} - \Gamma \psi_1 z_1)
\]
(10)
Note that
\[|z_1 \Delta_1| \leq |z_1| (\delta_1 \parallel (x_1, x_2) \parallel_2 + d_1)\]
\[
\leq \sum_{i=1}^{2} \frac{1}{4 \varepsilon_{1,i}} z_i^2 + (\varepsilon_{1,1} \delta_1 \parallel (x_1, x_2) \parallel_2)^2
\]
\[
+ (\varepsilon_{1,2} d_1)^2
\]
(11)
With (11) we can get
\[
\dot{V}_1 \leq z_1 z_2 - k_1 z_1^2 - l_1 z_1^2 x_1^2 + (\varepsilon_{1,1} \delta_1 \parallel (x_1, x_2) \parallel_2)^2
\]
\[
+ (\varepsilon_{1,2} d_1)^2 - \dot{\theta}^T \Gamma^{-1} (\dot{\theta} - \tau_1)
\]
(12)
where turning function is
\[
\tau_1 = \Gamma \psi_1 z_1
\]
(13)
Step 2: From (5) and (6) the derivative of \(z_2\) is
\[
\dot{z}_2 = \dot{x}_2 - y_r^{(2)} - \dot{\alpha}_1
\]
\[
= bu + f_2(x_1, x_2) + \psi^T_2(x_1, x_2) \theta
\]
\[
+ \Delta_2(x_1, x_2) - y_r^{(2)} - \dot{\alpha}_1
\]
(14)
Note that
\[
\dot{\alpha}_1 = \frac{\partial \alpha_1}{\partial x_1} (x_2 + f_1(x_1) + \psi^T_1(x_1) \theta + \Delta_1(x_1, x_2)) + \frac{\partial \alpha_1}{\partial \theta} \dot{\theta}
\]
(15)
We consider the following Lyapunov function
\[
V = V_1 + \frac{1}{2} \dot{\theta}^T \dot{\theta} + \frac{|\theta|}{\eta} \dot{\theta}
\]
(16)
where \(\tilde{\theta} = \theta - \hat{\theta}\) and \(\hat{\theta}\) is an estimate of unknown parameters \(\theta\). \(\eta\) is a positive parameter. We can chose the control law as follows:

Control laws:
\[
u = \hat{\theta} \dot{\theta};
\]
\[
\tilde{\theta} = -z_1 - k_2 z_2 - f_2(x_1, x_2) - \dot{\theta}^T \phi_2(x_1, x_2) + y_r^{(2)}
\]
\[
+ \frac{\partial \alpha_1}{\partial x_1} (x_2 + f_1(x_1)) + \frac{\partial \alpha_1}{\partial y_2} y_r^{(1)} + \frac{\partial \alpha_1}{\partial \theta} \dot{\theta}^T \phi_1(x_1)
\]
\[
- \sum_{i=1}^{2} \frac{1}{4 \varepsilon_{2,i}} z_2 - \sum_{i=1}^{2} \frac{1}{4 \varepsilon_{2,i}} \frac{\partial \alpha_1}{\partial x_1} \phi_2(x_1, x_2)
\]
\[
- l_2 z_2 x_2^2 + \frac{\partial \alpha_1}{\partial \theta} \tau_2 - \frac{\partial \alpha_1}{\partial \theta} \phi_2(x_1, x_2)
\]
where \(k_2 > 0, \varepsilon_{2,i} > 0, l_2 > 0, \eta > 0, \theta_0 > 0\) are design parameters. \(\tau_2\) is the tuning function and will be given bellow. Then the derivative of \(V\) is
\[
\dot{V} = \dot{V}_1 + z_2 \dot{z}_2 - \frac{|\theta|}{\eta} \dot{\theta}
\]
(18)
Note that
\[
u = \hat{\theta} \dot{\theta};
\]
\[
\tau_2 = b \tilde{\theta} \dot{\theta} = b (\theta - \hat{\theta}) \dot{\theta} = b \theta \dot{\theta} - \hat{\theta} \dot{\theta}
\]
(19)
Then we can get
\[
\dot{z}_2 \dot{z}_2 = z_2 \left[-z_1 - k_2 z_2 - b \tilde{\theta} \dot{\theta} - \dot{\theta}^T \phi_2 - \frac{\partial \alpha_1}{\partial x_1} \phi_1 \right]
\]
\[
- \frac{\partial \alpha_1}{\partial \theta} (\dot{\theta} - \tau_2 + l_0 (\theta - \theta_0)) - \sum_{i=1}^{2} \frac{1}{4 \varepsilon_{2,i}} z_2
\]
\[
- \sum_{i=1}^{2} \frac{1}{4 \varepsilon_{2,i}} \frac{\partial \alpha_1}{\partial x_1} \phi_2(x_1, x_2)
\]
\[
- \frac{\partial \alpha_1}{\partial \theta} \Delta_2(x_1, x_2)
\]
(20)
With
\[
|\dot{z}_2 \Delta_2| \leq \sum_{i=1}^{2} \frac{1}{4 \varepsilon_{2,i}} z_2^2 + (\varepsilon_{2,1} \delta_2 \parallel (x_1, x_2) \parallel_2)^2
\]
\[
+ (\varepsilon_{2,2} d_2)^2
\]
(21)
Then the derivative of \(V\) is
\[
\dot{V} \leq - \sum_{i=1}^{2} k_i z_i^2 - \dot{\theta}^T \Gamma^{-1} (\dot{\theta} - \tau_1 + \Gamma (\theta - \frac{\partial \alpha_1}{\partial x_1} \phi_1) z_2)
\]
Then we can get
\[
\hat{\theta} = \text{proj}(\tau_2 - l_0 \Gamma(\hat{\theta} - \theta_0))
\]
\[
\tau_2 = \tau_1 - \Gamma(\phi_2 - \frac{\partial \alpha_1}{\partial x_1})z_2
\]
\[
\hat{e} = -\eta \text{sign}(b)\hat{u}z_2 - l_e (\hat{e} - e_0)
\]

where \(l_e > 0, e_0 > 0\) are design parameters. \textit{proj}(\cdot) used in here can guarantee \(\hat{\theta}\) being bounded. With update laws given above, we have
\[
\dot{V} \leq - \sum_{i=1}^{2} k_i z_i^2 + \hat{\theta}^T l_0 (\hat{\theta} - \theta_0) - l_1 z_1^2 x_1^2 - l_2 z_2^2 x_2^2
\]
\[+ \Xi \| x \|_2^2 + |b| \hat{e} l_e (\hat{e} - e_0) + \bar{M}
\]

Note that
\[
l_0 \hat{\theta}^T (\hat{\theta} - \theta_0) \leq \frac{1}{2} l_0 ||\hat{\theta}||_2^2 + \frac{1}{2} l_0 ||\theta - \theta_0||_2^2
\]
\[
l_e \hat{e} (\hat{e} - e_0) \leq \frac{1}{2} l_e \hat{e}^2 + \frac{1}{2} l_e (e - e_0)^2
\]

Then we can get
\[
\dot{V} \leq - \sum_{i=1}^{2} k_i z_i^2 - \frac{1}{2} l_0 ||\hat{\theta}||_2^2 - \frac{1}{2} l_0 ||\theta - \theta_0||_2^2
\]
\[+ \Xi \| x \|_2^2 + M
\]

where
\[
M = \bar{M} + \frac{1}{2} l_0 ||\theta - \theta_0||_2^2 + \frac{1}{2} l_e (e - e_0)^2
\]

**Remark 4:** Note that \(l_0 \Gamma(\hat{\theta} - \theta_0)\) and \(l_0 \eta (\hat{e} - e_0)\) introduced in the parameter update laws (26) are employed to ensure system stability as detailed in the analysis given in stability analysis. \(\theta_0\) and \(e_0\) can be seen as the preliminary estimations of \(\theta\) and \(e\), respectively.

### IV. STABILITY ANALYSIS

From (29), we can establish our main result as stated in the following theorem.

**Theorem 1:** Consider the closed loop system consisting of system the system (4), controller (17) and update laws (26). Under the Assumption 1 to Assumption 3, to any \(\rho > 0\), for all \(z_1^2(t) + z_2^2(t) \leq \rho\) there exists a set of gains \(k_1, k_2\) and design parameters \(l_0, l_1, l_2\) such that \(V(t) \leq \rho\) \((\forall t > 0)\).

We will analysis the derivative of \(V\) on the set \(z_1^2(t) + z_2^2(t) = \rho\). In fact, semi-global bounded of control system can be easily obtained if the derivative of Lyapunov function \(V\) is non positive on \(z_1^2(t) + z_2^2(t) = \rho\). The following two cases are considered.

(1) \(z_1^2(t) \leq \sigma\)

With \(z_1 = x_1 - y_r\) and Assumption 3, we can get
\[
(x_1 - y_r)^2 \leq \sigma
\]

Clearly, we have \(x_1\) is bounded. Note that
\[
z_2 \geq \rho - \sigma
\]

Then we can get
\[
-l_2 \hat{z}_2^2 \leq -l_2 (\rho - \sigma) x_2^2
\]

With (29), we have
\[
\dot{V} \leq - \sum_{i=1}^{2} k_i z_i^2 - \frac{1}{2} l_0 ||\hat{\theta}||_2^2 - \frac{1}{2} l_0 \hat{e}^2
\]
\[+ l_2 (\rho - \sigma) x_2^2 - \Xi \| x \|_2^2 + \Xi \| u \|_2^2 + \Xi \| y \|_2^2 + \Xi \| z \|_2^2 + M
\]

From (31), we can get
\[
\|
\end{align}
\]

Then we can get
\[
\dot{V} \leq -\xi \| V \| (\rho - \sigma) x_2^2 + \Xi B_{x_1} + M
\]

where \(\xi = \frac{\xi_1}{\xi_2}\)
\[
\xi_1 = \min\{k_1, l_0, l_e\}
\]
\[
\xi_2 = \max\{\frac{1}{2}, \frac{\lambda_{\max}(\Gamma^{-1})}{2\eta}\}
\]

So we can select design parameters \(k_1, k_2, l_0, l_e, \eta, l_1, l_2\) such that
\[
l_2 (\rho - \sigma) > \Xi
\]
\[
P_\xi \leq \frac{\Xi B_{x_1} + M}{\rho}
\]

Then when \(V \geq \rho\), we have
\[
\dot{V} \leq -\xi \| V \| + \Xi B_{x_1} + M \leq 0
\]

(2) \(\sigma < z_1^2(t) \leq \rho\)
Easily we can get \( x_1 \) is bounded due to \( z_1 \) and \( y_r \) being bounded. In addition we have
\[
\frac{d}{dt} z_1^2(t) = \rho - z_1^2(t) \leq \rho - \sigma
\]  
(40)
So we have \( z_2 \) is bounded. Because \( x_1 \) is bounded and the boundedness of \( \dot{\theta} \) can be guaranteed by \( \text{proj}(-) \). Then \( \alpha_1 \) is bounded. Note that
\[
z_2 = x_2 - \alpha_1 - y_r^{(1)}
\]  
(41)
and with (9), we have \( x_2 \) is bounded too.
\[
z_1^2 > \rho \sigma
\]  
(42)
Then we can get
\[
-l_1 z_1^2 x_1^2 \leq -l_1 (\rho - \sigma) x_1^2
\]  
(43)
With (29), we have
\[
\dot{V} \leq - \sum_{i=1}^{2} k_i z_i^2 \frac{1}{2} l_b |\theta^1|^2 - \frac{1}{2} l_e \tilde{e}^2 - |l_1 | \sigma \xi \xi_1^2 + \Xi \xi_2^2 + M
\]  
(44)
Similar above, we use \( B_{s2} \) denotes the upper bound of \( x_2^2 \). Then we can get
\[
\dot{V} \leq - \xi V - |l_1 | \sigma \xi \xi_1^2 + \Xi B_{s2} + M
\]  
(45)
where
\[
\xi = \frac{\xi_1}{\xi_2}
\]
\[
\xi_1 = \min \left\{ k_1, k_2, \frac{l_b}{2}, \frac{l_e}{2} \right\}
\]
\[
\xi_2 = \max \left\{ \frac{1}{2}, \frac{\lambda_{\text{max}}(\Gamma^{-1})}{2}, \frac{|b|}{2\eta} \right\}
\]  
(46)
So when parameters \( k_1, k_2, l_b, l_e, \epsilon_{i,j}, l_1, l_2 \) are selected such that
\[
l_1 \sigma > \Xi
\]
\[
\xi \geq \frac{\Xi B_{s2} + M}{\rho}
\]  
(47)
Then when \( V \geq \rho \), we have
\[
\dot{V} \leq - \xi V + \Xi B_{s2} + M \leq 0
\]  
(48)
From the above two case, it is clear that the following inequality holds
\[
\dot{V} \leq - \xi V + \Xi B_{s2} + M \leq 0 \quad (B_{s} = \max B_{s1}, B_{s2})
\]  
(49)
when parameters \( k_1, k_2, l_b, l_e, \epsilon_{i,j}, l_1, l_2 \) are selected such that
\[
l_1 \sigma > \Xi
\]
\[
l_2 (\rho - \sigma) > \Xi
\]
\[
\xi \geq \max \left\{ \frac{\Xi B_{s1} + M}{\rho}, \frac{\Xi B_{s2} + M}{\rho} \right\}
\]  
(50)
Then we can get that if \( \frac{d}{dt} z_1^2(0) + z_2^2(0) \leq \rho \) and design parameters selected such that (50), all the signals of the closed-loop system are uniformly bounded.

Remark 5: Compared with the traditional semi global stability analysis which requires \( V(0) \leq \rho \), the proposed control scheme makes the semi global stability of system (4) to be hold only under the condition \( z_1^2(0) + z_2^2(0) \leq \rho \).

\[\text{FIGURE 1. Tracking.}\]
\[\text{FIGURE 2. Input u.}\]

V. SIMULATION STUDIES

In this section, the results of simulation are presented to verify the effectiveness of the proposed adaptive control law. Consider the following second-order system:
\[
\dot{x}_1 = x_2 + \cos x_1 + \Delta_1
\]
\[
\dot{x}_2 = u + \cos x_2 \sin^2 x_1 + \theta \cos x_2 + \Delta_2
\]  
(51)
where \( x_1 \) and \( x_2 \) are system states. \( u \) is input and \( y = x_1 \) is output signal. We set the value of unknown parameter \( \theta \) being 2. \( \Delta_i \) represents the non-triangular uncertainties and is given below:
\[
\Delta_1 = 0.1\sin(x_1^2 + x_2^2); \quad \Delta_2 = 0.2(x_1^2 + x_2^2)^{1/2}\cos x_1
\]

The design parameters are selected as \( k_1 = k_2 = 10, \epsilon_{t,1} = \epsilon_{t,2} = \epsilon_{t,2.1} = \epsilon_{t,2.2} = 1, l_1 = l_2 = 1, \theta_0 = 1, l_b = 0.1, \Gamma = 1 \). The initial values are chosen to be \( x_1(0) = 0.5, x_2(0) = 0, u(0) = 0 \). We make simulation to two different reference signals \( y_r = \sin 2t \) and \( y_r = 1 - e^{-t} \).

(1) When reference signal is taken as \( y_r = \sin 2t \).

The simulation results are shown in Fig.1 and Fig.2. Fig.1 shows the tracking performance including output,
reference signals and tracking errors. Fig.2 is the input signal. We can easily obtain that system is stable and the tracking performance can be realized perfectly under the proposed controller in this paper.

(2) When reference signal is taken as $y_r = 1 - e^{-t}$.

In order to further verify the effectiveness of the proposed adaptive control scheme shown in (17) and (26), another reference signal $y_r = 1 - e^{-t}$ and simulation results are given in Fig.3 and Fig.4. It is clear that stability and tracking performance of closed loop system can be achieved successfully.

VI. CONCLUSION

An adaptive control scheme is proposed by using backstepping techniques for a class of second order nonlinear systems with non-triangular uncertainties. The boundedness in the meaning of semi global stabilization of all signals can be guaranteed by the proposed control law and corresponding update laws. Finally, simulation studies also verify the effectiveness of this adaptive control scheme. A possible future perspective is to investigate the control scheme including state feedback and output feedback control laws for $n$-order nonlinear systems with non-triangular uncertainties.

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