I. INTRODUCTION

The phenomenon of localization of electronic states in disordered mesoscopic conductors, first predicted by P. W. Anderson [1], has attracted a lot of theoretical and experimental interest for several decades [2, 3]. This phenomenon arises due to quantum interference caused by the multiple elastic scattering events of the electrons in their motion along the sample, with randomly distributed impurities within the conductor [4]. In the presence of strong disorder all electronic states are known to be exponentially localized and the conductor behaves as an insulator. Furthermore, as a function of parameters like disorder strength, electric or magnetic fields, the sample can behave both as an insulator, localized phase, or as a conductor, delocalized phase. In addition, the sample can undergo a disorder-induced localized-delocalized transition. This transition is usually referred to as Anderson or metal-insulator transition (MIT). The understanding of the various phenomena that emerge at this transition has been the subject of an intense research activity in the last decades [2, 5–7] (for a recent review see also the Ref. [8] and the references therein). A very important characteristic feature of this critical point is that not only the electronic states, but also the spectra show unusual behavior. For the eigenstates it is reflected in multifractal behavior and strong amplitude fluctuations. These are usually described by an infinite set of critical exponents [4, 5, 9].

At the MIT not only the dimensionality, but also the symmetries present in the system play an important role.

For ordinary disordered samples, the random-matrix theory (RMT) has proved to be an effective tool in describing their statistical properties [10]. Dyson introduced three universal symmetry classes: the orthogonal class consisting of systems in the presence of time reversal invariance (TRI) and integer spin or TRI, half-integer spin, and rotational symmetry; the unitary class describing systems with broken time-reversal invariance; and the symplectic class for systems in the presence of TRI, half-integer spin, and no rotational symmetry. In the Dyson scheme these are labeled by the symmetry indices \( \beta = 1, 2, \) and 4, for the orthogonal, unitary, and symplectic class; respectively [11–13].

Up to now, the analysis and theoretical description of the multifractal properties of disordered systems at the MIT have been of great interest [4, 5, 8, 14–17]. However, because of the complexity in obtaining analytical expressions at this critical point, some of which are available only perturbatively, many investigations have mainly been focused on numerical analysis. In particular a widely used model that has attracted a lot of attention is the so-called power-law banded random matrix (PBRM) model [8, 18–20]. This is due to the fact that it captures all the key features of the Anderson critical point and is also well suited for numerical calculations.

More recently [21, 22], this model has been used to verify the validity of existing heuristic relations, established between the multifractal properties of eigenstates and their spectra at criticality. These relations have been proved to correctly describe the multifractal behavior of critical states for the PBRM model in the presence (\( \beta = 1 \)) [21] and absence (\( \beta = 2 \)) [22] of time reversal invariance, in a relatively wide range of the model parameters. Furthermore, it also accounts for the multifractal properties of other models showing critical behavior [21, 22]. However, the PBRM model corresponding to the third of the three classical Wigner-Dyson ensembles,
i.e., the symplectic case, has been left out. This model is of particular interest since it has been observed, using two-dimensional tight-binding models, the appearance of a localized-delocalized transition in systems that belong to the symplectic symmetry [8, 23–28].

Our purpose in this paper is to introduce the PBRM model for the symplectic class, i.e., for time reversal symmetric systems in the presence of spin-orbit interactions. For the sake of completeness, we also review the corresponding multifractal and statistical properties of systems in the presence and absence of time reversal invariance so that we can provide a full picture of the PBRM model for the three classical Wigner-Dyson ensembles. We show that the heuristic relations, proposed earlier [21, 22], for multifractal properties are also valid for the symplectic case, in certain range of the model parameters.

The paper is organized as follows. In next Section the PBRM model in the presence of the three Wigner-Dyson symmetries are described. There, we review the PBRM model for $\beta = 1$ and 2, and introduce the model for the symplectic class, i.e., the one with $\beta = 4$. In Sections III and III A we present the heuristic relations for the eigenstates statistics of the PBRM at criticality. The numerical results for the eigenstates statistics are presented in Section III B. The spectral analysis is the subject of Section IV and in Section IV C we discuss the results. Finally, we present our conclusions in Section V.

II. THE PBRM MODEL FOR THE WIGNER-DYSON ENSEMBLES

We start our study by defining the power-law banded random matrix (PBRM) model in the presence of three symmetry classes defined by the classical Wigner-Dyson ensembles. The PBRM model describes one-dimensional (1d) samples with random long-range hoppings. This model is represented by $N \times N$ real symmetric ($\beta = 1$), complex Hermitian ($\beta = 2$), or $2N \times 2N$ real quaternion Hermitian ($\beta = 4$) matrices whose elements are statistically independent random variables drawn from a normal distribution with zero mean and variance given by

$$\langle |H_{mm}|^2 \rangle = \beta^{-1}$$

$$\langle |H_{mn}|^2 \rangle = \frac{1}{2(1 + \delta_{m,n})} \frac{1}{1 + \left[ \sin \left( \frac{\pi |m-n|}{N} \right) / (\frac{\pi}{N}) \right]^{2\mu}}.$$ (1)

where $b$ and $\mu$ are the model parameters. The model of Eq. (1) is in its periodic version; i.e., the 1d sample is in a ring geometry. In this paper, the PBRM model for the symplectic case ($\beta = 4$) is introduced in the same line as the one originally proposed by Mirlin et.al. for $\beta = 1$ [18]. That is, a random matrix ensemble of the symplectic class with off-diagonal matrix elements decaying away from the diagonal in a power-law fashion with zero mean and variance given by Eq. (1). It is worth mentioning that such a model also preserves the quaternion structure of the Hamiltonian where each eigenvalue is two-fold degenerate due to Kramers degeneracy.

It has been proved [8, 18, 20, 29–33] that the PBRM model undergoes a transition at $\mu = 1$ from localized states for $\mu > 1$ to delocalized states for $\mu < 1$. This transition shows key features of the disorder-driven Anderson metal-insulator transition [8, 21, 22, 34–38], including multifractality of eigenstates and nontrivial spectral statistics. Thus the PBRM model possesses a line of critical points $b \in (0, \infty)$ at $\mu = 1$. By tuning the parameter $b$ from $b \ll 1$ to $b \gg 1$, the states cross over from strong multifractality which corresponds to localized-like, or insulator-like states, to weak multifractality showing rather extended, i.e., metallic-like states [8, 29]. Furthermore, at the true Anderson transition in $d = 3$ or at the integer quantum-Hall transition in $d = 2$, the states belong to the weakly multifractal regime, then the PBRM model allows for an investigation without such a limitation. In this paper we will focus on the PBRM model in the presence of the three symmetry classes labeled by $\beta = 1, 2,$ and $4$, at the critical point $\mu = 1$.

III. HEURISTIC RELATIONS

Recently, heuristic relations between the multifractal properties of eigenstates and their spectra at criticality have been proposed and verified for the PBRM model with $\beta = 1$ and 2 [21, 22]. Here, we review these relations and show that they also account for the multifractal properties and the spectra of the PBRM model in the presence of the symplectic symmetry ($\beta = 4$), proposed in Eq. (1).

A. Multifractality of electronic states

It is widely known that the spatial fluctuations of electronic states in disordered conductors at the Anderson transition show multifractal behavior [4, 5, 8, 39–41]. These fluctuations can be described by a set of multifractal dimensions $D_q$ defined by the scaling of the inverse mean eigenfunction participation numbers with the system size $N$:

$$\left\langle \sum_{i=1}^{N} |\Psi_i|^2 q \right\rangle \sim N^{-(q-1)D_q},$$ (2)

where $q$ is a parameter and $\left\langle \cdot \cdot \cdot \right\rangle$ stands for the average over some eigenstates within an eigenvalue window and over the ensemble. Notice that, for $q = 1$ Eq. (2) is not well defined. However, in the limit $q \rightarrow 1$ the so-called information dimension, $D_1$, is related to the information entropy of the eigenstates as

$$\left\langle -\sum_{i=1}^{N} |\Psi_i|^2 \ln |\Psi_i|^2 \right\rangle \sim D_1 \ln N.$$ (3)
A heuristic relation for the multifractal dimension is given by
\[ D_q \approx \left[ 1 + (\alpha_q b)^{-1} \right]^{-1} \tag{4} \]
where \( \alpha_q \) is a fitting constant. It has been shown to be valid in a wide range of the parameters \( q \) and \( b \) for the PBRM model of Eq. (1), in both the presence (\( \beta = 1 \)) [21] and absence (\( \beta = 2 \)) [22] of time reversal invariance.

In addition, an often employed quantity to characterize the spectral fluctuations is the level compressibility \( \chi \). In the metallic regime, where the states are extended, \( \chi \to 0 \); while in a strongly disordered conductor the levels are uncorrelated and lead to \( \chi \to 1 \). Furthermore, at criticality an intermediate statistics exists, \( 0 < \chi < 1 \), where the spectral and eigenstate statistics are supposed to be coupled [42, 43]. Analytical expressions, for \( \beta = 1 \) and 2, that describes qualitatively well the level compressibility, \( \chi \), in the small- and large-\( b \) limits are given by [8, 44, 45]
\[ \chi = \begin{cases} 
1 - 4b, & \beta = 1, \ b \ll 1, \\
1 - \pi\sqrt{2}b + \frac{1}{8}(2 - \sqrt{3})\pi^2b^2, & \beta = 2, \ b \ll 1, \\
\frac{1}{2\pi b}, & b \gg 1,
\end{cases} \tag{5} \]
which can be written in terms of the multifractal dimensions, \( D_q \), as [22]
\[ \chi \approx \frac{1 - D_q}{1 + (q - 1)D_q}. \tag{6} \]

Moreover, the multifractal dimensions, \( D_q \), can also be related to the information dimension, \( D_1 \), as [22]:
\[ D_q \approx \frac{D_1}{q + (1 - q)D_1}. \tag{7} \]

It is worth mentioning that Eqs. (4), (6), and (7) are valid only for \( q > 1/2 \). However, for the case in which \( q < 1/2 \) \( D_q \) and \( D_1 \) are related as [22]
\[ D_q \approx \frac{1 - 2q}{1 - q} + \frac{q}{1 - q} \frac{D_1}{1 + q(D_1 - 1)}. \tag{8} \]
Thus, from Eqs. (7) and (8) it is possible to explore the whole range of \( q \).

**B. Numerical results for multifractality**

We now verify the heuristic relations presented so far. For the numerical analysis of the eigenstates we use system sizes of \( N = 2^n \), with \( 8 \leq n \leq 13 \). The averages are taken over 12.5\% of the eigenvectors within an eigenvalue window around the band center of the spectrum, and over ensembles of sizes \( M = 2^{16-n} \). This guarantees that the statistics is fixed, i.e., the product \( N \times M = 2^{16} \) for any system and ensemble size. The reported error bars are the reduced root mean square (rms) of residuals between the numerical data and the analytical predictions.

In Fig. 1, panels (a)-(c), we show the logarithm of the mean generalized inverse participation numbers of Eq. (2) as a function of the logarithm of the system size, for several values of \( q \) (see inset). In all these cases the parameter \( b \) is fixed to one. We observe that Eq. (2) is in complete agreement with the numerical results from the PBRM model in the presence of the three symmetry classes shown in panels (a), (b), and (c), for \( \beta = 1, 2, \) and 4, respectively. Also, from these results we can extract the multifractal dimensions, \( D_q \), by performing a linear fitting to the numerical data, as is shown in dashed lines of the same figures. It is worth mentioning that for the special case \( q = 1 \), we used Eq. (3).

From these results, we are now able to calculate the multifractal dimensions \( D_q \). In Fig. 1, panels (d)-(f), we show \( D_q \) as a function of \( b \) for the same values of \( q \) considered previously. There, the dashed lines correspond to fittings to the heuristic relation given by Eq. (4). We observe that Eq. (4) is in agreement with the PBRM model with \( \beta = 1 \) and 2 [see panels (d) and (e)]. For the \( \beta = 4 \) case, we observe some deviation from Eq. (4) specially for values of \( b \) between 0.04 < \( b < 2 \) where \( D_q \) grows faster than expected. However, in the limiting cases \( b \ll 1 \) (insulator phase) and \( b \gg 1 \) (metallic phase) the behavior of \( D_q \) is well described by Eq. (4). These results have already been reported for the cases of \( \beta = 1 \) [21] and \( \beta = 2 \) [22].

The coefficients \( \alpha_q \) [see Eq. (4)], as a function of \( q \) are displayed in panels (g)-(i) of Fig. 1, as black-triangles. The cases \( \beta = 1 \) (g) and 2 (h) have been reported in Refs. [21] and [22], respectively. The \( \beta = 4 \) case is shown in panel (i). There, we observe an almost linear decay as \( q \) increases with values that lie between the cases \( \beta = 1 \) and \( \beta = 2 \). Also, in panels (g)-(i) of the same figure, we show the quantity \( \gamma_q = \alpha_1/\alpha_q \) as a function of \( q \) in blue-inverted-triangles. The red-dashed lines correspond to \( \gamma_q = q \). This is an interesting result since in the region \( 0 < q < 2.5 \) we can calculate these coefficients as \( \alpha_q \approx \alpha_1/q \) with high accuracy for the PBRM model in the presence of the three symmetry classes \( \beta = 1, 2, \) and 4, and then obtain very simplified recursive relations for several interesting quantities for instance the multifractal dimension \( D_q \) as given in Eq. (4).

In Fig. 2, panels (a)-(c), we show the level compressibility \( \chi \) as a function of \( b \), for several values of \( q \) as indicated in the inset. The blue-dotted line in panel (a) and the red-dashed one in panel (b) correspond to the analytical expression of Eq. (5), where for the sake of clarity we have added the subindex \( \beta = 1 \) and \( \beta = 2 \) according to the case considered. For comparison purposes, we have included both analytical expressions for \( \chi \) for the cases \( \beta = 1 \) (blue-dotted line) and \( \beta = 2 \) (red-dashed line), together with the numerical result for the symplectic case (dots) in panel (c). There, we can see that when \( b < 0.1 \) both curves, for \( \beta = 1 \) and 2, show the same behavior, which is different in the symplectic case (dots). Furthermore, when \( b > 1 \) the symplectic case behaves quite similar to \( \beta = 2 \), and when \( b > 10 \) the three models show the
same behavior, which is expected since they have reached the metallic phase. An interesting region is in the interval between $0.04 < b < 0.4$ where the three models show a different behavior. This matches with the region where $D_q$ changes fast [see Fig. 1, panels (d)-(f)]. In insets of panels (a)-(c) of Fig. 2 we show $q D_q (1 - D_q)^{-1}$ (dots) as a function of $b$. The red-dashed lines correspond to $\alpha_1 b$. From those figures we can see that $q D_q (1 - D_q)^{-1} \approx \alpha_1 b$ and therefore it does not depend on $q$. This explains why all numerical data shown in panels (a)-(c) of Fig. 2 fall almost on the same point for a given $b$.

Finally, we present the results for the multifractal dimension, $D_q$, for values $q < 1/2$ and $q > 1/2$ for different values of the parameter $b$. These are shown in panels (d)-(f) of Fig. 2 for the three symmetry classes $\beta = 1, 2, $ and 4, respectively. The red lines correspond to expression (8) while the black ones correspond to relation (7). Again, the results for $D_1$ are obtained from Eq. (3). As can be seen the results show a good agreement between the numerics and the theoretical expressions for the PBRM model, confirming the validity of those heuristic relations for the three symmetry classes here analyzed.

IV. SPECTRAL STATISTICS AT CRITICALITY

In the previous sections we have analyzed the eigenvector statistics of the PBRM model at the critical point. Such study was done through the multifractal dimension and the level compressibility. Another effective tool to distinguish between localized and extended states is given by the nearest level spacing distribution. This distribution also accounts for the symmetry class present in the system. Since the PBRM model allows a transition from
localized to extended states by tuning the parameter $b$, in this section we shall analyze the spectral statistics of the PBRM model at criticality as a function of $b$.

### A. The nearest-neighbor level spacing distribution

The spectrum of a disordered system in the localized regime (insulator phase) is uncorrelated, and follows a Poisson nearest-neighbor level spacing distribution [46]

$$ P_P(s) = \exp(-s), \quad (9) $$

where $s = |E_{i+1} - E_i|/\Delta$ being $E_i$ and $\Delta$ the eigenenergies and the mean level spacing, respectively. In contrast, the spectrum of a disordered system in the delocalized regime (metallic phase) follows a nearest-neighbor level spacing distribution of one of the three Wigner-Dyson distributions [47]

$$ P_{WD}(s) = \begin{cases} \frac{\pi}{2} s \exp\left(-\frac{\pi}{4} s^2\right), & \beta = 1, \\
\frac{32}{7\pi^2} s^2 \exp\left(-\frac{4}{7\pi} s^2\right), & \beta = 2, \\
\frac{218}{3\pi^3} s^4 \exp\left(-\frac{64}{9\pi^2} s^2\right), & \beta = 4,
\end{cases} \quad (10) $$

depending of the symmetry present in the system labeled by the Dyson symmetry index $\beta$. Now, the PBRM model allows us to study the signatures of the metal-insulator phase transition in its spectra. However, the spectrum of the PBRM model follows a more general distribution $P_c(s)$ which is neither Poissonian nor Wigner-Dyson, but in the limiting cases it agrees with equation (9) when $b \to 0$ and with equation (10) when $b \to \infty$. Since the spectral statistics shows a different behavior for small and large differences in energy, $s$, we divide our spectral analysis into two parts: the first one for $s \gg 1$ and the other one for $s \ll 1$.

In the case of large energy differences, $s \gg 1$, it has been derived analytically [48–50] and numerically proved [51] for $\beta = 1$ that $P_c(s)$ has the following asymptotic form

$$ P_c(s) \sim \exp\left(-As^\alpha\right), \quad s \gg 1, \quad (11) $$

where $A$ is a coefficient that depends only on the dimensionality of the system ($A \approx 1$ for $d = 1$ systems), and $\alpha$ is the critical exponent. For $\beta = 1$, $\alpha$ ranges in the interval $1 < \alpha < 2$ and can be fitted according to [51]

$$ \alpha = \begin{cases} 2 - a/b, & b \gg 1, \\
1 + cb, & b \ll 1.
\end{cases} \quad (12) $$
with $a$ and $c$ constants. In order to diminish the magnitude of the relative fluctuations here we consider not the nearest-neighbor level spacing distribution directly, but the cumulative level spacing distribution function

$$I(s) = \int_s^\infty P(s') ds'. \quad (13)$$

Meanwhile, in the case of small energy differences, $s \ll 1$, the nearest-neighbor level spacing distribution $P_c(s)$ behaves as [11]

$$P_c(s) \sim Cs^\beta, \quad s \ll 1 \quad (14)$$

where $C$ is a constant to be determined and $\beta$ is the Dyson symmetry index.

**B. The ratio of consecutive level spacings distribution**

The main disadvantage in calculating the nearest-neighbor level spacing distribution, Eq. (10), is the need to perform the unfolding procedure, which in some cases is not possible [10], for example in many-body problems. To overcome these difficulties new quantities have been proposed. For instance, for an ordered spectrum $\{e_n\}$ from a random matrix the nearest-neighbor spacing is $s_n = (e_{n+1} - e_n)$ and the ratio of consecutive level spacings is defined by [52]

$$\hat{r}_n = \frac{\min(s_n, s_{n-1})}{\max(s_n, s_{n-1})} = \min \left( r_n, \frac{1}{r_n} \right), \quad (15)$$

**FIG. 3.** (Color online) Spectral statistics of the PBRM model for $\beta = 1$ in left column, panels (a), (d), and (g); for $\beta = 2$ in middle column, panels (b), (e), and (h); and for $\beta = 4$ in right column, panels (c), (f), and (i). See text, for a detailed description.
In what follows, we present our numerical results for the spectral statistics for the PBRM model in the presence of the three symmetry classes, $\beta = 1, 2, \text{and} 4$.

### C. Numerical results for the spectral statistics

In this section, we present the numerical results concerned to the different aspects of the spectrum of the PBRM model in the presence of the three symmetry classes $\beta = 1, 2, \text{and} 4$, at the metal-insulator transition as defined in Eq. (1). For our numerical simulations we consider system sizes of $N = 2^n$, with $n = 6$ (red-diamonds), 7 (green-triangles), and 8 (blue-circles). The ensemble sizes $M$ are chosen such that the product $N \times M$ remains fixed to $5.12 \times 10^7$. Also, we consider eigenvalue windows within the 12.5% around the center of the spectrum. As before, the error bars in the different panels are the rms between the theoretical predictions and the numerical data.

In Fig. 3, panels (a)-(c), are plotted the minus logarithm of the integrated probability, Eq. (13), for the
PBRM model with $\beta = 1, 2$, and 4, respectively. In those panels we show the results for the following values of parameter $b$: 0.02, 0.1, 0.2, 0.4, 1, and 4, from bottom to top. The dashed lines correspond to the Poisson case, while the dotted lines are the corresponding Wigner-Dyson ensembles. The solid lines are fittings to Eq. (11). The result for $\beta = 1$ is in agreement with that reported in [51], while the results for the cases $\beta = 2$ and $\beta = 4$ have not been reported so far. Here, we verified that for both cases, $\beta = 2$ and 4, the coefficient $A \approx 1$ between the expected statistical error, and the fittings are done in the range of $s$ shown in each one of those panels, i.e., for instance, if $b < 0.2$ we have $2.5 < s < 5$ for $\beta = 2$, and $2 < s < 5$ for $\beta = 4$, and so on. With these results we confirm the asymptotic behavior of $P_c(s)$ predicted in Eq. (11).

The critical exponent, $\alpha$, as a function of $b$ [see Eq. (11)] is plotted in panels (d)-(f) of Fig. 3. The results for the $\beta = 1$ case, panel (d), are in agreement with those reported in [51]. In the same panel, the line corresponds to the fitting of Eq. (12) to the numerical data. For the cases $\beta = 2$ [Fig. 3 (e)] and $\beta = 4$ [Fig. 3 (f)] Eq. (12) gives an accurate description of the numerical data, for $b \ll 1$. However, for $b \gg 1$ some deviations appear. For $\beta = 1$ we obtain $\alpha \rightarrow 2.2$, while $\alpha \rightarrow 2.6$ for $\beta = 4$. This last result falls into the critical exponent predicted by using different $2d$ models in the presence of the symplectic symmetry [24, 55].

In the panels (g)-(i) of Fig. 3 we show the distribution $P_c(s)$ of Eq. (14), for $s \ll 1$. There, we present the results for three representative data set with $b = 0.04$, 0.1, and 4; from left to right. The dashed lines correspond to the Poisson distribution, Eq. (9), while the dotted ones are the Wigner-Dyson distribution, Eq. (10). The straight line segments are fittings between the numerical data to Eq. (14). In [51] the validity of Eq. (14), for $\beta = 1$, was proved. In the panel (g) we confirm that result. Additionally, here we proof and extend the validity of Eq. (14) to the cases $\beta = 2$ [panel (h)] and $\beta = 4$ [panel (i)], where a good correspondence between the numerics and Eq. (14) is obtained.

Now, we would like to apply the ideas presented in subsection IV B to our PBRM model. First, we calculate the average $\langle \tilde{r} \rangle$, Eq. (15), as a function of $b$. The results are displayed in Fig. 4, panels (a)-(c), for $\beta = 1, 2$, and 4, respectively. In those panels, the horizontal dashed lines correspond to the theoretical prediction of $\langle \tilde{r} \rangle$ for the Poisson case, while the horizontal dotted lines correspond to the respective theoretical values for the corresponding Wigner-Dyson ensembles [53]. We observe a transition from the Poisson case to the Wigner-Dyson cases as a function of $b$, as expected. Also, we can see that the localized regime is reached when $b \approx 0.002$, while the extended regime is reached when $b \approx 1$.

In Fig. 4, panels (d)-(f), are shown the PDF of $r$, Eq. (17), and that of $\tilde{r}$ in the insets. The dashed lines are the integrable cases [see Eqs. (19) and (20)], while the dotted ones correspond to the Wigner-Dyson like expression of Eq. (17). In all these panels, we can see the transition from the insulator to metallic phase, that is, when $b = 0.04$ the points are closer to dashed line while if $b = 4$ the dots are over the dotted line, additionally we are including an intermediate case with $b = 0.1$. It is clear that in the limit $r \to 0$, $P(r) \sim C r^\beta$. It is also clear that $P(r)$ as well as $P(\tilde{r})$ allow us to analyze the metal-insulator phase transition in the same way as can be done by using $P(s)$, as expected. However, the former has the advantage of being easier to compute and can be compared directly with data.

V. CONCLUSIONS

We have introduced a power-law banded random matrix model for the third of the three classical Wigner-Dyson symmetry classes. A detailed analysis of its eigenvectors and eigenenergies was presented. This model describes time reversal symmetric systems in the presence of strong spin-orbit interactions and shows key features of the driven Anderson metal-insulator transition. We showed that existing heuristic relations used to describe the multifractal properties of the PBRM model in the presence and absence of time reversal symmetry, i.e., the ones with $\beta = 1$ and 2, respectively, also accounts for the PBRM model with $\beta = 4$, for some ranges of the model parameters. From the statistical analysis, we showed that our results are in complete agreement with the corresponding analytical expressions and we also obtained a critical exponent that agrees with that previously reported, but using a different system with symplectic symmetry in two-dimensions. In order to present a complete picture of the PBRM model for the three classical Wigner-Dyson symmetry classes, we also reproduced previously reported results, for $\beta = 1$ and $\beta = 2$, and also derived some other results that have not been shown so far for these symmetry classes.

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