Abelian monopole condensation in lattice gauge theories

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We investigate the dynamics of lattice gauge theories in an Abelian monopole background field. By means of the gauge-invariant lattice Schrödinger functional we study the Abelian monopole condensation in U(1) lattice gauge theory at zero temperature and in SU(3) lattice gauge theory at finite temperature.

1. Introduction

The dual superconductivity mechanism to explain color confinement has been suggested since the early day of QCD [1]. The first evidences for the dual superconductivity was obtained by studying the dual Meissner effect [2]. More recently an alternative method to detect the dual superconductivity has been proposed by the Pisa Group [3]: it consists in measuring a disorder parameter given in terms of an operator with non zero magnetic charge and nonvanishing v.e.v. in the confined phase. In the case of non Abelian Gauge theories they need to perform the Abelian projection. Indeed the Pisa Group found evidence of Abelian monopole condensation in several gauges: plaquette gauge, butterfly gauge and Polyakov gauge [4].

The aim of this work is to investigate the dynamics of lattice gauge theories in an Abelian monopole background field in a gauge-invariant way. We use the gauge-invariant effective action for external background field defined by means of the lattice Schrödinger functional [5,6]

\[ \Gamma [\vec{A}^\text{ext}] = - \frac{1}{L_4} \ln \left\{ \frac{Z[\vec{U}^\text{ext}]}{Z[0]} \right\} \]

where \( Z[0] \) is the lattice Schrödinger functional without external background field (i.e. \( U^\text{ext}_\mu = 1 \)). Note that due to the manifest gauge invariance of the lattice background field effective action we do not need to fix the gauge.

2. U(1)

We are interested in the effective action with a Dirac magnetic monopole background field. In the continuum the Dirac magnetic monopole field with the Dirac string in the direction \( \vec{n} \) is:

\[ e\vec{b}(\vec{r}) = \frac{n_{\text{mon}}}{2} \frac{\vec{r} \times \vec{n}}{r(\vec{r} - \vec{r} \cdot \vec{n})} \]

where, according to the Dirac quantization condition, \( n_{\text{mon}} \) is an integer and \( e \) is the electric charge (magnetic charge = \( n_{\text{mon}}/2e \)). We consider the gauge-invariant background field action Eq. (2) where the external background field is given by the lattice version of the Dirac magnetic monopole field. In the numerical simulations we put the lattice Dirac monopole at the center of the time slice \( x_4 = 0 \). To avoid the singularity due to the Dirac string we locate the monopole between two neighbouring sites. We have checked that the results are not too sensitive to the precise position of the magnetic monopole. We introduce the disorder parameter for confinement:

\[ \mu = e^{-E_{\text{mon}}L_4} = \frac{Z[\text{mon}]}{Z[0]} \]
The monopole energy tends to the classical monopole action which behaves linearly in $\beta$. In order to obtain $\mu$ we perform the numerical integration of $E_{\text{mon}}'$:

$$E_{\text{mon}} = \int_0^\beta E_{\text{mon}}' d\beta'$$

We found that the disorder parameter $\mu$ is different from zero in the confined phase (i.e. the monopoles condense in the vacuum). Moreover $\mu \to 0$ when $\beta \to \beta_c$ in the thermodynamic limit (the precise determination of $\beta_c$ require a F.S.S. analysis). Our result is gauge-invariant for the manifest gauge invariance of the Schrödinger functional.

3. SU(3)

We have studied the Abelian monopole condensation in pure SU(2) lattice gauge theory at finite temperature. Here we restrict ourselves to the more interesting case of SU(3) gauge theory. In this case the maximal Abelian group is $U(1) \times U(1)$. Therefore we have two different types of Abelian monopole. Let us consider, firstly, the Abelian monopole field

$$g^{\delta a,3} = \frac{\delta a,3 \rho_{\text{mon}}}{2} \frac{\vec{r} \times \vec{n}}{r(\vec{r} - \vec{r} \cdot \vec{n})}$$

which we call the $T_3$-Abelian monopole. Now the functional integration constraint amounts on the lattice to fix the links belonging to the time slice $x_4 = 0$. In the present case the disorder parameter is defined as:

$$\mu = e^{-F_{\text{mon}}/T} = \frac{Z[\text{mon}]}{Z[0]}$$

where $T = 1/L_t$ is the temperature and $F_{\text{mon}}$ is the free energy per monopole. We measure $F_{\text{mon}} = \partial F_{\text{mon}}/\partial \beta$. Again this corresponds to measuring the difference between the average plaquette without and with the monopole field.

From Fig. 2 we see that in the thermodynamic limit the disorder parameter $\mu \sim 1$ in the confined phase, moreover $\mu \to 0$ when $\beta \to \beta_{c,\infty} = 5.6925(2)$ in the infinite volume limit.

The second type of Abelian monopole field is obtained from Eq. (6) replacing $\delta^{a,3}$ with $\delta^{a,8}$. A previous study [4] finds out that the disorder parameters for the two independent Abelian monopole defined by means of the Polyakov projection coincide with the usual disorder parameter $\mu$.
within errors. On the contrary, our numerical results show a dramatic difference for $F'_{\text{mon}}$. The peak of $F'_{\text{mon}}$ in the case of the $T_8$-Abelian monopole is about an order of magnitude greater than in the $T_3$-Abelian monopole case (see Fig. 3). Consequently, in the former case, the disorder parameter $\mu$ tends to zero more sharply.

4. Conclusions

We have studied the Abelian monopole condensation both in the Abelian gauge theory U(1) and finite temperature non Abelian gauge theories SU(2) and SU(3). We introduce a disorder parameter which signals the Abelian monopole condensation in the confined phase. Our definition of the disorder parameter is by construction gauge invariant. Our numerical results suggest that the disorder parameter is different from zero in the confined phase and tends to zero when the gauge coupling $\beta \to \beta_c$ in the thermodynamic limit. Our estimate of the critical couplings are in fair agreement with the ones in the literature. The precise determination of the critical couplings and the critical exponents in the infinite volume limit could be obtained by means of the finite size scaling analysis. In the case of the SU(3) gauge theory, there are two independent Abelian monopole fields related to the two diagonal generators of the gauge group. Remarkably we find that the non perturbative vacuum reacts strongly in the case of $T_8$-Abelian monopole. This seems to suggest that the vacuum monopole condensate is predominantly formed by $T_8$-Abelian monopoles.

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