Influence of relaxation on propagation, storage and retrieving of light pulses in electromagnetically induced transparency medium

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By solving the self-consistent system of Maxwell and density matrix equations to the first order with respect to nonadiabaticity, we obtain an analytical solution for the probe pulse propagation. The conditions for efficient storage of light are analyzed. The necessary conditions for optical propagation distance has been obtained.

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I. INTRODUCTION

Recent advances in quantum information science have shown that, on one hand, photons are ideal carriers of quantum information, and on the other hand, atoms represent reliable and long-lived storage and processing units. In recent years quantum light storage is one of the extensively studied tasks of quantum optics. Basic idea of light storage is electromagnetically induced transparency (EIT) \[1\].

Electromagnetically induced transparency is a coherent interaction process in which a coupling laser field is used to make the optical dense media transparent for the probe field. Since its discovery, a number of new effects and techniques for light-matter interaction have appeared \[2-6\]. Most notably, from the point of view of the work presented here, particular attention has been devoted to ultraslow light propagation and light storage techniques \[3-5\].

The key concept of EIT is the dark state and population trapping \[7\]. The dark state is a specific coherent superposition state which does not contain excited short-living atomic level due to destructive interference between two interaction paths. The dark-state is eigenstate of the light – atom interaction Hamiltonian, so the atom prepared in a dark-state can not be excited and cannot leave the dark-state if the interaction is adiabatic (Fig.1). The population trapping via applying strong coupling leads to the adiabatic formation of the dark-state. Since the interaction is realized by the light pulses the influence of nonadiabatic corrections may become important \[8, 9\]. This influence has been studied e.g. in \[8\]. In particular, the first nonadiabatic correction connects the dark state and the bright state, so the depletion of the bright state, because of optical pumping, can affect the dark state. Despite of large amount of experimental and theoretical papers concerning light storage (see \[2 and citations there\), and applications in quantum information science, the influence of decoherence level width on information carried by stored light is studied insufficiently.

In this work we present theoretical study which discusses and explains influence of all relaxations on probe propagation both analytically and numerically (it is essential in especially, solid state systems \[12\]). The goal is to study comprehensively how the depletion of bright state will affect the pulse propagation in an EIT media and light storage in particular. By solving the coupled system of Maxwell and density matrix equations to the first order of the nonstationary perturbation theory with respect to nonadiabaticity and decoherence we obtain analytical solution which completely describes the probe pulse propagation and is consistent with the recent light storage experiments.

The paper is organized as follows. In section II the basic equations are written down and the probe pulse propagation equation is derived and analyzed. In section III and Appendixes the analytical solution for counterraditive pulse switching order and for matched pulses are obtained and the asymptotic solutions discussed. Section IV deals with the physical consequences of the obtained solution, namely the necessary conditions of the pulse storage and retrieving, also the numerical results are demonstrated. In section V we consider the transverse relaxation of the coherence induced in the medium. Section VI concludes the paper.

II. BASIC EQUATIONS

Figure 1 shows a schematic diagram of the atomic system in the EIT basis: media of three level atoms interacting with two laser pulses \[E_p = A_p \cos (\omega_p t - k_p z + \varphi_p)\] (probe) and \[E_c = A_c \cos (\omega_c t - k_c z + \varphi_c)\] (coupling). The probe field resonantly connects the state \[|1\rangle\] to the state \[|3\rangle\] and the coupling field connects \[|2\rangle\] to \[|3\rangle\]. The Hamiltonian of the system in the rotating wave approximation is:

\[H = \hbar \Delta \sigma_{33} - \hbar \Omega_p \sigma_{31} - \hbar \Omega_c \sigma_{32} + H.c.,\]

where \[\Omega_{p,c} = \frac{A_{p,c} \Omega_{31}}{\hbar} (i = 1, 2)\] are the respective Rabi frequencies, \[\sigma_{ij} = |i\rangle\langle j|\] are the atomic transition operators, \[\Delta = \omega_p - \omega_{31} = \omega_c - \omega_{32}\] is the detuning of the pulse frequencies from the upper level and \[\mu_{31} (i = 1, 2)\] are the dipole moments of corresponding transitions.

We assume that: (i) the probe field is weak as compared to the coupling pulse field \[\Omega_p << \Omega_c; (ii) the in-\]
teration is adiabatic ($\Omega_c T >> 1$, where $T$ is the interaction duration). Then, the atomic density matrix equation may be written as

$$
\dot{\rho}_{31} = -\Gamma_1 \rho_{31} + i\Omega_p \rho_{31},
\dot{\rho}_{21} = 0,
\rho_{11} = 1,
\rho_{22} = \rho_{33} = \rho_{32} = 0,
$$

where $\Gamma$ is the width of the upper level which is the sum of the spontaneous decay and transverse relaxation rates. It is supposed that interaction is fast enough to neglect the decoherence between metastable levels (sec. III, IV), or to take it into account to the first order (sec. V).

The propagation of the pulses is governed by the Maxwell equation for slowly varying amplitudes,

$$
\left(\frac{i}{c} \frac{\partial}{\partial t} + \frac{1}{c} \frac{\partial}{\partial x'} \right) \Omega_p = i\rho_q \rho_{31},
\left(\frac{i}{c} \frac{\partial}{\partial t} + \frac{1}{c} \frac{\partial}{\partial t'} \right) \Omega_p = i\rho_q \rho_{32},
$$

where $q_t = \frac{2\pi \hbar c}{\mu_0 N}$, $N$ is the atomic number density.

System of equations (1) can be reduced to one equation for $\rho_{31}$

$$
\dot{\rho}_{31} - \dot{\rho}_{31} \frac{\Omega_p}{\Omega_c} + \rho_{31} \frac{\Omega_p^2}{\Omega_c} + \Gamma \left( \rho_{31} - \rho_{31} \frac{\Omega_p}{\Omega_c} \right) = i\Omega_c \theta
$$

where $\theta \equiv \frac{\Omega_p}{\Omega_c}$ is the common used notation for the so called mixing angle. Influence of the first two terms can be neglected if we confine to only first terms with respect to the nonadiabaticity (i.e. $(\Omega_c T)^{-2} \ll 1$ is neglected). Influence of the fourth term is essential parameter only under the assumption

$$
\Gamma T >> 1.
$$

The meaning of the condition (4) is obvious: under the condition of complete adiabatic relaxation does not affect the pulse propagation (dark-state), but taking into account first nonadiabatic correction, has essential influence. The relaxation can be neglected when $\Gamma T \lesssim 1$.

Finally, by substituting the Maxwell equation (2) into (3) one gets pulse propagation equation in wave variables $x = x', \ t = t - \frac{x}{c}$,

$$
\frac{1}{\Gamma_1} \frac{\partial^2 \theta}{\partial x^2} + \frac{q_t}{\Omega_c^2} \frac{\partial \theta}{\partial t} + \frac{\partial \theta}{\partial x} = 0,
$$

where notation $\Gamma_1 = \frac{\Omega_c^2}{\Gamma}$ is used. In this connection the coherence dynamics is governed by the following equation:

$$
\dot{\rho}_{21} = -\Gamma_1 \rho_{21} - \Gamma_1 \theta
$$

Thus $\Gamma_1$ is the coherence decay rate, or width of EIT resonance, due to applied coupling. Under the condition $\Gamma_1 T >> 1$ (7) equation (6) has the well known quasi-stationary solution $\rho_{21} = -\theta$, and the equation (4) passes to the dark state polariton propagation equation. The condition (4) means, that width of EIT resonance exceeds the spectral width of the probe.

III. Solution of Propagation Equation

The obtained probe pulse propagation equation (6) is solved by the method presented in [11]. Since (6) is linear in $\theta$ and $\Omega_c (t)$ is independent of $x$, it can be solved by using the Laplace transform with respect to $x$. The solution of (4) for $\theta$’s Laplace image can be found easily:

$$
\theta(s,t) = \int_{-\infty}^{t} \frac{\theta_0(t) \Gamma_1}{s + \Gamma} B(s,t,t_1) dt_1 + \mathcal{C}(s) B(s,t,-\infty)
$$

where $B(s,t,t_1) = \exp\left(-\frac{s}{s + \Gamma} \int_{t_1}^{t} \Gamma_1 dt'\right)$, $\mathcal{C}(s)$ is an integration constant that is determined by the initial condition, $\mathcal{C}(s) = \theta(s,-\infty)$. If pulses are switched in coutnerintuitive sequence (coupling turns on earlier than the probe does) then $\mathcal{C}(s) = 0$, since $\theta(z,-\infty) = \theta_0(-\infty) = 0$ (see appendix A).

Space time evolution of the probe pulse is obtained by implementing the reverse Laplace transform in (8).

$$
\theta(z,t) = \int_{-\infty}^{t} dt_1 \left( \theta_0(t) \Gamma_1 + \theta_0(t) \right) \times \times \exp\left(-z - \alpha (t_1,t)\right) I_0 \left(2\sqrt{z} \alpha (t_1,t)\right),
$$

where $z = \frac{q_0 x}{T}$ is propagation distance normalized to linear absorption factor and, for convenience, the notation $\alpha (t_1,t) = \int_{t_1}^{t} \Gamma_1 (t') dt'$ is used. By using the condition $z \alpha (t_1,t) >> 1$ one can substitute the modified Bessel function by its asymptote, so the solution (9) reduces to the following:

$$
\theta(z,t) = \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{t} dt_1 \left( \theta_0(t) \Gamma_1 (t_1) + \theta_0(t) \right) \times \times \exp\left(-\left(\sqrt{z} - \sqrt{\alpha (t_1,t)}\right)^2\right) \left(z \alpha (t_1,t)\right)^{-1/4}.
$$

Depending on optical propagation distance $z$ two simple asymptotes for (10) can be obtained (see appendix...
B). The first is the case where

$$\frac{\Gamma_{1m} T}{\sqrt{\zeta}} >> 4\sqrt{\ln 2}$$

(11)

$$\Gamma_{1m}$$ is the maximal value of $$\Gamma_1 (t)$$ (see also [2]). Under condition (11), solution (9) reduces to the dark-state polariton propagation solution with correction in (7):

$$\theta (z, t) = \theta_0 (\xi) + \frac{1}{\Gamma_1 (\xi)} \dot{\theta}_0 (\xi)$$

(12)

where $$\xi$$ is the non-linear time determined by

$$t \int \Omega_c^2 (t) dt_1 = q_p x$$

(13)

Note, that turning off the coupling $$\Omega_c (t)$$ does not reduce $$\Gamma_1 (\xi)$$ to zero, since $$\xi$$ retreats from $$t$$. In this case, as it will be shown below, the information stored in the medium can be well retrieved (see sec. IV).

In the case of condition reversed to (11),

$$\frac{\Gamma_{1m} T}{\sqrt{\zeta}} << 4\sqrt{\ln 2}$$

(13)

the solution (9) reduces to

$$\theta (z, t) = R \exp \left(- \left(\sqrt{\zeta} - \sqrt{\alpha (t_0, t)}\right)^2 \right) (z \alpha (t_0, t))^{-1/4}$$

where $$t_0$$ is the maximal value of $$\theta_0 (t)$$, $$R = \int_{-\infty}^{\infty} \Gamma_1 (t') \theta_0 (t') dt'$$ and does not depend on time. We emphasize that for propagation distances meeting the condition (13), the obtained pulse loses all the information about its initial temporal shape, since the right hand side in (13) does not contain time dependent $$\theta_0$$.

IV. DISCUSSION

In this section the probe pulse propagation dynamics obtained from the analytical solution (9) is presented. First of all we consider the case of constant coupling field. Shape of the initial pulse is chosen to be double-humped in order to visualize the propagation dynamics. For the small propagation distances when the condition (11) is met influence of $$\Gamma$$ is negligible (Fig. 2a). When the condition (11) is violated, the influence of upper level width becomes essential as one can see from Figs. 2b,c. Thus influence of $$\Gamma$$ breaks the adiabatic propagation regime.

As it was mentioned above, propagation over very long distances (13) leads to the lost of the information on the initial pulse temporal shape. This can be seen in Fig 3, where propagation over the same distance of two pulses with different temporal shapes but with the same initial area is depicted. By propagating over very long distance (13) they lose any information about their initial temporal shapes.

In Fig. 4 we show that the increase of $$\Gamma_1$$ suppresses the smearing of the probe. This is caused by the decrease of the bright state population and hence leads to the decrease the influence of $$\Gamma$$ on pulse propagation. Note, that in the dark-state propagation regime the pulse temporal shape does not depend on coupling field amplitude or on unstable level width.

Summarizing presented results one can see that to minimize the pulse smearing during its propagation one has to either increase $$\Gamma_1$$ or decrease the propagation distance $$z$$. The situation changes dramatically for the light storing and retrieving process ($$\Omega_c \neq \text{const}$$).

It is known, that pulse can be completely stored and retrieved from the medium if the medium length and $$\Gamma_1$$ meet the condition (see for example [2]):

$$z \gtrsim \Gamma_{1m} T.$$  

(15)

For efficient storage and retrieving the condition (11) also has to be met. Combining this two nonequalities one gets that to completely store and well retrieve the light pulse, $$\Gamma_1$$ has to meet the following condition:

$$\Gamma_{1m} T >> 16 \ln 2 >> 1.$$  

(16)

Therefore, influence of the second term in (19) is insufficient when the condition (16) is met.

Storage and retrieving of the light pulse for different propagation distances under the condition (16) is depicted in Fig 5. For small propagation distances when the condition (11) is violated only the falling edge of the pulse is stored and can be retrieved (Fig. 5a), because when this edge enters the medium, the leading edge emerges already. For larger propagation distances when the condition (16) is satisfied the whole pulse can be stored and then well retrieved by turning on the coupling field.

Let us now consider the case when the condition (16) is not met (Fig. 6). For small propagation distances when the condition (16) is satisfied but (11) is not, only the falling edge of the pulse can be stored and retrieved. Propagation over longer distances brings to satisfying of (16) and violation of (11). Thus the whole pulse can be stored but the retrieved pulse temporal shape is smeared.

We present finally comparison of the experimental results with our analytical solution. In Fig 7a the experimental data of storage and retrieving of light pulse from [6] are presented. Curve in Fig 7b is plotted from our analytical solution (9): all parameters correspond to the conditions of the experiment. One can see good consistency between experimental data and our analytical solution (Note that the storage in case of experiment (1) is incomplete as was discussed above).

V. CONSIDERATION OF $$\rho_{21}$$ TRANSVERSE DECAY

In this section we take into account the quantity $$\gamma T$$ in first order. This leads, instead of (11), to the equations.
\[
\begin{align*}
\dot{\rho}_{21} &= -\gamma \rho_{21} + i \Omega_e \rho_{21}, \\
\dot{\rho}_{31} &= -\Gamma \rho_{31} + i \Omega_p + i \Omega_e \rho_{21}, \\
\rho_{11} &= 1, \\
\rho_{22} &= \rho_{33} = \rho_{32} = 0,
\end{align*}
\]

Thus, probe pulse propagation equation is written as follows:

\[
\frac{1}{\Gamma_1} \frac{\partial^2 \theta}{\partial x \partial t} + \frac{\partial \theta}{\partial x} + \frac{q_p}{\Gamma_1 (\Gamma + \gamma)} \frac{\partial \theta}{\partial t} + \frac{q_p \gamma}{\Gamma_1 (\Gamma + \gamma)} \theta = 0,
\]

where \( \Gamma_1 (t) \) now is

\[
\Gamma_1 (t) = \frac{\Omega_e^2 + \gamma \left( \Gamma + \frac{\Omega_p}{\Omega_e} \right)}{\Gamma + \gamma}.
\]

As results from \( \text{(18)} \), to completely stop the light in the medium (\( \Gamma_1 = 0 \)) one should turn off the coupling field \( \Omega_e \) at the rate \( \Gamma \) (i.e., \( \Gamma + \frac{\Omega_p}{\Omega_e} = 0 \)).

By performing the stated above Laplace transform procedure one obtains analytical solution of the equation \( \text{(17)} \) in the form

\[
\theta (z, t) = \int_{-\infty}^{t} dt_1 \left( \theta_0 + \Gamma_1 \dot{\theta}_0 \right) \times
\]

\[
\times \exp \left( -z - \int_{t_1}^{t} \Gamma_1 dt' \right) I_0 \left( 2 \sqrt{z \alpha (t_1, t)} \right)
\]

where \( \alpha (t_1, t) = \int_{t_1}^{t} \Gamma_1 (t') - \gamma dt' \) and \( z = \frac{q_p x}{\Gamma + \gamma} \).

More detailed analysis of the expression \( \text{(17)} \) will be performed in a subsequent publication.

**VI. CONCLUSION**

We have considered the propagation, storage and retrieving of the light pulse in EIT media by taking into account all dephasing rates. From coupled system of Maxwell and density matrix equations we derive the probe pulse propagation equation, which in particular case passes into the dark-state polariton propagation equation. We find an analytical solution and analyzed its physical consequences. We derived a simple asymptotes of the solution, and showed strong dependence of light pulse temporal shape on optical propagation distance in the presence of relaxations. We demonstrated that an efficient storage of light is possible by choosing appropriate coupling intensities and optical propagation distances. Finally, we compared our solution with experimental data and showed that our solution is well consistent with the recent experiments.

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**APPENDIX A:**

To find the \( c (s) \) in general case we examine the equation \( \text{(2)} \).

From this equation one obtains \( \frac{\partial \theta}{\partial x} = i q_p \frac{\Omega_e}{\Omega_c} \rho_{31} (x, t) \), and the solution can formally be written as \( \theta (x, -\infty) = i q_p \frac{\Omega_e}{\Omega_c} \int_{-\infty}^{x} \rho_{31} (x', -\infty) dx' + \theta_0 (-\infty) \). This can be simplified by taking into account that there is no dipole moment induced before the interaction is turned on (\( \rho_{31} (x, -\infty) = 0 \)).

\[
\theta (x, -\infty) = \theta_0 (-\infty)
\]

For the counterintuitive order of pulse switching we have \( \theta_0 (-\infty) = 0 \), hence \( c (s) = 0 \). In the case of simultaneous pulse switching \( \theta_0 (-\infty) = \text{const} \), therefore \( c (s) = \text{const} \).

In the case \( \theta_0 = \text{const} \) from general solution \( \text{(8)} \) one easily obtains the so-called matched pulse propagation regime \( \theta = \theta_0 \) \( \text{(12)} \).

**APPENDIX B:**

The integrand function in \( \text{(10)} \) is the product of two time dependent bell-shaped functions. This kind of integral is evaluated easily when the temporal width of one function is much larger than the width of the other one. Then by the first order saddle-point technique the narrow function can be approximated by delta function.

The time widths of the first and second functions in \( \text{(10)} \) are, respectively, the interaction duration \( T \) and \( 4 \sqrt{z \ln (2)} \Gamma_{1m} \).

On the one hand, if the condition \( \text{(11)} \) is met, the integration of \( \text{(10)} \) gives the well known dark-state polariton solution \( \text{(12)} \). On the other hand, if the condition \( \text{(13)} \) is satisfied, integration of \( \text{(10)} \) by the saddle-point technique brings to the asymptote \( \text{(14)} \).
A.V.Turukhin, V.S. Sudarshanam, M.S. Shahriar, S.E.Harris, Phys. Rev. Lett., 70, 273 (2001). K.J. Boller, A.Immamoglu, and S.E. Harris, Phys. Rev. Lett. 83, 1767 (1991).

M.G.Raymer, J.Mostowski, Phys. Rev. A, 24, 1980 (1981).

M.D.Lukin, Rev. Mod. Phys. 75, 457 (2003). M.Fleischhauer, A.Immamoglu, and J.P. Marangos, Rev. Mod. Phys. in press (2005).

L.Hau,S.Harris, D.Dutton, C.Behroozi, Nature, 397, 594 (1999); D.Bulker, D.F. Kimball, S.M. Rochester, and V.V.Yashchuk, Phys. Rev. Lett. 83, 1767 (1999). M.D. Lukin, A.Immamoglu, Phys. Rev. Lett. 84, 1419 (2000). Michael M. Kush, Vladimir A. Sautenkov, Alexander S. Zibrov, L. Hollberg, George Welch, Mikhail D. Lukin, Yuri Rostovtsev, Edward S. Fry, and Marlan Scully, Phys. Rev. Lett. 82, 5229 (1999).

Ch.Liu, Z.Dutton, C.H.Behroozi, L.V.Hau Nature, 409, 457 (2001); M.G.Payne, L.Deng, Chris Schmitt, Shannon Anderson, Phys. Rev. A 66, 043802 (2002).

Fig.1 Schematic diagram of the three-level atomic system in the basis of EIT in the dressed-state (a) and the bare-state (b) representations. The field \( \Omega_c \) couples the \( |bright\rangle \) state to the upper level \( |3\rangle \), and another field with Rabi frequency \( \theta \) couples the \( |dark\rangle \) and \( |bright\rangle \) states.

Fig.2 Probe pulse temporal shape after propagation over different propagation distances in the case of constant coupling field; \( \Gamma T = 40 \). Initial temporal shape of the probe has been chosen as \( \Omega_p/\Gamma = 0.012 \exp \left( - (\Gamma t/100 - 7.5)^2 \right) + 0.01 \exp \left( - (\Gamma t/100 - 10)^2 \right) \). For small propagation distances when the condition \( \Omega_p/\Gamma < 1 \) is met the influence of upper level width is small (a). Violation of the condition \( \Omega_p/\Gamma > 1 \) increases the role of the upper level width: (b), (c). Here and below the dotted curve is the initial temporal shape of the probe.

Fig.3 Propagation of two probe pulses with different temporal shapes but with the same area over very long distances \( \Gamma T > 10^3 \) in the case of constant coupling field\( \Gamma T = 4 \). One can see that during propagation pulse loses all information about its initial temporal shape. The shape of the probe in the case (a) is the same as in Fig.2, in the case (b) it has been chosen as \( \Omega_p/\Gamma = 0.022 \exp \left( - (\Gamma t/100 - 8.5)^2 \right) \).

Fig.4 Probe pulse temporal shape after propagation over the same optical distance \( z = 20 \) for different constant coupling amplitudes. Increasing of coupling amplitudes decreases the influence of upper level width. The shape of the probe is the same as in Fig.2.

Fig.5 Storage and retrieving of the probe for different propagation distances, when the condition \( \text{10} \) is met.

The coupling field is chosen as \( \Omega_c/\Gamma = f (t) \) (a), where \( f (t) = 1 \) for \( \Gamma t < 1000 \) and \( \Gamma t > 2500 \); \( f (t) = \exp \left( - (\Gamma t/100 - 10)^2 \right) + \exp \left( - (\Gamma t/100 - 20)^2 \right) \) for \( 1000 \leq \Gamma t \leq 2500 \). For small optical propagation distances the falling edge of the pulse is stored and retrieved (b). For larger \( z \) the whole pulse can be stored and well retrieved (c). The shape of the probe is the same as in Fig.2.

Fig.6 Storage and retrieving of the probe for different propagation distances when the condition \( \text{10} \) is violated. The coupling field has been chosen as \( \Omega_c/\Gamma = f (t)/\sqrt{10} \). The coupling field has been chosen as \( \Omega_c/\Gamma = f (t)/\sqrt{10} \). Gamma is equal to 40 (a). Either only the falling edge of the pulse is stored and retrieved (b). For larger \( z \) the whole pulse can be stored and well retrieved (c). The shape of the probe is the same as in Fig.2.

Fig.7 Experimental data from \( \text{10} \), (a) and our analytical solution for parameters corresponding to the experimental conditions (b). The coupling field has been chosen as \( \Omega_c/\Gamma = f (t)/\sqrt{10} \). Gamma is equal to 40 (a). Either only the falling edge of the pulse is stored and retrieved (b). For larger \( z \) the whole pulse can be stored and well retrieved (c). The shape of the probe is the same as in Fig.2.
Fig. 4

Fig. 5
Fig. 6

Fig. 7