Spin-Temporal Interactions of Light

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Temporally varying electromagnetic media have been extensively investigated recently to unveil new means for controlling light. However, spin-dependent phenomena in such media have not been explored. Here, by introducing a temporal interface between bianisotropic chiral and dielectric media, we reveal the existence of a temporal analogue to the spin-Hall effect of light. In particular, we show theoretically and numerically that due to the material discontinuity in time, linearly polarized light is split into forward-propagating right-handed and left-handed circularly polarized waves having different angular frequencies and the same phase velocities. This salient effect allows complete temporal separation of the two spin states of light with high efficiency. In addition, a phenomenon of spin-dependent gain/loss is revealed. Furthermore, we show that when the dielectric medium is switched again to the original chiral medium, the right- and left-handed circularly polarized light waves (with different angular frequencies) merge to form a linearly polarized wave. Our findings extend spin-dependent interactions of light from space to space-time.
Introduction

Manipulating effective properties of electromagnetic systems in time provides an exceptional opportunity to control light and attain unique responses (1). Recently, by exploiting this approach, a multitude of wave phenomena and applications have been uncovered, including non-reciprocity (2–6), frequency conversion (7–9), time reversal (10), antireflection temporal coating (11), and more (12). However, spin-dependent interactions in time-varying media have not been contemplated. In addition, most studies have mainly focused on temporal manipulations of isotropic or anisotropic systems (13, 14), while less attention is given to time-varying bianisotropic media or systems (3). On one hand, it has been shown that a traveling wave space–time modulation that emulates a moving media creates bianisotropic-like coupling (15, 16). On the other hand, these works utilize temporal modulation to produce bianisotropic effects (magneto-electric coupling), which is fundamentally different from manipulating a bianisotropic medium in time. From this point of view, temporal manipulations of bianisotropic media is an unexplored area of research while it holds potential to induce spin-dependent interactions that we name as spin-temporal interactions of light (STIL).

To introduce STIL, it is helpful to refer to their spatial analogue: Spin-orbit interactions of light (17, 18) which has become an extremely active topic during recent years (19–22). A time-harmonic electromagnetic wave can be fully described by its intensity, wavevector, polarization state, and angular frequency. The intensity and wavevector of a wave represent its spatial degrees of freedom, while the angular frequency represents the temporal degree of freedom. Spin-orbit interactions take place when the spatial (orbital) degrees of freedom depend on the polarization (spin) of the propagating wave. In other words, the polarization state of the electromagnetic wave defines how the wave propagates through space. An example of spin-orbit interactions is the spin-Hall effect of light (23, 24), where a transverse spin-dependent
subwavelength shift takes place at a spatial planar interface. This effect is utilized to spatially split/decompose linearly polarized light into right-handed and left-handed circularly polarized (RHCP/LHCP) waves (25, 26). It appears possible to expect analogous effects at a temporal interface, where a spin-dependent frequency shift would take place, realizing a temporal analogue of the spin-Hall effect of light. In this case, the polarization state of the electromagnetic wave defines how the wave propagates through time. To investigate this possibility, we study abrupt changes of bianisotropic chiral media parameters as functions of time. One of the main motivations is a possibility to switch spatial mirror-inversion symmetry by varying some material parameters in time.

In this paper, we make an initial step in this direction and contemplate a nonstationary chiral medium. In particular, we consider a temporal interface between a chiral medium and a dielectric medium. It is useful to conceptualise the problem as a temporal interface between two symmetries. This is an interface between spatially mirror-inversion asymmetric and symmetric media, hence, the symmetry is switched in time. In a chiral medium, the RHCP and LHCP waves which form a linearly polarized propagating wave are associated with different phase velocities (27), which is the main property of chiral media. This property is not exclusive to chiral media, it is associated also with nonreciprocal magnetized and magneto-optical materials. In chiral media, such difference in the phase velocity of eigenwaves arises from spatial dispersion in materials with broken mirror-inversion symmetry. We show that due to this important characteristic, at a temporal interface between chiral and dielectric media the angular frequencies of the propagating RHCP and LHCP waves (composing a linearly polarized wave) are shifted to two different angular frequencies resulting in splitting the polarization states temporally. In addition, the energy density of the RHCP and LHCP waves experience a spin-dependent gain/loss effect. Such phenomena constitute examples of an unusual class of wave-matter interactions: spin-temporal interactions of light.
Moreover, if the dielectric medium is switched back to the original chiral medium, the decomposed RHCP and LHCP waves merge again and form a linearly polarized propagating wave. Hence, temporal discontinuities in bianisotropic chiral media can also be used to merge RHCP and LHCP waves (with different angular frequencies) to compose linearly polarized light. Finally, we calculate the amplitudes of forward and backward waves generated due to chiral-dielectric temporal discontinuities and prove that under certain conditions, the backward waves (reflected waves) vanish, meaning that only the forward waves are propagating after temporal discontinuities.

**Time-domain model of chiral media**

To study time-varying chiral media, time-domain constitutive relations are needed. However, the commonly used constitutive equations of chiral media (so called Post and Tellegen relations) (27) are applicable only in the frequency domain. This is due to the fact that electromagnetic chirality is a manifestation of spatial dispersion, leading to inevitable frequency dispersion of chirality parameters in both these models. For this reason, we use the Condon model, which connects the electric flux density to the time derivative of the magnetic field and the magnetic flux density to the time derivative of the electric field (28). This model, introduced in 1937, approximately models chirality effects with a non-dispersive parameter $g$, which is a crucial feature. The model is applicable at frequencies well below all resonances of chiral molecules or inclusions, where the rotatory power linearly decreases to zero at the limit of zero frequency (30).

In chiral media, a linearly polarized plane wave can be expressed as a combination of RHCP and LHCP waves having the same angular frequency but propagating at different phase velocities. Splitting the fields of a plane wave into RHCP and LHCP components, we write the
constitutive relations of isotropic chiral media as

\[
D^\pm = \left[\epsilon_{\text{eff}} E^\pm + g \frac{\partial H^\pm}{\partial t}\right], \\
B^\pm = \left[\mu_{\text{eff}} H^\pm - g \frac{\partial E^\pm}{\partial t}\right],
\]

(1a)

in which \(\epsilon_{\text{eff}}, \mu_{\text{eff}},\) and \(g\) are the non-dispersive effective permittivity, effective permeability, and the chirality parameter (or rotatory parameter as Condon called it). The ± superscripts mark the RHCP and LHCP wave components, respectively. We consider electric and magnetic fields of a linearly polarized plane wave propagating in the \(x\)-direction as

\[
E^\pm = E_0 \left[\hat{y} \pm j\hat{z}\right] e^{j(\omega t - \beta^\pm x)}
\]

and \(H^\pm = \frac{\hat{x} \times E^\pm}{\eta_{\text{eff}}},\) where \(\eta_{\text{eff}}\) is the medium effective intrinsic impedance, and \(E_0\) is the complex amplitude of the electric field. We use the electrical engineering convention for time-harmonic oscillations (i.e., \(\exp(j\omega t)\)). By substituting the fields into Eqs. (1a) and (1b), we arrive to the wavefield decomposition (see Supplementary Note 1)

\[
D^\pm = \epsilon_{\text{eff}} \left(1 \mp \Psi\right) E^\pm, \quad B^\pm = \mu_{\text{eff}} \left(1 \mp \Psi\right) H^\pm,
\]

(2)

in which \(\Psi = g\omega c,\) \(\omega\) is the angular frequency, and \(c = \frac{1}{\sqrt{\epsilon_{\text{eff}}\mu_{\text{eff}}}}.\) The wavenumbers of plane waves in the two equivalent isotropic media equal to

\[
\beta^\pm = \omega \sqrt{\epsilon_{\text{eff}}\mu_{\text{eff}}} (1 \mp \Psi) \quad (27).
\]

**Temporal interface between chiral and dielectric media**

Here, we consider a temporal interface between isotropic chiral and dielectric media having the same effective permittivity and permeability (Fig. 1). We note that it is possible to solve the problem in the general case when permittivity and permeability experience jumps, arriving to similar physical results. We contemplate a chiral medium supporting a linearly polarized plane wave at frequency \(\omega_1\) when the chirality parameter \(g\) rapidly changes to zero, that is, the medium becomes nonchiral. In his paper published in 1958, Morgenthaler showed that
Fig. 1: Schematic representation of a temporal interface between chiral and dielectric media. Spin-dependent frequency shift and spin-dependent gain/loss take place at the temporal interface.

The electric and magnetic flux densities are continuous at a temporal interface (29). Using that property, we write that $D_1^\pm = D_2^\pm$ and $B_1^\pm = B_2^\pm$, where the subscripts 1, 2 correspond to the fields before ($t = t_0^-$) and after ($t = t_0^+$) the temporal discontinuity, respectively ($t_0$ is the switching moment). According to Morgenthaler, after the temporal jump, there are forward and backward waves in analogy with a spatial interface at which we have transmitted and reflected waves. Keeping this in mind, we substitute Eq. (2) into $D_1^\pm = D_2^\pm$ and $B_1^\pm = B_2^\pm$. Assuming $t_0 = 0$ and after some mathematical manipulations, we find that (see Supplementary Note 2)

$$
\epsilon_{\text{eff}}(1 \pm \Psi_1) E_1^\pm = \epsilon_{\text{eff}} \left( \frac{E_2^\pm}{(\Gamma_{c||d}^+ + \Gamma_{c||d}^-)} \right) E_1^\pm, \\
\mu_{\text{eff}}(1 \pm \Psi_1) H_1^\pm = \mu_{\text{eff}} \left( \frac{H_2^\pm}{\Gamma_{c||d}^+ - \Gamma_{c||d}^-} \right) H_1^\pm,
$$

in which $\Psi_1 = \omega_1 gc$ depends on the angular frequency before the temporal discontinuity. The forward and backward propagation coefficients (for a temporal interface between chiral and
dielectric media) are denoted as $\Upsilon_{c||d}$ and $\Gamma_{c||d}$, respectively. The above expressions indicate that the polarization states and the phase constants are conserved at the temporal interface, as shown in Supplementary Note 2. Due to the conservation of the phase constant and by knowing that $\beta_1^\pm = \omega_1 \sqrt{\mu_{\text{eff}} \epsilon_{\text{eff}}} (1 \mp \Psi_1)$ and $\beta_2^\pm = \omega_2^\pm \sqrt{\mu_{\text{eff}} \epsilon_{\text{eff}}}$, we arrive to the following important relation:

$$\omega_2^\pm = \omega_1 \left(1 \mp \Psi_1\right). \quad (4)$$

This result shows that the RHCP and LHCP components have different angular frequencies after the temporal jump, meaning that the polarization states are separated temporally. In addition, from Eq. (3) we find that $\Gamma_{c||d}^\pm = 0$ and $\Upsilon_{c||d}^\pm = 1 \mp \Psi_1$. In other words, at fast transitions from a chiral medium to a dielectric one (while keeping the effective permittivity and permeability the same), no backward waves are generated.

The same result can be obtained using the Morgenthaler equations for forward and backward propagating waves at a temporal interface in dielectric media (29). These equations relate the forward and backward propagation coefficients to the permittivity and permeability of the medium before and after the temporal jump. Assume that the equivalent permittivity and permeability of the chiral medium for RHCP and LHCP equal $\epsilon_{\text{eq}}^\pm = \epsilon_{\text{eff}} \left(1 \mp \Psi_1\right)$ and $\mu_{\text{eq}}^\pm = \mu_{\text{eff}} \left(1 \mp \Psi_1\right)$, respectively. In this case, changing the chirality parameter in time is equivalent to changing the permittivities and permeabilities of two equivalent magnetodielectric media. Then, if we plug these parameters in the Morgenthaler equations as $\Upsilon_{c||d}^\pm = \frac{1}{2} \left[ \frac{\epsilon_{\text{eq}}^\pm + \sqrt{\epsilon_{\text{eq}}^\pm \mu_{\text{eq}}^\pm}}{\epsilon_{\text{eff}} + \sqrt{\epsilon_{\text{eff}} \mu_{\text{eff}}}} \right]$ and $\Gamma_{c||d}^\pm = \frac{1}{2} \left[ \frac{\epsilon_{\text{eq}}^\pm \mu_{\text{eq}}^\pm - \sqrt{\epsilon_{\text{eq}}^\pm \mu_{\text{eq}}^\pm}}{\epsilon_{\text{eff}} \mu_{\text{eff}}} \right]$, we get the same $\Upsilon_{c||d}$ and $\Gamma_{c||d}$ as the ones derived above.

Moreover, as the RHCP and LHCP waves propagate in media having different equivalent permittivities and permeabilities before the temporal interface, a wave having one of the polarization states exhibits loss and the other one exhibits gain, introducing spin-dependent gain/loss. The energy density gain/loss (defined as the ratio of the energy density before and after the temporal interface) is equal to $\frac{1}{2} \left(\frac{\epsilon_{\text{eq}}^\pm}{\epsilon_{\text{eff}}} + \frac{\mu_{\text{eq}}^\pm}{\mu_{\text{eff}}}\right)$ (29), which simplifies to $1 \mp \Psi_1$. 

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To better understand how spin of light interacts with bianisotropic temporal discontinuities, it is important to investigate what happens if the dielectric medium (after the temporal interface) is at a later moment of time switched back to the same chiral medium, representing a “temporal slab.” In this case the chiral medium is considered twice. For both cases the fields in the chiral medium are defined using the same parameters, and subscripts 1 and 3 are used to distinguish between them. Following the same analysis method as before, we find that the forward and backward propagation coefficients for the second temporal interface are

$$\Upsilon_{d||c}^\pm = \frac{1}{1 + \Psi_3^\pm}$$

and

$$\Gamma_{d||c}^\pm = 0,$$

respectively, where $$\Psi_3^\pm = \omega_3^\pm gc$$. Again, no backward propagating waves are created. Interestingly, the transmission coefficient is a function of the angular frequency after the second temporal interface. This angular frequency can be calculated from the conservation of wavenumbers

$$\beta_2^\pm = \omega_2^\pm \sqrt{\mu_{\text{eff}}\varepsilon_{\text{eff}}}$$

and

$$\beta_3^\pm = \omega_3^\pm \sqrt{\mu_{\text{eff}}\varepsilon_{\text{eff}}} (1 \mp \Psi_3^\pm).$$

As $$\beta_3^\pm$$ is a quadratic equation for $$\omega_3^\pm$$, equations $$\beta_2^\pm = \beta_3^\pm$$ have two solutions for frequencies $$\omega_3^\pm$$ after the second temporal interface, which read

$$\omega_3^\pm = \omega_1$$

and

$$\omega_3^\pm = \pm \frac{1}{cg} - \omega_1.$$ 

The first solution indicates that the created RHCP and LHCP waves have the same frequency as the initial wave before the first interface. Since $$\omega_3^\pm = \omega_1$$, the total forward propagation coefficient

$$\Upsilon_{d||c}^\pm \cdot \Upsilon_{c||d}^\pm = 1,$$

meaning that the amplitude of the created wave is equal to the initial amplitude. Thus, the initial linearly polarized wave is formed again according to the first solution. We note that this phenomenon can be used to merge two differently polarized waves (at different angular frequencies) into a linearly polarized wave. On the other hand, the second solution $$\omega_3^\pm = \pm \frac{1}{cg} - \omega_1$$ indicates that, after the second interface, additional waves can be created at high frequencies. The Condon model of chiral media is applicable only at enough low frequencies, well below the resonant frequencies of chiral particles forming the medium. However, for realistic values of $$g$$ and considering relatively small $$\omega_1$$ that is far from resonance, the term $$\frac{1}{cg}$$ is much larger than $$\omega_1$$, leading to very large $$\omega_3$$ (see Supplementary Note 2). Thus, the Condon model cannot be reliably used to get accurate results for the second solution. More broadband time-domain models of chiral media
need to be used to investigate the second solution, and for this reason we do not consider this solution here.

**Realization and numerical results**

As a specific realization, we consider canonical metal-wire chiral particles, where the particles are formed of two short straight wires (arm length \( l \)) connected to an electrically small loop (the loop area \( S \)) \((30)\). The electromagnetic fields defined above have transversal components in the \( \hat{y} - \hat{z} \) plane, thus, we consider an uniaxial chiral medium composed of two orthogonal arrays of small chiral particles (Fig. 2A). All the medium parameters are expressible in dyadic form as \( \vec{a} = a \vec{I} + b \hat{x} \hat{x} \), where \( a \) and \( b \) are scalars or pseudoscalars, \( \hat{x} \) is the unit vector of the preferred direction in the medium, and \( \vec{I} \) is the transverse unit dyadic. As there is magnetoelectric coupling only in the transverse plane, switching chirality induces temporal interface for transversal fields only. Hence, to the end of the paper, we solve the problem in the transverse plane, which reduces the mathematical representation. In this case, the chirality parameter dyadic \( \vec{g} = g \vec{I} \) reduces to pseudoscalar \( g \), similar to all the other parameter dyadics.

In the frequency domain, the effective material relations for chiral media are written as \( \begin{pmatrix} \vec{B} \\ \vec{D} \end{pmatrix} = \begin{pmatrix} \epsilon_{\text{eff}} & \alpha_{\text{eff}} \\ -\alpha_{\text{eff}} & \mu_{\text{eff}} \end{pmatrix} \cdot \begin{pmatrix} \vec{E} \\ \vec{H} \end{pmatrix} \), where \( \alpha_{\text{eff}} \) is the effective chirality parameter. The uniaxial particle polarizabilities can be estimated as \( \alpha_{ee} = \frac{1}{2} \frac{\mu C}{1-\omega^2 LC} \), \( \alpha_{mm} = \frac{1}{2} \frac{\mu^2 \omega^2 S^2 C}{1-\omega^2 LC} \), and \( \alpha_{em} = \alpha_{me} = \frac{1}{2} \frac{j\omega \mu S C}{1-\omega^2 LC} \) (see Supplementary Note 3), where \( \mu \) is the permeability of the background medium, \( C \) and \( L \) are the capacitance and inductance of the wire antenna and the loop respectively.

To estimate the effective material parameters of the composite, we use the Maxwell-Garnett model for mixtures of bianisotropic particles \((30, 31)\). Well below the particle resonance, the magnetic polarizability is negligible, being a second-order spatial dispersion effect, and a non-magnetic background medium is considered, thus, we have \( \mu_{\text{eff}} = \mu_0 \). Enough small number of particles per unit volume \( N \) is selected, so that switching chirality would not affect the effective
Fig. 2: (A) Uniaxial chiral composite having magnetoelectric coupling in the $\hat{y} - \hat{z}$ plane. (B) Relative effective permittivity as a function of frequency. (C) The chirality parameter $g$ as a function of frequency.
permittivity of the composite. This property is ensured if \( \frac{N^2 \alpha_{\text{em}}}{\epsilon \mu} \ll 1 \), where \( \epsilon \) is permittivity of the background medium (see Supplementary Note 3). In this case, the effective permittivity and effective chirality parameter can be estimated by the classical Maxwell-Garnett formula for electrically-polarizable particles as \( \epsilon_{\text{eff}} = \epsilon_0 + \frac{N \alpha_{\text{em}}}{1 - \frac{N \alpha_{\text{em}}}{\epsilon \mu}} \) and \( \alpha_{\text{eff}} = \frac{-N \alpha_{\text{em}}}{1 - \frac{N \alpha_{\text{em}}}{\epsilon \mu}} \), respectively (see Supplementary Note 3). To this point, the effective parameters are dispersive and resonant due to the denominator \( 1 - \omega^2 LC \). We design the chiral composite and choose the operating frequency such that we operate reasonably far from the resonance at all moments, before and after the temporal interface. Thus, \( 1 - \omega^2 LC \) is approximately constant at all moments of time, which is a condition that has to be met while designing the chiral composite obeying the Condon model. Keeping in mind that \( 1 - \omega^2 LC \) is constant, the non-dispersive chirality parameter \( g \) can be written in terms of the effective chirality parameter as \( g = \frac{\alpha_{\text{eff}}}{\omega} \). Transforming the effective chirality parameter to time domain leads to \( \alpha_{\text{eff}} = g \frac{\partial}{\partial t} \). The non-dispersive time-domain material relations can be written as \( \left( \begin{array}{c} \mathbf{B} \\ \mathbf{H} \end{array} \right) = \left( \begin{array}{c} \epsilon_{\text{eff}} g \frac{\partial}{\partial t} \\ -g \frac{\partial}{\partial t} \mu_{\text{eff}} \end{array} \right) \cdot \left( \begin{array}{c} \mathbf{E} \\ \mathbf{H} \end{array} \right) \), which is the Condon model. Finally, to modulate \( g \) without affecting \( \epsilon_{\text{eff}} \) or \( \mu_{\text{eff}} \), we modulate the loop area \( S \).

Consider the fields \( \mathbf{E}^{\pm} \) and \( \mathbf{H}^{\pm} \), which represent RHCP and LHCP plane waves that constitute a linearly polarized wave propagating in a chiral medium. We arbitrarily set \( E_0 = 2 \text{ V/m} \). While designing the chiral medium parameters, there are two conditions that have to be satisfied. Firstly, \( \Psi_1 \) should have a considerably large value to induce a considerable frequency shift, as the spin-dependent frequency shift is a function of \( \Psi_1 \). Secondly, the condition of operating reasonably far from the resonance should be met to keep \( 1 - \omega^2 LC \) constant. Taking these conditions in consideration, the arm length 11 mm, loop radius 2 mm, frequency \( \omega_1/(2\pi) = 3 \text{ GHz} \), background relative permittivity 2, and volume fraction 0.15 have been chosen, corresponding to \( g = 1.0348 \times 10^{-20} \text{ s}^2/\text{m} \), \( \Psi_1 = 0.035 \) and a considerable frequency shift of 105 MHz. Figures 2B and 2C show how \( \epsilon_{\text{eff}} \) and \( g \) depend on the frequency. It can be seen that the two conditions are inversely proportional, that is, increasing \( \Psi_1 \) requires getting closer
to the resonance. Hence, the frequency shift that can be achieved is limited if we stay within
the applicability of the Condon model. However, generally, the spin-dependent frequency shift
is not necessarily limited, it is limited in our case because of the limitations of the used models.
The effective parameters are approximately constant within the operating band that is defined
as $3 \pm 0.105$ GHz (Figs. 2B and 2C). Thus, both conditions are satisfied. Consequently, we have
$\mu_{\text{eff}} = \mu_0 \, \text{H/m}$ and $\epsilon_{\text{eff}} = 2.49 \times 10^{-11} \, \text{F/m}$. Accordingly, the equivalent parameters for the
wavefields in this chiral medium read $\epsilon_{\text{eq}}^+ = \epsilon_{\text{eff}} \left(1 \mp \Psi_1\right)$ and $\mu_{\text{eq}}^+ = \mu_{\text{eff}} \left(1 \mp \Psi_1\right)$. To finalize
the design, we check that the condition of negligible effect of varying chirality on the effective
permittivity is satisfied. Substituting the above values, we find that $N^2 \alpha_2 \approx 0.0001 \ll 1$. Thus,
all the validity conditions for used models are satisfied.

We verify the presented theory numerically using the time-domain solver of the commercial
software COMSOL Multiphysics®. In simulations we study two temporal discontinuities at
t_1 and t_2. At the first discontinuity at t_1, the medium properties change in time from those of
this chiral medium to a simple dielectric medium having $\mu_{\text{eff}} = \mu_0$, $\epsilon_{\text{eff}} = 2.49 \times 10^{-11} \, \text{F/m}$,
and $g = 0$, meaning that the medium has the same effective permittivity and permeability
before and after the time discontinuity. At the second discontinuity at t_2, the medium properties
change again in time to the same chiral medium with $g = 1.03 \times 10^{-20} \, \text{s}^2/\text{m}$. According to the
theoretical results presented above, there should be no generated backward waves ($\Gamma_{c|d}^+ = \Gamma_{d|c}^+ = 0$),
and the forward propagating waves should have the transmission coefficients and angular
frequencies given by $\Upsilon_{c|d}^+ = 1 \mp 0.035$ and $\omega_{d|c}^+ = 3(1 \mp 0.035) \, \text{GHz}$ after the first discontinuity,
while after the second discontinuity we should get $\Upsilon_{c|d}^+ = 1$ and $\omega_{d|c}^+/(2\pi) = 3 \, \text{GHz}$.

To simplify the data analysis, we simulate the RHCP and LHCP waves separately. The sim-
ulation domain is shown in Fig. 3A. Figures 3B and 3C show the y-component of electric field
for the incident, backward, and forward propagating waves. The amplitudes and frequencies
are in agreement with the theoretical predictions given above. Similar results are obtained for
Fig. 3: Numerical simulation of temporal interfaces between chiral and dielectric media using COMSOL Multiphysics. (A) Simulation domain. The chiral medium is associated with effective permittivity $\epsilon_{\text{eff}}$ and effective permeability $\mu_{\text{eff}}$, while the dielectric medium is associated with permittivity $\epsilon_{\text{eq}}$ and permeability $\mu_{\text{eq}}$. Three probes are used to measure the fields. Before the first temporal interface (before $t_1$), probe 1 measures the incident fields. On the other hand, after the first temporal interface (after $t_1$ and before $t_2$) probe 1 measures the backward propagating waves, and probe 2 measures the forward propagating waves. Similarly, after the second temporal interface (after $t_2$) probe 2 measures the backward propagating waves, and probe 3 measures the forward propagating waves. (B) $E_y(t)$ of the LHCP wave before and after the temporal interfaces. (C) $E_y(t)$ of the RHCP wave before and after the temporal interfaces.

the z-component of the electric field and for the magnetic field components (see Supplementary Note 4). It can be seen that there are no backward propagating waves, while the amplitudes and frequencies for RHCP and LHCP forward propagating waves are different after the first temporal interface. Hence, the first temporal interface efficiently splits the polarization states of the incident wave, as the RHCP and LHCP waves are propagating at considerably different angles.
lar frequencies, due to a frequency shift of approximately $10^3$ and $10^7$ MHz for the LHCP and RHCP waves, respectively (theory predicts $10^5$ MHz). In addition, the simulation also confirms that there is a spin-dependent gain/loss, as the amplitudes change by approximately $0.031$ and $0.039$ V/m for the LHCP and RHCP waves, respectively (theory predicts $0.035$ V/m). There are some insignificant differences between the theoretical and numerical results, which is expected due to switching the material parameters continuously in COMSOL, while the theory assumes step-wise changes. After the second temporal jump the RHCP and LHCP propagating waves combine again to constitute a linearly polarized wave, as both RHCP and LHCP propagate again at the same angular frequency. These numerical results confirm that temporal manipulations of bianisotropic chiral media control the spin of light and induce a spin-dependent frequency shift and spin-dependent gain/loss.

**Discussion and outlook**

This work contemplated a nonstationary chiral medium and unveiled spin-temporal interactions of light. Specifically, we showed that due to abruptly removing/inducing mirror-inversion asymmetry, a spin-dependent phenomena take place resulting in splitting/merging the spin states of light. Furthermore, we showed that temporal discontinuity of chiral media induce spin-dependent energy density gain/loss. Our results leverage time as one more degree of freedom for spin-controlled manipulations of light, which can lead to applications in electromagnetics, photonics, and quantum information processing. Potential applications include, for example, sensing and separating chiral particles and molecules in chemical production.

Our results serve as an initial study of spin-temporal interactions of light, paving the way to many opportunities. Presenting a general theory that takes into consideration the temporal dispersion of nonstationary chirality is an important direction in the future. In addition, scrutinizing periodic modulations of chirality and chiral time crystals is certainly promising
and intriguing. Also, expanding this work to other bianisotropic media (like omega medium) appears inevitable. Finally, experimental realizations of STIL is crucial. The proposed realization can be verified experimentally using switches that connect the dipole arms to the loop, and by disconnecting the switches, the magnetoelectric coupling can be switched off. Beside this proposal, STIL is not exclusive to nonstationary chirality, as nonstationary nonreciprocal magnetized and magneto-optical media should exhibit spin-dependent interactions too. Consequently, in addition to the system proposed in this article, experimental verification of STIL is also feasible using temporally switched magnetization.

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**Author contributions**

M.S.M conceived the original idea, M.H.M and S.A.T helped further formulating the concept. M.H.M performed the theoretical analysis with input from M.S.M and S.A.T. M.H.M performed the numerical analysis, and wrote the manuscript. M.S.M and S.A.T reviewed and edited the manuscript. S.A.T supervised the work.

**Competing interests**

All authors have no competing interests

**Data and materials availability**

All data is available in the manuscript or the supplementary materials.
Supplementary materials

Materials and methods

The time-domain solver of the commercial software COMSOL Multiphysics® has been used to numerically simulate temporal interfaces. The simulation setup (Fig. 3A) has dimensions equal to $14\lambda_d \times 0.7\lambda_d$, where $\lambda_d$ is the wavelength in the dielectric medium. A triangular mesh is implemented with minimum and maximum dimensions of $3 \times 10^{-5}$ m and $\lambda_d/20$. Periodic boundary conditions are applied at the top and bottom boundaries, to emulate plane waves. The boundary on the left is assigned to a scattering boundary behaving as a source, and the right boundary is assigned to a scattering boundary behaving as a perfect absorber. Electric field of circularly polarized plane waves propagating in the $x$-direction is excited using an analytical expression $E^\pm = E_0 \left[ \hat{y} \mp j \hat{z} \right] e^{j(\omega t - \beta^\pm x)}$. The temporal interfaces are modeled by inducing fast but smooth changes in the permittivity and permeability filling the rectangular box. Initially, the medium has permittivity $\epsilon_{eq}^\pm$ and permeability $\mu_{eq}^\pm$, then they change following a rectangular shape using analytical functions with smooth transitions with two continuous derivatives, while the duration of the transient period is $10^{-9}$ s.
Supplementary Note 1

Time-domain material relations in chiral media

To study time-varying chiral media, time-domain constitutive relations are needed. The commonly used constitutive equations of chiral media (so called Post and Tellegen relations) (27) are applicable only in the frequency domain. Because the chirality parameter in these formalisms is inherently frequency dispersive, transforming these equations to time domain would complicate the mathematical formulation, as the material relations would contain convolution integrals. Thus, Condon model is used (28), which approximately models non-resonant chirality effects with a non-dispersive parameter $g$. In chiral media, a linearly polarized plane wave can be expressed as a combination of RHCP and LHCP waves having the same angular frequency but propagating at different phase velocities. Splitting the fields of a plane wave into RHCP and LHCP components, and following the Condon model, we write the constitutive relations of isotropic chiral media as

$$ D = \left[ \epsilon_{\text{eff}} E^- + g \frac{\partial H^-}{\partial t} \right] + \left[ \epsilon_{\text{eff}} E^+ + g \frac{\partial H^+}{\partial t} \right], \quad \text{(S1a)} $$

$$ B = \left[ \mu_{\text{eff}} H^- - g \frac{\partial E^-}{\partial t} \right] + \left[ \mu_{\text{eff}} H^+ - g \frac{\partial E^+}{\partial t} \right], \quad \text{(S1b)} $$

in which $\epsilon_{\text{eff}}$, $\mu_{\text{eff}}$, and $g$ are the non-dispersive effective permittivity, effective permeability, and the chirality parameter (or rotatory parameter as Condon called it). Electric and magnetic fields of a linearly polarized plane wave propagating in $x$-direction is considered, and the fields read

$$ E^\pm = \frac{E_0}{2} \left[ \cos(\omega t - \beta^\pm x) \hat{y} \pm \sin(\omega t - \beta^\pm x) \hat{z} \right], \quad \text{(S2a)} $$

$$ H^\pm = \frac{E_0}{2\eta_{\text{eff}}} \left[ \mp \sin(\omega t - \beta^\pm x) \hat{y} + \cos(\omega t - \beta^\pm x) \hat{z} \right], \quad \text{(S2b)} $$

where $\eta_{\text{eff}}$ is the medium effective intrinsic impedance, and $E_0$ is the amplitude of the electric field. Considering the fields derivatives $\frac{\partial H^\pm}{\partial t} = \mp \omega \eta_{\text{eff}} E^\pm$ and $\frac{\partial E^\pm}{\partial t} = \pm \omega \eta_{\text{eff}} H^\pm$, Eqs. (S1a) and
\((\text{S1b})\) can be expressed as

\[
D = \left[ \varepsilon_{\text{eff}} E^- + \frac{g\omega}{\eta_{\text{eff}}} E^- \right] + \left[ \varepsilon_{\text{eff}} E^+ - \frac{g\omega}{\eta_{\text{eff}}} E^+ \right], \quad (\text{S3a})
\]

\[
B = \left[ \mu_{\text{eff}} H^- + g\omega\eta_{\text{eff}} H^- \right] + \left[ \mu_{\text{eff}} H^+ - g\omega\eta_{\text{eff}} H^+ \right], \quad (\text{S3b})
\]

which simplifies to

\[
D^\pm = \varepsilon_{\text{eff}} \left( 1 \mp \Psi \right) E^\pm, \quad (\text{S4a})
\]

\[
B^\pm = \mu_{\text{eff}} \left( 1 \mp \Psi \right) H^\pm, \quad (\text{S4b})
\]

where \(\Psi = g\omega c\), \(\omega\) is the angular frequency, and \(c = \frac{1}{\sqrt{\varepsilon_{\text{eff}} \mu_{\text{eff}}}}\). We note that the above relations are in a similar form as the relations for dispersive stationary chiral media, however, the models used to arrive to both relations are different and cannot replace each other.
Supplementary Note 2

Temporal boundary conditions

A. Chiral-dielectric temporal interface

We contemplate a chiral medium supporting a linearly polarized plane wave at frequency $\omega_1$ when the non-dispersive chirality parameter $g$ rapidly changes to zero, that is, the medium becomes nonchiral. In his paper published in 1958, Morgenthaler showed that the electric and magnetic flux densities are continuous at a temporal interface (29). Using that property, we write that $\mathbf{D}_1^\pm = \mathbf{D}_2^\pm$ and $\mathbf{B}_1^\pm = \mathbf{B}_2^\pm$, where the subscripts 1, 2 correspond to the fields before ($t = t_0^-$) and after ($t = t_0^+$) the temporal discontinuity, respectively ($t_0$ is the switching moment). The flux densities at $t_0^-$ equal $\mathbf{D}_1^\pm = \epsilon_{\text{eff}}(1 \mp \Psi) \mathbf{E}_1^\pm$ and $\mathbf{B}_1^\pm = \mu_{\text{eff}}(1 \mp \Psi) \mathbf{H}_1^\pm$, in which $\Psi = \omega_1 gc$. While at $t_0^+$ they equal $\mathbf{D}_2^\pm = \epsilon_{\text{eff}} \mathbf{E}_2^\pm$ and $\mathbf{B}_2^\pm = \mu_{\text{eff}} \mathbf{H}_2^\pm$. According to Morgenthaler, after the temporal jump, there are forward and backward waves in analogy with a spatial interface at which we have transmitted and reflected waves. As a result, the fields after the temporal interface can be written as

$$
\mathbf{E}_2^\pm = \Upsilon_{c||d}^\pm \frac{E_0}{2} \left[ \cos(\omega_2^\pm t - \beta_2^\pm x)\hat{y} \pm \sin(\omega_2^\pm t - \beta_2^\pm x)\hat{z} \right],
$$

$$
\mathbf{H}_2^\pm = \frac{\Upsilon_{c||d}^\pm E_0}{2\eta_{\text{eff}}} \left[ \cos(\omega_2^\pm t + \beta_2^\pm x)\hat{z} \mp \sin(\omega_2^\pm t + \beta_2^\pm x)\hat{y} \right],
$$

(S5a)

The forward and backward propagation coefficients (for a temporal interface between chiral and dielectric media) are denoted as $\Upsilon_{c||d}$ and $\Gamma_{c||d}$, respectively. The backward propagating wave is defined by the negative frequency component after the temporal interface. Interestingly,
both the forward and backward waves keep the same polarization state, resulting in handedness conservation along the temporal interface. Assuming that \( t_0 = 0 \), the fields reduce to

\[
\mathbf{E}^{\pm}_1 = \frac{E_0}{2} \left[ \cos(\beta_1^\pm x) \hat{\mathbf{y}} \mp \sin(\beta_1^\pm x) \hat{\mathbf{z}} \right],
\]

\[
\mathbf{E}^{\pm}_2 = \left( \Upsilon^{\pm}_{c|d} + \Gamma^{\pm}_{c|d} \right) \frac{E_0}{2} \left[ \cos(\beta_2^\pm x) \hat{\mathbf{y}} \mp \sin(\beta_2^\pm x) \hat{\mathbf{z}} \right],
\]

\[
\mathbf{H}^{\pm}_1 = \frac{E_0}{2\eta_{\text{eff}}} \left[ \cos(\beta^\pm x) \hat{\mathbf{z}} \pm \sin(\beta^\pm x) \hat{\mathbf{y}} \right],
\]

\[
\mathbf{H}^{\pm}_2 = \left( \Upsilon^{\pm}_{c|d} - \Gamma^{\pm}_{c|d} \right) \frac{E_0}{2\eta_{\text{eff}}} \left[ \cos(\beta^\pm x) \hat{\mathbf{z}} \pm \sin(\beta^\pm x) \hat{\mathbf{y}} \right].
\]

The spatial frequency is conserved at the temporal interface as no spatial boundaries are introduced, leading to \( \beta^\pm_1 = \beta^\pm_2 \). Then we find that \( \frac{\mathbf{E}^{\pm}_2}{\mathbf{E}^{\pm}_1} = \left( \Upsilon^{\pm}_{c|d} + \Gamma^{\pm}_{c|d} \right) \) and \( \frac{\mathbf{H}^{\pm}_2}{\mathbf{H}^{\pm}_1} = \left( \Upsilon^{\pm}_{c|d} - \Gamma^{\pm}_{c|d} \right) \).

From \( \mathbf{D}^{\pm}_1 = \mathbf{D}^{\pm}_2 \) and \( \mathbf{B}^{\pm}_1 = \mathbf{B}^{\pm}_2 \) we get \( \frac{\mathbf{E}^{\pm}_2}{\mathbf{E}^{\pm}_1} = 1 \mp \Psi_1 \) and \( \frac{\mathbf{H}^{\pm}_2}{\mathbf{H}^{\pm}_1} = 1 \mp \Psi_1 \). Finally, we see that \( \Upsilon^{\pm}_{c|d} + \Gamma^{\pm}_{c|d} = 1 \mp \Psi_1 \) and \( \Upsilon^{\pm}_{c|d} - \Gamma^{\pm}_{c|d} = 1 \mp \Psi_1 \), leading to \( \Gamma^{\pm}_{c|d} = 0 \).

**B. Chiral temporal slab**

To form a chiral temporal slab, another temporal interface between dielectric and chiral media has to take place. The material parameters after the second temporal interface are \( \epsilon_{\text{eq}} = \epsilon_{\text{eff}} (1 \mp \Psi_3) \), \( \mu_{\text{eq}} = \mu_{\text{eff}} (1 \mp \Psi_3) \), and \( g \neq 0 \), where \( \Psi_3 = g\omega^\pm_3 c \). The forward and backward propagation coefficients can be calculated by the Morgenthaler equations, leading to \( \Upsilon^{\pm}_{d|c} = \frac{1}{1 \mp \Psi_3} \) and \( \Gamma^{\pm}_{d|c} = 0 \). The angular frequencies \( \omega^\pm_3 \) can be calculated from the conservation of phase constants \( \beta^\pm_1 = \beta^\pm_2 = \beta^\pm_3 \), where \( \beta^\pm_3 = \omega^\pm_3 \sqrt{\mu_{\text{eff}} \epsilon_{\text{eff}}} (1 \mp \Psi_3^\pm) \). By solving for \( \omega^\pm_3 \) from \( \beta^\pm_2 = \beta^\pm_3 \) we get

\[
\omega^\pm_3 = \pm \frac{1 \pm \sqrt{1 \mp 4g\omega^\pm_2 c}}{2cg},
\]

(S7)
Substituting $\omega_2^\pm = \omega_1 (1 \mp \Psi_1)$ leads to

$$\omega_3^+ = \frac{1}{2} \pm \left| \frac{\Psi_1 - \frac{1}{2}}{cg} \right|,$$  \hspace{1cm} (S8a)

$$\omega_3^- = \frac{1}{2} \pm \left| \frac{\Psi_1 + \frac{1}{2}}{cg} \right|,$$  \hspace{1cm} (S8b)

in which $\pm$ represent the positive and negative branches of the square root. Considering $\Psi_1 < \frac{1}{2}$, we get

$$\omega_3^+ = \begin{cases} \frac{1}{2} + \frac{\Psi_1 - \frac{1}{2}}{cg}, & \text{for negative solution} \\ \frac{1}{2} - \frac{\Psi_1 + \frac{1}{2}}{cg}, & \text{for positive solution} \end{cases} \hspace{1cm} (S9a)$$

$$\omega_3^- = \begin{cases} \frac{1}{2} - \frac{\Psi_1 - \frac{1}{2}}{cg}, & \text{for negative solution} \\ \frac{1}{2} + \frac{\Psi_1 + \frac{1}{2}}{cg}, & \text{for positive solution} \end{cases} \hspace{1cm} (S9b)$$

which simplifies to

$$\omega_3^+ = \begin{cases} \omega_1^+, & \text{for negative solution} \\ \frac{1}{cg} - \omega_1^+, & \text{for positive solution} \end{cases} \hspace{1cm} (S10a)$$

$$\omega_3^- = \begin{cases} -\frac{1}{cg} - \omega_1^-, & \text{for negative solution} \\ \omega_1^-, & \text{for positive solution} \end{cases} \hspace{1cm} (S10b)$$

The chiral composite considered in the main text had chirality parameter $g = 1.0348 \times 10^{-20} \text{ s}^2/\text{m}$ and $\omega_1/(2\pi) = 3 \text{ GHz}$. Hence, the solution $| \pm \frac{1}{cg} - \omega_1^+ |$ results in $\omega_3^+$ being much larger than the resonance frequency. At such frequencies the time-domain Condon model is not applicable. Thus, only solution $\omega_3^+ = \omega_1^+$ is considered and discussed in the main text.
Supplementary Note 3

Effective parameters of uniaxial chiral composite

Here we consider microwave uniaxial chiral media composites in details. Consider a canonical metal-wire left-handed chiral particle, where the particle is formed by two short straight wires (the arm length $l$) connected to an electrically small loop (the loop radius $a$) (Fig. S1). For electrically small lossless wire antennas and loops the input impedances can be approximated as $Z_{\text{wire}} = \frac{1}{j\omega C}$ and $Z_{\text{loop}} = j\omega L$, where $C$ and $L$ are the capacitance of the wire and the inductance of the loop that are approximately equal to (30)

$$C = \frac{\pi l \epsilon}{\ln\left(\frac{2r_0}{a}\right)},$$  
(S11a)

$$L = \mu a \left[\ln\left(\frac{8a}{r_0}\right) - 2\right],$$  
(S11b)

where $\epsilon$ and $\mu$ are the permittivity and permeability of the background medium, and $r_0$ is the radius of the wire. The polarizability dyadics for this particle are expressed as

$$\overline{\alpha}_{ee} = \alpha_{xx}^{ee} \hat{z}\hat{z} + \alpha_{xx}^{ee} \hat{x}\hat{x} + \alpha_{yy}^{ee} \hat{y}\hat{y} + \alpha_{yx}^{ee} \hat{y}\hat{z} + \alpha_{xy}^{ee} \hat{z}\hat{y},$$  
(S12a)

$$\overline{\alpha}_{mm} = \alpha_{xx}^{mm} \hat{z}\hat{z},$$  
(S12b)

$$\overline{\alpha}_{me} = \alpha_{xx}^{me} \hat{z}\hat{z}.$$  
(S12c)

By choosing $2a \ll 2l$, $\alpha_{ee}^{xx}$ and $\alpha_{ee}^{yy}$ become negligible, and the cross-coupling components average to zero (30) in uniaxial composites, thus, the particle can be electrically polarized in the $\hat{z}$ direction only. The axial polarizabilities at frequencies well below the resonance equal to

$$\alpha_{ee}^{zz} = \frac{l^2}{j\omega(Z_{\text{wire}} + Z_{\text{loop}})},$$  
(S13a)

$$\alpha_{mm}^{zz} = \frac{-j\omega l^2 S^2}{(Z_{\text{wire}} + Z_{\text{loop}})},$$  
(S13b)

$$\alpha_{me}^{zz} = \frac{-\mu S l}{(Z_{\text{wire}} + Z_{\text{loop}})},$$  
(S13c)
which are simplified to

\[ \alpha_{ee}^{zz} = \frac{l^2 C}{1 - \omega^2 LC}, \quad (S14a) \]
\[ \alpha_{mm}^{zz} = \frac{\mu^2 \omega^2 S^2 C}{1 - \omega^2 LC}, \quad (S14b) \]
\[ \alpha_{em}^{zz} = -\alpha_{me}^{zz} = \frac{j\omega \mu S l C}{1 - \omega^2 LC}, \quad (S14c) \]

where \( S = \pi a^2 \) is the loop area. Next we estimate the effective material parameters of the uniaxial chiral composite using the Maxwell-Garnett model for mixtures of bianisotropic particles \((30, 31)\). By solving for the transversal components we get

\[ \epsilon_{\text{eff}} = \epsilon + \frac{1}{D_{\text{tr}}} \left[ N \alpha_{ee} - \frac{N^2}{3\mu} (\alpha_{ee} \alpha_{mm} + \alpha_{me}^2) \right], \quad (S15a) \]
\[ \mu_{\text{eff}} = \mu + \frac{1}{D_{\text{tr}}} \left[ N \alpha_{mm} - \frac{N^2}{3\epsilon} (\alpha_{ee} \alpha_{mm} + \alpha_{me}^2) \right], \quad (S15b) \]
\[ \alpha_{\text{eff}} = -\frac{N \alpha_{me}}{D_{\text{tr}}}, \quad (S15c) \]

where \( N \) is the number of particles per unit volume and

\[ \alpha_{ee} = \frac{\alpha_{ee}^{zz}}{2}, \quad (S16a) \]
\[ \alpha_{mm} = \frac{\alpha_{mm}^{zz}}{2}, \quad (S16b) \]
\[ \alpha_{me} = \frac{\alpha_{me}^{zz}}{2}, \quad (S16c) \]
\[ D_{\text{tr}} = \left( 1 - \frac{N \alpha_{ee}}{3\epsilon} \right) \left( 1 - \frac{N \alpha_{mm}}{3\mu} \right) + \frac{N^2 \alpha_{me}^2}{9\epsilon \mu}. \quad (S16d) \]

The uniaxial polarizabilities are divided by 2 as only half of the particles are polarized in one direction. Enough small \( N \) is selected, so that the expressions \( \frac{N^2}{3\mu} (\alpha_{ee} \alpha_{mm} + \alpha_{me}^2) \), \( \frac{N^2}{3\epsilon} (\alpha_{ee} \alpha_{mm} + \alpha_{me}^2) \), and \( \frac{N^2 \alpha_{me}^2}{9\epsilon \mu} \) become negligible. As a result, switching chirality does not affect the effective permittivity and permeability. In addition, well below the particle resonance the magnetic polarizability is negligible being a second-order spatial dispersion effect, and a non-magnetic
Fig. S1: Canonical metal-wire left-handed chiral particle formed by two short straight wires (arm length \( l \)) connected to an electrically small loop (the loop radius \( a \)).

background medium is considered, thus, we have \( \mu_{\text{eff}} = \mu_0 \). Considering these approximations, the effective parameters reduce to

\[
\begin{align*}
\epsilon_{\text{eff}} &= \epsilon + \frac{N\alpha_{\text{ee}}}{1 - \frac{N\alpha_{\text{ee}}}{3\epsilon}}, \\
\mu_{\text{eff}} &= \mu_0, \\
\alpha_{\text{eff}} &= -\frac{N\alpha_{\text{me}}}{1 - \frac{N\alpha_{\text{ee}}}{3\epsilon}}.
\end{align*}
\]
Supplementary Note 4

Extended numerical results

Figure S2 shows the $z$-component of electric field and the $y$- and $z$-components of the magnetic field. All field components get affected by the spin-dependent phenomenon, as expected.
Fig. S2: Extended numerical results for the numerical simulations presented in the main text. (A)-(B) $z$-component of electric field. (C)-(F) $y$- and $z$-components of the magnetic field.