In this paper, we investigate learning temporal abstractions in cooperative multi-agent systems, using the options framework (Sutton et al, 1999). First, we address the planning problem for the decentralized POMDP represented by the multi-agent system, by introducing a common information approach. We use the notion of common beliefs and broadcasting to solve an equivalent centralized POMDP problem. Then, we propose the Distributed Option Critic (DOC) algorithm, which uses centralized option evaluation and decentralized intra-option improvement. We theoretically analyze the asymptotic convergence of DOC and build a new multi-agent environment to demonstrate its validity. Our experiments empirically show that DOC performs competitively against baselines and scales with the number of agents.

Keywords Reinforcement learning; multi-agent learning; cooperative games: theory & analysis; temporal abstraction; common information

Introduction

Temporal abstraction refers to the ability of an intelligent agent to reason, act and plan at multiple time scales [1]. A standard way to include temporal abstraction in reinforcement learning agents is through the options framework [2]. In [3], the authors an approach for learning options, using a gradient-based approach.

Multi-agent systems present challenges due to the exacerbated curse of dimensionality and non-classical information structure. Cooperative multi-agent systems or dynamic team problems are decentralized control problems within which the participating agents share rewards and aim to accomplish a common goal, but have access to different information sets (see [4] and references therein for details). The decentralized nature of the information prevents the use of classical tools in centralized decision theory, such as dynamic programming, convex analytic methods, or linear programming. A common formulation of such systems is given by decentralized Markov Decision Processes (Dec-MDPs) and decentralized partially observable Markov Decision Processes (Dec-POMDPs). Dec-POMDPs offer a very general, sequential, synchronized decision-making framework, but finding the optimal solution for a finite-horizon Dec-POMDP is NEXP-complete, and the infinite-horizon problem is undecidable [5].

Related work

In this paper we study the option framework for a multi-agent system in a cooperative setting [6][7]. The difficulty pertaining to the combinatorial nature of Dec-POMDP can be mitigated by using the common information approach [8], in which the agents share a common pool of information, which they can use in addition to their own private information; a similar idea was presented recently in [9]. However, learning optimal policies in dynamic teams is still quite challenging and updating the common belief in a scalable way is a non-trivial problem. Omidsfahieic et al [10][11] discuss the problem of solving Decentralized Partially Observable Semi-Markov Decision Processes (Dec-POSMDPs), in which, like in the options framework, single time-step transitions are replaced by actions whose duration is stochastic and conditional on the state and action. Makar et al [12] attack the curse of dimensionality in cooperative multi-agent problems using the MAXQ framework for temporal abstraction [13] but their work requires a hand-designed decomposition of the problem based on prior knowledge, whereas we aim to learn this decomposition
We denote vectors by bold script. For any set $C$ as described in [18] the dynamics of the multi-agent system operates in discrete time, as given by:

$$\text{value function}$$

which is completely specified by the transition and observation model and the joint policy [19]. In case of $f$, where $f$ is the immediate reward of choosing action $r$ function (or critic) evaluates the performance of the agents. In this paper, we consider both communications, or always broadcasting) or intermittently (intermittent broadcasting). In a Dec-POMDP, the agents do not have complete knowledge of others’ states (and sometimes even their own states); instead, they share a common information which they update by communicating at every step (cheap talk or always broadcasting) or intermittently (intermittent broadcasting). In the cooperative setting, a centralized value function (or critic) evaluates the performance of the agents. In this paper, we consider both communications, $r^{\omega_j}(s_t)$ is the immediate reward of choosing action $a_t$ in state $s_t$. For reward independent Dec-POMDPs, such as ours, $r^{\omega_j}(s_t) = \sum_{j \in J} R^j(s_t, a_t^j, s_{t+1}^j)$, $p^{\omega}(s_t, s_{t+1})$ is the one-step transition probability from joint-state $s_t$ to $s_{t+1}$ under joint-action $a_t$. $\gamma \in (0, 1)$ is the discount factor.

Temporal abstraction with full observability

In this paper, we consider Markov options which execute in call-and-return way; we will now define these notions in the context of a multi-agent system (see [2] for more details).

In a fully observable multi-agent environment with $J$ agents, a Markov joint-option $\omega$ consists of a vector of component options for each agent, $\omega = (\omega^1, \ldots, \omega^J)$. It can initiate, if no other option is currently executing, at joint-state which is part of its initiation set $s \in \mathcal{I}^\omega$. If $\omega$ is executing at time $t$, it generates joint-action $a_t$ according to $a_t^j \sim \pi_t^j(\cdot | s_t^j)$. The environment then generates next joint-state $s_{t+1}$, where the option $\omega_t^j$ terminates with probability $\beta_t^j(s_{t+1}^j)$.
\[ \beta_{t+1}^{j}(s_{t+1}) \in (0, 1). \] If any of the component options terminates, then the joint option also terminates and a new joint-option has to be chosen. Otherwise, the joint-action selection process continues as above. We will denote by \( \mu \) the policy which chooses joint-options.

Let \( E(\omega_t^{j}, s_t^{j}) \), \( j \in J \) be the event that \( \omega_t^{j} \) is initiated at state \( s_t^{j} \) at time \( t \). Let \( m \) be a random variable indicating the time elapsed since \( t \). Then, the reward of Agent \( j \), \( r_{t}^{\omega_t^{j}}(s_t^{j}) \) until termination of \( \omega_t^{j} \) is:

\[
r_{t}^{\omega_t^{j}}(s_t^{j}) := E \left[ \sum_{\tau=t}^{t+m} \gamma^{\tau-t} R^j(s_t^{j}, a_t^{j}, s_{t+1}^{j}) \mid E(\omega_t^{j}, s_t^{j}) \right], \tag{2}
\]

where \( E[\cdot] \) denotes expectation, \( R^j \) is the reward of agent \( j \), actions \( a_t^{j} \) are generated according to the internal policy \( \pi_t^{\omega_t^{j}} \) of option \( \omega_t^{j} \). For ease of exposition, we write \( r_{t+1}^{j} = R^j(s_t^{j}, a_t^{j}, s_{t+1}^{j}) \). Note that (2) can be expanded recursively as follows:

\[
r_{t}^{\omega_t^{j}}(s_t^{j}) = \beta_{t}^{j}(s_t^{j}) + \gamma(1 - \beta_{t}^{j}(s_t^{j}))r_{t+1}^{j}(s_{t+1}^{j}),
\]

The total reward \( r_\omega(s_t) \) for joint option \( \omega_t = (\omega_1, \ldots, \omega_T) \) is given by

\[
r_\omega(s_t) := \sum_{j \in J} r_{t}^{\omega_t^{j}}(s_t^{j}). \tag{3}
\]

Next, let \( p^{\omega_t^{j}}(s, s') \) denote the probability of choosing joint-option \( \omega_t^{j} \) at state \( s \) and transitioning to state \( s' \), where \( \omega_t^{j} \) terminates, i.e., \( p^{\omega_t^{j}}(s, s') := P(s_{t+1} = s' \mid E(\omega_t^{j}, s_t) = s) \) for any \( t' > t \). Then

\[
p^{\omega_t^{j}}(s, s') := \sum_{m=1}^{\infty} p^{\omega_t^{j}}_{m}(s, s'), \tag{4}
\]

where \( p^{\omega_t^{j}}_{m}(s, s') \) is the probability that a joint-option \( \omega_t^{j} \) initiated in joint-state \( s \) at time \( t \) terminates in joint-state \( s' \) after \( m \) steps.

Let \( \beta^{\omega_t^{j}}_{\text{none}}(s_t) \) be the probability of no agent terminating in joint-state \( s_t \). From the independence of agents we have:

\[
\beta^{\omega_t^{j}}_{\text{none}}(s_t) = \prod_{j \in J} \left( 1 - \beta^{\omega_t^{j}}_{t}(s_t^{j}) \right). \tag{5}
\]

Then, \( p^{\omega_t^{j}}(s, s') \) can be expanded recursively as follows:

\[
p^{\omega_t^{j}}(s, s') = \gamma \sum_{a_t \in A^j} \pi^{\omega_t^{j}}(a_t = a | s_t = s) \sum_{s' \in S} P(s_{t+1} = s' \mid s_t = s, a_t = a) \beta^{\omega_t^{j}}_{\text{none}}(s) p^{\omega_t^{j}}_{m-1}(s', s').
\]

Let \( M \) be the space of Markov option-policies \( \mu_t : S \to \Delta(\Omega) \). We denote \( \mu_t(\omega_t | s_t) = \mu_t(\omega_t | s_t = s) \). Following (2), let \( U_\omega^{\mu_t}(s_t, \omega_t) \) be the option-value upon arrival at joint-state \( s_t \) using option-policy \( \mu_t \):

\[
U^{\mu_t}(s_t, \omega_t) := \beta^{\omega_t^{j}}_{\text{none}}(s_t) Q^{\mu_t}(s_t, \omega_t) + \gamma \left( \sum_{s_{t+1} \in S} p^{\mu_t}(s_t, s_{t+1}) U^{\mu_t}(s_{t+1}, \omega_t) \right),
\]

where we use a slight abuse of notation, \( \omega_t' \), to mean \( \omega_t = \omega_t' \). \( \Omega(T) \) denotes the set of options for agents in \( T \subseteq J \), where \( T \) is the set of the agents terminating their current options.

\( Q^{\mu_t} \) in (6) is the solution of the following Bellman update:

\[
Q^{\mu_t}(s_t, \omega_t) = \sum_{a_t \in A} \pi^{\omega_t^{j}}(a_t | s_t) \left[ r_\omega(a_t) + \gamma \sum_{s_{t+1} \in S} (p^{\mu_t}(s_t, s_{t+1}) U^{\mu_t}(s_{t+1}, \omega_t)) \right],
\]

where \( \pi^{\omega_t^{j}}(a_t | s_t) \) is the shorthand for the action-policy to choose joint-action \( a_t \) under joint-option \( \omega_t \) in joint-state \( s_t \). We denote by \( U^{\mu_t} \) and \( Q^* \) the corresponding optimal values.

The dynamic team problem that we are interested to solve is to choose policies that maximize the the infinite-horizon discounted reward: \( R^{\mu_t} \) as given by

\[
\sup_{\mu_t \in M} \sum_{\omega_t \in \Omega} \mu_t(\omega_t | s_t) E \left[ \sum_{t=0}^{\infty} \gamma^t r_{t+1} \mid E(\omega_0 \mu_0, s_0) \right], \tag{8}
\]

\[1\text{For agents with factored actions such as ours, } \pi^{\omega_t^{j}}(a_t | s_t) = \prod_j \pi^{\omega_t^{j}}(a_t^j | s_t^j). \]
Dec-POMDP planning with temporal abstraction

The Common Information Approach [20] is an effective way to solve a Dec-POMDP in which the agents share a common pool of information, updated eg via broadcasting, in addition to private information available only to each individual agent. A fictitious coordinator observes the common information and suggests a prescription - in our case the Markov joint-option policy \( \mu_t \). The joint-option \( \omega_t \) is chosen from \( \mu_t \) and is communicated to all agents \( j \), who in turn generate their own action \( a_t^j \) according to their local (private) information, and their own observation \( o_t^j \) : \( a_t^j \sim \pi_t^j(a_t^j|o_t^j) \). A locally fully observable agent chooses its action \( a_t^j \) based on its own state \( s_t^j \) or embedding \( e_t^j \) according to \( a_t^j \sim \pi_t^j(a_t^j|s_t^j) \). The notion of a centralized fictitious coordinator transforms the Dec-POMDP into an equivalent centralized POMDP, so one can exploit mathematical tools from stochastic optimization such as dynamic programming to find an optimal solution.

The common information-based belief on the joint-state \( s_t \in S \) is defined as:

\[
b_t^c(s) := P(s_t = s | I_t^c),
\]
where \( I_t^c \) is the common information at time \( t \).

Let \( \text{Broad}(o_t^j, \omega_t^j) = br^j \in \{0, 1\} \) be the broadcast symbol, where \( br^j = 1 \) if Agent \( j \) has broadcast and 0 otherwise. When Agent \( j \) decides to broadcast, its observation \( o_t^j \) is received by all other agents. Hence, the common information is \( \tilde{o}_t = (\tilde{o}_t^1, \ldots, \tilde{o}_t^j) \), where \( \tilde{o}_t^j, j \in J \) is given by

\[
\tilde{o}_t^j := \begin{cases} 
    o_t^j & \text{if Broad}(o_t^j, \omega_t^j) = 1 \\
    \emptyset & \text{otherwise}.
\end{cases}
\]

The coordinator observes \( \tilde{o}_{1:t} \), and generates \( \mu_t \), according to some coordination rule \( \psi \) such that \( \psi : (O \cup \{\emptyset\})^{t-1} \rightarrow \mathcal{M}, j \in J \),

\[
\mu_t = \psi(\tilde{o}_{1:t-1}, \mu_{1:t-1}),
\]

The options \( \omega_t^j \in \omega_t \), \( \omega_t \sim \mu_t \), are then communicated to all agents. Thus, \( I_t^c \) appearing in (9) is given by:

\[
I_t^c = \{\tilde{o}_{1:t-1}, \omega_{1:t-1}\},
\]
and thus, \( I_{t-1}^c \subseteq I_t^c \). Consequently, (9) can be rewritten as:

\[
b_t^c(s) := P(s_t = s | \tilde{o}_{1:t-1}, \omega_{1:t-1}).
\]

Upon receiving \( \omega_t^j \), Agent \( j \) uses the action-policy \( \pi_t^j \) and termination probability \( \beta_t^j \) corresponding to \( \omega_t^j \) and generates its action \( a_t^j \) using its local information \( o_t^j \) as per \( a_t^j \sim \pi_t^j(a_t^j|o_t^j) \).

From (12), \( b_t^c \) is measurable with \( (\tilde{o}_{1:t-1}, \mu_{1:t-1}) \), so also using (11), we can infer that there is no loss of optimality if we restrict attention to coordination rules \( \tilde{\psi} \) such that:

\[
\mu_t = \tilde{\psi}(b_t^c).
\]

The posterior of the common information based belief \( b_t^c \) can then be written as

\[
b_{t+1}^c(s) := P(s_{t+1} = s | \tilde{o}_{1:t}, \omega_{1:t}) = \sum_{s' \in S} p_{s'}(s) b_{t+1}^c(s).
\]

Using the argument of [20] Lemma 1, we can show that the coordinated system is a POMDP with prescriptions \( \mu_t \) and observations

\[
\tilde{o}_t = \tilde{h}_t(s_t, \mu_t),
\]

\footnote{For ease of exposition we use the notation for states but the same analysis applies to the embeddings.}

\footnote{In general there can be finite number of levels of broadcast, instead of binary levels. In this paper we use binary levels since that is sufficient for our purpose but the results are extendable to finite number of levels.}

\footnote{Bayesian filtering applies Bayesian statistics and Bayes’ rule in solving Bayesian inference problems including stochastic filtering problems. Iterative Bayesian learning was introduced by [21] (among others), which involves Kalman filtering as a special case. See [22] and references therein for details.}
Furthermore, define $o_{1:t}^\dagger := \bar{\sigma}_{1:t-1}$. Then:

$$P(o_{t+1}^\dagger \mid o_{1:t}^\dagger, \mu_t) = P(o_{t}^\dagger \mid o_{1:t}^\dagger, \mu_t),$$

(17)

where $o_{1:t}^\dagger$ denotes the realization of the sequence $\bar{\sigma}_{1:t}$, which behaves like a state.

This relies on showing equalities of conditional probability values by shedding off irrelevant information. Note that while computing the conditional probability in (17), the information captured in $o_{1:t}$ and $\bar{\sigma}_{1:t-1}$ is the same. So, $o_{1:t}^\dagger \bar{\sigma}_{1:t-1}$ can be considered redundant and can hence be removed from conditioning. The common-observation $\bar{\sigma}$ depends on the joint-state $s_t$ and the joint option-policy $\mu_t$ (through $\hat{h}_t$). So, when conditioned by $\mu_t, \bar{\sigma}_{1:t-1}$ does not give any additional information about $\bar{\sigma}$ and can thus be removed from conditioning as well.

The optimal policy of the coordinated centralized system is the solution of a suitable dynamic program which has a

$\text{The proof follows an argument similar to [23] for primitive actions. In particular,}$

$\text{1. $P(s_t \mid \bar{\sigma}_{1:t-1}, \omega_{1:t-1}) = P(s_t \mid b_{t-1}^\dagger)$}$

$\text{2. $P(b_{t+1}^\dagger \mid \bar{\sigma}_{1:t-1}, \omega_{1:t-1}) = P(b_{t+1}^\dagger \mid b_{t}^\dagger)$}$

$\text{3. $E[\omega_{t+1} \mid \bar{\sigma}_{1:t-1}, \omega_{1:t}] = E[\omega_{t+1} \mid b_{t}^\dagger, \omega_{t}]$}$

$\text{where $\omega_{t+1}$ is given by (26).}$

$\text{Proof}$

The proof follows an argument similar to [23] for primitive actions. In particular,

$\text{1. The equality of Part 1) readily holds from the fact that $b_t^\dagger$ is measurable by $\bar{\sigma}_{1:t-1}$ and so conditioning by $\bar{\sigma}_{1:t-1}$ is the same as conditioning by $b_t^\dagger$.}$

$\text{2. From (18) we can write}$

$P(b_{t+1}^\dagger \mid \bar{\sigma}_{1:t-1}) = P(s_{t+1} \mid \bar{\sigma}_{1:t-1}) \mathbb{I}(\bar{\sigma}_{1:t-1} = \bar{\sigma}_{1:t-1}).$}

$\text{Then the equality follows from the fact that $b_t^\dagger$ is measurable by $\bar{\sigma}_{1:t-1}$ and that conditioning on $b_t^\dagger$ is same as conditioning on $\bar{\sigma}_{1:t-1}$ (as is shown by part 1).}$

$\text{The optimal policy of the coordinated centralized system is the solution of a suitable dynamic program which has a fixed-point. In order to formulate this program, we need to show that $b_t^\dagger$ is an information state, i.e. a sufficient statistic to form, with the current joint-option $\mu_t$, a future belief $b_{t+1}^\dagger$. In other words:}$

$\text{Lemma 1 The common information based belief state $b_t^\dagger$ is an information state. In particular,}$

$\text{1. $P(s_t \mid \bar{\sigma}_{1:t-1}, \omega_{1:t-1}) = P(s_t \mid b_t^\dagger)$}$

$\text{2. $P(b_{t+1}^\dagger \mid \bar{\sigma}_{1:t-1}, \omega_{1:t-1}) = P(b_{t+1}^\dagger \mid b_t^\dagger)$}$

$\text{3. $E[\omega_{t+1} \mid \bar{\sigma}_{1:t-1}, \omega_{1:t}] = E[\omega_{t+1} \mid b_t^\dagger, \omega_{t}]$}$

$\text{where $\omega_{t+1}$ is given by (26).}$

$\text{Proof}$

The proof follows an argument similar to [23] for primitive actions. In particular,
3. We have by the definition of option
\[ E[r_\omega(s) \mid \tilde{o}_{1:t-1}, \omega] = \sum_{a \in \mathcal{A}} \pi^a(a \mid s) E[r^a(s) \mid \tilde{o}_{1:t-1}] \]
\[ = \sum_{a \in \mathcal{A}} \pi^\omega(a \mid s) \sum_{s_t \in S} r^a(s) P(s_t = s \mid \tilde{o}_{1:t-1}) \]
\[ \overset{(a)}{=} E[r_\omega^*(s) \mid b_t^\omega, \omega], \]
where \((a)\) holds by the definition of \(b_t^\omega\).

This completes the proof of the lemma.

For large systems, the common belief is intractable due to the combinatorial nature of joint state-space. One way to circumvent the combinatorial effect is to assume that the common belief is factored [9], i.e.,
\[ b_t^\omega(s) := P(s_t = s \mid \tilde{o}_{1:t-1}) \approx \prod_{j \in \mathcal{J}} P(s^j_t = s^j \mid \tilde{o}_{1:t-1}) =: \prod_{j \in \mathcal{J}} b_{t,j}^{\omega, \text{fact}}(s^j) =: b_t^\omega, \omega. \] (22)

Note that in situations where collision among agents is allowed, common belief becomes factored.

**Common-belief based option-value**

We can extend the notion of option-value with full observability, given by [6] and [7] to the case with partial observability. The **option-value upon arrival**, \(U^\mu\), and the **option-value**, \(Q^\mu\), are defined below:
\[ U^\mu_t(b_t^\omega, \omega_t) := \sum_{s_t \in S} U^\mu_t(s_t, \omega_t) b_t^\omega(s_t) \]
\[ = \sum_{s_t \in S} \left[ \beta^\omega_\text{none}(s_t) Q^\mu_t(s_t, \omega_t) b_t^\omega(s_t) + (1 - \beta^\omega_\text{none}(s_t)) \max_{T \in \text{Pow}(\mathcal{J})} \max_{\omega_t' \in \Omega(T)} Q^\mu_t(s_t, \omega_t') b_t^\omega(s_t) \right]. \] (23)

\(Q^\mu\) in [25] is the solution of the following Bellman update:
\[ Q^\mu_t(b_t^\omega, \omega_t) := \sum_{s_t \in S} Q^\mu_t(s_t, \omega_t) b_t^\omega(s_t) \]
\[ = \sum_{s_t \in S} \sum_{a_t \in \mathcal{A}} \left( \sum_{b_{t,j} \in \{0, 1\}^J} \sum_{a_t \in \mathcal{A}} \pi^{b_t, \omega}_{t,j}(b_t^\omega | a_t) \pi^{\omega}_{t,j}(a_t | s_t) f_t(a_t, s_t, \omega_t) \right) \]
\[ + \gamma \sum_{s_t+1 \in S} \sum_{b_{t+1}^{\omega}} \left( p^a_{t+1}(s_t, s_{t+1}) U^\mu_t(s_{t+1}, \omega_t') \right) b_t^\omega(s_t), \] (24)

where \(f_t(a_t, s_t, \omega_t)\) is given by [21] and \(p^a_{t+1}(s_t, s_{t+1})\) is the immediate reward of choosing action \(a_t\) and broadcast symbol \(b_{t,j}\) in state \(s_t\). The optimal values corresponding to [23] and [24] are defined as usual.

Define operators \(B^\mu\) and \(B^\ast\) as follows:
\[ [B^\mu Q^\mu_t](b_t^\omega, \omega_t) := \gamma \sum_{s_t \in S} \sum_{a_t \in \mathcal{A}} \left( \sum_{b_{t,j} \in \{0, 1\}^J} \sum_{a_t \in \mathcal{A}} \pi^{b_t, \omega}_{t,j}(b_t^\omega | a_t) \pi^{\omega}_{t,j}(a_t | s_t) \right) \]
\[ f_t(a_t, s_t, \omega_t-1) \sum_{s_t+1 \in S} b_t^{\omega}(s_t+1) \left( p^a_{t+1}(s_t, s_{t+1}) U^\mu_t(s_{t+1}, \omega_t') \right) b_t^\omega(s_t) \],
\[ [B^\ast Q^\ast](b_t^\omega, \omega_t) := \gamma \sum_{s_t \in S} \sum_{a_t \in \mathcal{A}} \left( \sum_{b_{t,j} \in \{0, 1\}^J} \sum_{a_t \in \mathcal{A}} \pi^{b_t, \omega}_{t,j}(b_t^\omega | a_t) \pi^{\omega}_{t,j}(a_t | s_t) \right) \]
\[ f_t(a_t, s_t, \omega_t-1) \sum_{s_t+1 \in S} b_t^{\omega}(s_t+1) \left( p^a_{t+1}(s_t, s_{t+1}) \max_{\omega_t' \in \Omega(T)} U^\mu_t(s_{t+1}, \omega_t') \right) b_t^\omega(s_t) \].

Then, \(Q^\mu\) and \(Q^\ast\) can be rewritten as
\[ Q^\mu_t(b_t^\omega, \omega_t) = r^\omega_t(b_t^\omega) + [B^\mu Q^\mu_t](b_t^\omega, \omega_t), \quad Q^\ast(b_t^\omega, \omega_t) = r^\omega_t(b_t^\omega) + [B^\ast Q^\ast](b_t^\omega, \omega_t), \] (25)
where
\[ r^{\omega_t}(b_t^*) := \sum_{s_t \in S} \sum_{o_t \in O} \sum_{b_{t+1} \in \Delta(S)} \sum_{a_t \in A} \pi_t^{\omega_t}(s_t | o_t, b_{t+1}) \pi_t^{\omega_t}(a_t | o_t) r^{a_t, b_{t+1}}(s_t, o_t, \omega_{t-1}) b_{t+1}^*(s_t). \] (26)

**Lemma 2** The operators \( B^{*} \) and \( B^{\mu_t} \) are contractions. In particular, for any \( \gamma \in (0, 1) \),
\[ \|B^{\mu_t} Q^{\mu_t}\|_\infty \leq \gamma \|Q^{\mu_t}\|_\infty, \quad \|B^{*} Q^{*}\|_\infty \leq \gamma \|Q^{*}\|_\infty \]
where \( \| \cdot \|_\infty \) is the sup-norm.

**Proof** We prove the contraction of \( B^{*} \). That \( B^{\mu_t} \) is a contraction can be shown similarly.

We begin by noting that the supremum in the definition of the sup-norm can be replaced by maximum since \( S \) is finite. Then, we have
\[ \|B^{*} Q^{*}\|_\infty \]
\[ = \gamma \max_{b_t^* \in \Delta(S)} \max_{s_t \in S} \sum_{a_t \in A} \pi_t^{\omega}(a_t | s_t) \times \left( \sum_{s_{t+1} \in S} b_{t+1}^*(s_{t+1}) p(a_t, s_{t+1}) U_t^*(s_{t+1}, o_t) \right) b_t^*(s_t) \]
\[ \leq (a) \max_{b_t^* \in \Delta(S)} \|Q^{*}\|_\infty \sum_{a_t \in A} \sum_{s_t \in S} \pi_t^{\omega}(a_t | s_t) \times \left( \sum_{s_{t+1} \in S} b_{t+1}^*(s_{t+1}) p(a_t, s_{t+1}) \right) b_t^*(s_t) \]
\[ \leq (b) \gamma \|Q^{*}\|_\infty, \]
where (a) follows from (15)–(16) by using Cauchy-Schwartz inequality and the definition of sup-norm. (b) holds due to the fact that \( \sum_{a_t \in A} \sum_{s_t \in S} \pi_t^{\omega}(a_t | s_t) \left( \sum_{s_{t+1} \in S} b_{t+1}^*(s_{t+1}) p(a_t, s_{t+1}) \right) b_t^*(s_t) \leq 1 \). The last inequality implies contraction since \( \gamma \in (0, 1) \). This completes the proof of the lemma.

Because \( B^{\mu_t} \) and \( B^{*} \) are contractions, (25) has a unique solution. Furthermore, since \( r^{a_t, b_{t+1}} \) is bounded, so is \( r^{\omega_t} \) and consequently so is \( Q^{*} \).

**Main result 1: Dynamic program**

The main result of this section is given by the following theorem, which provides a suitable dynamic program for the infinite horizon discounted reward dynamic program problem and establishes the optimality of the joint-option policy.

**Theorem 1** For the J-agent Dec-POMDP described above

1. The optimal state-value is the fixed point solution of the following dynamic program.
\[ V^{*}(b_t^*) := \max_{\mu_t \in \mathcal{M}^+} \sum_{\omega_t \in \Omega} \mu_t(\omega_t | b_t^*) \left[ r^{\omega_t}(b_t^*) + \gamma \sum_{\bar{a}_t \in \mathcal{O} \cup \{\emptyset\}} P(\bar{a}_t | b_t^*, \omega_t) V^{*}(b_{t+1}^*) \right], \] (27)
where \( \mathcal{M}^+ \) is the space of joint option-policies; \( r^{\omega_t}(b_t^*) \) is given by (26), \( P(\bar{a}_t | b_t^*, \omega_t) \), as given by (20) is the observation-model and \( b_{t+1}^* \) is given by (15).

2. Let \( \mathcal{M} \) denote the space of Markov joint-option policies. Then, there exists a time-homogeneous Markov joint-option policy \( \mu^{*} \in \mathcal{M} \) which is optimal, i.e.,
\[ \mu^{*} = \arg \max_{\mu_t \in \mathcal{M}} V^{\mu_t}(b_t^*), \]
where \( V^{\mu_t} \) is given by:
\[ V^{\mu_t}(b_t^*) = \sum_{\omega_t \in \Omega} \mu_t(\omega_t | b_t^*) \left[ r^{\omega_t}(b_t^*) \right] + \gamma \sum_{\bar{a}_t \in \mathcal{O} \cup \{\emptyset\}} P(\bar{a}_t | b_t^*, \omega_t) V^{\mu_t}(b_{t+1}^*). \] (28)

Then, \( V^{*}(b_t^*) = V^{\mu_t}(b_t^*) \), and furthermore, \( \mu^{*} \) is obtained using the common belief \( b_t^* \).
We are interested in individual agents learning independent policies and so we concentrate on learning the best factored which is polynomial in $J$. As a consequence of Theorem 1, we can now consider time-homogeneous Markov option policies (embedding) of all agents inferred from the common belief $\omega$ value by replacing its own component of the current joint option is a pure stationary (time-homogeneous) strategy that is optimal, for every discount factor close enough to one. An extension of $Q$ and intra-option value $s$ component of the common information (the joint-embedding option improvement distributed $\omega$ components).

Once any agent step, the centralized critic (coordinator) evaluates in agents. The agents learn to complete a cooperative task by learning in a model-free way. In the architecture [3], leverages the assumption of Distributed Option Critic (DOC; see Algo.1), employs the modified critic value uses a manner [26] the performance of all agents modified critic value uses a greedy manner using a joint-state (DOC; see the central option-critic $V$ option-value $\pi$ and $b_j$. Thus, we have:

$$V^*(b^*_j) = \max_{b \in M^{\psi,b^*_j}} V^\mu(b^*_j).$$

which completes the proof. 

As a consequence of Theorem 1 we can now consider time-homogeneous Markov option policies $\mu$. Subsequently, we use $\pi^\mu$, $\pi^\mu,b^\omega$ and $\beta^\mu$ in the rest of the paper.

Note that planning with a factored common belief reduces the exponential computation complexity to polynomial. Let the cardinality of a finite factored state space $S = S^1 \times \cdots \times S^J$ is $|S| = \prod_{j \in J} |S_j|$. Similarly, let the cardinality of a finite factored action space $A = A^1 \times \cdots \times A^J$ is $|A| = \prod_{j \in J} |A_j|$. Then, at each iteration of policy iteration the computational complexity is $O(|S|^2|A|(|S| + |A|))$, which is exponential in the number of agents $J$. In contrast, with factored agents and belief, the computational complexity becomes $O((|S|^2|A|)((J-1)|S| + J|A|))$ for fixed $j$, which is polynomial in $J$ and thus scalable.

Learning in Dec-POMDPs with options

We are interested in individual agents learning independent policies and so we concentrate on learning the best factored actor for a domain, even if it is suboptimal in a global sense. Also, for ease of readability, in this section we use full observability in the derivations, i.e., $\pi^\mu(b^\omega|s_t) = \pi^\mu(b^\omega|s^t_t)$ and $\pi^\omega(a_t|s_t) = \pi^\omega(a_t|s^t_t)$. However, our results hold even if the agents are not locally fully observable, where the observations depend on the states probabilistically (as is discussed in the previous sections) or deterministically (e.g., state-embedding as we use in our experiments).

Our proposed algorithm for learning options, Distributed Option Critic (DOC; see Algo.1), employs the option-critic architecture [3], leverages the assumption of factored actions of agents in the distributed policy updates and utilizes the sufficient statistic of common information to learn the critic. The centralized option evaluation is presented from the coordinator’s point of view, where the centralized option-value $Q$ and intra-option value $Q_{\text{intra}}$ using a joint-state (embedding) of all agents inferred from the common belief and the common information of options and actions of all agents. The agents learn to complete a cooperative task by learning in a model-free way. In the centralized option evaluation step, the centralized critic (coordinator) evaluates in temporal difference manner [26] the performance of all agents via a shared reward (plus a broadcast penalty in case of costly communication) using the common information. Once any agent $j$ terminates its own option $\omega^j$, it chooses a new option in a greedy manner using a modified critic value by replacing its own component of the current joint option $\omega$ with a new available option and keeping all other components $\omega^{-j}$ unchanged.

Each agent updates its parameterized intra-option policy, broadcast policy and termination function through distributed option improvement. In order to learn their policies, each agent $j$ uses a modified critic value, obtained by replacing $j$-th component of the common information (the joint-embedding $s_a$ sampled from the common belief $b^*_j$ in the critic $Q$ and intra-option value $Q_{\text{intra}}$ with their private information (their own embedding $s^t_j$) and keeping all other components $s^{-j}$ intact.

---

5 Blackwell optimality [24] states that, in any MDP with finitely many states, finitely many actions and discounted returns, there is a pure stationary (time-homogeneous) strategy that is optimal, for every discount factor close enough to one. An extension of Blackwell optimality holds for discounted infinite horizon POMDPs. See [25] Theorem 2.6.1 for details.

6 Algorithms for COE and DOI are in the appendix.
Algorithm 1: Distributed Option Critic (DOC)

**Input:** Set of goals $G$; broadcast penalty $B$ (for intermittent broadcast); learning rates $\alpha_\theta$, $\alpha_\epsilon$, $\alpha_\phi$, and $\alpha_Q$; pool of options $\Omega$; number of episodes $N_{\text{epi}}$.

**Output:** Estimate $Q$ of the optimal option-value $Q^*$

1. for episode in $N_{\text{epi}}$ do
   1. initialize: pool of available options $\Omega_{\text{avail}} = \Omega$; initial common belief $b_0$ (or initial common information $I_0$); parameters $\theta_j, \epsilon_j$, and $\phi_j, j \in J$
   2. for iteration $k = 1$ up to end of episode do
      3. Choose joint-option $\omega$ based on softmax or epsilon-greedy option-policy $\mu$. Denote the true current joint-state by $s$. Choose action $a_k = (a_1^k, \ldots, a_J^k)$ in true current joint-state $s$; $a_j^k \sim \pi_{\omega_j}^{\theta_j}, \theta_j$. Take a step through environment and get a reward $r$.
      4. Sample broadcast action $b^j, j \in J$ (for intermittent broadcast; otherwise $b^j = 1$).
      5. Get a new joint-observation $\tilde{o}_k$.
      6. Do a centralized option-value evaluation to compute $Q$. If an agent’s option terminates, choose a new option greedily keeping all other agents’ options frozen.
      7. Update action-policy, broadcast-policy and termination parameters ($\theta_j, \epsilon_j, \phi_j$) using distributed option improvement. To do so, replace component $s^j_k$ of $s_k \sim b^j_k$ by $s^j$, keep all other components $s_k^j$ frozen and plug into $Q$.
   8. return $Q$

---

Figure 1: TEAMGrid environments: (a) FourRooms, (b) Switch and (c) DualSwitch.

The action-policy, broadcast-policy and the termination function of Agent $j$ are parameterized by $\theta_j, \epsilon_j$ and $\phi_j$ respectively and are learnt in distributed manner in the Distributed Option Improvement step of DOC, through stochastic gradient descent.

**Main result 2: Convergence of DOC**

Using arguments for the convergence of the policy-gradient based algorithms (e.g., [27]) and the local optima achieved by distributed stochastic gradient descent [28 Theorem 1], we can show that DOC converges to the optimal option-value $Q^*$. The proof relies on first arguing that for factored agents, the distributed stochastic gradient leads to local optima in the dynamic cooperative game, and then showing that the expected value of the option-value update in DOC is a contraction, leading to convergence to the optimal option-value. We first state the following lemma.

**Lemma 3** Distributed gradient descent in a cooperative Dec-POMDP with options and with factored agents leads to local optima.

**Proof sketch:** According to [28 Theorem 1], for factored agents, distributed gradient descent is equivalent to joint gradient descent and thus achieves local optima. Then the lemma follows by [28 Theorem 1] due to the fact that DOC is a distributed gradient descent and so it leads to local optima.

**Proof (Convergence of DOC)** We now show that for the learning problem, intra-option $Q$-learning using common belief converges almost surely to the optimal $Q$-values, $Q^*$, for every joint-option $\omega_k \in \Omega$, regardless of what options are executed during learning, provided that every action gets executed in every state infinitely often. For every joint-option $\omega_k$, a joint-action $a_k$ and broadcast $b_{rk}$ is chosen according to action-policy $\pi^{\omega_k}$ and broadcast policy $\pi^{b_{rk}}$ respectively and then an off-policy one-step TD update is executed as follows.

$$Q(s_k^j, \omega_k) = Q(s_k^j, \omega_k) + \alpha Q \delta,$$
Figure 2: TEAMGrid results. (a) FourRooms average returns with 2 agents and 3 goals, (b) FourRooms average returns with 3 agents and 5 goals (c) FourRooms DOC: increasing number of options improved average returns, (d) FourRooms DOC average returns with always broadcasting (broadcast penalty 0.0) and intermittent broadcasting (broadcast penalty = -0.5), (e) Switch average returns, (f) DualSwitch average returns.

Figure 3: Intermittent broadcast: increase in broadcast penalty reduces the frequency of broadcast.

where \( \delta \) is the TD-error given by

\[
\delta = r^{\omega_k}(s) + \gamma U(s_{k+1}, \omega_k) - Q(s_k, \omega_k),
\]

where \( s \) is the true joint-state. At each step \( k \), the joint-states \( s_k \) and \( s'_{k+1} \) are sampled from the common beliefs \( b^k_c \) and \( b^k_{c+1} \) respectively. First we show that the expected value of \( \delta \) equals \( r^{\omega_k}(b^k_c) + \gamma E[U(b^k_{c+1}, \omega_k) | b^k_c] - Q(b^k_c, \omega_k) \).

Note that by definition as given by (15), \( b^k_{c+1} \) gives the belief of the true next joint-state \( s' \). Then, we have

\[
E[\delta | b^k_c] = \sum_{s \in S} \sum_{b^k_r \in \{0,1\}} \sum_{a_k \in A} \pi^{b^k_r, \omega_k}(br_k | s) \pi^{\omega_k}(a_k | s) r^{a_k, br_k}(s) b^k_c(s)
\]

\[
+ \sum_{s_k \in S} \sum_{b^k_r \in \{0,1\}} \sum_{a_k \in A} \pi^{b^k_r, \omega_k}(br_k | s_k) \pi^{\omega_k}(a_k | s_k) \left[ \gamma \sum_{s' \in S} p^{a_k}(s_k, s') U(s', \omega_k) - Q(s_k, \omega_k) \right] b^k_c(s_k)
\]

\[
\stackrel{(a)}{=} r^{\omega_k}(b^k_c) + \gamma E[U(b^k_{c+1}, \omega_k) | b^k_c] - Q(b^k_c, \omega_k),
\]

where \( (a) \) holds by the definitions of \( r^{\omega_k}(s), b^k_{c+1}, U(b^k_{c+1}, \omega_k) \) and \( Q(b^k_c, \omega_k) \).
Next, note that the by definition of intra-option $Q$-learning with full observability (e.g. see [2] Theorem 3), we have that for any $\varepsilon \in \mathbb{R}_{>0}$,
\[
\max_{s',\omega'} | Q(s'',\omega'') - Q^*(s'',\omega'') | < \varepsilon. \tag{29}
\]
The rest of the proof follows by showing that the expected value of $r^\omega b_k^t(s) + \gamma U(s_k^t+1, \omega_k)$ converges to $Q^*$, which is given as follows. For ease of exposition, we drop the subscript $k$ everywhere except for common beliefs in the following derivation.
\[
| r^\omega (b_k^t) + \gamma E[U(b_k^{t+1}, \omega)] - b_k^t | = \gamma \sum_{b_r \in \{0,1\}} \sum_{a \in A} \sum_{s' \in S} \pi^{b_r,\omega}(br|s)\pi^\omega(a|s) \left( \sum_{s'' \in S} p^a(s, s') \left[ \beta^\omega_{none}(s') \left( Q(s', \omega) - Q^*(s', \omega) \right) + (1 - \beta^\omega_{none}(s')) \left( \max_{T \in \text{Pow}(J), \omega' \in \Omega_{avail}(T)} Q(s', \omega') - \max_{T \in \text{Pow}(J), \omega' \in \Omega_{avail}(T)} \max_{s'' \in S} Q^*(s'', \omega'') \right) \right] b_k^t(s) \right) \leq \gamma \sum_{s \in S} \sum_{b_r \in \{0,1\}} \sum_{a \in A} \pi^{b_r,\omega}(br|s)\pi^\omega(a|s) \left( \sum_{s'' \in S} p^a(s, s') \left[ \max_{s'' \in S} Q^*(s'', \omega'') \right] b_k^t(s) \right) \leq \varepsilon \gamma.
\]
Note that since $J$ is finite, so is $\text{Pow}(J)$. Consequently, (a) holds since maximum over a finite set is bounded and since maximum over real line is convex. (b) holds by (29) and (c) holds since for fixed $a$ and $s$, $\sum_{s' \in S} b_k^t(s') p^a(s, s') \leq 1$; for a fixed $s$, $\sum_{br \in \{0,1\}} \sum_{a \in A} \pi^{b_r,\omega}(br|s)\pi^\omega(a|s) \leq 1$ and $\sum_{s \in S} b_k^t(s) = 1$. The last inequality implies convergence since $\varepsilon$ can be arbitrarily small and $\gamma \in (0,1)$.

The convergence of intra-option $Q$-learning in teams along with Lemma 3 ensures that the option-value $Q$ obtained by DOC converges to the optimal option-value $Q^*$.

**Experiments**

We empirically evaluate the merits of DOC in cooperative multi-agent tasks, and compare it to its single-agent counterpart, Option-Critic (OC), Advantage Actor-Critic (A2C) and Proximal Policy Optimization (PPO). We also extend A2C to the multi-agent setting by incorporating a centralized critic that uses common information. To the best of our knowledge, this is equivalent to Counterfactual Multi-Agent Policy Gradients (COMA) when the individual agents broadcast at every time step. We call this algorithm A2C2.

In all of our experiments, we use deep neural networks for actors and critics. These networks use two linear layers with 64 hidden units and tanh activation. The memory for both the actor and the critic are implemented with Long Short-Term Memory (LSTM) cells [29] as they allow a natural way to incorporate observations into a latent state. All neural networks were optimized using RMSProp [30] and Adam [31], which use adaptive learning rates for stochastic gradient descent. Experiments were run using CPU cores and the mean and variances were computed using 3 to 5 seeds over 40000 episodes (equiv. to 2-million frames) on TEAMGrid.

**TEAMGrid: A multi-agent extension of Minigrid**

We created TEAMGrid environments that extend Minigrid [17] to incorporate the multi-agent setting. An illustration of our environment is given in (Fig. 1). In each environment (a,b and c), the bigger frame to the right displays the global perspective of the environment where we see agents (triangles), goals (circles) and switches (yellow squares on the walls). The smaller frames to the left of each environment displays the local perspective of each agent’s field of view. Similar to Minigrid [17], the states of the agents are their positions, their observations are the cells within their fields of view and the available actions are Left, Right, Forward, Toggle, Pickup, Drop. The agents collect sparse rewards upon completing the task (e.g., discovering goals, picking up a ball, toggling a switch).

In TEAMGrid-FourRooms (Fig. 1), several agents try to find one or more goals while avoiding collision with each other (collision incurs a penalty). In TEAMGrid-Switch (Fig. 1), two agents are placed in two rooms. There is a goal...
object in the room on the right. The room on the right is dark until the switch in the room on the left is turned on. To maximize efficiency, one agent should go in the room on the right while the other turns on the switch in the room on the left. In TEAMGrid-DualSwitch (Fig. [1]), two agents are placed in two rooms, each with a switch and a goal. When one agent turns on a switch, the goal in the other room appears. The task is to get all goals.

Results

Fig. [2],b show that DOC outperforms the baselines in FourRooms, demonstrating the benefit of options in this environment. We also notice that DOC’s performance isn’t affected when we scale up the number of agents and goals. This is in contrast with A2C2’s performance. In Switch (Fig. [2]) DOC outperforms OC. In fact, OC never shows an increase in reward since this environment requires cooperation whereas DOC manages to capture this. In DualSwitch (Fig. [2]), PPO outperforms DOC. We suspect that since the agents act simultaneously, it was not particularly beneficial for temporal abstraction, the merit of which is mostly reflected in sequential action execution. However, PPO’s performance fluctuates quite significantly (we believe this is because the agents explore independently to get to the goal without any communication among themselves), while DOC performs competitively and its performance is comparatively more consistent. Both PPO and DOC do significantly better than A2C2 and OC.

We also ran experiments to investigate the effect that applying a broadcast penalty will have on both the frequency of broadcasts and performance. We find that the agents’ performance were heavily attenuated by intermittent broadcasting compared to always broadcasting (Fig. [2]d). Communication in distributed networks is a challenging problem mainly due to the losses in the channels and there exits a fundamental trade-off between communication cost and the estimation accuracy. The optimality of distributed communication depends to a great extent on generating a reliable estimate of the information states of the agents. Our agents learn to broadcast using estimates of others’ embeddings based on the common information and using the broadcast penalty as the feedback signal. In FourRooms, they were communicating 61% of times as opposed to 74% of times when the broadcast penalty changed from -0.01 to -0.5, as is shown in Fig. [3].

Finally, we study the effect of changing the number of options for DOC (Fig. [2]). We see that increasing in the number of options improves the overall performance. Interestingly, in practice we notice that changes to the number of options should be met with proportional changes to the amount exploration (i.e. entropy regularization) to see these improvements in performance. We believe this is due to the fact that having more options allows agents to increase the amount of targeted learning however to learn these abstract targets, more exploration in necessary.

Conclusion

In this paper, we extend the options framework for temporal abstraction to Dec-POMDPs for cooperative multi-agent systems. We leverage the common information approach in tandem with temporal abstraction and use it to convert the Dec-POMDP to an equivalent POMDP. We then show that the corresponding planning problem has a unique solution. We also propose DOC, a model free algorithm for learning options. We show that DOC leads to local optima and analyze its asymptotic convergence. The implication of Lemma 3 and the convergence of DOC is that DOC results in local optima \( \omega^* := (\omega^*_1, \ldots, \omega^*_J) \), where \( \omega^*_j \) is achieved by \( \pi^*_j, \pi^{b,j}_* \) and \( \beta_j^* \). We create a platform for gridworld environments facilitating multi-agent framework. Finally, our empirical results show that DOC performs competitively against the baselines.

As a future work, we would like to compare our method with the contemporary research on multi-agent temporal abstraction, some of which we have mentioned in the introduction. Also, we aim to test the performance of DOC in other environments suitable for multi-agent setting. Lastly, communication in a distributed environment is hard due to unreliability of the communication channels (e.g., packet drops in the channels) and so learning to communicate optimally is a non-trivial problem by itself. In our work the agents learn to broadcast to all other agents using a broadcast penalty. Learning to communicate only to neighbors, learning some characteristics of the channel (e.g. probability of packet drops) and communication with partial knowledge of the channel (e.g. with some side information about the channel) are interesting areas of future research.

Appendix: Centralized option evaluation and distributed option improvement

Algo. 2 describes centralized option evaluation and Algo. 3 describes distributed option improvement using policy gradient method.
Update the action policy, broadcast policy and termination parameters as follows. For all \( Q \):

Compute the TD-error \( \delta \) as follows:

\[
\delta = r.
\]

If \( s' \) is not terminal, do:

\[
\delta \leftarrow \delta + U(s'_k, \omega),
\]

where

\[
U(s'_k, \omega) := \beta_{\text{none}}^{\omega}(s')Q(s'_k, \omega) + (1 - \beta_{\text{none}}^{\omega}(s')) \max_{T \in \text{Pow}(T)} \max_{\omega' \in \Omega(T)} Q(s'_k, \omega'),
\]

where \( \Omega(T) \) denotes the set of options for agents in \( T \subseteq J \).

\( Q(s'_k, \omega') \) is given by:

\[
Q(s'_k, \omega') := \sum_{a_k \in A} \pi^{\omega'}(a_k | s_k') Q_{\text{intra}}(s'_k, \omega', a_k),
\]

and \( Q_{\text{intra}}(s'_k, \omega', a_k) \) is given by:

\[
Q_{\text{intra}}(s'_k, \omega', a_k) := r^{a_k}(s'_k) + \gamma \sum_{s'' \in S} p^{a_k}(s'_k, s'') U(s'', \omega').
\]

Update \( \delta \) as follows:

\[
\delta \leftarrow \delta - Q_{\text{intra}}(s_k, \omega, a_k)
\]

Update \( Q_{\text{intra}}(s_k, \omega, a_k) \) as follows:

\[
Q_{\text{intra}}(s_k, \omega, a_k) \leftarrow Q_{\text{intra}}(s_k, \omega, a_k) + \alpha Q \delta
\]

return \( Q_{\text{intra}}(s_k, \omega, a_k), Q(s_k, \omega) \)

---

**Algorithm 3: Distributed Option Improvement (DOI)**

function DOI(\( s_k, o, a_k, Q_{\text{intra}}, Q \))

Input : \( r, s_k, o \) and \( a_k; Q_{\text{intra}}, Q \)

Output : \( \theta^j, e^j, \phi^j \) for \( j \in J \)

Update the action policy, broadcast policy and termination parameters as follows. For all \( j \in J \),

\[
\theta^j \leftarrow \theta^j + \alpha_\theta \frac{\partial \log \pi^{\omega_i, \theta}(a_k^j | s_k^j)}{\partial \theta^j} Q_{\text{intra}}^j(s_k^j, \omega^j, a_k^j, br_k^j)
\]

\[
e^j \leftarrow e^j + \alpha_e \frac{\partial \log \pi^{\omega^j, \epsilon^j}(br_k^j | s_k^j)}{\partial \epsilon^j} Q_{\text{intra}}^j(s_k^j, \omega^j, a_k^j, br_k^j)
\]

\[
\phi^j \leftarrow \phi^j + \alpha_\phi \frac{\partial \beta^{\omega^j, \phi^j}(s_j^j)}{\partial \phi^j} \left( Q^j(s_k^j, \omega^j) - \max_{\omega^j \in \Omega} Q^j(s_k^j, \omega^j) \right)
\]

If \( \beta^{\omega^j, \phi^j} \) terminates in \( s_j^j \) then choose a new \( o^j \) according to \( \mu \) (softmax or \( \epsilon \)-greedy)

return \( \theta^j, e^j, \phi^j \) for \( j \in J \)
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