Since January 2020 Elsevier has created a COVID-19 resource centre with free information in English and Mandarin on the novel coronavirus COVID-19. The COVID-19 resource centre is hosted on Elsevier Connect, the company's public news and information website.

Elsevier hereby grants permission to make all its COVID-19-related research that is available on the COVID-19 resource centre - including this research content - immediately available in PubMed Central and other publicly funded repositories, such as the WHO COVID database with rights for unrestricted research re-use and analyses in any form or by any means with acknowledgement of the original source. These permissions are granted for free by Elsevier for as long as the COVID-19 resource centre remains active.
Integrated neuro-swarm heuristic with interior-point for nonlinear SITR model for dynamics of novel COVID-19

Muhammad Umar, Zulqurnain Sabir, Muhammad Asif Zahoor Raja, Fazli Amin, Tareq Saeed, Yolanda Guerrero Sanchez

PII: S1110-0168(21)00047-8
DOI: https://doi.org/10.1016/j.aej.2021.01.043
Reference: AEJ 2079

To appear in: Alexandria Engineering Journal

Received Date: 26 December 2020
Revised Date: 20 January 2021
Accepted Date: 23 January 2021

Please cite this article as: M. Umar, Z. Sabir, M. Asif Zahoor Raja, F. Amin, T. Saeed, Y. Guerrero Sanchez, Integrated neuro-swarm heuristic with interior-point for nonlinear SITR model for dynamics of novel COVID-19, Alexandria Engineering Journal (2021), doi: https://doi.org/10.1016/j.aej.2021.01.043

This is a PDF file of an article that has undergone enhancements after acceptance, such as the addition of a cover page and metadata, and formatting for readability, but it is not yet the definitive version of record. This version will undergo additional copyediting, typesetting and review before it is published in its final form, but we are providing this version to give early visibility of the article. Please note that, during the production process, errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

© 2021 Production and hosting by Elsevier B.V. on behalf of Faculty of Engineering, Alexandria University.
Integrated neuro-swarm heuristic with interior-point for nonlinear SITR model for dynamics of novel COVID-19

Muhammad Umar\textsuperscript{1, a}, Zulqurnain Sabir\textsuperscript{1, b}, Muhammad Asif Zahoor Raja\textsuperscript{2, 3, c}, Fazli Amin\textsuperscript{1, d}, Tareq Saeed\textsuperscript{4, e}, Yolanda Guerrero Sanchez\textsuperscript{5, f}

\textsuperscript{1}Department of Mathematics and Statistics, Hazara University, Mansehra, Pakistan
\textsuperscript{2}Future Technology Research Center, National Yunlin University of Science and Technology, 123 University Road, Section 3, Douliou, Yunlin 64002, Taiwan, R.O.C.
\textsuperscript{3}Department Electrical and Computer Engineering, COMSATS University Islamabad, Attock Campus, Attock 43600, Pakistan
\textsuperscript{4}Nonlinear Analysis and Applied Mathematics (NAAM)-Research Group, Department of Mathematics, Faculty of Science, King Abdulaziz University, 80203, Jeddah 21589, Saudi Arabia
\textsuperscript{5}Department of Dermatology, Stomatology, Radiology and Physical Medicine, University of Murcia, Spain

Abstract: The present study is related to present a novel design of intelligent solvers with a neuro-swarm heuristic integrated with interior-point algorithm (IPA) for the numerical investigations of the nonlinear SITR fractal system based on the dynamics of a novel coronavirus (COVID-19). The mathematical form of the SITR system using fractal considerations defined in four groups, ‘susceptible ($S$)’, ‘infected ($I$)’, ‘treatment ($T$)’ and ‘recovered ($R$)’. The inclusive detail of each group along with the clarification to formulate the manipulative form of the SITR nonlinear model of novel COVID-19 dynamics is presented. The solution of the SITR model is presented using the artificial neural networks (ANNs) models trained with particle swarm optimization (PSO), i.e., global search scheme and prompt fine-tuning by IPA, i.e., ANN-PSOIPA. In the ANN-PSOIPA, the merit function is expressed for the impression of mean squared error applying the continuous ANNs form for the dynamics of SITR system and training of these networks are competently accompanied with the integrated competence of PSOIPA. The exactness, stability, reliability and prospective of the considered ANN-PSOIPA for four different forms is established via the comparative valuation from of Runge-Kutta numerical solutions for the single and multiple executions. The obtained outcomes through statistical assessments verify the convergence, stability and viability of proposed ANN-PSOIPA.

Keywords: COVID-19; SITR system; Artificial neural networks; Treatment; Reference solutions; Particle swarm optimization; Diseases; Interior-point algorithm.

1. Introduction
The humanity faced numerous obstacles and challenges in the form of earthquake, diseases and floods, etc. To mention few spreading and dangerous diseases, like dengue feve caused by mosquito borne. It extensively spreads in the Southeast/South Asia, African countries, South/Middle America, Oceania areas, the Caribbean and the Eastern Mediterranean. It is one of the serious and perilous virus that wrapped half million people per year, approximately, but the recovered rate of this disease is so high and the majority of the illness people recovered in a number of days. Some key signs of this disease are vomiting, high fever, pain in joints and headache. Some other main infectious diseases are HIV, Lassa and Ebola that have achieved the supreme interest of the researchers. Therefore, numerous medical functions have been applied to avoid the spread of such dangerous diseases.

The world is currently facing a dangerous, deathly and people-to-people dispersal coronavirus (COVID-19) disease, which spreads respirational infection in an almost unrestrained way. This deathly disease is extremely transportable from one to another person via droplets [1]. A number of people diseased from this deathly COVID-19 with highly recovered ratio. The first case of COVID-19 is reported on December 31, 2019 in the province Hubei, Wuhan city, China [2]. At present, no specific vaccines, medicine or treatment for this virus and all the tricks have been failed to control the risk of this deathly virus. The government of China locked down the infectious areas and quarantined the infected people after this spreading disease. Currently, the whole world is wrapped around this disease and large number of cases report daily throughout the world. The important symptoms of this deathly COVID-19 are tired, flue, dry cough and fever and it affects badly older. The COVID-19 also affects those individuals who are suffering some sorts of medical illnesses like diabetes, heart problem, chronic respiratory, cancer and cardiovascular disease. Those individuals who have symptoms like difficulty of breathing, cough, flue and high temperature should follow immediately for the medical care. For the whole world, the COVID-19 spreading control has become a huge challenge and yet there is no proper medicine is discovered that can be used as a cure.

The goal of the proposed study is to evaluate SITR mathematical system for the novel COVID-19 dynamics using the Neuro-swarm heuristic solvers via artificial intelligent algorithms. The considerable prospective of neuro-swarm computing solvers is to exploit the stiff systems by operating the collective approximation capability of artificial neural networks (ANNs) together with the optimization of global particle swarm optimization (PSO) along with the local search interior-point approach (IPA), i.e., ANN-PSOIPA [3-7]. Recently, the stochastic numerical solvers have been used to exploit nonlinear prey-predator models [8], financial market forecasting [9], nonlinear functional differential model [10-11], computing approached contain nonlinear optics [12], Thomas-Fermi model [13], summer precipitation forecast for meteorological positions [14], higher order nonlinear multi-singular model [15], nonlinear Troesch’s problem [16], singular periodic boundary value problems [17], doubly stochastic differential equation [18], HIV infection model of CD4+ T cells [19], fourth-order nonlinear singular model [20], corneal shape model [21], nonlinear mosquito dispersal model [22] and heat distribution human head model [23].

The COVID-19 based SITR model is deliberated on the appliance of four categories, ‘susceptible (S) class’, ‘infected (I) class’, ‘treatment (T) class’ and ‘recovered (R) class’. The class ‘susceptible S’ is categories into two subgroups and the detail of each group is specified as:

- $S_i(\Psi)$: The group of those who are uninfected from COVID-19.
- $S_t(\Psi)$: The group of those who are also uninfected but they are suffering some major diseases or grew older.
- $I(\Psi)$: The group of those who are infected from COVID-19.
- $T(\Psi)$: The group for those individuals who started to take treatment.
- $R(\Psi)$: The group of those individuals who have been recovered from COVID-19.

The general form of the nonlinear mathematical SITR system governs by the dynamics of COVID-19 is written as [24]:

$$
\begin{align*}
S'(\Psi) &= B - (\alpha + \beta I(\Psi)) S(\Psi) - \beta \delta T(\Psi), & S(0) &= r_1,
S'_2(\Psi) &= B - (\alpha + \beta I(\Psi)) S_2(\Psi) - \beta \delta T(\Psi), & S_2(0) &= r_2,
I'(\Psi) &= \beta (S_1(\Psi) + S_2(\Psi)) I(\Psi) + \beta \delta T(\Psi) - (\alpha + \mu - \sigma) I(\Psi), & I(0) &= r_3,
T'(\Psi) &= \mu I(\Psi) - (\rho + \alpha - \psi - \varepsilon) T(\Psi), & T(0) &= r_4,
R'(\Psi) &= \rho T(\Psi) - \alpha R(\Psi), & R(0) &= r_5,
\end{align*}
$$

(1)

In the present research, the numerical solutions of the above SITR model are performed by integrating a neuro-swarm heuristic through ANNs, while its modifiable parameters are accomplished with the optimization of PSO along with the support of IPA, i.e., ANN-PSOIPA. The parameters together with specific descriptions of SITR model are tabulated in Table 1.

**Table 1:** Representations of state variables using the SITR nonlinear mathematical system.

| Parameter | Description |
|-----------|-------------|
| $\beta$   | Rate of contact |
| $\mu$     | Rate of recovery |
| $B$       | Natural birth ratio |
| $\rho$    | Infection ratio through treatment |
| $\psi$    | Healthy food ratio |
| $\delta$  | Reduce infection through treatment |
| $\sigma$  | Dry cough, tiredness, flue and fever rate |
| $\alpha$  | Death rate |
| $\varepsilon$ | Sleep factor rate |
| $r_k$, $k = 1$ to $5$ | Initial conditions |

The main geographies to solve the SITR system using the ANN-PSOIPA are concisely expressed as:

- The SITR nonlinear mathematical system for COVID-19 dynamics is solved by applying the intelligent Neuro-swarm ANN-PSOIPA solver.
- The obtained results through ANN-PSOIPA solver overlapped with Runge-kutta numerical solutions based on the SITR nonlinear mathematical model recognized the worth and value.
• The statistical performance is authenticated on multiple executions of ANN-PSOIPA via
Theil's inequality coefficient (TIC), semi interquartile (SI) range and mean absolute error
(MAE).
• Alongside the precise numerical solutions of nonlinear SITR mathematical system for the
dynamics of COVID-19, smooth operation, conceptual ease and consistency are other
significant perquisites.

The other parts of this study are provided as: Section 2 represents the proposed context of the
ANN-PSOIPA along with performance measures. Section 3 indicates the detail of the numerical
results for the SITR model. The conclusion and future search plans are described in the final
Section.

2. Methodology
The structure of the proposed ANN-PSOIPA solver to get the numerical findings of the SITR
nonlinear mathematical model on the basis of COVID-19 is presented in two segments:
• To exploit the model based on ANNs, an error based merit function (MF) is introduced.
• The optimization of the MF for the set of equation (1) using the combination of PSOIPA.

2.1 ANN modeling
The mathematical performance for the set of equations (1) is given using the continuous mapping
of ANN for the obtained numerical solutions \( \hat{S}_1(\Psi), \hat{S}_2(\Psi), \hat{I}(\Psi), \hat{T}(\Psi) \) and \( \hat{R}(\Psi) \) together with
the \( n^{th} \) order derivatives are written as:

\[
\begin{bmatrix}
\hat{S}_1(\Psi), \hat{S}_2(\Psi), \\
\hat{I}(\Psi), \hat{T}(\Psi), \\
\hat{R}(\Psi)
\end{bmatrix} = 
\begin{bmatrix}
\sum_{i=1}^{m} p_{S_{1,i}} Q(w_{S_{1,i}} \Psi + q_{S_{1,i}}), \\
\sum_{i=1}^{m} p_{S_{2,i}} Q(w_{S_{2,i}} \Psi + q_{S_{2,i}}), \\
\sum_{i=1}^{m} p_{I,i} Q(w_{I,i} \Psi + q_{I,i}), \\
\sum_{i=1}^{m} p_{T,i} Q(w_{T,i} \Psi + q_{T,i}), \\
\sum_{i=1}^{m} p_{R,i} Q(w_{R,i} \Psi + q_{R,i})
\end{bmatrix}
\]

(2)

\[
\begin{bmatrix}
\hat{S}_1'(\Psi), \hat{S}_2'(\Psi), \\
\hat{I}'(\Psi), \hat{T}'(\Psi), \\
\hat{R}'(\Psi)
\end{bmatrix} = 
\begin{bmatrix}
\sum_{i=1}^{m} p_{S_{1,i}} Q'(w_{S_{1,i}} \Psi + q_{S_{1,i}}), \\
\sum_{i=1}^{m} p_{S_{2,i}} Q'(w_{S_{2,i}} \Psi + q_{S_{2,i}}), \\
\sum_{i=1}^{m} p_{I,i} Q'(w_{I,i} \Psi + q_{I,i}), \\
\sum_{i=1}^{m} p_{T,i} Q'(w_{T,i} \Psi + q_{T,i}), \\
\sum_{i=1}^{m} p_{R,i} Q'(w_{R,i} \Psi + q_{R,i})
\end{bmatrix}
\]

where \( W \) represents the unidentified weight vector, written as:

\[ W = [W_S; W_S; W_I; W_I; W_R], \text{ for } W_S = [p_{S_1}, w_{S_1}, q_{S_1}]; \ W_S = [p_{S_2}, w_{S_2}, q_{S_2}]; \ W_I = [p_I, w_I, q_I], \]
\[ W_I = [p_I, w_I, q_I]; \ W_R = [p_R, w_R, q_R] \text{ and} \]
Using the log-sigmoid MF \( Q(\Psi) = (1 + e^{-\Psi})^{-1} \), the simplified form of the set (2) is given as:

\[
\begin{align*}
\begin{bmatrix}
\hat{S}_i(\Psi), \hat{S}_2(\Psi), \hat{I}(\Psi), \\
\hat{R}(\Psi), \hat{T}(\Psi)
\end{bmatrix}
& = \frac{\sum_{i=1}^{m} p_{S,i} e^{-w_{S,i} \Psi + q_{S,i}}}{\sum_{i=1}^{m} 1 + e^{-w_{S,i} \Psi + q_{S,i}}} \frac{\sum_{i=1}^{m} p_{S_2,i} e^{-w_{S_2,i} \Psi + q_{S_2,i}}}{\sum_{i=1}^{m} 1 + e^{-w_{S_2,i} \Psi + q_{S_2,i}}} \frac{\sum_{i=1}^{m} p_{I,i} e^{-w_{I,i} \Psi + q_{I,i}}}{\sum_{i=1}^{m} 1 + e^{-w_{I,i} \Psi + q_{I,i}}} \\
\begin{bmatrix}
\hat{S}_i'(\chi), \hat{S}_2'(\chi), \\
\hat{I}'(\chi), \hat{R}'(\chi), \\
\hat{T}'(\chi)
\end{bmatrix}
& = \frac{\sum_{i=1}^{m} w_{S,i} p_{S,i} e^{-w_{S,i} \Psi + q_{S,i}}}{\sum_{i=1}^{m} \left(1 + e^{-w_{S,i} \Psi + q_{S,i}}\right)^2} \frac{\sum_{i=1}^{m} w_{S,2,i} p_{S_2,i} e^{-w_{S_2,i} \Psi + q_{S_2,i}}}{\sum_{i=1}^{m} \left(1 + e^{-w_{S_2,i} \Psi + q_{S_2,i}}\right)^2} \frac{\sum_{i=1}^{m} w_{I,i} p_{I,i} e^{-w_{I,i} \Psi + q_{I,i}}}{\sum_{i=1}^{m} \left(1 + e^{-w_{I,i} \Psi + q_{I,i}}\right)^2} \\
\end{align*}
\]

Using the set (3), a MF is written as:

\[
E = E_1 + E_2 + E_3 + E_4 + E_5 + E_6,
\]

\[
E_1 = \frac{1}{N} \sum_{k=1}^{N} \left( (\hat{S}_i')_k - B + (\alpha + \beta \hat{I}_k)(\hat{S}_i)_k + \beta \delta \hat{T}_k \right)^2,
\]

\[
E_2 = \frac{1}{N} \sum_{k=1}^{N} \left( (\hat{S}_2')_k - B + (\alpha + \beta \hat{I}_k)(\hat{S}_2)_k + \beta \delta \hat{T}_k \right)^2,
\]

\[
E_3 = \frac{1}{N} \sum_{k=1}^{N} \left( \hat{I}_k - \beta \left( (\hat{S}_1)_k + (\hat{S}_2)_k \right) \hat{I}_k - \delta \beta \hat{T}_k + (\alpha + \mu - \sigma) \hat{I}_k \right)^2,
\]

\[
E_4 = \frac{1}{N} \sum_{k=1}^{N} \left( \hat{I}_k' - \mu \hat{I}_k + (\rho + \alpha - \psi - \varepsilon) \hat{I}_k \right)^2,
\]

\[
E_5 = \frac{1}{N} \sum_{k=1}^{N} \left( \hat{I}_k' - \mu \hat{I}_k \right)^2,
\]

\[
E_6 = \frac{1}{N} \sum_{k=1}^{N} \left( \hat{I}_k' - \mu \hat{I}_k + (\rho + \alpha - \psi - \varepsilon) \hat{I}_k \right)^2.
\]
\[
E_5 = \frac{1}{N} \sum_{k=1}^{N} \left( \hat{R}_k - \rho \hat{T}_k + \alpha \hat{T}_k \right)^2, 
\]
\[
E_6 = \frac{1}{S} \left( (\hat{S}_1)_0 - r_1 \right)^2 + (\hat{S}_2)_0 - r_2 \right)^2 + (\hat{I}_0 - r_3 \right)^2 + (\hat{T}_0 - r_4 \right)^2 + (\hat{R}_0 - r_5 \right)^2, 
\]

Here, \(\Psi_k = kh, (\hat{S}_1)_k = \hat{S}_1(\Psi_k), (\hat{S}_2)_k = \hat{S}_2(\Psi_k), \hat{I}_k = \hat{I}(\Psi_k), \hat{T}_k = \hat{T}(\Psi_k), Nh = 1\) and \(\hat{R}_k = \hat{R}(\Psi_k)\).

The estimated results for both subgroups of ‘susceptible’, ‘infected’, ‘treatment’ and ‘recovered’ are represented as \(\hat{S}_1, \hat{S}_2, \hat{I}, \hat{T}\) and \(\hat{R}\). Similarly, \(E_1, E_2, E_3, E_4, E_5\) and \(E_6\) are the MFs based on differential systems and initial values of the SITR nonlinear model of the COVID-19. The proposed solutions are obtained using the best weights for which the MF goes to zero, i.e., \(E \rightarrow 0\).

### 2.2 Optimization performance: PSOIPA

This section provides a complete detail of the PSOIPA to optimize the MF for solving the SITR nonlinear mathematical model that is designed for the dynamics of the COVID-19.

PSO is an optimization based global search scheme and applied as an alteration of genetic algorithm introduced by Kennedy and Eberhart in the past century [25]. The execution procedure of PSO is simple as well as easy to implement due to lesser requirements of the memory [26]. Recently, PSO is applied in nonlinear electric circuits [27], pitch control system of wind turbine [28], parameter approximation [29], reactive power dispatch generation [30], benchmark optimization models [31], tune an adaptive PID controller [32] and approximation of undrained shear soil strength [33].

In search space theory, a single standard result using the optimization procedure is known as a particle. In the optimization of PSO, the prime swarms spread in the wider area. To improve the PSO parameters, the scheme iteratively provides an optimal solution \(P_{LB}^{x-1}\) and \(P_{GB}^{x-1}\), i.e., the swarm’s position and velocity, mathematically given as:

\[
X_i^x = X_i^{x-1} + V_i^{x-1},
\]

\[
V_i^x = wV_i^{x-1} + \chi_1(P_{LB}^{x-1} - X_i^{x-1})r_1 + \chi_2(P_{GB}^{x-1} - X_i^{x-1})r_2,
\]

where \(w\) is the inertia vector, while the respective constant acceleration values are \(\chi_1\) and \(\chi_2\).

The global search PSO converges quickly to hybridize with the suitable local search approach and the PSO best values are applied as a prime weight. Therefore, an operative local search IPA is used for adjustment of the results based on the proposed optimization. Recently, IPA is used in active noise control models [34], simulation of aircraft parts riveting [35], mixed model of complementary monotone [36], economic load dispatch classification [37] and nonlinear identification models [38]. The present study is to present the solutions through the hybridization of PSOIPA to solve the SITR nonlinear mathematical model. The detailed pseudocode using the ANN-PSOIPA is provided in Table 2.
Table 2: Optimize performance through ANN-PSOIPA for solving the SITR nonlinear model

**PSO Starts**

1: **Initialization**: Produce the ‘initial swarms’ randomly. Transform the PSO parameters, as well as, routine of optimoptions.

2: **Fitness**: Calculate the fitness $E$ using set (4).

3: **Ranking**: Rank each individual for minimum values of the MF.

4: **Stopping Conditions**: Terminate if
   - Level of $E$ achieved.
   - Execute the selected flights.

When the above conditions meet, then go to phase 5

5: **Renewal**: For the equations (11) and (12), call the “position” & “velocity”.

6: **Improvement**: Repeat the steps (2)$\rightarrow$(6) up to the required flights are achieved.

7: **Storage**: Save the best $E$ designated as “best global values i.e., $W_{PSO}$

**PSO Ends**

Start the PSOIPA

**Inputs**: $W_{PSO}$

**Output**: The PSOIPA best values are designated as $W_{PI}$

**Initialize**: The start point is $W_{PSO}$

**Termination**: Stop if {$E = 10^{-19}$}, {Iterations=700}, {TolFun = $10^{-20}$}, {TolX = TolCon = $10^{-21}$}, {MaxFunEvals = 285000}

**While** {Terminate}

**Design of Fitness**: Use set (4) for the fitness values

**Adjustments**: Invoke the ‘fmincon’ for the IPA to adjust the weight vector.

Go to ‘fitness’ using the advanced form of $W$.

**Store**: Save the $W_{PI}$, time, iterations, $E$, and function evaluation.

**PSOIPA Ends**

2.3. Performance indices

The mathematical notations of the performances based on MAE and TIC for SITR nonlinear model designed using the sense of COVID-19 are described as:
The appropriate epidemic parameter values have been provided in Table 3, while the simplified SITR form of the model represented in the set (1) is written as:

$$\begin{align*}
\begin{bmatrix}
\text{MAE}_{S}, \text{MAE}_{S}, \text{MAE}_{I}, \\
\text{MAE}_{T}, \text{MAE}_{R}
\end{bmatrix}
&= \left[ \frac{1}{n} \sum_{k=1}^{n} \left| (S_{i})_{k} - (\hat{S}_{i})_{k} \right|, \frac{1}{n} \sum_{k=1}^{n} \left| (S_{2})_{k} - (\hat{S}_{2})_{k} \right|, \frac{1}{n} \sum_{k=1}^{n} \left| (I_{i})_{k} - \hat{I}_{k} \right|, \frac{1}{n} \sum_{k=1}^{n} \left| (T_{i})_{k} - \hat{T}_{k} \right|, \frac{1}{n} \sum_{k=1}^{n} \left| (R_{i})_{k} - \hat{R}_{k} \right| \right]
\end{align*}$$

(13)

$$\begin{align*}
\begin{bmatrix}
\text{TIC}_{S}, \text{TIC}_{S}, \\
\text{TIC}_{I}, \text{TIC}_{T}, \\
\text{TIC}_{R}
\end{bmatrix}
&= \left[ \frac{1}{n} \sum_{k=1}^{n} \left( I_{k} - \hat{I}_{k} \right)^{2}, \frac{1}{n} \sum_{k=1}^{n} \left( T_{k} - \hat{T}_{k} \right)^{2}, \frac{1}{n} \sum_{k=1}^{n} \left( R_{k} - \hat{R}_{k} \right)^{2}, \frac{1}{n} \sum_{k=1}^{n} \left( S_{i} - \hat{S}_{i} \right)^{2}, \frac{1}{n} \sum_{k=1}^{n} \left( S_{2} - \hat{S}_{2} \right)^{2} \right]
\end{align*}$$

(14)

3. Results and discussion

The comprehensive discussions of the SITR nonlinear mathematical model are provided in this section. The comparative investigations with the Runge-Kutta numerical results indicates the precision of ANN-PSOIPA solver. Moreover, statistical evaluations express the accuracy and precision of the designed approach.

3.1 SITR nonlinear mathematical model

The appropriate epidemic parameter values have been provided in Table 3, while the simplified SITR form of the model represented in the set (1) is written as:

**Table 3**: Indexes, parameter descriptions and assigned values for different values of the SITR nonlinear mathematical model

| Symbol | Description                                               | Values |
|--------|-----------------------------------------------------------|--------|
| \(B\)  | Natural birth ratio                                       | 0.3    |
| \(\beta\) | Contact rate                                              | 0.29   |
| \(\delta\) | Reduce infection through treatment                        | 0.3    |
| \(\mu\) | Recovery rate                                             | 0.1    |
| \(\sigma\) | Fever, flue, cough and tiredness rate                     | 0.005  |
| \(\alpha\) | Death rate                                                | 0.25   |
| \(\psi\) | Rate of healthy food                                      | 0.2    |
| \(\rho\) | Rate of infection through treatment                       | 0.3    |
| ε | Sleep rates | 0.1 |

**Scenario I:** Take the parametric values of Table 3 and $\beta=0.35$, the model 1 takes the form as:

\[
\begin{align*}
S'_1(\Psi) & = 0.3 - (0.25 + 0.35I(\Psi))S_1(\Psi) - 0.105T(\Psi), \\
S'_2(\Psi) & = 0.3 - (0.25 + 0.35I(\Psi))S_2(\Psi) - 0.105T(\Psi), \\
I'(\Psi) & = 0.35(S_1(\Psi) + S_2(\Psi))I(\Psi) + 0.105T(\chi) - 0.345I(\Psi), \\
T'(\Psi) & = 0.1I(\Psi) - 0.25T(\Psi), \\
R'(\Psi) & = -0.25R(\Psi) + 0.3T(\Psi),
\end{align*}
\]

The MF of the above set (15) is presented as:

\[
E = \frac{1}{N} \sum_{k=1}^{N} \left[ \left( \hat{S}'_1 \right)_k + (0.35I_k + 0.25)(\hat{S}_1)_k + 0.105T_k - 0.3 \right]^2 + \left( \hat{S}'_2 \right)_k + (0.35I_k + 0.25)(\hat{S}_2)_k + 0.105T_k - 0.3 \right]^2 + \\
\left( \hat{I}'_k - 0.105\hat{T}_k - 0.35((\hat{S}_1)_k + (\hat{S}_2)_k)\hat{I}_k + 0.345\hat{I}_k \right)^2 + \\
\left( \hat{T}'_k - 0.1\hat{I}_k + 0.25\hat{T}_k \right)^2 + \left( \hat{R}'_m - 0.3\hat{T}_m + 0.25\hat{R}_m \right)^2 + \\
\left( (\hat{S}_1)_0 - 0.65 \right)^2 + (\hat{S}_2)_0 - 0.15 \right)^2 + \\
\left( \hat{I}_0 - 0.75 \right)^2 + (\hat{R}_0 - 0.1)^2 \right].
\]

**Scenario II:** Take the parametric values of Table 3 and $\beta=0.3$, the model 1 takes the form as:

\[
\begin{align*}
S'_1(\Psi) & = 0.3 - (0.25 + 0.3I(\Psi))S_1(\Psi) - 0.09T(\Psi), \\
S'_2(\Psi) & = 0.3 - (0.25 + 0.3I(\Psi))S_2(\Psi) - 0.09T(\Psi), \\
I'(\Psi) & = 0.3(S_1(\Psi) + S_2(\Psi))I(\Psi) + 0.09T(\chi) - 0.345I(\Psi), \\
T'(\Psi) & = 0.1I(\Psi) - 0.25T(\Psi), \\
R'(\Psi) & = -0.25R(\Psi) + 0.3T(\Psi),
\end{align*}
\]

The MF of the above set (17) is presented as:

\[
E = \frac{1}{N} \sum_{k=1}^{N} \left[ \left( \hat{S}'_1 \right)_k + (0.3I_k + 0.25)(\hat{S}_1)_k + 0.09T_k - 0.3 \right]^2 + \left( \hat{S}'_2 \right)_k + (0.3I_k + 0.25)(\hat{S}_2)_k + 0.09T_k - 0.3 \right]^2 + \\
\left( \hat{I}'_k - 0.09\hat{T}_k - 0.3((\hat{S}_1)_k + (\hat{S}_2)_k)\hat{I}_k + 0.345\hat{I}_k \right)^2 + \\
\left( \hat{T}'_k - 0.1\hat{I}_k + 0.25\hat{T}_k \right)^2 + \left( \hat{R}'_m - 0.3\hat{T}_m + 0.25\hat{R}_m \right)^2 + \\
\left( (\hat{S}_1)_0 - 0.65 \right)^2 + (\hat{S}_2)_0 - 0.15 \right)^2 + \\
\left( \hat{I}_0 - 0.75 \right)^2 + (\hat{R}_0 - 0.1)^2 \right].
\]

**Scenario III:** Take the parametric values of Table 3 and $\beta=0.25$, the model 1 takes the form as:
The MF of the above set (19) is presented as:

\[
E = \frac{1}{N} \sum_{k=1}^{N} \left( \left[ (S_1')_k + (0.25I_k + 0.25)(S_1)_k + 0.075T_k - 0.3 \right]^2 + \left[ (S_2')_k + (0.25I_k + 0.25)(S_2)_k + 0.075T_k - 0.3 \right]^2 + \left[ \dot{I}_k - 0.075\dot{T}_k - 0.25 \left( (S_1)_k + (S_2)_k \right) \dot{T}_k + 0.345\dot{I}_k \right]^2 + \left[ \dot{T}_k - 0.1\dot{I}_k + 0.25\dot{T}_k \right]^2 + \left[ \dot{R}_m - 0.3\dot{T}_m + 0.25\dot{R}_m \right]^2 \right) \right) + \frac{1}{5} \left( \left[ (S_1)_0 - 0.65 \right]^2 + \left[ (S_2)_0 - 0.15 \right]^2 + \left[ T_0 - 0.35 \right]^2 + \left[ I_0 - 0.75 \right]^2 + \left[ R_0 - 0.1 \right]^2 \right) \right)
\]

\( (20) \)

**Scenario IV:** Take the parametric values of Table 3 and \( \beta = 0.25 \), the model 1 takes the form as:

\[
\begin{align*}
S_1'(\Psi) &= 0.3 - (0.25 + 0.1I(\Psi))S_1(\Psi) - 0.03T(\Psi), \\
S_2'(\Psi) &= 0.3 - (0.25 + 0.1I(\Psi))S_2(\Psi) - 0.03T(\Psi), \\
I'(\Psi) &= 0.1(S_1(\Psi) + S_2(\Psi))I(\Psi) + 0.03T(\Psi) - 0.345I(\Psi), \\
T'(\Psi) &= 0.1I(\Psi) - 0.25T(\Psi), \\
R'(\Psi) &= -0.25R(\Psi) + 0.3T(\Psi), \\
S_1(0) &= 0.65, \\
S_2(0) &= 0.15, \\
I(0) &= 0.75, \\
T(0) &= 0.35, \\
R(0) &= 0.1,
\end{align*}
\]

**Scenario IV:** Take the parametric values of Table 3 and \( \beta = 0.25 \), the model 1 takes the form as:

\[
\begin{align*}
S_1'(\Psi) &= 0.3 - (0.25 + 0.1I(\Psi))S_1(\Psi) - 0.03T(\Psi), \\
S_2'(\Psi) &= 0.3 - (0.25 + 0.1I(\Psi))S_2(\Psi) - 0.03T(\Psi), \\
I'(\Psi) &= 0.1(S_1(\Psi) + S_2(\Psi))I(\Psi) + 0.03T(\Psi) - 0.345I(\Psi), \\
T'(\Psi) &= 0.1I(\Psi) - 0.25T(\Psi), \\
R'(\Psi) &= -0.25R(\Psi) + 0.3T(\Psi), \\
S_1(0) &= 0.65, \\
S_2(0) &= 0.15, \\
I(0) &= 0.75, \\
T(0) &= 0.35, \\
R(0) &= 0.1,
\end{align*}
\]

\( (21) \)

**Scenario IV:** Take the parametric values of Table 3 and \( \beta = 0.25 \), the model 1 takes the form as:

\[
E = \frac{1}{N} \sum_{k=1}^{N} \left( \left[ (S_1')_k + (0.25I_k + 0.1)(S_1)_k + 0.03T_k - 0.3 \right]^2 + \left[ (S_2')_k + (0.25I_k + 0.1)(S_2)_k + 0.03T_k - 0.3 \right]^2 + \left[ \dot{I}_k - 0.03\dot{T}_k - 0.1 \left( (S_1)_k + (S_2)_k \right) \dot{T}_k + 0.345\dot{I}_k \right]^2 + \left[ \dot{T}_k - 0.1\dot{I}_k + 0.25\dot{T}_k \right]^2 + \left[ \dot{R}_m - 0.3\dot{T}_m + 0.25\dot{R}_m \right]^2 \right) \right) + \frac{1}{5} \left( \left[ (S_1)_0 - 0.65 \right]^2 + \left[ (S_2)_0 - 0.15 \right]^2 + \left[ T_0 - 0.35 \right]^2 + \left[ I_0 - 0.75 \right]^2 + \left[ R_0 - 0.1 \right]^2 \right) \right)
\]

\( (22) \)

The SITR nonlinear model is optimized by the hybrid-framework of PSOIPA for 35 independent executions along with 5 numbers of neurons. The best weight vectors set of ANN by PSOIPA are plotted in Fig. 1 and these best weights are functional to accomplish the estimated solutions of the SITR nonlinear system (1). The approximate values for all the scenarios using the ANN-PSOIPA are given as:
Approximate solutions for Scenario I

\[
\hat{S}_1(\Psi) = \frac{-2.5688}{1 + e^{-(0.5024\Psi + 0.4074)}} - \frac{0.7282}{1 + e^{-(1.4310\Psi + 5.4559)}} + \ldots + \frac{5.9508}{1 + e^{-(0.2248\Psi + 1.0373)}},
\]
\[
\hat{S}_2(\Psi) = \frac{-2.1955}{1 + e^{-(0.5023\Psi - 1.6958)}} + \frac{9.4641}{1 + e^{-(0.0001\Psi - 2.8791)}} + \ldots + \frac{0.5518}{1 + e^{-(0.5143\Psi - 5.6907)}},
\]
\[
\hat{I}(\Psi) = \frac{0.3744}{1 + e^{-(0.8629\Psi - 6.8415)}} + \frac{0.5465}{1 + e^{-(0.6734\Psi - 3.0504)}} + \ldots + \frac{1.5757}{1 + e^{-(0.0915\Psi - 0.2872)}},
\]
\[
\hat{T}(\Psi) = \frac{0.4559}{1 + e^{-(0.4939\Psi - 0.6679)}} + \frac{0.4549}{1 + e^{-(0.4939\Psi - 0.6679)}} + \ldots - \frac{1.8869}{1 + e^{-(0.0972\Psi + 6.3025)}},
\]
\[
\hat{R}(\Psi) = \frac{-1.3072}{1 + e^{-(0.3319\Psi + 7.4327)}} - \frac{0.4168}{1 + e^{-(0.6361\Psi - 2.0537)}} + \ldots + \frac{7.6812}{1 + e^{-(0.2181\Psi + 3.3883)}},
\]

The equations (23-27) are implemented to assess the solutions of the SITR nonlinear model using the ANN-PSOIPA and results are plotted in the figures 1-3 for 5 neurons. Figure 1 is drawn on the basis of the trained set for both the groups of susceptible $S_1$ and $S_2$, infected $I$, treatment $T$ and recovery $R$, respectively. Figure 2 indicates the comparison of the best, mean and exact outcomes for the SITR nonlinear system. It is shown that all the results overlapped over one another for the SITR system. Figure 3 represents the best and mean absolute error (AE) values for all parameters of the SITR nonlinear system. It is seen that the best and mean AE values for both the groups of susceptible $S_1$ and $S_2$ lie around $10^{-05}$-$10^{-07}$ and $10^{-03}$-$10^{-05}$, respectively. The best and mean AE values for the infected, treatment and recovered groups lie $10^{-06}$-$10^{-08}$ and $10^{-02}$-$10^{-04}$, respectively. One can conclude that the designed scheme is accurate on the behalf of these AE values. The statistical performance through TIC and MAD values for all the groups of the SITR nonlinear system together with histogram plots is plotted in figures 4 and 5. It is observed that the TIC and MAD values lie $10^{-08}$-$10^{-10}$ and $10^{-04}$-$10^{-06}$, respectively. These optimal values enhance the worth and accuracy of the designed ANN-PSOIPA approach.

For more precision, the statistical presentation of SITR nonlinear system for both the subgroups of susceptible $S_1(\Psi)$ and $S_2(\Psi)$, infected $I(\Psi)$, treatment $T(\Psi)$ and recovered $R(\Psi)$ groups, respectively. The results of statistical gages Minimum (Min), Maximum (Max), Mean, Median, semi interquartile range (SIR) and standard deviation (ST.D) have been performed. The error values based on Max and Min are the worst and best trials, respectively. However, the operator SIR is one-half of the difference of $3^{rd}$ quartile and $1^{st}$ quartile. The small and reliable values of these matrices validate the stability, accuracy and performance of the proposed ANN-PAOIPA for solving the SITR nonlinear model based on coronavirus.
Figure 1: Set of best weight for each group of the SITR nonlinear system for scenario I.
Figure 2: The values of the AE for each group of the SITR nonlinear system for scenario I
Figure 3: Best and Mean AE values for each parameter of the SITR nonlinear system for scenario I
Analysis of convergence through TIC for each parameter of the SITR nonlinear system

Figure 4: Convergence through TIC values for each group of the SITR nonlinear system together with histogram plots for scenario I
Analysis of convergence through MAD for each parameter of the SITR nonlinear system

Figure 5: Convergence through MAD values for each parameter of the SITR nonlinear system together with histogram plots for scenario I
| Ψ | $S_1(\Psi)$ | $S_2(\Psi)$ | $I(\Psi)$ |
|---|---|---|---|
| | Min | Max | Mean | Median | SIR | ST.D | Min | Max | Mean | Median | SIR | ST.D |
| 0 | 1.7530E-08 | 1.6466E-02 | 5.0691E-04 | 4.9525E-06 | 6.2660E-06 | 2.7791E-03 | 4.8274E-09 | 7.7844E-03 | 3.3633E-04 | 4.0418E-06 | 9.7350E-06 | 1.4066E-03 |
| 0.1 | 2.1577E-07 | 1.2509E-02 | 4.0203E-04 | 7.6276E-06 | 9.3840E-06 | 2.1102E-03 | 3.6974E-08 | 5.1352E-04 | 1.8456E-05 | 1.7621E-05 | 2.1164E-05 | 6.5228E-04 |
| 0.2 | 6.1336E-07 | 9.1563E-03 | 3.3030E-04 | 1.5846E-05 | 2.2546E-05 | 1.5462E-03 | 1.3148E-07 | 1.2059E-04 | 1.7628E-05 | 2.1027E-05 | 2.5673E-05 | 8.7486E-04 |
| 0.3 | 2.5044E-08 | 6.4022E-03 | 2.6777E-04 | 1.5596E-05 | 2.9449E-05 | 1.0987E-03 | 1.4424E-07 | 1.2516E-04 | 1.4481E-05 | 1.4481E-05 | 1.0762E-03 | 5.9534E-04 |
| 0.4 | 9.7671E-08 | 4.2454E-03 | 2.1920E-04 | 1.8024E-05 | 2.8698E-05 | 7.8287E-04 | 1.2493E-05 | 1.5083E-04 | 1.5083E-05 | 1.5083E-05 | 1.0762E-03 | 5.9534E-04 |
| 0.5 | 1.1367E-06 | 2.6870E-03 | 1.8978E-04 | 1.8885E-05 | 3.0799E-05 | 6.1827E-04 | 1.3751E-07 | 3.0297E-04 | 1.6177E-05 | 1.3662E-05 | 1.0762E-03 | 5.9534E-04 |
| 0.6 | 2.8205E-07 | 3.1573E-03 | 1.7672E-04 | 1.4481E-05 | 2.6253E-05 | 5.9534E-04 | 1.3751E-07 | 3.0297E-04 | 1.6177E-05 | 1.3662E-05 | 1.0762E-03 | 5.9534E-04 |
| 0.7 | 4.9851E-07 | 3.6726E-03 | 1.8456E-04 | 1.8024E-05 | 2.8698E-05 | 7.8287E-04 | 1.3751E-07 | 3.0297E-04 | 1.6177E-05 | 1.3662E-05 | 1.0762E-03 | 5.9534E-04 |
| 0.8 | 2.0231E-07 | 4.1638E-03 | 2.1253E-04 | 2.3569E-05 | 2.4289E-05 | 7.4432E-04 | 1.3751E-07 | 3.0297E-04 | 1.6177E-05 | 1.3662E-05 | 1.0762E-03 | 5.9534E-04 |
| 0.9 | 1.7644E-07 | 4.6206E-03 | 2.5378E-04 | 2.1027E-05 | 2.5673E-05 | 8.7486E-04 | 1.3751E-07 | 3.0297E-04 | 1.6177E-05 | 1.3662E-05 | 1.0762E-03 | 5.9534E-04 |
| 1 | 1.3751E-07 | 5.0297E-03 | 3.0050E-04 | 1.6177E-05 | 1.3662E-05 | 1.0762E-03 | 1.3751E-07 | 3.0297E-04 | 1.6177E-05 | 1.3662E-05 | 1.0762E-03 | 5.9534E-04 |
Table 7: Statistics performance of the SITR nonlinear system for the treatment $T(\Psi)$

| $\Psi$ | Min         | Max         | Mean        | Median      | SIR          | ST.D         |
|-------|-------------|-------------|-------------|-------------|--------------|--------------|
| 0     | 3.6117E-09  | 5.3960E-02  | 1.7208E-03  | 4.5416E-06  | 1.7703E-05  | 9.1413E-03  |
| 0.1   | 5.6484E-07  | 5.3463E-02  | 1.7144E-03  | 1.3942E-05  | 1.3187E-05  | 9.0573E-03  |
| 0.2   | 3.9416E-07  | 5.3138E-02  | 1.7396E-03  | 1.9095E-05  | 2.4662E-05  | 9.0113E-03  |
| 0.3   | 1.6968E-06  | 5.2861E-02  | 1.7520E-03  | 2.1147E-05  | 3.9186E-05  | 8.9719E-03  |
| 0.4   | 9.1735E-08  | 5.2554E-02  | 1.7204E-03  | 1.1359E-05  | 3.8164E-05  | 8.8475E-03  |
| 0.5   | 2.3735E-07  | 5.2168E-02  | 1.7240E-03  | 1.1359E-05  | 3.8164E-05  | 8.8475E-03  |
| 0.6   | 2.9090E-06  | 5.1679E-02  | 1.6950E-03  | 1.9999E-05  | 4.8312E-05  | 8.7531E-03  |
| 0.7   | 1.6433E-06  | 5.1075E-02  | 1.6645E-03  | 1.8584E-05  | 4.4472E-05  | 8.3726E-03  |
| 0.8   | 1.4395E-07  | 5.0353E-02  | 1.6274E-03  | 3.1143E-05  | 6.1514E-05  | 8.5131E-03  |
| 0.9   | 2.8921E-07  | 4.9517E-02  | 1.5849E-03  | 2.7373E-05  | 7.0198E-05  | 8.6407E-03  |
| 1     | 2.6048E-08  | 4.8574E-02  | 1.5447E-03  | 1.6984E-05  | 2.0258E-05  | 8.2210E-03  |

Table 8: Statistics performance of the SITR nonlinear system for the recovered $R(\Psi)$

| $\Psi$ | Min         | Max         | Mean        | Median      | SIR          | ST.D         |
|-------|-------------|-------------|-------------|-------------|--------------|--------------|
| 0     | 2.312E-08   | 1.538E-01   | 4.443E-03   | 3.287E-06   | 7.180E-06   | 2.599E-02   |
| 0.1   | 2.783E-07   | 1.551E-01   | 4.485E-03   | 7.036E-06   | 1.730E-05   | 2.621E-02   |
| 0.2   | 8.234E-08   | 1.571E-01   | 4.520E-03   | 1.598E-05   | 1.812E-05   | 2.656E-02   |
| 0.3   | 1.870E-07   | 1.580E-01   | 4.591E-03   | 1.854E-05   | 2.323E-05   | 2.669E-02   |
| 0.4   | 2.652E-07   | 1.589E-01   | 4.555E-03   | 1.714E-05   | 3.400E-05   | 2.684E-02   |
| 0.5   | 2.645E-08   | 1.608E-01   | 4.738E-03   | 2.160E-05   | 4.354E-05   | 2.716E-02   |
| 0.6   | 2.858E-07   | 1.623E-01   | 4.794E-03   | 1.822E-05   | 3.522E-05   | 2.741E-02   |
| 0.7   | 6.606E-07   | 1.629E-01   | 4.816E-03   | 2.560E-05   | 3.564E-05   | 2.752E-02   |
| 0.8   | 1.020E-07   | 1.643E-01   | 4.849E-03   | 3.178E-05   | 3.292E-05   | 2.775E-02   |
| 0.9   | 2.126E-06   | 1.672E-01   | 4.912E-03   | 2.688E-05   | 2.881E-05   | 2.824E-02   |
| 1     | 2.449E-06   | 1.712E-01   | 4.992E-03   | 1.683E-05   | 1.853E-05   | 2.893E-02   |

The AE for the other three scenarios $\beta=0.30,0.25$ and 0.1 is provided in figure 6(a-c). It is seen that all the AE values for the scenario II, III and IV lie in good ranges for all the parameters of the SITR nonlinear models. One can observe on the basis of these AE values that the designed approach ANN-PSOIPA is precise and accurate.
Figure 6: AE values for each parameter of the SITR nonlinear system for scenarios II, III and IV

4. Conclusions
A novel design of stochastic computational intelligent framework ANN-PSOIPA is designed to solve the SITR based nonlinear system based COVID-19 using the approximation competency of ANN combined with the optimization of global and local search schemes, i.e., PSOIPA, respectively. The nonlinear SITR system based COVID-19 is efficiently evaluated by the proposed ANN-PSOIPA numerical scheme with single input, single output and single hidden layers based structure of neural networks with 5 neurons. Four different scenarios of SITR have been studied based on the contact rates and the description of first cases is provided for the best weights, absolute error, comparison of results and statistical analysis through TIC and MAD using 35 multiple independent trials of ANN-PSOIPA, while the graphs of absolute error are also plotted for the other three scenarios of the SITR model to decipher the inferences more evidently. The correctness/exactness is proven by finding the absolute error in the range of $10^{-5}$ to $10^{-8}$ based on difference of proposed results of ANN-PSOIPA from numerical outcomes of Runge-Kutta scheme for all the scenarios of the SITR nonlinear model. Furthermore, the statistical analysis based on the TIC and MAD for 35 independent trials of ANN-PSOIPA validate the worth, accuracy and convergence of the designed scheme. Similarly, the statistical performance using different statistical gages Min, Max, Mean, Median, SIR and ST.D has been performed for each groups of SITR nonlinear model with error values for Max are also found in reasonable good ranges, while the rest of the statistical assessments validate the stability, accuracy and robustness of the proposed ANN-PSOIPA for solving the SITR nonlinear model for coronavirus, see Tables 4-8 and Figures 1-6.

In future, the proposed scheme ANN-PSOIPA is a promising alternative to be investigated for problems arising in fluids models [39-41], two-dimensional Boussinesq equations [42], higher order functional model [43], fractional differential equations [44-46], energy [47], prediction differential model [48] and biological systems [49-52].

**Funding:** This paper has been also partially supported by Ministerio de Ciencia, Innovación y Universidades grant number PGC2018-0971-B-100 and Fundación Séneca de la Región de Murcia grant number 20783/PI/18.

**Acknowledgements:** Not applicable

**Competing Interest:** The authors declare that they have no competing interests

**Authors’ contribution:** YG and ZS has worked in the model design. MU and TS has worked in the model resolution. MAZR and FA has worked in the writing and revision of the paper.

**Availability of data and material:** Data sharing not applicable to this article as no datasets were generated or analysed during the current study.
References

[1] China virus death toll rises to 41, more than 1,300 infected worldwide. CNBC. 24 January 2020. Archived from the original on 26 January 2020. Retrieved 26 January 2020. Retrieved 30 January 2020.

[2] Khan, M.A. and Atangana, A., 2020. Modeling the dynamics of novel coronavirus (2019-nCov) with fractional derivative. Alexandria Engineering Journal.

[3] Sabir, Z., Amin, F., Pohl, D. and Guirao, J.L., Intelligence computing approach for solving second order system of Emden–Fowler model. Journal of Intelligent & Fuzzy Systems, (Preprint), pp.1-16.

[4] Zúñiga-Aguilar, C.J., Romero-Ugalde, H.M., Gómez-Aguilar, J.F., Escobar-Jiménez, R.F. and Valtierra-Rodríguez, M., 2017. Solving fractional differential equations of variable-order involving operators with Mittag-Leffler kernel using artificial neural networks. Chaos, Solitons & Fractals, 103, pp.382-403.

[5] Sabir, Z., et al., 2020. Novel design of Morlet wavelet neural network for solving second order Lane–Emden equation. Mathematics and Computers in Simulation, 172, pp.1-14.

[6] Zúñiga-Aguilar, C.J., Coronel-Escamilla, A., Gómez-Aguilar, J.F., Alvarado-Martínez, V.M. and Romero-Ugalde, H.M., 2018. New numerical approximation for solving fractional delay differential equations of variable order using artificial neural networks. The European Physical Journal Plus, 133(2), p.75.

[7] Raja, M.A.Z., Khan, J.A., Chaudhary, N.I. and Shivanian, E., 2016. Reliable numerical treatment of nonlinear singular Flierl–Petviashivili equations for unbounded domain using ANN, GAs, and SQP. Applied Soft Computing, 38, pp.617-636.

[8] Umar, M., et al., 2019. Intelligent computing for numerical treatment of nonlinear prey–predator models. Applied Soft Computing, 80, pp.506-524.

[9] Bukhari, A.H., et al., 2020. Fractional Neuro-Sequential ARFIMA-LSTM for Financial Market Forecasting. IEEE Access.

[10] Sabir, Z., et al., 2020. Neuro-swarm intelligent computing to solve the second-order singular functional differential model. The European Physical Journal Plus, 135(6), p.474.

[11] Sabir, Z., Raja, M.A.Z., Umar, M. and Shoaib, M., 2020. Neuro-swarm intelligent computing to solve the second-order singular functional differential model. The European Physical Journal Plus, 135(6), p.474.

[12] Ahmad, I., et al., 2018. Neuro-evolutionary computing paradigm for Painlevé equation-II in nonlinear optics. The European Physical Journal Plus, 133(5), p.184.

[13] Sabir, Z., Manzar, M.A., Raja, M.A.Z., Sheraz, M. and Wazwaz, A.M., 2018. Neuro-heuristics for nonlinear singular Thomas-Fermi systems. Applied Soft Computing, 65, pp.152-169.

[14] Bukhari, A.H., et al., 2020. Neuro-fuzzy modeling and prediction of summer precipitation with application to different meteorological stations. Alexandria Engineering Journal, 59(1), pp.101-116.
[15] Sabir, Z., Raja, M.A.Z., Umar, M. and Shoaib, M., 2020. Design of neuro-swarming-based heuristics to solve the third-order nonlinear multi-singular Emden–Fowler equation. The European Physical Journal Plus, 135(6), pp.1-17.

[16] Majeed, K., et al., 2017. A genetic algorithm optimized Morlet wavelet artificial neural network to study the dynamics of nonlinear Troesch’s system. Applied Soft Computing, 56, pp.420-435.

[17] Sabir, Z., Raja, M.A.Z., Guirao, J.L. and Shoaib, M., A neuro-swarming intelligence based computing for second order singular periodic nonlinear boundary value problems.

[18] Saouli, M.A., 2020. Existence of solution for Mean-field Reflected Discontinuous Backward Doubly Stochastic Differential Equation. Applied Mathematics and Nonlinear Sciences, 5(2), pp.205-216.

[19] Umar, M., Sabir, Z., Amin, F., Guirao, J.L. and Raja, M.A.Z., 2020. Stochastic numerical technique for solving HIV infection model of CD4+ T cells. The European Physical Journal Plus, 135(6), p.403.

[20] Sabir, Z., et al. 2020. Integrated intelligent computing with neuro-swarming solver for multi-singular fourth-order nonlinear Emden–Fowler equation. Computational and Applied Mathematics, 39(4), pp.1-18.

[21] Umar, M., Amin, F., Wahab, H.A. and Baleanu, D., 2019. Unsupervised constrained neural network modeling of boundary value corneal model for eye surgery. Applied Soft Computing, 85, p.105826.

[22] Umar, M., Raja, M.A.Z., Sabir, Z., Alwabli, A.S. and Shoaib, M., 2020. A stochastic computational intelligent solver for numerical treatment of mosquito dispersal model in a heterogeneous environment. The European Physical Journal Plus, 135(7), pp.1-23.

[23] Raja, M.A.Z., Umar, M., Sabir, Z., Khan, J.A. and Baleanu, D., 2018. A new stochastic computing paradigm for the dynamics of nonlinear singular heat conduction model of the human head. The European Physical Journal Plus, 133(9), p.364.

[24] Sanchez, Y.G., Sabir, Z. and Guirao, J.L., 2020. Design of a nonlinear SITR fractal model based on the dynamics of a novel coronavirus (COVID).

[25] Shi, Y. and Eberhart, R. C., 1999. Empirical study of particle swarm optimization. In Proceedings of the 1999 Congress on Evolutionary Computation-CEC99 (Cat. No. 99TH8406) (Vol. 3, pp. 1945-1950). IEEE.

[26] Engelbrecht, A. P., 2007. Computational intelligence: an introduction. John Wiley & Sons.

[27] Mehmood, A., Zameer, A., Aslam, M.S. et al. Design of nature-inspired heuristic paradigm for systems in nonlinear electrical circuits. Neural Comput & Applic 32, 7121–7137 (2020). https://doi.org/10.1007/s00521-019-04197-7

[28] Kamarzarrin, M. and Refan, M.H., 2020. Intelligent Sliding Mode Adaptive Controller Design for Wind Turbine Pitch Control System Using PSO-SVM in Presence of Disturbance. Journal of Control, Automation and Electrical Systems, pp.1-14.
[29] Özsoy, V.S., Ünsal, M.G. and Örkcü, H.H., 2020. Use of the heuristic optimization in the parameter estimation of generalized gamma distribution: comparison of GA, DE, PSO and SA methods. Computational Statistics, pp.1-31.

[30] Monteiro, M.R., Rodrigues, Y.R., de Souza, A.C.Z. and Ribeiro, P.F., 2020. Particle swarm optimization applied to reactive power dispatch considering renewable generation. In Decision Making Applications in Modern Power Systems (pp. 247-267). Academic Press.

[31] Duary, A., Rahman, M.S., Shaikh, A.A., Niaki, S.T.A. and Bhunia, A.K., 2020. A new hybrid algorithm to solve bound-constrained nonlinear optimization problems. Neural Computing and Applications, pp.1-26.

[32] El-Gendy, E.M., Saafan, M.M., Elksas, M.S., Saraya, S.F. and Areed, F.F., 2020. Applying hybrid genetic–PSO technique for tuning an adaptive PID controller used in a chemical process. Soft Computing, 24(5), pp.3455-3474.

[33] Pham, B.T., Qi, C., Ho, L.S., Nguyen-Thoi, T., Al-Ansari, N., Nguyen, M.D., Nguyen, H.D., Ly, H.B., Le, H.V. and Prakash, I., 2020. A Novel Hybrid Soft Computing Model Using Random Forest and Particle Swarm Optimization for Estimation of Undrained Shear Strength of Soil. Sustainability, 12(6), p.2218.

[34] Raja, M.A.Z., Aslam, M.S., Chaudhary, N.I. and Khan, W.U., 2018. Bio-inspired heuristics hybrid with interior-point method for active noise control systems without identification of secondary path. Frontiers of Information Technology & Electronic Engineering, 19(2), pp.246-259.

[35] Stefanova, M., Yakunin, S., Petukhova, M., Lupuleac, S. and Kokkolaras, M., 2018. An interior-point method-based solver for simulation of aircraft parts riveting. Engineering Optimization, 50(5), pp.781-796.

[36] Sicre, M.R. and Svaiter, B.F., 2018. A $O(1/k^{3/2})$ hybrid proximal extragradient primal–dual interior point method for nonlinear monotone mixed complementarity problems. Computational and Applied Mathematics, 37(2), pp.1847-1876.

[37] Raja, M.A.Z., Ahmed, U., Zameer, A., Kiani, A.K. and Chaudhary, N.I., 2019. Bio-inspired heuristics hybrid with sequential quadratic programming and interior-point methods for reliable treatment of economic load dispatch problem. Neural Computing and Applications, 31(1), pp.447-475.

[38] Umenberger, J. and Manchester, I.R., 2018. Specialized Interior-Point Algorithm for Stable Nonlinear System Identification. IEEE Transactions on Automatic Control, 64(6), pp.2442-2456.

[39] Sabir, Z., Ayub, A., Guirao, J.L., Bhatti, S. and Shah, S.Z.H., 2020. The Effects of Activation Energy and Thermophoretic Diffusion of Nanoparticles on Steady Micropolar Fluid along with Brownian Motion. Advances in Materials Science and Engineering, 2020.

[40] Wahab, H.A.et al., 2019. Numerical Treatment for the Three-Dimensional Eyring-Powell Fluid Flow over a Stretching Sheet with Velocity Slip and Activation Energy. Advances in Mathematical Physics, 2019, pp.1-12.
[41] Sabir, Z., Akhtar, R., Zhiyu, Z., Umar, M., Imran, A., Wahab, H.A., Shoaib, M. and Raja, M.A.Z., 2019. A Computational Analysis of Two-Phase Casson Nanofluid Passing a Stretching Sheet Using Chemical Reactions and Gyrotactic Microorganisms. Mathematical Problems in Engineering, 2019.

[42] Sharifi, M. and Raesi, B., 2020. Vortex Theory for Two Dimensional Boussinesq Equations. Applied Mathematics and Nonlinear Sciences, 5(2), pp.67-84.

[43] Sabir, Z., Gujarhan, H. and Guirao, J.L., 2020. On a new model based on third-order nonlinear multisingular functional differential equations. Mathematical Problems in Engineering, 2020.

[44] Onal, M. and Esen, A., 2020. A Crank-Nicolson Approximation for the time Fractional Burgers Equation. Applied Mathematics and Nonlinear Sciences, 5(2), pp.177-184.

[45] Touchent, K.A., Hammouch, Z. and Mekkaoui, T., 2020. A modified invariant subspace method for solving partial differential equations with non-singular kernel fractional derivatives. Applied Mathematics and Nonlinear Sciences, 5(2), pp.35-48.

[46] Sabir, Z., Raja, M.A.Z., Shoaib, M. and Aguilar, J.G., 2020. FMNEICS: fractional Meyer neuro-evolution-based intelligent computing solver for doubly singular multi-fractional order Lane–Emden system. Computational and Applied Mathematics, 39(4), pp.1-18.

[47] Xue, D., Wang, J. and Zhu, Z., 2020. Impact of Environmental Information Disclosure on Certified Public Accountant Audit of Chinese Listed Companies in the Energy Industry. Applied Mathematics and Nonlinear Sciences, 5(2), pp.377-390.

[48] Sabir, Z., Guirao, J.L., Saeed, T. and Erdoğan, F., 2020. Design of a Novel Second-Order Prediction Differential Model Solved by Using Adams and Explicit Runge–Kutta Numerical Methods. Mathematical Problems in Engineering, 2020.

[49] de Araujo, A.L., Fassoni, A.C. and Salvino, L.F., 2020. An analysis of a mathematical model describing the growth of a tumor treated with chemotherapy. Applied Mathematics and Nonlinear Sciences, 5(2), pp.185-204.

[50] Umar, M. et al., 2020. A stochastic numerical computing heuristic of SIR nonlinear model based on dengue fever. Results in Physics, 19, p.103585.

[51] Umar, M. et al., 2020. A Stochastic Intelligent Computing with Neuro-Evolution Heuristics for Nonlinear SITR System of Novel COVID-19 Dynamics. Symmetry, 12(10), p.1628.

[52] Evirgen, F., Uçar, S. and Özdemir, N., 2020. System analysis of HIV infection model with CD4+ T under non-singular kernel derivative. Applied Mathematics and Nonlinear Sciences, 5(1), pp.139-146.
Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: