Chromofields of Strings and Baryons

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Received: date / Revised version: date

Abstract. We calculate color electric fields of quark/antiquark (\bar{qq}) and 3 quark (qqq) systems within the chromodielectric model (CDM). We explicitly evaluate the string tension of flux tubes in the \bar{qq}–system and analyze their profile. To reproduce results of lattice calculations we use a bag pressure \[ B = (320\text{MeV})^4 \] from which an effective strong coupling constant \( \alpha_s \approx 0.3 \) follows. With these parameters we get a Y shaped configuration for large \( \bar{qq} \)-systems.

PACS. 11.10.Lm Field theory; Nonlinear or nonlocal theories and models – 11.15.Kc Gauge field theories; Classical and semiclassical techniques – 12.39.Ba Phenomenological quark models; Bag model

1 Introduction

Quantum chromodynamics (QCD) is the widely accepted theory for the dynamics of quarks and gluons. Despite its success in the regime of high momentum transfer it remains an outstanding task to explain the low energy behavior of hadrons within QCD. Only in the last 10 years lattice QCD (lQCD) has found detailed evidence for the confinement of quarks in hadrons [1] but it still fails to give a dynamical description of this phenomenon. It is therefore necessary to rely on models, capable to describe confinement dynamically on the one hand and to reproduce static results of lQCD on the other hand.

In this talk we present static calculations within the Chromodielectric Model [2,3,4], namely the detailed analysis of quark–antiquark strings and three–quark configurations.

2 Phenomenology of the Model

In the Chromodielectric Model (CDM) it is assumed, that the vacuum of QCD behaves in the long range limit as a perfect color dielectric medium with vanishing dielectric constant \( \kappa = 0 \). The medium is generated through the non-abelian part of the gluonic sector of QCD which is represented in CDM as a scalar color singlet field \( \sigma \). The remaining two abelian gluon fields are able to propagate through this medium. The scalar field \( \sigma \) is driven by a scalar potential \( U(\sigma) \) (see fig. [1]) which exhibits two (quasi) stable points, separating the non-perturbative, perfect dielectric phase where \( \sigma = \sigma_{\text{vac}} \) from the perturbative phase with \( \kappa = 1 \), where the color fields can propagate freely and \( \sigma = 0 \).

In our description quarks are treated classically and the gluons are coupled to the quark current \( j_{\mu}^{\alpha} \). This results in the following Lagrangian

\[
\mathcal{L} = \mathcal{L}_q + \mathcal{L}_g + \mathcal{L}_\sigma
\]

\[
\mathcal{L}_q = -\sum_k m_k \sqrt{1 - \mathbf{u}_k^2} \, w(\mathbf{x} - \mathbf{x}_k(t)) - g_s \, j_{\mu}^{\alpha} A_{\mu}^{\alpha}
\]

\[
\mathcal{L}_g = -\frac{1}{4} \kappa(\sigma) F_{\mu\nu}^a F^{\mu\nu,a}
\]

\[
\mathcal{L}_\sigma = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - U(\sigma)
\]

\[ F_{\mu\nu,a} = \partial_\mu A_{\nu}^{\alpha} - \partial_\nu A_{\mu}^{\alpha}, \quad a \in \{3, 8\} \]

\[ j_{\mu}^{\alpha} = \sum_k q_k^a \, \mathbf{u}_k^\mu \, w(\mathbf{x} - \mathbf{x}_k(t)) = (\rho^a, J^a) \]

with \( u_k^\mu \) being the 4-velocity of particle \( k \) with classical charge \( q_k^a \) (see fig. [1]) and extension \( w(\mathbf{x} - \mathbf{x}_k(t)) \). The scalar potential \( U(\sigma) \) is chosen to be of a quartic form and is shown in fig. [1]. In this work \( U(\sigma) \) has no relative maximum between \( \sigma = \sigma_{\text{vac}} \) and \( \sigma = 0 \) and \( U \) is determined through the bag pressure \( B = U(0) \) and \( \sigma_{\text{vac}} \) alone.

The dielectric function is of the form \( \kappa(\sigma) = \exp\left(-\frac{\sigma}{\sigma_0}\right) \) for \( \sigma > 0 \) and \( \kappa(\sigma) = 1 \) else and has \( \kappa(\sigma_{\text{vac}}) \equiv \kappa_{\text{vac}} \ll 1 \).

In the static case, the equations of motion for the electric potentials \( \Phi^a \) and for the confinement field \( \sigma \) following from eq. (11) are:

\[
\nabla \cdot (\kappa(\sigma) \nabla \Phi^a) = -g_s \rho^a
\]

and

\[
\nabla^2 \sigma = U'(\sigma) - \frac{1}{2} \kappa(\sigma) \left( D^3 \cdot D^3 + D^8 \cdot D^8 \right)
\]

where \( D^a = \kappa(\sigma) \nabla \Phi^a \) denotes the color electric displacement. The energy (neglecting quark masses) is given by:

\[
E = E_{\sigma} + E_g
\]
In this section we study the field configurations of color flux tubes stretching from a quark \( q \) to an antiquark \( \bar{q} \). We start by showing the electric field \( E^a \) and \( D^a \) in fig. 2. It is seen that the electric displacement vanishes outside the cavity. The flux tube can be characterized by the profile function, i. e. the component of \( D \) parallel to the string axis along the center line perpendicular to the string axis. This profile has been studied within lQCD in [6]. Note that in CDM the \( D \) field is confined and we compare it to the \( E \) field of reference [1].

The value of \( \sigma_{\text{vac}} \) controls the surface of the bag. Decreasing its value leads to a sharper surface. In our simulations the detailed form of the dielectric function (see fig. 1) has little effect on the profile.

With the parameters given in tab. 1 we reproduce the results of lQCD [1] as can be seen in fig. 4.

Using the same parameters we can calculate the string tension of the flux tube. We vary the \( q-\bar{q} \) distance \( r \) and
\[ E_c(r) = E_0 + \tau r - \frac{\alpha_{\text{eff}}}{r}. \] 

(12)

where the linear term reflects the confinement behavior for large \( \bar{q}q \)-separations and the Coulomb term describes the gluon exchange dominant at small \( r \). The constant term \( E_0 = 560 \text{ MeV} \) is due to electric self energies included in eq. (9). We find a string tension \( \tau = 988 \text{ MeV/fm} \) and a value \( \alpha_{\text{eff}} = 0.291 \) which is to be compared to IQCD results where \( \alpha_{\text{eff}} = 0.295 \) \( \text{(1)} \). It should be noted, that due to the high bag pressure \( B \) the electric fields are not strong enough to expel totally the non-perturbative vacuum out of the string. The confinement field only drops to \( \sigma \approx 0.5\sigma_{\text{vac}} \), i. e. the dielectric function rises to \( \kappa \approx 0.5 \). However, confinement is still achieved as the energy of the color fields does not leak into the outside.

4 Baryons

In this section we study color fields of baryon like \( qqq \)-configurations. Given that the energy scales linearly with the \( \bar{q}q \)-separation, one can argue that configurations with 3 quarks sitting on the corner of an equilateral triangle will form strings with minimal total string length. This would be a configuration with a central Steiner point, called a Y configuration. However, if only two quark interactions are dominant, one might expect strings stretching pairwise from one quark to another, which would be the \( \Delta \) configuration. In IQCD the \( qqq \) potential has been studied and there are indications for both the \( \Delta \) baryon \( \text{[17]} \) and the Y baryon \( \text{[8]} \). However in the Gaussian Stochastic Vacuum model \( \text{[9]} \) clear evidence for the Y ansatz is found.

In fig. 4 we plot the electric field distribution for the baryon with the parameters given in tab. 1. The quarks are separated a distance \( \ell = 0.7(1.3) \text{ fm} \) from the Steiner point, i.e. the \( qq \) distance is \( L = \sqrt{3}\ell \approx 1.2(1.7) \text{ fm} \).

The field is clearly different from a simple superposition of 3 flux tubes between the quarks (see fig. 2). The electric energy is pushed towards the center of the baryon, and a Y shaped configuration (at least for large quark separations) is seen.

5 Summary

We have analyzed the \( \bar{q}q \) string within CDM and have reproduced the geometric profile function as well as the potential. With a bag constant \( B = (320 \text{ MeV})^4 \) and \( \sigma_{\text{vac}} = 1.5 \text{ fm}^{-1} \) we get a string tension \( \tau = 988 \text{ MeV/fm} \) and an effective strong coupling \( \alpha_{\text{eff}} = 0.29 \). \( qqq \) configurations with large \( qq \) separations tend to show a Y shaped geometry.

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