Summary: Given $d \in \mathbb{N}$, we establish sum-product estimates for finite, nonempty subsets of $\mathbb{R}^d$. This is equivalent to a sum-product result for sets of diagonal matrices. In particular, let $A$ be a finite, nonempty set of $d \times d$ diagonal matrices with real entries. Then, for all $\delta_1 < 1/3 + 5/5277$,

$$|A + A| + |A \cdot A| \gg_d |A|^{1+\delta_1/d},$$

which strengthens a result of M.-C. Chang [“Additive and multiplicative structure in matrix spaces”, Combin. Probab. Comput. 16, No. 2, 219–238 (2007; Zbl 1154.15020)] in this setting.

MSC:

11B30 Arithmetic combinatorics; higher degree uniformity
11C20 Matrices, determinants in number theory
15A30 Algebraic systems of matrices

Keywords:
arithmetic combinatorics; sum-product estimates

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