Quasiparticle picture of black holes and the entropy–area relation

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We propose an effective description of 0-brane black holes, in which the black hole is modeled as a gas of non-interacting quasi-particles in the dual quantum mechanics. This simple model is shown to account for many of the static thermodynamic properties of the black hole. It also accounts for dynamical properties, such as the rate at which energy gets thermalized by the black hole. We use the model to show that the entropy of the quantum mechanics is proportional to the black hole horizon area in Planck units.
1 Introduction

The Hawking-Bekenstein relationship \cite{1} between entropy and area is a fundamental property of quantum gravity. In recent years it has become possible to build microscopic models of black holes, and show that the entropy – area relationship indeed holds \cite{2, 3, 4, 5, 6, 7}. But in these calculations the underlying reason for a universal connection between entropy and area remains obscure, since the black hole microstates are only explicitly constructed in a limit where the geometry is singular.

The advent of the AdS/CFT correspondence \cite{8} and its generalizations \cite{9} should make it possible to better understand the relationship between entropy and area. In the AdS/CFT framework the entropy of the black hole can be identified with the thermal entropy of the CFT, while the area of the horizon should in principle be calculable within the CFT. However to date not much progress has been made in this direction. The problem is that we do not understand in detail how quasi-local bulk gravitational physics emerges from the dual CFT. This makes the direct CFT definition of horizon area difficult.

We will work in the context of the duality between 0-brane quantum mechanics and 0-brane black holes \cite{9}. In the regime where the black holes are well-described by supergravity the quantum mechanics is strongly coupled. In \cite{10, 11, 12} we developed a mean-field approximation scheme for the strongly-coupled quantum mechanics, and showed that it captures some of the essential physics of the dual gravity theory. Similar approximations have been used in \cite{13} in the framework of the IKKT matrix model \cite{14}.

In the end the mean-field approximation suggests the following very simple picture of the quantum mechanics. The finite-temperature quantum mechanics has an effective description in terms of a collection of non-interacting quasi-particles \footnote{The quasi-particles we consider have nontrivial two-point functions, but no multiparticle interactions.}, which we identify with the individual matrix elements appearing in the matrix fields $X(t)$. There are a total of $N^2$ quasi-particles. Some of the quasi-particles have an energy of order the temperature and can be thermally excited. The remaining quasi-particles are too heavy to be thermally excited. The light quasi-particles are unstable, and decay with a...
characteristic lifetime of order the inverse temperature.

We would like to stress that the validity of the quasi-particle picture we have described is compatible with, but does not rely on, the validity of any particular mean-field approximation. But given our mean-field results, the quasi-particle properties can be extracted as follows. The imaginary time two-point function \( \langle X(\tau)X(0) \rangle \) has a spectral representation, which we study in ref. [12] and appendix A of the present work. There we find that the spectral density \( \rho(\omega) \) has two peaks. One peak is concentrated at a frequency of order the temperature and the other is concentrated at a frequency of order one in 't Hooft units [12]. Moreover in appendix A we show that the low-frequency peak has a width of order the temperature, which implies that the lifetime of the light quasi-particles is approximately \( 1/T \). There are a total of \( N^2 \) quasi-particles. But at temperature \( T \), only

\[
N_{\text{eff}} = N^2 \int_{\text{first peak}} d\omega \rho(\omega)
\]

of the quasi-particles can be thermally excited. The number of light quasi-particles depends on the temperature. In [11] we computed the entropy of the quantum mechanics in the mean-field approximation and found agreement with the black hole entropy up to factors of order unity. Thus we take the number of light quasi-particles to be about equal to the entropy of the black hole, \( N_{\text{eff}} \approx S_{\text{BH}} \) up to constant factors.

To summarize, our picture of the quantum mechanics is that it consists of \( N_{\text{eff}} \approx S_{\text{BH}} \) light quasi-particles, each with an energy \( \sim T \) and a lifetime \( \sim 1/T \). We propose that these light quasi-particles give a holographic description of the dual black hole. That is, in the dual gravitational description we take the quasi-particles to correspond to the stretched horizon degrees of freedom, which provide a holographic description of the black hole together with its low-energy excitations. We take the stretched horizon to be located where the proper temperature is equal to the Planck temperature. As we will see in section 3.3, this is necessary to have a consistent stretched horizon description of the low-energy excitations of the black hole.

We should point out that the picture of black holes we are advocating draws heavily on previous work. It is closely related to the membrane paradigm description of black holes [15]. It also has significant overlap with the work of Susskind and others on black hole complementarity [16, 17, 18, 19, 20, 21, 22].
An outline of this paper is as follows. In section 2 we show that the quasi-particle picture correctly reproduces the equilibrium thermodynamic properties of the black hole, including a relationship between the entropy and radius of the black hole. In section 3 we compute the thermalization rate of the black hole, and use this to derive the relationship between entropy and horizon area. Section 4 contains some comments and conclusions. In appendix A we relate the quasi-particle picture of 0-brane quantum mechanics to our previous results on the mean-field approximation and deduce the quasi-particle lifetime. In appendix B we study quasi-normal modes of a scalar field in the black hole background, and show that their lifetime matches that of the quasi-particles.

2 Equilibrium properties

In this section we study four equilibrium thermodynamic properties of the quasi-particle gas, namely the entropy, energy, specific heat and radius. We will argue that they have the same qualitative behavior on both sides of the duality.

First let’s consider the quantum mechanics side. We model the quantum mechanics as made up of $N_{\text{eff}}$ harmonic oscillators with frequency $\omega \sim T$. Thus at temperature $T$ we have an entropy

$$S_{\text{QM}} = \left(1 - \beta \frac{\partial}{\partial \beta}\right) \log Z \approx N_{\text{eff}},$$

an energy

$$E_{\text{QM}} = -\beta \frac{\partial}{\partial \beta} \log Z \approx N_{\text{eff}} T,$$

and a specific heat

$$c_{\text{QM}} = \beta^2 \frac{\partial^2}{\partial \beta^2} \log Z \approx N_{\text{eff}}.$$

On the black hole side the entropy is given by

$$S_{\text{BH}} \sim N^2 (T/\lambda^{1/3})^{9/5}$$

where $\lambda = g_{YM}^2 N$ is the ’t Hooft coupling of the quantum mechanics. Thus the entropies agree provided the number of quasi-particles is about the same
as the entropy of the black hole, \( N_{\text{eff}} \approx S_{\text{BH}} \). At the moment we must simply assume that this relationship holds, although up to factors of order one, it can be shown to follow from the mean-field approximation \([11]\). The energy of the black hole is

\[ E_{\text{BH}} \sim N^2 \lambda^{1/3} (T/\lambda^{1/3})^{14/5} \approx TS_{\text{BH}} \]

in agreement with the quasi-particle prediction. Finally the specific heat of the black hole is

\[ c_{\text{BH}} \sim N^2 (T/\lambda^{1/3})^{9/5} \approx S_{\text{BH}} \]

again in agreement with the quasi-particle prediction.

Now let us consider the size of the black hole in the quantum mechanics. The matrix fields \( X(t) \) are related to a collection of canonically normalized harmonic oscillators by \( X = (\lambda N)^{1/2} Y \). So we identify

\[ R^2_{\text{h}} = \frac{1}{N} \langle \text{Tr} X^2 \rangle = \frac{\lambda}{N^2} \langle \text{Tr} Y^2 \rangle \]  

with the squared radius of the black hole. Following Susskind \([23]\) we suppress high-frequency contributions to the expectation value, so that only the \( N_{\text{eff}} \) light degrees of freedom contribute to the trace in \((2)\). This procedure is made possible by the clear separation of scales in the double-peaked spectral density. Thus

\[ R^2_{\text{h}} \approx \frac{\lambda}{N^2} N_{\text{eff}} \langle x^2 \rangle \approx \frac{\lambda N_{\text{eff}}}{N^2 T} \]

where we have used the fact that at temperature \( T \) a single harmonic oscillator with \( \omega \approx T \) has \( \langle x^2 \rangle \approx 1/T \). That is, the quantum mechanics predicts a relation between the horizon radius and the entropy \([12]\)

\[ R^2_{\text{h}} \approx \frac{\lambda S}{N^2 T}. \]  

(3)

Rather remarkably, this relationship indeed holds for 0-brane black holes \([10]\), since the supergravity expression for the horizon radius is\(^2\)

\[ R_{\text{h}} \sim \lambda^{1/3} \left( \frac{T}{\lambda^{1/3}} \right)^{2/5}. \]

\(^2\)denoted \( U_0 \) in \([9]\)
3 Horizon area and thermalization time

3.1 Measuring the area

Our original motivation for this paper was to gain insight into the relationship between entropy and area. In this section we report on our progress in this direction.

The first step is to find a meaningful way of measuring the area of the horizon. For black holes in asymptotically flat space a simple way to measure the horizon area is to perform a scattering experiment. The classical absorption cross section (geometric optics cross section) for high-energy particles is proportional to the area of the black hole. However this is not true for black holes in asymptotically AdS-like spaces. For example in string frame a null geodesic with energy $E$ and angular momentum $\ell$ in the near-horizon geometry (14) obeys

$$\left(\frac{dU}{d\tau}\right)^2 + \frac{\ell^2 U^5}{c\lambda} \left(1 - \frac{U^7}{U^7_0}\right) = E^2.$$  

Here $c = 240\pi^5$, $\lambda$ is the 't Hooft coupling, and $U$ is a radial coordinate with the horizon located at $U = U_0$. All geodesics fall into the horizon, and there are no classical scattering states. Another possibility is to work at low energy. For minimally coupled scalars in asymptotically flat space the absorption cross section at zero frequency is exactly the area of the black hole [24]. In asymptotically AdS-like spaces one must regard scattering as a tunneling problem and use non-normalizable solutions to the wave equation to compute ‘absorption’ cross-sections [25], which in the low-energy limit do turn out to be proportional to the horizon area. It would be interesting to apply this approach to 0-brane black holes. Unfortunately by working in a low energy limit one loses any intuitive connection between cross section and area.

We therefore turn to a different way of measuring horizon area, namely the fact that a hot object with surface area $A$ will emit blackbody radiation at a rate proportional to $A$.\textsuperscript{3} Following [15, 16] we model the black hole as

\textsuperscript{3}A related procedure (in a low-frequency limit) was used to define horizon area in [26], for black holes in asymptotically flat space.
made up of a set of degrees of freedom living at a stretched horizon which is located just outside the true event horizon. The Stefan-Boltzmann law gives the rate at which the stretched horizon emits energy in outgoing Hawking radiation.

\[ \frac{dE_{\text{out}}}{dt_{\text{proper}}} \sim AT_{\text{proper}}^{10}. \]

Here \( E_{\text{proper}} \), \( t_{\text{proper}} \) and \( T_{\text{proper}} \) are proper energies, times and temperatures measured at the stretched horizon. It is convenient to multiply this equation by \(-g_{tt}\), to get a relation between the corresponding Schwarzschild quantities.

\[ \frac{dE_{\text{out}}}{dt} = AT_{\text{proper}}^8 T^2. \]  

(4)

Again this is the rate at which the stretched horizon emits energy.\(^4\) In thermal equilibrium, of course, this is balanced against an equal and opposite flux of infalling energy. From the membrane paradigm point of view, the energy flux \( \overline{\Pi} \) is the rate at which energy leaves the membrane, turns around, and eventually falls back onto the membrane. Thus in thermal equilibrium it measures the rate at which energy gets redistributed among the quasi-particle degrees of freedom which live on the stretched horizon. We take the stretched horizon to be located at a radius where the proper temperature is equal to the Planck temperature.\(^5\) The rate at which energy is redistributed on the stretched horizon is then

\[ \frac{dE}{dt} = \frac{A}{\ell_P^3} T^2. \]  

(5)

We will use this to measure the horizon area in Planck units.

### 3.2 Relating entropy to area

The quantum mechanics has on average \( N_{\text{eff}} \) quasi-particles each with an energy \( \sim T \) and a lifetime \( \sim 1/T \). In thermal equilibrium as these quasi-particles decay new quasi-particles are continually created. The rate at which  

\(^4\)For black holes in asymptotically flat space only a tiny fraction of this energy flux ever reaches infinity, as most of the outgoing radiation eventually falls back onto the horizon \[15\].

\(^5\)For more on this choice see section 3.3 and the discussion section.
the total energy gets redistributed by this process is

$$\frac{dE}{dt} \approx N_{\text{eff}}T^2.$$ (6)

Comparing this to the black hole result (5) we are led to identify

$$N_{\text{eff}} \sim \frac{A}{\ell_P^8}.$$ (7)

But the entropy of the quantum mechanics $S_{\text{QM}} \sim N_{\text{eff}}$. Thus

$$S_{\text{QM}} \sim \frac{A}{\ell_P^8}.$$ (8)

This gives a simple direct connection between the area of the black hole as measured in Planck units and the entropy of the dual quantum mechanics. Up to a numerical coefficient of order one it agrees with the Hawking–Bekenstein formula.

3.3 Thermalization time

We conclude by computing the thermalization time of the quasi-particle gas, and showing that it matches the thermalization time of the black hole.

First let us consider the black hole side. In thermal equilibrium the rate at which energy is radiated by the stretched horizon (4) is balanced against an equal and opposite flux of infalling energy. But suppose we perturb the temperature of the black hole $T \rightarrow T + \Delta T$, while keeping the temperature outside the stretched horizon fixed. Then there will be a net flux of energy out of the black hole, given by

$$\frac{d\Delta E}{dt} \sim A T_{\text{proper}}^8 T \Delta T.$$

But the perturbation to the energy is $\Delta E = c\Delta T$, where $c$ is the specific heat, so

$$\frac{d\Delta E}{dt} \sim \frac{1}{c} A T_{\text{proper}}^8 T \Delta E.$$
Recall that for these black holes the specific heat is proportional to the entropy, \( c \sim S \sim A T_{\text{proper}}^8 \). Thus the thermalization time of the black hole is

\[
\tau_{\text{BH}} \sim 1/T. \tag{9}
\]

Now consider the quantum mechanics side. The thermalization time of the quantum mechanics is set by the quasi-particle lifetime, so

\[
\tau_{\text{QM}} \sim 1/T
\]

in agreement with the black hole.

Note that the result (9) relies on putting the stretched horizon at the Planck temperature. In appendix B we show that (9) matches the lifetime of quasi-normal excitations of the black hole background. This matching is expected if the stretched horizon degrees of freedom provide a holographic description of the black hole together with low-energy excitations of the space-time fields around the black hole. This shows that for a consistent holographic description we must take the proper temperature at the stretched horizon to be equal to the Planck temperature.

4 Discussion and conclusions

We have proposed a quasi-particle description of 0-brane quantum mechanics, and shown that certain simple properties of the quasi-particle gas are responsible for the entropy–area relationship. These properties are just the energy \( \sim T \) and lifetime \( \sim 1/T \) of the individual quasi-particles. The quasi-particle picture correctly reproduces many properties of the black hole; this strongly suggests that the quasi-particle picture is correct. In appendix A we give further evidence that the quasi-particle description can be derived from a mean-field approximation to the quantum mechanics.

Let us make a few comments on our main result (9). First, the Stefan-Boltzmann argument depends crucially on putting the stretched horizon at the Planck temperature. This is necessary to have a consistent holographic description, as we saw for example in section 3.3. Nonetheless it is worthwhile examining other motivations for this choice. We want to use (9) to extract the thermalization rate of the quantum mechanics. For this purpose one
must get close enough to the horizon. One might think that staying one string length outside the horizon is the right thing, but this is not the case. The energy stored in closed string modes outside a sphere one string length away from the horizon is of order one in 't Hooft units, while the black hole mass is of order $N^2$. Thus one cannot expect to pick up enough degrees of freedom if one stops at the string length. In contrast it is well known from brick wall models [27] that at one Planck length outside the event horizon the energy stored in closed string modes becomes of order the black hole mass.

Second, the use of the Stefan–Boltzmann law to describe radiation from the stretched horizon is only an approximation. The relation of Stefan–Boltzmann to a more precise calculation is as follows. Consider a scalar field in the Hartle–Hawking vacuum. In terms of Schwarzschild modes the Hartle–Hawking vacuum looks thermal, with an outgoing flux of radiation from the horizon. There is an energy flux associated with this outgoing radiation. For large black holes, where the near-horizon geometry is approximately Rindler, the flux of energy is given by the Stefan–Boltzmann law [28].

We should point out similarities of our picture with previous ideas. In a gauge-fixed formalism we identify the quasi-particles with the entries in the matrix fields $X(t)$.\(^\text{6}\) We take these to be a convenient set of degrees of freedom to describe the black hole. Of course gauge invariant operators are formed from traces of products of the matrices $X(t)$, but such operators do not form a convenient set. This reminds us of the picture that black hole energy and entropy are encoded in open strings (or half closed strings) stuck to the horizon of the black hole [29]. Our introduction of a stretched horizon is suggested by the membrane paradigm. Note that the timescale for linear response of a classical membrane has been shown to be of order the inverse temperature [15], in agreement with our results. Finally, our choice of a special distance just outside the event horizon is motivated by the brick wall model [27].

Acknowledgements

We are grateful to Gary Horowitz, Samir Mathur and Lenny Susskind for valuable discussions. NI and DK are supported by the DOE under contract DE-FG02-92ER40699. The research of DL is supported in part by DOE grant DE-FE0291ER40688-Task A. DK, DL and GL are supported in part

\(^6\)For a discussion of gauge symmetry in the mean-field approximation see [11].
by US–Israel Bi-national Science Foundation grant #2000359.

**A Mean-field approximation**

In [10, 11] we developed and applied a mean-field approximation scheme to the quantum mechanics of $N$ D0-branes at finite temperature. Working in the imaginary-time formalism, the approximation gives a set of two-point correlators evaluated at Matsubara frequencies. In this section we will focus on a particular scalar propagator, denoted $\Delta^2(k)$ in [11]. In this section we argue that the mean-field approximation gives rise to a quasi-particle picture of the quantum mechanics which is compatible with the properties that we have been discussing.

To extract quasi-particle properties from mean-field correlators, it is useful to introduce a spectral representation for the correlators. Thus we write the Euclidean propagator as

$$\Delta^2(k) = \int_0^\infty d\omega \rho(\omega) \frac{1}{k^2 + \omega^2}$$

where the spectral density is given by

$$\rho(\omega) = \frac{1}{Z} \sum_m e^{-\beta E_m} \sum_{n>m} |\langle n | \phi | m \rangle|^2 2\omega \left( 1 - e^{-\beta \omega} \right) \delta(\omega - E_n + E_m).$$

One can regard $\rho(\omega)d\omega$ as the number of single-string states with energy between $\omega$ and $\omega + d\omega$. In principle, the spectral density is uniquely determined by the Euclidean propagator evaluated at the Matsubara frequencies

$$k_l = 2\pi l / \beta \quad l \in \mathbb{Z}$$

Togetheer with some information about behavior at infinity [30]. But in practice it is extremely difficult to invert (10) to solve for $\rho(\omega)$.

Similar spectral representations were used in [12], where mean-field methods were applied to study the dynamics of a 0-brane probe of the black hole background. In that work we analytically continued the probe gap equations to determine the probe propagators in between the Matsubara frequencies.
This additional information made it possible to uniquely determine the spectral density of the probe using standard inverse-problem methods. A striking feature found in [12] was that, for small probe radius $R$, the spectral density consists of two well-defined and well-separated peaks. Once $R$ becomes smaller than the radius corresponding to the string-scale stretched horizon, the probe becomes indistinguishable from the 0-branes that make up the black hole. We therefore expect the spectral density for the black hole background to have two well-defined peaks.

To determine the spectral density for the propagators describing the black hole background, we could proceed as in [12], and analytically continue the gap equations of [11] away from the Matsubara frequencies. But rather than carry out this rather involved procedure, we begin by applying inverse-problem methods to the propagator evaluated only at the Matsubara frequencies.\footnote{We determine the propagator by solving the gap equations given in (32) – (40) of [11].} We apply the Tikhonov regularization method described in [12] to obtain the spectral density subject to a positivity constraint. In figure 1 we show the results for the spectral density at inverse temperature $\beta = 2$. A two-peak structure of the density of states clearly emerges. We also see that the low-frequency peak is symmetrical, and extends down to close to zero frequency. This behavior seems to persist as we lower the temperature, at least to $\beta = 4$. Thus we take the width of the low-frequency peak to be of order its mass.

We should mention a number of caveats in this analysis. It is possible that the peak width observed here is an artifact of evaluating the propagator only at Matsubara frequencies. However, the convergence of the regularization method, and its independence of the frequency resolution used, are signs that the density of states obtained this way is reliable. For $\beta > 4$ the method does not give convergent results. A more precise determination of the widths will be presented in future work on the continued gap equations.

To determine the dependence of the masses and widths on temperature, we proceed to make an ansatz for the form of the propagator, and determine the parameters appearing in the ansatz by fitting to the propagator evaluated at Matsubara frequencies. This will allow us to analyze inverse temperatures $\beta > 4$. Motivated by the results of [12], we expect the retarded propagator...
to have two pairs of poles in the lower half plane.

\[ G_R(k) = -\frac{A_1}{(k + i\Gamma_1/2)^2 - m_1^2} - \frac{A_2}{(k + i\Gamma_2/2)^2 - m_2^2}. \]

This corresponds to two species of quasi-particles, with masses \( m_1, m_2 \) and lifetimes \( 1/\Gamma_1, 1/\Gamma_2 \). The corresponding spectral density is

\[ \rho(\omega) = \frac{2\omega}{\pi} \operatorname{Im} G_R(\omega) = \frac{\omega}{2\pi} \sum_{i=1,2} \frac{A_i\Gamma_i}{m_i} \left( \frac{1}{(\omega - m_i)^2 + \Gamma_i^2/4} - \frac{1}{(\omega + m_i)^2 + \Gamma_i^2/4} \right) \]

which in turn implies that our ansatz for the Euclidean Green’s function is

\[ G_E(k) = \frac{A_1}{(|k| + \Gamma_1/2)^2 + m_1^2} + \frac{A_2}{(|k| + \Gamma_2/2)^2 + m_2^2}. \] (12)

Note that the Euclidean propagator is non-analytic at zero momentum.

The spectral density satisfies the sum rule \( \int_0^\infty d\omega \rho(\omega) = 1 \) which implies that \( A_1 + A_2 = 1 \). This leaves five parameters to be determined. Unfortunately, it does not seem possible to reliably extract the values of all
five parameters just given the propagators at Matsubara frequencies. We therefore fix the widths by hand, and minimize

$$\chi^2 = \sum_{l=0}^{\infty} \left( \Delta^2(2\pi l/\beta) - G_E(2\pi l/\beta) \right)^2$$

to determine the best-fit values of the remaining three parameters $m_1$, $m_2$, $A_1 = 1 - A_2$. Without loss of generality we take $m_1 < m_2$.

Results for $m_1(T)$ and $m_2(T)$ are shown in Fig. 2, where we have allowed $\Gamma_1$ and $\Gamma_2$ to vary over the ranges

$$T/2 < \Gamma_1 < m_1 \quad T/2 < \Gamma_2 < m_2/2.$$  \quad (13)

We find that we get quite good fits over these ranges. As can be seen in the figure, the results for $m_1$ are not particularly sensitive to our choice for
the widths. The three curves for \( m_1 \) shown in Fig. 2 are all well fit by \( m_1 \approx 1.5T \). This linear dependence of \( m_1 \) on temperature is consistent with the density of states from Tikhonov regularization. Likewise the range of widths \( \Gamma_1 \) compatible with a given \( m_1 \) are consistent with the regularization results that indicate a linear relation between width and mass.

The area under the first peak in the spectral density \( A_1 \) does not seem to be particularly sensitive to our choice for the widths. At very low temperatures \( A_1 \) falls off like a power law and is well fit by \( A_1 \approx 1.2\beta^{-0.9} \). \( A_1 \) should really fall off faster with \( \beta \) in order to reproduce the black hole entropy \( S_{\text{BH}} \approx \beta^{-1.8} \). This is a shortcoming of the particular gap equations we have used, which only correctly reproduce the entropy up to \( \beta \approx 4 \). This difficulty is discussed in more detail in [11].

For \( \beta > 2.5 \) the ansatz (12) provides a very good fit to the propagators. \( \chi^2 \) increases with \( \beta \), but even at \( \beta = 25 \) for widths in the range \( \Gamma_1 \) we have

\[
\chi^2 \approx 7 \times 10^{-3}
\]

\[
\max_l \left| \Delta^2(2\pi l/\beta) - G_E(2\pi l/\beta) \right| \approx 0.05 \quad \text{(off by about 10\%)}
\]

Given that the goodness-of-fit is quite insensitive to our choice of the widths we cannot determine the widths with any accuracy. At high temperatures, roughly \( \beta < 0.5 \), the propagator is well fit by a spectral density with a single sharp peak at a frequency \( m \approx 1.8/\beta^{1/4} \). The double Lorentzian ansatz does not give a good fit to the propagators in the range \( 0.5 < \beta < 2.5 \). We expect that only the more general inverse-problem methods will be useful in pinning down the spectral density in this regime.

### B Quasinormal modes

In this appendix we compute the thermalization time of the black hole by studying quasi-normal excitations of the black hole background. Note that this computation is independent of the considerations leading to (9).

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8This follows from the high-temperature analysis in [11]. In the notation of that paper we are setting \( m^2 \approx 1/\Delta_0^2 \approx m_\Delta^2 \).
Quasinormal modes for black holes in asymptotically AdS spaces were studied in \[32, 33, 34, 35, 36\]. The starting point is the near-horizon Einstein frame metric

\[
d s^2 = \text{const.} U^{21/8} \left[ -h(U)dU^2 + h^{-1}(U)dU^2 + \frac{c^{1/2}(g_YM^2N)1/2}{U^{3/2}}d\Omega_s^2 \right]
\]

\[
h(U) = \frac{U^{7/2}}{c^{1/2}(g_YM^2N)^{1/2}} \left( 1 - \frac{U^7_0}{U^7} \right) .
\]

(14)

Here \(c = 240\pi^5\) and \(g_YM\) is the coupling constant of the dual gauge theory. The horizon is located at \(U = U_0\), while the ‘boundary of space’ is at \(U = \infty\). The dilaton, for example, obeys the minimal scalar wave equation \(\nabla^2 \phi = 0\). Separating variables \(\phi(t, U, \Omega) = e^{-i\omega t} \phi(U)Y_\ell(\Omega)\) where \(Y_\ell\) is a spherical harmonic on \(S^8\) leads to the radial wave equation

\[
\frac{\partial_U}{U^8 \left( 1 - \frac{U^7_0}{U^7} \right)} \frac{\partial_U \phi}{U} + \left( \frac{cg_YM^2N\omega^2U}{1 - \frac{U^7_0}{U^7}} - \ell(\ell + 7)U^6 \right) \phi = 0 .
\]

Introduce a new dimensionless radial coordinate \(x = -\log \left( 1 - \frac{U^7_0}{U^7} \right)\). The horizon is at \(x = \infty\), while the boundary of space is at \(x = 0\). The wave equation takes the canonical form

\[
\left( \frac{d^2}{dx^2} + \frac{\rho^2}{\left( 1 - e^{-x} \right)^9/7} - \frac{\ell(\ell + 7)}{49} \frac{e^{-x}}{\left( 1 - e^{-x} \right)^2} \right) \phi = 0
\]

where the dimensionless parameter \(\rho\) is

\[
\rho = \frac{c^{1/2}(g_YM^2N)^{1/2}\omega}{7 U_0^{5/2}} .
\]

Quasinormal frequencies are determined by requiring ingoing waves at the future horizon, while as in \[32\] we will impose Dirichlet boundary conditions at \(x = 0\).

\[
\phi(t, x) \sim e^{i\rho x - i\omega t} \quad \text{as } x \to \infty \quad \text{with } x - \omega t/\rho \text{ fixed}
\]

\[
\phi = 0 \quad \text{at } x = 0 .
\]

(15)

The main point now follows simply from dimensional analysis. The boundary conditions (15) can only be satisfied for discrete complex values of \(\rho\), labeled by a radial quantum number \(n\) and an angular quantum number \(\ell\). Thus

\[
\rho_{n\ell} = f(n, \ell)
\]

15
or equivalently
\[ \omega_{n\ell} \sim \frac{U_0^{5/2}}{(g_{YM}^2 N)^{1/2}} f(n, \ell) \sim T f(n, \ell). \]
This shows that both the real and imaginary parts of the quasinormal frequencies are proportional to the Hawking temperature of the black hole. In the dual picture this means that the thermalization time of the quantum mechanics is proportional to \(1/T\). A similar scaling argument for conformal \(p\)-branes was presented in [32]. Related scaling arguments work for the near horizon geometry of all \(p\)-branes, and show that the thermalization time for all these theories is proportional to \(1/T\) [37].

Determining the quasinormal frequencies requires some numerical analysis. It is convenient to work in terms of a new radial variable \(w = e^{-x}\), which puts the horizon at \(w = 0\) and the boundary of space at \(w = 1\). For s-waves the radial wave equation becomes
\[
\left( w \frac{d}{dw} w \frac{d}{dw} + \frac{\rho^2}{(1-w)^9/7} \right) \phi = 0.
\]
This equation can be solved in a power series about \(w = 0\). The solution which is ingoing at the horizon has the expansion
\[
\phi(w) = w^{-i \rho} \left( 1 - \frac{9 \rho^2}{7 - 14 i \rho} w + \cdots \right).
\]
As in [32] we truncate the series at some finite order \(N\); imposing the Dirichlet condition \(\phi|_{w=1} = 0\) then gives an algebraic equation for \(\rho\). We have carried out this procedure keeping terms up to order \(w^{19}\), which gives the lowest quasinormal frequency
\[
\rho = 0.56 - 0.93i.
\]
This result is fairly stable; the lowest quasinormal frequency does not change significantly with the truncation order provided \(N > 10\).

These quasi-normal frequencies will show up as poles in the complex frequency plane of correlators of supergravity excitations [35]. We may map such correlators into correlation functions of scaling operators in the quantum mechanics, following the dictionary worked out in [38]. Such correlators reduce to convolutions of products of the elementary quasi-particle two-point functions. The width of the first peak in the quasi-particle spectral density should therefore exhibit the same temperature dependence as the quasi-normal mode frequencies.
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