Chaos in magnetic flux ropes

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Abstract
Magnetic flux ropes immersed in a uniform magnetoplasma are observed to twist about themselves, writhe about each other and rotate about a central axis. They are kink unstable and smash into one another as they move. Each collision results in magnetic field line reconnection and the generation of a quasi-separatrix layer. Three-dimensional magnetic field lines are computed by conditionally averaging the data using correlation techniques. Conditional averaging is possible for only a number of rotation cycles as the field line motion becomes chaotic. The permutation entropy can be calculated from the time series of the magnetic field data (this is also done with flows) and is used to calculate the positions of the data on a Jensen–Shannon complexity map. The location of data on this map indicates if the magnetic fields are stochastic, or fall into regions of minimal or maximal complexity. The complexity is a function of space and time. The Lyapunov and Hurst exponents are calculated and the complexity and permutation entropy of the flows and field components are shown throughout the volume.

Keywords: flux ropes, chaos, magnetic field line reconnection, complexity, entropy, laboratory experiments related to solar physics, solar corona

(Some figures may appear in colour only in the online journal)

1. Introduction
Magnetic flux ropes are twisted bundles of electrical current and magnetic field which can exist in magnetized plasmas [1]. There are quite a few situations where multiple flux ropes exist simultaneously. If the magnetic ropes have strong enough magnetic fields they can mutually interact. The interaction results in the ropes twisting about each other, colliding and sometimes merging. In instances when they do collide magnetic field can be partially converted to other forms of energy such as flows, heat, fast particles and waves. Flux ropes routinely occur near the surface of the Sun [2] and in all probability every other star. Sometimes large magnetic ropes are ejected from the Sun and if the timing and ejection are right they may come all the way to the Earth [3] where they are detected by probes on satellites. Their signature caused by light emitted by electrons moving along magnetic field lines have been recorded in ultraviolet (UV) and soft x-ray satellite photographs of the Sun [4]. Current sheets (which can be considered to be flux rope slabs), such as those observed in the Earth’s magnetotail can become unstable and tear into a series of nearly linear ropes which can interact and braid with one another [5]. Under the right conditions individual ropes can be subject to the kink instability [6]. The resulting motion can be violent enough to make them collide and produce bursts of magnetic field line reconnection. The study of these objects is so rich that it has involved solar and space plasma physicists, experimentalists, those doing simulations, and topologists [7].

Despite a great deal of interest in this field there have been very few experiments on flux ropes. The reason for this is simple, they are difficult to set up, and diagnose. Broadly speaking experiments fit into two classes. The first involves the generation of arched ropes to simulate what is happening near the surface of the sun. Using a plasma arc in a horse shoe shaped magnetic field unstable flux ropes have been studied by Bellan et al [8]. There was no background plasma and the rope plasma was generated by the arc. Much of the diagnostics involved imaging by fast cameras but extensive probe measurements of the fields was not possible because of shot-to-shot reproducibility. These
experiments, nevertheless, generated a great deal of interest because the ropes were seen to erupt as coronal mass ejections (CMEs) do. Experiments at UCLA with a high repetition rate and a background plasma independent of the ropes allowed for magnetic field measurements. Plasma when injected at the rope footprints trigger the eruption of the flux rope [9]. Magnetosonic waves were radiated into the background plasma when this occurred [10]. An experiment at LANL, which produced two kink unstable flux ropes using plasma guns to generate them, revealed magnetic field line reconnection when the ropes collided [11]. There was no background plasma but probes were used to measure magnetic fields, temperature and density in the device and the results were compared with models. The first UCLA experiment, done over 20 years ago studied two ropes that were not kink unstable [12]. The ropes twisted about one another because of their mutual $\mathbf{J} \times \mathbf{B}$ forces and then merged into a force free state. Experiments on two and three flux ropes resumed at UCLA in 2005 and have been previously reported [13–15]. These experiments have identified a quasi-separatrix layer [25] (QSL) for the first time. Ion and electron heating of the ropes was documented and multiple QSL’s were identified when three ropes were present. Experiments with two ropes of different diameters and energy were done and three-dimensional (3D) plasma flows measured. A detailed description of the ropes is in the references cited and the morphology of the flows and quantities associated with them such as the vorticity and cross-helicity will be discussed in a forthcoming publication [16]. The main point of this work is to discuss the chaotic behavior of the ropes.

The ropes are kink unstable and fast framing camera movies reveal that their motion is complex. Although the average fields can be graphed (some examples will be given) is there chaotic behavior as well? How can quantities related to chaos be evaluated and is there anything about the physical processes we can learn from it? This is the subject of this paper.

To our knowledge the quantitative study of chaotic systems began with the classic paper by Lorenz [17] who modeled flow with a system of deterministic ordinary nonlinear equations. The discovery he made was that the flow was nonperiodic and unstable. Two systems with states that are close may be far from alike at future times. This did not bode well for long-term weather forecasting. A recent article in Physics Today [18] celebrates the subject of chaos 50 years after the Lorenz paper. The history of the subject with many examples is well reviewed by Sprott [19]. Simple systems such as iterative maps (many of which are described at the end of Sprott’s book), and differential equations have chaotic solutions. The hallmark of these is trajectories of an orbit in phase space associated with these never entirely repeat themselves although they can orbit about fixed locations called strange attractors. Simple physical systems with only several degrees of freedom such as simple electrical circuits [20] (which mimic dripping faucets), can exhibit intermittent chaos. This is also so in the current voltage relation in plasma discharges [21] when a small oscillating voltage is applied on the dc anode–cathode voltage. There are many others, and as this is not meant to be a review paper we will deal with the topic of magnetic flux ropes. One can ask if it is possible to make quantitative measurements of chaos from data, the answer is yes. If chaos is characterized is it possible to work backwards and deduce something about the physics of the system? Here the answer is maybe. If the chaos parameters are the same as that of a known map does the system have the same properties? Is the fluctuation spectrum of a quantity such as magnetic field unique for a given nonlinear process?

2. Experimental setup

The flux rope experiments were done in the Large Plasma Device (LaPD) at UCLA. The plasma is produced by a dc discharge between a barium oxide coated cathode [22] and mesh anode. The bulk of the plasma carries no net current and is therefore quiescent. The background plasma parameters are $\delta n/n \approx 3\%$, $0.05\, eV \leq T_e \leq 5\, eV$, $10^{10}\, cm^{-3} \leq n \leq 2 \times 10^{12}\, cm^{-3}$, $T_i < 1\, eV$ for He and Ar plasmas. The background plasma is pulsed at 1 Hz with typical plasma duration of 15 ms. The machine can run continuously for approximately four months before the cathode has to be cleaned and recoated. A schematic of the machine indicating how the background plasma and flux ropes are generated is shown in figure 1.

The background plasma is generated by an oxide coated cathode shown on the right of figure 1. A transistor switch [23] in series with a capacitor bank is used to pulse a voltage between the negatively biased cathode and a molybdenum anode located 50 cm away. The background plasma is 18 m in length and 60 cm in diameter. A second, high emissivity lanthanum hexaboride cathode [24] (LaB$_6$) is inserted in the machine. The flux ropes cathode is masked with a carbon sheet so it can only emit from the unmasked areas. Cathode masks capable of making two different types of rope pairs are inserted in the LAPD plasma are shown in figure 2. When the ropes are pulsed on there is additional ionization and the plasma density in the ropes doubles and the electron temperature rises to over 10 eV in helium and 8 eV in argon. At this point the plasma in the ropes is very highly ionized. The electrical systems, which generate the ropes and background plasma are independent of one another. A second transistor switch, capacitor bank and charging supply is used for the ropes.

The timing for the flux rope experiment is as follows. First the dc discharge which produces the background plasma is initiated and after a few ms a quiescent background plasma is formed. When the background plasma reaches a steady state the flux ropes are switched on for up to 10 ms. Then both transistor switches open and the experiment is terminated. Probe data (three-axis magnetic probes (sensitive to dB/dt), Mach (flow) probes and Langmuir probes (plasma density and electron temperature) are digitized with a bank of digitizers. A computer controlled data acquisition system also controls the motion of the probes. The experiment runs around the clock; a data run (volumetric measurement of one set of flux ropes) lasts two to three weeks.

The presence of a background plasma is a matter of importance as it would have been possible to generate the flux ropes without it. The background plasma supports a variety of waves (for example drift waves driven by the ropes) and return
currents if reconnection demands it. These can, in turn drive instabilities, which will show up in the chaos analysis.

3. Flux rope characteristics: case 1 'moon ropes'

In case 1 the emission regions resembled half moons which were constructed to have the largest possible emission area \( A = 28 \text{ cm}^2/\text{rope} \) and small separation \( d = 1 \text{ cm} \). The discharge voltage between the cathode and anode was 110 V (90 A) for 2 ms and then ramped up to 210 V (105 A) for 2 ms longer. Magnetic field and flow data were acquired on coarse (outside of the ropes) and fine grids (including and between the ropes) on 16 \( Z \) planes \( (dz = 64.5 \text{ cm}) \) and 4282 locations on each plane for a total of 68,512 points. Data were acquired at 10,240 time steps \( (dt = 0.64 \mu s) \).

A movie of optical emission of the ropes made with a fast framing camera \( (30,000 \text{ frames s}^{-1}) \) shows the ropes spin about one another in the first 2 ms and when the voltage doubles the spin rate approximately doubles as well as the excursions the ropes make on the far away anode. The dynamics of the ropes is reflected in He II light at 303 Å (figure 3) acquired with a vacuum ultraviolet spectrometer which collected light from both ropes 8 m from their source.

In either case the flux ropes are kink unstable satisfying the criterion \[ I_{\text{rope}} > \frac{\pi a^2 B_z c \sqrt{1 - M^2}}{L} \] (1)
where \( M \) is the Mach number of the plasma flow along the magnetic field, \( a \) the radius of a rope and \( B_z \) the axial magnetic field. \( I_{\text{rope}} \) is in cgs units. For the case of a typical flux rope in this experiment \( (B_z = 330 \text{ G}, M = 0.2, a = 2 \text{ cm} \) \( L = 11 \text{ m} \) the rope currents are three-to-five times larger than the threshold.

The oscillations in the emitted light reflect the motion of the ropes as they rotate about one another and kink. The magnetic field was measured with three-axis differential pickup loops [25]. The magnetic probes are sensitive to \( \partial B / \partial t \) and the signals are integrated after they are acquired. The six differentially wound probes are 3 mm in diameter. They were tested with a network analyzer and have a flat frequency response to 5 MHz. The signals (for these and other probes)
were digitized with 16 bit analog-to-digital converters, and amplified with broadband low noise amplifiers constructed for the project. The measurement accuracy is greater than 100 µG. Standard swept Langmuir probes with 1 mm² tips were used to measure the density and temperature in several planes. The profiles are very similar to data shown in [14]. In this experiment the probe sizes are much smaller than the ion gyroradius.

Figure 4 shows the density and temperature profiles of a single flux rope in helium. The Langmuir probe was moved to 1800 positions on the plane and 10 current–voltage characteristic curves were averaged at each position. The flux rope electrons are about three times as hot and twice as dense as the background plasma. The density is higher as the hot electrons in the core ionize the background gas. The center of the ropes are fully ionized. When more than one rope is present the rope positions change in space and time as illustrated in [15].

The magnetic field at one location and for each discharge voltage is shown in figure 5. The background plasma is highly reproducible but the onset of the kink instability occurs at a different time for every shot. The same thing was done with 2 three-axis Mach probes. Each axis of the Mach probe has two oppositely directed faces (area = 1 mm², on ceramic stalk 2.5 cm in diameter) and the Mach number of the flow \( \mathbf{M} = u_{\text{ion}, x,y,z}/c_s \) (\( c_s \) is the ion sound speed) can be derived from the difference of the currents to the faces [26]. To generate temporal sequences a second three-axis magnetic probe as well as a three-axis Mach probe were fixed in space for the entire data run. They acquired data every shot while their twin probes were moved throughout the machine. A correlation technique is used to determine a new \( t = 0 \), that is the time the flux ropes start oscillating rather than the time the rope currents are switched on. This works well except there are instances (shots) in which the temporal signal is not highly correlated and therefore is not included in the average. This is the principle of conditional averaging. Figure 6 shows several time sequences of the magnetic field, acquired at two spatial locations every shot. The shots that are not counted are generally between the rope currents where reconnection occurs and a QSL forms.

The first (figure 6(a)) is a well-correlated case and the second (figure 6(b)) is not. The data is shown for the argon plasma but the result is the same in both cases. We emphasize that the flux rope current in the external circuit is identical for the volume as well as the average plasma flows can be derived from the conditionally averaged data.

4. Flux rope characteristics: case 2 ‘two ropes—argon’

In this geometry two side by side flux ropes each 3 cm in diameter and 2 cm apart (see figure 2) were used. The data (magnetic field, density and flow) were acquired and analyzed in the same way as for the moon ropes (data acquired on 14 planes, 1681 positions per plane for a total of 23,534 locations, 12,288 time steps at 0.64 µs per step, \( I_{\text{ropes}} = 182.5 \) A, total, with a rope discharge voltage of 160 V). The major difference was that this experiment was done in an argon plasma. The background magnetic field was 330 G (as in the He case) and the ion gyroradius in the background plasma was 1.9 cm, however the ions in the ropes heat up to 5 eV [14] and \( R_\parallel \) increases to 4.4 cm. The ions in the ropes are not magnetized. The flux ropes and flows of the conditionally averaged data in case 2 look remarkably like that in case I and are shown in figure 7. The magnetic field lines are colored in red and blue and are followed from locations in the center of the current channels. The plasma flow is seen to meander around and between the ropes. The current density on a plane 1.28 m from the cathode source is displayed on the right is close to the origin of the ropes. The largest current density in each channel is 8.63 A cm⁻². The total current in each rope is 91 A.

The magnetic field lines writhe about themselves and twist about one another as previously observed [13–16]. When the kink mode drives the ropes into one another there are bursts of reconnection and at that time a QSL [27, 28] forms. QSL’s are regions in which the magnetic field connectivity changes rapidly but continuously across a narrow spatial region. Two field lines which are very close to one another entering a QSL wind up spatially far apart when they emerge. The definition for the QSL is given by

\[
Q = \frac{\left( \frac{\partial X}{\partial x} \right)^2 + \left( \frac{\partial X}{\partial y} \right)^2 + \left( \frac{\partial Y}{\partial x} \right)^2 + \left( \frac{\partial Y}{\partial y} \right)^2}{\left| \frac{B_z(z_0)}{B_z(z_1)} \right|}. \tag{2}
\]

Consider field lines that pass through a plane at \( z = z_0 \) with transverse coordinates \( x, y \) and then through plane \( Z_1 \) with transverse coordinates \( X, Y \) when the average background magnetic field is perpendicular to both these planes. Trace two field lines with separation \( dx \) and \( dy \) on the \( z_0 \) plane. If there is a reconnection region between planes \( z_0 \) and \( Z_1 \) then the distances between them \( dX, dY \) can become quite large. Therefore the quantity \( \partial X/\partial x \) is a measure of how far the lines have diverged in \( X \) if they are closely spaced by \( dx \) in the first plane. The other terms are a variation of this. If there is no
Figure 4. (a) and (b) are the plasma density in the vicinity of a single rope. (b) is a line-out through the line indicated in (a). The right hand side is the electron temperature (c) and a line-out of $T_e$ (d). Note that the background He plasma is 60 cm in diameter and the flux rope is far from the edge.

Figure 5. Two components of the magnetic field measured at a single location for the two voltages between the cathode and anode. The fields were generated using a conditional averaging technique with 15 ‘shots’ at each location. The transition between the two cases is not shown. For each abscissa $t = 0$ corresponds to the time the given voltage is switched on.

Figure 7 and 8 were generated from the conditionally averaged data. As shown in figure 6 not all shots could be

change in the field line separation the cross terms are zero, and the remaining terms are 1, therefore the smallest possible value of the numerator is 2. The numerator of this expression indicates by how much the end points in the $Z_1$ plane change relative to small movements in the $z_0$ plane. The denominator is a factor that scales away changes in connectivity simply due to changes in the background field as you might have in a mirror geometry. It also insures that the value of $Q$ is invariant under reversal of the boundary planes, so in effect we can assign a value of $Q$ to each field line. Then, regions in the magnetic field where $Q \gg 2$ define the QSLs. The larger the value of $Q$ the further away they stray. The existence of a QSL is an indicator for the presence of magnetic field line reconnection. In some sense the QSL mirrors the concept of the Lyapunov exponent, $\lambda$, which for chaotic systems is a measure of how rapidly predicability is lost. We evaluate this quantity for flux ropes later in this manuscript. The QSL ($Q = 59.5$) is shown in figure 8 in which the plasma current density is displayed as lines, where the current is derived from $\vec{J} = (c/4\pi) \nabla \times \vec{H}$. Also shown in figure 8 is $\vec{E}_{\text{ind}} = -(1/c)(\partial \vec{A}/\partial t)$. The inductive electric field appears only when a QSL is present and is another indication that reconnection occurs.

The vector potential was calculated by using

$$\vec{A}(\vec{r}) = \frac{1}{c} \int \frac{\vec{J}(\vec{r}')}{|\vec{r} - \vec{r}'|} \, d^3\vec{r}'$$

in the Coulomb gauge.

(3)

We caution, however, that the total electric field being $\vec{E} = -(1/c)(\partial \vec{A}/\partial t) - \nabla \phi$ has not been measured and that a substantial contribution, from potential gradients, which could serve to reduce the total field is possible [29, 30]. The inductive component of the vector potential is shown as a transparent brown surface for $\vec{E}_\text{ind} = -36 \, \text{V m}^{-1}$. It points in a direction that would induce currents to flow in the opposite direction of the rope currents. This back-electromotive force (EMF) is caused by reconnection. Although this is a large electric field it is less than the Dreicer runaway field [31] ($E_D = 3 \times 10^5 \, \text{V m}^{-1}$). As mentioned the net electric field could be much smaller because of space charges.

Figures 7 and 8 were generated from the conditionally averaged data. As shown in figure 6 not all shots could be
Figure 6. Two cases showing success and failure of temporal data alignment. (a) $B_x$ at $(x, y, z) = (-3.5, y = -2.5, dz = 383.4 \text{ cm})$. Four shots (out of 15 collected are shown) after they have been temporally lined up using a correlation with a fixed probe at $(x_f, y_f, z_f) = (-6.0, 0.0, 672 \text{ cm})$. The dark solid line is their average which is used in constructing field lines. (b) $B_x$ at $(x, y, z) = (-3.5, -2.5, dz = 702.9 \text{ cm})$. At this location the magnetic field wildly fluctuates from shot to shot and the average field does not show the large excursions that the individual traces do. In the upper trace conditional averaging is possible because if a few of the 15 shots are very different they can be excluded. This is not true for the case in (b); argon plasma, $B_0 = 330 \text{ G}$.  

Figure 7. Flux ropes field lines, current and flows. The plane on the right ($dz = 1.28 \text{ m}$, $z = 0$ is on the LaB$_6$ cathode) shows the plasma current density. The value in the center spot (orange) is $8.63 \text{ A cm}^{-2}$ and the average value $5.75 \text{ A cm}^{-2}$. Blue is zero. The flux ropes field lines were started in the high current regions. Magnetic field lines are colored red and blue and the plasma flow, with a cross-hatched pattern is observed to snake between and around them. Three-axis marker arrow ‘a’ is located at $dz = 5.28 \text{ m}$ and arrow ‘b’ at $dz = 8.28 \text{ m}$. The background magnetic field ($B_0 = 330 \text{ G}$, Ar) is in the same direction as the blue marker arrow. The extent of the $x$ and $y$ planes is $dx = dy = 16 \text{ cm}$. The data was acquired at $t = 4.18 \text{ ms}$ after the flux rope turn on.
used to do this. There are instances where the topology is significantly different than in the average case and these are invisible. The only way to capture them would be with the use of tens of thousands of very small probes, coupled to digitizers to record the event in its entirety. Experiments often use multiple probes but they are too large and not numerous enough to do this. The probes would have to be microscopically small (MEMS) probes [32–34], however producing them in the quantity required and coupling them with digitizers remains a daunting problem. One day it will be possible.

In previous work the QSLs were studied, the magnetic helicity evaluated and the topology of the reconnection site was studied. Here we turn our attention to chaos within the flux ropes and evaluate it using mathematical tools to calculate the permutation entropy and complexity of the data.

5. Complexity and entropy—introduction

Fortunately advances in mathematics driven by the study of chaos in a number of systems, both purely theoretical (iterative maps) and real (fluid flow, weather systems, plasmas, etc) make it possible to go beyond the average data that has been presented and study its complexity. If one starts with a time series of data, such as a magnetic field component, and a single wave is present it is obvious in the data and its Fourier transform. If the fast Fourier transform (FFT) of the data stream has several sharp lines embedded in a continuous spectrum one could surmise that there are waves in a sea of noise. But what does the noise itself represent? Is it due to a completely random process, chaos? If it is chaotic, is the chaos due to deterministic processes such as a driven oscillator or a van der Pol system or something else? Recently, in the past decade, mathematicians working on the subject have introduced methods for placing time series data on maps. The position on the map offers a clue to the type of chaos present. The permutation entropy, $S$, of Bandt and Pompe [35] allows for calculation of the entropy in a time series of any data in the presence of noise.

$$ p_j(\pi) = \frac{\# \{ t_j | t_j \leq T - n, (x_{t_j}, \ldots, x_{t_j+n}) \text{ has permutation type } \pi \}}{T - n + 1}. $$ (4)
The Bandt–Pompe permutation entropy is

$$S(P) = -\sum_{\pi} p_j(\pi) \ln(p_j(\pi)), \quad N = n!. \quad (5)$$

Here the sum runs over all $n!$ permutations $\pi$ of order $n$. Note that this is of the same form as the standard thermodynamic definition of entropy where $p$ would be the number of states. Bandt and Pompe go on to show that the permutation entropy $S$ is similar to the Lyapunov exponent for chaotic system such as the logistic map and tent map. The permutation entropy was evaluated for the Lorentz model by Lehman et al [36].

The definition of $S$ is best shown by example. To calculate $S$ the data, $(x_t), t \leq T$ is broken into $n$ bins, where $n$ of necessity is smaller than the number of values in the time series. A set of the first $n$ numbers of the time series are first chosen. The number of permutations of the first set of numbers is $n!$. If $n = 5$ there are 120 possible permutations. In one permutation each successive number in the time series is larger than the one which occurred earlier. Call this case 1. If case 1 occurs 30 times in the time series of 1000 numbers then its contribution to the permutation entropy is $S(\text{case1}) = -(30/996) \log(30/996)$. The log is taken to the base 2.

Bandt and Pompe define the permutation entropy by first finding all permutations for series $j$ ($j$ is as large as 120 and $n = 5$). Note that $n \geq 2$.

The Shannon entropy may be normalized to lie between 0 and 1.

$$H(P) = \frac{S(P)}{\ln(N)}. \quad (6)$$

Here $N$ is the total number of possible permutations.

The method to distinguish noise from chaos depends upon the location of the data on a complexity–entropy plane. The abscissa is $H(P)$ and the ordinate the Jensen–Shannon complexity defined by [37]

$$C_{js} = -\frac{1}{N+1} \sum_{N} \ln(N+1) - 2 \ln(2N) + \ln(N) \quad (7)$$

where $P_e$ is the maximum entropy state, that for which every member of the probability distribution have the same value, $1/N$. Rosso et al [37] showed that a number of chaotic maps as well as fractional Brownian motion [38] appear at specific locations in the complexity plane. There are a number of statistical methods for dealing with data on turbulent processes. For example structure functions are often employed to estimate the slope of power spectra [39] when they are displayed on log–log plots. There is a long standing effort to relate temporal spectra to power spectra (this involves assumptions) and comparison of the exponential drop off to theory [40]. In this work we use an entirely different method with no assumptions. The Jensen–Shannon complexity coupled with the Bandt–Pompe entropy ($C–H$ diagram) comes from information theory. $C–H$ diagrams can discern between different degrees of periodicity associated with routes to chaos, can distinguish stochastic noise from chaos, and are sensitive to correlational structures. They are a promising tool for use in plasma physics.

Zunino et al [41] studied both the effect of adding increasing amounts of white noise, which is a contamination, and the embedding delay on distinguishing that type of dynamics using the $C–H$ plane. If a series has $T$ members one can choose to use every point, every other point and so on to calculate $H$ and $C_{js}$ so long as the decimated series has enough values such that $T_{\text{kept}} \gg n!$. Here $T_{\text{kept}} = T/D$ where $D$ is the embedding delay ($D = 1, 2, 3 \ldots$). By choosing every member of the original time series (small embedding delays) detection of uncorrelated stochastic dynamics is emphasized, for larger $D$ stochastic dynamics are optimally discriminated and for large $D$ the data is under-sampled. The presence of white noise and the optimum value of $D$ can be detected by plotting data on the $C–H$ plane as a function of $D$. Zunino et al [41] used this to study an ammonia laser (chaotic), flow in a river (chaotic), the North American atmospheric oscillation (stochastic), crude oil and gold price dynamics (stochastic rather than deterministic), and human posture dynamics (noisy and chaotic). Chaos theory has been used to study plasma physics. Horton et al [42] calculated several quantities such as the Lyapunov exponent, fractional dimension and bifurcation diagrams in a model of the solar wind–magnetosphere–ionospheric system. The first application of the permutation analysis methodology to plasma physics was done by Maggs and Morales [43]. The data was time series of the electron temperature measured in an unstable filament of heat that was generated in a magnetoplasma [44]. The data indicate that the filament which is stable for a short time (roughly an ion gyro-period) exhibits chaotic behavior when it becomes unstable. Frequency spectra can be associated with time series and different chaotic processes have differences (sometimes subtle) in their spectra. In the case of the filament the spectra is exponential, showing a linear relation when plotted on a log$(f(w))$ vs linear$(w)$ plot. It was shown [45], that the presence of Lorentzian pulses in the time series data are responsible for the exponential spectrum. In this work this methodology is extended to volumetric data on magnetic flux ropes. The complexity and entropy are evaluated at tens of thousands of spatial positions and compared to data sets which are conditionally averaged (and field lines drawn) and data sets containing time series excluded from the average. The complexity in space and time are evaluated.

6. Complexity and entropy—flux ropes

The complexity and entropy was calculated for both the helium (2 moon ropes, figure 2, left) and argon (2 circular ropes, figure 2, right) experiments. In the moon rope experiment there were 10 shots at each location with 10 240 times steps acquired and in the 2 ropes in argon (case 2) the 12 288 time steps of acquired data in each shot was broken into 8 segments of 1562 time steps ($d_t = 1.0$ ms). For the 2 moon ropes 2000 sample windows were used as in both cases the data was not correlated for longer times. Calculations of $H$ and $S$, $C_{js}$, were done on the short segments, which corresponded to field line and flow maps, as well as the entire temporal sequences.
Figure 9. The $C$–$H$ plane, case 1 of the two moon ropes, is shown in (b) for the data that was correlated, that is it could be conditionally averaged, and (a) for all of the data. The time series was subsampled by $2 (D = 2)$ and $n = 5$. Helium plasma $B_0 = 330 \text{ G}$. $V_0 = 110 \text{ V}$. (a) All data is displayed. (b) Spatial positions are displayed for all axial positions by color coding the dots. Here fBm, the black line, is where fractional Brownian motion falls. The ordinate is the Jensen–Shannon complexity and the abscissa, the $H$ axis, is the Bandt–Pompe entropy. The lines bounding the data correspond to minimum and maximum complexity. Points on the curve are associated with chaotic processes. ($X$—Lorenz 3D chaotic map), (+—Henon quadratic map), ($\Delta$—Henon map), (<—Lorenz map).

The choice of the embedding delay $D$ and the permutation number $n$ is important. If $D$ is too small noise in the data can dominate and if $n$ is too small $C_{js}$ can be overestimated [46].

Data on a plane 3.8 m from the source of the flux ropes in argon was used to explore this. This location is where the ropes collide and reconnection is observed. A series of 5585 time steps at a number of locations with 15 instances of the 3 vector components of the magnetic field was used. The embedding delay was varied from 1–4 and the permutation number $n$ varied from 2–6. There is one case in which an embedding delay of 16 was used to eliminate a high frequency mode. This will be discussed in detail. There was little high frequency noise in the argon data and little difference with embedding delays of $D = 1$–3 in $C_{js}$ or $H$ for a given $n$. When the permutation number, $n$, was varied it was seen that the complexity was overestimated for $n = 2$. For $n > 3$ $C_{js}$ and $H$ oscillated by 10% about a fixed value (which was position dependent). Permutation numbers $n$, of 4 and 5 and embedding delay $D$ of 1, 2 were used in this analysis with no noticeable difference in the $C_{js}$–$H$ diagrams. Since the temporal data sequences on conditionally averaged data had 1000–2000 points it made no sense to use $n = 6 (6! = 720)$. The work by Riedl et al [46] in analysis of the Bandt–Pompe entropy comes to the general conclusions the number of terms $T$ in the sequence under analysis should be $T > 5n!$ The data sequences analyzed had from 1200 to 2000 time steps and this is satisfied for an embedding number of 5 (or 4). In the data in figure 8 $D = 2$ and the use of the largest possible permutation entropy, $n$, is advised for data that has an inherent cycle in it. This is the case of the flux ropes.

In a separate test the embedding number $D$ was varied from 1 to 64. For intermediate embedding numbers this is somewhat like smoothing and high frequency noise is eliminated. For larger values of $D$ low frequency coherent modes are also eliminated and the map produced can be misleading. Coherent modes have low entropy and small complexity and reside on the lower left part of the $C$–$H$ diagram. For any value of $n$ there are minimum and maximum complexity curves and all data in the $C$–$H$ plane must lie somewhere between them [47].

Zero or low entropies reflect highly ordered processes, such as a monotonic series or a single wave. Stochastic processes are marked with large entropy but small complexity. The processes exhibiting the largest chaos are close to the maximum complexity line and at mid-entropy, close to the Henon map. The data clearly exhibits regions which are chaotic, even in the conditionally averaged data, however points near the top of the curve appear only when the time sequences which had to be excluded from the average are included in the complexity calculation. That which is potentially most interesting cannot be seen.

Figure 10. Complexity diagrams for the moon ropes in (a) the raw data and (b) the conditionally averaged data; $D = 16$, $n = 4$. The complexity of the raw data is for the most part above the fractional Brownian motion curve and has larger values of $C_{js}$. It exhibits chaos. The color bar indicates the axial locations where the points were evaluated. The comparison maps are as in figure 9.
What is responsible for the chaos in the upper part of figure 9(a)? The frequency spectra of the noise in the plane furthest from the source of the ropes indicates there were waves present at about 100 kHz. The initiation of the waves was independent of the ropes but there was an interaction of waves with the ropes. This will be addressed later, however the waves can be eliminated from the data by increasing $D_t$, the embedding delay. Figure 10 shows the raw data and conditionally averaged data of the moon ropes with an embedding delay of 16 and embedding dimension $n = 4$.

In figure 10 the conditional averaging brings out lower entropies, which can be associated with the rotating flux ropes. In both cases the complexity grows as one moves away from the source of the ropes ($z = 0$).

We next study the temporal development of these maps. The flux rope data for argon (12,288 time steps ($dt_{long} = 7.86$ ms), per data sequence) was broken into seven steps of 1562 and the temporal evolution of the complexity examined. Figure 11(a) is the temporal signal of one component of the magnetic field ($B_y(t)$) showing the intervals the time sequence was broken into.

In the first segment the background plasma is on but the flux ropes have not been switched on. The noise level is small but enough to dominate the calculated $C_{js}$ and $H(n = 5, D = 2)$. The rope currents grow in interval 2 and the kink instability begins. The $C–H$ diagram is shown for four of the intervals displayed in figure 11(a). The switch on, a period before the kink instability kicks in, shown in figure 11(b) has a large entropy but relatively small complexity and occupies the part of the diagram that fractional Brownian motion is seen to exist. The background noise appears stochastic as it straddles the fBm curve. In the next two time intervals displayed in figures 11(c) and (d) the complexity increases and straddles the diagram between minimum and maximum complexity. It is close to the Gissenger (Earth’s chaotic magnetic field reversals) and Hindemarsh (neuronal bursting model) processes. Figure 11(e) shows the complexity and entropy after the flux rope source is turned off. The behavior
Figure 12. Plasma currents shown as colored lines for the two flux ropes at $t = 4.18$ ms after the ropes are switched on. The leftmost plane ($z = 1.28$ m) has the magnitude of the current density and the source of the ropes is clearly visible. Also shown is a QSL present at this time ($Q = 36$). $C_{ij}$ of the $y$ component of the magnetic field, $B_y$, is calculated for the last temporal segment (which starts at 4.18 ms), $D = 2$, $n = 5$ is displayed on five planes which are transverse to the background magnetic field. The planes are located at $z = (2,3,4,5,6$ m) from the cathode flux rope source. A reflection in a virtual mirror is shown on the bottom.

of the ropes with a discharge voltage $V_D = 220$ V has different complexity plane mappings and will be discussed in a future publication.

The $C-H$ diagrams for argon ropes show chaos but not as large as that observed in the helium flux ropes. In argon the value of $C_{ij}$ is never greater than 0.32, there is nothing comparable to the ‘hook like’ region in figure 9(a). Regions of greater chaos [41] lie in the region of the $C-H$ plane where the complexity is large and the normalized entropy of order 0.5. There are two differences between the cases. The flux ropes are larger in the helium case and the ion gyroradius is also larger, at least by a factor of 2.5 in argon. The argon ion gyroradius is 2 cm when the ropes are formed which is the initial distance between the ropes and nearly their diameter (figure 2). The ions rapidly heat to to 6 eV [14] at which time $R_i = 4$ cm.

Since the Jensen–Shannon complexity, $C_{ji}$, was calculated and stored at every spatial location one can present the data throughout the volume and this is shown for $B_y$ in figure 12. Here the flux rope current and a QSL surface are shown as well. The key feature in figure 12 is that close to the source of the flux ropes, $C_{ij}$ is largest on both sides of the ropes and smaller between the ropes. At a larger distance ($z > 4$ m) the reverse occurs and the complexity is largest where the ropes reside. This spatially coincides with the axial location at which magnetic field line reconnection occurs. Note that the QSL is large between the ropes. The QSL passes through the reconnection region but extends throughout the volume (therefore it cannot be used to axially pinpoint the reconnection site). The maximum measured complexities are lower far away from the source, though. This could be due to a spatial smearing of the complexity because of the rotation of the flux ropes. The entropy and complexity are calculated for a time length spanning several rotation periods of the ropes; therefore temporal/spatial smearing is anticipated. High complexities around the flux ropes close to the source (where the rotation and smearing is negligible) could indicate that the chaotic signatures are due to gradient driven modes as in [44, 45]. The $C-H$ diagrams display entropy as well as complexity and entropy is absent from figure 12.

It is instructive, therefore to show the complexity and entropy on different planes and this is shown in figure 13, which is for magnetic field data on a plane at $dz = 1.28$ m, and for a time series 1 ms long and centered at $t = 4.2$ ms. The three components of the magnetic field do not evince the same complexity or entropy. The transverse components have some similarities especially close to the source of the ropes ($dz = 0$), as will be demonstrated next, however the axial
component is always much different. \( B_y \) shows both large entropy and complexity between the flux ropes, \( B_y \) is more complicated but at this location and time \( C_y \), and \( H \) are largest at the same locations. The axial component of \( B \) has a different structure. The complexity is largest on a ring surrounding the ropes, and the entropy is small which translate to points on the left-hand side of the \( C-H \) diagram (figure 9). There is some structure in \( CH(B_y) \) where the flux ropes are but in general \( C_y \) is large and \( H \) is low.

The complexity diagram is also a function of space, indicating that at any one time the chaotic motion grows along the flux ropes this is evident in figure 9, where the dots on the \( C-H \) diagram correspond to data acquired along the axis. The actual spatial morphology of the ropes is shown in figures 13 and 14. Close to the source of the ropes (figure 13) the axial current clearly shows the position of the ropes. Negative current density is consistent with electrons flowing away from the LaB\(_6\) cathode and the current antiparallel to the background field. In the transverse plane the current flows around the ropes in the electron diamagnetic direction. The perpendicular field. In the transverse plane the current flows around the component of \( B_y \) which are related to the axial current \( J_z = \frac{\pi}{C_y} (\partial B_y/\partial x - \partial B_x/\partial y) \).

The magnetic field is not the only quantity that exhibits complex motion. In both the helium and argon experiments the 3D flow was measured throughout the plasma volume at the same spatial locations that the magnetic fields were acquired. Figure 6 is an example. \( C_y \) and \( H \) were also evaluated for the flows and they are shown in figure 15. The \( C-H \) diagram at the rear of the figure is unlike that for the magnetic fields. Far from the source more of the points (red dots) hug the fractional Brownian motion line but closer in the diagram bifurcates, one grouping with complexities between 0.3 and 0.4 and, a higher chaos group which runs through the X corresponding to the Lorenz 3D chaotic map. The flow lines indicate flow circulating around the ropes and they are colored by their complexity. The flows, which will be discussed in a great more detail in an upcoming publication [16] range from \(-0.6 \leq M_y \leq 0.6, \alpha = x, y, z \) with the axial flows generally smaller than the transverse ones.

Another tool for analysis of temporal spectra is Fourier analysis. Plasma turbulence studies, in both the laboratory and space, have used spectra as a primary tool in analysis. There are many theories which predict a power law associated with the spectra in the hope that the power law is a unique indicator of what is going on. The spectra of the magnetic field fluctuations in the moon rope experiment is shown in figure 16. This corresponds to the complexity map of the raw data in figure 9(a).

In the helium experiment two peaks are visible at 112 and 110 kHz. These modes are not caused by the flux ropes or the currents associated with them, instead they are believed to be a manifestation of an Alfvén wave maser that can exist under these conditions [49]. The center curve corresponds to \( V_0 = 110 \) V (see figures 4 and 5). There are additional low frequency peaks corresponding to the rotating flux ropes and greatly enhanced noise from 20–100 kHz. At the largest discharge voltage (upper curve, \( V_0 = 220 \) V) there is a greater number of low frequency peaks. In figures 4 and 5 the overall frequency is obviously higher at higher discharge voltage but it is not purely sinusoidal. At higher voltage there are additional high frequency sidebands, roughly 70% of the ion cyclotron frequency, suggesting an interaction between the flux ropes and the maser. These are conditions under which the very chaotic signal in figure 9(a) is observed. Conditional averaging disallows the most chaotic, and perhaps the most interesting data from being plotted in space. Chaos involving the maser will be the subject of future work. Finally the dashed line on this log-linear frequency plot indicates that aside from the peaks the spectrum is exponential. Exponential spectra are associated with Lorentzian structures in the plasma [50]. Inspection of the time series data in the flux rope experiment shows that these structures are present. They are not waves, rather they are a nonlinear byproduct of what is going on. Exponential spectra have been observed in a great variety of situations [51] suggesting that these structures are a fundamental quantity in nature. The spectra in figure 16 is shown for the furthest data plane with respect to the start of the ropes. As one moves away from the cathode source of the background plasma, and toward the source of the ropes (see figure 2) the maser signals dies away and the region of largest complexity fades away as well. This is shown in figure 17 which is a 3D rendering of the \( C-H \) diagram, the third axis is axial distance from the source of the ropes (at \( dz = 0 \)). What stands out is that the \( x \) and \( y \) components of the magnetic field (red and green dots) do not occupy the same region as \( B_z \) (blue dots). The right side of the diagram corresponds to low entropy (the \( H \) axis is reversed to that shown in figure 8). Close to the source of the ropes the complexity is low and entropy high which corresponds to stochastic processes. Far from the LaB\(_6\) cathode data points appear at high complexity and entropy of order 0.6. This is the highly chaotic region also home to the Henon map.

The permutation entropy analysis of the previous sections is complemented by the calculation of the Hurst exponent and the largest Lyapunov exponent, two other important quantities in the nonlinear dynamics field. Calculations were done for the argon flux ropes at two axial locations, i.e. at \( z = 1.28 \) m and at \( z = 9.60 \) m. Results can be compared to the permutation entropy and complexity analysis in figures 13 and 14.
Figure 13. The complexity $C_{js}$ (top row) and permutation entropy ($H$) (bottom row) for three components of the magnetic field on a plane located 1.28 m from the source of the flux ropes. The time series which has 1562 samples starting at $t = 4$ ms acquired for 15 shots at each position was used in the calculation. $C_{js}$ and $H$ were averaged over the shots. Here $n = 5$, $D = 2$; argon ropes with $B_{0z} = 330$ G. For comparison the axial and perpendicular currents at this time are shown on the right. The legend on the bottom shows the range for $C_{js}$ and $H$ used in the figure.

Figure 14. The complexity $C_{js}$ (top row) and permutation entropy ($H$) (bottom row) for three components of the magnetic field on a plane located 9.56 m from the source of the flux ropes. The time series which has 1562 time series starting at $t = 4$ ms, acquired for 15 shots at each of 1681 transverse positions was used in the calculation. $C_{js}$ and $H$ were averaged over the shots. Here $n = 5$, $D = 2$; argon ropes with $B_{0z} = 330$ G. For comparison the axial and perpendicular currents at this time are shown on the right. The legend on the bottom shows the range for $C_{ij}$ and $H$ used in the figure.
Figure 15. Flow field lines for an argon plasma acquired at $dt = 4.18$ ms. The flow is colored by the Jensen–Shannon complexity. The transverse plane is $dx = dy = 16$ cm. Two markers show the distance along the $z$-axis. The blue arrow points along the background field (330 G). The $C–H$ diagram for the entire volume is projected on the rear. The symbols and color map (in which the dots are colored according to their distance from the source) is the same as those in figures 10 and 11.
The Hurst exponent [52] was developed by Hurst in the context of determining the optimal size of water reservoirs on the river Nile in Egypt. The Hurst exponent, $0 < H < 1$, measures the persistence ($0.5 < H < 1$) or anti-persistence ($0 < H < 0.5$) of time series, and thereby quantifies long-term memory of a time series. For persistent time series an increase will most likely be followed by an increase in the short term, and a decrease will most likely be followed by a decrease in the short run. Anti-persistent time series will switch between increases and decreases in the short run, and show this behavior for long times. An uncorrelated time series will have $H = 0.5$.

The estimation of the Hurst exponent follows from a rescaled range analysis of a time series. The rescaled range is defined as $R/s$, where $R$ is the range of the cumulative sums of the time series and $s$ is the standard deviation. The rescaled range $R/s$ has a power law dependence on time, i.e. $R/s \sim n^H$, where $H$ is the Hurst exponent. To determine the Hurst exponent a time series of length $T$ is subdivided in arrays of length $n$, with $n$ ranging from 8 to $T$. For each $n$ the rescaled range is calculated and averaged over all $T/n$ arrays of length $n$. The Hurst exponent is then obtained by fitting a straight line to $\langle \ln(R/s) \rangle$ plotted versus $\ln(n)$. The Hurst exponent was estimated for each time trace in two-dimensional (2D) planes during the steady state of the argon flux ropes. 4096 time samples were retained corresponding to 2.62 ms of data. Figures 18(a) and (b) show the Hurst exponent averaged over 15 plasma shots in 2D cross sections of the plasma at $z = 1.28$ m and $z = 9.60$ m. At 1.28 m the signature of the two flux ropes is clearly visible. The Hurst exponent is lowest around the flux ropes, with values approaching 0.6, meaning that the magnetic field time traces around the flux ropes are the least predictable, and approach Hurst exponents of uncorrelated time series ($H = 0.5$). At 9.60 m the Hurst exponent is lowest in a large area where the flux ropes reside.

![Figure 16](image16.png)

**Figure 16.** The magnetic field spectra in the ‘two moon’ helium rope experiment. The Fourier transform of all three components of $B$ and at every spatial location in the furthest plane from the source of the ropes ($dz = 9.6$ m) were averaged. The other trace is the spectra of background field fluctuations (figure 5). The ratio of the frequency to the ion cyclotron frequency at 330 G is shown at the top. All the peaks shown are well above the measurement noise level.

![Figure 17](image17.png)

**Figure 17.** The $C-H$ diagram ($D = 1, n = 5$) as a function of axial position in the device. The flux ropes are born at $dz = 0$. The first data plane where measurements were made was at $dz = 1.28$ m from the source of the ropes. The furthest plane measurements were made was at $dz = 9.56$ m. The minimum and maximum complexity curves become surfaces above and below the dots. A different color is used for each component of the field. The $z$ component of the magnetic field is represented by the blue spheres.
Figure 18. (a) Hurst exponent at $dz = 1.28$ m and (b) $dz = 9.60$ m. These should be compared to the $z$ component of the complexity and entropy in figures 13 and 14. (c) Lyapunov exponent at $dz = 1.28$ m and (d) Lyapunov exponent at $dz = 9.60$ m.

Hurst exponents are not as low as at 1.28 m from the source, but this is likely due to a temporal/spatial averaging of the flux ropes. At large distances from the source the flux ropes rotate around the machine axis and partially merge. The Hurst exponents obtained here were estimated using time series spanning several rotation periods, and therefore spatial smearing of the Hurst exponent is expected.

The other quantity of interest is the largest Lyapunov exponent. In chaos the sensitive dependence on initial conditions is of crucial importance. The spectrum of Lyapunov exponents quantifies this dependence by measuring the exponential divergence of initially close phase-space states. In this study we will only calculate the largest Lyapunov exponent as it is the most important one and as it will dominate the dynamics of the system.

Chaotic systems yield positive Lyapunov exponents and the value measures the average rate at which predictability is lost. Higher positive Lyapunov exponents therefore represent more chaotic systems.

The Rosenstein algorithm [19, 53] for determination of the largest Lyapunov exponent was used. This algorithm provides a practical method which can be used for small data sets, and is robust to changes in embedding dimension and noise level. The embedding dimension was kept the same as in the permutation entropy analysis, i.e. $n = 5$. The algorithm splits the data up in vectors of length $n$, wherein each sample of the vector is separated by a lag $J$, and $J$ is chosen as the lag at which the auto-correlation of the time trace drops below $1 - 1/e$. For each of these vectors of length $m$ the nearest neighbors is found. The separation $d$ between nearest neighbors is followed for a number of time samples. The largest Lyapunov exponent is found by fitting a line to the separation $d$ averaged over all vectors of length $m$, $(\ln d) \sim \lambda t$.

The Lyapunov exponents were normalized to the ion cyclotron frequency of argon ($w_{ci-AR} = 7.9 \times 10^4$ rad s$^{-1}$). Figures 18(c) and (d) show the largest Lyapunov exponents at $z = 1.28$ m and $z = 9.60$ m respectively, averaged over the 15 plasma shots obtained at each spatial location. The largest Lyapunov exponent was calculated for each time trace at each location in the 2D planes during the steady state of the argon flux ropes. Two thousand time samples were retained corresponding to 1.28 ms of data. At 1.28 m the largest Lyapunov exponents are found around each of the flux ropes, signaling that the most chaotic behavior is found at the edges of the ropes. This agrees with the complexity analysis of figure 12 and with the Hurst exponents analyzed above. Far away from the source, at $z = 9.60$ m, the most chaotic time series are found in the large region where the flux ropes rotate around the magnetic axis, again in agreement with figure 14 and the Hurst exponent analysis.

7. Summary and conclusions

The concept of permutation entropy and Jensen–Shannon complexity have been applied to the analysis of experiments involving magnetic flux ropes. This analysis has been used for the most part in the study of chaotic maps and processes mentioned in [19, 37]. This has previously been used in analysis of a plasma physics experiment [43], but here, for the first time, it is applied to fully three-dimensional data on magnetic flux ropes. The analysis shows that in a quiescent argon plasma, before the ropes are switched on, the magnetic noise is stochastic, somewhat like fractional Brownian motion. After the ropes are switched on the distribution in the C–H plane moves to regions that indicate moderate chaos in the presence of coherent modes. The ropes in a helium plasma
become highly chaotic partly due to the presence of a mode (Alfvén maser) that was present at the time the ropes were switched on. At larger rope discharge voltage additional peaks develop in the spectra suggesting nonlinear coupling between the two systems.

The complexity diagram is sensitive to all sources of fluctuations. When the maser is filtered out by binning the data, or by conditionally averaging, the ropes exhibit chaos nevertheless. The plasma flows become chaotic although they occupy a different space on the $C-H$ diagram. In all cases the frequency spectra become exponential over much of their range and Lorentzian structures were observed in the time varying magnetic fields. When plotted in the $C-H$ plane the magnetic field and flow can coincide with the location of several chaotic maps.

Suppose experimental data on the $C-H$ plane occurs at the same position as that for a known chaotic process. Do they have anything in common? One characteristic of a process is its frequency spectra. Is it a power law or exponential? Does it also have a number of spectral lines at discrete frequencies? These all could be clues to what is going on. We examined the spectral data for the Lorentz, Gissenger and logistic maps to that of experimental data that occupy the same position on the $C-H$ plane. The spectra associated with the magnetic fields did not look like those calculated for these maps. There may be other quantities such as the fractional dimension that are shared but these were not evaluated.

Associating data with locations on the $C-H$ plane is a fast and excellent way of identifying the existence of chaos. Its $C-H$ location can roughly evaluate the type of chaos it might be but, thus far, it can say nothing about the physical processes which causes it. Signals which occupy space in the $C-H$ plane associated with both chaos and coherent waves suggest that the cause could be deterministic, that is nonlinear coupling between several modes. This experiment revealed that close to their source the flux ropes were not chaotic and had low entropy. The plasma surrounding them was more chaotic suggesting instabilities driven by rope pressure gradients. Further away along the magnetic field the ropes collide. There is reconnection [14] and the region of large complexity is in the QSL. This is a region where there is a large back-EMF associated with reconnection. The complexity there could be the result of current driven instabilities. The Hurst and Lyapunov exponents, two parameters long used to characterize turbulence were also calculated and their values make sense when compared to the complexity and entropy. The Lyapunov exponent is a measure of the average rate at which predictability is lost and it is derived using a time series. The QSL is analogous in the spatial domain. It measures how rapidly field lines wander in the spatial domain. When there is reconnection $Q$ is large and field lines which are initially close to one another move far apart after encountering a reconnection site. It is not clear however if reconnection is a major player or a minor character actor in this interaction. Front and center are the flux ropes.

To verify any of the conjectures between the relationship of a location in the $C-H$ plane and the process which puts the data there requires an investigation using all the tools available to an experimentalist. The study of chaos and the evaluation of the Bandt–Pompe entropy and Jensen–Shannon complexity along with other tools under development by that community may one day enable us to say more about the chaotic processes which underlie it. This is all related to the reconstruction of the topology of what is going on in a chaotic situation. Three-dimensional maps of the flow and magnetic field were calculated in this experiment from conditionally averaged data. Using averaged data to do this is commonplace in this and other fields. Without a huge ($10^4$–$10^5$) number of detectors to capture everything that happens in any one occurrence one loses something. This could be displayed by making field lines or other displayed quantities fuzzy in proportion to the standard deviation. Can we do better?

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