Large Direction-of-Arrival Mismatch Correction for Adaptive Beamforming

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ABSTRACT Direction-of-arrival (DOA) mismatch can degrade the performance of adaptive beamforming algorithms. Thus, a projection method is proposed to correct this mismatch. In a beamforming algorithm, the DOA error is usually regarded as a steering vector error which is corrected using a steering vector optimization algorithm. This approach can provide an optimal steering vector but ignores the actual DOA estimate. The proposed algorithm provides correction after DOA estimation but before beamforming to improve both the DOA estimation accuracy and beamforming gain. First, the signal-to-noise ratio (SNR) of the signal is estimated and used to regularize the covariance matrix. Then, an estimated steering vector with DOA close to the true value is determined based on a minimum number of projections. Numerical results are presented to verify the effectiveness of the proposed method for DOA estimation correction. In most cases, this method improves the performance of the beamforming algorithms without changing them.

INDEX TERMS Direction-of-arrival (DOA) mismatch correction, adaptive beamforming, covariance matrix regularization, projection method.

I. INTRODUCTION

Adaptive beamforming is an important technique in array signal processing. It has been widely applied in radar [1], sonar [2], wireless communication [3], microphone array speech signal processing [4], medical imaging [5], and radio astronomy [6]. Compared with traditional beamforming which does not depend on the received data, adaptive beamforming optimizes the weight vector based on the received data and an optimality criterion. Moreover, this vector can be adjusted according to the environment to suppress both noise and interference. However, errors exist in practical antenna arrays due to the effect of the signal-of-interest (SOI) on the training data and a small number of snapshots can severely degrade performance. Several adaptive beamforming algorithms have been proposed to solve these problems including diagonal loading [7]–[9], eigenspace projection [10]–[12], uncertainty constraint [13], [14], steering vector estimation [15]–[18], covariance matrix reconstruction [19]–[21], and a combination of the latter two [22]–[24].

Diagonal loading is a simple robust adaptive beamforming (RAB) method that has low computational complexity. However, it is difficult to choose an appropriate loading level. To deal with this issue, some schemes select the loading level automatically [25], [26]. Low computational complexity and simplicity make eigenspace projection methods popular for real-time applications. While they can deal with arbitrary errors in the steering vector, the number of subspaces can impact performance. Further, the performance is poor when the SNR is low. Uncertainty set constraint beamformers such as worst-case (WC) [13] can solve the problems with eigenspace projection methods, but the performance is degraded at high SNRs. Combined methods which employ steering vector estimation and covariance matrix reconstruction can be employed to overcome these issues. They are
robust to many types of array errors and effective over a wide SNR range, but the computational complexity is high. Thus, there is a tradeoff between complexity and performance.

Steering vector estimation algorithms [18] require an initial SOI estimate. With the approaches in [19], [20], the steering vector of the interference signal is needed a priori to reconstruct the interference-plus-noise covariance (INC) matrix. The DOA is required to form the steering vector, and this can be determined using a DOA estimation algorithm or approximate orientation prediction. However, a large DOA mismatch in the received signal will result in significant errors in the steering vector [18], [23]. This will also make it difficult to determine the INC matrix [19], resulting in performance degradation. Conventional algorithms such as multiple signal classification (MUSIC) [27] and Capon [28] are commonly used in DOA estimation, but the estimation performance is poor when the SNR is low and with gain-phase uncertainties. Several techniques have been proposed to improve the performance of these algorithms such as sparse array configurations [29], DOA estimation with uncertain gain-phase sensors [30], and modified MUSIC algorithms [31].

To reduce DOA mismatch and make beamforming more robust, the DOA is corrected in this paper based on a minimum number of projections of the steering vector in the observation region. The received signal characteristics when the SNR is low and high differ. Thus, an SNR estimation algorithm is designed to distinguish between low and high SNRs. Since the noise level is large when the SNR is low, the influence of noise should be reduced in this case by adjusting the signal covariance matrix. One approach employs a tridiagonal matrix, but this requires selecting an appropriate loading level. Therefore, we combine the estimated SNR with the desired signal steering vector and signal covariance matrix to adaptively determine this level. In beamforming algorithms, the DOA error is usually regarded as a steering vector error which is corrected by a steering vector optimization algorithm. Here, a modified DOA is determined based on the minimum number of projections of a presumed steering vector onto the eigenvalues of the modified covariance matrix. This requires only the received signal, the array geometry, and the approximate DOA or angular sector of the desired signal. This method can improve beamforming performance in most cases by correcting the DOA mismatch without changing the algorithm.

The rest of this paper is organized as follows. Section 2 presents the background and introduces the signal model for adaptive beamforming. The proposed method is given in Section 3. Numerical results are presented and discussed in Section 4, and finally some conclusions are given in Section 5.

II. SIGNAL MODEL AND BACKGROUND

Consider a uniform linear array (ULA) composed of \( M \) omnidirectional sensors that receive \( N \) uncorrelated narrowband signals from far-field sources. These \( N \) signals consist of one SOI and \( N - 1 \) interference signals impinging on the array from directions \( \theta_1, \ldots, \theta_N \), respectively. The length \( M \) received signal vector at time \( k \) can be expressed as

\[
x(k) = x_s(k) + x_i(k) + x_n(k)
\]

where \( x_s(k) = s_1(k)a_1, x_i(k) = \sum_{j=2}^{N} s_j(k)a_j \) and \( x_n(k) \) is the desired signal, interference, and noise, respectively. The desired signal waveform is \( s_1(k) \) and the corresponding steering vector is \( a_1 \). The interference signal waveforms are \( s_j(k), j = 2, 3, \ldots, N \) and the corresponding steering vectors are \( a_j \). The received signal vector at time \( k \) is complex white Gaussian noise with zero mean and variance \( \sigma_n^2 \).

The steering vector of the array can be formulated as

\[
a(\theta) = [1, e^{-j2\pi d \sin \theta / \lambda}, \ldots, e^{-j2\pi (M-1)d \sin \theta / \lambda}]^T
\]

where \( (\cdot)^T \) denotes transpose, \( \lambda \) is the carrier wavelength, \( d \) is the distance between adjacent sensors, and the signal angle is \( \theta \). The output of the beamformer at time \( k \) can be expressed as

\[
y(k) = w^H x(k)
\]

where \( (\cdot)^H \) denotes Hermitian transpose and \( w = [w_1, w_2, \ldots, w_M]^T \) is the complex weight vector. The optimal weight vector can be obtained by maximizing the output signal-to-interference-plus-noise ratio (SINR) which is defined as

\[
SINR = \frac{\sigma_1^2 |w^H a_1|^2}{\bar{w}^H R_{in} \bar{w}}
\]

where \( \sigma_1^2 \) is the power of the desired signal and \( R_{in} \) is the INC matrix given by

\[
R_{in} = E \{ (x_s(k) + x_i(k))(x_s(k) + x_i(k))^H \} = \sum_{j=2}^{N} \sigma_j^2 a_j a_j^H + \sigma_n^2 I_M
\]

\( \sigma_j^2 \) and \( \sigma_n^2 \) are the power of the \( j \)-th interference signal and noise, respectively, \( E \{ \cdot \} \) denotes statistical expectation, and \( I_M \) is the \( M \times M \) identity matrix.

The SINR in (4) can be maximized by minimizing the output interference-plus-noise power. Thus, the weight vector optimization problem is

\[
\min_w \bar{w}^H R_{in} \bar{w} \quad \text{s.t.} \quad \bar{w}^H a_1 = 1
\]

known as the minimum variance distortionless response (MVDR) beamformer. The corresponding solution is given by

\[
w = \frac{R_{in}^{-1} a_1}{a_1^H R_{in}^{-1} a_1}
\]
The total received signal covariance matrix $\mathbf{R}$ is usually used instead of $\mathbf{R}_{in}$ because an estimate of $\mathbf{R}_{in}$ is difficult to obtain in practice. $\mathbf{R}$ can be expressed as

$$\mathbf{R} = E\{ (\mathbf{x}_i(k) + \mathbf{x}_n(k))(\mathbf{x}_i(k) + \mathbf{x}_n(k))^H \} = \sigma_i^2 \mathbf{a}_i\mathbf{a}_i^H + \sum_{j=2}^{N} \sigma_j^2 \mathbf{a}_j\mathbf{a}_j^H + \sigma_n^2 \mathbf{I}_M$$

$$= \mathbf{R}_s + \mathbf{R}_{in}$$

where $\mathbf{R}_s$ is the desired signal covariance matrix. The eigenvalue decomposition (EVD) of $\mathbf{R}$ is

$$\mathbf{R} = \sum_{i=1}^{M} \lambda_i \mathbf{e}_i\mathbf{e}_i^H$$

where $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_M = \sigma_n^2$ are the eigenvalues of $\mathbf{R}$, and $\mathbf{e}_i$, $i = 1, 2, \ldots, M$, are the corresponding eigenvectors. In practical applications, $\mathbf{R}$ is also unavailable and so is commonly replaced by the sample covariance matrix (SCM)

$$\hat{\mathbf{R}} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{x}(k)\mathbf{x}^H(k)$$

where $K$ is the number of samples. Then, the MVDR beamformer becomes the sample covariance inversion (SMI) beamformer and the corresponding weight vector is

$$\mathbf{w}_{SMI} = \frac{\hat{\mathbf{R}}^{-1}\mathbf{a}_1}{\mathbf{a}_1^H \hat{\mathbf{R}}^{-1}\mathbf{a}_1}$$

### III. PROPOSED METHOD

#### A. DIRECTION-OF-ARRIVAL CORRECTION

DOA estimation mismatch will affect the steering vector according to (2). Further, if the beamformer weight vector $\mathbf{w}$ is obtained using (7) or (11), SOI steering vector mismatch will affect this weight vector. Moreover, if the reconstruction of $\mathbf{R}_{in}$ in (7) requires the interference steering vectors, then DOA mismatch of the interference signals will also influence $\mathbf{w}$. In this section, an algorithm is introduced for DOA mismatch correction (DC).

The goal of DC is to determine the most suitable steering vector using the projection of each steering vector on the eigenvectors in the observation region. From [32], the mismatch between the actual steering vector $\mathbf{a}$ and the presumed steering vector $\hat{\mathbf{a}}$ is small. Thus, the eigenvectors related to the projection of $\hat{\mathbf{a}}$ on the eigenvectors $\mathbf{e}_i$ can be used to construct the desired signal subspace. This projection can be represented as

$$p_i = |\mathbf{e}_i^H \hat{\mathbf{a}}|^2 \quad i = 1, 2, \ldots, M$$

Now rearrange the $p_i$ in descending order so that $p_{[M]} \geq p_{[M-1]} \geq \ldots \geq p_{[1]}$ where the subscript $[M]$ denotes the largest of the $M$ values and $[1]$ denotes the smallest. Then, the corresponding eigenvectors in descending order are $\mathbf{e}_{[M]}, \mathbf{e}_{[M-1]}, \ldots, \mathbf{e}_{[1]}$. From [18], [32], if

$$\frac{p_{[M]} + p_{[M-1]} + \cdots + p_{[1]}}{\sum_{i=1}^{M} p_i} > \rho$$

then the eigenvectors $[\mathbf{e}_{[M]}, \mathbf{e}_{[M-1]}, \ldots, \mathbf{e}_{[1]}]$ span a new estimated desired signal subspace where $\sum_{i=1}^{M} p_i = M$, $0 < \rho < 1$ is the energy threshold, and $m$ is the largest value such that (13) is satisfied.

Different from [18], [32], the purpose of the proposed DC method is not to find the desired signal subspace, but to find the minimum number of projections, i.e. the maximum value of $m$, that satisfies (13) in a given angle interval. In theory, $\mathbf{a}$ and $\mathbf{e}_i$ are orthogonal for $i = N + 1, \ldots, M$. $|\mathbf{e}_i^H \mathbf{a}|^2 = 0$, so $|\mathbf{e}_i^H \hat{\mathbf{a}}|^2 = 0$ if $\hat{\mathbf{a}}$ is equal to $\mathbf{a}$, i.e. $p_i = 0$ for $i = N + 1, \ldots, M$ and $p_i > 0$ for $i = 1, \ldots, N$. The closer $\hat{\mathbf{a}}$ is to $\mathbf{a}$, the more $p_i$ are zero and the fewer $p_i$ are nonzero. Since the sum of the $p_i$ is constant, an increase in the number of nonzero $p_i$ will reduce their average value, so the number of elements required in the numerator to satisfy (13) will also increase.

Thus, considering only look direction error, different signal angles $\theta$ will result in different $\hat{\mathbf{a}}$ and each will have a corresponding value of $m$. When there is no look direction error ($\hat{\mathbf{a}} = \mathbf{a}$), let $p_{[M]} + p_{[M-1]} + \cdots + p_{[1]} / \sum_{i=1}^{M} p_i > \rho$. Then, if there is a look direction error ($\hat{\mathbf{a}} \neq \mathbf{a}$), for an $m_2$ that satisfies $p_{[M]} + p_{[M-1]} + \cdots + p_{[m_2]} / \sum_{i=1}^{M} p_i > \rho$, it can be concluded that $m_1 \geq m_2$. Therefore, for $\theta$ located in the angular sector $\Theta_\theta$ of the desired signal, the $\theta$ corresponding to the maximum value of $m$, $m_{\text{max}}$, can be considered to reconstruct the steering vector. In practice, $\mathbf{R}$ is replaced by $\hat{\mathbf{R}}$, $\mathbf{e}_i$ is derived from the EVD of $\hat{\mathbf{R}}$ and $|\mathbf{e}_i^H \mathbf{a}|^2 \neq 0$ for $i = N + 1, \ldots, M$. Since the value of $|\mathbf{e}_i^H \mathbf{a}|^2$ is small for $i = N + 1, \ldots, M$, the proposed DC method is still applicable.

Fig. 1 presents the average value of $m$ versus the estimated angle $\theta$ without mismatch when the true DOA is $5^\circ$ for 500 Monte Carlo trials and $K = 30$. It is assumed that the signal angular sector is $\Theta_s = [\hat{\theta} - 5^\circ, \hat{\theta} + 5^\circ]$ with an angle interval $0.5^\circ$ where $\hat{\theta} = 5^\circ$ is the presumed DOA and $\theta \in \Theta_s$. These results show that the maximum value of $m$ is not unique when $\theta$ is near the real DOA. The projection in (12) can be rewritten as

$$p_i = |\mathbf{e}_i^H \mathbf{d}(\theta)|^2$$

where $\mathbf{d}(\theta)$ is the steering vector corresponding to $\theta$ and $\theta$ is located in the angular sector $\Theta_\theta$ of the desired signal. $m_{\text{max}}$ is obtained by substituting $\theta$ in (14) and determining the corresponding largest value of $m$ satisfying (13).

In practice, the modified DOA $\hat{\theta}$ can be obtained by averaging $\theta_{\text{min}}$ and $\theta_{\text{max}}$

$$\hat{\theta} = \left( \theta_{\text{min}}(m_{\text{max}}) + \theta_{\text{max}}(m_{\text{max}}) \right) / 2$$

where $\theta_{\text{min}}$ is the minimum angle for which $m$ is maximum and $\theta_{\text{max}}$ is the maximum angle for which $m$ is maximum. If there is only one maximum for $m$ in $\Theta_\theta$, then $\hat{\theta} = \theta_{\text{min}} = \theta_{\text{max}}$. For an input SNR of $-20$ dB, $-10$ dB, 0 dB, 10 dB, and 20 dB, Fig. 1 indicates that the value of $\hat{\theta}$ estimated using the DC method is $2.5^\circ$, $4^\circ$, $4.5^\circ$, $5^\circ$, and $5^\circ$, respectively.

Fig. 2 shows the average value of $m$ versus the estimated DOA angle $\theta$ when the direction mismatch is uniformly distributed in $[-4^\circ, 4^\circ]$ when the true DOA is $5^\circ$(so $\hat{\theta}$ is uniformly distributed in $[1^\circ, 9^\circ]$), for 500 Monte Carlo trials.
B. MODIFIED COVARIANCE MATRIX

Figs. 1, 2, and 3 show that the DOA estimation error increases as the SNR decreases. This performance degradation is mainly due to noise. In order to reduce the influence of noise when the SNR is low, the SNR of the desired signal must first be estimated. The input SNR of the received signal can be estimated as

$$SNR_{input} = 10 \log_{10} \frac{\sigma^2_t}{\sigma_n^2}$$  \hspace{1cm} (16)

For input SNR values $-20$, $-10$, $0$, $10$ and $20$ dB, $\hat{\theta}$ is $3.5^\circ$, $4.75^\circ$, $5^\circ$, $4.5^\circ$, and $5^\circ$, respectively. The root-mean-square-error (RMSE) of the DOA estimates with and without the DC method versus SNR is shown in Fig. 3 with $K = 30$ and look direction mismatch uniformly distributed in $[-4^\circ, 4^\circ]$ for 500 Monte Carlo trials. Figs. 1, 2, and 3 show that the DC method can reduce the DOA estimation mismatch when the SNR is high but large errors still exist when the SNR is low. Thus, an algorithm for low input SNR is developed in the next section.

Note that both $\sigma_t^2$ and $\sigma_n^2$ are unknown. Thus, the SNR is approximated as

$$SNR_{approx} = 10 \log_{10} \frac{0.1 \lambda_{[M]}}{\bar{\sigma}_n^2}$$  \hspace{1cm} (17)

The estimated noise power is then

$$\bar{\sigma}_n^2 = \frac{1}{M - N} \sum_{i=N+1}^{M} \lambda_i$$  \hspace{1cm} (18)

The input SNR is considered low if $SNR_{approx} < 0$. Define $\beta = 0.1 \lambda_{[M]} / \bar{\sigma}_n^2$ where $\lambda_{[M]}$ is the eigenvalue corresponding to the eigenvector $e_{[M]}$ obtained from $|e_{[M]} \hat{a}|^2 = P_{[M]}$. This expression was determined based on extensive simulations. In this paper, a tridiagonal loading algorithm is used to reduce the DOA estimation mismatch when the input SNR is low. This is an effective way to mitigate beamforming issues caused by noise and/or model errors [33]. The loaded SCM can be expressed as

$$\hat{R}_L = \hat{R} + \tau T$$  \hspace{1cm} (19)

where $\tau$ is the loading level and the loading matrix $T$ is a simple tridiagonal Toeplitz matrix given by

$$T = \begin{bmatrix} 1 & -2 & 0 & \cdots & 1 & 1 \\ -2 & 1 & -2 & \cdots & 0 & 1 \\ 0 & -2 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 0 & 0 & \cdots & 1 & -2 \\ 1 & 1 & 0 & \cdots & -2 & 1 \end{bmatrix}$$  \hspace{1cm} (20)

The loading level in (19) is given by

$$\tau = (\|\hat{a}\|)^{-1} a^H \hat{R} \hat{a}$$  \hspace{1cm} (21)

where the weighted steering vector is

$$\hat{a} = \beta \odot \tilde{a}$$  \hspace{1cm} (22)
IV. PERFORMANCE EVALUATION

In this section, numerical results are presented to validate the effectiveness of the proposed method. Consider a uniform linear array with $M = 10$ omnidirectional sensors spaced half a wavelength apart. The noise is modeled as complex Gaussian which is temporally and spatially white with zero mean and unit covariance. Each result is the average of 500 Monte-Carlo trials.

A. DOA ESTIMATION PERFORMANCE

1) DOA Correction with Random Look Direction Errors

The performance with large DOA errors is now considered. The desired signal impinges on the array from the direction $\theta_1 = 5^\circ$, and the direction mismatch is randomly and uniformly distributed in $[-4^\circ, 4^\circ]$. Fig. 4 presents the RMSE of the DOA estimate versus SNR for $K = 30$. The red dotted line is the RMSE with no correction, the blue dotted line is the RMSE with correction using the DC method, and the solid green line is the RMSE with correction using the proposed method. This shows that the proposed algorithm effectively corrects the DOA error even at low SNRs. Fig. 5 presents the RMSE versus the number of snapshots for an input SNR of 20 dB (high SNR). This shows that the number of snapshots has a negligible effect on the RMSE. Note that for a high SNR, $\mathbf{R}_c$ in the DC method is the same as that in the proposed method, so the blue dotted and solid green lines coincide.

2) RMSE Performance

DOA estimation algorithms with the proposed method are now examined. The RMSE differences with and without the proposed method are used to evaluate the performance. The MUSIC and Capon DOA estimation methods as well as the rational invariance (ESmusic) [34], propagator method (PM) [35], modified propagator method (MPM) [36], and DOA estimation with uncertain gain-phase sensors (DSS) [30] techniques are considered. The desired signal impinges on the array from the direction $\theta_1 = 5^\circ$.

Fig. 6 presents the RMSE of the DOA estimation algorithms versus the input SNR under ideal conditions with $K = 30$, and Fig. 7 gives the corresponding RMSE versus the number of snapshots under ideal conditions with SNR $= -10$ dB. These results show that the RMSE is large at low SNRs and small at high SNRs, so the angle range considered should be based on the SNR. As a consequence, the signal angle sector is set according to the input SNR as shown in Table 1. Taking the second row as an example, when the input SNR is between $[-15, -5]$, the angle sector is $\Theta_s = [\theta_1 - 5^\circ, \theta_1 + 5^\circ]$ with angle interval $1^\circ$.

| Input SNR (dB) | Angle sector | Angle interval |
|----------------|--------------|----------------|
| $[-15, -5]$    | $[\theta_1 - 5^\circ, \theta_1 + 5^\circ]$ | $1^\circ$     |
| $[-5, 0]$      | $[\theta_1 - 2^\circ, \theta_1 + 2^\circ]$ | $0.2^\circ$   |
| $[0, 10]$      | $[\theta_1 - 1^\circ, \theta_1 + 1^\circ]$ | $0.1^\circ$   |
| $[10, 15]$     | $[\theta_1 - 0.5^\circ, \theta_1 + 0.5^\circ]$ | $0.05^\circ$  |
The RMSE difference is shown in Fig. 8 versus the SNR. A difference less than 0 indicates that the proposed method reduces the RMSE of the algorithm and makes it better. These results show that when the input SNR is less than 10 dB, the proposed method can reduce the estimation error of the Capon, ESmusic, PM, and MPM algorithms. For the MUSIC and DSS methods, this difference is slightly greater than 0 when the SNR is in the range $[-5, 5]$. When the SNR is greater than 10 dB, the effect of the proposed algorithm is small because the DOA estimation algorithms already provide accurate results. The RMSE difference versus the number of snapshots when the input SNR is $-10$ dB is given in Fig. 9. This shows that when the number of snapshots is greater than 20, the DOA estimation algorithms combined with the proposed method outperform the algorithms alone.

3) RMSE Performance with Array Uncertainties

In this section, the amplitude, phase, and sensor location errors are considered as array uncertainties. The amplitude and phase errors of each sensor are randomly distributed according to $N(0, 0.1^2)$ and $N(0, 0.25\pi^2)$, respectively, and the sensor location errors are uniformly distributed in the interval $[-0.1, 0.1]$ measured in sensor space. The $n$th element of the steering vector can then be expressed as

$$(1 + \Delta g)e^{-j2\pi(n+\Delta d)\sin \theta / \lambda}e^{-j\Delta \phi}, \quad n = 0, 1, \ldots, M - 1 \quad (24)$$

where $\Delta g$ is the gain error, $\Delta \phi$ is the phase error, and $\Delta d$ is the sensor location error. The errors were changed each trial but remain constant over the corresponding snapshots. The angle sector is chosen as previously. The RMSE difference is presented in Fig. 10. These results show that when the SNR is lower than 10 dB, the proposed algorithm can still reduce the RMSE of the original DOA estimation methods. Fig. 11 presents the RMSE difference for different numbers of snapshots. This indicates that when the number of snapshots is greater than 20, the proposed algorithm can still reduce the RMSE.
B. BEAMFORMER PERFORMANCE

In this section, the performance of the proposed method is evaluated using several beamformers widely used in the literature. One desired signal and two interference signals impinge on the array from directions $\theta_1 = 5^\circ$, $\theta_2 = -50^\circ$, and $\theta_3 = -20^\circ$, respectively. The interference-to-noise ratio (INR) for both signals is set to 30 dB. The following beamforming algorithms are considered. INC matrix reconstruction and steering vector estimation (ICM) [23], INC matrix reconstruction and steering vector via subspace estimation (SICM) [22], combined INC matrix reconstruction [19] with desired signal steering vector estimation [18] (SRNSV), worst-case-based beamformer (WC) [13], minimum sensitivity eigenspace-based beamformer (MSESB) [12], maximum entropy method (MEPS) [37], tridiagonal loading beamformer (TLBF) [33], and SMI [38]. The angle sector of the desired signal is set to $\Theta_s = [\bar{\theta} - 5^\circ, \bar{\theta} + 5^\circ]$ with angle interval $0.5^\circ$ for the methods in [22] and [23], and the proposed method. The energy threshold for the method in [18] and the proposed method is set to $\rho = 0.9$. The number of sampling points for the MEPS method are set to $L = 50$ and $S = 10$. The weight $w_{SLL}$ for the TLBF method [33] is set to 0.05, the number of dominant eigenvectors in [22] is set to 7, and $\epsilon$ in [13] is set to $0.3M$. The optimization problem is solved using CVX [39]. Fig. 12 presents the output SINR versus input SNR under ideal conditions with $K = 30$ and Fig. 13 gives the output SINR versus the number of snapshots under ideal conditions with $SNR = 20$ dB.

1) Beamforming with Random Look Direction Errors

In this section, the influence of the proposed algorithm on the beamforming algorithms is evaluated considering signal look direction errors with random direction mismatch uniformly distributed in $[-4^\circ, 4^\circ]$. Thus, the DOA estimates of the SOI and two interference signals are uniformly distributed in $[1^\circ, 9^\circ], [-54^\circ, -46^\circ]$ and $[-24^\circ, -16^\circ]$, respectively. The direction errors of the three signals are changed each trial but remain constant over the corresponding snapshots. Fig. 14 presents the output SINR of the beamformers versus input SNR with $K = 30$. Comparing these results with Fig. 12
indicates that the output SINR of the beamformers decreases in the presence of look direction errors. Fig. 15 gives the output SINR difference of the beamforming algorithms with and without the proposed method versus the input SNR. These results show that the proposed method can improve beamformer performance when there are DOA estimation errors. The improvement with the SRNSV and SICM methods is significant while the improvement with the SMI and TLBF algorithms is greater when the input SNR is high.

In the SRNSV [19] [18], ICM [23], SICM [22], TLBF [33], and SMI [38] methods, \( w \) is obtained using (7) and the output SINR is improved with the proposed algorithm. In ICM, the estimated INC matrix is reconstructed based on the revised SOI interval, and the steering vector optimization algorithm uses the steering vector from the proposed method. With MEPS, an improved estimate for the desired signal steering vector is obtained using the revised SOI interval. With SICM, the revised direction interval is needed to calculate the steering vector of the desired signal. In addition, the DOA of the interference signals must be estimated because the INC matrix is reconstructed based on the interference signal steering vector. Therefore, correcting the signal DOAs will improve the output SINR. The INC matrix reconstruction in SRNSV [19] requires the interference steering vector, and optimization of the steering vector of the desired signal in SRNSV [18] is based on constraints on the estimated steering vector of the desired signal. Determining these steering vectors requires the corresponding DOAs. As a result, DOA errors will affect both INC matrix reconstruction and steering vector estimation, which will decrease the output SINR. For TLBF, using the proposed method in estimating the steering vector of the desired signal and the loaded SCM regularization will improve the performance.

The performance of the SMI algorithm can be degraded when the input SNR of the desired signal is high, but using the modified \( a \) in (11) obtained with the proposed method can alleviate this problem. The WC algorithm is improved for input SNR in the range 0 to 30 dB, but there is minimal improvement with MSESBN for all SNRs. The weight vector \( w \) with WC [13] beamforming is obtained based on the worst-case constraint and so is robust to steering vector errors. However, when the input SNR is in the range 0 to 30 dB, the performance of this algorithm is poor. The proposed algorithm was shown to improve this performance. With the MSESBN method [12], the projection of the steering vector in the signal-interference subspace is needed to obtain \( w \). It is robust to steering vector errors and the subspace projection of the steering vector will reduce the effect of DOA correction. Thus, the proposed method provides little improvement for this method.

Fig. 16 presents the output SINR of the beamformers versus the number of snapshots without the proposed method for SNR = 20 dB. The corresponding output SINR difference with and without the proposed method is given in Fig. 17. These results show that the proposed method improves the SINR when there are DOA estimation errors, and the improvement in SINR is greatest for the SMI and SRNSV algorithms.

2) Beamforming with Amplitude, Phase and Sensor Location Errors

In this section, the random DOA mismatch, amplitude, phase, and sensor location errors are the same as in Section IV-A3. Fig. 18 gives the output SINR difference with and without the proposed method versus the input SNR for the beamformer algorithms. These results show that the proposed algorithm improves the performance of all algorithms. SMI and TLBF are significantly improved when the SNR is high, and ICM, SICM, MEPS and SRNSV are also better. However, WC is only improved in the range SNR = 0 to 30 dB, and there is little improvement for MSESBN.

Fig. 19 presents the output SINR difference versus the number of snapshots for the beamformer algorithms with and without the proposed method. This shows that using the proposed method improves the performance regardless of the number of snapshots. Further, these results indicate that...
the proposed method can improve beamforming performance with a variety of errors.

3) Beamforming with Fast Wavefront Distortion
In this section, the case when the signal steering vector is distorted by wave propagation is considered. Fast wavefront distortion occurs when the signal propagates in a long and inhomogeneous medium. It is assumed that the phase distortion is independent and Gaussian distributed according to $N(0, 0.1^2)$, and the distortion changes each trial and from snapshot to snapshot. The random DOA mismatch is uniformly distributed in $[-4^\circ, 4^\circ]$. Fig. 20 presents the output SINR difference for the beamformers with and without the proposed method versus the input SNR. This shows that the performance of the ICM, SRNSV, SMI, SICM, MEPS and TLBF algorithms with the proposed method is better over the entire SNR range, while WC is improved for SNR = 0 to 35 dB. Fig. 21 gives the output SINR difference for the beamformer algorithms with and without the proposed method for different numbers of snapshots. This indicates that the output SINR of these algorithms is improved with the proposed method regardless of the number of snapshots.

4) Beamforming with Incoherent Local Scattering Errors
In this section, the case when the signal steering vector is affected by incoherent local scattering errors is considered. The desired signal is time-varying so the steering vector is modeled as

$$\hat{a}_1(k) = s_0(k)a_1 + \sum_{p=1}^{4} s_p(k)\bar{a}(\theta_p)$$

(25)

where $s_p(k), p = 0, 1, 2, 3, 4$ are independent random variables with distribution $N(0, 1)$, and $\bar{a}(\theta_p)$ are the corresponding incoherent scattering paths. The angles $\theta_p, p = 1, 2, 3, 4$ are independent random variables with distribution $N(5^\circ, 4^\circ)$. $s_p(k)$ changes both from trial to trial and snapshot to snapshot, the directions of arrival $\theta_p$ change from trial to trial but
is constant over the corresponding snapshots. The random DOA mismatch is uniformly distributed in $[-4^\circ, 4^\circ]$. The model in (25) corresponds to the case of incoherent local scattering [40]. Under this assumption, the signal covariance matrix $\mathbf{R}_s$ is no longer a rank-one matrix and the output SINR has the form

$$SINR_{opt} = \frac{\mathbf{w}^H \mathbf{R}_s \mathbf{w}}{\mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w}}$$

(26)

which is maximized by the weight vector [41]

$$\mathbf{w}_{opt} = \mathbb{P}\{\mathbf{R}^{-1}_{i+n} \mathbf{R}_s\}$$

(27)

where $\mathbb{P}\{\cdot\}$ denotes the principal eigenvector of a matrix.

Fig. 22 gives the output SINR difference for the beamformer algorithms with and without the proposed method versus the input SNR. This shows that the differences for SMI and MSESBB are close to 0 dB, and the proposed method improves the output SINR of the other algorithms. Fig. 23 gives the output SINR difference for the beamformer algorithms with and without the proposed method for different numbers of snapshots. This indicates that except for SMI and MSESBB, the proposed method improves the performance regardless of the number of snapshots.

V. CONCLUSION

In this paper, a DOA correction method was presented for array signal processing. This method provides a new approach to SNR estimation and adaptive covariance matrix modification so it can be used for a wide range of input SNRs. A modified DOA was determined based on the relationship between the eigenvectors of the covariance matrix and the actual steering vector, and this DOA can be used to construct the steering vector. The proposed approach can be utilized with any beamforming algorithm and only requires an estimated angle range for the observed signal. It can also be used after DOA estimation to correct inaccurate estimation results. Numerical results were presented which show that the proposed method can not only correct DOA estimation errors, but can cope with large DOA errors and improve the performance of beamforming algorithms which require the DOA of the signal to reconstruct the INC matrix or
estimate the steering vector. Future work includes improving the robustness of the algorithm to adapt to more complex environments and employing it for signal detection.

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