UV LUMINOSITY FUNCTIONS AT \( z \approx 4, 5, \) AND 6 FROM THE HUBBLE ULTRA DEEP FIELD AND OTHER DEEP HUBBLE SPACE TELESCOPE ACS FIELDS: EVOLUTION AND STAR FORMATION HISTORY

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ABSTRACT

We use the ACS \( BVIz \) data from the HUDF and all other deep \( HST \) ACS fields (including the GOODS fields) to find large samples of star-forming galaxies at \( z \approx 4 \) and \( \sim 5 \) and to extend our previous \( z \approx 6 \) sample. These samples contain 4671, 1416, and 627 \( B, I, \) and \( i \)-dropouts, respectively, and reach to extremely low luminosities \([0.01-0.04]L_\odot\) or \( M_{UV} \approx -16 \) to \(-17\), allowing us to determine the rest-frame UV LF and faint-end slope \( \alpha \) at \( z \approx 4-6 \) to high accuracy. We find faint-end slopes \( \alpha = -1.73 \pm 0.05, -1.66 \pm 0.09, \) and \(-1.74 \pm 0.16 \) at \( z \approx 4, 5, \) and \( \sim 6 \), respectively, suggesting that the faint-end slope is very steep and shows little evolution with cosmic time. We find that \( M_{UV} \) brightens considerably in the 0.7 Gyr from \( z \approx 6 \) to \( \sim 4 \) (by \(-0.7 \) mag from \( M_{UV} = -20.24 \pm 0.19 \) to \(-20.98 \pm 0.10 \)). The observed increase in the characteristic luminosity over this range is almost identical to that expected for the halo mass function, suggesting that the observed evolution is likely due to the hierarchical coalescence and merging of galaxies. The evolution in \( \phi^* \) is not significant. The UV luminosity density at \( z \approx 6 \) is modestly lower than \( (0.45 \pm 0.09 \) times) that at \( z \approx 4 \) (integrated to \(-17.5 \) mag) although a larger change is seen in the dust-corrected SFR density. We thoroughly examine published LF results and assess the reasons for their wide dispersion. We argue that the results reported here are the most robust available. The extremely steep faint-end slopes \( \alpha \) found here suggest that lower luminosity galaxies play a significant role in reionizing the universe. Finally, recent search results for galaxies at \( z \approx 7-8 \) are used to extend our estimates of the evolution of \( M^* \) from \( z \approx 7-8 \) to \( z \approx 4 \).

Subject headings: galaxies: evolution — galaxies: high-redshift

1. INTRODUCTION

The luminosity function (LF) represents a key observable in astronomy. It tells us how many galaxies at some epoch emit light of a given luminosity. Comparisons of the LF with other quantities like the halo mass function provide critical insight into galaxy formation by establishing the efficiency of star formation at different mass scales (van den Bosch et al. 2003; Vale & Ostriker 2004). At ultraviolet (UV) wavelengths, this LF has been of keen interest because of its close relationship with the star formation rate (SFR). With the exception of galaxies with the largest SFRs and therefore likely significant dust extinction (e.g., Wang & Heckman 1996; Adelberger & Steidel 2000; Martin et al. 2005b), UV light has been shown to be a very good tracer of this SFR. Studies of the evolution of this LF can help us understand the physical processes that govern star formation. Among these processes are likely gas accretion and hierarchical buildup at early times, supernova (SN) and active galactic nucleus (AGN) feedback to regulate this star formation, and gravitational instability physics.

Over the past few years, there has been substantial progress in understanding the evolution of the rest-frame UV LF across cosmic time, building significantly on the early work done on these LFs at \( z \approx 3-4 \) from Lyman break galaxy (LBG) selections (Madau et al. 1996; Steidel et al. 1999) and work in the nearby universe \((z \leq 0.1; \) e.g., Sullivan et al. 2000). At lower redshift, progress has come through deep far-UV data from the Galaxy Evolution Explorer \((GALEX; \) Martin et al. 2005a), which have allowed us to select large samples of LBGs at \( z \approx 1.5 \) (Arnouts et al. 2005; Schiminovich et al. 2005) and thus derive the LF at the same rest-frame wavelength \((\sim 1600 \) Å) as higher redshift samples. At the same time, there has been an increasing amount of very deep, wide-area optical data available from ground and space to select large dropout samples at \( z \approx 4-6 \) (e.g., Giavalisco et al. 2004b; Bunker et al. 2004; Dickinson et al. 2004; Yan & Windhorst 2004b; Ouchi et al. 2004; Bouwens et al. 2006, hereafter B06; Yoshida et al. 2006). This has enabled us to determine the UV continuum LF across the entire range \( z \approx 0-6 \) and attempt to understand its evolution across cosmic time (Shimasaku et al. 2005; B06; Yoshida et al. 2006; Tresse et al. 2007).

Although there has been an increasing consensus on the evolution of the LF at \( z \approx 2 \) (Arnouts et al. 2005; Gabasch et al. 2004; Dahlen et al. 2007; Tresse et al. 2007), it is fair to say that the evolution at \( z \approx 3 \) is still contentious, with some groups claiming that the evolution occurs primarily at the bright end (Shimasaku et al. 2005; B06; Yoshida et al. 2006), others claiming it occurs at the faint end (Iwata et al. 2003, 2007; Sawicki & Thompson 2006a), and still other teams suggesting that the evolution occurs in a luminosity-independent manner (Beckwith et al. 2006). Perhaps the most physically reasonable of these scenarios and the one with the broadest observational support (Dickinson et al. 2004; Shimasaku et al. 2005; B06; Bouwens & Illingworth 2006; Yoshida et al. 2006) is the scenario where evolution happens primarily at the bright end of the LF. In this picture, fainter galaxies are established first and then the brighter galaxies develop later through hierarchical buildup. Observationally, this buildup is seen as an increase in the characteristic luminosity as a function of cosmic time (Dickinson et al. 2004; B06; Yoshida et al. 2006). Less evolution is apparent in the normalization \( \phi^* \) and faint-end slope \( \alpha \) (B06; Yoshida et al. 2006).

Despite much observational work at the bright end of the LF at high redshift, the observations have not provided us with as strong of constraints on what happens at the faint end of the LF. Most...
large-scale surveys for galaxies at $z \sim 3–6$ have only extended to ~27 mag (e.g., Yoshida et al. 2006; Giavalisco et al. 2004b; Ouchi et al. 2004; Sawicki & Thompson 2005), which is equivalent to ~$0.3L^*_{z=3}$ at $z \sim 4–5$. This is unfortunate since galaxies beyond these limits may be quite important in the overall picture of galaxy evolution, particularly if the faint-end slope $\alpha$ is steep. For faint-end slopes $\alpha$ of ~1.6, lower luminosity galaxies ($\geq 0.3L^*_{z=3}$) contribute nearly 50% of the total luminosity density, and this fraction will even be higher if the faint-end slope is steeper yet. Since these galaxies will almost certainly play a more significant role in the luminosity densities and SFRs at very early times, clearly it is helpful to establish how the LF is evolving at lower luminosities. This topic has been of particular interest recently due to speculation that lower luminosity galaxies may reionize the universe (Lehnert & Bremer 2003; Yan & Windhorst 2004a, 2004b; B06; Stark et al. 2007a; Labbé et al. 2006).

With the availability of deep optical data over the Hubble Ultra Deep Field (HUDF; Beckwith et al. 2006), we have the opportunity to extend current LFs to very low luminosities. The HUDF data are deep enough to allow us to select dropout samples to ~29.5 mag, which corresponds to an absolute magnitude of roughly ~16.5 mag at $z \sim 4$, or ~$-0.01L^*$, which is ~5 mag below $L^*$. This is almost 2 mag fainter than has been possible with any other data set and provides us with unique leverage to determine the faint-end slope. Previously, we have used an $\phi$-dropout selection over the HUDF to determine the LF at $z \sim 6$ to very low luminosities ($\sim 17.5$ mag), finding a steep faint-end slope $\alpha = -1.73 \pm 0.21$ and a characteristic luminosity $M^* \sim -20.25$ that was ~0.6 mag fainter than at $z \sim 3$ (B06; see also work by Yan & Windhorst 2004b; Bunker et al. 2004; Malhotra et al. 2005). Beckwith et al. (2006) also considered a selection of dropouts over the HUDF and used them in conjunction with a selection of dropouts over the wide-area Great Observatories Origins Deep Survey (GOODS) fields (Giavalisco et al. 2004a) to examine the evolution of the LF at high redshift. Beckwith et al. (2006) found that the LFs at $z \sim 4–6$ could be characterized by a constant $M^* \sim -20.4$, steep faint-end slope $\alpha \sim -1.6$, and evolving normalization $\phi^*$. Bunker et al. (2004) and Yan & Windhorst (2004b) also examined the evolution of the LF from $z \sim 6$ to ~3, interpreting the evolution in terms of a changing normalization $\phi^*$ and faint-end slope $\alpha$, respectively.

It is surprising to see that even with such high-quality selections as are possible with the HUDF, there is still a wide dispersion of results regarding the evolution of the UV LF at high redshift. This emphasizes how important both uncertainties and systematics can be for the determination of the LF at these redshifts. These include data-dependent uncertainties like large-scale structure and small number statistics to more model-dependent uncertainties (or systematics) like the model redshift distribution, selection volume, and $k$-corrections. In light of these challenges, it makes sense for us (1) to rederive the LFs at $z \sim 4–6$ in a uniform way using the most comprehensive set of Hubble Space Telescope (HST) data available, while (2) considering the widest variety of approaches and assumptions.

To this end, we make use of a comprehensive set of multicolor ($BViz$) HST data to derive the rest-frame UV LFs at $z \sim 4–5$, and ~6. These data include the exceptionally deep HUDF data, the two wide-area GOODS fields, and four extremely deep Advanced Camera for Surveys (ACS) pointings that reach to within ~1–0.5 mag of the HUDF. The latter data include two deep ACS parallels (~20 arcmin$^2$) to the UDF NICMOS field (HUDF-Ps; Bouwens et al. 2004b; Thompson et al. 2005) and the two HUDF05 fields (~23 arcmin$^2$; Oesch et al. 2007). Although these data have not been widely used in previous LF determinations at $z \sim 4–5$, they provide significant statistics faintward of the GOODS probe, provide essential controls for large-scale structure, and serve as an important bridge in linking ultradepth HUDF selections to similar selections made over the much shallower GOODS fields. By deriving the LFs at $z \sim 4$ and ~5, we fill in the redshift gap left by our previous study (B06) between $z \sim 6$ and ~3. We also take advantage of the additional HST' data now available (i.e., the two HUDF05 fields) to refine our previous determination of the LF at $z \sim 6$ (B06). In doing so, we obtain an entirely self-consistent determination of the UV LF at $z \sim 4$, ~5, and ~6. This allows us to make a more direct assessment of the evolution of the LF from $z \sim 6$ to $z \sim 3–4$ than we were able to make in our previous comparison with the LF at $z \sim 3$ from Steidel et al. (1999). It also puts us in a position to evaluate the wide variety of different conclusions drawn by different teams in analyzing the evolution of the LF at very high redshift (Bunker et al. 2004; Yan & Windhorst 2004b; iwata et al. 2003, 2007; Beckwith et al. 2006; Yoshida et al. 2006). While deriving these LFs, we consider a wide variety of different approaches and assumptions to ensure that the results we obtain are as robust and broadly applicable as possible.

We begin this paper by describing our procedures for selecting our $B$, $V$, and $i$-dropout samples (§ 2). We then derive detailed completeness, flux, and contamination corrections to model our shallower HUDF05, HUDF-Ps, and GOODS selections in a similar fashion to the way we model the HUDF data. We then move onto a determination of the rest-frame UV LFs at $z \sim 4$, ~5, and ~6 (§ 3). In § 4 we assess the robustness of the current LF determinations, comparing the present results with those in the literature and trying to understand the wide dispersion of previous LF results. Finally, we discuss the implications of our results (§ 5) and then include a summary (§ 6). Where necessary, we assume $\Omega_0 = 0.3, \Lambda_0 = 0.7$, and $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$. Although these parameters are slightly different from those determined from the WMAP three-year results (Spergel et al. 2007), they allow for convenient comparison with other recent results expressed in a similar manner. Throughout we use $L^*_{z=3}$ to denote the characteristic luminosity at $z \sim 3$ (Steidel et al. 1999). All magnitudes are expressed in the AB system (Oke & Gunn 1983).

2. SAMPLE SELECTION

2.1. Observational Data

A detailed summary of the ACS HUDF, HUDF-Ps, and GOODS data we use for our dropout selections is provided in our previous work (B06). Nevertheless, a brief description of the data is included here. The ACS HUDF data we use are the version 1.0 reductions of Beckwith et al. (2006) and extend to 5 $\sigma$ point-source limits of ~29–30 in the $B_{435}$, $i_{775}$, $z_{850}$ bands. The HUDF-Ps reductions we use are from B06 and take advantage of the deep (~72 orbit) $BViz$ ACS data fields taken in parallel with the HUDF NICMOS program (Thompson et al. 2005). Together the parallel data from this program sum to create two very deep ACS fields that we can use for dropout searches. While of somewhat variable depths, the central portions of these fields (12–20 arcmin$^2$) reach some 0.6–0.9 mag deeper than the data in the original ACS GOODS program (Giavalisco et al. 2004a). Finally, for the ACS GOODS reductions, we use an updated version of those generated for our previous $z \sim 6$ study (B06). These reductions not only take advantage of all the original data taken with the ACS GOODS program but also include all the ACS data associated with the SN search (Riess et al. 2007), GEMS (Rix et al. 2004), HUDF NICMOS (Thompson et al. 2005), and HUDF05 (Oesch et al. 2007) programs. The latter data (particularly the SN search data) increase the depths of the $i_{775}$- and $z_{850}$-band images by ~0.2 and ~0.5 mag.
respectively, over that available in the GOODS version 1.0 reductions (Giavalisco et al. 2004a).

Finally, we also take advantage of two exceptionally deep ACS fields taken over the NICMOS parallels to the HUDF (called the HUDF05 fields; Oesch et al. 2007). Each field contains 10 orbits of ACS $V_{606}$-band data, 23 orbits of ACS $I_{775}$-band data, and 71 orbits of ACS $z_{850}$-band data. As such, these fields are second only to the HUDF in their total $z_{850}$-band exposure time. Although these data were taken to search for galaxies at $z > 6.5$ (e.g., Bouwens & Illingworth 2006), they provide us with additional data for the UV LF determinations at $z \sim 5$–6. These data were not available to us in our previous study on the LF at $z \sim 6$ (B06). The ACS data over these fields were reduced using the ACS GTO pipeline Apsis (Blakeslee et al. 2003). Apsis handles image alignment, cosmic-ray rejection, and the drizzling process. To maximize the quality of our reductions, we median stacked the basic postcalibration data after masking out the sources and then subtracted these medians from the individual exposures before drizzling them together to make the final images. The reduced fields reach to $\sim 29$ mag at $5 \sigma$ in the $V_{606}$, $I_{775}$, and $z_{850}$ bands using $0.2''$ diameter apertures. This is only $\sim 0.4$ mag shallower than the HUDF in the $z_{850}$ band. A detailed summary of the properties of each of our fields is contained in Table 1.

### 2.2. Catalog Construction and Photometry

Our procedure for doing object detection and photometry on the HUDF, HUDF-Hu, HUDF05, and GOODS fields is very similar to that used previously (Bouwens et al. 2003b; B06). Briefly, we perform object detection for $B_-, V_-$, and $i$-dropout selections by constructing $\chi^2$ images (Szalay et al. 1999) from the $V_{606}$, $I_{775}$, and $z_{850}$-band data, $i_{775}$- and $z_{850}$-band data, and $z_{850}$-band data, respectively. The $\chi^2$ images are constructed by adding together the relevant images in quadrature, weighting each by $1/\sigma^2$, where $\sigma$ is the rms noise on the image. SExtractor (Bertin & Arnouts 1996) was then run in double-image mode using the square root of the $\chi^2$ image as the detection image and the other images to do photometry. Colors were measured using Kron-style (Kron 1980) photometry (MAG_AUTO) in small scalable apertures (Kron factor 1.2, with a minimum aperture of 1.7 semimajor [semiminor] axis lengths). These colors were then corrected up to total magnitudes using the excess light contained within large scalable apertures (Kron factor 2.5, with a minimum aperture of 3.5 semimajor [semiminor] axis lengths). We measured these corrections off the square root of the $\chi^2$ image as the detection image and the other images to do photometry. The median diameter of these apertures was $\sim 0.6''$ for the faintest sources in our samples. An additional correction was made to account for light outside of our apertures and on the wings of the ACS Wide Field Camera (WFC) point-spread function (PSF; Sirianni et al. 2005). Typical corrections were $\sim 0.1$–0.2 mag.

To assess the quality of our total magnitude measurements, we compared our measurements (which are based on global backgrounds) with those obtained using local backgrounds and found that our total magnitude measurements were $\sim 0.04$ mag brighter in the mean. Comparisons with similar flux measurements made available from the GOODS and HUDF teams (Giavalisco et al. 2004a; Beckwith et al. 2006) also showed good agreement (roughly $\pm 0.2$ mag scatter), although our total magnitude measurements were typically $\sim 0.08$ mag brighter. We believe that this offset is the result of the $\sim 0.1$ mag correction we make for light on the PSF wings (Sirianni et al. 2005).

While constructing our dropout catalogs, one minor challenge was in the deblending of individual sources. The issue was that SExtractor frequently split many of the more asymmetric, multi-component dropout galaxies in our samples into more than one distinct source. This would have the effect of transforming many luminous sources in our selection into multiple lower luminosity sources and thus bias our LF determinations. To cope with this issue, we experimented with a number of different procedures for blending sources together based on their colors. In the end, we settled on a procedure whereby dropouts were blended with nearby sources if (1) they lay within 4 Kron radii and (2) their colors did not differ at more than 2 $\sigma$ significance. Since SExtractor does not allow for the use of color information in the blending of individual sources, it was necessary for us to implement this algorithm outside the SExtractor package. We found that our procedure nearly always produced results that were in close agreement with the choices we would make after careful inspection.

### 2.3. Selection Criteria

We adopted selection criteria for our $B_-$, $V_-$, and $i$-dropout samples that are very similar to those used in previous works. Our selection criteria are

\[
(B_{435} - V_{606} > 1.1) \land (B_{435} - V_{606} > (V_{606} - z_{850}) + 1.1)
\]

\[
\land (V_{606} - z_{850} < 1.6)
\]
for our B-dropout sample,
\[
\left\{ \left[ V_{606} - i_{775} > 0.9 (i_{775} - z_{850}) \right] \lor \left( V_{606} - i_{775} > 2 \right) \right\} \\
\land \left( V_{606} - i_{775} > 1.2 \right) \land \left( i_{775} - z_{850} < 1.3 \right)
\]

for our V-dropout sample, and
\[
\left( i_{775} - z_{850} > 1.3 \right) \land \left\{ \left( V_{606} - i_{775} > 2.8 \right) \lor \left( S/N(V_{606} < 2) \right) \right\}
\]

for our i-dropout sample, where \( \land \) and \( \lor \) represent the logical AND and OR symbols, respectively, and \( S/N \) represents the signal-to-noise ratio. Our V- and i-dropout selection criteria are identical to those described in Giavalisco et al. (2004b) and B06, respectively. Meanwhile, our B-dropout criteria, while slightly different from those used by Giavalisco et al. (2004b), are now routinely used by different teams (e.g., Beckwith et al. 2006).

We also required sources to be clearly extended (SExtractor stellar indices less than 0.8) to eliminate intermediate-mass stars and AGNs. Since the SExtractor stellarity parameter rapidly becomes unreliable near the magnitude limit of each of our samples (see, e.g., the discussion in § D.4.3 of B06), we do not remove point sources faintward of the limits \( i_{775, AB} > 26.5 \) (GOODS), \( i_{775, AB} > 27.3 \) (HUDF-Ps/HUDF05), and \( i_{775, AB} > 28 \) (HUDF) for our B-dropout sample and \( z_{850, AB} > 26.5 \) (GOODS), \( z_{850, AB} > 27.3 \) (HUDF-Ps/HUDF05), and \( z_{850, AB} > 28 \) (HUDF) for our V- and i-dropout samples. Instead, contamination from stars is treated on a statistical basis. Since only a small fraction of galaxies faintward of these limits appear to be stars (<6% of the dropout candidates brightward of 27.0 are unresolved in our GOODS selections and <1% of the dropout candidates brightward of 28.0 are unresolved in our HUDF selections), these corrections are small and should not be a significant source of error. Sources that were not 4.5 \( \sigma \) detections in the selection band (0.3’’ diameter apertures) were also removed to clean our catalogs of a few spurious sources associated with an imperfectly flattened background. Finally, each dropout in our catalog was carefully inspected to remove artifacts (e.g., diffraction spikes or low surface brightness features around bright foreground galaxies) that occasionally satisfy our selection criteria.

In total, we found 711 B-dropouts, 232 V-dropouts, and 132 i-dropouts over the HUDF and 3828 B-dropouts, 888 V-dropouts, and 365 i-dropouts over the two GOODS fields. This is similar to (albeit slightly larger than) the numbers reported by Beckwith et al. (2006) over these fields. We also found 283 B-dropouts over the HUDF-Ps (12 arcm \(^2\)) and 332 V-dropouts and 160 i-dropouts over the HUDF-Ps and HUDF05 fields (32 arcm \(^2\)). Altogether, our catalogs contain 4671, 1416, and 627 unique detections in the HUDF, HUDF05, and CDFS GOODS fields, respectively. Table 2 provides a convenient summary of the properties of our B-, V-, and i-dropout samples. Figure 1 compares the surface density of dropouts found in our compilation with those obtained in the literature (Giavalisco et al. 2004b; Beckwith et al. 2006). With a few notable exceptions (see, e.g., Fig. 12 below), we are in good agreement with the literature.

### 2.4. Flux/Completeness Corrections

The above samples provide us with an unprecedented data set for determining the LFs at high redshift over an extremely wide range in luminosity. However, before we use these samples to determine the LFs at \( z \sim 4 - 6 \), we need to understand in detail how object selection and photometry affect what we observe. These issues can have a significant effect on the properties of our different selections, as one can see in Figure 1 by comparing the surface density of dropouts observed in the HUDF, HUDF05, HUDF-Ps, and GOODS fields, where clear differences are observed at faint magnitudes due to obvious differences in the completeness of these samples at such magnitudes.

To accomplish these aims, we use a very similar strategy to what we employed in previous examinations of the rest-frame UV LF at \( z \sim 6 \) (B06). Our strategy is to derive transformations that correct the dropout surface densities from what we would derive for noise-free (infinite S/N) data to that recoverable at the depths of our various fields. These transformations are made using a set of two-dimensional matrices, called transfer functions. These functions are computed for each dropout selection and field under consideration here (HUDF, HUDF05, HUDF-Ps, and GOODS). We describe the derivation of these transfer functions in detail in § A1. A summary of the properties of these functions is also provided in this section.

### 2.5. Contamination Corrections

Dropout samples also contain a small number of contaminants. We developed corrections for three types of contamination: (1) intrinsically red, low-redshift interlopers; (2) objects entering our samples due to photometric scatter; and (3) spurious sources.
We estimated the fraction of intrinsically red objects in our samples as a function of magnitude using the deep $K_S$-band data over the Chandra Deep Field–South (CDF-S) GOODS field (Vandame et al. 2008, in preparation). Contaminants were identified in our $B$, $V$, and $i$-dropout selections with an $(i_{775} - K_s)_{AB} > 2$, $(z_{850} - K_s)_{AB} > 2$, and $(z_{850} - K_s)_{AB} > 1.6$ criterion, respectively. The contamination rate from photometric scatter was estimated by performing selections on degradations of the HUDF. Section D4.2 of B06 provides a description of how we previously calculated this at $z \sim 6$. The contribution of these two contaminants to our samples was relatively small, on order $\sim 2\%$, $\sim 3\%$, and $\sim 3\%$, respectively, although this contamination rate is clearly magnitude dependent and decreases toward fainter magnitudes. The contamination rate from spurious sources was determined by repeating our selection on the negative images (e.g., Dickinson et al. 2004; B06) and found to be completely negligible ($\lesssim 1\%$).

2.6. Number Counts

Before closing this section and moving on to a determination of the UV LF at $z \sim 4–6$, it is useful to derive the surface density of $B$-, $V$-, and $i$-dropouts by combining the results from each of our samples and implementing each of the above corrections. Although we make no direct use of these aggregate surface densities in our derivation of the rest-frame UV LF, direct tabulation of these surface densities can be helpful for observers who are interested in knowing the approximate source density of high-redshift galaxies on the sky or for theorists who are interested in making more direct comparisons to the observations. We combine the surface densities from our various fields using a maximum likelihood procedure. The surface densities are corrected for field-to-field variations using the factors given in Table 16 (see Appendix B). Both incompleteness and flux biases are treated using the transfer functions that take our selections from HUDF depths to shallower depths. Our final results are presented in Table 3.

3. DETERMINATION OF THE UV LF AT $z \sim 4–6$

The large $B$-, $V$-, and $i$-dropout samples we have compiled permit us to determine the rest-frame UV LFs at $z \sim 4$, $\sim 5$, and $\sim 6$ to very faint UV luminosities (AB magnitudes roughly $-16$, $-17$, and $-17.5$, respectively), with significant statistics over a wide range in magnitude. This provides us with both the leverage and statistics to obtain an unprecedented measure of the overall shape of the LF for galaxies at $z \sim 4$, $\sim 5$, and $\sim 6$.

To maximize the robustness of our LF results, we consider a wide variety of different approaches to determining the LF at $z \sim 4$, $\sim 5$, and $\sim 6$. We begin by invoking two standard techniques for determining the LF in the presence of large-scale structure (both modified for use with apparent magnitudes). The first technique is the Sandage et al. (1979, hereafter STY79) approach, and the second is the stepwise maximum likelihood (SWML) method.
We then expand our discussion to consider a wide variety of different LF both in stepwise form and using a Schechter parameterization. For our approach and various assumptions we make about the form of the luminosity functions at 1600 Å, and 9. We begin by estimating the rest-frame UV LF from our B-, V-, and i-dropout samples using a Schechter parameterization

\[
\phi^*(\ln 10/2.5)10^{-0.4(M-M^*)/(\alpha + 1)}e^{-10^{0.4(M-M^*)}}
\]

and the maximum likelihood procedure of STY79. The parameter \(\phi^*\) is the normalization, \(M^*\) is the characteristic luminosity, and \(\alpha\) is the faint-end slope in the Schechter parameterization. The STY79 procedure has long been the technique of choice for computing the LF over multiple fields because it is insensitive to the presence of large-scale structure. The central idea behind this technique is to consider the likelihood of reproducing the relative distribution of dropouts in magnitude space given an LF. Because only the distribution of sources is considered in this measure and not the absolute surface densities, this approach is only sensitive to the shape of the LF and not its overall normalization. This makes this approach immune to the effects of large-scale structure and our LF fit results very robust.

It is worthwhile to note, however, that for our particular application of this approach, our results are not completely insensitive to large-scale structure. This is because, lacking exact redshifts for individual sources in our samples, we need to consider the apparent magnitudes of individual galaxies in computing the likelihoods and not the absolute magnitudes. This makes our results slightly sensitive to large-scale structure along the line of sight due to the effect of redshift on the apparent magnitudes. However, as we demonstrate in Appendix C, the expected effect of this structure is extremely small, introducing \(1 \sigma\) variations of \(\sim 0.05\) mag in the value of \(M^*\) and \(\sim 0.02\) in the value of the faint-end slope \(\alpha\).

To use this approach to evaluate the likelihood of model LFs, we need to compute the surface density of dropouts as a function of magnitude \(N(m)\) from the model LFs, so we can compare these numbers against the observations. We use a two-stage approach for these computations, so we can take advantage of the transfer functions we derived in §A1. These functions provide us with a very natural way of incorporating the effects of incompleteness and photometric scatter into our comparisons with the observations, so we will want to make use of them. In order to do this, we first need to calculate the surface density of dropouts appropriate for our deepest selection (the HUDF). Then, we correct this surface density to that appropriate for our shallowest field using the transfer functions.

The nominal surface densities in our HUDF selections \(N(m)\) are computed from the model LFs \(\phi(M)\) as

\[
\int_z \phi[M(m, z)]P(m, z)\, \frac{dV}{dz}\, dz = N(m),
\]

where \(dV/dz\) is the cosmological volume element, \(P(m, z)\) is the probability of selecting star-forming galaxies at a magnitude \(m\) and redshift \(z\) in the HUDF, \(M\) is the absolute magnitude at 1600 Å, and \(m\) is the apparent magnitude in the \(i_{775}, z_{850}\), or \(z_{850}\) band depending on whether we are dealing with a B-, V-, or i-dropout selection. Note that the \(i_{775}\) and \(z_{850}\) bands closely correspond to rest-frame 1600 Å at the mean redshift of our B- and V-dropout samples (\(z \sim 3.8\) and \(\sim 5.0\), respectively), whereas for our \(z \sim 6\) i-dropout selection, the \(z_{850}\) band corresponds to rest-frame 1350 Å.

With the ability to compute the surface density of dropouts in our different fields for various model LFs, we proceed to determine the LF that maximizes the likelihood of reproducing the observed counts with model LFs at \(z \sim 4, 5,\) and \(6\). The
formulae we use for computing these likelihoods are given in § A2, along with the equations we use to evaluate the integral in equation (2) and implement the transfer functions from § A1. We compute the selection efficiencies $P(m, z)$ through extensive Monte Carlo simulations, where we take real $B$-dropouts from the HUDF, artificially redshift them across the redshift windows of our samples, add them to our data, and then reselect them using the same procedure we use on the real data. A lengthy description of these simulations is provided in § A3, but the following are some essential points: (1) The $H$-B dropout galaxy profiles used in our effective volume simulations for each of our dropout samples are projected to higher redshifts assuming a $(1 + z)^{-1.1}$ size scaling (independent of luminosity) to match the size evolution observed at $z \sim 2$–6 (B06). (2) The distribution of UV continuum slopes in our $z \sim 4$ $B$-dropout effective volume simulations is taken to have a mean of $-1.5$ and $1 \sigma$ scatter of 0.6 for UV-luminous $L^*$ star-forming galaxies. For our higher redshift samples and at lower UV luminosities, the mean UV continuum slope is taken to be roughly $-2$. In all cases, these slopes were chosen to match that found in the observations (Meurer et al. 1999, Stanway et al. 2005; B06; R. J. Bouwens et al. 2008, in preparation). (3) To treat absorption from neutral hydrogen clouds, we have implemented an updated version of the Madau (1995) prescription so that it fits more recent $z \gtrsim 5$ Lyman forest observations (e.g., Songaila 2004) and includes line-of-sight variations (e.g., as performed in Bershady et al. 1999). In calculating the equivalent absolute magnitude $M$ for an apparent magnitude $m$ at $z \sim 6$, we use an effective volume kernel $V_{m,k}$ to correct for the redshift-dependent absorption from the Lyman forest on the observed $z_{21}$-band fluxes ($\S A2$). For our $z \sim 4$ LF, we restrict our analysis to galaxies brighter than $i_{75, AB} = 29.0$ since we found that our fit results were moderately sensitive to the color distribution we used to calculate the selection volumes (see Fig. 18 in Appendix A; see also § B4).

The best-fit Schechter parameters are $M_{1600, AB}^{*} = -21.06 \pm 0.10$ and $\alpha = -1.76 \pm 0.05$ at $z \sim 4$ for our $B$-dropout sample, $M_{1600, AB}^{*} = -20.69 \pm 0.13$ and $\alpha = -1.69 \pm 0.09$ at $z \sim 5$ for our $V$-dropout sample, and $M_{1580, AB}^{*} = -20.29 \pm 0.19$ and $\alpha = -1.77 \pm 0.16$ at $z \sim 6$ for our $i$-dropout sample. Since $z \sim 6$ galaxies appear to be very blue ($\beta = -2$; Stanway et al. 2005; B06), we expect $M_{1600, AB}^{*}$ at $z \sim 6$ to be almost identical ($\leq 0.1$ mag) to the value of $M_{1580, AB}^{*}$. To determine the equivalent normalization $\phi^*$ for our derived values of $\alpha$ and $M^*$, we compute the expected number of dropouts over all of our fields and compare that with the observed number of dropouts in those fields. Following this procedure, we find $\phi^* = 0.0011 \pm 0.0002$ $\text{Mpc}^{-3}$ for our

| Dropout Sample | $(z)$ | $M^*_{AB}$ | $(10^{-3}$ $\text{Mpc}^{-3}$) | $\alpha$ |
|----------------|------|------------|-----------------|--------|
| $B^b$ | 3.8 | -21.06 \pm 0.10 | 1.1 \pm 0.2 | -1.76 \pm 0.05 |
| $V^b$ | 5.0 | -20.69 \pm 0.13 | 0.9 \pm 0.3 | -1.69 \pm 0.09 |
| $i^b$ | 5.0 | -20.29 \pm 0.19 | 1.2 \pm 0.4 | -1.77 \pm 0.16 |

$^a$ Values of $M^*_{AB}$ are at 1600 $\AA$ for our $B$- and $V$-dropout samples and at $\sim 1350$ $\AA$ for our $i$-dropout sample. Since $z \sim 6$ galaxies are blue ($\beta = -2$; Stanway et al. 2005; B06), we expect the value of $M^*$ at $z \sim 6$ to be very similar ($\leq 0.1$ mag) at 1600 $\AA$ to the value of $M^*$ at 1350 $\AA$.

$^b$ Parameters determined using the STY79 technique (§ 3.1) not including evolution across the redshift window of the samples (see Table 7 for the parameters determined including evolution).

Figure 2.—Redshift distributions computed for our HUDF $B$-, $V$-, and $i$-dropout samples (blue, green, and red lines, respectively) using our best-fit Schechter parameters (Table 4) from the STY79 approach and the selection efficiencies given in Fig. 18. The mean redshift for our HUDF $B$-, $V$-, and $i$-dropout selections is 3.8, 5.0, and 5.9, respectively.

B-dropout sample, $\phi^* = 0.0009 \pm 0.0003$ $\text{Mpc}^{-3}$ for our $V$-dropout sample, and $\phi^* = 0.0012 \pm 0.0004$ $\text{Mpc}^{-3}$ for our $i$-dropout sample. We present these LF values in Table 4. The clearest evolution here is in the characteristic luminosity $M^*$, which brightens significantly across this redshift range: from roughly $-20.3$ at $z \sim 6$ to roughly $-21.1$ at $z \sim 4$. In contrast, both the faint-end slope $\alpha$ and normalization $\phi^*$ of the LF remain relatively constant, with $\alpha \approx -1.74$ and $\phi^* \approx 0.001$ $\text{Mpc}^{-3}$. For context, we have computed the redshift distributions for our HUDF $B$-, $V$-, and $i$-dropout selections using these best-fit LFs and presented them in Figure 2.

We plot the likelihood contours for different combinations of $\alpha$ and $M^*$ in Figure 3. These contours were used in our error estimates on $\alpha$ and $M^*$. For our estimates of the uncertainties on the normalization $\phi^*$, we first calculated the field-to-field variations expected over an ACS GOODS field ($\sim 150$ arcmin$^2$). Assuming that our $B$-, $V$-, and $i$-dropout selections span a redshift window of $dz = 0.7$, 0.7, and 0.6, respectively, have a bias of 3.9, 3.4, and 4.1, respectively (Lee et al. 2006; O"verzier et al. 2006), and using a pencil beam geometry for our calculations, we derive field-to-field variations of $\sim 22\%$ rms, $\sim 18\%$ rms, and $\sim 22\%$ rms, respectively. These values are similar to those estimated by other studies (Somerville et al. 2004; B06; Beckwith et al. 2006; cf. Stark et al. 2007c). With these estimates, we were then able to derive likelihood contours in $\phi^*$ by marginalizing over $\alpha$ and $M^*$, using the relationship between $\phi^*$ and the other Schechter parameters and supposing that $\phi^*$ has a $1 \sigma$ uncertainty equal to the rms values given above divided by $\sqrt{2}$ (to account for the fact that each GOODS field provides us with an independent measure of the volume density of high-redshift galaxies).

3.2. SWML

As a second approach, we parameterize our derived LF in a stepwise fashion, with 0.5 mag intervals. This approach is commonly known as the SWML method (Efstathiou et al. 1988) and allows us to look at the evolution of the LF in a more model-independent way than would be possible if we considered Schechter parameterizations alone. As with our STY79 determinations, we maximize the likelihood of reproducing the observed surface densities of dropouts in our different fields given an LF. Similar to that technique, this approach is robust to the presence of large-scale structure. In order to match the magnitude interval used in our stepwise LF, we bin the number counts $N_m$,
effective volume kernels \( V_{m,k} \), and transfer functions \( T_{m,l} \) on 0.5 mag intervals (see \( \S \) 2.2). We compute the surface densities from the model LFs in the same way as for the STY79 method, using equation (A4) from \( \S \) 2.2. The likelihoods are computed using equation (A5). Errors on each of the parameters \( \phi_k \) are derived using the second derivatives of the likelihood \( \mathcal{L} \). We normalize our stepwise LFs \( \phi(M) \) by requiring them to match the total number of dropouts over all of our search fields. Our stepwise determinations are tabulated in Table 5 and also included in the bottom panel of Figure 4. All LFs are Schechter-like in overall shape, as one can see by comparing the stepwise determinations with the independently derived Schechter fits (dashed lines).

### 3.3. Robustness of Schechter Parameter Determinations

It seems legitimate to ask how robust the Schechter parameters are that we derived in \( \S \) 3.1 using the STY79 method. There are a number of different approaches to treating large-scale structure uncertainties, for example, and we could have easily adopted a different approach (i.e., matching up the counts from each of our surveys and then deriving the LFs through a direct approach as we did in B06). By the same token, we also could have chosen to derive the LFs using a different set of SED templates, different assumptions regarding the \( \text{Ly}\alpha \) equivalent widths, different opacity models for absorption from neutral hydrogen clouds, or even different dropout criteria. To ensure that our LF determinations were not unreasonably affected by these choices, we repeated the present determinations of the LF at \( z \sim 4, \sim 5, \) and \( \sim 6 \) adopting a wider variety of different approaches. A detailed description of each of these determinations is provided in Appendix B. The corresponding Schechter parameters are summarized in Table 6. In general, these other determinations are in reasonable agreement with our fiducial STY79 determinations, although it is clear that there are a few variables that can have a small (\( \pm 20\% \)) effect on the derived parameters.

The following are our most significant findings: (1) We found less evolution in the value of \( M^* \) from \( z \sim 6 \) to \( z \sim 4 \) when making the measurement at a bluer rest-frame wavelength (i.e., \( \sim 1350 \) Å) than we did when making this measurement at \( \sim 1600 \) Å. This is likely the result of the fact that \( L_{\text{g}} \) galaxies at \( z \sim 4 \) (Ouchi et al. 2004) are much redder than they are at \( z \sim 5 \)–6 (Lehnert & Bremer 2003; Stanway et al. 2005; B06). (2) The inclusion of \( \text{Ly}\alpha \) emission lines in the SEDs of the model star-forming galaxies (assuming that 33\% of the sources have rest-frame

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**TABLE 5**

| \( M_{1600,AB}^a \) | \( \phi_k \) (Mpc\(^{-3}\) mag\(^{-1}\)) |
|-----------------|-----------------|
| \( B \)-Dropouts (\( z \sim 4 \)) |
| \( -22.26 \)        | 0.00001 \pm 0.00001 |
| \( -21.76 \)        | 0.00011 \pm 0.00002 |
| \( -21.26 \)        | 0.00025 \pm 0.00003 |
| \( -20.76 \)        | 0.00067 \pm 0.00004 |
| \( -20.26 \)        | 0.00106 \pm 0.00006 |
| \( -19.76 \)        | 0.00169 \pm 0.00008 |
| \( -19.26 \)        | 0.00285 \pm 0.00012 |
| \( -18.76 \)        | 0.00542 \pm 0.00055 |
| \( -18.26 \)        | 0.00665 \pm 0.00067 |
| \( -17.76 \)        | 0.01165 \pm 0.00123 |
| \( -17.26 \)        | 0.01151 \pm 0.00148 |
| \( -16.76 \)        | 0.02999 \pm 0.00375 |
| \( -16.26 \)        | 0.02610 \pm 0.01259 |

| \( I \)-Dropouts (\( z \sim 5 \)) |
| \( -21.66 \)        | 0.00003 \pm 0.00001 |
| \( -21.16 \)        | 0.00012 \pm 0.00001 |
| \( -20.66 \)        | 0.00031 \pm 0.00003 |
| \( -20.16 \)        | 0.00062 \pm 0.00004 |
| \( -19.66 \)        | 0.00113 \pm 0.00007 |
| \( -19.16 \)        | 0.00179 \pm 0.00020 |
| \( -18.66 \)        | 0.00203 \pm 0.00022 |
| \( -18.16 \)        | 0.00506 \pm 0.00057 |
| \( -17.66 \)        | 0.00530 \pm 0.00134 |
| \( -17.16 \)        | 0.00782 \pm 0.00380 |

| \( r \)-Dropouts (\( z \sim 6 \)) |
| \( -22.13 \)        | 0.00001 \pm 0.00001 |
| \( -21.63 \)        | 0.00001 \pm 0.00001 |
| \( -21.13 \)        | 0.00007 \pm 0.00002 |
| \( -20.63 \)        | 0.00013 \pm 0.00004 |
| \( -20.13 \)        | 0.00054 \pm 0.00012 |
| \( -19.63 \)        | 0.00083 \pm 0.00018 |
| \( -18.88 \)        | 0.00197 \pm 0.00041 |
| \( -17.88 \)        | 0.00535 \pm 0.00117 |

\(^a\) The LF is tabulated at 1350 Å at \( z \sim 6 \).
equivalent widths of 50 Å (see § B5) has a modest effect on the selection volumes computed for our three dropout samples and results in a modest decrease in $\phi^*$ at $z \sim 4$ (by 10%) but an increase in $\phi^*$ at $z \sim 5$ and $\sim 6$ (by $\sim 10\%$). (3) At $z \sim 4$, we found that our LF fit results could be somewhat sensitive to the distribution of UV colors used, depending on the faint-end limit we adopted in our analysis. As a result, we restricted ourselves to galaxies brighter than 29 mag in our $z \sim 4$ LF fits above to improve the overall robustness of the fit results. (4) We found that the Schechter parameters for our high-redshift LFs only show a slight ($\leq 10\%$) dependence on the model we adopted for the opacity coming from neutral hydrogen clouds. (5) If we allow for evolution in $M^*$ across the redshift window of each sample (by 0.35 mag per unit redshift as we find in our fiducial STY79 determinations), we recovered a slightly fainter value of $M^*$ (by $\sim 0.06$ mag), a higher value of $\phi^*$ (by $\sim 10\%$), and a shallower faint-end slope $\alpha$ (by $\sim -0.02$) for all three LFs. (6) In each and every analysis we considered, we found a significant ($\sim 0.5$--0.9 mag) brightening of $M^*$ from $z \sim 6$ to $\sim 4$, suggesting that this evolutionary finding is really robust. We also consistently recovered a very steep ($\alpha \lesssim -1.7$) faint-end slope. We would consider both of these conclusions to be quite solid.

Of all the issues considered in this section, the only issue that would clearly bias our LF determinations and for which we can accurately make a correction is the issue of evolution across the redshift selection windows of our dropout samples. Since this issue only has a minimal effect on the LF fit results (i.e., $\Delta M \sim 0.06$ mag, $\Delta \phi^*/\phi^* \sim 0.1$, $\Delta \alpha \sim 0.03$) and an even smaller effect on integrated quantities like the luminosity density, we do not repeat much of the analysis done thus far to include it. Instead, we simply adopt the results of the STY79 approach including this evolution in $M^*$ with redshift (Table 6; see § B8) hereafter as our preferred determinations of the Schechter parameters at $z \sim 4$, $\sim 5$, and $\sim 6$ (see Table 7).

3.4. Faint-End Slope

It is worthwhile to spend a little time reemphasizing how robust the current determination of a steep faint-end slope really is and how readily this result can be derived from the data. In fact, we could have determined the faint-end slope $\alpha$ at $z \sim 4$ simply from our HUDF $B$-dropout selection alone. At a rudimentary level, this can be seen from the number counts, which in our HUDF $B$-dropout sample increase from surface densities of 3 sources arcmin$^{-2}$ at $i_{775, AB} \sim 25.5$ to 30 sources arcmin$^{-2}$ at $i_{775, AB} \sim 29$, for a faint-end slope of $\sim -0.3$ dex mag$^{-1} \sim 0.7$ (red line in Fig. 5). Since the selection volume is largely independent of magnitude over this range, one can essentially “read off” the faint-end slope from the number counts and find that it is steep, roughly $\sim -1.7$. Use of our LF methodology on our HUDF selections permits a more rigorous determination and yields $\alpha = -1.76 \pm 0.07$ at $z \sim 4$. We should emphasize that these results are robust and are not likely to be sensitive to concerns about large-scale structure (the counts are drawn from a single field), small number statistics (the HUDF contains $\gtrsim 700$ $B$-dropout sources), or contamination (all known contaminants have shallower faint-end slopes). Even the model selection volumes are not a concern for our conclusion that the faint-end slope is steep since we can derive this conclusion from simple fits to the number counts (i.e., the red line in Fig. 5) as argued above and the inclusion of realistic selection volumes (which decrease toward fainter magnitudes) would only cause the inferred faint-end slope to be steeper. Similarly steep slopes are obtained from independent fits to the $B$-dropouts in our other fields (HUDF-Ps and both GOODS fields) and our other dropout selections, suggesting that a steep (roughly $\sim -1.7$) faint-end slope is really a generic feature of high-redshift LFs (see also Beckwith et al. 2006; Yoshida et al. 2006; Oesch et al. 2007).

3.5. Luminosity/SFR Densities

Having derived the rest-frame UV LF at $z \sim 4$, $\sim 5$, and $\sim 6$, we can move on to establish the luminosity densities at these epochs. The luminosity densities are of great interest because of their close link to the SFR densities. But, unlike the SFR densities inferred from luminosity density measurements, the luminosity densities are much more directly relatable to the observations themselves, requiring fewer assumptions. As such, they can be more useful when it comes to comparisons between different determinations in the literature, particularly when these determinations are made at the same redshift.

It is common in determinations of the luminosity density to integrate the LF to the observed faint-end limit. Here we consider two faint-end limits: 0.04$L^*$ (to match the limits reached by our LF at $z \sim 6$) and 0.3$L^*$ (to match the limits reached at $z \sim 7$--10; Bouwens et al. 2004c, 2005; Bouwens & Illingworth 2006). For convenience, we have compiled the calculated luminosity densities for our $z \sim 4$ and $\sim 5$ UV LFs in Table 8. We have also
| Method                        | $B$-Dropouts ($z \sim 4$) | $V$-Dropouts ($z \sim 5$) | $i$-Dropouts ($z \sim 6$) |
|-------------------------------|---------------------------|---------------------------|---------------------------|
|                              | $M_{UV}^{*}$ | $\phi^*$ | $\alpha$ | $M_{UV}^{*}$ | $\phi^*$ | $\alpha$ | $M_{UV}^{*}$ | $\phi^*$ | $\alpha$ |
| STY79 (mean $\beta = -1.4$)$^b$ | $-21.20 \pm 0.14$ | $0.9 \pm 0.2$ | $-1.76 \pm 0.05$ | $-20.69 \pm 0.13$ | $0.9 \pm 0.05$ | $-1.69 \pm 0.09$ | $-20.29 \pm 0.19$ | $1.2 \pm 0.6$ | $-1.77 \pm 0.16$ |
| STY79 (mean $\beta = -2.1$)$^b$ | $-21.16 \pm 0.10$ | $0.9 \pm 0.2$ | $-1.79 \pm 0.05$ | $-20.65 \pm 0.12$ | $1.1 \pm 0.05$ | $-1.70 \pm 0.09$ | $-20.26 \pm 0.19$ | $1.2 \pm 0.6$ | $-1.73 \pm 0.16$ |
| STY79 (LyC contribution)$^b$ | $-21.05 \pm 0.10$ | $1.0 \pm 0.2$ | $-1.76 \pm 0.05$ | $-20.70 \pm 0.13$ | $1.0 \pm 0.3$ | $-1.68 \pm 0.09$ | $-20.31 \pm 0.19$ | $1.3 \pm 0.6$ | $-1.76 \pm 0.16$ |
| STY79 (SFR contribution)$^b$ | $-20.97 \pm 0.13$ | $1.0 \pm 0.2$ | $-1.81 \pm 0.06$ | $-20.57 \pm 0.11$ | $1.3 \pm 0.3$ | $-1.63 \pm 0.08$ | $-20.39 \pm 0.23$ | $1.0 \pm 0.5$ | $-1.78 \pm 0.17$ |
| STY79 (Madau opacities)$^b$ | $-21.06 \pm 0.10$ | $1.1 \pm 0.2$ | $-1.75 \pm 0.05$ | $-20.66 \pm 0.12$ | $1.0 \pm 0.3$ | $-1.71 \pm 0.09$ | $-20.32 \pm 0.19$ | $1.3 \pm 0.6$ | $-1.76 \pm 0.16$ |
| STY79 (Evolving $M^*$)$^b$ | $-20.98 \pm 0.10$ | $1.3 \pm 0.2$ | $-1.73 \pm 0.05$ | $-20.64 \pm 0.13$ | $1.0 \pm 0.3$ | $-1.66 \pm 0.09$ | $-20.24 \pm 0.19$ | $1.4 \pm 0.6$ | $-1.74 \pm 0.16$ |

$^a$ Values of $M_{UV}^{*}$ are at 1600 Å for our $B$- and $V$-dropout samples and at $\sim 1350$ Å for our $i$-dropout sample. Since $z \sim 6$ galaxies are blue ($\beta \sim -2$; Stanway et al. 2005; B06), we expect the value of $M^*$ at $z \sim 6$ to be very similar (±0.1 mag) at 1600 Å to the value of $M^*$ at 1350 Å.

$^b$ LF determinations considered in §§ B1–B8, respectively.

$^c$ Only galaxies brighter than 28 mag are used in the fit results (see § B4).

$^d$ Adopted determinations of the Schechter parameters; see Table 7.
and "(L)," respectively) assuming simple evolution in the past. The values of \( M' \) and \( \alpha \) fixed at the values derived for these parameters at \( z \approx 6 \) (see Table 5.4).

Since both \( \phi' \) and \( \alpha \) show no significant evolution over the interval \( z \approx 6 \), we assume that this holds at even earlier times and that \( \phi' = 0.0014 \text{ Mpc}^{-3} \) and \( \alpha = -1.74 \). These determinations are only mildly sensitive to the assumed values of \( \phi' \) and \( \alpha \). Steeper values of \( \phi' \) (i.e., \( \alpha \approx -2 \)) yield \( M' \) values that are \( \approx 1 \text{ mag} \) brighter, and shallower values of \( \alpha \) (i.e., \( \alpha \approx -1.4 \)) yield \( M' \) values that are \( \approx 0.5 \text{ mag} \) fainter. Changing \( \phi' \) by a factor of 2 only changes \( M' \) by \( 0.3 \text{ mag} \).

\[ \phi' \approx 0.0014 \text{ Mpc}^{-3} \quad \alpha \approx -1.74 \]

### Table 7

| Dropout Sample \( \langle z \rangle \) | \( M'_{\text{UV}}^a \) (10\(^{-3}\) Mpc\(^{-3}\)) | \( \phi' \) | \( \alpha \) |
|-------------------------------------|------------------------------------------|---------|--------|
| \( B^b \)                         | 3.8 ± 0.08                              | 1.3 ± 0.2 | -1.73 ± 0.05 |
| \( I^b \)                         | 5.0 ± 0.06                              | 1.0 ± 0.3 | -1.66 ± 0.09 |
| \( z^c \)                         | 7.4 ± 0.3                               | (1.4)    | (-1.74) |
| \( z \)                           | 7.4 ± 0.3                               | (1.4)    | (-1.74) |

Notes: Based on LF parameters in Table 7. At \( z \approx 7.4 \), the luminosity densities are based on the search results for the Bouwens et al. (2006) conservative selection (\( z \approx 5.4 \)).

- Values of \( M'_{\text{UV}} \) are at 1600 Å for our \( B \)- and \( V \)-dropout samples, at \( \approx 1350 \text{ Å} \) for our \( i \)-dropout sample, and at \( \approx 1900 \text{ Å} \) for our \( z \)-dropout sample. Since \( z \leq 6 \) galaxies are blue (\( \approx 3 \text{ to } 2 \; \text{Stanway et al. 2005; B06} \)), we expect the value of \( M' \) at \( z \approx 6 \) to be very similar (±0.1 mag) at 1600 Å to the value of \( M' \) at 1350 Å. Similarly, we expect \( M' \) at \( z \approx 7 \) to be fairly similar at 1600 Å to the value at \( z \approx 1900 \text{ Å} \).
- Parameters are determined using the STY79 technique (\( z \approx 1 \)), including evolution across the redshift window of the samples (\( z \approx 4 \)), and therefore differ from those in Table 4, which do not.
- \( M'_{\text{UV}} \) are derived from both the conservative and less conservative 2500-Å dropout search results of Bouwens et al. (2006)[denoted here as "(C)"] and "(L)", respectively] assuming simple evolution in \( M' \) and keeping the values of \( \phi' \) and \( \alpha \) fixed at the values we derived for these parameters at \( z \approx 6 \) (see Table 5.4).
- Since both \( \phi' \) and \( \alpha \) show no significant evolution over the interval \( z \approx 6 \), we assume that this holds at even earlier times and that \( \phi' = 0.0014 \text{ Mpc}^{-3} \) and \( \alpha = -1.74 \). These determinations are only mildly sensitive to the assumed values of \( \phi' \) and \( \alpha \). Steeper values of \( \phi' \) (i.e., \( \alpha \approx -2 \)) yield \( M' \) values that are \( \approx 1 \text{ mag} \) brighter, and shallower values of \( \alpha \) (i.e., \( \alpha \approx -1.4 \)) yield \( M' \) values that are \( \approx 0.5 \text{ mag} \) fainter. Changing \( \phi' \) by a factor of 2 only changes \( M' \) by \( 0.3 \text{ mag} \).
The dust corrections of Schiminovich et al. (2005), and at $z \sim 6$ we use a dust correction of $-0.18$ dex (factor of $-1.5$), which we derived from the $\beta$-values observed for $z \sim 6$ i-dropouts (Stanway et al. 2005; Yan et al. 2005; B06) and the IRX-$\beta$ relationship (Meurer et al. 1999). The IRX-$\beta$ relationship provides a fairly good description of the dust extinction at $z \sim 0$ (e.g., Meurer et al. 1999) and $z \sim 2$ (Reddy & Steidel 2004; Reddy et al. 2006). At redshifts of $z \sim 4$--5, we interpolate between the dust extinctions estimated at $z \sim 2$--3 and those at $z \sim 6$. The results of these calculations are shown in Figure 7 for the luminosity densities integrated down to 0.04$L^{*}_{z=3}$ (the faint-end limit for our $z \sim 6$ searches) and 0.3$L^{*}_{z=3}$ (the faint-end limit for our $z \sim 7$--10 searches). These SFR densities are also tabulated in Table 9. At $z \sim 6$, the SFR density is just $-0.3$ times the SFR density at $z \sim 4$ (integrated to $-17.5$ mag). Clearly the SFR density seems to increase much more rapidly from $z \sim 6$ to $\sim 4$ than the UV luminosity density does. This is a direct result of the apparent evolution in the dust obscuration over this redshift interval.

4. ROBUSTNESS OF LF RESULTS

In the previous section we used our very deep and wide-area $B_7$, $V_5$, and i-dropout selections to determine the UV continuum LF at $z \sim 4$, $\sim 5$, and $\sim 6$ to $\sim 3$--5 mag below $L^*$. This is fainter than all previous probes not including the HUDF data. Since these determinations reach such luminosities with significant statistics and over multiple fields, they have the promise to provide us with a powerful measure of how galaxies are evolving at early times. However, given the considerable spread in LF results to date and significant differences in interpretation, it is important first to discuss the robustness of the current LF results. We devote some effort to this issue because the wide dispersion in observational results is really limiting their value.

![Fig. 6.—Rest-frame UV continuum luminosity density integrated to $0.3L^{*}_{z=3}$ (top) and $0.04L^{*}_{z=3}$ (bottom) as a function of redshift. The equivalent SFR density is also shown assuming no extinction correction. The rest-frame UV continuum luminosity density is converted to an SFR density assuming a constant $\sim 10^8$ yr star formation model and a Salpeter (1955) IMF (Madau et al. 1998). The present determinations are shown as large red circles, with 1 $\sigma$ errors. Also shown are the luminosity density determinations by Schiminovich et al. (2005; black hexagons), Steidel et al. (1999; green crosses), Giavalisco et al. (2004b; black diamonds), Ouchi et al. (2004; magenta circles), Yoshida et al. (2006; black circles), Beckwith et al. (2006; black crosses), Reddy et al. (2007; magenta crosses), Bouwens & Illingworth (2006; red pentagons), and Bouwens et al. (2005; red square, shown with its 1 $\sigma$ upper limit). The dotted hexagon in the bottom panel shows the inferred luminosity density at $z \sim 7.4$ assuming our fit results for the Bouwens & Illingworth (2006) conservative selection (§ 5.4; Table 7).

![Fig. 7.—SFR density of the universe integrated down to $0.3L^{*}_{z=3}$ (top) and $0.04L^{*}_{z=3}$ (bottom). This SFR density is shown both with and without a correction for dust extinction (upper and lower set of points, respectively). This is also indicated with the shaded red and blue regions, where the width of the region shows the approximate uncertainties estimated by Schiminovich et al. (2005). Symbols for the data points are the same as for Fig. 6. At $z \leq 3$, the dust corrections we assume are 1.4 mag and are intermediate between the high and low estimates of Schiminovich et al. (2005; 1.8 and 1.0 mag, respectively). At $z \sim 6$, the dust corrections are 0.4 mag as determined from the steep UV continuum slopes (B06). At $z \sim 4$--5, the dust corrections are interpolations between the $z \sim 3$ and $\sim 6$ values.

Table 9: Inferred Star Formation Rate Densities

| Dropout Sample | $\langle z \rangle$ | $L > 0.3L^{*}_{z=3}$ | $L > 0.04L^{*}_{z=3}$ |
|----------------|---------------------|----------------------|----------------------|
| Uncorrected    |                     |                      |                      |
| $B$            | 3.8                 | $-1.81 \pm 0.05$    | $-1.48 \pm 0.05$    |
| $V$            | 5.0                 | $-2.15 \pm 0.06$    | $-1.78 \pm 0.06$    |
| $i$            | 5.9                 | $-2.31 \pm 0.08$    | $-1.83 \pm 0.08$    |
| $z$            | 7.4                 | $-3.15 \pm 0.48$    | $-2.32$             |
| Dust Corrected |                     |                      |                      |
| $B$            | 3.8                 | $-1.38 \pm 0.05$    | $-1.05 \pm 0.05$    |
| $V$            | 5.0                 | $-1.85 \pm 0.06$    | $-1.48 \pm 0.06$    |
| $i$            | 5.9                 | $-2.14 \pm 0.08$    | $-1.65 \pm 0.08$    |
| $z$            | 7.4                 | $-2.97 \pm 0.48$    | $-2.14$             |

Notes.—Based on LF parameters in Table 7 (see § 3.5). At $z \sim 7.4$, the luminosity densities are based on the search results for the Bouwens & Illingworth (2006) conservative selection.
4.1. Completeness of Current Census

In this work, our goal was to derive rest-frame UV LFs that was representative of the star-forming galaxy population at $z \sim 3.5-6.5$. However, since our LFs were based on sample color selections, it seems legitimate to ask how complete these selections are, and whether our selection might miss a fraction of the high-redshift galaxy population. Such concerns have become particularly salient recently given claims from spectroscopic work that LBG selections may miss a significant fraction of the high-redshift galaxy population that is UV bright at $z \gtrsim 3$ (e.g., Le Fèvre et al. 2005; Pallottini et al. 2007). We refer our readers to Franx et al. (2003), Reddy et al. (2005), and van Dokkum et al. (2006) for an excellent discussion of these issues at slightly lower redshifts ($z \sim 2-3$).

Figure 8 shows a color-color diagram illustrating our $z \sim 4$ $B$-dropout and $z \sim 5$ $V$-dropout selections. The expected colors of galaxies with different UV continuum slopes are plotted as a function of redshift to show how our selection depends on the UV color. To illustrate how the observed distribution of dropout colors compares with these selections, a small sample of bright dropouts are overplotted on these diagrams. We elected to only include the bright dropouts on this diagram because it is only at bright magnitudes that we can efficiently select dropouts over a wide range of UV continuum slopes. Since all high-redshift galaxies will become quite red in their Lyman break colors ($B - V$ for $z \gtrsim 4$ galaxies and $V - i$ for $z \gtrsim 5$ galaxies), it seems clear that the only way galaxies will miss our selection is if they are too red in their UV continuum slopes. As is evident in the figure, the majority of the dropouts in our $B$- and $V$-dropout selections are significantly bluer than our selection limits in $(V_{606} - z_{850})_{AB}$ and $(i_{775} - z_{850})_{AB}$, respectively. Unless there is a distinct population of star-forming galaxies that are much redder than these limits (i.e., the UV color distribution is bimodal), we can conclude that our selection must be largely complete at bright magnitudes. Another way of seeing this is to compare the distribution of observed UV continuum slopes $\beta$ (calculated from the $i_{775} - z_{850}$ colors) for bright ($i_{775, AB} < 24.6$) $B$-dropouts from our sample with the selection limit (Fig. 8, inset), and it is again apparent that the bulk of our sample is significantly bluerward of the selection limit.

Independent evidence for the $z \sim 4$ galaxy population having very blue UV continuum slopes is reported by Brammer & van Dokkum (2007). By applying a Balmer break selection to the Faint Infrared Extragalactic Survey (FIRES) data (Labbé et al. 2003; Förster Schreiber et al. 2006), Brammer & van Dokkum (2007) attempt to isolate a sample of $z \sim 4$ galaxies with sizeable breaks. Since almost all ($\gtrsim 90\%$) of the galaxies in their $z \sim 4$ sample have measured UV continuum slopes bluer than 0.5 (and none having UV continuum slopes redder than 1.0), this again argues that the $z \sim 4$ galaxy population is very blue in general. The key point to note in the Brammer & van Dokkum (2007) analysis is that, in contrast to our LBG selection, their Balmer break selection should not be significantly biased against galaxies with very red UV continuum slopes. Therefore, unless there is a distinct population of UV-bright galaxies with minimal Balmer breaks and very red UV continuum slopes (which seems unlikely given that galaxies with redder UV colors have more dust, which in turn suggests a more evolved stellar population), it would appear that our census of UV-bright galaxies at $z \sim 4-6$ is largely complete. Apparently, the very red $\beta \sim 1-2$ population seen at $z \sim 2-3$ (e.g., van Dokkum et al. 2006) has not developed significantly by $z \sim 4$.

4.2. Cosmic Variance

One generic concern for the determination of any LF is the presence of large-scale structure. This structure results in variations in
the volume density of galaxies as a function of position. For our
dropout studies, these variations are mitigated by the large comov-
ing distances surveyed in redshift space (\(\sim 300–500\) Mpc for a
\(\Delta z \sim 0.7\)) for typical selections (see, e.g., Fig. 18 in Appendix A).
Since these distances cover \(\sim 40–100\) correlation lengths, typical
field-to-field variations of \(\sim 16–35\%\) are found in the surface
density of dropouts (Somerville et al. 2004; Bunker et al. 2004; B06; Beckwith et al. 2006).

Fortunately, these variations should only have a very minor ef-
fect on our results, and this effect will largely be on the normali-
ization of our LFs. It should not have a sizeable effect on the shape
of our LF determinations because of our use of the STY79 and
SWML techniques, which are only mildly sensitive to these vari-
ations of our LFs. It should not have a sizeable effect on the shape
of the expected variations expected over each GOODS field (22% rms
uncertainty in the overall normalization. We incorporated this
field-to-field variations of \(\sim 16–35\%\) for our
bright magnitudes, but they diverge somewhat from these determinations at fainter magnitudes.

4.3. Comparison with Previous Determinations

at \(z \approx 4, 5, \text{ and } z \approx 6\)

It is helpful to compare LFs with several previous determina-
tions to put the current results in context and provide a sense for
their reliability. We structure this section somewhat in order of
depth, beginning with a discussion of all pre-HUDF determina-
tions of the UV LF at \(z \approx 4\) and \(\sim 5\) before moving onto more
recent work involving the HUDF (Beckwith et al. 2006). We post-
pose a discussion of the UV LF at \(z \approx 6\) until the end of this sec-
tion because we had included a fairly comprehensive discussion of
previous \(z \approx 6\) determinations in B06.

4.3.1. Comparison at \(z \approx 4\)

At \(z \approx 4\), there had already been a number of notable deter-
ninations of the UV LF (Steidel et al. 1999; Ouchi et al. 2004; Gabasch et al. 2004; Sawicki & Thompson 2006a; Giavalisco 2005; Yoshida et al. 2006; Paltani et al. 2007; Tresse et al. 2007). These include a determination of the \(z \approx 4\) LF from Steidel et al. (1999) based on an early imaging survey for \(G\) dropouts, a deter-
mination based on a \(B\)-dropout search over deep wide-area im-
aging (1200 arcmin\(^2\)) available over the Subaru XMM-Newton
Deep Field and Subaru Deep Field (SDF; Ouchi et al. 2004), a
determination based on a \(G\)-dropout search over \(\sim 180\) arcmin\(^2\) of
imaging over the three Keck Deep Fields (Sawicki & Thompson
2006a), an earlier determination based on the two wide-area
(316 arcmin\(^2\)) ACS GOODS fields (Giavalisco 2005; Giavalisco
et al. 2004b), a determination based on a \(B\)-dropout search over a
deeper version of the SDF (Yoshida et al. 2006), and several deter-
ninations based on the VVDS spectroscopic sample (Paltani et al.
2007; Tresse et al. 2007). A comparison of these determinations is
in Figure 9 and Table 10.

We split our discussions between the bright and faint ends of
the \(z \approx 4\) LF. At bright magnitudes, our LF is in good agreement
with most previous determinations. Although there is a fair amount
of scatter between the individual LFs, the observed differences
seem consistent with originating from small systematics in the
photometry (\(\pm 0.1\) mag). Our LFs agree less well with the LFs de-

TABLE 10

| Reference | \(M_V\) (\(10^{-18}\) \(\text{Mpc}^{-3}\)) | \(\phi^*\) (\(10^{-3}\) \(\text{Mpc}^{-3}\)) | \(\alpha\) |
|-----------|-----------------|-------------------|---------|
| This work | \(-20.98 \pm 0.10\) | \(1.3 \pm 0.2\) | \(-1.73 \pm 0.05\) |
| Yoshida et al. (2006) | \(-21.14 \pm 0.14\) | \(1.5 \pm 0.4\) | \(-1.82 \pm 0.09\) |
| Beckwith et al. (2006) | \(-20.7\) | \(1.3\) | \(-1.6\) (fixed) |
| Sawicki & Thompson (2006a) | \(-21.0 \pm 0.4\) | \(0.9 \pm 0.5\) | \(-1.26 \pm 0.36\) |
| Giavalisco (2005) | \(-21.20 \pm 0.04\) | \(1.20 \pm 0.03\) | \(-1.64 \pm 0.10\) |
| Ouchi et al. (2004) | \(-21.0 \pm 0.1\) | \(1.2 \pm 0.2\) | \(-2.2 \pm 0.2\) |
| Steidel et al. (1999) | \(-21.2\) | \(1.1\) | \(-1.6\) (assumed) |
of bright galaxies at \( z \approx 3-4 \) whose colors are somewhat different from those typically used to model LBG selections (although there is some skepticism on this front; see, e.g., Reddy et al. 2007).

While such a population would need to be large to match the Paltani et al. (2007) numbers, it is interesting to ask what the effect of such a population would be on our derived UV LFs. To investigate this, we have replaced the bright points in our \( z \approx 4 \) LF with the Paltani et al. (2007) values (from their \( z \approx 3-4 \) LF) and then refitted this LF to a Schechter function. We find \( M^* \approx -21.88, \phi^* = 0.0005 \text{ Mpc}^{-3}, \) and \( \alpha = -1.82 \). Not surprisingly, the characteristic luminosity \( M^* \) is brighter than measured from our LBG selection, and the faint-end slope \( \alpha \) a little steeper, but these changes only result in a slight (<14%) increase in the overall luminosity density at \( z \approx 4 \) to our faint-end limit (−16 mag). This being said, the reduced \( \chi^2 \) (3.2) for the fit is poor, so we should perhaps not take these best-fit Schechter parameters too seriously.

At fainter magnitudes, differences with respect to other LFs become much more significant. At the one extreme, there are the Ouchi et al. (2004), Giavalisco (2005), and Yoshida et al. (2006) determinations, which exceed our determination by factors of \( \approx 2-3 \) lower. For the two most discrepant LFs, the difference in volume densities is nearly a factor of \( \approx 4 \). What could be the source of such a significant disagreement? Although it is difficult to be sure, there are a number of factors that could contribute to this large dispersion (e.g., the assumed Ly\( \alpha \) equivalent width distribution, the assumed SED template set, the assumed SED distribution, large-scale structure errors; see Appendix B). Perhaps the most problematic, however, are the incompleteness, contamination, and flux biases present near the detection limit of these probes. Since these effects can be quite challenging to model and may result in modest to significant errors (factors of \( \approx 1.5-2 \) in the volume density), it is quite possible that some systematics have been introduced in performing the corrections. By contrast, we would expect our own determinations to be essentially immune to such large errors (to at least an AB magnitude of \( \approx 28-28.5 \)) given that our deepest data set, the HUDF, extends some \( \approx 2.5 \) mag deeper than the data used in most previous determinations (the deep determinations of Beckwith et al. [2006] are discussed below). Even in our shallowest data sets, systematics should be much less of a concern in this magnitude range since we are able to make use of the significantly deeper HUDF, HUDF-Ps, and HUDF05 data to determine the completeness, flux biases, and contamination through degradation experiments (see §A1). In conclusion, because of this greater robustness of our selection at faint magnitudes, we would expect our LF to be the most accurate in these regimes.

### 4.3.2. Comparison at \( z \approx 5 \)

Now we compare our results with several determinations of the LF at \( z \approx 5 \) using moderately deep data (Iwata et al. 2003, 2007; Ouchi et al. 2004; Giavalisco 2005; Yoshida et al. 2006). Iwata et al. (2003) made their determination from deep Subaru data (\( \approx 575 \text{ arcmin}^2 \)) they had around the larger HDF-North, Giavalisco (2005) from the wide-area (\( \approx 316 \text{ arcmin}^2 \)) ACS GOODS data, Ouchi et al. (2004) from the deep wide-area (\( \approx 1200 \text{ arcmin}^2 \)) Subaru data they had over the Subaru XMM-Newton Deep Field and SDF, Yoshida et al. (2006) from an even deeper imaging over the SDF, and Iwata et al. (2007) from the ~1290 arcmin\(^2\) Subaru data around the HDF-North and J053+1234 region. A comparison of these LF determinations is provided in Figure 10 and Table 11.

Our \( z \approx 5 \) results are in excellent agreement with many previous studies (Yoshida et al. 2006; Ouchi et al. 2004), particularly at fainter magnitudes \( z_{500, AB} > 25 \). However, we are not able to reproduce the large number density of bright galaxies found by Iwata et al. (2003, 2007) and Giavalisco (2005). We are unsure of why this might be, since field-to-field variations should not produce such large differences, but it has been speculated that a significant fraction of the candidates in the probes deriving the higher volume densities (e.g., Iwata et al. 2003, 2007) may be contaminants (e.g., Ouchi et al. 2004). While Iwata et al. (2007) have argued, however, that such contamination rates are unlikely for their bright samples given the success of their own spectroscopic follow-up campaign (\( \geq 6 \) out of 8 sources that they followed up at \( 24 < z_{AB} < 24.5 \) were at \( z \geq 4 \)), we were only able to partially verify this success over the HDF-North GOODS field, where our searches overlapped. Of the three bright (\( z_{AB} \leq 24.5 \)) sources cited by Iwata et al. (2007) with spectroscopic redshifts, one (GOODS J123647.96+620941.7) appears to be an AGN. This suggests that a modest fraction of the sources in the Iwata et al. (2007) bright selection may be pointlike contaminants like AGNs (we note that Iwata [2007, private communication] report that they removed this particular AGN from their bright sample). We continue to regard our determination of the volume densities of the LF at \( z \approx 5 \) as the most robust due to the superb resolution and photometric

| Reference | \( M_{UV} \) (\( 10^{-3} \text{ Mpc}^{-3} \)) | \( \phi^* \) | \( \alpha \) |
|-----------|----------------|-----|-----|
| This work | \(-20.64 \pm 0.13\) | \(1.0 \pm 0.3\) | \(-1.66 \pm 0.09\) |
| Oesch et al. (2007) | \(-20.78 \pm 0.21\) | \(0.9 \pm 0.3\) | \(-1.54 \pm 0.10\) |
| Iwata et al. (2007) | \(-21.28 \pm 0.38\) | \(0.4 \pm 0.3\) | \(-1.48 \pm 0.32\) |
| Yoshida et al. (2006) | \(-20.72 \pm 0.16\) | \(1.2 \pm 0.4\) | \(-1.82 \) (fixed) |
| Beckwith et al. (2006) | \(-20.55\) | \(0.9\) | \(-1.6\) (fixed) |
| Giavalisco (2005) | \(-21.06 \pm 0.05\) | \(0.83 \pm 0.03\) | \(-1.51 \pm 0.18\) |
| Iwata et al. (2003) | \(-21.4\) | \(0.4\) | \(-1.5\) |
| Ouchi et al. (2004) | \(-20.7 \pm 0.2\) | \(1.4 \pm 0.8\) | \(-1.6\) (fixed) |

### Table 11

Determinations of the best-fit Schechter parameters for the rest-frame UV LFs at \( z \approx 5 \).
Fig. 11.—Comparison between the present determination of the LF at $z \sim 6$ and other determinations in the literature. Included in these comparisons are the LFs by Dickinson et al. (2004; dashed light blue line), Bouwens et al. (2004b; dotted green line), Yan & Windhorst (2004b; solid magenta line), Bunker et al. (2004; solid blue line), and Malhotra et al. (2005; red dot-dashed line). For Beckwith et al. (2006), we present both the LF derived from a fit to the number counts (solid line) and that obtained by applying a simple offset to the counts (dotted black line). The present determination of the $z \sim 6$ LF is a slight refinement on our previous determination (B06) and includes $\sim 100$ additional i-dropouts identified over the two very deep HUDF05 fields (reaching to within 0.4 mag of the HUDF in the $\lambda_{350}$ band).

Finally, we discuss the UV LF at $z \sim 6$. Already, there have been quite a significant number of LF determinations at $z \sim 6$ (e.g., Dickinson et al. 2004; Bouwens et al. 2004b; Yan & Windhorst 2004b; Bunker et al. 2004; Malhotra et al. 2005; B06; Beckwith et al. 2006). See Figure 11 and Table 12 for these comparisons. Most of these determinations have been made using some combination of i-dropouts selected from the HUDF, HUDF-Ps, and GOODS data. Since almost all of these determinations have already received significant discussion in our $z \sim 6$ study, we only comment here on the two most recent determinations (B06; Beckwith et al. 2006). One of these determinations is our own and based on a slightly smaller data set (the B06 determination did not include the $\sim 100$ i-dropouts available over the second and third deepest i-dropout search fields: HUDF05-1 and HUDF05-2). In general, the present determination is in good agreement with the previous one (B06), although somewhat ($\sim 30\%$) lower in normalization. The latter change is consistent with our previous determination within the errors and

quality of the GOODS data set (which allowed us to very effectively pull out high-redshift galaxies from our photometric samples and to reject both stars and AGNs).

Having discussed previous LFs at $z \sim 4-5$ based on shallower data, we compare our LF determinations with those obtained by Beckwith et al. (2006) at $z \sim 4$ and $\sim 5$ using the HUDF data and Oesch et al. (2007) at $z \sim 5$ using the HUDF+HUDF05 data. We begin with the results of Oesch et al. (2007). Oesch et al. (2007) based their LFs on large $V$-dropout selections over the HUDF+HUDF05 fields and then combined their results with the Yoshida et al. (2006) results to derive best-fit Schechter parameters. Compared to our $z \sim 5$ LF results (which also take advantage of data from the GOODS, HUDF-Ps, and HUDF05-2 fields), the Oesch et al. (2007) LF appears to be in good overall agreement, albeit a little ($\sim 20\%$–$30\%$) lower at the faint end. These differences appear to be attributable to (1) the larger ($\sim 20\%$) contamination corrections made by Oesch et al. (2007) and (2) Oesch et al. (2007) not correcting their fluxes for the light lost on the wings of the PSF (typically a $\sim 0.1$–$0.25$ mag correction for the small Kron apertures appropriate for faint galaxies; Sirianni et al. 2005).

Beckwith et al. (2006) based their LFs on large $B$- and $i'$-dropout samples derived from the ACS HUDF and GOODS fields and used nearly identical selection criteria to those considered here. They also considered an LF fit that included several previous determinations (Steidel et al. 1999; Ouchi et al. 2004; Sawicki & Thompson 2006a) to demonstrate the robustness of their results. Their results are plotted in Figures 9 and 10 with the black crosses. Both LFs seem to be fairly similar to our own in their overall shape but appear to be shifted to slightly lower volume densities. At the faint end of the LF, this shift is the most prominent. After careful consideration of the Beckwith et al. (2006) results, it appears that this occurs because Beckwith et al. (2006) do not include the modest incompleteness (see Fig. 18 in Appendix A) that occurs at fainter magnitudes near the upper redshift end of the selection (i.e., $z \geq 4$ and $z \geq 5.2$) due to photometric scatter. In addition, at $z \sim 5$, the faint end of the Beckwith et al. (2006) LF is derived from the HUDF, which as we show in § B1 (see Table 14 there) is underdense in $V_{606}$-dropouts (see also Oesch et al. 2007). Since Beckwith et al. (2006) do not use an approach that is insensitive to field-to-field variations (e.g., STY79 or SWML), we would expect this underdensity in $z \sim 5$ $V$-dropouts in the HUDF to propagate directly into the Beckwith et al. (2006) LF and therefore the faint end of their $z \sim 5$ LF to be low. Together these two effects appear to account for the differences seen.

### 4.3.3. Comparison at $z \sim 6$

Finally, we discuss the UV LF at $z \sim 6$. Already, there have been quite a significant number of LF determinations at $z \sim 6$ (e.g., Dickinson et al. 2004; Bouwens et al. 2004b; Yan & Windhorst 2004b; Bunker et al. 2004; Malhotra et al. 2005; B06; Beckwith et al. 2006). See Figure 11 and Table 12 for these comparisons. Most of these determinations have been made using some combination of i-dropouts selected from the HUDF, HUDF-Ps, and GOODS data. Since almost all of these determinations have already received significant discussion in our $z \sim 6$ study (B06), we only comment here on the two most recent determinations (B06; Beckwith et al. 2006). One of these determinations is our own and based on a slightly smaller data set (the B06 determination did not include the $\sim 100$ i-dropouts available over the second and third deepest i-dropout search fields: HUDF05-1 and HUDF05-2). In general, the present determination is in good agreement with the previous one (B06), although somewhat ($\sim 30\%$) lower in normalization. The latter change is consistent with our previous determination within the errors and

### Table 12

**Determinations of the Best-Fit Schechter Parameters for the Rest-Frame UV LFs at $z \sim 6$**

| Reference | $M_{UV}$ | $\phi^* (10^{-3} \text{ Mpc}^{-3})$ | $\alpha$ |
|-----------|----------|-------------------------------|--------|
| This work.............................................. | $-20.24 \pm 0.19$ | $1.4^{+0.6}_{-0.4}$ | $-1.74 \pm 0.16$ |
| B06....................................................... | $-20.25 \pm 0.20$ | $2.0^{+0.9}_{-0.8}$ | $-1.73 \pm 0.21$ |
| Beckwith et al. (2006).......................... | $-20.5$ | $0.7$ | $-1.6$ (fixed) |
| Malhotra et al. (2005).......................... | $-20.83$ | $0.4$ | $-1.8$ (assumed) |
| Yan & Windhorst (2004)...................... | $-21.03$ | $0.5$ | $-1.8$ |
| Bunker et al. (2004)........................ | $-20.87^a$ | $0.2$ | $-1.6$ |
| Dickinson et al. (2004)........................ | $-19.87^a$ | $5.3$ | $-1.6$ (fixed) |
| Bouwens et al. (2004b)........................ | $-20.26$ | $1.7$ | $-1.15$ |

* Since the quoted LF was expressed in terms of the $z \sim 3$ LF (Steidel et al. 1999), which is at rest-frame 1700 Å, it was necessary to apply a $k$-correction ($\sim 0.2$ mag) to obtain the equivalent luminosity at 1350 Å to make a comparison with the other LFs given here.
occurred as a result of a lower surface density of dropouts in the two HUDF05 fields (see Tables 15 and 16 in Appendix B) and the different SED templates and opacity model we assume. We explore the effect of these assumptions on our LF results in Appendix B.

Beckwith et al. (2006) also made a determination of the UV LF at $z \sim 6$ using the same methodology they used at $z \sim 4$ and $\sim 5$. We consider the Beckwith et al. (2006) $z \sim 6$ determination obtained from the fit to their number counts (i.e., $M^* = -20.5$, $\phi^* = 0.0007$ Mpc$^{-3}$, $\alpha = -1.65$).\footnote{Beckwith et al. (2006) also presented a stepwise determination of the $z \sim 6$ LF obtained directly from the number counts assuming a distance modulus and selection volume. We do not make a comparison against that determination since the Beckwith et al. (2006) assumption of a simple distance modulus leads to substantial biases in the reported LF. Note the significant differences between the solid and dotted black lines in Fig. 11.} A comparison with both our previous (B06) and updated determination is provided in Figure 11. While the Beckwith et al. (2006) LF is in excellent agreement with the present determinations at bright magnitudes, at fainter magnitudes the Beckwith et al. (2006) LF is markedly lower ($\approx 2$ times) than our results. Why might this be? A comparison of the total number of galaxies in the Beckwith et al. (2006) HUDF catalog shows only $54\%$ as many sources as our catalog to the same faint limit and only $25\%$ as many sources over the interval $28.0 < z_{850,AB} < 28.7$ (Fig. 12). While one might imagine that the differences might be due to differing levels of incompleteness, Beckwith et al. (2006) estimate that only $\sim 35\%$ of the galaxies are missing at $28 < z_{850,AB} < 28.7$ (see Fig. 13 from Beckwith et al. 2006), which is much smaller than the $\sim 75\%$ we estimate empirically through a comparison with our counts.

What then is the probable cause for this discrepancy? We suspect that it is due to the systematic differences between the $z_{850}$-band photometry Beckwith et al. (2006) use to select their sample (which appears to come from the photometric catalog initially provided with the HUDF release since an application of the Beckwith et al. (2006) criteria to that catalog yields precisely the same set of $i$-dropouts as are found in their paper) and that used in our analysis, which as shown in the inset to Figure 12 are systematically brighter by $\sim 0.4$ mag near the HUDF magnitude limit (red crosses). Although such significant differences may be cause for concern, it is interesting to note that the $z_{850}$-band magnitudes provided by Beckwith et al. (2006) for $i$-dropouts in the HUDF (Table 8 from that work) are also typically $\sim 0.3$ mag brighter than that initially provided with the HUDF release (black crosses). So it would appear that Beckwith et al. (2006) quote different $z_{850}$-band magnitudes for $i$-dropouts in the HUDF than they initially provided with the HUDF release and that they used to select their $i$-dropout sample!

4.4. State of the LF at $z \sim 6$, $z \sim 5$, and $z \sim 4$

Not surprisingly, there has already been a great deal of discussion regarding how the UV LF evolves at high redshift ($z \sim 3$–6) based on previous determinations, with some studies arguing for an evolution in the faint-end slope ($\gamma$; Yan & Windhorst 2004b), some studies advocating an evolution in $\phi^*$ (Beckwith et al. 2006), other studies suggesting an evolution in the characteristic luminosity (B06; Yoshida et al. 2006), and yet other studies arguing for an evolution at the faint end of the LF (Iwata et al. 2003, 2007; Sawicki & Thompson 2006a).

In this paper we found strong evidence for (1) an increase in the characteristic luminosity $M^*$ as a function of cosmic time, from roughly $-20.2$ at $z \sim 6$ to roughly $-21.1$ at $z \sim 3$, and (2) a steep faint-end slope $\alpha \sim -1.7$ at $z \sim 4$–6. While this agrees with the evolution found by some groups (B06; Yoshida et al. 2006; M. Giavalisco et al. 2008, in preparation), it is in significant contradiction with others (Iwata et al. 2007; Sawicki & Thompson 2006a; Beckwith et al. 2006). We find it quite disturbing that there are a wide variety of different conclusions being drawn by different teams.\footnote{The diversity of conclusions drawn in high-redshift LF studies certainly illustrates how difficult it is to accurately control for systematics. Of course, one additional complicating factor is clearly the extremely steep faint-end slopes possessed by high-redshift LFs. This makes it very difficult to locate the "knee" in the LF and therefore distinguish evolution in $\phi^*$ from evolution in $M^*$.} However, we think that our large data set, unprecedented in both its size and leverage (both in redshift and luminosity), should allow us to come to more robust conclusions than have previously been obtained. We are encouraged by the fact that one of the most recent studies using the deep wide-area (636 arcmin$^2$) Subaru Deep Field (Yoshida et al. 2006) obtains similar values for $M^*$ and $\alpha$ to what we find at $z \sim 4$ and $\sim 5$ and derives almost essentially the same evolution in $M^*$ over this interval ($\sim 0.35$ mag).
Similar results are obtained by Ouchi et al. (2004) using somewhat shallower data over the Subaru Deep Field and by M. Giavalisco et al. (2008, in preparation) using an independent analysis of the HUDF+GOODS data.

One of the most notable of several previous studies to differ from the present conclusions is that conducted by Beckwith et al. (2006). The Beckwith et al. (2006) analysis is notable because while Beckwith et al. (2006) use a very similar data set to own (our data set also includes four deep intermediate depth ACS fields, i.e., the two HUDF05 and two HUDF-Ps fields), they arrive at significantly different conclusions from our own. Beckwith et al. (2006) argue that the evolution in the UV LFs at $z \sim 4-6$ can be most easily explained through an evolution in $\phi^*$ and cannot be explained through an evolution in $M_*$. What could be the cause of these different conclusions? After a careful analysis of the Beckwith et al. (2006) results, we have three significant comments. First of all, Beckwith et al. (2006) determine their LFs using the surface density of galaxies binned according to their flux in passbands affected by absorption from the Ly$_\alpha$ forest (i.e., $V_{606}$ for their $z \sim 4$ LF, $i_{775}$ for their $z \sim 5$ LF, and $z_{850}$ for their $z \sim 6$ LF). This is worrisome since the Ly$_\alpha$ forest absorption is quite sensitive to the redshift of the sources, and therefore any systematic errors in the model redshift distributions (or forest absorption model) will propagate into the luminosities used for deriving their LFs. While we understand that Beckwith et al. (2006) used this procedure to determine the LF at $z \sim 4$, $\sim 5$, and $\sim 6$ in a self-consistent way, in doing so they have introduced unnecessary uncertainties into these determinations at $z \sim 4$ and $\sim 5$. These LFs can be derived from UV continuum fluxes not subject to these uncertainties.

Second, the value of $M^*$ that Beckwith et al. (2006) derive at $z \sim 4$ (alternatively quoted as $-20.3$, $-20.5$, and $-20.7$ depending on the fitting procedure) is significantly fainter than the values (i.e., $M^* \lesssim -21.0$) that have been derived in previous studies (Steidel et al. 1999; Sawicki & Thompson 2006a; Paltani et al. 2007; see Table 10). While these differences will partially result from Beckwith et al. (2006) determining the LF at $z \sim 1400$ Å ($L^*$ galaxies at $z \sim 4$ are somewhat redder in their UV continuum slopes $\beta$ than $-2.0$ and thus yield fainter values of $M^*$ at 1400 Å than they do at 1600 Å; § 3B), probably the biggest reason for these differences is one of procedure. Beckwith et al. (2006) derive their LFs using the surface density of dropouts binned in terms of the flux in bands affected by Lyman forest absorption ($\sim 0.2-1.0$ mag), while other analyses use UV continuum fluxes where this absorption has no effect. As discussed in the paragraph above, analyses that are much less sensitive to modeling this absorption would seem to be more reliable than those which are more sensitive. If the value of $M^*$ in the Beckwith et al. (2006) analysis is systematically too faint (and $\phi^*$ too high) for these reasons, this would shift the evolution from $M^*$ (which is what we believe the data suggest) to $\phi^*$ (which is what Beckwith et al. [2006] report).

Third, at $z \sim 6$, we disagree with the value of $\phi^*$ and $M^*$ obtained by Beckwith et al. (2006). Our basic disagreement hinges on the assessment we made of the Beckwith et al. (2006) HUDF i-dropout selection at faint magnitudes ($28 < z_{850, AB} < 28.7$; see § 4.3.3 and Fig. 12) and our suspicion that this selection may be somewhat incomplete due to a flux bias (Fig. 12). If indeed this incompleteness was not properly accounted for in the Beckwith et al. (2006) analysis, it would effectively lower their value of $\phi^*$ and brighten $M^*$. Again, this would shift the evolution in the LF from $M^*$ to $\phi^*$.

5. DISCUSSION

The unprecedented depth and size of current $B_*$, $V_*$, and $i$-dropout samples, along with the great experience represented in the previously determined LFs from the literature, have enabled us to establish what we think are the most robust $z \sim 4$, $\sim 5$, and $\sim 6$ LFs to date. These LFs extend significantly fainter than has been possible in all previous efforts that have not included the ultradeep HUDF data, providing us with unique leverage for constraining the evolution at the faint end of the LF. These deep LFs put us in a strong position to discuss a number of issues that are of current interest in studies of galaxy evolution.

5.1. Evolution of the Rest-Frame UV LF

Having established the evolution of the LF from $z \sim 6$ to $\sim 4$, it is interesting to compare this evolution with that found at lower redshifts (Steidel et al. 1999; Arnouts et al. 2005; Wyder et al. 2005). We look at this evolution in terms of the three Schechter parameters $\phi^*$, $M^*$, and $\alpha$ (Figs. 13 and 14). This may give us some clue as to the physical mechanisms that are likely to be at work in global evolution of the galaxy population. The clearest trend seems to be present in the evolution of $M^*$, which brightens rapidly at early times, reaches a peak around $z \sim 4$, and then fades

![Figure 13](image-url)
Over most of the redshift range $z \sim 0–6$ probed by current LF determinations, we observe no significant evolution in the normalization $\phi^*$ and only a modest amount of evolution in the faint-end slope $\alpha$. The evolution in $\phi^*$ and $\alpha$ becomes more substantial at the lowest redshifts being probed here, as $\phi^*$ evolves from $10^{-3}$ Mpc$^{-3}$ at $z \sim 1–6$ to $4 \times 10^{-2}$ Mpc$^{-3}$ at $z \sim 0$ (Wyder et al. 2005) and $\alpha$ evolves from $-1.74$ at $z \sim 4$ to roughly $-1.2$ at $z \sim 0$ (Wyder et al. 2005). Broadly, we expect some flattening of the faint-end slope $\alpha$ with cosmic time to match that predicted for the halo mass function. We would also expect $\phi^*$ to be somewhat higher at early times to account for the large population of lower luminosity galaxies predicted to be present then. At late times, we expect the value of $\phi^*$ to increase to compensate for the evolution in $M^*$ and thus keep the population of lower luminosity galaxies (which appear to evolve more slowly with cosmic time; e.g., Noeske et al. 2007) more constant. While we observe this increase in $\phi^*$ at late times, it is unclear at present whether $\phi^*$ is really higher at very early times ($z \gtrsim 6$). Progress on this question should be possible from ongoing searches for galaxies at $z \gtrsim 7$ (e.g., Bouwens & Illingworth 2006; Mannucci et al. 2007; Stark et al. 2007b).

5.2. Interpreting the Observed Evolution in $M^*$

We have already remarked that one probable interpretation for the observed brightening in $M^*$ is through the hierarchical coalescence and merging of galaxies into larger halos. We can look at the hypothesis in detail by comparing the observed brightening with the mass buildup seen in the halo mass function (Sheth & Tormen 1999) over this range. We assume that we can characterize the growth in the mass function by looking at the mass of halos with a fixed comoving volume density of $10^{-2.5}$ Mpc$^{-3}$ and that there is a fixed conversion from mass to UV light (halo mass to apparent SFR). A volume density of $10^{-2.5}$ Mpc$^{-3}$ corresponds to that expected for halos near the knee of the LF assuming a duty cycle of $\sim 25\%$ (see Stark et al. 2007c; Verma et al. 2007) and $\phi^*$ of $10^{-3}$ Mpc$^{-3}$, which is the approximate volume density of $L^*$ galaxies in the observations. The duty cycle tells us the approximate fraction of halos that have lit up with star formation at any given point in time. This analysis effectively assumes that $\phi^*$ is fixed as a function of time, which we assume to match the observations (Fig. 14). We plot the predicted brightening in Figure 13 with the solid line. We note that these predictions are only modestly sensitive to the volume densities chosen to make these comparisons. At volume densities of $\sim 10^{-2.5}$ Mpc$^{-3}$, the predicted brightening is 0.6 mag from $z \sim 6$ to $\sim 4$, while at $\sim 10^{-3}$ Mpc$^{-3}$, the predicted brightening is 0.9 mag. Surprisingly, the growth in the mass function is in striking agreement with the evolution we observe in $M^*$, even out to $z \sim 7.4$ where we derive our values of $M^*$ from the Bouwens & Illingworth (2006) search results (see § 5.4). This remarkable agreement strongly suggests that hierarchical buildup may contribute significantly to the evolution we observe.

While this is surely an interesting finding in itself, the overall level of agreement we observe here is surprising since we make a fairly simple set of assumptions about the relationship between the halo mass and the UV light in galaxies hosted by these halos, supposing that it is constant and nonevolving. Had we assumed that this ratio evolves with cosmic time, we would have made considerably different predictions for the evolution of the LF. This is interesting since there are many reasons for thinking that the mass-to-light ratio might be lower at early times and therefore that the evolution in $M^*$ might be less rapid with cosmic time. For one, the efficiency of star formation is expected to be higher at early times. The universe would have a higher mean density then,
and therefore the gas densities and SFR efficiencies should be higher. In addition, the cooling times and dynamical times should be less at early times. All this suggests that the evolution in the LF should much more closely resemble that predicted by Stark et al. (2007c), who also model the evolution in the LF using the mass function but assume that the star formation timescales evolve as $H(z)^{-1} \sim (1+z)^{-3/2}$. As a result of these star formation timescales, the Stark et al. (2007c) model predicts a mass-to-light ratio that evolves as $\sim(1+z)^{-3/2}$. This model yields significantly different predictions for how $M^*$ evolves with redshift (shown as the dot-dashed line in Fig. 13). The latter predictions appear to fit our data somewhat less well than for the simple toy model we adopted above assuming no evolution in the mass-to-light ratio. This suggests that this mass-to-light ratio may not evolve that dramatically with cosmic time. One possible explanation for this would be if SN feedback played a significant role in regulating the star formation within galaxies at these times, keeping it from reaching the rates theoretically achievable given the timescales and gas densities expected. Of course, while it is interesting to note the possible physical implications of our observational results, we should be cautious about drawing too strong of conclusions based on these comparisons. Our treatment here is crude, and the observational uncertainties are still quite large.

5.3. Comparisons with Model Results

Given the success of our simple toy model for reproducing the observed evolution in $M^*$, it is interesting to ask if this success is maintained if we consider more sophisticated treatments like those developed in the literature (Finlator et al. 2006; Oppenheimer & Davé 2006; Nagamine et al. 2004; Night et al. 2006; Samui et al. 2007). The most complicated of these models include a wide variety of physics from gravitation to hydrodynamics, shocks, cooling, star formation, chemical evolution, and SN feedback (see, e.g., Springel & Hernquist 2003). We examined two different models produced by leading teams in this field and which we suspect are fairly representative of current work in this area. These models are the momentum-driven wind “vzzw” model of Oppenheimer & Davé (2006) and the model of Night et al. (2006), which appears to be similar to the constant wind model of Oppenheimer & Davé (2006). Since LFs in these models more closely resemble power laws in overall shape than they do Schechter functions, we were not able to extract a unique value of $M^*$ from the model LFs. We were, however, able to estimate an evolution in $M^*$ by comparing the model LFs at a fixed number density and looking at the change in magnitude. In doing so, we effectively assume that the value of $\phi'$ is fixed just like we find in the observations (Fig. 14). To improve the S/N with which we measure this evolution from the models, we looked at this evolution over a range of number densities (i.e., $10^{-3.2}$ to $10^{-1.5}$ Mpc$^{-3}$). We plot the derived evolution from these models in Figure 13, and it is apparent that our observed evolution is in good agreement with the momentum-driven wind models of Oppenheimer & Davé (2006) but exceeds that predicted by the Night et al. (2006) model. The fact that our results agree with at least one of the two models is encouraging, since it suggests that the evolution we infer is plausible. Moreover, the fact that the two model results disagree suggests that we may be able to begin to use our observational results to begin constraining important aspects of the theoretical models. Particularly relevant on this front are the implications for the feedback prescription, which differ quite significantly between the two models considered here. For the momentum-driven wind models, feedback is much more important at early times than it is for the Night et al. (2006) model. This feedback effectively suppresses star formation at early times and therefore results in a much more rapid brightening of $M^*$ with cosmic time, in agreement with the observations.

5.4. Evolution of UV Luminosity at $z > 6$

The present determinations of the LF at $z \sim 4$–6 should provide us with a useful guide to the form of the LF at even earlier times and should be helpful in interpreting current searches for very high redshift ($z > 6$) galaxies. Currently, the most accessible regime for such probes lies just beyond $z\sim 6$, at $z \sim 7$–8, and can be probed by a $z$-dropout search. At present, the most comprehensive such search was performed by our team using $\sim$19 arcmin$^2$ of deep NICMOS data over the two GOODS fields (Bouwens & Illingworth 2006; see also Mannucci et al. 2007). In that work, we applied a very conservative ($z_{850} - J_{110} \sim 1.3$, ($z_{850} - J_{110} > 1.3 \pm 0.4(J_{110} - H_{160})_{AB}$), ($J_{110} - H_{160}$)$_{AB} < 1.2$ $z_{850}$-dropout criterion to those data and found only one plausible $z$-dropout, but we expected $\sim 10$ sources assuming no evolution from $z \sim 6$. We also applied a slightly less conservative $z$-dropout criterion and found three other possible candidates. From this, we concluded that the volume density of bright ($\gtrsim 3.6 z_{\text{mag}}$) galaxies at $z \sim 7.4$ was just $0.1_{-0.07}^{+0.29}$ (and $2.4_{-0.12}^{+0.20}$ times the volume density of bright sources at $z \sim 6$ for our conservative and less conservative criteria, respectively. Both large-scale structure and Poissonian statistics are included in the estimated errors here. For both selections, the result was significant and suggested to us that there was substantial evolution from $z \sim 7$–8 to $z \sim 6$. Given the sizeable evolution we had observed in $M^*$ between $z \sim 6$ and $z \sim 3$ (B06; see also Dickinson et al. 2004), it made sense for us to model our $z \sim 7$–8 search results in terms of an evolution of $M^*$, keeping $\phi'$ and $\alpha$ fixed. We also considered a model where changes in $M^*$ were offset by changes in $\phi'$ such as to keep the total luminosity density fixed. Using these two sets of assumptions, we estimated that $M^*$ was $1.1 \pm 0.4$ and $1.4 \pm 0.4$ mag fainter at $z \sim 7.4$ than it was at $z \sim 6$.

With our current work on the LFs at $z \sim 4$–6, we have been able to demonstrate more clearly than before that the most significant change in the LF occurs through a brightening of $M^*$ from $z \sim 4$ to $z \sim 3$ (see Yoshida et al. 2006). This strengthens the underlying motivations behind the Bouwens & Illingworth (2006) decision to model the evolution of the LF in terms of a change in $M^*$. The parameter $\phi'$ is consistent with being constant, although it may also decrease with time, as suggested by hierarchical buildup. Unfortunately, there are still too many uncertainties in the data to be sure about the trends in $\phi'$, and so it is difficult to significantly improve on the $M^*$ estimates made in Bouwens & Illingworth (2006) for our most conservative selection.

Nevertheless, we update our estimates for $M^*$ at $z \sim 7$–8 based on our conservative selection to be consistent with the present determinations for $\phi'$ and $\alpha$ at $z \sim 6$ while taking the evolution in the UV LF at $z \gtrsim 6$ to simply be in luminosity ($M^*$). With these assumptions (i.e., $\alpha = -1.74$ and $\phi' = 0.0014$ Mpc$^{-3}$), we find a value of $M^*_{1700} = -19.3 \pm 0.4$ for our UV LF at $z \sim 7$–8. It also makes sense to estimate the value of $M^*$ at $z \sim 7$–8 using the results of the less conservative selection of Bouwens & Illingworth (2006). We did not consider this selection in our original estimates of $M^*$ in Bouwens & Illingworth (2006) to avoid possible concerns about contamination and thus simplify the discussion. However, the contamination is not likely to be larger than 25% (see Bouwens & Illingworth 2006), and this selection offers much better statistics than for our conservative selection (four sources vs. one source), as well as a larger selection window that should make our selection volume estimates more reliable. Repeating the determination of $M^*$ using the results of our less conservative selection [$\rho(z = 7.4)/\rho(z = 6) = 0.24^{+0.20}_{-0.12}$] and assuming
simple evolution in $M^*$, we find $M^*_{UV} = -19.7 \pm 0.3$. The normal-ization $\phi^+$ and faint-end slope $\alpha$ were kept fixed at $1.4 \times 10^{-3} \text{ Mpc}^{-3}$ and $-1.74$, the values preferred at $z \sim 6$, for this modeling. Although it seems probable that the faint-end slope $\alpha$ may be quite steep at earlier times, this does not have a big effect on the derived values for $\phi^+$ and $M^*$. For example, making a $\Delta \alpha = 0.4$ change in the assumed faint-end slope only results in a 0.1 mag change in $M^*$. We added this determination of $M^*$ to Figure 13 as an open red circle, and it is in remarkable agreement with some of the theoretical predictions, as well as simple extrapolations of our lower redshift results ($\S$5.1–5.3). We include the Bouwens & Illingworth (2006) search results in Figure 15 along with a comparison with the LFs at $z \sim 4–6$. The Mannucci et al. (2007) search results for very luminous (brighter than $-21.5$ mag) $z \sim 7$ galaxies are also included in this figure.

5.5. Reionization

Finally, it seems worthwhile to discuss the implications of the current LF determination on the ionizing flux output of $z \geq 4$ galaxies. There has been a great deal of interest in the ionizing radiation output of high-redshift galaxies since it was discovered that hydrogen remains almost entirely ionized since a redshift of $z \sim 6$ (Becker et al. 2001; Fan et al. 2002, 2006; White et al. 2003) and that galaxies are the only obvious candidates to produce this radiation. The situation has even become more interesting now with the availability of the WMAP results, indicating that the universe may have been largely ionized out to redshifts as early as $10.9^{+3.3}_{-3.3}$ (Spergel et al. 2007; Page et al. 2007; cf. Shull & Venkatesan 2007).

Yet, despite galaxies being the only obvious source of ionizing photons at high redshift, there has been some controversy about the ability of galaxies to keep the universe reionized at high redshift. Much of the controversy has centered around the fact that the escape fraction is observed to be very low for galaxies at $z \sim 0–3$ (Leitherer et al. 1995; Hurwitz et al. 1997; Deharveng et al. 2001; Giallongo et al. 2002; Fernández-Soto et al. 2003; Malkan et al. 2003; Inoue et al. 2005; Shapley et al. 2006; cf. Steidel et al. 2001), and therefore while high-mass stars in galaxies may be efficient producers of ionizing photons, only a small fraction of these photons succeed in making it out into the intergalactic medium (IGM). This has led some researchers to question whether high-redshift galaxies are even capable of keeping the universe ionized (e.g., Stanway et al. 2003; Bunker et al. 2004). We must emphasize, however, that the escape fraction is still relatively poorly understood and that the true value may still be quite appreciable (e.g., Shapley et al. 2006).

Fortunately, it appears that there may be several ways of resolving this situation, even for relatively low values of the escape fraction. One of these is to suppose that the traditional assumptions about the IGM are not quite right and that one should use a smaller value for the clumping factor (e.g., Bolton & Haehnelt 2007; Sokasian et al. 2003; Iliev et al. 2006; Sawicki & Thompson 2006b) or higher temperature for the IGM (e.g., Shull & Venkatesan 2004) than has been assumed in many previous analyses of the ionization balance (i.e., Madau et al. 1999). Another possible solution is to suppose that there has been a change in the metallicities or IMF of stars at early times, such that these objects have a much higher ionizing efficiency than sources at lower redshift (Stiavelli et al. 2004). One final solution has been to assume a significant contribution to the ionizing flux from very low luminosity galaxies (e.g., Lehner & Bremer 2003; Yan & Windhorst 2004a, 2004b; B06).

The present determination of the LFs at $z \sim 4–6$ and, in particular, the steep faint-end slopes $\alpha = -1.73 \pm 0.05$ ($z \sim 4$), $\alpha = -1.66 \pm 0.09$ ($z \sim 5$), and $\alpha = -1.74 \pm 0.16$ ($z \sim 6$) provide significant support for the idea that lower luminosity galaxies contribute significantly to the total ionizing flux (see also Beckwith et al. 2006). Previously, there was some support for the idea that lower luminosity galaxies may have been important from the steep faint-end slopes obtained at $z \sim 6$ (B06; Yan & Windhorst 2004b) and at lower redshift (e.g., Steidel et al. 1999; Arnouts et al. 2005; Yoshida et al. 2006). However, this conclusion was a little uncertain due to the sizeable uncertainties on the faint-end slope $\alpha$ at $z \sim 6$, as well as some conflicting results at lower redshift (Gabasch et al. 2004; Sawicki & Thompson 2006a). Now, with the present LF determinations (see also Yoshida et al. 2006; Beckwith et al. 2006; Oesch et al. 2007), it seems quite clear that the faint-end slope $\alpha$ must be quite steep (i.e., roughly $-1.7$) at $z \sim 4$, although it is still difficult to evaluate whether this slope evolves from $z \sim 6$ to $\sim 4$ due to considerable uncertainties on this slope at $z \sim 6$.

We can use the stepwise LF at $z \sim 4–5$, and $\sim 6$ to look at the contribution that galaxies of various luminosities make to the total ionizing flux. Assuming a luminosity-independent escape fraction, we can examine this contribution by plotting the UV luminosity densities provided by galaxies at different absolute magnitudes (Fig. 16). Clearly, the lower luminosity galaxies provide a sizeable fraction of the total.

What fraction of the total flux would be provided by galaxies faintward of the current observational limits ($\sim 16$ mag), assuming that the present LFs can be extrapolated to very faint levels? With no cutoff in the LF, this fraction is $0.31$, $0.27$, and $0.40$ for our $z \sim 4$, $\sim 5$, and $\sim 6$ LFs, respectively (from Table 7). However, for the more physically reasonable situation that the LF has a cutoff (at a fiducial limit of $\sim 10$ mag; e.g., Read et al. 2006: see § 3.5), the fraction is $0.27$, $0.24$, and $0.34$, respectively. In all cases, this fraction is substantial and suggests that a significant fraction of the total ionizing flux may come from galaxies at very low luminosities. In fact, even if we suppose that our high-redshift LFs cut off just below the observational limit of our HUDF selection (i.e., $\sim 16$ mag), $\geq 50\%$ of the total ionizing flux would still arise from galaxies fainter than $\sim 19$ mag. Since $\sim 19$ mag is comparable to or fainter than the observational limits relevant for
most previous studies of high-redshift galaxies (i.e., Figs. 9 and 10), this shows that most previous studies do not come close to providing a complete census of the total UV light or ionizing radiation at high redshift. Ultradeep probes (such as are available in the HUDF) are necessary.

6. SUMMARY

Over its years in operation, the HST ACS has provided us with an exceptional resource of ultradeep, wide-area, multiwavelength optical (B/Vi) data for studying star-forming galaxies at high redshift. Such galaxies can be effectively identified in these multiwavelength data using a dropout criterion, with B−, Vi−, and i−dropout selections probing galaxies at a mean redshift of z ~ 3.8, ~5, and ~5.9, respectively. Relative to previous observations, deep ACS data reach several times fainter than ever before and do so over large areas. This allows us to investigate the properties of high-redshift star-forming galaxies at extremely low luminosities in unprecedented detail.

Here we have taken advantage of the historic sample of deep, wide-area ACS fields (HUDF, HUDF05, HUDF−Ps, and the two GOODS fields) to identify large, comprehensive selections of very faint, high-redshift galaxies. Our collective sample of B−, Vi−, and i−dropouts over these fields totalled 4671, 1416, and 627 unique sources, respectively. Putting together our deepest probe (HUDF) with our widest area probe (GOODS), our samples cover a 6−7 mag range with good statistics (factor of ~1000 in luminosity), extending from −23 mag to −16 or −17 mag. Through detailed simulations, we have carefully modeled the completeness, photometric scatter, contamination, and selection functions for each of our samples. We then put together the information from our combined sample of B−, Vi−, and i−dropouts to derive LFs at z ~ 4, ~5, and ~6. To ensure that our LF determinations are robust, we considered a wide variety of approaches and assumptions in the determinations of these LFs and made extensive comparisons with other determinations from the literature. Here are our principal conclusions:

1. Best-fit LFs.—We find that the rest-frame UV LFs at z ~ 4−6 are well fitted by a Schechter function over a ~5−7 mag (factor of ~100−1000) range in luminosity, from −23 to −16 mag (see also Beckwith et al. 2006). The best-fit parameters for our rest-frame UV LFs are given in Table 13. The present z ~ 6 LF determination is in reasonable agreement with those from B06 (see Table 12) but is slightly more robust at the faint end. The most salient finding from the individual LF determinations is that the faint-end slope α is very steep (roughly −1.7) at all redshifts considered here (see § 3.4).

2. Completeness of z ~ 4 B-dropout census.—The bulk of the bright B-dropouts we identify over the GOODS have β-values of ~1−0.5 (§ 4.1; see Fig. 8). Since our z ~ 4 B-dropout selection should be effective in identifying UV-bright galaxies as red as β ~ 0.5, the fact that we do not find many such galaxies in our selection in the range from β ~ −0.5 to ~0.5 suggests that this selection is largely complete (≥90%) at bright magnitudes. This supposition would appear to be supported by complementary selections of galaxies at z ~ 4 with the Balmer break technique (Brammer & van Dokkum 2007), which also find that galaxies have very blue UV continuum slopes (≥90% of the galaxies in the Brammer & van Dokkum [2007] selection had β-values ≤0.5). Since Balmer break selections do not depend on the value of the UV continuum slope, this again suggests that the bulk of the star-forming galaxy population at z ~ 4 is quite blue and will not be missed from our bright B-dropout selection.

3. Evolution of the LF.—Comparing our best-fit Schechter parameters determined at z ~ 6, ~5, and ~4, we find little evidence for evolution in the faint-end slope α or φ* from z ~ 6 to ~4. On the other hand, the characteristic luminosity for galaxies M_{UV} brightens by ~0.7 mag from z ~ 6 to ~4 (see also Yoshida et al. 2006).

### Table 13

**Summary of Key Results**

| Dropout Sample | (z) | M_{UV}^{b} | φ* | α | L > 0.3L_{B} | L > 0.04L_{B} | L > 0.3L_{C} | L > 0.04L_{C} |
|----------------|-----|-----------|----|----|------------|------------|------------|------------|
| B……………….. | ~3.8 | −20.98 ± 0.10 | 1.3 ± 0.2 | −1.73 ± 0.05 | −1.81 ± 0.05 | −1.48 ± 0.05 | −1.38 ± 0.05 | −1.05 ± 0.05 |
| V……………….. | ~5.0 | −20.64 ± 0.13 | 1.0 ± 0.3 | −1.66 ± 0.09 | −2.15 ± 0.06 | −1.78 ± 0.06 | −1.85 ± 0.06 | −1.48 ± 0.06 |
| I……………….. | ~5.9 | −20.24 ± 0.19 | 1.4 ± 0.6 | −1.74 ± 0.16 | −2.31 ± 0.08 | −1.83 ± 0.08 | −2.14 ± 0.08 | −1.65 ± 0.08 |
| z……………….. | ~7.4 | −19.3 ± 0.4 | (1.4) | (−1.74) | −3.15 ± 0.48 | −2.32 | −2.97 ± 0.48 | −2.14 |

Notes.—These LF determinations are based on the STY’79 technique, including evolution in M* across the redshift window of each sample (see Table 7 and § B8). They therefore differ from those given in Table 4, which do not include evolution.

* Values of M_{UV} are at 1600 Å for our B− and V−dropout samples, at ~1350 Å for our i−dropout sample, and at ~1900 Å for our z−dropout sample. Since z ~ 6 galaxies are blue (i ~ −2; Stanway et al. 2005; B06), we expect the value of M* at z ~ 6 to be very similar (±0.1 mag) at 1600 Å to the value of M* at 1350 Å. Similarly, we expect M* at z ~ 7−8 to be fairly similar at ~1600 Å to the value at ~1900 Å.
4. **UV luminosity/SFR densities.**—The UV luminosity densities and SFR densities we infer at \(z \sim 4, 5\), and \(\sim 6\) are summarized in Table 13. The UV luminosity density we derive at \(z \sim 6\) is modestly lower (0.45 \pm 0.09 times) than that at \(z \sim 4\) (integrated to \(-17.5\) mag). Taking into account the likely evolution in dust properties of galaxies across this interval suggested by the apparent change in mean UV continuum slope (e.g., B06), we infer a much more significant change in the dust-corrected SFR densities over this same interval of cosmic time; i.e., the SFR density at \(z \sim 6\) appears to be just \(-0.3\) times this density at \(z \sim 4\) (integrated to \(-17.5\) mag).

5. **Galaxies at \(z \sim 7-8\).**—By quantifying the evolution of the UV LF from \(z \sim 6\) to \(\sim 3\), we were able to better interpret the results of recent \(z_{850}\)-dropout searches of Bouwens & Illingworth (2006) in terms of an evolution of the LF (see § 5.4). Supposing that the evolution of the UV LF is simply in \(M^*\) (as observed from \(z \sim 6\) to \(\sim 4\)), we estimated that \(M^*_{UV}\) at \(z \sim 7.4\) was equal to \(-19.3 \pm 0.4\) and \(-19.7 \pm 0.3\) mag using the conservative and less conservative search results of Bouwens & Illingworth (2006), respectively (see § 5.4).

6. **Comparison with models.**—The brightening we observe in \(M^*\) from \(z \sim 6\) to \(\sim 4\) (and plausibly from \(z \sim 7.4\)) is almost identical to what one finds in the evolution of the halo mass function over this range (see also Stark et al. 2007c) assuming a constant proportionality between mass and light (see § 5.2). This suggests that hierarchical buildup largely drives the evolution in \(M^*\) over the redshift range probed by our samples. It also may indicate that there is no substantial evolution in the ratio of halo mass to UV light over this range. Since we might expect this ratio to evolve significantly due to changes in the mean gas density of the universe and therefore star formation efficiency, this suggests that feedback may be quite important in regulating the star formation of galaxies at early times. Of course, given the considerable uncertainties in the value of \(M^*\) at very high redshift (\(z \geq 6\)), it seems worthwhile to emphasize that these conclusions are still somewhat preliminary. Our observational results are also in reasonable agreement with that predicted by the momentum-driven wind models of Oppenheimer & Davé (2006).

7. **Implications for reionization.**—The very steep faint-end slopes \(\alpha\) of the UV continuum LF (roughly \(-1.7\)) suggest that lower luminosity galaxies provide a significant fraction of the total ionizing flux at \(z \gtrsim 4\) (see also discussion in Lehner & Bremer 2003; Yan & Windhorst 2004a, 2004b; B06; Sawicki & Thompson 2006b). Assuming that the escape fraction is independent of luminosity and that the high-redshift LFs maintain a Schechter-like form to a very faint fiducial limit (\(-10\) mag) and cut off beyond this limit, we estimate that 27\%, 24\%, and 34\% of the total flux comes from galaxies faintward of \(-16\) mag for our \(z \sim 4, 5\), and \(\sim 6\) LFs, respectively (see § 5.5).

The recent failure of the ACS aboard HST is a great loss for studies of galaxies. Even with the installation of WFC3, future HST observations will require approximately three times the telescope time that ACS required to obtain comparable constraints on the faint, \(z \sim 4-6\) population. As a result, it would appear that for the near to distant future the current probes of the UV LF at very high redshift will remain an important standard, until future facilities with superior surveying capabilities like the James Webb Space Telescope come online (or unless ACS is repaired).

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**APPENDIX A**

**LF DETERMINATIONS**

**A1. MODELING INCOMPLETENESS AND PHOTOMETRIC SCATTER**

To compare the expectations of the model LFs with the surface densities of dropouts observed in the different fields under study here, we need to be able to include the effect of incompleteness and photometric scatter in our calculations. We accomplish this by computing corrections that transform the surface density of dropouts from that recoverable in noise-free (infinite S/N) data to that recoverable in each of the fields considered in our study. We employ a two-part strategy: (1) deriving corrections necessary to transform the dropout surface densities from what we would recover for noise-free data to that recoverable in our HUDF selections, and (2) deriving corrections to transform these surface densities from HUDF depth data to that recoverable in even shallower data. Our use of a two-part strategy enables us to ensure that the corrections we derive for the shallower selections are extremely model independent (the most notable corrections being derived from degradation experiments).

Both corrections are implemented using a set of transfer functions, which correct the surface density of dropouts recoverable in deeper data to that recoverable in shallower data. We express these transfer functions as two-dimensional matrices, with the rows and the columns of these matrices indicating specific magnitude bins in the deeper and shallower data, respectively. Elements in these matrices indicate the fraction of galaxies with specific magnitudes in the deeper data recovered to have some other magnitude in the shallower data (see below). These transfer functions can then be applied to the surface density of dropouts in a given field, expressed as one-dimensional vectors, through simple matrix multiplication. For our \(B\)- and \(V\)-dropout selections, the axes of these matrices are given in terms of the \(i_{775}\)- and \(z_{850}\)-band magnitudes, respectively. These bands most closely correspond to flux at an approximately constant rest-frame wavelength (1600 Å) at the mean redshift of our samples (\(z \sim 3.8\) and \(\sim 5\), respectively) and are not affected by attenuation from the \(\text{Ly}\alpha\) forest. For our \(i\)-dropout selections, we express these transfer functions in terms of the total magnitude in the \(z_{850}\) band, which corresponds to rest-frame 1350 Å.
As noted, our first set of corrections is designed to correct the surface density of dropouts from what we would recover with noiseless (infinite depth) data to what we would recover in our HUDF selections. We restrict these corrections to a modeling of the flux biases and photometric scatter, since completeness is handled separately using a separate factor $P(m, z)$ (see eq. [2] in § 3.1). Modeling this scatter is important because of the tendency for fainter, lower significance sources to scatter into our selection through a Malmquist-like effect. To quantify this effect, we ran a series of simulations where we took $B$-dropout galaxies from the HUDF, artificially redshifted them across the redshift selection windows of our samples using our well-tested cloning software (Bouwens et al. 1998a, 1998b, 2003a), measured their photometry off of the simulated frames, and finally reselected these sources using our dropout criteria. By comparing the input magnitudes with those recovered, we were able to construct the transfer functions, which successfully incorporated the photometric scatter present in the real data. The assumptions we use in these simulations (e.g., size-redshift scalings, colors) are the same as those given in § A3.

Now we derive corrections to transform selections made with the HUDF data to similar selections made with shallower data. We accomplish this through a straightforward procedure, degrading the HUDF data to the depths of our shallower data and then repeating our selection and photometry at both depths. We perform these experiments for all three dropout samples and between the HUDF and all of our shallower fields (GOODS, HUDF-Ps, HUDF05). Again, we express the results of these experiments as transfer functions, which successfully incorporated the photometric scatter present in the real data. The assumptions we use in these simulations (e.g., size-redshift scalings, colors) are the same as those given in § A3.

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A2. EVALUATING THE LIKELIHOOD OF MODEL LFs

In this paper (§ 3) we evaluate candidate LFs by comparing the predicted dropout counts from these LFs with those found in our different fields. We compute the dropout counts from the LFs using a two-step procedure: first calculating the number of galaxies we would expect in our deepest selection the HUDF using equation (2), and then correcting this for photometric scatter and incompleteness using the transfer functions we derived in § A1.

To perform the integral in equation (2), we recast it in discrete form

$$\Sigma_k \phi_h V_{m,k} = N_m,$$  \hspace{1cm} (A1)
where $N_m$ is the number counts binned in 0.1 mag intervals $\int_{m-0.05}^{m+0.05} N(m') dm'$, $\Sigma \phi_k W(M - M_k)$ is the LF binned on 0.1 mag intervals, and $V_{m,k}$ is an effective volume-type kernel that can be used to calculate the number counts $N_m$ given some LF. It is calculated as

$$V_{m,k} = \int_z \int_m^{m+0.05} W[M(m', z) - M_k]P(m', z) \frac{dV}{dz} dm' dz,$$

where

$$W(x) = \begin{cases} 0, & x < -0.05, \\ 1, & -0.05 < x < 0.05, \\ 0, & x > 0.05. \end{cases}$$

Because of the minimal $k$-correction required in using the $i_{775}$-band fluxes of $z \sim 4$ B-dropouts to derive luminosities at rest-frame $\sim$1600 Å and in using the $z_{850}$-band fluxes of $z \sim 5$ $V$-dropouts to derive luminosities at $\sim$1600 Å (no Lyman forest absorption to consider), there is a fairly tight relationship between apparent and absolute magnitudes in our $z \sim 4$ and $\sim 5$ determinations (the only sizable differences are due to small changes in the distance modulus; see Fig. 18). The only elements that are nonzero in the kernel $V_{m,k}$ span a small range in magnitude ($\Delta m \sim 0.3$ mag). At $z \sim 6$, there is no deep wide-area imaging that probes rest-frame $\sim$1600 Å for $i$-dropouts, and therefore we must resort to examining galaxy luminosities at a slightly bluer wavelength (i.e., $\sim$1350 Å) using the $z_{850}$-band fluxes of $i$-dropouts. Since the $z_{850}$-band flux is affected by attenuation from the Lyman forest, the relationship between the apparent and absolute magnitudes is considerably less tight (see Fig. 18), so the nonzero elements in the kernel $V_{m,k}$ span a much wider range in magnitude (i.e., $\Delta m \gtrsim 1.5$ mag; see Fig. 7 of B06).

To incorporate the effects of incompleteness and photometric scatter on our results, we need to modify equation (A1) to include the transfer functions we computed in § A1. The resultant formula is

$$\Sigma_{l,k} \phi_l T_m lV_{l,k} = N_m,$$

where $T_m l$ are the transfer functions we derived in § A1 to take galaxies from a true total magnitude of $l$ to an observed total magnitude of $m$. This is the equation we use throughout our analysis in computing the surface density of dropouts in a given field from a model LF.

With the ability to calculate the number counts $N(m)$ given an LF, we need some means to decide which model LF fits our data the best. Our two primary approaches, STY79 and SWML, accomplish this by maximizing the likelihood of reproducing the distribution of galaxies as a function of magnitude. Since we consider the surface density of galaxies over multiple fields in our analysis, we express this likelihood $\mathcal{L}$ as a simple product

$$\mathcal{L} = \Pi_{\text{field}}[\Pi_i p(m_i)],$$

where

$$p(m_i) = \left( \frac{n_{\text{expected}, i}}{\sum_j n_{\text{expected}, j}} \right)^{n_{\text{observed}, i}},$$

and $n_{\text{observed}, i}$ is the number of sources observed in the magnitude interval $i$ and $n_{\text{expected}, j}$ is the number of sources expected in the magnitude interval $j$. In equation (A5), note that we only include magnitude intervals $i$ where $n_{\text{observed}, i}$ is positive. The value of $n_{\text{expected}, i}$ has no bearing on whether a magnitude interval $i$ is included or not.
In the determinations of the LF we performed in this paper, it was essential for us to account for the efficiency with which we can select dropouts in our data. We computed this efficiency as a function of redshift $z$ and the apparent magnitude $m$ of the star-forming galaxy in question. We establish these selection efficiencies for galaxies in the HUDF since we reference our shallower selections to the HUDF through transfer functions ($\S$ A1). The apparent magnitudes here are in the same passband as we use to bin our dropout samples, i.e., the $i_{775}$ band for our $B$-dropout sample, the $z_{580}$ band for our $V$-dropout sample, and the $z_{580}$ band for our $i$-dropout sample.

We estimate the selection efficiencies $P(m, z)$ using our well-tested cloning software (Bouwens et al. 1998a, 1998b, 2003a; R. J. Bouwens et al. 2008, in preparation) to project individual sources from our $z \sim 4$ HUDF $B$-dropout sample across the redshift range of our high-redshift samples. In calculating the selection efficiencies $P(m, z)$ for our $z \sim 4 B$-dropout selection, our projected $B$-dropout sample was taken to have mean UV continuum slopes $\beta$ of $-1.5$ at $L_{z=3}$ UV luminosities, but steeper mean UV continuum slopes $\beta$ of $-2.1$ at lower UV luminosities ($<0.1L_{z=3}$), while at intermediate luminosities the mean $\beta$ is varied smoothly between these two extremes. This is to account for the fact that UV-luminous galaxies at high redshift ($z \sim 2-4$) are found to have redder UV continuum slopes (Adelberger & Steidel 2000; Ouchi et al. 2004) than lower luminosity galaxies at these redshifts (Meurer et al. 1999; Beckwith et al. 2006; Iwata et al. 2007; R. J. Bouwens et al. 2008, in preparation). For our $z \sim 5 V$-dropout and $z \sim 6 i$-dropout selections, the mean UV continuum slope of galaxies was taken to be $-2.0$ to match the bluer observed colors for these sources (Lehnert & Bremer 2003; Stanway et al. 2005; B06; Yan et al. 2005). The $1 \sigma$ scatter in the $\beta$ distribution was taken to be 0.6, which gives a good fit to the observed colors. Instead of using simple power laws to represent model SEDs of given UV continuum slope $\beta$, we elected to use $10^8$ yr continuous star formation models (Bruzual & Charlot 2003) where the dust extinction (Calzetti et al. 1994) is varied to reproduce the model slopes. This should provide for a slightly more realistic representation of the SEDs of star-forming galaxies at $z \sim 4-5$ than can be obtained from simple power-law spectra. The sizes of $B$-dropouts in our simulations are scaled as $(1+z)^{-1.1}$ (for fixed luminosity) to match the observed size-redshift relationship (B06; see also Bouwens et al. 2004a; Ferguson et al. 2004).

We include the opacity from the Lyman series line and continuum absorption from neutral hydrogen using the Monte Carlo approach of Bershady et al. (1999). With this approach, absorbers are randomly laid down along the line of sight to each model galaxy according to a distribution of H i column densities and then the colors computed based on the net opacity in a given passband. For the distribution of column densities, we adopt that given in equation (10) of Madau (1995), but modified so that the volume densities of absorbers varied much more rapidly with redshift, i.e., as $\sim(1+z)^3$ instead of $\sim(1+z)^2$. The latter change was necessary to match the substantial Lyman decrements measured by Songaila (2004) for very high redshift ($z \gtrsim 5$) quasars.

The resultant selection functions $P(m, z)$ for our $B$, $V$, and $i$-dropout samples are presented in Figure 18.

### APPENDIX B

#### ALTERNATE DETERMINATIONS OF THE UV LF AT $z \sim 4-6$

To test the robustness of our LF determinations against the many significant uncertainties (e.g., large-scale structure and the model $k$-corrections) that can affect our results, it is useful to consider a variety of different approaches in the determination of these LFs.

In this appendix we consider seven such approaches. Our first two approaches employ alternative techniques to cope with large-scale structure uncertainties and to explore the resulting uncertainties. Our third approach explores possible uncertainties related to measuring the rest-frame UV LF at a bluer rest-frame wavelength where Lyman forest absorption is a concern. Our fourth and fifth approaches examine the dependence of our LF results on the assumptions we make about the form of SED templates and Lyα emission. Our sixth approach explores the dependence of these LF results on different selection criteria. Finally, with our final approach, we investigate the effect that an inherent evolution in $M^*$ across the selection windows of each of our samples would have on our results. A summary of the LF determinations is provided in Table 6.

### B1. $\chi^2$ METHOD (LSS CORRECTION)

One of the most significant uncertainties in the determination of the LF is the effect of large-scale structure ("cosmic variance"). Large-scale structure can result in significant variations in the effective normalization of the LF as a function of position or line of sight. In this paper we cope with these variations by fitting for the shape of the LF (i.e., $\alpha$ and $M^*$) in each of our fields using the STY79 maximum likelihood procedure. Since the normalization of the LF $\phi^*$ does not factor into the fits, our determinations of $M^*$ and $\alpha$ should be robust to the presence of large-scale structure.

An alternate approach is to establish the relative normalization of the LF in each of our fields and then correct for field-to-field variations directly. The relative normalization is established through a two-stage process, where we first establish the relative normalization of dropouts in the UDF to our intermediate depth fields (HUDF-Ps, HUDF05) and second establish the relative normalization of the intermediate depth fields to the GOODS fields. In each step, we establish the relative normalization by degrading our deeper fields down to the depth of our shallower fields, reapplying our selection procedure, and then comparing the surface densities to those found in the shallower field. To maximize the significance of these measurements of the relative normalization, we repeated these degradation experiments 10 times and then took an average. Appendix B of B06 provides a detailed description of our degradation procedure. The numbers and surface densities found for each of our degraded and observed fields are presented in Tables 14 and 15. Then, using these results and the same procedure presented in § 3.6 of B06, we estimated the relative normalization of dropouts in each of our fields. We scaled the surface density of dropouts in these fields by the reciprocal of the tabulated factors to make them consistent with the GOODS fields, which sample the largest comoving volume and therefore should provide us with the best estimate of the cosmic average.

After normalizing the surface density of dropouts in each of our fields to the GOODS areas, we computed the LF by comparing the expected counts with the surface densities (binned in 0.5 mag intervals) observed in each of our fields, computing $\chi^2$, and then
calculating the corresponding likelihood. To account for the uncertainties in the LF that result from the uncertain normalizations of our various fields (Table 14), we ran a series of simulations to compute the effect on the Schechter parameters we derive. This suggests that large-scale structure variations only have a modest effect on the Schechter parameters we derive.

In our STY79 determinations (§ 3.1) and the above determination (§ B1), we considered two different methods for computing the LF at $z \sim 4$ in the presence of large-scale structure. In the first approach (§ 3.1), we attempted to treat large-scale structure by using the STY79 fitting procedure, and in the second (§ B1), we accomplished this by renormalizing the surface density of dropouts found in the HUDF, HUDF05, and HUDF-Ps fields to match the GOODS fields. Although both approaches should provide us with an effective means of dealing with large-scale structure, it is also interesting to determine the LF at $z \sim 4$ ignoring these considerations altogether (and thus implicitly assuming that each survey field is representative of the cosmic average). This will allow us to better assess the impact that large-scale structure could have on the current LF determinations.

B2. $\chi^2$ METHOD (NO LSS CORRECTION)

In our STY79 determinations (§ 3.1) and the above determination (§ B1), we considered two different methods for computing the LF at $z \sim 4$ in the presence of large-scale structure. In the first approach (§ 3.1), we attempted to treat large-scale structure by using the STY79 fitting procedure, and in the second (§ B1), we accomplished this by renormalizing the surface density of dropouts found in the HUDF, HUDF05, and HUDF-Ps fields to match the GOODS fields. Although both approaches should provide us with an effective means of dealing with large-scale structure, it is also interesting to determine the LF at $z \sim 4$ ignoring these considerations altogether (and thus implicitly assuming that each survey field is representative of the cosmic average). This will allow us to better assess the impact that large-scale structure could have on the current LF determinations. Using the same $\chi^2$ methodology as we described in § B1, we repeat our determination of the LFs without making any large-scale structure corrections to the observed surface densities. The results are presented in Table 6 and are quite consistent with our fiducial STY79 determinations. This suggests that large-scale structure variations only have a modest effect on the Schechter parameters we derive.

B3. STY79 METHOD (AT $\sim 1350$ Å)

Thus far we have presented two alternate determinations of the rest-frame UV LFs at $z \sim 4$–6. Each determination offered a different approach for dealing with the uncertainties that arise from large-scale structure. However, in both the $z \sim 4$ and $z \sim 5$ determinations, we have derived the LFs using the surface density of dropouts binned as a function of their magnitude at the same approximate rest-frame wavelength ($\sim 1600$ Å). For our $z \sim 4$ $B$-dropout sample, dropouts were binned according to their $i_{775}$ band magnitudes, and for our

### Table 14

| FIELD                  | Surface Density$^a$ (arcmin$^{-2}$) |
|------------------------|--------------------------------------|
|                        | $B$-Dropouts | $V$-Dropouts | $i$-Dropouts |
| HDFN GOODS             | 8.05 ± 0.22 | 2.23 ± 0.12 | 0.49 ± 0.06 |
| CDFs GOODS             | 8.67 ± 0.23 | 2.06 ± 0.11 | 0.67 ± 0.06 |
| HUDF01                 | 7.97 ± 1.09 | 1.56 ± 0.46 | 0.56 ± 0.25 |
| HUDF02                 | 6.66 ± 1.11 | 3.00 ± 0.80 | 0.15 ± 0.15 |
| HUDF05-1               | ...         | 2.92 ± 0.53 | 0.49 ± 0.22 |
| HUDF05-2               | ...         | 2.55 ± 0.52 | 0.55 ± 0.24 |
| HUDF                   | 8.09 ± 0.79 | 1.45 ± 0.32 | 0.83 ± 0.26 |

* Note.—As observed in these fields after degrading the imaging data to the depth of the GOODS fields and reselecting dropouts in the same way as performed on the GOODS data.

### Table 15

| FIELD                  | $B$-Dropouts | $V$-Dropouts | $i$-Dropouts |
|------------------------|--------------|--------------|--------------|
|                        | Observed     | HUDF$^a$     | Observed     | HUDF$^a$     | Observed     | HUDF$^a$     |
| HUDF01                 | 127          | 137          | 46           | 34           | 34           | 31           |
| HUDF02                 | 78           | 88           | 35           | 19           | 10           | 19           |
| HUDF05-1               | ...          | ...          | 130          | 96           | 53           | 63           |
| HUDF05-2               | ...          | ...          | 113          | 74           | 28           | 49           |

**Notes.—** Only $B$-dropouts, $V$-dropouts, and $i$-dropouts to a depth $i_{775, AB} < 27$, $2850, AB < 28$, and $2850, AB < 28$ are included in the quoted surface densities. We chose 27.0 mag as a limit here because our GOODS dropout selections are still 75% complete to this limit.
z \sim 5 V$-dropout sample, dropouts were binned according to their $z_{850}$-band magnitudes. These two bands are sufficiently redward of Ly$\alpha$ (1216 Å) that they are not contaminated by absorption from the Ly$\alpha$ forest. This makes the determination of the UV LF relatively straightforward using approaches like the effective volume technique of Steidel et al. (1999).

Unfortunately, when moving to our highest redshift $z \sim 6$ $i$-dropout sample, it simply has not been possible to determine the LF in the same manner as at $z \sim 4$ due to the lack of deep near-infrared ("J" band) data to obtain coverage at ~1600 Å. Consequently, in our determinations of the $z \sim 6$ LF (here and in B06), we had to resort to use of the flux in the $z_{850}$ band (rest-frame ~1350 Å) as a measure of the UV continuum luminosity. The difficulty with this is that since the $z_{850}$ band extends below 1216 Å for galaxies at $z \gtrsim 5.7$, flux in this band is significantly attenuated by the Ly$\alpha$ forest, and so it was necessary for us to carefully model the redshift distribution of $i$-dropouts in our sample to remove this effect.

Although the latter procedure should be effective in treating the effects of the Ly$\alpha$ forest, it is not obvious that it will not result in any significant systematics in our determination of the LF. After all, the results will clearly depend somewhat on the rest-frame wavelength at which the LF is determined, as well as the model redshift distributions and assumed forest absorption model (see §3.3 A3 and B7). To verify that no large systematics are introduced, it is useful to repeat the determinations of the rest-frame UV LF at $z \sim 4$ and $z \sim 5$ but instead compiling the dropout surface densities in terms of their magnitudes in the optical passband just redward of the dropout band (i.e., the $V_{606}$ band for our $B$-dropout samples and the $i_{775}$ band for our $V$-dropout samples) to parallel use of the $z_{850}$ band for our $i$-dropout samples. In this way, we obtain a determination of the rest-frame UV LF at $z \sim 4$ and $z \sim 5$ at ~1350 Å to match our determination at $z \sim 6$. The best-fit parameters obtained using this approach are as follows: $\phi^* = (1.4 \pm 0.3) \times 10^{-3}$ Mpc$^{-3}$, $M_{1500} = -20.84 \pm 0.10$, and $\alpha = -1.81 \pm 0.05$ for our $z \sim 4$ $B$-dropout samples and $\phi^* = (0.8 \pm 0.4) \times 10^{-3}$ Mpc$^{-3}$, $M_{1500} = -20.73 \pm 0.26$, and $\alpha = -1.68 \pm 0.19$ for our $V$-dropout samples. Here the value of $M^*$ at $z \sim 4$ is somewhat fainter than in our fiducial STY79 determination. However, to make a fair comparison, it is necessary to account for the $k$-correction from 1350 to 1600 Å. The typical L$^*$ galaxy at $z \sim 4$ has an approximate UV continuum slope $\beta$ of $-1.5$ (e.g., Ouchi et al. 2004), but at $z \sim 5$–6, the UV continuum slope is much bluer, i.e., $\lesssim -2.0$ (Lehnert & Bremer 2003; Stanway et al. 2005; B06; Yan et al. 2005). This results in a typical $k$-correction of roughly $-0.14$ mag for $z \sim 4$ galaxies and $-0$ mag for $z \sim 5$–6 galaxies, resulting in an approximate value of $M^*$ at 1600 Å of $-20.9$ at $z \sim 4$ and $-20.7$ at $z \sim 5$. These values are in good agreement with our other determinations (Table 6), particularly when one considers the fact that the results of this approach are sensitive to the forest absorption model, large-scale structure along the line of sight, and an accurate model of the redshift distributions for each of our dropout samples.

B4. STY79 METHOD (ALTERNATE SED TEMPLATES)

Throughout this paper we have modeled the spectra of LBGs with $10^8$ yr constant star formation systems with varying amounts of dust extinction. We have used these model spectra to estimate the selection volumes of star-forming galaxies in our $B_r$, $V$, and $i$-dropout selections. For our $z \sim 4$ $B$-dropout selections, the model SEDs were taken to have mean UV continuum slopes of $-1.5$ at higher UV luminosities, while at lower UV luminosities (see §3 A3) the model SEDs were taken to have much bluer mean UV slopes in accordance with the observations (Meurer et al. 1999; R. J. Bouwens et al. 2008, in preparation). At $z \sim 5$ and $z \sim 6$, the model SEDs were assumed to have UV continuum slopes of $-2$ to match that present in the observations (Lehnert & Bremer 2003; Stanway et al. 2005; B06).

However, it is legitimate to ask how much our estimated selection volumes may depend on the form of the SED templates. For example, we could just as easily have modeled high-redshift galaxies using different star formation histories, dust content, or metallicities, even electing to model these systems as power laws $f_{\lambda} \propto \lambda^\alpha$. Fortunately, these choices can largely be constrained by the observed colors of our sample galaxies, and in fact in our simulations of the HUDF $B_r$, $V$, and $i$-dropout data (§3) we find excellent agreement between our model results and the observed colors. Even so, different SED templates only have a modest effect ($\lesssim 20\%$) on the selection volumes of our dropout samples (see, e.g., Tables 9 and 10 of Beckwith et al. 2006), particularly if we ignore concerns about the limited S/N of the data and photometric scatter. Within $\sim 1$–2 mag of the selection limit, however, the limited S/N of the data becomes a real concern and the selection volume can often be quite different. This makes it necessary to run detailed Monte Carlo simulations like those described in §3 A3 (Fig. 18) to compute these selection volumes.

To test the sensitivity of our LF determinations to the precise assumptions we make about the color and UV continuum slopes of high-redshift galaxies, we repeated our determination of the LF at $z = 4$, $z = 5$, and $z = 6$ assuming a mean UV continuum slope of $-1.4$ and $-2.1$, with $1 \sigma$ scatter of $0.6$. As in our fiducial STY79 determinations, we use $10^8$ yr constant star formation models (Bruzual & Charlot 2003) with the extinction (Calzetti et al. 1994) varied to match these UV continuum slopes. In general, we found Schechter parameters (Table 6) consistent with our fiducial determinations. One important exception was in our determinations of the $z = 4$ LF assuming the redder $\beta = -1.4$ UV continuum slopes. In that case, we found a significantly steeper faint-end slope $\alpha$ (i.e., roughly $-2.1$) than we obtained in our fiducial determinations. A quick investigation indicated that this resulted from the fact that red galaxies...
have a significantly more difficult time satisfying our \((B_{435} - V_{606})_{AB} > (V_{606} - z_{850})_{AB} + 1.1\) dropout criterion than blue galaxies, and therefore it is much more difficult to select red galaxies to fainter magnitudes than blue galaxies. To see whether our \(z \sim 4\) \(\beta = -1.4\) LF fit results were driven by the selection efficiency of faint (\(\geq 28\) mag) galaxies, we repeated our LF determination but restricted ourselves to galaxies brighter than 28.0 mag. In this case, we recovered Schechter parameters that were in good agreement with our fiducial STY79 determinations (Table 6).

**B5. STY79 METHOD (SIGNIFICANT CONTRIBUTION OF Ly\(\alpha\) EMISSION TO BROADBAND FLUXES)**

Another significant uncertainty in modeling the SEDs of high-redshift star-forming galaxies, and therefore estimating their selection volumes, is the distribution of Ly\(\alpha\) equivalent widths. At \(z \sim 3\), it is known that only a small fraction (\(< 25\%)\) of star-forming galaxies show significant Ly\(\alpha\) emission, i.e., \(\text{EW}(\text{Ly} \alpha) > 20\) Å (Shapley et al. 2003). At \(z > 3\), the incidence of Ly\(\alpha\) emission is thought to increase, in both strength and overall prevalence, although the numbers remain somewhat controversial. Some groups, using a narrow-band selection, claim that \(\geq 80\%\) of star-forming galaxies at the high-redshift end of our range (\(z \sim 5.7\)) have Ly\(\alpha\) equivalent widths of \(\geq 100\) Å (Shimasaku et al. 2006), while spectroscopic follow-up of pure dropout selections indicate that the fraction is closer to \(< 32\%\), with typical Ly\(\alpha\) equivalent widths of 30–50 Å (Dow-Hygelund et al. 2007; Stanway et al. 2004; Vanzella et al. 2006). These results suggest a modest to substantial increase in the fraction of Ly\(\alpha\)-emitting galaxies from \(z \sim 3\) to \(< 6\).

It is interesting to model the effect such emission would have on our computed selection volumes and thus overall determinations of the LF at \(z \sim 4\) and \(< 5\). We do this using the same procedure as we used in §3, but we assume that 33\% of the star-forming galaxies at \(z \sim 4\)–5 have Ly\(\alpha\) equivalent widths of 50 Å. This fraction exceeds slightly the findings of the Dow-Hygelund et al. (2007) study above and was chosen partially as a compromise with the Shimasaku et al. (2006) work. The Schechter parameters we find following this procedure are presented in Table 6 for our \(B_-, V_-,\) and \(i\)-dropout samples. At \(z \sim 4\), these LFs have slightly lower \(\phi^*\) values than similar LF determinations assuming no such emission. At \(z \sim 5\) and \(< 6\), however, the derived \(\phi^*\) values are higher. This owes to the fact that Ly\(\alpha\) lies outside of the dropout band at the lower redshift end of our \(B\)-dropout selections, but inside this band at the lower redshift end of our \(V\)- and \(i\)-dropout selections. Note that we did not include such emission in the SEDs for our fiducial STY79 determinations since (1) Ly\(\alpha\) can also be seen in absorption, not just in emission (which would counteract this effect somewhat), and (2) the overall distribution of Ly\(\alpha\) equivalent widths in star-forming galaxies at \(z \sim 4\)–6 still has not been firmly established.

**B6. STY79 METHOD (WITH ALTERNATE SELECTION CRITERIA)**

The present dropout selections rely on the presence of a two-color selection to isolate a sample of high-redshift star-forming galaxies at \(z \sim 4\) and \(< 5\) and a one-color criterion at \(z \sim 6\). These color criteria were chosen to maximize our sampling of high-redshift galaxies, while minimizing contamination by low-redshift galaxies. However, we could have just as easily chosen a different set of color criteria for our \(B_-, V_-,\) and \(i\)-dropout selections and computed our LFs on the basis of those criteria. To test the robustness of the present LFs, we elected to modify the present selection criteria slightly and repeat our determination of the \(z \sim 4\), \(< 5\), and \(z \sim 6\) LFs using the methodology laid out in §3 and 2. The criteria we chose were \([B_{435} - V_{606}] > 1.2\) \& \([B_{435} - V_{606}] < 1.2\) for our alternate \(B\)-dropout selection, \([V_{606} - i_{775}] > 0.9(i_{775} - z_{850}) \vee (V_{606} - i_{775} > 1.8) \& (V_{606} - i_{775} > 1.2) \& (i_{775} - z_{850} < 1.3)\) for our alternate \(V\)-dropout selection, and \((i_{775} - z_{850} > 1.4) \& (V_{606} - i_{775} > 2.8) \vee (S/N(V_{606}) < 2)\) for our alternate \(i\)-dropout selection. The \(B\)-dropout criterion above is the same as used in Giavalisco et al. (2004b) and results in a sample about half the size of the present one, with a narrower selection window in redshift and similar mean redshift. The \(V\)-dropout criterion is similar to that used in our primary selection, except that the \((V_{606} - i_{775})\) color cut was lowered to make our selection more complete at the higher redshift end of the \(i\)-dropout selection window. The best-fit Schechter parameters for these selections are presented in Table 6 and are in reasonable agreement with our fiducial STY79 determinations.

**B7. STY79 METHOD (MADAU OPACITIES)**

In this work we use the Monte Carlo procedure of Bershady et al. (1999) to model the effects that \(H_i\) line and continuum absorption have on the colors of high-redshift galaxies (§A3). We adopted this approach rather than the more conventional approach of using the Madau (1995) opacities to better account for the stochastic effects that line-of-sight variations have on the colors of high-redshift galaxies and to take advantage of advances in our knowledge of \(H_i\) column densities at \(z \geq 5\) (e.g., from Songaila 2004). This should make the present determinations of the LF slightly more accurate overall than we would have obtained had we not made these refinements. This being said, it is useful nevertheless to compare our LF results with what we would have obtained using the wavelength- and redshift-dependent opacities compiled by Madau (1995). This allows us to ascertain what the effects of these changes are on the present results. Repeating our determination of the selection efficiencies of \(B_-, V_-,\) and \(i\)-dropouts with the Madau (1995) opacities (§A3), we find that our \(V\)- and \(i\)-dropout selection windows are shifted to slightly higher redshifts in general, by \(\Delta z \sim 0.05\), but overall look very similar. The LFs we derive using these assumptions are presented in Table 6 and are quite similar to our fiducial STY79 determinations, except at \(z \sim 5\)–6 \(M^*\) is \(< -0.05\) mag brighter and at \(z \sim 6\) the value of \(\phi^*\) is \(< 10\%\) higher.

**B8. STY79 METHOD (WITH AN EVOLVING \(M^*\))**

In our fiducial STY79 determinations of the LF for each dropout sample, we assume that the LF does not evolve in redshift across the selection window of each sample. Since we observe significant evolution in the LF over the redshift range probed by our LFs (from \(z \sim 6\) to \(\sim 4\)), this assumption clearly cannot be correct in detail. To investigate whether our determinations may have been affected by this assumption, we repeated our determination of the LF for each of our samples, but assumed that \(M^*\) evolves by 0.35 mag per unit redshift. This evolution in \(M^*\) is a good match to the evolution we observe in the UV LF from \(z \sim 6\) to \(\sim 4\). The values of \(M^*, \phi^*,\) and \(\alpha\)
we derive at $z \approx 3.8$, $\approx 5$, and $\approx 5.9$ assuming an evolving $M^*$ are presented in Table 6. Encouragingly enough, the values we obtain including evolution are very similar to those recovered without evolution. This suggests that the overall Schechter parameters we have derived here are quite robust. Nonetheless, there do appear to be small systematic changes in the best-fit Schechter parameters if evolution is included. Accounting for evolution, the $M^*$ values recovered are $\approx 0.06$ mag fainter, the $\phi^*$ values recovered are $\approx 10\%$ higher, and the faint-end slopes $\alpha$ are marginally shallower (by $\approx 0.02$). Since the inclusion of evolution in the determination of the LF is presumably a better assumption than not including this evolution, the LF parameters we adopt in this paper (Table 7) will be from this section.

**APPENDIX C**

**EFFECT OF LARGE-SCALE STRUCTURE VARIATIONS ALONG THE LINE OF SIGHT ON OUR RESULTS**

The standard SWML and STY79 maximum likelihood approaches allow us to determine the shape of the LF in a way that is insensitive to the presence of large-scale structure. Unfortunately, since we do not have exact redshift information for the galaxies in our samples, we cannot determine the absolute magnitudes for individual galaxies in our sample and therefore we must modify the SWML and STY79 maximum likelihood approaches slightly so that the likelihoods are expressed in terms of the apparent magnitude for individual sources (instead of the absolute magnitude). Since the apparent magnitudes are related to the absolute magnitudes via the redshift and the distribution of redshifts is uncertain due to the presence of large-scale structure along the line of sight, our LF fit results will show some sensitivity to this structure.

To determine the effect of this structure on the derived values of $M^*$, $\phi^*$, and $\alpha$, we ran a number of Monte Carlo simulations where we introduced large-scale structure variations on a canonical mock catalog of dropouts for each dropout sample that we generated using the Schechter parameters given in Table 7. Our use of one standard mock catalog for each sample was necessary to ensure that variations in the best-fit parameters only resulted from large-scale structure fluctuations and not Poissonian-type fluctuations (which would arise if we regenerated these catalogs for each trial in our Monte Carlo simulations). We then proceeded to introduce large-scale structure fluctuations into this catalog. Within redshift slices of size $\Delta z = 0.05$, we calculated the density variations expected for each of our dropout samples assuming the values of the bias given in $\S$ 3.1, made random realizations of these density variations, applied these variations to our mock catalogs, and then recomputed the Schechter parameters using our implementation of the STY79 method. Repeating this process several hundred times for each dropout sample, we computed the $1\sigma$ rms variations in $\phi^*$, $M^*$, and $\alpha$ expected to result from large-scale structure along the line of sight. For our $z \approx 4$ $B$-dropout sample we found $1\sigma$ rms variations of 0.07 mag, 13%, and 0.01 in $M^*$, $\phi^*$, and $\alpha$, respectively; for our $z \approx 5$ $V$-dropout sample we found 1 $\sigma$ rms variations of 0.05 mag, 12%, and 0.01, respectively; and for our $z \approx 5.9$ $i$-dropout sample we found 1 $\sigma$ rms variations of 0.05 mag, 16%, and 0.04, respectively. Since the nominal errors from the STY79 method on $M^*$ and $\alpha$ are typically at least 2–3 times as large as this, this structure only increases the uncertainties on $M^*$ and $\alpha$ by a minimal $\approx 10\%$.

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