A scheme for symmetrisation verification

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Abstract
We propose a scheme for symmetrisation verification in two-particle systems, based on
one-particle detection and state determination. In contrast to previous proposals, it does not
follow a Hong–Ou–Mandel-type approach. Moreover, the technique can be used to generate
superposition states of single particles.

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(Some figures in this article are in colour only in the electronic version.)

1. Introduction
Non-factorizable states play a fundamental role in quantum
theory. There are two classes of non-separable states:
etangled and (anti)symmetrized ones. In recent years, there
has been a boom in the theoretical and experimental study of
the former, especially concerning their generation, verification
and potential applications in the fields of quantum information
and conceptual foundations of quantum theory. Here, we are
mainly interested in the question of verification. Two good
introductions to the problem are [1, 2].

In contrast, there are not so many studies concerning the
same issue for (anti)symmetrized states of massive particles.
In quantum theory, two identical particles whose wave
functions overlap at a given instant are indistinguishable at
any subsequent time. If we localize one of the particles at
any instant after the overlap, it is impossible to identify it
with any of the initial ones. In order to correctly describe
this impossibility, the wave function of the two-particle
system cannot be a product state (even for non-interacting
particles), but must be symmetrized in the case of bosons and
antisymmetrized in the case of fermions.

In quantum optics, the standard test for determining
identicalness, i.e. for testing if two photons are really in
indistinguishable states (and consequently if it is necessary
to symmetrize the state), is based on the Hong–Ou–Mandel
(HOM) arrangement [3]. This technique is based on the
interference of the photons at a beam splitter [4].

Several authors have proposed the use of the same
approach for massive particles [5–8]. In [5, 6], mesoscopic
electron beam splitters based on electron beam lithography on
a GaAs high-mobility two-dimensional electron gas system
have been experimentally used to test the antibunching
of fermions, an unequivocal demonstration of their
antisymmetrisation. Several other arrangements can be
used as beam splitters for beams of massive particles [9, 10],
also providing an alternative basis for feasible massive
HOM-type tests. Unfortunately, their manipulation is much
more delicate than their counterparts for light beams, making
the application of the technique more difficult to massive
particles.

The importance of having methods able to verify
(anti)symmetrisation can be easily understood. For instance,
in [11] it has been suggested to study massive two-particle
interferometry by diffraction gratings. One expects novel
effects when identical particles arrive in (anti)symmetrized
states. In order to experimentally test such effects, we must
know in advance if the particles incident on the diffraction
grating are in (anti)symmetrized states. If not, a negative result
in the experiment could be imputed either to the absence of
the effect or to the state of the particles (being in a product state
instead of a (anti)symmetrized one). In contrast, if we know
for sure that the particles are in an (anti)symmetrized state a
negative result could only be interpreted as a demonstration of
the absence of the effect.

We propose here a scheme able to verify
(anti)symmetrisation in two-particle systems, based only
on detection of one of the two particles and determination
of the state of the surviving one. To be concrete, if the state
of the two-particle system is (anti)symmetrized, the mode
distribution of the surviving particle will be a combination of
the initial distributions of the two particles. The scheme is,
in contrast to previous proposals, a test of identicalness not
based on an HOM-type approach.

As an interesting byproduct, the technique also provides
an efficient method for generating one-particle superposition
states. The surviving particle is in a superposition of the initial states of the two particles. The technique is especially well suited for generating superpositions of multi-mode states peaked around very different momentum values.

2. The arrangement

We consider the same arrangement as that in [12], where the process of detection of one of the particles in a system of two particles in multi-mode states is analyzed, with special emphasis on the one-particle interferences present in the detection pattern.

Two identical particles are prepared in the state

\[ |2_{fg}⟩ = \int d^3p \int d^3q f(p)g(q)\hat{a}_p^\dagger \hat{a}_q^\dagger |0⟩, \]

where \(f\) and \(g\) are the mode distributions, which we assume to be normalized, \(\int d^3p |f(p)|^2 = 1 = \int d^3q |g(p)|^2\). In equation (1), \(\hat{a}_p^\dagger\) is the creation operator of the mode \(p\) and \(|0⟩\) is the vacuum state. The creation and annihilation operators obey the usual relations \([\hat{a}_p, \hat{a}_q^\dagger] = \delta^3(p-q)\). In the double sign expressions, the upper one refers to bosons and the lower one to fermions. Note that the above state is not normalized. We shall work with a non-normalized state (see in [12] how this state is normalized), postponing the normalization to the final stage of the calculation.

In all the above expressions, we have not included the spin indexes. In order to simplify the presentation, we assume the two particles to be in the same spin state making superfluous the presence of the indexes. In terms of the quantum description, this assumption is equivalent to having for the spin part of the state the form \(|s⟩_1|s⟩_2\), i.e. a product state with 1 and 2 labeling the two particles. We could also have (for both bosons and fermions) a symmetric state for the spin variables, \(|(s⟩_1|s′⟩_2 + |s′⟩_1|s⟩_2)/\sqrt{2}\). With any of the above two choices, the part of the state related to the spatial variables must be symmetric for bosons and antisymmetric for fermions. We make our presentation in this framework, symmetrized states for the spatial or momentum wave functions of bosons, and antisymmetrized for fermions. The extension to the case of particles in antisymmetrized spin states is straightforward, as it is only necessary to consider antisymmetric and symmetric spatial states for, respectively, bosons and fermions.

The two-particle system now interacts with a detector. We consider only the cases where only one detection occurs. Moreover, we assume that in the detection process the particle is removed, for instance, by absorption. In order to postselect these cases, one must use a detector able to distinguish events with one- or two-detection events (see [13, 14] for an optical detector with this property).

Next, we show that the detection process is described by the action of the Schrödinger field operator. From a mathematical point of view, this operator \(\hat{\psi}(R, t)\) represents the annihilation of a particle at point \(R\) and time \(t\) [15, 16].

If the initial state of the system is an \(n\)-particle state \(|n⟩\), the action of the operator leaves an \(n-1\) particle state, \(\hat{\psi}|n⟩ → |n-1⟩\). Thus, this operator represents in a natural way processes in which particles are removed. Intuitively, one can identify this removal process with the destructive detection assumed above. With this identification \(\hat{\psi}(R, t)|2_{fg}⟩\) describes the state of the resultant one-particle system. At this point a criticism of this identification could be made. As the detection process is a non-unitary one, perhaps we should use a mixture instead of a pure state to describe the final one-particle system. We argue here in favor of the pure state choice. Our argument is based on the analogy to detection through photomultipliers in quantum optics [4] (also see [17] for a detailed treatment of absorption in multi-particle massive beams and its description in the framework of the Glauber scheme). In this type of destructive detection, the photons are absorbed by the atoms composing the photomultiplier. A description of the interaction between detection device and radiation is done in terms of the positive frequency part of the electromagnetic field operator \(\hat{E}^+\). If we denote by \(|R⟩\) the initial state of the radiation field, its final state is assumed to be \(\hat{E}^+|R⟩\), a pure state, not a mixture. From that pure state, one can evaluate the probability of transition to a state \(|R_\text{F}⟩\), as \(|\langle R_\text{F}|\hat{E}^+|R⟩|^2\). By analogy to the extensively verified scheme of detection through photomultipliers, we assume that after the destructive detection we have the pure state \(\hat{\psi}(R, t)|2_{fg}⟩\). As we will see in the following sections, this assumption gives rise to some conclusions that can be experimentally tested, making the assumption verifiable.

In the expression for the field operator, \(R\) refers to the point of detection. We use this notation to emphasize that we are considering the detector to be at a fixed position in each repetition of the experiment. We also want to remark that it does not act as a physical variable (as is the case for \(r\)), but acts as a parameter. This is a necessary consistency condition for the calculation, because in the second quantization formalism there cannot be explicit dependence on the spatial variables, unless it is in a parametric form [15].

The field operator is given by \(\hat{\psi}(R, t) = \int d^3p \psi_p(R)\hat{a}_p(t)\), with \(\psi_p\) being a complete set of orthonormal stationary wave functions. The most common choice for this set is that of plane waves \(\psi_p(R) = (2\pi\hbar)^{-3/2}\exp(\langle p \cdot R\rangle/\hbar). The time dependence is carried by the annihilation operator. For planes waves it is given by \(\hat{a}_p(t) = \hat{a}_p \exp(-iE_p t/\hbar)\) with \(\hat{a}_p\) being the annihilation operator at time \(t = 0\) and \(E_p = p^2/2m\) the energy.

From now onwards, for the sake of simplicity, we shall restrict our considerations to stationary situations. Consequently, we can omit the time variables from all the expressions.

3. Mode distribution of the surviving particle

In our case, the final state after one detection is

\[ \hat{\psi}(R)|2_{fg}⟩. \]

The evaluation of the state of the surviving particle is simple:

\[ \hat{\psi}(R)|2_{fg}⟩ = \int d^3p(\psi_f(R)|p⟩f(R)g(p))\hat{a}_p^\dagger|0⟩ = \psi_f(R)|1_f⟩ ± \psi_g(R)|1_f⟩ \]

with \(\psi_f(R) = \int d^3p f(p)\psi_p(R)\) and \(|1_f⟩ = \int d^3p f(p)\hat{a}_p^\dagger|0⟩.\]
The above state has not yet been normalized. We denote by $|1_h\rangle$ the normalized state of the surviving particle, which is given by $|1_h\rangle = \psi(R)/2_{fg}/N$, with
\[
N^2(R) = |\psi_f(R)|^2 + |\psi_g(R)|^2 \pm 2\text{Re}(\psi_f^*(R)\psi_g(R)|1_f\rangle|1_f\rangle).
\tag{4}
\]
Finally, the mode distribution of the surviving particle is
\[
h(p) = \frac{\psi_f(R)}{N(R)}g(p) \pm \frac{\psi_g(R)}{N(R)}f(p).
\tag{5}
\]
The mode distribution is neither $f$ nor $g$, but a combination of them. The coefficients in the combination are given by some functions depending on the values of the initial wave functions at the point of detection.

Note that only in the cases where $\psi_f(R) = 0$ or $\psi_g(R) = 0$, the surviving particle has a mode distribution similar to one of the initial ones $|1_e\rangle$ or $|1_f\rangle$. This behavior can be easily understood. Let us consider one of these nodal points, for instance, $\psi_f(R) = 0$. Then we know for sure that the particle detected is of the type $f$ and, consequently, that the surviving one is of type $g (h(p) = g(p))$.

In the points where simultaneously we have $\psi_f(R) = 0$ and $\psi_g(R) = 0$, the expression for $|1_h\rangle$ is undefined, reflecting the property that at these points the probability of detection of the particles is null and it does not make sense to speak about the surviving one.

The fact that the mode distribution of the surviving particle becomes a combination of the initial ones is a clear manifestation of a superposition process. This follows directly from the last term in equation (3), but can easily be visualized if we consider the first quantization version of the problem, where the wave function is given by
\[
\psi_h(r) = \int d^3p h(p) \exp(ip \cdot r/h)
= \frac{\psi_f(R)}{N(R)}\psi_g(r) \pm \frac{\psi_g(R)}{N(R)}\psi_f(r),
\tag{6}
\]
showing the superposition of $\psi_f$ and $\psi_g$. The wave function of the surviving particle is a superposition of the two initial wave functions. This superposition can be physically understood in terms of the indistinguishability of the alternatives available to the system, such as stated by the general criterion for quantum superpositions. This criterion states that when the evolution of a quantum system is compatible with several indistinguishable alternatives, the system representing the system is a superposition of them. In our case, outside the nodal points, the detector is unable to provide information on the mode distribution of the particle measured, making the superposition, detected particle with mode distribution $f$ and detected particle with mode distribution $g$, indistinguishable.

When the particles are not in (anti)symmetrized states the above results do not hold. In the second quantization formalism used in this paper, the absence of (anti)symmetrization translates into the use of (anti)commuting creation and annihilation operators $\hat{a}$ and $\hat{b}$. The initial state (now normalized) is
\[
|2_{fg}\rangle_{NS} = \int d^3p \int d^3q f(p)g(q)\hat{a}_p^\dagger\hat{b}_q^\dagger|0\rangle.
\tag{7}
\]

4. The scheme

The above results provide the basis of a scheme for (anti)symmetrization verification between the two initial particles. The scheme has two steps. In the first stage, the two particles interact with the detector. We postselect all the cases of one-detection events. Detectors able to distinguish one- and two-detection events would greatly simplify the task. As indicated before, in the optical case such types of detectors are available [13, 14]. In the second step, we deal with the postselected surviving particles. We must test if the particles are in one of the two initial states or in a superposition of them. To this end, we can use standard tomography techniques or study the spatial detection patterns of the surviving particles. If the detectors used in the first step are unable to discriminate between one- and two-detection events, the postselection must also be carried out in the second stage. Then we choose the cases where, in each repetition of the process, there is an event in the detector and another in the device testing the superposition. As the efficiency of the detectors of massive particles is, in general, high, the scheme seems to be, in principle, an effective one. These are the general lines of the scheme. Now we discuss in detail a particular realization that is, in principle, viable with present-day technology. We present it in figure 1.

We first consider the stage of the preparation of the state. We consider an arrangement where each particle is
emitted by a different source. Two main types of sources have been used in the study of (anti)bunching effects in massive particles: magnetic traps \[18, 19\] and optical lattices \[20, 21\]. Sometimes the traps and lattices use previously generated Bose–Einstein condensates (BEC), and on other occasions they use gases produced by evaporative and sympathetic cooling. In \[19–21\], the atoms are released at switch-off of the potentials retaining the atoms in the trap or lattice. A number of atoms of the order of \(10^4–10^5\) are released, making the technique useless for our purpose of preparing two-particle systems. From our point of view, more interesting is the method of \[18\], where a weak continuous microwave field is applied to the BEC in the magnetic trap for output coupling an atom laser. Some atoms inside the trap are spin-flipped and no longer experience the magnetic potential, leaving the trap. The number of released atoms is much smaller than in the previous techniques. As a matter of fact, in \[18\] a mean atom number very close to 2 was reported, making possible individual detection of the atoms. With this method, adequate values of the microwave fields and synchronization of the two fields acting on different traps, it seems possible to create two-particle states.

For different parameters of the two traps (different cooling, etc) we can have particles with different momentum distributions. Another way to generate different momentum distributions is to collimate the released beam via delimiters of different apertures. The uncertainty relation tell us that if the width of the aperture is \(l\) of different apertures. The uncertainty relation tell us that if the number of released atoms is much smaller than in the previous techniques. As a matter of fact, in \[18\] a mean atom number very close to 2 was reported, making possible individual detection of the atoms. With this method, adequate values of the microwave fields and synchronization of the two fields acting on different traps, it seems possible to create two-particle states.

In a different run of experiments, we obtain \(|\psi_f (r_{\text{det}})|^2\), \(|\psi_g (r_{\text{det}})|^2\), \(|\psi_f (R)|^2\) and \(|\psi_g (R)|^2\) by considering situations where only one of the sources emits particles. Then we compare \(|\psi_h (r_{\text{det}})|^2\) with \(|\psi_f (R)|^2|\psi_g (r_{\text{det}})|^2 + |\psi_f (r_{\text{det}})|^2 |\psi_g (R)|^2|\psi_f (r_{\text{det}})|^2\) in order to see if both distributions coincide (there is no superposition of the initial states in the surviving particle, corresponding to the situation in a mixture) or not (there is a superposition associated with a pure state). Note that we do not need to determine the normalization factor \(N(R)\), because it is not necessary to see if both distributions have the same analytical form.

With the above procedure we can determine the presence of the superposition in the surviving particle and, consequently, determine if it is in a pure state. If it is, we can complete the tomographic process by position measurements of the particle in different planes after the main detector (a detector array at each of these planes).

It must be noted that if we only want to know whether the two-particle state was (anti)symmetrized previous to the interaction with the main detector, it is enough to carry out the above procedure of comparing the \(|\psi_h|^2\), \(|\psi_f|^2\) and \(|\psi_g|^2\) distributions, without the need of a much more demanding complete process of tomography. In effect, when the surviving particle is in a superposition state the initial two-particle wave function necessarily had to be in a (anti)symmetric state. The situation is different when, in addition to determining the presence of exchange processes in the two-particle system previous to the interaction with the main detector, we want to know whether the spatial part of the state of the system was in a symmetric or antisymmetric state. As discussed before, the bosonic and fermionic particles can be in a symmetric or antisymmetric spatial state, depending on the antisymmetric or symmetric character of the spin part of the state. The symmetric or antisymmetric character of the spatial part of the two-particle wave function can be determined from the positive or negative sign in the superposition of the surviving particle. However, the value of this sign cannot be determined by the method of comparison of \(|\psi_h|^2\), \(|\psi_f|^2\) and \(|\psi_g|^2\) presented in previous paragraphs. We would need to know the complete expression of \(\psi_h\) (or equivalently, have knowledge of \(\psi_f\) and \(\psi_g\) and the use of equation (9)). We would need a complete tomography process.

5. Conclusions

We have presented in this paper a scheme for (anti)symmetrization verification. Our proposal is not based on HOM-type arrangements. The ideas presented here seem, in principle, to be accessible to experimental scrutiny.

The scheme could also be used to generate superposition states in one-particle systems, starting from two-particle ones. If the initial two-particle state is (anti)symmetrized, detection of one of the members of the pair outside the nodal points leaves the other in a superposition of the two initial states. We have generated superposition via (anti)symmetrization and detection. An interesting application of the method would be to generate a multi-mode superposition state of two very different central momenta. We only need (i) to prepare the initial particles with momentum distributions \(f(p)\) and \(g(p)\)
peaked around two values $p_f$ and $p_g$ with $|p_f - p_g| \gg \sigma$, where $\sigma$ is the typical width of the distributions, (ii) to get a non-negligible overlapping of the two particles in order to have an (anti)symmetrized state and (iii) to postselect the cases with one-detection events.

Finally, we must consider the possibility of applying the scheme to quantum optics. The HOM technique with beam splitters is very efficient for optical systems, making superfluous the existence of other verification schemes. Nevertheless, our proposal could be interesting to generate superpositions of one-photon states. A simple calculation shows that a result similar to that obtained for the distribution of the surviving massive particle holds in quantum optics. Moreover, in this field there is much experience in dealing with two-photon states, even in multi-mode states [4].

References

[1] van Enk S J, Lütkenhaus N and Kimble H J 2007 Phys. Rev. A 75 052318
[2] Plenio M B and Virmani S 2007 Quantum Inf. Comput. 7 1
[3] Hong C K, Ou Z and Mandel L 1987 Phys. Rev. Lett. 59 2044
[4] Loudon R 2000 The Quantum Theory of Light (Oxford: Oxford Science Publications)
[5] Liu R C, Odom B, Yamamoto Y and Tarucha S 1998 Nature 391 263
[6] Oliver W D, Kim J, Liu R C and Yamamoto Y 1999 Science 284 299
[7] Bose S and Home D 2002 Phys. Rev. Lett. 88 050401
[8] Lim Y L and Beige A 2005 New J. Phys. 7 155
[9] Adams C S, Sigel M and Mlynek J 1994 Phys. Rep. 240 143
[10] Croning A D, Schmiedmayer J and Pritchard D E 2009 Rev. Mod. Phys. 81 1051
[11] Sancho P 2010 J. Phys. B: At. Mol. Opt. Phys. 43 065504
Sancho P 2010 Phys. Rev. A 82 033814
[12] Sancho P 2004 J. Phys. A: Math. Gen. 37 11003
[13] Golovin V and Saveliev V 2004 Nucl. Instrum. Methods Phys. Res. 518 560
[14] Eraerds P, Legré M, Rochas A, Zbinden H and Gisin N 2007 Opt. Express 15 14539
[15] Landau L D and Lifshitz E M 1965 Quantum Mechanics (Reading, MA: Addison-Wesley)
[16] Baym G 1969 Lectures on Quantum Mechanics (New York: Benjamin)
[17] Sancho P 2008 Ann. Phys. 323 1271
[18] Öttl A, Ritter S, Köhl M and Esslinger T 2005 Phys. Rev. Lett. 95 090404
[19] Jeltes T et al 2007 Nature 445 402
[20] Fölling S, Gerbier F, Widera A, Mandel O, Gericke T and Bloch I 2005 Nature 434 481
[21] Rom T, Best Th, van Oosten D, Schneider U, Fölling S, Paredes B and Bloch I 2006 Nature 444 733
[22] Lvovsky A I and Raymer M G 2009 Rev. Mod. Phys. 81 299