FORM FACTORS AND EXCLUSIVE PROCESSES –
INTRODUCTION AND OVERVIEW

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Form factors, polarizabilities and excitation spectra are different aspects of many-body structures. These aspects are constrained by sum rules based on general principles like relativity, gauge invariance, causality and unitarity. Several such structure effects of baryons and mesons will be reviewed, with particular emphasis on the neutron charge form factor, the strangeness content of the nucleon, polarizabilities, and the Gerasimov-Drell-Hearn and related sum rules.

1 Introduction

Hadrons are complex systems with many internal degrees of freedom. In principle, their constituents are current quarks and gluons interacting by the laws of QCD. Due to these laws the colour interaction increases with decreasing energy and momentum involved, and as a consequence hadrons exist as highly correlated many-body systems. As in any such system, it becomes important to find the proper “effective” degrees of freedom, which in this case are the low-energy realizations of quarks and gluons, the colourless mesons and baryons.

Internal degrees of freedom have two immediate consequences, a finite size of the object and an excitation spectrum. In the case of the nucleon the finite size effect may be described by the electric (E) and magnetic (M) Sachs form factors,

\[ G_N^E(Q^2) = e_N - \frac{Q^2}{6} < r^2 >_N^E + [Q^4], \]
\[ G_N^M(Q^2) = (e_N + \kappa_N) \left( 1 - \frac{Q^2}{6} < r^2 >_N^M + [Q^4] \right), \]

with normalizations \( e_N = 1 \) or 0 for \( N=\text{proton or neutron} \) respectively, and \( \kappa_N \) the anomalous magnetic moment of the nucleon. The form factors are functions of 4-momentum transfer, \( q^2 = (k_f - k_i)^2 = -Q^2 \), where \( k_f \) and \( k_i \) are the 4-momenta of the leptons in the final and initial states, respectively. The quantity \( Q^2 \) has been defined to be positive for lepton scattering, which leads to the exchange of a “space-like” virtual photon \( (q^2 < 0) \). Pair production, on the other side, requires the exchange of a “time-like” virtual photon.
and becomes physically possible for \( q^2 \geq 4m_N^2 \), whenever the mass \( 2m_N \) of the nucleon-antinucleon pair can be produced. The mathematically most interesting part of the form factors lies in the unphysical region \( 0 < q^2 < 4m_N^2 \), where the function is strongly increasing and rapidly fluctuating due to the vicinity of the vector meson poles in the complex plane.

A particle with an anomalous magnetic moment can only exist if it has a finite size at the same time, elsewise one would encounter serious divergencies in the high-energy limit. Similarly the existence of a finite size has the consequence that the particle can be polarized by a (quasi)static electromagnetic field. A physical process to determine the polarizabilities of the nucleon is Compton scattering of photons with sufficiently low energy \( \omega \). The leading term of the Compton amplitude is the Thomson term, which depends only on “global” properties of the target. The subleading term of order \( \omega^2 \) is proportional to the electric (\( \alpha \)) and magnetic (\( \beta \)) polarizabilities describing the deformation under the influence of an external electromagnetic field. For the neutron the expansion starts with the second term. The corresponding cross section is given by the square of the scattering amplitude, i.e. it scales with \( \omega^4 \). This is the case of Rayleigh scattering, also observed in the scattering of light off neutral atoms leading to the blue sky.

We can visualize the relation between the polarizability and Compton scattering \( h(\gamma, \gamma')h' \) as follows: The incoming photon \( \gamma \) polarizes the hadron \( h \) whose charges arrange in an energetically more favourable distribution. The new equilibrium is described by a coherent superposition of ground and excited state wave functions, which is subsequently analyzed by studying the angular distribution of the outgoing photon.

Recently it has also become possible to observe so-called generalized polarizabilities in the reaction \( p(e, e'\gamma)p' \) with virtual incident photons. These generalized polarizabilities depend on the 4-momentum transfer, i.e. \( \alpha = \alpha(Q^2) \), and their Fourier transforms describe the spatial distribution of the polarization densities.

The reaction \( \vec{e} + \vec{N} \rightarrow \text{anything} \) is parametrized by 4 response functions \( \sigma_T, \sigma_L, \sigma'_TT \) and \( \sigma'LT \), which are functions of energy and momentum transfer. The indices \( T \) and \( L \) refer to the transverse and longitudinal polarization of the virtual photon, and the latter two structure functions can only be studied by double-polarization experiments. Several energy-weighted integrals over these structure functions are related to static and quasistatic properties of the hadronic system. These “sum rules” are based on Lorentz and gauge invariance, analyticity and unitarity of the scattering amplitudes. They can be derived by combining the results of low-energy theorems with dispersion relations and the optical theorem. Typical examples are the sum rules of Baldin,
2 Form Factors

The form factors of the nucleon, in particular the evasive charge form factor of the neutron, are now being studied by double polarization experiments, $\vec{e} + \vec{N} \rightarrow e' + N'$ or $\vec{e} + \vec{N} \rightarrow e' + \vec{N}'$. The differential cross section for these reactions takes the form

$$\frac{d\sigma}{d\Omega} = \cdots \left( E_E^2 + \cdots G_M^2 \right) + \cdots P_e P_{N\parallel} G_E G_M + \cdots P_e P_{N\perp} G_E^2 .$$

(2)

The ellipses in this equation indicate known kinematical factors. Since the Sachs form factors in the first term add in squares, it is difficult if not impossible to determine the (small) charge form factor of the neutron by an unpolarized experiment. The last two terms can be measured with longitudinally polarized incident electrons (polarization $P_e$) and polarized targets (or recoil polarization), $P_{N\parallel}$ and $P_{N\perp}$ referring to polarizations perpendicular and parallel to the momentum transfer. Except for known kinematical factors, the ratio of the asymmetries, $A_{\perp}/A_{\parallel}$, is proportional to $G_E/G_M$, i.e. $G_E$ can be determined in a model-independent way once $G_M$ is known. The observable $G_E^n$ was recently studied at MAMI by means of two different experiments, scattering of polarized electrons off a polarized $^3\text{He}$ target, and off an unpolarized $^2\text{H}$ target but analyzing the recoil polarization of the ejected neutron. The results will be presented by H. Schmieden and P. Grabmayr. There are indications for two surprises:

- The reaction was previously assumed to be quite independent of final-state interactions, at least in quasifree kinematics and for perpendicular polarization. It has now been shown that in the case of $^2\text{H}$ the corrections increase dramatically for smaller momentum transfer. Therefore, the observed discrepancy between the $^3\text{He}$ and $^2\text{H}$ results could well be due to an uncomplete treatment of final-state interactions and two-body currents in the case of $^3\text{He}$.

- All models (Nambu-Jona-Lasinio, solitons, constituent quarks, chiral and cloudy bags, etc.) predict a core with positive charge inside and a negative cloud outside. However, the predictions differ substantially in absolute value and zero-crossing of the charge distribution. The recent deuteron data indicate a zero-crossing at a small radius of about 0.7 fm, close to the
axial radius, which is determined by the quark bag radius in chiral bag models. The previously preferred “Platchkov fit” had the zero-crossing at about 0.9 fm, close to the proton radius. At the same time the values for $G_E^n$ and $|\rho_E^n|$ increase by as much as a factor of 2.

R. Baldini will report on the FENICE experiment $e^+e^- \rightarrow n\bar{n}$, which probes the neutron form factors in the time-like region. The main results are the following:

- The time-like form factors are at about twice the level of the space-like ones at the same values of $|q^2|$. This is a clear sign that asymptotia will only be reached at much higher momentum transfer.
- The neutron form factors at $q^2 > 0$ are about twice the value of the proton form factors, i.e. there is considerable absorptive strength of isovector-vector mesons in and above the threshold region.
- There are strong indications of a resonance structure near threshold, and some indications that both $G_E^n$ and $G_M^n$ could become small in this region (note: $G_E(4m^2) = G_M(4m^2)$ by definition).

The finite size of the nucleon is also addressed in the following contributions:

- The surprising results of the FENICE experiment are described by S. Dubnicka, A.-Z. Dubnicková et al. by a new dispersion fit to the data. In particular this fit needs a $\rho^{IV}(2.5$ GeV) with large width.
- R. Bijker and A. Leviatan have calculated the form factors in the framework of the algebraic model developed in collaboration with Iachello, and S. Boffi will present the results of the chiral constituent quark model. In both cases the data require an intrinsic quark form factor corresponding to a quark radius of about 0.6 fm, which is close to the radius of the wave function. This finite-size constituent quark can be visualized as a point-like (current) quark surrounded by a pion cloud. However, there are two immediate consequences: The virtual pion surrounding the quark can become real whenever sufficient energy is supplied, i.e. the quark size will also influence the excitation spectrum, and the crowded environment of large constituent quarks in a small bag will create strong exchange currents, leading to sizeable corrections for all observables as shown by G. Wagner and A. Buchmann.
- H. Haberzettl et al. have shown the need to include finite size effects for a consistent description of meson photoproduction.
I. Eschrich will present the latest results of the SELEX collaboration at Fermilab, a pion radius in good agreement with previous experiments, and for the $\Sigma^-$ a value of $< r^2 >_{\Sigma^-} = (0.60 \pm 0.08 \pm 0.08) \text{fm}^2$, markedly smaller than for the proton.

3 Strangeness

A sizeable strange sea in the nucleon is predicted from both the $\sigma$ term (extracted from pion-nucleon scattering) and deep inelastic lepton scattering (DIS). New and independent information can be obtained from parity-violating electron scattering, $N(\vec{e}, e')N'$, by studying the interference term of virtual photon ($\gamma^*$) and $Z^0$ exchange. Since the $Z^0$ can couple to both vector and axial vector currents, part of this interference term changes sign under an helicity flip of the incident electron. The resulting asymmetry is of order $\mathcal{A} \approx 10^{-4}Q^2/\text{GeV}^2$ and provides information on the electric ($\tilde{G}_E$), magnetic ($\tilde{G}_M$) and axial ($\tilde{G}_A$) form factors as seen by the $Z^0$.

Within the standard model and assuming isospin symmetry, these additional form factors determine the contributions of $u$, $d$ and $s$ quarks,

\[
\begin{align*}
\gamma^* p &\rightarrow G^p = \frac{2}{3} G_u - \frac{1}{3} G_d - \frac{1}{3} G_s \\
\gamma^* n &\rightarrow G^n = \frac{2}{3} G_d - \frac{1}{3} G_u - \frac{1}{3} G_s \\
Z^0 p &\rightarrow \tilde{G}^p = \left( \frac{1}{4} - \frac{2}{3} \sin^2 \theta_W \right) G_u + \left( -\frac{1}{4} + \frac{1}{3} \sin^2 \theta_W \right) (G_d + G_s) , \quad (3)
\end{align*}
\]

with $\theta_W$ the Weinberg angle and $\sin^2 \theta_W \approx 0.2325$.

The pioneering experiment was performed in 1997 at MIT/Bates. Much to the surprise of most theorists, the experiment yielded a positive value of the strange magnetic moment, however with large error bars. In April and May this year, also the HAPPEX collaboration at Jefferson Lab obtained a first result for the asymmetry. Another experiment is being prepared at MAMI by the A4 collaboration.

The status of the field will be reviewed by M. Pitt, and R. Michaels will report on HAPPEX. The latter experiment was performed at $Q^2 = 0.48 \text{ GeV}^2$ and yielded an asymmetry $\mathcal{A}_{\text{raw}} = -5.64 \pm 0.75$ in ppm. However, most of this asymmetry is due to $u$ and $d$ quarks, and the result for the strange sea is still compatible with zero, $G_E + 0.39 G_M = 0.019 \pm 0.035 \pm 0.023 \pm 0.026$, with errors due to the statistics, systematical uncertainties, and the presently bad knowledge of the neutron form factor.

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In the framework of chiral perturbation theory, $G_M(0) = \mu_p^s$ cannot be predicted, because it involves an unknown low-energy constant. However, T. Hemmert will report on a recent calculation in HBChPT that determines the slope of $G_M$ at $Q^2 = 0$. For the central value of the SAMPLE experiment, the extrapolation leads to $0.03 \leq \mu_p^s \leq 0.18$.

4 Polarizabilities

The electric ($\alpha$) and magnetic ($\beta$) polarizabilities of a macroscopic system describe its response to external quasistatic electric ($\vec{E}$) and magnetic ($\vec{H}$) fields. The system rearranges its charge and magnetization distributions, which results in a lowering of the total energy,

$$\Delta E = -\frac{1}{2} \alpha \vec{E}^2 - \frac{1}{2} \beta \vec{H}^2.$$  \hspace{1cm} (4)

For a metal sphere the electric polarizability $\alpha$ is essentially given by the volume $V$, because the charges can easily move to the surface. For a dielectric sphere an additional factor ($\epsilon - 1$) appears, and since the dielectric constant $\epsilon$ is close to 1, $\alpha$ is substantially reduced. In a similar way, the nucleon is a very rigid object. It is difficult to deform its charge distribution, which leads to a small ratio $\alpha/V \approx 2 \cdot 10^{-4}$.

Due to its spin, the nucleon has 4 additional vector (or spin) polarizabilities labeled $\gamma_1$ to $\gamma_4$, in addition to the 2 scalar polarizabilities $\alpha$ and $\beta$. These 6 observables can be measured in Compton scattering by determining the low energy expansion of 6 independent Compton amplitudes. In particular the forward Compton amplitude takes the form

$$T(\omega, \theta) = \vec{e}' \cdot \hat{e} f(\omega) + i \vec{\sigma} \cdot (\vec{e}' \times \hat{e}) g(\omega),$$ \hspace{1cm} (5)

where $\vec{\sigma}$ is the spin of the nucleon, and $\hat{e}$ and $\vec{e}'$ are the polarizations of the photons in the initial and final states, respectively. Since the amplitude has to obey the crossing symmetry, $f(\omega)$ is an even function of $\omega$ and $g(\omega)$ is odd.

As has been stated before, the leading term in $\omega$ is given by static properties of the target, and gauge invariance determines the subleading term in a model-independent way. The dynamical properties of the system appear at relative order $\omega^2$, and are parametrized by the polarizabilities,

$$f(\omega) = -\frac{e^2}{m} + 4\pi(\alpha + \beta) \omega^2 + [\omega^4],$$  \hspace{1cm} (6)
\[ g(\omega) = -\frac{e^2 \kappa^2 \omega}{2m^2} + 4\pi(\gamma_1 - \gamma_2 - 2\gamma_4) \omega^3 + \cdots. \]

In order to determine all 6 polarizabilities independently, it will be necessary to perform double polarization experiments such as \( \vec{\gamma} + \vec{p} \rightarrow \gamma' + p' \). Of particular current interest are the forward and backward spin polarizabilities, given by \( \gamma_0 = \gamma_1 - \gamma_2 - 2\gamma_4 \) and \( \gamma_\pi = \gamma_1 + \gamma_2 + 2\gamma_4 \), respectively. It has been shown that \( \gamma_0 \) is determined by the difference of s-wave pion production (multipole \( E_{0+} \)) and \( \Delta(1332) \) excitation (multipole \( M_{1+} \)), \( \gamma_0 \approx 2.5 \) (s-wave) \(-3.0(\Delta) - 0.1 \approx -0.6 \) (here and in the following in units of \( 10^{-4} \text{fm}^4 \)). While it is difficult to measure \( \gamma_0 \) by Compton scattering, it can be determined from the exclusive structure function \( \sigma'_{TT} \) by a sum rule (see chapter 5). From a combined analysis of (unpolarized) Compton scattering and pion photoproduction, Tonnison et al. have found a value of \( \gamma_\pi = -27.1 \pm 3.4 \)\( \text{fm}^4 \). The theoretical predictions, however, are in the range of \(-40 < \gamma_\pi < -34\), the bulk contribution of about \(-44\) due to the close-by pion pole \( (\pi^0 \rightarrow 2\gamma, \text{Wess-Zumino-Witten term or triangle anomaly}) \). Within the framework of backward dispersion relations, A. L’vov and A. Nathan have included the \( \pi N \) and \( \pi \pi N \) contributions in the \( s \) channel, and \( \pi^0, \eta, \eta' \) in the \( t \) channel, and predicted \( \gamma_\pi = -39.5 \). The puzzle should be solved by polarized Compton scattering. The asymmetry defined by parallel vs. antiparallel spin projections of photon and proton is very large and varies between values of 50% and 80%, depending on the size of \( \gamma_\pi \).

Virtual Compton scattering (VCS), \( \gamma^* + p \rightarrow \gamma + p' \), is realized by the reaction \( e + p \rightarrow e' + p' + \gamma \), with radiation from both the nucleon and the electron. The latter is the Bethe-Heitler process determined by QED. Due to the small mass of the electron it is the dominant process, particularly in the directions of the incident and scattered electrons. Assuming that the information can be read off the interference term between the Bethe-Heitler process and VCS, P. Guichon has expressed the reaction in terms of generalized polarizabilities (GPs), \( \alpha(Q^2) \) etc. These can be obtained by coupling the transverse electric, transverse magnetic and longitudinal multipoles of the incident virtual photon with the transverse electric and magnetic multipoles of the emitted real photon. There are 3 possibilities to couple these multipoles to zero and 7 possibilities to obtain one, resulting in 3 scalar and 7 vector GPs. Since there exist 4 model-independent relations between these GPs, only 6 GPs are independent. However, the unpolarized experiment yields only 3 independent observables, and it will take polarization degrees of freedom to determine all 6 GPs, e.g. the reaction \( e + p \rightarrow e' + p' + \gamma \).

First results of a pilot experiment at MAMI will be shown by N. d’Hose, and J. Friedrich will report on the experimental details. The experiment has...
been performed at fixed momentum of the virtual photon and for real photons with momenta $33.6 \text{ MeV} < q' < 111.5 \text{ MeV}$. As should be expected, the cross sections at low values of $q'$ are essentially determined by QED, but with increasing $q'$ the GPs become visible. Since these effects are of the order of $10^{-38} \text{ cm}^2/\text{MeV sr}^2$ in the differential cross section, the experiment requires a very careful analysis of higher order QED corrections and systematical errors.

Finally, there are two contributions to the Conference regarding the structure of the pion. L.V. Fil'kov and V.L. Kashevarov predict new values for the pion polarizabilities from dispersion relations and discuss the possibility to measure these quantities by the reaction $\gamma + p \rightarrow \pi^+ + \gamma' + n$ at MAMI. Such an experiment was performed at the Lebedev Institute in the 70's with the result $\alpha - \beta = 40 \pm 24$, while ChPT predicts $5.3$ (in units of $10^{-4}$ fm$^3$). Similar and other problems occur in the analysis of two other reactions, radiative pion scattering off a heavy nucleus and pion pair annihilation.

As will be reported by M. A. Moinester, the SELEX/E781 collaboration at Fermilab is investigating the reactions $e\pi \rightarrow e'\pi'\gamma$ and $e\pi \rightarrow e'\pi'\pi^0$ for incident pions of 600 GeV. The experiment is expected to provide new information on the excitation spectrum of the pion, in particular on the GPs of the pion and the chiral anomaly $\gamma^* \rightarrow 3\pi$.

5 Sum Rules

Assuming that $f(\omega)$ is an analytical function as required by causality, one can derive a once-subtracted dispersion relation,

\begin{equation}
\text{Re } f(\omega) = f(0) + \frac{2\omega^2}{\pi} \int \frac{\text{Im } f(\omega')}{\omega'\omega^2} d\omega',
\end{equation}

\begin{equation}
= f(0) + \frac{2}{\pi} \int \frac{\sigma_T(\omega')}{\omega'^2} d\omega' \cdot \omega^2 + [\omega^4],
\end{equation}

involving integration from one-pion threshold, $\omega' = \omega_{thr}$, to infinity. The second line of Eq. (7) follows from unitarity of the $S$ matrix and a Taylor expansion in $\omega$. By comparing this result with the LET, Eq. (6), we find

\begin{equation}
\alpha + \beta = \frac{1}{2\pi^2} \int \frac{\sigma_T(\omega)}{\omega'^2} d\omega,
\end{equation}

which is Baldin’s sum rule \cite{22}. Of course, the power series of Eq. (7) converges only for $\omega < \omega_{thr}$, for larger values of $\omega$ the function turns complex and has an infinite number of branch cuts due to multiple particle production.
The spin-flip amplitude $g$ can be studied by double polarization experiments. The corresponding absorption cross section will be denoted by $\sigma_{1/2}$ and $\sigma_{3/2}$ for antiparallel and parallel spins of photon and target nucleon, respectively. The transverse responses are then given by $\sigma_T = (\sigma_{1/2} + \sigma_{3/2})/2$ and $\sigma'_T = (\sigma_{3/2} - \sigma_{1/2})/2$, and the dispersion relation for $g(\omega)$ may be cast into the form

\[
Re \ g(\omega) = \frac{2\omega}{\pi} \int \frac{Im \ g(\omega')}{\omega'^2 - \omega^2} d\omega' = -\frac{2}{\pi} \int \frac{\sigma'_T(\omega')}{\omega'} d\omega' \cdot \omega - \frac{2}{\pi} \int \frac{\sigma'_T(\omega')}{\omega'^3} d\omega' \cdot \omega^3 + [\omega^5].
\]

Comparing again with Eq. (6), we obtain the relations

\[
I = \int \frac{\sigma_{1/2}(\omega) - \sigma_{3/2}(\omega)}{\omega} d\omega = -\pi e^2 \frac{\kappa^2}{2m^2} \quad (10)
\]

and

\[
\gamma_0 = \frac{1}{4\pi^2} \int \frac{\sigma_{1/2}(\omega) - \sigma_{3/2}(\omega)}{\omega^3} d\omega. \quad (11)
\]

The former relation is the famous GDH sum rule, the second one provides a recipe to calculate the forward spin polarizability. While the pion photoproduction multipoles add in squares in the case of the total absorption cross section $\sigma_T(\omega)$, they carry alternating signs in the case of the spin-flip amplitude,

\[
\sigma_{1/2} - \sigma_{3/2} \sim |E_{0+}|^2 - |M_{1+}|^2 + 6|E_{1+}M_{1+}| + 3|E_{1+}|^2 \pm \ldots \quad (12)
\]

Therefore the sum rules of Eqs. (10) and (11) are very sensitive to small changes in the individual multipoles. Due to the weight factors $\omega^{-1}$ and $\omega^{-3}$, the s-wave threshold amplitude is particularly enhanced.

In the past the GDH integral was evaluated by using the pion photoproduction multipoles of Eq. (12) and some model description for the more-pion channels. The first such calculation was performed by Karliner. In units of $\mu b$ she predicted $I^p = -261$ and $I^n = -183$, at variance with the GDH result $I^p = -205$ and $I^n = -233$. The discrepancy is particularly obvious for the isoscalar-isovector interference $I^p - I^n$, in which case the prediction differs in sign and by a factor of 3. The situation became even more puzzling when Sandorfi et al. evaluated the integral using the SAID multipoles. The result
was $\nu - \nu = -129$, to be compared with +28 according to GDH. As was shown recently, however, a large fraction of the difference is due to the fact that the s-wave multipoles used in the calculation miss the threshold value of the Kroll-Ruderman theorem by about 20%.\[329\]

The pioneering experiment on the helicity structure of photoabsorption was recently performed at MAMI, and H.J. Arends will report on the preliminary results obtained by the GDH collaboration in the energy range $200 \text{ MeV} < \omega < 800 \text{ MeV}$. The data clearly indicate an opposite sign of the $E_{0+}$ vs. $M_{1+}$ contributions below the $\Delta$ resonance and an excess of two-pion production over the one-pion prediction above the $\Delta$. The data collection will be continued at ELSA in order to study whether and how the GDH integral (weighting factor $\omega^{-1}$) saturates at the higher energies. However, already the present data should give a good value for the forward pion polarizability $\gamma_0$ due to weighting factor $\omega^{-3}$.

The sum rules and integrals for real photons can be generalized to the case of virtual photons. The total virtual cross section $\sigma_v$ is given by a flux factor $\Gamma$ and the four response functions mentioned above,

$$\sigma_v = \Gamma \left[ \sigma_T + \varepsilon_L \sigma_L + P_z P_x \sqrt{2 \varepsilon_L (1 - \varepsilon)} \sigma'_{LT} + P_x P_z \sqrt{1 - \varepsilon^2} \sigma'_{TT} \right]. \quad (13)$$

These 4 responses can be separated by varying the (transverse) polarization $\varepsilon$ of the virtual photon as well as the polarizations of the electron ($P_e$) and proton ($P_z$ parallel and $P_x$ orthogonal to the virtual photon, in the scattering plane). The relations between the virtual photon cross sections and the quark spin structure functions $g_1$ and $g_2$ can be read off the following equations, which define a possible generalization of the GDH integral and the BC sum rule,

$$I_1(Q^2) = \frac{2m^2}{Q^2} \int g_1 \, dx = \frac{m^2}{2\pi e^2} \int \frac{(1 - x)}{\nu} \left[ \sigma_{1/2} - \sigma_{3/2} + \frac{Q}{\nu} \sigma'_{LT} \right] \, d\nu , \quad (14)$$

$$I_2(Q^2) = \frac{2m^2}{Q^2} \int g_2 \, dx = \frac{m^2}{2\pi e^2} \int \frac{(1 - x)}{\nu} \left[ -\sigma_{1/2} + \sigma_{3/2} + \frac{\nu}{Q} \sigma'_{LT} \right] \, d\nu .$$

The integrals run over the Bjorken variable $x = Q^2 / 2m\nu$ from 0 to inelastic threshold $x_{thr} < 1$, and over the lab energy $\nu$ from inelastic threshold $\nu_{thr}$ to infinity. In the real photon limit the transverse-longitudinal function $\sigma'_{LT}$ does not contribute to $I_1$, which is then given by the GDH sum rule, $I_1(0) = -\kappa^2 / 4$. In the limit of large $Q^2$, the quark structure functions are supposed to scale, $g_1(x, Q^2) \to g_1(x)$, and $I_1 \to 2m\Gamma_1 / Q^2$ with $\Gamma_1$=const. In the case of the proton, DIS scattering experiments have established that $\Gamma_1 > 0$. Therefore,
the integral \( I_1 \) starts at the (large) negative GDH value for \( Q^2 = 0 \) and approaches (small) positive values for large \( Q^2 \).

The second integral of Eq. (14) can be expressed by the Sachs form factors,

\[
I_2(Q^2) = \frac{G_M(Q^2)(G_M(Q^2) - G_E(Q^2))}{4(1 + \frac{Q^2}{4m^2})},
\]

which is the less familiar BC sum rule. The integral vanishes as \( Q^{-10} \) for large momentum transfer and approaches \( I_2(0) = \frac{1}{4}\mu_k \) for real photons.

There exists an extensive literature on model calculations of the GDH sum rule and its generalizations. For reasons of brevity only a few can be mentioned at this point. The evolution of the generalized GDH integral was studied first by Anselmino in the framework of vector meson dominance

\[\text{Burkert and Ioffe}\]

combined this model with the available information on electroproduction cross sections in the resonance region, and Soffer and Teryaev pointed out the importance of the structure functions \( \sigma'_{LT} \) and \( g_2 \) at intermediate values of \( Q^2 \). The integrals of Eq. (14) were recently calculated in the framework of a gauge invariant and unitarized resonance model that describes the existing electroproduction data for \( \nu \leq 1.1 \text{ GeV} \) and \( Q^2 < 3 \text{GeV}^2 \) quite well.

In particular, the model has the appropriate threshold behaviour as required by the LET, agrees with the data for the \( \Delta \) multipole \( M_{1+}(Q^2) \) and shows the right helicity structure for the second and third resonance regions, the rapid change of the asymmetry from \( \sigma_{3/2} \) to \( \sigma_{1/2} \) dominance with increasing \( Q^2 \). The calculations show that even the small \( \Delta \) multipoles \( E_{1+} \) and \( S_{1+} \), the “bag deformation” in a simple quark model, could affect the integrals of Eq. (14) at intermediate values of \( Q^2 \). These multipoles are now being studied at Jefferson Lab, ELSA, MIT/Bates and MAMI.

Returning to the evolution of the generalized GDH integral \( I_1(Q^2) \), we find the zero-crossing of the integral at \( Q^2_0 \approx 0.8 \text{ GeV}^2 \) if we include the one-pion contribution only. This value is lowered to about 0.5 GeV\(^2\) if we also add the \( \eta \) production and, in a crude model, the more-pion channels. In this intermediate region the recent SLAC experiment yields \( I_1(0.5 \text{ GeV}^2) = 0.10 \pm 0.06 \), while our result is only slightly positive. There are at least two reasons for this (minor) deviation. First, the strong dependence of the zero-crossing on the higher production channels gives rise to uncertainties in our...
model. Second, the error of the experiment is only the statistical one, and systematical errors are estimated to be of equal size. In particular the lack of data points near the $\Delta$ resonance could easily lead to sizeable systematical errors.

The evolution of the generalized GDH integral is of considerable current interest. Its dependence on $Q^2$ describes the transition between the resonance dominated coherent process for small virtuality and the incoherent process of DIS off the constituents at large $Q^2$. Several experiments are planned and partially scheduled at Jefferson Lab to explore the spin structure of the nucleon in the transition region. Their outcome will be quite invaluable for our understanding of the nonperturbative phase of QCD.

References

1. R. G. Arnold et al., Phys. Rev. C 23, 363 (1981).
2. See contributions of H. Schmieden and P. Grabmayr in these proceedings.
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