Recent progress on the $h_1^{q\bar{q}}(x, Q^2)$ Distributions and the Nucleon Tensor Charges

J. Soffer

Centre de Physique Théorique, CNRS Luminy
Case 907, 13288 Marseille Cedex 9, France

Abstract

We recall the definitions and the basic properties of the transversity distributions $h_1^{q\bar{q}}(x, Q^2)$ and the corresponding nucleon tensor charges $\delta q(Q^2)$. We briefly comment on different estimates from several phenomenological models and on the future possible measurements with the polarized $pp$ collider at RHIC-BNL. Recent works on the $Q^2$-evolution of $h_1^{q\bar{q}}(x, Q^2)$ are also discussed and their implications on a very useful positivity bound.

In high-energy processes, the nucleon structure is described by a set of parton distributions, some of which are fairly well known and best determined by means of Deep Inelastic Scattering (DIS). In particular unpolarized DIS yields the quark distributions $q(x)$, for different flavors $q = u, d, s, \text{etc}...$, carrying the fraction $x$ of the nucleon momentum. They are related to the forward nucleon matrix elements of the corresponding vector quark currents $\bar{q}\gamma^\mu q$, and likewise for antiquarks $\bar{q}(x)$. Similarly from longitudinally polarized DIS, one extracts the quark helicity distributions $\Delta q(x) = q_+(x) - q_-(x)$, where $q_+(x)$ and $q_-(x)$ are the quark distributions with helicity parallel and antiparallel to the nucleon helicity. Clearly the spin-independent quark distribution $q(x)$ is $q(x) = q_+(x) + q_-(x)$. We recall that for each flavor, the axial charge is defined as the first moment of $\Delta q(x) + \Delta \bar{q}(x)$ namely,

$$\Delta q = \int_0^1 dx \left[ \Delta q(x) + \Delta \bar{q}(x) \right]$$

(1)

and in terms of the matrix elements of the axial quark current $\bar{q}\gamma^\mu \gamma^5 q$, it can be written in the form

$$2\Delta q s^\mu = \langle p, s | \bar{q}\gamma^\mu \gamma^5 q | p, s \rangle,$$

(2)

where $p$ is the nucleon four-momentum and $s^\mu$ its polarization vector. In addition to $q(x)$ and $\Delta q(x)$, for each quark flavor, there is another spin-dependent distribution for quarks, called the transversity distribution $h_1^q(x)$ related to the matrix elements of the tensor quark current $\bar{q}\sigma^{\mu\nu} i\gamma^5 q$. The $h_1^q$ distribution measures the difference of the number of quarks with transverse polarization parallel and antiparallel to the proton transverse polarization and similarly $h_1^\bar{q}(x)$ for antiquarks. One also defines the tensor charge as the first moment

$$\delta q = \int_0^1 dx \left[ h_1^q(x) - h_1^{\bar{q}}(x) \right],$$

(3)
which receives only contributions from the valence quarks, since those from sea quarks and antiquarks cancel each other due, to the charge conjugaison properties of the tensor current.

The existence of \( h_1^q(x) \) was first observed in a systematic study of the Drell-Yan process with polarized beams \( \square \) and some of its relevant properties were discussed later in various papers \( \square \square \square \). We recall that \( q(x), \Delta q(x) \) and \( h_1^q(x) \), which are of fundamental importance for our understanding of the nucleon structure, are all leading-twist distributions. Due to scaling violations, these quark distributions depend also on the scale \( Q \) and their \( Q^2 \)-behavior is predicted by the QCD evolution equations. They are different in the three cases and we will come back later to this important question. On the experimental side, a vast programme of measurements in unpolarized DIS has been undertaken for more than twenty five years. It has yielded an accurate determination of the \( x \) and \( Q^2 \)-dependence of \( q \) (and \( \bar{q} \)) for various flavors. The \( e\bar{p} \) collider HERA at DESY is now giving us access to a much broader kinematic range for \( x \) down to \( 10^{-4} \) or smaller and for \( Q^2 \) up to \( 5.10^4 GeV^2 \) or so. From several fixed-targets polarized DIS experiments operating presently at CERN, SLAC and DESY, we also start learning about the different quark helicity distributions \( \Delta q(x, Q^2) \), in some rather limited \( x \) and \( Q^2 \) ranges, \( i.e. \), \( 0.005 < x < 0.7 \) and \( < Q^2 > \) between 2 and \( 10 GeV^2 \). Concerning \( h_1^q(x, Q^2) \) (or \( h_1^q(x, Q^2) \)), they are not simply accessible in DIS because they are in fact chiral-odd distributions, contrarely to \( q(x, Q^2) \) and \( \Delta q(x, Q^2) \) which are chiral-even \( \square \). They can be best extracted from polarized Drell-Yan processes with two transversely polarized proton beams. For lepton pair production \( pp \rightarrow \ell^+ \ell^-X \ (\ell = e, \mu) \) mediated by a virtual photon \( \gamma^* \), the double transverse-spin asymmetry \( A_T^{\gamma*} \) reads

\[
A_T^{\gamma*} = \frac{\sum_q e_q^2 h_T^q(x, M^2) h_T^q(x, M^2) + (a \leftrightarrow b)}{\sum_q e_q^2 q(x, M^2) \bar{q}(x, M^2) + (a \leftrightarrow b)},
\]

where \( \hat{a}_{TT} \) is the partonic asymmetry calculable in perturbative QCD and \( M \) is the dilepton mass. The rapidity \( y \) of the dilepton is \( y = x_a - x_b \), and for \( y = 0 \) one has \( x_a = x_b = M/\sqrt{s} \), where \( \sqrt{s} \) is the center-of-mass energy of the \( pp \) collision. Note that this is a leading-order expression, which can be used to get a first estimate of \( A_T^{\gamma*} \) from different theoretical results for \( h_1^q \) and \( h_1^q \). If the lepton pair is mediated by a \( Z \) gauge boson, one has a similar expression for \( A_T^{Z} \), namely

\[
A_T^{Z} = \frac{\sum_q (b_q^2 - a_q^2) h_T^q(x, M_Z^2) h_T^q(x, M_Z^2) + (a \leftrightarrow b)}{\sum_q (b_q^2 + a_q^2) q(x, M_Z^2) \bar{q}(x, M_Z^2) + (a \leftrightarrow b)},
\]

where \( a_q \) and \( b_q \) are the vector and axial couplings of the flavor \( q \) to the \( Z \). However in the case of \( W^\pm \) production one expects \( A_T^{W} = 0 \), because the \( W \) gauge boson is a pure left-handed object (\( i.e. \), \( a_q = b_q \)), which does not allow a left-right interference effect associated to the existence of \( h_1^{q\bar{q}} \). Such experiments will be undertaken with the polarized \( pp \) collider at RHIC-BNL \( \square \), but so far, we have no direct experimental information on the shape, magnitude and \( Q^2 \)-evolution of these quark and antiquark transversity distributions. This is badly needed considering the fact that several theoretical models give rather different predictions for the transversity distributions. For example the MIT bag model \( \square \) leads to \( h_1^q(x) \), which is small for \( x \) near zero and has a maximum value of \( \sim 1.8 \) for \( x \sim 0.4 \). This is in contrast to the QCD sum rules calculations \( \square \), which predict a rather flat behavior for \( h_1^q(x) \) around the value 0.6 for \( 0.2 \leq x \leq 0.5 \). Let us also mention the chiral chromodielectric model \( \square \) which assumes for simplicity that \( h_1^q(x, Q_0^2) \approx \Delta q(x, Q_0^2) \) for a very small scale \( Q_0^2 \), \( e.g. \), \( Q_0^2 = 0.16 GeV^2 \). In this case the shape of \( h_1^q(x) \) is similar to that of the MIT bag with a larger maximum value of 3.8 or so for \( x \sim 0.3 \), as shown in Fig.1. For the \( d \) quark, \( h_1^d(x) \) is negative and smaller in magnitude, following the trend of the corresponding helicity distribution \( \Delta d(x) \). Similarly we expect all the
antiquark transversity distributions $h_1^q$ to be one order of magnitude smaller (see for example Fig.4). The isovector contributions of $h_1^q$ and $h_1^\bar{q}$ have been also calculated in the SU(3) chiral quark-soliton model [3].

Concerning the axial charges and the tensor charges defined above, there are various numerical estimates. In the non-relativistic quark model, they must be equal as a consequence of rotational invariance. For example by using the SU(6) proton wave function one finds,

$$\Delta u = \delta u = 4/3, \quad \Delta d = \delta d = -1/3, \quad \text{and} \quad \Delta s = \delta s = 0.$$  \hspace{1cm} (6)

So in this case the sum of the spin quarks (and antiquarks) is equal to the proton spin at rest since we have

$$\Delta \Sigma \equiv \Delta u + \Delta d + \Delta s = 1,$$ \hspace{1cm} (7)

but we get a wrong value for the axial-vector coupling $g_A = \Delta u - \Delta d = 5/3$. Of course in polarized DIS, one is probing the proton spin in the infinite momentum frame and the above result is surely no longer true. One can evaluate the relativistic effects by making use of the Melosh rotation [10] and one finds for the axial charges [11, 12],

$$\Delta u = 1, \quad \Delta d = -1/4 \quad \text{and} \quad \Delta s = 0.$$ \hspace{1cm} (8)

In this case $g_A$ becomes 5/4, in very good agreement with the experimental value and $\Delta \Sigma$ gets also reduced from 1 to 3/4. Although this shift goes in the right direction, this value is still too
large compared to the data, $\Delta \Sigma \sim 0.3$ or so, and it is very likely that the discrepancy is due to a large contribution from polarized gluons.

The effects of the Melosh rotation on the tensor charges have been calculated in ref. [13] and lead to

$$\delta u = \frac{7}{6} \quad \text{and} \quad \delta d = -\frac{7}{24},$$

in remarkable agreement with the values obtained in the MIT bag model [14]. However in ref. [9] they obtain

$$\delta u = 1.12 \quad \text{and} \quad \delta d = -0.42,$$

but the large $N_c$ behavior is expected to generate in this model, large theoretical uncertainties, mainly for the $d$ quark.

![Diagram](image)

Figure 2: The striped area represents the domain allowed by positivity (see eq.(12)).

Now let us turn to a model-independent result. If we consider quark-nucleon scattering, it can be shown that in the parton model $q(x)$, $\Delta q(x)$ and $h_i^q(x)$ are simply related to the imaginary parts of the three helicity amplitudes $\phi_1$, $\phi_2$ and $\phi_3$ which are the only ones to survive in the forward direction. From the positivity constraints among the $Im\phi_i(0)$’s ($i = 1, 2, 3$), one finds on the one hand the trivial bounds

$$q(x) \geq 0 \quad \text{and} \quad q(x) \geq |\Delta q(x)|,$$

(11)
and on the other hand, the following less obvious inequality

\[ q(x) + \Delta q(x) \geq 2|h^q_1(x)|. \]  

Clearly eq.(12) is more restrictive than the rather trivial bound which has been proposed in ref.[4] similar to eq.(11), namely

\[ q(x) \geq |h^q_1(x)|, \]  

which does not involve \( \Delta q(x) \). We show in Fig.2, the region allowed by eq.(12) which is half the region obtained by assuming eq.(13) instead. Indeed, in the very special situation where \( \Delta q(x) = q(x) \), eqs.(12) and (13) coincide, but it is not generally the case.

![Figure 3: The striped area represents the domain allowed for \( h^q_1(x) \), using eq.(16) and ref.[16].](image)

Needless to say that eq.(12) holds for all quark flavor \( q = u, d, s \) etc..., and as well as for their corresponding antiquarks. Obviously any theoretical model should satisfy these constraints and we shall give some examples. In a toy model [2] when the proton is composed of a quark and a scalar diquark, one obtains the equality in eq.(12). In the MIT bag model, let us recall that these three distributions are expressed in terms of two quantities, namely one has [4]

\[ q(x) = f^2(x) + g^2(x), \quad \Delta q(x) = f^2(x) - 1/3g^2(x) \quad \text{and} \quad h^q_1(x) = f^2(x) + 1/3g^2(x), \]  

so in this case again the inequality (12) is saturated. To illustrate further the practical use of eq.(12), let us assume, as an example, the simple relation

\[ \Delta u(x) = u(x) - d(x) \]  

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proposed in [16] and which is well supported by polarized DIS data. It is then possible to obtain the allowed range of the values for \( h_1^u(x) \) in terms of unpolarized \( u \) and \( d \) quarks distributions since eq.(12) reads now

\[
u(x) - 1/2d(x) \geq |h_1^u(x)|. \tag{16}\]

The allowed region is shown in Fig.3 and one can check, for example, that for \( x \sim 0.4 \) and \( Q^2 = 4 GeV^2 \) we get \( |h_1^u| \leq 1 \), which must be obeyed by any phenomenological model.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{The u quark helicity and transversity distributions \( \Delta u \) and \( h_1^u \), versus \( x \) at the input scale \( Q_0^2 = 0.23 GeV^2 \) and evolved up to \( 25 GeV^2 \). The same for \( \bar{u} \). (taken from ref.[8]).}
\end{figure}

The positivity bound (12) has been rigorously proved in the parton model so one may ask if it could be spoiled by QCD radiative corrections. Some doubts have been expressed in ref. [17], where the authors claim that the status of eq.(12) is similar to that of the Callan-Gross relation [18] which is known to be invalidated by QCD radiative corrections and becomes an approximate equality at finite \( Q^2 \). We will come back to this objection, which will turn out to be not relevant, but meanwhile we want to discuss what is known about the \( Q^2 \) evolution of \( h_1^{qq}(x, Q^2) \) and the corresponding tensor charge \( \delta q(Q^2) \). The Altarelli-Parisi equation for the QCD evolution of \( h_1^{qq}(x, Q^2) \) at order \( \alpha_s \) is \( (t \equiv \log Q^2/\mu^2) \)

\[
\frac{dh_1^{qq}(x,t)}{dt} = \frac{\alpha_s(t)}{2\pi} \int_x^1 \frac{dz}{z} P_h(z)h_1^{qq}(x/z,t) \tag{17}
\]

where the leading order (LO) splitting function \( P_h(z) \), which has been obtained in ref. [2], reads

\[
P_h(z) = \frac{4}{3} \left[ \frac{2}{(1-z)_+} - 2 + \frac{3}{2} \delta(z-1) \right] = P_{qq}^{(0)}(z) - \frac{4}{3}(z-1). \tag{18}\]
Here $P^{(0)}_{qq}(z)$ denotes the unpolarized LO quark-to-quark splitting function calculated in ref. [19] which is also equal to the longitudinally polarized LO splitting function $\Delta_L P^{(0)}_{qq}(z)$ due to helicity conservation. As a consequence of eq.(17) we see that $\Delta q(x, Q^2)$ and $h_1^q(x, Q^2)$ have different $Q^2$ behaviors. In particular, if for a given input scale $Q^2_0$ we have $\Delta q(x, Q^2) \approx h_1^q(x, Q^2)$ after some evolution to $Q^2 > Q^2_0$, one finds that mainly for $x < 0.1$, $h_1^q(x, Q^2)$ rises less rapidly than $\Delta q(x, Q^2)$, as shown for example in Fig.1. This is a general property and in Fig.4, we show for illustration, the difference in the $Q^2$ evolution between $\Delta u, \Delta \bar{u}$ and $h_1^u, h_1^\bar{u}$, for another set of distributions.

![Figure 5: The Drell-Yan double transverse-spin asymmetry $|A_{TT}/a_{TT}|$ (see eq.(4)) for $pp$ collisions at $\sqrt{s} = 100 GeV$, as a function of $x_a - x_b$ (Solid line $M^2 = 25 GeV^2$ and dot-dashed line $M^2 = 100 GeV^2$). For comparison the double helicity asymmetry $|A_{LL}/a_{LL}|$ is shown for $M^2 = 25 GeV^2$ (Dashed line). (taken from ref.[8]).](image)

A further consequence of eq.(18) is the $Q^2$ dependence of the moments of $h_1^q(x, Q^2)$ and in particular the tensor charge which is driven by the anomalous dimension $\gamma_h^b = -2/3$. Actually one finds that, unlike the axial charge $\Delta q(Q^2)$ which remains constant, the tensor charge $\delta q(Q^2)$ decreases with $Q^2$ since we have

$$\delta q(Q^2) = \delta q(Q^2_0) \left[ \frac{\alpha_s(Q^2)}{\alpha_s(Q^2_0)} \right]^{-4/27}.$$  \hspace{1cm} (19)

If one assumes as in ref. [8] that at $Q^2_0 = 0.16 GeV^2$ one has the input tensor charges given by
eq.(5), one gets at \( Q^2 = 25\text{GeV}^2 \)

\[
\delta u = 0.969 \quad \text{and} \quad \delta d = -0.25.
\]  

(20)

The next-to-leading order (NLO) evolution of \( h_1^q(x, Q^2) \) has been obtained in three very recent papers \cite{20, 21, 22}. The results of these two-loops calculations agree and show that, at NLO the tensor charge decreases with increasing \( Q^2 \) even faster than at LO (see Fig.9 in ref. \cite{21}).

Let us now come back to the \( Q^2 \) evolution of the inequality eq.(12). In a recent paper \cite{23}, it was argued, by using eq.(18), that a sufficient condition to insure the validity of eq.(12) at \( Q^2 > Q_0^2 \), if it is valid at \( Q_0^2 \), is that

\[
\frac{|h_1^q|}{dt} < \frac{q_+}{dt},
\]  

(21)

where \( q_+ = 1/2[q + \Delta q] \). Strictly speaking the argument fails because \( P_h(z) \) is not definite positive, but in a recent work \cite{24}, by means of a general mathematical method, it was shown that from the LO and NLO \( Q^2 \) evolutions, if the positivity bound eq.(12) holds at a given \( Q_0^2 \), it is preserved at any \( Q^2 > Q_0^2 \). The same conclusion was reached in ref. \cite{23}, using a numerical method.

Finally some estimates can be made for the double transverse asymmetry in dilepton production (see eq.(4)). Clearly at fixed energy, \( A^{TT}_{TT}/\hat{a}_{TT} \) increases with increasing dilepton mass \( M \), as shown in Fig.5, where we see that at RHIC energies, it will be at most 4\% for \( \sqrt{s} = 100\text{GeV} \) and \( M \sim 10\text{GeV} \). These predictions are confirmed in ref.\cite{25}, also in the case of the \( Z \) production and this small size is due to the small magnitude assumed for \( h_1^q \). Larger estimates (\( \sim 10\% \) or so) have been obtained in ref. \cite{26}, but of course one must wait for the polarized \( pp \) collider at RHIC-BNL to be turned on by year 2000.

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