Massive Spin-2 Supermultiplets

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Abstract

In this paper we construct explicit Lagrangian formulation for the massive spin-2 supermultiplets with $N = k$ supersymmetries $k = 1, 2, 3, 4$. Such multiplets contain $2k$ particles with spin-3/2, so there must exist $N = 2k$ local supersymmetries in the full nonlinear theories spontaneously broken so that only $N = k$ global supersymmetries remain unbroken. In this paper we unhide these hidden supersymmetries by using gauge invariant formulation for massive high spin particles. Such formulation, operating with the right set of physical degrees of freedom from the very beginning and having non-singular massless limit, turns out to be very well suited for construction of massive supermultiplets from the well known massless ones. For all four cases considered we have managed to show that the massless limit of the supertransformations for $N = k$ massive supermultiplet could be uplifted to $N = 2k$ supersymmetry. This, in turn, allows one to investigate which extended supergravity models such massive multiplets could arise from. Our results show a clear connection of possible models with the five-dimensional extended supergravities.

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Introduction

The problem of constructing a consistent interacting theory for massive spin-2 particles is an old but still unsolved one. Massive particle has more degrees of freedom than massless one and as a result even small graviton mass could give observables consequences. So an interesting and important question is how these additional degrees of freedom interact with matter. One of the possible ways in this direction is the investigation of supermultiplets containing massive spin-2 particle. Supersymmetry (especially extended) is a very restrictive symmetry so even the structure of free theories could give interesting and useful information. Surprisingly, there exist quite a few results on this subject. In [1, 2] all states of the first massive level of four-dimensional superstring with \( N = 1 \) supersymmetry was considered and superfield formulation for massive spin-2 \( N = 1 \) supermultiplet appeared in [3].

Massive spin-2 supermultiplets with \( N = k \) supersymmetry contains \( 2^k \) spin-3/2 particles [4, 17]. As consistent description of every spin-3/2 particle requires local supersymmetry there must exist \( N = 2k \) local supersymmetries spontaneously broken so that only \( N = k \) global supersymmetries remain unbroken. In this paper we consider Lagrangian formulation for all massive spin-2 supermultiplets for \( k = 1, 2, 3, 4 \) (multiplets with \( k > 4 \) will contain particles with spin greater than 2). We unhide these hidden extended supersymmetries by using gauge invariant description of massive high spin particles [5, 6]. Such description operating with right number of physical degrees of freedom from the very beginning and having non- singular massless limit turns out to be well suited for the investigation of massive high spin particles and their possible interactions. It could be easily generalized to the higher dimensions as well as (Anti)de Sitter space (e.g. [7]).

Here we use a straightforward generalization to the case of massive supermultiplets. One start with appropriate set of massless supermultiplets, then adds all possible low derivative mass terms to the Lagrangian as well as additional terms to the fermionic supertransformation laws (bosonic supertransformations do not contain derivatives, so there are no corrections to them). The requirement the whole Lagrangian to be invariant under the supertransformations fixes all the unknown coefficients in the Lagrangian and supertransformations. By saying appropriate set of massless supermultiplets we mean not only their number and particle content. The presence of vector fields leads to the possibility to make duality transformations mixing different supermultiplets. The existence of such dual versions of extended supergravities plays very important role in the problem of spontaneous supersymmetry breaking. As we will see for the construction of massive spin-2 supermultiplets it turns out absolutely necessary to choose the correct mixing of vector fields.

In all four cases considered we have managed to show that the massless limit of supertransformations of massive \( N = k \) supermultiplet could be uplifted up to \( N = 2k \) supersymmetry. This allows one to investigate which extended supergravities such massive supermultiplets could arise from and we give explicit examples of such theories having the correct structure of vector fields mixing as well as global symmetries related with the scalar fields. All examples turns out to be the theories that can be obtained by dimensional reduction from the corresponding five- dimensional supergravities.

In the following four sections we carry on such a program for massive spin-2 supermultiplets with \( N = 1, 2, 3, 4 \) correspondingly. Our notations, conventions and some useful formulas are collected in the Appendix.
For \( N = 1 \) supersymmetry massive spin-2 supermultiplet contains \([4, 17]\) four massive fields \((2, 3/2, 3/2, 1)\). As is well known, in the massless limit massive spin-2 particle breaks into the massless ones with spins 2, 1 and 0, massive spin-3/2 particle — into the massless spin-3/2 and spin-1/2, at last, massive spin-1 — into massless spin-1 and spin-0 particles. It is easy to see that in this limit one gets just four massless \( N = 1 \) supermultiplets, namely, gravity multiplet \( h_{\mu\nu}, \Psi_\mu \), spin-3/2 multiplet \( \Phi_\mu, A_\mu \), vector multiplet \( B_\mu, \chi \) and chiral multiplet \( \lambda, \varphi, \pi \). Note that chiral multiplet contains scalar and pseudoscalar fields, which in the massive case play the role of the Goldstone ones, so one of the vector fields has to be axial-vector.

The most general \( N = 1 \) linear global supertransformations leaving the sum of free massless Lagrangians invariant have the form:

\[
\delta h_{\mu\nu} = i(\bar{\Psi}_{(\mu} \gamma_{\nu)} \eta) \\
\delta \Psi_\mu = -\sigma^{\alpha\beta} \partial_\alpha h_{\beta\mu} \eta \\
\delta \Phi_\mu = -\frac{i}{2\sqrt{2}}(\sigma(\cos \theta A - \sin \theta \gamma_5 B)) \gamma_\mu \eta \\
\delta A_\mu = \sqrt{2}\cos \theta (\bar{\Phi}_\mu \eta) + \sin \theta (\bar{\chi} \gamma_\mu \eta) \\
\delta B_\mu = \sqrt{2}\sin \theta (\bar{\Phi}_\mu \gamma_5 \eta) + \cos \theta (\bar{\chi} \gamma_\mu \gamma_5 \eta) \\
\delta \chi = -\frac{1}{2}(\sigma(\sin \theta A + \cos \theta \gamma_5 B)) \eta \\
\delta \lambda = -i\partial^2(\varphi + \gamma_5 \pi) \eta \\
\delta \varphi = (\bar{\lambda} \eta) \\
\delta \pi = (\bar{\lambda} \gamma_5 \eta)
\]

One of the very important points here is the possibility to have a mixing between vector field from the spin-3/2 multiplet and axial-vector one from the vector multiplet, which is a manifestation of the general duality symmetry of supersymmetric theories. As we will see later, the construction of massive supermultiplet turns out to be possible for one concrete value of mixing angle \( \theta \) only.

Another very essential point is the requirement that the model be invariant under the whole \( U(N) \) \( R \)-symmetry of the superalgebra. In the \( N = 1 \) case it is just axial \( U(1) \)-symmetry, the axial charges of all fields being as follows:

| field   | \( \eta, \Psi_\mu, \chi \) | \( \Phi_\mu, \lambda \) | \( h_{\mu\nu}, A_\mu, B_\mu, \varphi, \pi \) |
|---------|----------------------------|-----------------------|----------------------------------|
| axial charge | +1                        | -1                    | 0                                |

Now let us add to the Lagrangian the most general gauge invariant mass terms (see Appendix) compatible with the axial \( U(1) \) invariance:

\[
\frac{1}{m} \mathcal{L}_1 = \sqrt{2}[h^{\mu\nu} \partial_\mu A_\nu - h(\partial A)] - \sqrt{3}A^\mu \partial_\mu \varphi - B^\mu \partial_\mu \pi - \bar{\Phi}_\mu \sigma^{\mu\nu} \Psi_\nu + i\kappa_1(\bar{\Psi} \gamma) \chi + i\kappa_2(\bar{\Phi} \gamma) \lambda + \kappa_3 \bar{\chi} \lambda \\
\frac{1}{m^2} \mathcal{L}_2 = -\frac{1}{2}(h^{\mu\nu} h_{\mu\nu} - h^2) - \sqrt{3}h \varphi + \varphi^2 + \frac{1}{2}B_\mu^2
\]

Note, in particular, that axial \( U(1) \) invariance dictates the Dirac mass term for the gravitini. To make complete Lagrangian invariant under the supertransformations one has to add new
terms to the fermionic transformation laws:

\[ \frac{1}{m} \delta' \Psi = (\alpha_1 A_\mu + \alpha_2 \sigma_{\mu\nu} A^\nu + \alpha_3 \gamma_5 B_\mu) \eta \]
\[ \frac{1}{m} \delta' \Phi = (i \alpha_4 h_{\mu\nu} \gamma^\nu + i \alpha_5 \varphi \gamma_\mu + i \alpha_6 \pi \gamma_5 \gamma_\mu) \eta \]
\[ \frac{1}{m} \delta' \chi = (\beta_1 \varphi + \beta_2 \gamma_5 \pi) \eta \]
\[ \frac{1}{m} \delta' \lambda = (i \beta_3 \hat{A} + i \beta_4 \hat{B} \gamma_5) \eta \]

Simple calculations show that the invariance fixes the mixing angle \( \sin \theta = \sqrt{3}/2, \cos \theta = 1/2 \), as well as all coefficients \( \alpha \) and \( \beta \):

\[ \kappa_1 = \kappa_2 = \sqrt{\frac{3}{2}} \quad \kappa_3 = -2 \]
\[ \alpha_1 = \alpha_2 = -\frac{1}{\sqrt{2}} \quad \alpha_3 = -\sqrt{\frac{3}{2}} \quad \alpha_4 = 1 \quad \alpha_5 = -\frac{3}{2\sqrt{6}} \quad \alpha_6 = \frac{3}{2\sqrt{6}} \]
\[ \beta_1 = -\frac{1}{2} \quad \beta_2 = -\frac{3}{2} \quad \beta_3 = \sqrt{3} \quad \beta_4 = 1 \]

Apart from the \( N = 1 \) global supertransformations given above the Lagrangian is invariant under two local (spontaneously broken) supertransformations:

\[ \delta \Psi = \partial \xi_1 \quad \delta \Phi = \frac{im}{2} \gamma_\mu \xi_1 \quad \delta \chi = \sqrt{\frac{3}{2}} m \xi_1 \]
\[ \delta \Phi = \partial \xi_2 \quad \delta \Psi = \frac{im}{2} \gamma_\mu \xi_2 \quad \delta \lambda = \sqrt{\frac{3}{2}} m \xi_2 \]

One can use these transformations to bring the fermionic laws to more simple and convenient form. For example, making field dependent \( \xi_1 \) transformation with \( \xi_1 = \sqrt{3/2} \gamma_5 \pi \eta \) we obtain:

\[ \delta \Psi = -\sigma^{\alpha\beta} \partial_\alpha h_{\beta\mu} \eta + \sqrt{\frac{3}{2}} \gamma_5 D_\mu \pi \eta - m \gamma_\mu \hat{A} \eta \]
\[ \delta \Phi = -\frac{i}{4\sqrt{2}} (\sigma(A - \sqrt{3} \gamma_5 B)) \eta + im h_{\mu\nu} \gamma^\nu \eta - \frac{3im}{2\sqrt{6}} \varphi \gamma_\mu \eta \]
\[ \delta \chi = -\frac{1}{4} (\sigma(\sqrt{3} A + \gamma_5 B)) \eta - \frac{m}{2} \varphi \eta \]
\[ \delta \lambda = -i \gamma^\mu (\partial_\mu \varphi + \gamma_5 D_\mu \pi) \eta + im \sqrt{3} \hat{A} \eta \]

where \( D_\mu \pi = \partial_\mu \pi - m B_\mu \).

A few comments are in order.

Now the (pseudo)scalar field \( \pi \) enters the Lagrangian and supertransformation laws through the derivative \( \partial_\mu \pi \) only. So one can construct a dual formulation of such model where this field is replaced by the skew-symmetric tensor \( C_{[\mu\nu]} \). Analogous construction for the massive spin-3/2 supermultiplet was considered in [8]. It seems that such formulation could be closer to the results of [3].
As usual in gauge invariant formulation of massive high spin fields one can use local gauge transformations to exclude all the Goldstone fields setting the gauge $A_\mu = \chi = \lambda = \varphi = \pi = 0$. In such a gauge one deals with four physical massive fields $h_{\mu\nu}, \Psi_\mu, \Phi_\mu$ and $B_\mu$ only, but the supertransformation leaving such Lagrangian invariant would be the combination of usual supertransformations given above and (higher derivative) field dependent gauge transformations restoring the gauge. We prefer to work with the complete gauge invariant formulation because it has a non-singular massless limit and so it is very well suited for the investigation of supergravity models which such massive supermultiplets could arise from.

In the massless limit the set of fields used perfectly combines into just two $N=2$ supermultiplets, namely $N=2$ supergravity multiplet and one vector multiplet. Let us stress however that it does not mean that $N=1$ vector and chiral multiplets should belong to the same $N=2$ supermultiplet. Recall that very similar situation appears when one consider the partial super-Higgs effect in $N=2$ supergravity. The massless limit of $N=1$ supergravity plus massive $N=1$ spin-3/2 supermultiplets gives exactly the same set of fields. But to construct the whole interacting theory one has to assign $N=1$ vector and chiral multiplets to two different $N=2$ supermultiplets, namely to vector and hypermultiplet.

Nevertheless, it is interesting to check the existence of minimal model of $N=2$ supergravity with just one vector multiplet having desired properties, the most important of which being the correct mixing of vector and axial-vector fields and global symmetries related with the scalar fields $\varphi$ and $\pi$. Now we will show that such a model really exists. First of all we take massless limit of our $N=1$ supertransformations and uplift them up to $N=2$ supertransformations:

\begin{alignat}{2}
\delta e_{\mu a} &= i(\bar{\Psi}_\mu^i \gamma_a \eta_i) \\
\delta \Psi_{\mu i} &= 2D_\mu \eta_i - \frac{i}{4\sqrt{2}} \varepsilon_{ij}(\sigma(A - \sqrt{3}\gamma_5 B))\gamma_\mu \eta^j - \sqrt{\frac{3}{2}} \gamma_5 \partial_\mu \pi \eta_i \\
\delta A_\mu &= \frac{1}{\sqrt{2}} \varepsilon^{ij}(\bar{\Psi}_{\mu i} \eta_j) + i \frac{3}{2}(\bar{\chi}^i \gamma_\mu \eta_i) \\
\delta B_\mu &= \frac{1}{\sqrt{2}} \varepsilon^{ij}(\bar{\Psi}_{\mu i} \gamma_5 \eta_j) + \frac{i}{2}(\bar{\chi}^i \gamma_\mu \gamma_5 \eta_i) \\
\delta \chi_i &= -\frac{1}{4}(\sigma(\sqrt{3}A + \gamma_5 B))\eta_i - i\varepsilon_{ij}\partial(\varphi + \gamma_5 \pi)\eta^j \\
\delta \varphi &= \varepsilon^{ij}(\bar{\chi}_i \eta_j) \quad \delta \pi = \varepsilon^{ij}(\bar{\chi}_i \gamma_5 \eta_j)
\end{alignat}

where $\Psi_{\mu i} = (\Psi_\mu, \Phi_\mu)$, $\chi_i = (\chi, \lambda)$. Now one can use straightforward Noether procedure to obtain complete nonlinear interacting Lagrangian. The bosonic part of this Lagrangian appears to be:

\begin{alignat}{2}
\mathcal{L}_B &= -\frac{1}{2}R + \frac{1}{2}\partial_\mu \varphi \partial_\mu \varphi + \frac{1}{2}\Phi^{-4} \partial_\mu \pi \partial_\mu \pi - \frac{1}{4} \Phi^6 A_{\mu\nu}^2 - \frac{1}{4} \Phi^2 (B_{\mu\nu} - \sqrt{2}\pi A_{\mu\nu})^2 - \\
&\quad - \frac{1}{4\sqrt{6}} \pi [B_{\mu\nu} \tilde{B}_{\mu\nu} - \sqrt{2}\pi A_{\mu\nu} \tilde{B}_{\mu\nu} + \frac{2}{3} \pi^2 A_{\mu\nu} \tilde{A}_{\mu\nu}] \\
\end{alignat}
while bilinear in fermionic fields part looks as follows:

$$\mathcal{L}_{2F} = \frac{i}{2} \varepsilon^{\mu \nu \alpha \beta} \bar{\Psi}^{i} \gamma_{\mu} D_{\alpha} \Psi_{\beta i} + \frac{i}{2} \chi^{i} \gamma_{\mu} D_{\mu} \chi_{i} -$$

$$\frac{1}{4 \sqrt{2}} \varepsilon^{ij} \bar{\Psi}^{i} \{ \Phi^{3} (A^{\mu \nu} - \gamma_{5} \tilde{A}^{\mu \nu}) + \sqrt{3} \Phi (\gamma_{5} B^{\mu \nu} + \tilde{B}^{\mu \nu}) - \sqrt{6} \Phi \pi (\gamma_{5} A^{\mu \nu} + \tilde{A}^{\mu \nu}) \} \Psi_{j} +$$

$$+ \frac{i}{8} \sqrt{2} \gamma^{\mu} (\sigma [\sqrt{3} \Phi^{3} A + \gamma_{5} \Phi B - \sqrt{2} \gamma_{5} \Phi A]) \Psi_{\mu i} -$$

$$- \frac{1}{4 \sqrt{6}} \varepsilon^{ij} \bar{\chi}_{i} (\sigma [\sqrt{3} \Phi^{3} A + \gamma_{5} \Phi B - \sqrt{2} \gamma_{5} \Phi A]) \chi_{j} -$$

$$- \frac{1}{2} \varepsilon^{ij} \bar{\chi}_{i} \gamma^{\mu} \gamma^{\nu} (\partial_{\mu} \varphi + \gamma_{5} \Phi^{-2} \partial_{\mu} \pi) \Psi_{j} -$$

$$- \frac{i}{4} \Phi^{-2} [\sqrt{3} \varepsilon^{\mu \alpha \beta} \bar{\Psi}^{i} \gamma_{\mu} \Psi_{\beta i} + \frac{1}{\sqrt{6}} \chi^{i} \gamma^{\alpha} \gamma_{5} \chi_{i}] \partial_{\alpha} \pi$$

(7)

Here $\Phi = \exp(-\frac{1}{\sqrt{6}} \varphi)$. This Lagrangian is invariant under the following $N = 2$ local supertransformations:

$$\delta e_{\mu a} = i (\bar{\Psi}^{i} \gamma_{a} \eta_{i})$$

$$\delta \Psi_{\mu i} = 2 D_{\mu} \eta_{i} - \frac{i}{4 \sqrt{2}} \varepsilon^{ij} (\sigma [\Phi^{3} A - \sqrt{3} \gamma_{5} \Phi (B - \sqrt{2} \pi A)]) \gamma_{j} \eta^{i} - \frac{3}{2} \gamma_{5} \Phi^{-2} \partial_{\mu} \pi \eta_{i}$$

$$\delta A_{\mu} = \frac{1}{\sqrt{2}} \Phi^{-3} \varepsilon^{ij} (\bar{\Psi} \gamma_{5} \eta_{j}) + i \frac{3}{2} \Phi^{-1} (\bar{\chi} \gamma_{5} \eta_{i})$$

$$\delta B_{\mu} = \sqrt{3} \frac{1}{2} \Phi^{-1} \varepsilon^{ij} (\bar{\Psi} \gamma_{5} \eta_{j}) + i \frac{2}{3} \Phi^{-1} (\bar{\chi} \gamma_{5} \eta_{i}) + \sqrt{2} \pi \delta A_{\mu}$$

$$\delta \chi_{i} = - \frac{1}{4} (\sigma [\sqrt{3} \Phi^{3} A + \gamma_{5} (B - \sqrt{2} \pi A)]) - i \varepsilon^{ij} \gamma^{\mu} (\partial_{\mu} \varphi + \gamma_{5} \Phi^{-2} \partial_{\mu} \pi) \eta^{j}$$

$$\delta \varphi = \varepsilon^{ij} (\bar{\chi} \eta_{j}) \quad \delta \pi = \Phi^{2} \varepsilon^{ij} (\bar{\chi} \gamma_{5} \eta_{j})$$

Apart from the local supertransformations this Lagrangian is invariant under two global transformations. One of them is a translation:

$\pi \rightarrow \pi + \tilde{\Lambda} \quad B_{\mu} \rightarrow B_{\mu} + \sqrt{2} A_{\mu} \tilde{\Lambda}$

(9)

while another one is a scale transformation:

$$\Phi \rightarrow e^{\Lambda} \Phi \quad A_{\mu} \rightarrow e^{-3 \Lambda} A_{\mu} \quad B_{\mu} \rightarrow e^{-\Lambda} B_{\mu} \quad \pi \rightarrow e^{2 \Lambda} \pi$$

(10)

In extended supergravities the only way to obtain nontrivial scalar field potential, symmetry breaking, masses and so on is the gauging of (part of) the global symmetries. The gauging of the translations like (3) is now rather well known mechanism playing very important role in the problem of spontaneous supersymmetry breaking without a cosmological term [11, 12, 13, 14, 15, 16, 17, 18, 19]. But the gauging of scale transformation like (10) in the usual Poincare supergravities is an open question that requires further investigations.

The model of $N = 2$ supergravity with vector multiplet constructed here is one of the simplest examples of the so called no-scale models [20] tightly connected with the reduction
of $N = 2$ $D = 5$ supergravities [21]. Such connection with $D = 5$ supergravities could hardly be a surprise. In the same way as massive $D = 4$ particles can be constructed by the reduction from massless $D = 5$ ones, massive $D = 4$ supermultiplets could arise from the reduction of appropriate $D = 5$ ones. For the massive spin-3/2 supermultiplet such procedure was considered in [22]. The possibility to obtain full nonlinear massive spin-2 $D = 4$ theory from massless $D = 5$ supergravity by some kind of compactification (for example radial compactification a la [23]) is again an open question.

$$\delta h_{\mu\nu} = i(\bar{\Psi}_{(\mu} i\gamma_\nu)\eta_{i})$$

$$\delta \Psi_{\mu} = -\sigma^{\alpha\beta} \partial_\alpha h_{\beta\mu} \eta_i - \frac{i}{4}(\sigma(C_4 - \gamma_5 C_5))\varepsilon_{ij} \gamma_\mu \eta^j$$

$$\delta C_{4\mu} = \varepsilon^{ij}(\bar{\Psi}_{\mu} \gamma_j \eta_i) + \frac{i}{\sqrt{2}}\varepsilon^{ij}(\bar{\lambda}_i \gamma_\mu \eta_j)$$

$$\delta C_{5\mu} = \varepsilon^{ij}(\bar{\Psi}_{\mu} \gamma_5 \gamma_j \eta_i) + \frac{i}{\sqrt{2}}\varepsilon^{ij}(\bar{\lambda}_i \gamma_\mu \gamma_5 \eta_j)$$

$$\delta \lambda^i = -\frac{1}{2\sqrt{2}}(\sigma(C_4 + \gamma_5 C_5))\varepsilon^{ij} \eta_j - i\gamma_\mu \partial_\mu (\varphi + \gamma_5 \pi) \eta_i$$

$$\delta \varphi = (\bar{\lambda}_i \eta_i), \quad \delta \pi = (\bar{\lambda}_i \gamma_5 \eta_i)$$

Now let us turn to the spin-3/2 supermultiplets. In order to have a possibility to introduce $SU(2)$ invariant (Dirac) mass term for all four gravitini the two spin-3/2 fields of this multiplets should transformed as a doublet under $SU(2)$. In this, vector fields are transformed as a triplet and a singlet. As in the $N = 1$ case it is crucial for the whole construction that one can introduce a mixing between this singlet vector field and a vector field from the remaining vector supermultiplet. Thus our second building block is the doublet of spin-3/2 supermultiplet mixed with one vector supermultiplet:

$$\delta \Phi^i_\mu = -\frac{i}{4}(\sigma C)^a \gamma_\mu (\tau^a)_{ij} \eta^j - \frac{i}{4}(\sigma (\sin \theta A - \cos \theta \gamma_5 B)) \gamma_\mu \eta_i$$

$$\delta C^a_\mu = (\bar{\Phi}_i^a (\tau^a)_{ij} \eta^j) + \frac{i}{\sqrt{2}}(\bar{\lambda}_i \gamma_\mu (\tau^a)_{ij} \eta^j)$$
\[ \delta A_\mu = \sin\theta (\Phi^i \eta_i) + \frac{i}{\sqrt{2}} \sin\theta (\lambda_i \gamma_\mu \eta_i) + i \cos\theta (\bar{\chi}_i \gamma_\mu \eta_i) \]

\[ \delta B_\mu = \cos\theta (\Phi^i \gamma_\mu \eta_i) - \frac{i}{\sqrt{2}} \cos\theta (\bar{\lambda}_i \gamma_\mu \gamma_5 \eta_i) + i \sin\theta (\bar{\chi}_i \gamma_\mu \gamma_5 \eta_i) \]  

(12)

\[ \delta \chi_i = -\frac{1}{2\sqrt{2}} (\sigma C)^a (\tau^a)^j \eta_j - \frac{1}{2\sqrt{2}} (\sigma (\sin\theta A - \cos\theta \gamma_5 B)) \eta_i \]

\[ \delta \bar{\chi}_i = -\frac{1}{2} (\sigma \cos\theta A + \sin\theta \gamma_5 B) \eta_i - i \varepsilon_{ij} \gamma^\mu \partial_\mu (z_4 + \gamma_5 z_5) \eta^j \]

\[ \delta z_4 = \varepsilon^{ij} (\bar{\chi}_i \eta_j) \quad \delta z_5 = \varepsilon^{ij} (\bar{\chi}_i \gamma_5 \eta_j) \]

here \( a = 1, 2, 3 \), and \( (\tau^a)^j \)antihermitian \( 2 \times 2 \) matrices, normalized so that \( Sp(\tau^a \tau^b) = -2 \delta^{ab} \).

The last block is just the hypermultiplet which we choose in the formulation with doublet of spinor fields and triplet and singlet of scalars (so called linear multiplet):

\[ \delta \chi^i = -i \gamma^\mu \partial_\mu (\bar{\varphi} \delta^j + z^a (\tau^a)^j) \eta_j \]

\[ \delta \varphi = (\bar{\chi}^i \eta_i) \quad \delta z^a = (\bar{\chi}^i (\tau^a)^j j) \eta_j \]  

(13)

The structure of these supertransformations unambiguously fixes the axial charges of all fields:

| field            | \( \eta_i, \Psi^i, \lambda_i, \chi_i \) | \( \Phi^i, \lambda^i, \chi^i \) | \( C_4 + \gamma_5 C_5, z_4 + \gamma_5 z_5 \) | others | axial charge |
|------------------|-----------------------------------|---------------------------------|---------------------------------|--------|-------------|

By using \( SU(2) \) and \( U(1) \) properties of all fields we can choose which (combination of) fields will play the role of the Goldstone ones. For the massive spin-2 \( h_{\mu \nu} \) it has to be vector field \( A_\mu \) and some combination of two scalars \( \varphi \) and \( \bar{\varphi} \). For spin-3/2 field \( \Psi^i \) it could be combination of spinors \( \lambda_i \) and \( \chi_i \), while for the \( \Phi^i \) — combination of \( \lambda^i \) and \( \chi^i \). At last for vector fields \( C_\mu^a, C_4 \mu, C_5 \mu, B_\mu \) it will be \( z^a, z_4, z_5 \) and \( \pi \) correspondingly. Having made this assignment we can write the most general mass terms compatible with the \( U(2) \) symmetry:

\[ \frac{1}{m} L_1 = \sqrt{2} [h_{\mu \nu} \partial_\mu A_\nu - h (\partial A)] - \sqrt{3} A^a \partial_\mu \varphi_1 - B_\mu \partial_\mu \pi - C_\mu^a \partial_\mu z^a - C_4 \partial_\mu z_4 - C_5 \partial_\mu z_5 - \Phi^i \sigma^{\mu \nu} \Psi_{\nu i} + i \kappa_1 \bar{\Phi}^i \gamma^\mu \chi_i + i \kappa_2 \bar{\Phi}^i \gamma^\mu \lambda_i + i \kappa_3 \bar{\Phi}^i \gamma^\mu \lambda_i + i \kappa_4 \bar{\Phi}^i \gamma^\mu \lambda_i + i \kappa_5 \bar{\Phi}^i \gamma^\mu \lambda_i + i \kappa_6 \bar{\Phi}^i \gamma^\mu \lambda_i + \kappa_7 \bar{\lambda}^i \lambda_i + \kappa_8 \bar{\lambda}^i \lambda_i \]  

(14)

\[ \frac{1}{m^2} L_2 = -\frac{1}{2} (h_{\mu \nu} h^{\mu \nu} - h^2) - \sqrt{3} h \varphi_1 + \varphi_1^2 + \frac{1}{2} B_\mu^2 + \frac{1}{2} (C_\mu^a)^2 + \frac{1}{2} (C_4)^2 + \frac{1}{2} (C_5)^2 - \frac{1}{2} \varphi^2 \]  

(15)

where \( \varphi_1 = \cos \alpha \varphi + \sin \alpha \bar{\varphi}, \varphi_2 = -\sin \alpha \varphi + \cos \alpha \bar{\varphi} \). To restore the invariance under the global \( N = 2 \) supertransformations we have add appropriate terms to the fermionic transformation laws (see later). In this, supersymmetry fixes both mixing angles: \( \cos \theta = \sin \theta = 1/\sqrt{2}, \cos \alpha = 1/\sqrt{3}, \sin \alpha = \sqrt{2}/3 \), all the coefficients for the mass terms in the Lagrangian:

\[ \kappa_1 = 1, \quad \kappa_2 = \kappa_5 = \frac{1}{\sqrt{2}}, \quad \kappa_3 = \kappa_4 = -\sqrt{2}, \quad \kappa_6 = -\kappa_7 = 1, \quad \kappa_8 = 0 \]
as well as all the coefficients in the fermionic transformation laws. The structure of the fermionic mass terms corresponds to two pairs of local gauge symmetries:

\[
\delta \Psi_\mu = \partial_\mu \eta_i \quad \delta \Phi_\mu^i = \frac{im}{2} \gamma_\mu \xi^i \quad \delta \lambda_i = \frac{m}{\sqrt{2}} \eta_i \quad \delta \chi^i = m \eta_i
\]

\[
\delta \Psi_\mu = \frac{im}{2} \gamma_\mu \xi^i \quad \delta \Phi_\mu^i = \partial_\mu \xi^i \quad \delta \lambda^i = \frac{m}{\sqrt{2}} \xi^i \quad \delta \chi^i = m \xi^i
\]

One can use these freedom to make (field dependent) local transformations to bring supertransformations of fermions to most simple and convenient form:

\[
\delta \Psi_\mu = -\sigma^{\alpha \beta} \partial_\alpha h_{\beta \mu} \eta_i - \frac{i}{4} (\sigma (C_4 - \gamma_5 C_5)) \epsilon_{ij} \gamma_\mu \eta^j + D_\mu z^a (\tau^a)^{ij} \eta_j + \frac{1}{\sqrt{2}} \gamma_5 D_\mu \pi \eta_i - \frac{m}{\sqrt{2}} \gamma_\mu \hat{A} \eta_i
\]

\[
\delta \Phi_\mu^i = -\frac{i}{4} (\sigma C)^a \gamma_\mu (\tau^a)^{ij} \eta_j - \frac{i}{4 \sqrt{2}} (\sigma (A - \gamma_5 B)) \gamma_\mu \eta_i + D_\mu (z_4 + \gamma_5 z_5) \epsilon^{i j} \eta_j + \imath m h_{\mu \nu} \gamma^\nu \eta - \frac{i}{2} \gamma_\mu (\frac{1}{\sqrt{2}} \varphi + \bar{\varphi}) \eta_i
\]

\[
\delta \lambda_i = -\frac{1}{2 \sqrt{2}} (\sigma C)^a (\tau^a)^{ij} \eta_j - \frac{1}{4} (\sigma (A - \gamma_5 B)) \eta_i + \frac{m}{2} \varphi \eta_i - \frac{m}{\sqrt{2}} \bar{\varphi} \eta_i
\]

\[
\delta \chi^i = -\frac{1}{2 \sqrt{2}} (\sigma (A + \gamma_5 B)) \eta_i - \imath \epsilon_{ij} \gamma_\mu \eta^j + \frac{m}{\sqrt{2}} \varphi \eta_i
\]

\[
\delta \chi_i = -\frac{1}{2 \sqrt{2}} (\sigma (C_4 + \gamma_5 C_5)) \epsilon^{i j} \eta_j - \imath \gamma^\mu (\partial_\mu \varphi + \gamma_5 D_\mu \pi) \eta_i + \imath m \hat{A} \eta_i
\]

where \( D_\mu z^a = \partial_\mu z^a - m C_{\mu \alpha}^a \) and so on.

Now let us turn to the question which \( N = 4 \) supergravity theory such a massive supermultiplet could originate from. As we already mentioned at the beginning of this section in the massless limit we have the following massless \( N = 2 \) supermultiplets: \( N = 2 \) supergravity, doublet of spin-3/2 supermultiplets, two vector and one hypermultiplet. Certainly, to have \( N = 4 \) supergravity one has to combine \( N = 2 \) supergravity, two spin-3/2 and one vector supermultiplet. This leaves us with one vector and one hypermultiplet. Not in any way evident that they have to come from one and the same \( N = 4 \) vector supermultiplet, but it is the simplest possibility, so we restrict ourselves to the models of \( N = 4 \) supergravity with one vector supermultiplet. As is well know due to \( O(6) \approx SU(4) \) there exist \( SU(4) \)-invariant formulation of \( N = 4 \) supergravity \( \text{[24]} \). Moreover, it could consistently couples to the arbitrary number of vector supermultiplets, the scalar field geometry being \( SO(6, n)/SO(6) \otimes SO(n) \). But the most general coupling of \( N = 4 \) supergravity with vector supermultiplets constructed in \( \text{[23]} \) heavily depends on the possibility to have a dual mixing of vector fields from the matter supermultiplets with the graviphotons. In this, though the scalar fields geometry remains to be the same, the global symmetry of the whole Lagrangian is in general lower than \( SO(6, n) \). As we have seen above the construction of massive spin-2 supermultiplet requires the mixing of matter vector field with one of the graviphotons. So as a maximum we could have \( SO(5) \approx USp(4) \) symmetry. Now we will show that such a formulation of \( N = 4 \) supergravity with one vector supermultiplet does exist.
First of all, we combine fermions to quartets: \( \Psi_{\mu i} = (\Psi_{\mu i}, \Phi_{\mu i}) \), \( x_i = (x_i, \chi^i) \), \( \lambda^i = (\lambda^i, \lambda_i) \), where \( i = 1, 2, 3, 4 \). Then we introduce quintet of vector fields \( C_\mu = (C_\mu^a, C_\mu^4, C_\mu^5) \), \( a = 1, 2, 3, 4, 5 \), as well as quintet of scalars \( z^a = (z^a, z_4, z_5) \). At last we construct five skew symmetric \( 4 \times 4 \) matrices \( \tau^a_{ij} \):

\[
\tau_{1,2,3} = \begin{pmatrix}
0 & \tau_{1,2,3} & 0 \\
-\tau_{1,2,3} & 0 & 0 \\
0 & 0 & 0
\end{pmatrix},
\tau_4 = \begin{pmatrix}
\varepsilon_{ij} & 0 & 0 \\
0 & -\varepsilon_{ij} & 0 \\
0 & 0 & -\varepsilon_{ij}
\end{pmatrix},
\tau_5 = \begin{pmatrix}
-\varepsilon_{ij} & 0 & 0 \\
0 & -\varepsilon_{ij} & 0 \\
0 & 0 & -\varepsilon_{ij}
\end{pmatrix}
\]

Now the massless limit of supertransformations obtained above could be uplifted to \( N = 4 \) (omitting hats):

\[
\delta \epsilon_{\mu
u} = i(\Psi_{\mu i}^i \gamma_\mu \eta_i)
\]
\[
\delta \Psi_{\mu i} = 2D_\mu \eta_i - i\frac{1}{4}(\sigma C)^a \gamma_\mu (\bar{\tau}^a)_{ij} \eta_j - i\frac{1}{4\sqrt{2}}(\sigma (A - \gamma_5 B)) \gamma_\mu \Omega^{ij} \eta_j + \\
+ \partial_\mu z^a \Omega_{ij} (\bar{\tau}^a)_{jk} \eta_k + \frac{1}{\sqrt{2}} \gamma_5 \partial_\mu \tau_i \eta_i
\]
\[
\delta C_\mu^a = 1 \sqrt{2}(\Psi_{\mu i} \Omega^{ij} \eta_j) + \frac{i}{\sqrt{2}}(\bar{\lambda}_i \gamma_\mu (\bar{\tau}^a)^{ij} \eta_j)
\]
\[
\delta A_\mu = \frac{1}{2\sqrt{2}}(\bar{\Psi}_{\mu i} \Omega^{ij} \eta_j) + \frac{i}{2}(\bar{\lambda}_i \gamma_\mu \Omega^{ij} \eta_j) + \frac{i}{\sqrt{2}}(\bar{\chi}^i \gamma_\mu \eta_i)
\]
\[
\delta B_\mu = \frac{1}{2\sqrt{2}}(\bar{\Psi}_{\mu i} \gamma_5 \Omega^{ij} \eta_j) - \frac{i}{2}(\bar{\lambda}_i \gamma_\mu \gamma_5 \Omega^{ij} \eta_j) + \frac{i}{\sqrt{2}}(\bar{\chi}^i \gamma_\mu \gamma_5 \eta_i)
\]
\[
\delta \chi^i = -\frac{1}{2\sqrt{2}}(\sigma C)^a (\bar{\tau}^a)_{ij} \eta_j - \frac{1}{4}(\sigma (A - \gamma_5 B)) \Omega^{ij} \eta_j - i\gamma_\mu \partial_\mu (\phi + \gamma_5 \pi) \eta_i
\]
\[
\delta \varphi = (\bar{\lambda}_i \eta_i) \delta \pi = (\bar{\chi}_i \gamma_5 \eta_i) \delta \bar{\varphi} = \Omega^{ij}(\bar{\chi}_i \eta_j) \delta z^a = (\bar{\chi}_i (\bar{\tau}^a)^{ij} \eta_j)
\]

Here \( \Omega^{ij} \) is skew symmetric \( USp(4) \) invariant tensor.

Now it is straightforward task by using standard Noether procedure to construct full nonlinear interacting theory. The bosonic part of the Lagrangian turns out to be:

\[
\mathcal{L}_B = -\frac{1}{2} R + \frac{1}{2}(\partial_\mu \varphi)^2 + \frac{1}{2} \Phi^{-4}(\partial_\mu \pi)^2 + \frac{1}{2}(\partial_\mu \bar{\varphi})^2 + \frac{1}{2} \Phi^{-2}(\partial_\mu z^a)^2 - \frac{1}{4} \Phi_2 \Phi^2 A_{\mu \nu}^2 - \frac{1}{4} \bar{\Phi} \Phi^{-2}(B_{\mu \nu} + \sqrt{2} \pi A_{\mu \nu})^2 - \frac{1}{4} \Phi^2 (C_{\mu \nu}^a - \sqrt{2} z^a A_{\mu \nu})^2 + \\
+ \frac{\pi}{2\sqrt{2}} (C_{\mu \nu}^a - \sqrt{2} z^a A_{\mu \nu})(\bar{C}_{\mu \nu}^a - \sqrt{2} \bar{z}^a \bar{A}_{\mu \nu}) - \frac{\bar{z}^a}{\sqrt{2}} C_{\mu \nu}^a \bar{B}_{\mu \nu} + \frac{(z^a)^2}{2} A_{\mu \nu} \bar{B}_{\mu \nu}
\]

where \( \Phi = exp(\frac{1}{\sqrt{2}} \varphi) \), \( \bar{\Phi} = exp(-\bar{\varphi}) \). In this, bilinear in fermionic fields part of the Lagrangian looks as follows:

\[
\mathcal{L}_{2F} = \frac{1}{4} \bar{\Psi}_{\mu i} (V^{\mu \nu} - \gamma_5 V^{\mu \nu})^a (\bar{\tau}^a)^{ij} \Psi_{\nu j} - \frac{1}{4\sqrt{2}} \bar{\Psi}_{\mu i} (U_{\mu \nu} - \gamma_5 \bar{U}_{\mu \nu}) + \Omega^{ij} \Psi_{\nu j} + \\
+ \frac{i}{4\sqrt{2}} \bar{\lambda}_i \gamma^\mu (\sigma V)^a (\bar{\tau}^a)^{ij} \Psi_{\nu j} + \frac{1}{8} \bar{\lambda}_i \gamma^\mu (\sigma U)^{-1} \Omega^{ij} \Psi_{\nu j} + 
\]
where we introduced the following notations:

\[
U_{\mu\nu}^\pm = \tilde{\Phi}[\Phi A_{\mu\nu} \pm \gamma_5 \Phi^{-1}(B_{\mu\nu} + \sqrt{2} \pi A_{\mu\nu})]
\]

\[
V_{\mu\nu}^a = \Phi(C_{\mu\nu}^a - \sqrt{2} z^a A_{\mu\nu})
\]

This Lagrangian is invariant under the following \( N = 4 \) local supertransformations:

\[
\delta e_{\mu r} = i(\bar{\Psi}^i \gamma_r \eta_i)
\]

\[
\delta \Psi_{\mu i} = 2 D_\mu \eta_i - \frac{i}{4} (\sigma V)^a \gamma_{\mu}(\tilde{\pi}^a)^{ij} \eta_j - \frac{i}{4 \sqrt{2}} (\sigma U)^{-\gamma_{\mu} \Omega^{ij}} \eta_j +
\]

\[
+ \tilde{\Phi}^{-1} \partial_\mu z^a \Omega_{ij} (\tilde{\pi}^a)^{jk} \eta_k + \frac{1}{\sqrt{2}} \Phi^{-2} \gamma_5 \partial_\mu \pi \eta_i
\]

\[
\delta C_{\mu}^a = \Phi^{-1}[(\bar{\Psi}^i \gamma_{\mu}(\tilde{\pi}^a)^{ij} \eta_j) + \frac{i}{\sqrt{2}} (\bar{\chi}_i \gamma_{\mu}(\tilde{\pi}^a)^{ij} \eta_j)] + \sqrt{2} z^a \delta A_{\mu}
\]

\[
\delta A_{\mu} = \Phi^{-1} \Phi^{-1} \left[ \frac{1}{\sqrt{2}} (\bar{\Psi}^i \gamma_{\mu} \Omega^{ij} \eta_j) + \frac{i}{2} (\bar{\chi}_i \gamma_{\mu} \Omega^{ij} \eta_j) + \frac{i}{\sqrt{2}} (\bar{\chi}^i \gamma_{\mu} \eta_i) \right]
\]

\[
\delta B_{\mu} = \Phi \tilde{\Phi}^{-1} \left[ \frac{1}{\sqrt{2}} (\bar{\Psi}^i \gamma_{\mu} \gamma_5 \Omega^{ij} \eta_j) - \frac{i}{2} (\bar{\chi}_i \gamma_{\mu} \gamma_5 \Omega^{ij} \eta_j) + \frac{i}{\sqrt{2}} (\bar{\chi}^i \gamma_{\mu} \gamma_5 \eta_i) \right] - \sqrt{2} \pi \delta A_{\mu}
\]

\[
\delta \lambda^i = \frac{1}{4} (\sigma U)^{-\gamma_{\mu} \Omega^{ij}} \eta_j - \frac{1}{2 \sqrt{2}} (\sigma V)^a (\tilde{\pi}^a)^{ij} \eta_j - i \gamma^\mu (\partial_\mu B_{\nu} + \gamma_5 \Phi^{-2} \partial_\mu \pi) \eta_i
\]

\[
\delta \chi_i = - \frac{1}{2 \sqrt{2}} (\sigma U)^{-\eta_i} - i \gamma_\mu (\partial_\mu \bar{\phi} \Omega^{ij} + \tilde{\Phi}^{-1} \partial_\mu z^a (\tilde{\pi}^a)^{ij}) \eta_j
\]

\[
\delta \varphi = (\bar{\lambda} \eta_i) \quad \delta \pi = \Phi^2 (\bar{\chi}_i \eta_i) \quad \delta \bar{\varphi} = \Omega^{ij} (\bar{\chi}_i \eta_j) \quad \delta z^a = \Phi (\bar{\chi}_i (\tilde{\pi}^a)^{ij} \eta_j)
\]

The scalar fields geometry of this model is rather peculiar. Out of eight scalar fields six are related with global translations:

\[
z^a \rightarrow z^a + \Lambda^a \quad C_{\mu}^a \rightarrow C_{\mu}^a + \sqrt{2} A_{\mu} \Lambda^a \quad \pi \rightarrow \pi + \Lambda_0 \quad B_{\mu} \rightarrow B_{\mu} - \sqrt{2} A_{\mu} \Lambda_0
\]

while the remaining two correspond to two "scale" transformations:

\[
\Phi \rightarrow e^{\Lambda} \Phi \quad A_{\mu} \rightarrow e^{-\Lambda} A_{\mu} \quad B_{\mu} \rightarrow e^{\Lambda} B_{\mu} \quad C_{\mu}^a \rightarrow e^{-\Lambda} C_{\mu}^a \quad \pi \rightarrow e^{2\Lambda} \pi
\]

\[
\tilde{\Phi} \rightarrow e^{\tilde{\Lambda}} \tilde{\Phi} \quad A_{\mu} \rightarrow e^{-\tilde{\Lambda}} A_{\mu} \quad B_{\mu} \rightarrow e^{-\tilde{\Lambda}} B_{\mu} \quad z^a \rightarrow e^{\tilde{\Lambda}} z^a
\]

As in the previous case, global symmetries of the Lagrangian, including \( USp(4) \) one, clearly shows that such a model is tightly connected with the \( N = 4 D = 5 \) supergravity \[28, 19\].
3 $N = 3$

In this case massive spin-2 supermultiplet contains the following states: $(2, 6 \times 3/2, 15 \times 1, 20 \times 1/2, 14 \times 0)$. In the massless limit one get additionally $(1, 6 \times 1/2, 16 \times 0)$. Altogether we have $(2, 6 \times 3/2, 16 \times 1, 26 \times 1/2, 30 \times 0)$ that corresponds exactly to the spectrum of $N = 6$ supergravity. But to construct massive $N = 3$ supermultiplet we have to start with $N = 3$ massless supermultiplets. It is easy to see that we have $N = 3$ supergravity multiplet $(2, 3 \times 3/2, 3 \times 1, 1/2),\) triplet of spin-3/2 supermultiplet $3 \times (3/2, 3 \times 1, 3 \times 1/2, 2 \times 0)$ and four vector supermultiplets $4 \times (1, 4 \times 1/2, 6 \times 0)$.

As is well known usual $N = 3$ supergravity is invariant under the real $SO(3)$-subgroup of the whole $U(3)$ $R$-symmetry group only. But as has been shown long time ago [13] there exist dual version of $N = 3$ supergravity with three vector supermultiplets invariant under the whole $U(3)$ group. It turned out that such a version admit the spontaneous supersymmetry breaking with three arbitrary scales, including partial super-Higgs effects $N = 3 \to N = 2$ and $N = 3 \to N = 1$. Moreover this theory can be coupled to arbitrary number of vector supermultiplets [14]. So as our first building block we choose right this system of $N = 3$ supergravity multiplet mixed with three vector supermultiplets:

$$
\begin{align*}
\delta h_{\mu\nu} &= i(\bar{\Psi}_\mu \gamma_\nu \eta_i) \\
\delta \Psi_{\mu i} &= -\sigma^{\alpha\beta} \partial_\alpha \lambda_{\beta \mu} \eta_i + \frac{i}{4} \varepsilon_{ijk}(\sigma(A - \gamma_5 B))_j \gamma_\mu \eta_k \\
\delta A_{\mu j} &= -\varepsilon_{ijk}(\bar{\Psi}_{\mu i} \eta_k) + \frac{i}{\sqrt{2}}(\bar{\chi}^i \gamma_\mu \eta_k) + \frac{i}{\sqrt{2}}(\bar{\rho} \gamma_\mu \eta_j) \\
\delta B_{\mu j} &= -\varepsilon_{ijk}(\bar{\Psi}_{\mu i} \gamma_5 \eta_k) + \frac{i}{\sqrt{2}}(\bar{\chi}^i \gamma_\mu \gamma_5 \eta_k) - \frac{i}{\sqrt{2}}(\bar{\rho} \gamma_\mu \gamma_5 \eta_j) \\
\delta \chi_{ij} &= -\frac{1}{2\sqrt{2}}(\sigma(A + \gamma_5 B))_{ij} \eta_i - \frac{i}{\sqrt{3}} \varepsilon_{ijk} \gamma_\mu \partial_\mu (\varphi + \gamma_5 \pi) \eta_k - \frac{i}{\sqrt{2}} \varepsilon_{ijk} \gamma_\mu \partial_\mu (\varphi_{ij}^j + \gamma_5 \pi_{ij}^j) \eta_l \\
\delta \rho &= -\frac{1}{2\sqrt{2}}(\sigma(A + \gamma_5 B))_{ij} \eta_i \\
\delta \lambda^i &= -\frac{i}{\sqrt{3}} \gamma_\mu \partial_\mu (\varphi - \gamma_5 \pi) \eta_i - \frac{i}{\sqrt{2}} \gamma_\mu \partial_\mu (\varphi_{ij}^j - \gamma_5 \pi_{ij}^j) \eta_j \\
\delta \varphi &= \frac{1}{\sqrt{3}}(\bar{\lambda}^i \eta_i) + \frac{1}{\sqrt{3}} \varepsilon_{ijk}(\bar{\chi}_{ij} \eta_k) \\
\delta \pi &= -\frac{1}{\sqrt{3}}(\bar{\lambda}^i \gamma_5 \eta_i) + \frac{1}{\sqrt{3}} \varepsilon_{ijk}(\bar{\chi}_{ij} \gamma_5 \eta_k) \\
\delta \varphi^a &= \frac{1}{\sqrt{2}}(\bar{\lambda}^i (\tau^a)_{ij} \eta_j) + \frac{1}{\sqrt{2}} \varepsilon_{ijk}(\bar{\chi}_{ij} (\tau^a)_{k} \eta_j) \\
\delta \pi^a &= -\frac{1}{\sqrt{2}}(\bar{\lambda}^i \gamma_5 (\tau^a)_{ij} \eta_j) + \frac{1}{\sqrt{2}} \varepsilon_{ijk}(\bar{\chi}_{ij} \gamma_5 (\tau^a)_{k} \eta_j)
\end{align*}
$$

\begin{tabular}{|c|c|c|c|c|c|}
\hline
field & $\Psi_{\mu i}, \eta_i$ & $\chi_{ij}, \lambda^i$ & $(A_{\mu} + \gamma_5 B_{\mu})_i$ & $\rho$ & others \\
\hline
axial charge & +1 & -1 & -2 & +3 & 0 \\
\hline
\end{tabular}

Now let us turn to the spin-3/2 multiplets. In order to have a possibility to introduce $U(3)$ invariant (Dirac) mass terms for all six gravitini the spin-3/2 fields of these multiplets
should transformed as antitriplet $\Phi_i^a$ under $SU(3)$. But then vector fields are transformed as an octet $C_\mu^a$ and a singlet $B_\mu$ leaving us with the possibility to make dual mixing of this singlet vector with the vector fields of the remaining vector supermultiplet. So our second (and the last) building block will be (anti)triplet of spin-3/2 supermultiplet mixed with one vector supermultiplet:

$$
\delta \Phi_i^a = \frac{i}{4}\gamma_5(\sigma C)^i_j\gamma_\mu\eta^j - \frac{i}{2\sqrt{6}}(\sigma(\sin\theta A - \gamma_5\cos\theta B))\gamma_\mu\eta^i
$$

$$
\delta C_\mu^a = (\Phi_i^a\gamma_5(\tau^a)_i^j\eta^j) + \frac{i}{\sqrt{2}}(\chi_{ij}\gamma_\mu\gamma_5\varepsilon^{ijkl}(\tau^a)_k^i\eta^l)
$$

$$
\delta A_\mu = \frac{2}{\sqrt{6}}\sin\theta(\Phi_i^a\eta^i) + \frac{i}{\sqrt{3}}\sin\theta(\chi_{ij}\gamma_\mu\varepsilon^{ijk}\eta^k) + i\cos\theta(\lambda_\mu\gamma_5\eta^i)
$$

$$
\delta B_\mu = \frac{2}{\sqrt{6}}\cos\theta(\Phi_i^a\gamma_5\eta^i) - \frac{i}{\sqrt{3}}\cos\theta(\chi_{ij}\gamma_\mu\gamma_5\varepsilon^{ijk}\eta^k) + i\sin\theta(\lambda_\mu\gamma_5\eta^i)
$$

$$
\delta \lambda_i = -\frac{1}{2}(\sigma(\cos\theta A + \gamma_5\sin\theta B))\eta^i - i\gamma_\mu\partial_\mu\varepsilon^{ijk}\bar{z}_i\eta^j
$$

$$
\delta \bar{\rho} = -i\gamma_\mu\partial_\mu\bar{z}_i\eta^i \quad \delta \varphi^i = (\bar{\chi}^i_\eta^i) \quad \delta \bar{\pi}^i = (\bar{\chi}^i_\gamma_5\eta^i)
$$

$$
\delta \bar{\varphi}^i = (\bar{\rho}_i\eta^i) + \bar{\gamma}^{ijk}(\bar{\lambda}_i\eta^i) \quad \delta \bar{\pi}^i = -(\bar{\rho}_i\gamma_5\eta^i) + \bar{\gamma}^{ijk}(\bar{\lambda}_i\eta^i)
$$

the axial charges being:

| field | $\Phi_i^a$ | $\chi^{ij}$, $\lambda_i$ | $z_i$, $\bar{z}_i$ | $\bar{\rho}$ | others |
|-------|------------|-------------------------|-----------------|----------|--------|
| axial charge | -1 | +1 | -2 | -3 | 0 |

Now by using transformation properties of all fields we could decide which fields will play the role of the Goldstone ones. For massive spin-2 it will be $A_\mu$ and $\varphi$, for vector fields $B_\mu$, $C_\mu^a$ and $(A_\mu + \gamma_5 B_\mu)^a$ — scalar fields $\pi$, $\pi^a$ and some combination of $z_i$ and $\bar{z}_i$, correspondingly. As for the gravitini fields, for the triplet $\Psi_\mu^i$, it could be some combination of $\lambda_i$ and $\chi^{[ij]}$, while for the antitriplet $\Phi_i^a$ that of $\lambda^i$ and $\chi^{[ij]}$. Then the most general mass terms compatible with the $U(3)$ symmetry could be written as follows:

$$
\frac{1}{m^2}\mathcal{L}_1 = \sqrt{2}[h^{\mu\nu}\partial_\mu A_\nu - h(\partial A)] - \sqrt{3}A_\mu^a\partial_\mu\varphi - B_\mu\partial_\mu\pi + C_\mu^a\partial_\mu\pi^a - A_\mu^i\partial_\mu\varphi^i - B_\mu^i\partial_\mu\pi^i - \nabla_\mu \Phi_i^a\sigma^{\mu
u}\Psi_{\nu i} + i\kappa_1 \Phi_i^a\gamma^\mu\lambda_i + i\kappa_2 \Phi_i^a\gamma^\mu\varepsilon^{ijk}\lambda^j + i\kappa_3 \Phi_i^a\gamma^\mu\varepsilon^{ijk}\lambda^j + i\kappa_4 \Phi_i^a\gamma^\mu\varepsilon^{ijk}\lambda^j + i\kappa_5 \chi^{ij}\lambda_{ij} + i\kappa_6 \chi^{ij}\lambda_{ij} + i\kappa_7 \varepsilon^{ijk}\bar{\chi}^{ij}\lambda_k + i\kappa_8 \varepsilon^{ijk}\bar{\chi}^{ij}\lambda_k + i\kappa_9 \lambda_i^i + i\kappa_{10} \bar{\rho}\bar{\rho}
$$

$$
\frac{1}{m^2}\mathcal{L}_2 = -\frac{1}{2}(h^{\mu\nu}h_{\mu\nu} - h^2) - \sqrt{3}h_\varphi + \varphi^2 + \frac{1}{2}B_\mu^2 + \frac{1}{2}(C_\mu^a)^2 + \nabla_\mu (A_\mu^i)^2 + \nabla_\mu (B_\mu^i)^2 - \frac{1}{2}(\varphi^a)^2 - \frac{1}{2}(\varphi^2)^2 - \frac{1}{2}(\pi^2)^2
$$

here $\varphi^i = \cos\alpha\varphi^i + \sin\alpha\bar{\varphi}^i$, $\varphi_2^i = -\sin\alpha\varphi^i + \cos\alpha\bar{\varphi}^i$ and analogously for $\pi_1^i$ and $\pi_2^i$.

As usual one has to add also additional terms to the fermionic transformation laws (see later). The requirement that the whole massive theory be invariant under the $N = 3$
The structure of the gravitini mass terms obtained corresponds to the invariance under the two triplets of local gauge transformations:

\[
\delta \Psi^i = \partial^i \eta_i, \quad \delta \Phi^{ij}_\mu = \frac{im}{2} \gamma^i \eta_i, \quad \delta \chi^{ij} = \frac{m}{\sqrt{2}} \varepsilon^{ijk} \eta_k, \quad \delta \lambda_i = \frac{m}{\sqrt{2}} \eta_i
\]

Using these symmetries we can by making fields dependent local transformations to bring the fermionic supertransformation laws to most simple form:

\[
\begin{align*}
\delta \Psi^i &= -\sigma^{\alpha^i} \partial^i h_{\alpha^i} + \frac{i}{4} \varepsilon^{ijk} (\sigma(A - \gamma_5 B)) j \gamma^j \eta^k + \\
&\quad + \frac{1}{6} \gamma_5 D_\mu \pi \eta_i - \gamma_5 D_\mu \pi^a (\lambda^a)^j \eta_j - \frac{m}{\sqrt{2}} \gamma^i \hat{A} \eta_i \\
\delta \chi^{ij} &= -\frac{1}{2\sqrt{2}} (\sigma(A + \gamma_5 B)) j \eta_i - \frac{i}{\sqrt{2}} \varepsilon^{ijk} \gamma^j (\partial_\mu \varphi^a + \gamma_5 D_\mu \pi^a) (\lambda^a)^k \eta_j + \\
&\quad - \frac{i}{\sqrt{3}} \varepsilon^{ijk} \gamma^j (\partial_\mu \varphi^a + \gamma_5 D_\mu \pi^a) (\lambda^a)^k \eta_j + im \hat{A} \varepsilon^{ijk} \eta_k + \frac{m}{2} (z_i - \bar{z}_i) \eta_j \\
\delta \chi^{ij} &= -\frac{1}{2\sqrt{2}} \varepsilon^{ijk} \gamma_5 (\sigma C)^j \eta_i - \frac{1}{4\sqrt{3}} (\sigma(\sqrt{3}A - \gamma_5 B)) \varepsilon^{ijk} \eta_k - i \gamma^j D_\mu \varphi^a (\lambda^a)^i \eta_j + \\
&\quad - \frac{m}{2\sqrt{2}} \varepsilon^{ijk} \varphi \eta_k + \frac{m}{\sqrt{2}} \varepsilon^{ijk} \varphi^a (\lambda^a)^i \eta_j \\
\delta \lambda_i &= -\frac{1}{4} (\sigma(A + \sqrt{3} \gamma_5 B)) \eta_i - i \gamma^j D_\mu \varphi^a (\lambda^a)^i \eta_j - \frac{m}{2\sqrt{3}} \varphi \eta_i + \frac{m}{2} \varphi^a (\lambda^a)^i \eta_j \\
\delta \lambda^i &= -\frac{i}{\sqrt{3}} \gamma^j (\partial_\mu \varphi^a - \gamma_5 D_\mu \pi^a) \eta_j - \frac{i}{\sqrt{2}} \gamma^j (\partial_\mu \varphi^a - \gamma_5 D_\mu \pi^a) (\lambda^a)^i \eta_j + \\
&\quad + im \hat{A} \eta_i - \frac{m}{2} \varepsilon^{ijk} (z_j - \bar{z}_j) \eta_k \\
\delta \rho &= -\frac{1}{2\sqrt{2}} (\sigma(A - \gamma_5 B)) \eta_i - \frac{m}{2} (z^i - \bar{z}^i) \eta_i \quad \delta \bar{\rho} = -i \gamma^i D_\mu \bar{z}^i \eta_i
\end{align*}
\]

where \(D_\mu \pi = \partial^i \pi - m B_\mu\) and so on.

As we have already mentioned at the beginning of this section the massless limit of such model gives exactly the set of fields corresponding to \(N = 6\) supergravity. But usual \(N = 6\) supergravity is invariant under the real \(SO(6)\) subgroup of the whole \(SU(6)\) \(R\) symmetry group, in this vector fields are transformed as 15-plet and a singlet under \(SO(6)\). The supertransformations given above clearly shows that it is necessary to have dual mixing of this singlet vector with one of the fifteen vector fields. Now we will show that this corresponds
to dual version of $N = 6$ supergravity invariant under $USp(6)$ group with the vector fields transforming as $14 + 1 + 1$. First of all we combine fermions into the representations of $USp(6)$: $\Psi_{\mu} = (\Psi_{\mu}^i, \Phi_{\mu}^i)$, $\lambda_i = (\lambda_i, \lambda^i)$, $\chi_{ijkl} = (\rho, \chi_{ij}, \chi^j, \bar{\rho})$, where $i = 1, 2, 3, 4, 5, 6$. Then we introduce 14-plet of vector fields $C_{\mu} = ((A_{\mu} + \gamma_5 B_{\mu})^i, C_{\mu}^a)$, where $a = 1, 2, ..., 14$. Analogously, we combine scalar fields into two 14-plets: $\varphi^a = (z_1^i, \varphi^a)$ and $\pi^a = (z_2^i, \pi^a)$, where $z_1, z_2 = \frac{1}{\sqrt{2}} (z^i \pm \bar{z}^i)$. At last we introduce 14 skew symmetric $6 \times 6$ matrices:

$$\Gamma^j = \left( \begin{array}{cc} -\varepsilon_{ijk} & 0 \\ 0 & \varepsilon^{ijk} \end{array} \right), \quad \Gamma^{3+j} = \left( \begin{array}{cc} \varepsilon_{ijk}\gamma_5 & 0 \\ 0 & \varepsilon^{ijk}\gamma_5 \end{array} \right), \quad \Gamma^{6+a} = \left( \begin{array}{cc} 0 & \gamma_5(\lambda^a) \\ -\gamma_5(\lambda^a) & 0 \end{array} \right)$$

satisfying the relation $\Omega\Gamma^\alpha + \Gamma^\alpha\Omega = 0$, where $\Omega$ is skew symmetric invariant tensor of $USp(6)$. Using this variables one can uplift massless limit of supertransformatons obtained above up to full $N = 6$ supersymmetry (omitting hats):

$$\delta e_{\mu\nu} = i(\Psi_{\mu}^i \gamma^i \eta_k)$$

$$\delta \Psi_{\mu}^i = 2D_{\mu}\eta_i - \frac{i}{4}(\sigma C)^a \gamma_\mu (\Gamma^a)^{ij} \eta_j - \frac{i}{4\sqrt{6}}[(\sigma(\sqrt{3}A - \gamma_5 B))\gamma_\mu \Omega^{ij}\eta_j - \partial_\mu \pi \Omega_{ij}\eta_k + \frac{1}{\sqrt{6}}\gamma_5 \partial_\mu \pi \eta_i]$$

$$\delta A_\mu = \frac{1}{\sqrt{2}}(\bar{\Psi}_{\mu} \Omega^{ij}\eta_j) - \frac{i\sqrt{3}}{4}(\chi^{ijk}\gamma_\mu \Omega_{ij}\eta_k) + \frac{i}{2}(\bar{\lambda}^i \gamma_\mu \eta_i)$$

$$\delta B_\mu = \frac{1}{\sqrt{6}}(\bar{\Psi}_{\mu} \gamma_5 \Omega^{ij}\eta_j) + \frac{i}{2\sqrt{2}}(\chi^{ijk}\gamma_\mu \gamma_5 \Omega_{ij}\eta_k) + \frac{i\sqrt{3}}{2}(\bar{\lambda}^i \gamma_\mu \gamma_5 \eta_i)$$

$$\delta C^a_{\mu} = (\bar{\Psi}_{\mu} (\Gamma^a)^{ij}\eta_j) - \frac{i\sqrt{3}}{2}(\chi^{ijk}\gamma_\mu (\Gamma^a)^{ij}\eta_k)$$

$$\delta \chi_{ijk} = \frac{\sqrt{3}}{4}(\sigma C)^a (\Gamma^a)_{[ij} \eta_{k]} + \frac{1}{4\sqrt{2}}[(\sigma(\sqrt{3}A - \gamma_5 B)) \Omega_{[ij} \eta_{k]} - \frac{i}{\sqrt{2}}\gamma_\mu \partial_\mu (\varphi + \gamma_5 \pi) \Omega^{ij} \Omega^{kl} \eta_l + \frac{i\sqrt{3}}{2}\gamma_\mu \partial_\mu (\pi^a + \gamma_5 \varphi^a) (\Gamma^a)^{ij} \Omega^{kl} \eta_l]$$

$$\delta \lambda_i = -\frac{1}{4}(\sigma(A + \sqrt{3}\gamma_5 B)) \eta_i - \frac{i}{\sqrt{3}}\gamma_\mu \partial_\mu (\varphi - \gamma_5 \pi) \Omega^{ij} \eta_j + \frac{i}{\sqrt{2}}\gamma_\mu \partial_\mu (\pi^a - \gamma_5 \varphi^a)(\Gamma^a)^{ij} \eta_j$$

$$\delta \varphi = \frac{1}{\sqrt{3}}(\bar{\lambda}_i \Omega^{ij} \eta_j) + \frac{1}{\sqrt{2}}(\bar{\chi}_{ijk} \Omega^{ij} \Omega^{kl} \eta_l)$$

$$\delta \pi = -\frac{1}{\sqrt{3}}(\bar{\lambda}_i \gamma_5 \Omega^{ij} \eta_j) + \frac{1}{\sqrt{2}}(\bar{\chi}_{ijk} \gamma_5 \Omega^{ij} \Omega^{kl} \eta_l)$$

$$\delta \varphi^a = \frac{1}{\sqrt{2}}(\bar{\lambda}_i \gamma_5 (\Gamma^a)^{ij} \eta_j) - \frac{\sqrt{3}}{2}(\bar{\chi}_{ijk} \gamma_5 (\Gamma^a)^{ij} \Omega^{kl} \eta_l)$$

$$\delta \pi^a = -\frac{1}{\sqrt{2}}(\bar{\lambda}_i (\Gamma^a)^{ij} \eta_j) - \frac{\sqrt{3}}{2}(\bar{\chi}_{ijk} (\Gamma^a)^{ij} \Omega^{kl} \eta_l)$$

In principle by straightforward calculations one can construct full nonlinear interacting model corresponding to such dual version of $N = 6$ supergravity. In this, the scalar fields
\( \pi^a \) will be related with the global translations \( \pi^a \rightarrow \pi + \Lambda^a \), while the scalar fields \( \varphi^a \) will realize the nonlinear \( \sigma \)-model \( SU(6)^* / USp(6) \). Ones again, these properties together with the \( USp(6) \) symmetry show the connection of such theory with the \( N = 6 \) \( D = 5 \) supergravity.

4 \( N = 4 \)

For \( N = 4 \) massive spin-2 supermultiplet contains the following fields: \((2, 8 \times 3/2, 27 \times 1, 48 \times 1/2, 42 \times 0)\). In the massless limit one has additionally \((1, 8 \times 1/2, 28 \times 0)\) which together gives exactly the \( N = 8 \) supergravity multiplet \((2, 8 \times 3/2, 28 \times 1, 56 \times 1/2, 70 \times 0)\). In terms of massless \( N = 4 \) supermultiplets it corresponds to \( N = 4 \) supergravity multiplet \((2, 4 \times 3/2, 6 \times 1, 4 \times 1/2, 2 \times 0)\), four spin-3/2 multiplets \( 4 \times (3/2, 4 \times 1, 7 \times 1/2, 8 \times 0) \) and six vector supermultiplets \( 6 \times (1, 4 \times 1/2, 6 \times 0) \).

As is well known usual \( N = 4 \) supergravity as a maximum could have \( SU(4) \) symmetry. But as we have shown long time ago \([13]\) there exist dual version of the system \( N = 4 \) supergravity plus six vector supermultiplets with the complete \( U(4) \) symmetry. It has been shown that such theory admits spontaneous supersymmetry breaking with four arbitrary scales and without a cosmological term, including all partial super-Higgs effects \( N = 4 \rightarrow N = 3 \), \( N = 4 \rightarrow N = 2 \) and \( N = 4 \rightarrow N = 1 \). Moreover, this theory could be coupled to arbitrary number of vector supermultiplets \([3]\). So our first building block will be this dual version of \( N = 4 \) supergravity multiplet mixed with six vector supermultiplet (see Appendix for notations):

\[
\begin{align*}
\delta h_{\mu \nu} &= i(\overline{\Psi} (\gamma_\mu \gamma_\nu) \eta_i) \\
\delta \Psi^{a} &= -\sigma^{a \beta} \partial_{\alpha} h_{\beta \mu} \eta_i - \frac{i}{4\sqrt{2}} (\sigma V^{a \beta}) \gamma_\mu (\overline{\pi}^{a \beta}) \eta_i \\
\delta A_\mu^{a} &= \frac{1}{\sqrt{2}} (\overline{\Psi} (\gamma_5 \overline{\pi}^{a \beta}) \eta_i) - \frac{i}{\sqrt{2}} (\overline{\chi}^{a \beta} \gamma_\mu \eta_i) - \frac{i}{2} (\overline{\lambda}^{a \beta} \gamma_\mu \gamma_5 \eta_i) \\
\delta B_\mu^{a} &= \frac{1}{\sqrt{2}} (\overline{\Psi} (\gamma_5 \overline{\pi}^{a \beta}) \eta_i) - \frac{i}{\sqrt{2}} (\overline{\chi}^{a \beta} \gamma_\mu \gamma_5 \eta_i) - \frac{i}{2} (\overline{\lambda}^{a \beta} \gamma_\mu \gamma_5 \eta_i) \\
\delta \chi_i^{a} &= \frac{1}{2\sqrt{2}} (\sigma V)^{a \beta} \eta_i - i \gamma^{\mu} \partial_\mu z^{a b} (\overline{\pi}^{b \beta}) \eta_i \\
\delta \lambda^i &= -\frac{i}{4} (\sigma V^{a \beta}) \eta_i - i \gamma^{\mu} \partial_\mu z^{a b} (\overline{\pi}^{b \beta}) \eta_i \\
\delta \varphi &= (\overline{\lambda}^{a \beta} \eta_i) \\
\delta \pi &= (\overline{\lambda}^{a \beta} \gamma_5 \eta_i) \\
\delta z^{a b} &= (\overline{\lambda}^{a \beta} (\overline{\pi}^{b \beta}) \eta_i)
\end{align*}
\]

where \( V_{\mu \nu}^{a} = A_{\mu \nu}^{a} + \gamma_5 B_{\mu \nu}^{a} \).

The remaining fields are just four spin-3/2 supermultiplets. By the same reasoning as in the \( N = 2 \) and \( N = 3 \) cases we choose gravitini to be transformed as \( \Phi_\mu^i \) under \( SU(4) \). In this, vector fields will be transformed as \( 15 \)-plet and a singlet. There are no any other vector supermultiplets left, so we can not make any dual mixing with this singlet vector field, but there exist two possibilities when this singlet field is a vector or axial vector one. Having in mind the role of this field as the (only) candidate for the role of the Goldstone field for massive spin-2, we choose it to be a vector. Then the supertransformations for all fields of
these four spin-3/2 supermultiplets look as follows:

\[
\begin{align*}
\delta\Phi^i_\mu &= -\frac{i}{8}(\sigma C)^{ab}\gamma_\mu(\Sigma^{ab})^i_\eta_j - \frac{i}{4\sqrt{2}}(\sigma A)\gamma_\mu\eta_i \\
\delta C_{\mu}^{ab} &= \frac{1}{2}(\Phi^i_\mu(\Sigma^{ab})^i_\eta_j) + \frac{i}{4}(\Omega^c_\mu\gamma_\mu(\Sigma^{ab})^i_\eta_j) \\
\delta A_\mu &= \frac{1}{\sqrt{2}}(\Phi^i_\mu\eta_i) - \frac{i}{2\sqrt{2}}(\Omega_i^a\gamma_\mu(\tau^a)^i_\eta_j) \\
\delta \Omega^{ia} &= -\frac{1}{8}(\sigma C)^{bc}(\Sigma^{bc})^i_j(\tau^a)^j_k\eta_k + \frac{1}{4\sqrt{2}}(\sigma A)(\tau^a)^i_k\eta_j - \\
&\quad -i\gamma_\mu\partial_\mu\frac{1}{2\sqrt{2}}(\tau^b)^i_j(\tau^c)^j_kz^b + \frac{1}{4\sqrt{3}}(\Gamma^{bcd})^i_j(\tau^a)^j_kz^{bcd}\eta_k
\end{align*}
\]

(28)

In this, all nonzero axial charges will be:

| field         | $\eta_i$, $\Psi_{\mu i}$, $\Omega^{ia}$, $\Phi^i_\mu$, $\chi_i^a$ | $(A_\mu + \gamma_5B_\mu)^a$, $z^a$, $\lambda^i$, $\rho_i$, $z$ |
|---------------|---------------------------------------------------------------|
| axial charge  | $+1$                                                          | $-1$ $-2$ $+3$ $-3$ $-4$ |

Accordingly, the role of Goldstone fields will be played by $A_\mu$ and $Tr(\varphi^{ab})$ for massive spin-2, by $z^a$ and $\pi^{ab}$ for $(A_\mu + \gamma_5B_\mu)^a$ and $C_{\mu}^{ab}$, and by $(\tau^a)^i_j\Omega^{ja}$ and $(\tau^a)^i_j\chi^a$ for $\Psi_{\mu i}$ and $\Phi^i_\mu$, correspondingly. That leads to the following most general form of the mass terms in the Lagrangian:

\[
\frac{1}{m}L_1 = \sqrt{2}[h^{\mu\nu}\partial_\nu A_\mu - h(A\partial A)] - \sqrt{3}A^a\partial_\mu\varphi_0 + C_\mu^{ab}\partial_\mu\pi^{ab} - A_\mu^a\partial_\mu\varphi^a - B_\mu^a\partial_\mu\pi^a - \Phi^i_\mu\sigma^{\mu\nu}\Psi_{\nu i} + i\kappa_1\Phi^i_\mu\gamma_\mu(\tau^a)^i_j\Omega^{ja} + i\kappa_2\Phi^i_\mu\gamma_\mu(\tau^a)^i_j\chi^a + \kappa_3\Omega^i_a\chi^a_i + \kappa_4\Omega^i_a(\tau^a)^i_j\chi^b_j + \kappa_5\lambda^i \rho_i
\]

(29)

\[
\frac{1}{m^2}L_2 = -\frac{1}{2}(h^{\mu\nu}h_{\mu\nu} - h^2) - \frac{3}{2}h\varphi_0 + \varphi_0^2 + \frac{1}{2}(C_\mu^{ab})^2 + \frac{1}{2}(A_\mu^a)^2 + \frac{1}{2}(B_\mu^a)^2 - \frac{1}{2}(\varphi^{ab})^2 - \frac{1}{2}(z^{abc})^2 - \frac{1}{2}z^*z
\]

(30)

where $\varphi_0 = Tr(\varphi^{ab})$, $\varphi^{ab} = \varphi^{ab} - 1/6\delta^{ab}\varphi_0$. By adding appropriate terms to the fermionic transformation laws (see later) global $N = 4$ supersymmetry could be restored, which requires:

$$\kappa_1 = \frac{1}{2}, \quad \kappa_2 = \kappa_4 = -\frac{1}{2}, \quad \kappa_3 = \kappa_5 = -1$$

16
ϕ could be organized into the singlet
Using these notations one can obtain the following
and fixes all the coefficients in the supertransformations. The str ucture of the gravitini mass
terms corresponds to the invariance under the two quartets of local gauge transformations:

$$\delta \Psi_{\mu i} = \partial_\mu \eta_i \quad \delta \Phi^i = \frac{im}{2} \gamma_\mu \eta_i \quad \delta \Omega^{ia} = -\frac{m}{2} (\tau^a)_{ij} \eta_j$$

$$\delta \Psi_{\mu i} = \frac{im}{2} \gamma_\mu \xi^i \quad \delta \Phi^i = \partial_\mu \xi^i \quad \delta \chi^a = \frac{m}{2} (\tau^a)_{ij} \xi^j$$

As usual, we use this freedom to bring the supertransformation laws to most simple form:

$$\delta \Psi_{\mu i} = -\sigma^{a\beta} \partial_\alpha h_{\beta \mu} \eta_i - \frac{i}{4\sqrt{2}} (\sigma V^a) \gamma_\mu (\tau^a)_{ij} \eta_j - \frac{D_\mu \pi^{ab}}{2} (\Sigma^{ab})_{ij} \eta_j - \frac{m}{\sqrt{2}} \gamma_\mu \hat{A} \eta_i$$

$$\delta \Phi^i = -\frac{i}{8} (\sigma C)^{ab} \gamma_\mu (\Sigma^{ab})_{ij} \eta_j - \frac{1}{4\sqrt{2}} (\sigma A) \gamma_\mu \eta_i + \frac{1}{\sqrt{2}} D_\mu \phi^a (\tau^a)_{ij} \eta_j +$$

$$+im \gamma^{\mu \nu} \gamma^{\alpha \beta} \hat{A} (\tau^a)_{ij} \eta_j - \frac{m}{\sqrt{2}} \gamma^{\mu \nu} \gamma^{\alpha \beta} \eta_i$$

$$\delta \chi^a = \frac{1}{2\sqrt{2}} (\sigma V^a) \eta_i - i \gamma_\mu \partial_\mu \phi^a (\tau^a)_{ij} \eta_j +$$

$$-i \gamma_\mu \gamma^{\mu \nu} (\tau^a)_{ij} \eta_j - \frac{1}{\sqrt{2}} (\sigma A) \gamma_\mu \eta_i + \frac{1}{\sqrt{2}} D_\mu \phi^a (\tau^a)_{ij} \eta_j +$$

$$-i \gamma_\mu \gamma^{\mu \nu} (\tau^a)_{ij} \eta_j - \frac{1}{\sqrt{2}} (\sigma A) \gamma_\mu \eta_i + \frac{1}{\sqrt{2}} D_\mu \phi^a (\tau^a)_{ij} \eta_j +$$

$$+m \gamma^{\mu \nu} \gamma^{\alpha \beta} \eta_i$$

$$\delta \Omega^{ia} = -\frac{1}{8} (\sigma C)^{bc} (\Sigma^{bc})_{ij} (\tau^a)_{jk} \eta_k + \frac{1}{4\sqrt{2}} (\sigma A) (\tau^a)_{ij} \eta_j -$$

$$-i \gamma_\mu \gamma^{\mu \nu} (\tau^a)_{ij} D_\mu \phi^a (\tau^a)_{ij} \eta_j +$$

$$-i \gamma_\mu \gamma^{\mu \nu} (\tau^a)_{ij} \eta_j - \frac{1}{\sqrt{2}} (\sigma A) \gamma_\mu \eta_i + \frac{1}{\sqrt{2}} D_\mu \phi^a (\tau^a)_{ij} \eta_j +$$

$$+m \gamma^{\mu \nu} \gamma^{\alpha \beta} \eta_i$$

As we have already noted the set of fields corresponds to $N = 8$ supergravity multiplet.
Now we consider massless limit of the supertransformations obtained and try to uplift them
upto $N = 8$ supertransformations to see which dual version of $N = 8$ supergravity corre-
sponds to our case. First af all we combine eight gravitini into octet $\Psi_{\mu i}, \ i = 1, 2, ... 8,$ vector
fields $A_\mu^a, B_\mu^a$ and $C_\mu^{ab}$ into 27-plet $C_\mu^A, A = 1, 2, ... 27,$ leaving $A_\mu$ as a singlet, all spinor
fields — into completely skew symmetric third rank tensor $\chi^a_{ijk}.$ As for the scalar fields they
could be organized into the singlet $\Phi_0, 27$-plet $z^A$ and completely skew symmetric fourth rank
\Omega-traceless tensor $\Phi_{ij[kl]},$ where $\Omega[ij] — skew symmetric invariant tensor of USp(8). At last
we introduce 27 skew symmetric matrices $(\Gamma^A)_{ij}$:

$$\Gamma^a = \begin{pmatrix} 1 \sqrt{2} \tau^a & 0 \\ 0 & -\frac{1}{\sqrt{2}} \tau^a \end{pmatrix} \quad \Gamma^{6+a} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \gamma_5 \tau^a & 0 \\ 0 & -\frac{1}{\sqrt{2}} \gamma_5 \tau^a \end{pmatrix} \quad \Gamma^{12+(ab)} = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} \Sigma^{ab} \\ -\frac{1}{\sqrt{2}} \Sigma^{ab} & 0 \end{pmatrix}$$

Using these notations one can obtain the following USp(8)-invariant supertransformations
(omitting hats):

$$\delta e_{\mu r} = i (\hat{\Psi}^i_{\mu} \gamma_r \eta_i)$$
\[
\delta \Psi_{\mu i} = 2D_{\mu} \eta_i - i \frac{4}{d} (\sigma C)^{A} \gamma_{\mu} (\bar{\Gamma}^{A})^{ij} \eta_j - i \frac{d}{4 \sqrt{2}} (\sigma A) \gamma_{\mu} \Omega^{ij} \eta_j + \partial_{\mu} z^{A} \Omega_{ij} (\bar{\Gamma}^{A})^{jk} \eta_k
\]

\[
\delta C^{A}_{\mu} = (\bar{\Psi}_{\mu i} (\bar{\Gamma}^{A})^{ij} \eta_j) + i \frac{\sqrt{3}}{2} (\bar{x}^{ijk} \gamma_{\mu} (\Gamma^{A})_{ij} \eta_k)
\]

\[
\delta A_{\mu} = \frac{1}{\sqrt{2}} (\bar{\Psi}_{\mu i} \Omega^{ij} \eta_j) + \frac{i \sqrt{6}}{4} (\bar{x}^{ijk} \gamma_{\mu} \Omega_{ij} \eta_k)
\]

\[
\delta x_{ijk} = - \frac{\sqrt{3}}{4} (\sigma C)^{A} (\Gamma^{A})_{[ij} \eta_k] - \frac{\sqrt{3}}{8} (\sigma A) \Omega_{[ij} \eta_k] - i \gamma^{\mu} \partial_{\mu} [\varphi_0 \Omega^{[ij} \Omega^{kl]} + z^{A} (\bar{\Gamma}^{A})^{[ij} \Omega^{kl]} + \Phi^{ijkl}] \eta_k
\]

\[
\delta \varphi_0 = (\bar{x}^{ijk} \Omega^{ij} \Omega^{kl} \eta_l) \quad \delta z^{A} = (\bar{x}^{ijk} (\bar{\Gamma}^{A})^{ij} \Omega^{kl} \eta_l)
\]

The full interacting version of $N = 8$ supergravity with all required properties ($USp(8)$-invariance, vector fields in 27-plet and a singlet, 27-plet of scalar fields $z^{A}$ entering through the derivatives $\partial_{\mu} z^{A}$ only and 42-plet $\Phi^{ijkl}$, realizing non-linear $\sigma$-model $E_{6,6}/USp(8)$) is already known [27, 28, 18]. As in all previous cases it arises from the dimensional reduction of five-dimensional supergravity.

**Conclusion**

Let us summarize briefly the results of our work. First of all we have seen that the framework based on gauge invariant description of massive gauge particles allows one easily construct massive supermultiplets out of the massless ones. We have seen also that in all cases the possibility to make dual transformations on vector fields plays a very important role. One of the properties of all the models constructed is that the Lagrangian and supertransformations turn out to be invariant under the whole $U(N) R$ symmetry group of $N$ extended superalgebra. As a consequence, the mass terms for the gravitini appears to be the Dirac ones. It resembles very much the situation in supersymmetric gauge theories. As is very well known, the superpartners for the massless vector fields such as photon and gluons are Majorana spinors, while superpartners for the massive one such as $W$ and $Z$ bosons are the Dirac spinors. Analogously, we have shown that the superpartners for massive graviton are Dirac gravitini.

For the massless limits of all four cases $N = k, k = 1, 2, 3, 4$, we have managed to uplift supertransformations up to $N = 2k$ supersymmetry. We did not try to give an exhaustive classification of all possible extended supergravity models which such massive supermultiplets could in principle originate from. But we have shown that there exist at least examples of such theories having desired properties. All this examples are the ones that can be obtained by dimensional reduction from five-dimensional supergravities.

**A Notations and some useful formulas**

In this Appendix we collect our notations, conventions and some useful formulas.

For the flat Minkowski space we use the metric $g_{\mu\nu} = diag(+, -, -, -)$. Throughout the paper we use Majorana representation of Dirac gamma matrices $\gamma_{\mu}$ in which all of them
are imaginary, the charge conjugation matrix being just $\gamma_0$. In this, combinations $\gamma_0\gamma_\mu$ and $\gamma_0\sigma_{\mu\nu}$ are symmetric in their spinor indices, while $\gamma_0$, $\gamma_0\gamma_5$ and $\gamma_0\gamma_5\gamma_\mu$ — skew symmetric. In
this representation Majorana spinors are real, so the $U(N)$ $R$ symmetry group is a so called
Majorana $U(N)$, where imaginary unit $i$ is replaced by $\gamma_5$. In particular, the simplest $U(1)$
symmetry is nothing else but axial transformations acting on spinor fields as

$$\Psi \rightarrow e^{q\gamma_5\Lambda} \Psi$$

where $q$ — axial charge of this field.

A.1 Massive spin-2 particle

In order to have gauge invariant description of massive spin-2 particle with a non-singular
massless limit one has to introduce three fields: symmetric tensor $h_{(\mu\nu)}$, vector $A_\mu$ and scalar
$\varphi$. We start with the sum of standard free massless Lagrangians:

$$L_0 = \frac{1}{2} \partial^a h_{\mu\nu} \partial_{a} h_{\mu\nu} - (\partial h)^{\mu}(\partial h)_\mu + \frac{1}{2} \partial^\mu h \partial_\mu h - \frac{1}{4} (A_{\mu\nu})^2 + \frac{1}{4} \partial^\mu \varphi \partial_\mu \varphi$$

where $A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Now by adding terms with one and no derivatives to the
Lagrangian and appropriate corrections to the gauge transformations one can easily obtain
gauge invariant formulation:

$$L_1 = m\sqrt{2}[h_{\mu\nu}A_\nu - h(\partial A)] - m\sqrt{3}A^\mu \partial_\mu \varphi - \frac{m^2}{2} (h_{\mu\nu}h_{\mu\nu} - h^2) - m^2 \sqrt{3}h \varphi + m^2 \varphi^2$$

in this, the total Lagrangian is invariant under the following gauge transformations:

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu + \frac{m}{\sqrt{2}} g_{\mu\nu} \Lambda \quad \delta A_\mu = \partial_\mu \Lambda + m\sqrt{2} \xi_\mu \quad \delta \varphi = m\sqrt{3} \Lambda$$

A.2 Massless supermultiplet $(2, 3/2)$

As is well known, in supergravity theories (and in general in all gravity theories with spinor
fields) one usually uses so called tetrad formulation of gravity in terms of tetrad $e^a_\mu$ and
Lorentz connection $\omega^{ab}_\mu$. But at the level of free models considered here it turns out to
be convenient to use simply symmetric tensor $h_{\mu\nu}$. In this, the global supertransformations
leaving a sum of massless spin-2 and spin-3/2 Lagrangians invariant has the form:

$$\delta h_{\mu\nu} = i(\bar{\Psi}_{(\mu} \gamma_{\nu)} \eta) \quad \delta \Psi_\mu = -\sigma^{\alpha\beta} \partial_\alpha h_{\beta\mu} \eta$$

One can easily check that the commutator of two supertransformations gives:

$$[\delta_1, \delta_2] h_{\mu\nu} = -2i(\bar{\eta}_2 \gamma^\alpha \eta_1) \partial_\alpha h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

where $\xi_\mu = 2i(\bar{\eta}_2 \gamma^\alpha h_{\alpha\mu} \eta_1)$. 

A.3 Dirac mass term for gravitini

It is very well known that by using Goldstone spinor fields one can construct gauge invariant formulation for one Majorana spin-3/2 particle (gravitino). Indeed, the Lagrangian:

$$L = \frac{i}{2} \varepsilon^{\mu \nu \alpha \beta} \bar{\Psi} \gamma_5 \gamma_{\nu} \partial_{\alpha} \Psi + \frac{i}{2} \tilde{\chi} \gamma^\mu \partial_{\mu} \chi - \frac{m}{2} \bar{\Psi} \sigma^{\mu \nu} \Psi \nu + \text{im} \sqrt{\frac{3}{2}} \bar{\Psi} \gamma^\mu \chi + \bar{\chi} \chi$$

is invariant under the gauge transformations:

$$\delta \Psi = \partial_{\mu} \eta + \frac{\text{im}}{2} \gamma_{\mu} \eta \quad \delta \chi = m \sqrt{\frac{3}{2}} \eta$$

But if we have two gravitini with equal masses there exist two possibilities. The first one is just the sum of two copies of the Lagrangian given above. Another one is a Dirac mass term $m \bar{\Psi} \sigma^{\mu \nu} \Phi$, which could be diagonalized into $-\frac{m}{2} \bar{\Psi} \sigma^{\mu \nu} \Psi \nu + \frac{m}{2} \Phi \sigma^{\mu \nu} \Phi$ (note the sign difference). The Lagrangian for such case can also be constructed in a similar manner:

$$L = L_0(\Psi, \Phi, \chi, \lambda) - m \bar{\Psi} \sigma^{\mu \nu} \Phi \nu + \text{im} \sqrt{\frac{3}{2}} (\bar{\Psi} \gamma) \chi + \text{im} \sqrt{\frac{3}{2}} (\bar{\Phi} \gamma) \lambda - 2 \bar{\chi} \lambda$$

which is invariant under the two gauge transformations:

$$\delta \Psi = \partial_{\mu} \eta + \frac{\text{im}}{2} \gamma_{\mu} \xi \quad \delta \Phi = \partial_{\mu} \xi + \frac{\text{im}}{2} \gamma_{\mu} \eta \quad \delta \chi = m \sqrt{\frac{3}{2}} \eta \quad \delta \lambda = m \sqrt{\frac{3}{2}} \xi$$

In all massive spin-2 supermultiplets we have pairs of gravitini with opposite axial charges so the Dirac mass term turns out to be the only possibility.

A.4 $SO(6) \approx SU(4)$ matrices

For $N = 4$ supersymmetry we use six skew symmetric matrices $(\tau^a)_{[ij]}$, $a = 1, 2, 3, 4, 5, 6$, $i, j = 1, 2, 3, 4$, satisfying the relation:

$$(\tau^a)_{ij}(\bar{\tau}^b)^{jk} + (\tau^b)_{ij}(\bar{\tau}^a)^{jk} = -2 \delta_i^k \delta^{ab}$$

where $(\bar{\tau}^a)^{ij} = \frac{1}{2} \varepsilon^{ijkl}(\tau^a)_{kl}$. The commutator $\tau$-matrices defines 15 antihermitian matrices:

$$(\Sigma^{[ab]})_{i}^{k} = \frac{1}{2}[(\tau^a)_{ij}(\bar{\tau}^b)^{jk} - (\tau^b)_{ij}(\bar{\tau}^a)^{jk}]$$

which play the role of $SO(6) \approx SU(4)$ generators. Besides, we need 20 symmetric matrices:

$$(\Gamma^{[abc]})_{(ij)} = (\tau^a)_{ik}(\bar{\tau}^b)^{jl}(\tau^c)_{lj}$$

which are self-dual in a sense that:

$$\Gamma^{abc} = \frac{1}{6} \gamma_5 \varepsilon^{abcdef} \Gamma^{def}$$

Let us give here some useful formulas with these matrices:

$$(\bar{\tau}^a)^{ij}(\tau^b)^{jk}(\bar{\tau}^a)^{kl} = 4(\bar{\tau}^b)^{il}$$

$$(\bar{\tau}^c)^{ij}(\Sigma^{ab})_{j}^{k}(\tau^c)_{kl} = 2(\Sigma^{ab})_{i}^{i}$$

$$(\bar{\tau}^d)^{ij}(\Gamma^{abc})_{jk}(\bar{\tau}^d)^{kl} = 0$$

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