Qualitative evaluation of product innovation on the basis of pattern recognition theory

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Abstract. The article systematizes the quality indicators of innovative products and proposes an approach to assessing the error of product quality levels.

1. Introduction
In a modern economy, innovation is a key tool to increase the competitive advantage of a company. In economic science, research is currently underway in the field of innovation management in conjunction with business development strategies and effective management systems, and, above all, development innovations are aimed at improving quality.

The development of innovative projects requires not only a review of existing systems and quality management standards, but also the development and implementation of new ones. In the current conditions, in order to obtain market advantages, in addition to strictly observing the requirements of working standards and standards, it is also necessary to take into account the ever-increasing requirements of customers of design services in terms of cost, timing, manufacturability, compliance with environmental requirements, etc. In modern market relations, one of the most significant components of product competitiveness is its quality. Various methods and tools have been developed for quality control. First of all, these are checks, control charts, Pareto diagrams, time series analysis, correlation and regression analysis, etc. However, all of them, firstly, are of a private nature, and secondly, they do not allow one and the same approach to be applied. assessment of the quality of various products, especially high technology [1].

Each of the groups of indicators characterizing quality includes the parameters of purpose, reliability, and others related to the technical, economic, service, and economic and commercial properties of the products. In addition, at each stage of the product life cycle, its features should be taken into account and these groups should be supplemented with relevant indicators. For example, a group of technical indicators of manufacturing quality characterizing the actual performance of products should be replenished with coefficients of stability of quality, defectiveness, etc. a group of operational and technical indicators characterizing the actual operational characteristics during the period of operation, - indicators of manufacturability of repairs and maintenance, anti-corrosion coating coefficients; and a
group of technical indicators of the quality of utilization - the coefficients of manufacturability of utilization, recycling, etc [2].

Therefore, assessing the quality of products on a wide range of its indicators, and ultimately its competitiveness (level of competitiveness) involves the development of modern scientifically based methods.

The problem of choosing quality indicators is one of the key in the formulation of the problem of assessing product quality (KP). A preliminary analysis of groups of indicators of product competitiveness made it possible to determine the main factors affecting product quality, namely: technical, organizational, economic and social (table) [3].

**Table 1. The influence of key factors on specific indicators product quality.**

| Factors       | Technical                                | Organizational                          | Economic                                  | Social                                    |
|--------------|------------------------------------------|------------------------------------------|-------------------------------------------|-------------------------------------------|
| Type of manufactured products and serial production. Status of technical documentation. The quality of technological equipment, tools. The condition of the test equipment. The quality of measurement and control. The quality of raw materials, raw materials, components | Provision with materials, raw materials, etc. Maintenance of equipment, tooling, etc. Systematic and rhythmic work. Organization of work with suppliers. Organization of information support. Scientific organization of labor, production culture. | The form of remuneration and the amount of wages. Bonus for high quality work and products. Retention for marriage. The ratio between KP, cost and price. Organization and carrying out cost accounting | The state of educational work. Selection, placement and movement of frames. Organization of study and advanced training. Organization and conduct of the competition. The relationship in the team. Housing conditions. Organization of rest |

2. Method

Depending on who the indicated groups of indicators will be used by (manufacturer or consumer), the composition of the groups and the range of indicators may be different, but in general they can be used in determining any of the quality levels.

Each of the main groups of indicators of product quality, classified by the homogeneity of the characteristics described, contains a number of subgroups of indicators: purpose; reliability; economical use of resources, energy; ergonomics; aesthetics; environmental friendliness; security; patent law indicators; standardization and unification; manufacturability; transportability; reuse or disposal (destruction); service indicators.

These quality indicators mainly characterize high-tech products, which are inherent in all stages of the life cycle.

In the future, we will present quality in a compact form in the form of a vector, the components of which are individual quality indicators.

Denote quality indicators by the following symbols: \( x_1 \) - purpose; \( x_2 \) - reliability; \( x_3 \) - economical use of resources, energy; \( x_4 \) - ergonomics; \( x_5 \) - aesthetics; \( x_6 \) - environmental friendliness; \( x_7 \) - security; \( x_8 \) - patent law indicators; \( x_9 \) - standardization and unification; \( x_{10} \) - manufacturability; \( x_{11} \) - transportability; \( x_{12} \) - secondary use or disposal (destruction); \( x_{13} \) - service indicators (not included in the level of trade and technical maintenance).

The state vector in our case has dimension \( m = 13 \) in the total number of indicators. Then quality as a vector quantity can be written in the form:

\[
x = [x_1, x_2, ..., x_{13}]^T.
\]

The number of components of the vector \( x \) can be any, it depends on the specific type or type of product. Therefore, in the general case, we assume that the vector \( x \) has a dimension equal to \( m \), i.e.

\[
x = [x_1, x_2, ..., x_m]^T.
\]
The values \( x_j, j = 1, m \), of the components of the vector \( x \) are random in nature. Firstly, these values are determined by objects from random samples; secondly, the measurements themselves or the calculations of individual quality indicators are random in nature.

From the entire list of indicators described above for a particular type of product, the most informative are selected by a comparative analysis of their weighting or significance coefficients, which are calculated, for example, by expert assessment.

We represent the quality scale in the form of intervals, which in the future will be called quality levels. Note that intervals can be intersecting. In this case, the task of assessing product quality is the statistical identification of its level of quality.

It is proposed to evaluate product quality based on the theory of pattern recognition. We will consider quality levels (excellent, good, satisfactory, unsatisfactory) as recognition objects, and quality indicators (signs) of the object as signs of recognition objects. Often reliable information on a priori information is missing. In this case, as a decisive classification rule, according to which the calculated values of the feature vector are compared with the reference ones, it is most advisable to choose a rule based on the criteria for the maximum likelihood function.

The recognition system is based on the reference features of objects for each class. The probabilistic nature of the features requires knowledge of the mathematical expectation of the feature vector and the scatter or relationships within the class defined by the covariance matrix. From a priori data, a complete set of hypotheses for classes is formed. Design features are compared with the reference. It is possible to assess the values of the reference parameters for each level of product quality based on expert assessments. At a minimum of the distance between the measured feature vector and the reference one, a decision is made on whether the observation belongs to a hypothesis [4].

The mathematical model for observing the feature vector is taken in the form:

\[
\tilde{x} = x + \xi
\]  

where
\( \tilde{x} \) - observing a feature vector, for example, \( \tilde{x} = (\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_m) \);
\( \xi \) - centered random error vector.

The problem of identifying the state vector of the control object \( x \) is formulated as follows.

From the only implementation \( \tilde{x} \) the additive model (1), it is necessary to determine the mathematical expectation vector \( \mathbf{M}(x/\Omega) \) and the covariance matrix \( \mathbf{R}(x/\Omega) \) of the vector \( x \), then choose the most probable hypothesis.

In general, we define the complete set of hypotheses

\[ G = \{ \Omega_0, ..., \Omega_k, ..., \Omega_l \}, \ k = 0, ..., (l-1) \]

it is assumed that conditional multidimensional distribution density of the vector of reference features is equal to

\[
P(x/\Omega_k) \sim \mathcal{N}\left[ \mathbf{M}(x/\Omega_k), \mathbf{R}(x/\Omega_k) \right]
\]  

and depends on the hypothesis (\( \Omega_k \)), which takes place during the measurement. In distribution (2) with \( \mathbf{M}(x/\Omega_k) \) the vector of mathematical expectations of reference features is indicated, and with \( \mathbf{R}(x/\Omega_k) \) - covariance matrix of the feature vector.

Let an arbitrary hypothesis take place during measurements \( \Omega_k \in G \), then \( x \) can be represented as follows:

\[
\tilde{x} = \mathbf{M}(x/\Omega_k) + \xi.
\]

Thus, the random vector \( x \) belongs to one of \( l \) sets (hypotheses), and \( x \) is distributed normally with a density

\[
P(x/\Omega_k) = (2\pi)^{-m/2} | \mathbf{R}(x/\Omega_k) |^{-1/2} \exp\left[ -\frac{1}{2} (x - \mathbf{M}(x/\Omega_k))^\top \mathbf{R}^{-1}(x/\Omega_k) (x - \mathbf{M}(x/\Omega_k)) \right].
\]  

(3)
It is reasonable to attribute the observation \( \tilde{x} \) to the hypothesis for which the likelihood function is maximal. The maximum likelihood function is achieved by minimizing the functional

\[
J_k = \| \tilde{x} - M(x/\Omega_k) \|^2_{R(x/\Omega_k)}^{-1}
\]

To solve the problem, the minimum functional is found \( J_k \). The most likely vector of mathematical expectations \( M(x/\Omega_k) \) and the most probable covariance matrix \( M(x/\Omega_k) \) determined by the most likely hypothesis \( (\Omega_k) \), selected as a result of the operation \( \min J_k \).

In what follows, we will call the estimated function \( J_k \) the estimated functional, or simply the functional:

\[
J_k = (\tilde{x} - M_k)^T R_k^{-1}(\tilde{x} - M_k),
\]

where \( R_k = R(x/\Omega_k), M_k = M(x/\Omega_k) \).

Since the quality scale is presented in the general case in the form of intersecting intervals, errors in the adoption of a specific hypothesis arise in the quality assessment process.

Let us evaluate the error in the classification of quality levels with the average probability of error, which allows one to estimate in advance the quality of features and the decision rule for the accepted number of hypotheses. To calculate the average probability of error \( P_{err} \), consider the following quantities: \( P_{kq} \) - conditional probabilities that in the presence of a hypothesis \( \Omega_q \) hypothesis accepted \( \Omega_k \); \( L_{kq} \) - sign of loss of wrong decision.

We write the mathematical expectation of losses

\[
R_{\text{losses}} = M(L_{kq}) = \sum_{k=0}^{l-1} \sum_{q=0}^{l-1} L_{kq} P_{kq}
\]

which is usually called medium risk. For a simple feature function, we can now assume

\[
L_{kq} = \begin{cases} 
0, & k = q \\
1, & k \neq q
\end{cases}
\]

Then the risk is equal to the average probability of error \( P_{err} \), wrong decision

\[
P_{err} = \sum_{k=0}^{l-1} \sum_{q=0}^{l-1} P_{kq}, \text{ at } k \neq q
\]

If conditional joint distribution is known \( P(J/\Omega_q) \) valuation function vectors \( J = (J_0, ..., J_{k-1}) \), then

\[
P_{kq} = \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} P(J/\Omega_q) dJ_0 \ldots dJ_{k-1} dJ_{k-1} \ldots dJ_{l-1} dJ_k
\]

The value of arbitrary functionality \( J_k \) (4) we will consider it as a random variable.

To calculate the integral (6), we translate the coordinates of the vector \( J \) into an orthonormal basis using the Karunen - Loev transformation [5]: \( I = CJ \), where the matrix \( C \) is composed of columns of orthogonalized eigenvectors of the covariance matrix \( K_J \) (conditional covariance matrix of a vector \( J \)).

\[
K_J = [K_{k_{n/q}}] = [M((J_k^q - M_k^q)(J_n^q - M_n^q))^T]
\]

Important that the matrix \( \Lambda = CK_J C^T \) - diagonal, and on the diagonal are the eigenvalues of the matrix \( K_J \). Let them \( \lambda_{k/q} \),
5

\( k = 0, 1, 2, \ldots, (l-1) \).

Probability \( P_{kq} \) defined as

\[
P_{kq} = \frac{1}{\sqrt{2\pi\lambda_{k/q}}} \int_{-\infty}^{\infty} \prod_{s=0}^{l-1} \Phi \left( \frac{l_k - \bar{M}_{s/q}}{\sqrt{\lambda_{s/q}}} \right) e^{-\frac{(l_k - \bar{M}_{s/q})^2}{2\lambda_{s/q}}} \, dl_k =
\]

\[
= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-y^2/2} \prod_{s=0}^{l-1} \Phi \left( \frac{y\sqrt{\lambda_{s/q}} + \bar{M}_{s/q} - \bar{M}_{k/q}}{\sqrt{\lambda_{s/q}}} \right) \, dy,
\]

(7)

Where \( \bar{M}_{s/q} = [C(M_q - M_s)]^T [CRC^T]^{-1} [C(M_q - M_s)] \);

\( \bar{M}_{k/q} = [C(M_q - M_k)]^T [CRC^T]^{-1} [C(M_q - M_k)] \);

\( \lambda_{s/q} \) – eigenvalues of the conditional covariance matrix \( K_j \);

\( y = \frac{l_k - \bar{M}_{k/q}}{\lambda_{k/q}} \)

\( \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp \left( -\frac{t^2}{2} \right) \, dt \)

3. Results

Expressions (5) - (7) can be used to assess the levels of the indicator of product quality and their significance. The dependence of the average probability of error on the number of tested hypotheses and the number of signs is presented in figure 1 and 2.

**Figure 1.** Dependence of the probability of erroneous recognition on the number of tested hypotheses.

**Figure 2.** The dependence of the probability of erroneous recognition on the number of signs.
The abscissa on the graphs shows the value

$$M/\sigma = \sum_{q=0}^{l-1} \sum_{s=0}^{l-1} \sum_{s \neq k} \frac{(|\bar{M}_{s,q} - \bar{M}_{k,q}|)}{\sqrt{\lambda_{s,q}}}.$$  

From the graphs shown in figure 1, it is seen that with an increase in the number of hypotheses tested, the probability of erroneous recognition increases. Dependencies in figure 2 show the feasibility of using several features (indicators of product quality).

4. Conclusions
In conclusion, we note that the calculation of particular indicators $x_j$, $j = 1, m$, in the general case, it is a rather complicated task, since the latter are multiparametric. To determine them, the approach described above may be applicable. The reference features for each group of indicators of product quality are one of the known methods, for example, the method of expert assessments.

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