RELATIVISTIC ELECTRON SHOCK DRIFT ACCELERATION IN LOW MACH NUMBER GALAXY CLUSTER SHOCKS

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Received 2011 April 5; accepted 2011 August 23; published 2011 November 3

ABSTRACT

An extreme case of electron shock drift acceleration (SDA) in low Mach number collisionless shocks is investigated as a plausible mechanism for the initial acceleration of relativistic electrons in large-scale shocks in galaxy clusters, where the upstream plasma temperature is of the order of 10 keV and the degree of magnetization is not too small. One-dimensional electromagnetic full particle simulations reveal that, even when a shock is rather moderate, a part of the thermal incoming electrons are accelerated and reflected through relativistic SDA and form a local non-thermal population just upstream of the shock. The accelerated electrons can self-generate local coherent waves and further be back-scattered toward the shock by those waves. This may be a scenario for the first stage of the electron shock acceleration occurring at the large-scale shocks in galaxy clusters, such as CIZA J2242.8+5301, which have well-defined radio relics.

Key words: acceleration of particles – galaxies: clusters: general – plasmas – radio continuum: galaxies – relativistic processes – shock waves

Online-only material: color figures

1. INTRODUCTION

In galaxy clusters, the presence of relativistic electrons with energies of around GeV has been implied by radio synchrotron emission (e.g., Willson 1970; Govoni & Feretti 2004; Fujita & Sarazin 2001). Although some possible acceleration mechanisms for those relativistic electrons have been proposed, they are still controversial topics (for a recent review, see Ferrari et al. 2008). The diffuse radio emission in galaxy clusters is categorized into several types, such as jet robes from active galactic nuclei, radio halos located at the center of galaxy clusters, and radio relics located at the cluster periphery. In particular, the origin of radio relics has been a major mystery. Recent observations suggest that they are associated with large-scale shocks, which are also thought to be possible sources of the ultra-high-energy cosmic rays above $\sim 10^{18.5}$ eV (Kang et al. 1997; Inoue et al. 2007). The existence of shocks in the radio relics implies that the high-energy electrons are most likely produced through the diffusive shock acceleration process (Enßlin et al. 1998; Miniati et al. 2001; Gabici & Blasi 2003; Keshet et al. 2004). In fact, radio shells showing the spectral softening due to the synchrotron and the inverse-Compton coolings are observed in CIZA J2242.8+5301 (van Weeren et al. 2010), while a temperature jump is found in A3667 (Finoguenov et al. 2010). Observations suggest that the temperature jump is $T \approx 9$ keV, the magnetic field strength is $B \approx 5 \mu$G, and the Mach number is $M \approx 4.5$ for CIZA J2242.8+5301, and $T \approx 2$–5 keV, $B \gtrsim 3 \mu$G, $M \approx 2$, and the number density is $n \approx 10^{-5}$–10$^{-4}$ cm$^{-3}$ for A3667.

According to the standard theory of structure formation, galaxy clusters result from mergers of smaller structures. High Mach number shocks ($M > 10$) are formed in accreting small halos and around the virialized core of the galaxy cluster. They can accelerate cosmic rays to ultra-high energies with a harder energy spectrum. On the other hand, low Mach number shocks ($M \sim 3$–4) are formed in the virialized core. During the mergers and accretions, the most kinetic energy of the accreting matter is dissipated by relatively low Mach number shocks with $M \lesssim 4$ (Miniati et al. 2000; Ryu et al. 2003; Kang et al. 2007). These shocks with $M \approx 3$–4 are needed in order to explain spectral indices of radio relics (Gabici & Blasi 2003). In fact, the shock velocity in the radio relics of the galaxy cluster is of the order of $10^3$ km s$^{-1}$ and the corresponding Mach numbers are about $M \approx 2$–4.5 (van Weeren et al. 2010; Finoguenov et al. 2010). The shock velocity is similar to what is observed for young supernova remnant (SNR) shocks, which generate galactic cosmic-ray electrons (Koyama et al. 1995) as well as nuclei (e.g., Ohira et al. 2011). However, the Mach number of a galaxy cluster shock is much smaller than that of an SNR shock because of a high upstream temperature as well as the so-called magnetization parameter, $\sigma \equiv B^2/4\pi n_1 m_e c^2$. Here, $B_1$, $n_1$, $m_e$, and $c$ denote upstream magnetic field, plasma density, electron rest mass, and the speed of light, respectively. Then, for the galaxy clusters, $T \approx 1$–10 keV and $\sigma \sim 0.1$ with $B_1 = 3 \mu$G and $n_1 = 10^{-4}$ cm$^{-3}$. Such small values of Mach numbers are of the same order as those of the shocks observed in the heliosphere, i.e., the terrestrial bow shock and interplanetary shocks in which electron acceleration is rarely observed (Shimada et al. 1999; Oka et al. 2006, 2009).

The diffusive shock acceleration is one of the most plausible acceleration mechanisms of charged particles at a shock front (e.g., Blandford & Eichler 1987), where a resultant energy spectrum of the high-energy particles obeys a power law. For SNR shocks, galactic cosmic rays are thought to be accelerated through this process. It is not resolved, however, as to how a part of the particles originally composing a thermal background plasma is embedded in the diffusive shock acceler-
tron injection is thought to be more difficult than ion injection because of cross-shock potential. Nevertheless, relativistic electrons are believed to be accelerated in SNR shocks (Koyama et al. 1995; Bamba et al. 2003; Pannuti et al. 2010). Although a variety of theoretical as well as simulation studies have revealed that microinstabilities in the shock transition region play a crucial role, the electron injection process, even in extremely high Mach number shocks, has not yet been understood completely (e.g., Papadopoulos 1988; Cargill & Papadopoulos 1988; Levinson 1992; Shimada & Hoshino 2000; Dieckmann et al. 2000; Hoshino & Shimada 2002; Ohira & Takahara 2007, 2008; Umeda et al. 2008, 2009; Amano & Hoshino 2007, 2009, 2010; Morlino 2009; Riquelme & Spitkovsky 2010).

In the above context, electron injection in low Mach number shocks is inferred to be even harder, since free energy for microinstabilities in the shock transition region should be much smaller than that in high Mach number shocks. Although a variety of microinstabilities get excited in some helsiospheric shocks (e.g., Wu et al. 1984), it is known, for instance, that the saturation level of the modified two-stream instability, which is one of the most plausible instabilities in shocks observed near the Earth, is not as high as that of the Buneman instability in SNR shocks (Matsukiyo 2010). Given this perspective, an injection process of electrons in the galaxy cluster shocks is shrouded in mystery. Indeed, this problem has never been addressed. Here, we note that the upstream temperature of the cluster shocks is much higher than those of SNR and helsiospheric shocks. As can be seen in the next section, the high upstream temperature enables some electrons to be accelerated to relativistic energies via the so-called shock drift acceleration (SDA).

In this paper, we investigate a limiting case of the electron SDA and associated microprocesses which may be responsible for the injection of electrons into the diffusive shock acceleration in the galaxy cluster shocks. In Section 2, the SDA process is briefly reviewed and extended to an extreme case. Simulation settings and results are shown in Section 3. Then, discussions and a summary are given in Section 4.

2. SHOCK DRIFT ACCELERATION

2.1. Some Basic Features

SDA is one of the efficient acceleration mechanisms of charged particles in the transition region of an oblique collisionless shock. The motion of non-relativistic electrons in this process was analyzed in detail previously (e.g., Leroy & Mangeney 1984; Wu 1984; Krauss-Varban et al. 1989; Krauss-Varban & Wu 1989; Krauss-Varban & Burgess 1991). In the so-called normal incidence frame (NIF), where the upstream flow direction is parallel to a shock normal, the acceleration occurs while an electron stays in the shock transition region and drifts along the shock surface due to a finite gradient of magnetic field strength. The electron gains energy since the direction of the drift motion is anti-parallel to the motional electric field.

Assuming that the coplanar magnetic field for a one-dimensional shock is in the x–z plane and the shock normal is along the x-axis, then

\[ \dot{y} = \frac{\mathbf{p} \cdot \dot{\mathbf{p}}}{m_e c^2 y} = -\frac{e}{m_e c^2} \mathbf{v} \cdot \mathbf{E} = -\frac{e}{m_e c^2} \left( -\frac{\partial \Phi}{\partial x} + v_y E_0 \right). \]  

(1)

Here, \( \gamma, \mathbf{p}, \mathbf{v}, \) and \( e \) are the Lorentz factor, momentum, velocity and elementary charge, respectively, \( \mathbf{E} \) is the electric field, and the dots denote the time derivative. Furthermore, \( \mathbf{E} = -(\partial \Phi/\partial x, E_0, 0) \) has been used, where \( \Phi \) and \( E_0 \) denote shock potential and the motional electric field. This results in

\[ \dot{W} = \frac{d}{dt} \left( \gamma - \frac{e \Phi}{m_e c^2} \right) = -\frac{e}{m_e c^2} v_y E_0. \]

(2)

Some incoming electrons gain significant energy during the drift motion and are reflected backward from the shock. These reflected electrons become a non-thermal component in the upstream plasma frame. The reflection occurs due to the magnetic mirror effect. The de Hoffmann–Teller frame (HTF), where the upstream flow is along the magnetic field, is convenient to consider the motion of a particle because the motional electric field disappears in this frame. The particle velocity in HTF \((\mathbf{v}^\prime)\) and NIF \((\mathbf{v})\) is related as \( \mathbf{v}^\prime = \mathbf{v} + \mathbf{V}_{HT} \), where \( \mathbf{V}_{HT} = (V_1 \times B_1) / \mathbf{n} / (B_1, \mathbf{n}) \) is the de Hoffmann–Teller velocity. \( \mathbf{n} \) is the unit vector normal to the shock, and \( \mathbf{V}_1 \) and \( B_1 \) are the upstream flow velocity and the magnetic field in the NIF, respectively. In the HTF, the right-hand side of Equation (2) becomes zero so that \( \dot{W} \) is conserved. Moreover, if the motion of the electron is assumed to be adiabatic, magnetic moment, \( \mu = p^2 / 2m_e B^2 \), is also conserved. Here, the prime denotes a quantity measured in the HTF. This assumption is valid while the spatial scale of a shock transition region is sufficiently larger than the Larmor radius of the electron. From these two restrictions, for instance, if the motion to the electron to be mirror reflected, the initial pitch angle, \( \alpha' = \tan^{-1}(p_{11}^\prime / p_{12}^\prime) \), should satisfy the following condition (cf. Feldman et al. 1983 for a non-relativistic version):

\[ \alpha' > \sin^{-1} \frac{\sqrt[2]{B_1^2 (\gamma' + \Delta \Phi)^2 - 1}}{\gamma'}. \]

(3)

Here, \( B^\prime \) denotes the magnetic field strength, \( \gamma' = p'/m_e c \), \( \Delta \Phi' = e(\phi' - \phi)/m_e c^2 \), and the subscripts 1 and 2 indicate upstream (initial) and downstream quantities, respectively. If an overshoot exists, however, \( B_1^\prime \) and \( \phi' \) should be defined there.

The energy gain in the SDA process is obtained by integrating Equation (2) in the NIF, \( m_e c^2 W(t) = -e \int E dt \), as the energy gained by an electron during its drift in the transition region along the direction anti-parallel to the motional electric field (for details see Krauss-Varban & Wu 1989). If one moves in the HTF, the energy gain is explained by the well-known Fermi acceleration mechanism (Wu 1984). If a particle velocity, before it’s reflected, is written as \( \mathbf{v}_{pre}, \mathbf{v}_{pre}' = \mathbf{v}_{pre} + \mathbf{V}_{HT} \), then after being reflected, the parallel velocity is reversed so that \( \mathbf{v}_{post}' = -(\mathbf{v}_{pre}' - \mathbf{V}_{HT}) \). Then, moving back in the NIF results in \( \mathbf{v}_{post}' = \mathbf{v}_{pre}' - \mathbf{V}_{HT} \). Now, \( \mathbf{V}_{HT} = (0, 0, U_1 \tan \Theta_{GR}) \) when \( V_1 = U_1 \mathbf{n} \) and \( \mathbf{n} \) is along the x-axis. Hence, the acceleration occurs mainly in the z-direction. The more \( \mathbf{V}_{HT} \) increases, the larger the energy gain of the particle becomes. According to Krauss-Varban et al. (1989), the acceleration time through this process is typically of the order of an ion gyro period, \( \sim \Omega_i^{-1} \), which will also be confirmed in our simulation later. For a typical intra-cluster magnetic field of \( \mu G \), this corresponds to about a few 100 s, which is much shorter than any timescales of incoherent loss processes.

If \( \Delta \Phi' = 0 \) is assumed, \( \alpha' > \alpha'' \equiv \sin^{-1}(B_1^\prime / B_1) \) is the necessary condition for an electron to be reflected. When a magnetic overshoot is neglected for simplicity, \( B_1^\prime \) or the critical pitch angle called a loss-cone angle, \( \alpha'' \), can be determined.
The bottom panel shows a schematic of a loss-cone in the center corresponds to the bulk velocity of the upstream plasma in space is indicated as the broken circle in the bottom panel. Its radius of 3 \( \alpha \) electron beta values with \( \sigma = 10^{-4} \) (top panel) and \( \sigma = 3^{-2} \) (middle panel). The top two panels in Figure 1 show regions in the normal and upstream magnetic field (Tidman & Krall 1971). Similarly, in the dark (light) gray area some electrons can be additionally reflected by assuming \( \Theta_{Bn} \) becomes large, the broken line gets more vertical and the center of the shell walks away from the origin. Then the whole shell again enters a loss-cone. The possible reflection areas indicated in the top two panels are defined like this by assuming the shell radius \( v_r = 3 \eta v_{te} \). The area expands with increasing \( \beta_e \) as expected. On the other hand, it does not depend much on \( \sigma \). In particular when \( \sigma < 10^{-3} \), distributions of the areas are almost exactly the same as the top panel. It is mainly due to the light speed limit of a shell radius that the areas look contracted for \( \sigma = 3^{-2} \) in the middle panel, because increasing \( \sigma \) with constant \( \beta_e \) results in an increase of the thermal velocity. (The actual calculation is performed by taking relativistic effects into account so that the shell is given in momentum space with \( p_r = 3 \beta_e v_{te} \).) It is speculated that in extremely high Mach number shocks, like an SNR shock whose Mach number is typically \( M_A \sim 10^{-3} \), electron reflection hardly occurs unless there are no preheating mechanisms as discussed by Amano & Hoshino (2007, 2010). On the other hand, electrons can be relatively easily reflected when \( M_A < 10 \), although acceleration may be weak because of small \( V_{HT} \). In practice, one should note that the finite shock potential may reduce the reflection rate (Wu 1984).

2.2. An Extreme Case

As mentioned above, in low Mach number shocks (\( M_A < 10 \)) electrons are relatively easily reflected, although the resultant acceleration is not so efficient in general. However, if there are some electrons on the velocity shell with sufficiently large \( v_r \), they may be reflected even at large \( \Theta_{Bn} \) where \( V_{HT} \sim O(c) \). The reflected electron energy in such a case would become relativistic.

The condition that an upstream electron on a velocity shell with radius \( v_r \) can be reflected is \( v_r' > (\sqrt{\eta v_{te}/c} + U_1)/\beta_e \). When \( V_{HT} \gg U_1 \), this leads to \( v_r' > V_{HT}\sqrt{B_1^2/B_2^2} \), where the effects of shock potential have been again neglected. If \( v_r = \eta v_{te} \), the following condition is finally obtained:

\[
\eta > \eta_r \equiv \frac{U_1}{c} \Theta_{Bn} \left( \frac{B_1/B_2}{\beta_e \sigma/2} \right)^{1/2}.
\]  

(4)

The inequality (4) implies that reflected electrons can be present if there are some electrons on a velocity shell with \( v_r > \eta_r v_{te} \). The reflected electrons will have relativistic energies when \( V_{HT} = U_1 \tan \Theta_{Bn} \sim O(c) \). In this limit \( \eta_r \to \eta_r \), where

\[
\eta_r = \left( \frac{B_1/B_2}{\beta_e \sigma/2} \right)^{1/2} = \sqrt{\frac{B_1}{B_2}} \frac{v_{te}}{c}.
\]

(5)

In the solar wind or interstellar plasmas \( \eta_r \) is extremely large, since the electron temperature is \( \sim 100 \) eV at the highest and the NIF, \( (U_1, 0) \), and the radius of the shell is \( v_r \). Let us consider whether or not some electrons on this velocity shell are mirror reflected (in this schematic picture, relativistic distortion of the shell has been neglected). If \( V_{HT} \) is added in \( v_r \), the center of the shell shifts along the thick gray vertical arrow. Now a broken line connecting the origin with the new center of the shell is parallel to an upstream bulk velocity in the HTF, which is along the upstream magnetic field, \( B_1 \). Two solid lines symmetrical with respect to the broken line denote a loss-cone. Therefore, the electrons outside this loss-cone indicated by the black solid arcs can be reflected or otherwise transmitted. When \( U_1 \) or a Mach number increases, the center of the shell shifts along the broken line and finally a whole shell lies inside the loss-cone. In this case no electrons are reflected. Similarly, when \( \Theta_{Bn} \) becomes large, the broken line gets more vertical and the center of the shell walks away from the origin. Then the whole shell again enters a loss-cone. The possible reflection areas indicated in the top two panels are defined like this by assuming the shell radius \( v_r = 3 \eta v_{te} \). The area expands with increasing \( \beta_e \) as expected. On the other hand, it does not depend much on \( \sigma \). In particular when \( \sigma < 10^{-3} \), distributions of the areas are almost exactly the same as the top panel. It is mainly due to the light speed limit of a shell radius that the areas look contracted for \( \sigma = 3^{-2} \) in the middle panel, because increasing \( \sigma \) with constant \( \beta_e \) results in an increase of the thermal velocity. (The actual calculation is performed by taking relativistic effects into account so that the shell is given in momentum space with \( p_r = 3 \beta_e v_{te} \).) It is speculated that in extremely high Mach number shocks, like an SNR shock whose Mach number is typically \( M_A \sim 10^{-3} \), electron reflection hardly occurs unless there are no preheating mechanisms as discussed by Amano & Hoshino (2007, 2010). On the other hand, electrons can be relatively easily reflected when \( M_A < 10 \), although acceleration may be weak because of small \( V_{HT} \). In practice, one should note that the finite shock potential may reduce the reflection rate (Wu 1984).

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\]

(5)

In the solar wind or interstellar plasmas \( \eta_r \) is extremely large, since the electron temperature is \( \sim 100 \) eV at the highest and
\[ \sigma \sim 10^{-4.6} \ (\beta_e \sim O(1)) \]. Therefore, satisfying the above condition for a shock in an environment such as in Earth’s bow shock or SNR shocks is almost hopeless. However, it may be possible if the electron temperature becomes \( \sim 10 \text{ keV} \) or \( \sigma \sim 10^{-(1-2)} \) and \( \beta_e \sim 0.1-1 \) as in some large-scale shocks in galaxy clusters (van Weeren et al. 2010; Nakar et al. 2008; Fujita et al. 2007).

### 3. ONE-DIMENSIONAL PIC SIMULATION

In this section, a one-dimensional particle-in-cell (PIC) simulation is performed to reproduce the relativistic SDA of electrons as discussed above in a self-consistent manner. In the simulation, a shock is produced by the so-called injection or reflecting wall method. An upstream magnetized plasma is continuously injected from the left-hand boundary. The plasma is reflected at the right-hand boundary and the mixture of the incoming and reflected plasmas results in a downstream medium. The shock is produced at a boundary between the upstream and downstream plasmas. Since the simulation frame is the downstream rest frame, the shock propagates in time from right to left. The simulation is done in the NIF so that the injection flow is parallel to the shock normal, which is along the \( x \)-axis. The upstream magnetic field is in the \( x-z \) plane. The size of a spatial grid is \( \Delta x \approx 0.24 \lambda_{De} \) where \( \lambda_{De} \) denotes the electron Debye length, and the number of super particles per cell is \( N_{sp} = 200 \) for both electrons and ions. Time resolution is \( \Delta t = 0.06875 \omega_{pe}^{-1} \) where \( \omega_{pe} \) is the electron plasma frequency. Other physical parameters are shown in Table 1. For Runs A–C, all injection parameters are common except for the shock angle, \( \Theta_{sh} \). The injection Alfvén Mach number is \( M_{Alf} = 5 \), which results in \( M_A \approx 7.8 \) or \( U_1 \approx 0.061c \) in the shock frame for Run A, for instance, where \( U_1 \) denotes the upstream bulk velocity. This value roughly corresponds to the fast Mach number \( M_f \approx 2.9 \) by the definition of ion acoustic velocity as \( C_i^2 = (3T_i + T_e)/m_i \). The magnetization parameter is \( \sigma = 1/9 \) and the plasma beta is \( \beta = 3(\beta_e + \beta_i) \) so that the upstream electron temperature is \( \sim 43 \text{ keV} \). An ion-to-electron mass ratio \( m_i/m_e = 1836 \) is realistic. In the last two columns, the maximum value of the magnetic field roughly measured in the overshoot relative to the upstream value, \( B_{os}/B_1 \), and the associated \( \eta_c \) calculated from Equation (4) are denoted for reference. Note that \( \eta_c \) may in fact be larger if non-negligible potential effects are taken into account (Wu 1984). The most efficient electron acceleration is observed in Run A as expected since \( U_{1f} \tan \Theta_{sh} \) is the largest, although in all cases reflected electrons are seen. Run D is performed with more realistic parameters where \( M_{Alf}(=3) \) and \( \sigma(=1/16) \) are reduced. The results of Run D are essentially the same as those of Run A and are discussed in Section 4. In the following, the results of Run A are discussed.

### Table 1

| Run | \( M_A \) | \( M_f \) | \( U_1/c \) | \( \sigma \) | \( \beta_e = \beta_i \) | \( T_e/c^2 \) | \( m_i/m_e \) | \( \Theta_{sh} \) | \( B_{os}/B_1 \) | \( \eta_c \) |
|-----|---------|---------|---------|---------|----------------|--------|---------|--------|----------|--------|
| A   | 7.8     | 2.9     | 0.061   | 1/9     | 1.5            | 42.6   | 1836    | 85     | 6        | 0.99   |
| B   | 7.7     | 2.9     | 0.060   | 1/9     | 1.5            | 42.6   | 1836    | 80     | 6        | 0.48   |
| C   | 7.4     | 2.8     | 0.058   | 1/9     | 1.5            | 42.6   | 1836    | 60     | 6        | 0.14   |
| D   | 5.0     | 1.9     | 0.029   | 1/16    | 1.5            | 24.0   | 1836    | 87     | 3.5      | 1.4    |

In Figure 2, spatiotemporal evolutions of color coded \( B_z \) (upper) and \( E_x \) (lower) field components and a trajectory of a typical reflected electron. Right: energy (upper) and momentum (lower) histories of the traced electron. (A color version of this figure is available in the online journal.)

#### 3.1. Run A

In Figure 2, spatiotemporal evolutions of \( B_z \) and \( E_x \) fields are shown in the left panels. The structure of the shock is more or less time stationary, although it represents weak breathing features (Comisel et al. 2011). An identical trajectory of one of the reflected electrons is denoted as the black solid lines. Its energy (upper) and momentum (lower) time histories are plotted in the right panels. Here, a rate of output data points has been significantly reduced, otherwise \( p_x \) (blue) and \( p_z \) (green) lines fill in the area because of rapid gyro motions. The electron gains energy mainly through \( p_x \), which is almost parallel to the magnetic field during its stay in the shock transition region. The time the electron stays in the transition region during the reflection process is \( \sim \Omega_{it}^{-1} \). These are typical features of SDA discussed by Krauss-Varban & Wu (1989) and Krauss-Varban et al. (1989), while energy of the reflected electron here becomes relativistic.

The first three panels from the top in Figure 3 show ion \( v_i - x \), electron \( p_x - x \), and electron \( p_z - x \) phase spaces at \( \omega_{pe}t = 64350(\Omega_{it} \approx 11.7) \), which is indicated as the dashed lines in Figure 2. It is clear in the third panel that some electrons are reflected basically along the magnetic field and have relativistic energies. The fourth panel represents the \( p_z - p_i \) phase space of the electrons surrounded by the black square in the third panel. Their energy distribution function is plotted as the black line in the bottom panel. The reflected electrons show a ring-beam feature leading to the non-thermal part of the distribution function. A fraction of the bulk energy density of the incoming ions carried by the reflected electrons, \( \epsilon \), is estimated as \( \epsilon \approx 0.014 \). The gray line in the bottom panel is a downstream distribution function corresponding to the region surrounded by the gray square in the third panel. The high-energy part (\( \gamma e - 1 > 1 \)) looks non-thermal. Its origin is the
electrons also having large negative $p_z$ around $x/\rho_i \sim 11$ and 13 in the third panel. They are produced in the second and third magnetic overshoots at $x/\rho_i \sim 12$ and 14 seen in Figure 2. By comparing the upstream and downstream energy distribution functions, the highest energy electron is upstream. This implies that in the simulation, the downstream region is still very limited for seeing an equilibrium state and this is clear also from the top three panels.

It is often thought that reflected electrons produce a loss-cone distribution because of their adiabatic behaviors (Lobzin et al. 2005). However, no clear loss-cone is seen in the fourth panel. This is probably due to the effects of small-scale waves generated in the transition region. To avoid getting involved here with a detailed analysis of this problem, we just show evidence of the presence of such small-scale waves. Figure 4 shows $B_y$ field fluctuations in a transition region (upper panel) and its Fourier spectrum at $\omega_{pe}t = 44220$ (lower panel) indicated by the solid line in the upper panel. Tiny streaks in the transition region are visible in the upper panel. They appear in the lower panel as spectral peaks between $0.1 < k\lambda_e < 1$ where $\lambda_e = c/\omega_{pe}$ denotes the electron inertial length. Corresponding structures are also seen in the ion $v_\| - x$ phase space indicating that reflected ions destabilize some kind of microinstability (although not shown).

3.2. Injection of Reflected Electrons into Further Acceleration Process

Here, some possible behaviors of the accelerated electrons after being reflected are discussed. Since the reflected electrons form the non-equilibrium distribution function, some local instabilities may be present. However, they may not have been properly treated in the previous section, because there wave vectors only are allowed along the $x$-axis. In the following, possible instabilities and associated wave-particle interactions are discussed by performing further one-dimensional simulations for various wave propagation angles with periodic boundary conditions. A plasma is assumed to be composed of three components; i.e., incoming electrons and ions and reflected electrons. The reflected electrons are assumed to form a ring-beam distribution with a bulk momentum $(p_\parallel/m_e c, p_{\perp 0}/m_e c) = (2, 1)$ and a relative density $n_r/n_i = 0.1$, where $n_i$ denotes the density of the incoming ions that are at rest on average. Hence, the simulation is done in the upstream plasma frame of the reference. A density and a bulk momentum of the incoming electrons are

![Figure 3](image1.png)

**Figure 3.** Phase space densities at $\omega_{pe}t = 64350$. The top three panels represent ion $v_\| - x$, electron $p_\| - x$ and $p_\perp - x$ phase spaces. The fourth and fifth panels show the $p_{\perp} - p_{\|}$ phase space and a corresponding energy distribution function of electrons surrounded by the squares in the third panel. (A color version of this figure is available in the online journal.)

![Figure 4](image2.png)

**Figure 4.** Small-scale fluctuations of $B_y$ in the shock transition region (upper panel) and its Fourier spectrum at $\omega_{pe}t = 44220$ (lower panel).
decided to satisfy charge and current neutral conditions. For all three components, initial temperatures are equal and isotropic in their proper frames. $T/m_e c^2 = \beta_e \sigma/2$, where $\beta_e$ and $\sigma$ are chosen to be the same as the values of the upstream plasma in the previous section so that $\beta_e = 1.5$ and $\sigma = 1/9$. The system size is $L/\lambda_e = 819.2$, the number of spatial grids is $N_x = 8192$ corresponding to $\Delta x = 0.11 \lambda_e \approx 0.35 \Delta t_{De}$, the number of super particles per cell is $N_p = 400$, and the time step is $\omega_{pe} \Delta t = 0.1$, respectively. With these parameters fixed, four different runs are performed with various wave propagation angles ($\theta_Bk = 0^\circ, 30^\circ, 60^\circ$, and $80^\circ$) with respect to the magnetic field, which is in the $x-z$ plane. Note that $\theta_Bk$ is independent of the shock angle, $\Theta_Bn$.

Before discussing simulation results, we first briefly note how the number of super particles per cell, $N_p$, is crucial in reproducing a long time evolution of the system. Figure 5 represents evolution of spatially averaged magnetic field energies as a function of time in the $\theta_Bk = 60^\circ$ case for three different values of $N_p$: thick black lines correspond to $N_p = 400$, thin gray lines to $N_p = 40$, and thick black lines to $N_p = 400$, respectively. The solid and broken lines indicate $B_x$ and $B_z$ components. Because of the large noise level, $N_p = 4$ causes totally different results from others. For $N_p = 40$, early development of an instability appears to be calculated properly. However, nonlinear evolution of the system ($\omega_{pe} t > 1000$) may not be well described. In contrast to the case with $N_p = 400$, the $B_z$ component settles in a constant value after $\omega_{pe} t = 1600$ and this at last results in the same level of magnitude as the $B_y$ component. For such an oblique propagation angle, $B_z$ can easily couple with electrostatic fluctuations. The lower limit of $B_z$ field energy in this nonlinear stage may be an influence of electrostatic noise. We confirmed that the results are qualitatively common for $N_p \geq 100$, at least up to the time to be discussed below. In the following, results for $N_p = 400$, which we believe to be reliable, will be discussed.

Figure 6 represents the time histories of field energies (top) and effective electron temperature anisotropy (second), the $\omega-k$ spectrum of the $E_x$ field for $0 \leq \omega_{pe} t \leq 204.8$ (third), and a final electron distribution in the $p_{\perp} - p_{\parallel}$ space (bottom) for $\theta_Bk = 0^\circ$. (A color version of this figure is available in the online journal.)

- **Figure 5.** Time histories of $B_x$ (solid lines) and $B_z$ (broken lines) field energies for different numbers of super particles per cell, $N_p$, in the case of $\theta_Bk = 60^\circ$. The black thick, gray thin, and black thick lines denote $N_p = 4, 40$, and 400 cases, respectively.

- **Figure 6.** Time histories of field energies (top) and effective electron temperature anisotropy (second), the $\omega-k$ Fourier spectrum of the $E_x$ field for $0 \leq \omega_{pe} t \leq 204.8$ (third), and a final electron distribution in the $p_{\perp} - p_{\parallel}$ space (bottom) for $\theta_Bk = 0^\circ$. (A color version of this figure is available in the online journal.)
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Figure 7. Time histories of field energies (top) and effective electron temperature anisotropy (second) for $\omega - k$ Fourier spectrum of the $B_x$ field for $0 < \omega_{pe} t < 3276.8$ (third), and a final electron distribution in the $p_\perp - p_\parallel$ space (bottom) for $\theta_{BL} = 60^\circ$.

(A color version of this figure is available in the online journal.)

The evolution of the pitch angle cosine (top right) and the trajectory in the $x$-direction (bottom) of the black particle in the top left panel are also shown.

Figure 8. Trajectories of some electrons back scattered by self-generated waves with the background gray scale of amplitudes of magnetic fluctuations (top left). The evolution of the pitch angle cosine of the electron corresponding to the black solid line is plotted in the top right panel and its trajectory in the $p_\perp - p_\parallel$ space is indicated in the bottom panel. Major changes in the pitch angle cosine occur in two bounded time domains where the electron encounters large amplitude wave packets ($\omega_{pe} t \sim 500$ and $\sim 1800$). In most of the remaining time, the electron propagates in one direction, although rapid changes in its pitch angle exist. These features are common for all other trajectories in the top left panel.

For $\theta_{BL} = 80^\circ$, a non-resonant instability is again dominant (not shown). In such large $\theta_{BL}$ generated waves are almost non-propagating pure growing modes, which are the basic features of the Weibel instability. However, the growth time is much longer and the relaxation of the effective electron temperature anisotropy is less efficient than in the $\theta_{BL} = 60^\circ$ case.

4. SUMMARY AND DISCUSSIONS

It was shown by using a one-dimensional electromagnetic full particle simulation that the relativistic SDA of electrons is feasible even though the quasi-perpendicular shock is rather moderate when the upstream electron temperature becomes of the order of 10 keV or the magnetization parameter $\sigma$ is not too small (Equation (5)). Such a condition may be realized in large-scale shocks of galaxy clusters, cosmic-ray-modified subshocks of SNRs, etc. For instance, van Weeren et al. (2010) showed evidence of strong acceleration of relativistic electrons forming radio relics in the merging galaxy cluster CIZA J2242.8+5301, where an average temperature of the intracluster medium is estimated as $\sim 9$ keV. Their polarization analysis indicates that the observed shock in the radio relics is quasi-perpendicular.

The maximum energy of the reflected electrons is affected by the de Hoffmann–Teller velocity, $V_{HT} = U_{HT} \tan \Theta_{BT}$, which is a strong function of the shock angle. The Lorentz transformations derive the energy of a reflected electron as $\gamma_{ref} = \gamma_{HT}^2 (1 + \beta_{HT}^2)$.
2\beta_{\text{HT}} v_z/c), where $\gamma$ and $v_z$ denote the electron’s initial NIF Lorentz factor and velocity component in the $z$-direction in the shock frame, and $\Gamma_{\text{HT}} = (1 - \beta_{\text{HT}}^2)^{-1/2}$, $\beta_{\text{HT}} = v_{\text{HT}}/c$, respectively. When the non-relativistic limit with a large $\Theta_Bn$ ($\sim 90^\circ$) is considered, the above expression reduces to Equation (16) in Krauss-Varban & Wu (1989). Observed values of the maximum Lorentz factors of upstream electrons for Runs A–C are $\sim 10.0$, $\sim 4.1$, and $\sim 2.7$, respectively. They roughly coincide with the values derived from the above estimate, $\sim 9.6$, $\sim 3.6$, and $\sim 2.3$, where $\gamma \approx 1.9$ (at a maximum) and $v_z/c \sim -0.8$ are again from the simulation. Possible reasons for the underestimation may be neglecting a temporal variation of the shock velocity and/or non-adiabatic features based on the small-scale waves in the transition region.

The parameter dependence of the maximum attainable energy through the SDA is shown in Figure 9. The energies of the reflected particles along the edges of the possible reflection areas in the top two panels in Figure 1 are plotted. Each line color corresponds to different $\beta_e$ (black: 0.1; dark gray: 0.5; light gray: 1.5). The upper panels show energies of the reflected electrons normalized to upstream electron bulk flow energies. The dashed lines denote the bulk energy of the ion flow. The normalized energies indicate apparent $M_A^{-2}$ dependence for a wide range of $M_A$. This is consistent with the result that the obtained energies do not basically depend on $M_A$ as confirmed in the lower panels. This is due to the fact that $v_{\text{HT}}$ is roughly constant along the edges of the possible reflection area. However, the maximum energy strongly depends on $\sigma$ implying that the magnetic field strength is crucial. Indeed, it should be noted that the vertical axes in the lower right panel are three orders of magnitude larger than those in the lower left panel, while a ratio of $\sigma$ between these two panels is also $\sim 10^3$.

The reflected electrons are accelerated mainly in the $z$-direction, which is almost along the magnetic field, forming the non-thermal part of the energy distribution function upstream of the shock. The efficiency of the energy transfer to the reflected electrons is highest in Run A. If an injection rate, $\epsilon$, is defined as the fraction of energy density carried by the reflected electrons with respect to the bulk energy density of the incoming ions, $\epsilon \approx 1.4\%$ in this case. The corresponding relative number density of the reflected electrons, or the reflection ratio, is 2.6%. These values are to be compared with observations. Finoguenov et al. (2010) estimated that the total energy of radio-emitting electrons in the radio relic in A3667 is 0.15% of the shock kinetic energy, where they adopt the field strength $B \approx 3 \mu G$ to explain dim inverse-Compton emission in the X-ray band. Another observational estimation of the injection efficiency is obtained assuming the equipartition between the energies of the magnetic field and the radio-emitting electrons. Using the observed quantities given in Finoguenov et al. (2010), we derive that 4% of the shock kinetic energy goes into that of radio-emitting electrons. The injection efficiency of a few percent obtained by our simulation result is comparable to the observational estimation (0.1%-4%), which, however, contains various uncertainties. It should be noted that the result here is originated from the one-dimensional simulations only for the particular parameter set and that the injection efficiency depends on various parameters such as the electron temperature, the shock normal angle, etc. To estimate the injection efficiency more accurately, we should perform a detailed survey on parameters as well as two- or three-dimensional simulations, which will be addressed in future works.

The accelerated relativistic-reflected electrons form a non-equilibrium local ring-beam-like distribution function upstream of the shock. It is possible to generate large amplitude waves through a variety of microinstabilities. Local simulations with periodic boundary conditions reveal that the electrostatic electron beam instability is rapidly destabilized basically along the magnetic field. This instability quickly saturates by forming a plateau of the distribution function so that the effective temperature anisotropy ($T_{\parallel}/T_{\perp} > 1$) remains. Afterward, the oblique non-resonant instability grows slowly, leading to efficient relaxation of the temperature anisotropy. In this process some of the reflected electrons are scattered back toward the shock by self-generated coherent wave packets. A sequence of the above process, i.e., the reflection of some incoming electrons through the SDA process followed by excitation of the microinstabilities and the backscattering of a part of the reflected electrons by the self-generated waves, may be the first step of injection into the diffusive shock acceleration process.

In Runs A, B, and C, $M_A$ and $\sigma$ may be somewhat higher than those of typical large-scale shocks in galaxy clusters. Therefore, an additional Run D is performed to confirm that a similar process can work also for more realistic parameters. Here,
The relativistic SDA works and associated ring-beam electrons are produced upstream. However, the maximum Lorentz factor of the reflected electrons, $\gamma_{\text{ref}}$, is not as large as in Run A, since $V_{\text{IRF}}$ and the initial $\gamma$ are a little less because of the small $M_A$ and $T_e$ here (Figure 10). In addition, the reflection rate of the incoming electrons becomes smaller, because $\eta_\gamma \approx 1.4$, which is larger than $\eta_\gamma \approx 0.99$ in Run A. Nevertheless, the relativistic SDA can work in such a parameter regime too. In practice, the process may occur even for more moderate and lower temperature shocks if a halo electron component is present in an intracluster medium as in the solar wind (Šverák et al. 2009; Louarn et al. 2009).

In this paper, the effects of higher spatial dimensions have been excluded. We expect that the processes discussed separately in Sections 3.1 and 3.2 are simultaneously observed in two- or three-dimensional systems. However, details of the competing processes among them have been unknown. For example, the electron beam instability is accompanied by pitch angle scattering through multidimensional wave-particle interactions in its nonlinear stage (e.g., Pavan et al. 2009). If this process proceeds more rapidly than the growth of the oblique non-resonant instability, the relaxation of the effective temperature anisotropy and associated backscattering of a part of the reflected electrons might occur. We are also curious to know some other effects like rippling and upstream turbulence (Guo & Giacalone 2010). All these are future issues.

We thank Hiroki Akamatsu and Susumu Inoue for useful comments and discussions. This work was supported by Grant-in-Aid for Scientific Research on Innovative Areas 21200050, Grant-in-Aid for Scientific Research on Priority Areas 19047004 (R.Y.), and Grant-in-Aid for Young Scientists (B) 21740184 (R.Y.) and 22740323 (S.M.).

Note added in proof. Dr. Yutaka Fujita kindly pointed out that upstream temperature may be 1 keV or slightly less near the periphery of the cluster. However, even in this case, $\sqrt{\sigma p_e}$ is around unity, so that the electron drift acceleration may occur according to the discussion in Section 2.2.

REFERENCES

Amano, T., & Hoshino, M. 2007, ApJ, 661, 190
Amano, T., & Hoshino, M. 2009, ApJ, 690, 244

Amano, T., & Hoshino, M. 2010, Phys. Rev. Lett., 104, 181102
Bamba, A., Yamazaki, R., Ueno, M., & Koyama, K. 2003, ApJ, 589, 827
Blandford, R. D., & Eichler, D. 1987, Phys. Rep., 154, 1
Bret, A. 2009, ApJ, 699, 990
Cargill, P. J., & Papadopoulos, K. 1988, ApJ, 329, L29
Comisel, H., Scholer, M., Soucek, J., & Matsukiyo, S. 2011, Ann. Geophys., 29, 263
Dieckmann, M. E., McClements, K. G., Chapman, S. C., Dendy, R. O., & Drury, L. O. C. 2000, A&A, 356, 377
Enßlin, T. A., Biermann, P. L., Klein, U., & Kohle, S. 1998, A&A, 332, 395
Feldman, W. C., Anderson, R. C., Baner, S. J., et al. 1983, J. Geophys. Res., 88, 96
Ferrari, C., Govoni, L., Schindler, S., Bykov, A. M., & Rephaeli, Y. 2008, Space Sci. Rev., 134, 93
Finoguenov, A., Sarazin, G. L., Nakazawa, K., Wick, D. R., & Clarke, T. E. 2010, ApJ, 715, 1143
Fujita, Y., Kohri, K., Yamazaki, R., & Kino, M. 2007, ApJ, 663, L61
Fujita, Y., & Sarazin, C. L. 2001, ApJ, 563, 660
Gabici, S., & Blasi, P. 2003, ApJ, 583, 695
Govoni, F., & Feretti, L. 2004, Int. J. Mod. Phys. D, 13, 1549
Guo, F., & Giacalone, J. 2010, ApJ, 715, 406
Hoshino, M., & Shimada, N. 2002, ApJ, 572, 880
Inoue, S., Sigl, G., Miniati, F., & Armengaud, E. 2008, in Proceedings of the 30th ICRC (2007), ed. R. Caballero et al. (Vol. 4; Mexico City: Universidad Nacional Autonoma de Mexico), 555
Kang, H., Rachen, J. P., & Biermann, P. L. 1997, MNRAS, 286, 257
Kang, H., Ryu, D., Cen, R., & Ostriker, J. P. 2007, ApJ, 669, 729
Keshet, U., Waxman, E., & Loeb, A. 2004, ApJ, 617, 281
Koyama, K., Petro, R. G., Gottelf, E., et al. 1995, Nature, 378, 255
Krauss-Varban, D., & Burgess, D. 1991, J. Geophys. Res., 96, 143
Krauss-Varban, D., Burgess, D., & Wu, C. S. 1989, J. Geophys. Res., 94, 15089
Krauss-Varban, D., & Wu, C. S. 1989, J. Geophys. Res., 94, 15367
Leroy, M. M., & Mangeney, A. 1984, Ann. Geophys., 2, 449
Levinson, A. 1992, ApJ, 401, 73
Lobzin, V. V., Krasnoselskikh, V. V., Schwartz, S. J., et al. 2005, Geophys. Res. Lett., 32, L18101
Louarn, P., Dieval, C., Génot, V., et al. 2009, Sol. Phys., 259, 311
Matsukiyo, S. 2010, Phys. Plasmas, 17, 042901
Miniati, R., Jones, T. W., Kang, H., & Ryu, D. 2001, ApJ, 562, 233
Miniati, R., Ryu, D., Kang, H., et al. 2000, ApJ, 542, 608
Morlino, G. 2009, Phys. Rev. Lett., 103, 121102
Nakar, E., Milosavljević, M., & Nagai, D. 2008, ApJ, 675, 126
Ohira, Y., Murase, K., & Yamazaki, R. 2011, MNRAS, 410, 1577
Ohira, Y., & Takahara, F. 2007, ApJ, 661, L171
Ohira, Y., & Takahara, F. 2008, ApJ, 688, 320
Oka, M., Terasawa, T., Fujimoto, M., et al. 2009, Earth Planets Space, 61, 603
Oka, M., Terasawa, T., Seki, Y., et al. 2006, Geophys. Res. Lett., 33, L24104
Pannuti, T. G., Allen, G. E., Filipović, M. D., et al. 2010, ApJ, 712, 1492
Papadopoulos, K. 1988, Astrophys. Space Sci., 144, 535
Pavan, J., Ziebell, L. F., Gacler, R., & Youn, P. H. 2009, J. Geophys. Res., 114, A01106
Riquelme, M. A., & Spitkovsky, A. 2010, ApJ, 717, 1054
Ryu, D., Kang, H., Hallman, E., & Jones, T. W. 2003, ApJ, 593, 599
Shimada, N., & Hoshino, M. 2000, ApJ, 543, L67
Shimada, N., Terasawa, T., Hoshino, M., et al. 1999, Astrophys. Space Sci., 264, 481
Šverák, K., Maksimovic, M., Trávníček, P., et al. 2009, J. Geophys. Res., 114, A05104
Tidman, D., & Krall, N. A. 1971, Shock Waves in Collisionless Plasmas (New York: Wiley-Interscience)
Umeda, T., Yamao, M., & Yamazaki, R. 2008, ApJ, 681, L85
Umeda, T., Yamao, M., & Yamazaki, R. 2009, ApJ, 695, 574
van Weeren, R. J., Röttgering, H. J. A., Brüggen, M., & Hoeft, M. 2010, Science, 330, 347
Williamson, M. A. G. 1970, MNRAS, 151, 1
Wu, C. S. 1984, J. Geophys. Res., 89, 8857
Wu, C. S., Winske, D., Zhou, Y. M., et al. 1984, Space Sci. Rev., 37, 63

Figure 10. Electron phase space density in $p_e - x$ at $\omega_{pe} t = 85200$ for Run D. (A color version of this figure is available in the online journal.)