Unimagined Imaginary Parts in Heavy Quark Effective Field Theory

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Abstract
We argue that the imaginary parts of the anomalous dimensions in the multiparticle sectors of heavy quark effective field theory may be removed by a suitable redefinition of the multiparticle states. The connection between the imaginary parts of the anomalous dimensions and the interquark potential is pointed out.

1 Introduction
Heavy quark effective field theory (HqEFT) has proven to be a useful tool for the systematic analysis of systems containing heavy quarks.

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This approach is based on the heavy mass limit in which the Hilbert space decomposes into superselection sectors labelled by the velocities of the heavy quarks. Corrections to the leading behavior may be calculated systematically and we shall focus in this work on the strong interaction corrections. These are treated in renormalization group improved perturbation theory, resumming logarithms of the heavy quark masses. Weak interactions will introduce operators connecting different velocity sectors and in general their anomalous dimensions will depend on the velocities of the heavy quarks. The lowest order expressions have been given in [6, 7].

In the limit $m_Q \to \infty$ particle and antiparticle number are separately conserved and most of the applications discussed up to now deal with the one (anti)particle sector of HqEFT. However, in the two particle sector some of the anomalous dimensions develop imaginary parts. In the full theory imaginary parts of Green’s functions emerge from analytic continuation and some of these imaginary parts contain logarithms of the masses. In the effective theory these logarithms are reproduced by the renormalization group. Therefore the anomalous dimensions have to pick up imaginary parts because of analyticity of the full theory.

Complex anomalous dimensions are somewhat unusual and the purpose of the present letter is to shed some light on their origin. We shall argue that the imaginary parts may be removed by a suitable redefinition of the multiparticle states of HqEFT. We shall construct these redefined multiparticle states to order $\alpha_s$ explicitly in the next section and discuss the extension of this method to all orders.

In the two particle state the imaginary parts of the anomalous dimensions lead to phases of the Wilson coefficients. Upon closer inspection, these phases turn out to be the nonabelian generalizations of the Coulomb phases familiar from QED [8, 9]. In a similar way as it may be done for the QED case, one may derive an interquark potential suitable for the physics at the renormalization scale. This is done up to two loops in section 4.

2 Complex Anomalous Dimensions

Since HqEFT conserves particle and antiparticle number separately, we shall begin our discussion in the particle-particle and particle-antiparticle sectors.
An effective Hamiltonian

$$H_{\text{eff}} = \sum_{i=1}^{n} \eta_i(\mu) \mathcal{O}_i(\mu)$$  \hspace{1cm} (1)

having matrix elements with two heavy particles in the final state will be expanded in a basis of hermitian operators $\mathcal{O}_i$, $i = 1, \ldots, n$ with Wilson coefficients $\eta_i$. They are calculated from matching at the heavy quark mass scale and subsequent renormalization group running. In such a basis the anomalous dimensions will be given by a specific $n \times n$ matrix $\gamma$. The hermiticity of the effective interaction requires real entries for the matrix $\gamma$. However, in HqEFT imaginary parts appear as coefficients of logarithmic divergences as soon as the corresponding operator has matrix elements with a two- or more heavy particle state $[7]$. This hints at a possible problem for the interpretation of HqEFT for these final states. The imaginary parts of these anomalous dimensions have been calculated to leading order in $[7]$. The generic expression is given by

$$\text{Im} \gamma(\alpha, vv') = -\alpha \frac{vv'}{\sqrt{(vv')^2 - 1}} T^a \otimes T^a.$$  \hspace{1cm} (2)

The color matrix $T^a \otimes T^a$ acts on the two heavy quarks in the final state. Coupling the two final state heavy quarks to definite color, it will be sufficient to consider the following two sets of operators. In the particle-antiparticle sector we have a color singlet and a color octet operator

$$J^1 = (\bar{h}_\alpha(v)\psi)(\phi h_\alpha(v'))$$  \hspace{1cm} (3)
$$J^{8,a} = (\bar{h}_\alpha(v)\psi)T^a_{\alpha\beta}(\phi h_\beta(v'))$$  \hspace{1cm} (4)

and similarly in the particle-particle sector there is a color antitriplet and a color sextet

$$J^3_{\alpha\beta} = (\bar{h}_\alpha(v))(\phi h_\beta(v')) - (\bar{h}_\alpha(v))(\phi h_\beta(v'))$$  \hspace{1cm} (5)
$$J^6_{\alpha\beta} = (\bar{h}_\alpha(v))(\phi h_\beta(v')) + (\bar{h}_\alpha(v))(\phi h_\beta(v')).$$  \hspace{1cm} (6)

In these equations, $\alpha, \beta = 1, \ldots, 3$ and $a = 1, \ldots, 8$ are the color indices. The Dirac spinors $\psi$ and $\phi$ are determined by the specific form of the operators $\mathcal{O}_i$; they may contain fields corresponding to the light degrees of freedom and possibly another heavy quark in the initial state.
In this basis the color matrix $T^a \otimes T^a$ is diagonal which means that the imaginary part of the anomalous dimension is diagonal and may be written as
\[ \text{Im} \gamma(\alpha, vv') = \alpha \frac{vv'}{\sqrt{(vv')^2 - 1}} K_C, \]
where the color factor $K_C$ is determined by the action of $T^a \otimes T^a$ on the operators $J_{1,8,3,6}$, yielding the eigenvalues $4/3, -1/6, 2/3, \text{and} -1/3$.

Thus the imaginary parts in fact depend only on the total color of the heavy quarks in the final state. This suggests that a suitable redefinition of the final state may render the anomalous dimensions real.

3 Redefinition of the Multiparticle States

Before we consider the nonabelian case, we shall briefly review the exactly soluble abelian case which has been extensively studied in the context of the QED infrared problem [10, 9]. The interaction in the abelian case is given by
\[ H_I(x_0) = g \int d^3 \vec{x} j_\mu(x) A^\mu(x), \] (7)
where the current of a fermion $Q$ is given by
\[ j_\mu(x) = \bar{Q}(x) \gamma_\mu Q(x). \] (8)

In the nonrecoil limit corresponding to the heavy mass limit for $Q$ the current (8) may be rewritten [8]
\[ j_\mu(x) \rightarrow J_\mu(x) = \int d^3 \vec{v} v_\mu \delta^3(\vec{x} - \vec{v} x_0/v_0) n(v), \] (9)
where $v = p_Q/m_Q$ is the velocity of the heavy particle and
\[ n(v) = \sum_{s = \pm} \left( b^\dagger(v, s) b(v, s) - d^\dagger(v, s) d(v, s) \right) \] (10)
is the charge density operator with $b(v, s), d(v, s)$ being the annihilation operators for particles and antiparticles of spin $s$ moving with velocity $v$ respectively. The dynamics of the system governed by the Hamiltonian
\[ \mathcal{H}_I(x_0) = g \int d^3 \vec{x} J_\mu(x) A^\mu(x) \] (11)
is exactly soluble. The operator $U$ transforming a free state into an interacting one is given by

$$U = \exp(iR) \exp(i\Omega),$$

(12)

with the radiation operator

$$R = -g \int d^4x \Theta(-x_0) J_\mu(x) A^\mu(x)$$

(13)

and phase operator

$$\Omega = \frac{g^2}{2} \int d^4x d^4y \Theta(-x_0) \Theta(x_0 - y_0) [J_\mu(x) A^\mu(x), J_\nu(y) A^\nu(y)],$$

(14)

This model is exactly soluble because in the abelian case the current commutes with itself for all $x$ and $y$

$$[J_\mu(x), J_\nu(y)] = 0.$$ 

Thus the phase operator may also be written as

$$\Omega = \frac{g^2}{2} \int d^4x d^4y \Theta(-x_0) \Theta(x_0 - y_0) J_\mu(x) [A^\mu(x), A^\nu(y)] J_\nu(y).$$

(15)

Normal ordering of the two currents introduces a mass renormalization term which is dealt with as usual. Inserting the $c$-number commutator of the $A$ fields yields for the phase operator

$$\Omega = \frac{g^2}{8\pi} \int d^3\vec{v} d^3\vec{v}' \frac{vv'}{(vv')^2 - 1} :n(v)n(v'):\int \frac{dx_0}{x_0}.$$ 

(16)

The divergent $x_0$ integration is cut off at small $x_0$ by the inverse of the mass $m_Q$ of the heavy particle and at large $x_0$ by a scale $\mu$ which will turn out to be the renormalization scale $\mu$. Thus we have

$$\Omega = -\frac{\alpha}{2} \int d^3\vec{v} d^3\vec{v}' \frac{vv'}{(vv')^2 - 1} :n(v)n(v'):\ln \left( \frac{m_Q}{\mu} \right).$$

(17)

In the abelian case this operator will contribute to $U$ as a phase factor multiplying the multiparticle states. The transformation

$$U_\Omega = \exp(i\Omega)$$
is unitary by itself and yields the correct Coulomb phases familiar from QED if applied to a multiparticle state [3].

On the other hand, one may also calculate the anomalous dimensions in the abelian version of HqEFT with no light quarks which exactly corresponds to the above model. From the exact solution (12) we read off that the anomalous dimensions are given by the one loop result. The $\beta$ function is trivial ($\beta \equiv 0$) since there is no self coupling of the gauge field and no light particles. However, the anomalous dimensions in this simple model develop an imaginary part similar to the one in HqEFT which we now trace back to the operator $U_\Omega$.

In fact, the imaginary part of the anomalous dimensions yields a phase factor in the Wilson coefficient. This phase factor for a matrix element $\mathcal{M}(vv')$ with two heavy particles moving with velocities $v$ and $v'$ respectively in the final state is governed by a renormalization group equation with only the imaginary part of the anomalous dimension [7]

\[
\left(\mu \frac{\partial}{\partial \mu} + i \text{Im} \gamma\right) \mathcal{M}(vv') = 0. \tag{18}
\]

The solution of (18) yields the Coulomb phase which is also obtained from the operator $U_\Omega$ acting on the two particle final state. In fact, the operator $U_\Omega$ satisfies the renormalization group equation (18)

\[
\left(\mu \frac{\partial}{\partial \mu} + iG\right) U_\Omega = 0. \tag{19}
\]

with the imaginary part of the anomalous dimension replaced by the operator $G$

\[
G = -\frac{\alpha}{2} \int d^3 \vec{v} d^3 \vec{v}' \frac{vv'}{\sqrt{(vv')^2 - 1}} n(v) n(v') . \tag{20}
\]

Note that the hermiticity of $G$ leads to a unitary operator $U_\Omega$. This makes manifest the connection of the Coulomb phases with the imaginary parts of the anomalous dimensions. In particular, this argument shows that it is appropriate to transfer the phases generated by the imaginary parts of the anomalous dimensions from the Wilson coefficients to the states by applying the operator $U_\Omega$ to the states.
Unfortunately, life is not that simple in the nonabelian case. The currents live in the adjoint representation of the gauge group and are given by

\[ j^a_\mu(x) = \bar{Q}_\alpha(x)T^a_{\alpha\beta}\gamma_\mu Q_\beta(x), \quad (21) \]

where \( T^a \) is the generator of color \( SU(3) \) in the fundamental representation. They become in the nonrecoil approximation

\[ j^a_\mu(x) \rightarrow J^a_\mu(x) = \int d^3\bar{v}v_\mu \delta^3(\vec{x} - \vec{v}x_0/v_0) n^a(v), \quad (22) \]

where now

\[ n^a(v) = \sum_{s=\pm} \left( b^1_\alpha(v,s)T^a_{\alpha\beta}b^s_\beta(v,s) - d^1_\alpha(v,s)T^a_{\alpha\beta}d^s_\beta(v,s) \right). \quad (23) \]

Due to the color structure these currents do not commute any more and an exact solution is not possible. Nevertheless we shall proceed along the lines suggested by the abelian case and define a nonabelian counterpart of the operator \( G \) which is given to order \( \alpha_s \) by

\[ G = -\frac{\alpha_s}{2} \int d^3\bar{v}d^3\bar{v}' \frac{vv'}{\sqrt{(vv')^2 - 1}} :n^a(v)n^a(v'):. \quad (24) \]

This reproduces the imaginary parts of the anomalous dimensions discussed in the previous section (cf. 3) when applied to a two particle state. Furthermore, \( G \) is a hermitian operator and a renormalization group equation analogous to (20) may be used to define the nonabelian generalization \( U_\Omega \) of the operator \( U_\Omega \). However, the self-coupling of the gluons and the existence of light quarks imply a nontrivial \( \beta \) function. Thus the equation determining \( U_\Omega \) in the nonabelian case is

\[ \left( \mu \frac{\partial}{\partial \mu} + \beta_\alpha \frac{\partial}{\partial \alpha} + iG \right) U_\Omega = 0. \quad (25) \]

As in the abelian case the operator \( U \) is a unitary operator because \( G \) is hermitian. The solution of (25) may be obtained in closed form and is given by

\[ U_\Omega = \exp \left\{ -i\frac{\alpha_s(\mu^2)}{2} \int d^3\bar{v}d^3\bar{v}' \frac{vv'}{\sqrt{(vv')^2 - 1}} :n^a(v)n^a(v'):\ln \left( \frac{m_Q}{\mu} \right) \right\}. \quad (26) \]
This operator again defines a unitary transformation on the multiparticle states. Accordingly this nonabelian generalization of the Coulomb phase may be shifted from the Wilson coefficients to the states, rendering the anomalous dimensions real to order $\alpha_s$.

Unlike the abelian case this is not the end of the story since there are higher order contributions to the anomalous dimensions. In general, the anomalous dimensions will contain imaginary parts in all orders of $\alpha_s$. Similar to the lowest order case one may define an anomalous dimension operator by replacing the color factors by appropriate density operators in the following way. The color factor of some set of diagrams is given by a tensor product of $T^a$’s corresponding to heavy quark-gluon vertices. However in the nonrecoil approximation every $T^a$ may be assigned to a fixed velocity $v$. Thus an anomalous dimension operator may be defined by replacing the tensor product of the $T^a$’s by a normal ordered product of the density operators $n^a(v)$. The generalization of $\mathcal{G}$ to all orders may be obtained by picking out the antihermitian part of this operator, which exactly corresponds to the imaginary part of the anomalous dimension. The hermitian operator $\mathcal{G}$ is now obtained by multiplying the antihermitian part of this anomalous dimension operator by $(-i)$.

The kernel $\mathcal{G}$ is put into the renormalization group equation (25) and the solution defines a unitary operator $U_\Omega$ which may be used to redefine the states, thereby removing the imaginary parts of the anomalous dimension to any desired order.

$\mathcal{G}$ may be expanded in powers of $n^a(v)$ corresponding to two-, three-, ..., $n$-particle operators

\[
\mathcal{G} = \frac{1}{2!} \int d^3\vec{v}_1d^3\vec{v}_2 f_{(2)}^{ab}(v_1, v_2):n^a(v_1)n^b(v_2): + \frac{1}{3!} \int d^3\vec{v}_1d^3\vec{v}_2d^3\vec{v}_3 f_{(3)}^{abc}(v_1, v_2, v_3):n^a(v_1)n^b(v_2)n^c(v_3): + \cdots.
\]  

The kernels $f_{(n)}^{a_1...a_n}$ of these operators are power series in $\alpha_s$ with the property

\[
f_{(n)}^{a_1...a_n} = O(\alpha_s^{n-1}).
\]  

Furthermore, the $f_{(n)}^{a_1...a_n}$ may be decomposed into rank $n$ invariant $SU(3)$
tensors; thus the lowest term \( f_{(2)}^{ab} \) must be proportional to \( \delta^{ab} \)

\[
f_{(2)}^{ab} = -\delta^{ab}\alpha_s \frac{vv'}{\sqrt{(vv')^2 - 1}} + \cdots,
\]

(29)

where the dots indicate higher order terms. Up to now only the leading and next-to-leading contributions to \( f_{(2)}^{ab} \) have been calculated \[7, 11\].

4 Potentials and Phases

Finally we shall point out the relation between the phase operator \( U_\Omega \) and the nonrelativistic interquark potential. In order to do this we consider a final state consisting of a heavy quark and a heavy antiquark in the singlet state with velocities \( v \approx v' \). The generalization to other two heavy quark final states is obvious since they differ only by color factors.

We shall consider the nonrelativistic limit, in which \( v_0, v'_0 \approx 1 \) and we have

\[
\frac{v \cdot v'}{\sqrt{(v \cdot v')^2 - 1}} \approx \frac{1}{|\vec{u}|} = \frac{1}{u},
\]

(30)

where \( \vec{u} \) is the velocity of one quark in the rest frame of the other.

In this limit one obtains a simple result for the imaginary part of the anomalous dimension up to two loops for this final state which reads \[11\]

\[
\text{Im} \, \gamma(u) = \alpha_s C_F \left[ 1 + \alpha_s \frac{31}{9} C_A - \frac{10}{9} n_f \right] \frac{1}{u} + \cdots,
\]

(31)

where the dots represent terms which are finite as \( u \to 0 \). \( C_F = 4/3 \) and \( C_A = 3 \) are the eigenvalues of the Casimir operator in the fundamental and the adjoint representation respectively. This imaginary part of the anomalous dimension will create a phase of the Wilson coefficient which may be absorbed into the states by applying the operator \( U_\Omega \) defined in the last section.

The connection between the potential and the phases appearing in the Wilson coefficient proceeds along the lines well known from the QED case \[8, 9\]. Here the Coulomb phases may be traced back to the long range part of the potential. In order to connect the phases with the potential we consider
a nonrelativistic system of two heavy particles interacting by a spherically symmetric potential. The Hamiltonian is

$$H = H_0 + H_I = \frac{\vec{p}_1^2}{2m_1} + \frac{\vec{p}_2^2}{2m_2} + V(|\vec{r}_1 - \vec{r}_2|).$$  \hspace{1cm} (32)$$

Separating the cms motion, the time evolution for the relative motion is given in the interaction picture by

$$U(t, t_0) = \exp \left( -i \int_{t_0}^t d\tau V(r_{12}(\tau)) \right),$$  \hspace{1cm} (33)$$

where $r_{12}(t)$ is the operator for the relative coordinate, again in the interaction picture.

Since the two particles are very heavy, we can have the particles simultaneously in a state of definite velocity and definite position. We choose

$$r_{12}(t) = ut,$$  \hspace{1cm} (34)$$

where $u$ is the velocity operator. When acting on states containing particles with definite velocities, the time evolution becomes a pure phase

$$U(t, t_0) = \exp \left( -i \int_{t_0}^t d\tau V(u\tau) \right).$$  \hspace{1cm} (35)$$

Introducing the Fourier transform of the potential

$$V(r) = \frac{1}{2\pi^2} \int_0^\infty dq \hat{V}(q) \sin(qr)$$  \hspace{1cm} (36)$$

we can rewrite (35) as

$$U(t, t_0) = \exp \left( -i \int_{t_0}^t d\tau \int_0^\infty dq \frac{\hat{V}(q)}{\tau} \sin(qu\tau) \right).$$  \hspace{1cm} (37)$$

For a potential falling off only like $1/r_{12}$, the $\tau$ integration is divergent for $t \to \infty$ as well as for $t_0 \to 0$. However, as argued before, $t_0$ has to be cut off at times of the order of $1/m_Q$, since the effective theory is not valid for shorter times/larger scales. The other cut off is related to the renormalization point $\mu$.
by \( t \sim 1/\mu \). These cut offs may be shifted from the time integration to the integration over the variable \( q \)

\[
\int_{t_0}^t d\tau \int_0^\infty dq \frac{q \tilde{V}(q)}{\tau} \sin(q u \tau) \mapsto \int_{1/t_0=m_Q}^{1/\mu} dq \int_0^\infty d\tau \frac{q \tilde{V}(q)}{\tau} \sin(q u \tau)
\]  

(38)
and the \( \tau \) integration may be performed to yield

\[
U(\mu; m_Q) = \exp \left( -\frac{i}{4\pi u} \int_\mu^{m_Q} dq \frac{q^2 \tilde{V}(q)}{q} \right).
\]  

(39)

The exponent is a dimensionless quantity and hence \( q^2 \tilde{V}(q) \) is also dimensionless. Calculated from QCD it is thus some function

\[
\phi(\alpha_s) = q^2 \tilde{V}(q).
\]  

(40)

However, \( \alpha_s \) is scale dependent and this dependence will render the potential nontrivial. Differentiation with respect to the scale \( \mu \) yields the renormalization group equation \([25]\) with the anomalous dimension being purely imaginary

\[
\text{Im } \gamma(\alpha_s) = -\frac{1}{4\pi u} \phi(\alpha_s).
\]  

(41)

This equation finally relates the imaginary part of the anomalous dimension to the long range interquark potential for heavy nonrelativistic quarks.

Including the scale-dependence of \( \alpha_s \), the potential may then be defined as

\[
\tilde{V}(q) = \frac{\phi(\alpha_s(q^2))}{q^2} = -\lim_{u \to 0} \frac{4\pi u \text{Im } \gamma(\alpha_s(q^2))}{q^2}
\]  

(42)
and we obtain from perturbation theory up to second order in the particle-antiparticle singlet channel

\[
\tilde{V}(q) = -\frac{4\pi \alpha_s(q^2)}{q^2} C_F \left[ 1 + \frac{\alpha_s(q^2)}{4\pi} \left( \frac{31}{9} C_A - \frac{10}{9} n_f \right) \right],
\]  

(43)

where the two loop running coupling constant has to be used. The QCD potential between two static color sources has been calculated long ago \([12, 13]\). Our expression for the potential (43), valid in the \( \overline{\text{MS}} \) scheme, is in complete agreement with these calculations.
Finally we point out that for dimensional reasons nontrivial behavior of the potential can only be generated by the scale dependence of the strong coupling constant. For instance, the linear confinement potential $V_{\text{lin}}(r) \propto r$, which may be obtained from lattice calculations and quarkonia-spectroscopy, corresponds to nonperturbative $\beta$-functions. The Fourier-transform of the linear potential is

$$\tilde{V}_{\text{lin}}(q) = C \frac{1}{q^2} \frac{\Lambda^2}{q^2},$$  \hspace{1cm} (44)

where $C$ is some numerical constant. At this point an additional scale $\Lambda^2$ has to enter the game, which has to be generated by dimensional transmutation from the scale dependence of the coupling constant. In fact, the general renormalization group equation

$$\mu^2 \frac{\partial}{\partial \mu^2} \alpha(\mu^2) = \beta(\alpha(\mu^2))$$  \hspace{1cm} (45)

has the solution

$$\frac{\mu^2}{\Lambda^2} = \exp \left( \int_{\alpha(\Lambda^2)}^{\alpha(\mu^2)} \frac{d\alpha}{\beta(\alpha)} \right).$$  \hspace{1cm} (46)

The potential (44) can now be expressed in terms of the $\beta$-function

$$\tilde{V}_{\text{lin}}(q) = C \frac{1}{q^2} \exp \left( - \int_{\alpha(\Lambda^2)}^{\alpha(\mu^2)} \frac{d\alpha}{\beta(\alpha)} \right),$$  \hspace{1cm} (47)

and we can use equation (42) backwards to deduce the imaginary part of the anomalous dimension corresponding to the linear potential

$$\text{Im} \gamma_{\text{lin}} = - C \frac{1}{4\pi^2 u} \exp \left( - \int_{\alpha(\Lambda^2)}^{\alpha(\mu^2)} \frac{d\alpha}{\beta(\alpha)} \right).$$  \hspace{1cm} (48)

In principle, we could now reconstruct the real part of the anomalous dimension from a dispersion relation. However, since the nonrelativistic potential approach breaks down for large $v v'$, we do not have any information in this region.

5 Conclusions

The imaginary parts of the anomalous dimensions of HqEFT may be removed by a suitable redefinition of the multiparticle states. The redefinition
of the multiparticle states is achieved by applying a unitary operator, similar
to the Coulomb-phase operator in QED. While this phase operator can be
constructed exactly in the QED case, we can only give a perturbative con-
struction of the nonabelian analogue of the Coulomb-phase operator. Since
the phase operator removes the imaginary parts of the anomalous dimen-
sions, it depends on the renormalization scale; the dependence is governed
by a renormalization group equation with the imaginary parts of the anom-
alous dimensions only.

Furthermore we have elucidated the connection between the imaginary
parts of anomalous dimensions and the long range inter-quark potential. The
potential derived from the HqEFT calculation in the two-particle sector up
to two loops coincides with earlier perturbative calculations.

We believe that we have settled the problem of the apparently complex
anomalous dimensions in the multiparticle sector of HqEFT. The imaginary
parts are an artifact of an improper definition of the states; after a suitable
redefinition of the states there is no such thing as a complex anomalous
dimension.

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