Nonextremal Kerr/CFT on a stretched horizon

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Abstract

We study the conformal symmetry described by $SL(2, R)_L \times SL(2, R)_R$ in the nonextremal Kerr black hole case. We calculate the central charges $c_{L,R}$ separately on a stretched horizon by the Hamiltonian gravity. In order to get two sets of conformal symmetry, we consider two Killing vectors each of which depends on either one of the two independent coordinates separated in the spirit of point-splitting from the angular coordinate of the corotating frame at the horizon. Then, we obtain the Bekenstein-Hawking entropy by the Cardy formula.
I. INTRODUCTION

Since the successful description of the holographic duality between the geometry near the horizon of extremal Kerr black hole and the conformal field theory [1–5], there have been many studies for the holographic duality in various black holes [6–23]. The holographic duality has been also developed for near extremal black holes by introducing the extremal parameter [15, 17] or calculating the graybody factor [24–27]. And the hidden conformal field theory described by the $SL(2, R)_L \times SL(2, R)_R$ in the Kerr black hole has been studied from the scalar wave equation [28]. However, in this case the values of the central charges $c_{L,R}$ have been just assumed to be the same as in the extremal black hole case. Up to now, the left and right central charges $c_{L,R}$ have been calculated only in the near extremal case [15]. There have been many extended studies of the hidden conformal symmetry for various black holes [29–41], but no calculation of the both central charges has been done. Recently, for the nonextremal Kerr black hole, the central charge has been obtained successfully by using the Hamiltonian gravity on a stretched horizon, but it describes only $SL(2, R)_L$ symmetry, not $SL(2, R)_L \times SL(2, R)_R$ [42–46]. There also have been studies for extremal black holes using the Hamiltonian gravity [47, 48].

In this paper, we investigate the conformal symmetry described by $SL(2, R)_L \times SL(2, R)_R$ on a stretched horizon in the nonextremal Kerr black hole case. Adopting the method of Carlip [45, 46], we obtain the central charges $c_{L,R}$ separately. The paper is organized as follows. In Sec. II, we review briefly the Hamiltonian gravity. In Sec. III, we obtain the central charges $c_{L,R}$, the temperatures $T_{L,R}$, and the total entropy $S$ on a stretched horizon in the nonextremal case with the Hamiltonian gravity. In order to have two Virasoro algebras we consider two Killing vectors which are functions of either one of the two coordinates split from the angular coordinate of the corotating frame at the horizon. This splitting was done in the spirit of point-splitting. Then the central charges and the total entropy are evaluated. Finally, in Sec. IV, we conclude with discussion.

II. BRIEF REVIEW ON HAMILTONIAN GRAVITY

In order to obtain the central charges, we evaluate the central terms in the Hamiltonian gravity in this work. Here, we briefly review the related ingredients in the Hamiltonian
gravity for this calculation. The detailed review can be found in Ref. [45]. We consider a stationary nonextremal black hole, whose metric can be written in the ADM form as follows.

$$ds^2 = -N^2 dt^2 + q_{ij}(dx^i + N^i dt)(dx^j + N^j dt).$$

(1)

Under a diffeomorphism generated by a vector field $\xi^\mu$, the metric (1) transforms as

$$\delta \xi N = \bar{\partial}_t \xi^\perp + \hat{\xi}^i \partial_i N,$$

(2)

$$\delta \xi N^i = \bar{\partial}_t \hat{\xi}^i - N q^{ij} \partial_j \xi^\perp + q^{ik} \partial_k N \xi^\perp + \hat{\xi}^j \partial_j N^i,$$

(3)

$$\delta \xi q_{ij} = q_{ik} \left( \partial_j \hat{\xi}^k - \frac{\partial_i N^k}{N} \xi^\perp \right) + q_{jk} \left( \partial_i \hat{\xi}^k - \frac{\partial_i N^k}{N} \xi^\perp \right) + \frac{1}{N} \xi^\perp \bar{\partial}_t q_{ij} + \hat{\xi}^k \partial_k q_{ij},$$

(4)

where the convective derivative $\bar{\partial}_t$ is defined by

$$\bar{\partial}_t \equiv \partial_t - N^i \partial_i,$$

(5)

and the Killing vector $\xi^\mu$ is redefined as

$$\xi^\perp \equiv N \xi^t, \quad \hat{\xi}^i \equiv \xi^i + N^i \xi^t,$$

(6)

which are called the “surface deformation parameters”. The symmetries of canonical general relativity are generated by the Hamiltonian

$$H[\xi] = \int d^3 x \left( \xi^\perp \mathcal{H} + \hat{\xi}^i \mathcal{H}_i \right),$$

(7)

with

$$\mathcal{H} = \frac{1}{\sqrt{q}} \left( \pi^i \pi_{ij} - \pi^2 \right) - \sqrt{q} \mathcal{R}, \quad \mathcal{H}^i = -2 D_j \pi^{ij},$$

(8)

where $q_{ij}$, $\pi^{ij}$, and $\mathcal{R}$ are the spatial metric, the canonical momentum, and the spatial curvature scalar, respectively, and $D_i$ is the spatial covariant derivative with respect to $q_{ij}$. $\mathcal{H}$ and $\mathcal{H}^i$ are the Hamiltonian and momentum constraints, respectively. In Refs. [45, 49], a new generator with a well-defined variation and with no boundary terms is defined by

$$\bar{H}[\xi] = H[\xi] + B[\xi],$$

(9)

where $B[\xi]$ depends only on the fields and parameters at the boundary, and is chosen to cancel the boundary terms in the variation of $H[\xi]$. The Poisson bracket of $\bar{H}[\xi]$ can be written as

$$\{ \bar{H}[\xi], \bar{H}[\eta] \} = \bar{H}[\{ \xi, \eta \}_{SD}] + K[\xi, \eta],$$

(10)
where the central term is given by

\[
K[\xi, \eta] = B[\{\xi, \eta\}_{\text{SD}}] - \frac{1}{8\pi G} \int_{\partial \Sigma} d^2x \sqrt{\sigma} \eta^k \left[ \frac{1}{\sqrt{q}} \pi_{ik} \{\xi, \eta\}_{\text{SD}} - \frac{1}{2\sqrt{q}} (\hat{\xi}_k \eta^i - \hat{\eta}_k \xi^i) \mathcal{H} 
+ (D_i \hat{\xi}_k D^i \eta^k - D_i \hat{\eta}_k D^i \xi^k) - (D_i \hat{\xi}^i D_k \eta^k - D_i \hat{\eta}^i D_k \xi^k)
+ \frac{1}{\sqrt{q}} (\hat{\eta}_k \pi^{mn} D_m \hat{\xi}_n - \hat{\xi}_k \pi^{mn} D_m \hat{\eta}_n) \right].
\] (11)

Here, the surface deformation brackets are given by

\[
\{\xi, \eta\}_{\text{SD}} = \hat{\xi}^i D_i \eta^i - \hat{\eta}^i D_i \xi^i,
\]

\[
\{\xi, \eta\}^i_{\text{SD}} = \hat{\xi}^k D_k \hat{\eta}^i - \hat{\eta}^k D_k \hat{\xi}^i + q^{ik} (\xi^l D_k \eta^l - \eta^l D_k \xi^l).
\] (12)

We can calculate the central charges from the central term \(K[\xi, \eta]\) given by Eq. (11). When a Virasoro subalgebra of the group of surface deformation is found in the following form, \(\{\xi, \eta\}_{\text{SD}} = \xi \eta' - \eta \xi'\) where the prime denotes the derivative with respect to the rotating angle \(\varphi\), then the boundary contribution \(B[\{\xi, \eta\}_{\text{SD}}]\) can be ignored for our purpose [45]. This is because that the central charge of the Virasoro algebra is given by the coefficient of the expression \(\int d\varphi (\xi' \eta'' - \eta' \xi'')\) extracted from \(K\) in Eq. (11).

### III. CFT IN NONEXTREMAL ROTATING BLACK HOLE

We now consider the nonextremal Kerr black hole described by

\[
ds^2 = -\frac{\Delta}{\Sigma} \left( dt - a \sin^2 \theta \, d\varphi \right)^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \frac{\sin^2 \theta}{\Sigma} \left[ a dt - (r^2 + a^2) d\varphi \right]^2,
\] (13)
in the Boyer-Lindquist coordinates, with

\[
\Delta(r) = r^2 + a^2 - 2Mr,
\]

\[
\Sigma(r, \theta) = r^2 + a^2 \cos^2 \theta,
\] (15)

where \(M\) is the mass of the black hole and \(a\) is a parameter related to the angular momentum \(J\) of the black hole as \(J = aM\). The nonextremal Kerr black hole (13) can be written in the ADM form as

\[
ds^2 = -N^2 dt^2 + d\rho^2 + q_{\varphi \varphi} (d\varphi + N^2 dt)^2 + q_{\theta \theta} d\theta^2,
\] (16)

where \(\rho\) is the proper distance from the horizon. In general, the line element of the nonextremal stationary rotating black hole can be expressed as in Eq. (16). From now on, we
consider the nonextremal stationary rotating black hole in four dimensions. Near the horizon of the black hole, the lapse function $N$ and the shift vector $N\varphi$ in the metric (16) can be expanded as

$$N = \kappa_H \rho + \frac{1}{3!} \kappa_2(\theta) \rho^3 + \cdots, \quad (17)$$

$$N\varphi = -\Omega_H - \frac{1}{2} \omega_2(\theta) \rho^2 + \cdots, \quad (18)$$

$$q_{\varphi\varphi} = q_{\varphi\varphi}^{(H)}(\theta) + \frac{1}{2} q_{\varphi\varphi}^{(2)}(\theta) \rho^2 + \cdots, \quad (19)$$

$$q_{\theta\theta} = q_{\theta\theta}^{(H)}(\theta) + \frac{1}{2} q_{\theta\theta}^{(2)}(\theta) \rho^2 + \cdots, \quad (20)$$

where $\kappa_H$ and $\Omega_H$ are the surface gravity and the angular velocity on the event horizon, respectively. For convenience, we introduce a parameter $\varepsilon$ defined by $\varepsilon \equiv -\frac{1}{2} \omega_2(\theta) \rho^2$ so that $N\varphi \approx -\Omega_H + \varepsilon$ up to the order of $\rho^2$. The shift vector $N\varphi$ becomes the angular velocity near horizon, when $\rho \to 0$.

When the black hole geometry is given by Eq. (16), the diffeomorphism given by (2)-(4) becomes

$$\xi^\rho = -\rho \partial_t \xi^t, \quad (21)$$

$$\tilde{\partial}_t \xi^\rho = \kappa_H^2 \rho^2 \partial_\rho \xi^t, \quad (22)$$

$$\tilde{\partial}_t \dot{\xi}^\varphi = \kappa_H^2 \rho^2 q_{\varphi\varphi} \partial_\varphi \xi^t - \frac{2\varepsilon}{\rho} \xi^\rho, \quad (23)$$

$$\partial_\rho \dot{\xi}^\varphi = -q_{\varphi\varphi}^{-1} \partial_{\varphi} \xi^\rho + \frac{2\varepsilon}{\rho} \xi^t, \quad (24)$$

where the convective derivative $\tilde{\partial}_t$ in Eq. (5) becomes

$$\tilde{\partial}_t = \partial_t - N\varphi \partial_\varphi \approx \partial_t + (\Omega_H - \varepsilon) \partial_\varphi. \quad (25)$$

Note that (16) can be rewritten as

$$ds^2 = q_{\varphi\varphi}(d\varphi - \Omega_+ dt)(d\varphi - \Omega_- dt) + d\rho^2 + q_{\theta\theta} d\theta^2, \quad (26)$$

where

$$\Omega_{\pm} \equiv -N\varphi \pm \frac{N}{\sqrt{q_{\varphi\varphi}}}. \quad (27)$$

Introducing a new parameter,

$$\bar{\varepsilon} \equiv \kappa_H \rho / \sqrt{q_{\varphi\varphi}}, \quad (28)$$

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we can write $\Omega_\pm$ as

$$\Omega_\pm = \Omega_H - \varepsilon \pm \bar{\varepsilon},$$  \hspace{1cm} (29)

up to the order of $\rho^2$.

We choose our stretched horizon such that its maximum and minimum angular velocities are given by $\Omega_+$ and $\Omega_-$, respectively, defined by (29). One can easily check that our new Killing vectors $\chi_\pm \equiv \partial_t + \Omega_\pm \partial_\phi$ are null. For the purpose of studying the conformal field theory, we introduce a new coordinate system defined by

$$\varphi_\pm = 2(\varphi - \Omega_\pm t).$$  \hspace{1cm} (30)

We assign the coordinates $\varphi_+$ and $\varphi_-$ as the right and left conformal coordinates, respectively. The two coordinates are slightly separated from the angular coordinate of the corotating frame at the horizon, $\bar{\varphi} = \varphi - \Omega_H t$. We may consider these two coordinates as two-fold split of the angular coordinate of the corotating frame at the horizon in the spirit of point-splitting [50].

Now, we assume the existence of two Killing vectors, which are functions of either $\varphi_+$ or $\varphi_-$, dubbed as the right (+) and left (−) handed Killing vectors. We thus consider the right/left handed conformal field theory (R/LCFT) where the Killing vectors $\xi_\pm$ are the functions of $\varphi_\pm$, respectively. In that case, the expressions for the diffeomorphism invariance given by Eqs. (21), (22), (23), and (24) become

$$\xi_\rho = \pm \rho \bar{\varepsilon} \partial_\varphi \xi_\pm,$$

$$\rho \partial_\rho \xi_\pm = -\bar{\varepsilon}^2 \kappa_H \partial_\varphi^2 \xi_\pm,$$  \hspace{1cm} (32)

$$\dot{\xi}_\varphi = \mp \bar{\varepsilon} \xi_\pm,$$  \hspace{1cm} (33)

$$\partial_\rho \dot{\xi}_\pm = \mp \rho \bar{\varepsilon} q^{\varphi \varphi} \partial_\varphi^2 \xi_\pm + \frac{2\varepsilon}{\rho} \xi_\pm,$$  \hspace{1cm} (34)

up to the leading order. From Eq. (11), the terms that contribute to the central charge are given by

$$K[\xi_\pm, \eta_\pm] = \pm \frac{\bar{\varepsilon}^3}{8\pi G\kappa_H} \cdot \frac{A}{2\pi} \int d\varphi \left( \partial_\varphi \xi_\pm^t \partial_\varphi^2 \eta_\pm^t - \partial_\varphi \eta_\pm^t \partial_\varphi^2 \xi_\pm^t \right),$$  \hspace{1cm} (35)

where $A$ denotes the area of the stretched horizon.
We obtain the surface deformation brackets \( \{ \xi_\pm, \eta_\pm \}_t \) for the generators up to the leading order as follows.

\[
\{ \xi_\pm, \eta_\pm \}_t^{SD} = \mp 2\varepsilon (\xi_\pm \partial \varphi \eta_\pm^t - \eta_\pm \partial \varphi \xi_\pm^t), \\
\{ \xi_\pm, \eta_\pm \}_t^\rho^{SD} = \pm \rho \varepsilon \partial \varphi \{ \xi_\pm, \eta_\pm \}_t^{SD}, \\
\{ \xi_\pm, \eta_\pm \}_t^{\varphi}^{SD} = \mp \varepsilon \{ \xi_\pm, \eta_\pm \}_t^{SD}.
\]

The commutator between the right and left handed generators are obtained as

\[
\{ \xi_+, \eta_- \}_t^{SD} = \{ \xi_+, \eta_- \}_t^\rho^{SD} = \{ \xi_+, \eta_- \}_t^{\varphi}^{SD} = 0,
\]

up to the leading order. Here we would like to note that (37) is obtained when our new parameter \( \tilde{\varepsilon} \) is defined by (28). Eq. (37) shows that the two Virasoro algebras of \( \xi_+ \) and \( \xi_- \) are independent of each other, and the two Virasoro algebras contribute separately to the entropy. One can also see from (36) that \( \xi_\pm^t \) have to be normalized as

\[
\xi_\pm^t = \mp \frac{1}{2\varepsilon} \tilde{\xi}_\pm^t,
\]

where the normalized ones are denoted by \( \tilde{\xi}_\pm^t \). Taking the above normalization into account, the central charge can be read as

\[
c_{R/L} \equiv c_\pm = \frac{3\varepsilon A}{2\pi G\kappa_H}.
\]

The temperatures, both the right and left handed ones, can be calculated by comparing the Boltzmann factor. For a scalar field in the Frolov-Thorne vacuum, the wave function in the eigenmodes of energy \( \omega \) and angular momentum \( m \) can be written as \( \Phi \sim e^{-i\omega t + im\varphi} \). Then, using the relation

\[
e^{-i\omega t + im\varphi} = e^{-in_- \varphi_- - im_+\varphi_+},
\]

we obtain the eigenmodes in the new coordinate system of \( \varphi_\pm \) as follows

\[
n_\pm = \frac{1}{4\varepsilon} (\omega - m\Omega_\mp).
\]

In terms of these variables, the Boltzmann factor can be written as

\[
e^{-(\omega - m\Omega_H)/T_H} = e^{-n_+/T_+ - n_-/T_-}.
\]
where the dimensionless left and right temperatures are given by
\[ T_{R/L} \equiv T_{\pm} = \frac{T_H}{2\varepsilon}, \quad (43) \]
with \( T_H = \kappa_H/2\pi \).

Finally, we calculate the entropy with the help of the Cardy formula. From Eqs. (39) and (43), the entropy of the right/left part is given by
\[ S_{R/L} = \frac{\pi^2}{3} c_{R/L} T_{R/L} = \frac{A}{8G}. \quad (44) \]
Thus we obtain the total entropy of the system as
\[ S = S_R + S_L = \frac{A}{4G}, \quad (45) \]
which agrees with the Bekenstein-Hawking entropy.

**IV. CONCLUSION**

In this paper, we obtain the entropy of the nonextremal Kerr black hole by considering two copies of the Virasoro algebras. In order to have two copies of the Virasoro generators, we introduce the left and right handed coordinates that correspond to the corotating coordinates of the minimum and maximum angular velocities of the stretched horizon, respectively, by adopting the point-splitting idea. We choose our new parameter \( \varepsilon \) such that the surface deformation brackets between the two Virasoro generators commute. Showing their independence, we calculate the central charges of the two Virasoro algebras and the corresponding entropies independently using the Cardy formula. In our setup, each part contributes the same amount of entropy, \( \frac{A}{8G} \), making the total entropy agree with the Bekenstein-Hawking entropy.

Now we make a final comment on the null condition for the Killing vectors on the stretched horizon. In Refs. [45, 46], \( \varepsilon \) has been chosen such that the Killing vector \( \tilde{\chi}^\mu = \partial_t^\mu + \tilde{\Omega} \partial_\varphi^\mu \) is null on the stretched horizon. Notice that \( \tilde{\Omega} \) agrees with our \( \Omega_\_ \) up to the first order in \( \rho \). There the condition was set by imposing the null condition of the Killing vector up to the leading order in \( \rho \):
\[ 0 = \tilde{\chi}^2 = -N^2 + q_{\varphi\varphi}(N^\varphi + \tilde{\Omega})^2 \]
\[ = -\kappa_H^2 \rho^2 + q_{\varphi\varphi}^2 \varepsilon^2 + O(\rho^3). \quad (46) \]
Thereby \( \varepsilon \) is set to satisfy \( \varepsilon^2 = \kappa_H^2 \rho^2 / q_{\varphi \varphi}^{(H)} \), which agrees with our definition. However, if we replace \( \bar{\Omega} \) with \( \Omega_\pm \) in the above defining relation of \( \bar{\chi} \), we end up with our new Killing vectors, \( \chi_\pm = \partial_t + \Omega_\pm \partial_\varphi \), which satisfy the null condition up to the subleading order in \( \rho \).

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