Alternative to Dzyaloshinskii-Moriya interaction for monolayer Fe$_3$GeTe$_2$ and other two-dimensional ferromagnets with trigonal prismatic symmetry

I.A. Ado,$^{1,2}$ Gulnaz Rakhmanova,$^3$ Dmitry A. Zezyulin,$^3$ Ivan Iorsh,$^3$ and M. Titov$^1$

$^1$Radboud University, Institute for Molecules and Materials, 6525 AJ Nijmegen, The Netherlands
$^2$Institute for Theoretical Physics, Utrecht University, 3584 CC Utrecht, The Netherlands
$^3$ITMO University, Faculty of Physics, Saint-Petersburg, Russia

We suggest a possible origin of noncollinear magnetic textures in ferromagnets (FMs) with the $D_{3h}$ point group symmetry. Suggested mechanism is different from the Dzyaloshinskii-Moriya interaction (DMI). Considered symmetry class is important because a large fraction of all single-layer intrinsic FMs should belong to it. In particular, so does a monolayer Fe$_3$GeTe$_2$. At the same time, DMI vanishes identically in materials described by this point group, in the continuous limit. We use symmetry analysis to derive all possible contributions to the free energy density that are of the fourth order with respect to the unit vector $n$ of the local magnetization direction and linear with respect to spatial derivatives of $n$. There are precisely seven such contributions. However, up to boundary terms, only two of them can be considered as independent. Moreover, for a two-dimensional system, one of these two necessarily vanishes! We investigate whether the remaining contribution can serve as a source of spin spirals and skyrmions recently observed in Fe$_3$GeTe$_2$. We also address a possible stabilization of bimerons by the same contribution.

Successful isolation of graphene in 2004[1, 2] attracted a remarkable interest to the field of purely two-dimensional (2D) materials. The field has been growing ever since and a large variety of other stable atomically thin crystals has been discovered. It is not only the fundamental interest [3-6] that drives the research on 2D materials, but the potential applications as well. Of a particular importance for the latter is the fact that low-dimensional systems can be tuned in a much more effective way than their bulk counterparts [7, 8]. It is also important that one can combine properties of different 2D materials by stacking them in heterostructures [9-11].

Potential applications of heterostructures and monolayers include, among others, spin-based computer logic and new ways to store information [12, 13]. Possibility to stabilize and effectively manipulate magnetic order is clearly an absolute necessity for the first direction, but it is often considered also in the context of the second. In particular, it is assumed that noncollinear magnetic textures, like skyrmions and miniature domain walls, might become the basis for future memory devices [14-16]. However, for more than a decade no atomically thin intrinsic magnets have been realized in experiments. This happened only in 2017 when magnetic order was reported in two-dimensional van der Waals materials Cr$_2$Ge$_2$Te$_6$ [17] and CrI$_3$ [18]. Soon they were accompanied by a metallic itinerant ferromagnet Fe$_3$GeTe$_2$ [19, 20] (FGT).

Recently, Néel-type skyrmions were observed in two different heterostructures based on FGT [21, 22], and spin spirals were reported in thin multilayers of this material [23]. However, identification of the origin of such textures runs into issues. Basically, it is unclear whether noncollinear magnetic order in FGT can be correctly explained by the Dzyaloshinskii-Moriya interaction [24, 25] (DMI). The reason for this is the following. Bulk FGT has an inversion symmetry center, and thus smooth textures cannot originate in associated with DMI contributions to the free energy. Monolayer FGT, on the other hand, does lack the inversion symmetry. But its point group $D_{3h}$ is still so symmetric that any contribution to the free energy density of the form $n_i \nabla_j n_k$ can affect magnetic order only at the sample boundaries. This fact was coined in Ref. [26] and repeated in a recent paper [27] with quite an illustrative title: ‘Elusive Dzyaloshinskii-Moriya interaction in monolayer Fe$_3$GeTe$_2’.” Some of us also mentioned this in Ref. [28]. Here by $n$ we denote the unit vector of the local magnetization direction.

In the context of magnetism in two spatial dimensions, there is another important class of materials — transition metal dichalcogenides (TMDs). Many of them, when thinned down to a monolayer, may also be intrinsically magnetic [29]. Typically, 2D TMDs are formed in 1T and 2H phases [29, 30]. Since the latter is again characterized by the point group $D_{3h}$, DMI in such materials can be considered as similarly elusive. Let us also mention here recently predicted 2D chromium pnictides [31], which are half-metallic ferromagnets with very high Curie temperatures. These materials, if realized in practice, can be particularly promising for spin-based logic and other spintronics applications. Needless to say, 2D chromium pnictides are also characterized by $D_{3h}$ — the group of symmetries of a triangular prism. Evidently, it is an important group in the field of intrinsic 2D FMs.

The two papers we mentioned earlier, Refs. [21, 22], suggest that textures addressed in these works are stabilized by DMI stemming from the interfaces. In this Letter, we discuss another possible origin of noncollinear magnetic textures in ferromagnets (FMs) with the $D_{3h}$ point group symmetry.

Such textures are commonly believed to be a result of a competition between a strong symmetric exchange in-
teraction and DMI that is normally weaker. In the continuous limit, the latter is expressed by antisymmetric contributions \( n_i \nabla_j n_k - n_k \nabla_j n_i \) to the free energy density (in FMs). Similar symmetric terms are represented by full derivatives \( \nabla_j (n_i n_k) \) and therefore are relevant only close to the edges of the sample. In \( D_{3h} \), all antisymmetric contributions vanish. It is thus worthwhile to consider terms of the fourth order with respect to the vector \( n \) that are also linear in its derivatives: \( n_i n_j n_k \nabla_l n_p \). Physically they can correspond, for example, to interactions between four spins [32, 33].

Let us assume that the 3-fold rotation axis is \( z \), while \( x \) is one of the 2-fold rotation axes (the other two are at the angles \( \pm 2\pi/3 \) in the \( xy \)-plane, which is also a mirror plane). Using standard symmetry analysis [34, 35] we can now identify all fourth order contributions to the free energy density that are allowed in \( D_{3h} \). There are precisely seven such contributions, and we collect them in Table II. Surprisingly, up to boundary terms, five of them are not independent. In order to prove this, one should take into account the constraint \( n^2 = 1 \) (see also Table I). We can choose

\[
\begin{align*}
  w_{||} &= n_x \left( n_x^2 - 3n_y^2 \right) (\nabla_x n_x + \nabla_y n_y), \\
  w_{\perp} &= n_x \left( n_x^2 - 3n_y^2 \right) \nabla_z n_z
\end{align*}
\]

as the only independent invariants. Obviously, for a 2D system, the second one can be disregarded. Hence, if the effects of boundaries are negligible, we are left with a single fourth order term: \( w_{||} \). This is a rather unexpected simplification of the theory.

It is interesting to relate the structure of \( w_{||} \) to the lattice geometry of a typical 2D crystal described by the point group \( D_{3h} \). One can notice that

\[
\begin{align*}
  w_{||} &\propto (\mathbf{n} \cdot \mathbf{d}_1) (\mathbf{n} \cdot \mathbf{d}_2) (\mathbf{n} \cdot \mathbf{d}_3) (\nabla_x n_x + \nabla_y n_y), \\
\end{align*}
\]

where \( \mathbf{d}_i \) represent the nearest neighbour vectors along the angles \( 0, 2\pi/3, \) and \( 4\pi/3 \) in the \( xy \)-plane. These angles correspond to the three armchair directions of a typical hexagonal lattice generated by \( D_{3h} \) (see the top part of Fig. 1). We also note that the combination of derivatives in Eq. 2 coincides with the expression for the emergent Rashba field [36] \( B_R \propto (\nabla_x n_x + \nabla_y n_y) e_z \). This, however, is just an amusing coincidence.

Now let us find out whether this new term \( w_{||} \) that we have introduced can stabilize spin spirals observed in FGT. In order to do this, we consider a conical ansatz

\[
\mathbf{n}(r) = m \cos \alpha + m_\theta \cos (\mathbf{k} r) + m_\phi \sin (\mathbf{k} r) \sin \alpha
\]

that parameterizes the transition from a collinear state, \( \sin \alpha = 0 \), to a helix, \( \cos \alpha = 0 \) (if \( \mathbf{k} \neq 0 \)). Here

\[
\begin{align*}
  m &= (\cos \phi \sin \theta, \sin \phi \sin \theta, \cos \theta), \\
  m_\theta &= (\cos \phi \cos \theta, \sin \phi \cos \theta, -\sin \theta), \\
  m_\phi &= (-\sin \phi, \cos \phi, 0)
\end{align*}
\]

is a standard basis in spherical coordinates. Vectors \( m_\theta \) and \( m_\phi \) correspond to oscillations, while \( m \) points in the direction of the average magnetization.

We substitute this ansatz into a model that accounts for the symmetric exchange and magnetic anisotropy,

\[
f = A \left[ (\nabla_x n)^2 + (\nabla_y n)^2 \right] + Kn^2 + 8Dw_{||},
\]

and average the total free energy \( F = \int d^2 r f \) over a large volume. Oscillating terms with \( kr \) then vanish and the averaged density \( (\bar{f}) \) becomes a quadratic function of \( \mathbf{k} \). Therefore, we straightforwardly minimize it with respect to the wave vector and find:

\[
\frac{A}{D^2} (\bar{f}) = -
\begin{align*}
  &\frac{9}{64} (\sin \alpha + 5 \sin 3\alpha)^2 \sin^4 \theta \cos^2 \theta \\
  &+ \frac{AK}{2D^2} (\sin^2 \theta \sin^2 \alpha + 2 \cos^2 \theta \cos^2 \alpha),
\end{align*}
\]

where the combination \( AK/D^2 \) is dimensionless and further minimization with respect to \( \theta \) and \( \alpha \) is needed. The wave vector is expressed as

\[
\begin{align*}
  \left( \frac{k_x}{k_y} \right) &= -
  \begin{align*}
  &\frac{3D}{2A} (5 \cos^2 \alpha - 1) \sin^2 \theta \cos \theta \left( \sin 2\phi \cos 2\phi \right).
\end{align*}
\end{align*}
\]

Before we proceed with the minimization, it is interesting to note that states described by Eq. (6) are degenerate with respect to \( \phi \). In other words, their free energy does not depend on the azimuthal angle of “the average magnetization vector” \( m \). At the same time, for spiral-like textures with a finite wave vector, the angles between \( \mathbf{k} \) and \( m \) are different for different values of \( \phi \):

\[
\mathbf{k} \cdot m \propto \sin 3\phi.
\]

Thus, by controlling the direction of the average magnetization (vector \( m \)), one should also be able to control the propagation direction of the spiral. Such control can
be achieved by an application of a small external magnetic field. The latter couples only to \( \mathbf{m} \), therefore it can be used to set the desired value of \( \phi \). As we see from Eq. (8), when the in-plane component \( \mathbf{m}_\parallel \) of the vector \( \mathbf{m} \) lies along one of the armchair directions of the lattice (\( \phi = 0, 2\pi/3, 4\pi/3 \)), then \( \mathbf{k} \) is orthogonal to it. When \( \mathbf{m}_\parallel \) is along a zigzag direction (\( \phi = \pi/6, 5\pi/6, 3\pi/2 \)), then \( \mathbf{k} \) (or \( -\mathbf{k} \)) points in the same direction (see the bottom part of Fig. 1). We wonder whether such control of the wave vector direction can be realized experimentally.

Let us proceed with the analysis of the functional defined by Eq. (6). Since it is a cubic function of both \( \sin^2 \theta \) and \( \sin^2 \alpha \), one can, in principle, find its minimum analytically. But a perturbative analysis with respect to small \( AK/D^2 \) provides almost a perfect fit in the entire range of parameters. States with \( \mathbf{k} \neq 0 \) can exist when \(-0.98 \lesssim AK/D^2 \lesssim 2.18 \), and for such states we find

\[
\sin^2 \theta = \frac{2}{3} + \frac{9}{128} \frac{AK}{D^2} + \ldots, \tag{9a}
\]

\[
\sin^2 \alpha = \frac{4}{15} \left( \frac{9}{32\sqrt{10}} \frac{AK}{D^2} \right)^2 + \ldots, \tag{9b}
\]

where only the leading and the subleading order terms are shown. For other values of the parameter \( AK/D^2 \), the state is collinear: out-of-plane for \( K < 0 \) and in-plane for \( K > 0 \) (see Fig. 2). This resembles a typical situation with magnetic textures determined by DMI: when the absolute value of the DMI strength \( D \) exceeds some critical value \( D_c \propto \sqrt{A/K} \), the system is found in a helical ground state, while for \( |D| < D_c \) the uniform magnetization is favoured.

Helical states described by Eqs. (9) are in a reasonable agreement with spirals found in FGT. For \( \phi = 0 \) and large enough \( |D| \), we have a helical texture with finite \( k \) pointing along the \( y \)-direction. The components \( n_x \) and \( n_z \) of this texture oscillate with a phase difference of \( \pi/2 \) (see Eqs. (3), (4)). This is precisely what has been reported in Ref. [24]. In addition, however, we have an oscillating \( n_x \) component, with a slightly smaller amplitude that is, basically, equal to \( \cos \theta \). It is not clear whether this is a crucial disagreement with the experiment or something that was not seen in it. We should also note that \( x \) and \( y \) axes in Ref. [24] describe the coordinates of detectors, not the crystal axes. Hence the \( x \) component of magnetization in that paper can indeed correspond to our \( n_y \).

Situation with skyrmions is less appealing. For a standard ansatz \( n(r) = (\cos \Phi \sin \Theta, \sin \Phi \sin \Theta, \cos \Theta) \), with \( \Phi = Q\phi + \delta \) and \( \Theta = \Theta(r) \), the fourth order term \( w_{||} \) in the free energy vanishes after integration over \( \phi \) when \( Q \) is an integer. Thus, \( w_{||} \) cannot stabilize circular skyrmions. There exist predictions of skyrmions with more complex axial symmetry [39], including trigonal [40, 41]. Such symmetry is natural for \( D_{3h} \) and we have checked that indeed, for “trigonal skyrmions”, \( w_{||} \) is generally finite after the angle integration. It is however very unlikely that this can explain the experiments of Refs. [21, 22].

At the same time, for \( K > 0 \) (easy plane anisotropy) our model can stabilize bimerons — the in-plane ana-

FIG. 1. Hexagonal lattice typical for many materials with trigonal prismatic symmetry. Panels show three particular directions of the spiral wave vector \( \mathbf{k} \) (up to a sign) and the in-plane component \( \mathbf{m}_\parallel \) of the vector \( \mathbf{m} \). The latter corresponds to the average magnetization direction and can be controlled by an external magnetic field. Note that directions of both \( \mathbf{k} \) and \( \mathbf{m}_\parallel \) are set by the angle \( \phi \) (see Eqs. (4a), (7)).

FIG. 2. Acute angles that correspond to the global minimum of the functional given by Eq. (6). In fact, for \( AK/D^2 \gtrsim 2.18 \), the configuration with \( \sin^2 \alpha = 1, \sin^2 \theta = 0 \) has the same energy as the one with \( \sin^2 \alpha = 0, \sin^2 \theta = 1 \) shown here. However, both configurations describe the same collinear state with \( \mathbf{n} \) lying in the \( xy \)-plane (i.e., with \( \pm \mathbf{n} = \mathbf{m} = \mathbf{m}_\parallel \)). Dashed curves represent the expressions of Eqs. (9).
log of skyrmions [12][14]. As we have mentioned previously, a collinear state is preferred over spin spirals when \( D^2 \lesssim AK/2.18 \approx 0.46AK \) (we now consider \( K > 0 \)). In this case, a bimeron can exist as a metastable transition from one in-plane state to another. In order to demonstrate this, let us consider a parameterization

\[
\begin{align*}
n(r) &= \mathcal{R}_z [\phi_0] (\cos \Theta, \cos \Phi \sin \Theta, \sin \Phi \sin \Theta), \\
\Phi &= Q \phi + \delta, \\
\Theta &= \Theta(r),
\end{align*}
\]

(10a)

(10b)

with the boundary conditions \( \Theta(0) = \pi, \Theta(\infty) = 0 \). In Eqs. (10), \( Q \) is the bimeron’s integer topological charge, and \( \mathcal{R}_z [\phi_0] \) denotes a matrix of rotation by an arbititary angle \( \phi_0 \) with respect to \( z \). The inset in the bottom panel of Fig. 3 provides an illustration of a bimeron that corresponds to \( \delta = \pi/2, \phi_0 = 0, Q = 1 \).

We substitute Eqs. (10) with \( Q = 1 \) into Eq. (9), integrate over \( \phi \), and compute a functional derivative of the result. This brings us to the Euler-Lagrange equation

\[
\begin{align*}
\Theta''(r) + \frac{\Theta'(r)}{r} - &\frac{\sin 2\Theta(r)}{2r^2} - \frac{K \sin 2\Theta(r)}{4A} \\
- &\frac{3D \sin (2\phi_0 + \delta)}{2Ar} \sin^2 \Theta(r) \left[ 5 \cos^2 \Theta(r) - 1 \right] = 0.
\end{align*}
\]

(11)

It is very similar to the equation that describes skyrmions in the presence of the DMI term \( \phi_0 \sin (2\Theta) \) [38, 45]. As we have metioned previously, a collinear state is preferred over spin spirals when \( D^2 \lesssim AK/2.18 \approx 0.46AK \) (we now consider \( K > 0 \)). In this case, a bimeron can exist as a metastable transition from one in-plane state to another. In order to demonstrate this, let us consider a parameterization

\[
\begin{align*}
n(r) &= \mathcal{R}_z [\phi_0] (\cos \Theta, \cos \Phi \sin \Theta, \sin \Phi \sin \Theta), \\
\Phi &= Q \phi + \delta, \\
\Theta &= \Theta(r),
\end{align*}
\]

(10a)

(10b)

with the boundary conditions \( \Theta(0) = \pi, \Theta(\infty) = 0 \). In Eqs. (10), \( Q \) is the bimeron’s integer topological charge, and \( \mathcal{R}_z [\phi_0] \) denotes a matrix of rotation by an arbititary angle \( \phi_0 \) with respect to \( z \). The inset in the bottom panel of Fig. 3 provides an illustration of a bimeron that corresponds to \( \delta = \pi/2, \phi_0 = 0, Q = 1 \).

We substitute Eqs. (10) with \( Q = 1 \) into Eq. (9), integrate over \( \phi \), and compute a functional derivative of the result. This brings us to the Euler-Lagrange equation

\[
\begin{align*}
\Theta''(r) + \frac{\Theta'(r)}{r} - &\frac{\sin 2\Theta(r)}{2r^2} - \frac{K \sin 2\Theta(r)}{4A} \\
- &\frac{3D \sin (2\phi_0 + \delta)}{2Ar} \sin^2 \Theta(r) \left[ 5 \cos^2 \Theta(r) - 1 \right] = 0.
\end{align*}
\]

(11)

It is very similar to the equation that describes skyrmions in the presence of the DMI term \( n_z (\nabla \cdot n) - (n \cdot \nabla) n_z \). From Refs. [35][45] we know that the radial profile of such skyrmions can be very well approximated by a domain wall with two parameters. We can use the approach of these papers to analyze solutions of Eq. (11) as well.

The only difference between the Euler-Lagrange equation for skyrmions and Eq. (11) is the presence of the term \( 5 \cos^2 \Theta(r) - 1 \) in the latter. At small values of \( r \), its effect on the solutions is minimal. But for larger \( r \), when \( \cos \Theta(r) \approx \pm 1/\sqrt{5} \), this term becomes important. Therefore, we can expect that the bimeron profile \( \Theta(r) \) is a superposition of a domain wall and some additional structure. Being optimistic, one can hope that at least some properties of \( \Theta(r) \) can be captured from the analysis of its domain wall “component” alone. It turns out that this is indeed the case.

We employ the ansatz of Ref. [45]:

\[
\Theta_{dw}(r) = 2 \arctan \left[ \frac{\sinh \left( \frac{R}{\Delta} \right)}{\sinh \left( \frac{r}{\Delta} \right)} \right],
\]

(12)

where \( \Delta \) is the width of the domain wall, and \( R \) is the profile radius: \( \Theta_{dw}(R) = \pi/2 \). Assuming \( R \gg \Delta \) and repeating the considerations of Ref. [45], we can estimate the free energy \( F \) of this ansatz as

\[
F \approx 4\pi \left[ A \left( \frac{R}{\Delta} + \frac{\Delta}{R} \right) + \frac{KR\Delta}{2} + \frac{3\pi DR}{16} \sin (2\phi_0 + \delta) \right].
\]

Based on this result, we can argue that the minimal energy corresponds to \( \sin (2\phi_0 + \delta) = -\text{sign} \, D \). Alternatively, we can see this from the direct minimization of the above expression (using the fact the \( R \) should be positive). Either way, we minimize our expression for \( F \) with respect to both \( R \) and \( \Delta \) to obtain

\[
\Delta = \frac{3\pi |D|}{16K}, \quad R = \frac{\Delta}{\sqrt{1 - K\Delta^2/2A}}.
\]

(13)

For \( D^2/\Delta K \lesssim 0.46 \), the square root can be safely ignored and we are left with \( R = \Delta = 3\pi |D|/16K \).

This result obviously contradicts the initial assumption \( R \gg \Delta \). Nevertheless, it works astonishingly well when \( r \lesssim R \), as can be seen from Fig. 3. There we plot numerical solutions of Eq. (11) that was supplemented with the condition \( \sin (2\phi_0 + \delta) = -\text{sign} \, D \). For \( \Theta(r) \gtrsim \pi/2 \), the ansatz \( \Theta_{dw}(r) \) correctly reproduces the shape of the bimeron (top panel) and allows us to get a good estimate of its radius (bottom panel). Out of curiosity, we also solved Eq. (11) for \( D^2/\Delta K = 4 \) (inset of the top panel). The result clearly looks like a superposition of two domain walls that match at \( \cos \Theta(r) \approx 1/\sqrt{5} \) (as could be
expected). This is of course by far the “spiral-region” of our model.

To conclude, we used symmetry analysis to obtain all fourth order contributions to the free energy density of the form $n_1 n_2 n_3 n_4 \nabla^4 n_5$ that are allowed by the point group $D_{4h}$. There are exactly seven such contributions. Only two of them can be chosen as independent if boundary terms are ignored, and only one of these two does not vanish in a 2D system. We showed that the fourth order contribution $w_4 = n_2 (n_3^2 - 3n_4^2) (\nabla_x n_1 + \nabla_y n_2)$ to the free energy density looks compatible with spin spirals observed in a recent experiment on FGT. It does not stabilize circular skyrmions, but for FM with an easy axis plane it can stabilize bimerons. We estimated the radius that can stabilize bimerons, and energy of such bimerons analytically and calculated numerically. We showed that the fourth order terms are ignored, and only one of these two does not vanish in a 2D system. We showed that the fourth order contributions to the free energy density of our model.

To conclude, we used symmetry analysis to obtain all fourth order contributions to the free energy density of the form $n_1 n_2 n_3 n_4 \nabla^4 n_5$ that are allowed by the point group $D_{4h}$. There are exactly seven such contributions. Only two of them can be chosen as independent if boundary terms are ignored, and only one of these two does not vanish in a 2D system. We showed that the fourth order contribution $w_4 = n_2 (n_3^2 - 3n_4^2) (\nabla_x n_1 + \nabla_y n_2)$ to the free energy density looks compatible with spin spirals observed in a recent experiment on FGT. It does not stabilize circular skyrmions, but for FM with an easy axis plane it can stabilize bimerons. We estimated the radius that can stabilize bimerons, and energy of such bimerons analytically and calculated numerically. We showed that the fourth order terms are ignored, and only one of these two does not vanish in a 2D system. We showed that the fourth order contributions to the free energy density of our model.

We are grateful to Marcos H. D. Guimarães for answering approximately one million of questions about the experiment of Ref. [23] and to Alexander Rudenko for insights regarding 2D FMs. This research was supported by the JTC-FLAGERA Project GRANSPORT.

[1] K. S. Novoselov, A. K. Geim, S. V. Morozov, D. Jiang, Y. Zhang, S. V. Dubonos, I. V. Grigorieva, and A. A. Firsov, *Science* **306**, 666 (2004).
[2] K. S. Novoselov, D. Jiang, F. Schedin, T. Booth, V. Khotkevich, S. Morozov, and A. K. Geim, *Proc. Natl. Acad. Sci. USA* **102**, 10451 (2005).
[3] K. S. Novoselov, A. K. Geim, S. V. Morozov, D. Jiang, M. I. Katsnelson, I. V. Grigorieva, S. V. Dubonos, and A. A. Firsov, *Nature* **438**, 197 (2005).
[4] Y. Zhang, Y.-W. Tan, H. L. Stormer, and P. Kim, *Nature* **438**, 201 (2005).
[5] H. L. Zhuang, P. R. C. Kent, and R. G. Hennig, *Phys. Rev. B* **93**, 134407 (2016).
[6] D. L. Cortie, G. L. Causer, K. C. Rule, H. Fritzsche, W. Kreuzpaintner, and F. Klose, *Adv. Funct. Mater. Mat. Sci. Adv. 4*, eaar2030 (2018).
[7] K. S. Burch, D. Mandrus, and J.-G. Park, *Nature* **563**, 47 (2018).
[8] A. K. Geim and I. V. Grigorieva, *Nature* **499**, 419 (2013).
[9] K. Novoselov, A. Mishchenko, A. Carvalho, and A. C. Neto, *Science* **353**, 9ac9439 (2016).
[10] M. Gibertini, M. Koperski, A. F. Morpurgo, and K. S. Novoselov, *Nat. Nanotech.* **14**, 408 (2019).
[11] A. Soumyanarayanan, N. Reyren, A. Fert, and C. Panagopoulos, *Nature* **539**, 509 (2016).
[12] C. Gong and X. Zhang, *Science* **363**, eaav4450 (2019).
[13] S. S. Parkin, M. Hayashi, and L. Thomas, *Science* **320**, 190 (2008).
[41] A. K. Behera, S. S. Mishra, S. Mallick, B. B. Singh, and S. Bedanta, J. Phys. D: Appl. Phys. 51, 285001 (2018).
[42] X. Zhang, M. Ezawa, and Y. Zhou, Sci. Rep. 5, 9400 (2015).
[43] Y. A. Kharkov, O. P. Sushkov, and M. Mostovoy, Phys. Rev. Lett. 119, 207201 (2017).
[44] B. Gobel, A. Mook, J. Henk, I. Mertig, and O. A. Tretyakov, Phys. Rev. B 99, 060407(R) (2019).
[45] X. Wang, H. Yuan, and X. Wang, Commun. Phys. 1, 31 (2018).