Single-Photon Image Classification

Abstract

Quantum computing-based machine learning mainly focuses on quantum computing hardware that is experimentally challenging to realize due to requiring quantum gates that operate at very low temperature. Instead, we demonstrate the existence of a lower performance and much lower effort island on the accuracy-vs-qubits graph that may well be experimentally accessible with room temperature optics. This high temperature “quantum computing toy model” is nevertheless interesting to study as it allows rather accessible explanations of key concepts in quantum computing, in particular interference, entanglement, and the measurement process.

We specifically study the problem of classifying an example from the MNIST and Fashion-MNIST datasets, subject to the constraint that we have to make a prediction after the detection of the very first photon that passed a coherently illuminated filter showing the example. Whereas a classical set-up in which a photon is detected after falling on one of the $28 \times 28$ image pixels is limited to a (maximum likelihood estimation) accuracy of 21.27% for MNIST, respectively 18.27% for Fashion-MNIST, we show that the theoretically achievable accuracy when exploiting inference by optically transforming the quantum state of the photon is at least 41.27% for MNIST, respectively 36.14% for Fashion-MNIST.

We show in detail how to train the corresponding transformation with TensorFlow and also explain how this example can serve as a teaching tool for the measurement process in quantum mechanics.

1 Introduction

Both quantum mechanics and machine learning play a major role in modern technology, and the emerging field of AI applications of quantum computing may well enable major breakthroughs across many scientific disciplines. Yet, as the majority of current machine learning practitioners do not have a thorough understanding of quantum mechanics, while the majority of quantum physicists only have an equally limited understanding of machine learning, it is interesting to look for “Rosetta Stone” problems where simple and widely understood machine learning ideas meet simple and widely understood quantum mechanics ideas. It is the intent of this article to present a setting in which textbook quantum mechanics sheds a new light on a textbook machine learning problem, and vice versa, conceptually somewhat along the lines of Google’s TensorFlow Playground (Smilkov et al. [2017]), which was introduced as a teaching device to illustrate key concepts from Deep Learning to a wider audience.

Specifically, we want to consider the question what the maximal achievable accuracy on common one-out-of-many image classification tasks is if one must make a decision after the detection of the very first quantum of light (i.e. photon) that passed a filter showing an example image from the test set. On the MNIST handwritten digit dataset (LeCun and Cortes [2010],) the best one can do classically in such a case is to detect a photon that lands on one of the $28 \times 28$ pixels and pick the most likely digit class using the per-pixel probability (i.e. light intensity) distribution observed on the training set. This requires to rescale the brightness of every example image to a unit sum, to obtain a
probability distribution. For MNIST, this yields a classical accuracy of 21.27%, substantially higher than random guessing (10%). The most likely digit class per pixel is shown in figure 2(b).

Quantum mechanics allows us to substantially beat this threshold, if one is permitted to apply a learned transformation to the quantum state of the photon between the image and the detector. This can be achieved with passive linear optical elements like beam splitters and phase shifters that produce a (hologram-like) interference pattern. Maximum likelihood estimation is then based on which region of the interference pattern the first photon lands on. This illustrates the quantum principle that the probability amplitude of a single quantum interferes with itself. It is not necessary to illuminate a scene with many photons at the same time in order to produce interference.

Conceptually, exploiting interference to enhance the probability of a quantum experiment producing the sought outcome is the essential idea underlying all quantum computing. The main difference between this problem and modern quantum computing is that the latter tries to perform calculations by manipulating quantum states of multiple “entangled” constituents, typically coupled two-state quantum systems called “qubits,” via “quantum gates” that are controlled by parts of the total quantum system’s quantum state. Building a many-qubit quantum computer hence requires delicate control over the interactions between constituent qubits. This usually requires eliminating thermal noise by going to millikelvin temperatures. For the problem studied here, the quantum state can be transformed with conventional optics at room temperature: the energy of a green photon is 2.5 eV, way above the typical room temperature thermal radiation energy of \( kT \approx 25 \text{ meV} \). The price to pay is that it is challenging to build a device that allows multiple photons to interact in the way needed to build a many-qubit quantum computer. Nevertheless, Knill, Laflamme, and Milburn (Knill et al. [2001]) devised a protocol to make this feasible in principle, avoiding the need for coherency-preserving nonlinear optics (which may well be impossible to realize experimentally) by clever exploitation of ancillary photon qubits, boson statistics, and the measurement process. In all such applications, the basic idea is to employ coherent multiphoton quantum states to do computations with multiple qubits.

In the problem studied here, there is only a single photon, the only relevant information that gets processed is encoded in the spatial part of its wave function (i.e. polarization is irrelevant), so the current work resembles the “optical simulation of quantum logic” proposed by Cerf et al. [1998] where a N-qubit system is represented by \( 2^N \) spatial modes of a single photon. Related work studied similar “optical simulations of quantum computing” for implementing various algorithms, in particular (small) integer factorization (Clauser and Dowling [1996], Sumhammer [1997]), but to the best of the present authors’ knowledge did not consider machine learning problems.

This work can be described as belonging to the category of machine learning methods on quantum non-scalable architectures. Alternatively, one can regard it as a quantum analogue of recent work that demonstrated digital circuit free MNIST digit classification via classical nonlinear optics, for instance via saturable absorbers (Khoram et al. [2019]).

Apart from providing an easily accessible and commonly understandable toy problem for quantum and ML experts, this simple-quantum/simple-ML corner also may be of interest for teaching the physics of the measurement process (often referred to as “the collapse of the wave function”) in a more accessible setting. Whereas explanations of the measurement process are forced to remain vague where they try to model “the quantum states of the observer” (typically unfathomably many that cannot be captured in any expression that one could ever hope to evaluate to actual numbers,) using machine learning as a sort-of cartoon substitute for high level mental processes can actually allow us to come up with fully concrete toy models of the measurement process on low-dimensional (such as: fewer than 1000 dimensions) Hilbert spaces that nevertheless capture many of the essential aspects. Specifically, if one considers a situation where an “atom” quantum system which in isolation is described with a low-dimensional Hilbert space interacts with a “measurement apparatus” or “observer” that is too complex to permit full quantum modeling, it is feasible to make the description fully tractable even at the numerical level if we allow replacing the role of a high level mental concept such as “observer sees a shoe” with a crude approximation such as “ML classifies the measurement as showing the image of a shoe”.

2 The measurement process

How well one can one solve a mental processing task, such as identifying a handwritten digit, if one is permitted to only measure a single quantum? This question leads to a Hilbert space basis
factorization that parallels the factorization needed to study the quantum mechanical measurement process. Let us consider a gedankenexperiment where our quantum system (see [Feynman et al. 2010], Landau and Lifshitz 1981) for an introduction to quantum mechanics) is a single atom that has two experiment-relevant quantum states, ‘spin-up’ and ‘spin-down’,

$$\psi_{\text{Atom}} = c_0 |\psi_{\text{Atom}=\uparrow}\rangle + c_1 |\psi_{\text{Atom}=\downarrow}\rangle. \quad (1)$$

This atom undergoes a measurement by interacting, over a limited time period, with an apparatus. The measurement process may involve for instance an atom emitting a photon that is detected by a camera, and it may include a human observing the result. We describe a quantum state in the potentially enormous Hilbert space of apparatus states with the vector $|\psi_{\text{Apparatus}}\rangle$. If, in this gedankenexperiment, we actually assume that we have maximal information about the (quantum) state of the measurement apparatus (which, however, in practical terms would be unfathomably complicated) at the beginning of the experiment, then the full quantum state of the initial system is the tensor product

$$|\psi_{\text{System, initial}}\rangle = |\psi_{\text{Atom, initial}}\rangle \otimes |\psi_{\text{Apparatus, initial}}\rangle. \quad (2)$$

This factorization implies that atom and apparatus states are independent before the interaction.

Without interaction between the apparatus and the atom, the time-evolution of the total system factorizes. A measurement requires an interaction between the apparatus and the atom, the solution of the Schrödinger equation is equivalent to the application of a unitary operator $U$ to the state $|\psi_{\text{System, initial}}\rangle$. This has the effect of combining the state components of the atom and the apparatus and, as a consequence, the joint time evolution no longer can be factorized. The overall state is $|\psi_{\text{System, final}}\rangle = U |\psi_{\text{System, initial}}\rangle$ and can be always decomposed in the sum:

$$|\psi_{\text{System, final}}\rangle = \alpha |\psi_{\text{Atom}=\uparrow}\rangle \otimes |\psi_{\text{Apparatus, final}=\uparrow}\rangle + \beta |\psi_{\text{Atom}=\downarrow}\rangle \otimes |\psi_{\text{Apparatus, final}=\downarrow}\rangle, \quad (3)$$

where the apparatus states $|\psi_{\text{Apparatus, final}=\uparrow}\rangle$ and $|\psi_{\text{Apparatus, final}=\downarrow}\rangle$ represent the state of the apparatus after the measurement for the two basis states of the atom. Therefore, the apparatus is in a different state for the two cases, which leads to the apparent “collapse” of the wave function. The apparatus in the state $|\psi_{\text{Apparatus, final}=\uparrow}\rangle$ perceives the “collapse” because the atom seems to have taken the state $|\psi_{\text{Atom}=\uparrow}\rangle$. The state of the apparatus includes also the representation of the thought process of a possible human observer, for instance asking herself at what instant the atom took a well determined state. This thought process disregards the superposed state $|\psi_{\text{Apparatus, final}=\downarrow}\rangle$ which represents the alternative reality, where the apparatus observed a different outcome.

Considering that a mental process could be seen as a measurement on the environment, one would naturally be inclined to think that high level mental concepts never would naturally lend themselves to a description in terms of some Hilbert space basis that has tensor product structure $|\psi_{\text{general concept}}\rangle \otimes |\psi_{\text{details}}\rangle$. Machine learning is now making the question to what extent this may nevertheless work quantitatively testable for some simple cases, if we consider it as providing reasonably good models for mental concepts.

Let us consider the spatial part of a single photon’s quantum state as it traveled through a mask that has the shape of a complicated object. For instance, let’s assume that the mask is obtained from a random sample of the Fashion-MNIST dataset (Xiao et al. 2017), where each sample represents an object such as a shirt, a trouser, etc. One would generally expect that any sort of transformation that connects a highly regular and mathematically simple description of such a quantum system, such as in terms of per-picture-cell (“pixel”) amplitudes, with a description in human-interpretable terms, such as “the overall intensity pattern resembles a shirt,” would unavoidably involve very complicated entanglement, and one should not even remotely hope to be able to even only approximately express such photon states in terms of some factorization

$$|\psi_{\text{ photon}}\rangle \approx \sum_{\text{shape class } C \in \{\text{shirt, trouser,...}\}} \sum_{\text{style } S} c_{CS} |\psi_{\text{shape class } C}\rangle \otimes |\psi_{\text{style } S}\rangle, \quad (4)$$

since one would not expect the existence of a basis of orthonormal quantum states that can be (approximately) labeled $|\psi_{\text{shirt}}\rangle$, $|\psi_{\text{shoe}}\rangle$, etc. Using machine learning, we can quantitatively demonstrate that, at least for some simple examples, precisely such a factorization does indeed work surprisingly well, at least if we content ourselves with the concept of a “shirt shape” being that of a one-out-of-many machine learning classifier, so not quite that of a human. In any case, it is reassuring to see that even near-future quantum computers might be able to express high level machine learning concepts rather well.
3 Single-Quantum Object Classification

Our gedankenexperiment starts with a single photon passing from faraway through a programmable LCD screen, which we here consider to consist of $N \times N$ pixels and show an image. For the MNIST handwritten digit dataset of LeCun and Cortes [2010] and the Fashion-MNIST dataset, we would have $N = 28$. Let us consider the situation that the size of the screen is sufficiently small for the photon’s quantum state to be at the same phase as it reaches each individual pixel. This does not mean that the screen has to be small in comparison to the wavelength. Rather, the light source and optical system must provide highly collimated illumination.

Ignoring some irrelevant quantum mechanics fine print, the relevant spatial part of the photon’s quantum state is described by an element of a $N \times N$-dimensional complex vector space, and we can choose a basis for this Hilbert space such that the quantum state of a photon that managed to pass through the screen (rather than getting absorbed) has the form

$$|\psi_{\text{photon}}\rangle = \sum_{\text{row } j, \text{ column } k} c_{jk} |\psi_{jk}\rangle$$

(5)

where the $|\psi_{jk}\rangle$ basis functions correspond to a photon that went through pixel $(j, k)$, and the coefficients $c_{jk}$ are real, non-negative, proportional to the square roots of the image’s pixel-brightnesses, and are normalized according to $\sum_{j,k} c_{jk}^2 = 1$. Let us embed this $N^2$-dimensional Hilbert space into a larger Hilbert space with dimensionality $M$ divisible by the number of object classes $C$, i.e. $M = C \cdot S$. This amounts to padding the image with always-dark pixels (that may not form complete rows).

As demonstrated by Reck et al. [1994], quantum mechanics allows us to implement any unitary transform $|\psi_{\text{photon}}\rangle \rightarrow U_\text{pad}|\psi_{\text{photon}}\rangle$ on this enlarged Hilbert space with beam splitters, phase shifters, and mirrors. For a problem $P$ such as handwritten digit recognition, can we engineer a single unitary operator $U_P$ in such a way that we can meaningfully claim:

$$U_P|\psi_{\text{photon}}\rangle = |\psi_{\text{photon}'}\rangle \approx \sum_{\text{example class } c} \sum_{\text{style } s} c_{cs}|\psi_{\text{class is } c}\rangle \otimes |\psi_{\text{style variant is } s}\rangle?$$

(6)

Specifically, for each individual example image $E$, we would like to have

$$U_P|\psi_{\text{Photon, } E}\rangle \approx |\psi_{C(E)}\rangle \otimes \sum_{\text{style } s} c_s|\psi_{\text{style variant is } s(E)}\rangle,$$

(7)

where $C(E)$ is the ground truth label of the example in a supervised learning setting.

An experimental realization would hence consist of beam splitters, mirrors, etc. after the screen that transform the photon quantum state $|\psi_{\text{photon}}\rangle$ into $U_\text{pad}|\psi_{\text{photon}}\rangle$ before the photon hits a detector array that can discriminate $M = C \cdot S$ quantum states which are labeled $|\psi_{\text{digit is a 0}}\rangle \otimes |\psi_{\text{style variant 1}}\rangle$, $|\psi_{\text{digit is a 0}}\rangle \otimes |\psi_{\text{style variant 2}}\rangle$, $|\psi_{\text{digit is a 3}}\rangle \otimes |\psi_{\text{style variant 57}}\rangle$, $|\psi_{\text{digit is a 9}}\rangle \otimes |\psi_{\text{style variant 50}}\rangle$. If we detect the photon in any of the $|\psi_{\text{digit is a 7}}\rangle \otimes \ldots \text{cells}$, the classifier output is a "7", and likewise for the other digits. One may think of such a device $D_P$ as an optical system that produces a hologram-like transformation of the input amplitudes (but sans the reference beam), and the detectors then interpreting light that falls onto different regions of that hologram differently.

From a machine learning perspective, the trainable parameters are the complex entries of the matrix $U_P$, which has to satisfy an unitarity constraint, $U_P U_P^\dagger = I$. For MNIST, where examples have $28 \times 28$ pixels, the most obvious choice is padding to a $M = 790$-dimensional input vector. While one could implement the unitarity constraint in terms of a (regularizer) loss-function contribution that measures the degree of violation of unitarity, it here makes more sense to instead use a parametrization of $U_P$ that automatically guarantees unitarity. In this work, we use $U_P = \exp(i H_P)$ with $H_P$ a hermitian matrix that is obtained from the trainable weights of a $790 \times 790$ matrix $W_P$ as $i H_P = (W_P - W_P^\dagger) + i(W_P + W_P^\dagger)$. The weights $W_P$ are adjusted by the machine learning training process. Every unitary matrix can be obtained in such a way. This approach slightly over-parametrizes the problem, since, in the tensor-product basis that we are transforming to, we can freely re-define the basis on each of the ten $790/10 = 79$-dimensional style subspaces. This means that 10% of the parameters are redundant.
Figure 1: (a) The two shapes of the toy example. The four gray pixels correspond to a photon arrival probability of $1/4$, i.e. a probability amplitude of $1/2$. (b) The per-pixel photon arrival probability after the orthogonal transformation is applied. The dark gray pixels correspond to a probability of $1/8$ and the light gray pixels to $1/2$.

Model accuracy has to be evaluated with caution: as we need to make a prediction after detection of the first photon, accuracy is the quantum probability of the correct label, averaged over all examples. This is observed to differ substantially from the probability, averaged over all examples, for the correct label to have highest quantum probability for the individual example. In other words, probabilities are uncalibrated, and the (non-linear “deep learning”) transformation that would be required to calibrate them cannot be expressed in terms of a unitary operator.

Let us consider a radically simplified example that illustrates why this method works. We want to discriminate between only two different shapes (with no further shape variation) on a $2 \times 4$ pixel screen where each pixel is either “on” or “off”, using only one photon. Specifically, let us consider the two Tetris “T” shapes represented in figure 1(a).

For both shapes, the probability that the single photon arrives on one of the “on” pixels is $1/4$; therefore, taking into account that for two pixels the correct shape is identified exactly and for two with 50% probability, we conclude that the baseline accuracy is $1/2 + 1/4 = 75\%$. Instead, we can apply a unitary transformation to reshape the probability amplitude distribution.

Let us consider the simple but not optimal transformation of the photon amplitude that replaces the pair of amplitudes $(a, b)$ in each 2-pixel column with $((a-b)/\sqrt{2}, (a+b)/\sqrt{2})$, i.e. creates destructive interference in the top row and constructive interference in the bottom row. This gives the detection probability patterns shown in figure 1 (b). Maximum likelihood estimation here gives an accuracy of $1/2 + 3/8 = 87.5\%$.

Obtaining the maximum achievable accuracy will require a more complicated all-pixel amplitude transformation, obtained as follows. the quantum amplitude transformation is angle-preserving, and the angle $\alpha$ between the two amplitude quantum states $q_1$, $q_2$ is given by $\cos \alpha = \langle q_1 | q_2 \rangle = 0.5$. Hence, we can rotate these two states to lie in the plane of the first two Cartesian coordinate axes of the Hilbert space, and at the same angle from their bisector. Identifying these coordinate axes with the correct labels, the accuracy is the cosine-squared of the angle between the transformed state and the corresponding axis, i.e. $\cos^2(\pi/4 - \alpha/2) = (\sqrt{3} + 2)/2 \approx 93.30\%$.

While the performance measure that we care about here is the probability for a correct classification, one observes that model training is nevertheless more effective when one instead minimizes cross-entropy, as one would when training a conventional machine learning model. Intuitively, this seems to make sense, as a gradient computed on cross-entropy loss is expected to transport more information about the particular way in which a classification is off than a gradient that is based only on maximizing the correct classification probability.

Overall, this task is somewhat unusual as a machine learning problem for three reasons: First, it involves complex intermediate quantities, and gradient backpropagation has to correctly handle the transitioning from real to complex derivatives where the loss function is the magnitude-squared of a complex quantity. TensorFlow is at the time of this writing the only widely used machine learning framework that can handle this aspect nicely. Second, we cannot simply pick the class for which the predicted probability is highest as the predicted class. Rather, the probability for the single-photon measurement to produce the ground truth label sets the accuracy. Third, while most machine learning architectures roughly follow a logistic regression architecture and accumulate per-class evidence which gets mapped to a vector of per-class probabilities, we here have the probabilities as
Table 1: Results for the Fashion-MNIST and MNIST datasets. The “classic” accuracy and information refer to the observation of a single photon, while the “quantum” quantities are obtained after applying the quantum transformation.

| Dataset   | Entropy [bits] | Accuracy (classic) | Information (classic) [bits] | Accuracy (quantum) | Information (quantum) [bits] |
|-----------|----------------|-------------------|------------------------------|-------------------|------------------------------|
| Fashion-MNIST | 3.32          | 18.27%            | 0.8694                       | 36.14%            | 1.853                        |
| MNIST     | 3.32          | 21.27%            | 1.09                         | 40.56%            | 2.02                         |

(a) Fashion-MNIST most likely class given detection of a single photon at the corresponding pixel coordinates. Here, the classes are: 0=T-shirt/top, 1=Trouser, 2=Pullover, 3=Shirt, 4=Coat, 5=Sandal, 6=Shirt, 7=Sneaker, 8=Bag, 9=Ankle Boot. (b) Most likely digit-class given detection of a single photon for MNIST.

Figure 2: (a) Fashion-MNIST most likely class given detection of a single photon at the corresponding pixel coordinates. The “classic” accuracy and information refer to the observation of a single photon, while the “quantum” quantities are obtained after applying the quantum transformation.

4 Results

If we use the detection of a single photon on a $N \times N$ pixel array as the input to a conventional classifier, the best achievable performance is obtained by working out for each pixel what the most probable class is for this pixel, weighting the examples in the training set by this pixel’s brightness. This is shown as a function of the pixel coordinates for the Fashion-MNIST and the MNIST in figure[2]. The choice of the most likely class gives the classic accuracy reported in the third column of table[1]. Note that, as pointed out by Sun et al. [2007], the Fashion-MNIST dataset contains many mislabeled instances which affect both classic and quantum results. We can compute the amount of information provided by the photon by computing the difference between the class entropy, i.e. $-\log_2(0.1) = 3.32$, since there are 10 classes, and the entropy associated to the classification errors, i.e. the accuracy. The mutual information for the classical classifier is given in the fourth column of table[1].
In contrast, training a unitary $U(790)$ quantum transformation that gets applied after the photon passed the image filter and before it hits a bank of 790 detectors allows boosting accuracy for both the Fashion-MNIST and MNIST datasets, as reported in the fifth column of table 1. Equivalently, one can say that the observation of the photon after the transformation provides a higher amount of mutual information with respect to the classical configuration. The values of mutual information in this case are given in the last column of table 1. Explicit matrices to perform the transformation for the two data sets have been made available with the supplementary material.

The quantum transformation $U_P$ allows us to define the pixel-space projection operators:

$$P_{\text{class}C} := U_P^{-1} (|\psi_C\rangle\langle\psi_C| \otimes I_{\text{style}}) U_P$$

with which we can decompose any example into contributions that are attributable to the different classes. Here, one must keep in mind that such separation is done at the level of probability amplitudes, so while we can compute intensities/probabilities from these components, which are mutually orthogonal as quantum states, summing these per-component per-pixel intensities will not reproduce the example’s per-pixel intensities. This shows most clearly when considering the decomposition of an example “Trouser” from the Fashion-MNIST dataset’s test set with the model we trained for this task, as shown in figure 4(a). The dark vertical line between the legs in the original image mostly comes from destructive interference between a bright line from the “Trousers” component and a matching bright line from the “Dress” component.

Due to the intrinsic quantum nature of this set-up, care must be taken when interpreting confusion matrices. Naturally, we never can claim of any single-photon classifier that it would ‘classify a particular example correctly’, since re-running the experiment on the same example will not see the photon always being counted by the same detector! So, strictly speaking, for any single-photon classifier realized as a device, the “confusion matrix” could be determined experimentally only in the
Figure 4: Projection of probability amplitudes for some samples of the Fashion-MNIST (a) and the MNIST (b) datasets. The first image shows the original sample probability and the following images show the probability amplitudes for each class. We visualize the complex amplitude by using brightness to represent magnitude and hue for phase (the colormap for the phase is shown on the right of each row.)

statistical sense, leaving uncertainty in the entries that decreases with the number of passes over the test set. Confusion matrices are shown in figure 3.

Our factorization ansatz appears to contain a hidden constraint: we are forcing each image class to use the same number of style-states. One could imagine, for instance, that a classifier might achieve even higher accuracy by treating image classes unevenly. This case can always be embedded into padding to some larger Hilbert space. Numerical experiments suggest that this is not the case. For instance, going to 1000 dimensions does not appreciably increase the achievable accuracy substantially for the classification problems studied here.

5 Discussion

In summary, we demonstrated that, at least for the considered datasets, the space of the single observed photon state can be factorized surprisingly well by a product of the example class space and a space collecting the remaining variables, such as the style. This factorization can be obtained easily by the proposed method and is experimentally realizable with optical elements placed in front of the sensor.

A device implementing the proposed system would be a demonstration of a high-temperature and low-effective-qubit quantum computer. With respect to other experimental approaches to quantum computing, this device has the limitation that it is built for the specific classification problem and cannot be reconfigured easily. It would be interesting to see whether an advanced quantum protocol along the lines of Knill et al. [2001] might enable the realization of more advanced intermediate-scale high temperature quantum machine learning in a way that mostly bypasses the need for quantum logic.

6 TensorFlow Code

TensorFlow2 code to reproduce the experiments of this work and all the figures is provided in the ancillary files together with the computed unitary transforms for MNIST and Fashion-MNIST.
References

Daniel Smilkov, Shan Carter, D. Sculley, Fernanda B. Viégas, and Martin Wattenberg. Direct-manipulation visualization of deep networks. CoRR, abs/1708.03788, 2017. URL http://arxiv.org/abs/1708.03788

Yann LeCun and Corinna Cortes. MNIST handwritten digit database. 2010. URL http://yann.lecun.com/exdb/mnist/

E. Knill, Laflamme, R., and G. Milburn. A scheme for efficient quantum computation with linear optics. Nature, 409:46–52, 2001. URL https://doi.org/10.1038/35051009

N. J. Cerf, C. Adami, and P. G. Kwiat. Optical simulation of quantum logic. Phys. Rev. A, 57:R1477–R1480, Mar 1998. doi: 10.1103/PhysRevA.57.R1477. URL https://link.aps.org/doi/10.1103/PhysRevA.57.R1477

John F. Clauser and Jonathan P. Dowling. Factoring integers with young’s n-slit interferometer. Phys. Rev. A, 53:4587–4590, Jun 1996. doi: 10.1103/PhysRevA.53.4587. URL https://link.aps.org/doi/10.1103/PhysRevA.53.4587

Johann Summhammer. Factoring and fourier transformation with a mach-zehnder interferometer. Phys. Rev. A, 56:4324–4326, Nov 1997. doi: 10.1103/PhysRevA.56.4324. URL https://link.aps.org/doi/10.1103/PhysRevA.56.4324

Erfan Khoram, Ang Chen, Dianjing Liu, Lei Ying, Qiqi Wang, Ming Yuan, and Zongfu Yu. Nanophotonic media for artificial neural inference. Photon. Res., 7(8):823–827, Aug 2019. doi: 10.1364/PRJ.7.000823. URL http://www.osapublishing.org/prj/abstract.cfm?URI=prj-7-8-823

Richard Phillips Feynman, Robert Benjamin Leighton, and Matthew Sands. The Feynman lectures on physics: New millennium ed. Basic Books, New York, NY, 2010. URL https://cds.cern.ch/record/1494701

L. D. Landau and L. M. Lifshitz. Quantum Mechanics Non-Relativistic Theory, Third Edition: Volume 3. Butterworth-Heinemann, 3 edition, January 1981. ISBN 0750635398. URL http://www.worldcat.org/isbn/0750635398

Han Xiao, Kashif Rasul, and Roland Vollgraf. Fashion-mnist: a novel image dataset for benchmarking machine learning algorithms. arXiv, 2017.

Michael Reck, Anton Zeilinger, Herbert J. Bernstein, and Philip Bertani. Experimental realization of any discrete unitary operator. Phys. Rev. Lett., 73:58–61, Jul 1994. doi: 10.1103/PhysRevLett.73.58. URL https://link.aps.org/doi/10.1103/PhysRevLett.73.58

J. Sun, F. Zhao, C. Wang, and S. Chen. Identifying and correcting mislabeled training instances. In Future Generation Communication and Networking (FGCN 2007), volume 1, pages 244–250, 2007.