Resolving $B$-CP Puzzles in QCD Factorization

Hai-Yang Cheng,$^{1,2}$ and Chun-Khiang Chua$^3$

1 Institute of Physics, Academia Sinica
Taipei, Taiwan 115, Republic of China

2 Physics Department, Brookhaven National Laboratory
Upton, New York 11973

3 Department of Physics, Chung Yuan Christian University
Chung-Li, Taiwan 320, Republic of China

Abstract

Within the framework of QCD factorization (QCDF), power corrections due to penguin annihilation can account for the observed rates of penguin-dominated two-body decays of $B$ mesons and direct $CP$ asymmetries $A_{CP}(K^-\pi^+)$, $A_{CP}(K^*^-\pi^+)$, $A_{CP}(K^-\rho^0)$ and $A_{CP}(\pi^+\pi^-)$. However, the predicted direct $CP$-violating effects in QCDF for $B^-\to K^-\pi^0, K^-\eta, \pi^-\eta$ and $\bar{B}^0\to \pi^0\pi^0$ are wrong in signs when confronted with experiment. We show that subleading $1/m_b$ power corrections to the color-suppressed tree amplitude due to spectator scattering or final-state interactions will yield correct signs for aforementioned $CP$ asymmetries and accommodate the observed $\pi^0\pi^0$ and $\rho^0\pi^0$ rates simultaneously. Implications are discussed.
1. In the heavy quark limit, hadronic matrix elements can be expressed in terms of certain nonperturbative input quantities such as light cone distribution amplitudes and transition form factors. Consequently, the decay amplitudes of charmless two-body decays of $B$ mesons can be described in terms of decay constants and form factors. However, the predicted rates for penguin-dominated $B \to PP, VP, VV$ decays ($P$ and $V$ denoting pseudoscalar and vector mesons, respectively) are systematically below the measurements (see the second column of Table I for a review, see [1]). Moreover, the calculated direct $CP$ asymmetries for $B^0 \to K^-\pi^+, K^+\pi^-$, $B^- \to K^-\rho^0$ and $\bar{B}^0 \to \pi^+\pi^-$ are wrong in signs when confronted with experiment as shown in the same Table. This implies the necessity of taking into account $1/m_b$ power correction effects. In the QCD factorization (QCDF) approach [2], power corrections often involve endpoint divergences. For example, the hard spectator scattering diagram at twist-3 order is power suppressed and possesses soft and collinear divergences arising from the soft spectator quark and the $1/m_b$ annihilation amplitude has endpoint divergences even at twist-2 level. Since the treatment of endpoint divergences is model dependent, subleading power corrections generally can be studied only in a phenomenological way. While the endpoint divergence is regulated in the pQCD approach by introducing the parton’s transverse momentum [3], it is parameterized in QCD factorization as

$$X_A \equiv \int_0^1 dy \frac{dy}{y} = \ln \frac{m_B}{\Lambda_b} (1 + \rho_A e^{i\phi_A}),$$

(1)

for penguin annihilation contributions with $\Lambda_b$ being a typical scale of order 500 MeV.

In the so-called “S4” scenario of QCDF [4] with some appropriate choice of the parameters $\rho_A$ and $\phi_A$, the above-mentioned discrepancies are resolved in the presence of power corrections due to the penguin annihilation topology. However, a scrutiny of the QCDF predictions reveals more puzzles in the regard of direct $CP$ violation. When power corrections due to penguin annihilation are turned on, the signs of $A_{CP}$ in $B^- \to K^-\pi^0, K^-\eta, \pi^-\eta$ and $\bar{B}^0 \to \pi^0\rho^0$ will also get flipped in such a way that they disagree with experiment (see the third column of Table I). The so-called $K\pi CP$-puzzle is related to the difference of $CP$ asymmetries of $B^- \to K^-\pi^0$ and $\bar{B}^0 \to K^-\pi^+$. This can be illustrated by considering the decay amplitudes of $B \to K\pi$ in terms of topological diagrams

\begin{align*}
A(\bar{B}^0 \to K^-\pi^+) &= P' + T' + \frac{2}{3}P'^C_{EW} + P'_A, \\
A(\bar{B}^0 \to K^0\pi^0) &= -\frac{1}{\sqrt{2}}(P' - C' - P'_{EW} - \frac{1}{3}P'^C_{EW} + P'_A), \\
A(B^- \to K^-\pi^-) &= P' - \frac{1}{3}P'^C_{EW} + A' + P'_A, \\
A(B^- \to K^-\pi^0) &= \frac{1}{\sqrt{2}}(P' + T' + C' + P'_{EW} + \frac{2}{3}P'^C_{EW} + A' + P'_A),
\end{align*}

(2)

where $T, C, A, P_{EW}$ and $P'^C_{EW}$ are color-allowed tree, color-suppressed tree, $W$-exchange, $W$-annihilation, color-allowed and color-suppressed electroweak penguin amplitudes, respectively, and $P_A$ is the penguin-induced weak annihilation amplitude. We use unprimed and primed symbols to denote $\Delta S = 0$ and $|\Delta S| = 1$ transitions. We notice that if $C'$, $P'_{EW}$ and $A'$ are negligible compared with $T'$, it is clear from Eq. (2) that the

1 We have included chirally enhanced but power suppressed penguin contributions. Numerically, they are of order $1/m_b^0$. 

\[1/m_b^0,\]
decay amplitudes of $K^-\pi^0$ and $K^-\pi^+$ will be the same apart from a trivial factor of $1/\sqrt{2}$. Hence, one will expect that $A_{CP}(K^-\pi^0) \approx A_{CP}(K^-\pi^+)$, while they differ by 5.3σ experimentally, $\Delta A_{K\pi} \equiv A_{CP}(K^-\pi^0) - A_{CP}(K^-\pi^+) = 0.148 \pm 0.028 \pm \frac{3}{2}$. We also notice that the decay $B^- \to K^-\eta$ has a world average $-0.37 \pm 0.09$ for $A_{CP}(K^-\eta)$ different from zero by 4.1 standard deviations.

Since in the heavy quark limit, CP asymmetries of the $K^-\pi^0$, $K^-\eta$, $\pi^-\eta$, $\pi^0\pi^0$ modes have the correct signs when compared with experiment, the $B$-$CP$ puzzles mentioned here are relevant to QCDF and may not occur in other approaches such as pQCD. In this work, we shall show that soft power corrections to the color-suppressed tree amplitude will bring the signs of $A_{CP}$ back to the right track. As a bonus, the rates of $\bar{B}^0 \to \pi^0\pi^0$, $\rho^0\pi^0$ can be accommodated.

2. The aforementioned direct $CP$ puzzles indicate that it is necessary to consider subleading power corrections other than penguin annihilation. For example, the large power corrections due to $P_A'$ cannot explain the $\Delta A_{K\pi}$ puzzle as they contribute equally to both $B^- \to K^-\pi^0$ and $\bar{B}^0 \to K^-\pi^+$. The additional power correction should have little impact on the decay rates of penguin-dominated decays but will manifest in the measurement of direct $CP$ asymmetries. Note that all the "problematic" modes receive a contribution from $c' = C' + P_{EW}'$. Since $A(B^- \to K^-\pi^0) \propto t' + c' + p'$ and $A(\bar{B}^0 \to K^-\pi^+) \propto t' + p'$ with $t' = T' + P_{EW}'$ and $p' = P' - \frac{1}{3}P_{EW}' + P_A'$, we can consider this puzzle resolved, provided that $|c'/t'|$ is of order 1.3 $\sim$ 1.4 with a large negative phase (naïvely $|c'/t'| \sim 0.9$). There are several possibilities for a large $c'$: either a large color suppressed $C'$ or a large electroweak penguin $P_{EW}'$ or a combination of them. Various scenarios for accommodating large $C'$ [8, 9, 10, 11, 12, 13, 14, 15] or $P_{EW}'$ [16, 17] have been proposed. To get a large $C'$, one can appeal to spectator scattering or final-state rescattering (see discussions below). However, the general consensus for a large $P_{EW}'$ is that one needs New Physics beyond the Standard Model. In principle, one cannot tell the difference of these two possibilities in penguin-dominated decays as it is always the combination $c' = C' + P_{EW}'$ that enters into the decay amplitude except for the decays involving $\eta$ and/or $\eta'$ in the final state where both $c'$ and $P_{EW}'$ present in the amplitudes [18]. Nevertheless, the two scenarios can lead to very distinct predictions for tree-dominated decays where $P_{EW} \ll C$ as the electroweak amplitude here does not get a CKM enhancement. The decay rates of $\bar{B}^0 \to \pi^0\pi^0, \rho^0\pi^0$ will be substantially enhanced for a large $C$ but remain intact for a large $P_{EW}'$. Since $P_{EW} \ll C$ in tree-dominated channels, $CP$ puzzles with $\pi^-\eta$ and $\pi^0\pi^0$ cannot be resolved with a large $P_{EW}'$. Therefore, it is most likely that the color-suppressed tree amplitude is large and complex. Motivated by the above observation, in this work we shall consider the possibility of a large complex $a_2$, the parameter for describing the color-suppressed tree topology, and parameterize power corrections to $a_2$, as

$$a_2 \to a_2^{\text{NLO}} (1 + \rho_C e^{i\phi_C}),$$

(3)

with the unknown parameters $\rho_C$ and $\phi_C$ to be inferred from experiment. The reader is referred to [22] for details. We shall first consider soft corrections to weak annihilation dictated by the parameters $\rho_A$ and $\phi_A$. A fit to the data of two-body hadronic decays of $B^0$ and $B^-$ mesons

\[2\] We use NLO results for $a_2$ in Eq. (3) as a benchmark to define power corrections. The NNLO calculations of spectator-scattering tree amplitudes and vertex corrections at order $a_2^2$ have been carried out in [19] and [20], respectively. While NNLO corrections can in principle push the magnitude of $a_2(\pi\pi)$ up to the order of 0.50 by lowering the value of the $B$-meson parameter $\lambda_B$, the strong phase of $a_2$ relative to $a_1$ cannot be larger than 15° [21].
within QCDF yields the values

\[
\rho_A^0 \approx 1.10, \quad 1.07, \quad 0.87, \\
\phi_A^0 \approx -50^\circ, \quad -70^\circ, \quad -30^\circ,
\]

(4)

for \( B \rightarrow PP, PV, VP \) respectively, where the superscript “0” of \( \rho_A \) and \( \phi_A \) indicates that they are the default values we shall use in this work. Basically, this is very similar to the “scenario S4” presented in [4]. For the annihilation diagram we use the convention that \( M_1 (M_2) \) contains an antiquark (a quark) from the weak vertex. Since the penguin annihilation effects are different for \( M_1 = P \) and \( M_1 = V \), the parameters \( \rho_A \) and \( \phi_A \) are thus different for \( B \rightarrow PV \) and \( B \rightarrow VP \).

Branching fractions and direct CP asymmetries for some selective \( B \rightarrow PP \) decays are shown in Table I. The theoretical errors correspond to the uncertainties due to variation of (i) the Gegenbauer moments, the decay constants, (ii) the heavy-to-light form factors and the strange quark mass, and (iii) the wave function spectator interactions described by the parameters \( \rho_{A,H}, \phi_{A,H} \), respectively. To obtain the errors shown in Table I, we first scan randomly the points in the allowed ranges of the above nine parameters (specifically, the ranges \( \rho_A^0 - 0.1 \leq \rho_A \leq \rho_A^0 + 0.1, \phi_A^0 - 20^\circ \leq \phi_A \leq \phi_A^0 + 20^\circ, 0 \leq \rho_H \leq 1 \) and \( 0 \leq \phi_H \leq 2\pi \) are used in this work) and then add errors in quadrature. More specifically, the second error in the table is referred to the uncertainties caused by the variation of \( \rho_{A,H} \) and \( \phi_{A,H} \), where all other uncertainties are lumped into the first error. Power corrections beyond the heavy quark limit generally give the major theoretical uncertainties.

For \( \rho_C \approx 1.3 \) and \( \phi_C \approx -70^\circ \), we find that all the CP puzzles in \( B \rightarrow PP \) decays are resolved as shown in fourth column of Table I. The corresponding \( a_2 \)'s are

\[
a_2(\pi\pi) \approx 0.60 e^{-i55^\circ}, \quad a_2(K\pi) \approx 0.51 e^{-i58^\circ}.
\]

(5)

They are consistent with the phenomenological determination of \( C(\pi\pi)/T(\pi\pi) \sim a_2/a_1 \) from a global fit to the available data [13]. Due to the interference between the penguin and the enhanced color-suppressed amplitudes with a sizable strong phase, it is clear from Table I that theoretical predictions for \( A_{CP} \) now agree with experiment in sign even for those modes with the measured \( A_{CP} \) less than \( 3\sigma \) in significance. As first emphasized by Lunghi and Soni [23], in the QCDF analysis of the quantity \( \Delta A_{K\pi} \), although the theoretical uncertainties due to power corrections from penguin annihilation are large for individual asymmetries \( A_{CP}(K^-\pi^0) \) and \( A_{CP}(K^-\pi^+) \), they essentially cancel out in their difference, rendering the theoretical prediction more reliable. We find \( \Delta A_{K\pi} = (12.3^{+2.2+2.1}_{-0.9-4.7})\% \), while it is only \( (1.9^{+0.5+1.6}_{-0.4-1.0})\% \) in the absence of power corrections to the topological amplitude “C” or \( a_2 \).

For the direct CP asymmetry of \( B^0 \rightarrow \bar{K}^0\pi^0 \), we predict \( A_{CP}(\bar{K}^0\pi^0) = (\overline{-10.6^{+2.7+5.5}_{-3.7-4.3}})\% \). Experimentally, the current world average \(-0.01 \pm 0.10 \) is consistent with no CP violation because the BaBar and Belle measurements, \(-0.13 \pm 0.13 \pm 0.03 \) [24] and \(0.14 \pm 0.13 \pm 0.06 \) [25] respectively, are opposite in sign. Nevertheless, there exist several model-independent determinations of this asymmetry: one is the SU(3) relation \( \Delta \Gamma(\pi^0\pi^0) = -\Delta \Gamma(\bar{K}^0\pi^0) \) [26], and the other is the approximate sum rule for CP rate asymmetries [27]

\[
\Delta \Gamma(K^-\pi^+) + \Delta \Gamma(\bar{K}^0\pi^-) \approx 2[\Delta \Gamma(K^-\pi^0) + \Delta \Gamma(\bar{K}^0\pi^0)],
\]

(6)

based on isospin symmetry, where \( \Delta \Gamma(K\pi) \equiv \Gamma(\bar{B} \rightarrow \bar{K}\pi) - \Gamma(B \rightarrow K\pi) \). This sum rule allows us to extract \( A_{CP}(\bar{K}^0\pi^0) \) in terms of the other three asymmetries in \( K^-\pi^+, K^-\pi^0, \bar{K}^0\pi^- \) modes that have been measured. From the current data of branching fractions and CP asymmetries, the above SU(3) relation and
TABLE I: CP-averaged branching fractions (in units of $10^{-6}$) and direct CP asymmetries (in %) of some selective $B \to PP$ decays obtained in QCD factorization for three distinct cases: (i) without any power corrections, (ii) with power corrections from penguin annihilation, and (iii) with power corrections to both penguin annihilation and color-suppressed tree amplitudes. The parameters $\rho_A$ and $\phi_A$ are taken from Eq. (4), $\rho_C = 1.3$ and $\phi_C = -70^\circ$. Sources of theoretical uncertainties are discussed in the text.

| Modes | W/o $\rho_{A,C}, \phi_{A,C}$ | With $\rho_A, \phi_A$ | With $\rho_{A,C}, \phi_{A,C}$ | Expt. [6] |
|-------|-----------------------------|----------------------|-----------------------------|----------|
| $\mathcal{B}(\bar{B}^0 \to K^- \pi^+)$ | 13.1$^{+5.8+0.7}_{-3.5-0.7}$ | 19.3$^{+7.9+8.2}_{-4.8-6.2}$ | 19.3$^{+7.9+8.2}_{-4.8-6.2}$ | 19.4$^{+0.6}_{-0.6}$ |
| $\mathcal{B}(B^0 \to K^0 \pi^0)$ | 5.5$^{+2.8+0.3}_{-1.7-0.3}$ | 8.4$^{+3.8+3.8}_{-2.3-2.9}$ | 8.6$^{+3.8+3.8}_{-2.2-2.9}$ | 9.8$^{+0.6}_{-0.6}$ |
| $\mathcal{B}(B^- \to K^0 \pi^0)$ | 14.9$^{+6.9+0.9}_{-6.0-1.0}$ | 21.7$^{+9.2+9.0}_{-6.0-6.9}$ | 21.7$^{+9.2+9.0}_{-6.0-6.9}$ | 23.1$^{+1.0}_{-1.0}$ |
| $\mathcal{B}(B^- \to K^- \pi^0)$ | 9.1$^{+3.6+0.5}_{-2.3-0.5}$ | 12.6$^{+4.7+4.8}_{-3.0-3.8}$ | 12.5$^{+4.7+4.9}_{-3.0-3.8}$ | 12.9$^{+0.6}_{-0.6}$ |
| $\mathcal{B}(B^- \to K^- \eta)$ | 1.6$^{+1.1+0.3}_{-0.7-0.4}$ | 2.4$^{+1.8+1.3}_{-1.1-1.0}$ | 2.4$^{+1.8+1.3}_{-1.1-1.0}$ | 2.3$^{+0.3}_{-0.3}$ $^a$ |
| $\mathcal{B}(B^0 \to \pi^+ \pi^-)$ | 6.9$^{+0.6-0.4}_{-0.6-0.6}$ | 7.0$^{+0.4+0.7}_{-0.7-0.7}$ | 7.0$^{+0.4+0.7}_{-0.7-0.7}$ | 5.1$^{+0.2}_{-0.2}$ |
| $\mathcal{B}(B^0 \to \pi^0 \pi^0)$ | 0.42$^{+0.29+0.18}_{-0.11-0.08}$ | 0.52$^{+0.26+0.21}_{-0.10-0.10}$ | 1.1$^{+1.0+0.7}_{-0.4-0.3}$ | 1.55$^{+0.19}_{-0.19}$ $^b$ |
| $\mathcal{B}(B^- \to \pi^- \pi^0)$ | 4.9$^{+0.9+0.6}_{-0.5-0.3}$ | 4.9$^{+0.9+0.6}_{-0.5-0.3}$ | 5.9$^{+2.2+1.4}_{-1.1-1.1}$ | 5.5$^{+0.4}_{-0.4}$ |
| $\mathcal{B}(B^- \to \pi^- \eta)$ | 4.4$^{+0.6+0.4}_{-0.3-0.3}$ | 4.5$^{+0.6+0.5}_{-0.3-0.3}$ | 5.0$^{+1.2+0.9}_{-0.6-0.7}$ | 4.1$^{+0.3}_{-0.3}$ $^a$ |
| $A_{CP}(\bar{B}^0 \to K^- \pi^+)$ | 4.0$^{+0.6+1.1}_{-0.7-1.1}$ | -7.4$^{+1.7+4.3}_{-1.5-4.8}$ | -7.4$^{+1.7+4.3}_{-1.5-4.8}$ | -9.8$^{+1.2}_{-1.1}$ |
| $A_{CP}(B^0 \to K^0 \pi^0)$ | -4.0$^{+1.2+3.5}_{-1.8-3.0}$ | 0.75$^{+1.88+2.56}_{-0.94-3.32}$ | -10.6$^{+2.7+5.6}_{-3.8-4.8}$ | -1$^{+0}_{-1}$ |
| $A_{CP}(B^- \to K^0 \pi^-)$ | 0.72$^{+0.06+0.05}_{-0.05-0.05}$ | 0.28$^{+0.03+0.09}_{-0.03-0.10}$ | 0.28$^{+0.03+0.09}_{-0.03-0.10}$ | 0.9$^{+0.5}_{-0.5}$ |
| $A_{CP}(B^- \to K^- \pi^0)$ | 7.3$^{+1.6+2.3}_{-1.2-2.7}$ | -5.5$^{+1.3+4.9}_{-1.8-4.6}$ | 4.9$^{+2.1+5.4}_{-3.9+4.4}$ | 5.0$^{+2.5}_{-2.5}$ |
| $A_{CP}(B^- \to K^- \eta)$ | -22.1$^{+7.7+14.0}_{-16.7-7.3}$ | 12.7$^{+7.7+13.4}_{-5.0-15.0}$ | -11.0$^{+8.4+14.9}_{-21.6-10.1}$ | -37$^{+9}_{-9}$ $^a$ |
| $A_{CP}(B^0 \to \pi^+ \pi^0)$ | -6.2$^{+0.5-1.8}_{-0.5+2.0}$ | 17.0$^{+1.2+4.3}_{-1.2-8.7}$ | 17.0$^{+1.2+4.3}_{-1.2-8.7}$ | 38$^{+6}_{-6}$ |
| $A_{CP}(B^0 \to \pi^0 \pi^0)$ | 33.4$^{+6.8+3.4}_{-1.0-37.7}$ | -26.9$^{+8.4+48.5}_{-6.0-37.5}$ | 57.2$^{+14.8+30.3}_{-20.8-34.6}$ | 43$^{+25}_{-24}$ |
| $A_{CP}(B^- \to \pi^- \pi^0)$ | -0.06$^{+0.01+0.01}_{-0.01-0.01}$ | -0.06$^{+0.01+0.01}_{-0.01-0.01}$ | -0.11$^{+0.01+0.06}_{-0.01-0.06}$ | 6$^{+5}_{-5}$ |
| $A_{CP}(B^- \to \pi^- \eta)$ | -11.4$^{+1.1+2.3}_{-1.0-27}$ | 11.4$^{+0.9+4.5}_{-0.9-9.1}$ | -5.0$^{+2.4+8.4}_{-3.4-10.3}$ | -13$^{+7}_{-7}$ $^a$ |

$^a$We have taken into account the new measurement of $B^- \to (K^-, \pi^-)\eta$ [7] to update the average.

$^b$This is the average of 1.83$^{+0.21}_{-0.13}$ by BaBar [31] and 1.1$^{+0.3}_{-0.1}$ by Belle [32]. If an S factor is included, the average will become 1.55$^{+0.35}_{-0.35}$.

CP-asymmetry sum rule lead to $A_{CP}(\bar{K}^0 \pi^0) = -0.073^{+0.042}_{-0.042}$ and $A_{CP}(\bar{K}^0 \pi^0) = -0.15 \pm 0.04$, respectively. An analysis based on the topological quark diagrams also yields a similar result $-0.08 \sim -0.12$ [28]. All these indicate that the direct CP violation $A_{CP}(\bar{K}^0 \pi^0)$ should be negative and has a magnitude of order 0.10. As for the mixing-induced asymmetry $S_{\pi^0 K^*}$, it is found to be enhanced from 0.76 to 0.79$^{+0.06+0.04}_{-0.04-0.04}$ when $\rho_C$ and $\phi_C$ are turned on, while experimentally it is 0.57$^{+0.17}_{-0.17}$ [6]. The discrepancy between theory and experiment for $S_{\pi^0 K^*}$ is one of possible hints of New Physics [29]. Our result for $S_{\pi^0 K^*}$ is consistent with [11, 12, 13] where soft corrections to $a_2$ were considered, but not with [14] where $S_{\pi^0 K^*} \sim 0.63$ was obtained. A correlation between $S_{\pi^0 K^*}$ and $A_{CP}(\bar{K}^0 K_S)$ has been investigated recently in [30]. For the mixing-induced asymmetry in $B \to \pi^+ \pi^-$, we find $S_{\pi^+ \pi^-} = -0.69^{+0.08+0.19}_{-0.10-0.09}$ in accordance with the world average of $-0.65 \pm 0.07$ [6]. From Table, we see that power corrections to the color-suppressed tree amplitude have almost no impact.
on the decay rates of penguin-dominated decays, but will enhance the color-suppressed tree dominated decay $B \to \pi^0\pi^0$ substantially owing to the enhancement of $|a_2| \sim O(0.6)$. Notice that the central values of the branching fractions of this mode measured by BaBar [31] and Belle [32] are somewhat different as noticed in Table II. It is generally believed that direct CP violation of $B^- \to \pi^-\pi^0$ is very small. This is because the isospin of the $\pi^-\pi^0$ state is $I = 2$ and hence it does not receive QCD penguin contributions and receives only the loop contributions from electroweak penguins. Since this decay is tree dominated, SM predicts an almost null CP asymmetry, of order $10^{-3} \sim 10^{-4}$. What will happen if $a_2$ has a large magnitude and strong phase? We find that soft corrections to the color-suppressed tree amplitude will enhance $A_{\text{CP}}(\pi^-\pi^0)$ substantially to the level of 2%. Similar conclusions were also obtained by the analysis based on the diagrammatic approach [18]. However, one must be very cautious about this. The point is that power corrections will affect not only $a_2$, but also other parameters $a_i$ with $i \neq 2$. Since the isospin of $\pi^-\pi^0$ is $I = 2$, soft corrections to $a_2$ and $a_i$ must be conspired in such a way that $\pi^-\pi^0$ is always an $I = 2$ state. As explained below, there are two possible sources of power corrections to $a_2$: spectator scattering and final-state interactions. For final-state rescattering, it is found in [33] that effects of FSIs on $A_{\text{CP}}(\pi^-\pi^0)$ are small, consistent with the requirement followed from the CPT theorem. In the specific residual scattering model considered by one of us (CKC) [11], $\pi^-\pi^0$ can only rescat into itself, and as a consequence, direct CP violation will not receive any contribution from final-state interactions. Likewise, if large $\rho_H$ and $\phi_H$ are turned on to mimic Eq. (5), we find $A_{\text{CP}}(\pi^-\pi^0)$ is at most of order $10^{-3}$. This is because spectator scattering contributes to not only $a_2$ but also $a_i$ and the electroweak penguin parameters $a_{7-10}$. Therefore, a measurement of direct CP violation in $B^- \to \pi^-\pi^0$ still provides a nice test of the Standard Model and New Physics.

In order to explain CP violation in the decay $B^- \to K^-\eta$, we shall elaborate it in more detail. Its decay amplitude is given by [4]

$$\sqrt{2}A(B^- \to K^-\eta) = A_{K\eta}\left[\delta_{\mu\nu}\alpha_2 + 2\alpha_5^p + \frac{1}{2}\alpha_{3,\text{EW}}^p\right] + \sqrt{2}A_{K\eta'}\left[\delta_{\mu\nu}\beta_2 + \alpha_3^p + \alpha_4^p - \frac{1}{2}\alpha_5^p - \frac{1}{2}\alpha_{3,\text{EW}}^p - \frac{1}{2}\alpha_{4,\text{EW}}^p + \beta_3^p + \beta_{3,\text{EW}}^p\right] + \sqrt{2}A_{\eta',\bar{\eta}}\left[\delta_{\mu\nu}\alpha_2 + \alpha_3^p + \alpha_4^p\right] + A_{\eta,\bar{\eta}}\left[\delta_{\mu\nu}(\alpha_1 + \beta_2) + \alpha_3^p + \beta_3^p + \beta_{3,\text{EW}}^p\right],$$

where the flavor states of the $\eta$ meson, $a\bar{a} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$, $s\bar{s}$ and $c\bar{c}$ are labelled by the $\eta_q$, $\eta_s$ and $\eta_c$, respectively. The reader is referred to [4] for other notations. The physical states $\eta$, $\eta'$ and $\bar{\eta}$ can be expressed in terms of flavor states $\eta_q$, $\eta_s$ and $\eta_c$. Since the two penguin processes $b \to s\bar{s}$ and $b \to sq\bar{q}$ contribute destructively to $B \to K\eta$, the penguin amplitude is comparable in magnitude to the tree amplitude induced from $b \to us\bar{u}$, contrary to the decay $B \to K\eta'$ which is dominated by large penguin amplitudes. Consequently, a sizable direct CP asymmetry is expected in $B^- \to K^-\eta$ but not in $K^-\eta'$ [34].

Quantities relevant to the calculation are the decay constants $f^q_{\eta}$, $f^s_{\eta}$ and $f^c_{\eta}$ defined by $\langle 0|\gamma_{\mu}\gamma_{\nu}|\pi^-\eta\rangle = i f^q_{\eta}q_\mu$ and $\langle 0|\bar{c}\gamma_{\mu}\gamma_5|\pi^-\eta\rangle = i f^c_{\eta}q_\mu$, respectively. A straightforward perturbative calculation gives [35]

$$f^c_{\eta} = -\frac{m^2_{\eta}/12m^2_c}{\sqrt{2}} f^q_{\eta}$$

For the decay constants $f^q_{\eta}$ and $f^s_{\eta}$, we shall use the values $f^q_{\eta} = 107$ MeV and $f^s_{\eta} = -112$ MeV obtained in [34] with the convention of $f_\pi = 132$ MeV. Although the decay constant $f^c_{\eta} \approx -2$ MeV is much smaller than
$f_1^{q,s}$, its effect is CKM enhanced by $V_{cb}V_{cs}^*/(V_{ub}V_{us}^*)$. In the absence of power corrections to $a_2$, $A_{CP}(K^-\eta)$ is found to be 0.127 (see Table I). When $p_C$ and $\phi_C$ are turned on, $A_{CP}(K^-\eta)$ will be reduced to 0.004 if there is no intrinsic charm content in the $\eta$. When the effect of $f_1^{q,s}$ is taken into account, $A_{CP}(K^-\eta)$ finally reaches at the level of $-11\%$ and has a sign in agreement with experiment. Hence, $CP$ violation in $B^- \rightarrow K^-\eta$ is the place where the charm content of the $\eta$ plays a role.

We add a remark here that the pQCD prediction for $A_{CP}(K^-\eta)$ is very sensitive to $m_{qq}$, the mass of the $\eta_q$, which is generally taken to be of order $m_{\pi}$. It was found in [37] that for $m_{qq} = 0.14, 0.18$ and $0.22$ GeV, $A_{CP}(K^-\eta)$ becomes $0.0562, 0.0588$ and $-0.3064$, respectively. There are two issues here: (i) Is it natural to have a large value of $m_{qq}$? and (ii) The fact that $A_{CP}(K^-\eta)$ is so sensitive to $m_{qq}$ implies that the pQCD prediction is not stable. Within the framework of pQCD, the authors of [38] rely on the NLO corrections to get a negative $CP$ asymmetry and to avoid the aforementioned issues. At the lowest order, pQCD predicts $A_{CP}(K^-\eta) \approx 9.3\%$. Then NLO corrections will change the sign and give rise to $A_{CP}(K^-\eta) = (-11.7^{+8.4}_{-11.4})\%$ [38].

As for the decay $B^- \rightarrow \pi^-\eta$, it is interesting to see that penguin annihilation will flip the sign of $A_{CP}(\pi^-\eta)$ into a wrong one without affecting its magnitude (see Table II). Again, soft corrections to $a_2$ will bring the $CP$ asymmetry back to the right track. Contrary to the previous case, the charm content of the $\eta$ here does not play a role as it does not get a CKM enhancement relative to the non-charm content of the $\eta$. Our result of $A_{CP}(\pi^-\eta) = -0.05^{+0.09}_{-0.11}$ is consistent with the measurement of $-0.13 \pm 0.07$. For comparison, the pQCD approach predicts $-0.37^{+0.09}_{-0.07}$ [39] and SCET gives two solutions [40], $0.05 \pm 0.29$ and $0.37 \pm 0.29$ with signs opposite to the data.

3. What is the origin of power corrections to $a_2$? There are two possible sources: spectator scattering and final-state interactions. The flavor operators $a_i^\rho$ are basically the Wilson coefficients in conjunction with short-distance nonfactorizable corrections such as vertex corrections, penguin contractions and hard spectator interactions. In general, they have the expression [2, 4]

$$a_i^\rho(M_1M_2) = \left(c_i + \frac{c_i}{N_c}\right) N_f(M_2) + \frac{c_i}{N_c} \frac{C_F a_s}{4\pi} \left[V_i(M_2) + \frac{4\pi^2}{N_c} H_i(M_1M_2)\right] + P_i^\rho(M_2),$$

(9)

where $i = 1, \cdots, 10$, the upper (lower) signs apply when $i$ is odd (even), $c_i$ are the Wilson coefficients, $C_F = (N_c^2 - 1)/(2N_c)$ with $N_c = 3$, $N_f(M_2) = 0$ for $i = 6, 8$ and equals to 1 otherwise, $M_2$ is the emitted meson and $M_1$ shares the same spectator quark with the $B$ meson. The quantities $V_i(M_2)$ account for vertex corrections, $H_i(M_1M_2)$ for hard spectator interactions with a hard gluon exchange between the emitted meson and the spectator quark of the $B$ meson and $P_i(M_2)$ for penguin contractions. A typical hard spectator term $H_i(M_1M_2)$ has the expressions [2, 4]:

$$H_i(M_1M_2) = \int_0^1 dx dy \frac{1}{\lambda_B} \frac{m_B}{X(\overline{B}M_1,M_2)} \left(\Phi_{M_1}(x)\Phi_{M_2}(y) + r_{M_1}^\rho \Phi_{M_1}(x)\Phi_{M_2}(y)\right),$$

(10)

for $i = 1 - 4, 9, 10$, where $X(\overline{B}M_1,M_2)$ is the factorizable amplitude for $\overline{B} \rightarrow M_1M_2$, $\tilde{x} = 1 - x$, $\lambda_B$ is the fist inverse moment of the $B$ meson light-cone wave function and

$$r_{M_1}^\rho(\mu) = \frac{2m_B^2}{m_\rho(\mu)(m_2 + m_1)(\mu)}, \quad r_{M_1}^\rho(\mu) = \frac{2m_B^2}{m_\rho(\mu)} \frac{f_{M_1}^\rho(\mu)}{f_{M_2}^\rho(\mu)}.$$

(11)

Power corrections from the twist-3 amplitude $\Phi_{M_1}$ are divergent and can be parameterized as

$$X_H = \int_0^1 \frac{dy}{y} = \ln \frac{m_B}{\Lambda_H} (1 + \rho_H e^{\phi_H}),$$

(12)
The value are usually small except for the branching fractions and annihilation. Soft corrections to the deficit for penguin-dominated decays can be accounted by the subleading power corrections from penguin the same topology as the color-suppressed tree diagram \[33\]. One of us (CKC) has studied the FSI effects in \[\bar{K} \rightarrow \rho\] a factor of 2 followed by the rescattering of \(K_- \eta'\) into \(K^0\pi^0\). This has the same topology as the color-suppressed tree diagram.

![Contributions to the color-suppressed tree amplitude of \(B^− \rightarrow K^−\pi^0\)](image)

FIG. 1: Contribution to the color-suppressed tree amplitude of \(B^− \rightarrow K^−\pi^0\) from the weak decay \(B^− \rightarrow K^−\eta'\) followed by the final-state rescattering of \(K^− \eta'\) into \(K^0\pi^0\). This has the same topology as the color-suppressed tree diagram.

Since \(c_1 \sim \mathcal{O}(1)\) and \(c_9 \sim \mathcal{O}(1.3)\) in units of \(\alpha_{em}\), it turns out that spectator scattering contributions to \(a_i\) are usually small except for \(a_2\) and \(a_{10}\) which are essentially governed by hard spectator interactions \[41\]. The value \(a_2(K\pi) \approx 0.51e^{-i58^\circ}\) corresponds to \(\rho_H \approx 4.9\) and \(\phi_H \approx -77^\circ\). Therefore, there is no reason to restrict \(\rho_H\) to the range \(0 \leq \rho_H \leq 1\).

A sizable color-suppressed tree amplitude also can be induced via color-allowed decay \(B^− \rightarrow K^−\eta'\) followed by the rescattering of \(K^− \eta'\) into \(K^−\pi^0\) as depicted in Fig. 1. Recall that among the 2-body \(B\) decays, \(B \rightarrow K\eta'\) has the largest branching fraction, of order \(70 \times 10^{-6}\). This final-state rescattering has the same topology as the color-suppressed tree diagram \[33\]. One of us (CKC) has studied the FSI effects through residual rescattering among \(PP\) states and resolved the \(B\)-CP puzzles \[11\].

4. Power corrections to \(a_2\) for \(B \rightarrow VP\) and \(B \rightarrow VV\) are not the same as that for \(B \rightarrow PP\) as described by Eq. (5). From Table II we see that an enhancement of \(a_2\) is needed to improve the rates of \(B \rightarrow \rho^0\pi^0\) and the direct \(CP\) asymmetry of \(B^0 \rightarrow K^{*0}\eta\). However, it is constrained by the measured rates of \(\rho^0\pi^-\) and \(\rho^-\pi^0\) modes. This means that \(\rho_C(VP)\) is preferred to be smaller than \(\rho_C(PP) = 1.3\). In Table II we show the branching fractions and \(CP\) asymmetries in \(B \rightarrow VP\) decays for \(\rho_C(VP) = 0.8\) and \(\phi_C(VA) = -80^\circ\). The corresponding values of \(a_2(VP)\) are

\[
\begin{align*}
  a_2(\pi\rho) &\approx 0.40e^{-i51^\circ}, & a_2(\rho\pi) &\approx 0.38e^{-i52^\circ}, \\
  a_2(\rho K^*) &\approx 0.36e^{-i52^\circ}, & a_2(\pi K^*) &\approx 0.39e^{-i51^\circ}.
\end{align*}
\]

(13)

It is clear from Table II that in the heavy quark limit, the predicted rates for \(\bar{B} \rightarrow \bar{K}^+\pi\) are too small by a factor of \(2 \sim 3\), while \(\mathcal{B}(B \rightarrow K\rho)\) are too small by \((15 \sim 50)%\) compared with experiment. The rate deficit for penguin-dominated decays can be accounted by the subleading power corrections from penguin annihilation. Soft corrections to \(a_2\) will enhance \(\mathcal{B}(B \rightarrow \rho^0\pi^0)\) to the order of \(1.3 \times 10^{-6}\), while the BaBar and Belle results, \((1.4 \pm 0.6 \pm 0.3) \times 10^{-6}\) \[46\] and \((3.0 \pm 0.5 \pm 0.7) \times 10^{-6}\) \[47\] respectively, differ in their central values by a factor of 2. Improved measurements are certainly needed for this decay mode. As

---

\(^3\) As pointed out in \[21, 42\], a smaller value of \(\lambda_B\) of order 200 MeV can enhance the hard spectator interaction [see Eq. (10)] and hence \(a_2\) substantially. However, the recent BaBar data on \(B \rightarrow \gamma\ell\nu\) \[43\] seems to imply a larger \(\lambda_B\) (> 300 MeV at the 90% CL). In this work we reply on \(\rho_C\) and \(\phi_C\) to get a large complex \(a_2\).
TABLE II: Same as Table I except for some selective $B \rightarrow VP$ decays with $\rho_C = 0.8$ and $\phi_C = -80^\circ$.

| Modes | W/ $\rho_{PA}, \phi_{AC}$ | With $\rho_A, \phi_A$ | With $\rho_{AC}, \phi_{AC}$ | Expt. [6] |
|-------|-----------------|------------------|-----------------|---------|
| $\mathcal{B}(\bar{B}^0 \rightarrow K^- \rho^+)$ | 6.5$^{+5.4+0.4}_{-2.6-0.4}$ | 8.6$^{+5.7+7.4}_{-2.8-4.5}$ | 8.6$^{+5.7+7.4}_{-2.8-4.5}$ | 8.6$^{0.9}_{-1.1}$ |
| $\mathcal{B}(\bar{B}^0 \rightarrow K^0 \rho^0)$ | 4.7$^{+3.3+0.3}_{-1.8-0.3}$ | 5.5$^{+5.1+4.3}_{-1.7-2.8}$ | 5.4$^{+3.3+4.3}_{-1.7-2.8}$ | 5.4$^{0.9}_{-1.0}$ |
| $\mathcal{B}(B^- \rightarrow \bar{K}^0 \rho^-)$ | 5.5$^{+6.1+0.7}_{-2.8-0.5}$ | 7.8$^{+6.3+7.3}_{-2.9-4.4}$ | 7.8$^{+6.3+7.3}_{-2.9-4.4}$ | 8.0$^{0.9}_{-1.4}$ |
| $\mathcal{B}(B^- \rightarrow K^0 \rho^0)$ | 1.9$^{+2.5+0.3}_{-0.9-0.6}$ | 3.3$^{+2.6+2.9}_{-1.1-1.7}$ | 3.5$^{+2.9+2.9}_{-1.2-1.8}$ | 3.8$^{0.8}_{-0.9}$ |
| $\mathcal{B}(\bar{B}^0 \rightarrow K^+ \pi^+)$ | 3.7$^{+3.0+0.4}_{-0.5-0.4}$ | 9.2$^{+3.0+1.7}_{-0.1-0.3}$ | 9.2$^{+1.0+3.7}_{-0.1-0.3}$ | 10.3$^{0.1}_{-0.1}$ |
| $\mathcal{B}(\bar{B}^0 \rightarrow \bar{K}^0 \pi^0)$ | 1.1$^{+2.0+0.2}_{-0.2-0.2}$ | 3.5$^{+3.0+1.8}_{-0.1-0.5}$ | 3.5$^{+3.0+1.8}_{-0.1-0.5}$ | 2.4$^{0.7}_{-0.7}$ |
| $\mathcal{B}(B^- \rightarrow \bar{K}^0 \pi^-)$ | 4.0$^{+0.7+0.6}_{-0.9-0.6}$ | 10.4$^{+1.3+4.3}_{-1.5-3.9}$ | 10.4$^{+1.3+4.3}_{-1.5-3.9}$ | 9.9$^{0.8}_{-0.9}$ |
| $\mathcal{B}(B^- \rightarrow \bar{K}^0 \pi^-)$ | 3.2$^{+2.0+0.3}_{-0.4-0.3}$ | 6.8$^{+0.7+2.3}_{-0.7-2.2}$ | 6.7$^{+0.7+2.3}_{-0.7-2.2}$ | 6.9$^{0.2}_{-0.3}$ |
| $\mathcal{B}(\bar{B}^0 \rightarrow \rho^0 \pi^0)$ | 0.76$^{+0.5+0.4}_{-0.3-0.1}$ | 0.58$^{+0.8+0.6}_{-0.3-0.2}$ | 0.58$^{+0.8+0.6}_{-0.3-0.2}$ | 2.0$^{0.5}_{-0.4}$ |
| $\mathcal{B}(\bar{B}^0 \rightarrow \rho^0 \pi^0)$ | 0.76$^{+0.5+0.4}_{-0.3-0.1}$ | 0.58$^{+0.8+0.6}_{-0.3-0.2}$ | 0.58$^{+0.8+0.6}_{-0.3-0.2}$ | 2.0$^{0.5}_{-0.4}$ |
| $\mathcal{B}(B^- \rightarrow \rho^+ \rho^-)$ | 11.6$^{+0.1+0.0}_{-0.9-0.5}$ | 11.8$^{+1.3+1.0}_{-0.9-0.6}$ | 11.8$^{+1.3+1.0}_{-0.9-0.6}$ | 10.9$^{0.4}_{-1.5}$ |
| $\mathcal{B}(B^- \rightarrow \rho^+ \rho^-)$ | 8.2$^{+1.8+1.2}_{-0.9-0.6}$ | 8.5$^{+1.8+1.2}_{-0.9-0.6}$ | 8.5$^{+1.8+1.2}_{-0.9-0.6}$ | 8.3$^{1.3}_{-1.4}$ |
| $\mathcal{B}(B^- \rightarrow \rho^+ \rho^-)$ | 15.3$^{+1.0+0.5}_{-1.5-0.5}$ | 15.9$^{+1.1+0.9}_{-1.5-1.1}$ | 15.9$^{+1.1+0.9}_{-1.5-1.1}$ | 15.7$^{0.8}_{-1.8}$ |
| $\mathcal{B}(B^- \rightarrow \rho^+ \rho^-)$ | 8.4$^{+0.4+0.3}_{-0.7-0.5}$ | 9.2$^{+0.4+0.5}_{-0.7-0.5}$ | 9.2$^{+0.4+0.5}_{-0.7-0.5}$ | 7.3$^{1.2}_{-1.2}$ |

$^{a}$If an $S$ factor is included, the average will become 2.0$\pm0.7$.

$^{b}$This is the average of 10$\pm40$ by BaBar [44] and $-49$ by Belle [45].

for direct $CP$ asymmetries, we see that penguin annihilation will flip the sign of $A_{CP}(K^- \rho^0)$ into the right direction. Power corrections to the color-suppressed tree amplitude are needed to improve the prediction for $A_{CP}(K^0 \eta)$. Our prediction is of order 0.035 to be compared with the experimental value of 0.19$\pm0.05$. The pQCD prediction of $A_{CP}(K^0 \eta) \sim 0.0057$ [48] is too small, while the S2E result of $-0.017$ [49] has a wrong sign. For $A_{CP}(K^0 \rho^0)$, it gets a sign flip after including soft effects on $\alpha_s$. Our prediction is $(8.7^{+8.8}_{-6.9})^\circ$, while it is 0.06$\pm0.20$ experimentally. Defining $\Delta A_{K^+ \pi} = A_{CP}(K^+ \pi^+) - A_{CP}(K^+ \pi^+)$ in analog
to $\Delta A_{K\pi}$, we predict that $\Delta A_{K^{\pi}} = (13.7^{+2.9+3.6}_{-1.4-6.9})\%$, while it is naively expected that $K^+ \pi^0$ and $K^- \pi^+$ have similar $CP$-violating effects. It is of importance to measure $CP$ asymmetries of these two modes to test our prediction. For mixing-induced $CP$ violation, we obtain $\Delta S_{\phi K_3} = 0.022^{+0.004}_{-0.002}$, $\Delta S_{\omega K_3} = 0.17^{+0.06}_{-0.08}$ and $\Delta S_{\rho' K_3} = -0.17^{+0.09}_{-0.18}$ [22], where $\Delta S_f = -\eta_f S_f - \sin 2\beta$. It turns out that soft corrections to $a_2$ have significant effects on the last two quantities.

As for $B \rightarrow VV$ decays, we notice that the calculated $B^0 \rightarrow \rho^0 \rho^0$ rate in QCDF is $\beta(B^0 \rightarrow \rho^0 \rho^0) = (0.88_{-0.41}^{+1.46+1.06}) \times 10^{-6}$ for $\rho_C = 0$ [50], while the world average is $(0.73^{+0.27}_{-0.28}) \times 10^{-6}$ [4]. Therefore, soft power correction to $a_2$ or $\rho_C(VV)$ should be small for $B^0 \rightarrow \rho^0 \rho^0$. Consequently, a pattern follows: Effects of power corrections on $a_2$ are large for $PP$ modes, moderate for $VP$ ones and very small for $VV$ cases. \footnote{This is consistent with the observation made in [13] that soft power correction dominance is much larger for $PP$ than $VP$ and $VV$ final states. It has been argued that this has to do with the special nature of the pion which is a $q\bar{q}$ bound state on the one hand and a nearly massless Nambu-Goldstone boson on the other hand [13]. The two seemingly distinct pictures of the pion can be reconciled by considering a soft cloud of higher Fock states surrounding the bound valence quarks. From the FSI point of view, since $B \rightarrow \rho^+ \rho^-$ has a rate much larger than $B \rightarrow \pi^+ \pi^-$, it is natural to expect that $B \rightarrow \pi^0 \pi^0$ receives a large enhancement from the weak decay $B \rightarrow \rho^+ \rho^-$ followed by the rescattering of $\rho^+ \rho^-$ to $\pi^0 \pi^0$ through the exchange of the $\rho$ particle. Likewise, it is anticipated that $B \rightarrow \rho^0 \rho^0$ will receive a large enhancement via isospin final-state interactions from $B \rightarrow \rho^+ \rho^-$. The fact that the branching fraction of this mode is rather small and is consistent with the theory prediction implies that the isospin phase difference of $\delta_0^B$ and $\delta_2^B$ and the final-state interaction must be negligible [51].}

5. $B$-$CP$ puzzles arise in the framework of QCD factorization because power corrections due to penguin annihilation, that account for the observed rates of penguin-dominated two-body decays of $B$ mesons and direct $CP$ asymmetries $A_{CP}(K^- \pi^+), A_{CP}(K^- \pi^+), A_{CP}(K^- \rho^0)$ and $A_{CP}(\pi^+ \pi^-)$, will flip the signs of direct $CP$-violating effects in $B^- \rightarrow K^- \pi^0, B^- \rightarrow K^- \eta, B^- \rightarrow \pi^- \eta$ and $B^0 \rightarrow \pi^0 \pi^0$ to wrong ones when confronted with experiment. We have shown that power corrections to the color-suppressed tree amplitude due to hard spectator interactions and/or final-state interactions will yield correct signs again for aforementioned $CP$ asymmetries and accommodate the observed $\pi^0 \pi^0$ and $\rho^0 \pi^0$ rates simultaneously. $CP$-violating asymmetries of $B^- \rightarrow K^- \eta$ can be understood as a consequence of soft corrections to $a_2$. $A_{CP}(K^0 \rho^0)$ is predicted to be of order $-0.10$, in agreement with that inferred from the $CP$-asymmetry sum rule, or SU(3) relation or the diagrammatical approach. For direct $CP$ violation in $B^- \rightarrow K^+ \eta, \pi^- \eta$, our predictions are in better agreement with experiment than pQCD and SCET. For $B^0 \rightarrow K^0 \rho^0$, we obtained $A_{CP}(K^0 \rho^0) = 0.087^{+0.088}_{-0.069}$. We argued that the smallness of $CP$ asymmetry of $B^- \rightarrow \pi^- \pi^0$ is not affected by the soft corrections under consideration. For the $CP$ asymmetry difference in $K^+ \pi$ modes de-

\footnote{Since the chiral factor $r^V_{\rho}$ for the vector meson is substantially smaller than $r^S_{\rho}$ for the pseudoscalar meson (typically, $r_{\rho}^V = O(0.8)$ and $r_{\rho}^S = O(0.2)$ at the hard collinear scale $\mu = \sqrt{\Lambda m_b}$), one may argue that Eq. (10) naturally explains why the power corrections to $a_2$ is smaller when $M_1$ is a vector meson, provided that soft corrections arise from spectator rescattering. Unfortunately, this is not the case. Numerically, we found that, for example, $H(K^+ \pi)$ is comparable to $H(K \pi)$. This is due to the fact that $\int_0^1 dx r^M_{\phi}(x)/\bar{x} = x H r^M_{\phi}$ for $M = P$ and approximated to $3(\chi_H - 2)r^V_{\chi}$ for $M = V$.}
fined by $\Delta A_{K^+\pi} \equiv A_{CP}(K^+\pi^-\pi^0) - A_{CP}(K^+\pi^+\pi^-)$, we predict that $\Delta A_{K^+\pi} \sim 14\%$, while these two modes are naively expected to have similar direct $CP$-violating effects. For mixing-induced $CP$ violation, we found $\Delta S_{\pi^0 K^0} = 0.12^{+0.07}_{-0.06}$, $\Delta S_{\phi K^0} = 0.02^{+0.04}_{-0.02}$, $\Delta S_{\omega K^0} = 0.17^{+0.06}_{-0.08}$ and $\Delta S_{\rho K^0} = -0.17^{+0.09}_{-0.18}$.

**Acknowledgments**

We are grateful to Chuan-Hung Chen, Cheng-Wei Chiang, Hsiang-nan Li, Tri-Nang Pham and Amarjit Soni for useful discussions. One of us (H.Y.C.) wishes to thank the hospitality of the Physics Department, Brookhaven National Laboratory. This research was supported in part by the National Science Council of R.O.C. under Grant Nos. NSC97-2112-M-001-004-MY3 and NSC97-2112-M-033-002-MY3.

[1] H.Y. Cheng and J. Smith, Annu. Rev. Nucl. Part. Sci. 59, 215 (2009) [arXiv:0901.4396 [hep-ph]].
[2] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Phys. Rev. Lett. 83, 1914 (1999); Nucl. Phys. B 591, 313 (2000).
[3] Y.Y. Keum, H.-n. Li, and A.I. Sanda, Phys. Rev. D 63, 054008 (2001).
[4] M. Beneke and M. Neubert, Nucl. Phys. B 675, 333 (2003).
[5] Particle Data Group, C. Amsler et al., Phys. Lett. B 667, 1 (2008).
[6] E. Barberio et al. (Heavy Flavor Averaging Group), [arXiv:0704.3575 [hep-ex]] (2007) and online update at http://www.slac.stanford.edu/xorg/hfag.
[7] B. Aubert et al. [BABAR Collaboration], [arXiv:0907.1743 [hep-ex]].
[8] Y. Y. Charng and H. n. Li, Phys. Rev. D 71, 014036 (2005); H.-n. Li, S. Mishima, and A.I. Sanda, Phys. Rev. D 72, 114005 (2005).
[9] C. S. Kim, S. Oh and C. Yu, Phys. Rev. D 72, 074005 (2005) [arXiv:hep-ph/0505060].
[10] M. Gronau and J. L. Rosner, Phys. Lett. B 644, 237 (2007) [arXiv:hep-ph/0610227].
[11] C. K. Chua, Phys. Rev. D 78, 076002 (2008) [arXiv:0712.4187].
[12] M. Ciuchini, E. Franco, G. Martinelli, M. Pierini and L. Silvestrini, Phys. Lett. B 674, 197 (2009).
[13] M. Duraisamy and A. L. Kagan, [arXiv:0812.3162 [hep-ph]].
[14] H. n. Li and S. Mishima, [arXiv:0901.1272 [hep-ph]].
[15] S. Baek, C. W. Chiang, M. Gronau, D. London and J. L. Rosner, [arXiv:0905.1495 [hep-ph]].
[16] T. Yoshikawa, Phys. Rev. D 68, 054023 (2003); S. Mishima and T. Yoshikawa, Phys. Rev. D 70, 094024 (2004); A.J. Buras, R. Fleischer, S. Recksiegel, F. Schwab, Phys. Rev. Lett. 92, 101804 (2004); Y.L. Wu and Y.F. Zhou, Phys. Rev. D 72, 034037 (2005); S. Baek, P. Hamel, D. London, A. Datta and D. A. Suprun, Phys. Rev. D 71, 057502 (2005); S. Baek and D. London, Phys. Lett. B 653, 249 (2007); T. Feldmann, M. Jung, and T. Mannel, JHEP 0808, 066 (2008).
[17] W. S. Hou, H. n. Li, S. Mishima and M. Nagashima, Phys. Rev. Lett. 98, 131801 (2007) [arXiv:hep-ph/0611107]; A. Soni, A. K. Alok, A. Giri, R. Mohanta and S. Nandi, [arXiv:0807.1971 [hep-ph]].
[18] C. W. Chiang, M. Gronau, J. L. Rosner and D. A. Suprun, Phys. Rev. D 70, 034020
(2004) [arXiv:hep-ph/0404073]; C. W. Chiang and Y. F. Zhou, JHEP 0612, 027 (2006) [arXiv:hep-ph/0609128].
[19] M. Beneke and S. Jager, Nucl. Phys. B 751, 160 (2006) [arXiv:hep-ph/0512351]; N. Kivel, JHEP 0705, 019 (2007) [arXiv:hep-ph/0608291]; V. Pilipp, Nucl. Phys. B 794, 154 (2008).
[20] G. Bell, Nucl. Phys. B 795, 1 (2008) [arXiv:0705.3127 [hep-ph]] [arXiv:0902.1915 [hep-ph]].
[21] M. Beneke, talk presented at the FPCP2008 Conference on Flavor Physics and CP violation, May 5-9, 2008, Taipei, Taiwan.
[22] H.Y. Cheng and C.K. Chua, in preparation.
[23] E. Lunghi and A. Soni, JHEP 0709, 053 (2007) [arXiv:0707.0212 [hep-ph]].
[24] B. Aubert et al. [BABAR Collaboration], Phys. Rev. D 79, 052003 (2009) [arXiv:0809.1174 [hep-ex]].
[25] I. Adachi et al. [Belle Collaboration], arXiv:0809.4366 [hep-ex].
[26] A. Kusaka et al. [Belle Collaboration], Phys. Rev. Lett. 98, 221602 (2007).
[27] A. Kusaka et al. [Belle Collaboration], Phys. Rev. Lett. 93, 051802 (2004) [arXiv:hep-ex/0311049].
[28] Z. Q. Zhang and Z. J. Xiao, arXiv:0807.2024 [hep-ph].