Bosonization of World-Sheet Fermions
in Minkowski Space-Time.

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ABSTRACT

We propose a way of bosonizing free world-sheet fermions for 4-dimensional heterotic string theory formulated in Minkowski space-time. We discuss the differences as compared to the standard bosonization performed in Euclidean space-time.

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1. In the Neveu-Schwarz-Ramond approach to superstring theory [1] any model with $D$ flat space-time dimensions will contain the coordinate fields $X^\mu$ and their world-sheet superpartners, the set of Majorana fermion fields $\psi^\mu$. The latter can have either Neveu-Schwarz (NS) or Ramond (R) boundary conditions around any non-contractible loop on the world-sheet. In order to obtain string scattering amplitudes one has to compute correlation functions of these fermions on arbitrary Riemann surfaces. The standard way of doing this is by bosonization. Bosonization is usually carried out in Euclidean space-time, and thus requires that we first rotate the metric from Minkowski to Euclidean space-time, then perform all calculations and only at the end rotate back to Minkowski space-time.

On the other hand, it could be convenient to formulate the theory and make all computations directly in Minkowski space-time. For example, the whole concept of unitarity requires that we are in Minkowski space-time, even though, to obtain amplitudes with the correct analytic properties, a careful procedure of analytic continuation in the momentum invariants is needed, as discussed for example in ref. [2]. Likewise, an important quantity such as the time-reversal operator $T$ only assumes its true physical significance in Minkowski space-time. Accordingly, if we want to prove the CPT-invariance of the $S$-matrix in a given string theory at any order in perturbation theory, as was done in ref. [3] for the 10-dimensional heterotic models, it would be most natural to define the CPT operator and make the proof entirely in Minkowski space-time. So, in some situations it might be useful if the bosonization could be carried out directly in Minkowski space-time, without having to rotate the metric.

In this letter we propose a bosonization procedure in Minkowski space-time. In section 2 we consider the simplest possible case, corresponding to a single pair of Majorana fermions transforming in the vector representation of $SO(1,1)$, and in sections 3 and 4 we then discuss how to incorporate other fermions, including the proper treatment of cocycles. To be explicit we consider the 4-dimensional heterotic string models of Kawai, Lewellen and Tye (KLT) [4], but our procedure should apply to other models as well.

2. We start by reviewing very briefly the well-known case of bosonization in $D = 2$ Euclidean space-time, that is, we consider two free chiral Majorana fermions on the world-sheet, transforming as vectors under $SO(2)$. In terms of a local complex coordinate $z$ they
are represented by two hermitean chiral conformal fields \( \psi^\mu(z) \) of dimension \( 1/2 \), with operator product expansion (OPE)

\[
\psi^\mu(z)\psi^{\nu}(w) = g^{\mu\nu}\frac{1}{z-w} + \ldots \quad \mu, \nu = (0,1) ,
\]

with \( g^{\mu\nu} = \delta^{\mu\nu} \).

(In general the hermitean conjugate field \( \phi^*_\Delta \) of a primary conformal field \( \phi_\Delta(z) \) of dimension \( \Delta \) is defined by the relation

\[
(\phi_\Delta(z))\dagger = (\frac{1}{z^*})^{2\Delta} \phi^*_\Delta (\frac{1}{z^*}) ,
\]

where \( z^* \) denotes the complex conjugate of \( z \), and we say that \( \phi_\Delta \) is hermitean (anti-hermitean) if \( \phi^*_\Delta = \phi_\Delta \) (\( \phi^*_\Delta = -\phi_\Delta \)).)

In Euclidean space-time we bosonize by introducing an anti-hermitean scalar field \( \phi \), defined by \( j_{01}(z) = -i\psi^0\psi_1(z) = \partial \phi(z) \), whose mode operators give rise to a Fock space of states with positive definite norm. We may then identify \( \psi^\pm = e^\pm \phi \) where \( \psi^\pm \) are two complex fermions formed out of \( \psi^0 \) and \( \psi^1 \). The situation is summarized in Table 1.

Notice that in this case the “momentum” operator \( N \) is hermitean. In particular, if we define \( |N = r\rangle = \lim_{z \to 0} e^{r\phi(z)}|0\rangle \), we find by conservation of the “momentum”

\[
\langle N = r | N = r' \rangle = \lim_{\zeta \to 0} \lim_{z \to 0} \langle e^{r\phi(\zeta)}\dagger e^{r'\phi(z)} \rangle = \delta_{r-r'} \quad (\text{Euclidean case}) .
\]

In Minkowski space-time, the metric \( g \) appearing in the OPE (1) becomes \( g = \text{diag}(2)(-1,1) \). To bosonize \( \psi^\mu \) in this case we note that the Kač-Moody current \( j^{01} = -i\psi^0\psi^1 \) remains hermitean but now has the opposite sign in the OPE compared to the Euclidean case. This means that the scalar field \( \phi \), just like the time coordinate field \( X^0 \), is forced to have the “wrong” sign in the OPE and thus gives rise to a Fock space containing states of negative norm. We choose to define \( j^{01}(z) = -i\partial \phi(z) \) so that \( \phi \) is now hermitean (rather than anti-hermitean) \(^2\) and then the OPE with the “wrong” sign is

\[
\phi(z)\phi(w) = +\log(z-w) + \ldots .
\]

\(^1\) Products (and exponentials) of operators at the same point are always normal ordered. We do not adopt the \( : \) notation.

\(^2\) It is also possible, although not convenient, to choose \( \phi \) anti-hermitean, again with the “wrong” sign in the OPE, \( \phi(z)\phi(w) = -\log(z-w) + \ldots . \)
| Euclidean | Minkowski |
|-----------|-----------|
| $\psi^0 = \frac{1}{\sqrt{2}} (e^\phi - e^{-\phi})$ | $\psi^0 = \frac{1}{\sqrt{2}} (e^\phi - e^{-\phi})$ |
| $\psi^1 = \frac{1}{\sqrt{2}} (e^\phi + e^{-\phi})$ | $\psi^1 = \frac{1}{\sqrt{2}} (e^\phi + e^{-\phi})$ |
| $e^{\pm \phi} = \psi^\pm = \frac{1}{\sqrt{2}} (\psi^1 \pm i\psi^0)$ | $e^{\pm \phi} = \psi^\pm = \frac{1}{\sqrt{2}} (\psi^1 \pm \psi^0)$ |
| $\psi^- = (\psi^+)^*$ | $(\psi^\pm)^* = \psi^\pm$ |
| $j^{01}(z) = \partial \phi(z)$ | $j^{01}(z) = -i\partial \phi(z)$ |

$$\phi(z)\phi(w) = + \log(z-w) + \ldots$$
$$\phi(z) = x + N \log z + \sum_{n\neq 0} \frac{\alpha_n}{n} z^{-n}$$

$[\alpha_n, \alpha_m] = n\delta_{n+m}$, \quad $[N, x] = 1$

| $\phi$ anti-hermitean | $\phi$ hermitean |
|------------------------|-------------------|
| $(\alpha_n)^\dagger = +\alpha_{-n}$ | $(\alpha_n)^\dagger = -\alpha_{-n}$ |
| $x^\dagger = -x$ | $x^\dagger = +x$ |
| $N^\dagger = +N$ | $N^\dagger = -N$ |
| $\langle N = r|N = r'\rangle = \delta_{r-r'}$ | $\langle N = r|N = r'\rangle = \delta_{r+r'}$ |

**Table 1:** Comparison of the bosonization in Euclidean/Minkowski space-time for two free chiral Majorana fermions.

The bosonization proceeds as before (see Table 1), but the hermiticity properties of the operators are different. The operators $e^{\pm \phi} \equiv \psi^\pm$ are now hermitean and have to be identified with hermitean linear combinations of $\psi^0$ and $\psi^1$. Also, the “momentum” operator $N$ is anti-hermitean. At first sight this seems to lead to an inconsistency: Generically an anti-hermitean operator should have imaginary eigenvalues. Instead, the states in the Neveu-Schwarz (NS) sector (i.e. those that are created from the conformal vacuum by operators obtained from the fields $\psi^\mu$ and their derivatives) involve only real integer values of the “momentum” $N$. Similarly, in the Ramond (R) sector, the eigenvalues are half-integer real numbers. An anti-hermitean operator can have nonzero real eigenvalues only if the corresponding eigenstates have zero norm. But this is exactly the case! If we redo the computation leading to equation (3) then, since $\phi$ is now hermitean rather than anti-hermitean, we obtain $\langle N = r|N = r'\rangle = \delta_{r+r'}$ for any real numbers $r, r'$. In particular
As is well known there are two ground states in the Ramond sector, $| \pm 1/2 \rangle$, created from the conformal vacuum by the spin field operators

$$S_{\pm 1/2}(z) \equiv e^{\pm \phi(z)/2},$$

which transform in the spinor representation of $SO(1,1)$. The corresponding Clifford algebra is generated by the matrices $\gamma^\mu$ defined by the OPE

$$\psi^\mu(z)S_\alpha(w) = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{z-w}} (\gamma^\mu)_\alpha^\beta S_\beta(w) + \ldots .$$

Explicitly one finds $\gamma^0 = -i\sigma_2$ and $\gamma^1 = \sigma_1$.

This concludes our discussion of the bosonization for a single pair of chiral $SO(1,1)$ Majorana fermions. In passing we note that all correlation functions on an arbitrary Riemann surface are the same whether $\phi$ is assumed to be hermitean (as in the Minkowski case) or anti-hermitean (as in the Euclidean case). This is because the correlation functions are uniquely determined by the boundary conditions (i.e. the spin structures) together with the short distance behaviour (i.e. the OPE) which are the same in both cases. Thus, the $N$-point $g$-loop vertex of ref. [5] can be used also in the Minkowski case.

3. When there are other fermions present besides $\psi^0$ and $\psi^1$, it is necessary to introduce cocycles in order to ensure that different fermions anti-commute even after they have been bosonized [6]. Furthermore, the typical spin field neither commutes nor anti-commutes with other operators but picks up a phase that is a fractional power of $-1$, and the cocycles serve to keep track also of this.

A priori there are many ways of choosing the cocycle operators, but the choices are limited by a number of consistency conditions. For the sake of being definite we will consider four-dimensional heterotic string models of the type described by Kawai, Lewellen and Tye [4], but the generalization to other models is straightforward. In ref. [7] we discussed in great detail how to bosonize a generic 4-dimensional heterotic KLT model in Euclidean space-time and here we briefly summarize the main points.

In a 4-dimensional heterotic KLT model in Euclidean space-time, there are 22 left-moving complex fermions $\bar{\psi}_{\bar{l}}^+(z)$, $\bar{l} = \bar{1}, \ldots, \bar{22}$, and eleven right-moving complex fermions $\psi_l^+$, $l = 0, 1, \ldots, 10$, with $(\bar{\psi}_{\bar{l}}^+)^* = \bar{\psi}_{\bar{l}}^-$ and $(\psi_l^+)^* = \psi_l^-$. As usual, a set of hermitean
Minkowski

\[
\psi^\mu(z)\psi^\nu(w) = \frac{\delta^{\mu\nu}}{z-w}, \quad \mu, \nu = (0, \ldots, 3)
\]

\[
\psi^\mu = \frac{1}{\sqrt{2}} \left\{ \frac{1}{3}(\psi^+ - \psi^-), \psi^+ + \psi^- \right\}
\]

\[
\Psi^-_L = (\Psi^+_L)^* \quad L = 1, \ldots, 33
\]

\[
\Psi^\pm_L = (\Psi^\pm_L)^* \quad L = 34
\]

| Euclidean | Minkowski |
|-----------|-----------|
| \[ C^\mu(\psi_L) = 22 + l \] | \[ C^\mu(\psi_L) = \frac{g^\mu
\nu}{z-w}, \quad g = (-, +, +, +) \] |
| \[ \psi^\mu = \frac{1}{\sqrt{2}} \left\{ \frac{1}{3}(\psi^+ - \psi^-), \psi^+ + \psi^- \right\} \] | \[ \psi^\mu = \frac{1}{\sqrt{2}} \left\{ (\psi^+ - \psi^-), (\psi^+ + \psi^-) \right\} \] |
| \[ \Psi^-_L = (\Psi^+_L)^* \quad L = 1, \ldots, 33 \] | \[ \Psi^-_L = (\Psi^+_L)^* \quad L = 33, 34 \] |

| \[ \Psi^\pm_L = e^\pm \Phi(L) (C(L))^{\pm1} \] | \[ \Phi(L)(z)\Phi(K)(w) = \eta_{L,K} \log(z-w) + \ldots \quad \eta_{L,K} = \text{diag}(34)(+,,+,+) \] |
| \[ [J^L_0, \Phi(K)] = \delta_{L,K}, \] | \[ (J^L_0)_{34} = -2 \] |
| \[ C(L) = C^{(L)}_g \cdot e^{i\pi e(L) Y^E J_0} \] | \[ C(L) = C^{(L)}_g \cdot e^{i\pi e(L) Y^M J_0} \] |
| \[ (J^L_0)^\dagger = J^L_0 \quad L = 1, \ldots, 33 \] | \[ (J^L_0)^\dagger = J^L_0 \quad L = 1, \ldots, 32 \] |
| \[ (C(L))^{\dagger} = (C(L))^{-1} \quad L = 1, \ldots, 33 \] | \[ (C(L))^{\dagger} = (C(L))^{-1} \quad L = 1, \ldots, 32 \] |

Table 2: Comparison of the bosonization in Euclidean/Minkowski space-time

for a 4d KLT heterotic string model.

fermions \( \psi^\pm_{11} \) is introduced when we fermionize the superghosts in the usual way, \( \beta = \partial \xi_{11} \) and \( \gamma = \psi^+_{11} \eta \).

To define the cocycles we have to introduce an ordering for the 33 complex fermions and \( \psi^+_{11} \), i.e. number them by integers \( L \) running from 1 to 34. The most natural example of an ordering is \( (1, 2, \ldots, 22; 0, 1, \ldots, 10; 11) \) where the left-moving fermions \( \tilde{\psi}_{11}^\pm \) are represented by the number \( L = l \), while the right-moving fermions \( \psi^+_{11} \) are represented by the number \( L = 22 + l \) and the superghost fermions \( \psi^+_{11} \) by \( L = 34 \).

Given such an ordering it is then convenient to denote the fermions corresponding to the integer \( L \) by \( \Psi^\pm_L \).

The most relevant bosonization formulae are summarized in Table 2, and more details can be found in ref. [7]. In the expression for the cocycle \( C(L) \) the 34 dimensional vector \( e(L) \) is given by \( e(L) = \delta_{L,K} \) and \( J_0 \) is the 34-component vector of number operators \( J^{(L)}_0 \).
In terms of the mode expansion given in Table 1 we have $J_0^{(L)} = N_0^{(L)}$ except for the superghosts where $J_0^{(\text{superghost})} = -N_0^{(\text{superghost})}$. Finally, the $34 \times 34$ matrix $Y^E$ has all elements in the diagonal and the upper triangle equal to zero, that is $Y^E_{KL} = 0$ for $K \leq L$, while $Y^E_{KL} = \pm 1$ for $K > L$. It is at this point, i.e. in the definition of the $Y$ matrix, that we make use of the ordering chosen for the fermions: The fermion $\Psi_{(L)}$ carries a cocycle factor involving the number operator $J_0^{(K)}$ if and only if $L > K$.

(The cocycle factor $C^{(L)}_{gh}$ involves the number operators of the reparametrization ghosts and of the $(\eta, \xi)$-system and is given explicitly in ref. [7]. For the present purposes it is sufficient to notice that $(C^{(L)}_{gh})^\dagger = C^{(L)}_{gh} = (C^{(L)}_{gh})^{-1}$ where the first equality sign holds if we exclude the $\xi$ zero mode and the second is a triviality, since the operator only takes values $\pm 1$ on any string state.)

Although it seems that the choice of ordering is totally arbitrary, this is not so. Indeed, the anomalous behaviour

$$\left(J_0^{(\text{superghost})}\right)^\dagger = -J_0^{(\text{superghost})} - 2$$

under hermitean conjugation forces us to assign the highest number to the superghost-related fermions, i.e. $L = 34$, so that $J_0^{(34)}$ never appears in the cocycle operators. This is because to be consistent with the fact that $\Psi^{\pm}_{(L)} = (\Psi^{\dagger}_{(L)})^*$ for $L = 1, \ldots, 33$, we must have $(C_{(L)})^\dagger = (C_{(L)})^{-1}$.

Notice, however, that the superghost bosonization formula is not consistent with hermitean conjugation, since $\Psi^{\pm}_{(34)}$ as well as $e^{\pm \Phi_{(34)}}$ are hermitean operator fields, but $(C_{(34)})^\dagger = (C_{(34)})^{-1}$. There seems to be no way to avoid this problem in the Euclidean formulation, but we will see that it is removed when we reformulate the bosonization in Minkowski space-time.

4. In Minkowski space-time, as it is clear from Table 1, the fermions $\psi^{\pm}_{(0)}$ are hermitean and they now play a role quite analogous to that of the superghost-related fermions $\Psi^{\pm}_{(34)}$. The number operator $N_{(0)}$ is anti-hermitean, rather than hermitean, and therefore, if the bosonization formulae for the 32 complex fermions beside $\psi^{\pm}_{(0)}$ are to retain the correct hermiticity properties, we must arrange for their cocycle operators to depend neither on $J_0^{(34)}$ nor on the number operator corresponding to $\psi^{\pm}_{(0)}$. To ensure this, in the Minkowski case we always choose an ordering for the fermions such that $\psi^{\pm}_{(0)}$ is given the number $L = 33$, retaining $L = 34$ for $\psi^{\pm}_{(11)}$. A natural choice for such an ordering is $(\bar{1}, \ldots, \bar{22}; 2, \ldots, 10; 1, 0; 11)$.
As long as we do not consider hermitian conjugation, the only change as compared to the Euclidean case is a reshuffle in the ordering of the fermions. In particular it follows that the consistency conditions discussed in ref. [7]—which constrain the choices of the 561 independent signs in the $Y$ matrix—are exactly the same in the Euclidean and Minkowski case. 3

The last question that remains to be addressed is whether the bosonization formula is also consistent with hermitian conjugation in the cases $L = 33$ and $L = 34$. Since $\Psi^\pm(K)$ and $e^{\pm \Phi(K)}$ are hermitian for $K = 33, 34$, we would like $C_{(33)}$ and $C_{(34)}$ to be hermitian too. A priori this does not seem to be the case, since

$$ (C_{(K)})^\dagger = C_{(K)} \exp \left\{ -2\pi i \sum_{L=1}^{32} Y_{KL}^M \sigma_0^{(L)} \right\} \equiv C_{(K)} F(K) \quad \text{for} \quad K = 33, 34 \quad (8) $$

However, when acting on any string state in the theory, the factor $F(K)$ appearing in eq. (8) is effectively equal to one. It is sufficient to check this on the ground states since any raising operator has $J_0^{(L)} = \text{integer}$. A generic ground state is created by a vertex operator having the form

$$ V_h(z, \bar{z}) = S_h(z, \bar{z}) e^{i k \cdot X(z, \bar{z})} , \quad (9) $$

where $S_h \equiv \prod_L S_{hL}^{(L)}$ and $S_{hL}^{(L)} = \exp \{ A_L \Phi(L) \} (C_{(L)})^{hL}$. Acting on such a ground state, $F(K)$ assumes the eigenvalue

$$ F(K) = \exp \left\{ -2\pi i \sum_{L=1}^{32} Y_{KL}^M A_L \right\} = \exp \left\{ -2\pi i \left( \varphi_K[A] - \tilde{Y}_{K,33}^M A_{33} - \tilde{Y}_{K,34}^M A_{34} \right) \right\} \quad \text{for} \quad K = 33, 34 \quad (10) $$

where

$$ \varphi_K[A] \equiv \sum_{L=1}^{34} \tilde{Y}_{KL}^M A_L \mod 2 \quad (11) $$

and the matrix $\tilde{Y}_{KL}^M$ is defined by $\tilde{Y}_{KL}^M = Y_{KL}^M$ for $K > L$ and $\tilde{Y}_{KL}^M = -Y_{LK}^M$ for $K < L$, while for $L = K$ we take $\tilde{Y}_{LL}^M = \pm \epsilon$, choosing $+\epsilon$ if $L$ corresponds to one of the fermions $\psi_l$, $l = 0, 1, \ldots, 10$, and $-\epsilon$ otherwise. Here $\epsilon = \pm 1$ keeps track of the phase encountered in the OPE $S_{hL}^{(L)}(z, \bar{z}) S_{BL}^{(L)}(w, \bar{w})$ when writing $(z - w) = e^{i \epsilon \pi} (w - z)$. The quantity $\varphi_K[A]$ keeps track of the statistics of the operator $S_h$, as encoded in the formula

$$ S_{BL}^{(L)}(z, \bar{z}) S_h(w, \bar{w}) = S_h(w, \bar{w}) S_{BL}^{(L)}(z, \bar{z}) e^{i \pi B_L \varphi_L[A]} . \quad (12) $$

3 The two matrices $Y^E$ and $Y^M$ are indeed related just by an interchange of rows and columns in the matrix $\tilde{Y}$ introduced in ref. [7] (see also below).
As was shown in ref. [7], one of the consistency conditions on $Y^M$—following from the requirement that the picture changing operator should always satisfy Bose statistics—is that

$$\varphi_{34}[\hat{A}] \mod 2 = \varphi_{33}[\hat{A}] = \text{integer} \quad (13)$$

for all vertex operators of the type (9) existing in the theory. Furthermore, world-sheet supersymmetry implies that either $A_{33}$ and $A_{34}$ are both integer (if the sector describes space-time bosons) or both half-integer (if the sector describes space-time fermions). It thus follows that the factor $F_{(K)}$ in eqs. (8), (10) is effectively equal to one.

Notice that this argument, ensuring well-defined hermiticity properties of the bosonized expression for $\Psi_{(34)}^\pm$, does not work in the Euclidean case. There all 33 fermion number operators appearing in $C_{(34)}$ are hermitean, and therefore one finds instead of (8)

$$ (C_{(34)})^\dagger = C_{(34)} \exp \left\{ -2\pi i \sum_{L=1}^{33} Y_{34,L}^E J_0^L \right\} \quad (\text{Euclidean case}) \quad (14) $$

By repeating the argument above we see that $C_{(34)}$ is actually hermitean between states with integer value of $A_{34}$ but anti-hermitean between states with half-integer values of $A_{34}$. From this point of view, bosonization seems more well-defined in the Minkowski case.

5. We have presented a prescription for bosonizing the free world-sheet fermions of a string theory embedded in Minkowski space-time.

To summarize, we have seen that the bosonization in Minkowski space-time differs from the one in Euclidean space-time in two ways. First, the world-sheet fermions $\psi_{(0)}^\pm$, which are related to the time direction in space-time, are assigned the label $L = 33$, whereas in the Euclidean case they could be assigned any value $L \in \{1, \ldots, 33\}$. This reordering of the fermions implies that the explicit representation of the gamma matrices, as well as other group theoretical objects, could be different in the Minkowski and Euclidean formulation. But as long as all quantities satisfy the correct group theoretical properties, as ensured by the cocycle consistency conditions [7], the final result in the computation of an amplitude should be independent of what representations we happen to be using.

More important, the hermiticity properties of the fields $\psi_{(0)}^\pm$ are different in the Euclidean and Minkowski formulations. This means that the map between $|\text{“in”}\rangle$ and $\langle \text{“out”}|\text{states}$ is different in the two cases. This should not come as a surprise, since it is well known that whereas the spinor representation is unitary in the Euclidean case it is not in the Minkowski case.
The bosonization prescription allows us to compute string scattering amplitudes without the need to make a rotation to Euclidean space-time. In string models based on free fields, such as the KLT models, all correlation functions involved in the computation can be evaluated explicitly. As a simple example, and a check of our prescription, we have considered the one-loop 3-point amplitude that was computed in Euclidean space-time in ref. [7] and we have redone this computation entirely in Minkowski space-time. As expected, the result agrees with what we obtain by just Wick rotating the Euclidean result, although the explicit form of the gamma matrices, the mass matrix and the generalized charge conjugation matrix turns out to be different in the two cases, as does the precise relation between the vertex operators describing incoming and outgoing “electrons”.

References

[1] M.B. Green, J.H. Schwarz and E. Witten, *Superstring Theory*, Cambridge University Press, 1987.
[2] E. D’Hoker and D.H. Phong, “The box graph in superstring theory”, preprint Columbia/UCLA/94/TEP/39, [hep-th/9410152](https://arxiv.org/abs/hep-th/9410152).
[3] H. Sonoda, Nucl.Phys. **B326** (1989) 135.
[4] H. Kawai, D.C. Lewellen and S.-H.H. Tye, Nucl.Phys. **B288** (1987) 1.
[5] P. Di Vecchia, M.L. Frau, K. Hornfeck, A. Lerda, F. Pezzella and S. Sciuto, Nucl.Phys. **B322** (1989) 317.
[6] V.A. Kostelecky, O. Lechtenfeld, W. Lerche, S. Samuel and S. Watamura, Nucl.Phys. **B288** (1987) 173.
[7] A. Pasquinucci and K. Roland, “On the computation of one-loop amplitudes with external fermions in 4d heterotic superstrings”, preprint NBI-HE-94-47, [hep-th/9411015](https://arxiv.org/abs/hep-th/9411015). Nuclear Physics **B** in press.