Thermal Equilibrium Solutions of Black Hole Accretion Flows: Outflows versus Advection

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Abstract

Observations and numerical simulations have shown that outflows generally exist in the accretion process. We revisit the thermal equilibrium solutions of black hole accretion flows by including the role of outflows. Our study focuses on the comparison of the cooling rate of outflows with that of advection. Our results show that, except for the inner region, outflows can dominate over advection in a wide range of flows, which is in good agreement with previous numerical simulations. We argue that an advection-dominated inner region together with an outflow-dominated outer region should be a general radial distribution for both super-Eddington accretion flows and optically thin flows with low accretion rates.

Unified Astronomy Thesaurus concepts: Accretion (14); High energy astrophysics (739); Black holes (162); Hydrodynamics (1963)

1. Introduction

Three well-known models have been widely investigated in black hole accretion systems, namely, the standard thin disk (Shakura & Sunyaev 1973; hereafter SSD), the slim disk (Abramowicz et al. 1988), and the optically thin advection-dominated accretion flow (Narayan & Yi 1994, 1995a; hereafter ADAF). The SSD is optically thick, geometrically thin, and radiative-cooling-dominated. This model has been proposed to be the central engine of luminous active galactic nuclei (AGNs) and black hole X-ray binaries (BHXBs) in the high/soft state (e.g., Kato et al. 2008). For the slim disk, it has a high mass accretion rate (super-Eddington); thus the radiation diffusion timescale is larger than the viscous timescale. This disk is optically thick and geometrically thin, which may work for ultraluminous X-ray sources, bright microquasars, and narrow-line Seyfert 1 galaxies (e.g., Wang & Zhou 1999; Mineshige et al. 2000; Done et al. 2007; Ku et al. 2017; Kosec et al. 2018). The ADAF is radiatively inefficient, where the viscous heating is mainly balanced by the advective cooling. The flow is geometrically thick and its gas temperature is close to the virial temperature. This model has been applied to describe sources such as the supermassive black hole in our Galactic center, Sagittarius A* (Sgr A*), low-luminosity AGNs, and BHXBs in the low/hard and quiescent states (e.g., Narayan & McClintock 2008; Yuan & Narayan 2014). The above models assume that the accretion rate remains constant during the accretion process.

Recent observations showed that outflows exist in the geometrically thin disks (e.g., King & Pounds 2015; Diaz Trigo & Boirin 2016; Homan et al. 2016), the super-Eddington accretion disks (e.g., Gladstone et al. 2009; Middleton et al. 2011; Du et al. 2015), and the radiatively inefficient accretion flows (e.g., Wang et al. 2013; Cheung et al. 2016; Homan et al. 2016; Ma et al. 2019; Muñoz-Darias et al. 2019). This means that the accretion rate is no longer a constant. Apart from observations, outflows were also found in the hydrodynamical and magnetohydrodynamical numerical simulations of the geometrically thin disks (e.g., Ohsuga & Mineshige 2011; Nomura et al. 2016, 2020), the super-Eddington accretion disks (e.g., Ohsuga et al. 2005; Ohsuga & Mineshige 2011; Jiang et al. 2014; Sadowski et al. 2014; Sadowski & Narayan 2015; Kitaki et al. 2017, 2018; Zahra Zeraatgari et al. 2020), and the radiatively inefficient accretion flows (e.g., Stone et al. 1999; Narayan et al. 2012; Yuan et al. 2012a, 2012b, 2015).

Outflows as a signature of the super-Eddington accretion were discussed by Shakura & Sunyaev (1973). In this pioneering paper, they supposed that outflows are inevitable when the luminosity of the disk exceeds the Eddington limit, since the radiation force is greater than the gravity. More importantly, Piran (1978) investigated the stability of accretion disks and proposed that wind escaping from the disk surface can have a stabilizing effect. Later, Abramowicz (1981) showed that the innermost parts of accretion disks are thermally and secularly stable owing to the general relativistic effect, where the physical picture is analogous to the case of Roche-lobe overflow in close binaries. On the other hand, Narayan & Yi (1994, 1995a) argued that outflows are likely to occur because the Bernoulli parameter is positive in some regions. Blandford & Begelman (1999) constructed the adiabatic inflow–outflow solutions to describe outflows. They assumed that the accretion rate is a function of radius $M \propto r^p$, where $p$ is in the range [0, 1]. Xie & Yuan (2008) also used a global method to show the influence of the outflows on the disk structure by this relationship. Further numerical simulations showed the power-law index $p$ in a range of [0.5, 1] (e.g., Stone et al. 1999; Ohsuga et al. 2005; Narayan et al. 2012; Yuan et al. 2012b; Bu & Gan 2018). In the case of Sgr A*, however, a relatively low value for the index $p$ is preferred, such as $p = 0.25$ (Quataert & Narayan 1999), 0.27 (Yuan et al. 2003), and 0.37 (Ma et al. 2019). Dotan & Shaviv (2011) proposed a model for super-Eddington accretion flows so that the disk remains slim and a significant wind is accelerated. Some works also demonstrated that the super-Eddington accretion (Gu & Lu 2007; Cao & Gu 2015; Gu 2015; Feng et al. 2019) and the optically thin ADAF (Gu 2015) ought to have outflows.
In this work, we revisit the thermal equilibrium solutions of black hole accretion flows by including the role of outflows. Our study focuses on the comparison of the cooling rate of outflows with that of advection. The paper is organized as follows. The basic equations for our model are described in Section 2. Numerical results and analyses are shown in Section 3. Conclusions and discussion are presented in Section 4.

2. Basic Equations

In this section, we describe the basic equations of our model. We consider a steady-state axisymmetric accretion flow, and use the pseudo-Newtonian potential $\Phi = -GM_{BH}/(R-R_g)$, where $M_{BH}$ is the mass of the black hole and $R_g$ is the Schwarzschild radius. The vertical scale height of the flow is $H = c_s/\Omega_K$, where $\Omega_K$ is Keplerian angular velocity, and $c_s = (P/\rho)^{1/2}$ is the isothermal sound speed, with $P$ and $\rho$ being the pressure and mass density, respectively. The kinematic viscosity coefficient is expressed as $v = \alpha c_s H$, where $\alpha$ is the constant viscosity parameter.

The basic equations describing the flow contain the continuity, radial momentum, azimuthal momentum, and energy equations. The continuity equation is

$$\frac{1}{R} \frac{d}{dR} \left( R \Sigma V_R \right) + \frac{1}{2\pi R} \frac{dM_{\text{w}}}{dR} = 0,$$

where $\Sigma$ is the surface density defined as $\Sigma \equiv 2\rho H$, and $V_R$ is the radial velocity, which is defined to be negative when the flow is inward. The outflow mass-loss rate $M_{\text{w}}$ is \citep{k1999}:

$$M_{\text{w}}(R) = \int_{R_{\text{in}}}^{R} 4\pi R' \dot{m}_{\text{w}}(R') dR',$$

where $R_{\text{in}}$ denotes the radius at the inner edge of the disk and $\dot{m}_{\text{w}}$ is mass-loss rate per unit area from each disk face.

Due to the influence of outflows, we assume that the accretion rate $\dot{M}$ varies with radius as follows \citep{blandford1979}:

$$\dot{M} = -2\pi R \Sigma V_R = M_{\text{outer}} \left( \frac{R}{R_{\text{outer}}} \right)^\alpha,$$

where $M_{\text{outer}}$ is the mass accretion rate at the outer boundary $R_{\text{outer}}$.

Some numerical simulations showed that outflows in super-Eddington accretion cases and ADAFs are stronger than that in SSDs \citep[e.g.,][]{ohsuga2011, ohsuga2014}, which can be physically understood as follows. The effective cooling of radiation in SSDs leads to a low temperature of the disk, i.e., a negative Bernoulli parameter of the flow. Thus, only relatively weak outflows may be produced by SSDs. On the contrary, for super-Eddington accretion cases, even though the temperature of the disk is only slightly higher than that of SSDs, a large amount of photons trapped in the disk result in high radiation pressure, which can contribute to strong outflows. In addition, for ADAFs, the extremely high temperature of the disk due to energy advection causes a positive Bernoulli parameter \citep[e.g.,][]{narayan1994, narayan1997}, which can also contribute to strong outflows. In summary, energy advection is helpful to produce strong outflows, no matter whether the physics of advection is related to photons or gas. It is known that the dimensionless thickness $H/R$ of the disk describes the strength of advection well, i.e., $H/R \ll 1$ for SSDs and $H/R \gtrsim 1$ for slim disks and ADAFs. We therefore assume that the power-law index $p$ is proportional to $H/R$ of the disk, i.e., $p = \lambda (H/R)$, where $\lambda$ is a constant. In our opinion, such an assumption is more appropriate than a fixed value of $p$ for different accretion models.

Using Equations (1)–(3), we obtain this relation,

$$\dot{m}_{\text{w}} = \frac{M_{\text{w}}}{4\pi R^2}.$$

The integrated radial momentum equation and the azimuthal equation of motions can be respectively written as

$$V_R \frac{dV_R}{dR} + \left( \Omega_K^2 - \Omega^2 \right) R + \frac{1}{\rho} \frac{dP}{dR} = 0,$$

$$-\frac{1}{R} \frac{d}{dR} \left[ R^2 \Sigma V_R \Omega \right] + \frac{1}{R} \frac{d}{dR} \left[ R^2 \Sigma \frac{d\Omega}{dR} \right] - \frac{(IR)^2 \Omega}{2\pi R} \frac{dM_{\text{w}}}{dR} = 0,$$

where the last term on the left-hand side of Equation (6) represents angular momentum carried by the outflowing materials. Here, $l = 0$ corresponds to a nonrotating outflow, and $l = 1$ corresponds to the outflowing materials carrying away the specific angular momentum at the point of ejection. The cases with $l > 1$ correspond to centrifugally driven magnetic disk winds that extract more angular momentum from the disk \citep{k1999}.

The pressure $P$ is the sum of gas and radiation pressure:

$$P = \frac{\mu \kappa}{\mu m_p} (T_i + T_e) + \frac{Q_{\text{rad}}}{4\epsilon} \left( \tau + \frac{2}{\sqrt{3}} \right),$$

where $T_i$ and $T_e$ are the ion temperature and the electron temperature, respectively, and $T_e = \min\left(T_e, 6 \times 10^9\text{ K}\right)$; $\mu = 0.617$ is the mean molecular weight, and $\tau = (\kappa_{\text{es}} + \kappa_{\text{abs}}) \rho H$ is the total optical depth, where $\kappa_{\text{es}} = 0.34\text{ cm}^2\text{ g}^{-1}$ and $\kappa_{\text{abs}} = 0.27 \times 10^{25} \rho_e^{1.5}\text{ cm}^{-3}\text{ g}^{-1}$ \citep[e.g.,][]{abramowicz1996}.

The energy equation is written as

$$Q_{\text{vis}} = Q_{\text{adv}} + Q_{\text{rad}} + Q_{\text{w}},$$

where $Q_{\text{vis}}$, $Q_{\text{adv}}$, and $Q_{\text{rad}}$ are the viscous heating rate, the advective cooling rate, and the radiative cooling rate, respectively. Their expressions are as follows,

$$Q_{\text{vis}} = \nu \Sigma \left( R \frac{d\Omega}{dR} \right)^2,$$

$$Q_{\text{adv}} = \Sigma V_R T_s \frac{dS}{dR} = \Sigma V_R \left( \frac{1}{\gamma - 1} \frac{dc_s}{dR} - \frac{c_s^2}{\rho} \frac{dP}{dR} \right),$$

$$Q_{\text{rad}} = 8\sigma T_e^4 \left( \frac{3\pi}{2} + \sqrt{3} + \frac{8\pi T_e^4}{Q_{\text{br}}} \right)^{-1}.$$

Equation (11) is valid in both optically thin and optically thick regimes \citep{narayan1995b}. The bremsstrahlung cooling is given by \citep[e.g.,][]{abramowicz1995}:

$$Q_{\text{br}} = 1.24 \times 10^{33} H \rho^2 T_e^4 \text{ erg s}^{-1}\text{ cm}^{-2}.$$
The quantity $Q_w$ in Equation (8) represents the energy taken away by outflows, which is expressed as

$$Q_w = 2 f_{\text{in}} \dot{m} V_K^2,$$  

where a factor of 2 represents the outflow energy that is emitted from both sides of the accretion disk; $\eta$ is an outflow energy parameter and $V_K$ is the Keplerian velocity. By using Equation (1) and integrating Equation (6), we have

$$\nu \Sigma = \frac{M_{fg}^{-1}}{3\pi} \left( 1 - \frac{f^2 p}{p + \frac{1}{2}} \right),$$

where $g = -(2/3)(d \ln \Omega_K / d \ln R)$ and the factor $f = 1 - (\Omega / \Omega(R))(3R_0/R)^p + 2$. For $p = 0$, Equation (14) returns to Equation (2.1) of Chen et al. (1995).

Following some previous works (e.g., Narayan & Yi 1994; Gu & Lu 2000), we adopt the self-similar assumptions and set $\gamma = 1.5$. Then Equations (9)–(11) are reduced to the following algebraic forms:

$$\frac{1}{2} V^2 + \frac{5}{2} \bar{c}^2_s + (\Omega^2 - \Omega_K^2)R^2 = 0,$$  

$$Q_{\text{vis}} = \frac{3M \Omega^2 f g}{4\pi} \left( 1 - \frac{f^2 p}{p + \frac{1}{2}} \right),$$  

$$Q_{\text{adv}} = \frac{1}{4\pi} \frac{M_{fg}^{-2}}{R^2}.$$  

Finally, substituting Equation (4) into Equation (13), we have

$$Q_w = \frac{f_{\text{in}} f \dot{m} M_{fg}^{-2}}{2\pi}.$$  

By solving the five equations, Equations (3), (7)–(8), and (14)–(15), for the five variables $\rho$, $g$, $c_s$, $\Omega$, and $V_K$ with given parameters $M_{\text{BH}}$, $\alpha$, $M$, and $l$, we obtain the thermal equilibrium solutions of accretion flows. In the following calculations, we fix $M_{\text{BH}} = 10 M_\odot$, $\alpha = 0.1$, and $l = 1$.

3. Numerical Results

In this section, we describe the numerical results of the accretion flows with outflows. Figure 1 shows thermal equilibrium solutions at $R = 10 R_g$ in the $\log \dot{m} - \log \Sigma$ plane, where $\dot{m}$ is the accretion rate normalized by the Eddington accretion rate $M_{\text{Edd}} = 64 \pi G M_{\text{BH}} / c^2 c_{\text{es}}$. The black line represents the solutions under the no-outflow assumptions (e.g., Abramowicz et al. 1995; Chen et al. 1995; Takeuchi & Mineshige 1998; Gu & Lu 2000). The curve on the left is composed of two branches, of which the upper one is for ADAFs and the lower one is for Shapiro–Lightman–Eardley disks (Shapiro et al. 1976). The right S-shaped curve is composed of three branches, of which the upper one is for slim disks, the middle one for radiation-pressure-supported SSDs, and the lower one for gas-pressure-supported SSDs. The red, blue, and green lines represent the solutions with $(\lambda = 0.5, \eta = 1)$, $(\lambda = 0.5, \eta = 2)$, and $(\lambda = 1, \eta = 1)$, respectively. It is seen from the figure that the maximal accretion rate of the left curve decreases with increasing cooling effects of outflows. However, the change of the right S-shaped curve is quite slight. The obtained values of $p$ of ADAFs and slim branches are $p \sim 0.23$ for $\lambda = 0.5$ and $p \sim 0.39$ for $\lambda = 1$. For comparison, we have $p \sim 0.003$ for gas-pressure-supported SSDs. For the ADAF branch, the values of $p$ are in good consistency with the fitting results of the Sgr A* observations under the radiatively inefficient accretion model (e.g., Quataert & Narayan 1999; Yuan et al. 2003; Ma et al. 2019).

In order to quantitatively understand the influence of outflows at different radii, we fix $\dot{m} = 100$, $\eta = 1$, $\lambda = 0.5$, and $R_{\text{outer}} = 10^4 R_g$. We then obtain variations of $f_{\text{adv}}(Q_{\text{adv}} / Q_{\text{vis}})$, $f_{\text{rad}}(Q_{\text{rad}} / Q_{\text{vis}})$, and $f_{\text{in}}(Q_{\text{w}} / Q_{\text{vis}})$ with radius. It is seen from Figure 2 that in the inner regions cooling is dominated by advection ($f_{\text{adv}}$, the red line), and in the middle regions cooling is dominated by outflows ($f_{\text{in}}$, the black line). Since the gravitational force in the inner regions is greater than the radiation force, and the radial velocity is large (i.e., the viscous timescale is short), only a weak outflow can form. In the middle regions (denoted by the green dashed line), however, the situation is reversed as outflow dominance.
Such an outflow-dominated region varies significantly with varying accretion rates, which is presented in Figure 3.

A main purpose of this work is to compare the cooling effects of outflows with that of advection. Figure 3 is a description of thermal equilibrium solutions of accretion flows with outflows in the $\dot{m}$–$R$ plane. For a comparison, Figure 3(a) is under the no-outflow assumptions. The $\dot{m}$–$R$ plane is divided into three regions by two curves. The region above the upper curves ($\dot{f}_{\text{adv}} = 1/2$) represents advective-cooling-dominated solutions ($\dot{f}_{\text{adv}} > \dot{f}_{\text{rad}}$) owing to the photon trapping, which corresponds to the super-Eddington accretion cases. The middle region represents solutions with cooling dominated by radiation ($\dot{f}_{\text{rad}} > \dot{f}_{\text{adv}}$). The region below the lower curve also represents advective-cooling-dominated solutions ($\dot{f}_{\text{adv}} > \dot{f}_{\text{rad}}$), but the physics of advection is related to the internal energy of accreted gas, which corresponds to the ADAF cases. Here, the maximal critical mass accretion rate $\dot{m}_{\text{crit}}$ of ADAF is assumed to be around $0.01 M_{\text{Edd}} (\dot{m}_{\text{crit}} \sim \alpha^2 M_{\text{Edd}})$ for the inner part with $R \lesssim R_{\text{in}}$, where $R_{\text{in}}$ is around $10^{-2} \sim 10^{-3} R_g$ for $\alpha = 0.1$. For $R > R_{\text{in}}$, $\dot{m}_{\text{crit}}$ decreases with increasing $R$ (see, e.g., Figure 8 of Narayan et al. 1998).

Similar to Figure 2, Figure 3(b) is also for $\eta = 1$ and $\lambda = 0.5$, which shows that there exists a boundary radius $R_0$ (red lines) that separates an inner “advection” region and an outer “outflow” region. By comparing Figures 3(a) and (b), it is seen that for $R > R_0$, the original region with cooling dominated by advection is replaced by a region with cooling dominated by outflows. The green dashed line denotes the outflow-dominated region of the example solution with $\dot{m} = 100$ in Figure 2. It is seen from Figure 3(b) that the outflow-dominated region increases with increasing accretion rates for the super-Eddington accretion cases. In addition, for the optically thin cases, Figure 3(b) shows that a relatively wide outflow-dominated region exists for ADAFs, and this region increases with decreasing accretion rates. Thus, we argue that an advection-dominated inner region together with an outflow-dominated outer region should be a general radial distribution for both super-Eddington accretion flows and optically thin flows with low accretion rates.

Figure 4 shows the variation of $R_0$ with the parameter $\eta$. The red solid (dashed) line represents solutions with $\dot{m} = 0.001$ and $\lambda = 1$ ($\lambda = 0.5$), where $p$ is less than 0.4 (0.25). The blue solid (dashed) line represents solutions with $\dot{m} = 100$ and $\lambda = 1$ ($\lambda = 0.5$), where $p$ is less than 0.33 (0.22). Since the temperature of ADAFs is close to the virial value, the outflows are stronger and therefore the boundary $R_0$ has a larger span in the radial direction. In addition, the figure shows that, for a certain $\lambda$, when $\eta \gtrsim 1$, the boundary $R_0$ is located between the inner stable circular orbit $(3 R_g)$ and $\sim 10 R_g$. The physical reason is that the gravitational force in the inner region is very strong and therefore the outflows are restrained. In other words, outflows from inside $\sim 10 R_g$ are quite weak, which is consistent with previous numerical simulations (e.g., Yuan et al. 2012a).

4. Conclusions and Discussion

In this work, we have revisited the thermal equilibrium solutions of black hole accretion flows by including the role of outflows. By comparing the cooling rate of outflows with that of advection, we have found that advection is important only in the inner regions and outflows play a key role in balancing the viscous heating in the outer regions (Figure 3). We argue that an advection-dominated inner region together with an
outflow-dominated outer region should be a general radial distribution for both super-Eddington accretion flows and optically thin flows with low accretion rates. In addition, we have also obtained the boundary $R_b$ as a function of $n$ and $\eta$. Our results are physically well understood, and agree well with observations and numerical simulations.

The present work is based on $\alpha = 0.1$ and $l = 1$. However, there exists a critical viscosity parameter $\alpha_{\text{crit}}$ for the structure of thermal equilibrium solutions (Chen et al. 1995). For $\alpha > \alpha_{\text{crit}}$, a new topological type of equilibria appears where the ADAF branch can smoothly connect to the slim disk branch. Similarly, critical viscosity parameters should also exist if the effects of outflows are taken into account. Thus, a new topological type of equilibria may also exist for large values of $\alpha$. In addition, the present study is based on $l = 1$. In fact, outflows can extract more angular momentum from the disk, which corresponds to $l > 1$, such as the centrifugally driven magnetohydrodynamic winds (Blandford & Payne 1982). This means more energy carried away by the outflows, which may have significant effects on the disk structure.

A fundamental difference between neutron stars and black holes is that the former has a hard surface whereas the latter has an event horizon. For the ADAF case, most of the energy due to the viscous heating process in the inner region of a disk has an essential contribution to the total radiation. According to our results, since outflows in the inner region are quite weak, in our opinion, the effects of outflows on this issue may not be significant.

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![Figure 4. Variations of the boundary radius $R_b$ with $\eta$ for a typical ADAF with $n = 0.001$ and a typical slim disk with $n = 100$. The solid and dashed curves correspond to the cases with $\lambda = 1$ and $\lambda = 0.5$, respectively.](https://example.com/fig4.png)
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