The effects of laminar separation on heat transfer in flow past an elliptic cylinder

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Abstract. This article presents the numerical studies on laminar forced convective heat transfer from two-dimensional elliptic cylinders of various axis ratios ($AR$). The local Nusselt number contribution on the two parts of elliptical cylinder surface that split up at the separation position is separately studied in detail. The effects of $AR$ and $Re$ on the average Nusselt number above the surface before the separation position and one of behind of the point are separately detailed analysed. The average Nusselt number described with the two items is in good agreement with existing experimental and numerical data.

1. Introduction

Forced convection heat transfer around a bluff body is of great interest because of its wide range of engineering applications and fundamental significance in nature. In particular, heat transfer around circular cylinder is has been a lot of investigations [1,2,3]. Elliptic is an inherited shape among bluff bodies and derives various structures between circle and flat plate with axis ratios. Unlike laminar flow and heat transfer around circular cylinder and finite-length flat plate, literatures on forced convective heat transfer from an elliptic cylinder very limited.

Many authors have studied heat transfer from an elliptic cylinder theoretically and numerically. The data on heat transfer around elliptical cylinder in laminar region focused on elliptical cylinder with one axis ratios 0.2 [4], 0.1, 0.5 [5], and 1:3 [6]. W. A. Khan et al. [7] employed an integral method of boundary-layer analysis to calculation of total drag and average heat transfer for flow across an elliptical cylinder under isothermal and iso-flux thermal boundary conditions. The results are obtained for the whole laminar range of Reynolds numbers on the assumption that there is no appreciable increase in the local heat transfer after the separation point. Immanuel Paul et al. [8] presented the numerical study of laminar forced convective heat transfer from elliptic cylinders of various axis ratios ($AR$=0.1, 0.4, 0.6, 0.8, and 1.0) and Reynolds numbers ($Re$=50, 100, 150, and 200).

These studies worked mainly out local heat transfer features on an elliptical and overall Nusselt number on an elliptical cylinder of various axis ratios in the laminar flow. A few literatures clarified the effects of the wakes behind the elliptical cylinder. There were no studies reported on the heat transfer characteristics on an elliptical behind the boundary layer separating. In this work, the average Nusselt number above the surface in front of the separation position ($\tilde{Nu}_b$) and one of behind the point ($\tilde{Nu}_b$) are calculated numerically with the wake patterns including the creeping flow, the symmetrical vortexes, and vortex shedding in the laminar flow past an elliptic cylinder. An attempt is made to describe the
average Nusselt number \( \overline{Nu_{a||}} \) on the elliptical cylinder with \( \overline{Nu_b} \) and \( \overline{Nu_t} \). The results have been compared with the previous studies wherever possible.

2. Simulation Methodology

The present simulation considers incompressible flow of fluid with a uniform velocity \( V^\infty \) and temperature \( T^\infty \), past an elliptical cylinder of the aspect ratio, \( AR = \frac{b}{a} \) oriented with its long axis along the flow. The surface of an elliptical cylinder is maintained at a constant temperature \( T_w \). The governing equations, schematic of computational domain and boundary conditions are detailly described in literature [9].

According to our results, the effects on the wall Nusselt number are negligible when an annular computational domain having an outer diameter of 50D (D is the sum of a and b) is chosen to simulate the unbounded flow past the cylinders. A grid system with 360 nodes in the circumferential direction and 360 nodes in the radial direction is acceptable from study on mesh independence.

Numerical simulations are worked out employing the laminar model at \( Re=0\sim20000 \) and \( AR=0.2, 0.4, 0.6, 0.8 \) and 1. All results reported here are for simulations using 288,000 \((360 \times 360)\) cells in a computational domain with outer boundary having a radius of 50D.

![Graphs showing Nusselt number distribution](image)

**Figure 1.** The local Nusselt number distribution on elliptical cylinder at \( AR=0.8 \): (a) \( Re=6 \) without separating, (b) \( Re=50 \) with two symmetrical eddies, (c) \( Re=8200 \) with vortex shedding.
3. Local Nusselt number distribution on the elliptical cylinder

The local Nusselt number distributions on cylinder with AR=0.8 at three flow states are represented in Figure 1. It is very clear in Figure 1 (a) that the Nusselt number on the elliptical cylinder decreases along the surface at Re=6 when no separate flow. The boundary layer becomes more and more thinker along the cylinder surface from the leading point to end station. Conduction is the main way of heat transfer in the laminar boundary layer. The value of Nu is smaller where the boundary layer is thinker.

The case with a pair of asymmetrical vortexes at Re=50 is shown in Figure 1 (b). The velocity at the separate point closed to cylinder is zero. The local Nu reaches the minimum value close to and behind of the separate point, then rises along the surface of the elliptical cylinder because of the symmetrical steady vortexes. Figure 1 (c) reveals the local Nu distribution at Re=8200 at the arbitrary time. The parameters of two sides of the cylinder are unsymmetrical. The either side local Nu before the separate point has still the same trend to the other flow patterns. While, the Nu on the surface behind of then separation is out of order because of the big and small vortexes.

The trend of the Nu distribution before separate point at the three wake states in Figure 1 while the other parts different. The two parts on the elliptical cylinder could be respectively expressed.

4. Overall Nusselt number on the elliptical cylinder

The local Nu on the various ARs has the same trend before separating. The overall average Nu \( \overline{\text{Nu}_{\text{all}}} \) consists of the average Nu before the separate point \( \overline{\text{Nu}_b} \) and one behind of the point \( \overline{\text{Nu}_h} \), handed as equation (1).

\[
\overline{\text{Nu}_{\text{all}}} = \overline{\text{Nu}_b} + \overline{\text{Nu}_h}
\]  

(1)

The results of numerical simulations for Re=0~20000 and AR=0.2, 0.4, 0.6, 0.8 and 1 are presented in Figure 2. The fluid flows along the surface from the forestage point to the bake before separating. The similar velocity distributions in the boundary layer for elliptical cylinders at various Re lead to the near average Nu in Figure 2 (a). The linear relationship is obvious in the present coordinate system. On the other land, the average Nu curses behind of the separate point \( \overline{\text{Nu}_h} \) differ considerably from these ARs in Figure 2 (b) because that the streamlines and boundary layers have great deference. All the results about \( \overline{\text{Nu}_b} \) and \( \overline{\text{Nu}_h} \) are respectively proposed to obtain functional form in equations (2) and (3).

\[
\overline{\text{Nu}_b} = 0.60969 \text{Re}^{0.47356} \text{Pr}^{1/3}
\]  

(2)
These expressions are substituted equation (1), overall average Nu formula of the elliptical cylinder in the laminar flow region is determined in equation (4). It is clear from the equation that the relationships of Nu and the parameter Nu, AR, Re in the two parts. The average Nu are a simple function expression about only Re and Pr except for AR in the leading part. In the other part, the streamlines and the velocity the boundary layer near the surface of the elliptical cylinder change greatly with Re, so the average Nu relational expression depending on Re, Pr and AR is more complex.

\[
\overline{Nu_{h}} = (0.01002AR^{2.7861})Re^{0.41573AR^{2.7861}}Pr^{0.17536}-
\]

\[
(0.00201AR^{2.7861})Re^{(-0.08734AR+1.9138)}Pr^{1/3}
\]  

\[
(3)
\]

The results from the equation are compared with the experimental results of Qi Shou-liang et al. [10], Terukazu et al. [6], and A. A. Zukauskas et al. [11], besides the numerical results of W. A. Khan et al. [7] in the Figure 3. The present results are in good agreement with the experimental and numerical results in the laminar region. The expression method is also adopted to the case of the middle and high Re in flow across an elliptical cylinder.

![Figure 3. Comparison of the average Nu with the literature values for flow around elliptical cylinder: (a) the experimental results, (b) the numerical results.](image)

5. Conclusion
The separate flow effects the heat transfer on the elliptical cylinder. The local Nu distributions of surface divided by separate point are different. The equations for the average Nu are respectively available. The overall average Nu of flow across an elliptical cylinder is expressed as addition the average Nu before the separate point and one behind of the point. The results are in good agreement with existing experimental and numerical data.

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