Criterion for local distinguishability of arbitrary orthogonal states

Ping-Xing Chen\textsuperscript{1,2,\ast} and Cheng-Zu Li\textsuperscript{1}

1. Department of Applied Physics, National University of Defense Technology, Changsha, 410073, P. R. China.

2. Laboratory of Quantum Communication and Quantum Computation, University of Science and Technology of China, Hefei, 230026, P. R. China

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Abstract

In this paper we present a necessary condition of distinguishability of orthogonal multi-partite quantum states. With this condition one can discuss some especial cases for distinguishability further. We also present a necessary condition of distinguishability of bipartite quantum states which is simple and general. With this condition one can get many cases of indistinguishability. The conclusions may be useful in understanding the essence of nonlocality and calculating the distillable entanglement and the bound of distillable entanglement.

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\*E-mail: pxchen@nudt.edu.cn
One of the interesting features of non-locality in quantum mechanics is that a set of orthogonal quantum states cannot be distinguished if only a single copy of these states is provided and only local operations and classical communication (LOCC) are allowed, in general. Taking the bipartite states as an example, the procedure of distinguishing quantum states locally is: Alice and Bob hold a part of a quantum system, which occupies one of $m$ possible orthogonal states $|\Psi_1\rangle$, $|\Psi_2\rangle$, ..., $|\Psi_i\rangle$, ..., $|\Psi_m\rangle$. Alice and Bob know the precise form of these states, but don’t know which of these possible states they actually hold. To distinguish these possible states they will perform some operations locally: Alice (or Bob) first measures her part. Then she tell the Bob her measurement result, according to which Bob measure his part. With the measurement results they can exclude some possibilities of the system [1]. Briefly speaking, the procedure of distinguishing quantum states locally is to exclude all or some possibilities by measurement on the system. Many authors have considered some schemes for distinguishing locally between a set of quantum states [1,2,3,4,5,6,7], both inseparable and separable. Bennett et al showed that some orthogonal product states cannot be distinguished by LOCC [2]. Walgate et al showed that any two states can be distinguished by LOCC [1]. For two-qubit systems (or $2 \otimes 2$ systems), any three of the four Bell states:

$$|A_1\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|A_2\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$|A_3\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$|A_4\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

cannot be distinguished by LOCC if only a single copy is provided [4]. The distinguishability of quantum states has some close connections [8] with distillable entanglement [3] and the information transformation [10]. On one hand, using the upper bound of distillable entanglement, relative entropy entanglement [11] and logarithmic negativity [12], the authors in Ref [4] proved that some states are indistinguishable. On the other hand, using the rules on distinguishability one can discuss the distillable entanglement [8]. So the further analysis for distinguishability is meaningful. In this paper, we will give a necessary condition of distinguishability of multi-partite quantum states. Then we present a necessary condition of distinguishability of bipartite quantum states which is simple and general. With this condition one can get many cases of indistinguishability. The conclusions may be useful in understanding the essence of nonlocality and calculating the distillable entanglement and the bound of distillable entanglement.

Consider $m$ possible orthogonal states shared between Alice and Bob. Any protocol
to distinguish the \( m \) possible orthogonal states can be conceived as successive rounds of measurements and communication by Alice and Bob. Let us suppose Alice is the first person to perform a measurement (Alice goes first [3]), and the first round measurement by Alice can be represented by operators \( \{ A_{1j} \} \), where \( A_{1j}^+A_{1j} \) is known as a POVM element realized by Alice [13,14], and \( \sum_j A_{1j}^+A_{1j} = I \). If the outcome \( 1_j \) occurs, then the given \( |\Psi\rangle \) becomes \( A_{1j} |\Psi\rangle \), up to normalization. After communicating the result of Alice’s measurement to Bob, he carries out a measurement and obtains outcome \( 1_k \). The given possible state \( |\Psi\rangle \) becomes \( A_{1j} \otimes B_{1k}(1_j) |\Psi\rangle \), where \( B_{1k}(1_j) \) is an arbitrary measurement operator of Bob which depend on the outcome \( 1_j \) of Alice’s measurement. After \( N \) rounds of measurements and communication, there are many possible outcomes which correspond to many measurement operators acting on the Alice and Bob’s Hilbert space. Each of these operators is a product of the \( N \) sequential and relative operators, \( A_{1j} \otimes B_{1k}(1_j) A_{2j}(1_j, 1_k) \otimes B_{2k}(1_j, 1_k, 2_j) ... A_{Nj}(1_j, 1_k, ..., (N-1)j) \otimes B_{Nk}(1_j, 1_k, ..., (N-1)j, N_k) \), carried out by Alice and Bob. We denote these operators as \( \{ A_i \otimes B_i \} \), where, \( A_i \otimes B_i \) denotes one of these operators, which represent the effects of the \( N \) measurements and communication. If the outcome \( i \) occurs, the given \( |\Psi\rangle \) becomes:

\[
A_i \otimes B_i |\Psi\rangle
\]  

(2)

The probability \( p_i \) Alice and Bob gain outcome \( i \) is

\[
p_i = \langle \Psi | A_i^+ \otimes B_i^+ A_i \otimes B_i |\Psi\rangle,
\]  

(3)

and

\[
\sum_i A_i^+ \otimes B_i^+ A_i \otimes B_i = I
\]  

(4)

Suppose we define:

\[
E_i = A_i^+ \otimes B_i^+ A_i \otimes B_i,
\]  

(5)

then \( E_i \) is a positive operator and that \( \sum_i E_i = I \). \( E_i \) is similar to the POVM element \( A_i^+ A_i \). We can regard \( E_i \) as a generalized POVM (GPOVM) element, which has same property as known POVM element. In fact, \( A_i \) can be written in the form [14]

\[
A_i = U_{A2} f_{Ai} U_{A1},
\]  

(6)

where \( f_{Ai} \) is a diagonal positive operator, \( U_{A2}, U_{A1} \) are two unitary operators, and similarly for \( B_i \). If each of \( N \) Alice’s operators denoted by \( A_i \) and each of \( N \) Bob’s operators denoted by \( B_i \) are projectors, the final operators \( A_i \otimes B_i \) are also projectors, i.e., \( A_i \otimes B_i A_j \otimes B_j = \delta_{ij} A_i \otimes B_i \), and \( \{ E_i \} \) is a set of projective measurement.
The discuss above means that: whatever Alice and Bob choose to do, including they decide to involve an ancillary system; they perform local unitary operators and measurements; they use one-way or two-way communication, and do many rounds of measurements and communication, their final actions will be described by a set positive operators \( \{ E_i \} \). The probability of a given possible state \( |\Psi\rangle \) yielding a certain outcome \( i \) is 
\[
\langle \Psi | A_i^+ \otimes B_i^+ A_i \otimes B_i | \Psi \rangle.
\]

Since a GPOVM element \( E_i \) has similar property as a POVM element, \( E_i \) can be represented in the form
\[
E_i = (a_i^i | \varphi_1^i \rangle_A \langle \varphi_1^i | + \cdots + a_m^i | \varphi_m^i \rangle_A \langle \varphi_m^i | + \cdots) \otimes
\]
\[
(b_i^i | \eta_1^i \rangle_B \langle \eta_1^i | + \cdots + b_m^i | \eta_m^i \rangle_B \langle \eta_m^i | + \cdots)
\]
\[
0 \leq a_m^i \leq 1, 0 \leq b_m^i \leq 1; 1 \leq m_a \leq N_a, 1 \leq m_b \leq N_b
\]
where \( \{ |\varphi_1^i \rangle, ..., |\varphi_m^i \rangle \} \), \( \{ |\eta_1^i \rangle, ..., |\eta_m^i \rangle \} \) is a set of bases of Alice’s and Bob’s, respectively; \( N_a, N_b \) is the dimensions of Alice’s and Bob’s Hilbert space, respectively.

**Theorem 1.** If a set of \( m \) orthogonal states \( \{|\Psi_i\rangle\} \) is reliably locally distinguishable, there is surely a set of product vectors such that each state \( |\Psi_i\rangle \) is a superposition of some of these product vectors as follows:
\[
|\Psi_i\rangle = |\varphi_1^i \rangle_A |\eta_1^i \rangle_B + \cdots + |\varphi_m^i \rangle_A |\eta_m^i \rangle_B + |\phi_1^i \rangle_A |\xi_1^i \rangle_B + \cdots + |\phi_m^i \rangle_A |\xi_m^i \rangle_B
\]
where \( \langle \eta_k^i | \eta_j^k \rangle_B = 0 \), for all \( i \neq j, 1 \leq k \leq m_1 \); \( \langle \phi_k^i | \phi_j^k \rangle_A = 0 \), for all \( i \neq j, 1 \leq k \leq m_2 \). \( m_1, m_2 \) are positive integral number. The set of states \( |\varphi_1^i \rangle_A, ..., |\varphi_m^i \rangle_A \), and the set of states \( |\xi_1^i \rangle_B, ..., |\xi_m^i \rangle_B \) is not necessary to be a set of orthogonal bases of Alice’s and Bob’s, respectively.

**Proof:** The proof follows from the following facts:

**Fact 1:** If a set of states is reliably locally distinguishable, there must be a set of GPOVM element \( \{ E_i \} \) representing the effect of all measurements and communication, such that if every outcome \( i \) occurs Alice and Bob know with certainty that they were given the state \( |\Psi_i\rangle \). Note that because the classical communications between Alice and Bob are allowed, some GPOVM elements in \( \{ E_i \} \) can be not orthogonal to others, even some have same form. For example, suppose Alice carried out a set of POVM, \( A_1, A_2, ..., A_n \). When Alice get a outcome \( i \) and tell Bob her result of measurement (they gain the classical information), to distinguish the \( m \) possible states they should choose appropriate sequential operates which correspond to \( i^{th} \) group of GPOVM. Given that in each round of measurement and communication different outcome (or the different classical information gained) may result in different groups of GPOVM elements, so there many groups of GPOVM elements. It is possible that there is a same GPOVM element \( \chi \) in some different groups. But if we
consider that there is different classical informations in different groups, we can distinguish \( \chi \) in different groups. So in a simple way we can say that a element \( E_i \) with the classical information can and only can “indicate” \( |\Psi_i\rangle \).

Fact 2: Since each element \( E_i \) with the classical information only indicate a state \( |\Psi_i\rangle \), the rank of \( E_i \) should be less than \( N_aN_b \). Otherwise, \( E_i \) will indicates all states \( \{ |\Psi_i\rangle \} \).

Without loss of generality, we suppose \( a_1^i, ..., a_{ma}^i, b_1^i, ..., b_{mb}^i \) in Eq. (7) are nonzero, the other coefficient are zero, then the state \( |\Psi_i\rangle \) should have all or part of the component (if \( |\Psi\rangle = |0\rangle |0\rangle + |1\rangle |1\rangle \), we say \( |\Psi\rangle \) has component \( |0\rangle |0\rangle \) and \( |1\rangle |1\rangle \))

\[
|\varphi_1^i\rangle_A |\eta_1^i\rangle_B + \cdots + |\varphi_{ma}^i\rangle_A |\eta_{ma}^i\rangle_B + \cdots + |\varphi_{ma}^i\rangle_A |\eta_{ma}^i\rangle_B + \cdots + |\varphi_{mb}^i\rangle_A |\eta_{mb}^i\rangle_B. 
\]  

(10)

The probability of each component emerges in the state \( |\Psi_i\rangle \) depend on the state \( |\Psi_i\rangle \) and the element \( E_i \). The effect of the operator \( E_i \) is project out the component in Eq. (11)

Because of the completeness of \( \{ E_i \} \) ( which assures that each component in all possible states can be indicated by a GPOVM element) and the necessity of reliably distinguishing the possible states (which asks a GPOVM element with the classical information only indicate a component of a possible states), each state of the \( m \) possible states can be a superposition of many component each of which can be indicated by a GPOVM element with the classical information. Some component may have same form owing to the fact that some GPOVM elements in \( \{ E_i \} \) can be not orthogonal to others, even some have same form.

On the other hand, if a operator \( E_i \) with the classical information only indicate a state, then \( E_i \) can be replaced by a set of operators

\[
E_{i1} = |\varphi_1^i\rangle_A \langle \varphi_1^i | \otimes |\eta_1^i\rangle_B \langle \eta_1^i|; \cdots; E_{im_b} = |\varphi_{ma}^i\rangle_A \langle \varphi_{ma}^i | \otimes |\eta_{mb}^i\rangle_B \langle \eta_{mb}^i|; 
\]

(11)

\[
E_{im_a m_b} = |\varphi_{ma}^i\rangle_A \langle \varphi_{ma}^i | \otimes |\eta_{mb}^i\rangle_B \langle \eta_{mb}^i|, 
\]

(12)

each of which also only indicates the same state as \( E_i \) does. The effect of each operator \( E_{ij}(j = 1, ..., m_a m_b) \) is to project out a product vector component. For example, operator \( E_{i1} \) project out the component \( |\varphi_1^i\rangle_A |\eta_1^i\rangle_B \). Thus all product vectors in the \( m \) possible states can be indicated by a set of operators \( \{ E_{ij} \} \) with the classical information. So each state \( |\Psi_i\rangle \) must be the superposition of some product vectors each of which is indicated by a GPOVM element with classical information.

Fact 3: During the procedure to distinguish the \( m \) possible states, after each round measurement and gaining a outcome the \( m \) possible states collapse into \( m'(m' \leq m) \)
possible locally distinguishable new states. According to the fact 2, Alice and Bob can choose the last round measurement such that after which the \( m \) possible states collapse into a product vector of a possible state. There are two cases: 1. Alice carries out the last round measurement, i.e., after Alice and Bob gain the outcome of Alice’s they achieve the procedure of distinguishing the all possible states; 2. Bob carries out the last measurement. Before Alice (Bob) carries out the last measurement, the \( m \) possible states should collapses into a few of locally distinguishable product vectors which can be distinguished by only Alice’s a round measurement (if the \( m \) possible states should collapse into a few of locally distinguishable entangled vectors, Alice and Bob can choose the further measurement so that the entangled states collapses into a set of locally distinguishable product vectors as shown in fact 2). So these product vectors can be written thus

\[
|\Psi_i\rangle = |i\rangle_A |\eta\rangle_B; \quad \text{where } \langle i |i'\rangle_A = 0, \text{ for } i \neq i'
\]

which correspond to the form \( |\Phi_i^k\rangle_A |\xi_i^k\rangle_B \) in Eq. (9). Because all possible last measurements belong to the two cases above, addition to the Fact 1 and Fact 2 we can follow that the \( m \) possible states have the form in the theorem 1. This completes the proof.

From theorem 1 it follows that the operator to distinguish the states can be always described as: First, Alice and Bob choose a person to go first to do measurement; After measurement, their Hilbert space collapse into a subspace. According to the outcome, they know that the \( m \) possible states collapses into \( m' (m' \leq m) \) distinguishable new states, and then choose a person to do measurement once more, and so on. After many rounds of measurements and classical communication, they may get a final product state which only belongs to one of the possible states, and the Hilbert space collapses into a one-dimension subspace. In each round of measurements and communication, Alice and Bob must choose an appropriate person to do measurement. The different round of measurement many need different person to do first, in general. For example, to distinguish six states in a \( 4 \otimes 4 \) system,

\[
|\Psi_1\rangle = |0\rangle_A |0\rangle_B; |\Psi_2\rangle = |1\rangle_A (|0\rangle + |1\rangle)_B; |\Psi_3\rangle = |0\rangle_A |1\rangle_B + |1\rangle_A (|0\rangle - |1\rangle)_B \quad (14)
\]

\[
|\Psi_4\rangle = |2\rangle_A |0\rangle_B; |\Psi_5\rangle = (|2\rangle + |3\rangle)_A |1\rangle_B; |\Psi_6\rangle = |3\rangle_A |0\rangle_B + (|2\rangle - |3\rangle)_A |1\rangle_B \quad (15)
\]

Ailce and Bob must first choose Alice to do measurement with Alice’s bases

\[
E_1 = |0\rangle_A \langle 0| + |1\rangle_A \langle 1|; \quad E_1 = |2\rangle_A \langle 2| + |3\rangle_A \langle 3|
\]

if the outcome is \( E_1 \) they must choose Alice to go first to do the sequential measurement; if the outcome is \( E_2 \) they must choose Bob to go first.

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If a set of states are distinguishable only by one person, for example Bob, doing the last measurement, the states can be written with a part of Eq. (14) as follows:

\[ |\Psi_i\rangle = \sum_{i=1}^l \sqrt{p_i} |\phi_i\rangle_A |\eta_i\rangle_B \quad \sum_{i=1}^l p_i = 1 \tag{16} \]

The distinguishability of states in $2 \otimes n$ systems is a special example of the theorem 1 above.

Now we consider the generalization of the theorem 1 to multi-partite states. The GPOVM element, the Fact 1 and the Fact 2 are fit to multi-partite cases obviously. Furthermore, if we consider more cases in the Fact 3, i.e., Alice, Bob or Charle et al carries out the last measurement, respectively, we can generalize the theorem 1 to multi-partite states.

Before giving theorem 2 in this paper, we define a concept of *Schmidt number*. If a pure state $|\Psi\rangle$ have following Schmidt decomposition:

\[ |\Psi\rangle = \sum_{i=1}^l \sqrt{p_i} |\phi_i\rangle_A |\eta_i\rangle_B \] \tag{17} \]

where $|\phi_i\rangle_A$ and $|\eta_i\rangle_B$ are orthogonal bases of Alice and Bob, respectively, we say $|\Psi\rangle$ has *Schmidt number* $l$.

**Theorem 2:** If the dimensions of Hilbert space of Alice’s part and Bob’s part are $N_a$ and $N_b$, respectively, one cannot distinguish deterministically a set of orthogonal states for which the sum of Schmidt numbers is more than $N_a N_b$ when only a single copy is provided. This can be expressed briefly as: one cannot distinguish a set of orthogonal states the sum of Schmidt number of which is more than the dimensions of whole Hilbert space of the quantum system.

From the theorem 2 one can get the following interesting cases:

**Case 1:** For $n \otimes n$ systems one cannot distinguish deterministically $n + 1$ states, each of which has Schmidt number $n$. For example, one can at most distinguish two entangled states in $2 \otimes 2$ systems.

**Case 2:** For $n \otimes n$ systems, if one can distinguish $n^2$ orthogonal states, these states must be orthogonal bases.

**Proof of theorem 2:** We choose an arbitrary set of Alice’s bases (or Bob’s bases) $|1\rangle_A, ..., |N_a\rangle_A$ in these bases every possible state $|\Psi_i\rangle$ can be written as:

\[ |\Psi_i\rangle = |1\rangle_A |\nu_i\rangle_B + \cdots + |N_a\rangle_A |\nu_i\rangle_B \] .

If we divide the Alice’s bases into arbitrary groups, such as two groups, $\{ |1\rangle_A , |2\rangle_A , ..., |l\rangle_A \}$ and $\{ |l + 1\rangle_A , ..., |N_a\rangle_A \}$, each of which corresponding to a subspace. Then

\[ |\Psi_i\rangle = |\Psi_i^1\rangle + |\Psi_i^2\rangle \tag{18} \]
where,

$$|\Psi_i^1\rangle = |1\rangle_A |\nu^1_i\rangle_B + \cdots + |l\rangle_A |\nu^l_i\rangle_B ;$$

$$|\Psi_i^2\rangle = |l+1\rangle_A |\nu^{l+1}_i\rangle_B + \cdots + |N_a\rangle_A |\nu^{N_a}_i\rangle_B$$

are the project of the $m$ possible states in subspace 1 and 2, respectively. The dimensions of Hilbert subspace 1 and 2 is $lN_b$ and $(N_a - l)N_b$, respectively. It is obvious that the sum of the Schmidt numbers of the states $|\Psi_i^1\rangle$ and $|\Psi_i^2\rangle$ is not less than the Schmidt number of the state $|\Psi_i\rangle$. So if the sum of Schmidt numbers of the $m$ possible states is more than the dimensions of whole Hilbert space of the quantum system, there must be a subspace in which the project of the $m$ possible states satisfies that the sum of the Schmidt numbers of these projective states is more than the dimensions of the Hilbert subspace. On the other hand, an arbitrary POVM element, $A_iA_i^+ = a_1 |1\rangle_A \langle 1| + \cdots + a_l |l\rangle_A \langle l|$, carried out by Alice (or Bob) can be regarded as a projector with change of relative weights of bases $|1\rangle_A , \ldots , |l\rangle_A$. After measurement, the $m$ possible states collapses into new possible states

$$|\Psi_i\rangle = a_1 |1\rangle_A |\nu^1_i\rangle_B + \cdots + a_l |l\rangle_A |\nu^l_i\rangle_B .$$

When we change the values of $a_1, \ldots, a_l$ in the realm $(0,1]$ the Schmidt numbers of $|\Psi_i\rangle$ have no change. This is because that the Schmidt numbers of $|\Psi_i\rangle$ is the number of linearly independent vectors in the set of states $\{|\nu^1_i\rangle_B, \ldots , |\nu^l_i\rangle_B\}$. There is same number of linearly independent vectors in the set of states $\{|\nu^1_i\rangle_B, \ldots , |\nu^l_i\rangle_B\}$ and states $\{a_1 |\nu^1_i\rangle_B , \ldots , a_l |\nu^l_i\rangle_B\}$. So there is a POVM element which results that the $m$ possible states collapses into new possible states, and the sum of Schmidt numbers of the new possible states is more than the dimensions of the Hilbert subspace. To distinguish the new possible states, Alice and Bob continue to do measurements until the whole Hilbert space collapses into a subspace in which the $m$ possible states collapses into locally distinguishable product states $\{ |\varphi^i\rangle_A |\eta^i_B , i = 1, \ldots, m\}$ or $\{ |\xi^i\rangle_B |\Phi^i_A , i = 1, \ldots, m\}$, as shown in the theorem 1 above. In each round of measurements there exists the possibility that the sum of Schmidt numbers of the new possible states gained is more than the dimensions of the Hilbert subspace. So if the sum of Schmidt numbers of the $m$ possible states is more than the dimensions of the Hilbert space, there must be nonzero probability that before Alice or Bob do the last measurement the $m$ possible states collapses into a set of product states $\{ |\varphi^i\rangle_A |\eta^i_B , i = 1, \ldots, m\}$ or $\{ |\xi^i\rangle_B |\Phi^i_A , i = 1, \ldots, m\}$, the numbers (or Schmidt numbers) of which is more than the dimensions of the Hilbert subspace. The Alice part or Bob part of the product states belongs to is a one-dimension Hilbert subspace. So not all states $\{ |\eta^i_B , i = 1, \ldots, m\}$ or $\{ |\Phi^i_A , i = 1, \ldots, m\}$ are orthogonal to each other. Thus the $m$ possible states cannot have the form as in the theorem 1, i.e., are not reliably locally distinguishable. This completes the proof theorem 2.
According to the theorem 2 we can also discuss completely the case for $2 \otimes 2$ systems, as be shown in Ref [3]. Here we omit the discussion.

In summary, we present a necessary condition of distinguishability of multi-partite quantum states. With this condition one can discuss some especial cases of distinguishability further. We also present a necessary condition of distinguishability of bipartite quantum states which is simple and general. With this condition one can get many cases of indistinguishability. These results come directly from the limits on local operations, not from the upper bound of distillable entanglement [4]. So we believe that they may be useful in calculating the distillable entanglement or the bound of distillable entanglement. The further works may be the applications of these results.

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REFERENCES

[1] J.Walgate, A.J.Short, L.Hardy and V.Vedral, Phys.Rev.Lett.85,4972 (2000).

[2] C.H. Bennett, D.P. DiVincenzo, C.A. Fuchs, T.Mor, E.Rains, P.W. Shor, J.A. Smolin, and W.K. Wootters, Phys. Rev. A 59,1070 (1999) or quant-ph/9804053.

[3] J. Walgate and L. Hardy, quant-ph/0202034, to published in Phys. Rev. Lett

[4] S.Ghosh, G.Kar, A.Roy, A.Sen and U.Sen, Phys.Rev.Lett.87, 277902 (2001); S. Ghosh, G.Kar, A.Roy, D.Sarkar, A.Sen(De) and U.Sen, quant-ph/0111136 (2001); M. Horodecki, A. Sen (De), U. Sen and K. Horodecki, quant-ph/0204116 (2002)

[5] S. Virmani, M.F. Sacchi, M.B. Plenio and D. Markham, Physics Letters A 288, 62-68 (2001);

[6] M. Horodecki, P. Horodecki, and R. Horodecki, Acta Physica Slovaca, 48, (1998) 141, or quant-ph/9805072

[7] Y.-X.Chen and D.Yang, Phys.Rev.A 64, 064303 (2001);

[8] P.-X Chen and C.-Z Li, quant-ph/0202163

[9] C. H. Bennett, G. Brassard, S. Popescu, B. Schumacher, J. A. Smolin, and W. K. Wootters, Phys. Rev. Lett 76 722 (1996); C. H. Bennett, D. P. Divincenzo, J. A.Smolin, and W. K.Wootters, Phys. Rev. A 54, 3824 (1996)

[10] J. Eisert, T. Felbinger, P. Papadopoulos, M.B. Plenio and M. Wilkens, Phys. Rev. Lett. 84, 1611 (2000); L.Henderson and V.Vedral, Phys.Rev.Lett 84, 2263 (2000).

[11] V.Vedral, M.B.Plenio, M.A.Rippin and P.L.Knight, Phys.Rev.Lett. 78, 2275 (1997); V.Vedral and M.B.Plenio, Phys.Rev.A 57, 1619 (1998)

[12] G.Vidal and R.F.Werner, quant-ph/0102117 (2001);

[13] H. Barnum, M. A. Nielsen and B. Schumacher, Phys.Rev.A 57, 4153 (1998);

[14] N. Linden, S. Massar and S. Popescu, Phys.Rev.Lett 81, 3279 (1998)