Features of the scattering of exchange spin waves by layer and superlattice of biaxial ferromagnets

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Abstract. The paper studies the scattering of exchange spin waves at the interface of biaxial magnetic materials with a uniform ground state and describes a method for finding the reflection and transmission coefficients from an isolated boundary, layer, and multilayer structure of such materials. The breaking of the axial symmetry of the precession of magnetization leads to the formation of surface exchange waves at the interface. These waves correspond to the imaginary roots of the characteristic equation and have the opposite to the bulk waves chirality of precession. Taking them into account aligns the number of boundary conditions and unknown amplitudes. The scattering coefficients are obtained by expanding over a small ellipticity parameter in the zero and first approximations. A comparison with the exact solution was made. It is shown that the frequency dependence of the reflection coefficient can be non-monotonic, and it can turn to zero, which corresponds to the equality of the wave impedances of the adjacent media.

1 Introduction

Devices based on exchange and magnetostatic spin waves can compete with devices of classical electronics now. This is associated with relatively small losses for Joule heat and low speed of their distribution (about a kilometer per second), which allows to significantly reduce their size.\cite{1,2}.

In addition, spin waves (SW), in contrast to electromagnetic ones, have a certain chirality of precession, since SWs cannot be linearly polarized. This leads to a more complicated spectrum compared to electromagnetic waves due to the large number of dynamic variables.

In\cite{3,4}, the problem of scattering of exchange spin waves was solved in the approximation of geometric optics. The purpose of this work is to find the scattering coefficients without approximations for a magnetic structure with a broken symmetry of the magnetization precession. It is shown that in this case the requirement of compatibility of the system of linearized boundary conditions for dynamic magnetization leads to the necessity of taking into account the imaginary roots of the characteristic equation.

2 Model

Let us consider the scattering of the exchange spin wave (ESW) by an isolated flat interface between semi-infinite biaxial ferromagnetic media. The anisotropy axes in both environments coincide in direction with each other. Between the materials it is assumed the presence of absolutely rigid
ferromagnetic coupling. In this case, the ground state will be uniform in each medium, i.e. -inhomogeneous transition structures of the domain wall type are absent on the border.

We choose the geometry so that the equilibrium magnetizations are directed along the $x$ axis. The normal to the interface along which the ESW is distributed coincides with the $z$ axis. The energy density in the selected geometry is written as:

$$ w = \begin{cases} w_0(z), & z < 0 \\ w_0(z), & z > 0 \end{cases} $$

where [5]

$$ w_p(z) = \frac{\lambda_p^2}{z} \left( \frac{d \tilde{M}}{dz} \right)^2 + \frac{\beta_{py}}{z} \left( \tilde{n}_y \tilde{M}_p \right)^2 + \frac{\beta_{px}}{2} \left( \tilde{n}_z \tilde{M}_p \right)^2, \quad p = a, b. $$

(2)

$\lambda_p$ – exchange length of magnetic material, $\beta_{py,zy} > 0$ – its anisotropy constants, $\tilde{M}_p$ – magnetization in each of the adjacent media, $\tilde{n}_y, \tilde{n}_z$ – unit vectors in directions $y$ and $z$, respectively.

In view of (2), the Landau – Lifshitz equation in each medium has the form:

$$ \dot{\tilde{M}}_p = \gamma_p \left( \tilde{M}_p \times -\frac{\lambda_p^2}{z} \frac{d^2 \tilde{M}}{dz^2} \tilde{M}_p + \beta_{py} \tilde{n}_y (\tilde{n}_y \tilde{M}_p) + \beta_{pz} \tilde{n}_z (\tilde{n}_z \tilde{M}_p) \right). $$

(3)

where $\gamma_p$ – the absolute value of the gyromagnetic ratio in p-th environment.

The boundary conditions for the components of the dynamic magnetization are obtained from (3) by integration over a vanishingly small neighbourhood of the boundary. Considering also that the border spins are parallel to each other, we get:

$$ \lambda_a^2 \tilde{M}_a \times \frac{d \tilde{M}_a}{dz} = \lambda_b^2 \tilde{M}_b \times \frac{d \tilde{M}_b}{dz}, $$

(4)

$$ \tilde{M}_a \times \tilde{M}_b = 0. $$

(5)

We linearize equation (3) by small deviations $\tilde{m}_p$ from the ground state $M_p \tilde{n}_x$

$$ \tilde{M}_p = M_p \tilde{n}_x + \tilde{m}_p. $$

(6)

The linearized boundary conditions (4) - (5) in the projections have the form:

$$ \mu_{a,y/z} \bigg|_{z=0} = \mu_{b,y/z} \bigg|_{z=0}, $$

(7)

$$ A_a \frac{d \mu_{a,y/z}}{dz} \bigg|_{z=0} = A_b \frac{d \mu_{b,y/z}}{dz} \bigg|_{z=0}. $$

(8)

where $A_p = \frac{\lambda_p^2 \mu_p}{z}$ – exchange constant, $\mu_p = \frac{m_p}{M_p}$ – normalized dynamic magnetization.

We will seek a solution to the Landau-Lifshitz equation in the form:

$$ m_{py} = C_p e^{i(k_p^z - \omega t)}, \quad m_{pz} = i D_p e^{i(k_p^z - \omega t)}. $$

(9)

Substituting (6) and (9) into (3), we get:

$$ -i \omega \tilde{m}_p = \gamma_p M_p \left( \lambda_p^2 k_p^2 \tilde{n}_x \times \tilde{m}_p + \beta_{py} \tilde{n}_y (\tilde{n}_y \tilde{m}_p) - \beta_{pz} \tilde{n}_z (\tilde{n}_z \tilde{m}_p) \right). $$

(10)

Equation (10) is an algebraic system:

$$ \omega C_p = \gamma_p M_p \left( \lambda_p^2 k_p^2 + \beta_{pz} \right) D_p, $$

$$ \omega D_p = \gamma_p M_p \left( \lambda_p^2 k_p^2 + \beta_{py} \right) C_p. $$

(11)

Multiplying equations (11), we find the spectrum:

$$ \omega^2 = \gamma_p^2 M_p^2 \left( \lambda_p^2 k_p^2 + \beta_{pz} \right) \left( \lambda_p^2 k_p^2 + \beta_{py} \right). $$

(12)

Expressing from here the wave vector of each medium, we obtain the biquadratic equation ($\omega_p = \gamma_p M_p$):

$$ \left( \lambda_p^2 k_p^2 \right)^2 + \lambda_p^2 k_p^2 \beta_{pz} + \beta_{py} + \left( \beta_{pz} \beta_{py} - \frac{\omega}{\omega_p} \right)^2 = 0, $$

(13)

it has four roots, two of which are real and correspond to running waves, and the other two are pure imaginary. In an unbounded magnetic environment, imaginary roots should be discarded as corresponding to unlimitedly increasing solutions at infinity. However, in an environment with at least one boundary, one of them describes the nonuniform ESW, decaying from the boundary into the
environment and must be taken into account, because otherwise there will be a discrepancy between
the number of boundary conditions and unknown amplitudes.

Solving the biquadratic equation (13), we find its roots, which are the possible values of the wave
vector:

$$k_{pv} = \pm \frac{1}{\kappa_p} \left[ \sqrt{\frac{\omega}{\omega_p}}^2 + \delta \beta_p^2 - \beta_p \right], \quad \kappa_p = \frac{1}{\lambda_p} \left[ \sqrt{\frac{\omega}{\omega_p}}^2 + \delta \beta_p^2 + \beta_p \right]$$

where $\beta_p = \beta_p \pm \beta_p^2$.

Note that for bulk waves, the frequency of the incident wave must be greater than the activation
frequency in this medium $\omega_{min} = \Omega_p (\beta_p^2 - \delta \beta_p^2) = \Omega_p \beta_p \beta_p^2$. Value $|k_{ps}|$ near this frequency is
of the order of reciprocal exchange length.

In the case of oblique incidence in expressions (15), a planar component of the wave vector $k$ will
appear, the same for both media:

$$\lambda_p k_{pv} = \sqrt{\frac{\omega}{\omega_p}}^2 + \delta \beta_p^2 - \beta_p - \lambda_p^2 k^2, \quad \lambda_p |k_{ps}| = \sqrt{\frac{\omega}{\omega_p}}^2 + \delta \beta_p^2 + \beta_p + \lambda_p^2 k^2$$

which leads to an increase in the frequency of activation of the bulk wave.

Defining the ellipticity of each wave as $\eta = \frac{D}{C}$, from the system (11) we find:

$$\eta_{p,v/s} = \frac{\lambda^2 k_{v/s}^2 + \beta_p - \delta \beta_p}{\omega_{p,v/s}} = \pm \sqrt{\frac{\omega_{p,v/s}}{\omega}}^2 + \frac{\delta \beta_p^2 + \delta \beta_p}{\omega_{p,v/s}}$$

wherefrom, in particular, should $\eta_v \eta_s = -1$. This means that the ellipses of the bulk and surface
waves are the same, but rotated on 90 degrees, and the precession in them is carried out with different
chirality.

### 3 Scattering by an isolated boundary

Let the ESW falls from the medium $a$ to the medium $b$. The components of magnetization in each
medium are sought in the form:

$$\mu_{av} = \eta_{av} e^{ik_{av} z} + \eta_{av} e^{-ik_{av} z} + \tau_s e^{ik_{as} z}$$

$$\mu_{az} = i \left( \eta_{av} (1 + e^{ik_{av} z} + \eta_{av} e^{-ik_{av} z}) + \eta_{as} \tau_s e^{ik_{as} z} \right)$$

$$\mu_{by} = t_v e^{ik_{by} z} + t_s e^{-ik_{by} z}$$

$$\mu_{bz} = i \left( \eta_{bv} \tau_v e^{ik_{bv} z} + \eta_{bs} \tau_s e^{ik_{bs} z} \right)$$

Then from the boundary conditions (7) - (8) we obtain a system for determining the coefficients
ESW scattering:

$$1 + r_v + r_s = t_v + t_s$$

$$\eta_a (1 + r_v) - \eta_{a}^{-1} r_s = \eta_b t_v - \eta_{b}^{-1} t_s$$

$$A_a (k_{av} (1 - r_v) - ik_{av} r_s) = A_b (k_{bv} t_v + ik_{bv} t_s)$$

$$A_a (k_{av} \eta_a (1 - r_v) + ik_{av} \eta_{a}^{-1} r_s) = A_b (k_{bv} \eta_{b} t_v - ik_{bv} \eta_{b}^{-1} t_s)$$

where the wave vectors and ellipticities are given by formulas (15) and (16).

In the limit of uniaxial ferromagnets $\delta \beta_p \to 0$, the ellipticities are equal to unity and, as follows
from system (18), the amplitudes of the surface waves vanish. In this case, we obtain the reflection and
transmission coefficients of the ESW:

$$r_{v0} = \frac{1-Z}{1+Z}, \quad t_{v0} = \frac{Z}{1+Z}.$$
which have the form of Fresnel. Here
\[ Z = \frac{A_b k_{bw}}{A_a k_{aw}} = D \sqrt{\frac{\omega - \beta_a \omega_b}{\omega - \beta_b \omega_a}} \] - relative impedance of two media, \( D = \frac{\gamma_a}{\gamma_b} \frac{A_b \omega_p}{A_a \omega_a} \).

From (19) it follows the conservation of the energy flow of ESW:
\[ r_{v0}^2 + Z t_{v0}^2 = 1, \quad (20) \]
A graphical comparison of the frequency dependence of the modules and phases of the reflection and transmission of the ESW for the cases of circular (\( \delta \beta_p = 0 \)) and elliptical (\( \delta \beta_p \neq 0 \), exact solution of system (18)) magnetization precession is shown in figure 1. Expressions (19) with increasing frequency monotonically and asymptotically tend to
\[ r_{v0}(\omega_0) = \frac{1-D}{1+D}, \quad t_{v0}(\omega_0) = \frac{2}{1+D}. \quad (21) \]
Consequently, the transmitted wave is amplified due to scattering of the reflected wave in antiphase. In this case, at a frequency of
\[ \omega_0 = \frac{\omega_a - D^2 \omega_b}{1-D^2} > \omega_a \quad (22) \]
the reflection coefficient vanishes, and its phase changes abruptly by \( \pi \). Obviously, this frequency corresponds to the equality of the unit relative impedance of the adjacent media.

![Graphical comparison of the frequency dependence of the modules and phases of the reflection and transmission of the ESW](image)

**Figure 1.** Dependences of the modules (a) and phases (b) of the reflection and transmission coefficients of the ESW for the parameter values \( \omega_b = 0.5 \omega_a, \beta_b = \beta_a = 1, D = 0.8 \). The abscissa is the frequency in units of the frequency of the FMR layer \( \beta_a \omega_a \). Pink is the exact solution of the system (18), built for \( \delta \beta_a = 0.3, \delta \beta_b = -0.6 \).

### 4 Scattering of ESW by a superlattice

Let the ESW falls normally from a semi-infinite medium \( a \), passes through a superlattice of \( n \) layers and dissipates into a semi-infinite medium \( b \). The parameters of the layers will be denoted by the index \( n \). The field of dynamic magnetization in each layer is represented as
\[ \mu_{ny} = \left( C_{v+n} e^{j k_{nl}(z-d_{n+1})} + C_{v-n} e^{-j k_{nl}(z-d_{n-1})} + C_{s+n} e^{j k_{nl}(z-d_{n-1})} + C_{s-n} e^{-j k_{nl}(z-d_{n-1})} \right), \]
\[ \mu_{nz} = i \left( \eta_n (C_{v+n} e^{j k_{nl}(z-d_{n+1})} + C_{v-n} e^{-j k_{nl}(z-d_{n-1})}) - \eta_n^{-1} (C_{s+n} e^{j k_{nl}(z-d_{n-1})} + C_{s-n} e^{-j k_{nl}(z-d_{n-1})}) \right), \quad (23) \]

\( D_{n+1} < z < D_n, \quad D_n = \sum_{k=1}^{n} d_k \)

Then the system of boundary conditions on the border of \( n-1 \) and \( n \)-th layer is written in the form:
\[ \hat{P}_{n-1}^{(r)} C_{n-1}^2 = \hat{P}_n^{(l)} C_n^2 \quad (24) \]

where
\[ \hat{P}_n^{(l)} = \begin{pmatrix} 1 & 1 & -\eta_n^{-1} & 1 \\ iA_n k_{n,v} & iA_n k_{n,v} & A_n |k_{ns}| & -A_n \eta_n^{-1} |k_{ns}| \\ iA_n \eta_n k_{n,v} & -iA_n \eta_n k_{n,v} & -A_n \eta_n^{-1} |k_{ns}| & A_n \eta_n^{-1} |k_{ns}| \end{pmatrix} \quad (25) \]
\[
\hat{\mathbf{r}}_n^{(r)} = \begin{pmatrix}
    e^{ik_n d_n} & e^{-ik_n d_n} & e^{ik_{ns} d_n} & e^{-ik_{ns} d_n} & e^{ik_{ns} d_n} & e^{-ik_{ns} d_n} \\
    \eta_n e^{ik_n d_n} & \eta_n e^{-ik_n d_n} & -\eta_n^{-1} e^{ik_{ns} d_n} & -\eta_n^{-1} e^{-ik_{ns} d_n} & A_n |k_{ns}| e^{ik_{ns} d_n} & A_n |k_{ns}| e^{-ik_{ns} d_n} \\
    iA_n k_n e^{ik_n d_n} & -iA_n k_n e^{-ik_n d_n} & -iA_n \eta_n k_n e^{ik_{ns} d_n} & -iA_n \eta_n k_n e^{-ik_{ns} d_n} & A_n |k_{ns}| e^{ik_{ns} d_n} & -A_n |k_{ns}| e^{-ik_{ns} d_n} \\
    iA_n \eta_n k_n e^{ik_n d_n} & -iA_n \eta_n k_n e^{-ik_n d_n} & A_n \eta_n^{-1} |k_{ns}| e^{ik_{ns} d_n} & A_n \eta_n^{-1} |k_{ns}| e^{-ik_{ns} d_n} \\
\end{pmatrix}
\]

- left and right T-matrices, a

\[
\hat{C}_n = \begin{pmatrix}
    C_{v,n} & C_{v,n+1} & C_{s,n} & C_{s,n+1} \\
\end{pmatrix}
\]

– vector column of the amplitude coefficients of the \( n \)-th layer.

Then, taking the amplitude of the incident bulk wave as a unit and introducing the notation for the amplitude coefficients

\[
\tilde{C}_a = \begin{pmatrix}
    1 \\
    r_v \\
    0 \\
    0 \\
\end{pmatrix}, 
\tilde{C}_b = \begin{pmatrix}
    t_v \\
    0 \\
    t_s \\
    0 \\
\end{pmatrix}
\]

we obtain a matrix equation for finding of amplitude coefficients of scattering:

\[
\hat{T} \tilde{C}_b = \tilde{C}_a,
\]

where

\[
\hat{T} = \left( \hat{T}_a^{(l)} \right)^{-1} \left( \prod_{k=1}^{N} \hat{T}_k^{(l)} \right) \left( \hat{T}_b^{(r)} \right)^{-1}
\]

From (29) it is easy to find amplitudes of reflected and transmitted waves:

\[
t_v = \frac{T_{a4}}{T_{a4}T_{a4} - T_{a4}T_{a4}} , 
\]

\[
t_s = -\frac{T_{a1}}{T_{a4}T_{a4} - T_{a4}T_{a4}}
\]

\[
r_v = \frac{T_{a4}T_{a4} - T_{a4}T_{a4}}{T_{a4}T_{a4} - T_{a4}T_{a4}}, 
\]

\[
r_s = \frac{T_{a4}T_{a4} - T_{a4}T_{a4}}{T_{a4}T_{a4} - T_{a4}T_{a4}}
\]

where \( T_{ik} \) – the elements of matrix \( \hat{T} \).

As an example, we present the calculated frequency dependences of the modules of the amplitude reflection and transmission coefficients of the bulk and surface waves, which are shown in figure 2.

![Figure 2](image-url)

**Figure 2.** The dependences of the modules of reflection and transmission coefficients of bulk (a) and surface (b) ESW for parameter values \( \omega_b = 0.5\omega_a, \beta_b = \beta_a = 1, D = 0.7, \delta \beta_a = 0.3, \delta \beta_b = -0.6 \) and layer thickness equal to 10 exchange lengths its material. The abscissa axis is the frequency in units of the FMR frequency of the layer \( \alpha \beta_a \omega_a \) starting from the activation frequency.

It can be seen from the graphs shown that surface ESW play the most significant role at relatively low frequencies close to the activation frequency. With increasing of frequency the polarization of the waves tends to be circular and the amplitudes of the surface waves are negligible. The frequencies,
corresponding to the zero reflection coefficient, are not equidistant due to the nonlinear nature of the spectrum.

5. Conclusion
In this work the coefficients of reflection and transmission of ESW from the boundary of biaxial ferromagnets in the presence of an absolutely rigid interlayer bond with normal incidence are obtained. Such waves are characterized by the elliptical precession of magnetization, as a result of which, when matching the incident and scattered waves at the boundary, it becomes essential to take into account the nonuniform ESW, the exponentially decaying solutions of the Landau–Lifshitz equations. Depending on the ratio of the frequencies of homogeneous FMR in contacting ferromagnets, the frequency dependence of the reflection coefficient may be monotonic or have a minimum. The origin of the latter can be traced to the condition of equality of wave impedances in the limiting case of circular precession. At the same time, the introduction of wave impedances into consideration in the case of elliptical precession is not possible. The frequency dependence of the transmission coefficient always remains monotonous.

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