Avalanches, Barkhausen Noise, and Plain Old Criticality

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Abstract

We explain Barkhausen noise in magnetic systems in terms of avalanches near a plain old critical point in the hysteretic zero-temperature random-field Ising model. The avalanche size distribution has a universal scaling function, making non-trivial predictions of the shape of the distribution up to 50% above the critical point, where two decades of scaling are still observed. We simulate systems with up to $1000^3$ domains, extract critical exponents in 2, 3, 4, and 5 dimensions, compare with our 2d and $6-\epsilon$ predictions, and compare to a variety of experimental Barkhausen measurements.

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When materials are pushed, they can yield in different ways. Some crackle: they transform through a series of pulses or avalanches. In many systems, the behavior of these avalanches is unaffected by thermal fluctuations: one domain triggers, pushing some of its neighbors to trigger, in a deterministic process dependent on the static, quenched disorder in the material (and on the stress history). The statistics of Barkhausen noise (the avalanches seen in magnetic materials as the external magnetic field is ramped up and down) has been extensively studied experimentally [1,2,3,4,5,6,7,8,9,10,11]. We suggest that the zero-temperature random-field Ising model [12,13] provides a universal, quantitative explanation for many of these experiments.

A typical experiment will collect a histogram of pulse sizes, times, or energies. The distribution will follow a power law, which cuts off after two to several decades — much broader than any observed morphological feature in the materials. An explanation for the experiment must involve collective motion of many domains; it must provide an explanation for the observed power-law scaling regions, and it must provide an explanation for the cutoff.

Figure 1 shows the distribution $D_{int}(S,R)$ of avalanche sizes for our model in 3d (discussed already in reference [12]), at several values of the microscopic disorder $R$. The model is a collection of domains $s_i = \pm 1$ coupled to an external field $H$, a local random field $f_i$ chosen from a distribution $\rho(f) = \exp(-f^2/2R^2)/\sqrt{2\pi R}$ of standard deviation $R$, and to its nearest neighbors $s_j$ with an energy of strength $J = 1$. The domain $s_i$ flips over when the net local field $F_i \equiv f_i + H + J \sum_{nn} s_j$ seen at site $i$ changes sign. Due to the nearest-neighbor interaction, a flipping spin often causes one or more neighbors to flip also, thereby spawning a whole avalanche of spin flips. Figure 1 shows the avalanches found by integrating as the external field $H(t)$ is raised adiabatically from $-\infty$ to $\infty$ (the field is thus constant during the individual avalanches).

Notice three things about figure 1. (1) The distributions follows a power law, which cuts off after two to several decades. (2) The cutoff appears to diverge at a critical value of the disorder $R_c$, which we estimate in three dimensions to be $2.16J$. (3) The critical region is large! While the true power-law distribution is only obtained at $R_c = 2.16$, we get
avalanches with more than a hundred domains all the way up at $R = 4$. This suggests that experiments can see decades of scaling without working hard to find the critical disorder. Several decades of scaling without tuning a parameter need not be self-organized criticality; it can be vague proximity to a plain old critical point.

Notice four more things about figure 1. (4) The straight line lying askew below the numerical data is our prediction for the asymptotic power law $D_{int}(S, R_c) \sim s^{-(\tau+\sigma\beta\delta)}$. The obvious experimental method of taking the slope on the log-log plot gives the wrong answer until many, many decades of scaling are obtained! (5) The inset shows the collapse of the data onto a scaling function

$$D_{int}(s^\sigma(R - R_c)/R) = \lim_{R \to R_c} s^{\tau+\sigma\beta\delta} D_{int}(S, R)$$

(1)

which is a universal prediction of our model: real experiments rescaled in the same way should look the same (apart from overall vertical and horizontal scales). This scaling function is quite unusual: it grows by a factor of over ten before cutting off. This bump is the reason that the experiments take so long to converge to the asymptotic power law. To make a definite prediction, we have phenomenologically fit our curve

$$D_{int}(x) = (0.021 + 0.002x + 0.531x^2 - 0.266x^3 + 0.261x^4) \exp(-0.789x^{1/\sigma})$$

(2)

where we guess the error in the curve to be less than 10% within the range $0.2 < x < 1.2$. (6) In the main figure, the scaling form passes through our data quite well, even far from $R_c$. The scaling theory is predictive for curves with only two or three decades of scaling. The critical region starts when the correlation length (and hence the avalanche size cutoff) becomes large — not only when the pure power law takes over. Using equations (1) and (2) and the values $\sigma = 0.24 \pm 0.02$ and $\tau + \sigma\beta\delta = 2.03 \pm 0.03$, an experimentalist should be able to fit any single histogram of avalanches, shifting only the overall vertical and horizontal scale factors. (7) Widom scaling (equation (1)) forms a powerful tool: only by varying $R$ were we able to extract the correct critical behavior. We suggest that experimentalists try
varying some parameter of their system (annealing time or temperature, grain size, impurity concentration, ...) and observe the resulting cutoff dependence. Any family of curves thus generated should, near the critical point, be fit with three parameters (including $R_c$ in (1) and (2)).

A comparison of our predicted exponents with power laws extracted from a number of experiments on magnetic Barkhausen noise in bulk three-dimensional systems is shown in Table 1. One of the experiments [8] varied the annealing time, and saw a shift in the cutoff, but did not extract a critical annealing time or do scaling collapses. The range of values for $\tau + \sigma \beta \delta$ observed in the experiments is compatible with the range of log-log slopes we observe due to the unusual scaling form for the integrated avalanche size distribution $D_{int}$ discussed above.

The largest set of experiments measure the avalanche size distribution in a narrow range of fields (i.e., without averaging over the entire loop): their power laws fits are a measure of the critical exponent $\tau$. Integrating over the hysteresis loop changes the power law, because only near a critical value $H_c$ of the external field do large avalanches occur. The cutoff in the avalanche size at $R_c$ goes as $|H - H_c|^{1/\sigma \beta \delta}$: $\sigma$ as discussed above gives the cutoff dependence with $R - R_c$, and $\beta$ and $1/\delta$, as usual [12], give the singularities of the magnetization with $R - R_c$ and $H - H_c$ respectively (the exponent for any quantity varying $H$ at $R_c$ is given by multiplying $1/\beta \delta$ by the exponent for the singularity varying $R$ at $H_c$). The scaling function for the non-integrated avalanche size distribution [12], we find, does not have a bump. The experimental measurements for $\tau$ are close to our numerical estimate.

The other experiments (pulse durations, power spectra, and pulse energies) introduce a new exponent combination $\sigma \nu z$. The correlation length exponent $\nu$ gives the divergence of the characteristic spatial extent of avalanches with $R - R_c$, and $\nu z$ gives the divergence of the avalanche durations with $R - R_c$. The critical exponent $z$ occasionally can depend on the details of the spin dynamics [15]; it is not even clear whether our simulation must have the same value of $z$ as our $\epsilon$-expansion. Nonetheless, the agreement between our predictions and the measured values are about as good as the agreement between the different measurements.
We have also investigated the application of our model to other systems. Many experiments are done in effectively two-dimensional systems (magnetic hysteresis \[16\], avalanches as the field is swept in superconductors \[17\], and avalanches as helium is injected into Nuclepore \[18\]); our 2d explorations are still rather preliminary. Our model does not fit the avalanche size distributions measured in 3d martensitic transitions as the temperature is ramped \[19\]: their measurement of the pulse duration exponent \((\tau + \sigma \beta \delta - 1)/\sigma \nu z + 1 \sim 1.6\) is significantly different from our prediction of \(2.81 \pm 0.11\). We expect that the long-range anisotropic elastic fields in martensites likely change the universality class; similarly the long-range antiferromagnetic demagnetization fields could affect experiments in certain geometries (see \[10\]). Full explanations about the various exponent combinations measured in different systems \[20\] and detailed discussions of experiments and systems \[21\] are forthcoming.

Figure 2 shows the results for five of our exponents \((\tau + \sigma \beta \delta, \tau, 1/\nu, \sigma \nu, \text{ and } \sigma \nu z)\), measured in 2, 3, 4, and 5 dimensions. (From these one can get \(\beta\) and \(\delta\) separately using the exponent equality \[20\] \(\tau + \sigma \beta \delta = 2 + \sigma \beta\).) We measure the exponents in the (unphysical) dimensions four and five in order to test our renormalization-group predictions \[13, 20\] for the behavior near six dimensions. First, notice that the numerical values converge nicely to the mean-field predictions as the dimension approaches six, and that the predictions of our \(6 - \epsilon\) expansion do remarkably well. (The primary role of the renormalization-group treatment, of course, is to explain why scaling and universality might be expected in these systems.) The predictions for \(1/\nu\) are to fifth order in \(\epsilon\): by mapping our model to all orders \[20\] onto the regular Ising model in two lower dimensions \[22\], we have been able to use \[23\] the series known to order \(\epsilon^5\) for \(\nu\). The other exponents shown have no equivalent in the equilibrium model: we have developed \[20\] a new method for calculating these avalanche exponents using two replicas of the system. The dashed lines show a Borel resummation \[23\] of the series for \(1/\nu\), and the predictions to first order for the other variables.

Second, notice the exponents in two dimensions. We here conjecture that the 2d exponents will be \(\tau + \sigma \beta \delta = 2, \tau = 3/2, 1/\nu = 0, \text{ and } \sigma \nu = 1/2\). It is likely that two is the
“lower critical dimension” for our system, below which all avalanches will be finite except at zero disorder. At the lower critical dimension, the critical exponents are often ratios of small integers, and it is often possible to derive exact solutions. For us, using the fact that there can be at most one system-spanning avalanche in two dimensions, one can derive a special exponent relation $1/\sigma \nu = d - \beta/\nu = 2 - \beta/\nu$: this “hyperscaling” relation is false in mean-field theory, and definitely ruled out numerically in four and five dimensions, and probably in three [24,14]. Folklore in the field [25] give us two other likely 2d relations: one each for the exponent giving the decay in space of the cluster correlation function $\bar{\eta} = \beta/\nu + 4 - d = 2 - d$ and of the avalanche correlation function $\eta = 2 + \beta/\nu - \beta\delta/\nu = 1$. These relations hold in the lower critical dimension for the Ising model, the Heisenberg model, and the equilibrium, thermal random-field Ising model. The first of these relations is equivalent to the statement that the avalanches are compact ($1/\sigma \nu = d = 2$).

We must mention that our firm conjectures about the exponents in two dimensions must be contrasted with our lack of knowledge about the proper scaling forms. At the lower critical dimension, the correlation length typically diverges exponentially as one approaches the critical point ($\nu \to \infty$). Some combinations of critical exponents stay finite (hence $\sigma \nu = 1/2$), but those which diverge and those which go to zero usually must be replaced by exponents and logs, respectively. We have used three different RG-scaling ansätze to model the data in two dimensions. (1) We used the traditional scaling form $\xi \sim (R - R_c)^{-\nu}$, deriving $\nu = 5.3 \pm 1.4$ and $R_c = 0.54 \pm 0.04$. These collapses worked as well as any, but the standard form has more free parameters to fit with. Also, the large value for $\nu$ (and larger values still for $1/\sigma = 10 \pm 2$) makes one suspicious. (2) We used a scaling form suggested by Bray and Moore [26] in the context of the equilibrium thermal random-field Ising model where $R_c = 0$: if they assume that $R$ is a marginal direction, then by symmetry the flows must start with $R^3$, leading to $\xi \sim \exp(A/(R - R_c)^2) \equiv \exp(A/R^2)$. This form had the fewest free parameters, and most of the collapses were about as good as the others (except notably for the finite-size scaling of the moments of the avalanche size distribution, which did not collapse well once spanning avalanches became common). (3) We developed another possible
scaling form, based on a finite $R_c$ and $R$ marginal, which generically has a quadratic flow under coarse-graining: here $\xi \sim \exp(A/(R-R_c))$. Here again the moments did not scale well; we find $R_c = 0.54 \pm 0.04$, quite compatible with the traditional scaling collapse. This is not a surprise: it is always hard to distinguish large power laws from exponentials. Amazingly enough, the exponents plotted in figure 2 were largely independent of which scaling form we used! The error bars shown span all three ans"atze, and are compatible with our conjectures above.

We are not the only ones to model avalanche behavior in disordered magnets. There has been much work on depinning transitions and the motion of individual interfaces [27,28]; our system, with many interacting interfaces, perversely seems much simpler to analyze. Many have studied related models with random bonds [29,30] and random anisotropies; random fields are actually rather rare in experimental systems. We now believe on symmetry grounds that all these systems are in the same universality class (as argued numerically [30] and previously shown for depinning [28]). The external field $H_c$ at the critical point breaks the rotational and up-down symmetries of these models (and of the experiments!), and the spins which flip far from the critical point (roughly $M(H_c)$) act as random fields. On the other hand, we ignore long-range forces (discussed above) and long-range correlations in the disorder (e.g., dislocation lines and grain boundaries): these likely will lead to closely related but distinct universality classes.

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### TABLE I. Critical exponents obtained from numerical simulations [{12,14}] and experiments on Barkhausen noise in different magnetic materials (Fe, alumel, metglass [3], NiS [4], SiFe [5,6,7], 81%NiFe [8], AlSiFe [9], and FeNiCo [10]). The sample shapes were mostly wires. The quoted exponents were experimentally obtained from the pulse-area distribution in a small bin of the magnetic field $H$ (exponent $\tau$), the pulse-area distribution integrated over the entire hysteresis loop $(\tau + \sigma \beta \delta)$, the distribution of pulse durations in a small bin of $H$ $((\tau - 1)/\sigma \nu z + 1)$, the distribution of pulse durations integrated over the loop $((\tau + \sigma \beta \delta - 1)/\sigma \nu z + 1)$, the power spectrum of the pulses in a small bin of $H$ $((3 - \tau)/\sigma \nu z)$, the power spectrum of the pulses integrated over the hysteresis loop $((3 - (\tau + \sigma \beta \delta))/\sigma \nu z)$, and the distribution of pulse energies in a small bin of $H$ $((\tau - 1)/(2 - \sigma \nu z) + 1)$. Notice that these experiments are mostly done in geometries which minimize the effects of demagnetization fields (deadly to our model).
FIGURES

FIG. 1. Avalanche size distribution curves in 3 dimensions integrated over the external field. From left to right, the first three curves are for system size $320^3$ and disorders $4.0$, $3.2$, and $2.6$. They are averages over different initial random field configurations. The last curve is a $1000^3$ run at $R = 2.25$, where $R_c = 2.16$ and $r = (R - R_c)/R$. The inset shows the scaling collapse of curves in 3d. The disorders range between $2.25$ and $3.2$; the top curves at $R = 3.2$ show noticeable $10\%$ corrections to scaling. In the main figure, the avalanche size distribution curves obtained from the fit to this data (thin lines) are plotted alongside the raw data (thick lines). Notice that the scaling theory is predictive up to $R = 3.2$, $50\%$ above $R_c$. The long-dashed straight line is the expected asymptotic power-law behavior $s^{-2.03}$. Notice that it does not agree with the measured slope of the raw data.

FIG. 2. Numerical values (filled symbols) of the exponents $\tau + \sigma \beta \delta$, $\tau$, $1/\nu$, $\sigma \nu z$, and $\sigma \nu$ (circles, diamond, triangles up, squares, and triangle left) in 2, 3, 4, and 5 dimensions. The empty symbols are values for these exponents in mean field (dimension 6). Note that the value of $\tau$ in 2d was not measured. The empty diamond represents the conjectured value (see text). We have simulated sizes up to $7000^2$, $1000^3$, $80^4$, and $50^5$, where for $320^3$ for example, more than 700 different random field configurations were measured. The long-dashed lines are the $\epsilon$ expansions to first order for the exponents $\tau + \sigma \beta \delta$, $\tau$, $\sigma \nu z$, and $\sigma \nu$. The short-dashed line is the Borel sum for $1/\nu$ to fifth order in $\epsilon$. The error bars denote systematic errors in finding the exponents from collapses of curves at different values of disorder $R$. Statistical errors are smaller.
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which need not determine a unique underlying function (J. Zinn-Justin “Quantum Field Theory and Critical Phenomena”, 2nd edition, Clarendon Press, Oxford (1993)). Our model likely has non-perturbative corrections (as did the equilibrium, thermal random-field Ising model: G. Parisi, lectures given at the 1982 Les Houches summer school XXXIX “Recent advances in field theory and statistical mechanics” (North Holland), and references therein). For obvious but perhaps foolish reasons, we did not fix the pole at $\epsilon = 3$ as our model should have its pole at $\epsilon = 4$; we are currently trying to duplicate the variable-pole analysis with the correct fifth order term.

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Exponents vs. Dimension (d)

- $\tau + \sigma \beta \delta$
- $\tau$
- $1/\nu$
- $\sigma \nu_z$
- $\sigma \nu$

Dimension (d):
- 2
- 3
- 4
- 5
- 6

Exponents:
- 0.0
- 1.0
- 2.0