Study of Instability of Liquid Jets Under Gravity

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Abstract

Breakup of water jets under gravity is a common-place phenomenon. The role of surface tension in the instability of water jets was recognized by Rayleigh and the theory propounded goes by the name of Plateau-Rayleigh theory. The necks and bulges down along the jet-length that are created by perturbation waves of wavelengths larger than a certain value keep growing with time and ultimately cause the jet to breakup into drops. The effect of perturbation waves have been investigated experimentally and found to confirm the essentials of the theory. However, there is no unanimity about the origin of these perturbation waves. Recently, the idea of recoil capillary waves as an important source of the perturbation waves has been emphasized. The recoil of the end point of the remaining continuous jet at its breakup point is considered to travel upward as a recoil capillary wave which gets reflected at the mouth of the nozzle from which the jet originates. The reflected capillary wave travels along the jet downward with its Doppler shifted wavelength as a perturbation wave. We set up an experiment to directly verify the existence and effect of the recoil capillary waves and present some preliminary results of our experiment.

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I. INTRODUCTION

A continuous water jet falling gently downward through a tap and its subsequent breakup into discontinuous drops is an everyday-witnessed common phenomenon. There have been many attempts theoretically as well as experimentally to understand the physical mechanism behind the breaking up of jets into drops instead of continuing to keep reducing its cross-sectional area forever. However, the complete understanding of this macroscopic phenomenon has eluded so far.

The scientific study of this phenomenon has a long history. Savart (1833) experimentally observed the instability of liquid jets that clearly indicated that the liquid column (jet) undergoes changes before breaking up into disjoint droplets. Savart’s experimental observations suggested that the liquid column develops necks and bulges in the form of waves which ultimately leads to the formation of disjoint droplets out of the continuous column. This problem of instability of liquid jets was taken up by Plateau and then by Rayleigh (1879) and developed a theory which came to be known as Plateau-Rayleigh theory. Rayleigh showed that perturbations of wavelengths larger than a certain value (larger than the jet diameter) grow rapidly with time which ultimately make the column unstable (Rayleigh instability) against formation of droplets based on surface energy considerations of inviscid liquids. Chandrasekhar later extended the theory to viscous liquids. Many experimental investigations have been conducted to examine the validity of Plateau-Rayleigh theory. The investigations on the instability of water jet and related phenomena have been extensively reviewed. The study continues to be of current interest.

The Plateau-Rayleigh theory, describes how the response of perturbations of certain wavelengths keep growing with time whereas those with lower wavelengths decay with time. It is the fastest growing perturbation that ultimately breaks the jet. However, the theory is silent about the origin of the perturbations. The experiment of Goedde and Yuen, for example, applied external perturbations to study the length of the liquid jet (measured from the root of the jet) before it breaks up. However, even if no external perturbations are applied, one obtains a finite deterministic length of the liquid jet. In this case, obviously, the perturbation must have an origin at the root of the jet at the mouth of the nozzle, through which the liquid flows out to form the jet. Naturally, the perturbations will have amplitudes of random sizes. As a consequence, the length of the jet should have random values. However,
on the average, the length turns out to be a deterministically fixed number depending on the conditions of the experiment. In some recent works, in order to overcome this difficulty of fixing the origin of the perturbations, Umemura and co-workers\[9, 10\] emphasized the idea that the perturbations get generated and sustained self-consistently. This is based on the observed fact that soon after the jet breaks up (at one of the points on the lowermost neck) the tip of the remaining column contracts to make its shape round once again to minimize the surface energy. The tip contraction gives rise to an upstream propagating capillary wave which upon reflection at the mouth of the nozzle moves downstream with Doppler modified wavelengths. Some of these waves with the right wavelength (as discussed by Rayleigh) cause the liquid column to break producing another contraction of the tip of the column and the process repeats once again. We set up and conduct an experiment to verify the existence and effect of recoil capillary waves on the length of continuous water jet. This we accomplish by damping the recoil capillary wave using an earlier used method\[11\].

II. EXPERIMENTAL SET UP AND EXPERIMENT

![FIG. 1: Photograph of the experimental set up.](image)

The experimental set up (Figs.1,2) consists of two water tanks, the first one (Tank 1) is placed at a lower height to maintain the constant level of water (i.e., to keep same pressure throughout the experiment) and the second one (Tank 2) is placed at a higher height to supply water at a constant rate to the Tank 2. Both of these tanks are fitted with one tap
FIG. 2: The diagramatic sketch of the experimental set up.

each. The tap of the Tank 2 is connected to the inlet of Tank 1 by a rubber pipe. A glass nozzle is connected to a tap of the Tank 1 by a rubber pipe through a flow control valve, which permits us to control the flow rate of water. The excess water in the Tank 1 flows out through the level outlet to the lower reservoir tank (Tank 3) (this tank is kept at the bottom level). The water from the Tank 3 is pumped up to the Tank 2 by using a water pump. A transparent rectangular beaker with a level outlet on one of its vertical sides is placed vertically below the nozzle so that the water jet falls directly on the water kept in the beaker. The water level in the beaker is maintained fixed by letting the excess water flow out through the level outlet of the beaker. The beaker is placed on a horizontal platform fitted to a travelling microscope so that the beaker can be smoothly moved vertically up and down and its position measured. A laser-pointer-and-detector arrangement is also fitted to the platform so that the horizontal laser beam incident normally on the vertical surface of the beaker and passes through the path of the water jet and then through the opposite surface of the beaker before it is collected by the detector. The laser-detector arrangement can be moved vertically and fixed as desired and the vertical position of the horizontal laser beam measured. A six digit counter mounted with a 24-hour clock is connected to the detector to count the number of drops and also to pin-point the first initial breaking point of the jet. Our experimental set up is similar in essentials to that of Goedde and Yuen (compare Fig.1 of Ref.[4]). The ambient condition of temperature and humidity is controlled and allowed to settle down at desired values before the experiment is conducted and the condition is kept
fixed throughout the duration of experiment.

The water is issued (jetted) vertically downward through a long glass nozzle (of length larger than about ten times its internal diameter, $d$) into the water in the beaker through the stagnant ambient air atmosphere. As long as the jet remains continuous the detector remains quiescent. However, a breakup into a drop (disjoint of the jet) is detected as a count. The vertical distance between the nozzle exit and the laser beam gives the breakup length. Initially, the laser beam is made to face the continuous jet by moving the platform up and then the platform is gradually lowered so that the beam position distance from the nozzle exit increases. At every position of the laser-detector arrangement the counts are recorded for two minutes for several times and their average calculated to obtain the average number of drop-counts per second. The platform is slowly and gradually lowered by small steps and at each position the experiment is repeated to obtain the average count rate. Naturally, the count rate begins from zero, reaches a threshold at which the rate just begins to show nonzero value and then gradually keeps increasing as the platform is slowly lowered in small steps. The process continued till the rate reaches a maximum (saturation) value. Throughout the above process the water flow rate is kept fixed. The same process is then repeated for several values of flow rates. Note that after each change of flow rate the flow and the jet is allowed to become steady before the measurement process is begun.

For our purpose, we perform two distinct kinds (sets) of experiments. In one we let the laser beam pass grazing the water level in the beaker (just about 0.2 cm above the water surface on the beaker). In the other the beam is kept at a height of about 1.5 cm above the water surface. In the first set of experiments, if the point of separation between a drop and the tip of the remaining jet is such that before the moving tip recoils it just touches the water surface, the process of jet-tip recoil is prevented and thus the recoil capillary wave propagating up the jet length is damped whereas in the other set of experiments this situation never occurs.

The experiment is performed in an enclosure where the air current is minimized and, as stated earlier, the temperature and humidity are kept fixed for all sets of measurements. Thus, the enclosure inside the lab is made sure to be stagnant and the external disturbance on the jet is minimized.
FIG. 3: Count rate (s$^{-1}$) as a function of distance of the laser beam from the nozzle exit at the flow rate of 55.5 cc/min and $d = 1.71$ mm for the two sets of experiments (laser beam close to the water surface and far from the surface. The inset shows a magnified graph to show the first (jet) breakup point.

Figure 3 shows the average number of counts (drops) per second, for a water flow rate of 55.5 cc/min and the nozzle inner diameter of 1.71 mm, as the vertical distance of the laser beam from the lower tip of the nozzle is increased. Each point on the graph corresponds to an average of three set of counts for a duration of two minutes each. The counts begin from zero (indicating continuous jet) to a saturation value (indicating the finality of the breakup process). From this data we calculate the distribution, $\rho$, of breakup lengths. The probability density of breakup lengths is roughly calculated as the derivative of the count rate with respect to breakup length, Fig. 4. In Fig.4 the distribution $\rho$ is not normalized. The normalized distribution $\rho_n$ will be obtained as $\rho_n = \rho/100$. The distribution provides information about the mean value of breakup lengths. The mean values are plotted in Fig.5 for the same nozzle of inner diameter 1.71 mm but for different flow rates for the two sets of experiments (with and without damping). The two sets of mean values lie in a band. The approximate equality of the two mean values is understandable, however, because once the water level in the observation beaker is well below the breakup point, it is as good as being far below and equivalent to a point of the set without damping.
FIG. 4: Count rate \( (s^{-1}) \) as a function of distance of the laser beam from the nozzle exit (same as Fig.3 but only for the set close to the surface). It also shows the corresponding unnormalized breakup length distribution.

The inset of Fig.3 gives a magnified diagram of the curves of Fig. 3 in the lower range of the distance of laser beam from the nozzle exit. From this graph one can obtain the first (initial) breakup lengths in the two cases (with and without damping) and are plotted in Fig. 5 as a function of water flow rates. The figure clearly shows that the first breakup length with the damped recoil capillary wave is larger than that without the damping for all values of flow rates measured. This indicates that the recoil capillary waves do exist and they aid in making the water jet unstable against breakup.

The mean breakup lengths plotted in Fig.5, though not exactly identical to the earlier reported results (for example, p. 104 of Ref.[7]), the qualitative features are very similar, showing various regimes of jet breakup. However, information on the first breakup length is entirely new. It shows that the first breakup lengths peak at a smaller Reynolds number (mean Re=1079.3) and Weber number (We=7.54) than in case of the mean breakup lengths (Re=1392.6, mean We=12.51). Our measurements are done at the temperature of \( (25 \pm 5)^\circ C \) and at relative humidity of \( 80 \pm 3\% \). However, in order to calculate Re and We, we have used the tabulated values of surface tension of water \( \sigma = 72 \times 10^{-3} \text{Nm}^{-1} \) and coefficient of viscosity \( \mu = 8.9 \times 10^{-4} \text{kg m}^{-1}\text{s}^{-1} \).
FIG. 5: First (lower set of two curves) and mean (upper set of two curves) breakup lengths (b.l.) as a function of water flow rate through the same nozzle of internal diameter, d=1.71 mm.

of the distance of laser beam from the nozzle exit. From this graph one can obtain the first (initial) breakup lengths in the two cases (with and without damping) and are plotted in Fig. 5 as a function of water flow rates. The figure clearly shows that the first breakup length with the damped recoil capillary wave is larger than that without the damping for all values of flow rates measured. This indicates that the recoil capillary waves do exist and they aid in making the water jet unstable against breakup.

IV. CONCLUSION

We have performed a very simple experiment the results of which may be considered to directly give a proof of the existence of recoil capillary waves and their effect on the jet breakup length. Though the experiment is not so precise and sophisticated the qualitative features shown are unmistakable. A high speed photographic measurement may help in arriving at a result with more confidence. We hope to obtain a conclusive result once we have the required facility.

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