Research on Prediction Accuracy of Mathematical Modeling Based on Big Data Prediction Model

Rui Guo*
Beijing Jiaotong University Beijing, China

*Corresponding author: ruiguo888@bjtu.edu.cn

Abstract. At present, people's food demand in the late stage of the new coronavirus is affected by many aspects and various factors, and the current prediction method is affected by redundant data, which leads to poor accuracy of the prediction results. In addition, in order to prevent the mismatch between food supply and demand in the event of major health events, the prediction model was selected in the big data environment and the prediction accuracy of the mathematical model was studied. Through the improvement of various prediction models and the comparison of prediction accuracy, it is found that the improved grey Verhulst has higher accuracy and is more suitable for short-term prediction.

1. Introduction
During the epidemic, affected by the epidemic, a large number of users flooded into the new e-commerce platform. Fresh electricity platform new users maintain an upward trend, the overall cost of the platform has decreased, and at the same time, a large number of "incremental users" are difficult to reach offline. Therefore, ensuring the supply of fresh food during and after the epidemic period is the current problem of fresh food e-commerce, which is also an opportunity for fresh food e-commerce to retain users and release growth potential. However, people's demand for food in the late stage of the epidemic is affected by many factors from many aspects. Therefore, in the context of big data, it is particularly important to select an appropriate prediction model through comparative analysis of the accuracy of the prediction model.

2. Literature Review

2.1. BP neural network method
Lin Lian and Lin Hua (2009) improved the traditional BP neural network model to improve its convergence speed, aiming at the defect of slow convergence speed. Liu Meilian and Zhu Meihua (2012) took the total import and export volume of foreign trade, the total output value of the primary industry and the total output value of the tertiary industry as the input variables of the BP neural network and the port throughput as the output variable, and established the BP neural network prediction model to predict the port throughput. Gao Xiuchun et al. (2014), based on BP network model, established a scientific and operational port logistics demand prediction model by analyzing it. Cai Wanzhen and Huang Han (2019) proposed a combined forecasting model based on BP and RBF neural network, and prove that the prediction precision of combination forecast model than the single forecasting model has higher
prediction accuracy, the probability of a larger can effectively reduce the error, the predicted results closer to the actual situation and provide the basis for port logistics development planning.

2.2. Exponential smoothing method
Xie Xiaoyan et al. (2013) made short-term prediction of logistics demand based on the actual situation of logistics demand in Huhu, Baotou and Hubei triangle and combined with the exponential smoothing prediction model. Yu Bo et al. (2018) made use of the two indexes of the quantity of goods and the turnover of goods in Yunnan Province from 2009 to 2017 and predicted the logistics demand of Yunnan Province in the next few years based on the forecasting method of exponential smoothing method. Xiao Yunmei et al. (2019) selected and determined the predicted types of typical agricultural products, investigated the production of agricultural products in Changsha-Zhuzhou-Xiangtan region from 2013 to 2018, and used the quadratic exponential smoothing method to forecast and analyze the typical agricultural products in Changsha-Zhuzhou-Xiangtan region from 2019 to 2023.

2.3. Grey theory
Chen Haodong and Duanmulingfeng (2018) established a logistics demand prediction model based on the grey theory, considering the grey prediction theory and the actual logistics demand of Henan Province. The case analysis and simulation are carried out to the established model. Liu Zhimin and Zhao Xingna (2019) took the effective supply of agricultural graduates from 2013 to 2017 as the original sequence, established the time series GM (1,1) model of the demand for agricultural talents, and predicted the total demand for agricultural talents and its structural level demand in China from 2020 to 2025. Wu Qian (2019) considers economic development, industrial structure, energy consumption structure, national consumption level, a variety of factors such as environmental policy, combining with related data availability and uniformity, the gross national product, per capita consumer spending, the added value of the second industry, coal import and export volume, urban population, the total coal production, the total energy consumption, ratio of the average annual precipitation PH 5.6 or less cities in our country for impact factor. The coal consumption is selected as the index of logistics demand forecast. The grey model was combined with BP neural network, and the new fruit fly algorithm optimization model was used to improve the combination model for the slow convergence speed of neural network and easy to fall into local optimum.

2.4. Gray Verhulst model
Considering that it is difficult to collect the demand data of Ding Dong food during the forecast period in this paper, the research idea in the paper of Lan Hongjie and Ru Yihong (2008) on the forecast analysis of the demand of food cold chain logistics for Beijing Olympic Games was adopted, and the total demand of fresh products was replaced by the product of the per capita daily demand and the total number of people in demand. For the prediction of the total number of people in demand, according to the trend analysis of the predicted objects during the research period and the limitations of data collection, this paper chooses the grey Verhulst model for prediction. Some scholars at home and abroad have applied this model to the demand prediction of different things.

Gross T J et al. (2011) discretized the corresponding differential equations of the gray Verhulst model through Euler numerical integration method. Anton Bezuglov and Gurcan Comert (2016) studied the accuracy level of first-order univariate grey model, GM (1,1) model with Fourier error correction and grey Verhulst model with Fourier error correction in the prediction of short-term traffic speed and travel time. C Zhang et al. (2017) established an optimized grey discrete Verhulst model using the metabolic method, and the change from the continuous form to the discrete form reduced the error from the differential equation to the difference equation in the modeling. Wang Zheng-xin and Li Qin (2019) used particle swarm optimization algorithm to optimize the structural parameters of the model and deduced the non-equidistance grey Verhulst model.

Long Zhao et al. (2015) used arithmetic mean estimation or geometric mean estimation to solve the four parameters in the model on the basis of optimizing grey action, and proposed a grey Verhulst direct
modeling model. Lu Minrong et al. (2017) proposed a grey Verhulst model with improved initial and background values. Zhang Kan et al. (2017) proposed the nonlinear residual grey Verhulst model for research, and selected EGA algorithm to establish the grey Verhulst econometric combination prediction model. Wang Haicheng et al. (2017) improved and optimized the grey derivative of the grey Verhulst model and proposed the optimal estimation formula for the model parameters. He Zhenggang and Huang Juan (2018) used particle swarm optimization (PSO) to optimize the parameter values of the gray Verhulst model.

3. Research Thought

From the perspective of fresh APPs on the current market in China, Hema Xiansheng, Daily Youxian, Jingdong Jiaju, Ding Dong Buy Food, Duandian and Meituan Buy Food have a wide audience. However, considering that the operation scope of some APPs is not only fresh products, for example, Hema Fresh Food can also deliver cooked food, and Multipoint is the combination of fresh products and supermarket distribution. However, the Ding Dong food platform basically only contains the sales of fresh products, so this paper predicts the fresh demand caused by the demand of Ding Dong food users (demand subject) for fresh products (demand object) in the late period of the epidemic (April 15, BBB 0, May 15). For the demand object, according to the definition of fresh products, it should include the demand for aquatic products, meat, eggs, milk and fruits and vegetables.

The forecast idea of this paper is that the total daily demand for fresh is equal to the product of the total number of daily demand subjects and the per capita daily demand for fresh. Therefore, this paper first predicted the number of daily active users of Dingdong Maimai, and then predicted the total daily demand of fresh products of Dingdong Maimai APP at the later stage of the epidemic based on the predicted results and combined with the daily demand of fresh products per capita and the number of copies purchased.

Before explaining the calculation method, this paper first makes the following assumptions about the model:

Given the platform during the outbreak of order data are difficult to collect, therefore, based on the average daily active users buy vegetables tinkling (this data is to heavy) as the number of orders, namely the user login platform will place the order, and an order to buy all the fresh products, and not to tear open sheet, this assumption is consistent and outbreak.

After some customers place an order with APP, the fresh quantity ordered is the demand of a whole family, that is, a customer corresponds to two or even three per capita daily demand. Therefore, considering the mismatch between the order number and the per capita daily demand and the family structure in most of China, this paper introduces the correlation coefficients \( a, b \) and \( c \) to represent the per capita daily demand for one, two and three orders, respectively.

For the per capita daily demand, according to the relevant data in China Statistical Yearbook, and using the moving average method, we forecast the per capita daily demand for fresh products of the national residents during the epidemic period in 2020.

4. Demand Forecasting

4.1. Forecast of average daily active population

We counted the user activity of the Ding Dong Food App on January 15, solstice and April 1. For the forecast of the demand of the next fresh electricity supplier, in order to find its statistical law, the relevant data are sorted out and the graph of user activity on January 15 solstice on April 1 is drawn, as shown in the figure:
Figure 1. January 15 solstice April 1 Ding Dong grocery shopping average daily active number.

Through the ding-dong shopping APP users, active statistical data analysis, you can see that at the beginning of the outbreak, the APP, the rapid growth of the number of active presents exponential, as epidemic situation under control, people's easing of travel restrictions to some extent, day gradually reduce the number of active growths, overall change trend of the saturated "S" shape, showed a strong regularity.

Through the above analysis, it can be seen that, firstly, there is not a lot of data information to refer to about the surge in demand caused by the epidemic. The statistical yearbook data are generally the results of previous years, and the statistical data of various e-commerce platforms only describe the growth trend roughly, without specific demand data. To predict the demand of fresh e-commerce platforms under the condition of the net increase of epidemic demand, it is not enough to rely on the data and growth trend of previous years. Therefore, the demand of fresh e-commerce platform during the epidemic period is characterized by "poor information" and "small sample"; Secondly, the active number of users who bought vegetables in Ding Dong during the epidemic period showed the characteristics of "S" shape, with a relatively fixed law. Therefore, in the construction of the prediction model, the two characteristics should be comprehensively considered. The grey Verhulst model is a special form of GM (1,1) exponent, which is mainly used to describe the process with saturation state, that is, the whole data feature is saturated "S" shape. Based on the above analysis, the grey Verhulst model is constructed in this section to predict the number of users in Ding Dong's grocery shopping from April 15 to May 15.

4.1.1. Overview of grey prediction theory. Grey system forecasting theory is a factor in people's relationship is not entirely clear, system mechanism is not understood completely, system factor is not entirely clear, the structure of the system is not fully know the grey system, by a small amount of, incomplete information, using the gray accumulation or b-b operator to deal with the original data, and generate a certain sequence to find the rule of system changes make the system of grayscale decreases gradually increase, white degree, and then establish a grey differential model forecast things in the future development trend of science. For the data with the characteristics of "small sample" and "poor information", the grey prediction model only needs to select the corresponding grey prediction model based on a small amount of basic data to simulate and forecast the future short-term and medium-term data.

Grey Verhulst prediction model is an important part of grey prediction theory. It was derived by German biologist Verhulst in 1837 when he studied the law of biological reproduction. The prediction
model is a single sequence first-order nonlinear dynamic model. At the same time, the grey Verhulst model describes a process with saturation state. Things show exponential rapid growth at the beginning, and as time goes by, the growth rate slows down and finally stabilizes at a fixed value due to the limitation of the surrounding external environment. That is to say, the grey Verhulst prediction model is mainly used to describe the grey dynamic change process when the whole data is saturated with "S" shape characteristics. Up to now, this model has been widely used in population prediction, animal reproduction prediction, biological growth prediction and product economic life prediction. However, the classical grey Verhulst prediction model is not very accurate in some aspects, so it needs to be improved and optimized. Therefore, many experts and scholars have carried out a lot of academic research on the optimization and improvement of the gray Verhulst model by establishing the corresponding gray Verhulst discrete model, modifying the initial value of the model, optimizing the parameter estimation method of the model and improving the gray derivative of the model.

4.1.2. Modeling principal analysis of grey Verhulst prediction model. Let the original data sequence be $x^{(1)}$, $x^{(0)}$ be the first-order reduction generation sequence of $x^{(1)}$, and $z^{(1)}$ be the generation sequence adjacent to the mean value of $x^{(1)}$. The exponential model is $GM(1,1)$.

Adjacent to the mean of $x^{(1)}$ generation, order

$$z^{(1)}_k = a x^{(1)}_k + (1 - a) x^{(1)}_{k-1}$$  \(\text{(2)}\)

According to the grey system modeling method, the nonlinear dynamic differential equation of Equation (2) can be obtained as follows:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b(x^{(1)})^2$$  \(\text{(3)}\)

Equation (3) is the whitening equation of the grey Verhulst model.

The least square method can be used to estimate the parameters $a$ and $b$ in the formula. Let's say:

$$B = \begin{bmatrix} -z_2^{(1)} & (z_2^{(1)})^2 \\ -z_3^{(1)} & (z_3^{(1)})^2 \\ \vdots & \vdots \\ -z_6^{(1)} & (z_6^{(1)})^2 \end{bmatrix}, \quad Y = \begin{bmatrix} x^{(0)}_2 \\ x^{(0)}_3 \\ \vdots \\ x^{(0)}_6 \end{bmatrix}$$  \(\text{(4)}\)

Then the least squares parameter of parameter column $\hat{a} = (a, b)^T$ of the gray Verhulst model is estimated as:

$$\hat{a} = (B^T B)^{-1} B^T Y$$  \(\text{(5)}\)

The solution of Verhulst whitening equation can be obtained as follows:

$$x^{(1)}(t) = \frac{ax^{(1)}(0)}{bx^{(1)}(0) + (a - bx^{(1)}(0))e^{-kt}}$$  \(\text{(6)}\)

The solution of Verhulst whitening equation can be obtained as follows:
4.1.3. Model solving

Table 1. Ding Dong Buy Vegetables Daily Average Active Number Statistics.

| date       | 1.15 | 1.31/2.1 | 2.15 | 2.29/3.1 | 3.15 | 3.31/4.1 |
|------------|------|----------|------|----------|------|----------|
| Number of living persons per day (ten thousand) | 770  | 1165     | 2200 | 2207.3   | 2203 | 2206     |

The average daily active number of Ding Dong food buyers in 2020 from 1.15 to 3.31 is shown in Table 1. The original data are taken to establish the Verhulst model and forecast (simulation). The specific modeling process is as follows.

a) Set $x_1^{(1)}$ as the original data list of the average active number of Ding Dong food buyers from January 15 to 3.31, i.e

$$x_1^{(1)} = (x_1^{(1)}, x_2^{(1)}, \ldots, x_6^{(1)}) = (70,165,200,207.3,203,206)$$

b) A is generated by $x_1^{(1)}$ degradation (1-IAGO), which is composed of

$$x_k^{(0)} = x_k^{(1)} - x_{k-1}^{(1)}, k = 2,3,\ldots,6$$

$$x^{(0)} = (x_1^{(0)}, x_2^{(0)}, \ldots, x_6^{(0)}) = (70,95,35,7.3,-4.3,3)$$

c) For $x^{(1)}$ is close to the average generation, order

$$z_k^{(1)} = \alpha x_k^{(1)} + (1 - \alpha)x_{k-1}^{(1)}, k = 2,3,\ldots,6$$

Let's say $\alpha = 0.5$ is equal to

$$z^{(1)} = (z_2^{(1)}, z_3^{(1)}, \ldots, z_6^{(1)}) = (117.5,182.5,203.65,205.15,204.5)$$

So

$$B = \begin{bmatrix} -z_2^{(1)} & (\frac{z_2^{(1)}}{x_2^{(0)}})^2 \\ -z_3^{(1)} & (\frac{z_3^{(1)}}{x_3^{(0)}})^2 \\ \vdots & \vdots \\ -z_6^{(1)} & (\frac{z_6^{(1)}}{x_6^{(0)}})^2 \end{bmatrix}, Y = \begin{bmatrix} x_2^{(0)} \\ x_3^{(0)} \\ \vdots \\ x_6^{(0)} \end{bmatrix}$$

$$d) The least squares estimation of parameter column \( \hat{\alpha} = (a, b)^T \) is obtained

$$\hat{\alpha} = (B^T B)^{-1} B^T Y = \begin{bmatrix} 1.87331288 \\ -0.00913104 \end{bmatrix}$$

e) The hulst model is
\[
\frac{dx^{(1)}}{dt} = 1.87331288x^{(1)} = -0.00913104 \left(x^{(1)}\right)^2
\]

(15)

Its time response is

\[
\frac{dx_k^{(1)}}{dt} = \frac{\beta x_k^{(1)}}{b x^{(1)} + (a - b x_k^{(1)}) e^{ak}} = \frac{131.13190167}{0.63917285 + 1.23414003 e^{-1.87331288k}}
\]

(16)

f) The predicted value of \( x^{(1)} \) is \( \hat{x}^{(1)} \)

\[
\hat{x}(1) = (\hat{x}_1^{(1)}, \hat{x}_2^{(1)}, \ldots, \hat{x}_6^{(1)}) = \begin{pmatrix}
70.15822778858, 196.21854684, 203.73281595, \\
204.93840863, 205.12486983
\end{pmatrix}
\]

(17)

g) Model accuracy check

The residual difference as

\[
\epsilon^{(0)} = (\epsilon_1, \epsilon_2, \ldots, \epsilon_6) = (x_1^{(1)} - \hat{x}_1^{(1)}, \ldots, x_6^{(1)} - \hat{x}_6^{(1)}) = \begin{pmatrix}
0.677221142, 3.78145316, 3.56718405, \\
-1.93840863, 0.875130167
\end{pmatrix}
\]

(18)

Then the mean and variance of \( x^{(1)} \) are respectively

\[
\bar{x} = \frac{1}{6} \sum_{k=1}^{6} x_k^{(1)} \approx 173.04
\]

\[
S_1^2 = \frac{1}{6} \sum_{k=1}^{6} (x_k^{(1)} - \bar{x})^2 \approx 2427.74
\]

(19)

Then the mean and variance of \( \epsilon^{(0)} \) are respectively

\[
\bar{\epsilon} = \frac{1}{6} \sum_{k=1}^{6} \epsilon_k \approx 2.1763
\]

\[
S_2^2 = \frac{1}{6} \sum_{k=1}^{6} (\epsilon_k - \bar{\epsilon})^2 \approx 8.1656
\]

(20)

The mean square deviation ratio is

\[
C = \frac{S_2}{S_1} \approx 0.0580
\]

(21)

Therefore, the model has a high precision of level 1 and can be used for demand forecasting.

h) Comparison of forecast results

According to the daily active population data of Ding Dong Maicai between 1.15 and 3.31, the corresponding GM (1,1) model (gray model) can also be obtained, with a mean square deviation ratio of about 3.998\%. The accuracy of the model is level 2, which is higher than that of Verhulst model. By comparing the two prediction results, it is found that the predicted value of the GM (1,1) model increases rapidly in the last three periods, while the Verhulst model shows an S-shaped curve, which is closer to people's demand for fresh electricity suppliers after the epidemic situation is stable.
4.1.4. Ding Dong buy food daily active number forecast. The grey Verhulst model was used to predict the average daily active users of Dingdong Maimai, and the average daily active users of 4.15, 4.30/5.1 and 5.15 in the next three time points were obtained, as shown in Table 2.

Table 2. Predicted value of the average daily active users of dingdong shopping in the next three time points.

| date   | 4.15     | 4.30     | 5.15     |
|--------|----------|----------|----------|
|        | 205.1535 | 205.1579 | 205.1586 |

4.2. Forecast of daily demand per capita
As for the per capita daily demand, according to the above hypothesis, the relevant data in China Statistical Yearbook and the moving average method are used to forecast the per capita daily demand for fresh products of national residents during the epidemic period in 2020. The per capita daily demand is obtained by dividing the trading volume of fresh products in the trading market by the national resident population in that year, and then dividing by 365 (see Table 1).

Table 3. National per capita daily demand for fresh products.

| date       | 4.15     | 4.30     | 5.15     |
|------------|----------|----------|----------|
| Demand (tons) | 3674.012 | 3674.091 | 3674.103 |

Since the data in this paper have strong stability and only need to forecast the data in 2020, that is, the next year, the one-time moving average method is used to make the prediction. According to the data characteristics, N is set as 2. The prediction formula is:

$$\bar{x}_{t+1} = M^{(1)}_t(2) = \frac{1}{2} (x_t + x_{t-1})$$  \hspace{1cm} (22)

The forecast results are shown in Table 4. In 2020, the per capita daily demand for fresh products will be 0.593 kg.

Table 4. National per capita daily demand for fresh products.

| year     | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 | 2020 |
|----------|------|------|------|------|------|------|------|------|
| Demand (kg) | 0.544 | 0.551 | 0.564 | 0.582 | 0.586 | 0.590 | 0.596 |      |
| Predicted quantity (kg) | 0.547 | 0.557 | 0.573 | 0.584 | 0.588 | 0.593 |      |      |
4.3. Coefficient estimation of the number of copies purchased

According to the above hypothesis, active users may order one, two or three daily fresh demand per capita through Ding Dong. The corresponding coefficients of the preliminary hypothesis are a, b and c.

According to the 2010 national census data, the size of the average household in China is shrinking, and the composition of the household is also changing. Compared to 2000, 1, 2 people of miniature family number is increased, 2010 tiny family is close to 40% of all households, 3-4 people seriously reduce the number of small family homes, drop of 5-6 people medium-sized family accounted, accounting for the proportion of the country's households had fallen to 14.2%, more than 7 people and large number of households is also reduced.

According to the first Chinese Family Development Report released by the National Health and Family Planning Commission, China has about 430 million households and their families are getting smaller, with the average household size of 3.02 people.

Considering that the epidemic situation is slightly stable at the present stage, users' demand for storage will be relatively reduced, and each active user will purchase food for the whole family. Here, the order quantity of each active user is recorded as 3.02 pieces of fresh demand per person per day.

To sum up, combined with the calculation formula: cold chain fresh demand = daily active users × per capita daily demand × number of servings purchased, the cold chain fresh demand for Ding Dong vegetables in the next three time points can be obtained, as shown in Table 5.

| Table 5. Demand for cold chain fresh vegetables in the next three time points |
|-----------------|-----|-----|-----|-----|-----|-----|-----|
| year            | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 |
| Demand (kg)     | 0.544 | 0.551 | 0.564 | 0.582 | 0.586 | 0.590 | 0.596 |

It can be seen from the forecast results that the demand for cold chain fresh food in Ding Dong will be stable in the next three time points, and the growth rate will be significantly lower than that in the early stage of the epidemic, which is in line with the actual situation of consumers. Therefore, it is suggested to use the gray Verhulst model to make short-term prediction, and analyze and forecast according to recent historical data, so as to provide reference for enterprises to deploy cold-chain related strategies.

5. Summary and Reflection

In the context of big data, this paper takes the fresh demand of Dingdong buy food e-commerce platform as the research object. Using historical data, this paper adopts a variety of big data prediction models for comparative analysis, mainly to compare the accuracy of the models. The model is also improved to improve the calculation speed and prediction accuracy. However, this paper mainly carries out mathematical modeling and analysis on the three models of neural network model, grey model and grey Verhulst model, so there are still some limitations.

References

[1] Tadeu Junior Gross, Taufik Abrão, Paul Jean Etienne Jeszensky. Distributed power control algorithm for multiple access systems based on Verhulst model[J]. AEUE - International Journal of Electronics and Communications, 2010, 65(4).

[2] A. Ismail, D-S. Jeng. Modelling load-settlement behaviour of piles using high-order neural network (HON-PILE model) [J]. Engineering Applications of Artificial Intelligence, 2011, 24(5).

[3] Anton Bezuglov, Gurcan Comert. Short-term freeway traffic parameter prediction: Application of grey system theory models [J]. Expert Systems With Applications, 2016, 62.

[4] Ding, S; Dang, YG; Xu, N; Chen, D; Cui, J. The optimization of grey Verhulst model and its application [J]. J grey Syst, 2015

[5] Chuang Zhang, Jian-zhong Li, Yong HE. Application of optimized grey discrete Verhulst–BP
neural network model in settlement prediction of foundation pit[J]. Environmental Earth Sciences, 2019, 78(15).

[6] T. Fikret Kurnaz, Ugur Dagdeviren, Murat Yildiz, Ozhan Ozkan. Prediction of compressibility parameters of the soils using artificial neural network[J]. SpringerPlus, 2016, 5(1).

[7] Nejad, FP; Jaksa, MB Load-settlement behavior modeling of single piles using artificial neural networks and CPT data[J]. Comput Geotech, 2017.

[8] Zheng-Xin Wang, Qin Li. Modelling the nonlinear relationship between CO2 emissions and economic growth using a PSO algorithm-based grey Verhulst model[J]. Journal of Cleaner Production, 2019, 207.

[9] Mostafa Yousefi Ramandi, Nooshin Bigdeli, Karim Afşar. Stochastic economic model predictive control for real-time scheduling of balance responsible parties[J]. International Journal of Electrical Power and Energy Systems, 2020, 118.

[10] Zhang, C; Peng, ZB; Peng, WX Application of optimized grey discrete Verhulst model in settlement prediction of foundation pit[J]. J Cent South Univ, 2017.

[11] Zheng-Xin Wang, Dan-Dan Li, Hong-Hao Zheng. Model comparison of GM(1,1) and DGM(1,1) based on Monte-Carlo simulation[J]. Physica A: Statistical Mechanics and its Applications, 2020, 542.

[12] Tarawneh, B; Imam, R Regression versus artificial neural networks: predicting pile setup from empirical data[J]. KSCE J Civ Eng, 2014.