A first estimate of $\eta/s$ in Au+Au reactions at $E_{lab} = 1.23 \ A GeV$

Tom Reichert$^1$, Gabriele Inghirami$^{2,5,6}$, Marcus Bleicher$^{1,2,3,4}$

$^1$ Institut für Theoretische Physik, Goethe Universität Frankfurt, Max-von-Laue-Strasse 1, D-60438 Frankfurt am Main, Germany
$^2$ GSI Helmholtzzentrum für Schwerionenforschung GmbH, Planckstr. 1, 64291 Darmstadt, Germany
$^3$ John von Neumann-Institut für Computing, Forschungszentrum Jülich, 52425 Jülich, Germany
$^4$ Helmholtz Research Academy Hesse for FAIR, GSI Helmholtz Center, Campus Frankfurt, Max-von-Laue-Str. 12, 60438 Frankfurt, Germany
$^5$ University of Jyväskylä, Department of Physics, P.O. Box 35, FI-40014 University of Jyväskylä, Finland and
$^6$ Helsinki Institute of Physics, P.O. Box 64, FI-00014 University of Helsinki, Finland

The HADES experiment at GSI has recently provided data on the flow coefficients $v_1, ..., v_4$ for protons in Au+Au reactions at $E_{lab} = 1.23 \ A GeV$. This data allows to estimate the shear viscosity over entropy ratio, $\eta/s$ at low energies via a coarse graining analysis of the UrQMD transport simulations of the flow harmonics in comparison to the experimental data. By this we can provide for the first time an estimate of $\eta/s \approx 2 \pm 0.4$ (or $25 \pm 5 \ (4\pi)^{-1}$) at such low energies.

I. INTRODUCTION

Relativistic heavy ion collisions provide an excellent laboratory to explore the properties of strongly interacting matter in great detail. While at ultra-relativistic collision energies, e.g. at RHIC and LHC, Quantum Chromodynamics (QCD) predicts the creation of a deconfined state of matter, called the Quark Gluon Plasma (QGP), one expects to explore a super dense baryonic system at lower collision energies (RHIC-BES, FAIR/NICA, or GSI). One of the key results of the exploration of QCD matter in collider experiments has been the precise determination of the QCD transport coefficients. In particular, the shear viscosity $\eta$ which is usually scaled by the entropy density $s$ of the matter is in the center of interest. At ultra-relativistic energies, the shear viscosity $\eta$ can be rather directly extracted by analyzing the Fourier expansion of the radial flow distribution. Especially, the second Fourier coefficient $v_2$ can be measured at midrapidity (e.g. at RHIC) with great precision and allows to extract $\eta/s$ from the data by comparison to hydrodynamic simulations. Typical values of $\eta/s$ at collider energies are $\eta/s = 2 - 4 \ (4\pi)^{-1}$. These values are near the conformal bound and indicate that the QGP is among the most perfect liquids ever created. Although $\eta/s$ can not be smaller than the KSS bound in equilibrium, recent AdS/CFT calculations point out that out-of-equilibrium values of $\eta/s$ can exceed the boundary.

Unfortunately, the methods used at collider energies can not easily be applied at very low energies. The reason is the following: At very high energies, the initial conditions for the hydrodynamic evolution can be straightforwardly defined, e.g. at midrapidity by a time independent spatial profile with a fixed eccentricity $\epsilon$ in space. The shape of the spatial profile can be extracted from Monte-Carlo Glauber calculations or from a saturation model. This initial state then expands into vacuum and creates a finally observable $v_2$ in momentum space out of the initial eccentricity. The response of the momentum space to the spatial eccentricity is governed by the viscosity. I.e. typically one has $v_n \sim \epsilon_n \exp[-\eta/s \cdot n^2]$. Thus comparing $\epsilon_2$ with $v_2$ allows to extract the viscosity.

At lower energies, such an approach is not possible due to the intricate time dependence of the geometry of the colliding and expanding matter:

- First of all, the expected viscous correction might be too high to allow for a meaningful hydrodynamic simulation. Estimates of the shear viscosity over entropy ratio of a moderately hot hadron gas suggest $\eta/s = 10 - 100 \ (4\pi)^{-1}$ in the low energy regime.

- Secondly, a simple initial state (i.e. fixed in time and with a geometry inferable from the initial nuclei) for the hydrodynamic expansion can not be defined. The reasons for this problem are: I) The compression stage lasts very long ($\approx 3 - 10 \ fm$), which means that the initial baryon currents do not quickly decouple as they do at very high energies, hindering the definition of a time independent initial state with a fixed eccentricity $\epsilon_n$. II) Due to the slow movement of the spectators, the expanding matter sees a time dependent boundary for its expansion, the well known spectator shadowing. This leads to an emission time dependent in-plane and out-of-plane component in $v_2$. Both effects prohibit the construction of a simple hydrodynamic initial state at low energies.

To overcome these problems one can either employ a viscous multi-fluid approach to include the space and time dependent source terms for the midrapidity fluid, or one can rely on transport simulations to extract the shear viscosity to entropy density ratio.

In the present paper we take the second route, because viscous multi-fluid hydrodynamics is not yet available. In the UrQMD model has recently demonstrated its ability to describe the directed, elliptic and triangular flow data from HADES. This means that the
implemented interactions can be used to calculate a reliable viscosity value that is in line with the observed elliptic and higher flow harmonics without the need to rely on the initial eccentricity. This allows for the first time, to extract the η/s ratio in Au+Au collisions at \( E_{\text{lab}} = 1.23 \) AGeV directly from a dynamical hadron cascade simulation. The idea is supplemented by the investigation of the η/s ratio in the transverse plane at different times to illustrate the relevant viscosities at the different expansion stages.

II. MODEL DESCRIPTION

For the present analysis, the Ultra-relativistic Quantum-Molecular-Dynamics (UrQMD) model [18, 19] is used to investigate the properties of the hadronic matter created in Au+Au reactions at 1.23 AGeV beam energy. At this energy, the relevant degrees of freedom are hadrons, and their interaction is based on nuclear potentials and scattering cross sections. A detailed description of the model is provided in [18, 19]. UrQMD has been shown to provide a very good description of the collision dynamics and especially of the flow characteristics [13–15] which is relevant for the present analysis. So far, the extraction of η/s from transport models generally utilizes infinite matter simulations which are run sufficiently long to allow for local thermal equilibration and then employs the Green-Kubo formalism [20, 21]. In this setup, the shear viscosity is expressed in terms of the zero-frequency slope of spectral densities of stress tensor-stress tensor correlations. This method has been successfully applied to the UrQMD model in [11] and the SMASH model [22, 23]. It was found that the η/s ratio can be well described by a simple relation extracted from relativistic kinetic theory [17], i.e. η/s(T) = N(Tm)^{1/2}σ^{-1}T^{-3}, with \( N \) being a proportionality constant, \( T \) being the temperature and \( \sigma \) the cross section of the interacting particles. For the present analysis, we extract a constant value for \( Nm^{1/2}σ^{-1} = 0.0029 \) GeV^{-2.5} from the UrQMD simulations [24, 25] for the UrQMD hadron gas in equilibrium for temperatures below 150 MeV.

To relate this quantity to the dynamics encountered in the collision, we employ the UrQMD coarse-graining approach [26–32]. It consists in computing the temperature and the baryo-chemical potential from the average energy-momentum tensor and net baryon current of the hadrons formed in a large set of heavy ion collision events with the same collision energy and centrality. The computation is done in cells of a fixed spatial grid at constant intervals of time. In the present study, the cells are four-cubes with spatial sides of length \( \Delta x = \Delta y = \Delta z = 1 \text{ fm} \) and \( \Delta t = 0.25 \text{ fm} \) length in time direction. First, we evaluate the net-baryon four current \( j^0_B(t, \mathbf{r}) \) and the energy momentum tensor \( T^\mu_\nu(t, \mathbf{r}) \) in the Eckart’s frame definition [33]. Then, we perform a Lorentz transformation of the net-baryon current and of the energy momentum tensor into the Local Rest Frame (LRF) and compute the local baryon density \( \rho_B \) and the energy density \( \varepsilon \) as:

\[
\rho_B(t, \mathbf{r}) = j_0^B,_{\text{LRF}}(t, \mathbf{r}), \quad \varepsilon(t, \mathbf{r}) = T^{00}_{\text{LRF}}(t, \mathbf{r}).
\]

The final step in the coarse graining procedure consists in associating to each cell of the coarse grained grid the temperature \( T(\varepsilon(t, \mathbf{r}), \rho_B(t, \mathbf{r})) \) and the baryo-chemical potential \( \mu_B(\varepsilon, \rho_B) \) through the interpolation of a tabulated Hadron Resonance Gas EoS [34, 34].

III. RESULTS

In the following, we use the relation \( \eta/s(T) = 0.0029 \cdot T^{-2.5} \) as extracted above from the UrQMD calculations in equilibrium [11, 22]. We apply it to the coarse grained simulation data of \( T(\varepsilon(t, \mathbf{r}), \rho_B(t, \mathbf{r})) \) and extract the space and time dependent η/s ratio during the evolution of the reaction.

A. Time evolution of η/s

Let us start by exploring the time evolution of the \( \langle \eta/s \rangle \) ratio in the central cube, \( V = (10 \text{ fm})^3 \), for Au+Au reactions at \( E_{\text{lab}} = 1.23 \) AGeV as shown in Figure 1. Cold cells \( T < 40 \text{ MeV} \) are explicitly excluded. We observe that η/s has a pronounced time dependence, which reflects the temperature and density evolution of the system. I.e. during the initial stage of the nuclear medium the shear viscosity to entropy density ratio is large, but decreasing, reflecting the increase in temperature. At full overlap \( (t = 9 \text{ fm}) \) η/s develops a plateau in line with a rather constant temperature until expansion takes over and η/s increases again. The time of full overlap, i.e. maximal compression is clearly visible at \( t \approx 5 \text{ fm} \), which corresponds to the calculated time of full overlap of \( t = 9 \text{ fm} \).

Next we turn to the temperature evolution. Figure 2 shows \( \langle \eta/s \rangle \) in dependence of the temperature again averaged in the central cube with side lengths of 10 fm. The time at which the average is calculated is denoted by the color scale on the r.h.s. We observe a sharp decrease of \( \langle \eta/s \rangle \) during the compression stage in line with rising temperature. In this phase, the entropy production increases stronger than the shear viscosity. After the maximal temperature of \( \approx 80 \text{ MeV} \) is achieved the expansion rate rises and η/s increases again moderately.

Figure 3 shows the evolution of \( \langle \eta/s \rangle \) in the same volume as a function of baryon density (normalized to the

1 Here, the HG EoS is used because of the absence of QGP at the investigated energy. When applying this method to higher energies, the employed EoS should be adjusted as well.

2 We define \( \langle \eta/s \rangle \) as the volume average of the local η/s(t,r) at each time: \( \langle \eta/s \rangle(t) = \langle \Delta V \rangle^{-1} \int_V d^3r \frac{\eta(s(t,r))}{s(t,r)} \) with \( V = (10 \text{ fm})^3 \).
FIG. 1. [Color online] Time evolution of \( \langle \eta/s \rangle \) in central Au+Au reactions at a beam energy of 1.23 AGeV.

FIG. 2. [Color online] Evolution of \( \langle \eta/s \rangle \) in central Au+Au reactions at a beam energy of 1.23 AGeV as a function of temperature \( T \). The time evolution is encoded in the color, see scale on the right hand side.

FIG. 3. [Color online] Evolution of \( \langle \eta/s \rangle \) in central Au+Au reactions at a beam energy of 1.23 AGeV as a function of the scaled baryon density \( \rho_B/\rho_0 \). The time evolution is encoded in the color, see scale on the right hand side.

B. \( \eta/s \) in the transverse plane

Now that the difference between the compression phase and the expansion phase in central Au+Au reactions at 1.23 AGeV beam energy is elaborated, we will take a look into \( \eta/s \) in the transverse plane at different times of the reaction. Fig. 4 shows the distribution of \( \eta/s(t,r) \) in the transverse plane (x-y plane) for \( z = 0 \) fm. The top left figure shows the beginning of the reaction (\( t = 5 \) fm), the top right figure indicates the \( \eta/s \) distribution at the time of maximal compression (i.e. after \( \approx 10 \) fm), while the bottom figures show the transverse distribution of \( \eta/s \) in the expansion stage, i.e. after 15 fm and 20 fm. Here we observe that the shear viscosity is rather uniformly distributed within the transverse plane. However, with further distance from the center of the fireball the \( \eta/s \) ratio increases rapidly. As expected, the region of low \( \eta/s \) evolves with elapsing time from an elliptic to a circular shape. As a side remark, we want to point out that the spatial gradients in \( \eta/s \) might influence the...
evolution of the vorticity $\omega$ via the vorticity transport equation $D\omega/Dt = (\omega \cdot \nabla)u + \eta \nabla^2 \omega$, with $u$ being the fluid velocity. This may allow to use $\Lambda$ and $\bar{\Lambda}$ polarization measurements to explore the radial dependence of $\eta/s$ using the different spatial emission regions of $\Lambda$ and $\bar{\Lambda}$ as suggested in [36].

FIG. 4. [Color online] Distribution of $\eta/s$ in cells in the transverse ($x$-$y$) plane with $-1 \leq z \leq 1$ fm at $t=5$ fm (top left), $t_{\text{overlap}}$ (top right), $15$ fm (bottom left) and $20$ fm (bottom right) in central Au+Au reactions at a beam energy of $E_{\text{lab}} = 1.23$ AGeV from UrQMD. The $\eta/s$ values are encoded in the color, see scale on the right hand side.

C. Energy dependence

Finally, we put the present result at very low energies in perspective to the existing estimates of $\eta/s$ at higher collision energies.

In Fig. 5 we show the dependence of $\eta/s$ on the center of mass energy $\sqrt{s_{\text{NN}}}$ in nucleus-nucleus collisions. The red square (this work) is taken at full overlap. The blue triangles-up are extracted from [24] at full overlap, while the black triangles-down are from [38] and the green circles are from [37]. The orange star denotes an estimate from [39], the dotted black line shows the KSS boundary and the dashed magenta line depicts the fit of $\eta/s$ from kinetic theory to the data.

The proportionality constant is calculated by fitting the derived dependence to the available data up to $\sqrt{s_{\text{NN}}} = 40$ GeV. We find a numerical value of $\alpha = 1.7$.

\[ \frac{\eta}{s}(\sqrt{s_{\text{NN}}}) = \alpha \left[ \frac{\sqrt{s_{\text{NN}}}}{2m_p} \right]^{-5/4}. \]

\[ \text{FIG. 5. [Color online] Dependence of } \frac{\eta}{s} \text{ from } \sqrt{s_{\text{NN}}}. \]  

The red square (this work) is taken at full overlap. The blue triangles-up are extracted from [24] at full overlap, while the black triangles-down are from [38] and the green circles are from [37]. The orange star denotes an estimate from [39], the dotted black line shows the KSS boundary and the dashed magenta line depicts the fit of $\eta/s$ from kinetic theory to the data.
IV. SUMMARY

In this article we have used the Ultra-relativistic Quantum Molecular Dynamics (UrQMD) transport approach combined with a coarse graining procedure to extract for the first time the local shear viscosity to entropy density ratio in Au+Au reactions in the FAIR energy regime. We showed that the $\eta/s$ ratio is space and time dependent. We found that $\eta/s$ is decreasing rapidly during the compression phase and levels of around the maximum compression of $\rho_B/\rho_0 = 2.3$, corresponding to a temperature of $T = 80$ MeV. During the expansion phase, the shear viscosity to entropy density ratio increases again. We further showed the distribution of $\eta/s$ in the transverse plane and discussed how the spatial distribution of $\eta/s$ might be explored with $\Lambda, \bar{\Lambda}$ polarization measurements. We suggest that at the discussed FAIR energies different flow components receive different contributions from the shear viscosity to entropy density ratio because they are created at different times of the collision. i.e. $v_1$ is mainly created during the maximal compression phase where $\eta/s$ is lowest, while $v_2$ gains further contributions during the expansion phase where $\eta/s$ moderately increases. Finally, we related our result to previous estimates on $\eta/s$ as a function of $\sqrt{s_{NN}}$ showing that the extracted value of $\eta/s \approx 2$ is compatible with estimates of kinetic theory.

ACKNOWLEDGMENTS

This work was supported by Helmholtz Forschungsakademie Hessen (HFHF), and in the framework of COST Action CA15213 THOR. G. Inghirami is supported by the Academy of Finland, Project no. 297058. Computational resources were provided by the Center for Scientific Computing (CSC) of the Goethe University.

REFERENCES

[1] K. H. Ackermann et al. [STAR], Phys. Rev. Lett. 86, 402-407 (2001) doi:10.1103/PhysRevLett.86.402 [arXiv:nucl-ex/0009011 [nucl-ex]].

[2] S. S. Adler et al. [PHENIX Collaboration], Phys. Rev. C 69, 034909 (2004) doi:10.1103/PhysRevC.69.034909 [nucl-ex/0307022].

[3] P. Huovinen, P. F. Kolb, U. W. Heinz, P. V. Ruuskanen and S. A. Voloshin, Phys. Lett. B 503, 58 (2001) doi:10.1016/S0370-2693(01)00219-2 [hep-ph/0101136].

[4] H. Song and U. W. Heinz, Phys. Lett. B 658, 279 (2008) doi:10.1016/j.physletb.2007.11.019 [arXiv:0709.0742 [nucl-th]].

[5] M. Luzum and P. Romatschke, Phys. Rev. C 78, 034915 (2008) Erratum: [Phys. Rev. C 79, 039903 (2009)] doi:10.1103/PhysRevC.78.034915, 10.1103/PhysRevC.79.039903 [arXiv:0804.4015 [nucl-th]].

[6] P. Romatschke and U. Romatschke, Phys. Rev. Lett. 99, 172301 (2007) doi:10.1103/PhysRevLett.99.172301 [arXiv:0706.1522 [nucl-th]].

[7] P. Kovtun, D. T. Son and A. O. Starinets, Phys. Rev. Lett. 94, 111601 (2005) doi:10.1103/PhysRevLett.94.111601 [arXiv:hep-th/0405231 [hep-th]].

[8] M. F. Wondrak, M. Kaminski and M. Bleicher, arXiv:2010.10575 [hep-ph].

[9] R. A. Lacey, A. Taranenko, N. N. Ajitanand and J. M. Alexander, arXiv:1105.3782 [nucl-ex].

[10] E. Shuryak and T. Zahid, Phys. Rev. C 88, no. 4, 044915 (2013) doi:10.1103/PhysRevC.88.044915 [arXiv:1301.4470 [hep-ph]].

[11] N. Demir and S. A. Bass, Phys. Rev. Lett. 102, 172302 (2009) doi:10.1103/PhysRevLett.102.172302 [arXiv:0812.2422 [nucl-th]].

[12] P. Batyuk et al., Phys. Rev. C 94, 044917 (2016) doi:10.1103/PhysRevC.94.044917 [arXiv:1608.00965 [nucl-th]].

[13] P. Hillmann, J. Steinheimer, T. Reichert, V. Gaebel, M. Bleicher, S. Sombun, C. Herold and A. Limphirat, J. Phys. G 47, no. 5, 055101 (2020) doi:10.1088/1361-6471/ab16f4 [arXiv:1907.04571 [nucl-th]].

[14] P. Hillmann, J. Steinheimer and M. Bleicher, J. Phys. G 45, no. 9, 095101 (2018) doi:10.1088/1361-6471/aac96f [arXiv:1802.01051 [nucl-th]].

[15] P. Hillmann, J. Steinheimer, T. Reichert, V. Gaebel, M. Bleicher, S. Sombun, C. Herold and A. Limphirat, Astron. Nachr. 340, no. 9-10, 996 (2019). doi:10.1002/asna.201913750.

[16] J. Adamczewski-Musch et al. [HADES Collaboration], arXiv:2005.12217 [nucl-ex].

[17] J. M. Torres-Rincon, doi:10.1007/978-3-319-04425-9 [arXiv:1205.0782 [hep-ph]].

[18] S. A. Bass et al., Prog. Part. Nucl. Phys. 41, 255 (1998) [Prog. Part. Nucl. Phys. 41, 225 (1998)] doi:10.1016/S0146-6410(98)00058-1 [nucl-th/9803035].

[19] M. Bleicher et al. J. Phys. G 25, 1859 (1999) doi:10.1088/0954-3899/25/9/308 [hep-ph/9904047].

[20] R. Kubo, Reports on Progress in Physics 29, 255 (1966) doi:10.1088/0034-4885/29/1/306.

[21] M. S. Green, The Journal of Chemical Physics 22, 398 (1954) doi:10.1063/1.1740852.

[22] H. Petersen, J. Steinheimer, G. Burau, M. Bleicher and H. Stocker, Phys. Rev. C 78, 044901 (2008) doi:10.1103/PhysRevC.78.044901 [arXiv:0806.1695 [nucl-th]].

[23] J.-B. Rose, J. M. Torres-Rincon, A. Schäfer, D. R. Oliinchenko and H. Petersen, Phys. Rev. C 97, no. 5, 055204 (2018) doi:10.1103/PhysRevC.97.055204 [arXiv:1709.03826 [nucl-th]].

[24] M. Teslyk, L. Bravina, D. T. Son, O. Vitiuk and E. Zabrodin, Phys. Rev. C 101, no. 1, 014904 (2020) doi:10.1103/PhysRevC.101.014904 [arXiv:1910.06293 [nucl-th]].

[25] E. Zabrodin, M. Teslyk, O. Vitiuk and L. Bravina, Phys. Scripta 95, no. 7, 074009 (2020). doi:10.1088/1402-4896/ab9035.

[26] G. Inghirami, H. van Hees, S. Endres, J. M. Torres-Rincon and M. Bleicher, Eur. Phys. J. C 79, no. 1, 52 (2019) doi:10.1140/epjc/s10052-019-6537-6 [arXiv:1804.07751 [hep-ph]].
[27] S. Endres, H. van Hees and M. Bleicher, Phys. Rev. C 93, no. 5, 054901 (2016) doi:10.1103/PhysRevC.93.054901 [arXiv:1512.06549 [nucl-th]].

[28] S. Endres, H. van Hees, J. Weil and M. Bleicher, Phys. Rev. C 92, no. 1, 014911 (2015) doi:10.1103/PhysRevC.92.014911 [arXiv:1505.06131 [nucl-th]].

[29] S. Endres, H. van Hees, J. Weil and M. Bleicher, Phys. Rev. C 91, no. 5, 054911 (2015) doi:10.1103/PhysRevC.91.054911 [arXiv:1412.1965 [nucl-th]].

[30] P. Huovinen, M. Belkacem, P. J. Ellis and J. I. Kapusta, Phys. Rev. C 66, 014903 (2002) doi:10.1103/PhysRevC.66.014903 [nucl-th/0203023].

[31] G. Inghirami, P. Hillmann, B. Tomášík and M. Bleicher, J. Phys. G 47, no. 2, 025104 (2020) doi:10.1088/1361-6471/ab53f1 [arXiv:1909.00643 [hep-ph]].

[32] T. Reichert, G. Inghirami and M. Bleicher, Eur. Phys. J. A 56, no.10, 267 (2020) doi:10.1140/epja/s10050-020-00273-y [arXiv:2007.06440 [nucl-th]].

[33] C. Eckart, Phys. Rev. 58, 919 (1940). doi:10.1103/PhysRev.58.919

[34] D. Zschiesche, S. Schramm, J. Schaffner-Bielich, H. Stoecker and W. Greiner, Phys. Lett. B 547, 7 (2002) doi:10.1016/S0370-2693(02)02736-3 [nucl-th/0209022].

[35] J. Steinheimer, M. Lorenz, F. Becattini, R. Stock and M. Bleicher, Phys. Rev. C 93, no. 6, 064908 (2016) doi:10.1103/PhysRevC.93.064908 [arXiv:1603.02051 [nucl-th]].

[36] A. Ayala et al., Phys. Lett. B 810, 135818 (2020) doi:10.1016/j.physletb.2020.135818 [arXiv:2003.13757 [hep-ph]].

[37] J. Auvinen, J. E. Bernhard, S. A. Bass and I. Karpenko, Phys. Rev. C 97, no.4, 044905 (2018) doi:10.1103/PhysRevC.97.044905 [arXiv:1706.03666 [hep-ph]].

[38] I. A. Karpenko, P. Huovinen, H. Petersen and M. Bleicher, Phys. Rev. C 91, no. 6, 064901 (2015) doi:10.1103/PhysRevC.91.064901 [arXiv:1502.01978 [nucl-th]].

[39] S. Ryu, J. F. Paquet, C. Shen, G. S. Denicol, B. Schenke, S. Jeon and C. Gale, Phys. Rev. Lett. 115, no.13, 132301 (2015) doi:10.1103/PhysRevLett.115.132301 [arXiv:1502.01675 [nucl-th]].

[40] N. Astrakhantsev, V. Braguta and A. Kotov, JHEP 04, 101 (2017) doi:10.1007/JHEP04(2017)101 [arXiv:1701.02266 [hep-lat]].