Implementation of the Operational Modal Analysis technique in a power transmission shaft

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Abstract. In this paper, we investigate the application of modal identification techniques used in civil structures, in which the excitation forces are not known, and extrapolate them to mechanical elements in operating conditions. A methodology is proposed to identify the modal parameters of a power transmission shaft under rotation. The methodology is based on the comparison of theoretical and experimental results. First, a simulation of the element under study is carried out, using a numerical model based on the Finite Element Method, to evaluate its theoretical dynamic characteristics. Then, an Experimental Modal Analysis (EMA) is performed and the theoretical model is verified by comparing theoretical and experimental results. Moreover, an algorithm based on the non-parametric technique "Peak Picking" of an Operational Modal Analysis (OMA) is developed and, from the records of accelerations obtained in the bearings, we identify the modal parameters of the shaft. These results are compared with the theoretical values, calculated previously and, thus, verify the effectiveness of the implemented technique.

1. Introduction

The research in engineering has proposed models and methodologies to study the dynamic response of structures, and their operation conditions. Some methodologies have been developed to detect damages in metallic structures by Experimental Modal Analysis (EMA) [1, 2]. The EMA uses excitation forces that can be controlled or measured. The researchers whose are focus on modal testing have improved this technique, mainly to civil engineering applications, field in which the excitation forces are operational and difficult to measure. To solve this difficulty an Operational Modal Analysis (OMA) technique has been developed.

The Operational Modal Analysis has been widely used for the modal characterization of structures. The excitation forces used for this process are environmental, such as wind currents, traffic, waves, micro-earthquakes, etc. The studies carried out have reported applications of the OMA in the characterization of bridges [3, 4, 5], buildings [6, 7, 8], scale models of civil structures [9] and for modal characterization of space rockets [10].

The OMA has been the basis of studies aimed at the inspection and determination of the level of damage of structures. Rahmatalla et al. [11] used the method to validate the predictability of a FEM model developed to identify the high-stress areas of a bridge. Wójcicki et al. [12] characterized the dynamic behavior of the foundations of a ball mill. The information was used for the tuning of the FEM model of the structure. In the literature, the versatility of the method is demonstrated in applications on large structures. Peeters et al. [13] and Cigada et al. [14]
focused on applying the OMA as part of the strategy to monitor the state of the stadiums, and ensure the safety of the spectators who attend the matches.

The OMA has not only been applied to the modal characterization of civil structures, but other types of structures such as the stator of a hydroelectric generator [15], in combine cutting platforms [16], in cargo ships [17], and even in the characterization of vehicle wheels [18]. Previous results reinforce the versatility of the method and open the door to the applications that OMA can have in other types of elements. Hence, this paper reports the development of a methodology, based on the OMA, for the characterization of the shaft of a rotating machine in operating conditions.

The remaining sections are summarised as follows. Section 2 describes the operational modal analysis technique, used to extract the modal parameters. Section 3 is devoted to explain the methodology used for the modal identification of the problems described in Sections 4. Then, in Section 5 we present the discussion of the results, followed by the concluding remarks in Section 6.

2. Operational Modal Analysis

The modal analysis that can be called “traditional” uses the information extracted experimentally (input and output signals), in the form of frequency response functions (FRF), to characterize a component or structure. This corresponds to the well-known Experimental Modal Analysis (EMA). The input signals, or system excitation forces, are generated in a controlled manner. Both the excitation and the response of the system are measured and treated to obtain the FRF ($H(\omega)$), according to Equation (1).

$$|H(\omega)| = \frac{|X(\omega)|}{|F(\omega)|}$$

In those structures or elements on which generating a controlled excitation is unfeasible, mainly due to the dimensions of the equipment required to generate the excitation, it is necessary to make some modification in the traditional method, EMA. Thus, Operational Modal Analysis (OMA) emerges as a response to this need, as a viable method for the modal characterization of large structures.

The Operational Modal Analysis proposes to use the environmental forces, or operating conditions, as forces of excitation of the structure. Therefore, the method starts from not knowing the input forces, fundamental information to obtain the frequency response functions. There are different methods to extract the modal information from the structure while ignoring the excitation forces. One of them is Peak Picking (PP), a method focused on determining the natural frequencies and modal damping of a structure excited by environmental forces [19]. Although it is a relatively simple method to apply, it generates good results in structures that do not have close frequency modes or that are not highly damped [20].

The PP technique uses the Power Spectral Density (PSD) to identify the natural frequencies of the system, recalling that the Fourier transform allows to represent a time domain signal in the frequency domain, see (2).

$$y_i(\omega) = \int_{-\infty}^{\infty} y_i(t)e^{-i\omega t}dt$$

The PSD can be obtained from:

$$S_{yy}(\omega) = y_i(\omega)y_j^*(\omega)$$

where $y(t)$ is the signal in time, $y(\omega)$ the signal in the frequency domain and $S_{yy}$ the auto-spectral density. Finally, the PSD considering cross-spectral and auto-spectral densities can be written as, see (4):
The PP method considers that, in the vicinity of a resonance, the contribution of one mode of vibration predominates, and considers that the contribution of the other modes is negligible. Thus, a model of multiple degrees of freedom can be treated as single models of an independent degree of freedom, and in this way obtain the modal parameters of the system. The natural frequencies coincide with the peaks identified in the PSD, and the damping value $\xi_n$ is calculated from Equation (5):

$$\xi_n = \frac{\omega_2^2 - \omega_1^2}{2\omega_n^2}$$

(5)

where $\omega_1$ and $\omega_2$ correspond to frequencies on both sides of the natural frequency $\omega_n$.

There are other methods of extracting the modal parameters through the OMA. On the one hand, there are the methods based on the frequency domain, like the PP, such as: Frequency Domain Decomposition (FDD) [21, 22] and the Enhanced Frequency Domain Decomposition (EFDD) [23]. On the other hand, there are methods in the time domain, among which we can mention: the Stochastic Subspace Identification (SSI) [24], Natural Excitation Technique (NExT) [25] and Eigensystem Realization Algorithm (ERA) [26]. The present work focuses on applying Peak Picking as a method to obtain the modal parameters of the system.

3. Methodology

The methodology begins with the modal characterization of the shaft through the application of the Experimental Modal Analysis, in the condition of free suspension, to have initial information on the natural frequencies and vibration modes of the rotor. In the same way, the characterization of the supports is done, elements whose natural frequencies and ways of vibrating will not be considerably affected when the system is in operation. For this characterization, the EMA is also used.

Once the main components of the system are characterized, the OMA is applied to the operating shaft. For this, the vibration measurements are recorded using accelerometers located in the supports while the shaft is at operating speed. One of the sensors will be used as a reference sensor.

With the information acquired, the PSDs are calculated. The results corresponding to the normalized average with respect to the reference sensor are displayed, i.e., the Normalized Power Spectral Density (NPSD). In this graph, the vibration frequencies are identified and the associated damping is calculated. The evaluation of the coherence functions validates the different energy peaks.

Finally, for the application and the verification of the methodology, two case studies have been implemented. The first focuses on a shaft simply supported at its ends and in operating conditions. The second case is a shaft simply supported at its ends and in a state of failure that induces a noise level in the data processing, also a disc is coupled to unbalance the shaft.

4. Case study: shaft supported at its ends

To reproduce the operating conditions, the test bench of Figure 1 is used. The system comprises the shaft, bearings at their ends and the coupling to the motor. To better identify the frequencies, the test bench is aligned and balanced.
Figure 1: Test bench, case study.

4.1. **Natural frequencies of the shaft with free-free condition**

A finite element (FE) model of the shaft was developed to obtain the natural frequencies. Then, the EMA technique is applied to the shaft to obtain experimental values. The Table 1 shows the results of the numerical and experimental modal analysis performed on the shaft, considering free-free condition.

| Modes | Natural frequency numerical model (Hz) | Natural frequency EMA (Hz) | Error (%) |
|-------|--------------------------------------|---------------------------|-----------|
| 1     | 127.0                                | 130.2                     | 2.52      |
| 2     | 349.5                                | 359.9                     | 2.98      |
| 3     | 683.4                                | 701.9                     | 2.70      |
| 4     | 1125.9                               | 1156.4                    | 2.70      |

4.2. **Natural frequencies of the shaft supported on the bearings**

The EMA technique is applied to the shaft supported in the two bearings to analyze the influence of these on the dynamical response of the shaft. Figure 2 shows the EMA procedure.
For the present case, the natural frequencies obtained for the first four vibration modes increase due to the change in the stiffness induced by the supports, compared to the measurements in the free-free condition of the shaft. The identified frequencies are 156.75 Hz, 475.82 Hz, 844.96 Hz and 1185.32 Hz. Figure 3 shows the natural frequencies of the shaft under the two conditions: supported on its bearings vs. suspended shaft.

Figure 3: Natural frequencies of the EMA in condition of free suspension and supported in the bearings.

4.3. Modal analysis of the supports
The support of the shaft consists of a metallic base and bearings joined together using screws, see Figure 4.

Figure 4: Support set of the shaft. Figure 5: Experimental Modal Analysis of the support set in free-free condition.

Figure 5 shows the procedure of the EMA in the support set of the shaft. The values of the natural frequencies obtained from this procedure are shown in Table 2.
Table 2: Natural frequencies of the support set obtained from the EMA.

| Modes | Natural frequency (Hz) |
|-------|------------------------|
| 1     | 408.5                  |
| 2     | 563.3                  |
| 3     | 611.3                  |
| 4     | 817.9                  |
| 5     | 1634.8                 |

4.4. Use of the Operational Modal Analysis for the shaft under operating conditions.

Figure 6 shows the ANPSD of the first case study. This result is obtained by normalizing the PSDs generated by the signal on the two supports of the shaft and taking one of them as a reference. In Figure 7, the coherence function is presented.

![Figure 6: Averaged Normalized ANPSD.](image)

![Figure 7: Coherence function of the ANPSD.](image)

Evaluation of natural frequencies: The first peaks of Figure 6, whose frequency values are in an approximate range of 100 Hz to 370 Hz, have a coherence above 90%, as well as the frequencies of 474.1 Hz, 770.8 Hz, and 830 Hz, see Figure 7. Thus, given the methodology explained, Table 3 shows the results of the first two natural frequencies for the shaft, since they are the first two peaks of greatest amplitude in the averaged normalized power spectral density (ANPSD), see Figure 6. Registered coherence is 0.9969 and 0.9839 for the two peaks, respectively, as shown in Figure 7.

Table 3: Natural frequencies of the shaft applying OMA.

| Modes | Natural frequency (Hz) |
|-------|------------------------|
| 1     | 118.4                  |
| 2     | 355.6                  |

Figure 6: Averaged Normalized ANPSD.
Evaluation of the damping ratio: In the present work, the bandwidth method is used, as explained in [27]. The method relates the predominant frequency in the ANPSD with its neighboring frequencies according to Equation (5). In Figure 8, the application of this method for the first natural frequency obtained from the ANPSD of the case I is shown.

![Figure 8: Determination of the damping ratio in case I. ANPSD of the peak of the first mode of vibration.](image)

The damping results calculated for the first two modes of the shaft are presented in Table 4.

| Modes | Damping (ξ) |
|-------|-------------|
| 1     | 0.211%      |
| 2     | 0.028%      |

5. Discussion
With the results obtained from the numerical model, the EMA tests (on the free shaft, supported shaft, and support set for the case I), and the OMA, the performance of the OMA method for determining the natural frequencies of the shaft under operating conditions is verified.

5.1. Case Study
Natural frequencies: Figure 9 shows the ability of the algorithm to record the first 4 natural frequencies of the shaft, considering that the information was analyzed in a bandwidth of 10 Hz to 1200 Hz. The methodology implemented can identify the first 2 natural frequencies with an error of 6.77% and 1.74%, respectively.

On the other hand, in Figure 9, within the NPSD, frequencies are detected that do not correspond to the vibration modes of the shaft. These frequencies belong to the adjacent elements of the shaft in the test bench. This is confirmed by performing the EMA test on the shaft, supported by its bearings. Some of the frequencies that differ with respect to the natural frequencies recorded by the EMA (see Figure 2) correspond to the natural frequencies of the support set.
Figure 9: Natural frequencies by ANPSD and EMA.

Figure 10 reaffirms the above statement. Notice that in the range of 400 Hz to 650 Hz, the influence of the supports on the shaft analysis is evidenced, since they are approximately natural frequencies of the support, and that the OMA technique could detect. The frequency of 830 Hz detected in the ANPSD corresponds to a natural frequency of the support. In addition, the peaks of 474.1 Hz, 770.6 Hz, and 830 Hz have a coherence above 90%, validating this statement.

**Damping ratios:** For the comparison of the damping ratios calculated through the bandwidth method, the coefficients calculated by the EMA were taken as reference values. The comparison of the obtained values is presented in Table 5.

### Table 5: Comparison of the damping ratios in case study: EMA vs OMA.

| Modes | OMA (%) | EMA (%) | Error (%) |
|-------|---------|---------|-----------|
| 1     | 0.2111  | 0.214   | 1.36      |
| 2     | 0.0281  | 0.033   | 14.8      |

6. **Conclusions**

- A methodology based in OMA was implemented for the modal characterization of a machine element in operating conditions.
- The Peak Picking technique was used to determine the natural frequencies and damping ratios of a shaft in the operating state.
- The first two natural frequencies and damping ratios were successfully found for the case study.
- The effectiveness of the algorithm was demonstrated to detect the first natural frequency of a shaft in operating state, simply supported, either in an admissible operating state.

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