How is Binary Radio-Pulsars with Black Holes Population Rich?

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ABSTRACT

Using "Scenario Machine" we have carried out population synthesis of radio pulsar with black hole binaries (BH+Psr) in context of the most wide assumptions about star mass loss during evolution, binary stars mass ratio distribution, kick velocity and envelope mass lost during collapse. Our purpose is to display that under any suppo-
sitional parameters of evolution scenario BH+Psr population have to be abundant in Galaxy. It is shown that in the all models including models evolved by Heger et al. (2002), Woosley et al. (2002), Heger et al. (2003) expected number of the black holes paired with radio pulsars is sufficient enough to discover such systems within the next few years.

Key words: stars: abundances – binaries: close – binaries: general – X-rays: binaries – pulsars: general – black hole physics.

1 INTRODUCTION

Binary radio pulsars with black holes (BH+Psr) revelation would be of fundamental significance for evidence of black holes existence and for investigation precision general relativity effects (Narayan et al. 1991; Lipunov et al. 1994). In such systems parameters of black holes - such as mass, Kerr parameter - will be measured with precision orders of magnitudes higher than indirect estimations of candidates in black holes masses in X-ray/BH-candidates binaries (Blandford & Teukolsky 1973; Brumberg et al. 1973). Moreover, if mutual disposition is apt, it might be able to observe propagation of radio emission arbitrarily near to an event horizon. First accounts of possible number of BH+Psr binaries conducted ten years ago displayed the systems might be observed by the modern radio-astronomical instruments (Lipunov et al. 1994).

However total observed radio pulsars number have increased twice for the last 10 years and ran up to $N_{\text{obs}} \approx 1500$ but none of them paired with black hole (Manchester et al. 1972; Taylor & Manchester 1993; Manchester et al. 2001; Lewandowski et al. 2004; ATNF psr cat. 2005). Besides during these 10 years our conceptions of evolution of stars which are able to produce black holes appreciably change. In particular, considerations in favour of greater mass loss for these stars were obtained, detailed numerical computations considering new factors appeared (Heger et al. 2002; Woosley et al. 2002; Heger et al. 2003).

We have carried out population synthesis of binary stars using "Scenario Machine". Description of it’s working principles may be found in (Lipunov et al. 1996).

Since binary radio pulsars with black holes have to be generated by massive binary stars we relied on the observable statistics of the candidates for black holes paired with OB-stars (BH+OB).

In the paper (Lipunov et al. 1994), it is supposed that any black hole paired with OB-supergiant must reveal itself as a system Cyg X-1 type. But, as it was shown by Karpov & Lipunov (2001), powerful X-ray radiation is able to originate only if accretion disk around black hole has formed. Accordingly, in this paper, we term the Cyg X-1 type systems as the subclass of such BH+OB binaries which has accretion disks.

Evolutionary scenario which results in BH+Psr forming may be roughly outlined as the next. We start calculation of massive binary system. When primary (more massive) star fills it’s Roche lobe mass transfer takes place and helium...
star remains instead of the primary star. As a rule black hole forms first, its companion - OB-star - is sufficiently separated from BH. At that in wide systems which can survive even after second mass transfer disk does not form (stellar wind velocity is too high near black hole and the moment of rotation of grasped matter is too little to form a disk). When second component fills its Roche lobe CE-stage begins and only wide systems (which number is greater than number of close binaries) produce BH+Psr (of course, anisotropic kick plays important role in this process, it leads not only disruption of a system, but, if kick velocity and direction are apt, it can bound a binary during supernova explosion). In scenarios with high mass loss rate CE stage might not to begin, because mass of optical star is not enough greater than BH mass. Another scenario of BH+Psr creation is possible: when neutron star forms before black hole. At that time pulsar has long and still purely explained history. Fortunately, part of these evolutionary tracks is small (see below).

Note that evolutionary scenario contains quite a number of key parameters poorly explained by theory (stellar wind magnitude, initial mass of a star which is able to form black hole, initial binary stars mass ratio distribution, kick velocity during relativistic stars formation, common envelope stage efficiency, part of mass falling into BH during collapse). Although it is possible to reduce them significantly (Lipunov et al. 1994, 1999) in present work we suppose them as independent parameters. Qualitative influences of these parameters on stellar evolution scenario have been investigated more than once in previous works (de Jager 1984, Shore et al. 1994, Lipunov et al. 1996, 1997, Woosley et al. 2002) and it is possible to briefly delineate them in the following way.

Stellar wind magnitude essentially influences scenario for the two reasons. The first consists in that spherically symmetric wind leads increasing of component separation. The second is in that total mass loss of a star by wind may cause change of it’s remnant type (it may produce neutron star instead black hole).

Increasing initial mass $M_{\text{min}}$ of a star which is able to form black hole one decreases possible number of black holes due to Salpeter power law (1).

Initial binary stars mass ratio distribution is important because BH+Psr binaries have massive progenitors, so in case of more flat ($q \sim 0$) distribution probability of BH+Psr systems birth decreases.

Anisotropic kick during compact star formation is detailed investigated by Lipunov et al. (1994). Increase of kick velocity leads to total decay of a binary stars including relativistic companion. It was shown that average kick velocity $v \sim 150 - 180$ km s$^{-1}$ accords to the observable number of binary radio pulsars. For the last few years two component kick velocity distribution with characteristic velocities 90 km s$^{-1}$ and 500 km s$^{-1}$ were suggested (Arzoumanian et al. 2000).

Effectiveness of CE stage is described by parameter $\alpha_{CE} = \Delta E_b/\Delta E_{orb}$, where $\Delta E_b = E_{\text{grav}} - E_{\text{thermal}}$ is the binding energy of the ejected envelope matter and $\Delta E_{orb}$ is the drop in systems orbital energy during spiral-in (Shore et al. 1994). Our population synthesis results shows very weak dependence on CE stage efficiency due to flat initial distribution of major semi-axis of binaries (in logarithmic scale). If one is decreasing $\alpha_{CE}$ close binary systems are originating from more and more wide binaries, which number is not changing on account of flat initial distribution (Shore et al. 1994). We suggest $\alpha_{CE} = 0.5$ (Lipunov et al. 1997), that is satisfactory very close binary systems to form even in case of a very high stellar wind. Henceforward we do not investigate any dependencies on this parameter.

Part of mass falling into BH during collapse is denoted as $k_{bh} = M_{bh}/M_{\text{pre-SN}}$, where $M_{bh}$ is black hole mass, $M_{\text{pre-SN}}$ - pre-supernova mass. It is important value because binary can experience possible decay concerned with quantity of mass ejected by a star during supernova explosion, i.e. the smaller $k_{bh}$ the smaller chance a system has to survive in the cataclysm.

We emphasize that purpose of this work is not to find optimal model(s) of stellar evolution (another our work will be concentrated on it), but it is to display that radio pulsar with black hole systems number in Galaxy have to be sufficient enough under any suppositional parameters of evolution scenarios to observe them within the next few years. So we have obtained multiple modelling using various appropriate parameters.

At that, as in the work Lipunov et al. (1994), were calculating ratio of binary radio pulsars with black holes number to the total number of pulsars (practically the last number is the number of single pulsars). It allows to be saved from many selection effects concerned with our lack of knowledge of average polar pattern of the pulsars, their lifetime, magnetic field decay time, distribution of the velocity of the pulsars and their spatial distribution emerging at calculations of absolute number as long as we suppose that physical parameters of radio pulsars paired with black holes have no differences relative to average characteristics of the pulsars of the field. This natural suggestion is righteous because all of them have the same progenitors - massive ($M > 10 M_\odot$) stars and, as it was shown by our calculations, part of non-terminal evolutionary tracks (when binary radio pulsars parameters might be dissimilar to parameters of single radio pulsars, for instance, due to effect of recycling) in BH+Psr binaries and any other radio pulsars (binary and, of course, single) is negligible (in most of models this quantity is smaller than 5% and never higher than 35%).

Since BH+Psr systems have not been observed yet we suggest that observational limit is $\frac{BH+Psr}{Psr} \lesssim 10^{-3}$, where 1500 is rounded number of observed radio pulsars - binary and single (ATNF psr cat. 2002).

2 MODELS DESCRIPTION

Common parameters for all of the models are Lipunov et al. (1996): initial mass distribution (Salpeter function) (1), where $M_1$ - initial mass of more massive star, mass ratio function (2), where $q = M_2/M_1$ - initial component’s mass ratio and $\alpha$ takes on values 0 and 2, semi-major axis distribution (3), where $a$ is semi-major axis within the limits of $10 R_\odot < a < 10^3 R_\odot$ (Krajcheva et al. 1981):

$$f(M) = M_1^{-2.35},$$

$$f(q) = q^2.$$


\[ f(a) = \frac{1}{a^2} \]  

(3)

Kick velocity along with mass loss rate is one of the crucial and bad fixed parameters affecting population synthesis results. Higher kick leads to reduction of number of binary systems containing relativistic companions (Lipunov et al. 1997). Although information about kick for neutron stars maybe received from the observable binary radio pulsars statistics (Lyne & Lorimer 1994; Hansen & Phinney 1997) and from single pulsars velocities, neutron stars kick is still under discussion (Willems et al. 2004; Murphy et al. 2004; Podsiadlowski et al. 2004).

In this work we have assumed the Maxwellian distribution of the kick velocity \( v \) during neutron star and black hole formation:

\[ f(v) \sim \frac{v^2}{v_0^3} e^{-\frac{v^2}{v_0^2}}, \]  

(4)

where \( v_0 \) is characteristic kick velocity.

Even less is known of probable black hole kick, that is why we vary this quantity under discussion (Willems et al. 2004; Murphy et al. 2004; Podsiadlowski et al. 2004).

Scenario A has a weak stellar wind. Mass loss rate \( \dot{M} \) during main sequence (MS) stage (de Jager 1984) is:

\[ \dot{M} \sim L/V_\infty, \]

(6)

where \( L \) - star luminosity, \( V_\infty \) - wind velocity at infinity.

For giants we take maximum between \( 4 \) and result obtained by Lamers (1981):

\[ \dot{M} \sim L^{1.42} R^{0.61} / M^{0.99}, \]

(7)

where \( R \) is stellar radius, \( M \) - it’s mass.

And for red supergiants we take maximum between \( 5 \) and Reimers’s formula (Kudritzki & Reimers 1984):

\[ \dot{M} \sim L R / M, \]

(8)

Mass change \( \Delta M \) in wind type A during one stage is no more than \( 0.1(M - M_{\text{core}}) \), where \( M \) is star’s mass in the beginning of a stage, \( M_{\text{core}} \) - it’s core mass. Mass loss during Wolf-Rayet (WR) star stage is parametrized as \( 0.1 \cdot M_{\text{WR}} \), where \( M_{\text{WR}} \) - maximum star mass during WR stage. We used for calculations of stellar wind type A core masses obtained by Varshavskii & Tutukov (1973); Iben & Tutukov (1985, 1987).

Scenario B uses calculations of single-star evolution by Schaller et al. (1992). According to these calculations, a massive star loses most of its mass because of the action of stellar wind, down to \( \approx 8 - 10 M_\odot \) before collapse, practically independent of it’s initial mass.

In scenario C stellar evolution model is based on the results of Vanbeveren (1998), which reproduce most accurately the observed galactic WR star distributions and stellar wind mass loss in massive stars. Mass loss by a star calculations were conducted if we used the next formula:

\[ \Delta M = (M - M_{\text{core}}), \]

(9)

where \( M_{\text{core}} \) is stellar core mass \( (1) \). If maximum star’s mass (usually it is initial mass of a star, but mass transfer in binary system is able to increase it’s mass over initial value) \( M_{\text{max}} > 15 M_\odot \) mass of core in main sequence stage is determined as \( (10) \), in giant and in supergiant stages as \( (11) \). In Wolf-Rayet star stage, if \( M_{\text{WR}} < 2.5 M_\odot \) and \( M_{\text{max}} \leq 20 M_\odot \) it is described as \( (10) \), if \( M_{\text{WR}} > 2.5 M_\odot \) and \( M_{\text{max}} \leq 20 M_\odot \) as \( (10) \), if \( M_{\text{max}} > 20 M_\odot \) - as \( (10) \).

\[ M_{\text{core}} = \begin{cases} 1.62 M_\odot^{0.83} \\ 10^{-3.051+4.21 \log M_{\text{max}}-0.93(\log M_{\text{max}})^2} \\ 0.83 M_{\text{WR}}^{0.36} \\ 1.3 + 0.65(M_{\text{WR}} - 2.4), \\ 3.03 M_{\text{max}}^{0.342} \end{cases} \]

(10)

Scenario C has high mass loss during the WR stage, it may reach 50% of a star mass or more here. Mass loss in other stages (MS, giant, supergiant) for stars with masses higher than \( 15 M_\odot \) (for less massive stars this scenario equals to type A wind) may reach \( \approx 30\% \) of mass of a star. Total mass loss \( \Delta M \) during all stages always is smaller than in scenario B and greater than in scenario A.

W model is made in two types: with moderate (Wc) and high (Wb) stellar wind. Pre-supernova, helium core and compact remnant masses according to the initial star mass to calculate parameter \( k_{bh} \) were taken from Woosley et al. (2002), fig. 16, Heger et al. (2002), fig. 2. As it should be we calculate \( k_{bh} \) as the next ratio: \( M_{bh}/M_{\text{preSN}} \), where \( M_{\text{preSN}} \) is mass of pre-supernova star which has produced black hole with mass \( M_{bh} \). We made our calculations in assumption that model Wc has type C wind, for model Wb we used type B wind. This suggestion is quite right because Schaller et al. (1992) made his calculations using stellar wind obtained by Nieuwenhuijzen & de Jager (1990) (this type of wind was used by Woosley et al. (2002), Heger et al. (2002) as high mass loss type). Vanbeveren (1998) - including mass loss rate by Hamann & Koesterke (1998) (this type of wind was used by Woosley et al. (2002), Heger et al. (2002) as reduced mass loss type).

Parameter \( k_{bh} \) in A, B and C model is an adjusted constant value for all supernova explosions, in Wc and Wb models it is various quantity dependent on initial mass of a star which is able to produce black hole.

Finally it is necessary to introduce minimal mass of black hole \( M_{\text{min}} \) (i.e. minimal pre-supernova star mass which is able to form black hole multiplied by \( k_{bh} \) ) which have been used for calculations in models A, B and
C. We vary this parameter within very wide bounds: 
\[ 2.5M_\odot \leq M_{\text{min}} \leq 10M_\odot. \]

3 OBSERVATIONAL FOUNDATION

To estimate probable number of binary pulsars with black holes there is a need to be guided by observable quantity of black holes candidates in our galaxy. From this viewpoint the nearest black hole paired with radio pulsar relation is Cyg X-1 type system - black hole paired with blue supergiant. Namely these binaries (BH+OB) are radio pulsars paired with black holes progenitors. As it is well known we observe only the one such source in the Galaxy for the time present - Cyg X-1. Although it is not by chance - very close binary system in which the regime of disk accretion on the black hole has been realized. It is not by chance - LMC X-1, LMC X-3 - convinces us of this.

In the Figure 1 models for the mass loss scenarios A, B, C we everywhere assume that the system Cyg X-1 type binary consisting of black hole and blue supergiant but it is a very close binary system in which the regime of disk accretion on the black hole has been realized. It is not by chance - very low stellar wind velocity is necessary to form accretion disk (Lipunov 1992):

\[ V < V_{cr} \approx 320(4\eta)^{2}m^{2}T_{10}^{2}R_{8}^{-2/3}(1 + \tan^{2} \beta)^{-2/3}, \quad (11) \]

where \( \eta \) - averaged over z-coordinate dynamic viscous coefficient, \( m = M_{/M_\odot}, M_{x} - \) relativistic star mass, \( T_{10} = T/10, T - \) orbital period in days, \( R_{8} = R_{\text{min}}/10^{8} \text{cm}, R_{\text{min}} - \) minimal distance from the compact object up to which free Keplerian motion is still possible, \( \beta - \) accretion axis incline angle with respect to the radial direction. For black holes \( R_{\text{min}} = 3R_{g}, \) where \( R_{g} = 2GM_{\odot}/c^{2}. \)

Inasmuch as stellar wind velocity rises with increasing of distance \( R \) from the normal star approximately as follow (de Jager 1980):

\[ V = V_{\infty}(1 - R/a)^{1/2}, \quad (12) \]

where \( a \) is characteristic radius of the star, accretion disk does not form in the most cases. Actually, velocities of stellar winds are not precisely measured (de Jager 1980). But (12) is quite good approximation for this work. A spherically symmetric accretion could not set a bright source on the sky (Karpov & Lipunov 2001). Thus in the present article we everywhere assume that the system Cyg X-1 type is a very close binary including blue supergiant with mass higher than 10 masses of the Sun in which disk accretion regime has been realized by the data (11).

Let us estimate (11) and (12) for the Cyg X-1.

Davis & Hartmann (1983) obtained \( V_{\infty} = 2300 \pm 400 \text{ km s}^{-1} \) for this system. It gives lower limit for wind velocity near black hole \( V \approx 1240 \text{ km s}^{-1} \) (12). Parameters in (11) and (12) for Cyg X-1 has the next values (Abubekerov et al 2004) - \( R/a \approx 0.57, T_{10} \approx 0.56, M = 10, R_{8} = 0.1, \) \( \beta \approx 0.1, \) also we suppose that \( (4\eta)^{2} \approx 1. \) So, critical velocity \( V_{\text{crit}} \approx 1170 \text{ km s}^{-1}. \) Taking into account that observational data have great uncertainties about \( V_{\infty} \) and (11) has approximate character \( (V < V_{\text{crit}} \approx ... \) this coincidence can be considered as quite good. Hence our theoretical estimation (11) and (12) is in approximate agreement with observational data about accretion disk forming conditions in the Cyg X-1 system.

We especially note that the Cyg X-1 is not the direct forerunner of the binary radio pulsar with black hole, most likely this system will merge after Roche lobe infill and common envelope stage (Bethe & Brown 1999). However, it is clear that Cyg X-1 type systems are very similar to BH+Psr progenitors which becomes apparent in correlation between their abundances during population synthesis (see below).

During population synthesis we were picking out only the pulsars with black holes having observable orbital periods (less than 10 years) (Lamb & Lamb 1976; Manchester & Taylold 1977; Johnston et al 1992; ATNF psr cat 2003): it is may be hard to find larger periods and the most known binary radio pulsar orbital period is less than 4 years.

4 RESULTS

The radio pulsar with black hole binaries quantity depending on Cyg X-1 number \( N_{\text{CygX-1}} \) numerical modelling results are presented in the two figures. Note that we calculate number of BH+Psr systems as the next ratio - \( 1500 \cdot N_{\text{BH+Psr}}/N_{\text{Psr}}, \) where 1500 is rounded number of known radio pulsars, \( N_{\text{BH+Psr}} \) is the number of BH+Psr binaries and \( N_{\text{Psr}} \) is the total number of radio pulsars appeared during population synthesis.

There were no Cyg X-1 type systems in all of the models with type B wind including Wb model, so all of these scenarios must be declined right away because of we observe the object and do not regard it as a statistical ejection.

In the Figure 1 models for the mass loss scenarios A and C at a very wide changing of all of the unknown parameters are shown. Since we know only one Cyg X-1 type X-ray source (namely Cyg X-1) and purpose of this work is to display that under any suppositional parameters BH+Psr systems have to be observable, we have presented in Figure 1 and Figure 2 only such models in which number of Cyg X-1 type systems is no more than 4 and number of BH+Psr binaries is no more than 3. "Forbidden zone" in these figures is highlighted area where there are no any models with conceivable appropriate parameters.

Black squares (mod.1, Figure 1) denote the model calculated with \( k_{bh} = 0.45, \) minimal black hole mass \( M_{\text{min}} = 7M_\odot, \) mass loss type A and uniform initial mass ratio distribution \( (a_{q} = 0). \) Characteristic kick velocity \( v_{0} \) for neutron stars and parameter \( v_{0}^{bh} \) for black holes take on a values 0, 180, 360, 720 km s\(^{-1}\). We change these velocities for NS and BH both to give a demonstration of it’s influence on dependence between BH+Psr and Cyg X-1 quantities. Number of BH+Psr systems if \( v_{0} = 0 \) and \( v_{0}^{bh} = 0 \) is much more than 3, so this point is not presented. Models in which characteristic kick velocity for neutron stars and parameter \( v_{0}^{bh} \) for black holes take on a values 720, 360, 180 km s\(^{-1}\) are shown in order of increasing of Cyg X-1 type systems number (higher kick reduces number of binaries including relativistic companion).

Grey circles (mod.2, Figure 1) depict models with stellar wind type A, uniform initial mass ratio distribution
During calculations quantities $k = 4.0, \ldots, 9.0, 10.0$ for neutron stars and parameter $v_0$ for black holes take on a values 0, 180, 360, 720 km s$^{-1}$. Number of BH+Psr systems if $v_0 = 0$ and $v_0^{bh} = 0$ is much more than 3, so this point is not presented. Models in which characteristic kick velocity for neutron stars and parameter $v_0$ for black holes take on a values 720, 360, 180 km s$^{-1}$ are shown correspondingly in order of increasing of Cyg X-1 type systems number. Grey circles (mod.2) depict models with stellar wind type A, uniform initial mass ratio distribution ($\alpha = 0$), characteristic kick velocity $v_0 = 180$ km s$^{-1}$ for neutron stars and parameter $v_0^{bh} = 180$ km s$^{-1}$ for black holes. During calculations quantity $k_{bh}$ has been varied within the limits 0.1 and 1 every 0.1, black hole minimal mass takes on a values 2.5, 3.0, 4.0, ..., 9.0, 10.0 $M_\odot$. Circles in the Figure 1 evidently group in some vertical lines. Each line corresponds to one value of $k_{bh}$, minimal black hole masses change along lines. Cyg X-1 systems usually are products of evolution of very massive binaries, so difference between their number in case of $M_{min} = 10 M_\odot$ and in case of $M_{min} = 2.5 M_\odot$ is negligible. Otherwise number of binary radio pulsars with black holes has strong dependence on minimal black hole mass - the more $M_{min}$ the less BH+Psr systems number. So the bottom-up sequence order of points in lines which depict models with various $k_{bh}$ is: $M_{min} = 10, 9, \ldots, 4, 3, 2.5 M_\odot$. Parameter $k_{bh}$ influences on Cyg X-1 and BH+Psr numbers both, the more $k_{bh}$, the more number of binaries including relativistic companion. Note that in many cases quantities of Cyg X-1 or BH+Psr systems are higher than limits in Figure 1. That is why in this figure presented only that models in which $k_{bh} \leq 0.6$ (for mod.2) and not all points in vertical lines. So in mod.2 vertical lines correspond (in order of decreasing $N_{Cyg X-1}$) to the next $k_{bh}$ values: 0.6, 0.5, 0.4, 0.3. Models with $k_{bh} = 0.1, 0.2$ has no Cyg X-1 type binaries and are merged with many other models without such systems in this figure. Open squares (mod.3) describe models with stellar wind type C and quadratic initial mass ratio distribution ($\alpha = 2$). Quantity $k_{bh}$, minimal black hole mass $M_{min}$, characteristic kick velocity for neutron stars $v_0$ and parameter $v_0^{bh}$ for black holes in mod.3, mod.4 and mod.5 take on the same values as in mod.2. Also in these models as in mod.2 points group into some vertical lines corresponding their $k_{bh}$, minimal BH mass change along lines. Maximum $k_{bh}$ for the presented models (mod.3) is 0.7. Black triangles (mod.4) mark models with stellar wind type A and quadratic initial mass ratio distribution ($\alpha = 2$). Maximum $k_{bh}$ for the presented models is 0.6. Black diamond formations (mod.5) designate models with stellar wind type C and uniform initial mass ratio distribution ($\alpha = 0$). Maximum $k_{bh}$ for the presented models is 0.6. "Forbidden zone" is highlighted area where there are no any models with conceivable appropriate parameters of stellar evolution.

\begin{figure}
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\includegraphics[width=\textwidth]{figure1.png}
\caption{Dependence between BH+Psr binaries and Cyg X-1 type systems quantities. Black squares (mod.1) denote the model calculated with $k_{bh} = 0.45$, minimal black hole mass $M_{min} = 7 M_\odot$, mass loss type A and uniform initial mass ratio distribution ($\alpha = 0$). Characteristic kick velocity $v_0$ for neutron stars and parameter $v_0^{bh}$ for black holes take on a values 0, 180, 360, 720 km s$^{-1}$. Number of BH+Psr systems if $v_0 = 0$ and $v_0^{bh} = 0$ is much more than 3, so this point is not presented. Models in which characteristic kick velocity for neutron stars and parameter $v_0$ for black holes take on a values 720, 360, 180 km s$^{-1}$ are shown correspondingly in order of increasing of Cyg X-1 type systems number. Grey circles (mod.2) depict models with stellar wind type A, uniform initial mass ratio distribution ($\alpha = 0$), characteristic kick velocity $v_0 = 180$ km s$^{-1}$ for neutron stars and parameter $v_0^{bh} = 180$ km s$^{-1}$ for black holes. During calculations quantity $k_{bh}$ has been varied within the limits 0.1 and 1 every 0.1, black hole minimal mass takes on a values 2.5, 3.0, 4.0, ..., 9.0, 10.0 $M_\odot$. Circles in the Figure 1 evidently group in some vertical lines. Each line corresponds to one value of $k_{bh}$, minimal black hole masses change along lines. Cyg X-1 type systems usually are products of evolution of very massive binaries, so difference between their number in case of $M_{min} = 10 M_\odot$ and in case of $M_{min} = 2.5 M_\odot$ is negligible. Otherwise number of binary radio pulsars with black holes has strong dependence on minimal black hole mass - the more $M_{min}$ the less BH+Psr systems number. So the bottom-up sequence order of points in lines which depict models with various $k_{bh}$ is: $M_{min} = 10, 9, \ldots, 4, 3, 2.5 M_\odot$. Parameter $k_{bh}$ influences on Cyg X-1 and BH+Psr numbers both, the more $k_{bh}$, the more number of binaries including relativistic companion. Note that in many cases quantities of Cyg X-1 or BH+Psr systems are higher than limits in Figure 1. That is why in this figure presented only that models in which $k_{bh} \leq 0.6$ (for mod.2) and not all points in vertical lines. So in mod.2 vertical lines correspond (in order of decreasing $N_{Cyg X-1}$) to the next $k_{bh}$ values: 0.6, 0.5, 0.4, 0.3. Models with $k_{bh} = 0.1, 0.2$ has no Cyg X-1 type binaries and are merged with many other models without such systems in this figure. Open squares (mod.3) describe models with stellar wind type C and quadratic initial mass ratio distribution ($\alpha = 2$). Quantity $k_{bh}$, minimal black hole mass $M_{min}$, characteristic kick velocity for neutron stars $v_0$ and parameter $v_0^{bh}$ for black holes in mod.3, mod.4 and mod.5 take on the same values as in mod.2. Also in these models as in mod.2 points group into some vertical lines corresponding their $k_{bh}$, minimal BH mass change along lines. Maximum $k_{bh}$ for the presented models (mod.3) is 0.7. Black triangles (mod.4) mark models with stellar wind type A and quadratic initial mass ratio distribution ($\alpha = 2$). Maximum $k_{bh}$ for the presented models is 0.6. Black diamond formations (mod.5) designate models with stellar wind type C and uniform initial mass ratio distribution ($\alpha = 0$). Maximum $k_{bh}$ for the presented models is 0.6. "Forbidden zone" is highlighted area where there are no any models with conceivable appropriate parameters of stellar evolution.
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Open squares (mod.3, Figure 1) describe models with stellar wind type C and quadratic initial mass ratio distribution ($\alpha_q = 2$). Quantity $k_{bh}$, minimal black hole mass $M_{min}$, characteristic kick velocity for neutron stars $v_0$ and parameter $v_0^{bh}$ for black holes take on the same values as in mod.2. Also as in mod.2 points group into some vertical lines corresponding their $k_{bh}$, minimal BH mass change along lines. Maximum $k_{bh}$ for the presented models is 0.7.

Black triangles (mod.4, Figure 1) mark models with stellar wind type A and quadratic initial mass ratio distribution ($\alpha_q = 2$). Quantity $k_{bh}$, minimal black hole mass $M_{min}$, characteristic kick velocity for neutron stars $v_0$ and parameter $v_0^{bh}$ for black holes take on the same values as in mod.2. Also as in mod.2 points group into some vertical lines corresponding their $k_{bh}$, minimal BH mass change along lines. Maximum $k_{bh}$ for the presented models is 0.6.

Black diamond formations (mod.5, Figure 1) designate models with stellar wind type C and uniform initial mass ratio distribution ($\alpha_q = 0$). Quantity $k_{bh}$, minimal black hole mass $M_{min}$, characteristic kick velocity for neutron stars $v_0$ and parameter $v_0^{bh}$ for black holes take on the same values as in mod.2. Also as in mod.2 points group into some vertical lines corresponding their $k_{bh}$, minimal BH mass change along lines. Maximum $k_{bh}$ for the presented models is 0.6.

As one can see from the Figure 1 in the value area containing at least one black hole candidate being in Cyg X-1 type system all of the models are complying with the condition $1500 \cdot (N_{BH+Psr})/N_{Psr} \gtrsim 0.4$. In zone which has limits $1 \leq N_{CygX-1} \leq 3$ lower limit of number of binary radio pulsars with black holes is between $\approx 0.4$ and $\approx 1.75$ in all models.

In the Figure 2 two kinds of Wc models are shown (Wb model is declined because it has no Cyg X-1 type systems).

Upright crosses (mod.6, Figure 2) describe model with uniform initial mass ratio distribution ($\alpha_q = 0$). During calculations characteristic kick velocity $v_0$ for neutron stars and parameter $v_0^{bh}$ for black holes take on a values 0, 180, 360, 720, 1000 km s$^{-1}$ in both cases. If $v_0 = 0$ and $v_0^{bh} = 0$ quantity of Cyg X-1 systems and BH+Psr binaries are more than limits of the Figure 2 and appropriate point is not presented. So upright crosses depict mod.6 and crossbucks depict mod.7 in which characteristic kick velocity for neutron stars and parameter $v_0^{bh}$ for black holes take on a values 180, 360, 720, 1000 km s$^{-1}$ are shown correspondingly in order of decreasing of Cyg X-1 type systems number.

It is evidently from the Figure 2 that in the value area containing at least one black hole candidate being in Cyg X-1 type system all of the models are complying with a condition $1500 \cdot (N_{BH+Psr})/N_{Psr} \gtrsim 0.4$. In zone which has limits $1 \leq N_{CygX-1} \leq 3$ lower limit of number of binary radio pulsars with black holes is between $\approx 0.4$ and $\approx 1.75$ in all models.

5 CONCLUSIONS

Lipunov et al. [1994] made a conclusion that observable by modern techniques BH+Psr binaries have to be in Galaxy. Despite new theories concerning evolution of massive stars appearance we confirm this conclusion about a possibility to discover binary radio pulsars with black holes. We suggest that expected value of pulsars paired with black holes comparative abundance may be found within the limits $0.4 \lesssim N_{BH+Psr}/N_{Psr} \lesssim 2$. We also confirm the conclusion made by Lipunov et al. [1994] about high eccentricity of such systems (Figure 3) and about sufficient number of system close enough to observe (Figure 4). Distribution of eccentricities shows evident peak at $e \approx 1$ that is the consequence of mass loss and kick during second supernov explosion (Kornilov & Lipunov [1984]).

Our calculations are important for estimation of merging of BH+BH and BH+NS systems. Results of
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0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
5 10 15 20 25 30

Figure 3. Eccentricity distribution of BH+Psr binaries in Wc model with quadratic initial mass ratio distribution ($\alpha_q = 2$), characteristic kick velocity for neutron stars $v_0$ and parameter $v_{bh}^0$ for black holes take on a value 180 km s$^{-1}$.

Figure 4. Orbital period distribution of BH+Psr binaries in Wc model with quadratic initial mass ratio distribution ($\alpha_q = 2$), characteristic kick velocity for neutron stars $v_0$ and parameter $v_{bh}^0$ for black holes take on a value 180 km s$^{-1}$.

Lipunov & Panchenko (2003) shows that merging rate can increase by $\approx 5-7$ times with respect to previous computations made by Lipunov et al. (1997). New estimations of merging rate of relativistic systems will be carried out in one of our future work.

In closing we emphasize again that results of this work does not depend on optimal evolutionary scenario parameters: radio pulsars paired with black holes have to be in Galaxy and might be discovered within the next few years. We accentuate that relative number of BH+Psr systems to the total number of pulsars have been calculated in this work and most of the pulsars do not experience recycling effect. Radio pulsars in BH+Psr binaries originated in our calculations have no differences with more than 90% of radio pulsars which have already been observed.

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Table 1. Possible evolutionary track which produces BH+Psr.

| Stage of primary star (1) | Stage of secondary star (2) | $M_1/M_\odot$ | $M_2/M_\odot$ | $a/R_\odot$ | Time $10^6$yr | CE-stage |
|---------------------------|-----------------------------|----------------|----------------|------------|---------------|-----------|
| MS                        | MS                          | 69.5           | 5.1            | 700        | 0.0           | –         |
| SG                        | MS                          | 66.4           | 5.0            | 730        | 3.3           | –         |
| Rov                       | MS                          | 63.5           | 5.0            | 760        | $\approx$ 3.6 | +         |
| WR                        | Rov                         | 37.9           | 5.1            | 13         | $\approx$ 3.6 | –         |
| SN                        |                             |                |                |            |               |           |
| BH                        | Rov                         | 16.4           | 4.9            | 410        | 3.7           | –         |
| BH                        | WR                          | 16.4           | 4.9            | 14         | 4.9           | –         |
| BH                        | Psr                         | 16.4           | 1.34           | 21         | 26.3          | –         |

Table 2. Possible evolutionary track which produces Psr+BH.

| Stage of primary star (1) | Stage of secondary star (2) | $M_1/M_\odot$ | $M_2/M_\odot$ | $a/R_\odot$ | Time $10^6$yr | CE-stage |
|---------------------------|-----------------------------|----------------|----------------|------------|---------------|-----------|
| MS                        | MS                          | 23.9           | 11.5           | 450        | 0.0           | –         |
| SG                        | MS                          | 22.4           | 11.4           | 480        | 6.3           | –         |
| Rov                       | MS                          | 21.0           | 11.4           | 500        | $\approx$ 6.9 | –         |
| WR                        | MS                          | 8.5            | 23.9           | 690        | $\approx$ 6.9 | –         |
| SN                        |                             |                |                |            |               |           |
| Psr                       | MS                          | 1.34           | 23.9           | 790        | 7.1           | –         |
| Ej                        | SG                          | 1.34           | 22.3           | 830        | 13.0          | –         |
| SN                        |                             |                |                |            |               |           |
| Psr                       | BH                          | 1.34           | 9.0            | 80         | 14.0          | –         |

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APPENDIX: POSSIBLE EVOLUTIONARY TRACKS WHICH PRODUCE BH+PSR BINARIES

We have presented possible evolutionary track which produces BH+Psr binary in Table 1 and possible evolutionary track which produces Psr+BH system in Table 2. Note that they do not depict all possibilities and evolution of concrete binary strongly depends on evolutionary scenario. In both cases we used stellar wind type A, parameter $k_{bh} = 0.43$.

Marks in the tables 1 and 2 depict the next stages: MS - main sequence stage, SG - super-giant stage, Rov - Roche lobe overflow stage, WR - Wolf-Rayet star stage, BH - black hole, Psr - radio pulsar, Ej - ejecting neutron star which does not appear itself as radio pulsar due to free-free absorption of radio emission in component’s stellar wind (detailed nomenclature of neutron stars may be found in book written by Lipunov (1992)), SN - supernova explosion.

First and second columns in each table contain information about primary (more massive) and secondary companion current stage correspondingly, third and fourth - about their masses, fifth contains value of major semi-axis of the system, in sixth we have presented time elapsed from the moment of binary system birth. Seventh column answer the question - is the system in common envelope stage or not?

Tracks which produce neutron star before black hole can lead to so-called “recycled” radio pulsars forming. In general, we calculate evolution of rotation of a neutron star and can include these pulsars into consideration. Nevertheless, in this case results of our work would be dependent on many extra assumptions: distribution of neutron stars on their magnetic field, magnetic field dissipation time, influence of accretion on neutron star’s magnetic field, etc. We prefer not to use any additional hypothesis since part of such tracks is negligible (see Introduction).