Switched-Capacitor Circuit Implementation of the Chaotic Neural Network Reservoir

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Abstract

A chaotic neural network reservoir has been proposed as an effective method for introducing various dynamics in the reservoir neural network. In this paper, we propose a method for implementing the chaotic neural network reservoir with a switched-capacitor (SC) circuit technique. Such a circuit consists of three SC chaotic neurons with discrete elements. We then confirm the functionality of the circuit through a discrimination task in which the network distinguishes sinusoidal and chaotic waveforms.

1. Introduction

The neural network reservoir, a model inspired by the information processing in the brain, has attracted attention as a neural network model, in particular, suitable for time-series processing [1], [2]. The reservoir has a simple network structure and can be learned using a simple algorithm; thus, it would be suitable for hardware implementation. One of the key properties of the neural network reservoir is its internal dynamics. One method to increase the diversity of the reservoir dynamics is to introduce chaotic dynamics. However, the conventional method, i.e., introduction of the network instability by changing the network connections, does not provide consistency, i.e., stable responses as a network.

To introduce high-dimension chaotic dynamics into the reservoir without changing its connection, we proposed the chaotic neural network reservoir using a chaotic neural network model [3]–[5]. For real world applications, compact and low-power circuit implementation of such a chaotic neural network reservoir is mandatory.

In this paper, we propose a method for implementing a chaotic neural network reservoir with a switched-capacitor (SC) circuit technique. Through experiments, we confirmed the functionality of the SC chaotic neural network reservoir circuit through a discrimination task in which the network distinguishes sinusoidal and chaotic waveforms.

2. Chaotic Neural Network Reservoir

Figure 1 shows the general neural network reservoir composed of the input layer, reservoir layer, and output layer. As shown in the figure, input connection weight \( W_{ij}^{in} \) between the \( i \)-th neuron in the input layer and the \( j \)-th neuron in the reservoir layer, and inner connection weight \( W_{ij} \) between \( i \)-th and \( j \)-th neurons in the reservoir layer are sparse and set randomly. Output connection weight \( W_{ij}^{out} \) between the \( i \)-th neuron in the reservoir layer and the \( j \)-th neuron in the output layer is adjusted using a simple learning rule such as linear regression.

The chaotic neural network reservoir [3]–[5] is realized by substituting the chaotic neuron [7] in place of the ordinary neuron in the reservoir layer. The state update equations of the \( i \)-th neuron in the reservoir layer is given by

\[
x_i(t + 1) = k x_i(t) + \sum_{j=1}^{L} W_{ij}^{in} u_j(t + 1) + \sum_{j=1}^{N} W_{ij} f(x_j(t)) - \alpha f(x_i(t)) + \theta
\]

(1)

\[
y_i(t + 1) = f(x_i(t + 1))
\]

(2)

\[
f(x) = \frac{1}{1 + \exp(-x/\epsilon)}
\]

(3)

where \( x_i(t) \) and \( y_i(t) \) are the internal state and output of the \( i \)-th neuron at time \( t \), respectively; \( W_{ij}^{in} \) is the input weight...
from the \( j \)-th neuron in the input layer to the \( i \)-th neuron in the reservoir layer; \( W_{ij} \) is the connection weight from \( j \)-th neuron to the \( i \)-th neuron in the reservoir layer; \( u_j(t) \) is the \( j \)-th external input; \( \alpha \) and \( \kappa \) are refractoriness parameters; \( L \) is the number of input units; \( N \) is the number of neurons in the reservoir; \( \theta \) is the external bias; and \( f(\cdot) \) is the output function given by Eq. (3) where \( \epsilon \) is the gain of the output function.

In addition, \( y_i^{\text{out}}(t + 1) \) is the output of the \( i \)-th neuron in the output layer given by

\[
y_i^{\text{out}}(t + 1) = f\left( \sum_{j=1}^{K} W_{ij}^{\text{out}} y_j(t + 1) \right) 
\tag{4}
\]

where \( W_{ij}^{\text{out}} \) is the output weight from the \( j \)-th neuron in the reservoir layer to the \( i \)-th neuron in the output layer, and \( K \) is the number of neurons in the output layer. Output weight matrix \( W^{\text{out}} \) is adjusted by teacher signal vector \( y^{\text{teach}} \) by using an appropriate learning rule.

### 3. SC Chaotic Neural Network Circuit

We implement Eq. (1)–(3) in the reservoir layer using the SC circuits technique. In the following text, we illustrate an SC chaotic neural network with three neurons. Figure 2 shows the circuit that realizes internal state \( x_1(t) \) of the first neuron in the network [8].

In the circuit shown in Fig. 2, \( V_{x_1}(t + 1) \) corresponds to \( x_1(t + 1) \) in Eq. (1). The circuit equations for the network with three neurons are given as follows.

\[
V_{x_1}(t + 1) = \left( 1 - \frac{C_k}{C_i} \right) V_{x_1}(t) - \frac{C_f}{C_i} V_{y_1}(t) + \frac{C_{l_{x_1}}}{C_i} V_{y_1}(t) - \frac{C_{l_{y_2}}}{C_i} V_{y_2}(t) \tag{5}
\]

\[
V_{x_2}(t + 1) = \left( 1 - \frac{C_k}{C_i} \right) V_{x_2}(t) - \frac{C_f}{C_i} V_{y_2}(t) + \frac{C_{l_{x_2}}}{C_i} V_{y_2}(t) - \frac{C_{l_{y_3}}}{C_i} V_{y_3}(t) \tag{6}
\]

\[
V_{x_3}(t + 1) = \left( 1 - \frac{C_k}{C_i} \right) V_{x_3}(t) - \frac{C_f}{C_i} V_{y_3}(t) + \frac{C_{l_{x_3}}}{C_i} V_{y_3}(t) - \frac{C_{l_{y_2}}}{C_i} V_{y_2}(t) \tag{7}
\]

where \( V_{y_j}(t + 1) \) corresponds to \( y_j(t + 1) \) in Eq. (2).

The connection weight matrix can be set according to the capacitor ratios as

\[
W^{\text{in}} = \begin{bmatrix} W_{11}^{\text{in}} & W_{12}^{\text{in}} & W_{13}^{\text{in}} \end{bmatrix} = \begin{bmatrix} \frac{C_{l_{x_1}}}{C_i} & \frac{C_{l_{y_1}}}{C_i} & \frac{C_{l_{y_2}}}{C_i} \end{bmatrix} \tag{8}
\]

\[
W = \begin{bmatrix} W_{11} & W_{12} & W_{13} \\
W_{21} & W_{22} & W_{23} \\
W_{31} & W_{32} & W_{33} \end{bmatrix} = \begin{bmatrix} 0 & \frac{C_{l_{x_1}}}{C_i} & \frac{C_{l_{y_2}}}{C_i} \\
0 & \frac{C_{l_{x_2}}}{C_i} & \frac{C_{l_{y_3}}}{C_i} \\
0 & \frac{C_{l_{x_3}}}{C_i} & 0 \end{bmatrix} \tag{9}
\]

In addition, the refractoriness parameters in Eq. (1) can be determined as

\[
k = 1 - \frac{C_k}{C_i} \tag{10}
\]

\[
\alpha = \frac{C_f}{C_i} \tag{11}
\]

Figure 3 shows the output function circuit of a neuron, as represented in Fig. 2. Figure 4 shows the measured characteristic of the output function circuit in Fig. 3 with \( R_1 = 22 \, \text{k}\Omega, R_2 = 980 \, \text{m}\Omega, R_3 = 11 \, \text{k}\Omega, R_4 = 82 \, \text{k}\Omega, \) and \( R_5 = 22 \, \text{k}\Omega. \)
4. Numerical Simulations

In this study, we use a discrimination task of periodic waveform \( p(t) \) given by Eq. (12) and chaotic waveform \( c(t) \) given by Eq. (13). The network parameters are summarized in Table 1.

\[
p(t) = 0.9 \sin^2(t/5) \tag{12}
\]

\[
c(t) = 3.8c(t-1)(1-c(t-1)), c(0) = 0.2 \tag{13}
\]

\( p(t) \) is used as the input signal for the first 1000 steps and \( c(t) \) is used as the signal after \( t = 1000 \). The teacher signal is \(-0.9\) and \(0.9\) when the input is \( p(t) \) and \( c(t) \), respectively. The values of input weight \( W^{in} \) and inner weight \( W \) are determined as follows.

\[
W^{in} = \begin{bmatrix}
-0.8641 & -0.9331 & 0 \\
0.5787 & 0 & -0.0203 \\
-0.8800 & -0.1780 & 0
\end{bmatrix} \tag{14}
\]

\[
W = \begin{bmatrix}
0 & 0.2670 & -0.7049 \\
0.5787 & 0 & -0.0203 \\
-0.8800 & -0.1780 & 0
\end{bmatrix} \tag{15}
\]

The learning rule used is the linear regression given by

\[
W^{out} = f^{-1}(y_{teach})y^T_{out}(y_{out}y^T_{out})^{-1} \tag{16}
\]

Figure 5a plots the input signal \( u(t) \), while Fig. 5b shows network output \( y^{out}(t) \). Figure 5b confirms that the chaotic reservoir network with three neurons can discriminate waveforms represented by Eqs. (12) and (13).

5. Experimental Results

We implement the SC chaotic neural network reservoir with three neurons in the reservoir layer with circuit parameters shown in Table 2.
Figure 6: Results of measurement

Figure 6a shows input signal $V_u(t)$ to the circuit. In Fig. 6a, the first 1000 steps represent the periodic wave of Eq. (12), and the next 1000 steps represent the chaotic waveform of Eq. (13). Figures 6b to 6d show the output voltages of each neuron. These figures confirm that the SC chaotic neural network reservoir circuit distinguishes the periodic and chaotic waveforms.

Furthermore, Fig. 7 shows the output of the network, $V_{y_{out}}(t)$, which corresponds to $y_{out}(t)$. As shown in the figure, the discrimination was estimated as 68.4%.

These results confirm that the chaotic neural network reservoir can be implemented using the proposed SC circuits.

6. Conclusions

In this paper, we proposed an SC chaotic neural network reservoir circuits. We then constructed the preliminary reservoir network with three neurons using discrete elements. The experimental results confirmed that the proposed SC circuit can be used in the chaotic neural network reservoir. As a future problem, we will construct a large-scale chaotic neural network reservoir using an integrated circuit technology.

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