Covert Wireless Communication with Artificial Noise Generation

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Abstract

Covert communication conceals the transmission of the message from an attentive adversary. Recent work on the limits of covert communication in additive white Gaussian noise (AWGN) channels has demonstrated that a covert transmitter (Alice) can reliably transmit a maximum of $O(\sqrt{n})$ bits to a covert receiver (Bob) without being detected by an adversary (Warden Willie) in $n$ channel uses. This paper focuses on the scenario where other “friendly” nodes distributed according to a two-dimensional Poisson point process with density $m$ are present in the environment. We propose a strategy where the friendly node closest to the adversary, without close coordination with Alice, produces artificial noise. We show that this method allows Alice to reliably and covertly send $O(\min\{n, m^{\gamma/2}/\sqrt{n}\})$ bits to Bob in $n$ channel uses, where $\gamma$ is the path-loss exponent. Moreover, we also consider a setting where there are $N_w$ collaborating adversaries uniformly and randomly located in the environment and show that in $n$ channel uses, Alice can reliably and covertly send $O\left(\min\left\{n, \frac{m^{\gamma/2}\sqrt{n}}{N_w} \right\}\right)$ bits to Bob when $\gamma > 2$, and $O\left(\min\left\{n, \frac{m^{\gamma/2}\sqrt{n}}{N_w \log^2 N_w} \right\}\right)$ when $\gamma = 2$. Conversely, under mild restrictions on the communication strategy, we demonstrate that no higher covert throughput is possible for $\gamma > 2$.

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This work has been supported, in part, by the National Science Foundation under grants CNS-1018464, ECCS-1309573, and CNS-1564067. The preliminary version of this work has been presented at the 52nd Annual Allerton Conference on Communication, Control, and Computing, Allerton, Monticello, IL, October 2014 [1].

This work has been submitted to the IEEE for possible publication. Copyright may be transferred without notice, after which this version may no longer be accessible.
Keywords: Covert Communication, Wireless Communication, Artificial Noise Generation, Covert Wireless Communication, Low Probability of Detection, LPD, Covert Channel, Covert Wireless Network, Wireless Network, Single-hop Communication, Additive White Gaussian Noise, AWGN, Information Theory.

I. INTRODUCTION

Covert communication hides the presence of a message from a watchful adversary. This is crucial in scenarios in which the standard method of secrecy, which hides the message content but not its existence, is not enough; in other words, there are applications where, no matter how strongly the message is protected from being deciphered, the adversary discerning that the communication is taking place results in penalties to the users. Examples of such scenarios include military operations, social unrest, and tracking of people’s daily activities. The Snowden disclosures \cite{2} demonstrate the utility of “meta-data” to an observing party and, thus, motivate hiding the presence of the message.

Security provisioning has emerged as a critical issue in wireless communications, where the signal is not restricted physically to a wire and, therefore, it is more difficult to hide the existence of the communication. Although spread spectrum approaches have been widely used in the past \cite{3}, the fundamental limits of covert communication were only recently established by a subset of the authors \cite{4}, \cite{5}, who presented a square root limit on the number of bits that can be transmitted securely from the transmitter (Alice) to the intended receiver (Bob) when there is an additive white Gaussian noise (AWGN) channel between Alice and each of Bob and the adversary (Warden Willie). In particular, by taking advantage of positive noise power at Willie, Alice can reliably transmit $O(\sqrt{n})$ bits to Bob in $n$ channel uses while lower bounding Willie’s error probability $P_e^{(w)} = P_{FA} + P_{MD} \geq \frac{1}{2} - \epsilon$ for any $0 < \epsilon < \frac{1}{2}$ where $P_{FA}$ is the probability of false alarm and $P_{MD}$ is the probability of mis-detection. Conversely, if Alice transmits $\omega(\sqrt{n})$ bits in $n$ uses of channel, either Willie detects her or Bob suffers a non-zero probability of decoding error as $n$ goes to infinity. Covert communications recently has been studied in many scenarios such as binary symmetric channels (BSCs) \cite{6}, multi-path noiseless networks \cite{7}, bosonic channels with thermal noise \cite{8}, and noisy discrete memoryless channels (DMCs) \cite{9}. Furthermore, higher throughputs are achievable when Alice can leverage Willie’s ignorance of her transmission time \cite{10}, and/or the adversary has uncertainty about channel characteristics \cite{11}, \cite{12}. These works, along with \cite{13}, \cite{14}, present a comprehensive characterization of the fundamental limits of covert communications over DMC and AWGN channels and have also motivated studying the fundamental limits of covert techniques for packet channels \cite{15}, \cite{16}.

In this paper, we turn our attention to the wireless network case, where a collection of nodes work to establish covert communication between a collection of source and destination pairs. The goal is
to establish an analog to the line of work on scalable low probability of intercept communications [17]–[20], which considered the extension of [21], [22] to the secure multipair unicast problem in large wireless networks. Here, in analog to [17], we investigate how we can improve security between Alice and Bob when there are a number of other nodes present in the environment. In [12], Sobers et al. consider the improvement of the covert throughput by leveraging Willie’s ignorance of the channel characteristics in a fading environment or when a jammer with varying power is present. However, in this paper, we assume that Willie knows his channel characteristics, as he knows the constant powers of the jammers.

Assume Alice attempts to communicate covertly with Bob without detection by Willie, but also in the presence of other (friendly) network nodes, which can assist the communication by producing background chatter to inhibit Willie’s ability to detect Alice’s transmission. We model the locations of the friendly nodes by a two-dimensional Poisson point process of density $m$, and that Alice and Bob share a secret (codebook) unknown to Willie. For this scenario, described in more detail in Section II, we show in Section III that turning on the closest friendly node to Willie enables Alice to covertly transmit $O(\min \{n, m^{\gamma/2}/\sqrt{n}\})$ bits to Bob in $n$ channel uses while keeping Willie’s error probability $P_e^{(w)} \geq 1/2 - \epsilon$ for any $\epsilon \geq 0$, where $\gamma$ is the path-loss exponent. Conversely, if Alice attempts to transmit $\omega(m^{\gamma/2}/\sqrt{n})$ bits to Bob in $n$ channel uses, there exists a detector that Willie can use to either detect her with arbitrarily low error probability $P_e^{(w)}$ or prevent Bob from decoding the message with arbitrarily low probability of error.

Next, we extend the scenario to the case of multiple Willies, and we show that when $N_w$ collaborating Willies are uniformly and independently distributed in the unit box (see Fig. 1), we can still turn on the closest friendly node to each Willie to improve the covert throughput. However, as $N_w \to \infty$, we observe two effects that reduce the covert throughput: (1) with high probability, there exists a Willie very close to Alice who receives a high signal power from her, thus making Alice employ a lower power to hide the transmission; (2) with high probability, there exists a Willie very close to Bob whose closest friendly node generates additional noise for Bob, hence reducing his ability to decode Alice’s message. We explore this scenario in Section IV in detail. Finally, we discuss the results in Section V and present conclusions in Section VI.

II. SYSTEM MODEL, DEFINITIONS, AND METRICS

A. System Model

Consider a source Alice ($A$) wishing to communicate with receiver Bob ($B$) located a unit distance away from her in the presence of adversaries (Warden Willies) $W_1, W_2, \ldots, W_{N_w}$, who are distributed
Fig. 1. System Configuration: Source node Alice wishes to communicate reliably and without detection to the intended receiver Bob at distance one (normalized) with the assistance of friendly nodes (represented by yellow nodes in the figure) distributed according to a two-dimensional Poisson point process with density $m$ in the presence of adversary nodes $W_1, W_2, \ldots, W_{N_w}$ located in the dashed box ($N_w = 3$ in the figure).

independently and uniformly in the unit square (Fig. 1) and seek to detect any transmission by Alice. When there is only a single Willie, we omit the subscript and denote it by $W$. Also present are friendly nodes $F_1, F_2, \ldots$ allied with Alice and Bob, who help hide Alice’s transmission by generating noise. We model the locations of friendly nodes by a two-dimensional Poisson point process with density $m$. The adversaries try to detect whether Alice transmits or not by processing the signals they receive and applying hypothesis testing on them, as discussed in the next subsection. We consider two scenarios: a single Willie ($N_w = 1$) and multiple Willies ($N_w > 1$). We assume all channels are discrete-time AWGN with real-valued symbols. Alice transmits $n$ real-valued symbols $s_1, s_2, \ldots, s_n$. Each friendly node is either on or off according to the strategy employed. Let $\theta_j$ denote the state of the $j^{th}$ friendly node $F_j$; $\theta_j = 1$ if $F_j$ is “on” (transmits noise) and $\theta_j = 0$ (silent) otherwise. If $F_j$ is on, it transmits symbols $\{s_i^{(j)}\}_{i=1}^{\infty}$, where $\{s_i^{(j)}\}_{i=1}^{\infty}$ is a collection of independent and identically distributed (i.i.d.) zero-mean Gaussian random variables, each with variance (power) $P_f$. The locations of all the parties are static and known to everyone. One implication of this assumption is that friendly nodes can determine which friendly node is the closest to each Willie.

Recalling that the distance between Alice and Bob is normalized to unity, Bob receives $y_1^{(b)}, y_2^{(b)}, \ldots, y_n^{(b)}$ where $y_i^{(b)} = s_i + z_i^{(b)}$ for $1 \leq i \leq n$. The noise component is $z_i^{(b)} = z_{i,0}^{(b)} + \sum_{j=1}^{\infty} \theta_j z_{i,j}^{(b)}$, where $\{z_{i,0}^{(b)}\}_{i=1}^{n}$
is an i.i.d. sequence representing the background noise of Bob’s receiver with \( z^{(b)}_{i,0} \sim \mathcal{N}(0, \sigma_{b,0}^2) \) for all \( i \), and \( \left\{ z^{(b)}_{i,j} \right\}_{i=1}^{\infty} \) is an i.i.d. sequence of zero-mean Gaussian random variables characterizing the chatter from the \( j \)th friendly node when it is “on”, each element of the sequence with variance \( \frac{P}{d_{b,j}} \), where \( d_{x,y} \) is the distance between nodes \( X \) and \( Y \), and \( \gamma \) is the path-loss exponent which in most practical cases satisfies \( 2 \leq \gamma \leq 4 \).

Similarly, the \( k \)th Willie observes \( y^{(k)}_1, y^{(k)}_2, \ldots, y^{(k)}_n \) where \( y^{(k)}_i = \frac{s_i}{d_{w,k}} + z^{(k)}_{i,0} + \sum_{j=1}^{\infty} \theta_j z^{(k)}_{i,j} \) where \( \left\{ z^{(k)}_{i,0} \right\}_{i=1}^{\infty} \) is an i.i.d. sequence representing the background noise at Willie’s receiver, where \( z^{(k)}_{i,0} \sim \mathcal{N}(0, \sigma_{w,k,0}^2) \) for all \( i \), and \( \left\{ z^{(k)}_{i,j} \right\}_{i=1}^{\infty} \) is an i.i.d. sequence characterizing the chatter from the \( j \)th friendly node when it is “on”; thus, \( \mathcal{N}(0, P_f/d_{w,k,f}) \). For a single Willie scenario, we omit the superscripts on \( y^{(k)}_i, z^{(k)}_{i,0}, \) and \( z^{(k)}_{i,j} \), and we denote the Willie by \( W \), and the closest friendly node to Willie by \( F \).

We assume Alice and the friendly nodes, while having a common goal, are not able to synchronize their transmissions; that is, the friendly nodes set up a constant power background chatter but are not able to, for example, lower their power at the time Alice transmits. In [12], the assumption is that a single jammer with varying power is present or the channel fading leads to uncertainty in Willie’s received power when Alice is not transmitting. Such uncertainty is not present here.

**B. Definitions**

Willie’s hypotheses are \( H_0 \) (Alice does not transmit) and \( H_1 \) (Alice transmits). We denote by \( \mathbb{P}_{FA} \) the probability of rejecting \( H_0 \) when it is true (type I error or false alarm), and \( \mathbb{P}_{MD} \) the probability of rejecting \( H_1 \) when it is true (type II error or mis-detection). We assume that Willie uses classical hypothesis testing with equal prior probabilities and seeks to minimize his probability of error, \( \mathbb{P}_{e} = \frac{\mathbb{P}_{FA} + \mathbb{P}_{MD}}{2} \); the generalization to arbitrarily prior probabilities is available in [5].

When there is only a single Willie in the scenario, he applies a hypothesis test to his received signal to determine whether or not Alice is communicating with Bob. We denote the probability distribution of Willie’s \( (W_k) \) collection of observations \( \left\{ y^{(k)}_i \right\}_{i=1}^{n} \) by \( \mathbb{P}_1 \) when Alice is communicating with Bob, and the distribution of the observations when she does not transmit by \( \mathbb{P}_0 \). For a scenario with multiple collaborating Willies (Theorems 2 and 3), they jointly process the signals they receive to arrive at a single collective decision as to whether Alice transmits or not.

**Definition 1.** (Covertness) Alice’s transmission is covert if and only if she can lower bound Willies’ average probability of error \( (\mathbb{E}_{F,W} \left[ \mathbb{P}_{e}^{(w)} \right] = \mathbb{E}_{F,W} \left[ \mathbb{P}_{FA} + \mathbb{P}_{MD} \right] ) / 2 \) by \( \frac{1}{2} - \epsilon \) for any \( \epsilon > 0 \), asymptotically [5]. The expectation is with respect to the locations of the friendly nodes as well as those of the Willie(s).
Definition 2. (Reliability) Alice’s transmission is reliable if and only if the desired receiver (Bob) can decode her message with arbitrarily low average probability of error $\mathbb{P}_e^{(b)}$ at long block lengths, where the expectation is over node locations. In other words, for any $\zeta > 0$, Bob can achieve $\mathbb{P}_e^{(b)} < \zeta$ as $n \to \infty$.

In this paper, we use standard Big-O, Little-Omega, and Big-Theta notations [23, ch. 3].

III. SINGLE WARDEN SCENARIO

In this section, we consider the case where there is only one Willie (W) located uniformly and randomly on the unit square shown as a dashed box in Fig. 1. To hide the presence of Alice’s transmission, we turn on the friendly node closest to Willie and then analyze Willie’s ability to detect Alice’s transmission. This allows us to derive a bound on Alice’s power so as to maintain covertness. The achievability proof concludes by considering the rate at which reliable decoding is still possible when Alice uses the maximum possible power. Then, we present a converse under mild restrictions on the signaling scheme.

Theorem 1. When there is one warden (Willie) located randomly and uniformly over the unit square, $m > 0$, and $\gamma > 0$, Alice can reliably and covertly transmit $O(\min\{n, m^{\gamma/2}/\sqrt{n}\})$ bits to Bob in $n$ channel uses. Conversely, if only the closest friendly node to Willie is on and Alice attempts to transmit $\omega(m^{\gamma/2}/\sqrt{n})$ bits to Bob in $n$ channel uses, there exists a detector that Willie can use to either detect her with arbitrarily low error probability $\mathbb{P}_e^{(w)}$ or Bob cannot decode the message with arbitrarily low error probability $\mathbb{P}_e^{(b)}$.

Proof. (Achievability)

Construction: Alice and Bob share a codebook that is not revealed to Willie. For each message transmission of length $L$ bits, Alice uses a new codebook to encode the message into a codeword of length $n$ at rate $R = \frac{L}{n}$. To build a codebook, we use random coding arguments; that is, codewords $\{C(M_l)\}_{l=1}^{2^nR}$ are associated with messages $\{M_l\}_{l=1}^{2^nR}$, where each codeword $C(M_l) = \{C^{(u)}(M_l)\}_{u=1}^{n}$, for $l = \{1, 2, \cdots, 2^nR\}$, is an i.i.d. zero-mean Gaussian random sequence; that is, $C^{(u)}(M_l) \sim \mathcal{N}(0, P_a)$ where $P_a$ is specified later. Receiver Bob employs a maximum-likelihood (ML) decoder.

Alice and Bob turn on the closest friendly node to Willie and keep all other friendly nodes off, whether Alice transmits or not. Therefore, Willie’s observed noise power is given by

$$\sigma_w^2 = \sigma_{w,0}^2 + \frac{P_f}{d_{w,f}^2},$$
where $\sigma^2_{w,0}$ is Willie’s noise power when none of the friendly nodes are transmitting and $d_{w,f}$ is the (random) distance of the closest friendly node to Willie; hence, $\sigma^2_w$ is a random variable that depends on the locations of the friendly nodes.

**Analysis:** *(Covertness)* Recall that $P_0$ is the joint probability density function (pdf) for Willie’s observations under the null hypothesis $H_0$ (Alice does not transmit), and $P_1$ be the joint pdf for corresponding observations under the hypothesis $H_1$ (Alice transmits). Observe

$$P_0 = P^n_w,$$

$$P_1 = P^n_s,$$

where $P_w = \mathcal{N}(0, \sigma^2_w)$ is the pdf for each of Willie’s observations when Alice does not transmit, and $P_s = \mathcal{N}(0, \sigma^2_w + \frac{P_a}{d^\gamma_{a,w}})$ is the pdf for each of the corresponding observations when Alice transmits.

When Willie applies the optimal hypothesis test to minimize $P_e(w)$:

$$P_e^{(w)} \geq \frac{1}{2} - \sqrt{\frac{1}{8} D(P_1||P_0)},$$

(1)

where $D(P_1||P_0)$ is the relative entropy between $P_1$ and $P_0$. Next, we lower bound $E_{F,W}[P_e^{(w)}]$, where $E_{F,W}[\cdot]$ denotes expectation over locations of the friendly nodes ($F_1, F_2, \ldots$), and the location of Willie (W). Taking the expected value of both sides of (1) yields:

$$E_{F,W}[P_e^{(w)}] \geq \frac{1}{2} - E_{F,W} \left[ \sqrt{\frac{1}{8} D(P_1||P_0)} \right].$$

For the given $P_0$ and $P_1$:

$$D(P_1||P_0) = \frac{n}{2} \left( \frac{P_a}{d^\gamma_{a,w}\sigma^2_w} - \ln \left( 1 + \frac{P_a}{d^\gamma_{a,w}\sigma^2_w} \right) \right) \leq n \left( \frac{P_a}{2d^\gamma_{a,w}\sigma^2_w} \right)^2,$$

(2)

where the last inequality follows from

$$\ln(1 + x) \geq x - \frac{x^2}{2}, \text{ for } x \geq 0.$$

(3)

By (1) and (2)

$$P_e^{(w)} \geq \frac{1}{2} - \sqrt{\frac{n}{8} \frac{P_a}{\sigma^2_w d^\gamma_{a,w}}},$$

(4)

If Alice sets her average symbol power

$$P_a \leq \frac{cm^{\gamma/2}}{\sqrt{n}},$$

(5)

where $c = \epsilon \left( \frac{\Gamma(\gamma/2+1)}{4\sqrt{2}\psi\gamma P_{a,n}^{\gamma/2+1}} \right)^{-1}$ is a constant independent of $n$, and $\psi = \sqrt{\frac{1}{2\pi}}$. Then, (4) becomes

$$P_e^{(w)} \geq \frac{1}{2} - \sqrt{\frac{1}{8} \frac{cm^{\gamma/2}}{2\sigma^2_w d^\gamma_{a,w}}},$$

(6)
To account for the singularity at \( d_{a,w} = 0 \), we define the event \( d_{a,w} > \psi \) and we lower bound \( \mathbb{E}_{F,W}[\mathbb{P}_e^{(w)}|d_{a,w} > \psi] \).

Next, we take the conditional expected value of both sides of (9) with respect to the locations of friendly nodes and location of Willie:

\[
\mathbb{E}_{F,W}[\mathbb{P}_e^{(w)}|d_{a,w} > \psi] \geq \frac{1}{2} - \sqrt{\frac{1}{8}} \mathbb{E}_{F,W} \left[ \frac{cm^{\gamma/2}}{2\sigma_w^2 d_{a,w}} \right] \left( d_{a,w} > \psi \right).
\]

Note that when \( d_{a,w} > \psi \), \( 1/d_{a,w}^2 \leq 1/\psi^2 \). Therefore,

\[
\mathbb{E}_{F,W}[\mathbb{P}_e^{(w)}|d_{a,w} > \psi] \geq \frac{1}{2} - \frac{cm^{\gamma/2}}{4\sqrt{2}\psi^\gamma} \mathbb{E}_{F,W} \left[ \frac{1}{\sigma_w^2} \right] \left( d_{a,w} > \psi \right) = \frac{1}{2} - \frac{cm^{\gamma/2}}{4\sqrt{2}\psi^\gamma} \mathbb{E}_{F,W} \left[ \frac{1}{\sigma_w^2} \right].
\]

where the last step is true because friendly nodes are distributed according to a Poisson point process over the entire plane, and thus Willie’s noise characteristics are independent of his location.

Next we upper bound \( \mathbb{E}_F[1/\sigma_w^2] \) in (7). From the properties of Poisson point processes, the pdf of \( d_{w,f} \) is

\[
f_{d_{w,f}}(x) = 2m\pi x e^{-m\pi x^2}.
\]

Therefore,

\[
\mathbb{E}_F \left[ \frac{1}{\sigma_w^2} \right] = \mathbb{E}_F \left[ \frac{1}{\sigma_{w,0}^2 + P_t/d_{w,f}^\gamma} \right] \leq \frac{2m\pi}{P_t} \int_0^\infty x^{\gamma+1} e^{-m\pi x^2} dx = \frac{\Gamma(\gamma/2 + 1)}{2P_t\pi^{\gamma/2+1}m^{\gamma/2}},
\]

where \( \Gamma(\cdot) \) is the Gamma function. Thus, (7) and (9) yield

\[
\mathbb{E}_{F,W}[\mathbb{P}_e^{(w)}|d_{a,w} > \psi] \geq \frac{1}{2} - \frac{c\Gamma(\gamma/2 + 1)}{8\sqrt{2}\psi^\gamma P_a\pi^{\gamma/2+1}} = \frac{1}{2} - \frac{\epsilon}{2},
\]

where the last step is true by substituting the value of \( c \). Next, we lower bound \( \mathbb{E}_{F,W}[\mathbb{P}_e^{(w)}] \) using (10).

Since \( \psi \leq \frac{1}{2} \), \( \mathbb{P}(d_{a,w} > \psi) = 1 - \pi\psi^2/2 \). The law of total expectation yields

\[
\mathbb{E}_{F,W}[\mathbb{P}_e^{(w)}] \geq \mathbb{E}_{F,W}[\mathbb{P}_e^{(w)}|d_{a,w} > \psi] \mathbb{P}(d_{a,w} > \psi) \geq \left( \frac{1}{2} - \frac{\epsilon}{2} \right) \left( 1 - \frac{\pi\psi^2}{2} \right),
\]

\[
= \left( \frac{1}{2} - \frac{\epsilon}{2} \right) \left( 1 - \frac{\epsilon}{4} \right) > \frac{1}{2} - \epsilon.
\]

Thus, \( \mathbb{E}_{F,W}[\mathbb{P}_e^{(w)}] > \frac{1}{2} - \epsilon \) for all \( \epsilon > 0 \), as long as \( P_a = \mathcal{O}(m^{\gamma/2}/\sqrt{\pi}) \).

Note that Alice does not use the locations of the friendly nodes to select the transmission power (and thus, per below, the corresponding rate). Rather, she selects a power and corresponding rate for a scheme that is covert when averaged over the locations of the friendly nodes.

(\textbf{Reliability}) First, we analyze Bob’s decoding error probability conditioned on \( \sigma_w^2 = \sigma_{b,0}^2 + \frac{P_t}{d_{b,f}^\gamma} \), which we denote \( \mathbb{P}_e^{(b)}(\sigma_w^2) \), where \( d_{b,f} \) is the distance from Bob to the friendly node closest to Willie. Then, we upper bound Bob’s average decoding error probability \( \mathbb{P}_e^{(b)} = \mathbb{E}_{F,W}[\mathbb{P}_e^{(b)}(\sigma_w^2)] \).
For a given $\sigma_b^2$, we can upper bound Bob’s decoding error probability by:

$$\mathbb{P}_e^{(b)}(\sigma_b^2) \leq 2^{-nR - \frac{n}{2} \log_2 \left(1 + \frac{P_a}{2\sigma_b^2}\right)}, \quad (12)$$

$$= 2^{-nR - \frac{n}{2} \log_2 \left(1 + \frac{cm\gamma/2}{2\sqrt{n}\sigma_b^2}\right)}, \quad (13)$$

where the last step is obtained by having Alice set $P_a = \frac{cm\gamma/2}{\sqrt{n}}$ to satisfy (5), and the inequality (12) results from an application of [5, Eqs. (5)-(9)]. Let $\phi = \sqrt{\frac{c}{2\pi}}$. Since the right hand side (RHS) of (13) is a monotonically non-decreasing function of $d_{b,f}$, when $d_{b,f} > \phi$

$$\mathbb{P}_e^{(b)}(\sigma_b^2) \leq 2^{-nR - \frac{n}{2} \log_2 \left(1 + \frac{cm\gamma/2}{2\sqrt{n}(\sigma_{b,0}^2 + P_f/\phi^2)}\right)}, \quad (14)$$

We set Alice’s rate to $R = \min\{1, R_0\}$ where

$$R_0 = \frac{1}{4} \log_2 \left(1 + \frac{cm\gamma/2}{2\sqrt{n}(\sigma_{b,0}^2 + P_f/\phi^2)}\right). \quad (15)$$

By (14), (15), $\mathbb{P}_e^{(b)}(\sigma_b^2) \leq 2^{-n(R - 2R_0)}$. Note that $R \leq R_0$ and thus $R - 2R_0 \leq -R_0$. Consequently

$$\mathbb{P}_e^{(b)}(\sigma_b^2) \leq 2^{-nR_0} = \left(1 + \frac{cm\gamma/2}{2\sqrt{n}(\sigma_{b,0}^2 + P_f/\phi^2)}\right)^{-\frac{n}{4}} \leq \left(1 + \frac{cm\gamma/2\sqrt{n}}{8(\sigma_{b,0}^2 + P_f/\phi^2)}\right)^{-1}, \quad (16)$$

where (16) follows from the following inequality provided $n \geq 4$:

$$(1 + x)^{-r} \leq (1 + rx)^{-1} \text{ for any } r \geq 1 \text{ and } x > -1. \quad (17)$$

Thus,

$$\mathbb{E}_{F,W}[\mathbb{P}_e^{(b)}(\sigma_b^2)|d_{b,f} > \phi] \leq \left(1 + \frac{cm\gamma/2\sqrt{n}}{8(\sigma_{b,0}^2 + P_f/\phi^2)}\right)^{-1}. \quad (18)$$

Next, we upper bound Bob’s average decoding error probability $\mathbb{P}_e^{(b)}$ using (18). The law of total expectation yields

$$\mathbb{P}_e^{(b)} = \mathbb{E}_{F,W}[\mathbb{P}_e^{(b)}(\sigma_b^2)] \leq \mathbb{E}_{F,W}[\mathbb{P}_e^{(b)}(\sigma_b^2)|d_{b,f} > \phi] + \mathbb{P}(d_{b,f} \leq \phi). \quad (19)$$

By (18), $\lim_{n \to \infty} \mathbb{E}_{F,W}[\mathbb{P}_e^{(b)}(\sigma_b^2)|d_{b,f} > \phi] = 0$. Now, consider $\mathbb{P}(d_{b,f} \leq \phi)$. Since the event $\{d_{b,f} \geq \phi\}$ is a subset of the event that the distance between Bob and the closest friendly node to him is larger than $\phi$, $\mathbb{P}(d_{b,f} \geq \phi) \leq 1 - \frac{\pi\phi^2}{2} = 1 - \zeta/4$, and thus $\lim_{n \to \infty} \mathbb{P}_e^{(b)} < \zeta$ for any $0 < \zeta < 1$.

(Number of Covert Bits) Now, we calculate $nR$, the number of bits that Bob receives. If $R_0 \geq 1$, then $R = 1$ and thus $nR = n$. Now, consider $R_0 < 1$ which implies $R = R_0$. By (15),

$$nR = n \frac{1}{4} \log_2 \left(1 + \frac{cm\gamma/2}{2\sqrt{n}(\sigma_{b,0}^2 + P_f/\phi^2)}\right).$$
By (15), when $R_0 < 1$, it must be that \( \frac{cm^\gamma/2}{2\sqrt{n}(\sigma_{b,0}^2 + P_f/\phi)} < 15 \). Therefore, we can use
\[
\log_2 (1 + x) \geq \frac{x}{4} \text{ for } 0 < x \leq 15,
\]
to show that
\[
nR \geq \frac{\sqrt{n}cm^\gamma/2}{8(\sigma_{b,0}^2 + P_f/\phi)}.
\]
Thus, Bob receives $\mathcal{O}(\min\{n, m^\gamma/2\sqrt{n}\})$ bits in $n$ channel uses.

(Converse) We present the converse assuming that only the closest friendly to Willie is on and Willie knows this. Suppose Willie uses a power detector on his collection of observations \( \{y_i\}_{i=1}^n \) to form \( S = \frac{1}{n} \sum_{i=1}^n y_i^2 \) and performs a hypothesis test based on $S$ and a threshold $t$. If $S < \sigma_w^2 + t$, Willie accepts $H_0$ (Alice does not transmit); otherwise, he accepts $H_1$ (Alice transmits). Recall that when $H_0$ is true, $y_i = z_{i,0} + z_{i,1}$, where \( \{z_{i,0}\}_{i=1}^n \) is an i.i.d. sequence representing the background noise with \( z_{i,0} \sim \mathcal{N}(0, \sigma_{w,0}^2) \), and \( \{z_{i,1}\}_{i=1}^n \) is an i.i.d. sequence characterizing the chatter from the closest friendly node with \( \mathcal{N}(0, P_f/d_{w,f}^\gamma) \). Since the two sources of noise are independent, we can model Willie’s total noise by a Gaussian noise with $y_i \sim \mathcal{N}(0, \sigma_w^2)$, where $\sigma_w^2 = \sigma_{w,0}^2 + P_f/d_{w,f}^\gamma$. Therefore [5],
\[
\mathbb{E}_Y[S | H_0] = \sigma_w^2,
\]
\[
\text{Var}_Y[S | H_0] = \frac{2\sigma_w^4}{n},
\]
where \( \mathbb{E}_Y[\cdot] \) and \( \text{Var}_Y[\cdot] \) denote the expectation and variance with respect to Willie’s received signal.

When $H_1$ is true, Alice transmits a codeword $C(M_l) = \{C^{(u)}(M_l)\}_{u=1}^n$ and Willie observes \( \{y_i\}_{i=1}^n \) which contains i.i.d. samples of mean shifted noise $y_i \sim \mathcal{N}\left(\frac{s_i}{d_{a,w}^\gamma}, \sigma_{w,1}^2\right)$, where $s_i$ is the value of Alice’s transmitted symbol in the $i$th channel use, and each $s_i$ is an instantiation of a Gaussian random variable $\mathcal{N}(0, P_a)$. Therefore [5],
\[
\mathbb{E}_Y[S | H_1] = \sigma_w^2 + \frac{P_a}{d_{a,w}^\gamma},
\]
\[
\text{Var}_Y[S | H_1] = \frac{4\frac{P_a}{d_{a,w}^\gamma}\sigma_w^2 + 2\sigma_w^4}{n}.
\]

We assume that Willie knows $\sigma_w^2$, the jamming scheme, and the distance to the closest friendly node. We show that Willie can choose the threshold $t$ independent of locations of the friendly nodes such that if Alice transmits $\omega\left(m^\gamma/2\sqrt{n}\right)$ bits to Bob, he can achieve arbitrarily small average error probability. Bounding $\mathbb{P}_{FA}$ by using Chebyshev’s inequality yields [5]:
\[
\mathbb{P}_{FA} \leq \frac{2\sigma_w^4}{nt^2}.
\]
Therefore, the law of total expectation yields
\[
\mathbb{E}_{F,W}[P_{FA}] = \mathbb{E}_{F,W}[P_{FA}|d_{w,f} \leq \eta_1] \mathbb{P}(d_{w,f} \leq \eta_1) + \mathbb{E}_{F,W}[P_{FA}|d_{w,f} > \eta_1] \mathbb{P}(d_{w,f} > \eta_1),
\]
\[
\leq \mathbb{P}(d_{w,f} \leq \eta_1) + \mathbb{E}_{F,W}[P_{FA}|d_{w,f} > \eta_1] \leq \left(1 - e^{-m \pi \eta_1^2}\right) + \frac{2 \left(\sigma_{w,0}^2 + \frac{P_t}{n_1}\right)^2}{nt^2}.
\]

\(\forall \eta_1 > 0\). Let Willie choose threshold \(t = \frac{2\sqrt{\lambda}}{\sqrt{n_1}} \left(\sigma_{w,0}^2 + P_t/\eta_1^2\right)\) where \(\eta_1 = \sqrt{\frac{\ln \left(\frac{1}{\xi}\right)}{m \pi}}\). Then
\[
\mathbb{E}_{F,W}[P_{FA}] \leq \left(1 - \left(1 - \frac{\lambda}{4}\right)\right) + \frac{2n\pi}{8n} = \frac{\lambda}{2}.
\]

Since \(d_{a,w} \leq 2\), Willie can upper bound \(P_{MD}\) \([5]\), Eq. (16)
\[
P_{MD} \leq \frac{4P_a\sigma_w^2 + 2\sigma_w^4}{n \left(\frac{P_a}{d_{a,w}} - t\right)^2} = \frac{4P_a\sigma_w^2 + 2\sigma_w^4}{n \left(\frac{P_a}{2n} - t\right)^2}.
\]
(21)

Then, \(\forall \eta_2 > 0\), the law of total expectation yields
\[
\mathbb{E}_{F,W}[P_{MD}] \leq \mathbb{P}\left\{d_{w,f} \leq \eta_2\right\} \cup \left\{d_{a,w} \leq \eta_3\right\} + \mathbb{E}_{F,W}[P_{MD}|\{d_{w,f} > \eta_2\} \cap \{d_{a,w} > \eta_3\}]\),
\[
\leq \left(1 - e^{-m \pi \eta_2^2}\right) + \frac{\pi^2}{2} \eta_3^2 + \frac{4P_a}{n} \left(\sigma_{w,0}^2 + \frac{P_t}{n_2}\right) + \frac{2 \left(\sigma_{w,0}^2 + \frac{P_t}{n_2}\right)^2}{n \left(\frac{P_a}{2n} - t\right)^2}.
\]

\(\forall \eta_2, \eta_3 > 0\). We now set \(\eta_2 = \sqrt{\frac{\ln \left(\frac{1}{4\lambda + 3\pi}\right)}{m \pi}}\), where \(0 < \lambda' < \lambda\), and \(\eta_3 = \sqrt{\frac{\lambda}{2n}}\). Since \(t = \Theta\left(\frac{m^{7/2}}{\sqrt{n}}\right)\), if Alice sets her average symbol power \(P_a = \omega\left(\frac{m^{7/2}}{\sqrt{n}}\right)\), then there exists \(n_0 > 0\) s.t. \(\forall n > n_0(\lambda') \)
\[
\mathbb{E}_{F,W}[P_{MD}] \leq \frac{\lambda - \lambda'}{4} + \frac{\lambda}{4} + \frac{\lambda'}{2} = \frac{\lambda}{2} + \frac{\lambda'}{4} < \lambda.
\]

Therefore, for any \(\lambda > 0\)
\[
\mathbb{E}_{F,W}[P_{e,w}] = \frac{\mathbb{E}_{F,W}[P_{FA} + P_{MD}]}{2} \leq \frac{3\lambda}{4} < \lambda,
\]

Consequently, Alice cannot send any codeword with average symbol power \(\omega\left(\frac{m^{7/2}}{\sqrt{n}}\right)\) covertly. Thus, to avoid detection of a given codeword, she must set the power of that codeword to \(P_{d,t} = O\left(\frac{m^{7/2}}{\sqrt{n}}\right)\).

Suppose that Alice’s codebook contains a fraction \(\xi > 0\) of codewords with power \(P_{d,t} = O\left(\frac{m^{7/2}}{\sqrt{n}}\right)\).

For such low power codewords, we can lower bound Bob’s decoding error probability by ((20) in \([5]\))
\[
P_{e,t}^d \geq 1 - \frac{P_{d,t}}{2\sigma_w^2} + \frac{1}{\log_2 \xi} + R,
\]
(22)

Since Alice’s rate is \(R = \omega\left(\frac{m^{7/2}}{\sqrt{n}}\right)\) bits/symbol, \(P_{e,t}^d\) is bounded away from zero as \(n \to \infty\).
In this section, we consider the case when there are $N_w$ collaborating Willies located independently and uniformly in the unit square (see Fig. 1). We present Theorem 2 for $\gamma > 2$ in Section IV-A and Theorem 3 for $\gamma = 2$ in Section IV-B. Analogous to the single warden scenario, Alice and Bob's strategy is to turn on the closest friendly node to each Willie and keep all other friendly nodes off, whether Alice transmits or not.

A. $\gamma > 2$

**Theorem 2.** Assume friendly nodes are independently distributed according to a two-dimensional Poisson point process with density $m = \omega(1)$, and $N_w$ collaborating Willies are uniformly and independently distributed over the unit square shown in Fig. 1. If $N_w = o\left(\min\left\{ \frac{m}{\log m}, \frac{n^{\gamma/2}}{\sqrt{m}} \right\} \right)$, then Alice can reliably and covertly transmit $O\left(\min\left\{ n, \frac{m^{\gamma/2}}{n} \right\} \right)$ bits to Bob in $n$ channel uses. Conversely, if the closest friendly node to each Willie is on and Alice attempts to transmit $\omega\left(\frac{\sqrt{m^{\gamma/2}}}{N_w} \right)$ bits to Bob in $n$ channel uses, there exists a detector that Willie can use to either detect her with arbitrarily low error probability $P_e(w)$ or Bob cannot decode the message with arbitrarily low error probability $P_e(b)$.

We present the proof assuming $N_w = \omega(1)$, as the proof for a finite $N_w$ follows from it.

**Proof.** (Achievability)

**Construction:** The codebook construction is the same as that in Theorem 1.

**Analysis:** (Covertness) By (1), when Willie applies the optimal hypothesis test to minimize his error probability,

$$P_e(w) \geq \frac{1}{2} - \sqrt{\frac{1}{8} D(P_1||P_0)}. \tag{23}$$

Here, $P_0$ and $P_1$ are the joint probability distributions of Willies’ channels observations for the $H_0$ and $H_1$ hypotheses, respectively; in other words

$$P_0 = \left[ P_0^{(1)}T P_0^{(2)}T \ldots P_0^{(N_w)}T \right] T,$$

$$P_1 = \left[ P_1^{(1)}T P_1^{(2)}T \ldots P_1^{(N_w)}T \right] T,$$

where $P_0^{(k)}$ is the vector probability distribution of the channel observation of the $k^{th}$ Willie ($W_k$) when $H_0$ is true and includes $n$ elements with the same probability distribution $P_{w_k} \sim \mathcal{N}(0, \sigma_{w_k}^2)$. In addition, $P_1^{(k)}$ is the channel observation of $W_k$ when $H_1$ is true and includes $n$ elements, each with the same
probability distribution \( P_{sk} \sim \mathcal{N} \left( 0, \sigma_{w_k}^2 + \frac{P_a}{d_{a,w_k}} \right) \). The relative entropy between two multivariate normal distributions \( P_1 \) and \( P_0 \) is [24]:

\[
D(\mathbb{P}_1 || \mathbb{P}_0) = \frac{1}{2} \left( \text{tr} \left( \Sigma_0^{-1} \Sigma_1 \right) + (\mu_0 - \mu_1)^T \Sigma_0^{-1} (\mu_0 - \mu_1) - \dim(\Sigma_0) - \ln \left( |\Sigma_1| / |\Sigma_0| \right) \right),
\]

where \( \text{tr}(\cdot) \), \(|\cdot|\), and \( \dim(\cdot) \) denote the trace, determinant and dimension of a square matrix respectively, \( \mu_0 = 0, \mu_1 = 0 \) are the mean vectors, and \( \Sigma_0, \Sigma_1 \) are nonsingular covariance matrices of \( \mathbb{P}_0 \) and \( \mathbb{P}_1 \), respectively, given by

\[
\Sigma_0 = S \otimes I_{n \times n},
\]

\[
\Sigma_1 = \left( S + P_a U U^T \right) \otimes I_{n \times n},
\]

where \( S = \text{diag}(\sigma_{w_1}^2, \ldots, \sigma_{w_{N_w}}^2) \), \( \otimes \) denotes the Kronecker product between two matrices, \( I_{n \times n} \) is the identity matrix of size \( n \), and \( U \) is a column vector of size \( N_w \) given by

\[
U = \left[ 1/d_{a,w_1}^{\gamma} \ 1/d_{a,w_2}^{\gamma} \ \ldots \ 1/d_{a,w_{N_w}}^{\gamma} \right]^T.
\]

Next, we calculate the relative entropy in (24). The first term on the RHS of (24) is:

\[
\text{tr} \left( \Sigma_0^{-1} \Sigma_1 \right) = n \sum_{k=1}^{N_w} \frac{1}{\sigma_{w_k}^2} \left( \sigma_{w_k}^2 + \frac{P_a}{d_{a,w_k}} \right) = nN_w + n \sum_{k=1}^{N_w} \frac{P_a}{d_{a,w_k}^2}.
\]

Then,

\[
|\Sigma_0| = |S \otimes I_{n \times n}| = |S|^n |I_{n \times n}|^{N_w} = |S|^n = \left( \prod_{k=1}^{N_w} \sigma_{w_k}^2 \right)^n,
\]

where \((a)\) is true from the determinant of the Kronecker product property presented in [25] p. 279]. Because \( \sigma_{w_k}^2 > 0 \), \( S \) is nonsingular. Therefore,

\[
|\Sigma_1| = |S + P_a U U^T|^{n} |I_{n \times n}|^{N_w} = |S + P_a U U^T|^{n} = |S|^n |I + P_a S^{-1} U U^T|^{n},
\]

\[
\overset{(b)}{=} |S|^n \left( 1 + P_a S^{-1} U U^T \right)^n = |S|^n \left( 1 + \sum_{k=1}^{N_w} \frac{P_a}{d_{a,w_k}^2} \right)^n,
\]

where \((b)\) is due to Lemma 1.1 in [26]. Therefore,

\[
\ln \left( \frac{|\Sigma_1|}{|\Sigma_0|} \right) = n \ln \left( 1 + \sum_{k=1}^{N_w} \frac{P_a}{d_{a,w_k}^2} \right).
\]

Thus,

\[
D(\mathbb{P}_1 || \mathbb{P}_0) = \frac{n}{2} \left( \sum_{k=1}^{N_w} \frac{P_a}{d_{a,w_k}^2} - \ln \left( 1 + \sum_{k=1}^{N_w} \frac{P_a}{d_{a,w_k}^2} \right) \right). \tag{25}
\]
Fig. 2. Event $\mathcal{A}$ is true when there is no Willie in the semicircular region with radius $\kappa$ shown above. Alice is only able to communicate covertly with intended receiver Bob if $\mathcal{A}$ is true.

By (3) and (25),

$$D(P_1 || P_0) \leq \frac{n}{4} \left( \sum_{k=1}^{N_w} \frac{P_a}{d_{a,w_k} \sigma_{w_k}^2} \right)^2. \quad (26)$$

Observe that $D(P_1 || P_0)$ in (26) has a singularity at $d_{a,w_k} = 0$ for all $k$. To account for this, we consider a small semicircular region with radius

$$\kappa = \sqrt{\frac{\epsilon}{4N_w}}$$

around Alice and define the event (see Fig. 2)

$$\mathcal{A} = \bigcap_{i=1}^{N_w} \{d_{a,w_k} > \kappa\}.$$

In other words, $\mathcal{A}$ is true when all of the Willies are outside of the semicircular region. Next, we lower bound $E_{F,W}[P_e^{(w)} | \mathcal{A}]$ and show that when $n$ is large enough, $P(\mathcal{A}) \geq 1 - \frac{\epsilon}{2}$. Then, we show that Alice can achieve $E_{F,W}[P_e^{(w)}] \geq \frac{1}{2} - \epsilon$ for arbitrarily $\epsilon > 0$. Suppose Alice sets her average symbol power so that

$$P_a \leq \frac{c m^{\gamma/2}}{\sqrt{nN_w^{\gamma/2}}}, \quad (27)$$

where

$$c = \frac{P_t \epsilon^{\gamma/2} (\gamma - 2) \pi^{\gamma/2}}{2^{\gamma - 0.5} \Gamma(\gamma/2 + 1)}. \quad (28)$$
We show in the Appendix A that for any \( \epsilon > 0 \) Alice can achieve:

\[
E_{F,W} \left[ \mathbb{P}_e^{(w)} \mid \mathcal{A} \right] \geq \frac{1}{2} (1 - \epsilon). \tag{29}
\]

Next consider \( \mathbb{P}(\mathcal{A}) \). Since \( \kappa < 1/2 \),

\[
\mathbb{P}(\mathcal{A}) = \left( 1 - \frac{\pi \kappa^2}{2} \right)^{N_w (c)} \geq 1 - \frac{\pi N_w \kappa^2}{2} \geq 1 - 2N_w \kappa^2 = 1 - \frac{\epsilon}{2}, \tag{30}
\]

where \( (c) \) is true since (17) is true. By (29), (30), and the law of total expectation

\[
E_{F,W}[\mathbb{P}_e^{(w)}] \geq E_{F,W} \left[ \mathbb{P}_e^{(w)} \mid \mathcal{A} \right] \mathbb{P}(\mathcal{A}) = \left( \frac{1}{2} - \frac{\epsilon}{2} \right) \left( 1 - \frac{\epsilon}{2} \right) \geq \frac{1}{2} - \epsilon,
\]

and thus communication is covert as long as \( P_a = O \left( \frac{m \gamma}{\sqrt{N_w \gamma}} \right) \).

(Reliability) Next, we calculate the number of bits that Alice can send to Bob covertly and reliably.

Consider arbitrarily \( \zeta > 0 \). We show that Bob can achieve \( \mathbb{P}_e^{(b)} < \zeta \) as \( n \to \infty \), where \( \mathbb{P}_e^{(b)} \) is Bob’s ML decoding error probability averaged over all possible codewords and the locations of friendly nodes and Willies. Bob’s noise power is

\[
\sigma_b^2 \leq \sigma_{b,0}^2 + \sum_{k=1}^{N_w} \frac{P_f}{d_{b,f_k}^2}, \tag{31}
\]

where \( d_{b,f_k} \) is the distance between Bob and the closest friendly node to Willie \( W_k \). Note that (31) becomes equality when each Willie has a distinct closest friendly node. By (12) and (27),

\[
\mathbb{P}_e^{(b)} (\sigma_b^2) \leq 2^{-nR - \frac{n}{2} \log_2 \left( 1 + \frac{\epsilon' m \gamma}{2 \sqrt{\pi \sigma_b^2 N_w \gamma}} \right)}, \tag{32}
\]

Suppose Alice sets \( R = \min \{ R_0, 1 \} \), where

\[
R_0 = \frac{1}{4} \log_2 \left( 1 + \frac{c' m \gamma}{4N_w \sqrt{n}} \right), \tag{33}
\]

\[
c' = c \zeta^{\gamma/2 - 1} \left( \gamma - 2 \right) \frac{1}{2^{\gamma+3} \pi^{\gamma/2}},
\]

and \( c \) is defined in (28). By the law of total expectation,

\[
\mathbb{P}_e^{(b)} = E_{F,W}[\mathbb{P}_e^{(b)} (\sigma_b^2)] \leq E_{F,W} \left[ \mathbb{P}_e^{(b)} (\sigma_b^2) \left| \frac{c' \sigma_b^2}{c N_w \gamma} \leq 1 \right] \right] + \mathbb{P} \left( \frac{c' \sigma_b^2}{c N_w \gamma} > 1 \right). \tag{34}
\]

Consider the first term on the RHS of (34). We show in the Appendix B that since \( m = \omega(1) \), \( N_w = \omega(1) \), and \( N_w = o \left( n^{1/2} \sqrt{m} \right) \),

\[
\lim_{n \to \infty} E_{F,W} \left[ \mathbb{P}_e^{(b)} (\sigma_b^2) \left| \frac{c' \sigma_b^2}{c N_w \gamma} \leq 1 \right] \right] = 0. \tag{35}
\]

To account for the singularities of \( \sigma_b^2 \) at \( d_{b,f_k} = 0 \) for all \( k \), we consider the following event (see Fig. 3)

\[
B = \bigcap_{k=1}^{N_w} \left\{ \{d_{w,k} \leq \delta \} \cap \{d_{b,k} > 2\delta \} \right\}, \tag{36}
\]

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Fig. 3. Event $B$ is true when there is no Willie in the semicircular region with radius $2\delta$ around Bob, and the distance between each Willie and the closest friendly node to him is smaller than $\delta$, i.e., $\{2d_{w_k,f_k} \leq \delta\} \cap \{d_{b,w_k} > 2\delta\}$ for $1 \leq k \leq N_w$.

where

$$\delta = \sqrt{\frac{\zeta}{4\pi N_w}}.$$  

This event is true when there is no Willie in the semicircular region with radius $2\delta$ around Bob, and the distance between each Willie and the closest friendly node to him is smaller than $\delta$. We show that Bob’s probability of error goes to zero when $B$ is true. Next, we show that $P(B) = 1 - \zeta/2$, as $n \to \infty$, and then we use the law of total expectation to upper bound Bob’s probability of error by $\zeta$ for any $0 < \zeta < 1$.

Consider the second term on the RHS of (34). The law of total probability yields

$$P\left(\frac{c'\sigma_b^2}{cN_w^{\gamma/2}} > 1\right) \leq P\left(\frac{c'\sigma_b^2}{cN_w^{\gamma/2}} > 1 \mid B\right) + P\left(\overline{B}\right).$$

Then, we show in the Appendix C that since $N_w = \omega(1)$,

$$\lim_{n \to \infty} P\left(\frac{c'\sigma_b^2}{cN_w^{\gamma/2}} > 1 \mid B\right) = 0,$$

and in the Appendix D that since $N_w = \omega(1)$ and $N_w = o(m/\log m)$,

$$\lim_{n \to \infty} P\left(\overline{B}\right) = \frac{\zeta}{2}.$$  

Thus, (34)-(38) yield $\lim_{n \to \infty} P_e^{(b)} < \zeta$ for any $0 < \zeta < 1$. 

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(Number of Covert Bits) Similar to the analysis of Theorem 1, we can show that when $\gamma > 2$, Bob receives $O \left( \min \left\{ n, \frac{m^{\gamma/2} \sqrt{n}}{N_w} \right\} \right)$ bits in $n$ channel uses.

(Converse) We present the converse assuming that the closest friendly node to each Willie is on and the Willies know this. We show that the signal received by the closest Willie to Alice is sufficient to detect Alice’s communication. Intuitively, the Willie closest to Alice has the best signal-to-noise ratio (SNR) and is the best Willie to detect Alice’s communication.

Denote Willie with minimum distance to Alice by $W_1$. We assume that $W_1$ knows $\sigma^2_{w_1}$ and the jamming scheme, in particular the position of the closest friendly node and its transmit power. $W_1$ uses a power detector on his collection of observations $\left\{ y_i^{(1)} \right\}_{i=1}^n$ to form $S = \frac{1}{n} \sum_{i=1}^n \left( y_i^{(1)} \right)^2$, picks a threshold $t$, and performs a hypothesis test based on $S$. If $S < \sigma^2_{w_1} + t$, he chooses $H_0$ (Alice does not transmit), otherwise, $H_1$ (Alice transmits).

Observe

$$\sigma^2_{w_1} \leq \sigma^2_{w_{1,0}} + \sum_{k=1}^{N_w} \frac{P_f}{d_{w_1, f_k}^\gamma},$$

(39)

where $\sigma^2_{w_{1,0}}$ is Willie’s noise power when all of the friendly nodes are off, i.e., AWGN, and $d_{w_1, f_k}$ is the distance between $W_1$ and the closest friendly node to $W_k$. Note that (39) becomes equality when all of the Willies have a distinct closest friendly node. Similar to the converse in Theorem 1, we can show that

$$\mathbb{E}_Y [S | H_0] = \sigma^2_{w_1},$$

(40)

$$\text{Var}_Y [S | H_0] = \frac{2 \sigma^4_{w_1}}{n},$$

(41)

$$\mathbb{E}_Y [S | H_1] = \sigma^2_{w_1} + \frac{P_a}{d_{a, w_1}^\gamma},$$

(42)

$$\text{Var}_Y [S | H_1] = \frac{4P_a \sigma^2_{w_1}}{n d_{a, w_1}^\gamma} + \frac{2 \sigma^4_{w_1}}{n}.$$  

(43)

If $S < \sigma^2_{w_1} + t$, $W_1$ accepts $H_0$; otherwise, he accepts $H_1$. By Chebyshev’s inequality [5]:

$$\mathbb{P}_{FA} \leq \frac{2 \sigma^4_{w_1}}{nt^2}.$$  

(44)

To account for the singularities of $\sigma^2_{w_1}$ at $d_{w_1, f_k} = 0$, we define the following event as depicted in Fig. 4:

$$\mathcal{C} = \bigcap_{k=2}^{N_w} \left\{ \left\{ d_{w_k, f_k} \leq \beta \right\} \cap \left\{ d_{w_1, w_k} > 2\beta \right\} \right\} \bigcap \left\{ d_{w_1, f_1} > \nu \right\},$$
Fig. 4. When $C$ is true: (1) there is no friendly node in a disk of radius $\nu$ centered at $W_1$; (2) the closest friendly node to each of $W_2, W_3, \ldots, W_N$ is in the disk of radius $\beta$ centered at it; and, (3) the distance between $W_1$ and any other Willie is larger than $2\beta$. Here, $N_w = 3$.

where

$$\nu = \sqrt{\frac{\ln \frac{1}{1-\lambda/8}}{m\pi}},$$

(45)

$$\beta = \sqrt{\frac{\lambda}{32\pi N_w}}.$$  

(46)

and $\lambda > 0$ is an arbitrarily constant. The law of total expectation yields

$$\mathbb{E}_{F,W}[\mathbb{P}_{FA}] \leq \mathbb{P}(C) + \mathbb{E}_{F,W}[\mathbb{P}_{FA}|C].$$  

(47)

We show in the Appendix that since $N_w = \omega(1)$, $m = \omega(1)$, and $N_w = o(m/\log m)$,

$$\lim_{n\to\infty} \mathbb{P}(C) \leq \lambda/4.$$  

(48)

Now, consider $\mathbb{E}_{F,W}[\mathbb{P}_{FA}|C]$. By (44),

$$\mathbb{E}_{F,W}[\mathbb{P}_{FA}|C] \leq \frac{2\mathbb{E}_{F,W}[\sigma_{w_1}^4|C]}{nt^2}.$$  

(49)

Next, we derive in the Appendix that:

$$\mathbb{E}_{F,W}[\sigma_{w_1}^4|C] \leq \sigma_{w_1,0}^4 + \mathbb{E}_{F,W}\left[ \frac{P_f^2}{d_{w_1,f_1}} \right] + N_w\mathbb{E}_{F,W}\left[ \frac{P_f^2}{d_{w_1,f_2}} \right] + N_w\mathbb{E}_{F,W}\left[ \frac{2P_f^2}{d_{w_1,f_1}d_{w_1,f_2}} \right]$$

$$+ N_w^2\mathbb{E}_{F,W}\left[ \frac{P_f^2}{d_{w_1,f_2}} \right] + \frac{2\sigma_{w_1,0}^2P_f}{\nu^2} + N_w\mathbb{E}_{F,W}\left[ \frac{2\sigma_{w_1,0}^2 P_f}{d_{w_1,f_2}} \right] C.$$  

(50)
Consider $\mathbb{E}_{F,W} \left[ \frac{1}{d_{w_1,t_2}} \right] C$ and $\mathbb{E}_{F,W} \left[ \frac{1}{d_{w_1,t_2}^{2\gamma}} \right] C$ in (50). We show in the Appendices G and H that:

$$
\mathbb{E}_{F,W} \left[ \frac{1}{d_{w_1,t_2}^{\gamma}} \right] C \leq \frac{8\pi}{(\gamma - 2)\beta^{\gamma - 2}}, \tag{51}
$$

$$
\mathbb{E}_{F,W} \left[ \frac{1}{d_{w_1,t_2}^{2\gamma}} \right] C \leq \frac{8\pi}{(2\gamma - 2)\beta^{2\gamma - 2}}. \tag{52}
$$

Because $\beta = \Theta(1/\sqrt{N_w})$, $\nu = \Theta(1/\sqrt{m})$, $N_w = \omega(1)$, and $N_w = o(m/\log m)$, for large enough $n$, (50)-(52) yield

$$
\mathbb{E}_{F,W} \left[ \sigma^2_{w_1} | C \right] \leq \frac{2P^2}{\nu^{2\gamma}}. \tag{53}
$$

This means that the noise generated by the closest friendly node to $W_1$ dominates the noise generated by the friendly nodes closest to the other Willies. By (49) and (53)

$$
\mathbb{E}_{F,W} \left[ \mathbb{P}_{FA} | C \right] \leq \frac{4P^2_t}{nt^2\nu^{2\gamma}} = \frac{4P^2_t \left( \frac{\pi m}{\ln \frac{1}{1-\lambda/8}} \right)^{\gamma}}{nt^2},
$$

where the last step follows from substituting in the value of $\nu$ in (46). Choose

$$
t = \frac{4P_t}{\sqrt{n\lambda}} \left( \frac{\pi m}{\ln \frac{1}{1-\lambda/8}} \right)^{\gamma/2}.
$$

Since $m = \omega(1)$, $N_w = \omega(1)$,

$$
\lim_{n \to \infty} \mathbb{E}_{F,W}[\mathbb{P}_{FA} | C] \leq \lambda/4. \tag{54}
$$

By (47), (48), and (54):

$$
\lim_{n \to \infty} \mathbb{E}_{F,W}[\mathbb{P}_{FA}] \leq \lambda/2. \tag{55}
$$

Now, consider $\mathbb{P}_{MD}$. Similar to the approach leading to (21), we obtain

$$
\mathbb{P}_{MD} \leq \frac{4P_{a,w_1} - \sigma^4_{w_1} + 2\sigma^4_{w_1}}{n \left( \frac{P_a}{d_{a,w_1}} - t \right)^2}. \tag{56}
$$

Define the event

$$
\mathcal{E} = C \cap \{ \ell \beta' \leq d_{a,w_1} < \beta' \},
$$

where

$$
\beta' = \sqrt{\frac{2\ln (8/\lambda)}{\pi N_w}},
$$

$$
\ell = \sqrt{\frac{\ln (1-\lambda/8)}{\ln (\lambda/8)}}. \tag{57}
$$
The law of total expectation yields

$$\mathbb{E}_{F,W}[P_{MD}] \leq \mathbb{E}_{F,W}[P_{MD}|E] + \mathbb{P}(\overline{E}).$$  

(58)

We show in the Appendix I that

$$\mathbb{E}_{F,W}[P_{MD}|E] \leq \frac{4P_a \sqrt{\frac{\beta}{\nu} \sqrt{\mathbb{P}(E)}}}{n^2} + \frac{4P_f^2}{n^2},$$  

(59)

and since $m = \omega(1)$, and $N_w = \omega(1)$,

$$\lim_{n \to \infty} \mathbb{P}(\overline{E}) \leq \lambda/2.$$  

(60)

Consider $\frac{1}{\mathbb{P}(E)}$ in (59). By (60), $\lim_{n \to \infty} \frac{1}{\mathbb{P}(E)} \leq \frac{1}{1-\lambda/2}$. On the other hand, since $t = \Theta(m^{\gamma/2}/\sqrt{n})$, $\nu = \Theta(1/\sqrt{m})$, $\beta' = \Theta(1/\sqrt{N_w})$, and $m = \omega(1)$, $N_w = \omega(1)$, if Alice sets her average symbol power $P_a = \omega\left(\frac{m^{\gamma/2}}{\sqrt{N_w}}\right)$, $\mathbb{E}_{F,W}[P_{MD}|E] = 0$ as $n \to \infty$. By (58) and (60)

$$\lim_{n \to \infty} \mathbb{E}_{F,W}[P_{MD}] \leq \lambda/2.$$  

(61)

Combined with (55), $\mathbb{E}_{F,W}[P_{FA} + P_{MD}] \leq \lambda$ for any $\lambda > 0$.

Thus, to avoid detection for a given codeword, Alice must set the power of that codeword to $P_{\text{fd}} = O\left(\frac{m^{\gamma/2}}{\sqrt{n^{N_w}/2}}\right)$. Suppose that Alice’s codebook contains a fraction $\xi > 0$ of codewords with power $P_{\text{fd}} = O\left(\frac{m^{\gamma/2}}{\sqrt{n^{N_w}/2}}\right)$. Similar to converse in Theorem 1, Bob’s decoding error probability of such low power codewords is lower bounded by (see (22))

$$\mathbb{P}_{e}^{\text{fd}} \geq 1 - \frac{P_{\text{fd}}}{2\sigma_b^2} + \frac{1}{n} \log_2 \frac{\xi}{R}.$$  

Denote the closest Willie to Bob by $W_4$. Since Bob’s noise is lower bounded by the noise generated from the closest friendly node to $W_4$, $\sigma_b^2 \geq \frac{P_1}{\sigma_{b,t_4}^2}$,

$$\mathbb{P}_{e}^{\text{fd}} \geq 1 - \frac{P_{\text{fd}}}{2\sigma_b^2} + \frac{1}{n} \log_2 \frac{\xi}{R}.$$  

Define the event

$$\mathcal{F} = \left\{ d_{b,t_4} < \sqrt{\frac{8 \ln(1/\tau)}{\pi N_w}} \right\},$$

where $0 < \tau < 1$. The law of total expectation yields

$$\mathbb{E}_{F,W}[\mathbb{P}_{\text{e}}^{\text{fd}}] \geq \mathbb{E}_{F,W}[\mathbb{P}_{\text{e}}^{\text{fd}}|\mathcal{F}] \mathbb{P}(\mathcal{F}).$$  

(62)
Consider \( P(\mathcal{F}) \). We show in the Appendix \[ \] that since \( m = \omega(1) \), \( N_w = \omega(1) \), and \( N_w = o(m/\log m) \),

\[
\lim_{n \to \infty} P(\mathcal{F}) = 1 - \tau. \tag{63}
\]

Now, consider \( E_{F,W}[P_{UE}|\mathcal{F}] \) in (62).

\[
E_{F,W}[P_{UE}|\mathcal{F}] \geq 1 - E_{F,W}\left[\frac{P_U e^{i2\pi m \gamma/2}}{2P_t} \left( \frac{2\ln \frac{1}{\tau}}{\sqrt{N_w}} \right)} + \frac{1}{n} \right] F, \tag{d}
\]

\[
= 1 - E_{F,W}\left[\frac{P_U \left( \frac{2\ln \frac{1}{\tau}}{\sqrt{N_w}} \right)}{2P_t} \left( \frac{2\ln \frac{1}{\tau}}{\sqrt{N_w}} \right)} + \frac{1}{n} \right] F, \tag{64}
\]

where (d) is true since \( F \) occurs. Suppose Alice desires to transmit \( \omega\left( \sqrt{m\gamma/2} \right) \) covert bits in \( n \) channel uses. Therefore, her rate (bits/symbol) is \( R = \omega\left( \sqrt{m\gamma/2} \right) \). Since \( P_U = O\left( \frac{m\gamma/2}{\sqrt{N_w}} \right) \), \( m = \omega(1) \), and \( N_w = \omega(1) \),

\[
\lim_{n \to \infty} E_{F,W}[P_{UE}|\mathcal{F}] = 1. \tag{64}
\]

By (62), (63), and (64), for any \( 0 < \tau < 1 \), \( \lim_{n \to \infty} E_{F,W}[P_{UE}] \geq 1 - \tau \), and thus \( E[P_{UE}] \) is bounded away from zero.

**B. \( \gamma = 2 \)**

**Theorem 3.** Assume friendly nodes are independently distributed according to a two-dimensional Poisson point process with density \( m = \omega(1) \), and \( N_w \) collaborating Willies are uniformly and independently distributed over the unit square shown in Fig. 1. If \( N_w = o\left( \min\left\{ \frac{m}{\log m}, \frac{m\sqrt{n}}{\log (m\sqrt{n})} \right\} \right) \), then Alice can reliably and covertly transmit \( O\left( \frac{m\sqrt{n}}{\sqrt{N_w} \log N_w} \right) \) bits to Bob in \( n \) channel uses.

**Proof.** *(Achievability)*

**Construction:** The construction is similar to that of Theorem 2.

**Analysis:** *(Covertness)* The difference between the results for \( \gamma > 2 \) and \( \gamma = 2 \) originates from the following integral necessary in the proofs:

\[
\int \frac{dx}{x^{\gamma-1}} = \begin{cases} 
  x^{2-\gamma}/(2-\gamma) + c_0, & \gamma > 2 \\
  \ln x + c_0', & \gamma = 2
\end{cases},
\]

where \( c_0 \) and \( c_0' \) are constants. Therefore, the analysis for \( \gamma = 2 \) follows similarly with a few minor modifications. Alice sets her average symbol power \( P_a \leq \frac{em}{\sqrt{mN_w} \ln N_w} \) where

\[
c = 4\sqrt{2}\pi P_t. \tag{65}
\]
Next, we modify (73) to
\[ \mathbb{E}_W \left[ \frac{1}{d_{a,w_k}} \mid d_{a,w_k} > \kappa \right] \leq \pi \ln (N_w). \]

Then, we can show that Alice achieves (29) and thus her communication is covert as long as \( P_a = O \left( \frac{m}{(\sqrt{nN_w \log N_w})} \right) \).

*(Reliability)* Similar to the approach in the reliability for \( \gamma > 2 \), we can show that if Alice sets \( R = \min \{ 1, R_0 \} \), where
\[
R_0 = \frac{1}{4} \log_2 \left( 1 + \frac{c'm}{4N_w^2(\ln N_w)^2\sqrt{n}} \right),
\]
(66)

and \( c' \) is defined in (65), then \( m = \omega(1), \ N_w = \omega(1), \) and \( N_w = o \left( \min \left\{ \frac{m}{\log m}, \frac{m\sqrt{n}}{\log (m\sqrt{n})} \right\} \right) \) yield
\[
\lim_{n \to \infty} \mathbb{P}_b^a < \zeta \text{ for any } 0 < \zeta < 1.
\]

*(Number of Covert Bits)* Similar to the analysis for \( \gamma > 2 \), we can show that by (66), Bob receives
\[
O \left( \min \left\{ n, \frac{m^{\gamma/2}\sqrt{n}}{N_w^2 \log^2 N_w} \right\} \right) \text{ bits in } n \text{ channel uses.}
\]

*(Converse)* The approach used for \( \gamma > 2 \), which involved choosing the closest Willie to Alice to decide whether Alice communicates with Bob or not, does not yield a tight result for \( \gamma = 2 \). Using this approach, we can show that if Alice sets her average symbol power \( P_a = \omega \left( \frac{m}{\sqrt{nN_w}} \right) \), then Willie detects her with arbitrarily small sum of error probabilities. However, from the achievability, we expect that \( P_a = \omega \left( \frac{m}{\sqrt{nN_w \log N_w}} \right) \) results in detection. This suggests that Willies have to consider their signals received collectively to detect Alice’s communication, as we expect for \( \gamma = 2 \) the signal decays slowly with distance.

V. DISCUSSION

A. Assumption of \( m = \omega(1) \) in Theorems 2 and 3

In Theorems 2 and 3, we assumed \( m = \omega(1) \); however, we can relax this assumption as in Theorem 1 and we only used it to simplify the proof when \( N_w = \omega(1) \). Furthermore, \( m = \omega(1) \) becomes plausible when the single hop communication scheme presented in this paper is extended to covert multi-hop communication over large wireless networks [21], as the number of friendly nodes grows with the size of the network. An example of employing artificial noise generation with growing density in a large wireless network is presented in [27], where authors analyze the throughput of key-less secure communication in a cell of size \( \sqrt{n} \times \sqrt{n} \). In particular, transmitter and receiver nodes are distributed according to a Poisson point process with density one in the cell, and each node is allowed to generate artificial noise.
VI. CONCLUSION

In this paper, we have considered the first step in establishing covert communications in a network scenario. We establish that Alice can transmit $O\left(\min\{n, m^{\gamma/2} \sqrt{n}\}\right)$ bits reliably to the desired recipient Bob in $n$ channel uses without detection by an adversary Willie if randomly distributed system nodes of density $m$ are available to aid in jamming Willie; conversely, no higher covert rate is possible, if the nearest node to Willie is used to jam his receiver. The presence of multiple collaborating adversaries inhibits communication in two separate ways: (1) increasing the effective SNR at the adversaries’ decision point; and (2) requiring more interference, which inhibits Bob’s ability to reliably decode the message. We established that in the presence of $N_w$ Willies, Alice can reliably and covertly send $O\left(\min\{n, \frac{\sqrt{m^{\gamma/2}}}{N_w}\}\right)$ bits to Bob when $\gamma > 2$, and $O\left(\min\{n, \frac{\sqrt{m}}{N_w^{3/2} \log^2 N_w}\}\right)$ when $\gamma = 2$. Conversely, under mild restrictions on the communication strategy, no higher covert throughput is possible for $\gamma > 2$.

Future work consists of proving the converse for $\gamma = 2$ and embedding the results of this single-hop formulation into large multi-hop covert networks.

APPENDIX

A. Proof of (29): By (26),

$$\mathbb{E}_{F,W} \left[ \sqrt{\frac{1}{8} D\left(\mathbb{P}_1 || \mathbb{P}_0\right)} \right] \leq \mathbb{E}_{F,W} \left[ \frac{\sqrt{n}}{4\sqrt{2}} \sum_{k=1}^{N_w} \frac{P_a}{d_{a,w_k}^\gamma \sigma_{w_k}^2} \right],$$

where (67) is true because $D\left(\mathbb{P}_1 || \mathbb{P}_0\right)$ is upper bounded in (26), (68) is true because $P_a$ is upper bounded in (27), (69) is true because the locations of friendly nodes are independent of the locations of Willies, and $\mathbb{E}_W[\cdot]$ denotes expectation with respect to the locations of Willies. Consider $\mathbb{E}_F \left[ \frac{m^{\gamma/2}}{\sigma_{w_k}^2} \right]$ in (69).

Similar to the approach leading to (9), we can show that for all $k$,

$$\mathbb{E}_F \left[ \frac{m^{\gamma/2}}{\sigma_{w_k}^2} \right] \leq \frac{\Gamma \left(\gamma/2 + 1\right)}{2P_f \pi^{\gamma/2 + 1}}.$$  

Now, consider $\mathbb{E}_W \left[ \frac{1}{d_{a,w_k}^\gamma} \right]$ in (69). Since Willies are distributed independently,

$$\sum_{k=1}^{N_w} \mathbb{E}_W \left[ \frac{1}{d_{a,w_k}^\gamma} \right] = \sum_{k=1}^{N_w} \mathbb{E}_W \left[ \frac{1}{d_{a,w_k}^\gamma} | d_{a,w_k} > \kappa \right] = N_w \mathbb{E}_W \left[ \frac{1}{d_{a,w_k}^\gamma} | d_{a,w_k} > \kappa \right].$$  

23
Next we find an upper bound for the pdf of $d_{a,w_k}$ given $d_{a,w_k} > \kappa$, $g(x)$, and then upper bound $\mathbb{E}_W \left[ \frac{1}{d_{a,w_k}} \right] d_{a,w_k} > \kappa$. Consider a circle of radius $x$ centered at Alice. As shown in Fig. 5 we can partition this circle into two regions: the yellow region whose area is $\mathbb{P}(\kappa \leq d_{a,w_k} \leq x)$ and the red region whose area is denoted by $h(x)$. Note that $h(x)$ is a monotonically increasing function of $x$. Therefore, $\frac{dh(x)}{dx} > 0$. Consequently,

$$g(x) = \frac{d}{dx} \mathbb{P}(\kappa \leq d_{a,w_k} \leq x) = \frac{d}{dx}(\pi x^2 - h(x)) = 2\pi x - \frac{dh(x)}{dx} \leq 2\pi x. \quad (72)$$

Hence,

$$\mathbb{E}_W \left[ \frac{1}{d_{a,w_k}^\gamma} \right] d_{a,w_k} > \kappa \leq \int_{x=\kappa}^\infty \frac{2\pi x}{x^\gamma} dx = 2\pi \frac{\kappa^{2-\gamma}}{\gamma - 2}. \quad (73)$$

Consequently, (71) becomes

$$\sum_{k=1}^{N_w} \mathbb{E}_W \left[ \frac{1}{d_{a,w_k}^\gamma} \right] \mathcal{A} \leq N_w 2\pi \frac{\kappa^{2-\gamma}}{\gamma - 2}. \quad (74)$$

Thus, (69), (70), and (74) yield

$$\mathbb{E}_W \left[ \sqrt{\frac{1}{8} D\left(\|P_1\|\|P_0\|\right)} \mathcal{A} \right] \leq \frac{c}{4\sqrt{2}N_\omega^{\gamma/2}} \frac{\Gamma(\gamma/2 + 1)}{2^{\gamma/2 + 1}} N_w \frac{2\pi \kappa^{2-\gamma}}{\gamma - 2} = \varepsilon, \quad (75)$$

where the last step is true since $c = \frac{F_1 e^{\gamma/2}(\gamma-2)\pi^{\gamma/2}}{2\gamma^{-0.5} \Gamma(\gamma/2+1)}$ and $\kappa = \sqrt{\frac{\varepsilon}{4N_w}}$. By (23) and (75), (29) is proved.
B. Proof of (33): Assume \( \frac{c'_b \sigma_b^2}{cN_w^2} \leq 1 \). Since the RHS of (32) is a monotonically increasing function of \( \sigma_b^2 \), (32) yields
\[
\mathbb{P}^{(b)}_e(\sigma_b^2) \leq 2^{-\frac{nR}{2} \log_2 \left( 1 + \frac{c'm^{\gamma/2}}{2N_w \sqrt{n \sigma_b^2}} \right)}.
\] (76)
By (33) and (76), \( \mathbb{P}^{(b)}_e(\sigma_b^2) \leq 2^{nR-2nR_0} \). Since \( R = \min\{1, R_0\} \leq R_0 \),
\[
\mathbb{P}^{(b)}_e(\sigma_b^2) \leq 2^{-nR_0} \leq 2^{-\frac{n}{2} \log_2 \left( 1 + \frac{c'm^{\gamma/2}}{2N_w \sqrt{n \sigma_b^2}} \right)} = \left( 1 + \frac{c'm^{\gamma/2}}{2N_w \sqrt{n \sigma_b^2}} \right)^{-\frac{n}{2}}. \tag{77}
\]
By (77), \( N_w = o \left( n^{\frac{1}{25}} \sqrt{m} \right) \), \( m = \omega(1) \), and \( N_w = \omega(1) \),
\[
\mathbb{E}_{F,W} \left[ \mathbb{P}^{(b)}_e(\sigma_b^2) \right] \leq 1 \leq \left( 1 + \frac{c'm^{\gamma/2}}{2N_w \sqrt{n \sigma_b^2}} \right)^{-\frac{n}{2}}(e) \leq \left( 1 + \frac{\sqrt{\pi}m^{\gamma/2}}{8N_w \sigma_b^2} \right)^{-1} \rightarrow 0 \text{ as } n \rightarrow \infty, \tag{78}
\]
where (e) is true since (17) is true.

C. Proof of (37): When \( B \) is true, \( d_{b,w_k} > 2\delta \) and \( 2\delta > 2d_{w_k,f_k} \). Thus, \( -d_{w_k,f_k} > -\frac{d_{b,w_k}}{2} \). On the other hand, the triangle inequality yields \( d_{b,f_k} \geq d_{b,w_k} - d_{w_k,f_k} \). Thus,
\[
d_{b,f_k} > \frac{d_{b,w_k}}{2}. \tag{79}
\]
Now, consider \( \frac{c'_b \sigma_b^2}{cN_w^2} \). Recall that \( \sigma_b^2 \leq \sigma_{b,0}^2 + \sum_{k=1}^{N_w} P_t \frac{d_{b,f_k}}{d_{b,w_k}}. \) When \( B \) is true,
\[
\frac{c'_b \sigma_b^2}{cN_w^2} \leq \frac{c'_b \sigma_{b,0}^2}{cN_w^2} + \frac{c'_b \sigma_b^2}{cN_w^2} \sum_{k=1}^{N_w} P_t \frac{d_{b,f_k}}{d_{b,w_k}} < \frac{c'_b \sigma_{b,0}^2}{cN_w^2} + \frac{c'_b \sigma_b^2}{cN_w^2} \sum_{k=1}^{N_w} \frac{P_t 2^\gamma}{d_{b,w_k}} \tag{80}
\]
\[
= \frac{c'_b \sigma_{b,0}^2}{cN_w^2} + \frac{\gamma - 2}{2^{5-\gamma} \pi} \frac{1}{N_w} \sum_{k=1}^{N_w} \frac{\delta^{\gamma-2}}{d_{b,w_k}^2} \tag{81}
\]
where (80) is true since \( B \) implies (79), and (81) is true since \( c' = c\frac{c_{\gamma/2-1}(\gamma-2)}{2^{\gamma+3}P_t \pi^{\gamma/2}} \) and \( \delta = \sqrt{\frac{\xi}{4\pi N_w}}. \) By (81),
\[
\mathbb{P} \left( \frac{c'_b \sigma_b^2}{cN_w^2} > 1 \mid B \right) \leq \mathbb{P} \left( \frac{c'_b \sigma_{b,0}^2}{cN_w^2} + \frac{\gamma - 2}{2^{5-\gamma} \pi} \frac{1}{N_w} \sum_{k=1}^{N_w} \frac{\delta^{\gamma-2}}{d_{b,w_k}^2} > 1 \mid B \right). \tag{82}
\]
Consider \( \frac{c'_b \sigma_{b,0}^2}{cN_w^2} \) in the above equation. Since \( N_w = \omega(1) \), for large enough \( n \), \( \frac{c'_b \sigma_{b,0}^2}{cN_w^2} \leq \frac{1}{2} \). Thus,
\[
\lim_{n \to \infty} \mathbb{P} \left( \frac{c'_b \sigma_b^2}{cN_w^2} > 1 \mid B \right) \leq \lim_{n \to \infty} \mathbb{P} \left( \frac{1}{2} + \frac{\gamma - 2}{2^{5-\gamma} \pi} \frac{1}{N_w} \sum_{k=1}^{N_w} \frac{\delta^{\gamma-2}}{d_{b,w_k}^2} > 1 \mid B \right),
\]
\[
= \lim_{n \to \infty} \mathbb{P} \left( \frac{\gamma - 2}{2^{5-\gamma} \pi} \frac{1}{N_w} \sum_{k=1}^{N_w} \frac{d_{b,w_k}^2}{d_{b,w_k}^2} > \frac{1}{2} \mid B \right),
\]
\[
= \lim_{n \to \infty} \mathbb{P} \left( \frac{1}{N_w} \sum_{k=1}^{N_w} \frac{\delta^{\gamma-2}}{\gamma - 2} > \pi^{2^{5-\gamma}} \mid B \right). \tag{83}
\]
Next, we upper bound \( \alpha = \mathbb{E}_{F,W} \left[ \frac{\delta_i \gamma - 2}{d_{b,w}} \right] \) and then apply the weak law of large numbers (WLLN) to show that \( (83) \) is equal to zero. Since the locations of Willies are independent of the locations of friendly nodes,

\[
\alpha = \mathbb{E}_{F,W} \left[ \frac{\delta_i \gamma - 2}{d_{b,w}} \right] \leq \delta \cap d_{b,w} > 2\delta = \mathbb{E}_{F,W} \left[ \frac{\delta_i \gamma - 2}{d_{b,w}} \right] = \mathbb{E}_{F,W} \left[ \frac{\delta_i \gamma - 2}{d_{b,w}} \right] \text{.}
\]

Similar to the arguments that leads to \( (73) \) we can show that

\[
\alpha = \mathbb{E}_W \left[ \frac{\delta_i \gamma - 2}{d_{b,w}} \right] \leq \frac{\pi^{23 - \gamma}}{2 - \gamma}.
\] (84)

Thus, \( \alpha \) is finite. By the WLLN and \( N_w = \omega(1) \), for all \( \epsilon' > 0 \), \( \mathbb{P} \left( \frac{1}{N_w} \sum_{k=1}^{N_w} \delta_i \gamma - 2 \leq \alpha \right) = 0 \), as \( n \to \infty \). Let \( \epsilon' = \alpha \),

\[
\lim_{n \to \infty} \mathbb{P} \left( \frac{1}{N_w} \sum_{k=1}^{N_w} \delta_i \gamma - 2 \geq 2\alpha \right) = 0. \] (85)

Using the upper bound on \( \alpha \) presented in \( (84), (85) \) yields

\[
\lim_{n \to \infty} \mathbb{P} \left( \frac{1}{N_w} \sum_{k=1}^{N_w} \delta_i \gamma - 2 \geq \frac{\pi^{23 - \gamma}}{2 - \gamma} \right) = 0. \] (86)

By \( (83) \) and \( (86), (37) \) is proved.

D. Proof of \( (38) \): Since \( \mathcal{B} \) is the union of \( \bigcup_{k=1}^{N_w} \{ d_{b,w} \leq 2\delta \} \) and \( \bigcup_{k=1}^{N_w} \{ d_{w,k} > \delta \} \),

\[
\mathbb{P} \left( \mathcal{B} \right) \leq \sum_{k=1}^{N_w} \mathbb{P} \left( d_{b,w} \leq 2\delta \right) + \sum_{k=1}^{N_w} \mathbb{P} \left( d_{w,k} > \delta \right) = N_w \mathbb{P} \left( d_{b,w} \leq 2\delta \right) + N_w \mathbb{P} \left( d_{w,k} > \delta \right). \] (87)

Because Willies are distributed uniformly, \( \mathbb{P} \left( d_{b,w} \leq 2\delta \right) \leq 2\pi \delta^2 \), and by \( (8) \), \( \mathbb{P} \left( d_{w,k} > \delta \right) = e^{-m \pi \delta^2} \). Therefore, \( (87) \) becomes \( \mathbb{P} \left( \mathcal{B} \right) \leq 2\pi N_w \delta^2 + N_w e^{-m \pi \delta^2} \). Since \( \delta = \sqrt{\frac{\zeta}{4\pi N_w}} \),

\[
\mathbb{P} \left( \mathcal{B} \right) \leq \zeta/2 + N_w e^{-m \pi \delta^2} = \zeta/2 + e^{\ln N_w - m \pi \delta^2}. \] (88)

Consequently, \( N_w = o(m/\log m) \), \( N_w = \omega(1) \), and \( m = \omega(1) \) yield \( \lim_{n \to \infty} \mathbb{P} \left( \mathcal{B} \right) = \zeta/2 \).

E. Proof of \( (48) \): Observe

\[
\mathbb{P}(\overline{C}) \leq \mathbb{P}(d_{w_1, f_1} \leq \nu) + \sum_{k=2}^{N_w} \mathbb{P}(d_{w_k, f_k} > \beta) + \sum_{k=2}^{N_w} \mathbb{P}(d_{w_1, w_k} \leq 2\beta), \] (89)

\[
= \mathbb{P}(d_{w_1, f_1} \leq \nu) + (N_w - 1) \mathbb{P}(d_{w_k, f_k} > \beta) + (N_w - 1) \mathbb{P}(d_{w_1, w_k} \leq 2\beta),
\]

\[
\leq \mathbb{P}(d_{w_1, f_1} \leq \nu) + N_w \left( \mathbb{P}(d_{w_2, f_2} > \beta) + \mathbb{P}(d_{w_1, w_2} \leq 2\beta) \right), \] (90)

where \( W_2 \) is an arbitrary Willie rather than \( W_1 \). Consider \( \mathbb{P}(d_{w_1, f_1} \leq \nu) \) in \( (90) \). By \( (8) \), \( \mathbb{P}(d_{w_1, f_1} \leq \nu) = 1 - e^{-m \pi \nu^2} \). Since \( \nu = \sqrt{\frac{\ln (1 - 2/\sqrt{8})}{m \pi}} \),

\[
N_w \mathbb{P}(d_{w_1, f_1} \leq \nu) = \frac{\lambda}{8}. \] (91)
Now, consider \( \mathbb{P}(d_{w_k,f_k} > \beta) \) in (90). By (87), \( \mathbb{P}(d_{w_k,f_k} > \beta) = e^{-m\pi \beta^2} \). Since \( \beta = \sqrt{\frac{\lambda}{32\pi N_w}} \), \( N_w = \omega(1) \), \( m = \omega(1) \), and \( N_w = o(m/\log m) \),

\[
\lim_{n \to \infty} N_w \mathbb{P}(d_{w_k,f_k} > \beta) = \lim_{n \to \infty} N_w e^{-m\pi \beta^2} = \lim_{n \to \infty} N_w e^{-\frac{\lambda m}{32\pi N_w}} = \lim_{n \to \infty} e^{\ln N_w - \frac{\lambda m}{32\pi N_w}} = 0. \tag{92}
\]

Now, consider \( \mathbb{P}(d_{w_1,w_2} \leq 2\beta) \) in (90). Note that \( \mathbb{P}(d_{w_1,w_2} \leq 4\beta^2) \leq 4\beta^2 \) and the equality holds when the distance between \( W_1 \) and the closest side of the unit square is not smaller than \( 2\beta \). Substituting the value of \( \beta \) yields

\[
N_w \mathbb{P}(d_{w_1,w_2} \leq 2\beta) \leq 4\pi N_w \beta^2 = \frac{\lambda}{8}. \tag{93}
\]

By (90)-(93), (48) is proved.

**F. Proof of (50):** Since \( \sigma_{w_1}^2 \leq \sigma_{w_1,0}^2 + \sum_{k=1}^{N_w} \frac{P_f}{d_{w_1,f_k}^2} \),

\[
\sigma_{w_1}^4 \leq \sigma_{w_1,0}^4 + \sum_{k,k'=1}^{N_w} \frac{P_f^2}{d_{w_1,f_k}^2 d_{w_1,f_{k'}}^2} + 2\sigma_{w_1,0}^2 \sum_{k=1}^{N_w} \frac{P_f}{d_{w_1,f_k}^3}. \tag{94}
\]

Consider the second term on the RHS of (94)

\[
\mathbb{E}_{F,W} \left[ \sum_{k,k'=1}^{N_w} \frac{P_f^2}{d_{w_1,f_k}^2 d_{w_1,f_{k'}}^2} \right] = \mathbb{E}_{F,W} \left[ \frac{P_f^2}{d_{w_1,f_1}^2} \right] + \mathbb{E}_{F,W} \left[ \frac{P_f^2 (N_w - 1)}{d_{w_1,f_2}^2} \right] + \mathbb{E}_{F,W} \left[ \frac{(2N_w - 2)P_f^2}{d_{w_1,f_1}^2} \right] + \mathbb{E}_{F,W} \left[ \frac{P_f^2 (N_w - 1)(N_w - 2)}{d_{w_1,f_2}^2} \right],
\]

\[
\leq \mathbb{E}_{F,W} \left[ \frac{P_f^2}{d_{w_1,f_1}^2} \right] + \mathbb{E}_{F,W} \left[ \frac{N_w P_f^2}{d_{w_1,f_2}^2} \right] + \mathbb{E}_{F,W} \left[ \frac{2N_w P_f^2}{d_{w_1,f_2}^2} \right] + \mathbb{E}_{F,W} \left[ \frac{N_w^2 P_f^2}{d_{w_1,f_2}^2} \right]. \tag{95}
\]

Recall that when \( C \) is true, \( d_{w_1,f_1} > \nu \). Therefore, conditioning the expected values in (95) on \( C \) and applying \( \mathbb{E}_{F,W}^2 [1/d_{w_1,f_1}^2 | C] \leq 1/\nu^2 \gamma \) and \( \mathbb{E}_{F,W}^2 [1/d_{w_1,f_1} | C] \leq 1/\nu \gamma \) yields:

\[
\mathbb{E}_{F,W} \left[ \sum_{k,k'=1}^{N_w} \frac{P_f^2}{d_{w_1,f_k}^2 d_{w_1,f_{k'}}^2} | C \right] \leq \frac{P_f^2}{\nu^2} + \mathbb{E}_{F,W} \left[ \frac{N_w^2 P_f^2}{d_{w_1,f_2}^2} | C \right] + \frac{1}{\nu^2} \mathbb{E}_{F,W} \left[ \frac{2N_w P_f^2}{d_{w_1,f_2}^2} | C \right] + \mathbb{E}_{F,W} \left[ \frac{N_w^2 P_f^2}{d_{w_1,f_2}^2} | C \right]. \tag{96}
\]

Now, consider the third term on the RHS of (94).

\[
\mathbb{E}_{F,W} \left[ \sum_{k=1}^{N_w} \frac{2\sigma_{w_1,0}^2 P_f}{d_{w_1,f_k}^3} | C \right] = \mathbb{E}_{F,W} \left[ \frac{2\sigma_{w_1,0}^2 P_f}{d_{w_1,f_1}^3} | C \right] + (N_w - 1) \mathbb{E}_{F,W} \left[ \frac{2\sigma_{w_1,0}^2 P_f}{d_{w_1,f_2}^3} | C \right],
\]

\[
\leq \mathbb{E}_{F,W} \left[ \frac{2\sigma_{w_1,0}^2 P_f}{d_{w_1,f_1}^3} | C \right] + N_w \mathbb{E}_{F,W} \left[ \frac{2\sigma_{w_1,0}^2 P_f}{d_{w_1,f_2}^3} | C \right],
\]

\[
\leq \frac{2\sigma_{w_1,0}^2 P_f}{\nu^2} + N_w \mathbb{E}_{F,W} \left[ \frac{2\sigma_{w_1,0}^2 P_f}{d_{w_1,f_2}^3} | C \right], \tag{97}
\]

where the last step is true because \( \mathbb{E}_{F,W}^2 [1/d_{w_1,f_1}^3 | C] \leq 1/\nu^2 \). By (94), (96), the proof is complete.
G. Proof of (51): When $C$ is true, \( \{d_{w_2,f_2} \leq \beta < d_{w_1,w_2}/2\} \). Thus, $-d_{w_2,f_2} \geq -d_{w_1,w_2}/2$. The triangle inequality yields $d_{w_1,f_2} \geq d_{w_1,w_2} - d_{w_2,f_2}$. Hence, $d_{w_1,f_2} \geq d_{w_1,w_2}/2$. Consequently:

\[
\mathbb{E}_{F,W} \left[ \frac{1}{d_{w_1,w_2}^\ell} \right] \leq \mathbb{E}_{F,W} \left[ \frac{2\gamma}{d_{w_1,w_2}^\ell} \right] = \mathbb{E}_{F,W} \left[ \frac{2\gamma}{d_{w_1,w_2}} \right] \left[ d_{w_1,w_2} > 2\beta \right].
\] (98)

Then, similar to the arguments leading to (72), we can show that the pdf of $d_{w_1,w_2}$ given $d_{w_1,w_2} > 2\beta$ is upper bounded by $2\pi x$. Therefore,

\[
\mathbb{E}_{F,W} \left[ \frac{2\gamma}{d_{w_1,w_2}^\ell} | d_{w_1,w_2} > 2\beta \right] = \int_{x=2\beta}^{\infty} \frac{2\gamma f_{\omega}(x)}{x^\gamma} \, dx \leq \int_{x=2\beta}^{\infty} \frac{2\gamma+1}{x^{\gamma+1}} \, dx = \frac{8\pi}{(\gamma - 2)\beta^{\gamma - 2}}.
\] (99)

H. Proof of (52): The proof is similar to that of (51).

I. Proof of (59): Consider the RHS of (56). Since $E$ implies $\ell\beta' \leq d_{a,w_1} < \beta'$, we replace $d_{a,w_1}$ in the numerator with $\ell\beta'$ and in the denominator with $\beta'$ to achieve

\[
\mathbb{E}_{F,W}[P_{MD} | E] \leq \frac{4 P_{0}^\beta}{\ell^{\gamma}} \mathbb{E}_{F,W}[\sigma_{w_1}^2] \frac{\nu^\gamma}{n} + \frac{2 \mathbb{E}_{F,W}[\sigma_{w_1}^4] \mathbb{P}(C)}{\nu^\gamma} \mathbb{P}(E).
\] (100)

Consider $\mathbb{E}_{F,W}[\sigma_{w_1}^4 | E]$ in (100).

\[
\mathbb{E}_{F,W}[\sigma_{w_1}^4 | E] = \mathbb{E}_{F,W}[\sigma_{w_1}^4 | E] \frac{\mathbb{P}(C)}{\mathbb{P}(E)} = \mathbb{E}_{F,W}[\sigma_{w_1}^4 | E] \frac{\mathbb{P}(C)}{\mathbb{P}(E)}.
\] (101)

By (53), since $m = \omega(1)$ and $N_w = \omega(1)$, for large enough $n$, (101) becomes

\[
\mathbb{E}_{F,W}[\sigma_{w_1}^4 | E] \leq \frac{2P_{0}^2}{\nu^{2\gamma}} \mathbb{P}(C) \leq \frac{2P_{0}^2}{\nu^{2\gamma}} \mathbb{P}(E).
\] (102)

Now, consider $\mathbb{E}_{F,W}[\sigma_{w_1}^2 | E]$ in (100). By (102) and the Jensen’s inequality, for large enough $n$:

\[
\mathbb{E}_{F,W}[\sigma_{w_1}^2 | E] \leq \mathbb{E}_{F,W}[\sigma_{w_1}^2 | E] \leq \sqrt{\mathbb{E}_{F,W}[\sigma_{w_1}^2 | E]} \leq \sqrt{\frac{2P_{0}^2}{\nu^{2\gamma}} \mathbb{P}(E)} = \frac{P_{0}^2}{\nu^{\gamma} \mathbb{P}(E)}.
\] (103)

By (100)-(103), (59) is proved. Consider $\mathbb{P}(E)$. Since $\beta' = \Theta(1/\sqrt{N_w})$, and $N_w = \omega(1)$, for large enough $n$, $\beta'$ becomes small such that the semicircular region around Alice with radii $\beta'$ and $\ell\beta'$ are inside the unit square, and thus $\mathbb{P}(d_{a,w_1} \geq \beta') = (1 - \pi\beta'^2/2)^{N_w}$ and $\mathbb{P}(d_{a,w_1} \geq \ell\beta') = (1 - \pi\ell\beta'^2/2)^{N_w}$. Hence:

\[
\mathbb{P}(E) \leq \mathbb{P}(E) + \mathbb{P}(d_{a,w_1} \geq \beta') + 1 - \mathbb{P}(d_{a,w_1} \geq \ell\beta') = \mathbb{P}(E) + (1 - \pi\ell\beta'^2/2)^{N_w} + 1 - (1 - \pi\ell\beta'^2/2)^{N_w}.
\] (104)

Since $m = \omega(1)$, and $N_w = \omega(1)$, $\beta' = \sqrt{\frac{2\ln(8/\lambda)}{\pi N_w}}$, and (48) is true, taking the limit of both sides of (104) as $n \to \infty$ yields

\[
\lim_{n \to \infty} \mathbb{P}(E) \leq \frac{\lambda}{4} + e^{-\frac{\pi\ell\beta'^2 N_w}{2}} + 1 - e^{-\frac{\beta'^2 N_w}{2}} = \frac{\lambda}{4} + \lambda/8 + 1 - (\lambda/8)^{\ell^2}.
\] (105)

By (57), $\lambda/8 + 1 - (\lambda/8)^{\ell^2} = \lambda/4$. Therefore, (105) becomes

\[
\lim_{n \to \infty} \mathbb{P}(E) \leq \frac{\lambda}{4} + \lambda/4 = \lambda/2.
\]
J. Proof of (63): Define the event:

\[ G = \left\{ d_{w_4 f_4} < \sqrt{\frac{2 \ln (1/\tau)}{N_w \pi}} \right\} \cap \left\{ d_{b_4 w_4} < \sqrt{\frac{2 \ln (1/\tau)}{N_w \pi}} \right\}. \]

From the triangle inequality, when \( G \) occurs, \( d_{b_4 f_4} < d_{w_4 f_4} + d_{b_4 w_4} < 2 \sqrt{\frac{2 \ln (1/\tau)}{\pi N_w}} \). Hence, \( P(F|G) = 1 \).

By the law of total probability:

\[ P(F) = P(F|G) P(G) + P(F|\overline{G}) P(\overline{G}) \geq P(G). \]  

(106)

Consider \( P(G) \). Since the locations of Willies are independent of the locations of friendly nodes,

\[ P(G) = P\left(d_{w_4 f_4} < \sqrt{\frac{2 \ln (1/\tau)}{N_w \pi}}\right) P\left(d_{b_4 w_4} < \sqrt{\frac{2 \ln (1/\tau)}{N_w \pi}}\right). \]  

(107)

Consider the first term on the RHS of (107). By (8), \( m = \omega(1) \), \( N_w = \omega(1) \), and \( N_w = o(m/\log m) \),

\[ P\left(d_{w_4 f_4} < \sqrt{\frac{2 \ln (1/\tau)}{\pi N_w}}\right) = 1 - e^{-\frac{2m \ln (1/\tau)}{N_w}} \to 1 \text{ as } n \to \infty. \]  

(108)

Next, consider the second term on the RHS of (107). Note that when \( x < \frac{1}{2} \), \( P(d_{b_4 w_4} < x) = 1 - \left(1 - \frac{x^2}{2}\right)^{N_w} \). Since \( N_w = \omega(1) \), for large enough \( n \), \( \frac{2m \ln (1/\tau)}{N_w} < 1/2 \) and thus

\[ P\left(d_{b_4 w_4} < \sqrt{\frac{2 \ln (1/\tau)}{\pi N_w}}\right) = 1 - \left(1 - \frac{\ln (1/\tau)}{N_w}\right)^{N_w} \to 1 - \tau \text{ as } n \to \infty. \]  

(109)

By (106)-(109),

\[ \lim_{n \to \infty} P(F) \geq 1 - \tau. \]  

(110)

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