Dynamic Packed Compact Tries Revisited

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Abstract

Given a dynamic set $\mathcal{K}$ of $k$ strings of total length $n$ whose characters are drawn from an alphabet of size $\sigma$, a keyword dictionary is a data structure built on $\mathcal{K}$ that provides locate, prefix search, and update operations on $\mathcal{K}$. Under the assumption that $\alpha = \lg_\sigma n$ characters fit into a single machine word $w$, we propose a keyword dictionary that represents $\mathcal{K}$ in $n \lg \sigma + O(kw)$ bits of space, supporting all operations in $O(m/\alpha + \lg \alpha)$ expected time on an input string of length $m$ in the word RAM model. This data structure is underlined with an exhaustive practical evaluation, highlighting the practical usefulness of the proposed data structure.

1 Introduction

A keyword $K$ is a string that is uniquely associated with an integer called the identifier of $K$. A keyword dictionary is a data structure that maintains a dynamic set of keywords $\mathcal{K}$, and provides the following operations for a string $S$ on it:

- $\text{insert}(S)$ makes $S$ a keyword, inserts $S$ into $\mathcal{K}$, and returns its identifier.
- $\text{locate}(S)$ returns the identifier of $S$ if $S \in \mathcal{K}$, or returns the invalid identifier $\perp$ otherwise.
- $\text{predecessor}(S)$ and $\text{successor}(S)$ return the identifier of the predecessor and successor of $S$ in $\mathcal{K}$, respectively.
- $\text{delete}(K)$ removes the keyword $K$ from $\mathcal{K}$.
- $\text{locatePrefix}(S)$ returns an iterator on the set of identifiers of all keyword in $\mathcal{K}$ having $S$ as a prefix. The iterator can report the next occurrence in constant time.$^1$

1.1 Related Work

Keyword dictionaries are an integral data structure with a plethora of applications (e.g., $n$-gram language models [25], compression [14], input method editors [21], query auto-completion [17], or range query filtering [32]). As a well-studied abstract data type they also have many representations. We refer to standard literature like [27, Chapter 5.2], [22, Chapter 28], or [23, Chapter 8.5.3] for an introduction to common representations like tries. Here, we highlight some of the most recent representations. For the analysis, we assume that the dynamic set $\mathcal{K}$ consists of $k$ keywords with a total length of $n := \sum_{K \in \mathcal{K}} |K|$. For the operations we take an input string of length $m$.

$^1$We return an iterator instead of this set, since most of the later explained data structures support all operations in the same time $O(t)$ for some $t$, while this operation would take $O(t + s)$ time, if the returned set has size $s$. 

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The HAT-trie [3] is a practically optimized version of the burst trie [16]. It suppresses the number of trie nodes by selectively collapsing subtries into cache-conscious hash tables of strings [4]. Although there is no discussion of prefix searches in [3], the implementation of Tessil\(^2\) supports locatePrefix. We are unaware of any theoretical results regarding space or time.

The Bonsai trie [10] is a trie whose nodes are maintained in a compact hash table [9]. Modern variants [26] use \(O(n \log \sigma)\) bits of space in expectancy, and perform insert and locate in \(O(m)\) expected time. However, it is not clear how to perform locatePrefix efficiently.

Grossi and Ottaviano [15] proposed a cache-friendly trie dictionary through path decomposition [13]. An operation can be carried out in \(O(m + h \log \sigma)\) time, where \(h\) is the height of the path-decomposed trie. The data structure is stored in compressed space by exploiting text compression techniques and succinct data structures; however, it is static. Kanda et al. [20] proposed a dynamic variant by means of incremental path decomposition. The dynamic trie supports insert and locate in \(O(m)\) expected time. However, there is no discussion about prefix searches. Actually, as Kanda’s trie is based on the Bonsai trie, it faces the same problem for locatePrefix.

The double array [1] simulates a trie by using two integer arrays to find a child in constant time, and thus can perform locate in \(O(m)\) time. Although the double array includes some vacant slots and consumes \(\Omega(n \log n)\) bits, those vacant slots have a negligible memory effect in practical implementations such as the Cedar trie [31]. In the static setting, Kanda et al. [19] proposed a practically compressed data structure for the two arrays. However, for any of these data structures, it is not clear to us what time is needed for answering locatePrefix.

The (dynamic) z-fast trie is a keyword dictionary of Belazzougui et al. [6], which uses \(n \log \sigma + O(k \log n)\) bits of space, and supports all operations in \(O(m/\alpha + \log m + \log \log \sigma)\) expected time.

Takagi et al. [29] proposed the dynamic packed compact trie, whose name we abbreviate to packed c-trie. The packed c-trie uses \(n \log \sigma + O(kw)\) bits of space, and supports all operations in \(O(m/\alpha + \log \log n)\) expected time.

The following keyword dictionaries are static, but share common traits with our proposed data structure:

- Marisa trie, developed by Yata [30], is a static trie that consists of recursively compressed Patricia tries stored in the LOUDS representation [18]. It recursively encodes edge labels in a Patricia trie using another Patricia trie. Yata’s implementation\(^3\) supports prefix searches.
- Arz and Fischer [2] proposed a static compressed trie by adapting the LZ78 parsing to basic dictionary operations such as locate. It represents \(K\) in \(O(k + n/\log \sigma n)\) words of space by leveraging the LZ78 compression. It can answer locate in \(O(m)\) expected time. However, we are not aware of whether this data structure supports efficient prefix searches.
- Bille et al. [7] presented a static keyword dictionary using \(O(n)\) space and \(O(n)\) time to represent \(K\). It supports queries in \(O(m/\alpha + \log m + \log \log \sigma)\) time.
- A recent approach is due to Bille et al. [8], who proposed a static keyword dictionary with \(O(n/\log \sigma n)\) words of space using \(O(\min(m \log \sigma, m + \log n))\) time for an operation in the pointer machine model.

1.2 Notions and Model of Computation

Let \(\log\) denote the logarithm to the base two. Our model of computation is the standard word RAM model of word size \(w\). We can read and process \(O(w)\) bits in constant time.

\(^1\)https://github.com/Tessil/hat-trie
\(^2\)https://github.com/s-yata/marisa-trie
\(^3\)https://github.com/s-yata/marisa-trie
Let $n$ be a natural number with $n = \mathcal{O}(2^w)$. Let $\Sigma$ be an integer alphabet of size $\sigma := n^{\mathcal{O}(1)}$. An element of $\Sigma^*$ is called a string. The length of a string $T$ is denoted by $|T|$. We write $T[i]$ for the $i$-th character of $T$, for $1 \leq i \leq |T|$. The empty string is the string with length zero. For a string $T = XYZ$, $X$, $Y$, and $Z$ are called a prefix, substring, and suffix of $T$, respectively. The word RAM model allows us to process\footnote{$\sigma = \mathcal{O}(1)$.}$\alpha = \mathcal{O}(w/\lg \sigma)$ characters in constant time.

As in Section 1.1, we fix a dynamic set $\mathcal{K}$ consisting of $k$ keywords with a total length of $n = \sum_{K \in \mathcal{K}} |K|$. In this setting, the main result of this article on the theoretical side can be stated as follows:

**Theorem 1.** There is a keyword dictionary representing $\mathcal{K}$ in $n \lg \sigma + \mathcal{O}(kw)$ bits of space. It supports all keyword dictionary operations in $\mathcal{O}(m/\alpha + \lg \alpha)$ expected time on an input string of length $m$.

## 2 Keyword Dictionary c-trie++

![Macro Trie](image1.png)

*Figure 1: The macro trie (a) and a micro trie (b) of c-trie++. Left: Micro tries are represented by shaded triangles (cf. [29] Fig. 2). Hollow circles are nodes stored exclusively in a micro trie. Cross-hatched circles are implicit nodes that are represented as an entry in the hash table of the macro trie of the packed c-trie [29]. Circles filled with black color are nodes that are explicitly represented in the macro trie. A node is an explicit macro trie node if it is a leaf or the root node of a micro trie. Right: An alphabet-aware z-fast trie built on our running example $\mathcal{K} = \{K_1 = \text{braureibäute}, K_2 = \text{brauen}, K_3 = \text{brauchbare}, K_4 = \text{brausendes}, K_5 = \text{brauerei\text{b}}\}$. A leaf $u$ storing number $i$ is associated with the identifier $i$, i.e., extent($u$) = $K_i$. In this example, the node $v$ storing the extent brauerei has two children $w_1$ and $w_2$, which are determined by their keys key($w_1$) = $i$ and key($w_2$) = $r$, respectively. If we assume that eight characters fit into a computer word, then the extent of $v$ is outside of the micro trie containing the root node. This fact is symbolized by the dashed line separating the eighth and the ninth character of extent($v$).*

Our approach, called c-trie++ for improved compact trie, is a hybrid of the z-fast trie and the packed c-trie. Like these two trie representations, the trie is decomposed in a macro trie storing micro tries. Fig. 1a captures this schematically. Here, we use the trie decomposition of the packed c-trie for the macro trie. Our micro tries are alphabet-aware z-fast tries.

The z-fast trie proposed by Belazzougui et al. [8] works on binary strings. Their results on micro trees work for binary strings up to length $\mathcal{O}(w)$. However, it is easy to modify these micro trees [8, Thm. 1] to work with strings on the alphabet $\Sigma$ up to length $\mathcal{O}(w/\lg \sigma) = \mathcal{O}(\alpha)$ by packing $\mathcal{O}(\alpha)$ characters in a constant number of machine words:

**Lemma 1.** There is a keyword dictionary storing keywords of length $\mathcal{O}(\alpha)$ that takes $n \lg \sigma + \mathcal{O}(kw)$ bits of space and supports all keyword dictionary operations in either $\mathcal{O}(\lg \alpha)$ expected time or $\mathcal{O}(\lg \alpha \lg w/\lg \lg w)$ deterministic time.
**Proof.** The main difference is that the original micro trie is a binary tree as its edge labels are drawn from a binary alphabet. Since the edge labels in our alphabet-aware variant are characters drawn from \( \Sigma \), traversing from a node to a specific child now costs \( O(\sigma) \) time. We improve this time by augmenting each node with a data structure maintaining its children such that, given a node \( v \) and a character \( c \), we can navigate from \( v \) to its child connected with the edge starting with \( c \) by querying this data structure. This data structure can be realized with a hash table with constant expected time, or with a predecessor data structure like \( \mathcal{P} \) with \( O(\log \log w) \) deterministic time.

An operation with a string of length \( m \) with \( m = \Omega(\alpha) \) (but with \( m = O(2^w) \)) involves the traversal of the macro tree, which is done in \( O(m/\alpha) \) expected time for all keyword dictionary operations [29]. Combining the operations in the macro trie and in micro tries gives \( O(m/\alpha + \log \alpha) \) total time, and concludes Theorem [1]

For explaining our implementation of \( c\text{-trie}' \) in the subsequent section, we briefly review the \( z\text{-fast} \) trie under the light of our alphabet-aware variant. We say that a node \( v \) is **associated with the identifier** of a keyword \( K \) if we can read \( K \) by following the path from the root to \( v \). The alphabet-aware \( z\text{-fast} \) trie is a compact trie in which each leaf \( v \) is associated with the identifier of a keyword. An internal node has at least two children unless it is also associated with the identifier of a keyword. If the set of keywords \( \mathcal{K} \) is prefix-free, then there are no nodes with a single child.

Fig. [1b] shows an instance of such a trie. The figure also depicts the following definitions that are substrings or nodes associated to each node of an alphabet-aware \( z\text{-fast} \) trie.

- **key**(\( v \)) is the first character in label of the edge connecting \( v \) with its parent. It is undefined if \( v \) is the root.
- **extent**(\( v \)) is the string obtained by concatenating the edge labels of the path from the root node to \( v \).
- **exit**(\( S \)) is the unique node \( v \) for which, among all other nodes, the longest common prefix between \( S \) and **extent**(\( v \)) is the longest.
- **parex**(\( S \)) is the parent node of **exit**(\( S \)), or a special symbol \( \perp \) with **extent**(\( \perp \)) = 1 if **exit**(\( S \)) is the root node.

It is left to explain for what **handle**(\( v \)) stands in the figure. For that we need the notion of 2-fattest numbers [6, Def. 1]. The **2-fattest number** of an interval \([\ell..r]\) of positive integers \( 0 < \ell < r \) is the integer in \([\ell..r]\) with the most trailing zeros in its binary representation. Given a node \( v \) with its parent \( u \), we can compute the 2-fattest number \( f \) of \([\text{extent}(u)] + 1..\text{extent}(v)\] to determine the handle of \( v \), which is **handle**(\( v \)) = **extent**(\( v \))[1..f]. In case that \( v \) is the root, we set **handle**(\( v \)) to the empty string.

For supporting the keyword dictionary operations, we need operations to descend in a micro tree. For that, as already described in the proof of Lemma [1], each internal node \( u \) stores a dictionary \( \text{DicChild}_u \) to access one of its child nodes \( v \) by the character **key**(\( v \)). Additionally, the trie maintains a dictionary \( \text{DicHandle}_u \) that can address each internal node \( u \) by its handle **handle**(\( u \)).

For the algorithmic part, we follow Algorithm 1 and Section 3.3 of [6]. Given a pattern \( P \) of length \( O(\alpha) \), this algorithm locates **exit**(\( P \)) and **parex**(\( P \)). Having **exit**(\( P \)) and **parex**(\( P \)), we can perform all keyword dictionary operations as in the \( z\text{-fast} \) trie. The idea of the algorithm is to perform a search on the interval \([\ell..r]\), which is set to \([1..|P|]\) at the beginning to try to find the lowest node whose handle is a prefix of \( P \). The search handles this similarly to a binary search. For explanation, the algorithm is divided in rounds. In each round, it (a) either enlarges \( \ell \) or shrinks \( r \), (b) computes the 2-fattest number \( f \) of \([\ell..r]\), and (c) queries \( \text{DicHandle}_u \) with the handle \( P[1..f] \). If there is a node \( v \) with **handle**(\( v \)) = \( P[1..f] \), the algorithm has matched \( P[1..f] \) with this node and simulate the descending to this trie node by setting \( \ell \leftarrow \text{extent}(v) \). Otherwise (there is no such node \( v \)), the algorithm sets \( r \leftarrow f - 1 \) to aim for jumping to a node whose extent is less than \( f \). The algorithm stops when it finds either **exit**(\( P \)) and **parex**(\( P \)) [6, Thm. 3], which is after \( O(|P|) \) rounds. If **exit**(\( P \)) is found, it has previously already computed **parex**(\( P \)). Otherwise, it takes that child of **parex**(\( P \)) whose edge connected to **parex**(\( P \)) leads us to **exit**(\( P \)). For finding this child, the algorithm uses \( \text{DicChild}_{\text{parex}(P)} \).
In the context of the example of Fig. [1b] this algorithm applied to \( P = \text{brauereibock} \) gives us the node exit\((P)\), which is the node \( v \) visualized in Fig. [1b]. From there, we can query \( \text{DicChild}_{\text{exit}(P)} \) for the predecessor (resp. successor) with the character \( c \) to find the predecessor (resp. successor) of \( P \), which is \( K_5 \) (resp. \( K_1 \)).

Analyzing the time complexity, we can compute the handle of a node from its extent in constant time, since a 2-fattest number in \([\ell..r]\) is the integer \(((\ell - 1) \oplus r) \& r\), where \( \oplus \) and \( \& \) denote the bitwise exclusive-OR and the bitwise AND operators, respectively \[\text{Footnote 4}\]. Moreover, we need to query \( \text{DicHandle} O(\lg |P|) \) times and \( \text{DicChild}_{\text{parent}(P)} \) at most one time. Choosing a suitable representation for \( \text{DicHandle} \) and \( \text{DicChild} \) is the major task of the next section dealing with practical aspects of c-trie++.

3 Implementation Techniques

Micro Tries Each node \( v \) stores its extent \( \text{extent}(v) \), which can be represented in a constant number of computer words. From \( \text{extent}(v) \) we can deduce \( \text{handle}(v) \) and \( \text{key}(v) \) in constant time. Therefore, the dictionaries \( \text{DicChild} \) and \( \text{DicHandle} \) have no need to store the keys of their entries, as they both only have to maintain the nodes with which a dictionary can restore the respective keys on demand. That said, a lookup of a node \( v \) with a key \( \text{handle}(v) \) (resp. \( \text{key}(v) \)) needs to compute \( \text{handle}(w) \) (resp. \( \text{key}(w) \)) of each node \( w \) in question for comparison. By conducting this check, the benefits of current processors featuring large cache lines become negligible in this context. Here, we focus on theoretical results that are strong in the pointer machine model, namely (a) binary search in sorted lists and (b) cuckoo hashing \[24\].

(a) Sorted Lists If the number of elements to store is marginal, we represent a dictionary as a sorted list. The lists by themselves are represented as arrays to avoid storing additional pointers to the next/previous item of each element of the list. A list is always sorted and always full, such that it reverses just as much space as its elements need. This all works in constant time when the number of managed elements is constant. If this number surpasses a certain threshold, we exchange the sorted list with a cuckoo hash table.

(b) Cuckoo Hashing Our cuckoo hash table \( H \) uses either two or three hash function. For faster hashing, we restrict the hash table size \( |H| \) to be a power of two. This allows us to map a hash value to \([1..|H|]\) more quickly by using bit shifts instead of a modulo operation (cf. the discussion in \[28\] Sect. 1). An insertion collision occurs if each of the entries located by the hash functions is already occupied. Given such a collision on inserting an element \( e \), we start a random walk by selecting the \( i \)-th hash functions \( h_i \) for a random \( i \), swapping \( H[h_i] \) with \( e \) and recurse. If this walk is unsuccessful after a certain number of steps, the hash table doubles its size. To keep the memory requirement at minimum, the chosen hash functions are determined at startup and are the same across all cuckoo hash tables. The hash functions are based on three xorshift operations borrowed from MurmurHash\[3\] and two multiplications with different 64-bit integer seeds. Unwisely chosen seeds can result in a failure of the data structure, as the hash functions are immutable (changing would cause to rehash all cuckoo hash table instances). However, this was not a problem in our experiments. While insertions take \( O(1) \) expected time for a sufficiently small load factor, i.e., the maximum ratio between the number of stored elements and \( |H| \) before doubling the size of \( H \), a lookup takes \( O(1) \) worst case time.

Practical Considerations Preliminary experiments revealed that the cuckoo hash table with three hash functions has faster insertion times, whereas the lookup is slightly slower than the table operating with only two hash functions. The load factor does not have much influence on the final size, since a higher load factor makes it more probable that an insertion collision exceeds the threshold of maximal iterations. Setting this threshold to a smaller value boosts the insertion speed at the expense of a higher risk of creating an unnecessarily large table. However, preliminary experiments were in favor for a small threshold around 100 iterations. For the experiments in the following section, we fixed it to 100, and set the load factor to 0.8.

\footnote{https://github.com/aappleby/smhasher/wiki/MurmurHash3}
Table 1: Characteristics of our keyword sets. The size is in megabytes, i.e., \( n/10^6 \). The number of keywords is \( k \). The average and maximum length of a keyword is written in the columns \( \varnothing \text{len} \) and \( \text{max-len} \), respectively. The columns \( \varnothing \text{LCP} \) and \( \text{max-LCP} \) show, respectively, the average length and the maximal length of the longest common prefixes of all keywords. The column \( \text{c-trie nodes} \) is the number of nodes a compact trie stores. The packed \( \text{c-trie}, \text{z-fast trie}, \) and \( \text{c-trie}^{++} \) have the same number of nodes.

| \( K \)   | size  | \( \sigma \) | \( k \)    | \( \varnothing \text{len} \) | \( \text{max-len} \) | \( \varnothing \text{LCP} \) | \( \text{max-LCP} \) | \( \text{c-trie nodes} \) |
|----------|-------|-------------|-----------|-----------------|-----------------|-----------------|-----------------|-----------------|
| proteins | 864.14| 26          | 2,982,092 | 302.8           | 36,805          | 38.8            | 16,190          | 5,778,282       |
| urls     | 1,365.31 | 98        | 18,564,704 | 76.1           | 2,048           | 60.9            | 2,006           | 35,342,984      |
| dblp.xml | 164.89| 96          | 2,950,237 | 57.6           | 685             | 34.4            | 104             | 5,899,908       |
| geographic | 109.28| 134         | 7,308,054 | 14.6           | 151             | 8.5             | 247             | 12,801,677      |
| commoncrawl | 118.16| 113         | 1,995,402 | 61.0           | 1,194,988       | 12.9            | 119,276         | 3,740,064       |
| vital    | 233.11| 203         | 494,483   | 493.3          | 9,794           | 12.7            | 1,806           | 986,415        |

Node Factory In our setting, we assume that \( k \) is much small than \( n \). Otherwise, \( \text{c-trie}^{++} \) becomes unfavorable with respect to other trie data structures like the Bonsai trie. That is because our trie data structure contains \( O(k) \) nodes in total (instead of \( O(n) \) nodes as in common trie representations). However, using \( w \) bits for a pointer to a node is wasteful. Instead, we store each node in a global two-dimensional array that assigns each node an integer with a smaller bit width, which we set to 32 bits for the experiments. By storing these 32-bit integers instead of the pointers, using 64-bit on commodity computers, in \text{DicChild} \) and \text{DicHandle}, we can roughly halve the memory requirement of these dictionaries.

4 Experiments

Finally, we analyze the empirical performance of \( \text{c-trie}^{++} \) with respect to time and memory consumption. In particular, we are interested in the running time of \text{insert}, \text{locate}, \) and \text{locatePrefix}. For that, we implemented \( \text{c-trie}^{++} \) in C++. Our implementation is available at [https://gitlab.com/habatakita/ctriepp](https://gitlab.com/habatakita/ctriepp). For the experiments, we set up a machine equipped with CentOS 6.10, with an Intel Xeon X5560 processor running at 2.80 GHz, and with 198GB of main memory.

4.1 Datasets

For an objective evaluation, we took a variety of data sets having different characteristics (cf. Table 1):

- **proteins** contains different sequences of amino acids.
- **dblp.xml** is part of the XML dump of the [dblp.org](http://dblp.org) website.
- **urls** is a crawl of webpages of the .uk domain from the WebGraph framework\(^5\).
- **geographic** contains names of different geographic locations collected by the GeoNames database\(^6\). Our keywords are extracted from the ascii name column.
- **commoncrawl** is a web crawl containing the ASCII-encoded content (without HTML tags) of random web pages extracted from Common Crawl.
- **vital** is the main text extracted from the most vital Wikipedia articles.

\(^5\)[http://law.di.unimi.it/webdata/uk-2002]  
\(^6\)[http://download.geonames.org/export/dump/allCountries.zip]
Table 2: Insertion of all keywords in random order. We measured (a) the time and (b) the memory needed for inserting all keywords of the respective data set. The (a) fastest time and the (b) lowest memory footprint for each keyword set is highlighted in bold font. For each instance, we measured the maximal virtual memory resident set size (VmrRSS), which is the second integer in the file /proc/self/statm.

The data sets proteins and dblp.xml are from the Pizza&Chili Corpus. The data sets commoncrawl and vital are provided by the tudocomp framework [12].

We interpreted each data set as a single string on the byte alphabet. We partitioned this string into keywords by splitting it either at newline characters or at full stops, and removed all duplicates afterwards. The resulting keyword sets are the input of our experiments.

### 4.2 Dictionary Representations of c-trie++

Tables 6a and 6b in the appendix show a geometric distribution of the relation between the number of dictionary instances and their sizes. This distribution justifies our selection of a lightweight data structure with worse asymptotic behavior (sorted lists) for small instances, and the use of a more heavyweight data structure for large instances (cuckoo hashing). We also did experiments with unsorted lists storing newly inserted elements at their end. These experiments showed that unsorted lists feature a small speed-up for tiny instances while becoming early slow after a number of insertions. For the following experiments, we did a simplification by always selecting the sorted list and the cuckoo hash table for DicChild and DicHandle, respectively.

### 4.3 Contestants

We compared c-trie++ with the following keyword dictionary representations featuring also a low memory footprint.

- **CT**: a compact trie without word packing.
- **DA**: the double array implementation of the Cedar library [8].

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7 http://pizzachili.dcc.uchile.cl
8 http://www.tkl.lis.u-tokyo.ac.jp/~ynaga/cedar/
Table 3: Insertion of all keywords in lexicographical order. Except to the ordering of the keywords, the setting is the same as in Table 2.

| \( \mathcal{K} \) | CT   | DA   | HAT-T\(_{16} \) | HAT-T\(_{32} \) | PCT\(_{\text{bit}} \) | PCT\(_{\text{hash}} \) | ZFT  | c-trie\(+\) |
|----------------|------|------|-----------------|-----------------|-----------------|-----------------|------|------------|
| proteins      | 39,716.6 | 1,225.7 | 861.8 | 850.4 | 38,547.4 | 48,384.0 | 1,601.7 | 1,001.7 | 909.6 |
| urls          | 9,849.2 | 610.8 | **489.2** | 498.5 | 6,398.6 | 4,786.9  | 1,750.9 | 624.5  |
| dblp.xml      | 7,736.4 | 452.1 | **375.0** | error | 8,012.2 | 9,930.3 | 997.0  | 643.3  |
| geographic    | 2,342.1 | 237.0 | 255.7 | 275.9 | 2,089.6 | 2,605.7 | 921.4  | 742.8  |
| commoncrawl   | 8,419.2 | 498.5 | **375.0** | error | 8,012.2 | 9,930.3 | 997.0  | 643.3  |
| vital         | 63,719.1 | 1,314.0 | 1,278.2 | 1,263.6 | 65,684.8 | 90,066.2 | 1,409.0 | **693.8** |

(a) Time in Nanoseconds

| \( \mathcal{K} \) | CT    | DA   | HAT-T\(_{16} \) | HAT-T\(_{32} \) | PCT\(_{\text{bit}} \) | PCT\(_{\text{hash}} \) | ZFT  | c-trie\(+\) |
|----------------|------|------|-----------------|-----------------|-----------------|-----------------|------|------------|
| proteins      | 2,889.31 | 1,780.34 | 890.14 | 897.18 | 2,889.32 | 4,376.2 | 549.8  | 681.4  |
| urls          | 8,533.40 | **1,016.79** | 1,302.22 | 1,346.34 | 8,533.41 | 10,027.4 | 3,731.1 | 3,645.6 |
| dblp.xml      | 1,445.39 | 172.09 | **141.59** | 148.67 | 1,445.40 | 1,850.2 | 552.1  | 567.7  |
| geographic    | 3,029.50 | 252.68 | **159.23** | 176.02 | 3,029.51 | 4,952.8 | 1,204.0 | 376.9  |
| commoncrawl   | 1,023.88 | 174.66 | error | **139.60** | 1,023.87 | 1,598.8 | 330.0  | 376.9  |
| vital         | 695.96 | 261.43 | 238.09 | 239.24 | 695.97 | 1,098.4 | 84.2   | 102.9  |

(b) Memory in Megabytes

- **HAT-T**: the HAT-trie [3] implementation of Tessil.\(^9\) This implementation exploits that keywords have a small length in practice. The default implementation assumes that all these lengths can be stored in 16 bits, which is not true for the data set commoncrawl (where we marked a measurement with error in the respective table). We therefore have two flavors HAT-T\(_{16}\) and HAT-T\(_{32}\) representing the lengths of the keywords in 16 and 32 bits, respectively.

- **PCT\(_{\text{bit}}\)**: a packed c-trie using bit parallelism to compare compact words.

- **PCT\(_{\text{hash}}\)**: a packed c-trie using additionally the hash table implementation unordered_map of the C\(+\) standard library as a dictionary in each micro trie for retrieving a node by its extent (it is similar to our DicHandle, but uses the extents instead of the handles as keys).

- **ZFT**: our z-fast trie portation from an implementation in Java\(^10\) to C\(+\).

The implementations of the compact trie and the packed c-tries are due to Takagi et al. [29]. The implementations PCT\(_{\text{bit}}\) and PCT\(_{\text{hash}}\) pack characters in 32-bit integers, whereas all other implementations use 64-bit integers, which reflect the machine word size of commodity computers nowadays. All implementations are written in C\(+\), and compiled with gcc-8.2.0 in the highest optimization mode -O3.

### 4.4 Evaluation of the Construction

In the first experiment, we measured the time it takes to insert all keywords of a data set into a keyword dictionary. This experiment is conducted in two variants, where we either inserted the keywords in random order (Table 2) or in lexicographically sorted order (Table 3). While the space requirement in both variants is nearly the same for each keyword dictionary, a lexicographically sorted insertion speeds up the construction of all of them. Both tables also reveal that the construction of c-trie\(+\) is faster than the construction of every packed trie (i.e., CT, PCT\(_{\text{bit}}\), PCT\(_{\text{hash}}\), and ZFT). Except for ZFT, its final size is also an improvement to

\(^9\)https://github.com/Tessil/hat-trie
\(^10\)This implementation is part of Vigna’s Sux4J library, located at https://github.com/vigna/Sux4J
### Table 4: Average time for locate($K$) in nanoseconds. We created a list $L$ storing all keywords $K \in \mathcal{K}$, and either (a) shuffled it or (b) sorted its contents lexicographically. We measured the time of a linear scan over $L$ during which we locate each visited keyword in the respective trie created either in the shuffled setting (a) of Table 2 or in the lexicographically sorted setting (b) of Table 3, and divided this time by $|K|$, which yields the average times shown in this table.

| $\mathcal{K}$  | CT    | DA    | HAT-T<sub>16</sub> | HAT-T<sub>32</sub> | PCT<sub>bit</sub> | PCT<sub>hash</sub> | ZFT   | c-trie<sup>++</sup> |
|----------------|-------|-------|---------------------|---------------------|-------------------|-------------------|-------|-------------------|
| proteins       | 42,199.7 | 1,419.9 | 614.4              | 615.7              | 33,678.0          | 20,011.6          | 2,505.2 | 1,314.6           |
| urls           | 14,411.8 | 2,627.4 | 560.3              | 568.7              | 13,887.0          | 10,279.9          | 2,190.5 | 2,290.4           |
| dblp.xml       | 10,454.9 | 983.6  | 439.5              | 446.5              | 8,990.6           | 6,869.7           | 2,190.5 | 975.8             |
| geographic     | 4,764.3  | 431.6  | 244.5              | 251.6              | 5,016.1           | 2,726.1           | 1,410.6 | 641.6             |
| commoncrawl    | 10,667.9 | 640.9  | error              | 298.7              | 9,071.7           | 7,219.6           | 1,616.5 | 697.6             |
| vital          | 71,552.6 | 1,130.1 | 684.1              | 685.6              | 52,391.1          | 29,774.7          | 2,774.5 | 1,150.6           |

(a) Random Order

| $\mathcal{K}$  | CT     | DA    | HAT-T<sub>16</sub> | HAT-T<sub>32</sub> | PCT<sub>bit</sub> | PCT<sub>hash</sub> | ZFT   | c-trie<sup>++</sup> |
|----------------|--------|-------|---------------------|---------------------|-------------------|-------------------|-------|-------------------|
| proteins       | 39,357.5 | 395.3 | 255.2              | 257.2              | 30,758.8          | 18,392.0          | 1,728.9 | 384.3             |
| URLs           | 10,296.2 | 142.8 | 141.0              | 140.7              | 6,372.7           | 4,481.0           | 1,146.3 | 194.6             |
| dblp.xml       | 7,957.8  | 136.0 | 113.8              | 140.7              | 6,372.7           | 4,481.0           | 1,146.3 | 194.6             |
| geographic     | 1,839.2  | 65.3  | 45.4               | 75.1               | 1,925.0           | 1,436.4           | 703.1  | 156.2             |
| commoncrawl    | 8,273.8  | 97.4  | error              | 99.2               | 6,555.0           | 4,294.6           | 915.4  | 201.3             |
| vital          | 69,059.5 | 528.5 | 344.5              | 348.0              | 49,871.5          | 28,850.4          | 2,037.7 | 415.1             |

(b) Lexicographically Sorted Order

the sizes of those data structures. If the average keyword length is sufficiently large, c-trie<sup>++</sup> is also superior to DA and HAT-T in both time and space while being inferior when maintaining mostly short keywords.

### 4.5 Evaluation of the Queries

Our next and final experiments measure the performance of locate and locatePrefix queries.

**Locate Queries** The results for locate are collected in Table 4. In all instances, c-trie<sup>++</sup> answered locate queries faster than all packed tries. However, HAT-T, followed by DA, provide the fastest solutions for answering locate.

**Locate Prefix Queries** A major highlight is the time needed for locatePrefix($S$) queries shown in Figs. 2 and 3. Instead of returning an iterator to a set as requested at the beginning of this article, we require each keyword dictionary to return the complete set of all keywords having $S$ as a prefix. In this setting, c-trie<sup>++</sup> dominates most of the time. Interestingly, DA becomes faster for longer prefixes. This effect can be explained as follows: First recognize by Table 4 that DA has competitive locate times, allowing the trie to match a pattern at high speed. The matching locates the lowest node $v$ whose extent is a prefix of $S$. After locating $v$, it resorts to exploring the entire subtree of $v$. If $v$ is a deep node, chances are that its subtree size is rather small, enabling DA to process $v$’s subtree quickly.

### 4.6 Future Work

We can speed up the insertions of keywords that share long prefixes with other keywords by vectorization. That is because the word packing approach for comparing two strings interpreted as two packed strings can
be vectorized. Recent instruction sets like AVX feature instructions for this task. An application\(^{11}\) shows that the computation time roughly halves for long enough common prefixes when exploiting the AVX2 instruction set.

Table 6a in the appendix shows that some instances of `DicHandle` grow extremely large while most of the other instances maintain only few entries. For the large ones, we can use a compact hash table\(^{12}\) that stores quotients instead of the values, where a quotient has bit length \(v - \lg |H|\) if the values can be represented in \(v\) bits (we set \(v\) to 32 bits in Sect. 3).

Considering different hash table layouts, we conducted an experiment with the linear probing hash table of Rigtorp\(^{13}\) storing nodes along with the (redundant) keys. While using much more space, this hash table performed only slightly better than the cuckoo hash table, even with a load factor of 0.5. Dropping the keys, a hash table with linear probing will likely be outperformed by our cuckoo hash table.

Table 6 in the appendix reveals that none of our data sets is prefix-free. In a more enhanced evaluation, we would like to conduct our experiments after a preprocessing step in which we discard every keyword that is a prefix of another keyword.

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\(^{11}\) [https://github.com/koeppl/packed_string](https://github.com/koeppl/packed_string)

\(^{12}\) e.g., [https://github.com/koeppl/separate_chaining](https://github.com/koeppl/separate_chaining). It is easy to change this hash table to a bucketized cuckoo hash table like [11].

\(^{13}\) [https://github.com/rigtorp/HashMap](https://github.com/rigtorp/HashMap)
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Figure 2: Time for answering locatePrefix($S$). The x-axis is the length of the input string $S$. The y-axis is the amount of time in nanoseconds (logarithmic scale). We took prefixes whose lengths are 60%, 70%, 80%, 90%, and 100% of the average keyword length. HAT-T is the minimum time needed by HAT-T_{16} and HAT-T_{32} (both operate at roughly the same speed).
Figure 3: Time for answering locatePrefix when the data structures are built and queried with the keywords in lexicographical sorted order. The setting is, except from the different order, the same as in Fig. 2.
A Additional Experiments

In the following, we present some additional statistics and evaluations.

Statistics
The statistics in the main paper only sketch the characteristics of the used keyword sets. Here, we like to present a more profound analysis by showing different distributions in Tables 5 and 6. We see that the lengths have a distribution that is more Gaussian, and by no means uniform. The lengths have also an impact on the sizes and shapes of the dictionaries, as can be seen in Table 6.

Missing Evaluations
In the main article, we covered the cases to create a trie (a) on lexicographically sorted keywords or (b) on keywords sorted in a random order $R$, and subsequently queried the trie with the keywords (a) lexicographically sorted or (b) in another random order $R'$. However, one might question whether other possibilities like building a keyword dictionary with (b) shuffled keywords and querying it (a) in lexicographical order is advantageous. Here, we present a Cartesian product of these possibilities, shown in Table 7 for locate, and in Figs. 4, 5, and 6 for locatePrefix. We see a remarkable speedup of the query operations of all keyword dictionary implementations when they are fed with keywords in lexicographically order. The best bets can be placed on the setting of Table 7b and Fig. 3. A slightly slower variant is to query in random order (Table 7b and Fig. 4). The execution times of the keyword dictionaries fed in random order follow with a large gap. Here, the order in which the queries are executed has again only a slight impact on the execution times. We obtain the fastest execution times when querying the keywords in the same order as we built a keyword dictionary (Table 7c and Fig. 4). ZFT and c-trie++ can take advantage of the case when the queries are in lexicographic order (Table 7a and Fig. 5), while the other implementations are slightly faster in the random case (Table 4a and Fig. 2).

Original z-fast trie
The original implementation of the z-fast trie of Vigna is written in Java as part of his Sux4J library. As a supplement, we conducted our experiments of this implementation on the same machine. However, we could not build this trie for the keyword set vital. The time and space needed for the trie construction are given in Table 8. Its time for locate and locatePrefix are shown in Table 9 and Fig. 7, respectively.
Table 5: Histogram of (a) keyword lengths and (b) the lengths of the longest common prefixes (LCPs) of the keywords. While Table 1 captures the average and maximal lengths of the keywords and their LCPs, these tables give an insight in the distributions of the lengths and the LCPs. A length is counted in the $i$-th row if $i = 1$ and $i = 2$, or belongs in $[2^{-i} + 1..2^{i-1})$ for $i \geq 3$. 

(a) $\#len \leftrightarrow |len|$ Histogram

| $i$ | proteins | urls | dblp.xml | geographic | commoncrawl | vital |
|-----|-----------|------|----------|------------|-------------|------|
| 1   | 19        | 85   | 2        | 11         | 97          | 39   |
| 2   | 132       | 851  | 1        | 262        | 1,546       | 26   |
| 4   | 5,485     | 7,888| 0        | 31,036     | 31,931      | 131  |
| 8   | 36,973    | 25,921| 5       | 1,270,765  | 137,074     | 726  |
| 16  | 75,796    | 24,188| 25      | 3,899,303  | 636,922     | 2,298|
| 32  | 66,530    | 197,634| 395,244 | 1,838,186  | 445,153     | 4,932|
| 64  | 130,527   | 8,620,706| 1,801,952 | 263,086    | 369,674     | 12,007|
| 128 | 481,117   | 8,463,502| 723,011  | 5,398,121  | 255,830     | 32,038|
| 256 | 818,538   | 1,100,909| 29,782   | 7           | 61,018      | 75,871|
| 512 | 955,403   | 100,867| 213     | 0           | 36,936      | 166,775|
| 1,024|343,983   |19,207|0      |0           |11,627       |165,169|
| 2,048|57,653    |2,946 |0      |0           |4,464        |33,599|
| 4,096|8,691     |0     |0      |0           |1,878        |857   |
| 8,192|1,145     |0     |0      |0           |795          |14    |
| 16,384|83       |0     |0      |0           |256          |1     |
| 32,768|15       |0     |0      |0           |99           |0     |
| 65,536|2        |0     |0      |0           |52           |0     |
| 131,072|0      |0     |0      |0           |39           |0     |
| 262,144|0      |0     |0      |0           |5            |0     |
| 524,288|0      |0     |0      |0           |3            |0     |
| 1,048,576|0    |0     |0      |0           |2            |0     |
| 2,097,152|0   |0     |0      |0           |1            |0     |

(b) $\#LCP \leftrightarrow |LCP|$ Histogram

| $i$ | proteins | urls | dblp.xml | geographic | commoncrawl | vital |
|-----|-----------|------|----------|------------|-------------|------|
| 0   | 22        | 91   | 2        | 84         | 101         | 111  |
| 1   | 490       | 2,633| 19       | 2,225      | 6,012       | 1,850|
| 2   | 9,014     | 11,115| 20      | 19,636     | 50,615      | 6,079|
| 4   | 470,608   | 29,492| 5       | 635,924    | 306,121     | 28,013|
| 8   | 1,432,010 | 26,723| 2,663    | 3,838,361  | 574,787     | 118,627|
| 16  | 203,019   | 76,180| 556,906  | 2,457,041  | 780,370     | 240,884|
| 32  | 207,474   | 1,450,143| 862,179 | 319,203    | 173,276     | 92,179|
| 64  | 205,067   | 10,668,966| 1,398,593| 34,830    | 77,137      | 4,730|
| 128 | 204,307   | 5,814,357| 129,715  | 749        | 21,026      | 1,043|
| 256 | 155,849   | 429,835| 134      | 0          | 3,870       | 559  |
| 512 | 73,927    | 37,058| 0        | 0          | 1,247       | 309  |
| 1,024|17,440    |8,263 |0      |0           |507          |93    |
| 2,048|2,468     |847   |0        |0           |193          |5     |
| 4,096|335       |0     |0        |0           |48           |0     |
| 8,192|60        |0     |0        |0           |18           |0     |
| 16,384|1       |0     |0        |0           |70           |0     |
| 32,768|0      |0     |0        |0           |0            |0     |
| 65,536|0      |0     |0        |0           |0            |0     |
| 131,072|0   |0     |0        |0           |3            |0     |
| $i$ | proteins | urls | dblp.xml | geographic | commoncrawl | vital |
|-----|----------|------|----------|------------|-------------|-------|
| 1   | 692,786  | 1,996,651 | 233,983 | 474,823 | 180107 | 57649 |
| 2   | 72,926   | 419,911  | 46,975  | 126,163 | 36273 | 13791 |
| 4   | 26,863   | 278,813  | 27,291  | 70,145  | 19255 | 8641  |
| 8   | 7,696    | 143,852  | 16,392  | 30,265  | 8500  | 4097  |
| 16  | 1,705    | 66,594   | 1,1386  | 13,357  | 3449  | 1651  |
| 32  | 420      | 27,161   | 6,411   | 6,424   | 1195  | 618   |
| 64  | 89       | 11,108   | 3,214   | 2,952   | 488   | 254   |
| 128 | 24       | 4,574    | 1,152   | 1,241   | 194   | 105   |
| 256 | 5        | 1,633    | 302     | 472     | 100   | 25    |
| 512 | 1        | 580      | 110     | 191     | 38    | 13    |
| 1024| 1        | 75       | 37      | 68      | 37    | 3     |
| 2048| 0        | 0        | 18      | 21      | 1     | 0     |
| 4096| 0        | 0        | 8       | 9       | 0     | 0     |
| 8192| 0        | 0        | 4       | 0       | 0     | 0     |
| 16384| 0        | 1        | 0       | 1       | 0     | 0     |
| 32768| 0        | 1        | 0       | 0       | 0     | 0     |
| 65536| 0        | 0        | 0       | 0       | 0     | 1     |
| 131072| 0        | 0        | 1       | 0       | 0     | 0     |
| 262144| 0        | 0        | 0       | 0       | 0     | 0     |
| 524288| 0        | 0        | 0       | 0       | 1     | 0     |
| 1048576| 1       | 0        | 0       | 0       | 0     | 0     |

(a) $\text{DicHandle} \leftrightarrow |\text{DicHandle}|$ Histogram

| $i$ | proteins | urls | dblp.xml | geographic | commoncrawl | vital |
|-----|----------|------|----------|------------|-------------|-------|
| 1   | 27,933   | 189,554 | 106      | 204,565    | 30,276     | 279   |
| 2   | 1,220,896| 3,939,539| 808,559  | 1,644,531  | 468,330    | 164,154|
| 4   | 231,439  | 1,594,225| 313,011  | 716,500    | 175,809    | 54,646|
| 8   | 86,483   | 886,825| 116,020  | 288,994    | 69,654     | 19,579|
| 16  | 42,571   | 507,437| 53,258   | 104,526    | 47,619     | 6,272 |
| 32  | 13,894   | 34,609| 14,298   | 28,445     | 6365       | 1,466 |
| 64  | 0        | 1,221  | 656      | 301        | 1,201      | 283   |
| 128 | 0        | 5      | 7        | 8          | 124        | 7     |

(b) $\text{DicChild} \leftrightarrow |\text{DicChild}|$ Histogram

Table 6: Histogram of (a) micro tries or (b) internal micro trie nodes storing a specific number of (a) child nodes or (b) internal nodes representing the sizes of (a) all $\text{DicHandle}$ instances or (b) all $\text{DicChild}$ instances. A (a) micro trie or (b) internal node is counted in the $i$-th row if the number of its stored nodes is $i$ for $i = 1$ and $i = 2$, or in $[2^{i-2} + 1..2^{i-1}]$ for $i \geq 3$. None of the keyword sets is prefix-free, as can be seen by the fact that there are nodes with only a single child.
Table 7: Average time for `locate(K)` in nanoseconds. We create a trie by inserting keywords contained a list $L$ whose elements are (a) lexicographically sorted or (b-c) in a random order $R$. We stick to the setting of Table [7] where we used $L$ for the queries. However, before the querying, we (a) shuffled $L$, (b) sorted the elements in $L$ lexicographically, or (c) kept $L$ as it is.

| $\mathcal{K}$ | CT     | DA     | HAT-T$_{16}$ | HAT-T$_{32}$ | PCT$_{\text{bit}}$ | PCT$_{\text{hash}}$ | ZFT    | c-trie$_{++}$ |
|---------------|--------|--------|--------------|--------------|---------------------|----------------------|--------|--------------|
| proteins      | 40,154.2 | 694.1  | **311.1**    | 318.3        | 31,487.0            | 18,817.7             | 2,066.2 | 893.2        |
| urls          | 10,725.0 | 373.0  | **179.7**    | 187.9        | 9,287.6             | 6,376.8              | 1,584.4 | 685.6        |
| dblp.xml      | 8,194.8  | 306.7  | 164.1        | **163.2**    | 6,609.8             | 4,781.5              | 1,439.4 | 521.5        |
| geographic    | 2,054.8  | 163.2  | **85.0**     | 88.4         | 2,130.7             | 1,376.0              | 1,100.3 | 434.0        |
| commoncrawl   | 8,697.6  | 276.6  | error        | **128.4**    | 6,892.4             | 4,220.5              | 1,292.8 | 582.1        |
| vital         | 71,081.0 | 835.0  | **412.4**    | 679.2        | 53,701.4            | 29,366.9             | 2,471.7 | 930.5        |

(a) Sorted - Order $R$

| $\mathcal{K}$ | CT     | DA     | HAT-T$_{16}$ | HAT-T$_{32}$ | PCT$_{\text{bit}}$ | PCT$_{\text{hash}}$ | ZFT    | c-trie$_{++}$ |
|---------------|--------|--------|--------------|--------------|---------------------|----------------------|--------|--------------|
| proteins      | 42,934.5 | 1,083.2 | **630.0**    | 631.2        | 33,231.1            | 19,988.6             | 2,392.7 | 1,109.7      |
| urls          | 14,563.1 | 1,741.2 | **578.5**    | 585.1        | 12,500.3            | 9,321.5              | 2,470.8 | 1,996.0      |
| dblp.xml      | 10,180.9 | 747.2  | **457.9**    | 461.0        | 8,702.2             | 6,496.8              | 2,037.6 | 916.8        |
| geographic    | 4,408.0  | 361.4  | **249.2**    | 255.5        | 4,665.0             | 3,746.0              | 1,263.5 | 576.0        |
| commoncrawl   | 10,370.6 | 474.8  | error        | **305.5**    | 8,761.5             | 6,016.1              | 1,534.9 | 637.8        |
| vital         | 71,992.7 | 1,051.7 | **686.4**    | 710.8        | 53,526.8            | 30,583.7             | 2,751.9 | 1,061.7      |

(b) Order $R$ - Sorted

| $\mathcal{K}$ | CT     | DA     | HAT-T$_{16}$ | HAT-T$_{32}$ | PCT$_{\text{bit}}$ | PCT$_{\text{hash}}$ | ZFT    | c-trie$_{++}$ |
|---------------|--------|--------|--------------|--------------|---------------------|----------------------|--------|--------------|
| proteins      | 42,134.8 | 1,219.0 | **604.4**    | 611.0        | 33,626.8            | 19,904.1             | 2,286.6 | 1,004.6      |
| urls          | 14,329.6 | 2,502.0 | **553.7**    | 566.3        | 13,008.3            | 10,187.0             | 2,406.1 | 1,931.0      |
| dblp.xml      | 10,398.2 | 866.6  | **435.8**    | 444.1        | 8,938.6             | 6,801.3              | 1,957.1 | 735.8        |
| geographic    | 4,703.1  | 397.5  | **244.5**    | 248.6        | 4,966.8             | 2,644.0              | 1,268.2 | 426.2        |
| commoncrawl   | 10,624.8 | 557.5  | error        | **295.6**    | 9,040.9             | 5,353.6              | 1,453.2 | 502.7        |
| vital         | 71,523.2 | 961.6  | **672.3**    | 678.3        | 52,342.0            | 29,680.5             | 2,554.9 | 849.5        |

(c) Order $R$ - Order $R$
Figure 4: Time for answering locatePrefix when the data structures are built with the keywords in lexicographical sorted order, but queried with the keywords in random order. The setting is, except from the different orders, the same as in Fig. 2.
Figure 5: Time for answering locatePrefix when the data structures are built with the keywords in random order, but queried with the keywords sorted in lexicographical order. The setting is, except from the different orders, the same as in Fig. 2.
Figure 6: Time for answering locatePrefix when the data structures are built with the keywords in a random order $O$, and queried with the keywords in the same order $O$. The setting is, except from the different order, the same as in Fig. 2.
Table 8: Inserting of all keywords in the z-fast trie Java-implementation.

| $\mathcal{K}$ | Random | Sorted |
|---------------|--------|--------|
| proteins      | 3,896.6| 2,764.8|
| urls          | 3,056.7| 2,038.7|
| dblp.xml      | 2,727.1| 1,693.6|
| geographic    | 2,802.9| 1,831.0|
| commoncrawl   | 2,883.3| 1,714.9|
| vital         | error  | error  |

(a) Time in Nanoseconds

| $\mathcal{K}$ | Random | Sorted |
|---------------|--------|--------|
| proteins      | 1,629.60| 1,630.81|
| urls          | 2,764.73| 2,341.69|
| dblp.xml      | 989.38  | 1,026.62|
| geographic    | 1,043.65| 1,075.91|
| commoncrawl   | 244.94  | 245.73  |
| vital         | error   | error   |

(b) Memory in Megabytes

Table 9: Average time for answering `locate(K)` with the z-fast trie Java-implementation. Times are in nanoseconds. The table covers the settings of Tables 4a (Order $R$ - Order $R'$), 4b (Sorted - Sorted), 7a (Order $R$ - Sorted), 7b (Sorted - Order $R$), and 7c (Order $R$ - Order $R$), where $R$ and $R'$ are two different random orderings.

| $\mathcal{K}$ | R-R' | S-S | S-R | R-S | R-R |
|---------------|------|-----|-----|-----|-----|
| proteins      | 5,093.5 | 4,798.0 | 5,403.4 | 5,136.4 | 5,052.2 |
| urls          | 2,384.2 | 1,655.2 | 2,615.3 | 1,730.9 | 2,438.1 |
| dblp.xml      | 1,778.6 | 1,322.2 | 1,848.7 | 1,265.9 | 2,165.1 |
| geographic    | 1,254.7 | 749.2  | 1,416.0 | 1,154.6 | 1,233.8 |
| commoncrawl   | 2,032.3 | 1,351.9 | 1,870.8 | 1,404.8 | 1,648.0 |
| vital         | error  | error | error | error | error |


Figure 7: Time for answering locatePrefix with the z-fast trie Java-implementation. The plots cover the settings of Figs. 2 (Order $R$ - Order $R'$), 3 (Sorted - Sorted), 4 (Order $R$ - Sorted), 5 (Sorted - Order $R$), and 6 (Order $R$ - Order $R$), where $R$ and $R'$ are two different random orderings.