Structure identification of fractional-order complex-variable network with complex coupling

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Abstract Fractional-order complex-variable dynamical network with complex coupling is considered in this paper. The topological structures and system parameters are assumed to be unknown. As we know, the topological structure and system parameters play a key role on the dynamical behavior of complex network. Thus, how to effectively identify them is a critical issue for better studying the network. Through designing proper controllers and updating laws, two corresponding network estimators are constructed. Based on the Lyapunov function method and Gronwall-Bellman integral inequality, the results are analytically derived. Finally, two numerical examples are performed to illustrate the feasibility of the theoretical results.

Keywords Structure identification · Fractional-order · Complex-variable network · Complex coupling

1 Introduction

In real world, many large-scale systems can be described by the complex dynamical networks, such as social networks, communication networks, and biological neural networks [1–3]. Complex network consists of nodes and edges, where the nodes denote individuals, and the edges represent the interconnections among nodes. It has attracted increasing attention from researchers as the potential value in practical application. In many theoretical research of complex network, pinning control problem is a study-worthy topic [4–9]. Because we can achieve the goal of reducing cost and improving efficiency in practical application by selecting and then pinning some nodes to control the entire network. In [4], Song and Cao studied the problem that how to choose the pinned nodes in directed networks, found that the nodes should be selected as pinned candidates if their out-degrees are bigger than in-degrees. In [5], Wang and Chen investigated both the randomly and specially pinning control schemes of scale-free networks, and suggested that pinning the specific nodes with large degrees will need much less controllers in comparison to the randomly pinning.

Obviously, the above discussions about pinning control are based on the assumption that topology structure in network is known in advance. However, it is difficult to know the system parameters and topology structure in advance in many practical application. Therefore, we need to identify them at first for making full use of these structural information to study complex network. In fact, structure identification has emerged as a hot subject. Researchers have done much research on it and obtained many valuable results [10–18]. In [10], Zhou and Lu utilized the adaptive feedback control strategy to identify the exact topology of weighted complex network. In [11], Tang et al. studied the structure identification issue of general uncertain complex network, and constructed a slave network to effectively estimate the system parameters and topology structure of the master network. When these structural information are successfully identified, we can better understand and more effectively control the whole networks.

It can be seen that the above literatures about the structure identification problem mainly investigated the real-variable networks. In fact, many physical systems include both the real and complex variables. Naturally, it will be more accurate by choosing the complex-variable network to model them. For instance, Gibbon and McGuinness used the complex Lorenz sy-
tem to model and simulate the rotating fluids and detuned laser in [19]. Consequently, researchers studied the complex-variable network [20, 21]. In [20], Wu and Fu designed the suitable network estimators to identify the unknown topological structure and node dynamics of uncertain dynamical network coupled with complex-variable chaotic systems. In the above-mentioned works, the node dynamics are complex-variable systems, but the outer and inner coupling matrices are considered as the real matrices. It means that the real part of one state variable is affected only by the real parts of other state variables. However, in most situations, the real part is influenced by both the real and imaginary parts. So does the imaginary part. Therefore, researchers investigated the dynamical networks with complex coupling [22, 23], i.e., the coupling matrices are complex matrices. Especially, Yan et al. discussed the structure identification of complex-variable dynamical networks with complex coupling in [22].

Noticably, the above literatures [4–18,20–23] mainly studied the traditional integer-order complex networks. On the other hand, as an extension of the integer-order calculus, the fractional-order calculus has received a lot of attention in recent years. And according to the theoretical studies, researchers found that it has some good properties (such as the infinite memory and genetic characteristics) and more degrees of freedom compared with the integer-order calculus [34,35]. Therefore, many researchers have been investigating the fractional-order dynamical networks [24–33]. In [24], Si et al. considered the structure identification problem of fractional-order dynamical networks, and proposed a novel method to effectively identify the system parameters and network topology based on the stability analysis of fractional-order system. In [25], Li et al. investigated the parameter identification issue of uncertain fractional-order complex networks by achieving the finite-time synchronization between the drive and response networks. However, there is little research about how to achieve structure identification in the fractional-order complex-variable dynamical networks with complex coupling, and it deserves detailed discussions.

Motivated by the above discussions, this paper investigates the problem about structure identification of uncertain fractional-order complex-variable dynamical networks with complex coupling. The main contributions of this paper are as follows: Firstly, only the coupling matrices are considered to be unknown. Then the exact topology will be identified by designing an effective network estimator with proper updating laws to trace the evolution of network nodes. Secondly, the case of both the system parameters and coupling configuration matrices are assumed to be unknown is also discussed. And the corresponding network estimator is constructed to successfully identify them.

This paper is organized as follows: In Section 2, the model of fractional-order dynamical network is introduced and some definitions, lemmas and assumptions are given. In Section 3, two effective network estimators are designed to identify the structural information of uncertain fractional-order complex-variable dynamical networks with complex coupling. In Section 4, numerical simulations are given to show the feasibility of theoretical results. Finally, the conclusions are given in Section 5.

2 Model and preliminaries

Throughout the paper, the Caputo derivative as a main research tool is introduced into our fractional-order differential systems, because it has a clearly interpretable physical meaning in practical applications. In the following, some basic definitions, necessary lemmas and assumptions are given.

Definition 1 [34] The fractional integral of order \( a \) for function \( f(t) \) is defined as:

\[
D_{t_0}^{-a} f(t) = \frac{1}{\Gamma(a)} \int_{t_0}^{t} (t - \tau)^{a-1} f(\tau) d\tau,
\]

where \( t \geq t_0, \alpha > 0 \) and \( \Gamma(\cdot) \) is the Gamma function.

Definition 2 [34] The \( a \)-th-order Caputo fractional derivative of the given function \( f(t) \) is defined as:

\[
cD_{t_0}^{a} f(t) = \frac{1}{\Gamma(m-a)} \int_{t_0}^{t} (t - \tau)^{m-a-1} f^{(m)}(\tau) d\tau,
\]

where \( m-1 < \alpha < m \in \mathbb{Z}^+ \).

In general, the initial time \( t_0 \) is often considered that \( t_0 = 0 \). Then, for the sake of simplicity, the \( D_{0}^{a} \) is denoted by \( I^a \) and \( cD_{0}^{a} \) by \( D^{a} \).

Lemma 1 [36] If \( z(t) \in C^1[0, b] \) and \( 0 < \alpha < 1 \), then:

1. \( D^{\alpha} I^{\alpha} z(t) = z(t), \quad 0 < t < b \);
2. \( I^{\alpha} D^{\alpha} z(t) = z(t) - z(0), \quad 0 < t < b \).

Lemma 2 [37] Let \( x(t) \in C^1 \) be a differentiable complex-valued vector. Then, \( \forall t \geq t_0, \forall \alpha \in (0, 1], \) the following inequality holds

\[
D^{\alpha} (\bar{x}^T(t) P x(t)) \leq \bar{x}^T(t) P D^{\alpha} x(t) + (D^{\alpha} \bar{x}^T(t)) P x(t),
\]

where \( P \in \mathbb{C}^{n \times n} \) is a constant positive definite Hermitian matrix.
Lemma 3 [38] (Gronwall-Bellman integral inequality). If \( z(t) \) satisfies \( z(t) \leq \int_0^t a(\tau) z(\tau) \, d\tau + b(t) \) with \( a(t) \) and \( b(t) \) being known real functions, then

\[
z(t) \leq \int_0^t a(\tau) b(\tau) \exp \left( \int_\tau^t a(\tau) \, d\tau \right) \, d\tau + b(t).
\]

If \( b(t) \) is differentiable, then

\[
z(t) \leq b(0) \exp \left( \int_0^t a(\tau) \, d\tau \right) \, b(t).
\]

In particular, if \( b(t) \) is a constant, it immediately follows that

\[
z(t) \leq b(0) \exp \left( \int_0^t a(\tau) \, d\tau \right).
\]

Consider a fractional-order dynamical network consisting of \( N \) nodes, which is described by

\[
D^\alpha x_k(t) = g(x_k(t), \phi) + \sum_{l=1}^N a_{kl} \Gamma_1 x_l(t) + \sum_{l=1}^N b_{kl} \Gamma_2 x_l(t), \quad k = 1, 2, \ldots, N,
\]

where \( 0 < \alpha < 1, \ x_k = (x_{k1}, x_{k2}, \ldots, x_{k,m+n})^T \) is the state vector of node \( k, \ \Gamma_1 = (\gamma_1^k, \ldots, \gamma_m^k, 0, \ldots, 0) \) and \( \Gamma_2 = \text{diag}(0, \ldots, 0, \gamma_1^k, \ldots, \gamma_n^k) \) are the inner coupling matrices, \( A = (a_{kl}) \in \mathbb{C}^{N \times N} \) and \( B = (b_{kl}) \in \mathbb{R}^{N \times N} \) are the zero-row-sum coupling configuration matrices, defined as: if there is a connection from node \( l \) to node \( k \), then \( a_{kl} \neq 0 \) and \( b_{kl} \neq 0 \); otherwise, \( a_{kl} = b_{kl} = 0 \).

In this paper, we assume that the \( g \) is a linear function about \( \phi \), namely

\[
g(x_k(t), \phi) = h(x_k(t)) + G(x_k(t))\phi. \tag{2}
\]

Assumption [20] Suppose that there exists a positive constant matrix \( \Delta = \text{diag}(\delta_1, \delta_2, \ldots, \delta_{m+n}) \) such that the complex vector function \( g \) satisfies

\[
(\hat{x} - x)^T [g(\hat{x}, \phi) - g(x, \phi)] + (\hat{x} - x)^T (g(\hat{x}, \phi) - g(x, \phi)) \leq (\hat{x} - x)^T \Delta (\hat{x} - x) \tag{3}
\]

for any \( \hat{x}, x \) and \( k = 1, 2, \ldots, N \).

Assumption [20] Denote \( G(x_k) = (G^1(x_k), \ldots, G^q(x_k)) \). Suppose that \( \{G^1(x_k), \ldots, G^q(x_k)\}_{k=1}^N \) are linearly independent on the orbits \( \{x_k(t)\}_{k=1}^N \) for \( t > 0 \).

3 Main Results

In this section, two suitable network estimators are constructed to identify the unknown parameters and topology structure of fractional-order dynamical network (1).

Firstly, only the coupling matrices are assumed to be unknown. Then, design the following network estimator

\[
D^\alpha \hat{x}_k(t) = g(\hat{x}_k(t), \phi) + c \sum_{l=1}^N \hat{a}_{kl}(t) \Gamma_1 \hat{x}_l(t) + \sum_{l=1}^N \hat{b}_{kl}(t) \Gamma_2 \hat{x}_l(t) + u_k(t), \tag{4}
\]

where \( \hat{x}_k(t) = (\hat{g}^T_k(t), \hat{z}_k^T(t))^T \) is the state variable of node \( k, \ \hat{g}_k(t) \in \mathbb{C}^m \) and \( \hat{z}_k(t) \in \mathbb{R}^n \) are the complex and real components respectively, \( \hat{a}_{kl}(t) \) is the controller, \( \hat{A}(t) = (\hat{a}_{kl}(t)) \in \mathbb{C}^{N \times N} \) and \( \hat{B}(t) = (\hat{b}_{kl}(t)) \in \mathbb{R}^{N \times N} \) are the estimations of the unknown coupling configuration matrices \( A \) and \( B \).

Define \( e_k(t) = \hat{x}_k(t) - x_k(t) \), then we derive the following error dynamical system

\[
D^\alpha e_k(t) = g(\hat{x}_k(t), \phi) - g(x_k(t), \phi) + c \sum_{l=1}^N a_{kl} \Gamma_1 e_l(t) + c \sum_{l=1}^N b_{kl} \Gamma_2 e_l(t) + c \sum_{l=1}^N (\hat{a}_{kl}(t) - a_{kl}) \Gamma_1 \hat{x}_l(t) + c \sum_{l=1}^N (\hat{b}_{kl}(t) - b_{kl}) \Gamma_2 \hat{x}_l(t) + u_k(t). \tag{5}
\]

Theorem 1 Suppose that Assumptions 1 and 2 hold. Then the unknown coupling matrices \( A \) and \( B \) of network (1) are well identified by the \( \hat{A}(t) \) and \( \hat{B}(t) \) of network estimator (4) with the following adaptive controllers \( u_k(t) \) and updated laws

\[
u_{kl}(t) = \frac{\omega_k(t) e_k(t)}{\omega_k(t)}, \quad D^\alpha \omega_k(t) = u_k(t) e_k^T(t) e_k(t), \tag{6}
\]

where \( k, l = 1, 2, \ldots, N, \ \nu_k > 0, \rho_{kl} > 0, \sigma_{kl} > 0 \) are the adaptive gains.
The fractional-order derivative of $\frac{V(t)}{\w(t)}$ is

where $\hat{\omega}_k (k = 1, 2, \ldots, N)$ are the arbitrarily positive constants to be determined.

The fractional-order derivative of $V(t)$ with respect to the trajectories of (5) and (6) is

with Lemma 1, we derive

By integrating both sides of the inequality (7) in fractional-order, we get

With Lemma 1, we derive

In addition, due to $e^T(t)e(t) \leq V(t)$, naturally

From Lemma 3, we have

Therefore, $e(t) \rightarrow 0$ as $t \rightarrow +\infty$. When the synchronization is achieved, namely $\hat{x}_k(t) \rightarrow x_k(t)$ as $t \rightarrow +\infty$, the error system (5) can be rewritten as follows

According to Assumption 2, we have $\hat{a}_{kl}(t) \rightarrow a_{kl}$ and $\hat{b}_{kl}(t) \rightarrow b_{kl}$ as $t \rightarrow +\infty$. This completes the proof. \[\square\]

Remark 1 If we take the derivative order $\alpha = 1$, the fractional-order complex dynamical networks (1) and (4) will degenerate into the following conventional integer-order complex dynamical networks:

The detailed discussion of this special case about the structure identification of uncertain integer-order complex variable dynamical network with complex couplings can be seen in the Theorem 1 in Ref.[22].
Secondly, both the system parameters and coupling matrices are assumed to be unknown. Then, design the following network estimator

\[ D^\alpha \hat{x}_k(t) = h(\hat{x}_k(t)) + G(\hat{x}_k(t))\varphi(t) + c \sum_{l=1}^{N} \hat{\alpha}_{kl}(t) I_{\Gamma_1} \hat{x}_l(t) \]

\[ + c \sum_{l=1}^{N} \hat{\beta}_{kl}(t) I_{\Gamma_2} \hat{x}_l(t) + u_k(t), \]

where \( \hat{A}(t) = (\hat{\alpha}_{kl}(t)) \in \mathbb{C}^{N \times N}, \hat{B}(t) = (\hat{\beta}_{kl}(t)) \in \mathbb{R}^{N \times N} \) and \( \varphi(t) \) are the estimations of \( A, B \) and \( \phi \) respectively.

Subtracting (1) from (8), we derive the following error dynamical system

\[ D^\alpha e_k(t) = g(\hat{x}_k(t), \phi) - g(x_k(t), \phi) + G(\hat{x}_k(t))(\varphi(t) - \phi) \]

\[ + c \sum_{l=1}^{N} a_{kl} I_{\Gamma_1} e_l(t) + c \sum_{l=1}^{N} b_{kl} I_{\Gamma_2} e_l(t) \]

\[ + \sum_{l=1}^{N} (\hat{\alpha}_{kl}(t) - a_{kl}) I_{\Gamma_1} \hat{x}_l(t) \]

\[ + \sum_{l=1}^{N} (\hat{\beta}_{kl}(t) - b_{kl}) I_{\Gamma_2} \hat{x}_l(t) + u_k(t). \]

(9)

**Theorem 2** Suppose that Assumptions 1 and 2 hold. The unknown coupling matrices \( A, B \) and system parameter \( \phi \) of network (1) are well identified by the \( \hat{A}(t), \hat{B}(t) \) and \( \varphi(t) \) of network estimator (8) with the following adaptive controllers \( u_k(t) \) and updated laws

\[ u_k(t) = -\omega_k(t) e_k(t), \]

\[ D^\alpha \omega_k(t) = \nu_k e_k(t) e_k(t), \]

\[ D^\alpha \varphi(t) = -\eta \sum_{k=1}^{N} G^2(\hat{x}_k(t)) e_k(t), \]

\[ D^\alpha \hat{\alpha}_{kl}(t) = -\rho_{kl} I_{\Gamma_1} e_k(t), \]

\[ D^\alpha \hat{\beta}_{kl}(t) = -\sigma_{kl} I_{\Gamma_2} e_k(t), \]

where \( k, l = 1, 2, \ldots, N, \nu_k > 0, \eta > 0, \rho_{kl} > 0, \sigma_{kl} > 0 \) are the adaptive gains.

**Proof** Consider the following Lyapunov function:

\[ V(t) = \sum_{k=1}^{N} e_k^2(t) \]

\[ + \frac{1}{\nu_k} (\omega_k(t) - \hat{\omega}_k(t))^2 \]

\[ + \frac{1}{\eta} (\varphi(t) - \hat{\varphi}(t))^T (\varphi(t) - \hat{\varphi}(t)) \]

\[ + \sum_{k=1}^{N} \sum_{l=1}^{N} c (\hat{\alpha}_{kl}(t) - a_{kl}) (\hat{\alpha}_{kl}(t) - a_{kl}) \]

\[ + \sum_{k=1}^{N} \sum_{l=1}^{N} c (\hat{\beta}_{kl}(t) - b_{kl})^2, \]

where \( \hat{\omega}_k(k = 1, 2, \ldots, N) \) are the arbitrarily positive constants to be determined.

Calculating the fractional-order derivative of \( V(t) \) with respect to the trajectories of (9) and (10), we can derive:

\[ D^\alpha V(t) \leq \sum_{k=1}^{N} (D^\alpha e_k^2(t)) e_k^2(t) \]

\[ + \sum_{k=1}^{N} \frac{2}{\nu_k} (\omega_k(t) - \hat{\omega}_k(t)) (D^\alpha \omega_k(t)) \]

\[ + \frac{1}{\eta} (D^\alpha \varphi(t) (\varphi(t) - \hat{\varphi}(t)) + (\varphi(t) - \hat{\varphi}(t))^T D^\alpha \varphi(t)) \]

\[ + \sum_{k=1}^{N} \sum_{l=1}^{N} \frac{c}{\rho_{kl}} ((D^\alpha \hat{\alpha}_{kl}(t))(\hat{\alpha}_{kl}(t) - a_{kl}) \]

\[ + (\hat{\alpha}_{kl}(t) - a_{kl})^T (D^\alpha \hat{\alpha}_{kl}(t))) \]

\[ + \sum_{k=1}^{N} \sum_{l=1}^{N} 2c (\hat{\beta}_{kl}(t) - b_{kl}) (D^\alpha \hat{\beta}_{kl}(t)) \]

\[ \leq \sum_{k=1}^{N} e_k^2(t) (\Delta - 2\hat{\omega}_k I_{m+n} e_k(t) \]

\[ + c \sum_{k=1}^{N} \sum_{l=1}^{N} (a_{kl} e_l^2(t)) I_{\Gamma_1} e_k(t) + c \sum_{k=1}^{N} \sum_{l=1}^{N} b_{kl} e_l^2(t) I_{\Gamma_2} e_k(t) \]

\[ = e^2(t) (I_N \otimes \Delta - 2 \Omega \otimes I_{m+n} + c(A^T \otimes \Gamma_1 + B \otimes \Gamma_2) e(t) \]

\[ + \sum_{k=1}^{N} (\hat{\alpha}_{kl}(t) - a_{kl}) I_{\Gamma_1} x_k(t) \]

\[ + \sum_{k=1}^{N} (\hat{\beta}_{kl}(t) - b_{kl}) I_{\Gamma_2} x_k(t) = 0, \]

where \( k, l = 1, 2, \ldots, N \). According to Assumption 2, we have \( \varphi(t) \rightarrow \hat{\varphi}(t) \rightarrow a_{kl} \) and \( b_{kl} \rightarrow b_{kl} \) as \( t \rightarrow +\infty \). This completes the proof.

\[ \square \]

4 Numerical illustrations

Choose the node dynamics as the following hyper-chaotic fractional-order Lü systems [39]:

\[ D^\alpha x_{k1} = \phi_1(x_{k2} - x_{k1}) + x_{k4}, \]

\[ D^\alpha x_{k2} = \phi_2 x_{k2} - x_{k1} x_{k3} + x_{k4}, \]

\[ D^\alpha x_{k3} = (x_{k1} x_{k2} + x_{k1} x_{k2})/2 - \phi_3 x_{k3}, \]

\[ D^\alpha x_{k4} = (x_{k1} x_{k2} + x_{k1} x_{k2})/2 - \phi_4 x_{k4}, \]
where $k = 1, 2, \cdots, 5$, $\alpha = 0.95$, $x_{k1}$, $x_{k2}$ are the complex functions, $x_{k3}$, $x_{k4}$ are the real functions, and 
$\phi = (\phi_1, \phi_2, \phi_3, \phi_4)^T = (42, 25, 6, 5)^T$. Similar to the discussions about Chen system in Ref. [6], there exist four positive constants $M_p$ $(p = 1, 2, 3, 4)$ such that $|x_{kp}| \leq M_p$, $k = 1, 2, \cdots, 5$. Then we have

$$
(\bar{x}_k - x_k)^T (g(\bar{x}_k, \phi) - g(x_k, \phi))
$$

$$
+ (\bar{x}_k - x_k)^T (g(\bar{x}_k, \phi) - g(x_k, \phi))
$$

$$
= - (\phi_1 + \bar{\phi}_1)x_{k1}\bar{e}_{k1} + (\phi_2 + \bar{\phi}_2)x_{k2}\bar{e}_{k2} - 2\phi_3 e_{k3}^2
$$

$$
- 2\phi_4 e_{k4}^2 + (\bar{e}_{k1}\phi_1 + \bar{e}_{k1}\phi_2) + (\bar{e}_{k2}\phi_1 + \bar{e}_{k2}\phi_2)
$$

$$
- (\bar{e}_{k2}(\bar{x}_{k1}\bar{x}_{k3} - x_{k1}x_{k3}) + \bar{e}_{k3}(\bar{x}_{k1}\bar{x}_{k4} - x_{k1}x_{k4})))
$$

$$
+ e_{k3}(\bar{x}_{k1}\bar{x}_{k2} + \bar{x}_{k1}\bar{x}_{k2} - \bar{x}_{k1}x_{k2} - x_{k1}x_{k2})
$$

$$
+ e_{k4}(\bar{x}_{k1}\bar{x}_{k2} + \bar{x}_{k1}\bar{x}_{k2} - \bar{x}_{k1}x_{k2} - x_{k1}x_{k2})
$$

$$
\leq (2 - (\phi_1 + \bar{\phi}_1) + M_2^2 + 2M_2^2)e_{k1}\bar{e}_{k1}
$$

$$
+ (3 + (\phi_2 + \bar{\phi}_2) + |\phi_1|^2 + 2M_2^2)e_{k2}\bar{e}_{k2}
$$

$$
+ (2 - 2\phi_3 + M_2^2)e_{k3}^2 + (4 - 2\phi_4)e_{k4}^2.
$$

Let $\delta_1 = \max_{1 \leq k \leq 5} |2 - (\phi_1 + \bar{\phi}_1) + M_2^2 + 2M_2^2|$, $\delta_2 = \max_{1 \leq k \leq 5} |3 + (\phi_2 + \bar{\phi}_2) + |\phi_1|^2 + 2M_2^2|$, $\delta_3 = \max_{1 \leq k \leq 5} |2 - 2\phi_3 + M_2^2|$, $\delta_4 = \max_{1 \leq k \leq 5} |4 - 2\phi_4|$, then we can choose $\Delta = \text{diag}(\delta_1, \delta_2, \delta_3, \delta_4)$ such that Assumption 1 holds.

Firstly, we verify Theorem 1. Choose $c = 0.1$, $I_1 = \text{diag}(1, 1, 0, 0)$, $I_2 = \text{diag}(0, 0, 0, 1, 1)$,

$$
A = \begin{bmatrix}
-5 + 3j & 0 & 2 + 1j & 0 & 3 + 2j \\
2 + 1j & -4 + 3j & 0 & 2 + 2j & 0 \\
0 & 3 + 4j & -1 - 2j & -2 - 2j & 0 \\
1 + 2j & 2 + 3j & 0 & -3 - 5j & 0 \\
-3 - 1j & 0 & 0 & 4 + 2j & -1 - 1j
\end{bmatrix},
$$

and

$$
B = \begin{bmatrix}
-5 & 0 & 2 & 0 & 3 \\
2 & -4 & 0 & 2 & 0 \\
0 & 3 & -1 & -2 & 0 \\
1 & 2 & 0 & -3 & 0 \\
-3 & 0 & 0 & 4 & -1
\end{bmatrix}.
$$

In the numerical simulations, choose the adaptive gains $\nu_k = 24$, $\rho_{kl} = 5$, $\sigma_{kl} = 10$, the initial values $\tilde{e}_{k1}(0) = 50$, $\tilde{e}_{k2}(0) = 1$ for $k, l = 1, 2, \cdots, 5$, and the initial values of the state variable $x_{k1}(0) = (1 + 0.5kj, 2 + 0.5kj, 3 + 0.5kj, 4 + 0.5kj)^T$ and $\bar{x}_{k1}(0) = (10 + 0.5k)j, 9 + 0.5k, 7 + 0.5k)^T$. Figures 1-3 show some orbits of $\tilde{a}_{k1}(t)$, $\tilde{a}_{k2}(t)$, and $\tilde{b}_{kl}(t)$ respectively.

Secondly, we verify Theorem 2. Choose $c = 0.1$, $I_1 = \text{diag}(1, 1, 0, 0)$, $I_2 = \text{diag}(0, 0, 2, 3)$ and the same $A$ and $B$. In the numerical simulations, choose the adaptive gains $\nu_k = 60$, $\eta = 3$, $\rho_{kl} = 5, \sigma_{kl} = 3.3$, the initial values $\tilde{e}_{k1}(0) = 50$, $\varphi(0) = (41, 24, 5, 4)^T$, $\tilde{a}_{kl}(0) = \text{(color online) Some orbits of $\tilde{a}_{k1}(t)$}.$
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5 Conclusions

In this paper, we consider the structure identification problem of fractional-order complex-variable dynamical network with complex coupling. Firstly, we assume that only the topological structures corresponding to the complex and real components are unknown. Then, we design a network estimator with proper adaptive feedback controllers and updating laws to identify the unknown real and complex coupling matrices simultaneously. Secondly, we assume that both the topological structures and system parameters are unknown. And we designing a corresponding network estimator as well. Based on the Lyapunov function method and Gronwall-Bellman integral inequality, we provide the analytical proof. Finally, we perform two numerical examples to verify the effectiveness of the obtained results.

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Conflict of interest

The authors declare that they have no conflict of interest.
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