Proton structure and tensor-gluons

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Abstract
We consider the possibility that inside the proton and, more generally, inside the hadrons there are additional partons—tensor-gluons—that can carry part of the proton momentum. The tensor-gluons have zero electric charge, like gluons, but have a larger spin. Inside the proton, the nonzero density of the tensor-gluons can be generated by the emission of tensor-gluons by gluons. The last mechanism is typical for non-Abelian tensor gauge theories, in which there exists a gluon-tensor-tensor vertex of order $g$. Therefore, the number of gluons changes not only because a quark may radiate a gluon or because a gluon may split into a quark–antiquark pair or into two gluons, but also because a gluon can split into two tensor-gluons. The process of gluon splitting suggests that part of the proton momentum that was carried by neutral partons is shared between vector and tensor-gluons. We derive evolution equations for the parton distribution functions that take into account these new processes. The momentum sum rule allows us to find the tensor-gluons’ contribution to the Callan–Simanzik beta function and to calculate the corresponding anomalous dimensions. This contribution changes the behavior of the structure functions, and the logarithmic correction to the Bjorken scaling becomes milder. This also influences the unification scale at which the coupling constants of the standard model merge, shifting its value to lower energies of the order of 40 TeV.

Keywords: gauge theory, renormalization group, Callan–Simanzik beta function, splitting functions, DGLAP equation

1. Introduction

It was predicted that Bjorken scaling could be broken by logarithms of transverse momentum $Q^2$ and that these deviations from the scaling law can be computed for the deep inelastic structure functions [1–12]. In the leading logarithmic approximation, the results can be formulated in the parton language [13] by assigning the well-determined $Q^2$ dependence to the parton densities [1–3, 14, 15].
In this article, we shall consider a possibility that inside the proton and, more generally, inside the hadrons there are additional partons—tensor-gluons—that can carry part of the proton momentum. Tensor-gluons have zero electric charge, like gluons, but have a larger spin. Inside the proton, a nonzero density of the tensor-gluons can be generated by the emission of tensor-gluons by gluons [16–19]. Therefore, the number of gluons changes not only because a quark may radiate a gluon or because a gluon may split into a quark–antiquark pair or into two gluons [3, 14, 15], but also because a gluon can split into two tensor-gluons [16–21]. The process of gluon splitting into tensor-gluons suggests that part of the proton momentum that was carried by neutral partons is shared between vector and tensor-gluons.

The proposed model was formulated in terms of a field theory Lagrangian [16–19]. The Lagrangian describes the interaction of the gluons with their massless partners of higher spin [16–19]. We shall call them tensor-gluons. The characteristic property of the model is that all interaction vertices between gluons and tensor-gluons have dimensionless coupling constants in four-dimensional space-time. That is, the cubic interaction vertices have only first-order derivatives and the quartic vertices have no derivatives at all. These are familiar properties of the standard Yang–Mills field theory. In order to understand the physical properties of the model, it was important to study the tree level scattering amplitudes. A very powerful spinor helicity technique [22–39] was used to calculate high-order tree level diagrams with the participation of tensor-gluons in [20].

These tree level scattering amplitudes describe a fusion of gluons into tensor-gluons [20]. They are generalizations of the Parke–Taylor scattering amplitude to the case of two tensor gauge bosons of spin $s$ and $(n - 2)$ gluons. The result reads [20]:

$$M_n(1^+,\ldots, 1^-,\ldots, k^+,\ldots, j^-,\ldots, n^+) = g^{n-2} \prod_{r=1}^{n} \frac{\langle ij \rangle^d}{\langle r(r+1) \rangle} \frac{\langle ij \rangle^{2s-2}}{\langle kl \rangle^2}, \quad (1.1)$$

where $n$ is the total number of particles and the dots stand for the positive-helicity gluons. Here $i$ is the position of the negative-helicity gluon, while $k$ and $j$ are the positions of the particles with helicities $+s$ and $-s$, respectively. For $s = 1$, this expression reduces to the well-known result for the MHV amplitude [27]. The scattering amplitudes (1.1) were used to derive the amplitudes of splitting of a gluon into tensor-gluons in [21].

Here, we shall use the splitting amplitudes in order to derive the generalization of the DGLAP evolution equations for the parton distribution functions, which take into account the processes of emission of tensor-gluons by gluons. The momentum sum rule allows us to find the tensor-gluons’ contribution to the one-loop Callan–Simanzik beta function, which takes the following form:

$$\alpha(t) = \frac{\alpha}{1 + ab \tau}, \quad b = \frac{(12s^2 - 1)C_2(G) - 4n_f T(R)}{12\pi}, \quad s = 1, 2, \ldots \quad (1.2)$$

where $s$ is the spin of the gauge bosons. The matrix of anomalous dimensions for the twist-two operators $\gamma_n$ can also be computed. The spin-dependent term in the Callan–Simanzik beta function coefficient (1.2) changes the $t = \ln (Q^2/Q_0^2)$ behavior of the structure functions, demonstrating that if the tensor-gluons exist, the logarithmic correction to the Bjorken scaling changes. This also influences the unification scale at which the coupling constants of the standard model merge [47, 48], shifting its value to lower energies of the order of $M \sim 40$ TeV.
\[
\ln \frac{M}{\mu} = \pi \left( \frac{1}{\alpha_e(\mu)} - \frac{8}{3} \frac{1}{\alpha_s(\mu)} \right). 
\]

(1.3)

where \(\alpha_e(\mu)\) and \(\alpha_s(\mu)\) are the electromagnetic and strong coupling constants at scale \(\mu\).

This paper is organized as follows. In section 2, the basic formulae for scattering amplitude and splitting functions are recalled, and definitions and notations are specified. In section 3, the generalized evolution equations that describe the \(Q^2\) dependence of parton densities are derived, and the physical interpretation of the new terms is presented. In section 4, we obtain the one-loop Callan–Simanik beta function for tensor-gluons and the anomalous dimensions for the singlet and nonsinglet scaling functions. In section 5, we discuss the unification of coupling constants of the standard model including the one-loop contribution of the tensor-bosons. Section 6 contains concluding remarks and the appendix contains details of the regularization scheme.

2. Splitting functions

It is convenient to represent the scattering amplitude for the massless particles of momenta \(p_i\) and polarization tensors \(\epsilon_i (i = 1, \ldots, n)\), which are described by irreducible massless representations of the Poincaré group and are classified by their helicities \(h = \pm s\), in the following form:

\[
M_n = M_n(p_1, \epsilon_1; p_2, \epsilon_2; \ldots; p_n, \epsilon_n).
\]

Furthermore, by representing the momenta \(p_i\) and polarization tensors \(\epsilon_i\) in terms of spinors, the scattering amplitude \(M_n\) can be considered as a function of spinors \(\lambda_i, \tilde{\lambda}_i\) and helicities \(h_i\) [22–39]:

\[
M_n = M_n(\lambda_1, \tilde{\lambda}_1, h_1; \ldots; \lambda_n, \tilde{\lambda}_n, h_n).
\]

(2.4)

The advantage of the spinor representation is that by introducing a complex deformation of the particles momenta one can derive a general form of the three-particle interaction vertices [20, 34]:

\[
M_3(1^{h_1}, 2^{h_2}, 3^{h_3}).
\]

The dimensionality of the three-point vertex \(M_3(1^{h_1}, 2^{h_2}, 3^{h_3})\) is \([mass]^{D=2(h_1+h_2+h_3)}\).

In the generalized Yang–Mills theory [16–19], all interaction vertices between high-spin particles have \textit{dimensionless coupling constants}, which means that the helicities of the interacting particles in the vertex are constrained by the relation \(D = \pm (h_1 + h_2 + h_3) = 1\). Therefore, the interaction vertex between massless tensor-gluons, the TTT-vertex, has the following form [20]:

\[
M_3 = g f^{abc} < 1, 2 >^{2h_1-2h_2-1} < 2, 3 >^{2h_1+1} < 3, 1 >^{2h_2+1}, \quad h_3 = -1 - h_1 - h_2, \\
M_3 = g f^{abc} [1, 2]^{2h_1+2h_2-1}[2, 3]^{-2h_1+1}[3, 1]^{-2h_2+1}, \quad h_3 = 1 - h_1 - h_2,
\]

(2.5)

where \(f^{abc}\) are the structure constants of the internal gauge group, G. In particular, considering the interaction between a gluon of helicity \(h_1 = \pm 1\) and a tensor-gluon of helicity \(h_2 = \pm s\), the GTT-vertex, one can find from (2.5) that

\[
h_3 = \pm s - 2, \pm s, \pm (s + 2),
\]

(2.6)
and the corresponding gluon-tensor-tensor interaction vertices GTT have the following form:

\[
M_3^{\alpha_1\alpha_2\alpha_3}(1^{-}, 2^{-}, 3^{+}) = g f_{\alpha_1\alpha_2\alpha_3} \cdot \frac{<1.2>^4}{<1.2><2.3><3.1>^2} (2.3),
\]

\[
M_3^{\alpha_1\alpha_2\alpha_3}(1^{-}, 2^{+}, 3^{-}) = g f_{\alpha_1\alpha_2\alpha_3} \cdot \frac{<1.3>^4}{<1.3><2.3><3.1>^2} (2.3). (2.7)
\]

These are the vertices that reduce to the standard triple gluon vertex when \( s = 1 \). Using these vertices, one can compute the scattering amplitudes of gluons and tensor-gluons. The color-ordered scattering amplitudes involving two tensor-gluons of helicities \( h = \pm \), one negative helicity gluon and \( n \) (\( n = 3 \)) gluons of positive helicity were found in \([20]\):

\[
\hat{M}_n(1^{+}...i^{-}...k^{+}...j^{-}...n^{+}) = ig^{\mu - \nu}(2\pi)^4\delta^{(4)} \left( P_{\mu} \right)^{<ij>} \prod_{i=1}^{n} \frac{<ii>}{<ll+1>} \left( <kk> \right)^{2s-2}, (2.8)
\]

where \( n \) is the total number of particles and the dots stand for any number of positive helicity gluons, \( i \) is the position of the negative-helicity gluon, while \( k \) and \( j \) are the positions of the gluons with helicities \( +s \) and \( -s \), respectively. The expression \( (2.8) \) reduces to the famous Parke–Taylor formula \([27]\) when \( s = 1 \). In particular, the five-particle amplitude takes the following form:

\[
\hat{M}_5(1^{+}, 2^{-}, 3^{+}, 4^{+}, 5^{-}) = ig^{\mu - \nu}(2\pi)^4\delta^{(4)} \left( P_{\mu} \right)^{<25>} \prod_{i=1}^{n} \frac{<ii>}{} \left( <24> \right)^{2s-2}, (2.9)
\]

where \( P_{\mu} = \sum_{m=1}^{n} a_{\mu} a_{\mu}^{\dagger} \) is the total momentum. The scattering amplitudes \( (2.8) \) and \( (2.9) \) can be used to extract the splitting amplitudes of gluons and tensor-gluons \([21]\). The collinear behavior of the tree amplitudes has the following factorized form \([26–28, 38, 39]\):

\[
\frac{M_n^{\text{tree}}(\ldots, a^{\lambda}, b^{\lambda}, \ldots) a \parallel b}{\rightarrow \sum_{i=1}^{n} \text{Split}_{\text{tree}}(a^{\lambda}, b^{\lambda}) \times M_{n-1}^{\text{tree}}(\ldots, P_{\lambda} \ldots)}, (2.10)
\]

where \( \text{Split}_{\text{tree}}(a^{\lambda}, b^{\lambda}) \) denotes the splitting amplitude and the intermediate state \( P_{\lambda} \) has momentum \( k_{\lambda} = k_{a} + k_{b} \) and helicity \( \lambda \). Considering the amplitude \( (2.9) \) in the limit when the particles 4 and 5 become collinear, \( k_{4//5}, k_{5//4} \), that is, \( k_{4} = zk_{P}, k_{5} = (1 - z)k_{P}, k_{P} \rightarrow 0 \) and \( z \) describes the longitudinal momentum sharing, one can deduce that the corresponding behavior of the spinors is \( \lambda_{4} = \sqrt{z} \lambda_{P}, \lambda_{5} = \sqrt{1 - z} \lambda_{P} \), and that the amplitude \( (2.9) \) takes the following factorization form \([21]\):

\[
M_5(1^{+}, 2^{-}, 3^{+}, 4^{+}, 5^{-}) = A_{4}(1^{+}, 2^{-}, 3^{+}, P^{-}) \times \text{Split}_{4}(a^{+}, b^{-}),
\]

where

\[
\text{Split}_{4}(a^{+}, b^{-}) = \left( \frac{1 - z}{z} \right)^{1/2} \frac{1}{\sqrt{z(1 - z) <a, b>}}. (2.11)
\]

In a similar way, one can deduce that

\[
\text{Split}_{4}(a^{-}, b^{+}) = \left( \frac{z}{1 - z} \right)^{1/2} \frac{1}{\sqrt{z(1 - z) <a, b>}}. (2.12)
\]
Considering different collinear limits \( k_1 \parallel k_5 \) and \( k_3 \parallel k_4 \), one can get [21]

\[
\text{Split}_{+s}(a^+, b^-) = \frac{(1 - z)^{s+1}}{\sqrt{z(1 - z)}} \frac{1}{<a, b>}, \quad \text{Split}_{-s}(a^-, b^+) = \frac{z^{s+1}}{\sqrt{z(1 - z)}} \frac{1}{<a, b>}
\]  

(2.13)

and

\[
\text{Split}_{+s}(a^+, b^+)= \frac{z^{s+1}}{\sqrt{z(1 - z)}} \frac{1}{<a, b>}, \quad \text{Split}_{-s}(a^+, b^-) = \frac{(1 - z)^{s+1}}{\sqrt{z(1 - z)}} \frac{1}{<a, b>}
\]  

(2.14)

The set of splitting amplitudes (2.11)–(2.14) \( G \rightarrow TT, T \rightarrow GT \) and \( T \rightarrow TG \) reduces to the full set of gluon splitting amplitudes [26–28, 38, 39] when \( s = 1 \).

Since the collinear limits of the scattering amplitudes are responsible for parton evolution [3], we can extract from the above expressions the Altarelli–Parisi splitting probabilities for tensor-gluons. Indeed, the residue of the collinear pole in the square (of the factorized amplitude (2.10)) gives the Altarelli–Parisi splitting probability

\[
P_z = C_2(G) \sum_{h, h'} |\text{Split}_{-h}(a^{h'}, b^{h})| \left| s_{ab} \right|^2. \tag{2.15}
\]

where \( s_{ab} = 2 k_a \cdot k_b = <a, b> > [a, b] \). The invariant operator \( C_2 \) for the representation \( R \) is defined by the equation \( t^{(i)} {t^a}^i = C_2(R) I \) and \( tr (t^a t^b) = T(R) \delta^{ab} \). Substituting the splitting amplitudes (2.11)–(2.14) into (2.15), we get

\[
R_{TG}(z) = C_2(G) \left[ \frac{z^4}{z(1 - z)} \left( \frac{z}{1 - z} \right)^{2r-2} + \frac{(1 - z)^4}{z(1 - z)} \left( \frac{1 - z}{z} \right)^{2s-2} \right],
\]

\[
R_{GT}(z) = C_2(G) \left[ \frac{1}{z(1 - z)} \left( \frac{1}{1 - z} \right)^{2r-2} + \frac{(1 - z)^4}{z(1 - z)} \left( \frac{1}{z} \right)^{2s-2} \right],
\]

\[
R_{TT}(z) = C_2(G) \left[ \frac{z^4}{z(1 - z)} z^{2r-2} + \frac{1}{z(1 - z)} \left( \frac{1}{z} \right)^{2r-2} \right]. \tag{2.16}
\]

The momentum conservation in the vertices is clearly fulfilled because these functions satisfy the relations

\[
P_{TG}(z) = P_{GT}(1 - z), \quad P_{GT}(z) = P_{TT}(1 - z), \quad z < 1. \tag{2.17}
\]

In the leading order, the kernel \( P_{TG}(z) \) denotes the variation per unit \( t \) of the probability density of finding a tensor-gluon inside a gluon, \( P_{GT}(z) \) denotes the same for finding a gluon inside a tensor-gluon and \( P_{TT}(z) \) denotes the same for finding tensor-gluon inside a tensor-gluon. For completeness, we shall also present quark and gluon splitting functions [3]:

\[
P_{qG}(z) = C_2(R) \frac{1 + z^2}{1 - z},
\]

\[
P_{Gq}(z) = C_2(R) \frac{1 + (1 - z)^2}{z},
\]

\[
P_{qG}(z) = T(R) \left[ z^2 + (1 - z)^2 \right],
\]

\[
P_{GG}(z) = C_2(G) \left[ \frac{1}{z(1 - z)} z + \frac{z^4}{z(1 - z)} \right] + \frac{(1 - z)^4}{z(1 - z)} \right]. \tag{2.18}
\]

where \( C_2(G) = N, C_2(R) = \frac{N^2 - 1}{2N}, T(R) = 1/2 \) for the SU(N) groups.
We have to note that the limit $s \to 1/2$ in the scattering amplitudes (2.8) and (2.9) reduces them to the tree level gluon scattering amplitudes into a quark pair. Thus the formula has a larger validity area than the area in which it has been initially derived [20]. This fact should be visible in the splitting probabilities as well. Indeed, let us consider the tensor splitting functions (2.16) and take the limit to the half-integer spin $s \to 1/2$. One can see that the tensor-gluon splitting probabilities (2.16) reduce to the quark–gluon splitting probabilities (2.18)

$$P_T(z) \to P_{qg}(z), \quad P_{GT}(z) \to P_{qg}(z), \quad P_G(z) \to P_{qG}(z). \quad (2.19)$$

Taking into account the remark that we made after the formula (2.14), one can see that the substitution $s \to 1$ gives a gluon–gluon splitting function as well:

$$\frac{1}{2}(P_G(z) + P_{GT}(z) + P_T(z)) \to P_{GG}(z). \quad (2.20)$$

Having in hand the new set of splitting probabilities for tensor-gluons (2.16), we can hypothesize that a possible emission of tensor-gluons should produce a tensor-gluon ‘shower’ of neutral partons inside the proton in addition to the quark and gluon ‘shower’. Our goal is to derive DGLAP equations [3–10] that will take into account these new emission processes.

3. Generalization of the DGLAP equation

It is well known that the deep inelastic structure functions can be expressed in terms of parton quark densities. If $q_i(x)$ is the density of quarks of type $i$ (summed over colors) inside a proton target with fraction $x$ of the proton longitudinal momentum in the infinite momentum frame, then the scaling structure functions can be represented in the following form:

$$2F_i(x) = F_2(x)/x = \sum_i Q_i^2 \left[ q_i(x) + \bar{q}_i(x) \right].$$

The scaling behavior of the structure functions is broken and the results can be formulated by assigning a well-determined $Q^2$ dependence to the parton densities. This can be achieved by introducing the integro-differential equations that describe the $Q^2$ dependence of quark $q_i(x, t)$ and gluon densities $G_i(x, t)$, where $t = \ln (Q^2/Q_o^2)$ [3–10].

Let us see what will happen if one supposes that there are additional partons—tensor-gluons—inside the proton. In accordance with our hypothesis, there is an additional emission of tensor-gluons in the proton, therefore one should introduce the corresponding density $T(x, t)$ of tensor-gluons (summed over colors) inside the proton in the $P_{tc}$ frame. We can derive the integro-differential equations that describe the $Q^2$ dependence of parton densities in this general case. They are:

$$\frac{dq^i(x, t)}{dt} = \frac{\alpha(t)}{2\pi} \int_1^1 \frac{dy}{y} \left[ \sum_{j=1}^{2n} q^j(y, t) P_{q^jq^i} \left( \frac{x}{y} \right) + G(y, t) P_{q^jG} \left( \frac{x}{y} \right) \right],$$

$$\frac{dG(x, t)}{dt} = \frac{\alpha(t)}{2\pi} \int_1^1 \frac{dy}{y} \left[ \sum_{j=1}^{2n} q^j(y, t) P_{Gq^j} \left( \frac{x}{y} \right) + G(y, t) P_{GG} \left( \frac{x}{y} \right) + T(y, t) P_{GT} \left( \frac{x}{y} \right) \right],$$

$$\frac{dT(x, t)}{dt} = \frac{\alpha(t)}{2\pi} \int_1^1 \frac{dy}{y} \left[ G(y, t) P_{TG} \left( \frac{x}{y} \right) + T(y, t) P_{TT} \left( \frac{x}{y} \right) \right]. \quad (3.21)$$
The $\alpha(t)$ is the running coupling constant ($\alpha = g^2/4\pi$). In the leading logarithmic approximation, $\alpha(t)$ is of the form

$$\frac{\alpha}{\alpha(t)} = 1 + b\alpha_t t,$$

(3.22)

where $\alpha = \alpha(0)$ and $b$ is the one-loop Callan–Simanzik coefficient, which, as we shall see below, receives an additional contribution from the tensor-gluons. Here, the indices $i$ and $j$ run over quarks and antiquarks of all flavors. The number of quarks of a given fraction of momentum changes when a quark loses momentum by radiating a gluon, or a gluon inside the proton may produce a quark–antiquark pair [3]. Similarly, the number of gluons changes because a quark may radiate a gluon or because a gluon may split into a quark–antiquark pair or into two gluons or into two tensor-gluons. This last possibility is realized, because, as we have seen, in non-Abelian tensor gauge theories there is a triple vertex GTT (2.7) of a gluon and two tensor-gluons of order $g$ [16–19]. This interaction should be taken into consideration, and we added the term $T(y, t)P_G(z)$ in the second equation (3.21). The density of tensor-gluons $T(x, t)$ changes when a gluon splits into two tensor-gluons or when a tensor-gluon radiates a gluon. This evolution is described by the last equation (3.21).

In order to guarantee that the total momentum of the proton, that is, of all partons, is unchanged, one should impose the following constraint:

$$\int_0^1 dz \int_0^1 d\nu \left[ \sum_{i=1}^{2n_f} q_i(z, t) + G(z, t) + T(z, t) \right] = 0.$$  

(3.23)

Using the evolution equations (3.21) one can express the derivatives of the densities in (3.23) in terms of kernels and see that the following momentum sum rules should be fulfilled:

$$\int_0^1 dz \int_0^1 d\nu \left[ P_{qq}(z) + P_{qg}(z) \right] = 0,$$

$$\int_0^1 dz \int_0^1 d\nu \left[ 2n_f P_{qG}(z) + P_{GG}(z) + P_{TH}(z) \right] = 0,$$

$$\int_0^1 dz \int_0^1 d\nu \left[ P_{GT}(z) + P_{TT}(z) \right] = 0.$$  

(3.24)

Before analyzing these momentum sum rules, let us first inspect the behavior of the tensor-gluon kernels (2.16) at the end points $z = 0,1$. As one can see, they are singular at the boundary values, similarly to the case of the standard kernels (2.18). Nevertheless, there is a difference here: the singularities are of higher order compared to the standard case [3]. Therefore one should define the regularization procedure for the singular factors $(1 - z)^{-2s+1}$ and $z^{-2s+1}$, reinterpreting them as the distributions $(1 - z)^{-2s+1}$ and $z^{-2s+1}$, similar to the Altarelli–Parisi regularization [3]. We shall define them in the following way:
\[
\int_0^1 \frac{dz}{(1 - z)^{2s} + 1} f(z) = \int_0^1 \frac{dz}{(1 - z)^{2s} + 1} f(z) - \sum_{k=0}^{2s-2} \frac{(-1)^k f^{(k)}(1)}{1!} (1 - z)^k
\]

\[
\int_0^1 \frac{dz}{z^{2s} + 1} f(z) = \int_0^1 \frac{dz}{z^{2s} + 1} f(z) - \sum_{k=0}^{2s-2} \frac{1}{1!} f^{(k)}(0) z^k
\]

\[
\int_0^1 \frac{dz}{z^k (1 - z)} = \int_0^1 \frac{dz}{z^k (1 - z)} (1 - z)f(0) - zf(1)
\]

(3.25)

where \(f(z)\) is any test function that is sufficiently regular at the end points and, as one can see, the defined subtraction guarantees the convergence of the integrals. Using the same arguments as in the standard case [3], we should add the delta function terms into the definition of the diagonal kernels so that they will completely determine the behavior of the \(P_{gg}(z)\), \(P_{GG}(z)\) and \(P_{TT}(z)\) functions. The first equation in the momentum sum rule (3.24) remains unchanged because there is no tensor-gluon contribution into the quark evolution. The second equation in the momentum sum rule (3.24) will take the following form (see the appendix for details):

\[
\int_0^1 dzz \left[ 2n_f P_q G(z) + P_{GG}(z) + P_G(z) + b_G \delta(z - 1) \right]
\]

\[
= \int_0^1 \left[ 2n_f T(R) \left[ z^2 + (1 - z)^2 \right] + C_2(G) \left[ \frac{1}{z(1 - z)} + \frac{z^4}{z(1 - z)} + \frac{(1 - z)^4}{z(1 - z)} \right] \right]
\]

\[
+ C_2(G) \left[ \frac{z^4}{z(1 - z)} \left( \frac{z}{1 - z} \right)^{2s} + (1 - z)^4 \left( \frac{z}{1 - z} \right)^{2s} \right] + b_G
\]

\[
= \frac{2}{3} n_f T(R) - \frac{11}{6} C_2(G) - \frac{12s^2 - 1}{6} C_2(2) + b_G = 0.
\]

(3.26)

From this result we can extract an additional contribution to the one-loop Callan–Symanzik beta function for gluons, \(b^G_G\), arising from the tensor-gluon loop. Indeed, the first beta-function coefficient enters into this expression because the momentum sum rule (3.24) implicitly comprises unitarity and thus the one-loop effects [3]. In (3.26), we have three terms that come from quark loops:

\[
b^G_G = -\frac{2n_f}{3} T(R),
\]

(3.27)

from the gluon loop:

\[
b^G_G = \frac{11}{6} C_2(G)
\]

(3.28)

and from the gauge boson loop of spin \(s\):

\[
b^{s\alpha}_G = \frac{12s^2 - 1}{6} C_2(G), \quad s = 1, 2, 3, 4, \ldots
\]

(3.29)

It is a very interesting result because at \(s = 1\) we are rediscovering the asymptotic freedom result [1, 2]. For larger spins, the tensor-gluon contribution into the Callan–Symanzik beta function has the same signature as for the standard gluons, which means that tensor-gluons ‘accelerate’ the asymptotic freedom (3.22) of the strong interaction coupling constant \(\alpha(t)\).
The contribution is increasing quadratically with the spin of the tensor-gluons—that is, at large transfer momentum the strong coupling constant tends to zero faster than in the standard case:

\[ \alpha(t) = \frac{\alpha}{1 + b\alpha t}, \]  

(3.30)

where

\[ b = \frac{(12s^2 - 1)C_2(G) - 4n_j T(R)}{12\pi}, \quad s = 1, 2, \ldots \]  

(3.31)

In particular, the presence of the spin-two tensor-gluons in the proton will give

\[ b = \frac{58C_2(G) - 4n_j T(R)}{12\pi}. \]  

(3.32)

Surprisingly, a similar result based on the parametrization of the charge renormalization taken in the form \( b = (-1)^s(A + Bs^2) \) was conjectured by Curtright [40]. Here \( A \) represents an orbital contribution and \( Bs^2 \) represents the anomalous magnetic moment contribution [41–43]. The unknown coefficients \( A \) and \( B \) were found by comparing the suggested parametrization with the known results for \( s = 0, 1/2 \) and \( 1 \).

It is also possible to consider a straightforward generalization of the result obtained for the effective action in Yang–Mills theory long ago [41–44] to the higher spin gauge bosons. With the spectrum of the tensor-gluons in the external chromomagnetic field \( \lambda = (2n + 1 + 2s)gH + k_q^2 \), one can perform a summation of the modes and get an exact result for the one-loop effective action similar to [41, 44]:

\[ \epsilon = \frac{H^2}{2} + \frac{(gH)^2}{4\pi} b \left[ \ln \frac{gH}{\mu^2} - \frac{1}{2} \right], \]  

(3.33)

where

\[ b = -\frac{2C_2(G)}{\pi} \zeta(-1, \frac{2s + 1}{2}) = \frac{12s^2 - 1}{12\pi} C_2(G), \]  

(3.34)

and \( \zeta(-1, q) = -\frac{1}{2} (q^2 - q + \frac{1}{2}) \) is the generalized zeta function. Because the coefficient in front of the logarithm defines the beta function [41, 42], one can see that (3.34) is in agreement with the result (3.29). It is also interesting to mention that the spectrum of the open strings in the background magnetic field has a similar spectrum with the gyromagnetic ratio equal to two, as was pointed out by Bachas and Fabre in [45].

---

1 The generalized zeta function is defined as \( \zeta(q, \varphi) = \sum_{n = 0}^{\infty} \frac{1}{(n + \varphi)^q} = \frac{1}{\Gamma(q)} \int_0^{\infty} dt \ t^{q-1} e^{-nt}, 1 - e^{-t}. \)
The third equation in the momentum sum rule (3.24) will take the following form:

\[
\int_0^1 \, dz \left[ P_{TT}(z) + P_{GT}(z) + b_T \delta(z - 1) \right] = C_2(G) \int_0^1 \, dz \left[ \frac{z^{2s+1}}{1 - z} + \frac{1}{(1 - z) z^{2s-1}} + \frac{1}{z} \frac{(1 - z)^{2s-1}}{z} \right] + b_T
\]

\[
\Rightarrow \text{Reg} = C_2(G) \int_0^1 \, dz \left[ \frac{z^{2s+2}}{1 - z} + \frac{1}{z} + (1 - z)^{2s+1} \right] + b_T
\]

\[
= -C_2(G) \sum_{j=1}^{2s+1} \frac{1}{j} + b_T = 0.
\]

And again we can extract the one-loop coefficient of the Callan–Symanzik beta function now for a tensor-gluon of spin \(s\), which has the form

\[
b_{TT} = C_2(G) \sum_{j=1}^{2s+1} \frac{1}{j}, \quad s = 1, 2, 3, 4, \ldots
\]

As one can see, at \(s = 1\) we have

\[
C_2(G) \left( 1 + \frac{2}{3} + \frac{1}{3} \right) = \frac{11}{6} C_2(G),
\]

and it coincides with the one loop contribution of the gluons (3.28). The coefficient grows as \(\ln s\). This growth is slower than \(s^2\) in (3.31). The reason is that, as we shall discuss at the end of this section, the splitting of tensor-gluons into pairs of tensor-gluons was discarded in the derivation of the evolution equation. In this article, we limit ourselves to considering only emissions that always involve the standard gluons and lower-spin tensors, ignoring the infinite ‘stair’ of transitions between tensor-gluons.

In summary, we have to add \(\delta(z - 1)\) to the diagonal kernels \(P_{qq}(z), P_{GG}(z)\), and \(P_{TT}(z)\) with the coefficients that have been determined by using the momentum sum rule (3.24) guaranteeing the total momentum conservation of a hadron:

\[
P_{qq}(z) = C_2(R) \left[ \frac{1}{1 - z} + \frac{3}{2} \delta(z - 1) \right],
\]

\[
P_{GG}(z) = 2C_2(G) \left[ \frac{z}{(1 - z)^s} + \frac{1 - z}{z} + z(1 - z) \right] + \frac{(12s^2 - 1) C_2(G) - 4n_f T(R)}{6} \delta(z - 1),
\]

\[
P_{TT}(z) = C_2(G) \left[ \frac{z^{2s+1}}{1 - z} + \frac{1}{(1 - z) z^{2s-1}} + \sum_{j=1}^{2s+1} \frac{1}{j} \delta(z - 1) \right].
\]

Thus we completed the definition of the kernels appearing in the evolution equations (3.21).

At the end of this section, we shall discuss what type of processes could also be included into the evolution equations (3.21). In (3.21) we ignore contribution of the high-spin fermions \(\tilde{q}\) of spin \(s + 1/2\), which are the partners of the standard quarks [16–19], supposing that they are even heavier than the top quark. That is, all kernels \(P_{\tilde{q}\bar{q}}, P_{\tilde{q}\tilde{q}}, P_{\tilde{q}\bar{q}}, P_{\tilde{q}\tilde{q}}, P_{\tilde{q}\tilde{q}}, P_{\tilde{q}\tilde{q}}\), and \(P_{\tilde{q}\tilde{q}}\) with the emission of \(\tilde{q}\) are taken to be zero. These terms can be included in the case of very high energies, but at modern energies it seems safe to ignore these contributions. In the evolution equation for tensor-gluons in (3.21), one could also include the kernels \(P_{TT}\) that describe the emission of tensor-gluons by tensor-gluons, the \(TTT^*\) vertex (2.5) [16–19].
also can be done, and the number of evolution equations in that case will tend to infinity. We shall consider a more general case in a separate work.

It is also natural to ask what will happen if one takes into consideration the contribution of tensor-gluons of all spins into the beta function. One can suggest two scenarios. In the first one, the high spin gluons, let us say of $s \geq 3$, will acquire a large mass and therefore can be ignored at a given energy scale. In the second case, when all of them remain massless, then one can suggest the Reimann zeta function regularization, similar to the Brink–Nielsen regularization [46], which gives:

\[
\begin{align*}
  b &= C_2(G) \sum_{s=1}^{\infty} \left( \frac{12s^2 - 1}{12\pi} \right) = C_2(G) \left[ \frac{1}{\pi} \zeta(-2) - \frac{1}{12\pi} \zeta(0) \right] = \frac{1}{24\pi} C_2(G),
\end{align*}
\]

where $\zeta(-2) = 0$, $\zeta(0) = -1/2$ and the theory remains asymptotically free with a smaller beta coefficient, but this summation requires further justification.

### 4. Moments of the scaling structure functions

The moments of the scaling structure functions measure the Fourier transform of the coefficients $C^{(n)}(Q^2, g)$ of Wilson’s operator-product expansion of currents [15]:

\[
\int_0^1 dx x^{n-1} F_i(x, Q^2) \sim Q^2 \to \infty C_i^{(n)}(t) \left< n \left| O^{(n)} \right| n \right>.
\]

Their logarithmic deviations from Bjorken scaling are obtainable from the calculation of anomalous dimensions $\gamma_n$ of the corresponding operators and the beta function coefficient $b$ in (3.31):

\[
C_i^{(n)}(t) = C_i^{(n)}(0) \left[ \frac{\alpha}{\alpha(t)} \right]^{\gamma_i/2ab}.
\]

Therefore one should calculate the matrix of relevant anomalous dimensions of the twist-two operators. They are represented in terms of moments of the kernels in (3.21):

\[
\gamma_n = \int_0^1 dzz^{n-1} \begin{pmatrix}
  P_{qq}(z) & 2n f_G P_{G}(z) & 0 \\
  P_{qq}(z) & P_{GT}(z) & P_{G}(z) \\
  0 & P_{GT}(z) & P_{TT}(z)
\end{pmatrix} = \begin{pmatrix}
  \gamma_n^{QQ} & 2nf_G \gamma_n^{QG} & 0 \\
  \gamma_n^{GG} & \gamma_n^{GG} & \gamma_n^{GT} \\
  0 & \gamma_n^{TG} & \gamma_n^{TT}
\end{pmatrix}.
\]

I would like to thank John Iliopoulos and Constantin Bachas for raising this question.
where the corresponding integrals for quarks and gluons are well known [1]:

\[
\int_0^1 \! \! dzz^{n-1} P_{qq}(z) = C_2(R) \left[ -\frac{1}{2} + \frac{1}{n(n+1)} - 2 \sum_{j=2}^{n} \frac{1}{j} \right],
\]

\[
\int_0^1 \! \! dzz^{n-1} P_{qg}(z) = C_2(R) \left[ \frac{2 + n + n^2}{n(n^2 - 1)} \right],
\]

\[
2n_f \int_0^1 \! \! dzz^{n-1} P_{gG}(z) = 2n_f T(R) \left[ \frac{2 + n + n^2}{n(n+1)(n+2)} \right],
\]

\[
\int_0^1 \! \! dzz^{n-1} P_{gG}(z) = C_2(G) \left[ -\frac{1}{6} + \frac{2}{n(n-1)} + \frac{2}{(n+1)(n+2)} \right.
\]

\[
\left. - 2 \sum_{j=2}^{n} \frac{1}{j} - \frac{2n_f T(R)}{3C_2(G)} \right].
\]

(4.40)

We have to calculate the new terms in the matrix of anomalous dimensions, which are the contribution of tensor-gluons. For the tensors-gluons kernel \( P_{TG}(z) \), they are:

\[
\int_0^1 \! \! dzz^{n-1} P_{TG}(z) = C_2(G) \left[ \sum_{k=2s-1}^{2s+n} \frac{(-1)^k(2s + n)!}{k!(2s + n - k)! k - 2s + 2} \right]
\]

\[
+ C_2(G) \left[ \sum_{k=2s-n}^{2s+1} \frac{(-1)^k(2s + 1)!}{k!(2s + 1 - k)! k + 1 + n - 2s} \right], \quad n \leq 2s - 1
\]

\[
\int_0^1 \! \! dzz^{n-1} P_{TG}(z) = C_2(G) \left[ \sum_{k=2s-1}^{2s+n} \frac{(-1)^k(2s + n)!}{k!(2s + n - k)! k - 2s + 2} \right]
\]

\[
+ C_2(G) \left[ \frac{\Gamma(2s + 2)!}{\Gamma(n + 3)} \Gamma(n - 2s + 1) \right], \quad n \geq 2s,
\]

(4.41)

for the gluons–tensors kernel \( P_{GT}(z) \):

\[
\int_0^1 \! \! dzz^{n-1} P_{GT}(z) = C_2(G) \left[ \frac{\Gamma(2s + 2)!}{\Gamma(n + 2s + 1)} \right], \quad n \leq 2s
\]

\[
\int_0^1 \! \! dzz^{n-1} P_{GT}(z) = C_2(G) \left[ \sum_{k=2s-1}^{n-2} \frac{(-1)^k(n - 2)!}{k!(n - k - 2)!} \frac{1}{k - 2s + 2} \right]
\]

\[
+ \frac{\Gamma(2s + 2)!}{\Gamma(n + 2s + 1)} \Gamma(n - 1) \right], \quad n \geq 2s + 1
\]

(4.42)
and for the tensors–tensors kernel $P_{TT}(z)$:

$$
\int_0^1 dz z^{n-1} P_{TT}(z) = -C_2(G) \sum_{j=2+2s}^{n+2s} \frac{1}{j}, \quad n \leq 2s, \\
\int_0^1 dz z^{n-1} P_{TT}(z) = -C_2(G) \left[ \sum_{j=2+2s}^{n+2s} \frac{1}{j} + \sum_{j=1}^{n-2s} \frac{1}{j} \right], \quad n \geq 2s + 1
$$

(4.43)

where $s = 2, 3, 4, \ldots$. For the nonsinglet piece of the structure functions, the relevant anomalous dimension is $\gamma_{q\pi}^{nS}$, and it does not have any additional contribution from tensor-gluons, therefore for nonsinglet pieces of the structure functions one can get

$$
\int_0^1 dx x^{n-2} F^{NS}(x, Q^2) \sim \infty C_{NS}^{(g)} \left[ \frac{\alpha}{\alpha(t)} \right] A_{NS}^{nS},
$$

(4.44)

where

$$
A_{NS}^{nS} = \frac{\gamma_{nS}}{2\pi b} = -\frac{3C_2(R)}{(11 + 12s^2 - 1)C_2(G) - 4sT(R)} \\
\times \left[ 1 - \frac{2}{n(n+1)} + 4 \sum_{j=2}^{n} \frac{1}{j} \right].
$$

(4.45)

The difference between it and the standard case comes from the value of the coefficient $b$ in the Callan–Sinzuk beta function (3.31). This result shows that if tensor-gluons exist in the strong interaction, the logarithmic correction to the Bjorken scaling becomes milder. For the $SU(3)$ ($C_2(G) = 3$, $C_2(R) = 4/3$, $T(R) = 1/2$, $n_f = 3$) and spin-two gluons $s = 2$, we get

$$
A_2^{NS} = -\frac{2}{63}, \quad A_3^{NS} = -\frac{25}{504}, \ldots, \quad A_n^{NS} = -\frac{1}{21} \ln n
$$

(4.46)

instead of $A_2^{NS} = \frac{16}{27}$, $A_3^{NS} = \frac{25}{27}$, $\ldots$, $A_n^{NS} = -\frac{8}{27} \ln n$ in the standard case [15].

For the singlet pieces of the structure functions, one should take into account the mixing of gluon, fermion and tensor operators, so that

$$
\int_0^1 dx x^{n-2} F^{S}(x, Q^2) \sim \infty C_{S}^{(g)} \left[ \frac{\alpha}{\alpha(t)} \right] A_{S}^{nS},
$$

(4.47)

where $A_n^{S} = \gamma_n/2\pi b$ are the eigenvalues of the matrix (4.39).

5. Unification of coupling constants of the standard model

It is interesting to know how the contribution of tensor-gluons changes the high-energy behavior of the coupling constants of the standard model [47, 48]. The coupling constants are evolving in accordance with the formulae

$$
\frac{1}{\alpha_i(M)} = \frac{1}{\alpha_i(\mu)} + 2b_i \ln \frac{M}{\mu}, \quad i = 1, 2, 3,
$$

(5.48)
where we shall consider only the contribution of the lower $s = 2$ tensor-bosons:

$$2b = \frac{58C_2(G) - 4n_f T(R)}{6\pi}. \tag{5.49}$$

For the $SU(3)_c \times SU(2)_L \times U(1)$ group with its coupling constants $\alpha_3$, $\alpha_2$ and $\alpha_1$ and six quarks $n_f = 6$ and $SU(5)$ unification group, we will get

$$2b_3 = \frac{1}{2\pi} 54, \quad 2b_2 = \frac{1}{2\pi} \frac{104}{3}, \quad 2b_1 = -\frac{1}{2\pi} 4,$$

so that by solving the system of equations (5.49) one can get

$$\ln \frac{M}{\mu} = \frac{\pi}{58} \left( \frac{1}{\alpha_3(\mu)} - \frac{8}{3} \frac{1}{\alpha_s(\mu)} \right). \tag{5.50}$$

where $\alpha_3(\mu)$ and $\alpha_s(\mu)$ are the electromagnetic and strong coupling constants at scale $\mu$. If one takes $\alpha_3(M_Z) = 1/128$ and $\alpha_s(M_Z) = 1/10$ one can get that the coupling constants have equal strength at energies of order

$$M \sim 4 \times 10^4 \text{ GeV} = 40 \text{ TeV},$$

which is much smaller than the scale $M \sim 10^{14} \text{ GeV}$ in the absence of the tensor-gluons contribution. The value of the weak angle $[47, 48]$ remains intact:

$$\sin^2 \theta_W = \frac{1}{6} + \frac{5}{9} \frac{\alpha_3(M_Z)}{\alpha_s(M_Z)}, \tag{5.51}$$

and the coupling constant at the unification scale remains of the same order $\tilde{a}(M) = 0, 01$.

6. Conclusion

Let us summarize the physical consequences of the tensor-gluons hypothesis. Among all parton distributions, the gluon density $G(x, t)$ is one of the least constrained functions since it does not couple directly to the photon in deep-inelastic scattering measurements of the proton $F_2$ structure function. Therefore it is only indirectly constrained by scaling violations and by the momentum sum rule, from which it follows that only half of the proton momentum is carried by charged constituents—the quarks—and that the other part is carried by the neutral constituents. As it was suggested in the article, the process of gluon splitting leads to the emission of tensor-gluons and therefore the part of the proton momentum carried by the neutral constituents here is shared between gluons and tensor-gluons. The density of neutral partons in the proton is therefore given by the sum of two functions: $G(x, t) + T(x, t)$, where $T(x, t)$ is the density of the tensor-gluons. To disentangle these contributions and to decide which piece of the neutral partons is the contribution of gluons and which one is of the tensor-gluons, one should measure the helicities of the neutral components, which seems to be a difficult task. The other test of the proposed model will be the consistency of the mild $Q^2$ behavior of the moments of the structure functions with the experimental data.

The gluon density can be directly constrained also by jet production [49]. In the suggested model, the situation is such that the standard quarks cannot radiate tensor-gluons (such a vertex is absent in the model [16–19]), therefore only gluons are radiated by quarks. A radiated gluon then can split into a pair of tensor-gluons without obscuring the structure of the observed three-jet final states. Thus it seems that there is no obvious contradiction with the
existing experimental data. Our hypotheses may be wrong, but the uniqueness and simplicity of the suggested extension could be a reason for its serious consideration.

In the last section, we also observed that the unification scale at which standard coupling constants are merging is shifted to lower energies, telling us that it may be that a new physics is round the corner. Whether all these phenomena are consistent with experiment is an open question.

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7. Note added

In supersymmetric extensions of the standard model [50, 51], the gluons and quarks have natural partners—gluinos of spin $s = 1/2$ and squarks of spin $s = 0$. If the gluinos appear as elementary constituents of the hadrons, then the theory predicts the existence of new hadronic states, the R-hadrons [52, 53]. These new hadronic states can be produced in ordinary hadronic collisions and decay into ordinary hadrons with the radiation of massless photino—the massless partner of the photon, which takes out a conserved R-parity. Depending on the model, the gluinos may be massless or massive, depending on the remaining unbroken symmetries and the representation content of the theory. The existing experimental data give evidence that most probably they have to be very heavy [54, 55].

It seems that the phenomenological limitation on the existence of the R-hadrons is much stronger than the limitation on the existence of the tensor-gluons inside the ordinary hadrons. This is because gluinos change the statistics of the ordinary hadrons: the proton has to have a partner—the R-proton, which is a boson and is indeed a new hadron. The existence of tensor-gluon partons inside the proton does not predict a new hadronic state—a proton remains a proton. The tensor-gluons change the proton dynamical properties—its structure functions. The experimental limitations on those dynamics should be studied in detail.

I would like to thank Pierre Fayet for illuminating discussions of the supersymmetry phenomenology and physics of R-hadrons, and John Iliopoulos for helpful discussions of the experimental data.

Appendix A

The momentum sum rule integrals (3.24), (3.26) and (3.35) can be calculated in two different ways: with direct regularization of the integrals near $z = 1$ and near $z = 0$ as defined in (3.25), or by using the substitution $w = 1 - z$ in the second term of the integrand and then by regularizing the resulting integrand near $z = 1$. As one can be convinced, both methods give identical results. Therefore we shall first calculate the integral over $P_{TG}(z)$ in (3.26) by direct regularization (3.25) as follows:

$$
C_2(G) \int_0^1 dz \left[ \frac{z^{2s+2}}{(1 - z)^{2s-1}} + \frac{(1 - z)^{2s+1}}{z^{2s+2}} \right] \equiv \text{Reg}
$$

$$
= C_2(G) \int_0^1 dz \left[ \frac{z^{2s+2}}{(1 - z)^{2s-1}} + \frac{(1 - z)^{2s+1}}{z^{2s+2}} \right] = -(-1)^{2s} \frac{12s^2 - 1}{6} C_2(G).
$$

(A.1)
We shall now calculate the same integral using the substitution \( w = 1 - z \) in the second term of the integrand and evaluating the sum

\[
C_2(G) \int_0^1 dz \left[ \frac{z^{2r+2}}{(1 - z)^{2r-1}} + \frac{z^{2r+1}}{(1 - z)^{2r-2}} \right] = C_2(G) \int_0^1 dz \frac{z^{2r+1}}{(1 - z)^{2r-1}} \tag{A.2}
\]

and then one should regularize the integral near \( z = 1 \), as defined in (3.25)

\[
C_2(G) \int_0^1 dz \frac{z^{2r+1}}{(1 - z)^{2r-1}} = - (-1)^{2r+1} \frac{12z^2 - 1}{6} C_2(G) \tag{A.3}
\]

which is identical to the previous result.

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