On the first eigenvalue of the Laplacian for polygons

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Abstract: In 1947, Polya proved that if $n = 3, 4$ the regular polygon $P_n$ minimizes the principal frequency of an $n$-gon with given area and suggested that the same holds for larger values of $n$. In 1951, Polya and Szego discussed the possibility of counterexamples. Recently, I constructed explicit $(2n - 4)$–dimensional polygonal manifolds and proved for $n$ large that there exists an explicit non-empty set $A_n$ such that $P_n$ has the smallest principal frequency among $n$–gons in $A_n$. The techniques involve a partial symmetrization, tensor calculus, the spectral theory of circulant matrices, and $W^{2,p}$ estimates. An application is given in the context of electron bubbles.