A Peculiar Dynamically Warped Theory Space

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Abstract. We study a supersymmetric deconstructed gauge theory in which a warp factor emerges dynamically, driven by Fayet-Iliopoulos terms. The model is peculiar in that it possesses a global supersymmetry that remains unbroken despite nonvanishing D-term vacuum expectation values. Inclusion of gravity and/or additional messenger fields leads to the collective breaking of supersymmetry and to an unusual phenomenology.

INTRODUCTION

In this talk\(^1\), we consider a four-dimensional, linear “moose” model that deconstructs [1] a slice of five-dimensional Anti-de Sitter (AdS) space [2]. The profile of link field vacuum expectation values (vevs) along the moose can be chosen to replicate the effects of a warp factor in the higher-dimensional theory. The question we study is whether the necessary profile can be generated dynamically and naturally. In Ref. [3], we present a number of examples where a monotonically varying warp factor is obtained by assuming a translational symmetry along the moose and specific choices for boundary conditions at its ends. Here, we focus on a supersymmetric example in which a warp factor is driven by Fayet-Iliopoulos (FI) terms. The model is peculiar in that it possesses a global supersymmetry that remains unbroken despite nonvanishing D-term vevs. Inclusion of gravity and/or additional messenger fields leads to the collective breaking of supersymmetry, with interesting consequences. Other applications of deconstruction in model building can be found in Refs. [4, 5].

THE MODEL

The model we consider is a 4D \( \mathcal{N}=1 \) SUSY U(1)\(^n\) moose theory. The link fields consist of chiral multiplets \( \phi_i \) with charges \( (q_i, q_{i+1}) = (+1, -1) \), where \( i \) labels the gauge group factor. Conjugate superfields \( \bar{\phi}_i \) are included to cancel anomalies.

\(^1\) Presented at SUSY06, the 14th International Conference on Supersymmetry and the Unification of Fundamental Interactions, Irvine, California, USA 12-17 June 2006. WM-06-109.
The scalar potential for the link fields is given by

$$V_D = \sum_{i=1}^{n} D_i^2,$$

(1)

where

$$D_i = g \left( |\phi_i|^2 - |\phi_{i-1}|^2 - |\bar{\phi}_i|^2 + |\bar{\phi}_{i-1}|^2 \right) + \xi_i,$$

(2)

and where we define $\phi_0 = \phi_n = 0$. Here $g$ is the common gauge coupling and $\xi_i$ is the FI term for the $i^{th}$ group. This potential is minimized when

$$\langle \phi_i \rangle \left( \langle D_i \rangle - \langle D_{i+1} \rangle \right) = 0,$$

(3)

which implies that the vacua of interest generically have equal $D$-terms,

$$\langle D_i \rangle = \frac{\sum_j g \xi_j}{n} \equiv D.$$

(4)

The scalar vevs $v_i$ and $|\mathbf{v}_i|$ satisfy the recursion relation

$$\left( |v_{i+2}|^2 - |\mathbf{v}_{i+2}|^2 \right) - 2\left( |v_{i+1}|^2 - |\mathbf{v}_{i+1}|^2 \right) + \left( |v_i|^2 - |\mathbf{v}_i|^2 \right) = (\xi_{i+1} - \xi_{i+2}),$$

(5)

which is a discretized form of

$$\frac{\partial^2 |\phi(y)|^2}{\partial y^2} = -\frac{\xi(y)}{a},$$

(6)

where $a = 1/(gv_1)$ is the lattice spacing. Integrating this result and expressing the link profile $\phi(y)$ in terms of the warp factor, one finds

$$\frac{\partial e^{-f(y)}}{\partial y} = (-g^2 \xi(y) + gD) a,$$

(7)

where $D/g = \int_0^R \! dy \xi(y)/R$.

Notice that any desired warp factor can be obtained by setting

$$\xi(y) = \tilde{\xi}(y) + D/g = \tilde{\xi}(y) + \int_0^R \! dy \xi(y)/R,$$

(8)

and choosing an appropriate function $\tilde{\xi}(y)$. However, Eq. (8) is self-consistent only if

$$\int_0^R \! dy \tilde{\xi}(y) = 0.$$

(9)

A monotonically varying warp factor is possible provided that $\tilde{\xi}(y)$ receives an opposite sign contribution at the boundary.
THE SPECTRUM

The Kaluza-Klein spectrum of this model is surprising, given the non-vanishing $\langle D_i \rangle$ in Eq. (4). The masses of the vector and chiral multiplets originate from the kinetic terms

$$\mathcal{L} \supset \int d^4 \theta \sum_i \Phi_i^\dagger \exp \left[ g(V_i - V_{i+1}) \right] \Phi_i + \bar{\Phi}_i^\dagger \exp \left[ g(-V_i + V_{i+1}) \right] \bar{\Phi}_i.$$  \hspace{1cm} (10)

One finds that the gauge boson mass matrix is

$$m_{\text{gauge}}^2 = 2g^2 \begin{pmatrix} v_1^2 & -v_1^2 & -v_2^2 & \cdots & -v_{n-1}^2 \\ -v_1^2 & v_1^2 + v_2^2 & v_2^2 + v_3^2 & \cdots & v_{n-1}^2 \\ -v_2^2 & v_2^2 + v_3^2 & \cdots & \cdots & v_{n-1}^2 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ -v_{n-1}^2 & v_{n-1}^2 & \cdots & \cdots & v_{n-1}^2 \end{pmatrix}.$$  \hspace{1cm} (11)

The mass matrix for the link field fermions and the gauginos is such that

$$M_{\text{fermions}}^2 = 2g^2 \begin{pmatrix} \Theta \Theta^\dagger \\ \Theta^\dagger \Theta \end{pmatrix},$$  \hspace{1cm} (12)

where the $n \times (n-1)$ dimensional matrix $\Theta$ is given by

$$\Theta = \begin{pmatrix} v_1 & v_2 & \cdots & v_{n-2} & v_{n-1} \\ -v_1 & v_2 & \cdots & v_{n-2} & v_{n-1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -v_{n-2} & v_{n-1} & \cdots & v_{n-2} & v_{n-1} \end{pmatrix}.$$  \hspace{1cm} (13)

Clearly, $2g^2 \Theta \Theta^\dagger \equiv m_{\text{gauge}}^2$, so that half of the fermion spectrum coincides with the gauge boson spectrum. The scalar spectrum, on the other hand, may be obtained by expanding Eq. (1) about its minimum. One finds

$$\mathcal{L} \supset \frac{1}{2}(\varphi_i^\dagger | \varphi_i) g^2 \begin{pmatrix} \Theta^\dagger \Theta & \Theta^\dagger \Theta \\ \Theta^\dagger \Theta & \Theta^\dagger \Theta \end{pmatrix} \begin{pmatrix} \varphi_i \\ \varphi_i^\dagger \end{pmatrix}.$$  \hspace{1cm} (14)

The imaginary modes $(\varphi_i - \varphi_i^\dagger)/\sqrt{2}$ have vanishing masses, and correspond to the would-be Goldstone bosons of the spontaneous symmetry breaking $U(1)^n \rightarrow U(1)$. The real modes $(\varphi_i + \varphi_i^\dagger)/\sqrt{2}$ have the mass matrix

$$M_{\text{scalars}}^2 = 2g^2 \Theta^\dagger \Theta,$$  \hspace{1cm} (15)

which coincides precisely with the remaining massive fermion modes. Finally, the $\tilde{\phi}$ scalars and their fermionic partners remain massless. Although $n$ FI terms are present, we conclude that the KK spectrum remains exactly supersymmetric.

This peculiar result can be understood by considering a simpler theory: a 4D $\mathcal{N} = 1$ SUSY $U(1)$ gauge theory with no matter, plus an FI term. This theory
also has an exactly supersymmetric spectrum. The sole effect of the FI term is to introduce a cosmological constant, which is irrelevant if gravity is not included. Precisely the same is true in our model. One can show that the potential Eq. (1) has a non-vanishing vacuum energy density \((\sum \xi_i)^2/n\).

The effects of SUSY breaking reappear in the particle spectrum if the model is coupled to another sector. Imagine that we introduce a vector-like pair of chiral superfields that are charged only under the first \(U(1)\) factor. The nonvanishing \(D_1\) vev will split the squared masses of their scalar components by \(\pm 2 \langle D_1 \rangle\). If these fields are also charged under the gauge groups of the minimal supersymmetric standard model (MSSM), then SUSY-breaking effects will be gauge mediated to the observable sector. Interestingly, the scale of SUSY breaking that is relevant to gauge mediation is determined by a single D-term vev, \(D_1\), while the scale relevant to gravity-mediation is set by all \(n\) non-vanishing D-terms. Gravity-mediated SUSY-breaking effects therefore scale with the size of the moose. It is possible in such a model to have competing effects from the gauge and gravity mediation of supersymmetry breaking and a heavier gravitino than in other D-term supersymmetry breaking scenarios.

**CONCLUSIONS**

We have presented a supersymmetric \(U(1)\) gauge theory that deconstructs a warped extra dimension and dynamically generates a warp factor. The warping is accomplished via Fayet-Iliopoulos D-terms that force the squares of the link field vevs to grow by an additive factor as one moves along the moose. In its simplest form, the model has the peculiar feature that supersymmetry breaking appears only via the generation of a cosmological constant, while the spectra of the physical gauge and link states remains supersymmetric. In the case where the moose is allowed to couple to additional matter, the delocalization of supersymmetry breaking implies that fields localized at a single site experience a source of supersymmetry-breaking, \(D_i^2\), that is \(1/n\) as strong as the full amount available for gravity mediation leading, for example, to a heavy gravitino. In addition, supersymmetry breaking is supersoft [6] in this scenario. These features may make our \(U(1)^n\) model distinctive if it is applied as a secluded supersymmetry-breaking sector for the minimal supersymmetric standard model.

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