Where does the hot electroweak phase transition end?

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We give the nonperturbative phase diagram of the four-dimensional hot electroweak phase transition. A systematic extrapolation $a \to 0$ is done. Our results show that the finite temperature SU(2)-Higgs phase transition is of first order for Higgs-boson masses $m_H < 66.5 \pm 1.4$ GeV. The full four-dimensional result agrees completely with that of the dimensional reduction approximation. This fact is of particular importance, because it indicates that the fermionic sector of the Standard Model (SM) can be included perturbatively. We obtain that the Higgs-boson endpoint mass in the SM is 72.4 $\pm$ 1.7 GeV. Taking into account the LEP Higgs-boson mass lower bound excludes any electroweak phase transition in the SM.

The observed baryon asymmetry is finally determined at the electroweak phase transition (EWPT) [6]. The perturbative approach breaks down for the physically allowed Higgs-boson masses (e.g. $m_H > 70$ GeV) [7]. Since merely the bosonic sector is responsible for the bad perturbative features (due to infrared problems) the simulations are done without the inclusion of fermions on four-dimensional lattices [3], [4]. Another approach is the simulations of the reduced model in three-dimensions [5], [6]. The comparison of the results obtained by the two techniques is not only a useful cross-check on the perturbative reduction procedure but also a necessity because the fermions must be included perturbatively.

The line separating the symmetric and broken phases on the $m_H - T$ plane has an endpoint, $m_{H,c}$. In four dimension at $m_H \approx 80$ GeV the EWPT turned out to be extremely weak, even consistent with the no phase transition scenario on the 1.5-$\sigma$ level [6]. Three-dimensional results show that for $m_H > 95$ GeV no first order phase transition exists [6] and more specifically that the endpoint is $m_{H,c} \approx 67$ GeV [6]. In this paper we present the analysis of the endpoint on four dimensional lattices in the continuum limit and transform the result to the full SM.

We will use different spacings in temporal ($a_t$) and spatial ($a_s$) directions. The asymmetry of the lattice spacings is given by the asymmetry factor

$$\xi = a_s/a_t.$$ The action reads

$$S[U, \varphi] =$$

$$\beta_s \sum_{sp} \left( 1 - \frac{1}{2} Tr U_{pl} \right) + \beta_t \sum_{tp} \left( 1 - \frac{1}{2} Tr U_{pl} \right)$$

$$- \kappa_s \sum_{\mu=1}^3 Tr (\varphi_{x+\mu}^+ U_{x,\mu} \varphi_x) - \kappa_t Tr (\varphi_{x+4}^+ U_{x,4} \varphi_x)$$

$$+ \sum_x \left[ \frac{1}{2} Tr (\varphi_x^+ \varphi_x) + \lambda \left[ \frac{1}{2} Tr (\varphi_x^+ \varphi_x) - 1 \right]^2 \right],$$

where $U_{x,\mu}$ denotes the SU(2) gauge link variable, $U_{sp}$ and $U_{tp}$ the path-ordered product of the four $U_{x,\mu}$ around a space-space or space-time plaquette, respectively; $\varphi_x$ stands for the Higgs field. The anisotropies $\gamma_3^2 = \beta_t/\beta_s$ and $\gamma_K^2 = \kappa_t/\kappa_s$ are functions of the asymmetry $\xi$. These functions have been determined perturbatively [10] and non-perturbatively [11]. In this paper we use the asymmetry parameter $\xi = 4.052$, which gives $\gamma_K = 4$ and $\gamma_3 = 3.919$

We have performed our simulations on finer and finer lattices, moving along the lines of constant physics (LCP). In our case there are three bare parameters ($\kappa$, $\beta$, $\lambda$). The bare parameters are chosen in a way that the zero temperature renormalized gauge coupling $g_R$ is held constant and the mass ratio for the Higgs- and W-bosons $R_{HW} = m_H/m_W$ corresponds to the Higgs mass at the endpoint of first order phase transitions: $R_{HW,c}$. These two conditions determine a LCP as a one-dimensional subspace in the original space

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of bare parameters. The position on the LCP gives the lattice spacing $a$. As the lattice spacing decreases $R_{\text{HW},c} \to R_{\text{HW},\text{cont.}}$.

Since our theory is a bosonic one we assumed that the corrections are quadratic in the lattice spacings; thus an $a^2$ fit has been performed for $R_{\text{HW},c}$ to determine its continuum value.

We have carried out $T \neq 0$ simulations on $L_t = 2, 3, 4, 5$ lattices and tuned $\kappa$ to the transition point. This condition fixes the lattice spacings: $a_t = \alpha_s/\xi = 1/(T_t L_t)$ in terms of the transition temperature $T_t$ in physical units. The third parameter $\lambda$, finally specifying the physical Higgs mass in lattice units, has been chosen in a way that the transition corresponds to the endpoint of the first order phase transition subspace.

In this paper $V = L_t \cdot L_5^3$ type four-dimensional lattices are used. For each $L_t$ we had 8 different lattices, each of them had approximately twice as large lattice-volume as the previous one. The smallest lattice was $V = 2 \cdot 5^3$ and the largest one was $V = 5 \cdot 50^3$. Reweighting was used to obtain information in the vicinity of a simulation point.

The determination of the endpoint of the finite temperature EWPT is done by the use of the Lee-Yang zeros of the partition function $Z$ [12]. Near the first order phase transition point the partition function reads $Z = Z_s + Z_b \propto \exp(-Vf_s) + \exp(-Vf_b)$, where the indices s(b) refer to the symmetric (broken) phase and $f$ stands for the free-energy densities. Near the phase transition point we also have $f_b = f_s + \alpha (\kappa - \kappa_c)$, which shows that for complex $\kappa$ values $Z$ vanishes at $\text{Im}(\kappa) = \pi \cdot (n-1/2)/(V\alpha)$ for integer $n$. In case a first order phase transition is present, these Lee-Yang zeros move to the real axis as the volume goes to infinity. In case a phase transition is absent the Lee-Yang zeros stay away from the real $\kappa$ axis. Denoting $\kappa_0$ the lowest zero of $Z$, i.e. the position of the zero closest to the real axis, one expects in the vicinity of the endpoint the scaling law $\text{Im}(\kappa_0) = c_1(L_t, \lambda) V^{\nu} + c_2(L_t, \lambda)$.

In order to pin down the endpoint we are looking for a $\lambda$ value for which $c_2$ vanish. In practice we analytically continue $Z$ to complex values of $\kappa$ by reweighting the available data. Also small changes in $\lambda$ have been done by reweighting. As an example, the dependence of $c_2$ on $\lambda$ for $L_t = 3$ is shown in fig. 2. To determine the critical value of $\lambda$ i.e. the largest value, where $c_2 = 0$, we have performed fits linear in $\lambda$ to the nonnegative $c_2$ values.

Having determined the endpoint $\lambda_{\text{crit.}}(L_t)$ for each $L_t$ we calculate the $T = 0$ quantities $(R_{\text{HW}}, g^2_H)$ on $V = (32L_t) \cdot (8L_t) \cdot (6L_t)^3$ lattices, where $32L_t$ belongs to the temporal extension, and extrapolate to the continuum limit. Having established the correspondence between $\lambda_{\text{crit.}}(L_t)$ and $R_{\text{HW}}$, the $L_t$ dependence of the critical $R_{\text{HW}}$ is easily obtained. Fig. 3 shows the dependence of the endpoint $R_{\text{HW}}$ values on $1/L_t^2$. A linear extrapolation in $1/L_t^2$ yields the continuum limit value of the endpoint $R_{\text{HW}}$. We obtain $66.5 \pm 1.4$ GeV, which is our final result.

Comparing our result to those of the 3d analyses [11] one observes complete agreement. Since the error bars on the endpoint determinations are on the few percent level, the uncertainty of the dimensional reduction procedure is also in this range. This indicates that the analogous perturbative inclusion of the fermionic sector results also in few percent error on $M_H$.

Based on our published data [4,11] and the present results we are able to draw the phase diagram of the SU(2)-Higgs model in the $(T_c/m_H - R_{\text{HW}})$ plane. This is shown in fig. 4. The continuous line – representing the phase-boundary – is a quadratic fit to the data points.

Finally, we determine what is the endpoint value in the full SM. Our nonperturbative anal-
Figure 2. Dependence of $R_{HW,cr}$, i.e. $R_{HW}$ corresponding to the endpoint of first order phase transitions on $1/L^2_t$ and extrapolation to the infinite volume limit.

Figure 3. Phase diagram of the SU(2)-Higgs model in the $(T_c/m_H - R_{HW})$ plane.

Analysis shows that the perturbative integration of the heavy modes is correct within our error bars. Therefore we use perturbation theory [13] to transform the SU(2)-Higgs model endpoint value to the full SM. We obtain $72.4 \pm 1.7$ GeV, where the error includes the measured error of $R_{HW,cont}$, $g_\rho^2$ and the estimated uncertainty [14] due to the different definitions of the gauge couplings between this paper and [13]. The dominant error comes from the uncertainty on the position of the endpoint.

The present experimental lower limit of the SM Higgs-boson mass is $89.8$ GeV [13]. Taking into account all errors (in particular those coming from integrating out the heavy fermionic modes), our endpoint value excludes the possibility of any EWPT in the SM. This also means that the SM baryogenesis in the early Universe is ruled out.

For details of this analysis see [16].

Simulations have been carried out on the Cray-T90 at HLRZ-Jülich, on the APE-Quadrics at DESY-Zeuthen and on the PMS-8G PC-farm in Budapest. This work was supported by OTKA-T016240/T022929 and FKP-0128/1997.

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