Obliquity evolution of the minor satellites of Pluto and Charon

Alice C. Quillen\(^1\), Fiona Nichols-Fleming\(^1\), Yuan-Yuan Chen\(^{1,2}\) and Benoit Noyelles\(^3\)

\(^1\)Department of Physics and Astronomy, University of Rochester, Rochester, NY 14627 USA
\(^2\)Key Laboratory of Planetary Sciences, Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008, China
\(^3\)Department of Mathematics and the Namur Centre for Complex Systems (naXys), University of Namur, 8 Rempart de la Vierge, Namur B-5000 Belgium

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ABSTRACT

New Horizons mission observations show that the small satellites Styx, Nix, Kerberos and Hydra, of the Pluto-Charon system, have not tidally spun-down to near synchronous spin states and have high obliquities with respect to their orbit about the Pluto-Charon binary (Weaver et al. 2016). We use a damped mass-spring model within an N-body simulation to study spin and obliquity evolution for single spinning non-round bodies in circumbinary orbit. Simulations with tidal dissipation alone do not show strong obliquity variations from tidally induced spin-orbit resonance crossing and this we attribute to the high satellite spin rates and low orbital eccentricities. However, a tidally evolving Styx exhibits intermittent obliquity variations and episodes of tumbling. During a previous epoch where Charon migrated away from Pluto, the minor satellites could have been trapped in orbital mean motion inclination resonances. An outward migrating Charon induces large variations in Nix and Styx’s obliquities. The cause is a commensurability between the mean motion resonance frequency and the spin precession rate of the spinning body. As the minor satellites are near mean motion resonances, this mechanism could have lifted the obliquities of all four minor satellites. If so the high obliquities of Pluto and Charon’s minor satellites imply that this system experienced orbital migration and all satellites were at one time captured into mean motion resonances.

1 INTRODUCTION

The five satellites of Pluto are Charon, Styx, Nix, Kerberos and Hydra, in order of distance from Pluto (Weaver et al. 2006; Showalter et al. 2011, 2012). The satellite system is nearly coplanar with orbital periods near ratios of 1:3:4:5:6, but sufficiently distinct from integer ratios relative to Charon’s orbital period that the small satellites are not presently in mean-motion resonances with Charon (e.g., Buie et al. 2013). As the masses of Pluto and Charon vastly exceed the masses of the other satellites, we refer to Pluto and Charon as a binary and Styx, Nix, Kerberos and Hydra as minor satellites of the Pluto-Charon binary.

Over 1–10 Myr, tidal evolution should have synchronized the rotation of Pluto and Charon and then circularized their orbit, expanding the binary to its present separation (Farinella et al. 1979). Tidal evolution of Pluto-Charon would lead to capture of the minor satellites into mean motion orbital resonances. However numerical integrations have shown that this often causes such wide-scale dynamical instability that resonant transport (migration) of minor satellites to their current location probably did not take place (Cheung et al. 2014). Alternatively the smaller satellites could have formed from a circumbinary disk, and after the formation, spin synchronization and circularization of the Pluto-Charon binary (Lithwick & Wu 2008; Kenyon & Bromley 2014).

Prior to the arrival of the New Horizons’ mission at Pluto, Showalter & Hamilton (2015) explored possible spin states for the minor satellites. They speculated that the minor satellites would have tidally spun down and so would be slowly spinning, with angular spin rate \(\omega\) similar to the orbital mean motion \(n_o\). A tumbling body is one with angular momentum spin vector that is not aligned with a principal body axis (an eigenvector of the body’s moment of inertia tensor). Showalter & Hamilton (2015) speculated that perturbations from Charon could affect the rotation state of the minor satellites, contributing to chaotic tumbling, in analogy to how an orbital resonance between Titan and Hyperion could affect the rotational state of Hyperion (Showalter & Hamilton 2015). Spin-spin resonance (e.g., Batygin & Morbidelli 2015; Jafari Nadoushan & Assadian 2016), where spin \(w\) is commensurate with the binary mean motion \(n_B\), might arise from Charon’s periodic per-
turbations. However, new Horizons observations showed that the low mass satellites are spinning faster than considered by Showalter & Hamilton (2015), with angular spin rates \( w \gtrsim 6a_o \), many times greater than their orbital mean motions, implying that despinning due to tidal dissipation has not yet taken place (Weaver et al. 2016).

The angle between the spin vector of a minor satellite and the orbit normal of the Pluto-Charon binary can be called an obliquity. Weaver et al. (2016) measured the obliquities of Pluto’s minor satellites and found that all four have obliquities near 90 degrees, with spin vector lying nearly in their orbital planes. These surprising obliquities are a puzzle to explain.

As a satellite despins due to tidal dissipation, it may be captured in spin-orbit resonant states (Peale 1977; Wisdom et al. 1984; Celletti 2010). However, a body that is only very slowly spinning down due to tidal dissipation could cross spin-orbit resonance or spin-spin resonance if there is a drift in the satellite’s semi-major axis, known as ‘orbital migration’. Attitude instability, leading to obliquity variations and chaotic behavior, is common within spin-orbit resonance (Wisdom et al. 1984; Melnikov & Shevchenko 2008, 2010). There may be a connection between the minor satellite obliquities and previous episodes of spin-orbit or spin-spin resonance crossing or capture.

External to spin-orbit resonance, tidal dissipation causes a spherical body initially at low obliquity and \( w/n_o \gtrsim 6 \) to slowly increase in obliquity (Goldreich & Peale 1970; Ward 1975; Gladman et al. 1996). However, the obliquity drift rate due to tidal dissipation is slower than but a similar size as the tidal spin-down rate (Goldreich & Peale 1970). Our numerical integrations have confirmed that this remains true for elongated non-spherical bodies. If the minor satellites have not spun down, then neither should their obliquities have approached 90°. It is unlikely that the high minor satellite obliquities in the Pluto-Charon system are due to tidal secular (non-resonant) obliquity evolution alone.

With near integer orbital period ratios between satellites, the Pluto-Charon satellite system is near orbital mean motion resonances and may have crossed or been captured into these resonances in the past. Migration could have take place due to tidal evolution of Pluto and Charon but also due to interactions with a previous and now absent circumbinary disk. The minor satellites themselves could have been embedded in a disk and migrated by driving spiral density waves into the disk. Inwards or outwards migration could have taken place (e.g., Lubow & Ida 2010).

Planetary orbits have small inclinations and undergo precession (a rate of change of the longitude of the ascending node) due to mutual planet/planet gravitational perturbations. A similarity in a body’s spin precession rate and a precession rate of its orbit or the orbit of a perturber can cause obliquity variations, (Ward & Hamilton 2004; Correia et al. 2016), a mechanism described as ‘secular spin-orbit resonance’. Secular spin-orbit resonant mechanisms may have operated in the Pluto-Charon system. Capture in a mean motion resonance with the Pluto-Charon binary can lift the obliquity evolution of a non-spherical body in orbit about a binary system. Because of their simplicity and speed, compared to more computationally intensive grid-based or finite element methods, mass-spring computations are an attractive method for simulating deformable bodies. By including spring damping forces they can model viscoelastic tidal deformation. We previously used a mass-spring model to study tidal encounters (Quillen et al. 2016a), measure tidal spin down for spherical bodies over a range of viscoelastic relaxation timescales (Frouard et al. 2016), and spin-down of triaxial bodies spinning about a principal body axis aligned with the orbit normal (Quillen et al. 2016b). Here we use the same type of simulations to study longer timescale obliquity and spin evolution. Like Mardling & Lin (2002); Boné & Laskar (2009), we compute torques on spinning bodies that are in orbit about point masses. Rather than averaging over body shape or orbit, we can take into account viscous dissipation in the body directly (using our damped springs). The four minor satellites are not round so we simulate bodies with body axis ratios based on the observed values. For other simulation techniques integrating orbits and body rotation see Showalter & Hamilton (2015); Correia et al. (2016); Boué (2016); Hou et al. (2016).

Before we begin our numerical study we tabulate and estimate parameters for Pluto and Charon and their minor satellites. We first reexamine estimates for the tidally induced spin-down time, then estimate the wobble decay time and an asphericity parameter used to characterize the strength of spin-orbit resonances. We also compute precession frequencies for the spin axis and the orbital longitude of the ascending node. These parameters will help us interpret our simulations. Our simulations are described in section 3. Simulations with tidal dissipation alone are described in section 4 and those allowing the binary to drift in section 5. A possible mechanism for lifting the minor satellite obliquities is identified from the simulations and discussed further in our final section 6.

2 PARAMETERS, TIME-SCALES AND FREQUENCIES

2.1 Tidal spin-down times

Spin-down timescales are often computed for spherical bodies in orbit about a single mass. We start with a spherical body of radius \( R \), mass \( M \) in orbit with orbital semi-major axis \( a_o \) about a body of mass \( M_o \). We use \( a_o \) to denote

![Figure 1. We simulate a resolved spinning satellite about the Pluto-Charon binary. Pluto and Charon are modeled as point masses whereas the minor satellite is modeled with masses and springs.](Image)
orbital semi-major axis and $a$ to denote body semi-major axis. After discussing the tidal spin-down timescales for a spherical body in orbit about a point mass, we will consider non-spherical bodies described by the radius of the equivalent volume sphere, $R_v$, and in orbit about a binary rather than single mass.

The secular part of the semi-diurnal ($l = 2$) term in the Fourier expansion of the perturbing potential (e.g., see appendix by Frouard et al. 2016) gives a tidally induced torque on the spherical body

$$T = \frac{3}{2a_o} GM^2 R^2 \dot{\nu}_2(\sigma_1) \sin \epsilon_2(\sigma_1)$$

(1)

(see also Kaula 1964; Goldreich 1963; Goldreich & Peale 1968; Murray & Dermott 1999; Efroimsky & Makarov 2013), with $G$ the gravitational constant. Here the tidal frequency \(\sigma_2 = 2(n_o - \dot{\nu}) = 2(n_o - w)\) where $n_o$ is the orbital mean motion. The angular spin of the body (when spinning about a principal axis) $\dot{\theta} = \dot{w}$ and the body is oriented with spin axis perpendicular to the orbital plane. The quality function is $K_2(\sigma_1) \sin(\epsilon_2(\sigma_1))$ and is often approximated as $2k_2/Q$ with $Q$ a tidal dissipation factor (e.g., Kaula 1964) and $k_2$ a Love number.

The Love number for an incompressible homogeneous elastic body

$$k_2 = \frac{3/2}{1 + \bar{\mu}}$$

(2)

$$\bar{\mu} = \frac{19\mu_s}{2\rho g R} \approx \frac{38\pi \mu_s}{3} e_g$$

(3)

(Murray & Dermott 1999; Burns 1977) where $\rho$ is the mean density, $g = GM/R^2$ is the gravitational acceleration at the surface, and $\bar{\mu}$ is the elastic shear modulus. We use

$$e_g = \frac{GM^2}{R^3} = 1.2 \text{GPa} \left(\frac{R}{1000\text{km}}\right)^2 \left(\frac{\rho}{1\text{ g cm}^{-3}}\right)^2$$

(4)

for a unit of central pressure or gravitational energy density. It is common to estimate $\mu_s \approx 4 \times 10^9 \text{ m}^{-2} = 4 \text{ GPa}$ for icy bodies (e.g., see Nimmo & Schenk 2006; Murray & Dermott 1999). Inserting equation 3 into equation 2 for a small icy body ($R < 1000$ km; $\bar{\mu} > 1$) gives

$$k_2 \approx 0.03 \frac{e_g}{\mu_s}$$

(5)

The spin-down time can be estimated from the body’s moment of inertia, $I$, and an initial spin $w_{	ext{init}}$ giving $t_{\text{despin}} \sim I w_{\text{init}} / \bar{\tau}$ or

$$t_{\text{despin}} \approx \frac{I w_{\text{init}} a_o}{3GM^2} \left(\frac{a_o}{R}\right)^5 \frac{Q}{K_2}$$

(6)

(see equation 9 by Gladman et al. 1996; Peale 1977). Using the moment of inertia for a spherical body, $I = \frac{2}{5}MR^2$, and setting the initial spin to be that of centrifugal breakup $w_{\text{init}} = \sqrt{GM/R^3}$, the spin-down time

$$\frac{t_{\text{despin}}}{P_o} \approx \frac{1}{15\pi} \frac{M}{M_o} \left(\frac{a_o}{R}\right)^{3/2} \frac{Q}{K_2}$$

(7)

where $P_o$ is the orbital period. Here the spin-down time is estimated for a spherical body with prograde spin axis perpendicular to the orbit plane.

To approximate the spin down times for non-spherical bodies, we replace the body radius with the radius of the equivalent volume sphere, $R_v$ (see Quillen et al. 2016b). It is convenient to define a gravitational timescale

$$t_g = \sqrt{\frac{R^3}{GM}} = \sqrt{\frac{3}{4\pi G\rho}}$$

(8)

$$= 2000 \text{ s} \left(\frac{\rho}{1 \text{ g cm}^{-3}}\right)^{-\frac{1}{2}}.$$

Following Quillen et al. (2016b) our parameter $e_g$ (equation 4) for non-spherical bodies has radius $R$ replaced by $R_v$.

To estimate spin down times for objects in orbit about a binary rather than single mass, we replace $M$ with $M_B$, the total mass of the binary. The orbital semi-major axis $a_o$ of the spinning body is computed with respect to the center of mass of the binary (see Figure 1) and using the total binary mass. It can be called an osculating orbital element (Renner & Sicardy 2006). The osculating mean motion $n_o$ approximates $\sqrt{GM_B/a_o^3}$.

We tabulate properties of the Pluto-Charon binary in Table 1. For the binary or Charon’s period, $P_C$, binary or Charon’s semi-major axis, $a_C$, mass of the binary, Charon to Pluto mass ratio, and binary reduced mass are based on those measured by Brozovic et al. (2015).

In Table 2 we list properties of the the 4 minor satellites of the Pluto-Charon system based on measurements by Showalter & Hamilton (2015) and improved measurements by Weaver et al. (2016). The radii of the equivalent volume spheres, $R_v$, are listed, and these we compute using the body axis diameters from Table 2 by Weaver et al. (2016). The spin angular rotation rates, $\omega$, are computed from the spin periods and are given in units of $t_g$ for a density of $1 \text{ g cm}^{-3}$ (see equation 8). If the actual satellite densities are higher then the spins in dimensionless units would be lower. The time $t_{\text{despin}}^{-1}$ is equal to the angular rotation rate of a particle just grazing the surface of a spherical body with radius $R_v$. In dimensionless units we notice that only Hydra is spinning rapidly (at 1/3 of centrifugal breakup), compared to a maximum value of approximately 1. In Table 2 we also list the ratio of the orbital to spin periods, $P_o/P_s$. Kerberos and Styx have $P_o/P_s \sim 6$ whereas Nix and Hydra have ratios

| Table 1. Pluto-Charon binary |
|-----------------------------|
| $GM_{PC}$ | 975.5 ± 1.5 km$^3$ s$^{-2}$ |
| $GM_C$ | 105.88 ± 1.0 km$^3$ s$^{-2}$ |
| $M_C/M_P$ | 0.12 |
| $\mu_{PC}/M_{PC}$ | 0.0967 |
| $P_C$ | 6.3872 days |
| $a_C$ | 19596 km |
| $\mu_B a_C^2 / m_{PC}$ | $3.715 \times 10^7$ km$^2$ |
of 13.6 and 89. All four of the small satellites are spinning much more rapidly than $w \sim n_{\text{sat}}$, or $P_s/P_e \sim 1$.

The masses of the small satellites are not well constrained. In Table 2 the mass ratios $M/M_{\text{PC}}$ (satellite mass divided by Pluto-Charon binary mass) are computed using preferred (or boldface) values from Table 1 by Showalter & Hamilton (2015). These are masses within 1 standard deviation of their dynamical mass constraints (based on their orbit integrations).

For the minor satellites, we compute values for energy density $e_g$, Love number $k_2$ and tidal spin-down times $t_{\text{despin}}$ and list them in Table 2. The energy density $e_g$ is computed using equation 4 (but with $R_e$ replacing $R$), the volumetric radii $R_n$ listed in Table 2 and assuming a density of 1 g/cc. We computed the Love number $k_2$ using our values for $e_g$, a shear modulus for ice of $\mu = 4$ GPa and equation 5.

Tidal spin down times $t_{\text{despin}}$ are computed using equation 7, a tidal dissipation factor $Q = 100$, the $k_2$, orbital periods and mass ratios listed in the Table. The estimated spin down times (see bottom of Table 2) exceed the age of the Solar system and imply that the satellite spins should not have significantly decreased due to tidal dissipation. This contrasts with the expectation by Showalter & Hamilton (2015) that the minor satellites of Pluto would be spinning slowly enough to be tumbling, where the body is not spinning about the maximum principal body axis of its moment of inertia tensor and due to chaotic behavior associated with spin orbit resonance overlap (Wisdom et al. 1984). The high spin values found by Weaver et al. (2016) led them to conclude that tidal spin down has not yet taken place and the spin down times we have computed support this conclusion.

The actual spin-down times would be shorter than those listed in Table 2 if $Q < 100$, corresponding to higher dissipation. The times would be longer for higher density bodies; $\rho > 1\text{g/cc}$ and for stronger bodies. The spin down times are estimated for spheres but the torque would only be about twice as large for bodies with the axis ratios of these satellites (see Quillen et al. 2016b). During a previous epoch of higher orbital eccentricity the torque might have been higher. The spin-down time scale estimate also neglects perturbations by the Pluto-Charon binary and assumes that the spin rate starts near the maximum value. The current spin values are 23, 13, 39 and 3 times slower than $t_{\text{despin}}^{-1}$ for Styx, Nix, Kerberos and Hydra, respectively. The satellite with the shortest spin down time is Nix at $t_{\text{despin}} \sim 10^{11.6}$ years exceeding the age of the Solar system, even if we divide this by 10 to take into account that the spin may have originally been 10 times lower than $t_{\text{despin}}^{-1}$. Our estimated spin down time is for a spinning body at zero obliquity. Our simulations (not presented here but similar to those presented by Quillen et al. 2016b) show that spin-down times are longer at higher obliquity (approximately an order of magnitude higher for a prolate body with axis ratio 0.5 at obliquity 90°) so if the body spends much time at high obliquity it would not have spun down as far.

### 2.2 Wobble decay times

It is common to assume that bodies are spinning about a principal axis because the timescale for wobble to decay is much shorter than the spin-down time (see Burns & Safronov 1973; Peale 1977; Harris 1994). Due to tidal dissipation the wobble decays on a timescale

$$t_{\text{wobble}} = \frac{3GQ}{w_{\text{init}}k_2R^5}$$

(equation 8 by Gladman et al. 1996) where $C$ is the maximum moment of inertia about a principal body axis. The ratio of the wobble decay to spin-down time

$$t_{\text{wobble}}/t_{\text{despin}} \approx 9(n_c/w_{\text{init}})^4$$

(the ratio of equation 9 and 6; Gladman et al. 1996). We compute $t_{\text{wobble}}/t_{\text{despin}}$ from the ratio of current spin and orbital periods finding $t_{\text{wobble}}/t_{\text{despin}} \sim 6 \times 10^{-3}$ for Styx, and Kerberos, $3 \times 10^{-4}$ for Nix and $10^{-7}$ for Hydra. The wobble decay timescales, computed using the breakup angular rotation rate, are also listed Table 2. Hydra and Nix are spinning fast enough that their wobble decay times are much shorter than the age of the Solar system. Taking spins near their current values (rather than a near breakup value), reduces the spin-down time by 20 for Styx and Kerberos. This reduces the wobble decay time for Styx to near the age of the Solar system, but not Kerberos. Kerberos could be wobbling. Only Nix and Hydra are certain to be spinning about a principal body axis. If tumbling in one of the minor satellites was excited by an encounter (collision or tidal encounter) or a spin resonance at some time well after formation, the long wobble decay timescales imply that the body could still be tumbling today (and this is particularly relevant for Kerberos).

### 2.3 Asphericity

The width of spin-orbit resonances depends on an asphericity parameter

$$\alpha = \sqrt{3(B - A)/C}$$

(Wisdom et al. 1984) where $A < B \leq C$ are the three moments of inertia (eigenvalues of the moment of inertia matrix). The frequency $\omega_{ss} = \alpha n_s$ is the frequency of small-amplitude oscillations (librations) of a satellite in synchronous resonance (see Wisdom et al. 1984). For a triaxial ellipsoid with body axes $a > b > c$, the moments of inertia are $C = \frac{4}{5}(a^2 + b^2)$, $B = \frac{4}{5}(a^2 + c^2)$, and $A = \frac{4}{5}(b^2 + c^2)$ giving

$$\alpha = \sqrt{3(a^2 - b^2) \over a^2 + b^2} = \sqrt{3(1 - (b/a)^2) \over 1 + (b/a)^2}.$$  

This only depends on the body axis ratios in the orbital plane (assuming a body with parallel spin and orbital normal and spinning about a principal axis). An oblate body has $\alpha = 0$. We compute asphericities for Pluto’s minor satellites from the body axis ratios by Weaver et al. (2016), and they are listed in Table 2. The minor satellites are all sufficiently elongated that $\alpha \gtrsim 1$.

### 2.4 Spin Precession Frequency

The spin axis of a non-spherical body spinning about its principal axis in orbit about a central mass precesses. Using
Table 2. Pluto and Charon’s minor satellites

|           | Styx            | Nix             | Kerberos | Hydra           |
|-----------|-----------------|-----------------|----------|-----------------|
| Size (km) | $16 \times 9 \times 8$ | $50 \times 35 \times 33$ | $19 \times 10 \times 9$ | $65 \times 45 \times 25$ |
| Body axis ratio $b/a$ | 0.56           | 0.70            | 0.53     | 0.69             |
| Body axis ratio $c/a$ | 0.50           | 0.66            | 0.47     | 0.38             |
| Volumetric Radius $R_v$ (km) | 5.2            | 19.3            | 6.0      | 20.9             |
| Orbital period $P_o$ (days) | 20.16155 ± 0.00027 | 24.85463 ± 0.00003 | 32.16756 ± 0.00014 | 38.20177 ± 0.00003 |
| Orbital Semi-major axis $a_o$ (km) | 42.656    | 48.694          | 57.783   | 64.738           |
| Spin period $P_s$ (days) | 3.24 ± 0.07    | 1.829 ± 0.009   | 5.31 ± 0.10 | 0.4295 ± 0.0008 |
| Period ratio $P_o/P_s$ | 6.2            | 13.6            | 6.06     | 88.8             |
| Period ratio $P_C/P_s$ | 1.97           | 3.49            | 1.20     | 14.2             |
| Period ratio $P_C/P_C$ | 3.1566         | 3.8913          | 5.0963   | 5.9810           |
| Obliquity $\epsilon_B$ (deg) | 91             | 123             | 96       | 110              |
| Spin $\omega$ | 0.0424         | 0.0752          | 0.026    | 0.320            |
| Mass ratio $M/M_{PC}$ | $1 \times 10^{-7}$ | $3 \times 10^{-6}$ | $8 \times 10^{-7}$ | $4 \times 10^{-6}$ |
| $\epsilon_B$ (GPa) | 3.2 × 10^{-5}  | 4.5 × 10^{-4}   | 4.3 × 10^{-5} | 5.2 × 10^{-4} |
| Love number $k_2$ | 3 × 10^{-7}    | 4 × 10^{-6}     | 4 × 10^{-7} | 5 × 10^{-6}      |
| $\log_{10} \frac{\mu_{\text{spin}}}{\text{yr}}$ | 12.7           | 11.6            | 14.4     | 12.2             |
| $\log_{10} \frac{\mu_{\text{oblate}}}{\text{yr}}$ | 10.5           | 8.0             | 12.2     | 5.4              |
| Asphericity $\alpha$ | 1.25           | 1.01            | 1.30     | 1.03             |
| Oblateness parameter $q_{eff}$ | 0.31           | 0.21            | 0.33     | 0.40             |
| Binary quad ratio $(\mu_{PC}/M_{PC})(a/b/a_0)^2$ | 0.0204         | 0.0157          | 0.0111   | 0.0089           |

The body sizes are diameters 2$a$, 2$b$, 2$c$ where $a$, $b$, $c$ are body semi-major axes. Sizes, orbital periods and spin rotation periods are from Table 2 by Weaver et al. (2016). The radii of the equivalent volume sphere are computed from the body semi-major axes as $R_v = (abc)^{1/3}$. Here, satellite obliquity, $\epsilon_B$, is given with respect to Pluto/Charon’s north (spin and orbital axes), has errors of about $10^\circ$ and are measured by Weaver et al. (2016). The spin angular rotation rates $\omega$ are computed from the spin periods and are given in units of $t_o$ for a density of 1 g/cc (see equation 8). The mass ratio $M/M_{PC}$ is given in units of the total mass of the Pluto-Charon binary and computed using preferred values from Table 1 by Showalter & Hamilton (2015). The ratio of the spin orbital period to the period of the Pluto-Charon binary $P_o/P_C$ is computed using the orbital rotation periods listed here for the minor satellites and the orbital period for Charon in Table 9 by Brozovic et al. (2015). The orbital semi-major axes are those listed in Table 2 by Weaver et al. (2016). Energy densities, $\epsilon_B$, are computed using equation 4 and assuming a density of 1 g/cc. Love numbers are computed using equation 5, the shear modulus of ice $\mu_s = 4$ GPa, the volumetric radii listed here and assuming a density of 1 g/cc. Tidal spin down times are computed using equation 7, $Q = 100$, and the volumetric radii, spin periods, orbital semi-major axes and $k_2$ values listed here. Asphericity, $\alpha$ and oblateness parameters $q_{eff}$ are computed using equations 12, and 15 and the body axis ratios listed in the table. The ratio $(\mu_{PC}/M_{PC})(a/b/a_0)^2$ is computed using the semi-major axes listed here and the masses and semi-major axis listed in Table 1.

equation 1 by Gladman et al. (1996) the tidally induced precession rate

$$\dot{\Omega}_s \approx \frac{S}{\omega} \cos \epsilon_o, \quad (13)$$

where $\epsilon_o$ is the obliquity (angle between spin axis and orbit normal) and

$$S \equiv \frac{3}{2} \epsilon_o^2 C - (A + B)/2 \quad C.$$  \quad (14)

When the body is spinning about a principal body axis, the angles $\Omega_s, \epsilon_o$ are Euler angles and the precession rate $\dot{\Omega}_s$ is the rate the spin vector precesses about the orbit normal. It is convenient to define a parameter related to an effective oblateness of the body averaged over its spin (when spinning about the maximum principal axis)

$$q_{eff} \equiv \frac{C - (A + B)/2}{1 + (b/a)^2}$$

where we have used ellipsoidal body semi-major axis ratios $a, b, c$. The parameter $q_{eff}$ for each of Pluto and Charon’s minor satellites are also listed in Table 2 and range from 0.2 (Nix) to 0.4 (Hydra). Using $S$ and $q_{eff}$ the body spin precession rate divided by the orbit mean motion

$$\frac{\Omega_s}{n_o} \approx \frac{3}{2} q_{eff} \frac{n_o}{\omega} \cos \epsilon_o \approx \frac{3}{2} \frac{P_s}{P_o} \cos \epsilon_o$$

and on the right we have written the precession rate in terms of the ratio of spin and orbit periods where the spin period $P_s = 2\pi/\omega$ and the orbital period $P_o \approx 2\pi/n_o$. At low obliquity and for $q_{eff} = 0.3$, the precession rate is rapid, at $\frac{\Omega_s}{n_o} \approx 1.2 \frac{\pi}{n_o}$. For a spinning body orbiting about a binary rather than a point mass, equation 16 likely underestimates the precession rate by a factor that depends on the quadrupole moment of the averaged binary gravitational potential. This moment is given in the next subsection and is small for Pluto and Charon’s satellites (only 1–2% of the monopole) so equation 16 is a pretty good estimate.

The spin precession rate is independent of body size or $R_o/a_o$. For numerical convenience we can adjust the spin in units of $t_o$. As long as we maintain the period ratios (orbital to binary to spin) the secular frequencies are not modified.

2.5 Orbit Precession Frequencies

Resonance location often depends on secular frequencies such as the precession rate of the longitude of the ascending node. The gravitational potential of the binary averaged over its orbit is similar to an oblate body. It has a quadrupole gravitational moment giving an effective gravitational har-
monic coefficient

\[ J_{2,f.b}^{\text{eff,B}} = \frac{1}{2} \frac{\mu_B}{M_B} \]  

(17)

(see problem 6.3 by Murray & Dermott 1999) where \( \mu_B \) is the reduced mass of the binary and \( M_B \) is the total mass of the binary. Inserting this \( J_2 \) into equations for orbital precession frequencies (Greenberg 1981) and taking lowest order terms

\[ n \approx n_o \left[ 1 + \frac{3}{8} \left( \frac{a_B}{a_o} \right)^2 \frac{\mu_B}{M_B} \right] \]  

(18)

\[ \dot{\Omega}_o = -n_o \frac{3}{4} \left( \frac{a_B}{a_o} \right)^2 \frac{\mu_B}{M_B} \]  

(19)

\[ \dot{\varpi}_o = n_o \frac{3}{4} \left( \frac{a_B}{a_o} \right)^2 \frac{\mu_B}{M_B} \]  

(20)

with

\[ n_o \equiv \sqrt{\frac{GM_B}{a_o^3}}. \]  

(21)

Here \( \dot{\Omega}_o \) is the precession rate for the longitude of the ascending node, \( \dot{\varpi}_o \), and \( \dot{\varpi}_o \) is the apsidal precession rate or the precession for the longitude of pericenter, \( \varpi_o \). We have subscripted orbital parameters and frequencies with an \( o \) so as to differentiate them from spin related quantities, but we also use the subscript to denote quantities that are based on osculating orbital elements. Here \( n \) is intended to approximate the sidereal mean motion whereas the osculating \( n_o \) is dependent on the osculating semi-major axis \( a_o \). For a particle in the binary orbital plane, the orbital period is computed from \( n \) not \( n_o \). The osculating orbital semi-major axis, \( a_o \), is computed for a particle in the binary plane assuming a Keplerian orbit and with respect to the mass and center of mass of the binary (see discussion by Greenberg 1981 and Renner & Sicardy 2006). As the orbital period and orbital precession frequencies depend on the ratio \( \frac{\mu_B}{M_B} \left( \frac{a_B}{a_o} \right)^2 \) we have computed this for the four minor satellites using values for the binary in Table 1 and semi-major axes of the satellites in Table 2 and we include the computed values for this ratio at the bottom of Table 2.

3 SIMULATIONS OF THE SPINNING MINOR SATELLITES

We begin by describing our simulation technique. Two types of simulations are carried out, simulations with tidal dissipation alone (discussed in section 4) and simulations with a slowly drifting apart central binary (discussed in section 5). In both settings we track the spin and orbital evolution of a spinning satellite in orbit about a binary that represents Pluto and Charon.

3.1 Description of mass-spring model simulations

To simulate tidal viscoelastic response of non-spherical bodies we use a mass-spring model (Quillen et al. 2016a; Frouard et al. 2016; Quillen et al. 2016b) that is built on the modular N-body code rebound (Rein & Liu 2012). Springs between mass nodes are damped and so the spring network approximates the behavior of a Kelvin-Voigt viscoelastic solid with Poisson ratio of 1/4 (Kot et al. 2015). Frouard et al. (2016); Quillen et al. (2016b) considered a binary in a circular orbit. One of the masses was a spinning body resolved with masses and springs. The other body (the tidal perturber) was modeled as a point mass. Here we consider three bodies, a spinning body resolved with masses and spring in orbit about a binary comprised of two point masses (representing Pluto and Charon); see Figure 1. The total binary mass is \( M_B \) and the ratio of the smaller to larger mass in the binary, \( q_B \). For our simulations we set the mass ratio \( q_B = 0.12 \) to be equal to the Charon to Pluto mass ratio. The total binary mass is set to \( 10^8 \) in units of \( M \) and is about the right order of magnitude for the ratio of a minor satellite to the sum of Pluto and Charon’s mass, though Nix and Hydra are more massive than Kerberos and all three more massive than Styx (see Table 2 for mass ratios). Each simulation only tracks three masses, the binary and the resolved spinning body, so our simulations neglect dynamical interactions between the minor satellites themselves and tidal interaction between Pluto and Charon.

The mass particles in the resolved spinning body are subjected to three types of forces: the gravitational forces acting on every pair of particles in the body and with two massive point mass companions, and the elastic and damping spring forces acting only between sufficiently close particle pairs. Springs have a spring constant \( k_s \) and a damping rate parameter \( \gamma_s \). When a large number of particles is used to resolve the spinning body the mass-spring model behaves like a continuum solid. The number density of springs, spring constants and spring lengths set the shear modulus,
whereas the spring damping rate, $\gamma_s$, allows one to adjust the shear viscosity, $\eta$, and viscoelastic relaxation time, $\tau_{relax} = \eta/\mu$. The tidal frequency in units of the relaxation time $\chi = | \sigma | / \tau_{relax}$ (see section 2.3 by Frouard et al. 2016) and for $\chi < 1$, the quality function for our mass-spring model has $k_2 | \sigma | \approx k_2 \chi$. In our simulations we chose $\gamma$ so as to remain in the linear regime where the quality function is proportional to $\chi$ (and approximately giving a constant time lag tidal dissipation model).

Our previous studies (Frouard et al. 2016; Quillen et al. 2016b) were restricted to bodies spinning about their principal axes and with parallel spin and orbital axes. Here we allow the spinning object to have an initial non-zero obliquity. We measure the tilt of the spinning body in two ways. The obliquity $\epsilon_B$ is the angle between the spin angular momentum vector of the resolved body and its orbit normal (the direction of orbital angular momentum). The orbit normal and orbital elements for the spinning body such as its inclination, eccentricity and semi-major axis, $a_o$, are measured with respect to the center of mass of the Pluto-Charon binary (and using the vector between the center of mass of the spinning body and the center of mass of the binary). The obliquity $\epsilon_B$ is the angle between the spin angular momentum vector of the resolved body and the binary orbit normal. The binary’s mean motion, $n_B$, and semi-major axis, $a_B$, are computed neglecting the much lower mass spinning body. The orbital semi-major axis is computed using the total mass of the binary and coordinates measured from the center of mass of the binary. The two sets of coordinates are essentially Jacobi coordinates (see Figure 1).

In our previous studies, we measured instantaneous tidal torques and so we resolved the spinning body with numerous particles. Here we aim to explore longer timescale behavior. Instead of maintaining or increasing the number of particles in the resolved body, we decrease it. Rather than a thousand or more particles in the resolved body we typically have only 40. The number is not fixed as particle positions are randomly generated for each simulation. A simulation snapshot is shown in Figure 2. The small number of particles or mass nodes allows us to run for many thousands of spin rotation periods. Frouard et al. (2016) measured a 30% difference between the drift rate computed from the simulations and that computed analytically. We do not try to resolve this discrepancy here but instead study the long timescale evolution of spin and obliquity with the goal of understanding processes that have affected minor satellite obliquity.

We work with mass in units of $M$ , the mass of the spinning body, distances in units of volumetric radius, $R_v$, the radius of a spherical body with the same volume, time in units of $t_g$ (equation 8) and elastic modulus $E$ in units of $\epsilon_B$ (equation 4) which scales with the gravitational energy density or central pressure. Initial node distribution and spring network are those of the triaxial ellipsoid random spring model described by Quillen et al. (2016b). For the random spring model, particle positions are drawn from an isotropic uniform distribution but only accepted into the spring network if they are within the surface bounding a triaxial ellipsoid, $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$, and if they are more distant than $d_f$ from every other previously generated particle. Here $a, b, c$ are the body’s semi-major axes. Once the particle positions have been generated, every pair of particles within $d_f$ of each other are connected with a single spring. Springs are initiated at their rest lengths. The body is initially a triaxial ellipsoid and if it were not rotating it would remain a triaxial ellipsoid. The ellipticity of the body is not due to its rotation. The body Young’s modulus, $E$, is computed using equation 29 by Frouard et al. (2016) and using the entire triaxial ellipsoid volume (bounded by $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$), however with only 40 or so particles this value is approximate. With only about 40 particles and about 400 springs, the spinning body does not well approximate a continuum solid with its material properties. However our simulation technique can show spin-orbit resonance capture and tumbling and it accurately computes time dependent torques arising from the Pluto-Charon binary. A fairly low value of the Young’s modulus (in units of $\epsilon_B$) was used so that the body was soft, reducing the integration time required to see tidal drifts in orbital semi-major axis, spin and obliquity due to dissipation in the springs. The viscoelastic relaxation timescale and associated tidal frequency $\chi$ are computed as done previously (Frouard et al. 2016; Quillen et al. 2016b). Each simulation required a few hours of computation time on a 2.4 GHz Intel Core 2 Duo from 2010.

### Table 3. Common simulation parameters

| Binary mass $M_B$ | $10^6$ |
|-------------------|--------|
| Time step $dt$    | 0.004  |
| Simulation outputs $t_{print}$ | 100 |
| Total integration time $T_{int}$ | $1.3 \times 10^6$ |
| Minimum particle spacing $d_f$ | 0.47 |
| Maximum spring length $d_s$ | 2.55$d_f$ |
| Spring constant $k_s$ | 0.05 |
| Number of particles $N$ | $\sim 40$ |
| Number of springs $N_g$ | $\sim 375$ |
| Young’s modulus $E/\epsilon_B$ | $0.6$ |

Notes. Mass and volume of the spinning body are the same for all simulations. Distances $d_s$ and $d_f$ and spring constant $k_2$ are used to generate the random spring network (see Quillen et al. 2016b). $N$ and $N_g$ refer to the number of particle nodes and springs in the resolved spinning body and vary by a few between simulations as the initial particle positions are generated randomly. Points in our subsequent figures are separated in time by $t_{print}$.

### Table 4. Simulation parameters for each simulated satellite

| Simulated satellite | Styx | Nix | Kerberos |
|---------------------|------|-----|----------|
| Body axis ratio $b/a$ | 0.56 | 0.70 | 0.53 |
| Body axis ratio $c/a$ | 0.47 | 0.67 | 0.47 |
| Initial spin $\omega_{i\text{init}}$ | 0.5 | 0.72 | 0.5 |
| Initial orbital semi-major axis $a_o$ | 600 | 830 | 600 |
| Initial mean motion $n_o$ | 0.068 | 0.042 | 0.068 |
| Initial orbital period $P_{o\text{init}}$ | 93.0 | 151.1 | 92.7 |

The orbital period was computed using equation 18 and takes into account the quadrupole moment of the binary. The mean motion is computed without correction and is based solely on the osculating semi-major axis $a_o$ (equation 21).
Table 5. Parameters for simulations with tidal dissipation alone

| Simulation name | Styx-t1 | Styx-t2 | Styx-t3 | Styx-t4 | Nix-t1 | Nix-t2 | Ker-t1 | Ker-t2 |
|-----------------|---------|---------|---------|---------|--------|--------|--------|--------|
| Spring damping rate | $\gamma_s$ | 0.01 | 0.01 | 0.04 | 0.01 | 0.1 | 0.1 | 0.03 | 0.1 |
| Tidal frequency | $\bar{\chi}$ | 0.002 | 0.002 | 0.002 | 0.002 | 0.04 | 0.04 | 0.002 | 0.02 |
| Initial binary semi-major axis | $q_B$ | 290 | 290 | 185 | 305 | 340 | 326 | 206 | 206 |
| Binary mass ratio | $q_B^*$ | 0.12 | $10^{-9}$ | 1 | 0.12 | 0.12 | 0.12 | 0.12 | 0.12 |

Initial obliquities for these simulations is $\epsilon_{\text{init}} = 20^\circ$.

3.1.1 Description of Simulation Figures

We begin with simulations intended to be near current orbital period and spin rates and with spinning body exhibiting tidal spin-down due to dissipation in our simulated springs. Common simulations parameters listed in Table 3. Our simulations are labelled according to which body is simulated and parameters that depend upon which body is simulated are listed in Table 4. The different body semi-major axis ratios, appropriate for Styx, Nix and Kerberos are the same as in Table 2. The simulations have different initial spins and semi-major axes. The spin values are chosen so the initial spin to orbit periods are similar to (but slightly higher than) the values listed in Table 2 and measured by Weaver et al. (2016). As time is given in units of $t_g$, Table 4 also lists the initial orbital period of the minor satellite (the resolved spinning body) about the center of mass of the binary.

We set the orbital semi-major axes in units of minor satellite radius $R_o$, smaller than the actual ratios so as to decrease the timescale required for tidal evolution to take place and allow us to see long timescale phenomena associated with spin-down in the simulations. This means that the spin rates in units of $t_g^{-1}$ are larger than their actual values though we approximately maintain the ratio of spin and orbital period. Because $w/n_o$ is larger for Nix than for Kerberos and Styx, the Nix simulations required larger initial semi-major axis (in units of $R_o$) so as to keep the ratios $w/n_o$ and $n_o/n_B$ similar to the actual values for this satellites. We don’t simulate Hydra because of its high actual spin value. To maintain its spin to orbit period rate and keep its spin below breakup we would require a large semi-major axis and much longer numerical integration times.

Parameters that differ for simulations with tidal dissipation alone are listed in Table 5. Dissipation is set by the spring damping parameter, $\gamma_s$, and adjusted so that the tidal frequency $\bar{\chi}$ (listed in Table 5) is less than 1, so as to remain in a linear regime where the quality function is proportional to $\bar{\chi}$ (a constant time lag regime, but not constant dissipation Q or constant phase lag regime). The Styx simulations have initial orbital period to binary period ratio about 3, that for Nix about 4 and those for Kerberos about 5.

In Figure 3 we plot quantities as a function of time measured from the outputs of a simulation for Styx. Points are plotted each simulation output, $(t_{\text{print}}$ in Table 3). The horizontal axes for each panel are the same and in units of $t_g$. Orbital elements for the spinning satellite are computed using the initial orientation of the binary as a fixed reference frame (with the binary orbital plane defining zero inclination) and using the mass of the binary. We compute the osculating orbital elements at each simulation output using the center of mass position and velocity of the spinning satellite measured with respect to the center of mass of the binary. Orbital inclination in degrees is shown in the third (from top)-left panel, and orbital eccentricity is shown on the second-right panel of the figure. The top right panel shows the ratio of the orbital period (for the spinning body) divided by that of the binary.

The angular momentum of the spinning body, $L_s$, is computed at each simulation output by summing the angular momentum of each particle node, using node positions and velocities measured with respect to the center of mass of the body. The moment of inertia matrix of the spinning body, in the fixed reference frame, $I$, is similarly computed. The instantaneous spin vector $w$ is computed by multiplying the spin angular momentum vector by the inverse of the moment of inertia matrix, $w = I^{-1}L_s$. The spin $w = |w|$ is the magnitude of this spin vector and shown divided by the osculating orbital mean motion, $n_o$, in the second-left panel. Black horizontal lines show the location of spin-orbit resonances on the same panel. The spin itself is also shown in the third-right panel (in units of $t_g$) with the location of the nearest integer multiple of the binary mean motion, $n_B$, in brown.

The dot product of the spinning body’s spin angular momentum vector and current orbit normal gives $\cos \epsilon_o$ where $\epsilon_o$ is an obliquity with respect to the orbit normal. This is shown with black dots in the top, left panel. The obliquity computed from the dot product of the spin angular momentum vector and binary orbit normal, $\epsilon_B$, is plotted with blue dots on the same panel. A comparison of the two obliquities can determine whether the object is in a Cassini state where the body’s spin precession rate matches that of the longitude of the ascending node, $\Omega_s \sim \Omega_o$.

At each simulation output we diagonalize the moment of inertia matrix and identify the eigenvectors (directions) associated with each principal body axis. The angle between the body’s spin angular momentum and the direction of the the maximum principal axis in degrees is shown with red dots on the bottom right panel. Maximum here refers to the maximum eigenvalue of the moment of inertia matrix rather than maximum body semi-major axis. This angle, $J$, is sometimes called the non-principal angle and it is one of the Andoyer-Deprit variables (Celletti 2010). The angle between the spin angular momentum and the direction of the
Figure 3. Tidal evolution for Styx and showing the Styx-t1 simulation with parameters listed in Tables 3, 4 and 5. Each sub-panel shows evolution of a quantity. The horizontal axes are the same and in units of time $t_g$ or about 2000 s. The orbital period of the spinning body is listed in Table 5 and is about 90. The top left panel shows obliquity evolution of the spinning body. The black dots show obliquity measured with respect to the body’s orbit normal. The blue dots show obliquity measured with respect to the orbit normal of the binary. The second from top left panel shows the body spin divided by the orbital mean motion with black horizontal lines giving the location of spin-orbit resonances. The third-left panel shows orbit inclination. The bottom left panel shows the spin precession rate (in red) and the precession rate of the longitude of the ascending node of the orbit (in blue) both divided by $n_o$. The top right panel shows the ratio of the orbital period (for the spinning body) divided by that of the binary. The second panel (from top) on the right shows orbital eccentricity. The third right panel shows body spin (in black) and the brown line on this plot shows the location of the nearest commensurability with the binary, where $w$ is an integer multiple of $n_B$. Also plotted in this panel (in orange) is the component of the spin vector in the direction of the maximum principal axis. This only differs from the spin when the body is tumbling. The bottom right panel shows in red the non-principal angle $J$ (the angle between spin angular momentum and the axis of the body’s maximum principal axis of inertia) and in green the angle between spin angular momentum and the minimum principal body axis. Both angles are in degrees and they only differ from 0 or 90° when the body is tumbling. This simulation illustrates intermittent obliquity variations, episodes of tumbling and a small jump in spin associated with crossing a spin-spin resonance where $w \sim 2n_B$

At each simulation output we compute the osculating orbital longitude of the ascending node, $\Omega_o$. The time derivative of this (computed from the simulation output differences) is used to compute instantaneous measurements of the orbit precession rate $\dot{\Omega}_o$. The precession rate divided by the initial mean motion $\dot{\Omega}_o/n_{o,init}$ is plotted with blue dots in the bottom left panel where $n_{o,init}$ is the osculating mean motion at the beginning of the simulation. At each simulation output the body spin angular momentum is projected into the initial orbital plane of the binary and the time derivative of the angle in this plane used to compute the spin precession rate $\dot{\Omega}_s$. The angle $\Omega_s$ is that between the x-axis and the spin angular momentum $L_s$ vector after projection into the binary orbital plane, $\Omega_s = \text{atan2}((L_s \cdot \hat{y}), (L_s \cdot \hat{x}))$.

The minimum principal axis is plotted on the same panel with green dots. When the body is spinning about the principal axis $J = 0$ and the other angle is 90°. Only when the body is tumbling do these angles differ from 0 and 90°. Also plotted in the third-right panel (in orange) is the component of the spin vector in the direction of the maximum principal axis. Only when the satellite is spinning about the maximum principal axis is this equal to the spin.
This simulation illustrates that at the spin rate of Nix, spin-orbit resonances do not affect the spin or obliquity of Nix.

where $x, y$ coordinates span the binary’s orbital plane. The spin precession rate divided by $n_{\text{init}}$ is plotted with red dots in the same panel as $\Omega_s$. When the two precession rates coincide, the body is in a Cassini state. The body spin precession rate curve (red line in bottom left panel) usually resembles the obliquity trajectory because the body precession rate is sensitive to obliquity, though $|\dot{\Omega}_s|$ also slowly increases as the body spins down (see equation 16).

When the body is spinning about a principal body axis, the angles $\Omega_s, \epsilon_B$ are Euler angles in a coordinate system defined by the binary orbital plane and using the initial binary orientation to give the $x$ direction.

4 TIDAL SPIN DOWN ALONE

In Figures 3 - 5 we show simulations for tidal evolution (spin-down) for Styx, Nix and Kerberos. The spinning bodies are begun at an obliquity near $20^\circ$ and at small orbital inclinations of a few degrees. Tidal dissipation, set by the spring damping rate $\gamma_s$, is chosen to be low enough that the bodies do not spin down completely. Nix has a higher spin with $w/n_o \sim 14$. Figure 4a shows that Nix crosses spin-orbit resonances without much affect on body spin or obliquity (see second from top left panel). Neither is a spin-spin resonance with the binary important (see third from top right panel). Obliquity oscillations arise due to precession of the spinning body with respect to the quadrupole potential of the binary, the small but non-zero orbital inclinations and the proximity to the 4:1 resonance with the binary.

4.1 Styx’s obliquity intermittency

With spin rate lower than for Nix, Styx (see Figure 3) might be affected by passage through spin-orbit resonances. The ratio of spin to mean motion exhibits kinks (see second left panel) near spin-orbit resonances. The body also experiences episodes of tumbling (see bottom two right panels) though these might be due to resonant perturbation exciting a tumbling or nutation frequency rather than a spin-orbit resonance or an instability associated with a secular spin resonance.

Obliquity variations are intermittent in the Styx-t1 simulation. The regime of weak resonance overlap is often associated with intermittent chaotic behavior. By comparing this simulation to similar ones, we attempt to identify the
Figure 5. Tidal evolution for Kerberos and showing a) the Ker-t1 and b) the Ker-t2 simulations. These simulations differ only in the level of their tidal dissipation. Similar to Figure 3 and with parameters listed in Tables 3 - 5. The Ker-t1 simulation (figure a) illustrates a temporary spin resonance capture that also causes some tumbling. The Ker-t2 simulation (figure b) illustrates a jump in spin due to crossing the $w \sim n_B$ spin-spin resonance and a long lived spin-orbit resonance capture that causes the obliquity to rise. This spin orbit resonance $w/n_o \sim 4$ is at a spin value well below that of Kerberos’s current value.
source of the chaotic behavior. We first checked that a similar simulation but starting at lower obliquity shows same phenomena as the Styx-t1 simulation. It does but it takes longer to reach high obliquity.

Does the binary play a role in causing Styx’s obliquity intermittency? A simulation with identical parameters but with an extremely low mass binary (the Styx-t2 simulation with mass ratio $q_B = 10^{-9}$) does not show intermittent obliquity variations and the simulation lacks kinks in the spin decay slope as spin-orbit resonances are crossed.

The binary also affects secular precession frequencies so perhaps the weak spin-orbit and spin-spin resonances and strong obliquity variations imply that that multiple secular resonances affecting spin are responsible for Styx’s high current obliquity. To test this possibility we the Styx-t3 simulation, similar to the Styx-t1 simulation but with a higher binary mass ratio $q_B = 1$ and a smaller binary semi-major axis so that binary quadrupole moment is the same as in the Styx-t1 simulation. The binary is more compact in the Styx-t3 simulation and $n_B$ by about twice that of the Styx-t1 simulation. This simulation was dull, lacking chaotic behavior. Spin-orbit resonances when crossed did not affect spin or body orientation, though because the binary period was shorter, the spin-spin resonance with $w \sim n_B$ was noticeable; it caused a jump in spin as it was crossed. We conclude that secular perturbations alone (due to only the quadrupole term) do not account for the obliquity intermittency in the Styx-t1 simulation and that frequency of the binary perturbations or proximity to the mean motion resonance are important.

We note that the period to binary ratio for Styx is near $3$. Two degree oscillations in the orbital inclination are apparent in the third from top panel in Figure 3 that we attribute to proximity to the 3:1 orbital resonance. The 3:1 resonance has $3n_o \sim n_B$. Expansion of the disturbing function gives a series of arguments that depend on $3\lambda_o - \lambda_B$ and also include some angles $\varpi_B$, $\varpi_o$, $\Omega_B$, $\Omega_o$ (e.g., Murray & Dermott 1999). Here $\lambda_B, \varpi_B, \Omega_B$ are the mean longitude, longitude of pericenter and longitude of the ascending node of the binary and $\lambda_o, \varpi_o, \Omega_o$ are similar orbital elements but for the satellite. The binary induced $J_2$ causes $\Omega_o$ to be negative and $\varpi_o$ to be positive (see equations 19, 20). The resonance term with argument

$$\phi_{a2} = 3\lambda_o - \lambda_B - 2\Omega_o$$

is sometimes called the $I^2$ resonance as it is second order in orbital inclination (e.g., section 8.12 of Murray & Dermott 1999). It has frequency $\dot{\phi} = 3n_o - n_B - 2\dot{\Omega}_o$ giving commensurability where

$$\frac{P_s}{P_B} \sim 3 - 2\frac{\dot{\Omega}_o}{n_o}.$$  \hspace{1cm} (23)

As $\dot{\Omega}_o < 0$ this implies that the inclination sensitive resonant subterm is encountered at $P_s/P_B > 3$, whereas as $\dot{\varpi}_o > 0$ the eccentricity subterms are encountered at $P_s/P_B < 3$.

Styx, with $P_s/P_B \approx 3.15$, is near the inclination sensitive part of the resonance as $\dot{\Omega}_o/n_o \sim 0.015$ (using equation 19 and Styx’s value for $\mu_B/M_B(a_B/a_o)^2$ from Table 2). The oscillations in orbital inclination in Figure 3 are from that resonance.

Are the intermittent obliquity variations caused by proximity to the 3:1 resonance, and in particular proximity to the inclination sensitive part of the resonance and so on the sign of $P_s/P_B - 3$? We ran an additional simulation Styx-t4, similar to the Styx-t1 simulation but with a somewhat wider binary. This simulation has larger eccentricity oscillations because the system is outside the 3:1 resonance, rather than inside it. $P_s/P_B - 3$ is negative rather than positive, and so the satellite is near the eccentricity sensitive region of the resonance rather than the inclination sensitive region. The trajectory of the spin/mean-motion ratio exhibits some waveness but there are no strong obliquity variations. We also ran a simulation identical to the Styx-t1 simulation but starting with zero orbital inclination. This simulation exhibits few degree variations in inclination, consistent with proximity to the inclination sensitive part of the 3:1 mean motion resonance but lacks intermittency and large swings in the obliquity. The obliquity intermittency seen in the Styx-t1 simulation requires proximity to the inclination sensitive parts of the 3:1 orbital resonance with the binary and a few degrees of orbital inclination.

### 4.2 Kerberos

In terms of the ratio $w/n_o$, Kerberos is spinning about as fast as Styx. We show a simulation for its spin-down evolution in Figure 5. Two simulations are shown, the first (Ker-t1) with a slower rate of tidal dissipation than the second. The Ker-t1 simulation (Figure 5a) shows a short spin resonance capture in the middle the simulation, evident from the leveling of $w/n_o$ in the second-left panel at $t \sim 1.8 \times 10^6$. During the event, the body started tumbling (see bottom right two panels) and the body was not stable in attitude; the obliquity increased by about $5^\circ$. Inspection of third right panel shows that the spin itself was not exactly constant, in fact the spin increased while the obliquity increased. Also plotted in the third-right panel (in orange) is the component of the spin vector in the direction of the maximum principal axis. While the spin increases, this component on average is level, implying that a resonance maintained the magnitude of this component rather than the total spin. The tumbling episode appears to be associated with a spin-orbit resonance as this occurred at $w/n_o \sim 6$. However, as we will discuss below this resonance should be extremely weak due to the low orbital eccentricity. Possibly the event was associated with another type of commensurability, involving spin or tumbling, precession rate and orbital period.

The spin down rate was higher in the Ker-t2 simulation (Figure 5b) and the spin dropped to $w/n_o = 4$ where it was captured into spin orbit resonance. This spin value is much lower than Kerbero’s current spin rate, but we include the simulation here to illustrate a long lived spin-orbit resonance capture event where the obliquity increased significantly, in this case to $60^\circ$. Also interesting is the kink in the spin at $t \sim 3 \times 10^5$ where the body crossed a spin-spin resonance (see third-right panel). Kerbero’s spin rate is similar to the binary mean motion and this is the strongest of the spin-spin resonances. These two simulations show that long lived spin-orbit resonance capture (and associated large obliquity variation) are unlikely for $w/n_o \sim 6$ at Kerbero’s current spin value.

Because the spin to mean motion ratio $w/n_o$ for Styx and Kerberos are similar, the Styx and Kerberos simulations listed in Table 5 have almost identical parameters. The body
axis ratios are also similar. The primary difference between the Styx-t1 and Ker-t1, t2 simulations is in the binary semi-major axis (compared to the orbital semi-major axis) placing Styx near the 3:1 mean motion resonance and Kerberos near the 5:1 resonance. Relative to \( R_\star \), the size of the spinning body, the binary semi-major axis is larger for the Styx simulations than the Kerberos simulations and so the binary quadrupole moment is larger and the binary is a stronger perturber. The secular precession frequencies (in the longitude of ascending node) induced by the binary quadrupole also differ in the two simulations. We ran a series of simulations for Kerberos varying the binary semi-major axis ratio and proximity to and side of the 5:1 resonance, but none exhibited the obliquity intermittent variations of the Styx-t1 simulation.

### 4.3 More on Nix

The simulation Nix-t1 has orbit to binary period ratio slightly smaller than 4, similar to that observed. In contrast the orbit to binary period ratio of Styx is greater than 3, and we suspect that obliquity variation in the Styx-t1 simulation is related to the proximity of the 3:1 inclination resonance with the binary. We are curious to see if a Nix simulation on the other side of the 4:1 resonance might exhibit larger obliquity variation. The Nix-t2 simulation is similar to the Nix-t1 simulation but has a smaller binary semi-major axis. This simulation exhibits obliquity oscillations of about 6 degrees in amplitude that seem to be coupled with variations in inclination. However large swings in obliquity are not seen and spin-orbit and spin-spin resonances have no visible effect on the satellite spin when they are crossed.

### 4.4 Spin-orbit resonances

Inspection of the second from top and left most panels in Figures 3 - Figure 5 shows that spin-orbit resonances predominantly do not cause large jumps in spin. At the high spin rates of Pluto’s minor satellites the spinning body is not likely to be captured into one of them. We describe \( \vartheta \) as an angle describing the orientation of the satellite’s long body axis an angle \( \gamma = \vartheta - p \mu \omega \), with \( \mu \omega \) the orbital mean anomaly and \( p \) a half integer the equation of motion for \( \gamma \) is

\[
\frac{d^2 \gamma}{dt^2} + \frac{n_0^2 \alpha^2}{2} \sum_p H(p, e) \sin(2\gamma) = T
\]  

(Goldreich & Peale 1968; Wisdom et al. 1984) where \( T \) is the time averaged torque from tides (averaged over the orbit) and \( \alpha \) is the asphericity defined in equation 12. The coefficient \( H(p, e) \) is a power series in orbital eccentricity \( e \) and for \( p > 1 \), the coefficient \( H(p, e) = O(e^{p-2}) \) (Cayley 1859; also see Celletti 2010 Table 5.1). The \( p \)-th spin orbit resonance width can be described with the frequency

\[
\omega_{so,p} = n_0 \alpha \sqrt{H(p, e)}.
\]  

(Wisdom et al. 1984). This frequency also characterizes the size of a jump in spin for a system crossing the resonance and influences the likelihood of resonance capture. If \( p \) is large and the eccentricity is low then the resonance is narrow and weak and the system unlikely to be captured into resonance.

Pluto and Charon’s minor satellites have \( w/n_0 \) spin to mean motion ratios greater than 6 requiring half integer index \( p > 12 \) and resonance widths \( \propto e^{p/2} \) or higher. We can attribute the unimportance of the spin-orbit resonances to the high satellite spins requiring high orders in eccentricity for spin-orbit resonance strength making them weak.

### 4.5 Spin-Spin Resonances

Spin-spin resonances, where the body rotation rate is commensurate with the binary mean motion (Batygin & Morbidelli 2015) depend on an interaction between the binary and the spinning body. The quadrupole term in the gravitational potential of the binary is of order \( G\mu_B 2^2 \) at distance \( r \) from the binary’s center of mass. Here \( \mu_B \) is the reduced mass of the binary and \( a_B \) the binary separation. A term dependent on the relative alignment of binary and body orientation angles in an expansion of the gravitational potential energy is

\[
U_{ss} = \frac{A_{ss}}{2} \cos(2(\vartheta - n_B t)),
\]  

with amplitude

\[
A_{ss} \sim G\mu_B \left( \frac{a_B}{r} \right)^2 \frac{B - A}{r^3} \sim n_0^2 \frac{\mu_B}{a_B} \left( \frac{a_B}{r} \right)^2 (B - A),
\]  

(see equation 7 by Batygin & Morbidelli 2015) and where we have used \( B - A \) as the size-scale of the spinning body’s moments of inertia. The size-scale for \( A_{ss} \) arises from a product of the quadrupole moment tensor of the binary and the quadrupole moment tensor of the spinning body. This expression should be generalized to include additional angles (all three Andoyer-Deprit angles), however the complexity of the gravitational potential \( (\text{Ashenberg 2007; Boué 2016}) \) makes this a daunting prospect. Taking an approximate equation of motion for the angle \( \gamma = \vartheta - n_B t \)

\[
C\ddot{\gamma} + A_{ss} \sin(2\gamma) = 0,
\]  

with \( C \) the smallest moment of inertia giving a libration frequency of order

\[
\omega_{ss,1} \sim n_0 \alpha \sqrt{\frac{\mu_B}{M_B} \frac{a_B}{a_0}} \sim n_0 \alpha \sqrt{\frac{\mu_B}{M_B} \left( \frac{P_B}{P_0} \right)^2}
\]  

in the 1:1 spin/spin resonance where \( w \sim n_B \). Here \( P_B \) is the binary orbital period. This libration frequency (and resonance strength) does not depend on orbital eccentricity, as do spin-orbit resonances, and falls only weakly with orbital radius or semi-major axis. As a consequence spin-spin resonances might be fairly strong in the Pluto-Charon system.

Pluto and Charon’s minor satellites have asphericity \( \alpha \sim 1 \) and \( \mu_B/M_B \sim 0.1 \) giving \( \omega_{ss,1}/n_0 \sim 0.1 \), far exceeding the high \( p \) spin-orbit resonance strengths that depend on high powers of the eccentricity. Kerberos currently has ratio of binary to spin period of 1.2 and so is near this particular resonance and Figure 5b shows evidence of this resonance affecting satellite spin with a small jump in spin as this resonance was crossed. The jump in spin was about \( \Delta \omega \sim 0.02 \), at \( w \sim 0.33 \) and \( w/n_0 \sim 5 \) giving \( \Delta \omega \sim 0.02 \times 5/0.33 \sim 0.3 \). The jump size in spin should be and is approximately the same size as the estimated libration frequency, as expected.
Higher order spin-spin resonances would arise from octupole and higher terms in an expansion of the potential energy for the interaction between Pluto-Charon binary and spinning satellite body. Each higher order term decays more quickly with radius, having one higher power of $a_B/r$. A $k:1$ spin-spin resonance with spin $w \sim k \Omega_B$ would have libration frequency of order
\[
\omega_{ss,k} \sim n_B \alpha \sqrt{\frac{M_B}{M_H}} \left( \frac{a_B}{a_S} \right)^{\frac{k+1}{2}}.
\]  
(30)

The increased sensitivity to distance from the binary makes the higher order spin-spin resonances weaker but perhaps not as weak as high index spin-orbit resonances that depend on high powers of orbital eccentricity. The Nix simulation Nix-t1 shown in Figure 4 shows the body crossing a 4:1 resonance with $w \sim 4 \Omega_B$ but with no effect on any quantity we measured during the simulation. The 2:1 spin-spin resonance with $w \sim 2 \Omega_B$ might have affected Styx’s spin at $t \sim 2 \times 10^6$ in the simulation Styx-t1 with a small jump in spin at that time (see Figure 3, third-right panel).

5 SIMULATIONS WITH AN OUTWARDS MIGRATING BINARY

With tidal dissipation alone and a moderate level of spin-down we can only explain the high obliquity of Styx. We now explore the possibility that the satellites could have been captured into mean-motion resonance due to outward migration of Charon with respect to Pluto. Slow separation of the Pluto-Charon binary could have been caused by tidal interaction between Pluto and Charon (Farinella et al. 1979). Mean motion resonant capture can also take place if a minor satellite migrates inward, and this could have occurred via interaction with a circumstellar disk.

To migrate the binary (slowly separate Pluto and Charon) we apply small velocity kicks to each body in the binary using the recipe for migration given in equations 8-11 by Beaugé et al. (2006). The kicks are applied so as to keep the center of mass velocity fixed. The migration rate, $\dot{a}_B$, depends on an exponential timescale $\tau_n$ (the parameter $A$ in the equation 9 by Beaugé et al. 2006). The migration rate depends on $\tau_n^{-1}$ with $\dot{a}_B \sim a_B \tau_n^{-1}$. We adopt a convention $\tau_n > 0$ corresponding to outward migration which allows an external minor satellite to be captured into mean-motion resonance.

We ran a series of simulations with a migrating binary and with parameters listed in Tables 3, 4, and 6 that are shown in Figures 6 – 7. Initial conditions are similar to those listed in Table 5 except the initial binary semi-major axis is smaller so as to let the binary approach the current satellite values. The body axis ratios are identical, but the dissipation in the spinning body is lower, and the initial obliquities are $5^\circ$.

Figure 6 – 7 show that as the binary separates, the minor satellite captured into mean motion resonance. In the Styx-b1,b2 simulations it is the 3:1 mean motion resonance and initially only the inclination increases, whereas for the Nix simulations the mean motion resonance is the 4:1 and both eccentricity and inclination increase. The 4:1 resonance is a third order resonance (in eccentricity) and its lowest order resonant arguments all contain the longitude of pericenter, $\omega$ (see the appendices by Murray & Dermott 1999). All resonant subterms affect the eccentricity. The 3:1 resonance is second order and does contain subterms with arguments that lack the longitude of pericenter and so one of these causes an increase in the inclination and not the eccentricity. In both cases the minor satellite obliquity is lifted to high values, near $90^\circ$. The lift in obliquity for Nix is particularly interesting because it is a mechanism for lifting obliquity that functions even at Nix’s high spin rate. The mechanism also works for Styx even though we found previously that Styx can undergo intermittent obliquity without mean motion resonance capture.

We ran similar simulations for Kerberos drifting the binary apart, but none of our simulations illustrated capture into 5:1 mean motion resonance. Kerberos is more easily captured into mean motion resonance by Nix and that could have lifted its orbital inclination. We suspect that a similar obliquity lifting mechanism might work for Kerberos but it would involve at least four 4 bodies; Nix to capture Kerberos into mean motion resonance and a simultaneous commensurability with Kerberos’s spin precession rate and the Pluto-Charon binary to lift Kerberos’s obliquity.

5.1 Obliquity increase near mean-motion resonance

Because Nix is spinning more rapidly than Styx, its spin precession rate, $\dot{\Omega}_s$, is closer to $\dot{\Omega}_s$, its precession rate of the longitude of the ascending node. The Nix-t1 simulation (Figure 4a) begins with the spinning body in a Cassini state with $\Omega_n \approx \Omega_s$ and the system remains in a Cassini state except for a brief period near $t = 1.7 \times 10^6$. That $\Omega_n \approx \Omega_s$ means that in mean motion resonance the spin precession rate lines up with the mean motion resonant angle and this coupled with the binary inclination is likely to account for the large obliquity variations. In resonance the binary perturbations are in phase with the tilt angle of the body. A similar simulation but with an initial obliquity of $20^\circ$, the Nix-t2 simulation, (Figure 7b) starts outside Cassini state ($\Omega_n \neq \Omega_s$) but moves into it after mean-motion resonance capture, probably because of the orbital inclination increase caused by the mean motion resonance. In that simulation the satellite exits mean motion resonance leaving the body in a state similar to that at the end of the Nix-b1 simulation (Figure 7a) with high obliquity oscillations.

The Styx-b1 simulation (Figure 6a) also begins in a Cassini state. As the system approaches mean motion resonance there is a large increase in obliquity. Mean motion resonance is entered about $t = 0.8 \times 10^6$ whereas the obliquity increase begins at about $t = 0.6 \times 10^6$ at which time the satellite also exits Cassini state. Outside of the Cassini state the body spin precession rate is about 3 times higher (in amplitude) than the rate of precession of the longitude of the ascending node. As the mean motion resonance is approached the body spin precession rate would be commensurate with the mean motion resonance resonant angle before the longitude of ascending node. In other words $2\Omega_n \sim 3\Omega_n - n_B$ prior to $2\Omega_n \sim 3n_B$. The increase in obliquity prior to entering the mean motion resonance is more clearly seen in the Styx-b2 simulation (Figure 6b) as it only enters Cassini state after capture into the mean motion resonance.

We suspect that the obliquity increases prior to enter-
ing mean motion resonance are due to a commensurability involving the mean motion resonance and Euler angle $\Omega$. To check this possibility we plot inclination and obliquity for the Styx simulations along with resonant angles in Figure 8 for simulations Styx-b1 and Styx-b2. In these figures three resonant arguments are plotted:

$$\phi_1 = 3\lambda - \lambda_B - \Omega_s - \Omega_o$$
$$\phi_2 = 3\lambda - \lambda_B - 2\Omega_o$$
$$\phi_3 = 3\lambda - \lambda_B - 2\Omega_o,$$

where $\lambda$, $\lambda_B$ are the mean longitude of satellite and binary, respectively, $\Omega$ is the longitude of the ascending node of the satellite and $\Omega$ is the precession angle of the spinning satellite. The last of these arguments, $\phi_3$, is the argument associated with the $I^2$ (inclination squared) part of the 3:1 mean motion resonance. The top two arguments involve the precession angle of the satellite. Figure 8a, showing the Styx-b1 simulation, shows $\phi_1$ freezing (or librating about a fixed value) during the same time period that the obliquity increases, whereas Figure 8b showing the Styx-b2 simulation, shows $\phi_2$ freezing (librating about 0) during the time period that the obliquity freezes. The freezing of these angles in the simulations suggests that these resonances are responsible for the large obliquity variations. In both Styx-b1,b2 simulations, $\phi_3$, associated with the mean motion resonances, is only librating when the orbital inclination increases and after the obliquity has reached a high value.

The simulations shown in Figures 6 - 8 with an outward drifting binary suggest that obliquity variation is associated with an increase in orbital inclination and proximity or capture into mean motion resonance. However the obliquity increases tend to take place just before entering resonance implying that a commensurability between mean motion resonance and spin-precession frequency is responsible. Since this type of resonance is associated with the spin precession we could call it a three-body secular resonance, except with such elongated bodies as Pluto’s minor satellites the spin precession frequency at low obliquity is not particular slow. And since such a commensurability involves a mean motion resonance, in terms of its orbital properties it is not secular (it depends on mean longitudes which are usually averaged for secular resonances). The resonance (involving spin precession and mean motions) might be important precisely because the spin precession frequencies are fast.

We lack a model for resonances with arguments given by $\phi_1$, $\phi_2$ in equation 31, but we can estimate a timescale associated with obliquity increase in resonance. We suspect that a torque in resonance would be similar to the torque from a spin-spin resonance (see equation 27 and based on equation 7 by Batygin & Morbidelli 2015). The torque should be a few times lower because we must average the effect of the spin-spin resonance over the orbit while in mean motion resonance and it should depend on the obliquity. Ignoring these dependencies the torque is of order $T \sim \eta B^2 \frac{\mu B}{M} \frac{\Omega}{a_0^2}$. $I$ with moment of inertia $I$. We estimate a timescale for obliquity change with $t_{obl} \sim \frac{I}{w/T}$ giving

$$t_{obl\eta} \sim \frac{w}{\eta o} \left[ \frac{\mu B}{M} \left( \frac{\eta B}{a_0} \right)^2 \right]^{-1}. \tag{32}$$

Taking $w/\eta o \sim 6$ and $\frac{\mu B}{M} \left( \frac{\eta B}{a_0} \right)^2 \sim 0.01$ (from the bottom of Table 2) we estimate a timescale for a large obliquity change $t_{obl\eta} \sim 600$ or about 100 orbital periods. In units of $t_B$ (our simulations) an orbital period is about 100 giving $t_{obl\eta} \sim 10^4$. The timescale for obliquity change seen for Styx (see Figures 6) is about $10^5$ and for Nix, with higher spins, is slower a few times $10^5$ (see Figures 7). This an order of magnitude higher than estimated with equation 32. The discrepancy is comfortably wide, wide enough that drift within resonance, reduction in strength from averaging over fast angles and body angular orientation (obliquity) and inclination dependence can probably be taken into account still giving the resonance enough strength to lift the obliquity. The comparison suggests that this type of resonance is capable of lifting the obliquities on the timescales seen in the simulations.

Our simulations do not exhibit obliquity $\epsilon_B$, with respect to the binary orbit, greater than $90^\circ$, corresponding to retrograde spin. Styx and Kerberos have near $90^\circ$ obliquities whereas Nix and Hydra have higher obliquities of $125^\circ$ and $110^\circ$, respectively. We notice from Table 2 that Styx and Kerberos have period ratio $P_j/P_C > j > 0$ where $j$ is the nearest integer (3 and 5 respectively), whereas Nix and Hydra have period ratio subtracted by the nearest integer $j$ (4 and 6 respectively) less than zero. Styx and Kerberos have $j\dot{\eta}_s > \eta B$ whereas $j\eta_o < \eta B$ for Nix and Hydra. The two satellites with the highest, and retrograde obliquities also have positive $j\dot{\eta}_s - \eta B$. Perhaps there is a connection between the obliquity and the side of the orbital resonance. With retrograde spin ($\epsilon_B > 90^\circ$), the spin precession rate $\Omega_s > 0$ rather than negative (as is true for Styx and Kerberos). Thus Nix and Hydra could be near commensurability with fixed or librating resonant argument $\phi_2$ or $\phi_1$.

With initial conditions at low obliquity, our simulations did not ever exhibit retrograde obliquities, but perhaps with additional orbital migration retrograde spins could be in-

### Table 6. Parameters for simulations with a migrating binary

| Simulation name | Styx-b1 | Styx-b2 | Nix-b1 | Nix-b2 |
|-----------------|---------|---------|--------|--------|
| Spring damping rate $\gamma_s$ | 0.001 | 0.001 | 0.1 | 0.1 |
| Tidal frequency $\tilde{\chi}$ | $2 \times 10^{-4}$ | $2 \times 10^{-4}$ | 0.04 | 0.04 |
| Initial obliquity $\epsilon_{init}$ | $5^\circ$ | $20^\circ$ | $5^\circ$ | $20^\circ$ |
| Initial binary semi-major axis $a_B$ | 275 | 275 | 324 | 324 |

For these simulations the binary mass ratio $q_B = 0.12$ and the binary semi-major axis drift rate $\tau_a^{-1} = 5 \times 10^{-8}$. 

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duced. The one simulation where the system leaves mean motion resonance and crosses to the side of mean motion resonance that Nix and Hydra are on is the Nix-b2 simulation (Figure 7b). However, in this simulation, the satellite did not stay in spin resonance when exiting the mean motion resonance though the satellite did stay in a Cassini state. The spin state exhibits high obliquity swings and perhaps further evolution could induce retrograde spin, or there may be a diversity of ways that the body can exit the mean motion resonance in the full N-body system including all minor satellites.

We reexamine the intermittent obliquity variations shown in simulation Styx-t1 (Figure 3) lacking binary migration. The obliquity panel (top left) shows that when the obliquity is high, the body is in a Cassini state and when the obliquity is low it is not in one. When the obliquity is low, the precession rate is high enough that it might be commensurate with the 3:1 mean motion resonant angle. To see if this is the case we created a figure similar to Figure 8 but for the Styx-t1 simulation. Resonant angles $\phi_{1},\phi_{2}$ show librating regions and these do occur during obliquity variations. We might attribute the intermittent chaotic obliquity evolution to perturbations from multiple terms involving the mean motion resonance and the body’s spin precession angle.

6 SUMMARY AND DISCUSSION

Estimates of tidal spin-down time suggest that none of Pluto and Charon’s minor satellites have had time during the age of the Solar system to reach near spin-synchronous states and this is consistent with the observed spin rates (Weaver et al. 2016). Mass spring model simulations with tidal dissipation (but allowing only moderate spin-down) show that only minor changes in minor satellite obliquity are caused by crossing spin orbit resonances, though our simulation method does exhibit spin-orbit resonance capture at lower spin rates. The high spin rates make the spin-orbit resonances sensitive to orbital eccentricity to a high power and so very weak. Spin-spin resonances with the binary depend on the binary quadrupole moment and so could remain strong at the high spin rates. Nevertheless, only small jumps in spin are seen when crossing these and with dozens of simulations we have found that even temporary capture into them is rare. Only Styx experiences large and intermittent obliquity variations when evolving tidally. Proximity to the 3:1 mean motion resonance and few degree orbital inclination seem to be required for Styx to show large and intermittent obliquity variations.

Simulations allowing the Pluto-Charon binary to slowly drift apart cause Styx to be captured into 3:1 mean motion resonance with the binary and Nix to be captured into 4:1 resonance. Inclination sensitive parts of these resonances are encountered first, and these increase the orbital inclination. The satellite obliquities are lifted to near 90° either on approach to or in mean motion resonance, depending upon whether the satellite is in a Cassini state or not. We suspect that the obliquity increases are caused by a commensurability between the mean motion resonance argument frequency and the satellite spin precession rate. This resonance is likely because the satellites are sufficiently elongated that the spin precession rates at low obliquity are fairly fast. Re-examination of the Styx simulation showing intermittent obliquity variations suggests that this type of resonance could contribute to Styx’s chaotic behavior. The mechanism for lifting obliquity, involving mean motion resonance and spin precession, functions for both Nix and Styx even though Nix is spinning much faster than Styx.

We have explored only 3 body integrations, the binary and a single resolved elongated spinning body. Our mechanism lifting Styx and Nix’s obliquities was not effective in our simulations for Kerberos that failed to capture Kerberos into 5:1 mean motion resonance. Simulations involving 4 or more satellites might succeed in lifting Kerberos’s obliquity with a similar mechanism. Kerberos’s orbital inclination could be increased via capture into 5:4 resonance with Nix and its obliquity lifted at the same time. Since this mechanism is not strongly dependent on the satellite spin rate, it may also work for the more rapidly spinning Hydra, perhaps via a 3:2 mean motion resonance with Nix. As all four satellites are near mean motion resonances, a mechanism that involves mean motion resonances and lifts obliquities (via matching spin precession) could operate effectively on all four minor satellites perhaps explaining why all of them are near 90°.

Styx and Kerberos, both inside mean motion resonance with Charon, have lower obliquities than Nix and Hydra, that are outside of mean motion resonance with Charon. There may be a connection between the direction of spin, prograde or retrograde, and the side of mean motion resonance. While our simulations did not induce retrograde spins, perhaps later orbital migration or tidal evolution in the full N-body system (with all 4 minor satellites) could induce these spin end-states.

We started our simulations with minor satellite spinning along a principal body axis, however Kerberos’s tumbling decay timescale might be so long that it could experience orbital evolution before its wobble decays. In future we could explore spin evolution of initially tumbling states; perhaps Kerberos is more likely to capture into spin resonances if it is tumbling.

If capture into mean motion resonance is required to account for Pluto and Charon’s high minor satellite obliquities then we could infer that all of the minor satellites were previous captured into mean motion resonance. However this would conflict with dynamical studies of the orbital evolution showing that this causes instability (Cheung et al. 2014). Perhaps migration, resonance capture and associated obliquity increases could have taken place when a circumbinary disk was still present that could damp inclinations and eccentricities and stabilize the orbits – or the system may have actually experienced episodes of instability and reformation.

We lack simple dynamical models for phenomena seen in our simulations, such as excitation of tumbling. The complexity of the potential (dependence on at least 3 angles) for quadrupole/quadrupole body gravitational interactions (Ashenberg 2007; Boué 2016) implies that constructing a more general model for spin-spin resonance (beyond Batygin & Morbidelli 2015; Jafari Nadoushan & Assadian 2016) that includes obliquity and tumbling would be challenging. Though we suspect a spin-precession/mean motion reso-
Figure 6. Simulations of Styx with a slowly separating binary. a) the Styx-b1 simulation starting at an obliquity of 5°. b) the Styx-b2 simulation, starting at an obliquity of 20°. Simulation parameters are listed in Tables 3, 4 and 6. Slow separation of the binary captures the minor satellite into 3:1 mean motion resonance which lifts the orbital inclination. The obliquities are lifted to near 90°.
Figure 7. Simulations of Nix with a slowly separating binary. a) the Nix-b1 simulation starting at an obliquity of $5^\circ$. b) the Nix-b2 simulation, starting at an obliquity of $20^\circ$. Slow separation of the binary captures the minor satellite into 4:1 mean motion resonance which lifts the orbital inclinations and obliquities. Simulation parameters are listed in Tables 3, 4 and 6.
nance mechanism for obliquity increase, we lack a dynamical model that would allow us to assess the resonance strength.

Our mechanism seems to require past orbital misalignments with non-zero but few degree satellite orbital inclinations with respect to the binary. Future observations or ongoing analysis of New Horizons observations should determine whether such inclinations are currently ruled out. After escape from resonance in the Nix-b2 simulation (Figure 7b) Nix exhibits large obliquity swings from near 0 to near 90° at a period of about \( t = 10^5 \) or corresponding to order 10^3 orbital periods or a few years in real time. If Nix or Styx is currently in such a spin state, large obliquity variations might be observed.

The obliquity lifting mechanism seen here involves fast precession rates (due to body) elongation and mean motion resonance with a massive binary. It would be interesting to explore other settings where a similar resonance might operate. Uranus is more nearly spherical but in the past might have been in mean motion resonance with Saturn or Jupiter. Perhaps Uranus’s high obliquity could have been lifted because of its past interaction with mean motion resonance, as seen here, rather than due to secular resonance (Rogoszinski & Hamilton 2016) or collisions (e.g., Parisi & Brunini 1997).

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Figure 9. Resonant arguments for the Styx-t1 simulation with tidal dissipation but lacking binary migration. Similar to Figure 8 but for the simulation shown in Figure 3. Intermittent obliquity evolution may be caused by resonances involving arguments $\phi_{s1} = 3\lambda_o - \lambda_B - \Omega_s - \Omega_o$ and $\phi_{s2} = 3\lambda_o - \lambda_B - 2\Omega_s$, shown in the second and third panels.

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