Numerical Simulation Research Based on Plasma Line-Tied Instability

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Abstract: In order to study the linear instability and turbulence of the boundary plasma, this paper uses the BOUT++ numerical simulation tool. Mainly analyze the driving mechanism of the instability of the ideal balloon model, and the instability suppression mechanism of the ion diamagnetic effect, and compare with the analysis results of the dispersion relationship. Subsequently, the integral dispersion relationship is used to analyze the suppression mechanism of the shear flow. In addition, the dispersion relationship is not suitable for analyzing the global effect of the shear flow. Due to the locality of the dispersion relationship, the integral dispersion relationship uses the numerical integration of the mode structure to resolve the dispersion relationship. Used to analyze shear flow. Then, use the numerical integration of kinetic energy in the whole space to study the contribution of these effects to free energy. Finally, using the above linear analysis method, the physical mechanism of these effects under the EAST divertor configuration is studied.

1. Introduction
Station-based plasma instability has an important influence on the energy loss of ELM bursts. Non-linear simulation is a common method to study the instability of station-based instability. Non-linear simulation results are often combined with the dispersion relationship to analyze the instability [1]. However, the dispersion relationship can only qualitatively explain part of the physical effects in the nonlinear simulation. Therefore, this article expands the analysis method of the dispersion relationship. The dispersion relationship is solved by numerical integration of the mode structure, so as to obtain the contribution of different physical effects to the instability. In addition, the numerical integration of the modular structure is also used to calculate the kinetic energy of different physical items in the whole space, and then the ratio of the total kinetic energy to the growth rate of the energy form of the physical item is calculated, so as to analyze the contribution of different physical effects to energy.

In this paper, the three-field model of BOUT++ is used to simulate and analyze the instability of the platform-based plasma. A new numerical analysis method is proposed to study the instability of the ideal balloon model, as well as the influence of the diamagnetic effect and shear flow on the ideal balloon model. Numerical nonlinear analysis shows that the balloon mode instability is driven by the pressure gradient coupled curvature. The diamagnetic effect suppresses the balloon mode by suppressing the curvature drive. The hoop rotation can also suppress the curvature drive, and the strong shear rate leads to a strong suppression effect. In addition, the nonlinear analysis of the energy form shows that the curvature drive provides free energy, shear Alfven waves, diamagnetic effects and shear flow absorb free energy.
2. Integral Dispersion Relation
Before introducing the integral dispersion relationship, first introduce the simulation settings and the characteristics of the balanced configuration. In this paper, a three-field model is used to simulate the instability of platform-based plasma. Non-linear simulation uses circular configuration. This balance is based on JET parameters and is formed by TOQ code. Figure 1 shows the circular configuration balance pressure and safety factor profile. The pressure gradient of the circular platform base is relatively steep, and the balloon mold is relatively unstable, which is often used for theoretical analysis [2].

![Figure 1 Profile of equilibrium pressure and safety factor of circular configuration](image_url)

The dispersion relation is obtained by the Fourier transform of the time partial derivative ($\partial / \partial t$) of the disturbance variable and the spatial gradient operator (B), which represents the local nonlinear growth rate. Under normal circumstances, the dispersion relationship can qualitatively analyze the partial global simulation results, and for physics with strong global effects, consistent results may not be obtained [3]. Therefore, this paper proposes the integral dispersion relation to study the physics with strong global effect. In order to distinguish the integral dispersion relationship from the growth rate obtained from the simulation results, first introduce the calculation of the nonlinear growth rate in the simulation, and define the time Fourier transform form of the disturbance variable as:

$$P - \tilde{P}e^{\gamma t}$$  \hspace{1cm} (1)

$P$ is the perturbation pressure and $\tilde{P}$ is the initial perturbation. In the nonlinear simulation, the model increases exponentially. It is easy to obtain the calculation formula of the nonlinear growth rate in the simulation:

$$\gamma = \frac{\partial (\ln |P|)}{\partial t}$$  \hspace{1cm} (2)

After the simulation converges, the remaining variables $\sigma, A_l$ etc. can be substituted into the above formula to obtain the same growth rate.

The following describes the integral dispersion relation to calculate the nonlinear growth rate. First, write the simulated three-field model 2.18-2.20 in matrix form:
For the global simulation, after the simulation converges, all variables have the same non-linear growth rate, and the non-linear growth rate does not change with the spatial position [4]. Using this property, the Fourier transform of the above formula is performed in time, and the dispersion relation is solved by numerical integration of the modular structure in space, so as to obtain the growth rate of the integral dispersion relation. The expression is:

$$\begin{bmatrix}
\frac{\partial}{\partial t} 
\begin{pmatrix}
\sigma_m \\
P \\
A_m 
\end{pmatrix}
= \begin{pmatrix}
0 & 2h_0 \times \kappa \cdot \nabla & -B^2 \nabla B_{\parallel} \\
0 & 0 & 0 \\
-B_0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\sigma_m \\
P \\
A_m 
\end{pmatrix}
\end{bmatrix}$$

$$= \tilde{L} \begin{pmatrix}
\sigma_m \\
P \\
A_m 
\end{pmatrix}$$

(3)

The line-bending term also represents the shear Alfven wave. Figure 2 shows the nonlinear growth rate of the ideal balloon model. The total growth rate $\gamma'$ of the model obtained by this method is the sum of each growth rate $\gamma'_{\text{cur}} + \gamma'_{\text{LB}}$. Note that the contribution of the other two in formula 5 to the
growth rate is almost 0 [6]. The MINERVA code gets the contribution of each item to energy, but does not calculate the growth rate of each item. \( \gamma_{\text{cur}} \) is the curvature driving term. This coupled pressure gradient drives the balloon mode instability. From the figure, it can be seen that the growth rate of the balloon mode is mainly contributed by this contribution. \( \gamma_{\text{LB}} \) is the Line-bending term, which means that the Alfvén wave propagates along the lines of magnetic force, and this contributes little to the growth rate. In addition, the frequency of the curvature term and the line-bending term in the ideal balloon mode is 0, and the total mode frequency is also 0 [7].

\[
\begin{align*}
\gamma = & \gamma_{\text{cur}} + \gamma_{\text{LB}} \\
& \approx \gamma_{\text{cur}} + \gamma_{\text{LB}}
\end{align*}
\]

\[ (8) \]

\[ (9) \]

Dispersion relationship has been Fourier transformed the disturbance variable in time and space, reflecting the local nonlinear characteristics of a certain spatial position. Equation 9 means that the ideal balloon mode is driven by the curvature coupled pressure gradient, and the nonlinear growth rate increases as the toroidal modulus \( n \) increases, which is consistent with the growth rate driven by the curvature term in Fig. 2, which means the integral dispersion relationship and the dispersion relationship The results describing the balloon model are consistent [9].

3. **Nonlinear Analysis of the Influence of Diamagnetic Effect on Balloon Model**

In order to study the influence of the diamagnetic effect on the balloon mode, the ion diamagnetic term
is extracted from the vorticity, and the vorticity formula is redefined as:

\[
\frac{\partial \mathbf{\omega}}{\partial t} = \frac{n_m}{B_0} \nabla^2 (V \cdot \nabla P_e) + B^2 \nabla \left( \frac{J_e}{B} \right) + 2B \times \kappa \cdot \nabla P_e, \mathbf{\omega} = \frac{n_m}{B_0} \nabla^2 \phi
\]  

(10)

The growth rate of the diamagnetic term is:

\[
\gamma_{\text{diam}} = \int \frac{n_m}{B_0} \nabla^2 (V \cdot \nabla P_e) dV
\]

\[
\gamma_{\text{diam}} = \int \left( \sigma_s \sigma_n + P_e P_n + A_{\text{lin}} A_{\text{nlin}} \right) dV
\]  

(11)

Figure 3 shows the analysis result of the integral dispersion relationship after adding the diamagnetic effect.

\[\text{Figure 3 Non-linear comparison model}\]

In Figure 3(a), the growth rate (green line) of the mode after the diamagnetic effect is added is significantly smaller than that of the ideal balloon mode, so the diamagnetic effect can suppress the instability of the balloon mode. The ideal balloon model is mainly driven by curvature. After adding the diamagnetic effect, the contribution of the curvature drive term to the growth rate is close to 0, which means that the diamagnetic stabilization effect mainly suppresses the curvature drive. The contribution of the line-bending term to the growth rate is also close to 0, and the effect of this term
may be related to curvature driving. The contribution of the diamagnetic term to the growth rate is close to the total growth rate, which means that this item plays a major role in the simulation of the diamagnetic effect. The total growth rate $\gamma_{\text{cur}} + \gamma_{\text{th}} + \gamma_{\text{dia}}$ obtained by the integral dispersion relation is approximately equal to the growth rate $\gamma$ of the mode, which is similar to the analysis result of the ideal balloon mode. Figure 3(b) shows the real frequency of the diamagnetic effect. After the diamagnetic effect is added, the mode frequency is approximately the frequency of the diamagnetic term, and the frequency of the ideal balloon mode is 0, which means that the ion diamagnetic current drives the balloon mode to rotate, and the mode rotates. The direction of is the diamagnetic drift direction [10].

Comparing the difference between the results of the integral dispersion relationship and the dispersion relationship after the diamagnetic effect is added to the ideal balloon model, the dispersion relationship after adding the diamagnetic effect is:

$$\gamma = \frac{w_{\text{dia}}}{2i} + \sqrt{\gamma_{\text{ideal}}^2 - \frac{w_{\text{dia}}^2}{4}}$$ (12)

$$w_{\text{dia}} = -\frac{1}{B_0N_0e}B_0 \times \nabla P_{\text{io}}k_{\perp}$$ (13)

Fig. 3 The simulation of ideal balloon mode with diamagnetic effect, (a) the integral dispersion relationship obtains the nonlinear growth rate of different terms, in addition, $\gamma$ is the growth rate of the mode after adding diamagnetic effect, and $\gamma_{\text{ideal}}$ is the growth rate of ideal balloon mode. (b) The real frequency of each term in the diamagnetic effect.

Among them, $w_{\text{dia}}$ is the diamagnetic frequency. Formula 12 shows that the diamagnetic effect can reduce the growth rate $\gamma_{\text{ideal}}$ of the ideal balloon mode, which is consistent with the diamagnetic effect obtained from the integral dispersion relationship that suppresses the curvature drive. In addition, the real frequency produced by the diamagnetic effect is the ion diamagnetic frequency $w_{\text{dia}} / 2$, and the diamagnetic frequency is the frequency of the wave propagating along the diamagnetic drift direction (Equation 13). This is consistent with the contribution that the mode frequency is mainly the diamagnetic term in the integral dispersion relation.

4. Nonlinear Analysis of the Influence of Annular Rotation on Balloon Mode

BOUT++ simulation shows that hoop rotation can suppress mid-to-high modulus balloon modes, but it can stabilize low-n balloon modes, which is more obvious for low-n QH modes. The MINERVA code uses the dispersion relation and the energy principle to non-linearly analyze the influence of the hoop rotation in the JT-60U. The simulation shows that the destabilization effect of the hoop rotation is caused by the difference between the mode frequency and the hoop rotation frequency. In this paper, the integral dispersion relation is used to analyze the influence of hoop rotation, in which hoop rotation is generated by radial electric field, which is formed by radial transport, NBI injection and other factors. The expression of the net flow generated by the radial electric field in the Field-aligned coordinate system is:

$$v_0 = \frac{B_0}{B_R} \frac{d\phi}{dy} e^y - \frac{B_R R}{B_0 h_y} \frac{d\phi}{dy} e^y - \frac{d\phi}{dy} e^y$$ (3-14)

Among them, $\phi_0$ is the equilibrium electrostatic potential, and the flow velocity generated by the radial electric field can be divided into the velocity along the magnetic field line ($e^y$) and the velocity
in the ring direction ($\hat{e}_z$) in the Field-aligned coordinate system. In addition, according to the formula 2.79, $\hat{e}_z = R\hat{e}_z$, the angular velocity of the hoop rotation is:

$$\Omega = -\frac{d\phi_0}{dy} \text{ (rad / s)}$$

(15)

In order to study the influence of hoop rotation, construct the hoop rotation angular velocity:

$$\frac{d\phi_0}{dy} = 0.5D_s \left[1 - \tan h(D_s(x - x_0))\right] + c$$

(16)

Among them, $x$ is the normalized angular magnetic flux, $D_s$ determines the shear flow gradient, $D_0$ determines the flow amplitude, and $c = 300 \text{rad / s}$ is a constant.

Figure 4(a) is the annular rotation angular velocity profile, and Figure 4(b) is the simulation result of the parallel flow $v_{0y}$ or the annular flow $v_{0z}$ being added to the ideal balloon mold.

![Figure 4](image)

**Figure 4 Comparison of Circumferential Flow and Parallel Flow Models**

When there are circular flow and parallel flow, parallel flow has almost no effect on the nonlinear growth rate. As shown in Figure 4(b), when there is no circular rotation and only parallel flow, the parallel flow has almost no effect on the growth rate, only the parallel flow is increased by 10 times, and the ideal balloon mold has a destabilizing effect. Therefore, in the field-linked coordinate system,
the parallel flow formed by the radial electric field has a relatively small effect on the balloon mode.

Figure 4 (a) Angular velocity profile of hoop rotation (Equations 15 and 16), where \( D_\phi = 130 \text{krad/s} \) and \( D_x = 20 \). (b) The non-linear growth rate of the circular flow or parallel flow \( \nu_{by} \). The growth rate of 10 times the parallel flow is the non-linear growth rate of the ideal balloon model.

In the three-field model, the toroidal rotation only appears in the convection term \( \nu \cdot \nabla \). In addition, according to the fluid reduction, the parallel disturbance is much smaller than the vertical disturbance \( k_z \). The magnitude of each term in the convection term is estimated: \( \sum \) (17)

The first term is about one to two orders of magnitude smaller than the second term. Therefore, the parallel part \( \nu_{by} \) of the shear flow has almost no effect on the balloon mode, and it has a certain effect after multiplying by 10 times. Therefore, the hoop rotation in the \( E \times B \) shear flow plays a dominant role in the instability.

5. Conclusion
This paper introduces two numerical methods to analyze the influence of diamagnetic effect and shear flow on the ideal balloon model, and analyze the influence of different physical effects on the nonlinear growth rate. Based on the dispersion relation, the dispersion relation is solved by numerical integration of the whole space of the modulus structure, and the nonlinear growth rate of the modulus is equal to the sum of the growth rates of different physical items. Based on the disturbance kinetic energy in the whole space, the growth rate is calculated by the ratio of the energy contributed by different physical items to the total energy, and the modulus growth rate is also equal to the sum of the growth rates of each item. The two methods have their own characteristics. The integral dispersion relationship is based on mathematics, which can well reflect the contribution of different physical items to the growth rate and real frequency. The nonlinear analysis method of energy form is based on kinetic energy, which has obvious physical meaning and reflects the contribution of different physical mechanisms to energy. These two methods are suitable for the analysis of plasma instability based on EAST station.

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