Dynamic error analysis based on flexible shaft of wind turbine gearbox

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Abstract. In view of the asynchrony issue between excitation and response in the transmission system, a study on the system dynamic error caused by sun axis which suspended in the gear box of a 1.5MW wind turbine was carried out considering flexibility of components. Firstly, the numerical recursive model was established by using D'Alembert's principle, then an application of MATLAB was used to simulate and analyze the model which was verified by the equivalent system. The results show that the dynamic error is not only related to the inherent parameter of system but also the external load imposed on the system; the module value of dynamic error are represented as a linear superposition of synchronization error component and harmonic vibration component and the latter can cause a random fluctuations of the gears, However, the dynamic error could be compensated partly if the stiffness coefficient of the sun axis is increased, thereby it is beneficial to improve the stability and accuracy of transmission system.

1. Introduction
Most mechanical systems including shaft system are sensitive to operating environment such as manufacturing error, excessive applied torque or installation problems. The sun gear shaft is a critical set which transmits the torque between planetary gear stage and parallel stage in wind turbine gearbox. When the running speed of the mechanical accelerates, dynamic error of drive train system may occur, and the flexibility of the sun gear shaft cannot be ignored. The transmission error of the system is influenced by a variety of factors. That is the reason why there has been numerous researches works focusing on the planetary gear transmission system in recent decades. In these factors, the inherited parameters of the system were highly investigated due to their determinative roles in the error. So, research topics on the analysis of mesh stiffness [¹,²], natural modes [³], and transmission error [⁴] were mainly concentrated in the previously published documentations.

Most of the papers were to expand the study from the point of the static error and the structural dynamics. However, fewer published literatures are available for dynamic error caused by the axis elastic deformation from the view of institutional dynamics point. The nonlinear dynamic model of translation-torsion coupling of gearbox system was investigated by Lin [⁵] with this model and the transmission error of drive train system caused by instability parameters were discussed, similarly, the robot system is also more sensitive to transmission error, a method for measuring the dynamic error of
the flexible member of a robot was carried on by R M Qian[6]. For the stability and accuracy issues of transmission system required further study, kinetic model was considered more factors such as mesh stiffness, spacing and backlash-related nonlinear dynamics in recent years. An interactive method has been developed for analyzing dynamic loads in a lightweight basic planetary gear system by August[7], the effects of fixed, semi-floating, and fully floating sun gear conditions had been emphasized. [8] by taking the gear mesh and manufacturing error into consideration. And it was further refined by Q W Zhang [9] with taking the Load fluctuation into account. While for the dynamic error caused by the axis elastic deformation from the view of institutional dynamics, the analytical models are seldom found.

Based on above research, the main objective of this paper is to develop the recursive dynamics model on dynamic error of 1.5MW wind turbine gearbox considering the shafting elastic deformation based on the Lagrange equation, we introduced the dynamic error of elastic members and derived the mathematical model for dynamic error. This is expected to supply the theoretical information for the stable operation of the system.

2. Dynamics model

2.1. Model hypothesis

In order to consider the effect which caused by the torsional deformation of sun gear shaft on the output motion of high-speed shaft, we ignored the influence of any other factors, for the sake of completeness they are made here. The gear mesh model is a linear time-invariant model; Variable stiffness of tooth pairs in contact is assumed negligible and therefore the transmission ratio is constant; Drive shaft is deemed to be torsion spring and the quality of the shaft does not take into account; the flexibility of support in structure is neglected as well as any other possible damping in the system.

The parallel stage in wind turbine gearbox is a common tandem gear transmission. For easy to analyze the dynamic error caused by the torsional deformation of sun gear shaft, based on the above assumptions and the conservation laws of energy, making the conversion of the load and the moment of parallel shaft to the sun gear shaft, so that the tandem gear transmission system can be equivalent to a single-axis system(figure.1).

2.2. Kinematic analysis

The angular displacement of each shaft showed in figure 1, each gear engaged with a fixed speed ratio in the case of excluding teeth elastically deformable, by analyzing the law of motion of the mechanism only from the point of institutions geometric position. By selecting \( q_1 = \theta_1 \) and \( q_2 = \theta_2 \) as the vector of the generalized co-ordinates, the displacement of the sun is \( \theta_3 = (z'_2 + z_3) q_1 / z_3 \), where \( z'_2 \) and \( z_3 \) are the teeth number of the gear 2 and 3. Based on above relationship, the following calculation can further simplify the model of figure 1(a).

Calculated transmission ratio for each axis according to the number of teeth in meshing gear
Coordinate Transformation

\[ \theta'_{e4} = \frac{\omega'_{4}}{i_{II,I}}, \quad \theta'_{e5} = \frac{\omega'_{5}}{i_{III,I}} \]

Equivalent moment of inertia

\[ J_{e3} = J_3, \quad J_{e4} = J_4 + J_4' i_{II,J}^2, \quad J_{e5} = J_5 + J_5' i_{III,J}^2 \]

The generalized force

\[ Q_1 = T_1, \quad Q_2 = T_2 \cdot i_{III,I} \]

According to equivalent conversion, equivalent dynamic model of the transmission system is simplified as the form of figure 1 (b), and therefore the two series gear transmission system which composed of triaxial system simplifies uniaxial system.

2.3. To establish the error equation

2.3.1. Dynamic equations of system. Based on the equivalent dynamic model, the dynamic error equations are established. In this article the dynamic-static force principle is used to obtain the dynamic equations of drive train system of wind turbine. The dynamic equations of each module in the system are

\[ [J_s + 3J_2 + 3(1 - Z_2' / Z_s')J_s + (1 + Z_2' / Z_s')J_s] k_s (q_s - q_s) = Q_s \tag{1} \]

\[ [J_s + (J_s' + J_s) i_{II,J}^2 + J_4' i_{II,J}^2] k_s (q_s - q_s) = Q_s \tag{2} \]

2.3.2. Mathematical model of dynamic error. Assuming \( s(\theta) \) is the input displacement of gearbox system, and \( y \) is the output, according to the transmission ratio \( N \) of the gearbox, synchronism output of gearbox should be \( N \cdot s(\theta) \). However, it is inconsistent of mechanism between actual output and theory output because of elastic deformation of the sun gear shaft. Therefore the transmission error is generated. The dynamic error of the system is defined as the difference between the output \( y \) and the actual input \( s(\theta) \), so the dynamic error of the gearbox can be expressed as \( \delta = y - s(\theta) \).

It should keep the synchronous relationship between output and input under rigid condition, this implies that the error is zero; When the elastic deformation of sun gear shaft is took into account, the conversion of the dynamic error equation to a generalized coordinate system is

\[ \delta = q_s / i_{II,I} - i_{III,I} q_s \tag{3} \]

2.4. Parameter determination dynamic error calculation

2.4.1. Parameters of unit. By taking a 1.5MW wind turbine as the research object, the relevant parameters of the drive system are selected according to design parameters, rated Power 1.5MW, rotor diameter 70.5m, design speed of rotor 19.8 r/min, total ratio \( N=96.59 \) and the effective length of sun gear shaft 715mm, the parameters of the gear in each step are listed in Table 1.

| Gear | 2 | 2' | 3 | 4 | 4' | 5 | 5' |
|------|---|----|---|---|----|---|----|
| Module /m | 12 | 12 | 12 | 10 | 10 | 6 | 6 |
| Tooth number /z | 43 | 111 | 23 | 99 | 23 | 104 | 27 |
Electro-magnetic torque of generator is obtained by equation (4)

\[ T = \frac{9500 \cdot P}{n} \]  

(4)

Where \( P \) is the rated power of generator, \( n \) is the design speed. For rated power of wind turbine, aerodynamic moment of the rotor can be calculated by equation (5)

\[ d_s = \frac{1}{2} \rho v^2 r d_s (1 + \cot^2 \theta)(C_L \sin \theta - C_D \cos \theta) \]  

(5)

Where \( ds = t \cdot dt \), that is the product of blade chord with the thickness of span wise, \( \rho \) is the air density, \( v \) is the speed of the air flow, \( \theta \) is the inclination of blade element.

2.4.2. Stiffness coefficient \( k \) of sun gear shaft. The sun shaft has the structure of special shape pipes and hollow shaft. The stiffness coefficient of shaft can be calculated according to the calculated (6)

\[ \frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots \]  

(6)

2.4.3. Numerical solution for error. The mathematical model of the gearbox system of wind turbine has established involves two degree of freedom (DOF), the numerical method are often implicated to solve the numerical solution in engineering, thus analyzing the response of the internal structure of the system. For the differential equations (1) and (2) are coupled by two DOFs, by means of the four-order Runge-kutta's method, the numerical solution of \( q_1 \) and \( q_2 \) are calculated, then the result are introduced into the error equation (3), the dynamic error is obtained (figure.2).

By this token, the amplitude value of dynamic error which caused by the elastic deformation of the sun gear shaft simply became larger and larger with the passage of time, and the effect was more significant as the angular displacement of rotor increased. On the whole, the curve shows a form of free vibration, and its amplitude increases gradually.

![Dynamic error curve of system.](image)

**Figure 2.** Dynamic error curve of system.

3. Discussion and analysis

The dynamic response of the elastic member is often associated with the operation speed of mechanism, especially, if the error becomes systemic incentive, it will produce unpredictable vibration response and affect the performance of the transmission system. For finding out the basic reasons for causing the error, a simplification of system is made according to the law of conservation of energy, the sun and the former part are regard as driving member of system, and the latter are driven member, at the same time, an equivalent transfer of mass and moment of inertia of various components is made to sun gear shaft (figure.3).
3.1. Model hypothesis

In order to consider the effect of torsional deformation of sun gear shaft on the output motion of high-speed shaft ignoring the effect on the system of any other factors, for the sake of completeness they are in the list of assumptions made here.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3.png}
\caption{Model of equivalent system.}
\end{figure}

Where $J_e$ is the equivalent moment of inertia of system, $\lambda$ is the input of system, $\theta$ is the angular displacement of sun, $y(t)$ is the law of output motion of high-speed shaft. The differential equation describing the torsional vibrations of the equivalent system is

$$\ddot{y} + \frac{k_s}{J_e} y = \frac{1}{J_e} \dot{\lambda}(\theta)$$

(7)

By regarding $\dot{\lambda}(\theta)$ as the known quantity, the analytical solutions of equation (7) are derived and then used to analyze the relationship between system parameters and the dynamic error of system. The solution for $y(t)$ is

$$y(t) = -\sqrt{k_s J_e} \cdot \dot{\lambda}(\theta) \cdot \sin\left(\frac{k_s}{J_e} \cdot t\right) + \frac{1}{k_s} \lambda(\theta)$$

(8)

In order to convenient for analyzing the effect on the input and output of gearbox caused by elastic deformation of sun gear shaft, a coordinate transformation is carried out from original coordinate system to $\theta_1$, that is $\lambda(t) = i_{31} \omega \theta_1$. Therefore, the equation (8) is changed into (9)

$$y(t) = -\sqrt{k_s J_e} \cdot i_{31} \dot{\theta}_1 \cdot \sin\left(\sqrt{k_s / J_e} \cdot t\right) + (i_{31} / k_s) \dot{\theta}_1$$

(9)

After the coordinate transformation the dynamic error model can be expressed as

$$\delta(t) = \sqrt{k_s J_e} \cdot i_{31} \dot{\theta}_1 \cdot \sin\left(\sqrt{k_s / J_e} \cdot t\right) + (1 - i_{31} / k_s) \dot{\theta}_1$$

(10)

It can be seen, the dynamic error can be resolved into two linear superposition components: one can be named as synchronization error which is synchronized with the input $\theta_1$, and it can be expressed by the equation $\delta_1 = (1 - i_{31} / k_s) \cdot \dot{\theta}_1$, the change law of the synchronization error have a form of linear distribution is shown in figure.4 ($\delta_1$). The value of this part is affected by the drive ratio $i_{31}$ and $k_s$, the formulae $1 - i_{31} / k_s$ is the instantaneous rate of change. However, in some condition, the value of dynamic error can be minimized by optimizing the instantaneous rate of change. That is the elastic deformation of sun gear shaft can be ignored when the stiffness of sun gear shaft reaches to infinity, therefore, this component of dynamic error is synchronized with the input, and $\delta_1 = \dot{\theta}_1$. In other words, the synchronization error can be compensated through increasing the rigidity of shaft or optimizing the transmission form of sun gear shaft.

Another component of dynamic error can be expressed as a equation

$$\delta_2 = \sqrt{k_s J_e} \cdot i_{31} \cdot \omega \cdot \sin\left(\sqrt{k_s / J_e} \cdot t\right)$$

named free vibration error, the production of $\delta_2$ due to the free vibration of system natural frequency, that a simple harmonic motion which frequency is the natural frequency superimpose on the motion of original design is shown in figure.4 ($\delta_2$), where $\omega = \sqrt{k_s / J_e}$. Owing to $J_e$ is the moment of inertia of various components, so $\delta_2$ is the result of vibration system included sun gear shaft and the transmission parts of parallel shaft. Although its vibration form is manifested as free vibration caused
by impact load, but the vibration amplitude can be reduced due to the existence of damp. However, the vibration problem is difficult to solve because of the periodic motion of shaft and the random excitations. More importantly, the gear meshing impact may be induced by this vibration and it will affect the performance of the gearbox. Therefore, the kinetic parameters of system such as the moment of inertia and the structural stiffness reasonably. Furthermore, obtaining the system's optimization design.

![Simulation of dynamic error](image)

**Figure 4.** Simulation of dynamic error.

### 4. Conclusions

We introduce the dynamic error of elastic members; the model and numerical verification of sun gear shaft which caused by elastic deformation are discussed from the view of mechanism dynamics: 

Based on the D'Alembert's principle, the mathematical model of dynamic error is established, the type and the main influencing factors of dynamic error are also discussed. The results showed that the dynamic error is consists of synchronization error and free vibration error, the former is synchronized with the input of system, and the latter caused by impact load. On the other hand, the types of the dynamic error are determined by the structural parameters, and the external excitations determine the amplitude of error and how long it will work.

Aiming at the kinds of dynamic error, this paper analyzed the role of system parameters in dynamic error. It is helpful to compensate dynamic error and improve the control system.

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