Research on designing a multiloop planar linkage

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Abstract. The paper presents the way of determining the dimensions of the elements of a planar linkage when imposing certain conditions regarding the movement of a component piston, aiming at obtaining a mechanism whose total mass is minimal. A computer program that simulates the kinematics of the analyzed mechanism has been developed by the authors using Maple programming environment. The NLPSolve function included in the Optimization Package of Maple has been used for establishing the dimensions of the elements that assure the minimal mass of the linkage. Finally, some simulation results are presented.

1. Introduction

The design of linkages is a complex process that aims at establishing an optimal functional constructive solution that fulfils the conditions imposed by the task to be accomplished. Many times in such a process it starts from a certain structural configuration of the linkage and the dimensions of the component elements are determined by solving a problem of estimating the extreme values of a function that expresses certain characteristic parameters [1-6]. From a mathematical point of view, this implies calculating the minimum or maximum value of this function in the presence of certain constraints expressing the conditions imposed by the functioning of the mechanism.

In the paper is presented the way of determining the dimensions of the elements of a planar linkage, consisting of seven links and three independent contours, when imposing certain conditions regarding the movement of a component piston, aiming at obtaining a mechanism whose total mass is minimal. A computer program that simulates the kinematics of the analyzed mechanism has been developed by the authors using Maple programming environment [7-8]. The NLPSolve function included in the Optimization Package of Maple has been used for establishing the dimensions of the elements that assure the minimal mass of the linkage.

2. Theoretical considerations

Figure 1 shows the mechanism to be analyzed. As it can be seen from the associated graph, shown in figure 2, the mechanism consists of three independent contours: 0-1-2-3-0, 0-1-2-4-5-0 and 0-5-6-7-0. The kinematics of the mechanism has been studied by projecting the vector circuits corresponding to the independent contours on the axes of the (Oxy) coordinate system [9-10].

Thus, by projecting the vector equation: \( \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BO} = 0 \), corresponding to the first independent contour 0-1-2-3-0, on the axes (Ox) and (Oy), we obtain the following system of equations:

\[
\begin{align*}
I_1 \cos \phi_1 + I_2 \cos \phi_2 &= 0 \\
I_1 \sin \phi_1 + I_2 \sin \phi_2 - s_3 &= 0
\end{align*}
\] (1)
where: \( l_1 = OA \), \( l_2 = AB \) and \( s_3 = OB \).

From the system of equations (1), the angle \( \varphi_2 \) may be calculated from the equation:

\[
\cos \varphi_2 = -\frac{l_1}{l_2} \cos \varphi_1
\]

Then, the displacement \( s_3 \) may be determined with the relationship:

\[
s_3 = l_1 \sin \varphi_1 + l_2 \sin \varphi_2
\]

For the second contour 0-1-2-4-5-0, by projecting the corresponding vector equation: \( \overrightarrow{OA} + \overrightarrow{AC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EO} = 0 \) on the axes \( (Ox) \) and \( (Oy) \), the following system of equations is obtained:

\[
\begin{align*}
& l_1 \cos \varphi_1 + l_{21} \cos \varphi_2 + l_4 \cos \varphi_4 + l_{51} \cos \varphi_5 - x_E = 0 \\
& l_1 \sin \varphi_1 + l_{21} \sin \varphi_2 + l_4 \sin \varphi_4 + l_{51} \sin \varphi_5 - y_E = 0
\end{align*}
\]

where: \( l_{21} = AC \), \( l_4 = CD \), \( l_{51} = DE \) and \( x_E \) , and \( y_E \) are the coordinates of point \( E \).

By solving the system of equations (4), the angle \( \varphi_4 \) may be calculated from the equation:

\[
C \cdot \cos \varphi_4 + D \cdot \sin \varphi_4 = E
\]

where:

\[
\begin{align*}
C &= 2l_4(l_1 \cos \varphi_1 + l_{21} \cos \varphi_2 - x_E) \\
D &= 2l_4(l_1 \sin \varphi_1 + l_{21} \sin \varphi_2 - y_E) \\
E &= l_{51}^2 - l_4^2 - (l_1 \cos \varphi_1 + l_{21} \cos \varphi_2 - x_E)^2 - (l_1 \sin \varphi_1 + l_{21} \sin \varphi_2 - y_E)^2
\end{align*}
\]

Figure 1. Planar mechanism.
The angle $\varphi_3$ may be determined with the relationship:

$$\varphi_3 = \text{ATAN2}(-(l_4 \sin \varphi_4 + l_1 \sin \varphi_1 + l_{21} \sin \varphi_2 - y_E),-(l_4 \cos \varphi_4 + l_1 \cos \varphi_1 + l_{21} \cos \varphi_2 - x_E))$$  \hspace{1cm} (7)

where ATAN2($y,x$) calculates arctan($y/x$) by taking into account the signs of $x$ and $y$ [9].

**Figure 2.** The associated graph.

The vector equation: $\overrightarrow{EF} + \overrightarrow{FG} + \overrightarrow{GE} = 0$ corresponding to the third contour 0-5-6-7-0, projected on the axes ($Ox$) and ($Oy$), leads to obtaining the following system of equations:

$$\begin{cases} l_{52} \cos \varphi_5 + l_6 \cos \varphi_6 - s_7 = 0 \\ l_{52} \sin \varphi_5 + l_6 \sin \varphi_6 - (y_G - y_E) = 0 \end{cases}$$  \hspace{1cm} (8)

where: $l_{52} = EF$, $l_6 = FG$ and $s_7 = x_G - x_E$ ($x_G$ and $y_G$ are the coordinates of point $G$).

By solving the system of equations (8), the angle $\varphi_6$ may be calculated from the equation:

$$\sin \varphi_6 = -\frac{1}{l_6}(l_{52} \sin \varphi_5 - y_G + y_E)$$  \hspace{1cm} (9)

and the displacement $s_7$ may be determined with the relationship:

$$s_7 = l_{52} \cos \varphi_5 + l_6 \cos \varphi_6$$  \hspace{1cm} (10)

**3. Simulation results**

Using Maple programming environment [7-8] a computer program that simulates the kinematics of the analyzed mechanism has been developed. The initial dimensions of the component elements have the following values: $l_1 = 0.15$ m; $l_2 = 1.05$ m; $l_4 = 0.5$ m; $l_5 = 0.8$ m; $l_{21} = f_2 l_2$, where $f_2 = 0.2$ and $l_{51} = f_5 l_5$, where $f_5 = 0.5$. The coordinates of point $E$ are: $x_E = 0.55$ m and $y_E = 0.35$ m and the $y$ coordinate of point $G$ is $y_G = 0.5$ m.

The angular speed $\omega_1$ is considered to have the value of 15 rad/s. The elements 1, 2, 4, 5 and 6 (figure 1) are made of steel bars with circular section having the following radii: $r_1 = 0.04$ m; $r_2 = 0.025$ m; $r_6 = 0.04$ m; $r_5 = 0.03$ m and $r_6 = 0.025$ m. The mass of the piston 3 is 5 kg and the mass of the piston 7 is also 5 kg.

We analyzed several cases in which it has been imposed the value of the stroke of the piston 7 and the values of the crank angle $\varphi_1$ for which the speed of the piston 7 is null, provided the total mass of the mechanism be minimal.
Further are presented some simulation results in the case when it was sought to determine the lengths of the bar-shaped components such that the piston 7 have zero velocity for the positions of the mechanism corresponding to the crank angle $\varphi_1 = 30^\circ$ and $\varphi_1 = 230^\circ$ and its stroke $(s_7(230^\circ) - s_7(30^\circ))$ be equal to 0.3 m, provided that the total mass of the linkage be minimal.

In this scope has been used the NLPSolve function included in the Optimization Package of Maple. The NLPSolve function solves problems involving the minimization or maximization of a continuous real nonlinear objective function possibly subject to constraints.

The NLPSolve command uses various methods implemented in a built-in library and the solvers are iterative in nature and require an initial point. In this case, the Sequential Quadratic Programming method has been used and the iterative calculus has been started with the initial values of the lengths of the component elements.

The NLPSolve function interrupts the calculation when one of the constraints is not fulfilled or complex numbers appear during the calculations. When solution of the problem is not identified after the calculations, a message appears specifying that no improved point could be found.

To avoid the occurrence of complex numbers during computations, the variation of the lengths of the component elements, less than $l_4$, was considered to be between 0.98 and 1.04 of the values corresponding to the previous step of the iterative process.

For $l_4$ it was considered that its value may vary between 0.98 and 1.3 of the value corresponding to the previous step.

After five iterations the program displayed the message that no improved point could be found. The values obtained for the lengths of the component elements are: $l_1 = 0.138 \text{ m}$; $l_2 = 0.968 \text{ m}$; $l_3 = 0.595 \text{ m}$; $l_5 = 0.461 \text{ m}$; $l_6 = 0.738 \text{ m}$; $l_2 = f_2/l_3$, where $f_2 = 0.228$ and $l_5 = f_2/l_5$, where $f_5 = 0.475$. For the coordinates of point $E$ have been obtained the following values: $x_E = 0.511 \text{ m}$ and $y_E = 0.346 \text{ m}$ and for the $y$ coordinate of point $G$ the value: $y_G = 0.463 \text{ m}$. The total mass of the mechanism is 75.568 kg.

In figures 3, 4 and 5 are represented the variation of the displacement $s_7$, of the speed and of the acceleration of the piston 7, during a cinematic cycle, beginning with $\varphi_1 = 30^\circ$. The curves noted with 1 correspond to the initial mechanism and the curves 2 correspond to the mechanism obtained after the analyzed case.

![Figure 3. The variation of the displacement of the piston 7: curve 1 for the initial mechanism and curve 2 for the mechanism obtained after analysis.](image-url)
Figure 4. The variation of the speed of the piston 7: curve 1 for the initial mechanism and curve 2 for the mechanism obtained after analysis.

Figure 5. The variation of the acceleration of the piston 7: curve 1 for the initial mechanism and curve 2 for the mechanism obtained after analysis.

Figure 3 and figure 4 show that the imposed conditions for the stroke and for the speed of the piston 7 are fulfilled. Also, figure 4 and figure 5 show that the extreme values of the speed and of the acceleration of the piston 7 have not undergone significant changes as against the original situation.

4. Conclusions
In the paper a series of results concerning the synthesis of a planar linkage have been presented. In the development of the kinematics simulation program, the Maple programming environment has been used, which offers besides symbolic computing facilities also a package with extremely useful functions in optimization problems. In this sense, the NLPSolve function has proved to be extremely useful and easy to implement in the simulation program to solve the problem of optimizing the functioning of the mechanism studied in the paper. It is obvious that the way of realizing the synthesis of the analyzed linkage can be used in the case of any other planar linkage with different configuration. However, particular attention must be paid to choosing the intervals of variation of the values of the lengths of the component elements, as well as to the choice of the initial values of these lengths on each iteration.
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