In first-order phase transitions in the early universe, the bubble wall is expected to be significantly slowed-down by its interaction with the surrounding plasma. We examine the behaviour of the phase of the Higgs field after two-bubble collisions, and find that phase differences equilibrate much more quickly in slow-moving bubbles than in those which expand at the speed of light. This could lead to a significant reduction in the initial density of topological defects formed at a first-order phase transition.

1 Introduction

First-order phase transitions proceed by bubble nucleation and expansion. When three or more bubbles collide a phase winding of $2\pi n$ may be generated, forming a cosmic string in the region between them. In order to assess the cosmological significance of cosmic strings, it is important to be able to forecast their initial density. This depends on the behaviour of the phase of the Higgs field after bubbles collide and merge – in particular if the phase difference between two bubbles can equilibrate before the arrival of the crucial third bubble, there may be a strong suppression of the initial string density.

At any phase transition where particles acquire mass, those particles outside the bubble without enough energy to become massive inside bounce off of the bubble wall, retarding its progress through the plasma. The faster the bubble is moving, the greater the momentum transfer in each collision, and hence the stronger the retarding force. Thus a force proportional to the bubble-wall velocity appears in the effective equations of motion. Impeded by its interaction with the hot plasma, the bubble wall reaches a terminal velocity $v < c$ – for the (Standard Model) electroweak phase transition, the value $v \sim 0.1c$ was predicted. In this paper, we investigate the consequences of slow-moving bubble walls on phase equilibration in global- and local-symmetry models.
2 Phase Equilibration

Writing the Higgs field \( \Phi = \rho e^{i\theta} \) the equations of motion for the Abelian Higgs (\( U(1) \) gauge symmetry) model (which we consider, for simplicity) are

\[
\ddot{\rho} - \rho'' - (\partial_\mu \theta - eA_\mu)^2 \rho = -\frac{\partial V}{\partial \rho} \tag{1}
\]

\[
\partial^\mu \left[ \rho^2 (\partial_\mu \theta - eA_\mu) \right] = 0 \tag{2}
\]

\[
\ddot{A}_\nu - A_\nu'' - \partial_\nu (\partial \cdot A) = -2e\rho^2 \partial_\nu \theta. \tag{3}
\]

Taking, after Kibble and Vilenkin, our gauge-invariant phase difference between two points to be

\[
\Delta \theta = \int_B^A dx \left( \partial_\nu - ieA_\nu \right), \tag{4}
\]

it is possible to derive an analytic expression for the phase difference after time \( t \) between the centres of two bubbles nucleated at time zero with radius \( R \) and initial phase difference \( 2\theta_0 \)

\[
\Delta \theta = \frac{2R}{t} \theta_0 \left( \cos \eta (t - R) + \frac{1}{\eta R} \sin \eta (t - R) \right), \tag{5}
\]

that is, decaying phase oscillations take place – see [3] for numerical verification.

In order to model the interaction of the bubble wall with the plasma, we add a term \( \Gamma \dot{\rho} \) to the equation of motion for the modulus of the Higgs field, Eq. (1), as motivated in §1. If there are no gauge fields, this leads to a different kind of decaying phase oscillations. What happens in theories with a gauge symmetry, where the bubbles move with speeds less than that of light?

Eq. (5) was obtained by imposing \( SO(1,2) \) Lorentz symmetry on the field equations for the two-bubble problem. If the bubbles do not move at the speed of light, no such assumption is possible. This is because whilst the modulus \( \rho \) of the Higgs field is constrained to propagate at a speed \( v \), there is no such restriction on the phase \( \theta \) or the gauge fields. The problem must then be approached via numerical simulations.

3 Results

For the sake of clarity, we have chosen to present our results in terms of the evolution with time of the gauge-invariant phase difference \( \Delta \theta \) between the centres of the two bubbles, though the qualitative behaviour was found not to change when calculated between different points.
Figure 1 (a) shows the behaviour of the gauge-invariant phase difference for bubbles moving at the speed of light - the decaying oscillations calculated by Kibble and Vilenkin in the local case. In the global case, \( e = 0 \), we find that the phase does equilibrate, but on a much longer time-scale. Thus we would expect that for fast-moving bubbles, fewer defects are formed in local theories than global ones, since in order to form a defect a phase difference inside the two merged bubbles must still be present when a third bubble collides.

In Figure 1 (b) we plot \( \Delta \theta \) for slower-moving bubbles. For \( e = 0 \), we confirm in \( 3+1 \)-dimensions the decaying phase oscillations described by Ferrera and Melfo and observed by them in \( 2+1 \)-dimensions. These oscillations are killed by adding in gauge fields - for a fixed bubble-wall velocity, the stronger the gauge coupling, the less time the gauge-invariant phase difference is non-zero, and hence the less likely a third collision will occur in time for a defect to form. Thus we would expect a lower defect-formation rate in local theories with slower-moving bubble walls.

Figure 2 illustrates our findings - it shows a cross-section through a non-simultaneous three-bubble collision, after all three bubbles have merged. In each case, the bubbles of initial radius \( R = 5 \), centred at \(( \pm 8, 0, -10)\) and \((0, 0, 10)\), were given phases \( \theta = -\pi/2, 0 \) and \( 2\pi/3 \). For identical initial conditions, we see that in the fast-moving case a vortex is formed, but when the bubbles are slowed down, the phase difference between the two bubbles has equilibrated by the time the third bubble collides, and no defect is formed.

For a fuller discussion of these and other results, including the effect of taking into account the finite conductivity of the plasma, and the magnetic fields formed at collisions of fast- and slow-moving bubbles, see[5].

Acknowledgments

This work was done in collaboration with A.-C. Davis. We would like to thank O. Törnkvist for helpful discussions. Financial support was provided by PPARC and Fitzwilliam College, Cambridge. Computer facilities were provided by the UK National Cosmology Supercomputing Centre in cooperation with Silicon Graphics/Cray Research, supported by HEFCE and PPARC.

References

1. G.D. Moore and T. Prokopec, Phys. Rev. Lett. 75, 777 (1995).
2. T.W.B. Kibble and A. Vilenkin, Phys. Rev. D 49, 679 (1995).
3. E.J. Copeland, P.M. Saffin and O. Törnkvist, hep-ph/9907437.
4. A. Ferrera and A. Melfo, Phys. Rev. D 53, 6852 (1996).
5. A.-C. Davis and M.J. Lilley, hep-ph/9908398.
Figure 1: Gauge-invariant phase difference between (a) two bubbles moving at the speed of light, $\Gamma = 0$, and (b) two slow-moving bubbles, $\Gamma = 2$.

Figure 2: Phase plot and bubble walls after three-bubble collisions, with phases 0 (bottom left), $2\pi/3$ (top left) and $-\pi/2$ (right): (a) with $\Gamma = 0$ a vortex is formed at the centre, and (b) with identical initial conditions, but $\Gamma = 0.5$ there is no vortex.