Assessing Interaction Networks with Applications to Catastrophe Dynamics and Disaster Management

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Abstract

In this paper we present a versatile method for the investigation of interaction networks and show how to use it to assess effects of indirect interactions and feedback loops. The method allows to evaluate the impact of optimization measures or failures on the system. Here, we will apply it to the investigation of catastrophes, in particular to the temporal development of disasters (catastrophe dynamics). The mathematical methods are related to the master equation, which allows the application of well-known solution methods. We will also indicate connections of disaster management with excitable media and supply networks. This facilitates to study the effects of measures taken by the emergency management or the local operation units. With a fictitious, but more or less realistic example of a spreading epidemic disease or a wave of influenza, we illustrate how this method can, in principle, provide decision support to the emergency management during such a disaster. Similar considerations may help to assess measures to fight the SARS epidemics, although immunization is presently not possible.

Key words: Master equation, interaction network, excitable media, supply chain management, robustness of graphs, causality network, catastrophe dynamics, disaster preparedness

1 Introduction

Natural disasters\,[1]\ have occurred since earliest times, and despite the development of science and technology, they still cause many victims each year. One reason for this is the increased world population. Nowadays, more people can be affected by a disaster than in former centuries. Apart from this, more people affect their environment, which challenges its stability and triggers disasters such as famines.
One common feature of many disastrous events is the so-called domino or avalanche effect. It means that one critical situation triggers another one and so on, so that the situation worsens even more. One famous example is a mountain slide that fell into a lake and caused very high waves (Vajont, Italy 1963 [2]). Other examples are fires and failures of water and electricity supply caused by earthquakes or, on a larger timescale, a disease harming the economy and social life of the affected area, which leaves the people and country even poorer and the medical system less effective, causing further fatalities (e.g. AIDS in Central Africa [3] or the plague in former ages).

Therefore, a great effort is necessary to prevent the emergence of disasters (which is not always possible) and to improve the management of catastrophes. Physics and other natural scientists have traditionally contributed a lot to understanding the laws behind catastrophes. For example, we mention the extensive work on forest fires [4] and earthquakes [5], which relate to concepts of self-organized criticality used to describe avalanche effects [6]. But there is also work on floodings [7], landslides [8], and volcanos [9]. A considerably attention has also been devoted to epidemics [10].

Some of the physical disciplines involved into the study of these subjects are the theory of catastrophes and bifurcations [11], non-equilibrium phase transitions [12], self-organized criticality and scaling laws [6], percolation theory [13], the statistical physics of networks [14] and extreme events [15], stochastic processes [16] and noise-induced transitions [17], but mechanics, fluid-dynamics, and other fields play an important role as well [18].

In this paper, we like to develop a flexible semi-quantitative method allowing

- to assess the suitability of alternative measures of emergency management, i.e. to give decision support,
- to estimate the temporary development of catastrophes, and
- to give hints when to take certain actions in an anticipatory way.

For that purpose, it is necessary to take into account all factors which are relevant during the catastrophe and all direct and indirect interactions between them. This method is in the tradition of systems theory [19]. It extends the concept of the causality diagram in Sec. 2, while a dynamical generalization is developed in Sec. 4.

In the next section, we start with a static analysis of interaction networks. Section 3 is intended to illustrate its usefulness for disaster management. There, we will calculate in a semi-fictitious example the effects of measures taken to combat an epidemic catastrophe. Section 4 contains the extension to a dynamic description, which is connected to the discrete master equation [16]. It allows to determine the probability and order of events as well as their most likely occurrence in time. In Secs. 5 and 7, we will develop a dynamical model
of disaster management, which specifies some parameters in the master equation. It relates to models of excitable media [20] and supply networks [21,22]. Some analytical results for this model are presented in Sec. 6, while Sec. 8 summarizes our results and closes with an outlook.

2 Assessment of Interaction Networks

In this section, we want to develop a simple method to reflect the approximate influence of different factors or sectors on each other. Such factors may, for example, be energy supply, public transport, or medical support. In principle, it is a long list of variables $i$, which may play a role for the problem under consideration. If we represent the influence of factor $j$ on factor $i$ by $A_{ij}$, we may summarize these (directed) influences by a matrix $A = (A_{ij})$. However, in practical applications, one faces the following problems:

(1) The number of possible interactions grows quadratically with the number of variables or factors $i$. It is, therefore, difficult to measure or even estimate all the influences $A_{ij}$.

(2) While it appears feasible to determine the direct influence $M_{ij}$ of one variable $j$ on another one $i$, it is hard or almost impossible to estimate the indirect influences over various nodes of the graph, which enter into $A_{ij}$ as well. However, feedback loops may have an important effect and may neutralize or even overcompensate the direct influences.

Problem (i) can be partially resolved by clustering similar variables and selecting a representative one for each cluster. The remaining set of variables should contain the main explanatory variables. Systematic statistical methods for such a procedure are, in principle, available, but intuition may be a good guide, when the quantitative data required for the clustering of variables are missing.

Problem (ii) may be addressed by estimating the indirect influences due to feedback loops based on the direct influences $M_{ij}$, which can be summarized by a matrix $M = (M_{ij})$. One may use a formula such as

$$A' = A'_\tau = \frac{1}{\tau} \sum_{k=1}^{\infty} (\tau M)^k = \frac{1}{\tau} \sum_{k=1}^{\infty} \tau^k M^k = \sum_{k=1}^{\infty} \frac{1}{\tau^k - 1} M^k,$$

but as this converges only for small enough values of $\tau$, we will instead use the formula

$$A = A_\tau = \frac{1}{\tau} \sum_{k=1}^{\infty} \frac{\tau^k M^k}{k!} = \frac{1}{\tau} [\exp(\tau M) - 1],$$

as this converges for all values of $\tau$. In practice, one may use smaller values of $\tau$ or apply approximation techniques to handle the case of larger values of $\tau$. The resulting matrix $A_\tau$ reflects the approximate indirect influences due to feedback loops.
where \(1\) denotes the unity matrix. The expression \(M^k\) reflects all influences over \(k - 1\) nodes and \(k\) links, i.e. \(k = 1\) corresponds to direct influences, \(k = 2\) to feedback loops with one intermediate node, \(k = 3\) to feedback loops with two intermediate nodes, etc. The prefactor \(\tau^k\) is not only required for convergence, but with \(\tau < 1\), it also allows to reflect that indirect interactions often become weaker, the more edges (nodes) are in between.

A further simplification can be reached by restricting to a few discrete values to characterize the influences. We may, for example, restrict ourselves to

\[
M_{ij} \in \{-3, -2, -1, 0, 1, 2, 3\},
\]

where \(M_{ij} = \pm 3\) means an extreme positive or negative influence, \(M_{ij} = \pm 2\) represents a strong influence, \(M_{ij} = \pm 1\) a weak influence, and \(M_{ij} = 0\) a negligible influence. Of course, a finer differentiation is possible, whenever necessary. (For an investigation of stylized relationships, it can also make sense to choose \(M_{ij} \in \{-1, 0, 1\}\), where \(M_{ij} = \pm 1\) represents a strong positive or negative influence, then.)

The matrix \(A = (A_{ij})\) will be called the assessment matrix and summarizes all direct influences \((M)\) and feedback effects \((A - M)\) among the investigated factors. It allows conclusions about

- the resulting strength of desirable and undesirable interactions, when feedback effects are included,
- the effect of failures of a specific sector (node),
- the suitability of possible measures to reach specific goals or improvements,
- the side effects of these measures on other factors.

This will be illustrated in more detail by the example in Sec. 3.

One open problem is the choice of the parameter \(\tau\). It controls how strong the indirect effects contribute in comparison with the direct effects. A small value of \(\tau\) corresponds to neglecting indirect effects, i.e.

\[
\lim_{\tau \to 0} A_\tau = M,
\]

while increasing values of \(\tau\) reflect a growing influence of indirect effects. This is often the case for catastrophes, as these are frequently related to bifurcations or phase transitions, to avalanches or percolation effects [5,4]. By variation of \(\tau\), one can study different scenarios.

Note that \(\tau\) may be interpreted as time coordinate: Defining

\[
\vec{X}(\tau) = \exp(\tau M)\vec{X}
\]
for an arbitrary vector $\vec{X}$, we find $\vec{X}(0) = \vec{X}$,

$$\frac{\vec{X}(\tau) - \vec{X}(0)}{\tau} = \frac{1}{\tau} [\exp(\tau M) - 1] \vec{X}(0) = A \tau \vec{X}(0)$$

and

$$\frac{d\vec{X}}{d\tau} = \lim_{\tau \to 0} \frac{\vec{X}(\tau) - \vec{X}(0)}{\tau} = M \vec{X}(0).$$

From this point of view,

$$\vec{X}(\tau) = (\tau A + 1) \vec{X}(0)$$

(6)

describes the state of the system at time $\tau$, and $M_{ij}$ the changing rates. $\vec{X} = \vec{0}$ is a stationary solution and corresponds to the normal (everyday) state. An initial state $\vec{X}(0) \neq \vec{0}$ may be interpreted as perturbation of the system by some (catastrophic) event. We should, however, note that the linear system of equations (6) is certainly a rough description of the system dynamics. It is expected to hold only for small perturbations of the system state and does not consider damping effects due to disaster management. Such aspects will be considered later on (see Secs. 4 and 7), after discussion of an example illustrating how to apply interaction matrices to cope with catastrophes.

3 Optimization of Interaction Networks: A Simple Example

One advantage of our semi-quantitative approach to catastrophes is that it allows to estimate the impact of certain actions on the whole set of factors. Usually, during a disaster the responsibilities have only dissatisfactory information and short time to decide, so in many cases they will take into account only direct impacts on other factors. In the worst case, this may lead to the opposite than the desired result, if the feedback effects exceed the direct influence. Therefore, it would be better to know the implications on the whole system. As we have argued before, all direct and indirect effects are summarized by the matrix $A$, which is determined from the matrix $M$ of direct interactions. Different measures taken by the responsibilities are reflected by different matrices $M$.

As an example we consider the spreading of a disease. For illustrative reasons, we will restrict to the discussion of five factors only:

(1) the number of infected persons,
(2) the quality of medical care,
(3) the public transport,
(4) the economic situation and
(5) the disposal of waste.
These factors are not independent from each other, as illustrated by Fig. 1.

The corresponding matrix of the assumed direct influences among the different factors is

\[
M = \begin{pmatrix}
0 & -2 & +2 & 0 & -1 \\
-2 & 0 & +1 & +2 & +1 \\
-1 & 0 & 0 & +2 & 0 \\
-1 & 0 & +2 & 0 & +1 \\
-1 & 0 & +1 & +2 & 0
\end{pmatrix}
\] (7)

The right choice of the sign of the direct influence \( M_{ij} \) of factor \( j \) on factor \( i \) is plausible: We assume a positive sign if the factor \( i \) increases with an increase of factor \( j \), while we assume a negative sign when factor \( i \) decreases with the growth of factor \( j \). However, the determination of the absolute value of \( M_{ij} \) requires empirical data, expert knowledge, or experience. We have argued as follows:

- A growing number of infected persons affects all other factors in a negative way (see first column), as these will not continue to work. That is, there will be problems to maintain a good economic situation, public transport, or the disposal of waste. Health care is affected twice, as not only the medical personnel may be infected, but also a higher number of patients needs to be treated, and capacities are limited. Therefore, we have chosen the value \(-2\) in this case, but \(-1\) for the other factors.
- A well operating health system (second column) can reduce the number
of infected persons efficiently, so that we have chosen a value of $-2$. The
influence of the health system on the economic situation and other factors
was assumed to be of indirect nature, by reducing the number of ill persons.

- Public transport (third column) contributes to a fast spreading of the in-
fecction assumed here. Therefore, we have selected a value of 2. Transport
is also an important factor for economic prosperity (therefore the value of
2), and it is required to get medical personnel and workers in the disposal
sector to work (which is reflected by a value of 1).
- The economic situation (fourth column) has a significant effect on the qual-
ity of the health system, public transport, and disposal, so that we have
chosen a value of 2 in each case.
- Waste may contribute to the spreading of the disease, if it is not properly
removed. Therefore, a good disposal system (fifth column) may reduce the
number of infections (therefore the value of $-1$). It is also required for a
functioning health system and economic production. That is, why we have
assumed a value of 1.

Depending on the respective situation, the concrete values of the direct in-
fluences $M_{ij}$ may be somewhat different. For their specification, it can also
be helpful to check the resulting values of $A_{ij}$ of the overall direct plus indi-
rect influence for their plausibility, and to compare the size of second-order
or third-order interactions. For example, we see that the third-order feedback
loop “number of infected persons $\rightarrow$ economic situation $\rightarrow$ quality of the health
system $\rightarrow$ number of infected persons” is proportional to $(-1) \cdot (+2) \cdot (-2) = 4$.
The same indirect influence is found for the feedback loop “number of infected
persons $\rightarrow$ economic situation $\rightarrow$ public transport $\rightarrow$ number of infected persons”.
Moreover, according to our assumptions, the second-order autocatalytic in-
crease of the number of infected persons via its impact on the health system
is four times as large as the one via its impact on the waste disposal. One
surprising observation is that the number of infected persons is reduced via
its impact on public transport. In fact, once less busses are operated (because
the bus drivers are ill), the spreading rate of the disease is reduced. This may
inspire responsibilities to reduce public transport or even stop it. Later on, we
will discuss the effect of this possible measure.

Before, let us have a look at the resulting overall interaction matrix

$$
A = (A_{ij}) = \begin{pmatrix}
0.9 & -2.2 & 1.3 & -0.8 & -1.6 \\
-3.4 & 1.1 & 1.5 & 3.5 & 2.3 \\
-1.7 & 0.6 & 0.5 & 2.5 & 0.8 \\
-2.0 & 0.6 & 2.1 & 1.5 & 1.6 \\
-2.0 & 0.6 & 1.5 & 2.9 & 0.9
\end{pmatrix}
$$
For its calculation, we have chosen the value $\tau = 0.4$, which will also be used later on to assess alternative actions to fight the spreading of the disease. In order to discuss a certain scenario, we will assume that $X_j$ reflects the perturbation of factor $j$. Because of Eq. (6), the quantities

$$Y_i = \sum_j (\tau A_{ij} + \delta_{ij}) X_j$$

will be used to characterize the potential response of the system in the specific scenario described by the perturbations $X_j$ (and without the damping effects by disaster management discussed in later sections of this contribution). Here, $\delta_{ij}$ denotes the Kronecker function, which is 1 for $i = j$ and 0 otherwise. We will assume $X_1 = 1.0$, as the number of infected persons is higher than normal, and $X_2 = X_3 = X_4 = X_5 = -0.1$, as the other factors are reduced by the spreading of the disease:

$$(X_1, X_2, X_3, X_4, X_5) = (1.0, -0.1, -0.1, -0.1, -0.1).$$

Moreover, if we attribute a weight of $Z_1 = 0.5$ to the number of infected persons, a weight $Z_4 = 0.3$ to the economic situation, and weights of $Z_2 = Z_3 = 0.1$ to the quality of the medical care and public transport, while we do not care about waste in our evaluation (i.e. $Z_5 = 0$), the resulting value of

$$F = F_\tau = \left( \sum_i Z_i Y_i^2 \right)^{1/2}$$

will be used to assess the overall situation of the system. In the stationary (normal) system state, $F$ would be zero. Therefore, we want to find a strategy which brings $F$ close to zero. For our basic scenario, we find

$$(Y_1, Y_2, Y_3, Y_4, Y_5) = (1.5, -1.8, -1.0, -1.1, -1.1) \quad \text{and} \quad F = 1.4.$$  

These reference values will be compared with the values for alternative scenarios which correspond to different actions taken to fight the catastrophe.

For example, let us assume to have limited stocks of vaccine for immunization. Should we use these to immunize 1) the transport workers, 2) the medical staff, or 3) the disposal workers? In the first case, we have the modified matrix

$$M = \begin{pmatrix}
0 & -2 & +2 & 0 & -1 \\
-2 & 0 & +1 & +2 & +1 \\
0 & 0 & 0 & +2 & 0 \\
-1 & 0 & +2 & 0 & +1 \\
-1 & 0 & +1 & +2 & 0 \\
\end{pmatrix},$$

(13)
which implies

\[
A = \begin{pmatrix}
1.2 & -2.3 & 1.4 & -0.8 & -1.7 \\
-3.2 & 1.1 & 1.5 & 3.5 & 2.2 \\
-0.5 & 0.1 & 0.8 & 2.4 & 0.5 \\
-1.6 & 0.5 & 2.2 & 1.5 & 1.5 \\
-1.7 & 0.6 & 1.6 & 2.9 & 0.9 \\
\end{pmatrix}, \tag{14}
\]

\[
(Y_1, Y_2, Y_3, Y_4, Y_5) = (1.6, -1.7, -0.5, -1.0, -1.0), \quad \text{and} \quad F = 1.4. \tag{15}
\]

In the second case, when we immunize the medical staff, we find

\[
M = \begin{pmatrix}
0 & -2 & +2 & 0 & -1 \\
-1 & 0 & +1 & +2 & +1 \\
-1 & 0 & 0 & +2 & 0 \\
-1 & 0 & +2 & 0 & +1 \\
-1 & 0 & +1 & +2 & 0 \\
\end{pmatrix}, \tag{16}
\]

which implies

\[
A = \begin{pmatrix}
0.5 & -2.1 & 1.3 & -0.7 & -1.5 \\
-2.3 & 0.7 & 1.8 & 3.4 & 2.0 \\
-1.7 & 0.6 & 0.5 & 2.5 & 0.8 \\
-1.9 & 0.6 & 2.1 & 1.5 & 1.6 \\
-1.9 & 0.6 & 1.6 & 2.9 & 0.9 \\
\end{pmatrix}, \tag{17}
\]

\[
(Y_1, Y_2, Y_3, Y_4, Y_5) = (1.3, -1.3, -0.9, -1.1, -1.1), \quad \text{and} \quad F = 1.2. \tag{18}
\]

In the third case, when the disposal workers are immunized, we expect

\[
M = \begin{pmatrix}
0 & -2 & +2 & 0 & -1 \\
-2 & 0 & +1 & +2 & +1 \\
-1 & 0 & 0 & +2 & 0 \\
-1 & 0 & +2 & 0 & +1 \\
0 & 0 & +1 & +2 & 0 \\
\end{pmatrix}, \tag{19}
\]
which implies

\[
A = \begin{pmatrix}
0.6 & -2.1 & 1.3 & -0.7 & -1.6 \\
-3.1 & 1.1 & 1.6 & 3.5 & 2.2 \\
-1.6 & 0.6 & 0.5 & 2.5 & 0.8 \\
-1.7 & 0.6 & 2.1 & 1.5 & 1.5 \\
-0.8 & 0.2 & 1.9 & 2.8 & 0.6
\end{pmatrix}, \quad (20)
\]

\[
(Y_1, Y_2, Y_3, Y_4, Y_5) = (1.4, -1.7, -0.9, -1.0, -0.7), \quad \text{and} \quad F = 1.3. \quad (21)
\]

While the immunization of the public transport staff has almost no effect on the overall situation in the system, the last two measures can improve it. We see that it is more effective to immunize the medical staff than the disposal workers, and the best would be to immunize both groups. This corresponds to

\[
M = \begin{pmatrix}
0 & -2 & +2 & 0 & -1 \\
-1 & 0 & +1 & +2 & +1 \\
-1 & 0 & 0 & +2 & 0 \\
-1 & 0 & +2 & 0 & +1 \\
0 & 0 & +1 & +2 & 0
\end{pmatrix}, \quad (22)
\]

and we obtain

\[
A = \begin{pmatrix}
0.2 & -2.0 & 1.2 & -0.7 & -1.5 \\
-1.9 & 0.6 & 1.9 & 3.4 & 1.9 \\
-1.6 & 0.5 & 0.5 & 2.5 & 0.8 \\
-1.7 & 0.6 & 2.1 & 1.5 & 1.5 \\
-0.8 & 0.2 & 1.9 & 2.8 & 0.6
\end{pmatrix}, \quad (23)
\]

\[
(Y_1, Y_2, Y_3, Y_4, Y_5) = (1.2, -1.2, -0.9, -1.0, -0.6), \quad \text{and} \quad F = 1.1. \quad (24)
\]

Other measures do not change the interactions in the system, but correspond to a change of the effective impact $\tilde{X}$ of the catastrophe. For example, we may consider to reduce public transport. With (8) and

\[
(X_1, X_2, X_3, X_4, X_5) = (1.0, -0.1, -1.0, -0.1, -0.1), \quad (25)
\]

we find

\[
(Y_1, Y_2, Y_3, Y_4, Y_5) = (1.0, -2.4, -2.0, -1.9, -1.7) \quad \text{and} \quad F = 1.6. \quad (26)
\]
We see that the number of infections could, in fact, be reduced. However, the overall situation of the system has deteriorated, as the economic situation and all the other sectors were negatively affected, because many people could not reach their workplace. Therefore, let us consider the option to increase the number of disposal workers. With (8) and

\[(X_1, X_2, X_3, X_4, X_5) = (1.0, -0.1, -0.1, -0.1, 0.5),\]  

we find

\[(Y_1, Y_2, Y_3, Y_4, Y_5) = (1.1, -1.3, -0.8, -0.8, -0.3) \text{ and } F = 1.0.\]  

(27)

(28)

In conclusion, increasing the hygienic standards can be surprisingly efficient.

Finally, let us assume an improved waste disposal together with the immunization of both, the medical staff and the disposal workers. In that case the interactions of the relevant factors are characterized by matrix (22), whereas the starting vector is again (27). The resulting response is

\[(Y_1, Y_2, Y_3, Y_4, Y_5) = (0.8, -0.7, -0.7, -0.6, 0.1) \text{ and } F = 0.75.\]  

(29)

Only this combination of measures manages to actually reduce the infections compared to the initial state, i.e. \(Y_1 < X_1\). However, we can also see that a negative impact on the economic situation and other factors cannot be avoided. In any case, we can assess which measures are reasonable to take, which impact they will have on the system, and which measures need to be combined in order to control the spreading of the disease (or other problems in different scenarios).

The simple example in this section was chosen to illustrate the procedure how to assess interaction networks and potential optimization measures. In an on-going project, we do now investigate the interaction network among a large number of factors for the floodings in Germany during August 2002 and other catastrophes. This involves considerably more detailed and much larger matrices \(M\), where nobody would be able to assess the feedback loops without a method such as the one proposed above. It also involves other aspects such as human forces fighting the catastrophe and the availability of technical or other equipment etc, which will be modeled in the following sections.

4 Impact of the Interaction Network on Catastrophe Dynamics

Before, we have used the interaction network predominantly for the static assessment of the influence of different factors on each other. We will now try
to extend this method step by step in a way that allows a semi-quantitative analysis of the time-dependence of catastrophes for the purpose of anticipation, which helps to prepare for the next step in catastrophe management or prevention. We are particularly interested in the domino or avalanche effects of particular events such as the failure of a certain factor or sector in the interaction network. We will assume that this failure spreads along and in the order of the direct connections in the interaction network (causality graph). In terms of example in Sec. 3, a failure of medical care would first affect the number of infected persons, and in a second step the economic situation, public transport, and the disposal of waste.

For a description of the catastrophe dynamics, let us assume that $P_i(\tau)$ denotes the impact on factor $i$ at time $\tau$ and $W_{ji}$ the rate at which this impact spreads to factor $j$, while $D_i$ is a damping rate describing the mitigation of the catastrophic impact on factor $i$ by disaster management. In this case, it is reasonable to assume the dynamics

$$\frac{d\vec{P}}{d\tau} = (W - D)\vec{P}(\tau) = L\vec{P}(\tau)$$

with $D = (\delta_{ij}D_i)$, $L = (L_{ij}) = (W_{ij} - \delta_{ij}D_i)$, and $\vec{P}(\tau) = (P_i(\tau))$. The symbol $\delta_{ij}$ represents the Kronecker function, i.e. it is 1 for $i = j$ and otherwise 0. When no better information is available, we may assume that the spreading rate $W_{ij}$ is proportional to the strength $|M_{ij}|$ of the direct influence of factor $j$ on factor $i$. With a constant proportionality factor $c$ this means

$$W_{ij} \approx c|M_{ij}|.$$  

(31)

The formal solution of equation (30) for a time-independent matrix $L$ is given by

$$\vec{P}(\tau) = \exp(L\tau)\vec{P}(0) = \sum_{k=0}^{\infty} \frac{\tau^k}{k!} L^k \vec{P}(0) = B(\tau)\vec{P}(0).$$

(32)

That is, $B(\tau)$ describes the spreading of an event in the causality network (interaction network) in the course of time $\tau$, while $\vec{P}(0)$ reflects the initial impact of a catastrophic event. A random series of catastrophic events can be described by adding a random variable $\xi(t)$ to the right-hand side of Eq. (30).

When we assume

$$D_i = \sum_j W_{ji},$$

(33)

equation (30) is related to the Liouville representation of the discrete master equation. In this case, we can apply all the solution methods developed for it. This includes the so-called path integral solution [23], which allows one to calculate the occurrence probability of specific spreading paths. This has some interesting implications. For example, the danger that the impact on sector $i_0$
affects the sectors $i_1, i_2, \ldots, i_n$ in the indicated order is quantified by

$$P(i_0 \to i_1 \to \cdots \to i_n) = \frac{|P_{i_0}(0)|}{D_{i_0}} \prod_{l=0}^{n-1} \frac{W_{i_l,i_{l+1}}}{D_{i_l}} \approx c^n \frac{|P_{i_0}(0)|}{D_{i_0}} \prod_{l=0}^{n-1} \frac{|M_{i_l,i_{l+1}}|}{D_{i_l}}.$$  

(34)

Moreover, the average time at which this series of events has occurred can be calculated as

$$T(i_0 \to i_1 \to \cdots \to i_n) = \sum_{l=0}^{n} \frac{1}{D_{i_l}},$$  

(35)

and the variance of this time is determined by

$$\Theta(i_0 \to i_1 \to \cdots \to i_n) = \sum_{l=0}^{n} \frac{1}{(D_{i_l})^2}.$$  

(36)

That is, Eq. (30) does not only allow to assess the likelihood of certain series of events rather accurately, but also their approximate appearance times. In other words, we have a detailed picture of the potential catastrophic scenarios and of their time evolution, which allows for a specific preparation and disaster management.

In the following, we do not want to restrict to the case (33). If

$$D_i < \sum_j W_{ji}$$  

(37)

for all $i$, the damping is weak and the solutions $P_i(\tau)$ are expected to grow more or less exponentially in the course of time, which describes a scenario where control is lost and the catastrophe spreads all over the system. In many cases, we will have

$$D_i > \sum_j W_{ji}$$  

(38)

for all $i$, i.e. the impact of the catastrophe on the system decays in the course of time, and $\lim_{\tau \to 0} P_i(\tau) \to 0$. This determines, how strong the damping effects, i.e. the means counteracting the catastrophe have to be chosen. In terms of Sec. 7, this concerns the specification of the parameters $V_{ik}$.

Finally, it may also happen that $D_i > \sum_j W_{ji}$ for some factors $i$, but $D_i < \sum_j W_{ji}$ for others. In such situations, everything depends on the initial impact $\vec{P}(0)$ and on the matrix $B(\tau)$. However, in all these cases, Eqs. (34) to (36) remain valid.

5 An Excitable Media Model of Disaster Management

The damping effects $D_i$ are, to a large extent, related to the forces counteracting the catastrophe. Therefore, we will now develop a dynamical model for
these, while our previous considerations assumed some more or less constant value of $D_i$. Let us denote by $N_k$ the quantity of human forces (e.g. police, fire fighters, or military) ready for action, or the quantity of materials (e.g. technical or medical equipment) ready for use to fight the catastrophe. The index $k$ distinguishes different kinds of forces or required materials. We will assume the following equation:

$$
\frac{dN_k}{d\tau} = \frac{R_k(\tau)}{T_k^R} \pm \lambda_k N_k^\pm e^{-\lambda_k \tau} - \sum_i |P_i| V_{ik} N_k(\tau) A_k^l(\tau).
$$

(39)

The first term on the right-hand side describes the quantity $R_k$ of (human) forces and materials, which were exhausted or not usable, but become available again after an average time period of $T_k^R$. The second term delineates reserve forces of quantity $N_k^\pm$, which are activated from the “standby mode” at a rate $\lambda_k$ after the occurrence of a catastrophe (plus sign), while they are removed after the recovery from the disaster (minus sign). In most cases, $N_k^- \leq N_k^+$, due to possible fatalities. The third term describes the activation of the forces $k$ to fight the problems with factor or sector $i$. For simplicity, it is here assumed to be proportional to the strength $|P_i|$ of the catastrophic impact on factor $i$, with proportionality factors $V_{ik}$ which reflect the priorities in disaster management and the speed with which the forces or materials $k$ become available for $i$. The exponent $l$ allows to distinguish different cases: When the impact of the catastrophe on factor $i$ is known, we may assume $l = 0$. However, when the active forces are assumed to order more forces, an exponent $l > 0$ can make sense as well.

The quantities $A_k$ of forces in action or materials in use change in time according to the equation

$$
\frac{dA_k}{d\tau} = \sum_i |P_i| V_{ik} N_k(\tau) A_k^l(\tau) - \frac{A_k(\tau)}{T_k^A} - \sum_i |P_i| \nu_{ik} A_k(\tau).
$$

(40)

The first term on the right-hand side is due to the available forces $k$, which are activated for fighting the catastrophe, while the second term describes a reduction of the active forces, as they become exhausted or damaged and require rest or repair after an average time period of $T_k^A$. The third term describes unrecoverable losses such as fatalities or unrecoverable damage of materials, which are assumed to occur with a rate $|P_i| \nu_{ik}$ proportional to the catastrophic impact $|P_i|$.

The quantity of exhausted or damaged forces is described by the following differential equation:

$$
\frac{dR_k}{d\tau} = \frac{A_k(\tau)}{T_k^A} - \frac{R_k(\tau)}{T_k^R}.
$$

(41)

Herein, $1/T_k^A$ is the rate at which the forces $k$ become exhausted or damaged, while $1/T_k^R$ is the recovery or repair rate.
The above model already contain many effects which are typically relevant in practical situations. We should note that it is related to models of excitable media developed to describe chemical waves, the propagation of electrical pulses in heart tissue, LaOla waves in human crowds in stadia [20], or the spreading of forest fires [4]. These models typically contain three different states: an excitable one, an active one, and a refractory one. In our model of disaster management, refractory states are described by the variables $R_k$, active states by the variables $A_k$, and excitable states by the variables $N_k$. Due to this analogy, we expect to find certain pattern formation phenomena for our model of disaster management. In a forthcoming paper, this aspect shall be investigated in more detail.

6 Some Analytical Results

In the following, we will try to get an idea of the possible behavior of the excitable media model of disaster management suggested in Sec. 5. The stationary state of this model is, for $\nu_{ik} = 0$, given by

$$\frac{R_k}{T_R} = \frac{A_k}{T_A} = \sum_i |P_i| V_{ik} N_k A_k^l = \text{const.}$$  \hspace{1cm} (42)

In order to investigate the sensitivity with respect to small perturbations, we will carry out a linear stability analysis of the simplified model with one sector $i$, $|P_i| \approx \text{const.}$, $\nu_{ik} = 0$, and one kind $k$ of forces. Dropping the subscripts and defining $P = |P_i|$, the resulting coupled set of differential equations is:

$$\frac{dN}{d\tau} = \frac{R}{T_R} - PVA_k^l,$$ \hspace{1cm} (43)

$$\frac{dA}{d\tau} = PVNA_k^l - \frac{A}{T_A},$$ \hspace{1cm} (44)

$$\frac{dR}{d\tau} = \frac{A}{T_A} - \frac{R}{T_R}.$$ \hspace{1cm} (45)

Its stationary solution is given by $N(\tau) = N_0$, $R(\tau) = R_0$, and $A(\tau) = A_0$ with

$$\frac{R_0}{T_R} = \frac{A_0}{T_A} = PV N_0 A_0^l.$$ \hspace{1.5cm} (46)

It is stable with respect to disturbances, if all eigenvalues $\lambda$ are non-positive. These eigenvalues can be calculated in the usual way. Assuming

$$N(t) = N_0 + \delta N e^{\lambda \tau}, \quad A(t) = A_0 + \delta A e^{\lambda \tau}, \quad \text{and} \quad R(t) = R_0 + \delta R e^{\lambda \tau},$$ \hspace{1cm} (47)
we find the following eigenvalue problem for the amplitudes \( \delta N \), \( \delta A \), and \( \delta R \) of the deviations from the stationary values \( N_0 \), \( A_0 \), and \( R_0 \):

\[
\lambda \begin{pmatrix} \delta N \\ \delta A \\ \delta R \end{pmatrix} = \begin{pmatrix} -PA_0^l & -PN_0A_0^{l-1} & 1/TA \\ PA_0^l & PN_0A_0^{l-1} - 1/TA & 0 \\ 0 & 1/TA & -1/TR \end{pmatrix} \begin{pmatrix} \delta N \\ \delta A \\ \delta R \end{pmatrix}.
\]

The eigenvalues \( \lambda \) are the solutions of the characteristic equation

\[
(-PA_0^l - \lambda)(PN_0A_0^{l-1} - 1/TA - \lambda)\left(-1/TR - \lambda\right) = 0.
\]

The three solutions are

\[
\lambda_{1/2} = \frac{PA_0^{l-1}(lN_0 - A_0)}{2} - \frac{1}{2TA} - \frac{1}{2TR} \\
\pm \sqrt{\left(\frac{PA_0^{l-1}(lN_0 - A_0)}{2} + \frac{1}{TA} - \frac{1}{TR}\right)^2 - \frac{PA_0^l}{TA}}
\]

and \( \lambda_3 = 0 \). Taking into account Eq. (46), which implies \( PA_0^{l-1} = 1/(N_0TA) \), this becomes

\[
\lambda_{1/2} = \frac{1}{2TA}\left(l - 1 - \frac{A_0}{N_0}\right) - \frac{1}{2TR} \pm \sqrt{\left[\frac{1}{2TA}\left(l - 1 - \frac{A_0}{N_0}\right) + \frac{1}{2TR}\right]^2 - \frac{A_0}{N_0(TA)^2}}.
\]

A detailed analysis of this expression shows the following: The system behaves unstable with respect to perturbations, when the real part of one of the above solutions becomes positive, which is the case for

\[
l - 1 - \frac{A_0}{N_0} > \min\left(\frac{A_0 TR}{N_0 T^A}, \frac{T^A}{TR}\right).
\]

Otherwise (apart from the case of marginal stability resulting for the equality sign), perturbations are damped, but one can distinguish two subcases: For

\[
-\frac{T^A}{TR} - 2\sqrt{\frac{A_0}{N_0}} < l - 1 - \frac{A_0}{N_0} < \min\left(\frac{A_0 TR}{N_0 T^A}, \frac{T^A}{TR}, -\frac{T^A}{TR} + 2\sqrt{\frac{A_0}{N_0}}\right),
\]

the resulting solution is complex, corresponding to damped oscillations, while the system behaves overdamped in the remaining case, where perturbations
fade away without any oscillations. For disaster management, the linearly unstable case and the case of damped oscillations are both unfavourable. Therefore, \( l \) should be small enough. Otherwise, if active forces recruit other forces, the resulting “autocatalytic effect” may cause instabilities or overreactions in the supply with forces and materials. This effect is most likely for disasters which nobody was prepared for, where the recruiting mechanism plays the most significant role. It may explain the suboptimal distribution of forces observed in these situations [24].

7 Connection with Supply Networks and Production Systems

Finally, we have to specify the influence of disaster management activities on the damping \( D_i \) of the impact \( P_i \), which a catastrophic event has on sector \( i \). Let us assume that we have \( K \) different kinds of forces, materials or technical equipment. With \( k \in K \), we will indicate that the forces or materials \( k \) can substitute each other, i.e. \( K \) summarizes equivalent forces or materials. On the other hand, certain actions require the simultaneous presence of different supplementary kinds of forces and materials. Let us assume that the quantities simultaneously required to reduce the problems with factor \( i \) are represented by the coefficients \( c_{iK} \). The units of \( c_{iK} \) shall be chosen in a way that the following equation holds:

\[
D_i(\tau) = (1 - L_i) \min_K \left\{ \frac{\sum_{k \in K} |P_i|V_{ik}N_k(\tau)A_k^i(\tau)}{c_{iK}} \right\}, \tag{54}
\]

where \( V_{ik}N_k(\tau)A_k^i(\tau) \) is the rate of activating forces \( k \) to mitigate the situation of factor \( i \). This equation reflects that, if only one of the required forces or materials is missing, no successful action can be taken. Moreover, there may be a loss \( L_i \) of efficiency, e.g. due to queueing or limited capacities \((0 \leq L_i \leq 1)\). The formula is analogous to that for production systems, where a product cannot be finished, as long as some required part or worker is missing, and where finite storage capacities may cause losses [21]. Therefore, this formula delineates the inefficiencies in disaster management, which occur when forces and materials are distributed in the wrong way. It has sometimes been reported, that too many forces have been located at some place, and missing at others [24]. Here, models designed to optimize supply networks could help to optimize the efficiency of disaster management [22].

Note that, for disaster management, a slight generalization of formula (54) is in place, as improvisation may cope with a lack of certain materials or forces.
It is reasonable to assume a generalized function $G_q$ with

$$D_i(\tau) = (1 - L_i)G_q\left(\sum_{k \in K} \frac{|P_i|V_{ik}N_k(\tau)A_k(\tau)}{c_{iK}}\right)$$

and

$$\min_K \left\{ \sum_{k \in K} \frac{|P_i|V_{ik}N_k(\tau)A_k(\tau)}{c_{iK}} \right\} \leq G_q\left(\sum_{k \in K} \frac{|P_i|V_{ik}N_k(\tau)A_k(\tau)}{c_{iK}}\right) \leq \sum_K \sum_{k \in K} \frac{|P_i|V_{ik}N_k(\tau)A_k(\tau)}{c_{iK}}.$$  

(56)

Herein, the minimum reflects the worst case, while the sum over $K$ describes the best case (if we neglect non-linearities, which may sometimes arise due to synergy effects). The specification

$$G_q\left(\sum_{k \in K} \frac{|P_i|V_{ik}N_k(\tau)A_k(\tau)}{c_{iK}}\right) = \left[\sum_K \left(\sum_{k \in K} \frac{|P_i|V_{ik}N_k(\tau)A_k(\tau)}{c_{iK}}\right)^q\right]^{1/q}$$

(57)

describes both extreme cases. The sum over $K$ corresponds to $q = 1$, while the minimum results for $q \to -\infty$. Hence, a variation of the parameter $q$ allows to investigate different possible scenarios lying between the best case and the worst case. In order to model synergy effects between different forces, one would have to add nonlinear terms, e.g., bilinear ones. However, this would introduce a large number of additional parameters, which are even harder to estimate than the first-order effects included in our model. Note that we already have non-linearities in our model, namely the products $|P_i(\tau)|N_k(\tau)A_k(\tau)$ and the function $G_q$. 

The framework of supply networks also allows one to move from the semiquantitative description of disaster management in Secs. 2 to 4 to a fully quantitative one, if the required data are available (while we work with assumption (31) otherwise): One simple case of the supply network model proposed in Refs. [21,22,25,26] corresponds to the dynamic input-output model

$$\frac{dN_i}{d\tau} = \sum_j (\delta_{ij} - c_{ij})Q_j(\tau)$$

(58)

with

$$\frac{dQ_j}{d\tau} = \frac{V_j(N_j) - Q_j(\tau)}{T_j},$$

(59)

where $N_i$ denotes the inventory (stock level) of factor or product $i$, $V_j(N_j)$ the desired and $Q_j \geq 0$ the actual throughput of sector $j$, $T_j$ the adaptation time, and $c_{ij} \geq 0$ the quantity of factor $i$ needed per throughput cycle. (For details see Ref. [22].) One could also say, $c_{ij}$ are the entries of the input-output
matrix measured in economics, and $c_{ij}Q_j$ is the flow of the quantity generated by factor $i$ to factor $j$. In the limit of short adaptation times $T_j \approx 0$, the above equations reduce to

$$\frac{dN_i}{d\tau} = \sum_j (\delta_{ij} - c_{ij})V_j(N_j). \quad (60)$$

Let us assume that the stationary state of this supply system is given by $N_i(t) = N_i^0$. Moreover let us denote the deviations from the stationary state by $\delta N_i(t) = N_i(t) - N_i^0$. With

$$V_j(N_j) \approx V_j(N_j^0) + \frac{dV_j(N_j^0)}{dN_j} \delta N_j = A_j - B_j \delta N_j,$$

the linearized version of Eq. (60) reads

$$\frac{d\delta N_i}{d\tau} = \sum_j (W_{ij} - B_j \delta_{ij}) \delta N_j(\tau), \quad (62)$$

where $A_j = V_j(N_j^0)$, $B_j = -dV_j(N_j^0)/dN_j > 0$, and $W_{ij} = c_{ij}B_j$. Here, we have used that, for the stationary solution $N_j^0$,

$$\sum_j (\delta_{ij} - c_{ij})(A_j - B_j N_j^0) = 0. \quad (63)$$

In Eq. (62), we have $B_i = \sum_j W_{ji}$ because of $\sum_j c_{ji} = 1$. Taking into account the additional contribution (55), we finally obtain the set of linear equations

$$\frac{d\bar{P}}{d\tau} = (W - D)\bar{P}(\tau) = L\bar{P}(\tau) \quad (64)$$

with $W = (W_{ij})$, $D = (\delta_{ij}(B_i + D_i))$, $L = W - D$, and $\bar{P}(\tau) = (P_i(\tau)) = (\delta N_i(\tau))$. Although $P_j$ can become negative due to some catastrophic impact, Eqs. (35) and (36) for the average occurrence times and their variance still remain meaningful, when $D_i$ is replaced by $(B_i + D_i)$. Hence, they can be used to estimate the time of impact on other factors or sectors in the supply network.

One particularly important aspect of supply networks is their sensitivity or robustness with respect to perturbations. It is, for example, known that supply chains may suffer from the so-called bullwhip effect, i.e. small temporal variations in the demand may cause large variations in the supply. This instability leads both to undesirable delays in delivery at some places and large stock levels at others [21,22,27]. In disaster management, this effect can have serious consequences. However, anticipation is known to efficiently stabilize the dynamics of supply networks [25,26]. As the formulas from Sec. 4 can be used to estimate the approximate time at which certain factors are likely to be affected, they can help to optimize the supply chain management, in particular to stabilize the supply of forces and materials in time. It is certainly
reasonable to have forces available in time to fight the spreading of the disaster to other sectors, rather than sending them all to the places which are already devastated. The philosophy is to reach an anticipative disaster management rather than having a responsive one. To model anticipation, the term \( \sum_i P_i(\tau) V_{ik} N_k(\tau) A_{kl}(\tau) \) has to be replaced by

\[
\sum_i |P_i(\tau + \Delta \tau)| V_{ik} N_k(\tau) A_{kl}(\tau), \tag{65}
\]

where \( \Delta \tau \) denotes the anticipation time horizon.

Even more interesting is the robustness of supply networks with respect to structural changes, e.g. when some supplier fails to work or to deliver. This may be investigated by changing the coefficients \( V_{ik} \) and \( c_{ik} \), which characterize the supply network. The influence of the topology of the supply network on its robustness, reliability, and dynamics is presently under investigation [26]. It has, for example, been noticed that supply ladders are more robust than linear supply chains or supply hierarchies (see Fig. 2). It is not surprising that the redundancy of supply ladders, i.e. the availability of alternative delivery channels, stabilizes the system compared to a linear supply chain. We note, however, that hierarchical systems are very common in disaster management, and better alternatives are expected to be found in a research project that we presently pursue.

Some insights regarding the robustness of networks have already been gained for the world wide web and other networks [14]. However, it is questionable whether the results for small-world, scale-free or random networks can directly be transferred to disaster management. Further research in this direction would be very helpful.

8 Summary and Outlook

In the past, physics has made significant contributions to the understanding of catastrophes. This concerns, for example, the statistics of extreme events and avalanche effects. In this paper, we have tried to indicate how physics could contribute to disaster management [28]. We have sketched a rather general, semi-quantitative approach for the assessment of interaction networks which can serve for decision support, as it allows to compare potential optimization measures including their side effects. The method takes into account feedback loops and can be generalized to a dynamical model, which is suitable to estimate likely sequences of events and the times at which they are expected to materialize. We have also described the dynamics of disaster management in a way similar to excitable media, distinguishing an excitable state ("ready to go"), an active state, and a refractory (exhausted) state requiring recovery.
Figure 2. Illustration of different supply networks: (a) linear supply chain, (b) “supply ladder”, and (c) hierarchical supply tree. The supply ladder is particularly robust because of its redundant links and nodes.

As successful actions need the simultaneous presence and/or action of several specialized forces and particular equipment, there was also a direct connection with the management of supply networks. We have pointed out that the management of disasters and supply chains can be considerably improved by anticipation, based on the formulas in this paper. Moreover, the robustness crucially depends on the structure of the supply network. In an on-going study, we investigate the optimal network structure to achieve robustness with respect to dynamical and structural perturbations. The statistical physics of networks and graphs is expected to make significant contributions to this. Some relevant aspects for the optimization of organizations and work groups have already been studied.

Our proposed approach connects to several methods and fields from statistical physics, such as the master equation, excitable media, and the dynamics of transport (supply) processes. It is also in the tradition of system dynamics, which has, with some success, been used to anticipate future problems of society. In such kinds of studies, it is reasonable to carry out a sensitivity analysis and to investigate the impact of random effects, which do, of course, play a significant role for the dynamics of catastrophes.
Apart from stochastic methods, one may in the future also apply elements of fuzzy logic [33] in order to describe the vague knowledge and soft facts, on which disaster management is often based. Insufficient, inconsistent, and uncertain information is one of the typical complications of disaster management, which makes it difficult to assess alternatives and to take the best decision, in particular under often very tense time constraints. In the future, information theory [34] is expected to make some valuable contributions to the design of decision support systems which can integrate inconsistent information and handle incomplete information [35].

In summary, statistical physics offers various promising concepts to develop and improve methods of disaster management. We think that the theory of self-organization [36] is particularly promising for this, having in mind principles such as synchronization [37], distributed control [38], and optimal self-organization [39]. It is expected that an application of these principles would lead to a more flexible, efficient, and robust disaster management compared to the present centralized or hierarchical concepts.

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