Reheating temperature from the CMB

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In the recent paper by Mielczarek et al. (JCAP 1007 (2010) 004) an idea of the method which can be used to put some constraint for the reheating phase was proposed. Another method of constraining the reheating temperature has been recently studied by Martin and Ringeval (Phys. Rev. D 82 (2010) 023511). Both methods are based on observations of the cosmic microwave background (CMB) radiation. In this paper, we develop the idea introduced in this first article to put constraint on the reheating after the slow-roll inflation. We restrict our considerations to the case of a massive inflaton field. The method can be, however, easily extended to the different inflationary scenarios. As a main result, we derive an expression on the reheating temperature \( T_{RH} \). Surprisingly, the obtained equation is independent on the unknown number of relativistic degrees of freedom \( g_\ast \), produced during the reheating. Based on this equation and the WMAP 7 observations, we find \( T_{RH} = 3.5 \cdot 10^9 \) GeV, which is consistent with the current constraints. The relative uncertainty of the result is, however, very high and equal to \( \sigma(T_{RH})/T_{RH} \approx 53 \). As we show, this uncertainty will be significantly reduced with future CMB experiments.

I. INTRODUCTION

The reheating \cite{1,2} is a hypothetical process in which the inflaton field \cite{3} is converted into the standard model particles. The mechanism of reheating is usually assumed to be a parametric production of particles \cite{4}. However, the considerations are purely speculative due to the lack of any possible empirical verification of the reheating phase. Up to now, only some weak constrains on the basic parameters of reheating are available. In particular, the reheating temperature \( T_{RH} \) can be constrained from the both sides. From bottom the constraint is given by the big bang nucleosynthesis (BBN), namely \( T_{RH} \gtrsim 10 \text{ MeV} \cite{5,6} \). From the top, the constraint comes from the energy scale at the end of inflation \( T_{RH} \lesssim 10^{16} \) GeV. Roughly 18 orders of magnitude remain to place the reheating temperature somewhere between. Worse, there is no observational window available at these energy scales. Such a window exists however at the energies of inflation. It is because the perturbations created during the inflation can be studied by its impact on the cosmic microwave background (CMB) radiation and subsequently the recombination can be found. As we show, this can be used to determine the reheating temperature. For simplicity, we assume the slow-roll inflation (described by a massive inflaton field), which is in good agreement with the CMB observations. After inflation, the inflaton field undergoes coherent oscillations at the bottom of a potential well. The reheating takes place when the Hubble parameter \( H \) falls to the value of the inflaton decay rate \( \Gamma_\phi \). We assume that the reheating is instantaneous. After reheating, the standard radiation phase takes place. The evolution of radiation is assumed to be adiabatic. During the reheating, the effective number of relativistic species produced is given by \( g_\ast \). The decay rate of the inflaton field can be related with the remaining two parameters \( g_\ast \) and \( T_{RH} \) by the Friedmann equation as follows

\[ \Gamma_\phi \simeq \frac{8\pi}{3m_{Pl}^2} g_\ast \frac{\pi^2 T_{RH}^4}{30}, \]

where \( m_{Pl} = 1.22 \cdot 10^{19} \) GeV. Therefore, only two from the parameters of reheating \( (\Gamma_\phi, T_{RH}, g_\ast) \) are independent. In this paper, we show that the reheating temperature can be determined independently on the remaining two parameters. Up to now, the constraints on \( T_{RH} \) were dependent on the value of \( g_\ast \). However, in the equation derived in this paper, the \( g_\ast \) factors surprisingly cancel out. Having \( T_{RH} \), the decay rate \( \Gamma_\phi \) can be expressed in terms of \( g_\ast \) only.

The considerations presented in this paper are restricted to the simplest setup in order to capture the essence of the method. However, extension to the different inflationary scenarios and to the more detailed models of reheating can be done straightforwardly.
II. METHOD

The main idea of the method can be understood by looking at Fig. 1. In this figure we schematically present evolution of the Hubble radius $R_H := 1/H$, together with the evolution of an arbitrary physical length scale $\lambda$. The present value of this length scale is equal to $\lambda_0$, what we call the pivot scale. The following values of the scale factor were distinguished:

- $a_0$ – the present value of the scale factor, we set $a_0 = 1$ for convenience.
- $a_1$ – the scale factor at the end of the radiation era.
- $a_2$ – the scale factor when the instantaneous reheating takes place (beginning of the radiation era).
- $a_3$ – the scale factor at the end of inflation. The inflation field starts to oscillate.
- $a_4$ – the scale factor at which the length scale of the present value $\lambda_0$ crossed the Hubble radius during inflation.

The total increase of the scale factor from $a_4$ to $a_0$ will be of particular importance. We call it $\Delta_{\text{tot}}$, which can be expressed as follows

$$\Delta_{\text{tot}} = \prod_{i=0}^{3} \Delta_i, \quad \text{where} \quad \Delta_i := \frac{a_i}{a_{i+1}}. \quad (2)$$

So, if we know durations of the four stages between the $a_4$ and $a_0$, the $\Delta_{\text{tot}}$ can be determined. This is, however, practically impossible to obtain because we do not know details of the intermediate periods as $\Delta_2$ and $\Delta_1$. Hopefully, there is an alternative method to determine $\Delta_{\text{tot}}$, which can be used to put constraints on $\Delta_2$ and $\Delta_1$. This method bases on the observation of the CMB radiation. In particular, on the measurements of the scalar power spectrum. The form of this spectrum is parameterized by the function

$$P_s(k) = A_s \left( \frac{k}{k_0} \right)^{n_s-1}. \quad (3)$$

The $A_s$ is an amplitude and $n_s$ is a spectral index of the scalar perturbations. The $k_0$ is some arbitrary fixed scale called pivot number. We can relate it to the pivot scale $\lambda_0$ introduced earlier by $\lambda_0 = 2\pi/k_0$. In particular, the WMAP collaboration choice is $k_0 = 0.002 \text{ Mpc}^{-1}$ (we also use this choice in this paper). For this value, the seven years of observations made by the WMAP satellite give the following values of the amplitude and spectral index of the scalar perturbations \(A_s\), \(n_s\)\):

$$A_s = 2.441_{-0.092}^{+0.088} \times 10^{-9}, \quad (4)$$
$$n_s = 0.963 \pm 0.012. \quad (5)$$

On the other hand, the well known prediction of the slow-roll inflation is

$$P_s(k) = \frac{1}{\pi\epsilon} \left( \frac{H}{m_{\text{Pl}}} \right)^2 \left( \frac{k}{aH} \right)^{n_s-1}, \quad (6)$$

where $\epsilon$ is the so-called slow-roll parameter equal to

$$\epsilon = \frac{m_{\text{Pl}}^2}{4\pi} \frac{1}{\phi'^2}. \quad (7)$$

For the considered massive slow-roll inflation $n_s = 1 - 4\epsilon$.

Let us now consider the power spectrum at the length scale $\lambda_0$ which corresponds to the pivot number $k_0$. From observation, an amplitude of the scalar perturbations at this scale is equal to $P_s(k_0) = A_s$. On the other hand, this amplitude is formed when $k \simeq aH$. Therefore, for the mode $k_0$ we have $S = A_s$.

Cosmological evolution of the pivot scale $\lambda_0$ is given by

$$\lambda(a) = \lambda_0 \frac{a}{a_0}. \quad (8)$$

This relation is represented by the red line in Fig. 1. The value of $\lambda$ was equal to the Hubble radius at $a_4$. Based on this, one can derive

$$\Delta_{\text{tot}} = \frac{a_0}{a_4} = \frac{\lambda_0}{\lambda(a_4)} = \frac{H}{k_0}, \quad (9)$$

where $H$ is the value of the Hubble parameter when the $\lambda$ crossed the horizon during the inflation. In the second equality, we have used relation $\lambda(a) = \frac{2\pi}{a_0}$, together with $k \simeq aH$ at the horizon crossing. Namely, $\lambda_0 = \frac{2\pi}{k_0}$ and $\lambda(a_4) = \frac{2\pi}{H a_0}$, where $a_0 = 1$. At the pivot scale, $S = A_s$, so

$$\frac{H}{m_{\text{Pl}}} = \sqrt{\pi\epsilon A_s}. \quad (10)$$
Expressing the $\epsilon$ from $n_s = 1 - 4\epsilon$, we find
\[ \Delta_{\text{tot}} = \frac{m_{\text{Pl}}}{2k_0} \sqrt{\pi(1 - n_s)} A_s. \] (11)

The essential conclusion derived from this equation is that: Based on the CMB observations, one can determine the total increase of the scale factor from the observed moment of inflation till now. In principle, from the WMAP 7 observations we determinate
\[ \Delta_{\text{tot}} = (8.0 \pm 1.5) \cdot 10^{51}. \] (12)

III. INFLATION AND REHEATING

The increase of a scale factor during the part of inflation from $a_4$ to $a_3$ is given by
\[ \Delta_3 = e^{N_{\text{obs}}}, \] (13)
where $N_{\text{obs}}$ is the $e$-folding number, which can be expressed as follows
\[ N_{\text{obs}} \approx -\frac{8\pi}{m_{\text{Pl}}^2} \int_{\phi_{\text{obs}}}^{\phi} \frac{V(\phi)}{V'(\phi)} d\phi \]
\[ = 2\pi^2 \int_{\phi_{\text{obs}}}^{\phi} \frac{2}{1 - n_s}. \] (14)

We have used here $V(\phi) = \frac{2}{3^2} \phi^2$ and defined $\phi_{\text{obs}} = \phi(a_4)$. In particular, based on the WMAP 7 data one can find $N_{\text{obs}} = 54 \pm 18$. The uncertainty is high because of the strong sensitivity on the uncertainty of the spectral index $n_s$. This will later propagate to the uncertainty of the reheating temperature. As we show in Sec. V the uncertainty of $N_{\text{obs}}$ can be significantly reduced with the future CMB experiments.

During the slow-roll inflation, evolution of the inflaton field $\phi$ is well approximated by
\[ \dot{\phi}(t) = \phi_{\text{max}} - \frac{m}{\sqrt{12\pi}} t. \] (15)

From comparison with the numerical results, it can be seen that this approximation holds till the end of inflation, when $\phi \approx 0$. Therefore, the kinetic term
\[ \frac{\dot{\phi}^2}{2} \approx \frac{m^2 m_{\text{Pl}}^2}{24\pi}, \] (16)
is approximately constant during the inflation. This contribution to the total energy density is, however, dominated by the potential part during the slow-roll inflation. At the end of inflation, the contribution from the potential part falls to zero ($V(\phi = 0) = 0$), and the kinetic term dominates. One can therefore estimate that, at the end of inflation, the energy density is given by
\[ \rho(a_3) \approx \frac{m^2 m_{\text{Pl}}^2}{24\pi}. \] (17)
A validity of this approximation was confirmed by the numerical computations.

After inflation, the field starts to oscillate at the bottom of the potential well. During this evolution, the energy density drops as in the matter dominated universe
\[ \rho(a) \approx \frac{\rho(a_3)}{a_3^3}. \] (18)

This evolution holds till $a_2$, when $H \approx \Gamma_\phi$ and the reheating takes place. Then, the energy density
\[ \rho(a_2) = g_* \frac{2\pi^4}{30} T_{\text{RH}}^4, \] (19)
here $g_* = g(T_{\text{RH}})$ is the number of ultrarelativistic degrees of freedom generated during the reheating, where $g(T)$ is defined as follows
\[ g = \sum_{\text{boson}} g_B + \frac{7}{8} \sum_{\text{fermion}} g_F. \] (20)

In particular, for Glashow-Weinberg-Salam (GWS) model $SU(2)_L \otimes U_Y(1) \otimes SU_C(3)$, we have $g = 106.75$. Therefore one may expect that $g_\ast \geq 106.75$ if the temperature of reheating is greater than the electroweak energy scale, $T_{\text{RH}} \gtrsim 300$ GeV.

Based on (18) we have
\[ \Delta_2 = \frac{a_2}{a_3} = \left( \frac{\rho(a_3)}{\rho(a_2)} \right)^{1/3}, \] (21)
and applying (17) and (19) we derive
\[ \Delta_2 = \frac{1}{\pi} \left( \frac{5}{4} \frac{m^2 m_{\text{Pl}}^2}{g_* T_{\text{RH}}^4} \right)^{1/3}. \] (22)

This result will be useful in the subsequent section when deriving the expression on $T_{\text{RH}}$. However, before we proceed to this issue we can see what we already can say about the reheating temperature. Let us notice that the following condition $\rho(a_2) \leq \rho(a_3)$ must be fulfilled. Energy scale of reheating cannot be higher than energy at the end of inflation. In order to use this constraint one has to firstly determine inflaton mass in Eq. (17). It can be done by noticing that the Friedmann equation reduces to
\[ H^2 \approx \frac{8\pi}{3m_{\text{Pl}}^2} \frac{1}{m^2 \phi^2}. \] (23)
in the slow-roll regime ($\epsilon \ll 1$). Based on this and condition $S = A_s$, together with $n_s = 1 - 4\epsilon$, one can derive
\[ m = m_{\text{Pl}} \frac{1}{4} \sqrt{3\pi A_s (1 - n_s)}. \] (24)

Applying this expression to the WMAP 7 results we obtain
\[ m = (1.4 \pm 0.5) \cdot 10^{-6} m_{\text{Pl}}, \]
\[ = (1.7 \pm 0.6) \cdot 10^{13}\text{GeV}. \] (25)
With use of this value, the condition $\rho(a_2) \leq \rho(a_3)$ reduces to

$$g_*^{1/4} T_{RH} \leq 6.5 \cdot 10^{15} \text{ GeV.} \tag{26}$$

Based on this, part of the parameter space $(g_*, T_{RH})$ can be excluded. It was shown as the shadowed region above the thick line in Fig. 2. As we mentioned earlier, if $T_{RH} \geq 300 \text{ GeV}$ then $g_* \geq 106.75$. This constraint excludes another part of the parameter space. This was represented in Fig. 2 as the shadowed region constrained by the vertical line. Based on the above constraints, one can conclude that

$$T_{RH} \leq 2.0 \cdot 10^{15} \text{ GeV.} \tag{27}$$

This corresponds to the inflationary bound on the reheating temperature.

IV. REHEATING TEMPERATURE

After reheating, the Universe is filled by the relativistic plasma. Expansion of this relativistic gas is assumed to be adiabatic and the masses of particles are neglected. The adiabatic approximation is valid until the entropy transfer between the radiation and other components can be neglected. In turn, this second approximation is valid if the temperature is much higher than the masses of the particles. Then, $dS = 0$, which implies $sa^3 = \text{const}$, where the entropy density $s$ of radiation is given by

$$s = \frac{2\pi^2}{45} g T^3. \tag{28}$$

Based on this one can derive expression on the increase of the scale factor from reheating till the recombination

$$\frac{a_1}{a_2} = \frac{T_2}{T_1} \left( \frac{g_2}{g_1} \right)^{1/3}. \tag{29}$$

We have $T_2 = T_{RH}$ and $T_1$ is equal to the recombination temperature $T_{\text{rec}}$. During recombination $g_1 = g_\gamma = 2$ and during reheating $g_2 = g_*$, therefore

$$\Delta_1 = \frac{T_{RH}}{T_{\text{rec}}} \left( \frac{g_*}{2} \right)^{1/3}. \tag{30}$$

Finally, increase of the scale factor from recombination till now is given by

$$\Delta_0 = 1 + z_{\text{rec}}, \tag{31}$$

where $z_{\text{rec}}$ is the recombination redshift which can be determined from the CMB observations. It is worth mentioning that an intermediate stage other than recombination can be used here. In particular, the equilibrium point (end of the radiation epoch, where $\rho_{\text{rad}} = \rho_{\text{mat}}$) can be chosen. The corresponding value of redshift can be also determined from the CMB observations.

At this point, we have all required to find the expression on $T_{RH}$. We have found all $\Delta_i$ and $\Delta_{\text{tot}}$. Based on (24), the following relation is fulfilled

$$\Delta_{\text{tot}} = \Delta_3 \Delta_2 \Delta_1 \Delta_0. \tag{32}$$

Inserting (13), (22), (30) and (31) we obtain

$$\Delta_{\text{tot}} = e^{4 N_{\text{obs}}} \frac{1}{\pi} \frac{5}{4} \frac{m^2 m_{\text{Pl}}^2}{a_*^3 T_{RH}^3} \left( \frac{T_{RH}}{T_{\text{CMB}}} \right)^{1/3} \left( \frac{g_*}{2} \right)^{1/3}, \tag{33}$$

where we have used $T_{\text{rec}} = T_{\text{CMB}}(1 + z_{\text{rec}})$. The important observation is that $g_*$ factors cancel out. This is crucial, because the expression on $T_{RH}$ will be free from the dependence on the unknown $g_*$ parameter. With use of (11), (14) and (24), the above equation can be rewritten into the following form

$$T_{RH} = 15 \frac{m_{\text{Pl}}}{16 \cdot \pi^{7/2}} \sqrt{\frac{1 - n_s}{A_*}} \left( \frac{k_0}{T_{\text{CMB}}} \right)^3 \exp \left\{ \frac{6}{1 - n_s} \right\} \cdot \tag{34}$$

This equation is a main result of this paper. Taking the constant parameters $T_{\text{CMB}} = 2.725 \text{ K} = 2.348 \cdot 10^{-4} \text{ eV}$ and $k_0 = 0.002 \text{ Mpc}^{-1}$ (and reexpressing units: Mpc$^{-1}$ = $6.39 \cdot 10^{-30} \text{ eV}$) one can rederive Eq. (34) to the practical form

$$T_{RH} = 3.36 \cdot 10^{-68} \sqrt{\frac{1 - n_s}{A_*}} \exp \left\{ \frac{6}{1 - n_s} \right\} \text{ GeV.} \tag{35}$$

In Fig. 3 we show relation (35) as a function of the spectral index $n_s$. We also mark the regions excluded from the inflationary constraint and the BBN constraint. For the data from the WMAP 7 observations, Eq. (35) leads to

$$T_{RH} = 3.5 \cdot 10^6 \text{ GeV.} \tag{36}$$

The relative uncertainty of this result is

$$\frac{\sigma(T_{RH})}{T_{RH}} \approx 53. \tag{37}$$
Here a first order Taylor expansion was applied when calculating propagation of uncertainties:

$$\sigma(T_{RH}) \approx \sqrt{\left(\frac{\partial T_{RH}}{\partial n_s}\right)^2 \sigma^2(n_s) + \left(\frac{\partial T_{RH}}{\partial A_s}\right)^2 \sigma^2(A_s)}.$$  

(38)

However, due to the strong (exponential) dependence of $T_{RH}$ on $n_s$, the applied linear approximation may turn out to be insufficient. Therefore, one can expect greater uncertainty of $T_{RH}$ than obtained here. Future studies need to address this issue. The high relative uncertainty is mainly a result of the weakly determined value of $N_{\text{obs}}$, which is a function of $n_s$. In the next section, we will examine how this uncertainty can be reduced with the future CMB experiments.

As it was discussed in Refs. [11, 12], if $T_{RH} \sim 10^{-9}$ GeV, then it may be possible to measure $T_{RH}$ with the planned space-based laser interferometer experiments such as the Big Bang Observer. The value $T_{RH} = 3.5 \cdot 10^6$ GeV obtained here fulfills this condition. Our prediction has therefore chance to be verified in future.

Furthermore, based on Eq. (1), one can express the inflaton decay rate $\Gamma_\phi$ in terms of $g_*:

$$\Gamma_\phi \approx 1.7 \cdot 10^{-6} \sqrt{g_*} \text{ GeV},$$  

(39)

where the previously derived value of $T_{RH}$ was used. It is reasonable to expect that $\Gamma_\phi = \alpha \cdot m$, which comes from the Heisenberg uncertainty relation. The $\alpha$ is a dimensionless parameter. With use of the inflaton mass found earlier we find

$$\alpha \approx 10^{-19} \sqrt{g_*}.$$  

(40)

Based on this result, one can deduce that the inflaton decays into the very light particles comparing with its mass. This is also the reason why the reheating takes place at the relatively low energies. It becomes unstable only when the sufficiently low energies are reached. However, the origin of this low vale of decay rate $\Gamma_\phi$ cannot be understood without a deeper understanding of the inflationary cosmology.

V. FORECASTING

As we have shown in the previous section, the value of $T_{RH}$ is strongly dependent on $n_s$. Therefore, the method presented can be used effectively only if the value of $n_s$ is determined with high precision. The value of $n_s$ from the WMAP 7 observations is not determined sufficiently precise to obtain a strong prediction concerning the reheating temperature. However, it may change if the new observational data will be available. In this section, we predict how the uncertainty on $T_{RH}$ will be reduced with the future CMB experiments. In particular, we consider the Planck satellite [13] experiment which is currently on the stage of collecting data. We consider the ACTPol [14] ground-based experiment which is under construction at present. We also consider the planned CMBPol [15] satellite experiment.

The uncertainty of $T_{RH}$ comes mainly from $n_s$, therefore, in the considerations we fix the value of $A_s$. Following Ref. [16], the expected uncertainties of $n_s$ from the mentioned CMB experiments are the following

$$\sigma(n_s) = \begin{cases} 
0.0031 \quad \text{Planck} \\
0.0021 \quad \text{Planck+ACTPol} \\
0.0014 \quad \text{CMBPol}
\end{cases}$$  

(41)

Based on this, let us first see the resulting uncertainties of the $e$-folding number $N_{\text{obs}}$. We find

$$\sigma(N_{\text{obs}}) = \begin{cases} 
4.5 \quad \text{Planck} \\
3.1 \quad \text{Planck+ACTPol} \\
2.0 \quad \text{CMBPol}
\end{cases}$$  

(42)

This significant reduction of the uncertainty of $N_{\text{obs}}$ (with respect to the WMAP 7 results) will be crucial for determining the reheating temperature. Based on (41) with (42), we forecast

$$\frac{\sigma(T_{RH})}{T_{RH}} = \begin{cases} 
13.5 \quad \text{Planck} \\
9.2 \quad \text{Planck+ACTPol} \\
6.1 \quad \text{CMBPol}
\end{cases}$$  

(43)

Here, the values of the parameters $n_s$ and $A_s$ were set to be those obtained from the WMAP 7 observations.

In order to have $\sigma(T_{RH})/T_{RH}$ smaller than unity, the uncertainty of $n_s$ should be reduced by 2 orders of magnitude with respect to the WMAP 7 results. At present, there is however no experiment planned to reach such sensitivity. The uncertainty may be nevertheless additionally reduced by combining data from the different available experiments. This is possible because of the angular scale dependent sensitivity of the CMB experiments. In particular, the ground-based experiments can provide much better data of the CMB polarization at the small angular scales (high multipoles) than the space-based experiments can.
VI. SUMMARY

In this paper, we have developed a new method of constraining the reheating phase after inflation. The method bases on the observations of the cosmic microwave background radiation. In particular, the fact that the total increase of the scale factor from the observed part of inflation till now can be determined is used. Based on this, we have found the expression on the reheating temperature. The expression is free from the dependence on the unknown $g^\ast$ parameter. With use of the WMAP 7 results, we have determined \( T_{RH} = 3.5 \times 10^3 \) TeV. The relative uncertainty of this result is equal to \( \sigma(T_{RH})/T_{RH} \approx 53 \). This high uncertainty can be, however, significantly reduced with the future CMB data. One can expect the reheating temperature to be quite precisely determined (reaching \( \sigma(T_{RH})/T_{RH} = O(1) \)) within the present decade.

The value of reheating temperature determined in this paper is consistent with the known bounds, in particular, with the lower bound \( T_{RH} \gtrsim 6 \) TeV recently found in Ref. [7]. It is also interesting to note that within the supersymmetric extension of the standard model, the upper bound on the reheating temperature exists \( T_{RH} \lesssim 10^4 \) TeV (see Refs. [17, 19]). Our result is also in agreement within this condition. Finally, it is worth mentioning that the low value of the reheating temperature, as determined here, can have interesting implications on the phenomenology of primordial black holes [20].

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