The role of roughness amplitude on depth distribution of contact stresses

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Abstract. The normal pressure distribution on contact area and depth distribution of von Mises equivalent stress are of major importance in prediction of various failure phenomena (e.g. rolling contact fatigue, scuffing) of the contacting surfaces. The paper reveals the relationship between roughness amplitude and stress state developed in the loaded material. The study was performed on a double rows spherical roller bearing under three levels of radial loading, and three significant values for the surface roughness. The numerical simulation was used to generate Gaussian rough surfaces with imposed values for $R_a$ parameter. A semi-analytical method was used to solve the non-hertzian contact between rough surfaces. For the same loading level, the depth distribution of von Mises stress for rough surfaces is close to the distribution found for smooth surfaces, except the shallow layer close to the contacting surfaces. For medium and especially high loads, the contact between rough surfaces develops, inside this shallow layer, von Mises equivalent stresses higher than the fatigue limit stress.

1. Introduction
In a rolling bearing the contact loads are attained between two non-conformal rough surfaces. The micro-contact areas and the corresponding pressures distribution depend significantly upon the micro-topography of the interacting surfaces. The main functional parameters of rolling bearings possible to be influenced by micro-topography of the contacting surfaces are: rolling bearing life, friction and corresponding efficiency; the limit rotational speed, vibration and noise.

The roughness acts as stress concentration sites and induces stresses many times greater than in an equivalent smooth contact. Peeling is known to occur from cracks originating at a depth equal to 2.5 µm … 5 µm from the surface, and subsurface originated fatigue commonly occurs at depth of approximately 25 µm … 100 µm below the surface. Although the effect of the roughness is often very shallow, the magnitude of the stresses involved are considerable so that, as the quality of steels improved, the failure of rolling contacts observed to initiate from the surface, [1]. According to ISO 281:2007 [2] the methods of calculating the basic dynamic load rating of rolling bearings manufactured from high quality hardened steel are based on conventional design as regards profiles of rolling contact surfaces with good manufacturing practice as regards roughness of the contacting surfaces.

The objective of present paper is to investigate the influence of the roughness amplitude on depth distribution of contact stresses. A numerical simulation program was developed to obtain the pressures distributions due to various surface roughness under low, medium and high loading. These pressures...
distributions were further used to find in-depth distribution of stress tensor components and von Mises equivalent stress.

2. Operating conditions

2.1. Spherical roller bearing

The investigation was conducted on a spherical roller bearing type 24038 ECA/W33S1. For this bearing the main catalogue data are: inner diameter \( d = 190 \) mm, outer diameter \( D = 290 \) mm, width \( B = 100 \) mm, inner ring chamfer radius \( r_{ch,i} = 3 \) mm, basic dynamic load rating \( C_r = 978 \) kN, basic static load rating \( C_0 = 1800 \) kN, fatigue load limit \( C_u = 163 \) kN, reference speed \( n = 1300 \) rpm, limiting speed \( n_{lim} = 1500 \) rpm.

The concentrated contacts achieved inside the bearing, schematically presented in figure 1, are characterized by the following geometrical data:

- for spherical roller: roller diameter \( D_w = 24.5 \) mm, radius of roller profile \( R_{w2} = 133 \) mm, roller length \( L_w = 39.4 \) mm, roller end chamfer \( R_{w2, ch} = 1.0 \) mm, number of spherical rollers \( Z_w = 2 \times 26 \) and contact angle \( \alpha = 11.4^\circ \);
- curvature radii of the inner raceway: in transverse plane \( r_{ci,1} = 111 \) mm, and in axial plane \( r_{ci,2} = 136 \) mm;
- curvature radii of the spherical outer raceway: \( r_{co,1} = r_{co,2} = 135 \) mm.

![Figure 1. Contact geometry in double rows spherical roller bearing.](image)

2.2. Operating conditions

Three levels: low, medium and high were considered for the radial load applied on spherical roller bearing. The first step was to define what means a high loading. In order to avoid edge effects, plastic deformations and ratcheting phenomena a maximum Hertz pressure lower than 2.4 GPa is suggested.
to be created by the normal load, [1]. When spherical roller bearings are subjected to endurance test there are standards limitations for the radial load, \( F_r \leq 0.3 \cdot C_r \).

Accordingly, a value of 300 kN has been adopted as high level radial force.

The distribution of radial force on spherical rollers, as shown in figure 2, was calculated considering the value of the effective internal radial clearance which takes into account the measured clearance and its modifications due to both interference fits and temperature differences between the involved components.

3. Pressures distributions

3.1. Necessity for a semi-analytical method (SAM)

When edge effect manifests, the concentrated contact achieved in a spherical roller-raceway contact becomes a non-Hertzian concentrated contact. The concentrated contacts between bodies with rough surfaces are typical non-Hertzian contacts. For non-Hertzian concentrated contacts there is not an analytical solving method. To find the pressures distribution on the common contact area, and further the stresses states developed in the stressed materials, numerical methods and corresponding computing codes have to be used. In cases where concentrated contacts are involved the Finite Element Method (FEM) proved to be computing time consuming when accurate results were needed and causes the FEM approach unaffordable when a large number of case studies are required.

Essentially, the particular analytic equations developed in theory of concentrated contacts [3] - [5], are converted into numerical formulations to express a semi-analytical method, [6] - [13]. Using a today personal computer a non-Hertzian concentrated contact is solved thousands times faster by a SAM comparing with FEM.

3.2. Digital formulation of SAM

The real contact area can be considered as a system of isolated areas \( a_{ri} \).
A hypothetical rectangular contact area \( A_h \) is built on the common tangent plane around the initial contact point, and it is chosen large enough to overestimate the unknown real contact area \( A_h > A_r \).

The model of surface deformation is defined by three equations: the geometric equation of the elastic contact, the integral equation of the normal surface displacement of the elastic half space (Boussinesq formula) and the load balance equation. The constraints of non-adhesion and non-penetration are also considered.

The discrete formulation is represented by the system of algebraic equations (7) – (10):

\[
g_{ij} = h_{ij} - R_{ij} + w_{ij} - \delta_0
\]  \hspace{1cm} (7)

\[
w_{ij} = \sum_{k=0}^{N_x-1} \sum_{l=0}^{N_y-1} K_{i-k,j-l} \cdot P_{kl}
\]  \hspace{1cm} (8)

\[
(\Delta x \cdot \Delta y) \sum_{k=0}^{N_x-1} \sum_{l=0}^{N_y-1} p_{ij} = F
\]  \hspace{1cm} (9)

\[
g(x,y) = 0, \quad p(x,y) > 0, \quad (x,y) \in A_r
\]  \hspace{1cm} (10)

\[
g(x,y) > 0, \quad p(x,y) = 0, \quad (x,y) \notin A_r
\]

The influence function \( K_{ij} \) describes the deformation of the discretized surface due to the unit pressure acting in element \((k,l)\).

Details on the SAM used are given in [5], [10], [11].

### 3.3. Computation algorithms

For computations presented in this paper, the uniformly spaced rectangular arrays were built with \( N_x = 128 \), \( N_y = 128 \) and \( N_z = 128 \). A single-loop conjugate gradient method, (CGM), has been used to solve the mentioned algebraic system of equations, [5], [7] to [11]. To increase the efficiency of the numerical algorithm, a dedicated discrete fast Fourier transform routine for 3D contact problems, (DC-FFT), has been used to solve the convolution products implicated in the calculation of the contact pressure, internal stress field and surface displacements, [5], [11].

### 3.4. Pressure distributions inside the most loaded roller contacts - smooth surfaces case

For an external radial force of 300 kN and using the classical approach of smooth surfaces, the figure 3 and figure 4 present the pressures distributions between the most loaded roller and outer and inner raceways, respectively. For the case of outer raceway contact, the pressures distribution reveals an incipient alteration of the hertzian ellipsoid of pressures. On the outer raceway the length of major axis of hertzian contact ellipse is 39.934 mm while the effective contact length of spherical roller is 37.4 mm. For the inner raceway the hertzian contact ellipse has a major axis of 37.750 mm and consequently no significant edge effect manifests.
Figure 3. Pressures distribution achieved by the most loaded roller on the outer raceway.

Figure 4. Pressures distribution achieved by the most loaded roller on the inner raceway.

Figure 5 presents: (a)-pressures distribution and von Mises stress in the xOz plane, (b)- the main stress along Oz axis, (c)-Von Mises and maximum shear stress along Oz axis, as developed in the contact of the most loaded roller and inner raceway.
Figure 5. (a)-Pressures distribution and von Mises stress in xOz plane, (b)- the main stresses along Oz axis, (c)-Von Mises and maximum shear stress along Oz axis.

4. Simulation of rough surfaces

4.1. Method to generate a random roughness with a Gauss distribution

The target of the numerical simulation was to generate Gaussian rough surfaces ($S_k = 0$, $K = 3$) with imposed values for $Ra$ parameter.
To generate a rough surface which has a Gaussian distribution it was needed to transform a random input sequence into an output sequence which has similar values with those we wish to impose for skewness and kurtosis. To obtain the values of skewness ($Sk_z$) and kurtosis ($K_z$) parameters close to the required ones, the relationships proposed by Watson and Spedding [14] have been used.

When arbitrary skewness and kurtosis are set they must fulfill the following relationship:

$$\alpha_z - Sk_z^2 - 1 \geq 0.$$  \hspace{1cm} (6)

A numerical method based on 2D digital filter technique, as suggested by Hu and Tonder [15] has been chosen to change the input sequence $\eta'(k,l)$ to an output sequence $z(i,j)$. The flow chart of the computation algorithm is presented in figure 6. Details are given in [16] to [19].

4.2. Imposed data for roughness simulation

The surface roughness corresponding to “good manufacturing practice” was adopted as $Ra_i = 0.16 \mu m$, $Ra_w = 0.12 \mu m$ which determine the value $Ra_\Sigma = 0.25 \mu m$.

Finer surface roughness was generated with the values $Ra_i = 0.12 \mu m$ on inner raceway, $Ra_w = 0.08 \mu m$ on roller surface resulting the composed roughness $Ra_\Sigma = 0.18 \mu m$.

A coarser roughness was generated with the values of $Ra_i = 0.25 \mu m$ on inner raceway, $Ra_w = 0.134 \mu m$ on roller surface that imply $Ra_\Sigma = 0.35 \mu m$.

4.3. Load levels

For low and medium levels of radial load, the values $F_r = 0.05 \cdot C_r$ and $F_r = 0.1 \cdot C_r$ have been considered [5]. For the high level of radial load the value $F_r = 0.3 \cdot C_r$ has been applied. As was previously exposed, a higher value of the radial load would create edge pressure on the contact area achieved between spherical roller and outer raceway.
5. Results and comments
A number of nine case studies (figure 7) have been taken into account as combinations of the mentioned values for surface roughness and load levels. The particular value of the normal load achieved between the most loaded spherical roller and inner raceway was considered in each case study.

Figure 7. The depth evolutions of von Mises equivalent stresses.

For reasons of comparisons, for each case study the computation provided the depth evolutions of equivalent von Mises stresses referring to:
- rough surfaces,
- smooth surfaces,
- smooth surfaces and the fatigue load limit as radial load (\( F_r = C_u \)).

The results graphically summarized in figure 7 were obtained with the data normalized to the values corresponding to the smooth surfaces case.
For the same loading level, the depth distribution of von Mises stress for rough surfaces is close to the distribution found for smooth surfaces, except the shallow layer.

Details of the von Mises stresses inside the shallow layer are pointed out in figure 8.

For medium and especially high radial loads, the contact between rough surfaces develops, inside this shallow layer, von Mises equivalent stresses higher than the fatigue limit stress. For condition of poor lubrication, these findings explain the initiation of the rolling contact fatigue in the shallow layer close to contacting surfaces.

**Figure 8.** The distribution of von Mises equivalent stresses inside the shallow layer.

### 6. Conclusions

The paper reveals the relationship between roughness amplitude and stress state developed in the loaded material.

For the same loading level, the depth distribution of von Mises stress for rough surfaces is close to the distribution found for smooth surfaces, except the shallow layer close to the contacting surfaces.

For medium and especially high radial loads, the contact between rough surfaces develops, inside the shallow layer, von Mises equivalent stresses higher than the fatigue limit stress.
For condition of lack of lubricant or poor lubrication, these findings explain the initiation of the rolling contact fatigue in the shallow layer, close to contacting surfaces.

**Nomenclature**

- $b_H$: minor half-axis of the hertzian contact area, in millimeters
- $C_r$: basic dynamic radial load rating, in newtons
- $C_u$: fatigue load limit, in newtons
- $D_w$: roller diameter, in millimeters
- $F_r$: bearing radial load, in newtons
- $g_{ij}$: gap, in millimeters, between surfaces after deformation, at the grid point $(i, j)$
- $g(x, y)$: gap, in millimeters, between surfaces after deformation, at the point $(x, y)$
- $h_{ij}$: separation, in millimeters, between surfaces when no load is applied, at the grid point $(i, j)$
- $i$: number of rows of rolling elements
- $K$: kurtosis parameter
- $K_{i-l,j-k}$: influence coefficient
- $n$: speed of rotation, in revolutions per minute
- $P_{xx}$: pressure distribution along x-x axis, in megapascals
- $P_{xy}$: pressure distribution, in megapascals
- $P_{yy}$: pressure distribution along y-y axis, in megapascals
- $p_{ij}$: pressure, in megapascals, at the grid point $(i, j)$
- $p(x, y)$: pressure, in megapascals, at the point $(x, y)$
- $R_a$: arithmetic average of the surface roughness heights, in micrometers
- $R_{\Sigma}$: arithmetic average of the composed roughness, in micrometers,
  \[ R_{\Sigma} = 1.25 \cdot \left( R_{a_{raceway}}^2 + R_{a_{roller}}^2 \right)^{1/2} \]
- $R_{ij}$: surface roughness, in micrometers, at the grid point $(i, j)$
- $Q_j$: radial load on the roller $j$, in newtons
- $Q_{A,max}$: maximum radial load on the roller from the row A, in newtons
- $Q_{B,max}$: maximum radial load on the roller from the row B, in newtons
- $Q_{max}$: maximum radial load on the roller, in newtons
- $S_K$: skewness parameter
- $w_{ij}$: composite normal surface deflection, in millimeters, for the grid point $(i, j)$
- $Z$: number of spherical rollers
- $Z(x, y)$: height of a rough surface, in millimeters, at the point $(x, y)$
- $\alpha$: nominal contact angle, in degrees, of a bearing
- $\delta_0$: rigid bodies approach
- $\sigma_H$: maximum pressure in a hertzian distribution
- $\sigma_{vMises}$: von Mises equivalent stress, in megapascals
- $\sigma_{xx}$: normal stress tensor component along x-x axis, in megapascals
- $\sigma_{yy}$: normal stress tensor component along y-y axis, in megapascals
- $\sigma_{zz}$: normal stress tensor component along z-z axis, in megapascals
- $e$: subscript for outer ring
i subscript for inner ring
w subscript for roller

7. References
[1] Harris T A and Kotzalas M N 2007 Rolling Bearings Analysis – Advanced Concepts of Bearing Technology (CRC Taylor & Francis Group)
[2] ISO 281: 2007 Rolling bearings—dynamic load rating and rating life (Geneva: ISO)
[3] Solomon L 1968 Elasticité Lineaire (Paris: Masson et Cie)
[4] Johnson K L 1985 Contact Mechanics (London: Cambridge University Press)
[5] Crețu S 2009 Contactul Concentrat elastic-plastic (Iasi: Politehnium)
[6] Hartnett M J 1979 The analysis of contact stresses in rolling element bearings ASME Journal of Lubrication Technology 101 pp 105-109
[7] Polonsky I A and Keer M L 1999 A numerical method for solving rough contact problems based on the multi-level multi-summation and conjugate gradient techniques Wear 231 pp 206–219
[8] Polonsky I A and Keer M L 2000 Fast methods for solving rough contact problems ASME Journal of Tribology 122 pp 36-45
[9] Liu S, Wang Q and Liu G 2000 A versatile method of discrete convolution and FFT (DC-FFT) for contact analyses Wear 243 pp 101-111
[10] Crețu S and Antaluca E 2003 The study of non-hertzian concentrated contacts by a GC-DFFT technique Annals of University of Galati VIII pp 39-47
[11] Crețu S 2005 Pressure distributions in concentrated rough contacts Buletinul Institutului Politehnic Iasi LI pp 1-31
[12] Allwood J 2005 Survey and performance assessment of solution methods for elastic rough contact problems ASME Journal of Tribology 127 pp 10-23
[13] Nélias D, Antaluca E, Boucly V and Crețu S 2007 A 3D semi-analytical model for elastic-plastic sliding contacts ASME Journal of Tribology 129 pp 761-771
[14] Wattson W and Spedding T 1982 The time series modeling of non-Gaussian processes Wear 83 pp 215-231
[15] Hu Y Z and Tonder K 1992 Simulation of 3D random rough surface by 2D digital filter and Fourier analysis International Journal of Machine Tools Manufacturing 32 pp 83-90
[16] Crețu S 2006 The influence of the correlation length on pressure distribution and stress state in concentrated rough contacts Proceedings of ASME/ASLE/ITC-2006 15 paper 12339
[17] Tufescu A and Crețu S 2012 Analysis of stress state developed in contacting elements with non-Gauss rough surfaces Buletinul Institutului Politehnic din Iasi LVII (LXI) 2 pp 71–80
[18] Urzica A, Balan M R and Crețu S 2012 Pressure distributions and depth stresses developed in concentrated contacts between elements with non-Gaussian rough surfaces Proceedings of the ASME Biennial Conference on Engineering System Design and Analysis ESDA 2012, Nantes France pp 1-8
[19] Urzica A and Crețu S 2013 Simulation of the non-gaussian roughness with specified values for the high order moments Journal of the Balkan Tribological Association 19(3) pp 391-400