The authors propose a new method for finding the possible decompositions of a tensor, based on the notion of confinement and on catalecticant maps.

The possible decompositions of tensors of sub-generic rank lie into special sub-varieties, called contact loci, of the variety $X$ of rank-1 tensors. However, for tensors of generic rank, such sub-varieties are meaningless and new methods are needed.

For a symmetric tensor $f \in \text{Sym}^d \mathbb{C}^{n+1}$, the image of the catalecticant map $C_f : \text{Sym}^h(\mathbb{C}^{n+1})^\vee \to \text{Sym}^{d-h} \mathbb{C}^{n+1}$ lies into the span $\Lambda$ of the $(d-h)$-th powers of linear forms in which $f$ decomposes, thus the sub-variety confining the possible decompositions of $f$ is the intersection $Y := \Lambda \cap X_{d-h}^n$ of $\Lambda$ with the $(d-h)$-th Veronese variety $X_{d-h}^n$.

The authors extend the catalecticant method and apply it to study the simultaneous Waring decompositions of a given number of forms, or equivalently the decompositions of vectors of forms.

The main working setting is the so-called perfect case: given $X_{a_1,\ldots,a_r}^n = \{(l^{a_1},\ldots,l^{a_r}) \mid l \in \mathbb{P}^n\}$ the variety of rank-1 polynomial vectors in $\mathbb{P}(\text{Sym}^{a_1} \mathbb{C}^{n+1} \oplus \ldots \oplus \text{Sym}^{a_r} \mathbb{C}^{n+1}) \simeq \mathbb{P}^N$, $f$ is assumed to belong to the $k$-th secant variety $S^k(X_{a_1,\ldots,a_r}^n)$ of expected dimension $N = k(dim X_{a_1,\ldots,a_r}^n) + 1$. The set of parameters defining the setting is denoted by $(n,r; a_1,\ldots,a_r)$.

In Sections 3 and 4 the authors analyze the perfect case with parameters $(2,3;3,3,3)$: through a catalecticant approach they show that the confinement for a general $f \in (\text{Sym}^3 \mathbb{C}^{3})^\oplus 1$ is a sextic elliptic normal curve (Proposition 4.1), and as a consequence such $f$ admits two simultaneous Waring decompositions with $k = 6$ summands (Theorem 4.4).

In Sections 5 and 6 the case $(2,4;4,4,4,4)$ is studied: the catalecticant approach for a general $f \in (\text{Sym}^4 \mathbb{C}^{3})^\oplus 4$ leads to a general threefold section as confinement (Proposition 6.1). As a consequence, it is computationally obtained that a such general $f$ admits 18 simultaneous Waring decompositions with $k = 10$ summands (Theorem* 6.5), but this number is theoretically proved to be just a lower bound (Theorem 6.9).

Reviewer: Vincenzo Galgano (Povo)

MSC:

14N07 Secant varieties, tensor rank, varieties of sums of powers
14N05 Projective techniques in algebraic geometry
14P05 Real algebraic sets
14Q99 Computational aspects in algebraic geometry
15A69 Multilinear algebra, tensor calculus

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Waring decomposition; unidentifiable case; elliptic curve; catalecticant map

Software:

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