On the shape of a rapid hadron in QCD.

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Abstract

We visualize the fundamental property of pQCD: the smaller size of the colorless quark-gluon configurations leads to a more rapid increase of its interaction with energy. Within the frame of the dipole model we use the $k_t$ factorization theorem to generalize the DGLAP approximation and/or leading ln($x_0/x$) approximation and evaluate the interaction of the quark dipole with a target. In the limit of fixed $Q^2$ and $x \to 0$ we find the increase with energy of transverse momenta of quark(antiquark) within the q$\bar{q}$ pair produced by the strongly virtual photon. The average $p_t^2$ is evaluated analytically within the double logarithmic approximation. We demonstrate that the invariant mass $M^2$ of the q$\bar{q}$ pair increases with the energy as $M_0^2(x_0/x)^\lambda$, where $\lambda \sim 0.4\alpha_sN_c/\pi$ for transverse photons, and as $\sim M_0^2 \exp 0.17(4\alpha_sN_c/\pi)\log(x_0/x)^{1/2}$ for longitudinal photons, where $M_0^2 \approx 0.7Q^2$ at the energies of the order $s_0 \sim 10^4$ GeV$^2$ ($x_0 \sim 10^{-2}$). The magnitude of the effect depends strongly on the small $x$ behavior of the gluon distribution. Similar pattern of the energy dependence of $M^2$ is found in the LO DGLAP approximation generalized to account for the $k_t$ factorization. We discuss the impact of the found phenomenon on the dependence of the coherence length on the initial energy and demonstrate that the shape of the final hadron state in DIS has the biconcave form instead of the pancake. Some implications of the found phenomena for the hard processes in pp collisions are discussed.
I. INTRODUCTION.

A dipole model developed in ref. [1], cf. also [2, 3, 4, 5, 6] is the generalization of the parton model to the target rest frame description. It accounts for the effects of the $Q^2$ and $\ln(x_0/x)$ evolutions. It also provides the solution of the equations of QCD in the kinematics of fixed and not too small $x = Q^2/\nu$ but $Q^2 \to \infty$. The characteristic feature of this solution is the approximate Bjorken scaling for the structure functions of DIS, i.e. the two dimensional conformal invariance for the moments of the structure functions. In this approximation as well as within the leading $\ln(x_0/x)$ approximation, the transverse momenta of quarks within the dipole produced by the local electroweak current are restricted by the virtuality of the external field:

$$\lambda^2_{QCD} \leq p_t^2 \leq (Q^2)/4. \quad (1.1)$$

The aim of the present paper is to demonstrate that the transverse momenta of (anti)quark of the $q\bar{q}$ pair produced by a local current also increase with energy and become larger than $Q^2$ at sufficiently large energies. Technically this effect follows from the more rapid increase with the energy of the pQCD interaction for smaller dipole and $k_t$ factorization. Let us note that this phenomena is different from the well known Lipatov diffusion. The latter means that within the leading log $\alpha_s \ln(x_0/x)$ approximation the parton transverse momenta are increasing with energy in the center of rapidity as $\ln^2(p_t^2/p_{t0}^2) \propto \ln(s/s_0)$ as a result of the diffusion in the space of transverse momenta\[7\].

Within the double logarithmic approximation we evaluate analytically both the maximum in the distribution over the invariant masses of the $q\bar{q}$ pair which contribute to the transverse and logitudinal total cross section of DIS, and the corresponding average transverse momenta squared.

Consider first the case of the longitudinal photons. Then the position of the maximum increases with energy as

$$M^2 = M_{1L}^2(s/s_0)^{\alpha_s(N_c/\pi)/9}. \quad (1.2)$$

Eq.1.2 is derived in the approximation $Q^2 \ll M^2 \ll s$ which is self-consistent at sufficiently high energies. One can see from this expression that that transverse momenta of quarks increase with the energy since $M^2 = p_t^2/z(1-z)$, and the configurations with $z = 1/2$ dominate at sufficiently high energies.

The dependence of the average quark transverse momenta on energy is calculated below numerically within the double logarithmic approximation and/or within the LO DGLAP and BFKL approximations. For certainty we define average transverse momentum of quark as corresponding to the median of integral for the total cross section. Within the double logarithmic approximation to the cross section initiated by longitudinal photon we obtain:

$$M_{2L}^2 \sim 4p_t^2 \sim 0.7Q^2 \exp(0.17((\alpha_sN_c/\pi) \log(x_0/x))^{0.55}). \quad (1.3)$$

Here $x_0 \sim 0.01$. The analysis was done in the interval $s = 10^4$ to $s = 10^{11}$ GeV$^2$. Note that the derived rate of the increase with the energy of the characteristic scale does not depend on the external virtuality $Q^2$. However, $M_{2L}^2$ depends on the normalisation point in $x_0$ and $Q_0^2$. It is worth emphasizing that since we are interested here in the proof of the rise of the transverse momenta in the current fragmentation region, we carry for the illustration, the calculations over a very wide spectrum of energies $s \sim 10^4 \div 10^{11}$ GeV$^2$. The detailed calculations for the realistic energies have been carried in the LO approximation using the CTEQ5L gluon pdfs [9, 10]. Qualitatively they produce similar results although depended
on chosen extrapolation to small $x$. In particular the CTEQ6L parametrization leads to a significant suppression of the effects discussed in the paper.

Similar results were obtained for the transverse photons. In this case we were able to carry out an analytical calculation for the invariant mass distribution maximum for the symmetric configurations and found that it rapidly increases with energy:

$$M_{1T}^2 \sim M_0^2(s/s_0)^{\alpha_s(N_c/\pi)/4}. \quad (1.4)$$

The analytical results has been obtained in the kinematics: $Q^2 \ll M^2 \ll s$. It is well known however that in the case of the transverse photons a major role in a wide kinematical region is played by $q\bar{q}$ configurations where one of the partons carries most of the plus component of the photon momentum. With increase of the energy the role of asymmetric configurations is reduced since their contribution grows with energy more slowly. In order to take into account the asymmetric configurations we have made a numerical calculation of a transverse cross-section in the interval $s = 10^4 - 10^{11}$ GeV$^2$, and obtained:

$$M_T^2 \sim 0.7Q^2(x_0/x)^{0.4\alpha_s N_c/\pi}, \quad (1.5)$$

$x_0 \sim 0.01$.

If we take into account the increase of the transverse momenta of the dipole $p_t^2$ with energy within the framework of the dipole model and the $k_t$ factorization theorem we are lead to the generalization of the DGLAP [8] and BFKL [7] approximations which is done in the paper within the LO approximation.

The rapid increase of the characteristic transverse scales in the fragmentation region has been found first in ref. [11, 12, 13, 14], within the black disk (BD) regime. Our new result is the prediction of the increase with energy of the jet transverse momenta in the fragmentation region, in the kinematical domain where methods of pQCD are still applicable. This effect could be considered as a precursor of BD regime indicating the possibility of smooth matching between two regimes.

As the application of obtained results we obtain that in pQCD

$$\sigma_L(x, Q^2)/\sigma_T(x, Q^2) \propto (Q^2/4p_t^2) \propto (Q^2/s)^\lambda. \quad (1.6)$$

i.e. this ratio should decrease as the power of energy instead of being $O(\alpha_s)$.

The increase of the parton momenta in the DIS in the current fragmentation region leads to the change of many characteristics of high energy processes. We find that the coherence length of the DIS processes increases with energy within pQCD as

$$\propto (1/2m_N)(s/Q^2)^{1-\lambda}, \quad (1.7)$$

i.e. slower than in the parton model $(1/2m_Nx$ - the Ioffe length). This is the because the coherence length for a given process follows from uncertainty principle:

$$l_c = (s/2m_N)/(M^2(s) + Q^2), \quad (1.8)$$

where $M^2(s) \propto p_t^2(s)$ is the typical $M^2$ important in the wave function of photon in the target rest frame and $p_t$ is the transverse momentum of constituents in the wave function of photon. This result has the implication for the space structure of the wave packet describing a rapid hadron. In the classical multiperipheral picture of Gribov a hadron has a shape of a pancake of the longitudinal size $1/\mu$ (where $\mu$ is the soft scale) which does not depend on
On the contrary, we find the biconcave shape for the rapid hadron in pQCD with the minimal longitudinal length for small impact parameter $b$ decreasing with increase of energy and being smaller for nuclei than for the nucleons.

The paper is organized in the following way. In section 2 using the technique first introduced in QED by V.Gribov [16], we rewrite the formulae of the dipole model for the inelastic cross-section of DIS in the form of the spectral representation over invariant masses for both longitudinal and transverse photons. $k_t$ factorization [17, 18] is explicitly fulfilled in this representation.

The analysis of these formulae predicts increase with energy of transverse quark momenta in the current fragmentation region. In section 3 we use the double logarithmic approximation for the amplitude for the interaction of quark dipole with the target, to evaluate the increase with the energy of the quark transverse momenta in the current fragmentation region. In section 4 we study the dependence of coherence length on the collision energy. In the section 5 we explain that in pQCD rapid hadrons and nuclei look like bi-concave lenses. Finally, in section 6 we discuss the possible applications of our results to pp, pA collisions at the LHC.

II. THE TARGET REST FRAME DESCRIPTION.

Within the LO approximation the QCD factorization theorem allows to calculate the total cross section of the longitudinally polarized strongly virtual photon scattering off a hadron target through the convolution of the virtual photon wave function calculated in the dipole approximation and the cross section of the dipole scattering off a hadron. In the target rest frame the cross section for the scattering of longitudinally polarized photon has the form [1, 5, 19, 20]:

\[ \sigma(\gamma^*_L + T \rightarrow X) = e^2 \frac{12}{12\pi^2}\alpha_s \int dp_t dz \bra \psi_{\gamma^*_L}(p_t, z) \right| \sigma(s, p_t^2) \left| \psi_{\gamma^*_L}(p_t, z) \ket. \]

(2.1)

Here $\sigma$ is the dipole cross-section operator:

\[ \sigma = (4\pi^2/3)\alpha_s(p_t^2)(-\Delta) \cdot xG(\bar{x} = (M^2 + Q^2)/s, M^2), \]

(2.2)

$\Delta$ is the two dimensional Laplace operator in the space of the transverse momenta, and

\[ M^2 = (p_t^2 + m_q^2)/z(1 - z), \]

(2.3)

is the invariant mass squared of the dipole. In the coordinate representation $\sigma$ is just a multiplication, but not a differential operator. In the leading $\ln(x_0/x)$ approximation a similar equation arises where the cross section is expressed in terms of convolution of impact factor and unintegrated gluon density. In practice, both equations should give close results. Using an integration over parts over $p_t$ it is easy to rewrite Eq. 2.1 within the LO accuracy in the form where integrand will be explicitly positive:

\[ \sigma(\gamma^*_L + T \rightarrow X) = e^2 \frac{12}{12\pi^2}\alpha_s \int d^2p_t dz \bra \nabla \psi_{\gamma^*_L}(p_t, z) \right| f(s, z, p_t^2) \left| \nabla \psi_{\gamma^*_L}(p_t, z) \ket, \]

(2.4)

where

\[ f = (4\pi^2/3)\alpha_s(p_t^2)xG(\bar{x}, M^2). \]

(2.5)
In the derivation we use boundary conditions that photon wave function is negligible at $p_t^2 \to \infty$ and that the contribution of small $p_t$ is the higher twist effect. The cross section of the interaction of the longitudinal photon can be rewritten in the form of spectral representation by explicitly differentiating the photon wave function:

$$
\sigma_L = 8\pi^2 \frac{\sum e_q^2 F^2 Q^2}{12} \int dM^2 \alpha_s(M^2/4) \frac{M^2}{(M^2 + Q^2)^4} \cdot g(\bar{x}, M^2).
$$

Here $F^2 = 4/3$ for the colorless dipoles build of color triplet constituents, and $F^2 = 9/4$ for the gluonic dipoles.

The spectral representation of the electro-production amplitude over $M^2$ is a general property of a quantum field theory at large energies where the coherence length significantly exceeds the radius of the target $T$ [21, 22]. The pQCD guarantees additional general property: the smaller size of the configuration in the wave function of projectile photon leads to the smaller interaction with the target but this interaction more rapidly increases with the energy. In the NLO approximation the structure of formulae should be the same except the appearance of the additional $q\bar{q}g, \ldots$ components in the wave function of photon due to the necessity to take into account the QCD evolution of the photon wave function [19].

The similar derivation can be made for the scattering of the spatially small transverse photon. In this case the contribution of small $p_t$ region (Aligned Jet Model contribution) is comparable to the pQCD one. To suppress AJM contribution we restrict ourselves in the paper by the region of large $p_t^2$ and sufficiently small $\bar{x}$ where pQCD contribution dominates because of the rapid increase of the gluon distribution with the decrease of $x$.

The pQCD contribution into the total cross-section initiated by the transverse photon has the form:

$$
\sigma_T = \pi^2 \frac{\sum e_q^2 F^2}{12} \times \left( \int_0^1 dz \int dM^2 \alpha_s(M^2 z(1 - z)) \frac{z^2 + (1 - z)^2}{z(1 - z)} \frac{(M^4 + Q^4)}{(M^2 + Q^2)^4} \cdot g(\bar{x}, 4M^2 z(1 - z)) \right).
$$

Here while doing the actual calculations we introduced a cut-off in the space of transverse momenta $M^2 z(1 - z) \geq u, u \sim 0.3$ GeV$^2$.

### III. THE DOUBLE LOGARITMIC APPROXIMATION.

In this section we analyze the new properties of the pQCD regime within the double log approximation. The advantage of this approximation is that it will enable us to perform some of the calculations analytically/semianalytically. We will present the detailed numerical results for the pattern observed in the complete LO/NLO in a later more detailed publication where we will demonstrate that qualitative though not numerically the results are the same (see however some preliminary results in LO below).

We find that at sufficiently large energies the characteristic invariant mass of the system of constituents produced by the electromagnetic current is not $Q^2$ but much larger-$M^2(s, Q^2)$. Thus in the calculations of of the high energy processes the effective virtuality is $M^2$ but not $Q^2$. 


In the double logarithmic approximation the structure functions are given by \[23\]
\[
g(x, Q^2) = \int \frac{dj}{2\pi i} (x/x_0)^{j-1} (Q^2/Q_0^2)^{\gamma(j)},
\]
(3.1)
where the anomalous dimension is
\[
\gamma(j) = \frac{\alpha_s N_c}{\pi (j - 1)}.
\]
To simplify the calculation we assume, the initial condition for the evolution with $Q^2$:
\[
g(x, Q_0^2) = \delta(x - 1).
\]
(3.2)
In the saddle point approximation one finds \[23\]:
\[
g(x, Q^2) = \log\left(\frac{Q^2}{Q_0^2}\right)^{1/4} \exp\left[\frac{4\alpha_s(Q_0^2)}{\pi} \log\left(\frac{Q^2}{Q_0^2}\right) \log\left(\frac{x}{x_0}\right)\right]
\]
(3.3)
Structure function of a hadron is given by the convolution of this kernel with nonperturbative structure function of hadron in the normalization point $Q^2 = Q_0^2$.

In the following we shall neglect the pre-exponential factor, since absolute value of $g$ as well as the pre-exponential factor weakly influence the transverse scale, and its evolution with energy:
\[
g(x, Q^2) = \exp\left[\frac{4\alpha_s(Q_0^2)}{\pi} \log\left(\frac{Q^2}{Q_0^2}\right) \log\left(\frac{x}{x_0}\right)\right].
\]
(3.4)

A. Energy dependence of the quark transverse momenta for fragmentation processes initiated by longitudinal photon.

We shall find analytically the scale of the transverse momenta in the limit when $s \gg M^2 \gg Q^2$. For certainty we restrict ourselves to the contribution of light quarks.

At large $Q^2$ the cross-section for the scattering of the longitudinal photon is dominated by the contribution of the spatially small dipoles, so it is legitimate to neglect the quark masses. In this limit the cross section is proportional to
\[
\sigma_L \propto Q^2 \int dM^2 n(M^2, s, Q^2),
\]
(3.5)
where the function $n(M^2, s, Q^2)$ is given by Eqs. 2.6, 2.7
\[
n(M^2, s, Q^2) = \alpha_s(M^2/4) \frac{M^2}{(M^2 + Q^2)^4}
\]
\[
\times \exp\left(\frac{4\alpha_s(Q_0^2)}{\pi} N_c \ln\left(M^2/M_0^2\right) \ln\left(s/(M^2 + Q^2)((M_0^2 + Q^2)/s_0)\right)\right).
\]
(3.6)
Here we keep only large terms depending on $M^2$ (we do not write here explicitly the $M^2$ independent overall normalization factor irrelevant for the calculations below).
Let us show that the maximum of \( n(M^2, s, Q^2) \) increases with the energy. At very high energies \( n \) is proportional to

\[
n \sim \exp(\ln \alpha_s(M^2/4) + \log(M^2/Q^2) - 4 \log((Q^2 + M^2)/Q^2) + \sqrt{4\alpha_s(N_c/\pi)(\ln(s/s_0) - \log((Q^2 + M^2)/(Q^2 + M_0^2)) \ln(M^2/M_0^2))}.
\]

(3.7)

In the limit of fixed \( Q^2 \) but very large energies, \( \log(s/s_0) \gg \log((Q^2 + M^2)/(Q^2 + M_0^2)) \). Let us assume that for the maximum: \( M^2 \gg Q^2 \). We can find the maximum of the expression \( n \sim \exp(\ln \alpha_s(M^2/4) - 3 \log(M^2/Q^2) + \sqrt{4\alpha_s(Q_0^2/(N_c/\pi))(\ln(s/s_0) \ln(M^2/M_0^2)}) \). (3.8)

Differentiating the argument of the exponent over \( \log(M^2/M_0^2) \) we obtain the equation for the maximum:

\[
1/\ln(M^2/4M_0^2) + 3 = (1/2)(1/\ln(M^2/M_0^2))\sqrt{4\alpha_s(Q_0^2/(N_c/\pi))\ln(s/s_0) / \ln(M^2/M_0^2)}.
\]

(3.9)

Neglecting the small first term we find:

\[
M^2 = M_0^2(s/s_0)^{\alpha_s(N_c/\pi)/9}.
\]

(3.10)

Here \( M_0^2 \sim Q^2 \) and \( s_0 \sim Q^2 \). We will refer to this extremum value of \( M^2 \) as \( M_1^2 \).

At the extremum \( n \propto (\alpha_s(M_1^2/4)/M_0^6)\exp((N_c/\pi)(\alpha_s/3) \ln(s/s_0)) \). Therefore

\[
\frac{d\sigma_L}{dM^2}|_{M^2=M_1^2} \approx \alpha_s(M_1^2/4)(Q_0^2/M_0^6)\exp((N_c/\pi)(\alpha_s(Q_0^2/3) \ln(s/s_0))
\]

(3.11)

However, the position of the maximum of the integrand is not sufficient to characterize the relevant transverse scales as a large range of \( M^2 \) is important in the integrand. (Calculation of second derivative shows that dispersion over \( M^2 \) is large.) The width of the distribution over \( \ln(M^2/M_0^2) \) is \( \sqrt{(2/3) \log(M^2/M_0^2)} \). Hence we need to determine \( M^2 \) range which gives most of the integrand support. For certainty, we define the range of \( M^2 \leq M_t^2 \) which provides a fixed, say, 50% fraction of the total perturbative cross-section. Let us estimate how this scale increases with the energy in the double log approximation. First, let us consider the total cross section. The upper limit \( u \) of integration over \( M^2 \) is determined by the condition \( M^2 \ll s \).

For certainty we choose upper limit of integration as

\[
M^2 \leq M_{max}^2 = 0.2s,
\]

(3.12)

although the result of numerical calculations is insensitive to the upper bound because essential \( M^2 \) are significantly smaller.

Let us first calculate the median scale semianalytically. Within the double logarithmic approximation, and assuming that the conditions \( \log(s/s_0) \gg \log((Q^2 + M^2)/(Q^2 + M_0^2)) \),
is still valid for the relevant $M^2$, the integral for the cross section can be written similar to Eq. 3.8

$$\sigma(u) = \left(\frac{Q^2}{M^4_0}\right) \int_0^{\log(u/M^2_0)} d\ln(M^2/M^2_0) \alpha_s(M^2/4) \exp(-2\ln(M^2/M^2_0))$$

$$+ \sqrt{(4\alpha_s N_c/\pi)\ln(M^2/M^2_0)\ln(s/s_0))}. \quad (3.13)$$

Here $u$ is the upper cut-off in the invariant masses. Introducing the new variable $t = \log(M^2/M^2_0)$, we obtain:

$$\sigma(u) = \left(\frac{Q^2}{M^4_0}\right) \int_0^{\kappa(u)} dt \alpha_s(tM^2_0/4) \exp(-2t + \sqrt{(4\alpha_s N_c/\pi)\log(s/s_0)t}), \quad (3.14)$$

where $\kappa(u) = \ln(u/s_0)$. The integral for the total cross-section is given by the equation similar to Eq. 3.14, with the upper integration limit being replaced by $\kappa(s) = \sqrt{\ln(0.2s/s_0)}$. The integral 3.14 is actually the error function [24], which can be easily evaluated numerically. Requiring that it gives one half of the cross section we find

$$M^2_t \sim M^2_0(s/s_0)^{0.28\alpha_s N_c/\pi}. \quad (3.15)$$

Evidently, for sufficiently large $s$ our initial assumption $\log(M^2/M^2_0) \gg \log(Q^2/Q^2_0)$ is fully self-consistent. This is because the decrease of $n$ with $M^2$ due to $1/M^6$ terms in the integrand of Eq. 2.6 is partially compensated by the rising exponential, giving a relatively slow decrease of $n$ to the right of its maximum.

Note that the rate of the increase of $M^2_t$ with $s$ is much higher than for $M^2_1$ due to the slow decrease of the integrand with $M^2$. The cross section of jet production with $M^2_1$ at this interval also increases with the energy as

$$\frac{d\sigma}{dM^2}_{M^2=M^2_1} \sim (s/s_0)^{0.24\alpha_s N_c/\pi}. \quad (3.16)$$

In order to understand the dependence of the median scale on both the energy and $Q^2$ quantitatively we made numerical calculation of the characteristic transverse momenta using the DGLAP double log structure function. We find that the increase rate of the transverse momenta indeed does not depend on the external virtuality $Q^2$. Considering the wide interval of energies and $s = 10^4 \div 10^{11}$ GeV$^2$, and $20 < Q^2 < 200$ GeV$^2$ we obtain the approximate formulae:

$$M^2_t \sim 0.7Q^2 \exp(0.17((4\alpha_s N_c/\pi)\log(x_0/x))^{0.55}). \quad (3.17)$$

We give this estimate only for illustrative purposes, since the double logarithmic approximation is semirealistic only. Still our results indicate that for external virtualities $Q^2 < 100$ GeV$^2$ and energies which can be reached at LHeC the onset of a new pQCD regime may take place.

The cross section of the jet production at this scale also increases with the energy as

$$\frac{d\sigma}{dM^2}_{M^2=M^2_1} \sim \frac{1}{Q^4} \frac{f(x)}{1 + 0.7f(x)} G(x(1 + 0.7f(x)), 0.7Q^2 f(x), \quad (3.18)$$

where

$$f(x) = \exp(0.17((4\alpha_s N_c/\pi)\log(x_0/x))^{0.55}). \quad (3.19)$$
B. Transverse photon: the characteristic transverse scale in the photon fragmentation region.

It is well known that the main difference between the longitudinal and transverse structure functions in the DIS is the presence of the strongly asymmetrical in $z$ configurations due to the presence of the $(z(1-z))^{-1}$ multiplier in the spectral density. As a result there is a competition between two effects. One is a slower decrease of the spectral function with $M^2$ (by the factor $M^2/Q^2$), leading to the more rapid increase of the characteristic transverse momenta for the symmetric configurations. The second effect is presence of the asymmetric ($z \to 0$) configurations which are characterized by the small transverse momenta $k_t^2$ for a given invariant mass $M^2$. For such configurations the rate of increase of the gluon structure function with energy is small.

Let us first show that the transverse momenta increase rapidly for symmetric configurations. The spectral representation for the transverse photon for symmetric configurations is proportional to

$$n(M^2, Q^2, s) \sim \frac{M^4 + Q^4}{(M^2 + Q^2)^4} \times \sqrt{4\alpha_s(N_c/\pi)(\ln(s/s_0) - \log((Q^2 + M^2)/(Q^2 + M_0^2)))\ln(M^2/M_0^2)}.$$  

(3.20)

Similar to the case of the longitudinal photon we obtain for high energies, when $M_1^2 \gg Q^2$, the dependence of the maximum of $n$ on energy:

$$M_1^2 \sim M_0^2(s/s_0)^{\alpha_s(N_c/\pi)/4},$$  

(3.21)

i.e. the increase rate is twice as fast as compared to the case of longitudinal photon. $M_1^2$ increases with $s$ at high energies and thus the condition $M^2 \gg Q^2$ is perfectly self-consistent at very high energies.

The jet cross section at the maximum of the curve also increases as

$$\frac{d\sigma}{dM^2} |_{M^2=M_1^2} \sim (s/s_0)^{\alpha_s N_c/2\pi}.$$  

(3.22)

In addition we calculate the total cross-section in the same approximation semi-analytically getting the error function and obtain the rate of increase $(s/s_0)^{0.14(4\alpha_s N_c/\pi)}$, which is twice that for the longitudinal case.

However, as we mentioned above, a considerable contribution of the nonsymmetrical configurations has the opposite effect. In order to take these configurations into account we performed a numerical calculation using the gluon distribution function within the double log approximation. The result is that the characteristic median scale $M_t^2$ increase like

$$M_0^2(s/s_0)^{0.1(4\alpha_s N_c/\pi)}.$$  

(3.23)

The value of the exponent is 0.12 for the beginning of the studied energy range $s \sim 10^4 \div 10^{11}$ GeV$^2$, and decreases to 0.09 at the upper end (for typical $\alpha_s = 0.25$. Thus the rate of the increase with the energy is approximately the same as for longitudinal photons for not very
high energies. For very high energies the symmetric configurations win over asymmetric ones, leading to twice as rapid increase of the transverse momenta than in the longitudinal case.

The precise determination of the scale $M_0^2(Q^2)$ is beyond the accuracy of this paper. Effectively we obtain the dependence $M_0^2 \sim 0.7Q^2(x_0/x)^{0.1(4\alpha_s N_c/\pi)}$.

One can also estimate the rate of the increase of the jet production cross section:

$$d\sigma_T/dM_2^2 \sim \frac{1}{Q^4}(1 + 0.5h(x))^2 G(x(1+h(x)), Q^2(1+h(x)),$$

$$h(x) = (x_0/x)^{0.1(4\alpha_s N_c/\pi)}$$

We found a rapid increase of the jets multiplicity. Thus the rate of the increase with energy of the transverse momenta of quarks in the current fragmentation region for transversely polarized photon is significantly more rapid. Consequently we find that $\sigma_L/\sigma_T \approx Q^2/M_2^2$ being numerically small should slowly decrease with energy at sufficiently high energies.

We conclude that it is possible to show analytically that for very high (asymptotic) energies the relevant invariant masses extend well beyond $Q^2$ and increase with the energy.

The direct numerical calculation of the $M_t^2$ scale shows that the rate of increase is independent of external virtuality.

C. The leading logarithmic approximation.

The above results were obtained in the double logarithmic approximation. It is also possible to carry out the numerical calculation in LO approximation using the CTEQ5L gluon distribution functions \[25\]. In this approximation the median scale still increases as $M_t^2 \sim 0.7Q^2(x_0/x)^{\lambda}$, where $x_0 \sim 10^{-2}$, and $\lambda \sim 0.06$ for longitudinal and $\lambda \sim 0.08$ for transverse photons. Although this increase is quite slow, the rise of momenta is not negligible: for energy increase from $10^4$ to $10^7$ GeV$^2$ the scale increases by a factor $\sim 1.5$. The use of CTEQ6L will decrease the considered effects.

IV. THE COHERENCE LENGTH.

In the previous sections we determined the energy dependency of the effective transverse scale at high energies which allowed us to evaluate coherence length. The coherence length $l_c$ corresponds to the life-time of the dipole fluctuation at a given energy in the rest frame of the target. The original suggestion of the existence of the coherence length in the deep inelastic scattering was first made by Ioffe, Gribov and Pomeranchuk \[21, 26\]. It was found already in the sixties within the parton model approximation by Ioffe \[27\] that the coherence length at moderate $x_B$ is $l_c \sim 1/2m_N x_B$ i.e. it linearly increases with energies. In pQCD coherence length

$$L_c = (1/2m_N x)(s_0/s)^{\lambda}$$

Less rapid increase of $L_c$ with energy has been found before in the numerical calculations of structure functions in the target rest frame accounting for $Q^2$ evolution of structure functions \[29, 30\].

It is worth noting that the discussed pattern of the energy dependence of the coherence length leads to a change of the structure of the fast hadron wave function as compared
to the Gribov picture [15] where the longitudinal size of the hadron is determined by the wee parton cloud and energy independent $L_z \sim 1/\mu$. Here $\mu \sim .3 \div 0.4 \text{GeV/c}$ is the soft mass scale. On the other hand a slower than $1/m_N x$ rate of the increase of the coherent length with energy leads to a decrease of the longitudinal size of the hadron with energy. The typical size is determined by the BD momentum at a given impact parameter for a particular energy. Moreover since the BD momentum is larger for small impact parameters the nucleon has a form of a double concave lens. It is of interest also that for the zero impact parameter the longitudinal size of a heavy nucleus is smaller than for a nucleon.

V. THE FORM OF NUCLEON, NUCLEUS IN DIS

Our results have the important consequences for the transverse structure of the hadrons and nuclei.

Let us consider the longitudinal distribution of the partons in a fast hadron. As it was already mentioned in the previous section, in the parton model the longitudinal spread of gluonic cloud is $L_z \sim 1/\mu$ for the wee partons and is much smaller than for harder partons, with $L_z \sim 1/xP_t$ for partons carrying a finite $x$ fraction of the hadron momentum [15]. The picture is changed qualitatively in the limit of very high energies when interactions reach BD regime for $k_t \gg \mu$. In this case the smallest possible characteristic momenta in the frame where hadron is fast are of the order $k_t(BDR)$ which is a function of both initial energy and transverse coordinate, $b$ of the hadron. Correspondingly, the longitudinal size is $\sim 1/k_t(BDR) \ll 1/\mu$. Since the gluon parton density decreases with the increase of $b$ the longitudinal size of the hadron is larger for large $b$, so a hadron has a shape of biconcave lens, see Figs. 1, 2.

We depict the typical transverse structure of the fast nucleon in Fig. 1. We see that it is drastically different from the naive picture of a fast moving nucleon as a flat narrow disk with small constant thickness.

Consider now the case of the DIS on the nuclei.

Consider first the case of external virtualities of the order of several GeV. In this case the shadowing effects mostly cancel the $A^{1/3}$ for a given impact parameter, $b \ [19]$ and the gluon density in the nuclei is comparable to that in a single nucleon for $b \sim 0$. Consequently over the large range of the impact parameters the nucleus longitudinal size is approximately the same as in the nucleon at $b \sim 0$.

Consider now the case of the large external virtualities $Q^2 \geq 40 \text{GeV}^2$. In this case the leading twist shadowing is small, and the corresponding gluon density unintegrated over $b$ is given by a product of a nucleon gluon density and the nuclear profile function:

$$T(b) = \int dz \rho(b, z), \quad (5.1)$$

where the nuclear three-dimensional density is normalized to $A$. We use standard Fermi step parametrization [28]

$$\rho(r) = C(A) \frac{A}{1 + \exp((r - R)/a), \quad R = 1.1A^{1/3} \text{fermi}, \quad a = 0.56 \text{fermi.} \quad (5.2)}$$

Here $r = \sqrt{z^2 + b^2}$, and $A$ is the atomic number. $C(A)$ is a normalization factor, that can be calculated numerically from the condition $\int d^3 r \rho(r) = A$. At the zero impact parameter $T(b) \approx 0.5A^{1/3}$ for large $A$. 

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The dependence of the thickness of a fast nucleus as a function of the transverse size is depicted in Fig. 2 for a typical high energy $s = 10^7$ GeV$^2$, $Q^2 = 40$ GeV$^2$. We see that the nuclei also has a form of a biconcave lens instead of a flat disk.

Note that this picture is very counterintuitive. We see that the nuclei is thinner than a single nucleon, i.e. the thickness of a nuclei is smaller than for a nucleon although we have $\sim A^{1/3}$ nucleons at the central impact parameter. In other words the longitudinal extend of a system is smaller than the the longitudinal extend of its constituents. The explanation of this BDR phenomenon is straightforward. For a given impact parameter $b$, the longitudinal size of a heavy nucleus $1/k_t^{(A)}(BDR) < 1/k_t^{(p)}(BDR)$ since the gluon distribution function in the nuclei $G_A(x,b) > G_N(x,b)$. So a naive classical picture of a system build of the constituents being larger than each of the constituents is grossly violated. This situation is in some respects analogous to the phenomenon of color transparency/existence of the point-like configurations). Thus we have a paradox: in the fast reference frame the nuclei is much thinner than any of its constituents.

![3D image of the fast nucleon at $s = 10^7$ GeV$^2$ and $Q^2 = 40$ GeV$^2$.](image)

The resolution of the paradox in the BD regime is quite simple: the soft fields of individual nucleons destructively interfere cancelling each other. So a naive classical picture of a system build of the constituents being larger than each of the constituents is grossly violated.

VI. EXPERIMENTAL CONSEQUENCES.

The current calculations of cross-sections of hard processes at the LHC are based on the use of the DGLAP parton distributions and the application of the factorization theorem. Our results imply that the further analysis is needed to define the kinematic regions where
FIG. 2: 3D image of the fast heavy nucleus (gold) at $s = 10^7 \text{GeV}^2$ and $Q^2 = 40\text{GeV}^2$.

one can use DGLAP distributions. We showed in the paper that for DIS at high energies there are kinematic regions where one is forced to use a $k_t$ factorization and the dipole model instead of the direct use of DGLAP. A similar analysis must be made for the $pp$ collisions at LHC. The expected effect is the increase with energy of the probability of the small dipoles in the wave function of proton $[32]$. Quantitative analysis of this problem will be presented elsewhere.

The hard processes initiated by the real photon can be directly observed in the ultraperipheral collisions $[31]$. The processes where a real photon scatters on a target, and creates two jets with an invariant mass $M^2$, can be analyzed in the dipole model by formally putting $Q^2 = 0$, while $M^2$ is an invariant mass of the jets. In this case with a good accuracy the spectral density discussed above will give the spectrum of jets in the fragmentation region. Our results show that the jet distribution over the transverse momenta will be broad with the maximum moving towards larger transverse momenta with increase of the energy and centrality of the $\gamma A$ collision.

Finally, our results can be checked directly, if and when the LHeC facility will be built at CERN. One of us B. Blok thanks S. Brodsky for the useful discussions of the results obtained in the paper. This work was supported in part by the US DOE Contract Number DE-FG02-93ER40771 and BSF.
[1] S. J. Brodsky, L. Frankfurt, J. F. Gunion, A. H. Mueller and M. Strikman, Phys. Rev. D50 (1994) 3134.
[2] H. Abramowicz, L. Frankfurt and M. Strikman, Surveys in High Energy Physics, 11 (1997) 51.
[3] L. Frankfurt, G.A. Miller, M. Strikman, Ann. Rev. Nucl. Part. Sci., 44 (1994) 501.
[4] B. Blaettel, G. Baym, L. Frankfurt and M. Strikman, Phys. Rev. Lett., 70 (1993) 896.
[5] A. H. Mueller Nucl. Phys. B 415, 373 (1994).
[6] J.C. Collins, L. Frankfurt and M. Strikman, Phys. Rev. D56 (1997) 2982.
[7] E. Kuraev, V. Fadin, L. Lipatov, Sov. Phys.-JEP, 44 (1976) 443; 45 (1977) 199. I. Balitsky and L. Lipatov, Sov. J. Nucl. Phys., 28 (1978) 822.
[8] G. Altarelli and G.Parisi, Nucl. Phys., B126 (1977) 298; V.N. Gribov and L. N. Lipatov, Sov. J. of Nucl. Phys., 15 (1972) 438,672; Yu.L. Dokshitser, Sov. Phys. JETP 46 (1977) 641.
[9] B. Blok, L. Frankfurt and M. Strikman, in preparation.
[10] B. Blok, L. Frankfurt and M. Strikman, arXiv:0808.2006 (hep-ph) Talk at small x conference, Columpari, Crete, July 2008.
[11] V. Guzey, L. Frankfurt, M. Strikman, M.McDermott, Eur. J. of Physics, C16 (2000) 641.
[12] L. Frankfurt, M. Strikman and C. Weiss, Ann.Rev.Nucl.Part.Sci. 55 (2005) 403-465.
[13] T. C. Rogers, A. M. Stasto and M. I. Strikman, Unitarity Constraints on Semi-hard Jet Production in Impact Parameter Space, arXiv:0801.0303 [hep-ph].
[14] T. C. Rogers and M. I. Strikman, Hadronic interactions of ultra-high energy photons with protons and light nuclei in the dipole picture,” J. Phys. G 32, 2041 (2006)
[15] V. N. Gribov, Space-time description of hadron interactions at high-energies. In *Moscow 1 ITEP school, v.1 'Elementary particles*, 65,1973. e-Print: hep-ph/0006158
[16] V.N.Gribov The theory of complex angular momenta: Gribov lectures on theoretical physics. Cambridge, UK: Univ. Pr. (2003)
[17] S. Catani, M. Ciafaloni, F. Hauptmann, Nucl. Phys. B366 (1991) 135.
[18] J. Collins, K. Ellis, Nucl. Phys., B360 (1991) 3.
[19] L. Frankfurt and M. Strikman, Phys. Rept., 160 (1988) 235.
[20] L. Frankfurt, A. Radyushkin, M. Strikman, Phys. Rev. D55(1997) 98.
[21] V. N. Gribov, Sov. Phys. JETP 30 (1970) 709.
[22] T.H. Bauer, R.D. Spital, D.R. Yennie , F.M. Pipkin, Rev.Mod.Phys.50 (1978) 261, Erratum-ibid.51(1979) 407.
[23] Yu. Dokshitzer,D. Diakonov and S. Troyan, Phys. Reports, 58 (1980) 269.
[24] M. Abramowitz and I. Stegun, Handbook of special functions, Dover Publications, New York,1964.
[25] B. Blok, L. Frankfurt and M. Strikman, in preparation.
[26] L. B. Ioffe, V. Gribov, I. Pomeranchuk, Sov. J. of Nucl. Phys., 2 (1966) 549.
[27] B. L. Ioffe, Phys. Lett., B30 (1969) 123.
[28] A. Bohr and B.R. Mottelson, Nuclear structure, v.1, W.A. Benjamin, New York, 1969.
[29] Y. Kovechegov and M. Strikman, Phys. Lett., B516(2001) 314.
[30] B.Blok and L.Frankfurt Phys.Lett.B630 (2005) 49-57.
[31] L. Frankfurt and M. Strikman, in Phys. Reports, 455 (2008) 105.
[32] We are indebted to S.Brodsky for emphasizing this point.