WHAT DO SOLAR MODELS TELL US ABOUT SOLAR NEUTRINO EXPERIMENTS?

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Abstract

If the published event rates of the chlorine and Kamiokande solar neutrino experiments are correct, then the energy spectrum of neutrinos produced by the decay of $^8$B in the sun must be different from the energy spectrum determined from laboratory nuclear physics measurements. This change in the energy spectrum requires physics beyond the standard electroweak model. In addition, the GALLEX and SAGE experiments, which currently have large statistical uncertainties, differ from the predictions of the standard solar model by $2\sigma$ and $3\sigma$, respectively.

At the conference, I presented a review of recent improvements in the calculations of neutrino fluxes from solar models and then used the most recent results to draw some conclusions about what we have learned by comparing the results of solar neutrino experiments with calculations from solar models of the neutrino fluxes. The analysis of solar models has now been published in detail, so there is no need to repeat that material here. My main goal at the conference was, in any event, not to elucidate technical issues in solar model theory, but rather to clarify and make quantitative the conclusions that follow from the confrontation of the solar model calculations with the four operating experiments. I will therefore take the model results as given and concentrate here on what they teach us about the four solar neutrino experiments.

The first point to recognize is that the individual rates of the four solar neutrino experiments tell us nothing about the possibility of new physics until these rates are compared with solar models. The analogy to an accelerator experiment is clear: we need to know what the beam intensity is, as well as the flavor composition and energy spectrum, in order to know if we are surprised or not by the experimental rates.

The standard solar model predicts the absolute fluxes from each of the important nuclear fusion reactions and furthermore says that all solar neutrinos are $\nu_e$’s. What is more, to an accuracy of one part in $10^5$, the energy spectrum of the $^8$B solar neutrinos must have the same shape as the spectrum determined from laboratory nuclear physics experiments.

The invariance of the energy spectrum allows us to compute the rate of neutrino capture in the chlorine experiment—indeed of any considerations of solar models—provided only that we know from the Kamiokande experiment the flux of the higher energy ($> 7.5$ MeV) $^8$B neutrinos. In this process, we ignore the expected contributions to the chlorine experiment, which has a threshold of only 0.8 MeV (an order of magnitude less than the Kamiokande
experiment), from $^7$Be, CNO, and pep neutrinos. Using the empirical result obtained for the Kamiokande experiment, one finds that the predicted rate from $^8$B neutrinos alone in the chlorine experiment is 6.20 SNU (from the standard model)×0.48 (from the Kamiokande measurement), or

$$<\phi\sigma>_{\text{Cl; Kamiokande only}} = [3.0 \pm 0.3(1\sigma) \pm 0.4(\text{syst})] \, \text{SNU}. \quad (1)$$

This minimum rate, which ignores the contributions of all other neutrino sources to the chlorine experiment, exceeds by 2$\sigma$ the observed chlorine rate,$^5$

$$<\phi\sigma>_{\text{Cl exp}} = (2.2\pm0.2) \, \text{SNU}, \quad 1\sigma \text{ error}. \quad (2)$$

Moreover, the lower-energy contributions from $^7$Be and pep neutrinos—which together amount to about 1.4 SNU—are much more reliably determined by the theoretical calculations than is the contribution from $^8$B neutrinos. If a fraction equal to 0.48 of the less-reliably calculated high energy $^8$B neutrinos are detected, then presumably more than 0.7 SNU of the expected 1.4 SNU from pep and $^7$Be neutrinos should be added to the minimum rate of 3.0 SNU calculated above. On the basis of this comparison, Hans Bethe and I concluded$^6$ that, if the chlorine and Kamiokande experiments are both correct, then physics beyond the standard electroweak model is required to change the $^8$B neutrino energy spectrum.

More recently, Hans and I have sharpened this argument$^7$ using a detailed Monte Carlo simulation of how the sun works. The basis for our investigation is a collection of 1000 precise solar models$^2$ in which each input parameter (the principal nuclear reaction rates, the solar composition, the solar age, and the radiative opacity) for each model was drawn randomly from a normal distribution with the mean and standard deviation appropriate to that variable. The uncertainties in the neutrino cross sections$^2$ for chlorine and for gallium were included by assuming a normal distribution for each of the absorption cross sections with its estimated mean and error.

We know that Monte Carlo simulations are necessary to understand the results of complicated experiments in nuclear and particle physics. It should therefore seem natural to physicists that Monte Carlo simulations are necessary to interpret the results of solar neutrino experiments; the sun may be as complicated as a terrestrial particle accelerator or detector.

The Monte Carlo study automatically takes account of the nonlinear relations among the different neutrino fluxes that are imposed by the coupled partial differential equations of stellar structure and by matching the stringent boundary conditions of reproducing the observed solar luminosity, the heavy element to hydrogen ratio, and the effective temperature at the present solar age. Attempts to simulate the uncertainties using average scaling laws of the dependence of fluxes upon a single parameter, the central temperature, can lead to serious errors. A full Monte Carlo calculation is required to determine the interrelations and absolute values of the different solar neutrino fluxes. For example, the fact that the $^8$B flux may be crudely described as $\phi(8B) \propto T_{\text{central}}^{18}$ and $\phi(7Be) \propto T_{\text{central}}^8$ does not specify whether the two fluxes increase and decrease together or whether their changes are...
out of phase with each other.

Figure 1 shows the number of solar models with different predicted event rates for the chlorine solar neutrino experiment. The solar model with the best input parameters predicts an event rate of about 8 SNU. None of the 1000 calculated solar models yields a capture rate below 5.8 SNU. Therefore, none of the 1000 solar models is within $16\sigma$ of the observed rate. The discrepancy that is apparent in Figure 1 was for two decades the entire “solar neutrino problem.” We can conclude from Figure 1 that something is wrong with either the standard solar model or the standard electroweak description of the neutrino.

The largest and the most uncertain contribution to the predicted chlorine rate is the $^8$B neutrino flux. This quantity is completely unimportant for all astronomical purposes since the reaction by which it is produced is extremely rare. Suppose therefore some mistake has been made in calculating the $^8$B neutrino flux and we normalize this flux, as before, by using the empirical determination in the Kamiokande experiment. What do we obtain for the 1000 solar models when we replace—for each model—the calculated flux by a value determined by the Kamiokande experiment?

Figure 2 provides a quantitative expression of the difficulty in reconciling the Kamiokande and chlorine experiments by changing solar physics, i.e., by arbitrarily changing the $^8$B neutrino flux. We constructed Figure 2 using the same 1000 solar models as were used in constructing Figure 1, but for Figure 2 we artificially replaced the $^8$B flux for each standard model by a value drawn randomly for that model from a normal distribution with the mean and the standard deviation measured by Kamiokande. The peak of the resulting distribution is moved to 4.7 SNU (from 8 SNU) and the full width of the peak is decreased by about a factor of three. The peak is displaced because the measured (i.e., Kamiokande) value of the $^8$B flux is smaller than the calculated value. The width of the distribution is decreased because the error in the Kamiokande measurement is less than the estimated theoretical uncertainty ($\approx 12.5\%$) and because $^8$B neutrinos constitute a smaller fraction of each displaced rate than of the corresponding standard rate.

Figure 2 was constructed by assuming that something is seriously wrong with the standard solar model, something that is sufficient to cause the $^8$B flux to be reduced to the value measured in the Kamiokande experiment. Nevertheless, there is no overlap between the distribution of fudged standard model rates and the measured chlorine rate. None of the 1000 fudged models lie within $3\sigma$ (chlorine measurement errors) of the experimental result.

The results presented in Figures 1–2 suggest that new physics is required beyond the standard electroweak theory if the existing solar neutrino experiments are correct within their quoted uncertainties. Even if one abuses the solar models by artificially imposing consistency with the Kamiokande experiment, the resulting predictions of all 1000 of the “fudged” solar models are inconsistent with the result of the chlorine experiment (see Figure 2).

Figures 3a–3b show the number of solar models with different predicted event rates for gallium detectors and the recent measurements by the SAGE$^8$ ($58^{+17}_{-24} \pm 14$ (syst) SNU) and GALLEX$^9$ ($83 \pm 19(1\sigma) \pm 8$ (syst) SNU) collabor-
orations. Figure 3a compares the gallium experimental results with the “unfudged” histogram of standard solar model calculations and Figure 3b compares the results when the $^8$B neutrino flux is taken from the Kamiokande measurement. Unlike the chlorine case (cf. Figures 1 and 2), in which almost 80% of the predicted event rate is from $^8$B neutrinos, Figures 3a and 3b are not qualitatively different because $^8$B neutrinos contribute very little (only about 10%) to the predicted event rate in the gallium experiments.

With the current large statistical errors, the gallium measurements differ from the best-estimate theoretical value of 132 SNU by approximately 2 $\sigma$ (GALLEX) and 3.5 $\sigma$ (SAGE). The gallium results provide modest support for the existence of a solar neutrino problem, but by themselves do not constitute a definitive conflict with standard theory.

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