Hard exclusive processes, such as deep electroproduction of photons and mesons off nuclear targets, could give access, in the coherent channel, to nuclear generalized parton distributions (GPDs). Here, a realistic microscopic calculation of the unpolarized quark GPD $H_3^q$ of the $^3\text{He}$ nucleus is reviewed. In Impulse Approximation, $H_3^q$ is obtained as a convolution between the GPD of the internal nucleon and the non-diagonal spectral function, describing properly Fermi motion and binding effects. The obtained formula has the correct limits. Nuclear effects, evaluated by a modern realistic potential, are found to be larger than in the forward case. In particular, they increase with increasing the momentum transfer and the asymmetry of the process. Another feature of the obtained results is that the nuclear GPD cannot be factorized into a $\Delta^2$-dependent and a $\Delta^2$-independent term, as suggested in prescriptions proposed for finite nuclei. The dependence of the obtained GPDs on different realistic potentials used in the calculation shows that these quantities are sensitive to the details of nuclear structure at short distances.

1. Introduction

Generalized Parton Distributions (GPDs) parametrize the non-perturbative hadron structure in hard exclusive processes. Their measurement would represent a unique way to access several crucial features of the nucleon (for a comprehensive review, see, e.g., Ref. 2). According to a factorization theorem derived in QCD, GPDs enter the long-distance dominated part of exclusive lepton Deep Inelastic Scattering (DIS) off hadrons. In particular, Deeply Virtual Compton Scattering (DVCS), i.e. the process $eH \rightarrow e'H'\gamma$ when $Q^2 \gg m_H^2$, is one of the the most promising to access GPDs (here and in the following, $Q^2$ is the momentum transfer between the leptons $e$ and $e'$, and $\Delta^2$ the one between the hadrons $H$ and $H'$). Therefore, relevant experimental efforts to measure GPDs by means of DVCS off hadrons are likely to take place in the next few years. Recently, the issue of measuring GPDs for nuclei has been addressed. The first paper on this sub-
ject \textsuperscript{5}, concerning the deuteron, contained already the crucial observation that the knowledge of GPDs would permit the investigation of the short light-like distance structure of nuclei, and thus the interplay of nucleon and parton degrees of freedom in the nuclear wave function. In standard DIS off a nucleus with four-momentum $P_A$ and $N$ nucleons of mass $M$, this information can be accessed in the region where $A x_B J \simeq \frac{Q^2}{2 P_A \cdot q} > 1$, being $x_B J = Q^2/(2 P_A \cdot q)$ and $\nu$ the energy transfer in the laboratory system. In this region measurements are difficult, because of vanishing cross-sections. As explained in Ref. 5, the same physics can be accessed in DVCS at lower values of $x_B J$. Since then, DVCS has been extensively discussed for nuclear targets. Calculations, have been performed for the deuteron\textsuperscript{6} and for finite nuclei \textsuperscript{7}. The study of GPDs for $^3$He is interesting for many aspects. In fact, $^3$He is a well known nucleus, for which realistic studies are possible, so that conventional nuclear effects can be safely calculated. Strong deviations from the predicted behaviour could be ascribed to exotic effects, such as the ones of non-nucleonic degrees of freedom, not included in a realistic wave function. Besides, $^3$He is extensively used as an effective neutron target, in DIS, in particular in the polarized case \textsuperscript{8,9}. Polarized $^3$He will be the first candidate for experiments aimed at the study of GPDs of the free neutron, to unveil details of its angular momentum content. In this talk, the results of an impulse approximation (IA) calculation\textsuperscript{10} of the quark unpolarized GPD $H_q^3$ of $^3$He are reviewed. A convolution formula is discussed and numerically evaluated using a realistic non-diagonal spectral function, so that Fermi motion and binding effects are rigorously estimated. The proposed scheme is valid for $\Delta^2 \ll Q^2$, $M^2$ and despite of this it permits to calculate GPDs in the kinematical range relevant to the coherent, no break-up channel of deep exclusive processes off $^3$He. In fact, the latter channel is the most interesting one for its theoretical implications, but it can be hardly observed at large $\Delta^2$, due to the vanishing cross section. The main result of this investigation is not the size and shape of the obtained $H_q^3$ for $^3$He, but the size and nature of nuclear effects on it. This will permit to test directly, for the $^3$He target at least, the accuracy of prescriptions which have been proposed to estimate nuclear GPDs\textsuperscript{7}, providing a tool for the planning of future experiments and for their correct interpretation.

2. Formalism

The formalism introduced in Ref. 11 is adopted. If one thinks to a spin 1/2 hadron target, with initial (final) momentum and helicity $P(P')$ and $s(s')$, \ldots
respectively, two GPDs \( H_q(x, \xi, \Delta^2) \) and \( E_q(x, \xi, \Delta^2) \), occur. If one works in a system of coordinates where the photon 4-momentum, \( q^\mu = (q_0, \vec{q}) \), and \( \bar{P} = (P + P')/2 \) are collinear along \( z \), \( \xi \) is the so called “skewedness”, parametrizing the asymmetry of the process, is defined by the relation

\[
\xi = -\frac{n \cdot \Delta}{2} = -\frac{\Delta^+}{2P^+} = \frac{x_{Bj}}{2 - x_{Bj}} + O \left( \frac{\Delta^2}{Q^2} \right),
\]

where \( n \) is a light-like 4-vector satisfying the condition \( n \cdot \bar{P} = 1 \). One should notice that the variable \( \xi \) is completely fixed by the external lepton kinematics. The values of \( \xi \) which are possible for a given value of \( \Delta^2 \) are \( 0 \leq \xi \leq \sqrt{-\Delta^2}/\sqrt{4M^2-\Delta^2} \). The well known natural constraints of \( H_q(x, \xi, \Delta^2) \) are: i) the so called “forward” limit, \( P' = P \), i.e., \( \Delta^2 = \xi = 0 \), where one recovers the usual PDFs \( H_q(x, 0, 0) = q(x) \); ii) the integration over \( x \), yielding the contribution of the quark of flavour \( q \) to the Dirac form factor (f.f.) of the target:

\[
\int dx H_q(x, \xi, \Delta^2) = F_{q1}(\Delta^2);
\]

iii) the polynomiality property\(^ {11} \).

In Ref. 10, specifying to the \(^3\)He target the procedure developed in Ref. 12, an IA expression for \( H_q(x, \xi, \Delta^2) \) of a given hadron target, for small values of \( \xi^2 \), has been obtained.

\[
H_q^N(x', \xi', \Delta^2) = \sum_N \int dE \int d\bar{p} [P^N_\bar{q}(\bar{p}, \bar{p} + \bar{\Delta}, E) + O(\bar{p}^0/M^2, \bar{\Delta}^2/M^2)]
\times \frac{\xi'}{\xi} H_q^N(x', \xi', \Delta^2) + O(\xi^2) .
\]

In the above equation, the kinetic energies of the residual nuclear system and of the recoiling nucleus have been neglected, and \( P^N_\bar{q}(\bar{p}, \bar{p} + \bar{\Delta}, E) \) is the one-body off-diagonal spectral function for the nucleon \( N \) in \(^3\)He:

\[
P^N_\bar{q}(\bar{p}, \bar{p} + \bar{\Delta}, E) = \frac{1}{(2\pi)^3} \frac{1}{2} \sum_{M, R, s} |\bar{p}^\mu| (\bar{p}^\mu - \bar{p}) S_{R, s} (\bar{p} + \bar{\Delta}) s
\times \langle (\bar{p} - \bar{p}) S_{R, s} \bar{p} | \bar{p}^\mu | \bar{p} S_{R, s} \rangle \delta(E - E_{min} - E_{R}) .
\]

Besides, the quantity \( H_q^N(x', \xi', \Delta^2) \) is the GPD of the bound nucleon \( N \) up to terms of order \( O(\xi^2) \), and in the above equation use has been made of the relations \( \xi' = -\Delta^+/(2\bar{p}^+ + \Delta^+) \), and \( x' = (\xi'/\xi)x \).

The delta function in Eq. (2) defines \( E \), the so called removal energy, in terms of \( E_{min} = |E_{3H_e}| - |E_{2H}| = 5.5 \) MeV and \( E_{2H}^* \), the excitation energy of the two-body recoiling system. The main quantity appearing in
the definition Eq. (3) is the overlap integral

\[ \langle \bar{P} M | \bar{P} R S_R, \bar{p} s \rangle = \int d\vec{y} e^{i\vec{p} \cdot \vec{y}} \langle \chi^s, \Psi^{S_R}_R(\vec{x}) | \Psi^M_M(\vec{x}, \vec{y}) \rangle , \tag{4} \]

between the eigenfunction \( \Psi^M_M \) of the ground state of \( ^3\text{He} \), with eigenvalue \( E_{^3\text{He}} \) and third component of the total angular momentum \( M \), and the eigenfunction \( \Psi^{S_R}_R \), with eigenvalue \( E_R = E_2 + E_R^* \) of the state \( R \) of the intrinsic Hamiltonian pertaining to the system of two interacting nucleons. Since the set of the states \( R \) also includes continuum states of the recoiling system, the summation over \( R \) involves the deuteron channel and the integral over the continuum states. Eq. (2) can be written in the form

\[ H^3_q(x, \xi, \Delta^2) = \sum_N \int_x^1 \frac{dz}{z} h^3_N(z, \xi, \Delta^2) H^N_q \left( \frac{x}{z}, \frac{\xi}{z}, \Delta^2 \right) , \tag{5} \]

where

\[ h^3_N(z, \xi, \Delta^2) = \int dE \int d\vec{p} P^3_N(\vec{p}, \vec{p} + \vec{\Delta}) \delta \left( z + \xi - \frac{p^+}{\bar{p}^+} \right) . \tag{6} \]

In Ref. 10, it is discussed that Eqs. (5) and (6) or, which is the same, Eq. (2), fulfill the constraint \( i) - iii \) previously listed.

3. Numerical Results

\( H^3_q(x, \xi, \Delta^2) \), Eq. (2), has been evaluated in the nuclear Breit Frame.

The non-diagonal spectral function Eq. (3), appearing in Eq. (2), has been calculated along the lines of Ref. 14, by means of the overlap Eq. (4), which exactly includes the final state interactions in the two nucleon recoiling system, the only plane wave being that describing the relative motion between the knocked-out nucleon and the two-body system. The realistic wave functions \( \Psi^M_M \) and \( \Psi^{SN}_R \) in Eq. (4) have been evaluated using the AV18 interaction and taking into account the Coulomb repulsion of protons in \( ^3\text{He} \). In particular \( \Psi^M_M \) has been developed along the lines of Ref. 16. The other ingredient in Eq. (2), i.e. the nucleon GPD \( H^N_q \), has been modelled in agreement with the Double Distribution representation. In this model, whose details are summarized in Ref. 10, the \( \Delta^2 \)-dependence of \( H^N_q \) is given by \( F_q(\Delta^2) \), i.e. the contribution of the quark of flavour \( q \) to the nucleon form factor. It has been obtained from the experimental values of the proton, \( F^p_1 \), and of the neutron, \( F^n_1 \), Dirac form factors. For the \( u \) and \( d \) flavours, neglecting the effect of the strange quarks, one has \( F_u(\Delta^2) = \frac{1}{4} (2F^p_1(\Delta^2) + F^n_1(\Delta^2)) \), \( F_d(\Delta^2) = 2F^n_1(\Delta^2) + F^p_1(\Delta^2) \). The contributions of
Figure 1. For the $\xi$ values which are allowed at $\Delta^2 = -0.15$ GeV$^2$, $H_3^q(x_3, \xi_3, \Delta^2)$, evaluated using Eq. (5), is shown for $0.05 \leq x_3 \leq 0.8$.

the flavours $u$ and $d$ to the proton and neutron f.f. are therefore $F_{q}^p(\Delta^2) = \frac{4}{3} F_{u}(\Delta^2)$, and $F_{d}^p(\Delta^2) = -\frac{1}{3} F_{u}(\Delta^2)$ and $F_{q}^n(\Delta^2) = \frac{2}{3} F_{d}(\Delta^2)$, $F_{d}^n(\Delta^2) = -\frac{2}{3} F_{u}(\Delta^2)$, respectively. For the numerical calculations, use has been made of the parametrization of the nucleon Dirac f.f. given in Ref. 18. Now the ingredients of the calculation have been completely described, so that numerical results can be presented. If one considers the forward limit of the ratio

$$R_q(x, \xi, \Delta^2) = \frac{H_3^q(x, \xi, \Delta^2)}{2H_p^q(x, \xi, \Delta^2) + H_n^q(x, \xi, \Delta^2)},$$

(7)

where the denominator clearly represents the distribution of the quarks of flavour $q$ in $^3$He if nuclear effects are completely disregarded, i.e., the interacting quarks are assumed to belong to free nucleons at rest, the behaviour which is found, shown in Ref. 10, is typically $EMC$–like, so that, in the forward limit, well-known results are recovered. In Ref. 10 it is also shown that the $x$ integral of the nuclear GPD gives a good description of ff data of $^3$He, in the relevant kinematical region, $-\Delta^2 \leq 0.25$ GeV$^2$. As an illustration, the result of the evaluation of $H_3^q(x, \xi, \Delta^2)$ by means of Eq. (2) is shown in Fig. 1, for $\Delta^2 = -0.15$ GeV$^2$ as a function of $x_3 = 3x$ and $\xi_3 = 3\xi$. The GPDs are shown for the $\xi$ range allowed and in the $x_3 \geq 0$
region. Let us now discuss the quality and size of the nuclear effects. The

![Figure 2](image_url)

*Figure 2.* In the left panel, the ratio Eq. (9) is shown, for the u flavour and $\Delta^2 = -0.15$ GeV$^2$, as a function of $x_3$. The full line has been calculated for $\xi_3 = 0$, the dashed line for $\xi_3 = 0.1$ and the long-dashed one for $\xi_3 = 0.2$. The symmetric part at $x_3 \leq 0$ is not presented. In the right panel, the same is shown, for the flavour d.

full result for the GPD $H_q^{3,0}(x, \xi, \Delta^2)$, Eq. (2), will be now compared with a prescription based on the assumptions that nuclear effects are completely neglected and the global $\Delta^2$ dependence can be described by the f.f. of $^3$He:

$$H_q^{3,0}(x, \xi, \Delta^2) = 2H_q^{3,p}(x, \xi, \Delta^2) + H_q^{3,n}(x, \xi, \Delta^2),$$  

(8)

where the quantity $H_q^{3,N}(x, \xi, \Delta^2) = \tilde{H}_q^N(x, \xi)F_q^3(\Delta^2)$ represents the flavor $q$ effective GPD of the bound nucleon $N = n, p$ in $^3$He. Its $x$ and $\xi$ dependences, given by the function $\tilde{H}_q^N(x, \xi)$, is the same of the GPD of the free nucleon $N$, while its $\Delta^2$ dependence is governed by the contribution of the quark of flavor $q$ to the $^3$He f.f., $F_q^3(\Delta^2)$.

The effect of Fermi motion and binding can be shown through the ratio

$$R_q^{(0)}(x, \xi, \Delta^2) = \frac{H_q^3(x, \xi, \Delta^2)}{H_q^{3,0}(x, \xi, \Delta^2)}$$  

(9)

i.e. the ratio of the full result, Eq. (2), to the approximation Eq. (8). The latter is evaluated by means of the nucleon GPDs used as input in the calculation, and taking $F_q^3(\Delta^2) = \frac{10}{13} F_{q_{ch}}^3(\Delta^2)$, $F_q^3(\Delta^2) = -\frac{4}{13} F_{q_{ch}}^3(\Delta^2)$, where
Figure 3. Left panel: the ratio $R^{(0)}$, for the $d$ flavor, in the forward limit $\Delta^2 = 0, \xi = 0$, calculated by means of the AV18 (full line) and AV14 (dashed line) interactions, as a function of $x_3 = 3x$. The results obtained with the different potentials are not distinguishable. Right panel: the same as in the left panel, but at $\Delta^2 = -0.25$ GeV$^2$ and $\xi_3 = 3\xi = 0.2$. The results are now clearly distinguishable.

$F_{ch}^3(\Delta^2)$ is the f.f. which is calculated within the present approach. The coefficients $10/3$ and $-4/3$ are simply chosen assuming that the contribution of the valence quarks of a given flavour to the f.f. of $^3$He is proportional to their charge. The choice of calculating the ratio Eq. (9) to show nuclear effects is a very natural one. As a matter of fact, the forward limit of the ratio Eq. (9) is the same of the ratio Eq. (7), yielding the EMC-like ratio for the parton distribution $q$ and, if $^3$He were made of free nucleon at rest, the ratio Eq. (9) would be one. This latter fact can be immediately realized by observing that the prescription Eq. (8) is exactly obtained by placing $z = 1$, i.e. no Fermi motion effects and no convolution, into Eq. (2).

Results are presented in Fig. 2, where the ratio Eq. (9) is shown for $\Delta^2 = -0.15$ GeV$^2$ as a function of $x_3$, for three different values of $\xi_3$, for the flavours $u$ and $d$.

Some general trends of the results are apparent:

i) nuclear effects, for $x_3 \leq 0.7$, are as large as 15 % at most.  
ii) Fermi motion and binding have their main effect for $x_3 \leq 0.3$, at variance with what happens in the forward limit.  
iii) nuclear effects increase with increasing $\xi$ and $\Delta^2$, for $x_3 \leq 0.3$.  
iv) nuclear effects for the $d$ flavour are larger than for the $u$ flavour.

The behaviour described above is discussed and explained in Ref. 10. It is known that the point $x = \xi$ gives the bulk of the contribution to hard
exclusive processes, since at leading order in QCD the amplitude for DVCS and for meson electroproduction just involve GPDs at this point. In Ref. 10 it is shown that also in this crucial region nuclear effects are systematically underestimated by the approximation Eq. (8). In Fig. 3, it is shown that nuclear effects are found to depend on the choice of the NN potential10, at variance with what happens in the forward case. Nuclear GPDs turn out therefore to be strongly dependent on the details of nuclear structure.

The issue of applying the obtained GPDs to calculate DVCS off $^3$He, to estimate cross-sections and to establish the feasibility of experiments, is in progress. Besides, the study of polarized GPDs will be very interesting, due to the peculiar spin structure of $^3$He and its implications for the study of the angular momentum of the free neutron.

References
1. D. Müller, D. Robaschik, B. Geyer, F.M. Dittes, and J. Hořejší, Fortsch. Phys. 42, 101 (1994); hep-ph/9812448; A. Radyushkin, Phys. Lett. B 385, 333 (1996); X. Ji, Phys. Rev. Lett. 78, 610 (1997).
2. M. Diehl, Phys. Rept. 388, 41 (2003).
3. J.C. Collins, L. Frankfurt and M. Strikman Phys. Rev. D 56, 2892 (1997).
4. P.A. Guichon and M. Vanderhaeghen, Prog. Part. Nucl. Phys. 41, 125 (1998).
5. E.R. Berger et al., Phys. Rev. Lett. 87, 142302 (2001).
6. F. Cano and B. Fire, Nucl. Phys. A711, 133c (2002); Nucl. Phys. A721, 789 (2003); Eur. Phys. J. A19, 423 (2004).
7. V. Guzey and M.I. Strikman, Phys. Rev. C 68, 015204 (2003); A. Kirchner and D. Müller, Eur. Phys. J. C 32, 347 (2003).
8. J.L. Friar et al. Phys. Rev. C 42, 2310 (1990).
9. C. Ciofi degli Atti et al. Phys. Rev. C 48, 968 (1993).
10. S. Scopetta, Phys. Rev. C 70, 015205 (2004).
11. X. Ji, J. Phys. G 24, 1181 (1998).
12. S. Scopetta and V. Vento, Phys. Rev. D 69, 094004 (2004).
13. C. Ciofi degli Atti, E. Pace, and G. Salmè, Phys. Lett. B 141, 14 (1984).
14. A. Kievsky, E. Pace, G. Salmè, and M. Viviani, Phys. Rev. C 56, 64 (1997).
15. R.B. Wiringa, V.G.J. Stocks, and R. Schiavilla, Phys. Rev. C 51, 38 (1995).
16. A. Kievsky, M. Viviani, and S. Rosati, Nucl. Phys. A 577, 511 (1994).
17. A.V. Radyushkin, Phys. Lett. B 449, 81 (1999);
18. M. Gari and W. Krümpelmann, Phys. Lett. B 173, 10 (1986).
19. S. Scopetta, nucl-th/0410057