Orthogonal coding for noise immunity’s increase with the fixed code rate

A V Rabin¹

¹ Saint-Petersburg State University of Aerospace Instrumentation (SUAI), Bolshaya Morskaya str., 67, lit. A, St. Petersburg, 190000, Russia

E-mail: alexey.rabin@guap.ru

Abstract. The objective of the research, which results are presented in this paper, consists in a development and a research of the offered by the author orthogonal coding as a way of noise immunity’s increase using the example of the information communication systems with phase-shift keying. It is shown that the orthogonal coding allows one to provide the required quality of communication with a smaller energy cost. The energy gain in a signal-to-noise ratio is provided by more effective use of energy of transmitted signals with the fixed code rate without increase in complexity and cost of transmitting/receiving devices. The orthogonal coding is an analogue of convolutional coding over the rational numbers’ field. Properties of orthogonality of proposed codes allow one to strengthen significance of a useful signal with simultaneous weakening of influence of AWGN. Such increase of a signal-to-noise ratio is preserved in various fading channels. Technical realization of orthogonal coding is characterized by low complexity: at each step a decoding process is reduced to calculation of several dot products and comparison with fixed (zero, in this case) threshold. For this reason, the offered way of coding and design of receiving and transmitting devices can be implemented in various communication systems.

1. Introduction
Noise immunity is one of the most important characteristic parameters of modern communication systems. Research and development of approaches to its greater increase at the fixed transfer rate by use of special mathematical methods is a relevant scientific problem. This article is devoted to a solution of this problem.

In modern ways of noise immunity’s increase a channel is fixed as a rule. Thus, transitional probabilities of output signals (with the fixed probabilities of input signals) don’t change [1]. As a result, the number of phases in modulation in such systems doesn’t change also. For the reason specified above there is no channel parameters dependence on the applied way of channel coding. The feature of signals’ processing in receivers in telecommunication systems is the basis of the offered orthogonal coding: the transmitted signals are chosen according to the decision of developers, and processing of the transmitted signals and noise is made with use of a special approach [2]. If in this case orthogonal codes are applied, increase of transmitted signals and decrease of noise are provided. This characteristic exists not only in the channels with additive noise, but also in the fading channels [3-6].

For formation of orthogonal codes, it is required to implement the synthesis of matrices with an equal number of lines and columns. The product of these matrices is the unitary matrix multiplied by the monomial characterizing the correcting ability of a code.

This problem is solved with the condition that elements of the matrices are polynoms of the degree 1
As a result, the class of the matrices allowing to solve practical problems of noise immunity’s increase was synthesized.

Orthogonal coding as a special case of convolutional coding is defined by square matrices, which elements are polynomials in the delay variable $D$ with integer coefficients. Codewords are given by multiplication of an information polynomial in $D$ by an encoding matrix, and decoding is performed by multiplication by a decoding matrix.

It is required that these matrices shall satisfy the following condition:

$$G(D) \cdot H(D) = \rho \cdot D^i \cdot I,$$

where $I$ denotes the identity matrix. The multiplier $\rho \cdot D^i$ shows that the amplitude of the input signal increased $\rho$ times, and the delay of reception of symbols is $i$ time units.

By use of orthogonal coding with differential phase-shift keying (DPSK) we will consider only an application of the matrix $H(D)$ with polynomials in variable $D$ of degree 1.

2. Encoding and decoding matrices of orthogonal codes and their parameters

The algorithm of synthesis of matrices satisfying the condition (1) is proposed in [7].

The algorithm of synthesis of matrices of the size $(n \times n)$ is the following. It starts with synthesis of the matrix $H(D)$. First $z = 2k$, $z \leq n$, elements of the main diagonal get values $1 + D$, and the other elements of the main diagonal are 1. Even number $z$ is named as the depth of the matrix [7]. Elements outside the main diagonal get the following values: elements of odd rows at the right and odd columns downwards from the main diagonal are equal to $1 - D$; elements of even rows at the right and even columns downwards from the main diagonal are equal to $1 + D$.

For example, the decoding matrix of order 8 and depth 4 can be written as:

$$H(D) = \begin{pmatrix} 1 + D & 1 - D & 1 - D & 1 - D & 1 - D & 1 - D & 1 - D & 1 - D \\ 1 - D & 1 + D & 1 + D & 1 + D & 1 + D & 1 + D & 1 + D & 1 + D \\ 1 - D & 1 + D & 1 + D & 1 - D & 1 - D & 1 - D & 1 - D & 1 - D \\ 1 - D & 1 + D & 1 - D & 1 + D & 1 + D & 1 + D & 1 + D & 1 + D \\ 1 - D & 1 + D & 1 - D & 1 + D & 1 - D & 1 - D & 1 - D & 1 - D \\ 1 - D & 1 + D & 1 - D & 1 + D & 1 - D & 1 - D & 1 - D & 1 - D \\ 1 - D & 1 + D & 1 - D & 1 + D & 1 - D & 1 - D & 1 - D & 1 - D \end{pmatrix}.$$  

The corresponding encoding matrix $G(D)$ is given by multiplication of matrix $H^{-1}(D)$ by the least common multiple of denominators of the $H^{-1}(D)$ elements:

$$G(D) = \begin{pmatrix} 17 + 17D & -17 + 17D & 0 & 0 & 0 & 0 & 0 & 0 \\ -17 + 17D & -51 + 17D & 34 & 34 & 0 & 0 & 0 & 0 \\ 0 & 34 & 0 & -34 & 0 & 0 & 0 & 0 \\ 0 & 34 & -34 & -48 & 28 & 12 & 4 & 4 \\ 0 & 0 & 0 & 28 & 12 & -24 & -8 & -8 \\ 0 & 0 & 0 & 12 & -24 & -20 & 16 & 16 \\ 0 & 0 & 0 & 4 & -8 & 16 & 28 & -40 \\ 0 & 0 & 0 & 4 & -8 & 16 & -40 & 28 \end{pmatrix}.$$  

Multiplication of the matrices $G(D)$ and $H(D)$ gives:
By use of the orthogonal coding the number of constellation points of DPSK increases [8]. It depends on a kind of the encoding matrix used for generation of the orthogonal code. Parameters of some matrices are listed and the numbers of phases of DPSK (M) for systems, designed on the basis of corresponding matrices, specified in table 1 [9].

**Table 1. Main parameters of encoding and decoding matrices.**

| H[n, z] | G(D) | H(D) | M  | H[n, z] | G(D) | H(D) | M  | H[n, z] | G(D) | H(D) | M  |
|---------|------|------|----|--------|------|------|----|--------|------|------|----|
| 2,2     | 4D   | 9    | 7, 2 | 164D   | 629  | 12, 6 | 396D | 2377   |
| 3,2     | 4D   | 21   | 7, 4 | 12D    | 141  | 12, 8 | 68D  | 409    |
| 4,2     | 12D  | 45   | 7, 6 | 4D     | 25   | 12, 10 | 12D  | 73     |
| 4,4     | 4D   | 21   | 8, 2 | 396D   | 1517 | 12, 12 | 4D   | 25     |
| 5,2     | 28D  | 109  | 8, 4 | 68D    | 73   | 16, 2 | 456972D | 174948 |
| 5,4     | 4D   | 25   | 8, 6 | 12D    | 341  | 16, 4 | 78404D | 392021 |
| 6,2     | 68D  | 261  | 8, 8 | 4D     | 25   | 16, 8 | 2308D | 13849  |
| 6,4     | 12D  | 61   | 12, 2 | 13452D | 51501 | 16, 12 | 68D  | 409    |
| 6,6     | 4D   | 25   | 12, 4 | 2308D | 11541 | 16, 16 | 4D   | 25     |

3. Use of orthogonal coding in the AWGN channel

On the basis of the received relations the detailed analysis of efficiency of joint application of orthogonal coding and DPSK was made.

Information symbols were chosen from \{+1, −1\} [1]. For orthogonal codes with a small order of the generator, matrix estimations of bit error rate were received analytically and by imitating modeling. For the orthogonal codes with a bigger order of the generator matrix estimations of bit error rate were received only as a result of imitating modeling. Estimations of quantities of coding gain in the AWGN channel (e.g. from 3.0 dB to 4.5 dB at a bit error rate of \(10^{-6}\)) are shown in figure 1.
Figure 1. Bit error rates in the AWGN channel for BDPSK and for schemes with the orthogonal coding on the basis of matrices of orders 4, 8, 16 and 32.

For example, by use of orthogonal coding OC-32, a bit error rate of $10^{-4}$ is assured by the signal-to-noise ratio $E_b/N_0 = 6.14$ dB, which is 3.1 dB less than in case of BDPSK without coding. By use of orthogonal coding OC-32 bit error rate $10^{-6}$ is assured by the signal-to-noise ratio $E_b/N_0 = 6.71$ dB, which is 4.5 dB less, than in case of BDPSK without coding.

Investigation of noise immunity's characteristics in the AWGN channel with application of orthogonal coding and with joint use of orthogonal and error correcting codes, block codes (BCH (63, 57) and (63, 30) [5]), as well as convolutional codes (codes of the planetary standard (2,1,3) and (3,1,3) with the soft-output Viterbi-decoder, SOVA [6]) was also fulfilled.

Estimations of coding gain’s values in the AWGN channel by combination of orthogonal and error correcting codes (e.g. from 1.32 dB to 3.03 dB for BCH codes and from 0.8 dB to 1.55 dB for convolutional codes at bit error rate $10^{-6}$), received by imitating modeling, are presented in figures 2-5.

Figure 2 shows bit error rates in the AWGN channel for BDPSK without coding, with the BCH code (63,57) and for DPSK with BCH code (63,57) and the orthogonal coding OC-4, OC-8, OC-16.

For example, using orthogonal coding OC-16 with the BCH code (63,57) bit error rate $10^{-6}$ is assured by the signal-to-noise ratio $E_b/N_0 = 5.41$ dB, which is 1.65 dB less than in case of orthogonal coding OC-16 without error correcting coding, which is 3.03 dB less than in case of BCH code (63,57) without orthogonal coding, and which is 5.8 dB less than in case of BDPSK without coding.
Figure 2. Bit error rates in the AWGN channel for BDPSK without coding, with the BCH code (63,57) and for DPSK with BCH code (63,57) and the orthogonal coding OC-4, OC-8, OC-16.

Figure 3 shows bit error rates in the AWGN channel for BDPSK without coding, with the BCH code (63,30) and for DPSK with BCH code (63,30) and the orthogonal coding OC-4, OC-8, OC-16.

For example, by use of orthogonal coding OC-16 with the BCH code (63,30) bit error rate $10^{-6}$ is assured by the signal-to-noise ratio $E_b/N_0 = 4.81 \text{ dB}$, which is 2.25 dB less than in case of orthogonal coding OC-16 without error correcting coding, which is 2.6 dB less than in case of BCH code (63,30) without orthogonal coding, and which is 6.4 dB less than in case of BDPSK without coding.
Figure 3. Bit error rates in the AWGN channel for BDPSK without coding, with the BCH code (63,30) and for DPSK with BCH code (63,30) and the orthogonal coding OC-4, OC-8, OC-16.

Figure 4 shows bit error rates in the AWGN channel for BDPSK without coding, with the convolutional code (2,1,7) and for DPSK with code (2,1,7) and the orthogonal coding OC-4, OC-8, OC-16.

For example, by use of orthogonal coding OC-16 with the convolutional code (2,1,7) bit error rate $10^{-6}$ is assured by the signal-to-noise ratio $E_b/N_0 = 4.18$ dB, which is 1.55 dB less than in case of convolutional code (2,1,7) without orthogonal coding, which is 2.88 dB less than in case of orthogonal coding OC-16 without error correcting coding, and which is 7.03 dB less than in case of BDPSK without coding.
Figure 4. Bit error rates in the AWGN channel for BDPSK without coding with the convolutional code (2,1,7) and for DPSK with code (2,1,7) and the orthogonal coding OC-4, OC-8, OC-16.

Figure 5 shows bit error rates in the AWGN channel for BDPSK without coding with the convolutional code (3,1,7) and for DPSK with code (3,1,7) and the orthogonal coding OC-4, OC-8, OC-16.

For example, by use of orthogonal coding OC-16 with the convolutional code (3,1,7) bit error rate $10^{-6}$ is assured by the signal-to-noise ratio $E_b/N_0 = 3.73$ dB, which is 1.54 dB less than in case of convolutional code (3,1,7) without orthogonal coding, which is 3.03 dB less than in case of orthogonal coding OC-16 without error correcting coding, and which is 7.18 dB less than in case of BDPSK without coding.
Figure 5. Bit error rates in the AWGN channel for BDPSK without coding with the convolutional code (3,1,7) and for DPSK with code (3,1,7) and the orthogonal coding OC-4, OC-8, OC-16.

Properties of orthogonality of proposed codes allow one to strengthen significance of a useful signal with simultaneous weakening of influence of AWGN. Such increase of a signal-to-noise ratio is preserved in various fading channels.

4. Use of orthogonal coding in flat slow fading channels

Let's consider the transmission of digital signals in multipath channels with fading and show that the losses in the signal-to-noise ratio can be significantly reduced by applying of orthogonal coding. The main focus in this paper is made on frequency non-selective and slow fading, described by the Rayleigh probability density function. We do this because of the wide distribution of this model to describe the effects of fading and because of the relative simplicity of its study.

A channel is said to be frequency non-selective, if the coherence band $f_c > \frac{1}{T_s}$, where $T_s$ is the symbol transmission time [3]. In this case, all spectral components of the signal will be exposed to the same effects from the channel (for example, will experience the same magnitude of fading).

Slow fading occurs in the channel, if the signal transfer rate $\frac{1}{T_s}$ is greater than the fading rate $\frac{1}{T_c}$. In other words, the time interval, during which the channel behavior is correlated, is long compared with the time required to transmit a symbol. Therefore, it can be expected that the state of the channel will be almost unchanged during the transmission time of the symbol [4].

Let's explore the noise immunity during a binary transmission in the channel with AWGN and frequency non-selective slow fading, consider the use of orthogonal coding in such a channel and using simulation modeling we will obtain estimates of the energy gain when using orthogonal coding. If only binary symbols from the set {$+1, -1$} are fed to the input of an encoder, for example, when using the decoding matrix of order 4 of depth 2 and the corresponding encoding matrix, signals from the set {$-22, -21, ..., 21, 22$} will appear at the output of the encoder.
In figure 6 it is shown that the use of OC-4 orthogonal coding in the AWGN channel and frequency non-selective slow fading provides an energy gain compared to using binary PSK and DPSK without coding.

For example, when using OC-4 orthogonal coding, the probability of bit error $10^{-3}$ is provided for the signal-to-noise ratio $E_b/N_0 = 16.62$ dB, which is 7.63 dB less than in the case of the binary PSK without coding and 10.62 dB less than in the case of binary DPSK without coding.

![Figure 6. Bit error rates for BPSK, BDPSK and for scheme with the orthogonal coding OC-4 in flat slow fading channels.](image)

Figure 7 shows the graph of bit error probability versus a signal-to-noise ratio in the AWGN channel and frequency non-selective slow fading when using binary DPSK and graphs of bit error probability versus signal-to-noise ratio as a result of simulation modeling for processing schemes using DPSK together with orthogonal coding on the basis of encoding and decoding matrices of order 4, 8, 16 and 32.

As it is shown in figure 7, the use of orthogonal coding based on encoding and decoding matrices of order 8, 16 and 32 gives, with an increase in the signal-to-noise ratio, a greater energy gain than orthogonal coding based on encoding and decoding matrices of order four. Let us note that by increasing of matrices’ order, the coding gain grows also. For example, using orthogonal coding OC-32 bit error rate $10^{-4}$ is assured in the flat slow fading channel by the signal-to-noise ratio $E_b/N_0 = 12.63$ dB, which is 22.74 dB less than in case of BDPSK without coding [10].
Figure 7. Bit error rates for BDPSK and for schemes with the orthogonal coding OC-4, OC-8, OC-16, OC-32 in flat slow fading channels.

5. Conclusion

Transmission of digital signals in AWGN channel and fading channels is considered and it is shown that losses in a signal-to-noise ratio can be significantly reduced by use of orthogonal coding. In the given paper only orthogonal codes which contain almost no redundancy have been considered. Of course, there are orthogonal codes that have significant redundancy. It is assumed that these codes provide a greater gain in the signal-to-noise ratio, but they are not considered in this paper.

Technical realization of orthogonal coding is simple enough. On each step decoding process is reduced to calculation of several dot products and comparison with fixed (zero, in this case) threshold. For this reason, the offered way of coding and design of receiving and transmitting devices can be implemented in various communication systems.

Parameters of encoding and decoding matrices assure additional gain in signal-to-noise ratio. This gain is got as a result of a more effective use of energy of transmitted signals. For transmission of one symbol energy of several symbols is accumulated.

Thus, the proposed way of orthogonal coding can be considered as a variety of reception in a whole of signals of M-ary DPSK with an optimum choice of a modulation code. Optimization is reached by averaging of error rate on all digits of the M-ary code.

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