A simple variational approach to the quantum Frenkel-Kontorova model

Choon-Lin Ho\textsuperscript{1,2} and Chung-I Chou\textsuperscript{3}

\textsuperscript{1}Department of Physics, Tamkang University, Tamsui 25137, Taiwan
\textsuperscript{2}Theory Division, Institute of Particles and Nuclear Studies, KEK, Tsukuba, Ibaraki 305, Japan
\textsuperscript{3}Institute of Physics, Academia Sinica, Taipei 11529, Taiwan

Abstract

We present a simple and complete variational approach to the one-dimensional quantum Frenkel-Kontorova model. Dirac’s time-dependent variational principle is adopted together with a Hatree-type many-body trial wavefunction for the atoms. The single-particle state is assumed to have the Jackiw-Kerman form. We obtain an effective classical Hamiltonian for the system which is simple enough for a complete numerical solution for the static ground state of the model. Numerical results show that our simple approach captures the essence of the quantum effects first observed in quantum Monte Carlo studies.
1. Introduction. The Frenkel-Kontorova (FK) model \cite{1,2} is a simple one-dimensional model used to study incommensurate structures appearing in many condensed-matter systems, such as charge-density waves, magnetic spirals, and adsorbed monolayers \cite{3}. These modulated structures arise as a result of the competition between two or more length scales. The FK model describes a chain of atoms connected by harmonic springs subjected to an external sinusoidal potential. In an important development in the study of the classical FK model, Aubry \cite{4} first made use of the connection between the FK model, the so-called “standard map”, and the Kolmogorov-Arnold-Moser (KAM) theorem to reveal many interesting features of the FK model. Particularly, he showed that when the mean distance (also called the winding number) between two successive atoms is rational, the system is always pinned. But when the winding number is irrational, there exits a critical external field strength below (above) which the system is unpinned (pinned). This transition is called by Aubry a “transition by breaking of analyticity”, and is closely connected with the breakup of a KAM torus. It is very analogous to a phase transition, and various critical exponents and questions of universality have been extensively studied in the past.

In recent years, the FK model has been applied to the study of transmission in Josephson junction and atomic-scale friction-nanoscale tribology \cite{5}. In these cases, quantum effects are very important. Unlike the classical case, study of quantum FK models is rather scanty. It was first considered in a quantum Monte Carlo (QMC) analysis in \cite{6}. Their main observation is that the map appropriate to describe the quantum case is no longer the standard map, but rather a map with a sawtooth shape. An explanation of this phenomenon was later attempted in \cite{7} using a mean field theory in which the inclusion of the contribution from quasidegenerate states is essential. But the mean field computations with these states are rather involved, and the quantum map that is to replace the classical standard map in different quantum regimes is not clearly identified. More recently, a less complicated approach was proposed in \cite{8} which uses the squeezed state function to demonstrate that the sawtooth behavior is simply the result of quantum fluctuations. In our opinion, the approach adopted in \cite{8} is very appealing in principle. However, we believe that some difficulties in
this work need to be overcome before it could be considered satisfactory. First, the assumed
squeezed state many-body ground state is general enough so as to include the correlations
of the positions of the atoms, expressed by the covariances \( G_{ij} = \langle (x_i - \bar{x}_i)(x_j - \bar{x}_j) \rangle \) \((i \neq j)\),
where \( x_i \) is the position of the \( i \)th atom, \( \langle \cdots \rangle \) is the expectation value in a given quantum
state, and \( \bar{x}_i = \langle x_i \rangle \). However, to find the equilibrium state of the model, one has to solve
a system of coupled equations of the variables \( x_i \) and the \( G_{ij} \). The equations obtained are
so complicated that the task of solving them within a single numerical framework is very
difficult. In fact, in \[8\] a hybrid numerical analysis was adopted in which the equations
for the \( G_{ij} \) were not solved. Instead, the values of \( G_{ij} \) were taken from QMC data. These
values were then treated as initial conditions in solving the equations for the atomic posi-
tions \( x_i \)’s. Technically, such hybrid analysis is not satisfactory. Second, the covariance terms
\( G_{ij} (i \neq j) \) are constrained by the values of the fluctuation terms \( G_{ii} \) and \( G_{jj} \) through the
Cauchy-Schwarz inequality. These constraints guarantee the boundedness from below of the
effective Hamiltonian \[9\]. But then this also calls for a proper variational principle that has
to take care of the interdependence of the \( G_{ij} \) terms.

In this letter we shall show that all the essential features observed in the QMC studies can
be obtained from an independent-particle picture of the many-body ground state without
the covariance terms. In the independent-particle picture the many-body trial wavefunction
are factorizable into single-particle states. One can assume the single-particle state to have
the form of a squeeze state. For the quantum FK model, a simpler and, in our view, more
elegant approach is to use the Jackiw-Kerman (JK) function \[10\] as the single particle state.
We shall show that this simple independent-particle approach produces an effective classical
Hamiltonian which is bounded below, admits simple numerical solution of the ground state
without recourse to QMC analysis, and reproduces the essential features observed in QMC
studies.

2. Effective Hamiltonian. The Hamiltonian of the quantum FK model is given by

\[
\mathcal{H} = \sum_i \left[ \frac{\dot{p}_i^2}{2m} + \frac{\gamma}{2} (\dot{q}_{i+1} - \dot{q}_i)^2 - V \cos(l_0 \dot{q}_i) \right].
\]  

(1)
Here \( \hat{q}_i \) and \( \hat{p}_i \) are the position and momentum operators, respectively, of the \( i \)th atom, \( \gamma \) the elastic constant of the spring, \( V \) and \( 2\pi/l_0 \) are the strength and the period of the external potential. As in [7], it is convenient to use the dimensionless variables \( \hat{Q}_i = l_0 \hat{q}_i, \hat{P}_i = l_0 \hat{p}_i/\sqrt{m\gamma} \), and \( K = Vl_0^2/\gamma \). With these new variables, we obtain the following dimensionless Hamiltonian \( H \)

\[
H = \sum_i \left[ \frac{\hat{P}_i^2}{2} + \frac{1}{2} \left( \hat{Q}_{i+1} - \hat{Q}_i \right)^2 - K \cos(\hat{Q}_i) \right].
\]  

We have \( H = \gamma H/l_0^2 \). The effective Planck constant is \( \tilde{\hbar} = \hbar l_0^2/\sqrt{m\gamma} \). For the classical FK model, the Aubry transition occurs at the critical value \( K_c = 0.971635 \cdot \cdot \cdot \).

To study the ground state properties of the quantum FK model in (2), we adopt here the time-dependent variational principle pioneered by Dirac [11]. In this approach, one first constructs the effective action \( \Gamma = \int dt \langle \Psi, t | i\hbar \partial_t - \mathcal{H} | \Psi, t \rangle \) for a given system described by \( \mathcal{H} \) and \( |\Psi, t\rangle \). Variation of \( \Gamma \) is then the quantum analogue of the Hamilton’s principle. The time-dependent Hatree-Fock approximation emerges when a specific ansatz is made for the state \( |\Psi, t\rangle \). We now assume the trial wavefunction of the ground state of our quantum FK system to have the Hatree form \( |\Psi, t\rangle = \prod_i |\psi_i, t\rangle \), where the normalized single-particle state \( |\psi_i, t\rangle \) is taken to be the JK wavefunction [10]:

\[
\langle Q_i | \psi_i, t \rangle = \frac{1}{(2\pi\tilde{\hbar}G_i)^{1/4}} \times \exp \left\{ -\frac{1}{2\tilde{\hbar}} (Q_i - x_i)^2 \left[ \frac{1}{2} G_i^{-1} - 2i\Pi_i \right] + \frac{i}{\tilde{\hbar}} p_i (Q_i - x_i) \right\}.
\]

The real quantities \( x_i(t), p_i(t), G_i(t) \) and \( \Pi_i(t) \) are variational parameters the variations of which at \( t = \pm \infty \) are assumed to vanish. The JK wavefunction can be viewed as the \( Q \)-representation of the squeeze state [12]. We prefer to use the JK form since the physical meanings of the variational parameters contained in the JK wavefunction are most transparent, as we shall show below. Furthermore, the JK form is in the general Gaussian form so that integrations are most easily performed.

It is not hard to check that \( x_i \) and \( p_i \) are the expectation values of the operators \( \hat{Q}_i \) and \( \hat{P}_i \): \( x_i = \langle \Psi | \hat{Q}_i | \Psi \rangle, p_i = \langle \Psi | \hat{P}_i | \Psi \rangle \). Also, one has \( \langle \Psi | (\hat{Q}_i - x_i)^2 | \Psi \rangle = \tilde{\hbar} G_i \), and \( \langle \Psi | i\hbar \partial_t | \Psi \rangle = \sum_i (p_i \dot{x}_i - \tilde{\hbar} G_i \dot{\Pi}_i) \), where the dot represents derivative with respect to (w.r.t.) time \( t \). It is
now clear that $\hat{h} G_i$ is the mean fluctuation of the position of the $i$-th atom, and that $G_i > 0$. With these expectation values, the (rescaled) effective action $\Gamma$ for the dimensionless $H$ can be worked out to be $\Gamma(x, p, G, \Pi) = \int dt \left[ \sum_i \omega_0^{-1} (p_i \dot{x}_i + \hat{h} \Pi_i \dot{G}_i) - H_{\text{eff}} \right]$, where $\omega_0 = \sqrt{\gamma/m}$ is the angular frequency of the spring, and $H_{\text{eff}} = \langle \Psi | H | \Psi \rangle$ is the effective Hamiltonian given by

$$H_{\text{eff}} = \sum_i \frac{1}{2} \left[ p_i^2 + \hat{h} \left( \frac{1}{4} G_i^{-1} + 4 \Pi_i^2 G_i \right) \right]$$

$$+ \sum_i \frac{1}{2} (x_{i+1} - x_i)^2$$

$$+ \sum_i \hat{h} (G_{i+1} + G_i)$$

$$- \sum_i K \exp \left( -\frac{\hat{h}}{2} G_i \right) \cos x_i . \quad (4)$$

The last term in (4) can be very easily obtained from $\langle \Psi | F(Q_i) | \Psi \rangle = \sum_{m=0}^{\infty} F^{(2m)}(x_i)(\hat{h} G_i)^m/(2m)!!$, where $F^{(n)}(x) = \partial^n F(x)/\partial x^n$, and $n!! \equiv n(n-2)(n-4) \cdots 1$. Eq. (4) is bounded from below. One sees from the form of the effective action $\Gamma$ that $\Pi_i$ is the canonical conjugate of $G_i$.

Varying $\Gamma$ w.r.t. $x, p, G$ and $\Pi$ then gives the equations of motion in the Hatree-Fock approximation. Since we are mainly concerned with the static properties of the ground state of the quantum FK model, we must set the time derivatives of these variables to zero. This gives the equations which determine the values of variational parameters corresponding to the equilibrium states (which include the ground state). Equivalently, we can obtain the equations for the equilibrium states by directly varying the effective Hamiltonian $H_{\text{eff}}$ w.r.t. the variables. Varying $H_{\text{eff}}$ w.r.t. $p_i, \Pi_i, x_i$ and $G_i$ give, respectively,

$$p_i = 0 , \quad 4 \Pi_i G_i = 0 , \quad (5)$$

$$x_{i+1} - 2x_i + x_{i-1} = K \exp \left( -\frac{\hat{h}}{2} G_i \right) \sin x_i , \quad (6)$$

$$\frac{1}{4} G_i^{-2} - K \exp \left( -\frac{\hat{h}}{2} G_i \right) \cos x_i - 2 = 4 \Pi_i^2 . \quad (7)$$

The second equation in (5) implies $\Pi_i = 0$ as $G_i > 0$. This in turn means that the right hand side of eq. (6) is equal to zero:
\[
\frac{1}{4}G_i^{-2} - K \exp \left( -\frac{\hbar}{2}G_i \right) \cos(x_i) = 2 = 0 . \tag{8}
\]

In the limit \( \hbar = 0 \), eq.(8) is equivalent to the standard map. We note that eq.(8) was also obtained in [8]. This is because in the formulation in [8] the covariances \( G_{ij} \)'s decoupled from the \( x_i \) and the fluctuations \( G_{ii} \) (\( G_i \) in our case) in the variation of their Hamiltonian w.r.t \( x_i \). Unlike our case, of course, these covariance terms do actually influence the solutions of (8) through other equations obtained by variation of the Hamiltonian w.r.t. the \( G_{ii} \) and \( G_{ij} \). And it is these equations that caused the difficulties mentioned in the Introduction. In particular, the values of the \( G_{ii} \) were input from the QMC data in order to solve for the \( x_i \) in (8). Our simple approach, on the other hand, allows us to solve for both the values of \( x_i \) and \( G_i \) coupled by eqs.(8) and (8) consistently by a single numerical method.

From \( p_i = \Pi_i = 0 \) and (8), we see that the problem of finding the static ground state of the quantum FK model reduces to the problem of minimizing w.r.t. to \( x_i \) and \( G_i \) the following effective potential

\[
V_{\text{eff}} = \sum_i \frac{1}{2} (x_{i+1} - x_i)^2 + \sum_i \frac{\hbar}{8} G_i^{-1} + \sum_i \frac{\hbar}{2} (G_{i+1} + G_i)
- \sum_i K \exp \left( -\frac{\hbar}{2}G_i \right) \cos(x_i) . \tag{9}
\]

Eq.(8) and (8) are just the conditions \( \partial V_{\text{eff}}/\partial x_i = 0 \) and \( \partial V_{\text{eff}}/\partial G_i = 0 \), respectively.

3. Numerical results. We numerically solve for the set of variables \( x_i \) and \( G_i \) which characterize the ground state using the Newton method [13]. In all our numerical computations the winding number \( P/Q = 610/987 \), which is an approximation of the golden mean winding number \( (\sqrt{5} - 1)/2 \), is used with the periodic boundary condition \( x_{i+Q} = x_i + 2\pi P \). This winding number is much more accurate than those used in previous works to approximate the golden mean number, thus giving us better accuracy in the computations of physical quantities related to the ground state. We emphasize that all values of \( x_i \) and \( G_i \) are determined by the same numerical method consistently. In particular, we do not have to input
the values of $G_i$ from quantum Monte Carlo results in order to solve for the $x_i$.

Having obtained the values of $x_i$ which give the mean positions of the quantum atoms in the chain, we can compare the results with the classical configuration, following [6], in two ways: (1) by the quantum hull function, which is the plot of $x_i$ (mod $2\pi$) of the atoms against their unperturbed positions $2\pi i P/Q$ (mod $2\pi$); (2) by the so-called $g$-function, defined by

$$g_i \equiv K^{-1} (x_{i+1} - 2x_i + x_{i-1})$$

versus the actual atomic positions $x_i$. From (8), we also have

$$g_i = \exp\left(-\frac{\tilde{\hbar}^2 G_i}{2}\right) \sin x_i.$$  

Here $G_i$ is related to $x_i$ by eq.(8). We see from this equation that quantum fluctuations $G_i$ will modify the shape of the classical $sine$-map. In addition to these two types of graphs, we also plot the graph of $G_i$ against the unperturbed and the actual positions. The formal graph was first introduced in [6] to show the strong correlation of the fluctuations of atoms’ positions with their unperturbed positions. We introduce the latter type of graphs here since we think that such graphs provide better picture about how the quantum fluctuations of the atoms are related to their actual positions.

In Fig. 1 we show the four graphs mentioned above with different values of $\tilde{\hbar}$ for the supercritical case $K = 5$. Fig. 1(a) shows the quantum hull functions. For small values of $\tilde{\hbar}$ the quantum hull function consists of a countable set of steps discontinuities, just as in the classical case: the atoms are in a pinning phase. In fact, the atoms are more likely to be located near the valley of the external potential well, namely, near $x_i = 0$ (mod $2\pi$). As the quantum effect increases, i.e., for increasing values of $\tilde{\hbar}$, the quantum hull function gradually changes into a monotonic analytic function, signifying that the system is entering the depinning phase. There exists a critical value, approximately $\tilde{\hbar}_c = 6.58$ for $K = 5$, above which the quantum hull function changes from an nonanalytic function to an analytic one. This is a quantum analogue of the Aubry transition in the classical case, and can therefore be called the quantum Aubry transition.
Next in Fig. 1(b) we show the graphs of the $g$-function. The curve defined by (11) with $G_i$ satisfying (8) are shown here as dashed curves for different $\tilde{h}$. In the classical limit ($\tilde{h} = 0$) this curve is simply the standard map (sine-curve). As $\tilde{h}$ increases, the amplitude of the curve decreases. For sufficiently large $\tilde{h}$, the curve resembles more closely a “sawtooth” shape. This is first noted in QMC study in [6]. Here we see that it comes out very naturally from the equation of motion (8) and (11). In the supercritical case ($K = 5$), when $\tilde{h} < \tilde{h}_c$, the positions $x_i$ of the atoms cover only a subset of the $g$-curves. This is in accord with the fact that the atoms are in the pinning phase [cf. Fig. 1(a)]. As $\tilde{h}$ increases, the points begin to spread along the $g$-curve. When $\tilde{h} > \tilde{h}_c$, the $g$-graph is completely covered as the system has entered the depinning phase.

Fig. 1(c) shows the quantum fluctuations $G_i$ plotted against the actual atomic positions $x_i$. The dashed curves represent the curves of eq.(8) for different $\tilde{h}$. For small $\tilde{h}$, the atoms are located near $x_i = 0 \pmod{2\pi}$ with small values of $G_i$ which means, from (8), that the wavefunctions are highly peaked at these positions. As the quantum effect increases, the external potential is so modified that now the atoms could be found at other positions, but with atoms at $x_i = \pi \pmod{2\pi}$ having the largest value of $G_i$. This indicates that wavefunctions of the atoms near the top of the potential are more extended with smaller amplitudes. Again, when $\tilde{h} > \tilde{h}_c$, the curves of (8) are completely covered by the solutions $x_i$. To compare with the results in [6], we plot the values of $G_i$ against the unperturbed positions in Fig. 1(d). One sees that the values of $G_i$ are strongly correlated with the unperturbed positions, as first noted in [6]. For $\tilde{h} < \tilde{h}_c$ the graphs consists of steps discontinuities, and for $\tilde{h} > \tilde{h}_c$ the graphs are continuous. This is correlated with the graphs of the quantum hull function in Fig. 1(a), since from (8) any fixed value of $x_i$ correspond to a fixed value of $G_i$.

Next we show in Fig. 2 the corresponding graphs for the case $K = 1.5$. This represents the situation which is slightly over the critical classical case. The general trends of the behavior of the graphs are the same as those in Fig. 1. As expected, quantum Aubry transition takes place at a smaller $\tilde{h}_c = 1.17$. We note here that the shape of the $g$-function at large $\tilde{h}$ in
this case is intermediate between a \textit{sine} and a sawtooth map.

We have also checked the subcritical cases with $K < K_c$. The classical system is already in the depinning phase in this regime. Quantum fluctuations only enhance the trend of depinning. The $g$-function is found to be closer to a \textit{sine}-shape with smaller amplitude for higher $\tilde{h}$. This is consistent with the QMC results [6].

Finally, we note here that, while we have reproduced the essential features first observed in the QMC studies of the quantum FK model, there is also slight discrepancy between the results of these two approaches. The difference is that, for a fixed value of $K$, the QMC results [3] indicated that the sawtooth shape of the $g$-function appeared at a lower value of $\tilde{h}$, and that the atoms began to spread along the $g$-curve also at a smaller $\tilde{h}$. For example, at $K = 5$ the QMC results showed that the above situation already appeared at $\tilde{h} = 0.2$, while our results (cf. Fig. 1(b)) indicate that at $K = 5$ and at a higher $\tilde{h} = 2$ the system is still closer to the classical case. We believe this could be explained as follows. First, our independent-particle wavefunction is only the lowest order approximation of the many-body wavefunction of the quantum FK system. A more accurate description of the system will require a better assumption of the wavefunction than that assumed here. This presumably may require the inclusion of the effects of the covariance terms as advocated in [8], but with a more appropriate variational principle to circumvent the difficulties already mentioned in the Introduction. Second, our results are obtained at zero temperature, while those in the QMC analysis were obtained, by the nature of the method itself, at small but finite temperatures (temperature $T=0.0067$ as given in [6]). It is natural that thermal fluctuations will cause the atoms to spread away from their zero-temperature positions.

4. \textbf{Summary.} In conclusion, we have presented a simple and complete variational approach to the quantum FK model based on a Hatree-type many-body trial wavefunction of the JK form. The effective Hamiltonian obtained is bounded below, and is simple enough for a complete numerical solution for the static ground state of the model in various quantum regimes. Numerical results show that our simple approach captures the essence of the quantum effects first observed in QMC studies. The map appropriate for the quantum FK
model is well described by eq.(11) and (8). In contrast to previous approaches, we do not
require the existence of the complicated quasidegenerate states, or the partial help from
quantum Monte Carlo data in order to obtain these results.

**Acknowledgments**

This work is supported in part by the R.O.C Grant NSC 89-2112-M-032-004. Part of
the work was done while one of us (CLH) was visiting the Theory Division at KEK (Japan)
under the auspices of the exchange program between KEK (Japan) and the National Center
for Theoretical Sciences (Taiwan). He would like to thank the staff and members of the
theory group of KEK for their hospitality and support. After the paper was submitted, we
were kindly informed by Prof. B. Hu that he and W.M. Zhang in their previous incomplete
work [14] had obtained a Hamiltonian similar to our eq.(1), but that they had not studied
the ground state properties.
REFERENCES

[1] Y.I. Frenkel and T. Kontorova, Zh. Eksp. Teor. Fiz. 8, 1340 (1938); Sov. Phys.-JETP 13, 1 (1938).

[2] F. C. Frank and J. H. van der Merwe, Proc. R. Soc. London, Ser. A 198, 205 (1949).

[3] P. Bak, Rep. Prog. Phys. 45, 587 (1982).

[4] S. Aubry, in Solitons and Condensed Matter Physics, ed. A.R. Bishop and T. Schneider, (Springer-Verlag, Berlin, 1978); J. Phys. (Paris) 44, 147(1983); Physica 7D, 240 (1983); ibid, 8D 381 (1983).

[5] M. G. Rozman, M. Urbakh, and J. Klafter, Phys. Rev. Lett. 77 683 (1996); Phys. Rev. E 54, 6485 (1996); M. Weiss and F.-J. Elmer, Phys. Rev. B 53, 7539 (1996); T. Gyalog and H. Thomas, Europhys. Lett. 37, 195 (1996).

[6] F. Borgonovi, I. Guarneri and D. Shepelyansky, Phys. Rev. Lett. 63, 2010 (1989); Z. Phys.B 79, 133 (1990); F. Borgonovi, Ph.D dissertation, Universitá Degli Studi di Pavia, 1989 (unpublished).

[7] G. P. Berman, E. N. Bulgakov and D. K. Campbell, Phys. Rev. B 49, 8212 (1994).

[8] B. Hu, B. Li and W.-M. Zhang, Phys. Rev. E 58, 4068 (1998); B. Hu and B. Li, Quantum Frenkel-Kontorova model (to appear in Physica A).

[9] B. Hu and B. Li, private communications.

[10] R. Jackiw and A. Kerman, Phys. Lett. A 71, 158 (1979).

[11] P.A.M. Dirac, Proc. Cambridge Phil. Soc. 26, 376 (1930).

[12] Y. Tsue and Y. Fujiwara, Prog. Theor. Phys. 86, 443 (1991).

[13] H.J. Schellnhuber, H. Urbschat and A. Block, Phys. Rev. A33, 2856 (1986); H.J. Schellnhuber, H. Urbschat and J. Wilbrink, Z. Phys. B80, 305 (1990).
[14] B. Hu, “The Frenkel-Kontorova Model: classical generalizations and quantum glimpses” in *Proceedings of the 4th Drexel Conference on Quantum Nonintegrability*, eds. D.H. Feng and B.L. Hu, 1997. In a brief section of this paper a Hamiltonian very similar to our eq.(4) (but with a fixed $\tilde{\hbar} = 1$) was presented based on squeezed state approach.

**Figures captions**

**Fig. 1** Structure of the quantum ground state for $K = 5$ and winding number $P/Q = 610/987$ at $\tilde{\hbar} = 2$ (black dots), 6 (white dots) and 7 (black curve). (a) quantum hull function plotted against unperturbed atomic positions; (b) $g$-function plotted against actual atomic positions (the dashed curves represent eq.(11) with $G_i$ satisfying (8); (c) and (d) quantum fluctuations $G_i$ plotted against the actual and unperturbed positions, respectively. The dashed curves in (c) represent the curves of eq.(8) for different $\tilde{\hbar}$.

**Fig. 2** Same as Fig. 1 for $K = 1.5$ and $\tilde{\hbar} = 0.5$ (black dots), 1.0 (white dots) and 2 (black curve).
This figure "Fig1.JPEG" is available in "JPEG" format from:

http://arxiv.org/ps/cond-mat/0008142v1
This figure "Fig2.JPEG" is available in "JPEG" format from:

http://arxiv.org/ps/cond-mat/0008142v1