Exotic Statistics for Ordinary Particles in Quantum Gravity

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SUMMARY

Objects exhibiting statistics other than the familiar Bose and Fermi ones are natural in theories with topologically nontrivial objects including geons, strings, and black holes. It is argued here from several viewpoints that the statistics of ordinary particles with which we are already familiar are likely to be modified due to quantum gravity effects. In particular, such modifications are argued to be present in loop quantum gravity and in any theory which represents spacetime in a fundamentally piecewise-linear fashion. The appearance of unusual statistics may be a generic feature (such as the deformed position-momentum uncertainty relations and the appearance of a fundamental length scale) which are to be expected in any theory of quantum gravity, and which could be testable.
I. INTRODUCTION

The spin-statistics theorem [1] states that half-integer spin particle obey Fermi-Dirac statistics and the Pauli exclusion principle and integer spin particles obey Bose-Einstein statistics is well-known and has many derivations [2]. I argue in this essay that lifting the restrictions in the usual derivations of this theorem leads one to expect *generically* that it will not hold when quantum gravitational effects are included. In fact, I will give so many arguments for this that I argue that this may well be a *generic* feature of quantum theories of gravity [3]. Conversely, if quantum gravity effects leave the usual field commutators and spin-statistics connection untouched, then something very deep must be going on to protect them.

II. THE STANDARD APPROACHES TO THE SPIN-STATISTICS THEOREM

There are two main ways to approach the spin-statistics theorem. The first is to consider commutation relations for fields $\phi(t, x)$ and their conjugate momenta $\pi(t, x)$ and *assume* that it makes sense to think of them as being analogous to $x$ and $p$ of nonrelativistic quantum mechanics. Suppressing Lorentz indices, one has expressions of the form

$$[\phi(t, x), \phi(t, x')] = 0$$

$$[\pi(t, x), \pi(t, x')] = 0$$

$$[\phi(t, x), \pi(t, x')] = i\delta^{(3)}(x - x')$$

where the brackets can represent commutators or anticommutators. Ignoring interactions and assuming locality and flat Minkowski spacetime, one then finds that one runs into
problems if commutators are used for Dirac (half integer spin) fields or if anticommutators are used for Bose (integer spin) fields.

An alternative viewpoint makes the swap of \( x \) and \( x' \) more physical by arguing [4] that it is equivalent to a rotation (FIG. 1).

![FIG. 1. Two particles (small circles) swapped by a rotation (two arcs with arrows). Here no flux passes through the circle that defines the rotation, but if it did, one might expect additional phases to appear.](image)

If one thinks of two particles being swapped as equivalent to a rotation of \( 2\pi \), then one picks up a sign of \( e^{(i2\pi s)} \) for spin \( s \): a “+” for bosons and a “−” for fermions. Implicit in such an argument is that there are no gauge fields present which could have altered the phases of particles en route.

III. EFFECTS OF QUANTUM GRAVITY

The most obvious concern with equations (1), (2), and (3) above is that they are equal-time commutation relations with an obvious reliance on a 3+1 split – something which might be dealt with by considering Peierls brackets instead [7]. Of course one would still have to
worry about possible extensions (central or not) when replacing brackets with commutators in the quantum theory.

Rather general physical arguments suggest that the commutators between $x$ and $p$ should be modified by quantum gravitational effects [5], as should the $x$ commutators [6].

The momentum commutators are already nontrivial in curved spacetime.

For fields, the $\delta^{(3)}(x - x')$ of (3) might be replaced by a sharply-peaked function with width related to the Planck mass. It is worth noting that such scale-dependent effects arise naturally for composite particles. The idea that statistics of composite particles might be subtle goes back to Wigner [9] in 1929 and Ehrenfest and Oppenheimer [10] in 1931. For fermion pairs ("quasibosons" [11]) such as superfluid helium-4 and Cooper pairs a short-range Pauli effect $i\delta$ is present. Lipkin [12] notes that the energy gap for Cooper pairs "...would be absent if the fermions behaved like simple bosons." In other words, we already know of systems which behave more or less bosonic or fermionic as a function of scale.

The most convincing general arguments for changes to the basic commutators are perhaps those of Ahluwalia-Khalilova [13] based on the work of Mendes, Chryssomalakos and Okon [14]. The idea is to find the most general stable extension of the combined Poincaré and Heisenberg algebras. This leads to the Snyder-Yang-Mendes algebra [15], which has nontrivial commutators (not all commutation relations are shown) between coordinates $X_\mu$ and momenta $P_\mu$:

$$[X_\mu, X_\nu] = i\ell_p^2 J_{\mu\nu}$$ (4)

I would argue for nontrivial commutation relation for positions already without quantum gravity. [8]
\[ [P^\mu, P^\nu] = i \frac{\hbar^2}{\ell_C^2} J_{\mu\nu} \] (5)

\[ [P^\mu, X^\nu] = i\hbar \eta_{\mu\nu} F + i\hbar \beta J_{\mu\nu} \] (6)

These involve \( \hbar \) (explicit here), two length scales \( \ell_P \) (presumably of order the Planck length) and \( \ell_C \) (presumably of cosmological size), a new dimensionless constant \( \beta \), and the angular momentum \( J_{\mu\nu} \). \( F \) is a new operator having nontrivial commutation relations with \( P \) and \( X \). The triply special relativity of Kowalski-Glikman and Smolin [16] can be put in this form [13].

Given that one postulates field commutators by analogy with the commutation relations of \( x \) and \( p \), it is now by no means obvious that (1), (2) and (3) are correct.

One can also stick to the usual commutation relations for the Poincaré and Heisenberg group and ask questions at the level of the fields themselves. After all, physically (and in the spirit of noncommutative geometry [17]), one constructs spacetime from measurements involving fields.

Now consider the product \( \phi(x)\phi(y) \) which is needed to form commutators, anticommutators and propagators (\( x \) and \( y \) now commuting labels for spacetime points). The first problem is that \( \phi(x)\phi(y) \) is not gauge invariant. As noted long ago by Schwinger, \( \phi(x)\phi(y) \) would need to be multiplied by a phase \( \exp(i \int_x^y A_\mu dx^\mu) \) for whatever connection \( A \) is relevant. For a self-interacting charged particle in flat spacetime one finds [18] that the free infrared propagator \( 1/(p^2 - m^2) \) is raised to a fractional power \( 1 + \frac{\alpha}{\pi} \) and becomes non-local (a charged particle carries a long-range field). It has been argued that Newtonian gravitational self-interaction even in flat spacetime will give a similar sort of correction [18].

In a general curved space background [19], but ignoring self-interaction, DeWitt [20]
has given an exact representation for the Feynman propagator which is complicated and nonlocal \(^2\). Of course all these discussions have assumed adiabatic processes where particle numbers do not change as in the Unruh and Hawking effects \([19]\).

Since what originally looked like a well-defined product (or expectation value thereof) of two free fields in flat spacetime must be replaced for interacting fields in curved spacetime by a nonlocal object, it seems unlikely that their (anti-)commutators would suffer no modifications.

The foregoing two arguments, one kinematical and the other dynamical, suggest that the usual commutation relations for quantum fields (and thus the usual derivations of the spin-statistics connection) may not be generally valid. There have been arguments in the literature that there should be no unusual spin-statistics relationships in curved spacetime \([22]\). They require assumptions about the (anti-)commutation relations, as well as the existence of suitable flat regions of spacetime. Bardek et al. \([23]\) considered exotic statistics in curved spacetime and argued for their consistency and constancy in an expanding universe, while Scipioni has argued \([24]\) that transitions of statistics might occur under some conditions. A comprehensive list of references can be found in \([25]\).

At first glance, statistics describes exchanges of objects (at the particle level) and thus is about representations of the permutation group of \(n\) objects \(S_n\). If an exchange is equivalent to a rotation \([4]\), and one wants two exchanges to be the identity, then one is naturally led to the representation of a single exchange by multiplying a wavefunction by \(\pm 1\). This was formalized with path-integral arguments by Laidlaw and deWitt \([26]\) who ruled out any

\(^2\)Even then there are subtleties. Toms has argued \([21]\) that there is an ambiguity due to the choice of path integral measure.
other possibilities – but there are loopholes.

Extended objects are well-known to support exotic statistics. Topological geons [27] can violate the spin-statistics theorem [28]. Charged extremal black holes have been argued [29] to obey “infinite statistics” [30], where any representation of the permutation group can occur. Infinite statistics admit a Fock-like representation [31] in terms of deformed commutation relations for creation and annihilation operators of the form 

\[ a_k a_l^\dagger - qa_l^\dagger a_k = \delta_{kl}. \]

A theory of objects satisfying these deformed commutation relations has a Hamiltonian which is nonlocal and nonpolynomial in the field operators – something obviously suggestive of curved spacetime and the expected nonlocalities described above.

If the particles have internal degrees of freedom (i.e. their wavefunctions are sections of \( \mathbb{C}^N \), or something else) there can be inequivalent quantizations, labelled by irreducible representations of the braid group \( B_n(M) \). All sorts of novel statistics are then possible [32,33]. Intuitively, internal degrees of freedom can keep track of how many times particles have gone “around each other”.

In 2+1 dimensions, it is well-known that particles coupled to gauge fields can form composite objects (“anyons”) with unusual statistics [34]. The physical idea is easy to understand. If one takes a particle around a flux \( \Phi \), one picks up an Aharonov-Bohm type phase \( \exp(ig\Phi) \) where \( g \) represents the coupling of the particle to the field. The braid group \( B_n \) keeps track of how the flux lines twist around each other as the particles move in 2 dimensions. Gravitational anyons in 2+1 dimensions [35] could be physically relevant at the surfaces of black holes (perhaps even microscopic or virtual ones [36]).

It has been suggested [38] that exotic statistics for charged particles (i.e. some electrons in a white dwarf acting as bosons) could arise for particles whose angular momentum comes
partly from coupling to external electromagnetic fields. Similar effects might be anticipated from gravitational fields.

Strings open up completely new possibilities. For a space \( M \) like \( \mathbb{R}^3 \) with \( n \) points removed (say by black holes or some sort of spacetime foam) one could have \( \pi_1(M) = 0 \) but strings would probe the loop space \( \Omega M \), with \( \pi_1(\Omega M) = \pi_2(M) \neq 0 \).

Exotic statistics are also possible [39] for strings even in topologically trivial 3-dimensional manifolds, due to their ability to be linked and tangled. For this, a further generalization of braid statistics is needed involving the “loop braid group” \( \text{LB}_n \) [40]. Lest one imagine that such issues are purely academic, Niemi has argued for exotic statistics in “leapfrogging” vortex rings [41] in quantum liquids and gases which, like anyons, really exist in the physical world. Exotic statistics for strings in 4-D BF theory have also been discussed by Baez, Wise, and Crans [40]. Particles corresponding to states of strings might inherit exotic statistics which could test stringy models of particles. That said, stringiness can also be somewhat hidden in the nonlocality of theories of point particles coupled to long-range fields [42].

Gambini and Setaro [43] found fractional statistics for composites of charged one-dimensional objects and vortices. Fort and Gambini found Fermi-Bose transmutation [44] for point scalars and Nielsen-Olesen strings in the Maxwell-Higgs system, and fractional statistics [45] in a 3+1 dimensional system composed of an open magnetic vortex and an electrical point charge.
IV. EXOTIC STATISTICS FOR ORDINARY PARTICLES IN LQG

I now want to concentrate on a very physical reason why loop quantum gravity (LQG) [46] and any 3+1 formulation of a piecewise-linear (PL) [47] Regge-like [48,49] theory of gravity naturally supports exotic statistics.

The idea is very simple: in 3+1 dimensional Regge-type theories [50], the curvature is distributional, with support on edges where flat 3-simplices meet. A particle picks up a phases as it moves around a line of singular curvature and that phase can contribute to exotic statistics.

In LQG, spin-networks define natural dual PL simplicial geometries [52,51] of flat simplicial complexes with distributional curvature along 1-dimensional subspaces where they join. A spin network is a graph composed of lines carrying $SU(2)$ representation labels which give areas to surfaces they pierce. They meet at vertices labelled by intertwiners. If there are at least 4 edges which meet at a vertex, one can think of the vertex as enclosed by faces which get their areas from the edges that pierce them. The intertwiner determines the volume enclosed (see FIG. 2 for an example). Curvature is represented by the deficit angle along edges where simplices meet (see FIG. 3 for an example).

If the distributional lines of curvature are taken to represent matter, we have physical strings and all the arguments for the loop braid group also appear in this context.
FIG. 2. A portion of a spin network and an associated simplex. The areas of the faces of the simplex are determined by the SU(2) representations $j_i$ on the edges which pierce them and the volume by the intertwiner associated with the vertex.

FIG. 3. View of 2-simplices fitting together with a deficit angle (the edges there should be brought together to construct the curved 2-geometry with curvature at the central point). The 3-dimensional picture is analogous, but the 5 triangles are now replaced by 5 tetrahedra from a spin-network and the point where the curvature is becomes a line of curvature. One could think of this figure as looking “down” on the 5 flat triangular faces of 5 flat tetrahedra joined along a line of singular curvature perpendicular to the page.
While I cannot claim to know the correct replacement for the usual (anti-)commutation relations in field theory, I would argue that their modification is a logical and *generic* possibility in theories of quantum gravity and worthy of theoretical and experimental study. It is a natural extension of the now familiar ideas of looking for changes to the basic commutators of $x$ and $p$. Such modifications might only appear at very large or very small scales ($\ell_C$ or $\ell_P$) and could easily have avoided experimental limits so far [53].

I would like to conclude with some speculations connecting ideas discussed here with other topics of current research. If one thinks of local supersymmetry transformations as local changes of statistics of objects which can be fermions or bosons, then local changes of symmetry lead naturally to local translation invariance and thus to general coordinate invariance. Is there a connection here? Jackson [54] has argued that a suitable position-momentum commutator can describe many features of gravity. To make this plausible, recall that essentially all the interesting things about the geometry of phase space in quantum mechanics come from the one nontrivial commutator.

Braiding and exotic statistics may also have some bearing on interpretations of Standard Model particles in terms of framed spin networks [55,56]. Framings arise naturally in spin-networks with q-deformed groups [37] which are needed in LQG with a cosmological constant (nontrivial $[P_{\mu}, P_{\nu}]$). Also of interest is [57] in which spin and statistics for spacetime and “internal” exchanges are connected. It is also interesting that non-commutative geometries arise naturally together with q-deformed groups in the same situations where anyons appear [58], so it seems many ideas may be connected.
V. ACKNOWLEDGEMENTS

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Note added: The day after this work was completed, a preprint from Mark G. Jackson entitled “Spin-Statistics Violations from Heterotic String Worldsheet Instantons” appeared on Arxiv (ArXiv:0803.4472v1) which discusses possible violations of the spin-statistics theorem in heterotic string theory and also makes the point that should the true scale for quantum gravity be much lower than the usual Planck scale, such effects might be more readily observable in the near future.

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