Heavy-to-light form factors:
Symmetries at large recoil and calculation of $\alpha_s$ corrections

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Recently it has been shown that in the large-recoil limit new symmetries emerge which impose various relations on form factors that parametrise the decay of heavy $B$ mesons into light mesons. These symmetry relations are broken by radiative QCD corrections. We show that these corrections are perturbatively calculable, and present results to first order in the strong coupling constant $\alpha_s$.

1. Introduction

We study matrix elements of bilinear quark currents, that are parametrised by form factors, encoding the (long-distance) strong interaction effects in exclusive, semi-leptonic or radiative $B$ decays, such as $B \to \pi l \nu$, $B \to K^* \gamma$ etc. They also appear as non-perturbative parameters in the factorisation theorem for non-leptonic $B$ decays in the heavy quark mass limit [1]. The knowledge of these form factors therefore helps to determine the CKM coupling $|V_{ub}|$, and to predict CP violating asymmetries and other quantities in rare $B$ decays.

Charles et al. have shown that certain symmetries apply, when the momentum of the final light meson is large [2]. These symmetries reduce the number of independent form factors, but they are broken by radiative corrections. In this contribution, which is based on work done together with Martin Beneke [3], we give a brief derivation of the symmetry relations and then discuss the computation of symmetry-breaking corrections at first order in the strong coupling constant $\alpha_s$.

2. Large-recoil symmetries

Let us concentrate on the form factors for $\bar{B}$ decays into a pseudoscalar meson. They are defined by the following Lorentz decompositions of the matrix elements:

\begin{align}
\langle P(p')|\bar{q}\gamma^\mu b|\bar{B}(p)\rangle &= f_+(q^2) \left[ p^\mu + p'^\mu - \frac{M^2-m^2_P}{q^2} q^\mu \right] + \nonumber \\
&\quad + f_0(q^2) \frac{M^2-m^2_P}{q^2} q^\mu , \\
\langle P(p')|\bar{q} \sigma^{\mu\nu} q_\nu b|\bar{B}(p)\rangle &= i f_T(q^2) \left[ q^2 (p^\mu + p'^\mu) - (M^2-m^2_P) q^\mu \right],
\end{align}

where $M$ is the $B$ meson mass, $m_P$ the mass of the pseudoscalar meson and $q = p - p'$.

It is useful to recapitulate the implications of heavy quark symmetry, when the final meson $P$ is also heavy, for example a $D$ meson.

2.1. Heavy-heavy decays

As long as the velocity transfer to the $D$ meson remains of order 1, we may assume that the heavy quarks interact with the spectator quark (and other soft degrees of freedom) exclusively via soft gluon exchanges characterised by momentum transfers much smaller than the heavy quark masses. Any hard interaction would imply large momentum of the spectator quark in the $B$ meson or $D$ meson or both, and such a configuration is assumed to be highly improbable. The simplifications that occur when heavy quarks interact only with soft gluons are formalised as heavy quark effective theory (HQET) (see e.g. Ref. [4] and references therein) which implies the

[Contribution to IV. International Conference on Hyperons, Charm and Beauty Hadrons, Valencia 2000; hep-ph/0008272]
well-known spin and heavy flavour symmetries in the infinite quark mass limit \( \bar{\xi} \).

A consequence of these symmetries is that all form factors are related to a single function of velocity transfer, \( \xi (v \cdot v') \), whose absolute normalisation is known at zero recoil (\( \xi (1) = 1 \)). The heavy quark symmetries are violated by radiative corrections (as well as higher dimension operators in the effective Lagrangian), such as the one shown in Fig. 1b. The symmetry breaking effects are caused only by the short-distance part of Fig. 1b. They are accounted for by multiplicatively renormalising the heavy quark current in HQET.

Figure 1. Different contributions to the \( B \rightarrow P(V) \) transition. (a) Soft contribution (soft interactions with the spectator antiquark are not drawn). (b) Hard vertex renormalisation. (c,d) Hard spectator interaction.

Let us consider the large recoil limit and assume that the \( b \) quark and the energetic light quark, created in the heavy-to-light transition, interact with the spectator quark (and other soft degrees of freedom) exclusively via soft gluon exchanges. We may then continue to exploit the heavy quark symmetries for the \( b \) quark. A similar simplification occurs for the light quark. To leading order in \( \Lambda_{\text{QCD}}/E \), the interaction of light energetic quarks with soft gluons can be described by an effective Lagrangian \( \mathcal{L}_{\text{eff}} \).

\[
\mathcal{L}_{\text{eff}} = \bar{q}_n \frac{\gamma \cdot p}{2} (i n \cdot D) q_n + \mathcal{O}(1/E) ,
\]

where \( n^\mu \) are two light-like vectors \( (n_+ n_- = 2) \), and \( q_n(x) = e^{iE(n \cdot x) - i n \cdot q(x)} \). In the limit \( E \rightarrow \infty \) the Lagrangian \( \mathcal{L}_{\text{eff}} \) leads to new symmetries that, combined with the heavy-quark symmetries, imply non-trivial relations between the soft contributions to the form factors \( \mathcal{M} \). Using techniques familiar from HQET, we find

\[
\langle P(p') | \bar{b} \gamma_\mu b | B(p) \rangle = \text{tr} \left[ A_P(E) \mathcal{M}_P \mathcal{M}_B \right] \quad (4)
\]

where

\[
\mathcal{M}_P = (\gamma_5) \frac{\frac{\gamma \cdot p}{2}}{2} \gamma_5 ,
\]

\[
\mathcal{M}_B = 1 + \frac{\frac{\gamma \cdot p}{2}}{2} \gamma_5 , \quad (5)
\]

with \( p^\mu \simeq M v^\mu, \), \( p'^\mu \simeq E n^\mu \). The most general form of the function \( A_P(E) \) is

\[
A_P(E) = 2E \xi_P(E) ,
\]

with a conveniently chosen overall normalisation.

It follows that the three pseudoscalar meson form factors are all related to a single function \( \xi_P(E) \).

(Analogously, the seven form factors for decays into light vector mesons can be expressed in terms of only two unknown functions, \( \xi_L(E) \) and \( \xi_L(E) \), see also Ref. \( [3] \).) Performing the trace in Eq. (4), and comparing with the form factor definition (12), we obtain, for instance,

\[
f_+(q^2) = \frac{M}{2E} f_0(q^2) = \frac{M}{M + m_P} f_T(q^2) = \xi_P(E) \quad (7)
\]

There is an important distinction between the effective Lagrangian for heavy quarks and for energetic light quarks. A configuration where one
light quark carries most of the energy is atypical. Therefore the symmetries of the interaction are not realised in the hadronic spectrum. Furthermore, the probability that such an asymmetric parton configuration hadronises into a light meson depends on the energy of the meson. Hence, the soft contributions to the form factors are energy-dependent functions, whose absolute normalisation is not known. This is to be contrasted to the case of heavy-heavy form factors, for which the spin symmetry relates pseudoscalar and vector mesons, and the Isgur-Wise form factor \( \xi(v \cdot v') \) is independent of the heavy quark mass. The scaling law for the soft contribution can be derived in different ways, but all of them make use of the endpoint behaviour of the pion’s light-cone distribution amplitude \( \tau \). If one assumes that the distribution amplitude vanishes linearly with the longitudinal momentum fraction of the spectator quark, as is suggested by its asymptotic form, one finds

\[
\xi_P(E) \sim \frac{M^{1/2} \Lambda_{QCD}^{1/2}}{E^2}. \tag{8}
\]

As in the case of a heavy-to-heavy transition, there will be a vertex correction to Eq. (8) (of the type shown at one loop in Fig. 1b). The hard part of these diagrams does not respect the symmetry relations, but it can be accounted for in perturbation theory by multiplicatively renormalising the heavy-to-light current just as in the case of a heavy-to-heavy transition.

The important new element of the discussion is provided by hard spectator interaction in Figs. 1c and 1d. This allows the light meson to be formed in a preferred configuration, in which the momentum is distributed nearly equally between the two quarks. Since both quarks have momentum of order \( M \), and the gluon in Figs. 1c and 1d has virtuality of order \( (M \Lambda_{QCD}) \), this contribution can be computed within the hard-scattering approach to exclusive processes \( \pi, \eta \). The resulting scaling behaviour for the pseudoscalar meson form factors is (see, for instance, Refs. 9, 10)

\[
f_{1,\text{hard}}(q^2 \simeq 0) \sim \alpha_s \left( \frac{\Lambda_{QCD}}{M} \right)^{3/2}. \tag{9}
\]

We can summarise this discussion by the following factorisation formula for a heavy-light form factor at large recoil, and at leading order in \( 1/M \):

\[
f_i(q^2) = C_i \xi_P(E) + \Phi_B \otimes T_i \otimes \Phi_P, \tag{10}
\]

where \( i = \{+ \rightarrow 0, T \} \). Analogous formulas hold for the form factors that parametrise the decays into light vector mesons \( \eta \). Here \( \xi_P(E) \) is the soft part of the form factor, to which the symmetries discussed above apply; \( T_i \) is a hard-scattering kernel (with the endpoint divergence regulated in a certain manner), convoluted with the light-cone distribution amplitudes of the \( B \) meson and the light pseudoscalar meson; \( C_i \) contains the hard vertex renormalisation. Eq. (8) implies that the hard spectator interaction (Figs. 1c and 1d) is suppressed by one power of \( \alpha_s \) relative to the soft contribution (Fig. 1b). Hence the form factor relations like Eq. (8) are indeed correct at leading order in \( 1/M \) and \( \alpha_s \).

We are going to calculate the order \( \alpha_s \) corrections to Eq. (8) in the full theory and identify the soft contributions to be absorbed into \( \xi_P \). We will not explicitly match the effective theory onto QCD, but, instead, define a factorisation scheme by imposing the convenient condition

\[
f_+ \equiv \xi_P. \tag{11}
\]

In this way, also some hard contributions are absorbed into \( \xi_P \). In particular, we circumvent some subtleties, arising in the effective theory defined by Eq. (8), that have been addressed in Ref. 11.

### 3. Results

Calculating the vertex corrections from Fig. 1b, using \( \overline{\text{MS}} \) renormalisation, and comparing with Eqs. (10, 11), we obtain \( \langle C_+ = 0 \rangle \)

\[
C_0 = \frac{2E}{M} \left( 1 + \frac{\alpha_s \epsilon}{4\pi} (2 - 2L) \right), \tag{12}
\]

\[
C_T = \frac{M + m_P}{M} \left( 1 + \frac{\alpha_s \epsilon}{4\pi} (\ln \frac{M^2}{\mu^2} + 2L) \right) \tag{13}
\]

where we have introduced the abbreviation \( L = -\frac{2E}{M - 2\mu} \ln \frac{2E}{\mu} \). Note that the tensor form factors depend on the renormalisation scale \( \mu \).
The hard-scattering corrections are calculated from Figs. 1c,d by convoluting the hard-scattering amplitude with the light-cone wave functions of heavy and light mesons. With Eqs. (11) this yields $\Phi_B \otimes T_0 \otimes \Phi_P = 0$

$$\Phi_B \otimes T_0 \otimes \Phi_P = \frac{\alpha_s C_F}{4\pi} \frac{q^2}{2E M} \Delta F_P ,$$ (14)

$$\Phi_B \otimes T_T \otimes \Phi_P = \frac{\alpha_s C_F}{4\pi} \frac{M + m_P}{2E} \Delta F_P ,$$ (15)

where we have defined the abbreviation

$$\Delta F_P = \frac{8\pi^2 f_B f_P}{N_C M} \langle \ell^+_1 \rangle_B \langle \bar{u}^{-1} \rangle_P .$$ (16)

Here $l_+$ is the light-cone plus component of the spectator quark in the $B$ meson, and $\langle \ell^+_1 \rangle_B$ the corresponding moment with its light-cone distribution amplitude. The same moment appears in the leading-order contribution to $B \to \ell \nu \gamma$ decays [2], and determines the leading non-factorisable corrections to $B \to \pi \pi$ decays [3]. The moment $\langle \bar{u}^{-1} \rangle_P$, with $\bar{u}$ being the longitudinal momentum fraction of the spectator quark in the light meson, is accessible, for instance, from the analysis of $P \gamma$ transition form factors, see, for instance, the review in Ref. [3].

Analogous results are found for the form factors that parametrise decays into light vector mesons [3]. We quote only one example that is important for the analysis of the forward-backward asymmetry zero in the rare decay $B \to K^* \ell^+ \ell^-$. The form factor ratios that enter the relation between the Wilson coefficients $C_{\text{eff}}^2$ and $C_{\text{eff}}^3$ [14] receive the following $\alpha_s$ correction

$$\frac{M + m_V}{M} T_1 \frac{T_2}{M + m_V} = \frac{M}{A_1} =$$

$$1 + \frac{\alpha_s C_F}{4\pi} \ln \frac{M^2}{\mu^2} L + \frac{\alpha_s C_F}{4\pi} \frac{M}{4E} \Delta F_{\perp}$$ (17)

where a similar quantity as in Eq. (16),

$$\Delta F_{\perp} = \frac{8\pi^2 f_B f_\perp}{N_C M} \langle \ell^+_1 \rangle_B \langle \bar{u}^{-1} \rangle_{\perp} ,$$ (18)

enters, with $\langle \bar{u}^{-1} \rangle_{\perp}$ being a moment of the distribution amplitude of a transversely polarised vector meson. Inserting standard values for the parameters in Eqs. (17), the $\alpha_s$ corrections in Eq. (17) amount to about 5%.

In summary, we have shown that, in the kinematic region where the momentum transfer to the light meson is large, heavy-to-light form factors can be described by a factorisation formula (14) which is based on the symmetries that arise in the large-recoil/heavy-quark limit. This implies that $\alpha_s$ corrections to symmetry relations are calculable in a systematic way: i) Vertex corrections to the heavy-to-light current can be treated in an analogous way as in heavy quark effective theory. ii) Hard rescattering with the spectator quark is described by the hard-scattering approach which involves light-cone distribution amplitudes of the participating mesons. Typically, to first order in the strong coupling constant, these corrections amount to a few percent. Further details can be found in Ref. [3].

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