Causal inference for process understanding in Earth sciences

Adam Massmann, Pierre Gentine, Jakob Runge

May 4, 2021

Abstract

There is growing interest in the study of causal methods in the Earth sciences. However, most applications have focused on causal discovery, i.e. inferring the causal relationships and causal structure from data. This paper instead examines causality through the lens of causal inference and how expert-defined causal graphs, a fundamental from causal theory, can be used to clarify assumptions, identify tractable problems, and aid interpretation of results and their causality in Earth science research. We apply causal theory to generic graphs of the Earth system to identify where causal inference may be most tractable and useful to address problems in Earth science, and avoid potentially incorrect conclusions. Specifically, causal inference may be useful when: (1) the effect of interest is only causally affected by the observed portion of the state space; or: (2) the cause of interest can be assumed to be independent of the evolution of the system’s state; or: (3) the state space of the system is reconstructable from lagged observations of the system. However, we also highlight through examples that causal graphs can be used to explicitly define and communicate assumptions and hypotheses, and help to structure analyses, even if causal inference is ultimately challenging given the data availability, limitations and uncertainties.

Note: We will update this manuscript as our understanding of causality’s role in Earth science research evolves. Comments, feedback, and edits are enthusiastically encouraged, and we will add acknowledgments and/or coauthors as we receive community contributions. To edit the manuscript directly (recommended) you can fork the project’s repository and submit a pull request at https://github.com/massma/causality-earth-science or you can alternatively email us comments, questions, and suggestions and we will try to incorporate them into the manuscript.

*Corresponding author: akm2203@columbia.edu
1 Introduction

There is growing interest in the study of causal inference methods in the Earth sciences [e.g., Salvucci et al., 2002, Ebert-Uphoff and Deng, 2012, Kretschmer et al., 2016, Green et al., 2017, Barnes et al., 2019, Samarasinghe et al., 2020, Runge, 2018, Runge et al., 2019, Goodwell et al., 2020]. However, most of this work focuses on causal discovery, or the inference (using data) of causal structure: i.e., the “links” and directions between variables. In some cases, causal discovery can be used to estimate the structure of a causal graph and the relationships between variables when the graph is not known a priori. However in many if not most Earth system applications, the causal graph is already known based on physical insights. For instance the impact of El Niño on West American rainfall is known to be causal and the graph does not need to be discovered (even though using causal discovery for this problem is a useful sanity check of the method’s ability).

This paper looks at causality through a different, but complementary, lens to causal discovery and examines how assumed causal graphs [Pearl, 1995], a fundamental from causal theory, can be used to clarify assumptions, identify tractable problems, and aid interpretation of results in Earth science research. Our goal is to distill [e.g., Olah and Carter, 2017] the basics of the graphical approach to causality in a way that is relatable for Earth scientists, hopefully motivating more widespread use and adoption of causal graphs to organize analyses and communicate assumptions. These tools are relevant now more than ever, as the abundance of new data and novel analysis methods have inevitably led to more opaque results and barriers to the communication of assumptions.

Beyond their usefulness as communication tools, if certain conditions are met, causal graphs can be used to estimate, from data, the generalized functional form of relationships between Earth science variables [Pearl, 2009b]. Ultimately, deriving generalized functional relationships is a primary goal of science. While we know the functional relationships between some variables a priori, there are many relationships we do not know [e.g., ecosystem scale water and carbon fluxes; Massmann et al., 2019, Zhou et al., 2019b, a, Grossiord et al., 2020], or that we do know but are computationally intractable to calculate [e.g., clouds and microphysics at the global scale: Randall et al., 2003, Gentine et al., 2018, Zadra et al., 2018, Gagne et al., 2020]. In these types of applications, causal graphs give us a path toward new scientific knowledge and hypothesis testing: generalized functional relationships that were inaccessible with traditional tools.

The main contribution of this paper is to demonstrate how causal graphs, a fundamental tool of causal inference discussed in Section 2, can be used to communicate assumptions, organize analyses and hypotheses, and ultimately improve scientific understanding and reproducibility. We want to emphasize that almost any study could benefit from inclusion of a causal graph in terms of communication and clarification of hypotheses, even if in the end the results cannot be interpreted causally. Causal graphs also encourage us to think deeply in the initial stages of analysis about hypotheses and how the system is structured, and can identify infeasible studies early in the research process before time is spent on analysis, acquiring data, building/running models, etc. These points require some background and discussion, so the paper is divided into the following sections:

- Section 2 introduces and discusses causal graphs within the general philosophy of causality and its application in Earth science.
• Section 3: Using a simple relatable example we explain the problem of confounders and how causal graphs can be used to isolate the functional mapping between interventions on some variable(s) to their effect on other variable(s).
• Section 4: We draw on a real example that benefits from inclusion of a causal graph, in terms of communicating assumptions, and organizing and justifying analyses.
• Section 5: We turn to more generic examples of graphs that are generally consistent with a wide variety of systems in Earth science, to highlight some of the difficulties we confront when using causal inference in Earth science and how we may be able to overcome these challenges.

Throughout the discussion key terms will be emphasized in italics.

2 The graphical perspective to causality and its usefulness in the Earth sciences

While there are many different definitions and interpretations of “causality”, for this manuscript we view causality through the lens of causal graphs, as introduced in Pearl [1995] and discussed more extensively in Pearl [2009b]. We take this perspective because we believe causal graphs are useful in Earth science, rather than because of any particular philosophical argument for causal graphs as the “true” representation of causality.

Causal graphs are Directed Acyclic Graphs (DAGs) that encode our assumptions about the causal dependencies of a system. To make a causal graph, a domain expert simply draws directed edges (i.e., arrows) from variables that are causes to effects. In other words, to make a causal graph you draw a diagram summarizing the assumed causal links and directions between variables (e.g., Figure 1). Causal graphs are useful tools because they can be drawn by domain experts with no required knowledge of maths or probability, but they also represent formal mathematical objects. Specifically, underlying each causal graph are a set of equations called structural causal models: each node corresponds to a generating function for that variable, and the inputs to that function are the node’s parents. Parents are the other nodes in the graph that point to a node of interest (e.g., in the most simple graph \( X \rightarrow Y \); \( X \) is a parent of \( Y \)). So in reality, drawing arrows from “causes” to “effects” is synonymous with drawing arrows from function inputs to generating functions.

In this way, drawing a causal graph is another way to visualize and reason about a complicated system of equations, which is a very useful tool for the Earth scientist: we deal with complicated interacting systems of equations and welcome tools that help us understand and reason about their collective behavior. In some cases we may know a priori (from physics) the equations for a given...
function in a causal graph. However, in practice we often either do not know all of the functions a priori [e.g., plant stomata response to VPD; Massmann et al., 2019; Zhou et al., 2019b,a; Grossiord et al., 2020], or some functions are computationally intractable to compute [e.g., turbulence, moist convection, and cloud microphysics in large scale models; Zadra et al., 2018; Gentine et al., 2018]. In these scenarios the benefits of causal graphs are fully realized: based on the causal graph we can calculate from data, using the do-calculus [Pearl, 1994], the response of target variables (i.e. effects) to interventions on any other variables in the graph (i.e. causes). When combined with statistical modeling (i.e., regression), one can estimate the functional relationship between interventions on causes and effects [Pearl, 2009a; Shalizi, 2013; Shi et al., 2019; Mao et al., 2020]. By viewing causal graphs through this pragmatic lens of calculating the functional form of relationships that we do not know a priori, we simultaneously identify causal graphs’ value for Earth scientists while also side stepping philosophical arguments about the meaning of causality. Causal graphs are pragmatic because in the Earth sciences we often need to estimate how the system responds to interventions (prescribed changes to variables of interest, or “causes”). For example, sub-grid physical parameterizations in Earth system models (e.g., turbulence) require estimates of the time tendencies’ response to interventions on the large scale state and environment. We also may desire to calculate experiments: for example how changing land cover from forest to grasslands affects (the statistics of) surface temperature. Do-calculus is a method to calculate this response to interventions without relying on approximate numerical models or real world experimentation, which can be infeasible or unethical [as is the case for geoengineering; e.g., unilateral decisions to seed the oceans with iron, or spray aerosols in the atmosphere, Hamilton, 2013]. While we want to maintain this emphasis on causality as a method for calculating the generalized response to intervention (possibly using regression to calculate the functional form of that response), for consistency with the causal literature we will call the response variables “effects”, and the intervened-upon variables “causes”. For some, it may not be clear how the functional response to interventions is different from naive regression between observed variables. We will demonstrate in Section 3 how uninformed regression is just a functional mapping of associations between variables, and how this differs from the response to interventions, i.e. a causal mechanism. This is the problem that do-calculus solves: it identifies which data are needed and how we can use those data to calculate the response to interventions, rather than just associations that may be attributable to other processes entirely. Because we know the response to the intervention is attributable to the intervention and not other processes, we have greater confidence in the generalizability of the response to interventions. This generalizability of the response to interventions makes do-calculus especially relevant for scientists and engineers. Working through an example will clarify some of these claims.

3 A Toy Example: the problem of confounding and the necessity of do-calculus for calculating interventions

To demonstrate the problem of confounding and the necessity of causal graphs/do-calculus, we use a simple toy example involving clouds, aerosols, and surface solar radiation/sunlight. As shown in Figure 1, the causal graph consists of:
1. An edge from aerosols to clouds because aerosols serve as cloud condensation nuclei or ice nucleating particles, which affect the probability of water vapor conversion to cloud (condensates).

2. An edge from aerosols to surface solar radiation, because aerosols can reflect sunlight back to space and reduce sunlight at the surface.

3. An edge from clouds to sunlight, because clouds also reflect sunlight back to space and can reduce sunlight at the surface.

Causal graphs encode our assumptions about how the system behaves, and the nodes and edges that are missing from the graph often represent strong assumptions about the lack of functional dependence. For example, in the cloud-aerosol-sunlight example, clouds also affect aerosols; e.g., by increasing the likelihood that aerosol will be scavenged from the atmosphere during precipitation [e.g., Radke et al., 1980; Jurado et al., 2008; Blanco-Alegre et al., 2018]. By not including an edge from cloud to aerosol, we are making the assumption that we are neglecting the effect of clouds on aerosols, and also preventing the graph from containing any cycles (a path from a variable to itself) which is a requirement of the theory: graphs must be acyclic. This acyclic requirement may raise concerns for the reader; many problems in Earth science contain feedbacks that introduce cycles. However, any feedback can be represented as an acyclic graph by explicitly resolving the time evolution of the feedback in the graph (Section 5 contains examples of such graphs). Considering this example is intended to be pedagogical for introducing causal theory to the readers, we will continue with the graph as drawn in Figure 1 (we refer the reader to Gryspeerdt et al. [2019] for a more realistic treatment of aerosols and clouds).

Even though mathematical reasoning is not required to construct a causal graph, the resulting graph encodes specific causal meaning based on qualitative physical understanding of the system. Implicitly, the graph corresponds to a set of underlying functions, called a structural causal model, for each variable:

\[
\begin{align*}
\text{aerosol} & \leftarrow f_{\text{aerosol}}(U_{\text{aerosol}}) \\
\text{cloud} & \leftarrow f_{\text{cloud}}(\text{aerosol}, U_{\text{cloud}}) \\
\text{sunlight} & \leftarrow f_{\text{sunlight}}(\text{aerosol}, \text{cloud}, U_{\text{sunlight}})
\end{align*}
\]

where \( U \) are random variables due to all the factors not represented explicitly in the causal graph, and \( f \) are deterministic functions that generate each variable in the graph from their parents and corresponding \( U \).

The presence of the random variables \( U \) introduces a third meaning to the causal graph: they induce a factorization of the joint distribution between variables into conditional and marginal factors:

\[
P(A, C, S) = P(S | C, A) P(C | A) P(A),
\]

where \( A \) represents aerosol, \( C \) represents cloud, \( S \) represents surface solar radiation, and \( P(C | A) \) denotes conditional probability of \( C \) given \( A \) (Appendix A describes the notation used in this paper and a brief introduction to probability theory for unfamiliar readers). The inclusion of randomness
in causal graphs is a key tool: by positing a causal graph, we are not stating that the variables in the graph are the only processes in the system nor that the relationships are deterministic. Instead, we are stating that all other processes not included in the graph induce variations in the graph’s variables that are independent of each other (e.g., all $U$ in Equation (1) are independent). For example, sources of aerosol variability not considered in Figure 1 include anthropogenic aerosol emission, the biosphere, fires, volcanoes, etc. (e.g., Boucher, 2015). For cloud, this includes synoptic forcing or atmospheric humidity, etc. (e.g., Wallace and Hobbs, 2006). For radiation, this includes variability of top of atmosphere radiation, etc. (e.g., Hartmann, 2015). Figure 1 states that all these external, or exogenous, sources of variability are independent of each other (in very technical terms, this means the graph is “Markovian,” Pearl, 2009b).

We can now apply causal inference theory (e.g., Pearl, 1995, Tian and Pearl, 2002, Shpitser and Pearl, 2006) to the assumptions encoded in our causal graph to identify which distributions must be estimated from data in order to calculate the response of effect(s) (e.g. of sunlight) to an experimental intervention on the cause(s) (e.g. presence or absence of a cloud). The goal of causal inference is to derive the response to the intervention in terms of only observed distributions. This process of identifying the necessary observed distributions is formally termed causal identification (Pearl, 2009b, Ch. 3). If a causal effect is not identifiable (un-identifiable), for example if calculating a causal effect requires distributions of variables that we do not observe, then we cannot use causal inference to calculate a causal effect, even with an infinite sample of data.

A necessary condition for unidentifiability is the presence of an unblocked backdoor path from cause to effect (Pearl, 2009b, Ch. 3). Backdoor paths are any paths going through parents of the cause to the effect. We can block these paths by selectively observing variables such that no information passes through them (Geiger et al., 1990). If observations are not available for the variables required to block the path, the path will be unblocked. However, if we can observe variables along the backdoor paths such that all backdoor paths are blocked, then we have satisfied the back-door criterion (Pearl, 2009a) and we can calculate unbiased causal effects from data.

Understanding backdoor paths and the backdoor criterion is helped by an example. Returning to our toy example (Figure 1), we attempt to calculate the causal effect of clouds on sunlight. In other words, we want to isolate the variability of sunlight due to the causal link from cloud to sunlight (Figure 1). However, aerosols affect both cloud and sunlight (i.e., there is a backdoor path from cloud to aerosol to sunlight), so if we naively calculate a "causal" effect using correlations between sunlight and cloud, we obtain a biased estimate. To demonstrate this, consider simulated cloud, aerosol, and sunlight data from a set of underlying equations consistent with Figure 1 and Equation (1):

\[
\begin{align*}
aerosol &= U_{aerosol}; \\cloud &= \text{Cloudy if } U_{cloud} + \text{aerosol} > 1; U_{cloud} \sim \text{uniform (0, 1)} \\
sunlight &= \begin{cases} 
\text{Cloudy} : 0.6 \cdot \text{downwelling clear sky radiation} \\
\text{Clear} : \text{downwelling clear sky radiation}
\end{cases}
\end{align*}
\]
Average sunlight difference: -157.74 W/m² (true effect of cloud on sunlight: -68 W/m²)

Figure 2: A naive approach to estimating the “effect” of clouds on sunlight: bin observations by cloudy and clear day, and compare the values of sunlight. This approach yields an average difference of 157.74 W m⁻² between cloudy and clear days, and is a large overestimation of the true causal effect of clouds on sunlight (-68.0 W m⁻²) in these synthetic data.

\[
downwelling \text{ clear sky radiation} = U_{\text{sunlight}} \cdot (1 - \text{aerosol}); \quad U_{\text{sunlight}} \sim \text{Normal}(340 \text{ W m}^{-2}, 30 \text{ W m}^{-2})
\]

Now, consider not knowing the underlying generative processes, but instead just passively observing cloud and sunlight. If one were interested in calculating the effect of cloud on sunlight, and aerosol data were not available or one were not aware that aerosol could have an impact on clouds and aerosol, one direct but incorrect approach would be to bin the data by cloudy and clear conditions and compare the amount of sunlight between cloudy and clear observations (Figure 2). This approach would suggest that clouds reduce sunlight by, on average, 158 W m⁻²; this is is a strong overestimation of the true average effect of clouds (-68 W m⁻²), derived from Equation (5). This overestimation is due to aerosol-induced co-variability between cloud and sunlight that is unrelated to the causal link from cloud to sunlight. However if aerosols were constant (e.g. observed or not varying), any co-variability between cloud and sunlight would be attributable to the causal edge between cloud and sunlight (Figure [I]). In other words, conditional on aerosol, all co-variability between cloud and sunlight is only due to the causal effect of cloud on sunlight. We can mathematically encode this requirement that we must condition on aerosol to isolate the causal effect of cloud on radiation, and doing so derives the causal effect of cloud on sunlight by satisfying the backdoor criterion with adjustment on aerosol:

\[
P(S|do(C = c)) = \int_a P(S|C = c, A = a) \, P(A = a) \, da,
\]

where the do-expression (\(P(S|do(C = c))\)) represents the probability of sunlight if we did an
experiment where we intervened and set cloud to a value of our choosing (in this case \( c \), which could be “True” for the presence of a cloud, or “False” for no cloud). In the case that observations of aerosols are not available, our causal effect is not identifiable and we cannot generally use causal inference without further assumptions, no matter how large the sample size is of our data.

Causal graphs are therefore powerful analysis tools: after encoding our domain knowledge in a causal graph, we can analyze the available observations to determine whether a causal calculation is possible, \textit{without needing to collect, download, or manipulate any data}. For more complicated graphs, causal identification can be automated \cite{Tian and Pearl, 2002, Shpitser and Pearl, 2006, Huang and Valtorta, 2006, Bareinboim and Pearl, 2016, Textor et al., 2017, https://causalfusion.net, http://www.dagitty.net/}. We later use this theory to assess assumptions that lead to tractable causal analyses for generic Earth science scenarios (Section 5).

Once we have established that a causal effect is identifiable from data, we must estimate the required observational distributions (Equation (6)) from data. Often it may be more computationally tractable to calculate an average causal effect, rather than the full causal distribution \( P(S|\text{do}(C = c)) \), which might be difficult to estimate. Returning to our toy example (Figure 1), the average effect is defined as:

\[
\mathbb{E}(S|\text{do}(C = c)) = \int_s s \ P(S = s|\text{do}(C = c)) \ ds,
\]

where \( \mathbb{E} \) is the expected value. Substituting Equation (6) into Equation (7), and rearranging gives:

\[
\mathbb{E}(S|\text{do}(C = c)) = \int_a P(A = a) \ \mathbb{E}(S|C = c, A = a) \ da,
\]

Where \( \mathbb{E}(S|C = c, A = a) \) is just a regression of sunlight on cloud and aerosol. Estimating the marginal \( P(A) \) is difficult, but if we assume that our observations are independent and identically distributed (IID) and we have a large enough sample, we can use the law of large numbers to approximate Equation (8) as \cite{Shalizi, 2013}:

\[
\mathbb{E}(S|\text{do}(C = c)) \approx \frac{1}{n} \sum_{i=1}^{n} \mathbb{E}(S|C = c, A = a_i).
\]

Data or prior knowledge can inform the estimate of \( \mathbb{E}(S|C = c, A = a_i) \), but whatever regression method is used, it should be checked to ensure it is representative of the data and there is sufficient signal to noise ratio to robustly estimate the regression. It is important to note how this estimate is different from the naive association of \( \mathbb{E}(S|C = c) \); in \( \mathbb{E}(S|C = c, A = a_i) \) we are controlling for the impact of aerosol on sunlight by conditioning on \( A \) and including aerosol in the regression. As we will see, using the \textit{do}-calculus estimate results in an estimate of the effect of cloud on aerosol that is very different from the naive association, and close to the true effect.
Effect of cloud on sunlight as estimated from data: -67.69 W/m² (true effect: -68 W/m²)

Figure 3: A linear relationship between aerosol and sunlight, conditional on cloud. If we use this regression to calculate the average causal effect of cloud on sunlight, as in Equation (9), our result is very close to the true causal effect of -68.0 W m⁻².

In our simple example, a linear model conditional on cloud is a suitable choice for the regression function $E(S|C = c, A = a_i)$ (Figure 3). However, many problems in Earth science require nonlinear approximation methods like neural networks and/or advanced machine learning methods; for examples of such machine learning methods, we refer readers to [Bishop, 2006].

The causal effect of clouds on sunlight as calculated using Equation (9)) (e.g. $E(S|do(C = \text{cloudy})) - E(S|do(C = \text{clear}))$) is -67.69 W m⁻², which closely matches the true causal effect from Equation (5) of -68 W m⁻². This example demonstrates how causal inference and theory can be used to calculate unbiased average effects using regression, subject to the assumptions clearly encoded in the causal graph. Further, causal inference can be used to justify and communicate assumptions in any observational analyses employing regression. In the best case, the causal effect is identifiable from the available observations, and the regression analysis can be framed as an average causal effect. In the worst case that identification is not possible from the available observations, one may present the regression as observed associations between variables. However, presentation of a causal graph still aids the reader: the reader can see from the causal graph what the confounders and unobserved sources of covariability are between the predictors and the output. In all cases, the presentation of a causal graph makes explicit the assumptions about the causal dependencies of the system. Wherever possible, we recommend including causal graphs with any observation-based analyses.

In summary of the main points of this introduction to causal graphical models and do-calculus:

- Graphical causal models encode our assumptions about causal dependencies in a system (edges are drawn from causes to effects). “Causal dependencies” really just refer to functional dependencies between inputs (causes) and outputs (effect), which are useful in the
Earth sciences to reason about graphically.

- In order to calculate an unbiased causal effect from data, we must isolate the covariability between cause and effect that is due to the directed causal path from cause to effect. The presence of non-causal dependencies between the cause and effect can be deduced from the causal graph: the presence of an unblocked backdoor path from the cause to the effect leads to non-causal dependencies (and co-variation).

- The backdoor criterion identifies the variables that we must condition on in order to block all backdoor paths, remove non-causal dependence between the cause and effect, and calculate an unbiased causal effect from data.

- The average causal effect can be reliably approximated with a regression (Equation (9)) derived from the backdoor criterion. In this scenario, causal theory and graphs identify the variables that should be included in the regression in order to calculate an unbiased causal effect (however researchers should still ensure their choice of regression model is appropriate for the data). While not demonstrated by our example, causal theory and graphs also identify the variables that should not be included in the regression [Pearl, 2009a]; we can also bias a causal effect by including too many variables in a regression.

- The do-calculus and identification theory provide a flexible tool to determine whether an effect is identifiable and, if so, which distributions should be estimated from data, while making no assumptions about the forms of the underlying functions and distributions. However, parametric assumptions can be applied to make the calculation of those distributions from data more computationally tractable.

Here we focused on the backdoor criterion to block backdoor paths. An unblocked backdoor path from the cause to the effect is a necessary condition for unidentifiability. However, an unblocked backdoor path from the cause to the effect is not a sufficient condition for unidentifiability: there are other identification strategies like the front door criterion [see Section 3.5.2 in Pearl, 2009b] and instrumental variables [see Chapter 8 in Pearl, 2009b] that do not rely on observing variables along the backdoor path, and can be used in some cases where observations are not available to satisfy the backdoor criterion (also see Tian and Pearl [2002] for more discussion on sufficient conditions for unidentifiability). These are examples of how the do-calculus admits any strategy that frames the response to interventions in terms of observed distributions. We focus on the backdoor criterion because it is the most fundamental and direct method for adjusting for confounding, the most intuitive for an introduction to causality, and is the most relevant for the generic temporal systems present in the Earth sciences (Figure 5 in Section 5). However, causal identification through other methods like instrumental variables and the front door criterion can also be automated; we refer the reader to Pearl [2009b] for further discussion and software tools like http://www.dagitty.net/ and http://www.causalfusion.net for interactive exploration.

4 Beyond toys: causal graphs as communicators, organizers, and time-savers

In Section 3 we used a toy example to demonstrate a causal analysis starting with drawing a graph and ending with the successful calculation of the average response of sunlight (the effect) to an intervention on cloud (the cause). However, often we may not be able to ultimately estimate the
causal effects from the available data. In many cases there are serious challenges due to unobserved confounding in generic Earth science problems (Section 5), and for other cases we may lack enough samples, or samples could be too systematically biased, to estimate the necessary distributions (or regressions; e.g., Equation (9)) with sufficient certainty. However, we want to emphasize that even if calculating a causal effect might in some cases be impossible, drawing a causal graph at the beginning of an analysis still offers tremendous benefits in terms of organization and scientific communication. Investing time to reason about the functional structure of the problem at the outset can save scientists time in the long term, forcing us to clarify our thinking early, expose potential challenges, and identify intractable approaches.

Additionally, once the causal graph is drawn, we can use it as a communication tool and include it in presentations, papers, and discussions of our results. Making our assumptions about dependencies in the system explicit greatly improves the interpretability and reproducibility of our results. Perhaps our analysis and graph meet the standards for a causal interpretation, but even if they do not, the causal graph helps the rest of the community assess the sources of confounding in the graph that were not controlled for, and understand if their conceptualization of the graph structure matches the authors’ hypotheses. Often in research there are more assumptions being made than are communicated, and even when they are communicated, the assumptions do not always get discussed in a precise way. Including a causal graph allows the assumptions to be clearly known, and discussed in a precise and rigorous way [e.g., Hannart et al., 2016].

To support the idea that many analyses would benefit from a causal graph, we will detail how a past project benefits from a causal graph. This example also moves beyond the toy example of Section 3 and demonstrates causal graphs’ applicability to real problems in Earth science.

4.1 Causal graphs’ utility in a real example

In Massmann et al. [2017], the lead author of this manuscript participated in a field campaign designed to study the impact of microphysical rain regime (specifically the presence of ice from aloft falling into orographic clouds) on orographic enhancement of precipitation. This field campaign and analysis benefits from a causal graph and is a real-world example argument for the more common use of causal graphs as research tools. Our retrospective causal graph of orographic enhancement in the Nahuelbuta mountains under steady conditions clearly communicates our assumptions about the system (Figure 4).

Many of the variables in the graph, such as “synoptic forcing”, “wind”, and “orographic flow”, are quite general quantities. Keeping quantities general can lead to more intuitive and interpretable graphs by limiting the number of details and nodes that one must consider. However, if logic needs clarifying, the graph can become more explicit (e.g., differentiating wind into speed, direction, and spatial distribution, both horizontally and vertically). We also include unobserved variables explicitly in the graph, so we can reason about processes’ impact on the system even if they are not observed. As graphs become more complicated, one can leverage interactive visualization software, or use static graph abstractions like plates [Bishop, 2006], which are particularly well suited to representing repeated structure common to spatiotemporal systems.
Figure 4: A graph representing steady conditions for orographic enhancement during the Chilean Coastal Orographic Precipitation Experiment [CCOPE, Massmann et al., 2017]. The field campaign attempted to quantify the effect of “rain regime” on “orographic enhancement.” Observed quantities are represented by solid nodes, while unobserved quantities are represented by dashed nodes. Variables that must be observed to block all backdoor paths are shaded. The effect of rain regime on orographic enhancement is identifiable through adjustment on these shaded variables.
While the exact details of the graph and the assumptions it encodes are interesting (e.g., “wind”, “stability”, and “atmospheric moisture” all refer to upwind conditions, and we assume that these upwind conditions are the relevant “boundary condition” for the downwind orographic clouds and precipitation), the noteworthy feature of the graph is that the field campaign’s effect of interest, rain regime on orographic enhancement, is identifiable from the field campaign’s observations. This is subject to the assumptions encoded in the graph, but those assumptions are explicitly represented and communicated by the graph. The causal graph helps interpret the field campaign’s results, and in some sense proves that the design of the field campaign is sound.

Therefore, for field campaigns, causal graphs are particularly useful at the planning and proposal stage. Such a causal graph could be included in any field campaign proposal, improving communication about the system and also rigorously justifying the campaign’s observations as necessary for calculating the desired effect(s). Even before the proposal, one could start with a causal graph, and then analyze it to determine which observations are needed to meet the campaign’s goals. Building on this idea, one could attach costs associated with observing each variable in the graph, and automatically determine the set of observations that minimizes cost while still allowing us to calculate our effect(s) of interest.

While this is just one example, it demonstrates that causal graphs are useful beyond toy problems in the Earth system. Additionally, as we will see in Section 5, we can draw quite general graphs that are representative of many problems in Earth science. We hope the reader considers drawing a causal graph as a first step in their next project; they help structure, organize, and clarify our analysis and its assumptions.

5 Overcoming unobserved confounding and partial observation of Earth system state

So far we have focused on toy (Section 3) and specific (Section 4) examples. We now turn our attention to more general and generic problems in Earth science systems, the common challenges we may encounter when attempting causal inference, and how we can overcome these challenges.

Earth science systems and their components are (typically) dynamical systems evolving through time according to an underlying system state [Lorenz, 1963, 1996; Majda, 2012]. This offers both opportunities and challenges for causal inference. When constructing causal graphs we may benefit from the temporal ordering of events [Runge et al., 2019]: we know that future events can have no causal effect on the past. We can also use the time dimension to explicitly resolve feedbacks in the system, and transform cyclic graphs with feedbacks into directed acyclic graphs (DAGs) required for causal inference. While handling feedbacks and avoiding cyclic graphs is a challenge of causal inference in Earth science, resolving the time dimension is a generic path to overcome this challenge when observations of sufficient resolution are available.

However, confounding due to incomplete observation of the system’s state variables also introduces challenges; challenges that are not unique to this paper’s causal lens: incomplete observation of the system precludes the use of many “causal discovery” algorithms as well (see Runge et al., [2019] for a detailed review). Causal identification and tractable causal inference in Earth science
Figure 5: A generic graph of the Earth system state sequence. Unobserved nodes are outlined by dashed lines. We only observe the state space at certain times (e.g., no observations at $S(t-1/2)$), and at times with observations, we only partially observe the full state ($S(t), S(t-1)$). In the scenario that we are interested in calculating the causal effect of any portion of the state space at time $t$ on some effect ($E$) at time $t+1$, the causal effect will be confounded by the unobserved portions of the state space, and calculating the causal effect is impossible (un-identifiable) without additional assumptions.

requires assumptions about the unobserved portions of the state space that introduce this confounding, and how the unobserved portions of the state space affect observed variables. Without such assumptions the unobserved portions of the state space will introduce confounding for any causal effect of interest (Figure 5). For example, we generally do not observe the state space at every time (e.g. $S(t-1/2)$, Figure 5), and at any given time, we do not observe the state space at all locations and for all state variables (e.g. $S(t)$ and $S(t-1)$ in Figure 5). In other words, despite our impressive and growing array of satellite, remote sensing, and in situ observation systems, we are still very far from observing every relevant state variable at every location in time and space. So, if we are interested in the causal effect of any state variable at time $t$ on some variable at time $t+1$ (e.g., $E$ in Figure 5), then the causal effect will be confounded by the unobserved portions of the state space, and calculating a causal effect will be impossible (un-identifiable) without additional assumptions.

However, there are assumptions we can make that may be reasonable for many generic applications, which remove this problem of unobserved confounding due to partial observation of the state space. We elaborate upon these assumptions in the following sections, and for each assumption, we draw a graph, briefly discuss the scenario and assumptions, present the identification formula and how average effects can be estimated (for example, using regression to estimate $E(\cdot)$), and include some strategies for testing the assumption(s) with data.
Figure 6: A generic graph for the assumption that we observe all state variables that impact our effect(s) of interest. $C(t)$ is the cause of interest, $E(t + 1)$ is the future effect of interest, and $O(t)$ are all observations not including $C(t)$. The full state $S(t - 1)$ is not observed (dashed node). Observing $O(t)$ (grey shading) blocks the backdoor path from $C(t)$ to $E(t + 1)$.

**Assumption: we observe all state variables that impact our effect(s) of interest**

- **Discussion:** While we may not observe the entire state of the system, sometimes it is reasonable to assume that we observe the portion of the state space that affects our specific effect of interest ($E(t + 1)$ in Figure 6). In this case, we can calculate causal effects by blocking backdoor paths with the observed portion of the state space.

- **Identification formula:**

  \[
  P(E(t + 1) \mid do(C(t) = c)) = \int_o P(E(t + 1) \mid C(t) = c, O(t) = o) P(O(t) = o) \, do,
  \]

- **Average causal effect:**

  \[
  \mathbb{E}(E(t + 1) \mid do(C(t) = c)) \approx \frac{1}{n} \sum_{i=1}^{n} \mathbb{E}(E(t + 1) \mid C(t) = c, O(t) = o_i),
  \]

  where $C(t)$ is the cause of interest, $E(t + 1)$ is the effect of interest, and $O(t)$ are all observed variables not including $C(t)$.

- **Check:** There is no way to check this assumption with data. Therefore, this assumption requires strong physical justification well supported by the literature. Care is also required to insure that there are no interactions between $C(t)$ and $O(t)$; e.g., the observations at a given time are truly “simultaneous” and cannot causally affect each other.
Assumption: We can reconstruct the state at any given time using lagged observations of the system.

- **Discussion:** While we may not observe the entire state space, in some cases we may be able to reconstruct the state at any given time using lagged observations of the system [see Takens' theorem, Takens [1981]]. In this case, we can use the reconstructed state to block backdoor paths and examine the effect of any observed variable ($C(t)$) on future variables ($E(t+1)$) in Figure 7. Note that additionally controlling for $O(t)$ can also make the effect estimate more reliable. More generally, there may be more than one suitable set of adjustment co-variates and current research is targeted at finding optimal ones that yield the lowest estimation error [e.g., Witte et al. 2020].

- **Identification formula:**

\[
P(E(t+1) \mid do(C(t) = c)) = \int_s P(E(t+1) \mid C(t) = c, S'(t-1) = s) P(S'(t-1) = s) \, ds.
\]

- **Average causal effect:**

\[
\mathbb{E}(E(t+1) \mid do(C(t) = c)) \approx \frac{1}{n} \sum_{i=1}^{n} \mathbb{E}(E(t+1) \mid C(t) = c, S'(t-1) = s_i),
\]

where $C(t)$ is the cause of interest, $E(t+1)$ is the effect of interest, and $S'(t-1)$ is the reconstructed state using lagged observations.

- **Check:** One check on the success of the state space reconstruction is to test whether the observed variables are conditionally independent given the reconstructed state variable. This
Figure 8: A generic graph asserting an assumption that there are forcings external to the evolution of the state-space.

approach bears similarity to the deconfounder approach introduced by [Wang and Blei 2019], which argues that causal effects can be calculated for many problems when we can infer a latent variable that renders the (multiple) causes conditionally independent given the latent variable.

Assumption: the cause of interest is independent of the systems’ state evolution

- **Discussion:** In some cases, we may assume that the cause of interest is independent of the systems’ state evolution (Figure 8). While not generally true, in some cases a variable may behave independent of the system’s state while still causally affecting that state. For example, some human behavior may be approximated as independent of the climate state (e.g., city planning and land use decisions).
- **Identification formula:**
  \[ P(E(t+1) \mid do(C = c)) = P(E(t+1) \mid C = c) \]
- **Average causal effect:**
  \[ E(E(t+1) \mid do(C = c)) = E(E(t+1) \mid C = c) \]
- **Check:** To check this assumption, we can test if the cause/forcing is independent of the past state. If the cause is independent of the past state, we have stronger confidence that the assumption holds.

While this list of assumptions is certainly not exhaustive, it presents some approaches that may apply to many scenarios in Earth science. Also while the provided checks might identify when assumptions break down, there is no general way to “validate” assumptions using data. Physical and science-based justification and reasoning are ultimately always required.
6 Conclusions

In summary, causal graphs and causal inference are powerful tools to reason about problems in the Earth system, whether in models or observations. Specifically, this review aimed at showing that:

- Causal graphs concisely and clearly encode physical assumptions about causal/functional dependencies between processes. Including a causal graph benefits any observational or modeling analysis, including those that use regression.
- Whether a causal effect can be calculated from data is determined by the causal graph. Thus the tractability of a causal analysis, or the strength of assumptions necessary to make the analysis tractable, is determined and assessed before collecting, generating, or manipulating data (which can cost a tremendous amount in terms of researchers’ time or computational resources). We recommend early causal analysis to determine tractability during a project’s conception, before resources are spent obtaining or analyzing data.
- Calculated causal effects measure the response of target variables (i.e. effects) to interventions on other variables in the system (i.e. causes). With statistical modeling (i.e., regression) one can estimate the functional relationship between interventions on causes and effects. These functional relationships generalize because they map interventions onto the response: unlike observed associations (e.g., naive regression), we know the response is attributable to the function’s input, and not to some other process in the system (e.g., an unobserved common cause). This causal approach opens up a new path to calculate generalized functional relationships when we either do not know the functional form a priori, or it is too computationally intractable to calculate from models.
- Because the Earth system and its constituents are dynamical systems evolving through time, we can construct broadly applicable, generic Earth system causal graphs. We can use these graphs to calculate generalized functional relationships between processes of interest, which would not be possible with associations, correlations or simple regressions. However, causal inference in the Earth sciences also presents challenges as we only partially observe the state space of the system.
- These challenges can be alleviated by applying causal theory to generic causal graphs of the Earth system and identifying the assumptions that allow for causal inference from data (Section 5).

Here we focus on the fundamentals of calculating causal effects from data. However, causal inference is a thriving, active area of research, and there are many other causal inference techniques and abstractions that could benefit the Earth system research community. For example, there are techniques for representing variables observed under selection bias in the causal graph and analyzing whether a causal effect can be calculated (i.e. identified) given that selection bias [e.g., Bareinboim et al. 2014, Correa et al. 2018]. Selection bias, defined as a preferential sampling of data according to some underlying mechanism, is very relevant in Earth sciences. For example, satellite observations are almost always collected under selection bias (e.g. clouds obscure surface data, satellites sample at certain local times of the day which is connected to top of atmosphere solar forcing, or the sensors themselves could have a bias). Additionally, transportability [e.g., Bareinboim and Pearl 2012, 2016, Lee et al. 2019] identifies whether one can calculate a causal effect in a passively observed system called the “target domain”, by merging experiments from
other systems, called “source domains”, that may differ from the target domain. A potential application for transportability in Earth sciences would be to merge numerical model experiments (e.g., Earth system models) and formally transport their results to the real world. In this case, numerical models are the source domains that differ from the target domain (“real world”) due to approximations and different resolutions.

However, because these developments in causal theory are relatively new, applied domains have yet to establish these recent theoretical developments’ utility for applied analysis. While we encourage applied scientists to explore how these developments may apply to their domains, we recognize that many scientists prefer tools with established utility. To that end, we believe that the use of causal graphs to organize and structure analyses is mature and directly applicable to many projects, and can serve as a gateway to applying these more recent causal developments. We hope that drawing and including causal graphs in Earth science research becomes more common in our field, and that this manuscript provides some of the necessary foundation for readers to attempt using causal graphs in their future research.

Acknowledgments The authors want to thank Elias Bareinboim, Beth Tellman, James Doss-Gollin, David Farnham, and Masa Haraguchi for thoughtful feedback and comments that greatly improved an earlier version of this manuscript.

References

Elias Bareinboim and Judea Pearl. Transportability of causal effects: Completeness results. In Twenty-Sixth AAAI Conference on Artificial Intelligence, 2012.

Elias Bareinboim and Judea Pearl. Causal inference and the data-fusion problem. Proceedings of the National Academy of Sciences, 113(27):7345–7352, 2016. ISSN 0027-8424. doi: 10.1073/pnas.1510507113. URL https://www.pnas.org/content/113/27/7345.

Elias Bareinboim, Jin Tian, and Judea Pearl. Recovering from selection bias in causal and statistical inference. In Twenty-Eighth AAAI Conference on Artificial Intelligence, 2014.

Elizabeth A. Barnes, Savini M. Samarasinghe, Imme Ebert-Uphoff, and Jason C. Furtado. Tropospheric and stratospheric causal pathways between the mjo and nao. Journal of Geophysical Research: Atmospheres, 124(16):9356–9371, 2019. doi: https://doi.org/10.1029/2019JD031024. URL https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2019JD031024.

Christopher M Bishop. Pattern recognition and machine learning. springer, 2006.

Carlos Blanco-Alegre, Amaya Castro, Ana I. Calvo, Fernanda Oduber, Elisabeth Alonso-Blanco, Delia Fernández-González, Rosa M. Valencia-Barrera, Ana M. Vega-Maray, and Roberto Fraile. Below-cloud scavenging of fine and coarse aerosol particles by rain: The role of raindrop size. Quarterly Journal of the Royal Meteorological Society, 144(717):2715–2726, 2018. doi: 10.1002/qj.3399. URL https://rmets.onlinelibrary.wiley.com/doi/abs/10.1002/qj.3399.
Juan D Correa, Jin Tian, and Elias Bareinboim. Generalized adjustment under confounding and selection biases. In Thirty-Second AAAI Conference on Artificial Intelligence, 2018.

Imme Ebert-Uphoff and Yi Deng. Causal discovery for climate research using graphical models. *Journal of Climate*, 25(17):5648–5665, 2012. doi: 10.1175/JCLI-D-11-00387.1. URL https://doi.org/10.1175/JCLI-D-11-00387.1.

David John Gagne, Chih-Chieh Chen, and Andrew Gettelman. Emulation of bin microphysical processes with machine learning. In 100th American Meteorological Society Annual Meeting. AMS, 2020.

Dan Geiger, Thomas Verma, and Judea Pearl. d-separation: From theorems to algorithms. In Max HENRION, Ross D. SHACHTER, Laveen N. KANAL, and John F. LEMMER, editors, *Uncertainty in Artificial Intelligence*, volume 10 of *Machine Intelligence and Pattern Recognition*, pages 139 – 148. North-Holland, 1990. doi: https://doi.org/10.1016/B978-0-444-88738-2.50018-X. URL http://www.sciencedirect.com/science/article/pii/B978044488738250018X.

P. Gentine, M. Pritchard, S. Rasp, G. Reinaudi, and G. Yacalis. Could machine learning break the convection parameterization deadlock? *Geophysical Research Letters*, 45(11):5742–5751, 2018. doi: 10.1029/2018GL078202. URL https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2018GL078202.

Allison E. Goodwell, Peishi Jiang, Benjamin L. Ruddell, and Praveen Kumar. Debates—does information theory provide a new paradigm for earth science? causality, interaction, and feedback. *Water Resources Research*, 56(2):e2019WR024940, 2020. doi: 10.1029/2019WR024940. URL https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2019WR024940 e2019WR024940 10.1029/2019WR024940.

Julia K. Green, Alexandra G. Konings, Seyed Hamed Alemohammad, Joseph Berry, Dara Entekhabi, Jana Kolassa, Jung-Eun Lee, and Pierre Gentine. Regionally strong feedbacks between the atmosphere and terrestrial biosphere. *Nature Geoscience*, 10(6):410–414, may 2017. doi: 10.1038/ngeo2957. URL https://doi.org/10.1038\%2Fngeo2957.

Charlotte Grossiord, Thomas N. Buckley, Lucas A. Cernusak, Kimberly A. Novick, Benjamin Poulter, Rolf T. W. Siegwolf, John S. Sperry, and Nate G. McDowell. Plant responses to rising vapor pressure deficit. *New Phytologist*, 226(6):1550–1566, 2020. doi: https://doi.org/10.1111/nph.16485. URL https://nph.onlinelibrary.wiley.com/doi/abs/10.1111/nph.16485.

E. Gryspeerdt, T. Goren, O. Sourdeval, J. Quaas, J. Mülmenstädt, S. Dipu, C. Unglaub, A. Gettelman, and M. Christensen. Constraining the aerosol influence on cloud liquid water path. *Atmospheric Chemistry and Physics*, 19(8):5331–5347, 2019. doi: 10.5194/acp-19-5331-2019. URL https://acp.copernicus.org/articles/19/5331/2019/.
Clive Hamilton. No, we should not just ‘at least do the research’. *Nature*, 496(7444):139–139, 2013.

A. Hannart, J. Pearl, F. E. L. Otto, P. Naveau, and M. Ghil. Causal counterfactual theory for the attribution of weather and climate-related events. *Bulletin of the American Meteorological Society*, 97(1):99–110, 2016. doi: 10.1175/BAMS-D-14-00034.1. URL https://doi.org/10.1175/BAMS-D-14-00034.1.

Dennis L Hartmann. *Global physical climatology*, volume 103. Newnes, 2015.

Yimin Huang and Marco Valtorta. Identifiability in causal bayesian networks: A sound and complete algorithm. In AAAI, pages 1149–1154, 2006.

Elena Jurado, Jordi Dachs, Carlos M. Duarte, and Rafel Simó. Atmospheric deposition of organic and black carbon to the global oceans. *Atmospheric Environment*, 42(34):7931 – 7939, 2008. ISSN 1352-2310. doi: https://doi.org/10.1016/j.atmosenv.2008.07.029. URL http://www.sciencedirect.com/science/article/pii/S1352231008006286.

Marlene Kretschmer, Dim Coumou, Jonathan F. Donges, and Jakob Runge. Using causal effect networks to analyze different arctic drivers of midlatitude winter circulation. *Journal of Climate*, 29(11):4069 – 4081, 2016. doi: 10.1175/JCLI-D-15-0654.1. URL https://journals.ametsoc.org/view/journals/clim/29/11/jcli-d-15-0654.1.xml.

Sanghack Lee, Juan D Correa, and Elias Bareinboim. General identifiability with arbitrary surrogate experiments. 2019.

Edward N. Lorenz. Deterministic nonperiodic flow. *Journal of the Atmospheric Sciences*, 20(2):130–141, 1963. doi: 10.1175/1520-0469(1963)020<0130:DNF>2.0.CO;2. URL https://doi.org/10.1175/1520-0469(1963)020<0130:DNF>2.0.CO;2.

Edward N Lorenz. Predictability: A problem partly solved. In *Proc. Seminar on predictability*, volume 1, 1996.

Andrew J. Majda. Challenges in climate science and contemporary applied mathematics. *Communications on Pure and Applied Mathematics*, 65(7):920–948, 2012. doi: 10.1002/cpa.21401. URL https://onlinelibrary.wiley.com/doi/abs/10.1002/cpa.21401.

Chengzhi Mao, Amogh Gupta, Augustine Cha, Hao Wang, Junfeng Yang, and Carl Vondrick. Generative interventions for causal learning. *arXiv e-prints*, pages arXiv–2012, 2020.

Adam Massmann, Pierre Gentine, and Changjie Lin. When does vapor pressure deficit drive or reduce evapotranspiration? *Journal of Advances in Modeling Earth Systems*, 11(10):3305–3320, 2019. doi: 10.1029/2019MS001790. URL https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2019MS001790.

Adam K. Massmann, Justin R. Minder, René D. Garreau, David E. Kingsmill, Raul A. Valenzuela, Aldo Montecinos, Sara Lynn Fults, and Jefferson R. Snider. The chilean coastal orographic precipitation experiment: Observing the influence of microphysical rain regimes on coastal orographic precipitation. *Journal of Hydrometeorology*, 18(10):2723–2743, oct 2017. doi: 10.1175/jhm-d-17-0005.1. URL https://doi.org/10.1175%2Fjhm-d-17-0005.1.
Chris Olah and Shan Carter. Research debt. *Distill.*, 2(3):e5, 2017. doi: 10.23915/distill.00005.

Judea Pearl. A probabilistic calculus of actions. In Ramon Lopez de Mantaras and David Poole, editors, *Uncertainty Proceedings 1994*, pages 454 – 462. Morgan Kaufmann, San Francisco (CA), 1994. ISBN 978-1-55860-332-5. doi: https://doi.org/10.1016/B978-1-55860-332-5.50062-6. URL http://www.sciencedirect.com/science/article/pii/B978155860332500626

Judea Pearl. Causal diagrams for empirical research. *Biometrika.*, 82(4):669–688, 1995.

Judea Pearl. Causal inference in statistics: An overview. *Statist. Surv.*, 3:96–146, 2009a. doi:10.1214/09-SS057. URL https://doi.org/10.1214/09-SS057

Judea Pearl. Causality: models, reasoning, and inference. *Cambridge University Press. ISBN 0, 521(77362):8, 2009b.

Lawrence F. Radke, Peter V. Hobbs, and Mark W. Eltgroth. Scavenging of aerosol particles by precipitation. *Journal of Applied Meteorology*, 19(6):715–722, 1980. doi: 10.1175/1520-0450(1980)019<0715:SOAPBP>2.0.CO;2. URL https://doi.org/10.1175/1520-0450(1980)019<0715:SOAPBP>2.0.CO;2

David Randall, Marat Khairoutdinov, Akio Arakawa, and Wojciech Grabowski. Breaking the cloud parameterization deadlock. *Bulletin of the American Meteorological Society*, 84(11):1547–1564, 2003. doi: 10.1175/BAMS-84-11-1547. URL https://doi.org/10.1175/BAMS-84-11-1547

J. Runge. Causal network reconstruction from time series: From theoretical assumptions to practical estimation. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 28(7):075310, 2018. doi: 10.1063/1.5025050. URL https://doi.org/10.1063/1.5025050

Jakob Runge, Sebastian Bathiany, Erik Bollt, Gustau Camps-Valls, Dim Coumou, Ethan Deyle, Clark Glymour, Marlene Kretschmer, Miguel D Mahecha, Jordi Muñoz-Mari, et al. Inferring causation from time series in earth system sciences. *Nature Communications*, 10(1):2553, 2019.

Guido D Salvucci, Jennifer A Saleem, and Robert Kaufmann. Investigating soil moisture feedbacks on precipitation with tests of granger causality. *Advances in Water Resources*, 25(8):1309 – 1312, 2002. ISSN 0309-1708. doi: https://doi.org/10.1016/S0309-1708(02)00057-X. URL http://www.sciencedirect.com/science/article/pii/S030917080200057X

Savini M. Samarasinghe, Yi Deng, and Imme Ebert-Uphoff. A causality-based view of the interaction between synoptic- and planetary-scale atmospheric disturbances. *Journal of the Atmospheric Sciences*, 77(3):925 – 941, 2020. doi: 10.1175/JAS-D-18-0163.1. URL https://journals.ametsoc.org/view/journals/atmos/77/3/jas-d-18-0163.1.xml

Cosma Shalizi. Advanced data analysis from an elementary point of view, 2013. URL https://www.stat.cmu.edu/~cshalizi/ADAfaEPoV/

Claudia Shi, David M Blei, and Victor Veitch. Adapting neural networks for the estimation of treatment effects. *arXiv preprint arXiv:1906.02120*, 2019.
Ilya Shpitser and Judea Pearl. Identification of conditional interventional distributions. In R. Dechter and T. Richardson, editors, *Proceedings of the Twenty-Second Conference on Uncertainty in Artificial Intelligence*, pages 437–444, Corvallis, OR, 2006. AUAI Press.

Floris Takens. Detecting strange attractors in turbulence. In *Dynamical systems and turbulence*, *Warwick 1980*, pages 366–381. Springer, 1981.

Johannes Textor, Benito van der Zander, Mark S Gilthorpe, Maciej Liśkiewicz, and George TH Ellison. Robust causal inference using directed acyclic graphs: the R package ‘dagitty’. *International Journal of Epidemiology*, 45(6):1887–1894, 01 2017. ISSN 0300-5771. doi: 10.1093/ije/dyw341. URL https://doi.org/10.1093/ije/dyw341

Jin Tian and Judea Pearl. A general identification condition for causal effects. In *Aaai/iaai*, pages 567–573, 2002.

John M Wallace and Peter V Hobbs. *Atmospheric science: an introductory survey*, volume 92. Elsevier, 2006.

Yixin Wang and David M. Blei. The blessings of multiple causes. *Journal of the American Statistical Association*, 114(528):1574–1596, 2019. doi: 10.1080/01621459.2019.1686987. URL https://doi.org/10.1080/01621459.2019.1686987

Janine Witte, Leonard Henckel, Marloes H Maathuis, and Vanessa Didelez. On efficient adjustment in causal graphs. *Journal of Machine Learning Research*, 21(246):1–45, 2020.

Ayrton Zadra, Keith Williams, Ariane Frassoni, Michel Rixen, Ángel F. Adames, Judith Berner, François Bouyssel, Barbara Casati, Hannah Christensen, Michael B. Ek, Greg Flato, Yi Huang, Falko Judt, Hai Lin, Eric Maloney, William Merryfield, Annelize Van Niekerk, Thomas Rackow, Kazuo Saito, Nils Wedi, and Priyanka Yadav. Systematic Errors in Weather and Climate Models: Nature, Origins, and Ways Forward. *Bulletin of the American Meteorological Society*, 99(4):ES67–ES70, 05 2018. ISSN 0003-0007. doi: 10.1175/BAMS-D-17-0287.1. URL https://doi.org/10.1175/BAMS-D-17-0287.1

Sha Zhou, A. Park Williams, Alexis M. Berg, Benjamin I. Cook, Yao Zhang, Stefan Hagemann, Ruth Lorenz, Sonia I. Seneviratne, and Pierre Gentine. Land–atmosphere feedbacks exacerbate concurrent soil drought and atmospheric aridity. *Proceedings of the National Academy of Sciences*, 116(38):18848–18853, 2019a. ISSN 0027-8424. doi: 10.1073/pnas.1904955116. URL https://www.pnas.org/content/116/38/18848

Sha Zhou, Yao Zhang, A. Park Williams, and Pierre Gentine. Projected increases in intensity, frequency, and terrestrial carbon costs of compound drought and aridity events. *Science Advances*, 5(1), 2019b. doi: 10.1126/sciadv.aau5740. URL http://advances.sciencemag.org/content/5/1/eaa5740.
A Basic probability and syntax

In this paper we use capital letters to represent random variables (e.g., “X”). For example, $P(X)$ is the marginal probability distribution of a random variable $X$. $P(X)$ is a function of one variable that outputs a probability (or density, in the case of continuous variables) given a specific value for $X$. We represent specific values that a random variable can take with lowercase letters (e.g., $x$ in the case of $X$). $P(X)$ is shorthand; a more descriptive but less concise way to write $P(X)$ is $P(X = x)$ which represents the fact that $P(X)$ is a function of a specific value of $X$, represented by $x$. We use both notations, and $P(X)$ has the same meaning as $P(X = x)$.

For the unfamiliar reader, there are a few basic rules and definitions in probability that provide relatively complete foundations for building deeper understanding of probability. These are the sum rule:

$$P(X = x) = \sum_Y P(X = x, Y = y)$$

and the product rule:

$$P(X = x, Y = y) = P(X = x | Y = y)P(Y = y) = P(Y = y | X = x)P(X = x)$$

The joint probability distribution ($P(X = x, Y = y)$) is the probability that the random variable $X$ equals some value $x$ and the random variable $Y$ equals $y$. The joint distribution is a function of two variables, $x$ and $y$ which are values in the domains of the random variables $X$ and $Y$ respectively. The conditional probability distribution ($p(X = x | Y = y)$) is also a function of two variables $x$ and $y$, but it is the probability of observing $X$ equal to $x$, given that we have observed $Y$ equal to $y$. In other words, if we filter our domain to only values where $Y = y$, then $p(X = x | Y = y)$ is the probability of observing $X = x$ in this sub-domain where $Y = y$. The marginal probability distribution ($P(Y = y)$) is just the probability that $Y$ equals some value $y$, and is a function of only $y$. We can calculate the marginal probability from the joint distribution by summing over all possible values values of the other random variables in the joint (the “sum rule” - Equation (10)). Additionally, the joint distribution can factorize into a product of conditional and marginal distributions (“the product rule” - Equation (11)). These two simple rules can be used to build much of the theory and applications of probability theory (e.g., Bayes’ theorem $P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)}$).

While Equations (10) deals with probability distributions of discrete random variables, there is also a sum rule analog for continuous random variables and probability density functions (the syntax of the product rule is the same):

$$P(X = x) = \int_Y P(X = x, Y = y) dy$$

where $\int_Y$ represents an integral over the domain of $Y$ (e.g., $\int_{-\infty}^{\infty}$ if $Y$ is a Gaussian random variable).