Abstract We calculate the cross sections for the processes $\rho J/\psi \to D^0\bar{D}^0$, $\rho J/\psi \to D^0\bar{D}^{*0}$ ($D^{0s}\bar{D}^{0s}$) and $\rho J/\psi \to D^{0s}\bar{D}^{0s}$ by modifying the usual sum of two-body interaction through a four-body form factor whose parameters are fitted to the relevant lattice QCD simulations. We use the formalism of the resonating group method, with the Born approximation utilized to decouple the resulting integral equations. Cross sections with using the realistic Cornell potential for each of the two-body interactions in the Hamiltonian are reported and are compared to those obtained through the quadratic potential. It is found that using four-body form factor, there is a significant suppression in the cross sections as compared to the more popular sum of two-body interaction. We find that total dissociation cross section of $J/\psi$ by $\rho$ meson is of the order of 0.1 mb for the most reliable case of Cornell potential with four-body form factor.

1 Introduction

In the quark potential model the gluonic field energy in quantum chromodynamics (QCD) is modelled as the potential energy of valence quarks. This gluonic energy is obtainable from the unquenched lattice simulations of the gluonic state $\Sigma^+_c$ reported in, for example, Refs. [1,2]. For the quark dynamics, we can use the static quark–antiquark potential in a Schrödinger or a relativistic equation. In this paper, we extend the dynamical use of the potential to a two-quark–two-antiquark system with better utilizing the numerical simulations of QCD performed for the same number of quarks and antiquarks and for each pair. A definition for Born–Oppenheimer potential usable for a heavy–light two-quark–two-antiquark system is mentioned in Sect. IIIA of Ref. [3]. An example of such a system might be a $K\bar{K}$ molecule for which Weinstein and Isgur [4,5] used a simpler sum of two-body potentials. Barnes and Swanson used this sum with Born-order quark exchange diagrams to calculate [6] $\pi^+\pi^+$, $K^+K^+$ and $\rho^+\rho^+$ elastic scattering phase shifts and cross sections. Using the same approach, Barnes et al. calculated [7] $BB$ intermeson potentials and scattering amplitudes. The diagrammatic approach has been used [8,9] for systems with charm and light quarks, the flavour sectors we address in the present paper. This model was also used by Wong et al. in Refs. [10,11] to calculate the cross sections for the dissociation of $J/\psi$ and $\psi'$ by $\rho$ and $\rho$ for the related channels.

It is worthwhile asking if the sum of two-body potential model is an extension to multiquarks agreeing to the lattice simulations of QCD. Such a comparison [12,13] was made by the UKQCD collaboration and coworkers for a large number of geometries of two quarks and two antiquarks. They noted that if the well-known Cornell model is used for each pair and summed over pairs, this results in binding energy of two quarks and two antiquarks square geometry increasing with each quark separation whereas the lattice simulations show [14] a decrease. Phenomenologically, the sum of pair-wise interactions results in the long range van der Waal’s interaction between colour singlet mesons. But experimentally there is no evidence for any long range interaction between hadrons. References [15,16] model simulations of a two-quark–two-antiquark system by a double-Y potential flipping to a two-meson state when this has lower energy. The energies can only be numerically compared for the minimum. Thus such a potential does not allow any analytical handling for the nine position degrees of freedom of a four particle system.

A simpler and more manageable way [17] to incorporate multi-body effects is to multiply off-diagonal elements in the normalization and other matrices, in the relevant colour basis, by a space dependent factor $f$. This factor $f$ approaches 1 (the sum of two-body potentials) in the limit of all the
interquark distances tending towards zero and tending to zero in the limit of large inter-quark separations. The actual form of this gluonic field overlap factor \( f \) and fit of its parameter(s) to the relevant lattice simulations was improved through Refs. [13,18–20].

The \( f \) factor affects the linear independence of the colour configurations. But still a two-states basis was found [13,20] to be sufficient to well model the relevant computer simulations.

In incorporating the quark motion, we use two basis vectors and subsume relativistic effects into the parameters of a non-relativistic quark potential model. References [21–24] indicate that a non-relativistic form can be even a preferred one. As in Refs. [25,26], we employ the resonating group method to write the dependence of the wave function on the intra-cluster positions. For each such factor we took the functional form of these quark–antiquark wave functions as that for the quadratic confinement but fitted its parameters to the numerically calculated eigenfunctions of the realistic Cornell potential. In the multiquark Hamiltonian we have used Cornell potential for each pair while incorporating the spin and flavour dependence. We decouple the two resulting integrations by replacing the unknown dependence on the inter-cluster position by that in the absence of any meson–meson interaction. One usage of this (Born) approximation is Eq. (21) of Ref. [6]. This paper claims much evidence for high-order diagrams to be relatively unimportant in hadron spectroscopy and in low energy scattering and annihilation-free decays. This and related work [9–11,27–29] make an extensive use of Born diagrams for two-channels scattering. Reference [27] has a useful comparison of the quadratic confinement single channel Born-order \( T \)-matrix elements with those resulting from the original resonating group method calculations. The comparison away from threshold indicates that a neglect of higher terms in the Born series is a good approximation.

We thus report the cross sections for the processes \( \rho J/\psi \to D^0 \bar{D}^0, \rho J/\psi \to D^0 \bar{D}^{0s}, \rho J/\psi \to D^{0s} \bar{D}^0, \rho J/\psi \to D^{0s} \bar{D}^{0s} \) and \( D^{0s} \bar{D}^0 \to \rho J/\psi, D^{0s} \bar{D}^{0s} \to \rho J/\psi \) and \( D^{0s} \bar{D}^0 \to \rho J/\psi \) for both a pure sum of two-body interaction and its four-body modification fitted to the relevant QCD simulations. We report results with the use of the realistic Cornell potential for each pair and with replacing this by a quadratic potential. Cornell and quadratic potentials are used in Ref. [27] to calculate the intermeson potentials in the quark model. Reference [15], mentioned above for incorporating multi-body colour flux tube dynamics, uses a quadratic potential; see its Eq. (2). Earlier, a quadratic potential had been used to study meson–meson scattering [30] and nucleon–nucleon interaction [31,32]; Ref. [31] extends the model to meson–nucleon and nucleon–antinucleon scattering. Moreover, a quadratic potential had been used to calculate the mesonic and baryonic spectra [33]. In the Isgur–Karl model [34], the leading order quark antiquark interaction is quadratic. In Ref. [35] qualitative properties of QCD Benzene have been studied with quadratic confinement and compared with those using a linear confinement. Zahra Ghalenovi et al. in Ref. [36] used both linear and quadratic confinement to calculate mass spectra of heavy and light scalar tetraquarks. In Ref. [26], the cross sections of process \( \rho J/\psi \to D^0 \bar{D}^{0s} \) are calculated by finding exact solution of integral equations of two-quark and two-antiquark system using quadratic potential with \( f = 1 \) and Gaussian form of \( f \). Considering such a frequent use of the quadratic confinement, it is useful to find how good is this approximation in a variety of settings. And this we do through our comparisons of this with using the original Cornell potential.

The cross sections calculated in this work might be useful in the experimental studies of \( J/\psi \) production in heavy ion collisions. It is expected that formation of a quark–gluon plasma (QGP) can produce suppression of \( J/\psi \) due to colour Debye screening [37]. However, a part of this observed suppression can also occur due to interactions of \( J/\psi \) with comoving hadrons, mainly \( \pi \) and \( \rho \) mesons, in hadronic matter. It is also known that the effect of interactions with the comovers can be significant if the interaction cross section is a few mb [38,39]. To disentangle these two suppression mechanisms, we require accurate knowledge of \( J/\psi \) dissociation cross sections by \( \pi \) and \( \rho \) mesons.

The paper is organized as follows: In Sect. 2, we write the total Hamiltonian of the diquark–diantiquark system in the sum of two-body approach and total state vector of the system using the adiabatic approximation. In Sect. 3, we apply the Born approximation to decouple the integral equations obtained by the resonating group method. In Sect. 4, we discuss how the cross sections are extracted from the solutions of the integral equations. In Sect. 5, we discuss the phenomenological fitting of the parameters on which the cross sections depend. In Sect. 6, we present the resultant \( J/\psi \) dissociation cross sections by \( \rho \) meson and compare the results obtained using quadratic and Cornell quark–antiquark potentials with and without including Gaussian form of \( f \). A comparison with other work is also given there.

2 The Hamiltonian and Gluonic basis

In the sum of two-body potential model, the total Hamiltonian of the four quark system is defined as

\[
\hat{H} = \sum_{i=1}^{4} \left[ m_i + \frac{\vec{p}_i^2}{2m_i} \right] + \sum_{i<j} (v_{ij} + H_{\text{hyp}}^{ij}) \vec{F}_i \cdot \vec{F}_j,
\]

where

\[
H_{\text{hyp}}^{ij} = -\frac{8\pi\alpha_s}{3m_im_j} S_i S_j \delta(r_{ij}).
\]
Like the Cornell potential the parameters \( d \) and \( b \) are phenomenologically fitted separately for \( u \) and \( \bar{c} \). This gives

\[
v(r_{ij}) = v_{ij} = \frac{\alpha_s}{r_{ij}} - \frac{3}{4} p \ r_{ij} + c,
\]

where \( i, j = 1, 2, 3, 4 \), is string tension parameter, \( \alpha_s \) is strong coupling constant, and \( c \) is the self-energy constant. We incorporate the energy and hence flavor dependence of the strong coupling and the self-energy constant by taking their values to be \( \alpha_{s1} \) and \( c_1 \) for the mesons having quark contents \( u \bar{c} \), and \( \alpha_{s2}, c_2 \) and \( \alpha_{s3}, c_3 \) for the \( u \bar{u} \) and \( c \bar{c} \) clusters, respectively. The phenomenological fit of these parameters is given in Sect. 5. The inter-quark quadratic potential, which we use for comparison with the results of Cornell potential, is given by

\[
v(r_{ij}) = v_{ij} = Cr_{ij}^2 + \tilde{C}.
\]

Like the Cornell potential the parameters \( C \) and \( \tilde{C} \) are phenomenologically fitted separately for \( u \bar{c} \), \( u \bar{u} \), and \( c \bar{c} \) mesons as discussed in Sect. 5. The total state vector of our diquark–diantiquark system can be written as

\[
|\psi(r_1, r_2, r_3, r_4; g)\rangle = \sum_{k=1}^{2} \langle k | g | k \rangle |k_f\rangle |\psi(r_1, r_2, r_3, r_4)\rangle,
\]

where the sum over \( k \) corresponds to possible topologies of \( q \bar{q} q \bar{q} \) system shown in Fig. 1. We have neglected the third topology given in Fig. 1, because the lattice energies in Table IV of Ref. [40] are essentially unaffected by this truncation.

\[
|k\rangle = \left\{ \begin{array}{ll}
|P_{13}P_{24}\rangle_0 & \text{for } D^0 \bar{D}^0, \\
|V_{13}V_{24}\rangle_{0,1,or2} & \text{for } D^{0*} \bar{D}^{0*}, \\
|P_{13}V_{24}\rangle_1 & \text{for } D^{0} \bar{D}^{0*}.
\end{array} \right.
\]

These spin states are explicitly defined with their overlaps and \( S, S \) matrix elements in Appendix A. The flavor contents of first and second channel are

\[
\langle 1 | f \rangle = |u \bar{c}\rangle_{13} |c \bar{u}\rangle_{24},
\]

\[
\langle 2 | f \rangle = \frac{1}{\sqrt{2}} |u \bar{u} - d \bar{d}\rangle_{14} |c \bar{c}\rangle_{23},
\]

which gives

\[
f\langle 1 | 2 \rangle f = f\langle 2 | 1 \rangle f = \frac{1}{\sqrt{2}}.
\]

Fig. 1 Different topologies of diquark–diantiquark system

The four 3-vectors \( r_1, r_2, r_3 \) and \( r_4 \) can be replaced by their linear combinations, represented by \( R_c \) (centre of mass), \( R_d \) and \( y_k \) and \( z_k \) are defined in Fig. 1. This implies

\[
R_1 = \frac{(r_1 - r_3) + r_c (r_3 - r_2)}{1 + r_c},
\]

where \( r_c = \frac{m_c}{m_c - m_u} \). \( R_2 \) and \( R_3 \) are defined similarly. The spatial part of the complete state vector in Eq. (5) is factorized into parts which are separate functions of the inter-cluster \( (R_c) \) and the intra-clusters \( (y_k \) and \( z_k \) ) position coordinates. This makes the total state vector of the form

\[
|\psi(r_1, r_2, r_3, r_4; g)\rangle = \sum_{k=1}^{2} \langle k | g | k \rangle |k_f\rangle |\psi(R_c, R_d, y_k, z_k)\rangle,
\]

where \( \xi_k(y_k) \) and \( \zeta_k(z_k) \) are intra-cluster wave functions which are taken in Gaussian form as follows:

\[
\xi_k(y_k) = \frac{1}{(2\pi d_{k1}^2)^{3/4}} \exp\left(-\frac{y_k^2}{4d_{k1}^2}\right),
\]

\[
\zeta_k(z_k) = \frac{1}{(2\pi d_{k2}^2)^{3/4}} \exp\left(-\frac{z_k^2}{4d_{k2}^2}\right),
\]

where \( d_{k1} \) and \( d_{k2} \) are the corresponding mesonic sizes. Substituting Eq. (10) in

\[
\langle \delta \Psi | \hat{H} - E | \Psi \rangle = 0,
\]
and taking linearly independent variations only in $\chi_k(R_k)$ factor, along with performing the trivial $R_c$ integration using box normalization, we get

$$\sum_{l=1}^{2} \int d^3 y_k d^3 z_k \xi_k(z_k) f(k|l)[k|\mathcal{H} - E|l] s|l_f$$

where $k_f$ through matrix modified by the factor, along with performing the trivial $R_c$ integration using box normalization, we get

$$\sum_{l=1}^{2} \int d^3 y_k d^3 z_k \xi_k(z_k) f(k|l)[k|\mathcal{H} - E|l] s|l_f$$

which defines two integral equations for $k = 1$ and 2. The matrix elements of the potential energy part of the Hamiltonian of Eq. (1) in our spin basis are given by

$$V_{ij} = \frac{8\pi \alpha_s}{3m_i m_j} \delta(r_{ij}) s|k|S_i S_j/l_s.$$

Now using the matrix elements of $F_i, F_j$ operator in the colour basis [4] modified by the factor $f$, we write the matrix elements of Eq. (15) as follows:

$$V(f) = \{s<k|\hat{V}(f)|l_s\}$$

$$= \left(\begin{array}{c} \frac{4f}{9}(V_{12} + V_{34} - V_{13} - V_{24} - V_{14} - V_{23})_{1.2} \\ \end{array}\right).$$

Here $f$ is defined as [13]

$$f = \exp \left(-bk_f \sum_{i<j} r_{ij}^2 \right).$$

where $k_f$ is a parameter fitted to minimize energies obtained through $f$ model and lattice QCD. Since the colour basis is non-orthogonal, the corresponding non-diagonal overlap matrix modified by the $f$-model is given as

$$N(f) = \{(k|l)E\} = \left(\begin{array}{c} \sqrt{\frac{2}{3}} \\ \sqrt{\frac{1}{3}} \\ \end{array}\right).$$

The $k, l$ matrix elements of the kinetic energy part of Eq. (1) in a modified colour basis are given by

$$K(f) = \{s<k|\hat{K}|l_f\} = N(f)^{1/2} \left(\begin{array}{c} \frac{\nabla^2}{2m_i} \end{array}\right) N(f)^{1/2}.$$

3 Writing the coupled equations

We first write the integral equations for the processes $\rho J/\Psi \rightarrow D^0 \bar{D}^0$ and $D^0 \bar{D}^0 \rightarrow \rho J/\Psi$, in which the total spin is zero. To obtain the amplitudes of these processes we put $k = 1$ and 2, respectively, in Eq. (14). We use spin overlaps and matrix elements of spin operator $S_i S_j$ in spin basis for $s = 0$ given in Appendix A, insert the elements of the potential energy, normalization, and kinetic energy matrices from Eqs. (17), (19) and (20), respectively, in Eq. (14) and use the Born approximation, i.e., take the non-interacting form of $\chi_k(R_k) = \sqrt{2/\pi} e^{P_1 \cdot R_k}$ to decouple the coupled equations (14). At the end we take the Fourier transform with respect to $R_1$ and obtain the following equation for $k = 1$:

$$\left(k_1 + k_2 P_{12} - E + 2m_u(1 + r_c)\right) \chi_1(P_1) = h_{12},$$

where

$$k_1 = \int d^3 y_1 d^3 z_1 \xi_1(y_1) \xi_1(z_1) \left(-\frac{4}{3}\right)$$

$$\times \left[\alpha_s \left(\frac{1}{y_1} + \frac{1}{z_1}\right) - \frac{3}{4} b \left(y_1 + z_1\right) - \frac{8\alpha_s \sigma_3}{3\pi^{1/2}}\right]$$

$$\times \left\{\frac{1}{m_1 m_3} \left(-\frac{3}{4}\right) e^{-\sigma^2 y_1^2}\right.$$}

$$+ \left.\frac{1}{m_2 m_4} \left(-\frac{3}{4}\right) e^{-\sigma^2 z_1^2}\right\} + 2c_1 \left[\xi_1(y_1) \xi_1(z_1)\right].$$

$$k_2 = \int d^3 y_1 d^3 z_1 \xi_1(y_1) \xi_1(z_1) \left(-\frac{1}{2m_u}\right)$$

$$\times \left\{\xi_1(z_1) g_1 \nabla^2 y_1 \xi_1(y_1) + \xi_1(y_1) h_1 \nabla^2 z_1 \xi_1(z_1)\right\}.$$
In Eq. (22) $V_c$, $V_{\text{hyp}}$, $K_{12}$ and $N_{12}$ are defined as

$$V_c = -\frac{\sqrt{3}}{2} \left\{ \frac{\alpha s_1}{y_2} \left( \frac{1}{y_3} + \frac{1}{z_3} - \frac{1}{y_1} - \frac{1}{z_1} \right) - \frac{\alpha s_2}{y_2} - \frac{\alpha s_3}{z_2} \right\} - \frac{3}{4} \frac{h_1(y_3 + z_3 - y_1 - z_1 - y_2 - z_2) - c_2 - c_3}{4}, \tag{23}$$

$$V_{\text{hyp}} = \frac{\sigma^3}{\sqrt{3} \pi} \left\{ \frac{\alpha s_1}{m m c} + \frac{e^{-\sigma^2 y_1^2} e^{-\sigma^2 z_1^2}}{m m c} - \frac{3 e^{-\sigma^2 y_1^2}}{m m c} - \frac{3 e^{-\sigma^2 z_1^2}}{m m c} \right\} + \frac{\alpha s_2 e^{-\sigma^2 y_2^2}}{m m c} + \frac{\alpha s_3 e^{-\sigma^2 z_2^2}}{m m c}, \tag{24}$$

$$K_{12} = \frac{1}{2 \sqrt{3}} \left\{ \frac{1}{2} \frac{1}{2} \frac{1}{2} \left( \{ \frac{\alpha s_1}{y_2} \} f_2 \sqrt{f} e^{P_1 R_1} (\sqrt{f} e^{P_1 R_1})^2 \right) + e^{P_1 R_1} \xi_2 (y_2) \xi_2 (z_2) f_2 \sqrt{f} e^{P_1 R_1} \right\} \tag{25}$$

$$N_{12} = (E - 2 m_u (1 + r_c)) \left( \frac{1}{2 \sqrt{3}} \right) \sqrt{f} e^{P_1 R_1} (\sqrt{f} e^{P_1 R_1})^2 \tag{26}$$

where $f_2 = \frac{1}{2} (1 + r_c)$, $g_2 = 2$, and $h_2 = \frac{2}{3}$. We chose the $z$-axis along $P_1$ and $x$-axis in the plane containing $P_1$ and $P_2$. Naming the angle between $P_1$ and $P_2$ as $\theta$, we get $P_{21} = P_2 \sin \theta, P_{22} = P_2 \cos \theta, P_{2y} = 0$, along with $P_{1x} = P_{1y} = 0, P_{1z} = P_1$. Similarly, for $k = 2\ Eq. (14)$ is reduced to the following equation:

$$\left( k_3 + k_4 P_2^2 - E + 2 m_u (1 + r_c) \right) \chi_2 (P_2) = h_{21}, \tag{27}$$

where

$$k_3 = \int d^3 y_2 d^3 z_2 \xi_2 (y_2) \xi_2 (z_2) \left( \frac{4}{3} \right) \times \left\{ \frac{\alpha s_2}{y_2} + \frac{\alpha s_3}{z_2} - \frac{3}{4} b (y_2 + z_2) \right\} + c_2 + c_3$$

$$- \frac{2 \sigma^2}{3 \pi} \left\{ \frac{\alpha s_2}{m m} + \frac{e^{-\sigma^2 y_2^2}}{m m} + \frac{\alpha s_3}{m m} + \frac{e^{-\sigma^2 z_2^2}}{m m} \right\} \xi_2 (y_2) \xi_2 (z_2) + \int d^3 y_2 d^3 z_2 \xi_2 (y_2) \xi_2 (z_2) \left( - \frac{1}{2 m_u} \right) \times \xi_2 (z_2) f_2 \sqrt{f} e^{P_1 R_1} \xi_2 (y_2) f_2 \sqrt{f} e^{P_1 R_1} \xi_2 (z_2) \right\}; \tag{28}$$

$$k_4 = \int d^3 y_2 d^3 z_2 \xi_2 (y_2) \xi_2 (z_2) \left( \frac{1}{2 m_u} \right) \xi_2 (y_2) \xi_2 (z_2) f_2 \sqrt{f} e^{P_1 R_1} \xi_2 (y_2) f_2 \sqrt{f} e^{P_1 R_1} \xi_2 (z_2) \right\}, \tag{29}$$

$$h_{21} = \frac{2}{(2 \pi)^{3/2}} \frac{1}{2} \sqrt{f} \int d^3 y d^3 z \xi_1 (y_1) \xi_1 (z_1) \left( - \frac{4}{9} (V_c - V_{\text{hyp}}) \right) \sqrt{f} e^{P_1 R_1} \xi_1 (y_1) \xi_1 (z_1) + K_{21} - N_{21} \right\} \sqrt{f} e^{P_1 R_1} \xi_1 (y_1) \xi_1 (z_1) \right\}. \tag{30}$$

$V_c$ and $V_{\text{hyp}}$ are the same as in Eqs. (23) and (24) and $K_{21}$ and $N_{21}$ are defined as

$$K_{21} = \frac{1}{2 \sqrt{3}} \left( - \frac{1}{2 m_u} \right) \int d^3 y d^3 z \xi_1 (y_1) \xi_1 (z_1) f_1 \sqrt{f} e^{P_1 R_1} \xi_1 (y_1) \xi_1 (z_1) \right\} \sqrt{f} e^{P_1 R_1} \xi_1 (y_1) \xi_1 (z_1) \right\} \sqrt{f} e^{P_1 R_1} \xi_1 (y_1) \xi_1 (z_1) \right\}. \tag{31}$$

and all the remaining terms of Eqs. (22) and (28) are multiplied by a factor $-2 \sqrt{3}$ due to a modified spin overlap factor that is $1/\sqrt{3}$.

For the processes $\rho J/\psi \rightarrow D^0 \bar{D}^{0*}$ or $D^0 \bar{D}^{0*} \rightarrow \rho J/\psi$ the total spin $s = 1$. As a result the hyperfine term of Eq. (24) is modified as follows:

$$V_{\text{hyp}} = \frac{\sigma^3}{3 \sqrt{3} \pi} \left\{ \frac{\alpha s_1}{m m c} + \frac{e^{-\sigma^2 y_1^2}}{m m c} + \frac{e^{-\sigma^2 z_1^2}}{m m c} \right\} + \frac{\alpha s_2 e^{-\sigma^2 y_2^2}}{m m c} + \frac{\alpha s_3 e^{-\sigma^2 z_2^2}}{m m c}, \tag{32}$$

and all the remaining terms of Eqs. (22) and (28) are multiplied by a factor $1/\sqrt{3}$, because the corresponding spin overlap for $s = 0$ states is $-1/2$. For $s = 2$, the hyperfine term becomes

$$V_{\text{hyp}} = \frac{2}{(2 \pi)^{3/2}} \frac{1}{2} \sqrt{f} \int d^3 y d^3 z \xi_1 (y_1) \xi_1 (z_1) \left( - \frac{4}{9} (V_c - V_{\text{hyp}}) \right) \sqrt{f} e^{P_1 R_1} \xi_1 (y_1) \xi_1 (z_1) \right\} \sqrt{f} e^{P_1 R_1} \xi_1 (y_1) \xi_1 (z_1) \right\}. \tag{33}$$

and all the remaining terms of Eqs. (22) and (28) are multiplied by a factor $-2 \sqrt{3}$, as the corresponding spin overlap for $s = 2$ states is 1. For $s = 1$, spin overlap factors are zero, making the contribution vanish of all matrix elements but the hyperfine terms. Our calculations show that the cross section in this case is almost zero. This result is consistent with Fig. 8 of Ref. [11] where the cross section of this channel is also zero for the same reason. For a quadratic potential, only the potential part of Eqs. (22) and (28) is changed, i.e., $V_c$ is replaced by $V_q$ which is given as

$$V_q = \frac{-\sqrt{3}}{2} \left\{ C_1 (y_3 + z_3 - y_1 - z_1) - C_2 y_3^2 - C_3 z_3^2 - 2 \right\}, \tag{34}$$

where $f_1 = \frac{1}{2} (1 + r_c), g_1 = \frac{1}{2} (1 + r_c), h_1 = \frac{1}{2} (1 + r_c).$
where $C_1$ is a constant of the inter-quark potential (see Eq. (4)) of $c\bar{u}$ or $u\bar{c}$ mesons, $C_2$ and $C_3$ are for $u\bar{u}$ and $c\bar{c}$ mesons. When we use the quadratic potential in the coupled equations (14), the spin averaging is used to fit these parameters as discussed in the section below.

4 Finding the cross sections

The amplitudes $T_{12}$ ($T_{21}$) of the process $1 \rightarrow 2$ ($2 \rightarrow 1$) can be read off from Eqs. (22) and (28) [25], as they are proportional to the coefficients ($h_{12}$ and $h_{21}$) of the non-relativistic Green operators $-1/\Delta_1(P_1)$ and $-1/\Delta_2(P_2)$,

$$\Delta_1(P_1) = (k_1 + k_2 P_1^2 - E + 2m_u(1 + r_c)), \quad \Delta_2(P_2) = (k_3 + k_4 P_2^2 - E + 2m_c(1 + r_c)).$$

The transition amplitudes are given by

$$T_{12} = 2\mu_{12} \frac{\pi}{2} P_2 \sqrt{\frac{v_1}{v_2}} h_{12},$$

$$T_{21} = 2\mu_{34} \frac{\pi}{2} P_1 \sqrt{\frac{v_3}{v_1}} h_{21},$$

where $P_1$ and $P_2$ are the relative momenta of two clusters for channel 1 and 2 defined as

$$P_1 = \sqrt{2\mu_{12}(E - M_1 - M_2)}, \quad P_2 = \sqrt{2\mu_{34}(E - M_3 - M_4)},$$

where $M_i$, $i = 1, 2, 3, 4$, are the corresponding meson masses and $\mu_{ij}$ are the reduced masses of $i$ and $j$ mesons, and $v_1 = P_1/\mu_{12}, v_2 = P_2/\mu_{34}$. Finally we obtain the spin averaged cross sections as follows [41]:

$$\sigma_{ij} = \frac{4\pi}{P_j^2} \sum_j \frac{2J + 1}{(2s_1 + 1)(2s_2 + 1)} |T_{ij}|^2,$$

where $J$ is the total spin of the two outgoing mesons and $s_1$ and $s_2$ are the spins of the incoming mesons.

5 Fitting the parameters

The parameters $\alpha_s$ and $c$ of the Cornell potential are fitted by minimizing $\chi^2$ using the experimental values of the masses taken from PDG [42] and the mass spectrum generated from Cornell potential with a hyperfine correction. We find separate fits for three classes of mesons having the quark contents $u\bar{c}(c\bar{u})$, $u\bar{u}$, and $c\bar{c}$. Corresponding values are $\alpha_{s1} = 0.38 \pm 0.00002$, $c_1 = 0.732 \pm 0.0003$ GeV (for $u\bar{c}$), $\alpha_{s2} = 0.5 \pm 0.002$, $c_2 = 0.612 \pm 0.001$ GeV (for $u\bar{u}$), and $\alpha_{s3} = 0.5 \pm 0.0004$, $c_3 = 0.692 \pm 0.0003$ GeV (for $c\bar{c}$). The errors in the fitted values are calculated using the experimental errors in the meson masses taken from PDG [42]. It is found that the errors in the parameters produce an error of about 5% in the resultant cross sections calculated using the Cornell potential. The values of the constituent quark masses $m_u = 0.345$ GeV, $m_c = 1.931$ GeV, string tension $b = 0.18$ GeV$^{-2}$, and $\sigma = 0.897$ GeV$^{-2}$ are taken from Refs. [10,11], and the value of $k_f = 0.075$ is given in Ref. [13]. In the case of a quadratic potential the constituent quark masses $m_u$ and $m_c$, and the angular frequency parameter $\omega$ are fitted by minimizing the chi-square

$$\chi^2 = \sum_i ((E_i - \tilde{E}_i)/\bar{E}_i)^2,$$

where $E_i$ are the spin averaged experimental masses [42] and $\tilde{E}_i$ are the experimental masses [42]. It is found that the resultant error in the resultant error in the cross sections is about 15% in the case of quadratic potential. The relatively large error to cross sections in the case of a quadratic potential is mainly produced by the error in $\omega$ of $u\bar{u}$ system, as the cross sections are found to be very sensitive to this parameter. The meson sizes for both quadratic and Cornell potential are given in Table 1. It is noted that the meson sizes obtained through two potential models are quite different. The shapes of the quadratic and Cornell potentials are such that, for any given energy $E$, the value of the classical turning point $r_0 (E = V(r_0))$ is smaller for the Cornell than the quadratic potential. The statefunction beyond the classical turning, where $E < V(r)$, dampens quickly. Thus, a larger value of classical turning point for quadratic potential produces a larger r.m.s radius than the Cornell potential, as shown in Table 1.

Table 1 Mesons masses and their corresponding sizes for Cornell and quadratic potentials

| Meson | Mass (GeV) | $d$ (GeV$^{-1}$) (Cornell) | $d$ (GeV$^{-1}$) (Quadratic) |
|-------|-----------|---------------------------|-------------------------------|
| $\rho$ | 0.7755    | 2.3827                    | 4.9241                        |
| $D^0/\bar{D}^0$ | 1.8648 | 1.6543                    | 3.5288                        |
| $D^{*0}/\bar{D}^{*0}$ | 2.0070 | 1.9238                    | 3.5288                        |
| $J/\psi$ | 3.0969 | 0.9947                    | 1.5199                        |
6 Results and conclusions

The values of the cross sections of $\rho J/\psi \rightarrow D^0\bar{D}^0$, $\rho J/\psi \rightarrow D^0\bar{D}^{*0}$ ($D^{*0}\bar{D}^0$), $\rho J/\psi \rightarrow D^{*0}\bar{D}^{*0}$ and the corresponding inverse processes are given in Figs. 2, 3, 4, 5, 6, 7, 8 and 9 for the Cornell as well as the quadratic potentials. These cross sections are obtained for both $f = 1$ and the Gaussian form of $f$. $T_c = E_c - m_{D^0} - m_{\bar{D}^0}$ for the processes $\rho J/\psi \rightarrow D^0\bar{D}^{*0}$ and $D^{*0}\bar{D}^0 \rightarrow \rho J/\psi$ and $T_c = E_c - m_{\rho} - m_{J/\psi}$ for the remaining four processes, where $E_c$ is total centre of mass energy. The processes $\rho J/\psi \rightarrow D^0\bar{D}^0$, $\rho J/\psi \rightarrow D^0\bar{D}^{*0}$ ($D^{*0}\bar{D}^0$) and $D^{*0}\bar{D}^{*0} \rightarrow \rho J/\psi$ are exothermic whereas all the remaining processes are endothermic. The plots show that the values of the cross sections obtained using quadratic potential are higher as compared to the Cornell potential. For $f = 1$, the shapes of the cross sections of the $\rho J/\psi \rightarrow D^0\bar{D}^0$, $\rho J/\psi \rightarrow D^0\bar{D}^{*0}$ ($D^{*0}\bar{D}^0$) processes are also different for the quadratic and the Cornell potentials. The cross sections of these processes exhibit a dip close to threshold energy in

![Graph for cross sections versus $T_c$ for Cornell potential with $f = 1$](image1)

![Graph for cross sections versus $T_c$ for quadratic potential with Gaussian form of $f$](image2)

![Graph for cross sections versus $T_c$ for quadratic potential with $f = 1$](image3)

![Graph for cross sections versus $T_c$ for Cornell potential with $f = 1$](image4)

![Graph for cross sections versus $T_c$ for quadratic potential with $f = 1$ for reversed processes](image5)

![Graph for cross sections versus $T_c$ for Cornell potential with $f = 1$ for reversed processes](image6)

![Graph for cross sections versus $T_c$ for quadratic potential with $f = 1$ for reversed processes](image7)
the quadratic potential as shown in Fig. 3, but no dip is seen for the Cornell potential. However, when a Gaussian $f$ is included the dip structure disappears in plots of the cross sections and results for both potentials become of similar shape as shown in Figs. 4 and 5. Considering significant differences in the plots of the cross sections for the quadratic and the Cornell potentials, we conclude that, using a quadratic in replace of the more justified Cornell potential is not a good approximation for studying two-quark–two-antiquark systems.

We observe a significant suppression in the cross sections when a Gaussian form of $f$ is applied as compared to $f = 1$, i.e., a simple sum of the two-body approach. We can observe this suppression in all endothermic reactions where the peaks of these cross sections are suppressed by the factor of 7 to 20 after including the Gaussian $f$. For the process $\rho J/\psi \to D^0 D^0$ a Gaussian $f$ produces a suppression by the factor of 18 for the Cornell potential, and by the factor of 13 for the quadratic potential. Meanwhile, the suppression in the cross sections of the process $D^0 D^0 \to \rho J/\psi$ are by factors of 20 and 7 for the Cornell and the quadratic potentials, respectively, and for the process $D^0 D^0$ or $D^{*0} D^0 \to \rho J/\psi$ the suppressions are by the factors 20 and 18 for the Cornell and the quadratic potentials, respectively. It is noted that the Gaussian form of $f$ produces a larger suppression in the case of the Cornell as compared to the quadratic potential. To elucidate this effect we produce the plots of the potential energy and remaining terms that include kinetic energy and constant terms, versus $T_c$ for a specific process $\rho J/\psi \to D^0 D^0$. The resultant values of cross sections depend on a delicate cancellation in these terms. Figure 10 shows that in the case of the Cornell potential these terms are modified by the $f$ factor in such a way that cancellation is higher as compared to a quadratic potential.

Haglin and Gale [44] used a chiral Lagrangian approach for heavy–light mesons to study the cross sections of $\rho J/\psi \to D^* D^*$ and $\rho J/\psi \to D D$ processes. Figures 3 and 4 of Ref. [44] show that the isospin averaged cross sections of these processes without form factors are about 4.5 and 1 mb, respectively, near threshold. However, after including the form factors the values of these cross sections, reduced by the factor of about $10^{-3}$, are much smaller than our results. Lin and Ko [45] used an effective hadronic Lagrangian to obtain the cross sections of the processes $\rho J/\psi \to D^* D^*$ and $\rho J/\psi \to D D$. Figure 4 of Ref. [45] shows that the isospin average cross section of the process $\rho J/\psi \to D^* D^*$ with form factor varies in the range 5 to 12 mb depending upon the value of cutoff parameter in the form factor. This result is comparable with our result for quadratic potential with $f = 1$, shown in Fig. 3, after multiplying it with an isospin averaging factor of 2. Figure 4 of Ref. [45] shows that the isospin averaged cross section of the process $\rho J/\psi \to D D$ with form factor is rapidly decreasing and at $T_c = 0.1$ GeV it varies in the range 0.3 to 0.06 mb depending on the cutoff parameter. Given the large uncertainty in their results, it is difficult to make a comparison in this case. However, the results of Refs. [44,45] and ours show that the contribution of $\rho J/\psi \to D D$ in the total inelastic cross sections is relatively small. Oh et al. [46] also used effective Lagrangian for the $J/\psi$ dissociation by $\pi$ and $\rho$ mesons. In Fig. 7, they have reported isospin averaged cross sections without form factors for the processes $\rho J/\psi \to D D$, $\rho J/\psi \to D^* D^*$ and $\rho J/\psi \to D^* D^*$. Unlike our results and those of Refs. [44,45] their cross sections are rapidly increasing functions of energy except the process $\rho J/\psi \to D D$, which exhibit a dip at energy 4.1 GeV. In Fig. 8, they have reported the total inelastic cross sections with form factor for $\rho + J/\psi$ including the contributions of $\rho J/\psi \to D D$, $\rho J/\psi \to D^* D^*$ and $\rho J/\psi \to D^* D^*$ processes. The plots of their Fig. 8 show that total cross section varies in the range of 2–6 mb depending on the cutoff parameter. These values of cross sections are comparable to our results for Cornell potential with $f = 1$, shown in Fig. 2, after multiplying with isospin averaging factor. The effective Lagrangian approach used in Refs. [45,46] is somewhat unrealistic as it is based on SU(4) flavour symmetry, which is badly broken in QCD due to large
charm quark mass as compared to light quarks. In Ref. [11],
Wong et al. calculated the cross sections of $\rho J/\psi \rightarrow D\bar{D}$, $\rho J/\psi \rightarrow D^*\bar{D}(D\bar{D}^*)$ and $\rho J/\psi \rightarrow D^*\bar{D}^*$ using the quark exchange model of Barnes and Swanson [6] along with the Cornell potential and without including any $f$. Their results for $\rho J/\psi \rightarrow D\bar{D}$ and $\rho J/\psi \rightarrow D^*\bar{D}^*$ processes are comparable, in shape as well as values, to our results for Cornell potential with $f = 1$. For the process $\rho J/\psi \rightarrow D^*\bar{D}(D\bar{D}^*)$ the shape is quite different although the values are comparable.

Figures 2 and 4 show that total $\rho + J/\psi$ cross sections for Cornell potential with and without Gaussian form of $f$ are of order 0.1 and 2 mb, respectively. As mentioned in the introduction, dissociation of $J/\psi$ by a light meson is important if the cross section is few mb. Thus, we conclude that dissociation of $J/\psi$ by the $\rho$ meson is either unlikely or marginally relevant in relativistic heavy ion collisions.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors’ comment: The data is already deposited in manuscript in the form of graphs and table.]

Appendix A: The spin basis

For total spin 0 we have the following spin states:

$$|P_{1\bar{3}}P_{2\bar{4}}\rangle_0 = \frac{1}{2} \left[ \uparrow\downarrow\uparrow - \uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow - \downarrow\uparrow\uparrow \right]$$  \hspace{1cm} (A1)

$$|V_{1\bar{4}}V_{2\bar{3}}\rangle_0 = \sqrt{\frac{1}{12}} \left[ 2 \uparrow\uparrow\downarrow + 2 \uparrow\downarrow\uparrow - \uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow - \downarrow\uparrow\uparrow \right]$$  \hspace{1cm} (A2)

$$|V_{1\bar{3}}V_{2\bar{4}}\rangle_0 = \sqrt{\frac{1}{12}} \left[ 2 \uparrow\uparrow\uparrow + 2 \uparrow\uparrow\downarrow - \uparrow\uparrow\downarrow - \uparrow\downarrow\downarrow - \downarrow\uparrow\downarrow \right].$$  \hspace{1cm} (A3)

The spin overlaps are

$$0\langle P_{1\bar{3}}P_{2\bar{4}}|P_{1\bar{3}}P_{2\bar{4}}\rangle_0 = 0\langle V_{1\bar{4}}V_{2\bar{3}}|V_{1\bar{4}}V_{2\bar{3}}\rangle_0 = 1,$$  \hspace{1cm} (A4)

$$0\langle P_{1\bar{3}}P_{2\bar{4}}|V_{1\bar{4}}V_{2\bar{3}}\rangle_0 = 0\langle V_{1\bar{4}}V_{2\bar{3}}|P_{1\bar{3}}P_{2\bar{4}}\rangle_0 = -\frac{\sqrt{3}}{2},$$  \hspace{1cm} (A5)

$$0\langle V_{1\bar{3}}V_{2\bar{4}}|V_{1\bar{4}}V_{2\bar{3}}\rangle_0 = 0\langle V_{1\bar{4}}V_{2\bar{3}}|V_{1\bar{3}}V_{2\bar{4}}\rangle_0 = -\frac{1}{2}.$$  \hspace{1cm} (A6)

By using the formalism $S_iS_j = (\frac{\sigma_i}{2})_i(\frac{\sigma_j}{2})_j + (\frac{\sigma_i^*}{2})(\frac{\sigma_j^*}{2})_j$ where $(\sigma_i)_i \downarrow = \downarrow_i$, $i = \downarrow_i$, $(\sigma_i^*)_i \downarrow = -\downarrow_i$, $(\sigma_i)_i \downarrow = \downarrow_i$, $(\sigma_i^*)_i \downarrow = -\downarrow_i$ ($i = 1, 2, 3, 4$), the matrix elements of $S_iS_j$ in $|1\rangle_0$ and $|2\rangle_0$ basis were calculated to be

$$0\langle P_{1\bar{3}}P_{2\bar{4}}|P_{1\bar{3}}P_{2\bar{4}}\rangle_0 = \begin{pmatrix} S_1S_2 \\ S_1S_3 \\ S_1S_4 \\ S_2S_3 \\ S_2S_4 \\ S_3S_4 \end{pmatrix} = \begin{pmatrix} 0 \\ -3/4 \\ 0 \\ 0 \\ -3/4 \\ 0 \end{pmatrix}$$  \hspace{1cm} (A7)

$$0\langle V_{1\bar{4}}V_{2\bar{3}}|P_{1\bar{3}}P_{2\bar{4}}\rangle_0 = \begin{pmatrix} S_1S_2 \\ S_1S_3 \\ S_1S_4 \\ S_2S_3 \\ S_2S_4 \\ S_3S_4 \end{pmatrix} V_{1\bar{4}}V_{2\bar{3}} = \begin{pmatrix} \sqrt{3}/8 \\ 3\sqrt{3}/8 \\ -\sqrt{3}/8 \\ -\sqrt{3}/8 \\ 3\sqrt{3}/8 \\ \sqrt{3}/8 \end{pmatrix}$$  \hspace{1cm} (A8)

$$0\langle V_{1\bar{4}}V_{2\bar{3}}|V_{1\bar{4}}V_{2\bar{3}}\rangle_0 = \begin{pmatrix} S_1S_2 \\ S_1S_3 \\ S_1S_4 \\ S_2S_3 \\ S_2S_4 \\ S_3S_4 \end{pmatrix} V_{1\bar{4}}V_{2\bar{3}} = \begin{pmatrix} -1/2 \\ -1/2 \\ 1/4 \\ 1/4 \\ -1/2 \\ -1/2 \end{pmatrix}$$  \hspace{1cm} (A9)
The matrix elements of \( S_i \) calculated to be

\[
\langle 0| V_{13} V_{24} |0 \rangle = \begin{pmatrix} 1/2 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/2 \end{pmatrix}
\]  

(A10)

\[
\langle 0| V_{13} V_{24} |0 \rangle = \begin{pmatrix} 1/4 & 0 & 0 & 0 \\ 0 & 1/4 & 0 & 0 \\ 0 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 1/4 \end{pmatrix}
\]  

(A11)

For total spin 1 we have the following spin states:

\[
| P_{13} V_{24} \rangle = \frac{1}{\sqrt{6}} [ \uparrow \downarrow \uparrow \downarrow - \downarrow \uparrow \uparrow \downarrow + \frac{1}{\sqrt{2}} \\
(\uparrow \downarrow \downarrow \downarrow + \downarrow \uparrow \uparrow \uparrow - \downarrow \uparrow \downarrow \uparrow )
\]

(A12)

\[
| V_{13} V_{24} \rangle = \frac{1}{\sqrt{12}} [ \uparrow \downarrow \uparrow \downarrow + \uparrow \uparrow \uparrow \uparrow - \uparrow \uparrow \downarrow \uparrow - \downarrow \uparrow \downarrow \uparrow \\
+ \uparrow \downarrow \downarrow \downarrow - \uparrow \downarrow \uparrow \uparrow - \uparrow \downarrow \downarrow \uparrow + \frac{1}{\sqrt{2}} \uparrow \uparrow \uparrow \uparrow + \sqrt{2} \uparrow \uparrow \uparrow \uparrow 
\]

(A13)

\[
| V_{14} V_{23} \rangle = \frac{1}{\sqrt{12}} [ \uparrow \downarrow \uparrow \downarrow + \uparrow \uparrow \uparrow \uparrow - \uparrow \uparrow \downarrow \uparrow - \downarrow \uparrow \downarrow \uparrow \\
+ \uparrow \downarrow \downarrow \uparrow - \uparrow \downarrow \uparrow \downarrow - \uparrow \downarrow \uparrow \uparrow + \frac{1}{\sqrt{2}} \uparrow \uparrow \uparrow \uparrow + \sqrt{2} \uparrow \uparrow \uparrow \uparrow 
\]

(A14)

The spin overlaps are

\[
1 \langle P_{13} V_{24} | P_{13} V_{24} \rangle = 1 \langle V_{14} V_{23} | V_{14} V_{23} \rangle = 1
\]

(A15)

\[
1 \langle P_{13} V_{24} | V_{14} V_{23} \rangle = 1 \langle V_{14} V_{23} | P_{13} V_{24} \rangle = \frac{1}{\sqrt{2}}
\]

(A16)

\[
1 \langle V_{13} V_{24} | V_{14} V_{23} \rangle = 0
\]

(A17)

The matrix elements of \( S_j \) in \( |1\rangle \) and \( |2\rangle \) basis were calculated to be

\[
1 \langle P_{13} V_{24} | P_{13} V_{24} \rangle = \begin{pmatrix} 0 & 0 & 0 & 0 \\ -3/4 & 0 & 0 & 0 \\ 0 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 1/4 \end{pmatrix}
\]  

(A18)

\[
2 \langle V_{13} V_{24} | V_{13} V_{24} \rangle = 2 \langle V_{14} V_{23} | V_{14} V_{23} \rangle = \begin{pmatrix} 1/4 & 0 & 0 & 0 \\ 0 & 1/4 & 0 & 0 \\ 0 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 1/4 \end{pmatrix}
\]  

(A23)

For total spin 2 we have the following spin states:

\[
| V_{13} V_{24} \rangle = | V_{14} V_{23} \rangle = \frac{1}{\sqrt{5}} [ \uparrow \downarrow \downarrow \downarrow + \uparrow \uparrow \uparrow \uparrow + \frac{1}{\sqrt{2}} ( \uparrow \downarrow \downarrow \uparrow + \downarrow \uparrow \uparrow \uparrow + \uparrow \downarrow \uparrow \downarrow + \downarrow \uparrow \uparrow \downarrow + \uparrow \downarrow \uparrow \uparrow + \downarrow \uparrow \uparrow \uparrow )
\]

(A21)

The spin overlaps are

\[
2 \langle V_{13} V_{24} | V_{14} V_{23} \rangle = 2 \langle V_{14} V_{23} | V_{13} V_{24} \rangle = 1
\]

(A22)

The matrix elements of \( S_i \) in \( |1\rangle \) and \( |2\rangle \) basis were calculated to be

\[
1 \langle P_{13} V_{24} | P_{13} V_{24} \rangle = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 1/4 \end{pmatrix}
\]  

(A18)

\[
2 \langle V_{13} V_{24} | V_{13} V_{24} \rangle = 2 \langle V_{14} V_{23} | V_{14} V_{23} \rangle = \begin{pmatrix} 1/4 & 0 & 0 & 0 \\ 0 & 1/4 & 0 & 0 \\ 0 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 1/4 \end{pmatrix}
\]  

(A23)
\[ 2\langle V_{13} V_{23} | V_{14} V_{24} \rangle = \frac{1}{4} \]

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