Could time-symmetric interactions reconcile relativity and quantum non-locality?

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Abstract

We present a simple model, demonstrating that time-symmetric relativistic interactions can account for correlations violating the Bell inequalities while avoiding conspiracies as well as the commitment to instantaneous influences. We emphasize the essential virtues and problems of such an account and discuss its relation to Bell’s theorem.

1 Introduction

In his beautiful article “Speakable and unspeakable in quantum mechanics” John S. Bell, discussing the implications of his seminal non-locality theorem [3, 5, 8] concludes:

“For me then this is the real problem with quantum theory: the apparently essential conflict between any sharp formulation [of quantum theory] and fundamental relativity. That is to say, we have an apparent incompatibility, at the deepest level, between the two fundamental pillars of contemporary theory.... It may be that a real synthesis of quantum and relativity theories requires not just technical developments but radical conceptual renewal.” [6]

From today’s perspective, there’s only little to add to this assessment. For once, one could emphasize that it’s now well understood that quantum non-locality does not imply the possibility of faster-than-light signaling or any other way to violate the principles of relativity operationally. One could also note that in the nearly two decades which have passed since Bell’s statement, modest yet significant progress has been made towards generalizing sharp formulations of non-relativistic quantum mechanics to the relativistic regime. In fact, Bell himself in a later publication [7] suggested that the GRW collapse

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theory may lend itself to a precise relativistic formulation, a feat that was
indeed accomplished by Tumulka \[35\], albeit only in a non-interacting setting
(see \[19\] for a recent discussion). Also, Lorentz invariant generalizations of
Bohmian mechanics can be formulated, although for the price of introducing
a preferred foliation of space-time (which, however, can be generated by a
Lorentz invariant law and shown to be empirically undetectable \[17\]). All in
all, the understanding that has grown over the last few years is that there
is indeed not a contradiction but a distinct tension between relativity and
quantum non-locality and that this tension is not primarily characterized by
the thread of superluminal signaling (and the causal paradoxes that could
result from it \[38\]) but, more subtly, by the fact that relativistic space-time –
having no structure of absolute simultaneity and no objective temporal
order between space-like separated events – is not particularly hospitable to
the kind of instantaneous influences that, according to Bell’s theorem, the
explanation of certain non-local correlations seems to require.\[1\]

In this paper, we want to discuss the possibility to alleviate the tension
between relativity and non-locality by explaining non-local correlations as a
result of interactions that are both retarded, “propagating” at a finite speed
from past to future, and advanced, “propagating” at a finite speed from future
to past. Despite the technical difficulties inherent to such theories (the most
important of which we will highlight in our discussion) and the philosophical
reservations that one might have against retrocausality (but which shall not
concern us here), the great virtue of such models would be that they could
provide a complete physical account of quantum correlations while drawing
exclusively on the resources of relativistic space-time with particles interact-
ing along their past and future light-cones. In fact, it can be argued on the
basis of time-symmetry that advanced + retarded interactions are really the
generic case of relativistic interactions and that it’s the empirical violation
of this symmetry (e.g. the absence of advanced electromagnetic radiation)
that requires explanation. And it has been speculated that such an expla-
nation could parallel, or even reduce to, the statistical explanation of the
thermodynamic asymmetry, accounting for the absence of advanced effects
on macroscopic scales, while the time-symmetry of the fundament laws may
still reveal itself on microscopic scales in the quantum phenomena \[28\].

The idea of accounting for quantum non-locality by admitting some form
of retrocausality is not new, but has been advanced by various authors for
many decades.\[2\] Nevertheless, it is rarely acknowledged as a possible impli-

\[1\]For a comprehensive discussion of the issue, see \[26\], \[27\]. See \[21\] for a simple argu-
ment substantiating said tension.

\[2\]See, for instance, \[11\], \[12\], \[22\], \[13\], \[23\], \[25\], \[20\]. Some authors have proposed time-
symmetric formulations of (non-relativistic) quantum mechanics, most notably Aharanov
cation of Bell’s theorem and even rarer to pass the threshold from a logical possibility to a serious option. The reason, we believe, are persistent preconceptions about how incomprehensible or unbecoming any form of retro-causality must be – and a shortage in intuitively graspable models that could help to overcome them. This paper, we hope, can contribute to mitigate this deficiency. By means of a simple toy-model, tailored, in particular, to the EPRB experiment we will 1) demonstrate that time-symmetric relativistic interactions can account for violations of the Bell inequalities without being conspiratorial, 2) point out some of the virtues and difficulties of such an account and 3) understand how it fits into the framework of Bell’s theorem.

2 The model

Our discussion is based on a simple model postulating a hidden spin-variable for relativistic particles. We emphasize that this is only a toy-model, used to explore the range of logical possibilities restricted by Bell’s theorem. In particular, it is neither able nor intended to reproduce all quantum spin-statistics (for instance, it will fail to do so for repeated spin measurements in varying directions). The virtue of the model, on the other hand, is that while it’s concrete enough to serve as an intuition pump, it is also general enough to emphasize the micro-causal structure that would characterize time-symmetric relativistic accounts of non-locality in general. Moreover, it will lend itself to a statistical analysis which allows us to demonstrate the violation of Bell’s inequality in a quantitative manner.

The model is defined by the following assumptions:

1) The particles have an internal degree of freedom (a “hidden variable”) represented by a vector $\mathbf{S}$ on the 2-sphere $S^2$. We refer to this degree of freedom as the particle’s spin.

2) A spin measurement in the direction $\mathbf{a} \in S^2$ cannot determine the exact value of $\mathbf{S}$, but only its the orientation relative to $\mathbf{a}$. The result of a spin measurement is thus given by

$$\text{sgn}(\mathbf{a}, \mathbf{S}) \in \{\pm 1\},$$

where $\text{sgn}(x)$ denotes the sign of $x$.  

et.al. who developed a two-time formalism [1, 2] and J. G. Cramer, whose transactional interpretation stipulates that any quantum mechanical interaction involves advanced and retarded solutions of the wave-equation [12, 13]. Concerning the latter, see [26] for a critique and [25] for a recent review. Note also the time-symmetric formulation of Bohmian mechanics discussed by Sutherland [34], and other references therein.
We say that the particle has *a-spin up* if \( \langle a, S \rangle > 0 \), i.e. \( |\langle a, S \rangle| = +1 \) and *a-spin down* if \( \langle a, S \rangle < 0 \), i.e. \( |\langle a, S \rangle| = -1 \).

We can disregard the special case \( \langle a, S \rangle = 0 \), as it will have probability 0.

3) One of the crucial lessons of quantum mechanics is that a measurement is not a purely passive process, but an invasive interaction that will in general effect the state of the measured system. Here, in analogy to a projective measurement in quantum mechanics, we assume that a spin measurement influences the particle’s hidden spin state by projecting it onto the respective direction in which it’s being measured. That is, if the spin of the particle undergoing a spin measurement in the \( a \)-direction is \( S \), its spin immediately after measurement will be

\[
\text{sgn}(a, S) a = \frac{\langle a, S \rangle}{|\langle a, S \rangle|} a.
\]  

4) We are interested in the statistics of the EPRB-experiment concerning simultaneous measurements on a pair of entangled particles in the singlet state. To this end, we consider in our model an ensemble of pairs of particles whose initial spin-variables are prepared with opposite orientation in a random direction, which it equidistributed on the unit sphere \( S^2 \). For any pair, we denote

\[
S^A(t = 0) = -S^B(t = 0) = S_0.
\]  

5) The Spin state \( S \) is subject to a *pair interaction* whose effect is such that a particle continuously rotates the spin of its partner towards the orientation antipodal to its own. This effect is manifested by an *advanced and retarded* action of one particle on the other. This could mean e.g. that the interaction is transmitted by a medium – like a field or a massless particle – propagating with the speed of light towards past and future or that the particles interact directly along their past and future light-cones.

To acknowledge the fact that EPR correlations persist over very long distances [31] – indeed, if quantum mechanics is correct, over arbitrary long distances – we will have to assume that the interactions in a particle-pair are unattenuated, i.e. unaffected by distance, and discriminating, i.e. unfazed and unscreened by any other matter, thus realizing two essential properties of what Maudlin describes as the “quantum connection” [26, p. 22] or what, in other words, can be understood as entanglement.

Beyond that, our discussion will not rely on any more specific model realizing these assumptions, first because that would suggest and require a level
of precision that we cannot provide in our analysis and second because our main results will not depend on the details of the interactions. However, to have at least one concrete example in mind, we can consider an interaction of the following type:

For two unit vectors \( \mathbf{X}, \mathbf{Y} \in S^2 \subset \mathbb{R}^3 \), their distance-vector is given by

\[
D(\mathbf{X}, \mathbf{Y}) = \frac{\arccos(\mathbf{Y} \cdot \mathbf{X})}{\sqrt{1 - (\mathbf{Y} \cdot \mathbf{X})^2}} \left( \mathbf{Y} - (\mathbf{Y} \cdot \mathbf{X})\mathbf{X} \right) \in T_X S^2,
\]

where \( T_X S^2 \) denotes the tangent-bundle of the sphere at point \( \mathbf{X} \). Then, assuming the particles have world-lines \( z_i^\mu(t), z_j^\mu(t) \), we can set

\[
\frac{d}{dt} S_i(t) \propto D(S_i(t), -S_j(\tau_{\text{ret}})) + D(S_i(t), -S_j(\tau_{\text{adv}})),
\]

where \( \tau_{\text{ret}}, \tau_{\text{adv}} \) are the advanced and retarded time, i.e. the solutions of

\[
(z_i^\mu(t) - z_j^\mu(\tau)) \left( z_{i,\mu}(t) - z_{j,\mu}(\tau) \right) = 0.
\]

Hence, the spin of particle \( i \) at time \( t \) is “repelled” by the spin of particle \( j \) at the advanced and retarded times. This is an example of a direct particle interaction along past and future light cones (fig. 1) (c.f. the Wheeler-Feynman theory of electrodynamics [36, 37]).

![Figure 1: Direct interaction of two particles along past and future light-cones.](image)
3 A heuristic analysis of the time-evolution

Figure 2 shows a sketch of the space-time diagram of the EPRB-experiment as described by our model.

![Space-time diagram of the EPRB experiment]

Figure 2: Space-time diagram of the EPRB experiment. For the analysis of the interactions, we distinguish 4 increments of the particles world-lines in different slices of space-time.

A pair of particles prepared with opposite spin move in different directions towards an apparatus (a Stern-Gerlach magnet), where they undergo a spin measurement in directions $\mathbf{a}$ and $\mathbf{b}$, respectively. These parameters can be freely chosen by the experimentalists right before the measurement. Since the two measurements are assumed to happen simultaneously (in the laboratory frame), they occur in space-like separated regions of space-time, here denoted by $\mathcal{A}$ and $\mathcal{B}$. According to our model, the outcomes of these measurements are determined by the particles’ hidden spin state as they pass the device. We denote the corresponding values by $\mathbf{S}_A$ and $\mathbf{S}_B$, respectively. The result of the spin measurement on particle A in direction $\mathbf{a}$ is then given by $A := \text{sgn} \langle \mathbf{a}, \mathbf{S}_A \rangle$. The result of the spin measurement on particle B in direction $\mathbf{b}$ is $B := \text{sgn} \langle \mathbf{b}, \mathbf{S}_B \rangle$.

In the end, we are interested in the probabilities of the coincidences $A = B$, respectively the anti-coincidences $A \neq B$. We recall that the Bell
inequality (in its simplest version) reads

\[ P(A \neq B|a, b) + P(A \neq B|b, c) + P(A \neq B|a, c) \geq 1, \]

for arbitrary parameter-settings \(a, b, c\) \[22\]. This inequality is violated for the correlations observed in the EPRB-experiment (for certain choices of \(a, b, c\)) thus implying, according to Bell’s theorem, that they cannot be reproduced by any local theory.

Indeed, we can consider for starters the predictions of our hidden variable model \textit{without} advanced interactions. This will serve as our point of reference as a completely \textit{local} model. Without advanced interactions, the initial configuration is a stationary point of the (retarded) dynamics and we have \(S_A = S_0\) and \(S_B = -S_0\), that is, the outcomes of the spin measurement depend only on the orientation of the initial hidden spin \(S_0\) relative to those of the measurement devices. Concretely, we find:

\[
A = B \iff \text{sgn} \langle a, S_0 \rangle = \text{sgn} \langle b, -S_0 \rangle \\
A \neq B \iff \text{sgn} \langle a, S_0 \rangle = \text{sgn} \langle b, +S_0 \rangle.
\]

Of course, we now know from Bell’s theorem that there can be no probability distribution for \(S_0\) for which such a model will reproduce statistical correlations violating the above inequality. And indeed, a short calculation shows that for the assumed equidistribution of \(S_0\) on \(S^2\) and arbitrary angles \(a, b, c\):

\[
P(A \neq B|a, b) + P(A \neq B|b, c) + P(A \neq B|a, c) = 1,
\]

so that Bell’s inequality is always satisfied.\[3\]

Let’s now turn to the more interesting case and consider our model with retarded \textit{and} advanced interactions. The key difference, of course, is that now the particles \textit{post measurement}, whose states are affected by the external intervention of the measurement process, can have a retro-causal effect on the particles \textit{before} measurement. To extract statistical predictions for this case, we will have to understand how the final spin states \(S_A\) and \(S_B\), determining the experimental outcomes, result from the time-symmetric interactions between the particles. This will be essentially a combination of three different effects due to what is sometimes described as \textit{zigzag causality}:

1) \textit{Feed-forward}: Particle A after undergoing measurement in space-time region \(A\) exhibits an advanced action on its partner, reaching particle B in space-time region 3 (fig. 3). The change in the spin-variable that particle

\[\text{More precisely, one can see by a few geometric considerations that } P(A \neq B|a, b) = 1 - 2 \frac{\angle a, b}{360^\circ}, \text{ where } \angle a, b \in [0, +180^\circ] \text{ is the acute angle between the vectors } a \text{ and } b.\]
$B$ experiences as a result is thus directed towards $-a$ if the measurement on $A$ yields $a$-spin up and $+a$ if the measurement on $A$ yields $a$-spin down. The same holds vice-versa, with particle $B$ in $B$ exhibiting an advanced action on an earlier version of particle $A$, rotating its spin towards the direction opposite to $B$’s post-measurement state. We call this effect a *feed-forward* since, intuitively spoken, every particle receives a “feedback” of its partners future measurement result.

2) *Preinforcement*: After being affected by the *feed-forward*, the advanced back-reaction of particle $B$ in space-time region 4 on particle $A$ in region 2 will tilt the spin of particle $A$ slightly towards the direction in which it is going to be measured in the future. The advanced action of particle $A$ in space-time region 4 has an analogous effect on particle $B$ in space-time region 2. For further reference, we call this effect *preinforcement*—so to speak, a preemptive reinforcement of the future measurement result.

Of course, in a similar way, particle $A$ in space-time region 2 has an advanced interaction with an earlier version of particle $B$, and so on... But further down the worldlines, the advanced effects originating from the particles after measurement are more and more diluted and will affect the state of the early particles only marginally.
3) Inertia: Since the early spins are largely unaffected by the retro-causal influences, the effect of the *retarded* action exhibited by the particles in space-time regions 1 and 2 is essentially to rotate the spins *back* towards their initial values $S_0$ and $-S_0$, respectively.

In total, the spin variable of particle $A$ by the time it enters the measurement device will be of the form

$$S_A = \frac{\alpha S_0 - \beta B b + \gamma A a}{\|\alpha S_0 - \beta B b + \gamma A a\|},$$

(7)

with positive parameters $\alpha, \beta, \gamma$; $\alpha + \beta + \gamma = 1$, whose precise values would depend on the details of the interactions and the experimental setting. Since *feed-forward* will, in general, act only for a very brief period of time until the particles are absorbed in a detector and since *preinforcement*, in turn, is only the “echo” of feed-forwards, we assume $\alpha > \beta > \gamma$. Note that the sign in front of $\gamma$ corresponds to the measurement outcome $A$, while the sign in front of $\beta$ is opposite to the outcome $B$ of the measurement on particle $B$.

An analogous expression will describe $S_B$, and for simplicity we assume that the variables parametrizing this solution are the same as for particle $A$.

For the results of the spin measurements, we have to consider the projections of the final spins $S_A$ and $S_B$ onto the corresponding directions in which they are being measured, that is (neglecting the normalization constant):

$$\langle a, S_A \rangle = \alpha \langle a, S_0 \rangle - \beta B \langle a, b \rangle + A \gamma$$

$$\langle b, S_B \rangle = -\alpha \langle b, S_0 \rangle - \beta A \langle a, b \rangle + B \gamma,$$

yielding

$$A = \text{sgn}\left\{ \alpha \langle a, S_0 \rangle - B \beta \langle a, b \rangle + A \gamma \right\}$$

$$B = \text{sgn}\left\{-\alpha \langle b, S_0 \rangle - A \beta \langle a, b \rangle + B \gamma \right\}.$$

(8)

However, as we will see, this does not determine the outcomes unambiguously.

4 Statistical Analysis

4.1 ‘Self-fulfilling prophecies’ and the underdetermination of the time-evolution by Cauchy data

The main technical and conceptual difficulty arising in theories with advanced and retarded interactions is that the laws of motion, in general, cannot be formulated as *Cauchy problems*, meaning that the specification of
initial conditions at one single moment in time (respectively on a space-like hypersurface) is not sufficient to distinguish a unique solution and thus to determine the system’s complete time-evolution. In our model, this problem is manifested in the fact that while for any choice of \(a, b\) there is a range of initial states \(S_0\) determining unique final states \(S_A\) and \(S_B\), there are also values of \(S_0\) (that is when \(\langle a, S_0 \rangle\) and/or \(\langle b, S_0 \rangle\) are small compared to the effects of feed-forward and preinforcement) for which two or more different prescriptions for \(A\) and \(B\) correspond to consistent evaluations of equation (8) and thus to possible solutions of the equations of motion. Moreover, in these cases, there is a sense in which one could say that the measurement outcomes are retro-causally responsible for their own occurrence. For further reference, we will call this phenomenon a self-fulfilling prophecy (SFP).

We emphasize that this underdetermination of the time-evolution by initial data is not a result of our analysis being too coarse, but an intrinsic feature of theories admitting retro-causal influences. We also note that while the possibility of self-fulfilling prophecies may be mind boggling, it need not imply the possibility of logical paradoxes – there is nothing inconsistent about the time-evolutions we consider here – though the thread of potential inconsistencies is something to be addressed in the context of a more mature theory. Nevertheless, in addition to the philosophical headaches that might be caused by SFP, one very concrete difficulty that we have to face here is that theories in which solutions are not parametrized by Cauchy data are in general not statistically transparent, in the sense that there is no obvious notion of a state-space on which one could implement a statistical hypothesis or define a measure of typicality. More simply put, in our case, since the measurement outcomes \((A, B)\) are not unambiguously determined by the initial state \(\pm S_0\), their probabilities are not unambiguously determined by the statistical distribution of \(S_0\).

For this reason, we will have to resort to a more unconventional form of statistical analysis, leaving open the question what boundary conditions in addition to \(S_0\) should be used to determine the time-evolution and how the resulting statistical description should look like in detail. While we cannot assign to each \((A, B) \in \{\pm 1\}^2\) a set of initial conditions \(S_0\) sufficient to produce that outcome, thus implementing \((A, B)\) as a random variable on \(S^2\), we will consider for each possible outcome \((A, B) \in \{\pm 1\}^2\) the necessary conditions in terms of the initial configuration \(S_0\), thus determining upper and lower bounds on its probability.

\[4\]

For a mathematical discussion of this issue in the context of Wheeler-Feynman electromagnetism, see [36, 15, 16] and references therein.
4.2 A case-by-case analysis

To this end, we will restrict our attention to the interesting case $\langle a, b \rangle < 0$, for which quantum mechanics predicts a violation of the Bell inequality (6). Furthermore, observing that the problem is symmetric under $S_0 \leftrightarrow -S_0$ together with an exchange of the particle labels (and assuming that SFP respects this symmetry) we can w.l.o.g. assume $\langle a, S_0 \rangle > 0$. Now we can brake down all remaining cases corresponding to valid solutions of eq. (8) and thus to valid solutions of our assumed time-evolution. In the table below, we have listed for all $(A, B) \in \{\pm 1\}^2$ the range of initial conditions $S_0$ that can produce the corresponding outcome:

| (A,B) | $\langle a, S_0 \rangle > 0$ | $\langle a, S_0 \rangle > 0$ |
|-------|-----------------|-----------------|
|      | $\langle b, S_0 \rangle > 0$ | $\langle b, S_0 \rangle < 0$ |
| + +  | $\alpha|\langle b, S_0 \rangle| < \beta|\langle a, b \rangle| + \gamma$ | always possible |
| - -  | $\alpha|\langle a, S_0 \rangle| < \beta|\langle a, b \rangle| + \gamma$ | $\alpha|\langle a, S_0 \rangle| < \beta|\langle a, b \rangle| + \gamma$ |
|      | $\alpha|\langle a, S_0 \rangle| + \gamma > \beta|\langle a, b \rangle|$ | $\alpha|\langle a, S_0 \rangle| + \gamma < \beta|\langle a, b \rangle|$ |
|      | $\alpha|\langle b, S_0 \rangle| + \gamma > \beta|\langle a, b \rangle|$ | $\alpha|\langle b, S_0 \rangle| + \beta|\langle a, b \rangle| < \gamma$ |
|      | $\alpha|\langle a, S_0 \rangle| + \beta|\langle a, b \rangle| < \gamma$ | $\alpha|\langle a, S_0 \rangle| + \beta|\langle a, b \rangle| < \gamma$ |

Figure 4: Table of possible outcomes for $\langle a, b \rangle < 0$.

Admittedly, this may still seem quite confusing, but the dust will settle in a minute. First, we note that unless $\langle a, b \rangle$ is very close to 0 or the ratio $\gamma/\beta$ unreasonably large, we will always find that $\beta|\langle a, b \rangle|$ is greater or equal $\gamma$, meaning that we can disregard all the cases requiring $\beta|\langle a, b \rangle| < \gamma$. Furthermore, we recall that what we’re ultimately interested in are the probabilities of the coincidences $A = B$ and the anti-coincidences $A \neq B$. To this end, only the following cases remain to be distinguished:
1) If \( \text{sgn}\langle a, S_0\rangle \neq \text{sgn}\langle b, S_0\rangle \) then \( A = B \) occurs (almost surely).

2) If \( \text{sgn}\langle a, S_0\rangle = \text{sgn}\langle b, S_0\rangle \) then:

\[
A = B \text{ possible if } \left( \alpha|\langle a, S_0\rangle| < \beta|\langle a, b\rangle| + \gamma \lor \alpha|\langle b, S_0\rangle| < \beta|\langle a, b\rangle| + \gamma \right)
\]

\[
A \neq B \text{ possible if } \left( \alpha|\langle a, S_0\rangle| + \gamma > \beta|\langle a, b\rangle| \land \alpha|\langle b, S_0\rangle| + \gamma > \beta|\langle a, b\rangle| \right).
\]

Comparing this to the local model without advanced interactions, where \( A \neq B \) occurred if and only if \( \text{sgn}\langle a, S_0\rangle = \text{sgn}\langle b, S_0\rangle \), we see that now, in certain cases that would have been “on the edge”, i.e. where \( \langle a, S_0\rangle \) and \( \langle b, S_0\rangle \) have the same sign but either one of the terms is small in absolute value, the spin variable of the particles are rotated just enough by the retro-causal feed-forward to produce the coinciding event \( A = B \) instead.

Now a quantitative statistical analysis will, of course, require some information about the distribution of the parameters \( \beta \) and \( \gamma \), depending, possibly, on \( S_0 \) and the experimental settings \( a \) and \( b \). Going forward, we will make the simplest possible ansatz which is that \( \beta \) and \( \gamma \) are not only the same for both particles in each pair, but constant throughout the ensemble of pairs, or, at least, distributed independently of the initial state \( S_0 \) for fixed \( a \) and \( b \) (which is basically to say that the strength of the spin-interactions does not depend on the \( S^2 \)-distance, i.e. the angle, between these spins). While one may object that this assumption is not only simple but somewhat simplistic, it will be fairly obvious that, qualitatively, our results will not depend on it too strongly. In any case, under this assumption, we can now determine an upper and lower bound on the probability of the anti-coincidences \( A \neq B \) for given parameter choices \( a \) and \( b \) with \( \langle a, b\rangle < 0 \).

- The maximal probability \( \mathbb{P}_{\text{max}}(A \neq B|a, b) \) is the probability of \( A \neq B \) assuming that \( A \neq B \) will occur whenever it is possible for an initial configuration \( S_0 \) (or, in other words, that SFP always favors \( A \neq B \)).

- The minimal probability \( \mathbb{P}_{\text{min}}(A \neq B|a, b) \) is the probability of \( A \neq B \) assuming that it will occur only if \( A = B \) is impossible for an initial configuration \( S_0 \) (or, in other words, that SFP always favors \( A = B \)).

\[5\text{In particular, our model predicts perfect (anti)correlations if the spins are measured in the exact opposite (respectively the same) direction.}\]

\[6\text{We emphasize again that these parameters play no fundamental role in the model, but that it’s merely a feature of our two-dimensional state-space that the results of the interactions, however complicated, can be parameterized by two variables. The precise values of these parameters, in every single instance, would of course depend on the details of the interactions and the relevant boundary conditions.}\]
From eq. (9) we can conclude:

\[
\begin{align*}
P_{\text{max}}(A \neq B | a, b) &= 2 \mathbb{P} \left( \langle a, S_0 \rangle > \beta | \langle a, b \rangle | - \gamma, \langle b, S_0 \rangle > \frac{\beta | \langle a, b \rangle | - \gamma}{\alpha} \right) \\
P_{\text{min}}(A \neq B | a, b) &= 2 \mathbb{P} \left( \langle a, S_0 \rangle > \beta | \langle a, b \rangle | + \gamma, \langle b, S_0 \rangle > \frac{\beta | \langle a, b \rangle | + \gamma}{\alpha} \right),
\end{align*}
\]

where the factor of 2 accounts for the case \( \langle a, S_0 \rangle, \langle b, S_0 \rangle < 0 \). This can be evaluated by means of the following identity that we derive in the appendix:

\[
|S^2|^{-1} \left| \left\{ S \in S^2 : \langle a, S \rangle > c \land \langle b, S \rangle > c \right\} \right| = \frac{1}{\pi} \int_c \sqrt{1 + (a,b)} \sqrt{\frac{z^2 - c^2}{z^2 - c^4}} \, dz.
\]

(10)

For better illustration of the results, we will simplify things a bit further, still, by estimating the ratio of \( \beta \) to \( \gamma \), thus obtaining a parameterization of the final spin states \( S_A \) and \( S_B \) in terms of a single affine parameter \( \nu \in [0, 1] \).

This is to say that we differentiate the analysis only by the strength of the advanced effects – represented by the parameter \( \nu \) – rather than by their dependence on distance or the duration of action, which could be dissected in a more fine-grained analysis by fitting the parameters \( \beta \) and \( \gamma \) (cf. figure 7 below). Setting \( \beta = \nu \), a reasonable estimate is \( \gamma = \nu^2 \) since preinforcement is a result of two advanced interactions, and hence \( \alpha = 1 - (\beta + \gamma) = 1 - (\nu + \nu^2) \).

4.3 Results

In figure 6, we have plotted the minimum and maximum probabilities for varying values of \( \nu \) and parameter settings \( \langle a, b \rangle = 120^\circ \), i.e. \( \langle a, b \rangle = -\frac{1}{2} \), which is the case for which quantum mechanics predicts the greatest violation of the Bell inequality (6). The upper curve represents the highest possible probability, the lower curve the lowest possible probability of the anticoincidence \( A \neq B \), depending on the value of \( \nu \), and we see that both are decreasing with \( \nu \), i.e. as retro-causal effects get more pronounced. The shaded area in between thus corresponds to the range of possible probabilities that could result from the present model, depending on how SFP’s are fixed. The growing spread between \( P_{\text{min}} \) and \( P_{\text{max}} \) is due to the fact that with increasing \( \nu \), the effects of preinforcement become more and more relevant and self-fulfilling prophecies thus more and more likely.

We see that for \( \nu \equiv 0 \), i.e. in the absence of advanced interactions, the account reduces to the local model discussed in section 3. That is, \( P_{\text{min}} \) and
$P_{\text{max}}$ coincide (since $(A, B)$ now is a random variable on $S^2$) yielding, in particular, $P(A \neq B | \angle a, b = 120^\circ) = \frac{1}{3}$ and thus for $a = 0^\circ, b = 120^\circ, c = 240^\circ$:

$$P(A \neq B | a, b) + P(A \neq B | b, c) + P(A \neq B | a, c) = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1.$$  

Hence, the Bell inequality is, of course, satisfied, though it’s important to note that this case is already critical, i.e. that equality holds in eq. (6). For now, as we consider values of $\nu$ greater 0, we see that the statistical effect of the advanced interactions is to lower the probability of the anticoincidence $A \neq B$, thus leading to a violation of the Bell inequality. Notabene, the fact that the maximal probability $P_{\text{max}}$ already is smaller than $\frac{1}{3}$ shows that the violation of the Bell inequality is not because of self-fulfilling prophecies, i.e. is not achieved by exploiting the underdetermination of the outcome by the initial state, but would have to occur in any case, regardless of how this underdetermination is resolved and what the resulting statistical description would like in detail.

On a more quantitative note, we have to keep in mind that we’ve made a series of simplifications and assumptions along the way, so it may or may not be significant that we find the range of possible probabilities to be fairly close to the quantum mechanical prediction of 0.25 for reasonable values of $\nu$.

To settle on more precise predictions, we can consider the median of $P_{\text{min}}(A \neq B)$ and $P_{\text{max}}(A \neq B)$, corresponding to the probability of the anti-coincidence assuming that SFP is not biased between $A = B$ and $A \neq B$. These values are plotted in Figure 7 – now, for greater generality, with

![Figure 5: Possible values for $P(A \neq B | a = 0^\circ, b = 120^\circ)$.](image-url)
the parameters $\beta$ and $\gamma$ varying independently – to be compared with the predictions of the local model ($P = 1/3$) and of quantum mechanics ($P = 1/4$). One of the most interesting things to note about this plot is that it doesn’t look particularly interesting – the values are rather steady except for the upward slope as $\gamma$ and $\beta$ go to 0 and the downward slope as they become very large – indicating a remarkable robustness of the predictions of the time-symmetric model against the details of the interactions.

Finally, we note that there is no fixed choice of parameters $\beta$ and $\gamma$ for which this particular evaluation reproduces the predictions of quantum mechanics (for the spin-singlet state) for all values of $a$ and $b$, that is

$$\mathbb{P}(A \neq B \mid a, b) = \frac{1}{2} + \frac{1}{2} \langle a, b \rangle. \quad (11)$$

Of course, it would’ve been quite miraculous if it did, given the overall crudeness of our statistical hypothesis. An interesting question might thus be whether by taking into consideration the possible dependence of $\beta$ and $\gamma$ on the relevant physical variables, one could find a probability distribution $p(\beta, \gamma \mid S_0, a, b)$ for which the model reproduces eq. \([11]\) exactly.

In conclusion, what our analysis shows is that the quantum non-locality we observe in nature in form of statistical correlations violating Bell’s inequality could really be understood as the signature of retro-causal effects due to time-symmetric relativistic interactions, rather than instantaneous (or superluminal) influences between space-like separated events.
5 Retro-causality and Bell’s Theorem

Having seen that time-symmetric relativistic interactions can, in principle, account for the violation of the Bell inequality, it is very instructive to reflect on how exactly such a model would fit into the framework of Bell’s non-locality theorem. We recall that the most general derivation of a Bell inequality (more specifically the CHSH inequality [5, 29]) is based on two (and only two) assumptions:

i) The locality assumption: The statistical correlations

\[ P(A, B|a, b) \neq P(A|a)P(B|b) \]  

(12)

between the outcomes of the space-like separated measurement events in the EPR experiment are locally explainable. By Bell’s definition, a candidate theory provides a local explanation of the correlations if conditioning on all the physical data in the (common) past of \( A \) and \( B \) which, according to that candidate theory, could be relevant to the prediction of \( A \) and \( B \), will screen off the correlations (12), meaning that the specification of \( A \) and \( a \) becomes redundant for the prediction of the probability of \( B \), and vice versa. Formally, comprising all possible “common causes” of \( A \) and \( B \) in a set of variables \( \lambda \), the locality condition reads:

\[ P(A|B, a, b, \lambda) = P(A|a, \lambda) \]
\[ P(B|A, a, b, \lambda) = P(B|b, \lambda). \]  

(13)

ii) The no-conspiracy assumption: The explanation of the correlations must not be conspiratorial, meaning that the experimental parameters \( a \) and \( b \) can be chosen freely or randomly, independent of each other and of any other physical process that might be relevant to the system before measurement, hence independent of \( \lambda \). Formally:

\[ P(\lambda|a, b) = P(\lambda). \]  

(14)

Since our model violates the Bell inequality (as well as the CHSH inequality) it must violate at least one of these assumptions. In some discussions, a “retrocausal explanation” of the EPR correlations is understood virtually synonymous with a “conspiracy”. However, the description provided by our toy-model is, all things considered, reasonable enough to show that the issue
deserves a second look and indeed, on that second look things turn out to be a bit more subtle:

If, in our model, we condition the measurement outcomes on the relevant physical configurations in the common past of $A$ and $B$ – that is, on the particles’ initial spin-variables $\pm S_0$ – we find that the locality condition (13) (in form of “parameter-independence” [24]) is violated, i.e.

$$P(A|a, b, S_0) \neq P(A|a, S_0)$$
$$P(B|a, b, S_0) \neq P(B|b, S_0)$$

This initial data, in other words, is not sufficient to screen off the correlations between the measurement outcomes on one side of the experiment and the parameter settings on the other. Hence, a physicist seeking to explain the correlations between the distant events, as we usually do, by looking for a common cause in their past is bound to fail, and may reasonably conclude that there must be some sort of instantaneous influence between the two sides of the experiment. However, if we lived in a world guided, on the microscopic level, by time-symmetric relativistic laws, this physicist, in doing so, would literally miss half the story, since the physical laws were such that the outcomes of the spin measurements were actually determined by the physical configurations in both past and future of $A$ and $B$.

On the other hand, if we condition $(A, B)$ on the initial states $\pm S_0$ and the particle states after measurement – which, in a time-symmetric model like ours, can actually be regarded as “causing” the outcomes – the probability, trivially, factorizes and the locality condition is, formally, satisfied.

Note that no assumption about the localization of $\lambda$ actually enters the derivation of the CHSH inequality, though it is only in the case that $\lambda$ refers to configurations or events or quantities in the past of $A$ and $B$ that we...
would speak of a local explanation in Bell’s sense. Hence, while our model is non-local in Bell’s sense, it is local in the more general (and time-symmetric) sense that it involves no direct influences between space-like separated events.

In any case, from this point of view, our model violates the Bell inequalities – as it cannot be otherwise – by violating the no-conspiracy condition. Obviously, the physical variables screening off the correlations are not independent of the parameter settings, since they include the particle states post measurement which are collinear with the chosen orientations $a$ and $b$ of the measurement devices. However, we see no reason to deem such an account “conspiratorial” – at least not in the devastating sense argued by Bell [5, 8] and others [30] to essentially render futile the scientific enterprise. The fact that the state of the microscopic system after we have interacted with it reflects our experimental choices is hardly mysterious and in no way different from what we have anyway come to expect. More importantly, the advanced effects of the microscopic interactions do in no way infringe upon our freedom to choose the parameters of the experiment as we please (or make the choice completely random), nor on the possibility to prepare a system (it’s initial state, that is) according to a our liking and practical abilities. In other words, while formally violating the no-conspiracy condition, the account does not presuppose any dependence between the parameters associated with the preparation of the system and the parameters associated with the setup of the measuring apparatus. Hence, it involves no conspiracy.

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Appendix: Derivation of equation (10)

We want to compute the measure of the set \( \left\{ S \in S^2 \mid \langle a, S \rangle > c, \langle b, S \rangle > c \right\} \), for \( c > 0 \) and \( \langle a, b \rangle < 0 \). In spherical coordinates, the variable \( S \in S^2 \) is parameterized as

\[
S = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta), \quad \theta \in [0, \pi), \phi \in [0, 2\pi).)
\]

W. l. o. g, we can locate \( a \) and \( b \) in the \( x-y \)-plane and set

\[
a = (0, 1, 0); \quad b = (\sin \chi, \cos \chi, 0),
\]

where \( \chi \) is the angle between \( a \) and \( b \) and hence \( \langle a, b \rangle = \cos \chi \). We thus have

\[
\langle a, S \rangle = \sin \theta \sin \phi>c \quad \langle b, S \rangle = \sin \theta (\sin \chi \cos \phi + \cos \chi \sin \phi)>c.
\]

Since the set is symmetric under interchange of \( a \) and \( b \), it suffices to consider the case \( \langle b, S \rangle > \langle a, S \rangle \) (and then double the measure).

Since the set is symmetric under reflection on the \( x-y \)-plane, it suffices to consider the case \( \theta < \frac{\pi}{2} \), i.e. \( S_z > 0 \) (and then double the measure).

Then we have, for once,

\[
0 < \langle a, S \rangle < \langle b, S \rangle \iff 0 < \sin \phi < \sin \chi \cos \phi + \cos \chi \sin \phi
\]

which yields \( \cos \phi > 0 \) and, after a little bit of algebra,

\[
0 < \sin \phi < \sqrt{\frac{1}{2}(1 + \cos \chi) = \sqrt{\frac{1}{2}(1 + \langle a, b \rangle)} =: d.}
\]

And we compute for \( M := \{ S \in S^2 \mid \langle b, S \rangle > \langle a, S \rangle > c, S_z > 0 \} \subset S^2 \):

\[
|M| = \int_0^{\pi/2} \int_0^{2\pi} 1_M(\theta, \phi) \sin \theta \, d\theta \, d\phi = -\int_0^{2\pi} \int_0^1 1\{\sin \theta < \frac{c}{\sin \phi}, \sin \phi < d, \phi \in (0, \pi/2)\} \, d\cos \phi \, d\phi
\]

\[
= \int_0^{\pi/2} \int_0^d 1\{c < \sin \phi < d\} \sqrt{1 - \frac{c^2}{\sin^2 \phi}} \, d\phi = \int_c^d \sqrt{\frac{z^2 - c^2}{z^2 - z^4}} \, dz,
\]

where in the last step we substituted \( z := \sin \phi \). Together with \( |S^2| = 4\pi \), equation (10) follows.
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