Analytical and Monte Carlo study of two antidots in magnetic nanodisks with vortex-like magnetization

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Abstract

How stable vortex-like magnetization in magnetic nanodisks with small aspect ratio ($L/R \ll 1$) is affected by two antidots is investigated analytically and by Monte Carlo simulations. For suitable ranges of the physical parameters this vortex presents bistable states when pinned around the antidots. The hysteresis loop obtained shows a central loop associated to the pinning mechanism of the vortex at the antidots. Our results agree qualitatively well with those provided by experiments and micromagnetic simulations.
Recently, ferromagnetic disk-shaped nanostructures with submicrometer lateral dimension (thickness) \( L < 1 \mu m \) have been fabricated and investigated for their potential applications in a number of magnetoelectronic mechanisms. In particular, it has been observed that above the so-called single-domain limit, magnetic vortex states appear in these samples, exhibiting a planar-like arrangement of spins outside the core, where a perpendicular magnetization is observed[1]. As long as one could manipulate these states other possibilities would emerge. In fact, one way towards this control is obtained by removing some small portions of the magnetic nanodisk, in such a way that the cavities (antidots) so created work by attracting and eventually pinning the vortex around themselves \[2\] \[3\] \[4\] \[5\] \[6\] \[7\]. Based upon such an idea, Rahm and coworkers \[8\] have studied the cases of two, three and four antidots (each of them with diameter \( \sim 85 \text{ nm} \)) inserted in a disk with diameter \( \sim 500 \text{ nm} \), separated by around \( 150 \text{ nm} - 200 \text{ nm} \). Their experimental results confirmed the previous statement about vortex pinning and put forward the possibility of using these stable states as serious candidates for magnetic memory and logical applications as long as we could control vortex position, for example, applying a suitable external magnetic field which should shift the vortex core from one defect to another, and vice-versa. Basic logical operations have been obtained by means of bistable magnetic switching[9]. Although experimental results are provided for this system a suitable analytical analysis is still lacking. The latter is important, for instance, to provide the basic physics behind the mechanism of vortex pinning and switching processes, giving the relevant parameters for a better control of possible applications and also indicating the limitations. Our present analytical model and Monte Carlo (MC) simulations have been able of capturing the basics involved in this problem, namely, how the vortex experiences the effects of the defects and how hysteresis loops are sensitive to the latter.

Let us starting by considering a magnetic dot represented by a small cylinder of radius \( R \) and thickness \( L \) (so that its aspect ratio \( L/R < < 1 \)). In addition, we shall assume that along the axial direction (\( z \)-axis), the magnetization \( \vec{M} \) is uniform. Furthermore, if we introduce \( N \) isolated holes (each of them with height \( L \) and radius \( \rho << R \)) in the dot, the total magnetic energy of the nanodisk can be approximated, in the continuum limit, by

\[
E_{\text{mag}} = \frac{L}{2} \int \int_{D} \left[ A(\partial_{\mu} \vec{m}) \cdot (\partial^{\mu} \vec{m}) - M_s^2 \vec{m} \cdot (\vec{h}_m + 2\vec{h}_{\text{ext}}) \right] \prod_{i=1}^{N} U_i(\vec{r} - \vec{d}_i) \, d^2 r , \tag{1}
\]

where \( A \) is the exchange coupling, \( D \) is the area of the cylinder face, \( \vec{m} = \vec{M}/M_s \) is an unity vector describing magnetization along \( D \) (with \( M_s \) being the saturation magnetization), \( \vec{h}_m = \vec{h}_m(\vec{m}) \equiv \vec{H}_m/M_s \) is the demagnetizing field, \( \vec{h}_{\text{ext}} \) is an applied magnetic field (Zeeman term) and \( \mu = 1, 2 \). The potential \( U_i \) in turn, brings about the effect of the antidots distributed throughout the nanodisk, say, \( \prod_{i=1}^{N} U_i(\vec{r} - \vec{d}_i) = U_1(\vec{r} - \vec{d}_1) U_2(\vec{r} - \vec{d}_2) \ldots U_N(\vec{r} - \vec{d}_N) \), with

\[
U_i(\vec{r} - \vec{d}_i) = \begin{cases} 0 & \text{if } |\vec{r} - \vec{d}_i| < \rho \\ 1 & \text{if } |\vec{r} - \vec{d}_i| \geq \rho \end{cases} . \tag{2}
\]
Therefore, the system of a dot with \( N \) isolated antidots may be viewed as a cylinder of radius \( R \) and thickness \( L \) with \( N \) smaller cylindrical cavities with radius \( \rho \ll R \), each of them centralized at \( \vec{d}_i \). Here, we shall study explicitly the case \( N = 2 \) (the treatment for \( N > 2 \) may be performed in the same way). Experimental and numerical results are available for very similar systems, say, disks with \( L \sim 30 \text{ nm} \), \( R \sim 500 \text{ nm} \) whose antidots (up to four) have diameters \( 2\rho \sim 80 \text{ nm} \) separated by \( 150 \text{ nm} - 200 \text{ nm} \) \[8\].

Now, let us consider a cylindrically symmetric vortex-like magnetization throughout a dot, say, with the vortex core centralized at \( \vec{r} = \vec{0} \). This is the ground state of a nanodisk in the absence of holes and external magnetic fields. For that, it is convenient to write \( \vec{m} = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta) \), with \( \theta = \theta_v(r) \) and \( \varphi = \arctan(y/x) \pm \pi/2 \). The function \( \theta_v(r) \) may be approximated by \( \sin \theta_v(r) = 0 \) in the dot center (\( \vec{r} = \vec{0} \)), while \( \sin \theta_v(r) \rightarrow 1 \) far away the center, \( |\vec{r}| = r >> a \) (\( a = \sqrt{A}/M_s \) is the unit-cell element size or the exchange length; for most magnetic materials \( a = 5 - 6 \text{ nm} \)). In words, the magnetization consists of a small core where spins display out-of-plane components for regularizing the exchange energy, and an outer region where spins are practically confined to the dot plane face. In this case, the magnetic superficial charges in the lateral face of the dot and the magnetic volumetric charges (\( \nabla \cdot \vec{m} \)) identically vanish yielding no contribution to \( \vec{h}_m \). The antidots affect this picture as follows: from the point of view of the exchange term, antidots lead to a less exchange energy for the vortex. Then, the topological structure is attracted by the hole suffering modifications in its profile. In addition, the distribution of magnetic charges throughout the internal edges of the cavities (holes) and mainly along the external lateral face of the cylinder (whenever the antidot is not at the geometrical center of the dot) increase the magnetostatic energy due to a change in the product \( \vec{m} \cdot \vec{n}_s \) (\( \vec{n}_s \) are unit vectors normal to external lateral surface of the disk and internal surfaces of the cavities). Thus, the demagnetizing field, \( \vec{h}_m \), can be obtained from its associated potential \( \Phi_{m} = \Phi_V + \Phi_{edge} + \Phi_{edge}^i \), in the usual way, \( \vec{h}_m = -\nabla \Phi_m \). Here, \( \Phi_V \) is the magnetostatic potential related to the volumetric charges, while \( \Phi_{edge} \) and \( \Phi_{edge}^i \) comes about from the surface charges on the external and internal (holes) edges, respectively. The contributions of the volumetric potential can be neglected since the approximations considered above leads to \( \nabla \cdot \vec{m} = 0 \).

For simplicity, the antidots centers are assumed to lie along a straight line that crosses the origin of the dot, say, at \( d_1 \) and \( d_2 \), as shown in Figure [1]. We also assume that the vortex displacement \( \vec{l} \) from its equilibrium position is not too large, so that it experiences no appreciable change in its profile (‘rigid’ vortex behavior \[10\]). Thus, the exchange potential experienced by the vortex may be estimated as \[11\]

\[
V_{ex}(s, d_1, d_2) \simeq \frac{\pi A}{2} \ln \left[ (1 - s^2) (1 - f_1 - f_2 + f_1 f_2) \right], \quad (3)
\]

where \( s = \vec{l}/R \), supposed to be small (\( |s| \ll 1 \)), measures the relative shift of the vortex from the dot center; the functions \( f_i \) are defined as \( f_i = a^2/|\vec{d}_i - \vec{s}R|^2 + b^2 \), where \( b \equiv 1.147\rho \) is a constant introduced to avoid spurious divergences whenever the vortex is centralized at one of the defects \[5\] \[7\]. Note that each \( f_i \) is related to the attractive potential that each isolated hole induces on the vortex, while the product \( f_1 f_2 \) accounts for a competition between them \[11\]. Thus, if our system were large
enough, the vortex would displace towards one of the holes as shown in Ref. [11]. In a small magnet, things are much more interesting once the vortex experiences a modification in its profile and magnetization is no longer cylindrically symmetric. Then, the magnetostatic energy increases and a restoring force appears in order to pull the vortex back to the dot center. Indeed, for small displacements of the vortex we may estimate this energy shift analytically, like below [5, 10]

\[
V_{\text{mag}}(s, \vec{d}_1, \vec{d}_2, \rho) \cong 2\pi^2 M_s^2 (R^2 - 2\rho^2) \left\{ F_1 \left( \frac{L}{R} \right) s^2 + F_1 \left( \frac{\rho}{R} \right) \left[ \alpha (s - |\vec{d}_1|)^2 + \beta (s - |\vec{d}_2|)^2 + \gamma \left( \frac{\rho}{R} \right)^2 \right] \right\},
\]

in which the first term is related to the contribution of the disk envelop, while the remaining ones are associated to the pinning of the vortex by one of the defects. In addition, \( F_1(\xi) = \int_0^\infty \frac{J_1(t)}{t} \left( 1 - \frac{1-e^{-\xi t}}{\xi t} \right) dt \), where \( J_1(t) \) is the Bessel function. The possible three distinct situations are: i) the vortex is centered at the antidot 1, then \( \alpha = 1 \) and \( \beta = \gamma = 0 \); ii) it is centered at the antidot 2, thus \( \alpha = \gamma = 0 \) while \( \beta = 1 \); iii) the vortex is not pinned at any defect, what yields \( \alpha = \beta = 0 \) and \( \gamma = 1 \).

Figure 2 shows how the effective potential, \( V_{\text{eff}} = V_{\text{ex}} + V_{\text{mag}} \), behaves as function of \( s \). Bistable states corresponding to the vortex-antidot pinned configurations appear for suitable values of the parameters. Our analytical model predicts that three parameters are the most relevant in this case: the radius of each antidot \( \rho \), their center-to-center separation, \( D \equiv |\vec{d}_1 - \vec{d}_2| \), and the (characteristic) exchange length \( a = \sqrt{A/M} \). We have observed that the appearance of bistable states in \( V_{\text{eff}} \) are generally associated to larger values for \( \rho \) and \( a \) and to smaller \( D \), as illustrated in Fig. 2. Actually, it shows that our analytical results qualitatively agree with those obtained in experiments of Ref. [8], but there the bistable states are observed even for larger separations (\( \sim 200 \) nm) than those fitted by the present analysis (\( \sim 130 \) nm). This discrepancy is related to the small displacements of the vortex core assumed by our model.

In the presence of the two holes the vortex equilibrium position, \( s_0 = (x_0, y_0) \), can be easily determined by evaluating \( dV_{\text{eff}}/ds = 0 \). Clearly, depending on the relevant parameters, the geometry of our system implies that \( s_0 \) will always correspond to \( (|x_0| \geq 0, y_0 = 0) \) so that the general expression for the local magnetization along the dot face now reads:

\[
\begin{align*}
    m_x(x, y, s_0) &= \frac{\pm y}{\sqrt{(x - s_0 R)^2 + y^2}}, \\
    m_y(x, y, s_0) &= \frac{\pm (x - s_0 R)}{\sqrt{(x - s_0 R)^2 + y^2}},
\end{align*}
\]

where the upper (down) signs are associated to counterclockwise (clockwise) magnetizations. Since the system is antisymmetric under reflection against \( x \)-axis, the average magnetization along this axis vanishes, while the \( y \) component may be estimated like below (the factor 2 in the 2nd term accounts for 2 antidots):

\[
\langle M_y \rangle \cong M_s \left[ \frac{1}{\pi R^2} \int_{\text{disk}} m_y(x, y, s_0) \, dx \, dy - \frac{2}{\pi \rho^2} \int_{\text{antidots}} m_y(x, y, s_0) \, dx \, dy \right].
\]
In addition, if an external homogeneous magnetic field is applied along the $y$-axis, $\vec{h}_{\text{ext}} = \vec{H} / |\vec{M}_s| = h_y \hat{y}$, then, in the lowest order, $V_{\text{eff}}$ must be augmented by:

$$V_{\text{ext}}(s) = -\pi M_s^2 h_y (R^2 - \rho^2) (s - s_0),$$

(7)

where $s$ is the shift in the vortex center position caused by the field. So the total potential now reads $V_{\text{total}} = V_{\text{eff}} + V_{\text{ext}}$. In the presence of $h_{\text{ext}}$ the vortex equilibrium position is now $s_h \neq s_0$. For example, if one takes the counterclockwise magnetization (upper signs in eq. (5)), then for $h_y > 0$ ($< 0$) the vortex will be shifted to the left (right) of $s_0$. Thus, the equilibrium position of the vortex center in the presence of the applied external field, $s_h$, is simply a small shift of $s_0$ along the $x$-axis. Therefore, in this situation $\langle M_x \rangle = 0$ while $\langle M_y \rangle$ is calculated from eq. (6) with $m_y(x, y, s_h)$ instead of $m_y(x, y, s_0)$. How magnetization behaves as $h_{\text{ext}}$ is varied gives the hysteresis loop left by the vortex motion under such conditions.

Figure \textbf{3} displays the hysteresis loops for some values of the relevant parameters when an alternating-like magnetic field is applied along the $y$-axis. The central loop shown in this figure, on the left, is in qualitative agreement with the experimental results of Refs.\cite{8, 9}. In these works, such a mechanism was observed for larger cavities radii and separations ($\rho \sim 40$ nm and $D \sim 200$ nm). In our analysis, we could observe a similar fact only for much smaller values of these parameters. Again, such a discrepancy comes about once our model is strictly valid for small $s$. The central loop, taking place at magnetic fields weaker than that for vortex annihilation, manifests the most important physical feature of the system: the existence of two metastable vortex states with the equilibrium position of the vortex center pinned at each antidot. There, the jumps are due to vortex core switching from one state to another while the plateaux corresponds to pinned vortex states. The mechanism of switching can be explained in the following way: in the situation shown in Fig. \textbf{2(a)} the vortex core can be initially pinned in equilibrium at one of the antidots, for instance, at the left antidot. However, this configuration may be perturbed if a strong enough external magnetic field is applied, say, along the $y$-axis, so that the vortex core is shifted to the another antidot. Inverting the magnetic field the vortex comes back to the initial configuration, but leaving a hysteresis loop in the complete round. The plateaux observed in the central loop of experimental data has a pronounced inclination respective to the magnetization axis, which may be associated to the deformation of the vortex profile on the dot face. Consequently, our preceding approach could not fit this fact. At the attempt of understanding this mechanism, we have performed MC simulations in order to improve the range of validity of our former results. For that, we consider the xy-model on a finite flat disc (of radius $R$) supplemented by a strong enough surface border anisotropy. Although it cannot be considered as the actual magnetostatic energy we expect that it could imitate its role, say, increasing the total energy as long as the spins develop normal components at the borders of the nanodisk and of the holes (2nd and 3rd terms below; see Ref.\cite{12}). Indeed, this anisotropy has the effect of keeping the original cylindric-like profile of the vortex near these borders although allowing it to deform in other regions along the dot face. Therefore, Hamiltonian reads:
\[ H = -J \sum_{i,j} (S^x_i S^x_j + S^y_i S^y_j) + B \sum_{k \in \text{disk border}} (\vec{S}_k \cdot \hat{m}_k)^2 + B \sum_{h \in \text{holes border}} (\vec{S}_h \cdot \hat{m}_h)^2, \]  

(8)

where \( J > 0 \) is the exchange ferromagnetic integral, \( \vec{S}_i = (S^x_i, S^y_i, S^z_i) \) are the classical spin vectors specified at the lattice sites \( i \) and the first summation is over nearest-neighbor spins. The other term represents the surface anisotropy where the sum over \( k \) includes only the border sites of the disk and sum over \( h \) includes only the border sites of the holes. Consequently, the unit vectors \( \hat{m}_k \) are perpendicular to the circumference envelop of the disk and the unit vectors \( \hat{m}_h \) are perpendicular to the circumference border of the holes. There, \( B \) is a constant of single-ion surface border anisotropy so that for \( B > 0 \) \((<0)\) the border spins \( \vec{S}_k \) and \( \vec{S}_h \) tend to lie perpendicular \( (\)parallel\) to \( \hat{m}_k \) and \( \hat{m}_h \) direction respectively. Of course, here we chose the case \( B > 0 \). Among other features, such a term enables vortex deformation on the dot face whereas keep its configuration at the border \( (\)for a fixed \( B)\), so that we can now fit the inclination of the central loop discussed above.

In our MC simulations for hysteresis loop we have adopted Metropolis algorithm \[13\] with the initial configuration of centered vortex in disks with diameters \( 2R = 40a_0 \) \( (a_0 \) is the distance between two spins in a discrete lattice inside the disk). In addition, we have introduced two circular holes \( \)regions with vacancy of spins) where we chosen radius \( \rho = 2a_0 \) and center-to-center separation \( D = 10a_0 \), in such a way that \( \rho/D \sim 0.2 \) is close to that from experiments \( (\)see Refs.\[8,9\]). Besides, we have taken the temperature \( T/J = 0.05 \) and the parameter \( B/J = 0.03 \). This is the critical value for \( B/J \) so that, above it, non-centered vortices are energetically favorable in these disks \( (\)see Ref.\[12\]). Figure 4 shows a typical central hysteresis loop as the vortex center is switched from one antidot to another and back again \( (\)note particularly that its inclination is in good qualitative agreement with experiments of Ref.\[8\]).

In conclusion, our analytical as well as MC calculations have shown how two holes incorporated into the body of a magnetic nanodisk attract the remanent vortex-like magnetization to their centers, creating the possibility of bistable states of vortex-hole pinned configurations, as observed in experiments. The analytical model for the effective potential \( (\)exchange + magnetostatic\) have yielded to this picture in good agreement with experimental findings. However, concerning the hysteresis loops, the analytical calculations show only a qualitative concordance with experiments. Indeed, the appearance of a central loop in these curves, related to the switching of the vortex core between the two stable states is verified here for values of \( \rho \) and \( D \) very smaller than those considered in experiments. In other words, our results in this case would lead to a good agreement with experiments only for very small displacements of the vortex core around its equilibrium position, \( s_h \) \( (\)'rigid' vortex regime). It implies in small distance \( D \) between the holes in a nanodisk. For improving this scenario \( (\)considering as much as possible generic distances) we have also performed Monte Carlo simulations which take into account the deformation of the vortex and effects due to the border. Our simulations for the hysteresis loop are in agreement to the experimental results and exhibits bistable magnetic switching mechanisms. The approach developed here
also predicts the possibility of gyration of the vortex core about one of the three equilibrium positions (see Fig. 2) with characteristic frequencies that depend on the relevant parameters $R$, $\rho$ and $D$. The theoretical study of a quite recent observed gyrotropic frequency of magnetic vortex around antidots in nanosized structures\[14\] is under investigation and will be communicated elsewhere.

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1 Figure Captions

Figure 1: Top (left) and lateral (right) views of a nanodisk with radius $R$, containing two antidots lying along the $x$-axis, whose centers are $\vec{d}_1$ and $\vec{d}_2$ apart from the disk center.

Figure 2: Typical plots of $V_{eff}/A$ as a function of $s$. Here, we have taken $R = 250\,\text{nm}$, $L = 30\,\text{nm}$, $\rho = 43\,\text{nm}$, and $a = \sqrt{A}/M_s = 5.7\,\text{nm}$ (values considered in Ref. [9]). The defects are placed along $x$-axis at the positions: (solid curve) $\vec{d}_1 = \vec{d}_2 = -50\,\text{nm}$; (dashed curve) $\vec{d}_1 = -\vec{d}_2 = -80\,\text{nm}$. Note that the bistable states tend to disappear as long as the distance between the cavities becomes larger.

Figure 3: The central loop associated to the switching of the vortex between the two antidots (bistable states) is jeopardized as long as $D$ increases and/or $a$ gets lower. External loops (not depicted) were associated to the creation/annihilation of the vortex. (a) $|\vec{d}_1 - \vec{d}_2| \equiv D = 40\,\text{nm}$, $\rho = 10\,\text{nm}$, and $a = \sqrt{A}/M_s = 17\,\text{nm}$; (b) $D = 40\,\text{nm}$, $\rho = 10\,\text{nm}$, and $a = 5.7\,\text{nm}$.

Figure 4: Hysteresis loop for a disk with two antidots with radius $\rho = 2a_0$ and center-to-center separation $D = 10a_0$ obtained by Monte Carlo simulations.
2 Figures

Figure 1:

Figure 2:
