The spreading kinetics of Herschel-Bulkley fluids on horizontal solid substrates were theoretically studied. The equations of film thickness were derived in both gravitational and capillary regimes. The dynamic contact angle for the capillary regime was also derived. Finally, a limiting result for the case of $\tau_0 = 0$ was obtained, which was compared with the known solution for validation. The results show that the yield behavior of the fluids had a significant impact on the spreading kinetics in both cases. Only when stress was larger than the yield stress, would substantial flow occur. The spreading zone was divided into two parts by the yield surface: sheared zone and yield zone, which was completely different from common Newtonian fluids or power-law fluids. The thickness of the yield zone mainly depended on yield stress and pressure gradient along the $z$-direction. According to the final evolution, both the film thickness and dynamic contact angle were affected not only by the power-law index but also by the yield behavior.

Keywords: spreading, Herschel-Bulkley, film thickness, dynamic contact angle, non-newtonian
to the analytical approach of the spreading of power-law fluids, the film thickness equations are derived in both gravitational and capillary regimes and the dynamic contact angle is also discussed.

ASSUMPTIONS

To simplify the problem, the following assumptions are made [18, 19]. 1) The spreading process belongs to the complete wetting case. 2) The fluids are incompressible, which will ensure the Navier-Stokes equation is workable. 3) The gas viscosity is ignored. 4) The film is much thinner than the horizontal length, so the flow can be converted into 2D. 5) The Reynolds number is small enough to ignore inertial influence. And 6) complete wetting is applied here to ensure a small contact angle.

THEORY

The Constitutive Model of Herschel-Bulkley Fluids

The constitutive model of Herschel-Bulkley fluids is similar to Bingham fluids or power-law fluids. The specific constitutive equation is given as follows:

$$\begin{cases} \tau = \tau_0 + k\dot{\gamma}^n, & |\tau| > \tau_0 \\ \dot{\gamma} = 0, & |\tau| \leq \tau_0 \end{cases} \quad (1)$$

where $\tau$ is shear stress, $k$ is viscosity coefficient, $\dot{\gamma}$ is the shear rate, $n$ is the power-law index, and $\tau_0$ is the initial yield stress.

Thin Film Equation

The spreading schematic of the advancing system of Newtonian or power-law fluids is shown in Figure 1. The derivation of the Herschel-Bulkley case was also started in Figure 1. In Figure 1, $u$ is the horizontal velocity of the fluid, which is affected by time $t$ and position $x, z$. $U$ is the framing moving velocity of the contact line, $\theta(U)$ is the dynamic contact angle, and $h$ is the height of film thickness of any position $x$.

With the proposed assumption (3), the pressure along the $z$-direction will meet the following equation.

$$\frac{\partial p}{\partial z} = -\rho g \quad (2)$$

Applying the Young-Laplace equation to the free surface ($z = h$), there is

$$p = p_G + \sigma \left( \frac{1}{R_1} + \frac{1}{R_2} \right) = p_G - \sigma \frac{\partial^2 h}{\partial x^2} \quad (3)$$

where $p_G$ represents the atmospheric pressure and $\sigma$ is surface tension.

Integrating Eq. (2) with respect to the boundary condition listed in Eq. (3), the pressure can be further calculated as

$$p = \rho g (h - z) + p_G - \sigma \frac{\partial^2 h}{\partial x^2} \quad (4)$$

When $z = 0$, the stress on the horizontal substrate can be obtained from Eq. (4) as

$$\tau_s = \rho g h + p_G - \sigma \frac{\partial^2 h}{\partial x^2} \quad (5)$$

Consequently, when the initial yield stress $\tau_0$ is larger than $\tau_s$, substantial flow will not occur. Thus the approximate condition for substantial flow is

$$|\tau_0| < \rho g h + p_G - \sigma \frac{\partial^2 h}{\partial x^2} \quad (6)$$

Substantial flow will be discussed below. Owing to the initial yield stress, there must be a yield surface at $z = h_0$. Thus the spreading zone will be divided into two zones, the significant
shear exists only below the yield surface. The shear rate is zero at or above the yield surface. The viewpoint is consistent with the spreading of Bingham fluids proposed by Liu [20]. Therefore, the actual flow schematic of the advancing systems of Herschel-Bulkley fluids is shown in Figure 2. The dashed line separates the zone into sheared and yield zones respectively. The schematic figure is completely different from the Newtonian or power-law case.

For the Navier–Stokes equation, it can be written as

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial x} \left( \mu \frac{\partial u}{\partial x} \right)$$

(7)

where \( p \) is the pressure and \( x \) is the distance of one point from the \( z \)-axis. According to Eq. (1), when the stress is larger than the yield initial stress, the viscosity can be expressed as

$$\mu = \frac{\tau_0}{|\partial u/\partial x|} + k |\partial u/\partial z|^{\nu-1}$$

(8)

Substituting Eq. (8) into Eq. (7), there is

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial x} \left( \frac{\tau_0}{|\partial u/\partial x|} + k |\partial u/\partial z|^{\nu-1} \right) \frac{\partial u}{\partial x}$$

(9)

Based on the boundary condition of no shear at the yield surface \((h(x) = h_0, \partial u/\partial x = 0)\), integrating Eq. (9) with respect to \( z \) and the following equation can be acquired.

$$\frac{\partial u}{\partial z} = \text{sign} \left( \frac{\partial p}{\partial x} \right) \left( \frac{\tau_0}{|\partial u/\partial x|} + k |\partial u/\partial z|^{\nu-1} \right) (h_0 - z)^{1/\nu}$$

(10)

With the boundary condition of no-slip at the solid surface \((z = 0, u = 0)\), the expression of \( u \) can be obtained by integrating Eq. (10).

$$u = \frac{n}{n + 1} \left( \frac{1}{k} \right)^{1/\nu} \text{sign} \left( \frac{\partial p}{\partial x} \right) \left| \frac{\partial p}{\partial x} \right| (h_0^{\nu} - (h_0 - z)^{\nu})$$

(11)

Equation (11) shows that the maximum velocity \( u_p \) occurs at the yield surface \( z = h_0 \) that can be obtained as

$$u_p = \frac{n}{n + 1} \left( \frac{1}{k} \right)^{1/\nu} \text{sign} \left( \frac{\partial p}{\partial x} \right) \left| \frac{\partial p}{\partial x} \right| h_0^{\nu}$$

(12)

When \( z > h_0 \), the velocity is still equal to the maximum velocity \( u_p \). According to mass conservation, there is

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0$$

(13)

where \( q \) is the flow of fluids.

For Herschel-Bulkley fluids, \( q \) is made up of two parts as

$$q = \int_0^{h_0} uz \, dz + u_p (h - h_0)$$

(14)

The first term of the right side represents the shear zone below the yield surface and its detailed expression is

$$\int_0^{h_0} uz \, dz = \frac{n}{2n + 1} \text{sign} \left( \frac{\partial^3 h}{\partial x^3} - \rho g \frac{\partial h}{\partial x} \right) \left( \frac{k}{1/\nu} \right)^{1/\nu} \left( \frac{\partial^3 h}{\partial x^3} - \rho g \frac{\partial h}{\partial x} \right) h_0^{\nu}$$

(15)

Then the flow \( q \) will be acquired as follows:

$$q = \frac{n}{2n + 1} \text{sign} \left( \frac{\partial^3 h}{\partial x^3} - \rho g \frac{\partial h}{\partial x} \right) \left( \frac{k}{1/\nu} \right)^{1/\nu} \left( \frac{\partial^3 h}{\partial x^3} - \rho g \frac{\partial h}{\partial x} \right) h_0^{\nu} + \frac{n}{n + 1} \text{sign} \left( \frac{\partial^3 h}{\partial x^3} - \rho g \frac{\partial h}{\partial x} \right) \left( \frac{k}{1/\nu} \right)^{1/\nu} \left( \frac{\partial^3 h}{\partial x^3} - \rho g \frac{\partial h}{\partial x} \right) h_0^{\nu} (h - h_0)$$

(16)

In Fig. 2, the velocity along the x-direction is parabolic, which can also be explained by Eqs. (11)–(12). When it refers to the velocity, the case here is similar to classical Poiseuille flow. Both of them are driven by the pressure gradient, and they have the same initial and boundary conditions, thus the relationship between \( h \) and \( h_0 \) can be obtained based on our previous work [21–22] as

$$h_0 = h - \frac{\tau_0}{\rho g} = h - \frac{\tau_0}{\rho g}$$

(17)

Substituting Eq. (17) into Eq. (16), a new formula of flow \( q \) is generated as

$$q = \frac{n}{2n + 1} \text{sign} \left( \frac{\partial^3 h}{\partial x^3} - \rho g \frac{\partial h}{\partial x} \right) \left( \frac{k}{1/\nu} \right)^{1/\nu} \left( \frac{\partial^3 h}{\partial x^3} - \rho g \frac{\partial h}{\partial x} \right) h_0^{\nu} + \frac{n}{n + 1} \text{sign} \left( \frac{\partial^3 h}{\partial x^3} - \rho g \frac{\partial h}{\partial x} \right) \left( \frac{k}{1/\nu} \right)^{1/\nu} \left( \frac{\partial^3 h}{\partial x^3} - \rho g \frac{\partial h}{\partial x} \right) h_0^{\nu} (h - h_0)$$

(18)

Finally, substituting Eq. (18) into Eq. (13), the following equation is obtained as

$$\frac{\partial h}{\partial t} + \frac{n}{2n + 1} \text{sign} \left( \frac{\partial^3 h}{\partial x^3} - \rho g \frac{\partial h}{\partial x} \right) \left( \frac{k}{1/\nu} \right)^{1/\nu} \left[ \frac{\partial^3 h}{\partial x^3} - \rho g \frac{\partial h}{\partial x} \right] h_0^{\nu} = 0$$

(19)

According to Eq. (16), the pressure mainly depends on the gravity and capillary forces. In the following part, two limiting regimes will be considered further. One is only the gravitational regime taken into consideration by ignoring the capillary force, \( \rho g R^2 \gg \sigma \). The other is only the capillary force regime considered, ignoring the gravity, \( \sigma >> \rho g R^2 \).
Gravitational Spreading Regime

The capillary force is ignored here, so Eq. (19) can be simplified into the following equation.

$$\frac{\partial h}{\partial t} + \text{sign}\left( -\rho g \frac{\partial h}{\partial x} \right) k \frac{\partial}{\partial x} \left( \sigma \frac{\partial h}{\partial x} \right) h_0^2 \left( \frac{n}{2n+1} h_0 + \frac{n}{n+1} \tau_0 \right) = 0$$

(20)

When the contact line moves at velocity $U$, the above equation can be converted into the following form by introducing a new variable $\xi = x - Ut$.

$$-U \frac{dh_0}{d\xi} + \left( \frac{\rho g}{k} \right)^{1/2} d \left( \frac{dh_0}{d\xi} \right)^{1/2} h_0 \left( \frac{n}{2n+1} h_0 + \frac{n}{n+1} \tau_0 \right) = 0$$

(21)

Integrating the above equation with respect to $\xi$, there is

$$U^n = \left( \frac{n}{2n+1} \right)^{n \rho g} \left( \frac{h_0 + 2n + 1 \tau_0}{n + 1 \rho g} \right)^n$$

(22)

With the boundary condition: $\xi = 0$, $h = 0$, Eq. (22) can be solved as

$$\frac{1}{n+2} \left[ \left( h + \frac{n}{n+1} \frac{\tau_0}{\rho g} \right)^{n+2} - \left( \frac{n}{n+1} \frac{\tau_0}{\rho g} \right)^{n+2} \right] - \frac{2n+1}{n+1} = \frac{\tau_0}{\rho g} \left( h + \frac{n}{n+1} \frac{\tau_0}{\rho g} \right)^{n+1} - \left( \frac{n}{n+1} \frac{\tau_0}{\rho g} \right)^{n+1}$$

$$= \left( \frac{2n+1}{n+2} \right)^{n} k \frac{\tau_0}{\rho g} \xi$$

(23)

Equation (23) addresses the fact that film thickness in capillary spreading is determined by initial yield stress $\tau_0$ and power index $n$.

Capillary Spreading Regime

In the condition, the gravitational action is ignored, then Eq. (19) can be simplified as follows

$$\frac{\partial h}{\partial t} + \text{sign}\left( -\rho g \frac{\partial h}{\partial x} \right) k \frac{\partial}{\partial x} \left( \sigma \frac{\partial h}{\partial x} \right) h_0 \left( \frac{n}{2n+1} h_0 + \frac{n}{n+1} \tau_0 \right) = 0$$

(24)

Introducing the above variable $\xi$ and $h_0$ into Eq. (24),

$$-U \frac{dh_0}{d\xi} + \text{sign}\left( \sigma \frac{d^2 h_0}{d\xi^2} \right) \left( \frac{1}{k} \right)^{1/2} d \left( \frac{d^2 h_0}{d\xi^2} \right)^{1/2} h_0 \left( \frac{n}{2n+1} h_0 + \frac{n}{n+1} \tau_0 \right) = 0$$

(25)

Integrating the above equation with respect to $\xi$,

$$-U + \frac{n}{2n+1} \left( \frac{1}{k} \right)^{1/2} \sigma \frac{d^2 h_0}{d\xi^2} \left( h_0 + \frac{2n+1}{n} \frac{\tau_0}{\rho g} \right) = 0$$

(26)

Then it is supposed that $\frac{\tau_0}{\rho g} = \lambda h_0$, so Eq. (26) can be translated into

$$U^n = \left[ \frac{n}{2n+1} \frac{(2n+1)(n+1)}{n+1} \right]^{n} \sigma \frac{d^2 h_0}{d\xi^2} h_0^n$$

(27)

With the boundary condition: $\xi = 0$, $h = 0$, Eq. (27) can be solved as

$$h_0^n = \frac{k}{\sigma} \left[ \frac{U}{n(n(2n+1)+\lambda+1))} \right]^{n} \left( \frac{n+2}{3(2n+1)n-1} \right)^{\frac{n}{3}}$$

(28)

Further, the film thickness equation can be obtained by replacing $h_0$,

$$\left( h - \frac{\tau_0}{\rho g} \right)^{n+1} = \frac{k}{\sigma} \left[ \frac{U}{n(n(2n+1)+\lambda+1))} \right]^{n} \left( \frac{n+2}{3(2n+1)n-1} \right)^{\frac{n}{3}}$$

(29)

Based on Eq. (17), the variable $\lambda$ can be calculated as

$$\lambda = \frac{\tau_0}{\rho g} \frac{h - \tau_0}{\rho g}$$

(30)

In this way, the equation of film thickness $h$ can be obtained, which depends on the power index $n$ and initial yield stress $\tau_0$.

Dynamic Contact Angle

The inclination angle at $x = \xi$ can be calculated by differentiating film thickness $h$ as shown in Eq. (29) for the capillary spreading regime.
\[ \tan \theta = \frac{dh}{d\xi} = \frac{\frac{1}{2} U^n}{(h - \tau_0/\rho g) \left[ \frac{n(n+1)k\sigma}{(2n+3)(n+1)} \right]^{(n+2) (n+1)} \left( n + 1 \right)} \left( \frac{\tau_0}{\rho g} - \frac{n^2 k^2 \sigma}{(2n+3)(n+1)} \right) \xi^2 \ (n \neq 1) \] 

(31)

In previous studies, many researchers found that the local microscopic contact angle cannot be measured directly for boundary conditions. Thus dynamic contact angle \( \theta \) is taken as the replacement, which is equal to the inclination angle at \( x = x_m \). Furthermore, the dynamic contact angle can be obtained as

\[ \theta = \tan \theta = \frac{\frac{1}{2} U^n}{(h - \tau_0/\rho g) \left[ \frac{n(n+1)k\sigma}{(2n+3)(n+1)} \right]^{(n+2) (n+1)} \left( n + 1 \right)} \left( \frac{\tau_0}{\rho g} - \frac{n^2 k^2 \sigma}{(2n+3)(n+1)} \right) x_m^{-2} \ (n \neq 1) \] 

(32)

To obtain Eq. (32), it is considered that when the angle \( \theta \) is small enough, it is approximately equal to \( \sin \theta \) or \( \tan \theta \).

**EXAMPLE**

In a previous study [21, 22], the typical Herschel-Bulkley model was obtained as follows

\[
\begin{align*}
\tau & = 3.899 + 1.103\gamma^{0.633}, \quad \tau_0 \geq 3.899 \\
\dot{\gamma} & = 0, \quad \tau_0 < 3.899
\end{align*}
\] 

(33)

The rheological figure is shown in Figure 3. The model was taken as an example to explain the spreading results. Figure 4 gives the film thickness in the gravitational regime. The area below the dashed line is the sheared zone. In Figure 5, the figure of the contact angle vs. the moving velocity at different film thicknesses is presented. It can be found that a thinner film will lead to a smaller contact angle.

**DISCUSSION AND CONCLUSION**

The spreading of Herschel-Bulkley fluids over solid substrates for a complete wetting case was theoretically studied here. The film thicknesses in both the gravitational and capillary regimes were derived and the dynamic contact angles were also explored. There was an obvious difference between other kinds of fluids.
(Newtonian fluids and power-law fluids). The flow was dramatically affected by the yield behavior of Herschel-Bulkley fluids. According to the analysis, only when the stress on the substrate was larger than the initial yield stress \( \tau_0 \) would the flow be appreciable. Even if appreciable flow occurred, the spreading zone was also divided into two zones: one was the shear zone and the other was the yield zone. The dividing surface was named yield zone was also divided into two zones: one was the shear zone and

It is noteworthy that when initial yield stress \( \tau_0 = 0 \), the fluids become power-law fluids. The film thicknesses of the gravitational regime and the capillary regime will be obtained as follows

\[
h^{n+2} = (n + 2) \left( \frac{U^2 n + 1}{n} \right)^{\frac{n}{n}} k \xi \quad (34)
\]

\[
h^{n+2} = \frac{k}{\sigma} \left[ \frac{(n+2)^2}{(n+1)^{3-n}} \right] \frac{n}{n} \xi^2 \quad (n \neq 1) \quad (35)
\]

Equations (34) and (35) represent the gravitational case and the capillary case, respectively. Also, the dynamic contact angle \( \theta \) of the capillary regime will be

\[
\theta \approx \tan^{-1} \left( \frac{k U^m}{\sigma} \right) \left[ \frac{3^{n+1} (n+2)^{1-n}}{(2n+1)^{3-n}} \right] \frac{1-n}{m} \quad (n \neq 1) \quad (36)
\]

Substituting Eq. (35) into Eq. (36), then the specific equation of the dynamic contact angle can be obtained as

\[
\theta^{n+2} = \left( \frac{k U}{\sigma} \right) \left[ \frac{3^{n+1} (n+2)^{1-n}}{(2n+1)^{3-n}} \right] \frac{(n+2)^{1-n}}{(n+1)^{1-n}} \frac{1-n}{m} \quad (n \neq 1) \quad (37)
\]

It can be found that all of them are in agreement with the results obtained by Wang [5]. Another noticeable feature of the analysis is that some of the equations have a collective limitation \( n \neq 1 \), which means the theoretical results are not applicable for Bingham fluids, although the fluid corresponds to it when \( n = 1 \).

**DATA AVAILABILITY STATEMENT**

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

**AUTHOR CONTRIBUTIONS**

JZ: idea, HG: method, JS: calculation, BL: calculation, JZ: calculation, WW: validation and writing.

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**SUPPLEMENTARY MATERIAL**

The Supplementary Material for this article can be found online at: https://www.frontiersin.org/articles/10.3389/fphy.2020.609926/full#supplementary-material.

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**Conflict of Interest:** The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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GLOSSARY

Nomenclature Symbols

g Gravitational acceleration
h The height of film thickness at position x
h₀ The film thickness of yield surface
k Viscosity coefficient
n The power-law index
p Pressure
p₀ Atmospheric pressure
q Flow of fluids
t Time
u Velocity along the x-direction

u_p Maximum velocity of u
U Frame moving velocity
x/z Values at two directions in the Cartesian coordinate system
γ Shear rate
θ Dynamic contact angle
λ Variable
ξ Variable, ξ = x-Ut
ρ Density
σ Surface tension
τ Stress
τ₀ Initial yield stress
τ_s Stress on the substrate