A REMARKABLE ANGULAR DISTRIBUTION OF THE INTERMEDIATE SUBCLASS OF THE GAMMA-RAY BURSTS

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ABSTRACT

In the article a test is developed, which allows to test the null-hypothesis of the intrinsic randomness in the angular distribution of gamma-ray bursts collected at the Current BATSE Catalog. The method is a modified version of the well-known counts-in-cells test, and fully eliminates the non-uniform sky-exposure function of BATSE instrument. Applying this method to the case of
all gamma-ray bursts no intrinsic non-randomness was found. The test also
did not find intrinsic non-randomnesses for the short and long gamma-ray
bursts, respectively. On the other hand, using the method to the new in-
termediate subclass of gamma-ray bursts, the null-hypothesis of the intrinsic
randomness for 181 intermediate gamma-ray bursts is rejected on the 96.4%
confidence level. Taking 92 dimmer bursts from this subclass itself, we ob-
tain the surprising result: This ”dim” subclass of the intermediate subclass
has an intrinsic non-randomness on the 99.3% confidence level. On the other
hand, the 89 ”bright” GRBs show no intrinsic non-randomness.

Subject headings: cosmology: observations - gamma rays: bursts

1 INTRODUCTION

Two results of the last years in the statistics of the gamma-ray bursts (GRBs)
are doubtlessly remarkable.

The first one concerns the number of subclasses. Recently, two differ-
ent articles ([Mukherjee et al. 1998]; [Horváth 1998]) simultaneously suggest
that the earlier separation [Kouveliotou et al. 1993] of GRBs into short and
long subclasses is incomplete. (It is a common practice to call GRBs hav-
ing \( T_{90} < 2 \) s \( (T_{90} > 2 \) s) as short (long) GRBs, where \( T_{90} \) is the time during
which 90\% of the fluence is accumulated [Kouveliotou et al. 1993].) These
articles show that, in essence, the earlier long subclass alone should be fur-
ther separated into a new ”intermediate” subclass \( (2 \) s \( < T_{90} < 10 \) s) and
into a ”truncated long” subclass \( (T_{90} > 10 \) s). (In what follows, the long
subclass will contain only the GRBs with \( T_{90} > 10 \) s, and the intermediate
subclass will be considered as a new subclass.)

The second result concerns the angular distribution of GRBs. At the
last years several attempts ([Hartmann et al. 1991], [Briggs et al. 1996];
[Tegmark et al. 1996b]; [Balázs et al. 1998]; [Balázs et al. 1999]) were done
either to confirm or to reject the randomness in the angular sky distri-
bution of GRBs being collected at BATSE Catalog ([Fishman et al. 1994];
[Meegan et al. 1998]). Theoretically, if the intrinsic distribution of GRBs is
actually random, an observation of some non-randomness is still expected due
to the BATSE non-uniform sky-exposure function ([Fishman et al. 1994];
[Meegan et al. 1998]). Hartmann et al. (1991), Briggs et al. (1996) and
Tegmark et al. (1996b) did not find any statistically significant depart-
ture from the randomness. On the other hand, the existence of some non-
randomness was confirmed on the > 99.9% confidence level by Balázs et al. (1998). This behavior can be caused either purely by instrumental effects or the instrumental effects alone do not explain fully the detected behavior and some intrinsic non-randomnesses should also exist. Balázs et al. (1998, 1999) suggest the second possibility. This conclusion follows from the result that while the short subclass shows a non-randomness, the intermediate + long subclasses do not indicate it. It is difficult to explain such behavior of subclasses by the instrumental effects alone.

In this article we will again investigate the angular distribution of GRBs. Trivially, after the discovery of the new intermediate subclass, it is highly required to test the intrinsic randomness in the angular distribution of this new subclass, too. In addition, of course, new different tests, which exactly eliminate the effect of the sky-exposure function, are also required in order to complete the results of Balázs et al. (1998, 1999).

The aim of this article is to test the intrinsic randomness in the angular distribution of all GRBs and of the three subclasses separately, too. We will use a modification of the well-known counts-in-cells method. This is a standard and simple statistical test (see, cf., Mészáros (1997) and references therein). The advantage of this method is given by the fact that it allows to eliminate quite simply and exactly the sky-exposure function. The main result of paper will be the surprising conclusion that the intermediate subclass and only this subclass alone suggests a non-randomness on the 96.4% confidence level; its "dimmer" half even on the 99.3% confidence level.

The paper is organized as follows. In Section 2 the method is described. In Section 3 the results of test are presented. Section 4 discusses and summarizes the results of the article.

2 THE TEST

Assume for the moment that there is no non-uniform sky-exposure function. We separate the sky in declination into $m_{\text{dec}} > 1$ stripes having the same area $(4\pi/m_{\text{dec}} \text{ steradian})$. The boundaries of stripes are the declinations $\delta_k$, $k = 0, 1, ..., m_{\text{dec}}$, where $\delta_0 = -90$ degree and $\delta_{m_{\text{dec}}} = +90$ degree, respectively. The remaining values are analytically calculable, and appear symmetrically with respect to $\delta = 0$. One has: $\sin \delta_k = 2k/m_{\text{dec}} - 1$. (For example: If $m_{\text{dec}} = 3$, then $\delta_{1,2} = \pm 19.47$ degree; if $m_{\text{dec}} = 4$, then $\delta_{1,3} = \pm 30.00$ degree and $\delta_2 = 0.00$ degree; etc.) We also separate the sky in right ascension $\alpha$
into \( m_{ra} > 1 \) stripes. They are defined by boundaries \( \alpha = 360k'/m_{ra} \) degree; \( k' = 0, 1, \ldots, m_{ra} \). (Obviously, a trivial modification of this separation in right ascension is the case, when the boundaries are \( \alpha = 360(k' + p)/m_{ra} \) degree, where \( p \) is an arbitrary real number fulfilling \( 0 < p < 1 \).) All this means that we separated the sky into \( M = m_{dec} \times m_{ra} \) areas ("cells") having the same size \( 4\pi/M \) steradian. If there are \( N \) GRBs on the sky, then \( n = N/M \) is the mean of GRBs at a cell. Let \( n_i; \ i = 1, 2, \ldots, M \) be the observed number of GRBs at the \( i \)th cell (\( \sum_{i=1}^{M} n_i = N \)). Then

\[
var_M = (M - 1)^{-1} \sum_{i=1}^{M} (n_i - n)^2
\]

defines the observed variance. For the given cell structure with \( M \) cells, due to the Bernoulli distribution ([Mészáros 1997], [Balázš et al. 1998]), the measured variance \( var_M \) should be identical to the theoretically expected value \( n(1 - 1/M) \). This theoretical prediction should then be tested.

Note that this and similar methods (see, e.g., Mészáros (1997) for details and further references) are usual in astronomy. For example, this method was used already by Abell (1958) to reject the randomness in the sky-distribution of clusters of galaxies. The test is, compared with other statistical tests ("two-point angular correlation function", "nearest neighbor distances", etc.; [Peebles 1980]; [Diggle 1983]; [Pásztor 1993]), not the most sensitive one to detect non-randomnesses. Its importance for our purposes is given by the fact that it allows an extremely simple generalization to the case with non-zero sky-exposure function.

Now, we generalize the method to this case. This may easily be done by changing the boundaries of cells in order to have the same probability (and hence the same expected number \( n = N/M \)) for a given cell. The sky-exposure function is a function of declination only ([Fishman et al. 1994]; [Meegan et al. 1998]). Hence, the choice of equatorial coordinates is highly convenient, because then no changes of boundaries are necessary in right ascension. The new boundaries \( \delta_k, \ k = 0, 1, \ldots, m_{dec} \) in declination may be calculated analytically as follows. Clearly, \( \delta_{0,m_{dec}} = \pm 90 \) degree remain. In BATSE Catalog [Meegan et al. 1998] the exposure function \( f(\delta) \) is defined for 37 values of declination (for \( \delta_r = -90, -85, \ldots, +85, +90 \) degree; \( r = 0, 1, 2, \ldots, 36 \)). To obtain \( \delta_k \) we, first, calculate the value

\[
A = \frac{5\pi}{180} \sum_{r=1}^{35} f(\delta_r) \cos \delta_r ,
\]
where for the given \( r \) the corresponding declination is \( \delta_r = -90 + 5r \) degree.

(Remark that \( r = 0 \) and \( r = 36 \), respectively, need not be in the sum, because \( \cos(\pm 90) = 0 \).) Then, second, for \( m_{\text{dec}} \geq 2 \) we search for the values \( \delta_k; \ k = 1, 2, ..., (m_{\text{dec}} - 1) \) as follows. For the given \( k \) we search for the declination \( \delta_i \) fulfilling the condition

\[
\frac{\pi}{36 A} \sum_{j=1}^{i} f(\delta_j) \cos \delta_j \leq \frac{k}{m_{\text{dec}}} < \frac{\pi}{36 A} \sum_{j=1}^{i+1} f(\delta_j) \cos \delta_j.
\]

Having this we search, by linear interpolation between \( \delta_i \) and \( \delta_{i+1} \), for the exact value of \( \delta_k \). By this method \( \delta_k \) is well calculable. (For example: For \( m_{\text{dec}} = 3 \) we obtain \( \delta_1 = -19.51 \) degree, \( \delta_2 = 22.44 \) degree; for \( m_{\text{dec}} = 4 \) we obtain \( \delta_1 = -30.83 \) degree, \( \delta_2 = 1.51 \) degree, \( \delta_3 = 33.60 \) degree; etc.) Having these cells with these “shifted” boundaries in declination the variance may be calculated identically to the case with no sky-exposure function. This method will test the pure intrinsic randomness; the effect of BATSE sky-exposure function is exactly eliminated.

It is natural to probe different values of \( M \). In addition, for some \( M \) different cell structures are still possible (cf. \( M = 12 \) allows \( m_{\text{dec}} = 2, 3, 4, 6 \)). Hence, generally, several - say \( Q \) - cell structures may be probed for the same sample of GRBs. Having these \( Q \) cell structures (and hence \( Q \) means + \( Q \) measured variances) two questions arise. 1. How to calculate the confidence level for a given cell structure? 2. Having \( Q \) values of confidence levels, how to calculate the final confidence level? The answer for the first question seems to be quite clear: \( \text{var}_M/n \) seems to be identical to the \( \chi^2 \) value for \( M - 1 \) degree of freedom ([Trumpler & Weaver 1953]; [Kendall & Stuart 1969]; [Press et al. 1992]; the mean is obtained from the sample itself, and therefore the degree of freedom is \( M - 1 \)). Nevertheless, the situation is not so obvious, because the \( \chi^2 \) test needs \( n > 5 \) ([Trumpler & Weaver 1953]; [Kendall & Stuart 1969]; [Press et al. 1992]). In addition, some statistical text-books propose to use ”quadratic” cells only ([Diggle 1983], Chapt. 2.5.). If all these restrictions were taken into account, then \( \chi^2 \) tests would be possible only for \( 2m_{\text{dec}} = m_{\text{ra}}; \ M = 2m^2_{\text{dec}} \) and \( N > 5M = 10m^2_{\text{dec}} \). This would be a drastic truncation of the possible cell structures. But, not doing these restrictions, the estimation of the confidence level for a given cell structure must be done by more complicated procedures; e.g., by numerical simulations. Concerning the answer to the second question the situation is even less clear. As the reasonable search for the final confidence level only Monte Carlo simulations seem to be usable [Press et al. 1992].
Keeping all this in mind, we will proceed as follows. In the coordinate system with axes $x = 1/M$ versus $y = \sqrt{\text{var}_M/n} = (\text{var}/\text{mean})^{1/2}$ the $Q$ values of $(\text{var}/\text{mean})^{1/2}$ define $Q$ points (one point for any cell structure; $y_j = \sqrt{\text{var}_{M,j}/n}$, where $j = 1, 2, ..., Q$). Clearly, for these points one expects the theoretical curve $y = \sqrt{1-x}$. This theoretical expectation can straightforwardly be verified, e.g., by least squares estimation ([Press et al. 1992], Chapt. 15.2.; [Diggle 1983], Chapt. 5.3.1). Our estimator is the dispersion

$$\sigma_Q = \sum_{j=1}^{Q} (y_j - \sqrt{1 - 1/M})^2. \quad (4)$$

Obviously, smaller $\sigma_Q$ suggests that the theoretical curve is better fitted. Note still that, as the best choice, the square root of $\text{var}_M/n$ is proposed in this "var/mean" test ([Diggle 1983], Chapt. 5.4.).

The confidence level can then be estimated by Monte Carlo simulations in the following way: We throw 1000-times randomly $N$ points on the sphere, and repeat the above calculation leading to $\sigma_Q$ for every simulated sample. Then we compare the size of the $\sigma_Q$ obtained from this simulation with $\sigma_Q$ obtained from the actual GRB positions. Let $\omega$ be the number of simulations, when the obtained $\sigma_Q$ is bigger than the actual value of $\sigma_Q$. Then one may conclude that $(100 - \omega/10)$ is the confidence level in percentage. Clearly, this method does not need $n > 5$ and quadratic cells.

There is no commonly accepted confidence level in statistics, above which the null-hypothesis should already be rejected ([Trumpler & Weaver 1953; Kendall & Stuart 1969]). It is only a general agreement that confidence levels smaller than 95% should not be considered. Our opinion is (see also Kendall & Stuart 1969) that the confidence levels bigger than 95% can already be taken as "remarkable", "suspicious", "interesting", etc.; a higher than 99% confidence level may still mean the rejection of null-hypothesis, and such result must doubtlessly be announced. Hence, we will require that the confidence level be bigger than 95%. Thus, here it must be $\omega < 50$.

In this paper GRBs will be taken between trigger-numbers 0105 and 6963 from Current BATSE Catalog [Meegan et al. 1998] having defined $T_{90}$ (i.e. all GRBs detected up to August 1996 having measured $T_{90}$). From them we exclude, similarly to Pendleton et al. (1997) and Balázs et al. (1998), the faintest GRBs having a peak flux (on 256 ms trigger) smaller than 0.65 photon/(cm$^2$s). This truncation is proposed by Pendleton et al. (1997) in order to avoid the problems with the changing threshold. The 1284 GRBs
obtained in this way define the ”all” class. From them there were 339 GRBs with \( T_{90} < 2 \) s (the ”short” subclass), 181 GRBs with \( 2 \) s < \( T_{90} < 10 \) s (the ”intermediate” subclass) and 764 GRBs with \( T_{90} > 10 \) s (the ”long” subclass). We will study the all class and the three subclasses separately.

We will ad hoc choose \( m_{\text{dec}} = 2, 3, \ldots, 8 \) and \( m_{\text{ra}} = 2, 3, \ldots, 16 \). I.e. it will be \( Q = 105 \). Of course, this choice of \( Q \) is more or less subjective. Nevertheless, our choice is motivated by two concrete arguments. First, we would like to study only the angular scales much bigger than the positional errors. (The size of a cell will not be smaller than 22.5 degree. On these angular scales no problems should arise from the positional errors [Meegan et al. 1998].) Second, it is reasonable not to consider such high values of \( m_{\text{dec}} \), when \( 180/m_{\text{dec}} \) is already comparable or even smaller than 5 degree. (If this were not required then the elimination of sky-exposure function would be problematic due to its definition for declination intervals with widths 5 degree.)

3 THE RESULTS

Figure 1 collects the results of \( Q = 105 \) ”var/mean” tests of four different cases. It is obvious immediately that for the ”all” case the points follow well the theoretical curve. For the ”short” and ”long” subclasses, on the other hand, there is a slight tendency of points to be above the theoretical curve. The situation concerning the intermediate subclass seems to be the most unambiguous: mainly for small \( M \) (roughly below \( M \approx 40 \)) the points are clearly above the theoretical curve. This suggests an intrinsic non-randomness mainly in the sky distribution of intermediate subclass; such possibility for the short and long subclasses, respectively, cannot be excluded, too.

The results of Monte Carlo simulations support this expectation only in the case of intermediate subclass. We obtain \( \omega = 287 \) (\( \omega = 80, \omega = 36, \omega = 440 \)) for all GRBs (short, intermediate, long GRBs). Hence, the rejection of null-hypothesis is confirmed for the intermediate subclass only on the 96.4% confidence level. For the short and long subclasses, respectively, and also for all GRBs the null-hypothesis cannot be rejected on the > 95% confidence level. For the short subclass we have a 92% confidence level; for the remaining two cases even smaller levels.
4 DISCUSSION AND CONCLUSION

The most surprising result of paper concerns the intermediate subclass. The intrinsic non-randomness is confirmed on the confidence level $> 95\%$. This confidence level, as discussed in Section 2, is “remarkable”, but is not enough to reject the null-hypothesis of randomness.

The results concerning the 339 short GRBs should also be mentioned. Nevertheless, the 92% confidence level is clearly not enough to reject the confidence null-hypothesis. On the other hand, this result, together with Balázs et al. (1998, 1999), suggest that also for the short subclass itself the rejection of null-hypothesis of intrinsic randomness can also occur by further tests.

In the case of 764 long GRBs, and also of the 1284 all GRBs, there are no indications for the non-randomnesses. All this seems to be in accordance with the results of Balázs et al. (1998, 1999).

We think that the result concerning the intermediate subclass is highly surprising, because just this new subclass, having the smallest number of GRBs, has a remarkable ”proper” behavior.

A short further investigation of this subclass fully supports this conclusion.

There are 181 GRBs in this intermediate subclass. Be divided this subclass into two further subclasses; into the ”dim” and ”bright” ones. By chance the peak flux = 2 photons/(cm$^2$s) (on 0.256s trigger) is practically identical to the medium of peak flux for this subclass. Therefore, we consider the GRBs having smaller (bigger) peak flux 2 photons/(cm$^2$s) as the ”dim” (”bright”) subclass of the intermediate subclass. There are 92 GRBs at the ”dim” subclass, and 89 GRBs at the ”bright” one.

We provide the 105 ”var/mean” tests for these two parts, too. We obtain the surprising result that the ”dim” subclass has an intrinsic non-randomness on the 99.3% confidence level ($\omega = 7$). Contrary this, the ”bright” subclass can still be random ($\omega = 662$).

The sky distribution of 92 intermediate dim GRBs is shown on Figure 2. We mean that the behavior of the intermediate subclass of GRBs, quite independently, supports the correctness of the introduction of this new subclass ([Mukherjee et al. 1998]; [Horváth 1998]). Further investigations of this new subclass are highly required.

Three notes are still needed.

First, purely from the statistical point of view, it must be precised that
even the rejection of null-hypothesis of the intrinsic randomness would not mean a pure intrinsic non-randomness in the spatial angular distribution of GRBs. This is given by the fact that, up to now, it cannot be fully excluded that GRBs (or some part of them) are not unique phenomenons, and there can occur some repetitions, too. This question is studied intensively by several papers ([Meegan et al. 1995], [Quashnock 1995], [Quashnock 1996], [Tegmark et al. 1996a], [Graziani et al. 1998], [Hakkila et al. 1998]) concluding that repetition can still play a role.

Second, strictly speaking, the statistical counts-in-cells test is testing the "complete spatial randomness" (shortly the "randomness") of the distribution of GRB on the celestial sphere ([Diggle 1983], Chapt.1.3). Therefore, in this paper we have kept this terminology. In cosmology, on the other hand, the word "random" ("non-random") is rarely used, and the word "isotropic" ("anisotropic") is usual (for the exact definition of isotropy in cosmology see, e.g., Weinberg (1972), Chapt. 14.1). Of course, here we will not go into the details of these terminology questions (see, e.g., Peebles (1980) for more details concerning these questions). We note only that the "random-isotropic" ("non-random-anisotropic") substitution is quite acceptable on the biggest angular scales; on smaller angular scales the situation is not so clear. Therefore, in Balázs et al. (1998, 1999), where only the angular scales $\sim 90$ degrees and higher were studied, the words "isotropy" and "anisotropy" were quite usable. In this article, going down up to the scales $\sim (20-25)$ degree, the used terminology is more relevant.

Third, trivially, further studies are needed. They should test - by other different statistical methods - again the intrinsic randomnesses (more generally: the intrinsic spatial distributions [Lamb 1997]), both for all GRBs and for the subclasses. In addition, a test of the repetition alone, i.e. a test not being influenced by positions, is highly required.

As the conclusion, the results of paper may be summarized as follows.

- We developed a method, which can verify quite simply the intrinsic randomness alone in the angular distribution of GRBs, because the method eliminates exactly the non-zero sky-exposure function.

- We rejected the null-hypothesis of the intrinsic randomness in the angular distribution of 181 intermediate GRBs on the 96.4% confidence level.
- We rejected the null-hypothesis of the intrinsic randomness in the angular distribution of 92 ”dim” intermediate GRBs on the 99.3% confidence level.

- We did not reject the null-hypotheses of the intrinsic randomnesses in the angular distribution of the remaining two subclasses and of the all GRBs, respectively, on the > 95% confidence levels; the ”bright” intermediate GRBs seem to be distributed randomly, too.

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Figure 1. The results of 105 “var/mean” tests of four different cases drawn in the $1/M$ vs. $\sqrt{\text{var/mean}}$ frame. The theoretical curve $\sqrt{1 - 1/M}$ (solid line) is also shown. $M$ is the number of cells.
Figure 2. Sky distribution of 92 GRBs of ”dim” subclass of the intermediate subclass in equatorial coordinates.