A regularization method for solving the redundant problems in multibody dynamic system

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Abstract: Redundant constraints may cause great difficulties during the numerical simulation, when solving the differential algebraic equations (DAEs). The article presents a regularization method based on optimization form of the Gauss principle of least constraint and the Udwadia and Kalaba (UK) method, which expresses the solution of the dynamic motion in an explicit expression. Compared with Lagrange method, the proposed method does not need to solve the Lagrange multipliers any more for the complex multibody mechanical systems. In the end, a numerical example is given to illustrate that the proposed method not only has the ability to handle the problems with redundant constraints, but also has higher accuracy and keep a stable constraint violation level than the traditional methods.

1. Introduction
Different styles modeling methods of multi-body system dynamics greatly promote the development of modern technology. In the process of modeling and simulation automation, redundancy constraints are inevitably introduced, which brings a huge amount of work. Many theories held the opinion that the constraint equations were independent of each other when dealing with multi-body dynamics problems with redundant constraints [1]. This premise is nevertheless hard to meet.

According to the literature research, one traditional processing method is to modify the velocity constraint equations and the configuration constraint equations in order to keep the violation degree of the constraints within the controllable precision range. Then the original dynamic problems are transformed into solving a set of underdetermined equations. The methods of solving the underdetermined equations mainly include generalized the inverse methods [2], constraint stabilization approaches [3], coordinate partitioning methods [4], violation correction methods [5] and so on. A great deal of methods are raised to identify independent constraints, such as the linear independence between the row vector of the Jacobi matrix of constraints or the constraint forces [6]. Augmented Lagrange Formulation [7] is considered to be able to solve dynamics problems of multi-body systems with redundant constraints. However, for numerical examples with large simulation time, this method is easy to fail and the violation correction method is needed to add to the system to guarantee the satisfaction of constraint equations. Udwadia and Kalaba [2] and Fan [8] raised the dynamic motion of multi-body systems in a form of explicit equations of motion by employing the Moore–Penrose pseudoinverse for the solution.

The paper proposes a regularization method based on the Gauss principle (RMGP) extended by Yao
and the method proposed by Udwadia and Kalaba [2], which does not need to make violation correction and keep the constraint violation in a stable level.

2. Method

Consider an unconstrained mechanical system, by employing Lagrange equation the motion of the system can be expressed as

$$M\ddot{q} = Q_s$$

where $\ddot{q}$ is the acceleration of system, $M$ is the generalized mass matrix and $Q_s$ is a vector matrix related to the generalized forces.

In this paper, we consider that the system is constrained by $m$ holonomic or non-holonomic constraint equations

$$\Phi(q, \dot{q}, t) = 0$$

which, at velocity and acceleration level, the constraint can be described as

$$\Phi = A\dot{q} - c = 0$$
$$\ddot{\Phi} = A\ddot{q} - \dot{b} = 0$$

where $A$ is the constraint Jacobian matrix, $b = \partial \Phi / \partial t$ and $c = \partial \Phi / \partial t$.

UK method has the following expression, described as

$$\ddot{q} = \overline{M} \begin{bmatrix} (I - A^t A)(Q + C) \\ b \end{bmatrix} + (I - \overline{M}^t \overline{M}) \eta$$

where $\overline{M} = \begin{bmatrix} (I - A^t A)M \\ A \end{bmatrix}$, $C$ and $\eta$ are arbitrary vectors which satisfy certain conditions. In the paper, both $C$ and $\eta$ equal zero.

In order to overcome the overdetermined problem resulted by the redundant constraints, here the paper adds a regularization item in the right side of Gauss principle similar to constraint stabilization method explained in [3]. The new equation has the following form:

$$G_0 = \frac{1}{2}(M\ddot{q} - Q_s)^T M^{-1}(M\ddot{q} - Q_s) + \kappa \left\| \Phi + 2\Omega \mu \Phi + \Omega^2 \Phi \right\|_2$$

where $\kappa$ is constant coefficient, which respectively represent the value of the penalty factor, $\Omega$ is the natural frequency and $\mu$ is damping ratios of the fictitious penalty systems assigned to each constraint condition. In theory, it can be easily demonstrated that the solution of $\delta G = 0$ coincides with $\delta G_0 = 0$, provided that $\kappa \to \infty$. Eq. (5) is a convex quadratic programming problem and its optimal solution exists and is unique. Together with the acceleration-level expression of the kinematic constraints, the original problem of computing the motion of system is converted into the solution of the following equations

$$\begin{bmatrix} M + \kappa A^t A \\ A \end{bmatrix} \ddot{q} = \begin{bmatrix} Q_s - \kappa A^t \left[ -b + 2\Omega \mu (A\ddot{q} - c) + \Omega^2 \Phi \right] \\ b \end{bmatrix}.$$  

According to the result of the literature [2], we can solve Eq. (6) to get

$$\ddot{q} = \begin{bmatrix} (I - A^t A)M + \kappa A^t A \\ A \end{bmatrix} \begin{bmatrix} Q_s - A^t AQ_s - \zeta \\ b \end{bmatrix}$$

where

$$\zeta = \kappa A^t \left[ -b + 2\Omega \mu (A\ddot{q} - c) + \Omega^2 \Phi \right]$$

and $A'$ is the Moore-Penrose inverse of the matrix $A$. Eq. (7) is the general and explicit expression of the solution of dynamic equations for constrained multibody system with holonomic or non-holonomic kinematic constraints. And it is also allowed to be applicable to singular systems and redundant
constrained systems.

3. Numerical simulation and discussion
The paper takes a parallel five-bar mechanism as the numerical example, which has eight kinematic constraints without any redundant actuators shown in figure 1. In the example, links 3 or 4 are redundant because they constrain the same degrees of freedom and restrict the relative motion between links 1 and 2 in the same way. The system is allowed to move under gravity starting from the initial position $\theta_1=45^{\circ}$. The resulting solution is compared with UK method from the reference [2]. In the simulation, the constraint time step is set to be 0.002 s.

The model parameters are $m_1=1.5$ kg, $m_2=2$ kg, $m_3=3$ kg, $m_4=2.5$ kg, $l_1=l_2=0.5$ m (lengths of the links 1 and 2), $l_3=l_4=0.8$ m (lengths of the links 3 and 4), and $g=9.8$ m/s$^2$. The chosen coordinates of this mechanism are $q = [\theta_1, \theta_2, x_3, y_3, x_4, y_4, \theta_3]^T$, which are redundantly constrained by the following eight passive constraint equations:

$$
\Phi(q,t) = 
\begin{bmatrix}
x_3 - 0.5l_1 \cos \theta_1 - 0.5l_2 \cos \theta_2 - l_3 \\
x_3 + 0.5l_1 \cos \theta_1 - 0.5l_2 \cos \theta_2 - l_3 \\
y_3 - 0.5l_1 \sin \theta_1 - 0.5l_2 \sin \theta_2 \\
y_3 + 0.5l_1 \sin \theta_1 - 0.5l_2 \sin \theta_2 \\
x_4 - 0.5l_4 \cos \theta_3 + 0.5l_1 \cos \theta_1 \\
x_4 + 0.5l_4 \cos \theta_3 + 0.5l_1 \cos \theta_1 - l_4 \\
y_4 - 0.5l_4 \sin \theta_3 + 0.5l_1 \sin \theta_1 \\
y_4 + 0.5l_4 \sin \theta_3 + 0.5l_1 \sin \theta_1
\end{bmatrix}.
$$

It is easy to compute that the rank of Jacobian matrix of the constraint equation is 7, which demonstrates the mechanism is redundant.

Figure 1. Five-bar mechanism  \hspace{1cm} Figure 2. Generalized coordinates versus time
In process of simulation, the Newmark method was used for the integrated velocities and configurations. The parameter $\kappa$, $\Omega$ and $\mu$ are respectively set to be $10^6$, 10 and 5. Figure 2 shows the curve of the selected generalized coordinates $\theta_1$ (rad), $x_3$ (m) and $x_4$ (m) versus the simulation time. Both of the two methods expressed the same behavior in the early 6 s of simulation. Figure 3(a) displays the computing time with the two methods versus simulation time which states that the regularization method shows high simulation efficiency. The average of constraints violation at the acceleration level is plotted in figure 3(b), which indicate that the UK method can not simulate for a long time. In essence, the UK method fail to solve the dynamic motion with redundant constraints after 40 s of simulation, as the constraint violation has exceeded error range of $10^{-6}$. However, the regularization method can keep stable constraint violation level, which indicates that the regularization method has faster solving speed and higher accuracy over UK method.

4. Conclusion

Redundant constraints of the multibody dynamic system bring great difficulties to the numerical simulation work. The paper propose a regularization method to handle the redundant constraints problem based on the Gaussian principle. After comparing the different simulation results obtained by UK method and the regularization method, the following conclusions are drawn:

1. The regularization method is mathematically equivalent to the traditional method for dynamics without redundant constraints. When the system has redundant constraints, the regularization method has more obvious advantages than the reference method, such as Pseudoinverse-Based Approach. For example, it has higher simulation efficiency than the UK method.

2. The proposed method not only does not need to do some corrections to the velocity and acceleration constraints, but also it can keep the violation of the constraints stable at both the position and velocity levels.

3. As the method has an explicit expression of the dynamic motion and the leading matrix is always symmetric and positive definite, it can be applicable to the mechanical systems with singular mass matrix and singular configurations.

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