Resonance Model of $\pi\Delta \to YK$ for Kaon Production in Heavy Ion Collisions

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Abstract

The elementary production cross sections $\pi\Delta \to YK$ ($Y = \Sigma, \Lambda$) and $\pi N \to YK$ are needed to describe kaon production in heavy ion collisions. The $\pi N \to YK$ reactions were studied previously by a resonance model. The model can explain the experimental data quite well [8]. In this article, the total cross sections $\pi\Delta \to YK$ at intermediate energies (from the kaon production threshold to 3 GeV of $\pi\Delta$ center-of-mass energy) are calculated for the first time using the same resonance model. The resonances, $N(1710)\frac{1}{2}(1^+)$ and $N(1720)\frac{3}{2}(3^+)$ for the $\pi\Delta \to \Sigma K$ reactions, and $N(1650)\frac{1}{2}(1^-)$, $N(1710)\frac{1}{2}(1^+)\frac{3}{2}(1^+)\frac{3}{2}(3^+)$ and $N(1720)\frac{1}{2}(3^+)$ for the $\pi\Delta \to \Lambda K$ reactions are taken into account coherently as the intermediate states in the calculations. Also $t$-channel $K^*(892)\frac{1}{2}(1^-)$ vector meson exchange is included. The results show that $K^*(892)$ exchange is negligible for the $\pi\Delta \to \Sigma K$ reactions, whereas this meson does not contribute to the $\pi\Delta \to \Lambda K$ reactions. Furthermore, the $\pi\Delta \to YK$ contributions to kaon production in heavy ion collisions are not only non-negligible but also very different from the $\pi N \to YK$ reactions. An argument valid for $\pi N \to YK$ cannot be extended to $\pi\Delta \to YK$ reactions. Therefore, cross sections for $\pi\Delta \to YK$ including correctly the different isospins must be calculated to be included in simulation codes for kaon production in heavy ion collisions, where no experimental data are available. Parametrizations of the total cross sections $\pi\Delta \to YK$ for kaon production in heavy ion collisions are given based on this work.

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Due to a long mean free path the $K^+$ meson is a good probe for highly compressed nuclear matter formed in heavy ion collisions [1]. $K^+$ production is sensitive to the nuclear equation of state (EOS) [2]. Thus many studies of $K^+$ production in heavy ion collisions have been performed by theoretically and experimentally [1]-[7].

However, in most theoretical studies of kaon production by either the Vlasov-Uehling-Uhlenbeck approach (VUU) [3], or by “quantum” molecular dynamics (QMD) [10, 11], the kaon elementary production cross sections are not calculated. M. Ko [6], and by J. Cugnon and R. M. Lombard [7] have been used. In these works, the amplitudes relevant for the elementary kaon production cross sections are given by Pauli blocking. This relatively high amount of the $\Delta$ in heavy ion collisions is ascribed to the suppression of the decay $\Delta \rightarrow \pi N$, whereas the central cell of the heavy ion collisions make up about 25 % of the baryons, whereas the nucleons represent about 60 % to 70 %. This relatively high amount of the $\Delta$'s in heavy ion collisions is ascribed to the suppression of the decay $\Delta \rightarrow \pi N$ by Pauli blocking. This implies that the processes $\pi \Delta \rightarrow Y K$ must also be taken into account for kaon production in heavy-ion collisions.

However, no experimental data are available for the $\pi \Delta \rightarrow Y K$ reactions needed to simulate kaon production.

In this article, we will give parametrizations of the total cross sections $\pi N \rightarrow \Sigma K$ based on theoretical calculations [8]. The processes $\pi N \rightarrow Y K$ ($Y = \Sigma, \Lambda$) are the so-called secondary processes in heavy-ion collisions, which are known to give about a 30 % contribution to kaon production in heavy ion collisions [8]. It turned out that the model can explain the total cross sections $\pi N \rightarrow \Sigma K$ quite well [8].

On the other hand, it was shown by W. Ehehalt. et al. [12] that the $\Delta$'s in the central cell of the heavy ion collisions make up about 25 % of the baryons, whereas the nucleons represent about 60 % to 70 %. This relatively high amount of the $\Delta$'s in heavy ion collisions is ascribed to the suppression of the decay $\Delta \rightarrow \pi N$ by Pauli blocking. This implies that the processes $\pi \Delta \rightarrow Y K$ must also be taken into account for kaon production in heavy-ion collisions.

In this article, we will give parametrizations of the total cross sections $\pi \Delta \rightarrow Y K$ for the first time based on theoretical calculations.

According to the compilation of the “Review of Particle Properties” [13, 14], one can select the resonances $N(1710) I(J^P) = \frac{1}{2}(1^+)$ and $N(1720) \frac{1}{2}(3^+)$ for $\pi \Delta \rightarrow \Sigma K$, and the resonances $N(1650) \frac{1}{2}(1^+)$, $N(1710) \frac{1}{2}(3^+)$, and $N(1720) \frac{1}{2}(3^+)$ for $\pi \Delta \rightarrow \Lambda K$, as intermediate states giving the main contributions. For t-channel $K^*$ meson exchanges, we consider the lightest $K^*(892) \frac{1}{2}(1^-)$ for $\pi \Delta \rightarrow \Sigma K$, where no isospin $I = 1/2$ $K^*$ meson contributes to $\pi \Delta \rightarrow \Lambda K$.

Effective interaction Lagrangians relevant for the $\pi \Delta \rightarrow Y K$ reactions depicted in fig. 1 are used:

\[
\mathcal{L}_{\pi \Delta N(1650)} = \frac{g_{\pi \Delta N(1650)}}{m_{\pi}} \left( \bar{\pi}(1650) \gamma_5 \vec{T} \frac{1}{2} \Delta^\mu \cdot \partial_\mu \phi + \bar{\Delta} \gamma_5 N(1650) \cdot \partial_\mu \phi \right),
\]

\[
\mathcal{L}_{\pi \Delta N(1710)} = \frac{g_{\pi \Delta N(1710)}}{m_{\pi}} \left( \bar{\pi}(1710) \gamma_5 \vec{T} \frac{1}{2} \Delta^\mu \cdot \partial_\mu \phi + \bar{\Delta} \gamma_5 N(1710) \cdot \partial_\mu \phi \right),
\]

\[
\mathcal{L}_{\pi \Delta N(1720)} = -i g_{\pi \Delta N(1720)} \left( \bar{\pi}^\mu(1720) \gamma_5 \vec{T} \frac{1}{2} \Delta_\mu \cdot \phi + \bar{\Delta}_\mu \gamma_5 N(1720) \cdot \phi \right),
\]

\[
\mathcal{L}_{K \Sigma N(1710)} = -i g_{K \Sigma N(1710)} \left( \bar{N}(1710) \gamma_5 \vec{T} \frac{1}{2} K + \bar{K} \vec{T} \gamma_5 N(1710) \right),
\]

\[
\mathcal{L}_{K \Sigma N(1720)} = \frac{g_{K \Sigma N(1720)}}{m_K} \left( \bar{\pi}(1720) \vec{T} \partial_\mu K + (\partial_\mu K) \vec{T} N(1720) \right),
\]
\[ \mathcal{L}_{K^*(892)\Sigma} = -ig_{K^*(892)\Sigma} \left( \bar{K}^*_\mu(892) \slashed{\bar{T}} \gamma_5 \Delta^\mu + \bar{\Delta}^\mu \gamma_5 \slashed{T} K^*_\mu(892) \right), \quad (6) \]

\[ \mathcal{L}_{K^*(892)K\pi} = if_{K^*(892)K\pi} \left( \bar{K} \tau K^*_\mu(892) \cdot \partial^\mu \bar{\phi} - (\partial^\mu \bar{K}) \tau K^*_\mu(892) \cdot \bar{\phi} \right) + \text{h.c.}, \quad (7) \]

\[ \mathcal{L}_{K\Lambda N(1650)} = -g_{K\Lambda N(1650)} \left( \bar{N}(1650) \Lambda K + \bar{K} \Lambda N(1650) \right), \quad (8) \]

\[ \mathcal{L}_{K\Lambda N(1710)} = -ig_{K\Lambda N(1710)} \left( \bar{N}(1710) \gamma_5 \Lambda K + \bar{K} \gamma_5 \Lambda N(1710) \right), \quad (9) \]

\[ \mathcal{L}_{K\Lambda N(1720)} = \frac{g_{K\Lambda N(1720)}}{m_K} \left( \bar{N}(1720) \Lambda \partial_\mu K + (\partial_\mu \bar{K}) \Lambda N(1720) \right), \quad (10) \]

where spin 3/2 Rarita-Schwinger particle fields \( \psi^\mu = N^\mu(1720) \) and \( \Delta^\mu(1920) \) with mass \( m \) satisfy the set of equations [13],

\[ (i\gamma \cdot \partial - m)\psi^\mu = 0, \quad (11) \]

\[ \gamma_\mu \psi^\mu = 0, \quad (12) \]

\[ \partial_\mu \psi^\mu = 0. \quad (13) \]

\( \slashed{T} \) is the transition operator defined by

\[ \slashed{T}_{Mm} = \sum_{\ell = \pm 1, 0} (1i\ell \frac{1}{2} m \frac{3}{2} M) \hat{\epsilon}_{\ell}^*, \]

and \( \tau \) are the Pauli matrices. \( N, \Delta^\mu, N(1650), N(1710) \) and \( N^\mu(1720) \) stand for the fields of \( N(938), \Delta(1232), N(1650), N(1710) \) and \( N(1720) \) resonances. They are expressed by \( \bar{N} = (\bar{n}, \bar{n}) \), similarly to the nucleon resonances, and \( \Delta_\mu = \left( \Delta^+_{\mu} + \Delta^-_{\mu}, \Delta^0_{\mu}, \Delta^-_{\mu} \right) \). The other field operators appearing in the Lagrangians are related to the physical representations as follows: \( K^T = (K^+, K^0), \bar{K} = (\bar{K}^-, \bar{K}^0), K^*_\mu(892)^T = (K^*_\mu(892)^+, K^*_\mu(892)^0) \), \( \bar{K}^*_\mu(892) = (K^*_\mu(892)^-, K^*_\mu(892)^0) \), \( \pi^\pm = \frac{1}{\sqrt{2}}(\phi_1 \mp i \phi_2), \pi^0 = \phi_3, \Sigma^0 = \frac{1}{\sqrt{2}}(\Sigma_1 + i \Sigma_2), \Sigma^0 = \Sigma_3, \) where the superscript \( T \) means the transposition operation. Here the pseudoscalar meson field operators are defined annihilating (creating) physical particle (anti-particle) states. \( SU(2) \) isospin symmetry is assumed for each doublet or multiplet.

We use for the propagators \( S_F(p) \) of the spin 1/2 and \( G^{\mu\nu}(p) \) of the spin 3/2 resonances,

\[ S_F(p) = \frac{\gamma \cdot p + m}{p^2 - m^2 + im \Gamma_{\text{full}}}, \quad (14) \]

\[ G^{\mu\nu}(p) = \frac{P^{\mu\nu}(p)}{p^2 - m^2 + im \Gamma_{\text{full}}}, \quad (15) \]

with

\[ P^{\mu\nu}(p) = -\left( \gamma \cdot p + m \right) \left[ g^{\mu\nu} - \frac{1}{3} \gamma^\mu \gamma^\nu - \frac{1}{3m} (\gamma^\mu p^\nu - \gamma^\nu p^\mu) - \frac{2}{3m^2} p^\mu p^\nu \right], \quad (16) \]

where \( m \) and \( \Gamma_{\text{full}} \) stand for the mass and the full decay width of the corresponding resonance. In a previous study [8], we investigated the difference between the results using the energy dependent decay widths and the results using the energy independent decay widths. The two results show that the difference is not significant. Thus, in order
to avoid introducing extra ambiguities, we use here the energy independent full decay widths for the propagators of the resonances, since the form of energy dependent decay width is not uniquely established. The different ways to introduce decay widths into the propagators of spin 1/2 and spin 3/2 resonances, and also other problems concerning the propagators of spin 3/2 particles are discussed by Benmerouche et al. in detail \[16\].

Without factors arising from the isospin structure, we define the amplitudes $M_a, M_b, M_c$ and $M_d$ corresponding to each diagram (a), (b), (c) and (d) given in fig. 1 as follows:

$$M_a = \frac{-g\pi\Delta N(1650)gK\Lambda N(1650)}{m_\pi} p_{\pi\mu} \bar{u}_\Lambda(p_\Lambda) \left( \gamma \cdot p + m_{N(1650)} \right) \gamma_5 u_\Delta^\mu(p_\Delta) \left( p^2 - m_{N(1650)}^2 + i m_{N(1650)} \Gamma_{N(1650)}^{full} \right),$$

(17)

$$M_b = \frac{-g\pi\Delta N(1710)gK\Sigma N(1710)}{m_\pi} p_{\pi\mu} \bar{u}_Y(p_Y) \left( \gamma \cdot p + m_{N(1710)} \right) u_\Delta^\mu(p_\Delta) \left( p^2 - m_{N(1710)}^2 + i m_{N(1710)} \Gamma_{N(1710)}^{full} \right),$$

(18)

$$M_c = \frac{g\pi\Delta N(1720)gK\Sigma N(1720)}{m_K} \frac{p_K\mu \bar{u}_Y(p_Y) \Gamma_{N(1720)}^{full} p_\pi^\mu(p_\pi+p_K) \gamma_5 u_\Delta^\mu(p_\Delta) \left( p^2 - m_{N(1720)}^2 + i m_{N(1720)} \Gamma_{N(1720)}^{full} \right)}{m_{K^*(892)}},$$

(19)

$$M_d = \frac{i f_{K^*(892)K^*}(p_\Sigma - p_\Delta)^2 - m_{K^*(892)}^2}{(p_\Sigma - p_\Delta)^2 - m_{K^*(892)}^2} \bar{u}_\Sigma(p_\Sigma) \gamma_5 u_\Delta^\mu(p_\Delta)(p_\pi+p_K)^\nu \left( g_{\mu\nu} - \frac{(p_\Sigma - p_\Delta)_{\mu}(p_\Sigma - p_\Delta)_{\nu}}{m_{K^*(892)}^2} \right),$$

(20)

where $u_\Delta^\mu(p_\Delta)$, $u_\Lambda(p_\Lambda)$ and $u_Y(p_Y)$ are the (vector-) spinors of the $\Delta$, the $\Lambda$ and in general for the hyperons ($Y = \Lambda, \Sigma$) with the momenta $p_\Delta$, $p_\Lambda$ and $p_Y$, respectively. Note that the $N(1650)$ resonance contributes to $\pi\Delta \rightarrow \Lambda K$ but not to $\pi\Delta \rightarrow \Sigma K$. On the other side, t-channel $K^*(892)$ exchange contributes to the $\pi\Delta \rightarrow \Sigma K$ but not to the $\pi\Delta \rightarrow \Lambda K$ reactions.

Then each amplitude of the $\pi\Delta \rightarrow YK$ reactions is given by:

**For the $\pi\Delta \rightarrow \Sigma K$ reactions:**

$$-(M_b + M_c) \quad \text{for} \quad \pi^{-}\Delta^{++} \rightarrow \Sigma^{0}K^{+} \quad \text{and} \quad \pi^{+}\Delta^{-} \rightarrow \Sigma^{0}K^{0},$$

(21)

$$\mp \sqrt{\frac{2}{3}}(M_b + M_c) \quad \text{for} \quad \pi^{-}\Delta^{+} \rightarrow \Sigma^{-}K^{+} \quad \text{and} \quad \pi^{+}\Delta^{0} \rightarrow \Sigma^{+}K^{0},$$

(22)

$$-M_d \quad \text{for} \quad \pi^{0}\Delta^{++} \rightarrow \Sigma^{+}K^{+} \quad \text{and} \quad \pi^{0}\Delta^{-} \rightarrow \Sigma^{-}K^{0},$$

(23)

$$\pm \sqrt{\frac{2}{3}}(M_b + M_c + M_d) \quad \text{for} \quad \pi^{0}\Delta^{+} \rightarrow \Sigma^{0}K^{+} \quad \text{and} \quad \pi^{0}\Delta^{0} \rightarrow \Sigma^{0}K^{0},$$

(24)

$$\mp \sqrt{\frac{2}{3}}(M_b + 2M_c + M_d) \quad \text{for} \quad \pi^{0}\Delta^{0} \rightarrow \Sigma^{-}K^{+} \quad \text{and} \quad \pi^{0}\Delta^{+} \rightarrow \Sigma^{+}K^{0},$$

(25)

$$\pm \sqrt{\frac{2}{3}}M_d \quad \text{for} \quad \pi^{+}\Delta^{+} \rightarrow \Sigma^{+}K^{+} \quad \text{and} \quad \pi^{-}\Delta^{0} \rightarrow \Sigma^{-}K^{0},$$

(26)

$$\pm \frac{1}{\sqrt{3}}(M_b + M_c + 2M_d) \quad \text{for} \quad \pi^{+}\Delta^{0} \rightarrow \Sigma^{0}K^{+} \quad \text{and} \quad \pi^{-}\Delta^{+} \rightarrow \Sigma^{0}K^{0},$$

(27)

$$\pm \frac{1}{\sqrt{3}}(M_b + M_c + M_d) \quad \text{for} \quad \pi^{+}\Delta^{-} \rightarrow \Sigma^{-}K^{+} \quad \text{and} \quad \pi^{-}\Delta^{++} \rightarrow \Sigma^{+}K^{0},$$

(28)
For the $\pi\Delta \to \Lambda K$ reactions:

- $\mp (M_a + M_b + M_c)$ for $\pi^- \Delta^+ \to \Lambda K^+$ and $\pi^+ \Delta^- \to \Lambda K^0$, (29)
- $\sqrt{2}/3 (M_a + M_b + M_c)$ for $\pi^0 \Delta^+ \to \Lambda K^+$ and $\pi^0 \Delta^- \to \Lambda K^0$, (30)
- $\pm 1/\sqrt{3} (M_a + M_b + M_c)$ for $\pi^+ \Delta^0 \to \Lambda K^+$ and $\pi^- \Delta^+ \to \Lambda K^0$, (31)

where the upper and lower signs in front of the amplitudes should be assigned to the $K^+$ and $K^0$ channels, respectively.

Next we need to determine the coupling constants appearing in the Lagrangians eqs. (1) - (10). In order to determine the coupling constants and to perform the calculations, form factors (denoted by $F(q)$ and $F_{K^*(892)K\pi}(q)$ below) are introduced which represent the finite size of the hadrons. These form factors must be multiplied to each vertex of the interactions. Thus, the coupling constants are obtained from the branching ratios in the rest frame of the resonances:

$$\Gamma(N(1650) \to \Delta \pi) = 2g^2_{\pi\Delta N(1650)}F^2(q(m_{N(1650)}, m_{\Delta}, m_\pi))$$

$$\frac{m_{N(1650)}(E_\Delta - m_\Delta)}{m_\Delta^2 m_{\Delta}} q^3(m_{N(1650)}, m_{\Delta}, m_\pi), \quad (32)$$

$$\Gamma(N(1710) \to \Delta \pi) = 2g^2_{\pi\Delta N(1710)}F^2(q(m_{N(1710)}, m_{\Delta}, m_\pi))$$

$$\frac{m_{N(1710)}(E_\Delta + m_\Delta)}{m_\Delta^2 m_{\Delta}} q^3(m_{N(1710)}, m_{\Delta}, m_\pi), \quad (33)$$

$$\Gamma(N(1720) \to \Delta \pi) = 2g^2_{\pi\Delta N(1720)}F^2(q(m_{N(1720)}, m_{\Delta}, m_\pi))$$

$$\frac{(m_\Delta)}{m_{N(1720)}} \left[ \left( \frac{E_\Delta}{m_\Delta} \right) - 1 \right] \left[ 2\left( \frac{E_\Delta}{m_\Delta} \right)^2 - 2\left( \frac{E_\Delta}{m_\Delta} \right) + 5 \right], \quad (34)$$

$$\Gamma(N(1650) \to \Lambda K) = 2g^2_{K\Lambda N(1650)}F^2(q(m_{N(1650)}, m_{\Lambda}, m_K))$$

$$\frac{(E_\Lambda + m_\Lambda)}{m_{N(1650)}} q(m_{N(1650)}, m_{\Lambda}, m_K), \quad (35)$$

$$\Gamma(N(1710) \to Y K) = 2g^2_{KYN(1710)}F^2(q(m_{N(1710)}, m_Y, m_K))$$

$$\frac{(E_Y - m_Y)}{m_{N(1710)}} q(m_{N(1710)}, m_Y, m_K), \quad (36)$$
\[
\Gamma(N(1720) \rightarrow YK) = \frac{g_{KYN(1720)}^2 F^2(q(m_{N(1720)}, m_Y, m_K))}{12\pi} \cdot \frac{(E_Y + m_Y)}{m_{N(1720)}^2} q^3(m_{N(1720)}, m_Y, m_K),
\]
\[
\Gamma(K^*(892) \rightarrow K\pi) = 3 \frac{f_{K^*(892)K\pi}^2 F_{K^*(892)K\pi}^2(q(m_{K^*(892)}, m_K, m_\pi))}{4\pi} \cdot \frac{2}{3m_{K^*(892)}^2} q^3(m_{K^*(892)}, m_K, m_\pi),
\]

with
\[
F(q) = \frac{\Lambda_C^2}{\Lambda_C^2 + q^2}, \quad F_{K^*(892)K\pi}(q) = Cq \exp\left(-\beta q^2\right),
\]
\[
q(x, m_B, m_P) = \frac{1}{2x} \left[\left(x^2 - (m_B + m_P)^2\right)^2\right]^{1/2},
\]

where \(B^*, B\), and \(P\) in \(q(m_{B^*}, m_B, m_P)\) stand for the relevant resonance, the baryon and the pseudoscalar meson, respectively. \(q = q(m_{B^*}, m_B, m_P)\) satisfies \(q = |\vec{q}_B|\) with \(\vec{q}_B = -\vec{p}_B\) and \(\vec{E}_B = \sqrt{m_B^2 + \vec{p}_B^2}\). \(F(q)\) is the form factor with the cut-off parameter \(\Lambda_C\), and \(Y\) stands for either the \(\Sigma\) or the \(\Lambda\). The constant \(D\) in eqs. \((36)\) and \((37)\) should be assigned to \(D = 3\) for \(Y = \Sigma\) and \(D = 1\) for \(Y = \Lambda\), respectively. The \(K^*(892)K\pi\) vertex form factor is taken from ref. \([17]\). The calculated coupling constants and the experimental data used to determine them are given in Tables 1 and 2. We use the value of the \(g_{K^*(892)\Sigma\Delta}\) for the \(g_{K^*(892)\Sigma\Delta}\), which obtained by fitting the \(\pi^+p \rightarrow \Sigma^+K^+\) channel in the previous study \([8]\). Note that in this case an extra factor \(\sqrt{3}\) must be included due to the different normalization of the operator in isospin space \(\tau\) and \(\bar{\tau}\). In evaluating the cross sections in the center-of-mass frame of the \(\pi\Delta\) system, each coupling constants \(g_{PBB^*}, g_{K^*(892)\Sigma\Delta}\) and \(f_{K^*(892)\pi}\) appearing in eqs. \((17)\) - \((20)\) must be replaced by \(g_{PBB^*} \rightarrow g_{PBB^*} F(q(\sqrt{s}, m_B, m_P)), g_{K^*(892)\Sigma\Delta} \rightarrow g_{K^*(892)\Sigma\Delta} F((\vec{q}_f - \vec{q}_i))\) and \(f_{K^*(892)\pi} \rightarrow f_{K^*(892)\pi} F_{K^*(892)\pi}(1/2(\vec{q}_f - \vec{q}_i))\), where \(s\) is the Mandelstam variable, \(q(\sqrt{s}, m_B, m_P) = |\vec{q}_B|, \vec{p}_B = -\vec{p}_B, |\vec{q}_f| = q(\sqrt{s}, m_\Sigma, m_K)\) and \(|\vec{q}_i| = q(\sqrt{s}, m_\Delta, m_\pi)\). We use the same value for the cut-off parameter \(\Lambda_C\) fixed by the previous study of the \(\pi N \rightarrow \Sigma K\) reactions \([8]\), i.e. \(\Lambda_C = 0.8\) GeV for all resonances considered here, \(\Lambda_C = 1.2\) GeV for the \(K^*(892)\Sigma\Delta\) vertex. The parameters \(C\) and \(\beta\) in \(F_{K^*(892)\pi}\) are \(C = 2.72\) fm and \(\beta = 8.88 \times 10^{-3}\) fm\(^2\) used in ref. \([17]\).

Hereafter, we will discuss the \(K^+\) production channels only. Corresponding arguments for the \(K^0\) production channels should be valid as seen from eqs. \((21)\) - \((24)\).

The calculated total cross sections are displayed in figs. 2 (a), 2 (b) and 3, corresponding to the \(\pi^- \Delta^{++} \rightarrow \Sigma^0 K^+, \nu \pi^+ \Delta^- \rightarrow \Sigma^- K^+\) and the \(\pi^- \Delta^{++} \rightarrow \Lambda K^+\) reactions, respectively.

We discuss the results of the \(\pi\Delta \rightarrow \Sigma K\) reactions given in figs. 2 (a) and 2 (b) first. The solid lines show them without interference terms and the dashed lines with interference terms. There are four possibilities for the sign combination of the interference
terms. The largest and the smallest results for each relevant channel are shown among the four different sign combinations of the coupling constants. It turned out that the $K^*(892)$ exchange contribution is negligible for the $\pi\Delta \to \Sigma K$ reactions. Typically a square of the $K^*(892)$ exchange amplitude $|M_d|^2$ is more than one order of magnitudes smaller compared to the other contributions. (See eqs. (21) - (28).) This is different from $\pi N \to Y K$ reactions where $K^*(892)$ exchange gives the same order for the contributions as other resonances.

According to our previous study [3], total cross sections have in their peaks for $\pi N \to \Sigma K$ about 0.2 to 0.4 mb, except for $\pi^+p \to \Sigma^+K^+$, where the experimental data show about 0.75 mb. It is clear that the total cross sections for $\pi\Delta \to \Sigma K$ are in their peaks of the same order of those for $\pi N \to \Sigma K$. Furthermore, the energy dependence is also different from $\pi N \to \Sigma K$. Thus, the argument valid for $\pi N \to \Sigma K$ reactions cannot be extended to the $\pi\Delta \to \Sigma K$ reactions. Thus difference exists not only for the absolute value but also the energy dependence of the total cross sections.

Next, we discuss the results of $\pi^-\Delta^{++} \to \Lambda K^+$ displayed in fig. 3. They are also given for three cases: Without interference terms (the solid line), and with interference terms (the dashed lines). There are four possibilities for the sign combination of the interference terms. Again the largest and the smallest results among the four different sign combinations are shown in fig. 3. The total cross section at this peak position is of about a 30% of the largest channel for the $\pi N \to \Lambda K$ reactions, i.e. for $\pi^+n \to \Lambda K^+$. The energy dependence of the total cross sections for $\pi\Delta \to \Lambda K$ is rather similar to that of $\pi N \to \Lambda K$.

It should be emphasized again here that, the total cross sections for $\pi\Delta \to Y K$ to kaon production are not small at all. Furthermore, no quantitative argument for kaon production cross sections $\pi\Delta \to Y K$ so far has been given based on theoretical calculations, nor based on experimental data.

Now, we are in a position to give parametrizations of total cross sections $\pi^-\Delta^{++} \to \Sigma^0 K^+$, $\pi^0\Delta^0 \to \Sigma^- K^+$, $\pi^+\Delta^- \to \Sigma^0 K^+$ and $\pi^-\Delta^{++} \to \Lambda K^+$ which are enough to reproduce whole channel parametrizations given in eqs. (21) - (31). The channels only the $K^*(892)$ exchange gives contribution are omitted since they are negligible as mentioned before. Since the signs of interference terms cannot be fixed by experimental data, we parametrize the results obtained without interference terms (solid lines in the figures 2 (a) to 3). They are:

For $\pi\Delta \to \Sigma K$:

$$\sigma(\pi^-\Delta^{++} \to \Sigma^0 K^+) = \frac{0.004959(\sqrt{s} - 1.688)^{0.7785}}{(\sqrt{s} - 1.725)^2 + 0.008147} \text{ mb},$$ (41)

$$\sigma(\pi^0\Delta^0 \to \Sigma^- K^+) = \frac{0.006964(\sqrt{s} - 1.688)^{0.8140}}{(\sqrt{s} - 1.725)^2 + 0.007713} \text{ mb},$$ (42)

$$\sigma(\pi^+\Delta^- \to \Sigma^0 K^+) = \frac{0.002053(\sqrt{s} - 1.688)^{0.9853}}{(\sqrt{s} - 1.725)^2 + 0.005414} + \frac{0.3179(\sqrt{s} - 1.688)^{0.9025}}{(\sqrt{s} - 2.675)^2 + 44.88} \text{ mb},$$ (43)

$$\sigma(\pi^-\Delta^- \to \Sigma^- K^+) = \frac{0.01741(\sqrt{s} - 1.688)^{1.2078}}{(\sqrt{s} - 1.725)^2 + 0.003777} \text{ mb},$$ (44)
For $\pi\Delta \to \Lambda K$:

$$\sigma(\pi^{-}\Delta^{++} \to \Lambda K^{+}) = \frac{0.006545(\sqrt{s} - 1.613)^{0.7866}}{(\sqrt{s} - 1.720)^2 + 0.004852} \text{ mb,} \quad (45)$$

where, the parametrizations for $\sigma(\pi\Delta \to \Sigma K)$ and $\sigma(\pi^{-}\Delta^{++} \to \Lambda K^{+})$ given above should be understood to be zero below the thresholds $\sqrt{s} \leq 1.688$ GeV and $\sqrt{s} \leq 1.613$ GeV, respectively. These parametrizations are useful for codes which simulate kaon production since no experimental data are available. In earlier work [8] we gave already parametrizations for the reactions $\pi N \to \Sigma K$ based on similar calculations with intermediate resonances and $K^{*}(892)$ exchanges.

To summarize, we studied the $\pi\Delta \to YK$ reactions by a resonance model, and presented for the first time explicit parametrizations of the energy dependence of their total cross sections. It turned out that the t-channel $K^{*}(892)$ exchange contribution for $\pi\Delta \to \Sigma K$ is negligible. Furthermore, the contributions of the $\pi\Delta \to YK$ reactions to kaon production in heavy ion collisions are not only non-negligible, but also very different from the $\pi N \to YK$ contributions. Therefore, the $\pi\Delta \to YK$ contributions must be adequately included into the studies of kaon production in heavy ion collisions without relying on isospin arguments relating the cross sections for isospin $I = 1/2$ nucleons and the isospin $I = 3/2$ $\Delta$'s.

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Table 1: Coupling constants for the $\pi\Delta \rightarrow \Sigma K$ reactions

| $B^*$ (resonance) | $\Gamma (MeV)$ | $\Gamma_\Delta(\%)$ | $g_{B^*\Delta\pi}^2$ | $\Gamma_{\Sigma K}(\%)$ | $g_{B^*\Sigma K}^2$ |
|------------------|----------------|---------------------|----------------------|------------------------|---------------------|
| $N(1710)$        | 100            | 17.5                | $1.85 \times 10^{-2}$ | 6.0                    | $4.50 \times 10^1$  |
| $N(1720)$        | 150            | 10.0                | $1.12 \times 10^1$   | 3.5                    | 3.15                |

$6.89 \times 10^{-1}$ $6.08 \times 10^{-1}$
($\Gamma = 50$ MeV, $\Gamma_{K\pi} = 100\%$)

Table 2: Coupling constants for the $\pi\Delta \rightarrow \Lambda K$ reactions

| $B^*$ (resonance) | $\Gamma (MeV)$ | $\Gamma_\Delta(\%)$ | $g_{B^*\Delta\pi}^2$ | $\Gamma_{\Lambda K}(\%)$ | $g_{B^*\Lambda K}^2$ |
|------------------|----------------|---------------------|----------------------|------------------------|---------------------|
| $N(1650)$        | 150            | 5.0                 | $6.56 \times 10^{-1}$ | 7.0                    | $6.40 \times 10^{-1}$ |
| $N(1710)$        | 100            | 17.5                | $1.85 \times 10^{-2}$ | 15.0                   | $4.74 \times 10^1$   |
| $N(1720)$        | 150            | 10.0                | $1.12 \times 10^1$   | 6.5                    | 3.91                |

Table 1
The calculated coupling constants and the experimental data for the $\pi\Delta \rightarrow \Sigma K$ reactions.

Table 2
The calculated coupling constants and the experimental data for the $\pi\Delta \rightarrow \Lambda K$ reactions.
Figure captions

Fig. 1
The processes contributing to the $\pi\Delta \rightarrow YK$ ($Y = \Sigma, \Lambda$) reactions. The diagrams are corresponding to the different intermediate resonance states; (a) : $N(1650) I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$, (b) : $N(1710) \frac{1}{2}(\frac{3}{2}^+)$ and (c) : $N(1720) \frac{1}{2}(\frac{3}{2}^+)$, respectively.

Fig. 2 (a)
The calculated total cross sections for the $\pi^-\Delta^{++} \rightarrow \Sigma^0K^+ (\pi^+\Delta^- \rightarrow \Sigma^0K^0)$ reactions. The solid line and the dashed lines stand for the results without and with the inclusion the interference terms, respectively. Note that the largest and the smallest results are displayed for the four possibilities arising from the possible signs of the coupling constants and thus the interference terms.

Fig. 2 (b)
The calculated total cross sections for the $\pi^+\Delta^- \rightarrow \Sigma^-K^+ (\pi^-\Delta^{++} \rightarrow \Sigma^+K^0)$ reactions. See the caption of fig. 2 (a) for further explanations.

Fig. 3
The calculated total cross sections for the $\pi^-\Delta^{++} \rightarrow \Lambda K^+ (\pi^+\Delta^- \rightarrow \Lambda K^0)$ reactions. See the caption of fig. 2 (a) for further explanations.
Fig. 1

(a) \( \Lambda \rightarrow K \pi \rightarrow N(1650) \frac{1}{2}(1^-) \Delta \)

(b) \( \Lambda \rightarrow K \pi \rightarrow N(1710) \frac{1}{2}(1^+) \Delta \)

(c) \( \Lambda \rightarrow K \pi \rightarrow N(1720) \frac{1}{2}(3^+) \Delta \)

(d) \( \Sigma \rightarrow K \pi \rightarrow K^*(892) \frac{1}{2}(1^-) \Delta \)
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