Huge linear magnetoresistance due to open orbits in $\gamma$-PtBi$_2$

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Some single crystalline materials present an electrical resistivity which decreases between room temperature and low temperatures at zero magnetic field, as in a good metal, and switches to a nearly semiconducting-like behavior at low temperatures with the application of a magnetic field. Often, this is accompanied by a huge and non-saturating linear magnetoresistance which remains difficult to explain. Here we present a systematic study of the magnetoresistance in single crystal $\gamma$-PtBi$_2$. We observe that the angle between the magnetic field and the crystalline c-axis fundamentally changes the magnetoresistance, going from a saturating to a non-saturating magnetic field dependence. In between, there is one specific angle where the magnetoresistance is perfectly linear with the magnetic field. We show that the linear dependence of the non-saturating magnetoresistance is due to the formation of open orbits in the Fermi surface of $\gamma$-PtBi$_2$.

Magnetoresistance (MR) is the modification of the electrical resistance by a magnetic field. MR is a ubiquitous phenomenon in metals and semiconductors, although it is not expected to occur just considering free electrons without interactions. The electrical resistivity $\rho$ occurs due to scattering of electrons on a timescale $\tau$ and the main consequence of applying a magnetic field is spiralling the electron orbits with an angular velocity $\omega = eB/\hbar$ (with $e$ the electron charge and $m^*$ the electronic effective mass). When considering nearly free electrons, the simplest magnetic field dependence found for the magnetoresistance is quadratic, $\rho(B) \propto B^2$, obeying the Onsager’s reciprocity condition $\rho(B) = \rho(-B)$. Furthermore, the MR saturates in the high field limit ($\omega_c \tau \gg 1$) unless electron and hole numbers are close to compensate with each other, in which case it continues growing as $\rho(B) \propto B^2$. In metals with no or weak electronic correlations, a small quadratic MR saturating at high fields is indeed often observed$^{1-3}$, with a few exceptions such as the semimetal Bi and other electron-hole compensated metals$^4$. Recently, the observation of huge and non-saturating MR has been discussed in a number of materials. In particular, a linear MR has been observed in some compounds, often being susceptible to host topologically non-trivial electronic excitations.

Several narrow gap semiconducting systems show nontrivial properties in the band structure, and present topologically protected edge states$^7$. For instance, in the topological insulator Bi$_2$Se$_3$, a linear MR is observed when thinning down samples to nanometer dimensions, suggesting that the linear MR is due to surface states with linear dispersion$^8$. Bulk crystals of topological insulators with highly anisotropic surface states such as Ag$_{2+\delta}$Te/Se also show a linear MR$^{11}$. A linear dispersion under magnetic fields undergoes Landau quantization with energy levels $E_{\text{LL}}$ that depend on the square root of the magnetic field. The so-called quantum limit, where only the lowest Landau level is populated, can then be reached at less stringent conditions than with a quadratic dispersion (where $E_{\text{LL}}$ is linear with $B$) and leads to a linear MR$^{15}$. Furthermore, there are a number of metallic or semimetallic compounds showing large and sometimes linear magnetoresistances, including LaSb, LaBi, LaSb$_2$, LaAgSb$_2$, MoP, WC, NbP, TaP, NbSe$_2$, NbS$_2$, NbSb$_2$, TaSb$_2$, TmB$_4$, PtS$_2$, PdS$_2$, or WTe$_2$. Band structure calculations suggest the presence of unconventional electronic excitations in some of these systems. For instance, in absence of inversion symmetry, TaP, NbP and WTe$_2$ are proposed to be Weyl semimetals (type I in TaP and NbP with Weyl points at the same energy, type II in WTe$_2$ with tilted Weyl points$^{33,34}$).

The semimetal $\gamma$-PtBi$_2$ stands out among these compounds because of the extreme values of the magnetoresistance$^{35}$. Furthermore, band structure calculations suggest the presence of triply degenerate nodal points$^{34-38}$. This feature produces crossings with a finite Chern number and thus Fermi arcs at the surface. Such triple points were also proposed for MoP or WC$^{30,31,35,39}$, although at energies far from the Fermi level, whereas in $\gamma$-PtBi$_2$ these are expected very close to the Fermi level. Triple points have no counterpart in high energy physics, contrary to Dirac and Weyl fermions$^{37-39}$.

$\gamma$-PtBi$_2$ has a layered structure with trigonal symmetry (space group $P31m$, No.157, see Fig. 1(a)). Electronic band structure calculations$^{35,40}$ show that this compound has a Fermi surface containing multiple electron and hole sheets. ARPES$^{39,41}$ and quantum oscillation studies$^{35}$ have measured the band structure and the Fermi surface. In particular, the ARPES data$^{39}$ revealed a spin-polarized surface state with linear dispersion, which was associated to the linear MR reported in an early study$^{42}$.

Here we make detailed measurements of the angular dependent MR up to 22 T on a high quality single crystal of $\gamma$-PtBi$_2$. We apply the current in-plane ($\perp$ c-axis) and rotate the magnetic field away from the c-axis into
FIG. 1. (a) Crystal structure of the layered $\gamma$-PtBi$_2$. (b) In blue we show the X-Ray diffraction pattern of $\gamma$-PtBi$_2$ powder. Red bars show the positions of the peaks expected to appear in this compound. The asterisks mark the peaks associated to residual Bi and Bi oxides from flux growth. The field is applied at an angle $\theta = 8.3^\circ$, which is also the precise angle at which we find non-saturating linear magnetoresistance. The temperature dependence is very similar for all field orientations. There is a strong decrease in the resistivity with decreasing temperature at zero magnetic field, which turns into an increase when applying the magnetic field. The inset shows a scheme of the direction of the applied current and magnetic field.

de the plane. To analyze effects only due to the crystalline orientation to the magnetic field, we keep the field direction perpendicular to the electrical current. We observe a continuous evolution from a saturating sublinear MR for $B \parallel c$ to a non-saturating quadratic-like MR for field in the plane. The linear non-saturating MR is only observed for a specific angle of the magnetic field with respect to the $c$-axis. We show that such a near to linear MR appears at a specific angle in presence of open orbits.

We grew single crystals of $\gamma$-PtBi$_2$ using the Bi self-flux method described in Ref.35. We used the equipment described in Ref.44, in particular frit-crucibles. Powder X-ray diffraction of our crystals reveals the expected crystal structure (Fig.1(b)), together with some peaks corresponding to residual Bi flux and Bi oxides. We measured a neat $\gamma$-PtBi$_2$ single crystal platelet, oriented with the $c$-axis out of the plane (inset of Fig. 1(b)). We found a residual resistance ratio (RRR) of 100, showing excellent sample quality. To measure the MR, we used a cryostat capable of reaching about 1 K, and a fully superconducting 20+2 T magnet supplied by Oxford Instruments. We used a home-made mechanical rotator, described in the Supplementary, to modify the field angle. The current was applied perpendicular to the magnetic field (inset in Fig.1(c)). The rotator allowed an angular range covering from field along the $c$-axis ($\theta = 0^\circ$) to in plane ($\theta = 90^\circ$). The angle of the magnetic field was measured using a Hall probe.

We define $\text{MR} = \frac{\rho(B) - \rho(0)}{\rho(0)}$, with $\rho(0)$ being the resistivity at zero magnetic field and provide it in %. To find Shubnikov-de Haas oscillations, we obtain the oscillatory component $\Delta\text{MR}$ by fitting the MR data above 13 T to a low order polynomial and make the Fourier transform of $\Delta\text{MR}(1/B)$.

In Fig. 1(c) we show the resistivity as a function of temperature at different magnetic fields. We find a metallic behavior at zero field (resistivity strongly decreases with decreasing temperature) and a semiconducting-like behavior under magnetic fields (resistivity increases with decreasing temperature at low temperatures). The semiconducting-like increase saturates below about 8 K.

FIG. 2. MR up to 20 T for different field directions from $B \parallel c$ ($\theta = 0^\circ$) to $B \perp c$ ($\theta$ close to $90^\circ$). The inset shows the angular dependence of the exponent $\alpha$ obtained by fitting the MR at each angle with an empirical power law $\text{MR} = c + aB^\alpha$. The green broken line in the main figure shows as an example the fit for the data at $\theta = 89^\circ$. 
In Fig. 2 we show the transverse MR up to 20 T for different orientations of the magnetic field, from $B \parallel c$ ($\theta = 0^\circ$) to $B \perp c$ ($\theta = 90^\circ$). The highest value of the MR is of 5800% at 20 T for $\theta$ close to 90°. For magnetic fields below 5 T we always find a quadratic MR. For higher fields we observe an angular evolution of the MR, from a concave curvature (saturating MR) at $\theta = 0^\circ$ to a quadratic-like, convex curvature (non-saturating MR) at $\theta$ close to 90°. The magnitude of the MR changes by about a factor of five as a function of the angle at high magnetic fields. In between the concave and convex magnetic field dependencies we observe a perfectly linear MR at $\theta = 8.3^\circ$.

To see this, we fit the data above 5 T with an empirical power law $MR(B) = c + aB^n$. We find an increasing $\alpha$ with $\theta$ (inset of Fig. 2) up to $\alpha \sim 1.65$. At $\theta = 8.3^\circ$, $\alpha = 1$ and the MR changes from saturating to non-saturating behavior.

In Fig. 3(a) we show the quantum oscillation pattern in the MR for $\theta = 8.3^\circ$ as a function of the magnetic field. In Fig. 3(b-d) we plot the Fourier transform of the quantum oscillation signal as a function of $\theta$. Each quantum oscillation frequency $F$ is related to an extremal cross sectional area of the Fermi surface normal to the field, $A_\kappa$, through the Onsager relation, $F = (\hbar/2\pi e)A_\kappa$. In Fig. 3(e) we track the quantum oscillation frequencies as a function of the angle. At $\theta$ close to $0^\circ$ our result exactly coincides with the result in Ref.35 (taken along the same field direction). We can thus identify several frequencies, $F_\beta = 388$ T, $F_\kappa = 1225$ T and $F_\eta = 3012$ T, using the same notation. The results for finite $\theta$ are new and we use the same notation extrapolating from $\theta = 0^\circ$. We measure in a smaller field range than Ref.35. As a result two low frequency orbits (at 40 T and at 15 T) are not well defined in our data. On the other hand we can sweep the magnetic field much more slowly and thus we resolve better in our data. On the other hand we can sweep the magnetic field much more slowly and thus we resolve better in our data. On the other hand we can sweep the magnetic field much more slowly and thus we resolve better in our data.

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The calculations in Ref.35 reveal a $\gamma$ band that has spherical-like Fermi surface pockets arranged in a honeycomb lattice located in the plane and interconnected with each other through tilted necks. The lower inset of Fig. 4 presents schematically this open Fermi surface sheet. We mark by the red dashed lines two planes perpendicular to a magnetic field tilted from the $c$-axis. These contain open orbits. In each of the planes, the spheres located at one side of the honeycomb structure are connected to each other, but not with the spheres of the other side (upper left insets of Fig. 4). Electron on open orbits go into non-circular trajectories, instead of following cyclotron motion. This has a strong impact on the MR. In a metal with a single electronic band, the presence of open orbits leads to a quadratic enhancement of the MR at certain field angles, leaving a usual saturated MR at the angles where the open orbits are absent. The presence of multiple Fermi surface sheets, as in $\gamma$-PtBi$_2$, has not been considered in detail, to our knowledge. To set-up a MR model taking into account multiple sheets we first consider that, in a semimetal, the MR has a $B^2$ unsaturated behavior when the electron and hole number ($n_e$ and $n_h$) are compensated. We consider a two-band model and take electrons as free carriers with the same mobility $\mu$. The level of electron-hole compensation is given by $d = (n_e - n_h)/(n_e + n_h)$. We add a field-independent small contribution from the open orbits $\delta \sigma_0$ to the total conductivity tensor $\sigma_0$. More details are provided in the Supplementary. We can then write for the resistivity $\rho(B)/\rho_0 = \frac{\delta(1+\eta^2)^2+(1+\eta^2)^2}{\delta(1+\eta^2)^2+(1+\eta^2)^2}$, where $\eta = \mu B$.

We find that this reproduces nicely the linear MR at $8.3^\circ$ (Fig. 4) and at all other field orientations (see Supplementary). We use for this angle the parameters $d = 0.00849$, $\mu = 4630$ cm$^2/(V\cdot$s) and $|d| = 0.229$. The
(a) $\Delta MR$ (defined in the text) versus magnetic field up to 22 T at $T=1$ K and $\theta = 8.3^\circ$. (b,c) Fourier transform of $\Delta MR$ for a few values of $\theta$. Greek letters mark the oscillation frequencies giving peaks in the Fourier transforms. (d) Fourier transform of $\Delta MR$ as a function of the frequency $F$ for different $\theta$. Data are shifted along the y-axis for clarity, following the value of $\theta$. (e) Angular evolution of the quantum oscillations frequencies, with the corresponding orbits labelled by greek letters.

FIG. 4. We show as orange circles the MR at $\theta = 8.3^\circ$ up to 22 T. By the blue line we show the result of the model discussed in the text. The green dashed line shows the result of the same model, but without contribution from open orbits. Notice the saturation observed at high magnetic fields. In the lower right inset we show schematically the $\gamma$-band Fermi surface sheet. The shape is that of an undulated grid of spheres arranged in a honeycomb lattice perpendicular to the $c$ axis. The sheets are connected by necks oriented at an angle to the plane. Necks that lie behind the spheres are schematically represented by dashed lines. When the magnetic field direction is slightly tilted from the $c$ axis, open orbits may appear on two different planes (marked by red dashed lines). The corresponding open orbits are provided in the upper left panels.

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Supplemental Materials for “Huge linear magnetoresistance due to open orbits in $\gamma$-PtBi$_2$”

I. TEMPERATURE AND FIELD DEPENDENCE OF THE SHUBNIKOV-DE HAAS OSCILLATIONS

![Figure S1](attachment:image.png)

**FIG. S1.** (a,b) We show as points the amplitude of the quantum oscillations of different frequencies observed at $\theta = 8.3^\circ$ and $\theta = 89^\circ$ as a function of temperature. Lifshitz-Kosevich fits are shown as solid lines. Each of the insets shows one of the Fourier transform peaks in the quantum oscillations with increasing temperature, from $T=1$ K (red) to 6 K (blue). (c) We show as points the Dingle factor $R_D$ (in a logarithmic scale) as a function of $1/B$ for a few selected frequencies ($\gamma$ at $\theta = 8.3^\circ$, blue points, $\eta$ at the same angle, green points and $\beta$ at $\theta = 89^\circ$, light blue points). The red lines are linear fits to each magnetic field dependence. (d) Landau level index ($N_L$) as a function of $1/B$ extracted from the peaks (integer) and valleys (half integer) for one of the $\beta$ orbits at $89^\circ$. The solid line is a linear fit. The lower right inset shows the zero $1/B$ intercept, giving $N_{L,0} = 0.024 \pm 0.015$. The upper left inset shows the amplitude of quantum oscillations with frequency $F_\beta$ vs $1/B$ obtained by band-pass filtering the raw data (black points are the data and red dotted line the fit to LK formula) and used to obtain $N_L$.

To discuss the temperature and magnetic field dependence of the amplitude of the quantum oscillations, let us write down the Lifshitz-Kosevich formula:

$$\Delta MR \propto \sqrt{B} \frac{\Theta m^*_i T/B}{\sinh(\Theta m^*_i T/B)} \exp(-\Theta m^*_i T_D/B) \cos \left[ 2\pi \left( \frac{F}{B} + \gamma - \delta \right) \right]$$

(S1)

where $\Delta MR$ is the oscillatory component of the MR, $m^*_i$ the effective electron mass of band $i$, $\Theta = 2\pi^2 k_B T_D / e \hbar$ is a numerical factor and $T_D = \hbar / 2\pi k_B T_D$ is the Dingle temperature. The phase factor $\cos \left[ 2\pi \left( \frac{F}{B} + \gamma - \delta \right) \right]$ depends on the frequency $F$ (normalized to $B$), on the phase shift $\delta$, given by the dimensionality of the Fermi surface (equal to 0 in a 2D system and $\pm 1/8$ in 3D) and on the phase shift $\gamma$. $\gamma$ is related to the Berry phase $\Phi_B$ accumulated over the orbit as $\gamma = \frac{1}{2} - \Phi_B / 2\pi$.

In Fig. S1(a,b) we show the temperature dependence of the amplitude of the Fourier transforms of the quantum oscillations at $8.3^\circ$ and $89^\circ$, respectively. We obtain the quasiparticle effective mass $m^*_i$ of each orbit from the fits.
to equation S1. In Fig. S1(c) we show the Dingle factor \( R_D = \exp(-\Theta m^* T_D / B) \) as a function of \( 1/B \) for a few selected frequencies. From these fits, we obtain the quantum lifetime \( \tau_Q \) in each band. The resulting parameters are summarized in Table S1. The Fermi wavevector \( k_F \) is calculated from the frequency \( F \) with the Onsager relation and the assumption of a circular orbit: \( A = \pi k_F^2 \). We see that \( k_F \) spans a large part of the Brillouin zone. The electron effective mass \( m^* \) is very close to the free electron mass for nearly all orbits, except for the \( \beta \) orbit where it is considerably smaller, about \( 0.4 m_e \), in good agreement with previous quantum oscillation measurements\(^1\). The mean free path \( \ell \) ranges from 400 Å to about 4000 Å.

### Table S1. Fermi surface parameters obtained from the quantum oscillations for two particular angular orientations of the magnetic field. \( F \) is the frequency, \( k_F \) the Fermi wavevector, \( m^* \) the effective mass, \( \ell(\lambda) \) the quantum mean free path and \( \tau_Q \) the quantum lifetime.

| \( F(T) \) (nm\(^{-1} \)) | \( k_F \) (nm\(^{-1} \)) | \( m^*/m_e \) | \( \ell(\lambda) \) (nm) | \( \tau_Q \) (ps) |
|--------------------------|--------------------------|---------------------|--------------------------|
| 8.3 \( ^\circ \) \( \beta \) | 388 | 1.09 | 0.39\( \pm \)0.03 | 420 | 0.14 |
| \( \gamma \) | 1235 | 1.94 | 0.77\( \pm \)0.01 | 1500 | 0.51 |
| \( \eta \) | 3023 | 3.03 | 0.90\( \pm \)0.03 | 1020 | 0.26 |
| \( \tau \) | 4442 | 3.67 | 0.83\( \pm \)0.04 | 3770 | 0.73 |
| 89\( ^\circ \) \( \beta \) | 377 | 1.04 | 0.61\( \pm \)0.03 | 520 | 0.26 |
| \( \gamma' \) | 1323 | 2.00 | 1.00\( \pm \)0.01 | 1070 | 0.46 |
| \( \nu \) | 2132 | 2.55 | 0.52\( \pm \)0.01 | 870 | 0.15 |
| \( \eta' \) | 2615 | 2.82 | 0.87\( \pm \)0.03 | 1400 | 0.37 |
| \( \tau' \) | 4326 | 3.60 | 0.98\( \pm \)0.04 | 700 | 0.22 |

If a non-trivial Berry phase \( \pi \) is acquired over an orbit, a change in \( \gamma \) will occur and the phase of the quantum oscillations will be shifted. In such case, in the linear plot of the Landau index \( N_L \) versus \( 1/B \), the zero \( 1/B \) intercept of \( N_L \) will lie close to 0. In case of a topologically trivial band with no Berry phase accumulation, the zero \( 1/B \) intercept of \( N_L \) will lie inside the interval \([3/8,5/8] \). In the upper left inset of Fig. S1(d) we show the oscillation of the Landau index \( N_L \) at \( 89^\circ \) (\( F=377 \) T). Fig. S1(d) shows the Landau index \( N_L \) plotted as a function of the maximum and minimum position in \( 1/B \) for this orbit. The zero \( 1/B \) extrapolation (lower right inset) gives an intercept \( N_{L,0} = 0.024\pm0.015 \), a value quite close to zero.

### II. TWO-BAND isotropic model including contribution from open orbits

Let the \( z \)-axis be along the direction of the applied magnetic field. Semiclassically, for a free electron gas, the resistivity tensor in the transverse plane \((x,y)\) can be written as:

\[
\hat{\rho} = \begin{pmatrix}
\frac{1}{\sigma_e} & \frac{\eta_e}{\sigma_e} \\
-\frac{\eta_e}{\sigma_e} & 1
\end{pmatrix}
\]  

where \( \sigma_e \) is the zero field conductivity and \( \eta_e = \mu_e B \) with \( \mu_e \) the electron mobility. In Equation S2, \( \rho_{xx} = \rho_{yy} \), and it is independent of magnetic field. The corresponding conductivity tensor is the following:

\[
\hat{\sigma}_e = \begin{pmatrix}
\frac{1}{\sigma_e} & \frac{\eta_e}{\sigma_e} \\
\frac{\eta_e}{\sigma_e} & 1
\end{pmatrix}
\]

The conductivity tensor for an isotropic single band of hole carriers can be written in the same way, apart from an opposite sign in the off-diagonal terms. When both electrons and holes are present and considered as independent conduction channels, one needs to add them in the conductivity tensor:

\[
\hat{\sigma} = \hat{\sigma}_e + \hat{\sigma}_h = \begin{pmatrix}
\frac{1}{\sigma_e} & \frac{\eta_e}{\sigma_e} \\
\frac{\eta_e}{\sigma_e} & 1
\end{pmatrix} + \begin{pmatrix}
\frac{1}{\sigma_h} & \frac{\eta_h}{\sigma_h} \\
\frac{\eta_h}{\sigma_h} & 1
\end{pmatrix} = \begin{pmatrix}
\frac{1}{\sigma} & \frac{\eta}{\sigma} \\
\frac{\eta}{\sigma} & 1
\end{pmatrix}
\]

where \( \sigma = \sigma_e + \sigma_h \) and \( \eta = \eta_e + \eta_h \).
Inverting \( \hat{\sigma} \) leads to the field dependence of the resistivity \( \rho_{xx} \) (\( x \) being the current direction):

\[
\rho_{xx} = \frac{n_e \mu_e + n_h \mu_h + \mu_e n_h (n_e \mu_e + n_h \mu_h) B^2}{(n_e \mu_e + n_h \mu_h)^2 + (n_e - n_h)^2 \mu_e^2 \mu_h^2 B^2}
\]  \( \text{(S5)} \)

where we have used the Drude formula for conductivity, \( \sigma = n e \mu \). Equation S5 is a result discussed in Ref. 2. When the electron and hole carrier numbers come close to compensation, \( \rho_{xx} \) reaches a resonance and increases as \( B^2 \) without saturation.

In \( \gamma\)-PtBi\(_2\), we can approximate the band structure by a two-band model. In our model, we consider one band which is fundamentally different from the others, due to the topology of its Fermi surface. It has an open Fermi surface formed by sphere-like sheets interconnected with each other, giving a 2D grid in reciprocal space. Electrons can be driven into open, non-circular orbits in the grid. The open orbits extend in the reciprocal space along the intersection line (along \( x \), the current direction) between the plane perpendicular to the field (\( x,y \)), and the [001] plane. Electron dynamics in open orbits is governed only by the usual scattering processes that are present without magnetic field. Thus they lead to an additional field-independent term in \( \sigma_{yy} \). With this, we can write:

\[
\hat{\sigma} = \begin{pmatrix}
\frac{\sigma_0}{1 + \eta^2} & -\frac{d \eta \sigma_0}{1 + \eta^2} \\
\frac{d \eta \sigma_0}{1 + \eta^2} & \sigma_0 + \frac{\delta \sigma_0}{1 + \eta^2}
\end{pmatrix}
\]  \( \text{(S6)} \)

where \( \sigma_0 = \sigma_e + \sigma_h = (n_e + n_h) e \mu \) is the total conductivity at zero field. The two-band model is characterized by \( d = (\sigma_e - \sigma_h)/\sigma_0 = (n_e - n_h)/(n_e + n_h) \). The contribution from the open orbits, \( \sigma_{o.o.} = \delta \cdot \sigma_0 \) is small (\( \delta \ll 1 \)), as only a tiny proportion of the whole Fermi surface should be engaged in forming the open orbits. From Equation S6 we deduce the following expression used in the main text, by inverting the conductivity tensor:

\[
\rho(B)/\rho_0 = \frac{\delta (1 + \eta^2)^2 + (1 + \eta^2)}{\delta (1 + \eta^2) + 1 + d^2 \eta^2}
\]  \( \text{(S7)} \)

Fig. S2 (a) shows how Equation S7 can explain our MR data in the whole angular range. The classical two-band model without open orbit contributions (without \( \delta \) terms in Equation S7) gives a good account for the MR behavior at most of the angles. In particular, it explains well both the saturating behavior at \( \theta = 0^\circ \), and the quasi \( B^2 \) behavior at \( \theta \) close to \( 90^\circ \). However, at \( \theta \) around \( 8.3^\circ \), it is not possible to reproduce the observed linear or quasi linear MR with the two-band model alone. Notably, a pronounced saturating behavior in the high field limit would always appear in this model. Introducing a contribution from open orbits (\( \delta \) terms in Equation S7) successfully reproduces the MR. In particular the linear MR at \( \theta = 8.3^\circ \) is well explained. Fig. S2 (b) shows the angular evolution of the different parameters used in the fits. The steady increase of the MR with \( \theta \) in the whole angular range, together with the reversal of the MR curvature, can be associated to a steady decrease of \( |d| \) from 0.22 to 0.07 (the electron and hole number gradually approach compensation). The open orbits contribution to the conductivity \( \delta \) equals 0 except at \( \theta \) around \( 8.3^\circ \). The mobility \( \mu \) undergoes little change with the angle.

### III. ANGULAR DEPENDENCE OF THE MAGNETORESISTANCE

In Fig. S3 (a) we show the angular dependence of the MR at constant magnetic fields. We find a strong increase for \( \theta \) close to \( 0^\circ \) which ceases at about \( 8.3^\circ \), the angle at which the MR changes from a saturating to a non-saturating behavior. The dip turns into a MR(\( \theta \)) consisting of a smooth background and a small oscillatory component for \( \theta > 8.3^\circ \). We fit the background with a polynomial function of order 4 (dashed lines in Fig. S3(a)) and visualize the oscillatory dependence in Fig. S3(b). In Fig. S3(c) we plot the same data as a function of \( \tan(\theta) \). We observe that the magnitude of the oscillations in \( \Delta \text{MR}(\theta) \) increases with the magnetic field. Their position in \( \theta \), however, is independent of the magnetic field. Notice that Shubnikov-de Haas quantum oscillations may provide oscillations in the angular dependence of the MR. However, their positions in angle should depend on the field, which is not the case here. Moreover, the amplitude of quantum oscillations in our data is one order of magnitude smaller than the oscillating pattern in the angular dependence of the MR, as we can see in the inset of Fig. S3(c).

Angular dependent MR oscillations, whose position in \( \theta \) does not change with the magnetic field, have been obtained in a number of systems\(^3\text{–}11\). In particular, in layered materials, MR oscillations appear due to two-dimensional Fermi surface sheets with some warping\(^10,11\). Two-dimensional Fermi surface tubes without warping only have one orbit for the magnetic field applied exactly parallel to the tube. By contrast, tubes with a small amount of warping show
FIG. S2. (a) The open circles show the MR data at different field orientations. The dashed lines show the corresponding fits to Equation S7. (b) Angular evolution of the used parameters. The contribution from the open orbits $\delta$ appears at $\theta$ close to 8.3°. The steady increase of the MR with $\theta$ in the whole angular range is mostly associated to a steady decrease of $|d|$ from 0.22 to 0.07 (gradual approaching to an exact electron-hole compensation).

FIG. S3. (a) Angular dependence of the MR at different magnetic fields, at $T=1$ K. The vertical dotted line marks the angle where a linear MR is observed: $\theta = 8.3^\circ$. The black lines show pair polynomial fits of order 4 to remove the background and get the oscillating part in (b). (b) The oscillating part of the angular dependence of the MR at different magnetic fields, at $T=1$ K (shifted along the vertical axis for clarity). The dashed vertical lines are guides for the eyes, and show that the angles where we observe oscillations do not depend on the magnetic field. (c) $\Delta$MR as a function of $\tan(\theta)$, at different magnetic fields. The inset gives a zoom on the signal in the MR due to quantum oscillations, which is strongly magnetic field dependent.

exactly two orbits defining two extremal areas of the Fermi surface for all orientations of the magnetic field. Yamaji showed that the difference between these extremal areas becomes zero at integer values of $\tan(\theta)$. It was then shown that this causes maxima in the angular dependent MR at angles corresponding to integer values of $\tan(\theta)$. Of course, this assumes that warping follows a spherical or cosine-like form along the c-axis. In $\gamma$-PtBi$_2$, quantum oscillations do not show tube-like Fermi surface sheets associated with a 2D nature of a layered system, and there is no sign of them in calculations neither. Instead, they show 3D spherical-like Fermi surfaces sheets with complex geometries. Then the angular-dependent oscillations of the MR may come from a similar effect within a more complex band structure, associated with 3D Fermi surface sheets with a spherical-like geometry which provide orbits with exactly the same area for different angles $\tan(\theta)$. Thus, the origin of the angular dependent oscillations may come from coincidences in size between different orbits on 3D Fermi surface sheets at certain field orientations.
IV. DESCRIPTION OF THE MECHANICAL ROTATOR USED FOR THE ANGULAR DEPENDENT MAGNETORESISTANCE MEASUREMENTS

Available designs for rotators consist of gears operated through long tubes going to room temperature. Although the precision achieved with such rotators is very high, friction is often a problem and provides considerable heating at very low temperatures. A solution is to use sapphire contacts with very low friction coefficients, although high rotation speeds still produce a sizeable power. An additional difficulty is that mechanical design has to consider carefully differential contraction of the whole set-up to avoid clamping at low temperatures. Piezo-driven systems have the advantage of providing compact designs on very small sizes and are commercially available. However, the application of high frequency signals to the piezoelectrics is nearly unavoidable, which also implies heating. Particularly for high speeds, heating can be a considerable problem, easily reaching the mW range. In addition, piezomotors rely on stick-slip motion, which is difficult to achieve at low temperatures. In another design used in some laboratories, a rotator is operated through Kevlar strings attached to a spring and a shaft that goes to room temperature. This is similar to the wire based nanoscale sample positioning system described in Ref.20, which allows for precise cryogenic motion at low temperatures. Here we describe a rotating sample stage devoid of frictional heating. The rotator is operated by pulling on a string that goes to the top of the experiment, through a bellow at ambient temperature. The string is thermalized at each stage of the refrigerator and is maintained straight by using a spring connected to the support mechanism. At the level of the rotator, there is another spring connected to the support. It pulls the rotator back to the initial position when the tension in the string is released. The rotator is mounted with a copper cylinder as the rotation axis. The copper cylinder is firmly anchored at the end of the support. The pieces surrounding the cylinder are of teflon to allow for smooth rotation without generating heat. We have built rotators in Macor, Aluminum and plastic, details and drawings are provided in Ref.21.

To make the resistivity measurements, we use a four probe configuration and a conventional lock-in amplifier to measure the voltage. To apply the current, we use a Howland pump circuit. We apply a voltage to the sample through a resistor $R$. The voltage is sensed at the exit of $R$ by an operational amplifier and maintained constant, independently of the impedance of the sample, by a feedback driven through another operational amplifier. The current range can be modified by modifying $R_s$ with a switch. The current range is given by the input voltage and the resistor $R$. The impedance of the source increases as the inverse of the precision in the resistors used. Thus, the sensitivity to modifications in the current is considerably improved with respect to the use of a simple resistor in series and a voltage source. For instance, using 0.01% resistors implies source impedances multiplied by two orders of magnitude. The bandwidth of the circuit is limited by the operational amplifiers and a capacitor inserted parallel
to \( R \) and is in any case above 10kHz.

\[ \text{(1)} \]

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Drawings and circuit details are available at OSF.