Nonlinear correction of 2-D PSD for pulsive excitation based on steady-state response results

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Abstract. Position sensitive detectors (PSDs) have been spotlighted as effective position sensors in terminal navigation of laser semi-active guided weapons. However, it remains an ongoing challenge to improve linearity, especially under impulsive excitation. In this paper, the response of the PSD for continuous light excitation is proposed for the first time to correct the PSD response results under pulsive excitation. After establishing the mathematical model of the two-dimensional PSD, the response results under different illumination excitation are simulated and analysed, which theoretically proves the feasibility of the correction method in this paper. Finally, the experiment is carried out with PSD (S5990-01). The results reveal that the distortion degree of the PSD response results is more obvious with the decrease of pulse width under pulse illumination. The impulse response result can be linearly mapped into a steady-state response result with the range of slope values being 0.403~0.614 and intercept values being -0.098~0.106.

1. Introduction

Recently, position sensitive detectors (PSDs) are fundamental to the measurement of an object displacement, especially in hard-to-reach situations [1, 2, 3]. One of the greatest challenges in their practical application is the nonlinearity in the PSDs’ response character [4]. As such, improving the linearity of PSDs still require further research. The main correction methods may be classified into hardware and software improvement [5, 6, 7]. To be specific, hardware improvement mainly includes PSD structure optimization and parallel connection of linear elements in the circuit to improve signal linearity [5]. Software improvement works in the PSD output signal processing by varying algorithm. Compared to the former, this method has great promotion potential because it is cost-effective and can avoid other problems caused by complex structures. Typical correction methods include look-up table correction, interpolation correction, and BP neural network-based algorithm. These methods all need to collect a large amount of training data from the calibration process in advance, which is time-consuming. In addition, cumbersome processes and slow processing speed make them impractical in real implementations that require fast response like autonomous navigation.

The terminal guidance distance of laser semi-active guided weapons reaches 2~3km. Long-time irradiation to a target requires a large emission power of a continuous wave laser which is difficult to meet in practice. Therefore, it is generally wiser to use a pulsed laser in laser semi-active guidance systems. However, it has been demonstrated that in the impulse excitation, PSD display more serious
nonlinearity than in continuous light excitation [2], while it is feasible to carry out a non-linear correction on PSD under the condition of steady excitation.

In this paper, after a brief review of the 2-D tetra-lateral PSD mathematical model, numerical simulations are carried out to analyze the response characteristics for pulsed excitation. In order to further explore the relationship between the response characteristic in a real implementation, an experiment has been conducted, and the result shows the impulse response result can be linearly mapped into a steady-state response result. Finally, conclusions are drawn.

2. Mathematical model and simulation

The basic structure of a PSD is similar to that of a photodiode. The general manufacturing method is to diffuse or inject impurities on the surface of the semiconductor substrate to form a P-N junction [8]. When the photosensitive layer of the PSD is illuminated unevenly, a potential difference is formed in the direction parallel to junction plane due to the lateral photoelectric effect, and then photo-generated current is shunted in the diffusion layer. Once the current is collected by the four electrodes, the position of the incident spot can be easily calculated via an intuitive formula. The current output from electrodes is related to the center of gravity of the incident light spot [2]. Therefore, the center of gravity of incident light spot can be continuously and directly detected according to the output current.

The schematic of a 2-D PSD is shown in Fig. 1(a), and Fig. 1(b) shows the equivalent circuit diagram of the 2-D PSD under laser irradiation. For simplicity, we take a PSD micro-element as the research object which is modeled with RC-transmission line shown in Fig. 1(c). The circuit elements in the model are defined as follows: $R$ is the series resistance per unit length in the resistance layer, $R_s$ is the junction leakage resistance per unit length, $C$ is the junction capacitance per unit area of the depletion layer, and $I(x, y, t)$ is the photocurrent which is dependent on the wavelength and the intensity of the illuminated light. Assume this microelement is constructed between $x$ two $x + \Delta x$ and $y$ to $y + \Delta y$.

The voltage difference caused by the current $i$ flowing through the resistance $R$ can be written as

$$\frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} = -iR \quad (1)$$

In a similar manner, the difference in current between the two ends of the section can be:

$$-(\frac{\partial i}{\partial x} + \frac{\partial i}{\partial y}) = C \frac{\partial V}{\partial t} + \frac{V}{R_s} - I(x, y, t) \quad (2)$$

Equation (1) and (2) can be transformed into the following second-order differential equation:

$$\frac{\partial V}{\partial t} = \frac{1}{RC} \left( \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} \right) - \frac{V}{CR_s} + \frac{1}{C} I(x, y, t) \quad (3)$$

It is assumed that the load resistance at both terminals is negligible compared to the resistance $Rs$ and $R$. We can consider that four terminals are short circuit at all ends. So, the initial conditions and boundary conditions are:

$$(V)_{t=0} = 0, \quad (V)_{x=0,L} = 0 \quad \text{and} \quad (V)_{y=0,L} = 0 \quad (4)$$
The solution that satisfies the initial and boundary conditions can be expressed by Fourier series, and assume that the excitation function is odd and periodic over the length $L$. After derivation, when $R_S \gg R$, the final solution will be

$$V(x, y, t) = \frac{4}{C L} \left( \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sin \left( \frac{n \pi x}{L} \right) \sin \left( \frac{m \pi y}{L} \right) \times \int_0^L \exp \left[ -\frac{(n^2 + m^2)(\mu - t)}{C R L^2} \right] d\mu \times \int_0^L I(\lambda, \alpha, t) \sin \left( \frac{m \pi \lambda}{L} \right) \sin \left( \frac{n \pi \alpha}{L} \right) d\lambda d\alpha \right)$$

Which is an instantaneous voltage expression excited by a time and spatially dependent pulse photocurrent.

We consider the condition where the incident excitation covers only a very small fraction of the junction. In this case, the photocurrent can be written as:

$$I(x, y, t) = \left\{ I_0 \delta(x - x_i) \delta(y - y_i) [U(t) - U(t - T_p)] \right\}$$

Where $I_0$ is a constant maximum photocurrent, $T_p$ is the pulse duration, $(x_i, y_i)$ is the incident spot coordinates, and the function $U(t - T_p)$ is the function of $U(t)$ delayed by $T_p$ which is defined as:

$$U(t - T_p) = \begin{cases} 0 & t \leq T_p \\ 1 & t > T_p \end{cases}$$

The voltage at any position $(x, y)$ can be written as:

$$V(x, y, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{4 I_0 R}{(n^2 + m^2)\pi^2 \sin \left( \frac{n \pi x}{L} \right) \sin \left( \frac{m \pi y}{L} \right) \sin \left( \frac{m \pi y}{L} \right)} \times \left[ 1 - \exp \left( \frac{-i \omega_m \tau_0}{\kappa} \right) \right] U(t) \left( 1 - \exp \left( \frac{-i \omega_m \tau_0}{\kappa} \right) \right) U(t - T_p)$$

After derivation, these currents flowing through the $x$-axis and $y$-axis are:

$$i_x = \frac{1}{R} \int_0^L \int_0^L \left( \frac{\partial V}{\partial y} \right)_{x} \ dx dy = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{8 I_0}{(n^2 + m^2)\pi^2} \sin \left( \frac{n \pi x}{L} \right) \sin \left( \frac{m \pi y}{L} \right) \left[ 1 - \exp \left( \frac{-i \omega_m \tau_0}{\kappa} \right) \right] U(t) \left( 1 - \exp \left( \frac{-i \omega_m \tau_0}{\kappa} \right) \right) U(t - T_p)$$

$$i_y = \frac{1}{R} \int_0^L \int_0^L \left( \frac{\partial V}{\partial x} \right)_{y} \ dx dy = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{-8 I_0}{(n^2 + m^2)\pi^2} \sin \left( \frac{n \pi y}{L} \right) \sin \left( \frac{m \pi y}{L} \right) \left[ 1 - \exp \left( \frac{-i \omega_m \tau_0}{\kappa} \right) \right] U(t) \left( 1 - \exp \left( \frac{-i \omega_m \tau_0}{\kappa} \right) \right) U(t - T_p)$$

In order to simplify the calculation in practical application, the traditional Equation is given as follows:

$$x = \frac{L}{2} \left( 1 - \frac{i_1 - i_2}{i_1 + i_2} \right), \quad y = \frac{L}{2} \left( 1 - \frac{i_3 - i_4}{i_3 + i_4} \right)$$
In order to analyze the response of the PSD under different illumination conditions, we have done a numerical simulation analysis on Equation (9). Under continuous light excitation or $T_p >> T_c$, the response of the 2-D PSD can reach a stable value (Fig. 2(a)) when the collection time is long enough. Specifically, when the collection time $t$ is greater than or equal to the time constant $T_c$, the 2-D PSD can theoretically achieve a linear response. In the case of long pulse illumination ($T_p = 3T_c$), however, the current difference can reach a maximum value only within a certain period of time (Fig. 2(b)). But in the case of extremely short pulse illumination ($T_p = 0.01T_c$), the response displays obvious nonlinearity (Fig. 2(c)). This is because the current collection time is too short to allow the current to reach its maximum value.

![Figure 1](image_url)

**Figure.** 1 (a) the schematic of a 2-D PSD, (b) the equivalent circuit diagram of the 2-D PSD, (c) a PSD micro-element model.
Figure. 2 (a) the steady-state response of the 2-D PSD, (b). The steady-state response of the 2-D PSD when $T_p=3T_c$, (c) the steady-state response of the 2-D PSD when $T_p=0.01T_c$.

In addition, it is found that when the pulsed laser of 1064nm is used to illuminated the boundary of a tetra-lateral PSD (2.5mm×2.5mm), and adjust $T_p$ to 100us, 25us, 2.5us, 1us, 0.5us respectively, and the calculated value of the distance from the edge to the PSD center becomes smaller as the pulse width becomes smaller, which means the solution value has a large distortion. Therefore, it is necessary to study the nonlinear correction of the 2-D PSD under extremely short pulses.

3. Experimental setup

The exact position during the correction process is difficult to determine, but the steady-state response of the 2-D PSD shows higher linearity after correction. In order to correct the response under the condition of extremely short-pulse excitation, this paper presents use the response of PSD for continuous light excitation to correct the PSD response results under pulsive excitation.

The schematic diagram of the experiment is shown in Fig.3. In the experiment, we used a tetra-lateral PSD S5990-01 from Hamamatsu, Japan, which offers a photosensitive area of 2mm ×2mm. In the range that is 80% from the center to the edge, the position detector errors are typically ±70 μm and maximally ±150 μm according to the datasheet. In addition, we built up a two-axis rotary table, which can make the PSD connected with it to do pitch motion or yaw motion. Also, an amplifying circuit was designed to amplify four output current signals of the PSD, and then by connecting this circuit with an oscilloscope, we can achieve the visualization of the output current signals. A signal generator with adjustable duty cycle provided an input signal for a modulated laser with 650nm wavelength so that the laser can output continuous or pulse laser with different duty cycles.
Figure 3. The schematic diagram of the experiment.

The diagonal of the PSD photosensitive surface is kept parallel to the horizontal direction of the two-axis turntable, and the laser is fixed. In test 1, with the center of the photosensitive surface of the PSD as the reference, keep the pitch turntable at a fixed angle, and the yaw angle of the PSD is adjustable so that the laser can scan a straight line on the photosensitive surface of the PSD. By changing the input signal of the signal generator to the laser, continuous and pulsed light are respectively irradiated at each fixed position. By solving the output current of the PSD, we obtain two typical straight lines, as Fig.4(a). Similarly, in test 2, keep the yaw angle turntable at a fixed angle and change the pitch angle of the PSD. The light spot response results are shown in Fig.4(d).

As can be seen from Fig. 4(a) and Fig.4(d), the response range of the PSD under pulse excitation is about half narrower than that under continuous light excitation, which means the PSD's response to the pulsed laser is obviously indented compared with continuous laser. In order to further analyze this response indentation phenomenon, the x-values of the calculated light spot (Fig. 4(b) and Fig. 4(e)) under the two conditions of continuous light excitation and pulsed light excitation are studied. Similarly, Fig. 4(c) and Fig. 4(f) shows the corresponding relationship between the y-values of the light spots calculated under these two conditions.

The relationship between the x-values which is under continuous light excitation and those in pulsed light excitation shows a perfect linear relationship. This relation can also be found from the corresponding relation of y-values of light spots. The specific fitting results are shown in Table 1. True and $b_{true}$ respectively represent the slope and intercept of the linear fitting results to the experimental results. The other four parameters ($k_{max}$, $k_{min}$, $b_{max}$, and $b_{min}$) are the slopes and intercepts of the linear fitting results when considering the inherent error of the PSD. To be specific, the measurement error is $\pm 0.15$mm within 80% of the response center area. Assuming that the fitting straight line is $y=kan+b$ when the value range of $k$ is $(0.403–0.614)$ and the value range of $b$ is $(–0.098–0.106)$, the impulse response result can be better mapped into the response result of continuous light.
Figure 4. (a) position response results of light spots in test 1, (b) position response results of light spots in test 1 in x-direction, (c) position response results of light spots in test 1 in y-direction, (d) position response results of light spots in test 2, (e) position response results of light spots in test 2 in x-direction, (f) position response results of light spots in test 1 in y-direction.

Table 1. Fitting results

|        | k_max | k_true | k_min | b_max | b_true | b_min |
|--------|-------|--------|-------|-------|--------|-------|
| test1_x | 0.507 | 0.436  | 0.403 | 0.106 | -0.021 | -0.059 |
| test1_y | 0.614 | 0.510  | 0.422 | 0.030 | -0.021 | -0.083 |
| test2_x | 0.529 | 0.467  | 0.432 | -0.019| -0.031 | -0.098 |
| test2_y | 0.530 | 0.477  | 0.434 | 0.048 | -0.003 | -0.059 |

Figure 5. (a) Position response results of light spots, (b) position response results of light spots in x-direction, (c) position response results of light spots in y-direction.

To verify this law, we scanned a straight line from top to bottom in the left photosensitive surface of the PSD, as shown in Fig. 5(a). Theoretically, the response results of continuous light excitation and pulsed excitation in Fig. 5 (a) should be vertical straight lines, but due to installation errors, the scanning results are slightly tilted. In addition, two points deviate slightly from the straight line, which is caused
by measurement errors. Although the x values obtained under these two conditions are somewhat concentrated, we can still obtain a fitting straight line conforming to the above rules, where $k=0.476$, and $b=-0.035$. However, under the two different excitations, the outputs in the y-direction show a very good linear relationship, and the specific fitting result is $k=0.490$ and $b=0.012$.

4. Conclusion
In the terminal guidance phase of laser semi-active weapon, the PSD can be used as a position sensor to detect the position of the target. Considering the limiting factors such as emission power, pulse laser is often used. However, the response of the PSD to pulsed light is nonlinear due to the inherent error and installation error of the sensor. The position response results have a large distortion as the pulse width becomes smaller. However, the steady-state response of the PSD shows good linearity. In this paper, we proposed that the response result of the PSD under continuous light excitation can be used to correct the response result of pulsed light when it is not convenient to obtain the real position of light spot in practical engineering. And from the experiment it is proved that for the PSD model S5990-01, the response result of a pulsed laser with a pulse width of 1us and repetition frequency of 50 kHz is reduced by about half compared with that of continuous light excitation. Considering the actual measurement error, the impulse response result can be linearly mapped into a steady-state response result with the range of slope values being 0.403–0.614 and intercept values being -0.098–0.106.

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