Credit Constraints and the Inverted-U Relationship Between Competition and Innovation

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Empirical studies have uncovered an inverted-U relationship between product-market competition and innovation. This is inconsistent with the original Schumpeterian model, where greater competition always reduces the profitability of innovation and thus the incentives to innovate. We show that the model can predict the inverted-U if the innovators’ talent is heterogeneous and asymmetrically observable. When competition is low and profitability is high, talented innovators are credit-constrained, since untalented innovators are eager to mimic them. As competition increases and profitability decreases, untalented innovators become less eager to mimic, and talented innovators can invest more. This generates the increasing part of the relationship. When competition is high and profitability is low, credit constraints disappear, and the relationship is decreasing. Our theory generates additional specific predictions that are well borne out by the existing evidence.

INTRODUCTION

Several empirical studies have uncovered an inverted-U relationship between product-market competition and innovation.¹ This finding is inconsistent with the original Schumpeterian model (Aghion and Howitt 1992), where stronger competition always reduces the incentives to innovate because it reduces post-innovation rents (the so-called ‘Schumpeterian effect’). To address this inconsistency, Aghion et al. (2005) have modified the original model to allow for innovation by established firms, who also care about pre-innovation rents. These firms are crucial to explaining the increasing part of the inverted-U, because it is only for them that competition may strengthen the incentives to innovate (by decreasing pre-innovation rents more than it decreases post-innovation rents).

While the mechanism in Aghion et al. (2005) is intuitive and easy to believe, we suspect that it may not fully account for the existence of an inverted-U. On the one hand, a vast majority of innovations are actually realized by either new entrants or established firms innovating on entirely new product lines—for both of which competition in the target market should primarily affect post-innovation rents, as in the original Schumpeterian model.² This casts doubt on whether the increasing part of the inverted-U can entirely be explained by the actions of firms focused on pre-innovation rents. On the other hand, some evidence of a positive relationship between competition and innovation has also been found for start-ups,³ which again are less likely to fit the notion of firms focused on pre-innovation rents.

In this paper, we show that the Schumpeterian model can predict the inverted-U even under the original assumption that innovators focus on post-innovation rents. Only two reasonable ingredients must be added to that effect: heterogeneous talent of innovators, and asymmetric information on talent. We start from a standard version of the model with overlapping generations and a fringe of competitive producers (as in Aghion and Howitt 2009), and allow for innovators to be of two types (talented and untalented), and for this to be the innovators’ private information. We construct a separating equilibrium...
in which the talented innovators signal themselves to investors by contributing their entire wage in equity, and by limiting the amount that they borrow. We study the comparative statics of this equilibrium, and show that the relationship between the strength of competition and the probability of innovation is first increasing and then decreasing.

In more detail, our mechanism works as follow. At low levels of competition, when post-innovation rents are high, the talented innovators would like to invest a lot. However, they cannot borrow enough at favourable conditions, since the untalented innovators are eager to mimic them (given the high profitability of innovation). They then invest less than is optimal. As competition increases and post-innovation rents decrease, the untalented innovators become less eager to mimic. As this happens, the amount that the talented innovators can borrow at favourable conditions increases, leading them to invest more. This explains the increasing part of the curve. We call this effect the selection effect, because it leads to a higher weight of the talented innovators in overall investment. At high levels of competition, when post-innovation rents are low, the talented innovators would like to invest only a modest amount. Moreover, they can borrow a lot at favourable conditions, since the untalented agents are not eager to mimic them. They then invest their optimal amount, which by the Schumpeterian effect is decreasing in the strength of competition. This generates the decreasing part of the curve.

One attractive feature of our model is that it rests on reasonable assumptions. We have already argued that innovators focused on post-innovation rents are an empirically relevant group. In addition, talent heterogeneity and asymmetric information are recognized features of the market for innovation financing. Hubbard (1998) and Brown et al. (2009) argue that asymmetric information is likely to be a particularly severe problem for R&D-intensive firms. This is not just because of the inherent difficulty of evaluating frontier research, but also because innovative firms are reluctant to reveal their ideas to investors, reducing the quality of the signal that they can make about potential projects (Lerner and Hall 2010, p. 614). Both Lerner and Hall (2010) and Kerr and Nanda (2015) list asymmetric information on the quality of projects as one of several key sources of frictions in the market for innovation financing. Not only are talent heterogeneity and asymmetric information reasonable assumptions; it is also the case that the credit constraints that these frictions generate are important enough to explain macro patterns such as the inverted-U. For example, Brown et al. (2009) argue that most of the unprecedented 1990s R&D boom can be explained with a relaxation of the credit constraints of young R&D-intensive firms.

To provide corroborating evidence in support of our mechanism, we show that the equilibrium that we characterize has specific features that match the evidence well. First, the increasing relationship between competition and innovation should be more pronounced in industries where credit constraints are more prevalent. This prediction fits the finding in Aghion et al. (2004), which shows that the inverted-U shifts to the right if one focuses on firms that are under more debt-pressure. Second, credit constraints should be more severe in industries where profits are higher (either because competition is lower, or for other reasons). Indeed, several papers in the finance literature have documented a positive relationship between credit constraints and return on equity. Interestingly, Li (2011) shows that this relationship is particularly important among R&D-intensive firms, where our mechanism is also likely to be particularly important.

This paper contributes to Schumpeterian growth theory (for a survey, see Aghion et al. 2014). It complements the main existing explanation for the inverted-U relationship between competition and innovation (Aghion et al. 2005) by showing that

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Schumpeterian theory is consistent with the inverted-U under a broader (and as discussed above, more empirically relevant) set of assumptions. Other theoretical explanations of the inverted-U focus squarely on issues of industry organization and dynamics, leaving virtually no role for financial factors. Conversely, our model puts asymmetric information in financial markets at its front and centre. This simple, realistic addition to an otherwise standard model allows us to generate the ‘inverted-U’ pattern through an intuitive mechanism.

A number of other papers in the Schumpeterian tradition have placed financial features at centre stage. These papers differ from ours in their assumptions (usually, and most importantly, the nature of the financial frictions that they consider) as well as their subject matters and applications. For example, Diallo and Koch (2018) investigate the relationship between economic growth and bank concentration; Malamud and Zucchi (2016) study corporate cash management when firms face exogenous financing costs; and Sunaga (2019) extends the standard model to deal with moral hazard in financial markets and monitoring by intermediaries. Bryce Campodonico et al. (2016) and Plehn-Dujowich (2009) develop Schumpeterian growth models with adverse selection in financing, but use them to study optimal tax policy and to quantify the reduction in the rate of growth stemming from the presence of financial frictions, respectively. Finally, Ates and Saffie (2013) study a general equilibrium endogenous growth model in which financial intermediaries screen the quality of projects from a heterogeneous population of entrepreneurs. None of these papers concerns itself with the relationship between an industry’s degree of competition and its R&D outcomes, which is the main focus of the present paper.

Finally, the paper also relates to the burgeoning literature on the macroeconomic implications of financial frictions (see Brunnermeier et al. 2013), and more specifically the branch analysing their effects on countries’ economic development (see Levine 2005).

The paper is organized as follows. In Section I we present the baseline model. Section II introduces imperfect information in financial markets, and derives the inverted-U relationship between competition and innovation. Section III discusses the empirical validity of two predictions that are specific to our model. Finally, Section IV concludes.

I. BASELINE MODEL

The baseline model is a standard Schumpeterian model with overlapping generations and a fringe of competitive producers (as in Aghion and Howitt 2009, pp. 130–2, 90–1), which we generalize to allow for heterogeneous talent of innovators. A final good is produced competitively using labour and a continuum of intermediate goods, according to the production function

\[ \frac{Y_t}{X_{it}} = L^{1-\xi} \int_0^1 A_{it}^{1-\xi} X_{ii} d\xi, \]

where \( X_{it} \) is input of the latest version of intermediate \( i \), and \( A_{it} \) is its productivity. Each intermediate is produced and sold by a monopolist, who can produce one unit of the intermediate at the cost of one unit of the final good. However, in each industry, there is also a fringe of competitive firms that can produce the intermediate at a cost of \( 1/\kappa_i \) units of the final good per unit produced. The parameter \( \kappa_i \in [\xi, 1] \) measures the strength of
competition faced by the monopolist. As will become clear below, \( \kappa_i = \alpha \) denotes the case of no competition, while \( \kappa_i = 1 \) denotes the case of perfect competition. For simplicity, all industries have the same initial level of productivity, \( A_{t-1} \equiv \int_0^1 A_{\beta-1} \, d\beta \).

Agents live for two periods, are risk neutral, and have a discount factor equal to 1. There are two equally-sized cohorts alive in each period, the young and the old. The young work in the final good sector, where they earn a wage. Before turning old, one of them per industry (the ‘innovator’) tries to invent a new version of the intermediate good that is \( \gamma > 1 \) times more productive than the previous version. If successful, then she invests in the production of the new version, which she sells as the monopolist when she turns old in the next period. If unsuccessful, then a young agent is chosen at random to invest in the production of the previous version, and to sell it as the monopolist when he turns old. As for the old agents, there is one of them in each industry who is the current monopolist, while all others are idle consumers.

There are borrowers and lenders in this model. Borrowers include young agents undertaking an investment—be it innovation or production—that they cannot fund through the wage that they have earned. The lenders are all young agents, who may want to use part of the wage that they have earned to consume when they are old. While production is a risk-free activity, innovators pay back only if successful. Then the financing of innovation is the only interesting part of the financial market. We assume that the maximum supply of credit (the total wage bill) is greater than demand, so that the risk-free interest rate is equal to the discount rate (zero).

A monopolist faces isoelastic demand \( P_t = \alpha(A_{t-1}L/X_t)^{1-\alpha} \), given which her optimal price is \( 1/\alpha \). However, facing competition from the fringe, the monopolist is forced to charge \( 1/\kappa_i \leq 1/\alpha \) instead. Plugging back in the demand function, we find the optimal \( X_{it} \), which can then be multiplied by profit per unit, \( (1-\kappa_i)/\kappa_i \), to find total profits. These are (normalized by initial productivity)

\[
\pi(\kappa_i) = \frac{1 - \kappa_i}{\kappa_i} (\kappa/\alpha)^{1/(1-\alpha)} L
\]

in an industry that has not innovated, and \( \gamma \pi(\kappa_i) \) in an industry that has. It is easy to show that \( \pi \) is decreasing in \( \kappa_i \in [\alpha, 1] \): intuitively, the stronger the competition, the lower the monopolist’s profits.

Substituting optimal \( X_{it} \) in the production function, differentiating with respect to \( L \), and dividing by \( A_{t-1} \), we find the normalized wage:

\[
w = (1-\alpha)(\kappa\alpha)^{\alpha/(1-\alpha)},
\]

where \( \kappa \equiv \int_0^1 \kappa_i \, d\beta \) is the average level of competition in the economy.

Innovators can be of two types: a high type (H) and a low type (L). If an innovator of type \( J \in \{H,L\} \) invests a normalized amount \( z \) in research, she is successful with probability \( a'(\mu(z)) \), where \( \mu \) is an increasing and concave function satisfying standard conditions, and \( a^H > a^L \). In each generation, there is an equal share of high types and low types.

For now, we assume that an innovator’s type is perfectly observable to everyone. Then competing lenders demand interest rate \( 1/[a'(\mu(z))] - 1 \) from type \( J \), given which the innovator’s net present value is

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where $e$ is her normalized equity contribution. With perfect information, the innovator’s net present value does not depend on her choice of financing, since the expected cost of both equity and external financing is equal to the risk-free interest rate. Let $\tilde{z}_j^I$ denote optimal, perfect-information investment by type $J$ in industry $i$. This is the level of investment that maximizes the $npv_J^i(z)$ function, and is thus implicitly defined by the condition

$$a^I \mu(\tilde{z}_j^I) \gamma \pi(\kappa_i) = 1.$$  

Condition (5) clarifies that there are two (and only two) reasons why $z$ may vary across industries. First, industries may differ in the type of their innovators, and, ceteris paribus, the high types always invest more than the low types. For example, if industry $i$ differs from industry $j$ only for having an innovator of the high type, then the two industries will invest $\tilde{z}_H^i$ and $\tilde{z}_L^j$, and $\tilde{z}_H^i > \tilde{z}_L^j$. Second, industries may differ in terms of the strength of competition within them. For example, if $i$ and $j$ differ only in the fact that $\kappa_i > \kappa_j$, then $\tilde{z}_L^i < \tilde{z}_L^j$: investment in the more competitive industry will be lower. The latter is the well-known Schumpeterian effect of competition on innovation: by reducing the profitability of innovation, stronger competition reduces the incentives to innovate. Because the Schumpeterian effect is the only effect existing at all levels of competition, the original Schumpeterian model predicts a monotonically decreasing relationship between competition and innovation.

Figure 1 illustrates. We will use the functional form and parameters used in this figure (and reported in the caption) as a running example in the remainder of the paper. The three panels differ only in the value of $\kappa_i$, which is increasing from top to bottom as reported on the left of the figure. By our choice of $z$, $\kappa_i$ is required to be in the range 0.4–1. Panel (a) then represents the extreme of no competition (the successful innovator is a true monopolist), while panels (b) and (c) progressively increase competition. The $npv_J^i(z)$ functions are represented by the thin solid curves (all other curves should be ignored for now). Investment choices under perfect information, $\tilde{z}_H^i$ and $\tilde{z}_L^i$, maximize these functions, and the related payoffs are represented by solid dots. In all panels, the high types invest more than the low types, as illustrated by $\tilde{z}_H^i$ being to the right of $\tilde{z}_L^i$. The Schumpeterian effect is clearly visible from the figure: as we move from panel (a) to panel (c), due to increasing competition, the $npv_J^i(z)$ functions rotate inwards, and investment by both types decreases.

II. ASYMMETRIC INFORMATION IN FINANCIAL MARKETS

We now assume that the innovator’s type is the innovator’s private information. Lenders must then determine the interest rate based solely on the subset of information that is observable, that is, the sizes of the proposed investment ($z$) and equity contribution ($e$). In this section we describe, in an intuitive way, a specific separating equilibrium, which happens to exist when parameters are as in our running example (the one drawn in Figure 1). In the Appendix, we formally derive the separating equilibrium (first
Figure 1. Illustration of the separating equilibrium. The three panels differ only by the size of $\kappa$, which increases from top to bottom as indicated to the left of the figure. The functional form and other parameters used are $\alpha = 0.4, L = 100, \kappa = 0.7, \mu(z) = 0.22\sqrt{z}, a^H = 1, a^L = 0.4$. 

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subsection), we identify the parameter space such that the equilibrium exists (second subsection), and we show that, in that parameter space, the equilibrium has an attractive feature: its outcome is the only one that can ‘reasonably’ realize in a perfect Bayesian equilibrium that survives a standard refinement criterion (third subsection).

Description of the equilibrium

Refer again to our running example (Figure 1). To describe the equilibrium, we fix competition at the level of panel (a). Later, we will conduct comparative statics by increasing competition to the levels in panels (b) and (c).

Recall that with perfect information, the high and low types invest \( b_i^{HZ} \) and \( b_i^{ZL} \), respectively. It is easy to show that with imperfect information, the low types must continue to invest \( b_i^{ZL} \) at a separating equilibrium, contributing any \( e \leq \hat{z}_i^{ZL} \) in equity.\(^{10}\) However, the high types may now be forced to invest less than in the perfect-information case.

To see why, consider one reasonable scenario in which the high types would be able to invest \( \hat{z}_i^{ZH} \): suppose that the lenders believed that anyone offering to contribute their entire labour income in equity (\( w \)) is a high type. Such a belief would allow the high types to borrow \( \hat{z}_i^H - w \) at their perfect-information rate, which would mean that they are able to finance any level of investment at the risk-free interest rate (in expectations). Then their net present value would still be \( npv_i^H \), and they would choose to invest \( \hat{z}_i^H \) as with perfect information. However for this to be an equilibrium, the low types should not want to mimic the high types. But in the example of panel (a) of Figure 1, the low types would indeed want to mimic the high types.

To see why, note that by contributing \( w \) in equity, and borrowing \( z - w \) at the high types’ perfect-information rate, the low types would receive net present value

\[
npv_i^L(z) = a^L\mu(z) \left[ \gamma \pi(k_i) - \frac{1}{a^H\mu(z)} (z - w) \right] - w
\]

This formula generates the dashed lines appearing towards the bottom of each panel of Figure 1. Each of these lines is higher than \( npv_i^L \) in the range where it is defined, because by mimicking the high types, the low types can pay less than the risk-free interest rate (in expectations) on external borrowing. Then investment gives them a higher payoff than under perfect information. Given this higher payoff, it is optimal for them to mimic the high types and propose to invest \( \hat{z}_i^H \), even though this also requires them to propose to contribute \( w \) in equity. In terms of Figure 1, this can be seen from the fact that \( \bar{npv}_i^H(\hat{z}_i^H) > npv_i^H(\hat{z}_i^L) \). Since the low types would find it optimal to mimic the high types, this cannot be a separating equilibrium.

If not \( \hat{z}_i^H \), what amount can the high types invest at a separating equilibrium? Suppose that the lenders had different beliefs: that even those contributing \( w \) in equity, when they invest more than \( z_i^{sep} \), can be high or low types with equal probability.\(^{11}\) Given these beliefs, the high types must now pay a higher-than-fair interest rate to invest more than \( z_i^{sep} \), because they would be pooled together with the low types. In terms of Figure 1, their net present value is no longer \( npv_i^H(z) \), but rather the broken solid line. This differs from \( npv_i^H(z) \) to the right of \( z_i^{sep} \), where it equals

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\[ \tilde{npv}_i^H(z) = a^H \mu(z) \left[ \gamma \pi(\kappa_i) - \frac{1}{a \mu(z)} (z - w) \right] - w = a^H \mu(z) \gamma \pi(\kappa_i) - \frac{a^H}{a} (z - w) - w, \]

where \( a = (a^H + a^L)/2 \). In words, to the right of \( z_i^{sep} \), the high types must pay the ‘fair’ interest rate of a hypothetical average agent, \( 1/(a \mu(z)) \).

Given their new net present value function, the high types choose to invest \( z_i^{sep} \). They are credit-constrained, in the sense that a friction of the credit market (the non-observability of talent) forces them to borrow only \( z_i^{sep} - w \), which is less than they would ideally borrow \( (\tilde{z}_i^H - w) \). Crucially, the low types no longer want to mimic the high types, since the latter’s conservative choice of leverage makes mimicking no more attractive than investing \( \tilde{z}_i^L \). In terms of Figure 1, we have \( \tilde{npv}_i^L(z_i^{sep}) = npv_i^L(\tilde{z}_i^L) \) (of course, \( z_i^{sep} \) was chosen precisely to satisfy this indifference condition). Thus by contributing their entire income in equity and by choosing to invest less than is optimal, the high types are able to signal themselves as talented innovators to the uninformed lenders.

What we have just described is a separating perfect Bayesian equilibrium, since both types pay their perfect-information interest rate, innovators invest optimally given the lenders’ beliefs, and beliefs are correct in equilibrium. The payoffs corresponding to the investment choices at this separating equilibrium, \( \tilde{z}_i^L \) and \( z_i^{sep} \), are represented by empty circles in Figure 1.

**Key comparative statics**

We now come to the central result of the paper, which is to show that at the separating equilibrium just described, the relationship between product-market competition, \( \kappa_i \), and the ex ante probability of innovation,

\[ \mu_i = \frac{1}{2} a^H \mu(z_i^{sep}) + \frac{1}{2} a^L \mu(z_i^{sep}), \]

is first increasing and then decreasing.

Consider again the example of Figure 1. We begin by increasing the strength of competition from the level in panel (a) to the level in panel (b). As discussed above, the curves representing \( npv_i^L(z) \) and \( npv_i^H(z) \) rotate inwards. Then investment by the low types, which is the same as in the perfect-information case \( (\tilde{z}_i^L) \), still decreases by the Schumpeterian effect. However, investment by the high types \( (z_i^{sep}) \) now increases, as is clearly visible from the figure.

To make sense of this, note that at this equilibrium the investment decisions of the high types are driven not by incentives, but rather by credit constraints. Then the Schumpeterian effect does not apply, and what matters is the effect of competition on credit constraints. Our key result is that stronger competition reduces credit constraints, thus allowing the high types to invest more. This is because stronger competition discourages the mimickers: it makes investment less attractive for everyone, but particularly so for agents who are considering to invest more than they would normally do. Formally, recall that \( z_i^{sep} \) is what the high types must invest to make genuine and mimicking low types equally well off. But a fall in \( \pi(\kappa_i) \) penalizes the mimicker more than
the genuine agent, since the mimicker invests more \((z_{i}^{\text{sep}} > z_{j}^{L})\) and thus has a higher probability of innovating. To restore equality of payoffs, \(z_{i}^{\text{sep}}\) must then increase. In terms of Figure 1, if we fixed \(z_{i}^{\text{sep}}\) at the level of panel (a) and decreased \(\pi(\kappa_{i})\) to the level of panel (b), then the value \(npv_{i}^{L}(z_{i}^{\text{sep}})\) would fall more than \(npv_{j}^{L}(z_{j}^{L})\). For the two to remain equal, a higher \(z_{i}^{\text{sep}}\) is required.

In other words, stronger competition, by creating a tougher operating environment, leads to a better selection of innovators, in the sense that the high types can invest more and a greater share of available funds is allocated to them. We call this the selection effect of competition on innovation.

Now suppose that \(\kappa_{j}\) increases further, to the level in panel (c) of Figure 1. While investment by the low types continues to decrease by the Schumpeterian effect, investment by the high types continues to increase by the selection effect, to the point that it is now equal to the perfect-information level \(z_{j}^{H}\). Now, the high types are no longer credit-constrained, because strong competition has made \(z_{j}^{H}\) low enough relative to what they can borrow. It follows that the Schumpeterian effect kicks back in for the high types as well, and any further increase in competition must now decrease investment by both types.

This example suggests that at the separating equilibrium described in the previous subsection, and across industries where the innovator is of a high type, one should find an increasing and then decreasing relationship between the strength of competition and innovation. This result is formally stated as follows.

**Proposition 1** Consider any two industries \(i\) and \(j\) where the innovator is a high type, and such that competition is stronger in \(j\) than in \(i\), so \(\kappa_{i} < \kappa_{j}\). At the separating equilibrium described above, there exists \(\hat{\kappa} \in (\alpha, 1)\) such that if \(\alpha \leq \kappa_{j} < \kappa_{i} \leq \hat{\kappa}\), then industry \(j\) has a higher probability of innovating than industry \(i\), while if \(\hat{\kappa} \leq \kappa_{i} < \kappa_{j} \leq 1\), then industry \(j\) has a lower probability of innovating than industry \(i\).

**Proof** Note that \(npv_{i}^{L}(z)\) is concave, and reaches a maximum at \(z_{i}^{H}\). Let

\[
(7) \quad z_{i}^{\text{sep}} = \min \arg \left\{ a^{L} \mu(z_{i}^{\text{sep}})\gamma\pi(\kappa_{i}) - a^{L} \left( z_{i}^{\text{sep}} - w \right) = a^{L} \mu(z_{i}^{L})\gamma\pi(\kappa_{i}) - z_{i}^{L} \right\},
\]

or, if such \(z\) does not exist, then \(z_{i}^{\text{sep}} = z_{i}^{H}\). There are two possible cases: \(z_{i}^{\text{sep}} < z_{i}^{H}\) or \(z_{i}^{\text{sep}} = z_{i}^{H}\). Suppose that \(z_{i}^{\text{sep}} < z_{i}^{H}\), and consider an increase in \(\kappa_{i}\). The total differential of the equation in curly brackets in (7) is

\[
(8) \quad a^{L} \mu'(z_{i}^{\text{sep}})\gamma\pi'(\kappa_{i})dz_{i}^{\text{sep}} + a^{L} \mu(z_{i}^{\text{sep}})\gamma\pi'(\kappa_{i})d\kappa_{i} - a^{L} \frac{d}{dz_{i}^{\text{sep}}} dz_{i}^{\text{sep}} = a^{H} \mu(z_{i}^{H})\gamma\pi'(\kappa_{i})d\kappa_{i},
\]

which can be rearranged as

\[
\frac{dz_{i}^{\text{sep}}}{d\kappa_{i}} = a^{H} \frac{\mu(z_{i}^{H}) - \mu(z_{i}^{\text{sep}})}{a^{H} \mu'(z_{i}^{\text{sep}})\gamma\pi(\kappa_{i}) - 1} \gamma\pi'(\kappa_{i}) > 0.
\]
Since $z_{i}^{\text{sep}}$ is continuously increasing in $\kappa_{i}$, while $\hat{z}_{i}^{H}$ is continuously decreasing and

$0 \leftarrow \hat{z}_{i}^{H}$

as $\kappa_{i} \rightarrow 1$, there exists $\hat{\kappa} \in (\alpha, 1)$ such that for $\kappa_{i} < \hat{\kappa}$ we have $z_{i}^{\text{sep}} < \hat{z}_{i}^{H}$, while for $\kappa_{i} \geq \hat{\kappa}$ we have $z_{i}^{\text{sep}} = \hat{z}_{i}^{H}$. In the latter range, we have $dz_{i}^{\text{sep}}/d\kappa_{i} = d\hat{z}_{i}^{H}/d\kappa_{i} < 0$. The result follows immediately. Note that $\hat{\kappa}$ must be the same across industries, since $\kappa_{i}$ is the only parameter that varies across industries.

Proposition 1 finds, for industries where the innovator is of a high type, an increasing and then decreasing relationship between competition and innovation. The region $\alpha \leq \kappa_{i} < \hat{\kappa}$ is where the high types invest $z_{i}^{\text{sep}}$ (panels (a) and (b) in Figure 1), while the region $\hat{\kappa} \leq \kappa_{i} \leq 1$ is where they invest $\hat{z}_{i}^{H}$ (panel (c)). The threshold $\hat{\kappa}$ is defined as the unique level of competition such that $z_{i}^{\text{sep}} = \hat{z}_{i}^{H}$.

One shortcoming of Proposition 1 is that it finds an increasing and then decreasing relationship between competition and innovation across industries only where the innovator is of a high type, while the relationship is decreasing across all other industries. This does not answer our initial question about the relationship between $\kappa_{i}$ and $\mu_{i}$, the ex ante probability of innovation. Looking further into this, it is immediate to see that such a relationship will be decreasing in the region $\hat{\kappa} \leq \kappa_{i} \leq 1$, where both $\hat{z}_{i}^{H}$ and $\hat{z}_{i}^{L}$ are decreasing in $\kappa_{i}$ by the Schumpeterian effect. On the other hand, we are now going to show that the relationship between $\kappa_{i}$ and $\mu_{i}$ will be increasing in the region $\alpha \leq \kappa_{i} < \hat{\kappa}$, at least for $\kappa_{i}$ close enough to $\hat{\kappa}$. So at least in a subset of $[\alpha, 1]$, the model predicts an increasing and then decreasing relationship between competition and the ex ante probability of innovation.

This is shown formally in the following result.

**Proposition 2** Consider any two industries $i$ and $j$ such that competition is stronger in $j$ than in $i$, so $\kappa_{j} < \kappa_{i}$. At the separating equilibrium described above, there exists $\hat{\kappa} \in (\alpha, \hat{\kappa})$ such that if $\hat{\kappa} < \kappa_{i} < \kappa_{j} \leq \hat{\kappa}$, then industry $j$ has a higher ex ante probability of innovating than industry $i$, while for $\hat{\kappa} \leq \kappa_{i} < \kappa_{j} \leq 1$, industry $j$ has a lower ex ante probability of innovating than industry $i$.

**Proof** Suppose that $z_{i}^{\text{sep}} < \hat{z}_{i}^{H}$. Then

$$
\frac{d\mu_{i}}{d\kappa_{i}} = \frac{1}{2} a^{H} \mu'(z_{i}^{\text{sep}}) \frac{dz_{i}^{\text{sep}}}{d\kappa_{i}} + \frac{1}{2} a^{L} \mu'(z_{i}^{L}) \frac{dz_{i}^{L}}{d\kappa_{i}}.
$$

The total derivative $dz_{i}^{\text{sep}}/d\kappa_{i}$ was derived in (7), while $dz_{i}^{L}/d\kappa_{i}$ can be found by taking the total differential of (5) and rearranging:

$$
\mu''(\hat{z}_{i}^{L}) \gamma \pi(\kappa_{i}) d\hat{z}_{i}^{L} + \mu'(\hat{z}_{i}^{L}) \gamma \pi'(\kappa_{i}) d\kappa_{i} = 0,
$$

so

$$
\frac{dz_{i}^{L}}{d\kappa_{i}} = -\frac{\mu'(\hat{z}_{i}^{L}) \pi'(\kappa_{i})}{\mu''(\hat{z}_{i}^{L}) \pi(\kappa_{i})} < 0.
$$

Substituting for $dz_{i}^{\text{sep}}/d\kappa_{i}$ and $dz_{i}^{L}/d\kappa_{i}$ into the expression for $d\mu_{i}/d\kappa_{i}$, imposing $d\mu_{i}/d\kappa_{i} > 0$ and rearranging, we obtain

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As $\kappa_i \to \tilde{\kappa}$, we have $z_{i^\text{sep}} \to z_{i^H}^H$. As this happens, the left-hand side of the last inequality approaches infinity, while the right-hand side remains finite. Then there exists $\tilde{\kappa} \in (\kappa, \tilde{\kappa})$ such that for $\kappa_i \in (\tilde{\kappa}, \tilde{\kappa})$ we have $d\mu_i/d\kappa_i > 0$, while for $\kappa_i > \tilde{\kappa}$ we have $d\mu_i/d\kappa_i < 0$. The result follows immediately. Note that $\tilde{\kappa}$ must be the same across industries, since $\kappa_i$ is the only parameter that varies across industries.

To make sense of the increasing part of the curve, recall that this is driven by industries where the innovator is of a high type: in those industries, $z_{i^\text{sep}}$ must increase as $\kappa_i$ increases, to restore equality of payoffs between a genuine and a mimicking low type (given that the latter suffers more from a fall in profits). But as $\kappa_i$ approaches $\tilde{\kappa}$ and $z_{i^\text{sep}}$ approaches $z_{i^H}^H$, which is the maximum of the mimicker’s net present value function, the gain to the mimicker from an increase in $z_{i^\text{sep}}$ monotonically decreases to zero. It follows that as $\kappa_i$ approaches $\tilde{\kappa}$, the increase in $z_{i^\text{sep}}$ that follows from an increase in $\kappa_i$ must grow unboundedly, as greater and greater increases are required to compensate the mimicker. In contrast, in industries where the innovator is of a low type, the decrease in $z_{i^H}^H$ is always finite. In other words, as $\kappa_i$ approaches $\tilde{\kappa}$, the selection effect must always be stronger than the Schumpeterian effect. This also suggests that the relationship between $\kappa_i$ and $\mu_i$ should be convex as we approach its peak from the left, an intuition that is confirmed by the computational exercise in the next subsection.

We have derived a specific separating equilibrium in which the relationship between competition and innovation has an inverted-U shape. But when exactly will this equilibrium exist, and how many other plausible equilibria are there? In the Appendix, we show that our separating equilibrium exists as long as the wage is neither too high (or else the talented innovators would not need to borrow) nor too low (or else they would need to borrow so much that they would opt for being pooled with the untalented innovators). Moreover, we argue that in this parameter subspace, our equilibrium outcome is the only one that can ‘reasonably’ be realized in a perfect Bayesian equilibrium that survives a standard refinement procedure.

We conclude this subsection by discussing two simplifying tricks that we have used in this paper. First, the standard Schumpeterian growth model described in Section I is an infinite-horizon model with an overlapping generation structure, and yet the signalling game described in Section II is a static game. This combination is possible only under a carefully selected set of assumptions. For example, had we assumed that an innovator can invest more than once, or that she cares about future innovators who are also more likely to succeed, then the signalling game would become more complicated, as the current innovator would have had to consider the future impact of her investment decisions. Second, we have considered investment only at a hypothetical period $t$ in which initial productivity is the same across industries. But already in period $t+1$, as some industries innovate and others do not, this assumption would necessarily be invalid. Credit constraints would vary across industries, even keeping talent and competition constant. While our model can easily accommodate this additional dimension of heterogeneity (as we show in Section III), a full analysis would need to keep track of how credit constraints evolve over time. The role of these simplifications is obvious: they allow us to describe in a clearer way a mechanism that would exist even in more complicated settings.
Computational exercise

One limitation of Proposition 2 is that it concerns itself exclusively with values of \( k_j \) close to \( \hat{k} \). A computational exercise for our running example will show that our model is able to generate the inverted-U pattern for all values of \( k_j \in (z, 1) \). Moreover, the cross-industry differences in innovation rates (across industries characterized by different levels of competition) are both statistically and economically significant.

Panel (a) of Figure 2 plots the \textit{ex ante} probability of innovation for the entire economy, \( \mu_i \), for all feasible values of \( k_j \) (which by our choice of \( \alpha \) must range between 0.4 and 1).\(^{13}\) We see that the resulting function is indeed increasing and then decreasing, for all feasible values of \( k_j \). As expected, the curve is convex to the left of the peak, since the marginal effect of an increase in \( k_j \) on \( z_{ij}^{sep} \) becomes infinitely large as we approach the peak. Consistently with our earlier discussion, the peak is reached for a level of competition between 0.5 and 0.6 (i.e. between panels (b) and (c) of Figure 1). In more detail, the \textit{ex ante} probability of innovation increases from 0.08 for \( k_j = 0.40 \) to 0.10 for \( k_j = 0.54 \), and then decreases to 0.00 as \( k_j \) grows towards 1.00.

Once we have computed the predicted probabilities of innovation for both high and low types, we can simulate industry-level patenting behaviour, aggregating over a large number of industries for each level of competition, in order to generate a synthetic dataset that can be used to run Poisson regressions similar to those in Aghion \textit{et al.} (2005). Panel (b) of Figure 2 shows the results of this exercise. We plot the histogram resulting from 10,000 runs of the model. We then regress the number of patents over our measure of competition, as well as this latter coefficient’s square. The resulting regression curve is plotted as a solid line over the histogram, and clearly displays the inverted-U shape found by the empirical literature. An analysis of the \( p \)-values confirms the significance of all coefficients at the 0.01 level.

This exercise suggests that even though in theory we do not exactly find an inverted-U relationship between competition and innovation (but rather an increasing and convex, and then decreasing and concave relationship),\(^{14}\) the finding of an inverted-U by the empirical literature is consistent with the data-generating process being driven by our mechanism.

### III. Additional Results

We have shown that a Schumpeterian model allowing for heterogeneous talent of innovators and asymmetric information can predict the inverted-U, even under the original assumption of innovators focused on post-innovation rents. To provide corroborating evidence in support of our mechanism, we show in this section that the equilibrium that we characterize has specific features which match the evidence well.

First, if our mechanism was important to explain the inverted-U relationship between product-market competition and innovation, then we should expect that in industries in which credit constraints are more prevalent, the increasing part of the relationship should hold for a larger range of levels of competition. In other words, the peak of the inverted-U should be located more to the right. To see this formally, consider a general version of the model in which initial productivity, \( A_{it-1} \), is allowed to vary across industries.\(^{15}\) As we will show, credit constraints are more prevalent in high-productivity industries, and the peak of their inverted-U is located more to the right.

Consider first the case of perfect information. Very little changes relative to the baseline model. This is because profits are also linearly increasing in \( A_{it-1} \), so that in
high-productivity industries, a higher cost of investment is exactly offset by higher profits. Mathematically, the \( npv_i(x) \) functions are still as in equation (4), and optimal investment levels are unchanged. In terms of our example, the thin solid lines of Figure 1 are unchanged, and so are their maxima. Of course, high-productivity industries will invest more in absolute terms, \textit{ceteris paribus}. In our example, suppose that industries 1

---

**Figure 2.** Panel (a) plots the economy-wide, \textit{ex ante} probability of innovation \( \mu_i \). Panel (b) reports the realized number of innovations when the model is run 10,000 times per each level of \( k_i \) (bars) and a quadratic Poisson regression curve of this data (solid line). The functional form and parameters used are the same as in Figure 1. [Colour figure can be viewed at wileyonlinelibrary.com]
and 2 both have an innovator of the high type, and face competition $\kappa_i = 0.4$ as in panel (a) of Figure 1. Without loss of generality, assume that $A_{1t-1}/C_{01} > A_{2t-1}/C_{01}$. We have $Z^H_1 = Z^H_1/A_{1t-1} = Z^H_2/A_{2t-1} = Z^H_2$ under perfect information, which immediately implies that $Z^H_1 < Z^H_2$.

Consider now the case of imperfect information. Only credit-constrained industries are worth examining, since all other industries behave as under perfect information. Credit-constrained industries are those endowed with an innovator of the high type, and located on the increasing part of the inverted-U. Across these industries, normalized investment $z_{i}^{\text{sep}}$ is lower when $A_{it-1}$ is higher, and credit constraints $Z^H_i - z_{i}^{\text{sep}}$ are tighter. This is because the normalized wage $w_i$ (now with a subscript) is lower, and so is the amount of normalized equity that the innovator is able to contribute.\footnote{Intuitively, the same wage buys less innovation in high-productivity industries than in low-productivity ones. For example, consider again industries 1 and 2, which are now credit-constrained by virtue of the fact that they have a low level of competition. We have $w_1 < w_2$, which implies $z_1^{\text{sep}} < z_2^{\text{sep}}$ and thus $Z^H_1 - z_1^{\text{sep}} > Z^H_2 - z_2^{\text{sep}}$. This can be seen using panel (a) of Figure 1 in conjunction with equation (6). A fall in $w_i$ leaves the $npv^L_i(z)$ curves and $Z^H_i$ unchanged; however, it shifts the origin of curve $npv^L_i(z)$ to the right, and the entire curve up. Then condition $npv^L_i(z_i^H) = npv^L_i(z_i^{\text{sep}})$ must be reached for a lower value of $z_i^{\text{sep}}$.

So credit constraints are more prevalent in high-productivity industries. But the peak of their inverted-U must then be located more to the right. For suppose that competition in industries 1 and 2 was at the level that puts industry 2 at its peak (i.e. the minimum level such that $z_2^{\text{sep}} = Z^H_2$). It would be $z_1^{\text{sep}} < z_1^H$ at this point, which would imply that investment in industry 1 is still increasing in competition.

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To illustrate the empirical implications of this, we repeat the computational exercise of Section II but allowing for two different levels of initial industry productivity. Industries can then be divided into two groups, high-productivity and low-productivity. At any level of competition, credit constraints are more prevalent in the first group, at both the extensive and intensive margins. In Figure 3, we reproduce the inverted-U calculated in Figure 2, and overlay this with the same curve calculated separately for high- and low-productivity industries. As expected, the peak of the inverted-U is located more to the right for high-productivity industries. Their inverted-U is initially lower, to reflect the fact that tighter credit constraints are associated with lower normalized investment.

These results are consistent with a finding in Aghion et al. (2004). They identify the 40% of firms subject to higher debt-pressure, and plot their inverted-U separately. Consistently with our results, they find that their peak is located more to the right (see their Figures 6.6a and 6.6b). They also find that their curve is higher than for other firms, which at first seems inconsistent with our results. However, they use a citation-weighted patent count as a measure of innovation, while we use the simple patent count. To the extent that the patents of high-productivity industries are more likely to be cited than other patents (as would seem reasonable, given that these are more sophisticated industries that attract larger R&D investments), a trivial extension of our model would also predict a higher inverted-U for high-productivity industries.

A second prediction that is specific to our model is that in credit-constrained industries, expected profitability should be positively correlated with credit constraints. Our main comparative statics has provided an example of this, by showing that stronger competition, which is associated with lower expected profits, leads to weaker credit constraints in credit-constrained industries. To provide other examples, one could allow for cross-industry variation in the overall quality of projects (the scale of $a^H$ and $a^L$, call this $a$) or in the opportunity for technological upgrading ($\gamma$). Lower $a$ or $\gamma$, which are both associated with lower expected profits, would again make the $npv^i(z)$ curves rotate inwards, leading to a higher $z^{sep}_i$ and a lower $z^H_i - z^{sep}_i$ (weaker credit constraints).

This prediction is consistent with a finding in the finance literature according to which firms with the largest returns on equity are also those that face the tighter credit constraints (for a review, see Li 2011). Our interpretation of this finding is that industries with high returns are particularly attractive to ‘lemons’, exacerbating the adverse selection problem and making it harder for high-quality projects to be financed. Interestingly, Li (2011) finds that the positive relationship between returns and credit constraints is much stronger among R&D-intensive firms than among non-R&D-intensive firms. This is consistent with our interpretation, since it is precisely in R&D-intensive industries that you would expect asymmetric information to be a major issue.

IV. CONCLUSIONS

We have shown that a Schumpeterian model allowing for heterogeneous talent of innovators and asymmetric information can predict the inverted-U relationship between competition and innovation, even under the original assumption of innovators focused on post-innovation rents. When competition is low and innovation is highly profitable, investment in innovation is likely to be governed by credit constraints. Then an increase in competition may lead to a positive selection effect, increasing the rate of innovation.
even as it reduces the post-innovation rents. When competition is high, however, the low profitability of innovation makes it less likely that credit constraints will be important. Then the negative impact of an increase in competition on post-innovation rents should also result in less innovation.

The main contribution of our paper is to show that an inverted-U relationship between product market competition and innovation may also emerge among firms focused on post-innovation rents. Given the great importance of these firms in the innovation process, this seems a desirable addition to our theoretical understanding of the inverted-U. In addition, our model has two specific predictions. First, the positive relationship between competition and innovation should be more pronounced in industries where credit constraints are more prevalent. Second, the average level of credit constraints in credit-constrained industries should be decreasing in any factor (such as stronger product market competition) that decreases expected profitability. We have argued that these predictions are consistent with, and provide an original interpretation of, existing evidence in the growth and finance literature.

One key policy implication of our work is that, at least for low levels of competition, fostering competition is a substitute for reducing asymmetric information in financial markets. Since the government is unlikely to develop an informational advantage over private investors in the market for innovation, its efforts should focus on fostering competition. The alternative explanation of how an inverted-U between innovation and competition relationship occurs, by Aghion et al. (2005), is based on the dynamics of step by step innovation, and relies on the varying incentives of innovators based on how far advanced they are relative to others. These dynamics are also unlikely to be structurally affected by government policy. Hence both explanations drive toward a similar conclusion: policy should foster competition up to a point, and in particular in industries that exhibit certain properties. However, there is a clear advantage for policy to focus on asymmetric information rather than differences in technological advancement. Differences in technological advancement are practically hard to observe and must rely on unsatisfactory proxies such as patenting effort. Asymmetric information, on the other hand, leads to clear volatility in innovation outcomes in industries as a whole. By measuring whether that volatility become attenuated as a result of its policy efforts, the government can have a reasonable sense of whether its policy efforts are working.

APPENDIX

In this Appendix, we first formally derive the separating equilibrium discussed in the main text. We then identify the parameter subspace where the separating equilibrium exists, and challenge the robustness of the equilibrium to a standard refinement procedure. We conclude by providing more details on the computational exercise of Section II.

DERIVATION OF THE EQUILIBRIUM

This subsection is organized as follows. We begin, in Theorem 1, by showing that if \( \frac{\gamma_H}{\gamma_L} \leq w \), then the two types must invest \( \gamma_H \) and \( \gamma_L \) in any perfect Bayesian equilibrium (PBE). Based on this result, Theorems 2 and 3 focus on the case \( 0 < w < \frac{\gamma_H}{\gamma_L} \).

Theorem 2 defines the threshold \( \kappa \), establishes its properties as a function of \( w \), and then shows that if \( \gamma \leq \kappa \), then the two types must again invest \( \gamma_H \) and \( \gamma_L \) in any PBE. Based on this result, the theorem further restricts the focus to the case case \( x \leq \kappa < \kappa \).

Theorem 3 begins by formally defining the separating equilibrium described in Section II (points (a)–(d)). Subsequently (points (1)–(3)), it defines the threshold \( \kappa \), and shows that the
separating equilibrium exists if and only if \( k \leq k_i < \pi \). Then it shows that if \( \bar{w} \leq w < \bar{z}_i^{H} \big|_{k_i = \pi} \), then we always have \( z_i^{sep} = \bar{z}_i^{H} \) in the area where the separating equilibrium exists.

**Theorem 1** If \( \bar{z}_i^{H} \big|_{k_i = \pi} \leq w \), in any PBE, the two types invest respectively \( \bar{z}_i^{H} \) and \( \bar{z}_i^{L} \), any combination of equity and external financing being possible.

**Proof** Since the opportunity cost of equity financing is zero, the high types are always able to invest \( \bar{z}_i^{H} \) using only personal wealth, and the minimum rate that they can be offered on external financing is \( 1/[a^{H}(\mu(z))] \), the high types would never select a research effort different from \( \bar{z}_i^{H} \). Furthermore, they would never take on external financing at a rate greater than \( 1/[a^{H}(\mu(z))] \). This last fact implies that a pooling equilibrium does not exist. As shown in note 10, at any separating equilibrium, the low types must select \( \bar{z}_i^{L} \). Then there exists only a separating equilibrium in which the two types invest \( \bar{z}_i^{H} \) and \( \bar{z}_i^{L} \), respectively. If an innovator borrows any money at such an equilibrium, then this must be at a rate \( 1/[a^{H}(\mu(\bar{z}_i^{H}))] \) for the high types and \( 1/[a^{L}(\mu(\bar{z}_i^{L}))] \) for the low types. Then the innovator is indifferent as to the amount borrowed, and it is possible to construct an equilibrium with any combination of equity and external financing.

**Theorem 2** If \( 0 < w < \bar{z}_i^{H} \big|_{k_i = \pi} \), let

\[
\bar{w} \equiv \arg(\bar{z}_i^{H} = w),
\]

a threshold that continuously decreases from 1 to \( w \) as \( w \) increases from 0 to \( \bar{z}_i^{H} \big|_{k_i = \pi} \). Then if \( \bar{w} \leq k_i \leq 1 \), in any PBE, the two types invest, respectively, \( \bar{z}_i^{H} \) and \( \bar{z}_i^{L} \), any combination of equity and external financing being possible.

**Proof** The properties of \( \bar{w} \) as a function of \( w \) follow from the fact that \( \bar{z}_i^{H} \) is equal to \( \bar{z}_i^{H} \big|_{k_i = \pi} \) for \( k_2 = \pi \), is continuously decreasing in \( k_1 \) and is equal to 0 for \( k_1 = 1 \). Then for \( w = \bar{w} \), we must have \( k = 1 \), \( k \) must be continuously decreasing in \( w \), and for \( w = \bar{z}_i^{H} \big|_{k_i = \pi} \), we must have \( k = \pi \). The rest of the theorem can be shown in the same way as for Theorem 1.

**Theorem 3** If \( 0 < w < \bar{z}_i^{H} \big|_{k_i = \pi} \) and \( x \leq k_i < \pi \), then consider the following situation.

(a) Lenders believe that those who contribute \( w \) in equity and invest \( z \in (w, \bar{z}_i^{sep}) \) are high types, where \( \bar{z}_i^{sep} \) is the minimum \( z > w \) such that

\[
\text{(A1) } \bar{npv}_i^{L}(z) = npv_i^{L}(\bar{z}_i^{L}),
\]

or, if such \( z \) does not exist, then \( \bar{z}_i^{sep} = \bar{z}_i^{H} \). They also believe that those who contribute \( w \) in equity and invest \( z > \bar{z}_i^{sep} \) are high and low types with equal probability. Finally, they believe that everybody else is a low type.

(b) Lenders offer rate \( 1/[a^{H}(\mu(z))] \) to the first group, rate \( 1/[a^{L}(\mu(z))] \) to the second, and rate \( 1/[d^{L}(\mu(z))] \) to the third.

(c) The low types invest \( \bar{z}_i^{L} \) (any combination of equity and external financing being possible).

(d) The high types invest \( \bar{z}_i^{sep} \) (contributing \( w \) in equity).

Then there exist \( \bar{w} \) and \( \bar{w} \), with \( 0 < \bar{w} < \bar{w} < \bar{z}_i^{H} \big|_{k_i = \pi} \), such that the following hold.

(1) If \( \bar{w} \leq w < \bar{z}_i^{H} \big|_{k_i = \pi} \), then situation (a)–(d) is a PBE. We have \( \bar{z}_i^{sep} = \bar{z}_i^{H} \).

(2) If \( \bar{w} \leq w < \bar{w} \), then situation (a)–(d) is a PBE. There exists \( \bar{k} \in (x, \pi) \) such that \( \bar{z}_i^{sep} < \bar{z}_i^{H} \) for \( k_i \in [x, \bar{k}] \) and \( \bar{z}_i^{sep} = \bar{z}_i^{L} \) for \( k_i \in [\bar{k}, \pi] \).

(3) If \( 0 < w < \bar{w} \), then point (2) is still true, except that there exists \( \bar{k} \in (x, \pi) \) such that situation (a)–(d) is not a PBE if \( k_i \in [x, \bar{k}] \).
Proof  Step I (preliminary step). Situation (a)–(d) is a PBE if and only if

\[
(A2) \quad npv^L(z_{i}^{sep}) \geq \tilde{npv}^L(z) \quad \text{for all } z > z_{i}^{sep}.
\]

To show this, we proceed in two substeps.

Step I(i). If condition \((A2)\) does not hold, then situation (a)–(d) is not a PBE. This follows from the fact that the high types have a profitable deviation, since they can contribute \(w\) in equity and invest some \(z > z_{i}^{sep}\), and obtain a higher payoff.

Step I(ii). If condition \((A2)\) holds, then situation (a)–(d) is a PBE. This follows from the fact that the following three facts hold true. First, for every action that borrowers could take, the lenders’ action is optimal given their beliefs. Second, for actions that borrowers take in equilibrium, the lenders’ beliefs are correct. Third, borrowers do not have a profitable deviation. To see the last point, let \(z_{i}^{d} \) represent type \(J\)’s preferences, and let \((z', e')\) represent type \(J\)’s investment profile (where \(e'\) denotes the innovator’s equity contribution). Consider first the high types. Their equilibrium action, \((z_{i}^{sep}, w)\), gives payoff \(npv^H(z_{i}^{sep})\). We want to show that \((z_{i}^{sep}, w) \geq (z, e)\) for any feasible \((z, e)\). This follows from the fact that if \(z < z_{i}^{sep}\), then the high types can at best obtain payoff \(npv^H(z)\). But \(z < z_{i}^{sep} \leq z^H\) implies that \(npv^H(z) < npv^H(z_{i}^{sep})\). If \(z = z_{i}^{sep}\), then the only way in which \((z, e)\) may differ from \((z_{i}^{sep}, w)\) is if \(e < w\). But by deviating in this way, the high types are identified as low types, and receive payoff \(npv^H(z_{i}^{sep}) - (a'/d' - 1) |z - e| < npv^H(z_{i}^{sep})\). Finally, if \(z > z_{i}^{sep}\), then the high types can at best obtain payoff \(\tilde{npv}^H(z)\), but we have \(\tilde{npv}^H(z) \leq npv^H(z_{i}^{sep})\) by condition \((A2)\). Next, consider the low types. Their equilibrium action \((z_{i}^{sep}, e^H)\), where \(e^H \in [0, z^L]\), gives payoff \(npv^L(z_{i}^{sep})\). We want to show that \((z_{i}^{sep}, e^H) \geq (z, e)\) for any feasible \((z, e)\). This follows from the fact that if \(e < w\), or if \(z \leq w\), or if both conditions hold, then the low types obtain payoff \(npv^L(z) \leq npv^L(z_{i}^{sep})\). If \(e = w\), and \(z \in (w, z_{i}^{sep})\), then the low types obtain payoff \(npv^L(z)\), and, by definition of \(z_{i}^{sep}\), we have \(npv^L(z) < npv^L(z_{i}^{sep})\). If \(e = w\), and \(z > z_{i}^{sep}\), then the low types receive payoff \(\tilde{npv}^L(z)\). But condition \((A2)\) must hold for \(z\). Multiplying both sides of this condition by \(a'/d'\), we obtain

\[
d^L \mu(z_{i}^{sep}) \gamma \pi(k_i) - d^L \mu(z_{i}^{sep}) \geq d^L \mu(z_{i}^{sep}) - d^L \mu(z-w) - d^L \mu(w),
\]

then subtracting \(-\{(d^L/d')w\}\) from both sides gives \(\tilde{npv}^L(z_{i}^{sep}) \geq npv^L(z)\), or, by the definition of \(z_{i}^{sep}\), \(npv^L(z_{i}^{sep}) \geq npv^L(z)\).

Step II (preliminary step). There exist \(w\) and \(w^*\), with \(z_{i}^{l} |_{k_i=x} < w < w^* < z^H |_{k_i=x}\), such that if \(k_i = x\), then we have \(z_{i}^{sep} < z^H\) if \(w < w\), and \(z_{i}^{sep} = z^H\) otherwise; and condition \((A2)\) holds if and only if \(w \geq w^*\). We show these two points in two separate substeps.

Step II(i). There exists \(w^*\), with \(z_{i}^{l} |_{k_i=x} < w < z^H |_{k_i=x}\), such that if \(k_i = x\), then \(z_{i}^{sep} < z^H\) if \(w < w^*\), and \(z_{i}^{sep} = z^H\) otherwise. Suppose that \(k_i = x\). Recall the definition of \(z_{i}^{sep}\) provided in part (a) of the theorem. Note that the function \(\tilde{npv}^L(z)\) is decreasing in \(w\). Given \(w < z^H |_{k_i=x}\) and \(k_i = x\), by Theorem 2, we have \(w \leq z^H\). Then the function \(\tilde{npv}^L(z)\) (which is defined only for \(z > w\)) is concave, reaches a maximum at \(z = z^H\), and turns negative for \(z\) large enough. As for \(npv^L(z)^{\ast}\), it is positive and constant in both \(w\) and \(z\). It is easy to see that if \(w = z^L\), then we have \(npv^L(z^L) = npv^L(z^L)\), implying that \(\tilde{npv}^L(z^L) > npv^L(z^L)\). Furthermore, for \(w \to z^H\), we have \(npv^L(z^H) \to npv^L(z^H) < npv^L(z^H)\). Then there exists \(w\), with \(z_{i}^{l} |_{k_i=x} < w < z^H |_{k_i=x}\) such that if \(w < w^*\), then \(z_{i}^{sep} = z^H\). It is also the case that \(z_{i}^{sep} = z^L\) for \(w = z^L\), and \(z_{i}^{sep} > w\), increasing in \(w\) for \(w > w^*\).

Step II(ii). There exists \(w^*\), with \(z_{i}^{l} |_{k_i=x} < w < w^*\), such that if \(k_i = x\), then condition \((A2)\) holds if and only if \(w \geq w^*\). Suppose that \(k_i = x\). The function \(\tilde{npv}^L(z)\) is concave and maximum for
\( z^\text{pool}_i = \arg(\mu(z)\gamma \pi(k_i) = 1) \), and \( z^\text{pool}_i \in (\bar{z}_i^L, \bar{z}_i^H) \). Then from results in Step II(i), there exists \( \hat{w} \in (\bar{z}_i^L, \bar{w}) \) such that \( z^\text{sep}_i \geq z^\text{pool}_i \) if and only if \( w \geq \hat{w} \). In such a case, a sufficient condition for (A2) to hold is \( npv^H(z^\text{sep}_i) \geq \bar{npv}^H(z^\text{pool}_i) \), which is always true. If \( w < \hat{w} \), then a necessary and sufficient condition for (A2) to hold is

\[
npv^H(z^\text{sep}_i) \geq \bar{npv}^H(z^\text{pool}_i).
\]

There exists \( w \), with \( z^\text{sep}_i \mid_{k_i=x} < w < \hat{w} \), such that (A3) holds if and only if \( w \in [\bar{w}, \hat{w}) \). This can be shown in two steps. First, note that the expression \( npv^H(z^\text{sep}_i) - \bar{npv}^H(z^\text{pool}_i) \) is continuously increasing in \( w \) for \( w \in (0, \hat{w}) \). To see this, start from condition \( npv^H(z^\text{sep}_i) = npv^H(\bar{z}_i^L) \). Multiplying both sides by \( d\mu(z)/a\mu \) and rearranging, this can be rewritten as

\[
da^H \mu(z^\text{sep}_i)\gamma \pi(k_i) - \bar{z}_i^\text{pool} = da^H \mu(z^L_i)\gamma \pi(k_i) - da^H \bar{z}_i^L + \frac{da^H - a}{a^H}w,
\]

where the left-hand side is equal to \( npv^H(z^\text{sep}_i) \). Then the expression \( npv^H(z^\text{sep}_i) - \bar{npv}^H(z^\text{pool}_i) \) can be written as

\[
da^H \mu(z^\text{sep}_i)\gamma \pi(k_i) - \frac{da^H - a}{a^H}z^L_i + \frac{da^H - a}{a^H}w - \left[ da^H \mu(z^\text{pool}_i)\gamma \pi(k_i) - \frac{da^H - a}{a^H}z^\text{pool}_i + \frac{da^H - a}{a^H}w \right],
\]

which is increasing in \( w \) (note that \( z^\text{pool}_i \) does not depend on \( w \)). Second, note that the expression \( npv^H(z^\text{sep}_i) - \bar{npv}^H(z^\text{pool}_i) \) is negative for \( w = z^L_i \), and positive for \( w = \hat{w} \). The latter follows from the earlier discussion; to see the former, recall that by Step II(i) we have \( z^\text{sep}_i = \bar{z}_i^L \) for \( w = \hat{w} \).

Then we have

\[
\bar{npv}^H(z^\text{pool}_i) > \bar{npv}^H(z^\text{sep}_i) = \bar{npv}^H(w) = npv^H(w) = npv^H(z^\text{sep}_i).
\]

**Step III.** (Point (2) in the theorem.) Suppose that \( w \leq w < \hat{w} \).

**Step III(i).** If \( k_i \in (x, \bar{\kappa}) \), then situation \((a')-(d)\) constitutes a PBE. From Theorem 2, we have \( k_i = x \). If \( k_i = x \), then by Step II, condition (A2) holds. But the condition also holds for \( k_i \in (x, \bar{\kappa}) \), which by Step I proves the result. To see this, consider two cases. First, if \( z^\text{sep}_i \geq z^\text{pool}_i \) for \( k_i = x \), then such an inequality also holds for \( k_i \in (x, \bar{\kappa}) \). This is because \( z^\text{pool}_i \) is decreasing in \( z \), while \( z^\text{sep}_i \) is either increasing or equal to \( z^L_i > z^\text{pool}_i \). But \( z^\text{sep}_i > z^\text{pool}_i \) implies that a sufficient condition for (A2) to hold is \( npv^H(z^\text{sep}_i) \geq \bar{npv}^H(z^\text{pool}_i) \), which is always true. Second, if \( z^\text{sep}_i < z^\text{pool}_i \) for \( k_i = x \), then there exists \( \hat{k} \) such that this inequality also holds for \( k_i \in (x, \hat{k}) \), while \( z^\text{sep}_i \geq z^\text{pool}_i \) for \( k_i \in (\hat{k}, \bar{\kappa}) \). This follows from the fact that \( z^\text{pool}_i \) is decreasing in \( k_i \), while \( z^\text{sep}_i \) is increasing and reaches \( z^H_i > z^\text{pool}_i \) for some \( k_i < \bar{\kappa} \). In the first region, condition (A2) follows from the fact that it holds for \( k_i = x \) and, expression (A4) is increasing in \( k_i \). In the second region, it follows from the fact that a sufficient condition for (A2) to hold is \( npv^H(z^\text{sep}_i) \geq \bar{npv}^H(z^\text{pool}_i) \), which is always true.

**Step III(ii).** If \( k_i \in (x, \bar{\kappa}) \), then there exists \( \hat{k} \in (x, \bar{\kappa}) \) such that \( z^\text{sep}_i < z^H_i \) for \( k_i \in [x, \bar{\kappa}) \), and \( z^\text{sep}_i = z^H_i \) for \( k_i \in [\hat{k}, \bar{\kappa}) \). Given \( k_i < \bar{\kappa} \), by Theorem 2, we have \( w < z^H_i \). The function \( \bar{npv}^L(z) \) (which is defined only for \( z > w \)) is concave in \( z \), reaches a maximum at \( z^H_i > w \), and turns negative for \( z \) large enough. At the same time, given \( w < \bar{w} \) and \( w \geq w > z^L_i \), by Step II(i), if \( k_i = x \), then theorem (A1) admits two solutions \( z^\text{sep}_i \) and \( z^\text{sep}_i \), with \( 0 < w < z^\text{sep}_i < z^H_i < z^\text{sep}_i < \infty \). But note that \( z^H_i \) is decreasing in \( k_i \), and, as shown in the proof of Proposition 1, \( z^\text{sep}_i \) is increasing and \( \bar{npv}_L(z^\text{sep}_i) - npv_L(z^L_i) \) is decreasing in \( k_i \) (this can be seen by rearranging equation (8)). Furthermore, for \( k \rightarrow \hat{k} \), we have \( w \rightarrow z^H_i \), which by a result in Step II(i) implies that \( \bar{npv}_L(z^\text{sep}_i) - npv_L(z^L_i) \) converges to \( \bar{npv}_L(z^H_i) - npv_L(z^L_i) < 0 \). The result follows.
Step IV. Point (1) in the theorem. Suppose that $w \leq w < z_H^H\big|_{\kappa_i=\alpha}$. Step III(i) still holds, with the simplification that, given $w > \overline{w}$, by a result in Step II(ii), for $\kappa_i = \alpha$, we need to consider only the case $z_i^{sep} > z_i^{pool}$. Step III(ii) also still holds. Finally, given $w \geq \overline{w}$, by Step II, if $\kappa_i = \alpha$, then $z_i^{sep} = z_i^H$. Furthermore, by Step III(i), $n_{\alpha i}(\overline{z}^f_i) - n_{\alpha i}(z_i^H)$ is decreasing in $\kappa_i$. It follows that $z_i^{sep} = z_i^H$ for all $\kappa_i \in [\alpha, \overline{\kappa}]$.

Step V. Point (3) in the theorem. Suppose that $0 < w < \overline{w}$. Steps III(i) and III(ii) still hold. By Step II, if $\kappa_i = \alpha$, then condition (A2) does not hold. Furthermore, given $w < w < \overline{w}$, by a result in Step II(ii), if $\kappa_i = \alpha$, then $z_i^{sep} < z_i^{pool}$. There exists $\overline{\kappa} \in (\alpha, \overline{\kappa})$ such that the last inequality also holds for $\kappa_i \in (\alpha, \overline{\kappa})$, while $z_i^{sep} \geq z_i^{pool}$ for $\kappa_i \in (\overline{\kappa}, \overline{\kappa})$. This follows from the fact that $z_i^{pool}$ is decreasing in $\kappa$, while $z_i^{sep}$ is increasing and equal to $z_i^H > z_i^{pool}$ for $\kappa_i = \overline{\kappa}$. There then exists $\kappa \in (\alpha, \overline{\kappa})$ such that condition (A2) does not hold for $\kappa_i \in [\alpha, \overline{\kappa}]$, while it holds for $\kappa_i \geq \overline{\kappa}$. This follows from the facts that the condition does not hold for $\kappa_i = \alpha$, that expression (A4) is increasing in $\kappa_i$, and that condition (A2) holds for $z_i^{sep} \geq z_i^{pool}$. It follows that, by Step I, situation (a)–(d) is not a PBE if $\kappa_i \in [\alpha, \overline{\kappa}]$. Otherwise, Step III(i) still applies, replacing $\alpha$ with $\overline{\kappa}$ everywhere.

EXISTENCE OF THE EQUILIBRIUM

Figure A1 represents the $(\kappa_i, \kappa, \alpha)$ parameter space, by plotting $\kappa_i$ on the vertical axis and $w = (1-\alpha)(\kappa \alpha)^{1-\alpha}$ on the horizontal axis. Our comparative statics in this paper has consisted of increasing $\kappa_i$ for given $w$. However, we have tacitly focused on a central case ($w < w < \overline{w}$ in the figure), while the remaining cases must also be considered.

The term $z_i^H|_{\kappa_i=\alpha}$ represents optimal investment by the high types when the monopolist faces effectively no competition (it can charge price $1/\alpha$). It is the highest amount that the high types may ever want to invest. Then if $w \geq z_i^H|_{\kappa_i=\alpha}$, the high types can always finance their optimal investment purely out of equity contributions. The last statement must also be true if $0 < w < z_i^H|_{\kappa_i=\alpha}$ and $\kappa_i$ is high enough, since a high $\kappa_i$ pushes $z_i^H$ down to zero, so that it is $w \geq z_i^H$. This second case is represented by the area $\kappa_i \geq \overline{\kappa}$ in Figure A1, where $\overline{\kappa}$ is the unique value of $\kappa_i$ such that $w = z_i^H$, and is intuitively decreasing in $w$. In both cases, the separating equilibrium does not exist, if anything because the high types would never contribute $w$ in equity. We show in the previous subsection that in a PBE, innovators always invest $z_i^H$ in this area.

Consider next the area $0 < w < z_i^H|_{\kappa_i=\overline{\kappa}}$, $\kappa < \overline{\kappa}$. The separating equilibrium must also not exist if both $w$ and $\kappa_i$ are very low, that is, in the area $0 < w < \overline{w}$, $\alpha \leq \kappa_i < \overline{\kappa}$ in Figure A1 (the threshold $\overline{w}$ and $\overline{\kappa}$ are derived in the previous subsection). To see why, note that $z_i^H$ is much greater than $w$ in this case. It follows that $z_i^H$ must also be much greater than $z_i^{sep}$, or else the high types would be leveraging a lot at the separating equilibrium, and the low types would want to mimic them. In other words, there must be a large discrepancy between the high types’ optimal investment and the maximum that they can invest by borrowing at their fair rate. But then the high types will prefer to pay an adverse selection premium, borrow more, and invest more. This point can be illustrated

![Figure A1](https://example.com/figure-a1.png)
using panel (a) of Figure 1: if \( w \) was very low, then the maximum of the high types’ net present value would be not \( z_{i}^{\text{sep}} \), but rather a local maximum to the right of it.

In summary, the separating equilibrium does not exist outside of the striped area in Figure A1. We show in the previous subsection that in the striped area, it always exists. Note that this area does not perfectly overlap with the area where the model can predict an inverted-U relationship between competition and innovation in industries where the innovators are of a high type (the shaded area). This is for two reasons. First, in the area \( w \geq w^* \), \( k_j < k_i \), even if the separating equilibrium exists, the threshold \( \kappa \) does not, so that Propositions 1 and 2 do not hold. Intuitively, at such high wages, the high types can always invest \( z_{i}^{H} \) at the separating equilibrium, so that only the decreasing part of the relationship obtains. Second, in the area \( 0 < w < w^* \), consider industries \( i \) and \( j \) such that \( \kappa \leq k_i < k_j \leq 1 \). Suppose further that \( k_i < \pi \leq k_j \leq 1 \), or \( \pi \leq k_i < k_j \leq 1 \). While strictly speaking Propositions 1 and 2 do not apply to industry \( j \), or \( i \) and \( j \), since these industries cannot be at the separating equilibrium, their investment must still be \( z_{i}^{H} \) and \( z_{j}^{T} \). So it must still be true that innovation is higher in industry \( i \) than in industry \( j \), and the logic of Propositions 1 and 2 carries through.

**EQUILIBRIUM REFINEMENT**

In this subsection, we show that our equilibrium outcome is one of only two that can be realized in a PBE that survives a standard refinement procedure. We begin by describing this in intuitive terms, then do the technical analysis.

**OVERVIEW**

We refine beliefs using a standard, dominance-based criterion (see Mas-Colell *et al.* 1995, p. 469). Let action \( a = (z, e) \) be dominated for type \( J \) if there exists another action \( a' \) that gives them a strictly higher payoff, for any belief that the lenders might have in equilibrium. The refinement criterion requires that if an action is dominated for one type but not for the other, then lenders must attach zero probability to the event that the former type undertakes that action (see below for details). We investigate the set of all possible PBE that survive this refinement in the area of existence of our separating equilibrium (the striped area in Figure A1).

This analysis leads to two main results. First, the refinement exactly dictates that beliefs must be associated with certain actions. Most importantly, lenders must believe that only the high types would take actions of the type \((z \in [\bar{z}_i, \bar{z}_i^{\text{sep}}], w)\) and \((z \in [\bar{z}_i^{\text{sep}}, \bar{z}_i], w)\), where \( z_{i}^{\text{sep}} \geq z_{i}^{H} \) denotes the second point at which the mimicker’s payoff \( npv^{L}_i(z) \) cuts through the payoff of the genuine low types, \(^{23}\) and \( \bar{z}_i \in [w, z_{i}^{\text{sep}}] \) and \( z_{i} > z_{i}^{\text{sep}} \). This is because these actions are dominated for the low types — any action \((\bar{z}_i^{H}, e \leq z_{i}^{T})\) gives them a higher payoff, no matter what lenders believe in equilibrium — but not for the high types. The beliefs in our separating equilibrium must be changed slightly for the equilibrium to survive the refinement; however, the equilibrium outcome does not change.\(^{24}\)

Second, the above-described requirement on beliefs implies that the PBE must be a separating equilibrium in which the high types contribute \( w \) in equity, and invest either \( z_{i}^{\text{sep}} \) or \( z_{i}^{H} \). Intuitively, these beliefs make it suboptimal for the high types to take any other action in a separating equilibrium. They also rule out the existence of a pooling equilibrium, for the same reason why our separating equilibrium exists: given the possibility of investing \( z_{i}^{\text{sep}} \) and be identified as high types, the high types prefer this to another action that would pool them together with the low types, even if that other action would allow them to invest more. Of course, this logic works only inside the area of existence of the separating equilibrium, where \( w \) (and thus \( z_{i}^{\text{sep}} \)) is high enough.

Investing \( z_{i}^{\text{sep}} \) gives the high types exactly the same payoff as investing \( z_{i}^{H} \). Furthermore, the two thresholds behave in an exactly symmetric fashion. Then \( z_{i}^{\text{sep}} \) is decreasing in \( k_i \). It follows that the main result of the paper needs to be qualified, since across industries where the innovator is of a high type, and invests \( z_{i}^{\text{sep}} \), the model still predicts a monotonic, decreasing relationship between competition and innovation. Of course, such a relationship is due not to the Schumpeterian effect, but to the effect of competition on credit constraints.

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We think that, on balance, these results are good news for our theory. Most crucially, our key equilibrium outcome, \( z_i^{sep} \), is one of only two that may be realized in a refined PBE. And while the existence of \( z_i^{sep} \) as an alternative equilibrium outcome makes it in principle harder for the model to predict an inverted-U, one may reasonably question whether such outcome will ever be observed. After all, while \( z_i^{sep} \) and \( z_i^{sep} \) give exactly the same payoffs to both lenders and borrowers, \( z_i^{sep} \) always implies a lower debt, and thus a lower expected size of default. If there was any additional cost from default, which increased with the size of the default, then \( z_i^{sep} \) would always be the preferred choice.

**TECHNICAL ANALYSIS**

We closely follow the discussion in Mas-Colell et al. (1995, p. 469). We use to the second (and second-weakest) form of domination-based refinement discussed there (equation 13.AA.2). Let \( J \in \mathcal{J} = \{H,J\} \) denote the type of the innovator. Let \( a \in A = \{(z, e) : z \geq 0, 0 \leq e \leq z\} \) denote the choice of investment and equity contribution made by the innovator. Let \( \pi(J|a) \) denote the probability that lenders assign to the innovator being of type \( J \), conditional on observing action \( a \in A \), and let \( r \in R = \{r : r \geq 1\} \) be the interest rate that they require. Let \( u(a,r,J) \) denote the expected payoff to an innovator of type \( J \).

We will say that action \( a \) is strictly dominated for type \( J \) if there exists another action \( a' \) with

\[
\min_{r \in [1/[d^J(a)]}, 1/[d^J(a)]]} u(a', r, J)z > \max_{r \in [1/[d^J(a)]}, 1/[d^J(a)]]} u(a, r, J).
\]

Define the set \( \mathcal{T}(a) \subseteq \mathcal{J} \) as

\[
\mathcal{T}(a) = \{J : \text{there is no } a' \in A \text{satisfying (A5)}\}.
\]

Our definition of a PBE with reasonable beliefs is as follows.

**Definition 1** A PBE has reasonable beliefs if for all \( a \in A \) with \( \mathcal{T}(a) \neq \emptyset \), \( \mu(J|a) > 0 \) only if \( J \in \mathcal{T}(a) \).

In other words, if an action is dominated for type \( J \), and for type \( J \) only, then beliefs are said to be reasonable if and only if lenders attach a zero probability to the event that someone taking action \( a \) is of type \( J \).

We are now ready to present our refinement result.

**Theorem 4** If \( 0 < w < \hat{z}_i^H \mid \kappa = a \) and \( \kappa < \kappa_i < \kappa \), let \( z_i^{sep} \) and \( z_i^{sep} \) be the minimum and maximum \( z > w \) such that

\[
\bar{npv}^L_i(z) = npv^L_i(z^L_i),
\]

or, if such \( z \) is unique or does not exist, then \( z_i^{sep} = z_i^{sep} = \hat{z}_i^H \). Then any PBE that has reasonable beliefs in the sense of Definition 1 is a separating equilibrium where the low types invest \( \hat{z}_i^L \) (any contribution of equity and external financing being possible), and the high types invest either \( z_i^{sep} \) or \( z_i^{sep} \) (contributing \( w \) in equity).

**Proof** There exist \( \hat{z}_i \in [w, z_i^{sep}] \) and \( \hat{z}_i > z_i^{sep} \) such that at any PBE that has reasonable beliefs in the sense of Definition 1, for any \( a = (z, w) \) such that \( z \in (\hat{z}_i, z_i^{sep}) \cup (z_i^{sep}, \hat{z}_i) \), lenders must believe \( \mu(L|a) = 0 \). To see this, note that there exists \( a' = (\hat{z}_i, e), \) with \( e \leq \hat{z}_i^L \), such that
min \( r \in [1/|a^d(u(z^l_i))|,1/|a^d(u(z^l_j))|] \) \( u(a^l^d, r, L) = npV^l_i(z^l_i) \)

\[ \max \left( r \in [1/|a^d(u(z^l_i))|,1/|a^d(u(z^l_j))|] \right) \]

where the inequality follows from the definition of \( z_i^{sep} \) and \( z_i^{pool} \).

A PBE that has reasonable beliefs in the sense of Definition 1 cannot be a pooling equilibrium. To see this, proceed by contradiction. Suppose that the PBE was a pooling equilibrium, and let \((z_i^{pool}, e_i^{pool})\) be the action taken by both types in equilibrium. Distinguish two cases. If \( z_i^{pool} > z_i^{sep} \), then the payoff to the high types would be

\[ npV^h(z_i^{sep}) = a^H(z_i^{pool}) - a^H(z_i^{pool} - e_i^{pool}) - e_i^{pool} < npV^l_i(z_i^{sep} - e), \]

where \( e \) is a small enough number. The last inequality follows from Theorem 3 and from continuity: since situation (a)–(d) is a PBE in this parameter subspace, we must have \( npV^{h}(z_i^{sep}) \geq npV^{l}(z_i) \), and thus \( npV^{h}(z_i^{sep} - e) \geq npV^{l}(z_i) \), for all \( z > z_i^{sep} \) (where the strict inequality follows from the fact that we have assumed \( \kappa > \kappa \) instead of \( \kappa \geq \kappa \)). So the high types could increase their payoff by choosing \( z_i^{pool} = z_i^{sep} \), then the payoff to the high types would be

\[ a^H(z_i^{pool})\gamma(p(k_i)) - a^H(z_i^{pool} - e_i^{pool}) - e_i^{pool} \leq a^H(z_i^{pool})\gamma(p(k_i)) - a^H(z_i^{pool} - w) - w = npV^H(z_i^{pool}) - e_i^{pool} \]

\[ < npV^l_i(z_i^{sep} - e), \]

where the inequality follows from continuity, given that \( e \) is low enough. Again, the high types could increase their payoff by choosing \( (z_i^{sep} - e, w) \). Finally, if \( 0 \leq z_i^{pool} < z_i^{sep} \), then the payoff to the high types would be

\[ a^H(z_i^{pool})\gamma(p(k_i)) - a^H(z_i^{pool} - e_i^{pool}) - e_i^{pool} \leq a^H(z_i^{pool})\gamma(p(k_i)) - z_i^{pool} < npV^H(z_i^{sep} - e), \]

where the second inequality follows from the fact that \( 0 \leq z_i^{pool} < z_i^{sep} - e \leq z_i^{H} \) for \( e \) low enough. Once again, the high types could increase their payoff by choosing \( (z_i^{sep} - e, w) \).

Let \( a = (z,e) \) be such that either \( z \in \{ z_i^{sep}, z_i^{pool} \} \) and \( e < w \), or \( z \in (z_i^{sep}, z_i^{pool}) \) and \( e \leq w \). Then at any separating equilibrium, we must have \( \pi(H|a) < 1 \). To see this, proceed by contradiction. Suppose that \( \pi(H|a) = 1 \). Then the low types could take action \( a \), obtaining payoff

\[ \pi^l_i(z_i^{sep}) + \frac{a^H - a^L}{a^H} (w - e) > npV^l_i(z_i^{sep}). \]

But since, by note 10, the low types must be taking action \((z_i^{l_i}, e_i^{l_i})\) (with \( e_i^{l_i} \leq z_i^{l_i} \)) in a separating equilibrium, obtaining payoff \( npV^l_i(z_i^{sep}) \), they would have a profitable deviation, contradicting the notion that this is a PBE.

The theorem now follows. To see this, recall that it was shown in note 10 that in a separating equilibrium, the low types must be taking action \((z_i^{l_i}, e)\), with \( e \leq z_i^{l_i} \). As for the high types, they could not take an action \((z,e)\) such that either \( z \in \{ z_i^{sep}, z_i^{pool} \} \) and \( e < w \), or \( z \in (z_i^{sep}, z_i^{pool}) \) and \( e \leq w \), since if they did, by Step III of the proof of Theorem 3, the lenders’ beliefs would be incorrect. At the same time, they could not take an action such that \( z < z_i^{sep} \), since their payoff would at best be

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npv_H(z), and it would always be possible to find \( e > 0 \) small enough so that \( z < z_{sep}^i - e \). Since \( npv_H^i(z) < npv_H(z_{sep}^i - e) \), the high types could then increase their payoff by choosing \( (z_{sep}^i - e, \mathbf{w}) \).

Finally, by a symmetrical logic, the high types could not be choosing an action such that \( z > z_{sep}^i \). It follows that the high types must take either action \( (z_{sep}^i, \mathbf{w}) \) in a PBE, or action \( (z_{sep}^i, \mathbf{w}) \).

**DETAILS ON THE COMPUTATIONAL EXERCISE**

To generate panel (b) of Figure 2, using MATLAB, we first create a large matrix by taking 121 evenly spaced values of \( \kappa_i \), going from \( z = 0.4 \) to \( z = 1 \), and then stacking this vector 10,000 times. Each of the elements of the resulting matrix can be interpreted as one industry. Having \( \kappa_i \) as well as \( z \), we can easily compute the average level of competition, \( \kappa \), and profits, as defined by equation (2).

We assign one innovator’s type to each industry using standard pseudo-random draws, where the probability of each of the two possible outcomes is fixed at 0.5.

We then want to compute the probability of successful innovation for each industry, which is characterized by a competition-level innovator’s type pair. In doing so, we assume the specific equilibrium configuration described in the paper. This makes working with low types straightforward, characterized by a competition-level innovator’s type pair. In doing so, we assume the specific equilibrium probability of successful innovation.

We want \( \mu \) to be tractable and invertible, since we will need to work with its inverse in order to compute \( \tilde{z}_l \).

Throughout this computational exercise, we need to stay in the region of the parameter space where our model generates an inverted-U pattern.

As stated in the paper, \( \mu \) is increasing and concave.

We want \( \mu \) to be tractable and invertible, since we will need to work with its inverse in order to compute \( \tilde{z}_l \).

We choose \( \mu(z) = 0.22 \sqrt{z} \) for each low type industry, it is easy to compute the desired probabilities as \( d^l \mu(\tilde{z}_l) \).

Dealing with high type innovators is trickier, as they will choose \( \tilde{z}_H^i \) or \( z_{sep}^i \), following the logic described in Section I. For each high type industry, then, we effectively need to model the behaviour of both high and low type innovators. We thus start by computing \( \tilde{z}_H^i \) and \( npv_H^i(\tilde{z}_H^i) \), then we compute \( \tilde{z}_H^i \) using the same methodology (i.e. condition (5), this time with \( d^H = 1 \)). We then check for which high type industries \( npv_H^i(\tilde{z}_H^i) \) (computed using equation (6)) goes above \( npv_H^i(\tilde{z}_L^i) \). For these industries, we pick \( z_{sep}^i \) as the equilibrium level of investment. If instead \( npv_H^i(\tilde{z}_H^i) < npv_H^i(\tilde{z}_L^i) \) for all \( z \geq 0 \), then we pick \( z_{sep}^i \). Finally, we use the formula \( a^H \mu(\cdot) \) to compute the equilibrium probability of successful innovation.

Once we have computed the probability of successful innovation for each industry, we use it as a parameter of a binomial distribution, in order to simulate real-world patenting behaviour and generate a synthetic dataset. The resulting histogram, obtained by collecting into bins corresponding to the 121 values of \( \kappa_i \), all the patents secured by both high and low types, is plotted in panel (b) of Figure 2.

We also use this synthetic dataset to generate the solid curve overlayed onto the histogram of Figure 2. To do so, we follow the methodology used in Aghion et al. (2005) as closely as our synthetic dataset allows. Specifically, we compute a Poisson regression of the total number of patents on a constant \( (\hat{\beta}_0) \), our vector of \( \kappa_i \), and a vector containing \( \kappa_i^2 \) for each \( i \).

Let the vector containing all \( \kappa_i \) be denoted as \( \mathbf{K} \), the vector containing \( \kappa_i^2 \) as \( \mathbf{K}^2 \), and the corresponding regression coefficients as \( \hat{\beta}_1 \) and \( \hat{\beta}_2 \), respectively. The solid curve is then computed as

\[
\exp(\hat{\beta}_0 + \hat{\beta}_1 \mathbf{K} + \hat{\beta}_2 \mathbf{K}^2). 
\]

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NOTES

1. The pattern has been observed in the USA (Scherer (1967), but see Hashmi (2013) for contradicting results), the UK (Aghion et al. 2005), Japan (Michiyuki and Shunsuke 2013), the Netherlands (Polder and Veldhuizen 2012), Sweden (but only for specific measures of competition—see Tingvall and Karpaty 2011), France (but only for large firms—see Askenazy et al. 2013) and Switzerland (Peneder and Woerter 2014), and for the pharmaceutical industry in a panel of 26 countries studied by Qian (2007).

2. Akcigit and Kerr (2018) estimate a Schumpeterian model with multi-product incumbents and new entrants, using US data. They find that incumbents innovating on new product lines together with new entrants account for more than 80% of aggregate productivity growth. They argue that this finding is consistent with the empirical literature surveyed in Foster et al. (2001). The fact that established firms that do not reap pre-innovation rents face much the same incentives to innovate as new entrants is recognized by the literature; see, for example, Aghion et al. (2001, p. 469).

3. See, for example, Schoonhoven et al. (1990), and Katila and Shane (2005).

4. The general idea of ‘skin in the game as a screening device’ has emerged repeatedly in the academic Finance literature. Applications to the field of entrepreneurship—and its financing—go all the way back to Leland and Pyle (1977); more recent contributions include Kaplan and Stromberg (2004), Skeie (2007) and Conti et al. (2013), among others. DeMarzo and Duffie (1999) and DeMarzo (2005) provide examples of this principle in the context of security design.

5. For subsequent modifications of their model, see Askenazy et al. (2013) and Hashmi (2013).

6. Some of these papers focus on firms innovating on products that they currently produce, thus sharing the same empirical limitations as Aghion et al. (2005) (see Chernyshew 2016; Rauch 2008). Other papers rely on ad hoc modifications of the standard model: Mukoyama (2003) needs imitators to play an important role alongside innovators; Scott (2009) requires firms to have a different perception of competition at different levels; and Onori (2015) requires external learning from innovation to be more important than internal learning.

7. In a related contribution, Chiu et al. (2017) develop a model in which ‘entrepreneurs’ get new ideas randomly and without paying any R&D costs, but search frictions and the presence of financial intermediaries influence the process of technological transfer. That is, their focus is on the allocation of these new blueprints to the agents who have the most talent for developing them and bringing them to market.

8. This assumption, also made by Aghion and Howitt (2009), simplifies the model by ruling out that credit constraints are weaker in industries where initial productivity, and thus the size of investment, is lower. Since the wage is determined in the economy-wide labour market, it is a linear function of average (as opposed to industry-specific) productivity. The normalized wage is then \( w_{X_0} = \frac{w}{A_{X_0}} \).

9. This demand function can be found by taking the first derivative of equation (1) with respect to \( X_0 \).

10. To see this, suppose that the low types invested \( z \neq z_0^L \). Since this is a separating equilibrium, the low types would have to be asked an expected interest rate equal to the risk-free rate, and their payoff would have to be \( npv^L(z) \). But by choosing \( z_0^L \), they could have not been asked a higher expected rate (in equilibrium), and it follows that \( z \) is not the low types’ optimal choice: a contradiction.

11. More precisely, lenders believe that those contributing \( w \) in equity are high types if they invest an amount lower than or equal to \( z_0^{WP} \), and high or low types with equal probability if they invest more than \( z_0^{WP} \). Additionally, they believe that those contributing less than \( w \) in equity are low types.

12. For example, the future innovator could be someone who inherits the current innovator’s talent, and whose payoffs are important to the current innovator (e.g. a descendant or employee whom the current innovator coaches). Then her current innovator would worry about the reputation that she establishes in period 1. We refer to the fourth subsection of the Appendix for a complete description of the methodology used.

13. It is our understanding that the convexity to the immediate left of the peak in Figure 2, and the concavity to its right, are a general feature of the model, but we have not shown that in a formal proposition.

14. We assume that \( A_{X_0} \) is uncorrelated with talent, and with the level of competition.

15. Recall that profits are found by substituting the (constant) optimal price in the demand faced by the monopolist, \( P_{X_0} = a(A_{X_0}/L/X_{X_0})^{1-z} \), by solving for the equilibrium quantity \( X_{X_0} \), and by multiplying this by the (constant) profit per unit. Since \( X_{X_0} \) is linear in \( A_{X_0} \), so are profits. Normalized profits are then as in equation (2), and do not depend on \( A_{X_0} \).

16. Since the wage is determined in the economy-wide labour market, it is a linear function of average (as opposed to industry-specific) productivity. The normalized wage is then \( w_{X_0} = w/A_{X_0} \).
18. We have shown that if industries 1 and 2 are both credit-constrained, then credit constraints are tighter in industry 1 (intensive margin). It is also possible that industry 1 is credit-constrained while industry 2 is not, but not the opposite (extensive margin).

19. We are grateful to an anonymous referee for pointing this out to us.

20. Aghion et al. (2004) identify these firms as those with the highest debt-payments-to-cash-flow (D/C) ratio. In our model, the D/C ratio is \( \frac{z^H_i - W}{NPV^H_i(z^H_i)} = \frac{z^H_i - W}{A^H_i + \gamma} / \frac{z^H_i}{NPV^H_i(z^H_i)} \) for non credit-constrained industries, and \( \frac{z^H_i - W}{A^H_i + \gamma} / \frac{z^H_i}{NPV^H_i(z^H_i)} \) for credit-constrained ones. While the former is increasing in \( A^H_i \) (since \( z^H_i \) is unchanged), the latter can be increasing or decreasing (since \( z^H_i \) decreases). So our high-productivity industries are not necessarily those with the higher D/C ratio. This is counterintuitive, as you would expect industries that require greater investment to feature both a higher D/C ratio and tighter credit constraints. This undesirable feature of the model is the result of simplifying assumptions. With a unified labour market, a higher \( A^H_i \) only makes it more expensive to innovate, without increasing the innovator’s capacity to contribute (in other words, it only reduces \( w_0 \)). This results in much tighter credit constraints, and may result in a lower D/C ratio. However, if the wage was partially related to industry productivity, then credit constraints would still be tighter in high-productivity industries (and the peak of their inverted-U would still be located more to the right), and so would be the D/C ratio. To see this, consider an extreme case in which the wage increases almost linearly with \( j \) and the peak of their inverted-U would still be located more to the right), and so would be the D/C ratio. Related to industry productivity, then credit constraints would still be tighter in high-productivity industries. Then credit constraints \( z^H_i - z_i^{sep} \) would still be (marginally) tighter (since \( z^H_i \) is unchanged and \( z_i^{sep} \) still decreases), and the D/C ratio would be higher (since \( z_i^{sep} \) decreases only marginally).

21. In the context of our model, the return on equity is \( ROE^*_L = NPV^H(z^*_L) / w \) for the low types (given a risk-free interest rate equal to zero), and \( ROE^*_H = NPV^H(z^*_H) / w \) for the high types. It is easy to show that these are decreasing in \( \gamma \). For the low types, we have \( dROE^*_L / dw_0 = (1 / w_0) \alpha L \mu \gamma \pi(k) d\gamma / dw_0 < 0 \). For the high types, we have \( dROE^*_H / dw_0 = (1 / w_0) \alpha L \mu \gamma \pi(k) d\gamma / dw_0 > 0 \). This contradicts the fact that by definition of \( z_i^{sep} \) we must have \( d[NPV^L(z_i^{sep})] / d\gamma = d[NPV^H(z_i^{sep})] / d\gamma = dROE^*_L / dw_0 \) and \( dROE^*_H / dw_0 < 0 \).

22. This case must be considered, as there always exist admissible values of the other parameters of the model, \( \gamma \) and \( \alpha \), and admissible forms of the function \( \mu(\cdot) \), such that \( z^H_i \) is unchanged.

23. The existence of such a point can be gauged from panel (a) of Figure 1. The function \( NPV^H(z) \) is a parabola reaching its maximum at \( z^H \). It must then cut through the horizontal line passing through \( NPV^H(z^H) \) twice, to the left and to the right of \( z^H \).

24. Beliefs must be changed in the following way. First, lenders must believe that those taking actions of the type \( \left( z \in \{z^sep, z^H_i \}, w \right) \) must be high types. Second, they must believe the same for those taking actions of the type \( \left( z \in \{z^sep, z^H_i \}, w \right) \) or \( \left( z \in \{z^sep, z^H_i \}, w \right) \), where \( \epsilon > \eta \) and \( w = \eta \), where \( \epsilon > \eta \) are small enough numbers. No other belief must be changed. Since \( NPV^H(z^sep) \) gives the high types exactly the same payoff as investing \( z_i^{sep} \), it is easy to see that the outcome of the equilibrium does not change.

25. The value of \( z \) is chosen in order to be broadly consistent with the labour share of income as recorded in the USA in postwar years.

26. To compute profits we also need to assign a value to \( z \), and we choose \( L = 100 \).

27. This is discussed in the second subsection of this Appendix. What is particularly relevant for us at this stage is that the wage \( w \) needs to be between \( w \) and \( w \). The wage can be computed using equation (3). We can also compute closed-form solutions for both \( w \) and \( w \), namely,

\[
\bar{w} = \frac{1}{1 - (aL/aH)} \left[ \frac{d\mu(z^H_i) \pi(k) - dL \bar{z}_i^H - npv^H(z^H_i)}{aL \bar{z}_i^H - npv^H(z^H_i)} \right],
\]

where \( i \) is such that \( \gamma_i = \gamma \), and

\[
\bar{w} = \frac{1}{(aL/aH) - (aL/aH)} \left[ \frac{d\mu(z^H_i) \pi(k) - dL \bar{z}_i^H - npv^H(z^H_i)}{aL \bar{z}_i^H - npv^H(z^H_i)} \right],
\]

where again \( i \) is such that \( \gamma_i = \gamma \) and \( a = (aL + aH) / 2 \).

28. Of course, \( 0.22 / 2 \) goes to infinity as \( z \) grows, so there might be concern about the resulting probabilities of successful innovation being larger than 1. This does not happen in our simulations, where the probabilities are in fact rather small, and always smaller than 0.18.

29. As in Aghion et al. (2005), these vectors are demeaned.
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