Core excitations in exotic nuclei

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Abstract. The role of core excitations in exotic nuclei is discussed in the framework of a microscopic cluster model. This cluster approach is complemented by the $R$-matrix theory to take account of the long-range part of the wave functions. We briefly describe the model, and present two recent examples: the neutron-rich nucleus $^{16}\text{B}$, described by a $^{15}\text{B}+\text{n}$ structure, and the proton-rich nucleus $^{17}\text{Na}$, described by a $^{16}\text{Ne}+\text{p}$ structure. In both cases core excitations are shown to play an important role.

1. Introduction

Clustering is a well known effect in light nuclei (see Ref. [1] and references therein). This property is at the basis of cluster models [2], where the nucleus is described in terms of two (or more) cluster wave functions [3, 4, 5]. Several variants exist according to whether they exactly include antisymmetrization or not. A natural extension of cluster models to exotic nuclei is the description of halo states, where external nucleons are simply considered as clusters. Typical examples are the $^6\text{He}$ and $^{11}\text{Li}$ nuclei, which can be described by $\alpha+n+n$ [6] and $^9\text{Li}+n+n$ [7] configurations, respectively.

Simple variants of cluster models only include the ground state of each cluster and, in particular, of the core nucleus. However this approximation may be, in some cases, too simple, and the importance of core excitations needs to be addressed. Here we use a microscopic cluster model with an $A$-body hamiltonian. In the two-cluster variant, an approximate wave function of the system reads, in a schematic notation,

$$\Psi = A\phi_1\phi_2 g(\rho),$$

where $A$ is the antisymmetrization operator. In this definition, $\phi_1$ and $\phi_2$ are the internal wave functions of the two clusters and $g(\rho)$ the radial function depending on the relative coordinate $\rho$. This model has been applied to many light nuclei and nucleus-nucleus reactions, including reactions of astrophysical interest (see references in Refs. [8, 5]). Definition (1) is the standard RGM (Resonating Group Method) wave function [9, 3], and is fully antisymmetric against exchanges of nucleons. Further simplification can be performed by neglecting the internal structure of the clusters, and by choosing an appropriate nucleus-nucleus potential [10]. Wave function (1) can be generalized to multicluster systems [5]. It is well adapted to exotic nuclei.
[11], where the breakup threshold is low. In that case, the long-range part of the wave function must be accurately determined.

Here we focus on extensions of (1) to core excitations; in other words the total wave function can be generalized as

$$\Psi = \sum_c A \phi_1 c \phi_2 g_c (\rho),$$

where \(c\) represents the excitation level of cluster 1 (cluster 2 is assumed to remain in its ground state). Previous applications to weakly bound nuclei and to unbound systems have shown that core excitations may play an important role [12, 13, 14]. This phenomenon was recently confirmed by an experimental study of \(^{18}\text{Ne} + p\) elastic scattering, performed in parallel with the \(^{18}\text{Ne}(p, p')^{18}\text{Ne}(2^+)\) inelastic scattering [15]. This experiment shows a clear evidence for \(^{19}\text{Na}\) states with a dominant \(^{18}\text{Ne}(2^+)+p\) structure. The importance of \(^6\text{He}^* + n\) configurations in the \(^7\text{He}\) spectrum is also well established [16, 17].

The paper is organized as follows. In section 2, we briefly present the microscopic cluster model. Sections 3 and 4 are devoted to the recent examples of \(^{16}\text{B}\) and \(^{17}\text{Na}\), respectively. These systems are typical neutron- and proton-rich nuclei. Concluding remarks are presented in section 5.

2. Microscopic cluster theories
Microscopic models are based on fundamental principles of quantum mechanics, such as the treatment of all nucleons, with exact antisymmetrization of the wave functions. Neglecting three-body forces, the Hamiltonian of an \(A\)-nucleon system is written as

$$H = \sum_{i=1}^{A} T_i + \sum_{i<j}^{A} V_{ij},$$

where \(T_i\) is the kinetic energy of nucleon \(i\), and \(V_{ij}\) an effective nucleon-nucleon interaction [3]. Effective forces, such as the Volkov [18] or the Minnesota [19] interactions are adapted to the model wave functions. Both contain one adjustable parameter which can be tuned to reproduce an important property of the nucleus, such as a threshold breakup energy.

In practice, the Schrödinger equation associated with this Hamiltonian cannot be solved exactly when \(A > 4\). For very light systems (\(A \sim 4 - 5\)) efficient methods [20] exist, even for continuum states [21]. Recent developments of \textit{ab initio} models (see for example Refs. [22, 23, 24]) are quite successful for spectroscopic properties of nuclei up to \(A \approx 12\). These models make use of realistic interactions, which are fitted to many properties of the nucleon-nucleon system, and include three-body forces. However, a consistent description of bound and scattering states of an \(A\)-body problem remains a very difficult task [21], in particular for transfer reactions.

In cluster models, it is assumed that the nucleons are grouped in clusters [3, 13]. We present here the specific application to two-cluster systems. The internal wave functions of the clusters are denoted as \(\phi^{I, \nu_i}_i (\xi_i)\), where \((I_i, \nu_i)\) and \(\pi_i\) are the spin/projection and parity of cluster \(i\), and \(\xi_i\) represents a set of their internal coordinates. A channel function is defined as

$$\varphi^{J, M}_{\ell I} (\Omega_\rho, \xi_1, \xi_2) = \left[ V_\ell (\Omega_\rho) \otimes [\phi^{I, \pi_1}_1 (\xi_1) \otimes \phi^{I, \pi_2}_2 (\xi_2)] \right]^{JM},$$

where different quantum numbers show up: the channel spin \(I\), the relative angular momentum \(\ell\), the total spin \(J\) and the total parity \(\pi = \pi_1 \pi_2 (-)^\ell\).
The total wave function (1) of the \( A \)-nucleon system is therefore written in a more explicit way as

\[
\Psi^{J\pi} = \sum_{\alpha \ell I} \psi_{\alpha \ell I}^{JM\pi}
= \sum_{\alpha \ell I} A_{\alpha \ell I}^{J\pi}(\rho) \varphi_{\alpha \ell I}^{JM\pi}(\Omega_{\rho}, \xi_1, \xi_2),
\]  

(5)

where index \( \alpha \) refers to different two-cluster arrangements. In most applications, the internal cluster wave functions \( \phi_{i}^{I\pi;\nu_{i}} \) are defined in the shell model. The relative wave functions \( g_{\alpha \ell I}^{J\pi}(\rho) \) are to be determined from the Schrödinger equation which, in the RGM, is transformed into an integro-differential equation involving a non-local potential [9]. In most applications, this relative function is expanded over Gaussian functions [9, 3, 5], which corresponds to the Generator Coordinate Method (GCM). The wave function (5) is therefore rewritten as

\[
\Psi_{\alpha \ell I}^{JM\pi} = \int f_{\alpha \ell I}^{J\pi}(R) \Phi_{\alpha \ell I}^{JM\pi}(R),
\]  

(6)

where \( \Phi_{\alpha \ell I}^{JM\pi}(R) \) is a projected Slater determinant, and \( f_{\alpha \ell I}^{J\pi}(R) \) the generator function, which must be determined. The GCM is equivalent to the RGM, but is better adapted to numerical calculations, as it makes use of projected Slater determinants (see Refs. [9, 3, 5] for detail).

The main advantage of cluster models with respect to other microscopic theories is their ability to deal with reactions, as well as with nuclear spectroscopy. As mentioned before, the RGM radial wave functions are expanded over a Gaussian basis. The GCM is well adapted to numerical calculations, and to a systematic approach, but the Gaussian behaviour is not physical at large distances, and must be corrected. We use the Microscopic \( R \)-matrix Method (MRM) [25, 26] which is a direct extension of the standard \( R \)-matrix technique [27], based on the existence of two regions: the internal region (with channel radius \( a \), where the nuclear force and the nucleus-nucleus antisymmetrization are important, and the external region where they can be neglected. In the external region, the Gaussian expansion of the RGM radial function is replaced by Coulomb functions. Matching the internal and external components provides either the collision matrix (for scattering states) or the binding energy (for bound states).

3. Application to the nucleus \(^{16}\text{B}\)

The unbound nature of \(^{16}\text{B}\) has been first proposed by Bowman et al. [28] and by Langevin et al. [29], and later confirmed by Kryger et al. [30] in an analysis of the \(^{17}\text{C}\) breakup. The low-lying structure has been investigated by Kalpacheva et al. [31] in a heavy-ion multi-nucleon transfer reaction. The existence of a narrow peak above the \(^{15}\text{B+n}\) threshold (\( E = 40 \pm 40 \text{ keV} \)) with a width lower than 100 keV, together with a higher level at \( E = 2.40 \text{ MeV} \) (\( \Gamma = 0.15 \text{ MeV} \)) has been reported. More recently, the \(^{16}\text{B}\) spectrum has been studied by single-proton removal from a 35 MeV/nucleon \(^{17}\text{C}\) beam by Lecouey et al. [32]. In that experiment, a narrow resonant structure at \( E = 85 \pm 15 \text{ keV} \) above the \(^{15}\text{B+n}\) threshold, with a width of \( \Gamma \ll 100 \text{ keV} \), has been observed and confirms the experiment of Kalpacheva et al. This peak is interpreted as a narrow resonance which decays by \( d \)-wave neutron emission.

In this section, we investigate the \(^{16}\text{B}\) nucleus in a \(^{15}\text{B+n}\) model, involving several \(^{15}\text{B}\) excited states (see detail in Ref. [13]). Shell-model wave functions are built from all configurations with 3 protons in the \( p \) shell, and 2 neutrons in the \( sd \) shell. This shell-model description involves 1320 Slater determinants and provides \(^{15}\text{B}\) states from the diagonalization of several operators: the total angular momentum \( J^2 \) (and its projection \( J_z \)), the orbital momentum \( L^2 \) and the intrinsic spin \( S^2 \) [5]. The lowest states can be considered as physical, and high-energy levels correspond to pseudostates, which simulate the \(^{15}\text{B}\) distortion. To keep computer times and
memory requirements within reasonable limits, we have limited the shell-model basis to the intrinsic spin $S = 1/2$; this provides $87\,^{15}\text{B}$ states.

In Figure 1, we present the low-lying $^{15}\text{B}$ spectrum obtained with the present conditions of calculation, i.e. with shell-model wave functions limited to the $sd$ shell. This model predicts a $3/2^-, 5/2^-$ and $7/2^-$ level ordering consistent with the data. In particular, we suggest that the state at 3.48 MeV reported by Kalpachieva et al. [31] could be assigned $1/2^-$. 

First we address the importance of core excitations, by analyzing the evolution of the $^{16}\text{B}$ spectrum as a function of the number of channels. Figure 2 shows low-lying states of $^{16}\text{B}$ for different numbers of channels. We present the single-channel $^{15}\text{B}(3/2^-)+n$, two-channel $^{15}\text{B}(3/2^-,5/2^-)+n$, and four-channel $^{15}\text{B}(3/2^-,5/2^-,7/2^-,1/2^-)+n$ calculations, as well as the full multichannel approach. Figure 2 is limited to the first states obtained for different $J^\pi$ values from $0^-$ to $3^-$. 

The resonance energies are clearly sensitive to the size of the variational basis. In particular, the $0^-$ level, assumed to be the $^{16}\text{B}$ ground state [31, 32], is strongly sensitive to excited configurations. In the multichannel calculation, the $1^-$ and $2^-$ states converge near the $^{15}\text{B}+n$ threshold. The $1^-_1$ state becomes the $^{16}\text{B}$ ground state, and is even slightly bound, when the full basis is considered. All these results support the importance of a multichannel framework to describe the $^{16}\text{B}$ nucleus.

Figure 3 provides the full GCM spectrum, compared with the available experimental data. The shell-model results of Ref. [32] are also shown for comparison. The existence of a narrow $0^- (\ell = 2)$ narrow resonance just above the threshold is confirmed by the GCM. The GCM predicts $1^-$ and $2^-$ states at low energies with a dominant component in the $^{15}\text{B}(g.s)+n$ channel. We find that the $1^-$ level is even slightly bound, although it presents a large reduced width. Of course a microscopic model cannot be expected to provide a precision of a few keV, but the existence of this low-lying state is likely. A similar s-wave resonance was predicted by the

| $E_{\text{cm}}$ (MeV) | Exp. | Present |
|----------------------|------|---------|
| 0                   | 3/2^- | 3/2^-  |
| 1                   | 5/2^- | 7/2^-  |
| 2                   | 7/2^- | 5/2^-  |
| 3                   | 1/2^- |         |
| 4                   | 1/2   |         |
| 5                   | 7/2^- |         |
| 6                   | 5/2^- |         |

**Figure 1.** $^{15}\text{B}$ spectrum obtained with the present calculation, and compared to experiment [31, 33].

**Figure 2.** Evolution of $^{16}\text{B}$ low-lying states according to the number of channels. Labels '1 Ch.', '2 Ch.', '4 Ch.' and 'All' refer to the number of channels.
GCM in $^{13}$Be at low energies [34], and found experimentally later [35]. As a general conclusion the $^{16}$B spectrum is predicted to have several additional states, which have not been observed experimentally yet.

![Figure 3. $^{16}$B spectra. Experimental energies are taken from Refs. [31, 32]. Shell-model (SM) results are taken from Ref. [32].](image)

4. **Application to the nucleus $^{17}$Na**

Our aim here is to show that narrow states can also exist in the spectrum of the proton rich nucleus $^{17}$Na, which is the mirror analogue of $^{17}$C famous for its peculiar structure (see details in Ref. [12]). The neutron binding energy in the ground state $^{17}$C(3/2$^+$) is only 728 keV, typical of halo nuclei. However, knockout experiments have shown that the weakly bound $^{16}$C(0$^+$)+n configuration is suppressed in $^{17}$C(3/2$^+$) and that this state is mainly based on the $^{16}$C(2$^+$)+n configuration [36] where the neutron binding energy is 2.5 MeV. A similar structure should be expected in the mirror nucleus $^{17}$Na(3/2$^+$). Therefore, the decay branch $^{17}$Na(3/2$^+$) $\rightarrow^{16}$Ne(0$^+$)+p could be suppressed and, if energetically allowed, the main decay mode would be $^{17}$Na(3/2$^+$) $\rightarrow^{16}$Ne(2$^+$)+p. If its decay energy is below the Coulomb barrier then its width may be small. Since the decay product $^{16}$Ne is unstable with respect to 2p emission, $^{17}$Na should be a three-proton (3p) emitter.

The $^{16}$C and $^{17}$Na are studied in parallel with the Volkov and Minnesota interactions. The two $^{16}$C valence neutrons occupy the $0d_{5/2}$, $1s_{1/2}$ and $0d_{3/2}$ orbitals, which gives many excitations in $^{16}$C. We only consider the $2^+_1$, $3^+_1$ and $4^+_1$ states, motivated by neutron knockout experiments where they are strongly populated. The $^{17}$C spectrum is fairly well known from experiment, and will be used to estimate the precision of the model applied to the mirror $^{17}$Na nucleus.

Let us first discuss the $^{17}$C spectrum shown in Fig. 4, compared to experiment. The parameters involved in the nucleon-nucleon interaction have been chosen to reproduce both the $^{16}$C+n threshold in $^{17}$C and the excitation energy of $^{17}$C(1/2$^+_1$). With these parameters, the energies of the three low-lying states are in very good agreement with experiment. In particular the level ordering is correctly reproduced. For comparison, the shell model spectrum is also shown in Fig. 4.

The decay scheme of the lowest part of the $^{17}$Na spectrum is shown in Fig. 5. We find that the $^{17}$Na(7/2$^+_1$) level should be very narrow. It decays into the d-wave $^{16}$Ne(2$^+_1$)+p and s-wave $^{16}$Ne(4$^+_1$)+p channels with partial widths of $\Gamma(2^+_1) = 25$ keV and $\Gamma(4^+_1) = 98$ keV respectively. It should be noted that the theoretical value of the latter threshold is underestimated by 630 keV and, therefore, the energy in this channel is too large. Decreasing this energy by tuning the Majorana parameter $m$, we obtain a partial width $\Gamma(4^+_1) = 4$ keV. Thus, the $^{17}$Na(7/2$^+_1$) state should be as narrow as $^{17}$Na(3/2$^+_2$). A similar situation occurs for the $^{17}$Na(9/2$^+_1$) state. The partial width for the decay into $^{16}$Ne(2$^+_1$)+p is predicted to be 211 keV. A similar width is expected for the decay into the $^{16}$Ne(4$^+_1$)+p channel. Tuning the energy of this channel to reproduce the position of the $^{17}$C(9/2$^+_1$) state with respect to the $^{16}$Ne(4$^+_1$)+p threshold, similar to what has been done in the case of 7/2$^+_1$, we get a partial width $\Gamma(4^+_1) = 78$ keV.
Figure 4. $^{17}$C spectra calculated in the MCM with the V2 and MN interactions in comparison to the experimental spectrum and to shell model (SM) WBP predictions. Labels correspond to $2J$. The spin-parity assignment for observed unbound levels corresponds to that suggested by 3n transfer [37].

Figure 5. $^{17}$Na decay scheme. The main decay branches are indicated by arrows.

There should be at least four narrow states, $3/2^+$, $5/2^+$, $7/2^+$ and $9/2^+$ in the $^{17}$Na spectrum. The decay product of these states, $^{16}$Ne, is unstable with respect to two-proton emission. Therefore, $^{17}$Na is in fact a three-proton emitter with a decay path $^{17}$Na $\rightarrow$ $^{16}$Ne$^* + p \rightarrow ^{14}$O$+2p+p$. Consequently, $^{17}$Na states can be identified by detecting $^{14}$O+p+p+p events in coincidence. A multichannel algebraic study [38] confirms the importance of core excitations in $^{17}$C and $^{17}$Na, but finds some differences for the resonance properties of $^{17}$Na.

5. Conclusion

Microscopic cluster models represent a powerful tool to investigate the structure of light nuclei. With the help of the $R$-matrix method, unbound systems can be studied with a rigorous treatment of the asymptotic wave function. This is particularly important for broad resonances showing up in many nuclei close to, or beyond, the driplines. Another advantage is that core excitations can be introduced without further parameters. The couplings between the channels are directly determined from the nucleon-nucleon interaction.

In most exotic nuclei, core excitations play an important role. This property is supported by
several experiments, on $^7$He or $^{19}$Na, for example. The present work addresses core excitations in $^{16}$B and $^{17}$Na which are both unstable in their ground state. The microscopic calculation suggests that several states should exist in the $^{16}$B spectrum. Most of them are very sensitive to core excitations, and the ground state should be close to particle stability. A similar conclusion holds for $^{17}$Na. Due to the dominant $^{16}$Ne$^* + p$ configurations, several states have narrow widths. These low-lying states have long lifetimes, and are suggested to be three-proton emitters.

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