pre-saddle neutron multiplicity for fission reactions induced by heavy ions and light particles

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Abstract

Pre-saddle neutron multiplicity has been calculated for several fission reactions induced by heavy ions and light particles. Experimentally, it is impossible to determine the contribution of neutrons being emitted before the saddle point and those emitted between the saddle and the scission points. Determination of the pre-saddle neutron multiplicity in our research is based on the comparison between the experimental anisotropies and those predicted by the standard saddle-point statistical model. Analysis of the results shows that the pre-saddle neutron multiplicity depends on the fission barrier height and stability of the compound nucleus. In heavy ion induced fission, the number of pre-saddle neutrons decreases with increasing the excitation energy of the compound nucleus. A main cause of this behavior is due to a reduction in the ground state-to-saddle point transition time with increasing the excitation energy of the compound nucleus. Whereas in induced fission by light particles, the number of pre-saddle neutrons increases with increasing the excitation energy of the compound nucleus.

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I. INTRODUCTION

In the last two decades, much theoretical attention has directed towards understanding the dynamics of fission. According to reports, measuring the number of neutrons emitted during fission most likely gives information on the timescale of fission as well as on the nuclear dynamics. The transition state model of fission, based on appropriate level densities, predicts the widths (and thus lifetimes) of fission and neutron emission. This model is also suitable for determining the pre-fission neutron multiplicity if the calculated lifetimes are long compared to the dynamically constrained fission lifetime. Several groups have invested an extensive effort in measuring the number of emitted neutrons associated with fission reactions induced by heavy ions [1-17]. The measurement of emitted neutrons is usually limited to the measurement of pre-scission neutron multiplicity, post-scission neutron multiplicity, and therefore total neutron multiplicity. These measurements show that the transition state model of fission leads to an underestimation of the number of measured pre-scission neutrons emitted in heavy ion induced fission at high excitation energies. This discrepancy can be related to the viscosity of the hot nucleus [18]. Hence, the fission lifetime of a hot nucleus is substantially longer than that determined by statistical model of Bohr and Wheeler [19]. As a result, it is natural to expect that a dissipative dynamical model would provide an appropriate description of nuclear fission at high excitation energies [20].

Pre-scission neutrons \( \nu_{\text{pre}} \), can be emitted between the ground state of the compound nucleus and the saddle point (pre-saddle neutrons) \( \nu_{\text{gs}} \), or between the saddle and the scission points (saddle-to-scission neutrons) \( \nu_{\text{ss}} \). The number of pre-saddle neutrons as well as the number of saddle-to-scission neutrons can be determined by a combined dynamical statistical model (CDSM) [21-23]. The contributions \( \nu_{\text{gs}} \) and \( \nu_{\text{ss}} \) to the pre-scission neutron multiplicity are also estimated by a stochastic approach based on three-dimensional Langevin equation [24]. Recently, a more accurate four-dimensional Langevin model as an extension of the three-dimensional Langevin model by adding the fourth collective coordinate (the projection of the total spin about the symmetry axis of the fissioning nucleus) is used to calculate the pre-scission neutron multiplicity [25].

A common assumption in the calculation of the angular anisotropy of fission fragments using by the transition state model is that all pre-scission neutrons are emitted prior to reaching the saddle-point, since it is not straightforward to separate experimentally the contribution
of neutrons being emitted before the saddle-point and those emitted between the saddle and the scission points [26-35]. It is well known that the standard saddle-point statistical model (SSPSM) has become the standard theory of fission fragment angular distributions and received great success since it was proposed. The effect of neutron evaporation prior to reaching the saddle-point is to reduce the temperature of the fissioning nucleus which in turn increases the fission fragment anisotropy prediction by using this model. Only $\nu_{gs}$ has an influence over the prediction of angular anisotropy by using SSPSM. The upper limit of the angular anisotropy of fission fragments, based on the prediction of SSPSM is determined by assuming that all the pre-scission neutrons are emitted before the saddle-point.

The pre-saddle neutrons as a crucial quantity in determining the angular anisotropy of fission fragments by using SSPSM plays a main role, although any precise method to determine it has not been introduced. In this article, we calculate the number of pre-saddle neutrons by a novel method. In this method, the values of $\nu_{gs}$ for several induced fission reactions by light particles and heavy ions are determined by the fission fragments angular distribution method. This method is based on comparison between the experimental anisotropies and those predicted by the standard saddle-point statistical model. This method is limited to the calculation of pre-saddle neutrons in induced fission in which the angular anisotropy of fission fragments has a normal behavior, i.e., it is observed a good agreement between the angular anisotropy of fission fragments and that predicted by the SSPSM.

In order to make the present paper self-contained, we present in Sec. II, an brief description of the standard saddle-point statistical model as well as the calculating method of the pre-saddle neutron multiplicity on the basis of the SSPSM in detail. Section III is devoted to the results obtained in this study. Finally, the concluding remarks are given in Sec. IV.

II. METHOD OF CALCULATIONS

A. Standard saddle-point Statistical Model

The standard transition-state model has been used to analyze the angular anisotropy of fission fragments in fission. In the transition-state model, the equilibrium distribution over the K degree of freedom (the projection of total angular momentum of the compound nucleus (I) on the symmetry axis of the fissioning nucleus) is assumed to be established at
the transition state. Two versions of the transition-state models based on assumptions on the position of the transition state: standard saddle-point statistical model (SSPSM), and scission-point statistical model (SPSM) can be used for the prediction of fission fragment angular distributions. The basic assumption of the SSPSM is that fission proceeds along the symmetry axis of a deformed compound nucleus, and that the distribution of $K$ is frozen from the saddle point to the scission point. In this model, the fission fragment angular distribution $W(\theta)$ for the fission of spin zero nuclei is given by the following expression [36]

$$W(\theta) \propto \sum_{I=0}^{\infty} \frac{(2I+1)^2T_I\exp[-p\sin^2\theta]J_0[-ip\sin^2\theta]}{\text{erf}[\sqrt{2p}]},$$

(1)

Where $T_I$ and $J_0$ are the transmission coefficient for fission, and the zeroth-order Bessel function, $p = (I + \frac{1}{2})^2/(4K_0^2)$, and the variance of the equilibrium $K$ distribution $(K_0)$ is

$$K_0^2 = \frac{\Im_{eff}T}{\hbar^2},$$

(2)

here $\Im_{eff}$ and $T$ are the effective moment of inertia and the nuclear temperature of the compound nucleus at the saddle point, respectively.

The angular anisotropy of fission fragments is defined as

$$A = \frac{W(0^\circ)}{W(90^\circ)},$$

(3)

The nuclear temperature of the compound nucleus at the saddle point is given by

$$T = \sqrt{\frac{E_{ex}}{a}},$$

(4)

where $E_{ex}$ is the excitation energy of the fissioning system and $a$ is the nuclear level density parameter at the saddle point. $E_{ex}$ can be expressed by the following relation

$$E_{ex} = E_{c.m.} + Q - B_f(I) - E_R(I) - \nu_{gs}E_n.$$  

(5)

In this equation, $E_{c.m.}$, $Q$, $B_f(I)$, $E_R(I)$, $\nu_{gs}$, and $E_n$ represent the center-of-mass energy of the projectile, the $Q$ value, the spin dependent fission barrier height, the spin dependent rotational energy of the compound nucleus, the number of pre-saddle neutrons, and the average excitation energy lost due to evaporation of one neutron from the compound nucleus prior to the system reaching to the saddle point, respectively. In the case of $p \gg 1$, the
angular anisotropy of fission fragments by using Eq. (1) is given by the following approximate relation

\[ A \approx 1 + \frac{< I^2 >}{4K_0^2}. \]  

(6)

The prediction of angular anisotropy of fission fragments by using the SSPSM is valid only under restrictive assumptions. At high angular momentum, or at high fissility, the rotating liquid drop model (RLDM) predicts that the fission barrier height \( B_f(I) \) vanishes even for a spherical nucleus, which leads to \( K_0^2 \rightarrow \infty \). Subsequently, the distribution of \( K \) is uniform and hence the prediction of the SSPSM for the fission fragments angular anisotropy is nearly uniform by using Eq. (1). This predicted tendency toward isotropy for fission fragments at high angular momentum is not seen in the experiments. This discrepancy is taken as a clear indication that the width of the \( K \) distribution is not determined at the predicted spherical saddle point shape, but at a point where nucleus is more deformed. Therefore, it has been proposed that the standard saddle-point statistical model breaks down at high spin and/or large values of \( \frac{Z^2}{A} \) of the compound nucleus (CN), and the angular distribution of fission fragments is governed by an effective transition state different from saddle point transition state.

B. Pre-saddle Neutron multiplicity

It is clear that because of the hindrance to fission, a large number of particles more that those predicted by the statistical model are emitted from the fissioning system. In heavy ion fusion reactions, due to the formation of a heavy compound nucleus, the competition between neutron emission and fission describes the decay possibilities rather well. During the collective motion to the scission point, neutrons will be evaporated if energetically possible, and would be experimentally as pre-fission, or more correctly, pre-scission neutrons. A longer saddle to scission time due to the viscosity effect, will result in a higher pre-scission neutron multiplicity \([15]\). The calculation of pre-saddle neutrons in heavy ion induced reactions based on the comparison between the experimental data of angular anisotropy and those predicted by the SSPSM depends on the kinetic energy and the binding energy of evaporated neutron from the compound nucleus prior to the system reaching to the saddle point. The energy spectrum of evaporated neutrons is usually given by the following form (an evaporation
\frac{dN}{dE} = CE \exp(-\frac{E}{T}). \tag{7}

Hence, the average kinetic energy of the emitted neutron, $E_K$ is given by

$E_K = 2T. \tag{8}$

The average excitation energy lost due to evaporation of one neutron from the compound nucleus prior to the system reaching to the saddle point is given by

$E_n = B_n + 2T, \tag{9}$

where, $B_n$ denotes the average neutron separation energy.

In this work, the average energy lost by an emitted neutron over the energy range of the projectile is calculated by Eq. (9), for heavy ion induced fission reactions, as well as for induced fissions by light projectiles. The level density parameter, $a$ is taken $\frac{A_{\text{C.N.}}}{10}$ (Considering the level density parameter as $\frac{A_{\text{C.N.}}}{8}$, rather than $\frac{A_{\text{C.N.}}}{8}$, the number of pre-saddle neutrons varies at most by 10%). Hence, number of pre-saddle neutrons is not sensitive to the level density parameter selected in the calculation. $\Im_{\text{eff}}$, $B_f(I)$, and $E_R(I)$ are accounted by the use of rotating finite range model (RFRM) [38], while $\langle I^2 \rangle$ quantities are calculated by several models [39-44]. In the following sections, the determination of the number of pre-saddle neutrons, $\nu_{gs}$ for these systems is based on the comparison between the experimental data of angular anisotropies and those predicted by the SSPSM. In the present work, it is determined pre-saddle neutron multiplicities for several systems undergoing heavy ion induced fission in which fission fragments angular anisotropies have a normal behavior as well as those systems undergoing light particle induced fission. In order to determine number of pre-saddle neutrons in heavy ion reactions with anomalous angular anisotropies, it is necessary to predict the average contribution of non compound nucleus fission events [45].

III. RESULTS AND DISCUSSION

The calculated multiplicities of pre-saddle neutrons as a function of $E_{ex}$ for the two $^{16}\text{O} + ^{209}\text{Bi} \rightarrow ^{225}\text{Pa}$ and $^{19}\text{F} + ^{208}\text{Pb} \rightarrow ^{227}\text{Pa}$ reaction systems leading to Protactinium isotopes, are shown in Fig. 1(a). For above studied systems, the experimental data of
angular anisotropy are taken from literature [46, 47]. As illustrated in the figure, the number of pre-saddle neutrons decreases with increasing the excitation energy of the compound nucleus. This behavior is due to the fact that the fission barrier height (and thus the ground state-to-saddle point transition time) decreases with increasing the excitation energy of the compound nucleus, which can be lead to that $\nu_{gs}$ decreases with $E_{ex}$. In this figure, the general trend of the number of pre-saddle neutrons as a function of the excitation energy of the compound nucleus is represented by a line using the method of least squares. Fig. 1(b), shows a similar case for the $^{16}\text{O} + ^{208}\text{Pb} \rightarrow ^{224}\text{Th}$ reaction system. For this system, the experimental data of angular anisotropy are taken from literature [8]. Multiplicities of pre-saddle neutrons for the two $^{11}\text{B} + ^{237}\text{Np}$ and $^{16}\text{O} + ^{232}\text{Th}$ reaction systems, both populating the same compound nucleus $^{248}\text{Cf}$ are also shown in Fig. 1(c). For these two systems, the experimental data of $<A>$ are taken from literature [56-58]. It is interesting to note that for these two systems, as well as for the two $^{16}\text{O} + ^{209}\text{Bi} \rightarrow ^{225}\text{Pa}$ and $^{19}\text{F} + ^{208}\text{Pb} \rightarrow ^{227}\text{Pa}$ reaction systems as shown in Fig. 1(a), the number of pre-saddle neutrons at any given excitation energy appears to be nearly equal. As a result, the multiplicities of pre-saddle neutrons for heavy ion fusion reactions populating the same compound nucleus are nearly independent of the entrance channel asymmetry and depend on the mass number of the compound nucleus.

The ratio of the calculated pre-saddle neutron multiplicity, $\nu_{cal}^{gs}$ to experimental pre-scission neutron multiplicity $\nu_{exp}^{pre}$ [8], and also the ratio of theoretical pre-saddle neutron multiplicity to theoretical pre-scission neutron multiplicity [21] for the $^{16}\text{O} + ^{208}\text{Pb} \rightarrow ^{224}\text{Th}$ reaction system, are given in Table I. As can be seen from Table I, the calculated number of pre-saddle neutrons for the $^{16}\text{O} + ^{208}\text{Pb} \rightarrow ^{224}\text{Th}$ reaction system is greater than $\nu_{exp}^{pre}$ at $E \text{ c.m.} = 76.9$ MeV. This unexpected result can be related to the measured value of fission fragment angular anisotropy at low energy. It seems that the measured value of the angular anisotropy at E c.m. = 76.9 MeV is reported more than its actual value.

As the nucleus is heated, the excitation energy of the compound nucleus, $E_{ex}$ exceeds the fission barrier height, $B_f$. Hence, it becomes possible for the nucleus to fission after passing through excited states above the fission barrier (transient state) [51]. In this transient state picture, the fission width, $\Gamma_f$ depends on the level density above the fission barrier. The fission width and the neutron width can be shown to be approximately given by $\Gamma_f \propto \exp(-\frac{B_f}{T})$ and $\Gamma_n \propto \exp(-\frac{B_n}{T})$ ($B_n$ is the neutron binding energy), respectively.
TABLE I: Comparison between the calculated $\nu_{\text{gs}}^{\text{cal}}$, $\nu_{\text{gs}}^{\text{cal}}/\nu_{\text{gs}}^{\text{exp}}$ and $\nu_{\text{th}}^{\text{gs}}/\nu_{\text{th}}^{\text{pre}}$ [21] for the $^{16}$O + $^{208}$Pb $\rightarrow ^{224}$Th reaction system.

| $E_{\text{c.m.}}$ (MeV) | $E_{\text{ex}}$ (MeV) | $\nu_{\text{pre}}^{\text{exp}}$ | $\nu_{\text{cal}}^{\text{gs}}$ | $\nu_{\text{gs}}^{\text{cal}}/\nu_{\text{gs}}^{\text{exp}}$ | $\nu_{\text{th}}^{\text{gs}}/\nu_{\text{th}}^{\text{pre}}$ |
|------------------------|----------------------|------------------|------------------|------------------|------------------|
| 76.9                   | 22.7                 | 1.50             | 1.81             | 1.21             | 0.96             |
| 82.6                   | 27.6                 | 1.90             | 1.60             | 0.84             | 0.91             |
| 92.0                   | 32.0                 | 2.40             | 1.30             | 0.54             | 0.78             |
| 105.9                  | 42.5                 | 2.80             | 0.52             | 0.18             | 0.64             |
| 119.0                  | 55.0                 | 3.40             | 0.00             | 0.00             | 0.56             |

Therefore, the energy dependence of the ratio $\Gamma_{n}/\Gamma_{f}$ is expected to be dominated by the ratio of appropriate Boltzmann factors, i.e., $\Gamma_{n}/\Gamma_{f} \approx \exp[(B_{f} - B_{n})/T]$.

In general, in heavy ion induced fission, $B_{f}$ will be relatively high at low excitation energy or at low angular momentum, $I$, however as $I$, as well as $E_{\text{ex}}$ is increased, the larger moment of inertia of the elongated saddle point configuration causes its energy to increase less rapidly than that of the compact equilibrium deformation, so the barrier height falls to zero at some $I$. The ratio $\Gamma_{n}/\Gamma_{f}$ is known to decrease sharply as $E_{\text{ex}}$ increases in nuclei of the Lead-Bismuth region, and it is expected to do just the opposite for nuclei with the largest known atomic numbers [52]. For the lighter group of fissioning elements $B_{f} \gg B_{n}$, and for the very heavy ones, it is expected that $B_{n} \gg B_{f}$. For nuclei of intermediate mass like the Neptunium, $B_{n}$ and $B_{f}$ are nearly equal and one expects only a slow variation of $\Gamma_{n}/\Gamma_{f}$ with $E_{\text{ex}}$. In a heavy ion reaction, there is sufficient excitation energy to emit several neutrons, and fission can compete at each stage (if the excitation energy is greater than the fission barrier height), thus the fission probability and neutron evaporation probability at stage i, are given by $p_{f,i} = (\Gamma_{f}/\Gamma_{\text{tot}})_{i}$ and $p_{n,i} = (\Gamma_{n}/\Gamma_{\text{tot}})_{i} = 1 - (\Gamma_{f}/\Gamma_{\text{tot}})_{i}$, respectively. As a result, the total fission probability, $P_{f}$ is given by

$$P_{f} = \sum_{k=1}^{\nu} \prod_{i=1}^{k} (p_{f,i})(p_{n,i-1}),$$

(10)

where $\Gamma_{\text{tot}} = \Gamma_{f} + \Gamma_{n}$. The mean number of neutrons emitted before fission, $\nu_{\text{pre}}$ can be
derived by the following expression

\[ \nu_{\text{pre}} = \left( \frac{1}{P_f} \right) \sum_{k=1}^{\nu} (k - 1) \prod_{i=1}^{k} (p_{f,i}) (p_{n,i-1}). \]  

(11)

As \( I \) increases, the fission barrier height decreases, then \( p_{f,1} \) along the decay chain approaches unity, and steps with \( k > 1 \) become insignificant, and \( \nu_{\text{pre}} \rightarrow 0 \); thus fission is predicted to occur at the first step in the decay chain. It is obvious that as the projectile energy rises, \( \nu_{\text{pre}} \) will initially rises, due to more chances for fission, but should subsequently falls as the angular momentum reaches the value at which \( P_f \) nears unity. It is shown that, the transient time at the scission point, \( \tau_{\text{sci}} \) by using a diffusion model for the fission process is given by [53]

\[ \tau_{\text{sci}} \simeq \tau_{\text{sad}} + \tau = \beta^{-1} \ln(10B_f/T) + \tau, \]  

(12)

where, \( \tau_{\text{sad}} \), \( \tau \) and \( \beta \) are the transient time at the saddle point, the average traveling time between the saddle and scission points, and the nuclear friction, respectively. The time \( \tau \) is a function of the value of the nuclear friction, of the shape of potential and of the excitation energy. The above equation shows that \( \tau_{\text{sad}} \) depends sensibly on the nuclear friction \( \beta \) and on the excitation energy of the compound nucleus.

Earlier calculations of fission fragment anisotropies based on SSPSM have been corrected to include the effect of pre scission neutron emission. The calculation of fission fragment anisotropies with taking into account the effect of pre-scission neutron emission better compares with the SSPSM predictions with the experimental results. However, there is a small discrepancy between model predictions and the data at high excitation energies. A fraction of pre-scission neutrons is expected to be emitted between saddle to scission. These latter neutrons do not longer influence the prediction of angular anisotropy by SSPSM, since it is assumed that the SSPSM is decided at the saddle point. In Fig. 2, the effect of pre-saddle neutrons in the prediction of angular anisotropy by SSPSM is demonstrated for the \( ^{16}\text{O}+^{208}\text{Pb} \rightarrow ^{224}\text{Th} \) reaction system [21]. As it is shown in the figure, the discrepancy between the experimental data of angular anisotropies and the prediction of the SSPSM can be removed to a large extent by taking into account the pre-saddle neutron emission correction. We observe that for the above studied system, the ratio of the calculated pre-saddle neutron multiplicity to experimental pre-scission neutron multiplicity, \( \nu_{\text{cal}}^{\text{gs}}/\nu_{\text{exp}}^{\text{pre}} \approx 4.1 \) at \( B_f = 1 \) is in agreement with \( \frac{\tau_{\text{gs}}}{\tau_{\text{gs}} + \tau_{\text{ss}}} \approx \frac{1}{3.7} \) (where, \( \tau_{\text{gs}} \) and \( \tau_{\text{ss}} \) are ground-to-saddle and saddle-to-scission transition times, respectively) [9]. Hence, the neutron emission rate by
the compound nucleus in the transition from the ground state to the saddle point and then in the transition from saddle to the scission points are approximately uniform.

The calculated multiplicities of pre-saddle neutrons as a function of $E_{cx}$ for the $^{11}$B + $^{197}$Au, $^{209}$Bi, $^{235}$U, $^{237}$Np reaction systems are shown in Fig. 3(a). For these studied systems, the experimental data of angular anisotropies are taken from literature [46, 48-50, 54-56]. The values of $\nu_{gs}$ as a function of $E_{cx}$ for the $^{14}$N, $^{16}$O + $^{197}$Au and $^{14}$N, $^{16}$O + $^{209}$Bi reaction systems are also shown in Fig. 3(b). For these systems, the experimental data of angular anisotropies are taken from literature [46, 47]. The calculated multiplicities of pre-saddle neutrons as a function of the excitation energy of the compound nucleus for induced fission of the $^{209}$Bi target by using different projectiles are shown in Fig. 3(c).

The average values of $\nu_{gs}$, as well as ranges of pre-saddle neutron multiplicities for the fission reactions of different targets induced by the same projectile over the same projectile energy range are shown in Table II. In this Table, the quantity $V_b$ denotes the Coulomb barrier height. It can be observed that $\nu_{gs}$ decreases with increasing the mass number of the target.

The average values of $\nu_{gs}$, as well as the pre-saddle neutron multiplicity in the form of a range for the induced fission of the same target by different projectiles over the same projectile energy are also given in Table III. As can be seen in Table III, the quantity $\nu_{gs}$ decreases with increasing the mass number of projectile. All heavy ion induced reactions show that $\nu_{gs}$ falls quite rapidly with increasing the mass asymmetry, since it is partly due to a reduction of the dynamical fission time scale with the mass asymmetry.

We now attempt to estimate the pre-saddle neutron multiplicities in several fission reactions induced by light projectiles. We must pay attention to some important points expressing the difference between fission induced by light projectiles and heavy ions. In the fission induced by light projectiles, the energy in the center-of-mass framework, $E_{c.m.}$ is roughly the same as that in the laboratory framework, as well as due to the low weight of projectile, rotational energy, $E_R$ can be neglected. Fig. 4, shows calculated pre-saddle neutron multiplicities for the two $\alpha + ^{182}$W, and $p + ^{185}$Re reaction systems which are leading to similar $^{186}$Os compound nucleus, as well as for the two $p + ^{209}$Bi, and $\alpha + ^{206}$Pb that formed the same $^{210}$Po compound nucleus. For these systems, the experimental data of angular anisotropies
are taken from literature [57-59]. The values of $< I^2 >$ for these systems are given by [57]:

$$< I^2 > = \frac{\sum (2I + 1)T_I (I + 1)}{\sum (2I + 1)T_I}$$  \hspace{1cm} (13)

where $T_I$ is the entrance channel transmission coefficients and satisfy $T_I = 1$ for $I \leq I_{\text{max}}$ and $T_I = 0$ for $I > I_{\text{max}}$. If the maximum angular momentum is determined by the relation $< I^2 > = 1/2I_{\text{max}}^2$, the following relations give the values of the mean square angular momentum of the compound nucleus for the fission of pre-actinide nuclei induced by proton and $\alpha$ particle, respectively:

$$< I^2 > = 2.08E_p(\text{MeV}) - 15, \hspace{1cm} (14)$$

$$< I^2 > = 10.2E_\alpha(\text{MeV}) - 199. \hspace{1cm} (15)$$

In heavy ion induced fission at low bombarding energies, several neutrons are evaporated prior to the reaching to the saddle point, and at the highest bombarding energy essentially all the neutrons are evaporated by the fission fragments, i.e., the fission process is rapid compared to the time scale for neutron evaporation. However, the number of pre-saddle neutrons, $\nu_{gs}$ increases with increasing the excitation energy of the compound nucleus in fission induced by light projectiles. This behavior is mainly due to that in the induced fission by light projectile, the fission barrier height is higher than the neutron binding energy, as well as $B_f$ is approximately independent of the excitation energy of the compound nucleus. Therefore, the fission probability, $P_f = \frac{\Gamma_f}{\Gamma_{tot}}$ is negligible at low energies. When $E_{\text{ex}} < B_f$, it is impossible that the compound nucleus undergoes fission, but there is sufficient excitation energy to emit several neutrons. It is clear that the fission becomes significant if $E_{\text{ex}} > B_f$.

**IV. SUMMARY AND CONCLUSIONS**

We have presented in this paper the calculated pre-saddle neutron multiplicities for several heavy ion induced fission reactions, as well as for several fission reactions induced by light projectiles. The calculation by using the experimental data of fission fragment angular anisotropies, as well as the prediction of the SSPSM is a novel method, which has been carried out in this work for the first time. We have also considered the behavior of pre-saddle neutron multiplicities in fission reactions induced by heavy ions and light projectiles. In heavy ion induced fission, the number of pre-saddle neutrons decreases with increasing the
excitation energy of the compound nucleus. Whereas in fission induced by light particles, the number of pre-saddle neutrons increases with increasing the excitation energy of the compound nucleus. The fission barrier height in heavy ion fission reaction depends on the excitation energy of the compound nucleus. On the other hand, the fission barrier height (and thus ground-to-saddle transition time; $\tau_{gs} \propto \ln(10B_f/T)$) decreases with increasing the excitation energy of the compound nucleus. As a result, in heavy ion induced fission the number of pre-saddle neutrons decreases with increasing the excitation energy of the compound nucleus. Our results also shows that the emission rate of neutrons is approximately constant in transition from the ground state to the saddle point and then from the saddle to the scission points. On the contrary, in fission induced by light projectiles, the fission barrier height is greater than the neutron binding energy, and the fission barrier is approximately independent of the excitation energy. Hence, the compound nucleus does not undergo fission, unless the excitation energy of the compound nucleus exceeds the fission barrier. As a result, in fission induced by light projectiles, the number of pre-saddle neutrons exhibits an increasing function against the excitation energy of the compound nucleus as shown our calculations. The number of pre-saddle neutrons for reactions lead to the same compound nucleus at any given excitation energy appears to be nearly equal, since the number of pre-saddle neutrons depends only on the mass of the compound nucleus and it is independent of the entrance mass asymmetry parameter. This behavior of the number of the pre-saddle neutrons as a function of the projectile mass and/or of the target mass may also be related to the size of compound nucleus. We observe that the average number of pre-saddle neutrons decreases with increasing the mass number of projectile in fission reactions of the same target induced by different projectiles. A similar behavior in the multiplicities of pre-saddle neutrons is also observed in fission reactions of different targets induced by the same projectile. At the end, our results may provide useful information on the ground state-to-saddle and saddle-to-scission transition times.
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FIG. 1: Calculated multiplicities of pre-saddle neutrons (a) for the two $^{16}\text{O} + ^{209}\text{Bi} \rightarrow ^{225}\text{Pa}$ and $^{19}\text{F} + ^{208}\text{Pb} \rightarrow ^{227}\text{Pa}$ reaction systems. Thick and dotted lines represent the general trends of $\nu_{gs}$ against the excitation energy of the compound nucleus for the two $^{16}\text{O} + ^{209}\text{Bi} \rightarrow ^{225}\text{Pa}$ and $^{19}\text{F} + ^{208}\text{Pb} \rightarrow ^{227}\text{Pa}$ reaction systems, respectively. (b) For the $^{16}\text{O} + ^{208}\text{Pb} \rightarrow ^{224}\text{Th}$ reaction system. Thick line represents the general trend of $\nu_{gs}$ against the excitation energy of the compound nucleus, and (c) for the two $^{11}\text{B} + ^{237}\text{Np}$ and $^{16}\text{O} + ^{232}\text{Th}$ reaction systems, both populating the same compound nucleus $^{248}\text{Cf}$. Thick and thin lines represent the general trends of $\nu_{gs}$ against the excitation energy of the compound nucleus for the two $^{11}\text{B} + ^{237}\text{Np}$ and $^{16}\text{O} + ^{232}\text{Th}$ reaction systems, respectively.
FIG. 2: Experimental and calculated anisotropies in $^{16}\text{O} + ^{208}\text{Pb} \rightarrow ^{224}\text{Th}$ reaction system [8, 54]. Thick, dashed, dotted curves are theoretical analysis in the framework of the SSPSM without neutron emission correction, with pre-saddle neutrons $[\nu_{\exp, \text{pre}} / \nu_{\text{gs, th.}} / \nu_{\text{pre, th.}}]$ correction, and pre-scission neutrons correction $[\nu_{\exp, \text{pre}}]$, respectively.
FIG. 3: Calculated pre-saddle neutron multiplicities, (a) for the $^{11}$B+$^{197}$Au, $^{209}$Bi, $^{235}$U, $^{237}$Np reaction systems. Thin, dashed, thick, and dashed-dotted lines represent the general trends of the number of pre-saddle neutrons against the excitation energy of the compound nucleus, respectively. (b) for the $^{14}$N,$^{16}$O+$^{197}$Au and $^{14}$N,$^{16}$O+$^{209}$Bi reaction systems. Thin, thick, dashed, and dashed-dotted lines represent the general trends of the number of pre-saddle neutrons against the excitation energy of the compound nucleus for these systems, respectively., and (c) for the $^{11}$B,$^{12}$C,$^{14}$N,$^{16}$O+$^{209}$Bi reaction systems. Thick, dashed-dotted, thin, and dashed lines represent the general trends of the number of pre-saddle neutrons against the excitation energy of the compound nucleus for these systems, respectively.
| Reaction Systems | Projectile Energy (in $E_{c.m.}$) | $\nu_{gs}$ | $\overline{\nu}_{gs}$ |
|------------------|-----------------------------------|------------|------------------|
| $^{11}\text{B} + ^{197}\text{Au}$ | 1.4-1.9 | 3.1-1.4 | 2.0 |
| $^{11}\text{B} + ^{209}\text{Bi}$ | 1.4-1.9 | 1.5-0.8 | 1.6 |
| $^{12}\text{C} + ^{197}\text{Au}$ | 1.3-1.8 | 2.4-1.6 | 2.1 |
| $^{12}\text{C} + ^{209}\text{Bi}$ | 1.3-1.8 | 1.5-0.1 | 1.0 |
| $^{14}\text{N} + ^{197}\text{Au}$ | 1.2-1.7 | 3.0-0.5 | 1.9 |
| $^{14}\text{N} + ^{209}\text{Bi}$ | 1.2-1.7 | 1.6-0.1 | 0.9 |
| $^{16}\text{O} + ^{197}\text{Au}$ | 1.0-1.6 | 3.3-0.7 | 2.0 |
| $^{16}\text{O} + ^{208}\text{Pb}$ | 1.0-1.6 | 1.9-0.1 | 1.5 |
| $^{16}\text{O} + ^{209}\text{Bi}$ | 1.0-1.6 | 1.7-0.9 | 1.4 |

**TABLE II**: Comparison between the calculated pre-saddle neutron multiplicity in the form of a range, as well as $\overline{\nu}_{gs}$ for fission reactions of the different targets induced by the same projectile.
| reaction systems | Projectile energy (in $E_{\text{c.m.}}$) | $\nu_{gs}$ | $\overline{\nu}_{gs}$ |
|------------------|----------------------------------------|----------|----------------|
| $^{11}$B + $^{209}$Bi | 1.2-1.7 | 2.2-1.2 | 1.9 |
| $^{12}$C + $^{209}$Bi | 1.2-1.7 | 1.8-0.8 | 1.2 |
| $^{14}$N + $^{209}$Bi | 1.2-1.7 | 1.6-0.2 | 0.9 |
| $^{16}$O + $^{209}$Bi | 1.2-1.7 | 1.4-0.6 | 0.8 |
| $^{12}$C + $^{197}$Au | 1.2-1.6 | 2.8-1.8 | 2.4 |
| $^{14}$N + $^{197}$Au | 1.2-1.6 | 3.0-1.0 | 2.3 |
| $^{16}$O + $^{197}$Au | 1.2-1.6 | 2.5-0.7 | 2.0 |
| $^{16}$O + $^{208}$Pb | 1.1-1.6 | 1.9-1.0 | 1.9 |
| $^{19}$F + $^{208}$Pb | 1.1-1.6 | 1.4-0.4 | 1.4 |

TABLE III: Comparison between the calculated pre-saddle neutron multiplicity in the form of a range, as well as $\overline{\nu}_{gs}$ for fission reactions of the same target induced by different heavy ions.
FIG. 4: The values of \( \nu_{gs} \) for the two \( \alpha +^{206}\text{Pb} \), and \( p +^{209}\text{Bi} \) reaction systems which are leading to the similar \( ^{210}\text{Po} \) compound nucleus, as well as for the two \( \alpha +^{182}\text{W} \), and \( p +^{185}\text{Re} \) reaction systems that formed the same \( ^{186}\text{Os} \) compound nucleus. Thick, thin, dashed, and dashed-dotted lines represent the general trends of the pre-saddle neutrons against the excitation energy of the compound nucleus for these systems, respectively.