Abstract

The lunar South pole likely contains significant amounts of water in the permanently shadowed craters there. Extracting this water for life support at a lunar base or to make rocket fuel would take large amounts of power, of order Gigawatts. A natural place to obtain this power are the “Peaks of Eternal Light”, that lie a few kilometers away on the crater rims and ridges above the permanently shadowed craters. The amount of solar power that could be captured depends on how tall a tower can be built to support the photovoltaic panels. The low gravity, lack of atmosphere, and quiet seismic environment of the Moon suggests that towers could be built much taller than on Earth. Here we look at the limits to building tall concrete towers on the Moon. We choose concrete as the capital cost of transporting large masses of iron or carbon fiber to the Moon is presently so expensive that profitable operation of a power plant is unlikely. Concrete instead can be manufactured in situ from the lunar regolith. We find that, with minimum wall thicknesses (20 cm), towers up to several kilometers tall are stable. The mass of concrete needed, however, grows rapidly with height, from $\sim 760 \text{ mt}$ at 1 km to $\sim 4,100 \text{ mt}$ at 2 km to $\sim 10^5 \text{ mt}$ at 7 km and $\sim 10^6 \text{ mt}$ at 17 km.

Keywords: moon, lunar mining, lunar towers, lunar solar power, concrete structure
1. Introduction

The South pole of the Moon appears to harbor significant resources in the form of water and organic volatiles in the permanently shadowed regions [1]. There is considerable interest in harnessing these resources to support a lunar base [2] or to manufacture rocket fuel to resupply rockets at lower cost than bringing the fuel up from Earth [3]. However, extracting these resources is a power-intensive operation. Kornuta et al. estimate that extraction of 2450 tons/year of water from the permanently dark craters would require power at a level of 0.4 - 1.4 GW (their figure 17) [3].

A promising solution is the nearly continuous energy supply that is potentially available a few kilometers away [4] on the “Peaks of Eternal Light” [5]. The “Peaks” are exposed to sunlight for over 90% of the lunar cycle [6]. However, the illuminated area is only a few square kilometers and much of that area would be shadowed by other solar towers [7], limiting the available power. One way of increasing the potential power output is to build higher. The resulting added power is not just due to an increase in the area provided by tall towers; the illumination is also more continuous as the tower rises above local topography [8]. Ross et al. showed that for towers up to 20 m tall, the maximum power attainable was of order a few megawatts; instead, for towers from 0.5 - 2 km tall several Gigawatts are achievable [7]. Given that Kornuta et al. (Figure 17) estimate that extraction of 2450 tons/year of water from the permanently dark craters would require power at a level of 0.4 - 1.4 GW, a need for towers in the kilometer-high range is indicated [3]. For scale, the Eiffel Tower is 330 m tall [9] and the tallest building on Earth, the Burj Khalifa in Dubai, is 829 m tall [10]. Evidently building comparably tall lunar towers is a challenge. However, the 1/6 gravity on the Moon [11], combined with the lack of an atmosphere and so of winds, and the minimal levels of seismic activity \(10^{10} - 10^{14} \text{ J/yr}\) [12], suggest that kilometer-scale lunar towers are not ruled out.

Here we explore the limits to how tall moon-based solar towers could be using simple modeling. Determining the tallest structure that can be built
with a given material is a field with a history stretching back to Greenhill (1881) [13]. General solutions are hard to find, and modeling has to make simplifying assumptions [14]. We considered limits imposed by both compressive strength and buckling. We focused in this first study on towers made of concrete. Transporting materials to the Moon is currently very expensive, of order $0.5 million/kilogram [15]. This makes for an enormous capital cost for a kilometer-scale tower, of order billions At these prices lunar water mining would be hard to make into a profitable industry. Instead it has been shown that concrete can be made out of the loose lunar surface material (“regolith”) [16]. Doing so would greatly reduce the up-front capital cost as only the relatively lightweight photovoltaic panels would need to be supplied from Earth. Hence, we explore the possibilities for concrete towers on the Moon in this paper.

We used an analytic approach to estimating the stresses in the modeled towers. In this way we could expose the scalings of maximum tower height to the model parameters. We first describe the model in section 2. In section 3 we then describe the results after optimizing tower geometry and imposing a minimum wall thickness. We discuss the limitations of these calculations, and so the need for further work, in section 4. We present a summary and our conclusions in section 5.

2. Theory

To explore the structural limitations of a concrete tower, we modeled a circular structure that gets exponentially thinner with height. The cross-sectional area at a given height $x$ above the base is described by

$$A(x) = A_0 e^{-kx}, \quad (1)$$

where $A_0$ is the cross-sectional area at the base of the tower, $k$ is the exponent by which the tower cross-section shrinks ($k \geq 0$), and $x$ is the height above the base.
The thickness of the tower’s walls also decrease with the same exponent, \( k \). The cross-section of the concrete walls by height is given by

\[ A_c(x) = A_{c,0}e^{-kx}, \]  

(2)

where \( A_{c,0} \) is the cross-sectional area of the walls at the base of the tower. Furthermore,

\[ A_{c,0} = A_0(1 - b), \]  

(3)

where \( b \) is unitless and determines the fraction of the tower that is hollow.

2.1. Stress

At any point, the tower’s walls are under compressive stress by the weight of the concrete above the point. Because of the circular symmetry of the model, any point of equal height, i.e. any point of a given cross-section, essentially experiences the same amount of stress. As a function of height, the stress is therefore

\[ \sigma(x) = \frac{F(x)}{A_c(x)}, \]  

(4)

where \( F(x) \) is the weight of the tower section above acting on the cross-section.

\[ F(x) = ma = \rho g \int_x^L A_c(x)dx, \]  

(5)

where \( \rho \) is the density of concrete, \( g \) describes the gravity on the surface of the moon, and \( L \) is the total height of the tower.

Applying a safety factor \( f_s \) to the load, the resulting stress in the tower is

\[ \sigma(x) = \frac{f_s \rho g}{k} \left(1 - e^{k(x-L)}\right) \]  

(6)

For an infinitely tall tower, this reduces to \( \sigma(x) = f_s \rho g / k \), which makes the compressive stress independent of height \( x \), i.e. constant throughout the tower. The parameter \( k \) can be picked to fix the compressive stress in the structure and optimize the tower’s dimensions. It is also worth mentioning that the stress is independent of the base area.
Figure 1: Tower specifications: $L$ is the tower’s total height, $x$ the height of a considered cross section, and $A_c(x)$ the cross-sectional area of the concrete at height $x$. $F(x)$ is the total force applied on a given cross section of height $x$ by the weight of the above concrete.

2.2. Buckling

Buckling is the sudden change in shape of a structure under load. For columns that means bending or bowing under a compressive load. If an applied load reaches the so called critical load, the column comes to be in a state of unstable equilibrium - the slightest lateral force will cause the column to suddenly bend, which decreases the carrying capacity significantly and likely causes the column to collapse.

The tower in our model is essentially a column and as such, buckles under its own weight at a certain height, also known as self-buckling. To find this critical height, we need to derive the stability conditions for the tower’s specific geometry.

For towers with uniform cross section, that is towers that do not get thinner
with height \((k = 0)\), Greenhill [13] found that the critical self-buckling height is
\[
L_c \approx \left( \frac{7.8373 \cdot EI}{\rho g A_c} \right)^{1/3} \quad (7)
\]
where \(E\) is the elastic modulus, \(I\) is the second moment of area of the beam cross section, \(\rho\) is the density of the material, \(g\) is the acceleration due to gravity and \(A_c\) is the cross-sectional area of the body [13].

**Self-Buckling of a column of non-uniform cross-section \((k > 0)\)**

The Euler–Bernoulli theory, also known as the classical beam theory, provides means of calculating the deflection behaviour of beams. The theory entails the bending equation, which relates the bending moment of a beam or column to its deflection:

\[
M(x) = -EI \frac{d^2 y}{dx^2} \quad (8)
\]

where \(M(x)\) is the bending moment at some position \(x\),

\(E\) is the elastic modulus,

\(I(x)\) is the second moment of area of the beam’s cross-section at \(x\),

\(y(x)\) describes the deflection of the beam in the \(y\)-direction at \(x\).

For this specific model (see figure 2), we can define some useful quantities: The linear weight density of the column is given by \(w(x) = A_c(x) \rho g\), where \(A_c(x) = A_{c,0} e^{-kx}\) is the cross-sectional area of the concrete at any given height. The second moment of inertia is \(I(x) = \int_A y^2 dA = A_0^2 (1 - b^2) e^{-2kx}/4\pi\).

We define \(\xi\) to be the height above the base of a elementary mass weighing on the horizontal plane of interest at height \(x\). The moment at height \(x\) can be written as

\[
M(x) = \int_x^L (w(\xi) d\xi) (y(\xi) - y(x)) \quad (9)
\]

Substituting equation [9] into the bending equation, equation [8] gives

\[
EI \frac{d^2 y}{dx^2} = \int_x^L w(\xi) (y(\xi) - y(x)) d\xi \quad (10)
\]
Figure 2: Quantities and variables of the buckling model: \( L \) is the tower's total height, \( x \) the height of a considered cross section, and \( A_c(x) \) the cross-sectional area of the concrete at height \( x \). \( \xi \) is the height of an infinitesimally thin sliver of concrete, whose weight acts on a given cross section at height \( x \). \( y(x) \) and \( y(\xi) \) quantify the tower's horizontal displacement from the \( x \)-axis at heights \( x \) and \( \xi \), respectively.

Substituting the expressions \( w(\xi) \), \( I(x) \) and \( A_c(\xi) \) and simplifying yields

\[
\frac{d^3 y}{dx^3} - 2 \frac{d^2 y}{dx^2} - \alpha \left( e^{2kx-kL} - e^{kx} \right) \frac{dy}{dx} = 0 \quad (11)
\]

where the constant \( \alpha = \frac{4\pi\rho g}{(b+1)EA_0k} \).

Setting \( \frac{dy}{dx} = \eta \) and \( \gamma = kx \) results in the following ordinary differential equation:

\[
\frac{d^2 \eta}{d\gamma^2} - 2 \frac{d\eta}{d\gamma} - \beta \left( e^{2\gamma-\lambda} - e^{\gamma} \right) \eta = 0 \quad (12)
\]

where \( \lambda = kL \) and \( \beta = \alpha/k^2 = \frac{4\pi\rho g}{(b+1)EA_0k^3} \).

Since the tower is fixed against deflection (clamped column end) at its base and is unconstrained and therefore unbent at the top (free column end), we have the following boundary conditions: \( \eta(0) = 0 \) (clamped), \( \eta'(L) = 0 \) (free end).

Buckling will occur when equation (12) has a non-trivial solution. This requirement yields a critical \( \lambda \) (or \( L \)) for a given \( \beta \), at which the tower will
buckle and which can be calculated numerically. Applying a safety factor $f_b$ to the loads makes the normalized length $\lambda = kL$ and the normalized load $\beta = 4\pi f_b \rho g / (b + 1) E A_0 k^3$.

3. Results

3.1. Safety Factors

The model calculates an absolute maximum height for a tower before failure. In any realistic tower a safety factor (S.F.) is needed. The disturbances on the Moon are presently much lower than on Earth, but the vibrations created by the mining activity that these towers would support would make for additional stresses. The possibility of vehicle collisions with the towers, e.g. during maintenance operations, must also be considered.

For concrete structures S.F. of 1.2 are commonly applied to compressive loads [17]. Structures at risk of buckling, usually require much higher additional safety factors. Since the exact building environment is difficult to predict at this time and the construction would be an costly endeavour, more conservative S.F. between 3 and 4 are likely warranted [18].

At this time, it is difficult to say which exact S.F. would be appropriate, as there are no norms or examples of structural engineering on extraterrestrial bodies. The high cost of transporting building essentials to the moon, might lead to the use of lower S.F. to save material. On the other hand, it could also be a reason to raise the S.F. to guarantee the structure’s longevity. Until more details on future shuttles to the moon are clear, it is hard to predict an exact safety factor.

In our analysis, we are therefore using a safety factor of 1. The results give the absolute limiting geometry of the tower - building a tower any taller or otherwise differently shaped, could result in immediate failure. Once a reasonable S.F. is determined, those results can be recontextualized. Throughout the paper, the safety factors for compressive stress and buckling are denoted by $f_s$. 

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and \( f_b \), respectively, and can easily be adjusted. Varying \( f_b \) changes the \( k \)-value proportionally (see equation 13), which decreases the height limit based on wall thickness (section 3.3). The effect of safety factors on the buckling height is discussed in section 3.4.

The minimum thickness of the tower walls is determined by both the safety factors and the exact material characteristics of lunar concrete, neither of which are completely determined at this time. Throughout this paper a minimum thickness of 20 cm is used. This value already includes extensive safety factors as it is a building guideline intended for load bearing exterior walls in tsunami- and earthquake-prone environments (see e.g. the Caribbean Disaster Mitigation Project [19]).

3.2. Failure due to compressive stress

"Concrete" describes a range of material with compressive strengths ranging from under 10 MPa to over 100 MPa [20]. A sulfur based concrete that can be made out of lunar regolith has a compressive strength of about 30 MPa [21]. This is a realistic value to use in our calculations.

In our model, the stress throughout the tower is constant, given a fixed density and \( k \)-value (see equation 6). To take full advantage of the concrete’s capacity, we can plug \( \sigma_{max} = 30 \) MPa, the maximum allowed compressive stress, into equation 6 and solve for the appropriate \( k \)-value:

\[
k = \frac{f_s \rho g}{\sigma_{max}}
\]

where \( \rho = 2400 \) kg \( m^{-3} \),

\( g = 1.62 \) m \( s^{-2} \),

\( \sigma_{max} = 30 \) MPa,

\( f_s \) is the safety factor applied to the load.

Literature offers several possible densities for lunar concrete ranging from 2200 kg \( m^{-3} \) to [21] to 2600 kg \( m^{-3} \) [16]. We are therefore using a density of 2400 kg \( m^{-3} \), which is the same as that of typical terrestrial concretes [22].
For $f_s = 1$ we find $k = 0.00013\text{ m}^{-1}$. In the case of a tower on the scale of a lunar radius, this value is an underestimate, as acceleration due to gravity ($g$) decreases with height. Even for shorter towers of height $L$, the stresses will not be exactly uniformly at 30 MPa, but instead will decrease by $\Delta\sigma = (f_s\rho g/k)\exp[k(x - L)]$ at any height $x$ (from equation 6). $\Delta\sigma$ is smallest at the base of the tower and, for a 100 m tower, constitutes a 97% change in compressive stress there. This change decreases exponentially as the tower height increases (e.g. > 89% for 1 km, > 27% for 10 km, > 0.0002% for 100 km). For any tower height, the compressive stress is below 30 MPa everywhere and the tower is stable against compression.

Theoretically, a tower with no additional forces acting upon it is only limited in height by the stress capacity of the material. In practice, the walls of the modeled tower will ultimately become too thin, to support any secondary structures such as solar panels (see section 3.3).

Additionally, horizontal forces caused by impacts or vibrations cannot be ruled out. Because of this, we need to consider the risk of buckling and adjust the critical height accordingly (see section 3.4).

3.3. Wall thickness

The stress in the tower is independent of the wall thickness at the base area and stays (roughly) constant as the walls become thinner with height (see equation 6). Theoretically, then, the walls could be infinitely thin, and the tower would still be self-supporting. Realistically, however, the tower’s walls should always exceed a minimum thickness (here 20 cm) \[19\].

In our model the wall thickness is indirectly defined through the cross-sectional area of the walls, defined in equation 2 by

$$A_e(x) = (1 - b)A_0e^{-kx},$$

where $x$ is the height above the base,
$A_0$ is the base area,

$k = 0.00013 \, m^{-1}$ is the factor of decay optimized for our model,

$b$ is a real, positive number so that $b < 1$ and is the fraction of the tower cross-section that is hollow. Larger values of $b$ correspond to thinner walls.

How high a given tower can be while still exceeding a minimum wall thickness depends on the $b$-value and the base area $A_0$.

To demonstrate the resulting trends, figure 3 shows the limiting tower height as a function of $b$ and base area. Figures 4 and 5 show cross-sections of the figure 3 for a 500 m$^2$ base area and a 0.5 $b$-value.

Figure 3: Tower height limit based on wall thickness $L$ (km) versus $b$ ( ) and base area $A_0$ (m$^2$). $k = 0.00013 \, m^{-1}$. 
Figure 4: Tower height limit based on wall thickness $L$ (km) versus $b$ ( ) for a base area $A_0$ of 500 m$^2$. $k = 0.00013$ m$^{-1}$.

Figure 5: Tower height limit based on wall thickness $L$ (km) versus base area $A_0$ (m$^2$) for a $b$-value of 0.5. $k = 0.00013$ m$^{-1}$.

3.4. Buckling

If a structures buckling load is exceeded, any imperfection or perturbation, no matter how small, cause the building to buckle [23]. (Buckling might occur at lower loads for large disturbances; safety factors take this into consideration.)

The boundary value problem describing the tower’s buckling behavior is given by equation [12] and the corresponding boundary conditions outlined in section 2 - one clamped and one free. To simplify the problem it is conve-
Since it is convenient to use the normalized length \( \lambda = kL \), and the normalized load \( \beta = 4\pi f_b \rho g / (3 + (1 + b) EA_0 k^3) \). A numerical solution yields the \( \beta-\lambda \) values shown in figure 6.

The analysis uses \( g = 1.62 \text{ m s}^{-2} \) as the lunar acceleration due to gravity. This is accurate at all the heights considered here, as the maximum values are much smaller than the Moon’s radius (1737.1 km [11]). The density of concrete used here is 2400 kg m\(^{-3} \) [22].

![Figure 6: Normalized length, \( \lambda \) versus normalized load, \( \beta \), on logarithmic scales. The yellow line shows the \( \lambda \sim \beta^{-0.0094} \) power law fit for the range \( \beta = 0.05 - 0.5 \). The red line shows the \( \lambda \sim \beta^{-0.36} \) power law fit for the range \( \beta = 0.5 - 1000 \).](image)

Figure 7 shows the tower’s critical buckling height as a function of the base area for \( k = 0.00013 \text{ m}^{-1} \) and the safety factor \( f_b = 1 \). Tower heights of order tens of kilometers are achievable.

To assure safe results, an additional safety factor \( f_b \) can be considered. The normalized load, \( \beta \), is proportional to the safety factor and will therefore increase linearly with it. As a result the critical buckling height of the tower decreases.
Figure 7: Critical buckling height, $L_c$ (km) as a function of tower base area ($m^2$) for $k = 0.00013 m^{-1}$ and various values of $b$. A safety factor of $f_b = 1$ is used here.

The new theoretical buckling height, will be able to support $f_b$ times the actual load.

The factor by which the theoretical buckling height decreases depends on the relationship between $\lambda$ and $\beta$ for a given setup. Over a wide range ($\beta = 1 - 1000$) this relation is well modeled by a power-law relation: $\lambda = 2.307\beta^{-0.36} + 0.012$. (See figure 6.) The maximum tower height scales down by this $f_{b}^{-0.36}$ factor for $\beta$ between 0.5 and 1000, which roughly correlates to heights under 20 km for $k = 0.00013 m^{-1}$. For $\beta$ between 0.05 and 0.5, the maximum tower heights scale down by a factor of $f_{b}^{-0.0094}$ which roughly corresponds to heights above 20 km for $k = 0.00013 m^{-1}$. 
3.5. Optimizing the maximum height

The ideal tower, should be both tall and require as little concrete for construction as possible. To keep the required mass of building material low, the tower’s walls should be as thin as possible, that is, parameter $b$ should be maximized. As the $b$-value increases, however, the maximum height decreases (see figure 4). The buckling height, on the other hand, increases with $b$ (see section 3.4). This suggests a trade-off between maximizing the buckling height or the height limit based on wall thickness. The maximum tower height will be the smaller of the two heights. To optimize maximum height, $b$ must be picked carefully.

Figure 8 shows both the height at which the minimum wall thickness is reached and the buckling height for a given $b$-value. The point at which the two curves cross, at $b \sim 0.92$ is where the dominant limiting factor changes. For $b < 0.92$, buckling dominates, for $b > 0.92$, wall thickness dominates.

![Figure 8: Height at which the wall thickness reaches 20 cm (km) and buckling height $L_c$ (km) versus $b$ for $f_b = 1$ and $A_0 = 500 \, m^2$. The vertical dashed line at $b = 0.92$ marks the intersection.](image)

For a base area of 500 $m^2$, 0.92 is the ideal $b$-value. Similarly, $b$ can be found for other base areas. Figure 9 shows the relationship between the base area and the ideal $b$, so that the total maximum height is as great as possible. For values of $A_0 > 10 \, m^2$ and for $A_0 > 100 \, m^2$ the optimum value of $b$ is >0.8 and >0.9,
respectively. The optimal $b$ value approaches 1, as the base area increases. This relationship will be kept in mind when choosing $b$.

![Figure 9: Ideal $b$ \((\ )\) to maximize height versus base area $A_0$ \((m^2)\) for $f_b = 1$.](image)

### 3.6. Mass of maximum height tower

Given the model parameters we can calculate the mass of the concrete required to build a tower of height $L$.

$$M = \rho \int_0^L A_0 (1 - b) e^{-kx} dx = \frac{\rho A_0}{k} (1 - b)(1 - e^{-kL}).$$

(14)

where $b$ is the wall thickness parameter chosen in relation to $A_0$ based on figure 9.

The mass of interest is for a tower at the buckling height, with the smallest possible base area for a given height and the thinnest possible walls. Based on figures 7 and 9, the base area $A_0$ and the wall thickness parameter $b$ are optimized for each tower height $L$. These parameters give the minimum concrete mass requirement for realistic tower proportions.

Figure 10 shows the mass of concrete required against both the total height of the tower and the base area required. Note that the x-axis with the values for the base area is not linearly scaled. Rather, the scale is chosen, so that a given height matches up with the ideal base area.
Figure 10: Mass of concrete (mt) needed to build a tower of a given height (km) and base area $A_0$ (m$^2$), for $f_b = 1$. For every height-base area pair, the ideal $b$-value from figure 9 is used. Note that the x-scale on top is not linear, but chosen to reflect the relationship between height $L$ and base area $A_0$. Also note that the y-scale and lower x-scale are logarithmic; the mass grows rapidly with height.

4. Discussion

4.1. Tower geometry

It is important for any freestanding structure to support its own weight. For a tower made of lunar concrete that means that the compressive stress must not exceed 30 MPa [21]. In an equal stress structure, the stress state at all points of the body is the same, which is the most efficient use of building material.

Because of this, our model is so that an infinitely tall concrete tower under the Moon’s surface gravity is an equal stress structure at its stress capacity. Here, the cross-sectional area of the tower’s walls decreases exponentially with height by a factor of $k = 0.00013 \text{ } \text{m}^{-1}$. For finite heights, the stress distribution
is not perfectly uniform anymore, however, it is always below 30 MPa, allowing
the tower to still be self-supporting.

\[ k = 0.00013 \, m^{-1} \] is chosen for a hollow concrete structure. If the tower
were more complex, i.e. interior structures such as floors were added or multi-
ple building materials included, the ideal \( k \)-value would change to reflect that.
Changes to the \( k \)-value of order \( 10^{-4} \) do not change the maximum heights sig-
nificantly.

The tower’s maximum stress is independent of its cross-sectional area, as well
as that of its walls. Theoretically, the wall could therefore be infinitely thin,
and the tower would still be self-supporting. However, concrete is an aggregate
material with a range of particle sizes that do not allow arbitrarily thin walls.
This property sets a minimum practical concrete wall thickness.

According to the Caribbean Disaster Mitigation Project, a load bearing exte-
rior wall should be a minimum of 20 cm thick \cite{19}. This value already includes
extensive safety factors as it is a building guideline intended for government
buildings in tsunami- and earthquake-prone environments.

This sets a limit to the tower height, as the tower walls becomes thinner with
height, but may not fall short of the minimum wall thickness. The maximum
height based on wall thickness increases with the base area \( A_0 \) and decrease as
the hollow fraction \( b \) of the tower’s cross section increases (see figure 3).

4.2. Buckling

Next to compressive behaviour, it is important to consider their buckling
behaviour. The tower’s buckling behaviour for a fixed \( k \)-value is dependent on
the cross-sectional area of its base \( A_0 \) and the relative thickness of the walls
\( b \).

Theoretically the critical height due to buckling can be infinite, given a su-
ficiently big base area. The surface of the moon offers limited construction area,
though. This limits how big the base area and therefore the critical height can
From figure 7 we know that a tower with a thinning rate \( k = 0.00013 \text{ m}^{-1} \) with a 500 m\(^2\) base area has a buckling height between 12 km and 15 km depending on the wall thickness parameter \( b \). A solid tower has the lowest buckling height at 12 km. The buckling height increases as the walls get thinner.

As the wall thickness decreases, the buckling height increases. Therefore, \( b \) should be made as large as possible to keep the wall thickness low. The walls cannot be arbitrarily thin, though. In this analysis they should always exceed 20 cm. In this model, the walls become exponentially thinner with height and will therefore always fall short of the minimum thickness at some height. The height at which this happens decreases as \( b \) increases. The buckling height has the reversed relationship with the wall thickness and increases with \( b \).

For the final tower to be as high as possible, \( b \) must be chosen so that the height at which the minimum wall thickness is reached and the buckling height are equal. This is dependent on the base area. The ideal \( b \)-value increases with the base area and approaches 1. For base areas between 10 m\(^2\) and 1000 m\(^2\), the \( b \)-value falls in between 0.75 and 0.95. (See figure 9.)

A tower with a 500 m\(^2\) base, \( k = 0.00013 \text{ m}^{-1} \) and \( b = 0.92 \) would reach its maximum height at 14 km. Such a tower would require 520 thousand tons of concrete (see figure 10).

Figure 10 shows that a 1 km tall tower of a \( \sim 1 \text{ m}^2 \) base needs only \( \sim 760 \text{ mt} \) of concrete, while a tower of 2 km height and \( \sim 2 \text{ m}^2 \) base requires a mass of concrete of around 4,100 mt. The mass required grows rapidly with height; by 7 km (70 m\(^2\) base area) the mass is \( 10^5 \text{ mt} \), and by 17 km (700 m\(^2\) base area) has almost reached \( 10^6 \text{ mt} \).

Figure 10 demonstrates that the mass and volume of regolith that needs to be processed into concrete in a reasonable time is quite likely to be the limiting factor for some time. If we require a construction time of 1 year, then a 2 km tower would have to process 11 mt/day. A 1 km tower would require \( \sim 80\% \)
lower rates. These seem like plausible numbers for a decade or two from now.

4.3. Extra weight from solar panels

The weight of the solar panels is trivial for the maximum height as an extreme example makes clear. At their thinnest, the tower’s walls are 20 cm thick, which amounts to a cross-sectional area of at least 0.126 m$^2$ and a cross-sectional circumference of 128 cm. Since the ratio of the concrete’s cross-sectional area to the circumference is the greatest at this point, this is where the solar panels will have the greatest impact on the load.

The density of concrete is 2400 kg m$^{-3}$ and the mass of a state of the art triple junction solar panel for use in space is $\sim 2$ kg m$^{-2}$ [24]. At the tower’s thinnest part the concrete will have a mass of 310 kg m$^{-1}$. Solar panels will add to this load by 0.6%.

4.4. Future considerations

This paper is only intended to provide a first estimate of the height limitations of lunar concrete towers and is not an exhaustive analysis of possible designs and failure modes. There are many different ways to implement solar towers on the moon, all of which have slightly different factors to take into account (material properties, geometry, reinforcements, etc.) In this section, we outline some of the considerations that come with our chosen design and should be explored further in further studies.

4.4.1. Shell Buckling

To determine the buckling limit, we performed a beam buckling analysis based on the Euler–Bernoulli theory, which is useful in predicting the buckling behaviour of beams, columns, and other members. This formalism, however, neglects imperfections and second-order deformations that can lead to local buckling phenomena in thin-shell structures, i.e. shell buckling.

A shell is a thin, curved rigid structure, whose thickness is small compared to its other dimensions. Such structures have a significantly lower critical buckling load than the Euler–Bernoulli values [25].
Since this paper finds the optimized tower to be $\sim 90\%$ hollow (see section 3.4), shell buckling could be especially relevant [26] and might decrease the optimal height-to-mass ratio for a given base area by imposing additional height limitations.

Predicting a tower’s shell buckling behaviour is a complex issue, requiring sophisticated analyses beyond the scope of this paper. This is a topic for future detailed investigation.

4.4.2. Lunar concrete

Scientists have yet to make any true lunar concrete, whose properties could be studied. Some concepts for lunar soil-based building material have been studied using simulated lunar regolith [16, 21], but there is no guaranteed such materials are realizable with real lunar regolith.

In our analysis, we therefore rely on best estimates based on limited lunar regolith samples [16] and data on sulfur based concrete, which is considered a promising candidate for lunar construction [21]. Further work into the properties of lunar concrete is ongoing. Prospects of using lunar regolith samples from relevant areas e.g. ”Peaks of Eternal Light” [5] are growing [27, 28]. As these results come in, lunar tower designs can be modified accordingly.

Another issue to address is the possible erosion though blast ejecta by landers. Estimates predict that a 200 ton lunar lander will blow 1,000 tons of ejecta (including fist-sized rocks at 100 km h$^{-1}$), part of which will be blasted over 20 km away from the landing site [29]. The effects of these ejecta can be mitigated with landing pads.

By the time construction on the moon is feasible, there will undoubtedly be several prospective lunar-based building materials with a range of differing properties to choose from. Depending on the location of the landing sites and the quality of landing pads, the blast ejecta can have dramatic effects on the longevity of the towers. The resistance of each form of concrete to such erosion needs to be considered next to mass and material strengths when deciding on a
building material.

4.4.3. Transport and Infrastructure

Although much work is being done on the topic, we do not know yet what transportation to and infrastructure on the Moon will look like in the future. The actual limitations of lunar construction may lie in other factors than the strength of the towers. These limitations include: material and labor cost and availability, safety factors, mechanical limitations (e.g. rotating solar panels), and height limitations to avoid flight risks.

5. Conclusion

We studied the stability of concrete towers on the Moon against compressive failure and buckling, and estimate the mass of concrete needed to build them. The presumed source of concrete is the lunar regolith, which saves the cost of importing construction material from Earth. We assumed circular towers growing exponentially thinner with height until a minimum wall thickness is reached.

We find that the stress distribution in the tower is best for an exponent $k = 0.00013 \text{ m}^{-1}$. The maximum height is reached for a fraction $b$ of the tower cross-section that is hollow, which increases with the base area and lies in the 0.9 - 1 range for base areas above 100 m$^2$. The base area required to support the tower, and therefore the hollow fraction of the cross-section, increases drastically with height.

Kilometer-scale concrete towers on the Moon can be stable against both compressive failure and buckling. The mass of concrete needed to reach 1 km heights is $\sim 760$ mt. If we require a construction time of 1 year, then a 1 km tower would have to process $\sim 2$ mt/day. However the mass required grows rapidly with height. This is related to the drastically increasing base area. At 2 km a mass of $\sim 4,100$ mt is required. Adding solar panels to these towers, the obvious first use for such towers, adds negligible mass.
Future studies should consider metal truss frame towers as they are likely to require much less mass. At sufficiently low transport costs metal trusses may be cheaper than concrete. The trade space between methods can then be investigated.

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## Appendix: Nomenclature

| Variable | Units   | Description                                                                 |
|----------|---------|-----------------------------------------------------------------------------|
| $A_0$    | m$^2$   | Cross-sectional area of the tower’s base                                   |
| $b$      | -       | Fraction of the cross section that is hollow, thereby describing the        |
|          |         | thickness of the walls through reversed relationship, $0 \leq b < 1$        |
| $f_b$    | -       | Safety factor applied to loads in buckling calculations                    |
| $f_s$    | -       | Safety factor applied to loads in stress calculations                      |
| $g = 1.62$ | m s$^{-2}$ | Lunar acceleration due to gravity [30]                                      |
| $k$      | m$^{-1}$ | Factor of decay describing how the cross-sectional area changes with height |
| $L$      | m       | Total height of the tower                                                  |
| $M$      | mt      | Mass of concrete required to build tower                                   |
| $x$      | m       | Height above base                                                          |
| $\beta$ | -       | $\beta = 4\pi \rho g/((b + 1)EA_0k^3)$, normalized load used in           |
|          |         | the buckling analysis                                                      |
| $\lambda$ | -     | $\lambda = kL$, normalized length used in the buckling analysis            |
| $\rho = 2400$ | kg m$^{-3}$ | Density of concrete (see section 3.2)                                      |
| $\sigma$ | Pa      | Compressive stress                                                         |