Enhancement of the Josephson current by magnetic field in superconducting tunnel structures with paramagnetic spacer

V. N. Krivoruchko and E. A. Koshina
Donetsk Physics & Technology Institute NASU,
R.Luxemburg Str., 72, Donetsk-114, 83114 Ukraine

Abstract

The dc Josephson critical current of a (S/M)IS tunnel structure in a parallel magnetic field has been investigated (here S is a superconductor, S/M is the proximity coupled S and paramagnet M bilayer and I is an insulating barrier). We consider the case when, due to the Hund’s rule, in the M metal the effective molecular interaction aligns spins of the conducting electrons antiparallel to localized spins of magnetic ions. It is predicted that for tunnel structures under consideration there are the conditions when the destructive action of the internal and the applied magnetic fields on Cooper pairs is weakened and the increase of the applied magnetic field causes the field-induced enhancement of the tunnel critical current. The experimental realization of this interesting effect of the interplay between superconductivity and magnetism is also discussed.

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I. INTRODUCTION

In ferromagnetic (F) metals the exchange field $H_E$, acting on the spin of conducting electrons via the exchange interaction with magnetic moments of ions, is in general so large as to inhibit superconductivity. When an external magnetic field is applied, superconductivity is suppressed due to orbital and spin pair breaking effects, as well. However, there are magnetic metals, such as (EuSn)Mo$_6$S$_8$ [1,2] or HoMo$_6$S$_8$ [3], where the applied magnetic field can induce superconductivity. Several mechanisms that may enable superconductivity to develop in a ferromagnet or a paramagnet have been investigated in more or less detail (see [4,5] and references therein). One of them is the so-called Jaccarino-Peter effect [6]. It takes place in those para- and ferro-magnetic metals, in which, due to Hund coupling energy, the exchange interaction, $J_{SS}$, orients the spins $s$ of the conducting electrons antiparallel to the spins $S$ of rare earth magnetic ions. The effective filed acting on the spin of conduction electron is $\mu_B H + g\mu_B J < S >$ with $J < 0$ ($\mu_B$ is Bohr magneton, $g$ is g-factor). In such magnetic metals the exchange field $g\mu_B J < S >$ can be reduced by the external magnetic field $\mu_B H$, so that the destructive action of both fields on the conducting electrons can be weakened or even canceled. If, in addition, these metals posses an attractive electron-electron interaction, as, for example, in pseudoternary compounds [5], it is possible to induce bulk superconductivity by a magnetic field.

In this report, we consider the dc Josephson effect for a tunnel structure where one electrode is the proximity coupled bilayer of a superconducting film (S) and a paramagnet (M) metal, while the second electrode is an S layer. The system is under the effect of weak external magnetic field, which by itself is insufficient to destroy superconductivity. The dc critical current of such a junction has been calculated using approximate microscopic treatment based on Gor’kov equations. We discuss the case when in the M metal the localized paramagnetic moments of the ions, oriented by magnetic field, exert the effective interaction on spins of the conducting electrons $J_{SS}$. The latter, whether it arises from the usual exchange interaction or due to configuration mixing, according to Hund rules, is the antiferromagnetic type, i.e. $J < 0$. In particular, such M metal could be a layer of pseudoternary compounds like (EuSn)Mo$_6$S$_8$ or HoMo$_6$S$_8$. (While experimentally the Jaccarino-Peter phenomenon was observed [1-5] for paramagnets, this mechanism is applicable both to ferromagnetic and paramagnetic metals, and both type of the magnetic orders
will be assumed here.) We demonstrate that in the region where the destructive action of the fields on both tunnel electrodes is decreased, an increase of the magnetic field causes the enhancement of the Josephson critical current.

II. THE MODEL

The system we are interested in is the (S/M)IS layered structure of the superconducting S/M bilayer and S films separated by very thin insulating (I) barrier (see Fig.1). The S/M bilayer consists of the proximity coupled superconducting and paramagnet metals in good electric contact. It is assumed that the thicknesses of the S layers are smaller than the superconducting coherent length and that the thickness of the magnetic layer is smaller than the condensate penetration length, i.e., $d_S << \xi_S$ and $d_M << \xi_M$. Here $\xi_{S(M)}$ is the superconducting coherence length of the S(M) layer; $d_{S(M)}$ is the thickness of the S(M) layer. In this case, the superconducting order parameter may be regarded as being independent of the coordinates and the influence of the magnetic layer on superconductivity is not local. Other physical quantities characterizing the S/M bilayer are modified, as well. Such an approach was recently discussed in [7,8] for SFIFS structures, and, as was demonstrated, under these assumptions, a thin S/F bilayer is equivalent to a superconducting ferromagnetic film with homogeneous superconducting order parameter and an effective exchange field. Similarly, we can consider the S/M bilayer as a thin SM film which is characterized by the effective values of the superconducting order parameter $\Delta_{ef}$, the coupling constant $\gamma_{ef}$ and the exchange field $H_{Eef}$ that are determined by the following relations:

$$\Delta_{ef}/\Delta = \gamma_{ef}/\gamma = \nu_S d_S (\nu_S d_S + \nu_M d_M)^{-1}, \quad (1)$$

$$H_{Eef}/H_E = \nu_M d_M (\nu_S d_S + \nu_M d_M)^{-1}, \quad (2)$$

where $\nu_S$ and $\nu_M$ are the densities of quasiparticles states in the superconductor and magnetic metals, respectively; $\gamma$ is the coupling constant in the S metal. We emphasize that the superconductivity of the M metal is due to proximity effect. The applied magnetic field is too weak to induce the superconducting properties through the Jaccarino-Peter scenario, if the M metal is the pseudoternary compound. While in the latter case the M metal can posses a nonzero electron-electron interaction, we will neglect this interaction assuming for the M layer a vanishing value of the bare superconducting order parameter $\Delta^0_M = 0$, so
that relation (1) still remains valid.

The system is under the effect of parallel magnetic field \( H \). We will also assume that the thicknesses of the SM and S films are smaller than the London penetration depth \( \lambda_{SM} \) and \( \lambda_S \), correspondingly. Then the magnetic field is homogeneous in both electrodes. The conditions \( d_S << \xi_S \), \( d_M << \xi_M \) ensure that the orbital effects can be neglected, as well. The longitudinal dimension of the junction, \( W \), is supposed to be much less than the Josephson penetration depth, \( W << \lambda_J \), so that a flux quantum can not be trapped by the junction: \( HW(d_M + 2d_S + t) << \phi_O \), here \( \phi_O \) is the flux quantum, \( t \) is the thickness of the insulator.

If the transparency of the insulating layer is small enough, we can neglect the effect of a tunnel current on the superconducting state of the electrodes and use the relation of the standard tunnel theory [9], according to which the distribution of the Josephson current density \( j_T(x) \) flowing in the \( z \)-direction through the barrier (see Fig.1) takes the form \( j_T(x) = I_C \sin \varphi(x) \). Here \( \varphi(x) \) is the phase difference of the order parameter across the barrier, while the Josephson current density maximum \( I_C \) is determined by the properties of the electrodes. In this report we present the results of the calculation of the critical current \( I_C \) for the tunnel junction under consideration.

III. CRITICAL CURRENT

As far as the exchange field and the external magnetic field act only on the spin of electrons we can write the Gor’kov equations for the S and SM layers in the magnetic field in the form:

\[
(i\varepsilon_n + \xi - \sigma H_S(SM))\hat{G}_{\varepsilon S(SM)} + \hat{\Delta}_{\varepsilon S(SM)}\hat{F}^+_{\varepsilon S(SM)} = 1, \tag{3}
\]

\[
(-i\varepsilon_n + \xi - \sigma H_S(SM))\hat{F}_{\varepsilon S(SM)} + \hat{\Delta}_{\varepsilon S(SM)}\hat{G}_{\varepsilon S(SM)} = 0, \tag{4}
\]

where \( \xi = \varepsilon(p) - \varepsilon_F \), \( \varepsilon_F \) is the Fermi energy, \( \varepsilon(p) \) is the quasiparticle spectrum, \( \sigma = \pm 1 \), \( \varepsilon_n = \pi T(2n+1), n = 0, \pm 1, \pm 2, \pm 3, ... \) are Matsubara frequencies; \( T \) is the temperature of the junction (here and below we have taken the system of units with \( \hbar = \mu_B = k_B = 1 \)); \( H_{SM} = H_{Eef} - H \) is the resulting magnetic field in the SM bilayer (the subscript \( SM \)) and \( H_S = H \) is the magnetic field in the S layer (the subscript \( S \)) ; \( G_{\varepsilon} \) and \( F_{\varepsilon} \) are normal and anomalous Green functions. The equations are also supplemented with the well known
self-consistency equations for the order parameters. In the case of conventional singlet superconducting pairing, when $\hat{\Delta} = i\sigma_y \Delta$ ($\sigma_y$ is Pauli matrix), one can easily find (see, e.g., [8]):

$$
\ln \left( \frac{\Delta_0}{\Delta_{S(SM)}} \right) = \int_0^{\omega_D} \frac{dx}{\sqrt{x^2 + \Delta_{S(SM)}^2}} \left\{ \frac{1}{\exp[\beta \sqrt{x^2 + \Delta_{S(SM)}^2} - H_{S(SM)}] + 1} \right\} +
$$

$$
+ \frac{1}{\exp[\beta \sqrt{x^2 + \Delta_{S(SM)}^2} + H_{S(SM)}] + 1}
$$

where $\Delta_0 = \Delta(0, 0)$ is the BCS gap at zero temperature and in the absence of both the applied and the exchange fields; $\omega_D$ is the Debye frequency; $\beta = 1/T$; $\Delta_{S(SM)}(T, H_{S(SM)})$, $\Delta_S(T, H_S)$ are the superconducting order parameters of the SM and S electrodes, respectively. If $H_{S(SM)} = 0$, formula (5) is reduced to Eq. (16.27) of Ref. 10.

In accordance with the Green’s function formalism, the critical current of the SMIS junction can be written as follows:

$$
I_C = (2\pi T/eR_N)Sp \sum_{n,\sigma} f_{SM}(H_{SM})f_{S}(H_S),
$$

where $R_N$ is the contact resistance in the normal state and $f_{\epsilon SM(S)}$ are averaged over energy $\xi$ anomalous Green functions. From Eqs. (3) and (4) one can easily find that:

$$
f_{\epsilon SM(S)} = \Delta[(\epsilon_n + i\sigma H_{SM(S)})^2 + \Delta^2]^{-1/2}.
$$

Using Eqs. (6) and (7), after summation over spin index, we find for the reduced (i.e. $eR_N\{4\pi T\Delta_0^2\}^{-1}I_C$) quantity

$$
j_C(T, H) = \Delta_{SM}(T, H_{SM})\Delta_S(T, H)\Delta_0^{-2} \times
$$

$$
\text{Re} \sum_n \{[(\epsilon_n - i(H_{Eef} - H))^2 + \Delta_{SM}^2(T, |H_{Eef} - H|)][(\epsilon_n + iH)^2 + \Delta_S^2(T, H)]\}^{-1/2}
$$

The Josephson critical current of the junction, as function of the fields and temperature, can be calculated using formula (8) and self-consistency equation (5). In the general case, the dependence of the superconducting order parameter on effective field can be complex enough due to the possibility of transition to the nonhomogeneous (Larkin-Ovchinnikov-Fulde-Ferrell) phase [11,12]. We will not touch upon this scenario here, restricting the
consideration below to the region with the homogeneous superconducting state. Even in this case at arbitrary temperatures the values of the $\Delta_{SM}(T, |H_{Eef} - H|)$ and $\Delta_S(T, H)$ can be determined only numerically. The phase diagram of a homogeneous superconducting state in the $H - T$ plane has been obtained earlier (see, e.g., [8]). At finite temperatures, it is found that $\Delta(T, H)$ has a sudden drop from a finite value to zero at a threshold of $H$, exhibiting a first-order phase transition from a superconducting state to a normal state. Using these results, from Eq. (5) we take only one branch of solutions, corresponding to a stable homogeneous superconducting state. It should be also noted that, as far as $H_E \propto <S>$, a self-consistency equation should be used for $H_{Eef}$, as well. However, we will suppose that $H_{Eef}$, being much smaller than in isolated M film, is still larger than $\Delta_{SM}(T, |H_{Eef} - H|)$ for full temperature region of the homogeneous superconducting state. So, proceeding in the way to tackle the new physics, we will ignore the temperature dependence of the $H_{Eef}$ in Eq. (8).

Figures 2 and 3 show the results of numerical calculations of expression (8) for the Josephson critical current versus external magnetic field for the case of low $T = 0.1T_C$ and finite $T = 0.7T_C$ temperatures, and different values of the exchange field. To keep the discussion simple, for the SM and S layers we put $\Delta_{SM}(0, 0) = \Delta_S(0, 0) = \Delta_0$. As is seen in the figures, for some interval of the applied magnetic field the enhancement of the dc Josephson current takes place in comparison with the case of $H = 0$. Note that, the larger the effective field $H_{Eef}$ is, the larger growth of the critical current can be observed (compare, for example, the $j_C$ curves for $H_{Eef} = 0.4\Delta_0$ and $H_{Eef} = 0.6\Delta_0$ at $H = 0$ in Fig. 2). This behavior is also predicted by expression (8). A sudden break off in the $j_C(H)$ dependences in the presence of $H$ results due to a first-order phase transition from a superconducting state with finite $\Delta(T, H)$ to a normal state with $\Delta(T, H) = 0$.

IV. DISCUSSION

As is well known [13,14], due to the difference in energy between spin-up and spin-down electrons and holes under the exchange field of a ferromagnet, a singlet Cooper pair, adiabatically injected from a superconductor into a ferromagnet, acquires a finite momentum. As a result, proximity induced superconductivity of the F layer is spatially inhomogeneous and the order parameter contains nodes where the phase changes by $\pi$. Particularly, transport
properties of tunnel SF structures have turned out to be quite unusual. The $\pi$ state is characterized by the phase shift of $\pi$ in the ground state of the junction and is formally described by the negative critical current $I_C$ in the Josephson current-phase relation: $j(\varphi) = I_C \sin(\varphi)$.

The $\pi$-phase state of an SFS weak link due to Cooper pair spatial oscillation was first predicted by Buzdin et al., [15,16]. Experiments that have been performed by now on SFS weak links [17,18] and SIFS tunnel junctions [19] directly prove the $\pi$-phase superconductivity.

There is another interesting case of a thin F layer, $d_F << \xi_F$, being in contact with an S layer. As far as the thickness of the F layer $d_F$ is much less than the corresponding superconducting coherence length $\xi_F$ there is spin splitting but there is no order parameter oscillation in the F layer. Surprisingly, but it was recently predicted [7,8,20-24] that for SFIFS tunnel structures with very thin F layers one can, on condition of parallel orientation of the F layers magnetization, turn the junction into the $\pi$-phase state with the critical current inversion; if the F layers internal fields have antiparallel orientation, one can even enhance the tunnel current. It is obvious, that physics behind the inversion and the enhancement of the supercurrent in this case differs from that proposed by Buzdin et al. Namely, in this case the $\pi$-phase state is due to superconducting phase jump at the SF interface [21,24].

The exchange-field enhancement of the critical current for SFIFS tunnel structure can be qualitatively understood using the simple fact that the Cooper pairs consist of two electrons with opposite spin directions. Pair–breaking effects due to spin-polarized electrons are weaker in the antiparallel-aligned configuration since spin polarizations from the exchange fields of the F layers are of opposite signs and at some conditions can cancel each other. More formally, one can show that the maximum of the supercurrent is achieved exactly at those values of the exchange field when two singularities in the quasiparticle density of states overlap [23].

We emphasize that the scenario of the magnetic-field enhancement of the critical current discussed here, differs from those studied before for SFIFS tunnel structures. In our case the pair–breaking effect due to spin-polarized electrons is weakened in the SM electrode since the spin polarizations from the exchange field of the magnetic ions and the applied field are of opposite signs and reduce each other. On the other hand, the paramagnetic effect induced by the external field is increased for the Cooper pairs of the S electrode if the applied field is increased. Competition of these two opposite effects determines the critical current behavior for the SMIS junction in the magnetic field. In our case the mechanism described above is valid for full temperature region of the homogeneous superconducting
state (see, e.g., Fig. 3), while for the SFIFS system with antiparallel geometry - only at low temperature $T << T_C$ [7,8].

In conclusion, we calculate the dc critical current of the (S/M)IS tunnel structure, where one electrode is the proximity coupled bilayer of a superconducting film and a paramagnet metal, while the second electrode is an S layer. The structure is under the effect of weak parallel external magnetic field. In the magnetic metal the localized magnetic moments of the ions, oriented by the magnetic field, exert the effective interaction on spins of the conduction electrons $J_{SS}$. The latter, whether it arises from the usual exchange interaction or due to configuration mixing, according to the Hund rules, is the antiferromagnetic type, i.e. $J < 0$. In particular, such a film can be the layer of the pseudoternary compounds like (EuSn)Mo$_6$S$_8$, HoMo$_6$S$_8$, etc. There are no specific requirements on the superconductor, so that it can be any superconducting film proximity coupled with the magnetic metal. Using approximate microscopic treatment of the S/M bilayer and the S layer, we have predicted the effect of magnetic-field-induced supercurrent enhancement in the tunnel structure. This striking behavior contrasts with the suppression of the critical current by magnetic field. The idea to use a magnetic material in which the effective magnetic interaction aligns spins of the conducting electrons antiparallel to the localized spin of magnetic ions, in order to enhance superconductivity of superconductor-magnetic metal multilayered structures, has not been considered before and, to our best knowledge, is new. The existing large variety of magnetic materials, the ternary compounds in particular, should allow experimental realization of this interesting new effect of the interplay between superconducting and magnetic orders.

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Figure captions

FIG. 1. (S/M)IS system in a parallel magnetic field. Here S is a superconductor; M is a magnetic metal; I is an insulating barrier; W is longitudinal dimension of the junction.

FIG. 2. Critical current of the SMIS tunnel junction vs external magnetic field for $T = 0.1T_C$, $\Delta_{SM}(0,0) = \Delta_S(0,0) = \Delta_0$ and different values of the effective exchange field in the SM bilayer: $H_{Eef}/\Delta_0 = 0.3$, 0.4, 0.5 and 0.6 (curves 1, 2, 3 and 4, respectively).

FIG. 3. Critical current of the SMIS tunnel junction vs external magnetic field for $T = 0.7T_C$, $\Delta_{SM}(0,0) = \Delta_S(0,0) = \Delta_0$ and different values of the effective exchange field in the SM bilayer: $H_{Eef}/\Delta_0 = 0.2$, 0.25, 0.3 and 0.35 (curves 1, 2, 3 and 4, respectively).
Fig. 1

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Fig. 2

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\[ \frac{H_{\text{eff}}}{\Delta_0} = 0.2 \quad (1) \]
\[ 0.25 \quad (2) \]
\[ 0.3 \quad (3) \]
\[ 0.35 \quad (4) \]

\[ T = 0.7 T_C \]

\[ \dot{J}_C \]

\[ \frac{H}{\Delta_0} \]

Fig. 3

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