Investigation of Dark Matter in Minimal 3-3-1 Models

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Abstract: It is shown that the 3-3-1 model with the minimal lepton content can work as two-Higgs-triplet 3-3-1 models while leaving the other scalars as inert particles responsible for dark matter. We study two cases of dark matter corresponding to the doublet and singlet scalar candidates. We figure out the parameter spaces in the WMAP allowed region of the relic density. The indirect and direct searches for dark matter in both cases are investigated by using micrOMEGAs.

Keywords: 3-3-1 Model, Dark Matter, WMAP.

ArXiv ePrint: ...............
1 Introduction

Cosmological observations [1] suggest that there must exist cold dark matter contained approximately 27% of all energy density of the Universe. Dark matter is mysterious and an interesting subject in particle physics as well as in astrophysics. In the context of particle physics, the most popular dark matter candidates perhaps include the lightest supersymmetric particle, the lightest KK particle, the lightest $T$-odd particle, the axion, some form of sterile neutrinos, inert scalars, and the others [2].

The Standard Model is very successful in describing experimentally observed phenomena, but it leaves some unsolved problems, such as neutrino masses and mixing, matter-antimatter asymmetry, dark matter, dark energy and etc., which guides us to go beyond the Standard Model. One simple way to go beyond the Standard Model is that we extend the gauge group $SU(2)_L \otimes U(1)_Y$ to $SU(3)_L \otimes U(1)_X$ [3, 4]. The class of $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (3-3-1) models has many interesting characteristics since they can explain the number of fermion generations, the uncharacteristically-heavy top quark [5], the electric charge quantization [6], the light neutrino masses [7], and dark matter [8].

There are two main versions of the 3-3-1 model depending on which type of particles is located at the bottom of the lepton triplets. The minimal 3-3-1 model [3] uses ordinary charged leptons $e_R$, while the version with right-handed neutrinos includes $\nu_R$ [4]. There is no dark matter candidate in the original minimal 3-3-1 model, neither in the original 3-3-1 model with right-handed neutrinos since the new particles in these models are electrically charged or rapidly decay. A natural approach [9] is that the stability of dark matter is based on $W$ parity (similar to $R$ parity in Supersymmetry) by considering the baryon minus lepton numbers as a local gauge symmetry. However, this mechanism works only with the 3-3-1 model with neutral fermions ($N_R$) that possess $L(N_R) = 0$ and $B(N_R) = 0$. To solve
the issue of dark matter in the original 3-3-1 model, one might introduce a $Z_2$ symmetry so that one scalar triplet of the theory is odd, while all other fields are even under the $Z_2$ symmetry: The odd particles act as inert fields [10]. Therefore, the lightest and neutral inert particle is stable and can be a dark matter [11, 12].

The minimal 3-3-1 model originally works with three scalar triplets $\rho = (\rho_1^+, \rho_2^0, \rho_3^{++})$, $\eta = (\eta_1^0, \eta_2^-, \eta_3^+)$, $\chi = (\chi_1^-, \chi_2^+, \chi_3^0)$, and either with or without one scalar sextet $S$. In order to enrich the inert scalar sector responsible for dark matter, one can consider the reduced 3-3-1 model [13] by excluding $\eta$ and $S$, or the simple 3-3-1 model [11] by excluding $\rho$ and $S$. Unfortunately, the reduced 3-3-1 model gives large flavor-changing neutral currents as well as large $\rho$ parameter because the new physics scale is limited by a low Landau pole of around 5 TeV. The approach with the simple 3-3-1 model seems to be more realistic, except a discrepancy between the FCNC and $\rho$ parameter constraints (however, this has not really ruled the model out). Additional inert scalars can be a triplet $\rho$ or sextets ($S$, $\sigma$) or a replication of $\eta$ or of $\chi$. Among these proposals, the simple 3-3-1 model with inert scalar sextet $\sigma$ (that has $X = 1$ where $X$ is the charge of $U(1)_X$) or with the replication of $\eta$ or of $\chi$ can provide realistic dark matter candidates. Dark matter candidates for the model with inert $\sigma$ has already been studied in [11]. In this work, we focus on dark matters in the models with $\eta$ and $\chi$ replications.

Our paper is organized as follows: In section 2, we briefly describe minimal 3-3-1 models that behave as the simple 3-3-1 model and the versions with $\eta$ and $\chi$ replications. We also calculate the interactions of the inert particles with the normal matter sector. In section 3, we present the dark matter relic density and experimental searches for those two models. Finally, we summarize our work in section 4.

## 2 Brief description of minimal 3-3-1 models

### 2.1 The simple 3-3-1 model

The fermions of the simple 3-3-1 model are arranged as [3]

$$
\psi_{aL} \equiv \begin{pmatrix}
\nu_{aL} \\
e_{aL} \\
e_{aR}^c
\end{pmatrix} \sim (1, 3, 0),
$$

$$
Q_{aL} \equiv \begin{pmatrix}
d_{aL} \\
u_{aL} \\
_{aL}
\end{pmatrix} \sim (3, 3^*, -1/3), \quad Q_{3L} \equiv \begin{pmatrix}
u_{3L} \\
_{3L}
\end{pmatrix} \sim (3, 3, 2/3),
$$

$$
\begin{align*}
&u_{aR} \sim (3, 1, 2/3), & d_{aR} \sim (3, 1, -1/3), \\
&J_{aR} \sim (3, 1, -4/3), & J_{3R} \sim (3, 1, 5/3),
\end{align*}
$$

(2.1)

(2.2)

where $a = 1, 2, 3$ and $\alpha = 1, 2$ are family indices. The quantum numbers in parentheses are defined upon the gauge symmetries ($SU(3)_C, SU(3)_L, U(1)_X$), respectively. The electric charge operator has the form $Q = T_3 - \sqrt{3}T_8 + X$, where $T_i (i = 1, 2, ..., 8)$ and $X$ are the charges of $SU(3)_L$ and $U(1)_X$, correspondingly. The exotic quarks have electric charges different from the usual ones, $Q(J_3) = -4/3$ and $Q(J_3) = 5/3$. 


The model works well with two scalar triplets [13] as
\[
\eta = \left( \frac{1}{\sqrt{2}}(u + S_1 + iA_1) \right) \sim (1, 3, 0), \quad \chi = \left( \frac{\chi_1^-}{\chi_2^+} \right) \sim (1, 3, -1). \tag{2.3}
\]

The scalar potential is given by
\[
V_{\text{simple}} = \mu_1^2 \eta^\dagger \eta + \mu_2^2 \chi^\dagger \chi + \lambda_1 (\eta^\dagger \eta)^2 + \lambda_2 (\chi^\dagger \chi)^2 + \lambda_3 (\eta^\dagger \eta)(\chi^\dagger \chi) + \lambda_4 (\eta^\dagger \chi)(\chi^\dagger \eta), \tag{2.4}
\]
where \(\mu_{1,2}\) have dimension of mass, while \(\lambda_{1,2,3,4}\) are dimensionless. These parameters satisfy
\[
\mu_{1,2}^2 < 0, \quad \lambda_{1,2,4} > 0, \quad -2\sqrt{\lambda_1 \lambda_2} < \lambda_3 < \text{Min} \left\{ 2\lambda_1 (\mu_2/\mu_1)^2, 2\lambda_2 (\mu_1/\mu_2)^2 \right\}. \tag{2.5}
\]

The model contains four massive scalars with respective masses were obtained in [11] as follows
\[
h \equiv c_\xi S_1 - s_\xi S_3, \quad m_h^2 \simeq \frac{4\lambda_1 \lambda_2 - \lambda_3^2}{2\lambda_2} u^2,
\]
\[
H \equiv s_\xi S_1 + c_\xi S_3, \quad m_H^2 \simeq 2\lambda_2 \omega^2,
\]
\[
H^\pm \equiv c_\phi \eta_3^\dagger + s_\phi \chi_1^\dagger, \quad m_{H^\pm}^2 \simeq \frac{\lambda_1}{2} \omega^2,
\]
with denotation \(c_x = \cos(x), \ s_x = \sin(x), \ t_x = \tan(x)\) for any \(x\) angle. The mixing angles \(\xi, \theta\) are obtained as
\[
t_\theta = \frac{u}{\omega}, \quad t_\phi \simeq \frac{\lambda_3 u}{\lambda_2 \omega}. \tag{2.7}
\]

There are eight Goldstone bosons \(G_Z \equiv A_1, \ G_{Z'} \equiv A_3, \ G_{W^\pm}^\dagger = \eta_2^\pm, \ G_Y^{\pm\pm} \equiv \chi_2^{\pm\pm}\) and \(G_X^{\pm\pm} \equiv c_\phi \chi_1^\dagger - s_\phi \eta_3^\dagger\) eaten by eight massive gauge bosons \(Z, \ Z', \ W^\pm, \ Y^{\pm\pm}\) and \(X^{\pm\pm}\), correspondingly. In the limit \(u \ll \omega\), we have \(\xi, \theta \to 0\) thus
\[
h \simeq S_1, \quad H \simeq S_3, \quad H^\pm \simeq \eta_3^\pm, \quad G_X^{\pm\pm} \simeq \chi_1^{\pm\pm}. \tag{2.8}
\]

In the gauge sector, the gauge boson masses arise from the Lagrangian \(\sum_{\Phi = \eta, \chi} (D_\mu(\Phi))^\dagger (D^\mu(\Phi))\), where the covariant derivative is defined as \(D_\mu = \partial_\mu + ig_s t_i G_{i\mu} + ig T_i A_{i\mu} + ig_X X B_\mu\), in which the gauge coupling constants \(g_s, \ g\) and \(g_X\) and the gauge bosons \(G_{i\mu}, \ A_{i\mu}\) and \(B_\mu\) are associated with the 3-3-1 groups, respectively. The gauge bosons with their masses are respectively given by [11]
\[
W^\pm \equiv A_1 \mp i A_2 \sqrt{2}, \quad m_W^2 = \frac{g^2}{4} u^2,
\]
\[
X^\pm \equiv A_4 \mp i A_5 \sqrt{2}, \quad m_X^2 = \frac{g^2}{4} (\omega^2 + u^2), \tag{2.9}
\]
\[
Y^{\pm\pm} \equiv A_6 \mp i A_7 \sqrt{2}, \quad m_Y^{\pm\pm} = \frac{g^2}{4} \omega^2. \tag{2.10}
\]
and for the neutral gauge bosons

\[ A = s_W A_3 + c_W \left( -\sqrt{3} t_W A_8 + \sqrt{1 - 3 t_W^2} B \right), \quad m_A = 0, \]

\[ Z_1 \simeq c_W A_3 - s_W \left( -\sqrt{3} t_W A_8 + \sqrt{1 - 3 t_W^2} B \right), \quad m_{Z_1}^2 \simeq \frac{g^2}{4c_W^2} u^2, \]

\[ Z_2 \simeq \sqrt{1 - 3 t_W^2} A_8 + \sqrt{3} t_W B, \quad m_{Z_2}^2 \simeq \frac{g^2 c_W^2}{3(1 - 4 s_W^2)} \omega^2, \quad (2.11) \]

where \( s_W = e/g = t/\sqrt{1 + 4t^2} \), with \( t = g_X/g \), is the sine of Weinberg angle \([14]\). The photon field \( A_\mu \) is exactly massless. For the gauge bosons \( Z_1, Z_2 \) we have taken the limit \( u \ll \omega \). The \( Z_1 \) is identified as the Standard Model \( Z \). The VEV \( u \) is constrained by the mass of \( W \), thus \( u \simeq 246 \) GeV.

The Yukawa Lagrangian is given by

\[ \mathcal{L}_Y = h_{33} J Q_{3L} \chi J_{3R} + h_{d} \tilde{Q}_{aL} \eta \chi_d + h_{u} \tilde{Q}_{aL} \eta \chi_u R \]

\[ + \eta_u \tilde{Q}_{aL} \eta u + \tilde{Q}_{aL} \eta \chi u R \]

\[ + h_{d} \tilde{Q}_{aL} \eta^* d + \tilde{Q}_{aL} \eta^* \chi d R \]

\[ + h_{e} \tilde{Q}_{aL} \eta \chi u + \tilde{Q}_{aL} \eta \chi \]

\[ + h_{c} \tilde{Q}_{aL} \eta \chi \]

\[ + \frac{1}{2} \Lambda (\bar{\psi}_{aL} \eta \psi_{bL} \eta^*) + \text{H.c.}, \quad (2.12) \]

where the \( \Lambda \) is a new scale with the mass dimension.

Let us introduce a \( Z_2 \) symmetry and all fields of the simple 3-3-1 model are assigned even under the \( Z_2 \). Below, we consider replication of the simple 3-3-1 model by adding an extra scalar triplet, either \( \eta' \) or \( \chi' \) assigned as an odd field under the \( Z_2 \).

### 2.2 The simple 3-3-1 model with \( \eta \) replication

An extra scalar triplet that replicates \( \eta \) is defined as

\[ \eta' = \begin{pmatrix} \frac{1}{\sqrt{2}} (H_1^I + iA_1^I) \\ \eta_1^- \\ \eta_1^+ \end{pmatrix} \sim (1, 3, 0). \quad (2.13) \]

We notice that the \( \eta' \) and \( \eta \) have the same gauge quantum numbers. However, \( \eta' \) is assigned as an odd field under the \( Z_2 \), \( \eta' \rightarrow -\eta' \), so \( < \eta' >= 0 \).

The scalar potential includes the \( V_{\text{simple}} \) given in Eq. (2.4) and the terms contained \( \eta' \),

\[ V_{\eta'} = \mu_{\eta'}^2 \eta'^\dagger \eta' + x_1 (\eta'^\dagger \eta')^2 + x_2 (\eta'^\dagger \eta') (\eta'^\dagger \eta') + x_3 (\chi^\dagger \chi) (\eta'^\dagger \eta') + x_4 (\eta'^\dagger \eta') (\eta'^\dagger \eta') + x_5 (\chi^\dagger \eta') (\eta'^\dagger \chi) + \frac{1}{2} [x_6 (\eta'^\dagger \eta')^2 + \text{H.c.}]. \quad (2.14) \]

Here, \( \mu_{\eta'} \) has mass dimension, while \( x_i \) \( (i = 1, 2, 3, ..., 6) \) are dimensionless. All the \( x_6, u \) and \( w \) can be considered to be real.
The model requires \[12\]
\[\mu_{\eta'}^2 > 0, \quad x_{1,3} > 0, \quad x_2 + x_4 \pm x_6 > 0.\] (2.15)
The gauge states \(H'_1, A'_1, \eta^+_{2+} \equiv H'^+_{2+} \) and \(\eta^+_{3+} \equiv H'^+_{3+}\) by themselves are physically inert particles with the corresponding masses as follows
\[m^2_{H'_1} = M_{\eta'}^2 + \frac{1}{2}(x_4 + x_6)u^2, \quad m^2_{A'_1} = M_{\eta'}^2 + \frac{1}{2}(x_4 - x_6)u^2,\]
\[m^2_{H'^+_{2\pm}} = M_{\eta'}^2, \quad m^2_{H'^+_{3\pm}} = M_{\eta'}^2 + \frac{1}{2}x_3\omega^2,\] (2.16)
where \(M_{\eta'}^2 \equiv \mu_{\eta'}^2 + \frac{1}{2}x_2u^2 + \frac{1}{2}x_3\omega^2\). If \(H'_1\) (or \(A'_1\)) is the lightest inert particle (LIP), it can be the dark matter candidate.

Let us calculate the interactions of the inert particles with the normal ones. Due to the \(Z_2\) symmetry, the inert scalars interact only with normal scalars and gauge bosons, not with fermions.

The Higgs boson-inert scalar interactions are obtained by expanding the \(V_{\eta'}\) as follows
\[V_{\eta'} \supset x_1 \left[ \frac{1}{2}(H'^2 + A'^2) + H'^+H'^- + H'^+H'^- \right]^2 + x_2 \left[ \frac{1}{2}(u + h)^2 + H^+H^- \right] \times \left[ \frac{1}{2}(H'^2 + A'^2) + H'^+H'^- + H'^+H'^- \right] + x_3 \left( \frac{1}{2}(u + h)^2 + H'^+H'^- \right) \times \left[ \frac{1}{2}(u + h)(H'_1 - iA'_1) + H^+H'^- \right] + x_4 \left[ \frac{1}{2}(u + h)(H'_1 - iA'_1) + H^+H'^- \right] \times \left[ \frac{1}{2}(u + h)(H'_1 - iA'_1) + H^+H'^- \right] + x_5 \left( \frac{1}{2}(u + h)(H'_1 - iA'_1) + H^+H'^- \right) \times \left[ \frac{1}{2}(u + h)(H'_1 - iA'_1) + H^+H'^- \right] + x_6 \left[ \frac{1}{2}(u + h)(H'_1 - iA'_1) + H^+H'^- \right] \times \left[ \frac{1}{2}(u + h)(H'_1 - iA'_1) + H^+H'^- \right] + \text{H.c.}.\] (2.17)

All the interactions of the inert scalars with the normal Higgs bosons are listed in Table 1. Note that the symmetry factor and imaginary unit as imposed by the Feynman rules are not included in the tables (the interacting Lagrangian is understood as coupling times vertex, respectively).

The triple interactions of the two inert scalars with one gauge boson are given in
\[\mathcal{L}_{\text{gauge-}\eta'}^{\text{triple}} = -ig\eta'^\dagger(T_{\mu\lambda}A_{\mu})\partial^\nu\eta'] + \text{H.c.}\]
\[=-\frac{ig}{2} \left[ \frac{1}{c_W}Z_{1\mu} + \sqrt{\frac{1 - 3t_W^2}{3}} Z_{2\mu} \right] \frac{H'_1 - iA'_1}{\sqrt{2}} \frac{\partial^\mu H'_1 + iA'_1}{\sqrt{2}} -2s_W A_{\mu} - \frac{c_W}{c_W}Z_{1\mu} + \sqrt{\frac{1 - 3t_W^2}{3}} Z_{2\mu} \right] H'^+H'^- -ig \left[ s_W A_{\mu} - s_W t_W Z_{1\mu} - \sqrt{\frac{1 - 3t_W^2}{3}} Z_{2\mu} \right] H'^+H'^- -ig \left[ W_{\mu}^+ (H'_1 - iA'_1) \frac{\partial^\mu H'^+}{\partial^\mu H'^+} + X_{\mu} (H'_1 - iA'_1) \frac{\partial^\mu H'^+}{\partial^\mu H'^+} + \text{H.c.} \right],\] (2.18)
Table 1. Interactions of the inert scalars with the normal Higgs bosons in the $\eta'$–model.

| Vertex | Coupling | Vertex | Coupling |
|--------|----------|--------|----------|
| $hA_1 A_2'$ | $(x_1 + x_2 - x_3)\omega$ | $hH'_1 H'_3$ | $(x_2 + x_3 + x_6)\omega$ |
| $hH'_1 H'_{3}$ | $x_2 u$ | $hH'_3 H'_3$ | $x_2 u$ |
| $H A_1 A_1'$ | $\frac{x_1 u}{3}$ | $H H'_1 H'_3$ | $\frac{x_1 u}{3}$ |
| $H H'H'_3$ | $x_3 \omega$ | $H H'_3 H'_3$ | $(x_3 + x_5)\omega$ |
| $H'_1 H'_{3}$ | $\frac{(x_1 + x_6)\omega}{2}$ | $A_1^{'} H'^{3} H'$ | $\frac{(x_1 + x_6)\omega}{2}$ |
| $H'_1 H_1 H H$ | $\frac{x_1}{7}$ | $H'_1 H'_1 A_1 A'_1$ | $\frac{x_1}{7}$ |
| $A_1 A'_1 h H$ | $\frac{x_1 x_2}{2}$ | $A'_1 A'_1 H'_3 H'_3$ | $x_1$ |
| $A'_1 A'_1 h h$ | $\frac{x_2 + x_3 - x_4}{2}$ | $A'_1 A'_1 H'^3 H'^3$ | $x_1$ |
| $A'_1 A'_1 H'_3 H'_3$ | $x_1$ | $H'_1 H'_3 H'_{3}$ | $x_2 + x_3$ |
| $A'_1 H'^3 H'_{3}$ | $\frac{x_1}{2}$ | $H'_1 H'_3 H'_{3}$ | $x_2 + x_3$ |
| $hH'_1 H'_2 H'_2$ | $\frac{x_1 x_2}{2}$ | $H H'_1 H'_2 H'_2$ | $x_2$ |
| $H'^{+} H'_{2} H'_{2}$ | $x_2$ | $H'^{+} H'^{+} H'^{+} H'^{+}$ | $x_2$ |
| $H'_1 H'_3 H'_3$ | $x_1$ | $h h H'_3 H'_3$ | $x_2$ |
| $H H H'_3 H'_3$ | $\frac{x_1 + x_3}{2}$ | $H'^{+} H'^{+} H'^{+} H'^{+}$ | $x_2$ |

Table 2. Triple interactions of the inert scalars with gauge bosons in the $\eta'$–model.

| Vertex | Coupling | Vertex | Coupling |
|--------|----------|--------|----------|
| $Z_{1\mu} H_{1}^{' \mu}$ | $\frac{g}{2\sqrt{2}}$ | $Z_{2\mu} H_{1}^{' \mu}$ | $\frac{g\sqrt{1-4s_W^2}}{2\sqrt{3}c_W}$ |
| $W_{\mu}^- H_{1}^{' \mu}$ | $\frac{1g}{2}$ | $X_{\mu}^- H_{1}^{' \mu}$ | $\frac{1g}{2}$ |
| $W_{\mu}^+ A_1'$ | $-\frac{1}{2}$ | $X_{\mu}^+ A_1'$ | $-\frac{1}{2}$ |
| $A_{\mu} H_{2}^{' \mu}$ | $ig s_{W}$ | $Y_{\mu}^{' \mu}$ | $ig s_{W}$ |
| $Z_{1\mu} H_{2}^{' \mu}$ | $ig s_{W}$ | $Z_{2\mu} H_{2}^{' \mu}$ | $-ig s_{W}/\sqrt{3}c_{W}$ |
| $A_{\mu} H_{3}^{' \mu}$ | $ig s_{W}$ | $Z_{1\mu} H_{3}^{' \mu}$ | $-ig s_{W} t_{W}$ |
| $Z_{2\mu} H_{3}^{' \mu}$ | $-ig s_{W}/\sqrt{3}c_{W}$ | $Z_{2\mu} H_{3}^{' \mu}$ | $-ig s_{W}/\sqrt{3}c_{W}$ |

where we have denoted $A \partial \mu B = A(\partial \mu B) - (\partial \mu A)B$. 

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The quartic interactions of the two inert scalars with two gauge bosons are given by

\[
\mathcal{L}_{\text{quartic}}^{\text{gauge-}\eta'} = g^2 \eta'^4 (T_i A_{i\mu})^2 \eta' \]

\[
= \frac{g^2}{4} \left[ W^+\mu W^- + X^{+\mu} X^- + \frac{1}{2} \left( \frac{1}{c_W} Z_{1\mu} + \sqrt{\frac{1 - 3t^2_W}{3}} Z_{2\mu} \right) \right]^2 (H_1'^2 + A_1'^2) \\
+ \frac{g^2}{4} \left[ 2W^+\mu W^- + 2Y^{+\mu} Y^{-} + \left( 2s_W A_{i\mu} + \frac{c_2W}{c_W} Z_{1\mu} - \sqrt{\frac{1 - 3t^2_W}{3}} Z_{2\mu} \right) \right]^2 H_2'^+ H_2'^- \\
+ \frac{g^2}{4} \left[ 2X^{+\mu} X^- + 2Y^{+\mu} Y^{-} + 4 \left( s_W A_{i\mu} + s_W t_W Z_{1\mu} + \sqrt{\frac{1 - 3t^2_W}{3}} Z_{2\mu} \right) \right]^2 H_3'^+ H_3'^- \\
+ \frac{g^2}{4} \left( \sqrt{2} X^{-\mu} Y^{+} + 2W^+\mu (-s_W A_{i\mu} + s_W t_W Z_{1\mu} + \sqrt{\frac{1 - 3t^2_W}{3}} Z_{2\mu}) \right)^2 (H_1'^2 - iA_1') H_2'^- + \text{H.c.} \\
+ \frac{g^2}{4} \left( \sqrt{2} W^{+\mu} Y^{-} + X^{-\mu} (2s_W A_{i\mu} + \frac{c_2W}{c_W} Z_{1\mu} - \sqrt{\frac{1 - 3t^2_W}{3}} Z_{2\mu}) \right)^2 (H_1'^2 - iA_1') H_2'^+ + \text{H.c.} \\
+ \frac{g^2}{4} \left( 2W^{-\mu} X^- - \sqrt{2} Y^{-\mu} \left( \frac{1}{c_W} Z_{1\mu} + \sqrt{\frac{1 - 3t^2_W}{3}} Z_{2\mu} \right) \right) (H_2'^2 + H_3'^2 + \text{H.c.}). \tag{2.19}
\]

All the triple and quartic interactions of the two inert scalars with gauge bosons are presented in Table 2 and Table 3, respectively.

2.3 The simple 3-3-1 model with \( \chi \) replication

The \( \chi \) replication takes the form

\[
\chi' = \begin{pmatrix} \chi_1' \\ \chi_2' \end{pmatrix} \sim (1, 3, -1). \tag{2.20}
\]

The \( \chi' \) is assigned odd under the \( Z_2 \) symmetry that requires \( < \chi' > = 0 \). The additional potential into Eq. (2.4) due to the \( \chi' \) field is given as

\[
V_{\chi'} = \mu_{\chi'}^2 \chi'^\dagger \chi' + y_1 (\chi'^\dagger \chi')^2 + y_2 (\eta'^\dagger \eta') (\chi'^\dagger \chi') + y_3 (\chi'^\dagger \chi') (\chi'^\dagger \chi') \\
+ y_4 (\eta'^\dagger \chi') (\chi'^\dagger \eta') + y_5 (\chi'^\dagger \chi') (\chi'^\dagger \chi') + \frac{1}{2} |y_6 (\chi'^\dagger \chi')^2 + \text{H.c.}|. \tag{2.21}
\]

To make sure the scalar potential is bounded from below and the \( Z_2 \) is conserved by the vacuum, we impose

\[
\mu_{\chi'}^2 > 0, \quad y_{1,2} > 0, \quad y_3 + y_5 \pm y_6 > 0. \tag{2.22}
\]
Table 3. Quartic interactions of the inert scalars with gauge bosons in the $\eta'$-model.

| Vertex | Coupling | Vertex | Coupling |
|--------|----------|--------|----------|
| $H'_1 H'_1 W^+ W^-$ | $\frac{g_t^2}{4}$ | $H'_1 H'_1 X^+ X^-$ | $\frac{g_t^2}{4}$ |
| $H'_1 H'_1 Z_1 Z_1$ | $\frac{g_t^2}{8 W^2}$ | $H'_1 H'_1 Z_1 Z_2$ | $\frac{g_t^2 \sqrt{1/4s_t^2 W}}{4\sqrt{3}c_t^2 W}$ |
| $H'_1 H'_1 Z_2 Z_2$ | $\frac{g^2 (1-4 s_t'^2 W)}{24 s_t'^2 W}$ | $A'_1 A'_1 W^+ W^-$ | $\frac{g_t^2}{4}$ |
| $A'_1 A'_1 X^+ X^-$ | $\frac{g_t^2}{4}$ | $A'_1 A'_1 Z_1 Z_1$ | $\frac{g_t^2}{8 W^2}$ |
| $A'_1 A'_1 Z_1 Z_2$ | $\frac{g_t^2 \sqrt{1-4 s_t'^2 W}}{4\sqrt{3}c_t'^2 W}$ | $A'_1 A'_1 Z_2 Z_2$ | $\frac{g^2 (1-4 s_t'^2 W)}{24 s_t'^2 W}$ |
| $H'_1 H'_2^+ AW^-$ | $\frac{g^2 s_t W^2}{2}$ | $H'_1 H'_2^+ X^+ Y^-$ | $\frac{g^2}{2\sqrt{2}}$ |
| $H'_1 H'_2^+ Z_1 W^-$ | $\frac{g^2 s_t W^2}{2}$ | $H'_1 H'_2^+ Z_2 W^-$ | $\frac{g^2 \sqrt{1-4 s_t'^2 W}}{2\sqrt{3}c_t'^2 W}$ |
| $H'_1 H'_3^+ AX^-$ | $\frac{g^2 s_t W^2}{2}$ | $H'_1 H'_3^+ W^+ Y^-$ | $\frac{g^2}{2\sqrt{3}}$ |
| $H'_1 H'_3^+ Z_1 X^-$ | $\frac{g^2 c_t W^2}{4 W^2}$ | $H'_1 H'_3^+ Z_2 X^-$ | $\frac{g^2 \sqrt{1-4 s_t'^2 W}}{4\sqrt{3}c_t'^2 W}$ |
| $A'_1 H'_2^+ AW^-$ | $\frac{ig^2 s_t W^2}{2}$ | $A'_1 H'_2^+ X^+ Y^-$ | $\frac{ig^2}{2\sqrt{2}}$ |
| $A'_1 H'_2^+ Z_1 W^-$ | $\frac{ig^2 s_t W^2}{2}$ | $A'_1 H'_2^+ Z_2 W^-$ | $\frac{ig^2 \sqrt{1-4 s_t'^2 W}}{2\sqrt{3}c_t'^2 W}$ |
| $A'_1 H'_3^+ AX^-$ | $\frac{ig^2 s_t W^2}{2}$ | $A'_1 H'_3^+ W^+ Y^-$ | $\frac{ig^2}{2\sqrt{2}}$ |
| $A'_1 H'_3^+ Z_1 X^-$ | $\frac{ig^2 c_t W^2}{4 W^2}$ | $A'_1 H'_3^+ Z_2 X^-$ | $\frac{ig^2 \sqrt{1-4 s_t'^2 W}}{4\sqrt{3}c_t'^2 W}$ |
| $H'_1 H'_1' AA$ | $g^2 s_t W^2$ | $H'_1 H'_1' AZ_1$ | $g^2 c_t W^2 t_W$ |
| $H'_2^+ H'_2^+ AZ_2$ | $\frac{g^2 t_W^2 \sqrt{1-4 s_t'^2 W}}{2\sqrt{3}}$ | $H'_2^+ H'_2^+ W^+ W^-$ | $\frac{g^2}{2}$ |
| $H'_2^+ H'_2^+ Y^+ Y^-$ | $\frac{g^2}{2}$ | $H'_2^+ H'_2^+ Z_1 Z_1$ | $\frac{g^2 c_t W^2}{4 W^2}$ |
| $H'_2^+ H'_2^+ Z_1 Z_2$ | $\frac{g^2 c_t W^2}{2\sqrt{3}c_t'^2 W}$ | $H'_2^+ H'_2^+ Z_2 Z_2$ | $\frac{g^2 (1-4 s_t'^2 W)}{12 s_t'^2 W}$ |
| $H'_2^+ H'_3^+ AA$ | $g^2 s_t W^2$ | $H'_3^+ H'_3^+ AZ_1$ | $-2g^2 s_t W^2 t_W$ |
| $H'_3^+ H'_3^+ AZ_2$ | $\frac{2g^2 t_W^2 \sqrt{1-4 s_t'^2 W}}{\sqrt{3}}$ | $H'_3^+ H'_3^+ Z_1 Z_1$ | $\frac{g^2 c_t W^2}{4 W^2}$ |
| $H'_3^+ H'_3^+ Y^+ Y^-$ | $\frac{g^2}{2}$ | $H'_3^+ H'_3^+ Z_2 Z_2$ | $\frac{g^2 (1-4 s_t'^2 W)}{3\sqrt{3}c_t'^2 W}$ |

The physical inert scalars $H'_1$, $A'_3$, $\chi'_1 \equiv H'_1^\pm$ and $\chi'_2 \equiv H'_2^\pm$ by themselves with the respective masses are obtained as

$$m^2_{H'_1} = M^2_{\chi'} + \frac{1}{2} (y_5 + y_6) \omega^2, \quad m^2_{A'_3} = M^2_{\chi'} + \frac{1}{2} (y_5 - y_6) \omega^2,$$

$$m^2_{H'_2^\pm} = M^2_{\chi'}, \quad m^2_{H'_1^\pm} = M^2_{\chi'} + \frac{1}{2} y_4 u^2,$$  \hspace{1cm} (2.23)

where $M^2_{\chi'} \equiv \mu_{\chi'}^2 + \frac{1}{2} y_2 u^2 + \frac{1}{2} y_3 s^2 u^2$. If $H'_3$ (or $A'_3$) is the LIP, it can be the dark matter candidate.
Let us consider the interactions of the inert Higgs with the normal ones as well as the
gauge bosons. We remind that the inert scalars do not interact with fermions because of
the invariance under the $Z_2$ symmetry.

The Higgs boson–inert scalar interactions are obtained as follows

$$V_{\chi'} \supset y_1 \left[ H_1^{\prime+} H_1^{\prime-} + H_2^{\prime++} H_2^{\prime--} + \frac{1}{2} (H_3^2 + A_3^2) \right]^2$$

$$+ y_2 \left[ \frac{(u + h)^2}{2} + H^+ H^- \right] \times \left[ H_1^{\prime+} H_1^{\prime-} + H_2^{\prime++} H_2^{\prime--} + \frac{1}{2} (H_3^2 + A_3^2) \right]$$

$$+ \frac{y_3}{2} (\omega + H)^2 \times \left[ H_1^{\prime+} H_1^{\prime-} + H_2^{\prime++} H_2^{\prime--} + \frac{1}{2} (H_3^2 + A_3^2) \right]$$

$$+ \frac{y_4}{2} \left[ (u + h) H_1^{\prime+} + (H_3 + i A_3^\prime) H^- \right] \times \left[ (u + h) H_1^{\prime+} + (H_3 - i A_3^\prime) H^+ \right]$$

$$+ \frac{1}{4} (\omega + H)^2 \left[ (y_5 + y_6) H_3^2 + (y_5 - y_6) A_3^2 \right].$$

(2.24)

Interactions of the two inert scalars with one gauge boson are appeared in

$$L_{\text{gauge–}\chi'}^{\text{triple}} = -ig \left[ \chi' \left( T_1 A_{1\mu} - t B_{\mu} I \right) \bar{\partial}^\mu \chi' \right]$$

$$= \frac{ig}{2} \left[ -2 s_W A_\mu + \frac{1 + 2 s_W^2}{c_W} Z_{1\mu} + \frac{1 - 9 t_W^2}{\sqrt{3} \sqrt{1 - 3 t_W^2}} Z_{2\mu} \right] H_1^{\prime+} \bar{\partial}^\mu H_1^{\prime-}$$

$$- \frac{ig}{2} \left[ -4 s_W A_\mu - c_W (1 - 3 t_W^2) Z_{1\mu} + \frac{1 - 9 t_W^2}{\sqrt{3} \sqrt{1 - 3 t_W^2}} Z_{2\mu} \right] H_2^{\prime++} \bar{\partial}^\mu H_2^{\prime--}$$

$$+ ig \left[ \frac{1}{\sqrt{3} \sqrt{1 - 3 t_W^2}} Z_{2\mu} \right] \left[ H_3^\prime - i A_3^\prime \leftrightarrow \frac{\bar{\partial}^\mu}{\sqrt{2}} H_3^\prime + i A_3^\prime \right]$$

$$- \frac{ig}{\sqrt{2}} \left[ W_+^{\mu} H_1^{\prime+} \leftrightarrow \frac{\bar{\partial}^\mu}{\sqrt{2}} H_1^{\prime-} + X_+^{\mu} \frac{H_3^\prime - i A_3^\prime}{\sqrt{2}} \leftrightarrow \frac{\bar{\partial}^\mu}{\sqrt{2}} H_3^{\prime-} \right]$$

$$+ y_5^{++} \frac{H_3^\prime - i A_3^\prime}{\sqrt{2}} \leftrightarrow \frac{\bar{\partial}^\mu}{\sqrt{2}} H_3^{\prime-} + \text{H.c.} \right].$$

(2.25)
The quartic interactions of the two inert scalars with two gauge bosons are given by

\[
\mathcal{L}_{\text{quartic}} = g^2 \alpha^2 \left( C T_i A_{i\mu} - t B_{i\mu} I \right)^2 \chi^2
\]

\[
= \frac{g^2}{2} (W^{+\mu W^-_\mu} + X^{+\mu X^-_\mu}) H^{1+}_1 H^{1-}_1 + \frac{g^2}{2} (W^{+\mu W^-_\mu} + Y^{+\mu Y^-_\mu}) H^{1+}_2 H^{1-}_2
\]

\[
+ \frac{g^2}{4} \left( -2 s W A_\mu + \frac{1 + 2 s^2}{c W} Z_{1\mu} + \frac{1 - 9 t^2}{\sqrt{3} \sqrt{1 - 3 t^2}} Z_{2\mu} \right)^2 H^{1+}_1 H^{1-}_1
\]

\[
+ \frac{g^2}{4} \left( 4 s W A_\mu + c W (1 - 3 t^2) Z_{1\mu} - \frac{1 - 9 t^2}{\sqrt{3} \sqrt{1 - 3 t^2}} Z_{2\mu} \right)^2 H^{1+}_2 H^{1-}_2
\]

\[
+ \frac{g^2}{4} \left[ X^{+\mu X^-_\mu} + Y^{+\mu Y^-_\mu} + \frac{2}{3 (1 - 3 t^2)} Z^i Z_{2\mu} \right] \left( H^2_3 + A^2_3 \right)
\]

\[
+ \frac{g^2}{4} \left[ 2 \left( X^{-\mu Y^{++}_\mu} + \sqrt{2} W^{+\mu} [-3 s W A_\mu + 3 s W t W Z_{1\mu} + \frac{1 - 9 t^2}{\sqrt{3} \sqrt{1 - 3 t^2}} Z_{2\mu}] \right) H^{1+}_1 H^{1-}_2
\]

\[
+ \left( \sqrt{2} W^{+\mu Y^{-\mu}} + X^{-\mu} [-2 s W A_\mu + \frac{1 + 2 s^2}{c W} Z_{1\mu} - \frac{1 + 9 t^2}{\sqrt{3} \sqrt{1 - 3 t^2}} Z_{2\mu}] \right) H^{1+}_1 H^{1-}_3 + i A^2_A \right) H^{1+}_1 H^{1-}_2
\]

\[
\times H^{1+}_2 \left( H^3 + i A^3_A \right) + \text{H.c.} \right].
\]

The interactions of the inert scalars with the normal Higgs bosons in this model are given in Table 4, while the gauge-inert field interactions are listed in Table 5 and Table 6.

### 3 Dark matter in minimal 3-3-1 models

The simple 3-3-1 model with inert $\rho$ triplet, and the model with inert scalar sextet were previously considered in [11]. In this work, we study dark matter in the simple 3-3-1 model with $\eta$ replication (called $\eta'$—model for shortcut) and the model with $\chi$ replication (called $\chi'$—model) in details.

In order to calculate the relic density as well as indirect and direct searches for dark matter, we use micrOMEGAs [15, 16] after expanding the relevant interactions and implementing new model files into CalcHEP [17]. All possible annihilation and coannihilation channels are considered in the computation of the relic density. The coannihilation may reduce the relic density significantly if the mass of the inert particles exist within around 10 % or even 20 % of the LIP (lightest inert particle) mass [18].

Dark matter annihilation produces pairs of the Standard Model particles (or new particles in our model) that hadronize and decay into stable particles. Indirect search observes the signals of positrons, anti-protons, gamma-rays that are finally produced in
Table 4. Interactions of the inert scalars with the normal Higgs bosons in the \( \chi' \)–model.

| Vertex     | Coupling | Vertex     | Coupling |
|------------|----------|------------|----------|
| \( hH'_3H'_3 \) | \( \frac{y_2 u}{2} \) | \( hA'_3A'_3 \) | \( \frac{y_2 u}{2} \) |
| \( hH'_1^+H'_1^- \) | \( (y_2 + y_4)u \) | \( hH'_2^+H'_2^- \) | \( y_2 u \) |
| \( HH'_3^3H'_3^3 \) | \( \frac{y_3 + y_5 + y_6 + y_8}{2} \) | \( HA'_3A'_3 \) | \( \frac{y_3 + y_5 + y_6 + y_8}{2} \) |
| \( HH'_3^3H'_3^3 \) | \( y_3 \omega \) | \( H H'_2^+H'_2^- \) | \( y_3 \omega \) |
| \( H'_3H'_3H'_3H'_3 \) | \( \frac{y_4 u}{2} \) | \( A'_3H'_1^-H'_1^+ \) | \( \frac{y_4 u}{2} \) |
| \( H'_3H'_3H'_3H'_3 \) | \( \frac{y_4 u}{2} \) | \( H'_3H'_3H'_3H'_3 \) | \( \frac{y_4 u}{2} \) |
| \( H'_3H'_3H'_3H'_3 \) | \( \frac{y_4 u}{2} \) | \( H'_3H'_3H'_3H'_3 \) | \( \frac{y_4 u}{2} \) |
| \( H'_3H'_3H'_3H'_3 \) | \( \frac{y_4 u}{2} \) | \( H'_3H'_3H'_3H'_3 \) | \( \frac{y_4 u}{2} \) |
| \( H'_3H'_3H'_3H'_3 \) | \( \frac{y_4 u}{2} \) | \( H'_3H'_3H'_3H'_3 \) | \( \frac{y_4 u}{2} \) |

Table 5. Triple interactions of the inert scalars with gauge bosons in the \( \chi' \)–model.

| Vertex       | Coupling | Vertex       | Coupling |
|--------------|----------|--------------|----------|
| \( ZZ'_{\mu}H'_3^3 \partial_{\mu}A'_3 \) | \( -\frac{g_{SW}}{\sqrt{3}\sqrt{1-4s^2_W}} \) | \( X^-_{\mu}H'_3^3 \partial_{\mu}H'_1^- \) | \( -\frac{ig}{2} \) |
| \( Y'^{\pm}H'_3' \partial_{\mu}H'_2'^{\pm} \) | \( -\frac{ig}{2} \) | \( X^+_{\mu}A'_3 \partial_{\mu}H'_1^- \) | \( -\frac{g}{2} \) |
| \( Y'^{\pm}A'_3 \partial_{\mu}H'_2'^{\pm} \) | \( -\frac{ig}{2} \) | \( A'_3H'_1^+ \partial_{\mu}H'_1^- \) | \( ig_{SW} \) |
| \( W'^{\pm}_{\mu}H'_1^+ \partial_{\mu}H'_2'^{\pm} \) | \( -\frac{ig}{2} \) | \( Z_{1\mu}H'_1^+ \partial_{\mu}H'_1^- \) | \( -ig(1+2s^2_W) \) \( 2c_W \) |
| \( Z_{2\mu}H'_1^+ \partial_{\mu}H'_1^- \) | \( -ig(1-16s^2_W) \) \( 2\sqrt{3}c_W \sqrt{1-4s^2_W} \) | \( A'_3H'_2^+ \partial_{\mu}H'_1^- \) | \( 2ig_{SW} \) |
| \( W^-_{\mu}H'_2'^{\pm} \partial_{\mu}H'_1^- \) | \( -\frac{ig}{2} \) | \( Z_{1\mu}H'_2^+ \partial_{\mu}H'_2^- \) | \( \frac{ig(1-16s^2_W)}{2c_W} \) |
| \( Z_{2\mu}H'_2'^{\pm} \partial_{\mu}H'_1^- \) | \( -ig(1-16s^2_W) \) \( 2\sqrt{3}c_W \sqrt{1-4s^2_W} \) | \( A'_3H'_2^+ \partial_{\mu}H'_2^- \) | \( \frac{ig(1-16s^2_W)}{2c_W} \) |

dark matter annihilation processes. MicrOMEGAs computes the photon, positron, antiproton flux at a given energy \( E \) and the angle in the direction of observation, which can be the source for experiments PAMELA, Fermi, and etc.

In direct searches, one measures the recoil energy deposited by scattering of LIPs with the nuclei. In this work, both \( \eta' \)–model and \( \chi' \)–model provide Higgs dark matter that can only contribute to the spin independent interaction with nuclei. To derive the LIP-nucleus cross section we use the method, as mentioned in [16]. All interactions of the LIP with
Table 6. Quartic interactions of the inert scalars with gauge bosons in the $\chi'$-model.

| Vertex | Coupling | Vertex | Coupling |
|--------|----------|--------|----------|
| $H'_3H'_3Y'^+Y'^-$ | $\frac{g^4}{4}$ | $H'_3H'_3X^+X^-$ | $\frac{g^4}{4}$ |
| $H'_3H'_3Z_2Z_2$ | $\frac{g^4c_W}{6(1-4s_W)}$ | $A'_3A'_3Y'^+Y'^-$ | $\frac{g^4}{4}$ |
| $A'_3A'_3X^+X^-$ | $\frac{g^4}{4}$ | $A'_3A'_3Z_2Z_2$ | $\frac{g^4c_W}{6(1-4s_W)}$ |
| $H'_3H'_3AX^-$ | $-\frac{g^4s_W}{2}$ | $H'_3H'_3W^+Y^-$ | $-\frac{g^4}{2\sqrt{3}}$ |
| $H'_3H'_3Z'_1X^-$ | $\frac{g^4}{4}c_W$ | $H'_3H'_3Z'_2X^-$ | $-\frac{g^4(1+8s_W)}{4\sqrt{3}c_W\sqrt{1-4s_W}}$ |
| $H'_3H'_3W^+X^-$ | $\frac{g^4}{2\sqrt{3}}$ | $H'_3H'_3W^+Y^-$ | $-\frac{g^4}{2\sqrt{3}}$ |
| $H'_3H'_3Z'_1Y^-$ | $-\frac{g^4(1-4s_W)}{2c_W}$ | $H'_3H'_3Z'_2Y^-$ | $-\frac{g^4(1+8s_W)}{4\sqrt{3}c_W\sqrt{1-4s_W}}$ |
| $A'_3H'_3AX^-$ | $-\frac{ig^4s_W}{2}$ | $A'_3H'_3W^+Y^-$ | $-\frac{i\sqrt{3}}{2}$ |
| $A'_3H'_3Z'_1X^-$ | $\frac{ig^4(1+2s_W)}{2c_W}$ | $A'_3H'_3Z'_2X^-$ | $-\frac{i\sqrt{3}}{2}$ |
| $A'_3H'_3W^+X^-$ | $\frac{ig^4}{2\sqrt{3}}$ | $A'_3H'_3W^+Y^-$ | $-\frac{i\sqrt{3}}{2}$ |
| $A'_3H'_3Z'_1Y^-$ | $-\frac{ig^4(1-4s_W)}{2c_W}$ | $A'_3H'_3Z'_2Y^-$ | $-\frac{i\sqrt{3}}{2}$ |
| $H'_1^+H'_2^-AW^-$ | $\frac{3g^4s_W}{\sqrt{2}}$ | $H'_1^+H'_2^-X^-Y'^+$ | $\frac{g^4}{2\sqrt{3}}$ |
| $H'_1^+H'_2^-Z'_1W^+$ | $\frac{3g^4s_W}{\sqrt{2}}$ | $H'_1^+H'_2^-Z'_2W^+$ | $\frac{g^4(1-10s_W)}{\sqrt{6}c_W\sqrt{1-4s_W}}$ |
| $H'_1^+H'_1^-AA$ | $\frac{g^4}{2}$ | $H'_1^+H'_1^-AZ'_1$ | $-\frac{g^4}{2}$ |
| $H'_1^+H'_1^-AZ'_2$ | $\frac{g^4}{2}$ | $H'_1^+H'_1^-W^+W^-$ | $\frac{g^4}{2}$ |
| $H'_1^+H'_1^-X^+X^-$ | $\frac{g^4}{2}$ | $H'_1^+H'_1^-Z'_1Z_1$ | $\frac{g^4(1+2s_W^2)}{4c_W}$ |
| $H'_1^+H'_1^-Z'_1Z_2$ | $\frac{g^4(1+2s_W^2)(1-10s_W)}{2\sqrt{3}c_W\sqrt{1-4s_W}}$ | $H'_1^+H'_1^-Z'_2Z_2$ | $\frac{g^4(1-10s_W)}{12c_W^2(1-4s_W)}$ |
| $H'_1^+H'_2^-AA$ | $\frac{4g^4}{2}$ | $H'_2^+H'_2^-Z'_1Z_1+H'_2^{-}H'^{-}AZ'_1$ | $2g^4t_W(1-4s_W^2)$ |
| $H'_2^+H'_2^-AZ'_2$ | $\frac{2g^4t_W(1-10s_W)}{\sqrt{3\sqrt{1-4s_W}}}$ | $H'_2^+H'_2^-W^+W^-$ | $\frac{g^4}{2}$ |
| $H'_2^+H'_2^-Y'^+Y'^-$ | $\frac{g^4}{2}$ | $H'_2^+H'_2^-Z'_1Z_1$ | $\frac{g^4(1-4s_W^2)}{4c_W}$ |
| $H'_2^+H'_2^-Z'_1Z_2$ | $\frac{2g^4(1-10s_W)(1-10s_W)}{2\sqrt{3}c_W\sqrt{1-4s_W}}$ | $H'_2^+H'_2^{-}Z'_2Z_2$ | $\frac{g^4(1-10s_W)^2}{12c_W^2(1-4s_W)}$ |

quarks are input in the model files, CalcHEP then generates and calculates all diagrams for LIP - quark/anti-quark elastic scattering at zero momentum. The normalized cross section on a point-like nucleus is obtained as

$$
\sigma_{LIP-N}^{SI} = \frac{4\mu_{LIP}^2}{\pi} (Z\lambda_p + (A - Z)\lambda_n)^2, \tag{3.1}
$$

where $\mu_{LIP}$ is the LIP-nucleus reduced mass. $\mu_{LIP} = m_{LIP}m_{\text{nuclei}}/(m_{LIP} + m_{\text{nuclei}}) \approx m_{\text{nuclei}}$. $\lambda_p$ and $\lambda_n$ are the effective couplings of the LIP to protons and neutrons, respectively. The couplings $\lambda_{p,n}$ are connected to the coefficients $f_q^N$, which are linked to the pion-nucleon.
sigma term $\sigma_{\pi N}$ and the quantity $\sigma_0$ [16]. Recent analyses suggest that [19]

$$\sigma_{\pi N} = 55 - 73 \text{MeV}, \quad \sigma_0 = 35 \pm 5 \text{MeV}. \quad (3.2)$$

The direct rate does not change so much in the above ranges of $\sigma_{\pi N}$ and $\sigma_0$. The results on the relic density as well as search for dark matter in each model are presented in subsections below.

3.1 Dark matter in the simple 3-3-1 model with $\eta$ replication

The inert particles in the simple 3-3-1 model with $\eta$ replication are $H'_1, A'_1, H'_2^\pm, H'_3^\pm$. With the condition $x_6 < \min\{0, -x_4, (w/u)^2 x_5 - x_4\}$, $H'_1$ is the LIP and it can be a candidate for dark matter. All possible annihilation channels of $H'_1$ are listed in Fig. 1

The $\eta'$-model contains parameters: $\mu^2_{\eta'}, \omega, \lambda_{1,2,3,4}, x_{1,2,3,4,5,6}$. Let us chose some fixed ones as

$$\lambda_2 = \lambda_3 = \lambda_4 = 0.1, \quad x_1 = 0.01, \quad x_2 = 0.03, \quad x_3 = 0.01, \quad x_4 = 0.07, \quad x_5 = 0.08, \quad x_6 = -0.09. \quad (3.3)$$

The coupling $\lambda_1$ is constrained by the mass of the Standard Model Higgs, $m_h = 125$ GeV. From Eq. (2.16) the dark matter mass depends on the two parameters $\mu_{\eta'}$ and $\omega$.

Fig. 2 shows the relic density as a function of dark matter mass by varying $\mu_{\eta'}$ from 100 GeV to 5000 GeV for $\omega = 3$ TeV (red), $\omega = 4$ TeV (green), and $\omega = 5$ TeV (blue). For each value of $\omega$, there are two main regions of dark matter mass, $m_{H'_1}$ can be some hundreds GeV or at TeV scale. $m_{H'_1}$ should be heavier than 1.15 TeV for $\omega = 3$ TeV (or $m_{H'_1} > 1.6$ TeV for $\omega = 4$ TeV, or $m_{H'_1} > 2.05$ TeV for $\omega = 5$ TeV ) or $m_{H'_1}$ should be lighter than 600 GeV for the three values of $\omega$ in order to satisfy the bound of the WMAP results on dark matter relic density [20]. The mass region of dark matter is quite narrow for each value of $\omega$ to provide the right abundance

$$\Omega h^2 = 0.1120 \pm 0.0056.$$ 

The three lines coincide at the region below 1 TeV as shown in Fig. 2 that infers we can have dark matter with the mass below 600 GeV for $\omega = 3$ TeV or 4 TeV or 5 TeV. It is worth noticing that even though $\omega$ changes, we always can have $m_{H'_1} < 600$ GeV in agreement with the WMAP results on the relic density. This conclusion is more clearly shown in Fig. 3, in which we figure out the plane of $\omega - \mu_{\eta'}$ (left) and $\omega - m_{H'_1}$ (right) by varying both $\omega$ and $\mu_{\eta'}$ in the regions (3000 GeV $< \omega < 9000$ TeV) and (100 GeV $< \mu_{\eta'} < 3100$ GeV).

The color regions are in agreement with the requirement $\Omega h^2 < 0.1176$. The red regions satisfy $0.1064 < \Omega h^2 < 0.1176$. The lightest dark matter mass can be at electroweak scale ($m_{DM}(\text{min}) = 212.8$ GeV for $\omega = 3$ TeV, $\mu_{\eta'} = 1$ GeV). The figure on the right panel in Fig. 3 shows that the 570 GeV-dark matter supplies the right relic density when $\omega$ changes, corresponding to the red point-line in the region $\mu_{\eta'} < 600$ GeV on the left side. The mass of dark matter can be at TeV scale when the value of $\mu_{\eta'}$ is large enough.

Now let us consider the results on indirect and direct search for dark matter in detail. For example, with $\omega = 3$ TeV, $\mu_{\eta'} = 534$ GeV we get $m_{H'_1} = 574.7$ GeV and other inert
particles $m_{H_2'} = 575.2$ GeV, $m_{A_1'} = 579.4$ GeV, $m_{H_3'} = 831.2$ GeV. Since the mass square difference between $m_{H_1'}^2, m_{A_1'}^2, m_{H_2'}^2$ is in order of $x_4 u^2, x_6 u^2$, the mass of $A_1'$ and $H_2'\pm$ is very close to $m_{H_1'}$ for any values of $\omega$. That is why the co-annihilation contributes a lot to the main $\Omega h^2$. For the choice $\omega = 3$ TeV, $\mu_{\eta'} = 534$ GeV, we get $\Omega h^2 = 0.111$ and the main

**Figure 1.** Diagrams contributing to the annihilation of the $H_1'$ dark matter.
Figure 2. \( \Omega h^2 \) as a function of \( m_{H_1'} \) for \( \omega = 3 \) TeV (red), \( \omega = 4 \) TeV (green), and \( \omega = 5 \) TeV (blue). (The three curved lines are coincident at low mass region and separated at TeV scale for \( \omega = 3 \) TeV, \( \omega = 4 \) TeV, and \( \omega = 5 \) TeV, respectively from left to right. The dotted lines are rare regions for \( \omega = 3 \) TeV (left), \( \omega = 4 \) TeV (middle), and \( \omega = 5 \) TeV (right). The horizontal line is the WMAP limit on the relic density.)

Figure 3. Contour plot of the relic density on \( \omega - \mu_{\nu} \) plane (left) and \( \omega - m_{H_1'} \) plane (right) in agreement with the WMAP data. The red regions (darker fringe in black-white print) yield the right abundance, \( 0.1064 < \Omega h^2 < 0.1176 \).

Annihilation/co-annihilation channels are
\[
H_2^+H_2^- \to W^+W^-, \quad H_1^1H_1^i \to Z_1Z_1, \quad H_1^+H_1^- \to W^+W^-, \quad H_1^iH_1^\pm \to AW^\pm, \quad H_2^+H_2^- \to AA.
\]

In this case, the photon flux, positron flux and anti-proton flux are
\[
2.8 \times 10^{-14} \text{ (cm}^2\text{ sr s GeV)}^{-1}, \quad 1.8 \times 10^{-12} \text{ (cm}^2\text{ sr s GeV)}^{-1}, \quad 3.5 \times 10^{-11} \text{ (cm}^2\text{ sr s GeV)}^{-1},
\]
correspondingly, for the angle of sight 0.10 rad and energy \( E = 100 \) GeV. The \( H_1^i - p, n \)
cross section is $1.5 \times 10^{-47} \text{cm}^2$ and the total number of events is $2.2 \times 10^{-6}$ events/day/kg.

The dark matter mass can be at TeV scale if we chose $\mu_{\eta'} = 1171 \text{ GeV}$ for $\omega = 3 \text{ TeV}$. In this case the dominant channels of annihilation/co-annihilation can be heavy gauge bosons, such as $H'^+ H'^+ \rightarrow W^+ X^+, Y^{++} Z_1$. For the dark matter with the mass around 570 GeV, the results on the relic density as well as search for dark matter do not change when varying $\omega$ since the couplings in the dominant channels do not depend on $\omega$.

The plane $< \sigma . v_{\text{rel}} > - m_{H'^1}$ for the abundance below the experimental upper bound, $\Omega h^2(\text{max}) = 0.1176$, is shown in Fig. 4. For the right abundance of dark matter, the total annihilation cross section times the relative velocity of incoming dark matter particles and the dark matter mass is in order of $10^{-26} \text{cm}^3/\text{s}$, respectively for $m_{H'^1} < 2 \text{ TeV}$, and it decreases when $m_{H'^1}$ increases because the heavier dark matter is, the more the contribution of co-annihilation to the $\Omega h^2$ gets.

Fig. 5 shows the values of $\sigma_{\text{LIP-nucleon}}$ as a function of dark matter mass obtained from micrOMEGAs by fixing the nucleon form factors, $\sigma_0 = 30 \text{ MeV}$ and $\sigma_{\pi N} = 73 \text{ MeV}$. The value of $\sigma_{\text{LIP-nucleon}}$ is $5.4 \times 10^{-48} \text{cm}^2$ for Xe detector and the total number of events is $1.1 \times 10^{-8}$ events/day/kg for dark matter with mass around 2 TeV.

Let us calculate the direct dark matter search by hand and compare to the results achieved from micrOMEGAs. The dark matter scatters off the nuclei of a large detector via interaction with quarks confined in nucleons. Since the dark matter is closely non-relativistic, the process can be described by an effective Lagrangian [16],

$$\mathcal{L}_S = 2\lambda_q m_{H'^1} H'^1_H'^1 \bar{q}q. \quad (3.5)$$

Note that, for the real scalar field only spin-independent and even interactions are possible. There exist interactions of the pair $H'^1$ couple to $h$ and $H_0$. However, the dominant contributions to $H'^1$- quark scattering are done by the t-channel exchange of $h$. We obtain

$$\lambda_q = \frac{(x_2 + x_4 + x_6)m_q}{2m_{H'^1}m_h^2}. \quad (3.6)$$
The $H'_1$-nucleon scattering amplitude is taken as a summation over the quark level interactions with respective nucleon form factors. The $H'_1$-nucleon cross section is given as

$$
\sigma_{H'_1-N} = \frac{4m_r^2}{\pi}\lambda_N^2,
$$

where $N = p, n$ denotes nucleon, and

$$
m_r = \frac{m_{H'_1}m_N}{m_{H'_1} + m_N} \approx m_N,
$$

$$
\frac{\lambda_N}{m_N} = \sum_{u,d,s} f_{Tq}^N \frac{\lambda_q}{m_q} + \frac{2}{27} f_{TG}^N \sum_{c,b,t} \lambda_q m_q,
$$

where $f_{Tq}^N = 1 - \sum_{u,d,s} f_{Tq}^N$. The $f_{Tq}^N$ values were considered in [21],

$$
f_{Tu}^N = 0.014 \pm 0.003, \quad f_{Td}^N = 0.036 \pm 0.008, \quad f_{Ts}^N = 0.118 \pm 0.062.
$$

Taking $m_N = 1$ GeV and $m_h = 125$ GeV [22], we obtain

$$
\sigma_{H'_1-N} \approx \left[ \frac{x_2 + x_4 + x_6}{m_{H'_1}(\text{TeV})} \right]^2 \times 6.146 \times 10^{-44} \text{ cm}^2
$$

with notice that the $x_{2,4,6}$ given in Eq. (3.3). The $\sigma_{H'_1-N}$ got in Eq. (3.10) is inversely proportional to the square of the dark matter mass that is shown as a (blue) continuous line passed by the red region in Fig. 5. It implies that the direct search calculated by hand is in nice agreement with the result yielded by micrOMEGAs package.
3.2 Dark matter in the simple 3-3-1 model with \( \chi \) replication

The simple 3-3-1 model with \( \chi \) replication contains six inert particles \( H'_1^\pm, H'_2^{\pm\pm}, H'_3, A'_3 \). If we assume that \( y_6 < \text{Min}\{0, -y_5, (u/w)^2 y_4 - y_5\} \), \( H'_3 \) is the lightest inert particle and can be the dark matter candidate. The annihilation channels of \( H'_3 \) are almost similar to that of \( H'_1 \) by shifting \( H'_1 \rightarrow H'_3 \) and \( A'_1 \rightarrow A'_3 \). Fig. 6 only lists the channels which are different from ones due to the annihilation of \( H'_1 \). We see that there is only one possible diagram \( H'_3 H'_3 \rightarrow Z_1 Z_1; Z_1 Z_2; W^+ W^- \), less than the number of diagrams corresponding to \( H'_1 \) annihilation, while there are more possibility of \( H'_3 H'_3 \rightarrow Y^{++} Y^{--} \).

The parameters appeared in this model are \( \mu^2, \omega, \lambda_{1,2,3,4}, y_{1,2,3,4,5,6} \), in which the couplings \( \lambda_{1,2,3,4} \) are fixed as given in the \( \eta' \)-model. Now let us consider the results for the relic density and indirect search as well as direct search with a set of \( y_{1,2,3,4,5,6} \) in the same order

\[
y_1 = 0.01, y_2 = 0.04, y_3 = 0.058, y_4 = 0.01, y_5 = 0.05, y_6 = -0.06. \tag{3.11}
\]

The abundance is considered as a function of \( m_{H'_3} \) shown in Fig. 7 for \( \omega = 3 \text{ TeV} \)
Figure 8. Contour plot of the relic density on $\omega - \mu_{\chi'}$ plane (left) and $\omega - m_{H'_3}$ plane (right) in agreement with the WMAP data. The red regions (darker fringe in black-white print) yield the right abundance, $0.1064 < \Omega h^2 < 0.1176$.

Figure 8 shows the contour plot of the relic density on $\omega - \mu_{\chi'}$ plane (left) and $\omega - m_{H'_3}$ plane (right) in agreement with the WMAP data. The red regions (darker fringe in black-white print) yield the right abundance, $0.1064 < \Omega h^2 < 0.1176$. With the choice of parameters given in Eq. (3.11), the dark matter in the $\chi'$-model should be heavier than 580 GeV for $\omega = 3$ TeV (or 770 GeV for $\omega = 4$ TeV, or $m_{H'_3} > 990$ GeV for $\omega = 5$ TeV). It is illustrated more clearly in the $\omega - \mu_{\chi'}$ plane (left) and $\omega - m_{H'_3}$ plane (right) in Fig. 8. For each value of $\omega$, there is a lower bound on the value of $\mu_{\chi'}$ that results a respectively lower bound on $m_{H'_3}$ in order to satisfy the WMAP data. It is different from the $\eta'$-model that the doublet dark matter $H'_1$ in the $\eta'$-model can be at electroweak scale but the singlet one $H'_3$ in the $\chi'$-model should be heavier.

By varying $\omega$ and $\mu_{\chi'}$ in the ranges (3000, 9000) GeV and (100, 3000) GeV, correspondingly, we figure out the $<\sigma.v_{\text{rel}}>-m_{H'_3}$ plane in Fig. 9, in which the green regions satisfy the relic density $\Omega h^2 \leq 0.1064$, while the red ones yield the right abundance. The $<\sigma.v_{\text{rel}}>$ gets the typical value $\sim 10^{-26}$cm$^3$/s for the dark matter mass below 2 TeV that is similar in the $\eta'$-model. The direct search results depending on $m_{H'_3}$ are shown in Fig. 10. The $H'_3$-nucleon cross section is $2.1 \times 10^{-47}$cm$^2$ and the number of events is $8.7 \times 10^{-7}$ events/day/kg for $m_{H'_3} = 2$ TeV.

Here, we give an example for the dark matter at low energy. For $\omega = 3$ TeV, $\mu_{\chi'} = 361$ GeV, the dark matter with mass 589 GeV provides the abundance 0.11. The main annihilation/co-annihilation channels are

$$H'_3 H'_1^{\pm} \rightarrow Z_1 X^\pm, \quad H'_3 H'_2^{\pm\pm} \rightarrow Z_1 Y^{\pm\pm}, \quad H'_2^{++} H'_2^{--} \rightarrow hh, \quad H'_1^{++} H'_1^{--} \rightarrow hh, \quad H'_3 H'_3 \rightarrow hh.$$  \hspace{1cm} (3.12)

The photon flux, positron flux and anti-proton flux are $5.3 \times 10^{-16}$ (cm$^2$ sr s GeV)$^{-1}$, $2.4 \times 10^{-14}$ (cm$^2$ sr s GeV)$^{-1}$, $6.9 \times 10^{-13}$ (cm$^2$ sr s GeV)$^{-1}$, correspondingly, for the angle of sight 0.10 rad and energy $E = 100$ GeV. The $H'_3-p,n$ cross section is $2.3 \times 10^{-46}$cm$^2$ and the total number of events/day/kg is $3.3 \times 10^{-5}$. For
the same dark matter mass around 580 GeV, the signals in indirect search for dark matter in the \( \eta' \)-model are more sensitive but the direct search results are lower than that in the \( \chi' \)-model. This conclusion keeps the same if we test for the dark matter in TeV range.

Similarly, we can calculate the direct search by hand as analysis in the \( \eta' \)– model. The effective lagrangian takes the form

\[
\mathcal{L}'_{S} = 2\lambda'_{q}m_{H'_3}^{2}H'_3H'_3\bar{q}q.
\]  

We obtain

\[
\lambda'_{q} = \frac{y_{2}m_{q}}{2m_{H'_3}m^{2}_{h}}.
\]  

\[
\text{Figure 9.} \quad \langle \sigma \cdot v_{\text{rel}} \rangle \sim m_{H'_3} \quad \text{plane in agreement with the WMAP data. The red regions (darker regions in black-white print) yield the right abundance, } 0.1064 < \Omega h^{2} < 0.1176.
\]

\[
\text{Figure 10.} \quad \sigma_{H'_3 - N} \text{ (left) and the total number of events/day/kg (right) as functions of } m_{H'_3}. \quad \text{The (blue) continuous line on the left panel is obtained by hand for direct search.}
\]
and finally

$$\sigma_{H'_3-N} \simeq \left[ \frac{y_2}{m_{H'_3}(\text{TeV})} \right]^2 \times 6.146 \times 10^{-44} \text{ cm}^2. \quad (3.15)$$

The result given in Eq. (3.15) is depicted as a (blue) continuous line shown on the left side in Fig. 5 for $y_2 = 0.04$. The line goes through the red region that indicates the result calculated by hand is in nice agreement with the one yielded from micrOMEGAs.

4 Conclusion

We have considered the minimal 3-3-1 model behaved as the simple 3-3-1 model with two scalar triplets $\eta$ and $\chi$, and the simple 3-3-1 model with $\eta$ replication, namely $\eta'$—model as well as the version with $\chi$ replication, namely $\chi'$—model. The original simple 3-3-1 model does not contain dark matter. By introducing an odd Higgs triplet ($\eta'$ in the $\eta'$—model or $\chi'$ in the $\chi'$—model) under a $Z_2$ symmetry while all other fields are even, the odd scalars are inert and do not mix with the normal ones. Due to the $Z_2$ symmetry, the two extra triplets have zero VEV and they either interact with the normal Higgs via additional potential or couple to gauge bosons through Lagrangian gauge-inert. There is no interaction between the inert particles and fermions. The lightest and neutral inert particle is stable and it can be the dark matter. The doublet dark matter $H'_1$ in the $\eta'$—model as well as the singlet dark matter $H'_3$ in the $\chi'$—model have been studied in this work in detail.

We have calculated all relevant interactions, which can contribute to the annihilation/co-annihilation processes. The results on the relic density as well as experimental searches for dark matter have been investigated by using micrOMEGAs package after inputting new model files. It is interesting that in both $\eta'$ and $\chi'$ models, the dark matter masses can be a few hundreds GeV to several TeV and the region below 2 TeV gets the typical value of the thermally averaged annihilation cross section times velocity, $<\sigma v> \sim 10^{-26}\text{ cm}^3/\text{s}$. The dark matter-nucleon cross section yielded by micrOMEGAs and that calculated by hand are found to be coincident, which is experimentally acceptable. For a given value of $\omega$, the region of dark matter mass providing the right abundance is quite narrow. The different results between the two models are

- The doublet dark matter $H'_1$ in the $\eta'$—model can be at the electroweak scale but the singlet dark matter $H'_3$ in the $\chi'$—model should be much heavier. There is a lower bound on $m_{H'_3}$, for example $m_{H'_3} > 580$ GeV for $\omega = 3\text{TeV}$ (or $m_{H'_3} > 770$ GeV for $\omega = 4\text{TeV}$, or $m_{H'_3} > 990$ GeV for $\omega = 5\text{TeV}$).

- For all values of $\omega$, we always have the dark matter below 600 GeV in the $\eta'$—model but not in the $\chi'$—model.

- The indirect search (the particles fluxes) for the dark matter in the $\eta'$—model is more sensitive, while the direct search results such as the $\sigma_{\text{LIP-nucleon}}$, the total number of events/day/kg, are lower for the same dark matter mass in comparison with the signals in the $\chi'$—model.
Acknowledgments

This research is funded by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 103.01-2013.43, and by the National Research Foundation of Korea (NRF) grant funded by Korea government of the Ministry of Education, Science and Technology (MEST) (No. 2011-0017430) and (No. 2011-0020333).

References

[1] D. N. Spergel et al. (WMAP), Wilkinson Microwave Anisotropy Probe (WMAP) three year results: implications for cosmology, Astrophys. J. Suppl. 170, 377 (2007) [arXiv: astro-ph/0603449]; Planck Collaboration, P. A. R. Ade et al., Planck 2013 results. I. Overview of products and scientific results, A&A 571, A1 (2014) [arXiv:1303.5062 [astro-ph.CO]].

[2] G. Jungman, M. Kamionkowski and K. Griest, Supersymmetric dark matter, Phys. Rept. 267, 195 (1996); G. Bertone, D. Hooper and J. Silk, Particle dark matter: Evidence, candidates and constraints, Phys. Rept. 405, 279 (2005); H. Murayama, Physics Beyond the Standard Model and Dark Matter, Lectures at Les Houches Summer School - Session 86: Particle Physics and Cosmology: The Fabric of Spacetime [arXiv:hep-ph/0704.2276]; S. Dodelson, L. M. Widrow, Sterile neutrinos as dark matter, Phys. Rev. Lett. 72, 17 (1994); C. Boehm and P. Fayet, Scalar dark matter candidates Nucl. Phys. B 683, 219 (2004); D. Fargion and M. Yu. Khlopov, Tera-Leptons Shadows over Sinister Universe [arXiv:hep-ph/0507087]; S. B. Gudnason, C. Kouvaris and F. Sannino, Dark Matter from new Technicolor Theories, Phys. Rev. D 74, 095008 (2006); S. B. Gudnason, C. Kouvaris and F. Sannino, Towards working technicolor: Effective theories and dark matter, Phys. Rev. D 73, 115003 (2006); D. Fargion, M. Yu. Khlopov, C. A. Stephan, Cold dark matter by heavy double charged leptons?, Class. Quant. Grav. 23, 7305 (2006); M. Yu. Khlopov, Composite dark matter from 4th generation, JETP Lett. 83, 1 (2006); C. G. Boehmer, T. Harko, Can dark matter be a Bose-Einstein condensate?, JCAP 0706, 025 (2007); C. Kouvaris, Dark Majorana Particles from the Minimal Walking Technicolor, Phys. Rev. D 76, 015011 (2007); M. Yu. Khlopov and C. Kouvaris, Strong Interactive Massive Particles from a Strong Coupled Theory, Phys. Rev. D 77, 065002 (2008); K. Hamaguchi, S. Shirai and T. T. Yanagida, Composite messenger baryon as a cold dark matter, Phys. Lett. B 654, 110 (2007); G. Belanger, A. Pukhov, G. Servant, Dirac Neutrino Dark Matter, JCAP 0801, 009(2008); H. S. Cheng, J. L. Feng and K. T. Matchev, Kaluza-Klein dark matter, Phys. Rev. Lett. 89, 211301 (2002); G. Servant and T. M. P. Tait, Elastic scattering and direct detection of Kaluza-Klein dark matter, New J. Phys. 4, 99 (2002); Is the lightest Kaluza-Klein particle a viable dark matter candidate?, Nucl. Phys. B 650, 391 (2003); F. Fucito, A. Lionetto and M. Prisco, Extra-dimensions and dark matter, JCAP 0606, 002 (2006); K. Hsieh, R. N. Mohapatra and S. Nasri, Dark matter in universal extra dimension models: Kaluza-Klein photon and right-handed neutrino admixture, Phys. Rev. D 74, 066004 (2006); Mixed dark matter in universal extra dimension models with TeV scale W(R) and Z-prime, JHEP 0612, 067 (2006); M. Regis, M. Serone and P. Ullio, A Dark Matter Candidate from an Extra (Non-Universal) Dimension, JHEP 0703, 084 (2007); D. Hooper and S. Profumo, Dark matter and collider phenomenology of universal extra dimensions, Phys. Rept. 453, 29 (2007); S. Matsumoto, J. Sato, M. Senami and M. Yamanaka, Relic
abundance of dark matter in universal extra dimension models with right-handed neutrinos, Phys. Rev. D 76, 043528 (2007); B. A. Dobrescu, D. Hooper, K. Kong and R. Mahbubani, Spinless photon dark matter from two universal extra dimensions, JCAP 0710, 012 (2007); A. Martin, Dark Matter in the Simplest Little Higgs Model, arXiv:hep-ph/0602206; A. Birkedal, A. Noble, M. Perelstein and A. Spray, Little Higgs dark matter, Phys. Rev. D 74, 035002 (2006); C. S. Chen, K. Cheung and T. C. Yuan, Novel Collider Signature for Little Higgs Dark Matter Models, Phys. Lett. B 644, 158 (2007); M. Perelstein and A. Spray, Indirect Detection of Little Higgs Dark Matter, Phys. Rev. D 75, 083519 (2007); D. Hooper and G. Zaharijas, Distinguishing Supersymmetry From Universal Extra Dimensions or Little Higgs Models With Dark Matter Experiments, Phys. Rev. D 75, 035010 (2007).

[3] F. Pisano and V. Pleitez, An SU(3) × U(1) model for electroweak interactions, Phys. Rev. D 46, 410 (1992); P. H. Frampton, Chiral dilepton model and the flavor question, Phys. Rev. Lett. 69, 2889 (1992); R. Foot, O. F. Hernandez, F. Pisano and V. Pleitez, Lepton masses in an SU(3) × U(1)N gauge model, Phys. Rev. D 47, 4158 (1993).

[4] M. Singer, J. W. F. Valle and J. Schechter, Canonical neutral current predictions from the weak electromagnetic gauge group SU(3) × U(1), Phys. Rev. D 22, 738 (1980); R. Foot, H. N. Long and Tuan A. Tran, SU(3) × U(1)N and SU(4) × U(1)N gauge models with right-handed neutrinos, Phys. Rev. D 50, 34 (1994); J. C. Montero, F. Pisano and V. Pleitez, Neutral currents and GIM mechanism in SU(3) × U(1)N models for electroweak interactions, Phys. Rev. D 47, 2918 (1993); H. N. Long, SU(3) × U(1)N model for right-handed neutrino neutral currents, Phys. Rev. D 54, 4691 (1996); The 3-3-1 model with right-handed neutrinos, Phys. Rev. D 53, 437 (1996).

[5] D. Ng, Electroweak theory of SU(3) × U(1), Phys. Rev. D 49, 4805 (1994); D. G. Dumm, F. Pisano, and V. Pleitez, Flavor changing neutral currents in SU(3) × U(1)ν models, Mod. Phys. Lett. A 09, 1609 (1994); H. N. Long and V. T. Van, Quark family discrimination and flavour-changing neutral currents in the SU(3) × SU(3) × U(1) model with right-handed neutrinos, J. Phys. G 25, 2319 (1999).

[6] F. Pisano, A Simple solution for the flavor question, Mod. Phys. Lett. A 11, 2639 (1996); A. Doff and F. Pisano, Charge quantization in the largest leptoquark bilepton chiral electroweak scheme, Mod. Phys. Lett. A 14, 1133 (1999); C. A. de S. Pires and O. P. Ravinez, Electric charge quantization in a chiral bilepton gauge model, Phys. Rev. D 58, 035008 (1998); C. A. de S. Pires, Remark on the vectorlike nature of the electromagnetism and the electric charge quantization, Phys. Rev. D 60, 075013 (1999); P. V. Dong and H. N. Long, Electric charge quantization in SU(3) × SU(3) × U(1)N models, Int. J. Mod. Phys. A 21, 6677 (2006).

[7] M. B. Tully and G. C. Joshi, Generating neutrino mass in the 3-3-1 model, Phys. Rev. D 64, 011301 (2001); Alex G. Dias, C. A. de S. Pires, and P. S. Rodrigues da Silva, Naturally light right-handed neutrinos in a 331 model, Phys. Lett. B 628, 85 (2005); D. Chang and H. N. Long, Interesting radiative neutrino mass in an SU(3) × SU(3) × U(1) model with right-handed neutrinos, Phys. Rev. D 73, 053006 (2006); P. V. Dong, H. N. Long, and D. V. Soa, Neutrino masses in the economical 3-3-1 model, Phys. Rev. D 75, 073006 (2007); P. V. Dong and H. N. Long, Neutrino masses and lepton flavor violation in the 3-3-1 model with right-handed neutrinos, Phys. Rev. D 77, 057302 (2008); P. V. Dong, L. T. Hue, H. N. Long, and D. V. Soa, The 3-3-1 model with A4 flavor symmetry, Phys. Rev. D 81, 053004 (2010); P. V. Dong, H. N. Long, D. V. Soa, and V. V. Vien, The 3-3-1 model with S4 flavor symmetry, Eur. Phys. J. C 71, 1544 (2011); P. V. Dong, H. N. Long, C. H. Nam, and V. V. Vien, S3 flavor symmetry in 3-3-1 models, Phys. Rev. D 85, 053001 (2012); S. M. Boucenna,
S. Morisi, and J. W. F. Valle, *Radiative neutrino mass in 3-3-1 scheme*, Phys. Rev. D 90, 013005 (2014).

[8] D. Fregolente and M. D. Tonasse, *Selfinteracting dark matter from an SU(3)(L) × U(1)(N) electroweak model*, Phys. Lett. B 555, 7 (2003); H. N. Long and N. Q. Lan, *Self-interacting dark matter and Higgs bosons in the SU(3)C × SU(3)L × U(1)N model with right-handed neutrinos*, Europhys. Lett. 64, 571 (2003); S. Filippi, W. A. Ponce, and L. A. Sanches, *Dark matter from the scalar sector of 3-3-1 models without exotic electric charges*, Europhys. Lett. 73, 142 (2006); C. A. de S. Pires and P. S. Rodrigues da Silva, *Scalar Bilepton Dark Matter*, JCAP 0712, 012 (2007); J. K. Mizukoshi, C. A. de S. Pires, F. S. Queiroz, and P. S. Rodrigues da Silva, *WIMPs in a 3-3-1 model with heavy Sterile neutrinos*, Phys. Rev. D 83, 065024 (2011); J. D. Ruiz-Alvarez, C. A. de S. Pires, F. S. Queiroz, D. Restrepo, and P. S. Rodrigues da Silva, *On the Connection of Gamma-Rays, Dark Matter and Higgs Searches at LHC*, Phys. Rev. D 86, 075011 (2012); S. Profumo and F. S. Queiroz, *Constraining the Z mass in 331 models using direct dark matter detection*, Eur. Phys. J. C 74, 2960 (2014); C. Kelso, C. A. de S. Pires, S. Profumo, F. S. Queiroz, and P. S. Rodrigues da Silva, *A 331 WIMPy Dark Radiation Model*, Eur. Phys. J. C 74, 2797 (2014); P. S. Rodrigues da Silva, *A Brief Review on WIMPs in 331 Electroweak Gauge Models*, arXiv:1412.8633 [hep-ph].

[9] P. V. Dong, T. D. Tham, and H. T. Hung, *3-3-1-1 model for dark matter*, Phys. Rev. D 87, 115003 (2013); P. V. Dong, D. T. Huong, F. S. Queiroz, and N. T. Thuy, *Phenomenology of the 3-3-1-1 model*, Phys. Rev. D 90, 075021 (2014); D. T. Huong, P. V. Dong, C. S. Kim, and N. T. Thuy, *Inflation and leptogenesis in the 3-3-1-1 model*, arXiv:1501.00543 [hep-ph].

[10] N. G. Deshpande and E. Ma, *Pattern of symmetry breaking with two Higgs doublets*, Phys. Rev. D 18, 2574 (1978).

[11] P. V. Dong, N. T. K. Ngan, and D. V. Soa, *Simple 3-3-1 model and implication for dark matter*, Phys. Rev. D 90, 075019 (2014).

[12] P. V. Dong, T. Phong Nguyen, and D. V. Soa, *3-3-1 model with inert scalar triplet*, Phys. Rev. D 88, 095014 (2013).

[13] J. G. Ferreira Jr, P. R. D. Pinheiro, C. A. de S. Pires, and P. S. Rodrigues da Silva, *Minimal 3-3-1 model with only two Higgs triplets*, Phys. Rev. D 84, 095019 (2011).

[14] P. V. Dong and H. N. Long, *U(1)Q invariance and SU(3)C ⊗ SU(3)L ⊗ U(1)X models with beta arbitrary*, Eur. Phys. J. C 42, 325 (2005).

[15] G. Belanger, F. Bouclijema, P. Brun, A. Pukhov, S. Rosier-Lees, P. Salati, and A. Semenov, *Indirect search for dark matter with micrOMEGAs2.4*, Comput. Phys. Commun. 182, 842 (2011) [arXiv:1004.1092 [hep-ph]].

[16] G. Belanger, F. Bouclijema, A. Pukhov, and A. Semenov, *Dark matter direct detection rate in a generic model with micrOMEGAs2.2*, Comput. Phys. Commun. 180, 747 (2009) [arXiv:0803.2360 [hep-ph]].

[17] A. Pukhov, *CalcHEP 2.3: MSSM, structure functions, event generation, batchs, and generation of matrix elements for other packages*, arXiv:hep-ph/0412191.

[18] Kim Griest and David Seckel, *Three exceptions in the calculation of relic abundances*, Phys.Rev. D 43, 3191 (1991).

[19] M. M. Pavan, I. I. Strakovsky, R. L. Workman, and R. A. Arndt, *The pion-nucleon Sigma term is definitely large: results from a G.W.U. analysis of pion nucleon scattering data*, PiN Newslett. 16 (2002) 110.115 [arXiv:hep-ph/0111066].
[20] E. Komatsu, K. M. Smith, J. Dunkley, C. L. Bennett, B. Gold, G. Hinshaw, N. Jarosik, D. Larson, M. R. Nolta, L. Page, D. N. Spergel, M. Halpern, R. S. Hill, A. Kogut, M. Limon, S. S. Meyer, N. Odegard, G. S. Tucker, J. L. Weiland, E. Wollack, and E. L. Wright, 
*Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation*, Astrophys. J. Suppl. 192, 14, (2011) [arXiv:1001.4538 [astro-ph.CO]].

[21] J. Ellis, A. Ferstl, and K. A. Olive, *Re-evaluation of the elastic scattering of supersymmetric dark matter*, Phys. Lett. B 481, 304 (2000).

[22] J. Beringer et al. (Particle Data Group), *Review of Particle Physics*, Phys. Rev. D 86, 010001 (2012).