\[ \Theta^+ \text{ baryon, } N^*(1685) \text{ resonance, and } \pi N \text{ sigma term reexamined within the framework of a chiral soliton model} \]

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We reexamine the properties of the baryon antidecuplet \(\Theta^+\) and \(N^*\), and the \(\pi N\) sigma term within the framework of a chiral soliton model. It turns out that the measured value of the \(N^*\) mass, \(M_{N^*} = 1686\) MeV, is consistent with that of the \(\Theta^+\) mass \(M_{\Theta^+} = 1524\) MeV by the LEPS collaboration [T. Nakano et al. [LEPS Collaboration], Phys. Rev. C 79, 025210 (2009)]. The \(N^* \rightarrow N \gamma\) magnetic transition moments are almost independent of the \(\Theta^+\) mass. The ratio of the radiative decay width \(\Gamma_{n\pi^*}\) to \(\Gamma_{p\rho^*}\) turns out to be around 5. The decay width for \(\Theta^+ \rightarrow NK\) is studied in the context of the LEPS and DIANA experiments. When the LEPS value of the \(\Theta^+\) mass is employed, we obtain \(\Gamma_{\Theta^+NK} = (0.5 \pm 0.1)\) MeV. The \(\pi N\) sigma term is found to be almost independent of the \(\Theta^+\) mass. In addition, we derive a new expression for the \(\pi N\) sigma term in terms of the isospin mass splittings of the hyperon octet as well as that of the antidecuplet \(N^*\).

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1. Introduction

The baryon antidecuplet is the first excitation consisting of exotic pentaquark baryons [1–3]. Since the LEPS collaboration reported the first measurement of the pentaquark baryon \(\Theta^+\) [4], pentaquark baryons attracted a great deal of attention, before a series of CLAS experiments announced null results for \(\Theta^+\) [5–8], which cast doubt on the existence of pentaquarks [9,10]. On the other hand, the DIANA collaboration has continued searching for \(\Theta^+\) [11,12] and observed the formation of a narrow \(pK^0\) peak with a mass of \(1538 \pm 2\) MeV/c\(^2\) and a width of \(\Gamma = 0.39 \pm 0.10\) MeV in the \(K^+n \rightarrow K^0p\) reaction with higher statistical significance \((6\sigma \sim 8\sigma)\) [12]. The decay width was more precisely measured in comparison with the former DIANA measurement [13], the statistics being doubled. The SVD experiment has also found a narrow peak with the mass \(1523 \pm 2\) stat. \(\pm 3\) syst. MeV in the inclusive reaction \(pA \rightarrow pK^0 + X\) [14,15]. In 2009, the LEPS collaboration again announced evidence for \(\Theta^+\) [16]: The mass of \(\Theta^+\) was found to be \(M_{\Theta^+} = 1524 \pm 2 \pm 3\) MeV/c\(^2\) and the statistical significance of the peak turned out to be \(5.1\) \(\sigma\). The differential cross section was estimated to be \((12 \pm 2)\) nb/sr in the photon energy ranging from 2.0 GeV to 2.4 GeV in the LEPS angular range. While the statistics of the new LEPS data has been improved by a factor of 8 over the previous measurement [4], Ref. [17] has raised doubts about the \(\Theta^+\) peak found by the LEPS.
collaboration. Very recently, Amaryan et al. reported a narrow structure around 1.54 GeV in the process $\gamma + p \rightarrow p K^0 K_L$ via interference with $\phi$-meson production with the statistical significance 5.9 $\sigma$ [18], though the CLAS collaboration has not officially approved their analysis [19].

In addition to the $\Theta^+$ baryon, Kuznetsov et al. [20] have observed a new nucleon-like resonance around 1.67 GeV from $\eta$ photoproduction off the deuteron in the neutron channel. The decay width was measured to be around 40 MeV, the effects of the Fermi motion being not excluded [21]. On the other hand, this narrow resonant structure was not seen in the quasi-free proton channel [20]. The findings of Ref. [20] are consistent with the theoretical predictions [22, 23] of non-strange exotic baryons. Moreover, the narrow width and isospin asymmetry in the initial states, also called the neutron anomaly [24], are typical characteristics for photoexcitation of the non-strange antidecuplet baryons [25, 26]. New analyses of the free proton GRAAL data [24, 27–30] have revealed a resonance structure with a mass around 1685 MeV and width $\Gamma \leq 15$ MeV, though the data of Ref. [31] do not agree with those of Ref. [27]. For a detailed discussion of this discrepancy, we refer to Ref. [28]. The CB-ELSA collaboration [32–34] has also confirmed evidence for this $N^*$ resonance in line with that of GRAAL. Very recently, Kuznetsov and Polyakov have extracted a new result for the narrow peak: $M_{N^*} = 1686 \pm 7 \pm 5$ MeV with the decay width $\Gamma \approx 28 \pm 12$ MeV [35]. All these experimental facts are compatible with the results for transition magnetic moments in the chiral quark–soliton model ($\chi$QSM) [25, 26] and the phenomenological analysis of non-strange pentaquark baryons [36]. The $\gamma N \rightarrow \eta N$ reaction was studied within an effective Lagrangian approach [37, 38] that has described qualitatively well the GRAAL data. The present status of $N^*(1685)$ is summarized in Ref. [39], in which the reason why $N^*(1685)$ can be most probably identified as a baryon antidecuplet member was discussed in detail.

In the present work, we want to examine the relation between the $\Theta^+$ mass and other observables such as the mass of the $N^*$ ($M_{N^*}$), $N^* \rightarrow N \gamma$ transition magnetic moments ($\mu_{N N^*}$), the decay width of $\Theta^+$ ($\Gamma_{\Theta^+}$), and the $\pi N$ sigma term ($\sigma_{\pi N}$), in the context of the LEPS and DIANA experiments. In particular, we will regard the $N^*(1685)$ resonance with the narrow width as a member of the antidecuplet in this work. The mass splittings of the SU(3) baryons within a chiral soliton model ($\chi$SM) were reinvestigated with all parameters fixed unequivocally [40]. Since the mass of $\Theta^+$ observed by the LEPS collaboration is different from that observed by the DIANA collaboration, it is of great importance to examine carefully the relevance of the analysis in Ref. [40] with regard to the LEPS and DIANA experiments. We will show in this work that the decay width $\Gamma_{\Theta}$ obtained from the $\chi$SM [40] is consistent with these two experiments. We will also study the dependence of the $N^*$ mass on $M_{\Theta^+}$, which turns out to be compatible with the LEPS data. In addition, we also investigate the dependence of the $N^* \rightarrow N$ magnetic transition moment on $M_{\Theta^+}$, which is shown to be almost insensitive to the $\Theta^+$ mass. Finally, $\sigma_{\pi N}$ will be examined; this becomes one of the essential quantities in the physics of dark matter [41, 42]. Motivated by its relevance in dark matter, a great amount of effort has gone into the evaluation of $\sigma_{\pi N}$. For example, there are now various results from lattice QCD [43–46]. However, the value of $\sigma_{\pi N}$ still does not converge, but is known only with a wide range of uncertainties: 35–75 MeV. Thus, we will discuss $\sigma_{\pi N}$ in connection with the baryon antidecuplet and will show that it is rather stable with respect to the $\Theta^+$ mass. Moreover, its predicted value is smaller than that used in previous analyses [2, 47, 48].

The present work is organized as follows: In Sect. 2, the pertinent formulae for the baryon antidecuplet within a chiral soliton model are compiled. In Sect. 3, we discuss the results. The final section is devoted to a summary and a conclusion.
2. Baryon antidecuplet from a chiral soliton model

We first recapitulate briefly the formulae of mass splittings, magnetic moments, and axial-vector constants within the framework of the $\chi$SM. We begin with the collective Hamiltonian of chiral solitons, which have been thoroughly studied within various versions of the $\chi$SM, such as the chiral quark–soliton model [49,50], the Skyrme model [51], and the chiral hyperbag model [52]. The most general form of the collective Hamiltonian in the SU(3) $\chi$SM can be written as follows:

$$H = M_{cl} + H_{rot} + H_{sb},$$

(2.1)

where $M_{cl}$ denotes the classical soliton mass. $H_{rot}$ and $H_{sb}$ respectively stand for the $1/N_c$ rotational and symmetry-breaking corrections with the effects of isospin and SU(3) flavor symmetry breakings included [53]:

$$H_{rot} = \frac{1}{2I_1} \sum_{i=1}^{3} \hat{J}_i^2 + \frac{1}{2I_2} \sum_{p=4}^{7} \hat{J}_p^2,$$

(2.2)

$$H_{sb} = (m_d - m_u)(\sqrt{3/2} \alpha D_{38}^{(8)}(A) + \beta \hat{T}_3 + \frac{1}{2}\gamma \sum_{i=1}^{3} D_{3i}^{(8)}(A) \hat{J}_i)
+ (m_s - \bar{m})(\alpha D_{88}^{(8)}(A) + \beta \hat{Y} + \frac{1}{\sqrt{3}}\gamma \sum_{i=1}^{3} D_{8i}^{(8)}(A) \hat{J}_i)
- (m_u + m_d + m_s)(\alpha + \beta),$$

(2.3)

where $I_{1,2}$ represent the soliton moments of inertia that depend on the dynamics of specific formulations of the $\chi$SM. However, they will be determined by the masses of the baryon octet, $\Omega$ mass, and $\Theta^+$ mass. The $J_i$ denote the generators of the SU(3) group. The $m_u$, $m_d$, and $m_s$ designate the up, down, and strange current quark masses, respectively. The $\bar{m}$ is the average of the up and down quark masses. We want to mention that we do not need to know each current quark mass separately but only the ratio of them, i.e., $m_s/\bar{m}$, to determine the sigma $\pi N$ term. The $D_{ab}^{(7)}(A)$ indicate the SU(3) Wigner D functions. The $\hat{Y}$ and $\hat{T}_3$ are the operators of the hypercharge and isospin third component, respectively. The $\alpha$, $\beta$, and $\gamma$ are given in terms of the $\sigma_{\pi N}$ and soliton moments of inertia $I_{1,2}$ and $K_{1,2}$ as follows:

$$\alpha = -\left(\frac{\sigma_{\pi N}}{3\bar{m}} - \frac{K_2}{I_2}\right), \quad \beta = -\frac{K_2}{I_2}, \quad \gamma = 2\left(\frac{K_1}{I_1} - \frac{K_2}{I_2}\right).$$

(2.4)

Since $\alpha$, $\beta$, and $\gamma$ depend on the moments of inertia and $\sigma_{\pi N}$, they are also related to the details of the specific dynamics of the $\chi$SM. Note that $\alpha$, $\beta$, and $\gamma$ defined in the present work do not contain the strange quark mass, while those in Refs. [2,47] include it.

In the $\chi$SM, we have the following constraint for $J_8$:

$$J_8 = -\frac{N_c}{2\sqrt{3}} B = -\frac{\sqrt{3}}{2}, \quad \gamma' = \frac{2}{\sqrt{3}} J_8 = -\frac{N_c}{3} = -1,$$

(2.5)

where $B$ represents the baryon number. It is related to the eighth component of the soliton angular velocity, which is due to the presence of the discrete valence quark level in the Dirac-sea spectrum in the SU(3) $\chi$QSM [49,54], while it arises from the Wess–Zumino term in the SU(3) Skyrme model [55–57]. Its presence has no effect on the chiral soliton but allows us to take only the SU(3) irreducible representations with zero triality. Thus, the allowed SU(3) multiplets are the baryon octet ($J = 1/2$), decuplet ($J = 3/2$), and antidecuplet ($J = 1/2$), etc.
The baryon collective wave functions of \( H \) are written as SU(3) Wigner \( D \) functions in the representation \( \mathcal{R} \):

\[
\langle A|\mathcal{R}, B(Y T T_3, Y' J J_3) \rangle = \psi^{(\mathcal{R}'; YT T_3)}_{(\mathcal{R}^*; Y' J J_3)}(A) \\
= \sqrt{\text{dim}(\mathcal{R})}(-)^{J_3+Y'/2} D^{(\mathcal{R}^*)}_{(Y, T, T_3)(-Y', J, -J_3)}(A),
\]

where \( \mathcal{R} \) stands for the allowed irreducible representations of the SU(3) group, i.e. \( \mathcal{R} = 8, 10, \overline{10}, \ldots \) and \( Y, T, T_3 \) are the corresponding hypercharge, isospin, and its third component, respectively. The constraint of the right hypercharge \( Y' = 1 \) selects a tower of allowed SU(3) representations: the lowest ones, i.e., the baryon octet and decuplet, coincide with those of the quark model. This has been considered as a success of collective quantization and as a sign of certain duality between a rigidly rotating heavy soliton and a constituent quark model. The third lowest representation is the antidecuplet [2], which includes the \( \Theta^+ \) and \( N^* \) baryons.

Different SU(3) representations get mixed in the presence of the symmetry-breaking term \( H_{sb} \) of the collective Hamiltonian in Eq. (2.3), so that the collective wave functions are no longer in pure states but are given as the following linear combinations [49,58]:

\[
\begin{align*}
|B_8\rangle &= |8_{1/2}, B\rangle + e_{10}^B |10_{1/2}, B\rangle + c_{27}^B |27_{1/2}, B\rangle, \\
|B_{10}\rangle &= |10_{3/2}, B\rangle + a_{27}^B |27_{3/2}, B\rangle + a_{35}^B |35_{3/2}, B\rangle, \\
|B_{\overline{10}}\rangle &= |\overline{10}_{1/2}, B\rangle + d_{27}^B |27_{1/2}, B\rangle + d_{35}^B |35_{1/2}, B\rangle. 
\end{align*}
\]

Detailed expressions for the coefficients in Eq. (2.7) can be found in Refs. [47,49].

Since we take into account the effects of isospin symmetry breaking, we also have to introduce the EM mass corrections to the mass splitting of the SU(3) baryons, which are equally important. The EM corrections to the baryon masses can be derived from the baryonic two-point correlation functions. The corresponding collective operator has already been derived in Ref. [59]:

\[
M_{\mu}^{EM} = \langle B|\mathcal{O}^{EM}|B\rangle,
\]

where

\[
\mathcal{O}^{EM} = c^{(1)} D^{(1)}_{\Lambda \Lambda} + c^{(8)} (\sqrt{3} D^{(8)}_{\Sigma^0 \Lambda} + D^{(8)}_{\Lambda \Lambda}) + c^{(27)} (\sqrt{3} D^{(27)}_{\Sigma^0 \Lambda_{27}} + \sqrt{3} D^{(27)}_{\Sigma^0 \Lambda_{27}} + D^{(27)}_{\Lambda_{27} \Lambda_{27}}).
\]

The unknown parameters \( c^{(8)} \) and \( c^{(27)} \) are determined by the experimental data for the EM mass splittings of the baryon octet, while \( c^{(1)} \) can be absorbed in the center of baryon masses. The values of \( c^{(8)} \) and \( c^{(27)} \) have been obtained as

\[
c^{(8)} = -0.15 \pm 0.23, \quad c^{(27)} = 8.62 \pm 2.39
\]

in units of MeV [59].

The final expressions for the masses of \( \Theta^+ \) and \( N^* \) are given as

\[
\begin{align*}
M_{\Theta^+} &= M_{\overline{10}} + \frac{1}{4} \left( c^{(8)} - \frac{4}{21} c^{(27)} \right) - 2(m_s - \bar{m})\delta, \\
M_{N^*} &= M_{\overline{10}} + \frac{1}{4} \left( c^{(8)} - \frac{32}{63} c^{(27)} \right) T_3 + \frac{1}{4} \left( c^{(8)} + \frac{8}{63} c^{(27)} \right) \left( T_3^2 + \frac{1}{4} \right) \\
&\quad - (m_d - m_u) T_3 \delta - (m_s - \bar{m})\delta.
\end{align*}
\]
where $\overline{M}_{10}$ denotes the center of the mass splittings of the baryon antidecuplet and $\delta$ is a parameter defined as
\[
\delta = -\frac{1}{8} \alpha - \beta + \frac{1}{16} \gamma.
\] (2.12)

The collective operators for the magnetic moments and axial-vector constants can respectively be parameterized by six parameters that can be treated as free [60–62]:
\[
\begin{align*}
\hat{\mu} &= w_1 D^{(8)}_{X3} + w_2 dpq D^{(8)}_{Xp} \cdot \hat{J}_q + \frac{w_3}{\sqrt{3}} D^{(8)}_{X8} \hat{f}_3 \\
&\quad + \frac{u_4}{\sqrt{3}} dpq D^{(8)}_{Xp} D^{(8)}_{8q} + w_5 (D^{(8)}_{X3} D^{(8)}_{88} + D^{(8)}_{X8} D^{(8)}_{83}) + w_6 (D^{(8)}_{X3} D^{(8)}_{88} - D^{(8)}_{X8} D^{(8)}_{83}), \\
\hat{g}_A &= a_1 D^{(8)}_{X3} + a_2 dpq D^{(8)}_{Xp} \hat{J}_q + \frac{a_3}{\sqrt{3}} D^{(8)}_{X8} \hat{f}_3 \\
&\quad + \frac{u_4}{\sqrt{3}} dpq D^{(8)}_{Xp} D^{(8)}_{8q} + a_5 (D^{(8)}_{X3} D^{(8)}_{88} + D^{(8)}_{X8} D^{(8)}_{83}) + a_6 (D^{(8)}_{X3} D^{(8)}_{88} - D^{(8)}_{X8} D^{(8)}_{83}).
\end{align*}
\] (2.13)

where $\hat{J}_q$ ($\hat{J}_3$) stand for the $q$th (third) component of the spin operator of the baryons. The parameters $w_i$ and $a_i$ can be unambiguously fixed by using the magnetic moments and semileptonic decay constants of the baryon octet (Ref. [63] and G.-S. Yang and H.-Ch. Kim, manuscript in preparation). We refer to Ref. [63] for detailed expressions for the $N^* \to N$ transition magnetic moments and for the $\Theta^+$ magnetic moment and axial-vector constants for $\Theta \to K N$ decay.

3. Results and discussion

In order to find the masses of the baryon antidecuplet, we need to fix the relevant parameters. There are several ways to fix them. For example, Diakonov et al. [2] use the mass splittings of the baryon octet and decuplet, $\pi N$ sigma term, and the $N^*$ mass that was then taken to be around 1710 MeV. The $\pi N$ sigma term was taken from Ref. [64], i.e. $\sigma_{\pi N} \approx 45$ MeV. In addition, the ratio of the current quark mass $m_s / (m_u + m_d) \approx 12.5$ was quoted from Ref. [65] to determine the parameters:
\[
m_s \alpha \approx -218 \text{ MeV}, \quad m_s \beta \approx -156 \text{ MeV}, \quad m_s \gamma \approx -107 \text{ MeV}.
\] (3.1)

On the other hand, Ellis et al. [47] carried out the analysis for the mass splittings of the baryon antidecuplet, based on the experimental data of the $\Theta^+$ and $\Xi^-$ masses together with those of the baryon octet and decuplet. They predicted the $\pi N$ sigma term $\sigma_{\pi N} = 73$ MeV from the fitted values of the parameters:
\[
I_2 = 0.49 \text{ fm}, \quad m_s \alpha = -605 \text{ MeV}, \quad m_s \beta = -23 \text{ MeV}, \quad m_s \gamma = 152 \text{ MeV}.
\] (3.2)

Very recently, Ref. [40] reanalyzed the mass splittings of the SU(3) baryons within a $\chi$SM, employing isospin symmetry breaking. An obvious advantage of including the effects of isospin symmetry breaking is that one can fully utilize the whole experimental data of the octet masses to fix the parameters. Using the baryon octet masses, the $\Omega^-$ mass ($1672.45 \pm 0.29$) MeV [66] and the $\Theta^+$ mass ($1524 \pm 5$) MeV [16], both of which are isosinglet baryons, the key parameters were found to be:
\[
I_2 = (0.420 \pm 0.006) \text{ fm}, \quad m_s \alpha = (-262.9 \pm 5.9) \text{ MeV},
\]
\[
m_s \beta = (-144.3 \pm 3.2) \text{ MeV}, \quad m_s \gamma = (-104.2 \pm 2.4) \text{ MeV}.
\] (3.3)

In addition, the $\pi N$ sigma term was predicted as $\sigma_{\pi N} = (36.4 \pm 3.9)$ MeV. Since $\delta$ is defined in Eq. (2.12), let us compare its values from each piece of work mentioned above. The corresponding
The dependence of the $N^*$ mass on $M_{\Theta^+}$. The vertical shaded bars bounded with solid and dashed lines denote the measured values of the $\Theta^+$ mass with uncertainties from the LEPS and DIANA collaborations, respectively. The horizontal shaded region shows the values of the $N^*$ mass with the uncertainty taken from Ref. [24]. The sloping shaded region represents the present results of the $M_{\Theta^+}$ dependence of the $N^*$ mass.

The predicted masses of the $N^*(1685)$ resonance from the previous analyses deviate from the experimental data. Moreover, it is essential to take into account the effects of isospin symmetry breaking, in order to produce the mass of the $N^*$ resonance quantitatively [40]. Since there are, however, two different experimental values for the $\Theta^+$ mass from the LEPS and DIANA collaborations, it is necessary to examine carefully the dependence of the relevant observables on that of the $\Theta^+$ baryon rather than choosing one specific value of $M_{\Theta^+}$ to fit the parameters. Thus, in the present section, we discuss the dependence of relevant observables on $M_{\Theta^+}$, taking it as a free parameter.

In Fig. 1, we show the $N^*$ mass as a function of $M_{\Theta^+}$. The vertical shaded bars bounded with solid and dashed lines denote the measured values of the $\Theta^+$ mass with uncertainties from the LEPS and DIANA collaborations, respectively. The horizontal shaded region denotes the values of the $N^*$ mass with an uncertainty taken from Ref. [24]. The sloping shaded region shows the

\[
\begin{align*}
\text{results are given, respectively, as follows:} \\
&\quad m_s \delta = 177 \text{ MeV (Diakonov et al.)}, \quad m_s \delta = 108 \text{ MeV (Ellis et al.)}, \\
&\quad m_s \delta = 171 \text{ MeV (present work)}, \\
&\quad (3.4) \\
\text{with isospin symmetry breaking switched off. If we use the LEPS experimental data [16] for } M_{\Theta^+}, \text{ we can immediately obtain the corresponding masses for } N^*, \text{ respectively:} \\
&\quad M_{N^*} = 1700 \text{ MeV (Diakonov et al.)}, \quad M_{N^*} = 1631 \text{ MeV (Ellis et al.)}, \\
&\quad M_{N^*} = 1694 \text{ MeV (present work)}. \\
&\quad (3.5) \\
\text{If one employs the DIANA data [12], the } N^* \text{ mass is given as} \\
&\quad M_{N^*} = 1715 \text{ MeV (Diakonov et al.)}, \quad M_{N^*} = 1646 \text{ MeV (Ellis et al.)}, \\
&\quad M_{N^*} = 1708 \text{ MeV (present work)}. \\
&\quad (3.6) \\
\text{The comparison made above already indicates that the predicted masses of the } N^*(1685) \text{ resonance from the previous analyses deviate from the experimental data. Moreover, it is essential to take into account the effects of isospin symmetry breaking, in order to produce the mass of the } N^* \text{ resonance quantitatively [40]. Since there are, however, two different experimental values for the } \Theta^+ \text{ mass from the LEPS and DIANA collaborations, it is necessary to examine carefully the dependence of the relevant observables on that of the } \Theta^+ \text{ baryon rather than choosing one specific value of } M_{\Theta^+} \text{ to fit the parameters. Thus, in the present section, we discuss the dependence of relevant observables on } M_{\Theta^+}, \text{ taking it as a free parameter.} \\
\end{align*}
\]
dependence of the $N^*$ mass on $M_{\Theta^+}$. The $N^*$ mass increases monotonically as $M_{\Theta^+}$ increases. This behavior can be easily understood from Eq. (2.11): the mass of the $N^*$ resonance depends linearly on the parameter $\delta$. Interestingly, if we take the $M_{\Theta^+}$ value of the LEPS experiment, i.e., $M_{\Theta^+} = 1524$ MeV, we obtain $M_{N^*} \approx 1690$ MeV, which is in good agreement with the experimental data: $M_{N^*} = (1685 \pm 12)$ MeV [24]. On the other hand, if we use the value of $M_{\Theta^+}$ measured by the DIANA collaboration, the $N^*$ mass turns out to be larger than 1690 MeV. This implies that the $\Theta^+$ mass reported by the LEPS collaboration [16] is consistent with that of $N^*(1685)$ from recent experiments [20,32–35]—at least, in the present framework of a $\chi$SM with isospin symmetry breaking [40].

The parameters $w_i$ in Eq. (2.13) can be fitted by the magnetic moments of the baryon octet [60–62]. However, since the mixing coefficients appearing in Eq. (2.7) depend explicitly on $\alpha$ and $\gamma$, the parameters $w_i$ are also given as functions of $\sigma_{\pi N}$ through $\alpha$ and $\gamma$, as shown in Ref. [26]. As previously mentioned, since the mass parameters $\alpha$ and $\gamma$ as well as $\sigma_{\pi N}$ were unambiguously fixed in Ref. [40], we can derive the transition magnetic moments for the $N^* \to N\gamma$ decay unequivocally. Explicitly, the transition magnetic moments $\mu_{pp^*}$ and $\mu_{nn^*}$ are recapitulated, respectively, as follows [26]:

\begin{align}
\mu_{pp^*}^{(0)} &= 0,
\mu_{pp^*}^{(op)} &= -\frac{1}{27\sqrt{3}} w_4 - \frac{1}{18\sqrt{5}} \left( \frac{w_5 + 3}{2} w_6 \right),
\mu_{pp^*}^{(wf)} &= -\frac{5}{24\sqrt{3}} \left( w_1 + \frac{5}{2} w_2 - \frac{1}{2} w_3 \right) c_{12} - \frac{35}{72\sqrt{5}} \left( w_1 - \frac{11}{14} w_2 - \frac{3}{14} w_3 \right) c_{27}
+ \left[ \frac{1}{2\sqrt{5}} \left( w_1 - \frac{1}{2} w_2 + \frac{1}{6} w_3 \right) - \frac{7}{6\sqrt{5}} \left( w_1 - \frac{1}{2} w_2 - \frac{1}{14} w_3 \right) \right] d_8
+ \frac{1}{45\sqrt{5}} \left( w_1 + 2 w_2 - \frac{3}{2} w_3 \right) d_{27},
\mu_{nn^*}^{(0)} &= \frac{1}{6\sqrt{3}} \left( w_1 + w_2 + \frac{w_3}{2} \right),
\mu_{nn^*}^{(op)} &= -\frac{1}{54\sqrt{5}} w_4 + \frac{1}{18\sqrt{5}} \left( w_5 + \frac{3}{2} w_6 \right),
\mu_{nn^*}^{(wf)} &= \frac{7}{36\sqrt{5}} \left( w_1 - \frac{11}{14} w_2 - \frac{3}{14} w_3 \right) c_{27} + \frac{1}{2\sqrt{5}} \left( w_1 - \frac{1}{2} w_2 + \frac{1}{6} w_3 \right) d_8
- \frac{1}{90\sqrt{5}} \left( w_1 + 2 w_2 - \frac{3}{2} w_3 \right) d_{27}.
\end{align}

As already discussed in Ref. [26], $\mu_{pp^*}$ vanishes in the SU(3) symmetric case. Thus, $\mu_{pp^*}$ is only finite with the effects of SU(3) symmetry breaking included.

Figure 2 shows the transition magnetic moments for the $N^* \to N\gamma$ decay as functions of $M_{\Theta^+}$. While $\mu_{pp^*}$ is almost independent of $M_{\Theta^+}$, $\mu_{nn^*}$ decreases slowly as $M_{\Theta^+}$ increases. On the other hand, the magnitude of $\mu_{nn^*}$ turns out to be larger than that of $\mu_{pp^*}$, as has already been pointed out in Refs. [25,26,36].

In Table 1, we list each contribution to $\mu_{NN^*}$ as well as the radiativedecay widths for $N^* \to N\gamma$ using the mass of $\Theta^+$ from the LEPS experiment. Note that the sign of $\mu_{nn^*}$ is negative whereas
Fig. 2. The dependence of the transition magnetic moments for the $N^* \rightarrow N\gamma$ decay on $M_{\Theta^+}$. The vertical shaded bars bounded with solid and dashed lines denote the measured values of the $\Theta^+$ mass with uncertainties from the LEPS and DIANA collaborations, respectively. The horizontal shaded regions stand for the present results of the $M_{\Theta^+}$ dependence of the transition magnetic moments $\mu_{p^*p}$ and $\mu_{n^*n}$.

Table 1. The results of the $N^* \rightarrow N$ transition magnetic moments in units of the nuclear magneton $\mu_N$ and of the radiative decay widths in units of keV. The mass $M_{\Theta^+} = (1524 \pm 5)$ MeV is used as an input.

| $\mu_{NN^*}$ | $\mu^{(0)}_{NN^*}$ | $\mu^{(op)}_{NN^*}$ | $\mu^{(wi)}_{NN^*}$ | $\mu^{(total)}_{NN^*}$ | $\Gamma_{NN^*}$ (keV) |
|--------------|----------------------|----------------------|----------------------|-------------------------|----------------------|
| $\mu_{pp^*}$ | 0                    | 0.272 ± 0.051        | -0.125 ± 0.013       | 0.146 ± 0.053           | 17.7 ± 3.2           |
| $\mu_{nn^*}$ | -0.252 ± 0.077       | -0.159 ± 0.042       | 0.107 ± 0.003        | -0.304 ± 0.089          | 77.1 ± 11.3          |

that of $\mu_{pp^*}$ is positive. However, the previous result for $\mu_{nn^*}$ was positive [26]. The reason for this can be found in the different values of $w_i$. Let us compare closely the present results with those of Ref. [26], considering only the SU(3) symmetric part without loss of generality. In fact, $w_i$ derived in Ref. [26] depends on $\sigma_{\pi N}$:

$$w_1^{\text{old}} = -3.736 - 0.107 \sigma_{\pi N}, \quad w_2^{\text{old}} = 24.37 - 0.21 \sigma_{\pi N}, \quad w_3^{\text{old}} = 7.547.$$  (3.8)

If one takes the value of the $\pi N$ sigma terms as $\sigma_{\pi N} \approx 40$ MeV (70 MeV), one gets

$$w_1^{\text{old}} = -8.14(-11.44), \quad w_2^{\text{old}} = 15.97(9.67), \quad w_3^{\text{old}} = 7.547,$$  (3.9)

while the results in this work use the newly obtained values of $w_i$ (G.-S. Yang and H.-Ch. Kim, manuscript in preparation):

$$w_1 = -12.95 \pm 0.10, \quad w_2 = 5.388 \pm 0.933, \quad w_3 = 8.354 \pm 0.861.$$  (3.10)

Thus, the magnitude of the present $w_1$ is larger than those of $w_1^{\text{old}}$, whereas that of $w_2$ turns out to be smaller than those of $w_2^{\text{old}}$. Since $w_1$ and $w_2$ have different signs, as in Eq. (3.7), they destructively interfere each other, so that the sign of $\mu_{nn^*}$ becomes negative in the present case but is positive in Ref. [26]. However, the magnetic properties of the octet and decuplet baryons are almost intact because of the constructive interference of $w_1$ and $w_2$, even though we have different
Fig. 3. The dependence of the decay width $\Gamma_{N \Theta^+}$ for the $\Theta^+ \to KN$ decay on $M_{\Theta^+}$. The vertical shaded bars bounded with solid and dashed lines denote the measured values of the $\Theta^+$ mass with uncertainties from the LEPS and DIANA collaborations, respectively. The horizontal shaded region shows the values of the $N^*$ mass with an uncertainty taken from Ref. [24]. The sloping shaded region represents the present results of the $M_{\Theta^+}$ dependence of $\Gamma_{N \Theta^+}$.

The narrowness of the decay width is one of the peculiar characteristics of pentaquark baryons. For example, the decay width of $\Theta^+ \to KN$ vanishes in the nonrelativistic limit [67]. The decay width $\Gamma_{\Theta NK}$ has already been studied in chiral soliton models with SU(3) symmetry breaking taken into account. We refer to Refs. [69,70] for details. In Fig. 3, we examine the dependence of the decay width $\Gamma_{N \Theta}$ for $\Theta^+ \to KN$ on the $\Theta^+$ mass. Being different from the $N^*$ mass and the transition magnetic moments, the decay width $\Gamma_{N \Theta}$ increases almost quadratically as $M_{\Theta^+}$ increases. This can be understood from the fact that the decay width is proportional to the square of the $g_{NK\Theta^+}$ coupling constant, which depends linearly on $M_{\Theta^+}$. When the $\Theta^+$ mass is the same as the value measured by the LEPS collaboration, $\Gamma_{N \Theta}$ turns out to be about 0.5 MeV. However, at the value $M_{\Theta^+} \approx 1540$ MeV, corresponding to that of the DIANA experiment, the decay width $\Gamma_{N \Theta}$ is close to 1 MeV. We want to emphasize that the decay width of $\Theta^+$ is still below 1 MeV in the range of $M_{\Theta^+}: 1520 - 1540$ MeV. When we use the measured value of $M_{\Theta^+}$ from the LEPS collaboration, we obtain $\Gamma_{\Theta^+ \to NK} = 0.5 \pm 0.1$ MeV.

Figure 4 depicts the predicted values of the $\pi N$ sigma term as a function of the $\Theta^+$ mass. At first sight, the result is rather surprising. Firstly, it is almost insensitive to the $\Theta^+$ mass. Secondly, the value of $\sigma_{\pi N}$ is fairly small compared to those known from previous work on the baryon antidecuplet [47,48]. In order to understand the reason for this difference, we want to examine in detail the $\pi N$ sigma term in comparison with those discussed in previous work, in particular, with Ref. [48], where the $\pi N$ sigma term was extensively studied within the same framework. Since $\sigma_{\pi N}$ is expressed as

$$\sigma_{\pi N} = \bar{m} (\alpha + \beta),$$

we need to scrutinize the dependence of $\bar{m} \alpha$ and $\bar{m} \beta$ on $M_{\Theta^+}$.
Fig. 4. The dependence of $\sigma_{\pi N}$ on the $\Theta^+$ mass. The vertical shaded bars bounded with solid and dashed lines denote the measured values of the $\Theta^+$ mass with uncertainties from the LEPS and DIANA collaborations, respectively. The sloping shaded region represents the present results of the $M_{\Theta^+}$ dependence of $\sigma_{\pi N}$.

Fig. 5. The results of the parameters $-3\bar{m}\alpha$ and $-3\bar{m}\beta$ as functions of $M_{\Theta^+}$.

Figure 5 depicts the results of the parameters $-3\bar{m}\alpha$ and $-3\bar{m}\beta$ as functions of $M_{\Theta^+}$. Interestingly, while $-3\bar{m}\alpha$ increases monotonically as $M_{\Theta^+}$ increases, $-3\bar{m}\beta$ decreases at almost the same rate as $-3\bar{m}\alpha$. Consequently, $\sigma_{\pi N}$ remains rather stable. On the other hand, Schweitzer [48] expressed $\sigma_{\pi N}$ in terms of the mass splittings of each representation:

$$\frac{m_s}{m} \sigma_{\pi N} = 3(4M_{\Sigma} - 3M_{\Lambda} - M_N) + 4(M_{\Omega} - M_{\Delta}) - 4(M_{\Xi_{3/2}} - M_{\Theta^+}),$$

and determined it to be $\sigma_{\pi N} = (74 \pm 12)$ MeV, taking the experimental values of $M_{\Theta^+} = 1540$ MeV and $M_{\Xi_{3/2}} = 1862$ MeV [68] for granted at that time, and using the ratio of the current quark mass
\( m_s/m = 25.9 \). However, using the predicted value of \( M_{\Theta^+} \approx 2020 \text{ MeV} \) in Ref. [40], we get \( \sigma_{\pi N} \approx 45 \text{ MeV} \). Thus, the present result is not in contradiction with that of Ref. [48].

Taking the effects of isospin symmetry breaking into account, however, we can rewrite \( \sigma_{\pi N} \) in terms of the mass splittings of the isospin multiplets

\[
\sigma_{\pi N} = \frac{3m}{m_d - m_u} \left[ \frac{10}{3} (M_{\Sigma^0} - M_{\Sigma^+}) + \frac{5}{3} (M_{\Xi^-} - M_{\Xi^0}) - 4(M_{n^*} - M_{p^*}) \right].
\]

Plugging the ratio \((m_d - m_u)/(m_u + m_d) = 0.28 \pm 0.03 \) [71] into Eq. (3.14), considering the experimental data for the corresponding baryon octet masses [66], and using the values of \( M_{n^*} \) and \( M_{p^*} \) predicted in Ref. [40], we obtain \( \sigma_{\pi N} \approx 34 \text{ MeV} \), which is almost the same as that of Ref. [40].

4. Summary and conclusion

In the present work, we aimed to investigate various observables of the baryon antidecuplet \( \Theta^+ \) and \( N^* \), emphasizing their dependence on the \( \Theta^+ \) mass within a chiral soliton model. We utilized the mass parameters \( \alpha, \beta, \) and \( \gamma \), derived unequivocally in Ref. [40]. We first compared the present result of the \( N^* \) mass with those predicted by previous analyses [2,47]. We then examined the dependence of the \( N^* \) mass on the \( \Theta^+ \) one. We found that the measured value of the \( \Theta^+ \) mass by the LEPS collaboration turned out to be consistent with that of the \( N^* \) mass by Kuznetsov and Polyakov [24] within the present framework. We then scrutinized the transition magnetic moments of the radiative decay \( N^* \rightarrow N\gamma \). While \( \mu_{pp^*} \) is almost independent of the \( \Theta^+ \) mass, \( \mu_{nn^*} \) decreases slowly as \( M_{\Theta^+} \) increases. We also discussed the results of the \( N^* \rightarrow N\gamma \) transition magnetic moments with those of previous work. The decay width of \( \Theta^+ \) was studied and was found to be \( 0.5 \pm 0.1 \text{ MeV} \) when the LEPS data for \( M_{\Theta^+} \) were employed, which is compatible with the corresponding measured decay width from the DIANA collaboration. Finally, we analyzed the \( \pi N \) sigma term within the present framework. It turned out that \( \sigma_{\pi N} \) was almost independent of the \( \Theta^+ \) mass. We explained the reason why it was smaller than those in previous analyses, in particular, in Ref. [48]. In addition, we found a new expression for the \( \pi N \) sigma term in terms of the isospin mass splittings of the hyperon octet as well as that of the antidecuplet \( N^* \).

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References

[1] M. Praszałowicz, in Proceedings of the Workshop on Skyrmions and Anomalies, Kraków, Poland, 1987, eds. M. Jezabek and M. Praszałowicz (World Scientific, Singapore, 1987).
[2] D. Diakonov, V. Petrov, and M. V. Polyakov, Z. Phys. A 359, 305 (1997).
[3] M. Praszałowicz, Phys. Lett. B 575, 234 (2003).
[4] T. Nakano et al. [LEPS Collaboration], Phys. Rev. Lett. 91, 012002 (2003).
[5] M. Battaglieri et al. [CLAS Collaboration], Phys. Rev. Lett. 96, 042001 (2006).
[6] B. McKinnon et al. [CLAS Collaboration], Phys. Rev. Lett. 96, 212001 (2006).
[7] S. Niccolai et al. [CLAS Collaboration], Phys. Rev. Lett. 97, 032001 (2006).
[8] R. De Vita et al. [CLAS Collaboration], Phys. Rev. D 74, 032001 (2006).
[9] F. Close, Nature 435, 287 (2005).
[10] C. G. Wohl, in K. Nakamura et al. [Particle Data Group Collaboration], J. Phys. G: Nucl. Part. Phys. 37, 075021 (2010).
[11] V. V. Barmin et al. [DIANA Collaboration], Phys. At. Nucl. 70, 35 (2007).
[12] V. V. Barmin et al. [DIANA Collaboration], Phys. At. Nucl. 73, 1168 (2010).
[13] V. V. Barmin et al. [DIANA Collaboration], Phys. At. Nucl. 66, 1715 (2003).
[14] A. Aleev et al. [SVD Collaboration], arXiv:hep-ex/0509033.
[15] A. Aleev et al. [SVD Collaboration], arXiv:0803.3313 [hep-ex].
[16] T. Nakano et al. [LEPS Collaboration], Phys. Rev. C 79, 025210 (2009).
[17] A. Martinez Torres and E. Oset, Phys. Rev. C 81, 055202 (2010).
[18] M. J. Amaryan et al., Phys. Rev. C 85, 035209 (2012).
[19] M. Anghinolfi et al. [CLAS Collaboration], arXiv:1204.1105 [hep-ex].
[20] V. Kuznetsov et al. [GRAAL Collaboration], Phys. Lett. B 647, 23 (2007).
[21] A. Fix, L. Tiator, and M. V. Polyakov, Eur. Phys. J. A 32, 311 (2007).
[22] D. Diakonov and V. Petrov, Phys. Rev. D 69, 094023 (2004).
[23] R. A. Arndt, Y. I. Azimov, M. V. Polyakov, I. I. Strakovsky, and R. L. Workman, Phys. Rev. C 69, 035208 (2004).
[24] V. Kuznetsov and M. V. Polyakov, JETP Lett. 88, 347 (2008).
[25] V. V. Polyakov and A. Rathke, Eur. Phys. J. A 18, 691 (2003).
[26] H.-Ch. Kim, M. Polyakov, M. Praszałowicz, G.-S. Yang, and K. Goeke, Phys. Rev. D 71, 094023 (2005).
[27] V. Kuznetsov, M. Polyakov, T. Boiko, J. Jang, A. Kim, W. Kim, and A. Ni, arXiv:hep-ex/0703003.
[28] V. Kuznetsov et al., arXiv:0801.0778 [hep-ex].
[29] V. Kuznetsov et al., Phys. Rev. C 83, 022201 (2011).
[30] O. Bartalini et al. [GRAAL Collaboration], Acta Phys. Pol. B 39, 1949 (2008).
[31] I. Jaegle et al. [CBELSA and TAPS Collaborations], Phys. Rev. Lett. 100, 252002 (2008).
[32] I. Jaegle et al., Acta Phys. Pol. B 39, 1949 (2008).
[33] I. Jaegle et al., Eur. Phys. J. A 47, 89 (2011).
[34] V. Kuznetsov and M. V. Polyakov, AIP Conf. Proc. 1388, 284 (2011).
[35] Y. Azimov, V. Kuznetsov, M. V. Polyakov, and I. Strakovsky, Eur. Phys. J. A 25, 325 (2005).
[36] K. S. Choi, S. i. Nam, A. Hosaka, and H.-Ch. Kim, Phys. Lett. B 636, 253 (2006).
[37] K. S. Choi, S. i. Nam, A. Hosaka, and H.-Ch. Kim, J. Phys. G: Nucl. Part. Phys. 36, 015008 (2009).
[38] M. V. Polyakov, arXiv:1108.4524 [nucl-th].
[39] G.-S. Yang and H.-Ch. Kim, arXiv:1010.3792 [hep-ph].
[40] J. R. Ellis, K. A. Olive, and C. Savage, Phys. Rev. D 77, 065026 (2008).
[41] J. Ellis and K. A. Olive, arXiv:1202.3262 [hep-ph].
[42] D. B. Leinweber, A. W. Thomas, and R. D. Young, Phys. Rev. Lett. 92, 242002 (2004).
[43] H. Ohki et al., Phys. Rev. D 78, 054502 (2008).
[44] S. Durr et al., Phys. Rev. D 85, 014509 (2012).
[45] G. S. Bali et al. [QCDSF Collaboration], Phys. Rev. D 85, 054502 (2012).
[46] J. R. Ellis, M. Karliner, and M. Praszałowicz, J. High Energy Phys. 0405, 002 (2004).
[47] P. Schweitzer, Eur. Phys. J. A 22, 89 (2004).
[48] A. Blotz, D. Diakonov, K. Goeeke, N. W. Park, V. Petrov, and P. V. Pobylitsa, Nucl. Phys. A 555, 765 (1993).
[49] A. Blotz, K. Goeeke, N. W. Park, D. Diakonov, V. Petrov, and P. V. Pobylitsa, Phys. Lett. B 287, 29 (1992).
[50] H. Weigel, Lect. Notes Phys. 743, 1 (2008).
[51] B. Y. Park and M. Rho, Z. Phys. A 331, 151 (1988).
[52] A. Blotz, K. Goeeke, and M. Praszałowicz, Acta Phys. Pol. B 25, 1443 (1994).
[53] C. V. Christov et al., Prog. Part. Nucl. Phys. 37, 91 (1996).
[54] E. Witten, Nucl. Phys. B 223, 433 (1983).
[55] E. Guadagnini, Nucl. Phys. B 236, 35 (1984).
[56] S. Jain and S. R. Wadia, Nucl. Phys. B 258, 713 (1985).
[57] H.-Ch. Kim, A. Blotz, M. V. Polyakov, and K. Goeeke, Phys. Rev. D 53, 4013 (1996).
[58] A. K. Raghavan, A. R. Sood, and S. Wadia, Nucl. Phys. B 233, 433 (1983).
[59] E. Guadagnini, Nucl. Phys. B 236, 35 (1984).
[60] S. Jain and S. R. Wadia, Nucl. Phys. B 258, 713 (1985).
[61] H.-Ch. Kim, A. Blotz, M. V. Polyakov, and K. Goeeke, Phys. Rev. D 53, 4013 (1996).
[62] G.-S. Yang, H.-Ch. Kim, and M. V. Polyakov, Phys. Lett. B 695, 214 (2011).
[63] H.-Ch. Kim, M. Praszałowicz, and K. Goeeke, Phys. Rev. D 52, 2859 (1998).
[64] H.-Ch. Kim, M. Praszałowicz, M. V. Polyakov, and K. Goeeke, Phys. Rev. D 58, 114027 (1998).
[65] G.-S. Yang, *Dissertation*, Ruhr-Universität Bochum (2009).
[64] J. Gasser, H. Leutwyler, and M. E. Sainio, Phys. Lett. B 253, 252 (1991).
[65] H. Leutwyler, Phys. Lett. B 378, 313 (1996).
[66] K. Nakamura et al. [Particle Data Group], J. Phys. G 37, 075021 (2010).
[67] D. Diakonov, V. Petrov, and M. Polyakov, arXiv:hep-ph/0404212.
[68] C. Alt et al. [NA49 Collaboration], Phys. Rev. Lett. 92, 042003 (2004).
[69] G.-S. Yang, H.-Ch. Kim, and K. Goeke, Phys. Rev. D 75, 094004 (2007).
[70] T. Ledwig, H.-Ch. Kim, and K. Goeke, Phys. Rev. D 78, 054005 (2008).
[71] J. Gasser and H. Leutwyler, Phys. Rep. 87, 77 (1982).