Renormalization of Chiral-Even Twist-3 Light-cone Wave Functions for Vector Mesons in QCD

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Abstract:

We present the one-loop anomalous dimension matrices for the chiral-even twist-3 (nonsinglet) conformal operators, which govern the scale-dependence of the vector meson light-cone wave functions through the conformal expansion. It is clarified that the constraints from the charge-conjugation invariance and the chirality conservation allow only one independent anomalous dimension matrix for each conformal spin.

PACS numbers: 12.38.-t, 12.38.Bx

[Keywords: Light-cone wave function, Twist three, Chiral-even, Anomalous dimension matrix]

\textsuperscript{1}Supported in part by the Monbusho Grant-in-Aid for Scientific Research No. 09740215.
In a recent paper [1], we presented a systematic exploration on the twist-3 light-cone wave functions (distribution amplitudes) of the vector mesons. These wave functions are relevant for understanding preasymptotic corrections to various hard exclusive processes producing vector mesons in the final state [2, 3]. According to that study, two chiral-even wave functions \( g_\perp^{(v)}(u) \), \( g_\perp^{(a)}(u) \), and two chiral-odd ones \( h_\parallel^{(a)}(u) \), \( h_\parallel^{(t)}(u) \) constitute a complete set of the twist-3 quark-antiquark wave functions (with light-cone momentum fraction carried by quark). The renormalization of these wave functions has also been studied based on the (approximate) conformal invariance in massless QCD [4]: The wave function can be decomposed into “partial waves” of definite conformal spin, and the coefficients in this expansion are given as vacuum-to-meson matrix elements of the conformal operators.

Conformal symmetry allows renormalization mixing only among the conformal operators with the same conformal spin up to \( O(\alpha_s^2) \) correction. It has also been shown in [1] that the anomalous dimensions of those conformal operators can be expressed in terms of those for the nucleon’s structure functions (parton distributions) for inclusive processes. In particular, the “\( \mu^2 \)-evolution” of the chiral-odd wave functions \( h_\parallel^{(a)}(u, \mu^2) \) and \( h_\parallel^{(t)}(u, \mu^2) \) has been solved in terms of the anomalous dimension matrices for the nucleon’s chiral-odd twist-3 quark distribution functions \( e(x, \mu^2) \) and \( h_L(x, \mu^2) \) [3, 4, 7, 8]. On the other hand, the anomalous dimension matrices for the chiral-even wave functions have been only partially given in [1] based on the known results for the nucleon’s another chiral-even distribution, \( g_T(x, \mu^2) \) [9, 10].

The purpose of this letter is to complete the renormalization of the twist-3 conformal operators relevant to chiral-even wave functions \( g_\perp^{(v)}(u, \mu^2) \) and \( g_\perp^{(a)}(u, \mu^2) \). We give a convenient choice of the independent operator basis, and clarify the symmetry constraints on their renormalization mixing. We shall complete the relevant anomalous dimension matrices for all conformal spins.

Let us first introduce a twist-3 conformal operator basis relevant for the renormalization of the nonsinglet chiral-even wave functions, \( g_\perp^{(v)}(u, \mu^2) \) and \( g_\perp^{(a)}(u, \mu^2) \), and give precise formulation of our problem referring to the results of [1]. The QCD equations of motion allow us to reexpress the quark-antiquark wave functions \( g_\perp^{(v)}(u, \mu^2) \) and \( g_\perp^{(a)}(u, \mu^2) \) in terms of the twist-3 quark-antiquark-gluon wave functions. This can be performed explicitly order by order in conformal expansion. For the conformal partial wave of conformal spin \( \ell = n + 3/2 \) \((n = 2, 3, \ldots)\), two sets of twist-3 quark-antiquark-gluon conformal operators, \( \{R_{n,k}^+\} \) and \( \{R_{n,k}^-\} \), come into play:

\[
R_{n,k}^\pm = \Theta_{k,n-k-2}^V \pm \Theta_{k,n-k-2}^A + \Theta_{k,n-k-2}^A, \quad (k = 0, 1, \ldots, n - 2),
\]

(1)

\[
\Theta_{k,n-k-2}^V(0) \equiv \bar{\psi}(0)d^{n-k-2}\Delta g\bar{G}_\perp^{\perp}(0)\Delta A_\pm d^k\psi(0) + \text{(total derivatives)},
\]

(2)

\[
\Theta_{k,n-k-2}^A(0) \equiv \bar{\psi}(0)d^{n-k-2}\Delta g\bar{G}_\perp^{\perp}(0)\Delta A_\pm d^k\gamma_5\psi(0) + \text{(total derivatives)},
\]

(3)

where we introduced an auxiliary light-like vector \( \Delta^\mu \) with \( \Delta^+ = \Delta^\pm = 0 \), which guarantees the symmetrization of the \( n \) Lorentz indices and the subtraction of the trace terms, and \( d \equiv i\Delta\cdot D \) with \( D_\mu = \partial_\mu - igA_\mu \) the covariant derivative. For the purpose of renormalization,
it suffices to consider the flavor-diagonal operators as given above. The dual gluon field strength is defined as
\[ \tilde{G}_{\mu
u} = \frac{1}{2} \epsilon_{\mu
u\lambda\sigma} G^{\lambda\sigma} \] where \( \epsilon_{0123} = 1 \). (We follow the convention of [1] in this paper). In (2), ‘total derivatives’ stands for the terms consisting of
\[ (\Delta \cdot \partial)^{n-r} [\bar{\psi} d^{r-j-2} \Delta g \bar{G}^{\perp} \Delta \partial^{j} \psi], \quad (2 \leq r \leq n-1, \ 0 \leq j \leq r-2), \]
and similarly for those in (3). These total derivative terms are organized so that the operators (2) and (3) form irreducible representation of conformal spin \( j \) of the so-called collinear conformal group[4]. The complete expression for \( \{ \Theta_{V,A}^{k,n} - k - 2 \} \) is given in terms of the Appell polynomials, but it is irrelevant here. The suffix \( k \) of (1)-(3) labels many independent conformal operators having the same conformal spin, illustrating that three-particle representations of conformal group are degenerate; the number of independent operators increases with the spin. Conformal symmetry does not allow renormalization mixing in the leading logarithmic order between the operators with different \( n \); it does allow, however, mixing between the operators with different \( k \) for the same \( n \). Here we note that there exists a constraint from symmetry: \( \{ R_{n,k}^+ \} \) and \( \{ R_{n,k}^- \} \) have a definite charge-conjugation parity \( (-1)^n \) and \( (-1)^{n+1} \), respectively, and do not mix with each other under renormalization. If we define the anomalous dimension matrix for \( \{ R_{n,k}^\pm \} \) as \( \Gamma_n^\pm \), the renormalization group (RG) equation for \( \{ R_{n,k}^\pm \} \) is given by
\[ \mu \frac{d}{d\mu} R_{n,k}^\pm(0; \mu^2) = -\alpha_s \frac{n-2}{2\pi} \sum_{l=0}^{n-2} \left( \Gamma_n^\pm \right)_{k,l} R_{n,l}^\pm(0; \mu^2), \quad (4) \]
where \( \mu \frac{d}{d\mu} = \mu \frac{d}{d\mu} + \beta(g) \frac{d}{dg} \) with \( \beta(g) \) the \( \beta \)-function. This equation is solved to give the scale-dependence of \( R_{n,k}^\pm \) as
\[ R_{n,k}^\pm(0; Q^2) = \sum_{l=0}^{n-2} \left[ \left( \frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)} \right)^{\Gamma_n^\pm / b} \right]_{k,l} R_{n,l}^\pm(0; \mu^2), \quad (5) \]
where \( b = \frac{11}{3} N_c - \frac{2}{3} N_f \). The mixing matrix \( \Gamma_n^\pm \) is subject to our computation. The nucleon’s forward matrix elements of \( R_{n,k}^+ \) with even \( n \) is associated with the \( n \)-th moment of the transverse spin structure function \( g_2(x, Q^2) \) in the deep inelastic scattering. In this connection the renormalization of \( R_{n,k}^+ \) has been studied by several different approaches [3, 10] for even \( n \), while the result for odd \( n \) has not been considered seriously. On the other hand, there have been no places in inclusive processes where \( R_{n,k}^- \) appear, and their renormalization has never been discussed before.

In order to obtain the anomalous dimension matrix for the conformal operator basis \( \{ R_{n,k}^\pm \} \), it is convenient to work with their forward matrix elements: We imbed \( \{ R_{n,k}^\pm \} \) into the three-point function as
\[ \int d^4x d^4y d^4z e^{ipx - iqy + ig(p-q)} \langle 0 | T R_{n,k}^\pm(0) \psi(x) \bar{\psi}(y) A_c^\mu(z) | 0 \rangle. \quad (6) \]
Then, the total derivative terms in (2) and (3) drop out and the calculation becomes greatly simplified. For later convenience, we introduce two sets of operators, which replace $R_{n,k}^\pm$ in (3):

$$W_{n,k}^{(a)\sigma} = U_{(+)}^\sigma + U_{(-)}^\sigma_{n-k-2},$$  (7)$$
$$W_{n,k}^{(v)\sigma} = U_{(+)}^\sigma - U_{(-)}^\sigma_{n-k-2},$$  (8)

where $k = 0, 1, \ldots, n - 2$, and

$$U_{(\pm)k}^\sigma = \bar{\psi}d^{n-k-2}\Delta \left( g\tilde{G}\sigma^\lambda\Delta_\lambda \pm i\gamma_5 g\sigma^\lambda\Delta_\lambda \right) d^k\psi.$$  (9)

Note that $W_{n,n-k-2}^{(a)\sigma=\perp}$ and $W_{n,k}^{(v)\sigma=\perp}$ agree with $R_{n,k}^+$ and $R_{n,k}^-$, respectively, if one ignores the total derivatives from the latter. For the renormalization, we follow the method of [3, 4, 10]. We calculate the one-loop corrections for (6). To renormalize the composite operators involving massless fields consistently, we keep the quark and gluon external lines off-shell. We then find that the following twist-3 operators also participate: $\{W_{n,k}^{(a)\sigma}\}$ mix with $\{W_{n,k}^{(v)\sigma}\}$

$$W_{n,F}^{(a)\sigma} = \frac{1}{n+1} \left[ n\bar{\psi}\gamma_5 \sigma^\alpha d^n\psi - \sum_{k=1}^n \bar{\psi}\gamma_5 \Delta d^{k-1} iD^\sigma d^{n-k}\psi \right],$$  (10)$$
$$W_{n,E}^{(a)\sigma} = \frac{n}{2(n+1)} \left[ \bar{\psi}i\not{D}\gamma_5 \sigma^\alpha \Delta d^{n-1}\psi + \bar{\psi}\gamma_5 \sigma^\alpha \Delta d^{n-1} i\not{D}\psi \right],$$  (11)

while $\{W_{n,k}^{(v)\sigma}\}$ mix with

$$W_{n,F}^{(v)\sigma} = \frac{1}{n+1} i\epsilon^{\sigma \mu \nu \tau} \ell^\mu \Delta^\nu \left[ n\bar{\psi}\gamma^\tau d^n\psi - \sum_{k=1}^n \bar{\psi}\Delta d^{k-1} iD^\sigma d^{n-k}\psi \right],$$  (12)$$
$$W_{n,E}^{(v)\sigma} = \frac{n}{2(n+1)} i\epsilon^{\sigma \mu \nu \tau} \ell^\mu \Delta^\nu \left[ -\bar{\psi}i\not{D}\gamma_5 \sigma^\alpha \Delta d^{n-1}\psi + \bar{\psi}\gamma_5 \sigma^\alpha \Delta d^{n-1} i\not{D}\psi \right].$$  (13)

where we introduced another light-like vector $\ell^\mu$ with $\ell^- = \ell^\perp = 0$ and $\ell \cdot \Delta = 1$. In the following, we assume $\ell^\mu$ and $\Delta^\mu$ have mass dimensions +1 and −1, respectively. The forward matrix elements of the quark bilinear operators $W_{n,F}^{(a)\sigma}$ and $W_{n,E}^{(v)\sigma}$ coincide with those of the twist-3 quark-antiquark conformal operators of spin $j = n + 3/2$ for $g_{(a)}(u, \mu^2)$ and $g_{(v)}(u, \mu^2)$. It is instructive to note that they originate from the light-cone nonlocal operators $\bar{\psi}(0)\gamma_5 \sigma[0, \Delta]\psi(\Delta)$ and $\bar{\psi}(0)\gamma_5 \sigma[0, \Delta]\psi(\Delta)$, where $[0, \Delta] \equiv \exp \left\{ -ig \int_0^\Delta dt \Delta_\mu A^\mu(t\Delta) \right\}$ is the gauge link operator; Taylor expansion of these nonlocal operators at small quark-antiquark separations yields $\bar{\psi}(\gamma_5) \gamma^\sigma iD^{(\mu_1} \cdots iD^{\mu_n)} \psi$, and the subtraction of the twist-2 components,

2 The operators $W_{n,F}^{(a)\sigma}, \frac{1}{2(n+1)} W_{n,k}^{(a)\sigma}$, and $W_{n,E}^{(a)\sigma}$ coincide with a set of operators considered in [10] for the massless quark limit.  

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which are totally symmetric and traceless among \( \{ \sigma, \mu_1, \ldots, \mu_n \} \), leads to \( W^{R(a,v)\sigma}_{n,F} \) corresponding to antisymmetric pairs of indices \( \sigma \) and \( \mu_i \). \( W^{R(a,v)\sigma}_{n,E} \) are the so-called equation-of-motion (EOM) operators, which contribute as nonzero operators to the off-shell Green’s function. As it is well known, the above twist-3 operators are not independent, but obey the following relation:

\[
W^{R(a,v)\sigma}_{n,F} = \frac{1}{2(n+1)} \sum_{k=0}^{n-2} (k+1) W^{R(a,v)\sigma}_{n,k} + W^{R(a,v)\sigma}_{n,E}.
\]  

Therefore, we can conveniently choose (7) and (11) ((8) and (13)) as an independent basis for renormalization. A physical matrix element of \( W^{R(a,v)\sigma}_{n,E} \) vanishes, and does not affect the final results of the scale-dependence. However, it has to be kept in the intermediate step of the calculation to work out the renormalization [5, 7, 10].

Here we comment on the places where these operators appear in inclusive processes involving nucleons. The nucleon matrix element \( \langle PS|W^{(a,v)\perp}_{n,F}|Q^2\rangle|PS\rangle \) gives the twist-3 part of the \( n \)-th moment \( \int_{-1}^{1} dxx^n g_T(x,Q^2) \) with \( g_T(x,Q^2) \) the nucleon’s transverse parton distribution. (\( \langle PS\rangle \) is the nucleon state with momentum \( P^\mu \) and spin vector \( S^\mu \)). For even \( n \), this moment is associated with \( \int_{-1}^{1} dxx^n g_1(x,Q^2) \) measurable in the deep inelastic scattering, while both even and odd moments are relevant in the polarized Drell-Yan process (see e.g. [5]). In order to identify the role of \( W^{(v)\sigma}_{n,F} \) in inclusive processes, we recall that it is generated from the nonlocal light-cone operator \( \bar{\psi}(0)\gamma^\sigma[0,\Delta]\psi(\Delta) \) and consider the following decomposition of the nucleon matrix element,

\[
\int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle PS|\bar{\psi}(0)\gamma_\mu[0,\Delta]\psi(\Delta)|PS\rangle = 2 \left[ f_1(x)\ell_\mu + \epsilon_{\mu\nu\rho\sigma} \ell^\nu \Delta^\rho S^\perp_\sigma f_T(x) + M^2 \Delta_\mu f_4(x) \right],
\]

where \( P^\mu \) and \( S^\mu \) are decomposed as \( P^\mu = \ell^\mu + \frac{M^2}{2} \Delta^\mu \) and \( S^\mu = (S \cdot \Delta) \ell^\mu + (S \cdot \ell) \Delta^\mu + S^\perp_\mu \) with the nucleon mass \( M \) and \( P^2 = -S^2 = M^2 \). \( f_1(x) \) and \( f_T(x) \) are the spin-independent parton distribution functions of twist-2 and twist-4, respectively, as discussed in [5]. The new function \( f_T(x) \) is of twist-3 [12]. By comparing both sides of (13), it is easy to see that \( \int_{-1}^{1} dxx^n f_T(x,Q^2) \) is associated with \( \langle PS|W^{(v)\perp}_{n,F}|Q^2\rangle|PS\rangle \). However, the existence of \( f_T(x) \) violates time reversal invariance, and therefore \( \langle PS|W^{(v)\perp}_{n,F}|PS\rangle = \langle PS|W^{(v)\perp}_{n,k}|PS\rangle = 0 \). (The nonforward matrix elements like \( \langle 0|R^k_{n,F}|PS\rangle \) relevant to meson wave functions are not constrained to vanish by time reversal invariance.)

In order to work out the renormalization of \( W^{R(a,v)\sigma}_{n,k} \), we clarify the symmetry constraints on the renormalization mixing. We find:

(i) Two sets \( \{ U^{\sigma}_{(+k)} \} \) and \( \{ U^{\sigma}_{(-k)} \} \) in (4) do not mix with each other. To see this, we recall \( \sigma = \perp \) and introduce right- and left-handed circular polarization vectors for the gluon as \( R = (0,1,i,0)/\sqrt{2} \) and \( L = (0,1,-i,0)/\sqrt{2} \), respectively. Then we find \( U^{\sigma=R}_{(+k)} = -2i\bar{\psi}_L d_0^{-k-2} \Delta G^{R\mu} \Delta_\nu d^\nu \bar{\psi}_L \) and \( U^{\sigma=R}_{(-k)} = -2i\bar{\psi}_R d_0^{-k-2} \Delta G^{R\mu} \Delta_\nu d^\nu \bar{\psi}_R \) where \( \psi_{R,L} = (1 \pm \gamma_5)\psi/2 \). We also obtain similar expressions with \( R \leftrightarrow L \) for \( U^{L}_{(+k)} \) and \( U^{L}_{(-k)} \).
These results explain no mixing between \( \{ U_{(+)k}^\sigma \} \) and \( \{ U_{(-)k}^\sigma \} \), since the perturbative quark-gluon coupling preserves chirality of each quark line.

(ii) The anomalous dimension matrices for \( \{ U_{(+)k}^\sigma \} \) and \( \{ U_{(-)k}^\sigma \} \) are identical: From (i), the RG equation for \( \{ U_{(+)k}^\sigma \} \) can be written with its anomalous dimension matrix \( \Gamma_n \) as

\[
\mu \frac{d}{d\mu} U_{(+)k}^\sigma = -\frac{\alpha_s}{2\pi} \sum_{l=0}^{n-2} (\Gamma_n)_{k,l} U_{(+)l}^\sigma,
\]

where the terms due to the EOM operators are ignored assuming that we take a certain physical matrix element. By applying the charge conjugation \( \mathcal{C} \) on both sides, one obtains

\[
\mu \frac{d}{d\mu} U_{(-)n-k-2}^\sigma = -\frac{\alpha_s}{2\pi} \sum_{l=0}^{n-2} (\Gamma_n)_{k,l} U_{(-)n-l-2}^\sigma,
\]

where we used \( \mathcal{C} U_{(+)k}^\sigma C^{-1} = (-1)^n U_{(-)k}^\sigma \). Equations (16) and (17) explain the statement.

From (16) and (17), one obtains the RG equation for \( W_{n,k}^{(a,v)\sigma} \) as

\[
\mu \frac{d}{d\mu} W_{n,k}^{(a,v)\sigma} = -\frac{\alpha_s}{2\pi} \sum_{l=0}^{n-2} (\Gamma_n)_{k,l} W_{n,l}^{(a,v)\sigma}.
\]

By comparing this result with forward matrix element of (4), we can identify the anomalous dimension matrix \( \Gamma_n \) as

\[
(\Gamma_n^+)_{k,l} = (\Gamma_n)_{n-k-2,n-l-2}; \quad (\Gamma_n^-)_{k,l} = (\Gamma_n)_{k,l}.
\]

We also note that we can use whichever convenient set of the operators \( \{ W_{n,k}^{(a,v)\sigma} \}, \{ U_{(\pm)k}^\sigma \} \), in order to obtain \( \Gamma_n \) (see (10)–(18)). These features of the anomalous dimensions are in parallel with a familiar example of the twist-2 distributions \( f_1 \) and \( g_1 \) defined from the nonlocal operators

\[
\bar{\psi}(0)\gamma^\sigma[0,\Delta]\psi(\Delta) \quad \text{and} \quad \bar{\psi}(0)\gamma_5\gamma^\sigma[0,\Delta]\psi(\Delta): \quad \text{Their } n\text{-th moment is associated with the local operators}
\]

\[
\bar{\psi}d^n\Delta\psi = \bar{\psi}_R d^n\Delta\psi_R + \bar{\psi}_L d^n\Delta\psi_L \quad \text{and} \quad \bar{\psi}d^n\Delta\gamma_5\psi = \bar{\psi}_R d^n\Delta\gamma_5\psi_R - \bar{\psi}_L d^n\Delta\gamma_5\psi_L,
\]

respectively, and similar argument as above explains that the nonsinglet parts of \( f_1 \) and \( g_1 \) have the same anomalous dimensions.

Explicit calculation of the mixing matrix \( \Gamma_n \) is yet desirable for general (even and odd) \( n \). As noted above, one has to keep the EOM operators as nonzero operators during the course of the renormalization. We choose \( \{ W_{n,k}^{(a,v)\sigma}, W_{n,E}^{(a,v)\sigma} \} \) as a basis of the renormalization. (For completeness, we parallelly discuss two sets of the operators.) We imbued these operators into the three-point function \( (10) \). The relevant one-loop diagrams are the same as those considered in \( (10) \). We adopt the Feynman gauge, and obtain the mixing matrix \( \Gamma_n \) in the
MS scheme of the dimensional regularization. One technical difficulty in the computation is the complicated mixing with many gauge noninvariant EOM operators. To avoid this, we introduce a vector \( \Omega^\mu \) with the condition \( \Omega \cdot \Delta = 0 \), and contract the external gluon line of (6) with this vector [5]. Under this condition, the vertices for \( W_{n,k}^{(a,v)\sigma} \) and \( W_{n,E}^{(a,v)\sigma} \) can be written as

\[
W_{n,k}^{(a,v)\sigma} = \pm g(\hat{q} - \hat{p}) \left( \gamma^k \gamma^{n-k-2} \gamma^\sigma \gamma^\Delta \mp \gamma^k \gamma^{n-k-2} \gamma^\sigma \gamma^\Delta \right) t^c, \tag{20}
\]

\[
W_{n,E}^{(a,v)\sigma} = \mp \frac{n}{2(n+1)} g \left( \gamma^{n-1} \gamma^\sigma \gamma^\Delta \pm \gamma^{n-1} \gamma^\sigma \gamma^\Delta \right) t^c, \tag{21}
\]

where \( t^c \) is the color matrix normalized as \( \text{tr}(t^ct^{c'}) = \frac{1}{2} \delta^{cc'} \), and we introduced the notation \( \hat{p} = p \cdot \Delta \), etc. It is now straightforward to obtain the mixing matrix \( \Gamma_n \). The result reads

\[
(\Gamma_n)_{kl} = -C_G \left( \frac{(l+1)(l+2)}{(k+1)(k+2)(k-l)} \right.
+ \left(2C_F - C_G \right) \left[ (-1)^{k+l+1} n_{-1} C_l n + k - l \right. \frac{1}{n_{-1} C_k n(k-l)} + \left. (-1)^{l+1} \frac{1}{k+2} \right], \tag{22}
\]

\[
(\Gamma_n)_{kk} = C_G \left( \frac{1}{n - k - 1} + \frac{n - k}{n - k + 1} + \frac{1}{k + 2} + S_{n-k-1} + S_{k+1} \right)
+ \left(2C_F - C_G \right) \left[ \frac{1}{n} + \frac{2(-1)^{n-k}}{(n - k - 1)(n - k)(n - k + 1)} + \frac{(-1)^{k+1}}{k + 2} \right]
+ C_{F} \left( 2S_{k+1} + 2S_{n-k-1} - 3 \right), \tag{23}
\]

\[
(\Gamma_n)_{kl} = -C_G \left( \frac{(n-l)(n-l+1)}{(n-k)(n-k+1)(l-k)} + (2C_F - C_G) \left[ (-1)^{k+l+1} n_{-1} C_{l+1} n + l - k \right. \frac{1}{n_{-1} C_{k+1} n(l-k)}
+ \left. \frac{2(-1)^{n-l} n_{-1} C_{l-k}}{(n-k)(n-k+1)(n-k+1)} \right], \tag{24}
\]

where \( n_{C_k} = n!/\left[ k!(n-k)! \right] \), \( S_n = \sum_{j=1}^{n} 1/j \), and \( C_F = (N_c^2 - 1)/2N_c, C_G = N_c \) for \( N_c \) color. This result combined with [13] gives \( \Gamma_{n}^\pm \). The result for \( \Gamma_n^- \) is wholly new. For \( \Gamma_n^+ \), the result is now completely applicable also for the cases with odd \( n \) which has not received much attention.\footnote{\( \Gamma_{2}^+, \Gamma_{3}^+ \), and \( \Gamma_{4}^- \) give Eqs. (4.66), (4.67) and (4.68) of [1].} For example, if one replaces the term \((-1)^l/(n-l)\) proportional to \( 2C_F - C_G \) in Eq. (16) of [13] by \((-1)^{n-l+1}/(n-l)\) in their convention, their result in Eqs. (15)–(17) becomes consistent with the above (22)–(24) for all \( n \).

We make contact with the work of Balitsky and Braun [13], who also considered the renormalization of similar quark-antiquark-gluon operators. They introduced the nonlocal
light-cone operators

\[ S^\pm_\sigma(\alpha, \beta, \gamma) = g \psi(\alpha \Delta) \{\Delta, \beta \Delta\} \left( G_{\sigma\nu}(\beta \Delta) \pm \tilde{G}_{\sigma\nu}(\beta \Delta)i\gamma_5 \right) \Delta^\nu [\beta \Delta, \gamma \Delta] \psi(\gamma \Delta), \]

(25)

and computed the one-loop evolution kernels in the coordinate space representation. They found by explicit calculation that \( S^+_\sigma(\alpha, \beta, \gamma) \) and \( S^-_\sigma(\alpha, \beta, \gamma) \) are renormalized without mixing with each other. The reason for this phenomenon is made clear by examining the \( \sigma = R, L \) components of (23) similarly to (i) above. A direct relation between our results and those of [13] can be provided by expanding the operators (24) in powers of \( \Delta_\mu \) and taking the forward matrix element: Expansion of \( S^\pm_\sigma(\alpha, \beta, \gamma) \) generates a series of the local operators with increasing spin. In particular, the \( n \)-th term generates \( \Delta^\nu \epsilon^\lambda \epsilon_{\sigma\nu\lambda\mu} U^\mu_{(\pm)k} \) with \( k = 0, 1, \ldots, n-2 \), up to the twist-4 corrections. In this connection, we mention the following point: The kernels for \( S^+_\sigma(\alpha, \beta, \gamma) \) and \( S^-_\sigma(\alpha, \beta, \gamma) \), given by Eq.(6.2) of [13], have rather different forms, although we have shown that the anomalous dimension matrices for the corresponding two sets of local operators, \( \{U^\sigma_{(+)k}\} \) and \( \{U^\sigma_{(-)k}\} \), are identical with each other (see (16) and (17)). In fact, the equivalence of these two kernels can be easily proved by applying the parity transformation combined with the time-reversal transformation\(^4\) to the corresponding evolution equations, and therefore the results of [13] are consistent with ours.

We finally recall in our notation that the anomalous dimension matrix in (22)–(24) satisfies the relation in the large \( N_c \) limit, i.e. \( C_F \rightarrow N_c/2 \) in (22)–(24) [14]:

\[ \sum_{k=0}^{n-2} (k+1) (\Gamma_n)_{k,l} = (l+1) \gamma_n, \]

(26)

where \( \gamma_n \) is the lowest eigenvalue of the anomalous dimension matrix \( \Gamma_n \) in this limit, and is given by

\[ \gamma_n = 2N_c \left( \psi(n+1) + \gamma_E - \frac{1}{4} + \frac{1}{2(n+1)} \right). \]

(27)

Here \( \psi(n+1) = \sum_{k=1}^{n} 1/k - \gamma_E \) is the digamma function and \( \gamma_E \) is the Euler constant. This leads to a simple evolution equation in the large \( N_c \) limit (see [4], [13], and [13]):

\[ O_n(Q^2) = \left( \frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)} \right)^{\gamma_n/b} O_n(\mu^2), \]

(28)

where \( O_n(\mu^2) = \sum_{k=0}^{n-2} (n-k-1)R_{n,k}^+(0; \mu^2), \sum_{k=0}^{n-2} (k+1)R_{n,k}^-(0; \mu^2), \) and \( W_{n,F}^{(a,v)}(\mu^2) \). Similar relations (26) and (28) hold also for the limit of large conformal spin with \( \gamma_n \rightarrow \gamma_n + (4C_F - 2N_c)(\ln n + \gamma_E - 3/4) \) to \( O(\ln n/n) \) accuracy. As a result of (28), the vector meson wave functions \( g_{1}^{(a)}(u, \mu^2), g_{1}^{(v)}(u, \mu^2) \) as well as the nucleon parton distribution \( g_T(x, \mu^2) \) obey simple DGLAP-type evolution in the two limits, \( N_c \rightarrow \infty \) and \( n \rightarrow \infty \) [1, 14].

\(^4\) This transformation is more convenient for this purpose than the charge-conjugation transformation used in [16] and [17], since it connects \( S^+_\sigma(\alpha, \beta, \gamma) \) with \( S^-_\sigma(\alpha, \beta, \gamma) \) keeping the condition \( \alpha > \gamma \), which is assumed in Eq.(6.2) of [13].
To summarize, we have completed the renormalization of chiral-even twist-three non-singlet wave functions for the vector mesons. The result is presented in the form of the anomalous dimension matrices for the corresponding conformal operators \( \{ R_{n,k}^\pm \} (k = 0, 1, \cdots, n-2) \) for all conformal spins. Although \( \{ R_{n,k}^+ \} \) and \( \{ R_{n,k}^- \} \) are the independent operators, their anomalous dimension matrices coincide completely due to the charge-conjugation invariance and the chirality conservation.

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