A signal detection method based on matrix information geometric dimensionality reduction

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Abstract. Traditional signal detection methods have achieved satisfactory performance in many contexts of signal processing. However, these methods based on mathematical statistics show a drawback in dealing with the low SNR cases, which limits their practicability. To this end, inspired by image processing techniques, we first make use of the short time Fourier transform to generate sufficient 2-D spectrograms of the received data. Then we extract high dimensional features of these spectrogram to construct high dimensional covariance matrices, transforming into a binary classification problem lying on a symmetric positive definite (SPD) manifold. In addition, by reducing the dimensionality directly on the SPD manifold, these spectrograms are mapped into a more discriminative SPD manifold, which improves the separability between the two classes. The simulation experiment results demonstrate that our method achieve satisfactory signal detection performance in the task of signal detection under K distribution data, even in the case of SNR = -10.

1. Introduction

Signal detection is the crucial operation in any communication problems[1]. Only when a signal is detected, it is possible to proceed to the next step of signal processing. The basic task is to effectively identify the existence of useful signal for the received data mixed with noise and interference. Traditional methods in dealing with this issue mentally make use of the mathematical theory of probability and statistics, e.g., the Bayes criterion, the Neyman-Pearson lemma. They have made a series of achievements in many application fields, and also have solved many practical problems with satisfactory performance[2]. However, these probability statistical based methods show an evident drawback in case of weak signal, strong noise and many external interferences, which results in limits of practicability. Moreover, how to select the appropriate threshold is also a serious problem to be solved[3].

Nowadays, time-frequency analysis has been a powerful tool for analyzing noised signals and has been proved to be effective in signal detection tasks[4]. Among these time-frequency analysis methods, the short-time Fourier transform (STFT) is known to be the major method of research in time-frequency analysis, and has made many achievements in signal analysis[5]. However, the task of signal detection is still faced with many practical challenges. For instance, the traditional time-frequency analysis for signal detection is sensitive to noise, has serious cross-terms interference, and
has a large amount of computation. Consequently, the existing methods for signal detection are far from the actual need[6].

In recent years, due to the abundant edge information in the time-frequency spectrum after STFT, several researches tried to extract features of these 2-D spectrograms from the perspective of image processing, and these image processing based methods had been proven to be effective in fields of some specific signals[7-8], but they only extracted few features maintaining local information of the image, simply using low-dimensional information from the high-precision spectrogram. Clearly, the effect of these methods was very poor in the task of complex signal detection under a low signal-to-noise-ratio (SNR), which made these methods less practical.

On the other hand, data representation for Symmetric Positive Definite (SPD) manifold have been widely applied to many areas, and have been proven its great power and success [9-13]. From the perspective of the information geometry, signals can be naturally regarded as the manifold-valued data, and the 2-d spectrograms processed after STFT demonstrating the energy distribution of signals can be also known as points on a specific manifold. However, one of the drawbacks in using this method is that the calculation cost increases rapidly with the dimension of the manifold[14]. In this context, computation on the Riemannian manifold of high-dimensional SPD matrices comes at a high consumption, which limits the development of existing methods. Mehrtash Harandi in 2018[15] proposed a new DR technique dedicated to SPD manifolds. More specifically, it maps the high-dimensional manifold into a low-dimensional one by optimizations on the Grassmannian manifold. This technique will not change the important Riemannian structure, which is shown to overcome the limitations of Euclidean geometry.

The contributions in this paper are following:(i): Previous studies simply extracted local or edge information in the spectrogram. Clearly, the low-dimensional feature is bound to be less discriminative and powerful[16]. In our experiment, sufficient high-dimensional features are extracted from spectrograms. (ii): To reduce the redundancy of high dimensional information, an efficient and novel DR method on the SPD manifold based on information geometry is employed, which can be clearly seen in figure. 1.

\[\text{Figure 1. Conceptual illustration of manifold-DR based on information geometry.}\]

2. The Riemannian Geometry
Now we introduce some useful theories regarding the Riemannian geometry. A manifold is a set of points endowed with a flat local Euclidean structure. A tangent vector at \(p \in \mathcal{M}_H\) is defined as the equivalence class of all curves that have the same velocity at \(p\). Unlike the uniform Euclidean space, a Riemannian manifold exhibiting local homeomorphism is a differentiable topological space, in which the tangent space \(T_p\) of each point \(p\) on the manifold is defined by a smoothly varying inner product.

An important rule to define a Riemannian manifold is to give an appropriate metric \(\delta\) for a manifold \(\mathcal{M}_H\), then it becomes a valid Riemannian manifold. In this way, the distance and area on the Riemannian space can be measured under the Riemannian metric.

2.1. The Symmetric Positive Definite (SPD) Manifold
Let \(X_p\) denotes an \(n \times n\)-dimensional symmetric positive definite matrix, which lies on the SPD space \(S_p^{n+}\), forming the interior of a convex cone in the \(n(n + 1)/2\)-dimensional Euclidean space [17]. As a
consequence, the Riemannian structure of negative curvature is formed by a set of SPD matrices $X$. The SPD space becomes a Riemannian manifold when it is endowed with an appropriate Riemannian metric. Among these useful metrics, the Log-Euclidean metric is a common similarity measure comparing SPD matrices, which have the property that having a simple form and low computational complexity. By viewing SPD matrices as a Lie group, this efficient Riemannian metric maps the elements lying on the manifold to the tangent space so that classical Euclidean methods can be applied, which can be defined for $X,Y \in S_{++}^n, \delta_{LE}(X,Y) = \| \log(X) - \log(Y) \|_F$. Here $\log(\cdot)$ denotes the matrix logarithm map, and $\| \cdot \|_F$ means the Frobenius norm.

### 3. Region Covariance Matrix Descriptor

The region covariance matrix (RCM) descriptor is a powerful data representation method due to the sufficient information it provides, which can efficiently capture the second-order statistical characteristics of local information with less noise interference. In 2006, a fast method to construct RCM based on integral images is proposed[18]. Let $I$ be a $W \times H$ spectrogram, and $n$ represents the number of image pixels. $z_k$ is the characteristic value corresponding to the $k$-th pixel point, and $\mu = \frac{1}{n} \sum_{i=1}^{n} z_k$ is the mean value of the corresponding feature. Therefore, the RCM of the time-frequency spectrogram can be represented by:

$$C_R = \frac{1}{n-1} \sum_{k=1}^{n} (z_k - \mu)(z_k - \mu)^T$$ (1)

In this paper, the dense sift descriptor[19] is used to extract 128-dimensional information from each grid point of each spectrogram. More specifically, with Gaussian smoothing operation on the image in advance, the dense sift descriptor is obtained by sliding a specific window. Figure 2 shows the 128-dimensional descriptor of one grid in the spectrogram. In this way, the region covariance matrix of 128×128 can be obtained for each time-frequency spectrogram. The feature extraction advantage of this dense descriptor is significant that it can quickly extract densely distributed feature points with the unique sampling method. However, one of the drawbacks is obvious that it has a certain degree of redundancy. To address this problem, we employ the novel DR method based on information geometry to reduce the redundancy, which will be detailed in the next section.

![Figure 2. The 128-dimensional dense SIFT descriptor.](image)

### 4. Dimensionality Reduction on SPD Manifold

For a set of SPD matrices $X \in S_{++}^n$, our goal is to seek for a mapping that $f: S_{++}^n \times R^{n \times m} \rightarrow S_{++}^m$ with the learned parameter $W \in R^{n \times m}, m < n$. In fact, the constructed matrices are the basic elements lying on the SPD manifold, and the dimensionality reduction based on the SPD manifold geometry is employed to reduce the redundancy.
4.1. Affinity Graph Embedding

Clearly, the distance between the matrices of the within-class templates are closer and the between-class are further under an appropriate Riemannian metric. In this context, an affinity function $A(i,j)$ is added to this mapping to preserve geometric properties after dimensionality reduction. More specifically, let $\{(X_i,Y_i)\}_{i=1}^p$ denotes $p$ labelled spectrograms, here $X_i \in S_{+,+}^n$, $Y_i \in \{0,1\}$. In particular, we will be faced with the binary classification problem, that is, whether there exists signal in the spectrogram. In this way, the affinity function $A(i,j)$ can be established by the within-class similarity graph $G_w(i,j)$ and the between-class similarity graph $G_b(i,j)$. The two binary matrices are constructed by nearest neighbour graph, which is defined as follows:

$$G_w(i,j) = \begin{cases} 1, & \text{if } X_i \in N_w(X_j) \text{ or } X_j \in N_w(X_i) \\ 0, & \text{otherwise} \end{cases}$$

$$G_b(i,j) = \begin{cases} 1, & \text{if } X_i \in N_b(X_j) \text{ or } X_j \in N_b(X_i) \\ 0, & \text{otherwise} \end{cases}$$

(3)

Here $N_w(X_i)$ represents the neighbour of $X_i$ that belongs to the same class, $N_b(X_i)$ represents the neighbour of $X_i$ having different labels. Finally, the affinity function is:

$$A = G_w - G_b$$

Figure 3. Schematic illustration of nearest neighbour graph.

In this way, the dispersion degree of the between-class and the aggregation degree of the within-class can be increased while preserving the origin correlation between features of spectrograms after dimensionality reduction, which can be seen in figure 3.

4.2. Cost Function

In particular, to ensure that the new constructed SPD manifold is valid, that is, $W^T X W > 0, \forall X \in S_{+,+}^n$, the projection matrix $W$ is required to be full rank. Thus, an orthogonal constraint $W^T W = I_m$ is added. After learning the embedding affinity function $A(i,j)$, we confront a optimization problem, which can be formulated as:

$$\min \sum_{i,j} A(i,j) \delta_{IE}^2 (W^T X_i W, W^T X_j W)$$

s.t. $W^T W = I_m$

(5)

Here $\delta_{IE}$ denotes the Log-Euclidean metric. By Taylor expansion, $\log (W^T X W) \approx W^T \log (X) W$. And let

$$F(W) = \sum_{i,j=1}^p a(X_i, X_j)(\log (X_i) - \log (X_j))WW^T \times (\log (X_i) - \log (X_j))$$

(6)

In this way, the cost function can be rewritten as:
\[
\min_{W \in \mathbb{R}^{n \times m}} \text{Tr} \left( W^T F(W) W \right) \quad \text{s. t. } W^T W = I_m
\]  

The above lemma can be solved by eigen-decomposition. First, \( F(W) \) is fixed in the case assuming that \( F(W) \) does not rely on \( W \). With the help of the property that matric trace is equal to the sum of the eigenvalues, the current solution is obtained by taking the \( m \) smallest eigenvectors of \( F(W) \). Then we have a new \( W \) and update the corresponding \( F(W) \). These steps are repeated until convergence. And after several iterations, a more discriminative, and low dimensional SPD manifold can be learned.

5. Numerical Experiments

We use K distribution to simulate the clutter, and the target signal with Doppler frequency \( f_d = 0.15 \) Hz. By short time Fourier transform, 4000 time-frequency images are simulated for each SNR of -5 to -10. Then we resize the spectrograms to 200×200, using dense SIFT descriptor to extract features so that 128-dimensional covariance matrices are constructed. In this way, these spectrograms are mapped into an originally 128-dimensional SPD manifold. The flow chart of our experiment is shown in figure 4. In our experiment, 1000 spectrograms are used for training and others for test, classified by KNN algorithm. We take false alarm probability \( P_a \) and detection probability \( P_d \) to measure our detection performance, which are important evaluation criteria in signal detection.

\[
P_a = \frac{n_{01}}{m_0} \tag{8}
\]
\[
P_d = \frac{n_{11}}{m_1} \tag{9}
\]

Here \( m_0 \) refers the number of noise-only samples, i.e., the negative samples, while \( m_1 \) means the number of signal-contained sample, i.e., the positive samples; \( n_{01} \) represents the number of negative samples predicted to be positive, while \( n_{11} \) refers the number of positive samples that were successfully predicted to be positive.

![Figure 4. The flow chart of our experiment.](image)

Moreover, in the classification process, we assume that the parameter \( k \) in KNN classifier is fixed to contrast the signal detection performance before and after dimensionality reduction. And the parameter \( N_w \) and \( N_b \) are also fixed to evaluate the effect of dimension on detection performance. We note that these optimization parameters in dimensionality reduction are determined by valid validation and the final result is averaged. In order to evaluate the effect before and after dimension reduction, we refer to the three algorithms in our experiment as: KNN-LogE, KNN-LogE-DR and Radon transform detector based on image processing[20]. Specifically, the KNN-LogE algorithm represents the log-Euclidean metric-based K nearest neighbour classifier on the original SPD manifold, while the KNN-LogE-DR algorithm means log-Euclidean metric-based K nearest neighbour classifier on the lower-dimensional SPD manifold via geometric dimensionality reduction. Moreover, the Radon transform detector is used to detect lines in the image, such method obviously will not cause the false alarm.

In this experiment, the parameter \( k \) is set to 1, and the new dimension \( \text{dim} \) is 20. In addition, parameter \( N_w \) is set to 40 with \( N_b \) set to 20 in affinity graph embedding. From table 1, we can clearly see that our method in signal detection task can improve the detection performance, while reduce the dimension of the algorithm. In general, as SNR goes down, the false alarm probability \( P_a \) increases and the detection probability \( P_d \) decreases to a certain extent with these three algorithms. In particular, the Radon transform detector results in many missed detections, which is more serious in lower SNR cases. And the KNN-logE algorithm on the origin SPD manifold has higher false alarm probability than the other methods. Moreover, under each SNR of -5 to -10, KNN-LogE-DR achieves a satisfactory alarm probability while keeps a very high detection probability, which shows that the data
on SPD manifolds were more separable after dimensionality reduction so that the KNN-logE-DR algorithm is better than the other two algorithms.

| SNR  | Method             | $P_a$  | $P_d$  |
|------|--------------------|--------|--------|
| -5   | KNN-logE           | 0.0038 | 0.9961 |
|      | KNN-logE-DR        | 0      | 1      |
|      | Radon transform    | 0      | 0.9995 |
|      | KNN-logE           | 0.0042 | 0.9963 |
| -6   | KNN-logE-DR        | 0      | 1      |
|      | Radon transform    | 0      | 0.9925 |
|      | KNN-logE           | 0.0077 | 0.9935 |
| -7   | KNN-logE-DR        | 3.41E-04 | 1     |
|      | Radon transform    | 0      | 0.964  |
|      | KNN-logE           | 0.0073 | 0.9947 |
| -8   | KNN-logE-DR        | 8.38E-04 | 1     |
|      | Radon transform    | 0      | 0.87   |
|      | KNN-logE           | 0.0143 | 0.9873 |
| -9   | KNN-logE-DR        | 1.00E-03 | 1    |
|      | Radon transform    | 0      | 0.722  |
|      | KNN-logE           | 0.0185 | 0.985  |
| -10  | KNN-logE-DR        | 2.00E-03 | 0.999 |
|      | Radon transform    | 0      | 0.483  |

Then we try to find out the dimension effect on the signal detection performance. As mentioned earlier, the parameter $k$ in KNN classification is still fixed, and $N_w$ is set to 40 with $N_b$ 20. Moreover, the dimension changes from 20 to 100 with an interval of 20 to explore the signal detection performance in different dimensions. From Table 2, we can clearly see that the detection performance varies with the dimension. In general, with the increase of dimension, the detection performance decreases obviously. In particular, reducing the dimension to 20 or 40 works better than other dimensions by using our method. It is possible that the SPD manifolds of these dimensions are more discriminative so that the geometric distances between samples with the same label are closer.
Table 2. Signal detection performance under various dimensions.

| SNR | Method    | dim=20 | dim=40 | dim=60 | dim=80 | dim=100 |
|-----|-----------|--------|--------|--------|--------|---------|
|     |           | Pa     | Pd     | Pa     | Pd     | Pa      | Pd      |
| -5  | KNN-LogE  | 0.00382| 0.9962 | 0.00467| 0.9962 | 0.00382| 0.9962  |
|     | KNN-LogE-DR| 0      | 1      | 0      | 1      | 0       | 1       |
| -6  | KNN-LogE  | 0.00416| 0.9963 | 0.00516| 0.9963 | 0.00533| 0.9963  |
|     | KNN-LogE-DR| 0      | 1      | 3.35E-04| 0.9998| 1.0E-03| 0.999   |
| -7  | KNN-LogE  | 0.0077 | 0.9935 | 0.00616| 0.9955 | 0.0075 | 0.9947  |
|     | KNN-LogE-DR| 3.41E-04| 1      | 3.37E-04| 0.9997| 1.0E-03| 0.9992  |
| -8  | KNN-LogE  | 0.00734| 0.9947 | 0.00932| 0.9992 | 0.00850| 0.9917  |
|     | KNN-LogE-DR| 8.38E-04| 1      | 8.38E-04| 0.9999| 4.50E-03| 0.9973  |
| -9  | KNN-LogE  | 0.0143 | 0.9873 | 0.0122 | 0.9880 | 0.0140 | 0.9868  |
|     | KNN-LogE-DR| 1.00E-03| 1      | 6.67E-03| 2.17E-03| 3.50E-03| 6.17E-03|
| -10 | KNN-LogE  | 0.01849| 0.9855 | 0.01914| 0.9854 | 0.01718| 0.9858  |
|     | KNN-LogE-DR| 2.00E-03| 1      | 2.67E-03| 5.18E-03| 6.00E-03| 1.18E-02|

6. Conclusion
In this paper, we regard the task of signal detection as a binary classification problem. More specifically, from the perspective of image processing, the 2-D spectrogram transformed by short time Fourier transform reserves many useful information of the sample data. In this case, we first extract features from these spectrograms using a high dimensional descriptor, then we construct the region covariance matrices, and map these spectrograms into a high dimensional SPD manifold. After learning the affinity between each symmetric positive definite (SPD) matrices, we take advantage of the geometric dimensionality reduction method to reduce the complexity and improve the detection performance.

The experimental results clearly demonstrate our method works better than other signal detection method based on image processing. However, there are still some problems with our approach that need to be addressed. For future research, we may focus on how to reduce the false alarm probability in lower SNR cases, and more experiments on different types of signal detection in different clutter environments will be conducted.

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