Formulae for Calculating the Resistance Parameters of Irregular Through-silicon Via

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Abstract. The through-silicon via (TSV) is a key technology in the three-dimensional integrated circuit, and the accuracy of its parasitic parameter extraction is the important factor of ensuring accuracy of the integrated circuit simulation. Due to the influence of the production process, the through-silicon via (TSV) is not a cylinder with a smooth surface and regular shape, and with an increase in the frequency and decrease in the size of the TSV, the irregular shape has more and more influence on the parameter extraction of the TSV. To extract the resistance parameters of an irregular TSV more rapidly and accurately, we studied the high surface ratio trapezoid and rough surface TSV and put forward the analytic expressions for calculating the resistance. The two analytic expressions reflect the change rule of resistance parameters along with the size and frequency variation of the TSV. The error of analytic expression was within 5% when compared with the simulation results, which fully satisfies the requirement of accuracy.

Keywords: TSV, Parameter Extraction, 3D Integrated Circuit

1. Introduction

Silicon via technology can realize the interconnection of chips through making vertical conduction between chips and chips, and wafers and wafers, which can enable the 3D encapsulated chip to realize more functions on a certain size and simultaneously avoid the RC delay caused by high-density 2D encapsulation [1]. As the interconnected through-silicon via (TSV) is in the 3D integrated circuit, the parasitic resistance will directly affect the power consumption, time delay, noise, and other performances of the integrated circuit. Therefore, the resistance parameter of the TSV is of great significance in the successful design of a high-performance chip [2-3].

Under ideal conditions, a TSV is a smooth surface cylinder, but actually a TSV is not a perfect cylinder due to the errors in the manufacturing process [4]. In the etching process, because of the different size of the etching area, which leads to a different amount of etching reagent, and the angle between the lateral and bottom sides of the TSV groove which will be slightly changed as well, the actual shape of the TSV is a trapezium in most cases. As shown in Figure 1(a), compared with the cylindrical TSV, the trapezoid reduces the cross-sectional area of the wire, thereby the resistance of the TSV is increased. The surface of the actual interconnecting wire is rough due to the influence of...
chemical treatment and other methods. As shown in Figure 1(b), the roughness will have a significant impact on the resistance of the wire due to the skin effect of the classical electromagnetic theory.

Scholars at home and abroad have studied the parameter extraction of silicon through hole with irregular shape has been studied by. For trapezoidal TSV, Liang et al. deduced the analytical formula of its resistance, but this formula is only applicable to the case where the surface ratio is less than 10, and when the surface ratio is relatively large, the formula will show greater calculation error. For TSV with rough surface, Tanaka has conducted such experiments and the experimental results shows that the effective resistance of different copper conductors will increase by 50%-70% due to the rough surface. Wang's research shows that the insertion loss of the rough side wall TSV structure increases by 15 % compared to the smooth side wall TSV \[5\]. However, there is no report on the derivation of the analytical formula for calculating the resistance of TSV with rough surfaces.

The common methods of calculating resistance are numerical method, analytic method and so on. Numerical method can get accurate result, but there are some problems, such as large computational quantity, singular point, slow calculating, and so on, so it is difficult to meet the need of fast extracting parameters in large scale chip. The analytic model method is based on the results of theoretical analysis, experimental measurement or numerical simulation, and the analytic model formula is obtained by interpolation and fitting method \[6-7\]. The application of analytic formula in EDA software can greatly improve the computational efficiency and has important practical significance on the analysis of large scale integrated circuits.

For this reason, this paper took the trapezoid TSV and the rough surface TSV as the research object, analyzed the change law of the resistance parameter along with the size and frequency, and advanced two kinds of analytic formulae for calculating resistance as a result. The analytic formulae can quickly and accurately calculate the resistance value of the TSV and the error is within 5%.

\[\text{Fig.1} \quad (a) \text{ trapezoidal TSV} \quad (b) \text{ rough surface TSV}\]

2. Derivation of Formulae

According to the literature,\[4\] the resistance parameter of the TSV is correlated with the square root of the frequency and can be expressed as follows:

\[R(f) = (R_{1\text{GHz}} - R_{dc}) \cdot \sqrt{\frac{f}{f_{1\text{GHz}}}} + R_{dc}\]  \hspace{1cm} (1)

\(R_{dc}\) is a DC resistance, and the \(R_{1\text{GHz}}\) is the resistance value when the frequency is 1 GHz. It is clear that in formula (1) the \(R_{dc}\) and \(R_{1\text{GHz}}\) are the key parameters. The derivations of the resistance value of the trapezoid TSV and the rough surface TSV are as follows.
2.1. Derivation of the Trapezoidal TSV Model

The trapezoidal TSV model, as shown in Figure 2 (a) and (b) is the size of the TSV, where $\alpha$ is a dip angle of the trapezoidal TSV, $\beta$ is the tangent value of $\alpha$, and $b$ and $a$ are the radius of the upper and lower surfaces of the trapezoid, respectively. The relationships are as follows:

$$\tan \alpha = \beta = \frac{b-a}{L} \quad (2)$$

The trapezoid TSV is divided longitudinally, which can be regarded as several cylinders with a different radius. When the longitudinal height is $Z_1$, the radius size of the TSV can be expressed as follows:

$$r_{z_l} = a + \beta z_l \quad (3)$$

The current density distribution inside the trapezoidal TSV is divided into the axial current, $J_z$, and the radial current, $J_r$. When the vertical coordinate is set as $Z$, the $J_z$ and $J_r$ meet the current continuity equation.

$$\left. \frac{1}{r} \frac{\partial}{\partial r} (r J_z(r)) \right|_{z=z_l} + \left. \frac{\partial J_r(r)}{\partial z} \right|_{z=z_l} = 0 \quad (4)$$

The expressions of $J_z(r)$ and $J_r(r)$ are as follows:

$$J_z(r) \big|_{z=z_l} = \left. \frac{1}{L} \frac{c}{2 \pi r_{z_l} I_1(cr_{z_l})} I_0(cr) \right) \quad (5)$$
\[ J_r(r)|_{z=z_l} = \frac{1}{I} \int_0^{z_l} r \frac{\partial J_z(z)}{\partial z} \, dr \]
\[ = \frac{1}{I} \frac{\beta c I_0 (cr_{z_l})}{2 \pi r_{z_l} I_1 (cr_{z_l})} \cdot I_1 (cr) \] (6)

\( I_0 \) and \( I_1 \) are zero and first order category one modified Bessel functions, respectively. \( I \) is the total current of the TSV, and \( c \) is the constant that changes with the skin depth.

By integrating the current density, the resistance of the metal conductor in the TSV can be calculated.

\[ R = \frac{\int_0^L \int_0^{z_l} \left( |J_z|^2 + |J_r|^2 \right) 2 \pi r dr dz}{I^2 \sigma} \]
\[ = \frac{\int_{z_l} \int_0^L \int_0^{\sigma} 2 \pi |J_r|^2 \, dh \cdot \int_{z_l} \int_0^L \int_0^{\sigma} 2 \pi |J_z|^2 \, dh \cdot \frac{\sigma}{\pi}}{F \sigma} \] (7)

When the frequency is low, the resistance is DC resistance, as follows:

\[ R_{dc}^{tri} = R_z + R_r \approx \frac{L}{\sigma \pi a (a + L \beta)} \left( 1 + \frac{1}{2} \beta^2 \right) \] (8)

Considering \( \beta < 1 \) that, \( \frac{1}{2} \beta^2 \) can be ignored. Therefore

\[ R_{dc}^{tri} = \frac{L}{\sigma \pi a (a + L \beta)} \] (9)

When the frequency is higher, the skin depth is much smaller than the radius of the TSV (1), and the resistance formula under high frequency can be derived as follows:

\[ R_{tri}^{tri} = R_z + R_r \approx \frac{2 + \beta^2}{4 \pi \sigma \delta \beta} \cdot \ln \left( 1 + \frac{2 \beta L}{2 a - \beta} \right) \] (10)

To reduce the error between the formula (10) and the simulation value, the adjustment coefficient is added to the formula, and the adjusted formula is as follows:

\[ R_{\text{GIt}-\text{tri}}^{\text{tri}} = 5 \beta L \cdot \frac{2 + \beta^2}{4 \pi \sigma \delta \beta} \cdot \ln \left( 1 + \frac{2 \beta L}{2 a - \beta} \right) \] (11)

If formulae (8) and (11) are put into (1), the analytic formula for calculating the resistance of the trapezoidal TSV at a high surface ratio can be obtained.

\[ R_{\text{tri}}^{\text{tri}} (f) = \left( R_{\text{GIt}-\text{tri}}^{\text{tri}} - R_{dc}^{\text{tri}} \right) \cdot \frac{f}{f_{\text{GIt}-\text{tri}}} + R_{\text{GIt}-\text{tri}}^{\text{tri}} \] (12)
2.2. Derivation of the Resistance Parameter Formula of the Rough Surface TSV

The height distribution of the rough surface TSV is random. To simplify the calculation of the $s$, the surface is assumed to be a convex with regular changes. The model is as shown in Figure 3. The greater height of projection indicates a rougher surface. The roughness can be described by the root mean square of height.

$$RMS = \sqrt{\frac{1}{\text{length}} \int_0^{\text{length}} h^2(x) \, dx}$$  \hspace{1cm} (13)

where, $h$ is the height of the surface of the TSV.

According to the formula of direct current resistance of the cylindrical TSV, the parameter formula of direct current resistance of the rough surface TSV can be derived as follows:

$$R_{dc}^{\text{rough}} = \frac{L}{\sigma \pi r \left( r + RMS \cdot e^{-\sqrt{RMS}} \right)}$$  \hspace{1cm} (14)

where, $r$ is the radius of the TSV.

When the frequency is 1 GHZ, the analytic formula derived from the rough surface TSV resistance parameters is:

$$R_{GHz}^{\text{rough}} = \begin{cases} k_1 L / \sigma \pi \left[ r^2 - (r - \delta - h)^2 \right] & (\delta < r) \\ k_2 L / \sigma \pi \left[ r + RMS \cdot e^{-\sqrt{RMS}} \right] & (\delta \geq r) \end{cases}$$  \hspace{1cm} (15)

After the coefficients, $k_1$ and $k_2$, are adjusted, the expression is as follows:

$$\begin{cases} k_1 = 1.69^{0.05} + RMS^{0.02} + 1.96^{0.05} - e^{-h} & (\delta < r) \\ k_2 = 0.05^{0.05} + RMS^{1.20} + 1.96^{0.05} - e^{0.197} & (\delta \geq r) \end{cases}$$  \hspace{1cm} (16)

Then, the formula for the resistance parameter derived from the rough surface TSV is as follows:
\[
R_{\text{rough}}(f) = \left( R_{\text{dc}} - R_{\text{dc}}^{\text{rough}} \right) \sqrt{\frac{f}{f_{\text{dc}}}} + R_{\text{dc}}^{\text{rough}}
\]

(17)

3. Results
The formula (12) and (17) are the analytical expressions based on the two variables: frequency and size of the through silicon via. In order to verify the correctness of the analytical formulae, the analytical calculation results are compared with the simulation results of the Q3D software. Fig. 4(a) shows that when the diameter of the bottom edge is 20um and the surface ratio is 15, the resistance value of the trapezoidal TSV changes with frequency, and 4(b) is the error between the calculated value and the simulated value of the formula (12). Fig. 5 shows the resistance value of the rough surface TSV when the bottom diameter is 10um, the face ratio is 5, and RMS is 0.4 um and 0.14 um. And from the figure, it can be seen that the analytical calculation results are consistent with the simulation results. The error can be controlled within 5 %.

![Fig. 4](image1)
![Fig. 5](image2)

4. Conclusion
In order to calculate the resistance value of the irregular shaped TSV quickly and accurately, this paper deduced the analytical formulae of the two typical structures, the trapezoidal TSV and the rough surface TSV. In both of the two analytical formulae, the variables are the frequency and size of the
TSV. Compared with the simulation results, the calculation results of analytical formulae are accurate and the error is less than 5%.

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