Multi-scale analysis of rural and urban areas: A case study of Indian districts

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Abstract: It is well known that the urban systems, in particular cities, display scaling behaviour regarding socio-economic, infrastructural and individual basic services indicators. However, understanding urbanisation and the links between rural and urban areas is fundamental to making the most of the global transformations happening around the world. In this context, it is important to study the scaling laws based on both the urban and rural regions, going beyond cities. This paper explores the extension of the idea of allometric urban scaling law to study the scaling behaviour of Indian districts, with both the urban and rural population. To proceed, we have chosen districts (both rural and urban) of India, a relatively larger local administrative units, which are more or less independently functional within a country. This interdisciplinary work focus on the scaling analysis of various socio-economic indicators (SEIs) corresponding to the size (population) of four distinct urbanization classes, namely rural, semi-rural, semi-urban and urban districts. The scaling exponents (\(\beta\)) were estimated for each classes for the years 2001 and 2011 along with their goodness-of-fit measured by the \(R^2\) values. Our rigorous statistical analysis indicates that the scaling laws indeed exist even at the district level for most of the SEIs considered, related to education, employment, housing, health, etc.; the \(R^2\) values obtained for these SEIs are very high (often greater than 0.8 or 0.9) in both the the years. Moreover, linearity of the scaling factors have been statistically tested and it has been found, at 95% level of confidence, that not all the SEIs behave linearly (\(\beta = 1\)); some of them are characterized by super-linear (\(\beta > 1\)) behaviour and some behave sub-linearly (\(\beta < 1\)). Statistical hypothesis tests have also been performed to test the equality of two scaling factors corresponding to two distinct classes and two different years to understand the differences in scaling relationships among increasing urbanisation classes and their changes over time.

Keywords: Scaling Laws of Rural and Urban Population, Socio-Economic indicators; Estimation of Scaling Exponents; Statistical Hypothesis testing; Testing Linearity of the Scaling Exponents.

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I. INTRODUCTION

Scaling is one of the simplest rules that describes the underlying behaviour of a complex system [1]. Scaling, as a manifestation of the fundamental dynamics, has been conducive to scientists gather deeper insights about problems ranging across the entire spectrum of science and technology. The analytic framework of scaling may help one to model the dependence of aggregate properties of systems on their sizes. In the literature we can find numerous examples of scale invariance properties including earthquakes, clouds, networks, etc. [2–4]. Scale invariance seems to be widespread in natural systems. Correct evaluation of the scaling exponents is of primary importance in assessing the existence of universality classes [5]. It has been widely used in biological, ecology and physical complex systems [6].

Scaling has also been used to model urban growth in cities through self-similarity [7–16]; the essential properties of cities in terms of their infrastructure and socio-economics are functions of their population size in a scale invariant way and these scale transformations are common to all urban systems. Urban scaling shows how urban indicators change with the size of city (generally population) and predicts several underlying features like economic diversity, innovation, productivity and many more [17, 18]. It is important to study such scaling laws of various socio-economic indicators associated with the urbanisation in a country.

Urbanisation is a socio-economic transformation that converts rural settlements to urban settlements by building infrastructural facilities, increasing the population of urban areas in comparison to rural areas. Beyond such simplistic understanding, it is indeed an economic, physical, socio-cultural change and economic activities change giving way to agglomeration and subsequent urbanization. Even after cities emerged, however, a large majority of people lived and worked in rural areas. It has been projected that by 2050, the world’s population will be one-third rural and two-thirds urban, roughly the reverse of the situation in the mid-twentieth century [19].

It is interesting to note that Asia and Africa together cover 90% [20] of world’s rural population with the largest rural populated country being India (893 million). In recent years, however, Asian and African countries are portraying a rapid accelerating urbanisation. For example, the population residing in urban areas in India was 11.4% in 1901, which increased to 28.53% by the 2001 census [21]. In the year 2011, for the first time since independence, the absolute increase in population in India was more in urban areas than in rural areas. Level of urbanization increased from 27.81 % in 2001 Census to 31.16 % in 2011 Census [21]. This data inspired us to study a comparative scaling analysis of the distributional features of various socio-economic indicators associated with
the urban and rural population in India, a country with diverse geographical, economic and cultural background [22]. The effect of urbanization can be realized by the proper information of socio-economic trends which can, in turn, help us studying its sustainability over a longer period of time. To be specific, understanding the scaling features of various socio-economic indicators, associated with both the urban and rural population in a country, is essential to make sure that the appropriate resources and services are available where they are needed. This challenging study may provide a new prospect of urban scaling analysis.

It is well known that in the urban scaling analysis, scaling laws are mostly studied in respect of cities, where cities are used as independent functioning population units with the underlying rationale being the idea of agglomeration. Despite advances in the study of science in cities, we still lack a unified theoretical framework for the spatial distributions of various elements within cities and macroscopic scaling laws across cities. These results indicate the existence of high variability of scaling exponents with respect to variations in the definition of city or urban areas [23, 24].

There is another practical difficulty in studying the socio-economic properties of cities in respect of scaling analysis, since most official statistics on socio-economic indicators pertain to somewhat arbitrarily defined administrative units, which, at some level, are not cities at all. Examples include counties or census tracts in the USA, several forms of local authority in the UK, prefectures in Japan, prefecture and districts in China, districts in India and municipalities in many European and South American nations [25].

With this backdrop one may try to formulate a unified framework of scaling, comprising both the rural and urban population, implementing the idea of city based urban scaling analysis. It is important to perceive how the change in various rural- and urban- level factors with respect to the population-size can be instrumental in creation of a unified theory of scaling in the rural and urban areas. This inspired us to extend the idea of city based urban scaling analysis at the level of Indian districts consisting of both the urban and rural population. These studies may reveal the general characteristics of infrastructure and services in the urban as well as rural areas of a nation like India. There are a few studies on urban scaling in this direction. Recently urban scaling has been studied in Indian slums using Census data [26], the scaling law has been studied in intra-urban administrative units [27] and also there is a scaling analysis of rural housing land transition under China’s rapid urbanization [28]. However, it is unclear whether scaling law exists for a more general administrative unit which includes both rural and urban areas. In general, for the administrative purpose and the ease of working in a local manner, the states or provinces (first tier division i) in a large country, like China or India, are subdivided into more smaller
(second tier) strata called counties or districts. Although there is a wide heterogeneity among the administrative units (states or districts) in a country, the local administrative units can be seen as socially constructed strata [22], which serve as spatial scenarios for social and economic processes [29]. Such administrative divisions are often regions of a country that are granted a certain degree of autonomy and manage themselves through local governments. So, the scaling analysis done at the level of such local administrative units in a country (e.g., districts in India) is expected to be more comprehensive as well as challenging.

In this paper, our main goal is to empirically verify if the scaling law exists across somewhat different and larger administrative units (other than cities). For this purpose, we consider all the Indian districts, as administrative functioning units, which are surely independent, and investigate the existence of the scaling laws of a few important socio-economic variables in respect to their population. However, the geographic, cultural, economic or political diversity of Indian districts are well known; most districts in India have both rural and urban populations, except for a few large cities which are districts by themselves. And, it has recently been reported that consideration of all the districts, of all the states, in India do not show agglomeration economies as its geographical diversity is critical to emergence of the scaling law [30]. So, we appropriately classify the districts in more homogeneous classes with respect to their urbanisation status. Different classification criteria [31] have been proposed and widely applied to find the distinct types of urban, suburban regions of USA and China [32, 33]. In the context of urbanisation in India, features of classification may serve as an important tool in studying the diverse nature of Indian districts. Economic, demographic, and spatial structures may play an important role to find out the distinct classes. As our primary focus is to see the effect of urbanisation in Indian states, we set an urbanisation index, the ratio of urban and rural population of each district in India, as the classification parameter to find out four distinct classes, namely the rural, semi-rural, semi-urban and urban (see Section II B for details).

In the context of rapid urbanisation in India, it is very important and also challenging to study the scaling laws in respect to these four urbanization classes separately. The result of linear or non-linear scaling (super-linear or sub-linear) may help one to frame future unified policies for different districts in India.

In summary, the contributions of the present paper is many fold.

- Firstly, we statistically test for the existence of scaling laws at district level administrative units of India, comprising both the rural and urban population, by using the census data of the years 2011 and 2001 for various socio-economic indicators. As far as our knowledge
In the relevant literature we find no study with such a large amount of data in Indian context. Also the existing studies were mostly focused on cities whereas we empirically demonstrate the existence of scaling laws, in particular allometric scaling laws, across the districts of India within appropriate urbanisation classes, suggesting a multi-scale analysis in the spatial morphology leading to a new prospect for urban scaling analyses.

- Secondly, we estimate the scaling exponent $\beta$ for all the distinct urbanization classes. We adopt the method of statistical hypothesis testing to ensure the robustness of our analysis in estimating the power law exponent $\beta$. We statistically test for the linearity ($\beta = 1$) against sub or super linearity ($\beta < 1$ or $\beta > 1$) of the scaling index rather than deciding just based on their values. It has already been shown that the process of measuring the power law exponent needs a rigorous statistical approach [34]. It helps us to put confidence in our conclusion taking care of the sampling fluctuations and other sorts of distortion present in the empirical data.

- Finally, based on these results, we comment, with statistical confidence, on the allometric scaling laws of distinct urbanisation classes in India. Our scaling analysis portray the effects of urbanisation on different socio-economic parameters along with their changes from the year 2001 to 2011.

The rest of the paper is organized as follows. Section II is devoted to explain the data and the methodology that has been used for this study. Section III presents detailed discussion on the results of the analyses. Finally, we summarize and conclude the paper in IV.

II. MATERIALS AND METHODS

A. Research Data

We consider the data from Indian census which is conducted in every 10 years to capture a detailed picture of demographic, economic and social conditions of all persons in the country pertaining to that specific time. The raw census data for the years 2011 and 2001 are obtained from the Primary Census data and Digital library (www.censusindia.gov.in). Following the stratified administrative structure of India, we use the districts as the smaller second-tier units within each state (first-tier strata) of India. According to the final census in 2011, there were 29 states and 7 Union territories (UT) in India, which consist of the pool of strata in our analysis. It is important to note that some
states, created at a later time, were not present in the earlier round of census (at the year 2001). The total number of districts comprising all the states (and UTs) of India was 640 and 593 in the years 2011 and 2001, respectively.

The data shows wide inter-State disparity in urbanization level [35]. In terms of overall urban population, Maharashtra had the largest urban population of 50.8 million followed by Uttar Pradesh, which had an urban population of 44.5 million in 2011. If we look at the level of urbanization, defined as urban population as a proportion of total population, Goa was the most urbanized state with 62.17% urbanization in 2011 followed by Mizoram at 52.11% urban population. Among the Union Territories, Delhi had urbanization level of 97.50% followed by Chandigarh with an urbanization level of 97.25% in 2011. Himachal Pradesh had the lowest urbanization with only 10.03% population living in urban areas in 2011, followed by Bihar (11.29%).

According to the publicly available data, various socio-economic parameters of all the districts of India have been selected for our study. In particular, for a wide spread analysis, we have chosen 14 socio-economic indicators (SEIs) from sectors such as education, employment, housing, health, etc., keeping in mind the diversity of the indicators in different domains of life as well the availability of reliable data. We refer to these indicators as Total Literate, Informal-Literate, High-Literate, Not-Literate, Main-Worker, Marginal-Worker, Workplaces (offices, factory, workshop, work-shed, etc.), Residence-Places, Market-Places, Education-Places (school, colleges, etc.), Tourist-Places, Medical-Places (hospital, etc.), High-Literate-Women, Main-Worker-Women [36]. Explicit definitions of all these SEIs are provided in Appendix A.

B. Classification of Indian districts according to their urbanization status

Keeping in mind the wide inter-state disparity in the urbanization level across the country, in this new prospect of scaling analysis at the district level, corresponding to both the rural and urban population, we would like to appropriately classify the districts in homogeneous classes with respect to their urbanisation status. To start with, we first characterize the urbanisation in any district of India by the ratio of the urban and rural population in that district, which we call the Urban-Rural ratio (URR). Based on this urbanization index URR, all the districts of India may be divided into 4 distinct classes. Naturally, the small values of URR will tune with the Rural Class and the districts with URR $> 1$ will correspond to the Urban Class. The interim values of URR may be termed as Semi-Rural ans Semi-Urban districts. For our analysis, we have distinctly classified the rural, semi-rural, semi-urban and urban districts as:
1. Class-A (Rural): Rural districts are represented by URR in between 0 and 0.1. In this class, there are 108 and 126 districts in the year 2011 and 2001, respectively.

2. Class-B (Semi-rural): URR of these districts are in between 0.1 and 0.3. There are 271 districts in this class in 2011 and 242 districts in 2001.

3. Class-C (Semi-urban): The semi-urban districts correspond to URR in between 0.3 and 1. The census data show that there are 180 and 167 semi-urban districts in the years 2011 and 2001, respectively.

4. Class-D (Urban): URR of these districts are above 1 and are classified as urban districts of India. Accordingly, there are 81 and 55 urban districts in 2011 and 2001 respectively.

See Figure 1 for a spatial distributions of these four urbanisation classes in the year 2011. We may point out here that the cut-offs 0.1 and 0.3 used in the classification of districts are rather ad hoc; but they lead to the desired homogeneity within each of the resulting four classes in respect to the urban scaling as we will see in our subsequent analyses.

**Remark 1**: It has been verified that the main conclusions about the nature and validity of the scaling law across districts within these four urbanisation classes do not differ significantly if these cut-offs are changed a little.
FIG. 1. Spatial distribution of 4 distinct classes (classified according to the urbanization index URR) of Indian districts in the year 2011—Rural (108), Semi-Rural (271), Semi-Urban (180) and Urban (81).
C. From the Empirical Data to the Scaling Law

The urban scaling law encompasses a wide range of complex urban phenomena, including city fractals, Zipf’s distribution, allometric growth, 1/f noise, power-law distance decay, scale-free network, and self-similar hierarchy of cities with cascade structure. [See for example [37]]. However, for our analysis we use the common model, the allometric urban scaling law [30, 38, 39] that is mathematically represented as

\[ Y_i = Y_0 N_i^\beta e^{\zeta_i}, \]  

where \( N_i \) represents the population of the \( i \)-th district of a given urbanisation class at a given time point (2001 or 2011) and \( Y_i \) represents the measure of a targeted SEI corresponding to that district. It follows from Eqn. (1) that there is a linear relationship between \( \log Y_i \) and \( \log N_i \). Thus, the scaling factor \( \beta \) measures the average relative change of \( Y_i \) with respect to \( N_i \) in the logarithmic scale, and \( \zeta_i \) measures the deviance of the \( i \)-th individual district from the scaling law (average relationship pattern) of the corresponding class.

Based on our empirical data on population and any given SEI, we fit the scaling law (1) separately for the four urbanisation classes and for the years 2001 and 2011. For any given year and class, the corresponding values of the scaling factor \( \beta \) is estimated by the ordinary least square (OLS) method applied to the linear regression model of \( \log Y_i \)'s on \( \log N_i \)'s with \( i \) varying over all districts within that particular class. More precisely, the OLS estimate of \( \beta \) is obtained as

\[ \hat{\beta} = \frac{\langle \log Y_i \log N_i \rangle - \langle \log Y_i \rangle \langle \log N_i \rangle}{\langle \log N_i^2 \rangle - \langle \log N_i \rangle^2}, \]  

where \( \langle \cdot \rangle \) represents the expectation (mean) operator. The process also provides an estimate of \( Y_0 \) as

\[ \log \hat{Y}_0 = \langle \log Y_i \rangle - \hat{\beta} \langle \log N_i \rangle \]  

and that of the individual deviance \( \zeta_i \) as

\[ \hat{\zeta}_i = \log Y_i - \log \hat{Y}_0 - \hat{\beta} \log N_i \]  

for any \( i \). The overall model error variance \( (\sigma^2) \) can then be estimated by the variance of the deviances as

\[ \hat{\sigma}^2 = \frac{n}{n-2} \langle \zeta_i^2 \rangle, \]
where \( n \) is the number of districts in that particular class.

**Goodness-of-fit Measure**

After fitting the linear regression model, we need to determine how well our model fits the data. We use the \( R^2 \) goodness-of-fit measure for linear regression models and test the appropriateness of the scaling law for the given empirical data. The popular coefficient of variation (referred to as \( R^2 \)) measure with higher values (close to one) indicates better fit. The full estimation process is implemented by using the built-in functions in the software MATLAB.

**Remark 2** We would like to mention that, Shalizi [40] proposed using the per capita measures instead of the raw (extensive) values for the SEIs, which automatically show a “mass” effect. If we use the per capita values of the SEIs \( \frac{Y_i}{N_i} \) in place of \( Y_i \) in Equation (1), the resulting form of the scaling law would be

\[
\frac{Y_i}{N_i} = Y_0 N_\gamma e^{\zeta_i}.
\]

(6)

By comparing it with Equation (1), it can easily be seen that the scaling law in the per capita modeling must satisfy \( \gamma = \beta - 1 \). By the location equivariance of our estimation method (OLS), the estimate of \( \gamma \) would then be \( \hat{\beta} - 1 \) and all subsequent inference would be equivalent. Only the linearity of the scaling exponent \( (\beta = 1) \) in our formulation (1) would be equivalent to the zero value (non-significance) of the scaling exponent in the per-capita formulation of the problem. Due to such equivalence, we have decided to continue with the well-known form of the scaling law as given in (1) in our present paper.

**D. Scaling Exponent and Statistical Hypothesis Testing**

Hypothesis testing is a systematic method for deciding whether a parameter (e.g., scaling factor) in a population supports a specific theory based on our empirical (sample) data.

When there is an ambiguity in taking a decision, statistical tests give a quantitative indication helping to deduce a conclusion with certain level (often 95%) of confidence. In the present context, two types of hypothesis will be considered to (i) test the linearity of the scaling factor and to (ii) test the equality of the scaling factors.

**Test for linearity of Scaling factor:**

Firstly, we consider the null hypothesis that a given SEI (say \( Y_i \)) scale linearly, i.e., \( H_0 : \beta = 1 \)
for the scaling factor $\beta$ of that particular class, which we statistically test against the omnibus alternative $H_1 : \beta_1 \neq 1$. For this purpose, we use the test statistics ($t_{\text{stat}}$) computed as

$$t_{\text{stat}} = \frac{\hat{\beta} - 1}{SE(\hat{\beta})},$$

where the standard error of the estimated scaling factor $\hat{\beta}$ is given by

$$SE(\hat{\beta}) = \hat{\sigma} \left( \langle \log N_i^2 \rangle - \langle \log N_i \rangle^2 \right)^{-1/2}$$

The probability of getting the t-value equal or greater than $t_{\text{stat}}$ is known as the p-value, which can be easily computed under normality assumption (that holds for classes with large number of districts). The greater the value of $t_{\text{stat}}$, less the p-value is and, hence, the more affirmation against null hypothesis is obtained. More precisely, if the resulting p-value is less than 0.05, we reject the null hypothesis (linearity of the scaling factor) at 95% level of significance and infer the scaling law to follow a positive or negative scaling relation according to the sign of $(\hat{\beta} - 1)$. The scaling is termed as super-linear for $\beta > 1$ and sub-linear for $\beta < 1$.

**Test for equality of Two Scaling factors:**

The other interesting aspect is to compare the scaling law fitted to two classes (or at two time points) by testing for the equality of their respective scaling factors, say $\beta_1$ and $\beta_2$. The corresponding null and alternative hypotheses would be $H_0 : \beta_1 = \beta_2$ and $H_1 : \beta_1 \neq \beta_2$, respectively. Let us denote the estimated scaling factors in the two classes (or two time-points) by $\hat{\beta}_1$ and $\hat{\beta}_2$, respectively, which are obtained based on the data on $n_1$ and $n_2$ districts in the respective classes. For this testing procedure, we use the two-sample test statistics given by

$$t^{(2)}_{\text{stat}} = \frac{\hat{\beta}_2 - \hat{\beta}_1}{\sqrt{SE(\hat{\beta}_1)^2 + SE(\hat{\beta}_2)^2}},$$

where the standard errors (SEs) are obtained as before from the empirical data of the respective classes. Under appropriate distributional assumptions, the statistics $t^{(2)}_{\text{stat}}$ follows a $t$-distribution with degrees of freedom $df = \frac{(SE(\hat{\beta}_1)^2 + SE(\hat{\beta}_2)^2)^2}{\frac{SE(\hat{\beta}_1)^4}{n_1} + \frac{SE(\hat{\beta}_2)^4}{n_2}}$, which helps us to compute the corresponding p-values. We infer that the two classes behave according to significantly different scaling rules (at 95% level) if the resulting p-value is obtained to be less than 0.05.

For both the above-mentioned testing procedures, computation of the p-values is done using the software MATLAB.
III. RESULTS AND ANALYSIS

A. Allometric Scaling Laws for the Rural and Urban Districts

Following the methods described in Sec.II we have analysed the scaling features associated with various SEIs for both the years 2011 and 2001. Firstly, we have estimated the values of the scaling exponents for the pool of all Indian districts (640 in the year 2011 and 590 in the year 2001) and then for the four distinct classes—rural, semi-rural, semi-urban and urban separately. The estimated values of the (district level) scaling factors for each of the four urbanisation classes are presented in Table I, along with the same obtained for the pool of all districts of India, for both the years 2011 and 2001. Further, as a measure of the goodness-of-fit for the respective scaling laws of all the SEIs in different classes, the corresponding $R^2$ values are also presented in the same Table I. See Figure 2 below and Figures S1-S3 in the Supplementary Material for graphical representations of the fitted Scaling Laws for different SEIs in the years 2001 and 2011.

It is remarkable to note that our result shows the existence of the allometric scaling laws even at the level of Indian districts for all education related SEIs (Total-Literate, Informal-Literate, High-Literate, Not-Literate) as well as for the Main-Workers, Residence-Places and Medical-Places; the $R^2$ values obtained for these SEIs are very high (often greater than 0.8 or 0.9) in both the years 2011 and 2001. More interestingly, the scaling law behaves in a robust manner for all these SEIs (except for Medical-Places) yielding high $R^2$ (and hence greater validity for the fitted scaling law) at all the four urbanisation classes. However, class-wise scaling factors deviate from the average (all India level) scaling factor with their values being below the average in rural areas (Class A) for most SEIs.

For the SEIs, for which the validity of the scaling law is not very satisfactory (slightly lower $R^2$ values) at all India districts level, the existence of the scaling law becomes more prominent with greater validity (higher $R^2$ values) as we restrict ourselves within the urbanisation classes with greater urban populations. In particular, among the urban districts (Class D), all our SEIs (except only for Tourist-Places) exhibit strong evidences (higher $R^2$ values greater than 0.8) of the existence of scaling law in 2011. The values of $R^2$ was a bit lower for these SEIs in the year 2001. Thus, in general, the existence of scaling law in all Indian district level data is seen to become more prominent over time, and also for classes with higher URR at any given time-point. Particularly at the year 2011, the validity of scaling law weakens considerably as we move continuously from Class D to Class A (urban to rural) for High-Literate-Women, Main-Worker-Women and all SEIs.
**TABLE I.** Estimated scaling factors ($\beta$) for all SEIs at the district level, within four urbanisation classes (A–D) and also across all India. The $R^2$ values indicating the goodness-of-fit for the corresponding scaling law is given in the respective parenthesis.

| SEI               | All India | Class A | Class B | Class C | Class D |
|-------------------|-----------|---------|---------|---------|---------|
|                   | All Districts | Rural   | Semi-rural | Semi-urban | Urban   |
| Total-Literate    | 1.001 (0.97) | 0.955 (0.97) | 0.995 (0.98) | 0.981 (0.99) | 0.987 (1.00) |
| Informal-Literate | 1.103 (0.87) | 1.136 (0.90) | 1.068 (0.83) | 1.123 (0.86) | 1.068 (0.91) |
| High-Literate     | 1.117 (0.82) | 0.929 (0.82) | 1.076 (0.88) | 1.051 (0.92) | 1.088 (0.92) |
| Not-Literate      | 1.005 (0.93) | 1.072 (0.95) | 1.012 (0.95) | 1.045 (0.95) | 1.042 (0.96) |
| Main-Workers      | 0.964 (0.94) | 0.856 (0.94) | 0.934 (0.93) | 0.982 (0.96) | 1.018 (0.98) |
| Marginal-Worker   | 0.933 (0.77) | 1.015 (0.90) | 0.989 (0.86) | 0.994 (0.86) | 1.004 (0.84) |
| Workplaces        | 0.979 (0.70) | 0.746 (0.66) | 0.907 (0.74) | 0.971 (0.77) | 1.041 (0.77) |
| Residence-Places  | 0.998 (0.98) | 0.945 (0.97) | 0.998 (0.97) | 1.001 (0.98) | 1.010 (0.98) |
| Market-Places     | 0.987 (0.79) | 0.795 (0.70) | 0.978 (0.81) | 0.973 (0.84) | 0.953 (0.87) |
| Education-Places  | 0.722 (0.76) | 0.630 (0.75) | 0.749 (0.78) | 0.777 (0.81) | 0.815 (0.82) |
| Tourist-Places    | 0.751 (0.59) | 0.510 (0.48) | 0.701 (0.57) | 0.828 (0.67) | 0.714 (0.63) |
| Medical-Places    | 0.955 (0.85) | 0.704 (0.71) | 0.947 (0.87) | 0.976 (0.91) | 1.046 (0.92) |
| High-Literate-Women | 1.119 (0.73) | 0.858 (0.69) | 1.047 (0.81) | 1.052 (0.87) | 1.095 (0.90) |
| Main-Worker-Women | 0.866 (0.71) | 0.728 (0.69) | 0.799 (0.63) | 0.925 (0.73) | 1.004 (0.87) |

**Year 2001**

| SEI               | All India | Class A | Class B | Class C | Class D |
|-------------------|-----------|---------|---------|---------|---------|
|                   | All Districts | Rural   | Semi-rural | Semi-urban | Urban   |
| Total-Literate    | 0.993 (0.95) | 0.903 (0.94) | 0.999 (0.94) | 0.982 (0.97) | 0.989 (0.99) |
| Informal-Literate | 1.084 (0.76) | 1.066 (0.83) | 1.077 (0.69) | 1.168 (0.77) | 1.076 (0.72) |
| High-Literate     | 1.116 (0.81) | 1.010 (0.87) | 1.049 (0.86) | 1.068 (0.89) | 1.065 (0.87) |
| Not-Literate      | 1.009 (0.93) | 1.086 (0.97) | 1.001 (0.92) | 1.033 (0.95) | 1.038 (0.97) |
| Main-Workers      | 0.917 (0.90) | 0.779 (0.85) | 0.929 (0.92) | 0.965 (0.96) | 1.021 (0.84) |
| Marginal-Worker   | 0.918 (0.77) | 0.959 (0.88) | 0.941 (0.82) | 1.006 (0.88) | 0.998 (0.77) |
| Workplaces        | 1.003 (0.73) | 0.818 (0.71) | 0.973 (0.73) | 0.988 (0.75) | 1.064 (0.82) |
| Residence-Places  | 0.986 (0.97) | 0.950 (0.97) | 0.995 (0.96) | 0.991 (0.98) | 1.000 (0.99) |
| Market-Places     | 1.026 (0.79) | 0.881 (0.76) | 1.002 (0.78) | 1.030 (0.85) | 0.980 (0.89) |
| Education-Places  | 0.738 (0.77) | 0.629 (0.74) | 0.779 (0.74) | 0.819 (0.84) | 0.794 (0.84) |
| Tourist-Places    | 0.738 (0.55) | 0.453 (0.41) | 0.705 (0.45) | 0.834 (0.67) | 0.818 (0.75) |
| Medical-Places    | 0.989 (0.86) | 0.819 (0.77) | 1.002 (0.84) | 1.014 (0.92) | 1.054 (0.96) |
| High-Literate-Women | 1.126 (0.68) | 0.905 (0.73) | 1.013 (0.73) | 1.092 (0.81) | 1.076 (0.85) |
| Main-Worker-Women | 0.781 (0.62) | 0.604 (0.60) | 0.747 (0.49) | 0.866 (0.68) | 0.945 (0.76) |
related to places; their $R^2$ values indeed become less than 0.8 in Class A, except for the Residence-Places that still has high $R^2 = 0.97$ in Class A. An interestingly opposite trend is observed with Main-Workers, for which the degree of validity towards scaling law decreases from Class A (with $R^2 = 0.9$) to Class D ($R^2 = 0.84$). Figure 3 shows the plot of the $R^2$-values as a function of the estimated scaling exponent for both the years 2001 and 2011; we can clearly see that, in most cases, the fits of the scaling law are better, having a higher $R^2$ on average than others, for the SEIs which yield scaling exponents close to 1.
FIG. 3. Plot of the goodness-of-fit measures ($R^2$) vs. the estimated scaling exponent ($\beta$) for the years 2001 and 2011. $R^2$ values are higher (depicting good fit) for values of $\beta$ close to 1.

We may note the change in the $\beta$ values corresponding to the various SEIs in a decade (2001 to 2011) from the Table I. Interestingly, within this period, for most of the variables the change in $\beta$ is very small. To be specific, the changes in $\beta$ are in the range [0.002 - 0.059] only within the urban class (Class D) except for the Tourist Places where the change is 0.104. For the Class C districts, the change in $\beta$ is within the range of [0.001 - 0.059], whereas it is within [0.004 - 0.055] for the Class B. For the Rural population (Class A) the change in $\beta$ is a little heterogeneous; the range is [0.052 - 0.124] for most of the variables except for the Not literate, Residence Places, Education Places and Tourist Places where the change is small, within the range [0.001 - 0.014] only.

B. Linearity of the scaling index: a statistical test

It is stimulating to statistically examine the values of the estimated scaling exponents from Table I to see if the corresponding SEI deviates from the linear scaling law ($\beta = 1$) and follows a sub-linear ($\beta < 1$) or super-linear ($\beta > 1$) pattern. Actually, the super-linear allometric relation corresponds to positive allometric scaling relation, whereas the sub-linear allometric relation corresponds to the negative allometric scaling relation. The linearity of the scaling exponent is sometimes termed as isometric growth in the literature. (See for example, [39, 41]).
To adjust for the sampling fluctuations near the value one, we have statistically tested if the estimated scaling factor deviates significantly from one (at 95% level). Consistently, we comment on the nature (or behavior) of the scaling law for all SEIs in different urbanisation classes which are reported in Table II; the exact p-value for each test are provided in Table S1 of the Supplementary material. Recall that these p-values are obtained from a statistical test for the null hypothesis of linearity of the scaling exponent (as described in Section II.D), and hence, smaller the p-values we have larger evidence against the null hypothesis (linearity) so that we reject it in favour of non-linearity of the scaling exponent.

| SEI                  | Class A | Class B | Class C | Class D | Class A | Class B | Class C | Class D |
|----------------------|---------|---------|---------|---------|---------|---------|---------|---------|
|                      |         |         |         | Year 2011 |         |         | Year 2001 |         |
| Total-Literate       | Sub-linear | Linear | Sub-linear | Linear | Sub-linear | Linear | Linear | Linear |
| Informal-Literate    | Super-linear | Super-linear | Super-linear | Linear | Linear | Linear | Super-linear | Linear |
| High-Literate        | Linear | Super-linear | Super-linear | Super-linear | Linear | Linear | Linear | Linear |
| Not-Literate         | Super-linear | Linear | Super-linear | Linear | Super-linear | Linear | Linear | Linear |
| Main-Workers         | Sub-linear | Sub-linear | Linear | Linear | Linear | Linear | Linear | Linear |
| Marginal-Worker      | Linear | Linear | Linear | Linear | Linear | Linear | Linear | Linear |
| Workplaces           | Sub-linear | Sub-linear | Linear | Linear | Linear | Linear | Linear | Linear |
| Residence-Places     | Sub-linear | Linear | Linear | Linear | Linear | Linear | Linear | Linear |
| Market-Places        | Sub-linear | Linear | Linear | Linear | Linear | Linear | Linear | Linear |
| Education-Places     | Sub-linear | Sub-linear | Sub-linear | Sub-linear | Sub-linear | Sub-linear | Sub-linear | Sub-linear |
| Tourist-Places       | Sub-linear | Sub-linear | Sub-linear | Sub-linear | Sub-linear | Sub-linear | Sub-linear | Sub-linear |
| Medical-Places       | Sub-linear | Sub-linear | Linear | Linear | Linear | Linear | Linear | Linear |
| High-Literate-Women  | Sub-linear | Linear | Linear | Sub-linear | Linear | Linear | Linear | Super-linear | Linear |
| Main-Worker-Women    | Sub-linear | Sub-linear | Linear | Linear | Sub-linear | Sub-linear | Linear | Linear |

It is to be noted from Table II that in the urban Class D, most of the SEIs corresponding to individual basic needs (education, house) behave linearly with respect to the population in both the years 2011 and 2001, except for Education-Places, Tourist-Places, High-Literate and High-Literate-Women. However, Education-Places and Tourist-Places are seen to follow sub-linear scaling relationship in both the years 2001 and 2011. Interestingly, the scaling factor for SEI related to higher education, linear in the year 2001 but changes in 2011. SEIs connected to economic activities in a country, namely Main-Workers, Marginal-Workers and Workplaces scale linearly in urban Class D and becomes sub-linear as the proportion of rural population increases in our urbanisation classes; Main-Workers and Workplaces indeed scales sub-linearly within the rural Class A in both the years. It is good to note that the sub-linear scaling relationship of the variable Main-Workers...
in 2001 has become linear in 2011.

Tables I and II indicate that the allometric scaling analysis in all the four urbanization classes have been analysed from the gender perspective also. The last 2. rows of Table II depicts the nature of the linearity of the scaling factors for the SEIs related to higher education in Women and Women Main workers corresponding to the years from 2001 to 2011 in all the 4 urbanization classes.. The Table I shows that the estimated scaling factors, corresponding to High-Literate and High-Literate-Women, at all India level is observed to be around $\beta \sim 1.12$ for both the years 2001 and 2011, implying a good sign for urban development.

It is understood that the super-linear scaling (positive allometric scaling) represents faster increment of the indicators with respect to the system size while sub-linear scaling (negative allometric scaling) indicates slower returns. The results in the Tables II and may help the policy makers to build the strategies for the betterment of the rural, semi-rural, semi-urban and urban population of the Indian districts. Care may also be taken to improve the infra-structural facilities in the rural India. For example, care may be taken so that the super-linear scaling of Not Literate variable for the Class A people at least becomes linear. As the number of main-workers and number of factories/workshops are correlated with each other, policy makers should encourage for more industry (Workplaces which is now sub-linear) in rural areas.

C. Comparison of the scaling exponents across different classes

We compare the scaling factors of various SEIs of the four distinct urbanisation classes using appropriate statistical tests and the findings obtained at 95% level of significance (Detailed results reported in Table III, Appendix B). The exact p-values of all the pairwise comparison tests are provided in Table S2 of the Supplementary Material.

We note that, most interestingly, there is no significant difference between the scaling factors obtained in Class C (semi-urban) and Class D (urban) for all SEIs indicating that they behave similarly in both semi-urban and urban classes; this is indeed true for both the years 2001 and 2011. While comparing with rural and semi-rural classes as well, we see that all the four classes have statistically similar scaling law behaviours (no significant differences) in terms of Informal-Literate and Marginal-Workers in both the years (2001 and 2011). There was a statistically significant difference of the rural Class A from the rest of the classes (B, C, D) in terms of Total-Literate in 2001 but that difference has almost vanished in the year 2011. On the contrary, there was no significant difference in the scaling law of High-Literate across the urbanisation classes in 2001,
but a significant discrimination has been developed in 2011 departing the rural Class A from the rest. These indicate that, over the 10 years (from 2001 to 2011), there has been changes in the number of highly educated people in rural India, which needs to be investigated further for proper policy making. The slight differences in the scaling behavior of Not-Literate across the urbanisation classes, however, seem to further decrease over time.

The scaling exponents $\beta$ of the economical SEIs, Main-worker and Workplaces, of the rural Class A is seen to differ significantly from the rest of the classes which are rather statistically similar in the year 2001. In the year 2011, however, the variable Main-Workers follows significantly different scaling law even between the rural and semi-rural classes but the same for Workplaces are still similar between these two classes. In terms of other infrastructural SEIs related to different types of places, there has been a clear discrimination between the scaling factors of the rural class versus the rest (Classes B, C and D, which do not differ significantly among themselves) in both the years 2001 and 2011.

IV. DISCUSSION AND CONCLUSION

In recent years, concept of scaling has become one of the attentive topics in the area of urban-rural research. The study of scaling law is important as it provides a new way to analyze scale-free systems. As a first attempt to formulate a unified framework of scaling, going beyond cities, comprising both the rural and urban population, we considered larger district-like administrative units. This interdisciplinary work, a multi-scale analysis based on rural and urban population, sheds light on the understanding of scaling laws emerging in Indian districts. Here, we have investigated the existence of allometric scaling laws at the district (second tier administrative units) level corresponding to the urbanization in India. We study the allometric scaling relations of several socio-economic indicators corresponding to the population of all Indian districts as well as the population (referred to size of the class) of the four distinct classes—rural, semi-rural, semi-urban and urban classes of Indian districts separately for the years 2001 and 2011. Adopting a rigorous statistical hypothesis test we check the linearity of the corresponding scaling exponents $\beta$. The linearity test helps us to inspect whether the estimated scaling index ($\beta$) for any particular socio-economic indicator or infra-structural variable behaves in a linear, sub-linear or super-linear manner indicating the rates of growth of the underlying factor with the population of the districts within a given urbanisation class.

We have classified all Indian districts in four groups (URR classes) based on the proportion of urban
to rural populations within the districts. We then investigated the scaling laws of all the Indian districts as well as within such groups so that the analysis may help to reduce the heterogeneity between districts in terms of their urbanization structure. The scaling exponents, associated with various SEIs of the rural, semi-rural, semi-urban and urban population classes have been estimated. Subsequently, the linearity of the scaling exponent has been inspected through a statistical hypothesis testing. Moreover, to understand the change in the scaling behaviour with time, we have presented a comparative analysis with the data for the years 2001 and 2011. Our results indicate that not all the scaling factors behave linearly, some of them characterize a super-linear (a positive allometric relation) behaviour and some behave sub-linearly (negative allometric relation). These results clearly illustrate that more investigation along this line would be really useful to further understand the underlying process of urbanizations within districts. As India stands today at the inflection point of urban transformation, we believe, this data based comprehensive study of allometric scaling in Indian districts will help in making the urban transformation more effective and accelerate future work in this area of rural-urban research.

The present multi-scale analysis with rural and urban population may set-off future study in the area of fractal analysis of urban scaling. With the model presented here, one can study the fractal/multi-fractal features of the allometric scaling exponents [42–44] associated with various SEIs of the rural and urban districts. Also one can make quantitative predictions about the economic gain that can be expected with certain changes in various socio-economic variables and its population size. This may be be aided by the estimated super-linear and sub-linear scaling exponents appearing in various SEIs relative to urban, semi-urban, semi-rural and rural population of Indian districts and their changes over time.

Urban scaling laws are deeply related with the ways people live in an area. Generally, cities within a district, considered as areas of settlement of the urban population, have many facilities of livelihood, whereas semi-towns, towns or villages comprising the increasing proportion of rural population lack the requisite infrastructure. It is important to perceive the variation in different rural- and urban-level factors with respect to the population-size to understand the underlying dynamics of the urbanization process. We believe that the present study reflects the actual administrative/economic/physical activities of rural and urban regions and reveal the general characteristics of infrastructure and services in the urban as well as rural areas of India. This comparative scaling analysis of rural and urban India within a unified framework, a significant contribution in the literature of allometric urban scaling law, may, in turn, guide the policymakers to take appropriate measures to maintain sustainable developments of these important socio-economic factors.
along with the continuous urbanisation of India.

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Appendix A: Definitions of our Socio-Economic Indicators (SEIs)

In the following, we briefly describe the definitions of all the socio-economic variables considered in our analysis [36].

- Total Literate: Total number of literate persons. A person aged 7 years and above, who is able to both read and write, is defined as a literate person.
- Informal-Literate: Number of literate persons who never go to any educational institute like school.
- High-Literate: Total number of persons, having a degree of graduation and above.
- Not-literate: Total number of illiterates. A person, who cannot read and write or can read but cannot write, is considered to be as illiterate along with all children of age on or below 6 years.
- Main-Workers: Total number of main workers. A person working for an economic productivity [36] (production and self-consumption also considered as economic activity) for more than 6 months is defined as a main worker.
- Marginal-Workers: Total number of marginal workers, defined as the person working for less than 6 months.
- Workplaces: Total number of Factory, workshop, workshed etc.
- Residence-Places: Total number of census houses used only for residential purposes. A ‘Census house’ is a building or part of a building used or recognized as a separate unit because it has a separate main entrance from the road or common courtyard or staircase etc. It may be occupied or vacant and may be used for residential or nonresidential purposes or both.
- Market-Places: Total number of census houses used only for business purposes like shops and offices.
- Education-Places: Total number of school colleges, etc.
- Tourist-Places: Total number of Hotel, lodge, guest house, etc.
- Medical-Places: Total number of Hospital and Dispensary, etc
- High-Literate-Women: Total number of women, having a degree of graduation and above.
• Main-Worker-Women: Total number of women who are main workers.

**Appendix B: Comparison of estimated Scaling factors**

The following Table III shows the details of the comparison of the scaling factors for both the years.

TABLE III. Results of the statistical tests to compare the estimated scaling factors between any two urbanisation classes (among A, B, C, D). H1 indicates that the corresponding scaling factors are significantly different at 95% level of significance; whereas H0 indicates that there is no evidence to conclude their differences to be statistically significant.

| SEI                  | A–D | A–C | A–B | B–C | B–D | C–D | A–D | A–C | A–B | B–C | B–D | C–D |
|----------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Total-Literate       | H0  | H0  | H1  | H0  | H0  | H0  | H1  | H1  | H1  | H0  | H0  | H0  |
| Informal-Literate    | H0  | H0  | H0  | H0  | H0  | H0  | H0  | H0  | H0  | H0  | H0  | H0  |
| High-Literate        | H1  | H1  | H1  | H0  | H0  | H0  | H0  | H0  | H0  | H0  | H0  | H0  |
| Not-Literate         | H0  | H0  | H1  | H0  | H0  | H0  | H0  | H1  | H1  | H0  | H0  | H0  |
| Main-Workers         | H1  | H1  | H1  | H1  | H1  | H0  | H1  | H1  | H1  | H0  | H0  | H0  |
| Marginal-Worker      | H0  | H0  | H0  | H0  | H0  | H0  | H0  | H0  | H0  | H0  | H0  | H0  |
| Workplaces           | H1  | H1  | H1  | H0  | H0  | H0  | H1  | H1  | H1  | H0  | H0  | H0  |
| Residence-Places     | H1  | H1  | H1  | H0  | H0  | H0  | H1  | H1  | H1  | H0  | H0  | H0  |
| Market-Places        | H1  | H1  | H1  | H0  | H0  | H0  | H0  | H1  | H1  | H0  | H0  | H0  |
| Education-Places     | H1  | H1  | H1  | H0  | H0  | H0  | H1  | H1  | H1  | H0  | H0  | H0  |
| Tourist-Places       | H1  | H1  | H1  | H1  | H0  | H0  | H1  | H1  | H1  | H0  | H0  | H0  |
| Medical-Places       | H1  | H1  | H1  | H0  | H1  | H0  | H1  | H1  | H1  | H0  | H0  | H0  |
| High-Literate-Women  | H1  | H1  | H1  | H0  | H0  | H0  | H1  | H1  | H0  | H0  | H0  | H0  |
| Main-Worker-Women    | H1  | H1  | H0  | H1  | H1  | H0  | H1  | H1  | H1  | H0  | H1  | H0  |

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