Economic-mathematical model of choice of heating system development option

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Abstract. The large number of the mathematical models describing the production and technological party of economy and intended for acceptance of effective solutions is so far developed. A considerable part of the constructed models to some extent is used in practice. However, despite the considerable achieved success remains many problems requiring the solution. So, many models carry out only preprocessing of information and its storage, but do not provide to the person, receiving the decision (PRD), means for the analysis of a situation and decision-making.

1. Introduction
The choice of economically expedient option of a heat supply system of the area (SHSA) demands consideration and accounting of a large number factors and operating conditions of a system. The most effective will be impact on totals of work of SHSA on design stages, in the course of the multiple pre-project analysis of heat supply systems of the complex housing estate or reconstruction of quarters. At the same time for practical economic activity existence not of one, but several factors (indicators) affecting efficiency of the made decision on development of SHSA is characteristic. At the same time use of multicriteria methods is necessary.

2. Problem setup
At present, due to the awareness of the limitations of optimization methods, much attention has been paid to the simulation approach to the analysis of decision-making problems. The simulation approach has important advantages: it allows you to analyze sufficiently detailed models, and the number of indicators studied during the study can be large. However, the simulation approach is also not free from shortcomings: variant calculations allow you to evaluate the consequences of only individual solutions (design competing options), but cannot give a general picture of the potential impact on the studied SHSA system with any significant number of acceptable solutions. Therefore, none of the commonly used...
methods of analysis alone allows a decision to be made on the choice (or impact) of an economically complex SHSA based on an analysis of its mathematical model.

When using economic and mathematical modeling methods, a whole range of complex problems arise in deciding on the choice of the SHSA development option. To overcome this type of difficulty, a systematic approach to the decision-making problem is proposed, which has found its practical implementation in man-machine simulation decision-making systems.

From a methodological point of view, a systemic approach is warranted when the researcher is faced with problems of dealing with the whole problem, that is to say, constructing theories that combine heterogeneous factors, To see the links between the processes taking place in the system and other phenomena, to take account of their mutual conditional factors, to see the prospects for the development of a process. The system approach leads to the need to take into account all significant aspects of the problem (including those that are not well formalized) in the decision-making process and analyze all possible effects on the subject under study and select the most rational ones taking into account all the indicators of interest to the PRD.

The properties and adequacy of the real processes in the simulated system determine the type of economic and mathematical model.

Economic and mathematical models shall classify [1] according to different characteristics:

1) by target purpose- theoretical and analytical (for the research of general properties and patterns) and applied (to solve certain tasks);
2) by the degree of certainty of the initial economic information deterministic and stochastic (taking into account the random nature of the processes);
3) by time factor accounting method – static (unchanged, for one moment or period of time) and dynamic;
4) by type of described dependencies- linear and non-linear
5) by the degree of consideration of the transport factor – production (transport factor neglected) and production and transport (consideration of the significant impact of the transport factor);
6) by the degree of possible localization – one-stage(only production and transportation of manufactured products is considered) and multi-stage(prior stages of transportation of necessary material resources are also taken into account).

Taking into account the given classification of economic and mathematical models, the SHSA development model developed in this work is defined as a dynamic multi-stage model of the production and transport type.

3. Results and discussion

In a basis of the developed model, the principle of system approach is put, that is subject to design is considered as the system interacting with adjacent systems and the environment surrounding it. About three main stages are offered to carry out development of issues of development of SHSA: The I stage – a formulation of a problem; The II stage – realization of economic-mathematical model of development of SHSA (EMMD SHSA); The III stage – the analysis of model [2, 3, 4].

At the first stage, the goals and objectives of the study are formulated, as well as an assessment of the possibility of conducting applied research, a qualitative description of the object of the study is given.

One of the main features of applied research on this issue is the participation of a PRD in it – a certain person or organization that poses a problem to researchers and uses the results of the analysis (the customer acts as a PRD here).

Closely related to the construction of the list of questions to be answered by the inquiry is the problem of formulating the concept of the subject of the research. First of all, the issue of the scale of the problem is solved: about the boundaries of the studied system, about the duration of the simulated period of time. It
is specified for which type of research objects you need to answer the formulated questions: either this is a certain generalized object that reflects the properties of systems of some type, or the task is associated with some particular system. During this period, it is decided which elements will be considered in the study and which will be considered external, and their effects – exogenous. Finally, it is established which of the exogenous variables are under the control of the customer, that is, he can control these values (within certain limits) to achieve his own goals. In this case, a list of impacts is formulated and the limits of their change are described.

After a qualitative description of the object, an analysis of the possibility of mathematical research of problems formulated by the customer is carried out.

Evaluation of the possibility of conducting applied research allows you to answer the question: can the developed structure of connections between variables be turned into a mathematical model that adequately describes these connections.

Two points of principle are of particular importance. One is that there is no single model of this system, but there are many models, each of which, having characteristic mathematical properties, is used to study a certain class of questions. Another point is that using the same model you can obtain different scenarios for the development of the system depending on the goals set. The latter provision requires, in conducting research, the choice of the optimal (best) development option of the allocated system.

The purpose of the first stage of the research is to choose among the variety of problems of interest to the customer, such as in solving which the researcher can really help him. At this stage, the tasks and scope of work are outlined, according to which the research system is allocated (using a systemic approach).

The fundamental principle at this stage is that the more broadly formulated the task, the more comprehensively the question is raised with less limitations, the more effective the task is to be expected.

Based on the generated initial information, the developed EMMD SHSA system for computer calculations is implemented. At the same time, linear programming methods are used that have successfully proven themselves in solving a large number of technical and economic problems.

The purpose of the economic and mathematical calculations was to find the optimal version of the development of SHSA from among the many possible by the minimum cost criterion and additional criteria, taking into account the accepted restrictions.

At mathematical modeling of pipeline systems to which the systems of heat supply belong the concept of a hydraulic chain is most often used (sets of the devices and pipelines connecting them closed or open channels which are carrying out transportation of the compressed and incompressible liquids – waters, oil, gas, fuel oil, air, etc.). The hydraulic chain in this case is understood as a mathematical object - the mathematical model including two components:

- the settlement scheme of a chain which is geometrically displaying structure (configuration) of the studied system and a picture of the possible directions, mixtures and divisions of streams of the transported environment;
- set of the mathematical ratios describing interdependence of quantitative characteristics of elements of this scheme.

In any hydraulic system as a material object distinguish three of its main subsystems:

- the sources of an expense or pressure (for example, boiler, pump and compressor stations, etc.) providing inflows of the transported environment and introducing energy in a system;
- the pipeline or hydraulic network (in the form of set of the interconnected pipelines, air ducts and open channels) connecting sources to a great number of consumers and delivering them this Wednesday;
- subscriber subsystems (or simply-consumers).

At mathematical modeling all subsystems find corresponding reflection in the settlement scheme of a chain [5, 6]. Among parameters of knots (sources of heat supply, sources of an expense or consumers) and
branches (the sites of network including fittings, other local resistance and also pressure sources) of a hydraulic chain we will distinguish subsequently: technical characteristics, hydraulic parameters, boundary conditions.

The conditional scheme of a hydraulic chain – the graphic representation of the modeling system - is submitted us as set of two ordered sets: sets of hubs $l \text{ and } j$, $l = \{l : l = 1, ..., L\}$, $j = \{j : j = 1, ..., J\}$, the consumers consisting of subsets and subsets of sources; sets of the branches displaying the set communications (connections) between knots; sets of the conventional signs characterizing type and specific features of elements. The number of knots and branches are parameters of a hydraulic chain [7, 8, 9].

We will consider the movement of the transported environment one-dimensional, averaging the speed, density and pressure of a stream on the section of a pipe or the channel. The stationary hydraulic mode answering to some process of a current which was established in a system will be considered at the same time.

Except requirements of profitability of heat supply systems of areas it is necessary to impose the following communications and to execute restrictions for development and functioning of objects of SHSA: balance; fuel; technological; ecological; reliability conditions.

Initial data in the deterministic setting of the task:
- annual requirements for thermal energy of heating and hot water supply systems of the residential area according to the stages of the calculation period;
- hourly heat loads of consumers (by parts of the annual heat consumption schedule) for each stage of the calculation period;
- existing heat supply facilities and their technical and economic parameters and limit heat capacities;
- possible directions of heat supply routes (main, distribution and intra-quarter) and fuel supply (for gas supply networks – at different pressure levels in the network);
- types of fuel and other resources, their qualitative characteristics and values.

It is necessary to define:
- types of new heat supply sources;
- composition of the main thermal power equipment of these facilities;
- annual supply of thermal energy by heat supply facilities;
- type of fuel used;
- length of heat networks and fuel supply networks routes;
- capital investments in the reconstruction of existing and construction of new thermal power facilities and the costs of their operation;
- consumption of various fuels and electrical energy, as well as other material resources.

By optimization it is also necessary to consider discreteness of parameters of heat power objects, nonlinear dependences of technical and economic indicators of objects on their established thermal power, dynamics of development of SHSA, multi-mode of objects work and reliability of heat supply.

Mathematical statement of complex optimization for development of SHSA is determined - in it there is no ambiguity of initial information on future conditions of development of a system of heat supply.

Criterion function of optimization of development of SHSA is formulated as follows: it is required to define:

$$TC(x, q) = \min_{x, q} \sum_{l=1}^{L} \sum_{b=1}^{B} TC_{lb}(x_{l(b-1)}, x_{ub}, q_{ub})$$  \hspace{1cm} (1)$$

At the following communications and restrictions:
a) on balances of thermal power and energy in knots of consumers of areas
\[ W_h(D_h, Q_h, q_h) = 0 \] (2)

b) on the balances of consumption of fuel establishing connection with fuel and energy balances of the area of design and construction of SHSA

\[ V_h(x_h, Q_h, B_h) = 0 \] (3)
c) on limits of development of rated capacities of objects

\[ x_{h, \text{min}} \leq x_h \leq x_{h, \text{max}} \] (4)
d) on limits of thermal loads of objects

\[ q_h \leq q_{h, \text{max}} \] (5)
e) on volumes of consumption of fuel of various look

\[ B_h \leq B_{h, \text{max}} \] (6)
f) on emissions of harmful substances in the environment

\[ Y_h \leq Y_{h, \text{max}} \] (7)

For all \( h = 1 \ldots H \).

In formulas (1-7) \( TC_{ih} \) – the expenses brought to the h-th to a step of the settlement period to the i-th an object; \( x_h = [x_{ih}], i = 1, \ldots, I \) - a vector of conditions of objects on the h-th a step of the settlement period; \( q_h = [q_{ih}], i = 1, \ldots, I ; s = 1, \ldots, S \) – a vector of thermal loads of objects on the h-th a step of the estimated period in zone s of the schedule of loadings; H-number of steps (stages) of the settlement period; S-number of zones of the production schedule; D – the determined vector of basic data; \( B_h = [B_{ih}] \) – vector of an expense of the T-th type of fuel on the h-th step of the settlement period; \( O_h = [Q_{ih}] \) – a vector of development and the release of energy of thermal energy objects (sources, in knots) on the h-th a calculation step; \( x_{h, \text{min}} \sim x_{h, \text{max}} \) – respectively lower and top limits of a vector of states \( x_h; q_h(x_h) \) – the top limit of thermal power at a vector of states \( x_h \), defining the maximum thermal load of objects depending on the equipment installed on it; \( Y_h = [Y_{mh}] \) – the number of emissions of the m-th harmful substance on the h-th a step of the settlement period; \( Y_{h, \text{max}} \) – limits of emissions of harmful substances on the h-th a step of the settlement period [10, 11].

By drawing up initial mathematical model of complex optimization of development of SHSA the following basic principles of the description of territorial, technological and ecological communications and conditions are realized:

- balance conditions are uniform for all power objects (knots, heat supply sources) making thermal energy;
- the model has to be capable to consider development of a system of a fuel supply;
- for the description the variables determining volumes of emissions of harmful substances when burning different types of organic fuel are entered into models of ecological conditions (at the same time in this work attention is paid mainly to gaseous fuel as to the most eco-friendly and technological).

The structure of mathematical model of optimization of development of SHSA is given in figure 1.

On adequacy of economic-mathematical models, and, therefore, incompleteness and inaccuracy of the available initial information have a significant negative effect on reliability of the forecasts received with
their application [12, 13, 14]. At a stage of implementation of feasibility studies on options of development of SHSA a number of initial indicators is accepted with some assumptions or is characterized by some uncertainty. Uncertainty, in this case, is understood as incompleteness or inaccuracy information on conditions of implementation of the project, including on the related expenses and results.

**Figure 1.** Structure of mathematical model of optimization of development of SHSA.

For accounting of a uncertainty factor at assessment of the project efficiency in compliance the method of check of its stability is offered (sensitivity of the project to external factors change) [15, 16, 17]. It provides development of scenarios of implementation of the project in the most probable or most "dangerous" conditions.

In most cases, it is appropriate for SHSA to investigate the effects of the following factors:
- capital expenditures for the project implementation in the range ± 30%;
- production costs (operating costs) in the range ± 30%;
- inflation rate;
- prices of sale of excess thermal energy to third-party consumers in the range of 10-100% of the current tariff;
- fuel (gas) cost in the range from 50 to 400% of the initial value (the upper limit is accepted at the level of gas cost sold by RAO Gazprom enterprises);
- values of the maximum thermal load of the facility (area) within the range ± 30% (this takes into account the uncertainty of the initial technological indicators of development);
- costs for preparation of the territory, construction of foundations and external utilities (that is, costs that are not included in the cost of the main equipment of heat supply sources and heat networks; the range of their variation is based on the specific circumstances of the option).

4. Conclusions

When deciding on the choice of the SHSA development option, a set of complex interrelated problems arises, it is proposed to solve the tasks based on a system multivariable approach. In this case, it is possible to consider the problem as a whole, combine heterogeneous factors, see the connections of processes in the system with other phenomena, take into account their mutual conditionality, see the prospects for the development of a particular process.

System approach allows to consider all essential moments of the studied problem, including badly formalized, to analyze all potential impacts on the studied object for the purpose of the choice of the most effective (optimum) solution. For the purpose of search of an optimal solution in this work it is offered to apply an extreme and alternative method. The person making the decision (PMD) directly participates in this choice. Such approach finds the practical embodiment in creation of human-machine imitating systems of decision-making.

Thus, as a result of the conducted researches the procedure of development and choice for development effective option of a system heat supply of areas as is proved sequence for the following stages: statement of a problem; allocation of a research system; formation of factors characterizing set a state and development of the allocated system; development of the SHSA economic-mathematical model; formation of solution information base (database); development of the program of calculations on the COMPUTER; carrying out multiple machine calculations; multicriteria assessment of decisions versions and choice of best (optimum).

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