The vector meson mass in the large \( N \) limit of QCD

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Abstract

The vector meson mass is computed as a function of quark mass in the large \( N \) limit of QCD. We use continuum reduction and directly compute the vector meson propagator in momentum space. Quark momentum is inserted using the quenched momentum prescription.

Key words:
Large \( N \) QCD, Vector meson masses, Low energy constants

Meson masses remain finite in the ’t Hooft limit of large \( N \) QCD in four dimensions \cite{1}. Chiral symmetry is broken and the value of the chiral condensate has been measured on the lattice for overlap fermions using random matrix theory techniques \cite{2}. The result can be summarized as \cite{3}

\[
\frac{\Sigma(b)}{T_c^3(b)} = 0.828 \left[ \ln \frac{0.268}{T_c(b)} \right]^{\frac{1}{11}}.
\]  

(1)

where \( b = \frac{1}{g^2 N} \) is the bare ’t Hooft coupling on the lattice. The deconfining temperature, \( T_c(b) \), is also known from a lattice calculation \cite{4} and is given by

\[
b_I = b_c(b); \quad e(b) = \frac{1}{N} \langle \text{Tr} U_p(x) \rangle; \quad T_c(b) = 3.85 \left( \frac{48\pi^2 b_I}{11} \right)^{\frac{1}{11}} e^{-\frac{24\pi^2 b_I}{11}}.
\]  

(2)

Continuum reduction holds if \( L > \frac{1}{T_c(b)} \) \cite{4} and meson propagators can be directly computed in Euclidean momentum space without any finite volume effects. The pion mass as a function of quark mass, \( m_o \), was computed on the lattice using overlap fermions and the pion decay constant is given by \cite{3}

\[
\frac{f_\pi}{\sqrt{NT_c(b)}} = 0.269.
\]  

(3)

In this letter, we present results for the mass of the vector meson, \( m_\rho \), as a function of the quark mass, \( m_o \), using the same technique as the one used for the computation of the pion mass in \cite{3}. The \( \rho \) propagator is computed using

\[
M_{\mu\nu}(p, m_o) = \text{Tr} \left[ S_{\gamma\mu} G(U_\mu e^{\frac{i p\cdot x}{m_o}}, m_o) S_{\gamma\nu} G(U_\mu e^{\frac{-i p\cdot x}{m_o}}, m_o) \right].
\]  

(4)
• $G(U_\mu, m_\circ)$ is the lattice quark propagator computed using overlap fermions in a gauge field background given by $U_\mu$.

• The phase factors, $e^{\pm ip_\mu \frac{2}{N_L}}$, multiplying the gauge fields correspond to the force-fed momentum of the two quarks in the quenched momentum prescription.

• The meson momentum was chosen to be

$$p_\mu = \begin{cases} 0 & \text{if } \mu = 1, 2, 3 \\ \frac{2\pi k}{N_L} & \text{if } \mu = 4. \end{cases}$$

(5)

• $S$ smears the operator in the zero momentum directions using the inverse of the gauged Laplacian.

The $\rho$ meson is made up of two different quarks (say $u$ and $d$) with degenerate quark masses. Since the associated vector currents are conserved, the propagator, after averaging over gauge fields, will be of the form

$$M_{\mu\nu}(p, m_\circ) = A \left( \frac{p_\mu p_\nu - p^2 \delta_{\mu\nu}}{p^2 + m_\circ^2} \right),$$

(6)

assuming the propagator is dominated by the lowest vector meson state. Our numerical result is consistent with the above form. We found all off-diagonal ($\mu \neq \nu$) terms and the $\mu = \nu = 4$ term to be zero within errors for the specific choice of momentum in (5) and we also found the $\mu = \nu = 1, 2, 3$ terms to be the same within errors in our small test runs. Since the evaluation of the quark propagators is the computationally intensive part, we set $\mu = \nu = 1$ and obtained a value for the $\rho$ meson mass at six different quark masses by fitting it to the form in (6).

Four different couplings were used in [3] for the computation of the meson masses. We found two of those couplings to be too strong for the computation of the $\rho$ mass. We report here, the results for the $\rho$ mass at two couplings, namely, $b = 0.355$ and $b = 0.360$. Chiral perturbation theory for vector mesons [5] suggests that we fit the data to the form

$$M_\rho = \tilde{M}_8 + \Lambda_2 M + \delta M_\rho,$$

(7)

where

$$M_\rho = \frac{m_\rho}{T_c(b)}; \quad M = \frac{2m_0\Sigma}{T_c^4(b)};$$

(8)

are the mass of the $\rho$ meson and the renormalization group invariant quark mass measured in units of the deconfining temperature. The two coefficients

\footnote{\textit{M} denoted the sum of the two quark masses comprising the $\rho$ meson and hence the factor of 2 in the formula for \textit{M}.}
Table 1: Simulation parameters, critical box size, bare chiral condensate along with the estimates for the two coefficients $\bar{M}_8$ and $\Lambda_2$ in the mass term of the chiral lagrangian.

| $L$ | $N$ | $b$   | $T_c(b)$ | $\Sigma^{1/3}(b)$ | $\bar{M}_8$ | $\Lambda_2$ |
|-----|-----|-------|----------|------------------|-------------|-------------|
| 10  | 19  | 0.355 | 0.144    | 0.1265           | 5.39(54)    | 1.51(32)    |
| 11  | 17  | 0.360 | 0.125    | 0.1130           | 5.87(37)    | 1.76(20)    |

in (7) are two of the coefficients in the mass term in the chiral lagrangian, namely,

$$\bar{M}_8 = \frac{\bar{\mu}_8}{T_c(b)}; \quad \Lambda_2 = \frac{\lambda_2 T_c^3(b)}{\Sigma}.$$  \hfill (9)

The data is plotted in Fig. 1. The result for the chiral condensate in (11) was obtained such that the pion mass as a function of the quark mass scaled properly in the range of coupling from $b = 0.345$ to $b = 0.360$. It is not necessary that this eliminates finite lattice spacing effects on all quantities and we do see effects of finite lattice spacing effects in Fig. 1. A linear fit performs quite well at both the couplings to yield consistent estimates for $\bar{M}_8$ and $\Lambda_2$. The results are shown both in Fig. 1 and Table 1.

Chiral perturbation theory suggests that $\delta M_\rho$ in (7) should lead off as $M^{3/2}$ and the coefficient of this leading term should be negative. A fit with a $M^{3/2}$ term is shown in Fig. 1 and we see that the coefficient at $b = 0.360$ is consistent
with it being negative. The error in this coefficient is rather large.

Using the result for $\mathcal{M}_8$ in Table 4 for $b = 0.360$ and the result for $f_\pi$ in (3), we have

$$\bar{\mu}_8 = \frac{21.8 \pm 1.4}{\sqrt{N}} f_\pi. \quad (10)$$

If we use $f_\pi = 86$ MeV and $N = 3$, then we get $\bar{\mu}_8 = 1082 \pm 70$ MeV.

The vector meson masses have been computed in the quenched approximation for $N = 2, 3, 4, 6$ in [6, 7]. The couplings used in [6] and in [7] are roughly the same. The strongest and weakest coupling correspond to $b = 0.296$ and $b = 0.353$ respectively in the notation of this paper. There is a bulk transition on the lattice in the large $N$ limit that becomes a cross-over at finite $N$. The region between $b = 0.34$ and $b = 0.36$ is in the meta-stable region of this transition [4] and we need to be above $b = 0.34$ to be in the continuum phase of the large $N$ theory. Since the vector meson is heavy compared to the pion for small quark masses, finite lattice spacing effects are larger in the case of the vector meson. Our study at $b = 0.350$, not reported in this paper, does yield a value for $\mathcal{M}_8$ that is about 25% smaller than the one quoted here at $b = 0.360$ and consistent with the value obtained in [7].

Acknowledgments

A.H. and R.N. acknowledge partial support by the NSF under grant number PHY-055375. A.H. also acknowledges partial support by US Department of Energy grant under contract DE-FG02-01ER41172. R.N. would like to thank Joe Kiskis for some useful discussions.

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