From Super-AdS$_5 \times S^5$ Algebra to Super-pp-wave Algebra

Machiko Hatsuda, Kiyoshi Kamimura† and Makoto Sakaguchi

Theory Division, High Energy Accelerator Research Organization (KEK),
Tsukuba, Ibaraki, 305-0801, Japan
† Department of Physics, Toho University, Funabashi, 274-8510 Japan
E-mails: mhatsuda@post.kek.jp, kamimura@ph.sci.toho-u.ac.jp, Makoto.Sakaguchi@kek.jp

Abstract

The isometry algebras of the maximally supersymmetric solutions of IIB supergravity are derived by the Inönü-Wigner contractions of the super-AdS$_5 \times S^5$ algebra. The super-AdS$_5 \times S^5$ algebra allows introducing two contraction parameters; the one for the Penrose limit to the maximally supersymmetric pp-wave algebra and the AdS$_5 \times S^5$ radius for the flat limit. The fact that the Jacobi identity of three supercharges holds irrespectively of these parameters reflects the fact that the number of supersymmetry is not affected under both contractions.

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1 Introduction

Recently the supersymmetric pp-wave backgrounds, as well as the anti-de Sitter (AdS) backgrounds, have been widely studied as supergravity vacua. A maximally supersymmetric pp-wave solution for the eleven-dimensional supergravity theory was found in [1, 2, 3, 4], and one for the type IIB supergravity theory was found in [5]. Relations between the pp-wave background and the AdS background are crucial for the point of view of the AdS/CFT correspondence [7, 6, 8] where the pp-wave background is recognized as an approximation of the AdS background. The plane wave space is obtained as the limiting space from arbitrary space shown first by Penrose [9], and this idea is extended to supergravity and superstring theories [10, 11, 12].

The maximally supersymmetric solution of the IIB supergravity consists of the pp-wave metric and the self-dual null homogeneous 5 form flux [5]

\[
\begin{align*}
  ds^2 &= 2dx^-dx^+ - 4\lambda^2 \sum_{i=1}^{8} (x^i_+)^2 (dx^-)^2 + \sum_{i=1}^{8} (dx^i_+)^2 \\
  F_5 &= \lambda dx^- \wedge (dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4 + dx^5 \wedge dx^6 \wedge dx^7 \wedge dx^8)
\end{align*}
\] (1.1)

with a constant dilaton. It has 32 Killing spinors preserving the solution. Recently it was discussed in detail that the solution (1.1) is obtained as a Penrose limit of the AdS$_5 \times $S$^5$ background [11, 12]. Since the pp-wave solution (1.1) is obtained as a limiting case of the AdS$_5 \times $S$^5$ solution by the Penrose limit [9], the isometry algebra of the former should be also obtained from the latter by some limiting procedure. The Penrose limit is usually discussed as a mapping between metrics in terms of local coordinates. By using canonical symmetry generators written in terms of local coordinates, this mapping directly tells the relation of generators, thus of algebras.

In this paper we examine an Inönü-Wigner (IW) contraction [13] of algebra which is independent of choice of local coordinates. Then the Penrose limit maps not only the AdS$_5 \times $S$^5$ metric into pp-wave metric but also the super-AdS$_5 \times $S$^5$ algebra into the maximally supersymmetric pp-wave algebra\(^2\) as an IW contraction. The IW contraction from the AdS algebra to the pp-wave algebra for the bosonic case is discussed in the section 2, and the one for the supersymmetric case is in the section 3.

2 From AdS algebra to pp-wave algebra

The AdS$_d \times $S$^{D-d}$ algebra is given, in terms of dimensionless momenta $P$’s and rotations

\[^1\]This limiting procedure had appeared in the context of the Wess-Zumino-Witten model in [16].

\[^2\]We refer the isometry algebra of the pp-wave metric as the pp-wave algebra.
\( M \)'s, as
\[
[P_a, P_b] = M_{ab}, \quad [P_{a'}, P_{b'}] = -M_{a'b'}
\]
\[
[P_a, M_{bc}] = \eta_{ab}P_c - \eta_{ac}P_b, \quad [P_{a'}, M_{b'c'}] = \eta_{a'b'}P_{c'} - \eta_{a'c'}P_{b'}
\]
\[
[M_{ab}, M_{cd}] = \eta_{bc}M_{ad} + 3 \text{ terms}, \quad [M_{a'b'}, M_{c'd'}] = \eta_{b'c'}M_{a'd'} + 3 \text{ terms}
\]  
(2.1)

where \( a = 0, 1, ..., d - 1 \) and \( a' = d, ..., D - 1 \) stand for vector indices of AdS\(_d\) and S\(_{D-d}\) respectively. The symmetry group is isomorphic to \( \text{SO}(d - 1, 2) \times \text{SO}(D - d - 1) \) which has a flat limit to \( \text{ISO}(d - 1, 1) \times \text{ISO}(D - d) \) by the following IW contraction \([13]\).

1. Rescale the translation generators \( P \)'s using the radii of the AdS\(_d\) and S\(_{D-d}\), \( R \) and \( R' \) respectively, as
\[
P_a \rightarrow RP_a, \quad P_{a'} \rightarrow R'P_{a'}.
\]  
(2.2)

2. Then take \( R \rightarrow \infty \) and \( R' \rightarrow \infty \).

In the limit, \( P \)'s become the linear momenta and \( M \)'s are the Lorentz generators. Taking large limit of the radii, \( R \) and \( R' \), corresponds to reducing an AdS×S space into a flat space.

Besides the flat limit, as any other metrics the AdS×S metric allows a limit giving a plane wave metric (Penrose limit) \([4]\). The Penrose limit can be understood as an IW contraction of the AdS×S algebra into the pp-wave algebra.

1. Define the light cone components of the momenta \( P \) and boost generators \( P^* \) as
\[
P_{\pm} \equiv \frac{1}{\sqrt{2}}(P_{D-1} \pm P_0), \quad P_i = \left\{ \frac{P_i}{P_{\hat{i}}} \right\}, \quad P_i^* = \left\{ \frac{P_i^*}{P_{\hat{i}'}} \equiv M_{0\hat{i}} \right\}.
\]  
(2.3)

where \( \hat{i} = 1, ..., D - 2, \; i = 1, ..., d - 1, \; \hat{i}' = d, ..., D - 2 \).

2. Suppose the plane wave propagates with respect to \( x_+ \) time. The transverse translation and boost generators are rescaled with the dimensionless parameter \( \Omega \) as
\[
P_+ \rightarrow \frac{1}{\Omega^2}P_+, \quad P_i \rightarrow \frac{1}{\Omega}P_i, \quad P_i^* \rightarrow \frac{1}{\Omega}P_i^*.
\]  
(2.4)

3. Then take \( \Omega \rightarrow 0 \) limit.

In the limit, \( P_+ \) becomes a center, and it can be treated as a constant.
To see them explicitly the AdS×S algebra \((2.1)\) is rescaled following to \((2.2)\) and \((2.4)\)

\[
\begin{align*}
[P_+, P_i] &= \frac{\Omega^2}{\sqrt{2} R^2} P_i^* \\
[P_+, P_{i'}] &= -\frac{\Omega^2}{\sqrt{2} R^2} P_{i'}^* \\
[P_+, P_{i''}] &= -\frac{\Omega^2}{\sqrt{2} R^2} P_{i''} \\
[P_+, P_{i'''}] &= -\frac{\Omega^2}{\sqrt{2} R^2} P_{i'''} \\
\end{align*}
\]

\[
\begin{align*}
[P_-, P_i] &= -\frac{1}{\sqrt{2} R^2} P_i^* \\
[P_-, P_{i'}] &= -\frac{1}{\sqrt{2} R^2} P_{i'}^* \\
[P_-, P_{i''}] &= \frac{1}{\sqrt{2} R^2} P_{i''} \\
[P_-, P_{i'''}] &= \frac{1}{\sqrt{2} R^2} P_{i'''}
\end{align*}
\]

\[
\begin{align*}
\{P_i, P_j\} &= \frac{\Omega^2}{R^2} M_{ij} \\
\{P_i^*, P_j^*\} &= \Omega^2 M_{ij} \\
\{P_i, P_j^*\} &= -\frac{\Omega^2}{2 \sqrt{2}} \eta_{ij} (P_+ - \Omega^2 P_-) \\
\{P_i^*, P_j^*\} &= -\frac{\Omega^2}{2 \sqrt{2}} \eta_{ij} (P_+ + \Omega^2 P_-) \\
\{P_i, M_{j\bar{k}}\} &= \eta_{ij} P_{\bar{k}} \\
\{P_i^*, M_{j\bar{k}}\} &= \eta_{ij} P_{\bar{k}}^* \\
\{M_{ij}, M_{k\bar{l}}\} &= \eta_{ij} M_{k\bar{l}} + 3 \text{ terms} \\
\text{others} &= 0.
\end{align*}
\]

By taking \(\Omega \to 0\) limit, \((2.3)\) becomes the plane wave algebra,

\[
\begin{align*}
\{P_-, P_i\} &= \frac{1}{\sqrt{2}} P_i \\
\{P_-, P_{i'}\} &= -4 \sqrt{2} \lambda_i P_{i'}^* \\
\{P_i, M_{j\bar{k}}\} &= \eta_{ij} P_{\bar{k}} \\
\{P_i^*, M_{j\bar{k}}\} &= \eta_{ij} P_{\bar{k}}^* \\
\{M_{ij}, M_{k\bar{l}}\} &= \eta_{ij} M_{k\bar{l}} + 3 \text{ terms} \\
\text{others} &= 0
\end{align*}
\]

where

\[
\lambda_i = \left\{\begin{array}{c}
\lambda_i = \frac{1}{2 \sqrt{2} R} \\
\lambda_i' = \frac{1}{2 \sqrt{2} R'}
\end{array}\right.
\]

This is the symmetry algebra of the pp-wave metric \((14)\). \(P_+\) is a central element in \((2.0)\) allowing arbitrary rescaling \(^3\) with \(\lambda\).

Furthermore the flat limit is taken by \(R, R' \to \infty\) in \((2.3)\) or \(\lambda_i \to 0\) in \((2.0)\). Although this flat limit algebra is different from the direct flat limit from the AdS algebra, the \(D\)-dimensional Poincare group ISO\((D - 1, 1)\) can be recovered by supplying spontaneously broken generators consistently.

### 3 From Super-AdS\(_5\times S^5\) algebra to Super-pp wave algebra

We extend the previous analysis to the supersymmetric case and show the maximally supersymmetric IIB pp-wave algebra \(^4\) is obtained by a contraction of the super-

\(^3\)In \((14)\), \(P_+\) is rescaled as \(\lambda^2 P_+\), and \(P_i\)s and \(P^*\)s are \(e^i\)s and \(e^*\)s respectively. Or \(e^i\)s can be recognized as \(\lambda^{-2} e^*\) corresponding to \(P^*\), but this prohibits the flat limit \(\lambda \to 0\) in their superalgebra.
AdS$_5 \times S^5$ algebra. In this case the Jacobi identities of the superalgebra give a restriction on the scale parameters $R$ and $R'$ in (2.1), as $R = R'$. We start with the super-AdS$_5 \times S^5$ algebra following to the notation of Metsaev and Tseytlin [15]. In 10 dimensional IIB theory the supersymmetry generators $Q_A$ are chiral Majorana spinors and are SL(2,R) doublet ($A = 1, 2$). The gamma matrices $\Gamma_a$, ($\hat{a} = 0, 1, \ldots, 9$) in 10 dimensions are composed using those of AdS$_5$, $\gamma_a$, ($a = 0, 1, \ldots, 4$), and of $S^5$, $\gamma'_a$, ($a' = 5, \ldots, 9$), as

$$\Gamma_a = \gamma_a \otimes 1' \otimes \sigma_1$$

and satisfy

$$\{\Gamma_\hat{a}, \Gamma_b\} = 2\eta_{a\hat{b}}, \quad \eta_{a\hat{b}} = (- + + + +, + + + + +)$$

$$\{\gamma_a, \gamma_b\} = 2\eta_{ab}, \quad \eta_{ab} = (- + + + +)$$

$$\{\gamma_a', \gamma_{b'}\} = 2\eta_{a'b'}, \quad \eta_{a'b'} = (+ + + + +).$$

The supercharges $Q_A$’s are satisfying

$$Q_A = Q_A \frac{1 + \Gamma_{11}}{2}, \quad \Gamma_{11} = \Gamma_{0123456789} = 1 \otimes 1' \otimes \sigma_3. \quad (3.2)$$

The charge conjugation matrix $C$ in 10 dimensions is taken as

$$C = C \otimes C' \otimes i\sigma_2 \quad (3.3)$$

where $C$ and $C'$ are respectively the charge conjugations for AdS$_5$ and $S^5$ spinors.

The bosonic part of supersymmetric AdS$_5 \times S^5$ algebra is (2.1) with $d = 5, D = 10$. In addition to it the odd generators $Q_A$ satisfy

$$[Q_A, P_a] = \frac{i}{2} Q_B \gamma_a \epsilon_{BA}, \quad [Q_A, P_{a'}] = -\frac{1}{2} Q_B \gamma_{a'} \epsilon_{BA}$$

$$[Q_A, M_{ab}] = -\frac{1}{2} Q_B \gamma_{ab}, \quad [Q_A, M_{a'b'}] = -\frac{1}{2} Q_B \gamma_{a'b'}$$

$$\{Q_{\alpha', A}, Q_{\beta' B}\} = \delta_{AB} \left[-2iC'_{a'}{}^{\alpha'\beta'} (C' \gamma')_{\alpha'\beta'} P_a + 2C_{\alpha\beta} (C' \gamma')_{\alpha'\beta'} P_{a'} \right]$$

$$+ \epsilon_{AB} \left[C'_{a'}{}^{\alpha'\beta'} (C' \gamma')_{\alpha'\beta'} M_{ab} - C_{\alpha\beta} (C' \gamma')_{\alpha'\beta'} M_{a'b'} \right] \quad (3.4)$$

where we omit to write the chiral projection operators.

For the present purpose we rewrite this algebra in terms of 10-dimensional covariant gamma matrices, $\Gamma$’s, rather than $\gamma$’s. It is convenient to introduce following matrices

$$\Gamma_{01234} = 1 \otimes 1' \otimes -i\sigma_1 \equiv \Gamma_0 I$$

$$\Gamma_{56789} = 1 \otimes 1' \otimes \sigma_2 \equiv \Gamma_9 J \quad (3.5)$$

The AdS superalgebra (2.4) are rewritten as

$$[Q_A, P_\hat{a}] = -\frac{1}{2} Q_B \Gamma_0 I \Gamma_\hat{a} \epsilon_{BA}$$

$$[Q_A, M_{\hat{a}b}] = -\frac{1}{2} Q_B \Gamma_\hat{a}$$

$$\{Q_{\hat{a}, A}, Q_{\hat{b}, B}\} = -2i\delta_{AB} C \Gamma_{\hat{a}} P_\hat{a} + i\epsilon_{AB} \left[C \Gamma_{\hat{a}} \Gamma_0 I M_{\hat{a}b} + C \Gamma_{\hat{a}} \Gamma_{\hat{b}} \Gamma_9 J M_{a'b'} \right] \quad (3.6)$$

4
using with the useful relation in the first line
\[
\frac{1 + \frac{\Gamma_{11}}{2}}{2}(\Gamma_0 I) = \frac{1 + \frac{\Gamma_{11}}{2}}{2}(\Gamma_9 J) \ .
\] (3.7)

Associating with the rescaling of \(P\)'s by \(R\) as in (2.4) the dimensionless supercharge \(Q_A\) is rescaled as
\[
P_a \to RP_a, \ P_{a'} \to RP_{a'} \ , \ Q_A \to \sqrt{R} Q_A. \tag{3.8}
\]

Corresponding to the rescaling of bosonic generators (2.4) the components of the supercharges are rescaled with proper weights. The \(Q_A\) is decomposed as
\[
Q_A = Q_A + Q_A - A, \quad Q_{\pm A} \Gamma_\mp = 0 , \quad \Gamma_\pm = \frac{1}{\sqrt{2}}(\Gamma_9 \pm \Gamma_0) \ . \tag{3.9}
\]

The supercharges are rescaled as
\[
Q_+ \to \frac{1}{\Omega} Q_+ , \quad Q_- \to Q_- \tag{3.10}
\]
in the superalgebra of the \(AdS^5\times S^5\) to obtain the well defined Penrose limit.

The super-\(AdS^5\times S^5\) algebra (3.6) after the rescaling of the \(P\)'s, (2.2) with \(R = R'\) and (2.4), and those of \(Q\)'s , (3.8) and (3.10), becomes
\[
\begin{align*}
\{Q_{+,A}, P_-\} = & \frac{-i\Omega^2}{2\sqrt{2}R} Q_{+,B} I \epsilon_{BA}, \quad \{Q_{-,A}, P_+\} = \frac{\Omega^2}{2\sqrt{2}R} Q_{-,B} I \epsilon_{BA} \\
\{Q_{+,A}, P_i\} = & \frac{i\Omega^2}{2\sqrt{2}R} Q_{-,B} \Gamma_j \epsilon_{BA}, \quad \{Q_{-,A}, P_i\} = -\frac{1}{2\sqrt{2}R} Q_{+,B} \Gamma_i \epsilon_{BA} \\
\{Q_{+,A}, P_A^\ast\} = & \frac{i\Omega^2}{2\sqrt{2}R} Q_{-,A} \Gamma_i \epsilon_{BA}, \quad \{Q_{-,A}, P_A^\ast\} = -\frac{1}{2\sqrt{2}R} Q_{+,A} \Gamma_i \epsilon_{BA} \\
\{Q_{+,A}, P^\ast_i\} = & -\frac{i\Omega^2}{2\sqrt{2}R} Q_{-,A} \Gamma_j \epsilon_{BA}, \quad \{Q_{-,A}, P^\ast_i\} = -\frac{1}{2\sqrt{2}R} Q_{+,A} \Gamma_j \epsilon_{BA} \\
others = 0
\end{align*}
\]
\[
\begin{align*}
\{Q_{+,A}, Q_{+,B}\} = & -2i\delta_{AB} \epsilon_{ij} I P_+ - \frac{i\Omega^2}{2\sqrt{2}R} \epsilon_{ij} \left[ C \Gamma^+ \Gamma^+ I M_{ij} - C \Gamma^+ \Gamma^+ I M_{ij} \right] \\
\{Q_{+,A}, Q_{-,B}\} = & -2i\delta_{AB} \epsilon_{ij} \left[ -\frac{1}{R} P_i + \frac{2i}{R} \epsilon_{AB} \left[ C \Gamma^- \Gamma^- I P_i^\ast + C \Gamma^- \Gamma^- I P_i^\ast \right] \right] \\
\{Q_{-,A}, Q_{-,B}\} = & -2i\delta_{AB} \epsilon_{ij} \left[ C \Gamma^- \Gamma^- I M_{ij} + C \Gamma^- \Gamma^- I M_{ij} \right].
\end{align*}
\]

Here we have used relations following from (3.7),
\[
\Gamma_\pm I \frac{1 + \frac{\Gamma_{11}}{2}}{2} = \mp \Gamma_\pm J \frac{1 + \frac{\Gamma_{11}}{2}}{2} \ . \tag{3.13}
\]

It is important that negative power terms of \(\Omega\) disappear due to the presence of the chiral and the light cone projections. Therefore we can take the Penrose limit \(\Omega \to 0\) of the algebra to obtain
\[
\begin{align*}
\{Q_{A}, P_-\} = & -\lambda Q_B (I + J) \epsilon_{BA}, \quad \{Q_{A}, P_i\} = \lambda Q_B \Gamma_+ \Gamma_i \epsilon_{BA} \\
\{Q_{A}, P^\ast_i\} = & -\frac{1}{2\sqrt{2}R} Q_A \Gamma_i \epsilon_{BA}, \quad \{Q_{A}, P^\ast_i\} = \lambda Q_B \Gamma_+ \Gamma_i \epsilon_{BA} \\
\{Q_{A}, M_{ij}\} = & -\frac{1}{2} Q_A \Gamma_i \epsilon_{BA} \\
\end{align*}
\] (3.14)
\[
\{Q_A, Q_B\} = -2i \delta_{AB} \left[ C \Gamma^+ P_+ + C \Gamma^- P_- + C \Gamma^i P_i \right] \\
+ 4\sqrt{2} i \lambda \epsilon_{AB} \left[ C \Gamma^i P_i^* + C J \Gamma^i P_i^* \right] \\
+ 2i \lambda \epsilon_{AB} \left[ C \Gamma^- \Gamma^j M_{ij} + C \Gamma^- J \Gamma^{i'j'} M_{i'j'} \right]
\]

(3.15)

where \( \lambda = 1/(2\sqrt{2}R) \). This is the maximally supersymmetric pp-wave algebra obtained in [3].

Furthermore the flat limit is taken by \( R \to \infty \) in (3.11) and (3.12) or \( \lambda \to 0 \) in (3.15).

4 Summary and Discussions

In this paper we have derived the super-pp-wave algebra taking with an IW contraction of the super-AdS\(_5\times S^5\) algebra. It is stressed that the Jacobi identities of this algebra (2.3), (3.11) and (3.12) hold for any value of \( \Omega \) and \( \lambda \), and the algebra has well defined even in their zero limits. It explains naturally why the pp-wave and the flat supersymmetry are maximally supersymmetric as the super-AdS\(_5\times S^5\). The number of bosonic generators in AdS\(_5\times S^5\) and pp-wave algebra are same since their algebras are connected by the contraction.

The relation between the super-AdS background and the super-pp-wave background at algebraic level is practical to construct mechanical actions for branes in the super-pp-wave background. Any form field in the latter can be obtained from the former. This approach manifests symmetries of super-pp-wave systems whose dynamics and spectrum will be analyzed elegantly.

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