The Influence of Cognitive Style on Mathematical Communication of Prospective Math Teachers in Solving Problems

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Abstract. Communication is important to connect with others. It is one of the important aspects that must be mastered by the prospective math teacher as a tool of knowledge transfer. They are expected to have good mathematical communication skills to facilitate them in the mathematics learning process. Problem-solving in learning mathematics is something that cannot be ignored. Through problem-solving, students can have a better understanding of concepts because through mathematical problem solving, and students are required to be more active in using the concepts they have in a variety of new situations encountered. FD/FI cognitive style is the extent to which a person perceives parts of a field discrete from the surroundings rather than embedded in it as a whole. Based on differences from cognitive style, it certainly affects the way of communication, especially mathematical communication. So, the aim of this research was exploration the mathematical communication of prospective math teacher in solving problem viewed from FI/FD cognitive style. This type of research was descriptive research with a qualitative approach. The subjects of this research were 86 prospective math teachers. Data were collected through tests, assignments, and task-based interviews. The results of this study indicated that mathematical communication from FD and FI students were not different when solving routine problems, but were different when solving non-routine problems, which require strategy and decision making. FD students had difficulty communicating their strategies clearly and coherently. Meanwhile, FI students explained the strategies they used in more detail.

1. Introduction

Communication is important to connect with others. If a person cannot communicate his thoughts clearly then it can lead to misunderstanding. Especially for a teacher, communication is very important, it is important to express what is in the mind of the teacher, it is important to explain the material to students so that students are not misconceived, it is important to express what they want clearly so that students do not misunderstand. Pourdavood said that communication helps the teacher to recognize the student’s ability, understand their mathematical understanding, and then make the appropriate decisions to help them [1]. Based on some of the opinions above, it is clear that communication is important for teachers. However, communication is also important for students, better communication makes it easy for students to learn the material, convey ideas correctly and fluently following their mind so that teachers can understand their thinking. In other words, communication is important in every learning, including mathematics learning. NCTM also stated the importance of communication in learning mathematics. In Principles and Standards for School Mathematics, the
National Council of Teachers of Mathematics (NCTM) states: “Students who have opportunities, encouragement, and support for speaking, writing, reading, and listening in mathematics classes reap dual benefits: they communicate to learn mathematics, and they learn to communicate mathematically” [2]. Mathematical communication means organizing mathematical thinking coherently and clearly to the others and using the language of math to express mathematical ideas precisely. Mathematical communication includes a variety of cognitive skills because it is an exchange of ideas, which includes both listening, reading (understanding), speaking and writing (expressions) about mathematics, mathematical expressions include the representation of mathematical ideas in a non-linguistic way. Indicators of mathematical communication, according to NCTM are as follows:

1. skill to express mathematical ideas and demonstrating them and visualising them visually;
2. skill to understand, interpret, and evaluate mathematical ideas;
3. skill to use terms, mathematical notations and structures to present ideas, describe relation, and situational models.

Prospective math teachers are specially prepared to become qualified mathematics teachers. Teachers have an important role played in helping students to learn and affecting the educational outcomes-achievement and attitudes of their students. They are expected to have good mathematical communication skills to facilitate them in the mathematics learning process. So, it is interesting to study the mathematical communication of prospective math teachers.

The research about mathematical communication has been investigated in many studies. Wichelt and Kearney [3] conducted research about communication in mathematics learning using open-ended questions. They claimed that students’ mathematical vocabulary comprehension was better when using open-ended questions. Pantaleon et al [4] studied the communication of prospective teacher in proving problems. They found that in the proof of geometry problem, the subject explains what is understood, presents the idea in the form of drawing and symbols, and explains the content/value of a representation accurately and clearly, but the subject can’t convey the argument systematically and logically. Kabael [5] found out that junior high school math teachers had no communication skills in the mathematical language as expected of a mathematics teacher. Kabael put more highlighted on the skill of mathematical communication than process. In this study, the focus was on the written math communication of prospective math teachers.

Problem-solving in learning mathematics is something that cannot be ignored. Through problem-solving, students can have a better understanding of concepts because through mathematical problem solving, and students are required to be more active in using the concepts they have in a variety of new situations encountered. Problem-solving places the focus on the student making sense of mathematical ideas. When solving problems students are exploring the mathematics within a problem context rather than as an abstract. Liljedahl [6] argued that problem-solving not only requires logical and deductive reasoning but also requires extra-logical processes of creativity, insight and illumination to produce a solution. Lestari [7] studied the differences in the reasoning of prospective mathematics teachers in solving context problems in shipping companies. The result showed female prospective teachers were more capable of logical reasoning, using concepts, facts and procedures and algebraic operations to conclude, make interpretations and evaluations.

Many aspects affect a person’s thinking, reasoning, and communication, some of which are their beliefs, gender, cognitive style, and memory capacity. Muhtarom et al. [8] found that prospective teacher’s success in solving the problem is in line with their belief in solving the problem. Based on research conducted by Juniati [9], it was found that the strategies used by students in completing technology-based assignments varied and using the software in completing assignments made it easier for students to understand concepts and communicate their results better. Sun [10] studied the effects of thinking style on design strategies, carried out in Taiwan. The results described that the thinking style showed a positive correlation with frequency of substantial change in structure and the goal of strategy, it meant that thinking style affects strategy designed. FD/FI cognitive style is the extent to which a person perceives parts of a field discrete from the surroundings rather than embedded in it as a whole. A field-dependent learner is one who processes information globally. This kind of learner is less analytical, not attentive to detail, and sees the perceptual field as a whole. Meanwhile, a independent
field person can easily break the field down into its parts. He/she is typically not influenced by the existing structure and can make choices independent of the perceptual field. Based on differences from cognitive style, it certainly affects the way of communication, especially mathematical communication.

In this study, the communication aspects that will be explored from prospective math teacher were accuracy, coherence, completeness and clarity of written math in solving mathematical problems.

2. Method
The type of this research was descriptive research with a qualitative approach. A total of 86 prospective math teachers (Mathematics education students of Universitas Negeri Surabaya) participated in this study, 17 were male, and 69 were female students, aged 18-23 years old.

The instruments used in this study were: (1). The Group Embedded Figure Test (GEFT) by Witkin was used to classify subjects based on their cognitive style namely FI (Field Independence) and FD (Field Dependence). The GEFT test consists of 18 items, which are questions about finding a figure in more complex figures. The range of GEFT score was from 1 to 18, the subject was said to have an FD cognitive style if the GEFT score was less than 10 and if the score was 10 or more then the subject was classified as had FI cognitive style; (2). The Mathematical test was used to determine the mathematical abilities of subjects; and (3). The Problem-solving test was used to investigate the mathematical communication of subjects.

The research procedures carried out were: GEFT test was given to subjects and by their score of GEFT test, subjects were classified into FI and FD groups. Mathematics tests were given to subjects to determine the subject's mathematical abilities. Then problem-solving tests were given to the subjects to investigate the subject's mathematical communication in solving problems. The subjects' problem solving were analysed based on aspects of the accuracy in using mathematical symbols, mathematical terms, mathematical expressions and the coherence in explaining the mathematical ideas used, also the completeness and the clarity of the explanations used in solving problems. The results of mathematical communication subjects were grouped based on differences in FI and FD, and then two subjects with the same mathematical abilities were selected, namely one FI subject and one FD subject to be interviewed for further study of mathematical communication. The criterion of each aspect of mathematical communication used in this study were: the written results were said to be accurate if the notations, terms and formulas and the mathematical writing used are correct and mathematically accurate, was said to be coherent if the steps used, the reasons given were logical and interrelated, and said to be complete and clear, if all information needed in the process was given and the explanation given was clear.

3. Results and Discussion
From the results of the GEFT test given to 86 subjects, 11 FD and 75 FI subjects were obtained. The problem solving provided consists of integral problems, geometry problems and maximal strategy problems. Integral problem was the problem of determining the integral of a function if the derivative of the function was given. The geometry problem used is the problem of determining the circumference of the snowflake curve in the tenth iteration if the circumference of the first iteration is equal to 1. The problem of the maximum strategy was the problem of determining the maximum number of apples (in kg) sent to the store on the hill using a cart under certain conditions, subjects were asked to determine the strategy so that the apple sent to the store was maximum. The following general description of written mathematics communication was grouped by FI and FD:

3.1. Integral Problem
The results of the mathematical communication analysis of the work of the FI and FD groups for integral problems showed almost the same results, namely around 64% of the FD group and 71% of the FI group that wrote their answers accurately, the steps that were carried out coherently and logically and the information needed complete. Apart from groups that wrote answers accurately, coherently and completely, many subjects wrote integral forms without "dx", forgot to add constants in the process of integration, changed variables of functions for no apparent reason. Some of them got the function f only
by integrating the given function, even though the requested function was integral off when the derivative of f was known. They were not careful in understanding the problem because usually the questions they had encountered so far were determining the integral of a function or determining the derivative of a function.

3.2. Geometry Problem
Students were asked to calculate the circumference of the Koch curve (in Figure 1) in the 10th iteration when the circumference of the first iteration was 1.

Unlike the Integral problem which was solved by simply using the formula or integral knowledge of the subjects, the geometry problem required carefulness in analysing the change of the length of the sides of the geometry figure when iteration increased. After the scale changed of the side length was known when iteration increased, this problem could be solved either by using the growth of the circumference with the scale changed in the length of the sides directly or by using a geometric sequence formula.

Only 20% of the FD group and 40% of the FI group answered this geometry problem by writing out the steps that were understandable and logical. They wrote accurate mathematical terms, symbols and formulas and wrote down the information needed to solve the problem completely.

Most FD students had difficulty in determining the pattern of changing sides of a geometry figure as the iteration increases, and they focused more on the number of sides rather than the length. They could not explain or express what they thought, what the steps they took, and some of them just gave the signs in the form of fractions or numbers at the bottom of the figure without any explanation. The works of more than half FD students were incomplete, so it was incoherent and difficult to understand the way they thought. In general, FI students wrote the steps they used to solve this problem more clearly than FD students, although some of them were wrong in determining the ratio of change inside length, the symbols and mathematical formulas that they used were also accurate. However, some of them did not write down the information needed completely. They knew that the problem could be solved by geometric sequences, but the formula used without explanations, why and how they determined the first term and the ratio.

3.3. Maximal strategy problem
The problem is given:
Andre will send 100 kg apples using a cart to a store located on a hill 10km from his warehouse. The cart can carry a maximum of 10 kg, and each time is travelling 1km up the hill, Andre consumes 1 kg of apples to eat. Discover the strategy so that Andre can send as many apples to the store as possible and determine how many kg of apples reach the destination. At each km from the warehouse to the store, there is a stop so that Andre can temporarily leave the apples there if needed.

From the results of students FD work, no one got the optimal answer. 40% of them chose the strategy of sending apples directly so that the apples sent were 0 kg because they were consumed in shipping.

The other 40% used the strategy of trying two different ways, and they got the apples sent were 25 kg. 20% of them had difficulty determining the strategy, they just described the problem in a picture. Almost all FD students did not explain the strategy used coherently and clearly, and they only used the arrows and charts without explanation. 25% of FI students found the optimal results, that was 32 kg.
apples were sent to the store located in the hills, and they explained the strategies they used either by using words or in the form of schema with little information. Another 50% use a trial and error strategy, but they did not get the optimal results, while 25% could not determine the strategy to be taken.

The students thought a lot when solving problems related to finding strategies, but they could not communicate it by writing their ideas clearly and accurately.

The results of the mathematics test with the range score 0 - 100 showed that of 84 students who took the test, the minimum score of students was 20, and the maximum score was 100. Based on the results of the analysis, the researchers selected one FD subject and one FI subject of 84 students with moderate mathematics ability, that was between 65 to 75.

3.4. Mathematical communication of FD student
The following description was the mathematical communication profile in the integral, geometry and maximal strategy problems of one FD student.

3.4.1. Integral Problem
Figure 2 below shows the answer of FD subject on integral problem.

![Figure 2. Integral problem solving by the FD subject](image)

In solving the Integral problem, the FD subject initially determined the function $f$ by integrating the derivative of the function $f$, then reintegrated the function that had been obtained to get the integral of function $f$, but she did not integrate the constants part of the function. The steps used in solving problems understandable, but some of the information used was not equipped with explanations such as the use of the $f(t)$ function. Based on the interview, the subject said that the function $f(t)$ was intended as a function that must be found, but the subject could not provide a reason why he changed the variable from $x$ to $t$. Therefore, the mathematical communication of FD subjects on integral problem was coherent, less accurate and incomplete.

3.4.2. Geometry Problem
Figure 3 below shows the answer of FD subject on the geometry problem.

![Figure 3. Geometry problem solving by the FD subject](image)

From the answers of FD subjects, it was difficult to understand what she wants to explain. The FD subject only gave a mark on the side of the figure with a fraction and wrote “the circumference of the figure for each iteration is the same that is 1”. Then, on the right side she wrote the geometric sequence
formula and determined the 10th term with the initial term value was 3 and the ratio value was 4. When interviewed, the subject of FD said that the initial step she took was to determine the scale of the side changes for the next iteration. In the first iteration, because the circumference was 1, the side length was one third. In the second iteration, she counted the number of sides in the figure were 12, so she calculated the length of the sides was one-twelfth. Also, at the third iteration, the side length of one forty-eighth was obtained. Then she concluded that the circumference was always 1. However, she also remembered that this problem could also be solved by using the geometric sequence formula by determining the 10th term. She determined the initial term is the number of sides in the first figure, 3 and the ratio obtained from the number of sides of the second feature divided by the number of sides in the first figure so that it is obtained 4. She realised that she found two different answers but she did not know which one was true, and how to checked it. Therefore, the mathematical communication of FD subjects on geometry problem was incoherent, inaccurate and incomplete.

3.4.3. Maximal strategy problem
Figure 4 below shows the answer of FD subject on maximal strategy problem.

![Figure 4. Maximal strategy problem solving by FD](image)

The subject of FD wrote down the important points of the problem in diagrammatic form, she also wrote the three sequential numbers that satisfy the Pythagorean theorem, then made a chart that links the number of apples consumed with the distance travelled to send the apples. Then she concluded the distance between the warehouse and the store was 10 km and the apple consumed was 10 kg equal to the maximum apple that can fit in the cart so that the apple sent was 0 kg. In determining the maximum strategy for sending apples, the FD subject initially thought of using the Pythagorean theorem because it was related to "a store on the hill", but she finally realised there was no need to use it. She made a diagram of the relationship between the apple consumed, and the distance travelled to send the apple. She could not think of another way. Therefore, the mathematical communication of FD subjects on maximal strategy problem was incoherent, inaccurate and incomplete.

3.5. Mathematical communication of FI subject
The following description was the mathematical communication profile in the integral, geometry and maximal strategy problems of FI student.
3.5.1. Integral Problem

Figure 5 below shows the work of FI subject on the integral problem.

The FI subject solved the integral problem by integrating the function known, and she noted this by function \( f \). Then, she integrated the new function to find the answer. She wrote the solutions coherently so that the steps could be understood, and it’s just that there was a little inaccuracy when writing integrals at the end of the answer where the sign \( dx \) was missed. Therefore, the mathematical communication of FI subjects on integral problem was coherent, less accurate and complete.

3.5.2. Geometry Problem

Figure 6 below shows the work of FI subject on the geometry problem.

The steps used by FI student: at first, she calculated the length of the side of the triangle, that was \( \frac{1}{3} \), and then she determined the length of the side for the second iteration, and the third iteration respectively was \( \frac{1}{9} \) and \( \frac{1}{27} \). Then she counted the number of sides in the first, the second and the third iteration that she got respectively was 3, 4x3 = 12, 16x3 = 48. She noted the circumference of the object for the first iteration, the second and the third iteration, and from these, she found the pattern of the circumference. She used the formula of the pattern to find the circumference of the tent iteration the curve. She explained her way of thinking by writing down the strategies she used coherently, and used the marked in the side of the curve to show the length of the side but was not careful in entering the calculation results into formulas. When entering the value in the formula, she did not recheck with the data she had obtained so that she did not realise that there was an error in writing the exponent. Therefore, the mathematical communication of FI subjects on geometry problem was coherent, less accurate and incomplete.
3.5.3. Maximal strategy problem
Figure 7 below shows the work of FI subject on the geometry problem.

![Figure 7](image)

**Figure 7.** The answer to the maximal strategy problem by FI subject

FI subjects solved the problem by describing the stages of the strategy with a schema by writing down the number of times the apple was sent and the rest of the apple at the next stop. Even though the answer given was correct, it was the optimal result, but there was no written statement that it was an optimal result and the explanation about that. From students’ explanations, it was known that after experimenting with various strategy, she wrote the maximum answer, but she could not provide justification logically. Therefore, the mathematical communication of FI subjects on maximal strategy problem was coherent, less accurate and incomplete.

4. Conclusion
Based on the results of this study, it can be concluded that most prospective math teacher in solving problems could not express what was in their minds, could not explain the strategies used clearly, accurately and coherently. Especially if the problem-given was a non-routine problem. Because, to solve routine problems, not too many explanations were needed. This matter could be seen from the percentage of the number of students answering integral problems accurately and coherently was more significant than when answering geometry problems even more with the maximum strategy problem. Related to differences in mathematical communication between FI students and FD students, based on the results of data analysis, it was found that for the integral problems, FI and FD students were relatively similar, they could write their answers understandable, only that there was little inaccurate writing. While for geometry problem and the maximal strategy problem, FI students explained the strategies they did more coherently and clearly than FD students, although some of them used charts, tables to solve the problems. FI students were more detailed in explaining what they did than FD students, and this may be due to the nature of FD who thinks globally. This research was based on written mathematical communication from prospective math teachers, it is suggested that the oral mathematical communication of prospective math teacher also needs to be investigated considering one of the teacher’s tasks is to explain the subject matter to students.

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