Charge Qubit Storage and its Engineered Decoherence via Microwave Cavity

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We study the entanglement of the superconducting charge qubit with the quantized electromagnetic field in a microwave cavity. It can be controlled dynamically by a classical external field threading the SQUID within the charge qubit. Utilizing the controllable quantum entanglement, we can demonstrate the dynamic process of the quantum storage of information carried by charge qubit. On the other hand, based on this engineered quantum entanglement, we can also demonstrate a progressive decoherence of charge qubit with quantum jump due to the coupling with the cavity field in quasi-classical state.

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I. INTRODUCTION

The concept of quantum entanglement is crucial to the fundamental aspects of quantum mechanics, including the quantum measurement and quantum decoherence \cite{ref1}. It also dominates the development of quantum information science and technology. At this stage, implementing the realistic quantum computation and communication, one needs to create the maximally quantum entanglement for potentially physical carrier of quantum information.

As potential candidates of information carrier in quantum computation, some kinds of qubit based on Josephson junctions \cite{ref2}, such as charge qubit, phase qubit and flux qubit, have been implemented experimentally. Now single qubit operation and two qubit logic gate based on Josephson junctions demonstrate an important role to realize a scalable quantum computing. Recent experiments demonstrate the Rabi oscillation in a Cooper-pair box (charge qubit) \cite{ref3} and indicate existence of entangled two-qubit states \cite{ref4}. Up to now, the decoherence time which is of order 5µs has been reported in \cite{ref5}. In the process of quantum computing, we need qubits with long decoherence time, so we must find a longer life time medium to store the state of qubit. The microwave cavity is a good candidate which life time is of order 1ms \cite{ref6}. Actually quantum information storage is the central issues in the study of quantum information and computation \cite{ref7,ref8,ref9,ref10}.

In this paper, we describe a model of a charge qubit coupled to a single mode quantum cavity field. Under the rotating wave approximation (RWA), we demonstrate how to implement a dynamic process of quantum information storage in the qubit-cavity system. In addition, we consider the entanglement and engineered decoherence of the qubit-cavity system.

In fact, the integration of qubit and cavity QED \cite{ref11,ref12,ref13} has become the focus of quantum information storage and quantum computation. In order to investigate quantum coherence and information storage in the qubit-cavity system, we consider a single mode quantum cavity field with frequency $\omega \sim 30GHz$ (typically in the microwave domain) coupled to a charge qubit (with dc SQUID) in which the charging energy $E_C \sim 122\mu eV$ and the Josephson coupling energy $E_J \sim 34\mu eV$ \cite{ref14}.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{charge_qubit_cavity.png}
\caption{Schematic of the charge qubit-cavity system. Superconducting microwave cavity with parameter, R=2.55mm, L=0.5cm.}
\end{figure}

II. CHARGE QUBIT COUPLED TO CAVITY FIELD

The charge qubit considered in this paper is a dc SQUID consisting of two identical Josephson junctions enclosed by a superconducting loop. The Hamiltonian for an dc SQUID can be written as in \cite{ref15}:

$$H = 4E_C \left( n_g - \frac{1}{2} \right) \sigma_z - E_J \cos \left( \frac{\Phi}{\Phi_0} \right) \sigma_x + \omega a^\dagger a \quad (1)$$

where $E_C$ is the charging energy, $E_J$ Josephson coupling energy and we assume $E_C >> E_J$, and $\Phi$ the magnetic

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flux generated by controlled classical magnetic field and quantum cavity field. Quasi-spin operators

\[ \sigma_z = |0\rangle_q \langle 0|_q - |1\rangle_q \langle 1|_q, \sigma_x = |0\rangle_q \langle 1|_q + |1\rangle_q \langle 0|_q \]

are defined in the charge qubit basis \(|0\rangle_q \text{ and } |1\rangle_q\), \(\Phi_0 = \frac{\Phi_0}{\Phi_0}\) denotes the flux quanta.

Now consider the case that the magnetic flux threading the dc SQUID is generated by magnetic field \(B_c\) consisting of external classical magnetic field \(B_c\) and quantum cavity field \(B_f\), i.e., \[B = B_c + B_f.\]

So the total magnetic flux threading SQUID is also divided into two parts

\[ \Phi = \Phi_e + \Phi_f, \]

where \(\Phi_e = \int B_c \cdot dS\) is the external classical flux threading the dc SQUID, \(\Phi_f = \int B_f \cdot dS\) the cavity-induced quantum flux through the dc SQUID and \(S\) any area bounded by the dc SQUID.

We assume that there exists a single mode standing wave cavity field of frequency \(\omega\) \(\text{[16]}\). The quantum cavity field is assumed to be transverse with respect to the magnetic field \(B_f\) polarized in the y-direction, i.e.,

\[ B_y(z) = -i \left( \frac{\hbar \omega}{\epsilon_0 V c^2} \right)^{1/2} (a - a^\dagger) \cos \left( \frac{2\pi}{\lambda} z \right) \]

where \(a^\dagger(a)\) is the creation (annihilation) operator of the single mode cavity field and \(V\) the electromagnetic mode volume in cavity.

In Fig.1 the dc SQUID lies in the x-z plane and at the position of the antinode of standing wave field, i.e., \(z = 0\). Then the magnetic field \(B_y(z)\) will be

\[ B_y(0) = -i \left( \frac{\hbar \omega}{\epsilon_0 V c^2} \right)^{1/2} (a - a^\dagger) \]

and magnetic flux \(\Phi_f\) is given by

\[ \Phi_f = -i \left( \frac{\hbar \omega}{\epsilon_0 V c^2} \right)^{1/2} S (a - a^\dagger). \]

In this case, we can derive the spin-Boson Hamiltonian from Eq.1 in a straightforward way,

\[ H = 4E_C \left( n_q - \frac{1}{2} \right) \sigma_z - E_f \cos (\phi_e + \phi_f) \sigma_x + \omega a^\dagger a \]

where

\[ \phi_e = \frac{\pi \Phi_e}{\Phi_0}, \phi_f = -i \phi_0 (a - a^\dagger) \]

and

\[ \phi_0 = \frac{\pi S}{\Phi_0} \left( \frac{\hbar \omega}{\epsilon_0 V c^2} \right)^{1/2}. \]

As shown in Fig.1 two spherical mirrors form microwave cavity containing a single mode standing wave field and an external classical magnetic field is also injected into the cavity. In this paper, we adopt some appropriate parameters for the geometry of cavity, the curvature radius \(R = 2.55\) mm, the width between two mirrors \(L = 0.5\) cm. By some simple calculations, we get that the cavity field \(B = \left( \frac{\hbar \omega}{\Phi_0} \right)^{1/2} = 7.52 \times 10^{-11}\) (Tesla) and \(\phi_0 = \frac{\pi \omega}{\Phi_0} = 1.14 \times 10^{-5}\). In a low photon number cavity, we find that \(\phi_f \ll \phi_e\), thus there is only a weak polynomial nonlinearity in Eq.1.

### III. DYNAMICAL QUANTUM INFORMATION-STORAGE OF CHARGE QUBIT

Any quantum computer realized in a laboratory will be subject to the influence of environmental noise in the preparation, manipulation, and measurement of quantum-mechanical states which causes quantum decoherence. In the implementation of quantum computation, quantum information should be encoded in a quantum network formed by many qubits with longer decoherence time. Subsequently one can store, manipulate and communicate quantum information \(\text{[17]}\). So we are interested in study of quantum information-storage.

Now up to the first order of \(\phi_f\) in Eq.4, we consider quantum information-storage in the qubit-cavity system. To this end, we adopt the appropriate parameters, i.e., the frequency of microwave cavity at \(\omega \sim 30\) GHz and the charging energy of charge qubit at \(E_C \sim 29.5\) GHz \(\text{[13]}\). When we tune gate voltage of Josephson junction at \(n_q \sim 0.627\), the RWA is valid.

In this section, we consider the storage of quantum information of the charge qubit in the cavity mode. We derive the Hamiltonian for qubit-cavity system from Eq.4,

\[ H = \frac{\omega}{2} \sigma_z + i\eta (a^\dagger \sigma_+ - a \sigma_-) + \omega a^\dagger a. \]

It is just as same as a Jaynes-Cummings model \(\text{[18]}\) in optical cavity QED.

Now we demonstrate a process of quantum information transfer from one charge qubit to cavity field. We assume that the initial state of charge qubit is in an arbitrary state \(\alpha |0\rangle_q + \beta |1\rangle_q\) and we want transfer this state from charge qubit to cavity that its initial state is in the vacuum state \(|0\rangle_c\). The process of information storage from qubit to cavity can be described by

\[ (\alpha |0\rangle_q + \beta |1\rangle_q) \otimes |0\rangle_c \rightarrow |0\rangle_q \otimes (\alpha |0\rangle_c + \beta |1\rangle_c). \]

From Eq.6, we find that this information-storage process could be done with a transformation \(\text{[11]}\)

\[ |0\rangle_q |0\rangle_c \rightarrow |0\rangle_q |0\rangle_c, \]

\[ |1\rangle_q |0\rangle_c \rightarrow |0\rangle_q |1\rangle_c. \]
We can easily verify that the Hamiltonian in Eq. (5) holds the condition of information storage in Eq. (7), i.e.,

\[
H |0\rangle_q |0\rangle_c = -\frac{\hbar \omega}{2} |0\rangle_q |0\rangle_c \\
H |1\rangle_q |0\rangle_c = -\frac{\hbar \omega}{2} |1\rangle_q |0\rangle_c + i\eta |0\rangle_q |1\rangle_c
\]

Now the process of quantum state transfer from the charge qubit to microwave cavity can be realized in the following steps.

Firstly at the time \( t = 0 \), we turn off the external classical field and prepare the initial state of qubit in the pure state

\[
\alpha |0\rangle_q + \beta |1\rangle_q
\]

corresponding to the reduced density matrix of the qubit

\[
\rho_q (0) = \left( \begin{array}{cc} |\alpha|^2 & \alpha \beta^* \\
\alpha^* \beta & |\beta|^2 \end{array} \right)
\]

where \(|\alpha|^2 + |\beta|^2 = 1\), the cavity is prepared in the vacuum state \(|0\rangle_c\). The initial state of the qubit-cavity system can be written as

\[
|\psi(0)\rangle = (\alpha |0\rangle_q + \beta |1\rangle_q) \otimes |0\rangle_c. \quad (8)
\]

Secondly with time increasing, we turn on the external classical field and allow the qubit and microwave cavity to interact on resonance. Then the state of the qubit-cavity system evolves into

\[
|\psi(t)\rangle = |0\rangle_q \otimes (\alpha e^{i\phi t}/2 |0\rangle_c - \beta e^{-i\phi t/2} \sin \eta t |1\rangle_c) + \beta e^{-i\pi/2} \cos \eta t |1\rangle_q |0\rangle_c
\]

corresponding to the reduced density matrix of the cavity

\[
\rho_c (t) = \left( \begin{array}{cc} |\alpha|^2 + |\beta|^2 \cos^2 \eta t & -\beta^2 \alpha e^{i\pi t} \sin \eta t \\
-\alpha^* \beta e^{-i\pi t} \sin \eta t & |\beta|^2 \sin^2 \eta t \end{array} \right). \quad (9)
\]

Obviously, we find that when \( \eta t = 0 \),

\[
\rho_c (0) = \left( \begin{array}{cc} 1 & 0 \\
0 & 0 \end{array} \right) \quad (10)
\]

and the transfer of quantum information occurs when \( \eta t = \frac{\pi}{2} \) or \( t = \frac{\pi}{2\eta} \).

\[
\rho_c \left( \frac{\pi}{2\eta} \right) = \left( \begin{array}{cc} |\alpha|^2 & -\alpha^* \beta e^{-i \frac{\pi}{2\eta}} \\
-\beta^2 \alpha e^{i \frac{\pi}{2\eta}} & |\beta|^2 \end{array} \right)
\]

and the state of the qubit-cavity system evolves into the state

\[
|\psi(\frac{\pi}{2\eta})\rangle = |0\rangle_q \otimes (\alpha e^{i \frac{\pi}{2\eta}} |0\rangle_c - \beta e^{-i \frac{\pi}{2\eta}} |1\rangle_c), \quad (11)
\]

where the cavity is in the pure state

\[
\alpha e^{i \frac{\pi}{2\eta}} |0\rangle_c - \beta e^{-i \frac{\pi}{2\eta}} |1\rangle_c.
\]

From Eq. (8) and Eq. (11), we can say that at time \( t = \frac{\pi}{2\eta} \), the quantum information contained in the qubit has been stored in the microwave cavity.

As shown in Fig. 2, we set \( \alpha = \beta = \frac{1}{\sqrt{2}} \) and demonstrate the probability of that quantum information of the qubit is transferred to the microwave cavity

\[
P = \frac{1}{4} \left( \omega t - \frac{\omega \pi}{2\eta} \right) \sin \eta t + \frac{1}{4} \sin^2 \eta t. \quad (12)
\]

We find that the qubit transfers the quantum information to the microwave cavity at the time \( t = \frac{\pi}{2\eta} \sim 2.7 \mu s \).

FIG. 2: The probability of the system in the state \(|\psi(\frac{\pi}{2\eta})\rangle\).

The qubit transfer quantum information to the microwave cavity when the probability is equal to 1.

IV. ENTANGLEMENT AND ENGINEERED DECOHERENCE

In previous work on cavity QED \[16\], Raimond et al dealt with entanglement and decoherence for the cavity-atom system and show a reversible decoherence process of a mesoscopic superposition of field states. In this paper, our model is quite similar to a cavity QED model without the rotation-wave-approximation (RWA), which usually describes the single mode cavity interacting with an two-level atom \[21\]. In this cavity QED model, when the atoms in different states \(|0\rangle\) and \(|1\rangle\) will modify the states of cavity field in different ways and thus induce the quantum decoherence of atomic states superposition. In this section, we consider these issues about entanglement and decoherence in the qubit-cavity system.

Now we consider a single charge qubit (dc SQUID) coupled to a single-mode cavity. According to Eq. (13), we tune the gate voltage \( V_g \) such that \( n_g = \frac{1}{2} \) and then get a Hamiltonian for a standard quantum measurement model,

\[
H = -E_f \cos (\phi_c + \phi_f) \sigma_x + \omega a^\dagger a. \quad (13)
\]

By some simple calculations from Eq. (13), we derive an effective Hamiltonian,

\[
H = H_0 |0\rangle \langle 0| + H_1 |1\rangle \langle 1| \quad (14)
\]
which is diagonal with respect to $|0\rangle = |0\rangle_q + |1\rangle_q$ and $|1\rangle = |0\rangle_q - |1\rangle_q$ are eigenstates of quasi spin operator $\sigma_x$. The effective actions on cavity field are

$$H_k = - (-1)^k E_J \cos (\phi_e + \phi_f) \sigma_x + \omega a^\dagger a$$

(15)

for $k = 0, 1$ respectively. Obviously the Hamiltonian in Eq. (14) can create entanglement between charge qubit and cavity field. Since the different actions on the cavity field are exerted by the charge qubit in different quasi-spin state.

Technically the engineered decoherence process is described by the time evolution of the reduced density matrix of the coupled qubit-cavity system. To analyze it, we can derive the reduced density matrix for the time evolution of the qubit-cavity system. The decoherence process means that the off diagonal elements of the reduced density matrix of the qubit vanish, while the diagonal elements remain unchanged.

Here we get time evolution of density matrix for the qubit-cavity system

$$\rho(t) = |\psi(t)\rangle \langle \psi(t)|$$

(16)

and calculate the reduced density matrix of the qubit

$$\rho_s(t) = C_0^* C_0 |0\rangle \langle 0| + C_1^* C_1 |1\rangle \langle 1| + (d_1(t) |d_0(t)\rangle \langle 1| + h.c$$

(17)

where

$$\langle d_1(t) |d_0(t)\rangle \langle 1|$$

and $d_1(t)$ and $d_0(t)$ are the first order interaction strength and the second order interaction strength respectively. Obviously the Hamiltonian in Eq. (19) containing terms of $\delta$ which drives the cavity field into the coherent state and the interaction with terms of $\eta \phi_e$ drives the cavity field into the squeezed state.

In this paper, we control the interaction between qubit and cavity by changing the strength of the external classical field, i.e., $\phi_e$. From Eq. (19), if we turn off the external field, i.e., $\sin \phi_e = 0$, the above Hamiltonian only includes the second order interaction. If we turn on the external magnetic field, i.e., $\cos \phi_e = 0$, the above Hamiltonian only includes the first order interaction.

Now we choose the coherent state $|\alpha\rangle$ as the initial state of quantum cavity field. It is not difficult to prepare such quasi-classical state by a driving current source. Through some simple calculations, we find that the Hamiltonian in Eq. (19) containing terms of $aa$ and $a^\dagger a^\dagger$ will drive the cavity field into two different squeezed states

$$|d_k(t)\rangle = |\beta_k, \mu_k, \nu_k\rangle e^{i\delta_k}$$

(20)

where

$$\beta_k = \alpha + (-1)^k \frac{i\eta}{\Omega_k N_k} \left( 1 - e^{i\Omega_k t} \right)$$

$$\theta_k = \left( \frac{\eta^2}{\Omega_k N_k} + (-1)^k E_J \cos \phi_e \right) t$$

$$\mu_k = \cos \Omega_k t + i \frac{\omega + (-1)^k 2\delta}{\Omega_k} \sin \Omega_k t$$

$$\nu_k = -(-1)^k \frac{i2\delta}{\Omega_k} \sin \Omega_k t$$

$$\Omega_k = \sqrt{\omega^2 + (-1)^k 4\delta^2}$$

$$N_k = \frac{\sqrt{4\delta^2 + \omega^2}}{\omega}$$

and $\delta_k = \alpha$ when the time $t = 0$, for $k = 0, 1$ respectively. Here the squeezed states $|\beta_k, \mu_k, \nu_k\rangle$ is defined as in [16] for a new set of boson operators [21]

$$A_k = \mu_k a - \nu_k a^\dagger$$

(25)

for $k = 0, 1$ which hold

$$A_k |\beta_k, \mu_k, \nu_k\rangle = \beta_k |\beta_k, \mu_k, \nu_k\rangle$$

Then the time evolution of decoherence factor which characters the quantum coherence of the charge qubit is

$$D(t) = \frac{1}{\sqrt{\mu_0 \nu_0 - \nu_1 \mu_0}} \exp \left\{ -\frac{1}{2} \frac{|\beta_0|^2}{\mu_0^2} - \frac{1}{2} \frac{|\beta_1|^2}{\nu_1^2} + \frac{2\delta^2}{\mu_0^2} \frac{\beta_0^* \beta_1 + \beta_1^* \beta_0}{2} \right\}$$

We control the interaction between qubit and cavity by changing the strength of the external classical field, i.e., $\phi_e$. From Eq. (19), if we turn off the external field, i.e., $\sin \phi_e = 0$, the above Hamiltonian only includes the
FIG. 3: Time evolution of the decoherence factor $D(t)$ with $\phi_e = 0$ and different $\alpha = 0, 1, 2, 3$ respectively. When $\alpha \neq 0$, the second order interaction in Eq. (18) induces collapses and revivals of quantum coherence. When $\alpha = 0$, the qubit owns the least loss of quantum coherence.

In Fig. 4 when the interaction Hamiltonian only includes the second order interaction, i.e., at the value of $\phi_e$ for $\sin \phi_e = 0$, the period of oscillation for $D(t)$ is unchanged and the width of the peak becomes smaller with $\alpha = 1, 2, 3$ respectively. Specially at the value of $\phi_e$ for $\sin \phi_e = 0$ and $\alpha = 0$, $D(t)$ approaches one, i.e., the loss of coherence is negligible. In Fig. 4 when the interaction Hamiltonian only includes the first order interaction, i.e., $\cos \phi_e = 0$, we have the time evolution of the decoherence factor

$$D(t) = \exp \left[ \frac{8 \pi^2}{\Omega^2} \sin^2 \frac{\omega t}{2} \right]. \quad (26)$$

It means that $D(t)$ is independent of values of $\alpha$ and the amplitude of oscillation is very small.

V. DISCUSSIONS

In this paper, we have studied quantum entanglement in the charge qubit-cavity system and demonstrate a engineered decoherence process in the case of the second order approximation. In Fig. 3 and Fig. 4, we find that controlled parameter $(\phi_e, \alpha)$ exists two optimal values $(\sin \phi_e = 0, \alpha = 0)$ and $(\cos \phi_e = 0, \alpha)$ with the least loss of quantum coherence for the charge qubit.

On the other hand, we demonstrate a dynamical process of quantum information storage between the charge qubit and the microwave cavity. From the above results, we get the time of the information storage $\Delta t = \pi \Omega^2 \sim \sqrt{V/S}$. To implement faster operation of information storage, we can reduce the electromagnetic mode volume $V$ of microwave cavity and increase the bounded area $S$ of dc SQUID (charge qubit). We wish that our results in this paper will be helpful to experiments.

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