Experimental Verification of Thermal Structural Responses of a Flexible Rolled-Up Solar Array

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This study aims to verify an established study on the theoretical analysis of the thermal structural response of a flexible space structure through comparison with the experimental analysis results using an experimental configuration of a reduced asymmetric solar array model. The solar array model is composed of a blanket, two flexible booms and a rigid spreader bar. To reproduce the thermal structural response of a solar array model subjected to solar heat flux in space, the experiment is executed inside a vacuum chamber. The experimental results are analyzed by measuring the boom tip deflection, and are compared with theoretical analysis results under various experimental conditions. The theoretical results are obtained using quasi-static and dynamic response analyses. Through the comparison with experimental results, we are able to quantitatively and qualitatively verify the quasi-static response, and quantitatively verify the dynamic response.

Key Words: Thermal Structural Response, Solar Array Model, Experimental Verification, Vacuum Chamber

1. Introduction

Flexible space structures, such as solar arrays and antennas, are typically unfolding structures each composed of a boom, membrane and other parts. In space, deformation and vibration of flexible space structures occur owing to the rapid change of solar heat flux during night–day orbital transitions. This phenomenon results in the unstable dynamic response of a flexible space structure. For a typical model, we can consider the vibration problem that occurred on the Hubble Space Telescope (HST) solar array. This problem manifested itself as a significant vibration that combined with the bending and torsion modes of the solar array during night–day orbital transitions in April 1990, when the HST had just been launched into orbit.

In the past, much research has been targeted at HST solar arrays. Thornton and Kim presented an analytical approach to the thermally induced bending vibration of a symmetric HST solar array. They analyzed both the uncoupled and coupled thermal-structural dynamic response for the bending mode. Chung and Thornton studied torsional buckling and free vibration analysis of a symmetric HST solar array. They analyzed it in three parts: buckling in torsion, the effect of an initial imperfection during deployment, and free vibration analysis. Murozono and Thornton described the buckling and quasi-static thermal-structural analysis of an asymmetric HST solar array. They presented an analytical approach to the twisting behavior of the HST solar array. Thornton introduced a thermally induced vibration structural response analysis of an asymmetric HST solar array subjected to thermal load. He presented an analytical approach to quasi-static response, dynamic response and structural stability criteria.

Experimental verification would provided more credibility to this established research. Therefore, this study aims to verify the validity by comparing the theoretical analysis and the experimental results. The theoretical results are analyzed taking into consideration the thermal structural quasi-static and dynamic response of an asymmetric HST solar array model. The experimental results are obtained from a thermally induced vibration experiment in a vacuum chamber and in air using an approximate 1/10 reduction model of an HST solar array. In particular, this study verifies comparisons for various experimental conditions, such as the time records of boom tip deflections, variation of boom tip deflections as a function of the axial compressible force and the solar heat flux, and the influence on the experimental results of being either in the vacuum chamber or in the air.

2. Analysis Model

For the theoretical analysis, the analysis model is shown in Fig. 1. The analysis model consists of two booms, a spreader bar and a solar blanket, and is suddenly subjected to incident solar heat flux $s_0$ from the positive $z$ direction. The solar array length and the half spreader bar length are denoted by $L$ and $b$. The widths of the blanket from the centerline of the solar array to the inner and outer booms are denoted by $b_1$ and $b_2$. For the structural characteristic of the analysis model, the ends of the booms are connected to a spreader bar to which the solar blanket is attached, working under tension. Thus, during orbital operations, the blanket tension on the spreader bar exerts a compressive force on each boom.

The analysis model assumes that 1) the solar blanket is an inextensible membrane whose thermal expansions and contractions are neglected, 2) the solar blanket is subjected to
uniform tension in the $x$ direction, and the membrane tensile force $F_s$ per unit width is constant, 3) the inner and outer booms are identical cantilevered beams subjected to different axial compressive force $P_1$ and $P_2$, respectively, 4) torsional rotations are sufficiently small that the bending displacements of the booms occur only in the $x$-$z$ plane, 5) thermal expansions of the booms are neglected and 6) the spreader bar is a rigid member of length $2b$ and supports the tensile force of the membrane over length $b_1 + b_2$. For the determination of the temperature distributions, the booms are assumed to be thin-walled circular section beams.

3. Formulations

3.1. Thermal analysis

The solar array is subjected to an incident solar heat flux $s_0$ that varies as a step function with time from the positive $z$ direction. Therefore, the thermal bending moment of boom $M_T$ considering the conservation of energy including circumferential conduction and radiation from the boom’s external surface can be expressed as

$$ M_T = \frac{EI\alpha T_m(t)}{R} $$

(1)

where $EI$ is the bending stiffness, and $\alpha$ and $R$ are the coefficient of thermal expansion and radius of the boom.

Assuming the temperature distribution of the boom’s cross-section is represented by the sum of the average temperature and the amplitude of the perturbation temperature $T_m(t)$, the solution can be represented by the heat conduction equation using the equation of temperature distribution. Detailed discussion can be found in Refs. 3), 6), and 7).

3.2. Quasi-static response

The quasi-static response of the analysis model was developed originally in Ref. 4). When the solar blanket is subjected to a uniform tensile force $F_s$ per unit width, the inner and the outer booms are subjected to axial compressive forces $P_1$ and $P_2$, respectively. When the effect of the compressive axial force $P_1$ is considered, the partial differential equations and the boundary conditions for the bending and torsion vibrations of the booms are as follows.

$$ EI \frac{\partial^4 w_i}{\partial t^4} + P_1 \frac{\partial^2 w_i}{\partial x^2} + \frac{\partial^2 M_i}{\partial x^2} = 0 $$

$$ w_i(0, t) = 0, \quad \frac{\partial w_i}{\partial x}(0, t) = 0, \quad M_i(L, t) = 0, \quad i = 1, 2 $$

(2)

$$ E'I \frac{\partial^4 \theta_{ai}}{\partial t^4} - \left( GJ - \frac{P_i I_i}{A} \right) \frac{\partial^2 \theta_{ai}}{\partial x^2} = 0 $$

$$ \theta_{ai}(0, t) = 0, \quad \frac{\partial \theta_{ai}}{\partial x}(0, t) = 0, \quad \frac{\partial \theta_{ai}}{\partial x}(L, t) = 0, \quad i = 1, 2 $$

(3)

$$ F_s \frac{\partial^2 w_m}{\partial x^2} = 0 $$

$$ w_m(0, t) = 0, \quad w_m(L, y, t) = w_s(y, t) = w_{sd}(t) + y\theta_{sd}(t) $$

(4)

where $A$ is the cross-sectional area, $E'I$ is the warping stiffness, $GJ$ is the torsional stiffness, $I_i$ is the polar moment and $M_{ai}$ the bending moment about the boom. Assuming that the spreader bar is rigid, its deflection in the $z$-direction $w_s$ may be derived using the deflection of its center of mass $w_{sd}$ and the rotation angle of the spreader bar $\theta_{sd}$. Here, subscripts 1 and 2 denote the inner and the outer booms, respectively.

The deflection of boom in the $z$-direction, $w_s$, the angle of twist about the $x$-axis $\theta_{ai}$ and the deflection of the solar blanket $w_m$ are obtained by solving Eqs. (2)-(4). Detailed discussion on the quasi-static response can be found in Ref. 4).

3.3. Dynamic response

The dynamic response of the analysis model was developed originally in Ref. 8). If it is assumed that the thermally induced dynamic response can be expressed as the sum of the quasi-static deformation of the solar array and the vibrations about the quasi-static deformations, then we obtain the non-homogeneous equation and the homogeneous boundary conditions for the vibration response as follows.

$$ EI \frac{\partial^4 \ddot{w}_i}{\partial t^4} + P_1 \frac{\partial^2 \ddot{w}_i}{\partial x^2} + \rho A \frac{\partial^2 \ddot{w}_i}{\partial t^2} = -\rho A \frac{\partial^2 \ddot{w}_m Q}{\partial t^2} $$

$$ \ddot{w}_i(0, t) = 0, \quad \frac{\partial \ddot{w}_i}{\partial x}(0, t) = 0, \quad \frac{\partial^2 \ddot{w}_i}{\partial x^2}(L, t) = 0, \quad i = 1, 2 $$

(5)

$$ E'I \frac{\partial^4 \ddot{\theta}_{ai}}{\partial t^4} - \left( GJ - \frac{P_i I_i}{A} \right) \frac{\partial^2 \ddot{\theta}_{ai}}{\partial x^2} = -I_s \frac{\partial^2 \ddot{\theta}_{ai} Q}{\partial t^2} $$

$$ \ddot{\theta}_{ai}(0, t) = 0, \quad \frac{\partial \ddot{\theta}_{ai}}{\partial x}(0, t) = 0, \quad \frac{\partial \ddot{\theta}_{ai}}{\partial x}(L, t) = 0, \quad i = 1, 2 $$

(6)

$$ F_s \frac{\partial^2 \ddot{w}_m}{\partial x^2} - \sigma_m \frac{\partial^2 \ddot{w}_m}{\partial t^2} = \sigma_m \frac{\partial^2 \ddot{w}_m Q}{\partial t^2} $$

(7)

Here, $\rho$ is the density, $\sigma_m$ is the membrane mass per unit area and $w_{sd}$ includes the vibration component of the spreader bar deflection. Here, subscript $Q$ denotes the quasi-static deformations obtained by neglecting the effects of
the inertial force. The vibration response components are denoted with tildes.

As stated above, the vibration responses are dominated by the homogeneous boundary conditions similar to those used in the natural vibration analysis and non-homogeneous equations. We can then use the mode summation procedures\(^9\) based on the results of the natural vibration analysis to obtain the dynamic response. Therefore, we can obtain the vibration responses of deflection and rotation of the booms and the solar blanket deflection. Detailed discussion on the dynamic response can be found in Ref. 8).

4. Experimental Configuration

In this study, we construct the thermally-induced vibration experimental equipment to verify the theoretical analysis of the coupled thermal structural quasi-static and dynamic response of the HST solar array model when subjected to a thermal load. This is assumed to be independent of structural deformation.

Thermally induced vibration experiments in a vacuum chamber or in atmosphere using an approximate 1/10 scale reduction model of the HST solar array are performed to verify the influence of various experimental conditions, such as the time history of boom tip deflection and the variation of boom tip deflection according to the axial compressive force and solar heat flux.

In this study, the reduction model is considered only as a scale of reduction and does not consider the properties of actual HST solar array for the reason that is difficult to produce the reduced model including the properties of actual HST model at the laboratory level. However, verification of analytical model is considered sufficient.

4.1. Experimental equipment

To realize the structural characteristic of the test specimen, we constructed the experimental equipment shown in Fig. 2. A motor mounted on the top section is able to move the central shaft connected to the worm gear up and down. At the same time, we can apply any required tension to the blanket. Thus, the tension applies an axial compressive force to the boom. This axial compressive force is measured by a load cell. Here, we ignored the gravity effect. The reason is that the experimental model is considered to affect the tension of including the gravity effect from the spreader bar, which is installed in the direction that gravity acts, and the gravity is considered to have an affect in the perpendicular direction of movement of the spreader bar.

For the thermally-induced vibration experiment of the test specimen, two heaters are provided, one in front of the inner and the other in front of the outer boom. To reproduce the rapid irradiation change in orbital situations, a heat shutter is positioned between the heater and test specimen. In addition, the heat shutter eliminates the effects of the time-lag required for the heater to reach maximum output power if the power to the heater is switched on and off. As a consequence of the shutter operation, thermal deformation of the boom, similar to orbital conditions, occurs. The displacement of the boom tip is measured using a laser displacement sensor.

Figure 3 shows the experimental equipment in the vacuum chamber. To remove the influence of air turbulence, the experimental equipment of Fig. 2 is placed in the vacuum chamber. Thermally-induced vibration experiments are performed under atmospheric and vacuum conditions.

4.2. Test specimen

The test specimen is composed of two booms, a spreader bar and a blanket. Each boom has a stainless steel tube (SUS316) containing a 0.5 mm slit to reduce the torsional stiffness. Table 1 shows the physical properties of the test specimen. These physical properties are used in the theoretical analysis, but the bending stiffness property has the
shown in Fig. 4. Bar that passes through the bearing. Each configuration is tip, bottom joint 2 combines the bearing and the spreader screws. To satisfy the boundary conditions for the boom are joined by corresponding male and female threaded Araldite, as for the upper section. Bottom joints 1 and 2 lower section boom tip is attached to bottom joint 1 using Araldite. Top joint 1 is fixed to top joint 2 and boundary conditions for the upper section. In addition, the booms have a pair of thermocouples welded to their sides for measuring experimentally determined value based on the fixed state of the upper section of the boom. In addition, the booms have a pair of thermocouples welded to their sides for measuring the surface temperature.

The upper section of the boom is attached to top joint 1 using Araldite. Top joint 1 is fixed to top joint 2 and the frame using screws. This configuration satisfies the boundary conditions for the upper section. In addition, the lower section boom tip is attached to bottom joint 1 using Araldite, as for the upper section. Bottom joints 1 and 2 are joined by corresponding male and female threaded screws. To satisfy the boundary conditions for the boom tip, bottom joint 2 combines the bearing and the spreader bar that passes through the bearing. Each configuration is shown in Fig. 4.

Fig. 4. Composition of the top and bottom of the boom.

Table 1. Properties of the test specimen.

| Property                        | Value                        |
|---------------------------------|------------------------------|
| Modulus of elasticity           | $E = 193 \times 10^6$ Pa     |
| Modulus of rigidity             | $G = 74.23 \times 10^9$ Pa   |
| Density                         | $\rho = 7.54 \times 10^3$ kg/m³ |
| Bending stiffness               | $EI = 2.808 \times 10^{-1}$ N·m² |
| Warping rigidity                | $EI_t = 5.973 \times 10^{-7}$ N·m² |
| Torsional rigidity              | $GIJ = 1.560 \times 10^{-4}$ N·m² |
| Polar moment of inertia         | $I_E = 1.647 \times 10^{-12}$ m² |
| Moment of inertia of boom       | $I_t = 1.241 \times 10^{-8}$ kg·m |
| Radius                          | $R = 1.455 \times 10^{-3}$ m  |
| Solar array length              | $L = 6.0 \times 10^{-1}$ m    |
| Wall thickness                  | $h = 9.0 \times 10^{-3}$ m    |
| Solar blanket mass per unit area| $\sigma_m = 9.37 \times 10^{-2}$ kg/m² |
| Cross-sectional area            | $A = 7.778 \times 10^{-7}$ m² |
| Spreader bar mass               | $M_s = 0.324$ kg             |
| Spreader bar mass moment of inertia| $I_s = 2.11 \times 10^{-3}$ kg·m² |
| Coefficient of thermal expansion| $\alpha = 1.350 \times 10^{-5}$ 1/K |
| Thermal absorptivity            | $\alpha_t = 0.5$             |
| Thermal emissivity              | $\varepsilon = 0.13$         |
| Specific heat                   | $c = 4.605 \times 10^2$ J/(kg·K) |
| Thermal conductivity            | $k = 1.628 \times 10^4$ W/(m·K) |

The blanket is a 0.5-mm-thick polytetrafluoroethylene sheet. The mass per unit area of the sheet is $\sigma_m = 9.37 \times 10^{-2}$ kg/m². The upper section of the blanket is fixed to the blanket top bar, and the lower section is fixed to the spreader bar at the bottom by screws. The asymmetry and symmetry of the test specimen is influenced by the amount of offset of the blanket between the inner and outer booms. The inner and outer booms are the right and left booms with respect to the center of spreader’s length when the test specimen is viewed from the front.

4.3. Measurement system

The experimental measurements can be grouped into three major sets, axial compressive load on the boom, boom tip deflection and frequency of thermally induced vibration.

Figure 5 shows a schematic diagram of the measurement system in the experiment. The output voltage signals from the load cell, which measure the axial compressive load on the boom, are small and are amplified by a DC amplifier, before being recorded on a digital recorder. The boom tip displacement of the test specimen is measured using a laser displacement sensor, which has its own amplification unit that can directly interface with the digital recorder. Furthermore, the frequency is analyzed from the laser displacement sensor data using a fast Fourier transform (FFT) analyzer. Finally, the data from the FFT analyzer and digital recorder are processed via a computer. Table 2 shows the specifications for the heater.

Table 2. Specifications of the heater.

| Manufacturer   | Research Inc. |
|----------------|--------------|
| Product title  | Parabolic strip heater model 5305-25 |
| Length         | 757 mm       |
| Weight         | 2.09 kg      |
| Length of lamp | 635 mm       |
| Output         | 2.5 kW (480 V) |

Fig. 5. Schematic diagram of measurement system.

(a) Top of the boom (b) Bottom of the boom
the sum of the mass of the blanket and blanket top bar and the mass of the spreader bar.

For the boom tip deflection measurement, we use a non-contact laser displacement sensor that minimizes any effect on the vibration. To easily reflect the laser, a white painted aluminum plate is attached to the boom tip as shown in Fig. 6. Table 4 shows the specifications for the laser displacement sensor.

The frequency analysis of thermally induced vibration is measured in the constant static deflection state.

### 5. Results and Discussion

#### 5.1. Time records of boom tip deflection

Figure 7 shows the time records of the theoretical and experimental results. Each result shows the outer and inner boom tip deflection of a symmetric model between 0 and 45 s. The theoretical results are calculated using the same conditions as the experiment; the average axial compressive forces $P = 0.81$ N and the heat flux $s_0 = 1,900$ W/m$^2$. In each of the results, time 0 represents the moment when the heating starts. In both the theoretical and experimental results, only the bending modes are considered. This is justified because each boom is subjected to the same axial compressive force and heat flux.

Figure 8 shows the results of time records of the asymmetric model where the blanket leans towards the outer boom by $\Delta b$. As a result, each boom receives different axial compressive forces. As for the time recordings of the symmetric model, the theoretical results are calculated for the
experimental conditions; the average axial compressive forces $P = 1.54$ N and heat flux $s_0 = 1,900$ W/m$^2$. In case of the theoretical results, the inner and outer boom tip deflection shows the beat motion, but the experimental results do not verify this beat motion. The reason for the beat motion is thought to be that the bending mode and torsion mode frequencies are very close in value. Beat motion does not occur in the experiment. It is thought that this is because the beams were not subject to the steady axial compressive forces that cause the blanket’s contraction and expansion.

### 5.2. Frequency response

Figure 9 shows the frequency response using FFT analysis. The horizontal axis represents the frequency and the vertical axis the frequency spectrum. The FFT analysis is obtained using times from the theoretical and experimental results from Fig. 8 for the asymmetric model that has an average axial compressive force $P = 1.54$ N. For the FFT analysis, the data for the experimental results are sampled 512 times, which are the times for boom tip deflections between 6 and 31.5 s at 0.05 s intervals.

Here, the experimental and theoretical results show a smaller natural frequency than 1 Hz and seem to be almost identical. The frequency responses of the theoretical results show two prominent frequencies at 0.7129 and 0.7715 Hz, while the frequency responses of the experimental results show one prominent frequency at 0.8594 Hz. Furthermore, through theoretical natural frequency analysis using the same analysis model, we obtained natural frequencies for the first and second mode of 0.7134 and 0.7690 Hz. Comparing the first and second mode frequencies via FFT analysis of the theoretical results to those of the theoretical natural frequency analysis, there is an error of approximately 0.01%. In addition, the average axial compressive force is lower than 1.54 N, and we know, through the natural frequency analysis, that the first mode is dominated by buckling while the second mode is dominated by torsion. Therefore, the prominent frequency of 0.8594 Hz shown in Fig. 9 is considered to be the frequency of the buckling mode, and this result dominates the first mode. In the above we conclude that the theoretical and experimental results show a near match in frequency characteristics.

### 5.3. Influence of the axial compressive force

Figure 10 shows the results of the effect of the average axial compressive force in the asymmetric model, under a
steady external radiation heat of $s_0 = 1,900 \text{ W/m}^2$. The average value of the measured boom tip dynamic amplitude and static deflection are taken between 5 and 35 s after the start of heating. As shown in Fig. 10(a), the theoretical and experimental values do not agree, because the experimental results are dominated by the bending mode, although the theoretical analysis is based on the coupling of bending and torsion. As shown in Fig. 10(b), the calculated and measured static deflections show a trend for the boom tip deflection of the outer boom to be larger than the inner boom deflection as the average axial compressive force increases. Experimental results agree well with the theoretical results since the axial force of the boom is not large. These results show that the theoretical and experimental results nearly match for the quasi-static response, but the dynamic response is not in accord.

5.4. Influence of heat flux

Figure 11 shows the results for the effect of heat flux in the asymmetric model, under a steady axial compressive force $P = 1.0 \text{ N}$. As shown in Fig. 11(a), the calculated and measured boom tip amplitudes will continue to increase with increasing irradiation, but experimental results do not confirm the obvious difference seen in the theoretical results for the inner and outer booms. However, the static deflection shown in Fig. 11(b) does confirm the closeness of the theoretical and experimental results. In addition, the calculated and measured static deflections show a trend in the boom tip deflection of the outer boom being a little larger than the inner boom deflection as the irradiation increases. These results show that the theoretical and experimental results, quantitatively and qualitatively, confirm a match for the quasi-static response, while for the dynamic response, there was a qualitative match, but quantitatively they were not in accord.

5.5. Influence of vacuum and atmosphere

Figure 12 shows the results for the comparison between the experiment being performed under pressures of either atmosphere or vacuum, under a steady axial compressive force. Theoretical results in atmosphere and in a vacuum show increases with increasing irradiation. However, the experimental results in the vacuum are shown to be qualitatively consistent with the theoretical results, while the experimental results in atmosphere are not consistent.

6. Conclusions

In this study, theoretical analyses of quasi-static and dynamic responses for a simple asymmetric model of an HST solar array were presented. The theoretical analyses were based on the geometric asymmetry of the blanket and the coupled thermoelastic problem.

To verify the theoretical analysis, thermally-induced vibration experimental equipment was constructed using an approximate 1/10 scale reduction model of an HST solar array. The experimental results were compared with the results of theoretical analyses.

Experimental and theoretical results were analyzed by measuring and calculating the boom tip deflection, and the influence of various conditions were verified, such as the time record of boom tip deflections, variations of boom tip...
deflections according to the axial compressive force and solar heat flux, and variations due to vacuum and atmosphere conditions. The theoretical results were calculated for quasi-static and dynamic responses, and were compared with experimental results.

Through the comparison of theoretical and experimental analyses under the various conditions, the experimental results for the vacuum were more in accord with the theoretical results than the experimental results for the atmosphere condition. Furthermore, the quasi-static response was shown to be in accord with the experiment results, while the dynamic response was only quantitatively in accord. Therefore, we conclude that theoretical analysis for the dynamic response appears to lack credibility. However, through improvement of experimental equipment, it is expected that the credibility of the theoretical analysis will improve.

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