QCD SUM RULES AND VECTOR MESONS IN NUCLEAR MATTER

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Abstract

Based on an effective Lagrangian which combines chiral SU(3) dynamics with vector meson dominance, we have developed a model for the s-wave vector meson-nucleon scattering amplitudes. We use this as an input for the low energy part of the current-current correlation function in nuclear matter. Its spectrum enters directly in the “left hand side” of QCD sum rules. For the isovector channel we find a significant enhancement of the in-medium spectral density below the $\rho$ resonance while the mass of $\rho$ meson itself decreases only slightly. The situation is different in the isoscalar channel. Here the mass and therefore the peak position of the $\omega$ meson decreases strongly while the width increases less drastically than in the $\rho$ meson case. For the $\phi$ meson we find almost no mass shift, the width of the peak broadens due to the imaginary part of the scattering length. We find a remarkable degree of consistency with the operator product expansion of QCD sum rules in all three channels.

1 Introduction

At present there is a lively discussion about the behaviour of vector mesons in dense and hot hadronic matter. According to Brown and Rho [1] the vector meson masses should drop like the pion decay constant in baryonic matter (BR-scaling). Other scaling laws [2] using a bag model find a mass reduction of two thirds times that of the nucleon mass. Several analyses of the $\rho$ and $\omega$ meson masses in matter using QCD sum rules [3, 4] seem to confirm BR-scaling. On the other hand, model calculations of the density dependent two pion self-energy of the rho meson in nuclear matter [5, 6] suggest only marginal changes of the in-medium rho meson mass, but a strongly increased decay width instead. Additional BR scaling seems to be needed in order to match

\footnote{Work supported in part by GSI and BMBF}
the hadronic models with the QCD sum rule analysis \cite{7}. Phenomenologically, a dropping $\rho$ meson mass seems to help \cite{8, 9} understanding the enhanced dilepton yields seen at masses below the rho meson resonance in the CERES and HELIOS experiments at CERN (see the updated review by A. Drees at this workshop).

The QCD sum rule approach, at any practical level so far, has used only a caricature of the true isovector spectrum, namely a parameterization in terms of a delta function at the meson pole accompanied by a theta function type continuum at higher masses. While such a parameterization is valid in vacuum (as we shall confirm), this turns out not to be the case for the in-medium spectrum. We will show that using a hadronic model to calculate the vector meson-nucleon scattering lengths leads to correlation functions which are consistent with the QCD sum rule analysis without need for additional BR scaling.

In section 2 we review properties of the current correlation function in vacuum and give a brief account of the QCD sum rule approach. The current-current correlation functions in baryonic matter will be developed in section 3. The comparison with QCD sum rules at finite baryon density is made in section 4.

## 2 Vector mesons and QCD sum rules in the vacuum

Before we start looking into the in-medium properties of neutral vector mesons it is useful to give a brief reminder of their vacuum properties. Since the vector mesons are not stable they can only be seen as resonances in current-current (CC) correlation function,

$$\Pi_{\mu\nu}(q) = i \int d^4x \ e^{iq \cdot x} \langle 0 | \mathcal{T} j_\mu(x) j_\nu(0) | 0 \rangle$$

where $\mathcal{T}$ denotes the time-ordered product and $j_\mu$ is the electromagnetic current. It can be decomposed as

$$j_\mu = j_\mu^\rho + j_\mu^\omega + j_\mu^\phi$$

into vector currents specified by their quark content:

$$j_\mu^\rho = \frac{1}{2} (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d),$$

$$j_\mu^\omega = \frac{1}{6} (\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d),$$

$$j_\mu^\phi = -\frac{1}{3} (\bar{s}\gamma_\mu s).$$
Current conservation implies a transverse tensor structure

\[ \Pi_{\mu\nu}(q) = \left( g_{\mu\nu} - \frac{g_{\muq}g_{\nuq}}{q^2} \right) \Pi(q^2), \]  

(6)

with the scalar CC correlation function

\[ \Pi(q^2) = \frac{1}{3} g^{\mu\nu} \Pi_{\mu\nu}(q). \]  

(7)

Figure 1: Calculated spectra of current-current correlation functions. The solid lines show the vacuum spectra in the \( \rho \), \( \omega \) and \( \phi \) channels normalized such that they can be compared directly with the corresponding \( e^+e^- \rightarrow \text{hadrons} \) data. The dashed lines show the spectral functions in nuclear matter at densities \( \rho_0/2 \) and \( \rho_0 = 0.17 \, \text{fm}^{-3} \), as discussed in section 3.
The imaginary part of the correlation function is proportional to the cross section for $e^+e^- \rightarrow \text{hadrons}$:

$$R(s) = \frac{\sigma^{I=1}(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = -\frac{12\pi}{s} \text{Im}\Pi(s)$$

(8)

where $\sigma(e^+e^- \rightarrow \mu^+\mu^-) = 4\pi\alpha^2/3s$ with $\alpha = e^2/4\pi = 1/137$. The vector mesons can be distinguished by looking at different hadronic channels with corresponding flavour (isospin) quantum numbers. For example, G-parity demands that the isovector current describing the $\rho$ meson can only decay into even numbers of pions (see data in fig.1a). Similarly the annihilation of $e^+e^-$ into odd numbers of pions determines the isoscalar current describing the $\omega$ meson. This can be seen in figure 1b. Isospin violating processes are small but visible as $\rho\omega$ mixing corrections to the pion formfactor. The situation for the $\phi$ meson is more involved. The OZI rule forbids decays into pions, however a strong violation of about five percent is seen in the three-pion channel. (Some experimental data near the $\phi$ resonance have been left out in fig. 1b to emphasize the $\omega$-contribution given by the solid line). The $\phi$ meson decays mainly into OZI allowed channels such as $K^+K^-$ or $K_SK_S$ (see fig. 1c). However these channels also exist for the $\rho$ and $\omega$ mesons. The measured cross section of the annihilation into kaons includes an interference between all three vector mesons. (For more details see ref [11]). Nevertheless the $\phi$ meson still dominates the data in these channels.

Vector meson dominance (VMD) leads to the solid lines in figs.1a, b and c. In comparison with the experimental data for $e^+e^-$ annihilation into hadronic channels, we clearly see that the low energy region of the CC correlation function is very well described by VMD. The extended VMD model [11] gives:

$$\text{Im}\Pi(q^2) = \sum_V \frac{\text{Im}\Pi_{V}^{\text{vac}}(q^2)}{g_V^2} |F(q^2)|^2,$$

(9)

where

$$F(q^2) = \frac{(1 - a_V)q^2 - m_V^2}{q^2 - m_V^2 - \Pi_{V}^{\text{vac}}(q^2)}.$$ 

(10)

Here $m_V$ are the bare vector meson masses, $\Pi_{V}^{\text{vac}}$ the vacuum self energies and $g_V$ the strong SU(3) couplings of the vector mesons. The parameter $a_V$ is free, but usually close to one. It is identical to unity in case of “complete” VMD in which all of the hadronic electromagnetic interaction is transmitted through vector mesons. For the rho meson channel $F(q^2)$ is simply given by the pion
form factor
\[
F_\pi(q^2) = \left(1 - \frac{q^2}{\hat{g}_\rho} \right) \frac{q^2 - \hat{m}_\rho^2}{q^2 - \hat{m}_\rho^2 - \Pi_\rho(q^2)}, \tag{11}
\]
where \( g (\hat{g}_\rho) \) is the coupling of the \( \rho \) meson to the pion (photon) and \( \Pi_\rho \) is the \( \rho \) meson self energy which is dominated by the two-pion loop \[11\].

In the high energy region the measured correlation function approaches the asymptotic plateau predicted by QCD:
\[
R(s) = d \left(1 + \frac{\alpha_S}{\pi} \right) \Theta(s - s_0) \tag{12}
\]
for large \( s \), where \( d_\rho = 3/2, d_\omega = 1/6 \) and \( d_\phi = 1/3 \).

The basic idea of QCD sum rules is to compare two ways of determining \( \Pi(q^2) \) in the region of large spacelike \( Q^2 = -q^2 \gg 1 \text{ GeV} \). One way is to use a twice subtracted dispersion relation for each one of the channels \( V = \rho, \omega, \phi \):
\[
\Pi(q^2) = \Pi(0) + c q^2 + \frac{q^4}{\pi} \int ds \frac{\text{Im}\Pi(s)}{s^2(s - q^2 - i\epsilon)}, \tag{13}
\]
where the vanishing photon mass in the vacuum requires \( \Pi(0) = 0 \). The other one is to calculate the correlation function using the operator product expansion (OPE):
\[
\frac{12\pi}{Q^2} \Pi(q^2 = -Q^2) = \frac{d}{\pi} \left[ -\left(1 + \alpha_S(Q^2)/\pi \right) \ln \left(\frac{Q^2}{\mu^2}\right) \right. \tag{14}
\]
\[
\left. + \frac{c_1}{Q^2} + \frac{c_2}{Q^4} + \frac{c_3}{Q^6} + \ldots \right].
\]
Here the coefficients \( c_{1,2,3} \) incorporate the non-perturbative parts coming from the condensates such as the gluon condensate in \( c_2 \) or the four-quark condensate in \( c_3 \). The commonly used values \[13\] for the isoscalar and isovector channel are \( c_2^\rho = c_2^\omega = 0.04 \text{ GeV}^4 \) and \( c_3^\rho = c_3^\omega = -0.07 \text{ GeV}^6 \). The coefficient \( c_1 \) is proportional to the squared quark mass; due to the small mass of the up and down quark we can safely neglect those contributions and set \( c_1^{\rho,\omega} = 0 \). The situation is different in case of the \( \phi \) meson. Because of the large strange quark mass, \( c_2 \) changes and \( c_1 \) is no longer negligible. Here we take \( c_1^\phi = -0.07 \text{ GeV}^2, c_2^\phi = 0.17 \text{ GeV}^4 \) and \( c_3^\phi = -0.07 \text{ GeV}^6 \) \[13\].
A Borel transform is used in order to improve the convergence of the OPE series. Comparing eq. (13) and (14) after Borel transformation we end up with

\[
\frac{12\pi^2\Pi(0)}{d\mathcal{M}^2} + \frac{1}{d\mathcal{M}^2} \int_{0}^{\infty} ds R(s) \exp \left[ -\frac{s}{\mathcal{M}^2} \right] = (1 + \frac{\alpha S(\mathcal{M}^2)}{\pi}) + \frac{c_1}{\mathcal{M}^2} + \frac{c_2}{\mathcal{M}^4} + \frac{c_3}{2\mathcal{M}^6},
\]

(15)

where the Borel mass parameter should be chosen in the range \( \mathcal{M} \gtrsim 1 \) GeV in order to ensure convergence of the OPE side of eq. (15). The “left side” comes from the dispersion relation of eq. (13) and \( R \) represents the ratio (8), but now specified for each individual flavour channel with \( V = \rho, \omega, \phi \). The “right side” is determined by the OPE. In fig. 3a, b and c we show the comparison between the “left side” (solid line) and the “right side” (dashed line) for the various (vacuum) channels. The consistency between “left” and “right” sides is evidently quite satisfactory. Only for a Borel mass below 0.8 GeV higher order terms in the OPE become important and the convergence fails. Keeping this success of QCD sum rules in mind we now want to explore whether this still holds in the medium.

### 3 Current correlation functions in baryonic matter

The CC-correlation function in medium at temperature \( T = 0 \) is defined as

\[
\Pi_{\mu\nu}(\omega, \vec{q}; \rho) = i \int_{-\infty}^{+\infty} dt \int d^3 x e^{i\omega t - i\vec{q} \cdot \vec{x}}
\]

\[
* \langle \rho | T j_\mu(t, \vec{x}) j_\nu(0) | \rho \rangle,
\]

(16)

where we have replaced the vacuum by \( |\rho\rangle \), the ground state of infinite, isotropic and isospin symmetric nuclear matter with density \( \rho \). We assume matter as a whole to be at rest. This specifies the Lorentz frame that we will use in the following. For \( \vec{q} = 0 \), the case where the vector meson is at rest, only the transverse tensor structure survives and \( \Pi^{00} = \Pi^{0j} = \Pi^{i0} = 0 \). We write

\[
\Pi_{ij}(\omega, \vec{q} = 0; \rho) = -\delta_{ij}\Pi(\omega, \vec{q} = 0; \rho).
\]

(17)

As a first step we apply the low density theorem to both the correlation function and the self energy of the vector mesons. We have

\[
\Pi(\omega, \vec{q} = 0; \rho) = \Pi_{\text{vac}}(\omega^2) - \rho T(\omega) + \ldots,
\]

(18)
and
\begin{equation}
\Pi_V(\omega, \vec{q} = 0; \rho) = \Pi_{V, \text{vac}}^\omega(\omega^2) - \rho T_{V,N}(\omega) + \ldots,
\end{equation}
where $T(\omega)$ is the Compton amplitude and $T_{V,N}$ is the vector meson-nucleon scattering amplitude, both taken at $\vec{q} = 0$. On the other hand using eq. (19) we can extend eq. (9) to the in-medium correlation function in terms of VMD and write [10]
\begin{equation}
g_V^2 \Im \Pi(\omega, \vec{q} = 0; \rho) = \Im \left( \Pi_{V, \text{vac}}^\omega(\omega^2) - \rho T_{V,N}(\omega) \right)
\end{equation}
\begin{equation}
\times \left| \frac{1 - a_V}{\omega^2 - a_V^2 - (\Pi_{V, \text{vac}}^\omega(\omega^2) - \rho T_{V,N}(\omega))} \right|^2.
\end{equation}
Comparing the terms linear in density of eqs. (18) and (20) we find that the Compton amplitude translated into VMD language becomes:
\begin{equation}
T(\omega) = g_V^2 F(\omega^2) T_{V,N}(\omega) F(\omega^2).
\end{equation}
In order to determine the in-medium correlation function we are left with the task to calculate the vector meson-nucleon scattering amplitudes. We use an effective Lagrangian which combines chiral SU(3) dynamics with VMD. This Lagrangian has been applied successfully in the vacuum [11] and has been extended to incorporate meson-baryon interactions. For the baryons we include nucleons, hyperons and $\Delta$’s. The most important processes contributing to the scattering amplitudes are shown in fig. 2a and b for the $\rho$, $\omega$ and $\phi$ meson respectively. For the $\rho$ ($\phi$) meson we draw the diagrams which survive in the limit of large baryon mass (fig. 2a-d). The last one (fig. 2d) only contributes to the real part of the $\rho N$ ($\phi N$) scattering amplitude. While for the $\rho$-meson the $\pi N$ and $\pi \Delta$ loops govern the scattering amplitude, $K \Sigma$ and $K \Lambda$ loops dominate for the $\phi$ meson. For the $\omega N$ scattering amplitude we only show the dominant contributions to the imaginary part (figs. 3e, f). We evaluate the imaginary parts of the amplitudes by cutting the diagrams in all possible ways. The real parts are then determined by using a once subtracted dispersion relation, with the subtraction constant fixed by the Thomson limit. Evaluating those diagrams and using eq. (21) we plot the spectra of the correlation functions for various densities as shown by the dashed curves in fig. 2a, b and c. We observe important differences between the various channels. The $\rho$ meson mass decreases only slightly while the width increases very strongly. This causes the peak to shift downwards and broaden. We also see strong threshold contributions starting at the pion mass. On the other hand the mass of the $\omega$ meson decreases significantly. Its width also increases but not as strongly...
Figure 2: Dominant diagrams for the Compton amplitude $T(\omega)$ and the vector meson-nucleon amplitudes $T_{VN}(\omega)$. In order to get from $T$ to $T_{VN}$, replace the (wavy) photon line by the relevant vector mesons. Diagrams (e) and (f) operate, in particular, in the $\omega$ meson channel.

as for the $\rho$ meson. For the $\phi$ meson there is almost no change in the peak position while the width increases.

4 Comparison with QCD sum rules

We now proceed to test the consistency of these in-medium correlation functions with the OPE of QCD sum rules. Performing an OPE of the density dependent correlation functions the coefficients of eq.(14) become density dependent. We use the values $c_2^{\rho,\omega} = 0.04 + 0.018(\rho/\rho_0)$ GeV$^4$ and $c_3^{\rho,\omega} = -0.07 + 0.036(\rho/\rho_0)$ GeV$^6$ proposed by Hatsuda et al. for the $\rho$ and $\omega$ meson. They neglected higher twist condensates which are hardly known but might be important. For the four-quark condensates we assume as in ref. that ground state saturation holds to the same extent as in the vacuum. For the $\phi$ meson the coefficient $c_1$ remains unchanged and we take the values $c_2^{\phi} = 0.17 + 0.01(\rho/\rho_0)$ GeV$^4$ and $c_3^{\phi} = -0.07 + 0.006(\rho/\rho_0)$ GeV$^6$ as suggested by Asakawa and Ko. In figure 3 we have plotted the Borel transformed
OPE at normal nuclear matter (long dashed lines) and compared with the “left side” (dashed dotted line) as given by the dispersion relation representation of our calculated in-medium correlation function.

The consistency at normal nuclear matter is not as excellent as in the vacuum but still very impressive. We therefore conclude that QCD sum rules and our hadronic model of the in-medium current-current correlation function are mutually compatible. We also point out that assuming a simple pole ansatz for the spectrum at finite densities can lead to erroneous interpretation of the in-medium masses. This is very obvious in the isovector channel. While our model gives only a small change of the rho mass a simple pole ansatz leads to a reduction of $m_\rho$ by more than 10 percent at $\rho = \rho_0$.

Figure 3: Comparison of the “left” and “right” side of the QCD sum rules (15) in the vacuum (solid and dashed line) and at normal nuclear density (dot dashed and long dashed line) as a function of the Borel mass parameter $M$. 
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