Trapping and Mobilization of Residual Fluid During Capillary Desaturation in Porous Media

A. Lucian$^{1,2}$ and R. Hilfer$^{2,3}$

$^1$Inst. Atomic Physics, I.N.F.L.P.R., Lab. 22, P.O Box MG-7 76900 Bukarest, Romania
$^2$ICA-1, Universität Stuttgart, Pfaffenwaldring 27, 70569 Stuttgart
$^3$Institut für Physik, Universität Mainz, 55099 Mainz, Germany

We discuss the problem of trapping and mobilization of nonwetting fluids during immiscible two phase displacement processes in porous media. Capillary desaturation curves give residual saturations as a function of capillary number. Interpreting capillary numbers as the ratio of viscous to capillary forces the breakpoint in experimental curves contradicts the theoretically predicted force balance. We show that reploting the data against a novel macroscopic capillary number resolves the problem for discontinuous mode displacement.

PACS: 47.55Mh, 81.05Rm, 61.43Gt, 83.10Lk, (47.55Kf)
I. INTRODUCTION

A displacement of one fluid by another within a porous medium poses challenging problems of micro-to-macro-scale transitions which have received considerable attention from physicists in recent years [1–20]. For reviews the reader is referred to [12,21]. Apart from the omnipresent quenched correlated disorder and structural heterogeneity, fluid-fluid and fluid-solid interactions generate metastability and hysteresis phenomena which have resisted quantitative prediction and understanding. A central problem of great practical importance is the prediction of residual nonwetting fluid saturation after flooding the pore space with immiscible wetting fluid [22–24]. Obvious interest in this problem arises from enhanced oil recovery [25,26] or in situ remediation of soil contaminants [27,28].

Microscopically the laws of hydrodynamics governing the pore scale processes are well known. The complexity of the microscopic fluid movements and the lack of knowledge about the microstructure and wetting properties, however, renders a detailed microscopic treatment impossible. Instead one has to resort to a more macroscopic treatment.

Desaturation experiments [29–36] show that the conventional macroscopic description is incomplete [23,24]. We shall begin our discussion by reminding the readers of this incompleteness. We then exhibit another problem [37]. This arises from the fact that microscopic capillary numbers seemingly cannot represent the force balance in a desaturation experiment.

Given these problems our main objective is to show that the recent analysis of [37] gives the correct force balance between macroscopic viscous and capillary forces for continuous mode desaturation experiments. For so called discontinuous mode displacements it leads to bounds on the size of residual blobs.

II. PROBLEMS OF TWO-PHASE FLOW EQUATIONS

Let us begin our discussion with the standard macroscopic equations of motion for two-phase immiscible displacement. Macroscopic equations of motion describe multiphase flow on length scales large compared to a typical pore diameter. Hence they are applied to laboratory samples with linear dimensions on the order of centimeters as well as to whole reservoirs measuring kilometers and more. Consider a system having lengths \( L_x, L_y, L_z \) in the three spatial directions. The pore space is assumed to be filled with two immiscible fluids denoted generically as water (index \( w \)) and oil (index \( o \)). The equations read [38–40,37]

\[
\phi \frac{\partial S_w}{\partial t} = \nabla \cdot \left\{ \frac{K_w}{\mu_w} \left[ \nabla P_w - \rho_w g O \right] \right\}
\]

\[
- \phi \frac{\partial S_w}{\partial t} = \nabla \cdot \left\{ \frac{K_o}{\mu_o} \left[ \nabla (P_w + P_c) - \rho_c g O \right] \right\}
\]

and they are supplemented with the constitutive relationships

\[ k_w(x,t) = k_w(S_n(x,t)) \]

\[ k_o(x,t) = k_o(S_n(x,t)) \]

\[ P_c(x,t) = P_c(S_n(x,t)) \]

where

\[ S_n(x,t) = \frac{S_w(x,t) - S_{wi}}{1 - S_{wi} - S_{or}} \]
The variables in these equations are the pressure field of water denoted as $P_W$, and the water saturation $S_W$. The saturation is defined as the ratio of water volume to pore space volume. Pressures and saturations are averages over a macroscopic region much larger than the pore size, but much smaller than the system size. Their arguments are the macroscopic space and time variables $(x, t)$. The saturations obey $S_{wi} < S_W < 1 - S_{or}$ where the two numbers $0 \leq S_{wi}, S_{or} \leq 1$ are two parameters representing the irreducible water saturation, $S_{wi}$, and the residual oil saturation, $S_{or}$. The residual oil saturation gives the amount of oil remaining in a porous medium after water injection. The normalized saturation $S_n$ varies between 0 and 1 as $S_W$ varies between $S_{wi}$ and $S_{or}$. The permeability of the porous medium is given by the absolute (single phase flow) permeability tensor $K$. The porosity $\phi$ is the volume fraction of pore space. The two fluids are characterized by their viscosities $\mu_W, \mu_O$ and their densities $\rho_W, \rho_O$. The terms $\rho g \mathbf{e}$ represent the gravitational body force where $\mathbf{e}^T = (0, 0, -1)$ is a unit row vector pointing along the negative $z$-axis, and $g$ is the acceleration of gravity. The orthogonal matrix

$$O = \begin{pmatrix} \cos \alpha_y & 0 & \sin \alpha_y \\ 0 & 1 & 0 \\ -\sin \alpha_y & 0 & \cos \alpha_y \end{pmatrix}$$

(7)

describes an inclination of the system. Here a rotation around the $y$-axis with tilt or dip angle $\alpha_y$ was assumed.

The macroscopic capillary pressure $P_c$ is defined as the pressure difference between the oil and the water phase. The constitutive relation for $P_c$ assumes that the capillary pressure function $P_c$ depends only on the saturation $S_W$ [11]. In addition to $P_c$ the dimensionless relative permeabilities are assumed to be functions of saturation only, $k^r_W = k^r_W(S_W), k^r_O = k^r_O(S_W)$. They represent the reduction in permeability for one phase due to the presence of the other phase [32-34, 38, 39]. The three constitutive relations $k^r_W(S_W), k^r_O(S_W)$ and $P_c(S_W)$ are assumed to be known from experiment. Equations (1) and (2) are coupled nonlinear partial differential equations which must be complemented with large scale boundary conditions. For laboratory experiments the boundary conditions are typically given by a surface source on one side of the sample, a surface sink on the opposite face, and impermeable walls on the other faces. For a geosystem the boundary conditions depend upon the well configuration and the geological modeling of the reservoir environment.

The problem with the macroscopic equations of motion (1)-(2) arises from the experimental observation that the parameters $S_{or}$ and $S_{wi}$ are not constant and known, but depend strongly on the flow conditions in the experiment [24, 25, 26]. Hence they may vary in space and time. More precisely, the residual oil saturation depends strongly on the microscopic capillary number $Ca = \mu_W v / \sigma_{ow}$ where $v$ is a typical flow velocity and $\sigma_{ow}$ is the surface tension between the two fluids. The capillary desaturation curve $S_{or}(Ca)$ is shown in Figure 1 for unconsolidated glass beads and sandstones [32, 34]. Such curves contradict clearly to the assumption that the functions $k^r_W(S_W), k^r_O(S_W)$ and $P_c(S_W)$ depend only on saturation. Instead they show that $k^r_W(S_W), k^r_O(S_W)$ and $P_c(S_W)$ depend also on velocity [42, 43] and pressure. Hence they depend on the solution and cannot be considered to be constitutive relations characterizing the system. The dependence shows that the system of equations of motion is incomplete.

The breakpoint in a capillary desaturation curve marks the point where the viscous forces, which attempt to mobilize the oil, become stronger than the capillary forces, which try to keep the oil in place. For this reason the capillary desaturation curves are usually plotted against $Ca$ which represents the microscopic force balance between viscous and capillary forces. From Figure 1 it is seen that the theoretical force balance corresponding to $Ca = 1$ on the abscissa and the experimental force balances represented by the various breakpoints at $Ca \ll 1$ differ by several orders of magnitude. Plotting residual saturation against the correct ratio of viscous to capillary forces should result in a breakpoint at $Ca \approx 1$ [37, 39].

We now proceed to show that a partial understanding of the macroscopic force balance can already be obtained from the traditional equations of motion. The main result of this analysis is a preliminary experimental validation of the dimensionless considerations introduced in [37].
III. DISCONTINUOUS VS. CONTINUOUS MODE DISPLACEMENT

Before embarking on a discussion of the force balance during capillary desaturation it is crucial to emphasize an important difference in the way capillary desaturation curves are measured.

1. In the first method of measuring $S_{or}(Ca)$ the oil in a fully oil saturated sample is displaced with water at a very low $Ca$. After the oil flow at the sample outlet stops and several pore volumes of water have been injected without producing more oil the flow rate is increased. Again the oil flow is monitored until no more oil appears at the outlet. In this way the flow rate is increased iteratively until $S_{or}$ has fallen to zero. After the first injection the oil configuration is discontinuous (or disconnected) and it remains disconnected throughout the rest of the experiment. This mobilization mode will be called discontinuous.

2. In the second method of measuring capillary desaturation curves one again starts from a fully saturated sample, and performs a waterflood at a given value of $Ca$. After oil flow ceases at the outlet the residual saturation is determined. Then the sample is again saturated fully with oil, a new value of $Ca$ is chosen, and the injection is repeated. In this experiment the injection starts always with a connected oil phase contrary to the previous experiment where the oil phase is discontinuous at higher $Ca$. This mobilization mode will be called continuous.

The capillary number required to reach a given $S_{or}$ is known to be much lower in the continuous mode than in the discontinuous mode \[36,35\]. This is also seen in Figure 1 which shows continuous mode data and discontinuous mode data for the unconsolidated glass beads. From the fact that the equations of motion are only valid in the subinterval $S_{wi} < S_w < 1 - S_{or}$ it follows that they cannot be applied to discontinuous mobilization mode experiments. They should however be valid for the continuous mode to the extent that the equations themselves are correct. Therefore we discuss next the macroscopic force balance predicted by these equations for the continuous mobilization mode.

IV. MACROSCOPIC FORCE BALANCE

It was shown in \[37\] that the balance of macroscopic viscous and capillary forces is not represented appropriately by the microscopic capillary number $Ca$, and that a new macroscopic capillary number $\hat{Ca}$ should be used instead. Here we generalize those results to anisotropic and inclined porous media and apply them to replot the $S_{or}$ data obtained for the continuous mode displacement. To this end we rewrite the equations (1) and (2) in dimensionless form. Introducing the matrix

$$L = \begin{pmatrix} L_x & 0 & 0 \\ 0 & L_y & 0 \\ 0 & 0 & L_z \end{pmatrix}$$

(8)

and defining $l$ and $\hat{L}$ through

$$l = (\det L)^{1/3}$$

(9)

$$\hat{L} = l \hat{\hat{L}}$$

(10)

one defines dimensionless quantities

$$x = L \hat{x} = l \hat{\hat{L}} \hat{x}$$

(11)
Define \( k \) and \( \tilde{\mathbf{K}} \) through

\[
\mathbf{K} = \begin{pmatrix}
k_{xx} & k_{xy} & k_{xz} \\
k_{xy} & k_{yy} & k_{yz} \\
k_{xz} & k_{yz} & k_{zz}
\end{pmatrix}
\]  

(13)

\[k = (\det \mathbf{K})^{(1/3)}\]  

(14)

\[\mathbf{K} = k \tilde{\mathbf{K}}\]  

(15)

The time scale is normalized as

\[t = \frac{L_x L_y L_z \phi \hat{t}}{Q}\]  

(16)

using the volumetric flow rate \( Q \). Time is measured in units of injected pore volumes. The pressure is normalized using the macroscopic equilibrium capillary pressure as

\[P = P_b \hat{P}\]  

(17)

where

\[P_b = P_e ((S_{wi} - S_{or} + 1)/2)\]  

(18)

is the pressure at an intermediate saturation. \( P_e(S_w) \) is the equilibrium capillary pressure function.

With these definitions the dimensionless macroscopic capillary numbers for oil and water, defined as

\[\overline{Ca}_{\text{w}} = \frac{\mu_w Q}{P_b kl}\]  

(19)

\[\overline{Ca}_{\text{o}} = \overline{Ca}_{\text{w}} \frac{\mu_o}{\mu_w}\]  

(20)

give an expression of the balance between macroscopic viscous and capillary forces. The macroscopic gravity numbers

\[\overline{Gr}_{\text{w}} = \frac{\mu_w Q}{\rho_w g kl^2}\]  

(21)

\[\overline{Gr}_{\text{o}} = \overline{Gr}_{\text{w}} \frac{\mu_o \rho_w}{\mu_w \rho_o}\]  

(22)
express the viscous to gravity force balance. Finally the gravillary numbers

$$\overline{Gl}_W = \frac{\rho_\omega g l}{P_b}$$  \hspace{1cm} (23)$$

$$\overline{Gl}_O = \overline{Gl}_W \frac{\rho_c}{\rho_\omega}$$  \hspace{1cm} (24)$$

express the ratio between gravitational and capillary forces. The well known bond number, measuring the magnitude of buoyancy forces, is given as

$$Bo = \overline{Gl}_W - \overline{Gl}_O$$  \hspace{1cm} (25)$$

in terms of the gravillary numbers.

With these definitions the dimensionless equations of motion may be rewritten as

$$\frac{\partial S_w}{\partial t} = \nabla \cdot \left\{ \hat{A} k_w \left[ \overline{Ca}_w^{-1} \nabla \hat{P}_w - \overline{Gl}_W^{-1} \hat{g} \right] \right\}$$  \hspace{1cm} (26)$$

$$- \frac{\partial S_w}{\partial t} = \nabla \cdot \left\{ \hat{A} k_c \left[ \overline{Ca}_w^{-1} \nabla (\hat{P}_w + \hat{P}_c) - \overline{Gl}_O^{-1} \hat{g} \right] \right\}$$  \hspace{1cm} (27)$$

where the dimensionless matrix

$$\hat{A} = \hat{L}^{-1} \hat{K} \hat{L}^{-1} = \begin{pmatrix}
\frac{L_x^2}{k} & \frac{L_y^2}{k} & \frac{L_z^2}{k} \\
\frac{L_y L_x k}{k} & \frac{L_z L_y k}{k} & \frac{L_z L_x k}{k} \\
\frac{L_z L_x k}{k} & \frac{L_z L_y k}{k} & \frac{L_z L_x k}{k}
\end{pmatrix}$$  \hspace{1cm} (28)$$

contains generalized “aspect ratios”. The vector

$$\hat{g}^T = (\hat{L} \hat{O} \hat{e})^T = \left( -\frac{L_x}{l} \sin \alpha_y, 0, -\frac{L_z}{l} \cos \alpha_y \right)$$  \hspace{1cm} (29)$$

represents the effect of dip angle and geometric shape of the system on the gravitational driving force.

V. APPLICATION TO EXPERIMENT

We are now in a position to plot the capillary desaturation curves against the macroscopic capillary number $\overline{Ca}$ which represents the balance between macroscopic viscous and capillary forces. To do so we have searched the literature for capillary desaturation measurements and found Refs. \[22,25,29,31,45–48,35\]. Unfortunately none of the publications contains all the necessary flow and medium parameters to calculate $\overline{Ca}$. Measuring all the flow
parameters for a displacement process is costly and time consuming, and hence they are rarely available (see also [49]). While permeability, porosity and the fluid parameters such as viscosities and surface tensions are usually available capillary pressure data, relative permeabilities and residual saturations are not routinely measured. Hence we extract the required parameters from different publications assuming that they have been measured correctly and are reproducible anywhere and at all times. In spite of all the uncertainties it is well known that the capillary pressure curves of unconsolidated sands with various grain sizes can be collapsed using the Leverett-j-correlation [50,11,51]. The Leverett-j-correlation states essentially that

\[ P_c(S_w) = \sigma_{OW} \sqrt{\frac{\phi}{k}} j(S_w). \]  

(30)

Often the formula contains in addition an average contact angle at a three phase contact. We do not include the wetting angle as it is generally unknown, and including it would not change our results significantly. Similar to the capillary pressure data, the \( S_{or} \) data for unconsolidated sands seem to be well established [34,36] in spite of larger fluctuations of the results. Because most \( P_c \)- and \( S_{or} \)-data are available for unconsolidated sand and standard sandstones such as Berea or Fontainebleau we limit our analysis to these two cases.

The experimental \( S_{or} \) data analyzed here are taken from Ref. [32] for sandstones and from Ref. [36] for unconsolidated glass beads. In Figure 1 we show the capillary desaturation curve for oil-water displacement in a typical sandstone (sample No. 799 from [32]) using star symbols. These data were obtained in the continuous mode of displacement. The values of the surface tension and fluid viscosities for this experiment are given in Table 1. We also show continuous mode \( S_{or} \)-data for unconsolidated glass beads from Figure 6 in [36]. To calculate \( \text{Ca} \) we have used the values given in Table 1.

Plotting the data against \( \text{Ca} \) we obtain Figure 2. It is seen that the continuous mode displacements give a breakpoint for \( \text{Ca} \approx 1 \) while the discontinuous mode displacements have their breakpoint at a higher value. This is consistent with the idea that the traditional equations of motion (1) and (2) should be applicable to the continuous mode but not to the discontinuous case.

The result obtained here is consistent with the theoretical predictions from [37]. To fully validate our use of \( \text{Ca} \) as a correlating group for plotting continuous mode \( S_{or} \) data, however, it would be desirable to vary \( \text{Ca} \) by varying the system size \( l \). If the \( S_{or} \) curves obtained for different media and length scales \( l \) also show their breakpoint at \( \text{Ca} \approx 1 \) this would give further evidence for the applicability of the traditional equations of motion. We consider it possible, however, that deviations appear indicating a breakdown of the equations also for continuous mode displacement [24,21].

As stated above the equations of motion are not applicable to discontinuous mode displacements because of the constraint \( S_{wi} < S_w < 1 - S_{or} \). Nevertheless we can use the group \( \text{Ca} \) to estimate an upper bound for the size of residual blobs. As the flow rate is increased the residual blobs whose size is so large that the viscous drag forces on them exceed the capillary retention forces will break up and coalesce with other blobs which may again break up and coalesce further downstream [34,33]. The condition that the viscous forces dominate the capillary forces \( \text{Ca} \geq 1 \) predicts that after a flood with \( \text{Ca} \) the porous medium contains only blobs of linear size smaller than

\[ l_{blob} \leq \frac{\mu_w Q}{P_b k}. \]  

(31)

This result is of importance for microscopic models [32] of breakup and coalescence during immiscible displacement.

ACKNOWLEDGEMENT: One of us (R.H.) thanks Dr. P.E. Øren for many useful discussions. We are grateful to the Deutsche Forschungsgemeinschaft for financial support.
TABLE I. The sample and flow parameters used in the calculations. The permeabilities for the glass bead packs were obtained using the relation between their radius and permeabilities published in [23] on page 47. The capillary pressure was estimated using the Leverett-\(j\)-function correlation \[30\]. For sandstone the \(j\)-function from Ref. \[51\] was used and for bead packs the one from Ref. \[50\] was used. The \(j\)-function was evaluated close to \((S_{wi} - S_{or} + 1)/2\) according to \[18\].

| sample                  | permeability \(k\) \(10^{-12}\) m\(^2\) | porosity \(\phi\) | surface tension \(\sigma_{ow}\) (N/m) | cap. pressure \(P_b\) (Pa) |
|-------------------------|----------------------------------------|-------------------|--------------------------------------|--------------------------|
| sandstone 799 (Ref. \[32\]) | 0.14 (Ref. \[32\])                        | 0.28 (Ref. \[32\]) | \(3.37 \times 10^{-2}\) (Ref. \[32\]) | \(1.7 \times 10^4\) (Ref. \[32\]) |
| 70 \(\mu\)m bead pack (Ref. \[36\]) | 10 (Ref. \[36\])                            | 0.36 (Ref. \[36\]) | \(2.8 \times 10^{-3}\) (Ref. \[36\]) | \(2.4 \times 10^5\) (Ref. \[36\]) |
| 115 \(\mu\)m bead pack (Ref. \[36\]) | 30 (Ref. \[36\])                         | 0.36 (Ref. \[36\]) | \(1.18 \times 10^{-2}\) (Ref. \[36\]) | \(5.8 \times 10^2\) (Ref. \[36\]) |
FIGURE CAPTIONS

**Figure 1:** Experimentally measured capillary desaturation curves (capillary number correlations) for bead packs [36] (solid lines with circles and squares) and sandstone [32] (star symbols) as a function of microscopic capillary number $\text{Ca} = \frac{\mu_w v}{\sigma_{ow}}$. The dashed line is the desaturation curve for continuous mode displacement. The dash-dotted line marks the plateau value for sandstone.

**Figure 2:** Same as Figure 1 but plotted against the macroscopic capillary number $\overline{\text{Ca}} = \frac{\mu_w Q}{(P_{bkl})}$ from eq. (19). Note that the breakpoint for continuous mode displacement occurs around $\overline{\text{Ca}} \approx 1$. 
Figure 2: Graph showing the relationship between $S_{or}$ and $\overline{Ca}$ for different samples.

- Dashed line: bead pack, discontinuous oil
- Solid line: bead pack, continuous air
- Squares: bead pack, continuous oil
- Stars: sandstone, continuous oil

$S_{or}$ and $\overline{Ca}$ are plotted on the y and x axes respectively.
[1] M. Blunt and P. King, “Macroscopic parameters from simulations of pore scale flow,” Phys. Rev. A, vol. 42, p. 4780, 1990.
[2] U. Oxaal, “Fractal viscous fingering in inhomogeneous porous models,” Phys. Rev. A, vol. 44, p. 5038, 1991.
[3] U. Oxaal, F. Boger, J. Feder, T. Jossang, P. Meakin, and A. Aharony, “Viscous fingering in square lattice models with two types of bonds,” Phys. Rev. A, vol. 44, p. 6564, 1991.
[4] R. Hilfer, “Geometric and dielectric characterization of porous media,” Phys. Rev. B, vol. 44, p. 60, 1991.
[5] R. Hilfer, “Local porosity theory for flow in porous media,” Phys. Rev. B, vol. 45, p. 7115, 1992.
[6] R. Hilfer, “Geometry, dielectric response and scaling in porous media,” Physica Scripta, vol. T44, p. 51, 1992.
[7] M. Blunt, M. King, and H. Scher, “Simulation and theory of two-phase flow in porous media,” Phys. Rev. A, vol. 46, p. 7680, 1992.
[8] V. Frette, J. Feder, T. Jossang, and P. Meakin, “Buoyancy driven fluid migration in porous media,” Phys. Rev. Lett., vol. 68, p. 3164, 1992.
[9] P. Meakin, J. Feder, V. Frette, and T. Jossang, “Invasion percolation in a destabilizing gradient,” Phys. Rev. A, vol. 46, p. 3357, 1992.
[10] M. Kataja, K. Hiltunen, and J. Timonen, “Flow of water and air in a compressible porous medium: a model of wet pressing of paper,” J. Phys. D, vol. 25, p. 1053, 1992.
[11] B. Nest, B. Hansen, and E. Haslund, “Dielectric dispersion of composite materials,” Physica Scripta, vol. T44, p. 67, 1992.
[12] M. Sahimi, “Flow phenomena in rocks: From continuum models to fractals, percolation, cellular automata and simulated annealing,” Rev. Mod. Phys., vol. 65, p. 1393, 1993.
[13] M. Blunt and H. Scher, “Pore-level modeling of wetting,” Phys. Rev. E, vol. 52, p. 6387, 1995.
[14] V. Horvat and H. Stanley, “Temporal scaling of interfaces propagating in porous media,” Phys. Rev. E, vol. 52, p. 5166, 1995.
[15] S. Schwarzer, “Sedimentation and flow through porous media: Simulating dynamically coupled discrete and continuum phases,” Phys. Rev. E, vol. 52, p. 6461, 1995.
[16] L. Furuberg, K. Maloy, and J. Feder, “Intermittent behaviour in slow drainage,” Phys. Rev. E, vol. 53, p. 966, 1996.
[17] B. Virgin, E. Haslund, and R. Hilfer, “Rescaling relations between two- and three dimensional local porosity distributions for natural and artificial porous media,” Physica A, vol. 292, p. 1, 1996.
[18] H. Kytömaa, M. Kataja, and J. Timonen, “On the effect of pore pressure on the isotropic behavior of saturated porous media,” J. Appl. Phys., vol. 81, p. 7148, 1997.
[19] B. Berkowitz and H. Scher, “Anomalous transport in random fracture networks,” Phys. Rev. Lett., vol. 79, p. 4038, 1997.
[20] J. Andrade, M. Almeida, J. M. Filho, S. Havlin, B. Suки, and H. Stanley, “Fluid flow through porous media: The role of stagnant zones,” Phys. Rev. Lett., vol. 79, p. 3901, 1997.
[21] R. Hilfer, “Transport and relaxation phenomena in porous media,” Advances in Chemical Physics, vol. XCII, p. 299, 1996.
[22] G. Willhite, Waterflooding. 3rd ed. USA: Society of Petroleum Engineers, 1986.
[23] R. Hilfer, “Macroscopic equations of motion for two phase flow in porous media,” Physical Review E, vol. 58, p. 2090, 1998.
[24] H. Besserer and R. Hilfer, submitted for publication.
[25] L. Lake, Enhanced Oil Recovery. Englewood Cliffs: Prentice Hall, 1989.
[26] P. Oren, J. Billiotte, and W. Pinczewski, “Mobilization of waterflood residual oil by gas injection for water wet conditions,” SPE Formation and Evaluation, vol. March 1992, p. 70, 1992.
[27] J. Taber, J. Thovert, and P. Adler, “Reconstructed porous media and their application to fluid flow and solute transport,” Journal of Contaminant Hydrology, vol. 13, p. 3, 1993.
[28] R. Helmig, Multiphase Flow and Transport Processes in the Subsurface. Berlin: Springer, 1997.
[29] H. Dombrowski and L. Brownell, “Residual equilibrium saturation of porous media,” Industrial and Engineering Chemistry, vol. 46, p. 1207, 1954.
[30] J. Taber, “Dynamic and static forces required to remove a discontinuous oil phase from porous media containing both oil and water,” Soc.Petr.Eng. Journal, vol. 9, p. 3, 1969.
[31] J. Taber, J. Kirby, and F. Schroeder, “Studies on the displacement of residual oil: Viscosity and permeability effects,” AIChE Symp.Ser., vol. 69, p. 53, 1973.
[32] A. Abrams, “The influence of fluid viscosity, interfacial tension, and flow velocity on residual oil left by waterflood,” Soc. Petr. Eng. Journal, vol. 15, p. 437, 1975.
[33] I. Chatzis, M. Kuntanukkula, and N. Morrow, “Effect of capillary number on the microstructure of residual oil in strongly water wet sandstones,” SPE Reservoir Engineering, vol. August 1988, p. 902, 1988.
[34] N. Wardlaw and M. McKellar, “Oil blob populations and mobilization of trapped oil in unconsolidated packs,” The Canadian Journal of Chemical Engineering, vol. 63, p. 525, 1985.
[35] R. Larson, H. Davis, and L. Scriven, “Displacement of residual nonwetting fluid from porous media,” Chemical Engineering Science, vol. 36, p. 75, 1981.
[36] N. Morrow, I. Chatzis, and J. Taber, “Entrapment and mobilization of residual oil in bead packs,” *SPE Reservoir Eng.*, vol. Aug. 1988, p. 927, 1988.

[37] R. Hilfer and P. Øren, “Dimensional analysis of pore scale and field scale immiscible displacement,” *Transport in Porous Media*, vol. 22, p. 53, 1996.

[38] A. Scheidegger, *The Physics of Flow Through Porous Media*. Toronto: University of Toronto Press, 1974.

[39] F. Dullien, *Porous Media - Fluid Transport and Pore Structure*. San Diego: Academic Press, 1992.

[40] M. Sahimi, *Flow and Transport in Porous Media and Fractured Rock*. Weinheim: VCH Verlagsgesellschaft mbH, 1995.

[41] J. Bear, *Dynamics of Fluids in Porous Media*. New York: Elsevier Publ. Co., 1972.

[42] P. deGennes, “Dynamic capillary pressure in porous media,” *Europhys. Lett.*, vol. 5, p. 689, 1988.

[43] F. Kalaydijan, “Dynamic capillary pressure curve for water/oil displacement in porous media: Theory and experiment,” *SPE Proceedings, SPE 24813*, vol. 67th SPE Conference, Washington, p. 491, 1992.

[44] R. Hilfer and P. Øren, “Two phase flow and relative permeabilities,” 1993. Statoil Publ. Nr. F&U-LoU-94001.

[45] E. Donaldson, R. Thomas, and P. Lorenz, “Wettability determination and its effect on recovery efficiency,” *Soc.Petr.Eng. Journal*, vol. 9, p. 13, 1969.

[46] R. Ehrlich, H. Hasiba, and P. Raimondi, “Alkaline waterflooding for wettability alteration — evaluating a potential field application,” *J. Petroleum Technology*, vol. 26, p. 1335, 1974.

[47] E. L. duPrey, “Factors affecting liquid-liquid relative permeabilities of a consolidated porous medium,” *Soc.Petr.Eng. Journal*, vol. 13, p. 39, 1973.

[48] S. Gupta and S. Trushenski, “Micellar flooding — compositional effects on oil displacement,” *Soc.Petr.Eng. Journal*, vol. 19, p. 116, 1979.

[49] R. Hilfer and P. Øren, “Scaling of multiphase flow,” 1996. Statoil Publ. Nr. F&U-LoU-96015.

[50] M. Leverett, W. Lewis, and M. True, “Dimensional-model studies of oil field behaviour,” *Trans. AIME*, vol. 146, p. 175, 1942.

[51] J. Dumore and R. Schols, “Drainage capillary pressure function and the influence of connate water,” *Soc.Petr.Eng. Journal* (Oct. 1974), vol. 26, p. 437, 1974.

[52] L. Anton and R. Hilfer. unpublished first version of manuscript.

[53] P. Wong, J. Koplik, and J. Tomanic, “Conductivity and permeability of rocks,” *Phys. Rev. B*, vol. 30, p. 6606, 1984.