A critique of scaling behaviour in non-linear structure formation scenarios.

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ABSTRACT

Moments of the BBGKY equations for spatial correlation functions of cosmological density perturbations are used to obtain a differential equation for the evolution of the dimensionless function, \( h = -(v/\dot{a}x) \), where \( v \) is the mean relative pair velocity. The BBGKY equations are closed using a hierarchical scaling ansatz for the 3-point correlation function. Scale-invariant solutions derived earlier by Davis and Peebles are then used in the non-linear regime, along with the generalised stable clustering hypothesis \( (h \to \text{const.}) \), to obtain an expression for the asymptotic value of \( h \), in terms of the power law index of clustering, \( \gamma \), and the tangential and radial velocity dispersions. The Davis-Peebles solution is found to require that tangential dispersions are larger than radial ones, in the non-linear regime; this can be understood on physical grounds. Finally, stability analysis of the solution demonstrates that the allowed asymptotic values of \( h \), consistent with the stable clustering hypothesis, lie in the range \( 0 \leq h \leq 1/2 \). Thus, if the Davis-Peebles scale-invariant solution (and the hierarchical model for the 3-pt function) is correct, the standard stable clustering picture \( (h \to 1 \text{ as } \xi \to \infty) \) is not allowed in the non-linear regime of structure formation.

Subject headings: cosmology: theory — large scale structure of the Universe

1. Introduction

The cosmological BBGKY equations have been used on a number of occasions to study the evolution of non-linear density fluctuations in an expanding, flat \((\Omega = 1)\), background Universe (Davis & Peebles 1977; Ruamsuwan & Fry 1992; Yano & Gouda...
These equations have the advantage of dealing directly with statistical quantities, i.e. the spatial N-point correlation functions; however, they form an infinite hierarchy in which the equation for the N-point correlation function contains terms involving the (N+1)-point function. This necessitates the development of closure schemes in which the hierarchy is cut off at a finite number of equations by making some assumptions wherein the higher order correlation functions are written in terms of the lower order ones. Such a scheme was used by Davis & Peebles (1977; hereafter DP) to demonstrate the existence of a similarity solution to the equations; the assumptions used were the stable clustering hypothesis, the vanishing of velocity skewness, a hierarchical model for the 3-point correlation function, $\zeta \propto <\xi^2>$, where $\xi$ is the 2-point correlation function, and finally, a factorisation of the 2- and 3-body phase space distributions which gives a form for the 3-body weighted pair velocity. In this solution, an initial power spectrum of form $P_o(k) \sim k^n$ evolves to give a 2-point correlation function

$$\xi(r) \propto r^{-\gamma}, \quad \gamma = \frac{3(n+3)}{(n+5)}$$

in the strong clustering regime with $\xi \gg 1$. Ruamsuwan & Fry (1992; hereafter RF) showed that the assumption of vanishing velocity skewness was not a priori necessary but could be derived as a result of other, more general assumptions; the solution was also shown to be marginally stable to perturbations. Finally, Yano & Gouda (1997) found that the stability condition used by DP was satisfied for the unique case of vanishing skewness and the specific form for $\zeta$ in terms of the products of the 2-point correlation functions. The power index of the 2-point function in the strong clustering regime depends, in general, on the mean relative physical velocity, the skewness and the 3-point correlation function.

The present work considers the evolution of the function, $h \equiv -(<v>/\dot{a}x)$, i.e. the ratio of the mean relative peculiar velocity to the Hubble velocity. The BBGKY equations are not derived again; instead, we use the relevant moment equations from RF to proceed. The form for the 3-point correlation function is the same as in DP and RF. We derive an equation for the evolution of $h$ in terms of the mean 2-point correlation function, $\bar{\zeta}$, using the ansatz that $h$ is a function of $\bar{\zeta}$ alone (Hamilton et al. 1991; Nityananda & Padmanabhan 1994, hereafter NP). This ansatz is, however, not crucial to the later discussion. (Note, further, that the above equation is derived using only the zeroth and first moments of the second BBGKY equation and hence does not contain
any assumptions regarding the form of the velocity skewness or the higher velocity moments, excepting the fact that they be such as to yield the DP similarity solution for the two-point correlation function and the velocity dispersions; the assumption of vanishing velocity skewness is not used here. Of course, as shown by RF, this assumption is not necessary to arrive at the DP solution; in fact, zero skewness results as only one particular case in their closure scheme.) The DP solution is then substituted in the equation for $h$ and the generalised stable clustering hypothesis ($h \to \text{const.}$, for $\xi \gg 1$) used, to obtain an expression relating the asymptotic value of $h$ to $\gamma$, the power law index of clustering, and the radial and tangential pair velocity dispersions. This, and the requirement that $h$ is real, gives the constraint that tangential dispersions are larger than radial dispersions, in the non-linear regime. Finally, a stability analysis carried out by perturbing about the above solution shows that stable solutions are attained only for asymptotic values of $h$ in the range $0 < h < 0.5$. Thus, the standard stable clustering picture ($h \to 1$ as $\xi \to \infty$) is incompatible with the DP scaling solution.

2. The BBGKY hierarchy

The zeroth and first moments of the 2nd BBGKY equation are respectively (RF, equations (22) and (23))

$$
\frac{\partial \xi_{12}}{\partial t} + \frac{1}{a} \frac{\partial}{\partial x_{12}^i} \left[ <v_{12}^i> (1 + \xi_{12}) \right] = 0 \tag{2}
$$

$$
\frac{1}{a} \frac{\partial}{\partial t} \left[ a <v_{12}^i > (1 + \xi_{12}) \right] + \frac{1}{a} \frac{\partial}{\partial x_{12}^j} \left[ <v_{12}^i v_{12}^j> (1 + \xi_{12}) \right] + G \rho_b a \int d^3 x_3 (\xi_{13} + \xi_{23} + \xi_{123}) \left( \frac{x_{13}^i}{|x_{13}|^3} - \frac{x_{23}^i}{|x_{23}|^3} \right) = 0 \tag{3}
$$

where $x_{12}^i \equiv x_1^i - x_2^i$ denotes the separation between particles labelled 1 and 2 and $v_{12}^i \equiv v_1^i - v_2^i$ is their relative velocity ($i$ is a vector index). $\xi_{12}$ and $\xi_{123}$ are the usual 2- and 3-point correlation functions respectively while $\rho_b$ is the background density.

The number of independent vector components in equation (3) can be reduced by making use of various cosmological symmetries. The isotropy of the background
Universe implies that the mean relative pair velocity, \( <v_{12}^i> \), should be aligned with the pair separation, i.e.

\[
<v_{12}^i> = v(x_{12}) \hat{x}_{12}^i
\]  

(4)

where \( \hat{x}^i \) denotes a unit vector along \( x^i \), and the pair velocity dispersion should have longitudinal and transverse polarisations about the mean \( \Pi \) and \( \Sigma \) (RF).

\[
<\Delta v^i \Delta v^j> = \hat{x}^i \hat{x}^j \Pi(x) + (\delta^{ij} - \hat{x}^i \hat{x}^j) \Sigma(x)
\]  

(5)

We note that \( \Pi \) and \( \Sigma \) are peculiar velocity dispersions. Finally, \( \zeta_{123} \) is taken to be the hierarchical form (DP; RF)

\[
\zeta_{123} = Q(\xi_{12} \xi_{13} + \xi_{13} \xi_{23} + \xi_{12} \xi_{23})
\]  

(6)

This form for \( \zeta_{123} \) satisfies the necessary symmetry under the exchange of indices; further, it vanishes when any of the three points is removed to a large distance. Using the above ansatz, we obtain

\[
\frac{\partial \xi}{\partial t} + \frac{1}{ax^2} \frac{\partial}{\partial x} \left[ x^2 v(1 + \xi) \right] = 0
\]  

(7)

and

\[
\frac{1}{a} \frac{\partial}{\partial t} \left[ av(1 + \xi) \right] + \frac{1}{ax^2} \frac{\partial}{\partial x} \left[ x^2 (\Pi + v^2)(1 + \xi) \right] - \frac{2}{ax} \Sigma(1 + \xi)
\]

\[
+ \frac{2G\rho_0 a}{x^2} \int_0^x d^3z \xi(z) + 2GQ\rho_0 a \int d^3z \left[ \xi(x) + \xi(z) \right] \xi(z + x) \cos \theta \frac{z^2}{z^2} = 0
\]  

(8)

We next differentiate equation (7) with respect to \( t \) and equation (8) with respect to \( x \); the resulting equations can be combined to yield

\[
- \frac{\partial}{\partial t} \left[ a^2 \frac{\partial (x^2 \xi)}{\partial t} \right] + \frac{\partial^2}{\partial x^2} \left[ x^2 (\Pi + v^2)(1 + \xi) \right] - \frac{\partial}{\partial x} \left[ 2\Sigma x(1 + \xi) \right]
\]
\[ + \frac{\partial}{\partial x} \left[ 2G\rho_b a^2 \int_0^x d^3 z \xi(z) \right] + \frac{\partial}{\partial x} \left[ 2GQ\rho_b a^2 x^2 M \right] = 0 \quad (9) \]

where \( M \) is defined by

\[ M = \int d^3 z \left[ \xi(x) + \xi(z) \right] \xi(z-x) \frac{\cos \theta}{z^2} \quad (10) \]

Now, the mean 2-point correlation function, \( \bar{\xi}(x,a) \), is defined by

\[ \bar{\xi}(x,a) = \frac{3}{x^3} \int_0^x dx \xi(x,a)x^2 \quad (11) \]

Substituting for \( \xi \) in terms of \( \bar{\xi} \), we obtain

\[ -\frac{\partial}{\partial t} \left[ a^2 \frac{\partial}{\partial t} \left( \frac{x^3 \bar{\xi}}{3} \right) \right] + \frac{\partial^2}{\partial x^2} \left[ (\Pi + v^2) \frac{\partial}{\partial x} \left( \frac{x^3(1 + \bar{\xi})}{3} \right) \right] - \frac{\partial}{\partial x} \left[ 2\Sigma \frac{\partial}{\partial x} \left( \frac{x^3(1 + \bar{\xi})}{3} \right) \right] + \frac{\partial}{\partial x} \left[ \frac{8\pi G}{3} \rho_b a^2 x^3 \xi \right] + \frac{\partial}{\partial x} \left[ 2GQ\rho_b a^2 x^2 M \right] = 0 \quad (12) \]

After integrating over \( x \) and carrying out some algebra, this gives

\[ \frac{\partial^2 \bar{\xi}}{\partial A^2} + \frac{1}{2A} \frac{\partial \bar{\xi}}{\partial A} - 3\bar{\xi} - \left( h_\|^2 + h_\perp^2 \right) \left[ 4F + \frac{\partial F}{\partial X} \right] - F \frac{\partial}{\partial X} \left[ h_\|^2 + h_\perp^2 \right] + 2h_\perp^2 F = \frac{9QM e^{-X}}{4\pi} \quad (13) \]

In the above, we have defined \( F \) by

\[ F = x \frac{\partial \bar{\xi}}{\partial x} + 3(1 + \bar{\xi}) \quad (14) \]

, substituted \( X = \ln x \) and \( A = \ln a \), and defined

\[ h = -\frac{v}{\dot{a}x}, \quad h_\|^2 = \frac{\Pi}{\dot{a}^2 x^2} \quad \text{and} \quad h_\perp^2 = \frac{\Sigma}{\dot{a}^2 x^2} \quad (15) \]
However, $\partial \xi / \partial A = h F$ (NP). Hence,

$$F\left(\frac{\partial h}{\partial A} - h \frac{\partial h}{\partial X}\right) + \frac{h F}{2} - h^2 F - 3\xi - h^2 \left(4F + \frac{\partial F}{\partial X}\right) - F \frac{\partial h^2}{\partial X} + 2h^2 F = \frac{9QM e^{-X}}{4\pi} \quad (16)$$

Here, we make the ansatz, $h \equiv h(\xi)$ (Hamilton et al. 1991; NP; Mo, Jain & White 1995; Padmanabhan & Engineer 1998). This gives

$$\frac{\partial h}{\partial \xi} \left[\frac{\partial \xi}{\partial A} - h \frac{\partial \xi}{\partial X}\right] + \frac{h}{2} - h^2 - 3\xi - h^2 \left(4 + \frac{\partial \ln F}{\partial X}\right) - \frac{\partial h^2}{\partial X} + 2h^2 = \frac{9MQ e^{-X}}{4\pi F} \quad (17)$$

and, finally,

$$3h(1 + \xi) \frac{dh}{d\xi} + \frac{h}{2} - h^2 - 3\xi - h^2 \left(4 + \frac{\partial \ln F}{\partial X}\right) - \frac{\partial h^2}{\partial X} + 2h^2 = \frac{9MQ e^{-X}}{4\pi F} \quad (18)$$

where we have used (NP)

$$\frac{\partial \xi}{\partial A} - h \frac{\partial \xi}{\partial X} = 3h(1 + \xi) \quad (19)$$

Further, the ansatz, $h \equiv h(\xi)$ has been used to convert the partial derivative into a total derivative. Equation (18) governs the evolution of $h$ in terms of the mean 2-point correlation function $\xi$.

### 3. The non-linear regime

We will consider the small separation, strong clustering limit, $\xi \gg 1$. In this regime, the 2-point correlation function was found by DP to exhibit a scale-invariant, power-law behaviour, with the assumption of stable clustering. The latter is physically well-motivated as it seems reasonable to expect stable, bound systems to form under the influence of gravity. Such systems would neither expand nor contract and would hence have peculiar velocities equal and opposite to the Hubble expansion, i.e. $v^i_{pec} = -\dot{a}x^i$. The stable clustering ansatz has, however, not been deduced from any fundamental
considerations and might certainly be considered suspect if mergers of structures are
important (Padmanabhan et al. 1996). Also, while N-body simulations (Hamilton et
al. 1991) indicate that \( h \to 1 \) for \( \xi \gg 1 \), the results are, at best, inconclusive. The
argument can, however, be generalised (Padmanabhan 1997) as the function \( h \), which is
the ratio of two velocities, should tend to some constant value, if the virialised systems
have reached stationarity in the statistical sense. We will use this more general stability
condition, namely \( h \to \text{const.} \), to proceed; using this assumption, equation (4) reduces
to

\[
\frac{a}{a} \frac{\partial \xi}{\partial a} - \frac{h}{x^2} \frac{\partial}{\partial x} [x^3 \xi] = 0
\]

The above equation has the power-law solution, \( \xi \sim a^\beta x^{-\gamma} \) with \( \beta = (3 - \gamma)h \).
Dimensional analysis of equation (8) reveals that \( \Pi \) and \( \Sigma \) scale as \( \Pi \sim \Sigma \sim a^{(3-\gamma)h} x^{2-\gamma} \).
Now, for a flat (\( \Omega = 1 \)) Universe, \( a^2 \propto a^{-1} \). This implies that

\[
h_\parallel^2 = \frac{\Pi}{a^2 x^2} \propto a^{(3-\gamma)h} x^{-\gamma} = \Pi_o \xi
\]

and, similarly,

\[
h_\perp^2 = \Sigma_o \xi
\]

where \( \Pi_o \) and \( \Sigma_o \) are constants of proportionality. Also, (Yano & Gouda 1997) we can
write \( M = M' x \xi^2 \), where \( M' \) is another constant. In the non-linear limit, \( \xi \gg 1 \),

\[
F = (3 - \gamma) \xi + 3
\]

and

\[
\frac{\partial \ln F}{\partial X} = -\gamma \left[ 1 + \left( \frac{3}{3-\gamma} \right) \left( \frac{1}{\xi} \right) \right]^{-1}
\]
Substituting for $F$, $h_\parallel^2$, $h_\perp^2$ and $\xi$ in equation (18) and using the limit $\xi \gg 1$, we obtain

$$3h\xi \frac{dh}{d\xi} - h^2 + \frac{h}{2} - \left(\frac{3}{3 - \gamma}\right)D(\gamma) = \xi C(\gamma) + \vartheta\left(\frac{1}{\xi}\right)$$  \hspace{1cm} (25)$$

where $D(\gamma)$ and $C(\gamma)$ are defined by

$$D(\gamma) = 1 + \gamma \Pi_o - \frac{9QM'}{4\pi(3 - \gamma)}$$  \hspace{1cm} (26)$$

and

$$C(\gamma) = 2\Pi_o(2 - \gamma) - 2\Sigma_o + \frac{9QM'}{4\pi(3 - \gamma)}$$  \hspace{1cm} (27)$$

Equation (25) is exact (within the exact power law solutions for $\xi$, $h_\parallel^2$, $h_\perp^2$ and $M$) upto order constant, in the limit of large $\xi$, with terms of order $\vartheta(1/\xi)$ neglected.

Clearly, $C(\gamma)$ must be exactly zero, as otherwise, $h \propto \sqrt{\xi}$ for $\xi \gg 1$, which violates the stable clustering hypothesis. Note that $C(\gamma) \approx 0$ is not sufficient as, if this were the case, the term in $C(\gamma)$ would cause $h$ to grow with $\xi$, for sufficiently large $\xi$. Thus

$$2\Sigma_o - 2\Pi_o(2 - \gamma) = \frac{9QM'}{4\pi(3 - \gamma)}$$  \hspace{1cm} (28)$$

This equation is equivalent to equation (48) of RF, albeit in slightly different form. Equation (25) thus reduces to

$$3h\xi \frac{dh}{d\xi} - h^2 + \frac{h}{2} = A$$  \hspace{1cm} (29)$$

where $A = [3/(3 - \gamma)]D(\gamma)$.

Now, the ansatz of stable clustering implies that $\xi dh/d\xi \to 0$ as $\xi \to \infty$. Equation (29) then gives

$$h^2 - \frac{h}{2} + A = 0$$  \hspace{1cm} (30)$$
\[ h = \frac{1}{4} \left[ 1 \pm \sqrt{1 - 16A} \right] \]  

(31)

Since \( h \) is a real quantity, the above equation immediately yields

\[ A \leq \frac{1}{16} \]  

(32)

This gives

\[ 1 + \gamma \Pi_o - \frac{9QM'}{4\pi(3 - \gamma)} \leq \left( \frac{3 - \gamma}{48} \right) \]  

(33)

Replacing for \( 9QM'/4\pi(3 - \gamma) \) from equation (28), we obtain

\[ 2\Sigma_o \geq (4 - \gamma)\Pi_o + 1 - \left( \frac{3 - \gamma}{48} \right) \]  

(34)

RF have pointed out that various integrals in the BBGKY hierarchy do not converge unless \( 0 < \gamma < 2 \). This range of \( \gamma \) values implies that \( 1 - (3 - \gamma)/48 > 0 \) and \( (4 - \gamma) > 2 \). Thus, the inequality (34) gives

\[ \Sigma_o > \Pi_o \]  

(35)

Thus, the DP solution requires that tangential dispersions exceed radial dispersions in the non-linear regime. This is understandable on physical grounds, as tangential dispersions cause deviations from radial infall; stable structures would hence only be expected to form once these dispersions become comparable to or larger than the radial ones.

The general solution to equation (29) has the following three different forms depending on whether \( (1 - 16A) \) is positive, negative or zero. The constant of integration is denoted by \( B \) in each case.
1. For \((1 - 16A) < 0\), the solution is

\[
\ln \left( B\xi^{1/6} \right) = \ln \left[ \left( 2h^2 - h + 2A \right)^{1/4} \right] + \frac{1}{2} \sqrt{\frac{3 - \gamma}{\gamma + 45}} \tan^{-1} \left[ (4h - 1) \sqrt{\frac{3 - \gamma}{\gamma + 45}} \right]
\] (36)

2. Next, for \(A = 1/16\), the solution has the form

\[
B\xi^{2/3} = \left[ 2h^2 - h + 2A \right] \exp \left[ -\frac{2}{4h - 1} \right]
\] (37)

3. Finally, for \((1 - 16A) > 0\), the solution is

\[
B\xi^{2/3} = \left[ 2h^2 - h + 2A \right] \left[ \frac{p - 4h + 1}{p + 4h - 1} \right]^{1/p}
\] (38)

where we have defined \(p = \sqrt{1 - 16A}\). As mentioned earlier, \((1 - 16A)\) cannot be negative since \(h\) is a real quantity; equation (36) can hence be immediately ruled out as a possible solution. Further, although \(\xi dh/d\xi \to 0\), \(h = 0.25(1 \pm |p|)\) is certainly a solution of equation (29), it is not clear if this can be actually reached from the general solution embodied in equations (37) and (38). We hence perturb these solutions by writing \(h = h_o + \epsilon\), where \(\epsilon\) is the perturbation parameter and \(h_o\) satisfies the equation

\[
h_o^2 - \frac{h_o}{2} = \frac{3}{\gamma - 3}
\] (39)

i.e.

\[
h_o = 0.25 (1 \pm |p|)
\] (40)

We then attempt to impose the condition \(\epsilon \to 0\) as \(\xi \to \infty\); if this is possible, it clearly indicates that \(h \to h_o\) as \(\xi \to \infty\), i.e. a solution exists which satisfies the stable clustering hypothesis. We initially consider the case \(A = 1/16\) \((p = 0)\) and rewrite equation (37) as

\[
B\xi^{2/3} = \left[ 2h_o^2 - h_o + 2A + 4h_o\epsilon + \epsilon^2 - \epsilon \right] \exp \left[ -\frac{2}{4h_o - 1} \right]
\] (41)
Since $h_0 = 0.25$ for $p = 0$, this gives

$$B^2/3_\xi = \left[\epsilon^2\right] \exp \left[-\frac{1}{2\epsilon}\right] \tag{42}$$

One can impose the condition $\epsilon \to 0$ as $\xi \to \infty$, in the above equation, only if $\epsilon$ is negative. $A = 1/16$ is thus an allowed solution and $h \to 0.25$ from below, as $\xi \to \infty$. Finally, we consider the case $(1 - 16A) > 0$, i.e. $A < 1/16$. Equation (38) can be rewritten as

$$B^2/3_\xi = \left[2h_o^2 - h_o + 2\epsilon^2 + 4h_o (\epsilon - 1)\right] \left[p - 4h_o - 4\epsilon + 1\right]^{1/p} \left[p + 4h_o + 4\epsilon - 1\right]^{1/p} \tag{43}$$

Using equation (39) in the above and retaining terms upto first order in $\epsilon$, we obtain

$$B^2/3_\xi = \left[4h_o (\epsilon - 1)\right] \left[p - 4h_o - 4\epsilon + 1\right]^{1/p} \left[p + 4h_o + 4\epsilon - 1\right]^{1/p} \tag{44}$$

The two possible values of $h_o$, given by equation (40), are $h_o = 0.25 (1 \pm |p|)$. For $h_o = 0.25 (1 - |p|)$, we choose $p > 0$ in equation (44). This gives

$$B^2/3_\xi = -|p| \epsilon \left[\frac{2|p| - 4\epsilon}{4\epsilon}\right]^{1/|p|} \tag{45}$$

$$= -|p| \left[\frac{|p| - 2\epsilon}{2}\right]^{1/|p|} \epsilon^{1-1/|p|} \tag{46}$$

Equation (46) shows that one can satisfy the the condition $\epsilon \to 0$ as $\xi \to \infty$ if $1 < |p|^{-1}$ i.e. if

$$\sqrt{1 - 16A} < 1 \tag{47}$$

or, in other words, $A > 0$. (An equivalent result can be obtained for the solution $h_o = 0.25 (1 + |p|)$ by choosing $p < 0$ in equation (44)).
Since $A \geq 0$, equation (31) then implies that

$$h(h - \frac{1}{2}) \leq 0 \tag{48}$$

i.e. $0 \leq h \leq 1/2$. Thus, the asymptotic values of $h$, allowed by the DP solution and consistent with the stable clustering hypothesis, lie in the range $0 < h < 1/2$. In the standard stable clustering scenario, $h \to 1$ for $\bar{\xi} \gg 1$. The above analysis clearly rules out this value of $h$ as $\bar{\xi} \to \infty$. Next, the constraint $A \geq 0$ gives

$$1 + \gamma \Pi_o - \frac{9QM'}{4\pi(3-\gamma)} \geq 0 \tag{49}$$

Using equation (28), this gives

$$1 + (4 - \gamma) \Pi_o - 2\Sigma_o \geq 0 \tag{50}$$

i.e.

$$\Sigma_o \leq \frac{(4 - \gamma)}{2} \Pi_o + \frac{1}{2} \tag{51}$$

We have already shown that $\Sigma_o > [(4 - \gamma)/2] \Pi_o + 1/2 - (3 - \gamma)/96$. The above equation (51) indicates that only a very narrow range of values is permitted for $\Sigma_o$ (in terms of $\Pi_o$), in the non-linear regime. Thus, the DP solution imposes strong constraints on the relative values of radial and tangential dispersions; it is not obvious that the dynamics of the system will actually cause these constraints to be satisfied.

Finally, the preceding results do not appear to be influenced by the ansatz $h \equiv h(\bar{\xi})$ (although this ansatz is well-motivated, since it appears to be validated in N-body simulations; see, for example, [Hamilton et al. 1991]) as one could have instead carried out the analysis using the scaled variable $s \equiv xt^{-\alpha}$ (with $\gamma = 2/(\alpha + 2/3)$). In the non-linear regime, self-similar evolution implies that $h \equiv h(s)$ and $\bar{\xi} \equiv \bar{\xi}(s)$; one can clearly write $h \equiv h(\bar{\xi})$ in this regime.
4. Conclusions

In the present work, the BBGKY hierarchy of equations has been revisited and an equation derived for the evolution of the dimensionless function $h = -(v/\dot{a}x)$. The assumptions used (following DP) are a hierarchical model for the 3-point correlation function and the ansatz that $h$ is a function of the mean 2-point correlation function, $\bar{\xi}$, alone, i.e. $h \equiv h(\bar{\xi})$. No assumption is made regarding the vanishing of velocity skewness. In fact, the analysis only uses the zeroth and first moments of the second BBGKY equation; it is thus applicable to any form of the skewness (or the higher velocity moments) which yields the DP solution for $\bar{\xi}$ and the parallel and perpendicular velocity dispersions. The DP similarity solution is then substituted in this equation for the $h$ function, in the non-linear regime, and the generalised stable clustering hypothesis ($h \to \text{const.}$) used to obtain an expression for the asymptotic value of $h$, in terms of $\gamma$, the power law index of clustering and the tangential and radial velocity dispersions. The DP solution is found to require that tangential dispersions are larger than radial ones, in the strong clustering regime; this can be understood on physical grounds. Finally, stability analysis of the solution demonstrates that the allowed asymptotic values of $h$, consistent with the stable clustering hypothesis, lie in the range $0 \leq h \leq 1/2$. Thus, if the DP scale-invariant solution (and the hierarchical model for the 3-pt function) is correct, the standard stable clustering picture ($h \to 1$ as $\bar{\xi} \to \infty$) is not allowed in the non-linear regime of structure formation.

It is a pleasure to thank T. Padmanabhan for initially suggesting this problem to me. I also thank him and Kandu Subramanian for exceedingly useful discussions during the course of the present work, as well as for a critical reading of previous drafts of this paper. Finally, I thank an anonymous referee for his/her comments on and criticism of an earlier version.

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