Brane Mechanism of Spontaneously Generated Gravity

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Abstract—The braneworld scenario is studied and the effective action of 3-brane living in 5-dim. Minkowski space is constructed. This action is proved to be invariant under spontaneously broken scale and Weyl symmetries and to encode a model of quadratic gravity generalizing the Starobinsky model. The symmetry breaking generates the Hilbert–Einstein term with the Newton constant \( G_N \), where \( G_N \) is a mass scale equal to the mean curvature of the vacuum hyper-worldsheet of 3-brane. This result proposes a brane modification of the mechanism of spontaneously generated gravity.

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Observation data from Planck [1] and other high precision experiments (see [2] and refs. there) support the conception of inflation in cosmology. String/M-theory unifying QFT and gravity in 11-dimensional space-time are considered as a suitable basis for solution of this problem provided the extra dimensions are compactified to the Planck scale \( l_{pl} \sim 10^{-33} \) cm. The original idea of Kaluza–Klein has supposed that all fields in nature propagate in the same number of dimensions. Later this condition was weakened by the assumptions allowing trapping of matter or the existence of noncompact extra dimensions [3–5]. Studies in this direction resulted in interesting models solving the hierarchy problem between the electroweak and Planck scales, as well as explaining the observed weakness of the Newton gravity [6–9]. Their result was that fields of the Standard Model might be confined on a 3-brane embedded into 5-dim. space-time [10–15]. This confirmed the \( 1/r^2 \) Newton law corrected by the term \( \sim 1/r^4 \) that might in principle be tested at distances of about 1 mm, remembering that gravity was accurately measured at distances of \( \sim 1 \) cm. The fact that extra dimensions can be much larger than the electroweak scale \( \sim 10^{-16} \) cm makes the braneworld scenario promising for the unification of SM and gravity. This gives rise to the question about emergence of the Hilbert–Einstein gravity from a 3-brane identified with our Universe. Here we study the structure of non-linearities of 3-branes embedded into the 5-dim. Minkowski space on the basis of the Gauss–Cartan differential-geometric approach [16]. This approach treats strings and branes in terms of their hyper-worldsheets (hyper-ws) embedded into higher-dimensional spaces [17–21]. We build an effective action for the 3-brane and reveal that this action encodes a model of quadratic gravity [22–29] with the spontaneously broken Weyl and scale symmetries. This breaking creates the Hilbert–Einstein action with the Newton constant \( G_N \sim 1/\mu^2 \) in agreement with the mechanism of spontaneously generated gravity suggested in [23–25]. The above constant \( \mu \), equal to the mean curvature of the vacuum 4-dim hyper-ws embedded into \( R^{1,4} \), introduces a mass scale breaking the scale invariance.

1. We explore effective action for p-branes in terms of the metric \( g_{\mu\nu}(\xi) \) and extrinsic curvature \( l_{\mu\nu}(\xi) \) of their hyper-worldsheets \( \Sigma_{p+1} \) in the \( (p + 2) \)-dim. Minkowski space \( R^{1,p+1} \)

\[
S_p = \frac{1}{k_p} \int d^{p+1}\xi \sqrt{|g|} \times \left( \frac{1}{2} \nabla_\mu l_{\nu\rho} \nabla^\mu l^{\nu\rho} - \nabla_\mu l_{\nu\rho} \nabla_\rho l^{\nu\mu} + U_p(g,I) \right).
\] (1)

The tensors \( g_{\mu\nu}, l_{\mu\nu} \) define \( \Sigma_{p+1} \) modulo its global translations and rotations. For \( p = 3 \) and \( R^{1,4} \) we expect this action to be a generally covariant extension of a massless \( \phi^4 \) theory which has only one dimensionless coupling constant \( k_3 \) depending on 3-brane tension.
(see [30]). In this case the scalar potential $U_3$ is represented by the quartic polynomial

$$U_3 = \frac{2}{3}Tr l Tr(l^2) - \frac{1}{2}(Tr(l^2))^2 + b_2 Tr(l^2)(Tr l)^3 + b_4 Tr(l^4) + c_3,$$

where $b_2$, $b_4$, $b_3$ are dimensionless parameters. We use the canonical dimensions of the fields and coordinates in the units $\hbar = c = 1$, i.e. $[\xi] = [\text{length}]$, $[l_{\mu \nu}] = [\text{length}^{-1}]$.

The invariants of diffeomorphisms $Tr l Tr(l^2)$ are formed by covariant contractions of $n$ symmetric tensors $l_{\mu \nu}^{-1}$.

$$Tr l = l_{\mu \nu} g^{\mu \nu}, \quad Tr(l^2) = l_{\mu \nu} l^{\mu \nu}, \quad Tr(l^3) = l_{\mu \nu} l^{\mu \nu} l^{\rho \gamma}, \quad Tr(l^4) = l_{\mu \nu} l^{\mu \nu} l_{\rho \gamma} l^{\rho \gamma}.$$

When $p = 3$ and the cosmological constant $c_3 = 0$, the action (1) with the potential (2) becomes invariant under the global Weyl and scale transformations.

To prove this statement we choose the Weyl symmetry transformations for $g_{\mu \nu}$ as

$$\xi^{\mu} = \xi^{\mu} , \quad g_{\mu \nu} (\xi^{\prime}) = e^{2 \varphi} g_{\mu \nu} (\xi).$$

This gives the transformation law for the $(p + 1)$-dim. differential world-volume

$$d^{p+1} \xi \sqrt{|g(\xi)|} = e^{(p+1) \varphi} d^{p+1} \xi \sqrt{|g(\xi)|}.$$  \hspace{1cm} (5)

The change of (5) must be compensated by transformations of the Lagrangian density in (1). Using Eq. (5) we find that the kinetic term prescribes the transformation for $l_{\mu \nu}$

$$l_{\mu \nu}^{\prime} (\xi) = e^{-\frac{\varphi}{2}} l_{\mu \nu} (\xi).$$ \hspace{1cm} (6)

The potential term (with $c_\varphi = 0$) dictates the Weyl transformation law to be

$$l_{\mu \nu}^{\prime} (\xi) = e^{-\frac{\varphi}{2}} l_{\mu \nu} (\xi).$$ \hspace{1cm} (7)

The laws (6) and (7) coincide for $p = 3$. It proves the invariance of the effective action $S_3$ (1) with $c_3 = 0$ under the global Weyl transformations

$$\xi^{\mu} = \xi^{\mu} , \quad g_{\mu \nu} (\xi^{\prime}) = e^{2 \varphi} g_{\mu \nu} (\xi), \quad l_{\mu \nu}^{\prime} (\xi) = e^{\varphi} l_{\mu \nu} (\xi).$$ \hspace{1cm} (8)

The action (1) with $c_3 = 0$ is also invariant under the global dilatation symmetry.

In field theory a field $\varphi(x)$ of the dimension $q$ has the following scaling transformations

$$x^m = e^{-\lambda} x^m , \quad \varphi(x^\prime) = e^{\alpha q} \varphi(x),$$ \hspace{1cm} (9)

where $\lambda$ is a rescaling parameter. Using these rules we find the form variation $\delta \varphi(x)$ of a field $\varphi$ under infinitesimal transformations

$$\delta \varphi(x) = \delta \lambda q + x^m \partial_m \varphi(x),$$

$$\delta \varphi(x) = q \varphi(x) \delta \lambda.$$ \hspace{1cm} (10)

Keeping in mind that $g_{\mu \nu} (\xi^{\prime})$ is dimensionless we have $\delta g_{\mu \nu} (\xi^{\prime}) = \delta \lambda \xi^{\prime} \partial_\mu g_{\mu \nu} (\xi^{\prime})$ that gives

$$\xi^{\prime \mu} = e^{-\lambda} \xi^{\mu} , \quad g_{\mu \nu}^{\prime} (\xi^{\prime}) = g_{\mu \nu} (\xi) \rightarrow d^{p+1} \xi \sqrt{|g(\xi)|} = e^{-(p+1) \lambda} d^{p+1} \xi \sqrt{|g(\xi)|},$$ \hspace{1cm} (11)

respectively. Eqs. (11) prescribe the following transformation rule for a scalar potential $U_\rho$

$$U_\rho (g_{\mu \nu} (\xi), l_{\mu \nu} (\xi)) = e^{(p+1) \lambda} U_\rho (g_{\mu \nu} (\xi), l_{\mu \nu} (\xi)).$$ \hspace{1cm} (12)

As a result, we obtain the scaling transformation law for $l_{\mu \nu} (\xi)$

$$l_{\mu \nu}^{\prime} (\xi) = e^{-\frac{\lambda}{2}} l_{\mu \nu} (\xi).$$ \hspace{1cm} (13)

To find the law for $l_{\mu \nu} (\xi)$ prescribed by the kinetic part of the action (1) note that

$$\partial_\mu = e^{2 \lambda} \partial_\mu , \quad \Gamma_{\mu \nu \rho}^{\prime} (\xi) = e^{2 \lambda} \Gamma_{\mu \nu \rho} (\xi), \rightarrow \nabla^{\mu} = e^{2 \lambda} \nabla^{\mu},$$ \hspace{1cm} (14)

where $\Gamma_{\mu \nu \rho} (\xi)$ are the Christoffel symbols for $\Sigma_{p+1}$. It yields the following law for $l_{\mu \nu} (\xi)$

$$l_{\mu \nu}^{\prime} (\xi) = e^{-\frac{\lambda}{2}} l_{\mu \nu} (\xi).$$ \hspace{1cm} (15)

The transformation rules (13) and (15) coincide only if $p = 3$. This proves invariance of $S_3$ (1) under the global scaling transformations

$$\xi^{\mu} = e^{-\lambda} \xi^{\mu} , \quad g_{\mu \nu}^{\prime} (\xi^{\prime}) = g_{\mu \nu} (\xi), \quad l_{\mu \nu}^{\prime} (\xi) = e^{\varphi} l_{\mu \nu} (\xi).$$ \hspace{1cm} (16)

Thus, $S_3$ (1) with $c_3 = 0$ turns out to be invariant under the $U(1) \times U(1)$ abelian symmetry

$$\xi^{\mu} = e^{-\lambda} \xi^{\mu} , \quad g_{\mu \nu}^{\prime} (\xi^{\prime}) = e^{2 \varphi} g_{\mu \nu} (\xi), \quad l_{\mu \nu}^{\prime} (\xi) = e^{\varphi} l_{\mu \nu} (\xi).$$ \hspace{1cm} (17)

Equations (17) show that the abelian subgroup $U_+$ of $U(1) \times U(1)$ formed by $\alpha = \lambda$

$$\xi^{\mu} = e^{-\lambda} \xi^{\mu} , \quad g_{\mu \nu}^{\prime} (\xi^{\prime}) = e^{2 \varphi} g_{\mu \nu} (\xi), \quad l_{\mu \nu}^{\prime} (\xi) = e^{\varphi} l_{\mu \nu} (\xi)$$ \hspace{1cm} (18)

belongs to the group of diffeomorphisms of $\Sigma_3$ and it protects $U_+$ from breakdown. It means that breaking of the Weyl symmetry demands breaking of the scale symmetry of $S_3$ (1) with $c_3 = 0$. This observation hints that the scale symmetry can be broken without break-

In our previous papers we used the German abbreviation SpF for the TrF.
ing of the diffeomorphism symmetry by imposing the diff invariant condition

\[ Trl = \mu, \mu = \text{const} \neq 0 \]  \hspace{1cm} (19)

which also breaks the Weyl symmetry, but preserves the diff symmetry in view of

\[ Trl'(\xi) = e^\lambda Trl(\xi), \quad Trl''(\xi) = e^{-\alpha} Trl(\xi). \]  \hspace{1cm} (20)

The condition (19) introduces the new constant \( \mu \) with the dimension \( [\mu] = [l_{\mu\nu}] = (\text{length})^{-1} \) in addition to \( k_3 \) from (1). So, the condition (19) destroys the Weyl and scale symmetries of \( S_3 \), as well as its discrete \( Z_2 \) symmetry

\[ l'_{\mu\nu}(\xi) = -l_{\mu\nu}(\xi). \]  \hspace{1cm} (21)

Below we show that all these symmetries are spontaneously broken, because the condition (19) realizes the extremal of the potential \( U_3 \).

2. The equation of motion for \( l_{\mu\nu} \) following from \( S_3 \) (1) is

\[ \frac{1}{2} \nabla_\mu \nabla_\nu l^{\mu\nu} = -[\nabla_\mu, \nabla_\nu l^{\mu\nu}] + \frac{\partial U_3}{\partial l_{\mu\nu}}, \]  \hspace{1cm} (22)

whereas the evolution of \( g_{\mu\nu} \) is defined by the Gauss embedding conditions for \( S_3 \)

\[ R_{\mu\nu\alpha\beta}(g) = l_{\mu\alpha} l_{\nu\beta} - l_{\mu\beta} l_{\nu\alpha}. \]  \hspace{1cm} (23)

The latter combined with the Bianchi identities

\[ \{ \nabla_\mu, \nabla_\nu \} l^{\mu\nu} = R^{\mu\nu}_{\lambda\lambda} l^{\lambda\lambda} + R^{\mu\lambda}_{\nu\lambda} l^{\lambda\lambda}. \]  \hspace{1cm} (24)

permits to write the commutator in the r.h.s. of (22) in the form

\[ \frac{1}{2} \{ \nabla_\mu, \nabla_\nu \} l^{\mu\nu} = (l^2)^{\mu\nu} Trl - l^{\mu\nu} Tr(l^2), \]  \hspace{1cm} (25)

where \( Tr(l^2) := l_{\mu\rho} l^{\mu\rho} S^{\mu\nu} \). As a result, Eq. (22) transforms to the PDE for \( l_{\mu\nu} \)

\[ -\frac{1}{4} \nabla_\mu \nabla_\nu l^{\mu\nu} = (l^2)^{\mu\nu} Trl - l^{\mu\nu} Tr(l^2) - \frac{1}{2} \frac{\partial U_3}{\partial l_{\mu\nu}}. \]  \hspace{1cm} (26)

To clarify our approach we consider a simple potential \( U_3 \) with zero phenomenological constants \( b_2 = b_4 = 0 \) changing \( U_3 \rightarrow U \) with \( U \) given by

\[ U = \frac{2}{3} Trl(l^2) - \frac{1}{2} (Tr(l^2))^2. \]  \hspace{1cm} (27)

Then EOM (26) simplifies to the PDE

\[ \frac{1}{2} \nabla_\mu \nabla_\nu l^{\mu\nu} = \frac{2}{3} Tr(l^2) \frac{\partial Trl}{\partial l_{\mu\nu}}, \]  \hspace{1cm} (28)

which is the Euler—Lagrange equation generated by the Weyl and scale invariant action

\[ S = \frac{1}{k_3^2} \int d^4x \sqrt{|g|} \left( \frac{1}{2} \nabla_\mu l_{\nu\rho} l^{\mu\nu} l^{\rho\sigma} - \nabla_\mu l^{\mu\nu} \nabla_\nu l_{\rho\sigma} \right) + \frac{2}{3} Trl(l^3) - \frac{1}{2} \left( Tr(l^2) \right)^2. \]  \hspace{1cm} (29)

An extremal \( l_{\mu\nu}^{\text{opt}} \) of the reduced potential \( U \) (27) is found from the equation

\[ (l_{\mu\nu}^{\text{opt}})^{\mu\rho} Trl_{\rho\sigma} - l_{\mu\nu}^{\text{opt}} Trl_{\sigma\rho} = 0, \]  \hspace{1cm} (30)

which can be written as

\[ l_{\alpha\beta}(l_{\sigma\rho}^{\text{opt}} Trl_{\rho\sigma} - g_{\alpha\beta} Trl_{\sigma\rho}^{\text{opt}}) = 0. \]  \hspace{1cm} (31)

An evident solution of Eq. (31) is \( l_{\sigma\rho}^{\text{Mink}} = 0 \) which corresponds to usual 4-dim. Minkowski vacuum with \( g_{\mu\nu} = \eta_{\mu\nu} \). This vacuum solution breaks the Weyl symmetry simultaneously with the diff symmetry but it preserves the scale symmetry.

More appealing extremal given by a non-degenerate matrix \( l_{\alpha\beta}^{\text{opt}} \equiv g_{\alpha\beta} l_{\rho\sigma}^{\text{opt}} \) is

\[ l_{\mu\nu} = \frac{Trl_{\mu\nu}}{4} g_{\mu\nu}, \quad \det l_{\mu\nu} \neq 0. \]  \hspace{1cm} (32)

The solution (32) yields the relations

\[ (l_{\mu\nu}^{\text{opt}})^{\text{opt}} = \left( \frac{Trl_{\mu\nu}}{4} \right)^n g_{\mu\nu} \rightarrow Trl_{\mu\nu} = 4 \left( \frac{Trl_{\mu\nu}}{4} \right)^n. \]  \hspace{1cm} (33)

The vacuum (32) is infinitely degenerate (similarly to the vacuum of (anti)ferromagnetic [31] defined by its magnetization direction) being a linear representation of the symmetry groups of the action (29). However, degeneracy with respect to the Weyl and scale groups is removed by the requirement for extremal (32) to satisfy EOM (28) which r.h.s. vanishes on (32). This implies vanishing of the l.h.s. of (28) which can be achieved by the condition

\[ \nabla_\mu l_{\alpha\beta}^{\text{opt}} = 0 \rightarrow \nabla_\mu l_{\alpha\beta}^{\text{opt}} = \nabla_\mu Trl_{\alpha\beta} = \partial_\mu Trl_{\alpha\beta}. \]  \hspace{1cm} (34)

The substitution of the extremal \( l_{\mu\nu} \) (32) in (34) results in the condition

\[ \frac{1}{4} \partial_\mu Trl_{\alpha\beta} = \partial_\mu Trl_{\alpha\beta}, \]  \hspace{1cm} (35)

which requires \( Trl_{\alpha\beta} = \text{constant} \). So, we arrive at the earlier discussed condition (19): \( Trl_{\alpha\beta} = \mu \). The latter transforms the extremal (32) into the extremal

\[ l_{\mu\nu} = \frac{\mu}{4} g_{\mu\nu}. \]  \hspace{1cm} (36)

The vacuum (36) spontaneously breaks the Weyl, scale and \( Z_2 \) symmetries of the effective action (29) introducing the mass scale \( \mu \). Noticing that \( Trl \) is invariant under diffeomorphisms one can identify this...
trace with a massless scalar field $\phi$ having a non-zero vev $\langle \phi \rangle_0$

$$\phi := Tr l \rightarrow \phi_0 := \langle \phi \rangle_0 = Tr l_0 = \mu. \quad (37)$$

One can handle $\phi$ as dilaton similarly to the proposal [32], where a scale-invariant model of quadratic gravity, including a scalar field coupled with gravity, was studied.

The extremal (36) of the effective field theory (29) is associated with a 3-brane, because it satisfies the Peterson–Codazzi embedding equation

$$V_{\mu l} l_{\mu p} = 0 \quad (38)$$

(since $V_{\mu g_{\mu \nu}} = 0$) for a hyper-ws with codim 1. Equation (38) supplements Eq. (23) together with which they select the 3-brane sector in the space of solutions of (28). So, the explicit solution (36) permits to restore the vacuum hyper-ws $\Sigma^o_4$ using the Gauss theorem (23)

$$R_{\mu q \nu l} := l_{\mu q} l_{\nu l} - l_{\nu q} l_{\mu l}. \quad (39)$$

Then the substitution of (36) in (39) yields the vacuum Riemann tensor

$$R_{\mu q \nu l} = \left( \frac{\mu}{4} \right)^2 \left( g_{\mu q} g_{\nu l} - g_{\mu l} g_{\nu q} \right), \quad (40)$$

which shows that $\Sigma^o_4$ is the space of constant curvature

$$R_{\mu q \nu l} = \frac{3}{(4)^3} \mu^2 g_{\mu q \nu l}, \quad R_0 := g^{\mu \nu} R_{\mu q \nu l} = \frac{3}{4} \mu^2. \quad (41)$$

From (39) we find the Ricci tensor $R_{\mu q \nu l}$ and the scalar curvature $R_0$ of $\Sigma^o_4$

$$- R_{\nu l \lambda} = (l^q_0)_{\nu l} - \mu l_{\nu l} - R_0 = Sp(l^q_0) - \mu^2,$n$$

$$- R_{\nu l \lambda} = Sp(l^q_0) - \mu l_{\nu l} - R_0 = Sp(l^q_0). \quad (42)$$

Potential $U$ (27) on the extremal $l_{\mu q \nu l}$ (36) takes the form

$$U_0 := U_{|_{l_{\mu q \nu l} = 0}} = \frac{3}{2} \mu^3 - \mu (Tr l_0^2) = \frac{1}{2} (Tr l_0^2). \quad (43)$$

Relations (33) give $U_0 = \mu^4 / 96$. Next we observe that (43) can be written as

$$U_0 = -\frac{1}{2} R_0^2 + \frac{\phi_0^2}{3} R_0 - \frac{2}{3} R_{\nu l \lambda} l^{\nu l} \phi_0 + \frac{\phi_0^4}{6}. \quad (44)$$

Using (44) and vanishing of the kinetic terms in (29) on extremal (36) we obtain

$$S_0 = \frac{1}{k_0^2} \int d^4 x \sqrt{|g_0|}
\times \left\{ -\frac{1}{2} R_0^2 + \frac{\phi_0^2}{3} R_0 - \frac{2}{3} R_{\nu l \lambda} l^{\nu l} \phi_0 + \frac{\phi_0^4}{6} \right\}. \quad (45)$$

This shows that $S_0$ is the first term in the expansion of the quadratic gravity action

$$S = \frac{1}{k_0^2} \int d^4 x \sqrt{|g|} \left[ \frac{1}{2} \nabla \phi \nabla \phi - \nabla \phi \nabla \phi \right]$$

$$\times \left\{ -\frac{1}{2} R_0^2 + \frac{\phi_0^2}{3} R_0 - \frac{2}{3} R_{\nu l \lambda} l^{\nu l} \phi_0 + \frac{\phi_0^4}{6} \right\}, \quad (46)$$

around the minimum of the potential $U$ (27). In (46) we represent $l_{\mu \nu}$ as the sum

$$l_{\mu \nu} = T_{\mu \nu} + \frac{1}{4} g_{\mu \nu}, \quad Tr T = 0, \quad (47)$$

where $T_{\mu \nu}$ is the traceless part of $l_{\mu \nu}$, i.e. $g^{\mu \nu} T_{\mu \nu} = 0$, in correspondence with (37). Thus, the effective scale invariant action (29) of 3-brane realizes the mechanism of spontaneously generated gravity. The resulting Newton constant $G_N \sim \frac{1}{k_0^2 \mu^2}$ is defined by the vev $\phi_0$ equal to the mean curvature $\mu$ of the vacuum hyper-ws $\Sigma^o_4$.

**SUMMARY**

In the braneworld scenario we treat the action (46) as some action of our universe. This gives an example of building new scale-invariant $R^2$ models in which a Brans–Dicke-like scalar $\phi$ is changed by tensors similar to $g_{\mu \nu}$. In new models scalars generalizing $\phi$ will emerge in the form of composite fields such as the diff invariant $Tr l = g_{\mu \nu} l_{\mu \nu}$ with a non-zero vev $Tr l_0$ introducing a new mass scale similar to $\mu$. The fact that scale-invariant models with scalars well describe inflation, reheating and fit the modern observations stimulates their extensions which introduce $l_{\mu \nu}$ for analyzing the current experiments in cosmology.

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**REFERENCES**

1. P. A. R. Ade et al. (Planck Collab.), “Planck 2015 results. XX. Constraint on inflation,” Astron. Astrophys. 594 (2016); arXiv:1502.02114 [astro-ph].

2. J. E. Lidsey, “Inflation and braneworld,” arXiv: astro-ph/0305528 (2003).
3. V. A. Rubakov and M. Shaposhnikov, “Do we live inside a domain wall?,” Phys. Lett. B 125, 136–138 (1983).
4. M. Gel’fand and B. Zweibach, “Dimensional reduction of spacetime induced by nonlinear scalar dynamics and noncompact extra dimensions,” Nucl. Phys. B 260, 5 (1985).
5. G. Nicolai and C. Wetterich, “On the spectrum of Kaluza–Klein theories with non-compact internal spaces,” Phys. Lett. B 150, 347 (1985).
6. M. Gell-Mann and B. Zweibach, “Dimensional reduction of spacetime induced by nonlinear scalar dynamics and noncompact extra dimensions,” Nucl. Phys. B 260, 5 (1985).
7. G. Nicolai and C. Wetterich, “On the spectrum of Kaluza–Klein theories with non-compact internal spaces,” Phys. Lett. B 150, 347 (1985).
8. N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, “The hierarchy problem and new dimensions at a millimeter,” Phys. Lett. B 429, 263–272 (1998).
9. I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, “New dimensions at a millimeter to a Fermi and superstrings at a TeV,” Phys. Lett. B 436, 257–263 (1998).
10. L. Randall and R. Sundrum, “Large mass hierarchy from a small extra dimension,” Phys. Rev. Lett. 83, 3370 (1999).
11. L. Randall and R. Sundrum, “An alternative to compactification,” Phys. Rev. Lett. 83, 4690 (1999).
12. P. Horava and E. Witten, “Heterotic and type I string dynamics from eleven dimensions,” Nucl. Phys. B 460, 506 (1999).
13. G. Dvali and M. Shifman, “Domain walls in strongly coupled theories,” Phys. Lett. B 396, 64 (1997).
14. G. Shiu and S.-H. Tye, “TeV scale superstring and extra dimensions,” Phys. Rev. D: Part. Fields 58, 086007 (1998).
15. R. Sundrum, “Effective field theory for a three-brane universe,” Phys. Rev. D: Part. Fields 59, 085009 (1999).
16. A. Lukas, B. A. Ovrut, K. S. Stelle, and D. Waldram, “Universe as a domain wall,” Phys. Rev. D: Part. Fields 59, 086001 (1999).
17. G. Dvali, G. Gabadadze, and M. Porrati, “4D gravity on a brane in 5D Minkowski space,” Phys. Lett. B 485, 208–214 (2000); arXiv: hep-th/0005016
18. E. Cartan, Riemannian Geometry in an Orthogonal Frame (World Scientific, Singapore, 2001).
19. F. Lund and T. Regge, “Unified approach to strings and vortices with soliton solutions,” Phys. Rev. D: Part. Fields 14, 1524–1535 (1976).
20. R. Omnes, “A new geometric approach to the relativistic string,” Nucl. Phys. B 149, 269–284 (1979).
21. B. M. Barbashov and V. V. Nesterenko, Introduction to the Relativistic String Theory (World Scientific, Singapore, 1990).
22. A. A. Zheltukhin, “Classical relativistic string as an exactly solvable sector of SO(1,1)xSO(2) gauge model,” Phys. Lett. B 116, 147–150 (1982); A. A. Zheltukhin, “Gauge description and nonlinear string equations in d-dimensional space-time,” Theor. Math. Phys. 56, 785–795 (1983).
23. A. A. Zheltukhin, “Gauge theory approach to branes and spontaneous symmetry breaking,” Rev. Math. Phys. 29 (3), 1750009 (2017); A. A. Zheltukhin, “Phenomenological Lagrangians, gauge models and brane,” Phys. Part Lett. 14, 312–317 (2017).
24. A. A. Starobinsky, “A new type of isotropic cosmological models without singularity,” Phys. Lett. B 91, 99102 (1980).
25. L. Smolin, “Towards a theory of space-time structure at very short distances,” Nucl. Phys. B 160, 253–258 (1979).
26. A. Zee, “Spontaneously generated gravity,” Phys. Rev. D: Part. Fields 23, 858–866 (1980).
27. S. Adler, “Order-R vacuum action functional in scalar-free unified theories with spontaneous scale breaking,” Phys. Rev. Lett. 44, 1567 (1980).
28. E. S. Fradkin and A. A. Tseytlin, “Renormalizable asymptotically free quantum theory of gravity,” Phys. Lett. B 104, 377 (1981).
29. B. Zwiebach, “Curvature squared terms and string theories,” Phys. Lett. B 156, 315 (1985).
30. H. Lü, A. Perkins, C. N. Pope, and K. S. Stelle, “Spherically symmetric solutions in higher-derivative gravity,” Phys. Rev. D: Part. Fields 92 (124019) (2015).
31. K. S. Stelle, “Abdus Salam and quadratic curvature gravity: Classical solutions,” Int. J. Mod. Phys. A 32, 1741012 (2017).
32. A. A. Zheltukhin, “Inflation versus collapse in brane matter,” Mod. Phys. Lett. A 32, 1–9 (2017); A. A. Zheltukhin, “p-Branes with AdS_{p+1} vacuum as models of R^p gravity,” Eur. Phys. J. 79, 633 (2019).
33. M. Rinaldy and L. Vanzo, “Inflation and reheating in theories with spontaneous scale invariance symmetry breaking,” Phys. Rev. D: Part. Fields 94, 024009 (2016).