Critical State Model of Sand-Tire Derived Aggregate Mixtures Based on Triaxial Tests

Adolfo Foriero1 and Nima Ghafari1

Abstract
This study is part of an environmental experimental program on the use of scrap automobile tires for geotechnical applications. Different types of laboratory tests were conducted to determine the elastic, plastic, and creep parameters of tire derived aggregate (TDA)-sand granulated mixtures. However, this paper emphasizes the plasticity parameters via the development of a critical state model based on the results of triaxial tests. This was attained by considering loose sand specimens, at a predetermined TDA volumetric content, subject to three different confining pressures under a constant axial displacement rate. The calculated deviatoric stress versus axial strain curves, obtained via the modified Cam Clay model, captured the non-linear elastoplastic response obtained in the tests. Results indicated that the level of the shear strength is highly dependent on critical state friction angle which in turn depends on the TDA content. For the loose TDA-sand mixtures used in the present study, the effect of the TDA content demonstrates a reinforcement of the sand matrix. However this reinforcement diminishes as the TDA content increases.

Environmental as well as economic conditions have given rise to the use of recycled tires as construction materials. There are various reasons for this. One of them is that stockpiles of discarded tires are on the rise and constitute a real fire hazard, not to mention a breeding ground for rodents. Another reason is that natural resources like sand, clay, gravel, and other mineral aggregates are being depleted throughout the world because of the opening of new quarries. Finally, when shredded, tire derived aggregate (TDA) could be mixed with traditional construction aggregate materials. The basic characteristics of such mixtures are that they are lightweight and frictionally resistant precisely as a result of the TDA. Another important property of sand-TDA mixtures is their low thermal conductivity. This constitutes an economic and engineering advantage over traditional materials.

Granular material mixed with TDA is increasingly used in geotechnical applications. In many of these operations, sand is mixed with TDA, at various volumetric ratios, in areas of applications such as highway embankments and bridge abutments (3–5). In northern regions, TDA-granular soil mixtures are also employed in pavement design. They primarily serve as an insulating layer to prevent frost heave and therefore degradation of the road surface (12).

A review of related past studies exposes a lack of experimental as well as numerical investigations on long-term time-dependent constitutive behavior of sand-TDA mixtures. A recent study on sand-TDA mixtures clearly indicates a primary creep phase that rapidly transitioned into a secondary stationary creep phase, never attaining the tertiary phase (8). The magnitude of the creep strain was strongly affected by the TDA volume fraction content (21). This observation conducd the adoption of the Norton-Bailey law as a possible constitutive model for creep of TDA-sand mixtures. Other studies such as Wartman et al. examined the immediate and time-dependent compression of tire chips and shreds (22). The time-dependent deformation of the mixture was also shown to be inversely proportional to the sand content. In another study, the one-dimensional compression of specimens, composed of sand and granulated tire rubber, was investigated (16). There again, results indicated that the time-dependent deformation is significant. However, a complete constitutive model must consider, in addition...
to time-dependent deformations, the shear strength of the sand-TDA mixtures.

In the past decades, most of the laboratory research with regard to shear strength was conducted on sand reinforced with tire chips (generally from 10 to 300 mm in size) (1, 5, 6, 9, 11). The direct shear box test and the triaxial test were the apparatuses mostly used to test these mixtures. The overall research emphasized that the shear strength of sand is increased by the tire chips.

Direct shear box tests by Bosscher et al. and Foose et al. show an increase in shear strength with a rubber content of up to 50% (4, 9). Foose et al. also reported that the failure envelope of sand-rubber mixtures containing dense sand is non-linear (9). Yang et al. also observed a non-linear failure envelope of the shear stress, but in the cases of granulated rubber and not tire chips (23). However, most studies of sand mixed with granulated rubber, using the direct shear box, show inconsistencies in the results. For example, Anbazhagan et al. determined an increase of the shear strength when granulated rubber is mixed with uniform sand (2). However, the shear strength decreased when the granulated rubber was mixed with poorly graded sand. Ghazavi reported an insignificant increase in shear strength of the mix with an increase in granulated rubber (10).

Experimental studies on the shear strength measured in the triaxial apparatus also show considerable inconsistencies in the test results. A study by Noorzad and Raveshi, in which tire crumbs act as reinforcement of a sand-tire mixture, found that the shear strength decreases with increase in the amount of tire crumbs. A similar conclusion was obtained for mixtures of sand and granulated rubber by Youwai and Bergrado and Madhusudhan et al. (14, 24). On the other hand, a study by Venkatappa and Dutta determined some improvement in strength on additions of rubber inclusions (20). Tire chips with an aspect ratio of 2 showed the best improvement for low confining pressures and chip content.

Numerous laboratory investigations have considered sand reinforced with tire chips, whereas few have considered sand-TDA granular mixtures. None have considered a critical state model for interpretation of the test results. This will be attempted in the following.

**Sand-TDA Mixtures of the Triaxial Tests**

For the triaxial tests in this study, the specimens consisted of mixtures of two granular materials. One of these materials is Ottawa sand whose granulometric curve is shown in Figure 1. From the shape of this curve and with calculated values of $C_u = 2.1$ and $C_c = 1.26$, this sand is defined as a poorly graded sand (SP). The other material is TDA obtained at Shercom Industries Inc. in Saskatoon, Saskatchewan, Canada, with calculated values of $C_u = 1.69$ and $C_c = 1.03$. This rubber granular material would also be considered as poorly graded if classified according to the Unified Soil Classification System. A granulometric curve for this material is also shown in Figure 1.

For the purpose of the tests these two materials were combined at three different volumetric ratios: $\theta_p = 0.50, 0.75, 1.0$ where $\theta_p$ represents the volume fraction of TDA per total volume of the mixture. In Figure 2, a phase diagram for the sand-TDA mixture was used to determine the initial void ratios that correspond to the three volumetric ratios mentioned previously. This was possible because both the weights of the TDA and sand phases, as well as the total volume they
occupied were measured beforehand. For the calculations the specific gravity of the TDA ($G_{TDA} = 1.15$) and Ottawa sand ($G_{Sand} = 2.65$) were necessary. The phase volumes of the TDA and sand were obtained, respectively, as $V_{TDA} = \frac{W_{TDA}}{G_{TDA} \gamma_w}$ and $V_{Sand} = \frac{W_{Sand}}{G_{Sand} \gamma_w}$. Consequently, the initial void ratio, $e$, and initial unit weight $\gamma$ of the TDA-sand mixture were known at the start of all tests (Table 1).

**Table 1. Initial Void Ratio and Unit Weight of Ottawa Sand-Tire Derived Aggregate (TDA) Mixture**

| TDA (%) | Void ratio, $e_o$ | Unit weight, $\gamma$, kN/m$^3$ |
|---------|-------------------|-------------------------------|
| 50      | 0.78              | 11.02                         |
| 75      | 0.91              | 8.25                          |
| 100     | 1.05              | 5.5                           |

**Test Equipment and Experimental Procedure**

A standard triaxial apparatus, generally used for soils, was chosen for this study (Figure 3). The sand-TDA specimen measured 35.1 mm in diameter and 67.5 mm in height. A consistent procedure for preparing and testing dry samples of sand-TDA mixtures at a loose state was established. The required percentages of TDA were uniformly mixed with Ottawa sand in a dry condition. The sand was then poured into a rubber membrane inside a split mold former under vacuum. The sand-TDA specimens when sheared were subjected to confining cell pressures of 50, 100, and 150 kPa for each volumetric ratio $\theta_p = 0.50, 0.75, 1.0$. A conventional drained triaxial test displacement rate of 0.0061 mm/min was maintained throughout the tests.

**Test Results and Observations**

A total of nine triaxial tests on sand-TDA dry granular mixtures (at the three volumetric ratios and three confining pressures) were completed in the present study. Typical stress-strain curves for the cases pertaining to $\theta_p = 0.50$ and 0.75 are exhibited in Figures 4 and 5. Results show, as would be expected, that for a constant TDA volumetric ratio, the deviatoric stress increases with the confining stress. However, from these two figures, an increase in the TDA volumetric content tends to lower the shear strength of the sand-TDA mixture. Moreover, both Figures 4 and 5 depict a strain hardening behavior. This was confirmed by the barrel shape of the specimen at the end of the test (Figure 6) which occurred at an axial strain of approximately 23%.

An interesting result for the triaxial tests with pure TDA particles $\theta_p = 1.0$ is given in Figure 7. Here again, the deviatoric stress increases with the confining stress.
but for the lower confining stresses this increase is minimal. The stress-strain behavior in this instance and for practical purposes appears to be linear.

In all of these tests, the value of the failure strain must be defined. At approximately 23% of the axial strain, the sand-TDA specimen is severely distorted, as previously mentioned (Figure 6). Therefore, the critical state, which is reached when no further changes in shear stress and volume occur under continuous shearing, cannot be attained. Several studies on triaxial testing have defined failure based on a predetermined strain in the range of 10%–20% (17, 23). Thus, in this study, four different values of the axial strain ($e_a = 10\%, 15\%, 20\%, 23\%$) were considered (Figures 8–10) to examine the effect of the TDA content on the presumable deviatoric stress at failure. As seen in these figures, for a constant confining pressure, an increase in the TDA content lowers the deviatoric stress. Again, the deviatoric stress increases with the confining cell pressure at one particular TDA volumetric ratio.

The previous stress-strain curves (Figures 4, 5, and 7) showed a reduction in the stiffness of the sand-TDA mixtures as the TDA content increased. An average representation of stiffness for non-linear behavior is through the secant modulus. Figures 11–13 exhibit the secant modulus at the vertical failure strains previously cited. As confirmed by the previous results, the trend is analogous. The secant moduli decrease with an increase in TDA and a decrease of the confining pressure.

It is clear from the previous experimental results that the TDA volumetric ratio plays an important role as far as the behavior of the mixture is concerned. A constitutive equation is therefore needed that accounts for this. Consequently, the next section of this paper is devoted to the development of a simple framework capable of describing, interpreting, and anticipating the sand-TDA mixture response to loading.

**Figure 5.** Test results of deviatoric stress versus axial strain at 75% tire derived aggregate (TDA).

**Figure 6.** Sand-tire derived aggregate (TDA) mixture at the end of the triaxial test.

**Figure 7.** Test results of deviatoric stress versus axial strain at 100% tire derived aggregate (TDA).
A Critical State Model to Interpret TDA-Sand Mixtures

As a preliminary attempt in understanding TDA-sand mixtures, it is tacitly assumed that such mixtures are quasi-single-phased. This means that deformation causes no change, or negligible change, in the phase ratio per unit volume. In this particular case, it refers to the sand-TDA phase ratio, implying that the TDA does not flow out of the sand matrix. Consequently, the term effective stress shall be indiscriminately used from hereon.

The fundamental concept of a unique failure surface is adopted in the present approach. Moreover, the terms failure and critical state are taken as synonymous. From hereon, the failure line will be referred to the critical state line (CSL) (19). The yield surface is an ellipse whose equation in \((p_0, q, e)\) space is

\[
(p_0)^2/C_0 + q^2/M(u)^2 = 0 \tag{1}
\]

and is a function of the TDA volumetric content through the friction constant \(M\). The other variables \(p', q, p_0\) are, respectively, the effective mean stress, the maximum past effective mean stress, and the deviatoric stress. In geotechnical engineering this surface is better known as the modified Cam Clay (MCC) yield surface (18).

The Mohr–Coulomb failure criterion is written in relation to the previously mentioned stress invariants as

\[
q_f = M(0)p_f' \tag{2}
\]

where \(p'_f, q_f\) are, respectively, the effective mean stress and the deviatoric stress at failure. Since the triaxial testing program in the present study dealt with axisymmetric compression only, the expression for the friction parameter is specialized to

\[
M(0) = M(\theta) = \frac{6 \sin(\phi'_{cs}(\theta))}{3 - \sin(\phi'_{cs}(\theta))} \tag{3}
\]

where the critical state friction angle \(\phi'_{cs}\) is back-calculated from the present triaxial tests with

**Figure 8.** Deviatoric stress versus axial strain at failure for \(c_3 = 50\) kPa.

**Figure 9.** Deviatoric stress versus axial strain at failure for \(c_3 = 100\) kPa.

**Figure 10.** Deviatoric stress versus axial strain at failure for \(c_3 = 150\) kPa.
The equation for the CSL in $(e - p')$ space is represented by

$$
e_f = e_T - \lambda \ln(p')$$

where $e_f, e_T, \lambda, p'$ are, respectively, the void ratio at failure, the void ratio on the CSL when $\ln(p') = 1$, the slope of the CSL, and effective mean stress at failure. Then, $e_T$ is determined from the initial state of the soil with

$$
es_T = e_o + (\lambda - \kappa) \ln\left(\frac{p'}{2}\right) + \kappa \ln(p_o)$$  \hspace{1cm} (6)

where $\kappa$ is the unloading/reloading index or the recompression index.

To calculate the elastic response, the elastic modulus $E'$ is required. This modulus is obtained from the triaxial tests. This modulus can be approximated by a secant modulus (Figures 11–13) over the stress increment of interest. However, an estimate of this modulus is also possible using the critical state model through the bulk modulus. The bulk modulus is calculated with

$$K' = \frac{p'(1 + e_o)}{\kappa}$$  \hspace{1cm} (7)

And, since $E' = 3K'(1 - 2\mu')$, it is obtained that

$$E' = \frac{3p'(1 + e_o)(1 - 2\mu')}{\kappa}.$$  \hspace{1cm} (8)

Consequently, the elastic shear modulus, $G$, is also estimated as

$$G = \frac{3p'(1 + e_o)(1 - 2\mu')}{2\kappa(1 + \mu')}.$$  \hspace{1cm} (9)

Equations 7 and 8 indicate a non-linear elastic behavior because both are functionally dependent on the mean effective stress. Consequently, calculations must be carried out incrementally.

The total volumetric strain is the sum of the elastic and plastic volumetric strain and is written as

$$e_T = e_o + (\lambda - \kappa) \ln\left(\frac{p'}{2}\right) + \kappa \ln(p_o)$$  \hspace{1cm} (6)
where the superscripts $e$ and $p$ indicate the elastic and plastic components. If the soil yields at a void ratio of $e_1$ and a small increment of stress causes the yield surface to expand to a void ratio of $e_2$, then the corresponding total change in volumetric strain is given by

$$
\Delta e_v = \frac{\Delta e}{1 + e_o} = \frac{|e_2 - e_1|}{1 + e_o} = \frac{\lambda}{1 + e_o} \ln \left( \frac{p'_2}{p'_1} \right)
$$  

where $p'_1, p'_2$ are the applied mean effective stresses producing the overall stress increment. For the same stress increment the volumetric elastic strain increment is calculated with (7) as

$$
\Delta e_v^e = \frac{p'_2 - p'_1}{K}
$$  

or via

$$
\Delta e_v^e = \frac{\kappa}{1 + e_o} \ln \left( \frac{p'_2}{p'_1} \right)
$$

by considering the unloading/reloading line associated with the maximum mean effective stress for the yield surface on which unloading initiates. The change in volumetric plastic strain is now calculated as $\Delta e_v^p = \Delta e_v - \Delta e_v^e$ and is expressed as

$$
\Delta e_v^p = \left( \frac{\kappa - \lambda}{1 + e_o} \right) \ln \left( \frac{p'_2}{p'_1} \right).
$$

The shear strains are calculated from the representation of the yield surface. Furthermore, for the purposes of the present study, these strains are calculated by assuming that the plastic potential function and the yield function are the same. In other words, a normality condition is assumed. The resulting plastic deformation produces a volumetric and a deviatoric plastic strain component. The volumetric plastic deformation was given in (14), the deviatoric component of the volumetric plastic strain is obtained as

$$
d e_s^p = d e_s^p \frac{q}{(M(0))^2 (p' - p'_0)}
$$

by considering the normal to the yield surface. Finally, the elastic deviatoric strains are obtained with

$$
\Delta e_s^e = \frac{\Delta q}{3G}.
$$

**Calculation Procedure for the Stress-Strain Response of the Sand-TDA Mixture**

From the equations of the last section, it is possible to determine the stress-strain response and the volume changes from the initial stress state. The required parameters are $p'_o, e_o, p'_c, \lambda, \kappa, \theta, \phi_c(\theta)$, and $\mu'$. The procedure used in this study to simulate the stress-strain response of the present triaxial tests results is as follows: (a) determine the TDA volumetric content $\theta$ of the sand-TDA mixture; (b) determine the mean effective stress and the deviatoric stress at initial yield by finding the coordinates of the initial yield surface with the effective stress path; this is achieved numerically by finding the root of the resulting coupled Equation 1 and $p' = p'_o + \frac{q}{\lambda}$ of the particular test; (c) determine the mean effective stress and deviatoric stress at failure with $p'_f = \frac{3p'_f}{M(0)}$ and $q'_f = \frac{M(0)q'_f}{3-M(0)}$; (d) calculate the non-linear elastic modulus $G$ using (9); (e) calculate the initial elastic volumetric strain with (13) and initial elastic deviatoric strain using (16); (f) discretize the stress path from the initial stress point to the failure point into several equal sufficiently small stress increments; (g) determine the major axis of the ellipse, using the current mean effective stress, for each stress increment via (1) with $p'_c = p' + \frac{q^2}{(M(0))^2 p'}$; (h) calculate the volumetric strain increment for each stress increment using (11); (i) calculate the plastic volumetric strain for each stress increment using (14); (j) calculate the plastic deviatoric strain increment for each stress increment using (15); (k) calculate the elastic deviatoric strain increment for each stress increment using (16); (l) sum the elastic and plastic deviatoric strains increments to give the total deviatoric strain increment; (m) sum the total volumetric strain increments; (n) calculate $e_o = e_o + \Delta e_v, e'_o = \frac{e'_o}{\lambda} + p', \text{ and } e'_o = p' - \frac{q}{\lambda}$.

This procedure was coded and carried out numerically with the MATLAB software (7).

**Comparison of the Critical State Model with the Triaxial Test Results**

The TDA-sand mixture parameters $p'_o, e_o, p'_c, \theta$, and $\phi_c(\theta)$ were known for all of the triaxial tests carried out in this study. The parameters $\lambda, \kappa, \text{ and } \mu' \text{ were estimated.}$

In general, for soils $\frac{1}{6} < \frac{\kappa}{\lambda} < \frac{1}{3}$ and $0.1 < \mu' < 0.4$ (19). For the simulations, a ratio of $\frac{\kappa}{\lambda} = 0.15$ and a value of $\mu' = 0.3$ were taken. Numerous simulations showed that the level-value of the shear strength at the critical state was insensitive to the values of these parameters. It was also observed that these values affect mostly the initial portion of the stress-strain curve; the reason being that the critical state shear strength, obtained with the MCC model, is strongly affected by the critical state friction
angle $\phi_0^f$ through the Mohr–Coulomb relation $q_f = M(\theta)\phi_0^f$. On the other hand, the yield surface, represented by the elliptic locus, is responsible for the elastic wall at a particular void ratio thus affecting the stress-strain curve mostly before failure. Consequently, emphasis will be placed from hereon on the shear strength of sand-TDA mixtures.

As previously mentioned, in this study the failure state is synonymous with the critical state. Moreover, it is reiterated that establishment of the failure state of a sand-TDA mixture depends on a chosen strain criterion (17, 23). To determine a value for the critical state friction angle, four different values of the axial strain ($\varepsilon_a = 10\%, 15\%, 20\%, 23\%$) were considered at three different TDA contents. Figures 14–17 show typical determinations of the value of a mobilized friction angle $\phi_{mob}^f$ based on the triaxial test curves of Figures 5 and 6. These values were obtained, for a particular TDA content, by linear regression based on the picked-off $(p_f, q_f)$ pair at the four previously stated axial strains.

The critical state friction angle is determined from a plot of the mobilized friction angles versus the axial strains at which these mobilized values were obtained (Figures 18 and 19). As previously mentioned, the critical state was difficult to attain because of the excessive deformations achieved during the tests (Figure 6). The critical state friction angle is unique and generally obtained when the stress-strain curve attains a limiting value of the shear stress. Figures 18 and 19 show that a critical state friction angle is asymptotically approached at very high axial strains, where the value of the mobilized friction angles begin to level off. This leveling off of the mobilized friction angle depends on the TDA content. For a TDA volumetric ratio of $\theta = 50\%$ (Figure 18) a critical state friction angle greater than $38^\circ$ is realistic since this value continues to grow. On the other hand, Figure 19, for the case $\theta = 75\%$, shows that a value of $34^\circ$ could be taken as a lower bound. Consequently, in the simulations that follow, the critical state friction angle is bracketed to capture the measured shear strength of the triaxial tests.
The MCC yield surface, as well as the CSL, for Ottawa sand and a typical Ottawa sand-TDA mixture at \( \theta = 50\% \), are shown in Figure 20. It is clear from this figure that the TDA reinforces the sand matrix because the yield surface is expanded and the CSL assumes a greater inclination.

In all of the following simulations a sand-TDA sample is sheared at its current mean effective stress, \( p'_{0} \), by increasing the axial stress, while keeping the cell pressure \( \sigma_3 \) constant. The imposed effective stress path has a slope of \( \frac{q}{p'} = 3 \). The load is incremented along the effective stress path until the sand-TDA mixture fails.

Figures 21–24 show the results of four triaxial test simulations using the MCC. These simulations were conducted at confining cell pressures of 100 and 150 kPa and TDA contents of 50% and 75%.

Figures 21–24 depict, for each of the four tests, the calculated deviatoric stress versus axial strain, the void ratio versus axial strain, and the deviatoric strain versus the volumetric strain in comparison with the test results. The initial yield surface expands and the stress-strain response is a curved path because the sand-TDA mixture behaves elastoplastic. As previously stated, the level of the shear strength is highly dependent on critical state friction angle which in turn depends on the TDA content. In all of these cases, the deviatoric stresses obtained in the tests are bracketed by the theoretical curves generated with the MCC.

In all simulations, the curves of the void ratio versus axial strain (Figures 21–24) show an overall compression of the specimen, the initial void ratio attenuating toward a critical void ratio. Again, this behavior conforms with that of loose granular materials (15). The resulting barrel-shaped specimen (Figure 6) at the end of the test is a manifestation of this type of behavior.

Finally, the curves of the deviatoric strain versus volumetric strain confirm that distortion increases with the volumetric strain, the curve assuming a non-linear shape with no peaks. In other words, no dilation is predicted by the MCC because all of the sand-TDA mixtures were in a loose state at the start of the test.

Overall, the critical state model is satisfactory if it is wished to predict the overall magnitude of the shear

---

**Figure 17.** Determination of the frictional constant \( M \) at a tire derived aggregate (TDA) of 75% and \( \varepsilon_a = 23\% \).

**Figure 18.** Mobilized friction angle versus axial strain at tire derived aggregate (TDA) = 50%.

**Figure 19.** Mobilized friction angle versus axial strain at tire derived aggregate (TDA) = 75%.
strength of initially loose sand-TDA mixtures. The model is highly dependent on the critical state friction angle, \( \phi_{cs}^f \), which in turn depends on the TDA volumetric content. For the loose TDA-sand mixtures used in the present study, the effect of the TDA content demonstrates a reinforcement of the sand matrix. However, this reinforcement diminishes as the TDA content increases.

Finally, triaxial tests on dense sand-TDA specimens are warranted because such samples will sustain higher stresses than loose samples. Moreover, it is expected that there will be a peak value of the deviatoric stress at low axial strains followed by a decrease with increasing strain, attaining a critical deviatoric stress. The determination of the critical friction angle in such tests will be evident provided the critical state is attainable. Moreover, since it is well known in the literature that the MCC model performs less well when dilation is present, a new constitutive model capable of capturing this effect will be required (13).

**Conclusions**

Based on the triaxial test results obtained in this study, the following conclusions with regard to granular TDA arose. In general, TDA content reinforces a sand matrix when compared with the original sand. For loose sand-TDA mixtures, results show that, for a constant TDA volumetric ratio, the deviatoric stress increases with the confining stress. However, an increase in the TDA

Figure 20. Modified Cam Clay (MCC) model in \( p-q \) space for Ottawa sand. 
*Note:* CSL = critical state line; TDA = tire derived aggregate.

Figure 21. Deviatoric stress versus axial strain, void ratio versus axial strain, and deviatoric strain versus volumetric strain at 50% tire derived aggregate (TDA) and \( \sigma_3 = 100 \) kPa. 
*Note:* MCC = modified Cam Clay.
Figure 22. Deviatoric stress versus axial strain, void ratio versus axial strain, and deviatoric strain versus volumetric strain at 50% tire derived aggregate (TDA) and \( \sigma_3 = 150 \text{ kPa} \).
Note: MCC = modified Cam Clay.

Figure 23. Deviatoric stress versus axial strain, void ratio versus axial strain, and deviatoric strain versus volumetric strain at 75% tire derived aggregate (TDA) and \( \sigma_3 = 100 \text{ kPa} \).
Note: MCC = modified Cam Clay.
volumetric content tends to lower the shear strength of the sand-TDA mixture. A strain hardening behavior was confirmed by the stress-strain curves, and by the barrel shape of the specimen at the end of the test which occurred at an axial strain of approximately 23%. Consequently, the critical state is difficult to attain in such cases.

Test results also showed a reduction in the stiffness of the sand-TDA mixtures as the TDA content increased. In particular, the secant moduli decreased with an increase in TDA and a decrease of the confining pressure. It is clear from the previous experimental results that the TDA volumetric ratio plays an important role as far as the behavior of a mixture is concerned. A constitutive equation based on a critical state model—MCC—is satisfactory in predicting the level of the shear strength offered by a sand-TDA mixture. The calculated deviatoric stress versus axial strain curves, obtained via the model, captured the non-linear elastoplastic response obtained in the tests. The level of the shear strength is highly dependent on critical state friction angle which in turn depends on the TDA content. In all of these cases, the deviatoric stresses obtained in the tests are bracketed by the theoretical curves generated with the MCC.

In all simulations, the curves of the void ratio versus axial strain show an overall compression of the specimen, the initial void ratio attenuating toward a critical void ratio. Again, this behavior conforms with that of loose granular materials.

Finally, the curves of the deviatoric strain versus volumetric strain confirm that distortion increases with the volumetric strain. The curves assume a non-linear shape with no peaks. In other words, no dilation is predicted by the MCC because all of the sand-TDA mixtures were in a loose state at the start of the test.

**List of Symbols**

- \( e_0 \) = Initial void ratio
- \( e_f \) = Void ratio at failure
- \( e_T \) = Void ratio on the CSL when \( \ln(p_f) = 1 \)
- \( e_a \) = Total axial strain
- \( e_e \) = Total deviatoric strain
- \( e_{e}^e \) = Elastic deviatoric strain
- \( e_{e}^p \) = Plastic deviatoric strain
- \( e_v \) = Total volumetric strain
- \( e_{v}^e \) = Elastic volumetric strain
- \( e_{v}^p \) = Plastic volumetric strain
- \( \phi_{cs} \) = Friction angle at the critical state
- \( \gamma \) = Unit weight having units \( \frac{kN}{m^2} \)
- \( \kappa \) = Unloading/reloading index
- \( p_e \) = Mean effective stress in (kPa)
- \( p_{e0} \) = Initial mean effective stress in (kPa)
- \( p_{eY} \) = The mean effective stress at yield in (kPa)
Acknowledgments

The authors acknowledge and thank Shercom Industries Inc. in Saskatoon, Saskatchewan, Canada, for their provision of the TDA used in the laboratory tests. Finally, the authors also acknowledge the technical support of Christian Juncault (Technicien en travaux d’enseignement et de recherche) of Département de génie civil et de génie des eaux, Laval University.

Author Contributions

The authors confirm contribution to the paper as follows: study conception and design: A. Foriero, N. Ghafari; data collection: N. Ghafari, A. Foriero; analysis and interpretation of results: A. Foriero, N. Ghafari; draft manuscript preparation: A. Foriero, N. Ghafari; conception and design: A. Foriero, N. Ghafari; data collection: A. Foriero, N. Ghafari; approval of the final version of the manuscript: A. Foriero, N. Ghafari. All authors reviewed the results and approved the final version of the manuscript.

Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

References

1. Apex Companies. Tarrtown Bridge Project Case Study Guidance Document for Tire Derived Aggregate Embankment Construction and Design Strategic Recycling Contract No. 355R06 Pollution Prevention Section. Equad, LLC 269 Great Valley Parkway Malvern, Pennsylvania, 2008.
2. Bosscher, P. J., T. B. Edil, and N. Eldin. Construction and Performance of Shredded Waste Tire Test Embankment. Transportation Research Record, 1992. 1345: 44–52.
3. Bosscher, P., T. Edil, and S. Kuraoka. Design of Highway Embankments Using Tire Chips. Journal of Geotechnical and Geoenvironmental Engineering, Vol. 123, No. 4, 1997, pp. 295–304.
4. Humphrey, D. N., and R. A. Eaton. Field Performance of Tire Shreds as Subgrade Insulation for Rural Roads. Proc., 6th International Conference on Low-Volume Roads, Transportation Research Board, Vol. 2, 1995, pp. 77–86.
5. Foriero, A., and N. Ghafari. Laboratory Creep Parameter Determination of Sand-TDA Mixtures and Subsequent FEM Validation. Indian Geotechnical Journal, Vol. 50, 2020, pp. 710–725.
6. Ward, J. I., and J. Sweeney. An Introduction to the Mechanical Properties of Solid Polymers. 2nd ed. John Wiley and Sons, New York, NY, 2004.
7. Wartman, J., M. Natale, and P. Strenk. Immediate and Time-Dependent Compression of Tire Derived Aggregate. Journal of Geotechnical and Geoenvironmental Engineering, Vol. 133, No. 3, 2007, pp. 245–256.
8. Ngo, A., and J. Valdez. Creep of Sand-Rubber Mixtures. Journal of Materials in Civil Engineering, Vol. 19, No. 12, 2007, pp. 1101–1105.
9. AbdelRazek, A., R. M. El-Sherbiny, and H. A. Lofti. Mechanical Properties and Time-Dependent Behaviour of Sand-Granulated Rubber Mixtures. Geomechanics and Geoenvironmenting: An International Journal, Vol. 13, No. 4, 2018, pp. 1–13.
10. Edil, T. B., and P. J. Bosscher. Engineering Properties of Tire Chips and Soil Mixtures. Geotechnical Testing Journal, Vol. 14, No. 4, 1994, pp. 453–464.
11. Foose, G. J., C. H. Benson, and P. J. Bosscher. Sand Reinforced with Shredded Waste Tires. Journal of Geotechnical Engineering, Vol. 122, No. 9, 1996, pp. 760–767.
12. Ghasavi, M., and M. AmelSakhi. Influence of Optimized Tire Shreds on Shear Strength Parameters of Sand. International Journal of Geomechanics, Vol. 5, 2005, pp. 58–65.
13. Yang, S., R. A. Lohnes, and B. H. Kjartanson. Mechanical Properties of Shredded Tires. Geotechnical Testing Journal, Vol. 25, No. 1, 2002, pp. 44–52.
14. Anbazhagan, P., D. R. Monobar, and D. Rohit. Influence of Size of Granulated Rubber and Tyre Chips on the Shear Strength Characteristics of Sand-Rubber Mix. Geomechanics and Geoenvironmenting, Vol. 12, No. 4, 2017, pp. 266–278.
15. Ghasavi, M. Shear Strength Characteristics of Sand Mixed With Granular Rubber. Journal of Geotechnical and Geological Engineering, Vol. 22, 2004, pp. 401–416.
16. Madhusudhan, B. R., A. Boominathan, and S. Banerjee. Static and Large-Strain Dynamic Properties of Sand-Rubber Tire Shred Mixtures. Journal of Materials in Civil Engineering, Vol. 29, No. 10, 2017, p. 04017165.
17. Youwai, S., and D. T. Bergrado. Strength and Deformation Characteristics of Shredded Rubber Tire-Sand Mixtures. Canadian Geotechnical Journal, Vol. 40, No. 1, 2003, pp. 254–264.
18. Venkatappa, R., and R. K. Dutta. Compressibility and Stress Behaviour of Sand-Tyre Chip Mixture: Technical Note. Geotechnical and Geological Engineering, Vol. 24, 2006, pp. 711–724.
19. Foriero, A. “Exposé MATLAB”, Faculté des sciences et génies, Informatique pour l’ingénieur, 2019, Course Notes, Laval University, pp. 1–323.
20. Noorzad, R., and M. Raveshi. Mechanical Behaviour of Waste Tire Crumbs-Sand Mixtures Determined by Triaxial Tests. Geotechnical and Geoenvironmental Engineering, Vol. 35, 2017, pp. 1793–1802.
21. Schofield, A., and C. P. Wroth. Critical State Soil Mechanics. McGraw-Hill, London, 1968.
22. Roscoe, K. H., and J. B. Burland. On the Generalized Stress-Strain Behaviour of Wet Clay. In *Engineering Plasticity* (J. Heyman and F. Leckie, eds.), Cambridge University Press, Cambridge, 1968, pp. 535–609.

23. Mitchel, J., and K. Soga. *Fundamental of Soil Behaviour*. 3rd ed., Chap. 12. Wiley, Hoboken, NJ, 2005, pp. 465–521.

24. Jefferies, M. G., and K. Been. *Soil Liquefaction: A Critical State Approach*. Taylor & Francis, London and New York, 2006.