I discuss recent theoretical results for inclusive decays of quarkonia into a photon plus hadrons, and summarize the status of perturbative calculations.

Radiative decays of heavy quark bound states have been investigated in the framework of perturbative QCD and have been used to measure the QCD coupling at scales of the order of the heavy quark mass (for recent studies see Refs. [1-3], and references therein). Away from the boundaries of the phase space, the inclusive spectra are expected to be well described by first or second order perturbation theory. Near the phase space boundaries, effects associated with non-perturbative contributions and with high orders in perturbation theory are expected to become important. In this paper I focus on two of these effects. First, I describe the role of the fragmentation of gluons and light quarks into a photon. This is relevant to the shape of the inclusive photon spectrum at small $z$, where $z = 2 E_\gamma / M$ is the energy fraction carried by the photon, with $M$ being the quarkonium mass. Second, I consider the opposite end of the spectrum, $z \to 1$. I discuss the resummation of soft gluons in this region and its physical implications.

Consider the decay of a $^3S_1$ quarkonium state into a prompt photon plus hadrons:

$$X_{QQ} \to \gamma + \text{hadrons}$$

(1)

It was pointed out in Ref. [4] that this reaction receives contributions at leading logarithmic order (LO) both when the photon is coupled to highly virtual processes (“direct term”) and also when the photon is produced by collinear emission from light quarks (“fragmentation term”). As a consequence of the factorization theorem for collinear mass singularities, the decay width $d\Gamma/dz$ has the structure

$$\frac{d\Gamma}{dz}(z, M) = C_\gamma (z, \alpha_s(M)) + \sum_{a = q, \bar{q}, g} \int_z^1 \frac{dx}{x} C_a (x, \alpha_s(M)) \, D_{a\gamma} \left( \frac{z}{x}, M \right).$$

(2)

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The photon spectrum in $\Upsilon$ decay to leading order (LO). The solid curve is the full LO result. The dashed curve is the contribution from the direct term, while the dot-dashed curve is the contribution from the fragmentation term. We use units $4\pi\psi_0^2/M^2 = 1$, with $\psi_0$ being the bound state wave function at the origin, and $\alpha_s = 0.2$.

The first term in the right hand side denotes the direct contribution, while the second term denotes the fragmentation contribution. The functions $C_A$, with $A = \gamma, q, \bar{q}, g$, are short-distance coefficient functions, calculable as power series expansions in $\alpha_s$:

$$C_A = C_A^{(0)} + \alpha_s C_A^{(1)} + \cdots.$$  

(3)

The functions $D_{a\gamma}$ are the fragmentation functions for $a \rightarrow \gamma + X$, satisfying evolution equations of Altarelli-Parisi type. Through these functions, the process is sensitive to long-distance physics. In general, both the $C$’s and $D$’s depend on a factorization scale $\mu$. The separation between direct and fragmentation components depends on the choice of this scale. For simplicity, in Eq. (2) the factorization scale $\mu$ and the renormalization scale that appears in the running coupling have been set equal to $M$.

The direct term to LO, $C_{\gamma}^{(0)}$, stemming from the decay $X_{Q\bar{Q}} \rightarrow \gamma gg$, was calculated in Ref. [5]. It is shown by the dashed curve in Fig. 1 for the case of $\Upsilon(1S)$. The fragmentation terms to LO, $C_{a}^{(0)} \otimes D_{a\gamma}$, were calculated in
To this order, the fragmentation contribution comes entirely from the
 gluon channel ($a = g$), as coefficients in the quark channels ($a = q, \bar{q}$) vanish.
This contribution is shown by the dot-dashed curve in Fig. 1, where we use
the set of fragmentation functions given in Ref. [6]. The solid curve in Fig. 1
is given by the sum of the direct and fragmentation terms, and provides the
full LO result.

We see that the fragmentation component is suppressed with respect to
the direct component if $z$ is sufficiently large. As $z$ decreases, fragmentation
becomes important. In particular, to this order in perturbation theory one
finds the behavior in $1/z$ for $z \to 0$ characteristic of the soft bremsstrahlung
spectrum.

Fig. 1 suggests that, if one wishes to analyze the inclusive photon data
in $\Upsilon$ decay by using theoretical formulas based on direct production only, one
should restrict oneself to the region above some minimum value $z_{\text{min}}$. However,
the question of how big this value should be is also influenced by the size
of the next-to-leading terms. I will comment on this below. Alternatively,
and more interestingly, Fig. 1 suggests that, if the experimental errors on the
measurement of the spectrum at relatively low $z$ were reduced with respect to
their present very large values, one could use this process to perform a test of
the full QCD prediction, and to learn about gluon fragmentation into photons.

The theory has not been fully developed to the next-to-leading order
(NLO) yet. The NLO coefficients $C_A^{(1)}$ are not known at present. Only the
result of a calculation for the integral $\int_0^1 dz C_A^{(1)}(z)$ is available. Besides, there
have been studies of $C_\gamma(z)$ based on modeling certain classes of higher order
corrections. A calculation of $C_\gamma^{(1)}(z)$ is in progress. The full calculation
of the coefficients $C_A^{(1)}$ would be important. It could be combined with the
results of the next-to-leading evolution of fragmentation functions to give a
consistent NLO treatment of radiative $\Upsilon$ decays. In particular, note that in
NLO quark fragmentation starts to contribute. Quark fragmentation functions
are much harder than gluon fragmentation functions. As a result, the value of
the cut $z_{\text{min}}$ introduced above might be pushed significantly to the right with
respect to the value that can be read from Fig. 1.

The second aspect that I discuss in this paper concerns the region near the
endpoint $z = 1$ of the spectrum. This region is influenced by non-perturbative
QCD effects. The shape of the spectrum is expected to become very sensitive
to hadronization corrections as $z \to 1$. A way of dealing with such effects
was proposed in the Monte Carlo study of Ref. [8]. This calculation uses
a parton-shower description for the gluon radiation in higher orders, and an
independent-fragmentation model for hadronization. A different approach may
be found in Refs. [2, 12], based on parametrizing non-perturbative contributions in terms of an effective gluon mass \[^{13}\].

Further sensitivity to nonperturbative parameters may enter through higher Fock states in the quarkonium. The role of these states has been studied recently \[^{14}\]. Away from \( z = 1 \), the color-singlet Fock state dominates the decay, as color-octet contributions are suppressed by powers of the relative velocity \( v \) of the quark pair in the non-relativistic expansion. Near \( z = 1 \), however, this power counting may be overcome due to the form of the short-distance coefficient associated with the color-octet component. This coefficient formally starts with a \( \delta \)-function distribution in lowest order. The analysis of the hadronic smearing in Ref. [14] indicates that color-octet terms may become important within a range \( \Delta z \sim v^2 \) from the endpoint.

A crucial issue for the understanding of the endpoint spectrum is the behavior of high-order terms in perturbation theory as \( z \to 1 \). It is known that potentially large contributions in \( \ln(1 - z) \) may appear to all orders in \( \alpha_s \), associated with the unbalance between real and virtual emission of soft gluons near the phase space boundary. The problem then arises of summing these logarithmically enhanced terms. In particular, it is important to see whether this summation gives rise to a Sudakov damping factor of the form \( \sim \exp(-\alpha_s \ln^2(1 - z)) \) in the decay width.

This question is addressed in a perturbative analysis now in progress \[^{15}\]. This analysis shows that, in the color-singlet channel, the logarithms of \( (1 - z) \) cancel at each order in \( \alpha_s \). As a consequence, no Sudakov factor arises. In contrast, the color-octet channel does have a Sudakov suppression.

The reason for this behavior lies in the properties of coherence of color radiation. Qualitatively, the cancellation mechanism can be understood from an illustration at one-loop level. Consider the amplitude for the Born process in the singlet case, \( \Upsilon(P) \rightarrow \gamma(k) + g(k_1) + g(k_2) \), and consider the emission of an additional soft gluon from this amplitude. In the leading infrared approximation, the decay width has the factorized structure

\[
\frac{d\Gamma}{dz} \sim J^2 \left( \frac{d\Gamma}{dz} \right)_{\text{Born}} d\Phi \,, \quad (4)
\]

where \( J \) is the eikonal current for soft gluon emission, and \( d\Phi \) is the associated phase space. The standard power counting in terms of the soft gluon energy \( \omega \) gives

\[
d\Phi \sim \omega d\omega d\Omega \,, \quad J^2 \sim \frac{1}{\omega^2} \frac{k_1 \cdot k_2}{f(z; \text{angles})} \,, \quad (5)
\]

where \( d\Omega \) is the angular phase space, and \( f \) is a function of \( z \) and the angles
as $\omega \to 0$. Up to the first order in $\omega$ the gluon correlation is

$$k_1 \cdot k_2 \sim \frac{M^2}{(1 - z) + \frac{\omega}{M} g(z; \text{angles})}.$$  \hspace{1cm} (6)

That is, as $z \to 1$ the photon recoils against two almost-collinear hard gluons. In this configuration, the logarithmic integration $d\omega / \omega$ in Eq. (5) is canceled. One can show that this mechanism generalizes to all orders in $\alpha_s$.

An interesting spin-off of this analysis is that the coherence scale for the QCD radiation associated with the decay is not the heavy quarkonium mass $M^2$ but rather the final-state invariant mass $(k_1 + k_2)^2$. Correspondingly, destructive interference occurs outside a cone with opening $\theta^2 \propto 1 - z$. We note that, in contrast, the assumptions underlying the Monte Carlo model of Ref. [8] amount to fixing the coherence scale to be $M^2$, and $\theta^2 \sim 1$. This model therefore does not take account of the gluon radiation in higher orders correctly. It would be of interest to see how a Monte Carlo model in which coherence is correctly implemented compares with the inclusive photon data.

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