Bar Fraction in Early- and Late-type Spirals

Yun Hee Lee1, Hong Bae Ann2, and Myeong-Gu Park1,3

1 Department of Astronomy and Atmospheric Sciences, Kyungpook National University, Daegu 41566, Republic of Korea; mgp@knu.ac.kr
2 Pusan National University, Busan 46241, Republic of Korea
3 Research and Training Team for Future Creative Astrophysicists and Cosmologists (BK21 Plus Program), Kyungpook National University, Daegu 41566, Republic of Korea

Received 2018 August 17; revised 2019 January 13; accepted 2019 January 15; published 2019 February 14

Abstract

Bar fractions depend on the properties of the host galaxies, which are closely related to the formation and evolution of bars. However, observational studies do not provide consistent results. We investigate the bar fraction and its dependence on the properties of the host galaxies by using three bar classification methods: visual inspection, an ellipse fitting method, and Fourier analysis. Our volume-limited sample consists of 1698 spiral galaxies brighter than $M_e = -15.2$ with $z < 0.01$ from the Sloan Digital Sky Survey/DR7 visually classified by Ann et al. We first compare the consistency of classification among the three methods. Automatic classifications detect visually determined, strongly barred galaxies with the concordance of 74% to 85%. However, they have some difficulty in identifying bars, in particular in bulge-dominated galaxies, which affects the distribution of bar fraction as a function of Hubble type. We obtain, for the same sample, different bar fractions of 63%, 48%, and 36% by visual inspection, ellipse fitting, and Fourier analysis, respectively. The difference is mainly due to how many weak bars are included. Moreover, we find a different dependence of bar fraction on Hubble type for strong versus weak bars: SBs are preponderant in early-type spirals, whereas SABs are in late-type spirals. This causes a contradictory dependence on host galaxy properties when different classification methods are used. We propose that strong bars and weak bars experience different processes for their formation, growth, and dissipation by interacting with different inner galactic structures of early-type and late-type spirals.

Key words: galaxies: evolution – galaxies: formation – galaxies: photometry – galaxies: spiral – galaxies: structure

1. Introduction

Early theoretical studies showed that stellar disks lacking random motions are generally unstable, which rapidly leads to the formation of bars (Toomre 1964; Ostriker & Peebles 1973). On the other hand, spherical components such as bulges and halos stabilize the stellar disks and delay bar formation (Ostriker & Peebles 1973; Athanassoula & Sellwood 1986). However, once a bar forms, spherical components help the bar grow stronger (Combes & Sanders 1981; Athanassoula 2002; Athanassoula & Misiriotis 2002) by depriving the bars of angular momentum and energy (Kalnajs 1970; Lynden-Bell & Kalnajs 1972; Kormendy 1979; Sellwood 1980; Tremaine & Weinberg 1984; Weinberg 1985; Little & Carlberg 1991; Hernquist & Weinberg 1992; Athanassoula 1996, 2002, 2003; Debattista & Sellwood 1998, 2000).

Bars drive gas and stars into the galactic center by large-scale streaming motions (Roberts et al. 1979; Schwarz 1981, 1984; van Albada & Roberts 1981; Prendergast 1983; Combes & Gerin 1985; Athanassoula 1992; Sellwood & Wilkinson 1993). The bar-driven stars trapped in an elongated orbit induce the bar to become more eccentric (Combes & Gerin 1985; Athanassoula 2003), and gas inflow driven by the bar builds up the central mass concentration (CMC; Pfenniger & Norman 1990; Ann & Lee 2000; Athanassoula & Misiriotis 2002). Subsequently, the growth of the CMC broadens radial and vertical resonance regions, induces slow secular evolution, which creates bulge-like structures such as pseudobulges and peanut-shaped bulges, and dissolves the bar itself (Hasan & Norman 1990; Pfenniger & Norman 1990; Hasan et al. 1993; Kormendy 1993; Norman et al. 1996; Debattista et al. 2006). This is the general picture for bar formation, evolution, and destruction that we have learned from model and simulation studies.

We expect to find some clues to the formation, growth, and destruction of bars from the observed bar fraction in different galaxies, which is the integrated outcome of the processes bars have experienced. Bars, including both strong and weak bars, have been found in over 60% of disk galaxies in the local universe (de Vaucouleurs et al. 1991; Buta et al. 2010, 2015; Ann et al. 2015). The bar fraction has been widely known to depend strongly on the Hubble sequence, mass, color, and bulge prominence (Sheth et al. 2008). However, detailed observational tendencies are still controversial: bar fractions are reported to increase toward early-type spirals that are massive, red, gas-poor, and bulge-dominated (Sheth et al. 2008; Aguerri et al. 2009; Laurikainen et al. 2009; Cheung et al. 2013; Gavazzi et al. 2015; Consolandi 2016), or increase toward the opposite direction of the Hubble sequence, that is, toward late-type spirals that are less massive, blue, gas-rich, and disk-dominated (Barazza et al. 2008, 2009; Aguerri et al. 2009; Weinzirl et al. 2009; Buta et al. 2015; Yoshino & Yamauchi 2015; Erwin 2018). In addition, other studies have shown a bimodal distribution of the bar fraction, with each peak in early-type and in late-type spirals (de Vaucouleurs et al. 1991; Knapen 1999; Eskridge et al. 2000; Nair & Abraham 2010; Masters et al. 2011; Lee et al. 2012; Oh et al. 2012; Díaz-García et al. 2016).

How can we understand this apparent disagreement? Nair & Abraham (2010) have explained the reason for the discrepancy as the different mass ranges of the sample galaxies each study had used, while Erwin (2018) considered the angular resolution from the combination of the FWHM for the telescope and the distance to sample galaxies. In this paper, we investigate the effect of classification methods in detecting barred galaxies and the different bar definitions in the studies.
We suspect that different methods to identify bars may have contributed to the inconsistent results in bar fraction as a function of Hubble sequence. In previous studies, the high bar fractions in early-type spirals were mainly obtained by visual inspection or Fourier analysis (Sheth et al. 2008; Aguerri et al. 2009; Laurikainen et al. 2009; Cheung et al. 2013), whereas the opposite results were derived mainly by the ellipse fitting method (Barazza et al. 2008, 2009; Aguerri et al. 2009). Since the sample for the classification itself can greatly affect the outcome, we need to apply each bar detection method to the same sample and compare the results. We chose visual inspection, ellipse fitting, and Fourier analysis methods in this paper.

In addition, we cannot ignore the fact that the definition of barred galaxies could be different depending on the studies. The earlier visual inspections provided us with a bar frequency of about 60% when including SB and SAB galaxies (Nilson 1973; Sandage & Tammann 1987; de Vaucouleurs et al. 1991; Ann et al. 2015; Buta et al. 2015), while most of the recent visual inspections suggest a ∼30% bar fraction, which is consistent with the fraction of SB galaxies (Nair & Abraham 2010; Hoyle et al. 2011; Masters et al. 2011; Lee et al. 2012; Oh et al. 2012; Skibba et al. 2012; Cheung et al. 2013; Simons et al. 2014). Among the automatic methods, the ellipse fitting method yields a level of ∼45% for the bar fraction (Marinova & Jogee 2007; Reese et al. 2007; Barazza et al. 2008; Aguerri et al. 2009; Marinova et al. 2010), while Fourier analysis could not show consistent results for the bar fraction among the studies that used different criteria (Oh et al. 1990; Aguerri et al. 2000, 2009; Laurikainen et al. 2002, 2004).

In this paper, we introduce our sample, data reduction, and bar classification methodology in Section 2. Even though some studies used the same basic method to classify barred galaxies, they adopted different criteria. In Section 3, we compare the bar classification results obtained by applying the three methods and diverse criteria for the same sample. We report in Section 4 how the overall bar fractions are different depending on the classification methods and selection criteria used despite being applied to the same sample. Next, we show that the dependence of the bar fractions on the properties of galaxies also becomes different depending on the classification method in Section 5. Finally, we compare our observational results with previous observational and theoretical studies in the context of bar formation and evolution in Section 6. The conclusion is given in Section 7.

2. Data Reduction and Bar Classification

2.1. Sample and Data Reduction

We used the catalog of Ann et al. (2015, hereafter Ann15) to compare automatic classifications of barred galaxies against visual classification. The catalog provides detailed morphological types based on visual inspections for 5836 galaxies from the Sloan Digital Sky Survey (SDSS)/DR7 with bin in the B band (Jogee et al. 2002; Laine et al. 2002). We constructed B-band images according to the relation $m_B = m_g + 0.3$ (Rodgers et al. 2006) from SDSS g-band images. We confirmed that the relation $m_B = m_g + 0.3130 \times (m_g - m_r) + 0.2271$ also produces nearly the same results. We evaluated the position angle and ellipticity by fitting the ellipse in IDL, as mentioned in Section 2.2.1. We present the final deprojected image in Figure 1(f). After deprojection, we excluded from our sample 195 galaxies with inclination higher than 60° and 84 galaxies with frames smaller than R25. Ultimately, we have a subsample of 884 galaxies in the magnitude range $-21 \leq M_r \leq -15$ suitable for the automatic analysis.

2.2. Bar Classification Methodology

2.2.1. Ellipse Fitting

Wozniak et al. (1995) showed that the characteristics of bar signatures are increasing ellipticity $e$ and constant position angle (PA) in the radial profiles derived from ellipse fits. Jogee et al. (2004) proposed two criteria to identify bars: (1) $e$ must rise to a global maximum above 0.25 ($e_{bar} \geq 0.25$), while the PA remains relatively constant within $\pm 20^\circ$ along the bar ($\Delta P_{e_{bar}} \leq \pm 20^\circ$); (2) the transition occurs between the bar and the disk region, with $e$ dropping by more than 0.1 ($e_{disk} \geq 0.1$) and with PA changing by more than 10° ($\Delta P_{e_{disk}} \geq 10^\circ$). Therefore, this bar selection method is defined by criteria on four parameters, the ellipticity...
threshold for the bar ($\epsilon_{\text{bar}}$), the range of constant PA over the bar region ($\Delta P_{\text{bar}}$), and the change in ellipticity ($\Delta \epsilon_{\text{tri}}$) and PA ($\Delta P_{\text{tri}}$) at the transition between a bar and a disk. The threshold for bar ellipticity distinguishes bars from oval structures, and the constant PA distinguishes bars from spiral arms.

These criteria have been widely used with some variations. We list the criteria and the bar fraction results from previous

---

Table 1

| Author                   | $\Delta \epsilon_{\text{tri}}$ | $\Delta P_{\text{bar}}$ | $\Delta P_{\text{tri}}$ | $\epsilon_{\text{bar}}$ | $F_{\text{bar}}$(%) | Wavelength |
|--------------------------|---------------------------------|--------------------------|--------------------------|--------------------------|---------------------|------------|
| Wozniak et al. (1995)    | 0.02                            | 2°                       | const                    | ...                      | ...                 | B, V, R, I  |
| Laine et al. (2002)      | 0.1                             | ...                      | 10°±20°                  | ...                      | 17                  | optics     |
| Jogee et al. (2004)      | 0.1                             | 10°±20°                  | 0.4                      | 33                       | 36                  | NIR        |
| Menén dez-Delmestre et al. (2007) | 0.1                             | 10°±20°                  | $\epsilon_{\text{max}}$ > 0.2 | 59                       | $J + H + K$        |            |
| Marinova & Jogee (2007)  | 0.1                             | change                   | $\epsilon_{\text{max}}$ ≥ 0.25 | 44 ± 7                   | 60 ± 7              | optics     |
| Reese et al. (2007)      | 0.1                             | change                   | $\epsilon_{\text{max}}$ ≥ 0.25 | 48                       | 47                  | I band     |
| Barazza et al. (2008)    | 0.1                             | change                   | $\epsilon_{\text{max}}$ ≥ 0.25 | 48                       | 47                  | r band     |
| Sheth et al. (2008)      | 0.1                             | change                   | $\epsilon_{\text{max}}$ ≥ 0.25 | 48                       | 47                  | r band     |
| Aguerri et al. (2009)    | 0.08                            | 5°±10°                   | $\epsilon_{\text{max}}$ ≥ 0.25 | 48                       | 47                  | r band     |
| Marinova et al. (2009)   | 0.1                             | change                   | $\epsilon_{\text{max}}$ ≥ 0.25 | 48                       | 47                  | I band     |
| Marinova et al. (2010)   | 0.1                             | change                   | $\epsilon_{\text{max}}$ ≥ 0.25 | 48                       | 47                  | I band     |
| Marinova et al. (2012)   | 0.1                             | change                   | $\epsilon_{\text{max}}$ ≥ 0.25 | 48                       | 47                  | I band     |
| Consolandi (2016)        | 0.08                            | 20°±20°                  | ...                      | 36                       | 36                  | optics     |

Note. Column (1): previous studies using ellipse fitting method. Column (2): $\epsilon$ transition criterion between bars and disks. The values of Wozniak et al. (1995) in columns (2) and (3) are mean differences at transition, not criterion. Column (3): PA transition criterion between bars and disks. Column (4): range of constant PA over the bar region to distinguish between bars and spiral arms. Column (5): limit on $\epsilon$ to distinguish bars from oval structures. In particular, $\epsilon_{\text{max}}$ indicates that the ellipticity of the bar should be the global maximum. Jogee et al. (2004) suggested the criterion for weak bars as $0.25 \leq \epsilon_{\text{max}} < 0.4$. Column (6): resulting bar fraction from each study. Column (7): wavelength.
Bar Fractions and Different Criteria for the Fourier Analysis

| Author             | \( I_{m} / I_{0} \) | \( \Phi_{m} \) | \( F_{\text{full}}(\%) \) | \( F_{\text{full}}(z < 0.01)(\%) \) |
|--------------------|---------------------|--------------|-----------------|-----------------|
| Ohta et al. (1990) | \( I_{b} / I_{0b} > 2 \) | ...          | ...             | 72.29           |
| Aguerri et al. (2000) | \( I_{b} / I_{0b} > (\max - \min) / 2 + \min \) | ...          | ...             | 51.47           |
| Laurikainen et al. (2002) | \( I_{b} / I_{0b} > 0.3 \) | \( \phi_{b}, \phi_{d} \) | 40              | 36.31           |
| Laurikainen et al. (2004) | \( A_{2} > 0.12 \) (SB) | \( \phi_{b}, \phi_{d} \) | 65              | 32.47           |
| Laurikainen et al. (2009) | \( A_{2} > 0.09 \) (SAB) | \( \Delta(t_{b} / I_{0}) > 0.2 \) | \( \phi_{d} \) | 53.85           |

**Note.** Column (1): previous studies using Fourier analysis. Column (2): criteria for relative Fourier amplitude. Column (3): criteria for constant phase angle. Column (4): bar fraction from previous studies. Column (5): bar fraction obtained in our study by applying each criterion.

Table 2

Studies in Table 1. We want to emphasize that even if we use the ellipse fitting method to detect bars, the results may depend on the different criteria adopted. In fact, we see a wide range of bar fractions from previous studies in Table 1, even after considering the fact that they are affected by the different wavelength or sample properties.

Marinova et al. (2012) tested additional criteria for bars, whether \( e_{\text{bar}} \) should be a global maximum or not. Marinova & Jogee (2007) compared the difference between observed images and deprojected images. We tested various criteria and conditions such as wavelength and deprojection effect in order to find the optimal conditions to identify bars.

We calculated the ellipse fits following Davis et al. (1985) and Athanassoula et al. (1990) in IDL. It is also based on the Fourier decomposition along a given ellipse, \((x^2/a^2) + (y^2/b^2) = 1\), where \( x \) is in the direction of the major axis:

\[
I(a, \phi) = I_{0}(a) + \sum_{m=1}^{\infty} [A_{m}(a) \cos m\phi + B_{m}(a) \sin m\phi],
\]

where

\[
A_{m}(a) = \frac{1}{\pi} \int_{0}^{2\pi} I(a, \phi) \cos m\phi \, d\phi
\]

and

\[
B_{m}(a) = \frac{1}{\pi} \int_{0}^{2\pi} I(a, \phi) \sin m\phi \, d\phi.
\]

We analyzed the luminosity profiles \( I(a, \phi) \) constructed from initial parameters such as ellipticity \((\epsilon)\), position angle \((\theta)\), and center position \((x_c, y_c)\). From the initial solution, we obtained the Fourier coefficients \( A_{1}, B_{1}, A_{2}, B_{2} \) which are related to \( x_c, y_c, \epsilon, \) and PA (Young et al. 1979; Kent 1983). We adjusted the ellipse parameters that yield the largest \( A_{m} \) or \( B_{m} \) and accepted the final isophote when \( |A_{m}| < 10^{-3} \) and \( |B_{m}| < 10^{-3} \). We confirmed that our results are consistent with those from the IRAF/ellipse package based on Kent (1983) and Jedrzejewski (1987), who directly correct the ellipse parameters at each iteration by using the relation between coefficients and parameters. Our method converges to robust results, while being less affected by initial parameters than the IRAF/ellipse package.

We performed ellipse fitting with step sizes increasing by 10% from three pixels from the center. The center was taken as the centroid within a given isophote, generally the isophote at the intensity one-third of the peak intensity. We used the fixed center for all radii. Fitting with a fixed center is more efficient in finding the transition between a bar and a disk, although a varying center may reflect the real light distribution better. Our code finds transitions between bars and disks at first, and then checks if the bar candidates satisfy the criteria for ellipticity and PA.

2.2.2. Fourier Analysis

An alternative way to classify bars automatically is to utilize the Fourier coefficient directly (Ohta et al. 1990; Aguerri et al. 1998, 2000). Previous studies have used the deprojected images and analyzed the azimuthal luminosity profiles \( I(r, \theta) \) along concentric ellipses with Fourier decomposition, as shown in Equation (1).

To identify bars, some studies (Ohta et al. 1990; Aguerri et al. 2000) have used the ratio of bar intensity to interbar intensity, \( I_b / I_{0b} \), where the bar intensity \( I_b \) is defined as the sum of the even Fourier components, \( I_0 + I_2 + I_4 + I_6 \), and the interbar intensity \( I_{0b} \) is given by \( I_0 - I_2 + I_4 - I_6 \). On the other hand, others (Laurikainen et al. 2002, 2004; Aguerri et al. 2009) used the relative Fourier amplitude of the \( n \)th component, described as

\[
\frac{I_{m}(r)}{I_0(r)} = \frac{[A_{m}(r)^2 + B_{m}(r)^2]^{1/2}}{A_{0}(r)/2}.
\]

Generally, even terms, in particular \( m = 2 \), appear much larger than odd terms over the bar region (Ohta et al. 1990; Aguerri et al. 2000). However, other nonaxisymmetric structures such as spiral arms can produce the same effect. Therefore, some studies also demanded that the phases defined by

\[
\Phi_{m}(r) = \tan^{-1} \frac{B_{m}(r)}{A_{m}(r)}
\]

remain constant in order to distinguish bars from spiral arms (Laurikainen et al. 2002, 2004; Aguerri et al. 2009). In particular, Aguerri et al. (2009) utilized these parameters to investigate the transition between a bar and a disk as the way in the ellipse fitting method. We list their criteria for bar classification and the bar fractions they obtained in Table 2.

We reproduced all of their bar classifications using deprojected \( i \)-band images as in previous studies and compared them to each other. However, we did not exactly reproduce Laurikainen et al. (2004). They performed a two-dimensional
bar–bulge–disk decomposition before the deprojection process and applied the deprojection only to the disk and bar components because a spherical bulge is not, in principle, affected by projection. However, paradoxically, it needs the information about bar structure to decompose the image into bar, bulge, and disk before bar classification. They considered both cases in which galaxies host a bar or not, and we just applied their criterion to the wholly deprojected image.

3. Comparison of Bar Classification

3.1. Visual Inspection

We compared the results of the bar classifications done by different methods but applied to the same sample. At first, we compared the classifications by two independent visual inspections, the Ann15 and RC3 catalogs. Ann15 classified 5836 galaxies with Petrosian r magnitudes greater than 17.77 with $z < 0.01$. The RC3 catalog classified more than 23,000 galaxies larger than 1 arcmin and brighter than $B = 15.5$ with $z < 0.05$. The Ann15 classification system follows the RC3 classification. Both have classified spiral galaxies as not only SA and SB but also SAB. Ann15 found 361 strong bars (SB, 31.0%) and 365 weak bars (SAB, 31.4%) out of the final 1163 disk galaxies after rejecting 535 edge-on galaxies. The RC3 catalog provides the classification for 1707 spirals within $z = 0.01$. It comprises 24% SA, 28% SAB, and 48% SB for 1274 galaxies after excluding 433 uncertain or doubtful objects to determine whether they host a bar or not. We found that the two catalogs include 706 common spiral galaxies.

We investigated the agreement of classification for these 706 galaxies between the two catalogs. In Figure 2, we present the percentage of classification of Ann15 (y axis) against the classification of RC3 (x axis). A higher number and darker shade mean better agreement. We confirmed that the mutual concordances reach up to 87%, 66%, and 72% in the SA, SAB, and SB categories, respectively, after excluding edge-on and uncertain galaxies in each catalog. Ann15 classified only 4% (four galaxies out of 111) of SAs determined by RC3 as SBs and RC3 categorized just 9% (16 galaxies) of SAs by Ann15 as SBs. Therefore, the disagreement between SA and SB galaxies is less than 10%. However, when it comes to weak bars, we found that the agreement is not as good. Out of 155 SABs from RC3, 102 galaxies (66%) have been classified as SABs by Ann15. The remaining 53 SABs in RC3 are evenly classified either as SA (27 galaxies) or SB (26 galaxies) in Ann15. This is in a way expected because SAB is intermediate between SA and SB, and its classification would include more ambiguity than SA or SB.

We will use only the matched galaxies between the two studies to compare with the automatic classification in Section 3.2 and Section 3.3. Rejecting highly inclined galaxies and objects with a smaller image size than $R_{25}$ leaves only 211 galaxies. Therefore, the galaxies that we use to compare with automatic methods are 63 SA, 64 SAB, and 84 SB galaxies, which we call the “concordance sample.”

3.2. Ellipse Fitting Method

We compare the classification by ellipse fits to that by visual inspection using the concordance sample discussed above. As mentioned before, ellipse fitting methods have been applied with different criteria depending on the studies. In this paper, we tested each criterion in order to find the optimal criteria that yield the highest agreement with visual classification. We experimented with transition between the bar and the disk, most studies adopted 0.1 as $\Delta \epsilon_{\text{tra}}$ but a different value for $\Delta \epsilon_{\text{max}}$. Whereas some studies used 10° or 5°, others did not specifically constrain $\Delta \epsilon_{\text{tra}}$ and consider any change in PA profile as a signature of a bar. In Figure 3, we show how the value of $\Delta \epsilon_{\text{tra}}$ affects the agreement and disagreement with the visual classification. The matched percentages of classification by ellipse fitting versus visual classification are displayed as a function of $\Delta \epsilon_{\text{tra}}$. The solid lines and dotted lines show the agreement and disagreement with the visual classification, respectively. Although we distinguish weak bars from strong bars by their $\epsilon_{\text{bar}}$, 0.25 $\leq \epsilon_{\text{bar}} < 0.4$ for SAB and $\epsilon_{\text{bar}} \geq 0.4$ for SB (Jogee et al. 2004), the agreement rate of SAB with visual classification is very low.

We found that as we apply a higher threshold for $\Delta \epsilon_{\text{tra}}$, the agreement of SA (black solid line) increases, whereas those of SB (red solid line) and SAB (blue solid line) decrease. This is because a large $\Delta \epsilon_{\text{max}}$ would classify a bar aligned with a disk as a nonbarred galaxy, as noted earlier (Menéndez-Delmestre et al. 2007). However, if we deproject the images, we can reduce the effect of $\Delta \epsilon_{\text{tra}}$ as shown in Figures 3(b) and (c). Using deprojected images in general increases the match between the ellipse fitting method and visual classification and also lessens the dependence on the choice of $\Delta \epsilon_{\text{tra}}$. We found the optimal threshold of $\Delta \epsilon_{\text{tra}}$ in each case to increase the matched fraction and reduce the unmatched fraction for SB and SA against visual classification. We indicate them as black vertical lines in Figure 3 and present their consistency against visual classification in Figure 4 in the same way as in Figure 2.
In general, the ellipse fitting method classifies around 70% of visually determined SAs as SAs and 70% to 80% of visually classified SBs as SBs. The ellipse fitting method seems to be as effective in detecting barred galaxies as the two independent visual inspections described in Section 3.1. However, the confusion between SB and SA in the ellipse fitting method is somewhat higher than in the visual classification. We investigated missed SBs in the automatic classification and found that they are mostly early-type spirals such as S0/a, Sa, or Sab. A large bulge makes the ellipticity of the bar lower and dilutes the transition between a bar and a disk in ellipticity profiles. Aguerri et al. (2009) noted earlier that the ellipse fitting method more efficiently detects bars with a sharp end than those with a smooth transition by testing artificial galaxies. On the other hand, it sometimes classifies tightly wound spiral arms as bar structures, and they are usually Sc or Scd. Therefore, the ellipse fitting method can affect the bar fraction differently for different Hubble types, as will be discussed in Section 5.2.

Deprojecting g-band images can increase the agreement for SBs by 5% and reduce the missed SBs by 9% (Figure 4(b)). The deprojection process somewhat helps in disentangling the aligned bar from the inclined disk. It also helps a little in finding bars in early-type spirals with a large bulge. Therefore, using g-band deprojected images with the criterion of $\Delta PA_{\text{tra}} \geq 5^\circ$ and $\epsilon_{\text{bar}} \geq 0.25$ seems to be the optimal choice to construct the ellipse fitting method most consistent with the visual classification.

We had expected that i-band images are more favorable for detecting bars because near-infrared (NIR) images not only reflect old populations that make up the bar structure but also reduce the dust obscuration. Moreover, previous studies had presented a higher bar fraction in NIR (Esckeridge et al. 2000; Marinova & Jogee 2007; Buta et al. 2015). However, we found that using i-band images decreases the matched fraction of SB while increasing that of SA (Figure 3(c)). This is because i-band images have a smoother light distribution than g-band images, and it makes it harder for the algorithm to find the transition between a bar and a disk. We obtained a similar agreement with using g-band observed images when we applied the lower threshold of $\Delta PA_{\text{tra}} \geq 2^\circ$ and $\epsilon_{\text{bar}} \geq 0.2$ in i-band deprojected images (Figure 4(c)).

We experimented with the lower limits of 0.25 and 0.2 for the $\epsilon_{\text{bar}}$ in all cases. A lower threshold helps in detecting more visually determined SABs, but, at the same time, has a risk of misjudging bulges of nonbarred galaxies as weak bars. For the visual classification, our eyes distinguish bulges from weak bars considering not only the ellipticity but also other characteristics such as the light distribution of the bulge. Only for i-band images, the lower limit of 0.2 helps increase the matched fractions for both SA and SAB. That is why we selected different criteria of $\epsilon_{\text{bar}}$ for the g- and i-band images.

In addition, we tested the effect of the smoothing box size. Although we need to smooth images in order to minimize the effect of the artificial residuals after masking, we caution that a larger smoothing box can hide the transition between a bar and a disk. This is similar to a lower angular resolution, from the limited FWHM of the telescope or from the distance to the galaxy, decreasing the detection of bars. In our test, a smaller smoothing box increases the agreement of SBs with the visual inspection, while it decreases that of SAs. We found that the size 0.1$R_25$ is optimal for our sample galaxies in order to increase the matched fractions for both SB and SA.

### 3.3. Fourier Analysis

We also investigated the agreement between the automatic classification based on the Fourier analysis and that by visual inspection for the concordance sample. We applied five criteria listed in Table 2 and summarized the matched percentage
between the Fourier analysis and visual classifications in Table 3.

The earlier criteria (Ohta et al. 1990; Aguerri et al. 2000) constructed from very small samples seem to be very awkward in classifying barred galaxies. They consider most of the galaxies as barred galaxies or divide galaxies in half regardless of their morphologies. Their criteria are matched not only by bar structures but also by other diverse components. On the other hand, more recent studies that demand $\Phi_2$ and $\Phi_4$ be constant over a bar region greatly improve the agreement with the visual classification. In particular, the criterion of Laurikainen et al. (2015) yields an outstanding result in distinguishing barred galaxies from nonbarred galaxies. It measures nonbarred galaxies and barred galaxies with a matched fraction of 79% and 87% with visual inspections, respectively. This is the highest agreement with visual inspections among all of the methods and criteria we have tested. Nevertheless, they missed 11 visual SB galaxies (13%). All of them have gradually increasing or decreasing phase profiles ($\Phi_0(r)$) caused by a large bulge or spirals wrapped with the bar. On the other hand, they classified 13 visually selected SAs (21%) as barred galaxies. We found that three of them have bar signatures that previous visual inspections did not find. The rest of the misjudged SBs are located near the thresholds of the criteria, and half of them are early-type spirals with a large bulge. The inclined bulges are often confused with a bar. In other words, misclassifications by Fourier analysis against visual classifications are increasing in early-type spirals. A large bulge is very tricky to deal with in automatic classification, and our deprojection process cannot solve the inclination problem completely.

Laurikainen et al. (2004) used $A_2$ instead of relative amplitude, and $A_2$ missed more visually classified strong bars (29%) than before (13%). This is because many visual SBs, especially late-type spirals such as Sd, do not have $A_2$ larger than the threshold suggested by Laurikainen et al. (2004), despite satisfying the criterion on $I_2/I_0$ used by Lau02. Therefore, the relative amplitude would be more useful in determining a bar structure than the absolute amplitude. In contrast, the criterion of Aguerri et al. (2009) distinguishes only 46% of visually nonbarred galaxies from barred galaxies, whereas it shows a high agreement of 87% with visual classifications for SBs. We found that this is because they did not set any threshold on $I_2/I_0$ for bar structures to be distinct from the inclined bulge. In practice, the criterion by Aguerri et al. (2009) classifies many inclined bulges or weakly oval structures that have a constant phase as bars.

4. Bar Fraction

We measured the overall bar fractions from the final 884 sample galaxies using visual inspections, ellipse fits, and Fourier analysis. The results are listed in Table 4. The resulting bar frequencies become different depending on the methods or criteria used to detect bars even for the same sample galaxies. Furthermore, the wavelength band of the images, deprojection, and spatial resolution also influence the fraction of detected bars. It is apparent why researchers derived different frequencies of barred galaxies.

We calculated the bar fraction, which includes strong bars (30%) and weak bars (33%), from the visually determined catalog of Ann15. It is similar to the typical bar fractions of classical visual classifications such as UGC, RSA (Nilson 1973; Sandage & Tammann 1987), and RC3, which present the fraction of ~30% for SB and SAB, respectively. Buta et al. (2015) also obtained a similar bar fraction of 66% including SAB by visual inspection of 1160 galaxies in NIR wavelength. On the other hand, many recent visual inspections show a frequency around 30% (Nair & Abraham 2010; Hoyle et al. 2011; Masters et al. 2011; Lee et al. 2012; Oh et al. 2012; Skibba et al. 2012; Cheung et al. 2013; Simmons et al. 2014). They usually deal with obvious bars only, and almost all weak bars may have been excluded. Their bar fractions are consistent with the fraction of strong bars in the classical visual classification.

From the ellipse fitting method, we obtained the bar fraction in the range of 48% to 56%, depending on the detailed criteria or conditions. This is similar to the ranges of previous studies by ellipse fits (Marinova & Jogee 2007; Menéndez-Delmestre et al. 2007; Reese et al. 2007; Barazza et al. 2008; Aguerri et al. 2009; Marinova et al. 2010, 2012). The fraction of detected bars is higher than the fraction of strong bars only, yet less than that of all bars, strong and weak, from visual inspection.
Table 3
Comparison between the Classifications by Visual Inspection and Fourier Analysis

| Visual class | Fourier class | SA | SAB | SB | SA | SAB | SB | SA | SAB | SB | SA | SAB | SB | SA | SAB | SB |
|--------------|---------------|----|-----|----|----|-----|----|----|-----|----|----|-----|----|----|-----|----|
| Ohta et al. (1990) | 46.03 | 0 | 53.97 | 26.56 | 0 | 73.44 | 0 | 0 | 100. |
| Aguerri et al. (2000) | 57.14 | 0 | 42.86 | 50.00 | 0 | 50.00 | 34.52 | 0 | 65.48 |
| Lau02 | 79.37 | 0 | 20.63 | 56.25 | 0 | 43.75 | 15.48 | 0 | 84.52 |
| Laurikainen et al. (2004) | 79.37 | 4.76 | 15.87 | 65.62 | 1.56 | 32.81 | 27.38 | 3.57 | 69.05 |
| Aguerri et al. (2009) | 46.03 | 0 | 53.97 | 31.25 | 0 | 68.75 | 11.90 | 0 | 88.10 |

Note. The numbers indicate the matched percentage of classification by the Fourier analysis to that by the visual inspection. The weak bars have never been defined, except for by Laurikainen et al. (2004).

Table 4
Bar Fraction by Different Classification Methodologies

| Classification method | $F_{\text{bar}}$ | $F_{\text{SB}}$ | $F_{\text{SAB}}$ |
|-----------------------|-----------------|----------------|----------------|
| Visual inspection (Ann15) | 63% | 30% | 33% |
| Ellipse fitting ($g_{\text{bar}}+5^e+0.25$) | 48% | 41% | 7% |
| Ellipse fitting ($g_{\text{dep}}+5^e+0.25$) | 56% | 45% | 11% |
| Ellipse fitting ($i_{\text{dep}}+2^e+0.20$) | 52% | 35% | 17% |
| Fourier analysis (Lau02) | 36% | 36% | ... |

Note. Most bars calculated by ellipse fits fall under the strong bars, $e_{\text{bar}} > 0.4$, and a small fraction stay under $0.25 < e_{\text{bar}} < 0.4$ for weak bars.

5. Bar Fraction and Host Galaxies

Now we investigate the dependence of bar frequency on the physical properties of host galaxies for each classification method. We use four parameters to represent the properties of galaxies: numerical code $T$, $(g - r)$ color, $\text{frac}dV$, and inverse light concentration $C_{\text{in}}$. The parameter $T$ is the indicator of the Hubble stage, its number running from 0 to 9 for S0/a, Sa, Sb, Sbc, Sc, Scd, Sd, Sdm, and Sm (RC3). They are grouped into early-type ($0\text{a}/\text{Sb}$), intermediate-type ($\text{Sbc}/\text{Scd}$), and late-type ($\text{Sd}/\text{Sm}$) spirals (Ann15). The SDSS parameter $\text{frac}dV$ is the fraction of the light fit by a de Vaucouleurs profile versus an exponential profile (Masters et al. 2010, 2011). Therefore, $\text{frac}dV$ indicates the bulge-to-total ratio, and large values mean bulge-dominated systems that have mostly classical bulges. Galaxies with a small $\text{frac}dV$ are disk-dominated systems in which bulges are mostly pseudobulges or even bulgeless. The last parameter, inverse light concentration, is defined as $C_{\text{in}} = R_{50}/R_{90}$, where $R_{50}$ and $R_{90}$ are the Petrosian radii enclosing 50% and 90% of the total galaxy light (Petrosian 1976; Lee et al. 2012). It increases toward a less-concentrated system such as late-type spirals.

In fact, all these parameters are known to be related to the galaxy morphology (Kormendy 1993; Abraham et al. 1996; Conselice et al. 2000; Graham et al. 2001; Conselice 2003; Park & Choi 2005; Aguerri et al. 2009). We present the correlations among the parameters for our sample galaxies in Figure 5 and the correlation coefficients at the top left of each panel. Hubble type ($T$) shows comparatively good correlations with color, $\text{frac}dV$, and $C_{\text{in}}$ (Figure 5(a)–(c)). Despite the large scatter, as $T$ increases, the median values of $(g - r)$ and $\text{frac}dV$ decrease, and $C_{\text{in}}$ increases gradually. In particular, we note that early-type spirals (small $T$) span the whole range of $\text{frac}dV$ while late-type spirals span a limited range of $\text{frac}dV$ (Figure 5(b)). This implies that early-type spirals have not only classical bulges but also pseudobulges, whereas it is difficult for late-type spirals to have classical bulges. This is consistent with observational results that pseudobulges are discovered even in lenticular galaxies (Laurikainen et al. 2009). When we inspect the correlations among $(g - r)$, $\text{frac}dV$, and $C_{\text{in}}$, we find that $\text{frac}dV$ and $C_{\text{in}}$ are weakly anticorrelated with a coefficient of $-0.39$ even though both of them have often been used as an indicator of the bulge size (Figure 5(f)). Also, $\text{frac}dV$ has a relatively strong correlation with $(g - r)$, which reflects the stellar population or recent star formation (Park & Choi 2005), while the inverse light concentration $C_{\text{in}}$ has little correlation with $(g - r)$ (Figure 5(d)–(e)).

Stellar mass is another important parameter that affects bar formation and evolution (Barazza et al. 2008; Nair & Abraham 2010; Méndez-Abreu et al. 2012; Díaz-García et al. 2016;...
Erwin 2018). Our sample galaxies are distributed in the stellar mass range $10^8 \lesssim M_\star/M_\odot \lesssim 10^{11}$ following Bell et al. (2003):

$$\log \frac{M_\star}{M_\odot} = -0.306 + 1.097(g - r) - 0.4(M_r - M_r,0).$$  \hspace{1cm} (6)$$

However, we have to caution that this mass estimate may contain significant errors caused by the relatively large peculiar velocities, because our sample galaxies are within $z = 0.01$.

We used the values of $T$ and $(g - r)$ from the Ann15 catalog, and other information was obtained from the SDSS database. The color is corrected for Galactic extinction.

### 5.1. Bar Fraction by Visual Inspection

#### 5.1.1. Total Bar Fraction

We investigated the dependence of the fraction of barred galaxies classified by Ann15 on the properties of the host galaxies. In Figure 6, the top row shows histograms of the number of barred galaxies, and the bottom row shows the fractions of bars as a function of $T$, $(g - r)$, fracdeV, and $C_{in}$ from left to right. The black dot-dashed and gray solid lines represent the total galaxies and the total barred galaxies, respectively. Strong bars (SB) and weak bars (SAB) are represented by red solid and blue dotted lines. The uncertainties are $(f(1 - f)/N)^{1/2}$ for the fraction $f$ and number of galaxies $N$ in a given bin, representing statistical uncertainty in the fraction (Sheth et al. 2008). We reversed the $x$ axes of $(g - r)$ and fracdeV from right to left for easy comparison with other parameters. Accordingly, in all panels, the left side corresponds to early-type spirals with low $T$, high $(g - r)$, high fracdeV, and low $C_{in}$, while the right side corresponds to late-type spirals that have large $T$, small $(g - r)$, small fracdeV, and high $C_{in}$. We notice that our sample contains lots of galaxies of late type ($T \gtrsim 5$), blue color ($0.3 \lesssim g - r \lesssim 0.5$), small bulge (fracdeV $< 0.15$), and less concentration ($0.45 \leq C_{in} \leq 0.55$; Figure 6(a)–(d)).

The gray solid lines (bottom row of Figure 6) that represent the bar fractions of both strong (SB) and weak bars (SAB) do not show any significant trend as a function of $T$, $(g - r)$, or fracdeV. They seem to be roughly constant. Previous studies showed similar results when both strong and weak bars are considered (Knapen 1999; Eskridge et al. 2000; Li et al. 2017). In more detail, the overall bar fractions seem to show big or small double peaks on $T$, $(g - r)$, and fracdeV. One peak appears in each panel around early-type spirals with $T = 3$, $(g - r) \approx 0.75$, and fracdeV $\approx 0.9$. The other peak is located around late-type spirals or intermediate-type spirals with $T \gtrsim 7$, $(g - r) \approx 0.45$, and fracdeV $\approx 0.5$. This is consistent with recent studies by visual inspection dealing with only strong or obvious bars (Nair & Abraham 2010; Masters et al. 2011; Lee et al. 2012). They showed a more dramatic peak above $(g - r) \approx 0.7$ and a small peak below $(g - r) \approx 0.4$. We also see more conspicuous peaks around early-type spirals when we separate strong bars from weak bars (red solid lines in Figure 6(e)–(f)). When it comes to the light concentration (Figure 6(h)), the overall bar fraction obviously increases as host galaxies are less concentrated, showing no double peaks.

**Figure 5.** Correlation between properties of host galaxies: (a) $T$ and $(g - r)$, (b) $T$ and fracdeV, (c) $T$ and $C_{in}$ (d) $(g - r)$ and fracdeV, (e) $(g - r)$ and $C_{in}$, (f) fracdeV and $C_{in}$. Correlation coefficients are presented at the top left, and triangles show the median values.
However, this monotonous trend is the result of the steady increase in weak bars with $C_{in}$ (blue dotted line in Figure 6(h)), and strong bars show the highest peak in the most concentrated bin (red solid line in Figure 6(h)).

5.1.2. Different Dependence of Strong and Weak Bar Fractions on Host Galaxy Properties

Therefore, we need to analyze the bar fractions of SB and SAB galaxies separately. Counting SBs (red solid line) and SABs (blue dotted line) separately shows that $F_{SB}$ and $F_{SAB}$ have different distributions against the properties of host galaxies. The double peaks shown in the whole bar fractions are the combination of one peak by strong bars and another by weak bars: in Figure 6(e), strong bars are dominant in early-type spirals with $T \leq 3$, while weak bars are prominent in intermediate-type and late-type spirals with $T \geq 3$. When it comes to the $(g - r)$ color (Figure 6(f)), $F_{SB}$ has a peak at red spirals with $(g - r) = 0.75$, and $F_{SAB}$ is peaked at blue spirals with $(g - r) = 0.45$. As for $fraceV$ (Figure 6(g)), strong bars are frequent in bulge-dominated systems with $fraceV \geq 0.3$, while the weak bar fraction slowly increases as $fraceV$ decreases. We also found peaks of $F_{SB}$ and $F_{SAB}$ at different $C_{in}$ in Figure 6(h): strong bars increase steeply at the most concentrated systems (lowest $C_{in}$), whereas weak bars rise toward less-concentrated systems (higher $C_{in}$) and have a peak at the least-concentrated system. In other words, strong and weak bars have their own peaks at different ranges. Strong bars are more frequent in early-type, red, bulge-dominated, and the most concentrated spirals, while weak bars are frequent in intermediate-type and late-type, blue, disk-dominated, and less-concentrated spirals. The correlation between bar types and host galaxy properties can solve some contradictory results seen in previous studies. We will discuss it more in Section 6.1.

Our findings are consistent with lots of previous studies. An earlier study by Ann & Lee (1987) showed that early-type spirals with higher spheroid-to-disk ratio have stronger bars. Erwin (2005) mentioned that late-type spirals have weak bars twice as often as strong bars by analyzing the sample of Martin (1995). Abraham & Merrifield (2000) have shown that SBs and SAs increase toward early-type spirals and SABs are frequent in late-type spirals. Masters et al. (2011) also showed that the bar fraction increases with the value of $fraceV$ by using the Galaxy Zoo project, which mainly dealt with obvious bars, although it appeared to be more rapidly increasing than in our work. Similar dependencies have been reported for flat and exponential bars: flat bars are frequent in early-type spirals, while late-type spirals mainly have exponential bars (Elmegreen & Elmegreen 1985, 1989; Baumgart & Peterson 1986; Elmegreen et al. 1996; Hoyle et al. 2011). When it comes to the bar length, early-type or red spirals have longer bars than late-type or blue spirals (Erwin 2005; Lee et al. 2012). Early-type spirals showed a higher bar strength estimated by the bar luminosity.
(Ann & Lee 1987) and by the Fourier amplitude (Ohta et al. 1990; Laurikainen et al. 2009).

We noticed another interesting feature in Figure 6(e). The bar fraction declines abruptly at S0/a galaxies (T = 0) despite the fact that not only do early-type spirals have a high fraction of strong bars, but also bulge-dominated and highly concentrated systems exhibit a high fraction of strong bars. Although they are not exactly about S0/a, we often find previous studies that showed a lack of bars in S0 galaxies compared to spirals (Aguerri et al. 2009; Laurikainen et al. 2009; Buta et al. 2015; Li et al. 2017). Laurikainen et al. (2009) mentioned that S0 galaxies have a higher fraction of ovals or lenses, which might have been bars if they were not weakened by central concentration. We suspect another possibility that a lack of gas in S0/a and S0 galaxies makes it hard to drive bar instability if the formation of bars is delayed until gas is removed from these galaxies.

5.2. Different Tendencies in Bar Fraction by Automatic Classification

We display the dependence of bar fraction on host galaxy properties by using the ellipse fitting method (blue dotted lines) and Fourier analysis (red solid lines) in Figure 7, in the same manner as in Figure 6 except for the method to select barred galaxies. We chose the condition of $g_{\text{obs}} + 5^\circ + 0.25$ for ellipse fitting and the criterion of Lau02 for Fourier analysis in order to compare with previous studies. These are typical conditions used in previous studies and yield good agreement with visual classification (refer Section 3). Weak bars are not treated separately here because the fraction is too small in the ellipse fitting method (Table 4) and is not defined in Lau02.

At a glance, the distributions of bar fraction seem to be quite different depending on the method of selecting bars. The Fourier analysis shows higher bar fractions toward early-type spirals, which increase as the host galaxies are red, bulge-dominated, and more concentrated (red solid lines in Figure 7(a)–(d)). These tendencies are consistent with previous studies by Fourier analysis (Aguerri et al. 2009; Laurikainen et al. 2009). And the bar fractions from these studies resemble the distribution of $F_{\text{SB}}$ by visual inspection, although they are not exactly the same. The consistency can be understood because the criterion of Lau02 mainly detects strong bars. Actually, the overall bar fraction by Lau02 just stays around 36%, which is slightly higher than the frequency of visually detected strong bars. We, however, have to be cautious that the errors of the Fourier method are particularly large in early-type spirals with a large bulge. Fortunately, the errors do not seem to significantly influence the distribution of bar fraction as a function of Hubble type because the Fourier method sometimes misses barred galaxies and other times determines nonbarrad galaxies as barred galaxies in early-type spirals, as discussed in Section 3.3.

On the contrary, the bar fractions yielded by the ellipse fitting method increase toward late-type spirals, that is, in disk-dominated and less-concentrated systems. These tendencies agree with previous studies obtained by the ellipse fitting method (Barazza et al. 2008, 2009; Aguerri et al. 2009) and resemble the distribution of $F_{\text{SB}}$ found by visual inspection in some ways. But these look to be the opposite of those found by Fourier analysis. Consequently, though we reproduced the bar fraction as a function of galaxy properties consistent with previous studies by using ellipse fitting and Fourier analysis, we faced a contradiction that the distributions of bar fractions are different even for the same sample. Aguerri et al. (2009) also compared the two automatic classifications and found that the bar fraction from Fourier analysis is lower in late-type spirals than that from ellipse fitting. They reported that Fourier analysis is less efficient in detecting bars of the late-type spirals with lenses or strong spiral arms. Our analysis is slightly different from their findings. We suspect two effects as causes for the discrepancy. One is that two methods accept weak bars of different fractions as their barred galaxies: the ellipse fitting method classifies more visual SABs as barred galaxies compared to Fourier analysis. Second, the systematic errors of the ellipse fitting technique, which could miss bars mainly in early-type spirals with a large bulge, could influence the bar fraction as a function of galaxy properties, as expected in Section 3.2.

6. Discussion

6.1. Bar Fraction as a Function of Host Galaxy Properties

This work is partly motivated by the contradictory bar fraction results as a function of host galaxy properties shown in previous studies. We find a trend that studies with relatively higher bar fractions between 45% and 66% show frequent bars in late-type spirals (Barazza et al. 2008; Aguerri et al. 2009; Weinzirl et al. 2009; Buta et al. 2015; Yoshino & Yamauchi 2015; Erwin 2018),
while studies with lower bar fractions of around 30% report abundant bars in early-type spirals (Sheth et al. 2008; Aguerri et al. 2009; Masters et al. 2011; Lee et al. 2012; Oh et al. 2012; Cheung et al. 2013; Gavazzi et al. 2015; Consolandi 2016). In the previous section, we showed the different dependence of bar fraction on the Hubble type for strong versus weak bars. So we can understand that studies including more weak bars would show an increasing bar fraction toward late-type spirals, whereas studies that mainly deal with strong bars would show a prominent bar fraction in early-type spirals.

In fact, Erwin (2018) has also questioned the different Hubble type dependence for SBs and SABs. However, he could not find significantly distinct characteristics for \((g - r)\) and the stellar mass \(M_\text{a} \) and has shown the diverging bar fractions of SB and SAB only for very high gas mass ratios. Our analysis also confirms that the difference is more prominent for the Hubble type and bulge dominance rather than for \((g - r)\) and \(M_\text{a} \) in Figures 9 and 11. This suggests that \((g - r)\) and \(M_\text{a} \) have little effect on the different formation or evolution of strong versus weak bars, although they are important parameters for bar formation and evolution.

Nair & Abraham (2010) understood the discrepancy was caused by the different mass ranges of sample galaxies they used in previous studies (Barazza et al. 2008; Sheth et al. 2008). They showed a bimodal distribution of bar fraction as a function of host galaxy properties from a visually obvious bar fraction of \(\sim 30\%\). Masters et al. (2011, 2012) and Lee et al. (2012) have found similar trends of bar fraction with a high peak at red color and a very small peak at blue color from visually obvious bars, which constitute \(\sim 30\%\) of the spiral galaxies. They argued that the disagreement was caused by excluding red disk galaxies using a color cut in the study by Barazza et al. (2008).

In order to test the possible sample bias, we split our sample into two groups, bright \((M_r < -18)\) versus faint \((M_r > -18)\) galaxies, in Figure 8. They show a trend roughly similar to the total sample: roughly equal SBs in early and late types, and more SABs in late-type spirals. However, we note that another high peak of SBs appears at late-type and blue spirals when faint galaxies are excluded. This can explain the higher bar fraction at low-mass galaxies shown in Nair & Abraham (2010), as discussed by Gavazzi et al. (2015).

In addition, we emphasize that the method classifying barred galaxies causes the different bar fractions for the host galaxy properties. This is because they not only contain different fractions of weak bars but also often misclassify bulge-dominated galaxies. Besides, Erwin (2018) suspected the poor spatial resolution of SDSS as the reason for the discrepancy between previous studies. Cheung et al. (2013) provided another view that the mass dependence of the bar fraction again depends on the specific star formation rate.

Lastly, we investigate the bar fraction in terms of stellar mass. In Figure 9, we display the bar fraction as a function of mass by visual inspection and automatic classification. As for other properties, the fraction of strong bars by visual inspection increases with mass (red solid line in Figure 9(a)). This result is consistent with previous studies mainly dealing with obvious bars by visual inspection (Masters et al. 2012; Melvin et al. 2014; Gavazzi et al. 2015). On the other hand, weak bars are hardly influenced by mass (blue dotted line in Figure 9(a)). For the automatic classification, both methods yield strong trends that increase with mass (Figure 9(b)). Although this seems different from the tendency in other properties shown in Section 5.2, it agrees with previous studies by automatic classification (Gavazzi et al. 2015; Consolandi 2016). When it comes to mass, we do not find the opposite tendency between the Fourier analysis and the ellipse fitting method. However, we emphasize that the estimate of stellar mass for our galaxy sample may have significant errors, due to their peculiar velocities. Likewise, we need to be cautious about the results related to the galaxy mass in previous studies that deal with the nearest galaxies (Díaz-García et al. 2016; Erwin 2018).

6.2. Properties of Bars: SBs versus SABs

In general, earlier classifications by visual inspection showed SBs of 30% and SABs of 30% in nearby spiral galaxies (Nilson 1973; Sandage & Tammann 1987; de Vaucouleurs et al. 1991). More recently, Ann15 and Buta et al. (2015) also presented similar results. They usually inspect the shape, relative bar length, and contrast to distinguish SABs from SBs.

We find a similar dichotomy of bars in previous studies (Elmegreen & Elmegreen 1985, 1989; Baumgart & Peterson 1986; Elmegreen et al. 1996; Regan & Elmegreen 1997; Kim et al. 2015). They investigated two types of bars: flat and exponential profiles in surface brightness. Flat bars have nearly constant light distributions along the bar, whereas those of exponential bars decrease exponentially. They also differ in their structures and the intensity contrast between the bar and the disk: flat bars are longer, wider, and stronger than exponential bars and have a higher contrast than exponential bars. Besides, Athanassoula (1992) explained that flat bars could have roughly rectangular orbits around the end of the bar through the stellar orbit calculation. These properties can also be explained by the locations of resonances (Lynden-Bell 1979; Contopoulos & Papayannopoulos 1980; Sellwood 1981; Contopoulos et al. 1989; Athanassoula 1992; Skokos et al. 2002). A flat density profile develops in crowding stellar orbits between the inner 4:1 resonance and corotation radius (Combes & Elmegreen 1993; Elmegreen & Elmegreen 1985; Elmegreen et al. 1996). Exponential bars end near the inner Lindblad resonance and do not have such crowding orbits (Lynden-Bell 1979; Elmegreen & Elmegreen 1985; Elmegreen et al. 1996). Therefore, flat bars and exponential bars may be expected to have different pattern speeds based on the value of \(R = R_{CR}/R_{BAR}\), where \(R_{CR}\) and \(R_{BAR}\) are the radius of the corotation resonance and the bar, respectively (Debattista & Sellwood 2000; Valenzuela & Klypin 2003). Although some studies reported the observational lack of slow bars (Debattista & Sellwood 2000; Aguerri et al. 2015), others showed that the pattern speed of bars roughly depends on the Hubble type: fast bars in early-type spirals and slow bars in late-type spirals (Aguerri et al. 1998; Rautiainen et al. 2008). More observational data will help us understand the relation between the density profile and the pattern speed of bars.

Therefore, the shape, length, strength, and pattern speed of bars seem to be related to the dichotomy of flat and exponential bars. We investigated whether our SBs and SABs defined by their shape, rectangular or oval, are related to flat or exponential bars. We classified our sample galaxies into flat or exponential profiles. We investigated the bar intensity profiles of \(I_0 + I_2 + I_4 + I_6\) calculated in Section 2.2.2 (Oh et al. 1990; Aguerri et al. 2000) and classified galaxies that show a plateau in the bar intensity profile as flat bars (Elmegreen & Elmegreen 1985; Elmegreen et al. 1996; Kim et al. 2015), as shown in Figure 10. We described the percentage of flat and exponential
profiles for each class, SB, SAB, and SA, in Table 5. Even nonbarred galaxies have flat profiles. Flat profiles can be formed not only by bar structures but also by rings, winding spirals, and bright clusters around the center. Nevertheless, we confirm that flat profiles increase toward strong bars and exponential profiles increase toward nonbarred galaxies. Although there is no direct match between the bar type (SBs or SABs) and the bar luminosity profile (flat or exponential), flat bars are more dominant in SBs than in SABs.

Figure 8. Dependence of the fraction of barred galaxies from the Ann15 catalog on (a) Hubble sequence, (b) $g - r$, (c) fracdeV, and (d) $C_m$. The top row shows the subsample of brighter galaxies with $M_r \leq -18$, and the bottom row shows fainter galaxies with $M_r > -18$. We present total barred galaxies (gray solid line), strongly barred galaxies (red solid line), and weakly barred galaxies (blue dotted line).

Figure 9. Bar fraction as a function of stellar mass (a) by visual inspection and (b) by automatic classification.

Figure 10. Examples for a flat bar and an exponential bar. The left panels show images and the right panels the bar intensity and interbar intensity profiles. The bar intensity (solid line) is calculated by $I_b = I_1 + I_2 + I_3 + I_4$ from Fourier analysis and the interbar intensity (dotted line) by $I_{b0} - I_1 - I_2 - I_3 - I_4$. (a) SB and flat bar. The flat bar has a plateau in the bar intensity profile. (b) SB and exponential bar.
6.3. Bar Formation and Evolution in Early-type and Late-type Spirals

We carried out the Kolmogorov–Smirnov (K-S) test to estimate how SBs and SABs are differently distributed with respect to the host galaxy properties. Table 6 presents the probabilities of K-S tests between bar families. Small probability values mean that two distributions are statistically different. The cumulative distributions of histograms for $F_S$ (solid line), $F_{SAB}$ (dotted line), and $F_S$ (dashed line) are shown in Figure 11. We find that SBs have totally different distributions from those of SABs as a function of the Hubble sequence, as shown in Figure 6(e). This is consistent with the analysis of Abraham & Merrifield (2000). They argued that SABs would be the expanded form of nonbarred galaxies in late-type spirals.

On the other hand, the parameter that distinguishes SB from both SA and SAB is the bulge property represented by $\frac{\text{deV}}{g}$. SBs have characteristics similar to SAs when it comes to $\frac{\text{deV}}{g}$ but are distinguished from SAs by $C_{\text{in}}$. We cannot find any difference between bar families in $(g − r)$. Similar characteristics have appeared for the mean values of $T$, $(g − r)$, $\frac{\text{deV}}{g}$, and $C_{\text{in}}$ for SA, SAB, and SB galaxies (Table 7). We find that SB galaxies have higher mean values of $\frac{\text{deV}}{g}$ than SAs and SABs, whereas SAB galaxies have higher $T$ than SAs and SBs. Although we do not find significant differences in $(g − r)$ and $C_{\text{in}}$, SAs have slightly lower mean values in $(g − r)$ and SABs have slightly higher in $C_{\text{in}}$.

Accordingly, we surmise that strong and weak bars prefer different inner galactic structures for their formation and evolution. They would experience different processes in early-type and late-type spirals. Especially, we think that it is the bulge properties that make strong bars different from weak bars. These different bar properties in early- and late-type spirals have been reported similarly in previous studies: flat bars and buckled bars are dominant in early-type spirals, whereas exponential bars and unbarred bars are mainly found in late-type spirals (Elmegreen & Elmegreen 1985, 1989; Baumgart & Peterson 1986; Elmegreen et al. 1996; Erwin & Debattista 2017; Li et al. 2017). Different characteristics of bars seem to be closely related to the difference in the structure of galaxies.

Theoretical studies and numerical simulations have shown that bars form, grow, and weaken via active interactions with other components of host galaxies such as bulge, disk, halo, and gas components. For early-type spirals, their spheroidal components stabilize the disk and delay the formation of bars (Toomre 1964; Ostriker & Peebles 1973; Combes & Sanders 1981; Athanassoula & Sellwood 1986). However, their bars grow gradually by interaction with a bulge and a halo, which gain angular momentum from the disk and exert a dynamical friction on the bar (Sellwood 1980; Weinberg 1985; Athanassoula 2002, 2003). Consequently, the bar pattern speed continuously slows down, and the corotation radius moves toward outer regions, driving more particles to be trapped (Weinberg 1985; Little & Carlberg 1991; Hernquist & Weinberg 1992; Athanassoula 1996, 2003).

Debattista & Sellwood 1998). Combes & Elmegreen (1993) explained that early-type spirals with large bulge-to-disk ratios have a large maximum in the distribution of $\Omega − \frac{\kappa}{2}$ and a large pattern speed initially. Therefore, the corotation radius is located at a small radius, and bars can transfer the angular momentum to stars near the corotation and outer resonance regions. Enough interactions make the pattern speed slow down and intersect $\Omega − \frac{\kappa}{2}$. Eventually, inner Lindblad resonances occur, and it is possible to develop flat bars and ring structures.

On the other hand, in the case of late-type spirals, although they readily form bars because they have less-massive halos, bars cannot grow because there are few interacting stars (Combes & Sanders 1981; Combes & Elmegreen 1993; Athanassoula 2002; Athanassoula & Misiriotis 2002). They have a low $\Omega − \frac{\kappa}{2}$ and pattern speed. Thereby, the corotation radius resides far beyond the end of the bar. In this case, the disk scale length determines the bar length, and the bar has an exponential density profile (Combes & Elmegreen 1993).

Bars in early-type and late-type spirals also showed different results in their destruction by CMC, such as by a supermassive black hole, central disk, dense central stellar clusters, and bar-driven gas. First, bars in disk-dominated galaxies are totally destroyed, while bars in massive halo systems just weaken in strength with respect to the same CMC (Athanassoula et al. 2005), due to the extra angular momentum absorbed by the halo. Second, late-type spirals have abundant gas, and bars in late-type spirals seem to be more easily weakened. This is not only because the gas inflow can give angular momentum to bars but also because the gas inflow largely contributes to the massive CMC even when they have the same halos (Berentzen et al. 2007; Athanassoula et al. 2013). Finally, disk thickness also drives the difference in bars. The disk scale height is an indicator of the phase-space density in the central region. A thin disk with high phase-space density and smaller random velocity stimulates the build-up of the CMC. On the other hand, bars in thick disks with a larger random velocity lose more angular momentum and rotate slower in a way similar to the bulge (Klypin et al. 2009).

Consequently, bars in late-type spirals are not only easily generated and but also easily destroyed with rare growth. However, bars in early-type spirals gradually grow by interacting with their bulge, thick disk, and halo and are not easily destroyed. Therefore, bars in early-type spirals would be more able to survive, although they need a longer time to form compared to bars in late-type spirals. Nevertheless, we seem to find similar bar fractions of SBs and SABs in the local universe. We speculate that this is caused by the combination of frequent formation and short lifetime of weak bars and rare formation and longer lifetime of strong bars.
Properties $T$ $g - r$ fracdeV $C_{in}$
SA 4.7 0.53 0.26 0.47
SAB 5.6 0.57 0.24 0.49
SB 4.8 0.56 0.34 0.47

6.4. Bar Growth and Destruction by Bulge and CMC

The growth and destruction of bars seem to depend on two competitive effects between bar growth by the bulge or halo and bar destruction by the CMC. We have two parameters, fracdeV and $C_{in}$, which can roughly estimate the bulge dominance and CMCs, respectively. Although both of them have often been used as an indicator of bulge size (Abraham et al. 1996; Abraham & Merrifield 2000; Park & Choi 2005; Masters et al. 2011), we have shown not only that the correlation between the two parameters is relatively weak (Figure 5(f)) but also the dependence of bar fraction on the two parameters appears to be different (Figures 6(g) and (h)). If we consider that fracdeV obtained by profile model fitting would better reflect the bulge properties while $C_{in}$ calculated by comparing $R_{50}$ and $R_{90}$ could be representative for the CMC due to the supermassive black hole (SMBH), bar-driven gas, and bar itself as well as the bulge, we can obtain an observational hint for the effect of the bulge and the CMC on the evolution of bars. The parameter $C_{in}$ has been explained as the indicator of the CMC in Nair & Abraham (2010) and Lee et al. (2012).

A first look at Figure 6(g) shows that the distributions of $F_{SB}$ and $F_{SAB}$ have the opposite slope with respect to fracdeV: strong bars are abundant in bulge-dominated spirals, and weak bars gradually increase toward the spirals with small bulges. It seems hard for bars to grow stronger in galaxies with a small bulge. This confirms the simulations that bars grow stronger by interaction with bulges, which deprive the bars of angular momentum (Athanassoula 2002, 2003; Athanassoula & Misiriotis 2002).

The dependence of $F_{SB}$ and $F_{SAB}$ on $C_{in}$ seems to be more complex, as seen in Figure 6(h). Strong bars and weak bars have a maximum in bar fraction on the opposite-end bins: $F_{SB}$ is peaked at the most concentrated spirals, whereas $F_{SAB}$ is peaked at the least-concentrated spirals. If the most concentrated $C_{in}$ bin is excluded, both $F_{SB}$ and $F_{SAB}$ are slightly increasing toward less-concentrated spirals. It seems that the light concentration does not help all types of bars to form or survive. This observation is consistent with the theoretical expectation that the CMCs weaken or completely destroy bars (Hasan & Norman 1990; Pfenniger & Norman 1990; Hasan et al. 1993; Norman et al. 1996). Although more recent simulations showed that bars are robust structures, and the current masses of SMBHs and diffuse gas concentration are inefficient in destroying bars totally, they all found that the growth of CMCs decreases the bar strength (Shen & Sellwood 2004; Athanassoula et al. 2005; Debattista et al. 2006). Our observation also suggests that the bar growth still seems to be regulated by the CMC.

On the other hand, the distributions of $F_{SB}$ and $F_{SAB}$ on $C_{in}$ show some difference in the most concentrated bin (Figure 6(h)). In the most concentrated systems, the fraction of strong bars abruptly increases to a high ratio, whereas that of weak bars greatly decreases. Therefore, the distribution of $F_{SB}$ appears bimodal (Figure 6(h)), which is consistent with Nair & Abraham (2010), who dealt with strong bars. We find a hint to understand the unexpected increase in strong bars in the most concentrated systems from the correlation between fracdeV and $C_{in}$ in Figure 5. All of the most concentrated spirals with $C_{in} \leq 0.3$ are distributed in the region with a massive bulge of fracdeV $\geq 0.6$. Therefore, we may conclude that the bar growth by bulge is more prevalent than the weakening of bars by CMC in the most concentrated galaxies. This agrees with the simulation of Athanassoula et al. (2005), which showed that the effect of CMCs becomes weak in bulge-dominated systems.

In Figure 12, we compare the mean values of fracdeV of SBs and SABs for each $C_{in}$. Although the distribution of fracdeV in the same range of $C_{in}$ shows large scatter, we find two dominant features. First, as the system is more concentrated, its mean fracdeV largely increases. Although this is partly due to the effect of the weak correlation between the two parameters, we can speculate that galaxies need more massive bulges for bar survival when galaxies are more concentrated. Second, SB galaxies always have higher mean values of fracdeV compared to SABs for the same $C_{in}$ over all ranges of $C_{in}$. This supports the view that galaxies need a more developed bulge for strong bars compared to weak bars for a given CMC. Therefore, we confirm that bars are formed and supported by the balance between the bulge and CMC.
circles and open triangles indicate strong bars and weak bars, respectively. Mean frac\(\text{deV}\) of SBs and SABs as a function of Figure 12.

The Astrophysical Journal, C

on the other hand, we find the increasing bulge ratio of strong bars again at \(C_{\text{in}} > 0.55\) despite their low CMCs in Figure 12. There seems to be difficulties for the bars to grow stronger in galaxies with a very low concentration, and more prominent bulges are needed for strong bars. This would explain why \(F_{\text{SB}}\) does not increase any more when \(C_{\text{in}} > 0.55\) in Figure 6(h).

Lastly, we want to discuss the bulge component. When the value of frac\(\text{deV}\) is large, we expect the existence of a classical bulge, and small frac\(\text{deV}\) galaxies with pseudobulges or no bulges. Therefore, Figure 6(g) shows that strong bars are frequent in galaxies with classical bulges, while weak bars are found in galaxies with pseudobulges. In general, pseudobulges have been known to be formed as a result of secular evolution by bars. The bar-driven gas can broaden the vertical resonance, and it is believed to form pseudobulges, which are rotationally supported by a metal-rich and young stellar population (Kormendy 1982, 1993; Pfenniger & Norman 1990; Kormendy & Kennicutt 2004). They are abundant in late-type spirals, although early-type spirals also have pseudobulges (Andredakis & Sanders 1994; Andredakis et al. 1995; Carollo et al. 2002; Kormendy & Kennicutt 2004). Studies that showed the increasing bar fraction toward late-type spirals argued that the frequent pseudobulges in late-type spirals would be related to the high bar fraction in late-type spirals (Barazza et al. 2008). However, we want to consider the fact that oval structures such as weak bars are enough to drive gas inflow and pseudobulge formation (Kormendy & Kennicutt 2004; Kim et al. 2018), and the larger gas fraction helps grow the CMC (Berentzen et al. 2007; Athanassoula et al. 2013). Therefore, frequent weak bars and abundant gas in late-type spirals seem to explain the frequent pseudobulges seen in late-type spirals well.

7. Conclusions

We have studied the bar fraction and the relation between the fraction and the properties of their host galaxies for 884 spiral galaxies selected from a parent sample of 1876 spiral galaxies from the Ann15 catalog. It is a volume-limited sample with \(z < 0.01\) from SDSS/DR7, brighter than \(M_r = -15.2\), and an inclination \(i < 60^\circ\). Specifically, we have compared the result from each bar classification method, visual inspection (RC3 and Ann15 catalogs), ellipse fitting, and Fourier analysis, in order to understand the bias caused by different methods.

1. We confirm the consistency of around \(~80\%\) and inconsistency below 10\% between two independent visual inspections, RC3 and Ann15. The agreement of the ellipse fitting method with visual inspection reaches 70\% to 80\% in the SA and SB classes, and we obtain the best agreement when we apply the PA transition (\(\Delta P_{\text{in}}\)) of 5\(^\circ\) to \(g\)-band deprojected images. However, this method misses about 15\% of visually strong barred galaxies. We note that this is caused by the large bulge in early-type spirals that hides the transition between a bar and a disk. Therefore, the classification based on the ellipse fitting method produces a systematic effect in the bar fraction as a function of Hubble sequence. We can get the highest consistency of around 80\% with the visual inspection when we apply the combined criterion of the relative Fourier amplitude and constant phase suggested by Lau02. However, it also often makes mistakes in classification, in particular in early-type spirals, because of the large bulge. Large bulges have given variations on the phase, or inclined bulges have been confused as bar structures.

2. We obtain different bar fractions from the same sample galaxies when we apply different classification methods, criteria, or conditions. The visual inspection by Ann15 shows bar fractions of 30\% of SB and 33\% of SAB. The ellipse fitting method, in general, yields a bar fraction of over 48\%. The resultant bar fraction contains almost all visually determined SBs and one-half of visually determined SABs. The Fourier analysis method with the criteria of Lau02 yields a lower bar fraction of 36\%. It mainly finds strongly barred galaxies. Therefore, the range of barred galaxies can be different depending on the methods used to classify bars. In addition, even when we use the same method, different criteria and observing wavelength also influence the bar fraction.

3. The dependence on the host galaxy properties of bar fraction depends on bar types, strong and weak. The visual inspection yields a different correlation between bar types and host galaxy properties. Strong bars are more frequent in early-type, red, bulge-dominated, and most concentrated galaxies, while the fraction of weak bars increases toward late-type, blue, disk-dominant, and less-concentrated galaxies. We propose that strong and weak bars have experienced different processes for their formation and evolution within different types of galaxies, early- and late-type spirals. Their similar bar fractions of \(~30\%) are likely to be the result of a combination of frequent formation and dissipation of weak bars and the rare formation and longer survival of strong bars.

4. The dependence on the host galaxy properties of bar fraction depends on the methods used to select bars. For the same sample, we obtained the opposite dependence on the host galaxy properties when we used different classification methods. Bars classified by the ellipse fitting method are frequent in late-type spirals, which resemble those of weak bars by visual inspection. On the contrary, the fraction of bars identified by Fourier analysis increases toward early-type spirals, and they are similar to those of strong bars by visual inspection.
We suspect that this is due to the fact that the ellipse fitting method misses some of the bulge-dominant barred galaxies and the Fourier analysis finds strongly barred galaxies.

We thank the anonymous referee for comments that improved this paper. We acknowledge the support for Y.H. L. and M.G.P. by the National Research Foundation of Korea to the Center for Galaxy Evolution Research (2017R1A5A1070354).

**ORCID iDs**

Yun Hee Lee @https://orcid.org/0000-0003-2779-6793
Hong Bae Ann @https://orcid.org/0000-0002-9822-5608
Myeong-Gu Park @https://orcid.org/0000-0003-1544-8556

**References**

Abraham, R. G., & Merrifield, M. R. 2000, AJ, 120, 2835
Abraham, R. G., Tanvir, N. R., Santiago, B. X., et al. 1996, MNRAS, 279, L47
Aguerri, J. A. L., Beckman, J. E., & Prieto, M. 1998, AJ, 116, 2136
Aguerri, J. A. L., Méndez-Abreu, J., & Corsini, E. M. 2009, A&A, 495, 491
Aguerri, J. A. L., Méndez-Abreu, J., Falcón-Barroso, J., et al. 2015, A&A, 576, A102
Aguerri, J. A. L., Muñoz-Tuñón, C., Varela, A. M., & Prieto, M. 2000, A&A, 361, 841
Andredakis, Y. C., Peletier, R. F., & Balcells, M. 1995, MNRAS, 275, 874
Andredakis, Y. C., & Sanders, R. H. 1994, MNRAS, 267, 283
Ann, H. B., Lee, H. M. 2000, JKAS, 33, 1
Ann, H. B., Lee, S.-W. 1987, JKAS, 20, 49
Ann, H. B., Seo, M., & Ha, D. K. 2015, ApJS, 217, 27
Athanassoula, E. 2003, MNRAS, 341, 1179
Athanassoula, E., & Misiriotis, A. 2002, MNRAS, 330, 35
Athanassoula, E., & Sanders, R. H. 1994, MNRAS, 267, 283
Athanassoula, E., & Sellwood, J. A. 1986, MNRAS, 221, 215
Barazza, F. D., Jogee, S., & Marinova, I. 2008, ApJ, 675, 1194
Barazza, F. D., Jablonka, P., Desai, V., et al. 2009, A&A, 497, 173
Bentz, M. C., Krolik, J. H., & Volmer, M. 2009, ApJ, 699, L99
Bentz, M. C., Krolik, J. H., & Volmer, M. 2009, ApJ, 699, L99
Bentz, M. C., Krolik, J. H., & Volmer, M. 2009, ApJ, 699, L99
Bentz, M. C., Krolik, J. H., & Volmer, M. 2009, ApJ, 699, L99
Bentz, M. C., Krolik, J. H., & Volmer, M. 2009, ApJ, 699, L99
Bentz, M. C., Krolik, J. H., & Volmer, M. 2009, ApJ, 699, L99
Bentz, M. C., Krolik, J. H., & Volmer, M. 2009, ApJ, 699, L99
Bentz, M. C., Krolik, J. H., & Volmer, M. 2009, ApJ, 699, L99
Bentz, M. C., Krolik, J. H., & Volmer, M. 2009, ApJ, 699, L99
Bentz, M. C., Krolik, J. H., & Volmer, M. 2009, ApJ, 699, L99
Bentz, M. C., Krolik, J. H., & Volmer, M. 2009, ApJ, 699, L99
Schwarz, M. P. 1981, ApJ, 247, 77
Schwarz, M. P. 1984, MNRAS, 209, 93
Sellwood, J. A. 1980, A&A, 89, 296
Sellwood, J. A. 1981, A&A, 99, 362
Sellwood, J. A., & Wilkinson, A. 1993, RPPh, 56, 173
Shen, J., & Sellwood, J. A. 2004, ApJ, 604, 614
Sheth, K., Elmegreen, D. M., Elmegreen, B. G., et al. 2008, ApJ, 675, 1141
Simmons, B. D., Melvin, T., Lintott, C., et al. 2014, MNRAS, 445, 3466
Skibba, R. A., Masters, K. L., Nichol, R. C., et al. 2012, MNRAS, 423, 1485
Skokos, C., Patsis, P. A., & Athanassoula, E. 2002, MNRAS, 333, 861
Toomre, A. 1964, ApJ, 139, 1217

Tremaine, S., & Weinberg, M. D. 1984, ApJL, 282, L5
Valenzuela, O., & Klypin, A. 2003, MNRAS, 345, 406
van Albada, G. D., & Roberts, W. W., Jr. 1981, ApJ, 246, 740
Weinberg, M. D. 1985, MNRAS, 213, 451
Weinzirl, T., Jogee, S., Khochfar, S., Burkert, A., & Kormendy, J. 2009, ApJ, 696, 411
Wozniak, H., Friedli, D., Martinet, L., Martin, P., & Bratschi, P. 1995, A&AS, 111, 115
Yoshino, A., & Yamauchi, C. 2015, MNRAS, 446, 3749
Young, P. J., Sargent, W. L. W., Kristian, J., & Westphal, J. A. 1979, ApJ, 234, 76