LONG TIME BEHAVIOUR OF STRONG SOLUTIONS TO INTERACTIVE FLUID-PLATE SYSTEM WITHOUT ROTATIONAL INERTIA

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ABSTRACT. We study well-posedness and asymptotic dynamics of a coupled system consisting of linearized 3D Navier–Stokes equations in a bounded domain and a classical (nonlinear) full von Karman plate equations that accounts for both transversal and lateral displacements on a flexible part of the boundary. Rotational inertia of the filaments of the plate is not taken into account. Our main result shows well-posedness of strong solutions to the problem, thus the problem generates a semiflow in an appropriate phase space. We also prove uniform stability of strong solutions to homogeneous problem.

1. Introduction. We deal with a coupled system which describes an interaction of a homogeneous viscous incompressible fluid which occupies a domain $O$ bounded by the (solid) walls of the container $S$ and a horizontal (flat) part of the boundary $\partial O \cap \Omega$ on which a thin (nonlinear) elastic plate is placed. The motion of the fluid is described by linearized 3D Navier–Stokes equations. To describe deformations of the plate we use the full von Karman plate model. This is a rather general model based on Kirchhoff hypotheses that describes both longitudinal and transversal oscillations of the plate.

This fluid-structure interaction model assumes that large deflections of the elastic structure produce small effect on the fluid. This corresponds to the case when the fluid fills the container which is large in comparison with the size of the plate.

We note that the mathematical studies of the problem of fluid–structure interaction in the case of viscous fluids and elastic plates/bodies have a long history. We refer to [3, 5, 8, 11, 12, 13] and the references therein in the case of plates, see also the literature cited in these papers.

We note that the global (asymptotic) dynamics in nonlinear plate-fluid models was studied before in [5, 8, 9]. The article [5] deals with a class of fluid-plate interaction problems, when the plate, occupying $\Omega$, oscillates in longitudinal directions only. This kind of models arises in the study of blood flows in large arteries (see, e.g., [11] and the references therein). A fluid-plate interaction model, accounting for purely transversal displacement of the plate, was studied in [8]. A mathematical model, the most close to the current one, was considered in [9]. It takes into account both transversal and in-plane displacements. In contrast to the model in

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the present paper, it accounted for rotational inertia, and mechanical dissipation for the transversal component of the plate displacement was assumed.

Let $\mathcal{O} \subset \mathbb{R}^3$ be a bounded domain with a sufficiently smooth boundary $\partial \mathcal{O}$. We assume that $\partial \mathcal{O} = \Omega \cup S$, where

$$\Omega \subset \{x = (x_1; x_2; 0) : x' = (x_1; x_2) \in \mathbb{R}^2\}$$

with a smooth contour $\Gamma = \partial \Omega$, and $S$ is a surface which lies in the halfspace $\mathbb{R}^3_+ = \{x_3 \leq 0\}$. The exterior normal to $\partial \mathcal{O}$ is denoted by $n$. Evidently, $n = (0; 0; 1)$ on $\Omega$.

We consider the following linear Navier–Stokes equations in $\mathcal{O}$ for a fluid velocity field $v = v(x, t) = (v^1(x, t); v^2(x, t); v^3(x, t))$ and a pressure $p(x, t)$:

$$\begin{align*}
v_t - \nu \Delta v + \nabla p &= G_{fl} \quad \text{in} \quad \mathcal{O} \times (0, +\infty), \\
\text{div} v &= 0 \quad \text{in} \quad \mathcal{O} \times (0, +\infty),
\end{align*}$$

where $\nu > 0$ is the dynamical viscosity of the fluid and $G_{fl}$ is a volume force. We supplement (1) and (2) with the (non-slip) boundary conditions imposed on the velocity field $v = v(x, t)$:

$$v = 0 \text{ on } S; \quad v \equiv (v^1; v^2; v^3) = (u_1; u_2; w_t) \text{ on } \Omega,$$

where $u = u(x, t) \equiv (u^1; u^2; w)(x, t)$ is the displacement of the plate occupying $\Omega$. Here $w$ stands for transversal displacement, $\bar{u} = (u^1; u^2)$ — for lateral (in-plane) displacements.

These boundary conditions describe influence of the plate on the fluid. To describe influence of the fluid on the plate, we consider the surface force $T_f(v)$ exerted by the fluid, which is equal to $Tn|_\Omega$, where $n$ is the outer unit normal to $\partial \mathcal{O}$ at $\Omega$ and $T = \{T_{ij}\}_{i,j=1}^3$ is the stress tensor of the fluid,

$$T_{ij} \equiv T_{ij}(v) = \nu \left(v_{x_j}^1 + v_{x_i}^2\right) - p\delta_{ij}, \quad i, j = 1, 2, 3.$$

Since $n = (0; 0; 1)$ on $\Omega$, we have that

$$T_f(v) = (\nu(v_{x_3}^1 + v_{x_2}^2); \nu(v_{x_2}^2 + v_{x_3}^3); 2\nu\partial_x v^3 - p).$$

To describe the shell motion we use the full von Karman model which does not take into account rotational inertia of the filaments, but accounts for in-plane acceleration terms. Below for some simplification we assume that Young’s modulus $E$ and Poisson’s ratio $\mu \in (0, 1/2)$ are such that $Eh = 2(1 + \mu)$, where $h$ is the thickness of the plate. The corresponding PDE system is written in the form

$$\begin{align*}
w_{tt} + \Delta^2 w &= \text{div} \{C(P(u))\nabla w\} + G_3 - 2\nu\partial_x v^3 + p, \\
\bar{u}_{tt} &= \text{div} \{C(P(u))\} + \left(G_1 - \nu(v_{x_3}^1 + v_{x_2}^2) \right) + \left(G_2 - \nu(v_{x_3}^2 + v_{x_2}^3) \right)
\end{align*}$$

where $\bar{u} = (u^1; u^2)$, $G_{pl} = (G_1; G_2; G_3)$ is an external load, and $C(P(u))$ is the stress tensor with

$$\begin{align*}
C(\epsilon) &= 2(1 - \mu)^{-1} [\mu \text{trace} \epsilon \cdot I + (1 - \mu)\epsilon], \quad P(u) = \epsilon_0(\bar{u}) + f(\nabla w), \\
\epsilon_0(\bar{u}) &= \frac{1}{2}(\nabla \bar{u} + \nabla^T \bar{u}), \quad f(s) = \frac{1}{2}s \otimes s, \quad s \in \mathbb{R}^2.
\end{align*}$$

This form of the full von Karman system was used earlier by many authors in the case when the plate does not interacts with other objects ($v = 0$) (see, e. g., [14] and the references therein).
We impose the clamped boundary conditions on the plate
\[ u^1|_{\partial \Omega} = u^2|_{\partial \Omega} = w|_{\partial \Omega} = \frac{\partial w}{\partial n'}|_{\partial \Omega} = 0, \tag{7} \]
where \( n' \) is an outer normal to \( \partial \Omega \) in \( \mathbb{R}^2 \), and supply (1)–(7) with initial data for the velocity field \( v = (v^1; v^2; v^3) \) and the plate displacement vector \( u = (u^1; u^2; w) \)
\[ v|_{t=0} = v_0, \quad u|_{t=0} = u_0, \quad w|_{t=0} = u_1. \tag{8} \]
Here \( v_0 = (v^1_0; v^2_0; v^3_0), \quad u_j = (u^1_j; u^2_j; w_j), \quad j = 0, 1, \) are given vector functions subjected to some compatibility conditions which we specify later.

We note that (2) and (3) imply the following compatibility condition
\[ \int_{\Omega} w_t(x',t) \, dx' = 0 \quad \text{for all } t \geq 0. \tag{9} \]
This condition fulfills when
\[ \int_{\Omega} w(x',t) \, dx' = \text{const} \quad \text{for all } t \geq 0 \tag{10} \]
and can be interpreted as preservation of the volume of the fluid.

We emphasize, that even in the linear case we cannot split system (1)–(8) into two sets of equations describing longitudinal and transversal plate movements separately, i.e., we cannot reduce the model under consideration to the cases studied in [5, 8]. The point is that the surface force \( T_f(v) \) is not the sum of the corresponding loads in the models [5] and [8]. For the detailed discussion see [9], Remark 1.1 (C).

In this paper we deal with well-posedness of strong solutions to coupled system in (1)–(8) and their long-time dynamics. In the system under consideration rotational inertia is neglected, thus \( w_t \) has lower regularity (\( L^2(\Omega) \)), then is the case of rotational inertia accounted for, which was considered in [9]. Such regularity still allows us to prove existence of weak solutions satisfying the energy inequality the same way as in [9], but uniqueness of these solutions is still an open question. Sedenko’s method does not work here because the nonlinearity is strongly supercritical when rotational inertia is neglected.

Our main novelties are well-posedness of strong solutions and their uniform stability (in the case of zero external forces only). We face three main mathematical difficulties: (i) low regularity of time derivative of the transversal component of the plate displacement \( w_t \), (ii) a supercritical vector nonlinearity arising in the plate component, and (iii) a singular character of the surface force \( T_f(v) \), which is defined via boundary traces of the pressure and spatial derivatives of the fluid velocity field. The last two ones do not allow us to apply the methods developed for shell/plates with subcritical and critical nonlinearities (see, e.g., [6, 7] and the references therein), unlike [5, 8]. These difficulties were overcome in [9] for the system like (1)–(8) but with rotational inertia accounted for. However, in most of the proofs there \( H^1 \)-regularity of \( w_t \) is substantially used, that prevents application of such methods in this paper.

Our first main result (see Theorem 3.4) states well-posedness of problem (1)–(8) in the class of strong solutions. To achieve existence of strong solutions we prove additional smooth estimates for the first and second derivatives of the components of (1)–(8). Possibly they may be obtained by Sedenko’s method (see [18]), however, we prefer to use smoothing effect due to the linearized Navier-Stocks equations. The other issues of well-posedness are rather standard by now in view of the smoothing effect. We also show additional regularity of the strong solutions.
Our well-posedness result shows that the strong solutions to (1)–(8) generates an evolution semigroup $S_t$ on an appropriate state space.

Our second main result (see Theorem 4.2) deals with long-time dynamics and states uniform stability of the evolution semigroup $S_t$ in the case of zero external forces. First we show exponential stability of $S_t$ with respect to the energy norm for zero external forces. Then, using possibility to pick initial data of arbitrary small energy norm, we first prove boundedness of strong solutions and then their uniform stability. Regularizing effect of Navier-Stokes equations is substantially used in the proof. It allows us to discard any mechanical damping in the plate component $u$ and still have a stable system.

2. **Preliminaries.** In this section we introduce Sobolev type spaces we need and provide some results concerning the Stokes problem.

2.1. **Spaces and notations.** To introduce Sobolev spaces we follow approach presented in [21].

Let $D$ be a sufficiently smooth domain and $s \in \mathbb{R}$. We denote by $H^s(D)$ the Sobolev space of order $s$ on the set $D$ which we define as a restriction (in the sense of distributions) of the space $H^s(\mathbb{R}^d)$ (introduced via Fourier transform) to the domain $D$. We define the norm in $H^s(D)$ by the relation

$$
\|u\|_{s,D}^2 = \inf \left\{ \|w\|_{s,\mathbb{R}^d}^2 : w \in H^s(\mathbb{R}^d), \ w = u \text{ on } D \right\}.
$$

We also use the notation $\| \cdot \|_D = \| \cdot \|_{0,D}$ and $(\cdot, \cdot)_D$ for the corresponding $L_2$ norm and inner product. We denote by $H^s_0(D)$ the closure of $C_0^\infty(D)$ in $H^s(D)$ (with respect to $\| \cdot \|_{s,D}$) and introduce the spaces

$$
H^s_*(D) := \{ f|_D : f \in H^s(\mathbb{R}^d), \ \text{supp } f \subset \overline{D} \}, \ s \in \mathbb{R}.
$$

Below we need them to describe boundary traces on $\Omega \subset \partial \Omega$. We endow the classes $H^s_*(D)$ with the induced norms $\|f\|_{s,D}^* = \|f\|_{s,\mathbb{R}^d}$ for $f \in H^s_*(D)$. It is clear that

$$
\|f\|_{s,D} \leq \|f\|_{s,D}^*, \ f \in H^s_*(D).
$$

However, in general the norms $\| \cdot \|_{s,D}$ and $\| \cdot \|_{s,D}^*$ are not equivalent.

Understanding adjoint spaces with respect to duality between $C_0^\infty(D)$ and $[C_0^\infty(D)]'$ by Theorems 4.8.1 and 4.8.2 from [21] we also have that

$$
[H^s_*(D)]' = H^{-s}(D), \ s \in \mathbb{R}, \ \text{and } [H^s(D)]' = H^{-s}_*(D), \ s \in (-\infty, 1/2).
$$

Below we also use the factor-spaces $H^s(D)/\mathbb{R}$ with the naturally induced norm.

To describe fluid velocity field we introduce the following spaces.

Let $\mathcal{C}(\mathcal{O})$ be the class of $C^\infty$ vector-valued solenoidal (i.e., divergence-free) functions on $\overline{\mathcal{O}}$ which vanish in a neighborhood of $S$ and $C_0(\mathcal{O})$ be the class of $C_0^\infty(\mathcal{O})$ vector-valued solenoidal functions. We denote by $X$ the closure of $\mathcal{C}(\mathcal{O})$ with respect to the $L_2$-norm and by $V$ the closure of $\mathcal{C}(\mathcal{O})$ with respect to the $H^1(\mathcal{O})$-norm. Notations $V_0$, $X_0$ are used for the closure of $C_0(\mathcal{O})$ with respect to the $H^1(\mathcal{O})$-norm and $L_2(\mathcal{O})$-norm, respectively. One can see that

$$
X = \left\{ v = (v^1; v^2; v^3) \in [L_2(\mathcal{O})]^3 : \ \text{div } v = 0, \gamma_n v \equiv (v, n) = 0 \text{ on } S \right\}; \quad (11)
$$

and

$$
V = \left\{ v = (v^1; v^2; v^3) \in [H^1(\mathcal{O})]^3 : \ \text{div } v = 0, \ v = 0 \text{ on } S \right\}.
$$
We equip $X$ and $X_0$ with $L_2$-type norm $\| \cdot \|_\sigma$ and denote by $(\cdot, \cdot)_\sigma$ the corresponding inner product. We denote

$$V^\alpha = H^{1+\alpha}(\Omega) \cap V, \quad V_0^\alpha = H^{1+\alpha}(\Omega) \cap V_0, \quad \alpha \geq 0.$$ 

The spaces $V^\alpha$, $V_0^\alpha$ are endowed with the norm $\| \cdot \|_{V^\alpha} = \| \nabla \cdot \|_{\alpha, \sigma}$, and $V = V^0$. For the details concerning spaces of this type we refer to [20], for instance.

We also need the Sobolev spaces consisting of functions with zero average on the domain $\Omega$, namely we consider the space

$$\overset{\sim}{L}^4_2(\Omega) = \left\{ u \in L^2_2(\Omega) : \int_\Omega u(x')dx' = 0 \right\}$$

and also $\overset{\sim}{H}^s(\Omega) = H^s(\Omega) \cap \overset{\sim}{L}^4_2(\Omega)$ for $s > 0$ with the standard $H^s(\Omega)$-norm. The notations $\overset{\sim}{H}^s_0(\Omega)$ and $\overset{\sim}{H}^s_0(\Omega)$ have a similar meaning.

To describe plate displacement we use the spaces $W$ and denote by $\overset{\sim}{W}$ a subspace in $\mathcal{H}$ of the form

$$\mathcal{H} = \left\{ (v_0; u_0; u_1) \in X \times W \times Y : v_0^3 = u_3^3 \text{ on } \Omega \right\},$$

with the standard product norm.

We also denote by $\overset{\sim}{\mathcal{H}}$ a subspace in $\mathcal{H}$ of the form

$$\overset{\sim}{\mathcal{H}} = \left\{ (v_0; u_0; u_1) \in \mathcal{H} : w_0 \in \overset{\sim}{H}^2_0(\Omega) \right\},$$

where $w_0$ is the third component of the displacement vector $u_0$. Phase space for strong solutions will be defined later.

2.2. Stokes problem. In further considerations we need some regularity properties of the terms responsible for fluid–plate interaction. To this end we consider the following Stokes problem

$$-\nu \Delta v + \nabla p = g, \quad \text{div} v = 0 \text{ in } \Omega;$$

$$v = 0 \text{ on } S; \quad v = \psi = (\psi^1; \psi^2; \psi^3) \text{ on } \Omega,$$

where $g \in [L^2(\Omega)]^3$ and $\psi \in [L^2(\Omega)]^3 \times \overset{\sim}{L}^4_2(\Omega)$ are given. This type of boundary value problems for the Stokes equation was studied by many authors (see, e.g., [20] and references therein). We collect some properties of solutions to (16) in the following assertion.

Proposition 1. The following statements hold.

1. Let $g \in [H^{-1+\sigma}(\Omega)]^3$, and $\psi \in [H^{1/2+\sigma}(\Omega)]^3$ with $\int_\Omega \psi^3(x')dx' = 0$. Then for every $0 \leq \sigma \leq 1$ problem (16) has a unique solution $\{v; p\} \in [H^{1+\sigma}(\Omega)]^3 \times [H^\sigma(\Omega)/\mathbb{R}]$ such that

$$\|v\|_{[H^{1+\sigma}(\Omega)]^3} + \|p\|_{H^\sigma(\Omega)/\mathbb{R}} + \|T_f(v)\|_{[H^{1/2(\Omega)]^3} \leq c_0 \left\{ \|g\|_{[H^{-1+\sigma}(\Omega)]^3} + \|\psi\|_{[H^{3/2+\sigma(\Omega)]^3} \right\},$$

where $T_f(v)$ is defined by (4).
If \( g = 0, \psi \in [H^{-1/2+\sigma}(\Omega)]^3, 0 \leq \sigma \leq 1, \int_{\Omega} \psi^3(x')dx' = 0, \) then
\[
\|v\|_{H^s(\Omega)}^3 + \|p\|_{H^{-s+\sigma}(\Omega)/\mathbb{R}} \leq C_0 \|\psi\|_{[H^{-1/2+\sigma}(\Omega)]^3}.
\]
In particular, we can define a linear operator \( N_0 : [L^2(\Omega)]^2 \times \tilde{L}_2(\Omega) \to [H^{1/2}(\Omega)]^3 \) by the formula
\[
N_0\psi = w \iff \left\{ \begin{array}{l}
-\nu \Delta w + \nabla p = 0, \quad \text{div } w = 0 \quad \text{in } \Omega; \\
w = 0 \quad \text{on } S; \quad w = \psi \quad \text{on } \Omega,
\end{array} \right.
\]
for \( \psi \in [L^2(\Omega)]^2 \times \tilde{L}_2(\Omega) \) \((N_0\psi \text{ solves (16) with } g \equiv 0)\). It follows from two previous estimates that
\[
N_0 : [H^s_0(\Omega)]^2 \times \hat{H}^s_0(\Omega) \to [H^{1/2+s}(\Omega)]^3 \cap X \text{ continuously}
\]
for every \(-1/2 \leq s \leq 3/2\).

For the proof see [9].

2.3. Tensors. For convenience we collect here tensor properties which will be used in the paper. They can be easily verified by direct calculations.

Lemma 2.1. Let \( a(t), b(t) \) are vectors of length 2 and \( A \) is a symmetric tensor of order 4. Then
\[
\frac{d}{dt}(a(t) \otimes b(t)) = \frac{d}{dt} a(t) \otimes b(t) + a(t) \otimes \frac{d}{dt} b(t),
\]
\[
a(t) \otimes b(t) = (b(t) \otimes a(t))^T,
\]
\[
A \cdot (a(t) \otimes b(t)) = A \cdot (b(t) \otimes a(t)).
\]

For the stress tensor \( C \) the following properties hold.

Lemma 2.2. Let \( A, B \) are two tensors of order 4. Then
\[
C(A) \cdot B = C(B) \cdot A,
\]
\[
2\|A\|_{\Omega}^2 \leq \langle C(A), A \rangle_{\Omega} \leq \left( 2 + \frac{4\mu}{1 - \mu} \right) \|A\|_{\Omega}^2.
\]

3. Well-posedness theorem. To define weak (variational) solutions to (1)–(8) we need the following class \( \mathcal{L}_T \) of test functions \( \phi \) on \( \Omega \):
\[
\mathcal{L}_T = \left\{ \phi \in L_2(0, T; [H^1(\Omega)]^3), \phi_t \in L_2(0, T; [L^2(\Omega)]^3), \right. \\
\left. \text{div } \phi = 0, \phi|_{S} = 0, \phi|_{\Omega} = b = (b^1; b^2; b^3), \\
d \in L_2(0, T; \tilde{H}^2_0(\Omega)), b^j \in L_2(0, T; H^1_0(\Omega)), j = 1, 2, \\
d_t \in L_2(0, T; \tilde{L}_2(\Omega)), b^j_t \in L_2(0, T; L_2(\Omega)), j = 1, 2. \right\}
\]
We also denote \( \mathcal{L}_T^0 = \{ \phi \in \mathcal{L}_T : \phi(T) = 0 \} \).

Definition 3.1. A pair of vector functions \((v(t); u(t))\) with \( v = (v^1; v^2; v^3) \) and \( u = (u^1; u^2; u^3) \) is said to be a weak solution to the problem (1)–(8) on a time interval \([0, T]\) if
- \( v \in L_\infty(0, T; X) \cap L_2(0, T; V) \);
- \( u \in L_\infty(0, T; H^1_0(\Omega) \times H_0^1(\Omega) \times H^1_0(\Omega)) \);
- \((u^1_t; u^2_t; u^3_t) \in L_\infty(0, T; L^2(\Omega) \times L^2(\Omega) \times \tilde{L}_2(\Omega)) \) and \( u(0) = u_0 \);
- \( u_t \in L_2(0, T; [H^{1/2}(\Omega)]^3) \) and the compatibility condition \( v(t)|_{\Omega} = (u^1_t; u^2_t; u^3_t) \) holds for almost all \( t \in [0, T] \);
• for every $\phi \in L^0_T$ with $\phi|_\Omega = b = (b^1; b^2; d)$ the following equality holds:

$$ -\int_0^T (v, \phi_t)_\Omega dt + \nu \int_0^T E(v, \phi)_\partial \Omega dt - \int_0^T \left[ ([w_t, d_t])_{\Omega} + ([\bar{u}_t, \bar{b}_t])_{\Omega} \right] dt \\
+ \int_0^T (\Delta w, \Delta d)_\Omega dt + \int_0^T (C(P(u), \nabla \delta \otimes \nabla w + \epsilon_0(\bar{b}))_\Omega dt = \\
(\nu_0, \phi(0))_\partial \Omega + (w_1, d(0))_{\Omega} + ([\bar{u}_1, \bar{b}(0)])_{\partial \Omega} + \\
\int_0^T (G_{pl}, \phi)_\partial \Omega dt + \int_0^T (G_{pl}, b)_{\Omega} dt $$

(19)

where $\bar{u} = (u^1; u^2), \bar{b} = (b^1; b^2)$ and

$$ E(u, \phi) = \frac{1}{2} \sum_{i,j=1}^3 \left( v_{x_i}^2 + v_{x_j}^2 \right) \left( \phi_{x_i}^2 + \phi_{x_j}^2 \right). $$

Taking in (19) $\phi(t) = \int_0^T \chi(\tau) d\tau \cdot \psi$, where $\chi$ is a smooth scalar function and $\psi$ belongs to the space

$$ \tilde{V} = \left\{ \psi \in V \mid \psi|_\Omega = \beta \equiv (\beta^1; \beta^2; \delta) \in H^1_0(\Omega) \times H^1_0(\Omega) \times H^2_0(\Omega) \right\}, $$

one can see that the weak solution $(v(t); u(t))$ satisfies the relation

$$ \frac{d}{dt} [(v(t), \psi)_\partial \Omega + (w_t(t), \delta)_\Omega + (\bar{u}_t(t), \bar{\beta})] = \\
- [\nu E(v, \psi) + (\Delta w, \Delta \delta)_\partial \Omega + (C(P(u)), \nabla \delta \otimes \nabla w + \epsilon_0(\bar{b}))_{\partial \Omega} + \\
(G_{pl}(u), \psi)_\partial \Omega + (G_{pl}, \bar{b})_{\partial \Omega}.$$  

(20)

for all $t \in [0, T]$ and $\psi \in \tilde{V}$ with $\psi|_\Omega = \beta = (\beta^1; \beta^2; \delta)$ and $\bar{\beta} = (\beta^3; \beta^4)$.

The following theorem on existence of weak solutions can be proved the same way, as in [9].

**Theorem 3.2.** Assume that $U_0 = (v_0; u_0; u_1) \in H, G_{pl} \in V', G_{pl} = (G_1; G_2; G_3) \in [H^{-1/2}(\Omega)]^3$. Then for any interval $[0, T]$ there exists a weak solution $(v(t); u(t))$ to (1)-(8) with the initial data $U_0$. This solution possesses the following properties:

$$ U(t; U_0) \equiv U(t) \equiv (v(t); u(t); u_t(t)) \in L_\infty(0, T; X \times W \times Y), \\
||w_t(t)||_{H^{1/2}(\Omega)}^2 + ||u_t(t)||_{H^{1/2}(\Omega)}^2 + ||u_t^2(t)||_{H^{1/2}(\Omega)} \leq C||v(t)||_{\partial \Omega}^2 $$

(21)

for almost all $t \in [0, T]$.

The solution is bounded globally in $t$.

However, due to the strong supercriticality of the nonlinearity in full von Karman equation we didn’t manage to prove uniqueness of weak solutions. Therefore we build dynamical system in a space of more smooth functions.

To describe behaviour of the fluid component, we will use the space

$$ X_s = V^1_0 \oplus N_0(Y_s) $$

(22)

where $N_0(Y_s)$ is the image of $Y_s$ under $N_0$ in $V^{1/2}$. That is, $v \in X_s$ means $v = \hat{v} + N_0 u_1$, where $\hat{v} \in V^1_0$ and $u_1 \in Y_s$. The norm is defined as

$$ ||v||_{X_s}^2 = ||\hat{v}||_{V^1_0}^2 + ||N_0 u_1||_{3/2, \partial \Omega}. $$

For strong solutions we use the spaces

$$ \mathcal{H}_s = \{(v_0, u_0, u_1) \in X_s \times W_s \times Y_s : v_0|_\Omega = u_1 \}, \quad \tilde{\mathcal{H}}_s = \mathcal{H}_s \cap \mathcal{H}. $$

(23)
Definition 3.3. A pair of vector functions \((v(t); u(t))\) is said to be a strong solution to the problem (1)–(8) on a time interval \([0, T]\) if it is a weak solution to this problem on \((0, T)\) and
\[
(v; u; u_t)(t) \in L_{\infty}(0, T; X_s \times W_s \times Y_s).
\]

Remark 1. The phase space for strong solutions is in agreement with the domain of the generator of the semigroup in the problem of fluid-structure interaction that encounters for in-plane displacements of the plate only, see formula (22) and Remark 2.3 from [5].

Theorem 3.4. Assume that \(U_0 = (v_0; w_0; u_1) \in \mathcal{H}_s\), \(G_{fl} \in X\), \(G_{pl} \in [H^{1/2}(Ω)]^3\). Then for any interval \([0, T]\) there exists a unique strong solution \((v(t); u(t))\) to (1)–(8) with the initial data \(U_0\). This solution possesses the following properties:

- it is continuous with respect to \(t\) in the phase space, i.e.,
  \[
  U(t; U_0) \equiv U(t) \equiv (v(t); u(t); u_t(t)) \in C(0, T; X_s \times W_s \times Y_s),
  \]

- there exists \(C_T > 0\), depending on \(\|U_0\|_{\mathcal{H}_s}\), such that for all \(\varepsilon > 0\)
  \[
  \int_0^T dt \left( \|w_t(t)\|^2_{H^{3/2}(Ω)} + \|w_{tt}(t)\|^2_{H^{3/2}(Ω)} + \|w(t)\|^2_{H^{3/2}(Ω)} \right) \leq C_T
  \]

- The solutions depend continuously (in strong topology) on initial data in the space \(\mathcal{H}_s\).

- The energy balance equality
  \[
  E(v(t), u(t), u_t(t)) + \nu \int_0^t E(v, v) \, dt = E(v_0, u_0, u_1) + \int_0^t (G_{fl}, v) \, dt + \int_0^t (G_{pl}, u_t) \, dt
  \]
  is valid for every \(t > 0\), where the energy functional \(E\) is defined by
  \[
  E(v, u, u_t) = \frac{1}{2} \left( \|v\|^2_{2, Ω} + \|w_t\|^2_{2, Ω} + \|u_t\|^2_{2, Ω} + \|\Delta w\|^2_{2, Ω} + (C(P(u)), P(u))_{Ω} \right)
  \]

Remark 2. (A) \(H^{k+δ}(Ω) = H^k_δ(Ω) \cap H^{k+δ}(Ω)\) for \(δ \in [0, 1/2]\), but not for \(δ = 1/2\) [16]. Thus, extension by zero is not continuous from \(H^1_0(Ω) \cap H^{3/2}(Ω)\) to \(H^{3/2}(Ω)\) and the operator \(N_0\) does not map \(H^1_0(Ω) \cap H^{3/2}(Ω)\) into \(L^2(Ω)\). Therefore we cannot obtain \(v \in V^1\).

(B) For an uncoupled full von Karman plate Sedenko [17] obtained existence and uniqueness of strong solutions of the smoothness \((\tilde{u}; \dot{w})(t) \in L_{\infty}(0, T; [H^3(Ω) \cap H^1_0(Ω)]^2 \times H^4(Ω) \cap H^1_0(Ω)), (\ddot{u}; \ddot{w})(t) \in L_{\infty}(0, T; [H^2(Ω) \cap H^1_0(Ω)]^2 \times H^3(Ω))\). That is, \(\tilde{u} \in D(A^{1/2}), \ddot{u} \in D(A)\), where \(A\) is an operator generated by the form \(a(\cdot, \cdot)\), defined by (27) on \(H^1_0(Ω)\). But in our case, if we want to obtain \(v \in V^1\) or more smooth, we need to have \(u_t(t) \in L_{\infty}(0, T; [H^2_0(Ω)]^2)\). But \([H^2_0(Ω)]^2 \neq D(A)\), and Galerkin method cannot be used to prove, e.g., that \((v; w; u_t)(t) \in L_{\infty}(0, T; V^1 \times [H^3(Ω) \cap H^1_0(Ω)]^2 \times H^4(Ω) \cap H^1_0(Ω) \times [H^2_0(Ω)]^3)\), provided \((v; w; u_t)(0) \in V^1 \times [H^3(Ω) \cap H^1_0(Ω)]^2 \times H^4(Ω) \cap H^1_0(Ω) \times [H^2_0(Ω)]^3\). If such existence theorem has place, is an open question.

(C) The result takes place for hinged boundary conditions for the transversal component of the plate displacement as well. Whether uniform stability holds for another boundary conditions, e.g. in the case of free boundary conditions for the
transversal component or Neumann boundary conditions for the in-plane displacement, is an open question. These cases also require another phase space choice.

**Proof of theorem 3.4. Step 1.** Existence. The estimates below are obtained for approximate solutions. Construction method for approximate solutions is the same as in [9] (see proof of Theorem 3.3), and we describe it here for the sake of convenience only. This method is inspired by [3]. Let \( \{\psi_i\}_{i \in \mathbb{N}} \) be the orthonormal basis in \( X_0 \) consisting of the eigenvectors of the Stokes problem:

\[
-\Delta \psi_i + \nabla p_i = \mu_i \psi_i \quad \text{in } \mathcal{O}, \quad \text{div}\psi_i = 0, \quad \psi_i|_{\partial \mathcal{O}} = 0.
\]

Here \( 0 < \mu_1 \leq \mu_2 \leq \cdots \) are the corresponding eigenvalues. Denote by \( \{\xi_i\}_{i \in \mathbb{N}} \) the basis in \( \tilde{H}_0^2(\Omega) \) which consists of the eigenfunctions of the problem

\[
(\Delta \xi_i, \Delta w)_\Omega = \tilde{\kappa}_i (\xi_i, w)_\Omega, \quad \forall \ w \in \tilde{H}_0^2(\Omega),
\]

with the eigenvalues \( 0 < \tilde{\kappa}_1 \leq \tilde{\kappa}_2 \leq \cdots \) and such that \( (\xi_i, \xi_j)_\Omega = \delta_{ij} \). Further, let \( \{\eta_i\}_{i \in \mathbb{N}} \) be an orthogonal basis in \( H_0^1(\Omega) \times H_0^1(\Omega) \) such that \( (\eta_j, \eta_i)_\Omega = \delta_{ij} \), which consists of eigenfunctions of the problem

\[
a(\eta_i, w) = \tilde{\kappa}_i (\eta_i, w)_\Omega, \quad \forall \ w \in H_0^1(\Omega) \times H_0^1(\Omega),
\]

with the eigenvalues \( 0 < \tilde{\kappa}_1 \leq \tilde{\kappa}_2 \leq \cdots \) and \( ||\eta_i||_\Omega = 1 \). The form \( a(\eta, w) \) is given by

\[
a(\eta, w) = \sum_{i=1}^2 (\nabla \eta^i, \nabla w^j)_\Omega + \frac{1 + \mu}{1 - \mu} (\text{div} \eta^i, \text{div} w^j)_\Omega. \tag{27}
\]

Let \( \hat{\phi}_i = N_0(0;0;\xi_i) \) and \( \hat{\phi}_i = N_0(\eta_i;0) \), where the operator \( N_0 \) is defined by (17). By Proposition 1 we have that \( \phi_i, \hat{\phi}_i \in V^{1-\delta} \) for every \( \delta > 0 \).

In what follows we suppose \( \hat{\phi}_i = \hat{\phi}_i, \; \xi_i = (0;0), \; \phi_{n+i} = \hat{\phi}_i, \; \zeta_{n+i} = (0,0,\xi_i) \), for \( i = 1, \ldots, n \).

We define an approximate solution as a pair of functions \((v_{n,m};u_n)\):

\[
v_{n,m}(t) = \sum_{i=1}^m \alpha_i(t) \psi_i + \sum_{j=1}^{2n} \beta_j(t) \phi_j, \quad u_n(t) = \sum_{j=1}^{2n} \beta_j(t) \zeta_j + (0;0;(I-\hat{P})w_0),
\]

where \( \hat{P} \) is the projection on \( \tilde{H}_0^2(\Omega) \) in \( H_0^2(\Omega) \) which is orthogonal with respect to the inner product \((\Delta, \Delta)_\Omega\), which satisfy the relation (20)) and initial conditions

\[
v_{n,m}(0) = \Pi_m(v_0 - N_0 u_1) + N_0(R_n(u^1_1; u^2_1); P_n w_1), \quad u_n(0) = (R_n(u^1_0; u^2_0); P_n \hat{P} w_0 + (I-\hat{P})w_0), \quad \hat{u}_n(0) = (R_n(u^1_1; u^2_1); P_n w_1),
\]

where \( \Pi_m \) is the orthoprojector on \( \text{Lin}\{\psi_j : j = 1, \ldots, m\} \) in \( X_0 \), \( P_n \) is the orthoprojector on \( \text{Lin}\{\xi_i : i = 1, \ldots, m\} \) in \( L_2(\Omega) \) and \( R_n \) is the orthoprojector on \( \text{Lin}\{\xi_i : i = 1, \ldots, n\} \) in \( L_2(\Omega) \times L_2(\Omega) \).

Thus, we have for approximate (and weak) solutions (see [9])

\[
v_{n,m}(t) = \sum_{i=1}^m \alpha_i(t) \psi_i + N_0[\partial_t u_n(t)], \quad t \geq 0. \tag{28}
\]

In the arguments below we drop the subscripts for the sake of brevity.
Differentiating (20) with respect to $t$ and denoting $\tilde{v} = v_t$, $\tilde{u} = u_t$, $\tilde{w} = w_t$, we get that approximate solutions satisfy
\[
[\tilde{v}(t), \psi]_{\Omega} + \tilde{v}_t(t), \delta)_{\Omega} + (\tilde{u}_t(t), \tilde{\beta})_{\Omega} = \nu H(\tilde{v}, \psi) + (\Delta \tilde{w}, \Delta \tilde{\delta})_{\Omega} + (C(P(u, \tilde{u})), \nabla \delta \otimes \nabla w \epsilon_0(\tilde{\beta}))_{\Omega}
\]
(29)
where
\[
P(u, \tilde{u}) = \frac{d}{dt} P(u) = \epsilon_0(\tilde{u}) + \frac{1}{2} \left[ \nabla w \otimes \nabla \tilde{w} + \nabla \tilde{w} \otimes \nabla w \right].
\]
Setting in (29) $\psi = \tilde{v}, \tilde{\beta} = \tilde{u}_t, \delta = \tilde{w}_t$, we obtain
\[
\frac{d}{dt} \left[ ||\tilde{v}(t)||^2_{\Omega} + ||\tilde{u}_t(t)||^2_{\Omega} + ||\tilde{w}_t(t)||^2_{\Omega} + ||\Delta \tilde{w}||^2_{\Omega} \right] = -\nu H(\tilde{v}, \tilde{v})
\]
(30)
D\text{ue to the symmetry of } C(P(u)) \text{ we have}
\[
\left( \frac{d}{dt} C(P(u)), \nabla \tilde{w}_t \otimes \nabla w + \epsilon_0(\tilde{u}_t) \right)_{\Omega} = \\
\frac{1}{2} \left( \frac{d}{dt} C(P(u), ||\tilde{u}_t||^2_{\Omega}) - (C(P(u, \tilde{u})), \nabla \tilde{w} \otimes \nabla \tilde{w})_{\Omega},\right)
\]
(31)
We denote
\[
\tilde{E}(t) = \tilde{E}(t) = \tilde{E}(0) - \frac{1}{2} \left( ||\tilde{v}(t)||^2_{\Omega} + ||\tilde{u}_t(t)||^2_{\Omega} + ||\Delta \tilde{w}||^2_{\Omega} + (C(P(u, \tilde{u})), P(u, \tilde{u}))_{\Omega} \right),
\]
(31)
and after integration by $t$ get
\[
\tilde{E}(t) = \tilde{E}(0) - \frac{\nu}{2} \int_0^t H(\tilde{v}, \tilde{v})
\]
(32)
For the proof of well-posedness we need the following lemmas.

**Lemma 3.5.**

\[
||\tilde{u}||^2_{\Omega} \leq C(P(u, \tilde{u})), ||\Delta w||^2_{\Omega} + ||\Delta \tilde{w}||^2_{\Omega}.
\]

**Lemma 3.6** (Additional smoothness). In the following estimates $C^w_j$ are generic constants depending on $H$-norm of initial data and $f_j(u) = f_j(||\Delta w||_{\Omega}, ||\tilde{u}||_{1, \Omega})$ are generic functions that behave like $c_1||\Delta w||^2_{\Omega} + c_2||\tilde{u}||^3_{1, \Omega}$ near zero with $\alpha, \beta > 0$. $G$ means $(G_f, G_p)$.

- for every $\sigma \in [0, 1]$
  \[
  ||v||^2_{1, \Omega_{1+\sigma, \sigma}} \leq \frac{C}{1+\sigma, \sigma} \leq C(||\tilde{v}||^2_{1+\sigma, \sigma} + ||\tilde{u}||^2_{1/2+\sigma, \Omega});
  \]
- \[
  ||w||^2_{3, \Omega} \leq C^w_1[\tilde{E}(t) + ||G||^2 + f_1(u)];
  \]
- \[
  ||\tilde{u}||^2_{2, \Omega} \leq C^w_2[\tilde{E}(t) + ||G||^2 + f_2(u)];
  \]
• for every $\delta > 0$ and $\gamma > 0$

\[
\int_0^t d\tau |\vec{u}(\tau)|_{3/4,\Omega}^2 \leq \int_0^t d\tau E(\vec{v}, \bar{v}) + C_w \frac{1}{4\delta} \int_0^t d\tau [\vec{E}(\tau) + ||G||^2 + f_2(u)] \quad (36)
\]

\[
\int_0^t d\tau |w(\tau)|_{17/4,\Omega}^2 \leq C_w \int_0^t d\tau \left( \delta E(\vec{v}, \bar{v}) + (2 + \frac{1}{4\delta})\vec{E}(\tau) + ||G||^2 + f_3(u) \right) \quad (37)
\]

\[
\int_0^t d\tau |\vec{u}(\tau)|_{3/4,\Omega}^2 \leq \delta \int_0^t d\tau E(\vec{v}, \bar{v}) + C_w \left( 1 + \frac{1}{4\delta} \right) \int_0^t d\tau [\vec{E}(\tau) + ||G||_{1/4}^2 + f_2(u)] \quad (38)
\]

\[
\int_0^t d\tau |\vec{v}(\tau)|_{3/4,\Omega}^2 \leq 2\delta \int_0^t d\tau E(\vec{v}, \bar{v}) + \frac{C^w}{4\delta} \int_0^t d\tau \left[ \frac{C_w}{4\gamma^2} \vec{E}(\tau) + ||G|| + f_3(u) \right] \quad (39)
\]

\[
\int_0^t d\tau |w(\tau)|_{17/4,\Omega}^2 \leq \int_0^t d\tau \delta E(\vec{v}, \bar{v}) + C_w \left( 1 + \frac{1}{4\delta} \right) \int_0^t d\tau [\vec{E}(\tau) + ||G||_{1/4}^2 + f_3(u)] \quad (40)
\]

The lemma formulated above gathers all the smoothing effects, that fluid produce on the plate. These estimates are essentially used for the proof of the existence of strong solutions.

We'll prove the lemmas later, now we proceed with (32). First we estimate the third term of the right hand side in it. This term is a combination of terms of the form

\[
B_1 = \int_\Omega u_{ij}^t \vec{w}_{x_k} \vec{w}_{x_i} |_{0}^t, \quad B_2 = \int_\Omega w_x \vec{w}_{x_k} \vec{w}_{x_i} |_{0}^t,
\]

with $i, j, k, l = 1, 2$. Using Schwartz inequality and interpolation we obtain

\[
|B_1| \leq ||u_{ij}^t||_{L^2(\Omega)} ||\vec{w}_{x_k}||_{L^4(\Omega)} ||\vec{w}_{x_l}||_{L^4(\Omega)} ||\vec{w}_{x_i}||_{L^4(\Omega)} |_{0}^t \leq C ||\vec{u}||_{1,\Omega} ||\vec{w}_{x_k}||_{L^4(\Omega)} \Delta \vec{w}_{x_l} \Delta \vec{w}_{x_i} |_{0}^t,
\]

\[
|B_2| \leq ||\vec{w}_{x_k}||_{L^2(\Omega)} ||\vec{w}_{x_l}||_{L^4(\Omega)} ||\vec{w}_{x_i}||_{L^4(\Omega)} ||\vec{w}_{x_i}||_{L^4(\Omega)} |_{0}^t \leq C ||\vec{w}||_{1,\Omega} ||\vec{w}_{x_k}||_{L^4(\Omega)} \Delta \vec{w}_{x_l} \Delta \vec{w}_{x_i} |_{0}^t
\]

Noting, that the norm of solution in $H$ is bounded uniformly with respect to $t$ and using Young inequality with $p = 4/3$, $q = 4$, we obtain that

\[
(C(P(u)), \nabla \vec{w} \otimes \nabla \vec{w})_{\Omega} |_{0}^t \leq C^w_1 \vec{E}(0) + \frac{1}{4} ||\Delta \vec{w}(t)||_{1,\Omega}^2 + C^w_2.
\]

Now we turn to the last term of the right hand side in (32). It is a combination of terms of the form

\[
B_3 = \int_\Omega \vec{u}_{ij}^t \vec{w}_{x_k} \vec{w}_{x_i}, \quad B_4 = \int_\Omega w_x \vec{w}_{x_k} \vec{w}_{x_i},
\]

Using Hölder’s inequality, Galiardo-Nirenberg inequality, and estimate (39), we obtain

\[
|B_4| \leq \int_0^t ||w_x||_{L^4(\Omega)} ||\vec{w}_{x_j}||_{L^4(\Omega)} ||\vec{w}_{x_k}||_{L^4(\Omega)} ||\vec{w}_{x_i}||_{L^4(\Omega)} \leq
\]
we obtain \( H \) since every weak solution is globally bounded in \( X \).

In a similar way,

\[
\max_{\tau \in (0, T)} ||\Delta u||_{L^1}\int_0^t ||\tilde{u}||_{L^2}^2 d\tau \leq C_1^w \left( \int_0^t ||\tilde{v}||_{L^2}^2 d\tau \right)^{1/2} \left( \int_0^t ||\tilde{w}||_{L^2}^2 d\tau \right)^{1/2} \leq \]

\[
C_1^w \left( \int_0^t ||\Delta \tilde{w}||_{L^2}^2 d\tau \right)^{1/2} \left( \int_0^t ||\tilde{w}||_{L^2}^2 d\tau \right)^{1/2} \leq \]

\[
C_2^w \delta \int_0^t d\tau E(\tilde{v}, \tilde{v}) + C_2^w \int_0^t d\tau \left[ \frac{C^w_3}{4\gamma^2} \tilde{E}(\tau) + ||G|| + f_3(u) \right].
\]

Choosing \( \delta, \gamma \) small enough and substituting these estimates in (22), we get

\[
\frac{1}{2} \tilde{E}(t) = C_1^w \tilde{E}(0) - \frac{\nu}{2} \int_0^t E(\tilde{v}, \tilde{v}) + C_2^w \int_0^t d\tau \left[ \frac{1}{4\delta} \tilde{E}(\tau) + f_3(u) \right],
\]

since every weak solution is globally bounded in \( H \). Thus, using Gronwall’s lemma, we obtain

\[
\tilde{E}(t) \leq C_1^w(T) (\tilde{E}(0) + C_2^w).
\]

Now we need to prove that \( \tilde{E}(0) \) is bounded from above by the norm of initial data in \( H \) and this estimate does not depend on \( n, m \). Since \( \Pi_m, R_n \) and \( P_n \) are spectral projectors we have that

\[
\tilde{w}_n(0) = \partial_t w_n(0) \rightarrow w_1 \text{ strongly in } H^2_0(\Omega), n \rightarrow \infty,
\]

\[
\tilde{u}_n(0) = \partial_t \tilde{u}_n(0) \rightarrow \tilde{u}_1 \text{ strongly in } H^2_0(\Omega), n \rightarrow \infty.
\]

Substituting \( v_{n,m} \) in the form (28) in equation (1) and setting \( t = 0 \), we get

\[
\tilde{v}_{n,m}(0) = \partial_t v_{n,m}(0) = -A\tilde{v}_{n,m}(0) + \Pi_m G_{f1} + T_n G_{pl},
\]

where \( A \) is a Stokes operator, \( T_n \) is a projector onto \( Lin\{\phi_j, j = 1...n\} \), and \( \tilde{v}_{n}(0) = \Pi_n v_0 \). Thus, if \( v_0 \in H^2(\Omega) \cap X_0 \), then

\[
\tilde{v}_{n,m}(0) \rightarrow -A\tilde{v}_0 + G_{f1} \text{ strongly in } X_0.
\]

Further, equation (5) implies

\[
\partial_t w_n(0) = -\Delta^2 w_n(0) + P_n(\text{div}(C(P(u_n(0))))\nabla w_n(0) + G_{pl}^2) + P_n(T_f(\tilde{v}(0))) + P_n(T_f(N_0(\partial_t u_n(0)))).
\]

Since \( u_1 \in [H^2_0(\Omega)]^2 \times H^2_0(\Omega), \)

\[
P_n(T_f(N_0(\partial_t u_n(0)))) \rightarrow T_f(N_0u_1) \text{ strongly in } L^2(\Omega)
\]

due to Proposition 1. Thus, since \( u_0 \in W_s \) and \( \tilde{v}_0 \in V^1_0 \),

\[
\partial_t w_n(0) \rightarrow -\Delta^2 w_1 + (\text{div}(C(P(u_0))))\nabla w_0 + G_{pl}^2 + T_f(\tilde{v}_0) + T_f(N_0(u_1))
\]

strongly in \( L^2(\Omega) \). The same way, \( \partial_t \tilde{u}_n(0) \) converges strongly in \( [L^2(\Omega)]^2 \). Therefore

\[
\tilde{E}((\tilde{v}_{n,m}, \tilde{u}_n, \partial_t \tilde{u}_n)(0)) \leq C(||(v_0; u_0; u_1)||^2_{\tilde{H}_s}).
\]

This estimate together with Lemma 3.5, implies

\[
||v(t)||^2_{\tilde{H}_s} + ||u(t)||^2_{\tilde{H}_s} + ||\tilde{u}(t)||^2_{\tilde{H}_s} + ||\Delta w(t)||^2_{\tilde{H}_s} \leq C(T, ||(v_0; u_0; u_1)||^2_{\tilde{H}_s}) \quad \forall t \leq T.
\]
This estimate together with lemma 3.6 (after limit transition) give us existence of strong solutions.

**Step 2.** Improved regularity. Trace theorem implies \(|u_{tt}(t)|_{1/2,\Omega} \leq |\nabla v_t|_\Omega\), which together with equality (32) yield \(u_{tt}(t) \in L_2(0,T; H^{1/2}(\Omega))\). Since \(w \in H^4(\Omega)\), \(\text{div}(C(\nabla w \otimes \nabla w)) \in H^{1/2}(\Omega)\), and therefore \(\bar{u} \in L_2(0,T; H^{5/2}(\Omega))\). Due to this we can improve regularity results for the right-hand side of (5). It is enough to prove \(C(P(u))\nabla w \in H^{3/2}(\Omega)\). This expression consists of the terms of the form \(w_x, w_x, w_x, w_x\), where \(i, j, k\) can be 1 or 2. The multiples is contained at least in \(H^{3/2}(\Omega)\). Since for \(\Omega \subset \mathbb{R}^2\), \(H^{1+\varepsilon}(\Omega)\) is a multiplicative algebra, \(C(P(u))\nabla w \in L_2(0,T; H^{5/2}(\Omega))\) and thus \(w \in L_2(0,T; H^{9/2}(\Omega))\). We improve regularity of time derivatives using theorem on intermediate derivative [16]. In turn, this allows us improve regularity of \(v\) to \(V^{1-\varepsilon}\), \(\varepsilon > 0\), using Proposition 1. Estimate (25) is proved.

**Step 3.** Continuity with respect to \(t\). We use the standard scheme due to Lions [16] here. Aubin-Dubinsky theorem implies

\[
\begin{align*}
w &\in C(0,T; H^{4-\varepsilon}(\Omega)), \\
w_t &\in C(0,T; H^{2-\varepsilon}(\Omega)), \\
v &\in \Phi^{\varepsilon}(0,T; V_0^{1-\varepsilon}), \\
\bar{v} &\in C(0,T; H^{2-\varepsilon}(\Omega)), \\
\bar{u} &\in C(0,T; H^{1-\varepsilon}(\Omega)), \\
\bar{u}_t &\in C(0,T; H^{1-\varepsilon}(\Omega)), \\
N_0 u_t &\in C(0,T; V^{1/2-\varepsilon}).
\end{align*}
\]

Variational equality implies weak continuity:

\[
u_{tt} \in C_w(0,T; L_2(\Omega)), \quad v_t \in C_w(0,T; L_2(\Omega)).
\]

To use the standard techniques from [16], we need to prove “energy equality” (32) not only approximate solutions, but also for strong solutions. We can prove (32), using finite difference approximations of derivatives \(\tilde{w}_t, \tilde{u}_t\), the same way as in [14].

We denote

\[
D_h g(t) = \frac{1}{2h} (g(t+h) - g(t-h)), \quad h > 0
\]

where \(g(t) = g(0)\) for \(t < 0\) and \(g(t) = g(T)\) for \(t > T\). First we differentiate equations (1), (5), (6) with respect to \(t\). Multiplying the first equation by \(D_h w_t\) in \(L^2(\Omega)\), the second one by \(D_h w_t\) in \(L_2(\Omega)\), and the third one by \(D_h \bar{u}_t\), we obtain

\[
\begin{align*}
\int_0^t [\tilde{v}_t, D_h v]_\Omega + (\tilde{w}_t, D_h w_t)_\Omega + (\bar{u}_t, D_h \bar{u})_\Omega = & \\
\int_0^t [-\nu E(\tilde{v}, D_h v) - (\Delta \tilde{w}, \Delta D_h w_t)_\Omega + (C(P(u)), \nabla D_h w_t \otimes \nabla \tilde{w})_\Omega & \\
- (C(P(u, \tilde{u})), \nabla D_h w_t \otimes \nabla w + \epsilon_0(D_h \tilde{u}))_\Omega].
\end{align*}
\]

(44)

From variational equality (29) we get \(\tilde{w}_{tt} \in L_\infty(0,T; H^{-2}(\Omega))\), \(\tilde{u}_{tt} \in L_\infty(0,T; H^{-1}(\Omega))\), and \(\tilde{v}_t \in L_\infty(0,T; (V_0^1)' \cap (N_0(\bar{Y}))')\). Thus, using Proposition 4.3 form [14], we can perform limit transition in the left hand side of (44). Since \(u_t\) is weakly continuous with respect to \(t\) in \(\bar{Y}_s\), we can perform limit transition in two weakly weakly in the right hand side of (44). Thus,

\[
\frac{1}{2} ||v_t||^2_{\Omega} + ||\tilde{w}_t||^2_{\Omega} + ||\tilde{u}_t||^2_{\Omega} = -\frac{1}{2} ||\Delta \tilde{w}||^2_{\Omega} - \int_0^T [\nu E(\tilde{v}, \tilde{v}) - \\
- \lim_{h \to 0} \int_0^t (C(P(u, \tilde{u})), \nabla D_h w_t \otimes \nabla w + \epsilon_0(D_h \tilde{u}))_\Omega]
\]
\[
\lim_{h \to 0} \int_0^t \left( (C(P(u)), D_h w_t, \nabla \hat{w}) \right)_\Omega.
\]

The last term equals
\[
\lim_{h \to 0} \int_0^t (C(P(u)) \nabla \hat{w}, \nabla D_h w_t)_\Omega.
\]

Due to the smoothing effect (25) \( C(P(u)) \in L_2(0, T; H^{3/2}(\Omega)) \). Since \( \hat{w} \in L_2(0, T; H^0_0(\Omega)) \), \( C(P(u)) \nabla \hat{w} \in L_2(0, T; H^0_0(\Omega)) \). Proposition 4.3 [14] implies \( D_h \hat{w} \to \hat{w}_t \) weakly in \( L^2(0, T; L^2(\Omega)) \). Thus,
\[
\lim_{h \to 0} \int_0^t \left( (C(P(u)) \nabla \hat{w}, \nabla D_h w_t) \right)_\Omega = \int_0^t \left( (C(P(u)), \hat{w}_t, \nabla \hat{w}) \right)_\Omega.
\]

The term before last equals
\[
\int_0^t \left( (C(\epsilon_0(\tilde{u})), \epsilon_0(D_h \tilde{u})) \right)_\Omega + \int_0^t \left( (C(\epsilon_0(\tilde{u})), \nabla w, \nabla D_h \hat{w}) \right)_\Omega + \frac{1}{2} \int_0^t (C(\nabla w \otimes \nabla \hat{w} + \nabla \hat{w} \otimes \nabla w, \epsilon_0(D_h \tilde{u})) \right)_\Omega + \frac{1}{2} \int_0^t \left( (C(\nabla w \otimes \nabla \hat{w} + \nabla \hat{w} \otimes \nabla w) \nabla w, \nabla D_h \hat{w}) \right)_\Omega
\]

Limit transition is evident (Proposition 4.3 [14]) in all terms except the second. Due to smoothing effect \( C(\epsilon_0(\tilde{u})) \nabla w \in L_2(0, T; H^{1/2}(\Omega)) \) and \( D_h \hat{w} \to \hat{w}_t \) weakly in \( L^2(0, T; H^{1/2}(\Omega)) \). Thus,
\[
\lim_{h \to 0} \int_0^t \left( (C(\epsilon_0(\tilde{u})), \nabla w, \nabla D_h \hat{w}) \right)_\Omega = \int_0^t \left( (C(\epsilon_0(\tilde{u})), \nabla \tilde{w}_t \otimes \nabla w) \right)_\Omega.
\]

Performing calculations similar to that for obtaining (32), we conclude the proof of “Energy equality”.

Now we can use method of [16] to prove \( u_t \in C(0, T; L^2(\Omega)) \), \( u_t \in C(0, T; L_2(\Omega)) \), \( \tilde{u}_t \in C(0, T; H^1(\Omega)) \). Then theory of elliptic operators gives us desired continuity.

**Step 4.** Uniqueness. The proof cannot be done the same way, as in [17], because of coupling with fluid. We use here modification of the idea.

Let \((\tilde{v}; \tilde{u}) \) and \((\hat{v}; \hat{u})(t) \) — two strong solutions with the initial data \((\hat{v}_0; \hat{u}_0; \hat{u}_1) \) and \((\tilde{v}_0; \tilde{u}_0; \tilde{u}_1) \). Then their difference \((v; u)(t) = (\tilde{v}; \tilde{u})(t) - (\hat{v}; \hat{u})(t) \) satisfies
\[
v_t - \nu \Delta v + \nabla p = 0 \quad \text{in} \quad \Omega \times (0, +\infty), \tag{45}
\]
\[
\text{div} v = 0 \quad \text{in} \quad \Omega \times (0, +\infty), \tag{46}
\]
\[
v = 0 \quad \text{on} \quad \partial \Omega; \quad v \equiv (v^1; v^2; v^3) = (u^1_t; u^2_t; w_t) \quad \text{on} \quad \Omega, \tag{47}
\]
\[
w = 0 \quad \text{on} \quad \partial \Omega; \quad w^2 = \text{div} \left[ (C(P(\hat{u})) \nabla \hat{w} - C(P(\hat{u})) \nabla \hat{w}) \right] - 2 \nu \partial_{x^3} v^3 + p \quad \text{on} \quad \Omega, \tag{48}
\]
\[
\tilde{u}_t + C(\epsilon_0(\tilde{u})) = \text{div} \left( C(f(\nabla \hat{w}) - f(\nabla \hat{w})) \right) + \left( -\nu (v^1_s + v^3_s) \right). \tag{49}
\]

Multiplying the first equation by \( v \), the second one by \( w_t \) and the third one by \( \tilde{u}_t \) (since we consider strong solutions, we can perform this multiplication directly), we get
\[
\frac{d}{dt} \left[ \frac{1}{2} ||v(t)||^2_\Omega + ||w_t(t)||^2_\Omega + ||\tilde{u}_t(t)||^2_\Omega + (C(\epsilon_0(\tilde{u})), \epsilon_0(\tilde{u}))_\Omega + ||\Delta w||^2_\Omega \right] =
- \left[ \nu E(v, v) + (C(f(\nabla \hat{w}) - f(\nabla \hat{w})), \epsilon_0(\tilde{u}_t))_\Omega + (C(P(\tilde{u})) \nabla \hat{w} - C(P(\tilde{u})) \nabla \hat{w}, \nabla w_t)_\Omega \right]
\]
Noting that
\[
\frac{d}{dt} \left( \mathcal{C}(f(\nabla \bar{w}) - f(\nabla \bar{w})), \varepsilon_0(\bar{u}) \right)_\Omega = \left( \mathcal{C}(f(\nabla \bar{w}) - f(\nabla \bar{w})), \varepsilon_0(\bar{u}_t) \right)_\Omega +
\]
\[
(\mathcal{C}(\varepsilon_0(\bar{u})), \frac{1}{2}[\nabla w_t \otimes \nabla \bar{w} + \nabla w \otimes \nabla \bar{w}_t + \nabla \bar{w}_t \otimes \nabla w + \nabla \bar{w} \otimes \nabla w_t])_\Omega,
\]
after some calculations we obtain
\[
\mathcal{E}_t(v, u, u_t)(t) = - \frac{d}{dt} \left( \mathcal{C}(f(\nabla \bar{w}) - f(\nabla \bar{w})), \varepsilon_0(\bar{u}) \right)_\Omega + \frac{1}{2} (\mathcal{C}(\varepsilon_0(\bar{u})), \nabla w \otimes \nabla \bar{w}_t + \nabla \bar{w}_t \otimes \nabla w)_\Omega - \frac{1}{2} (\mathcal{C}(P(\bar{u}) + P(\bar{u})), \nabla w \otimes \nabla w_t)_\Omega - \frac{1}{2} (\mathcal{C}(f(\nabla \bar{w}) - f(\nabla \bar{w})), (\nabla \bar{w} + \nabla \bar{w}) \otimes \nabla w_t)_\Omega,
\]
where
\[
\mathcal{E}_t(v, u, u_t)(t) = \frac{1}{2} \|v(t)\|_\Omega + \|w_t(t)\|_\Omega + \|\bar{u}_t(t)\|_\Omega + (\mathcal{C}(\varepsilon_0(\bar{u})), \varepsilon_0(\bar{u})\Omega) + \|\Delta w\|_\Omega^2.
\]
In the following estimates of this step \(C_T\) is a generic positive constant depending on time interval \((0, T)\) and \(H_\varepsilon\)-norms of strong solutions \((\bar{v}, \bar{u})(t), (\bar{v}, \bar{u})(t)\) on this interval. The second term in (50) consists of the terms of the form
\[
\int_\Omega u_{x_j}^2 w_{x_k} w_{tx_1} \leq \|u_j\|_{L_4(\Omega)} \|w_{x_k}\|_{L_4(\Omega)} \|w_{tx_1}\|_{L_4(\Omega)} \leq C_T \|\bar{u}\|_{L_1(\Omega)} \|\Delta w\|_{L_4(\Omega)}.
\] 
(51)
The third term consists of the terms of the form
\[
B_2 = \int_\Omega \mathcal{C}(P(\bar{u}))_{ij} w_{x_k} w_{tx_1} = - \int_\Omega \partial_{x_j} \mathcal{C}(P(\bar{u}))_{ij} w_{x_k} w_t - \int_\Omega \mathcal{C}(P(\bar{u}))_{ij} w_{x_k} w_{tx_1} w_t
\]
for which the following estimates are valid
\[
\left| \int_\Omega \partial_{x_j} \mathcal{C}(P(\bar{u}))_{ij} w_{x_k} w_t \right| \leq \|P(\bar{u})\|_{L_1(\Omega)} \|w_{x_k}\|_{L_4(\Omega)} \|w_t\|_{L_4(\Omega)} \leq \delta E(v, v) + C_T/4\delta \|\Delta w\|_{\Omega}^2
\] 
(52)
\[
\left| \int_\Omega \mathcal{C}(P(\bar{u}))_{ij} w_{x_k} w_{tx_1} w_t \right| \leq \|w_{x_k} w_t\|_{L_1(\Omega)} \|P(\bar{u})\|_{L_4(\Omega)} \|w_t\|_{L_4(\Omega)} \leq \delta E(v, v) + C_T/4\delta \|\Delta w\|_{\Omega}^2.
\]
The similar way, for the last term holds
\[
|B_3| \leq C_T \|\Delta w\|_{\Omega} \|w_t\|_{\Omega}.
\] 
(53)
Using interpolation, for the first term we obtain
\[
\left| \int_0^t \frac{d}{dt} \left( \mathcal{C}(f(\nabla \bar{w}) - f(\nabla \bar{w})), \varepsilon_0(\bar{u}) \right)_\Omega \right| \leq C_T (\|\bar{u}(0)\|_{L_1(\Omega)}^2 + \|\Delta w(0)\|_{\Omega}^2) + \varepsilon (\|\bar{u}(t)\|_{L_4(\Omega)}^2 + \|\Delta w(t)\|_{L_4(\Omega)}^2) + C_T \varepsilon \|w(t)\|_{\Omega}^2
\] 
(54)
Thus, integrating (50) with respect to \(t\) and applying estimates (51)-(54), we obtain
\[
\mathcal{E}_t(v, u, u_t)(t) \leq C_T (\mathcal{E}_t(v, u, u_t)(0) + \|w(t)\|_{\Omega}^2) + \int_0^t (\mathcal{E}_t(v, u, u_t)(\tau)) - \frac{\nu}{2} \int_0^t E(v, v),
\]
To estimate $\|w(t)\|_{\Omega}^2$, we note that
\[
\|w(t)\|_{\Omega}^2 = \|w(0)\|_{\Omega}^2 + \frac{1}{2} \int_0^t d\tau (w(\tau), w_t(\tau))_{\Omega} \leq \|w(0)\|_{\Omega}^2 + \int_0^t d\tau \mathcal{E}_t(v, u, u_\tau)(\tau)
\]
which together with the previous estimate gives
\[
\mathcal{E}(v, u, u_t)(t) \leq C_T \mathcal{E}(v, u, u_t)(0).
\] (55)
Thus, uniqueness is proved.

**Step 5.** Continuity with respect to initial data. Estimate (55) gives us continuity with respect to initial data in $\mathcal{H}$-norm. Thus, the evolution operator $S_t$ maps a convergent strongly in $\mathcal{H}_s$ sequence to a convergent weakly in $\mathcal{H}_s$ sequence. To obtain continuity in $\mathcal{H}_s$ we can use method from [14], making use of “energy equality” (32).

Let
\[
w_{0n} \to w_0 \text{ strongly in } H^4(\Omega) \cap H^2_0(\Omega),
\]
\[
\bar{u}_{0n} \to \bar{u}_0 \text{ strongly in } H^2(\Omega) \cap H^1_0(\Omega),
\]
\[
w_{1n} \to w_1 \text{ strongly in } H^2(\Omega)
\]
\[
\bar{u}_{1n} \to \bar{u}_1 \text{ strongly in } H^1_0(\Omega),
\]
\[
v_{0n} = \bar{v}_{0n} + N_0u_{1n} \to \bar{v}_0 + N_0u_1 \text{ strongly in } X_s,
\]
which implies
\[
\partial_t u_{0n}(0) \to \partial_t u(0) \text{ strongly in } L_2(\Omega),
\]
\[
\partial_t v_{0n}(0) \to \partial_t v(0) \text{ strongly in } X.
\]
It follows from equations (1)–(8), that
\[
\partial_t u_{0n}(t) \to \partial_t u(t) \text{ weakly in } L_2(\Omega),
\]
\[
\partial_t v_{0n}(t) \to \partial_t v(t) \text{ weakly in } X.
\]
Thus, to prove continuity in the strong topology of $\mathcal{H}_s$, it is sufficient to prove
\[
\tilde{\mathcal{E}}((\tilde{v}_n, \tilde{u}_n, \partial_t \tilde{u}_n)(t)) \to \tilde{\mathcal{E}}((\tilde{v}, \tilde{u}, \partial_t \tilde{u})(t)) \text{ a.e.}
\]
“Energy equality” (32) implies
\[
\tilde{\mathcal{E}}((\tilde{v}_n, \tilde{u}_n, \partial_t \tilde{u}_n)(t)) = \tilde{\mathcal{E}}((\tilde{v}_n, \tilde{u}_n, \partial_t \tilde{u}_n)(0)) - \nu \int_0^t E(\tilde{v}_n, \tilde{v}_n) +
\]
\[
\frac{1}{2}(\mathcal{C}(P(u_n)), \nabla \tilde{w}_n \otimes \nabla \tilde{w}_n)_{\Omega}^T + 3 \int_0^t (\mathcal{C}(P(u_n, \tilde{u}_n)), \nabla \tilde{w}_n \otimes \nabla \tilde{w}_n)_{\Omega}.
\]
Due to weak convergence of $((v_n; u_n; \partial u_n)(t))$
\[
\mathcal{C}(P(u_n)) \to \mathcal{C}(P(u)) \text{ weakly in } L_2(\Omega)
\]
\[
\mathcal{C}(P(u_n, \tilde{u}_n)) \to \mathcal{C}(P(u, \tilde{u})) \text{ weakly in } L_2(\Omega)
\]
\[
\nabla \tilde{w}_n \otimes \nabla \tilde{w}_n \to \nabla \tilde{w} \otimes \nabla \tilde{w} \text{ strongly in } L_2(\Omega).
\] (56)
Variational equality (29) implies
\[
\tilde{v}_n \to \tilde{v} \text{ weakly in } L_2(0, T; V).
\]
Since $E(v, v)$ is equivalent norm on $V$,
\[
\lim_{n \to \infty} E(\tilde{v}_n, \tilde{v}_n) \geq E(\tilde{v}, \tilde{v}).
\] (57)
Estimates (56)-(57) give
\[ \lim_{h \to 0} \tilde{E}((\tilde{v}, n, \partial_t \tilde{u}_n)(t)) \leq \tilde{E}((\tilde{v}, u, \partial_t u)(t)) \]
which, together with weak convergence of \((\tilde{v}_n, \tilde{u}_n, \partial_t \tilde{u}_n)(t)\) and elliptical problems
theory give us the desired result.

Theorem 3.4 is proved. \(\Box\)

Proof of Lemma 3.6. Estimate (33) follows directly from Proposition 1.

Now we prove (34). Let’s denote by \(B\) the operator
\[ \Delta^2 : D(B) \subset L^2(\Omega) \to L^2(\Omega), \text{ with } D(B) = H^4(\Omega) \cap H_0^2(\Omega). \]
(Dirichlet boundary conditions). This operator is positive definite and self-adjoint,
thus it’s fractional powers are well-defined and \(D(B^{\frac{1}{2}}) = H^2(\Omega) \cap H_0^2(\Omega)\) and on
this space \(\| \cdot \|_{3,\Omega}\) is equivalent to \(\| B^{\frac{1}{2}} \cdot \|_\Omega \).
Multiplying equation (5) by \(B^{\frac{1}{2}} w\), we obtain
\[ \| B^{\frac{1}{2}} w \|_{\Omega}^2 \leq \| \Delta w \|_{\Omega}^2 (|\tilde{w}|_{L_2(\Omega)}|^2 + 2\nu \delta \xi_{x_j} + p)\| \Omega + |G_3| + |C(P(u))\| \| B^{\frac{1}{2}} w \|_{\Omega} \]
Using (33) with \(\sigma = 1/2\) and estimate(18), we get
\[ |T_f(v)|_{\Omega}^2 \leq C((1 + \| \Delta w \|_{\Omega}^2)\tilde{E}(t) + |G_{f1}|_{\Omega}^2). \] (58)

Now we need to estimate \(L_2\)-norm of \(C(P(u))\| \nabla w\). It consists of the terms of the
form \(B_1 = w_{x_j} w_{x_k}, B_2 = w_{x_x} w_{x_k}, i, j, k = 1, 2\). It is easy to verify that for every \(\varepsilon > 0\)
\[ \| B_1 \|_{\Omega}^2 \leq \| w_{x_j} \|_{L_\infty(\Omega)}^2 \| \tilde{u} \|_{H_1(\Omega)} \leq C \| w \|_{H_2(\Omega)}^2 \| \tilde{u} \|_{H_1(\Omega)}^2, \]
\[ \| B_2 \|_{\Omega}^2 \leq \| w_{x_x} \|_{L_\infty(\Omega)}^2 \| \nabla w \|_{L_4(\Omega)} \leq C \| w \|_{H_2(\Omega)}^2 \| \nabla w \|_{H_1(\Omega)} \| \Delta w \|_{\Omega}^2. \]
Using interpolation and Young inequality with \(p = 2/(1 + \varepsilon), q = 2/(1 - \varepsilon)\), we arrive
\[ \| C(P(u)) \| \nabla w \|_{\Omega} \| B^{\frac{1}{2}} w \|_{\Omega} \]
\[ \leq C \| \| \Delta w \|_{\Omega}^{1-\varepsilon} \| B^{\frac{1}{2}} w \|_{\Omega} |1 + \| \nabla w \|_{H_1(\Omega)} \| \Delta w \|_{\Omega}^2) \]
\[ \leq \frac{1+\varepsilon}{2} \| B^{\frac{1}{2}} w \|_{\Omega} + \frac{1-\varepsilon}{2} \| \Delta w \|_{H_1(\Omega)} + \| \nabla w \|_{H_1(\Omega)} \| \Delta w \|_{\Omega}^2 \]
This implies (34).

Having (34), we are in position to prove (35). If we consider (6) as an elliptic
problem with a known right hand side, we obtain
\[ \| \tilde{u} \|_{2,\Omega}^2 \leq C(\| \tilde{u} \|_{\Omega}^2 + ||T_f(v)||_{\Omega}^2 + ||B(w)||_{\Omega}^2 + |G_1|_{\Omega}^2 + |G_2|_{\Omega}^2), \]
where \(B(w) = \text{div} (C(f(\nabla w)))\). Since \(\| B(w) \|_{\Omega}^2 \leq \| \Delta w \|_{\Omega}^2 \| w \|_{3,\Omega}^2 \) and (58) holds,
(35) is proved.

Using trace estimate and Theorem on intermediate derivative [16], for every \(\delta > 0\) we obtain
\[ \int_0^T \| \tilde{u} \|_{3/4,\Omega}^2 \leq C \left( \delta \int_0^T E(\tilde{v}, \tilde{v}) dt + \frac{1}{4\delta} \int_0^T \| \tilde{u} \|_{2,\Omega}^2 \right), \]
which together with (35) implies (36).

This estimate allows us to get better smoothness of \(v\), and we can use (33) with
\(\sigma = 3/4\). Therefore the following estimate takes place:
\[ \int_0^t d\tau ||\tilde{u}||_{9/4,\Omega}^2 \leq C \int_0^t d\tau (||\tilde{u}||_{7/4,\Omega}^2 + ||T_f(v)||_{7/4,\Omega}^2 + ||B(w)||_{7/4,\Omega}^2 + |G|_{7/4,\Omega}^2). \]
Proposition 2. Denote $H$ and using subsequently (33) with $\sigma = 3/4$ and (36), we obtain (38).

Now we estimate $||w||_{4.\Omega}$. It follows from (5) that

$$||w||_{4.\Omega} \leq C(||u||_3^2 + ||\nabla (C(P(u))\nabla w)||).$$

If we expand the last term, we’ll see we need to estimate $K_1 = ||u_{x_jx_i}w_{x_k}||_\Omega$, $K_2 = ||w_{x_jx_i}w_{x_k}||_\Omega$, $K_3 = ||u_{x_j}^4w_{x_k}||_\Omega$. It is easy to see, that

$$K_2 \leq C||\Delta w||_\Omega||w||_{3.\Omega},$$

$$K_3 \leq C||u_{x_j}^4||_{4.\Omega}||w_{x_k}||_\Omega \leq C||\Delta w||_\Omega^{1/2}||w||_\Omega^{1/2}||\tilde{u}||_2^{1/2}||\tilde{u}||_2^{1/2}.$$ (61)

For $K_1$ we have

$$K_1 \leq ||w_{x_k}||_{L_\infty(\Omega)}||u_{x_j}^4||_\Omega ||w||_{2+\varepsilon,\Omega}||\tilde{u}||_{2,\Omega}$$ (62)

for any $\varepsilon > 0$. This implies $||w||_{4.\Omega}$ is bounded on $[0, T]$ provided $\tilde{E}(t)$ is bounded on $[0, T]$. However, the overall degree of smooth norms is too high to use this estimate in Gronwall inequality. Using interpolation, we get

$$K_1 \leq ||\Delta w||_\Omega^{1-\varepsilon}||w||_3^{1/4}||\tilde{u}||_1^{1/4}||\tilde{u}||_{4/3}^{1/4}.$$ (64)

Integrating (59) with respect to $t$ from 0 to $T$, using Young inequality in the last estimate for $K_1$ and applying estimate (38), we obtain (37).

We prove estimates (39) and (40) the same way, as (36) and (38). \qed

4. Asymptotic behaviour. Theorem 3.4 means that solutions to (1)-(8) generates dynamical system $(H_s, S_s)$ in the following way: $S_s(v_0; u_0; u_1) = U(t) = (v(t); u(t); u(t))$, where the couple $(v(t); u(t))$ solves (1)-(8).

Remark 3. Due to preservation of mean of transversal displacement $w$ (10) dynamical system $(H_s, S_s)$ cannot be dissipative. Therefore we consider DS on subspaces where $w$ has zero mean, namely, $(\tilde{H_s}, S_s)$. Spaces $\tilde{H}$ and $\tilde{H_s}$ are defined by (15) and (23), respectively.

In this section we assume that

$$G_{\tilde{H}} \equiv 0, G_1 = G_2 \equiv 0 \text{ and } G_3 \equiv g \in H^2(\Omega).$$ (63)

In this case set of stationary points of the dynamical system $(H_s, S_s)$ is non-empty and bounded, see Proposition 4.2 [9]. We also modify energy for this section, setting

$$E(v, u, u_1) = \frac{1}{2} ||v||_\Omega^2 + ||w||_\Omega^2 + ||\tilde{u}||_\Omega^2 + ||Dw||_\Omega^2 + (C(P(u)), P(u))_\Omega + 2(G_3, w)_\Omega$$

For a proof of dissipativity we need the following estimates.

Proposition 2. Denote $A(u) := ||\Delta w||_\Omega^2 + (C(P(u)), P(u))_\Omega$. Then

$$c_1||w||_W - c_2 \leq A(u) \leq ||w||_W^2 + ||\Delta w||_\Omega^2$$ (64)

$$||\tilde{u}||_\Omega^2 \leq c_3(C(P(u)), P(u))_\Omega + c_4||\nabla w||_\Omega^2||\Delta w||_\Omega^2.$$ (65)

Let $||u||_W \leq R$. Then

$$A(u) \geq \frac{||u||_W^2}{C(R^2 + 2)}.$$ (66)
Proof. The lemma can easily be proved by direct calculations with the help of Korn inequality.

Lemma 4.1. \((\hat{H}, S_t)\) is \((\hat{H}, \hat{H})\)-dissipative, i.e. there exists \(B_0\) bounded in \(\hat{H}\) such that for any \(B \subset \hat{H}\), there exists time \(t_0(B)\) such that for every \(t > t_0(B)\) \(S_t(B) \subset B_0\).

Proof. Let’s denote by \(\hat{H}_R\) the subset of \(\hat{H}\) such that \(E(v, u_0, u_1) \leq R^2\). We will prove, that dissipativity radius \(R_0\) is the same for every DS \((\hat{H}_R, S_t)\), that gives us the desired dissipativity.

Like in [5], we use as a Lyapunov function
\[
\Psi(v, u_0, u_1) = E(v, u_0, u_1) + \eta ((u_0, u_1) + (v, N_0 u_0)_\Omega)
\]  
where we chose appropriate \(\eta\) (depending on \(R\)). First we estimate full time derivative along the trajectory for the second term. Using Green formula and properties of \(N_0\), we obtain
\[
\frac{d}{dt} [(u, u_t) + (v, N_0 u)_\Omega] = \|u_t\|_{\Omega}^2 - \|\Delta w\|_{\Omega}^2 - (\bar{C}(P(u)), P(u))_\Omega +
\]
\[
E(v, N_0 u) + (G_3, w)_\Omega + (v, N_0 u_t) \leq
\]
\[
2\|u_t\|_{\Omega}^2 + C\|v\|_{\Omega}^2 - \|\Delta w\|_{\Omega}^2 - (\bar{C}(P(u)), P(u))_\Omega + C_{\delta_1} \|G_3\|_{\Omega}^2 +
\]
\[
\delta_1 \|w\|_{\Omega}^2 + C_{\delta_2} \|\nabla v\|_{\Omega}^2 + \delta_2 \|\Delta w\|^2_{\Omega} + \delta_2 \|\tilde{u}\|_{\Omega}^2.
\]
To compensate positive \(\|\tilde{u}\|_{\Omega}^2\) we use (65) and boundedness of the trajectory. Thus,
\[
\frac{d}{dt} [(u, u_t) + (v, N_0 u)_\Omega] \leq 2\|u_t\|_{\Omega}^2 - (1 - \delta_1 - \delta_2 (1 + c_4 R)) \|\Delta w\|_{\Omega}^2
\]
\[
- (1 - \delta_2 c_3) (\bar{C}(P(u)), P(u))_\Omega + C_{\delta_1} \|G_3\|_{\Omega}^2 + (C_{\delta_2} + C_{\delta_1}) \|\nabla v\|_{\Omega}^2
\]
Evidently, we can choose \(\delta_1\) not depending on \(R\) and \(\delta_2\) depending on \(R\) to obtain proper signs of the terms.

Now we need to choose \(\eta = \eta(R)\) to guarantee \(\Psi\) is bounded from below by a function depending on the \(\Omega\)-norm, which goes to infinity as the norm goes to infinity. Using (65) and properties of \(N_0\), we obtain
\[
\Psi(v, u_0, u_1) \geq \frac{1}{2} \left[ \|v\|_{\Omega}^2 + \|u_1\|_{\Omega}^2 + \|\Delta w_0\|_{\Omega}^2 + (\bar{C}(P(u_0)), P(u_0))_\Omega \right]
\]
\[
- \delta \|\Delta w_0\|_{\Omega}^2 - C_{\delta_1} \|G_3\|_{\Omega}^2 - \frac{\eta}{2} \left( \|u_1\|_{\Omega}^2 + \|v\|_{\Omega}^2 \right)
\]
\[
+ (1 + c) c_3 (\bar{C}(P(u_0)), P(u_0))_\Omega + (1 + c) (1 + c_4 R) \|\Delta w_0\|_{\Omega}^2.
\]
Setting \(\delta = 1/8\) and choosing \(\eta < 1/2\) such that \(\eta (1 + c_3) < 1/2, \eta (1 + c_3) (1 + c_4 R) < 1/4\), we obtain
\[
\Psi(v, u_0, u_1) \geq \frac{1}{4} \left[ \|v\|_{\Omega}^2 + \|u_1\|_{\Omega}^2 + \|\Delta w_0\|_{\Omega}^2 + (\bar{C}(P(u_0)), P(u_0))_\Omega \right] -
\]
\[
2 \|G_3\|_{\Omega}^2 \geq C\|\bar{C}(v; u_0; u_1)\|_{\Omega} - D,
\]
where constants \(C, D\) don’t depend on \(R\). Finally, using estimate (21), for every \(\eta : (2 \gamma_1 + C_{\delta_2} + C_{\delta_1}) < \nu /2\), we get
\[
\frac{d}{dt} \Psi((v; u; u_t)(t)) + C_{\delta_1} \eta \bar{E}((v; u; u_t)(t)) \leq \eta \bar{C}_{\delta_1} \|G_3\|^2_{\Omega}
\]
with \(C_1, C_{\delta_1}\) not depending on \(R\). This proves the lemma. The radius of dissipativity \(R_d\) is of the form \(c_2 + C \|G_3\|_{\Omega}^2\), where \(c_2\) is from (64).
Corollary 1. Let $G_3 = 0$. Then the dynamical system $(\hat{H}_s, S_t)$ is uniformly exponentially stable with respect to $\mathcal{H}$-norm.

Proof. This follows from Lemma 4.1 and estimate (66).

In proof of existence of strong solutions we estimate integrals with respect to $t$ of RHS. This prevents to build Lyapunov function in $\hat{H}_s$ at once, like in [4]. Instead, first we prove global boundedness of a strong solution in $\hat{H}_s$ and then prove stability of $(\hat{H}_s, S_t)$.

Theorem 4.2. If $G_3 = 0$, $(\hat{H}_s, S_t)$ is uniformly stable.

Proof. Let $B$ be a set of initial data bounded in $\hat{H}_s$, i.e., there exists $R > 0$ such that for all $U_0 \in B$ $||U_0||_{\hat{H}_s} \leq R$. Then due to Corollary 1 there exists $t_0 = t_0(B, R_d)$ such that for all $t \geq t_0$ $||S_t U_0||_{\hat{H}_s} \leq R_d$. Thus, we can chose initial data from the ball $B_{R_d}$ in $\mathcal{H}$ of arbitrary small radius $R_d$. Note, that $\hat{H}_s$-norm of $S_t U_0$ may increase, but due to Theorem 3.4 $||S_t U_0||_{\hat{H}_s} \leq C(t_0, ||U_0||_{\hat{H}_s})$.

In this proof we work with approximate solutions and make a limit transition in the very end of each step.

Step 1. Global boundedness of the trajectory. In this proof we utilize the function

$$\Psi(t) = \bar{E}(t) + \eta(\tilde{\bar{u}}(\tilde{\bar{u}}) + (\tilde{\bar{v}}, N_0 \tilde{\bar{u}})_{\Omega}],$$

where $\eta$ will be chosen later. To differentiate the second term, we use variational equation (29). Substituting $\delta = \tilde{\bar{w}}, \beta = \tilde{\bar{u}}, \psi = N_0 \tilde{\bar{u}}$ into resulting relation, we arrive

$$d(\tilde{\bar{u}} + \tilde{\bar{u}})_{\Omega} + (\tilde{\bar{v}}, N_0 \tilde{\bar{u}})_{\Omega} = ||\tilde{\bar{u}}||^2_{\Omega} + (\tilde{\bar{v}}, N_0 \tilde{\bar{u}})_{\Omega} - \nu E(\tilde{\bar{v}}, N_0 \tilde{\bar{u}}) + ||\Delta \tilde{\bar{w}}||^2_{\Omega} + (C(P(u, \tilde{\bar{u}}), P(u, \tilde{\bar{u}}))_{\Omega} + (C(P(u))_{\Omega} \tilde{\bar{v}} \tilde{\bar{w}} \tilde{\bar{w}} \tilde{\bar{w}} \tilde{\bar{w}} \tilde{\bar{w}})_{\Omega} \leq \nu E(\tilde{\bar{v}}, N_0 \tilde{\bar{u}}) + C_1 (1 + \nu) E(\tilde{\bar{v}}, \tilde{\bar{v}}) + C_2 ||\tilde{\bar{u}}||_{\Omega}^2 + C_3 ||C(P(u))_{\Omega}||_{\Omega} ||\tilde{\bar{w}}||_{\Omega}^{1/2} ||\Delta \tilde{\bar{w}}||_{\Omega}^{3/2}.$$

We chose $\eta < 1/2, \eta(1 + c_1)(1 + c_4 R_d) < 1/2$ to make functional $\Psi(t)$ bounded from below as in Lemma 4.1. Then we obtain the following estimate for $\Psi(t)$, integrating inequality above with respect to $t$ and using estimates (41), (43), (42), and Young inequality for the last term in the previous estimate:

$$\Psi(T) + (\eta - \varepsilon) \int_0^T \bar{E}(t) dt \leq C_{R_d} \Psi(0) - (\nu/2 - C_1 (1 + \nu)) \int_0^T E(\tilde{\bar{v}}, \tilde{\bar{v}}) + C_2 \int_0^T E(v, v) dt + C_3 \int_0^T C_{P_a} \int_0^T (E(\tilde{\bar{v}}, \tilde{\bar{v}}) + \frac{1}{4\delta} \bar{E}(t) + f_{3}(u)) dt dt,$$

where $C_{P_a}$ are generic constants which depend only on $R_d$ — the $\mathcal{H}$-radius of the ball, from which initial data are taken, and $C_{P_a} \to 0$ when $R_d \to 0$. The integral $\int_0^\infty f_{3}(u(t)) dt < C_{R_d} \to \infty$ due to the character of $f_{3}(u)$ and Corollary 1. Now we make $\eta$ to be smaller, then $\nu/(1 + \nu) \cdot 1/(4C_1)$, if needed. We need to choose $R_d$ and $\delta$ in such a way, that

$$\frac{C_{R_d}^2}{2\delta} < \eta, \quad C_{R_d}^2 \delta < \frac{\nu}{4}.$$
Multiplying the first inequality by the second, we get

\[
(C_{Rd}^2)^2 < \frac{\eta \nu}{2}, \quad \frac{C_{Rd}^2}{\eta} < \delta < \frac{\nu}{4C_{Rd}^2}
\]

Thus, we can chose \( R_d \) small enough to guarantee existence of \( \delta \) satisfying the second inequality.

Due to the energy equality (26)

\[
C_2 \int_0^T E(v, v) dt + 1/4|\Delta w(T)|_{\Omega}^2 \leq C\mathcal{E}(0).
\]

Finally, we arrive

\[
\tilde{\Psi}(T) + \mu_1 \int_0^T \tilde{\mathcal{E}}(t) dt + \mu_2 \int_0^T E(\tilde{v}, \tilde{v}) dt \leq C_{Rd}^2 \tilde{\Psi}(0) + C_{Rd}^2.
\]

This implies

\[
\|u_{tt}(t)\|_{1, \Omega}^2 + \int_0^t \|u_{tt}(\tau)\|_{1, \Omega}^2 d\tau \leq C(\|([v_0; u_0; u_1])\|_{\tilde{H}_s}) \tag{70}
\]

\[
\|\bar{u}_{t}(t)\|_{1, \Omega}^2 + \int_0^t \|\bar{u}_{t}(\tau)\|_{1, \Omega}^2 d\tau \leq C(\|([v_0; u_0; u_1])\|_{\tilde{H}_s}) \tag{71}
\]

\[
\|\Delta w(t)\|_{\Omega}^2 + \int_0^t \|\Delta w(t)\|_{\Omega}^2 d\tau \leq C(\|([v_0; u_0; u_1])\|_{\tilde{H}_s}) \tag{72}
\]

\[
\|v_{t}(t)\|_{\Omega}^2 + \int_0^t \|\nabla v_{t}(\tau)\|_{\Omega}^2 d\tau \leq C(\|([v_0; u_0; u_1])\|_{\tilde{H}_s}) \tag{73}
\]

where constants \( C(\|([v_0; u_0; u_1])\|_{\tilde{H}_s}) \) do not depend on \( t \). Using (35), we conclude

\[
\|\bar{u}(t)\|_{2, \Omega}^2 + \int_0^t \|\bar{u}(\tau)\|_{2, \Omega}^2 d\tau \leq C(\|([v_0; u_0; u_1])\|_{\tilde{H}_s}).
\]

Estimates (37) and (59)-(62) give

\[
\|\Delta^2 w(t)\|_{\Omega}^2 + \int_0^t \|\Delta^2 w(t)\|_{\Omega}^2 d\tau \leq C(\|([v_0; u_0; u_1])\|_{\tilde{H}_s}). \tag{74}
\]

Since \( v_{t}(t) = A \tilde{v}(t) \), we have similar estimate for \( v(t) \). Now we perform limit transition to justify estimates (70)-(74) for strong solutions with initial lata form \( B_{Rd} \subset \tilde{H} \). Thus, we prove that strong solutions are bounded globally in \( t \).

**Step 2.** Uniform stability. Now we are in position to prove uniform stability of \((\tilde{H}_s, S_1)\). In this step we use modified energy functional

\[
\tilde{\mathcal{E}}_{mod}(t) = \tilde{\mathcal{E}}(t) + \langle C(P(\bar{u}, \bar{u}), \nabla \bar{w} \otimes \nabla \bar{w}) \rangle_{\Omega}
\]

to eliminate terms \( B_1, B_2 \) (see proof of Theorem 3.4, step 1) from the estimate. Since

\[
|\langle C(P(\bar{u}, \bar{u}), \nabla \bar{w} \otimes \nabla \bar{w}) \rangle_{\Omega}| \leq C(\|\bar{u}\|_{1, \Omega} + \|\Delta w\|_{\Omega}^2)\|\Delta \bar{w}\|_{\Omega}^2,
\]

due to Corollary 1 we can chose initial data from \( B_{Rd} \subset \tilde{H} \) with radius small enough to make \( \tilde{\mathcal{E}}_{mod}(t) \) bounded from below. As in step 1, we chose \( \eta \) small enough to guarantee

\[
\tilde{\Psi}_{mod}(t) = \tilde{\mathcal{E}}_{mod}(t) + \eta [\bar{u}(t), \bar{u}(t)]_{\Omega} + (\tilde{v}, N_0 \bar{u})_{\Omega},
\]
is bounded from below. We do some modifications of previous estimate. In (68) we
do not estimate the last term and use interpolation for $||\hat{u}||_{1/2,\Omega}$. Thus,
\[
\frac{d}{dt}[(\hat{u}, \hat{u}_t)_\Omega + (\hat{v}, N_0 \hat{u})_\Omega] \leq \\
-||\Delta \hat{w}||^2_{\Omega} - (C(P(u, \hat{u})), P(u, \hat{u}))_\Omega - (C(P(u, \hat{u})), \nabla \hat{w} \otimes \nabla \hat{w})_\Omega + \\
C_1 E(\hat{v}, \hat{v}) + \delta||\hat{u}||^2_{1,\Omega} + \delta||\Delta \hat{w}||^2_{\Omega} + \frac{C_2}{\delta}||u_t||^2_{\Omega}.
\]

To estimate the time derivative of $\tilde{E}_{mod}(t)$, we need to estimate quantities $B_3, B_4$
not integrated with respect to $t$. Using interpolation, Holder inequality, and bound-
edness of the trajectory in $\hat{\mathcal{H}}_s$, we get
\[
\left| \int_{\Omega} w_{x_j} \hat{w}_{x_j, k} \hat{w}_{x_k} \right| \leq C \eta w_{x_j} ||\hat{w}_{x_j, k}|| ||\hat{w}_{x_k}|| ||\hat{w}_{x_k}|| \leq C^s ||\Delta \hat{w}||_{1,\Omega}^{3/4} ||\Delta \hat{w}||_{\Omega}^{1/4}, \\
\left| \int_{\Omega} \hat{u}^T\hat{w}_{x_j, k} \hat{w}_{x_k} \right| \leq C \eta ||\Delta \hat{w}||_{1,\Omega}^{1/4} ||\hat{u}|| ||\hat{w}_{x_k}|| ||\hat{w}_{x_k}|| \leq C^s ||\hat{u}||_{1,\Omega}^{3/4} ||\Delta \hat{w}||_{\Omega}^{1/4},
\]
where $C^s$ are constants that depend only on $||\{(v_0; u_0; u_1)\}||_{\hat{\mathcal{H}}}$.

Similarly as in step 1, choosing $\eta$ and $\delta$ small enough, we get
\[
\frac{d}{dt} \tilde{\Psi}_{mod}(t) \leq -\frac{\eta}{2} \left[ ||\Delta \hat{w}||_{\Omega}^{2} + (C(P(u, \hat{u})), P(u, \hat{u}))_\Omega + (C(P(u)), \nabla \hat{w} \otimes \nabla \hat{w})_\Omega - \\
\nu E(\hat{v}, \hat{v}) + C^s ||\Delta \hat{w}||_{1,\Omega}^{3/4} ||u_t||_{\Omega}^{1/4} + \eta C_{\delta} ||u_t||_{\Omega}^{2} \right] \leq \frac{\eta}{4}
\]
Due to Corollary 1 we can chose initial data from a ball $B_{R_4} \subset \mathcal{H}$ such that
\[
C^s ||\Delta \hat{w}||_{1,\Omega}^{3/4} ||u_t||_{\Omega}^{1/4} + \eta C_{\delta} ||u_t||_{\Omega}^{2} \leq \frac{\eta}{4}
\]
and, using corollary 1 for the last term, obtain
\[
\frac{d}{dt} \tilde{\Psi}_{mod}(t) + c_{\eta} \tilde{\Psi}_{mod}(t) \leq \eta C ||\{(v_0; u_0; u_1)\}||_{\hat{\mathcal{H}}}^{2} e^{-\mu t},
\]
where $\mu > 0$. We can chose $\mu < c_{\eta}/2$ without loosing of generality. Multiplying
this inequality by $e^{\mu t}$ and integrating with respect to $t$ from $0$ to $T$, we arrive
\[
\tilde{\Psi}_{mod}(T) \leq \tilde{\Psi}_{mod}(t_0) e^{c_{\eta}(t_0 - T)} + C ||\{(v_0; u_0; u_1)\}||_{\hat{\mathcal{H}}}^{2} e^{-\mu T}.
\]
This estimate implies that $||u_{tt}(t)||_{\Omega}^{2} + ||u_t(t)||_{\Omega}^{2} + ||v_t(t)||_{\Omega}^{2}$ decays exponentially
to zero. We add to (75) estimates of Lemma 3.6 and pass to the limit to conclude
the following: if the $\hat{\mathcal{H}}$-norm of initial data is small enough, the strong solution
is exponentially stable in the strong topology of $\hat{\mathcal{H}}_s$. Thus, Corollary 1 gives us
uniform stability of strong solutions in the strong topology of $\hat{\mathcal{H}}_s$. 

\textbf{Remark 4.} This proof of uniform stability of $(\hat{\mathcal{H}}_s, S_t)$ critically relies on the fact,
than $\hat{\mathcal{H}}$-norm of the solution will be arbitrary small after some time. The latter
takes place only for homogeneous problem. Whether there exists a strong global
attractor for a non-homogeneous problem, is an open question.
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