Stringy Corrections to Kaluza-Klein Black Holes

N. Itzhaki*

Raymond and Beverly Sackler Faculty of Exact Sciences
School of Physics and Astronomy
Tel Aviv University, Ramat Aviv, 69978, Israel

March 28, 2022

Abstract

We consider string theory corrections to 4D black holes which solve the 5D vacuum Einstein equations. We find that the corrections vanish only for the extremal electric solution. We also show that for the non-extremal electric black hole the mass corrections are related to the charge corrections. The implications to string states counting and the correspondence principle for black holes and strings are discussed.

*Email:sanny@post.tau.ac.il
1 Introduction

Although the curvature at the horizon of a large black hole is very small (of the order of $1/M^2$), there are strong gravitational interactions near the horizon due to the large red shift (for a recent review see [1]). Therefore, it is generally admitted that non-perturbative gravitational effects are involved in the full resolution, yet to be found, of the quantum black hole puzzle.

However, for certain classes of black holes it seems that string theory overcomes, at least partially, this difficulty. In particular, for certain classes of black holes string states counting at small coupling yields $e^{A/4}$ [2, 3]. This result is puzzling because Hawking-Bekenstein entropy is obtained at a region where the string size is much larger than the size of the horizon, which means that classically the area of the horizon, $A$, is meaningless at that region. A classical horizon is formed when the string coupling, $g$, is of order $1/M$. An explanation why the counting at $g = 0$ still works at $g \approx 1/M$ is needed. For extremal BPS black holes, super-symmetry non-renormalization arguments protect the counting as one increases $g$. Recently, non-renormalization arguments were provided for near-extremal black holes [4] in the dilute gas region [5]. Furthermore, it has been shown that the processes of absorption and emission by near-extremal D-branes configurations are not renormalized [6]. Still, we have not reached a full understanding of why and when string states counting at $g = 0$ yields $e^{A/4}$. For instance, the counting also yields $e^{A/4}$ for near extremal black holes which are not in the dilute gas region [3] and even for extremal non-supersymetric black hole in type I string theory [7]. Another interesting related issue is the success to relate the black hole horizon area to the string states counting, for some classes of non-extremal black holes [8]. The relation does not predict the numerical coefficient, but it does yield the correct dependence on the mass and charges.

The aim of the present paper is to examine these questions from a different point of view. Instead of increasing $g$ from 0 toward $1/M$ and analyzing the microscopic dynamics, we begin with the classical black hole solution and study the $\alpha'$ corrections to the macro-
scopic properties of the black hole. We start in a region where \( g \approx 1 \). The dimensionless expansion parameter is \( \lambda = \frac{\alpha'}{M} \), and we decrease \( g \) toward \( 1/M \). In 4D

\[
G \approx g^2 \alpha',
\]

(1)

where \( G \) is the Newton constant in four dimensions. Therefore, as \( g \) is decreased toward \( \approx 1/M \) \( \lambda \) is increased toward 1 (\( G \) is fixed). This means that in general at that region \( \alpha' \) corrections will modify the solution completely and hence the thermodynamical properties of the black hole will be changed drastically. But this cannot be the case for all black holes because for certain classes of black holes, string states counting at \( g = 0 \) gives the area of the black hole with no \( \alpha' \) corrections. For these classes of black holes, \( \alpha' \) corrections should be suppressed.

Put differently, if for a certain black hole the \( \alpha' \) corrections are suppressed, then one can decrease \( g \) without changing the black hole solution. Eventually, the size of the string will be larger than the size of the horizon\(^1\) and then string states counting at \( g = 0 \) can be trusted. Since the thermo-dynamical properties of the black hole are not modified as one changes \( g \), the counting should give \( e^{A/4} \).

For the non-extremal black holes string states counting at \( g = 0 \) does not yield \( e^{A/4} \) so \( \alpha' \) corrections are not expected to be suppressed. However, it has been recently shown\(^3\) that for a wide range of non-extremal black holes the string states counting leads to the correct dependence on mass and charge thought it does not predict the numerical coefficient. For these black holes we do not expect to find suppression of the \( \alpha' \) corrections but to find that the \( \alpha' \) corrections to the mass are related to the \( \alpha' \) corrections to the charge.

The paper is organized as follows: In Sec. 2 we describe the 4D Kaluza-Klein black holes which solve the 5-dimensional vacuum Einstein equations. In Sec. 3 we discuss string theory corrections to the vacuum Einstein equations in general and in Sec. 4 we demonstrate their effects on Schwarzschild solution. In Sec. 5 we analyze the corrections

\(^1\)The one example, for which the horizon does not become smaller as \( g \) is decreased is black hole with magnetic Neveu-Schwarz charge.
to the electric black holes. We present an explanation why the correspondence principle for black holes and string is valid for the electric solutions. In Sec. 6 the corrections to Pollard-Gross-Perry-Sorkin monopole are analyzed. In Sec. 7 we consider the corrections to the extremal Reissner-Nordström solution.

2 The Classical Solutions

In this section we review the 4-dimensional black holes which solve the 5-dimensional vacuum Einstein equations \[^{1}\]. A generalization can be found in \[^{10}\]. The starting point is therefore the 5-dimensional Einstein-Hilbert action

\[
S_5 = \frac{1}{16\pi G_5} \int d^5 x \sqrt{-g_5} R ,
\]

where \(G_5\) is the Newton constant in five dimensions and \(g_5\) is the five dimensional metric. Expanding this action around the vacuum \(M^4 \times S^1\), where \(S^1\) is a circle with radius \(R\), leads to

\[
S_4 = \int d^4 x \sqrt{-g_4} \left( \frac{R}{16\pi G} - \frac{1}{4} \exp(2\sqrt{3}\kappa\sigma) F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} g_4^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma \right) ,
\]

where \(G = G_5/2\pi R\), \(\kappa^2 = 4\pi G\) and \(g_4\) is the four dimensional metric.

The relation between the 5D metric and the 4D metric, gauge fields and scalar is

\[
ds_5^2 = e^{4\sqrt{3}\kappa\sigma} (dx^5 + 2\kappa A_\mu dx^\mu)^2 + e^{-2\sqrt{3}\kappa\sigma} g_\mu\nu dx^\mu dx^\nu ,
\]

where \(ds_5\) is the line element in five dimensions. The equations of motion derived from \(S_4\) are invariant under the duality transformation \[^{2}\]

\[
g_{\mu\nu} \rightarrow g_{\mu\nu}, \quad \sigma \rightarrow -\sigma, \quad e^{2\sqrt{3}\kappa\sigma} F_{\mu\nu} \rightarrow * F_{\mu\nu} .
\]

The spherically symmetric, time-independent solutions can be characterized by the ADM mass \(M\), the total electric charge \(Q\) and the total magnetic charge \(P\). The scalar

\[^{2}\]Quantum aspects of that duality (when \(\sigma = 0\)) were analyzed in \[^{11}\]. It was shown that the rate at which charged black holes are created is invariant under the Electric-Magnetic duality. The calculations are semi-classical, still they suggest that the duality is more then a symmetry of the classical equations of motion. See also \[^{12}\].
charge $\Sigma$ is related to $M, Q, P$ by

$$\frac{2}{3} \Sigma = \frac{Q^2}{\Sigma + \sqrt{3}M} + \frac{P^2}{\Sigma - \sqrt{3}M},$$

in units where $G = 1$.

The black holes solutions are the following

$$e^{4\sigma/\sqrt{3}} = \frac{B}{A},$$

$$A_\mu dx^\mu = \frac{Q}{B} (r - \Sigma) dt + P \cos \theta d\phi,$$

$$g_{\mu\nu} dx^\mu dx^\nu = -F/\sqrt{AB} dt^2 + \sqrt{AB}/F dr^2 + \sqrt{AB}(d\theta^2 + \sin^2 \theta d\phi^2),$$

where

$$F = (r - r_+)(r - r_-),$$

$$A = (r - r_{A+})(r - r_{A-}),$$

$$B = (r - r_{B+})(r - r_{B-}),$$

and

$$r_\pm = M \pm \sqrt{M^2 + \Sigma^2 - P^2 - Q^2},$$

$$r_{A \pm} = \frac{\Sigma}{\sqrt{3}} \pm \sqrt{\frac{2P^2\Sigma}{\Sigma + \sqrt{3}M}},$$

$$r_{B \pm} = -\frac{\Sigma}{\sqrt{3}} \pm \sqrt{\frac{2Q^2\Sigma}{\Sigma + \sqrt{3}M}}.$$

Under the duality transformation (Eq. (5)) the solutions are transformed in the following way

$$Q \leftrightarrow P, \quad \Sigma \leftrightarrow -\Sigma.$$

The coordinate singularities at $\theta = 0$ and $\theta = \pi$ can be simultaneously removed only if

$$P = l\frac{R}{\kappa},$$

where $l$ is an integer. The quantization of the momentum in the $y$ direction implies that

$$Q = m \frac{2\pi \kappa}{R}.$$
So Dirac relation
\[ QP = 2\pi n, \]  
(13)

is satisfied. The minimum ADM mass of the electric and magnetic charges are
\[ M_q = \frac{1}{R}, \quad M_p = \frac{R}{\kappa^2}. \]  
(14)

Therefore, from Eq.(13) it is clear that when \( g \to 0 \) the horizon in string units vanishes.

3 String theory corrections

In this section we shortly review the \( \alpha' \) corrections to Einstein vacuum equations. The low-energy effective action in string theory is given by a sum of classical, quantum (string loops) and non-perturbative contributions
\[ S = S_c + S_l + S_{np}. \]  
(15)

We consider only \( S_c \). It is important, therefore, to find the region where our approximation is valid. We neglect higher order corrections in \( g \) and consider only \( \alpha' \) corrections which means that \( g \ll 1 \). Our dimensionless expansion parameter is \( \frac{\alpha'}{M^2} \), where \( M \) is the mass of the black hole. From Eq.(13) it is clear that our approximation is valid only for a large black hole in the region where \( 1/M \ll g \ll 1 \) (\( M^2 \gg \alpha' \gg 1 \)). To leading order in \( \alpha' \)
\[ S_c = \frac{1}{16\pi G_D} \int d^Dx \sqrt{-g}e^{-2\phi} \left( R + 4(\nabla \phi)^2 + \frac{\lambda}{2} R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \right), \]  
(16)

where we dropped terms which are not relevant to our discussion \( \lambda = \frac{\alpha'}{2}, \frac{\alpha'}{4}, 0 \) for bosonic, heterotic and type II strings, respectively. \( G_D \) is the Newton constant in \( D \) dimensions.

We are after the \( \alpha' \) corrections in the Einstein frame, the frame at which the area is related to the entropy. The action in Einstein frame is obtained by redefining the metric by a conformal transformation involving the dilaton
\[ g_{\mu\nu} \to e^{4\phi/(D-2)} g_{\mu\nu}. \]  
(17)

To first order in \( \alpha' \) the full expression of \( S_c \), including \( B_{\mu\nu} \) terms, was calculated in [15].
The action (16) becomes
\[ S = \frac{1}{16\pi G_D} \int d^D x \sqrt{-g} \left( R - \frac{4}{D-2} (\nabla \phi)^2 + \frac{\lambda}{2} e^{-4\phi/(D-2)} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \right). \] (18)

The equations of motion are
\[ \Box \phi = \frac{\lambda}{4} e^{-4\phi/(D-2)} N, \] (19)
\[ R_{\mu\nu} = \lambda e^{-4\phi/(D-2)} N_{\mu\nu}, \]
where
\[ N = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}, \] (20)
\[ N_{\mu\nu} = R_{\mu\nu\rho\sigma} R^{\rho\sigma} - \frac{1}{2(D-2)} g_{\mu\nu} N = 0. \]

Terms which only contribute at higher orders were eliminated for simplicity [13]. The critical dimensions in string theories are 10, 26. Nevertheless, it is sufficient to consider only the non-compact space-time. Considering a larger $D$ will not modify the first order corrections to the metric of the non-compact space-time [13]. We illustrate that point in the next section.

## 4 Schwarzschild solution

In this section we study the $\alpha'$ corrections to the 4D Schwarzschild solution. These corrections were already discussed in details in [13] from the 4-dimensional point of view. Here we consider the corrections from the 5-dimensional point of view. We shall see why taking $D = 5$ in Eq.(20) leads to the same 4D result as taking $D = 4$.

From the 4D point of view the zeroth order line element is
\[ ds^2 = -(1 - \frac{2M}{r}) dt^2 + \frac{1}{1 - \frac{2M}{r}} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2), \] (21)
the zeroth order value of the dilaton is 0.

\footnote{Corrections for the Schwarzschild solution at $D \geq 4$ were considered in [13]. Corrections to the 4D solutions with dilaton and axion hairs were analyzed in [14].}
Since $N_{\mu\nu} = 0$ there are no corrections at the first order. Note that this is a result of the Gauss-Bonnet theorem in 4D: Schwarzschild solution is a solution to the 4D equations, $R_{\mu\nu} = 0$ and since in 4-dimensions one can replace the $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ term in Eq.(18) by $R^2 - 4R_{\mu\nu}R^{\mu\nu}$, there are no corrections at that order. There are corrections at higher orders which modify completely the solution at $g \approx 1/M$ ($\alpha' \approx M^2$). Therefore, string states counting at $g = 0$ cannot yields the Bekenstein-Hawking entropy. Namely, interactions plays an important role in the microscopic description of the Schwarzschild black hole entropy.

From the 5D point of view the zeroth order line element is

$$ds^2 = -(1 - \frac{M_s}{r})dt^2 + \frac{1}{1 - \frac{M_s}{r}}dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2) + dy^2, \quad (22)$$

where $y$ is the Kaluza-Klein direction. Eq.(22) is a solution to $R_{\mu\nu} = 0$ in 5D, but in 5D one cannot replace the $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ term in Eq.(18) by a linear combination of $R^2$ and $R_{\mu\nu}R^{\mu\nu}$. Therefore, $N_{\mu\nu}$ does not vanish and there are corrections. The non-vanishing components of $N_{\mu\nu}$ are

$$N_{tt} = U g_{tt}, \quad (23)$$
$$N_{rr} = U g_{rr},$$
$$N_{\theta\theta} = U g_{\theta\theta},$$
$$N_{\phi\phi} = U g_{\phi\phi},$$
$$N_{yy} = -2U g_{yy},$$

where

$$U = 4\frac{M_s^2}{r^6}. \quad (24)$$

Since in 4D there are no corrections, it is clear from Eq.(14) that the natural ansatz for the corrected metric is

$$ds^2 = \frac{1}{K} \left(-(1 - \frac{M_s}{r})dt^2 + \frac{1}{1 - \frac{M_s}{r}}dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)\right) + K^2dy^2, \quad (25)$$
where $K = 1 + \lambda f(r)$. At order $\alpha'$ Eq.(20) yields

$$ \ddot{f}(r - M_s) + \dot{f} (2r - M_s) = \frac{16M_s^2}{r^4}, $$

(26)

where $\dot{f} = \frac{df}{dr}$. The solution to this equation is

$$ f(r) = \frac{4M_s^2 + 3r M_s + 3r^2}{9r^3 M}, $$

(27)

where the boundary condition is such that $f$ is regular at the horizon and $f(\infty) = 0$. So the 5D metric is corrected in such a way that the 4D metric is not corrected. The only correction is that the radius of the compact dimension is no longer a constant but a decreasing function of $r$. For the dilaton Eq.(19) yields

$$ \frac{d}{dr} \left( r(r-2M) \frac{d\phi}{dr} \right) = 12 \lambda \frac{M^2}{r^4}, $$

(28)

the solution is $[13]$

$$ \phi = -\lambda \left( \frac{2M}{3r^3} + \frac{1}{2r^2} + \frac{1}{2Mr} \right), $$

(29)

5 Electric black holes

5.1 Extremal electric black hole

In this subsection we consider the solution with $Q = 2M$ and $P = 0$. The 5D line element is

$$ ds^2 = -\frac{r - 4M}{r} dt^2 + dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) + \frac{r + 4M}{r} dy^2 + \frac{M}{r} dt dy. $$

(30)

In $[15]$ it was shown that this is an exact solution in the bosonic (as well as in the supersymmetric) string theory. Indeed, for that solution we get

$$ N_{\mu \nu} = 0, $$

(31)

so there are no $\alpha'$ corrections at the first order in $\alpha'$. There is a general argument (which does not rest on string theory) why there are no $\alpha'$ corrections at any order: The physical
meaning of the solution is a 5D gravitational shock wave in the y-direction [16]. The gravitational shock wave is obtained by boosting Eq.(22) to infinity along the y direction while taking $M_s \to 0$ and keeping their product (the 4D mass) fixed. But in the rest frame (Eq.(22)) it is clear that when $M_s \to 0$, any higher order corrections vanish, because for $M_s = 0$ the solution is flat. Note that because $M_{Sc} \to 0$, the area of the black hole vanishes.

5.2 Non-extremal electric black hole

To obtain the electric black hole solution one can boost Eq.(22) along the y direction and reduce to four dimensions. The result is [16] a black hole with mass

$$M = M_s \left(1 + \frac{v^2}{2(1 - v^2)}\right),$$

and electric charge

$$Q = 2M \frac{v}{2 - v^2},$$

where $v$ is the velocity in the y direction.

The $\alpha'$ corrections to the electric black hole solution are the $\alpha'$ corrections to Eq.(22) boosted along the y direction and reduced to 4D. As a result there are two kinds of corrections:1- Corrections to the dimensional reduction, these corrections appear already at the first order in $\alpha'$ (Eq.(27)). 2- Corrections to $M_s$ which appear only at the second order in $\alpha'$. From the discussion in sec.4 it is clear that when $\alpha' \approx M_s^2$ the solution is completely modified at the horizon. So when $\alpha' \approx M_s^2$ the macroscopic properties of the black hole will be changed drastically. Thus, string state counting at small $g$ cannot yields the right numerical coefficient.

Nevertheless, there is one macroscopic property of the black hole with no $\alpha'$ corrections- the ratio between the mass and electric charge. The reason for this is simple; there are no $\alpha'$ corrections to the velocity boost. Therefore, although there are $\alpha'$ corrections to $M$ and $Q$ Eq.(33) is an exact relation between $M$ and $Q$. This is why string state counting...
for these black hole does not predict the right numerical coefficient but does give the
correct dependence on the mass and charge \[8\].

\section{Pollard-Gross-Perry-Sorkin monopole}

The PGPS monopole \[18\] is an extremal magnetic black hole \(P = 2M, Q = 0\). The 5D metric is

\[
ds^2 = -dt^2 + V \left( dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) + \frac{1}{V} (dy + 4M(1 - \cos \theta)d\phi)^2, \tag{34}
\]

\[
V = 1 + \frac{4M}{r}.
\]

The non-vanishing components of \(N_{\mu\nu}\) are

\[
N_{tt} = -2W g_{tt},
\]

\[
N_{rr} = W g_{rr},
\]

\[
N_{\theta\theta} = W g_{\theta\theta},
\]

\[
N_{\phi\phi} = W g_{\phi\phi},
\]

\[
N_{yy} = W g_{yy},
\]

\[
N_{y\phi} = W g_{y\phi},
\]

where

\[
W = -32 \frac{M^2}{(r + 4M)^6}. \tag{36}
\]

Notice that PGPS monopole is dual to the extremal electric solution, but, unlike the
extremal electric solution, there are \(\alpha'\) corrections to the PGPS monopole which break
the duality.

Setting \(dt = 0\) in Eq.(34), the resulting 4D metric satisfies \(R_{\mu\nu} = 0\). Therefore, just
like in the Schwarzschild solution, Gauss-Bonnet theorem in 4D implies that the first
order corrections in \(\alpha'\) to the 4D metric vanish. The difference is that now the 4D space
is the Euclidean space \((r, \theta, \phi, y)\) while in the Schwarzschild solution it was the Minkowski
space \((t, r, \theta, \phi)\). As a result Eq.(23,35) are related under \(t \leftrightarrow y\). This implies that the ansatz for the corrected 5D metric should be

\[
ds^2 = -H^{-2} dt^2 + H \left( V (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2) \right) + \frac{1}{V} (dy + 4M(1 - \cos \theta) d\phi)^2,
\]

where

\[H = 1 + \lambda h(r).\]  \hspace{1cm} (38)

To first order in \(\lambda\) we get from Eq.(19)

\[r h''(r) + 2 h'(r) = 64 \frac{M^2}{(r+4M)^5}.\]  \hspace{1cm} (39)

The boundary conditions are such that at the horizon \(h\) is regular and \(h(\infty) = 0\). The solution is

\[h(r) = -\frac{1}{12} \frac{48M^2 + 12rM + r^2}{M(r+4M)}.\]  \hspace{1cm} (40)

For the dilaton Eq.(19) yields

\[
\frac{d}{dr} \left( r^2 \frac{d\phi}{dr} \right) = 96 \lambda M^2 \frac{r}{(r+4M)^5},
\]

The solution is

\[\phi = \frac{-\lambda}{8M} \frac{48M^2 + 12rM + r^2}{(r+4M)^3}.\]  \hspace{1cm} (42)

Notice that qualitatively this solution behaves like the dilaton solution in Schwarzschild metric (Eq.(29)), it is a negative increasing function of \(r\).

### 7 The extremal Reissner-Nordström solution

The areas of the extremal solutions which we considered until now vanish. In this subsection we discuss the simplest extremal solution with non-vanishing area—the extremal Reissner-Nordström solution. This solution is self-dual at the zeroth order. In the previous section we saw that the \(\alpha'\) corrections break the duality, so we expect the self-duality
of the solution to be broken by the $\alpha'$ corrections. The values of the zeroth order four dimensional metric, Abelian gauge field and scalar are

$$ds^2 = -\frac{(r - r_+)(r - r_-)}{r^2}dt^2 + \frac{r^2}{(r - r_+)(r - r_-)}dr^2 + r^2d\Omega^2,$$

$$A_\mu dx^\mu = \frac{Q}{r} dt + P \cos \theta d\phi,$$

$$\sigma = 0,$$

where

$$r_\pm = M \pm \sqrt{M^2 - Q^2 - P^2},$$

and

$$P = Q = \frac{M}{\sqrt{2}}.$$

The calculation of the first order corrections of this solution relegated to the appendix. The result is a solution which is similar to Eqs. (43, 44) but with

$$M_g(r) = M - \lambda(r - m)\left[\frac{1}{70r^5M} \left(182r^2(r - M)^2 \log\left(\frac{r}{r - M}\right) - \frac{M^2(r - M)}{4r^4}\right)\right],$$

$$M_i(r) = M - \lambda(r - m)\left[\frac{1}{70r^5M} \left(182r^2(r - M)^2 \log\left(\frac{r}{r - M}\right) - \frac{M^2(r - M)}{4r^4}\right) + \frac{M^2(r - M)}{4r^5}\right],$$

$$Q(r) = \frac{M}{\sqrt{2}} - \lambda \frac{\sqrt{2}(r - m)}{140r^4M^2} \left[364 \log\left(\frac{r}{r - M}\right)(r^2(r - M)^2) - 122M^4 - 142M^3r + 545M^2r^2 - 421Mr^3\right],$$

$$P(r) = \frac{M}{\sqrt{2}},$$

$$\sigma(r) = \lambda \frac{1}{105\sqrt{3}r^4M^2} \left[1092r^2(r - M)^2 \log\left(\frac{r}{r - M}\right) + 1204r^3M - 1697r^2M^2 + 360rM^3 + 225M^4\right].$$
where $M_g$ and $M_i$ are the gravitational and inertial mass which are defined respectively as

$$-g_{tt} = 1 - \frac{2M_g}{r} + O\left(\frac{1}{r^2}\right), \quad g_{rr} = 1 + \frac{2M_i}{r} + O\left(\frac{1}{r^2}\right).$$

The boundary conditions that we choose are

$$M_g = M_i = \sqrt{2}P = \sqrt{2}Q = M. \quad (48)$$

It is convenient to work with these boundary conditions because the temperature remains zero so the black hole remains extremal. At infinity one finds the corrections to the total mass and charges

$$M_i(\infty) = M_g(\infty) = M + \frac{16}{35M}, \quad (49)$$

$$Q(\infty) = \frac{M}{\sqrt{2}} + \lambda\sqrt{2}\frac{57}{140M},$$

$$P(\infty) = \frac{M}{\sqrt{2}},$$

$$\sigma(\infty) = 0.$$

Note that due to the equivalence principle one gets $M_g(\infty) = M_i(\infty)$. We see, therefore, that there are $\alpha'$ corrections to the black hole extremality condition, which become of order one when $g \approx 1/M$ ($\lambda \approx M^2$). Thus for this black hole string states counting at small $g$ cannot yields the Bekenstein-Hawking entropy. Moreover,

$$M(\infty) \neq \sqrt{2}Q(\infty), \quad M(\infty) \neq \sqrt{2}P(\infty), \quad Q(\infty) \neq P(\infty),$$

(50)

so the correspondence principle for black holes and strings [8] is not expected to be valid for this black hole. Since $\alpha'$ corrections break the duality it is unlikely that the correspondence principle for black holes and strings will be valid for $p \neq 0$.

For the dilaton Eq.(19) yields

$$\frac{d}{dr} \left((r - m)^2 \frac{d\phi}{dr}\right) = 3\lambda M^2 \frac{8r^2 - 16rM + 7M^2}{2r^6}, \quad (51)$$
the solution is
\[ \phi = \frac{\lambda}{40M^2r^4} \left( 60r^4 \log\left( \frac{r}{r - M} \right) - 64r^3M - 34r^2M^2 - 24rM^3 + 21M^4 \right). \]

At \( r \gg M \) this solution, like the corrected dilatons in Schwarzschild and GPS solutions (Eq.(29,42)), is negative so \( \phi \) decreases as \( r \) is decreased. But, unlike Eq.(29,42) here at the horizon
\[ \lim_{r \to M^+} \phi = \infty, \quad \lim_{r \to M^-} \phi = -\infty. \]

The physical meaning of that solution can be understood from Eq.(18). Just outside the horizon the effective string scale \( \alpha' e^{-\phi/2} \) goes to zero, while just inside the horizon \( \alpha' e^{-\phi/2} \to \infty \). It is worthwhile to mention that for the non-extremal Reissner-Nordström solution the corrected dilaton is not singular at the horizon.

### 8 Summary

The black holes that we analyzed here are rather simple, from string theory point of view, because at the zeroth order \( B_{\mu\nu} = 0 \) and \( \phi = 0 \). As a result, the only solution with no \( \alpha' \) corrections is the extremal electric black hole \([15]\). At its neighborhood the \( \alpha' \) corrections are suppressed. All other solutions, even the extremal solutions, are completely modify when \( g \approx 1/M \), thus we do not expect that the microscopic description of these black holes at \( g = 0 \) will yield \( A/4 \). It would be interesting to consider the \( \alpha' \) corrections to solutions with \( B_{\mu\nu} \neq 0 \) and \( \phi \neq 0 \) at the zeroth order. Of special interest are the near-extremal black holes for which the string states counting at \( g = 0 \) gives \( A/4 \). For these solutions \( \alpha' \) corrections should be suppressed.

Another conclusion is that the correspondence principle for black holes and strings is not valid for all black holes solutions. In fact, our analysis of the \( \alpha' \) corrections for the Kaluza-Klein black holes implies that it is valid only for pure electric black holes. It would be interesting to compute the \( \alpha' \) corrections to other black holes for which the correspondence principle is valid \([8]\). It might provide us with an indication about the crucial properties of black holes for which the correspondence principle of \([8]\) is valid.
It is important to emphasis that our conclusions are valid only for bosonic and heterotic string theories. In type II theories there are corrections only at order \( \alpha' \). We did not perform the full calculations in that case but we expect to get the same qualitative result as the following argument implies. \( \alpha' \) corrections to Einstein equations in type II theories in the string frame lead in out case to \( R_{\mu\nu} = \alpha'^3 \nabla_\mu \nabla_\nu F(r) \). This term cannot be canceled by the dilaton contribution which is proportional to \( g_{\mu\nu} \). So locally \( \alpha' \) corrections are not suppressed. Still, there might be a global effect which would cancel the corrections at infinity to the ADM mass and charges.

**Acknowledgments** I would like to thank A. Casher and S. Yankielowicz for helpful discussions.
A Appendix

In this section we derive Eq.(46). There are many ways one can calculate the first order corrections to Eq.(43, 44). We choose to work with the 4D variables $M_g, M_i, Q$ and $P$. To the first order in $\alpha'$ the component $(r, \theta)$ of Eq.(20) yield

$$\dot{P} = 0,$$

where $\dot{f} = \frac{df}{dr}$. From Eq.(48) we get

$$P = \frac{M}{\sqrt{2}}$$

(55)

From the equivalence principle it is clear that one should get $M_g(\infty) = M_i(\infty)$, therefore it seems useful to work with

$$M_+ = \frac{1}{2}(M_g + M_i), \quad M_- = \frac{1}{2}(M_g - M_i),$$

(56)

Instead of $M_g$ and $M_i$. Gauss law implies that it might be helpful to work with

$$e(r) = \frac{(Q(r) - M/\sqrt{2})}{r},$$

(57)

and not with $Q(r)$. We also work with

$$k(r) = -\frac{2}{\sqrt{3}}\sigma(r)$$

(58)

The nontrivial independent components of Eq.(21) to $\lambda$ order are

$$(tt) : \quad \tilde{M}_+ \frac{(r - M)^2}{r^3} + \frac{\sqrt{2}M}{2r^4} \left(3r\ddot{e}(r - M)^2 + 4\dot{e}(r - M)^2\right)$$

$$+ \frac{1}{2r^6} \left(\dddot{k}r^2(r^2 - 2rM + 5M^2)(r - M)^2ight.$$  

$$+ 2\ddot{k}r(r - M)(r^3 - 2r^2M - rM^2 + 6M^3) + 6M^2k(M^2 + r^2)$$

$$+ \frac{1}{r^5(r - M)^2} \left(\dddot{M}_- r^2(r - M)^4 - 2rM\dot{M}_-(r + M)(r - M)^2ight.$$  

$$+ 2MM_{-}(Mr^2 + 3M^3 - M^2r + r^3)$$

$$= M^2 \frac{2r^4 - 36r^3M + 97r^2M^2 - 94rM^3 + 37M^4}{r^{10}}$$

$$17$$
\[(t\phi):\quad \frac{\sqrt{2}M}{r^3} \left( \ddot{r}(r-m)^2 + 2\dot{e}r^2 - 2rM - M^2 \right) + \frac{M^2}{r^4} \left( 2r\ddot{k}(r-M)^2 + \dot{k}(r^2 + 2rM - 3M^2) \right) - \frac{2M^2}{r^4(r-M)^2} \left( r\ddot{M}_{\pm}(r-M)^2 - M_{\pm}(3M^2 + r^2) \right) = -2M^3 \frac{6r^3 - 14r^2M + 10rM^2 - 5M^3}{r^9} \]

\[(rr):\quad -r\ddot{M}_{\pm} + \frac{M}{\sqrt{2}}(r\ddot{e} + 4\dot{e}) - \frac{1}{2r^2} \left( \dddot{r}^2(r-M)^2 + 2\dot{k}\dot{r}^2(r-M) + 6M^2k \right) - \frac{1}{r(r-M)^2} \left( r^2\ddot{M}_{\pm}(r-M)^2 + 2r\dot{M}_{\pm}(3rM - M^2 - 2r^2) \right) + 2M_{\pm}(2r^2 + rM - M^2) \]
\[= M^2\frac{2r^2 - 3M^2}{r^6} \]

\[(\theta\theta):\quad -2\ddot{M}_{\pm} + \sqrt{2}Me - \frac{1}{2r^2} \left( \dddot{r}^2(r-M)^2 + 2\dot{k}\dot{r}^2(r-M) - 6M^2k \right) + \frac{2(r + M)}{r(r-M)} \dddot{M}_{\pm} = M^2\frac{8r^2 - 20rM + 11M^2}{r^6} \]

\[(yy):\quad -\sqrt{8}Me + \dddot{r}(r-M)^2 + 2\dot{k}(r-M) + 4M_{\pm} \frac{M^2}{r(r-M)^2} = -M^2\frac{5M^2 - 16rM + 8r^2}{r^6} \]

Note that due to the Bianchi identities we have five equations but only four variables \((M_{\pm}, e, k)\). To solve this equations it is helpful to notice that the combination

\[ (t\phi) - \frac{M^2}{r^3} (yy) - \frac{r}{2} \left( (tt) + \frac{(r-M)^2}{r^4} (rr) \right), \quad (60) \]

gives the simple differential equation

\[ \frac{2}{r^4} \left( r\dddot{M}_{\pm}(r-M) - M_{\pm}(r+M) \right) = 2\frac{M^2}{r^9} (r-M)^3. \quad (61) \]

whose solution is

\[ M_{\pm} = \frac{M^2(r-M)^2}{4r^5} \quad (62) \]

Plugging this solution back into Eq.(59) one gets equations for \(M_{\pm}, e, k\) whose solution is Eq.(46).
References

[1] G. ’t Hooft, Int. J. Mod. Phys. A11 (1996) 4623.

[2] A. Strominger and C. Vafa, Phys. Lett. B379 (1996) 99;
   J. C. Breckenridge, R. C. Myers, A. W. Peet and C. Vafa, hep-th/9602065;
   J. Maldacena and A. Strominger, Phys. Rev. Lett. 77 (1996) 428;
   C. Johnson, R. Kuhri and R. Myers, Phys. Lett. B378 (1996) 78.

[3] C. Callan and J. Maldacena, Nucl. Phys. B475 (1996) 645;
   G. Horowitz and A. Strominger, Phys. Rev. Lett. 77 (1996) 2368;
   G. Horowitz, D. Lowe and J. Maldacena, Phys. Rev. Lett. 77 (1996) 430;
   J. C. Breckenridge, D. Lowe, R. C. Myers, A. W. Peet, A. Strominger and C. Vafa
   Phys. Lett. B381 (1996) 423;
   I. Klebanov and A. Tseytlin, Nucl. Phys. B475 91996) 179;
   V. Balasubramanian and F. Larsen, Phys. Lett. B388 (1996) 51.

[4] J. Maldacena, hep-th/9611125.

[5] G. Horowitz, J.Maldacena and A. Strominger, Phys. Lett. B383 (1996) 151.

[6] S. R. Das, hep-th/9703146.

[7] A. Dabholkar, hep-th/9702050.

[8] G. Horowitz and J. Polchinski hep-th/9612146 to be published in Phys. Rev D.

[9] G. W. Gibbons and D. L. Wiltshire, Ann. Phys. 167 (1987) 201 ,Erratum Ann. Phys.
   176 (1987) 393.

[10] M. Cvetic and D. Youm, Phys. Rev. Lett. 75 (1995) 4165.

[11] S. W. Hawking and S. F. Ross, Phys. Rev. D52 (1995) 5865.

[12] S. Deser, M. Henneaux and C. Teitelboim, Phys. Rev. D55 (1997) 826.
[13] C. G. Callan, R. C. Myers and M. J. Perry, Nucl. Phys. B311 (1988/89) 673.

[14] B. Campbell, M. Duncan, N. Kaloper and K. A. Olive, Phys. Lett. B251 (1990) 34;  
    B. Campbell, N. Kaloper and K. A. Olive, Phys. Lett. B263 (1991) 364;  
    B. Campbell N. Kaloper and K. A. Olive Phys. Lett. B285 (1992) 199;  
    P. Kanti and K. Tamvakis, Phys. Rev. D52 (1995) 3506;  
    S. Mignemi and N. R. Stewart, Phys. Rev. D47 (1993) 5259;  
    S. Mignemi, Phys. Rev. D51 (1995) 934;  
    P. Kanti, N. E. Mavromatos, J. Rizos, K. Tamvakis and E. Winstanley, Phys. Rev.  
    D54 (1996) 5049;  
    T. Torii, H. Yajima and K. Maeda, Phys. Rev. D55 (1997) 739.

[15] A. Tseytlin, hep-th/9410008. To be published in the proceedings of International  
    School of Astrophysics (D. Chalonge): 3rd Course: Current Topics in Astrofundamental  
    Physics, Erice, Italy, 4-16 Sep 1994.

[16] G. W. Gibbons, Nucl. Phys. B207 (1982) 337.

[17] R. Metsaev and A. Tseytlin, Nucl. Phys. B293 (1987) 385.

[18] D. Pollard, J. Phys. A16 (1983) 565;  
    D. J. Gross and M. J. Perry, Nucl. Phys. B226 (1983) 29;  
    R. Sorkin, Phys. Rev. Lett. 51 (1983) 87.

[19] D. J. Gross and E. Witten, Nucl. Phys. B277 (1986) 1;  
    M. T. Grisaru, A. E. M. Van de Ven and D. Zanon, Phys. Lett. B173 (1986) 423;  
    Nucl. Phys. B277 (1986) 388, 409;  
    M. D. Freeman and C. N. Pope, Phys. Lett. B174 (1986) 48.