The stochastic model of rice price fluctuation in Indonesia

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Abstract. Rice is a major food commodity in Indonesia. Based on its quality, the rice prices classified into premium, medium, and low. Besides as staple food, the production of rice involves millions of farmers. Therefore, the fluctuation of rice prices has a significant impact for national economy. The fluctuation of rice prices is random movement events that observed all the time. It can be viewed as stochastic processes. Then, Markov chain is a stochastic process where the outcome of the next event only depends on one previous event. This paper discusses transition probabilities, steady state probabilities and mean recurrence times properties of Markov chain. Then it will find the stochastic model for monthly rice price data in Indonesia from January 2013 to June 2018. There are two states in rice price assumed, i.e., increasing state and decreasing state. The discussion result showed that rice with premium, medium, and low quality, have almost similar price movements, and the probability of increasing price is greater than the probability decreasing price.

1. Introduction
Rice is a main food product that has strategic role in strengthening food security, economic security, and the political stability of a country [1]. The production of rice in the world ranks third of all cereal, and most rice consumers come from Asian countries. Indonesia is the third-largest rice producer in Asian and one of biggest rice consumers. Rice is a major food commodity in Indonesia. Based on its quality, rice prices are classified into premium, medium, and low. The fluctuation of rice prices has a direct impact on Indonesian’s daily life.

The fluctuation of rice prices is random movement events that observed from time to time as stochastic process. The part of stochastic process that usually used to analysis the random movement events is Markov chain. The using of Markov chain to analysis behaviour of market prices is popular because prospective investors are interested in market fluctuation which will guide to an optimum strategy of investment. According to [2] a similar study of Markov chain had done. It discussed that the stock prices followed a random movement implies that the price changes in two states. In 2009 [3] forecasted the China’s stock market based on Markov chain approach. Besides that, [4] analyzed Nigerian stock market price by determining transition probabilities between several states. In this paper, different from [2], [3] and [4], the Markov chain is used to find rice price fluctuation model in Indonesia. The rice price fluctuations have two states. They are increasing state and decreasing state.

2. The theoretical basis
In this section, we explain the definitions about stochastic process, state space and Markov chain. According to [5], a stochastic process is a family of random variables $X_t \in S$, for time index $t$, where $t \in \mathbb{N}$. A state space $S$ is a range of possible values for random variables $X_t$.

According to [5], a Markov Chain $\{X_t\}$ is a stochastic process with property that, given the value of $X_t$, the value of $X_s$ for $s > t$ are not influenced by the value of $X_u$ for $u < t$. Therefore, the probability of $X_{t+1}$ being in state $j$ given that $X_t$ is in state $i$ is called transition probability and is denoted by $p_{ij}$. That is,

$$p_{ij} = P(X_{t+1} = j \mid X_t = i)$$

where $\sum_{j=1}^{m} p_{ij} = 1$ for each $i = 1, 2, \ldots, m$. The transition probability is customary arranged in a matrix, as transition probability matrix. Transition probability matrix is denoted by $P$. It is a transition probability matrix $P$ with order $m \times m$

$$P = \begin{pmatrix}
    p_{11} & p_{12} & \cdots & p_{1m} \\
    p_{21} & p_{22} & \cdots & p_{2m} \\
    \vdots & \vdots & \ddots & \vdots \\
    p_{m1} & p_{m2} & \cdots & p_{mm}
\end{pmatrix}$$

The $n$-step transition probability $(p_{ij}^{(n)})$ of a Markov chain $\{X_t; t \in \mathbb{N}\}$ is a transition probability that it goes from state $i$ to state $j$ in $n$ step, and it is defined as

$$p_{ij}^{(n)} = P(X_n = j \mid X_1 = i)$$

where $i, j = 1, 2, \ldots, m$.

Based on [6], a Markov chain is irreducible if all the states communicate each other. A state $i$ with a period $d(i)$ is called aperiodic if $d(i) = 1$, where $d(i)$ is the greatest common divisor of $n$, for which $p_{ii}^{(n)} > 0$. If all states of a Markov chain are aperiodic, so the Markov chain is as an aperiodic Markov chain.

If the Markov chain is irreducible and aperiodic, then the chain has the long run invariant probability [7]. The long run invariant probability is denoted by $p_j = P(X_t = j)$ and can be arranged as a long run invariant probability vector, that is $p = (p_j)$ where $j = 1, 2, \ldots, m$. The long run invariant probability vector $p$, that is

$$pP = p$$

where $p = (p_j)$, with $0 < p_j < 1$ and $\sum_{j=1}^{m} p_j = 1$.

This probability vector can be viewed as the unique distribution of a random variable in the long run. Consequently, the mean recurrent times $\mu_j$ are given by

$$\mu_j = \frac{1}{p_j}$$

for each $j$.

3. Data description
The rice prices data were taken from Badan Pusat Statistik’s website (www.bps.go.id). The rice prices in Indonesia are classified in premium, medium, and low quality from January 2013 to June 2018. From that data, the rice price fluctuation is observed to count the transition frequencies. Generally, the rice price fluctuation shows that the price value of this month has increased or decreased from the previous month. According to [2], the changes in the rice prices were considered as two states. The state 1 is the rice price when it’s increasing, while state 2 is the rice price when it’s decreasing. Thus, the classification of two states are state 1 (increasing) and state 2 (decreasing). Increasing price means
the price value of this month is greater than the price value of the previous day, while decreasing price means the price value of this month is less than the price value of the previous day. The sequence of the monthly observations on the state can be known as Markov chain with state space \{1,2\}.

4. Result and discussion
The transition frequencies from the rice prices fluctuation with the premium, medium, and low quality are presented on table 1, table 2, and table 3 respectively.

| Table 1. The transition frequencies from the rice price fluctuation with the premium quality |
| State       | Increasing (1) | Decreasing (2) | Sum |
| Increasing (1) | 33             | 9              | 42  |
| Decreasing (2)  | 9              | 13             | 22  |

| Table 2. The transition frequencies from the rice price fluctuation with the medium quality |
| State       | Increasing (1) | Decreasing (2) | Sum |
| Increasing (1) | 32             | 8              | 40  |
| Decreasing (2)  | 8              | 16             | 24  |

| Table 3. The transition frequencies from the rice price fluctuation with the low quality |
| State       | Increasing (1) | Decreasing (2) | Sum |
| Increasing (1) | 29             | 11             | 40  |
| Decreasing (2)  | 11             | 13             | 24  |

From those tables, the transition probability $p_{ij}$ can be obtained. According to [8], the transition probability $p_{ij}$ is the relative frequencies, that is

$$p_{ij} = \frac{n_{ij}}{n_i}$$

where $n_{ij}$ is the number of transitions from state $i$ to state $j$ and $n_i = \sum_{j=1}^{m} n_{ij}$. From Table 1, the rice price fluctuation of the premium quality has the transition probabilities, those are $p_{11} = \frac{n_{11}}{n_1} = \frac{33}{42} \approx 0.79$, $p_{12} = \frac{n_{12}}{n_1} = \frac{9}{42} \approx 0.21$, $p_{21} = \frac{n_{21}}{n_2} = \frac{9}{22} \approx 0.41$, and $p_{22} = \frac{n_{22}}{n_2} = \frac{13}{22} \approx 0.59$. Those probabilities can be form as transition probability matrix ($\mathbf{P}$), that is

$$\mathbf{P} = \begin{pmatrix} 0.79 & 0.21 \\ 0.41 & 0.59 \end{pmatrix}$$  \(3\)

Each element of the matrix (3) is the transition probability for the change of the rice price fluctuation with premium quality in two successive months. The largest probability is $p_{11} = 0.79$, that means the largest probability is the probability that the price of premium quality rice increase in a month and will still increase in the next month. According to the transition probability matrix, all states communicate, so it is an irreducible Markov chain. All states are also aperiodic, so it is as an aperiodic Markov chain. The irreducible and aperiodic Markov chain has the long run invariant probability. The long run invariant probability vector ($\mathbf{p}$) can be obtained by counting (1), that is

$$\mathbf{p} = (0.66 \ 0.34).$$  \(4\)

From (4), the probability that the price of premium quality rice will increase is 0.66 and the probability that the price of premium quality rice will decrease is 0.34. So, the probability of the increasing price...
is larger than the decreasing price. Then, the long run invariant probability vector can be used to get the mean recurrence time vector by (2), and it is obtained

$$\mu = (1.52 \quad 2.91).$$  \hspace{1cm} (5)$$

From (5), the mean recurrence time of increasing price is 1.52 month and the mean recurrence time of decreasing price is 2.91 month.

In the same manner, for the price of medium quality rice, based on Table 2, the transition probability matrix, the long run invariant probability vector, and the mean recurrence time vector are obtained as follows

$$P = \begin{pmatrix} 0.8 & 0.2 \\ 0.33 & 0.67 \end{pmatrix}$$

$$p = (0.63 \quad 0.37).$$  \hspace{1cm} (7)$$

$$\mu = (1.6 \quad 2.67).$$  \hspace{1cm} (8)$$

From (6), the largest probability is $p_{11} = 0.8$, that means the largest probability is the probability where the price of medium quality rice increase in a month and will still increase in the next month. From (7) the probability that the price of medium quality rice will be increase is 0.63 and the probability that the price of medium quality rice will be decrease is 0.37. So, the probability of the increasing price is larger than the decreasing price. From (8), the mean recurrence time of increasing price is 1.6 month and the mean recurrence time of decreasing price is 2.67 month.

For the price of low quality rice, in the same manner, based on Table 3, the transition probability matrix, the long run invariant probability vector, and the mean recurrence time vector are obtained as follows

$$P = \begin{pmatrix} 0.73 & 0.27 \\ 0.46 & 0.54 \end{pmatrix}$$

$$p = (0.62 \quad 0.38)$$

$$\mu = (1.6 \quad 2.66).$$  \hspace{1cm} (11)$$

From (9), the largest probability is $p_{11} = 0.73$, that means the largest probability is the probability where the price of low quality rice increase in a month and will still increase in the next month. From (10) the probability that the price of low quality rice will be increase is 0.62 and the probability that the price of low quality rice will be decrease is 0.38. So, the probability of the increasing price is larger than the decreasing price. From (11), the mean recurrence time of increasing price is 1.6 month and the mean recurrence time of decreasing price is 2.66 month.

5. Conclusion

This paper has presented the stochastic model especially Markov chain on monthly rice price fluctuation in Indonesia. The rice has three qualities premium, medium and low quality. The fluctuation of the prices can be as increasing state and decreasing state. For the premium rice, the probability that the rice price will increase is 0.66 and the probability that the rice price will decrease is 0.34. For the medium rice, the probability that the rice price will increase is 0.63 and the probability that the rice price will decrease is 0.37. Then, for the low rice, the probability that the rice price will increase is 0.62 and the probability that the rice price will decrease is 0.38. The results of rice prices fluctuation with Markov chain are almost same for each quality. That is the probability of the increasing price is larger than the decreasing price.

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