An Impedance Transition Method to Verify the Reference Impedance of Multiline TRL Calibration

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Abstract—In this paper, we present a new technique for assessing the validity of the reference impedance in multiline thru-reflect-line (mTRL) calibration. When performing an mTRL calibration, it is assumed that all transmission line standards exhibit the same characteristic impedance. As a result, the reference impedance after calibration is set to the characteristic impedance of the transmission line standards used in the calibration. However, because of imperfections, these assumptions are prone to errors. The purpose of this paper is to assess the validity of the reference impedance after an mTRL calibration. The method we propose uses the reflection coefficient of an impedance transition segment as a verification metric. The verification is achieved by performing two mTRL calibrations. The first mTRL calibration is the one we desire to validate, while the second mTRL calibration is based on step impedance lines that create the impedance transition. We conclude that the mTRL calibration is valid if the resulting reflection coefficient falls within the expected 95% confidence interval. We demonstrate our proposed method with printed circuit board (PCB) measurements of microstrip lines up to 150 GHz. The advantage of our approach is that the reflection coefficient of an impedance transition is almost constant with respect to frequency for many types of transmission line, which makes this validation metric easy to interpret when errors are present.

Index Terms—calibration, microwave measurement, vector network analyzer, millimeter-wave, printed circuit board, transmission line, microstrip line

I. INTRODUCTION

THE accuracy of scattering parameters (S-parameters) measurement taken by a vector network analyzer (VNA) strongly depends on the accuracy of the applied calibration method. The multiline thru-reflect-line (mTRL) calibration procedure, first introduced by the National Institute of Standards and Technology (NIST) [1], sets the reference plane of the VNA with high precision to the characteristic impedance of the calibration line standards. The precision of the mTRL calibration comes from the fact that transmission line standards can be manufactured with high accuracy compared to their resistive standards counterparts. Furthermore, the mTRL algorithm presented in [2] improves the original algorithm by eliminating the need to assume a first-order perturbation in the error boxes.

A crucial aspect of any calibration technique is the ability to verify its validity after completing the calibration procedure. The mTRL method falls under the family of self-calibration techniques, where some of the applied calibration standards are not fully specified in advance. However, the mTRL algorithm does require consistency between the line standards. That is, the line standards should be identical in all aspects while differing only in their length. Inconsistency between line standards can lead to impedance mismatch [3]–[5]. The traditional techniques for calibration verification require complete knowledge of a set of reference standards. Often such verification techniques are used for coaxial or waveguide transmission lines, where traceable standards are available [6], [7]. However, mTRL calibration is not only applied in coaxial or waveguide configurations, but also for planar circuits, e.g., printed circuit boards (PCBs) or wafers. In such environments, the fabrication of traceable standards is generally more tricky. Traceable standards for on-wafer measurement have been demonstrated [8]. However, it is more challenging to establish traceable standards with PCB technology, as most dielectric materials used in PCB manufacturing are composite materials made from reinforced fiberglass with epoxy resin. Millimeter-wave (mm-wave) measurements performed on PCB are affected by uncertainties in the manufacturing process [9], [10]. Furthermore, dielectric materials based on fiberglass and epoxy resin can introduce a significant delay skew depending on the location of the transmission lines on the substrate [11], [12]. An illustration of microstrip lines on a PCB based on a fiberglass and epoxy resin mixture is depicted in Fig. 1.

Software implementation and measurements are available online: https://github.com/ZiadHatab/verification-multiline-trl-calibration

Fig. 1. An illustration of a cross-section of a PCB based on reinforced fiberglass in epoxy resin. The different placement of the microstrip line leads to variation in the microstrip lines’ effective permittivity. (a) microstrip line placed directly above the fiberglass, (b) between the glass yearns.

A common technique used to verify calibrations is based on characterizing reflect standards [13]–[15]. For mTRL calibration, it is assumed that the reflect standard is unknown yet symmetric at both ports. Therefore, we could use the calibrated measurement of reflecting standard as a verification metric. Ideally, the calibrated reflect standard should exhibit behavior similar to an ideal reflect standard (e.g., short or open). However, this technique of comparing the calibrated reflect standard has a couple of shortcomings. First, the parasitic behavior of the reflect standard at high frequencies can be challenging to predict, especially at mm-wave and
II. MOTIVATION: ANALYZING THE IMPACT OF IMPEDANCE VARIATION ON mTRL CALIBRATION

We start the analysis by defining the switch-term corrected error box model of a two-port VNA [22], as depicted in Fig. 2. Using the scattering transfer parameters (T-parameters), the measurement of a DUT with an uncalibrated VNA is given by

$$M_{dut} = \frac{k}{k_a k_b} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & 1 \end{bmatrix} T_{dut} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & 1 \end{bmatrix}, \quad (1)$$

where the matrices $A$ and $B$ are the T-parameters of the error boxes holding the first six error terms, and $k$ is the 7th term.

We can convert between T-parameters and S-parameters by the following conversion equations.

$$T = \frac{1}{S_{21}} \begin{bmatrix} S_{12}S_{21} - S_{11}S_{22} & S_{11} \\ -S_{22} & 1 \end{bmatrix}, \quad (2a)$$

$$S = \frac{1}{T_{22}} \begin{bmatrix} T_{12} & T_{11}T_{22} - T_{12}T_{21} \\ 1 & -T_{21} \end{bmatrix}. \quad (2b)$$

After performing an mTRL calibration, we obtain estimates for the calibration coefficients $A$, $B$, and $k$. Since measurements are never ideal, the estimates of the calibration coefficients are prone to error, which can be described by

$$\hat{A} = A \hat{A}^{-1} \frac{1}{k_a}, \quad \hat{B} = \frac{1}{k_b} \hat{B}^{-1} B, \quad (3)$$

where the matrices $\hat{A}$ and $\hat{B}$ summarize the residual error, which are defined by

$$\hat{A} = \begin{bmatrix} a_{11} & \tilde{a}_{12} \\ \tilde{a}_{21} & 1 \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} \tilde{b}_{11} & \tilde{b}_{12} \\ \tilde{b}_{21} & 1 \end{bmatrix}. \quad (4)$$

Since the calibration coefficient $k$ is defined as a common scalar from the error boxes, we can define its residual error as the product of the residual scalars from the error boxes, i.e., the terms $\tilde{k}_a$ and $\tilde{k}_b$ from (3). This leads to

$$\tilde{k} = \frac{\tilde{k}_a \tilde{k}_b}{k}. \quad (5)$$

Therefore, applying a non-ideal mTRL calibration to a DUT can be written as follows.

$$\hat{T}_{dut} = \frac{1}{k} \hat{A}^{-1} M_{dut} \hat{B}^{-1} = \tilde{k} \hat{A} T_{dut} \hat{B}. \quad (6)$$

Ideally, if there are no errors, the calibrated measurement of the DUT reduces to the actual DUT. For the discussion below, we assume that the residual error is introduced only by the impedance variation in the mTRL calibration standards [5]. Therefore, the residual errors reduce to an impedance transformation error, which can be described by [23],

$$\hat{T}_{dut} = \frac{1}{k} \begin{bmatrix} 1 & \Gamma \end{bmatrix} \Gamma^{-1} \begin{bmatrix} \hat{A} & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{B}^{-1} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\Gamma \\ 0 & 1 \end{bmatrix}, \quad (7)$$

where $\Gamma$ is the residual reflection coefficient defined by

$$\tilde{\Gamma} = \frac{Z_0 - \tilde{Z}}{Z_0 + \tilde{Z}}, \quad (8)$$

with $Z_0$ being the true reference impedance and $\tilde{Z}$ is the perturbed reference impedance. In an error free scenario, $Z_0 = \tilde{Z}$ and therefore, $\tilde{\Gamma} = 0$. On the other hand, under the assumption of random error, we can assign $\tilde{\Gamma}$ to be a random process described by its probability distribution. Generally, we might not exactly know the probability distribution of $\tilde{\Gamma}$. In such cases, we can assume a normal distribution, $\tilde{\Gamma} \sim \mathcal{N}(0, \Sigma_{\tilde{\Gamma}})$, where $\Sigma_{\tilde{\Gamma}}$ is the covariance matrix of $\tilde{\Gamma}$.
For an arbitrary two-port DUT, its S-parameters after calibration are determined by converting the T-parameters in (7) to S-parameters using (2b), which results in

\[
S_{\text{cal}} = \frac{S_{22} + \Gamma(\Delta + 1) + S_{11}}{S_{22} + \Gamma(\Delta + 1) + S_{11}},
\]

where \( \Delta = S_{22}S_{11} - S_{12}S_{21} \). To analyze the impact of \( \tilde{\Gamma} \) on the calibrated measurements, we compute the sensitivity of \( |\tilde{S}_{ij}| \) with respect to the real and imaginary parts of \( \tilde{\Gamma} \). Since the quantity \( |\tilde{S}_{ij}| \) is real-valued, we can derive its sensitivity equations by using the Wirtinger equations, relating between complex-valued and real-valued differentiation [24],

\[
\begin{bmatrix}
\frac{\partial |\tilde{S}_{ij}|}{\partial \Re(\tilde{\Gamma})} \\
\frac{\partial |\tilde{S}_{ij}|}{\partial \Im(\tilde{\Gamma})}
\end{bmatrix} = \begin{bmatrix}
1 & 1 \\
\frac{\partial |\tilde{S}_{ij}|}{\partial \Re(\tilde{\Gamma})} & \frac{\partial |\tilde{S}_{ij}|}{\partial \Im(\tilde{\Gamma})}
\end{bmatrix},
\]

where the derivatives \( \frac{\partial |\tilde{S}_{ij}|}{\partial \Re(\tilde{\Gamma})} \) and \( \frac{\partial |\tilde{S}_{ij}|}{\partial \Im(\tilde{\Gamma})} \) are the complex derivatives of \( |\tilde{S}_{ij}| \) with respect to \( \tilde{\Gamma} \) and its conjugate, which are computed by the following equations,

\[
\frac{\partial |\tilde{S}_{ij}|}{\partial \Re(\tilde{\Gamma})} = \frac{1}{2|\tilde{S}_{ij}|} \left( \tilde{S}_{ij}^* \frac{\partial \tilde{S}_{ij}}{\partial \Re(\tilde{\Gamma})} + \tilde{S}_{ij} \frac{\partial \tilde{S}_{ij}^*}{\partial \Re(\tilde{\Gamma})} \right),
\]

\[
\frac{\partial |\tilde{S}_{ij}|}{\partial \Im(\tilde{\Gamma})} = \frac{1}{2|\tilde{S}_{ij}|} \left( \tilde{S}_{ij}^* \frac{\partial \tilde{S}_{ij}}{\partial \Im(\tilde{\Gamma})} + \tilde{S}_{ij} \frac{\partial \tilde{S}_{ij}^*}{\partial \Im(\tilde{\Gamma})} \right),
\]

under the condition \( |\tilde{S}_{ij}| > 0 \). Since \( \tilde{S}_{ij} \) is a rational function in \( \tilde{\Gamma} \), it is straightforward to show that

\[
\frac{\partial \tilde{S}_{ij}}{\partial \Re(\tilde{\Gamma})} = \frac{\partial \tilde{S}_{ij}}{\partial \Im(\tilde{\Gamma})} = 0, \quad \frac{\partial \tilde{S}_{ij}^*}{\partial \Re(\tilde{\Gamma})} = \left( \frac{\partial \tilde{S}_{ij}}{\partial \Re(\tilde{\Gamma})} \right)^*,
\]

Accordingly, combining the results of (11) and (12) and inserting them into (10), we have the general sensitivity equations of \( |\tilde{S}_{ij}| \) with respect to \( \tilde{\Gamma} \) given by

\[
\begin{bmatrix}
\frac{\partial |\tilde{S}_{ij}|}{\partial \Re(\tilde{\Gamma})} \\
\frac{\partial |\tilde{S}_{ij}|}{\partial \Im(\tilde{\Gamma})}
\end{bmatrix} = \begin{bmatrix}
\Re \left( \frac{\tilde{S}_{ij}}{|\tilde{S}_{ij}|} \frac{\partial \tilde{S}_{ij}}{\partial \Re(\tilde{\Gamma})} \right) \\
\Im \left( \frac{-\tilde{S}_{ij}}{|\tilde{S}_{ij}|} \frac{\partial \tilde{S}_{ij}}{\partial \Im(\tilde{\Gamma})} \right)
\end{bmatrix} = \begin{bmatrix}
\Re \left( \frac{\tilde{S}_{ij}}{|\tilde{S}_{ij}|} \frac{\partial \tilde{S}_{ij}}{\partial \Re(\tilde{\Gamma})} \right) \\
\Im \left( \frac{\tilde{S}_{ij}}{|\tilde{S}_{ij}|} \frac{\partial \tilde{S}_{ij}}{\partial \Im(\tilde{\Gamma})} \right)
\end{bmatrix}.
\]

For calibration verification, we desire a standard that is most sensitive to mismatch error. Such a standard will allow us to identify the error in the reference impedance of the calibration. For example, we consider a one-port device, which is described by

\[
\tilde{S}_{11} = \frac{\tilde{\Gamma} + S_{11}}{\tilde{\Gamma}S_{11} + 1}, \quad \frac{\partial \tilde{S}_{11}}{\partial \tilde{\Gamma}} = \frac{1 - \tilde{S}_{11} \tilde{S}_{11}}{\tilde{\Gamma}S_{11} + 1}.
\]

The sensitivity of \( |\tilde{S}_{11}| \) is obtained by plugging (14) into (13). Fig. 2 depicts (13) for an arbitrary one-port device. It is clear from Fig. 2 that we achieve the highest sensitivity for \( \Re (\tilde{\Gamma}) \) when the standard is matched and at an integer multiple of half-wavelength. This evaluation also shows that we achieve the lowest sensitivity with highly reflective standards. The sensitivity to \( \Im (\tilde{\Gamma}) \) is maximized at an integer multiple of quarter-wavelength and by using a highly reflective standard. It is worth mentioning that for the majority of transmission lines implemented on PCBs we have \( |\Im (\tilde{\Gamma})| \ll |\Re (\tilde{\Gamma})| \). Therefore, a one-port device closely matched to the reference plane would be the best candidate to identify errors in the reference impedance. However, as mentioned in Section I, manufacturing and accurately characterizing load standards is challenging for mm-wave frequencies and beyond [25]. Therefore, using a one-port load standard is not ideal for calibration verification unless it is well characterized.

Another inconvenience of using one-port devices as verification standards is the inability to assess uncertainties in transmission terms, that is, \( S_{12} \) and \( S_{21} \). A commonly used two-port device for calibration verification is the step-line (also known as the Beatty line) [13]. This device is widely used in coaxial mTRL validation, where the standard is implemented as a traceable impedance mismatch airline. The S-parameters of an ideal step-line are given by

\[
S_{\text{step}} = \begin{bmatrix}
\frac{e^{\gamma l}(e^{2\gamma l}-1)}{e^{2\gamma l}-1} & \frac{e^{\gamma l}(1-e^{-2\gamma l})}{e^{2\gamma l}-1} \\
\frac{e^{\gamma l}(e^{2\gamma l}-1)}{e^{2\gamma l}-1} & \frac{e^{\gamma l}(1-e^{-2\gamma l})}{e^{2\gamma l}-1}
\end{bmatrix},
\]

where \( l \) and \( \gamma \) are the length and propagation constant of the step-line. The term \( \Gamma_{nm} \) is the reflection coefficient of the impedance transition from \( Z_n \) to \( Z_m \), which is given by

\[
\Gamma_{nm} = \frac{Z_m - Z_n}{Z_m + Z_n},
\]

Similarly to the one-port case, we can calculate the sensitivity of the calibrated step-line by inserting (15) into (9) and computing the derivatives with respect to \( \tilde{\Gamma} \) and evaluating (13). The expressions for the derivatives are lengthy and are not presented here. However, they were assessed using the Python symbolic library SymPy [26]. The sensitivities of \( |\tilde{S}_{11}| \) and \( |\tilde{S}_{21}| \) are shown in Fig. 3. From the figure, we see that \( |\tilde{S}_{21}| \) is not sensitive to impedance variation when the step-line is matched, regardless of the electrical length. Due to this reason, we cannot use the line standards used in the calibration as verification standards to identify impedance.
errors. Generally, the sensitivity of $|S_{11}|$ and $|S_{21}|$ varies as a function of electrical length and equals zero at an integer multiple of half-wavelength. Therefore, using a single step-line cannot cover all frequencies.

The frequency limitation that we observe in the sensitivity graph in Fig. 4 is similar to the frequency limitation of the TRL calibration. Therefore, we can cover a wide range of frequencies using multiple step-line standards of different lengths. To combine the result of every step-line, we propose a second mTRL calibration using the step-lines as the standards. In an ideal case, the error boxes between the reference mTRL calibration and the step-line calibration should correspond to the impedance transformation from the mTRL reference impedance to the step-line impedance. In Section III, we elaborate on extracting the reflection coefficient of the impedance transition from non-ideal measurements. Section IV highlights how to use the obtained results as a verification metric.

III. EXTRACTION OF THE IMPEDANCE TRANSITION

The core principle of our method is to perform two mTRL calibrations using line standards of different impedance. The objective is to measure the error boxes between the two mTRL calibrations, which correspond to the impedance transition segment. We can extract the reflection coefficient of the impedance transition from the error boxes. This reflection coefficient then becomes our verification metric, to which we define a confidence interval to validate the calibration (see Section IV). An illustration of a microstrip thru standard of both mTRL calibrations is shown in Fig. 5.

Similar to the discussion in the previous section, we use the T-parameters to describe the error box model of both mTRL calibrations. The error box model of the primary mTRL calibration is given by

$$M_1 = k_gA \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & 1 \end{bmatrix} T_{\text{dut}} \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix},$$

and the error box model of the second mTRL calibration is given by

$$M_2 = k_gk_B \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & 1 \end{bmatrix} T_{\text{dut}} \begin{bmatrix} d_{11} \\ d_{21} \end{bmatrix}. \tag{18}$$

From Fig. 5, we know that the error boxes of the second mTRL comprise both the error boxes of the first mTRL and the impedance transition segments. Therefore, by performing both mTRL calibrations, we can determine the matrices $A$, $B$ from the primary mTRL calibration, and the matrices $C$, and $D$ from the verification mTRL calibration. From these matrices, the T-parameters of the left impedance transition are determined by

$$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & 1 \end{bmatrix} = \frac{a_{11} - a_{21}a_{12}}{a_{11} - a_{21}c_{12}} A^{-1} C, \tag{19}$$

while the T-parameters of the right impedance transition are given by

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & 1 \end{bmatrix} = \frac{b_{11} - b_{12}b_{21}}{b_{11} - b_{12}d_{21}} D B^{-1}. \tag{20}$$

It should be noted that we have not considered the terms $k_g$ and $k_B$ in the above equations, because the transition segments are reciprocal devices (i.e., $S_{12} = S_{21}$) and the terms $k_g$ and $k_B$ are implicitly contained in $g_{11}$ and $h_{11}$ through the conversion relationship between S- and T-parameters, as stated in (2). Therefore, there is no mathematical benefit in including $k_g$ and $k_B$ in the derivation.
A. Modeling the impedance transition

To accurately extract the reflection coefficient of the impedance transition, we require a model to account for the non-ideality of such a structure. For simplicity sake, we focus only on the left impedance transition in the following discussion.

For the determination of the reflection coefficient of the left impedance transition, we propose three possible models to characterize the transition, as shown in Fig. 6. In general, the impedance transition can be split into four blocks:

1) Initial offset: This offset accounts for the case when the impedance discontinuity is realized at an offset from the primary mTRL calibration. Generally, this offset is known by the propagation constant $\gamma_1$ and the physical length $d_1$ of the offset.

2) Transition parasitic: In general, this non-ideal behavior of the impedance transition is accounted for by one of the three parasitic models presented in Fig. 6.

3) Ideal impedance transformer: This part accounts for the actual impedance transformation, which is defined by $\Gamma_{nm}$ from (16). For notation simplicity, we drop the indices, i.e., $\Gamma_{nm} = \Gamma$.

4) Second offset: The offset after the impedance transformation is an essential part of the transition design. We can account for this offset from knowledge of the propagation constant $\gamma_2$ and the physical length $d_2$.

![Fig. 6. Proposed models for an impedance transition segment between transmission lines. All matrices are given in T-parameters.](image)

It is worth mentioning that the placement of the model for the parasitic before or after the transformer is arbitrary, as it is assumed that the behavior of the parasitic transition is unknown, and hence scaling it with an impedance transformation makes no difference in the derivation. To solve for the unknowns, we write the cascaded matrices of Fig. 6 in relation to the matrix $G$ as follows:

$$k_y G = \frac{1}{\sqrt{1 - \Gamma^2}} L_1 P \begin{bmatrix} 1 & \Gamma \\ \Gamma & 1 \end{bmatrix} L_2,$$  \label{eq:matrix}

where $L_1$ and $L_2$ are the left and right offsets, respectively, and $P$ represents the parasitic transition, which is defined by the three models in Fig. 6 to equal:

$$P^{(1)} = \frac{1}{2} \begin{bmatrix} (1 - y)(1 - z) + 1 & (1 - y)(1 + z) - 1 \\ (1 + y)(1 - z) - 1 & (1 + y)(1 + z) + 1 \end{bmatrix}$$  \label{eq:model1}

$$P^{(2)} = \frac{1}{2} \begin{bmatrix} (1 - y)(1 - z) + 1 & (y + 1)(z - 1) + 1 \\ (y - 1)(z + 1) + 1 & (1 + y)(1 + z) + 1 \end{bmatrix}$$  \label{eq:model2}

$$P^{(3)} = \frac{1}{t} \begin{bmatrix} -r^2 + t^2 & r \\ -r & 1 \end{bmatrix}.$$  \label{eq:model3}

To be able to solve for the unknowns ($\Gamma$ and parasitic), we need to construct three equations relating to $G$. As $L_1$ and $L_2$ are assumed to be known from performing both calibrations, we can take their inverse on both sides of (21). Furthermore, since $k_y$ is not needed, we can normalize the equation by the fourth element of the matrix. Applying these two operations results in

$$\begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & 1 \end{bmatrix} = \begin{bmatrix} \Gamma_{e_1 e_1} + \Gamma_{e_2 e_2} & \Gamma_{e_1 e_2} \\ \Gamma_{e_2 e_1} & \Gamma_{e_2 e_2} \end{bmatrix} \begin{bmatrix} \Gamma_{e_1 e_1} + \Gamma_{e_2 e_2} & \Gamma_{e_1 e_2} \\ \Gamma_{e_2 e_1} & \Gamma_{e_2 e_2} \end{bmatrix},$$  \label{eq:gamma}

where $p_{ij}$ are the elements of the parasitic matrix $P$, and $\gamma_{ij}$ are defined as

$$\gamma_{11} = g_{11} e^{2\gamma_1 d_1 + 2\gamma_2 d_2},$$  \label{eq:gamma11}

$$\gamma_{21} = g_{21} e^{2\gamma_2 d_2},$$  \label{eq:gamma21}

$$\gamma_{12} = g_{12} e^{2\gamma_1 d_1},$$  \label{eq:gamma12}

with the parameters $\{\gamma_1, d_1\}$ and $\{\gamma_2, d_2\}$ being the propagation constant and the offset length of the offset line, as illustrated in Fig. 6.

From (23), we recognize that we have three equations in three unknowns, and thus can be solved for the three proposed models uniquely. The solution for the reflection coefficient from the first and second models is given by

$$\Gamma^{(1,2)} = \pm \frac{(\gamma_{11} + \gamma_{21} + \gamma_{12} + 1)^2 - 4(\gamma_{11} - \gamma_{21} \gamma_{12})}{(\gamma_{11} + \gamma_{21} + \gamma_{12} + 1)^2 + 4(\gamma_{11} - \gamma_{21} \gamma_{12})},$$  \label{eq:gamma12}

where the plus and minus signs are the solutions for the first and second models, respectively. Similarly, the reflection coefficient of the third model is given by

$$\Gamma^{(3)} = \frac{\gamma_{21} + \gamma_{12}}{\gamma_{11} + 1}. $$  \label{eq:gamma3}

Likewise, we can also derive a solution for the parasitic elements. For the first model, $y$ and $z$ are given by

$$y^{(1)} = \frac{-\gamma_{11} + \gamma_{21} - \gamma_{12} + 1}{\gamma_{11} + \gamma_{21} + \gamma_{12} + 1},$$  \label{eq:y1}

$$z^{(1)} = \frac{(\gamma_{12} + 1)^2 - (\gamma_{11} + \gamma_{21})^2}{4(\gamma_{11} - \gamma_{21} \gamma_{12})}.$$  \label{eq:z1}
For the second model, $y$ and $z$ are given by
\[
y^{(2)} = \frac{(\bar{g}_{12} - 1)^2 - (\bar{g}_{11} - \bar{g}_{21})^2}{4(\bar{g}_{11} - \bar{g}_{21}\bar{g}_{12})}, \tag{28a}
\]
\[
z^{(2)} = \frac{-\bar{g}_{11} - \bar{g}_{21} + \bar{g}_{12} + 1}{\bar{g}_{11} - \bar{g}_{21}\bar{g}_{12} + 1}. \tag{28b}
\]

Lastly, for the third model, $t^2$ and $r$ are given by
\[
t^2 = \frac{(\bar{g}_{11} - \bar{g}_{21}\bar{g}_{12})((\bar{g}_{11} + 1)^2 - (\bar{g}_{21} + \bar{g}_{12})^2)}{(\bar{g}_{11} - \bar{g}_{21}\bar{g}_{12} - \bar{g}_{12} + 1)^2}, \tag{29a}
\]
\[
r = \frac{-\bar{g}_{12} - \bar{g}_{11}\bar{g}_{21}}{\bar{g}_{11} - \bar{g}_{21}\bar{g}_{12} - \bar{g}_{21}^2 + 1}. \tag{29b}
\]

We should note that the equations for the parasitic elements of the three models are not used in the following discussion, as we mainly care about $\Gamma$. However, these equations could be used as an accurate approach for the general characterization of impedance transitions of various types of transmission lines. Furthermore, all equations we derived for the left impedance transition can be used for the right impedance transition by simply substituting $g_{ij}$ by the following relationships:
\[
g_{11} \longleftrightarrow h_{11}, \tag{30a}
\]
\[
g_{21} \longleftrightarrow -h_{12}, \tag{30b}
\]
\[
g_{12} \longleftrightarrow -h_{21}. \tag{30c}
\]

Additionally, since the impedance transition on the left and right sides is assumed to be equal, we can use this fact to build an average value for $g_{ij}$ from $h_{ij}$ as
\[
g_{ij}^{(avg)} = \frac{1}{2} (g_{11} + h_{11}), \tag{31a}
\]
\[
g_{21}^{(avg)} = \frac{1}{2} (g_{21} - h_{12}), \tag{31b}
\]
\[
g_{12}^{(avg)} = \frac{1}{2} (g_{12} - h_{21}). \tag{31c}
\]

B. Differences between proposed models

The most common transmission line impedance transition models assume constant inductance and capacitance [27]. The models presented in Fig. 6 are general and can vary with frequency. In the error-free case, all three models should give identical results, as all three are the exact solution to (23). However, in certain types of errors, they behave differently. For example, the first and second models in Fig. 6 are sensitive to length offset error, as the parasitic effects are modeled with lumped elements. In contrast, the third model can account for any symmetrical error. In fact, the sensitivity of the first and second models to length offset can be advantageous in identifying length offset errors, which could arise from the estimated propagation constant. For illustration purposes, we tested the three models to extract $\Gamma$ using electromagnetic (EM) simulation with the software ANSYS HFSS. The tested transmission line is a microstrip line with an impedance of approximately 50 Ω on one side and roughly 30 Ω on the other side. A port extension in the simulation compensates the offset length of the transition. Fig. 7 shows the result of the extracted $\Gamma$ for the three proposed models. Furthermore, we introduced an error of +0.03 mm offset at the 50 Ω line segment, whose effects can be seen in the results of the first and second models. The third model does not show a significant impact under small offset variation.

In general, the third model is the most reliable in capturing the impedance transition, as it does not enforce any assumptions on the type of parasitic behavior, except that the network must show a symmetric response. For verification purposes, it is better to use all three models. If the first and second models show deviation away from the third model, then we know that there is an error in the location of the reference plane, which could arise from an error in the estimation of the propagation constant.

IV. Defining Validation Bounds

From the previous section, we established an approach to determine the reflection coefficient of the impedance transition from the verification mTRL calibration kit. To verify the accuracy and consistency of the reference impedance of the primary mTRL, we require the reflection coefficient of the impedance transition to be within a confidence bound. For example, the confidence bound can be set to 95% of the coverage of a Gaussian distribution. The advantage of transmission line standards is that they can be fully characterized by knowing their cross-sectional geometry and material properties. Therefore, if we know the geometric and material parameters and their uncertainties, the reflection coefficient extracted from the measurement must remain within the confidence interval from the propagated uncertainties through the transmission line. For example, in Fig. 8, we illustrate a cross-section of a microstrip line with parameters showing uncertainties. The surface finish and roughness of the copper foil are not explicitly included in Fig. 8. Their impact can be incorporated into the uncertainty of the copper’s resistivity as an effective resistivity (or conductivity) [28].
The Jacobian matrix $J$ where $\mu$ and $\kappa$ condition: coefficient from the verification mTRL meets the following the discussion in Section II with Equation (13). Therefore, the result surpasses the confidence bounds, other types of error exist beyond an impedance variation.

$$\Sigma_{Z_i} = J_{Z_i}(\mu_\theta) \text{diag} \left( \begin{bmatrix} \sigma_{\theta_1}^2 & \sigma_{\theta_2}^2 & \cdots \end{bmatrix} \right) J_{Z_i}^T(\mu_\theta),$$

where $\theta$ is the vector that contains all parameters to which we know their mean value $\mu_\theta$ and their standard uncertainty $\sigma_\theta$. The Jacobian matrix $J_{Z_i}(\theta)$ is defined by

$$J_{Z_i}(\theta) = \frac{\partial \text{Re}(Z_i)}{\partial \theta_1} \quad \frac{\partial \text{Re}(Z_i)}{\partial \theta_2} \quad \cdots \quad \frac{\partial \text{Im}(Z_i)}{\partial \theta_1} \quad \frac{\partial \text{Im}(Z_i)}{\partial \theta_2} \quad \cdots.$$ 

The derivatives in the Jacobian matrix can be determined from analytical models of the transmission line or, more accurately, directly from the EM solver [30]. The standard uncertainty of the absolute value of the reflection coefficient is determined similarly by propagating the covariance matrices of the characteristic impedance of both calibrations from (32) through the absolute value of (16), which is determined by

$$\sigma_{|\Gamma|}^2 = J_{|\Gamma|}(\mu_{Z_1}, \mu_{Z_2}) \begin{bmatrix} \Sigma_{Z_1} & 0 \\ 0 & \Sigma_{Z_2} \end{bmatrix} J_{|\Gamma|}^T(\mu_{Z_1}, \mu_{Z_2}),$$

where $\mu_{Z_1}$ and $\mu_{Z_2}$ are the expected values of the characteristic impedance of both transmission lines. The Jacobian matrix $J_{|\Gamma|}(Z_1, Z_2)$ is given by

$$J_{|\Gamma|}(Z_1, Z_2) = \begin{bmatrix} \frac{\partial |\Gamma|}{\partial \text{Re}(Z_1)} & \frac{\partial |\Gamma|}{\partial \text{Im}(Z_1)} & \frac{\partial |\Gamma|}{\partial \text{Re}(Z_2)} & \frac{\partial |\Gamma|}{\partial \text{Im}(Z_2)} \end{bmatrix}.$$ 

The partial derivatives in (35) are determined following the discussion in Section II with Equation (13). Therefore, an mTRL calibration is validated if the extracted reflection coefficient from the verification mTRL meets the following condition:

$$\text{mTRL Validity} = \begin{cases} \text{True}, & -\kappa \sigma_{|\Gamma|} \leq |\Gamma| - \mu_{|\Gamma|} \leq \kappa \sigma_{|\Gamma|} \\ \text{False}, & \text{otherwise} \end{cases}$$

where $\mu_{|\Gamma|}$ is the expected value of the reflection coefficient, and $\kappa$ is the coverage factor of a Gaussian distribution. For 95% coverage, $\kappa = 2$, while for 99.7% coverage, $\kappa = 3$. If the result surpasses the confidence bounds, other types of error exist beyond an impedance variation.

V. PCB Measurement

A. Measurement setup

The setup comprises two microstrip-based mTRL calibration kits. The interface pads of the microstrip lines were designed as a tapered grounded coplanar waveguide (GCPW) to microstrip, which allowed us to measure the structures on a probe station with ground-signal-ground (GSG) probes. To perform the measurements, an Anritsu vector star VNA was integrated with a semi-automatic probe station from Formfactor. The VNA measurement heads cover a frequency range of 70 kHz to 150 GHz using 0.8 mm coaxial connectors. A photo of the PCB on the probe station is depicted in Fig. 9.

The PCBs were designed using four copper layers and three dielectric substrates, with top and bottom substrates being prepreg and the middle substrate is a core laminate. The measured structures were designed on the top prepreg. The other layers were only used for mechanical support. The initial dimensions used for the design of the microstrip lines were as follows: substrate thickness 0.05 mm, copper thickness (trace thickness) 0.02 mm, first trace width (primary mTRL) 0.107 mm, second trace width (verification mTRL) 0.220 mm. These dimensions corresponded to line standards of a characteristic impedance of approximately 50 $\Omega$ and 30 $\Omega$, respectively. For accurate modeling of the microstrip lines, a cross-section inspection was performed, as depicted in Fig. 10, which showed that the fabricated dimensions are within the expected manufacturing tolerances while differing slightly from the nominally designed values. From several cross-section photos and the information from the PCB manufacturer AT&S AG, the estimated dimensional parameters with their tolerances are presented in Table I.

We used the Megtron 7 R-5680(N) prepreg substrate from Panasonic with a fiberglass style 1027 and a resin content of 77% [31]. In the EM simulation, we treated the substrate’s loss tangent and the copper foil resistivity as ideal with the values in Table I, because we did not have reliable material.
measurements available with quantified uncertainties. For the dielectric constant estimation, the uncertainties covered a random placement of the microstrip line above the substrate. In some cases, the microstrip might be located on a fiberglass weave, while in other cases, the microstrip might be located on the resin. Generally, fiberglass has a higher dielectric constant than epoxy resin. From the reference [32], the “low Dk glass” Panasonic uses is expected to have a dielectric constant of around 2.6. Therefore, we can estimate the standard uncertainty in the dielectric constant seen by the microstrip lines using these maximum and minimum values as the 99.7% coverage of a Gaussian distribution. Under these conditions, we get a standard deviation for the dielectric constant given by [33],

$$\sigma_{\epsilon_r} = \frac{1}{6} (\max(\epsilon_r) - \min(\epsilon_r)) = \frac{5 - 2.6}{6} = 0.4. \quad (37)$$

The measurement was conducted in the frequency range from 2 GHz to 150 GHz. Both mTRL calibrations use six microstrip lines with lengths \{0, 0.5, 1, 3, 5, 6.5\} mm. The reference planes were set to the middle of the thru structure for both calibration kits (similar to the illustration in Fig. 5). The offset length of the impedance transition was chosen to be 0.5 mm on both sides (that is, \(d_1 = d_2 = 0.5\) mm in Fig. 6). The reflect standard was implemented as an offset-short with micro-VIA with an offset of 0.5 mm.

B. Results and discussion

The mTRL calibration was performed using the algorithm of [2]. In Figs. 11-13, we depict the parameters that are usually investigated after performing the mTRL calibration to identify any systematic error, e.g., user error or faulty standard. Fig. 11 shows the attenuation per-unit-length and effective relative permittivity of both microstrip mTRL kits. The primary mTRL has lower effective permittivity and losses than the verification mTRL because the latter has broader traces and is impacted more by the permittivity of the substrate and the roughness of the copper foil. Figs. 12 and 13 depict the calibrated response of the 3 mm line from both calibrations and the offset-short standard. From both figures, we can identify that the measurement above 110 GHz is more noisy, which can be attributed to the measurement instrument. Albeit the noise, the calibrated response of the lines and the offset-short indicate no apparent user error nor faulty standards.

![Cross-section photos of the fabricated transmission lines](image1)

![Cross-section photos of the fabricated transmission lines](image2)

**Fig. 10.** Cross-section photos of the fabricated transmission lines. (a) and (b) are the cross-section of the primary mTRL calibration, while (c) and (d) are from the second mTRL calibration (step-lines).

![Plot of the magnitude of S21](image3)

![Plot of the magnitude of S21](image4)

**Fig. 11.** The extracted (left plot) attenuation per-unit-length and (right plot) the effective relative permittivity of the microstrip lines of the primary and verification mTRL calibration kits.

![Plot of the magnitude of S21](image5)

![Plot of the magnitude of S21](image6)

**Fig. 12.** Plot of the magnitude of \(S_{21}\) of the calibrated 3 mm microstrip line from both mTRL kits.

Lastly, we computed the reflection coefficient of the impedance transition from the left and right error boxes following the procedures discussed in Section III. The result of the extracted reflection coefficient from both sides is shown...
in Fig. 14. The expected response and the validation bounds were determined by 2d EM simulation using the full-wave solver ANSYS HFSS. The values listed in Table I were used for the simulation and all necessary derivatives were calculated directly from the EM simulator [30]. From Fig. 14, we recognize that all proposed models exhibit similar response, which indicate no significant offset error present in the calibration. Furthermore, the extracted reflection coefficient of the impedance transition from the left and right error boxes agrees with the expected result and is within the 68% validation bounds. Nevertheless, the fluctuation we see in Fig. 14 does indicate a variation between the standards, which is anticipated due to non-uniform dielectric material and manufacturing tolerances.

In Fig. 15, we show the uncertainty budget of the reflection coefficient due to the individual parameters from the EM simulation. It is evident that the uncertainty in the dielectric constant is the dominant cause of impedance variation. In addition, we see that the thickness of the substrate also has a strong influence. One way to mitigate the uncertainties due to the dielectric substrate is to use a thicker substrate and a spread fiberglass cloth. It should be noted that in the EM simulation we assumed the substrate to be nondispersive due to missing specifications.

VI. CONCLUSION

This work presented a method to verify the mTRL calibration by performing an additional mTRL calibration using step-line standards. The reflection coefficient of the impedance transition structure between the two mTRL calibrations was extracted and used as a verification metric. We derived a confidence interval by which the calibration can be verified. We indicated that variation in the characteristic impedance among the line standards is typical in PCBs because of non-uniform dielectric substrates being constructed from reinforced fiberglass in epoxy resin. Manufacturing tolerances also introduce errors in the reference impedance of the calibration.

The advantage of our method is its ability to assess the accuracy of the reference impedance at the calibration plane across a wide range of frequencies without explicitly measuring the characteristic impedance of the line standards. Moreover, for many types of transmission lines, the reflection coefficient of the impedance transition rarely varies with frequency over a wide range of frequencies, making it an ideal validation metric that is easy to interpret. One disadvantage of the proposed method is the requirement of performing another calibration using multiple standards, which can be laborious. However, calibration could be automated if measurements are performed on a probe station.

ACKNOWLEDGMENT

The financial support by the Austrian Federal Ministry for Digital and Economic Affairs and the National Foundation for Research, Technology, and Development is gratefully acknowledged. The authors also thank AT&S AG for their support during the design and fabrication of the PCBs and for providing the cross-sectional images of the microstrip lines. The authors thank ebsCENTER for granting access to their measurement equipment.

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