Production of $f_2(1270)$ meson in $pp$ collisions at the LHC via gluon-gluon fusion in the $k_t$-factorization approach

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Abstract

We calculate inclusive cross section for $f_2(1270)$ tensor meson production via color singlet gluon-gluon fusion in the $k_t$-factorization approach with unintegrated gluon distribution functions (UGDFs). The process is maybe interesting in the context of searches for saturation effects. The energy-momentum tensor, equivalent to helicity-2 coupling, and helicity-0 coupling are used for the $g^*g^* \rightarrow f_2(1270)$ vertex. Some parameters are extracted from $\gamma\gamma \rightarrow f_2(1270) \rightarrow \pi\pi$ reactions. Different modern UGDFs from the literature are used. The results strongly depend on the parametrization of the $g^*g^* \rightarrow f_2(1270)$ form factor. Our results for transverse momentum distributions of $f_2$ are compared to preliminary ALICE data. We can obtain agreement with the data only at larger $f_2(1270)$ transverse momenta only for some parametrizations of the $g^*g^* \rightarrow f_2(1270)$ form factor. No obvious sign of the onset of saturation is possible. At low transverse momenta one needs to include also the $\pi\pi$ final-state rescattering. The agreement with the ALICE data can be obtained by adjusting probability of formation and survival of $f_2(1270)$ in a harsh quark-gluon and multipion environment. The pomeron-pomeron fusion mechanism is in addition discussed and results are quantified.
I. INTRODUCTION

The mechanism of $f_2(1270)$ meson production in proton-proton collisions at high energies was not carefully studied so far. Till very recently in the PYTHIA event generator $f_2(1270)$ is not produced in a primary fragmentation process but occurs only in decays, e.g. $J/\psi \to f_2(1270)\omega$, $D_s^\pm \to f_2(1270)\pi^\pm$, $B^\pm \to \tau^\pm v_\tau f_2(1270)$. The corresponding branching fractions are rather small so one cannot expect large contributions. On the other hand it is rather difficult to observe $f_2(1270)$ experimentally. The dominant, and relatively easy, decay channel is $f_2(1270) \to \pi^+\pi^-$. Then the signal is on a huge non-reducible $\pi^+\pi^-$ background. So far only STAR [1] and ALICE [2] undertook experimental efforts. Some time ago [3] it was suggested that the gluon-gluon fusion could be the dominant production mechanism. Certainly this interesting working hypothesis requires further elaboration and experimental confirmation.

In the present paper we follow the idea from [3] and try to shed new light on the situation. We will apply the $k_t$-factorization approach successfully used for $\chi_c$ quarkonium production [4, 5], for $\eta_c(1S/2S)$ production [6, 7], and recently for $f_0(980)$ production [8]. In this study, we focus on the production of $f_2(1270)$ meson. The $g^*g^* \to f_2(1270)$ vertex is not known a priori. We will use modern unintegrated gluon distributions from the literature. We will try to verify the hypothesis of the dominance of the helicity-2 component, the coupling of spin-2 mesons to the energy-momentum tensor [3], by comparing our results to the preliminary ALICE data [2].

The tensor-meson dominance for energy-momentum tensor (see e.g. [3]) is a possibility used already in $\gamma\gamma \to f_2(1270)$ subprocess [9] for two on-shell photons. In [10] two tensor structures corresponding to $F^{(0)}$ helicity-0 and $F^{(2)}$ helicity-2 couplings were found and their strength was determined from the comparison to the Belle data for the $\gamma\gamma \to \pi\pi$ reactions. The data with the on-shell photons require dominance of helicity-2 coupling over helicity-0 coupling; see Refs. [11–13]. The $\gamma^*\gamma \to f_2(1270)$ coupling was discussed in [14–17]. The differential cross section for the process $\gamma^*\gamma \to \pi^0\pi^0$ in $e^+e^-$ scattering up to $Q^2 = 30\text{ GeV}^2$ were studied by the Belle Collaboration [18]. The transition form factor of the $f_0(980)$ and helicity-0, -1, and -2 transition form factors of the $f_2(1270)$ were extracted there. We will also use tensorial structures for $\gamma^*\gamma^* \to f_2(1270)$ vertex from [19] (see also Ref. [20]). Recently, some numerical results for the helicity amplitudes of $\gamma^*\gamma^* \to \pi\pi$ that, include $f_2(1270)$, depending of the photon virtualities, were presented in [21, 22].

II. SOME DETAILS OF THE MODEL CALCULATIONS

A. $\gamma^*\gamma^* \to f_2(1270)$ vertex

1. Ewerz-Maniatis-Nachtmann vertex (EMN)

In [10] the $f_2\gamma\gamma$ vertex for ‘on-shell’ $f_2$ meson and real photons was considered; see Eq. (3.39) of [10] and the discussion in Sec. 5.3 there. Here we are interested in $\gamma^*(Q_1^2)\gamma^*(Q_2^2) \to f_2(1270)$ process, thus to describe the dependence on photon virtualities we should introduce the vertex form factors $F^{(0)}(Q_1^2, Q_2^2)$ and $F^{(2)}(Q_1^2, Q_2^2)$.

Then the $\gamma^*\gamma^* \to f_2(1270)$ vertex, including the form factors $F^{(\Lambda)}(Q_1^2, Q_2^2)$, can be
parametrized as

\[ \Gamma_{\mu\nu\kappa\lambda}^{(f_2\gamma\gamma)}(q_1, q_2) = 2a_{f_2\gamma\gamma}\Gamma_{\mu\nu\kappa\lambda}^{(0)}(q_1, q_2) F^{(0)}(Q_1^2, Q_2^2) \]

\[ -b_{f_2\gamma\gamma}\Gamma_{\mu\nu\kappa\lambda}^{(2)}(q_1, q_2) F^{(2)}(Q_1^2, Q_2^2), \]  

(2.1)

with two rank-four tensor functions,

\[ \Gamma_{\mu\nu\kappa\lambda}^{(0)}(q_1, q_2) = \left[ (q_1 \cdot q_2) g_{\mu\nu} - q_2\mu q_1\nu \right] \left[ q_1x q_2\lambda + q_2x q_1\lambda - \frac{1}{2} (q_1 \cdot q_2) g_{\kappa\lambda} \right], \]  

(2.2)

\[ \Gamma_{\mu\nu\kappa\lambda}^{(2)}(q_1, q_2) = \left( q_1 \cdot q_2 \right) \left( g_{\mu\kappa} g_{\nu\lambda} + g_{\mu\lambda} g_{\nu\kappa} \right) + g_{\mu\nu} \left( q_1x q_2\lambda + q_2x q_1\lambda \right) - q_1\nu q_2\lambda g_{\mu\kappa} - q_1\nu q_2x g_{\mu\lambda} - q_2\mu q_1\lambda g_{\nu\kappa} - q_2\mu q_1x g_{\nu\lambda} - \left[ (q_1 \cdot q_2) g_{\mu\nu} - q_2\mu q_1\nu \right] g_{\kappa\lambda}. \]  

(2.3)

see Eqs. (3.18)–(3.22) of [10].

To obtain \( a_{f_2\gamma\gamma} \) and \( b_{f_2\gamma\gamma} \) in (2.1) we use the experimental value of the radiative decay width

\[ \Gamma(f_2 \rightarrow \gamma\gamma) = (2.93 \pm 0.40) \text{ keV}, \]

helicity zero contribution \( \approx 9\% \) of \( \Gamma(f_2 \rightarrow \gamma\gamma) \),

(2.4)

as quoted for the preferred solution III in Table 3 of [13]. Using the decay rate from (5.28) of [10]

\[ \Gamma(f_2 \rightarrow \gamma\gamma) = \frac{m_{f_2}}{80\pi} \left( \frac{1}{6} m_{f_2}^6 |a_{f_2\gamma\gamma}|^2 + m_{f_2}^2 |b_{f_2\gamma\gamma}|^2 \right), \]

(2.5)

and assuming \( a_{f_2\gamma\gamma} > 0 \) and \( b_{f_2\gamma\gamma} > 0 \), we find

\[ a_{f_2\gamma\gamma} = \alpha_{em} \times 1.17 \text{ GeV}^{-3}, \]

(2.6)

\[ b_{f_2\gamma\gamma} = \alpha_{em} \times 2.46 \text{ GeV}^{-1}, \]

(2.7)

where \( \alpha_{em} = e^2 / (4\pi) \approx 1/137 \) is the electromagnetic coupling constant.

2. Pascalutsa-Pauk-Vanderhaeghen vertex (PPV)

In Refs. [19, 20, 23] it was shown that the most general amplitude for the process \( \gamma^*(q_1, \lambda_1) + \gamma^*(q_2, \lambda_2) \rightarrow f_2(\Lambda) \), describing the transition from an initial state of two virtual photons to a tensor meson \( f_2 \) \((J^{PC} = 2^{++})\) with the mass \( m_{f_2} \) and helicity \( \Lambda = \pm 2, \pm 1, 0 \), involves five independent structures (invariant amplitudes).
In the formalism presented in [19] the $\gamma^*\gamma^* \to f_2(1270)$ vertex was parameterized as

$$\Gamma_{\mu
u k \lambda}^{(f_2\gamma\gamma)}(q_1, q_2) = 4\pi\alpha_{em} \left\{ \left[ R_{\mu\nu}(q_1, q_2) R_{\nu\lambda}(q_1, q_2) + \frac{s}{8X} R_{\mu\nu}(q_1, q_2)(q_1 - q_2)_{\kappa}(q_1 - q_2)_{\lambda} \right] \times \frac{V}{m_{f_2}} T^{(2)}(Q_1^2, Q_2^2) \right. \\
+ R_{\nu k}(q_1, q_2)(q_1 - q_2)_{\lambda} \left( q_{1\mu} + \frac{Q_1^2}{V} q_{2\mu} \right) \frac{1}{m_{f_2}} T^{(1)}(Q_1^2, Q_2^2) \\
+ R_{\mu k}(q_1, q_2)(q_2 - q_1)_{\lambda} \left( q_{2\nu} + \frac{Q_2^2}{V} q_{1\nu} \right) \frac{1}{m_{f_2}} T^{(1)}(Q_2^2, Q_1^2) \\
+ R_{\mu\nu}(q_1, q_2)(q_1 - q_2)_{\kappa}(q_1 - q_2)_{\lambda} \frac{1}{m_{f_2}} T^{(0,T)}(Q_1^2, Q_2^2) \\
- \frac{1}{V} \left( q_{1\mu} + \frac{Q_1^2}{V} q_{2\mu} \right) \left( q_{2\nu} + \frac{Q_2^2}{V} q_{1\nu} \right) (q_1 - q_2)_{\kappa}(q_1 - q_2)_{\lambda} \frac{1}{m_{f_2}} T^{(0,L)}(Q_1^2, Q_2^2) \right\}, \quad (2.8)$$

where photons with four-momenta $q_1$ and $q_2$ have virtualities, $Q_1^2 = -q_1^2$ and $Q_2^2 = -q_2^2$, $s = (q_1 + q_2)^2 = 2\nu - Q_1^2 - Q_2^2$, $X = \nu^2 - q_1^2 q_2^2$, $\nu = (q_1 \cdot q_2)$, and

$$R_{\mu\nu}(q_1, q_2) = -g_{\mu\nu} + \frac{1}{X} \left[ \nu (q_{1\mu} q_{2\nu} + q_{2\mu} q_{1\nu}) - q_1^2 q_{2\mu} q_{2\nu} - q_2^2 q_{1\mu} q_{1\nu} \right]. \quad (2.9)$$

In Eq. (2.8) $T^{(\Lambda)}(Q_1^2, Q_2^2)$ are the $\gamma^*\gamma^* \to f_2(1270)$ transition form factors for $f_2(1270)$ helicity $\Lambda$. For the case of helicity zero, there are two form factors depending on whether both photons are transverse (superscript T) or longitudinal (superscript L).

We can express the transition form factors as

$$T^{(\Lambda)}(Q_1^2, Q_2^2) = F^{(\Lambda)}(Q_1^2, Q_2^2) T^{(\Lambda)}(0, 0). \quad (2.10)$$

In the limit $Q_{1,2}^2 \to 0$ only $T^{(0,T)}$ and $T^{(2)}$ contribute and their values at $Q_{1,2}^2 \to 0$ determine the two-photon decay width of $f_2(1270)$ meson.

Comparing two approaches given by (2.1) and (2.8)–(2.10) at both real photons ($Q_1^2 = Q_2^2 = 0$) and at $\sqrt{s} = m_{f_2}$ we found the correspondence

$$4\pi\alpha_{em} T^{(0,T)}(0, 0) = -a_{f_2\gamma\gamma} \frac{m_{f_2}^3}{2}, \quad (2.11)$$

$$4\pi\alpha_{em} T^{(2)}(0, 0) = -b_{f_2\gamma\gamma} 2m_{f_2}. \quad (2.12)$$

B. $g^*g^* \to f_2(1270)$ vertex

We will apply the formalism for the $\gamma^*\gamma^* \to f_2$ vertices discussed in Sec. IIIA. Because $f_2(1270)$ is extended, finite size object one can expect in addition a form factor $F(Q_1^2, Q_2^2)$ associated with the gluon virtualities for the $g^*g^* \to f_2$ vertex. In the present
letter the form factor is parametrized in different ways as:

\[ F(Q_1^2, Q_2^2) = \frac{\Lambda_M^2}{Q_1^2 + Q_2^2 + \Lambda_M^2}, \quad (2.13) \]

\[ F(Q_1^2, Q_2^2) = \left( \frac{\Lambda_D^2}{Q_1^2 + Q_2^2 + \Lambda_D^2} \right)^2, \quad (2.14) \]

\[ F(Q_1^2, Q_2^2) = \frac{\Lambda_1^2}{Q_1^2 + \Lambda_1^2 Q_2^2 + \Lambda_1^2}, \quad (2.15) \]

\[ F(Q_1^2, Q_2^2) = \frac{\Lambda_2^4}{(Q_1^2 + \Lambda_2^2)^2 (Q_2^2 + \Lambda_2^2)^2}, \quad (2.16) \]

where \( \Lambda \) is a parameter whose value is expected to be close to the resonance mass \[23\].

In the calculations for the factorised monopole (2.13) and dipole (2.14) forms we take \( \Lambda_M = \Lambda_D = m_{f_2} \). The values of \( \Lambda_1 \) in (2.15) and \( \Lambda_2 \) in (2.16) are expected to be of order of 1 GeV. The results for different forms of the form factors (2.13)–(2.16) are presented in Fig. 3. The results strongly depend on the parametrization chosen and the value of the corresponding parameter.

C. \( k_t \)-factorization approach

In Fig. 1 we show a generic Feynman diagram for \( f_2(1270) \) meson production in proton-proton collision via gluon-gluon fusion. This diagram illustrates the situation adequate for the \( k_t \)-factorization calculations used in the present paper.

![Feynman diagram](image)

**FIG. 1.** General diagram for inclusive \( f_2(1270) \) production via gluon-gluon fusion in proton-proton collisions.

The differential cross section for inclusive \( f_2(1270) \) meson production via the \( g^* g^* \rightarrow f_2(1270) \) fusion in the \( k_t \)-factorization approach can be written as:

\[ \frac{d\sigma}{dyd^2p} = \int \frac{d^2q_1}{\pi q_1^2} \frac{d^2q_2}{\pi q_2^2} \frac{F_g(x_1, q_1^2)}{F_g(x_2, q_2^2)} \delta^{(2)}(q_1 + q_2 - p) \frac{\pi}{(x_1x_2s)^2} |\mathcal{M}_{g^* g^* \rightarrow f_2}|^2. \quad (2.17) \]
Here \( q_1, q_2 \) and \( p \) denote the transverse momenta of the gluons and the \( f_2(1270) \) meson. The \( f_2 \) meson is on-shell and its momentum satisfies \( p^2 = m_{f_2}^2 \). \( M_{g^*g^* \rightarrow f_2} \) is the matrix element for off-shell gluons for the hard subprocess and \( \mathcal{F}_g \) are the gluon unintegrated distribution functions (UGDFs) for both colliding protons. The UGDFs depend on gluon longitudinal momentum fractions \( x_1, x_2 = m_T \exp(\pm y) / \sqrt{s} \) and \( q_1^2, q_2^2 \) entering the hard process. In principle, they can depend also on factorization scales \( \mu_{F,i}^2, i = 1, 2 \). It is reasonable to assume \( \mu_{F,1}^2 = \mu_{F,2}^2 = m_T^2 \). Here \( m_T \) is transverse mass of the produced \( f_2(1270) \) meson; \( m_T = \sqrt{p^2 + m_{f_2}^2} \). The \( \delta^{(2)} \) function in Eq. (2.17) can be easily eliminated by introducing \( q_1 + q_2 \) and \( q_1 - q_2 \) transverse momenta \([4]\).

The off-shell matrix element can be written as (we restore the color-indices \( a \) and \( b \))

\[
\mathcal{M}^{ab} = \frac{q_1^\mu q_2^\nu}{|q_1||q_2|} \mathcal{M}_{\mu\nu}^{ab} = \frac{q_1 + q_2}{|q_1||q_2|} n^\mu n^\nu \mathcal{M}_{\mu\nu}^{ab} = \frac{x_1 x_2 s}{2|q_1||q_2|} n^\mu n^\nu \mathcal{M}_{\mu\nu}^{ab}
\]

(2.18)

with the lightcone components of gluon momenta \( q_{1+} = x_1 \sqrt{s}/2, q_{2-} = x_2 \sqrt{s}/2 \). Here the matrix-element reads

\[
\mathcal{M}_{\mu\nu} = \Gamma_{\mu\nu\lambda\rho}^{(f_2\gamma\gamma)} (q_{1+}, q_{2+}) (\epsilon^{(f_2)\kappa\lambda}(p))^*,
\]

(2.19)

where \( \epsilon^{(f_2)} \) is the polarisation tensor for the \( f_2(1270) \) meson.

In the \( k_t \)-factorization approach in \([3]\) the matrix element squared (for energy-momentum tensor coupling) was written as:

\[
|\mathcal{M}_{g^*g^* \rightarrow f_2}|^2 = \frac{1}{(N_c^2 - 1)^2 \kappa^2} \sum_{a,b} \frac{q_{1+} q_{2+}}{q_{1-} q_{2-}} V_{ab}^{\alpha_1 \beta_1 \alpha_2 \beta_2} (q_{1+}, q_{2+}) P^{(2)}_{\alpha_1 \beta_1, \alpha_2 \beta_2} (p) \frac{q_{1+} \mu_2 q_{2+} \nu_2}{q_{1-} q_{2-}} \left( \frac{\sqrt{s}}{2} V_{ab}^{\alpha_2 \beta_2 \mu_2 \nu_2} (q_{1+}, q_{2+}) \right)^*
\]

\[
= \frac{1}{(N_c^2 - 1)^2 \kappa^2} P^{(2)}_{\alpha_1 \beta_1, \alpha_2 \beta_2} (p) H_{\perp}^{\alpha_1 \beta_1} (q_{1+}, q_{2+}) H_{\perp}^{\alpha_2 \beta_2} (q_{1+}, q_{2+}) \left( \frac{x_1 x_2 s}{2|q_1||q_2|} \right)^2,
\]

\[
(2.20)
\]

where \( N_c \) is the number of colors, \( V_{ab}^{\alpha_\mu \beta_\nu} \) is the \( gg \rightarrow f_2 \) vertex \([1]\) (see Eq. (A1) of \([3]\)), and \( \kappa \approx \mathcal{O}(0.1 \, \text{GeV}) \) is to be fixed by experiment. The explicit forms for the spin-2 projector \( P^{(2)} \) and \( H_{\perp}^{\alpha_\beta} \) functions (with transverse components) are given in \([3]\). In the above formula \((2.20)\) \( \alpha_s \) is not explicit but is hidden in the normalization constant. In our calculation we will make \( \alpha_s \) explicit, i.e. include its running with relevant scales. We have checked that the approach in \([3]\) is equivalent to the approach with the helicity-2 EMN vertex function \((2.3)\) when ignoring running of \( \alpha_s \) and vertex form factor \( F(Q^2_1, Q^2_2) \). Having \( F(Q^2_1, Q^2_2) \) is crucial for description of transverse momentum distribution of \( f_2(1270) \) as will be discussed in the result section.

The \( g^*g^* \rightarrow f_2(1270) \) coupling entering in the matrix element squared can be obtained from that for the \( \gamma^*\gamma^* \rightarrow f_2(1270) \) coupling by the following replacement:

\[
\alpha_{em}^2 \rightarrow \alpha_s^2 \frac{1}{4N_c(N_c^2 - 1)} \frac{1}{(\langle e_q^2 \rangle)^2}.
\]

(2.21)

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1 Please note that the order of Lorentz indices here (and in Ref. \([3]\)) is different than in Eq. \((2.3)\).
Here \( \langle e_q^2 \rangle^2 = 25/162 \) for the \( \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}) \) flavour structure assumed for \( f_2(1270) \).

In realistic calculations the running of strong coupling constants must be included. In our numerical calculations presented below the renormalization scale is taken in the form:

\[
\alpha_s^2 \rightarrow \alpha_s(\max \{ m_T^2, q^2 \}) \alpha_s(\max \{ m_T^2, q^2 \}).
\]

The Shirkov-Solovtsov prescription \cite{24} is used to extrapolate down to small renormalization scales relevant for the \( f_2(1270) \) production for the ALICE kinematics.

D. A simple \( \pi \pi \) final-state rescattering model

As will be shown in the present paper the \( g^* g^* \rightarrow f_2(1270) \) mechanism is insufficient at low \( f_2(1270) \) transverse momenta therefore we consider also a final-state rescattering of produced pions. The general diagram representing the \( \pi \pi \) rescattering is shown in Fig. 2. Both \( \pi^+ \pi^- \) and \( \pi^0 \pi^0 \) rescatterings may lead to the production of the \( f_2(1270) \) meson as an effect of final state resonance interactions.

The spectrum of pions will be not calculated here but instead we will use a Lévy parametrization of the inclusive \( \pi^0 \) cross section proposed in \cite{25} for \( \sqrt{s} = 7 \) TeV. At the ALICE energies and midrapidities we assume the following relation:

\[
\frac{d\sigma^{\pi^+}}{dydp_t}(y, p_t) = \frac{d\sigma^{\pi^-}}{dydp_t}(y, p_t) = \frac{d\sigma^{\pi^0}}{dydp_t}(y, p_t)
\]

(2.23)

to be valid.

Our approach here is similar in spirit to color evaporation approach considered, e.g., in \cite{8, 26}. In our approach here we do not include possible \( \pi \pi \) correlation functions. They are discussed usually at very small \(|\vec{p}_1 - \vec{p}_2|/2 \). For identical particles (\( \pi^0 \pi^0 \) in our case) this is discussed usually in the context of Bose-Einstein correlations. The non-identical particle correlations (\( \pi^+ \pi^- \) in our case) is less popular but also very interesting \cite{27, 28}. To form the resonance the two pions must be produced in an invariant mass window corresponding to the \( f_2(1270) \) meson and close in space one to each other. Including explicitly the second condition would require knowledge of the space-time development of the hadronization process and goes far beyond the present study devoted
to the $g^*g^* \to f_2(1270)$ mechanism. Instead we write the number of produced $f_2(1270)$ per event as

$$N = \int dy_1 dp_{1t} \int dy_2 dp_{2t} \int \frac{d\phi_1 d\phi_2}{2\pi} \frac{dN^\pi_1}{2\pi dy_1 dp_{1t}} \frac{dN^\pi_2}{2\pi dy_2 dp_{2t}} p_{\pi\pi \to f_2}, \quad (2.24)$$

where $dN^\pi_{/\p d p t}$ is number of pions per interval of rapidity and transverse momentum. Here we use the Tsallis parametrization of $\pi^0$ at $\sqrt{s} = 7$ TeV from [25]; see Eq. (2) of [25] and fit parameters in Table 3 therein. In Eq. (2.24) $P_{\pi\pi \to f_2}$ parametrizes probability of the $\pi^+\pi^-$ and $\pi^0\pi^0$ formation of $f_2(1270)$ as well as probability of its survival in a dense hadronic system. It will be treated here as a free parameter adjusted to the $f_2(1270)$ data from [2]. The distributions $dN^\pi_{/\p d p t}$ are obtained then by calculating $y$ and $p_t$ of the $f_2(1270)$ meson and binning in these variables.

The effect of hadronic rescattering in high-energy $pp$ collisions is also discussed in [29] and the application is being developed and will be implemented to PYTHIA event generator.

**III. NUMERICAL RESULTS**

To convert to the number of $f_2(1270)$ mesons per event, as was presented in Ref. [2], we use the following relation:

$$\frac{dN}{dp_t} = \frac{1}{\sigma_{\text{inel}}} \frac{d\sigma}{dp_t}. \quad (3.1)$$

The inelastic cross section for $\sqrt{s} = 7$ TeV was measured at the LHC and is:

$$\sigma_{\text{inel}} = 73.15 \pm 1.26 \text{ (syst.) \: mb}, \quad (3.2)$$

$$\sigma_{\text{inel}} = 71.34 \pm 0.36 \text{ (stat.)} \pm 0.83 \text{ (syst.) \: mb}, \quad (3.3)$$

as obtained by the TOTEM [30] and ATLAS [31] collaborations, respectively. In our calculations we take $\sigma_{\text{inel}} = 72.5 \text{ mb}$.

In Fig. 3 we present the $f_2(1270)$ meson transverse momentum distributions at $\sqrt{s} = 7$ TeV and $|y| < 0.5$ together with the preliminary ALICE data from [2]. Here, for the color-singlet gluon-gluon fusion mechanism, we used JH UGDF from [32]. We compare results for the monopole and dipole form factors for two different $g^*g^* \to f_2$ vertices discussed in Sec. II A. The result strongly depends on the parametrization of the form factors according to Eqs. (2.13)–(2.16) and (2.10).

Fig. 4 shows that there is some difference in the role of $\Lambda = 0, 2$ contributions for the EMN and PPV vertices. In the formalism of [19] [see the PPV vertex (2.8)] there is no interference between so-called $\Lambda = 0, T$ and $\Lambda = 2$ terms while the naive use of the formalism from [10] [see the EMN vertex (2.1)] generates some interference effects. Different couplings (independent invariant amplitudes) lead to different shapes of the transverse momentum distributions. The shape could be verified by experimental data.

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2 This type of UGD has been obtained by Hautmann and Jung [32] from a description of precise HERA data on deep inelastic structure function by a solution of the CCFM evolution equation [33–35]. This UGDF is available from the CASCADE Monte Carlo code [36]. We use “JH-2013-set2” of Ref. [32], which we label as “JH UGDF”. 

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FIG. 3. The $f_2(1270)$ meson transverse momentum distributions at $\sqrt{s} = 7$ TeV and $|y| < 0.5$. The preliminary ALICE data from [2] are shown for comparison. The results for the EMN (left panel) and PPV (right panel) $g^* g^* \rightarrow f_2(1270)$ vertex for different $F(Q_1^2, Q_2^2)$ form factor (2.13)–(2.16) are shown. In this calculation the JH UGDF was used.

FIG. 4. The $f_2(1270)$ meson transverse momentum distributions at $\sqrt{s} = 7$ TeV and $|y| < 0.5$ together with the preliminary ALICE data from [2]. Shown are the results calculated in the two approaches, EMN (left panel) and PPV (right panel) vertices, and the helicity-0 and -2 components separately and their coherent sum (total). In this calculation we used dipole form factor parametrization (2.14) with $\Lambda_D = m_{f_2}$. The dotted line corresponds to the Born-level result for the $pp \rightarrow pp f_2(1270)$ process via pomeron-pomeron fusion.
In the left panel of Fig 5, we show results for the KMR UGDF. The KMR UGDF (dashed lines) gives smaller cross section than the JH UGDF (solid lines). The results for both UGDFs coincide for large $p_t$. The larger the $f_2(1270)$ transverse momentum the larger the range of gluon transverse momenta $q_{1t}$ and/or $q_{2t}$ are probed. This means that at larger gluon transverse momenta one enters a more perturbative region.

To better illustrate the dependence of UGDFs on $q^2$, we present in the right panel of Fig 5 the results with the Gaussian smearing of collinear GDF, often used in the context of TMDs, for different smearing parameter $\sigma_0 = 0.25, 0.5, 1.0$ GeV. The GJR08VFNS(LO) collinear GDF was used for this purpose. As expected the shape of $d\sigma/dp_t$ strongly depends on the value of the smearing parameter $\sigma_0$ used in the calculation. The speed of $d\sigma/dp_t$ approaching to zero for $p_t \rightarrow 0$ strongly depends on the value of $\sigma_0$. It is impossible to describe simultaneously $p_t < 1$ GeV and $p_t > 1$ GeV regions with the same value of $\sigma_0$. This illustrates the generic situation with all UGDFs.

In Fig. 5, the $f_2(1270)$ meson transverse momentum distributions at $\sqrt{s} = 7$ TeV and $|y| < 0.5$ together with the preliminary ALICE data from [2]. In the left panel results for two different UGDFs, JH (solid lines) and KMR (dashed lines), are shown. In the right panel we show the dependence on the Gaussian smearing parameter $\sigma_0$ for the GJR08VFNS(LO) GDF [40]. Here the EMN vertex discussed in Sec. II A 1 and the dipole form factor (2.14) with $\Lambda_D = m_{f_2}$ were used.

In Fig. 6, we present $d^2\sigma/dq_{1t}dq_{2t}$ for the EMN (left panel) and PPV (right panel) $g^* g^* \rightarrow f_2(1270)$ vertices. Here the JH UGDF was used. The maximal contributions come from the region of rather small gluon transverse momenta $q_{1t}, q_{2t} < 1$ GeV. It is easy to check (numerically) that the larger-$p_t$ region ($p_t > 2$ GeV) is sensitive to $q_{1t}, q_{2t} > 1$ GeV where perturbative methods apply. At low $p_t$ there is a nonnegligible contribution from the nonperturbative region of UGDFs which is not under full theoretical control. Here the gluon saturation effects may be potentially important.

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3 Here we use a glue constructed according to the prescription initiated in [37] and later updated in [38, 39], which we label as “KMR UGDF”. The KMR UGDF is available from the CASCADE Monte Carlo code [36].
FIG. 6. Two-dimensional distributions in gluon transverse momenta for the JH UGDF and for two $g^*g^*f_2(1270)$ vertex prescription: EMN (left panel) and PPV (right panel). Here we used the dipole form factor (2.14) with $\Lambda_D = m_f$.

We have checked that

$$\frac{d^2\sigma_{EMN}}{dq_1dq_2} \left( \frac{d^2\sigma_{PPV}}{dq_1dq_2} \right)^{-1} \rightarrow 1, \text{ for } q_1 \rightarrow 0 \text{ and } q_2 \rightarrow 0,$$

i.e. the two vertices are equivalent for both on-shell photons.

In Fig. 7 we show auxiliary (normalized) distributions to discuss a possible role of the remaining terms in the $g^*g^*f_2$ PPV vertex (2.8) corresponding to helicities ($\Lambda = 0$, L) and ($\Lambda = 1$). Here we assumed the same $Q^2$ dependence of the form factor functions for all $\Lambda$ terms; see Eqs. (2.10) and (2.14). The dominance of $\Lambda = 2$ term over $\Lambda = 0$ and $\Lambda = 1$ terms is certainly maintained at small values of $Q^2_{\text{ave}}$ and of $p_t$. However, the situation changes drastically at large gluon virtualities, i.e., the ($\Lambda = 1$) and ($\Lambda = 0$, L) structures of the $g^*g^*f_2$ vertex become equally important for $p_t > 2$ GeV.

Note that from the analysis of the $\gamma^*(Q_1^2)\gamma^*(Q_2^2) \rightarrow \pi\pi$ processes performed in [21, 22] it is clear that in the $f_2(1270)$ resonance region the helicity-(0, T) amplitude gives the dominant contribution and the other helicity projections become increasingly important for larger virtualities. From Fig. 5 of [21] and Fig. 3 of [22] we can see that for $Q_1^2$ fixed the helicity-1 contribution increases with increasing $Q_2^2$ while helicity-(0, L) contribution only slightly decreases. The situation changes when both photon virtualities are identical $Q_1^2 = Q_2^2$ and large, i.e. then the helicity-(0, L) component increases with increasing virtualities and becomes even larger than the helicity-1 component. This observation is consistent with our results presented in Fig. 7.

The theoretical results for the color-singlet gluon-gluon fusion contribution underestimate the ALICE data especially for low-$p_t$ region, $p_t < 2$ GeV. Does it mean that other mechanism(s) is (are) at the game?
FIG. 7. Normalized distributions in averaged virtuality $Q_{\text{ave}}^2 = (Q_1^2 + Q_2^2)/2$ (left panel) and in the $f_2(1270)$ meson transverse momentum (right panel). Results for different $\Lambda = 0, 1, 2$ helicity terms in the $g^* g^* f_2$ vertex (2.8)–(2.10) using the same form of vertex form factors $F^{(\Lambda)}(Q_1^2, Q_2^2)$ (2.14) with $\Lambda_D = m_{f_2}$ are shown. In the calculation the JH UGDF was used.

In Fig. 8 we show the $\pi\pi$ rescattering contribution. Clearly the $\pi\pi \rightarrow f_2(1270)$ rescattering effect cannot describe the region of $p_t > 2$ GeV, where the $gg$-fusion mechanism is a possible explanation. In addition, we present the Born result (without absorptive corrections important only when restricting to purely exclusive processes) for the $pp \rightarrow pfp_2(1270)$ process proceeding via the pomeron-pomeron fusion mechanism calculated in the tensor-pomeron approach. For details regarding this approach we refer to [10, 41–43]. In the calculation we take the pomeron-pomeron-$f_2(1270)$ coupling parameters from [43].
FIG. 8. Results for the $\pi\pi$ rescattering mechanism (long-dashed line), for the $gg$-fusion mechanism (solid lines), and for the pomeron-pomeron fusion mechanism (dotted line) together with the preliminary ALICE data from [2]. We show maximal contribution from the $\pi\pi$ rescattering as described in the main text. The results for the $gg$-fusion contributions were calculated for the JHU GDF and for the PPV vertex [helicity-2 plus helicity-(0, T) terms] and for two form factor functions (2.15) (top solid line) and (2.14) (bottom solid line).

IV. CONCLUSIONS

In the present paper we have discussed production of $f_2(1270)$ tensor meson in proton-proton collisions. Two different approaches for the $\gamma^*\gamma^* \to f_2(1270)$ coupling have been considered. We have discussed their equivalence for both on-shell photons. The coupling constants have been fixed by the Belle data for $\gamma\gamma \to f_2(1270) \to \pi\pi$. The $g^*g^* \to f_2(1270)$ vertex has been obtained by replacing electromagnetic coupling constant by the strong coupling constant, modifying colour factors and assuming a simple flavour structure of the $f_2(1270)$ isoscalar meson. We have shown that the energy-momentum tensor vertex, proposed in [3], is equivalent to the dominant in $\gamma\gamma \to f_2(1270)$ $\Gamma^{(2)}$ tensorial structure [see Eq. (2.3)] discussed in [10].

We have performed our calculation of the cross section for $pp \to f_2(1270) + X$ within the $k_t$-factorization approach. Different unintegrated gluon distributions from the literature have been used. We have discussed corresponding uncertainties.

Our results have been compared to preliminary ALICE data presented in [2]. We have taken into account only the case when both photons are transverse. At low $f_2(1270)$ transverse momenta the helicity-2 ($\Lambda = 2$) contribution dominates, while the helicity-0 ($\Lambda = 0, T$) is small, almost negligible, but competes with the $\Lambda = 2$ and even dominates at larger transverse momenta of $f_2(1270)$. In the PPV formalism there could be also $\Lambda = 0, L$ and $\Lambda = 1$ contributions which are difficult to fix by available data.

It has been shown that the results strongly depend on the form of the vertex form factor. With the GVDM form factor used previously in $\gamma^*\gamma \to f_2(1270)$ fusion [14, 15]...
one cannot describe the preliminary ALICE data. We have tried also other choices. With plausible form factor [e.g., dipole ansatz (2.14) with $\Lambda_D \approx m_{f_2}$, factorized VDM ansatz (2.15) with $\Lambda_1 \approx 1 \text{ GeV}$] one can describe the data for $p_t > 2 \text{ GeV}$ but it seems impossible to describe the low-$p_t$ data. Clearly some mechanism at low-$p_t$ must be in the game there.

We have shown that the final state rescattering may be the missing candidate. A simple empirical model has been proposed. Adjusting corresponding probability for the $\pi\pi \rightarrow f_2(1270)$ rescattering and the $\Lambda_D$ parameter in the dipole form factor for the $g^*g^* \rightarrow f_2$ vertex we have been able to describe the preliminary ALICE data.

The gluon saturation is expected at low $x_1$ and $x_2$ i.e. automatically rather low transverse momenta of $f_2(1270)$ where most probably the $\pi\pi$ rescattering dominates, which enables observation of saturation.

We have calculated also the exclusive production of $f_2(1270)$ meson via the pomeron-pomeron fusion mechanism with the parameters found in our previous analysis where they were fixed to ‘purely’ exclusive data for the reaction $pp \rightarrow pp\pi^+\pi^-$. This contribution is concentrated at small $f_2(1270)$ transverse momenta but its role is rather marginal.

Our calculation suggest that the gluon-gluon fusion may be the dominant mechanism of the $f_2(1270)$ production at larger transverse momenta, $p_t > 3 \text{ GeV}$. Other mechanisms are of course not excluded but it is clear that the gluon-gluon fusion is a very important mechanism which cannot be ignored in the analysis.

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