Analyzing Complex Network User Arrival Patterns and Their Effect on Network Topologies

Michael Fire\textsuperscript{1,*,**} and Carlos Guestrin\textsuperscript{2*}

\textsuperscript{*}Department of Computer Science & Engineering, University of Washington

\textsuperscript{**}The eScience Institute, University of Washington

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Abstract
Complex networks have non-trivial characteristics and appear in many real-world systems. Due to their vital importance in a large number of research fields, various studies have offered explanations on how complex networks evolve, but the full underlying dynamics of complex networks are not completely understood. Many of the barriers to better understanding the evolution process of these networks can be removed with the emergence of new data sources.

This study utilizes the recently published Reddit dataset, containing over 1.65 billion comments, to construct the largest publicly available social network corpus, which contains detailed information on the evolution process of 11,965 social networks. We used this dataset to study the effect of the patterns in which new users join a network (referred to as user arrival curves, or UACs) on the network topology. Our results present evidence that UACs are a central factor in molding a network’s topology; that is, different arrival patterns create different topological properties. Additionally, we show that it is possible to uncover the types of user arrival patterns by analyzing a social network’s topology. These results imply that existing complex network evolution models need to be revisited and modified to include user arrival patterns as input to the models, in order to create models that more accurately reflect real-world complex networks.

1 Introduction

Complex networks are loosely defined as networks with non-trivial structure and dynamics, appearing in social, molecular, biological, ecological, and economical

\textsuperscript{1}fire@cs.washington.edu
\textsuperscript{2}guestrin@cs.washington.edu
real-world complex systems [11]. Many studies have provided models and algorithms to explain how complex networks evolve [6, 12, 18, 22, 24]. However, the full underlying dynamics of complex networks are not entirely comprehended, and many questions remain unanswered [1].

With the rapid advances in the field of data science in recent years, new algorithms, infrastructures, and techniques for data mining, data storage, data prediction, and data visualization have emerged [3, 10, 23, 33]. These new tools make it feasible to gain new insights from large quantities of data (also known as big data), and they can be utilized to tackle significant problems in research fields as diverse as movie recommendations [19], malware detection [29], whale detection [17], and disease identification [15].

In this study, we utilize data science tools to investigate the patterns in which new users join online communities and to study the impact of these arrival patterns on the network structure. Our study focuses on Reddit, a huge collection of online communities that discuss what is new and popular on the Internet. Namely, we utilize the recently published Reddit dataset, which contains over a terabyte of uncompressed data and consists of over 13.2 million users who posted over 1.65 billion comments published over a 7-year time period. This vast dataset contains information on the evolution process of more than 239,000 online communities and the connections among the community members (see Section 3). We used the Reddit dataset to construct a large corpus of selected 11,965 directed social networks, which contain over 97% of all the posted comments in the dataset (see Section 3). We assembled an unprecedented corpus of social networks with diverse topologies, ranging from social networks with 11 vertices up to social networks with over 4 million vertices (see Table 1).

Using the constructed social networks, we studied how the patterns in which users joined the network influence the communities’ social network topologies. We analyzed the networks and their corresponding UACs using regression analysis, machine learning algorithms, and graph theory algorithms, and we ran our analyses utilizing the AWS cloud computing infrastructure [3] and the GraphLab Create machine learning framework [23].

Our study’s results indicate that most of the analyzed UACs match polynomial functions. Specifically, we observed that most of these curves can be categorized into one of five main types of growth patterns (see Section 5.3 and Figure 1): (a) polynomial growth, (b) sublinear growth, (c) linear growth, (d) superlinear growth, and (e) sigmoidal growth. Moreover, we observed that different UACs influence the structure of the social networks. We observed that sigmoidal-like growth usually makes the network denser and with a higher average clustering-coefficient than polynomial growth (see Section 6).

These results indicate that different user-joining rates can considerably affect the structure of a complex network. Therefore, to better understand complex network dynamics, and to develop models that simulate complex network evolution, it is highly important to take into account the patterns of how new users join online communities.
join an emerging network. Until now, to the best of our knowledge, only a single proposed network evolution model [21] incorporated the users’ arrival times as part of the evolution model. Our study’s results should aid in developing new and more accurate evolution models, assist in evaluating existing models, and help in better understanding complex networks.

1.1 Contributions

To our knowledge, by several orders of magnitude this is the largest study to analyze real-world directed social networks. In this study, we present evidence that different users’ arrival patterns are a very important factor in the molding of the network’s topology. Additionally, we show that it is possible to uncover the types of social network users’ arrival patterns by analyzing the network’s topology.

Furthermore, this study contains the following contributions: First, we present a novel method to categorize UACs. Second, we empirically show that the majority of UACs on the Reddit dataset match polynomial functions, which provides a method to identify anomalous patterns that in many cases are influenced by real-world events. Third, we analyze the topological features of 11,965 real-world directed social networks and provide statistical analyses of these network features. Fourth, we share both our constructed social network datasets as well as a significant portion of the code used for this study in order to make this
1.2 Organization

The remainder of the paper is organized as follows: In Section 2, we provide an overview of various related studies. In Section 3, we give details on the Reddit dataset which was used throughout this study. Additionally, we present the methodology that we used to clean the Reddit dataset. Section 4 describes the methodology we used to construct the subreddit social networks and the social network topological features we utilized in this study. Next, in Section 5, we present the methods and algorithms we used to construct, categorize, and predict the subreddit UACs. Then, in Section 6, we discuss the obtained results. Lastly, in Section 7, we present our conclusions from this study and also offer future research directions.

2 Related Work

The multidisciplinary fields of complex networks have been deeply studied in recent years, ever since Barabási and Albert observed in 1999 that many real-world networks, such as social networks and the World Wide Web, are best described as networks with complex topologies. Barabási and Albert also pointed out that many of these large networks have a common property of having a scale-free, power-law distribution of their vertex connections [6]. Since then, a large number of studies have been devoted to identifying complex networks in various real-world systems [7, 25]. Additionally, many studies have provided models that explain how complex networks are created [6, 12, 18, 21, 24].

In this section, we present a focused overview of studies related to social networks and their evolution process. In addition, we give an overview of studies pertaining to online communities and the Reddit website, which are related to the dataset utilized in this study.

2.1 Complex Networks

The study of complex networks began over half a century ago, in 1965. While studying a network of citations among scientific papers, Price observed probably the first example of a network in which degree distribution followed a power law [26]. Later, in 1976, Price provided an explanation of the creation of these types of networks by pointing out that “Success seems to breed success. A paper which has been cited many times is more likely to be cited again than one which has been little cited” [27]. Using the rule that success breeds success, Price offered a method for the creation of networks in which their degree distribution follows a power law.

Over three decades later, in 1999, Barabási and Albert observed that networks with degree distributions that follow power laws exist in a variety of
networks, including the World Wide Web [6]. Barabási and Albert coined the term “scale-free networks” for describing such networks — ones having degree distributions that follow a power law distribution. Similar to Price’s method for creating scale-free networks [27], Barabási and Albert [6] suggested a simple and elegant model for creating random complex networks based on the rule that the rich are getting richer. In their model (also known as the Barabási and Albert model), a network starts with $m$ connected vertices. Then, each new vertex that is added to the network has a higher probability of connecting to pre-existing vertices with higher degree, where the probability of connecting to an existing $v$ is proportional to $v$’s degree [6]. As a result of the tendency of a new vertex to connect to vertices with higher degrees, “rich” vertices with high degrees tend to become even “richer” due to their connections with new vertices that join the graph.

Even though the Barabási and Albert model can explain some of the characteristics of real-world complex networks, the random networks created by the model were lacking in other network properties that were observed in real-world complex networks. Therefore, in recent years, other models have been suggested for the creation of complex networks which have additional characteristics that were observed in real-world complex networks.

In 2005, while studying the evolution of four real-world networks, Leskovec et al. [22] observed that in contrast to existing graph-generation models, most of the observed networks became denser over time, the average distance between vertices shrinking over time. To create random complex networks that aligned with their observations, Leskovec et al. introduced the Forest Fire model that is based on having new vertices attach to the network by “burning” through existing edges in epidemic fashion [22]. In the basic version of the model, vertices first join the network one at a time. Then, each new vertex $v$ creates an out-link to an existing vertex $u$. Vertex $v$ starts burning outward from $u$ to other vertices in the network, by linking with given probability to vertices that are connected to $u$ and recursively repeating the burning process with the new vertices that $v$ was just linked to.

In 2008, McGlohon et al. [24] studied 14 real-world weighted graphs. In their study, they observed the following patterns in the graphs: (a) non-giant connected components seem to stabilize in size; and (b) edges’ weights follow several power laws with surprising exponents. Using these observed patterns, McGlohon et al. presented the Butterfly model in which new nodes can behave like “social butterflies’ by choosing more than one starting point, or ‘host,’ in their interactions; meeting nodes in the vicinity of the host, out-linking to some of them, and flying away.” [24]

In the same year, Leskovec et al. [21] performed edge-by-edge analysis of four large-scale networks - Flickr, Delicious, Yahoo! Answers, and LinkedIn - with time spans ranging from four months to almost four years. By studying a wide variety of network formation strategies, they observed that edge locality plays a

\footnote{A detailed explanation of the differences between Price’s method and the Barabási and Albert model for constructing complex networks can be found in [25].}
critical role in the evolution of networks. Based on their finding, they offered a model which focused on microscopic vertex behavior. In their proposed model, vertices arrive at a pre-specified rate and choose their lifetimes. Afterwards, each vertex “independently initiates edges according to a ‘gap’ process, selecting a destination for each edge according to a simple triangle-closing model free of any parameters” [21]. They showed that their model can closely mimic the macroscopic characteristics of real social networks. Additionally, Leskovec et al., similar to this study, observed the arrival patterns of various vertices. Namely, they observed that (a) Flickr has grown exponentially over most of their network data; (b) Delicious has grown slightly superlinearly; (c) LinkedIn has grown quadratically; and (d) Yahoo! Answers has grown sublinearly. Due to these vertex arrival pattern observations, they concluded that vertex arrival functions needed to be part of their proposed model, and they subsequently used specific vertex arrival functions as part of their network construction process. However, their study did not analyze the implications of using different arrival functions.

In 2012, Gong et al. [12] utilized data of around 30 million Google+ users to study the evolution of social-attribute networks (SANs). They observed that in contrast to several other social networks, the assortativity of Google+ is neutral. In addition, they observed that the Google+ distinct phases (initial launch, invite only, public release) manifest themselves in the social and attribute structures. Gong et al. used their observations to develop a new generative model for social attribute networks, and they demonstrated that their proposed model could “reproduce SANs that accurately reflect the true ones with respect to various network metrics and real-world applications” [12].

In addition to the studies described above, a comprehensive review on various complex network properties and various evolution models can be found at [9, 25].

2.2 Online Communities

In this study, we analyze the evolution process of social networks created in Reddit’s online communities. In recent years, due to the sharp increase in online social network platform usage, researchers have begun to explore how online social networks and online communities develop over time.

In 2006, Backstrom et al. [4] utilized the DBLP and the LiveJournal datasets to explore the principles by which groups evolve. Additionally, by using decision-tree techniques, they studied the structural features that influence whether individuals will join a community, and whether a given group will grow significantly. Backstrom et al. discovered that the tendency of people to join communities, and of communities to grow rapidly, depends on the underlying network structure. Moreover, Backstrom et al. discovered that the tendency of individuals to join a community is not only influenced by the number of friends the individual has in the community, but also by how those friends are connected to each other. Furthermore, their obtained results indicated that communities with a higher percentage of fringe members were much more likely to grow significantly over the studied given period of time [4].

Two years later, in 2008, Backstrom et al. [5] utilized Yahoo! Groups, a
large-scale dataset, to study the social engagements and evolving relationships among members within online groups. They discovered that group members who would become heavily engaged in a group in the long term received “highly differentiated treatment from the very first message they post.” Additionally, they discovered that a message receives a response decline as users remain heavily engaged in the group. Moreover, they discovered that an average member of a small and private group will be considerably more active than an average member of a large and public group.

In 2012, Kairam et al. [16] utilized the Ning platform to study the factors contributing to the growth and longevity of groups. They developed models for predicting the growth and longevity of groups with different ages and sizes. Their utilized features captured the rate of growth and the proportion of that growth occurring due to diffusion processes, as well as due to network-based features. Kairam et al. study’s results indicated that groups with a higher proportion of growth as a result of diffusion have a higher probability of dying. Additionally, their results indicated that groups which contain relatively large cliques have significantly less likelihood of dying. Moreover, Kairam et al. study’s results indicated that groups that grow mainly from diffusion tend to reach smaller eventual sizes.

In addition to the general studies on online communities, there have been several studies which have analyzed and studied the Reddit community. These studies analyzed various data collected from Reddit.

To study the expected behavior within the Reddit community, Kelly Bergstrom [8] explored, as a case study, the story of “Grandpa Wiggly,” a Reddit community member who was accused of being a troll.

In 2013, Himabindu et al. [20] utilized image submissions to study the interplay among content, title, community, and posting time. They utilized images posted on Reddit – multiple times with different titles, to multiple communities at different times – to observe how well the same content performs when posted on different scenarios. Himabindu et al. succeeded in developing models that can help to “understand how to better target social media content: by using the right title, for the right community, at the right time.”

Recently, Max Woolf published a blog post [32] in which he gave several insights on the Reddit dataset we used in this study. In his blog post, he also provided an easy-to-follow tutorial on how to analyze the Reddit data using Google’s BigQuery web service.4

3 The Reddit Dataset

To analyze UACs, we chose to use data collected from the Reddit website. Reddit is a news aggregation website and online social platform, which was launched in 2005 by Steve Huffman and Alexis Ohanian [8]. Reddit users (also known as “redditors”) can submit links on the website, which are then commented upon, and upvoted or downvoted by other users in order to increase or decrease the

4https://cloud.google.com/bigquery/
submission visibility. Redditors can also create their own subreddit on a topic of their choosing, make it public or private, and let other redditors join it. This makes Reddit a collection of online communities, centered around a variety of topics such as books, gaming, science, and asking questions.\footnote{Each subreddit web page can be accessed at the following URL: \url{https://www.reddit.com/r/<SubredditName>/}, by replacing \textit{SubredditName} with the subreddit’s name.}

In this study, we utilized the Reddit dataset which was recently made public by Jason Michael Baumgartner (see Section 8). The Reddit dataset contains over 1.65 billion comments that were posted from October 2007 through May 2015. These posts were created by 13,213,173 users, with unique usernames, in 239,772 different subreddits. The dataset contains information on the exact time and date each comment was posted. Moreover, for each comment, the dataset contains the comment’s ID, as well as information on the user who posted it and the ID of the parent comment, i.e., the ID to which the current comment replied.

For this study, we cleaned the dataset by removing nonessential comments. Namely, we removed comments that were marked as deleted and those that did not include the information of the user who posted them. Additionally, we removed posts by users who with high probability were bots. Namely, we removed all the users who posted more than 100,000 comments each, and we removed 897 redditors whose comments appeared in the bots list published in the BotWatchman subreddit.\footnote{We downloaded the bots list from the BotWatchman subreddit \url{https://www.reddit.com/r/BotWatchman/} during December 2015.} After the removal of these posts, we were left with over 1.42 billion comments.

4 Subreddit Social Networks

In the following subsections, we give a detailed description of the methods that we used to construct and analyze the subreddit social networks. In Section 4.1, we introduce the methods we used to construct each subreddit’s social network. Then, in Section 4.2, we present the topological features we extracted from each network. Additionally, we provide a statistical overview of each topological feature across all social networks.

4.1 Social Network Construction

To perform the analysis of the subreddits’ underlying social networks, we first needed to construct these social networks. However, many of the subreddits did not contain enough users or were not active for a long enough time to extract meaningful UACs. For example, the median number of redditors in a subreddit was 3, while only 5\% of the subreddits consisted of 371 redditors or more. Therefore, for all subreddits in the clean dataset of over 1.42 billion comments, we selected only those subreddits that had at least 10 users, consisted of at least
1,000 comments, and were active\(^7\) for at least 1 year. Out of all the subreddits, 11,965 subreddits with over 1.38 billion posts (referred to as selected subreddits) fulfilled this criteria.

Next, for each selected subreddit, similar to the construction method used by Kairam et al. [16], we created the subreddit’s social network directed graph by connecting users who posted comments as replies to other posted comments. Namely, for a subreddit, we define the subreddit’s directed graph to be: \(G := \langle V, E \rangle\) where \(V\) is the set of vertices, representing all the subreddit’s users that posted at least a single comment in the subreddit, and \(e := (u, v) \in E\) is the list of all edges between the subreddit’s users \(u \in V\) and \(v \in V\). We define an edge between \(u\) and \(v\) to exist if there exists a comment on the subreddit posted by \(u\) to which \(v\) posted a reply on the same subreddit. Lastly, we used the Powerlaw Python package [2] and observed that most of the social networks’ vertex connection distributions match power law distributions with various exponent values. It important to notice that the constructed directed graph also includes single vertices of redditors who posted comments and did not receive any reply, as well as self-loop edges of redditors who posted a comment and then posted a comment as a reply to their own comment.

### 4.2 Calculating Topological Features

For each selected subreddit constructed social network graph, \(G := \langle V, E \rangle\), we calculated the following topological features:

- **Vertices number** - the number of vertices in the graph, defined as \(|V|\).
- **Edges number** - the number of edges in the graph, defined as \(|E|\).
- **Density** - the graph density, defined as \(D := \frac{|E|}{|V|(|V|−1)}\).
- **Number of self-loops** - the number of self-loops in the graph, defined as \(\text{Loops} := |\{(v, v) \in E | v \in V\}|\).
- **Number of triangles** - the number of triangles (denoted by \(|T|\)) in the graph [30].
- **Average clustering coefficient** - the graph’s average clustering coefficient (denoted by \(CC\)) [28].
- **Degree-based features** - for a vertex \(v \in V\), we defined the in-degree, out-degree, and total-degree of \(v\) to be \(d_{in}(v) := |\{u \in V | \exists(u, v) \in E\}|\), \(d_{out}(v) := |\{u \in V | \exists(v, u) \in E\}|\), and \(d_{tot}(v) := |\{u \in V | \exists(u, v) \in E\} or \exists(v, u) \in E\}|\), respectively. Using the vertex degree definitions, we can define the following graph degree features:
  - **avg-degree** and **avg-in-degree** - the graph’s average-degree and average-in-degree, defined as \(\text{avg-deg} := \frac{\sum_{v \in V} d_{tot}(v)}{|V|}\) and \(\text{avg-in-deg} := \frac{\sum_{v \in V} d_{in}(v)}{|V|}\) respectively.
  - **max-in-degree** and **max-out-degree** - the graph’s maximum in-degree and maximum out-degree, defined as \(\text{max-in-deg} := \max_{v \in V} (d_{in}(v))\) and \(\text{max-out-deg} := \max_{v \in V} (d_{out}(v))\) respectively.

\(^7\) Throughout this study, we considered the time in which a subreddit was active as the time difference between the first and last comments published in the subreddit.
Table 1: Subreddits Social Network Features Overview

| Feature       | Min  | Max  | Median | Mean  | Std  |
|---------------|------|------|--------|-------|------|
| Avg-deg       | 0.0  | 53.343 | 1.893 | 2.889 | 3.404 |
| Avg-in-deg    | 0.0  | 26.672 | 0.947 | 1.444 | 1.702 |
| CC            | 0.0  | 0.941 | 0.025 | 0.047 | 0.067 |
| D             | 0.0  | 0.936 | 0.001 | 0.003 | 0.017 |
| Days          | 365.690 | 2,785.483 | 1,271.786 | 1,312.678 | 570.675 |
| LC-Ratio      | 0.002 | 1 | 0.416 | 0.389 | 0.189 |
| Loops         | 0.0 | 73.791 | 7 | 88.058 | 931.786 |
| max-in-deg    | 0.0 | 24469 | 39 | 151.107 | 466.908 |
| max-out-deg   | 0.0 | 24113 | 40 | 162.092 | 484.829 |
| |E|LC|    |0.0 |4,2973,517 |768 |30,118.416 |48,4602.036 |
| |E|    |0.0 |42,980,043 |860 |30,287.708 |48,4771.671 |
| |Single|     |0.0 |164,374 |626 |4,037.163 |31,026.642 |
| |T|     |0.0 |73,148,002 |92 |70,057.296 |1,289,299,629 |
| |V|LC|    |1.0 |2,391,502 |341 |4,035,554 |39,425,021 |
| |V|    |1.0 |4,043,528 |1,110 |8,605,852 |70,357,225 |
| |WCC|   |1.0 |1,647,816 |663 |4,132.8 |31,247,404 |

and

max-out-deg := max_{v \in V}(d_{out}(v)), respectively, where the max function returns the maximum value in a set.

- Connected components-based features - we separated the graph into a set of weakly connected components (denoted by WCC) [31], in which WCC := \{H \leq G | H is subgraph of G\} and G = \sqcup_{H \in WCC} H. Using the WCC, we can also define the graph’s largest component (referred as LC) G_{LC} := \langle V_{LC}, E_{LC} \rangle, where G_{LC} \leq G, and \forall H := \langle V’, E’ \rangle \in WCC, |V_{LC}| \geq |V’|. Using the above definitions, we can define the following graph features:

  - Number of connected components - the number of weakly connected components, defined as —WCC—.
  - Largest component vertices number - the number of vertices in LC, defined as |V_{LC}|.
  - Largest component edges number - the number of edges in LC, defined as |E_{LC}|.
  - Largest component ratio - the ratio between the number of users in the largest component and all users in the subreddit, defined as LC-Ratio := \frac{|V_{LC}|}{|V|}.
  - The number of single components - the number of components in WCC that consist of only a single vertex (denoted by —Single—), defined as \{H := \langle V”, E” \rangle \in WCC || |V”| = 1 \}.

Additionally, we added an extra feature (denoted as Days), where we calculated the number of days that had passed between the post times of the first and last comments.

Table 1 presents an overview of the various calculated topology features. Notice that the full set of topological features of all 11,965 selected subreddits is available online (see Section 8).
5  Subreddit User Arrival Curves and Results

In the following subsections, we describe in detail the methods which were used to construct and analyze UACs. In Section 5.1, we define the UAC function and explain how we constructed the selected subreddit UACs. Next, in Section 5.2, we describe the process we utilized to match each UAC and its corresponding function. Afterwards, in Section 5.3, we present the methods used to categorize the different UACs. Lastly, Section 5.4 gives details on the methods used to predict the UAC categories based on the subreddits’ topologies.

5.1 User Arrival Curve Construction

For all the selected subreddits, we constructed the UACs using the following methodology: First, for each subreddit $S$, using the cleaned Reddit dataset, we calculated the number of weeks (denoted as $t_{end}^S$) between the first comment and last comment that were posted on the subreddit, according to the dataset. Afterwards, we defined $\text{Users-Number}_S(t)$ for $t \in [0, t_{end}^S]$ to be the number of users who joined the subreddit $t$ weeks since the first comments were posted on the subreddit. We also defined the overall number of users who joined the subreddit after $t_{end}^S$ weeks to be $\text{Total-Users}_S$. Then, using the above definitions, we defined $UAC : [0, t_{end}^S] \rightarrow [0, 1]$ as:

$$UAC_S(t) = \frac{\text{Users-Number}_S(t)}{\text{Total-Users}_S},$$

where $UAC_S(0)$ and $UAC_S(t_{end}^S)$ are always equal to 0 and 1, respectively. Lastly, to create the $S$ UACs, we calculated the $UAC_S$ value in 4-week intervals.\(^8\)

By using a time interval of 4 weeks, the number of samples of the UACs for each subreddit ranged from 15 to 101, with a median value of 47.

5.2 User Arrival Curve Regression Analysis

To better understand the 11,965 UACs that we created, we utilized CurveExpert software\(^9\) to match several selected UACs with their best-fit functions using regression analysis. Using CurveExpert, we obtained a list of best-fit candidate functions. In most cases, the best-fit function was a high-degree polynomial function. To avoid over-fitting of the fitted function, we used the python-fit package\(^9\) to find the polynomial function that was a best-fit for the majority of UACs and still had a relatively low degree. Using the python-fit package, we discovered that 11,273 (94.2\%) and 9,199 (76.9\%) of the UACs matched quartic function ($q(x) := a + bX + cX^2 + dX^3 + eX^4$) with $R^2 \geq 0.95$ and $R^2 \geq 0.99$, respectively. Figure 2 presents the regressions’ $R^2$ distribution values of these matched quartic functions, as well as the coefficient distributions of the 11,273 matched quartic polynomials (referred to as matched-polynomials).

\(^8\)In case $t_{end}^S$ does not divide evenly by 4, the time interval between the next-to-last and last UAC values will be less than 4 weeks.

\(^9\)https://pypi.python.org/pypi/python-fit/1.0.0
which matched subreddit UACs with $R^2$ values greater or equal to 0.95, as well as the regressions’ $R^2$ distribution values of these matched quartic functions.

Additionally, we performed regression analyses of the 692 UACs that did not match quartic functions. Out of these UACs, we observed that 274 matched the MMF model, $\frac{ab+cx^d}{b+x^d}$ [14], with $R^2 \geq 0.95$. The other 418 UACs (referred to as anomalous UACs) presented a wide range of patterns. By manually reviewing these patterns, we can estimate that at least 37% of the UACs seem to have unique shapes that we believe can be attributed to external events.

5.3 User Arrival Curve Categorization

After matching the UACs with their best-fit quartic functions, we could now categorize the different UACs. To achieve this goal, for each matched quartic function $q_S(x)$ of subreddit $S$, we defined the normalized area function $\text{norm-area} : q_S \rightarrow [0, 1]$ as:

$$\text{norm-area}(q_S) := \frac{narea(q_S)}{t_{end}^S - t_0^S}$$

By the distribution of the 11,273 subreddit norm-areas, we observed that the norm-area distributions were skewed to the right, with a minimal value of 0.065, a maximal value of 0.935, and a median value of 0.4, with a standard deviation of 0.159 (see Figure 3).

As a rule of thumb, we can foresee that fast growing subreddits, such as the Blogging subreddit (see Figure 1), will have a relatively small norm-area value of close to 0. On the other hand, subreddits that had their growth halted for relatively long time, such as the Eagle-Scouts subreddit (see Figure 1), will have a norm-area of about 1, and subreddits with constant growth rates will have a norm-area of about 0.5. With this rule of thumb and with the norm-area distributions in mind, we divided the matched-polynomials into five sets, according to their norm-area values: Set 1, $\{q_S|\text{narea}(q_S) \in [0, 0.24]\}$ (with 905 UACs); Set 2, $\{q_S|\text{narea}(q_S) \in [0.24, 0.4]\}$ (with 4,733 UACs); Set 3, $\{q_S|\text{narea}(q_S) \in [0.4, 0.56]\}$ (with 3,435 UACs); Set 4, $\{q_S|\text{narea}(q_S) \in [0.56, 0.72]\}$ (with 1,483 UACs); and Set 5, $\{q_S|\text{narea}(q_S) \in [0.72, 1]\}$ (with 717 UACs). For the five sets of matched-polynomials, we calculated the various coefficient distributions (see Figure 4), as well as manually viewed the UAC graph visualizations for each set.10 By examining these UAC graphs, and by analyzing the coefficient distributions in each set, we can observe the following:

1. The majority of the UACs with $\text{norm-area} \in [0, 0.24]$ (Set 1) have positive $d$ and $e$ coefficients. Therefore, in the majority of the cases these UACs will have a cubic or quartic growth rate (referred to as polynomial growth).
2. The majority of the UACs with $\text{norm-area} \in [0.24, 0.4]$ (Set 2) have a negative $e$ coefficient. Additionally, by manually reviewing the UACs, we can observe that in most cases these UACs have a sublinear growth rate.

10The UAC graphs of each are available for download; see Section 8.
Figure 2: Matched quartic polynomials’ coefficients and $R^2$ distributions.
In most cases the growth starts relatively slowly and then changes into linear growth.

3. The majority of the UACs with $\text{norm-area} \in [0.4, 0.56)$ (Set 3) have relatively large $c$ coefficients. However, in most cases the $d$ and $e$ coefficients have opposite signs; one is positive the other is negative. This indicates that most of the UACs do not grow at a fast rate. By manually reviewing the UACs in this set, we observed that in most cases these UACs have a nearly linear growth rate.

4. The majority of the UACs with $\text{norm-area} \in [0.56, 0.72)$ (Set 4) have a relatively high positive $b$ coefficient. However, in most cases the $d$ and $e$ coefficients have opposite signs. This indicates that most of the UACs do not grow at a polynomial rate. Indeed, by manually examining the UACs, we observed that most UACs in this set have a superlinear growth rate. Many of the UACs in this set started with a faster than linear growth rate and then dropped to a linear growth rate.

5. The majority of the UACs with $\text{norm-area} \in [0.72, 1)$ (Set 5) have relatively high positive $b$ and $d$ coefficients, and low negative $c$ and $e$ coefficients. This indicates that the UACs grow very fast and then slow down until the growth stops. Indeed, we observed a growth pattern in these UACs which is similar to the sigmoidal growth rate.

Additionally, we inspected the growth patterns of the 692 UACs that did not match quartic functions. The MMF model belongs to the family of sigmoidal...
models. Therefore, the 274 UACs that matched this model have a sigmoidal growth rate. A closer look at the content of the other 418 anomalous UACs revealed that many of them were related to external events that affected the growth, such as the launch of a new season of TV shows, special calendar dates, and the release of movies. We refer to this growth pattern as events-oriented growth.

5.4 User Arrival Curve Prediction

Our main goal of this study is to understand how various UAC growth rates affect the social networks’ topologies. To achieve this goal, using the 11,965 selected subreddits that matched a quartic function, we first calculated the Pearson correlations between the subreddit topological features (see Section 4.2) and the subreddit normalized-area values (see Section 5.3). Then, we used regression algorithms to construct models which could predict a subreddit’s normalized-area values. Lastly, we used different classification algorithms to construct classifiers that could predict UAC categories based on their topologies. In the rest of this subsection, we provide a detailed overview of each method we used and the obtained results.

5.4.1 Correlations

We calculated the Pearson correlations between the 11,965 subreddit social network topological features and their normalized areas. The obtained results indicate that there is a weak positive correlation between norm-area and the network’s average clustering-coefficient ($r = 0.23$), and also between the norm-area and the network’s density ($r = 0.21$). Additionally, there is a negligible negative correlation between the norm-area and the network’s maximal in-degree ($r = -0.12$), as well as between the norm-area and the maximal out-degree ($r = -0.14$). Moreover, there is a moderate negative correlation between the norm-area and the Days feature ($r = -0.47$).

These results indicate that subreddits that have relative smaller norm-areas, probably due to their rapid growth rates, will be active for longer periods of time (and vice-versa). Additionally, networks with relatively high norm-areas are, in general, denser and have higher clustering-coefficients.

5.4.2 Regression Analysis

We constructed regression prediction models, which can predict a subreddit’s UAC normalized area using GraphLab Create [23]. We evaluated three regression models: linear regression, Boosted Trees regression, and Random Forest regression. We created these models twice, one time using all the topological features described in Section 4.2 and the second time with all the features plus the Days feature. We evaluated the models using 10-folds cross validation and measuring the models’ average value of (a) mean absolute error ($MAE$), (b) mean square error ($MSE$), and (c) root mean squared error ($RMSE$).
Figure 4: Coefficient distributions in each set.
Out of these regression models, the Boosted Trees regression presented the best results, which were slightly better than the linear and Random Forest models, with $MAE = 0.125$, $MSE = 0.027$, and $RMSE = 0.163$, using only the topological features, and $MAE = 0.114$, $MSE = 0.232$, and $RMSE = 0.152$, using the topological features plus the $Days$ feature. Additionally, the linear regression model created using the 11,965 subreddits’ topology features presented an $R^2$ value of 0.125, using only the topological features, and an $R^2$ value of 0.262, using all the topological features plus the $Days$ feature. Moreover, in the linear regression model constructed with all the features, the $|CC|$, $D$, $|E|$, $|E_{LC}|$, $|V_{LC}|$, max-in-deg, Loops, $|T|$, and $|WCC|$ had positive coefficients. The Avg-deg, $|E|$, LC-Ratio, max-out-deg, $|Single|$, $|V|$, and $Days$ had negative coefficients.

### 5.4.3 Supervised Learning

We constructed supervised learning classifiers which can classify the category of the subreddit’s UAC, based on only the 11,965 selected subreddits’ topological features, in the following way: First, we created a labeled dataset with the six growth categories we defined in Section 5.3. Next, using the labeled dataset, we used WEKA [13] and constructed various classifiers using the following algorithms: OneR, J48 decision tree, Logistic, K-Nearest-Neighbors (KNN) with $K = 1$, Rotation Forest, and Random Forest. Then, we evaluated each classifier using the 10-folds cross validation method and calculated the classifier’s AUC (area under the ROC curve) values. Lastly, we repeated the construction and evaluation process, only this time with only two growth categories – polynomial growth and sigmoidal growth.

Out of all the trained classifiers, the Logistic classifier obtained the best results, in terms of AUC, on both datasets. On the first dataset, with six categories, the Logistic classifier obtained the highest weighted average AUC and the highest correct classification percentage of 0.64 and 41.2%, respectively. These results were considerably better than the simple OneR and KNN classifiers that obtained weighted average AUCs of 0.52 and 0.53, and correct classification percentages of 37.4% and 31.7%, respectively. On the second dataset, which consisted of subreddits with only polynomial growth and sigmoidal growth, the Logistic classifier obtained the highest AUC and highest true positive rate of 0.82 and 0.81, respectively. These results were considerably better than the simple OneR and KNN classifiers that obtained AUCs of 0.67 and 0.64, and true positive rate of 0.7 and 0.63, respectively.

To obtain an indication of the usefulness of various features, we analyzed their importance using WEKA’s information gain attribute selection algorithm. On the first dataset, with all six growth categories, the top five attributes with the highest rank were: $|V|$ (score of 0.101), $|WCC|$ (0.1), $|Single|$ (0.01), $D$ (0.088), and $|V_{LC}|$ (0.075).
6 Discussion

By analyzing the results presented in Sections 4 and 5, the following can be noted:

First, from the construction process results presented in Section 4.1, we can observe that most of Reddit’s online communities have an activity period of less than a year. Moreover, we can observe that the 11,965 selected subreddits, which are about 5% of all the subreddits in the dataset, contain 1.38 billion posts; these are 97.7% of all the posts in the constructed clean Reddit dataset. This indicates that most communities are short-lived and contain significantly less content than the content that can be found in other more active communities. Therefore, studies aiming to analyze online communities need to carefully select the communities they choose to study.

Second, from Table 1 we can observe that the Reddit dataset contains many types of communities with a wide range of size scales and with various topologies. However, from the results presented in Section 5.2, we can surprisingly observe that even though there are many different social networks and the reasons people join these communities are varied and complex, in most online communities the UACs match quartic functions. Moreover, we can notice that among the six identified patterns presented in Section 5.3, there are user-join patterns which are more common than others among the analyzed subreddits. Namely, we can see that sublinear growth is the most common growth pattern, which was identified in 39.5% of all studied UACs, while anomalous patterns, which do not match quartic or MMF functions, appeared in 3.5% of the UACs. Moreover, near linear growth patterns, which include UACs categorized with sublinear, linear, and superlinear growth, were observed in about 80% of the UACs (see Section 5.3).

Third, according to the high $R^2$ values presented in Figure 4, for most subreddits, the subreddits’ UACs match quartic functions. The matched quartic functions may be able to predict how the community will grow (or decline), and also be able to identify significant events in the community. This can provide valuable insights to the communities’ administrators, as well as to the web service provider.

Fourth, according the correlation results presented in Section 5.4.1, we can observe that, in general, UACs with higher normalized areas are probably denser and have higher clustering coefficients. Additionally, the negative correlation between the normalized area and the Days feature indicates that communities with smaller normalized areas, such as communities with polynomial growth, are active for longer periods of time. Furthermore, the linear regression coefficients presented in Section 5.4.2, in which the $D$ and $CC$ features have positive coefficients while the Days feature has a negative coefficient, support the above observations. Additionally, the coefficients’ signs in the obtained linear regression and the information gain results presented in Section 5.4 indicate that larger social networks with a higher number of users, higher number of links, and higher maximal degree features are usually more likely to have UACs with smaller normalized areas.
Fifth, by observing the regression results in Section 5.4.2, which presented models with relatively low RMSE values, and the classification results presented in Section 5.4, which presented promising results with AUC of 0.82, we can infer that analyzing a social network topology can, in many cases, predict the normalized area size of the UAC that created the network. Therefore, in many cases, we can predict the general pattern in which users joined the social network. Moreover, according to the classification results, the difference is noticeable between social networks created by polynomial growth and social networks created by sigmoidal growth.

Lastly, the results presented in Section 5 strongly indicate that the UAC has a critical influence on the topological structure of a social network. This has never been deeply studied before. If we take the social network of a school as an example, we can assume that different social connections will be established depending on whether new children join the school one by one or as a group. Of course, additional analysis would need to be performed to validate that there are no hidden causes that could influence both the community’s UAC and its topology. Nevertheless, developed complex network evolution models need to create social network which their topologies present different properties for different input UACs.

7 Conclusions

In this study, we analyzed a unique and large-scale dataset containing over 1.65 billion comments. From this dataset, we constructed 11,965 social networks (see Section 4), calculated various topological features of each social network (see Section 4.2), and created each network’s UACs (Section 5.1). Our methodology demonstrated that most UACs match one of six user-join patterns (see Section 5.3). Moreover, we classified the UAC category of the network, with a correct classification percentage of 41.2%, by utilizing only the social network’s topology. Moreover, we observed that networks with relatively larger normalized areas are denser and have higher clustering coefficients (see Sections 5.4.1 and 5.4.2), while networks with smaller normalized areas tend to have higher numbers of users and remain active for a longer time (see Sections 5.4.1, and 5.4.2).

These results indicate that complex network evolution models need to include UACs as part of the models’ input. Moreover, for different UAC categories, these models must create networks with different topological properties, which correlate with the topological properties observed in the subreddits’ social networks which have UACs with the same category.

In the future, we hope to develop a complex evolution model that provides topological structures similar to the ones observed in this study for different UACs. Another research direction we hope to pursue is to analyze the effect of different UACs on the topology of various complex networks, such as biological networks. A further possible research direction is to investigate how the content of a community influences the community’s topology, and whether communities
with similar content have similar topologies and UACs. Moreover, the large corpus of social networks created and released as a result of this study can greatly contribute to better understanding online communities and complex networks. According Albert-László Barabási, “If data of similar detail capturing the dynamics of processes taking place on networks were to emerge in the coming years, our imagination will be the only limitation to progress” [7].

8 Data and Code Availability

Instructions on how to download the raw Reddit dataset are available in the following Reddit post. Additionally, the dataset can be downloaded from this link. This study is reproducible research. Therefore, the social network datasets, a considerable part of the study’s code, are available for other researchers by contacting the paper’s first author. Moreover, in the upcoming weeks we intend to publish a website which will give researchers the ability to interactively explore and better understand the social networks in this study’s dataset (see Appendix A).

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References

[1] R. Albert and A.-L. Barabási. Statistical mechanics of complex networks. Reviews of modern physics, 74(1):47, 2002.

[2] J. Alstott, E. Bullmore, and D. Plenz. powerlaw: a python package for analysis of heavy-tailed distributions. PloS one, 9(1):e85777, 2014.

[3] M. Armbrust, A. Fox, R. Griffith, A. D. Joseph, R. Katz, A. Konwinski, G. Lee, D. Patterson, A. Rabkin, I. Stoica, et al. A view of cloud computing. Communications of the ACM, 53(4):50–58, 2010.

[4] L. Backstrom, D. Huttenlocher, J. Kleinberg, and X. Lan. Group formation in large social networks: membership, growth, and evolution. In Proceedings of the 12th ACM SIGKDD international conference on Knowledge discovery and data mining, pages 44–54. ACM, 2006.
[5] L. Backstrom, R. Kumar, C. Marlow, J. Novak, and A. Tomkins. Preferential behavior in online groups. In Proceedings of the 2008 International Conference on Web Search and Data Mining, pages 117–128. ACM, 2008.

[6] A.-L. Barabási and R. Albert. Emergence of scaling in random networks. science, 286(5439):509–512, 1999.

[7] A.-L. Barabási et al. Scale-free networks: a decade and beyond. science, 325(5939):412, 2009.

[8] K. Bergstrom. “don’t feed the troll”: Shutting down debate about community expectations on reddit. com. First Monday, 16(8), 2011.

[9] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D.-U. Hwang. Complex networks: Structure and dynamics. Physics reports, 424(4):175–308, 2006.

[10] M. Bostock, V. Ogievetsky, and J. Heer. D3 data-driven documents. Visualization and Computer Graphics, IEEE Transactions on, 17(12):2301–2309, 2011.

[11] E. Estrada. Journal of complex networks: Quo vadis? Journal of Complex Networks, 1(1):1–2, 2013.

[12] N. Z. Gong, W. Xu, L. Huang, P. Mittal, E. Stefanov, V. Sekar, and D. Song. Evolution of social-attribute networks: measurements, modeling, and implications using google+. In Proceedings of the 2012 ACM conference on Internet measurement conference, pages 131–144. ACM, 2012.

[13] M. Hall, E. Frank, G. Holmes, B. Pfahringer, P. Reutemann, and I. H. Witten. The weka data mining software: an update. ACM SIGKDD explorations newsletter, 11(1):10–18, 2009.

[14] D. Hyams. Curveexpert software. http://www.curveexpert.net, 2010.

[15] Kaggle. Identify signs of diabetic retinopathy in eye images. https://www.kaggle.com/c/diabetic-retinopathy-detection. [Online; accessed 25-January-2016].

[16] S. R. Kairam, D. J. Wang, and J. Leskovec. The life and death of online groups: Predicting group growth and longevity. In Proceedings of the fifth ACM international conference on Web search and data mining, pages 673–682. ACM, 2012.

[17] A. Karpištšenko. The marinexplore and cornell university whale detection challenge - summary of the competition. https://www.kaggle.com/c/whale-detection-challenge/forums/t/4472/summary-of-the-competition. [Online; accessed 25-January-2016].

[18] K. Klemm and V. M. Eguiluz. Growing scale-free networks with small-world behavior. Physical Review E, 65(5):057102, 2002.
[19] Y. Koren. The bellkor solution to the netflix grand prize. *Netflix prize documentation*, 81, 2009.

[20] H. Lakkaraju, J. J. McAuley, and J. Leskovec. What’s in a name? understanding the interplay between titles, content, and communities in social media. *ICWSM*, 1(2):3, 2013.

[21] J. Leskovec, L. Backstrom, R. Kumar, and A. Tomkins. Microscopic evolution of social networks. In *Proceedings of the 14th ACM SIGKDD international conference on Knowledge discovery and data mining*, pages 462–470. ACM, 2008.

[22] J. Leskovec, J. Kleinberg, and C. Faloutsos. Graphs over time: densification laws, shrinking diameters and possible explanations. In *Proceedings of the eleventh ACM SIGKDD international conference on Knowledge discovery in data mining*, pages 177–187. ACM, 2005.

[23] Y. Low, J. E. Gonzalez, A. Kyrola, D. Bickson, C. E. Guestrin, and J. Hellerstein. Graphlab: A new framework for parallel machine learning. *arXiv preprint arXiv:1408.2041*, 2014.

[24] M. McGlohon, L. Akoglu, and C. Faloutsos. Weighted graphs and disconected components: patterns and a generator. In *Proceedings of the 14th ACM SIGKDD international conference on Knowledge discovery and data mining*, pages 524–532. ACM, 2008.

[25] M. E. Newman. The structure and function of complex networks. *SIAM review*, 45(2):167–256, 2003.

[26] D. Price. Statistical studies of networks of scientific papers. In *Statistical Association Methods for Mechanized Documentation: Symposium Proceedings*, volume 269, page 187. US Government Printing Office, 1965.

[27] D. Price de Solla. A general theory of bibliometric and other cumulative advantage process. *Journal of the American Society of Information Science*, 27:292–306, 1976.

[28] J. Saramäki, M. Kivelä, J.-P. Onnela, K. Kaski, and J. Kertesz. Generalizations of the clustering coefficient to weighted complex networks. *Physical Review E*, 75(2):027105, 2007.

[29] J. Saxe and K. Berlin. Deep neural network based malware detection using two dimensional binary program features. *arXiv preprint arXiv:1508.03096*, 2015.

[30] T. Schank. Algorithmic aspects of triangle-based network analysis. *Phd in computer science, University Karlsruhe*, 3, 2007.

[31] E. W. Weisstein. Weakly connected component. [http://mathworld.wolfram.com/WeaklyConnectedComponent.html](http://mathworld.wolfram.com/WeaklyConnectedComponent.html). [Online; accessed 5-February-2016].

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A Visualizing the Impact of User Arrival Curves on the Social Network Topology

During this research, we observed that in many cases the topological differences of the social networks with different UAC categories can be easily observed using the social networks’ graph images. For example, in Figure A.5, we can notice that the fifa2013 social network, which has 5,657 vertices, 11,243 edges, and a sigmoidal growth UAC, is considerably denser and has fewer single vertices than the blogging social network, which has 2,301 vertices, 4,591 edges, and a polynomial growth UAC. To better understand how different UACs influence the structure of the subreddits’ social network, we created a web interface (see Figure A.6 and Section 8), which enables users to view each subreddit’s social network UAC and other features, as well as visualize a sample of a selected social network’s topology. We believe that using this interface will empower users to perform their own investigation of the social networks corpus.

Figure A.5: Blogging and fifa2013 social networks. It is noticeable that the fifa2013 social network is considerably denser than the Blogging social network

[32] M. Woolf. How to analyze every reddit submission and comment, in seconds, for free. http://minimaxir.com/2015/10/reddit-bigquery/. [Online; accessed 29-January-2016].

[33] M. Zaharia, M. Chowdhury, M. J. Franklin, S. Shenker, and I. Stoica. Spark: cluster computing with working sets. In Proceedings of the 2nd USENIX conference on Hot topics in cloud computing, volume 10, page 10, 2010.
Figure A.6: Web interface for viewing the selected subreddits’ information.