The Wroblewski Parameter from Lattice QCD

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Abstract. Enhancement of strangeness production has since long been proposed as a promising signal of quark-gluon plasma production. A convenient indicator for it is the Wroblewski parameter which has been shown to be about a factor two higher in heavy ion collisions. Using a method proposed by us earlier, we obtained lattice QCD results for the Wroblewski parameter from our simulations of QCD with two light quarks both below and above the chiral transition. Our first principles based and parameter free result compare well with the A-A data.

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1. Introduction

Amongst the many proposed signatures of quark-gluon plasma in relativistic heavy ion collisions, enhancement of strangeness production \[1\] is a promising signal. Proposed in the very early stages of this field, it was based on simple model considerations like many other signals. Exploiting the fact, that the strange quark mass is smaller than the expected transition temperature whereas the mass of the lowest strange hadron is significantly larger, it was argued that the production rate for strangeness in the QGP phase, \(\sigma_{QGP}(s\bar{s})\) is greater than that in the hadron gas phase, \(\sigma_{HG}(s\bar{s})\).

While this energy threshold argument for strangeness production in the two phases is qualitatively appealing, one has to face quantitative questions of details for any meaningful comparison with the data. Since the temperature of the plasma produced in RHIC, or even LHC, may not be sufficiently high for perturbative QCD to be applicable, estimation of strangeness production by lowest order processes like \(gg \rightarrow s\bar{s}\) could be misleading. Indeed, it is now well-known that even for charm production, the next order correction to \(gg \rightarrow c\bar{c}\) is equally large.

A variety of aspects of the strangeness enhancement have been studied and many different variations have been proposed. One very useful way of looking for strangeness enhancement is the Wroblewski parameter \[2\]. Defined as the ratio of newly created strange quarks to light quarks,

\[
\lambda_s = \frac{2\langle s\bar{s} \rangle}{\langle uu + dd \rangle} \tag{1}
\]

the Wroblewski parameter has been estimated for many processes using a hadron gas fireball model \[3\]. An interesting finding from these analyses is that \(\lambda_s\) is around 0.2 in most processes, including proton-proton scattering, but is about a factor of two higher in heavy ion collisions. An obvious question one can ask is whether this rise by a factor of two can be attributed to the strangeness enhancement due to quark gluon plasma and if yes, whether this can be quantitatively demonstrated. We show below how quark number susceptibilities, obtained from simulations of lattice QCD, may be useful in answering these questions.

2. \(\lambda_s\) from Quark Number Susceptibilities

Quark number susceptibilities (QNS) can be calculated from first principles using the lattice formulation. Assuming three flavours, \(u\), \(d\), and \(s\) quarks, and denoting by \(\mu_f\) the corresponding chemical potentials, the QCD partition function is

\[
Z = \int DU \exp(-S_G) \prod_{f=u,d,s} \text{Det} M(m_f, \mu_f) . \tag{2}
\]

Note that the quark mass and the corresponding chemical potential enter only through the Dirac matrix \(M\) for each flavour. Defining \(\mu_0 = \mu_u + \mu_d + \mu_s\) and \(\mu_3 = \mu_u - \mu_d\), the
baryon and isospin densities and the corresponding susceptibilities can be obtained as:

\[ n_i = \frac{T}{V} \frac{\partial \ln Z}{\partial \mu_i}, \quad \chi_{ij} = \frac{T}{V} \frac{\partial^2 \ln Z}{\partial \mu_i \partial \mu_j}. \]  

(3)

QNS in (3) are crucial for many quark-gluon plasma signatures which are based on fluctuations in globally conserved quantities such as baryon number or electric charge. Theoretically, they serve as an important independent check on the methods and/or models which aim to explain the large deviations of the lattice results for pressure \( P(\mu=0) \) from the corresponding perturbative expansion. Here we will be concerned with the Wroblewski parameter which we have argued can be estimated from the quark number susceptibilities:

\[ \lambda_s = \frac{2\chi_s}{\chi_u + \chi_d}. \]  

(4)

In order to use (4) to obtain an estimate for comparison with experiments, one needs to compute the corresponding quark number susceptibilities on the lattice first and then take the continuum limit. All susceptibilities can be written as traces of products of \( M^{-1} \) and various derivatives of \( M \) with respect to \( \mu \). With \( m_u = m_d \), diagonal \( \chi_{ii} \)'s can be written as

\[ \chi_0 = \frac{T}{2V} \left[ \langle O_2(m_u) \rangle + \frac{1}{2} \langle O_{11}(m_u) \rangle \right], \]  

(5)

\[ \chi_3 = \frac{T}{2V} \langle O_2(m_u) \rangle, \]  

(6)

\[ \chi_s = \frac{T}{4V} \left[ \langle O_2(m_s) \rangle + \frac{1}{4} \langle O_{11}(m_s) \rangle \right]. \]  

(7)

Here \( O_2 = \text{Tr} \ M^{-1} - \text{Tr} \ M^{-1} M' u - \text{Tr} \ M^{-1} M' u M^{-1} M' u, \) and \( O_{11}(m_u) = (\text{Tr} \ M^{-1} M' u)^2 \). The traces are estimated by a stochastic method: \( \text{Tr} \ A = \sum_{i=1}^{N_v} R_i^A R_i / 2N_v, \) and \( (\text{Tr} \ A)^2 = 2 \sum_{i>j=1}^{L} (\text{Tr} \ A)_i (\text{Tr} \ A)_j / L(L-1) \), where \( R_i \) is a complex vector from a set of \( N_v \), subdivided further in \( L \) independent sets.

Figure 1. Comparison of quenched and full QCD (left) results and typical continuum extrapolation results (right).

Left panel of Figure 1 displays results for the susceptibilities as a function of temperature in units of \( T_c \), where \( T_c \) is the transition temperature. Normalized to the corresponding ideal gas results on the same lattice, i.e, the infinite temperature
limit, results for QCD with two light dynamical quarks of mass 0.1 \( T_c \) are shown as points whereas the continuous curves correspond to the results in the quenched approximation. The valence quark mass \( m_v \), appearing in (5)-(7), is shown in the figure in units of \( T_c \). Note that \( T_c \) in these two cases differ by a factor of 1.6 but the results for the corresponding susceptibilities as a function of \( T/T_c \) differ by a few percent only. Encouraged by this, we investigated the continuum limit for the quenched case by increasing the temporal lattice size from 4 to 14, as shown in the right panel of Figure 1 for \( T = 2 T_c \). The continuum results for QNS thus obtained in the quenched approximation are exhibited in Figure 2 for small \( m_v \).

The strange quark susceptibility was obtained from the same simulations by simply choosing \( m_v/T_c = 1 \) (in both full and quenched QCD in view of Figure 1). Using (4), \( \lambda_s(T) \) can then be easily obtained, and extrapolated to \( T_c \) by employing simple ansätze. The resultant \( \lambda_s(T_c) \) in quenched QCD is shown in the right panel of Figure 2 along with the results obtained from the analysis of the RHIC and SPS data in the fireball model. The systematic error coming from extrapolation is not shown but is of approximately the same size as the shown statistical error. The agreement of the lattice results with those from RHIC and SPS is indeed very impressive.

The nice agreement needs to be treated cautiously, however, in view of the various approximations made. Let me list them in order of severity.
The result is based on quenched QCD simulations and extrapolation to $T_c$. As seen from Figure 1, the quark number susceptibilities, and hence $\lambda_s(T)$, are expected to change by only a few per cent. Since the nature of the phase transition does depend strongly on the number of dynamical quarks, a direct computation near $T_c$ for full QCD is desirable. We are currently making such a computation and have some preliminary results for full QCD with two light dynamical quarks for lattices with four sites in temporal direction. These are shown in Figure 3 along with the continuum quenched results for $\lambda_s(T)$ and the band for experimental results. While the emerging trend is encouraging, further exploration with varying strange quark mass, temporal lattice size (to obtain continuum results) and spatial volume is still necessary.

The experiments at RHIC and SPS have nonzero albeit small $\mu$ whereas the above result used $\mu = 0$. Based on both lattice QCD and fireball model considerations, $\lambda_s$ is expected to change very slowly for small $\mu$. This can, and should, be checked by direct simulations.

Lattice simulations yield real quark number susceptibility whereas for particle production its imaginary counterpart is needed. Assuming that the characteristic time scale of plasma are far from the energy scales of strange or light quark production, one can relate \[6\] the two to justify the use of lattice results in obtaining $\lambda_s$. Observation of spikes in photon production may falsify this assumption.

3. Summary

Quark number susceptibilities which can be obtained from first principles using lattice QCD contribute substantially to the physics of RHIC signals. In particular, the continuum limit of $\chi_u$ and $\chi_s$, obtained in quenched QCD, leads to $\lambda_s(T)$. Its extrapolation to $T_c$ appears to be in good agreement with results from SPS and RHIC. First full QCD results near $T_c$ confirm this as well, although many technical issues, e.g, finite lattice cut-off or strange quark mass, need to be sorted out still.

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