aspeed: Solver Scheduling via Answer Set Programming*

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Abstract

Although Boolean Constraint Technology has made tremendous progress over the last decade, the efficacy of state-of-the-art solvers is known to vary considerably across different types of problem instances and is known to depend strongly on algorithm parameters. This problem was addressed by means of a simple, yet effective approach using handmade, uniform and unordered schedules of multiple solvers in ppfolio, which showed very impressive performance in the 2011 SAT Competition. Inspired by this, we take advantage of the modeling and solving capacities of Answer Set Programming (ASP) to automatically determine more refined, that is, non-uniform and ordered solver schedules from existing benchmarking data. We begin by formulating the determination of such schedules as multi-criteria optimization problems and provide corresponding ASP encodings. The resulting encodings are easily customizable for different settings and the computation of optimum schedules can mostly be done in the blink of an eye, even when dealing with large runtime data sets stemming from many solvers on hundreds to thousands of instances. Also, the fact that our approach can be customized easily enabled us to swiftly adapt it to generate parallel schedules for multi-processor machines.

KEYWORDS: Algorithm Schedules, Answer Set Programming, Portfolio-Based Solving

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Boolean Constraint Technology has made tremendous progress over the last decade, leading to industrial-strength solvers. Although this advance in technology was mainly conducted in the area of Satisfiability Testing (SAT; (Biere et al. 2009)), it meanwhile also led to significant boosts in neighboring areas, like Answer Set Programming (ASP; (Baral 2003)), Pseudo-Boolean Solving (Biere et al. 2009, Chapter 22), and even (multi-valued) Constraint Solving (Tamura et al. 2009). However, there is a prize to pay. Modern Boolean constraint solvers are rather sensitive to the way their search parameters are configured. Depending on the choice of the respective configuration, the solver’s performance may vary by several orders of magnitude. Although this is a well-known issue, it was impressively illustrated once more during the 2011 SAT Competition, where 16 prizes were won by the portfolio-based solver portfolio (Roussel 2011). The idea underlying portfolio is very simple: it independently runs several solvers in parallel. If only one processing unit is available, three solvers are started. By relying on the process scheduling mechanism of the operating system, each solver gets nearly the same time to solve a given instance. We refer to this as a uniform, unordered solver schedule. If several processing units are available, one solver is started on each unit; however, multiple solvers may end up on the last unit.

Inspired by this simple, yet effective system, we devise a more elaborate, yet still simple approach that takes advantage of the modeling and solving capacities of ASP to automatically determine more refined, that is, non-uniform and ordered solver schedules from existing benchmarking data. The resulting encodings are easily customizable for different settings. For instance, our approach is directly extensible to the generation of parallel schedules for multi-processor machines. Also, the computation of optimum schedules can mostly be done in the blink of an eye, even when dealing with large runtime data sets stemming from many solvers on hundreds to thousands of instances. Despite its simplicity, our approach matches the performance of much more sophisticated ones, such as satzilla (Xu et al. 2008) and 3S (Kadioglu et al. 2011). Unlike both, our approach does not rely on the availability of domain-specific features of the problem instance being solved, which makes it easily adaptable to other domains.

The remainder of this article is structured as follows. In Section 2, we formulate the problem of determining optimum schedules as a multi-criteria optimization problem. In doing so, our primary emphasis lies in producing robust schedules that aim at the fewest number of timeouts by non-uniformly attributing each solver (or solver configuration) a different time slice. Once such a robust schedule is found, we optimize its runtime by selecting the best solver alignment. We then extend this approach to parallel settings in which multiple processing units are available. With these formalizations at hand, we proceed in two steps. First, we provide an ASP encoding for computing (parallel) timeout-minimal schedules (Section 3). Once such a schedule is identified, we use a second encoding to find a time-minimal alignment of its solvers (Section 4). Both ASP encodings are also of interest from an ASP modelling perspective, because they reflect interesting features needed for dealing with large sets of (runtime) data. Finally, in Section 5, we provide an empirical evaluation of the resulting system aspeed, and we contrast it with related approaches (Section 6). In what follows, we presuppose a basic acquaintance with ASP (see (Gebser et al. 2012) for a comprehensive introduction).
2 Solver Scheduling

In the following, we formulate the optimization problem of computing a solver schedule. To this end, we introduce robust timeout-minimal schedules for single-threaded systems that are extended by a solver alignment mechanism to minimize the used runtime. Furthermore, in order to exploit the increasing prevalence of multi-core processors, we consider the problem of finding good parallel solver schedules.

2.1 Sequential Scheduling

Given a set $I$ of problem instances and a set $S$ of solvers, we use function $t : I \times S \mapsto \mathbb{R}^+$ to represent a table of solver runtimes on instances. Also, we use an integer $\kappa$ to represent a given cutoff time. For illustration, consider the runtime function in Table 1; it deals with 6 problem instances, $i_1$ to $i_6$, and 3 solvers, $s_1$, $s_2$, and $s_3$.

Each solver can solve three out of six instances within the cutoff time, $\kappa = 10$; timeouts are indicated by ‘$\geq 10$’ in Table 1. The oracle solver, also known as virtual best solver (VBS), is obtained by assuming the best performance of each individual solver. As we see in the rightmost column, the oracle would be able to solve all instances in our example within the cutoff time; thus, if we knew beforehand which solver to choose for each instance, we could solve all of them. While we can hardly hope to practically realize an oracle solver on a single threaded system (at least in terms of CPU time), performance improvements can already be obtained by successively running each solver for a limited period of time rather than running a single solver until the cutoff is reached. For instance, by uniformly distributing time over all three solvers in our example, as done in ppfolio, we could solve 4 out of 6 instances, namely instance $i_1 \ldots i_4$. Furthermore, the number of solved instances can be increased further by running $s_1$ for 1, $s_2$ for 6, and $s_3$ for 2 seconds, which allows us to solve 5 out of 6 instances, as indicated in bold in Table 1. In what follows, we show how such an optimized non-uniform schedule can be obtained beforehand from given runtime data.

Given $I$, $S$, $t$, and $\kappa$ as specified above, a timeout-optimal solver schedule can be expressed as a function $\sigma : S \to [0, \kappa]$, satisfying the following condition:

$$\sigma \in \arg \max_{\sigma : S \to [0, \kappa]} \{i : \exists s \in S : t(i, s) \leq \sigma(s)\}$$

such that

$$\sum_{s \in S} \sigma(s) \leq \kappa$$

(1)

An optimal schedule $\sigma$ consists of slices $\sigma(s)$ indicating the (possibly zero) time allotted to each solver $s \in S$. Such a schedule maximizes the number of solved instances, or conversely, minimizes the number of timeouts. An instance $i$ is solved by $\sigma$ if there is a solver $s \in S$ that has

|    | $s_1$ | $s_2$ | $s_3$ | oracle |
|----|-------|-------|-------|--------|
| $i_1$ | 1     | $\geq 10$ | 3     | 1      |
| $i_2$ | 5     | $\geq 10$ | 2     | 2      |
| $i_3$ | 8     | 1     | $\geq 10$ | 1      |
| $i_4$ | $\geq 10$ | 2     |       | 2      |
| $i_5$ | $\geq 10$ | 6     | $\geq 10$ | 6      |
| $i_6$ | $\geq 10$ | 8     | $\geq 10$ | 8      |

Table 1: Table of solver runtimes on problem instances with $\kappa = 10$; ’$\geq 10$’ indicates a timeout.
an equal or greater time slice $\sigma(s)$ than the time needed by the solver to solve the instance, viz. $t(i,s)$. As a side constraint, the sum of all time slices $\sigma(s)$ has to be equal or less than the cutoff time $\kappa$.

The above example corresponds to the schedule $\sigma = \{ s_1 \mapsto 1, s_2 \mapsto 6, s_3 \mapsto 2 \}$; in fact, $\sigma$ constitutes one of nine timeout-optimal solver schedules in our example. Note that the sum of all time slices is even smaller than the cutoff time. Hence, all schedules obtained by adding 1 to either of the three solvers are also timeout-optimal. A timeout-optimal schedule consuming the entire allotted time is $\{ s_1 \mapsto 0, s_2 \mapsto 8, s_3 \mapsto 2 \}$.

In practice, however, the criterion in (1) turns out to be too coarse, that is, it often admits a diverse set of solutions among which we would like to make an educated choice. To this end, we make use of (simplified) $L^\infty$-norms as the basis for refining our choice of schedule. In our case, an $L^\infty$-norm on schedules is defined as $\Sigma_{s \in S, \sigma(s) \neq 0} \sigma(s)^n$. Depending on the choice of $n$ as well as whether we minimize or maximize the norm, we obtain different selection criteria. For instance, $L^0$-norms suggest using as few (or as many) solvers as possible, and $L^1$-norms aim at minimizing (or maximizing) the sum of time slices. Minimizing the $L^2$-norm amounts to allotting each solver a similar time slice, while maximizing it prefers schedules with large runtimes for few solvers. In more formal terms, for a given set $S$ of solvers, using an $L^n$-norm we would like to determine schedules satisfying the constraint

$$\sigma \in \arg\min_{\sigma : S \mapsto [0,\kappa]} \Sigma_{s \in S, \sigma(s) \neq 0} \sigma(s)^n,$$  \hspace{1cm} (2)

or the analogous constraint for $\arg\max$ (in case of maximization).

For instance, our example constraint $\sigma = \{ s_1 \mapsto 1, s_2 \mapsto 6, s_3 \mapsto 2 \}$ has the $L^n$-norms 3, 9, and 41 for $n = 0, 2$. In contrast, we obtain norms 3, 9, and 27 for the (suboptimal) uniform schedule $\{ s_1 \mapsto 3, s_2 \mapsto 3, s_3 \mapsto 3 \}$ and 1, 9, and 81 for a singular schedule $\{ s_3 \mapsto 9 \}$, respectively. Although empirically, we found that schedules for various $n$ as well as for minimization and maximization have useful properties, overall, we favor schedules with a minimal $L^2$-norm. First, this choice leads to a significant reduction of candidate schedules and, second, it results in schedules with a maximally homogeneous distribution of time slices, similar to $ppfolio$. In fact, our example schedule has the smallest $L^2$-norm among all nine timeout-optimal solver schedules.

Once we have identified an optimal schedule w.r.t. criteria (1) and (2), it is interesting to determine which solver alignment yields the best performance as regards time. More formally, we define an alignment of a set $S$ of solvers as a bijective function $\pi : \{1, \ldots, |S| \} \mapsto S$. Consider the above schedule $\sigma = \{ s_1 \mapsto 1, s_2 \mapsto 6, s_3 \mapsto 2 \}$. The alignment $\pi = \{ 1 \mapsto s_1, 2 \mapsto s_3, 3 \mapsto s_2 \}$ induces the execution sequence $(s_1, s_3, s_2)$ of $\sigma$. This sequence takes 29 seconds for all six benchmarks in Table 1; in detail, it takes 1, $1+2$, $1+2+1$, $1+2$, $1+2+6$, $1+2+7$ seconds for benchmark $i_k$ for $k = 1..6$, whereby instance $i_6$ could not be solved. For instance, benchmark $i_3$ is successfully solved by the third solver in the alignment, viz. $s_2$. Hence the total time amounts to the time allotted by $\sigma$ to $s_1$ and $s_3$, viz. $\sigma(s_1)$ and $\sigma(s_3)$, plus the effective time of $s_2$, viz. $t(i_3, s_2)$.

This can be formalized as follows. Given a schedule $\sigma$ and an alignment $\pi$ of a set $S$ of solvers, and an instance $i \in I$, we define the runtime $\tau(\sigma, \pi)$ of schedule $\sigma$ aligned by $\pi$ on $i$:

$$\tau_{\sigma, \pi}(i) = \begin{cases} \sum_{j=1}^{\min(P_{\sigma, \pi})-1} \sigma(\pi(j)) + t(\pi((\min(P_{\sigma, \pi})-1)+1),s) & \text{if } P_{\sigma, \pi} \neq \emptyset, \\ \kappa & \text{otherwise} \end{cases} \hspace{1cm} (3)$$

1 The common $L^n$-norm is defined as $\sqrt[n]{\Sigma_{x \in X} x^n}$. We take the simpler definition in view of using it merely for optimization.
where \( P_{\sigma, \pi} = \{ l \in \{1, \ldots, |S|\} \mid t(l, \pi(l)) \leq \sigma(\pi(l)) \} \) are the positions of solvers solving instance \( i \) in a schedule \( \sigma \) aligned by \( \pi \). If an instance \( i \) cannot be solved at all by a schedule, \( \tau_{\sigma, \pi}(i) \) is set to the cutoff \( \kappa \). For our example schedule \( \sigma \) and its alignment \( \pi \), we obtain for \( i_3 \):

\[
\min P_{\sigma, \pi} = 3 \quad \text{and} \quad \tau_{\sigma, \pi}(i_3) = 1 + 2 + 1 = 4.
\]

For a schedule \( \sigma \) of solvers in \( S \), we then define the optimal alignment of schedule \( \sigma \):

\[
\pi \in \arg \min_{\pi: \{1, \ldots, |S|\} \rightarrow S} \sum_{i \in I} \tau_{\sigma, \pi}(i)
\]

(4)

For our timeout-optimal schedule \( \sigma = \{ s_1 \rightarrow 1, s_2 \rightarrow 6, s_3 \rightarrow 2 \} \) w.r.t. criteria (1) and (2), we obtain two optimal execution alignments, namely \( (s_3, s_1, s_2) \) and \( (s_1, s_3, s_2) \), both of which result in a solving time of 29 seconds for the benchmarks of Table 1.

### 2.2 Parallel Scheduling

The increasing availability of multi-core processors makes it interesting to extend our approach for distributing a schedule’s solvers over multiple processing units. For simplicity, we take a coarse approach in binding solvers to units, thus precluding re-allocations during runtime.

To begin with, let us provide a formal specification of the extended problem. To this end, we augment our previous formalization with a set \( U \) of (processing) units and associate each unit with subsets of solvers from \( S \). More formally, we define a distribution of a set \( S \) of solvers as the function \( \eta: U \rightarrow 2^S \) such that \( \bigcap_{u \in U} \eta(u) = \emptyset \). With it, we can determine timeout-optimal solver schedules for several cores simply by strengthening the condition in (1) to the effect that all solvers associated with the same unit must respect the cutoff time. This leads us to the following extension of (1):

\[
\sigma \in \arg \max_{\sigma, S \rightarrow [0, \infty]} \{|i \mid \exists s \in S: t(i, s) \leq \sigma(s)\} \quad \text{such that} \quad \sum_{s \in \eta(u)} \sigma(s) \leq \kappa \quad \text{for each } u \in U
\]

(5)

For illustration, let us reconsider Table 1 along with schedule \( \sigma = \{ s_1 \rightarrow 1, s_2 \rightarrow 8, s_3 \rightarrow 2 \} \). Assume that we have two cores, 1 and 2, along with the distribution \( \eta = \{ 1 \rightarrow \{ s_2 \}, 2 \rightarrow \{ s_1, s_3 \} \} \). This distributed schedule is an optimal solution to the optimization problem in (5) w.r.t. the benchmarks in Table 1 because it solves all benchmarks within a cutoff time of \( \kappa = 8 \).

We keep the definitions of a schedule’s \( L^\infty \)-norm as a global constraint. However, for determining our secondary criterion, enforcing time-optimal schedules, we relativize the auxiliary definitions in (3) to account for each unit separately. Given a schedule \( \sigma \) and a set \( U \) of processing units, we define for each unit \( u \in U \) a local alignment of the solvers in \( \eta(u) \) as the bijective function \( \pi_u: \{1, \ldots, |\eta(u)|\} \rightarrow \eta(u) \). Given this function and a problem instance \( i \in I \), we extend the definitions in (3) as follows:

\[
\tau_{\sigma, \pi_u}(i) = \begin{cases} 
\left( \sum_{j=1}^{\min \{P_{\sigma, \pi}\}} \sigma(\pi_u(j)) \right) + t(i, \pi_u(\min \{P_{\sigma, \pi}\})) & \text{if } P_{\sigma, \pi} \neq \emptyset, \\
\kappa & \text{otherwise}
\end{cases}
\]

(6)

where \( P_{\sigma, \pi} = \{ l \in \{1, \ldots, |\eta(u)|\} \mid t(i, \pi_u(l)) \leq \sigma(\pi_u(l)) \} \).

The collection \((\pi_u)_{u \in U}\) regroups all local alignments into a global alignment. For a schedule \( \sigma \) of solvers in \( S \) and a set \( U \) of (processing) units, we then define an optimal global alignment:

\[
(\pi_u)_{u \in U} \in \arg \min_{(\pi_u: \{1, \ldots, |\eta(u)|\} \rightarrow \eta(u))_{u \in U}} \sum_{i \in I} \min_{u \in U} \tau_{\sigma, \pi_u}(i)
\]

(7)

For illustration, reconsider the above schedule \( \sigma = \{ s_1 \rightarrow 1, s_2 \rightarrow 8, s_3 \rightarrow 2 \} \) and distribution.
η = {1 ↦ {s2}, 2 ↦ {s1, s3}}, and suppose we chose the local alignments π₁ = {1 ↦ s₂} and π₂ = {1 ↦ s₁, 2 ↦ s₃}. This global alignment solves all six benchmark instances of Table 1 in 22 seconds wallclock time. In more detail, it takes 1₂, 1 + 2₁, 1 + 2₂, 6₁, 8₁ seconds for instance iₖ for k = 1..6, where the solving unit is indicated by the subscript.

Note that the definitions in (5), (6), and (7) correspond to their sequential counterparts in (1), (3), and (4) whenever we are faced with a single processing unit.

3 Solving Timeout-Optimal Scheduling with ASP

To begin with, we detail the basic encoding for identifying robust (parallel) schedules. In view of the remark at the end of the last section, however, we directly provide an encoding for parallel scheduling, which collapses to one for sequential scheduling whenever a single processing unit is used.

Following good practice in ASP, a problem instance is expressed as a set of facts. That is, Function \( t : I \times S \rightarrow \mathbb{R} \) is represented as facts of form time(i, s, t), where \( i \in I, s \in S \), and \( t \) is the runtime \( t(i, s) \), converted to a natural number with limited precision. The cutoff is expressed via Predicate kappa/1, and the number of available processing units is captured via Predicate units/1, here instantiated to 2 units. Given this, we can represent the contents of Table 1 as shown in Listing 1 below.

Listing 1: Facts

```prolog
kappa(10).
units(2).
time(i1, s1, 1). time(i1, s2, 11). time(i1, s3, 3).
time(i2, s1, 5). time(i2, s2, 11). time(i2, s3, 2).
time(i3, s1, 8). time(i3, s2, 1). time(i3, s3, 11).
time(i4, s1, 11). time(i4, s2, 11). time(i4, s3, 2).
time(i5, s1, 11). time(i5, s2, 6). time(i5, s3, 11).
time(i6, s1, 11). time(i6, s2, 8). time(i6, s3, 11).
```

The encoding in Listing 3 along with all following ones are given in the input language of gringo (Gebser et al.). The first three lines of Listing 3 provide auxiliary data. The set \( S \) of solvers is given by Predicate solver/1. Similarly, the runtimes for each solver are expressed by time/2 and each processing unit by unit/1. In addition, the ordering of instances by time per solver is precomputed; it is expressed via order/3.

Listing 2: I is solved immediately before J by solver S

```prolog
order(I, J, S) :-
time(I, S, T), time(J, S, V), (T, I) < (V, J),
not time(K, S, U) : time(K, S, U) : (T, I) < (U, K) : (U, K) < (V, J).
```

The above results in facts order(I, J, S) capturing that instance J follows instance I by sorting the instances according to their runtimes. Although this information could be computed via ASP (as shown above), we make use of external means for sorting (the above rule needs cubic time for instantiation, which is infeasible for a few thousand instances). Instead, we use gringo’s embedded scripting language lua for sorting.

The idea of Listing 3 is now to guess for each solver a time slice and a processing unit (in
Line 5). With the resulting schedule, all solvable instances can be identified (in Line 10 – 12), and finally, all schedules solving a maximal number of instances are selected (in Line 14).

Listing 3: ASP encoding for Timeout-Minimal (Parallel) Scheduling

```plaintext
1 solver(S) :- time(_,S,_) .
2 time(S,T) :- time(_,S,T) .
3 unit(1..N) :- units(N) .
4 {slice(U,S,T): time(S,T): T <= K: unit(U)} 1 :- solver(S), kappa(K) .
5 :- not [ slice(U,S,T) = T ] K, kappa(K), unit(U) .
6 slice(S,T) :- slice(_,S,T) .
7 solved(I,S) :- slice(S,T), time(I,S,T) .
8 solved(I,S) :- solved(J,S), order(I,J,S) .
9 solved(I) :- solved(I,_).  
10 #maximize { solved(I) @ 2 } .
11 #minimize [ slice(S,T) = T*T @ 1 ] .
```

In more detail, a schedule is represented by atoms `slice(U,S,T)` allotting a time slice `T` to solver `S` on unit `U`. In Line 5, at most one time slice is chosen for each solver, subject to the condition that it does not exceed the cutoff time. At the same time, a processing unit is uniquely assigned to the selected solver. The integrity constraint in Line 6 ensures that the sum over all selected time slices on each processing unit is not greater than the cutoff time. This implements the side condition in (5), and it reduces to the one in (1) whenever a single unit is considered. The next line projects out the processing unit because it is irrelevant when determining solved instances (in Line 8). In Lines 10 to 12, all instances solved by the selected time slices are gathered via predicate `solved/1`. Considering that we collect in Line 8 all time slices among actual runtimes, each time slice allows for solving at least one instance. This property is used in Line 10 to identify the instance `I` solvable by solver `S`; using it, along with the sorting of instances by solver performance in `order/3`, we collect in Line 11 all instances that can be solved even faster than the instance in Line 10. Note that at first sight it might be tempting to encode Lines 10 – 12 differently:

```plaintext
solved(I) :- slice(S,T), time(I,S,TS), T <= TS .
```

The problem with the above rule is that it has a quadratic number of instantiations in the number of benchmark instances in the worst case. In contrast, our ordering-based encoding is linear, because only successive instances are considered. Finally, the number of solved instances is maximized in Line 14, using the conditions from (5) (or (1), respectively). This primary objective is assigned a higher priority than the $L^2$-norm from (2) (priority 2 vs 1).

4 Solving (Timeout and) Time-Minimal Parallel Scheduling with ASP

In the previous section, we have explained how to determine a timeout-minimal (parallel) schedule. Here, we present an encoding that takes such a schedule and calculates a solver alignment per processing unit while minimizing the overall runtime according to Criterion (7). This two-phase approach is motivated by the fact that an optimal alignment must be determined among all
permutations of a schedule. While a one-shot approach had to account for all permutations of all potential timeout-minimal schedules, our two-phase approach reduces the second phase to searching among all permutations of a single timeout-minimal schedule.

We begin by extending the ASP formulation from the last section (in terms of kappa/1, units/1, and time/3) by facts over slice/3 providing the time slices of a timeout-minimal schedule (per solver and processing unit). In the case of our example from Section 2.2, we extend the facts of Listing 1 with the following obtained timeout-minimal schedule to create the problem instance:

```
Listing 4: Schedule Facts
slice(1,s2,8). slice(2,s1,1). slice(2,s3,2).
```

The idea of the encoding in Listing 5 is to guess a permutation of solvers and then to use ASP's optimization capacities for calculating a time-minimal alignment. The challenging part is to keep the encoding compact. That is, we have to keep the size of the instantiation of the encoding small, because otherwise, we cannot hope to effectively deal with rather common situations involving thousands of benchmark instances. To this end, we make use of #sum aggregates with negative weights (Line 23) to find the fastest processing unit without representing any sum of times explicitly.

The block in Line 1 to 10 gathers static knowledge about the problem instance, that is, solvers per processing unit (solver/2), instances appearing in the problem description (instance/1), available processing units (unit/1), number of solvers per unit (solvers/2), instances solved by a solver within its allotted slice (solved/3), and instances that could be solved on a unit given the schedule (solved/2). Note that, in contrast to the previous encoding (Listing 3), the solved instances (solved/3) can be efficiently expressed as done in Line 5 of Listing 5, because slice/3 are facts here. In view of Equation (6), we precompute the times that contribute to the values of $\tau_{\sigma,\pi,u}$ and capture them in capped/4 (and capped/3). A fact capped(U,I,S,T) assigns to instance $i$ run by solver $S$ on unit $U$ a time $T$. In Line 7, we assign the time needed to solve the instance if it is within the solver’s time slice. In Line 8, we assign the solver’s time slice if the instance could not be solved, but at least one other solver could solve it on processing unit $U$. In Line 9, we assign the entire cutoff to dummy solver $d$ (we assume that there is no other solver called $d$) if the instance could not be solved on the processing unit at all; this is to implement the else case in (6) and (3).

The actual encoding starts in Line 12 and 13 by guessing a permutation of solvers. Here, the two head aggregates ensure that for every solver (per unit) there is exactly one position in the alignment and vice versa. In Line 15 and 16, we mark indexes (per unit) as solved if the solver with the preceding index could solve the instance or if the previous index was marked as solved. Note that this is a similar “chain construction” used in the previous section in order to avoid a combinatorial blow-up.

In the block from Line 18 to 26, we determine the time for the fastest processing unit depending on the guessed permutation. The rules in Line 18 and 19 mark the times that have to be added up on each processing unit; the sums of these times correspond to $\tau_{\sigma,\pi,u}(i)$ in Equation (6) and (3). Next, we determine the smallest sum of times by iteratively determining the minimum. An atom min(U,I,S,T) marks the times of the fastest unit in the range from unit 1 to U to solve an instance (or the cutoff via dummy solver $d$, if the schedule does not solve the instance for the unit). To this end, we initialize min/4 with the times for the first unit in Line 20. Then, we add a rule in
Listing 5: ASP encoding for Time-Minimal (Parallel) Scheduling

1  solver(U,S) :- slice(U,S,1..N).
2  instance(I) :- time(I,_,_).
3  unit(1..N) :- units(N).
4  solvers(U,N) :- unit(U), N := \{solver(U,\_)}.
5  solved(U,S,I) :- time(I,S,T), slice(U,S,TS), T <= TS.
6  solved(U,I) :- solved(U,_,I).
7  capped(U,I,S,T) :- time(I,S,T), solved(U,S,I).
8  capped(U,I,S,T) :- slice(U,S,T), solved(U,I), not solved(U,S,I).
9  capped(U,I,d,K) :- unit(U), kappa(K), instance(I), not solved(U,I).
10  capped(I,S,T) :- capped(_,I,S,T).
11 { order(U,S,X) : solver(U,S) } 1 :- solvers(U,N), X = 1..N.
12 { order(U,S,X) : solvers(U,N) : X = 1..N } 1 :- solver(U,S).
13 solvedAt(U,I,X+1) :- solved(U,S,I), order(U,S,X).
14 solvedAt(U,I,X+1) :- solvedAt(U,I,X), solvers(U,N), X <= N.
15 mark(U,I,d,K) :- capped(U,I,d,K).
16 mark(U,I,S,T) :- capped(U,I,S,T), order(U,S,X), not solvedAt(U,I,X).
17 min(1,I,S,T) :- mark(1,I,S,T).
18 less(U,I) :- unit(U), unit(U+1), instance(I),
19 [min(U,I,S1,T1) = T1, mark(U+1,I,S2,T2) = -T2] 0.
20 min(U+1,I,S,T) :- min(U,I,S,T), less(U,I).
21 min(U,I,S,T) :- mark(U,I,S,T), not less(U-1,I).
22 #minimize \{min(U,\_\_,\_\_): not unit(U+1) = T\}.

Line 22 and 23 that, given minimal times for units in the range of 1 to \(U\) and times for unit \(U+1\), determines the faster one. The current minimum contributes positive times to the sum, while unit \(U+1\) contributes negative times. Hence, if the sum is negative or zero, the sum of times captured in \(\text{min/4}\) is smaller than or equal to the sum of times of unit \(U+1\), and therefore, the unit thus slower than some preceding unit, which makes the aggregate true and derives the corresponding atom over \(\text{less/2}\). Depending on \(\text{less/2}\), we propagate the smaller sum, which is either contributed by unit \(U+1\) (Line 25) or the preceding units (Line 26). Finally, in Line 28, the times of the fastest processing unit are minimized in the optimization statement, which implements Equation (7) and (4).

5 Experiments

After describing the theoretical foundations and ASP encodings underlying our approach, we now present the results from an empirical evaluation on representative ASP, CSP, MaxSAT, SAT and QBF benchmarks. The python implementation of our approach, dubbed \textit{aspeed}, uses the state-of-the-art ASP systems (Calimeri et al. 2011) of the potassco group (Gebser et al. 2011),
Table 2: Runtime data sets used in our experiments from the 2011 SAT Competition (1), the ASP benchmark repository asparagus (2), Kadioglu et al. 2011 (3), Gent et al. 2010 (4), Pulina and Tacchella 2009 (5) and Malitsky et al. 2013 (6).

| Source          | 3S-Set | CSP-Set | QBF-Set | MaxSAT-Set |
|-----------------|--------|---------|---------|------------|
| Cutoff (sec.)   | 5000   | 5000    | 3600    | 1800       |
| #Instances      | 5467   | 2024    | 1368    | 337        |
| #Solvers        | 37     | 2       | 5       | 11         |
| Source          | (3)    | (4)     | (5)     | (6)        |

namely the grounder gringo (3.0.4) and the ASP solver clasp (2.0.5). The sets of runtime data used in this work are freely available online. ²

5.1 Experimental setup

Our experiments are based on a set of runtime data obtained by running several solvers (or solver configurations) on a set of benchmark instances (similar to Table 1). To provide a thorough empirical evaluation of our approach, we selected eight large data sets of runtimes for five prominent and widely studied problems, ASP, CSP, MaxSAT, SAT and QBF; these are summarized in Table 2. The sets Random, Crafted and Application contain the authentic runtimes taken from the 2011 SAT Competition³ with a cutoff of 5000 seconds. We selected all non-portfolio, non-parallel solvers from the main phase of the competition, in order to provide a fair comparison with the portfolio-based SAT Solver satzilla (Xu et al. 2008), which has been evaluated based on the same data (Xu et al. 2012).

Also, we evaluated our approach on an ASP instance set (ASP-Set) based on different configurations of the highly parametric ASP solver clasp (Gebser et al. 2012), which is known to show excellent performance on a wide range of ASP instances. We used the complementary configuration portfolio of claspfolio (1.0.1) (Gebser et al. 2011) designed by the main developer of clasp, B. Kaufmann, and measured the runtime of clasp (2.1.0). Because the instance sets from recent ASP competitions are very unbalanced (Hoos et al. 2013) (most of them are either too easy or too hard for clasp), we select instances from the ASP benchmark repository Asparagus,⁴ including the 2007 (SLparse track), 2009 and 2011 ASP Competitions. gringo was not able to ground some instance from the 2011 ASP Competition within 600 CPU seconds and 2 GB RAM, and thus those instances were excluded. Our ASP-Set is comprised of the 2589 remaining instances.

The runtime measurements for our ASP-Set were performed on a compute cluster with 28 nodes,
each equipped with two Intel Xeon E5520 2.26GHz quad-core CPUs and 48 GB RAM, running Scientific Linux (2.6.18-308.4.1.el5). Since all clasp configurations used in our experiments are deterministic, their runtimes on all instances were measured only once.

Furthermore, we evaluated our approach on sets already used in the literature. The set of runtime data provided by Kadioglu et al. was part of the submission of their solver 3S (Kadioglu et al. 2011) to the 2011 SAT Competition. We selected this set, which we refer to as 3S-Set, because it includes runtimes of many recent SAT solvers on prominent SAT benchmark instances. The CSP-Set was used by Gent et al. (2010), the QBF-Set by Pulina and Tacchella (2009), and MaxSAT-Set by Malitsky et al. (2013), respectively.

The performance of aspeed was determined from the schedules computed for Encodings 3 and 5 with a minimization of the $L^2$-norm as second optimization criterion. Although we empirically observed no clear performance gain from the latter, we favour a schedule with a minimal $L^2$-norm: First, it leads to a significant reduction of candidate schedules and second, it results in schedules with a more uniform distribution of time slices, (resembling those used in ppfolio). All runtimes for the schedule computation were measured in CPU time rounded up to the next integer value, and runtime not allocated in the computed schedule was uniformly distributed among all solvers in the schedule.

Using the previously described data sets, we compared aspeed against

- single best: the best solver in the respective portfolio,
- uniform: a uniform distribution of the time slices over all solvers in the portfolio,
- ppfolio-like: an approach inspired by ppfolio, where the best three complementary solvers are selected with an uniform distribution of time slices in the sequential case,
- satzilla (Xu et al. 2012) and claspfolio (Gebser et al. 2011), prominent examples of model-based algorithm selection solvers for SAT and ASP, respectively,
- as well as against the oracle performance (also called virtual best solver)\(^5\).

The performance of satzilla for Random, Crafted and Application was extracted from results reported in the literature (Xu et al. 2012), which were obtained using 10-fold cross validation. In the same way, claspfolio was trained and cross-validated on the ASP-Set. In the following, the selection approach represents satzilla for the three SAT competition sets and claspfolio for the ASP-Set.

Unfortunately, aspeed could not be directly compared against 3S, because the tool used by 3S to compute the underlying model is not freely available and hence, we were unable to train 3S on new data sets. To perform a fair comparison between aspeed and 3S, we compare both systems in an additional experiment in the last part of this section.

### 5.2 Schedule Computation

Table 3 shows the time spent on the computation and the proof of the optimality of timeout-minimal schedules and time-minimal alignments on the previously described benchmark sets for sequential schedules (first two rows) and parallel schedules for eight cores (next two rows). For the Random, Crafted and CSP-Set benchmark sets, the computation of the sequential and parallel schedule always took less than one CPU second. Some more time was spent for the

\(^5\) The performance of the oracle is the minimal runtime of each instance given a portfolio of solvers and corresponds to a portfolio-based solver with a perfect selection of the best solver for a given instance.
### Table 3: Runtimes of clasp in CPU seconds to calculate an optimal schedule for one and eight cores.

| #cores | Opt. Step         | Random  | Crafted | Application | ASP-Set |
|--------|-------------------|---------|---------|-------------|---------|
| 1      | Schedule (sec)    | 0.54    | 0.45    | 119.2       | > 1d    |
| 1      | Alignment (sec)   | 0.04    | 0.23    | 0.07        | 0.50    |
| 8      | Schedule (sec)    | 0.28    | 0.05    | 61.65       | > 1d    |
| 8      | Alignment (sec)   | 0.02    | 0.006   | 0.07        | 0.50    |
| 1      | Combined (sec)    | > 1d    | 47/15   | > 1d        | MEMOUT  |

|        | 3S-Set | CSP-Set | QBF-Set | MaxSAT-Set |
|--------|--------|---------|---------|------------|
| 1      | > 1d   | 0.10    | 14.98   | 1.64       |
| 1      | Alignment (sec) | > 1d   | 0.04    | 0.75    | 0.02 |
| 8      | Schedule (sec) | > 1d   | 0.20    | 0.21    | 0.30 |
| 8      | Alignment (sec) | > 1d   | 0.12    | 0.27    | 0.02 |
| 1      | Combined (sec) | MEMOUT | 0.89    | 32.09    | > 1d |

Application, QBF-Set and MaxSAT-Set benchmark set but it is still feasible to find an optimal schedule. We observe that the computation of parallel time slices is faster than the computation of sequential schedules, except for the very simple CSP-Set. Given the additional processing units, the solvers can be scheduled more freely, resulting in a less constrained problem that is easier to solve. Furthermore, calculating a time-minimal alignment is easier in the parallel setting. In our experiments, we obtained fewer selected solvers on the individual cores than in the sequential case. This leads to smaller permutations of solvers and, in turn, reduces the total runtime. For the ASP-Set, we could not establish the optimal schedule even after one CPU day and for the 3S-Set, the calculation of the optimal schedule and optimal alignment was also impossible. However aspeed was nevertheless able to find schedules and alignments, and hence, was able to minimize the number of timeouts and runtime. Finally, it is also possible that aspeed found an optimal schedule but was unable to prove its optimality. Therefore, we limited the maximal runtime of clasp for these sets to 1200 CPU seconds in all further experiments, and used the resulting sub-optimal schedules and alignments obtained for this time.6

We also ran experiments on an encoding that optimizes the schedule and alignment simultaneously; this approach accounts for all permutations of all potential timeout-minimal schedules. The results are presented in the row labelled ‘Combined’ in Table 3. The combination increases the solving time drastically. Within one CPU day, clasp was able to find an optimal solution and proved optimality only for Crafted, CSP-Set and QBF-Set. In all other cases, we aborted clasp after one CPU day and then used the best schedules found so far. Nevertheless, we could find better alignments than in our two step approach (between 0.6% and 9.8% improvement), at the cost of substantially higher computation time and memory. Because this encoding has a very large instantiation, viz., more than 12 GB memory consumption, we were unable to run aspeed using it on the 3S-Set and ASP-Set.

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6 Note that in our experiments, the performance of unclasp (Andres et al. 2012), which optimizes based on unsatisfiable cores, did not exceed the performance of clasp in computing solver schedules.
5.3 Evaluation of Timeout-Minimal Schedules

Having established that optimal schedules can be computed within a reasonable time in most cases, we evaluated the sequential timeout-minimal schedule of aspeed corresponding to the first step of our optimization process (cf. Equation (1)). The number of timeouts for a fixed time budget assesses the robustness of a solver and is in many applications and competitions the primary evaluation criterion.

To obtain an unbiased evaluation of performance, we used 10-fold cross validation, a standard technique from machine learning: First, the runtime data for a given instance set are randomly divided into 10 equal parts. Then, in each of the ten iterations, 9/10th of the data is used as a training set for the computation of the schedule and the remaining 1/10th serves as a test set to evaluate the performance of the solver schedule at hand; the results shown are obtained by summing over the folds. We compared the schedules computed by aspeed against the performance obtained from the single best, uniform, ppfolio-like, selection (satzilla and claspfolio; if possible) approaches and the (theoretical) oracle. The latter provides a bound on the best performance obtainable from any portfolio-based solver.

Table 4 shows the fraction of instances in each set on which timeouts occurred (smaller numbers indicate better performance). In all cases, aspeed showed better performance than the single best solver. For example, aspeed reduced the number of timeouts from 1881 to 603 instances (less 23% of unsolved instances) on the 3S-Set, despite the fact that aspeed was unable to find the optimal schedule within the given 1200 CPU seconds on this set. Also, aspeed performed better than the uniform approach. The comparison with ppfolio-like and selection (satzilla and claspfolio) revealed that aspeed performed better than ppfolio-like in seven out of eight scenarios we considered, and better than satzilla and claspfolio in two out of four scenarios. We expected that aspeed would solve fewer instances than the selection approach in all four scenarios, because aspeed, unlike satzilla and claspfolio, does not use any instance features or prediction of solver performance. It is somewhat surprising that satzilla and claspfolio do not always benefit from their more sophisticated approaches, and further investigation into why this happens would be an interesting direction for future work.
Table 5: Ratios of the expected performance of a random alignment and alignments computed by \textit{aspeed}, \textit{heu-Opt} and \textit{heu-Min}; \textit{heu-Opt} sorts the solvers beginning with the solver with the minimal number of timeouts; \textit{heu-Min} begins with the solver with the smallest time slice. The expected performance of a random alignment was approximated by 10,000 samples for all sets marked with $^*$. 

5.4 Evaluation of Time-Minimal Alignment

After choosing the time slices for each solver, it is necessary to compute an appropriate solver alignment in order to obtain the best runtimes for our schedules. As before, we used 10-fold cross validation to assess this stage of \textit{aspeed}. To the best of our knowledge, there is no system with a computation of alignments to compare against. Hence, we use a random alignment as a baseline for evaluating our approach. Thereby, the expected performance of a random alignment is the average runtime of all possible alignments. Since the number of all permutations for \textit{ASP-Set} and \textit{3S-Set} is too large ($\gg 1000000000$), we approximate the performance of a random alignment by 10,000 sampled alignments.

Table 5 shows the ratio of the expected performance of a random alignment and alignments computed by \textit{aspeed}. Note that this ratio can be smaller than one, because the alignments are calculated on a training set and evaluated on a disjoint test set.

Also, we contrast the optimal alignment with two easily computable heuristic alignments to avoid the search for an optimal alignment. The alignment heuristic \textit{heu-Opt} sorts solvers beginning with the solver with the minimal number of timeouts (most robust solver), while \textit{heu-Min} begins with the solver with the smallest time slice.

As expected, the best performance is obtained by using optimal alignments within \textit{aspeed} (Table 5); it led, for example, to an increase in performance by a factor of 2.13 on \textit{MaxSAT-Set}. In all cases, the performance of \textit{heu-Min} was strictly better than (or equal to) that of \textit{heu-Opt}. Therefore, using \textit{heu-Min} seems desirable whenever the computation of an optimal alignment is infeasible.

The actual runtimes of \textit{aspeed} and the other approaches are quite similar to the results on the number of timeouts (Table 4) (data not shown). The penalized runtimes (PAR10) are presented in Figure 1 (a),(c) and (e) at $\#\text{cores} = 1$.

5.5 Parallel Schedules

As we have seen in Section 3, our approach is easily extendable to parallel schedules. We evaluated such schedules on \textit{Random}, \textit{Crafted}, \textit{Application} and \textit{ASP-Set}. The results of this experiment are
Fig. 1: Parallel Schedules (left) and reduced cutoff time (right), single best (+), uniform (×), ppfolio-like approach (+), aspeed (■), selection (□), oracle (○).
Table 6: Comparison of sequential and parallel schedules with 4 cores w.r.t. the number of timeouts and PAR10 score.

| Schedule Type   | #TO | PAR10 | #TO | PAR10 | #TO | PAR10 | #TO | PAR10 |
|-----------------|-----|-------|-----|-------|-----|-------|-----|-------|
| uniform-SP      | 1001| 9847  | 283 | 7077  | 357 | 10176 | 21  | 1470  |
| ppfolio-like-SP | 796 | 7662  | 283 | 7077  | 357 | 9657  | 10  | 731   |
| aspeed-SP       | 603 | 6001  | 275 | 6902  | 344 | 9272  | 7   | 516   |
| uniform-4P      | 583 | 5720  | 253 | 6344  | 316 | 8408  | 4   | 511   |
| ppfolio-like-4P | 428 | 4095  | 253 | 6344  | 316 | 8404  | 4   | 353   |
| aspeed-4P       | 204 | 2137  | 253 | 6344  | 316 | 8403  | 3   | 332   |
| oracle          | 0   | 198   | 253 | 6344  | 314 | 8337  | 0   | 39    |

Table 6: Comparison of sequential and parallel schedules with 4 cores w.r.t. the number of timeouts and PAR10 score.

presented in Figure 1 (a), (c), (e) and (g). These evaluations were performed using 10-fold cross validation and measuring wall-clock time.

In each graph, the number of cores is shown on the x-axis and the PAR10 (penalized average runtime) on the y-axis; we used PAR10, a commonly used metric from the literature, to capture average runtime as well as timeouts. (The sequential performance of aspeed can be read off the values obtained for one core.) Since the single best solver (+) and selection (□, satzilla resp. claspfolio) cannot be run in parallel, their performance is constant. Furthermore, the ppfolio-like approach (+) is limited to run at most three component solver on the first core with uniform time slices and one component solvers on each other core. This more constrained schedule is also computed with the ASP encodings presented in Section 3 by adding three more constraints.

As stated previously, the sequential version of aspeed (■) performed worse than satzilla (□) in Random and Application. However, aspeed turned out to perform at least as well as satzilla when using two or more cores, in terms of PAR10 scores as well in terms of average runtime (data not shown). For example, aspeed-4P – that is parallel aspeed using four cores – achieved a speedup of 1.20 over the sequential aspeed on Random (20 fewer timeouts), 1.10 on Crafted (9 fewer timeouts), 1.44 on Application (26 fewer timeouts) and 1.57 on ASP-Set (111 fewer timeouts); furthermore, aspeed-4P solved 4, 13, 17, 117 instances more on these sets than (sequential) satzilla and claspfolio, respectively. Considering the high performance of satzilla (Xu et al. 2012) and claspfolio (Gebser et al. 2011), this represents a substantial performance improvement.

Table 6 presents the performance of parallel aspeed with four cores (aspeed-4P), the parallel uniform and parallel ppfolio-like schedule, respectively, on 3S-Set, CSP-Set, QBF-Set and MaxSAT-Set. We decided to use only four cores because (i) CSP-Set and QBF-Set have two resp. five solvers, and therefore it is trivial to perform as well as the oracle with 4 or more cores, and (ii) we saw in Figure 1 that the curves flatten beginning with four cores, which is an effect of the complementarity of the solvers in the portfolio. The performance of aspeed-SP, i.e., sequential aspeed, is already nearly as good as the oracle on MaxSAT-Set and aspeed-4P was only able

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7 PAR10 penalizes each timeout with 10 times the given cutoff time (Hutter et al. 2009).
5.6 Generalization Ability of aspeed

The schedule computation of aspeed uses runtime data measurements, which require extensive computational resources. Therefore, we investigated the possibility to decrease the cutoff time on the training data to reduce the overall computational burden of training. The schedules thus obtained were evaluated on test data with an unreduced cutoff time. We note that only instances are considered for the computation of schedules that are solved by at least one solver in the portfolio. Therefore, using this approach with a lower training cutoff time, the computation of a schedule is based on easier and fewer instances than those in the test set used to ultimately evaluate it. Figures 1 (b), (d) and (f) show the results of evaluating the resulting schedules in the same way as in the experiments for parallel schedules with 10-fold cross validation but using only one processing unit. The cutoff time on the training set (shown on a logarithmic x-axis) was reduced according to a $2/3$-geometric sequence, from the maximal cutoff time of 5000 down to 195 CPU seconds for Random, Crafted and Application and 900 down to 52 CPU seconds for the ASP-Set. A flat line corresponds to the expected optimal case that the performance of a schedule does not suffer from a reduced cutoff time; the uniform approach ($\times$) does not rely on training data and therefore has such a constant performance curve.

Surprisingly, the reduced cutoff time had nearly no effect on the performance of aspeed (■) on Random (Figure 1b) and ASP-Set (Figure 1h). On the other hand, the selection of the single best solver (+) got worse with an increased cutoff time on the training data of Random. On the Crafted set (Figure 1d), the performance of aspeed was found to benefit from an increased cutoff time, but the improvement was small for a cutoff time longer than 2222 CPU seconds (4/9 of the maximal cutoff time). In contrast, the improvement of the ppfolio-like approach (+) was small on Crafted and Random; and the performance of aspeed, ppfolio-like approach and single best fluctuated on the Application set (Figure 1f). All three approaches benefited from the maximal cutoff time (5000 CPU seconds); however, the benefit was small in comparison to aspeed with the fully reduced cutoff time (195 CPU seconds). We conjecture that in the case of Crafted, the easy instances are not representative for the harder instances in the test set, unlike in the case of Random, where all instances were randomly generated and of similar structure. Consequently, on sets like Random, easier instances can be used for the computation of a schedule, even if the resulting schedule is ultimately applied to (and evaluated on) harder instances.

In an additional experiment, we assessed the performance of aspeed in the context of preparing for a competition. aspeed was trained on instances of the 2009 SAT Competition with the SAT solvers cryptominisat, clasp and tnm, which are the same solvers used by ppfolio, and evaluated on the instances of the 2011 SAT Competition; see Table 7. On the entire instance set, aspeed
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|            | Random | Crafted | Application | Complete |
|------------|--------|---------|-------------|----------|
| 3S         | 16415  | 23029   | 19817       | 18919    |
| aspeed-SP  | 22095  | 22180   | 24579       | 22737    |
| aspeed-4P  | 16380  | 20142   | 17164       | 17517    |

Table 8: PAR10 of 3S and aspeed, trained on the training data of 3S and evaluated on 2011 SAT Competition.

had a PAR10 of 21 196, in contrast to the single best solver with 32 457 (a factor 1.53 higher). Also, aspeed outperformed the single best solver on Random and Crafted, and it performed just as well as the single best solver on Application. This latter observation is due the fact that the performance of cryptominisat dominated on the Application set, and hence, aspeed was unable to obtain improved performance on Application.

5.7 Comparison with 3S

In our final experiment, we compared aspeed with the SAT solver 3S, which uses an approach similar to aspeed, but combines a static solver schedule with algorithm selection based on instance features (see Section 6). Since only the sequential version of the solver 3S is freely available but not the schedule building method, we could not train the models of 3S on new benchmark sets. Therefore, we trained aspeed on the same training runtime measurements used by the authors of 3S for training on the 2011 SAT Competition, namely the 3S-Set. We note that training of 3S, unlike aspeed, additionally requires a set of instance features. Using these versions of aspeed and 3S trained on the same set of instances, we measured the runtime of both solvers (utilizing a single processor SP or multi-processor environment with four parallel threads MP4) on the instances of the 2011 SAT Competition with the same cutoff of 5000 CPU seconds as used in the competition.

Table 8 shows the results based on the PAR10 of the runtime measurements. The results are similar to the comparison between satzilla and aspeed. The single processor version of aspeed, aspeed-SP, outperformed 3S on Crafted in the sequential case. This could indicate that the instance feature set, used by satzilla and 3S, does not sufficiently reflect the runtime behaviour of the individual solvers on these types of instances. Furthermore, aspeed with four cores, aspeed-4P, performed better than 3S on all three instance sets.

6 Related Work

Our work forms part of a long line of research that can be traced back to John Rice’s seminal work on algorithm selection (1976) on one side, and to work by Huberman, Lukos, and Hogg (1997) on parallel algorithm portfolios on the other side.

Most recent work on algorithm selection is focused on mapping problem instances to a given set of algorithms, where the algorithm to be run on a given problem instance $i$ is typically determined based on a set of (cheaply computed) features of $i$. This is the setting considered prominently in (Rice 1976), as well as by the work on SATzilla, which makes use of regression-based models of running time (Xu et al. 2007; Xu et al. 2008); work on the use of decision trees and case-base reasoning for selecting bid evaluation algorithms in combinatorial auctions (Guerri and Milano ; Gebruers et al. 2004); and work on various machine learning techniques for selecting algorithms
for finding maximum probable explanations in Bayes nets in real time (Guo and Hsu 2004). All these approaches are similar to ours in that they exploit complementary strengths of a set of solvers for a given problem; however, unlike these per-instance algorithm selection methods, aspeed selects and schedules solvers to optimize performance on a set of problem instances, and therefore does not require instance features.

It may be noted that the use of pre-solvers in satzilla, i.e., solvers that are run feature-extraction and feature-based solver selection, bears some resemblance to the sequential solver schedules computed by aspeed; however, satzilla considers only up to 2 pre-solvers, which are determined based on expert knowledge (in earlier versions of SATzilla) or by exhaustive search, along with the time they are run for.

cphydra is a portfolio-based procedure for solving constraint programming problems that is based on case-based reasoning for solver selection and a simple complete search procedure for sequential solver scheduling (O’Mahony et al. 2008). Like the previously mentioned approaches, and unlike aspeed, it requires instance features for solver selection, and, according to its authors, is limited to a low number of solvers (in their work, five). Like the simplest variant of aspeed, the solver scheduling in cphydra aims to maximize the number of given problem instances solved within a given time budget.

Early work on parallel algorithm portfolios highlights the potential for performance improvements, but does not provide automated procedures for selecting the solvers to be run in parallel from a larger base set (Huberman et al. 1997; Gomes and Selman 2001). ppfolio, which demonstrated impressive performance at the 2011 SAT Competition, is a simple procedure that runs between 3 and 5 SAT solvers concurrently (and, depending on the number of processors or cores available, potentially in parallel) on a given SAT instance. The component solvers have been chosen manually based on performance on past competition instances, and they are all run for the same amount of time. Unlike ppfolio, our approach automatically selects solvers to minimize the number of timeouts or total running time on given training instances using a powerful ASP solver and can, at least in principle, work with much larger numbers of solvers. Furthermore, unlike ppfolio, aspeed can allot variable amounts of time to each solver to be run as part of a sequential schedule.

Concurrently with our work presented here, Yun and Epstein (2012) developed an approach that builds sequential and parallel solver schedules using case-based reasoning in combination with a greedy construction procedure. Their RSR-WG procedure combines fundamental aspects of cphydra (O’Mahony et al. 2008) and GASS (Streeter et al. 2007); unlike aspeed, it relies on instance features. RSR-WG uses a relatively simple greedy heuristic to optimize the number of problem instances solved within a given time budget by the parallel solver schedule to be constructed; our use of an ASP encoding, on the other hand, offers considerably more flexibility in formulating the optimization problem to be solved, and our use of powerful, general-purpose ASP solvers can at least in principle find better schedules. Our approach also goes beyond RSR-WG in that it permits the optimization of parallel schedules for runtime.

Gagliolo and Schmidhuber consider a different setting, in which a set of algorithms is run in parallel, with dynamically adjusted timeshares (2006). They use a multi-armed bandit solver to allocate timeshares to solvers and present results using two algorithms for SAT and winner determination in combinatorial auctions, respectively. Their technique is interesting, but considerably more complex than aspeed; while the results for the limited scenarios they studied are promising, so far, there is no indication that it would achieve state-of-the-art performance in standardized settings like the SAT competitions.
For AI planning, Helmert et al. implemented the portfolio solver Stone Soup (Helmert et al. 2011; Seipp et al. 2012) which statically schedules planners. In contrast to aspeed, Stone Soup computes time slices using a greedy hill climbing algorithm that optimizes a special planning performance metric, and the solvers are aligned heuristically. The results reported by Seipp et al. (2012) showed that an uniform schedule achieved performance superior to that of Stone Soup. Considering our results about uniform schedules and schedules computed by aspeed, we have reason to believe that the schedules optimized by aspeed could also achieve performance improvements on AI planning problems.

Perhaps most closely related to our approach is the recent work of Kadioglu et al. on algorithm selection and scheduling (Kadioglu et al. 2011), namely 3S. They study pure algorithm selection and various scheduling procedures based on mixed integer programming techniques. Unlike aspeed, their more sophisticated procedures rely on instance features for nearest-neighbour-based solver selection, based on the (unproven) assumption that any given solver shows similar performance on instances with similar features (Kadioglu et al. 2010). (We note that solver performance is known to vary substantially over sets of artificially created, uniform random SAT and CSP instances that are identical in terms of cheaply computable syntactic features, suggesting that this assumption may in fact not hold.) The most recent version of 3S (Malitsky et al. 2012) also supports the computation of parallel schedules but is unfortunately not available publicly or for research purposes. We focussed deliberately on a simpler setting than their best-performing semi-static scheduling approach in that we do not use per-instance algorithm selection, yet still obtain excellent performance. Furthermore, 3S only optimizes the number of timeouts whereas aspeed also optimizes the solver alignment to improve the runtime.

7 Conclusion

In this work, we demonstrated how ASP formulations and a powerful ASP solver (clasp) can be used to compute sequential and parallel solver schedules. In principle, a similar approach could be pursued using CP or ILP as done within 3S (Kadioglu et al. 2011). However, as we have shown in this work, ASP appears to be a good choice, since it allows for a compact and flexible encoding of the specification, for instance, by supporting true multi-objective optimization, and can be applied to effectively solve the problem for many domains.

Compared to earlier model-free and model-based approaches (ppfolio and satzilla, respectively), our new procedure, aspeed, performs very well on ASP, CSP, MaxSAT, QBF and SAT – five widely studied problems for which substantial and sustained effort is being expended in the design and implementation of high-performance solvers. In the case of SAT, there is no single dominant solver, and portfolio-based approaches leverage the complementary strength of different state-of-the-art algorithms. For ASP, a situation exists with respect to different configurations of a single solver, clasp. This latter case is interesting, because we essentially use clasp to optimize itself. While, in principle, the kind of schedules we construct over various configurations of clasp could even be used within aspeed instead of plain clasp, we have not yet investigated the efficacy of this approach.

Our open-source reference implementation of aspeed is available online. We expect aspeed to work particularly well in situations where various different kinds of problem instances have to be solved (e.g., competitions) or where single good (or even dominant) solvers or solver configurations are unknown (e.g., new applications). Our approach leverages the power of multi-core and multi-processor computing environments and, because of its use of easily modifiable
and extensible ASP encodings, can in principle be readily modified to accommodate different constraints on and optimization criteria for the schedules to be constructed. Unlike most other portfolio-based approaches, aspeed does not require instance features and can therefore be applied more easily to new problems.

Because, like various other approaches, aspeed is based on minimization of timeouts, it is currently only applicable in situations where some instances cannot be solved within the time budget under consideration (this setting prominently arises in many solver competitions). In future work, we intend to investigate strategies that automatically reduce the time budget if too few timeouts are observed on training data; we are also interested in the development of better techniques for directly minimizing runtime.

In situations where there is a solver or configuration that dominates all others across the instance set under consideration, portfolio-based approaches are generally not effective (with the exception of performing multiple independent runs of a randomized solver). The degree to which performance advantages can be obtained through the use of portfolio-based approaches, and in particular aspeed, depends on the degree to which there is complementarity between different solvers or configurations, and it would be interesting to investigate this dependence quantitatively, possibly based on recently proposed formal definitions of instance set homogeneity (Schneider and Hoos 2012). Alternatively, if a dominant solver configuration is expected to exist but is unknown, such a configuration could be found using an algorithm configurator, for instance ParamILS (Hutter et al. 2007; Hutter et al. 2009), GGA (Ansótegui et al. 2009), F-Race (López-Ibáñez et al. 2011) or SMAC (Hutter et al. 2011). Furthermore, automatic methods, like hydra (Xu et al. 2010) and isac (Kadioglu et al. 2010), construct automatically complementary portfolios of solver configurations with the help of algorithm configurators which could be also combined with aspeed to further increase its performance.

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