A review on the Lorentz invariant treatment of neutrino spin and flavour oscillations in moving and polarized matter is presented. Within this approach it becomes possible to consider neutrino oscillations in arbitrary electromagnetic fields. It is also shown that neutrino effective potential in matter can be significantly changed by relativistic motion of matter.

In this paper I should like to give a short review on the recent studies of neutrino oscillations in moving and polarized matter, with an electromagnetic field being also imposed, that have been performed during the last few years by our research group at the Moscow State University. For about six years ago we, while studying neutrino oscillations in strong magnetic fields, to our much surprise realized that in literature there were no attempts to consider neutrino spin evolution in any electromagnetic field rather than constant in time and transversal in respect to the neutrino propagation magnetic field $\vec{B}_\perp$. The influence of the longitudinal component of magnetic field was previously neglected because this component in the rest frame of relativistic neutrino, $\vec{B}_\parallel$, can be suppressed in respect to the transversal component which acquire a factor $\gamma = (1 - \beta^2)^{-1/2}$ in the rest frame of the particle: $\vec{B}_\parallel = \gamma \vec{B}_\perp$ ($\beta$ is the neutrino speed).

Further more, in all of the studies of neutrino flavour oscillations in matter performed before 1995, the matter effect was treated only in the nonrelativistic limit that was found to be adequate in applications to the stellar collapse (see, for example, ref. 1) or to the solar environments accounting for weak currents, ref. 2.

The first our attempt to consider neutrino flavour oscillations in matter in the case when matter is moving with relativistic speed was made in 1995, ref. 3. In that our study we tried to apply the Lorentz invariant formalism for description of neutrino flavour oscillations and realized that the value of the matter term in the neutrino effective potential can be significantly changed if matter is moving with relativistic speed. However, we have continued our studies on evaluation
of the Lorentz invariant formalism in neutrino oscillations only in 1999 (see ref. and references therein) with investigation of neutrino oscillations in arbitrary electromagnetic fields. We start from the Bargmann-Michel-Telegdi (BMT) equation for evolution of the spin $S_\mu$ of a neutral particle with nonvanishing magnetic, $\mu$, and electric, $\epsilon$, dipole moments in electromagnetic field, given by its tensor $F_{\mu\nu} = (\vec{E}, \vec{B})$:

$$\frac{d\vec{S}}{d\tau} = 2\mu \left\{ F^{\mu\nu} S_\nu - u^\mu \left( u_\nu F^{\nu\lambda} S_\lambda \right) \right\} + 2\epsilon \left\{ \vec{F}^{\mu\nu} S_\nu - u^\mu \left( u_\nu \vec{F}^{\nu\lambda} S_\lambda \right) \right\},$$

(1)

This form of the BMT equation corresponds to the case of the particle moving with constant speed, $\vec{\beta} = \text{const}$, $u_\mu = (\gamma, \gamma \beta)$. The spin vector satisfies the usual conditions, $S^2 = -1$ and $S^\mu u_\mu = 0$. We generalize the BMT equation for the case of massive neutrino by including the effects of weak interactions with background matter and finally arrive to the following equation for the three dimensional neutrino spin vector $\vec{S}$:

$$\frac{d\vec{S}}{dt} = \left[ \vec{S} \times \left( \frac{2\mu}{\gamma} \vec{B}_0 + \frac{2\epsilon}{\gamma} \vec{E}_0 + \left( V - \frac{\delta m^2 A(\theta)}{2E} \right) \vec{n} \right) \right],$$

(2)

where $\delta m^2$ is the mass squared difference and $E$ is the energy of neutrino. The derivative in the left-hand side of Eq. (2) is taken with respect to time $t$ in the laboratory frame, whereas the values $\vec{B}_0$ and $\vec{E}_0$ are the magnetic and electric fields in the neutrino rest frame

$$\vec{B}_0 = \gamma \left( \vec{B}_\perp + \frac{1}{\gamma} \vec{B}_\parallel + \sqrt{1 - \gamma^{-2}} \left[ \vec{E}_\perp \times \vec{n} \right] \right), \quad \vec{E}_0 = \gamma \left( \vec{E}_\perp + \frac{1}{\gamma} \vec{E}_\parallel - \sqrt{1 - \gamma^{-2}} \left[ \vec{B}_\perp \times \vec{n} \right] \right), \quad \vec{n} = \frac{\vec{\beta}}{\beta},$$

(3)

where $\vec{F}_{\perp,\parallel} (F = B \text{ or } E)$ are fields components in the laboratory frame. The two parameters, $A(\theta)$ being a function of vacuum mixing angle and $V = V(n_{\text{eff}})$ being the difference of neutrino effective potentials in matter, depend on the nature of neutrino conversion processes in question. For specification of $A(\theta)$ and $V(n_{\text{eff}})$ for different types of the neutrino conversions see, for example, in ref.

Using the neutrino spin evolution equation we get the effective Hamiltonian that determines the evolution of the system $\nu = (\nu_R, \nu_L)$ in presence of electromagnetic field with given (in the laboratory frame) components $B_{\parallel,\perp}(t), E_{\parallel,\perp}(t)$:

$$H = (\bar{\sigma} \vec{n}) \left( \frac{\delta m^2 A(\theta)}{4E} - \frac{V}{2} - \frac{\mu B_\parallel + \epsilon E_\parallel}{\gamma} \right) - \mu \bar{\sigma} \left( \vec{B}_\perp + [\vec{E}_\perp \times \vec{n}] \right) - \epsilon \bar{\sigma} \left( \vec{E}_\perp - [\vec{B}_\perp \times \vec{n}] \right) + O\left( \frac{1}{\gamma^2} \right).$$

(4)

This Hamiltonian accounts for the direct interaction of neutrino with electromagnetic fields, for transversal fields it reproduces the result of ref. There could be also indirect influence of electromagnetic field on neutrino due to the matter polarization by the longitudinal magnetic field of the considered electromagnetic field configuration (see ref. and references therein). The former effect is included into the difference of neutrino potentials in matter $V$.

The obtained Hamiltonian enables us to consider neutrino spin procession in an arbitrary configuration of electromagnetic fields, including those that contains strong longitudinal components, and derive the corresponding resonance conditions for neutrino oscillations. As one of the examples, let us consider the new effect of the neutrino spin conversion $\nu_L \leftrightarrow \nu_R$ that could appear when neutrinos propagate in matter under the influence of a field of electromagnetic wave and a constant longitudinal magnetic field superimposed. We suppose that the neutrino speed is constant and denote by the unite vector $\vec{e}_3$ the axis that is parallel with $\vec{n}$ and by $\phi$ the angle between $\vec{e}_3$ and the direction of the wave propagation (for simplicity we shall neglect terms proportional to the neutrino electric dipole moment $\epsilon$). In this case the magnetic field in the neutrino rest frame is given by

$$\vec{B}_0 = \gamma \left[ B_1 (\cos \phi - \beta) \vec{e}_1 + B_2 (1 - \beta \cos \phi) \vec{e}_2 - \frac{1}{\gamma} B_1 \sin \phi \vec{e}_3 \right],$$

(5)
where \( \vec{e}_{1,2,3} \) are the unit orthogonal vectors. For the electromagnetic wave of circular polarization propagating in matter it is easy to get:

\[
B_1 = B \cos \psi, \quad B_2 = B \sin \psi,
\]

(6)

where \( B \) is the amplitude of the magnetic field in the laboratory frame and the phase of the wave at the point where the neutrino is located at given time \( t \) is

\[
\psi = g\omega t \left( 1 - \frac{\beta}{\beta_0} \cos \phi \right),
\]

(7)

where \( \omega \) is the electromagnetic wave frequency. The phase depends on the wave speed \( \beta_0 \) in matter (\( \beta_0 \leq 1 \)). The values \( g = \pm 1 \) correspond to the two types of the circular polarization of the wave.

In the adiabatic approximation the probability of neutrino conversion \( \nu_L \to \nu_R \) can be written in the form (\( x \) is the distance travelled by neutrino),

\[
P_{\nu_L \to \nu_R}(x) = \sin^2 2\theta_{\text{eff}} \sin^2 \frac{\pi x}{L_{\text{eff}}}, \quad \sin^2 2\theta_{\text{eff}} = \frac{E_{\text{eff}}^2}{E_{\text{eff}}^2 + \Delta_{\text{eff}}^2}, \quad L_{\text{eff}} = \frac{2\pi}{\sqrt{E_{\text{eff}}^2 + \Delta_{\text{eff}}^2}},
\]

(8)

where

\[
E_{\text{eff}} = 2\mu B (1 - \beta \cos \phi)
\]

(9)

(terms \( \sim O(\gamma^{-1}) \) are omitted here), and

\[
\Delta_{\text{eff}} = V - \frac{\delta m^2 A(\theta)}{2E} - g\omega \left( 1 - \frac{\beta}{\beta_0} \cos \phi \right) + 2\frac{\mu B}{\gamma}.
\]

(10)

The corresponding resonance condition now is:

\[
V - \frac{\delta m^2 A(\theta)}{2E} - g\omega \left( 1 - \frac{\beta}{\beta_0} \cos \phi \right) + 2\frac{\mu B}{\gamma} = 0.
\]

(11)

Thus, we predict the new type of resonances in neutrino oscillations \( \nu_L \leftrightarrow \nu_R \) that can exist in presence of the combination of electromagnetic wave and constant longitudinal magnetic field. It is easy to see that the longitudinal constant magnetic field can be switched off and similar analysis of neutrino oscillations in the field of circularly polarized electromagnetic wave is straightforward. Neutrino oscillations in the field of linearly polarized electromagnetic wave, the possibility of the parametric amplification of neutrino oscillations in electromagnetic fields and neutrino spin evolution in presence of general external fields were also considered in ref. 9.

Within the developed above approach to neutrino spin evolution we have focused mainly on description of influence of different electromagnetic fields, while modeling the matter we confined ourselves to the most simple case of nonmoving and unpolarized matter. Now we should like to go further, ref. 10, and to generalize the developed Lorentz invariant approach to the neutrino spin oscillations in arbitrary electromagnetic fields for the case of moving and polarized homogeneous background matter. The described below formalism is valid for accounting of matter motion and polarization for arbitrary (also relativistic) speed of matter. It should be noted here that effects of matter polarization in neutrino oscillations were considered previously in several papers (see, for example, refs. 11 and references therein). However, the used in refs. 11, procedure of accounting for the matter polarization effect does not enable one to study the case of matter motion with relativistic speed. Within our approach we can reproduce corresponding results of refs. 11, in the case of matter which is slowly moving or is at rest.

We start again with the BMT spin evolution Eq. (1) of electrodynamics and generalize it for the case when effects of various neutrino interactions (for example, weak interaction for which
$P$ invariance is broken) with moving and polarized matter are also taken into account. The Lorentz invariant generalization of Eq. (1) for our case can be obtained by the substitution of the electromagnetic field tensor $F_{\mu\nu} = (E, B)$ in the following way:

$$F_{\mu\nu} \to F_{\mu\nu} + G_{\mu\nu}. \tag{12}$$

In evaluation of the tensor $G_{\mu\nu}$ we demand that the neutrino evolution equation must be linear over the neutrino spin, electromagnetic field, the matter fermions currents $j_f^\mu$ and polarizations $\gamma_f^\mu$ (matter is composed of different fermions, $f = e, n, p, ...$)

$$j_f^\mu = (n_f, n_f \vec{v}_f), \quad \gamma_f^\mu = \left(n_f (\vec{\zeta}_f \vec{v}_f), n_f \vec{\zeta}_f \sqrt{1 - v_f^2} + \frac{n_f \vec{v}_f (\vec{\zeta}_f \vec{v}_f)}{1 + \sqrt{1 - v_f^2}} \right). \tag{13}$$

Here $n_f, \vec{v}_f,$ and $\vec{\zeta}_f$ ($0 \leq |\vec{\zeta}_f|^2 \leq 1$) denote, respectively, the number densities of the background fermions $f$, the speeds of the reference frames in which the mean momenta of fermions $f$ are zero, and the mean values of the polarization vectors of the background fermions $f$ in the above mentioned reference frames. The mean value of the background fermion $f$ polarization vector, $\vec{\zeta}_f$, is determined by the two-step averaging of the fermion relativistic spin operator over fermion quantum states in a given electromagnetic field and then over fermion statistical distribution density function. Thus, in general case of neutrino interaction with different background fermions $f$ we introduce for description of matter effects antisymmetric tensor

$$G^{\mu\nu} = \epsilon^{\mu\nu\rho\lambda} g^{(1)}_{\rho\lambda} u_\lambda - (g^{(2)}_{\mu\nu} u_\nu - u^\mu g^{(2)}_{\mu\nu}), \tag{14}$$

where

$$g^{(1)}_{\mu\nu} = \sum_f \rho_f^{(1)} j_f^\mu + \rho_f^{(2)} \lambda_f^\mu, \quad g^{(2)}_{\mu\nu} = \sum_f \xi_f^{(1)} j_f^\mu + \xi_f^{(2)} \lambda_f^\mu, \tag{15}$$

(summation is performed over the fermions $f$ of the background). The explicit expressions for the coefficients $\rho_f^{(1),(2)}$ and $\xi_f^{(1),(2)}$ could be found if the particular model of neutrino interaction is chosen. In the usual notations the antisymmetric tensor $G_{\mu\nu}$ can be written in the form,

$$G_{\mu\nu} = (- \vec{P}, \vec{M}), \tag{16}$$

where

$$\vec{M} = \gamma \{(g_0^{(1)} \vec{\beta} - g_0^{(1)}) - [\vec{\beta} \times g_0^{(2)}]\}, \quad \vec{P} = -\gamma \{(g_0^{(2)} \vec{\beta} - g_0^{(2)}) + [\vec{\beta} \times g_0^{(1)}]\}. \tag{17}$$

It worth to note that the substitution (12) implies that the magnetic $B$ and electric $E$ fields are shifted by the vectors $\vec{M}$ and $\vec{P}$: $\vec{B} \to \vec{B} + \vec{M}, \quad \vec{E} \to \vec{E} - \vec{P}$.

We finally arrive to the following equation for the evolution of the three-dimensional neutrino spin vector $\vec{S}$ accounting for the direct neutrino interaction with electromagnetic field $F_{\mu\nu}$ and matter (which is described by the tensor $G_{\mu\nu}$):

$$\frac{d\vec{S}}{dt} = \frac{2\mu}{\gamma} \left[\vec{S} \times (\vec{B}_0 + \vec{M}_0)\right] + \frac{2e}{\gamma} \left[\vec{S} \times (\vec{E}_0 - \vec{P}_0)\right]. \tag{18}$$

The influence of matter on the neutrino spin evolution in Eq. (18) is given by the vectors $\vec{M}_0$ and $\vec{P}_0$ which in the rest frame of neutrino can be expressed in terms of quantities determined in the laboratory frame

$$\vec{M}_0 = \gamma \beta (g_0^{(1)} - \frac{\vec{\beta} g_0^{(1)}}{1 + \gamma^{-1}}) - \vec{g}^{(1)}, \quad \vec{P}_0 = -\gamma \beta (g_0^{(2)} - \frac{\vec{\beta} g_0^{(2)}}{1 + \gamma^{-1}}) + \vec{g}^{(2)}. \tag{19}$$
Let us describe, for example, the electron neutrino propagation in moving and polarized electron gas. If we consider the case of the standard model supplied with $SU(2)$-singlet right-handed neutrino $\nu_R$ then neutrino effective interaction Lagrangian reads

$$L_{\text{eff}} = - f^\mu \left( \bar{\nu} \gamma^- \frac{1 + \gamma^5}{2} \nu + \bar{e} \gamma^- \frac{1 + \gamma^5}{2} e \right), \quad f^\mu = \frac{G_F}{\sqrt{2}} \left( (1 + 4 \sin^2 \theta_W) j_e^\mu - \lambda_e^\mu \right).$$  \hspace{1cm} (20)

For the coefficients $\rho_e^{(1),(2)}$ we get (it is supposed that $\epsilon = 0$, so that $\zeta_e^{(i)} = 0$)

$$\rho_e^{(1)} = \frac{G_F}{2\mu \sqrt{2}} (1 + 4 \sin^2 \theta_W), \quad \rho_e^{(2)} = - \frac{G_F}{2\mu \sqrt{2}}.$$  \hspace{1cm} (21)

Using expressions for the vector $\vec{M}_0$, Eqs. (13), (19), we find,

$$\vec{M}_0 = \beta \gamma \frac{n_0}{\sqrt{1 - v_e^2}} \left\{ \left( \rho^{(1)} + \rho^{(2)} \zeta_e \bar{v}_e \right) (1 - \beta \bar{v}_e) + \rho^{(2)} \sqrt{1 - v_e^2} \left[ \frac{(\zeta_e \bar{v}_e) (\beta \bar{v}_e)}{1 + \sqrt{1 - v_e^2}} \beta \bar{v}_e \right] + O(\gamma^{-1}) \right\}.$$  \hspace{1cm} (22)

It follows that the value of the matter effect in neutrino spin evolution depends on the values and correlations of the three vectors $\vec{v}, \vec{v}_e, \zeta_e$. In particular, the matter effect can be "eaten" by the relativistic motion of matter if matter is moving along the neutrino propagation and $1 - \beta \bar{v}_e \approx 0$.

Now let us discuss, within the Lorentz invariant approach, the neutrino flavour oscillations in moving and polarized matter, ref. [12]. For simplicity, we consider neutrino two-flavour oscillations, e.g. $\nu_e \leftrightarrow \nu_\mu$, in matter composed of only one component, electrons ($f = e$), moving with relativistic total speed. Generalizations for the cases of other types of neutrino conversions and different matter compositions and motions are straightforward.

The matter effect in neutrino oscillations occurs as a result of elastic forward scattering of neutrinos on the background fermions. In our case the difference $\Delta V$ between the potentials $V_e$ and $V_\mu$ for the two-flavour neutrinos is produced by the charged current interaction of the electron neutrino with the background electrons (the neutral current interaction is effective in oscillations between the active and sterile neutrinos). The corresponding part of the neutrino effective Lagrangian can be written now in the following form

$$L_{\text{eff}} = - f^\mu \left( \bar{\nu} \gamma^- \frac{1 + \gamma^5}{2} \nu + \bar{e} \gamma^- \frac{1 + \gamma^5}{2} e \right), \quad f^\mu = \sqrt{2} G_F (j_e^\mu - \lambda_e^\mu).$$  \hspace{1cm} (23)

This additional term in the Lagrangian modifies the Dirac equation for neutrino:

$$(\gamma_0 E - \vec{p} \cdot \vec{\beta} - m) \psi = (\gamma_\mu f^\mu) \psi.$$  \hspace{1cm} (24)

In the limit of weak potential $|\vec{f}| \ll p_0 = \sqrt{p^2 + m^2}$ we get for the effective energy of the electron neutrino in the moving and polarized matter

$$E = \sqrt{p^2 + m^2} + U \left\{ (1 - \zeta_e \bar{v}_e) (1 - \beta \bar{v}_e) + \sqrt{1 - v_e^2} \left[ \zeta_e \bar{v}_e - \frac{(\beta \bar{v}_e) (\zeta_e \bar{v}_e)}{1 + \sqrt{1 - v_e^2}} \right] \right\} + O(\gamma^{-1}),$$  \hspace{1cm} (25)

where in the considered case of the two-flavour neutrino oscillations $\nu_e \leftrightarrow \nu_\mu$ and one-component matter $U = \sqrt{2} G_F n_0 / \sqrt{1 - v_e^2}$, $n_0$ is the invariant matter (electron) density.

Thus, in the adiabatic limit the probability of neutrino conversion $\nu_e \rightarrow \nu_\mu$ can be written in the form

$$P_{\nu_e \rightarrow \nu_\mu} (x) = \sin^2 \theta_{\text{eff}} \sin^2 \frac{\pi x}{L_{\text{eff}}},$$  \hspace{1cm} (26)
where the effective mixing angle, $\theta_{\text{eff}}$, and oscillation length, $L_{\text{eff}}$, are given by

$$
\sin^2 2\theta_{\text{eff}} = \frac{\Delta^2 \sin^2 2\theta}{(\Delta \cos 2\theta - A)^2 + \Delta^2 \sin^2 2\theta}, \quad L_{\text{eff}} = \frac{2\pi}{\sqrt{(\Delta \cos 2\theta - A)^2 + \Delta^2 \sin^2 2\theta}}.
$$

(27)

Here $\Delta = \frac{\delta m^2_{\nu}}{2|\vec{p}|}$, $\vec{p}$ is the neutrino momentum, $\theta$ is the vacuum mixing angle and

$$
A = \sqrt{2}G_F\frac{n_0}{\sqrt{1 - v_e^2}}\left\{(1 - \beta \vec{v}_e)(1 - \vec{\zeta}_e \vec{v}_e) + \sqrt{1 - v_e^2}\left[\vec{\zeta}_e \vec{\beta} - \frac{(\vec{\beta} \vec{v}_e)(\vec{\zeta}_e \vec{v}_e)}{1 + \sqrt{1 - v_e^2}}\right]\right\}.
$$

(28)

One can see that the neutrino oscillation probability, $P_{\nu_e \rightarrow \nu_\mu}(x)$, the mixing angle, $\theta_{\text{eff}}$, and the oscillation length, $L_{\text{eff}}$, exhibit dependence on the total speed of electrons $\vec{v}_e$, correlation between $\vec{\beta}$, $\vec{v}_e$ and polarization of matter $\vec{\zeta}_e$. The resonance condition

$$
\left(\frac{\delta m^2_{\nu}}{2|\vec{p}|}\right) \cos 2\theta = A,
$$

(29)

at which the probability has unit amplitude no matter how small the vacuum mixing angle $\theta$ is, also depends on the motion and polarization of matter and neutrino speed. It also follows that the relativistic motion of matter could provide appearance of the resonance in the neutrino oscillations in certain cases when for the given neutrino characteristics, $\delta m^2_{\nu}$, $|\vec{p}|$ and $\theta$, and the invariant matter density at rest, $n_0$, the resonance is impossible. Analysis of the neutrino effective potential in moving and polarized matter for different particular cases (for different compositions, speeds and polarizations of matter) can be found in ref. [4].

We have developed the Lorentz invariant formalism for neutrino spin, flavour and spin-flavour oscillations in background matter and electromagnetic fields. Within this approach it becomes possible to describe neutrino spin oscillations in arbitrary electromagnetic fields. In particular, we predict new types of neutrino spin oscillations (and resonances) in the field of electromagnetic wave (e.m.w.) and the e.m.w. with the longitudinal magnetic field superimposed. We also generalized the Lorentz invariant approach to neutrino spin and flavour oscillations for the case of relativistic motion of background matter with effects of matter polarization are taken into account. It is shown that the matter terms in neutrino effective potentials depend on the values and correlations of the three vectors, the neutrino and matter speeds and matter polarization. In the case of relativistic motion of matter along (against) to neutrino propagation, matter effects in neutrino oscillations are suppressed (increased). These effects can lead to interesting consequences for environments with neutrino propagating through relativistic jets of matter.

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