One-Level Prosodic Morphology

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Abstract

Recent developments in theoretical linguistics have led to a widespread acceptance of constraint-based analyses of prosodic morphology phenomena such as truncation, infixation, floating morphemes and reduplication. Of these, reduplication is particularly challenging for state-of-the-art computational morphology, since it involves copying of some part of a phonological string. In this paper I argue for certain extensions to the one-level model of phonology and morphology (Bird & Ellison 1994) to cover the computational aspects of prosodic morphology using finite-state methods. In a nutshell, enriched lexical representations provide additional automaton arcs to repeat or skip sounds and also to allow insertion of additional material. A kind of resource consciousness is introduced to control this additional freedom, distinguishing between producer and consumer arcs. The non-finite-state copying aspect of reduplication is mapped to automata intersection, itself a non-finite-state operation. Bounded local optimization prunes certain automaton arcs that fail to contribute to linguistic optimisation criteria such as leftmostness of an infix within the word. The paper then presents implemented case studies of Ulwa construct state infixation, German hypocoristic truncation and Tagalog overapplying reduplication that illustrate the expressive power of this approach, before its merits and limitations are discussed and possible extensions are sketched. I conclude that the one-level approach to prosodic morphology presents an attractive way of extending finite-state techniques to difficult phenomena that hitherto resisted elegant computational analyses.
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1 Introduction

Prosodic morphology is the study of natural language phenomena in which the shape of words is to a major extent determined by phonological factors such as obedience to wellformed syllable or foot structure, adjacency to stress peaks or sonority extrema etc. Introductory examples from truncation, infixation, root-and-pattern morphology and reduplication are given in (1).

(1) a. German: hypocoristic i-truncation
   Petra > Pet-i, Andrea > And-i, Gorbatschow > Gorb-i

b. Ulwa: construct state suffixation/infixation
   bás-ka ‘his hair’, ás-ka-na ‘his clothes’, karás-ka-mak ‘his knee’

c. Modern Hebrew: vowel/∅ alternation in nonconcatenative verbal morphology
   gamar ‘finished’ (3sg.m), gˆm- ra (3sg.f),
   yi-gmor ‘will finish’ (3sg.m), yi-gmer-u (3pl)

d. Mangarayi: reduplicated plural etc.
   gabuji > gababuji ‘old people’, jimgan > jimgingan ‘knowledgeable people’,
   muygji > muygjuygji ‘having a dog’

Prosodic conditions govern the cutoff point in German hypocoristic truncation, serve to determine the placement of the floating -ka- suffix/infix within Ulwa possessive noun constructions, control the deletion of vowels in Modern Hebrew nonconcatenative verb morphology and determine position and length of the reduplicant copy in Mangarayi plurals. We will see later what some of these conditions are and how to devise computational analyses for such phenomena.

In his extensive overview of the state of the art in computational morphology, Sproat (1992) provides ample indication that there is still work to do with regard to these phenomena. Here is a relevant sample of Sproat’s comments:

Subtractive morphology – presumably since it is relatively infrequent – has attracted no attention. [p.170]
   ... computational models have been only partly successful at analyzing infixes. [p.50]
From a computational point of view, one point cannot be overstressed: the copying required in reduplication places reduplication in a class apart from all other morphology. [p.60]
   ... a morphological analyzer needs to use information about prosodic structure. [p.170]
   ... there is a tendency in some quarters of the computational morphology world to trivialize the problem, suggesting that the problems of morphology have essentially been solved simply because there are now working systems that are capable of doing a great deal of morphological analysis ... On should not be misled by such claims ... there are still outstanding problems and areas which have not received much serious attention ... [p.123]

There is still truth in Sproat’s words seven years after they were written.

The primary goal of this paper is therefore to start answering the challenge posed by Sproat’s comments and show how central linguistic insights into the way prosodic morphology works translate into an implemented finite-state model. The model is named ONE-LEVEL PROSODIC MORPHOLOGY (OLPM), because it was developed as an adaptation of One-Level Phonology (Bird & Ellison 1994, OLP) to prosodic morphology. Despite some initial attempts, it is probably fair to say that prosodic morphology as a branch of theoretical linguistics is still rather underformalized. While Bird & Ellison (1994, §5.4) does contain a very

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brief autosegmental analysis of just the Arabic verbal stem *kattab* – itself a legitimate piece of prosodic morphology –, we will see that the extensions introduced in this paper can be justified on the basis of a broader view of what prosodic morphology encompasses. The model shares the essential assumption of monostratality with One-Level Phonology, maintaining the restrictive postulate that there should only be one level of linguistic description. It thus inherits two key advantages of the former model, namely easy integrability into monostratal frameworks of grammar such as HPSG (Pollard & Sag 1994) and simplified machine learning of surface-only generalizations (Ellison 1992). By furthermore retaining the finite-state methodology of its predecessor it keeps what is computationally attractive about the former model’s properties. Combining these essential traits, it is clear that OLPM – like OLP – will be based on finite-state automata (FSA) with their characteristic combinatorial operation of regular set intersection, also known as automaton product. Thus, it contrasts with models employing two, three or more levels (Koskenniemi 1983, Touretzky & Wheeler 1990, Chomsky & Halle 1968), which are usually implemented with finite-state transducers (FSTs) and characteristically employ a composition operation to combine individual transducers into a single overall mapping.

The present work differs, however, in not following the specific representational assumptions of nonlinear autosegmental phonology (Goldsmith 1990) embodied in OLP, using a flat prosodic-segmental representation instead. This difference is not crucial to the task at hand. It mainly stems from our desire to avoid an additional layer of complexity that promises little return value for the immediate goal stated above. Needless to say, however, a representational variant of OLPM using autosegmental diagrams would not seem to be unthinkable.

There is another difference. Existing research in nonconcatenative finite-state morphology has primarily been concerned with templatic aspects of Semitic languages (e.g. Kataja & Koskenniemi 1988 for Akkadian, Beesley, Buckwalter & Newton 1989, Beesley 1996 and Kiraz 1994, Kiraz 1996 for Arabic and Syriac). Alternative (feature-)logical treatments share this phenomenological bias (Bird & Blackburn 1991 on Arabic, Klein 1993 on Sierra Miwok, Walther 1997 and Walther 1998 on Tigrinya and Modern Hebrew). In contrast, our focus is on the difficult rest of prosodic morphology, where infixation, circumfixation, truncation and in particular reduplication have not received elegant computational analyses so far.

The paper is organized as follows. Section 2 provides some background to the emergence of constraint-based models of prosodic morphology, while 3 lays out the range of data to be accounted for. As the presentation unfolds, a number of desiderata for formalization and implementation are formulated. The central proposals of the paper are contained in the following section 4, where I show which representational assumptions must be made and which new devices need to be incorporated into a comprehensive one-level model of prosodic morphology. In section 5 the proposals are evaluated in practice by developing detailed implementations of Ulwa infixation, German truncation and Tagalog reduplication phenomena. Section 6 concludes with a discussion of these proposals, evaluating their merits both on internal grounds and in comparison to other works.

2 Background

Since the beginning of the seventies it has been recognized that rule-based models of prosodic morphology lack explanatory adequacy, a fact that has come to be known as the ‘rule conspiracies’ problem (Kisseberth 1970). Kisseberth used the vowel/zero alternation patterns from inflected verb forms in Tonkawa to make his point (2). Tonkawa is an extinct Coahuiltecan language with CV(C) syllable structure.

(2) **TONKAWA VERB FORMS**

| ‘to cut’ | ‘to lick’ |
| --- | --- |
| #picn-o?# | #netle-o# |
| #we-pcen-o# | #we-ntal-o# |
| #picna-n-o# | #netle-n-o# |
| p(i)c(e)n(a) | n(e)l(f)(a)(e) |

3sg.obj.stem-3sg.subj. 3pl.obj.-stem-3sg.subj. 3sg.obj.stem-prog. stems
While trying to incorporate more and more affixation patterns, Kisseberth observed that the usual enlarge-corpus/modify-analysis cycle resulted in increasingly baroque levels of complexity for the vowel deletion rules. But more to the point, these rules ‘conspired’ to maintain a very simple, yet global invariant – sequences of three consecutive consonants are banned on the surface, symbolically *CCC (the word boundary # acts as a consonant). The reader may easily verify that no further deletion of vowels is possible in (2) without violating the invariant. Under the rule-based analysis, however, this global condition is nowhere expressed directly. According to Kisseberth the failure can be traced back to a defect of the derivational paradigm itself: By design each rule only sees the input given to it by a prior rule application.

Later developments have decomposed segment-level constraints such as +stem by referring to the prosodic concept of the syllable instead – complex syllable onsets and codas are disallowed in Tonkawa core syllables. At least since Kahn (1976) and Selkirk (1982) this idea of an independent level of syllable structure superseded the SPE (Chomsky & Halle 1968) conception of purely segmental strings in the generative literature. While the trend towards representationalism in phonology, marked by the advent of such frameworks as Autosegmental and Metrical Phonology (see Goldsmith 1990 for an overview), reduced the reliance on rules and further strengthened the role of prosodic structure in actual analyses, the fundamental defect that Kisseberth and others had recognized in derivational theories still awaited a principled solution.

That solution emerged at the beginning of the nineties, as constraint-based models of phonology were proposed to directly capture the missing ‘output orientation’ that plagued its derivational predecessors. Bird (1990) and Scobbie (1991) were among the first to use monotonic formal description languages to express surface-true *non-violable constraints, defining what has now become known collectively as Declarative Phonology (DP; Scobbie, Coleman & Bird 1996). In DP, both lexical items and more abstract generalizations are constraints, with constraint conjunction being the characteristic device for formalizing constraint interaction. Shortly thereafter Prince & Smolensky (1993) argued for ranked *violable constraints instead. Their proposal was named Optimality Theory (OT) and has since become a much-recognized new paradigm in theoretical phonology and beyond.

In OT constraints seek to capture conflicting universal tendencies while strict ranking imposes an extrinsic ordering relation on the set of constraints, expressing which one takes precedence for purposes of conflict resolution. According to the OT ideal, languages differ only in how they rank the common pool of constraints. Strictness of ranking means that, in contrast to arbitrarily weighted grammars, no amount of positive wellformedness of an input with respect to lower-ranked constraints can compensate for illformedness due to a higher-ranked constraint. Finally, although constraints may be gradiently violated by the set of structurally enriched candidates that is generated from the input, only candidates with the minimal number of violations are designated as grammatical. Note that, because of this powerful mechanism of global optimization, the OT analyst is free to propose constraints that are *never surface-true (an example would be the excessive-structure-minimizer *STRUC ‘Avoid structure’, Prince & Smolensky 1993, ch.3, fn.13; see Walther 1996, 13 for a formalization).

While DP paid considerable attention to proper formalization of phonology, the empirical domain of prosodic morphology so far has received much less attention than in OT, where the co-appearance of McCarthy & Prince (1993) with Prince & Smolensky (1993) marked the beginning of a continuous involvement with the subject. In particular, certain problems in the prosodic morphology of reduplication motivated an extension to classical OT known as Correspondence Theory (McCarthy & Prince 1995). Here constraints are allowed to simultaneously refer to both levels of two-level pairings for assessing gradient wellformedness, in what appears to be rather analogous to two-level morphology (Koskenniemi 1983). However, the range of correspondence-theoretic mappings – mediated by some abstract indexation scheme – goes beyond Koskenniemi’s original framework in that it includes intra-level instances such as base-redundant correspondence with the same word level in addition to classical cross-level mappings like so-called input-output (i.e lexical-surface) correspondence.

Despite this growing body of theoretical work the average level of formalization in concrete OT analyses has been rather low,3 which is a genuine problem in the light of Chomsky (1965, 4)’s definition of

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2 Witness e.g. the rewrite rule \[ \frac{V}{+\text{stem}} \rightarrow / \left\{ \begin{array}{c} V \quad C \\ C+ \quad CVC \end{array} \right\} C \frac{V}{+\text{stem}} \] from Kisseberth (1970).

3 Notable exceptions include Albro (1997) on Turkish vowel harmony, Eisner (1997b) on stress systems, Ellison (1994b) on Arabic
a generative grammar as one that must be “perfectly explicit” and “not rely on the intelligence of the understanding reader”.  

Note that this is not due to a lack of proposals for formalizing the abstract paradigm of (classical) OT itself, where Ellison (1994b), Walther (1996), Eissner (1997a), Frank & Satta (1998), and Karttunen (1998) have all made contributions within the framework of finite-state systems. Karttunen’s work is especially worth mentioning, because he was able to show that, under regularity assumptions for both the constraints and the input set, optimal outputs can be computed using only established finite-state operations such as transducer composition and automaton complement, without any overt optimization component. The impact of his results is that it places SPE-style rule cascades and OT-style constraint rankings into rather close proximity. Both represent ways to define finite-state transducers, albeit with very different high-level specification languages. Again the assumption of an underlying finite-state architecture was the key factor in establishing Karttunen’s findings.

Given the state of the field outlined above, it seems particularly attractive now to combine these separate strands of research. More precisely, the desire is to produce a model that can (i) capture theoretical insights into the analytical requirements of prosodic morphology without (ii) unduly compromising in the area of proper formalization while (iii) still ensuring effective computability with the help of finite-state methods. To this end I chose to extend One-Level Phonology, itself a finite-state incarnation of DP, than fleshing out one of the proposals for OT.

The principal reason for this choice has to do with the fact that there are much better prospects for automatic constraint acquisition than for its two-level competitors. Corpora usually contain the surface phonological form of words only and do not come equipped with pairs of surface and abstract underlying representations (SRs and URS). Theoretical linguistics offers no help either, as e.g. Kenstowicz (1994, §3.4) argues at length that no a priori restriction on possible URS suffices for all scenarios. The OT notion of ‘lexicon optimization’ (Prince & Smolensky 1993), while meant to address the same problem of deriving suitable URS, is still too vague to merit closer attention. This lack of either a principled or a natural, non-handcrafted source for two-level pairings means that there may be an arbitrarily large gap to bridge when trying to infer a finite-state mapping SR ↔ UR from surface-only data. It is thus no accident that e.g. the results of Ellison (1992) on learning a number of phonological properties in a typologically balanced sample of 30 languages were obtained by using one-level FSAs for the representation of inferred generalizations. Other results from the literature on machine learning of natural language seem to confirm this key advantage of monostratality (e.g. Belz 1998). For OT, on the other hand, no substantial result is known that addresses the hard problem of constraint acquisition. The prospects of remedying this situation without giving up elementary OT premises are not particularly good. As noted above, constraints are formally unrestricted and not required to be surface true for at least some pieces of data. Existing results on OT-based learning only deal with the much simpler problem of inferring the ranking of constraints given pairs of structurally annotated outputs and inputs (Tesar & Smolensky 1993, Boersma 1998, Boersma & Hayes 1999). This is probably also due to the fact that orthodox OT itself expresses disininterest in the question by assuming that all constraints are already given as part of Universal Grammar.

A second reason for extending OLP is that the lack of arbitrary mapping between levels plus the lack of global optimization forces a healthy reexamination of existing analytical devices. Maintaining the restrictive set of assumptions embodied in OLP often leads to the discovery of new surface-true generalizations. Starting with the one-level approach is more illuminating for investigating the precise nature of the trade-off between mono- or polystratal analyses. Finally, this point of departure promises better answers to the question of which set of theory extensions is absolutely necessary in order to cover the enlarged range of empirical phenomena under study. In what follows we will see that this approach of ‘starting small’ indeed yields some of these expected payoffs.

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4 Cf. also the verdict of Pierrehumbert & Nair (1996, 537), who write: “Any attempt to argue for a particular method of combining constraints without simultaneously formalizing the constraints is technically incoherent.”

5 But see Ellison (to appear) for strong arguments against the universalist interpretation of OT and Hayes (1999) for initial attempts at phonetically grounded constraint induction.

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3 The Problem

In this section, I want to give a brief survey of some of the challenges of prosodic morphology. The focus will be on what the relevant linguistic data suggest about minimal requirements for formalization and implementation. The topics to be presented in turn will be reduplication, discontiguity and partial realization of morphemes as well as cases of ‘floating’, i.e. positionally underspecified morphemes together with their directional behaviour.

3.1 Reduplication

Let us begin with some terminology. The original part of a word from which reduplication copies will be called the base, while the copy is also referred to as the reduplicant. The next bits of terminology arise as we further classify reduplicative constructions below. The impetus behind this classification is that it rules out some easy ways of avoiding the full complexity of the reduplication problem.

One such classificatory subdivision is between total reduplications and partial ones. The former are defined to be isomorphic to the formal language \( w^m w \), known to be context-sensitive, while the latter exhibit imperfect copying of one sort or another. A frequent case of imperfection has a truncated portion of the base as the reduplicant. Furthermore, there are unmodified and modified reduplications, where in the latter case reduplicant and base differ in the applicability of phonological alternations. In contrast to exfixing reduplications, where the reduplicant is a prefix or suffix, in the infixing variant the reduplicant interrupts the immediate adjacency relationships of the base. Also, there are discontiguous reduplicants in addition to the more usual contiguous ones. Figuratively speaking, discontiguity means that some segments of the base are skipped over in constructing the reduplicant. Whereas some of the most well-known reduplication instances like Indonesian plural are unbounded in the sense that the reduplicant length is a linear function of the length of the base, in bounded types of reduplication a finite, and often rather small, upper bound can be placed on the length of the reduplicant. Finally, there are purely reduplicative constructions versus their fixed melody-enriched counterparts. The former have reduplicants which are entirely constructed from copied base material (possible modified in the above sense), whereas the latter also contain base-independent segmental material as part of the construction.

Table (3) shows how constructions from four languages instantiate the classificational scheme outlined above, illustrating each opposition with at least one construction. Actual examples are supplied in (4) – (7). Reduplicants are marked with bold face and subscripts mark base-reduplicant correspondences where necessary.

| Language          | total (+) | partial (-) | unmodified (+) | modified (-) | contiguous (+) | discontiguous (-) | unbounded (+) | bounded (-) | purely reduplicative (+) | fixed melody parts (-) |
|-------------------|-----------|-------------|----------------|--------------|----------------|--------------------|---------------|-------------|--------------------------|-----------------------|
| Madurese plural   | +         | -           | +              | -            | +              | -                  | +             | -           | -                        | -                     |
| Mokilese progressive | -         | +           | +              | -            | -              | +                  | -             | +           | -                        | +                     |
| Nisgha, prefixing | -         | -           | -              | -            | +              | -                  | -             | -           | -                        | -                     |
| Koasati, infixing | -         | -           | (-)            | -            | +              | -                  | -             | -           | -                        | -                     |

The first example from Madurese (Malayo-Polynesian) shows a case of unbounded total reduplication (4): a rather familiar type that needs no further comment here.
Mokilese (Micronesian) illustrates the next case (5), namely prefixed reduplicants that consists of a partial copy of the base. Since the number of segments varies as a function of the base, simple templatic generalizations seem not to be available here.

(5) **Mokilese progressive** (Blevins 1996)

| Base    | Redupl. | Meaning              |
|---------|---------|----------------------|
| p-x dok | pdp-x dok | ‘plant/ing’          |
| niki d  | niki ni kid | ‘save/ing’           |
| wia [wi j a] | wii wia | ‘do/ing’              |
| soro k  | so so ro k | ‘tear/ing’            |
| on o p  | on on o p | ‘prepare/ing’         |
| an dip  | an an dip | ‘spit/ing’            |
| ur u ur | ur ur ur ur | ‘laugh/ing’          |

Nisgha (Salish), shown in (6), differs from Mokilese in that there may be phonological modifications in the reduplicant that do not affect their correspondence partners in the base.

(6) **Nisgha CVC prefixing reduplication** (Shaw 1987)

| Base                | Redupl. | Meaning   |
|---------------------|---------|-----------|
| mákʷ⁷⁷ₕ-s⁻kʷ⁷⁷ₕ      | m₁ is₂⁻m₁ ákʷ⁻s₂-kʷ⁷⁷ₕ | ‘be white’ |
| lílkʷ⁷⁷ₕ             | l₁ ux₂⁻l₁ ilkʷ⁷⁷ₕ | ‘to lace (shoes)’ |
| qo?ós               | q₁ as₂⁻q₁ ō?ós₂ | ‘to be cooked’ |

In (6).b we can see that spirantization has turned the velar stop /kʷ⁷⁷ₕ/ into a velar fricative /xʷ⁷⁷ₕ/. Furthermore, the vowel quality of the fixed-size CVC reduplicant is not copied from the base, but constitutes a fixed melody part instead. As it is only the first and last segment of the base that is copied, the reduplicant is discontiguous as well.

Finally, Koasati (Muskogean) has a infixing reduplicative construction depicted in (7), where the base-initial segment is copied to the interior of the base and followed by a fixed-melody element /o(ː)/.

(7) **Koasati infixing aspectual reduplication** (Kimball 1988)

| Base                | Punctual | Meaning      |
|---------------------|----------|--------------|
| taháspín             | t₁ahas⁻t₁ ō⁻pin | ‘to be light in weight’ |
| lapátkín             | l₁apat⁻l₁ ō⁻kin | ‘to be narrow’ |
| aklátlín             | a₁k⁻h₁ ō⁻látlin | ‘to be loose’ |
| okcákkkon             | o₁k⁻h₁ ō⁻cákkkon | ‘to be green or blue’ |

The facts are further complicated by the need to distinguish between consonantal left edges and vocalic ones, where in the latter case apparently /h/ – the voiceless equivalent of a vowel – serves as the modified copy.

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⁶Actually, it may be said to be semi-fixed, since apparently reduplicant vowel quality is determined by the flanking consonants. The claim to a fixed-melody construction remains valid, however, because the reduplicant vowel is not identical to the corresponding base vowel.
Two further remarks on properties of reduplication seem appropriate at this point. First, there are cases like reduplication in Chumash (8) which show that copying must be phonological – it is not in general sufficient to just repeat a morpheme!

(8) Chumash (Hokan) (Applegate 1976)

| s-RED-i kuk | sik-sikuk | 3sg-cont.-chop, hack |
|------------|-----------|----------------------|
| s-iš-RED-expeč | s-išex-šexpeč | 3pl-dual-cont.-sing |

Note that the copy includes not only some initial portion of the verb stem but also the last consonant of the immediately preceding affix, irrespective of its precise morphological function.

Second, there are reduplicants whose shape cannot be statically determined as a function of the base, but which crucially require knowledge of the reduplicant’s eventual surface position. A case in point is again represented by the Koasati construction from (7), where the reduplicant has a long vowel when forced to occupy the stressed (penultimate syllable) position in the word, but exhibits a short-vowelled allomorph when landing elsewhere. In fact, because of stress shifts in comparison with base-only words (7).a,b, base stress is a poor predictor of reduplicant position. The lesson of Koasati is that independent lexical precomputation of the shape of morphemes entering into a reduplicative construction cannot account for all cases.

Let us now step back from the individual cases and ask some obvious questions: What do these examples really tell us? How can the classification that captures their essential dimensions of variation help derive necessary and sufficient properties of a linguistically adequate one-level account of reduplication?

First of all, reduplications of the partial, modified, infixing, discontiguous and fixed melody type show that the model depicted in figure (9).a below is insufficient.

The modified type of reduplication shows that sometimes it is not enough to copy object-level segments from within a word’s surface realization, but that a more abstract notion of identity needs to be captured. While it makes no sense to place the copy device before the lexical transducer, in a multi-level approach one might envision to place the copy device in the sandwiched position of a 2-transducer cascade (9).b. One could then copy at the level where object identity still holds and carry out the necessary modifications in the post-copy part. Unfortunately the aforementioned problems of hybrid models carry over to this setting as well. With only one level, OLPM will of course have to employ rather different means to model modified reduplications.

Exfixing types of reduplication allow one to leave base linearizations untouched; no computational means for infixation need to be provided. As a further consequence, such constructions in principle can
support extensive arc sharing through the coexistence of simple and reduplicated realizations in the same finite-state network. In contrast, the infixing type with its disruption of base linearizations a priori permits no such memory efficiency and needs formal means to support infixation in the first place.

Discontiguity in general poses the problem of how to model the absence of some stretch of segmental material in specific reduplicative constructions when the same material is required to be present in all other realizations of bases. Moreover, the greater the length of the skipped-over substring of the base in discontiguous reduplicants, the more acute is the problem of handling even bounded-length variants (6) of such long-distance dependencies with finite-state automata (Sproat 1992, 91; Beesley 1998).

The bounded types of reduplications are special in that, given the assumptions of full-form lexica and sufficient storage space, they could, in principle, be precompiled into a finite-state network. In practice, however, this can yield prohibitively large networks for realistic fragments of natural language morphologies (Sproat 1992, 161). Also, it has been remarked that finite-state storage is very inefficient when it comes to simulating even the fixed amount of global memory needed to remember the copied portion of a base (Kornai 1996). In contrast, the unbounded type cannot be modelled by precompilation at all. No artificially imposed upper bound will do justice to the facts in a total-reduplication language like Bambara (Northwestern Mande) where facts such as wulu-o-wulu ‘whichever dog’, wulunyinina-o-wulunyinina ‘whichever dog searcher’, wulunyininafilele-o-wulunyininafilele ‘whoever watches dog searchers’, etc. can be arbitrarily extended, as Culy (1985)’s careful investigation shows. Positing separate models for the two types would seem to be highly problematic, since bounded and unbounded reduplications may occur in the same language. For example, Madurese, whose total reduplication appeared in (4), also has bounded final-syllable reduplication: tre-estre ‘wives’, bu-sembu ‘something increased’, wa-buwa ‘fruits’.

3.2 Discontiguity

It is a well-known fact that morphology is not always concatenative. Rather, various patterns of morpheme “overlap” can be observed, as shown in (10).

(10) PATTERNS OF MORPHEME OVERLAP

| Morphemes | [overlap] | [inclusion] | Language |
|-----------|-----------|-------------|----------|
| teil | bar | - | - | German |
| s a n a | hi t | + | - | Mod. Hebrew |
| b a s a | + | + | Tagalog |

In these diagrams, we depict the surface extent of a morpheme as the temporal interval between its first and last segment.7 Hence, overlap between two morpheme intervals implies that at least one of the ordering relationships between intramorphemic segments must be weakened from immediate to transitive precedence: the hallmark of a discontiguous morpheme.

While German teil-bar ‘divisible’ is a typical instance of purely concatenative arrangement of morphemes, the next example from Modern Hebrew illustrates a first case of deviation from the concatenative ideal, namely morpheme-edge metathesis. Here the conjugation class prefix /hit-/ (hitpael binyan), which ends in coronal /t/, partially overlaps verbal stems whose first segment is a coronal obstruent. Note that hit-

7This assumes an understanding of morphemes (and words) as totally ordered sets of segments. Suprasegmental morphemes like ‘nasalize word till first obstruent’ (cf. the Arawakan language Terena, Bendor-Samuel 1960) need a generalization that refers to observable phonological effects instead of segments. As far as I know, cases of improper inclusion or total overlap always correspond to such suprasegmentals, too, and can be exemplified by phenomena like nasalization, pharyngealization, tone marking etc.
and coronal-initial verbal roots are only discontiguous if cooccurring (*hit-fana, but me-fane ‘change’ (pres.) and hit-gala ‘appear’, *higtala, etc.).

In Tagalog infixation, the deviation is more severe, as the actor-trigger morpheme -um- is totally overlapped by the verb stem in b-um-asa ‘read’. In the dual case of circumfixation, also attested in Tagalog (e.g. ka-an, as in ka-bukir-an ‘fields’), affix and stem have simply changed roles in what amounts to the same pattern of total overlap. It is worth pointing out that Tagalog still has its share of purely concatenative morphology, even involving the same stems (e.g. makī-basa ‘read to somebody (by chance)’).

Even if a language allows discontiguity in regular morphology, it may nevertheless exhibit exceptional morphemes that forbid intrusion into their own material. For example, in Ulwa construct-state morphology (1).b there are noun stems like kililih ‘cicada’ where infixation *kili-ka-lih would be predicted, but only suffixation kililih-ka is possible. It must therefore be possible to control infixability in the Ulwa lexicon.

With so much focus on discontiguity, we should tackle a potential objection. Is productive discontiguous morphology perhaps limited to so-called ‘exotic’ languages? English, thought otherwise to be purely concatenative, allows us to give a negative answer to that question. The language has a productive process of expletive insertion which readily creates words like Kalama-goddam-zoo, in-fuckin-stantiate, kanga-bloody-ro, in-fuckin-possible, guaran-friggin-tee (Katamba 1993, 45), thereby breaking up stems that appear elsewhere as contiguous.

The challenge of discontiguity then is to come up with a generic formal solution that is ideally able to represent morphemes in a uniform manner, regardless of whether discontiguity is prominent, rare or nonexisting in a language. It should also capture the fact that immediate precedence of intramorphemic segments appears to be the default, giving way to transitive precedence only if immediate adjacency leads to ungrammaticality. Finally, the solution should allow for cases where discontiguity is either grammatically or lexically forbidden.

### 3.3 Partiality

Sometimes morphemes do not realize all their segmental material. We have already seen that in Tonkawa and Modern Hebrew, certain stem vowels are omitted by way of regular processes, depending on the affixation pattern. However, in these and many similar cases the number of potentially zero-alternating segments is strictly predictable. In Tonkawa, at most every stem vowel (and /h/, phonetically a voiceless vowel) can lose one mora (V, /h/ → 0, VV → V), whereas in Modern Hebrew there is a maximum of two alternating stem vowels per verb form. Furthermore, since in these languages omittable vowels are intercalated with stable consonants, the length of contiguous stretches of deletable material is also bounded by a small constant, often 1. Besides this type of bounded partial realization, however, there is a type of potentially unbounded partial realization, a case of which we have seen in productive German i-truncation (1).a. Here, the length of the deleted string suffix of a base noun is a linear function of its original length and therefore in principle unbounded (2, 3 and 5 segments in [pet:{:e}], [ʔand{ːe}], [gau{ːf}]). While it is true that Standard High German does not use truncation for ordinary grammatical processes such as pluralization, other languages like Tohona O’odham, Alabama, Choctaw and Koasati employ truncation for exactly this purpose (Anderson 1992, 65f).

Summing up, some natural desiderata for a generic formalization of partiality would be to allow for morpheme representations where a priori no segmental position must be realized, to provide for flexible control over actual realization patterns, and to account easily for the frequent case where no part of a morpheme is omitted.

### 3.4 Floating Morphemes and Directionality

The inherent assumption of the continuation-classes approach to morphotactics (Koskenniemi 1983) is that morphemes are always tied to a fixed position in the usual chain of affixes and stems that make up a word. However, clear counter-cases of so-called floating morphemes do exist, for example, in Huave (Huavean) and Afar (East Cushitic). In Afar (11), the same affix may flip between prefixal and suffixal position, depending on the phonology of the stem (Noyer 1993).

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*However, some German dialects do have limited truncation for plurals, e.g. Upper Hessian hando ‘dog (sg.)’ → hâm ‘(pl.)’ (Golston & Wiese 1996).
(11) **SECOND PERSON FLOATING AFFIX IN AFAR**

a. t-okm-è  
b. yab-t-à  
c. ab-t-è  
2-eat-perf  speak-2-impf  do-2-perf  
‘you ate’  ‘you speak’  ‘you did’

Descriptively, the second-person affix “t- occurs as a prefix before a nonlow [stem-initial, M.W.] vowel and elsewhere as a suffix” (Noyer 1993).

The placement of variable-position infixes has been analyzed in OT by attributing to them an inherent, directional ‘drift’ towards the left or right edge of some constituent such as the word. Analyses using this idea typically employ additional affix-independent constraints expressing e.g. prosodic wellformedness to control the ultimate devianve from the desired position. OT’s development of constraint-based directionality was originally illustrated with the Tagalog -um- infixation case that we briefly mentioned above. It led to the introduction of the EDGEMOST constraint (Prince & Smolensky 1993) and later to a family of generalized alignment constraints (McCarthy & Prince 1994, et seq) that seek to minimize the distance to some designated edge.

Adapting the gist of the idea for our present purposes, one could analyze Tagalog -um- by assuming leftward drift for the infix while simultaneously imposing a specific prosodic wellformedness condition on surface words: syllables must have onsets. One advantage of this analysis is that it explains why lexically onsetless affixes cannot become prefixes: *um.-ba.sa., !b-u.m-a.sa.  

We may proceed similarly for the Afar case. This time the leftward drift of (-)t- needs to be coupled with an affix-specific conditional constraint that demands a nonlow vowel to the right if landing in word-initial position. In addition, some formal way of ruling out discontiguous stems is needed to prevent infixed -t-. Note that if the second person affix cannot become a surface prefix, it indeed lands on the first position after the stem, thereby underscoring the leftward-drifting behaviour attributed to t-.

For motivation of rightward drift, let us look at some data from Nakanai (12), an Austronesian language cited in Hoeksema & Janda (1988, 213f).

(12) **NAKANAI SUFFIXING VC REDUPLICATION**

| Sanskrit  | Nakanai Suffixing VC Reduplication |
|----------|-----------------------------------|
| haro     | har-ar-o ‘days’                    |
| velo     | vel-el-o ‘bubbling forth’          |
| baharu   | bahar-ar-u ‘windows’               |
| abi      | ab-ab-i ‘getting’                  |
| kaiamo   | kaiam-am-o ‘residents of K. village’ |

Besides technical devices to model total substring reduplication \(XY \rightarrow X_iY_j - X_iY_j\) and a VC requirement for the reduplicant’s segmental content, a minimalist analysis only needs to add rightward-drifting behaviour for the reduplicant. The reader will find it easy to verify that the forms in the second column of (12) indeed optimally satisfy both the drift specification and the VC requirement, whereas any drift further to the right would violate the latter requirement: *haro-ro shows faithful copying and perfect suffixation, but has a CV reduplicant.  

9This analysis deviates from Prince & Smolensky’s original treatment, which admitted onsetless um- before roots like aral ‘teach’. Their analysis is rejected in Boersma (1998, 198). He points out that orthographically vowel-initial roots are actually pronounced with a leading glottal stop (\(^*\)t-aral), while forms like \(^*\)mag-aral ‘study’, with a proper prefix, show that this glottal stop is better assumed to be part of lexical representation. To prevent intrusion of -um- into complex-onset roots like gradwet, we may either assume that initial /m/ is presupposed to become a syllable onset or make sure that immediately adjacency in complex onset members is inviolable lexical information: \(^*\)g-um-rad.wet., !gr-u.m-ad.wet. ‘graduate’.

10Interestingly, the symmetrical case of leftward-drifting VC reduplication with otherwise identical conditions can be found in the Salish language Lushootseed (Hoeksema & Janda 1988, 214): st-duf > st-dubuf ‘man’, ?baw > ?b-baw ‘grandchildren’.

11One objection to the analysis just sketched might be that one could (perhaps generally) eliminate drift at the expense of prosodic subcategorization. In Nakanai – at least for the data in (12) – this would involve no more than the reduplicant’s requirement for adjacency to a word-final syllable nucleus. We will discuss the nature of the tradeoff involved in choosing between conventional versus drift-based analyses in more detail in §5.
A seemingly different kind of directionality is involved in choosing grammatical vowel/∅ realization patterns in the bounded-partiality morphologies of Modern Hebrew, Tonkawa and others. To account for the kind of left-to-right preference found in these patterns, Walther (1997) has proposed a so-called Incremental Optimization Principle, “Omit zero-alternating segments as early as possible”. The principle explains, inter alia, why in Tonkawa we- pik-en-o?: is grammatical (cf. (2)) but *we- pik-en-o?:, *we- pik-en-o?: are not. The latter two represent a missed chance to leave out /i/, which appears earlier in the speech stream than the second stem vowel /e/. Prosodic wellformedness alone does not distinguish between these forms since none of them violates the CV(C) syllable constraints.

To sum up, floating morphemes and the different kinds of directionality posit yet another challenge for formalization. The question here is how to express drift in lexical representations and whether it is possible to unify the seemingly diverse kinds of directionality within a single formal mechanism.

Having provided linguistic motivation for certain abstract requirements for formalization of prosodic morphology, we now turn to our central proposals that meet these requirements in the context of a finite-state model.

4 Extending Finite-State Methods to One-Level Prosodic Morphology

In this section I will present the extensions to OLP that are deemed necessary to formalize the range of prosodic morphology phenomena described above. A basic knowledge of formal languages, regular expressions and automata will be helpful in what follows, perhaps at the level of introductory textbooks on the subject (Hopcroft & Ullman 1979, Partee, ter Meulen & Wall 1990).

4.1 Technical Preliminaries

Bird & Ellison (1994, §3) proposed state-labelled automata as the formal basis for autosegmental phonology. In the present work, however, I will return to more conventional arc-labelled automata. The main reason for this choice is that it eases actual computer implementation, since existing FSA toolkits all rest on the arc-label assumption (van Noord 1997, Mohri, Pereira & Riley 1998). As Bird & Ellison themselves note, the choice has no theoretical consequences because the two automata types are equivalent (Moore-Mealy machine equivalence).

With respect to the actual content of the arcs, however, I will follow OLP by allowing sets as arc labels. Eisner (1997a), who employs the same idea for an implementation of OT-type phonological constraints, argues that this helps keep constraints small. The reason why we may gain a more compact encoding of automata for phonological and morphological purposes is because boolean combinations of finitely-valued features can be stored as a set on just one arc, rather than being multiplied out as a disjunctive collection of arcs. Again it must be emphasized that the choice is not crucial from a theoretical point of view, but simply convenient for actual grammar development.

Of course, sets-as-arc-labels require modifications to the implementation of various standard operations on finite-state automata. Our representation for these sets is in the form of bit vectors. For FSA intersection \( A \cap B \) this implies that the identity test \( (A.arc_i = B.arc_j) \) between two arc labels must be replaced with a refined notion of arc compatibility \( (A.arc_i \cap B.arc_j \neq \emptyset) \), which is efficiently implementable with bitwise logical AND and test-for-nonzero instructions. While FSA concatenation, union and reversal have operationalizations that are independent of the nature of arc symbols, forming the complement of a FSA involves determinization and completion, two operations which again require modification. Recall that a complete automaton is one that has a transition from every state for each element of the automaton alphabet.

Completion can be implemented for set-labelled automata by creating a new nonfinal state \( sink \) and adding for every state \( s \neq sink \) an extra arc pointing at \( sink \). This arc is labelled with the universal alphabet set \( \Sigma \) minus the union of all sets that label the outgoing arcs of state \( s \). Again, bitwise operations for complement and logical OR can be used here. Whereas for completion a direct realization seems preferrable, one way to realize FSA determinization instead involves a reduction to the conventional, identity-based version. It capitalizes on the insight that non-empty label intersection gives the same results as label
identity test iff all label sets bear the property of being either pairwise identical or disjoint. By replacing existing arcs with disjunctive arcs in such a way that this property is met in the entire automaton, a new automaton can be created that is then subject to conventional determinization, i.e. with bit vectors reinterpreted as simple integers. Hence, this scheme allows the reuse of existing, optimized software. An optional post-determinization step could then fold multiple disjunctive arcs connecting any two states back into a single arc bearing a disjunctive label. Of course, nothing precludes more efficient schemes which would presumably require somewhat more extensive modification of classical determinization algorithms to make the implementation efficient for bit vector arc labels.

Finally, FSA minimization is an important operation when it comes to the compact presentation of analysis results and also when memory efficiency is crucial. However, because reversal and determinization suffice to implement minimization (Brzozowski 1962), no further modifications are necessary to support set-based arc labels.

### 4.2 Enriched Representations

This section describes in detail some generic enrichments to conventional finite-state representation in the OLPM model, motivating in each case why inclusion of the proposed representational element is warranted for one-level analyses of prosodic morphology.

In what follows we will abbreviate set-based labels with mnemonic symbols for reasons of readability. Also, we will liberally apply set-theoretic notions to symbols, speaking e.g. of disjoint symbols when we really mean that the sets denoted by those symbols must be disjoint.

As it is actually done in the implementation, we will furthermore conceive of those symbols as types that are organized into a type hierarchy, allowing the grammar writer to express both multiple inheritance and type disjointness. However, the syntactic details will be suppressed in the text; rather, we will describe the essential parts of the type signature in prose for the sake of clarity. The semantics of the type system assumed here is extremely simple: the denotation of a parent type in the directed acyclic graph that constitutes a type inheritance hierarchy is defined as the union of the denotation of its children, whereas a type node without children denotes a unique singleton set (cf. Aït-Kaci, Boyer, Lincoln & Nasr 1989). Complex type formulae are permitted, using the Boolean connectives \( \& ; \sim \) for logical AND (intersection), OR (union) and NOT (complement), respectively. As mentioned before, all type formulae are ultimately represented by bit vectors.

Let us proceed to the first enrichment, which is a preparatory step for reduplication. It is a defining characteristic of reduplication that it repeats some part of a string and it would be nice if this property could somehow be encoded explicitly. Intuitively, in one sense repetition is just about moving backwards in time during the process of spelling out string symbols. (We momentarily disregard the second aspect of repetition, that of ensuring proper identity of symbols). Therefore, in an initial attempt to flesh out this intuition one could add designated ‘backjump’ arcs to an automaton. Backjump arcs \( S \xrightarrow{\text{backjump}} P_i \) would directly connect every state \( S \) to all of its predecessor states \( P_i \). A predecessor state \( P_i \) is defined as follows: \( P_i \) lies on a path \( p \) leading from a start state to \( S \) and there exists a non-empty proper subpath of \( p \) beginning at \( P_i \). The example in (13) shows the automaton for a Bambara word *malo* ‘rice’.
However, this potential solution has several shortcomings. It is not linear in the length of the string, adding $n(n+1)/2$ arcs for a string of length $n$. Given a single backjump symbol – which is desirable for reasons of uniform ‘navigation’ within a network –, it introduces too much nondeterminism, because at string position $i$, $0 < i \leq n$ there is a choice between $i$ possible backjumps. This nondeterminism in turn makes it cumbersome to specify a fixed-length backjump, a specification which would not be unusual in linguistic applications. Finally, we would require an unbounded memory of already produced states if lazy incremental generation of string arcs was desired for an efficient implementation.

As it turns out, however, we really need only a proper subset of the backjumps anyway, because with the kleene-star operator we already have sufficient means in our finite-state calculus to express iterated concatenation, i.e. nonlocal arc traversal. A better solution therefore breaks down nonlocal backjumping into a chain of local single-symbol backjumps. It consists of adding a reverse repeat arc $j \rightarrow i$ labelled with a new technical symbol repeat for every pair of states $<i, j>$ connected by at least one content arc $i \rightarrow j$. Content arcs are defined as arcs labelled with segmental and other properly linguistic information. Furthermore, content symbols must be disjoint from technical symbols. (14) has an example showing the Bambara word for rice under the new encoding.

(14) REPEAT ARCS: THE FINAL VERSION

To give a preview of how this can be used in reduplication: the regular expression

\[ \text{segment}^* \text{repeat repeat repeat repeat segment}^* \]

describes an automaton which – when intersected with (14) – will yield a string that contains two instances of m\textit{alo} separated by four repeat symbols. The type \textit{segment} used in the previous expression abbreviates the union of all defined segmental content symbols.
Summing up then, the repeat-arc solution is better than the previous one, because the additional amount of repeat arcs is linear in the length of the string, because there is no nondeterminacy in jumping backwards, because it is now trivial to add a local repeat arc upon lazy generation of a content arc, and because it has become easy to jump backwards a fixed amount of $k$ segmental positions by way of the regular expression \textit{repeat}^k.

The reader will have noted, however, that repeat arcs change the language recognized by the respective automaton, in particular by rendering it infinite through the introduction of a cycle within each arc-connected state pair $<i, j>$. Also, by referring to states, repeat-arc introduction was defined as manipulation of a concrete automaton representation rather than with respect to the language denoted by the automaton.\textsuperscript{12} Obviously, the minimization properties of automata containing repeat arcs are rather different from those of repeat-free automata as well. I will take up these and other issues related to repeat arcs later, when we have seen how the proposed enrichment interacts with intersection and synchronization marks to implement reduplication.

The next enrichment covers \textit{discontiguity}, with its characteristic property of transitive precedence between content symbols. Again, to be maximally generic we need to allow for intervening material at every string position, i.e. before and after every content symbol. Also, intervening material may itself be modified by other morphological processes, i.e. it may contain technical symbols such as repeat. We therefore add a \textit{self loop} to every state $i$, i.e. an arc $i \rightarrow i$ labelled with $\Sigma$, the union of content and technical symbols. The example this time displays the Tagalog word \textit{basa} (15).

(15) \textbf{SELF LOOPS: A REPRESENTATION FOR DISCONTIGUITY}

To preview a later use of this representation: when figuring as part of \textit{b-um-asa}, the self loop pertaining to state 1 will absorb the infixal material \textit{um}. The further issues and consequences that arise from this enrichment step are mostly the same as for the previous one, apart from the additional question of what to do with unused self loops like those emerging from states 0,2,3,4 in the case of singly-infixed \textit{b-um-asa}. Again, it seems best to treat those issues later on.

The third enrichment deals with \textit{partiality} and is especially useful for truncation. Here we want to be able to exercise fine control over the amount of material that gets realized or skipped over, and also have the option of leaving out an \textit{a priori} unbounded amount of segmental content. Therefore, the leading idea again is to use a local encoding, which consists of adding companion \textit{skip arcs} $S \xrightarrow{\text{skip}} T$ to all content arcs $S \xrightarrow{\text{ContentSymbol}} T$. Like its repeat counterpart, \textit{skip} is defined to be a new technical symbol. Example (16) illustrates how the automaton representation of German Pet\textit{ra} ‘proper (first) name’ looks like after the enrichment.

\textsuperscript{12}This way of constructing automata is sometimes discredited, with preference given to exclusively high-level algebraic characterizations of the underlying languages and relations (Kaplan & Kay 1994, 376). However, in line with van Noord & Gerdemann (1999) who argue convincingly for a more flexible overall approach, there are good reasons to deviate from Kaplan\&Kay’s advice in our case. Concretely, while it is possible to define the appropriate automaton for all strings of length 1 through the regular expression $\text{ContentSymbol} (\text{ContentSymbol repeat})^*$, concatenation of $n$ such expressions for a string $s, |s| = n$ does not define the same language as the repeat-enriched automaton corresponding to $s$. I conjecture that only the automaton-based approach can be compositional with respect to concatenation.
To give a simple idea of how skip arcs might be used, consider the following regular expression for picking out the base portion of the hypocoristic form Pet-i:

\[seg^* \text{skip skip}\]

When intersecting this expression with (16), the string \(pet \text{skip skip}\) results. A more principled analysis of German hypocoristic formation follows in 5.2.

Noting that in this solution technical and content arcs systematically share both source and target states, we can actually merge those two arc types and avoid the additional complexity introduced by separate skip arcs. Under a set-based labelling regime this works as follows: every content arc will now be labelled with the union of the old content symbol and the skip symbol (type formula: \(\text{ContentSymbol; skip}\)). Thus separate skip arcs would only be strictly necessary when for some reason set labels are not available.

4.3 Resource Consciousness

So far we have ignored a specific problem associated with self loops, the mechanism proposed in (15) to handle free infixation. The problem is that the same self loops that were designed to absorb infixed content may also absorb accompanying contextual constraints in unexpected ways. It is best to illustrate this unwanted interaction by way of an actual example that is already familiar, Tagalog -um- infixation. Since the infix lands just behind an obligatory word-initial stretch of syllable onsets, one particularly simple way of encoding this prosodic requirement is to attach it to the left side of the infix itself. In doing so we assume that segments are tagged with syllable role information by means of finite-state syllabification (not shown here).
However, upon intersection of (17).a\textsuperscript{13} with the self-loop-enriched representation of a stem like basa (17).b the resulting complex automaton overgenerates. It contains at least one ungrammatical placement of the affix-plus-left-context in absolute prefixal position: onset$^+u\text{mbasa}$ versus the correctly infixing $b\text{onset u m a s a}$. This is because the affix proper and its prosodic context are both consistent with basa’s initial self loop $0\xrightarrow{\Sigma}0$, causing a kind of vacuous self-fulfilment of the contextual constraint which now hallucinates onsets that were never provided by the stem itself.

Note that it is not possible to remove the initial self loop: ordinary prefixation using e.g. mag- (17).c. can apply to the same stems that take -um-. Therefore, unless we are willing to give up the overall idea of morphemes-as-constraints that are uniformly combined via intersection, we should use intersection in this case as well, with the initial self loop now playing host to the prefixal material. The argument becomes particularly compelling in the case of conditioned allomorphy. Ellison (1993) has argued that because such cases of contextually restricted morphemes exist even in agglutinative languages like Turkish and because intersection can enforce restrictions and simulate concatenation but not vice versa, it should become the preferred method of morpheme combination in a constraint-based setting.

What then is the cause of our problems in (17)? My diagnosis is that we lack an essential distinction between producers and consumers of information. Contextual constraints should only be satisfiable or ‘consumable’ when proper lexical material has been provided or ‘produced’ by independent grammatical resources. Now this notion itself is already familiar from other areas of computational linguistics, going back at least to LFG’s distinction between constraining and constraint equations (Bresnan & Kaplan 1982). Since then the general idea has gained some popularity under the heading of resource-conscious logics.\textsuperscript{14}

Here I propose to introduce resource consciousness into automata as well. Suppose we tag symbols

\textsuperscript{13}Note that -um- is specified as a contiguous morpheme by leaving out morpheme-medial self loops. This reflects the fact that the infix itself never gets broken up.

\textsuperscript{14}See e.g. the ‘glue logic’ approach to the syntax-semantics interface in LFG (Dalrymple 1999); asymmetric agreement under coordination (Bayer & Johnson 1995); Johnson’s (1997) development of a more thoroughly resource-based R-LFG version; Dahl, Tarau & Li (1997)’s Assumption Grammar formalism; Abrusci, Fouqueré & Vauzeilles (1999) logical formalization of TAGs.
with a separate producer/consumer bit (P/C bit) to formally distinguish the two kinds of information. P/C = 1 defines a producer, P/C = 0 its consumer counterpart. By convention, let us mark producers by bold print in both regular expressions and automata; see (17) for illustration. Suppose furthermore that we distinguish two modes of interpretation within our formal system. During open interpretation the intersection of two compatible arcs produces a result arc whose P/C bit is the logical OR of its argument bits, whereas in closed interpretation mode the result arc receives a P/C bit that is the logical AND of its argument bits. Drawing the analogy to LFG, open interpretation mode is somewhat akin to unification-based feature constraint combination during LFG parsing, while closed interpretation mode would correspond to checking the satisfiability of constraining equations against minimal models at the end of the parse. In open interpretation, producers are dominant in intersective combination, so that the only consumer arcs surviving after a chain of constraint intersections are those that never combined with at least one producer arc. This is similar to the behaviour found in intuitionistic resource-sensitive logics, where a resource can be multiply consumed, but must have been produced at least once.

Open versus closed interpretation imposes a natural two-phase evaluation structure on grammatical computation. After ordinary intersective constraint combination using open interpretation mode obtains in phase I, phase II intersects the resulting automaton with $\Sigma^*$ – the universal producer language – in closed interpretation mode. Because as a result only producer arcs survive, the second step effectively prunes away all unsatisfied constraints, as desired.\footnote{Because pruning in closed interpretation mode is based on examination of an arc’s content, specifically its P/C bit, our proposal differs from Bird & Ellison (1992)’s purely structural $\text{prune}(A)$ operation, which indiscriminately removes all self loops from a state-labelled automaton $A$.} Observe that all the real work happens in phase I, which is fully declarative. Phase II on the other hand is automatic and not under control of the grammar.\footnote{This is to be understood as a conceptual statement. For practical experimentation we have devised a macro whose use is visible in formal grammars. It follows that open interpretation mode is the default setting in our implementation.} To indicate that declarativity has only slightly been sacrificed, we will call the resulting grammatical framework that obeys two-phase evaluation and the open/closed interpretation distinction quasi-declarative, and speak of Quasi-Declarative Phonology etc.

It is now easy to see that our introductory problem of getting b-um-asa right has been solved: in the illformed alternative corresponding to word-initial position the consumer-only onset arcs that constitute -um–’s contextual constraint meet a consumer-only 0 $\xrightarrow{\Sigma} 0$ self loop. Since consumers intersecting with each other in phase I remain just what they are, they are immediately eliminated when intersecting with the universal producer language in phase II of grammatical evaluation. In later sections we will encounter further examples that underscore the utility of a resource-conscious style of grammatical analysis and demonstrate its wide applicability in prosodic morphology.

4.4 Copying as Intersection

In this section I will explain a generic method to describe reduplicative copying using finite-state operations. The germ of the idea already appeared in Bird & Ellison (1992, 48), where the authors noted that the product of automata, i.e. FSA intersection, is itself a non-regular operation with at least indexed-grammar power. In illustrating their claim they drew attention to the fact that odd-length strings of indefinite length like the one described by the regular expression $(a b c d e f g)^+$ can be repeated by intersecting them with an automaton accepting only strings of even length, yielding $(a b c d e f g a b c d e f g)^+$ in the example at hand.

Since we already know now that with intersection we have a promising operation in our hands to implement duplication, let us work out the details. First, we will show how to get total reduplication in a way that makes use of neither the odd-length assumption of Bird & Ellison’s toy example nor of a priori knowledge about the length of the string as in our preview of a possible application of repeat arcs (14). An initial step that improves on that previous account for reduplicating the repeat-encoded malo string dispenses with the length knowledge by using the following regular expression:

$$m \text{ seg}^* \circ \text{ repeat}^* \text{ seg}^*$$
Although we have now replaced 4 (= |malo|) consecutive repeats by \textit{repeat*}, an expression which encodes jumping back an indefinite amount of time, it is clear that something else is required to generalize beyond this example. In comparison to words like \textit{wulu} ‘dog’, \textit{malo} is special in that \textit{m} and \textit{o} are distinct symbols that occur only once in the string. Thus they are able to serve two functions at the same time, acting as ordinary content symbols \textit{and} as markers of left and right edges of the reduplicant.

That observation points to the crucial issue at hand, which is how to identify the edges of a reduplicant in a generic fashion. In \textit{malo} it just happened that the edges were already self-identifying. For the general case we may borrow freely from Bird & Ellison (1994, 68f)’s solution to an analogous problem arising in autosegmental phonology, that of modelling the synchronizing behaviour of association lines. Just like in their solution, let us assume a distinct \textbf{synchronization bit}. Here it will be added to all content symbols, being set to 1 for the edges we want to identify and receiving the value 0 elsewhere.\footnote{Ellison (1993) contains an earlier application of synchronization symbols to the problem of translating concatenation into intersection. Ellison’s comment that only a finite alphabet (\{0,1\}) is needed in the translation carries over into the present setting: synchronisation symbols need not be multiplied for triplication, quadruplication etc. would all be feasible using the same approach. Also, it is interesting to see how one bit actually suffices in this scheme to identify two kinds of edges in all strings of length > 1, exploiting the fact that concatenation is associative but not commutative.}

Adopting Bird & Ellison’s notation for combining content and synchronization information, we can draw the automaton for \textit{wulu} as follows.\footnote{In our typed setting, we actually use a new type \textit{synced}, writing \textit{ContentSymbol&synced} for \textit{ContentSymbol:1} and \textit{ContentSymbol&~synced} for \textit{ContentSymbol:0}. Type declarations ensure that each instance of \textit{ContentSymbol} is itself underspecified with respect to synchronization.}

\begin{figure}
\centering
\begin{tikzpicture}[node distance = 1.5cm, >=latex,]
  \node (0) [state, initial, accepting] {0};
  \node (1) [state, right of=0] {1};
  \node (2) [state, right of=1] {2};
  \node (3) [state, right of=2] {3};
  \node (4) [state, right of=3] {4};

  \draw [->] (0) edge [loop below] node {\textit{repeat}} (0);
  \draw [->] (0) edge [bend left] node {\textit{seg:1}} (1);
  \draw [->] (1) edge node {\textit{seg:0}} (2);
  \draw [->] (2) edge node {\textit{seg:1}} (3);
  \draw [->] (3) edge node {\textit{seg:0}} (4);
  \draw [->] (4) edge [loop below] node {\textit{seg:1}} (4);

  \draw [->] (1) edge [bend left] node {\textit{repeat}} (0);
  \draw [->] (2) edge [bend left] node {\textit{repeat}} (1);
  \draw [->] (3) edge [bend left] node {\textit{repeat}} (2);
\end{tikzpicture}
\end{figure}

Now the regular expression that describes total reduplication can be liberated from mentioning any particular segmental content:

\[
\textit{seg:1 seg:0* seg:1 repeat* seg:1 seg:0* seg:1}
\]

Figuratively speaking we start at a synchronized symbol representing the left edge, move right through a possibly empty series of unsynchronized segments to another synchronized symbol representing the right edge, then go back through a series of repeat arcs until we encounter a synchronized symbol again, which must be the left edge. Note how the same subexpression is used twice to identify ‘original’ and ‘copied’ occurrence of the reduplicative constituent. With more instances of \textit{seg:1 seg:0* seg:1}, triplication, quadruplication etc. would all be feasible using the same approach. Also, it is interesting to see how one bit actually suffices in this scheme to identify two kinds of edges in all strings of length > 1, exploiting the fact that concatenation is associative but not commutative.

To handle actual Bambara’s \textit{Noun-o-Noun} reduplication, we need to combine the enrichments of section 4.2 and 4.3, in particular using one of the self loops to provide space for the intervening /o/. Here then is the full encoding of both \textit{wulu} ‘dog’ and the reduplicative construction itself:
After intersection (19).a ∩ (19).b and pruning away of consumer-only arcs we get an automaton which is equivalent to

\[
\begin{align*}
w &\to l:0 \ u:0 \ \text{seg:0} \\
\text{seg:0} &\to \text{seg:1} \\
\text{seg:1} &\to \text{seg:1} \\
\text{seg:1} &\to \text{seg:1} \\
\text{seg:1} &\to \text{seg:1} \\
\end{align*}
\]

There was nothing special in intersecting with a singleton lexical set, hence it is trivially possible to extend the Bambara lexicon beyond the single example we have shown. All that is needed is to represent new lexemes as FSAs in the same vein as \textit{wulu}, and define the lexicon as the union of those FSA representations.

Also, it is easy to see that we can construct a variety of other reduplication automata that use different synchronization and repeat patterns or employ additional skip arcs. These would encode the various types of partial reduplications that we saw in section 3.1. I will indeed present one such case in section 5, but leave the others as an exercise to the reader for reasons of space.

4.5 Bounded Local Optimization

This section defines a final operation over enriched automata called Bounded Local Optimization (BLO), which can be understood as a restricted, non-global search for least-cost arcs in weighted automata. The operation will be shown to permit implementation of the Incremental Optimization Principle (IOP, p.11), while later sections illustrate that it can also be used for morpheme drift and longest-match behaviour.

To prepare the ground for such an operation it is best to return to the simple Tonkawa example \textit{we-pcen-oʔ} (cf. (2)) that was discussed previously in connection with the IOP. Given a stem representation \(p(i)c(e)n(a)\) that contains three zero-alternating vowels, the addition of two nonalternating affixes \textit{we-} and \textit{-oʔ} does not enlarge the resulting set of eight \(2^3\) word forms. Intersection of this set with some simplified prosodic constraints \*CCC and \*VV that ban sequences of at least three consecutive consonants and two adjacent vowels still leaves us with three remaining forms. Here the IOP steps in, preferring \textit{we-pcen-oʔ} over \textit{*we-picen-oʔ}, \textit{*we-picn-oʔ} because only the first form lacks the \(i\) that constitutes the earliest omittable vowel.

To implement this kind of behaviour, the first idea is to extend the FSA model once again, namely towards the inclusion of \textbf{local weights} on arcs. The weights are taken from a totally ordered set and represent the costliness of a particular choice. In our example, the realization of an alternating vowel should be more costly than its omission, e.g. by receiving a greater weight. Now this move in itself fits in with the recent gain in popularity that weighted automata and transducers have enjoyed, finding application in areas such as speech recognition, speech synthesis, optimality theory and others (Pereira & Riley 1996, Sproat 1996, Ellison 1994b). Usually, however, the theoretical assumption has been that the minimal weighted unit is the string, not the individual symbol. Taking advantage of this assumption, Mohri (1997) is able to both move
and modify individual weights between arcs in an operation called ‘pushing’ in order to prepare a weighted automaton for minimization. In our application, though, weights represent localized linguistic information which should not be altered. Therefore, I will at present pursue the alternative of weighting the symbols themselves. Also, for purposes of this paper it will actually suffice to introduce a small finite number of different weights.\footnote{Interestingly, Kiraz (1999) reports that actual grammars for Bell Labs text-to-speech applications also obey this restriction, e.g. the German module has 33 weights and the French module 12.} Hence the weights used here can equally well be formalized as part of the type hierarchy that structures label sets, and that is indeed what the current implementation does. While it would be definitely be worth trying to take the other route as well and make the transition to the general case of unrestricted, possibly numeric, weighting schemes that build upon the weighted-string assumption, the choice seems premature right now. Rather, it seems best to wait until the analysis of enough decisive phenomena has been carried out in the present framework and then evaluate what the ultimate consequences of each assumption are.

Here then is one automaton representation of the \{we-pcen-o?, we-picen-o?, we-picn-o?\} set, enriched with only two weights representing the marked realization case (depicted as /1) and the unmarked elsewhere case (depicted as /0). The ordering that will be assumed is marked > unmarked.

\begin{itemize}
  \item weights are represented by \(w\) (positive for marked, negative for unmarked)
  \item \(e\) is the empty string
  \item \(c\) is the consonant ‘root’ \(p - c - n\)
\end{itemize}

Now, how does one use the weights to implement the IOP? The crucial observation is that IOP’s pruning of costlier alternatives translates into local inspection of the arcs emanating from states like 3 and 10 in (20). In each of the choices \(3 \rightarrow 4\) or \(9, 10 \rightarrow 5\) or \(6\), there is an alternative between a marked arc and an unmarked one. Let us call such an arc \(A \in \text{State.arcs} \ a \ choice \ arc \ whenever \ |\text{State.arcs}| > 1\). By cutting away marked, i.e. non-optimal choice arcs, we arrive at an automaton which recognizes the single string \(we-pcen-o?\), as desired. Since it is desirable to abstract away from the specifics of this example, we will in the following develop a new operation called Bounded Local Optimization (BLO) which will encapsulate the locally-determined pruning of non-optimal arcs. To make it widely applicable, we introduce two generalizations into our example procedure.

The first generalization is to prune only those arcs from the set \(S.arcs\) whose weight is greater than the \textit{minimum} weight over this entire set. For example, \(3.arcs = \{3 \rightleftarrows 10, 3 \rightleftarrows 1, 3 \rightleftarrows 1, 9, 3 \rightleftarrows 4, 3 \rightleftarrows 1, 9\}\) has the associated minimum weight 0. As a consequence, preservation of non-choice arcs like \(4 \rightarrow 5\) is automatic, since the minimum over singleton weight sets is independent of the only element’s weight value. Also, the generalization means that even multiple choice arcs will survive pruning iff they are all weighted with the minimum cost, thus providing a way to maintain alternatives beyond the pruning step, e.g. to implement free variation.\footnote{One slight modification that is not pursued here (but will be assumed in §5.3) would add a mechanism to make designated arcs inert to minimum-based pruning. The rationale behind this move is that there are scenarios where e.g. technical arcs would be compared to content arc alternatives of various weight in a nonsensical way. A simple way to signal inertness is by negative weights: an arc is pruned whenever the length-\(k\) alternatives yield a \textit{positive} lower summed weight. With the help of a special value \(-\infty\) we can even prevent arc pruning \textit{independent} of \(k\) and the actual weight distribution in the alternatives.}

The second generalization is to parametrize the operation under development with a fixed look-ahead of \(k\) arcs, summing up weights over each individual \(k\)-length path extending from a given state. In our running example, \(k = 1\) was sufficient to detect gradient wellformedness differences in a maximally local fashion. In general, though, one might need to examine a greater number of consecutive arcs to discover the \(\text{(non-)}\)optimality of an alternative path. For example, if the arc labels of \(3 \rightarrow 9, 9 \rightarrow 10\) switched places...
and arc $10 \rightarrow 6$ was eliminated, we would need $k = 3$ to see that $3 \rightarrow 9$ initiates a costlier path, hence should be optimized away ($\sum 3 \rightarrow 9 \rightarrow 10 \rightarrow 5 = 2, \sum 3 \rightarrow 4 \rightarrow 5 \rightarrow 6 = 1$).

To sum up, calling BLO a kind of optimization is justified because non-minimally-weighted choice arcs are pruned; the optimization is boundedly local because no path of length greater $k$ influences the decision of which arcs to prune. In contrast, the (N-best) shortest-path algorithms (Dijkstra 1959, Tarjan 1983) used in most other applications of weighted automata constitute global optimization procedures which will use information from the entire automaton to determine their result. Although the latter optimization procedures could presumably be used as part of the aforementioned general exploration of the weighted-string alternative, pursuing the more restricted local variant is preferred here, because it incorporates an interesting hypothesis about what formal power is actually needed in processing prosodic morphology.\textsuperscript{21}

Another consequence of adopting BLO is that it makes the question of what maximal look-ahead is (perhaps universally) required a topic of promising empirical research, which could shed further light onto the resource-conscious structure of natural language patterns.

Let us now proceed from informal sketches to a more precise definition of BLO. First, I will give a different characterization of BLO as a function that maps between weighted formal languages, in order to ensure its representation-independent definability. Such a characterization is desirable, since for any fixed $k$ and suitable locally-weighted language $L$ one can construct ‘illbehaved’ automaton representations, e.g. by inserting epsilon transitions, so that non-optimal paths cannot be detected within a window of length $k$. Therefore, in a second step I will briefly consider the automaton-theoretic implementation of the BLO again, focussing in particular on the question of which automaton representation acts as wellbehaved input to it.

In the language-theoretic characterization of BLO, then, the idea is to sum over (the local weights of) $k$-length substrings, the equivalent of $k$-length arc paths. We discard a string $w$ when there is at least one position whose associated $k$-length sum exceeds the minimum sum for that position obtained through evaluation of other comparable strings, i.e. those which share a common prefix. For the case where only a substring of length $j < k$ exists, we simply define that its weight sum is the sum of the existing weights up to position $j$, or equivalently that non-existing positions have implicit weight 0. Here then is the formal version of BLO:

(21) Given an alphabet $\Sigma$ as a finite, nonempty set of symbols and a set of positive, real-valued weights $\mathbb{R}_+$, a locally weighted language $L$ is defined as follows: $L \subseteq (\Sigma \times \mathbb{R}_+)^*$. We will also speak of a locally weighted string $w$ whenever $w \in L$ for some locally weighted language $L$. Finally, let $w[i]$ pick out the $i$-th pair in $w$ for $0 \leq i < |w|$, let $w[0 \ldots i]$ denote the length-$i$ prefix of $w$ and let $\pi_2$ be the projection of the second element of a pair.

Then \textit{Bounded Local Optimization} $\text{BLO} : L \times \mathbb{N} \setminus \{0\} \mapsto L$ is defined as

$$\text{BLO}(L, k) = \{w \in L | \exists pos \in \mathbb{N}, 0 < pos + k \leq |w|, \exists v \in L: v[0 \ldots pos] = w[0 \ldots pos], \text{weight}_\text{sum}(pos, k, v) < \text{weight}_\text{sum}(pos, k, w)\},$$

and, with $k, pos \in \mathbb{N}$ and $w$ a locally weighted string,

$$\text{weight}_\text{sum}(pos, k, w) = \begin{cases} 0, & k = 1 \land pos \geq |w| \\ \pi_2(w[pos]), & k = 1 \land pos < |w| \\ \text{weight}_\text{sum}(pos, 1, w) + \text{weight}_\text{sum}(pos, k - 1, w), & k > 1 \end{cases}$$

\textsuperscript{21}Interestingly, Mohri, Riley & Sproat (1996, 96ff) also explore (different) incomplete optimization methods that visit only a subset of the states. However, their motivation is to solve a practical problem in speech recognition, namely that the enormous number of states prohibits plain application of single-source shortest-path algorithms on current hardware.
To illustrate the operation just defined, suppose

\[ L = \{w_1, w_2\}, \text{ with } w_1 = \langle a, 0\rangle \langle b, 1\rangle, w_2 = \langle c, 1\rangle \langle d, 0\rangle. \]

Then \( BLO(L, 1) = \{w_1\} \), because

\[ w_1[0 \ldots 0] = w_2[0 \ldots 0] = \epsilon \]

and

\[ \text{weight} \_\text{sum}(0, 1, w_2) = \pi_2(\langle c, 1\rangle) = 1 > 0 = \pi_2(\langle a, 0\rangle) = \text{weight} \_\text{sum}(0, 1, w_1), \]

together with the fact that examination of position 1 leaves the optimality of \( w_1 \) unchallenged,

thus no common prefix exists: \( w_1[0 \ldots 1] = \langle a, 0\rangle \neq w_2[0 \ldots 1] = \langle c, 1\rangle. \)

Thus, the example illustrates that BLO is in fact a ‘greedy’, directional type of optimization: weights are evaluated by position such that a string cannot compensate for an initial costly string portion by some cheaper suffix if an alternative with cheaper prefix exists. This is a welcome result since one application of BLO is to model the IOP, whose original formulation “Omit zero-alternating segments as early as possible” was intentionally defined in directional terms. Although BLO’s directionality is perhaps not immediately apparent from the declarative definition, it does in fact follow from the local examination of weights embodied in \( \text{weight} \_\text{sum} \) together with the common prefix requirement. Note also that \( BLO(L, k) = L \) for \( k > 1 \), because by \( \text{weight} \_\text{sum}(0, k, w_{1/2}) = 1 \) both strings are kept. Hence, we see that optimization results are not necessarily monotonic with respect to parameter \( k \). In particular, care must be taken to avoid look-ahead windows that are too big, because – as we have just seen – overstretching the bounds of locality can sometimes blur distinctions expressed by the weights.

There is a last peculiarity worth noting, which has to do with the entity over which BLO, or any other grammar-defining optimization approach, for that matter, should be applied. Suppose that in our illustrative example \( L \) the strings \( w_1 \) and \( w_2 \) actually represented different lexical items rather than realization alternatives of a single item. The result \( BLO(L, 1) = w_1 \) then means that with \( w_2 \) unfortunately a lexical item itself has been pruned, or in other words, that optimization cannot be safely applied over the entire lexicon. Rather, BLO must be applied on a per-item basis, at least conceptually. There are various ways to actually implement this requirement. One way would be to prefix each lexical item with string-encoded semantic and morphological information that is weighted with the same item-independent weight. The prefixes make item beginnings unique, so they will be preserved even in a minimized FSA version of the lexicon, and the uniform weights ensure that no item will be prematurely discarded, thus banning harmful interaction. However, one drawback of such a scheme would be that FSA minimization would have less chances of reducing the size of automata, as compared to the usual encoding of grammatical information at the end of phonological strings (Karttunen, Kaplan & Zaenen 1992). The latter encoding could be preserved in a second scheme where during generation one first intersected the lexicon automaton with \( \Sigma^* <\text{semantic/morphological features of desired word form}> \) and then applied BLO to the result. Finally, one might devise an algorithm to predict which arcs potentially participate in harmful interaction and prefix only those with disambiguating information, in what might be seen as an attempt to use the first method on a demand-driven basis for improved minimization behaviour.

Returning to the question of what machine representation \( \alpha \) of a locally weighted language \( L \) is sound input to an automaton-based implementation of BLO, I conjecture that a sufficient condition for soundness is that \( \alpha \) takes the form of the minimal deterministic automaton for \( L \) having a single start state. To see the plausibility of this conjecture recall that while a minimal automaton is defined to have the minimal number of states, it is also minimal in number of transitions (Mohri 1997, Corollary 1). Being minimal and deterministic, string prefixes are shared wherever possible and there are neither epsilon transitions nor is there useless nondeterminism. Thus the remaining choice arcs must encode non-reducible local choices. While the BLO algorithm considers each of the choice arcs emanating from a given state for pruning, it suffices to examine the summed weights of length-\( k \) paths starting with those arcs because the condition of common prefixes in definition (21) is already guaranteed through sharing. Because \( \alpha \) has a single start state, it follows that the condition is also met for the case of the empty prefix.

Given this clarification about the nature of BLO input, we are now in a position to present the algorithm in pseudocode for the automaton-based implementation in (22).
one-arc-per-disjunct assumption, at least in connection with BLO.

(22) BLO Algorithm

\[
\text{BoundedLocalOptimization}(\alpha, k) = \\
1. \beta.trans \leftarrow \text{visited} \leftarrow \emptyset \\
2. \text{states} \leftarrow \beta.start \leftarrow \alpha.start \\
3. \text{while states} \neq \emptyset \text{ do} \\
4. \quad q \leftarrow \text{DEQUEUE}(\text{states}) \\
5. \quad \text{visited} \leftarrow \text{visited} \cup \{q\} \\
6. \quad \text{if } |q.arcs| > 0 \\
7. \quad \text{then } (\text{nextstates, nextarcs}) \leftarrow \text{NextArcsOnMinimalPaths}(q, k) \\
8. \quad \text{ENQUEUE}(\text{states}, \text{nextstates} - \text{visited}) \\
9. \quad \beta.trans \leftarrow \beta.trans \cup \text{nextarcs} \\
10. \quad \text{if } q \in \alpha.final \\
11. \quad \text{then } \beta.final \leftarrow \beta.final \cup \{q\} \\
12. \text{return } \beta
\]

The algorithm takes a locally weighted automaton \( \alpha \) and the look-ahead constant \( k \) as input. Line 1 initializes the set of transitions of the result automaton \( \beta \) and the set of already \( \text{visited} \) states to empty. While line 2 copies the input start states to both the result start states and the set of unprocessed \( \text{states} \). While there are \( \text{states} \) to process (line 3), we remove a current state \( q \) from this set and mark it as \( \text{visited} \) (line 4–5). If that state has outgoing arcs (line 6), we examine all paths of maximum length \( k \) that originate at \( q \) and return those \( \text{nextstates} \subseteq \{\text{dest} | q \overset{\text{label}}{\rightarrow} \text{dest} \} \) and \( \text{nextarcs} \subseteq q.arcs \) that were found to lie on one of the paths with minimal weight sum (line 7).\(^\text{22}\) We then add the new, i.e. non-visited ‘minimal’ states found in this way to the set of unprocessed states (line 8). Note that this step is responsible for the local, incomplete exploration of the state set of \( \alpha \): ‘non-minimal’ states will not be considered in further iterations (unless, of course, other arcs happen to refer back to them). In the next step we add \( \text{nextarcs} \) – the pruned subset of \( q \)’s outgoing arcs – to the transition set of the result automaton (line 10). Finally, if the current state was final in the input, then so must it be in the result (line 11). After the loop over all ‘minimal’ states has been completed, we return the finished result automaton \( \beta \) in line 12.

It is easy to see that the algorithm in (22) must always terminate. The set of \( \text{states} \) that controls the only existing loop is initialized to a finite set of start states at the beginning. While line 8 from the loop body increases \( \text{states} \) by some amount which is bounded by \( \max_{q \in \alpha.states} (|q.arcs|) \), it simultaneously ensures that no state will be added more than once due to the subtraction of \( \text{visited} \) states (\( \text{visited} \) itself grows monotonically, line 5). Because by definition \( |\alpha.states| < \infty \), the total increase must be finite as well. Since each iteration unconditionally removes one element from \( \text{states} \), the nonemptiness condition in line 3 will evaluate to false after a finite number of iterations, as required for the proof of termination.

Similar ideas have been explored under the heading of locality in violable constraint evaluation by Tesar (1995) and in particular Trommer (1998, 1999). Trommer (1998, p.30,fn.12) explicitly acknowledges the intellectual debt to the Incremental Optimization Principle of Walther (1997), which is also the precursor of BLO; both of Trommers papers apply the local evaluation concept in an interesting way to Mende tone data. However, while the clearest exposition of his local optimization algorithm is in Trommer (1999), there are a number of differences and problems. Trommer’s \( \text{Optimize}(T) \) is defined as an algorithm over transducers only, there is neither a characterization in terms of regular relations comparable to our language-theoretic definition of BLO nor a discussion of the dependency on a suitable normal form for automata.

\(^{22}\)Note a slight complication that arises when type-based disjunctive arc labels are allowed: sometimes even a single arc like \( i \overset{\text{ak}0}{\rightarrow} j \) with type-encoded weights 0 and 1 must not be pruned altogether but rather have non-minimal disjuncts removed, as in \( i \overset{\text{ak}0}{\rightarrow} j \). The necessary generalization of \( \text{NextArcsOnMinimalPaths}(q, k) \) is easy: one simply collects the set of (summed) weights \( W \) for any given arc (path) using \( n \) type subsumption checks that test containment of each of the \( n \) weight types, and then \( \text{intersects} \) the arcs \( q.arcs \) with 0 \( \min(W) \) 1 to effect pruning. However, for expository purposes we will stick with the conventional one-arc-per-disjunct assumption, at least in connection with BLO.
His algorithm definition is both somewhat erroneous (lines 8,9) and not formulated as an incomplete search that directly exploits the computational advantage of locality. Finally, the generalization to a look-ahead $k > 1$ is missing, and $\text{Optimize}(T)$ is used at each step of a constraint cascade, in contrast to the restricted use of BLO as a one-step filter on the final result of autonomous automata intersections.

### 4.6 Flat representation of prosodic constituency

So far we have avoided to take any stand on the issue of which set of prosodic categories to assume, how to conceive of the relationships between such categories and how to represent these in a finite-state framework. For concreteness, we will now briefly consider the topic in a bit more detail. However, perhaps somewhat surprisingly, prosodic constituency above the level of sonority will not be strictly necessary in any of the three case studies under §5. Thus, the reader may skip this subsection on a first reading, resting assured on the principal result developed below, namely that the present framework freely allows for conventional prosodic constituency, albeit in a new representational format, whenever the empirical facts or different styles of analysis seem to warrant its inclusion.

Since at least Selkirk (1980) many authors have assumed that phonology above the segmental level is organised in a fashion much similar to syntax, employing hierarchical structure to represent prosodic constituency. In Selkirk’s work, for example, the categories of syllable $\sigma$, foot $\Sigma$ and prosodic word $\omega$ are proposed, together with subscripted s(strong)/w(weak) modifications to mark up subcategories and superscripted primes to tag supercategories. Hence, a word like English *sensational* receives the following prosodic representation (23).

\[
\begin{align*}
\text{ENGLISH} & \quad \text{sensational} \\
\omega & \quad \Sigma' \quad \Sigma_w \quad \Sigma_s \\
\sigma & \quad \sigma_s \\
\end{align*}
\]

Though not depicted by Selkirk, one might proceed similarly below the syllable level with syllabic roles that ultimately connect to segments (24).

\[
\begin{align*}
\text{ENGLISH} & \quad \text{sensational: possible syllabic structure} \\
O & \quad N \quad C \quad O \quad N \quad C \quad O \quad N \quad CO \quad N \quad C \\
\sigma & \quad \sigma_s \quad \sigma_w \quad \sigma_w \\
s & \quad e \quad n \quad s \quad e \quad j \quad f \quad a \quad n \quad a \quad l \\
\end{align*}
\]

In our case we have used the four roles O, N, C, CO for onset, nucleus, coda and codaonset, the latter being a representation for ambisyllabic segments and geminates (cf. Walther 1997, ch.3). Of course there are many competing proposals as to which categories to adopt and how to make best use of the dominance.
relationships. Yet all of these proposals share the common assumption of a finite category set; hence for formal purposes the examples just given suffice to illustrate our main point.

This main point is how to linearize such graph-structured representations in conventional finite-state models. In particular, a perspicuous lossless encoding of both dominance and immediate precedence relationships is needed. Moreover, we would ideally want a kind of distributed representation where the properties denoted by categories can be locally inspected rather than, say, demanding a nonlocal reference to some distant boundary symbol in a traditional bracketed notation.

Towards this goal, our leading idea will be to **reinterpret the transitive dominance relation as monotonically inheritable**. Now Bouma & Nerbonne (1994, 44ff) have pointed out that one of the restrictions of the inheritance relation, as it is usually defined, is its idempotency: inheritance cannot distinguish between structures that differ only in recursion level (e.g. anti-anti-missile ≠ anti-missile). Fortunately, the above diagrams – and most others in the phonological literature – contain no such recursion in the literal sense. Although a super-foot category \( \Sigma \) dominates an ordinary foot \( \sigma \) in (23), it is distinguished by a prime. To proceed with inheritance, we therefore demand that occurrences of any such pseudo-recursive categories must be pairwise distinct, which can be achieved by means of e.g. X-bar levels or primes that form part of the symbol at hand.

A second restriction imposed specifically by monotonically inheritable is commutativity under associativity: if \( C \) inherits from \( B \) and \( B \) inherits from \( A \), then the result is the same as if \( C \) inherits from \( A \) and \( A \) inherits from \( B \). This equivalence implies that for purposes of simulation-by-inheritance the order of dominance must not matter. One way to ensure this is to fix a particular order in advance. We therefore demand that the dominance relation of any constituent structure must be consistent with an a priori given total order \( \succ_{\text{dom}} \) over the set of categories: \( \forall x, y: \text{category}(x) \wedge \text{category}(y) \wedge \text{dominates}(x, y) \rightarrow x \succ_{\text{dom}} y \). In our examples, we would have \( \omega \succ_{\text{dom}} \Sigma_{s,w} \succ_{\text{dom}} \Sigma_{s,w} \succ_{\text{dom}} \sigma_{(s,w)} \succ_{\text{dom}} O, N, C, CO \). Hence, a diagram where e.g. \( \sigma \) dominated \( \Sigma_{w} \) would be ruled out as inconsistent.

With only a finite set of prosodic categories left that enter into formally non-recursive structures and moreover respect the dominance precedence relation \( \succ_{\text{dom}} \), the recipe for flattening a given structure is now quite simple to formulate (25).

(25) a. To prepare classification of occurrences of category \( X \), set up the following type hierarchy for each nonterminal \( X \in \text{CategorySet} \): 
\[
\begin{align*}
X & \quad | \
[ X ] & \quad | \
[ X ] & \quad | \
[ X ] & \quad | \
\end{align*}
\]

The intuition behind this is that category \( X \) is best modelled as a phonological event (Bird & Klein 1990), i.e. a temporal interval bearing the property \( X \). On the standard assumption that there are terminal categories whose concatenation forms the ‘terminal yield’ of a category-astemporal-interval, we then will be able to tag each of those terminal elements for their relative position within the interval (cf. also Eisner 1997a). In actual phonological practice, terminals will frequently be segments, but could also be features etc. Left or right brackets in boundary subtypes signal interval beginnings or endings whereas the underscore symbol as left or right part of a subtype stands for nonempty context, i.e. there is at least one terminal to the left or right.

b. Whenever terminal type \( T \), transitively dominated by category \( X \in \text{CategorySet} \), is found in initial or medial or final position of the terminal yield of \( X \), add a conjunct \([X \text{ or } X_{-} \text{ or } X]\) to \( T \). The ‘or’ is ‘inclusive OR’, in particular to cover length-1 terminal yields.

c. Whenever terminal type \( T \) is not transitively dominated by category \( X \in \text{CategorySet} \), add a conjunct \( \neg X \) to \( T \).

d. Add a conjunct \( \neg [X] \) to all terminals whose type formula is not maximally specific with respect to category \( X \). This step ensures full specification for boundary occurrences in those intervals which either contain more than one atom or are not multiply dominated.
To exemplify: (26) shows a flat representation of the joint diagrams of (23) and (24). (Note that for abbreviatory purposes we assume here that $\Sigma^{(2)}$ is the supertype of $\Sigma^{(1)}$ and $\Sigma^{(2)}_w$). The reader may verify that we can indeed recover the graph-structured version provided that the dominance precedence relation $>_dorn$ is known.

(26) **Flat representation of sensational**

\[
\begin{align*}
    s \land [O] & \land [\sigma_n] & \land [\Sigma_w] & \land \neg \Sigma \land [\omega] & \land [\omega_w] \\
    e & \land [N] & \land [\sigma] & \land [\Sigma_w] & \land \neg \Sigma \land [\omega] & \land [\omega_w] \\
    n & \land [C] & \land [\omega] & \land [\Sigma_w] & \land \neg \Sigma \land [\omega] & \land [\omega_w] \\
    s & \land [O] & \land [\sigma_n] & \land [\Sigma_w] & \land [\Sigma_w] & \land \omega_w \\
    r & \land [N] & \land [\sigma] & \land [\Sigma_w] & \land [\Sigma_w] & \land \omega_w \\
    j & \land [C] & \land [\sigma_s] & \land [\Sigma_w] & \land [\Sigma_w] & \land \omega_w
\end{align*}
\]

The impact of having a linearized distributed representation of (prosodic) constituency is twofold. First, we can now refine prosodically underspecified segmental strings with suitable constraints that specialize for each layer of the prosodic hierarchy. For example, given a finite-state version of declarative syllabification (Walther 1992, Walther 1995) for predicting syllabic roles from segmental information (itself using an intermediate layer of sonority difference information), the next layer would use local syllable role configurations to demarcate syllable boundaries $[\sigma, \sigma]$ and syllable interior $\sigma^{23}$, and so forth.

Second, we can freely use this locally encoded prosodic information to condition both generic and construction-specific constraints that must reflect some dependency on a given prosodic configuration. Eisner (1997a) illustrates, from the perspective of his Primitive Optimality Theory, just how appealing such local encodings can be for purposes of compact constraint formulation. Because in his results the emphasis is on locality in representational formats rather than on violability, one can be confident that their advantages will be preserved in the present framework.

### 4.7 Parsing

So far we have described OLPM from the perspective of generation only. Because reversibility is usually held to be an important property in practical applications of finite-state networks, we will now briefly consider how to do parsing under the new framework.

Disregarding optimization at first, parsing seems next to trivial. The central mechanism for constraint combination is automaton intersection, an associative operation that supports reversibility. Under this view one would simply intersect the string to be parsed with the FSA constituting grammar and lexicon; a nonempty result would then signal successful recognition. In the face of a structure-building grammar that adds e.g. syllable role information or other prosodic annotations, we of course should represent the parse as decorated surface forms.

However, an immediate complication is that in OLPM the technical arcs *skip*, *repeat* would prevent literal matching of even structurally underspecified surface strings with the grammar. If e.g. some reduplicated form is to be recognized, the grammar will assign several repeat arcs as part of the ‘surface’ string, and this decorated surface form will then fail to intersect with the plain, undecorated parse string. The solution is to employ a trivial preprocessing step at the interface between phonetics and phonology: enrich the automaton corresponding to an undecorated parse string with consumer self loops that tolerate exactly the set of technical symbols. Note that automata enriched in this way are still rather different from transducers.

\[\text{For example, by way of the following monotonic rules: } O \lor N \to [\sigma] \lor [\Sigma] \lor [\omega] \lor [\omega_w] \lor [C] \lor [\Sigma] \lor [\Sigma_w] \lor [\omega] \lor [\omega_w] \lor [C] \lor [\omega] \lor [\omega_w] \lor [C] \lor [\omega] \lor [\omega_w].\]

\[\text{On automata representations of monotonic rules, see Bird & Ellison (1992, 34f). The disjunctions in the preceding rules can even be eliminated with a suitable featural decomposition of syllable roles using the features [±onset] and [±coda] (Walther 1997, §3.4.3).}\]
even granting a simulation of composite arc symbols $x^{\text{input}}z^{\text{output}}$ as consecutive arcs $x^{\text{input}}y^{\text{output}}z$ in conventional automata (cf. fig. 9 in US patent 5,625,554 granted to Xerox on April 29, 1997). This is evident from the fact that, in contrast to their behaviour in the simulation, odd arcs do not consistently play the role of inputs and neither can even arcs be seen as corresponding outputs.

A second issue is that in parsing one would normally want a little more than mere recognition of grammatical forms (and rejection of ungrammatical ones), namely categorial information in the form of morphological, syntactic and semantic properties or features. Although we have been silent on this issue up to now, it is actually simple to represent the required annotations in grammar and lexicon by reserving one or more final arcs at the end of automata for appropriate category labels, just like in the transducer-based proposals of Karttunen, Kaplan & Zaenen (1992). (Categorial information is again supposed to be pairwise disjoint from technical and segmental type information.) However, unlike in the FST version, where one can map underlying categorial information to the empty string $\epsilon$ on the surface, in our one-level version this information would again be visible in the surface string. As a consequence, the above preprocessing step needs to be slightly modified to tolerate categorial information in those self loops that are attached to final states. Finally, the parse string itself constitutes an unconfirmed hypothesis that needs verification by independently produced grammatical and lexical resources. This means that – at least if self-loop enrichments are present in the grammar – it is necessary to formally mark each segment of the parse as a consumer. Only when parse-segments-as-consumer-hypotheses intersect with matching producer segments from the lexicon, will they survive phase II of the two-stage evaluation procedure outlined in §4.3.

The following definition of a parse operator in (27) accurately reflects the preceding discussion. Because it makes use of the notational format of the Prolog-based FSA toolbox that will only be introduced later in §5, the reader is urged to come back to this section on a second reading. Note in particular the interspersed self loops, defined via the Kleene star operator $*$ and the intersection $\&$ of the preprocessed ($\text{ParseString}$) with grammar and lexicon.

(27) **PARSING (IN THE ABSENCE OF OPTIMIZATION)**

```prolog
preprocessed([SurfaceSegment|RestSegments]) :=
    preprocess(RestSegments, SurfaceSegment).

preprocess([], LastSegment) :=
    [consumer(LastSegment \& segment),
     consumer((technical_symbols \& categorial_information)) *].

preprocess([CurrentSegment|RestSegments], PreviousSegment) :=
    [consumer(technical_symbols) *,
     consumer(PreviousSegment \& segment) |
     preprocess(RestSegment, CurrentSegment)].

parse(ParseString) :=
    closed_interpretation(preprocessed(ParseString) \&
     cache(grammar_and_lexicon)).
```

As is to be expected, extending the OLPM parsing task to cover optimization adds new complications. We can no longer be sure that a nonzero intersection with grammar and lexicon signals grammaticality; such a result merely means that the parse string is consistent with one member of the set of alternatives to be optimized over. To be sure, this is still a welcome improvement over the parsing problem that would obtain in an all-default framework like OT, where the notion of consistency plays no role at all. However, it means that a second step must be added to the parse step from (27).
That step consists first of the extraction of categorial information from the annotated parse string and then using that information to generate an optimal surface result via application of Bounded Local Optimization. If this optimal result and the preprocessed parse string intersect, fine; if not, it means that the parse string is ungrammatical. Extraction itself can be performed by composing (\( o \)) the annotated parse string with a simple transducer that maps segmental symbols to their maximally underspecified representatives and preserves the identity of all other symbols. With some caching of intermediate results we can prevent doing double work in our optimizing parser. Note also the use of Bounded Local Optimization \( \text{blo} \) which needs to know its LookaheadConstant. Here then again comes a code fragment that shows \( \text{optimizing\_parse} \) in all its glory:

\[
\text{(28) PARSING IN THE PRESENCE OF OPTIMIZATION}
\]

\[
\begin{align*}
\text{extraction} & := [ \{ \text{identity(consumer\_repeat))}, \ % \text{type identity} \\
& \text{identity(consumer\_skip))}, \ % \ ... \ \text{ditto} \\
& \text{identity(consumer\_segment))}\ % \ ... \ \text{ditto} \\
\} *, \\
\{@:\} * \ % \text{token identity} \\
\ % \text{elsewhere, i.e.} \\
\ % \text{in mapping} \\
\ % \text{categorial info!} \\
\}.
\end{align*}
\]

\[
\begin{align*}
\text{optimizing\_parse(String, LookaheadConstant)} & := \\
\text{blo( ( cache(parse(String)) } \\
\text{ o } \\
\text{extraction} \\
\text{ ) } \\
\text{ & grammar\_and\_lexicon, } \\
\text{LookaheadConstant) } \\
\text{ & parse(String).}
\end{align*}
\]

We could call the preceding proposal a kind of analysis-by-synthesis approach (cf. Walther 1998 for further discussion in a feature-logical setting). Given these initial results, there clearly is a need for further research into parsing under optimization. In particular, one should investigate its efficiency in realistic cases and conduct a careful implementation that makes use of lazy automaton intersection (Mohri, Pereira & Riley 1998).

5 Implemented Case Studies

In this section I will discuss worked examples from three languages that illustrate the interplay of the various enrichments and mechanisms proposed above. To be maximally concrete, snippets from the actual implementation are provided. The notational format is that of the FSA Utilities toolbox (van Noord 1997), a subset of which is depicted in (29).
(29) **Format of regular expression operators**

| Operator | Description |
|----------|-------------|
| `[]`     | empty string |
| `{}`     | empty language |
| **Lower:Upper** | pair |
| `[E1,E2, ...,En]` | concatenation of E1, E2, ..., En |
| `{E1,E2, ...,En}` | union of E1, E2, ..., En |
| `E*`     | Kleene closure |
| `E+`     | Kleene plus ([E, E*]) |
| `E^`     | optionality |
| `E1 \& E2` | intersection |
| `RelA o RelB` | composition |
| `identity(E)` | identity transduction |

Significantly for our purposes, FSA Utilities offers the possibility to define new regular expression operators. Departing from the original `macro(Head,Body)` notation I use the infix expression `Head := Body` – to be read as “Head is substituted by Body” – for reasons of better readability. Macro definitions may be parametrized with the help of Prolog variables in order to define new regular expression operators in terms of existing ones. Also, Prolog hooks in the form of definite-clause attachments are provided to help construct more complicated expressions which would be too cumbersome to build using the above facilities alone. Finally, it is sometimes of importance that regular expression operators are alternatively definable through direct manipulation of the underlying automata. Again, here the toolbox provides abstract data types that support access to alphabets, states, transitions etc.

I will take liberty in sometimes suppressing macro definitions whose details are not essential to the discussion at hand, resorting to descriptions in prose instead. Also, for the sake of brevity the type hierarchy that structures the alphabet will not be displayed separately, which can be justified on the ground of mnemonic type names that make it obvious what the hierarchy would be like.

### 5.1 Ulwa construct state infixation

Ulwa is an endangered Misumalpan language spoken in Eastern Nicaragua. The purpose of this section is to analyze the placement of possessive infixes in nouns, since “Ulwa serves as a nice example of a language in which infixation is clearly sensitive to prosodic structure” Sproat (1992, 49). While Ulwa data have been discussed in the literature for some time (e.g. McCarthy & Prince 1993 and Sproat 1992), an up-to-date descriptive reference has only recently become available (Green 1999). Green shows that Ulwa nouns can participate in a syntactic construction called *construct state*, “a cover term for an entire paradigm of genitive agreement inflection” (ibid., 78) where the head noun is marked morphologically by affixation. The affix shows inflection for person and number (30). The primary semantics expressed by the construct is possession.

(30) **Forms of the Construct-state Affix**

| Person | sg. | pl.   |   |
|--------|-----|-------|---|
| 1st    | -ki-| -ki-na| exclusive |
| 2nd    | -mi-| -mi-na| inclusive |
| 3rd    | -ka-| -ka-na| |

(31) shows some data for the third person singular affix (-)ka-, collected from McCarthy & Prince (1993, 105) and Sproat (1992, 49) and checked against the dictionary in appendix B of Green (1999).²⁴

²⁴Sproat additionally cites *gaad, gaad-ka* ‘God’, while Green’s dictionary completely lacks g-initial words; this is because he con-
Long vowels are represented as VV, which both simplifies the statement of heaviness and eases the actual analysis.

(31) **ULWA CONSTRUCT STATE SUFFIXATION/INFIXATION**

| N       | his/her/its N |
|---------|---------------|
| bas     | bas-ka       | ‘hair’ |
| kii     | kii-ka       | ‘stone, rock’ |
| taim    | taim-ka      | ‘time’ *(preferred: aakatka)* |
| sapaa   | sapaa-ka     | ‘forehead’ |
| suulu   | suu-ka-lu    | ‘dog’ |
| asna    | as-ka-na     | ‘clothes, dress’ |
| paumak  | pau-ka-mak   | ‘tomato’ |
| waiku   | wai-ka-ku    | ‘moon, month’ |
| siwanak | siwa-ka-nak  | ‘root’ |
| arakbus | arak-ka-bus  | ‘rifle, gun’ *(Spanish: arquebus)* |

According to the descriptions of McCarthy & Prince and Sproat, primary stress in Ulwa falls on the first syllable, if it is heavy, otherwise on the second syllable from the left. In Ulwa, the core syllable template is (C)V(V)(C) with a small set of exceptions that exhibit complex onsets or codas. Syllables count as heavy iff they are either closed off by at least one consonant *(bas)* or contain more than one vowel *(kii,pau,taim)*. Monosyllabic words are always heavy.

From this description and the data in (31) alone it would follow that the possessive affix is invariably located after the stressed syllable, emerging as a suffix after heavy monosyllables and as an infix otherwise. Note that the affix itself is never stressed. The immediate goals of the analysis to be developed below will then be to formalize both this stress distribution and the morphemes involved.

Before we can do that, however, we should note that the fuller picture that Green (1999) presents both for the infixation/suffixation behaviour and the stress facts does add some complications. There are many cases of free variation between suffixation and infixation *(kubalamh-ki ∼ kuba-ki-lamh ‘butterfly’)*. Stems show two-way exceptions, some taking suffixes exclusively although infixation should *a priori* be allowed *(tiwiliski-ka, *tiwi-ka-liski ‘sandpiper’)*, a small set also tolerating infixation although it should *a priori* be ruled out *(ta-ka-pas ‘mouth’)*. The same goes for stress which can sometimes oscillate *(‘baka, ba’kaa ‘child’)*, while on other occasions preceding *(‘sarīq ‘avocado’) or following *(taslaawan ‘needlefish’) the locus predicted above. Interestingly, construct state formation may involve accentuation of the affix in a few exceptional cases *(ma-ka-lnak ‘payment’), can even disambiguate alternating stress *(ba’kaa-ka, *baka-ka ‘child’) and cause stress shift in a number of pseudo-reduplicative root shapes *(ki’iliilih kili’ih-ka ‘cicada’)*. We refer the reader to Green’s extensive discussion for further study, concentrating on the core cases in the analysis to follow.

In a first step, the original, disjunctive formulation of the stress generalization can be simplified by stating that *the syllable containing the second mora from the left must be stressed*. According to moraic theory *(Hayes 1995)*, a mora μ is an abstract unit of syllabic weight which figures prominently in the analysis of stress systems of many of the world’s languages. Thus, reference to moras is wellfounded in our context. To exemplify: baσμμ, taσμμ, amσμμ, aμσμμ all receive stress on the first syllable, while saσμμ, paσμμ, kuσμμ, luσμμ, kσμμ are accented on the second syllable.

To facilitate identification of moras, we will take an intermediate step by tagging each segment with the *relative difference in sonority*. That is, we will mark whether sonority is rising, falling or level when comparing each segment with its right neighbour. Recall that sonority is an abstract measure of intrinsic prominence for speech sounds. While it is customary to employ sonority for determining full syllable

---

cludes that – given only a single native counterexample, *aaguguh ‘song,sing’ ~ /g/ is not a phoneme of Ulwa. There is no contradiction here because Green acknowledges that /g/ exists in a few obvious loan words. McCarthy & Prince erroneously cite the form *kula*ka-luk from pseudo-reduplicative kulu-ku-kuk ‘lineated woodpecker’, which Green (1999, 54f) marks as ungrammatical since “speakers seem to recognize them as reduplicative in form, making these stems resist the infixation process.”
structure, which in turn then serves as the input to foot structure and stress computation, it is possible to bypass all higher-level structure in the case at hand.\(^\text{25}\) Also, for current purposes we can conflate most of the distinctions of Blevins (1995, 211)’s nine-positional sonority scale, keeping only consonant ≪ vowel. Given that scale, each segment is tagged with one of \{up, down\}, where the tag depends on the sonority value of its right neighbour: if the right segment’s prominence is higher, up is used, while down is assigned in the case of lower or same sonority. The final segment, which has no natural right neighbour, is marked with down. To exemplify: the Ulwa word for ‘clothes’ will be tagged \(a_{\text{down}}s_{\text{down}}n_{\text{up}}a_{\text{down}}\) and ‘stone’ is marked as \(k_{\text{up}}i_{\text{down}}i_{\text{down}}\). The crucial observation now is that moraic segments are exactly those that are tagged with down.

This observation has obvious repercussions on the formalization of Ulwa stress below:

\[
\begin{align*}
\text{material\_is}(&\text{Spec}) := [\text{consumer}(\text{Spec}) \,*]. \\
\text{stress} := &\text{sonority\_differences} \& \\
&[\text{pre\_main\_stress}, \text{main\_stress} +, \text{post\_main\_stress} ^ ] . \\
\text{pre\_main\_stress} := &[\text{non\_moraic} *, \text{mora}, \text{non\_moraic} *] \& \\
&\text{material\_is}(\text{unstressed}). \\
\text{post\_main\_stress} := &[\text{non\_moraic}, \text{material\_is}(\text{anything})] \& \\
&\text{material\_is}(\text{unstressed}). \\
\text{non\_moraic} := &\text{consumer}(\text{up}). \\
\text{mora}(&\text{Spec}) := [\text{consumer}(\text{down} \& \text{Spec})]. \\
\text{mora} := &\text{mora}(\text{anything}). \\
\text{main\_stress} := &\text{mora}(\text{stressed}).
\end{align*}
\]

Unsurprisingly, stress is built upon computation of sonority\_differences. Note next how the stress pattern itself is initially decomposed into zero or more unstressed non-moraic onset segments followed by the first mora followed by more non-moraic material (pre\_main\_stress). Thereafter comes an obligatory stretch of stressed moraic material delimited by an optional block of post-stress segments whose start is signalled by a non-moraic segment. Observe that there will be multiple adjacent stress marks if the syllable hosting the second mora is not monomoraic. In other words: the whole rime of the accentuated syllable is formally marked as stressed (e.g. \(b_{\text{unstressed}}a_{\text{stressed}}s_{\text{stressed}}\)), in what amounts to the explicit linear equivalent of the stress feature percolation or structural referral to \(\sigma_s\), that is implicit in more traditional approaches. Also, stress is contingent on the availability of independently introduced lexical material, hence everything is encoded as consumer-type information.

With stress assignment already given, it is now fairly easy to define the affix itself.

\[
\begin{align*}
\text{possessive\_third\_singular} := \\
&\text{add\_repeats}(\text{contiguous}([\text{consumer}(\text{stressed}), \\
&\text{producer}(k \& \text{unstressed}), \text{producer}(a \& \text{unstressed})])).
\end{align*}
\]

Its segmental content \(ka\) is simultaneously marked as unstressed, in accordance with the surface facts (modulo the small number of exceptional words of type \(ta\-\text{kua-pas}\) mentioned above, for which an allomorph would have to be set up). Of course, this is producer information. Additionally, in this analysis the affix receives a prosodic subcategorization frame: its left context restriction mentions an immediately adjacent stressed segment. As a contextual requirement, it must be encoded using the consumer macro. The whole tripositional sequence is wrapped with two more macros: contiguous introduces edge-only self loops

\(^{25}\)As an aside, note that evidence for higher-level prosodic structure above the syllable role level is often surprisingly weak; in stark contrast to the wholesale adoption of the entire prosodic hierarchy (McCarty & Prince 1991) throughout most of the generative literature. In the case of Ulwa, for example, Green (1999, 64) admits that iterativity in noun stress – usually held to be a basic reflex of foot formation – rests on inconclusive evidence from three forms only.

Also, for a full phonological grammar encompassing syllabification, computation of relative sonority differences is independently needed, since it forms an essential first step in the declarative syllabification schemes of Walther (1993), Walther (1997). While these were originally couched in feature logic, a non-weighted finite-state version is both easy to implement and attractive due to its conceptual simplicity, as there is no need for a simulation of the Maximum Onset Principle (which complicated Mohri, Riley & Sproat (1996, 139)’s weighted finite-state syllabification for Spanish). However, the details are beyond the scope of this paper.
to permit infixal behaviour on the one hand while disallowing internal breakup of -ka- on the other hand.

Ob top of this add_repeats modifies the automaton as described in §4 to allow for possible uses in reduplication. According to Green (1999), Ulwa does indeed exhibit reduplicative constructions, although the details are beyond the scope of this section.

Encoding of stems is not complicated, yet contains some points worth noting:

\[
\text{discontiguous_lexeme}(L) := \\
\text{add_repeats} (\text{discontiguous}(\text{lexeme}(L) \& \text{stress})) . \\
hair := \text{discontiguous_lexeme}("bas"). \\
forehead := \text{discontiguous_lexeme}("sapaa"). \\
root := \text{discontiguous_lexeme}("siwanak"). \\
gun := \text{discontiguous_lexeme}("arakbus").
\]

The first point is that, of course, stems must tolerate discontiguity in order to host infixes: discontiguous therefore adds self loops at all positions, not only at the edges.\(^{26}\)

The innermost lexeme macro converts a Prolog string into a concatenation of producer-type segmental positions, working off the assumption that all symbols represent defined segmental types.

Now the most interesting second aspect is that the undecorated string automaton corresponding to the lexeme itself is intersected with the stress constraint. This is nothing but the constraint-based equivalent of a lexical rule application for stress assignment, applied to stems in isolation. The reason for assuming lexically stressed stems is that we want to rule out coalescence, i.e. amalgamation of segmental material, between affix and stem segments. As the affix contains the segments \(k\) and \(a\), it could in principle overlap the \(k\) in ‘gun’ (*ara-ka-bus) or the (pen)ultimate \(a\) in ‘forehead’ (*sap-ka-a, *sapa-ka). Note that such overlapping placement would still satisfy the affix’s prosodic requirement, as the immediately preceding segment in these examples is indeed the second mora from the left and therefore would be surface-stressed. To be sure, coalescence as such is attested in other languages (e.g. in Tigrinya, Walther 1997). However, it must be forbidden in our Ulwa construction. The solution is now easy to understand: by lexically stressing the stems, the left context of the infix in the ungrammatical coalescent realizations of ‘gun’ and ‘forehead’ is fixed to unstressed, hence will properly conflict with the stressed requirement of the possessive affix and lead to the elimination of the illformed disjuncts. Note that another, morphological solution to the coalescence problem would have been to tag affix and stems contrastively, e.g. as \(-k_a a-\) versus \(s_s a_s p_s a_s\) (cf. Ellison 1993). Interestingly, at least in this case such purely technical diacritics seem not be required; rather, we can profit from a clean phonological solution.

There is one last aspect which needs our attention, and that concerns banning discontiguous main stress. Even with lexical stress assignment, a heavy stem syllable such as \(á\)s receives two formal stress marks, and – being interruptible – could therefore satisfy the prosodic requirements of the possessive in two ways: \(*á-ka-š, \! láš-ka-\). Continuing with our phonological solution, the cure is again immediate: surface and lexical stress must not conflict! Since the formulation of stress ensures contiguous main stress by way of the kleene plus operator, and because it is descriptively true that lexical stem stress coincides with the word-based surface stress pattern (again modulo the exceptions noted above), we can simply impose the stress constraint once again on the whole infixed word to ensure full wellformedness of the Ulwa possessive construction:

\[
\text{word}(\text{Stem}) := \text{Stem} \& \text{possessive_third_singular} \& \text{stress}. \\
\text{stems} := \{\text{hair, forehead, root, gun}\}. \\
\text{possessive_nouns} := \text{closed_interpretation}(\text{word}(\text{stems})).
\]

Note that it is the intersection of all constraints that defines a word. After expanding that macro in the body of possessive_nouns with (the disjunction of) defined stems as actual argument together with pruning away of unsatisfied consumer arcs through closed_interpretation, we arrive at an automaton

\(^{26}\)Recall that this refers to the core cases; exceptions that ban infixation would require a different treatment. One way would be to parametrize discontiguous for the actual content of self loops – currently the entire alphabet \(\Sigma\) – which then could be restricted to technical symbols that are incompatible with the segmental content of affixes.
which contains exactly the desired grammatical surface strings \{baska, sapaaka, siwakanak, arakkabus\}, of course enriched with stress and sonority information.

At this point an additional remark seems appropriate. Thomas Green’s reference work summarizes the underlying cause of the construct-state phenomenon in derivational parlance as follows: “... the construct morphology itself does not receive stress, and does not cause shifts in the stresses of the material which follows it... it is as if the infixation takes place at a point in the derivation after the metrical structure [i.e., stress, M.W.] of the word has been determined” (Green 1999, 64f). I take it to be quite satisfying that the iterative process of formal grammar development, while seemingly being driven by technical problems very unlike those of a descriptive grammarian, nevertheless led to the same fundamental conclusions.

The observant reader will have noted that the analysis presented so far is not based on the notion of drift, hence does not need to make any use of optimization and the concomitant representational extensions. As promised in §4, we will now show what the drift-based alternative looks like. As it turns out, the experience is highly instructive and sheds new light on the pros and cons of optimization.

The first step in such an optimization-based analysis of the same facts of Ulwa construct-state infixation consists in distributing the appropriate weights to both the affix and the stem, which in turn is contingent on the direction of drift we would like to see in the data. Contrary to what McCarthy & Prince (1993, 107) suggested with their use of the RIGHTSMOSTNESS OT constraint, I would argue that the proper direction is leftward. The reasoning is as follows: with lexical stem stress given as before, a leftward-drifting affix will correctly ‘float’ towards the accented position. To prevent the affix from floating past that position – which would immediately cause disruption of the lexical stress pattern – we again simply impose the same stress constraint on the surface word form. In order to formalize leftward drift, the affix material must be weighted cheaper than the stem, hence we will use two weights-as-types unmarked ≪ marked to that effect.

Here then is the exchanged portion of the stem-defining macros:

\[
\text{discontiguous\_lexeme}(L) :=
\text{add\_repeats(\text{discontiguous}(\text{lexeme}(L) \& \text{material\_is(marked) \& stress}))}.
\]

The crucial difference to the previous analysis now is that the affix does not need to be prosodically subcategorized! Here is the new definition:

\[
\text{possessive\_third\_singular} :=
\text{add\_repeats(\text{contiguous}(\text{lexeme("ka") \& \text{material\_is(unmarked \& unstressed}))}.}
\]

The unmarked tagging of affixal tagging is the hallmark of leftward drift, as expected; no further mention of stress is needed. The definition of word does not need any changes, since as already noted the same need of avoiding surface discontiguities in main stress arises in the present analysis. The only remaining difference is, of course, that bounded local optimization (blo, with look-ahead 1) needs to be formally applied on the resulting, minimized (mb) automaton:

\[
\text{optimized\_possessive\_noun}(\text{Stem}) :=
\text{blo(mb(closed\_interpretation(word(Stem))),1)).}
\]

With stems like hair, gun etc. as actual parameters, the same results obtain as in the previous analysis. It is instructive to see what the automaton for Ulwa ‘gun’ – encoding \{*arakbuska, *arakbukas, arakkabus\} – looks like before optimization (32).
Note how at each bifurcation in the graph the immediate alternative is between an unmarked affix versus a marked stem segment; hence look-ahead \( k = 1 \) indeed suffices here. Also, as a byproduct of pruning arc \( 4 \rightarrow 5 \) first, the algorithm in (22) will never explore the set of states \( \{5, 7, 9, 10, 13\} \); thus the incomplete search embodied in BLO does indeed bear fruit in practical cases.

With both analyses in place, let us sum up. The drift-based optimizing analysis probably holds some special appeal to theoretical linguists because one can dispense with affix-specific prosodic subcategorization and cling to the ‘lean lexicon’ view that is popular in much of generative linguistics. Moreover, it could be argued to be slightly more explanatory in that it sees nondisruption of lexical stress, a global desideratum for words, as the sole prosodic factor which drives the placement of Ulwa construct-state affixes.\(^2\)

\(^2\)With only slight changes to the formulation of the stress constraint, one could go even further and claim that the unstressedness of the construct-state inflectional affixes itself is derivable as well: having less than two moras, CV syllables like \( ki, ka, ma \) will not
One disadvantage of this analysis from a formal point of view is that it requires a canonical, i.e. minimized automaton format for application of BLO. BLO itself must be counted as an additional ingredient in the analysis which furthermore precludes simple computation of a whole lexicon, as noted above.

The non-optimizing alternative, on the other hand, avoids all the problems associated with optimization, but at the expense of greater representational cost: the affix specification requires explicit mention of the left-adjacent stress peak. This in turn makes it harder to justify why exactly this subcategorization happens to be crucial in Ulwa morphology, and why e.g. unstressedness on the third segment to the right would not be an equally plausible candidate for a possible prosodic subcategorization. On the positive side one should realize that local, surface-detectable properties such as adjacency to a stress peak would probably be rather easily learnable for both child and machine. If proven to be feasible in future research, such a learnability result would have an important impact on the ongoing debate about richness of lexical representations versus the need for optimization.

5.2 German hypocoristic truncation

We briefly mentioned in (1) that German provides a productive form of truncation for hypocoristic forms of proper names. Taking up the subject again, let us look at a representative sample of the data in (33).

(33) **GERMAN HYPOCORISTIC I-TRUNCATION**
   a. Petra /ˈpɛtra/ > Pet-i
   b. Andreas /ˈanədʁəs/ > And-i
   c. Gabriele /ɡaˈbriːəl/ > Gab-i
   d. Patrizia /ˈpɑtrɪtsia/ > Patt-i
   e. Gorbatschow /ˈɡɔrbaʧəʊ̯/ > Chruschtschow /ˈkruʃtʃəʊ̯/ > Chruschtsch-i
   f. Gorb-i
   g. Imke /ˈɪmka/ > Imk-i
   h. Hans /ˈhɑnts/ > Hans-i

All derived forms in (33) end in -i, and all polysyllabic ones truncate some portion of their base (truncated part shown in boldface).

Previous analyses so far have sought to establish a connection between the cutoff point in truncation and syllable structure. Neef (1996) and Werner (1996) proposed that the initial portion of the base preceding the -i must form a ‘potential maximal syllable’. The qualification ‘potential’ is significant here, because – as Gabr.ile ‘female first name’ vs. Gorbatschow ‘Gorbatchev’ show – reference to actual base syllabifications would make wrong predictions (*Gor-i, *Gabr-i, because .Gorb. is a maximal syllable, but *Gabr. with reversed consonantal cluster is not). However, (33).f,g show that the maximal syllable approach misses some crucial data: *Chruschtsch., and *.Imk. are illformed as syllables of German, yet their i-suffixed versions are the correct hypocoristic forms. (33).f also rules out another proposal (Féry 1997), namely that the relevant criterion should instead be one of ‘simple second syllable onset’: Chruschs.i has a complex two-member onset /tʃ/. In contrast to these proposals I claim that the simplest correct analysis is again one which makes direct use of the subsyllabic concept of sonority. To see the plausibility of this claim, let us first assume that the sonority scale for German is as follows (Wiese 1995):

obstruents ≪ nasals ≪ laterals ≪ rhotics ≪ high vowels ≪ nonhigh vowels

**Graphing sonority over time for the critical example Chruschtschow /kruʃtʃəʊ̯/ in (34),**
receive primary stress when viewed in isolation, e.g. in their lexical entry form!
we note next that the last base segment to be retained in the hypocoristic form is the second /p/, which
is located at the first sonority minimum. A sonority minimum is defined as a segmental position at which
sonority goes upwards to the right while it does not rise from the left. Observe that the leftmostness ex-
pressed by ‘first’ is not redundant for bases with at least two syllables, because suffixing the characteristic
ending -i to the unaltered base would create another minimum. Inspection of the other forms in (33) reveals
that the sonority-minimum criterion in fact forms a surface-true generalization over German hypocoristic
truncations.28

Given our sonority-based generalization, we can now put the pieces together to create a formal analysis.
Here are high-level definitions for our example stem and the hypocoristic constraint:

\[
\text{chruschtschow} := \text{contiguous\_lexeme("kRUStSOf")}.
\]
\[
\text{hypocoristic} := \text{sonority\_differences} \&
\begin{align*}
&[\text{sonority\_based\_cutoff\_point}, \\
&\text{truncated\_part}, \\
&\text{characteristic\_ending}].
\end{align*}
\]

Although all we really ask of a truncatable representation is tolerance of skipping, chruschtschow is
defined as a contiguous\_lexeme for better reuse of existing macros. Stems will be intersected with
a hypocoristic constraint which – apart from tagging segments with sonority\_differences –
dissects a word into an initial portion up to a sonority\_based\_cutoff\_point, followed by a possibly
empty (\textit{Hans-i}) truncated part and finally an obligatory characteristic\_ending. These macros are
in turn defined as follows:

\[
\text{sonority\_based\_cutoff\_point} := \text{first\_}(\text{sonority\_minimum}).
\]
\[
\text{truncated\_part} := [\text{producer(skip)} \,*].
\]
\[
\text{characteristic\_ending} := \text{producer(i)}.
\]
\[
\text{sonority\_minimum} := [\text{consumer(segment & \textasciicircum up)}, \text{consumer(up)}].
\]
\[
\text{first\_}(X) := [\text{not\_contains}(X), X].
\]

28There are, however, three kinds of exceptions: (i) base portion is not string prefix of base word: \textit{Birgit} > \textit{Bigg-i}, \textit{Birg-i} (CIVIC2C3 \ldots \rightarrow CIVIC3 \ldots) \textit{Elisabeth} > \textit{Liss-i}, \textit{Barbara} > \textit{Babs-i} (ii) base portion extends past sonority minimum: \textit{Depressiver} > \textit{Depr-i} (\textit{?Depp-i}); \textit{Asphalt-i}, \textit{Bankrott-i}, \textit{Beideut-i}, \textit{Altemaiti-i}, \textit{Komp\textasciitildepost-i}, \textit{El\textasciitildegant-i}, \textit{Ersitzemai\textasciitildet-i} (iii) base portion stops before minimum: \textit{West/Ostdeutscher} > \textit{Wess-i/Oss-i}, *\textit{West-i/Ost-i}, \textit{Hunderter} > \textit{Hunn-i}. As far as I can tell, these exceptions
are a problem for all other published analyses as well, which is to be expected given the intimate connection between sonority graphs
and syllable structure. Neglected factors playing a role here seem to include the influence of morphological structure in the form of
(pseudo)compounding and avoidance of homonymy, the position of main stress, details of phonetic realization and recognizability of
the base. Therefore the proposed analysis should be viewed as the core component of a more complete account that integrates those
additional factors.
Of these, only the definition of \texttt{first} is not entirely straightforward. The idea here is to establish leftmostness by excluding via \texttt{not contains} any occurrences of a particular set of strings \texttt{X} in the – possibly empty – material \texttt{preceding} the realization of \texttt{X}.

Testing the definitions so far reveals an interesting deficiency of the analysis as it stands: it contains an unformalized hidden assumption of longest-match behaviour! To see this, consider in (35) the automaton that \texttt{closed interpretation(chruschtschow & hypocoristic)} evaluates to.

\begin{equation}
(35) \quad \text{UNOPTIMIZED AUTOMATON FOR HYPOCORISTIC Chruschtschow}
\end{equation}

Besides the correct realization, two alternative paths starting at nodes 3 and 4 mark unwanted realizations that truncate earlier within the medial consonant cluster \texttt{/fj}. These alternatives occur (i) because sonority differences are computed over the \texttt{truncated} surface form, (ii) the amount of truncation is left indeterminate by \texttt{truncated part}, and (iii) a stem-internal \texttt{(up)}\textsuperscript{n} up configuration like our \texttt{Udown plateau plateau up} provides \texttt{n} possible truncation points satisfying \texttt{sonority based cutoff point}, with no preference given to any of them. Now Karttunen (1996) reports an interesting simulation of longest-match behaviour for rewrite rules using only finite-state machinery. Unfortunately, it appears that his results are crucially dependent on the ability of finite-state transducers to describe two-level correspondences, precluding a transfer of his results to the present monstral setting. Fortunately, we already have another formal device to express preferences, namely Bounded Local Optimization. Observe that the unwanted alternatives are distinguished from their grammatical counterpart in that their length-2 subpaths \texttt{3 \to 5 \to 7, 4 \to 7 \to 8} all contain a \texttt{skip} – the hallmark of truncation – whereas the corresponding grammatical paths \texttt{3 \to 4 \to 6, 4 \to 6 \to 8} are ‘better’ insofar as they contain only proper segmental material. Therefore a weightscale \texttt{segment < skip} which feeds into look-ahead-2 application of Bounded Local Optimization \texttt{blo} solves our problem by effectively preferring late truncation. This leads us to the final formulation of \texttt{i_formation}:

\begin{verbatim}
i_formation :=
blo(mb(closed_interpretation(chruschtschow & hypocoristic)),2).
\end{verbatim}

At this point the alert reader may start to wonder whether German hypocoristic truncation admits a non-optimizing alternative analysis similar in spirit to Ulwa. The short answer is yes, but with some extra
subtleties. Again, such an alternative involves lexical constraint application, this time pertaining to the prespecification of sonority differences in stems.

The crucial observation leading to this is that too-short truncations necessarily entail a conflict between the sonority curves of isolated stems and their truncated remainder in the i-suffixed word forms: the vocalic suffix provides a new right context which affects the sonority difference of its consonantal left neighbour. For example, in $k_{up}^{down} plateau_{up}^{down} down$ the first $/i/ is on a sonority plateau because of neighbouring /t/ in the stem, but will be tagged with the conflicting value $up$ when higher-sonority /i/ forms the new right context: $*k_{up}^{down} f_{plateau}^{up} i_{down}$.

However, lexical tagging of sonority differences must be applied cautiously in our surface-true setting because of monosyllabic words like Hans $\sim$ Hans-i: since computation of individual sonority differences requires examination of the right context segment, the sonority difference of the last stem segment – which lacks a natural context – must be lexically underspecified to avoid a conflict $down \neq up$ whenever -$i$ adds some further word-level context. Fortunately, this demand corresponds well to a modular version of sonority differences, which separates the boundary condition responsible for tagging a word's last segment with down from the plain sonority differences that take care of the rest. Also, we must make sure that lexical tagging does see a non-truncatable, skip-free representation of the stem in order to avoid inadvertent tagging of the set of all possible truncations. The last concern is adressed by devising a separate add_skips regular expression operator which is applied on the result of lexical tagging of skip-free stem material. This much being said, we can now present the alternative analysis:

\[
\text{technical_symbols} := \text{material_is}\{(\text{skip};\text{repeat})\}.
\]

\[
\text{sonority_differences} :=
\begin{align*}
&\text{ignore} \left( \text{plain_sonority_differences} \& \text{boundary_conditions}, \\
&\text{technical_symbols} \right).
\end{align*}
\]

\[
\text{stringToSegments}([],[]).
\]

\[
\text{stringToSegments}(\text{[ASCII|Codes]}, \text{[producer(Segment)|Segments]}) :-
\begin{align*}
&\text{name(Segment, [ASCII])}, \\
&\text{stringToSegments}(\text{Codes, Segments}).
\end{align*}
\]

\[
\text{(stem(String) := add_repeats(contiguous(add_skips(}
\begin{align*}
&\text{Segments \& plain_sonority_differences))))) :-
\begin{align*}
&\text{stringToSegments(String, Segments)}.
\end{align*}
\]

\[
\text{lexicon} := \{ \text{stem("kRUSTSOf"), stem("hans") } \}.
\]

\[
\text{non_optimizing_i_formation} :=
\begin{align*}
&\text{closed_interpretation(lexicon \& hypocoristic)}.
\end{align*}
\]

In order to be surface-true despite occasional occurrences of interspersed technical_symbols, in particular skip, sonority_differences jumps over sequences of such symbols with the help of the built-in ignore operator. Note also that the stem macro makes use of the Prolog hook facilities (\text{:- stringToSegments(...)}) to synthesize a regular expression denoting the concatenation of $N$ producer-type segments from a more convenient String description of length $N$, transferring the result via another Prolog variable Segments. Finally, observe that the definition of hypocoristic has been left unchanged, hence in particular imposing its word-level view of the sonority_differences to fully specify even the last segment.

The pros and cons of the two alternative treatments of German hypocoristic truncation are pretty much the same as in the case of Ulwa. In particular, as indicated in the code fragment above, we can again compute the results of hypocoristc formation over the entire lexicon in one fell swoop. Like in Ulwa, careful lexical enrichment was the key to achieve the non-optimizing analysis for German. Although a full conclusion seems premature at this point, the need for much more careful argumentation is only too apparent when it comes to defending an all-optimization framework for phonological and morphological analysis.
5.3 Tagalog overapplying reduplication

Tagalog, an Austronesian language with 14,850,000 first language speakers, is the national language of the Philippines (Grimes 1996). The language is of particular interest to reduplication theorists because of its wide use of reduplication in productive word formation, coupled with interesting instances of overapplication of nasal assimilation and coalescence processes on a par with normal application of an intervocalic flapping rule. While the specific Tagalog instance was already noted by Bloomfield (1933, 221f), it was Wilbur (1973) who coined the generic term overapplication in her pioneering thesis. There she described a class of reduplicative processes interacting with phonological rule applications, where a particular change to the base effected by a phonological rule is mirrored in the reduplicant and vice versa, although only one of these constituents actually provides the context required by the rule. In derivational terms the rule is therefore said to overapply even where its context is not met, whereas with normal application this behaviour would be ruled out. (The converse case of underapplication exists as well, even in Tagalog, but will not be discussed further due to lack of space). With the advent of constraint-based theories the mechanisms for analyzing overapplicational reduplication have of course changed, but the terminology is still widely used for convenience.

For information on Tagalog reduplication in the context of various generative analyses, see in particular Carrier (1979), Marantz (1982), Lieber (1990, ch.4), McCarthy & Prince (1995). According to these sources, Tagalog has three major reduplication patterns, termed RA, R1, R2 in Carrier (1979). Type RA prefixes the initial CV portion of the base, accompanied by lengthening of the reduplicant vowel, while R1 insists on a short vowel in the same CV reduplicant shape. Type R2 copies longer portions of the base, sometimes the entire stem. These three patterns function in a variety of word formation processes, and their semantic contribution can only be evaluated with reference to the accompanying affixes and other aspects of morphological structure. For example, while any Tagalog verb can undergo RA reduplication to receive aspectual marking, RA is interpreted as causative aspect only together with the prefix na-ka-, while conveying future aspect in conjunction with certain subject-topic markers such as mag-, -um-. Note that Tagalog verb forms always require at least one topic marker. The actual array of permissible topic markers must be lexically specified for each verbal stem.

Because RA reduplication – due to lengthening – shows extra material not present in the base, and because it can exhibit both overapplication and normal application effects, it seems to have just the right amount of ‘real-life’ complexity for an illustrative implementation. Hence we will focus on this type in what follows.

In (36) we provide a relevant sample of the data (ST/DOT abbreviate Subject/Direct Object Topic markers).

(36) Tagalog CV: reduplication

|   |   |   |   |
|---|---|---|---|
| a. | mag-liinis ‘ST-clean’ | b. | mag-li-linis ‘ST-will clean’ |
| c. | mag-bukas ‘open’ | d. | mag-bu-bukas ‘will open’ |
| e. | na-bukas ‘opened’ | f. | na-bu-bukas ‘is/was opening’ |
| g. | na-bu-ka?-antok | h. | na-ka-ka?-antok ‘causing sleepiness’ |
| i. | ?i-pa-?i-paq-bilih ‘DOT-will sell’ | j. | ma-?i-?i-paq-bilih ‘will manage to clean for’ |
| | ?i-paq-li-bilih | ma-?i-?i-paq-liinis | ma-?i-paq-li-linis |
| k. | p-um-idit ‘one who compelled’ | l. | nag-pu-p-um-idit ‘one who makes extreme effort’ |

We can see in (36).a-f that RA reduplication has copied stem material and that its meaning contribution varies as a function of the segmental prefix. However, in (36).h-l we find that the same reduplicative process can also repeat affixal material, once again underscoring the phonological character of reduplicative copying. Most interestingly, with more than one prefix we get free variation as to which part is reduplicated (36).i,j, with only the leftmost affix being exempted from reduplication. To complete the case against a putative morphological characterization of RA reduplication, (36).l shows an example where the reduplicant...
freely combines stem and infix segments. As mentioned before, Tagalog also has an alternation known in derivational terms as Nasal Substitution (37).

(37) Nasal Substitution and Assimilation

| Example | Meaning |
|---------|---------|
| /maŋ-bili/ → manilih | ‘ST-shop’ |
| /maŋ-dikit/ → manikit | ‘ST-get thoroughly stuck’ |
| /maŋ-basah/ → mambasah | ‘ST-read’ |
| /maŋ-chukut/ → manchukut | ‘ST-pick pockets’ |
| /maŋ-kaŋ-dikit/ → maŋ-kan-dikit | ‘ST-get stuck accidently as a result of’ |

The first entries (37).a,b show an assimilation of the place features of the prefix-final nasal to the following stop, coupled with a coalescence of the two segmental positions into just one (the substitution). However, the coalescence part of the alternation is subject to two-way exceptions: not only do certain bases resist coalescence in the presence of coalesceable prefixes like /maŋ- (37).c,d, but also there are other prefixes like mag- which – although they share the final velar nasal – do not coalesce under concatenation with the right bases (37).e. We will have to take care of this lexical conditioning in the analysis to follow.

The interaction of Nasal Substitution and RA reduplication now produces the overapplication effects mentioned above (38).

(38) Reduplication and Overapplying Nasal Substitution

| Example | Meaning |
|---------|---------|
| /pa-mu-partul/ → pa-mu-mu-partul | ‘that used for cutting’ |
| /maŋ-kaʔilan/ → maŋ-ʔiʔilan | ‘ST-will need’ |
| /maŋ-pulah/ → maŋ-ʔiʔilan | ‘will turn red’ |

Although only the leftmost instance of the triggering plosive is local to the prefix-final nasal, its second occurrence must be realized as a place-assimilated nasal as well: nasal substitution overapplies. To complicate matters, however, it must be noted that this kind of long-distance dependency between base and reduplicant segments is restricted to certain segmental classes: like in the rest of morphology (39).a,b, two occurrences of dental stops created by one of the reduplication patterns can become dissimilated through intervocalic flapping, a normally applying phonological ‘rule’ (39).c-e.

(39) d ~ r: Normal Application of Intervocalic Flapping

| Example | Meaning |
|---------|---------|
| /dəntot/ | ‘stinginess’ |
| /ma-ramboj/ | ‘stingy’ |
| /man-da-ramboj/ | ‘bandit’ |
| /sumd-sum-in/ | ‘no gloss’ |
| /d-um-ʔiʔ-datij/ | ‘attends now and then’ |

With this last piece of evidence on Tagalog reduplication we have now assembled enough data and generalizations to proceed to the analysis itself.

---

29Bloomfield (1933, 221f) cites the contrasting case of [tæɔwa] ‘a laugh’, with [tætæɔwa] ‘one who will laugh’ turning into [tumactæɔwa] ‘one who is laughing’. Although here it appears as if RA reduplication would not always copy right-adjacent material – skipping over infixed -um- in this case – we will in fact assume that [tætæɔwa] and similar cases are lexicalized reduplications, i.e. new stems, to which -um- infixation regularly applies. If this assumption turned out to be wrong, one would have to make the RA reduplicant discontiguous and condition the contrastive behaviour of pairs like [maŋ-por-ʔiʔ-ʔiʔ] vs. [ʔi-um-ʔiʔ-tæɔwa] with the help of suitable morphological features.
Because our aim is to model RA reduplication, let us start with various macro specifications which help implement the necessary synchronisation of morpheme edges. The idea here is that all non-floating, i.e. more or less concatenative morphemes have their left edge synchronized (type synced) and their interior material unsynchronized (~synced), while only the right edge of the stem is synchronized as well; bound morphemes remain unsynchronized. Floating morphemes like Tagalog’s famous -um- would be special in that they would be underspecified for synchronization, to be compatible with whatever landing site their prosodic conditions demand; however, we disregard the additional complexity for the fragment under development. Here is the code portion for synchronisation (applications of it will appear in later code snippets):

```plaintext
synced_position(Spec) := consumer(synced & Spec).
synced_position := synced_position(anything).
synced_producer(Spec) := producer(synced & Spec).

unsynced_position(Spec) := consumer(~ synced).
unsynced_position := unsynced_position(anything).
unsynced_portion := [unsynced_position *].
left_synced_portion := [synced_position, unsynced_portion].
synced_constituent := [left_synced_portion, synced_position].
```

Next we need to turn to the representational needs of stems. They should of course be reduplicatable, internally discontiguous (for later tolerance of floating morphemes), and synchronized at both edges. While non-stop-initial stems like linis need only basic segmental specifications in addition to these needs to complete their definition, stems like pulah, dikit, ka’tilanan require additional means to implement the alternating behaviour of their initial plosives. Recall that in Declarative Phonology destructive processes like the one hinted at by ‘nasal substitution’ are impossible to represent literally, hence must give way to alternative representational treatments. The solution here is to underspecify the manner features signalling obstruenthood and (non-)voicing in one alternant of the initial segment’s specification, with a fully specified stop constituting the other alternant. In a cooperative fashion, coalesceable /C/-final prefixes will then supply the missing nasal manner feature to guarantee full specification after the intersection of all participating morphemes-as-constraints has been done. Finally, to model the default stand-alone realization as a plain stop, the fully specified alternant is encoded as a producer, whereas the ‘cooperative’, underspecified alternative is specified as a consumer. With these definitions in place, a wide range of stems can now receive their representations with ease:

```plaintext
discontiguous_stem(Segments) :=
    add_repeats(contiguous(internally_discontiguous(Segments) & synced_constituent)).
underspecified(BaseSpec,AddedForFullSpec) :=
    { producer(BaseSpec & AddedForFullSpec),
      consumer(BaseSpec) }.

bihi :=
    discontiguous_stem([underspecified(labial,obstruent & voiced),
```

Note that internally_discontiguous is a variant of the discontiguous operator introduced in earlier analyses that spares both start and final states from the self-loop enrichment. It follows that we can safely intersect such a representation with additional constraints like our synchronized constituent without fear of inadvertent constraining effects on peripheral material introduced outside of the morphemic domain under construction. In particular, here we want the edges of a morpheme to be synchronized, not the edges of the whole word. Upon wrapping the result with the previously defined contiguous operator, which introduces self loops to the start and final states only, we then have an appropriately conditioned partial description of an entire word with the required tolerance of other morphemes that may be present.
stringToSegments("ilih").

pulah := discontiguous_stem(
    [underspecified(labial, obstruent & ˜ voiced),
     stringToSegments("ulah")].

dikit := discontiguous_stem(
    [underspecified(dental, obstruent & voiced),
     stringToSegments("ikit")].

kaqilanan := discontiguous_stem(
    [underspecified(dorsal, obstruent & ˜ voiced),
     stringToSegments("aqilanan")].

basah := discontiguous_stem(stringToSegments("basah").

dukut := discontiguous_stem(stringToSegments("dukut").

guloh := discontiguous_stem(stringToSegments("guloh").

laakad := discontiguous_stem(stringToSegments("laakad").

linis := discontiguous_stem(stringToSegments("linis").

dambong := discontiguous_stem([producer(voiced & dental),
                                stringToSegments("amboN")].

With the previous remarks on underspecification as a suitable strategy for (many) seemingly destructive alternations, the reader will have no difficulty to understand the definition for dambong, not mentioned before; here again missing manner features will be supplied from later constraints to guarantee full specification of its initial voiced dental, fleshing it out either as a stop or as a flap.

With the definitions for stems in place, let us concentrate next on interesting prefixes, the foremost of which are the n-final ones. They present a good case for Ellison (1993)’s claim that intersective morpheme combination is often to be preferred over concatenative one, since part of their definition is the nasalhood imputed on suitable stem-initial consonants (producer(nasal)), i.e., an underspecified contextual restriction that must be realized outside of their own morphemic interval. Because they are also ordinary segmental prefixes, however, these morphemes are specified as contiguous synced constituents. What makes them special is that – even when coalescence of nasal and stop features is not possible, like in *ma-masah – the nasal must assimilate in place to the following obstruental stop, if any (otherwise receiving a default place, namely dorsal); hence the other prefix-final alternant labelled assimilated nasal obstruental sequence.31 A non-coalescing prefix like magkang differs minimally in dispensing with the underspecified nasal alternant, but retains the assimilation part, as demanded by the data in (37):

ng_final_prefix(String, SpecifiedAlternant) :=
    contiguous(synced_constituent &
               [stringToSegments(String),
                SpecifiedAlternant,

31A subtle detail needs our attention here: the conjunction of synced_constituent with the two-way alternating segmental specification correctly marks the rightmost segment as synced, irrespective of the ultimate length of the prefix (length 2 for the coalescent case, length 3 for the pure assimilation case). This rightmost segment is nothing else but the beginning of the stem, hence correct intermorphemic alignment can be ensured with the help of one additional constraint (to be detailed later) which governs the correct distribution of synchronisation marks.
The place-assimilating behaviour itself must be simulated in a piecewise fashion for all the possible place categories involved (place([labial, dental, dorsal])), because of the well-known lack of token identity in regular description languages. We can regain some expressivity, though, by making use of a recursive macro assimilation_for to off-line-synthesize the iterated disjunction from the list of places of articulation. The trick is to exploit the fact that Prolog is the host language of our finite-state toolkit, which means that we can use token identity at the description level, though not at the object level. Employing the power of logical variables we therefore distribute the variable SharedCategory to each disjunct (which will of course become bound at compile time).

Putting together the pieces in assimilated_nasal_obstruent_sequence again requires a judicious use of producer- and consumer-type information to distinguish the lexical contribution of the morpheme itself – which is the nasal part – from the contextual requirement participating in assimilation – which is the obstruent part. Finally, a default disjunct encodes the dorsal realization that is observed whenever a suitable obstruent stop is lacking:

Before we proceed to add the reduplicative part to our growing Tagalog fragment, let us test the definitions so far to obtain some highly instructive intermediate results. With a little additional code shown below to ensure wellformed synchronization, we can in fact form meaningful intersections like word(mang & bilih).

```prolog
matching_synchronisation :=
    not_contains2(two_synced_segments_in_a_row).

two_synced_segments_in_a_row :=
    [consumer(synced), consumer(synced)].

generic_word_constraints := matching_synchronisation.

word(Expr) :=
    closed_interpretation(Expr & generic_word_constraints).
```
However, the surprising result is that – with coalescable stems and alternating η-final prefixes – we actually get two results (40).

(40) /maη-bilih/: NASAL COALESCEENCE PROBLEM

Besides the desired coalescing alternant m (2 → 4) we get an unwanted alternative mb (2 → 3 → 4) that does not merge the two segmental positions. Some reflection reveals that this behaviour is unavoidable given our modelling assumptions about independent and compositional specification of morphemes. Since the prefix must combine with both the nasal-substituting bilih and the non-substituting basah, the sequence mb cannot be ruled out categorically, hence both alternants must remain in the denotation of the affix. Likewise, an alternating stem like bilih needs to tolerate non-nasal realizations of its first segment in addition to the coalescent nasal case, to cater for isolated pronunciation or other prefixal options. If we devise an abstract feature [± coalesce] to characterize both stems and prefixes, and relate the alternating case to an underspecified feature value [0 coalesce], we can illustrate the scenario with the following paradigm (41).

(41) PREFIX-STEM COMBINATION PARADIGM

| Prefix → Stem | [− coalesce] | [0 coalesce] |
|---------------|-------------|-------------|
| [− coalesce]  | magkam-basah| mam-basah   |
| [− coalesce]  | magkam-bilih| ma-milih    |
| [0 coalesce]  |              | [+ coalesce]|

The paradigm reveals that we actually demand full specification from the (intersective) combination of two underspecified features [0 coalesce], an impossibility in a monotonic setting where underspecification is equivalent to a (systematic) disjunction of fully specified disjuncts. This state of affairs has been noticed before: replacing ‘coalesce’ with δ, the featural part of the paradigm turns out to be a verbatim copy of the one discussed in (Ellison 1994a, p.31, ex. (15)) under the general rubric “paradigm[s] that” might be decomposable into morphemes only with the use of defaults.”! With Tagalog Nasal Substitution we have merely uncovered a first concrete instance of the abstract case predicted by Ellison. The default in our case would be [+ coalesce].

Fortunately, optimization again comes to our rescue to implement the required default behaviour. Note that the default preference for coalescence amounts to an eager choice of synchronized morpheme beginnings in our representational setting. This is illustrated well in the critical choice between unsynchronized m:0 → 3 and synchronized m:1 → 4 in (40). Therefore, Bounded Local Optimization parametrized with look-ahead 1 and a weight scale ¬sched > sched completely solves our little problem.

With Nasal Substitution handled on its own, let us move on towards a declarative formalization of type RA reduplication with overapplication. Because we again see reduplication as partial specification
of an entire word, we construct it as follows. Before the copy we need to expect at least one (prefixal) morpheme, possibly more (36). After it we must allow for the rest of the base and also tolerate whatever morphemic material finishes off the word (especially in the case where one of the prefixes is targeted for reduplication and the stem must still be incorporated). Within the reduplicant-base compound we need to enforce a kind of long-distance agreement between the quality of the first reduplicated segment and its base counterpart to model ‘overapplication’. However, as we have seen in the discussion of flapping, in a surface-true perspective this kind of agreement must be carefully constructed to hold only over certain subcategories of the segmental makeup and spare the dimensions involved in (non-)flapping.

On the face of it, the type RA_reduplicant itself copies the first two base segments – which happen to form a CV sequence – and lengthens the V. To do so formally, we can identify the initial base segment as a synced_position, hence its successor is of course an unsynced_position. For lengthening of the vowel, we have chosen a formalization which makes use of the repeat arcs to step back one position and then proceed forwards again by providing for an abstract vowel position, which can only be the vowel we have targeted already before. After that, a kleene-plus-wrapped producer(repeat) steps all the way back to the beginning of the base, which again is characterizable as a synchronized segment. However, to make sure that consumer-type underspecified alternants as contained in the first segment of bilih will survive closed interpretation, we have been careful here to specify this beginning of the base proper with a matching synced_producer. Like in the nasal assimilation instance, the segmental positions agreeing in an overapplicational manner are again abstracted out via the logical variable AgreeingFeatures. Enforcing the agreement in a reasonably perspicuous way is done via the Prolog hook mechanism of PSA Utilities; the respective predicate enforce_agreement_in_/3 constructs an iterated disjunction abbreviating the repeated realization of the abstract reduplicant parametrized for each of the five values of the overapplicational categories.

This extensionalization of agreement is indeed the price to pay for a framework that does not handle (possibly overridable) true token identity at the object level, as demonstrated by (Beesley 1998). That paper is relevant here because it contains a careful study of the problem of long-distance morphosyntactic dependencies in a finite-state framework together with potential remedies; one could indeed apply some of the solutions proposed there to our present task of modelling nonlocal phonological dependencies. In particular, the use of weakly non-finite-state enhancements like global registers that can be set (for initializing an agreeing feature) and tested against (an existing feature value) should be a profitable amendment, once residual problems with automaton transformations like minimization in the face of enhanced automata have been settled.

Here then is the central code portion responsible for Tagalog RA reduplication:

```prolog
some_morpheme := left_synced_portion.
pre_base_morphemes := [some_morpheme +].
rest_of_base := unsynced_portion.
rest_of_word := [some_morpheme *, synced_position].

( ra_reduplicated_word := [pre_base_morphemes, Reduplicant,
  rest_of_base, rest_of_word]) :-
  enforce_agreement_in_(type_RA_reduplicant,
  [ nasal, % 1: /m,n,N/
    " nasal & " voiced & labial, % 2: e.g. /p/
    " nasal & " voiced & " labial, % 3: e.g. /t/
    " nasal & voiced & labial, % 4: e.g. /b/
    " nasal & voiced & " labial], % 5: e.g. /d/
Reduplicant).

type_RA_reduplicant(AgreeingFeatures) :=
  [ synced_position(AgreeingFeatures),
    unsynced_position,producer(repeat),
    unsynced_position(vowel), producer(repeat) +,
    synced_producer(AgreeingFeatures) ].
```

45
enforce_agreement_in_(MacroName, [], `{}`).
enforce_agreement_in_(MacroName, [Cat|Cats],
    { MacroInstance, MoreMacroInstances }) :-
    MacroInstance =.. [MacroName,Cat],
enforce_agreement_in_(MacroName, Cats, MoreMacroInstances).

optimal_word(Expr) :=
    blo(mb(closed_interpretation(Expr & word)),1).

We are now in a position to show in (42) the actual outcome of an automaton specification such as
optimal_word(mang & bilih & ra_reduplicated_word)

(42) /maŋ-RA-bilih/ → [mamı:mılıh]: RA, OVERAPPLYING

With the addition of some ordinary prefixes we can even model the free variation in reduplication
arising from the presence of multiple prefixes. For ease of exposition these will be straightforwardly concatenated [mag, qi, pag, ... ] rather than being intersected. The intersective modelling variant would
make use of the method proposed in Ellison (1993) to simulate concatenation with the help of suitable
tagged representations.

prefix(String) :=
    add_repeats([stringToSegments(String) & some_morpheme,
                 material_is(anything)]).

qi := prefix("qi"). % 'q' denotes glottal stop
pag := prefix("pag").
ma := prefix("ma").

The result of word([ma, qi, pag, linis] & ra_reduplicated_word) is depicted in (43).
It turns out that we need to do something else to preserve this result under BLO application (optimal_word(...)). The reason is that we will otherwise lose one of the alternants due to a nonsensical comparison to a repeat-initiated alternative path, no matter what weight we assign to repeat: it cooccurs with both the larger ($4 \rightarrow 5$) and the smaller weight ($7 \rightarrow 9$) in the automaton of (43). This then is a perfect case for applying the slight modification to weight computation proposed on page 20, fn. 20: because we want repeat arcs to behave as inert with respect to BLO, we assign them infinite negative weight, thereby exempting them from ever getting pruned. Remember that this welcome result follows because the essence of the modification is that weight-sum comparison now only looks for better positive alternatives.

The final piece of the analysis handles the normal application of the flapping ‘rule’: only intervocalic voiced dentals are flapped [r], elsewhere they are pronounced as a stop [d]. In the context of reduplication the rule applies uniformly to both base and reduplicant, which translates into simple intersection of a surface-true constraint flap_distribution with the word. Here it is actually helpful that our framework is limited to identity, since the repetition of previous material will not copy particular allophonic choices in either base or reduplicant, allowing the flap constraint to rule freely. This constraint is defined below:

\[
\begin{align*}
\text{nonflapped_intervocalic_dental_stop} & := \\
& [\text{consumer(vowel)}, \text{consumer(d)}, \text{consumer(vowel)}]. \\
\text{preCflap} & := [\text{consumer(flap)}, \text{consumer(\neg vowel & segment)}]. \\
\text{postCflap} & := [\text{consumer(\neg vowel & segment)}, \text{consumer(flap)}]. \\
\text{no_peripheral_occurrence_of(Segment)} & := \\
& ([\text{consumer(segment & \neg Segment)}, [\text{material_is(segment)}, \\
& \text{consumer(segment & \neg Segment)}] ^\top ^\top). \\
\text{ignore_intervening_technical_symbols(Expr)} & := \\
& \text{ignore(Expr, material_is(misc))}. 
\end{align*}
\]
flap_distribution :=
    ignore_intervening_technical_symbols(
        not_contains3(nonflapped_intervocalic_dental_stop) &
        not_contains2(preCflap) &
        not_contains2(postCflap) &
        no_peripheral_occurrence_of(flap)).

Observe that the constraint is build up (in an admittedly somewhat roundabout way) from negative subconstraints banning intuitively sensible subconfigurations where a flap must not occur. This is done by way of parametrized constraints not contains2/3 that ban occurrences of its length-2/3 string arguments (details suppressed for the sake of readability). After that has been done, the resulting automaton is modified to ignore_intervening technical symbols, which will occur especially in reduplication.

With flap distribution in place, we have removed the last barrier to a fairly complete account of type RA reduplication, as we can now demonstrate even the correct outcome of

    optimal_word(mang & dambong & ra_reduplicated_word &
        flap_distribution),

which in compact regular expression notation is

    [m,a,n,d,a,repeat,a,repeat,repeat,repeat *, ’D’,a,m,b,o,’N’].

Note both the nonflapped d that starts off the reduplicant after the consonant-final prefix and the flapping of ’D’ in the base, which correctly happens even across intervening repeat symbols, because vowels a_a form its ultimate segmental context.

6 Discussion

As we have seen in previous sections, the one-level approach presented in this paper offers solutions to a wide range of analytical problems that arise in modelling prosodic phenomena in morphology. Enriched representations allow for repetition, skipping and insertion of segmental material. The non-finite-state operation of reduplicative copying is mapped to automata intersection, an operation whose formal power is also beyond finite-state, but which is independently needed as the most basic mechanism to combine constraints. The distinction between contextual requirements and genuine lexical contribution is reflected in differentiating between consumers and producers of information, a move that introduces resource consciousness into automata. Bounded local optimization supports the formulation of optimizing analyses that seem to be called for in a range of empirical cases. A generic recipe for flattening prosodic constituency provides the basis for maximally local access to higher phonological structure. The architecture supports both generation and parsing. Finally, the proposal proved its worth in three practical, computer-implemented case studies.

As is to be expected with any new proposal, however, there are areas that deserve further investigation, limitations that need to be overcame and alternative design decisions that should be explored.

First, the repeat-arc enrichment that facilitates reduplication assumed that base strings are separately enriched, and not their union as a whole, because only then can inadvertent repetition of parts of some other base be excluded. Somewhat loosely speaking we have relied on the fact that the weaker notion of type identity available in finite-state networks can still mimic token identity if there is only one type to consider, i.e. in moving back and forth in time one sees only one base string at a time. A drawback of this mode of enrichment is that the set of base strings cannot be compressed much further, as would be possible in other cases of finite-state modelling. However, recall that the repeat-arc encoding imposes a highly regular layer onto existing base lexicons. What is regular is also predictable, hence can be virtualized: one could keep a minimized base lexicon and devise an on-the-fly implementation of the add repeats regular expression operator to add repeat arcs on demand only to the currently pursued base hypothesis. With suitable diacritics one could even extend such an approach to languages where exceptions that fail to reduplicate must be taken into account. A variant of that approach would add global register-manipulating instructions to choice arcs.
in the minimized base lexicon, e.g. Beesley (1998)’s unify-test. Injecting a little bit of memory into automata in this way would serve to synchronize choices in the base and reduplicant part of the word form, acting as a kind of distributed disjunction. Because the same instruction would be used twice in repetition-as-intersection, the use of a (limited kind of) unification is indeed necessary here. It remains to be seen whether such a mixed approach would be feasible in practice.

Also, from the point of view of competence-based grammar, the proposed encoding for reduplication is not as restricted as one might wish, because it can also encode unattested types of reduplication such as mirror image copying $u w^R$, a context-free language. Here is the essential part of an actual piece of code showing how to do it:

\begin{verbatim}
realize_base := [boundarySymbol,interior_part,boundarySymbol].
mirror_image_total_reduplication :=
  [realize_base, [repeat,repeat,contentSymbol] *,
   repeat, repeat, boundarySymbol, skip *].
\end{verbatim}

When intersecting with an enriched version of a string like \([a,b,c,d]\) (bracketed with boundarySymbol), each character of the mirror-image part \([d,c,b,a]\) will be preceded by two repeat symbols. Moreover, because we will ultimately have stepped back to the beginning of the string, we finally need to skip over the entire base to avoid the realization of an additional conventional reduplicant. This additional effort suggest one way to explain the absence of mirror-image reduplication, namely by extending the general notion of resource consciousness: traversing an arc carries a certain cost and there will be a maximum cost threshold in production and acquisition. Under this view mirror-image reduplication fares significantly worse than its naturally occurring competitor because it needs three times the amount of technical arcs. Clearly, this sketch of a performance-based explanation of an important gap in natural language patterns of reduplication needs to be worked out more fully in future research, but the initial line of attack seems promising.

Another point worth noting is that the repeat-arc encoding is not intimately tied to one-level automata. This is to be expected given the fact that transducer composition $\circ$ can simulate intersection $A \& B$ via $\text{identity}(A) \circ \text{identity}(B)$. As demonstrated in appendix B, a version of our proposal that works for transducers is readily constructed for those who prefer to work in a multilevel framework.

Finally, one might worry about the apparent need for a greater number of online operations, especially runtime intersection. The radical alternative, relying completely on offline computations, seems rather unattractive in terms of storage cost and account of productivity for cases like reduplication in Bambara, Indonesian, etc. However, we believe that with lazy, on-the-fly algorithms for intersection etc. the runtime cost can still be kept at a moderate level. This is an interesting area for further experimentation, especially when representative benchmarking procedures can be found.

There is also room for variation with respect to the ideas about resource-conscious automata. First, instead of distinguishing between producers and consumers on the level of entire segmental symbol, one could employ a finer subdivision into feature-level producers and consumers. So e.g. a segment might produce a nasal feature but consume place-of-articulation features delivered from elsewhere.

Second, one might play with changing the logic of producer/consumer combination. For example, in a stricter setting corresponding to linear logic, resources could be consumed only once and leftover unconsumed resources would result in ungrammaticality. One potential application area might be to prevent coalescent overlap, signalled by a doubly produced resource at the position of coalescence. However, the intuitionistic behaviour embodied in the present proposal seems to fit more naturally with the demands of productive reduplication, where base segments are consumed two or even more times. In contrast, linear behaviour would require explicit re-production of consumed resources here. More analyses need to be undertaken to find out whether both types of behaviour (or even more) are needed to formulate elegant grammars of a wider range of phenomena, or whether a single uniform combinatory logic suffices.

Finally, there is a possibility that resource consciousness of the kind proposed here may be reducible to classical arc intersection via global grammar analysis/transformation. This is not altogether implausible given the fact that LFG’s bipartite setup of constraint and constraining equations is now seen as involving resource-conscious notions, while at the same time a theoretical reduction of LFG grammars to simpler
ones employing only conventional constraint equations (i.e. pure unification grammars) exists. To be sure, even if such a reduction would prove to be feasible in our case, the value of resource-consciousness as a high level concept would still be unaffected.

For Bounded Local Optimization, a brief note will do, namely that its local search for minimal-weight arcs can profitably be integrated with the elimination of consumer-only arcs in phase II of grammatical evaluation. As a result, we now need only one pass over (a fraction of) the automaton, with a subsequent gain in efficiency.

In conclusion, while there are certainly areas in need of future research, the one-level approach to prosodic morphology presented in this paper already offers an attractive way of extending finite-state techniques to difficult phenomena that hitherto resisted elegant computational analyses.

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A Alternative method: 
Lexical transducer plus intelligent copy module

Finite-state transducers are currently the most popular implementation device in application-oriented computational morphology, mainly for reasons of low time complexity, reversibility for generation and parsing and the existence of minimization methods. They are used to model both the basic lexicon and its additional morphological and phonological variation. However, to date there exists no proposal that can handle reduplicative morphology in general, apart from isolated attempts at simpler, rather local instances (cf. a Tagalog case modelled by Antworth 1990). In view of the fact that the copy language \textit{ww} is context sensitive, some compromise or approximation must obviously be found, at least when the finite-state assumption is to be maintained. The following ideas form part of such an approximative solution.

The first part of the solution separates the copying aspect from finite-state-based lexicon and underlying-to-surface variation.

(44) \textsc{separate copy} \\
$\quad\rightarrow$ lexicon + phonology/morphology (finite-state) $\rightarrow$ copy $\rightarrow$ surface form  \\

To preserve regularity, the copy must be made outside of the lexicon + phonology FSA/FST. Since there are languages where phonological rules apply to a reduplicated form, the copy should not be done before the phonology. Also, while conceptually lexicon and phonology may be separated, in practice they are normally composed to form a single lexical transducer (Karttunen 1994). The key advantage of this compilation step is that a possibly exponential growth of the rule transducer is prevented in practice through the limiting context provided by the lexicon. However, having only a single lexical transducer means that an intermediate stage is no longer available for copying before applying the phonology.

Having secured the proper place of a copy module after the lexical FST or FSA, we next need to devise a way to control copying. Since we cannot model the nested nonlocal dependencies exhibited by \textit{ww}-type reduplicative constructions directly, let us instead encode a segment-local promise to reduplicate, to be fulfilled by the copy module. In (45) we see a first example from German.

(45) segment-local encoding of multiple realisations

\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
\textit{surface} & \textit{techtelmechtel}  \\
\textit{encoding} & \textit{segment string} & t & e & c & h & t & e & l & m  \\
\hline
\# copies & & 1 & 2 & 2 & 2 & 2 & 2 & 2 & -1  \\
\hline
\end{tabular}

In a nutshell, the copy module now gets complex symbols, which consist of a segment proper and an annotation for the number of copies to be made. The module scans a string of complex symbols from left to right, outputting each segment that has a nonzero number of copies. By convention, whenever the number of copies is negative, a new scan is started after outputting the current segment. The new scan performs the same actions as before, except that it first decrements the value of the number of copies when positive while incrementing when negative. Thus, in our example we have:

(46) stepwise derivation of example (45)

\begin{tabular}{|c|c|c|}
\hline
\textit{scan} & \textit{input} & \textit{output}  \\
\hline
1 & segment string & techteilm  \\
& \# copies & 1 2 2 2 2 2 2 -1  \\
\hline
2 & segment string & echtel  \\
& \# copies & 0 1 1 1 1 1 0  \\
\hline
\end{tabular}
Note that rescans are only initiated upon encountering a negative number, so that incrementing the last occurrence of $-1$ suffices to stop rescanning. Here then is a more systematic tabulation of proper usage of the above encoding (47), followed by pseudocode of the algorithm interpreting it (48).

(47) **NUMBER-OF-COPIES ENCODING SCHEMA**

| output segment | Scan 1 | Scan 2 | #copies |
|----------------|--------|--------|---------|
| yes            | yes    |        | 2       |
| yes            | no     |        | 1       |
| no             | yes    |        | 0       |

(48) **COPY ALGORITHM**

```plaintext
input string of symbols Input[1 . . . N] condition length(Input) > 0
ScanPos ← TempPos ← LastRescanPos ← 0
AtEnd ← length(Input)
repeat
    ScanPos ← ScanPos + 1
    CurrentSymbol ← Input[ScanPos]
    if CurrentSymbol.NumCopies ≠ 0 then
        output CurrentSymbol.Segment
    endif
    if CurrentSymbol.NumCopies < 0 then
        Rescan ← true
        TempPos ← ScanPos
        ScanPos ← LastRescanPos
        LastRescanPos ← TempPos
        if CurrentSymbol.NumCopies = -1 then
            LastRescanPos ← TempPos
        endif
    else
        Rescan ← false
    endif
    if CurrentSymbol.NumCopies > 0 then
        CurrentSymbol.NumCopies ← CurrentSymbol.NumCopies − 1
    else
        CurrentSymbol.NumCopies ← CurrentSymbol.NumCopies + 1
    endif
until ScanPos = AtEnd ∧ ¬ Rescan
```

It is not difficult to see that the algorithm always terminates. Informally, because every input string is of finite length and the number of copies associated with each segmental positions must become positive in a finite number of steps (by way of the last if-then-else statement), the number of rescans initiated by negative number-of-copies annotations will be finite as well.

What are salient properties of the solution just proposed? It is

- **dynamic**, i.e. the surface form of a given input token can be computed by the algorithm, while the set of all such surface forms is not statically represented
- **not persistently linearised**, i.e. the complete surface linearization of the input cannot be read off the input easily using only finite-state mechanisms
- **not uniquely encoded**, i.e. there is a countably infinite number of possible input codes for any given surface form. The chief source of infiniteness comes from 0-annotated material that is outside...
of a rescan domain, for example \(<t_1e_2c_2b_2t_2e_2l_2m_1x_0x_0x_0 \ldots >\). In this way one may add an arbitrary amount of underlying material that will never be output by the copy algorithm.

Let us comment on some of these aspects. The dynamic aspect seems unavoidable as long as the finite-state assumption is upheld in its most restricted form, i.e. there are no online operations with greater-than-finite-state power like composition, intersection etc. during parsing and generation.

The next point about full linearization being performed outside of phonology proper, however, causes more concern. One reason is that there are cases where the shape of reduplicants depends on their surface position, as we have seen in Koasati (recall the contrast between penultimate heavy syllables in ak-ho-lat.lin vs. tahas-\(\text{-}t\)oo-\(\text{-}t\)in). Of course this is just an instance of the general fact that phonological alternations often depend on surface syllable structure. But those surface syllables may cut through the ‘folded’ encoding of inputs: e.g. \(<\text{tech},\text{tel},\text{mech},\text{tel}>\) has \(<\text{mech}>\) as the third syllable, which happens to be nonlocally represented under the input encoding \(<t_1e_2c_2b_2t_2e_2l_2m_1>\).

Also, the example of Washo shows that surface-derived syllable roles may differ between reduplicant and base, with concomitant phonological consequences (data taken from Wilbur 1973, 17):

\[
\text{(49) Surface Linearization and Washo Reduplication}
\]

\begin{align*}
\text{wetwedi} & \quad \text{‘it’s quacking’} \\
\text{fupjab} & \quad \text{‘he’s crying gently’} \\
\text{tum/sopso} & \quad \text{‘he’s splashing his feet’} \\
\text{b’akbagi} & \quad \text{‘he’s smoking’}
\end{align*}

In the data under (49) we can see that obstruents get devoiced in the coda. Crucially, however, the relevant codas are indirectly created by reduplication: a \(C_1VC_2\) reduplicant prefixed to the base results in a \(VCCV\) context that \(\text{surface-based syllabification resolves by assigning coda-onset roles to the } CC\text{ cluster. Since } C_2\text{ in the base ends up being in an onset position, it is not devoiced, resulting in the phonological difference between reduplicant and base underlined in (49)}.\)

Washo seems to present a problem for our proposed input encoding: when using a representation like \(<w_2c_2d_2i_1>\) that only covers the underlying base, the independence between base and reduplicant required for phonological differences is lost. However, the last aspect about the non-uniqueness of input encodings – which has not been illustrated so far – actually provides hope for such situations. Let us therefore have a look at some alternative encodings for reduplicative forms in (50). Example (50).a2 demonstrates an alternative, more local encoding for \(\text{techtelmechel}\) that brings original /t/ and its modification /m/ into close proximity. It also allows to syllabify the string /mechel/, as if it were a surface string – e.g. using a finite-state version of the proposals of Walther (1992) et seq. Unlike (50).a2 the critical /m/ will then become part of a complex onset, thus correctly receiving the same syllable role as in the linearized surface form. The examples (50).b1,2 show that locality vs nonlocality is not the only source of ambiguity in the linearization code: original-modification pairs such as \(<c_1a_0>\) may be reversed in reduplicative constructions without affecting the surface result. In general precedence relationships in the input string are only relevant if the corresponding annotations target the same scans. The next set of examples (50).c1-4 shows a broad range of options differing in the placement of fixed material. (50).c1 is approximately surface-true in so far as it has the /olo/ in the penultimate syllable position. However, it simultaneously serves to again show the potential for non-surface truth within individual syllable role assignments: /\&l/ will be assigned an onset role, but the surface realization has a coda instead. Example (50).c2 illustrates a different compromise in that it sees all of the reduplicative affix as a non-distributed prefix. Example (50).c3 shows a slightly odd analysis that denies reduplicative status to \(\text{tahastoo}\) in annotating an input suffix to end up as an infix after linearization. Finally, (50).c4 separates reduplicated and fixed-melody material, seeing only the latter as suffixed to the input base. The last two encodings share the property of leaving the base uninterrupted, which may be advantageous in terms of compression degree of the base lexicon. The last pair of examples (50).d1-2 illustrates how to proceed when syllable roles and segmental realization differ between base and copy: instead of incorrectly representing just one token (50).d1, it is sometimes possible to maintain two adjacent copies (50).d2 in such a way that the input encoding itself can be correctly syllabified.

As we have just seen, the encoding is quite versatile. In fact, it is not even limited to describing reduplicative patterns alone but can also handle infixations, circumfixations and truncations. (51) shows a few more examples from previous sections.
Non-Uniqueness in Encoding of Surface Forms

a1. techtelmechtel techtelm 1222222-1
a2. tmechtel 1022222-2
b1. schnickschnack schniack 2222102-2
b2. schniack 2222012-2
c1. tahastoopin tahasoopin 2111-111111

c2. tooahaspin 200111-1111

c3. tahasptoo tahasptoo 111100011-1

c4. tahaspiino tahaspiino 2111-10001-1

d1. *wedwededi wedi 22-21
d2. wetwedi wetedi 22-111

More Encoded Examples

| Encoding | Surface | Encoding | Surface |
|----------|---------|----------|---------|
| wulu     | wuluowulu | krandhi | kanikrandh |
| 2222-1   |         | 202200-1 |         |
| c?eet    | ctc?eet | umbasa  | bumas   |
| 2000-2   |         | 00-1111 |         |
| silin    | silsin  | berrgte  | gebergte |
| 21-211   |         | 00001-1111 |         |
| velo     | velelo  | kuroftjoji | karuftji |
| 12-21    |         | 111111001 |         |

The Bella Coola form silsin in (51) shows that straightforward use of the encoding undermines a succinct analysis of cases where segmental realizations are the same, although the syllable roles differ between a base segment and its corresponding partner in the reduplicant. Assuming the syllabification /silin/, the /l/ is in coda position in the reduplicative prefix (spelled out in scan 1), but in onset position in the base (spelled out in scan 2) – contrary to the unique onset position it receives when directly syllabifying the input /silin/. Of course, by introducing a redundant extra /l/ like in the Washo case we could rescue surface-like syllabification, albeit at the cost of introducing considerable redundancy and with no other motivation but to avoid technical difficulties. (Note that in this case, though, the systematic local copy of a single consonant $C_i;C_{i,\text{copy}}$ in each base would be technically feasible using a finite-state rule). To conclude, while all the information to ultimately deduce surface position – and thus, surface prosodic role – is formally present in the input, it is often difficult to exploit that information. Sometimes at least the individual solutions would seem to include construction-specific, non-surface-referring rules for finite-state syllabification that preclude maximal modularity and component reuse in analyses.

Thus far I have described the proposed local encoding method from a generation or production perspective only, where a single string of complex symbols gets spelled out on the surface. It remains to show that the method is also usable for parsing or perception, where matching against a finite-state-encoded set
of strings is required. Also, finite-state networks are attractive because minimization algorithms exist to
reduce storage requirements, and it is natural to ask whether these continue to produce good results under
the new encoding.

Turning first to the issue of parsing surface forms, let us assume as before that the lexicon takes the
form of an acyclic FSA or FST, using complex symbols to encode segments. In a loop the parsing algorithm
would then read a single surface segment and try to match it to the segment symbol of an arc emanating
from the current state of the network. However, encountering a symbol with zero annotation causes that
arc to be skipped, repeating the matching attempt with the following arc(s). For various reasons it may
become necessary to nondeterministically choose which arc from a set of possible arcs should be followed.
As the linearisation of encoded strings works off a single string only, and furthermore modifies its number-
of-copies annotations during consecutive scans, we need to copy at least the annotation information of the
partial path hypothesis that is currently pursued to a separate buffer. The copying can of course be done
incrementally. All rescans then will only exame this buffer, which contains the current state of number-
of-copies annotations, whereas the network itself remains unmodified. Also, the stored path hypothesis
must be retracted upon backtracking – to be initiated when it turns out that the current hypothesis cannot
be correct because a segment match fails – so that we actually need a stack of partial path buffers. A match
failure pops off one entry, whereas each nondeterministic choice records the arc chosen and provides a new
partial path buffer which initially contains a copy of the previous one. However, upon closer examination
it becomes clear that recording current scan number and ScanPos actually suffice to uniquely determine
the state of the algorithm in (48), hence we might trade (recomputation) time for space in this way. Details
remain to be worked out how to exactly define suitable data structures and the recognition algorithms itself,
but it seems reasonably clear that this can be done.

Actually, a second route to take would be to make the copy algorithm (48) itself reversible, instead of
implementing a separate version for recognition. Borrowing from existing work in Prolog implementation
it seems sufficient to implement a backtrackable destructive assignment operation on top of the backtracking
mechanism that is independently needed for traversing nondeterministic networks. Again, this sketch
obviously needs more detail to facilitate full evaluation.

The final issue worth discussing is minimization and storage requirements for networks encoded
with the proposed method. Obviously, if the same segments carry different number-of-copies annotations,
they are different when viewed as complex symbols. Without further means this simple fact diminishes
the chance for sharing of common substrings. This is illustrated in (52) using the two German words
techtelmechtel, technik.

33Note that with very large networks – such as that employed in AT&T speech applications –, the network data themselves may
reside on read-only disk and not fit into main memory at all. Rather, they will be swapped in on demand through memory-mapped
files. Similarly, networks might be stored in ROM, again necessitating RAM-resident copies to record on-the-fly modifications.

However, only large-scale experiments with lexicons containing reduplicative constructions can tell
whether the loss of sharing compared to a naive full-form lexicon is not outweighed in practice by the
saving that comes with not having to repeat reduplicative parts on the segmental level.

In the face of these initial difficulties, a natural question to ask is this: can we have the best of both
worlds, i.e. minimization results approaching that of non-complex-encoded networks and the space saving that potentially comes with the proposed method? To preview, it seems that only approximative solutions are possible. A first idea that comes to mind is to linearize the encoding, following each segment with the annotation that goes with it, e.g. `t1m0e2c2h2t2e2l2-l`. However, it is easy to see that – given a regular set $S$ containing only even-length strings – the ‘intercalated union’ $U$ of the projection $O$ of $S$ containing only the letters at odd positions (e.g. segments like `tnechtel`) with the projection $E$ containing only even string positions (e.g. annotations like `1022222`) is not equivalent to $S$ itself, thus precluding an approach that would couple separate offline minimization with parallel online traversal, i.e. alternating between the $O$ and $E$ machines depending on string position. In general $U$ is strictly larger than $S$, because choices in $E$ and $O$ are not synchronized. One could try to develop sophisticated schemes that implement a kind of distributed disjunctions to effect synchronization, while simultaneously taking into account that in our application $|E| \ll |U|$. However, in order to decide which synchronized choice to make in $E$ when having done a previous choice at a node $k \in O$, we in general require knowledge of the entire path $p_{k,O}$ travelled so far. Unfortunately, storing such paths and synchronization marks is likely to become costly again.

A second approach to improving minimization quality would consist of building a variant of the network that implements perfect hashing from each word $w_i$ into a natural number $i$ from $1 \ldots N \in \mathbb{N}$, again assuming an acyclic FSA/FST that contains a finite number $N$ of words (see e.g. Daciuk 1998 for an implementation, or US patent 5,551,026 granted August 27, 1996 to Xerox). We could then store the annotation vector $v_i$ for $w_i$ in a table indexed by $i$. Unfortunately, this will definitely increase the amount of search/backtracking in recognition, as the annotations are crucial in defining surface shape, but will not become available incrementally in this approach. It might also not help much in saving storage space unless clever compression is employed for the annotation table.

Another approach sees the annotations as weights in a weighted FSA/FST. Again each word $w_i$ has an annotation vector $v_i$, but now we can view the vectors as being composed from suitable smaller parts distributed over the network so as to facilitate maximal compression. For example, weights could be strings from $(-2|-1|0|1|2)^*$ and the weight-combining operator could be concatenation. There exist minimization methods for the weighted case that essentially comprise two phases (Mohri, Riley & Sproat 1996). The first phase pushes the weights to the beginning of the FSA/FST as far as possible, thus enabling greater similarity – and therefore greater shareability – towards the end. The second phase consists of ordinary FSA/FST minimization. This approach then promises to address the storage issue better than the previous one, although it is similar in that again one has to wait till the end of each input string to get the composed total weight that drives the inverse copy algorithm.

Finally, annotations could simply be output strings in an ordinary FST, more precisely a sequential transducer, where the associated input automaton is deterministic. Mohri (1994) has described an algorithm to make such transducers very compact, based on the closely related idea of pushing partial output strings (represented as members of an enlarged alphabet) towards the initial state of the FST, and then minimizing this FST in the sense of traditional automata minimization. Advantages and disadvantages of this approach seem to be quite similar to the previous one, except that by indentifying output strings with annotations instead of traditional surface strings we have perhaps lost the ability to use canonical underlying representations with morphological and other annotations.

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34Note that this linearized encoding generalizes to finite-state transducers, where Input : Output pairs can be sequentialized as Input : Output to convert the transducer into an ordinary automaton (cf. fig. 9 in US patent 5,625,554 granted to Xerox on April 29, 1997). Of course, application of underlying and surface strings must then skip odd and even arcs to match and also output the following or preceding arc as result.
B Alternative method: Reduplicative copying as transducer composition

```prolog
:- op(1200,xfx,'='). \% separates name and definition of macro
\% expand new macro notation into FSA Utilities format during file read-in
:- multifile (user:term_expansion/2).
user:term_expansion(':='(Head,Body), macro(Head,Body)).
user:term_expansion(':-'(':='(Head,Body), PrologGoals),
(macro(Head,Body) :- PrologGoals)).
\% the grammar proper
\% add_repeats/1 is best defined as manipulation of the underlying automaton:
\% for each noncylic arc A --> B, add B --[/repeat--> A
rx(add_repeats(E), FA) :-
(fsa_regex:rx(E, FA0)),
(fsa_regex:add_symbols([repeat], FA0, FA1)),
(fsa_data:copy_fa_except(transitions, FA1, FA, Transitions0, Transitions)),
findall(trans(From, []/repeat, To), ( member(trans(To, _, From), Transitions0),
To \== From ), RepeatTransitions),
append(Transitions0, RepeatTransitions, Transitions1),
sort(Transitions1, Transitions).
boundary := escape(#).
epsilon := [].
technical_symbols := { boundary, repeat }.
enriched_words([]) := {}.
enriched_words([Word|Words]) :=
    { add_repeats([ epsilon:boundary, word(Word), epsilon:boundary ]),
enriched_words(Words) }.
\% N.B. {} denotes the empty language, [] is the empty string/epsilon. The builtin
\% word(Atom) yields an automaton where the characters of Atom are concatenated
reduplicant := [boundary:epsilon, (? - technical_symbols) *, boundary:epsilon].
\% N.B. ? is the any (meta) symbol, A - B is set difference.
\% Regular languages are automatically coerced to relations/transducers
\% if necessary (e.g. in the context of composition o)
total_reduplication(WordList) :=
enriched_words(WordList)
    o
[reduplicant, repeat:epsilon *, reduplicant].
test := total_reduplication([orang,utan]).
```

% This code assumes FSA Utilities (http://www.let.rug.nl/~vannoord/Fsa/)
\% op(1200,xfx,'='). \% separates name and definition of macro
\% expand new macro notation into FSA Utilities format during file read-in
:- multifile (user:term_expansion/2).
user:term_expansion(':='(Head,Body), macro(Head,Body)).
user:term_expansion(':-'(':='(Head,Body), PrologGoals),
(macro(Head,Body) :- PrologGoals)).