In solving a number of practical problems, one has to work with various random processes that depend on time, such as, for example, conducting medical diagnostics based on the analysis of electrocardiograms (ECG). Different signals can be divided into two groups: stationary and non-stationary. Stationary signals have statistical and spectral characteristics that are constant over time, for a comprehensive description of which the Fourier transform is sufficient. ECG rhythmogram can be attributed to the class of chaotic processes. The characteristics of a chaotic process change over time, and other methods must be used to analyze their dynamics. One such method is wavelet transform. In the course of this work, we propose introducing a new method for assessing the fetal heart rhythm, based on extracting an informative feature from the wavelet bicoherence of the rhythmogram.

A number of recent scientific papers are aimed at developing a method for calculating multispectra using a frequency-time dynamic window [1] - [4]. However, there are several restrictions on the use of such algorithms; in particular, an exact determination of the characteristic time scale is required, taking into account its invariance in time. There is also considerable difficulty in normalizing the obtained multispectrum, since the influence of discretization and random errors leads to the need for regularization [5]. To overcome these shortcomings,
the wavelet bispectrum and bicoherence were introduced in [6] - [8] to study the interactions between individual local oscillators in a complex dynamic system. The generalization of the bispectrum to the wavelet transform makes it possible to analyze such fundamentally nonlinear phenomena as the temporal dynamics of the phase coupling between different harmonics in the signal, as well as to isolate short-lived structures in spatial-temporal data sets.

Information about the possible connection of various frequency components with each other is not detected in classical estimate of the power spectrum. To obtain this kind of information, it is necessary to use multispectral analysis, which allows you to detect the effects of the relationship of spectral components. In particular, the wavelet-bispectrum of the signal under study can be defined as [6]:

\[ B_W(f_1, f_2) = \int T W^*(f_3, b) W(f_2, b) W(f_1, b) db \]  

(1)

By the definition given in [2], a continuous wavelet transform of some signal \( f(x) \) has the following form:

\[ W_\psi(a, b) = \frac{1}{\sqrt{C_\psi}} \int_\mathbb{R} f(x) \psi_{ab}^*(x) dx \]  

(2)

where:

\[ \psi_{ab}(x) = \frac{1}{\sqrt{a}} \psi\left(\frac{x-b}{a}\right) \];

\( a \) and \( b \) are the scale and shift, respectively;

\( \psi \) is the wavelet-forming function.

When analyzing rhythmogram signals, we will use the concept of wavelet bicoherence, which we define in the form of the following normalized wavelet-bispectrum:

\[ b_W(f_1, f_2) = \frac{|B_W(f_1, f_2)|}{\int T |W(f_1, b)|^2 db \int T |W(f_3, b)|^2 db} \]  

(3)

The use of wavelet bicoherence (3) allows not only to reveal the internal fine structure of the time series determined by phase bonds, but also to evaluate the dynamics of changes in this structure over time. Using expression (3), we propose to search for the presence of pathology using the standard deviation \( S \) of the form:

\[ S = \frac{1}{21} \sum_{A=10}^{30} \frac{1}{T} \sum_{i=1}^{T} |B_i - \mu| \]  

(4)

where:

\( B_i \) is wavelet-bicoherence vector by \( A \) scale of wavelet transform;

\( A \) is the scale of wavelet transform.

The rhythmogram of a healthy fetus without pathologies is shown (fig. 1). The wavelet-bicoherence is shown in a top view (fig. 2) and in a side view (fig. 3). Similarly, the rhythmogram of the fetus with the presence of pathologies is shown (fig. 4) and the wavelet-bicoherence for the fetus is shown in the top (fig. 5) and side (fig. 6) views, respectively.

![Fig. 1. Fetal rhythmogram without pathologies](image-url)
Fig. 2. Wavelet-Bicoherence for fetal rhythmogram without pathologies (top view)

Fig. 3. Wavelet-Bicoherence for fetal rhythmogram without pathologies (side view)

Fig. 4. Rhythmogram of the fetus with the presence of pathologies
The table (tabl. 1) shows the results of calculating the sign of $S$ by using (4) for three fetus without the onset of abnormalities in the rhythm and for three fetus with arrhythmia.

| Patient Number | With rhythm disturbance, $S$ | No rhythm disturbance, $S$ |
|----------------|-----------------------------|---------------------------|
| 1              | $1,3 \times 10^{-3}$        | $4.8 \times 10^{-5}$     |
| 2              | $9.6 \times 10^{-4}$        | $5.1 \times 10^{-5}$     |
| 3              | $1.59 \times 10^{-4}$       | $4.64 \times 10^{-5}$    |

Mean $= 8.06 \times 10^{-4} = 80.6 \times 10^{-5}$
Mean $= 3.4 \times 10^{-5}$

[author’s development]
Conclusions. Thus, the process of detecting fetal heart rhythm disturbances using wavelet-bicoherence was shown in the work. As can be seen from the results (tabl. 1), the proposed feature in the presence of pathologies gives almost 23.7 times more value of parameter S than for a fetus that has no rhythm disturbances.

In the course of work, real signals were used, and the number of processed records was 13. In the future, it is planned to search for additional features in wavelet-bicoherence, which would indicate the presence of pathologies in the work of the heart. It is also planned to implement an automatic version of this method.

Currently, fragments of the signal were manually selected in which the doctor diagnosed the presence of a disturbance. It is also planned to make an automatic method based on wavelet-bicoherence, which will be able to identify the form of disturbance, and not just the presence. For example, arrhythmias, tachycardia, AV blockade, etc.

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