"4N MULTIPOLe PERIODICITY" OF THE GALAXy IMAGE IN THE WMAP DATA.

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ABSTRACT

We present a specific periodicity of the Galaxy images in the multipole domain which can be used for separation of the cosmic microwave background (CMB) signal from the Galaxy and foreground component. This method takes into consideration all the coefficients of the expansion of the signal from the sky \(\Delta T(\Theta, \phi)\) into spherical harmonics and dealing with combination of multipoles \(a_{l,m}\) and \(a_{l+\Delta,m}\) for each multipole mode \((\ell, -\ell \leq m \leq \ell)\) from the whole sky without galactic cut, masks or any dissection of the whole sky into disjoint regions. For the polar coordinate system we use particular values for \(\Delta = 4n\), \(n = 1, 2, \ldots\) which remove all bright point-like sources localized in the Galactic plane and strong diffused component down to the CMB level. To illustrate that significant correlations of the point like sources and foregrounds \(a_{l,m}\), \(a_{l+\Delta,m}\) coefficients we apply this method to the WMAP Q, V and W bands in order to remove these marks of Galaxy from the maps. We believe that our method would be useful for separation of the CMB signal from different kind of "noises" in the maps.

1. INTRODUCTION

Separation of the CMB signal and foregrounds (especially, Galactic foregrounds) is the major problem for all the CMB experiments, including ongoing the WMAP and future the PLANCK missions. The basics approach to solve this problem is related with implementation of different masks and disjoint regions of the whole sky maps (see, Bennett et al. (2003a), Bennett et al. (2003b) Tegmark, de Oliveira-Costa & Hamilton (2003),Eriksen et al. (2004) ). However, the question is how the Galactic region of the map can affect any estimators of the primordial CMB signal, particularly, the power spectrum \(C(l)\) and phases of the multipole representation of the CMB, if we try to process the data sets without any masks? This question closely related to the question, what statistical properties of the Galactic foreground could be?

In this letter we would like to present some of the results of investigation of the Galactic area of the WMAP sky using the \(K - W\) bands signals. Particularly, we will show that in the polar coordinate system the brightest area of the map related with the Galaxy can be removed down to the CMB level by simplest linear combination of the spherical harmonics coefficients \(a_{l,m}\)

\[
d_{l,m}^\Delta = a_{l,m} - \frac{|a_{l,m}|}{|a_{l+\Delta,m}|}a_{l+\Delta,m}
\]

(1)

where \(\Delta\) is free parameter, which corresponds to transition from the \(l\) multipoles to \(l + \Delta\) multipoles for the same \(|m| \leq l\) modes. As one can see from Eq. (1) our idea is to introduce new characteristics of the \(\Delta T(\Theta, \phi)\) field on the sphere

\[
D^\Delta(\Theta, \phi) = \sum_{l=2}^{l_{\text{max}}} \sum_{m=-l}^{l} d_{l,m}^\Delta Y_{l,m}(\Theta, \phi)
\]

(2)

(here \(Y_{l,m}(\Theta, \phi)\) are spherical harmonics) which can significantly decrease the Galactic plane signal in the K-W maps. In Section 1 we show, that this new estimator \(d_{l,m}^\Delta\) directly related with phases and power spectrum of the whole sky signal and can be used for separation of the CMB signal from foregrounds. Section 2 is devoted to practical implementation of the method to the WMAP data sets. In this section we will show that for \(d_{l,m}^\Delta\) with \(\Delta = 4n, n = 1, 2, \ldots\) the \(D^\Delta(\Theta, \phi)\) map derived from K-W maps do not have strong signal from the Galaxy. In Section 3 we discuss rotational invariance of \(\Delta = 4n\) correlations and show that they are specific for particular orientation of the Galaxy in the reference system. In the conclusion we summarize all the results and briefly discuss an application of the method for cleaning of the CMB maps from foregrounds.

2. PROPERTIES OF \(d_{l,m}^\Delta\) ESTIMATOR.

To understand the properties of \(d_{l,m}^\Delta\) estimator let us define the transformation of the \(\Delta T(\Theta, \phi)\) signal to spherical harmonic coefficients \(a_{l,m}\) as

\[
\Delta T(\Theta, \phi) = \sum_{l=2}^{l_{\text{max}}} \sum_{m=-l}^{l} |a_{l,m}| e^{i\Phi_{l,m}} Y_{l,m}(\Theta, \phi)
\]

(3)

where \(|a_{l,m}|\) and \(\Phi_{l,m}\) are the magnitudes and phases of \(l, m\) harmonics correspondingly. In terms of these variables our estimator \(d_{l,m}^\Delta\) now has a form

\[
d_{l,m}^\Delta = |a_{l,m}| e^{i\Phi_{l,m}} (1 - \exp[i(\Phi_{l,m} - \Phi_{l+\Delta,m})])
\]

(4)

One can see, that \(d_{l,m}^\Delta\) coefficients depend on the phase difference \(\Phi_{l,m} - \Phi_{l+\Delta,m}\), taking for \(l, l + \Delta\) multipoles. If the phase difference is small enough \(\Phi_{l,m} - \Phi_{l+\Delta,m} \ll \pi/2\), then

\[
d_{l,m}^\Delta \approx |a_{l,m}| e^{i(\Phi_{l,m} + \frac{\pi}{2})} (\Phi_{l,m} - \Phi_{l+\Delta,m})
\]

(5)

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So, for Gaussian CMB signal the phase difference is not especially small because of non-correlation of the phases, while for non-Gaussian signal these correlation should be significant (see, for example, Chiang et al, 2003, Naselsky et al, 2004). According to Eq. (5) their contribution to \( d_{l,m}^\Delta \) became to be vanish, if \( \Phi_{l,m} \) is almost the same as \( \Phi_{l+\Delta,m} \).

To illustrate that tendency, let us describe the simplest model (see Naselsky et al, 2004), when a bright point source is placed in the Galactic plane area, having coordinates \( \theta_j = \pi/2, \phi_j \) and amplitude \( A_j \). The phases of the \( a_{l,m} \) harmonics for that signal are (Naselsky et al, 2004)

\[
\tan \Psi_{l,m} = -\frac{\sin(m\phi_j)}{\cos(m\phi_j)}
\]

and they do not depend on \( l \). So, for single point source \( \Phi_{l,m} = \Phi_{l+\Delta,m} \) and that source do not contribute to the \( D^\Delta(\theta, \phi) \) map for all \( \Delta = 1, 2, \ldots \) in Eq(1). For combination of the sources \( \Delta T(\theta, \phi) = \sum_j \Delta T_j(\theta, \phi) \) the properties of the phases are more complex that for the single one (see Naselsky et al, 2004), but the tendency of "selfcleaning" of the phases are more complex that for the single one (see Naselsky et al, 2004)

In the next section we will show that for \( \Delta = 4n \), where \( n = 1, 2, \ldots \) this method provide the best result for the Galactic plane area, including not the point sources only, but diffused foreground as well.

3. "4n" PERIODICITY OF THE WMAP BANDS.

To investigate the properties of \( d_{l,m}^\Delta \) estimator we use the HEALPix package (Gorski, Hivon & Wandelt, 1999) to decompose each of the WMAP signal maps at K-W bands for the spherical harmonics coefficients \( a_{l,m}^{K-W} \) \((N_{side} = 512)\). Then we perform transformation of the \( a_{l,m}^{K-W} \) coefficients to \( d_{l,m}^\Delta \) for each band and obtain the following maps, shown in Fig.1. We reproduce the same analysis using the GLESP package (Doroshkevich et al, 2003) in order to show that corresponding properties of the maps do not depend on the sky pixelization.

To remove contribution of the CMB signal to the \( D^\Delta(\theta, \phi) \) map, in Fig.2 we plot \( D^{\Delta=4}(\theta, \phi) \) for difference between V and W maps, which contain the foregrounds and instrumental noise only.

In Fig.3 we show the \( D^\Delta(\theta, \phi) \) maps for W band and different \( l_{\text{max}} \), but for the same value \( \Delta = 4 \). Note that for \( l_{\text{max}} > 150 \) the properties of \( D^{\Delta=4}(\theta, \phi) \) map depend on the instrumental noise, while for \( l_{\text{max}} = 50 \) all the \( D^{\Delta=4}(\theta, \phi) \) maps have quite regular structure, without Galactic belt. Important remark is that if \( \Delta \neq 4n \) then \( d_{l,m}^\Delta \) estimator does not remove the strong signal in the Galactic plane.

4. ROTATIONAL INVARIANCE OF THE \( D^{\Delta=4}(\theta, \phi) \) MAP.

The \( d_{l,m}^{\Delta=4} \) estimators of Galactic signal are dealing with phases of \( a_{l,m} \) coefficients and consequently depend on the reference system of coordinates. Obviously, for different system of coordinates these \( a_{l,m} \) coefficients and corresponding phases are different for the same values \( l, m \). Practically speaking, are \( d_{l,m}^{\Delta=4} \) estimators rotationally invariant and if not, how significantly that non-invariance can transforms any conclusions about contribution of the Galactic signal? To answered to these questions we need to know how to transform a given set of \( a_{l,m} \) taking for given reference coordinate system to new coefficients \( b_{l,m} \) which corresponds to new coordinate system rotated by Euler angles \( \alpha, \beta, \gamma \). Follow to general method Varshalovich,Moskalev & Khersonskii (1988) and taking into account properties of spherical harmonics (see Coles, 2005, Naselsky et al, 2004) we get

\[
b_{l,m} = \sum_{m'} D_{m,m'}^l(\alpha, \beta, \gamma) a_{l,m'} \]

where \( D_{m,m'}^l(\alpha, \beta, \gamma) \) is a spherical harmonic decomposition of the Wigner function \( D(\alpha, \beta, \gamma) \). The coefficients \( D_{m,m'}^l(\alpha, \beta, \gamma) \) should preserve the modules \( \sum_m |b_{l,m}|^2 = \sum_m |a_{l,m}|^2 \) under transformation. Without lost of generality let assumes that for initial reference system of coordinates \( a_{l,m} = \exp(i\Phi_{l,m}) \) where \( \Phi_{l,m} \) are the phases for given \( l \) and \( |m| \leq l \). To obtain the trigonometric moments in the reference system after rotation we define a matrix of correlations

\[
G_{l,t,m,m'} = b_{l,m} b_{l,m'}^\ast = \sum_{q,p} D_{m,q}^l(\alpha, \beta, \gamma) D_{m',q}^l(\alpha, \beta, \gamma)
\]

Fig. 1.— From the top to the bottom : the \( D^{\Delta=4}(\theta, \phi) \) map for the V band for \( \Delta = 0 \), \( \Delta = 1 \) and \( \Delta = 4 \). Next tree maps are the same , but for W band. All the maps are taking for \( l \leq 250 \). Here and bellow the amplitudes of the signal are in micro Kelvin. For all \( D^{\Delta=4}(\theta, \phi) \) maps the range of temperature scale is \(-0.5, 0.5mK\).
\[ D_{m', p}^{\ell} (\alpha, \beta, \gamma) a_{\ell, q} a_{\ell', p} \]  

where \( \ell' = l + \Delta, \, m' = m \) and \( b_{\ell, m} = |b_{\ell, m}| \exp(i \phi_{\ell, m}) \) is the phase of \( l, m \) harmonics after rotation. So, under rotation the \( D^{\Delta=4}(\theta, \phi) \) map transforms by the following way:

\[
e^{i(\phi_{l, m} - \Phi_{l', m'})} = \frac{e^{i\alpha(m-m')}}{|b_{l, m} b'_{l', m'}|} \sum_{p, q} D_{l, p}^{\ell} (\alpha, \beta, \gamma) D_{m', q}^{\ell'} (\alpha, \beta, \gamma) e^{i(\Phi_{l', p} - \Phi_{l', p'})}
\]

Taking into account that (see Varshalovich, Moskalev & Khrusovskii (1988))

\[ D_{m', m''}^l = \exp(-i \alpha) d_{m', m''}^l (\beta) \exp(-i \beta) \gamma ; \]

where

\[
d_{m', m''}^l (\beta) = \left[ (l + m')!(l - m')!(l + m)!(l - m)! \right]^{\frac{1}{2}} \times
\]

\[
\sum_{k} (-1)^{l' - m' - k} \frac{(l - m - k)!}{(l - m' - k)!(k + m + m')!} \times (\cos \beta)^{2k + m + m'} (\sin \beta)^{2l - 2k - m - m'}
\]

we obtain the following formula :

\[
e^{i(\phi_{l, m} - \Phi_{l', m'})} = \frac{e^{i\alpha(m-m')}}{|b_{l, m} b'_{l', m'}|} \sum_{p, q} W_{l, m', p, q}^{\ell, \ell'} (\beta) e^{i(\Phi_{l', p} - \Phi_{l', p'}) + \gamma(p-q)}
\]

where

\[
W_{l, m', p, q}^{\ell, \ell'} (\beta) = d_{m, p}^{\ell'} (\beta) d_{m', q}^{\ell'} (\beta)
\]

The Eq(12) show that after rotation the properties of the phase correlation became to be more complicated than in the reference system. To estimate their analytically, we would like to point out that real and imaginary parts of the exponent in Eq. (12) now contains not phase difference \( \Phi_{l, p} - \Phi_{l', q} \), but additional terms \( \gamma(p-q) \) and \( \alpha(m-m') \). However, taking into account that in the reference system the major correlations corresponds to \( q = p \) and \( \ell' = l + 4n \), we can draw our attention on these modes only. As one can see from Eq. (12), for these modes dependency of exponent in the left hand side on \( \alpha \) and \( \gamma \) parameters is vanished, while dependency on \( \beta \) looks like a linear combination of exponent \( \Phi_{l, p} - \Phi_{l', q} \) convolved by the coefficients \( W_{l, m', p, q}^{\ell, \ell'+4n} (\beta) \). Obviously, that part of exponent is not rotationally invariant, while rotation of the map with \( \beta = 0 \) preserved rotationally invariance of the phase difference. To illustrate that effect in Fig.4 we plot the \( D^{\Delta=4}(\theta, \phi) \) map taking from the W band, and rotated by the angles \( \theta = 77^\circ, \phi = 38^\circ \) (which corresponds to \( \beta \neq 0 \)) relatively to the system of standard polar coordinates. As one can see from this fig., the Galactic belt is still the major source of the high amplitude signal. Thus, the effect of \( 4n \)-periodicity is rotationally invariant for particular choice of the Euler angles \( \alpha \neq 0, \gamma \neq 0, \beta = 0 \).
estimator should be non-correlated and uniformly distributed within interval \([0, 2\pi]\), while the power spectrum \(D^{\Delta=4}(l) = 1/(2l+1) \sum_m |d_{l,m}^{\Delta=4}|^2\) related to standard one \(C(l) = 1/(2l+1) \sum_m |c_{l,m}|^2\) through equation

\[
D^{\Delta=4}(l) = 2C(l) - \frac{2}{2l+1} \sum_{m=-l}^{m=l} |c_{l,m}|^2 \cos(\xi_{l+m} - \xi_{l,m})
\]

(13)

From Eq. (13) one can see that \(D^{\Delta=4}(l) \simeq 2C(l)\) with error about "cosmic variance" error. \((\sim \frac{1}{\sqrt{l+\frac{1}{2}}} )\). For non-Gaussian primordial CMB signal with correlated phases \(\xi_{l+m}, \xi_{l,m}\) the power of \(d_{l,m}^{\Delta=4}\) signal has a tendency to drop down to the level \(D^{\Delta=4}(l) \rightarrow 0\), if \(\xi_{l+m} \rightarrow \xi_{l,m}\). That tendency open a new possibility to test Gaussianity of the CMB signal trough power spectrum estimation, using \(a_{l,m}\) and \(d_{l,m}^{\Delta=4}\) coefficients and corresponding power spectra \(C(l)\) and \(D^{\Delta=4}(l)\). For Gaussian signal we should obtain \(D^{\Delta=4}(l) \simeq 2C(l)\), while for non-Gaussian signal it should be \(D^{\Delta=4}(l) < 2C(l)\) with "cosmic variance" error bars.

5. Conclusion.

In this paper the new estimator for the CMB maps has been presented. This estimator is based on the phase correlation between \(l + 4n, m\) and \(l, m\) harmonics and it decrease significantly contribution of the Galactic foregrounds to the map. Decovered \(4n, n = 1, 2...\) periodicity of the Galactic foregrounds phases seems to be important for accurate solution of the problem of separation the primordial CMB signal from the foregrounds and instrumental noise including whole sky signals. This method will be publish in a separate paper.

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