Strangeness nuclear physics – 2010

Overview of strangeness nuclear physics

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Selected topics in Strangeness Nuclear Physics are reviewed: \( \Lambda \)-hypernuclear spectroscopy and structure, multistrangeness, and \( K \) mesons in nuclei.

§1. Introduction

The properties of hypernuclei reflect the nature of the underlying baryon-baryon interactions and, thus, can provide tests of models for the free-space hyperon-nucleon (YN) and hyperon-hyperon (YY) interactions. The Nijmegen group has constructed a number of meson-exchange, soft-core models, using SU(3)_{f} symmetry to relate coupling constants and form factors.\(^{1}\) The Jülich group, in addition to YN meson exchange models,\(^{2}\) published recently leading-order chiral effective-field theory YN and YY potentials.\(^{3}\) Quark models have also been used within the \((3q)-(3q)\) resonating group model (RGM), augmented by a few effective meson exchange potentials of scalar and pseudoscalar meson nonets directly coupled to quarks.\(^{4}\) Finally, we mention recent lattice QCD calculations.\(^{5},\,6\)

On the experimental side, there is a fair amount of data on single-\( \Lambda \) hypernuclei, including production, structure and decay modes.\(^{7}\) Little is known on strangeness \( S=-2 \) hypernuclei. The missing information is vital for extrapolating into strange hadronic matter (SHM) for both finite systems and in bulk, and into neutron stars.\(^{8}\) Therefore, following a review of the spectroscopy of single-\( \Lambda \) hypernuclei in Sect. 2, I update in Sect. 3 what is known about \( \Lambda \Lambda \) hypernuclei and discuss in Sect. 4 the nuclear potential depths anticipated for other hyperons (\( \Sigma, \Xi \)) from YN interaction models, as well as from the scarce hypernuclear data available for these hyperons. Aspects of \( K \) nuclear interactions are reviewed in Sect. 5 highlighting the issue of kaon condensation.

§2. \( \Lambda \) hypernuclei

To test YN models against the considerable body of information on \( \Lambda \) hypernuclei, effective interactions for use in limited spaces of shell-model orbits must be evaluated. The \( \Lambda \) well depth resulting from soft-core Nijmegen nuclear-matter G-matrices\(^{10}\) can be brought to a reasonable agreement with the empirical value 28 MeV deduced in fitting binding energies of \( \Lambda \) single-particle (s.p.) states.\(^{10}\) However, the partial-wave contributions, in particular the spin dependence of the central interaction, vary widely in different models, and the \( \Lambda \)-nuclear spin-orbit splittings

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do not come sufficiently small in a natural way in most of the available models.\textsuperscript{2} The l.h.s. of Fig. 1 shows one of the most impressive examples of \( \Lambda \) s.p. structure. Although the structure of the \( f_\Lambda \) orbit in \(^{89}\Lambda Y\) may suggest a spin-orbit splitting of 1.7 MeV, a more careful shell-model analysis demonstrates consistency with a \( \Lambda \) spin-orbit splitting of merely 0.2 MeV, with most of the observed splitting due to mixing of \( \Lambda N^{-1} \) particle-hole excitations.\textsuperscript{12} Interesting hypernuclear structure is also revealed between major \( \Lambda \) s.p. states in \(^{12}\Lambda C\). This has not been studied yet with sufficient resolution in medium-weight and heavy hypernuclei, but data already exist from JLab on \(^{12}\Lambda C\) and other targets, with sub-MeV resolution, as shown on the r.h.s. of Fig. 1. Furthermore, even with the coarser resolution of the \((\pi^+, K^+)\) data shown in Fig. 1, most of the \(^{12}\Lambda C\) levels between the (left) \( 1s_\Lambda \) peak and the (right) \( 1p_\Lambda \) peak are particle-stable and could be studied by looking for their electromagnetic cascade deexcitation to the ground state.

A systematic program of \( \gamma \)-ray measurements\textsuperscript{15} has been carried out for light \( \Lambda \) hypernuclei at BNL and KEK in order to study the spin dependence of the effective \( \Lambda N \) interaction in the nuclear \( p \) shell\textsuperscript{16}

\[
V_{\Lambda N} = \tilde{V} + \Delta \vec{s}_N \cdot \vec{s}_A + S_A \vec{l}_N \cdot \vec{s}_A + S_N \vec{l}_N \cdot \vec{s}_N + T S_{12},
\]

specified here by four radial matrix elements: \( \Delta \) for spin-spin, \( S_A \) and \( S_N \) for spin-orbit, \( T \) for the tensor interaction. The most completely studied hypernucleus to date is \(^7\Lambda Li\) with five observed \( \gamma \)-ray transitions, allowing a good determination of these parameters in the beginning of the \( p \) shell.\textsuperscript{17}

\[
A = 7, 9: \quad \Delta = 430, \quad S_A = -15, \quad S_N = -390, \quad T = 30 \text{ (keV)}.
\]

The dominant spin-dependent contributions to \(^7\Lambda Li\) are due to \( \Delta \) for \( 1s_\Lambda \) inter-doublet spacings listed in Table II and to \( S_N \) for intra-doublet spacings.

\textsuperscript{1)} Nevertheless, it was suggested recently\textsuperscript{13} that \( \Lambda \to \Sigma \to \Lambda \) iterated one-pion exchange contributions overlooked in MF approaches cancel out the short-range \( \sigma + \omega \) MF contributions to the \( \Lambda \) nuclear spin-orbit potential.
Fig. 2. γ-ray spectra of Λ hypernuclei from BNL E930, see Tamura’s review. The observed twin peaks (in order left to right) result from the $\frac{5}{2}^+$ and $\frac{3}{2}^+$ levels in $^9\Lambda$Be separated by 43 keV, deexciting to the ground state, and from deexcitation of a $1^-$ level in $^{16}\Lambda$O to the ground-state doublet $0^-$ and $1^-$ levels separated by 26 keV.

A remarkable experimental observation of minute doublet spin splittings in $^9\Lambda$Be and in $^{16}\Lambda$O is shown in Fig. 2. The contributions of the various spin-dependent components in Eq. (2.1) to these and other doublet splittings are given in Table I using Eq. (2.2) for $^9\Lambda$Be and a somewhat revised parameter set for heavier hypernuclei which exhibit greater sensitivity, in the $p_{\frac{1}{2}}$ subshell, to the tensor interaction:

$$ A > 9 : \quad \Delta = 330, \quad S_{\Lambda} = -15, \quad S_N = -350, \quad T = 23.9 \text{ (keV).} \quad (2.3) $$

Listed also are $\Lambda\Sigma$ mixing contributions, from Ref. [17]. Core polarization contributions normally bounded by 10 keV are not listed. In $^9\Lambda$Be, since both $\Delta$ and $T$ are well controlled by data from other systems, it is fair to state that the observed $43 \pm 5$ keV doublet splitting provides a stringent measure of the smallness of $S_\Lambda$, the $\Lambda$ spin-orbit parameter for $1s_{\Lambda}$ states, consistently with the small $\Lambda$ spin-orbit splitting $152 \pm 54\text{(stat.)} \pm 36\text{(syst.)}$ keV observed in $^{13}\Lambda$C between the $1p_{\frac{1}{2}} \rightarrow$ g.s. and $1p_{\frac{3}{2}} \rightarrow$ g.s. γ-ray transitions. It is worth noting that some of the hypernuclear energy shifts observed, with respect to core states in the middle of the nuclear $1p$ shell, are poorly understood, requiring perhaps $\Lambda N N$ contribution beyond the substantial shifts occasionally provided by the induced nuclear spin-orbit parameter $S_N$.\[17\]

The spin dependence of the $\Lambda N$ interaction may also be studied, as reported recently by the FINUDA Collaboration, observing pionic weak decay spectra. This is demonstrated in Fig. 3 for two species. The $^{11}\Lambda B \rightarrow \pi^- + ^{11}\text{C}$ spectrum shown on the l.h.s. confirms the spin-parity assignment already established for $^{11}\Lambda B_{\text{g.s.}}$, $J^\pi(^{11}\Lambda B_{\text{g.s.}}) = \frac{3}{2}^+$, whereas the $^{15}\Lambda N \rightarrow \pi^- + ^{15}\text{O}$ spectrum shown on the r.h.s. suggests a spin-parity assignment $J^\pi(^{15}\Lambda N_{\text{g.s.}}) = \frac{3}{2}^+$, consistently with the positive value listed in Table II for the ground-state doublet splitting $^E(\frac{1}{2}^+) - ^E(\frac{3}{2}^+)$.\[17\]
Table I. Calculated 1s,1 doublet splitting contributions\textsuperscript{17} $E_{ij}$, $j$ running over $\Lambda \Sigma$ mixing and $AN$ spin-dependent interaction terms, using Eq. (2.2) for $^7\Lambda Li$ and $^9\Lambda Be$, and Eq. (2.3) for $A > 9$. The calculated total splittings $E_{calc}$ are compared with $E_{exp}$ from experiment\textsuperscript{15} (in keV).

| $^A\Lambda$ | $J_{upper}$ | $J_{lower}$ | $E_{\Lambda \Sigma}$ | $E_{\Delta}$ | $E_{S\Lambda}$ | $E_{S_N}$ | $E_T$ | $E_{calc}$ | $E_{exp}$ |
|------------|-------------|-------------|---------------------|-------------|-------------|----------|--------|------------|----------|
| $^7\Lambda Li$ | $^3\Sigma^+$ | $^1\Sigma^+$ | 72                  | 628         | −1          | −4       | −9     | 693       | 692      |
| $^7\Lambda Li$ | $^3\Sigma^+$ | $^3\Sigma^+$ | 74                  | 557         | −32         | −8       | −71    | 494       | 471      |
| $^9\Lambda Be$ | $^3\Sigma^+$ | $^5\Sigma^+$ | −8                  | −14         | 37          | 0        | 28     | 44        | 43 ± 5   |
| $^{11}\Lambda B$ | $^3\Sigma^+$ | $^1\Sigma^+$ | 56                  | 339         | −37         | −10      | −80    | 267       | 263      |
| $^{11}\Lambda B$ | $^3\Sigma^+$ | $^3\Sigma^+$ | 61                  | 424         | −3          | −44      | −10    | 475       | 504      |
| $^{12}\Lambda C$ | $2^-$ | $1^-$ | 61                  | 175         | −12         | −13      | −42    | 153       | 161      |
| $^{15}\Lambda N$ | $^1\Sigma^+$ | $^3\Sigma^+$ | 42                  | 232         | 34          | −8       | −208   | 92        |          |
| $^{15}\Lambda N$ | $^3\Sigma^+$ | $^1\Sigma^+$ | 65                  | 451         | −2          | −16      | −10    | 507       | 481      |
| $^{16}\Lambda O$ | $1^-$ | $0^-$ | −33                 | −123        | −20         | 1        | 188    | 23        | 26.4 ± 1.7 |
| $^{16}\Lambda O$ | $2^-$ | $1_2^-$ | 92                  | 207         | −21         | 1        | −41    | 248       | 224      |

Fig. 3. Left: $^{11}\Lambda B \rightarrow \pi^- + ^{11}\Lambda C$ weak decay spectrum, with spin-parity $5/2^+$ favored over $7/2^+$. Right: $^{15}\Lambda N \rightarrow \pi^- + ^{15}\Lambda O$ weak decay spectrum, with spin-parity $3/2^+$ favored over spin-parity $1/2^+$. Spectra taken by FINUDA\textsuperscript{19}.

§3. $\Lambda\Lambda$ hypernuclei

Several $\Lambda\Lambda$ hypernuclear assignments have been proposed based on $\Xi^-$ capture events observed in hybrid emulsion KEK experiments\textsuperscript{20} as listed in Table II. In particular, the Nagara event\textsuperscript{23} yields a unique assignment and a ground-state binding energy value for $^6\Lambda\Lambda$He which is the lightest particle stable $\Lambda\Lambda$ hypernucleus established so far; a claim for $^4\Lambda\Lambda$H from BNL-E906\textsuperscript{26} has been downgraded recently\textsuperscript{27}. Comprehensive stochastic variational calculations of $s$-shell $\Lambda$ and $\Lambda\Lambda$ hypernuclei by Nemura et al.\textsuperscript{28} using meson-exchange based phenomenological coupled-channel potentials to account for $\Lambda N - \Sigma N$ and $\Lambda\Lambda - \Xi N$ mixings, predict that $^4\Lambda\Lambda$H and
Table II. Compiled binding energies (in MeV) of $\Lambda\Lambda$ hypernuclei and as calculated fitting $V_{\Lambda\Lambda}$ to $B_{\Lambda\Lambda}(\Lambda^6\text{He})_{\exp}$. The Hida event is assigned either to $^{11}_{\Lambda}\Lambda\text{Be}$ or to $^{12}_{\Lambda}\Lambda\text{Be}$. Values of $\delta B_{\Lambda\Lambda}(\Lambda^A\Lambda Z) \equiv B_{\Lambda\Lambda}(\Lambda^A\Lambda Z)_{\exp} - B_{\Lambda\Lambda}(\Lambda^A\Lambda Z)_{\calc}$ (in MeV) are given in brackets for the calculation of Ref. (21).

| event         | $\Lambda^6\Lambda Z$ | $B_{\Lambda\Lambda}(\Lambda^A\Lambda Z)_{\exp}$ | $B_{\Lambda\Lambda}(\Lambda^A\Lambda Z)_{\calc}$ | $B_{\Lambda\Lambda}(\Lambda^A\Lambda Z)_{\calc}$ |
|---------------|-----------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|
| E373-Nagara(23) | $^{6}_{\Lambda}\Lambda\text{He}$ | 6.91 ± 0.16                                     | 6.91 (0.54)                                     | 6.91 ± 0.16                                     |
| Danysh et al.(23) | $^{10}_{\Lambda}\Lambda\text{Be}$ | 14.8 ± 0.4                                      | 14.74 (0.53)                                   | 15.08 ± 0.20                                   |
| E373-Hida(29)   | $^{11}_{\Lambda}\Lambda\text{Be}$ | 20.83 ± 1.27                                    | 18.23 (0.56)                                   | 18.39 ± 0.27                                   |
| E373-Hida(29)   | $^{12}_{\Lambda}\Lambda\text{Be}$ | 22.48 ± 1.21                                    | –                                             | 20.71 ± 0.21                                   |
| E17(25)         | $^{13}_{\Lambda}\Lambda\text{B}$  | 23.3 ± 0.7                                      | –                                             | 23.21 ± 0.21                                   |

$^{5}_{\Lambda}\Lambda\text{H} - ^{5}_{\Lambda}\Lambda\text{He}$ are also particle stable. These predictions depend quantitatively on the assumed value of $\Delta B_{\Lambda\Lambda}(\Lambda^6\text{He}) \equiv B_{\Lambda\Lambda}(\Lambda^6\text{He}) - 2B_{\Lambda}(\Lambda^6\text{He})$ which now stands on $0.67 \pm 0.17$ MeV(29) rather than slightly over 1 MeV in these calculations, and since $^{4}_{\Lambda}\Lambda\text{H}$ is calculated to be particle stable only by 2 keV, it is likely to be particle unstable in agreement also with a Faddeev-Yakubovsky (FY) calculation. In contrast, the particle stability of $^{5}_{\Lambda}\Lambda\text{H}$ and $^{5}_{\Lambda}\Lambda\text{He}$, which have not yet been discovered, appears theoretically robust as shown on the l.h.s. of Fig. 4 as a function of the strength assumed for $V_{\Lambda\Lambda}$. We therefore conclude that the onset of $\Lambda\Lambda$ hypernuclear binding is likely to occur for $V_{\Lambda\Lambda} = 5$.

![Graph](image-url)

Fig. 4. Left: FY relationship between $\Delta B_{\Lambda\Lambda}(\Lambda^6\text{He})$ and $\Delta B_{\Lambda\Lambda}(^{5}_{\Lambda}\text{H} or ^{5}_{\Lambda}\text{He})$ (Filiikhin and Ga(20)). Right: spectrum of $^{11}_{\Lambda}\Lambda\text{Be}$ in a 5-cluster calculation. (23)

Hiyama et al.(21,23) reported on cluster calculations of $\Lambda\Lambda$ hypernuclei beyond $^{6}_{\Lambda}\Lambda\text{He}$. The good agreement between $B_{\Lambda\Lambda}(^{11}_{\Lambda}\Lambda\text{Be})_{\exp}$ and $B_{\Lambda\Lambda}(^{10}_{\Lambda}\Lambda\text{Be})_{\calc}$ in Table II rests on the assumption that $^{10}_{\Lambda}\Lambda\text{Be}_{g.s.}$ was identified by its $\pi^-$ decay to $^{9}_{\Lambda}\Lambda\text{Be}^* (3$ MeV). It is consistent with the production of $^{10}_{\Lambda}\Lambda\text{Be}_{g.s.} (\sim 3$ MeV) in the Demachi-Yanagi event. (22) For $^{11}_{\Lambda}\Lambda\text{Be}$, a calculated spectrum is shown on the r.h.s. of Fig. 4. The $\approx 2\sigma$ discrepancy between $B_{\Lambda\Lambda}(^{11}_{\Lambda}\Lambda\text{Be})_{\exp}$ and $B_{\Lambda\Lambda}(^{11}_{\Lambda}\Lambda\text{Be})_{\calc}$ casts
doubts on this interpretation of the Hida event. For heavier species, for which only shell-model simple estimates are available, it is seen that the \(^{12}\text{Be}\) interpretation of Hida is as dubious as \(^{11}\text{Be}\). The good agreement between \(B_{\Lambda A}(^{13}\text{B})_{\text{exp}}\) and \(B_{\Lambda A}(^{13}\text{B})_{\text{calc}}\) in Table [II] rests on the assumption that \(^{13}\text{Be}\) was identified by its \(\pi^-\) decay to \(^{13}\text{C}^*(4.9\ \text{MeV})\).

§4. \(\Lambda, \Sigma, \Xi\) hyperon nuclear potential depths and SHM

A vast body of \((K^-,\pi^\pm)\) and \((\pi^-,K^+)\) data indicate a repulsive \(\Sigma\) nuclear potential, with a substantial isospin dependence, which for very light nuclei may conspire in selected configurations to produce \(\Sigma\) hypernuclear quasibound states. The most recent \((K^-,\pi^\pm)\) spectra plus the very recent \((\pi^-,K^+)\) spectra and related DWIA analyses suggest that \(\Sigma\) hyperons do not bind in heavier nuclei.

A repulsive component of the \(\Sigma\) nuclear potential arises also from analyzing strong-interaction shifts and widths in \(\Sigma^-\) atoms. In fact, \(\text{Re}V_{\text{opt}}^\Sigma\) is attractive at low densities outside the nucleus, as enforced by the observed ‘attractive’ \(\Sigma^-\) atomic level shifts, changing into repulsion on approach of the nuclear radius. The precise size of the repulsive component within the nucleus, however, is model dependent. This repulsion bears interesting consequences for the balance of strangeness in the inner crust of neutron stars, primarily by delaying to higher densities, or even aborting the appearance of \(\Sigma^-\) hyperons, as shown in Fig. 5.

\[ V^Y = V_0^Y + \frac{1}{A} V_1^Y T_A \cdot s_Y. \] (4.1)

In the latest Nijmegen ESC08 model, this repulsion is dominated by repulsion in the isospin \(T = 3/2\), \(^3S_1 - ^3D_1\) \(\Sigma N\) coupled channels where a strong Pauli
Table III. Isoscalar and isovector hyperon potentials, Eq. (4.1) in MeV, calculated for Nijmegen soft-core potential models\textsuperscript{[13]} denoted by year and version, at $k_F = 1.35\ \text{fm}^{-1}$ corresponding to nuclear-matter density. Excluded are $\text{Im} V^\Sigma$ due to $\Sigma N \to \Lambda N$ and $\text{Im} V^\Xi$ due to $\Xi N \to \Lambda\Lambda$.

|   | 97f | 04d | 06d | 08a | 08b | phenom. | Ref. |
|---|-----|-----|-----|-----|-----|---------|-----|
| $V_0^\Lambda$ | −31.7 | −44.1 | −44.5 | −35.6 | −34.0 | −28     | [10] |
| $V_1^\Lambda$ | −13.9 | −26.0 | −1.2  | +13.4 | +20.3 | +30 ± 20 | [30] |
| $V_0^\Xi$    | −30.4 | +30.4 | +52.6 | +64.5 | +85.2 | ≈ +80   | [10] |
| $V_1^\Xi$    | −18.7 | −20.2 | −32.4 | ≈ −14 |       |         | [11] |
| $V_2^\Xi$    | +50.9 | −40.4 | −69.7 |       |       |         |      |

This exclusion effect is suggested by SU(6) quark-model RGM calculations\textsuperscript{[11]}. A strong repulsion appears also in a recent SU(3) chiral perturbation calculation\textsuperscript{[23]} which yields $V_0^\Sigma > 60\ \text{MeV}$. Phenomenologically $V_0^\Sigma > 0$ and $V_1^\Sigma > 0$, as listed in the table, and both components of $V^\Sigma$ give repulsion in nuclei\textsuperscript{[23]}

![Fig. 6. $\Lambda$ and $\Xi$ nuclear matter potentials calculated in the quark model\textsuperscript{[2]} fss2 and in $\chi$EFT\textsuperscript{[10]} Figure adapted from Ref. [48].](image1)

![Fig. 7. Left: $\Sigma$ nuclear matter potentials calculated in the quark model\textsuperscript{[2]} fss2 and in $\chi$EFT\textsuperscript{[10]} Right: $T = 3/2$ $\Sigma N$ potentials in these same models. Figure adapted from Ref. [45].](image2)

Very little is established experimentally on the interaction of $\Xi$ hyperons with nuclei. Inclusive $(K^-, K^+)$ spectra\textsuperscript{[21]} on $^{12}\text{C}$ yield a somewhat shallow attractive

\textsuperscript{[1]} In the case of $^{3}\text{He}$, the only known quasibound $\Sigma$ hypernucleus\textsuperscript{[33]}\textsuperscript{[33]} the isovector term provides substantial attraction owing to the small value of A towards binding the $T = 1/2$ hypernuclear configuration, while the isoscalar repulsion reduces the quasibound level width\textsuperscript{[33]}.
potential, \( V^\Xi \approx -14 \text{ MeV} \), by fitting near the \( \Xi^- \) hypernuclear threshold. All of the Nijmegen soft-core potentials listed in Table III produce a somewhat stronger isoscalar attraction, \( V^0_0 \approx -25 \pm 7 \text{ MeV} \), while giving rise selectively, owing to the strong spin and isospin dependence which is reflected in the large size of the isovector \( V^1_1 \) potential, to predictions of quasibound \( \Xi \) states in several light nuclear targets, beginning with \( ^7\text{Li}^0 \). These predictions should be considered with a grain of salt since no phenomenological constraint exists on \( V^1_1 \). A ‘day-1’ experiment at J-PARC on a \(^{12}\text{C}^0 \) target has been scheduled.\(^4\)

It is worth noting that the main features provided by the Nijmegen potentials for hyperon-nuclear potentials also arise, at least qualitatively, in other models. This is demonstrated in Fig. 6 for \( \Lambda \) and \( \Xi \) nuclear potentials, and in Fig. 7 for \( \Sigma \) nuclear potentials. Whereas the \( \Lambda \) nuclear potential is essentially attractive, and as deep as \( \approx -30 \text{ MeV} \), the \( \Xi \) nuclear potential is weaker and could even turn repulsive. The \( \Sigma \) nuclear potential is repulsive, as argued above. The r.h.s. of Fig. 7 demonstrates the origin of this repulsion, for \( T = 3/2 \), due to the \( \Sigma N \to S_1 \) channel which is strongly dominated by the Pauli exclusion principle for quarks at short distances.

\( \Xi \) hyperons could become stabilized in multi-\( \Lambda \) hypernuclei once the decay \( \Xi N \to \Lambda \Lambda \), which releases \( \approx 25 \text{ MeV} \) in free space, gets Pauli blocked.\(^5\) The onset of \( \Xi \) particle-stability would occur for \( \Xi^6 \) He or for \( \Xi^7 \) He, depending on whether or not \( \Xi^5 \) He is bound, and by how much (if bound).\(^6\) It was shown that a large strangeness fraction, \(-S/A \approx 0.7\) could be reached upon adding \( \Xi \)s to multi-\( \Lambda \) hypernuclei in particle-stable configurations,\(^7\) which leads to the concept of Strange Hadronic Matter (SHM) consisting of equal fractions of protons, neutrons, \( \Lambda \), \( \Xi^0 \) and \( \Xi^- \) hyperons,\(^8\) with \( f_S = 1 \) as in Strange Quark Matter (SQM). Both SHM and SQM provide macroscopic realizations of strangeness, but SHM is more plausible phenomenologically, whereas SQM is devoid of any experimental datum from which to extrapolate.

§5. \( \bar{K} \) nuclear interactions and \( \bar{K} \) condensation

The \( \bar{K} \)-nucleus interaction near threshold comes strongly attractive and absorptive in fits to the strong-interaction shifts and widths of \( K^-\)-atom levels,\(^3\) resulting in deep potentials, \( \text{Re} V^\bar{K} (\rho_0) \sim -(150 - 200) \text{ MeV} \) at threshold.\(^5\) Chirally based coupled-channel models that fit the low-energy \( K^-p \) reaction data, and the \( \pi \Sigma \) spectral shape of the \( \Lambda(1405) \) resonance, yield weaker but still very attractive potentials, \( \text{Re} V^\bar{K} (\rho_0) \sim -100 \text{ MeV} \), as summarized recently in Ref. 51). A third class, of relatively shallow potentials with \( \text{Re} V^\bar{K} (\rho_0) \sim -(40 - 60) \text{ MeV} \), was obtained by imposing a Watson-like self-consistency requirement.\(^5\)

The onset of nuclear (quasi) binding for \( K^- \) mesons occurs already with just one proton: the \( \Lambda(1405) \) which is often represented by an \( S \)-matrix pole about \( 27 \text{ MeV} \) below the \( K^-p \) threshold. However, in chirally based models, the \( I = 0 \) \( \bar{K}N - \pi \Sigma \) coupled channel system exhibits also another \( S \)-matrix pole roughly \( 12 \text{ MeV} \) below threshold and it is this pole that enters the effective \( \bar{K}N \) interaction,\(^*\) With \( \approx 80 \text{ MeV} \) release in \( \Sigma N \to \Lambda N \), however, \( \Sigma \) hyperons are unlikely to stabilize.
affecting dominantly the $\bar{K}$-nucleus dynamics. The distinction between models that consider the twin-pole situation and those that are limited to the $A(1405)$ single-pole framework shows up already in calculations of $|\bar{K}(NN)_{I=1}J=1/2,J^\pi=0^+\rangle$, loosely denoted $K^-pp$, which is the configuration that maximizes the strongly attractive $I = 0$ $KN$ interaction with two nucleons. In Table IV which summarizes $K^-pp$ binding-energy calculations, the $I = 0$ $KN$ binding input to the first variational calculation is stronger by about 15 MeV than for the second one, resulting in almost 30 MeV difference in $B_{K^-pp}$. Furthermore, it is apparent from the ‘coupled-channel’ entries in the table that the explicit use of the $\pi\Sigma N$ channel adds about $20\pm5$ MeV to the binding energy calculated using effective $\bar{K}N$ potential within a single-channel calculation. It is fair to state that in spite of the wide range of binding energies predicted for $K^-pp$, its existence looks robust theoretically. The experimental state of the art in searching for a $K^-pp$ signal is rather confused, as discussed during HYP09 [Nucl. Phys. A 835 (2010)]. New experiments, at GSI with a proton beam and at J-PARC with pion and with kaon beams are underway.

Table IV. Calculated $B_{K^-pp}$, mesonic ($\Gamma_m$) & nonmesonic ($\Gamma_{nm}$) widths.

| $(\text{MeV})$ | $\bar{K}NN$ single channel variational | $\bar{K}NN - \pi\Sigma N$ coupled channels Faddeev | $\bar{K}NN - \pi\Sigma N$ coupled channels variational |
|---|---|---|---|
| $B_{K^-pp}$ | 48 | 17 $- 23$ | 50 $- 70$ | 60 $- 95$ | 40 $- 80$ |
| $\Gamma_m$ | 61 | 40 $- 70$ | 90 $- 110$ | 45 $- 80$ | 40 $- 85$ |
| $\Gamma_{nm}$ | 12 | 4 $- 12$ | | $\sim 20$ | |

Fig. 8. Missing mass spectra (left) and $\chi^2$ contour plots (right) for the inclusive reactions ($K^-, n$) (upper) and ($K^-, p$) (lower) at $p_{K^-} = 1$ GeV/c on $^{12}$C, from Ref. 60.

A fairly new and independent evidence in favor of deep $\bar{K}$-nucleus potentials is provided by ($K^-, n$) and ($K^-, p$) spectra taken at KEK on $^{12}$C, and very recently also on $^{16}$O at $p_{K^-} = 1$ GeV/c. The $^{12}$C spectra are shown in Fig. 8 where the solid lines on the left-hand side represent calculations (outlined in Ref. 61) using potential
depths in the range 160 – 190 MeV. The dashed lines correspond to using relatively shallow potentials of depth about 60 MeV which I consider therefore excluded by these data. Although the potentials that fit these data are sufficiently deep to support strongly-bound antikaon states, a fairly sizable extrapolation is required to argue for $\bar{K}$-nuclear quasibound states at energies of order 100 MeV below threshold, using a potential determined largely near threshold. Furthermore, the best-fit $\text{Im} V^{\bar{K}}$ depths of 40 – 50 MeV imply that $\bar{K}$-nuclear quasibound states are broad, as studied in Refs. 63, 64.

![Graph](image)

Fig. 9. Left: calculated neutron-star population as a function of density, from Ref. 65). The neutron density stays nearly constant once kaons condense. Right: calculated separation energies $B_{\bar{K}^-}$ in multi-$\bar{K}^-$ nuclei based on $^{40}\text{Ca}$ as a function of the number $\kappa$ of $\bar{K}^-$ mesons in several RMF models, for two choices of parameters fixed for $\kappa = 1$, from Ref. 66).

A robust consequence of the sizable $\bar{K}$-nucleus attraction is that $K^-$ condensation, when hyperon degrees of freedom are ignored, could occur in neutron star matter at about 3 times nuclear matter density, as shown on the l.h.s. of Fig. 9. Comparing it with the r.h.s. of Fig. 5 for neutron stars, but where strangeness materialized through hyperons, one may ask whether $\bar{K}$ mesons condense also in SHM. This question was posed and answered negatively long time ago for neutron star matter, but only recently for SHM in Ref. 66) by calculating multi-$\bar{K}$ nuclear configurations. The r.h.s. of Fig. 9 demonstrates a remarkable saturation of $K^-$ separation energies $B_{\bar{K}^-}$ calculated in multi-$\bar{K}^-$ nuclei, independently of the applied RMF model. The saturation values of $B_{\bar{K}^-}$ do not allow conversion of hyperons to $\bar{K}$ mesons through the strong decays $\Lambda \rightarrow p + K^-$ or $\Xi^- \rightarrow \Lambda + K^-$ in multi-strange hypernuclei, which therefore remain the lowest-energy configuration for multi-strange systems. This provides a powerful argument against $\bar{K}$ condensation in the laboratory, under strong-interaction equilibrium conditions. It does not apply to kaon condensation in neutron stars, where equilibrium configurations are determined by weak-interaction conditions. This work has been recently generalized to multi-$\bar{K}^-$ hypernuclei.47

4) This conclusion, for the $(K^-, p)$ spectrum, has been disputed recently by Magas et al.62

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References:

63) [Reference]
64) [Reference]
65) [Reference]
66) [Reference]
Acknowledgments

Thanks are due to John Millener for instructive correspondence on the analysis of the $\gamma$-ray experiments, and to the Organizers of the NFQCD10 Workshop at the Yukawa Institute, Kyoto, for their generous hospitality.

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