The statistical model assuming chemical equilibrium and local strangeness conservation describes most of the observed features of strange particle production from SIS up to RHIC. Deviations are found as the maximum in the measured $K^+ / \pi^+$ ratio is much sharper than in the model calculations. At the incident energy of the maximum, the statistical model shows that freeze out changes regime from one being dominated by baryons at the lower energies toward one being dominated by mesons. It will be shown how deviations from the usual freeze-out curve influence the various particle ratios. Furthermore, other observables exhibit also changes just in this energy regime.
1. Introduction

The NA49 Collaboration has performed a series of measurements of the production of strange particles in central Pb-Pb collisions at 20, 30, 40, 80 and 158 A GeV beam energies [1, 2, 3, 4, 5]. The most interesting result is the pronounced maximum in the $K^+ / \pi^+$ ratio observed around 30 A GeV. This ”horn” has initiated a lot of discussion related as to whether or not it indicates a phase transition. Indeed, this has been suggested in [6, 7]. A more conventional interpretation has been presented within the hadron gas model [8], yet this model did not reproduce the measured yields in a satisfactory manner. This description together with the recently published values from NA49 and earlier AGS and RHIC results [9, 10, 11, 12, 13, 14, 15, 16, 17, 18] are summarized in Fig. 1.

![Graph showing ratios of strange-to-non-strange mesons and baryons](image)

**Figure 1:** Ratio of strange-to-non-strange mesons (upper part) and the corresponding ratios for baryons (lower part) as a function of $\sqrt{s_{NN}}$.

It is important to remark that only the $K^+ / \pi^+$ ratio exhibits a sharp maximum while the $K^- / \pi^-$ ratio shows a continuous rise with $\sqrt{s_{NN}}$. The dashed and solid lines represent calculation within the statistical model [8] explained in the next section. Both trends are qualitatively described within this model.

In the lower part of Fig. 1 the corresponding ratios with strange baryons over pion are seen to exhibit also a maximum, most pronounced for the ratio $\Lambda / \pi^+$. For the other ratios the experimental
situation is less clear and more results are eagerly needed.

2. Maximum relative strangeness content around $\sqrt{s_{NN}} \approx 6 - 8$ GeV

The statistical model is very successful in describing particle yields from SIS up to RHIC energies with only two parameters $T$ and $\mu_B$. At the very low incident energies a canonical description with exact strangeness conservation is needed [19]. The extracted parameters $T$ and $\mu_B$ plotted in a “phase diagram” describe a smooth line which can be parameterized e.g. by the $E/N \approx 1$ GeV condition [20]. Figure 2 shows these values as a function of $\sqrt{s_{NN}}$ exhibiting for $T$ a rising curve which saturates above top SPS energies at a value of about 170 MeV. The other parameter $\mu_B$ decreases with incident energy from a value near the nucleon mass to zero for fully transparent collisions. The lines represent parameterizations

$$T(\mu_B) = a - b \mu_B^2 - c \mu_B^4,$$

(2.1)

where $a = 0.166 \pm 0.002$ GeV, $b = 0.139 \pm 0.016$ GeV$^{-1}$ and $c = 0.053 \pm 0.021$ GeV$^{-3}$, and

$$\mu_B(\sqrt{s}) = \frac{d}{1 + e \sqrt{s}},$$

(2.2)

with $d = 1.308 \pm 0.028$ GeV and $e = 0.273 \pm 0.008$ GeV$^{-1}$.

Figure 2: Energy dependence of the chemical freeze-out parameters $T$ and $\mu_B$. The curves have been obtained using a parametrization discussed in the text.

Based on this set of the $\sqrt{s_{NN}}$ dependence of the thermal parameters the dashed lines in Fig. 1 have been calculated. They described the observed trends qualitatively, but not the sharp maximum in $K^+/\pi^+$. Recently, the statistical model has been extended including higher resonances [21].
As they mostly decay into pions the strong drop of the $K^+ / \pi^+$ ratio towards RHIC energies as observed in the data, is now much better described.

To study whether strangeness has a maximum or not, it is more convenient to plot the Wroblewski factor $\lambda_s$ defined as

$$\lambda_s = \frac{2\langle s\bar{s}\rangle}{\langle u\bar{u} \rangle + \langle d\bar{d} \rangle}$$

where the quantities in angular brackets refer to the number of newly formed quark-antiquark pairs, i.e. $\lambda_s$ excludes all quarks that were present in the target and the projectile nuclei. Figure 3, left, shows as solid line (marked “sum”) the Statistical-Model calculations along the unified freeze-out curve [20] with the energy-dependent parameters $T$ and $\mu_B$ given above. From this figure we conclude that around $\sqrt{s_{NN}} = 6$ GeV corresponding to an incident energy of $20A$ GeV, the relative strangeness content in heavy-ion collisions reaches a clear and well pronounced maximum. The Wroblewski factor decreases towards higher energies and reaches a limiting value of 0.43. For details see Ref. [8].

The appearance of the maximum can be traced to the specific dependence of $\mu_B$ and $T$ on the beam energy as also pointed out in Ref. [23]. Figure 3, right, shows lines of constant $\lambda_s$ in the $T - \mu_B$ plane. As expected, $\lambda_s$ rises with increasing $T$ for fixed $\mu_B$. Following the chemical freeze-out curve, shown as a dashed line in Fig. 3, one can see that $\lambda_s$ rises quickly from SIS to AGS energies, then reaches a maximum at $\mu_B \approx 500$ MeV and $T \approx 130$ MeV. These freeze-out parameters correspond to $30 A$ GeV laboratory energy. At higher incident energies the increase in $T$ becomes negligible but $\mu_B$ keeps on decreasing and as a consequence $\lambda_s$ also decreases.

The importance of finite baryon density on the behavior of $\lambda_s$ is demonstrated in Fig. 3, left, showing separately the contributions to $\langle s\bar{s}\rangle$ coming from strange baryons, from strange mesons...
and from hidden strangeness, i.e. from hadrons like $\phi$ and $\eta$. The origin of the maximum in the Wroblewski ratio can be traced essentially to the contribution of strange baryons. The production of strange baryons dominates at low $\sqrt{s_{\text{NN}}}$ and loses importance at high incident energies when the yield of strange mesons increases. However, strange mesons also exhibit a maximum, yet less pronounced. This is due to the fact that strangeness production at the lower energies occurs via the associated production, i.e. $K^+$ are created together with hyperons $[24]$. Therefore the $K^+$ mesons are affected by the properties of the baryons, but the $K^-$ are not.

3. Transition from Baryonic to mesonic freeze out

While the Statistical Model cannot fully explain the sharpness of the peak in the $K^+/\pi^+$ ratio, there are nevertheless several phenomena giving rise to the rapid change which warrant a closer look at the model.

![Figure 4: The entropy density normalized to $T^3$ as a function of the beam energy as calculated in the Statistical Model using THERMUS $[25]$.](image)

It has been shown that $s/T^3 = 6$ is a fairly good criterium to describe the freeze-out curve $[26]$ and we use it here to describe the nature of the rapid change in the various ratios. We show in Fig. 4 the entropy density divided by $T^3$ as a function of beam energy as solid line. The separate contribution of mesons and of baryons to the total entropy is also shown in this figure by the dashed and dotted lines. There is a clear change of baryon to meson dominance around $\sqrt{s_{\text{NN}}} = 8 \text{ GeV}$. 

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Above this value the entropy is carried mainly by mesonic degrees of freedom. It is remarkable that the entropy density divided by $T^3$ is almost constant for all incident energies above the top AGS.

The separation line between meson dominated and baryon dominated areas in the $T - \mu_B$ plane is given in Fig. 5. In this figure the separation line crosses the freeze-out line at the stated $\sqrt{s_{NN}}$. This figure invites for further speculations as e.g. an existence of a triple point [27].

**Figure 5:** The line separating the $T - \mu_B$ plane into an area dominated by baryonic and one by mesonic freeze out as calculated in the Statistical Model using THERMUS [25].

**4. Deviations from the freeze-out curve**

In the previous section we have argued that the Statistical Model with unique freeze out for all particles can not fully quantify the sharpness of the $K^+/\pi^+$ ratio. In this section, we explore the possibility that freeze-out might happen earlier in this transition region. For this interpretation, we show in Fig. 6 the calculated values of the $K^+/\pi^+$ ratio for various combinations of $T$ and $\mu_B$ as contour lines with the corresponding values given in the figure. The thick solid line reflects the locations of the freeze out given by the condition of Ref. [20]. If freeze out happens around an incident energies of 30 $A$ GeV at higher $T$, then the ratio $K^+/\pi^+$ will be higher. This ratio can never exceed a value of 0.25 in an equilibrium condition.

It turns out that other particle ratios are less affected by a different freeze-out scenario, as their variation in the $T - \mu_B$ plane is very different [28].
5. Other observations in this transition regime

An early freeze-out is also supported by results from HBT studies [29]. Figure 7 shows the extracted volume as a function of $\sqrt{s_{\text{NN}}}$. Between top AGS and the lowest SPS energies a minimum can be seen. As the fireball is expanding, a smaller volume reflects an earlier time. The authors of Ref. [29] relate this minimum to a change in the interaction from $\pi N$ to $\pi\pi$. Indeed, assuming a mean free path length of about 1 fm nicely explains the observed trends. These studies have been continued and combined with the volume extracted from the statistical model fits [30, 31] and they all exhibit a change of sign in this energy regime.

Furthermore, the pion multiplicity per number of participating nucleons in heavy ion collisions crosses the results from pp collisions also in this energy regime.

6. Summary

It has been shown that the Statistical Model yields a maximum in the relative strangeness content around 30 $A$ GeV. This is due to a saturation in the temperature $T$ while the chemical potential keeps decreasing with incident energy. Since the chemical potential plays a key role, it is clear that baryons are strongly affected. Indeed, all hyperon/$\pi$ ratios yield maxima. In contrast, the $K^-/\pi^-$ ratio shows a continuously rising curve as expected from the arguments above. The
K⁺/π⁺ ratio, however, exhibits a maximum, as K⁺ mesons are sensitive to the baryo-chemical potential due to their associate production with hyperons occurring at the lower incident energies. The model predicts that for different hyperon/π ratios the maxima occur at different energies. If experiments prove this, the case for a phase transition is strongly weakened.

The energy regime around 30 A GeV seems to have specific properties. It is shown that the entropy production occurs below this energy mainly via creation of baryons, while at the higher incident energies meson production dominates.

Finally, we put attention on the impact of a change in the freeze-out condition which might lead to an early freeze-out, thus deviating from the usual freeze-out condition. Such a scenario would increase the K⁺/π⁺ while leaving other particle ratios essentially unchanged. HBT studies show that around 30 A GeV a minimum in the extracted volumina occurs. This could be interpreted as an earlier kinetic freeze-out and might indicate also another freeze-out for chemical decoupling.

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