Separation of time and length scales in spin-glasses: temperature as a microscope

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Abstract

We summarize the different puzzles raised by aging experiments of spin-glasses and their various interpretations. We try to reconcile the ‘real space’, droplet like pictures with the hierarchical pictures that have been proposed in the past. The basic ingredient is a strong separation of the time scales that govern the dynamics of the system on different length scales. Changing the temperature changes the length scale at which the system is observed, thereby allowing rejuvenation (that concerns short length scales) and memory (stored in long length scales) to coexist. We show that previous experiments can be reanalyzed in terms of vanishing energy barriers at the spin-glass transition, an important ingredient to obtain a fast separation of time scales. We propose to distinguish between ‘fixed landscape rejuvenation’, which is already present in simple two (or multi) level systems, from the ‘strong’ chaos effect on scales larger than an ‘overlap length’ conjectured in the context of the droplet model. We argue that most experiments can be accounted for without invoking the existence of an overlap length. New experiments are presented to test some recent predictions of the strong chaos scenario, with negative results.
La complexité de l’ensemble fait que tout ce qui peut leur arriver est vraiment, malgré l’expérience acquise, impossible à prévoir, encore plus à imaginer. Il est inutile de tenter de le décrire, car on peut concevoir n’importe quelle solution.

BORIS VIAN, in L’Automne à Pékin.

1 Facts and puzzles

Although spin-glasses are totally useless pieces of material, they constitute an exceptionally convenient laboratory frame for theoretical and experimental investigations [1, 2]. Theoretical concepts and experimental protocols relevant for more ‘useful’ glassy materials (polymer and molecular glasses, foams and pastes, etc.[3]) have been elaborated and tested on spin-glasses [4]. There are at least two reasons for this: (a) the theoretical models are conceptually simpler (although still highly non trivial) and (b) the use of very sensitive magnetic detectors allows one to probe in details the a.c and d.c spin dynamics of these systems down to very small external fields. The corresponding mechanical measurements in other glassy systems are much more difficult to control, although some recent progress have been made [5, 6], in particular concerning the measurement of age-dependent structure factors [7, 8, 9].

The aging dynamics of spin glasses has therefore been studied in glory details recently, and has revealed an extremely rich phenomenology [1, 2, 11, 12]. The most striking aspect is the role of small temperature changes, that we summarize as follows: (a) Superactivated behaviour: time scales grow faster than what would be expected from simple thermal activation when the temperature is decreased; (b) Rejuvenation and memory: after a small negative temperature jump (within the glass phase) the system behaves as if it had been quenched from above the glass transition temperature $T_c$ (rejuvenation). However, a perfect memory of the time spent at the initial temperature is somehow kept, as clearly demonstrated by the now well known ‘dip’ imprinting experiments (see Fig. 1) [11]; and (c) Weak cooling rate dependence: the a.c. susceptibility hardly depends on the thermal history, or actually only on the cooling rate at higher temperature, in particular when crossing $T_c$, is irrelevant, in strong contrast with Random Field like systems [11, 23].
Figure 1: Series of ‘dip’ imprinted on the a.c. susceptibility by successive stops at different temperatures while the system is cooled. Further cooling ‘rejuvenates’ the system (i.e. the susceptibility goes up). However, the dips are one by one remembered by the system when heated back. For more details, see [11].

This absence of cooling rate dependence is in fact another manifestation of rejuvenation.

It is now well established, both experimentally [13, 14] and numerically [15, 16, 17, 18] that a certain ‘coherence’ length is growing in an aging spin-glass. This was first predicted in the context of the ‘droplet model’ [19], but is presumably of much more general validity: larger length scales take a longer time to evolve. This is expected to be true even if the basic tenets of the droplet model turn out to be incorrect. The coherence length is found to grow as a power of time: \( \ell \sim t^{1/z} \), with an apparent exponent \( 1/z \) linear in temperature [16, 17, 18, 13]. This suggests an activated behaviour over barriers that grow as the logarithm of the length \( \ell \). Indeed, writing \( t(\ell) \sim \exp[\Delta(\ell)/k_BT] \) with \( \Delta(\ell) = \Delta_0 \log \ell \) leads to \( 1/z = k_BT/\Delta_0 \). This is confusing because (a) as mentioned above, experiments suggest superactivated behaviour; (b) barriers should grow with \( \ell \) faster than excitations.
energies, which are thought to grow as $\ell^\theta$ with $\theta = 0.2$ in the droplet model. (Note however that recent numerical simulations suggest that non compact excitations indeed correspond to $\theta = 0$ [21, 22]); and (c) the exponent $z$ determined numerically or experimentally can be written as $z(T) = z_c T_c / T$, where $z_c \sim 6$ is the critical exponent that governs the dynamics of the spin-glass at the critical point. This coincidence suggests that the system is somehow affected by critical fluctuations (as also suggested, on the basis of different arguments, in [22]).

The rejuvenation effect (and absence of cooling rate dependence) brings still more confusion. Activated barrier crossing is obviously easier at higher temperature: so why waiting longer around $T_c$ does not help to equilibrate the system, as happens in e.g. random field like systems? (see the e.g. the discussion in [23]). This should be even more crucial if the dynamics is super-activated. A way out of this contradiction is to invoke chaos in temperature [24, 19, 25]: if new patterns need to equilibrate when the temperature is changed, it is clear that the time spent at a higher temperature does not help much to equilibrate the system at the final temperature. However (a) no sign of chaos has been found in most recent static numerical studies of the 3d Edwards-Anderson model [22] (at variance with earlier studies [27]), nor in the theoretical analysis of the SK model [28]; (b) no rejuvenation in the dynamics upon temperature changes has been found either in numerical simulations [29]; and (c) the coherence length that has grown at one given temperature seems to carry on growing (although at a different rate) at another temperature, in contradiction with the chaos idea. This continuity has been established both numerically [17] and experimentally [14]. ‘Chaos’ should furthermore be compatible with memory: growing new patterns at a given temperature should not erase the patterns grown at higher temperatures if one wants to account for the ‘multi-dip’ experiment shown in Fig. 1. A possible scenario for this was suggested in [34].

In this paper, we wish to develop a consistent qualitative picture for the dynamics in spin-glasses that allows one to resolve the above apparent contradictions. This picture, as the original droplet model [20, 19], heavily relies on three basic ideas: (i) times grow as the exponential of the energy barriers; (ii) the energy barriers grow as a power of the length scales involved in the dynamics; and (iii) the energy barriers vanish at the critical temperature $T_c$. These ingredients are enough to understand that changing the temperature for a given observation time corresponds to changing the length scale at
which the system is probed. Therefore, the objects contributing to the aging dynamics are different at different temperatures. This gives a precise content to the ‘hierarchical’ picture advocated in many papers [30, 31, 32, 33]. Although the present picture is similar in spirit to the droplet model, we will assume (but this is not crucial) that the low energy metastable states exist on arbitrary length scales (i.e. $\theta = 0$). We will also introduce the idea of a fixed landscape rejuvenation, which concerns short length scales and is distinct from the large scale chaos that appears in the context of the droplet model. We reanalyze previous experimental data within this framework, which strongly suggest that energy barriers vanish at $T_c$, and allow us to extract estimates of the exponent $\psi$ that relates energy barriers and length scales.

2 Basic ingredients

Let us consider a large scale low lying excitation in a spin glass. This excitation is made of a large connected cluster of spins that is flipped with respect to the ground state. The surface of this cluster can be thought of as a ‘domain wall’ which happens to occupy a very favorable position since the overall energy of this surface is very small. Therefore, this wall is ‘pinned’ by the disorder and tends to adopt some special conformation. If $\theta = 0$, this wall has no overall tendency to disappear with time, and will be present in the system even after very long times. If $\theta > 0$, conversely, these large scale walls tend to fraction in smaller and smaller bubbles before disappearing in the equilibrium state. However, in some exceptional circumstances, the energy of these droplets is smaller than $k_B T$, and these walls survive in equilibrium. In the droplet picture, this occurs with probability $k_B T/\ell^\theta$ [19].

Now, there are many conformations of the domain wall which have approximately the same energy. One can flip clusters of spins that touch the domain wall at a small cost, corresponding to a local modification (‘blister’) of the conformation of the wall. These excitations can occur on all length scales: one can create blisters within blisters, etc. The situation here is not specific to spin-glasses but is also true for a domain wall in a disordered ferromagnet that can adopt many different metastable configuration. An important difference is that in a disordered ferromagnet, these domain walls have a positive energy and tend to disappear with time: this is the coars-
ening phenomenon. The evolution of the system is in this case a slow but irreversible march towards order \cite{13,33}. The evolution in phase space is biased by the fact that these walls cost energy.

It is useful to decompose the conformations of these pinned ‘domain walls’ on different length scales. Identical conformations on length scale $\ell_n$ may differ by the presence or absence of blisters of smaller sizes $\ell_{n-1}, \ell_{n-2}, \ldots$, where $\ell_n = b^n \ell_0$, and $b$ an arbitrary factor, say $b = 2$. Here again, we follow ideas developed in the context of the droplet model for spin-glasses \cite{19} or pinned domain walls \cite{36,37}. In the droplet model, the time needed to evolve the conformation on scale $\ell_n$ is taken to be:

$$t_n = t(\ell_n) \sim \tau_0 \exp \left( \frac{\Upsilon \ell_n^\psi}{k_B T} \right)$$  \hspace{1cm} (1)$$

where $\tau_0$ is a microscopic time, $\Upsilon$ a typical energy setting the scale of energy barriers between conformations, and $\psi$ the so-called barrier exponent. This form has an immediate consequence: the time needed to evolve the system on scale $\ell_n$ is extremely long compared to the time needed to evolve the system on scales $\ell_{n-1}, \ell_{n-2}, \ldots$. This means, as emphasized in \cite{38}, that on a time scale $t_n$, all excitations on scales $\ell_{n'}$ with $n' > n$ are essentially frozen, whereas all excitations on scales $\ell_{n'}$ with $n' < n$ are essentially equilibrated. Note also that short length scales are ‘slaved’ to large length scales: when a large length scale flips over, all the smaller length scales have to re-equilibrate in a new environment. In this sense, the dynamics is hierarchical (see a related discussion in \cite{32}).

We will choose to write Eq.\,(1) in a generalized form, more appropriate to describe the vicinity of the spin-glass transition:\footnote{In the following, all length scales are expressed in lattice size units.}

$$t_n = t(\ell_n, T) \sim \tau_0 \ell_n^{\nu} \exp \left( \frac{\Upsilon(T) \ell_n^\psi}{k_B T} \right),$$  \hspace{1cm} (2)$$

with $\Upsilon(T) = \Upsilon_0 [T_c - T/T_c]^{\psi\nu}$, $\nu$ being the critical exponent governing the divergence of the equilibrium correlation length $\xi(T)$ at $T_c$: $\xi(T) = |T_c - T|^{-\nu}$. Therefore, the term in the exponential can be rewritten as: $(\Upsilon_0/k_B T)[\ell_n/\xi(T)]^\psi$. Since $\Upsilon_0$ is expected to be of the order of $k_B T_c$, one sees that as long as $\ell_n$ is smaller than $\xi(T)$, barriers are small compared to
and the exponential term in Eq. (2) can be set to 1. This leads to the usual critical dynamics (non activated) relation:

\[ t_n \sim \tau_0 \ell_n^{z_c}. \]  

Conversely, for length scales larger than \( \xi \), the exponential becomes the dominant factor, and one recovers (to logarithmic accuracy) the Fisher-Huse relation (1) with the microscopic time \( \tau_0 \) replaced by the typical critical time scale \( \tau_0 \xi^{z_c} \). As we shall find below, the experiments are typically in a crossover regime where barriers are larger than, but comparable to \( k_B T \). Therefore, we keep the full form of Eq. (2), which correctly interpolates between the two regimes, to describe the intermediate regime.

Several interesting consequences of Eq. (2) are worth discussing.

- Fig. 2 shows \( \log_{10} \) of \( t_n \) versus \( \ell_n \) for a choice of parameters suggested by the experiments on AgMn: \( \psi = 1.5, \Upsilon_0/k_B T_c = 2, z_c = 5 \) and \( \nu = 1.3 \), and for different values of \( T/T_c \). The thick horizontal line corresponds to \( t/\tau_0 = 10^{15} \), corresponding to an experimental time scale of 1000 seconds. One sees that the associated length scales are very modest, in the range 10 to 100, and change appreciably when the temperature is changed only slightly. The horizontal dotted line corresponds to numerically accessible time scales. We see that in that case, length scales are extremely small and do not separate at all with temperature.

- Let us fix a certain length scale \( \ell \) and change the temperature from \( T_1 \) to \( T_2 = T_1 - \Delta T \). The associated time scale \( t_{n1} \) at temperature \( T_1 \) is changed into \( t_{n2} \) given by:

\[ \frac{t_{n2}}{\tau_\ell} = (\frac{t_{n1}}{\tau_\ell})^\beta \quad \beta = \frac{T_1}{T_2} \left( \frac{T_c - T_2}{T_c - T_1} \right)^{\psi \nu} \]  

with \( \tau_\ell = \tau_0 \ell^{z_c} \). This leads to superactivation effects. For example, with \( T_1 = 0.9T_c \) and \( T_2 = 0.8T_c \), \( \psi \nu = 2 \), one finds \( \beta \sim 4 \) ! As soon as \( t_{n1} \gg \tau_\ell \), the value of \( t_{n2} \) is astronomically large. This means that the separation of time scales when the temperature is lowered is extremely fast, and to a good approximation, those length scales that are aging at temperature \( T_1 \) become completely frozen at a slightly lower temperature. In this sense, temperature acts as a microscope. If \( \Upsilon \) was temperature independent, one would find \( \beta = T_1/T_2 = 1.125 \) for
Figure 2: Logarithm (base 10) of equilibration time versus length scales, as given by Eq. 2, for three different temperatures, corresponding to the experimental data of Table 1. The thick horizontal line corresponds to 1000 sec. (10^{15} in microscopic units), and the dotted horizontal line to typical numerical time scales. One should notice that for experimental time scales, length scales are rather modest, but do separate when the temperature is changed, at variance with numerical simulations.
the above choice of parameters. The separation of time scales would then only be mild.

- If one fits Eq. (2) with a temperature dependent power law over a restricted range of times, one finds an effective exponent:

\[
  z_{\text{eff}} = \frac{d \log t_n}{d \log \ell_n} = z_c + \psi \frac{\Upsilon(T)}{k_B T},
\]

that grows when the temperature is lowered. We will come back to this point later.

3 Old experiments revisited

3.1 Small temperature jumps

We now revisit two sets of experiments and show that the results are indeed compatible with Eq. (2). The first set of experiments concerns TRM relaxation. One first measures an array of curves corresponding to the standard protocol: cool the system from above \( T_c \) to \( T_1 \), leave a (small) magnetic field on for a certain waiting time \( t_{w1} \), and observe the relaxation at time \( t_{w1} + t \). One finds curves that scale approximately as \( t/t_{w1} \) once the stationary (fast) initial part has been subtracted. Then, a second protocol is followed: cool now to \( T_2 = T_1 - \Delta T \), and wait for a time \( t_{w2} \). Then heat up the system to \( T_1 \) and simultaneously cut the field. What effective waiting time \( t_{w1}^{\text{eff}} \) should one choose to match this second type of TRM with a standard, isothermal one? The experimental results show that as long as \( \Delta T \) is small enough, it is always possible to find such a \( t_{w1}^{\text{eff}} \) so that the curves match perfectly. This means that the objects involved in the dynamics are exactly the same for the two temperatures. For larger \( \Delta T \)'s, the curves are distorted and such a perfect matching is impossible. Typically, experiments were performed with very small \( \Delta T = 0.02 \) K \( (\Delta T/T_g \sim 0.2\%) \). Obviously, since the time spent at \( T_2 \) corresponds to a smaller time spent at \( T_1 \), one finds \( t_{w1}^{\text{eff}} < t_{w2} \), see Table. It was furthermore shown in [39] that the correspondence between \( t_{w1}^{\text{eff}} \) and \( t_{w2} \) could not be understood in terms of simple thermal activation. More precisely, one finds that \( \log t_{w1}^{\text{eff}}/\tau_0 < (T_2/T_1) \log t_{w2}/\tau_0 \), corresponding to superactivation (unless \( \tau_0 \) is chosen to be unphysically small). This was
Table 1: Effective waiting time $t^{\text{eff}}_{w1}$ at $T_1$ versus real waiting time $t_{w2}$ at $T_2 = T_1 - \Delta T$ for different initial temperatures. The sample is Ag Mn, with a spin-glass temperature of 10.4 K. From [39].

interpreted in [39] as indicating a divergence of the corresponding barrier at smaller temperatures. Note however that a barrier involving a finite number of spins cannot diverge at any temperature. Another interpretation is that barriers actually vanish at $T_c$, as in the droplet model. This is reasonable since the ‘domain walls’ (whatever their precise nature) become more and more loosely defined and can no longer be pinned at $T_c$ (this is also true in a disordered ferromagnet – but not in a random field system: see [11]).

We have therefore reanalyzed the very clean experimental data on Ag Mn of ref. [39] by postulating Eq. (2). We fix $z_c = 5$ and $\nu = 1.3$ to reasonable values and determine $\psi$ and $\Upsilon_0/k_BT_c$ such that the length scale
\( \ell_1 \) corresponding to the time \( t_{w1}^{\text{eff}} \) and the length scale \( \ell_2 \) corresponding to the associated time \( t_{w2} \) are as close as possible: since the shape of the TRM’s are made to coincide, the corresponding length scales should also coincide. Therefore, we choose \( \psi \) and \( \Upsilon_0 / k_B T_c \) such that the mean squared relative difference:

\[
E^2 = \sum_{i=1}^{N} \left( \frac{\ell_1^i - \ell_2^i}{\ell_2} \right)^2,
\]

summed over all experiments reported in Table 1, is as small as possible. The best values are found to be in a ‘crescent’ in the \( \psi, \Upsilon \) plane with for example \( \psi = 1.5 \) and \( \Upsilon_0 / k_B T_c = 2. \), corresponding to a rather small root mean square relative error of \( E = 0.48\% \), or \( \psi = 2 \) and \( \Upsilon_0 / k_B T_c = 0.3 \), corresponding to an error of \( E = 0.40\% \) (the experimental error on \( t_{w1}^{\text{eff}} \) corresponds to a relative error on \( \ell \) on the order of 0.5\%). The typical length scales obtained are shown in Fig. 2. The value \( \psi = 2 \) corresponds with the upper bound proposed by Fisher and Huse for an Ising spin glass. A similar range of values (\( \psi = 1.3 \) for \( \Upsilon_0 / k_B T_c = 1 \)) is found for the insulating CrIn compound \[40\], using the same procedure but on less precise data (larger \( \Delta T \)). The point here is not to claim a very good precision on the value of \( \psi \), but rather to show that the results are compatible with the idea that the barriers continuously vanish at \( T_c \). Note that more recent experiments on the role of small temperature jump on Ising spin-glasses confirm the present analysis, although the separation of time scales is much ‘milder’: the dynamics for \( T > 0.6T_c \) involve both activated events over temperature dependent barriers and critical dynamics that increases the effective value of the ‘trial time’ \( \tau_{\ell} = \tau_0 \ell^{z_c} \). However, the value of \( \psi \) for this Ising-like sample is significantly smaller (\( \psi \sim 0.3-0.5 \)) than is the less anisotropic sample reported above, a somewhat counter-intuitive result.

### 3.2 A time dependent length probed by magnetic field

We now turn to another set of more recent experiments \[13\], which exploits the fact that a small magnetic field acts as to reduce the energy barriers, suggested in \[41\]. If the number of spins involved in a re-conformation is \( N \), one can expect that a field \( H \) will perturb the barriers by an amount

\footnote{The fact that a rather large value of this trial time was needed to account for the experiments was also noticed in \[44\].}
proportional to $N\chi H^2$, where $\chi$ is the magnetic susceptibility. The typical relaxation time of the TRM aged for a time $t_w$ is therefore multiplied by $\exp(-\alpha N(t_w, T)\chi H^2/k_BT)$, where $\alpha$ is a numerical factor. By measuring this reduction factor for different waiting times $t_w$, one can estimate the typical number of spins involved in the dynamics as a function of the waiting time. Finally, by writing $N(t_w, T) \propto \ell^3$, one has access to a time dependent coherence length $\ell(t_w, T)$. As mentioned in the introduction, this dependence can be fitted, for three different types of spin glasses, by a power law with a temperature dependent exponent $z(T)$. Note that the estimated number of spins, in the range $10^5 - 10^6$, is compatible with the length scales reported in Fig 2.

Here, we want to reanalyze the data of [13] in the light of Eq. (2), and show that again, these experimental results suggest that barriers vanish at $T_c$. In order to do so, we have plotted the quantity:

$$G(t_w, T) = \left( \frac{\log t_w/\tau_0 - \frac{z}{3} \log N(t_w, T)}{\frac{T}{T_c} N(t_w, T)^{\psi/3}} \right)^{1/\psi}$$

as a function of $T/T_c$, for different spin-glasses. If Eq. (2) is correct, one should observe $G(t_w, T) = G_0(1 - T/T_c)$, where $G_0$ is a numerical factor. The vanishing of $G(t_w, T)$ is direct manifestation of the vanishing of the barriers. The results are shown in Fig 3. We have kept the values $z = 5$, $\psi = 1.5$ suggested above. A linear fit through the points is very reasonable; the most interesting point is that this linear fit is found to be $G_{fit}(t_w, T) = 0.58 (1.025 - T/T_c)$, very close to what is expected from Eq. (2). Note that the extrapolated value of $T/T_c$ is even closer to one if $z_c$ is chosen to be equal to 6. Therefore, the power-law dependence of $\ell$ reported in [13] (and also in the numerical work of [16, 17, 18]) might actually be an effective power law, as suggested by Eq. (5) above, which naturally matches the critical dynamics exponent when $T \rightarrow T_c$.

In summary, we have shown in this section that two completely independent sets of experiments can be interpreted consistently within the framework of section 2: time scales and length scales are related by Eq. (2). The most important aspect is the fact that energy barriers vanish at $T_c$. This means that (a) dynamics is superactivated and time scales separate extremely fast in spin-glasses, and (b) the critical point does significantly affect the dynamics by slowing down the ‘microscopic’ frequency.
Figure 3: Plot of the experimentally determined quantity $G(t_w, T)$ for different waiting times, temperatures, and three different spin-glasses with different $T_c$, plotted as a function of $T/T_c$. Also shown is a linear regression through all the data points, extrapolating to zero at $T/T_c = 1.025$. 
4 Fixed landscape rejuvenation and Memory

4.1 Qualitative ideas

Let us now come back in more details to the temperature cycling experiments of [11, 35] and show how these can be qualitatively interpreted within the general picture of section 2. The crucial observation is that for a given time scale $t_{w1}$ and temperature $T_1$, the aging dynamics is dominated by a characteristic length $\ell_1$ such that $t(\ell_1, T_1) = t_{w1}$. Larger length scales are essentially frozen and do not evolve on the time scale of the experiment, while shorter length scales are fully equilibrated (for a given larger scale conformation) and only contribute to the stationary part of the response function. Note that this picture is obviously a caricature because the energy barriers in disordered systems are expected to fluctuate in space. Therefore, some larger length scales might locally see an exceptionally low barrier, and vice versa.

Now, let the temperature change from $T_1$ to $T_2 = T_1 - \Delta T$. Because of the separation of time scales, a relatively small $\Delta T$ is sufficient to freeze completely the dynamics on scale $\ell_1$ (which will therefore retain the memory of the stay at $T_1$) and to slow down the initially fast dynamics on shorter length scales, selecting a particular one $\ell_2$ to be in the experimental time window. Since at $T_1$ this length scale is equilibrated, the different conformations appear in the course of time with their Boltzmann weights. As the temperature is changed, these Boltzmann weights are modified and the system has to evolve towards a new state. This is true even if the (free)-energy landscape does not significantly evolve between the two temperatures: this is what we call fixed landscape rejuvenation. Take for example a simple two-level (for example two conformations of a domain wall differing by a ‘blister’ of scale $\ell$), with an energy difference $E(\ell)$. The population difference between the two levels change significantly (say by at least 10%) if the change of temperature is such that:

$$\Delta T \frac{\partial \tanh \left( \frac{E(\ell)}{2k_B T} \right)}{\partial T} = \frac{\Delta T E(\ell)}{2k_B T^2} \frac{1}{\cosh^2 \left( \frac{E(\ell)}{2k_B T} \right)} \geq 0.1$$

(8)

The minimal value of $\Delta T$ for such a rearrangement to occur is therefore $\Delta T^*/T \approx 0.15$ which is corresponds to $E(\ell) \sim 2.4 k_B T$. (For multi-level systems, $\Delta T^*$ can be much smaller than this, see below and [47]). It is further-
more easy to show that when a two-level system is driven out-of-equilibrium by a rapid change of temperature, an excess low-frequency dissipation follows. The out of phase a.c. susceptibility is indeed given by:

\[ \chi''(\omega, t_w) \sim \exp\left[-t_w/t(\ell, T_2)\right] \]

where \( t(\ell) \) is the relaxation time of the two-level system. Therefore, cooling a disordered system can induce a strong increase in the out of phase susceptibility (rejuvenation) if there are metastable states such that \( t(\ell_2, T_2) \sim \omega^{-1} \) and \( E(\ell_2) \sim k_B T \).

Lower frequencies therefore probe larger length scales \( \ell_2 \). In the droplet model, \( E(\ell) \) is typically of order \( \Upsilon \ell^\theta \), and the probability to observe an ‘active’ droplet of energy of order \( k_B T \) is \( (k_B T / \Upsilon) \ell^{-\theta} \). If \( \theta \) is positive, we expect that rejuvenation should asymptotically disappear as \( \omega \to 0 \) since the probability of observing a large droplets goes to zero. However, since \( \ell_2 \) depends logarithmically on frequency and \( \theta \) is small (\( \leq 0.2 \)), this will in practice never happen and things will look very much as if \( \theta = 0 \). In other words, the fact that \( \ell \) cannot much exceed 100 in experiments and that \( \theta \) is so small, means that \( \ell^{-\theta} \geq 0.4! \) The influence of large scale, low energy excitations (of order \( k_B T \)), is therefore dominant in real spin-glasses. This feature is actually the basic outcome of mean-field models.

As emphasized above, the strong separation of time scales enables one to observe simultaneously rejuvenation and memory if \( \Delta T \) is large enough: the length scales that one observes at \( T_1 \) are totally frozen at temperature \( T_2 \) and therefore resume aging, unaffected by the long stay at smaller temperatures. Of course, the states on scale \( \ell_2 \) are now out of equilibrium at \( T_1 \); however, the time needed for them to equilibrate is very short, since it is given by:

\[ t(\ell_2, T_1) = \tau_{\ell_2} \left( \frac{t(\ell_2, T_2)}{\tau_{\ell_2}} \right)^{1/\beta} = \tau_{0}^{1-1/\beta} \ell_2^{1-1/\beta} \omega^{-1/\beta} \]

Taking for example \( \beta = 2, \ell_2 = 10 \), we find that \( \omega t(\ell_2, T_1) = 10^3 (\omega \tau_0)^{1/2} \sim 10^{-3} \) for \( \omega = 1 \) Hz. Therefore, these length scales have equilibrated far before the first oscillation of the a.c. field has taken place.

It is interesting to remark that in the scenario, memory cannot be observed after a positive temperature cycle \( T_1 \to T_1 + \Delta T \), if \( \Delta T \) is large enough to induce rejuvenation. This is because upon heating, length scales
larger than $\ell_1$ will now un-freeze and evolve. Since the shorter length scales are slaved to the larger length scales, the length scale $\ell_1$ will itself have to re-equilibrate completely at higher temperatures – this erases the memory of the stay at $T_1$. As will be discussed below, this is not necessarily true in the ‘chaos’ scenario. Note that the ‘slaving’ of small length scales to large length scales also explains qualitatively the ‘second noise spectrum’ experiments of Weissmann [32], which reflect the fact that when a large length scale ‘jumps’, all the smaller length scales have to re-equilibrate in a new environment, uncorrelated with the first [32, 33].

Before describing a more realistic model, let us summarize the above discussion by a schematic figure in the plane $\ell, \Delta T$, showing how the different length scales evolve during the temperature cycle. This figure allows to account qualitatively for the temperature cycling experiments, in particular the fact that memory is not perfect if $t_{w2}$ is large enough or $\Delta T$ small enough (see also [3]). If $\Delta T < \Delta T^*$, no rejuvenation is expected and aging continues, although at a slower rate. Rejuvenation and memory are made possible by the strong length and time scale separation as a function of temperature for a fixed time scale. As shown in Fig. 2 this is not true in numerical simulations, where one cannot probe the multi-scale dynamics of the system.

Note finally that the above discussion is not restricted to spin-glasses, and can be applied to describe the aging dynamics of pinned domain walls in ferromagnets [43], ferroelectrics [35], and glassy polymers [65] where indeed similar effects have been observed experimentally. There is a significant difference, though, which is due to domain coarsening. The dynamics of the system progressively gets rid of the domain walls; this corresponds to a cumulative aging effect, which is cooling rate dependent. As time evolves, the fraction of spins that belong to domain walls and contribute to aging dynamics systematically decreases [42], as observed in these systems [43, 35].

4.2 More sophisticated models

The simple two-level picture discussed above is obviously not sufficient to explain in details the experimental data. For example, the aging behaviour of $\chi''$ is well fitted by a power-law: $\chi''_{AG}(\omega, t_w) \propto (\omega t_w)^{-b}$, with $b \sim 0.2$. This is quite different from Eq. (9), although the latter correctly predicts a strong increase of $\chi''$ at low frequencies. On a given length scale $\ell$, there are actually many metastable conformations of the domain walls, between which the sys-
Figure 4: Schematic view of the plane $\ell, \Delta T$, for a certain time $t_{w1}$ spent at $T_1$ (corresponding to length $\ell_1$) and a certain (longer) time spent at $T_2 = T_1 - \Delta T$. For large enough $\Delta T > \Delta T^*$, strong rejuvenation is expected. However, very small length scales (below the dashed curve) are always in equilibrium. Large length scales (above the thick curve, the position of which depends on $t_{w2}$) do not evolve at all during the stay at $T_2$. Intermediate length scales (between the two curves at to the left of the vertical plain line) do age at $T_2$, but re-equilibrate at $T_1$ faster than $1/\omega$. These length scales are responsible for the coexistence of rejuvenation and perfect memory. Finally, for large enough $t_{w2}$ or small enough $\Delta T$, certain length scales (indicated by the arrow) continue to age at $T_2$ and re-equilibrate slowly at $T_1$, therefore destroying the perfect memory effect. This is indeed observed experimentally. A very similar picture would hold in the ‘chaos’ scenario, with the constant $\Delta T^*$ line replaced by a crossover line $\Delta T^* \sim \ell^{-1}$.
tem jumps. One expects that the energy barriers between these metastable states are distributed, over a certain energy scale $\gamma(T)\ell^\psi$. A simple model is to assume that energy barriers are independent random variables – this is the trap model \[48\]. There is a limit where this model has a sharply defined behaviour: when the distribution of barriers is exponential, of mean $T_c(\ell)$, then there is a well defined transition temperature which separates an equilibrium phase for $T > T_c(\ell)$ where all configurations contribute more or less equally to the equilibrium partition function, from an aging phase for $T < T_c(\ell)$ where the partition sum is dominated by a few metastable states only \[44, 45, 46\]. This means that a small temperature change for $T_c(\ell) + \Delta T/2$ to $T_c(\ell) - \Delta T/2$ completely changes the way the configuration space is explored. Correspondingly, a strong rejuvenation effect is expected upon crossing $T_c(\ell)$ (see \[47\] for details). Furthermore, the realized probability to find the system in a metastable state of lifetime $\tau$ after a waiting time $t_w$ is, for $T < T_c(\ell)$, given by \[48\]:

$$P_{\ell}(\tau, t_w) \sim \frac{t_w^{x_\ell - 1}}{\tau^{x_\ell}} \mathcal{F}\left(\frac{\tau}{t_w}\right)$$

where $x_\ell = \frac{T}{T_c(\ell)} < 1$, \[11\]

where $\mathcal{F}(u)$ is a cut-off function, decaying as $1/u$ for large arguments. One can therefore obtain the a.c. susceptibility by averaging the simple two-level contribution Eq. \[10\] over the realized distribution of trapping times:

$$\chi''(\omega, t_w) = \int_{\omega - 1}^{\infty} d\tau \ P_{\ell}(\tau, t_w) \frac{\exp[-t_w/\tau]}{\omega \tau} \propto (\omega t_w)^{-b},$$

with $b = 1 - x_\ell$ \[19\]. Therefore, the introduction of a large number of metastable states allows one to obtain both a strong rejuvenation effect at the transition temperature, and a realistic form for the decay of the a.c. susceptibility. Since there is a continuum hierarchy of length scales, each of which corresponding to a different transition temperature $T_c(\ell)$, the problem is actually a multilevel trap model of the type studied in \[33, 52\]. The contributions of the different length scales are intertwined, and one expects rejuvenation to occur at all temperatures provided the corresponding $\ell$ is somewhat larger than the microscopic length. \[\] The coexistence of strong

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\(3\) The a.c. susceptibility is however dominated at long times $t_w$ by the length scales such that $x_\ell \sim 1$, therefore explaining why $b = 1 - x_\ell$ is found to be small ($1/f$ noise) and nearly temperature independent.
rejuvenation and memory in this model has been numerically demonstrated very convincingly in [52], and confirm the above qualitative discussion.

There are many possible variants of the above trap model [50, 51]. The basic results still hold for more general barrier height distributions, with a weakly time dependent exponent $x_\ell$ [50]. One can also consider long-range correlated energy landscapes, such as the one dimensional Sinai model where barriers typically grow with the position of the representative point [53]. Many results on aging have been obtained for this model [54], which has a typical self-similar ‘valley within valley’ structure. Preliminary numerical studies also show that rejuvenation and memory effects are also present in the Sinai model [55]. The mechanism at work in these landscape models is very close to the hierarchical picture first advocated by experimentalists to explain rejuvenation and memory [30, 31]: a reduction of temperature reveals finer details of the energy landscape within which the system must equilibrate.

4.3 More on ‘temperature chaos’

4.3.1 Rejuvenation....

As mentioned in the introduction, the strong rejuvenation effect and the absence of cooling rate dependence have also been interpreted in terms of ‘temperature chaos’. Within the droplet model, the argument suggesting this behaviour is the following: the free energy of an excitation of length $\ell$ is small as a result of the compensation of its energy and entropy, both of them being much larger than $\Upsilon_\ell$. Therefore, a small change of temperature $\Delta T$ should ruin this subtle compensation on large length scale. This can be interpreted in terms of a complete re-shuffling of the dominant configuration beyond a certain overlap length $\ell^*$ that diverges when $\Delta T \to 0$. The physical mechanism is actually related to the discussion of the previous subsection: since the shorter length scales have to reorganize when the temperature changes, the free energy of the larger length scales will be strongly affected. So the description of large length scales in terms of simple two (or multi) level systems with a fixed energy landscape is inappropriate: the ‘landscape’ itself is temperature dependent.

Using the value of the exponents in the droplet model in three dimensions, one naively estimates $\ell^* \sim \Upsilon(T)/\Delta T$. Using the above results on $\Upsilon(T)$, one
finds that for $T = 0.7T_c$, $T_c = 15$ K and $\Delta T = 1$ K, one should have $\ell^* \sim 1$; therefore ‘chaos’ effects should be observable both numerically and experimentally. Since no sign of chaos was detected numerically for small systems \[26\], this suggests that numerical prefactors are perhaps large, and that actually the overlap length is, for practical purposes, larger than the numerically or experimentally relevant length scale. This scenario is supported by recent precise numerical studies of temperature chaos in a simpler problem \[58\] – the pinning of a one dimensional domain wall (the directed polymer problem). In this case, the overlap length indeed scales as expected with $\Delta T$, although rather large sizes are needed to observe the effect. The heuristic scaling arguments for chaos \[19, 24\] can furthermore, for this simplified problem, be made more precise \[58\], and suggest that the decorrelation beyond the overlap length only decays as a power law rather than an exponential \[60\]. The need to go to very large length scales appears to be true also for the 2D spin-glass \[61\]. Therefore, even if temperature chaos appears to be absent in the sk model, it is possible that this effect exists in three dimensions, although perhaps on very large length scales.

On the other hand, as we discussed above, a hierarchical landscape rejuvenation appears sufficient to account for most of the experimental data. The main distinction is the existence of a characteristic overlap length. In this respect, one reason to believe that ‘strong’ temperature chaos is perhaps not relevant is that rejuvenation appears to be a small scale, rather than large scale, phenomenon. When the temperature is changed sufficiently to induce partial rejuvenation, the aging a.c. susceptibility is made up of two contributions, a short time, rejuvenation part, and a long time contribution which is the continuation of aging at the first temperature, with a shifted effective age to account for the modification of time scales with temperature. This suggests that the length scale $\ell_1$ built at $T_1$ actually continues to grow at $T_2$, although by definition, since some rejuvenation is observed, one should be in a regime where $\ell^* < \ell_1$. The same effect is seen in ZFC (zero field cooled) experiments and in numerical simulations \[4\]: after a short transient, the length scale that grew at $T_1$ continues to grow (albeit at a different rate) at $T_2$.

Another argument against the relevance of an overlap length is provided by the quantitative analysis of the effect of a small temperature cycle on the a.c. susceptibility. If the length scales $< \ell^*$ are unaffected, one expects that the effective initial age $t^*$ of the a.c. susceptibility at $T_1 - \Delta T$ is such
that \( t(\ell^*, T_1 - \Delta T) = t^* \) (provided that the size of the domains grown at \( T_1 \) is larger than the overlap length, i.e. \( \ell_1 > \ell^* \)). We have extracted \( t^* \) from a set of experiments where \( t_{w1} \) and \( T_1 \) are fixed, and \( \Delta T \) is varied. For CrIn, we find that \( t^* \) behaves as \( \Delta T^{-a} \), with \( a \sim 1 \) for \( T/T_c = 0.72 \) and \( a \sim 1.5 \) for \( T/T_c = 0.84 \). This is incompatible with the assumption that \( \ell^* \sim \Delta T^{-1} \), which would lead to an extremely fast divergence of \( t^* \) for small \( \Delta T \), according to Eq. (2). A possibility would be that \( \ell^* \) behaves as \( \Delta T \) to a small negative power, but this violates bounds obtained within the framework of the droplet model [19].

4.3.2 ...and memory?

Strong chaos does obviously explain complete rejuvenation for \( \Delta T \) sufficiently large. In order to be compatible with memory, one should argue, as above, that the length scale \( \ell_2 \) growing at \( T_2 \) remains somewhat smaller than \( \ell_1 \); the fast separation of time scales is then used to account for memory – see the discussion in [34]. On the other hand, the magnetic field is known to lead to very strong ‘chaos’-like effects. For example, a small magnetic field cycle is sufficient to rejuvenate completely the system. Furthermore, chaos with magnetic field in the SK model was obtained long ago [59]. An interesting idea is then to perform a simultaneous temperature and magnetic field cycle, with a \( \Delta T \) such that rejuvenation and perfect memory are observed in the absence of magnetic field changes. In the temperature chaos scenario, adding a magnetic field should not change anything since the overlap length is already very small. In the fixed landscape scenario, the magnetic field should have an effect since the large scale structures built at \( T_1 \) will couple to the field (as was discussed in subsection 3.2) and therefore speed up their dynamics at \( T_2 \). Therefore, some loss of memory should be observed, as indeed suggested by the experiments: see Figure 5.

Following this line of thought, we have also performed an experiment to test a very spectacular prediction of [34]: a double temperature cycling where \( T_1 \to T_2 = T_1 - \Delta T \) is followed, after a certain time \( t_{w2} \) at \( T_2 \) by a second small quench \( T_2 \to T_3 = T_2 - \Delta T' \). If the system is re-heated rapidly to \( T_1 \) without stopping at \( T_2 \), the dramatic prediction of [34] is that the memory effect should be destroyed. Only if the system is allowed to ‘take its breath’ at \( T_2 \) will memory be preserved. The problem is that ‘re-heating rapidly’ cannot be achieved experimentally, since the fastest achievable temperature
Figure 5: Effect of a magnetic field on the memory. Here, the system is cooled from $T_1 = 14$ K to $T_2 = 12$ K, and heated back to 14 K. Two experiments have been performed: one where the system is unperturbed during the stay at $T_2$, the other where an extra magnetic field of $\Delta H = 60$ Gauss is imposed. This magnetic field is known to lead to a strong rejuvenation effect. The choice of parameters is such that memory is not perfect, even for $\Delta H = 0$. Here, we see that the effect of $\Delta H$ is noticeable, which shows that the purely thermal overlap length cannot be small.
ramps allows the system to stay at each temperature during a time which is huge in microscopic units. This might be enough (again due to the fast separation of time scales) to allow for memory conservation! Our idea was then to use the magnetic field to prevent the system from tracing back its previous history, which is the crucial ingredient, in the chaos scenario, to preserve the memory (see the detailed discussion in [34]). We have therefore applied a magnetic field while the system is re-heated: the target state at $T_2$ is therefore completely scrambled. In spite of this, the loss of memory when returning at $T_1$ is the same as for a single quench procedure. Therefore, the spectacular effect predicted within the ‘strong chaos’ scenario fleshand out in [34] is not observed.

Finally, note that in the chaos scenario, it should be possible to choose $\Delta T$, $t_{w1}$ and $t_{w2}$ in a positive temperature cycling such that $t_2 < t_1$. In this case, memory should be preserved in a positive cycling experiment, even if some rejuvenation takes place at $T_2$. To our knowledge, this has never been observed, but this might again be due to the fact that the time scales separate very quickly. Taking the same value of parameters as in Fig. 2, we find that for $T_2 = 0.77T_c$, $t_{w2} = 10$ sec., $t_2 \sim 20$. One would need to wait at least $t_{w1} \sim 10^5$ seconds (1.5 day) at $T_1 = 0.77T_c$ to reach the same length and allow memory to be preserved. Note that the fundamental asymmetry between small positive and small negative temperature jumps is probably the most clear cut difference between the ‘chaos’ scenario and the hierarchical landscape scenario. It would therefore be crucial to find an experimental situation where rejuvenation and memory in a positive $\Delta T$ cycle should in principle be observed. It would be interesting to study this issue in the Ising-like sample studied in [40], where the separation of time scales is milder.

5 Conclusion – Open problems

In this paper, we have summarized the different puzzles raised by aging experiments of spin-glasses and their different interpretations. We try to reconcile the ‘real space’, droplet like pictures and the hierarchical pictures that have been proposed in the past. The basic ingredient is a strong separation of the time scales that govern the dynamics of the system on different length scales. Changing the temperature changes the length scale at which the system is
Figure 6: Now, the system is cooled from $T_1 = 14$ K to $T_2 = 12$ K, and then to $T_3 = 10$ K in zero field. When the system is heated back, an extra magnetic field is imposed between 11 K and 13 K. In the ‘strong’ chaos scenario, this should prevent the system from ‘remembering’ its previous 12 K history, and should completely scramble the memory effect at 14 K. The comparison between this data and that of Figure 5 shows that the loss of memory is only partial, and nearly identical in the two cases. Therefore, the spectacular loss of memory in a double quench experiment suggested in [34], is not observed.
observed, thereby allowing rejuvenation (that concerns short length scales) and memory (stored in long length scales) to coexist. We have shown that previous experiments can be reanalyzed in terms of vanishing energy barriers at the spin-glass transition, which in turn leads to the ‘super-activated’ behaviour observed in several experiments. We have argued that the power-law dependence of the coherence length with time might actually reflect a slow crossover form critical to activated dynamics. Finally, we have tried to distinguish between a hierarchical landscape rejuvenation, which is already present in simple multi-level systems like the Random Energy Model [47], from a more sophisticated ‘strong’ chaos that arises because large length scales free energies are renormalized by small length scale fluctuations [24, 19]. We have argued that most experiments can be accounted for without invoking the existence of an overlap length. Some specific predictions of the strong chaos scenario have been tested and our not borne out by our new experimental results. Nevertheless, we believe that this effect should exist on sufficiently large length scales, but these are perhaps out of reach both from numerical and experimental possibilities. It should finally be noted that rejuvenation and memory effects have also been observed in other, very different systems, such as PMMA [65], where the relevance of temperature chaos is not clear, whereas a scenario based on multiscale dynamics is plausible.

From a theoretical point of view, the dynamics of mean field models corresponding to full replica symmetry breaking has been shown to exhibit rejuvenation and memory effects [56], and are actually very closely related to models of diffusion in self-similar landscapes such as the Sinai model [57], although the precise role of activation in these models is still rather obscure. However, these models are in principle incompatible with simple $t/t_w$ aging, as experimentally [2] and numerically [64] observed. It would be gratifying to understand whether or not these mean field models can correctly be interpreted in finite dimensions in terms of the simple trap models [19]. This question of course has a far more general scope and relevant for other glassy systems as well.

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