Gravity as a Quantum Effect on Quantum Space-Time

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The 3+1-dimensional Einstein-Hilbert action is obtained from the 1-loop effective action on non-commutative branes in the IIB or IKKT matrix model. The presence of compact fuzzy extra dimensions \( K \) as well as maximal supersymmetry of the model is essential. The E-H action can be interpreted as interaction of \( K \) with the space-time brane via IIB supergravity, and the effective Newton constant is determined by the Kaluza-Klein scale of \( K \). The classical matrix model defines a pre-gravity action with 2 derivatives less than the induced E-H action, governing the cosmological regime. The perturbative physics is confined to the space-time brane, which for covariant quantum space-times includes all dof of gravity, as well as a tower of higher-spin modes. The vacuum energy of the background is given in terms of the symplectic volume form, and hence does not act as cosmological constant.

I. INTRODUCTION

Classical gravity is well described by general relativity (GR), which arises from the Einstein-Hilbert action. However, this formulation is not well suited for quantization. Among the many approaches to reconcile quantum mechanics with gravity, string theory is perhaps distinguished by a sort of structural uniqueness. However, it leads to a vast “landscape” of possible compactifications to 3 + 1 dimensions. In this letter, we propose a possible resolution of this problem based on a certain matrix model, which was proposed as a constructive definition of string theory.

Our starting point is the maximally supersymmetric IIB or IKKT matrix model

\[
S = \frac{1}{g^2} \text{Tr} \left( [Y^{\dot{a}}, Y^{\dot{b}}][Y_{\alpha}, Y_{\beta}] + \nabla \Gamma_{\alpha}[Y^{\dot{a}}, \Psi] \right). \tag{1}
\]

Dotted indices transform under a global \( SO(9, 1) \), and are raised and lowered with \( g^{\dot{a} \dot{b}} \). Here \( Y^{\dot{a}}, \alpha = 0, ..., 9 \) are hermitian matrices or operators acting on some (finite-dimensional or separable) Hilbert space \( \mathcal{H} \), while \( \Psi \) is a matrix-valued Majorana-Weyl spinor of \( SO(9, 1) \). We will mostly ignore the fermions \( \Psi \), although their presence is crucial for quantization.

The action (1) is known to have a variety of critical points or backgrounds \( Y^{\dot{a}} \), which can be interpreted as branes in target space \( \mathbb{R}^{9,1} \) carrying some \( B \) field. The basic examples are flat branes described by Moyal-Weyl quantum planes \( \mathbb{R}^{2,9} \subset \mathbb{R}^{9,1} \), where \( [Y^{\dot{a}}, Y^{\dot{b}}] = i \theta^{\dot{a}\dot{b}} \mathbb{1} \). Fluctuations \( Y^{\dot{a}} \rightarrow Y^{\dot{a}} + A^{\dot{a}} \) of such backgrounds are governed by a non-commutative gauge theory \( \mathbb{1} \), where \( A^{\dot{a}} \) becomes a gauge field on the brane. We focus on backgrounds leading to a weakly interacting gauge theory. However, we consider more general backgrounds \( Y^{\dot{a}} \) interpreted as quantized embedding functions

\[
Y^{\dot{a}} \sim y^{\dot{a}} : \mathcal{M} \mapsto \mathbb{R}^{9,1} \tag{2}
\]

of a Poisson manifold \( \mathcal{M} \) in target space \( \mathbb{R}^{9,1} \), reducing to classical functions \( y^{\dot{a}} \) in the semi-classical (i.e. Poisson) limit. The \( Y^{\dot{a}} \) will generically not commute, and their commutator \( [Y^{\dot{a}}, Y^{\dot{b}}] \sim i \{ y^{\dot{a}}, y^{\dot{b}} \} \) is interpreted as quantized Poisson bracket on \( \mathcal{M} \). This will be considered as a (non-commutative) brane.

The quantization of this model is defined by the “matrix path integral”

\[
\langle Y...Y \rangle = \frac{1}{Z} \int dY \Psi_\alpha Y_{\beta} e^{iS} \quad Z = \int dY \Psi_\alpha e^{iS} .
\]

This oscillatory integral becomes absolutely convergent for finite-dimensional \( \mathcal{H} \) upon implementing the regularization

\[
S \rightarrow S + i \varepsilon \sum_\alpha Y_\alpha Y_\alpha , \tag{3}
\]

which amounts to a Feynman \( i \varepsilon \) term in the noncommutative gauge theory. For block-matrix configurations describing several branes, the path integral leads to interactions of the branes consistent with IIB supergravity \( \mathbb{1} \) \[4\] in target space \( \mathbb{R}^{9,1} \), thus providing a direct link with string theory.

We will focus on the perturbative physics around matrix configurations describing a single noncommutative brane \( \mathcal{M} \). Then the matrices \( \text{Mat}(\mathcal{H}) \sim C(\mathcal{M}) \) can be interpreted as quantized functions on \( \mathcal{M} \). Fluctuations around the background can thus be interpreted as fields on the brane, governed by a non-commutative gauge theory. In particular, the model leads to 3 + 1 dimensional physics on \( \mathcal{M}^{3,1} \) branes. There are no fields propagating in transversal directions, in contrast to standard string theory.

The propagation of all fluctuations on such a \( \mathcal{M}^{3,1} \) brane is governed by a universal dynamical metric specified below. We will demonstrate that the dynamics of this geometry is governed by an Einstein-Hilbert term which arises in the one loop effective action of the IKKT model, assuming the presence of fuzzy extra dimensions \( \mathcal{K} \). More specifically, we assume a background brane with
product structure\footnote{Such backgrounds are known to be solutions of the classical matrix model amended with quadratic and cubic terms \cite{10,11}. We assume that these terms can be replaced by loop corrections, as suggested by \cite{12,13}.} \[ \mathcal{M}^{3,1} \times \mathcal{K} \subset \mathbb{R}^{9,1}, \] where $\mathcal{K}$ is a quantized compact symplectic space (such as a fuzzy sphere) supporting finitely many dof, cf. \cite{10,11}.

II. KINEMATICAL SETUP

We consider matrix configurations $Y^\alpha = (Y^{\hat{a}}, Z^i)$ which describe a noncommutative brane $\mathcal{M} \times \mathcal{K} \subset \mathbb{R}^{9,1}$ embedded in target space. Here $M$ describes a quantized 3+1-dimensional space-time $Y^\alpha$ which correspond to the compact space $\mathcal{K}$ embedded along the first 4 coordinate directions, and $Z^i$ describe a compact quantum space embedded along the transversal directions. Dotted Latin indices $\dot{a}, \dot{i}$ indicate frame-like indices transforming under the $\mathcal{K}$-group and are local coordinates on $\mathcal{K}$. All fluctuations in the matrix model are governed by a kinetic term of the form

$$T^{\dot{a} \mu} := \{ \mathcal{Y}^{\dot{a}}, \mathcal{X}^\mu \} \sim \int \frac{d\delta x}{\sqrt{G}} G^{\mu \nu} \partial_\mu \phi \partial_\nu \phi$$

in the semi-classical limit $\Phi \sim \phi$. This defines the effective metric on $\mathcal{M}$:

$$G^{\mu \nu} = \rho^{-2} \gamma^{\mu \nu}, \quad \gamma^{\mu \nu} = \eta_{\dot{a} \dot{b}} E^{\dot{a} \mu} E^{\dot{b} \nu},$$

for a uniquely determined dilaton $\rho$ \cite{13}. To capture the geometry in the matrix model, it is useful to consider the following “torsion” tensor $T^{\dot{a} \mu}$ defined via $E^{\dot{a} \mu} E^{\dot{b} \nu} = \delta^\nu_\mu$ and obtain

$$T_{\sigma \kappa}^{\mu} = E^{\dot{a} \mu} (\partial_\kappa E^{\dot{a} \sigma} - \partial_\sigma E^{\dot{a} \kappa}).$$

This becomes more transparent in terms of the 2-form

$$T^{\dot{a}} = \frac{1}{2} T_{\mu \nu}^{\dot{a}} d x^\mu d x^\nu = d \theta^{\dot{a}}$$

where

$$\theta^{\dot{a}} := E^{\dot{a} \mu} d x^\mu$$

is the coframe one-form. Therefore the torsion 2-form is nothing but the exterior derivative of the coframe. In the matrix model framework, the first form in (15) is most relevant; for a more detailed discussion see \cite{10,11}.

a. Covariant quantum spaces. For 3+1-dimensional noncommutative branes $\mathcal{M}$, the tangential fluctuations cannot provide the most general metric dof. Transversal fluctuations are presumably suppressed in the presence of $\mathcal{K}$. Furthermore, the presence of an antisymmetric tensor field $\sigma^{\dot{a} \dot{b}}$ on space-time is problematic. These issues are resolved for covariant quantum spaces, which are twisted bundles over space-time with a 2-dimensional fuzzy sphere $S^2_\eta$ as fiber \cite{13,15},

$$\mathcal{M} \cong \mathcal{M}^{3,1} \times S^2_\eta.$$
embedded in the matrix model via 3+1 generators $Y_\alpha$. For example, a FLRW space-time is realized by

$$Y_\dot{\alpha} = \frac{1}{R} \mathcal{M}^{44}$$  \hspace{1cm} \text{(20)}$$

acting on a doubleton irrep $\mathcal{H}_a$ of $\mathfrak{so}(4,2)$. One can then expand the fluctuations into harmonics $\phi_{sm} = \phi_{sm}(y) Y_{sm}$ on $S_n^2$, so that \[ \square \mathcal{M} = \square \mathcal{M}^{a_{s+1}} + n_s^2, \quad m_s^2 = \frac{s(s-1)}{R^2}. \hspace{1cm} \text{(21)} \]

Here $R$ is a large (cosmological) scale parameter. Due to the twisted bundle structure, the $Y_{sm}$ turn out to be spin $s$ modes on space-time. This leads to a finite tower of higher-spin excitations for $s \leq n$, which provides all degrees of freedom of gravity in 3+1 dimensions [13]. The above discussion for frame and torsion generalizes easily, in terms of higher-spin valued frame and torsion. We will ignore their higher-spin contributions in the following, and focus on the geometric sector.

### III. ONE-LOOP EFFECTIVE ACTION

We wish to compute the 1-loop effective action on a background of the above type, in the weak coupling regime. We first ignore the internal fiber $S_n^2$ of covariant quantum spaces, which will be included later. The one-loop effective action of the IKKT model for some given matrix background $Y_\alpha$ is defined by

$$Z_{\text{1-loop}}[Y] = \int_{\text{1-loop}} dY d\psi e^{iS[Y,\psi]} = e^{i(S[Y]+\Omega_{1\text{loop}}[Y])}$$

in terms of an oscillatory Gaussian integral around $Y$. Taking into account the fermions and the ghost contributions, this leads to [11, 13, 16]

$$\Gamma_{\text{1-loop}}[Y] = \frac{i}{2} \text{Tr} \left( \log(\square + M^{(V)}_{\dot{a}\dot{b}})\Theta^{\dot{a}\dot{b}}, \right)$$

$$- \frac{1}{2} \log(\square + M^{(w)}_{\dot{a}\dot{b}})\Theta^{\dot{a}\dot{b}}, \right) - 2 \log \square \right). \hspace{1cm} \text{(22)}$$

Here the trace is over hermitian matrices in $\text{Mat}(\mathcal{H})$, and

$$M^{(V)}_{\dot{a}\dot{b}} = \frac{1}{4\pi}[\Gamma\dot{a}, \Gamma\dot{b}]$$

$$M^{(w)}_{\dot{a}\dot{b}} = i(\delta_{\dot{a}\dot{b}}\eta_{\dot{a}\dot{b}} - \delta_{\dot{a}\dot{b}}\eta_{\dot{a}\dot{b}}), \hspace{1cm} \text{(23)}$$

are $SO(9,1)$ generators acting on the vector or spinor representation, respectively. The $2 \log \square$ term arises from the ghost contribution. Using the expansion

$$\log(\square + \mathcal{O}) = \log \square + \sum_{n=0}^{\infty} \frac{1}{n} (\square^{-1} \mathcal{O})^n \hspace{1cm} \text{(24)}$$

and observing that the first 3 terms in this expansion cancel due to maximal supersymmetry, the leading non-trivial term is the 4th order term given by [16]

$$\Gamma_{\text{4-loop}}^{\text{1-loop}} = \frac{i}{8} \text{Tr} \left( \left(\square^{-1} M^{(V)}_{\dot{a}\dot{b}}\Theta^{\dot{a}\dot{b}}, \right)^4 - \frac{1}{2} \left(\square^{-1} M^{(w)}_{\dot{a}\dot{b}} \Theta^{\dot{a}\dot{b}}, \right)^4 \right) \hspace{1cm} \text{(25)}$$

dropping $O(\square^{-5})$ contributions. In these expressions, $\square \rightarrow \square - i\varepsilon$ arising from [3] is understood.

#### a. Evaluation of the trace.

We will evaluate the trace over $\text{Tr}_{\mathcal{M}} = \text{Tr}_{\text{Mat}(\mathcal{H}, \mathcal{M})}$ using the basic formula [5]

$$\text{Tr}_{\mathcal{M}} \mathcal{O} = \frac{1}{(2\pi)^4} \int_{\mathcal{M} \times \mathcal{M}} \Omega_x \Omega_y \langle \mathcal{O} \mid \mathcal{O}_y \rangle \hspace{1cm} \text{(26)}$$

where $\Omega = d^4x \rho_{\mathcal{M}}$ is the symplectic volume form. Here

$$\mid \mathcal{O}_y \rangle := |y\rangle \langle x| \in \text{Mat}(\mathcal{H}, \mathcal{M}) \hspace{1cm} \text{(27)}$$

are string modes in terms of coherent states $|x\rangle$ on the symplectic space $\mathcal{M}$. This formula is exact for homogeneous quantum spaces, and follows from group invariance [5]. As a symplectic manifold, we can assume that $\mathcal{M}$ is equivalent to a 4-dimensional quantized homogeneous space such as $\mathbb{R}^4_2$; the extra $S_n^2$ factor on covariant quantum spacetime will be taken into account later.

String modes are useful to evaluate the trace over $\text{Mat}(\mathcal{H}, \mathcal{M})$, because they enjoy approximate localization properties in both position and momentum [5, 17]. In particular,

$$\square(|y\rangle \langle x|) \sim |x - y|^2 + L_{NC}^2 |y\rangle \langle x|$$

$$\langle \Theta^{\dot{a}\dot{b}}, \mid |y\rangle \langle x| \rangle \sim \delta \Theta^{\dot{a}\dot{b}}(y,x) |y\rangle \langle x|$$

$$\delta \Theta^{\dot{a}\dot{b}}(y,x) = \Theta^{\dot{a}\dot{b}}(y) - \Theta^{\dot{a}\dot{b}}(x). \hspace{1cm} \text{(28)}$$

where $|x - y|$ is the distance in target space $\mathbb{R}^{9,1}$, and

$$L_{NC}^2 = \text{det}(\theta^{\dot{a}\dot{b}})^{1/4} \hspace{1cm} \text{(29)}$$

is the scale of noncommutativity. Thus $\square$ measures the length of the string modes, which are responsible for UV/IR mixing in non-SUSY models [13].

We can therefore evaluate the 1-loop integral approximately as follows

$$\Gamma_{\text{4-loop}}^{\text{1-loop}} = \frac{i}{4} \frac{1}{(2\pi)^4} \text{Tr}_\mathcal{K} \int_{\mathcal{M} \times \mathcal{M}} \Omega_x \Omega_y \frac{3V_4[\delta \Theta(x,y)]}{|x - y|^2 + L_{NC}^2} \hspace{1cm} \text{(30)}$$

where [5]

$$V_4[\delta \Theta] = \delta \Theta^{\dot{a}\dot{b}} \delta \Theta^{\dot{a}\dot{b}} \delta \Theta^{\dot{a}\dot{b}} \delta \Theta^{\dot{a}\dot{b}} - 4\delta \Theta^{\dot{a}\dot{b}} \delta \Theta^{\dot{a}\dot{b}} \delta \Theta^{\dot{a}\dot{b}} \delta \Theta^{\dot{a}\dot{b}} - 4(4\delta \Theta)^4 \hspace{1cm} \text{(31)}$$

Here $\delta \Theta^{ij} \rightarrow [\Theta^{ij}, \cdot]$ is understood for the contributions of $\mathcal{K}$. On a product space $\mathcal{M} \times \mathcal{K}$, this leads to the following contributions

$$V_4 = V_4^\mathcal{M} + V_4^\mathcal{K} + V_4^{\mathcal{M}\mathcal{K}}. \hspace{1cm} \text{(32)}$$

The most interesting term is

$$V_4^{\mathcal{M}\mathcal{K}} = 2\delta \Theta^{\dot{a}\dot{b}} \delta \Theta^{\dot{a}\dot{b}}[\Theta^{ij}, [\Theta^{ij}, \cdot]] \hspace{1cm} \text{(33)}$$
which is responsible for the interaction between $\mathcal{M}$ and $\mathcal{K}$, and gives rise to a gravity action on $\mathcal{M}$. There are no other contributions, since the mixed components of the Poisson tensor $[Y^\alpha, Y^\beta] = 0$ vanish. The contribution of $\mathcal{K}$ in \cite{25} can be evaluated for the product states \cite{9}, noting that
\[
[\Theta^{ij}, \phi_\lambda(y)\lambda_\lambda] = \phi_\lambda(y) [\Theta^{ij}, \lambda_\lambda]. \tag{34}
\]
We assume for simplicity that $\lambda_\lambda$ is a common eigenvector of both $\Box_\mathcal{K}$ and $[\Theta^{ij}, \lambda_\lambda]$ in $\text{Mat}(\mathcal{H}_\mathcal{K})$. Then
\[
[\Theta^{ij}, [\Theta^{ij}, \lambda_\lambda]] = m^2_k C^2_\lambda \lambda_\lambda,
\]
where $m^2_k$ is the KK mass scale, and $C^2_\lambda > 0$ are numerical constants depending on the structure of $\mathcal{K}$.

To compute $V^4_{4\mathcal{KM}}$, we need to evaluate $\delta\Theta^{ab} \delta\Theta^{ab}$ on the string states, recalling that $\delta\Theta^{ab}$ stands for $[\Theta^{ab}, \cdot]$. Since the integral \cite{30} is convergent due to the $\frac{1}{\sqrt{|x-y|^2}}$ behavior, only “short” string states with $|x-y| \leq L_{\text{NC}}$ contribute, which capture the low-energy sector of $\text{Mat}(\mathcal{H}_\mathcal{M})$. Then $[\Theta^{ab}, \cdot] \sim \{\Theta^{ab}, \cdot\}$ can be evaluated in the semi-classical regime, but the approximate relations \cite{25} need to be refined. This is achieved by recognizing that short string states correspond precisely to localized Gaussian wave packets on $\mathcal{M} \approx \mathbb{R}^d$:
\[
e^{\frac{1}{4} k^\mu k^\nu \delta_{\mu\nu}} e^{-\frac{1}{4} |x-y|^2} = \Psi_{k,x} \equiv \psi_{k,y}(x) = \frac{2}{\pi L^2_{\text{NC}}} e^{i k x y} e^{-\frac{1}{4} |x-y|^2}. \tag{36}
\]
Here $\tilde{k}$ is the KK mass scale, and $C^2_\lambda$ are Gaussian averages of the short string modes with size $L \gg L_{\text{NC}}$. These are approximately plane waves $\psi_k \propto e^{i k x}$, which allows to evaluate $\{\Theta^{ab}, \cdot\} \sim i \{\Theta^{ab}, \cdot\}$ as
\[
\{\Theta^{ab}, \psi_k\} = \{\Theta^{ab}, x^\mu\} \partial_\mu \psi_k \approx -i T^{ab}_\mu k_\mu \psi_k
\]
and
\[
\{\Theta^{ab}, \psi_{k,y}\} \approx -\{\Theta^{ab}, x^\mu\} \{\Theta^{ab}, x^\nu\} k_\mu k_\nu \psi_{k,y}.
\]
This is justified if the torsion $T^{ab}_\mu$ \cite{15} is approximately constant on length scales $L$, which can be assumed in the context of gravity. Putting this together, $V^4_{4\mathcal{KM}}$ acting on the $\psi_{k,y}$ depends on $C^2_\lambda$ is approximately diagonal, and reduces to
\[
V^4_{4\mathcal{KM}}[\psi_{k,y}] = 2m^2_k C^2_\lambda T^{ab}_\mu T^{\lambda\nu}_b k_\mu k_\nu \psi_{k,y} \lambda_\lambda.
\]
Hence the full trace reduces to the following local integral
\[
\Gamma^{K-M}_{\text{1loop}} = \frac{3i}{4} \text{Tr}(\frac{V^4_{4\mathcal{KM}}}{(4 - i \varepsilon)^4}) \approx 2(2\pi)^4 \int_{\mathcal{M}} d^4x \sqrt{G} \sum \frac{C^2_\lambda}{\sqrt{K}} \int d^4k \frac{T^{\alpha\beta\mu\nu} T^{\lambda\sigma\nu\rho} k_\mu k_\rho}{(k^2 + m^2_k - i \varepsilon)^4}
\]
where $k^2 = k_\mu k_\mu$, re-inserting the $i \varepsilon$. The integral over $k$ can be evaluated using contour integration as
\[
\int \frac{d^4k}{(k^2 + m^2_k - i \varepsilon)^4} = \frac{i\pi^2}{12m^2} \delta^{\mu
u} G_{\mu\nu}. \tag{39}
\]
Since the frame $E^\alpha$ and the torsion correspond to the metric $\gamma^{\mu\nu} = \rho^2 G^{\mu\nu}$ \cite{14}, it follows that
\[
\rho^{-4} T^{\alpha\beta\mu\nu} T^{\lambda\sigma
u\rho} G_{\mu\nu} = T^{\phi}_{\sigma\mu} T^{\rho}_{\nu\sigma} G^{\mu\nu}. \tag{40}
\]
Therefore
\[
\Gamma^{K-M}_{\text{1loop}} = -\frac{c^2_\mathcal{K}}{8} \int_{\mathcal{M}} d^4x \sqrt{G} \rho^{-2} m^2_k T^{\phi}_{\sigma\mu} T^{\rho}_{\nu\sigma} G^{\mu\nu}, \tag{41}
\]
where
\[
c^2_\mathcal{K} = \frac{\pi^2}{8} \sum_{\lambda} \frac{C^2_\lambda}{\mu^2_\lambda} > 0 \tag{42}
\]
is finite, determined by the dimensionless KK masses \cite{8}. This is recognized as gravitational action using the following identity obtained from (E.2) in \cite{12}
\[
\mathcal{R} = -\frac{1}{2} T^{\rho}_{\sigma\mu} T^{\rho}_{\mu\sigma} \rho G^{\sigma\nu} - \frac{1}{2} T^{\rho}_{\sigma\mu} G^{\mu\nu} + 2 \rho^2 G^{\mu\nu} \partial_\mu \rho \partial_\nu \rho - 2 \nabla^\mu (G^{\nu}(\rho^{-1} \partial_\nu \rho))
\]
Here $\mathcal{R}$ is the Ricci scalar of the effective metric $G_{\mu\nu}$, and $T^{\rho}_{\sigma\mu} = - * \left( \frac{1}{2} G_{\sigma\nu} T^{\rho}_{\mu\nu} dx^\sigma dx^\mu \right)$ is the Hodge-dual of the totally antisymmetric torsion, which reduces using the eom of the matrix model to a gravitational axion $\hat{\rho}$ \cite{12}
\[
T^{\rho}_{\mu} = \rho^{-2} \partial_\mu \hat{\rho}. \tag{43}
\]
Therefore we obtain the Einstein-Hilbert action with an extra contribution from $T_{\mu}$ and $\rho$:

$$\Gamma_{1\text{loop}}^{K-\mathcal{M}} = \int_{\mathcal{M}} d^4x \sqrt{|G| G_N} \left( R + \frac{1}{2} T_{\mu} T_\nu G^{\mu\nu} - 2\rho^{-2} \partial_\mu \rho \partial^\mu \rho + 2\rho^{-1} \partial_\mu \rho G_N^{-1} \partial^\mu G_N \right).$$  \tag{44}

with Newton constant set by the compactification scale

$$\frac{1}{G_N} = \frac{2\varepsilon_K^2}{\pi^2} \rho^{-2} m_K^2. \tag{45}$$

Hence the Planck scale is related to the Kaluza-Klein scale for the fuzzy extra dimensions $K$, which also serves as an effective UV cutoff. Since the 1-loop effective action is related to IIB supergravity, the gravity action in 3+1 dimensions can be interpreted as quasi-local interaction of $K$ and $\mathcal{M}$ via $9+1$-dimensional IIB supergravity.

It turns out that $\Gamma_{1\text{loop}}^{K-\mathcal{M}} = c^2 m_K^2 > 0$ \cite{11} is positive for the covariant FLRW space-time in \cite{13}. Combined with the bare action, the effective action has the structure

$$V(m_K^2) = c^2 m_K^2 + \frac{d^2}{g^2} m_K^4 \tag{46}$$

at weak coupling, which has a minimum for $m_K^2 > 0$. Since $m_K$ is set by the radius of $K$, this indicates that $K$ can be stabilized by quantum effects, providing some justification for \cite{4}.

b. Vacuum energy. The vacuum energy arising at 1 loop from $K$ is obtained using an analogous trace computation, leading to a result of structure

$$\Gamma_{1\text{loop}}^{K} = \frac{3i}{4} \mathcal{M} \left( \tilde{V}_4^{K} \right)^{-1} \sim -\frac{\pi^2}{8(2\pi)^3} \int_{\mathcal{M}} \Omega \rho^{-2} m_K^2 \sum_{\Lambda_s} \frac{V_{4,\Lambda}}{\mu_\Lambda^2} \tag{47}$$

assuming $\frac{1}{2\pi} \ll m_\Lambda^2 \sim m_K^2$. Here $V_{4,\Lambda}$ is determined by the structure of $K$, and could have either sign. Since the symplectic volume form $\Omega$ and independent of the metric, the 1-loop vacuum energy does not act as a cosmological constant. The present setup can therefore be viewed as a realization of induced gravity in the spirit of Sakharov \cite{20,21}, avoiding the associated cosmological constant problem. Finally, $V_{4\mathcal{M}}$ \cite{22} leads to a 4-derivative contribution $\Gamma_{1\text{loop}}^\mathcal{M}$, which is expected to be negligible for long wavelengths.

c. Covariant quantum space-time. The above computation extends straightforwardly to covariant quantum space-times \cite{19}, leading again to the gravity action \cite{44}, with $c_K$ modified as

$$c_K^2 = \frac{\pi^2}{8} \sum_{\Lambda_s} \frac{(2s + 1)C_A^2}{\mu_\Lambda^2 + \frac{a(s-1)}{R^2 m_K^2}} \tag{48}$$

assuming $R^2 m_K^2 \gg 1$. The vacuum energy \cite{47} is also slightly modified with the same qualitative features, and does again not gravitate. More details will be given in a forthcoming publication.

IV. DISCUSSION

We have shown that the 3+1-dimensional Einstein-Hilbert action arises at one loop in the IKKT matrix model on suitable 3+1-dimensional branes, in the presence of fuzzy extra dimensions $K$ but without target space compactification. The vacuum energy is large but does not gravitate, due to the symplectic structure of the brane. Combining the 1-loop contribution \cite{41} with the bare action, the effective action for gravity up to second derivatives in the frame has the form

$$S_{\text{grav}} = -\int \Omega \frac{1}{g^2} \Theta_{ab} \Theta^{ab} + \Gamma_{1\text{loop}}^{K-\mathcal{M}}. \tag{49}$$

The bare action has 2 derivatives less than the E-H action and is hence interpreted as “pre-gravity” \cite{12,19}, which should dominate at long (cosmic) scales. At shorter scales the E-H term will dominate, leading to a cross-over with GR. The mechanism works only in the maximally supersymmetric model, where the gauge theory on the brane is UV finite. The induced E-H action on $\mathcal{M}$ can be understood as quasi-local IIB supergravity interaction between $K$ and $\mathcal{M}$ with negative binding energy, suggesting that such configurations can be stable. It should be possible to verify (meta)stability of such configurations by numerical simulations in the matrix model, but this is very challenging, cf. \cite{22,23} for related numerical work.

Finally, the presence of $K$ leads to a non-trivial gauge theory on $\mathcal{M}$, and fermions coupling to chiral gauge theories can arise from self-intersections of $K$ \cite{7,9,24}. This may lead to physically interesting gauge theories coupled to the above “emergent” gravity in a consistent quantum framework.

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