Parameter Identification With Finite-Convergence Time Alertness Preservation

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Abstract—In this letter note we present two new parameter identifiers whose estimates converge in finite time under weak interval excitation assumptions. The main novelty is that, in contrast with other finite-convergence time (FCT) estimators, our schemes preserve the FCT property when the parameters change. The previous versions of our FCT estimators can track the parameter variations only asymptotically. Continuous-time and discrete-time versions of the new estimators are presented.

Index Terms—Estimation, discrete event systems, identification.

I. PARAMETER ESTIMATORS WITH ALERTNESS-PRESERVING FINITE-CONVERGENCE TIME

This letter is devoted to the study of continuous-time (CT) and discrete-time (DT) on-line estimators of the parameters \( \theta \in \mathbb{R}^q \) of a linear regression equation (LRE) of the form

\[ y = \phi^T \theta, \tag{1} \]

from the measurable quantities \( y \) and \( \phi \). This is a topic of central importance that has attracted the attention of researchers in control and systems theories for many years. The interested reader is referred to textbooks on identification and adaptive control, e.g., [1] and [2], respectively, for a detailed study of the topic and a long list of references.

The objective of this letter is to propose CT and DT estimators that should satisfy the following three specifications:

- **S1** Convergence of the estimates should be achieved in finite-time.
- **S2** Convergence is ensured under weak interval excitation (IE) conditions [3].
- **S3** The estimator should preserve its alertness to be able to estimate—still with finite-convergence time (FCT)—future variations of the unknown parameters.

Instrumental to solve this problem is the use of the dynamic regressor extension and mixing (DREM) procedure proposed in [4]. In particular, we adapt the FCT-DREM estimator proposed in [5] to incorporate the new feature of FCT alertness preservation (AP). A CT version of such a scheme was reported in [6, Sec. V]. In this note we give a DT version of it.

An early variation of the least-squares method, that converges in finite time, was proposed 32 years ago in [7]. In its initial stage the algorithm is akin to an off-line estimator, which involves a numerically sensitive matrix inversion and, as it converges to a standard least-squares, loses its alertness. More recent FCT estimators [8]–[10] monitor the incoming signals and select those that satisfy some “spanning” condition, that is akin to an off-line operation. For the sake of completeness we present comparative simulations of the proposed estimator with the FCT estimator reported in [11], which relies on the injection of high-gain via the use of fractional powers and/or relays in the estimator dynamics. See also [12], [13] where a similar high-gain approach is adopted.

**Notation:** \( \mathbb{R}_{>0}, \mathbb{R}_{\geq 0}, \mathbb{Z}_{\geq 0} \) and \( \mathbb{Z}_{>0} \) denote the positive and non-negative real and integer numbers, respectively. We denote \( |a|^q = |a|^q \text{sign}(a) \) for any \( a \in \mathbb{R} \) and \( a \in \mathbb{R}_{>0} \).

Continuous-time (CT) signals \( s : \mathbb{R}_{>0} \to \mathbb{R} \) are denoted \( s(t) \), while for discrete-time (DT) sequences \( s : \mathbb{Z}_{\geq 0} \to \mathbb{R} \) we use \( s(k) = s(kT_s) \), with \( T_s \in \mathbb{R}_{>0} \) the sampling time. When a formula is applicable to CT signals and DT sequences the time argument is omitted.

II. FCT ESTIMATORS: FORMULATION FROM SCALAR LRES VIA DREM

As it has been widely documented the powerful DREM estimator design procedure [4] allows us to generate, from the \( q \)-dimensional LRE (1), \( q \)-scalar LREs of the form

\[ Y_i = \Delta \theta_i, \quad i \in \bar{q} = \{1, 2, \ldots, q\} \tag{2} \]
where $\Delta$ is the determinant of an extended regressor matrix. In the remaining of the note, we will use these simple scalar LREs to design the AP-FCT parameter estimator.

### A. Continuous-Time FCT Estimator Using DREM

The following CT FCT-DREM estimator was reported in [5] and, as it constitutes the basis of our new AP-FCT, we repeat it for ease of reference. Also, to make the note self-contained we give a brief summary of the proof, referring the interested reader to [5] for further details.

**Proposition 1:** Consider the scalar CT LREs (2) and the gradient-descent estimators

$$
\hat{\theta}_i(t) = \gamma_i \Delta(t)[\gamma_i(t) - \Delta(t)\hat{\theta}_i(t)],
$$

with $\gamma_i > 0$. Define the FCT estimate

$$
\hat{\theta}^\text{FCT}_i(t) := \frac{1}{1 - w_i^2(t)}[\hat{\theta}_i(t) - w_i^2(t)\hat{\theta}_i(0)],
$$

where $w_i^2(t)$ is defined via the clipping functions

$$
w_i^2(t) = \begin{cases} 
\mu_i & \text{if } w_i(t) \geq \mu_i \\
1 & \text{if } w_i(t) < \mu_i.
\end{cases}
$$

$\mu_i \in (0, 1)$ are designer chosen parameters, and $w_i(t)$ is given by

$$
\dot{w}_i(t) = -\gamma_i \Delta^2(t)w_i(t), \quad w_i(0) = 1.
$$

Assume there exists a time $t^*_i > 0$ such that the IE condition

$$
\gamma_i \int_0^{t^*_i} \Delta^2(\tau) d\tau \geq -\frac{1}{\gamma_i} \ln(1 - \mu_i),
$$

is satisfied. Then,

$$
\dot{\hat{\theta}}^\text{FCT}_i(t) = \theta_i, \quad \forall t > t^*_i,
$$

that is, the estimator has the feature of FCT.

**Proof:** Define the parameter error $\hat{\theta}_i := \dot{\hat{\theta}}_i - \theta_i$. The scalar error equations are given by

$$
\dot{\hat{\theta}}_i(t) = -\gamma_i \Delta^2(t)\hat{\theta}_i(t),
$$

whose explicit solution is

$$
\hat{\theta}_i(t) = e^{-\gamma_i \int_0^t \Delta^2(\tau) d\tau}\hat{\theta}_i(0).
$$

Now, notice that the solution of (6) is

$$
w_i(t) = e^{-\gamma_i \int_0^t \Delta^2(\tau) d\tau}.
$$

The key observation is that, using the equation above in (9), and rearranging terms we get that

$$
[1 - w_i(t)]\theta_i = \hat{\theta}_i(t) - w_i(t)\hat{\theta}_i(0).
$$

Now, observe that $w_i(t)$ is a non-increasing function and, under the interval excitation assumption (7), we have that

$$
w_i^2(t) = w_i(t) < \mu_i < 1, \quad \forall t \geq t^*_i.
$$

Clearly, (10) and (11) imply that

$$
\frac{1}{1 - w_i^2(t)}[\hat{\theta}_i(t) - w_i^2(t)\hat{\theta}_i(0)] = \theta_i, \quad \forall t > t^*_i,
$$

completing the proof.

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1In the sequel, the quantifier $i \in \mathbb{Q}$ is omitted for brevity.

### B. Discrete-Time FCT Estimator Using DREM

In [5, Proposition 2] a DT DREM estimator was reported. We give below an FCT-version of this scheme and give a brief summary of the proof.

**Proposition 2:** Consider the scalar DT LREs defined by (2) with the DT gradient-descent estimator

$$
\hat{\theta}_i(k + 1) = \hat{\theta}_i(k) + \frac{\Delta(k)}{c_i + \Delta^2(k)}[\gamma_i(k + 1) - \Delta(k)\hat{\theta}_i(k)]
$$

with positive constants $c_i$. Define the dynamic extension

$$
w_i(k + 1) = \frac{c_i}{c_i + \Delta^2(k)}w_i(k), \quad w_i(0) = 1.
$$

and the clipping function

$$
w_i^2(k) = \begin{cases} 
\rho_i & \text{if } w_i(k) \in [\rho_i, 1] \\
w_i(k) & \text{if } w_i(k) \in [0, \rho_i),
\end{cases}
$$

where $\rho_i \in (0, 1)$ are designer chosen constants. Assume there exists a $k^*_i \in (0, \infty)$ such that the IE condition

$$
\prod_{i=0}^{k^*_i} \left[\frac{c_i}{c_i + \Delta^2(i)}\right] < \rho_i,
$$

is satisfied. Then,

$$
\hat{\theta}^\text{FCT}_i(k) := \frac{1}{1 - w_i^2(k)}[\hat{\theta}_i(k) - w_i^2(k)\hat{\theta}_i(0)],
$$

ensures

$$
\hat{\theta}^\text{FCT}_i(k) = \theta_i, \quad \forall k \geq k^*_i.
$$

**Proof:** From (2) and (12) we get the parameter error equation

$$
\hat{\theta}_i(k + 1) = \frac{c_i}{c_i + \Delta^2(k)}\hat{\theta}_i(k)
$$

whose explicit solution satisfies

$$
\hat{\theta}_i(k) = \psi_i(k)\hat{\theta}_i(0),
$$

where, for ease of future reference, we defined the scalar sequence

$$
\psi_i(k) := \prod_{j=0}^{k-1} \left[\frac{c_i}{c_i + \Delta^2(j)}\right]
$$

The solution of (13) is given by

$$
w_i(k) = \psi_i(k),
$$

whose replacement in (18) yields

$$
\hat{\theta}_i(k) = w_i(k)\hat{\theta}_i(0).
$$

Using the definition of the parameter error and rearranging terms we get that

$$
[1 - w_i(k)]\theta_i = \hat{\theta}_i(k) - w_i(k)\hat{\theta}_i(0).
$$

According to (14) we have that, under the assumption (15), $w_i(k) < \rho_i < 1$ for all $k \geq k^*_i$. Consequently, for $k \geq k^*_i$ we can write

$$
\theta_i = \frac{1}{1 - w_i(k)}[\hat{\theta}_i(k) - w_i(k)\hat{\theta}_i(0)].
$$

The proof is completed, from (16), noting that $w_i(k) = w_i^2(k)$ for all $k \geq k^*_i$. 

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III. NEW ESTIMATOR WITH FCT ALERTNESS PRESERVATION

There are two practical problems with the approach described above. First, the estimates at the current time are reconstructed from the knowledge of the initial estimate $\hat{\theta}_i(0)$, complicating the task of tracking variations of the true parameters after convergence of the estimates. Second, independently of the behavior of $\Delta$, the functions $w_i$ are monotonically non-increasing and converge to zero if $\Delta$ is not square summable (or integrable). In this case, $\hat{\theta}_i^{\text{FCT}} \to \tilde{\theta}_i$, and the FCT estimator converges to a standard gradient, losing its FCT feature. Therefore, to keep the FCT alertness of the estimator, i.e., to track parameter variations in finite-time upon the arrival of new excitation, it is necessary to reset the estimators—a modification that is always problematic to implement. A typical procedure is the so-called covariance resetting for least-squares algorithms, see [2, Sec. 2.4.2].

These drawbacks can be overcome with the new FCT-DREM estimators proposed below.

A. CT FCT-DREM With Alertness Preservation

For the sake of brevity, we present only the derivation of a relation similar to (10), from which we can easily construct the FCT estimator and prove that the new FCT estimator does not converge to the gradient one.

Proposition 3: Fix $T_D \in \mathbb{R}_{>0}$ and define

$$\dot{w}_i^2(t) = -\gamma_i \left[ \Delta^2(t) - \Delta^2(t - T_D) \right] w_i^2(t), \quad w_i^2(0) = 1.$$  

Then,

$$[1 - w_i^2(t)] \hat{\theta}_i = \tilde{\theta}_i(t) - w_i^2(t) \hat{\theta}_i(t - T_D).$$  

Moreover, $w_i^2(t)$ is bounded away from zero.

Proof: Without loss of generality we assume that $\Delta(t - T_D) = 0$ for $t < T_D$. Then, the solution of (22) is

$$w_i^2(t) = e^{-\gamma_i \int_{t-T_D}^{t} \Delta^2(s)ds}.$$  

From (24), and the fact that

$$\int_{t-T_D}^{t} \Delta^2(s)ds \leq \Delta_{\text{max}}^2 T_D,$$

where $\Delta_{\text{max}} = \| \Delta(t) \|_{\infty}$, we conclude that

$$w_i^2(t) \geq e^{-\gamma_i \Delta_{\text{max}} T_D} > 0.$$  

Now, from the solution of the parameter error equation (9) in the interval $[t - T_D, t]$ we get

$$\tilde{\theta}_i(t) = e^{-\gamma_i \int_{t-T_D}^{t} \Delta^2(s)ds} \tilde{\theta}_i(t - T_D).$$  

Hence, $\tilde{\theta}_i(t) = w_i^2(t) \tilde{\theta}_i(t - T_D)$. The proof of the claim is established by suitably grouping the terms of the equation above.

Remark 1: It is important to note that when $\Delta(t)$ decreases—that is, when we lose excitation—we have $w_i^2(t)$ grows towards one, and the alertness in lost. On the other hand, when new excitation arrives and $\Delta(t)$ grows, then $w_i^2(t)$ decays and the FCT condition $w_i^2(t) < \mu_i$ is satisfied. In this way, the new FCT estimator preserves its FCT property if the parameters change. This fact is illustrated in the simulations of Section IV. Notice also that, in contrast with the FCT estimator of Proposition 1, where the calculation of the $\tilde{\theta}_i^{\text{FCT}}$ in (4) is done using the initial parameter estimate $\tilde{\theta}_i(0)$, the FCT reconstruction of the estimated parameter is done in (23) using the estimate at time $t - T_D$, that is, $\hat{\theta}_i(t - T_D)$.

Remark 2: For the new FCT DREM estimator the interval excitation inequality becomes the existence of a time $t_f' \geq T_D$ such that

$$\int_{t_f'-T_D}^{t_f'} \Delta^2(s)ds \geq -\frac{1}{\gamma_i} \ln(1 - \mu_i).$$  

Recalling (24), it has the same interpretation as (7).

B. DT FCT-DREM With Alertness Preservation

Similarly to the CT case, for the sake of brevity, we present only the derivation of a relation similar to (21), from which we can easily construct the FCT estimator and prove that the new FCT estimator does not converge to the gradient one.

Proposition 4: Fix a positive integer $\tilde{d}$. Consider the DT, scalar LRE (2) and the gradient parameter update (12) with the dynamic extension (13) and the sequence

$$w_i^2(k) = \frac{w_i(k)}{w_i(k - \tilde{d} - 1)}.$$  

Then,

$$[1 - w_i^2(k)] \hat{\theta}_i = \tilde{\theta}_i(k) - w_i^2(k) \tilde{\theta}_i(k - \tilde{d}).$$

Proof: First, we make the observation that, for all finite $k$, the sequence $w_i(k)$ is bounded away from zero

$$w_i^2(k) = \prod_{i=k-\tilde{d}}^{k-1} \left[ \frac{c_i + \Delta^2(i)}{c_i} \right].$$

Hence, replacing the equation above in (18) we have that

$$\tilde{\theta}_i(k) = w_i^2(k) \tilde{\theta}_i(k - \tilde{d}).$$

The proof is completed by rearranging the terms of the identity above.

Remark 3: It is clear that the same behavior that is indicated in Remark 1 regarding the CT version is observed in this DT one (explaining the important FCT-AP property).

IV. SOME REMARKS ON THE CHOICE OF THE TUNING PARAMETERS

In the proposed algorithms there are several tuning parameters that need to be chosen by the designer. Since these gains severely affect the transient performance of the estimators, particular care must be taken in their selection. Unfortunately, in the absence of a theoretical analysis of the transient behavior, there are no clear-cut rules for their selection. This drawback is prevalent in all designs for nonlinear systems. In this subsection some pertinent remarks that provide guidelines for the commissioning of the estimators are given.

For the sake of brevity we restrict the discussion below to CT estimators, but as will become clear, similar remarks are applicable to the DT case.

2It should be underscored that the convergence claims of all propositions remain valid for all choices of these gains.
G1 From the definition of the IE conditions (7) (or (26)) we see the role played by the constants $\gamma_i$ and $\mu_i$. Indeed, the right hand side decreases increasing $\gamma_i$ or choosing $\mu_i$ close to one—and this in its turn decreases the convergence time $t'$. However, there are other considerations to take into account in the choice of these numbers. On one hand, $\gamma_i$ is the parameter adaptation gain that, as seen in (9), determines its speed of convergence, so one might be tempted to pick a large number. Nevertheless, from identification and adaptive control theories [2], it is well-known that selecting large values for it brings deleterious effects in the face of noise or unmoderated dynamics.

G2 Notice that in the time interval $[0, t'_1]$ the FCT estimated parameter (4) takes the form

$$\hat{\theta}_T^{\text{FCT}}(t) = \frac{1}{\mu_i} \left[ \hat{\theta}_i(t) - (1 - \mu_i)\hat{\theta}_i(0) \right].$$

Therefore, if we choose the constant $\mu_i$ close to zero there is a potential high-gain injection. Therefore, there is a compromise in the choice of these constants that—in the absence of clear guidelines—is usually done via trial-and-error.

G3 Regarding the choice of the parameter $T_D$ in the estimator of Proposition 3 we see from the definition of the IE condition (26) that it defines the window of integration of the key signal $\Delta^2(t)$, therefore choosing a small value for it imposes severe constraints on the excitation level. It’s role is, in some sense, similar to the role of the gains $\gamma_i$ and $\mu_i$ discussed in G1 above.

G4 The role of $T_D$ in the transient performance of the estimator of Proposition 3 is also clear in expression (25). Indeed, choosing a small value for it decreases the value of the exponential, reducing the convergence rate.

V. SIMULATIONS

In this section we present simulations illustrating the results for a single parameter. Moreover, for the sake of comparison, we show some simulation results of the high-gain FCT estimator proposed in [11].

A. CT FCT-DREM With Alertness Preservation

In this subsection we compare the FCT DREM of Proposition 1 and the new FCT DREM of Proposition 3. The objective of the simulation is to prove that the new FCT DREM is able to react when new excitation arrives. This is in contrast with the old FCT DREM estimator that, since $w_i(t) \rightarrow 0$, converges to the gradient estimator and loses its FCT alertness property.

We consider two scenarios: with and without excitation in $\Delta(t)$. For the first case we consider the PE signal $\Delta(t) = \sin(\frac{\pi}{10}t)$, and for the second one $\Delta(t) = \frac{1}{\sqrt{t+1}}$. Note that in the second case $\Delta(t) \rightarrow 0$, hence it is not PE. However, $\Delta(t) \notin \mathcal{L}_2$, hence it satisfies the conditions for convergence of the DREM estimator [4], [6].

To illustrate the FCT tracking capabilities of the estimators the unknown parameter $\theta$ is time-varying and given by

$$\theta(t) = \begin{cases} 10 & \text{for } 0 \leq t < 10, \\ 15 & \text{for } 10 \leq t < 20, \\ 15 - 0.5(t - 20) & \text{for } 20 \leq t < 30, \\ 10 & \text{for } t \geq 30, \end{cases}$$

i.e., it starts at 10, jumps to 15 at $t = 10$, and then linearly returns to 10.

For the simulations we set $\gamma = 2$, $\mu = 0.98$, and $T_D = 0.2$. These parameters have been chosen such that the transients of both FCT estimators coincide in the ideal case when $\theta$ is constant and the system is excited.

The transient of the estimators for $\Delta(t) = \sin(\frac{\pi}{10}t)$ are given in Fig. 1, where we plot the time-varying parameter $\theta$, the gradient estimate $\hat{\theta}^{\text{grad}}(t)$, as well as the old and the new FCT estimates, denoted in the plots as $\hat{\theta}^{\text{FCT}}(t)$ and $\hat{\theta}^{\text{FCT-D}}(t)$, respectively. We observe that, as expected, in the time interval $[0, 10]$ both FCT estimators are overlapped and converge in finite time, while the gradient converges only asymptotically. The difference between the old FCT estimator and the new one is clearly appreciated in the time interval $[10, 20]$, where we see that the new FCT estimator tracks the parameter variation in finite time, while the old one—now glued to the gradient—only does it asymptotically.

The behavior of the CT estimators for $t \in [20, 40]$ shows that, as predicted by the theory, the old FCT behaves as the gradient estimator and their trajectories coincide. On the other hand, the new estimator preserves FCT alertness after the first parameter jump and achieves fast tracking of the linearly time-varying $\theta(t)$. We also observe in the figure a bump in the estimates at $t = 30$ that coincides with the time of freezing of the true parameter.

For the non-PE case of $\Delta = 1/\sqrt{t+1}$, the transients of the CT estimators are given in Fig. 2. We observe that both FCT estimators, again, essentially coincide in the first few seconds and converge in finite time, while the gradient does it only asymptotically. After the first parameter change at $t = 10$ the old FCT and the gradient coincide, while the new FCT manages to track in finite time the parameter jump. However, during the ramp parameter change—because of the lack of excitation—neither one of the estimators can track the
Fig. 2. Transients of the CT parameter estimates with $\Delta(t) = \frac{1}{\sqrt{t+1}}$.

Fig. 3. Transients of the DT parameter estimates with $\Delta(t) = \sin(\frac{\pi}{10}t)$.

parameter variation but the new FCT estimator performs much better.

B. DT FCT-DREM With Alertness Preservation

In this subsection we present the simulation results for the DT estimators of Propositions 2 and 4. The estimator gains were set to $\gamma = 2$, $c = 1$, $d = 1$, $T_D = 1$ and $T = 0.5$.

The same simulation scenario of the CT schemes given above is reproduced here and the transient behaviors are shown in Figs. 3 and 4. Essentially the same remarks made for the CT schemes of the previous subsection are applicable in the DT case.

C. Comparison of the CT FCT-DREM With Two With Two High-Gain Schemes

Now, we compare the FCT-D algorithm with two of the schemes proposed in [11]. Namely,

1) Algorithm 1

$$\dot{\hat{\theta}}(t) = \gamma \Delta(t) [Y(t) - \Delta(t)\hat{\theta}(t)]^\alpha$$

where $\gamma > 0$ and $\alpha \in [0, 1)$.

2) Algorithm 3

$$\dot{\hat{\theta}}(t) = \gamma \text{sign}(\Delta(t)) [Y(t) - \Delta(t)\hat{\theta}(t)]^{\frac{\Delta(t)}{\Delta_{\text{max}}(t)}}$$

where $\gamma > 0$, $\varsigma > 1$, $\Delta_{\text{max}} = \max_{t \in [-T_0, T_0+T]}|\Delta(t)|$ for given $T_0 \in \mathbb{R}_+$ and $\hat{\theta}(t_0) = 0$.

It should be underscored that in [11] a third estimator was also proposed but this scheme is not well-defined when $\Delta(t) = 0$ and could not be simulated.

For the simulation of the CT FCT-D we set $\gamma = 2$, $\mu = 0.98$, and $T_D = 0.2$, while for Algorithm 1 (28) we set $\gamma = 5$ and $\alpha = 0.75$ and for Algorithm 3 (29) we set $\gamma = 5$, $\varsigma = 2$.

The same simulation scenario of the CT schemes given in Section V-A is reproduced here and the transient behaviors are shown in Figs. 5 and 6. As shown in the figures Algorithm 3 indeed achieves FCT and preserves its alertness.

The purpose of adding a simulation with noise was to try to exhibit the purported “sensitivity” of high-gain-based estimators to it. Surprisingly, this drawback was not observed in this particular instance.

It is often argued that control and estimation algorithms that rely on the injection of high-gain, for instance, with the use of fractional powers like (28) and (29) above, are sensitive to noise. To test the robustness of the algorithms with respect to noise, we repeated the simulations adding a signal $0.1 \sin(10t)$ to the measurement $Y(t)$. The results of the simulations are shown in Figs. 7 and 8. Interestingly, the behavior
of Algorithm 1 does no seem to be affected by the noise—but, from the simulations it seems that the tracking is only asymptotic. On the other hand, Algorithm 3 seems more sensitive to noise, but it achieves the desired FCT. For the PE case of Fig. 7 we see only a minor performance degradation due to the noise in all schemes that is further degraded for the non-PE case of Fig. 8.

VI. CONCLUSION

We have presented two new FCT-DREM parameter estimators with enhanced performance—in particular, with respect to their ability to track parameter variations in finite-time. CT and DT versions of the new estimators are given. The performance improvement of the proposed schemes was illustrated with representative simulations.

In closing this note we bring to the readers attention the fact that the FCT results reported in this letter are “trajectory dependent”—in the sense that they hold true only for the specific initial conditions imposed in the estimators. This important issue has been discussed in [6, Remark 7] and is elaborated in further detail in [14].

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