NON-ABELIAN SURPRISES IN GRAVITY

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ABSTRACT

We present a brief review of some recent results on non-abelian solitons and black holes in different theories.

1Lecture presented at the Simi-96, Tbilisi, Georgia, September 22-28, 1996. To be published in the Proceedings.
2On leave of absence from Tbilisi Mathematical Institute, 380093 Tbilisi, Georgia
1 Introduction

Since I plan to talk about “surprises”, let me first explain how do I understand this word. Surprise (in physics) is when you get something what you do not expect with an educated guess. The first surprise in the field, which I plan to review in the present lecture, came in 1988, when Bartnik and McKinnon [1] discovered a discrete family of globally regular, static, spherically symmetric solutions of the Einstein-Yang-Mills (EYM) theory. This was not expected from experience in lower dimensions (EYM theory in (2+1)dimension) [2], the Einstein-Maxwell case, pure non-abelian theory [3] and pure gravity.

The important step was to understand that the Bartnik-McKinnon (BK) solitons are classically unstable [4]. With guidance from this due it was shown [5, 6] that they are not ordinary particles, but rather gravitational analogues of the electroweak sphalerons [7]. That was beginning of the story. Now the BK paper gets more then 80 citations in the SLAC database. It is difficult to review everything, so I will bring to your attention selected topics from my perspective.

The plan of the lecture is as follows. I will begin with some general remarks about classical solutions. The third section starts with a discussion of solitons in YMH, EYM, EYMD, and EYMH theories. Next I will discuss stability analysis and present a brief comparison of different theories. In subsection 3.7 I will talk about non-abelian black holes in the abovementioned theories. All these are asymptotically flat solutions.

In the Section 4 I will talk about non-asymptotically flat solutions. Name-ly, I will present our results on the EYM theory with the cosmological constant. In my concluding remarks I will speculate about possible applications and I will mention some other interesting directions which are not covered in this presentation.

The discussion will be rather schematic, but I will try to compensate for this by providing a detailed list of references.

2 General remarks about classical solutions

A number of interesting classical solutions with definite physical meaning have been considered in the literature.

Quite “famous” are static, finite energy solutions: monopoles [8] and
two sphalerons \[\text{[1]}\]. Two types of euclidean solutions are known: an instanton \[\text{[2]}\] interpolating between vacua with different topological numbers and a bounce \[\text{[3]}\] describing the decay of a metastable vacuum. One can consider as well a namdvilon solution \[\text{[4]}\], which in real time interpolates between two sphalerons.

We believe that cosmological solutions of the Einstein equations describe the evolution of the Universe. One should not forget about the most spectacular objects predicted by General Relativity - black holes.

I do not plan to discuss here in detail the role and meaning of each of these solutions. The only point I would like to stress is that some of these solutions have an unstable modes. So, the instability of a solution does not automatically mean that the solution is bad (or good). It might be an essential property, part of the interpretation like in case of the electroweak sphaleron or bounce. The sphaleron is unstable since it is “sitting” at the top of the barrier separating neighboring vacua. The bounce has a single negative mode \[\text{[5]}\] which is essential to describe the decay of the metastable state.

To summarize: the first step is to find a classical solution, the next is to understand its role in the (quantum) theory.

3 Asymptotically flat solutions

3.1 YM-H sphaleron

Let us consider YM-H theory in flat space.

The action for the YMH theory has the form

$$S_{\text{YMH}} = \frac{1}{4\pi} \int \left( -\frac{1}{4g^2} F_{\mu\nu}^a F^{a\mu\nu} + (D^\mu \Phi)^\dagger D_\mu \Phi - \lambda (\Phi^\dagger \Phi - \frac{v^2}{2})^2 \right) d^4x, \quad (1)$$

where $F_{\mu\nu}^a$ is the $SU(2)$ gauge field strength, $F_{\mu\nu}^a = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + \epsilon^{abc} W^b_\mu W^c_\nu$, and $a = 1,2,3$ is the $SU(2)$ group index, $\mu, \nu = 0,1,2,3$ are space-time indices. Covariant derivatives are defined by $D_\mu \Phi = \partial_\mu \Phi - \frac{i}{2} \tau^a W^a_\mu \Phi$.

For the gauge field let us take the usual “monopole” ansatz

$$W^a_0 = 0, \quad W^a_\nu = \epsilon_{aij} \frac{n_j}{r} (1 - W(r)), \quad (2)$$

3
with $n_j = x_j/r$ and doublet ansatz for the Higgs field

$$\Phi = \frac{v}{\sqrt{2}} H(r) \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad (3)$$

The reduced action has the form

$$S_{YMH}^{\text{red}} = - \int \left[ \frac{1}{g^2} (W'^2 + \frac{(W^2 - 1)^2}{2r^2}) + \frac{\nu^2}{2} H'^2 + \frac{H^2(1 + W)^2}{4} + \frac{\lambda \nu^2}{4} (H^2 - v^2)^2 \right] dr \quad (4)$$

where a prime denotes $d/dr$. Out of the gauge coupling constant $g$, the Higgs vacuum expectation value $v$, and the Higgs self coupling constant $\lambda$ one can form two mass scales $M_W = \frac{1}{2} g v$ and $M_H = \sqrt{2} \lambda v$ which are the gauge boson mass and Higgs mass respectively.

Solutions with finite energy have to interpolate between

$$W = 1, \quad H = 0,$$

at $r \to 0$ and

$$W = -1, \quad H = v$$

for $r \to \infty$.

It was found [7] that such a solutions $\{W(r), H(r)\}$ indeed exist. They are called sphalerons.

After suitable rescaling, the energy of the sphaleron can be written as

$$E_{YMH} = -S_{YMH}^{\text{red}} = \frac{2M_W}{\alpha_W} B(M_H/M_W) \quad (5)$$

where $\alpha_W = g^2$ is the electroweak “fine structure” constant and the numerical value of the function $B$ varies from about 1.5 to 2.7 as $M_H$ varies from zero to infinity [8]. Since $\alpha_W = \frac{1}{29}$ and $M_W = 80$ Gev, the energy of the electroweak sphaleron is of order of 10 Tev.

The main properties of the sphaleron are as follows:

(i) it has finite energy
(ii) one can assign fractional topological charge
(iii) it is a saddle point of the action [13]
(iv) there are fermion zero modes in background of sphaleron solutions [14]
Solutions which I discuss in what follows are analogues of sphaleron.

If one takes a triplet Higgs, one gets a monopole \[8\]. Including gravity one obtains gravitating monopoles. I will not talk about gravitating monopoles. They are discussed in \[15\], \[16\], \[17\].

### 3.2 EYM theory

Let me now describe the discovery of Bartnik and McKinnon.

The action for the EYM theory has the form

\[ S_{\text{EYM}} = \frac{1}{4\pi} \int \left( -\frac{1}{4G} R - \frac{1}{4g^2} F^a_{\mu\nu} F^a{\mu\nu} \right) \sqrt{-g} \, d^4x \]  

(6)

where \( G \) is Newton’s constant.

A convenient parametrization for the metric turns out to be

\[ ds^2 = S^2(r) N(r) \, dt^2 - \frac{dr^2}{N(r)} - r^2 d\Omega^2 , \]  

(7)

where \( d\Omega^2 = d\theta^2 + \sin^2(\theta) d\phi^2 \) is the line element of the unit sphere.

For the \( SU(2) \) YM potential we make the usual ('magnetic') spherically symmetric ansatz Eq. (2). Substituting this ansatz into the action (6) we obtain the reduced action

\[ S_{\text{red EYM}} = - \int S \left[ \frac{1}{2G} (N + rN') - 1 \right] \left( NW'^2 + \frac{(1 - W^2)^2}{2r^2} \right) \, dr . \]  

(8)

Now let me explain what one means when one talks about Planck mass \( M_{\text{Pl}} = \sqrt{\hbar c/G} \) scale in relations to the BK solutions. This is subtle since the classical EYM action does not contain \( \hbar \) at all. For this purpose I will restore \( c \) and \( \hbar \). Out of two coupling constants with dimensions \( [G] = L^3 T^{-2} M^{-1} \) and \( [g] = L^{-1} T^{1/2} M^{-1/2} \) one can form a quantity with dimension of length \( l_{\text{EYM}} = \sqrt{G/cg^2} \) and mass \( m_{\text{EYM}} = \sqrt{c/Gg^2} \). Introducing the dimensionless “fine structure constant” \( \alpha_g = \hbar g^2 \), the mass scale can be written as \( m_{\text{EYM}} = M_{\text{Pl}}/\sqrt{\alpha_g} \). If we work in dimensionless variables \( \hat{r} = r/l_{\text{EYM}} \), the energy functional is

\[ E_{\text{EYM}} = -S_{\text{red EYM}} \frac{M_{\text{Pl}}}{\sqrt{\alpha_g}} S(\hat{r}) \]  

(9)
Now we keep in mind scaling (9), and put \( G = g = 1 \) and omit a hat over the \( r \) in the resulting field equations

\[
\begin{align*}
(NSW')' &= S \frac{W(W^2 - 1)}{r^2}, \\
N' &= \frac{1}{r} \left( 1 - N - 2 \left( NW'^2 + \frac{(1 - W^2)}{2r^2} \right) \right), \\
S^{-1}S' &= \frac{2W'^2}{r}.
\end{align*}
\]  

(10)

The field equations (10) have singular points at \( r = 0 \) and \( r = \infty \) as well as points where \( N(r) \) vanishes (horizon!). The regularity at \( r = 0 \) of a configuration requires \( N(r) = 1 + O(r^2) \), \( W(r) = \pm 1 + O(r^2) \) and \( S(r) = S(0) + O(r^2) \). \( W \) and \(-W\) are gauge equivalent and we choose \( W(0) = 1 \). Similarly we can assume \( S(0) = 1 \) since a rescaling of \( S \) corresponds to a trivial rescaling of the time coordinate. Inserting a power series expansion into (10) one finds

\[
\begin{align*}
W(r) &= 1 - br^2 + O(r^4), \\
N(r) &= 1 - 4b^2r^2 + O(r^4), \\
S(r) &= 1 + 4b^2r^2 + O(r^4),
\end{align*}
\]  

(11)

where \( b \) is an arbitrary parameter.

Similarly assuming a power series expansion in \( \frac{1}{r} \) at \( r = \infty \) for asymptotically flat solutions, one finds \( \lim_{r \to \infty} W(r) = \{ \pm 1, 0 \} \). It turns out that \( W(\infty) = 0 \) cannot occur for globally regular solutions. For the remaining cases one finds

\[
\begin{align*}
W(r) &= \pm \left( 1 - \frac{c}{r} + O\left( \frac{1}{r^2} \right) \right), \\
N(r) &= 1 - \frac{2m}{r} + O\left( \frac{1}{r^2} \right), \\
S(r) &= S_\infty \left( 1 + O\left( \frac{1}{r^4} \right) \right),
\end{align*}
\]  

(12)

where again \( c, m \) and \( S_\infty \) are arbitrary parameters and have to be determined from numerical calculations.

It was found \cite{1} that equations (11) admit a discrete sequence of finite-energy solutions \( \{ W_n, N_n, S_n \} \) which interpolate between the asymptotic behaviors (11) for \( r \to 0 \) and (12) for \( r \to \infty \). Solutions can be labeled by
an integer \( n \) the number of zeroes of the gauge amplitude \( W \). The energy (mass) of the solutions is given

\[
M_{EYM}^{(n)} = \frac{M_{Pl}}{\sqrt{\alpha_g}} m_n
\]  

where \( m_n \) takes values between 0.82 and 1.0 when \( n \) varies from one to infinity \[1, 18\].

### 3.3 EYMD theory

There are many reasons to believe that there is a dilaton field.

Introducing a dilaton field we naturally obtain a EYMD theory with the action

\[
S_{EYMD} = \frac{1}{4\pi} \int \left(-\frac{1}{4G} R + \frac{1}{2} (\partial \varphi)^2 - \frac{e^{2\kappa \varphi}}{4g^2} F^2 \right) \sqrt{-g} d^4x,
\]  

(14)

where \( \kappa \) denotes the dilatonic coupling constant. After proper rescaling this theory depends on a dimensionless parameter \( \gamma = \frac{\kappa}{g \sqrt{G}} \). The model (14) was analyzed \[19, 20, 21, 22\] and for any value of the dilaton coupling constant a discrete family of globally regular solutions of finite mass was found.

A few remarks are in order.

In the limit \( \gamma \to 0 \) one gets the EYM theory studied in \[1\].

The value \( \gamma = 1 \) corresponds to a model obtained from heterotic string theory. A very special situation occurs for this value of the dilaton coupling constant \( \gamma = 1 \). It was found \[19\] that for the \( n = 1 \) solution the parameter \( b \) is a rational number, \( b = \frac{1}{6} \). Another regularity found numerically is that an asymptotic coefficient \( c \) in the equation analogous to the Eq.(12) is related to the mass of the solution, \( c = 2m \). In addition one has one Bogomol’nyi type equation

\[
g_{00} = e^{2\gamma \varphi}.
\]  

(15)

We think these are arguments indicating that the lowest lying \( (n = 1) \) regular solution in the EYMD theory may be obtained in a closed form similarly to the “stringy instanton” and the “stringy monopole” \[23\].

In the limiting case \( \gamma \to \infty \) one obtains the YM-dilaton theory in flat space \[24, 25\]. The YMD system is quite different from EYM, but surprisingly one obtains again tower of solutions with properties similar to the BK solutions.
I would stress as a main difference with the EYM case the existence of magnetically charged limiting solution with infinitely many zeroes \[24, 25\].

Another important point is that the masses of YMD solutions (as well as EYMD for strong coupling \(\gamma \gg 1\)) are inverse proportional to the dilatonic coupling constant \(M_{YMD} = \frac{M_{Pl}}{\gamma/\sqrt{\alpha_g}} m_n\) (16).

### 3.4 EYMH theory

One can consider the combined EYMH theory. This theory was analyzed \[26\] and the results confirm what one can expect from EYM and YMH cases.

The theory has four coupling constants \(G, g, v\) and \(\lambda\) or three mass scales \(m_{YEM}, M_W, M_H\). After a suitable rescaling the EYMH theory depends on two dimensionless ratios \(\alpha = M_W/(\alpha_g m_{YEM})\) and \(\beta = M_H/M_W\). For any given \(\beta\) one gets 1-parameter family of solutions. It is important that for small \(\alpha \neq 0\) the solutions bifurcate: there are two different solutions with any given number of nodes. In fact there are two types of nodes: one is a BK type, with typical size \(l_{YEM}\) and the other a sphaleron type with \(l_W = 1/M_W\). With increasing \(\alpha\), the sphaleron type node moves inwards and at some value \(\alpha_{max}\) the two solutions merge and cease to exist for bigger values of \(\alpha\) \[26, 27, 17, 28, 29\].

### 3.5 Stability analysis

In order to analyze the stability of the above discussed solutions we have to consider the spectrum of (harmonically) time dependent perturbations in the background of a given solution. The existence of imaginary frequencies in the spectrum of the linearized equations (leading to an exponential growth of the initially small perturbations in time) indicates the instability of the background solution.

One should be careful in stability analysis. The point is that the most general spherically symmetric ansatz for the \(SU(2)\) Yang-Mills field \(W^a\) can be written (in the Abelian gauge) as \[30\]

\[
\begin{align*}
W^a_t &= (0, 0, A_0) , \\
W^a_\phi &= (\phi_1, \phi_2, 0) , \\
W^a_r &= (0, 0, A_1) , \\
W^a_\psi &= (-\phi_2 \sin \theta, \phi_1 \sin \theta, \cos \theta) .
\end{align*}
\] (17)
and contains four functions, whereas sphaleron like-solutions lie in a special ansatz (2) with only one gauge amplitude. One can consider perturbations within and outside of the ansatz. It turns out they form two different sectors [27, 31, 32]. Unstable modes of the first type we call gravitational instabilities since they have no analogue for the flat space sphalerons. Instabilities of the second type we call sphaleron instabilities, because they have the same nature as the instability found for the electroweak sphaleron [13].

The gravitational sector for the BK solutions was investigated already in 1990 [4] and n unstable modes were found for the n\textsuperscript{th} solution.

Recently the sphaleron sector was carefully analyzed [27, 33] and an extra n negative modes for the n\textsuperscript{th} solutions was found. Altogether the n\textsuperscript{th} solution has 2n negative modes within the most general spherically symmetric ansatz. This is true for the EYM [1, 27, 33], EYMD [14, 29], EYMH [31, 28] solutions.

The numerical values for the energies \(E = \omega^2\) of the negative modes of the first three BK solutions in the gravitational respectively sphaleron sectors are shown in Tables 1 and 2 [27]:

| n = 1  | n = 2  | n = 3  |
|--------|--------|--------|
| \(E_1 = -0.0525\) | \(E_1 = -0.0410\) | \(E_1 = -0.0339\) |
| \(E_2 = -0.0078\) | \(E_2 = -0.0045\) | \(E_3 = -0.0006\) |

Tab 1. Bound state energies for the \(n = 1, 2, 3\) BK solutions, (gravitational sector).

| n = 1  | n = 2  | n = 3  |
|--------|--------|--------|
| \(E_1 = -0.0619\) | \(E_1 = -0.0360\) | \(E_1 = -0.0346\) |
| \(E_2 = -0.0105\) | \(E_2 = -0.0037\) | \(E_3 = -0.0009\) |

Tab 2. Bound state energies for the \(n = 1, 2, 3\) BK solutions, (sphaleron sector).

### 3.6 Comparison of different theories

As mentioned earlier, there are no static solutions in the pure YM theory in (3 + 1) dimension [3]. The reasons are that pure YM theory is repulsive and has no scale. In order to have solutions with finite energy one needs some
extra field which breaks scale invariance and provides an attraction which compensates the YM repulsion. In the case of the electroweak sphaleron, this job is done by a Higgs field. As we discussed earlier, it can be done by gravity \[1\] and by the dilaton field \[24, 25\].

The YM configuration is essentially the same and the EYM and the YMD solutions share the main properties of the sphaleron. Namely, they have finite energy, fractional charge \[35, 36\] and there are fermion zero modes in the background of these solutions \[37, 36\].

The situation is summarized in the Tab.3.

| Properties of solitons in different theories | YMH | EYM | (E)YMD |
|--------------------------------------------|-----|-----|--------|
| Finite energy                              | 2B(M_H/M_W)M_W/\alpha_W | M_{Pl}/\gamma \sqrt{\alpha_g} | |
| Fractional charge                          | 1/2 | 1/2 | 1/2 |
| Negative mode(s)                           | yes | yes (2n!) | yes (2n!) |
| Existence of fermion zero modes            | yes | yes | yes |

Tab 3. Main properties of solitons in different theories.

### 3.7 Sphaleron black holes

There are corresponding black holes in EYM, EYMD, EYMH theories \[38, 13, 26\]. This is in a way a non-trivial fact since not all classical lumps allow a horizon \[39\].

One of the most interesting results of this activity is finding of a violation of the No-Hair conjecture. There was a widespread belief that black holes are completely characterized by their “quantum numbers” seen from infinity: mass, electric and magnetic charges, angular momentum. J.A. Wheeler put this statement as “black holes have no hairs”. There are No-Hair theorems for theories with scalar fields \[10, 11\] and Maxwell field \[12, 13\].

The non-abelian case shows that this conjecture is not valid. In the EYM and EYMD theory there are non-abelian black holes which have the same quantum numbers as Schwarzschild hole but are different from Schwarzschild!

Very unfortunately the sphaleron black holes share the instability of the globally regular solutions. Magnetically charged non-abelian black holes...
4 Non-asymptotically flat solutions. EYM theory with the cosmological constant

In this section I will briefly touch the situation in which the solutions are non-asymptotically flat \[44, 45, 46\].

The action for the EYM theory with the cosmological constant $\Lambda$ has the form:

$$S_{EY M\Lambda} = \frac{1}{4\pi} \int (-\frac{1}{4G}(R + 2\Lambda) - \frac{1}{4g^2}F^2)\sqrt{-g}d^4x. \quad (18)$$

It turns out that some of the solutions of the model (18) have critical points, and Schwarzschild-like coordinates (7) are no more suitable. A convenient system of coordinates in this case turns out to be

$$ds^2 = Q^2(\rho)dt^2 - \frac{d\rho^2}{Q^2(\rho)} - r^2(\rho)d\Omega^2. \quad (19)$$

The EYM$_\Lambda$ theory was analyzed and a whole bunch of solutions was found \[45\]. Close to $r = 0$ they can be parameterized as in BK case by the shooting parameter $b$. Fig.1 shows the shooting parameter $b$ versus $\Lambda$ for different solutions. For small $\Lambda$ one obtains solutions which generalize the BK solitons, but now are surrounded by a cosmological horizon and asymptotically approach the de Sitter geometry. Increasing $\Lambda$ one obtains solutions which have both equator (critical point where $dr/d\rho = 0$) and horizon. With increasing cosmological constant the horizon shrinks to zero size and the curves in Fig.1 end with regular solutions which are topological spheres.

In fact one can integrate the system of equations and obtain $n = 1$ regular solution in closed form:

$$Q = 1, \ r = \sqrt{2}\sin(\frac{\rho}{\sqrt{2}}), \ W = \cos(\frac{\rho}{\sqrt{2}}), \ \Lambda = \frac{3}{4} \quad (20)$$

A stability analysis of these solutions \[46\] is much more involved than in asymptotically flat case. The main source of difficulties is the existence of a critical point in the background solution. It was shown that pulsation equations can be brought to a form convenient for numerical analysis \[46\]. The
conclusion is again that the $n^{th}$ solution has $2n$ unstable modes. Since the system of pulsation equations in the gravitational sector is in a non-standard form, textbook theorems are not applicable, and mathematical questions about spectrum of bound states (that it is real and discrete) are still open.

5 Concluding remarks

To conclude, let me list what I think are main non-abelian surprises:
- Existence of nontrivial globally regular solutions in the EYM and YMD theory.
- Violation of the No-Hair conjecture in non-abelian case.
- Interplay between gravity and YM which leads to the solution Eq. (21).
- EYMD theory, string case, $n = 1, \gamma = 1$ analytic solution (?)

So far a whole zoo of solutions has been found. A natural question arises: what might be a possible application for these non-abelian “animals”. Let me mention a few ideas:
- Fermion number non-conservation.

One finds fermion zero modes in the background of EYM and YMD solitons leading to fermion number non-conservation. Can one hope about a possible generation of the baryon asymmetry? I think the answer is No. The masses are too high and they cannot “compete” with the electroweak sphaleron. Another problem is that the gravitational and dilatonic sphalerons have even number of negative modes. The EYM example shows (Tab.1 and Tab.2) that instabilities in the different sectors have comparable energies and therefore one cannot neglect gravitational instabilities and apply formulas which are designed for the case with a single negative mode. This question needs further investigation.

- Counterexamples for a No-Hair conjecture.

This is very important observation. There are some speculations about black holes as elementary particles. If the No-Hair conjecture had been true, there would have been no chance to assign lepton or baryon number to a hole (particle). Non-abelian black holes show that this is not totally excluded.

- Cosmological applications (?)

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3 This case I would rather call as an “anti-surprise” according my definition. There was hope (and strong indications!) that one can find solution in a closed form but till now nobody was able to find it.
-Interpretation as an instanton. Static solutions can be considered as an instantons in lower dimensional space. This might be a direction in which to think as well.

There are many interesting developments, which I could not cover in this presentation. Let me mention that the existence proof for EYM solitons and black holes has been given \[17, 18\], the dynamical evolution of the perturbed BK solutions was studied \[18, 19\] and the relation to the critical phenomenon in black hole collapse was investigated \[19\]. Also axially symmetric solutions of the EYMD theory have been constructed \[50\] recently.

6 Acknowledgments

This work was supported by the Tomalla Foundation.

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Figure 1: The shooting parameter $b$ versus cosmological constant $\Lambda$ for different solutions of the EYM$_\Lambda$ theory.