Optimization of UAV landing taking into consideration of limitation on control on the basis of solution of the boundary value problem by the parameter continuation

Nguyen Ngoc Dien, Ngo Van Toan
Le Quy Don technical university, Ha Noi, Social republic of Viet Nam
bauman252@mail.ru

Abstract. In this article, we investigate the optimal control of flying equipment, taking into account the constraints of UAV landing. When using the Pontryagin maximum principle the optimal control problem is shifted to the boundary problem. To solve the boundary problem, use the parameter continuation method (Parameter continuation method). Control signals are considered as tangential load factors (tangential load factor) and normal load factors (normal load factor). Main research results - The program for optimal control of UAV motion when landing with consideration of limitation of the control signals.

1. Introduction
The optimal UAV trajectory on landing has not lost its urgency in the past 10 years. Many published works by the authors [1, 6, 7, 8, 9, 10]. In optimum control using the Pontryagin maximum principle, it is possible to move from the optimal problem to the boundary problem. Solving the boundary problem encounters many difficulties related to the computation time, the choice of approximate parameters initially, and the convergence of solutions. A number of studies have suggested the use of the Newtonian method [1, 6, 8] However, when constrained with control signals, the solution encountered many difficulties such as the Newtonian method. Other studies have proposed continuous methods of solving parametric problems, but the authors have not applied the theory presented in specific problems [4, 11]. Thus, in this article, the author proposes the application of the parametric continuation method to the optimal plane landing problem. The optimal indicator is the Bolza index, which contains the precision that leads the UAV to the end point of the trajectory and the minimum energy consumption.

2. Statement of problem
We consider the motion of the UAV in the vertical plane. The UAV equation system has the form [1, 2, 6, 8, 10]:

\[
\begin{align*}
\dot{V} &= g(n_x - \sin \theta); \dot{\theta} = \frac{g}{V}(n_x - \cos \theta); \dot{x} = V \cos \theta; \dot{y} = V \sin \theta, \\
\end{align*}
\]

With \( V \) – velocity UAV; \( \theta \) - the flight-path angle; \( x \) – range of flight; \( y \) – flight altitude; free acceleration \( g = 9.80665 \text{ m/s}^2 \); \( X = [V, \theta, x, y] \) - The state vector of the UAV; \( n_x, n_y \) - tangential load factor and normal load factor.

Control signal selected \( u = [n_x, n_y]^T \). Quality indicators have the forms of Bolza indicators:
\[ J = 0.5 \rho_1 (x(t_f) - x_f)^2 + 0.5 \rho_2 (y(t_f) - y_f)^2 + 0.5 \rho_3 (\theta(t_f) - \theta_f)^2 + 0.5 \int_{t_i}^{t_f} u^T k^2 u \, dt; \]  
\[ \text{with } \rho_1, \rho_2, \rho_3 - \text{constants; } k^2 = \text{diag}(k_1^2, k_2^2) - \text{given coefficient; } t_i, t_f - \text{the beginning and end of the corresponding flight process; } x_f, y_f, \theta_f - \text{the given value at the end } t_f. \]

3. Optimization of UAV landing

Consider the motion of the UAV on a vertical plane. The system of equations of motion of the aircraft means [1, 2, 6, 8, 10]:

\[ V = g(n_x - \sin \theta); \dot{x} = \frac{g}{V} (n_x - \cos \theta); \dot{y} = V \cos \theta; \dot{\theta} = V \sin \theta; \]

The equations for the conjugate variables are as follows:

\[ \dot{P}_x = -\frac{\partial H}{\partial x} = P_\theta \frac{V}{V^2} (n_x - \cos \theta) - P_y \cos \theta - P_x \sin \theta; \]
\[ \dot{P}_\theta = -\frac{\partial H}{\partial \theta} = P_x g \cos \theta - P_x \frac{g}{V} \sin \theta + P_y V \sin \theta - P_y V \cos \theta; \]
\[ \dot{P}_y = -\frac{\partial H}{\partial y} = 0; \dot{P}_\theta = -\frac{\partial H}{\partial \theta} = 0. \]

With limited control signals the problem is reversed: Find the optimal control signal at each moment so that the Pontryagin \( H \) function reaches \( \min(\dot{H}) \), suppose \( n_x \) has no restrictions.

From optimum conditions, \( \frac{\partial H}{\partial n_x} = 0 \), we get the optimal control signal: \( n_x = -P_x g k_1^2; \)

We get a system of equations that push enough UAV motions to conjugate variables:

\[ \dot{V} = g(n_x - \sin \theta); \]
\[ \dot{x} = \frac{g}{V} (n_x - \cos \theta); \]
\[ \dot{y} = V \cos \theta; \]
\[ \dot{\theta} = V \sin \theta; \]
\[ \dot{P}_x = P_\theta \frac{g}{V^2} (n_x - \cos \theta) - P_y \cos \theta - P_x \sin \theta; \]
\[ \dot{P}_\theta = P_x g \cos \theta - P_x \frac{g}{V} \sin \theta + P_y V \sin \theta - P_y V \cos \theta; \]
\[ \dot{P}_y = 0; \]
\[ \dot{P}_\theta = 0. \]

It is necessary to find the initial value \( P_x(t_0), P_\theta(t_0), P_y(t_0), t_f \) to satisfy the boundary conditions \( x(t_f) = x_f, y(t_f) = y_f, \theta(t_f) = \theta_f, H(X, P, t_f) = 0 \). To solve the boundary problem, we apply the parameter continuation method.
4. Parameter continuation method

The essence of the parameter continuation method is the formal reduction of the considered boundary value problem to the Cauchy problem [3, 4, 5, 10]. The boundary problem for a dynamic system with boundary conditions can be represented as an equation for the residuals at the right end of the trajectory:

$$ f(z) = 0 \quad (6) $$

where, $z = [P_x(t_0), P_y(t_0), P_z(t_0), t_f]^T$ — vector of unknown parameters of the boundary value problem.

At some initial approximation for the unknown parameters of the boundary value problem $z_0$, we calculate the residual vector (6):

$$ f(z_0) = b \quad (7) $$

Consider the immersion of equation (6) in a one-parameter family:

$$ f(z) = (1 - \tau)b \quad (8) $$

where, $\tau$ is the continuation parameter, and we represent the vector $z$ as a function of this parameter: $z = z(\tau)$, moreover $z(0) = z_0$ from equation (7). We require equality (8) for any $0 \leq \tau \leq 1$. Obviously, for $\tau = 0$, equation (8) coincides with (7), and for $\tau = 1$ — the equation for residuals for the desired boundary value problem (6).

Differentiating equation (8) with respect to the continuation parameter $\tau$ and solving the resulting expression for the derivative $dz/d\tau$, we obtain a formal reduction of equation (6) to the Cauchy problem:

$$ f(z) = (1 - \tau)b \Rightarrow \frac{dz}{d\tau} = -\left(\frac{\partial f}{\partial z}\right)^{-1}b, \quad (9) $$

$$ z(0) = z_0, \quad 0 \leq \tau \leq 1. $$

Obviously, integrating (9) over $\tau$ from 0 to 1, by virtue of (7), (8), you can define the desired vector of unknown parameters of the boundary value problem (6) in the form $z = z(1)$.

$$ \int_0^1 \frac{dz}{d\tau} d\tau = -\int_0^1 \left(\frac{\partial f}{\partial z}\right)^{-1}b d\tau \Leftrightarrow z(1) = z(0) - \int_0^1 \left(\frac{\partial f}{\partial z}\right)^{-1}b d\tau $$

$z(1)$ — Solution of the original problem.

5. Results

After calculating the aerodynamic calculation of a single aircraft model, we have a range of the normal load factor $n_{n[-1,5,4]}$. It is assumed that $P_x(0) = 1; k_0 = 0,1; k_2 = 0,1$.

We will consider 2 cases. The first case: the flight velocity of the UAV with values $V(0) = 50, 70, 90, 100 \text{ m/s}$; Initial state of UAV: $\theta(0) = 0\text{rad}; x(0) = 0 \text{ m}; y(0) = 1000 \text{ m}$. Boundary conditions: $\theta(t_f) = 0\text{rad}; x(t_f) = 2000 \text{ m}; y(t_f) = 0 \text{ m}$.

The trajectory of the UAV and the dependence of the flight-path angle are shown in fig. 1 and fig. 2.
In fig. 2, the dependence of the flight-path angle of time.

In fig. 3 and fig. 4 shows the components of the overload. The values of the normal load factor from -1 to 1.5, as in a given range, and the tangential load factor change from -1.2 to 0.6.

In fig. 5 and fig. 6 shows the dependence of the flight velocity of time and the dependence of the Pontryagin function values of time. For different values of the flight velocity at the initial moment of flight, the flight velocity at the end of the trajectory can be increased or decreased to zero. In fig. 6, it is clear that the values of the Pontryagin function at the end of the trajectory are zero, which proves that the flight time is already optimal.

The second case: Flight range varies from 1000 m to 4000 m. $x(t_f) = 1000, 2000, 3000, 4000$ m; Initial state of UAV: $V(0) = 50$ m/s; $\theta(0) = 0$ rad; $x(0) = 0$ m; $y(0) = 1000$ m. Boundary conditions: $\theta(t_f) = 0$ rad; $y(t_f) = 0$ m.

The trajectory of the UAV and the dependence of the flight-path angle are shown in fig. 7 and fig. 8.
Figure 7. The trajectory of the UAV

Figure 8. The dependence of the flight-path angle of time

In fig. 9 and fig. 10 shows the components of the overload. The values of the normal load factor from -0 to 1.5, as in a given range, and the tangential load factor change from -1.2 to 1.05.

Figure 9. The dependence of the tangential load factor of time

Figure 10. The dependence of the normal load factor of time

In fig. 11 and fig. 12 shows The dependence of the flight velocity of time and The dependence of the Pontryagin function values of time. With a flight velocity of 1000 m/s, the flight velocity at the end of the trajectory decreases. In fig. 11, it is obvious that the greater the flight range, the higher the velocity at the end of the trajectory.

Figure 11. The dependence of the flight velocity of time

Figure 12. The dependence of the Pontryagin function values of time

6. Conclusion

In the article, the method of continuous solving by parameters is solved to solve the boundary problem. Carry out the analysis of results in two cases: with varying distances and with velocity at the initial time of change. The results indicate that, at a fixed value of the flight range, the higher the velocity at the initial moment of flight time, the easier it is to land. And equivalently, with a fixed value of flight velocity at the initial moment of time, the longer the flight range, the easier the landing.

We get the optimal UAV trajectory when landing. When the orbital realization improves the safety of an automatic UAV landing (safety of an automatic UAV landing).

References

[1] Aleksandrov A.A., Kabanov S.A.2008 Optimal control of UAV landing with consideration of limitation on control Mechatronics, automation, control 5 50-54
[2] Kim D. P2004 Theory of automatic control. Vol. 2. Multidimensional, nonlinear, optimal and adaptive systems, *Proc. allowance. M.: Fizmatlit*

[3] Kim D. P2004 Theory of automatic control. Vol. 2. Multidimensional, nonlinear, optimal and adaptive systems: *Proc. allowance. - M.: Fizmatlit*

[4] Konstantinov M.S., Nguyen Ngoc Dien 2016 The analysis of ballistic capabilities for countering disturbances associated with temporary emergency electric propulsion shutdown. *Solar system research*.

[5] Shalashilin V. I., Kuznetsov E. B1999 Parameter continuation method and the best parametrization., *M.: Editorial URSS224*

[6] Nguyen Ngoc Dien 2014 Optimization of the interplanetary trajectory of a spacecraft with an electric propulsion system within the framework of the four-body problem taking into account the influence of the second zonal harmonic, *the Earth Materials XLIX scientific readings in memory*, K. E. Tsiolkovsky. Kaluga 01

[7] Kabanov S. A., Aleksandrov A. A. 2007 Applied optimal control problems: *Proc. Guide to practical exercises. SPb.: Ed. Balt State Tech. University*

[8] Bondarev V. G.2012 Automatic landing of an aircraft on an aircraft carrier., *Scientific Herald of MSTU GA* 8124-131

[9] Kulifeev Yu. B., Afanasyev Yu. N., Algorithm for automatic landing of an aircraft. *Journal Trudy MAI, Russia, issue* 10 62

[10] Nguyen Ngoc Dien, Tran Quang Minh, Solutions for optimal control problem of aircraft landing on the basis of solving the boundary value problem by parameter continuation method, *Journal “Synergy nauk”, ISSN 2500-09509*.

[11] Dikusar V.V., Koshka M., Figura A.A2001 Parameter Extension Method for Solving Boundary Value Problems in Optimal Control Theory. *Differential Equations* 5 479–484.