Calibration of critical speed predictions using experimental measurements

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Abstract. Experimental characterization of structures provides information about the real
dynamic behavior of machineries. This results in a better estimation of dynamic stress levels,
used to predict lifespan, reliability and optimal operating ranges. Experimental results can also
be used to improve and calibrate simulations and calculations. In particular, calculation of shaft
bending frequencies is needed to define the critical rotation speed. However, natural frequency
computation requires a prior knowledge on many subcomponent properties, that are not
precisely known in practice. This leads to a great uncertainty on the predicted critical speed.
The purpose of this study is to use experimental in-situ measurements as a means to reduce
uncertainties on physical properties of the shaft line and increase confidence in the prediction
of natural frequencies and critical speed. This involves ambient modal analysis, calibration
techniques and statistical approaches to gain insight on the true shaft physical properties.

1. Introduction

Hydroelectric power plays an essential role in the energy market, as it has many advantages regarding
economic matters, environment and efficiency. To optimize production, maximize performances and
ensure safety, one of the keys is the prediction of radial loads to avoid any harmful situation. In
particular, an accurate computation of shaft bending modes is required to properly estimate the shaft
critical rotation speed. This speed is defined as the rotational speed that excites the first forward
whirling mode. In such case, radial loads can provoke either fatal damages or safety issues.

The critical speed is determined using the structural properties of shaft line, runner, generator and
bearings, in a conservative manner. In this approach, the rigidity and damping properties are assumed
constant and independent from the rotational speed. In practice, some structural parameters like
bearing rigidities are often not available, and are estimated within some interval ranges. This lack of
knowledge results in a poor estimation of the critical speed, possibly leading to unnecessary
conservatism to ensure the safety and reliability of the structure.

The purpose of the upcoming work is to avoid over conservative critical speed predictions and
very costly decision without compromise to safety. For this, it is proposed to reduce the level of
uncertainty in the modal predictions by using experimental measurements as a means to reduce
bearing stiffness uncertainty intervals. This should allow a better prediction of the natural frequencies,
and increase confidence in the critical speed determination. Such idea was already evoked by Loth [1], where a comparison of theoretical results and experimental modal analysis results was achieved. In the same vein, the purpose of this work is to calibrate the numerical simulations using experimental approaches to improve numerical results.

The first part of this paper presents the conservative approach generally used to make predictions. Then, a section introduces the protocol to identify whirling natural frequencies from experimental measurements. Next, the calibration of numerical simulations using experimental identification is done to reduce the uncertainty on bearing rigidities, and improve the accuracy of critical speed determination. Finally, this procedure is illustrated in a case study, where the critical speed of an existing medium head Francis turbine is investigated.

2. Conservative approach using finite elements

The prediction of critical speed relies on structural modal analysis. This analysis is done using Finite Element Analysis (FEM), a method used to discretize continuous structures, particularly relevant for complex geometries. The conservative equation of motion for the undamped system is expressed in a matrix form as follows [2]:

$$M\ddot{q}(t) + \omega(t)G\dot{q}(t) + Kq(t) = f(t)$$

The undamped equation is used because damping coefficients are not used in the critical speed prediction. In eq. (1), $q$ and $f$ are the displacement and force vector, and $\omega$ the rotational speed. $M$ and $K$ are the mass and stiffness matrices, respectively. $G$ is the skew-symmetric gyroscopic matrix, expressing the polar moment of inertia of the turbine. Modal frequencies are obtained using modal decomposition [3]. These frequencies are functions of matrices $M, K, G$, which depend on the structure’s physical characteristics.

Since bearing rigidities are not precisely known, $K$ is uncertain, as well as the predicted whirling frequencies. A safe design relies on a worst-case approach, where the critical speed is taken as the lowest whirling frequency allowed by the uncertain parameters. Although the method ensures the real critical speed to not be accidentally reached, the prediction can be over-conservative and needlessly alarming. The new methodology proposes to reduce the prediction bias by calibrating the modal analysis with experimental observations. The processing of ambient measurements allows identifying the real natural frequencies, which can be used to update the numerical model and improve the modal analysis.

3. Experimental characterization

Over the last decade, turbine field vibration measurement campaigns have become a standard procedure during commissioning or after maintenance sessions. This provides a wide database that can be used for identifying shaft whirling frequencies. Such frequencies should be found in shaft bending or bearing displacement measurements during asynchronous tests, like run-up or coast-down. When the runner speed varies, the rotating speed harmonics excite a wider range of frequencies, and are more likely to pass through resonances. As measurements are made during operational sessions, recorded signals must be treated in an output-only fashion, using operational modal analysis.

Order-Based Modal Analysis (OBMA) was proven effective for identifying resonances under harmonic excitation [4]. The modes are extracted using order domain angular resampling [7], and modal parameters are estimated using a Maximum Likelihood Estimator (MLE) [5, 6]. Since extracted whirling modes are assumed well separated, identification is done with a Single-Degree of Freedom (SDoF) model. The cost function related to the optimization problem is given in eq. (2), and is called Negative Log-Likelihood Function.
\[
\mathcal{L}(\theta) = N_g N_f \ln(\pi) + \sum_{k=1}^{N_f} \ln(|G_k(\theta)|) + \sum_{k=1}^{N_f} \hat{X}_k^* G_k^{-1}(\theta) \hat{X}_k
\]  

(2)

In eq. (2), \(N_g, N_f\) are the number of channels and frequency samples, respectively. \(\hat{X}_k\) is the frequency response obtained with Order Tracking (OT) [7]. \(G_k(\theta)\) is the theoretical Power Spectral Density (PSD) of the SDoF at each frequency sample \(\omega_k\). Finally, \(\theta\) is the modal parameter vector, containing the frequency, damping, modal force PSD and channel noise PSD (assuming a homoscedastic system). Because inverse and determinant of \(G_k\) need extensive computations to be determined, alternative formulations of eq. (2) exist, based on eigenvalue decompositions [6]. The bending frequency estimation, say \(f_0^{\text{exp}}\), is obtained with eq. (3).

\[
f_0^{\text{exp}} = \arg\min_{\theta} \mathcal{L}(\theta)
\]  

(3)

Even in presence of very noisy data, MLE generally results in a good estimate of the natural frequency. As mentioned earlier, the critical speed is defined with the first forward bending mode, i.e., the lowest whirling mode that rotates with the runner. To ensure the identified mode is forward, a phase-shift analysis between sensors could be advised. Last but not least, numerical models work in stationary coordinates. If the measurements come from rotating sensors, one should convert the identified frequency in the stationary coordinates. For a frequency identified at speed \(v_B\), this transform writes:

\[
f_B^{\text{exp}} = \begin{cases} 
  f_0^{\text{exp}} + v_0, &\text{forward mode} \\
  f_0^{\text{exp}} - v_0, &\text{backward mode}
\end{cases}
\]  

(4)

Finally, an experimental whirling frequency \(f_B^{\text{exp}}\) is obtained. An effective rotation speed \(v^{\text{exp}}\) is also obtained. This is the rotational speed of the excitation pattern that induced resonance. For instance, let consider a case where:
- The bending resonance is induced by the third harmonic of the rotation speed.
- The rotating speed at which resonance develops is 45 rpm.
- The resonance is detected on strain gauge shaft bending records, i.e., on the rotating part.

Then, the bending mode in the rotating coordinates is 3×45 = 135 rpm, or 2.25 Hz. In the stationary coordinates, the frequency turns out to be 3×45 + 45 = 180 rpm, or 3 Hz. The related effective rotational speed is 135 rpm.

4. Model updating

Let \(\mathcal{M}\) be the modal analysis driven by eq. (1). As bearing rigidity values are uncertain, a set \(\{K\}\) of possible stiffness matrices can be constructed. Then, the computation of the shaft whirling frequency at speed \(v_\tau\) returns an ensemble of frequency candidates:

\[
\{f_0^{\text{num}}|v_\tau\} = \mathcal{M}(M, \{K\}, G, v_\tau)
\]  

(5)

In the classical procedure, the worst-case approach chooses the lowest frequency among the frequency candidates. This protocol ensures safety, but can lead to unreal scenarios. Instead, it is proposed to use the experimental results to condense the set \(\{K\}\) into a calibrated subset \(\{K^{\text{exp}}\}\) satisfying eq. (6). This subset does not reduce to a single value as the model \(\mathcal{M}\) is generally not injective.

\[
f_0^{\text{exp}} = \mathcal{M}(M, \{K^{\text{exp}}\}, G, v^{\text{exp}})
\]  

(6)
The obtained \( \{ K_{\text{exp}} \} \) comprises all the possible combinations of bearing rigidities that produces the observed whirling mode at speed \( v_{\text{exp}} \). Submitting the results of eq.(6) in eq. (5) gives eq.(7), that is the calibrated model.

\[
\{ f_0^{\text{num}} | f_0^{\text{exp}}, v_r \} = M(M, \{ K_{\text{exp}} \}, G, v_r)
\]

Henceforth, there are two ways to determine the critical speed. First, it is possible to use another worst-case approach on the calibrated frequencies \( \{ f_0^{\text{num}} | f_0^{\text{exp}}, v_r \} \). This should reduce the existing bias without compromising safety. On the other hand, it is possible to infer the critical speed in a statistical way. In the statistical approach, the interval ranges of possible bearing rigidities are modeled with a probability law. A probability is assigned to each element of \( \{ K_{\text{exp}} \} \), as the probability of obtaining this element given the bearing distributions. This probability is propagated through model \( M \) with eq. (7), and a critical speed distribution is obtained in a frequentist way.

5. Case study

5.1. Numerical approach

The studied turbine is a medium-head Francis runner of specific speed \( n_k = 57 \), producing a power of 160MW. The shaft consists in a stainless steel hollow cylinder of infinite rigidity. The runner has an inertia \( I_n \) and a structural mass \( m_n \). An added mass comes from the fluid-structure interaction, denoted \( m_p \). It is computed from Ansys® with a full geometry including labyrinths and massive inflow. The alternator has an inertia \( I_q \) and a mass \( m_q \). The shaft is fastened by three bearings of rigidity \( K_i, i \in \{1, 2, 3\} \). Figure 1 introduces a schematic of the system. Each bearing is characterized by an interval range: \( K_i \in [K_{i,\text{min}}, K_{i,\text{max}}] \).

![Figure 1. Turbine model including generator, runner, shaft and bearings](image)

Since the generator rotor is modeled with an infinite rigidity structure that is not representative of the real behavior, it is proposed to allow a flexibility on the generator rotor inertia, following an equivalence rule. This allows introducing some variability in the rigidity. The effective inertia turns out to be included between 0.25 \( I_G \) and \( I_G \). Shaft structural mass distribution is precisely known, and no uncertainty is considered for this parameter.

The Finite Element model is processed into a modal analysis solver using the so-called worst-case scenario to obtain a first estimate of the critical speed. This computation is done with RBTS software, provided by ARMD. Results are drawn on Figure 2 (lower red line). According with this approach, the critical speed is 498 RPM (or 8.3 Hz). The upper red line represents the upper prediction for the
whirling frequencies. Parameters used in this approach are depicted in Table 1. In Figure 2, the first harmonics of the rotating speed are also shown. This provides a preliminary indication about the regions where whirling modes can be observed in the data.

Table 1. Parameters used in the worst-case conservative scenarios

| Properties         | Lower   | Upper   |
|--------------------|---------|---------|
| Bearing rigidities | $K_{l_{\text{lower}}}$ | $K_{l_{\text{upper}}}$ |
| Effective inertia  | 0.25 $I_G$ | $I_G$   |

Figure 2. Worst-case scenarios with conservative approach.

5.2. Calibration approach

In this case study, the data coming from two operational unit start-ups were used. Measurements recorded the shaft bending in two orthogonal axes. Figure 2 suggests that the target mode should be around $6 - 12 \text{Hz}$ in the stationary coordinates. After exploring the data, it is found that the fourth harmonic of the rotational speed passes through a resonance during the acceleration. Figure 3 gives an example of extracted singular value response spectrum. In the rotational cascade, MLE identifies a frequency $f_B^{\text{rot}} = 6.99 \text{Hz}$ at speed $v^{\text{exp}} = 1.75 \text{Hz}$. This gives $f_B^{\text{exp}} = 8.74 \text{Hz}$ in the stationary cascade.

The model given in eq. (6) is computed for several effective inertias included in $[0.25 I_G, I_G]$. Results are presented in Figure 4 for uniform interval distribution, and in Figure 5 for Gaussian intervals (each bound represents the 95% confidence interval). Calibrated stiffness combinations give a hyperplane embedded in the space parameter. This hyperplane has the shape of a curved 2D-plan in the three-dimensional space. The lower the generator inertia, the higher the plan elbow curvature. Each point of the hyperplane represents the coordinates of a bearing stiffness combination that produces the observed whirling frequency. To each of these combinations, a probability is assigned, based on the stiffness distributions. For uniform distributions, the combinations are naturally equally probable. For Gaussian distributions, it seems that the high probability regions are concentrated on the elbow curvature.
Eq. (7) is used to predict the critical speed. This leads to the results shown on Figure 6. Each black line represents the evolution of the predicted whirling frequency for a given calibrated stiffness matrix. Each line carries a different probability weight, based on the stiffness combination probabilities. Frequencies vary with the rotational speed since the gyroscopic effects plays a role in their prediction. As expected, all the frequencies match at the calibration speed $v^\text{exp}$, and are equal to $f_0^\text{exp}$. The red line represents the rotational speed. The critical speed prediction is made by picking up...
the intersection of the whirling frequency lines with the rotational speed. Results are shown in Figures 7 a. and b., for uniform and Gaussian distributions of the bearing rigidities. Densities are approximated using histograms. Uniform stiffness distributions results in a pseudo-uniform distribution of the critical speed prediction, while Gaussian distributions seem to give more statistical weight to lower critical speed predictions.

Figure 6. Whirling calibrated frequencies. Calibration point is at (1.75, 8.75) Hz.

Figure 7. Frequentist distribution of the calibrated critical speed.

As shown in Table 2, it is of main importance to take into account generator rotor flexibility, because lower inertia lowers the predicted whirling frequencies. In the studied case, the frequency is lowered down by 1 Hz. As shown in Table 2, the statistical definitions of the critical speed (with unilateral 5% threshold) and the worst-case approach give similar results.

From the case study, several conclusions can be made to minimize computation time and facilitate procedure. First, a full statistical approach is not needed, as critical speed distributions have small deviations, so that the study of tails is almost equivalent to a worst-case approach. Moreover, it is possible to restrict the investigation to the system with the lowest effective inertia, because this last will produce the lowest whirling frequencies. However, due to non-uniform mass distribution, the stiffness combination defining the critical speed is not merely the lowest values for each bearing, and cannot be trivially guessed. For instance, here, the critical speed is based upon a stiffness vector with
very high $K_3$, but low $K_1$ and $K_2$. Finally, the critical speed determined with this calibration approach is around 1 Hz above the previous critical speed prediction without experimental data available.

Table 2. Calibrated critical speed prediction (Hz).

|                      | 5% frequentist approach | Worst-case |
|----------------------|-------------------------|------------|
| Inf. rigidity        | 10.60                   | 10.34      | 10.26      |
| Flex. rotor          | 9.47                    | 9.35       | 9.28       |

6. Conclusion

In this study, a finite element modal analysis procedure was calibrated using experimental data to increase its accuracy in the first forward whirling frequency. As a result, the critical speed determination was revised upward, giving more flexibility in over-speed operation procedures. In the case study, the critical speed is very high and is not strictly speaking “critical”. But in some situations, a better assessment of this last should be welcomed. Such studies should be particularly relevant for turbines with overhaul speed which can get close to the critical speed, or for shaft lines with low whirling frequencies.

One weakness of the study is the only case study used for testing the model. For a more reliable validation, several other cases, including different characteristics and geometry should have been tested. However, this paper draws a general framework seems promising for calibration purposes. Since operational measurements are in their early stages, any contribution showing the relevancy of the process contributes to popularize this emergent topic.

In the case study, only the first whirling mode frequency was identified and used for calibrating the model. Future works could enrich this first calibration with further experimental whirling frequencies, to obtain smaller $K_C^{DE}$. Some works could be conducted about the optimization of experimental setup, in order to maximize the modal information with the lowest cost. This study is unable to provide such advices because only one case was studied.

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