SU(3) × SU(2) × U(1) Chiral Models from Intersecting D4-/D5-Branes

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We clarify RR tadpole cancellation conditions for intersecting D4-/D5-branes. We determine all of the D4-brane models that have D=4 three-generation chiral fermions with SU(3) × SU(2) × U(1)\(^n\) symmetries. For the D5-brane case, we present a solution to the conditions that gives exactly the matter content of the standard model with U(1) anomalies.

Intersecting D\(p\) -branes are useful tools to obtain chiral fermions existing on their intersections. Although configurations of type IIB D6-branes wrapped on T\(^6\) give rise to identically the matter content of the standard model, the D6-brane models do not solve the fine-tuning problem caused by its breakdown of supersymmetry. Contrastingly, this difficulty is avoided by using D4-/D5-branes wrapped on T\(^{2d}\) × R\(^{6-2d}\)/\(\mathbb{Z}_N\) through reduction of the string scale with a very large volume of the \(\mathbb{Z}_N\) orbifold. Intersecting D4-/D5-branes in both oriented and unoriented theories, we construct D=4 chiral models whose matter content transforms as (3, 2)\(_1\) + (3, 1)\(_{-\frac{1}{2}}\) + (3, 1)\(_3\) under SU(3) × SU(2) × U(1)\(_Y\).

We start with the definition of the spacetime geometry in the oriented theory. The compactified six dimensional space is the product of \(d\) rectangular two-tori T\(^I\)\(_1\) (1 ≤ I ≤ d ≤ 2) and a R\(^{6-2d}\)/\(\mathbb{Z}_N\) orbifold. The space is parameterized by the complex coordinates Y\(_I\) = X\(_{2I+2}\) + iX\(_{2I+3}\) with radii \(R_{2I+2}\) and \(R_{2I+3}\) for I = 1, 2, 3.

Let us consider K different stacks, each of which is a set of \(N_a\) coincident D4-/D5-branes. They are labeled by the index \(a (a = 1, \cdots, K)\) and wrapped around a 1-cycle denoted by \((n_a^{(I)}, m_a^{(I)})\) on each of the \(d\) two-tori, where \(I = 1\) for D4-branes and \(I = 1, 2\) for D5-branes. When the \(a\)-th stack and the \(b\)-th stack intersect, we refer to their intersections as an \(a-b\) sector. The intersection number of the \(a-b\) sector is given by

\[
I_{ab} = \prod_{I=1}^{d} I_{ab}^{(I)} = \prod_{I=1}^{d} (n_a^{(I)} m_b^{(I)} - n_b^{(I)} m_a^{(I)}).
\]

The \(\mathbb{Z}_N\) action is realized by powers of the twist generator represented by a shift vector \(v\) in the space of the SO(6 − 2d) Cartan subalgebra. In the configurations with intersecting D4-branes, modular invariance conditions for one-loop amplitudes

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restrict the form of the twist vector \( v = (v_1, v_2, v_3) \) to \((0,1,b_2)/N\), where \( b_2 \) is an odd integer. For the case of D5-branes, \( v = (0,0,2/N) \).

Since \( N_a \) coincident D-branes have \( U(N_a) \) gauge factors in their world-volume, open strings stretched between the \( a \)-th stack and the \( b \)-th stack give rise to states in the \( a-b \) sector that belong to bifundamental representations of \( U(N_a) \times U(N_b) \). The gauge degrees of freedom are expressed by the Chan-Paton (CP) factors as \(|ij\), where \( i \) and \( j \) correspond to a D-brane in the \( a \)-th and the \( b \)-th stack, respectively. We express the embedding of the \( Z_N \) action \( \theta^k \) using the unitary matrix \( \gamma_k \). We diagonalize \( \gamma_k \) so that

\[
\gamma_k = \text{diag}(\zeta^{l_1}1_{N_1}, \zeta^{l_2}1_{N_2}, \cdots, \zeta^{l_K}1_{N_K}),
\]

where \( \zeta = e^{2\pi i \frac{1}{N}} \) and \( l_a \) are integers. Then the \( N_a \) D-branes in the \( a \)-th stack have the same CP phase, \( \zeta^{l_a} \).

The RR tadpole cancellation conditions are

\[
\sum_{a=1}^K N_a \prod_{I=1}^d n_a^{(I)} = \sum_{a=1}^K N_a \prod_{I=1}^d m_a^{(I)} = 0, \quad (3)
\]

\[
\sum_{k=1}^{N-1} \left( \prod_{J=d+1}^3 |\sin\pi k v_J| \right) \left( \sum_{a=1}^K \prod_{I=1}^d n_a^{(I)} \text{Tr}_a \gamma_k \right)^2 = 0, \quad (4)
\]

\[
\sum_{k=1}^{N-1} \left( \prod_{J=d+1}^3 |\sin\pi k v_J| \right) \left( \sum_{a=1}^K \prod_{I=1}^d m_a^{(I)} \text{Tr}_a \gamma_k \right)^2 = 0, \quad (5)
\]

where \( \text{Tr}_a \) represents the trace over the CP factor on D-branes that belong to the \( a \)-th stack. For each shift vector \( v \) of \( Z_N \), we can obtain solutions for 1-cycles \((n_a^{(I)}, m_a^{(I)})\) of the \( a \)-th stack and CP phases \( \zeta^{l_a} \). The solution produces an open string spectrum that is invariant under the \( Z_N \) action.

For the D4-brane case, left-handed fermions in the \( a-b \) sector are given in bifundamental representations under \( U(N_a) \times U(N_b) \) as

\[
I_{ab}\{(N_a^{i}, N_b^{-1-b_2}) + (N_a^{i+1-b_2}, N_b^{-1}) + (N_a^{i+1-b_2}, N_b^{-1-b_2}) + (N_a^{i}, N_b^{i+1-b_2})\}, \quad (6)
\]

where the symbol \( N_a^{i} \) expresses the meaning that this is the representation \( N_a \) of \( U(N_a) \) and the CP phase of the \( a \)-th stack is \( \zeta^i \). A negative value of \( I_{ab} \) indicates a positive multiplicity of the field that transforms as the complex conjugate of each bifundamental representation. To obtain the three \((3,2)\) representations, we require \( N_1 = 3 \) and \( N_2 = 2 \) stacks and the intersection number \( I_{12} = 3 \). The two types of three-generation right-handed quarks require \( N_3 = 1 \) and \( N_4 = 1 \) stacks and intersection numbers \( I_{13} = \pm 3 \) and \( I_{14} = \pm 3 \). One more stack \( N_5 = 1 \) is necessary for the consistency conditions (3)–(5) to be satisfied. We have investigated all of these configurations and found two solutions, given in Table I.

Since the gauge symmetry \( U(1) \) of the stack \( N_a = 1 \) with 1-cycle \((n_a, 0)\) extends to \( U(1)^{\max} \), the gauge symmetry of the models is \( U(3) \times U(2) \times U(1)^5 = SU(3) \times
With this hypercharge, the spectrum (10) become
\[ SU(2) \times U(1)^7. \]

The fermion spectrum under the symmetry given by D4-1 in Table I is as follows:
\[ 3(3, 2)_{(1,-1,0,0,0,0)} + 3(3, 1)_{(-1,0,1,0,0,0)} + 3(3, 1)_{(-1,0,0,1,0,0)} + 3(1, 2)_{(0,1,0,0,-1,0,0)} + 3(1, 1)_{(0,0,-1,0,1,0)} + 3(1, 1)_{(0,0,0,0,-1,1,0)}, \]

where the underlines indicate permutation of indices. Computing the mixed anomalies which need to be cancelled by the Green-Schwartz mechanism, we find six non-anomalous U(1) linear combinations. They include the suitable hypercharge
\[ Q_Y = -\frac{1}{6}Q_2 + \frac{1}{3}Q_3 - \frac{2}{3}Q_4 + \frac{1}{3}(Q_5^{(1)} + Q_5^{(2)}) - \frac{2}{3}Q_5^{(3)}, \]

where \( Q_a \) is the generator of the \( a \)-th U(1). With this hypercharge, the fermion spectrum is
\[ 3(3, 2)_{\frac{1}{3}} + 3(3, 1)_{\frac{1}{3}} + 3(3, 1)_{-\frac{2}{3}} + 6(1, 2)_{-\frac{1}{2}} + 3(1, 2)_{\frac{1}{2}} + 6(1, 1)_{1} + 9(1, 1)_{0} + 3(1, 1)_{-1}. \]

When we set \( n_2 + n_3 = 1 \), D4-2 in Table I gives the following fermion spectrum:
\[ 3(3, 2)_{(1,-1,0,0,0,0)} + 3(3, 1)_{(-1,0,1,0,0,0)} + 3(3, 1)_{(-1,0,0,1,0,0)} + 3(1, 2)_{(0,1,0,0,-1,0,0)} + 3(1, 1)_{(0,0,-1,0,1,0)} + 3(1, 1)_{(0,0,0,0,-1,1,0,0)} \]

\[ + 6(1, 2)_{(0,-1,0,1,0,0)} + 6(1, 2)_{(0,-1,0,1,0,0)} + 3(1, 1)_{(0,0,1,0,-1,0,0)} \]

We choose the hypercharge from anomaly-free realizations of U(1) as
\[ Q_Y = \frac{1}{3}Q_1 + \frac{1}{6}Q_2 + \frac{2}{3}Q_3 - \frac{1}{3}Q_4 - \frac{1}{3}(Q_5^{(1)} + Q_5^{(2)}) + Q_5^{(3)}. \]

With this hypercharge, the spectrum (11) become
\[ 3(3, 2)_{\frac{1}{3}} + 3(3, 1)_{\frac{1}{3}} + 3(3, 1)_{-\frac{2}{3}} + 12(1, 2)_{-\frac{1}{2}} + 9(1, 2)_{\frac{1}{2}} + 12(1, 1)_{1} + 9(1, 1)_{0} + 9(1, 1)_{-1}. \]

When we set \( n_2 + n_3 = -1 \), we get the same spectrum (12) by exchanging \( Q_3 \) and \( Q_4 \) in the hypercharge (11). For the case \( n_2 + n_3 \neq \pm 1 \), we have not been able to obtain the solution with the correct hypercharge \( Q_Y \).
When the \(a\)-th stack and \(b\)-th stack have the same CP phase, there are \(I_{ab}\) tachyon fields in the \(a-b\) sector. In these models, the 2-3 and 2-4 sectors give rise to tachyon fields that transform as the \(SU(2)\) doublet. This indicates that the configurations of branes that have \(U(2)\) and \(U(1)\) symmetries are unstable. This instability suggests a stringy Higgs mechanism of electroweak symmetry breaking.

For the D5-brane case, the fact that there are many wrapping numbers \((a_i)\) with \(I = 1, 2\) makes it difficult to obtain general solutions. For this reason, in this paper, we content ourselves with a single example. We will return to this problem at the end of the investigation of the unoriented theory.

We now discuss the unoriented theory. We introduce an orientifold group written as \(Z_N + \Omega R Z_N\), where \(\Omega\) is the world sheet parity. \(R\) transforms \(Y_I\) to \(Y_I\) for \(1 \leq I \leq d\) and \(Y_I\) to \(-Y_I\) for \(d + 1 \leq I \leq 3\). The space-time is expressed as

\[
\mathcal{M}_4 \times \prod_{I=1}^d T_2^I \times \frac{(\prod_{I=1}^3 T_2^I/Z_N)}{\Omega R}.
\]

We express the \(\Omega R\) action on the CP factors using unitary matrices \(\gamma_{\Omega R}\). Since \(\Omega R \theta (\Omega R)^{-1} = \theta^{2k}\), we have

\[
\gamma_{\Omega R k} = \gamma_k \gamma_{\Omega R} = \pm \gamma_{2k} \gamma_{\Omega R k}^T.
\]  

This leads to \(\gamma_{\Omega R} = \pm \gamma_{\Omega R}^T\). In the following equations, the upper and lower sign correspond to the symmetric and antisymmetric \(\gamma_{\Omega R}\), respectively.

In addition to the \(a-b\) sector, there are intersections of the \(a\)-th stack and mirror branes of the \(b\)-th stack, which we refer to as \(a-b^*\) sector. The intersection number \(I_{ab^*}\) is obtained by changing \(m_b(I)\) to \(-m_b(I)\) in the expression (1).

The \(Z_N\) action with even \(N\) contains an order 2 element, so that the orientifold group contains an element that does not vary \(Y_I (I \geq d + 1)\). Then there will be D8-branes in the type IIA D4-brane theory. Since the concept of intersecting D-branes involves use of the same dimensional D-branes, we restrict ourselves to the case that the order \(N\) of \(Z_N\) is odd.

Tadpole divergences come from Klein bottle and Möbius strip amplitudes as well as cylinder amplitudes. The RR tadpole cancellation conditions common to configurations with intersecting D4-branes and configurations with D5-branes are given by

\[
\frac{K}{a=1} N_a \prod_{I=1}^d n_a(I) = \pm 16,
\]

\[
\frac{K}{a=1} N_a \prod_{I=1}^d m_a(I) = 0,
\]

\[
\frac{K}{a=1} \left( \prod_{I=1}^d m_a(I) \right) \text{Tr} \gamma_k = 0 \quad \text{for} \quad k = 1, \cdots, N - 1.
\]

There is one more condition on the product of the cycles \(n_a(I)\) and the trace of \(\gamma_k\).
When we consider a $Z_3$ orbifold, it is given by

$$\sum_{a=1}^{K} n_a^{(1)} \text{Tr} \gamma_k = \pm 4 \quad \text{for } k = 1, 2,$$

(17)

for the D4-brane case and

$$\sum_{a=1}^{K} n_a^{(1)} n_a^{(2)} \text{Tr} \gamma_k = \mp 8 \quad \text{for } k = 1, 2,$$

(18)

for the D5-brane case. For other $Z_N$ models with odd $N$, the condition is expressed by a linear summation of $\prod_{j=d+1}^{3} \sin \pi k v_j$ and $\prod_{j=d+1}^{3} \sin 2\pi k v_j$ over $k = 1, \ldots, N-1$ similar to (4), and we could not find any solution to 1-cycles and $\gamma$ matrices.

In order to satisfy the conditions (14) and (17) for unoriented D4-brane configurations giving a standard-like model, we must introduce many $U(1)$ stacks or an $N_2 = 2$ stack that has a 1-cycle $n_2^{(1)} \neq 1$. Some of these $U(1)$ stacks and the $N_1 = 3$ stack always have intersections that lead to non-standard $(3, 1)$ or $(\overline{3}, 1)$ representations under $SU(3) \times SU(2)$. The 1-cycles $n_2^{(1)} \neq 1$ in the unoriented theory produce fields that transform as a three-dimensional representation under $U(2)$.\[\]

Thus we were not able to obtain the matter content of the standard model.

For models with intersecting D5-branes, left-handed fermions in the $a$-$b$ sector and $a$-$b^*$ sector are given by\[\]

$$I_{ab}(N_a^i, N_b^{-i+1} + (N_a^{-i}, N_b^{-i+1})),$$

(19)

$$I_{ab^*}(N_a^i, N_b^{-i-1} + (N_a^{-i}, N_b^{-i+1})),$$

(20)

under $U(N_a) \times U(N_b)$.

The bifundamental representations of (21) for the $a$-$a^*$ sector change to $N_a^i \wedge N_a^{-i-1}$ and $N_a^i \wedge N_a^{-i+1}$ under $U(N_a)$. On-orientifold intersections of the $a$-$a^*$ sector give $4m_a^{(1)} m_a^{(2)}$ fermions in the representation of either an antisymmetric or symmetric tensor, depending on whether $\gamma_{OR}$ is antisymmetric or symmetric. Off-orientifold intersections of the $a$-$a^*$ sector produce $2m_a^{(1)} m_a^{(2)} (n_a^{(1)} n_a^{(2)} - 1)$ symmetric and antisymmetric representations. We require that any brane stack satisfy $m_a^{(1)} m_a^{(2)} = 0$ to avoid the appearance of exotic quantum numbers and to satisfy the RR tadpole cancellation conditions (15) and (16). Adding D5-branes with $m^{(1)} = m^{(2)} = 0$ will satisfy the conditions (14) and (18) without modifying the fermionic matter content.

Since $I_{12} = -I_{12^*}$ under 1-cycles for which $m_a^{(1)} m_a^{(2)} = 0$, we must set $I_{12} = \pm 3$ and choose CP factors yielding no fermions in the 1-2* sector to get 3(3, 2) representations of $U(3) \times U(2)$. This configuration makes it impossible to obtain just the standard model spectrum without $U(1)$-$U(1)$-$U(1)$ anomalies.\[\]

An example is given in Table II for the case in which the orbifold group is $Z_3$.

The spectrum under the gauge symmetry $SU(3) \times SU(2) \times U(1)^5$ is given by

$$3(3, 2)(1, -1, 0, 0, 0) + 3(\overline{3}, 1)(-1, 0, 1, 0, 0) + 3(\overline{3}, 1)(-1, 0, 0, 1, 0) + 3(1, 2)(0, 1, 0, 0, -1) + 3(1, 1)(0, 0, -1, 0, 1) + 3(1, 1)(0, 0, 0, -1, 1).$$

(21)
The orbifold group is $\mathbb{Z}_3$ and the parameter $\zeta$ is $e^{2\pi i/3}$.

| $N_a$ | $(n_a^{(1)}, m_a^{(1)})$ | $(n_a^{(2)}, m_a^{(2)})$ | CP phase |
|-------|-----------------|-----------------|----------|
| $N_1 = 3$ | $(n_1^{(1)}, 0)$ | $(n_1^{(2)}, 3/n_1^{(1)})$ | $\zeta^2$ |
| $N_2 = 2$ | $(n_2^{(1)}, m_2^{(1)})$ | $(1/m_2^{(1)}, 0)$ | $\zeta$ |
| $N_3 = 1$ | $(n_3^{(1)}, m_3^{(1)})$ | $(-1/m_3^{(1)}, 0)$ | $\zeta$ |
| $N_4 = 1$ | $(n_4^{(1)}, m_4^{(1)})$ | $(-1/m_4^{(1)}, 0)$ | $\zeta$ |
| $N_5 = 1$ | $(n_5^{(1)}, 0)$ | $(n_5^{(2)}, -3/n_5^{(1)})$ | $\zeta^2$ |

Although the spectrum has a $U(1)$-$U(1)$-$U(1)$ anomaly and $G^2$-$U(1)$ mixed anomalies that must be removed by some mechanism, two anomaly-free $U(1)$ linear combinations exist. We can define the hypercharge as

$$Y = \frac{1}{6}Q_1 + \frac{1}{2}(Q_3 - Q_4 + Q_5).$$

and obtain identically the standard model spectrum under $SU(3) \times SU(2) \times U(1)_Y$ gauge symmetry with three generations of right-handed neutrino.

We now consider the construction of D5-branes in the oriented theory. In this case, any matter fermions for the unoriented theory are obtained from the $a$-$b$ sector, not from the $a$-$b^*$ sector. Adding an appropriate number of $U(1)$ branes to the stacks in Table II, we obtain D5-branes for the oriented theory that satisfy the conditions on the 1-cycles and CP phases (3)–(5).

Configurations with intersecting D-branes cause the breakdown of supersymmetry. Then the string scale must be close to the weak scale to avoid the fine-tuning problem. The four dimensional Plank mass $M_p$ and string scale $M_s$ are related as

$$M_p \approx 4\pi^2 M_s^4 \sqrt{V_T V_{Z_N}/\lambda_{II}},$$

where $\lambda_{II}$ is the Type II string coupling, $V_T$ is the volume of the tori, and $V_{Z_N}$ is the volume of the orbifold. While a very large value of $V_T$ leads to small Yukawa and gauge couplings, we can give a very large volume to the transverse $Z_N$ orbifold. For models of D5-branes, for example, we can set $M_s \approx 1 \sim 10$ TeV and $V_T \sim 1/M_s^2$. In order to obtain very large value of the Plank mass, we should choose $\sqrt{V_{Z_N}} \approx 10^9 - 10^{11}$ (GeV)$^{-1}$, i.e., $10^{-2} - 10^{-4}$ cm.

In this paper, we have not investigated all of the standard-like spectra that accompany intersecting D5-branes. In particular, unoriented models may be obtained from 1-cycles for which $m_{a}^{(1)}m_{a}^{(2)} \neq 0$, $n_{a}^{(1)}n_{a}^{(2)} = 1$. In such models, the $a-a^*$ sector may produce the antisymmetric tensors of $N_a \wedge N_a$ under $U(N_a)$ for $N_a = 2, 3$ without symmetric tensors that are not present in the matter content of the standard model.

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