Sensitivity to new supersymmetric thresholds through flavour and $CP$ violating physics

Maxim Pospelov$^{(a,b)}$, Adam Ritz$^{(a)}$ and Yudi Santoso$^{(a)}$

$^{(a)}$Department of Physics and Astronomy, University of Victoria, Victoria, BC, V8P 1A1 Canada
$^{(b)}$Perimeter Institute for Theoretical Physics, Waterloo, ON, N2J 2W9, Canada

Abstract

Treating the MSSM as an effective theory below a threshold scale $\Lambda$, we study the consequences of having dimension-five operators in the superpotential for flavour and $CP$-violating processes. Below the supersymmetric threshold such terms generate flavour changing and/or $CP$-odd effective operators of dimension six composed from the Standard Model fermions, that have the interesting property of decoupling linearly with the threshold scale, i.e. as $1/(\Lambda m_{\text{soft}})$, where $m_{\text{soft}}$ is the scale of soft supersymmetry breaking. The assumption of weak-scale supersymmetry, together with the stringent limits on electric dipole moments and lepton flavour-violating processes, then provides sensitivity to $\Lambda$ as high as $10^7 - 10^9$ GeV. We discuss the varying sensitivity to these scales within several MSSM benchmark scenarios and also outline the classes of UV physics which could generate these operators.

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1 Introduction

Weak-scale supersymmetry (SUSY) is a theoretical framework that helps to soften the so-called gauge hierarchy problem by removing the power-like sensitivity of the dimensionful parameters in the Higgs potential to the square of the ultraviolet cutoff $\Lambda$. This feature, among others, has stimulated a large body of theoretical work on weak-scale supersymmetry, supplemented by continuing experimental searches, which now spans almost three decades. Yet the supersymmetrized version of the Standard Model (SM), the minimal supersymmetric Standard Model (MSSM), suffers from well known problems such as the large array of allowed free parameters responsible for soft SUSY breaking, and the consequent possibility of large flavour and $CP$ violating amplitudes. The absence of $CP$-violation at the $O(1)$ level in the soft-breaking sector of the MSSM, as suggested by the null results of electric dipole moment (EDM) searches and the perfect accord of the observed $K$ and $B$ meson mixing and decay with the predictions of the SM, implies that the soft-breaking sector of the MSSM somehow conserves $CP$ and does not source new flavour-changing processes. Whether or not such a pattern of soft-breaking masses is theoretically feasible is the subject of on-going studies addressing the mechanism of SUSY breaking and mediation (see, e.g. [1]). In this work, we will make the assumption that an (approximately) flavour-universal and $CP$-conserving soft-breaking sector is realized, and study the consequences of the presence of SUSY-preserving higher-dimensional operators on flavour and $CP$-violating observables.

These operators may be thought to emerge from new physics at some high-energy scale $\Lambda$, which is larger than the electroweak scale. Even though the field content of the MSSM may be perfectly ‘complete’ at the electroweak scale, it is clear that almost by construction the MSSM cannot be a fundamental theory because of the required high-energy physics responsible for SUSY breaking and mediation. In recent years there is also a more phenomenological motivation for a new threshold, namely the new physics responsible for neutrino masses (assuming they are Majorana) and mixings. Beyond these primary concerns, the possibility of new thresholds, intermediate between the weak and the GUT scales, is also suggested by the axion solution to the strong $CP$ problem, by the SUSY leptogenesis scenarios [2] and, more entertainingly, by the possibility of a lowered GUT/string scale arising from the large radius compactification of extra dimensions [3]. In summary, given the assumed existence of weak-scale supersymmetry, there seems ample motivation to expect additional new physics thresholds above the electroweak scale and possibly below the GUT scale. The presence of such thresholds will generically be manifest not just through corrections to relevant and marginal operators, but also through the presence of higher-dimensional operators.

As is easy to see, both Kähler terms and the superpotential can receive additional non-renormalizable terms at the leading dimension five level [4, 5]. Some of these operators are well-known and were studied in connection with baryon-number violating processes and also the see-saw mechanism for neutrino masses. However, to the best of our knowledge, an analysis of the full set of dimension-five operators with respect to flavour and $CP$-violating...
observables is still lacking. The purpose of this paper is thus to consider all possible
dimension five extensions of the MSSM superpotential and Kähler terms, concentrating on
those that conserve lepton and baryon number and are $R$-parity symmetric. We initiated
such a study recently [7], and will provide further details and extensions in the present work.
As we shall see, such operators can induce large corrections to flavour-changing and/or $CP$-
violating amplitudes and therefore can be efficiently probed with existing experiments and
future searches.

There is a clear parametric distinction between the effects induced by nonuniversal soft-
breaking terms and by the higher-dimensional extensions of the superpotential. Whereas
the former typically scale as $m_{\text{soft}}^{-2}$ times one or two powers of the flavour-mixing angle $\delta_{ij}$
in the squark(slepton) sector, the latter decouple as $(\Lambda m_{\text{soft}})^{-1}\delta_{ij}'$, where $\delta_{ij}'$ parametrizes
flavour violation in the dimension five operators. When $\Lambda$ is relatively large, and thus the
threshold corrections to the soft-terms are small, we may have scenarios where $\delta_{ij} \simeq 0$
while $\delta_{ij}'$ are significant, and the corrections to the superpotential can be the dominant
mechanism for SUSY flavour and $CP$ violation, providing considerable sensitivity to $\Lambda$.
At the same time, the additional $CP$ and flavour violation introduced in this way can be
rendered harmless by simply increasing $\Lambda$.

The layout of the paper is as follows. In the next section we list the possible operators
in the MSSM superpotential and the Kähler terms at dimension five level, including for
completeness those that violate $R$-parity. The relevant supersymmetric renormalization
group equations for the operators of interest are included in an Appendix. In section 3,
we perform the required calculations at the SUSY threshold to connect this extension of
the superpotential with the resulting Wilson coefficients in front of various effective SM
operators of phenomenological interest. Section 4 addresses the consequent predictions for
the most sensitive $CP$-odd and flavour-violating amplitudes and infers the characteristic
sensitivity to $\Lambda$ in each channel. In section 5, we perform this analysis within four SPS
benchmark scenarios [8] (see also [9]) in order to infer the dependence of this sensitivity on
the features of the SUSY spectrum. Section 6 contains a discussion and also a brief analysis
of the general classes of new physics which could be responsible for these operators, while
our conclusions are summarized in section 7.

## 2 Dimension-5 operators in the MSSM

In this section, we will enumerate all the allowed structures in the superpotential and Kähler
potential up to dimension 5 according to the standard symmetries of the MSSM (see, e.g.
[4, 5]). We begin by recalling in Table 1 the chiral superfields of the MSSM [10] along with
their gauge quantum numbers.

The matter parity, $P_M$, is defined in the usual way,

$$P_M \equiv (-1)^{3(B-L)}$$

(1)

where $B$ is the baryon number and $L$ the lepton number. This can be restated as $R$-parity,
defined as
\[ P_R = (-1)^{3(B-L)+2s} \] (2)

where \( s \) is the spin of the component field. All known Standard Model particles have \( P_R = +1 \), while their superpartners have \( P_R = -1 \). However, when using the superfield formalism it is often more convenient to use matter parity in which all fields belonging to the same superfield have the same value of \( P_M \).

The superpotential of the MSSM contains a number of dimensionless parameters, and one dimensionful parameter \( \mu \) or, in equivalent language, is composed from one dimension three and several dimension four operators:
\[
W^{(3)} = -\mu H_d H_u \\
W^{(4)} = Y_u UQH_u - Y_d DQH_d - Y_e ELH_d,
\]
where gauge and generation indices are suppressed. All these terms conserve \( R \)-parity. In counting the dimensions, one should keep in mind that we are implicitly including \( \dim[d^2 \theta] = 1 \). At the renormalizable level, there are additional terms that are forbidden by matter/\( R \) parity but allowed by gauge invariance,
\[
W_R^{(3)} = -\mu' LH_u \\
W_R^{(4)} = \lambda LLE + \lambda' LQD + \lambda'' UDD.
\]

Going beyond the renormalizable level, at dimension-five there are a number of operators allowed by symmetries. It is worth recalling that in the Standard Model, above the electroweak scale, there is only a single class of dimension-five operators allowed by symmetries – the seesaw operator – which can naturally provide a small Majorana mass for the active neutrinos. Within the MSSM, the list is only slightly longer. Suppressing a variety of gauge and generational indices, the collection of dimension five operators can be presented in the following schematic form:
\[
W^{(5)} = c_{qq} QUQD + c_{qe} QULE + c_{hb} H_u H_d H_u H_d \\
+ c_{ho} H_u LH_u L + c_{p1} UUDE + c_{p2} QQQL.
\]
The final two terms in this list violate baryon and lepton number by one unit, and therefore induce proton decay. Detailed studies of these operators induced by triplet Higgs exchange have been conducted over the years in the context of SUSY GUT models \[4, 5\] (for a recent assessment, see \[6\]). The \(H_uLH_dL\) operator is the superfield generalization of the SM see-saw operator and can be responsible for generating the neutrino masses and mixings. Assuming neutrinos are Majorana, the flavour structure of \(c_\nu\) is currently being determined in neutrino physics experiments (see \(e.g. \[11\]\).

Going over to \(R\)-parity violating terms (see \(e.g. \[12\]\)), one finds additional dimension five operators,

\[
W_R^{(5)} = c_1^R QU H_d E + c_2^R H_u H_d H_u L + c_3^R QQQH_d,
\]  

(8)

that can be obtained from (7) upon the simple substitution \(L \leftrightarrow H_d\).

If we now consider the Kähler potential, it is easy to see that at dimension four one has the standard \(\Phi^\dagger e^V \Phi\) operators, where \(\Phi\) represents a generic MSSM chiral superfield, and the additional dimension four operators \(Le^V H_d^\dagger\) that violate \(R\) parity. In all cases, \(V\) should be chosen as the correct linear combination of individual vector superfields to insure gauge invariance. At dimension five level, we have three additional structures that are allowed by all gauge symmetries and \(R\)-parity,

\[
K^{(5)} = c_u QU H_d^\dagger + c_d QDH_u^\dagger + c_e LE H_u^\dagger,
\]  

(9)

and several further operators that violate \(R\)-parity,

\[
K_R^{(5)} = c_{K1}^R EH_d H_u^\dagger + c_{K2}^R QUL^\dagger + c_{K3}^R UED^\dagger + c_{K4}^R QOD^\dagger.
\]  

(10)

At this point, it is important to recall that the equations of motion can be utilized within the effective Lagrangian to remove various redundancies in the full set of higher-dimensional operators listed above. We will work with tree-level matching at the \(\Lambda\)-threshold and thus, if one leaves aside corrections from SUSY breaking, all the structures in \(K^{(5)}\) can be reduced on the superfield equations of motion and absorbed into \(W^{(4)}\) and \(W^{(5)}\). Indeed, in the limit of exact SUSY, the superfield equation of motion for \(e.g. H_d^\dagger\) reads

\[
\bar{D}^2 H_d^\dagger \propto -\mu H_d + Y_u QU,
\]  

(11)

where \(\bar{D}\) is the spinorial derivative. Substituting this into the expression for \(K^{(5)}\), we observe that the operator \(LEH_u^\dagger\), for example, reduces to a linear combination of the usual Yukawa structure with \(\Delta Y_e = \mu c_e\) and the dimension-five superpotential term:

\[
\int d^4 \theta c_e LEH_u^\dagger \propto \int d^2 \theta c_e LE \bar{D}^2 H_u^\dagger \propto \int d^2 \theta (-c_e \mu LEH_d + c_e Y_u QUL E).
\]  

(12)

The inclusion of soft SUSY breaking terms in the equation of motion would change this analysis only slightly; new soft-breaking structures such as dimension-four four-sfermion interactions \(\bar{Q}UL E\) and new trilinear terms such as \(\bar{L} EH_u^\dagger\) \([13]\) would appear. Since the analysis of higher-dimensional soft-breaking terms goes beyond the scope of the present
paper, we choose to eliminate all Kähler higher dimensional terms via the equations of motion and analyze only the corrections to superpotential.

Comparing the $H_u L H_u L$-induced neutrino masses to the characteristic mass splitting $\sim (0.01 - 0.1) \text{eV}$ observed in neutrino oscillations, we deduce the corresponding range of the energy scales $\Lambda_\nu$:

$$ (0.01 - 0.1) \text{eV} \sim c_\nu \langle H_u^2 \rangle \quad \Rightarrow \quad \Lambda_\nu \sim c_\nu^{-1} \sim (10^{14} - 10^{16}) \text{GeV}. \quad (13) $$

The actual mass scale of the new states responsible for generating the effective term $H_u L H_u L$ can be lower than $\Lambda_\nu$. Indeed, in the see-saw scheme $c_\nu = Y_\nu^2 M_R^{-1}$, and the mass of the right-handed neutrinos $M_R$ can be smaller than $\Lambda_\nu$ if $Y_\nu$ is small. A considerably smaller energy scale for $M_R$ than $10^{14}$ GeV is indeed suggested by SUSY leptogenesis scenarios [2].

The mediation of proton decay by the $QQQL$ and $DUUE$ operators has been extensively studied over more than two decades in the context of SUSY GUT models. Typically, such operators are induced by the exchange of a colour-triplet Higgs superfield, and therefore the operators are proportional to the square of the Yukawa couplings. For this study, we will not go into the details of how such terms were generated, and simply deduce the sensitivity to $c_{p1}$ and $c_{p2}$. The absence of proton-decay at the level of $\Gamma^{-1} > 10^{32}$yr implies a rather stringent upper bound on the baryon and lepton number violating couplings $c_p$,

$$ \Lambda_p \sim c_p^{-1} > 10^{24} \text{GeV}, \quad (14) $$

which is well above the scale of quantum gravity, $10^{19}$GeV. The discrepancy in the scales $\Lambda_p$ and $\Lambda_\nu$ is somewhat problematic for SUSY GUTs, and is part of the doublet-triplet splitting problem. In any event, the disparity between $\Lambda_p$ and $\Lambda_\nu$ clearly illustrates the fact that the energy scales associated with the effective operators in (11) could be widely different, and thus motivates a dedicated study to determine the sensitivity to $c_{qq}$, $c_{qe}$ and $c_h$.

3 Induced operators at the SUSY threshold

We begin our analysis by making explicit the colour and flavour structure of the new dimension five operators. It is easy to see that the $SU(2)$ indices can be contracted in only one way, via the antisymmetric tensor $\epsilon_{ij}$. Therefore, we suppress these indices in the expression below:

$$ W = W_{\text{MSSM}} + \frac{y_h}{\Lambda_h} H_d H_u H_u H_d + \frac{Y_{ijkl}^{qe}}{\Lambda_{qe}} (U_i Q_j) E_k L_l 
+ \frac{Y_{ijkl}^{qq}}{\Lambda_{qq}} (U_i Q_j) (D_k Q_l) + \frac{\tilde{Y}_{ijkl}^{qq}}{\Lambda_{qq}} (U_i t^A Q_j) (D_k t^A Q_l). \quad (15) $$

Here $y_h$, $Y_{qe}$, $Y_{qq}$ and $\tilde{Y}_{qq}$ are dimensionless coefficients with the latter three also being tensors in flavour space, while the $\Lambda$’s are the corresponding energy scales. The parentheses (...) in
\[ (Q \ell^A U)(Q \ell^A D) \] reduces to \((QU)(QD)\) upon the use of the completeness relation for the generators of \(SU(3)\).

From the superfield formulation of Eq. \ref{eq:15}, one can easily move to the component form using the standard rules of supersymmetric field theories. However, the full interaction Lagrangian resulting from \ref{eq:15} is quite cumbersome, and we will quote only those terms that are \(\sim 1/\Lambda\) and of potential phenomenological importance, namely the terms in the Lagrangian that involve two SM fermions and two sfermions. As an example, the \(QULE\) operator in the superpotential generates the following semi-leptonic two fermion - two boson interaction terms:

\[
\int d^2 \theta \, QULE \supset \bar{U}Q \tilde{L} \tilde{E}^- + \bar{E} \tilde{Q} \tilde{L} - \bar{U}L \tilde{Q} \tilde{E}^+ + \bar{Q} \tilde{U} \tilde{E}^+ + \bar{E} \tilde{Q} \tilde{L} \tilde{U}^+.
\]  

(16)

In this expression, letters with a tilde atop denote sfermions, and four-dimensional spinors are used for fermions with \(Q\) being the left-handed quark doublet and \(Q^c\) its charge conjugate, etc. The generalization of \ref{eq:16} to the rest of the operators in \ref{eq:15} is straightforward.

At the next step, we integrate out the squarks and sleptons to obtain operators composed from SM fields alone, or to be more precise, from the fields of a type II two-Higgs doublet model. This procedure is facilitated by the observation that the first two terms in \ref{eq:16} have a close resemblance to the LR squark and slepton mixing terms, with the only difference being that instead of the usual \(m_{e(u)}\) \(\tan \pm 1/\beta\) and/or \(A_{u(e)}m_{u(e)}\) mixing coefficients one has dimension three fermion bilinear insertions \(\bar{U}Q\) and \(\bar{E}L\). It is then clear that \(\tilde{L} \tilde{E}^+\) and \(\tilde{Q} \tilde{U}^+\) can be integrated out straightforwardly encountering loop integrals that are common in the MSSM literature. Notice that only in the first two terms in \ref{eq:16} can the sfermions be integrated out at one loop, as the remaining terms contain a slepton and a squark, and so integrating them out requires at least two loops.

### 3.1 Corrections to the SM fermion masses

The SM operators of lowest dimension that are of phenomenological interest are the fermion masses. In Figure 1, we show the one-loop diagrams that lead to the logarithmic renormalization of the fermion masses. Cutting the ultraviolet divergence at the corresponding threshold \(\Lambda\), we arrive at the following expression for fermion masses corrected by the dimension five operators:

\[
(M_e)_{ij} = (M_e^{(0)})_{ij} + Y_{e_{kij}}^{qe}(M_u^{(0)})^*_{kl} \frac{3 \ln(\Lambda_{ge}/m_{sq})}{8\pi^2\Lambda_{ge}} (A^*_u + \mu \cot \beta) \\
(M_d)_{ij} = (M_d^{(0)})_{ij} + K_{kij}^{qq}(M_u^{(0)})^*_{kl} \frac{\ln(\Lambda_{qq}/m_{sq})}{4\pi^2\Lambda_{qq}} (A^*_u + \mu \cot \beta) \\
(M_u)_{ij} = (M_u^{(0)})_{ij} + Y_{i_{kji}}^{qe}(M_e^{(0)})_{kl} \frac{\ln(\Lambda_{ge}/m_{sl})}{8\pi^2\Lambda_{ge}} (A_e + \mu \tan \beta) \\
+ K_{ijkl}(M_u^{(0)})^*_{kl} \frac{\ln(\Lambda_{qq}/m_{sq})}{4\pi^2\Lambda_{qq}} (A^*_u + \mu \cot \beta)
\]  

(17)
with implicit summation over the repeated flavour indices, and we have also defined the combination,

\[ K^{qq} \equiv Y^{qq} - \frac{2}{3} \tilde{Y}^{qq}, \]  

that will reappear again below. \( M^{(0)}_{e,d,u} \) denote the unperturbed mass matrices arising from dimension four terms in the superpotential.

Some of the mass corrections in (17) correspond to new “non-holomorphic” operators such as \( \bar{U}QH_d^\dagger \), which break supersymmetry, and scale as \( \Delta m/m \sim (A/\Lambda) \times \log \Lambda \). The other set of corrections survive in the limit of unbroken SUSY, scaling as \( \Delta m/m \sim (\mu/\Lambda) \times \log \Lambda \). This is a correction to the standard mass term in the superpotential, \( UQH_u \) generated by the dimension five operator. Given the non-renormalization theorem for the superpotential \([14]\), it may look surprising that such corrections could arise at all. A more careful look at the diagram in Fig. 1 reveals that it is the dimension five Kähler term \( QUH_u^\dagger \) that receives a logarithmic loop correction, leading to the quark mass correction in (17) upon the use of the equation of motion \([12]\).

3.2 Dipole operators

Dimension five dipole operators first arise at two-loop order via integrating out the heavy-flavour squarks from \( \bar{E}L\tilde{U}^*\bar{Q} \), as in Fig. 2. The results for these diagrams can be deduced from the calculations of the two-loop Barr-Zee-type supersymmetric diagrams in the limit of large pseudoscalar mass \([15]\). In the charged lepton sector they result in

\[ \mathcal{L}_e = \frac{A_u + \mu \cot \beta}{\Lambda^{q_e} m_{sq}^2} \frac{e\alpha}{12\pi^3} (M_u)_{kl}^* Y^{qe}_{kl} \bar{E}_i (F\sigma) P_L E_j + (h.c.), \]  

where we treated \( LR \) squark mixing as a mass insertion, and used \( P_L = \frac{1-\gamma_5}{2} \) and \( (F\sigma) = F_{\mu\nu}\sigma^{\mu\nu} \). In the quark sector the corresponding results are more cumbersome to write down due to a large number of possible diagrams.
Figure 2: A representative of the two-loop SUSY threshold diagrams that generate dipole amplitudes and contribute to EDMs, $\mu \rightarrow e\gamma$, the anomalous magnetic moment of the muon, etc.

### 3.3 Semileptonic four-fermion operators

Going up another dimension, we now consider dimension-six four-fermion operators composed from the SM fermion fields generated by the operators [15]. Two representatives of the relevant one-loop diagrams are shown in Figure 3. The loop functions entering these calculations are identical to those found in the calculation of the corrections to the SM fermion masses arising from the SUSY threshold [16]. We will generalize the results of [7] by working with the full loop function [16],

$$I(x, y, z) = \frac{-xy\ln(x/y) + yz\ln(y/z) + zx\ln(z/x)}{(x - y)(y - z)(z - x)},$$

which satisfies

$$I(z, z, z) = \frac{1}{2z},$$

allowing us to consider several benchmark SUSY spectra later on. All the SUSY masses, $m_{sq}$, $m_{sl}$, $M_i$ and the $\mu$ parameter are considered to be somewhat larger than $M_W$, so that the effects of gaugino-Higgsino mixing in the chargino and neutralino sector are not particularly important for the values of the loop integrals.

Integrating out gauginos and sfermions at one-loop level, we find the following semileptonic operators, sourced by the $QULE$ term in the superpotential,

$$\mathcal{L}_{qe} = \frac{1}{\Lambda_{qe}} \left[ \frac{2\alpha_s}{3\pi} M_3^2 I(m_{u1}^2, m_{u2}^2, |M_3|^2) - \frac{\alpha_1}{4\pi} M_1^2 I(m_{e1}^2, m_{e2}^2, |M_1|^2) \right] Y^{qef} U_i Q_j \bar{E}_k L_l + (h.c.).$$

In this expression, we retained the gluino-squark contribution as the largest in the squark sector and the sfermion-bino contribution in the lepton sector. If all SUSY masses are approximately the same, then the second term in the square bracket of Eq. (22) is subdominant, but this may not be the case if the masses in the slepton-bino sector are significantly lighter than in the squark-gluino sector. Notice the absence of contributions from $SU(2)$ gauginos, that turn out to be suppressed by additional power(s) of $M_W/m_{soft}$. Finally, as expected the overall coefficient in front of the semileptonic operator (22) scales as $(\Lambda m_{soft})^{-1}$. 

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Figure 3: One-loop SUSY threshold diagrams that generate dimension six four-fermion operators composed from the SM fields. Diagram (a) is a squark-gluino loop giving rise to a semi-leptonic operator, and diagram (b) is a squark-Higgsino loop leading to a four-fermion operator in the down-quark sector.

3.4 Four-quark operators

Purely hadronic operators in Eq. (15) give rise to the following four-quark effective operators upon integrating out gluinos and squarks:

\[ \mathcal{L}_{qq} = \frac{1}{\Lambda_{qq}} \frac{\alpha_s}{6\pi} M^s_3 I(m^2_{\tilde{q}_1}, m^2_{\tilde{q}_2}, |M_3|^2) \]

\[ \times K^{qq} \left( \frac{8}{3} (\bar{U}Q)(\bar{D}Q) + (\bar{U}t^AQ)(\bar{D}t^AQ) \right) + (h.c.), \]  

where the summation over flavour is carried out exactly as in Eq. (15).

It is well-known that the strongest constraints on FCNC in the quark sector often arise from \( \Delta F = 2 \) amplitudes in the down-squark sector that contribute to the mixing of neutral \( K \) and \( B \) mesons. It is easy to see that such amplitudes are not present in Eq. (23) where two of the quarks are always of the up-type. Of course, they can be converted to down-type quarks at the expense of an additional loop with \( W \)-bosons, but this introduces an additional numerical suppression. In any event, the conversion of the right-handed quark field \( U \) into a \( D \) field would necessarily require additional Yukawa suppression. There is, however, a more-direct one-loop SUSY threshold diagram that can give rise to \( \Delta F = 2 \) amplitudes in the down-quark sector. As shown in Figure 3b, it consists of a Higgsino–up-squark loop. The result for this diagram,

\[ \mathcal{L}_{dd} = \frac{1}{\Lambda_{qe}} \frac{1}{8\pi^2} \mu^s I(m^2_{\tilde{u}_1}, m^2_{\tilde{u}_2}, |\mu|^2)(Y_u^*)_{ijm}(Y_d^*)_{nj} \]

\[ \times K^{qq}_{ijkl} \left[ \frac{1}{3} (\bar{Q}_m D_n)(\bar{D}_k Q_l) - (\bar{Q}_m t^A D_n)(\bar{D}_k t^A Q_l) \right] + (h.c.), \]  

inevitably contains additional suppression by the Yukawa couplings of the up and down type quarks originating from the Higgsino-fermion-sfermion vertices.

Notice that in the limit \( \mu \gg m_{sq} \) the overall coefficient in equation (24) scales as \((\Lambda\mu)^{-1}\log(\mu/m_{sq})\) and thus has only mild dependence on the soft-breaking scale. In this
case, the operator \( \mathbb{Q} \mathbb{D}^\dagger \mathbb{D} \mathbb{Q} \) must have an explicit superfield generalization. Indeed, it is easy to see that in this limit \( \mathbb{Q} \mathbb{Q} \mathbb{Q} \mathbb{Q} \mathbb{Q} \) corresponds to a dimension six Kähler term: \( Q^i D D^j Q \).

### 3.5 Modifications to the Higgs sector and sparticle spectrum

Thus far, we have not considered the consequences of the presence of the first operator in \( \mathbb{Q} \mathbb{Q} \mathbb{Q} \mathbb{Q} \mathbb{Q} \), which consists entirely of Higgs superfields. Its most obvious implication is a modification of the Higgs potential and the sparticle spectrum. The addition to the Higgs potential, linear in \( y_h \), has a simple form:

\[
\Delta V_h = -\frac{2\mu^* y_h}{\Lambda_h} \left[ (H_u^\dagger H_u) + (H_d^\dagger H_d) \right] (H_d H_u) + (h.c.).
\]  

(25)

If \( \mu^* y_h \) has a cumulative phase, this would create mixing between \( A \) and the \( h, H \) bosons that violate \( CP \) symmetry. However, its most important consequence for our study here will be an induced complex shift of the bilinear soft parameter \( m_{12}^2 \), which enters one-loop contributions for fermion EDMs.

The mixing of left- and right-handed sfermions is also affected by this term. In addition to the usual \( \mu \) or \( A \)-proportional mixing, we have the following contribution to the mixing matrix element of \( \tilde{u}_L \) and \( \tilde{u}_R \):

\[
\delta (M_{uu}^2)_{LR} = -m_u \frac{y_h v_{SM}^2}{\Lambda_h} \cos^2 \beta,
\]

(26)

and analogous formulae for \( \tilde{e} \) and \( \tilde{d} \) with the \( \cos \beta \to \sin \beta \) substitution. In this expression, \( v_{SM}^2 = 4M_W^2/g_W^2 \) corresponds to the SM Higgs v.e.v.

The neutralino mass matrix also receives two new (complex) entries, i.e. Majorana masses for the neutral components of \( \tilde{H}_u \) and \( \tilde{H}_d \), proportional to \( y_h \lambda_h^{-1} v_{SM}^2 \cos^2 \beta \) and \( y_h \lambda_h^{-1} v_{SM}^2 \sin^2 \beta \) respectively.

### 4 Phenomenological consequences and sensitivity to \( \Lambda \)

In this section, we estimate the sensitivity of various experimental searches to the energy scales \( \Lambda^{qe} \) and \( \Lambda^{qq} \). Of course, one of the most important issues is then the assumed flavour structure of the new couplings \( Y^{qe} \), \( Y^{qq} \) and \( \tilde{Y}^{qq} \). Since we are thinking of \( \Lambda \) as an intermediate scale and wish to explore the full reach of precision measurements, we will make the generous assumption that these coefficients are complex, of order one, and do not factorize into products of Yukawa matrices in the superpotential: \( Y^{qe} \neq Y_u Y_e \). It is clear that a much more restrictive assumption, e.g. minimal flavour violation, would dramatically reduce the sensitivity to these operators, but we will not explore this option here.
4.1 Naturalness bounds – fermion masses and the $\theta$-term

With the above assumption on flavour structure, we should first investigate the requirement that the corrections to masses of the SM fermions do not exceed their measured values, as otherwise we will face a new fine-tuning problem in the flavour sector. Taking $(M_u A_u)_{kl} = (M_u A_u)_{33}$ $\sim$ $m_t A_t$ $\sim$ 175 GeV $\times$ 300 GeV and using the expression for $\Delta m_e$ in (17), we arrive at the following estimate,

$$\Delta m_e \sim \frac{3m_t A_t}{8\pi^2 \Lambda_{qe}^2} \ln \left( \frac{\Lambda_{qe}}{m_{sq}} \right) \sim 1\text{MeV} \times \frac{10^7 \text{GeV}}{\Lambda_{qe}},$$

which clearly suggests that the ‘naturalness’ scale for the new physics encoded in semileptonic dimension five operators in the superpotential is on the order of $10^7$ GeV, while the analogous sensitivity in the squark sector is slightly lower, $\Lambda_{qq} \sim 10^6$ GeV. This high naturalness scale is simply a restatement of the small Yukawa couplings for the light SM fermions. However, perhaps surprisingly, we will see below that this sensitivity is not the dominant constraint on the threshold scale.

Before we proceed to estimate the effects induced by four-fermion operators, we would like to consider the effective shift of the QCD $\theta$-angle due to the mass corrections (17). Assuming an arbitrary overall phase for the $Y^{qq}$ matrices relative to the phases of the eigenvalues of $Y_u$ and $Y_d$, one typically finds the following shift of the $\bar{\theta}$ parameter,

$$\Delta \bar{\theta} \sim \frac{\text{Im}(m_d)}{m_d} \sim \frac{\text{Im}(Y^{qq} m_t A_t)}{4\pi^2 m_d \Lambda^{qq}} \ln \left( \frac{\Lambda_{qe}}{m_{sq}} \right) \sim 10^{-10} \times \frac{10^{17} \text{GeV}}{\Lambda^{qq}}.$$ 

This is a remarkable sensitivity of $\Delta \bar{\theta}$ to new sources of CP and flavour violation, and can translate into a strong bound on $\Lambda^{qq}$ depending on how the strong CP problem is addressed. If it is solved by an axion, there are no consequences of (28). However, in other possible approaches $\bar{\theta} \simeq 0$ is engineered by hand, using e.g. discrete symmetries at high energies [17]. In this case, dimension five operators can pose a potential threat up to the energies $\Lambda^{qq} \sim 10^{17}$ GeV, which by itself is very remarkable. Future progress in measuring the electric dipole moments (EDMs) of neutrons and heavy atoms [18], can clearly bring this scale up to the Planck scale and beyond.

4.2 Electric dipole moments from four-fermion operators

Electric dipole moments (EDMs) of the neutron [19] and heavy atoms and molecules [20, 21, 22, 23] are primary observables in probing for sources of flavour-neutral CP violation. The high degree of precision with which various experiments have put limits on possible EDMs translates into stringent constraints on a variety of extensions of the Standard Model at and above the electroweak scale (see, e.g. [24, 18]). Currently, the strongest constraints on CP-violating parameters arise from the atomic EDMs of thallium.
and mercury [21], and that of the neutron [19]:

\begin{align*}
|d_{\text{Tl}}| &< 9 \times 10^{-25} \text{e cm} \\
|d_{\text{Hg}}| &< 2 \times 10^{-28} \text{e cm} \\
|d_n| &< 3 \times 10^{-26} \text{e cm}
\end{align*}

(29)

When \( \theta \) is removed by an appropriate symmetry, the EDMs are mediated by higher-dimensional operators. Both (22) and (23) are capable of inducing the atomic/nuclear EDMs if the overall coefficients contain an extra phase relative to the quark masses. Restricting Eq. (15) to the first generation and dropping the \( U(1) \) contribution, we find the following CP-odd operator:

\begin{equation}
L_{\text{CP-odd}} = -\frac{1}{\Lambda_{qe}} \frac{\alpha_s}{3\pi} |M_3 Y^{uee}| I(m_{u1}^2, m_{u2}^2, |M_3|^2) \sin \delta \times [(\bar{u}u)(\bar{e}i\gamma_5 e) + (\bar{u}i\gamma_5 u)(\bar{e}e)],
\end{equation}

(30)

with the CP-violating phase \( \delta = \text{Arg}(M_3 Y^{uee}) \) in a basis with real \( m_e \) and \( m_u \). Taking into account the QCD running from the superpartner mass scale to 1 GeV, and upon the use of hadronic matrix elements over nucleon states, \( \langle N|\bar{u}u|N \rangle \) and \( \langle N|\bar{u}i\gamma_5 u|N \rangle \), we can make a connection to the \( C_S \) and \( C_P \) coefficients in the effective CP-odd electron-nucleon Lagrangian,

\begin{equation}
L = C_S \bar{N}N\bar{e}i\gamma_5 e + C_P \bar{N}N\bar{e}i\gamma_5 N\bar{e}.
\end{equation}

(31)

The isospin singlet part of the \( C_S \) coefficient is given by

\begin{align*}
C_S &= -\frac{4\alpha_s}{3\pi\Lambda_{qe}} \text{Im}(M_3^* Y^{uee}) I(m_{u1}^2, m_{u2}^2, |M_3|^2) \\
&\sim 2 \times 10^{-4} (1 \text{ GeV} \times \Lambda_{qe})^{-1},
\end{align*}

(32)

where in the latter equality we also assumed maximal violation of \( CP \), \( |Y^{uee}|(\Lambda_2) \sim \sin \delta \sim O(1) \), and chose the superpartner masses degenerate at 300 GeV. The quark matrix element, \( \langle N|\bar{u}u + \bar{d}d|2/N \rangle \sim 4 \), is in accord with standard values for the quark masses and the nucleon \( \sigma \)-term.

Using the same assumptions, and the pseudoscalar matrix element over the neutron, \( \langle n|\bar{u}i\gamma_5 u|n \rangle \sim -0.4(m_N/m_u)\bar{n}i\gamma_5 n \), we obtain a similar expression for the neutron \( C_P \) coefficient,

\begin{align*}
C_P &= \frac{\alpha_s}{6\pi\Lambda_{qe}} \left( \frac{0.4 m_n}{m_u} \right) \text{Im}(M_3^* Y^{uee}) I(m_{u1}^2, m_{u2}^2, |M_3|^2) \\
&\sim 4 \times 10^{-3} (1 \text{ GeV} \times \Lambda_{qe})^{-1}.
\end{align*}

(33)

Comparing (32) and (33) to the limits on \( C_S \) and \( C_P \) deduced from the bounds on the EDMs of Tl and Hg [28], we obtain the following sensitivity to the energy scale \( \Lambda_{qe} \),

\begin{align*}
\Lambda_{qe} &\gtrsim 3 \times 10^8 \text{ GeV} \quad \text{from Tl EDM} \\
\Lambda_{qe} &\gtrsim 1.5 \times 10^8 \text{ GeV} \quad \text{from Hg EDM}
\end{align*}

(34)
These are remarkably large scales, and indeed not far from the intermediate scale suggested by neutrino physics. In fact, the next generation of atomic/molecular EDM experiments have the chance of increasing this scale by two-three orders of magnitude which would put it close to the scales often suggested for right-handed neutrino masses.

Going over to purely hadronic $CP$-violating operators, e.g. $C_{ud} (d\bar{\gamma}_5 d)(\bar{u}u)$, we note that these would induce the EDMs of neutrons, and EDMs of diamagnetic atoms mediated by the Schiff nuclear moment $S(g_{\pi NN})$. In particular, we have for the $CP$-odd isovector pion-nucleon coupling,

$$g^{(1)}_{\pi NN} = -4 \times 10^{-2} \frac{C_{ud}}{m_d},$$

with

$$C_{ud} = -\frac{\alpha_s}{9\pi\Lambda_{qq}} \text{Im}(M_3^* Y_{uudd}) I(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2, |M_3|^2),$$

obtained as for the semileptonic operators above. The typical sensitivity to $\Lambda_{qq}$ in this case is somewhat lower than in the case of semi-leptonic operators,

$$\Lambda_{qq} \gtrsim 3 \times 10^7 \text{ GeV}\quad \text{from Hg EDM.}$$

Semileptonic operators involving heavy quark superfields are also tightly constrained by experiment, via the two-loop diagrams of Fig. 2. Assuming no additional $CP$ violation in the soft-breaking sector and taking into account only the stop loops, we obtain the following result for the EDM of the electron:

$$d_e = \frac{e}{12\pi^3} \frac{\alpha}{\Lambda_{qe}} \frac{\text{Im}(Y^{ttee})}{m_t} \frac{m_t |A_t - \mu^* \cot \beta|}{|m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2|} \ln \left( \frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2} \right),$$

where $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$ are the stop masses in the physical basis. Assuming maximal $CP$-odd phases and large stop mixing, we arrive at the following estimate for $d_e$,

$$d_e \sim \frac{10^8 \text{ GeV}}{\Lambda_{qe}} \times 10^{-27} \text{ e cm},$$

which together with the sensitivity to the electron EDM inferred from the constraint on $d_{T1}$, $|d_e| \lesssim 1.6 \times 10^{-27} \text{ e cm}$, translates to

$$\Lambda_{qe} \gtrsim 6 \times 10^7 \text{ GeV}.$$

Expressions similar to (38) can be obtained for the quark EDMs and color EDMs, furnishing similar sensitivity to $\Lambda_{qq}$.

### 4.3 Lepton flavour violation

Searches for lepton-flavour violation, such as the decay $\mu \rightarrow e\gamma$ and $\mu \rightarrow e$ conversion on nuclei have resulted in stringent upper bounds on the corresponding branching ratio.
and the rate of conversion normalized on capture rate \[30\]:
\[
\text{Br}(\mu \to e\gamma) < 1.2 \times 10^{-11}
\]
\[
R(\mu \to e^- \text{ on Ti}) < 4.3 \times 10^{-12}.
\]

Focussing first on $\mu \to e$ conversion, one can deduce the sensitivity of these searches to the energy scale of the semileptonic operators \[15\] as the conversion is mediated by the $(\bar{u}u)(\bar{e}i\gamma_5\mu)$ and $(\bar{u}u)(\bar{e}\mu)$ operators, and thus involves the same matrix elements as does $C_S$. Indeed, the characteristic amplitude for the scalar operator has the form
\[
\frac{G_F}{\sqrt{2}} \eta_{e\mu} = -\frac{4\alpha_s}{3\pi\Lambda_{qe}} \text{Im}(M_3 Y_{uue\mu}) I(m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2, |M_3|^2).
\]
(42)

Using the bounds on such scalar operators derived elsewhere (see e.g. \[32\]), we conclude that $\mu \to e$ conversion currently probes energy scales as high as 
\[
\Lambda_{qe} \gtrsim 1 \times 10^8 \text{ GeV from } \mu^- \to e^- \text{ on Ti}.
\]
(43)

However, it is important to note that the bound on the conversion rate is necessarily proportional to $(\eta_{e\mu})^2$ and thus these effects decouple as $(\Lambda_{qe})^{-2}$ in contrast to the linear decoupling of the EDMs.

A sensitivity to slightly lower scales arises from the two-loop–mediated $\mu \to e\gamma$ process. We have
\[
\text{Br}(\mu \to e\gamma) = 384 \pi^2 \frac{\mu_{e\mu}^2}{4G_F^2 m_{\mu}^2},
\]
(44)

with the transition amplitude generated in the same manner as $d_e$, 
\[
\mu_{e\mu} = \frac{\alpha}{12\pi^3} \frac{\text{Re}(Y^{tue}) m_e |A_t - \mu^* \cot \beta|}{\Lambda_{qe}} \ln \left( \frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2} \right),
\]
(45)

where once again the sensitivity in the branching fraction is weakened relative to the EDMs by quadratic decoupling with the threshold scale.

Future progress in lepton flavour violation searches should be able to extend the reach of these probes by one-two orders of magnitude. Disregarding a factor of a few between \[34\] and \[43\], we conclude that currently the EDMs and searches for lepton flavour violation probe these extensions of the MSSM up to similar energy scales of $\sim 10^8 \text{ GeV}$.

It is also worth noting that sensitivity to $\Lambda_{qe}$, that is somewhat more robust to changing assumptions on the flavour structure of $Y_{ue}$, can also be achieved through comparison of the two modes of charged pion decay into first and second generation leptons. The typical sensitivity to $\Lambda_{qe}$ in this case could be as large as
\[
\Lambda_{qe} \gtrsim 10^4 \text{ GeV from } \mu-e \text{ universality in } \pi^\pm \text{ decay.}
\]

\[1\] A recent announcement from the SINDRUM II collaboration suggests a slightly stronger constraint, $R(\mu \to e^- \text{ on Au}) < 7 \times 10^{-13}$ \[31\].
Finally, the two-loop amplitudes in Fig. 3 would also give corrections to the anomalous magnetic moments of $e$ and $\mu$. In the latter case, one can estimate the sensitivity to $\Lambda^{qq}$ as no higher than about 1 TeV.

4.4 $K$ and $B$ meson mass-difference

Often, the most constraining piece of experimental information comes from the contributions of new physics to the mixing of neutral mesons, $K$ and $B$. In the case of generic couplings $Y^{qq}$ and $\tilde{Y}^{qq}$, the four-fermion operators \[ \text{(24)} \] will contain \[ \langle \bar{s}_R d_L \rangle \langle \bar{s}_L d_R \rangle \] and \[ \langle \bar{b}_R d_L \rangle \langle \bar{b}_L d_R \rangle \] terms. Using a simple vacuum factorization ansatz, we find

\[
\langle K^0 | (\bar{d}_R s_L) (\bar{d}_L s_R) | K^0 \rangle = \left[ \frac{1}{24} + \frac{1}{4} \left( \frac{m_K}{m_s + m_d} \right)^2 \right] m_K f_K^2 \approx 2 m_K f_K^2,
\]

and therefore can neglect \[ \langle \bar{d}_R t^A s_L \rangle \langle \bar{d}_L t^A s_R \rangle \] due to its small matrix element. Taking into account the one-loop QCD evolution of the \[ \langle \bar{d}_R s_L \rangle \langle \bar{d}_L s_R \rangle \] operators from the SUSY threshold scale down to 1 GeV,

\[
\langle K^0 | (\bar{d}_R s_L) (\bar{d}_L s_R) | K^0 \rangle \approx 5 (\bar{d}_R s_L)(\bar{d}_L s_R)_M_Z,
\]

we can estimate the contribution of additional four-fermion operators to the mass splitting of $K$ mesons:

\[
\Delta m_K = -2 \text{Re} \langle K^0 | L_{dd} | K^0 \rangle \approx -5 \left( m_K f_K^2 | \mu |^2 \right) \frac{6 \pi^2 \Lambda^{qq}}{\mu^I_{ij} (m_1^2, m_2^2, |\mu|^2)} \mathcal{Y}_{ds},
\]

where we took $\mu$ to be real, and introduced the following notation for the relevant combination of Yukawa couplings:

\[
\mathcal{Y}_{ds} = \text{Re}(Y_u^*)_{i1} (Y_d^*)_{2j} K_{ij12}^{qq} + \text{Re}(Y_u)_{i2} (Y_d^*)_{1j} K_{ij21}^{qq*}.
\]

It is easy to see that the presence of $Y_u$ and $Y_d$ in \[ \text{(49)} \] results in a significant numerical suppression of the corresponding Wilson coefficients even with $Y^{qq} \sim O(1)$. The minimal suppression is realized with an intermediate stop-loop, in which case the first term in \[ \text{(49)} \] becomes of order $V_{ts} y_t y_s$, while the second term is $\sim V_{ts} y_t y_d$. Numerically, this corresponds to a suppression factor,

\[
\mathcal{Y}_{ds} \sim 3 \times 10^{-4} \times \frac{\tan \beta}{50},
\]

which is also $\tan \beta$-dependent.

Putting all the factors together, with the same assumption of degenerate soft mass parameters at 300GeV, we come to a disappointingly weak result:

\[
\Delta m_K \sim 3 \times 10^{-6} \text{eV} \times \frac{\tan \beta}{50} \times \frac{200 \text{GeV}}{\Lambda^{qq}},
\]

15
with the actual measured value of the mass splitting being $3.5 \times 10^{-6}$ eV. The calculation of $\Delta m_B$ results in a similar sensitivity level, prompting the conclusion that neither $\Delta m_K$ nor $\Delta m_B$ can probe the flavour structure of additional dimension five operators beyond the SUSY threshold. The $CP$-violating observable $\epsilon_K$ will obviously be more sensitive by almost three orders of magnitude, resulting in

$$\Lambda_{\text{eff}} \gtrsim 10^5 \text{GeV} \times \frac{\tan \beta}{50} \quad \text{from } \epsilon_K,$$

which is still clearly inferior to the sensitivity of EDMs and lepton flavour violation. Moreover, it is easy to see that if the complete theory at scales $\Lambda$ also provides new dimension-six operators in the Kähler potential, the possible consequences of those for $\Delta m_K$ and $\Delta m_B$ would be considerably more serious that of the dimension-five operators. We give an explicit example of this in the discussion section.

### 4.5 Constraining the Higgs operator

The strength of the constraints on $QULE$ and $QUQD$ comes primarily from the fact that such operators flip the chirality of light fermions without paying the usual price of small Yukawa couplings. This was a consequence of our assumption on the arbitrary flavour structure of the dimension-five operators. Should all transitions from $u_R$ to $u_L$ and $e_R$ to $e_L$ be suppressed by $m_f/v$, the constraints (34), (43) and (52) would be relaxed all the way to the weak scale and below. Therefore, it would come as no surprise if the effective operator in the Higgs potential were to have very weak implications for $CP$-violating physics and no consequences at all for the flavour-changing transitions.

We note first of all that the mixing of the left- and right-handed $d$-squarks is affected by the Higgs operator (26). This feeds into the one-loop $d$-quark EDM diagram, where this parameter behaves similarly to the insertion of the complex $A$-term, $A_d^{\text{eff}} \sim y_h v_{SM}^2 \Lambda_h^{-1}$ and leads to a contribution to $d_d$ that does not grow with $\tan \beta$. The typical sensitivity of the neutron and mercury EDMs to the imaginary part of the $A_d$ parameter [33], with the superpartner masses in the ballpark of $\sim 300$ GeV, then implies

$$\Lambda_h \gtrsim 1 \text{ TeV} \quad \text{maximal } CP \text{ violation, neutron EDM.}$$

Of course, a mere increase of the superpartner masses to around 1 TeV would completely erase this sensitivity.

Another possibly interesting $CP$-violating effect would come from the admixture of the pseudoscalar Higgs $A$ to the scalars $h$ and $H$ at tree level. Subsequent Higgs exchange would then induce the $C_S$ operator [34, 35], or contribute to the two-loop EDM of quarks and electrons [36]. The latter results in contributions to observable EDMs that are $\tan \beta$-dependent and furnish a sensitivity to $\Lambda_h$ up to a few TeV.

These are relatively minor effects. However, it turns out that significant sensitivity to this operator can indeed arise through its shift of the Higgs potential (25), and more
specifically the effective shift of the $m_{12}^2$ parameter,
\[
m_{12}^2H_uH_d \to \left( m_{12}^2 + \frac{\mu y_h v_{SM}^2}{\Lambda_h} \right) H_uH_d,
\]
assuming the reality of $\mu$. The quantity in the parentheses is an effective $m_{12}^2$ parameter, which is complex on account of $\text{Im}(y_h)$. Moreover, its complex phase is enhanced in the large $\tan \beta$ limit because $m_{12}^2 \simeq m_A^2 \tan^{-1} \beta$. The resulting phase affects the one-loop SUSY EDM diagrams. Assuming for simplicity a common mass scale $m_{\text{soft}}$ for sleptons, gauginos, and $\mu$, we have [33]:
\[
d_e \sim \frac{e m_e \tan \beta}{16\pi^2 m_{\text{soft}}^2} \left( \frac{5g_2^2}{24} + \frac{g_1^2}{24} \right) \sin \left[ \text{Arg} \left( \frac{\mu M_2}{(m_{12}^2)_{\text{eff}}} \right) \right].
\]
Expanding to leading order in $1/\Lambda_h$, and imposing the present limit on $d_e$, we find the sensitivity,
\[
\Lambda_h \gtrsim 2 \times 10^7 \text{ GeV} \left( \frac{\tan \beta}{50} \right)^2 \left( \frac{300 \text{ GeV}}{M_{\text{SUSY}}} \right) \left( \frac{300 \text{ GeV}}{m_A} \right)^2,
\]
which reaches impressively high scales for maximal $\tan \beta$.

5 Constraints within MSSM benchmark scenarios

A summary of the characteristic sensitivity to the threshold scale $\Lambda$ in different channels is given in Table 2, assuming generic and degenerate SUSY spectra. In this section, we will go somewhat further and examine the variation in sensitivity among a few benchmark SUSY scenarios, with different spectra chosen in order to satisfy other constraints, including requiring the correct relic LSP density to form dark matter. We have also included the leading one-loop SUSY evolution of the dimension-five operators from the $\Lambda$-scale to the soft threshold. These effects are generally on the order of 20-40%, and the relevant RG equations are included in the Appendix.

The benchmark spectra we have chosen are the following representative SPS points [3]:

- SPS1a – mSugra
- SPS2 – focus point
- SPS4 – large $\tan \beta$ in the funnel region
- SPS8 – gauge mediation

The results are shown in Figs. (4-6), with the observables normalized to their current experimental bound plotted against the threshold scale $\Lambda$. All of the scenarios exhibit broadly similar sensitivity to the degenerate spectrum utilized previously. However, there
Table 2: Sensitivity to the threshold scale. Note that the naturalness bound on $\text{Im}(Y^{qq})$ doesn’t apply to the axionic solution of the strong $CP$ problem, the best sensitivity to $\text{Im}(y_h)$ is achieved at maximal $\tan\beta$, and the Hg EDM constraint on $\text{Im}(Y^{qq})$ applies when at least one pair of quarks belongs to the 1st generation.

![Figure 4: Constraints on the benchmark scenarios from contributions to $d_{Tl}(C_S)$ and $d_{Hg}(\bar{g}_{\pi NN})$. In these plots and those below, SPS1a = red (solid), SPS2 = blue (dashed), SPS4 = green (dotted), SPS8 = brown (dot-dashed). The shaded region is above the current experimental bound.](image-url)
$R(\mu \rightarrow e)$

$\Lambda$ [10$^7$ GeV]

$Br(\mu \rightarrow e\gamma)$

$\Lambda$ [10$^5$ GeV]

Figure 5: Constraints on the benchmark scenarios from contributions to $\mu \rightarrow e$ conversion and $\mu \rightarrow e\gamma$.

$d_{TI}(d_e(2l))$

$\Lambda$ [10$^7$ GeV]

$d_{TI}(d_e(y_h))$

$\Lambda$ [10$^6$ GeV]

Figure 6: Constraints on the benchmark scenarios from contributions to $d_{TI}(d_e)$.
is still significant variation in terms of the level of sensitivity exhibited within different
benchmark spectra.

If we focus first on Fig. 4, the constraints on $C_S$ and $\bar{g}_{\pi NN}$ are most stringent within
SPS1a, while SPS2 exhibits least sensitivity simply through having a generically heavy
SUSY spectrum.

Fig. 5 shows the constraints imposed by lepton flavour violating observables, and the
quadratic sensitivity to $\Lambda$ is clearly evident in comparison to the EDM bounds. The
strongest constraints again arise within SPS1a and SPS4 due to having a generically lighter
SUSY spectrum. The differences in the constraints from $\mu \rightarrow e\gamma$ are particularly marked,
with SPS2 and SPS8 exhibiting rather minimal sensitivity. This can be understood from
the two-loop amplitude as due to the relatively small stop mixing in these cases.

Fig. 6 exhibits the constraints from the electron EDM. The constraints from the two-
loop amplitude on the left naturally exhibit very similar features to the constraints from
$\mu \rightarrow e\gamma$, given the similarity between the two dipole amplitudes. The constraints on the
Higgs operator on the right of Fig. 6 are most pronounced in SPS4 as one would expect due
to $\tan \beta$-enhancement. The weak constraint from SPS2 is primarily because of the large
value of $m_{12}^2$ which acts to suppress the effect of the additive complex shift.

6 Discussion

So far we have kept our discussion rather general within the context of effective field theory,
without concerning ourselves with the details of particular renormalizable UV models. In
the case of the see-saw operator and proton-decay operators, such models are well-studied.
We would now like to briefly provide an example of how the effective terms in the super-
potential studied in this paper can be generated by renormalizable interactions.

We will limit our discussion here to scenarios in which these operators can be generated
by tree level exchange of additional heavy states. As a basic example, consider the MSSM
with an expanded Higgs sector – an additional heavy singlet $S$ and heavy pair of doublets
$H'_u$ and $H'_d$. This is sufficient to generate all the operators we have considered assuming
renormalizable interactions of the form:

$$\mathcal{W} \sim \frac{1}{2} MS^2 + \kappa SU_u H_d - \mu H_d H_u - \mu' H'_d H'_u$$

$$+ UQ(Y_u H_u + Y'_u H'_u) - DQ(Y_d H_d + Y'_d H'_d) - EL(Y_e H_d + Y'_e H'_d).$$

Integrating out the singlet $S$ will clearly generate the operator $(H_u H_d)^2$. The complex
parameters $\mu$ and $\mu'$ are the eigenvalues of a $2 \times 2$ complex matrix of $\mu$ parameters that
can always be reduced to diagonal form by bi-unitary transformations in the $(H_u, H'_u)$
and $(H_d, H'_d)$ spaces. Assuming the hierarchy, $\mu' \gg \mu$, we can integrate out heavy Higgs
superfields, producing a set of dimension five operators,

$$\mathcal{W}^{(5)} = \frac{\kappa^2}{M} H_u H_d H_u H_d + \frac{Y'_e Y'_u}{\mu'} E L U Q + \frac{Y'_d Y'_d}{\mu'} (U Q)(D Q).$$
Comparing (58) with (15), we can make the identification $y_h/\Lambda_h = \kappa^2/M$, $\bar{Y}^{\eta \eta} = 0$, $Y^{\eta e}/\Lambda^{\eta e} = Y^{\eta}_{u} Y^{\eta}_{u}/\mu'$, and $Y^{\eta e}/\Lambda^{\eta e} = Y^{\eta}_{d} Y^{\eta}_{d}/\mu'$ and translate the sensitivity to the $\Lambda$'s into a sensitivity to the extra Higgs fields. Since a priori there is no correlation or dependence between the two sets of Yukawa matrices, one can expect novel flavour and CP violating effects induced by (58). More specific predictions could be made in models that predict or constrain the Yukawa couplings $Y$ and $Y'$, due e.g. to horizontal flavour symmetries, Yukawa unification, or discrete symmetries such as parity or CP.

All the effects which decouple as $1/\Lambda$, when put in the language of the model (58), probe the exchange of heavy Dirac fermions, namely Higgsino particles composed from $\bar{H}_u'$ and $\bar{H}_d'$. It is then natural to ask the question of whether the dimension six operators induced by the exchange of the heavy scalar Higgses could provide better sensitivity to $\mu'$. It is easy to see that in the case of arbitrary $Y'_{u,d} \sim O(1)$, the contribution of dimension six operators to the $K$ meson mass splitting is

$$\Delta m_K \sim \frac{0.25 \text{ GeV}^3}{\mu'^2} \implies \mu' \gtrsim 8 \times 10^6 \text{ GeV},$$

while $\epsilon_K$ is sensitive to scales $\sim 1 \times 10^8$ GeV. The reason why this dimension-six contribution dominates so dramatically over (51) and (52) is the suppression of the dimension five effects by loop and Yukawa factors (50).

We conclude that $\Delta F = 2$ processes mediated by dimension-six operators in the MSSM extended by an additional pair of Higgses comes very close in sensitivity to the estimates (58) and (51), with the latter being somewhat more stringent. This statement does of course depend on the SUSY mass spectrum, and having heavier squarks and gluinos would reduce the EDM sensitivity. In contrast we should also note that unlike the previous limits (51) and (43), the constraint (59) is essentially ‘static’, i.e. difficult to improve upon, as there is a limited extent to which new physics contributions to $\Delta m_K$ and $\epsilon_K$ can be isolated from SM uncertainties.

We will end this section with a few additional remarks on issues that we touched on in this work:

(i) Thus far, we have studied the subset of all possible dimension five operators neglecting, for example, $R$-parity violation. It is easy to see, however, that no strong constraints on the $R$-parity violating terms in (8) would arise at dimension-five level. Indeed, limits on $R$-parity violation usually come from SM processes which have to be bilinear in $R$-parity violating parameters. Thus, only a combination of two dimension-five terms, or a dimension-five term with a dimension-four term, would induce four-fermion operators for example. Since the dimension-four terms are tightly constrained (see, e.g. [12]), one would not expect the limits on dimension-five operators to be competitive with (34).

(ii) A primary goal of any theory of CP violation is to provide a solution to the strong CP problem. We have shown that the effective shift in $\theta$ can be quite significant even if higher-dimensional operators are suppressed by $10^{17}$ GeV. This has implications for solutions to the strong CP problem that do not employ the dynamical relaxation of $\bar{\theta}$. For example, if $\bar{\theta} = 0$ is achieved due to a new global symmetry that forces $m_u = 0$ at
the dimension-four level but is broken e.g. by quantum gravity effects, one could expect the emergence of Planck-scale suppressed operators in the superpotential and, remarkably enough, progress in neutron EDM measurements by just one or two orders of magnitude would directly probe such a scenario! Similarly, supersymmetric models that construct a small $\bar{\theta}$ using discrete symmetries can also be affected by these operators, with possible observable consequences for the neutron EDM.

(iii) Since the $CP$-odd effective interaction $C_S \bar{N}N\bar{e}i\gamma_5 e$ provides the leading sensitivity to the energy scale of new physics encoded in the semileptonic dimension-five operator in the superpotential, it is prudent to recall that the best constraint on $C_S$ comes from the EDM of the Ti atom, which is also used for extracting a constraint on $d_e$. To make both bounds independent of the possibility for mutual cancellations, one should use experimental information from other atomic EDM measurements. In this respect, the interpretation of promising new molecular EDM experiments that aim to improve the sensitivity to $d_e$ will require additional theoretical input on the exact dependence on $C_S$.

(iv) Finally, we would like to emphasize that the main result of this paper, namely the sensitivity to the high-energy scale in Eqs. (34) and (43), is quite robust in the sense that it has a mild dependence on the SUSY threshold as exhibited in the preceding section. For example, an increase of the average superpartner mass to 3 TeV would reduce the sensitivity to $\Lambda^{\theta\theta}$ and $\Lambda^{oo}$ by only a factor of 10, still probing scales of a few$\times 10^7$ GeV. Contrary to this, the dependence of the electron EDM on the Higgs operator is highly dependent on the details of the SUSY spectrum, as taking $\tan \beta \sim 5$ and $m_A \sim M_{\text{SUSY}} \sim 1$ TeV would reduce the sensitivity to $\Lambda^{h}$ to a few TeV.

7 Conclusions

Continuing progress in precision experiments searching for $CP$- and flavour-violation provides an increasingly stringent test for models of new physics beyond the electroweak scale, and supersymmetric theories in particular. In this paper, we have presented an analysis of flavour and $CP$ violating effects in a two-stage theoretical framework: assuming first that the SM becomes supersymmetric at or near the weak scale, and then that the MSSM gives way at some higher scale $\Lambda$ to a theory with additional degrees of freedom. If nature indeed chooses supersymmetry, both steps can clearly be justified. The first one follows from the solution to the gauge hierarchy problem offered by SUSY and the evidence for the second (third, etc.) energy scale comes from rather intrinsic features that are required by, but not contained within, the MSSM: mediation of SUSY breaking, neutrino masses, not to mention problems which require other solutions, i.e. baryogenesis, the strong $CP$ problem, etc.

We have examined new flavour- and $CP$-violating effects mediated by dimension five operators in the superpotential to show that sensitivity to these operators extends far beyond the weak scale, and indeed probes very high energies. The semi-leptonic operators that mediate flavour-violation in the leptonic sector and/or break $CP$ could be detectable
even if the scale of new physics is as high as $10^9 \text{ GeV}$. Since the effects studied here decouple linearly, an increase of sensitivity by just two orders of magnitude would already start probing the scales that are relevant for Majorana neutrino physics. It is also important to note that theoretically, should a major breakthrough in the precision of EDM measurements take place, there is ample room for the EDMs of paramagnetic atoms to probe $CP$-violating operators suppressed by the GUT or string scale without facing the SM background from the Kobayashi-Maskawa phase, which is known to induce tiny EDMs in the lepton sector [37].

The MSSM can contain a variety of new sources of flavour- and $CP$-violation related to the soft-breaking sector. This plethora of sources appears highly excessive given the rather minimalist pattern of $CP$ and flavour violation observed experimentally. A number of model-building scenarios have addressed this issue, often successfully, especially if supersymmetry is broken at a relatively low energy scale. Supposing that the wish of many theorists is granted, and a $CP$-symmetric, and flavour-conserving, pattern of soft SUSY breaking is achieved in a compelling manner, we may ask the following question: is there any new information about such a SUSY theory that could be provided by the continuation of the low-energy precision experimental program? This paper provides a clear affirmative answer to this question.

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Appendix A

In this appendix, we summarize the 1-loop renormalization group equations used to evolve the dimension five operators down to the soft threshold. This evolution of course arises purely from the Kähler terms.

In general, we have:

\[
\frac{d}{dt} y_h = y_h \left[ \frac{1}{16\pi^2} \left( \text{Tr}[6 y_u y_u^\dagger + 6 y_d y_d^\dagger + 2 y_e y_e^\dagger] - 6 g_2^2 - \frac{6}{5} g_1^2 \right) \right] + \ldots 
\]  

(60)

\[
\frac{d}{dt} Y_{ijkl}^{q e} = \frac{1}{16\pi^2} \left[ Y_{ijkl}^{q e} (y_e^\dagger y_e)_{ml} + Y_{ijkl}^{q e} (2 y_e y_e)_{mk} + Y_{ijkl}^{q e} (y_u^\dagger y_u + y_d^\dagger y_d)_{mj} 
\right. 
+ Y_{ijkl}^{q e} (2 y_u^\dagger y_u)_{mi} - Y_{ijkl}^{q e} \left( \frac{16}{3} g_3^2 + 3 g_2^2 + \frac{31}{15} g_1^2 \right) 
+ \ldots 
\]  

(61)

\[
\frac{d}{dt} Y_{ijkl}^{q q} = \frac{1}{16\pi^2} \left[ Y_{ijkl}^{q q} (y_u^\dagger y_u + y_d^\dagger y_d)_{ml} + Y_{ijkl}^{q q} (2 y_d y_d)_{mk} 
\right. 
+ Y_{ijkl}^{q q} (y_u^\dagger y_u + y_d^\dagger y_d)_{mj} + Y_{ijkl}^{q q} (2 y_u^\dagger y_u)_{mi} 
\left. - Y_{ijkl}^{q q} \left( \frac{32}{3} g_3^2 + 3 g_2^2 + \frac{11}{15} g_1^2 \right) \right] + \ldots 
\]  

(62)

and similarly for \( \tilde{Y}_{ijkl}^{q q} \), where \( y_u, y_d, y_e \) are 3 \( \times \) 3 Yukawa matrices, and the dots represent higher order terms.

In practice, since only the third generation Yukawa couplings are significant, we can make use of the simplified RGEs,

\[
\frac{d}{dt} y_h \approx y_h \left[ \frac{1}{16\pi^2} \left( 6 y_t y_t^\dagger + 6 y_b y_b^\dagger + 2 y_e y_e^\dagger - 6 g_2^2 - \frac{6}{5} g_1^2 \right) \right] 
\]  

(63)

\[
\frac{d}{dt} Y_{uuee}^{q e} \approx \frac{Y_{uuee}^{q e}}{16\pi^2} \left[ - \left( \frac{16}{3} g_3^2 + 3 g_2^2 + \frac{31}{15} g_1^2 \right) \right] 
\]  

(64)

\[
\frac{d}{dt} Y_{ttee}^{q e} \approx \frac{Y_{ttee}^{q e}}{16\pi^2} \left[ 3 y_t^* y_t + y_b^* y_b - \left( \frac{16}{3} g_3^2 + 3 g_2^2 + \frac{31}{15} g_1^2 \right) \right] 
\]  

(65)

\[
\frac{d}{dt} Y_{uuep}^{q e} \approx \frac{Y_{uuep}^{q e}}{16\pi^2} \left[ - \left( \frac{16}{3} g_3^2 + 3 g_2^2 + \frac{31}{15} g_1^2 \right) \right] 
\]  

(66)

\[
\frac{d}{dt} Y_{ttep}^{q e} \approx \frac{Y_{ttep}^{q e}}{16\pi^2} \left[ 3 y_t^* y_t + y_b^* y_b - \left( \frac{16}{3} g_3^2 + 3 g_2^2 + \frac{31}{15} g_1^2 \right) \right] 
\]  

(67)

\[
\frac{d}{dt} Y_{uadd}^{q q} \approx \frac{Y_{uadd}^{q q}}{16\pi^2} \left[ - \left( \frac{32}{3} g_3^2 + 3 g_2^2 + \frac{11}{15} g_1^2 \right) \right] 
\]  

(68)
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