Millimeter Wave MIMO out Door Channel Estimation and Precoding

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Abstract. Millimeter wave (mm Wave) communications is one of the technologies for 5G cellular systems. In the mm Wave communication, there is a lot of path loss can be reduced by Precoding. The channel state information (CSI) should be known at the transmitting station, in the design of precoding matrices and to get good accuracy in estimating sparse channels a Compressive sensing (CS) based recovery algorithms was used. Not only for good accuracy the algorithm is also used for mm Wave channel estimation for exploiting the mm Wave channel’s sparse in multi-path construction. Hence, in this paper, for mm Wave outdoor channel estimation, the CS recovery methods orthogonal matching pursuit (OMP) and compressive sampling matching pursuit (CoSaMP) are used. The singular value decomposition (SVD) precoding is developed using the estimated channel. By (MSE) mean square error and spectral efficiency which were the performance metrics in channel estimation and precoding were done by using MATLAB simulations to get the efficacy of the OMP and CoSaMP algorithm.

1. INTRODUCTION

The Millimeter wave (mm Wave) band communication is the spectrum crunch solving technology for 5G systems [1-7]. The enormous usage of data in the 6GHz sub-band is a challenging task to data centres and mobiles services due to limited available spectrum even by using the conventional cellular and WiFi-based solutions. The use of licensed spectrum on opportunistic basis [8] is also disruptive and limited by the channel bandwidth.

The recently opened unlicensed spectrum which works under the range of 57-71GHz is achieving a data rate of gigabit per second to Millimeter wave (mm Wave) [9]. To set high quality long distance communication links with increased power at receiver end, directional beam forming exploits a large antenna array at base station and mobile station [10-13] as mm Wave bands gives a high path loss characteristics. Analog beamforming is a technique in which MIMO and beamforming control is executed at the RF level. Phase shifters can be used to achieve analog beamforming. This technique is used to regulate the phases of earliest signals to achieve the maximal antenna array gain and sufficient SNR. Analog beam forming has a basic hardware structure that is easier to implement as shown in Figure 1.

However, this beam forming has low antenna gain and has a problem with severe performance loss, since only the phases of the transmit signals can be controlled but not their amplitudes.
Therefore, it is not directly adaptable for mm Wave communication systems. In the digital beam forming methods shown in Figure.2, the central process is performed using a digital signal processor that provides significant flexibility with more degrees of freedom to implement efficient beamforming mechanisms. This approach necessitates a distinct RF chain for each antenna element, resulting in a more complex architecture and higher power usage. Because of the utilization of a assigned RF chain for each antenna; it is not cost-effective for the mm Wave communication system, which employs a both the transmitter and receiver have a huge number of antennas.

CS theory which was introduced in [16-17] has made it possible for estimating channels with an improved accuracy even though the signals are compressible or sparse with less number of measurements. The OMP and CoSaMP algorithms are well known in the literature as compressive sensing recovery algorithms.

In this paper, on the basis of outdoor mm Wave channel model, sparse formulation of the mm Wave channel estimation is developed. Then, OMP and CoSaMP algorithms are employed for mm Wave outdoor channel estimation. Later, the SVD precoding developed using the estimated channels.

This paper is collocated as follows. In Section II, mm Wave outdoor channel model is presented. The sparse expression of the mm Wave channel estimation is given in Section III to exploit the use of compressive sensing recovery algorithms for channel estimation. Section IV Provides CS recovery algorithms OMP and CoSaMP for sparse channel estimation. SVD Precoding is discussed in Section V. The performance of the OMP and CoSaMP algorithms are compared through simulation results given in Section VI. This paper is concluded in Section VII.

2. MILLIMETER WAVE OUTDOOR CHANNEL MODEL

Consider a millimetre wave cellular system with a downlink where a base station with $N_{BS}$ transmission antennas located at the center of the cell and provides the coverage to the associated users in the network. By assuming a narrow band clustered channel model, the received signal at the mobile station (MS) is expressed as

$$y = Hx + \eta$$  \hspace{1cm} (1)

Where H represents the $N_{BS} \times N_{MS}$ millimeter wave channel matrix between base stations (BS) and related MS and $\eta$ is the additive white Gaussian noise.

A narrow band clustered channel with L scatters is considered based on the saleh-valenzuela, which perfectly incorporates the characteristics of a millimeter wave channel including limited scattering, severe path loss, and significance correlation among antennas due to nearly placed antenna arrays, etc. As such, H can be expressed as

$$H = \sqrt{\frac{N_{BS}N_{MS}}{\rho}} \sum_{l=1}^{L} \alpha_{l}a_{MS}(\theta_{l})a_{BS}^{*}(\phi_{l})$$  \hspace{1cm} (2)

Where $\rho$ stands for the average path loss and $\alpha_{l}$ is the lth path complex gain. The amplitudes of the paths are assumed are Rayleigh distributed with average power gain $\bar{P}$. $\phi_{l}, \theta_{l} \in [0, 2\pi]$ are the lth paths azimuth angles of departure or arrival (AODs/AOAs) of the BS and the lth paths azimuth angles of departure or arrival of the MS. All scattering occurs in azimuth only as the elevation is neglected. Hence, the two dimensional beam forming is to be implemented at the BS and MS. $a_{BS}(\phi_{l})$ and $a_{MS}(\theta_{l})$ are the BS and MS response vectors, respectively. If a uniform linear array (ULA)
is assumed \( a_{BS}(\phi_l) \), the following formula can be used:

\[
a_{BS}(\phi) = \frac{1}{\sqrt{N_{BS}}} \begin{bmatrix}
    1, e^{\frac{2\pi}{\lambda}i\sin(\phi)}, ..., e^{\frac{(N_{BS}-1)2\pi}{\lambda}i\sin(\phi)}
\end{bmatrix}^T
\]

(3)

Where \( d \) is the distance between antenna elements, and \( \lambda \) is the signal wavelength. At the MS, the array response vectors \( a_{MS}(\theta_l) \) can be written in a similar fashion. The channel in equation Eq.(2) is written in a more condensed manner as

\[
H = A_{MS} \text{diag}(\alpha) A_{BS}^H
\]

(4)

Where

\[
\alpha = \sqrt{\frac{N_{BS}N_{MS}}{\rho}} [\alpha_1, \alpha_2, ..., \alpha_L]^T
\]

The matrices

\[
A_{BS} = \begin{bmatrix}
    a_{BS}(\phi_1), a_{BS}(\phi_2), ..., a_{BS}(\phi_L)
\end{bmatrix}
\]

and

\[
A_{MS} = \begin{bmatrix}
    a_{MS}(\theta_1), a_{MS}(\theta_2), ..., a_{MS}(\theta_L)
\end{bmatrix}
\]

Contain the BS and MS array response vectors. In order to determine the receiver's channel, the millimeter wave channel estimation problem is formulated and the conventional compressive sensing (CS) based OMP and COSAMP algorithms are used to solve the estimation problem.

2.1 Sparse mm wave outdoor channel model

Assuming that the transmitted symbols are equal, with average power \( P \) namely, \( X = \sqrt{P} I \), Eq.(1) becomes

\[
Y = \sqrt{P} H + \eta
\]

(5)

By vectorising the resultant matrix \( Y \), we obtain

\[
y_v = \sqrt{P}(A_{BS} \circ A_{MS})\alpha + \eta_v
\]

(6)

Where \((A_{BS}^* \circ A_{MS})\) is an \( N_{MS}N_{BS} \times L \) matrix, with each column is the Kronecker product of the BS and MS array response vectors \((a_{BS}(\phi_l) \otimes a_{MS}(\theta_l)), l = 1, 2, ..., L\), associated with the AoA/AoD of the channel's \( l \)th path. The AoAs and AoDs are believed to be drawn from a uniform grid of \( N \) points, with \( N \ll L \). Eq.(6) can be approximated as

\[
y_v = \sqrt{P} A_D Z + \eta_v
\]

(7)

Where \( A_D \) is a \( N_{MS}N_{BS} \times N^2 \) dictionary matrix containing \( N^2 \) column vectors in the shape of \((a_{BS}(\phi_u) \otimes a_{MS}(\theta_v)), \) where \( \phi_u \) and \( \theta_v \) are the \( u \)th and \( v \)th points of the uniform grid's angles. The \( Z \) in Eq.(7) has only \( L \) non zero elements with \( L \ll N^2 \), and hence Eq.(7) represents the sparse mm Wave channel model. Now, defining \( \sqrt{P} A_D \) as the sensing matrix, CS recovery algorithms can be leveraged to estimate the no zero elements of \( Z \) with less number of required measurements \( N_{MS}N_{BS} \ll N^2 \).
3. Sparse Channel Estimation Algorithms

3.1. OMP Algorithm

Letting \( \hat{Z} \) be an estimate of \( Z, \psi = \sqrt{PA} \) is a reconstruction matrix/dictionary matrix, the stepwise process for the OMP algorithm \([16]\) is as follows: Let \( \Omega_k \) be a set of indices of the non-zero coefficients of \( Z, r \) is the residual, and \( k \) is the iteration count.

Step 1: Initialize \( \Omega_0 = 0 \), the residual \( r_0 = y_v \), the counter \( k=1 \)

Step 2: Find a column \( \lambda_k \) of \( \psi \) that is mostly correlated with the residual \( r_{k-1} \).

\[
\lambda_k = \arg \max_{j=1,\ldots,N} \psi^H r_{k-1}
\] (8)

Where \( \psi^H \) is Hermitian transpose of \( \psi \)

Step 3: Unite \( \lambda_k \) with \( \Omega_{k-1} \)

\[
\Omega_k = \Omega_{k-1} \cup \{ \lambda_k \}
\] (9)

Step 4: Obtain Least Squares Estimate of \( \hat{Z} \) using pseudo inverse

\[
\hat{Z}_k = (\psi_k^H \psi_k \Omega_k)^{-1} \psi_k^H y_v
\] (10)

Step 5: Update the residual:

\[
r_k = y_v - \psi \Omega_k \hat{Z}_k
\] (11)

Increment \( k \). Repeat steps 2 thru 5 until stopping criteria holds

Step 6: Return the vector \( Z \) with the components

\[
\hat{Z}(i) = \begin{cases} \hat{Z}_k(i) & \text{for } i \in \Omega_k \\ 0 & \text{Otherwise} \end{cases}
\] (12)

Step 7: Estimate the channel using

\[ \hat{H} = \psi \hat{Z} \] (13)

3.2. CoSaMP Algorithm

The stepwise procedure for CoSaMP algorithm \([17]\) is as follows:

Step 1: Initialize \( Z_0 = 0 \), the residual \( r_0 = y_v \), the counter \( k=1 \)

Step 2: Form proxy as

\[ e = \psi^H r_{k-1} \] (14)

Step 3: Identify large components

\[ \Omega = \text{supp}(e_{2K}) \] (15)

Where \( K \) stands for sparsity level.

Step 4: Merge the strongest support sets:

\[ T = \Omega \cup \text{supp}(Z_{k-1}) \] (16)

Step 5: Obtain least squares signal estimate

\[
b_T = (\psi_T^H \psi_T)^{-1} \psi_T^H y_v
\] (17)

\[
b_{T^c} = 0
\] (18)

Where \( T^c \) is compliment of \( T \)

Step 6: Prune \( Z_k \) and update the residual:

\[
Z_k = b_k
\] (19)

\[
r_k = y_v - \psi Z_k
\] (20)

Increment \( k \). Repeat steps 2 thru 6 until stopping criteria holds

Step 7: Estimate the channel using

\[ \hat{H} = \psi Z_k \] (21)
4. SVD PRECODING

The basic proposal of precoding is to decompose the channel into the transmitter precode setting matrix, equalizer matrix, and equivalent channel matrix that converts a MIMO is split into a series of parallel single-input single-output (SISO) channels.

The precoding based on SVD is used in this paper. The singular value decomposition of channel matrix $H$, can be written as

$$H = U \Sigma V^H$$

(22)

Where $H$ is the channel matrix, $U$ and $V^H$ are unitary matrices, and the diagonal elements of $\Sigma$ are Eigen values. The schematic block diagram of a MIMO system with precoding is shown in Figure.3

As the precoding matrix $F$ is given by $F = (V^H)^*$ the received signal can be expressed by

$$y = HFx + \eta = HVx + \eta$$

(23)

The channel can be equalized by multiplying $y$ with $U^H$ as shown in Fig.1. Then, the output $\tilde{y}$ is expressed

$$\tilde{y} = U^H HVx + U^H \eta$$

$$= U^H U \Sigma V^H Vx + U^H \eta$$

(24)

Since $U^H U = I$ and $V^H V = I$, $\tilde{y}$ can be rewritten as

$$\tilde{y} = \Sigma x + \tilde{\eta}$$

(25)

Where the noise $\tilde{\eta} = U^H \eta$

Eq.(25) indicates that each symbol is weighted by its Eigen value. After SVD on the estimated mm Wave channel, let

$$F_{est} = V(:,1:L)$$

(26)

$$W_{est} = U(:,1:L)$$

(27)

The number of paths is denoted by the letter L.

The precoding maximizes the rate, which can be expressed for mm Wave channel as

$$R = \log_2(\det(I_{L^2} + \frac{P}{L} W_{est}^H H_{est} F_{est}^H F_{est} H_{est} W_{est}))$$

(28)

5. RESULTS AND DISCUSSION

The performance of OMP and CoSaMP algorithms is given by MSE which is one of the performance metrics for outdoor mm Wave sparse channel estimation was evaluated through MATLAB simulations. In this simulation, $N_{BS} = 32, N_{MS} = 16$, $L=3$ $N=80$ are considered. The MSE performance of channel estimation using OMP and CoSaMP is shown in Figure.4. The spectral efficiency is evaluated using Eq. (28) for SVD precoding with OMP estimated channel and CoSaMP estimated channel. The performance of spectral efficiency for both estimated channels is shown in Figure.5. It is noticed from Figure.5 that the OMP algorithm is more accurate than the CoSaMP algorithm for channel estimation. Further, it is evident from Figure.5 that the OMP estimated channel yields better spectral efficiency than the CoSaMP estimated channel.

6. CONCLUSION

This paper has evaluated the performance of compressive sensing recovery algorithms OMP and CoSaMP through MATLAB simulations for mm Wave outdoor channel estimation. Simulation results have shown that the OMP algorithm outperforms the CoSaMP algorithm. Further, Spectral efficiency is evaluated using the OMP and CoSaMP estimated channels. It is observed that the OMP algorithm
realizes higher spectral efficiency than achieved with the CoSaMP algorithm.

7. FIGURES

![Analog beamforming](image1)

**Figure 1.** Analog beamforming

![Digital beamforming](image2)

**Figure 2.** Digital beamforming

![MIMO system with SVD precoding](image3)

**Figure 3.** MIMO system with SVD precoding

![SNR vs MSE performance Of OMP and CoSaMP for channel estimation](image4)

**Figure 4.** SNR vs MSE performance Of OMP and CoSaMP for channel estimation

![SNR vs Spectral efficiency](image5)

**Figure 5.** SNR vs Spectral efficiency
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