Parton showers with quantum interference

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Plan

- **Something old:**
  - A convenient notation for understanding parton showers.

- **Something new:**
  - A revised formulation at leading order.
• Use an example in which partons carry momenta, but no flavor, color, or spin.

• $\phi^3$ theory in six dimensions works for this.

• Also, just consider the evolution of the final state, as in electron-positron annihilation.

• For a generic description of shower MCs, use a notation adapted to classical statistical mechanics.
States

- State with $m$ final state partons with momenta $p$
  
  \[
  \{|p\}_m = |\{p_1, p_2, \ldots, p_m\}\rangle
  \]

- General state $|\rho\rangle$

- Cross section for the state to have $m$ partons with definite momenta ($\{p\}_m|\rho\rangle$)

- Completeness relation

  \[
  1 = \sum_m \frac{1}{m!} \int [d\{p\}_m] |\{p\}_m\rangle \langle \{p\}_m|
  \]
Measurement functions

- Measurement function \((F|)\)
- Cross section for \(F\)

\[
\sigma[F] = \sum_{m} \frac{1}{m!} \int [d\{p\}_m] (F|\{p\}_m)(\{p\}_m|\rho)
\]
- Totally inclusive measurement function \((1|)\)

\[
(1|\{p\}_m) = 1
\]
Evolution

- State evolves in resolution scale $t$.
- $t = 0$: hard; increasing $t$ means softer.
- Evolution follows a linear operator

$$|\rho(t)\rangle = U(t, t')|\rho(t')\rangle$$

- Evolution does not change the cross section

$$\langle 1|U(t, t')|\rho(t')\rangle = \langle 1|\rho(t')\rangle$$
Structure of evolution

\[ U(t_3, t_1) = \mathcal{N}(t_3, t_1) + \int_{t_1}^{t_3} dt_2 \, U(t_3, t_2) \, \mathcal{H}_I(t_2) \, \mathcal{N}(t_2, t_1) \]

\[ \mathcal{H}_I(t) = \text{splitting operator} \]

\[ \mathcal{N}(t', t) = \text{no change operator} \]

\[ \mathcal{N}(t', t)|\{p\}_m \rangle = \Delta(t, t'; \{p\}_m)|\{p\}_m \rangle \]
Probability conservation

\[ U(t_3, t_1) = \mathcal{N}(t_3, t_1) + \int_{t_1}^{t_3} dt_2 \, U(t_3, t_2) \, \mathcal{H}_I(t_2) \, \mathcal{N}(t_2, t_1) \]

\[ (1|U(t, t') = (1| \mathcal{N}(t', t)|\{p\}_m) = \Delta(t, t'; \{p\}_m)|\{p\}_m) \]

\[ 1 = \Delta(t_3, t_1; \{p\}_m) + \int_{t_1}^{t_3} dt_2 \, (1|\mathcal{H}_I(t_2)|\{p\}_m) \, \Delta(t_2, t_1; \{p\}_m) \]

\[ \frac{d}{dt_3} \Delta(t_3, t_1; \{p\}_m) = -(1|\mathcal{H}_I(t_3)|\{p\}_m) \, \Delta(t_3, t_1; \{p\}_m) \]

\[ \Delta(t_3, t_1; \{p\}_m) = \exp\left(-\int_{t_1}^{t_3} d\tau \, (1|\mathcal{H}_I(\tau)|\{p\}_m)\right) \]
\[ M(\{ \hat{p} \}_{m+1}) \approx M(\{ p \}_m) \times \frac{g}{2 \hat{p}_l \cdot \hat{p}_{m+1}} \]

\[ (\{ \hat{p} \}_{m+1} | \mathcal{H}_I(t) | \rho) = \sum_l \delta \left( t - \log \left( \frac{Q_0^2}{2 \hat{p}_l \cdot \hat{p}_{m+1}} \right) \right) \left[ \frac{g}{2 \hat{p}_l \cdot \hat{p}_{m+1}} \right]^2 (\{ p \}_m | \rho) \]
Showers with initial state

Showers develop in “hardness” time.

Real time picture

Shower time picture
• QCD is more complicated than scalar field theory.

• In typical parton shower algorithms, the main approximation is small angle or soft splitting.

• But color, spin, and quantum interference from soft gluon emissions not fully accounted for.
Approximations

- Interference between different graphs treated in “angular ordering” approximation.
- Color is treated in limit $1/N_c \to 0$.
- Average over parton spins.
Our aim

- Base parton shower on approximation of small angle or soft splitting.
- Eliminate the other approximations.
- Beware: there is no code and there are negative weights.
The matrix element

- The basic object is the quantum matrix element

\[ M(\{p, f\}_m)_{c_a, c_b, c_1, ..., c_m}^{c_a, c_b, c_1, ..., c_m} \]

- This is a function of the momenta and flavors and carries color and spin indices. Consider it as a vector in color and spin space

\[ |M(\{p, f\}_m)\rangle \]
The cross section

The cross section with a measurement function $F$ is then

$$\sigma[F] = \sum_m \frac{1}{m!} \int \left[ d\{p,f\}_m \right] \frac{f_{a/A}(\eta_a, \mu^2_F) f_{b/B}(\eta_b, \mu^2_F)}{4n_c(a)n_c(b) 2\eta_a\eta_b p_A \cdot p_B} \times \langle M(\{p,f\}_m) | F(\{p,f\}_m) | M(\{p,f\}_m) \rangle$$
The density matrix

\[ \sigma[F] = \sum_{m} \frac{1}{m!} \int [d\{p, f\}_m] \text{Tr}\{\rho(\{p, f\}_m)F(\{p, f\}_m)\} \]

where

\[ \rho(\{p, f\}_m) \]

\[ = \left| \langle M(\{p, f\}_m) \rangle \frac{f_{a/A}(\eta_a, \mu_f^2)f_{b/B}(\eta_b, \mu_f^2)}{4n_c(a)n_c(b)2\eta_a\eta_b p_{A} \cdot p_{B}} \right| \]

\[ = \sum_{s,c} \sum_{s',c'} \left| \langle s, c \rangle_m \rho(\{p, f, s', c', s, c\}_m) \langle s', c' \rangle_m \right| \]
Density matrix in “classical” notation

\[ \rho(\{p, f, s', c', s, c\}_m) = (\{p, f, s', c', s, c\}_m | \rho) \]

- For QCD partons have momenta and flavors.
- Furthermore, there are two sets of spin indices and sets of color indices.
- There are lots of indices, but the general formalism is the same as sketched earlier.
\[ M(\{\hat{p}, \hat{f}\}_{m+1}) \approx M(\{p, f\}_m) \approx H(\{\hat{p}, \hat{f}\}_{m+1}) + \cdots \]

this is an exact Feynman graph

approximation is here, the kinematics is an m body configuration
Soft gluon emission

Splitting includes interference graphs.

A soft gluon approximation is used for the splitting function.

Since we use the interference graphs, we do not need the angular ordering approximation.
We use a set of “string” basis states for color.

With this basis, splitting is simple.
Color approximations

- Implement a large $N_c$ approximation if desired.

An interference diagram, to be decomposed in basis states.

The leading contribution  
A subleading contribution.
Summary

The parton shower relies on the universal soft and collinear factorization of the QCD matrix elements. This should be the only approximation ...

... but we have some further approximations:

✗ Interference diagrams are treated approximately with angular ordering
✗ Color treatment is valid in the large $N_c$ limit
✗ Spin treatment is approximated.
✗ Usually very crude approximation for phase space
✗ “Hidden tricks”

Parton shower as classical statistical mechanics
Summary

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Parton shower as **Quantum statistical mechanics**

Parton shower as **classical statistical mechanics**