Abstract

The essential features underlying the top-down scenarios for UHECR are discussed, namely, the stability (or lifetime) imposed to the heavy objects (particles) whatever they be: topological and non-topological solitons, X-particles, cosmic defects, microscopic black-holes, fundamental strings. We provide an unified formula for the quantum decay rate of all these objects as well as the particle decays in the standard model. The key point in the top-down scenarios is the necessity to adjust the lifetime of the heavy object to the age of the universe. This ad-hoc requirement needs a very high dimensional operator to govern its decay and/or an extremely small coupling constant. The natural lifetimes of such heavy objects are, however, microscopic times associated to the GUT energy scale ($\sim 10^{-28}$ sec. or shorter). It is at this energy scale (by the end of inflation) where they could have been abundantly formed in the early universe and it seems natural that they decayed shortly after being formed.

Contents

1 Introduction 2
2 Topological solitons, non-topological solitons and heavy particles 2
3 Quantum Decay of Heavy Particles 3
4 Quantum Decay of Solitons 5
5 Quantum Decay of Fundamental Strings 5
6 Quantum Decay of Black Holes 6
1 Introduction

Ultrahigh Energy Cosmic Rays (UHECR) have been observed by a number of experiments at energies above $10^{20}$ eV [1]. The forthcoming cosmic rays detectors as the Auger array, the EUSO and OWL space observatories are expected to greatly improve our present knowledge on the UHECR gathered from Fly’s Eye, HiRes, AGASA and previous detectors [1, 2].

Top-down scenarii for UHECR are based on heavy relics from the early universe which are assumed to decay at the present time. In all cases, whatever the nature of the objects: heavy particles, topological and non-topological solitons, black-holes, microscopic fundamental strings, cosmic defects etc., one has to fine tune the lifetime of these objects to be the age of the universe.

We provide an unified description for the quantum decay formula of unstable particles which encompass all the above mentioned cases, as well as the particle decays in the standard model (muons, Higgs, etc). In all cases the decay rate can be written as,

$$\Gamma = \frac{g^2 m}{\text{numerical factor}}$$

where $g$ is the coupling constant, $m$ is the typical mass in the theory (it could be the mass of the unstable particle) and the numerical factor contains often relevant mass ratios for the decay process.

The key drawback of all top-down scenarii is the lifetime problem. The ad-hoc requirement of a lifetime of the order the age of the universe for the heavy particles implies an operator with a very high dimension describing the decay, and/or an extremely small coupling constant.

Heavy relics could have been formed by the end of inflation at typical GUT’s energy scales, but their natural lifetime would be of the order of microscopic times typically associated to GUT’s energy scales [20, 21].

UHECR may result from the acceleration of protons and ions by shock-waves in astrophysical plasmas (Fermi acceleration mechanism) [3]. Large enough sources can accelerate particles to the energies of the observed UHECR. Sources in the vicinity of our galaxy as hot spots of radio galaxies (working surfaces of jets and the inter galactic medium) and blazars (active galactic nuclei with relativistic jet directed along the line of sight) as BL Lacertae can evade the GZK bound [4, 5, 6].

2 Topological solitons, non-topological solitons and heavy particles

Stable solutions in classical field theory (as monopoles) become (heavy) particles in quantum field theory. There is no difference at the quantum level between heavy particles associated to a local field and those associated to classical stable solutions.
The stability of classical solutions in field theory is a highly nontrivial issue. There are basically two types of solutions: topological and non-topological. Topological classical solutions have associated a non-zero topological number (topological charge) which vanishes for the vacuum. If there is a lower bound for the energy of the solution involving this topological number the classical solution is stable. This is the case for kinks in one space dimension scalar theories, vortices in the two-dimensional Higgs model, monopoles in the three dimensional Georgi-Glashow model, Hopf solitons in appropriate three dimensional scalar models. In all known cases, classical stability comes together with quantum stability.

Gravitational analogues of these classical solutions exist in the euclidean (imaginary time) regime: they are black-holes in three space dimensions (with periodicity in the imaginary time), which are gravitational analogues of electric type monopoles and Taub-Nut’s in four space dimensions (gravitational analogues of magnetic type monopoles). The topological charges here are related to the temperature and magnetic charge of the solutions, respectively.

It must be stressed that the mere presence of a conserved topological number does not guarantee the stability of the corresponding classical solution. The energy must be related with the topological number in question such that a non-zero topological number implies a non-zero energy. Otherwise, a classical solution possessing non-zero topological number can decay into lighter particles.

In other words, the topological charge may be disconnected from the dynamics and it can decay in the course of the evolution. A topological soliton may collapse loosing its topological charge. This does not happen when the topological charge bounds the energy from below.

Non-topological solitons are stable thanks to a conserved \( U(1) \) charge of ‘electric’ type. Again, the mere presence of a conserved \( U(1) \) charge does not guarantee stability for charged particles except for the lightest one. Let us call \( m \) the mass of the lightest charged particle and let us take its \( U(1) \) charge as unit of charge. Assume that there are heavier particles with mass \( M > m \) and charge \( Q > 1 \) with \( M = M(Q) \). A sufficient condition for quantum stability is

\[
M(Q) < mQ,
\]

since a particle with charge \( Q \) and mass larger or equal than \( mQ \) can always decay into \( Q \) particles of mass \( m \) and unit charge respecting charge and energy conservation.

It must be stressed that in quantum theory all non-forbidden process do happen.

3 Quantum Decay of Heavy Particles

Typically, the decay of a heavy particle with mass \( m_X \) can be described by an effective interaction lagrangian formed by the local field \( X(x) \) associated to this heavy particle times the lighter fields in which it decays. Let us take the muon decay which is a well known case. Notice the mass of the muon \( m_\mu = 206.8 \, m_e \gg m_e \).

The effective Fermi lagrangian can be written as

\[
\mathcal{L}_I = -\frac{G_F}{\sqrt{2}} \bar{\psi}_\nu \gamma^\alpha (1 + \gamma_5) \psi_\mu \bar{\psi}_\nu \gamma_\alpha (1 + \gamma_5) \psi_\nu
\]

where \( \psi_\mu \) stands for the muon field, \( \psi_\nu \) and \( \psi_\nu \) for the electron neutrino and muon neutrino fields, respectively. The Fermi coupling \( G_F \) has the dimension of an inverse square mass.
The muon width $\Gamma_\mu$ describing the decay is then given by

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192 \pi^3}$$

The Fermi coupling can be related to the W-mass as follows

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 m_W^2}$$

where $g$, the standard model coupling, is dimensionless. Thus,

$$\Gamma_\mu = \frac{g^4 m_\mu}{6144 \pi^3} \left( \frac{m_\mu}{m_W} \right)^4$$  \hspace{1cm} (3)

As we shall see below, Eq.(3) has the generic structure for the decay width of an unstable particle.

For the muon decay, the monomial interaction in the effective lagrangian (2) has dimension six in mass units.

An analogous example is the Higgs decay into muons, neutrinos, $W^\pm$ and the $Z^0$. Notice that the Higgs mass $m_H$ must be higher than the $W^\pm$ mass $m_W$ and the $Z^0$ mass. The lagrangian as given by the standard model is here

$$2 g \sin \theta_W M_W H W^-_\mu W^\mu_\mu$$  \hspace{1cm} (4)

and a similar expression for the coupling with the $Z$. Here $\theta_W$ stands for Weinberg’s angle.

One finds for the Higgs decay rate[16],

$$\Gamma_{Higgs} = \frac{3 g^2}{128 \pi} m_H \left( \frac{m_H}{m_W} \right)^2$$  \hspace{1cm} (5)

where we consider for simplicity the case $M_H \gg M_W$. In this case the monomial interaction in the effective lagrangian (4) has dimension three in mass units.

Notice that in both cases, eq.(3) and eq.(5), the width grows as a positive power of the mass of the decaying particle.

Let us consider an effective lagrangian containing a local monomial of dimension $n$ (in mass units)

$$\mathcal{L}_I = \frac{g}{M^{n-4}} X \Theta.$$  \hspace{1cm} (6)

Here the field $X$ is associated to the decaying particle of mass $m_X$ and $\Theta$ stands for the product of fields coupled to it.

Then, the decay rate for a particle of mass $m_X$ takes the form

$$\Gamma = \frac{g^2}{\text{numerical factor}} m_X \left( \frac{m_X}{M} \right)^{|2n-8|}$$  \hspace{1cm} (7)

$\Gamma_\mu$ eq.(3) and $\Gamma_{Higgs}$ eq.(5) correspond to $n = 6$ and $n = 3$, respectively.
4 Quantum Decay of Solitons

The mass of classical soliton solutions (as magnetic monopoles in unified theories) are of the form

$$M_{sol} = \frac{\mu}{g^2}$$

where $\mu$ is the mass of the basic fields in the lagrangian and $g$ their dimensionless coupling. For small coupling these objects are much heavier than the particles associated to the basic fields in the lagrangian.

Quantum mechanically the soliton mass acquires corrections of order $g^0$ and higher. To one-loop level one finds

$$M_{sol} = \frac{\mu}{g^2} + \frac{1}{2} \sum_n \left[ \omega_n - \omega_0 \right], \quad (8)$$

where $\omega_n$ stands for the frequency of oscillations around the soliton. These oscillations are close but not identical to the frequency of oscillations around the vacuum $\omega_0$. The sum in eq.(8) yields a finite result proportional to $\mu$. \[10]\]

Now, if the classical solution is unstable some of the frequencies $\omega_n$ develops an imaginary part $i\mu \beta$ where $\beta$ is a pure number. Hence,

$$ImM_{sol} = \beta \mu \quad \text{and} \quad ReM_{sol} = \frac{\mu}{g^2} + \mathcal{O}(g^0) \quad (9)$$

and

$$\Gamma_{sol} = ImM_{sol} = g^2 \beta ReM_{sol} \quad (10)$$

We see that the width $\Gamma_{sol}$ has a similar structure than for heavy particles in the previous section.

The term $\mathcal{O}(g^0)$ in eq.(3) stand for the first quantum correction to the mass. Notice that we choose $\hbar = 1$ which is absorbed in $g^2$.

5 Quantum Decay of Fundamental Strings

The decay of closed strings in string theory has been computed to the dominant order (one string loop)[17]. Assuming the closed string in an $N$th excited state, it can decay into lower excited states including the graviton and the dilaton. The mass of this quantum string is given by

$$m^2 = 32\pi T N$$

where $T$ is the string tension $T = 1/(4\pi\alpha')$ and $\alpha'$ the string constant. The length of such string is given by $L = 2 \alpha' m$.

One finds for the total width for string decay[17],

$$\Gamma_{string} = \frac{\kappa^2 \sqrt{T N}}{\text{numerical factor}} \quad (11)$$

where the dimensionless coupling $\kappa$ is given by,

$$\kappa = 48\pi \sqrt{2GT}$$

The total width can be then rewritten as,

$$\Gamma_{string} = \frac{\kappa^2 m}{1083 \times \text{numerical factor}} \cdot (12)$$
This formula again has the same structure as the previous widths (7) and (10) once we identify \( g = \kappa, \quad m_X = ReM_S = m \).

Eq. (12) can be rewritten as,

\[
\Gamma_{\text{string}} = 42 \frac{GTm}{\text{numerical factor}} = 21 \frac{Gm^3}{16\pi \text{ numerical factor}} = 21\sqrt{2} \frac{\sqrt{N} G}{\alpha'^{3/2}} \quad (13)
\]

6 Quantum Decay of Black Holes

As it is known, in the context of field theory black holes decay semiclassically through thermal emission at the Hawking temperature\[18\]

\[
T_{BH} = \frac{\hbar c}{4\pi k_B} \frac{1}{R_s}, \quad R_s = \frac{2GM}{c^2}
\]

\((M\text{ being the black hole mass and } k_B\text{ the Boltzmann constant})\).

Black hole emission follows a ‘gray body’ spectrum (the ‘filter’ being the black hole absorption cross section \( \sim R_s^2 \)). The mass loss rate in this process can be estimated following a Stefan-Boltzmann relation,

\[
\frac{dM}{dt} = -\sigma R_s^2 T_{BH}^4 \sim T_{BH}^2
\]

where \( \sigma \) is a constant. Thus, the black hole decay rate is

\[
\Gamma_{BH} = \left| \frac{1}{M} \frac{dM}{dt} \right| \sim GT_{BH}^3 \sim \frac{G}{R_s^3}
\]

As evaporation proceeds, the black hole temperature increases till it reaches the string temperature\[19\]

\[
T_{\text{string}} = \frac{\hbar c}{k_B} \frac{1}{bL_s}, \quad L_s = \sqrt{\frac{\hbar \alpha'}{c}}
\]

\((L_s\text{ being the fundamental string length and } b\text{ a constant exclusively depending on the spacetime dimensionality and the string model chosen.})\). The black hole enters its string regime \( T_{BH} \rightarrow T_{\text{string}}, \quad R_s \rightarrow L_s \), becomes a string state and decays with a width

\[
\Gamma_{BH} \rightarrow G T_s^3 \sim \frac{G}{\alpha'^{3/2}} \sim \Gamma_{\text{string}}.
\]

Notice that this formula is similar to eq. (13) and again has the generic structure of the widths eqs. (7)–(10) and (12) if one identifies \( g = \kappa, \quad m_X = ReM_S = m \).

We consider here both fundamental strings and black holes since their decay rates can be nicely recasted as in eq. (9) independently of whether or not they may be considered as candidate sources of UHECR.

7 Particles Lifetime and the Age of the Universe

Heavy particles with masses in the GUT scale can be produced in large numbers during inflation and just after inflation\[20\]. The production mechanism is parametric or spinodal amplification in the inflaton field. That is, linear resonance of the quantum modes of the heavy field in the background or condensate of the inflaton. In addition, non-linear
quantum phenomena play a crucial role and can enhance the particle production. Such non-linear production is of the same order of magnitude as the gravitational production of particles by the time dependent metric.

Once these heavy particles are produced they must have a lifetime of the order of the age of the universe in order to survive in the present universe and decay into UHE cosmic rays. Only in the early universe the production of such heavy objects is feasible due to their large mass.

Moreover, in order to be the source of UHECR, these particles must have a mass of the observed UHECR, namely $m_X > 10^{21} \text{ eV} = 10^{12} \text{ GeV}$.

Let us assume that the effective lagrangian (6) describes the decay of the $X$ particles. Their lifetime will be given by eq.(7)

$$
\tau_X = \frac{\text{numerical factor}}{g^2} \frac{1}{m_X} \left( \frac{M}{m_X} \right)^{2n-8} = \frac{\text{numerical factor}}{g^2} \frac{1}{m_X} 10^{6(n-4)}
$$

where we set a GUT mass $M = 10^{15} \text{ GeV}$. The age of the universe is $\tau_{\text{universe}} \sim 2 \times 10^{10} \text{ years}$ and we have to require that $\tau_X > \tau_{\text{universe}}$. Therefore,

$$
10^{54} < \frac{\text{numerical factor}}{g^2} 10^{6(n-4)} \quad \text{or} \quad \log_{10} g < 3(n - 13) \quad (14)
$$

and we dropped the numerical factor in the last step.

For $g \sim 1$, eq.(14) requires an operator $\Theta$ with dimension at least thirteen in the effective lagrangian (6) which is a pretty high dimension.

That is, one needs to exclude all operators of dimension lower than thirteen in order to extremely suppress the decay. Clearly, one may accept lower dimension operators $\Theta$ paying the price of a small coupling $g$. For example: $g = 10^{-9}$ and $n = 10$ fullfil the above bound still being a pretty high dimension operator. Notice that a moderate $n$ as $n = 4$ lowers the coupling to $g \sim 10^{-27}$.

In summary, a heavy $X$-particle can survive from the early universe till the present times if one chooses

- an extremely small coupling $g$
  
  and/or

- an operator $\Theta$ with high enough dimension

None of these assumptions can be supported by arguments other than imposing a lifetime of the age of the universe to the $X$-particle. That is, the lifetime must be here fine tuned. That is, one has to build an ad-hoc lagrangian to describe the $X$-particle decay. Indeed, a variety of ad-hoc lagrangians have been proposed in the literature together with the symmetries which can adjust a wide variety of lifetimes.

It must be recalled that no known (weakly broken) symmetry protects the $X$-particle from decaying rapidly, except for supersymmetry. However, if supersymmetry would be invoked in this context, that would imply that the supersymmetry scale is at the GUT scale or beyond. It must be also noticed that the natural lifetime for particles of such a mass is the GUT scale, that is typically $10^{-28} \text{ sec.} - 10^{-35} \text{ sec.}$
8 Cosmic Defects and Heavy Particles

Closed vortices from abelian and non-abelian gauge theories are not topologically stable in 3+1 space-time dimensions. Static vortices in 3+1 space-time dimensions just collapse to a point since their energy is proportional to their length. They do that in a very short (microscopic) time.

It must be noticed that only a restricted set of spontaneously broken non-abelian gauge theories exhibit vortex solutions. For example, there are no topologically stable vortices in the standard $G = SU(3) \times SU(2) \times U(1)$ model in 3+1 space-time dimensions just because $\Pi_1(G)$ and $\Pi_2(G)$ are trivial for such group manifolds. [For a recent review see 25]. Grand unified theories may or may not possess vortex solutions in 2+1 space-time dimensions depending under which representations of the gauge group belong the Higgs fields.

The existence of cosmic string networks is not established although they have been the subject of many works. In case such networks would have existed in the early universe they may have produced heavy particles X of the type discussed before and all the discussion on their lifetime applies here. The discussion on the lifetime problem also applies to rotating superconducting strings which have been proposed as classically stable objects[24].

Cosmic strings are closed vortices of horizon size. In 3+1 space-time dimensions, strings collapse very fast except if they have horizon size in which case their lifetime would be of the order of the age of the universe. However, such horizon size cosmic strings are excluded by the CMB anisotropy observations and by the isotropy of cosmic rays. Such gigantic objects behaves classically whereas microscopic closed strings (for energies $< M_{\text{Planck}} = 10^{19} \text{ GeV}$) behave quantum mechanically.

In summary, a key point here is the unstability of topological defects in 3+1 space-time dimensions. Unless one chooses very specific models [8, 9, 11, 12, 13, 14], topological defects decay even classically with a short lifetime. They collapse to a point at a speed of the order of the speed of light in 3+1 space-time dimensions.

9 Decay Products of Heavy Particles and Final Conclusions

In summary, if the X-particles, whatever their origin and type could be made sufficiently stable to survive till now, then their decay products could provide the UHECR observed today. However, the X-particles lifetime of the order of the age of the universe must be imposed ad-hoc i.e. fine tuned while the natural lifetime for those particles should be extremely short about $10^{-28}$ sec at most.

Various GUTs contain candidates for X-particles of masses around the GUT scale ranging approximately from $10^{12} \text{ Gev}$ to $10^{16} \text{ Gev}$ depending on the model. These particles could have been produced naturally in the early universe typically by the end of inflation[20]. Analogously, topological defects, fundamental strings and primordial black holes could have been formed in the early universe. The hard job, however, is to have these heavy objects still present and decaying today. Instead of that, it seems more natural that the X-particles and the other heavy objects above mentioned decayed in the early universe shortly after being formed, having lifetimes corresponding to their respective energy scales. Their decay products will then form relic primordial backgrounds as graviton, neutrino and dilaton backgrounds, as we know now the relic photon CMB background. Those backgrounds could have characteristic detectable spectra and signatures containing
informations about the early universe.

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