Three-body model for the complete fusion of a two-cluster composite projectile with a heavy target

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The purpose of this work is to show that within a three-body description, the complete fusion process and the target excitation by the projectile can be taken into account by introducing a three-body optical potential in the formalism. We give a schematic description of such a potential and points to ways of testing the validity of the CDCC.

1. Introduction

The fusion of exotic nuclei with heavy targets has recently been the subject of intense experimental and theoretical investigation \cite{1}. Recently a formalism has been introduced \cite{2} in the study of the incomplete fusion process resulting in a better understanding of its physical characteristics as well as giving the possibility of unifying various, and in some cases somewhat conflicting approaches. In the model, consisting of a target $A$ and a projectile $a$ composed of two particles $b$ and $x$, the process is described by the optical potentials $U_{xA}$ and $U_{bA}$ of the systems $(xA)$ and $(bA)$, respectively, as well as the hermitian potential $V_{xb}$ which binds the structure-less particles $b$ and $x$ to form the projectile $a$. The incomplete fusion process appears directly connected to the absorptive (imaginary) parts of $U_{xA}$ and $U_{bA}$, denoted by $W_{xA}$ and $W_{bA}$, respectively. In this framework, the total absorption cross section is given by

$$\sigma_{abs} \propto \langle \phi_x^{(+)}|W_{xA}|\phi_x^{(+)} \rangle + \langle \phi_b^{(+)}|W_{bA}|\phi_b^{(+)} \rangle,$$

where each term of Eq. (1) represents the integration of the incomplete fusion cross section of observing the spectator particles $x$ and $b$. The wave functions $|\phi^{(+)}\rangle$ are source functions \cite{2}. The complete fusion cross section is obtained from $\langle \phi^{(+)}|W_{aA}|\phi^{(+)} \rangle$ after subtracting (1) and the elastic break-up contribution.

To be consistent, however, one should only consider $U_{xA}$ and $U_{bA}$, within a genuine three-body model \cite{3}. The three-body model, however, does not account for the complete fusion of the particle $a$ nor for the excitation of the target $A$ by the projectile. These processes are missing in the three-body model due, as we shall see, to the use of two-body optical potentials.

When calculating complete fusion, we have difficulty in evaluating consistently incomplete fusion (or inclusive breakup). The schematic figures below show the two processes.
On the other hand, when calculating incomplete fusion, one gets an underestimation of total fusion (= complete + incomplete fusion), because complete fusion is not included (difficult). Accordingly, calculation of fusion of a two-cluster projectile with a target nucleus is difficult.

2. Theories of Incomplete and Complete fusion

In the eighties, Austern et al. [4], Ichimura [5], Hussein and McVoy [6], Udagawa and Tamura [7] and others worked hard to formulate a practical and consistent three-body theory of inclusive breakup reactions (incomplete fusion). They did not address the question of complete fusion. Here we give some thoughts concerning this issue.

In the spectator model, cluster \( b \) in the projectile \( a = b + x \), only scatters optically from the target. It is proven by Austern et al. [4] that the incomplete fusion of the participant particle \( x \) is given by the energy and angle integrals of the spectator particle \( b \)'s spectrum

\[
\sigma_{IF} = \int d\Omega_b \frac{d^2\sigma}{d\Omega_b dE_b},
\]

(2)

where

\[
\frac{d^2\sigma}{d\Omega_b dE_b} = \rho(E_b) \int d\vec{r}_x \langle \Psi_{aA}^{(+)} | \chi_b^{(-)}(\vec{r}_x) W_{xA}(\vec{r}_x) (\chi_x^{(-)} | \Psi_{aA}^{(+)}) \rangle,
\]

(3)

\(|\Psi_{aA}^{(+)})\) is the full three-body wave function.

In the limit of \( \Psi_{aA}^{(+) \rightarrow \chi_{aA}^{(+) \rightarrow}} \), one gets the Hussein-McVoy theory [6] whose no-distortion limit (of \( b \)) is just the old Serber model. M.Ichimura clarified the connections among the different theories [5]. Notice that the source functions \(|\phi_x^{(+)}\rangle\) and \(|\phi_b^{(+)\rangle}\) of Eq.(1) are just \(\langle \chi_b^{(-)} | \chi_{aA}^{(+)}\rangle\) and \(\langle \chi_x^{(-)} | \chi_{aA}^{(+)\rangle}\).
In the Glauber limit,

\[ \sigma_{IF}^{(x)} = \frac{\pi}{k^2} \sum_{l_x} (2l_x + 1) \langle T_x(l_x)(1 - T_b(l_b)) \rangle \]  

(4)

Though not apparent, Yabana et al. hinges on the above (no b-target interaction) and thus they underestimate the total fusion.

To obtain complete fusion, one "guesses" the result using unitarity. This is not consistent three-body model of complete fusion.

On the other hand, to account for CF using Faddeev equations with non-hermitian coupling hamiltonians, one is bound to introduce an effective three-body optical potential. We turn to this question in what follows.

Within the Feshbach projection operator framework, the part of the optical potential that arises from the \( xA \) coupling is:

\[ \hat{U}_{xA} = V_{xA,x+}^+ G_{A+x} V_{xA,A+x} \]  

(5)

There is a similar potential for \( b \) (x taken as spectator)

\[ \hat{U}_{bA} = V_{bA,A+b}^+ G_{A+b} V_{bA,A+b} \]  

(6)

Accordingly a typical term in the \( a(=x+b) + A \) T-matrix:

Thus during the intermediate \( A + x \) or \( A + b \) propagation, \( x \) and \( b \) do not interfere with each other; \( x \) and \( b \) are never simultaneously absorbed by target: no complete fusion.
To include CF, the spectator particle (be it \(b\) or \(x\)), must be allowed to interact with the intermediate composite system. A possible candidate for three-body optical potential is:

\[
U_{Ax} = V_{xA}^+ G_{A+x}^+ V_{b}^+ G_{A+x+b} + V_{xA} V_{b} G_{A+x+b} G_{A+b} V_{b}^+ G_{A+b}^+ V_{b}^+ G_{A+b}^+ V_{b}.
\]  

Thus, with \(W_{xA}\), \(W_{bA}\) and \(W_{3B}\), being the imaginary potentials of the \(x + A\), \(b + A\) and \(x + b + A\) systems respectively, we have for incomplete fusion:

\[
\sigma_{IF} \propto \int \langle \Psi^+ | \chi_{b}^+ (\vec{r}_{x}) W_{xA} (\vec{r}_{x}) (\chi_{b}^- | \Psi^+) \rangle d\vec{r}_{x} \tag{8}
\]

or

\[
\sigma_{IF} \propto \int \langle \Psi^+ | \chi_{x}^+ (\vec{r}_{b}) W_{bA} (\vec{r}_{b}) (\chi_{x}^- | \Psi^+) \rangle d\vec{r}_{b} \tag{9}
\]

and the complete fusion

\[
\sigma_{CF} \propto \langle \Psi^+ | W_{3B} | \Psi^+ \rangle. \tag{10}
\]

Eqs.\(8\) and \(9\) have been analyzed in Ref.\cite{2}. It remains as a challenge the calculation of Eq.\(10\) \cite{9}.

3. Conclusions

Within a three-body, Faddeev, description of the fusion reaction one needs three-body optical potential (simplified version of \(\hat{U}_{3B}\)) to account for \(\sigma_{CF}\). Such a description will allow assessment of the CDCC model which consistently fails to account for the incomplete fusion \cite{10}.

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