Few-body reaction frameworks for the study of light nuclei

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Abstract. Three-body reaction formalisms, the exact Faddeev/AGS approach and the approximate reaction models such as CDCC, DWIA and Glauber multiple scattering approach are reviewed. The limits of the validity of the reaction approaches and recent developments and outlook are presented.

1. Introduction
One of the challenges of Nuclear Physics is the study of Nuclear Structure and its evolution along the nuclear landscape. Structure information from light nuclei has often been extracted from the study of its collision with light targets. It is now well established that for each target probe and for a given projectile energy, one has to identify what are the key excitation mechanisms and dynamical aspects that need to be incorporated in order to extract the relevant structure information from the data.

We consider here that the system under study is well described by a core (either in the ground or excited state) and a valence nucleon. The collision process reduces then to a three-body problem. Special attention will be payed to the case of breakup of light halo nuclei and nucleon knockout from the collision with a proton target. In this case one expects to extract structure information from both valence and inner shells.

In this contribution we shall make a review of the available state-of-the art three-body reaction frameworks and their application for the study of nucleon knockout from a stable/exotic nucleus and breakup of a loosely bound system such as a one-nucleon halo system. We shall give insight on their physics content and revise recent benchmark calculations.

2. The Faddeev/AGS multiple scattering framework
The standard Faddeev/Alt, Grassberger and Sandhas (Faddeev/AGS) [1, 2] is a three-body nonrelativistic reaction framework.

Let us consider the few-body scattering reaction where, in inverse kinematics, the composite system, a bound pair of particles (2,3) scatters from particle 1. In our main case of interest this corresponds to the scattering of a composite system (of a core C and a valence/knockout nucleon N) from a proton target (p).

The standard approach is formulated in the Hilbert space $H_{C+N+p}$ for $C+N+p$ free relative motion where the heavy fragment can only be either in the ground state or in a low-lying excited
state. Therefore, core dynamical excitations during the collision process, highly excited states above evaporation or multiple particle knockout (p, pNN' ..) are not taken into account in this truncated Hilbert space. This reaction formalism treats all open channels (elastic, breakup and transfer) simultaneously. In addition, all resonant and nonresonant states of each pair are automatically included.

In this approach, the transition amplitudes leading to the observables are the on-shell matrix elements of the operators $U^{\beta\alpha}$, calculated from the solution of the three-body AGS integral equations

$$U^{\beta\alpha} = \delta_{\beta\alpha} G_0^{-1} + \sum_{\gamma} \delta_{\beta\gamma} t_{\gamma} G_0 U^{\gamma\alpha},$$

(1)

with $\alpha, \beta, \gamma = (1, 2, 3)$. Here, $\delta_{\beta\alpha} = 1 - \delta_{\beta\alpha}$ and the two-body transition operator is

$$t_{\gamma} = v_{\gamma} + v_{\gamma} G_0 t_{\gamma},$$

(2)

with the free resolvent $G_0 = (E + i0 - H_0)^{-1}$, with $E$ the total energy of the three-particle system in the center of mass (c.m.) frame. The solution of the Faddeev/AGS equations at higher energies can be found by iteration

$$U^{\beta\alpha} = \delta_{\beta\alpha} G_0^{-1} + \sum_{\gamma} \delta_{\beta\gamma} t_{\gamma} G_0 G_0 U^{\gamma\alpha} + \sum_{\gamma} \delta_{\beta\gamma} t_{\gamma} \sum_{\xi} G_0 \delta_{\gamma\xi} t_{\xi} G_0 \delta_{\xi\alpha} + \cdots.$$  

(3)

The successive terms of this series can be considered as terms of zero order (which contribute only for rearrangement transitions), first order (single scattering), second order (double scattering) and so on in the transition operators $t_{\gamma}$. The $\beta = 0$ partition corresponds to three free particles in the continuum.

The breakup/nucleon knockout observables are calculated from the on-shell matrix elements of the AGS operators, $T^{\beta\alpha} = \langle \text{qp} | U^{\beta\alpha} | \psi_\alpha \rangle$, where particle $\alpha$ is the spectator in the initial state (in our case $\alpha$ is the proton target) and $\text{qp}$ are the Jacobi momenta in the final state

$$p = \frac{m_N K_p - m_p K_N}{m_p + m_N},$$

$$q = \frac{(m_p + m_N) K_C - m_C (K_p + K_N)}{M},$$

(4)
with $K_p$, $K_N$, $K_C$ ($m_p, m_N, m_C$) being the LAB momenta (masses) of the proton, valence/knockout nucleon and core in the exit system, and $M = m_p + m_N + m_C$. The kinematically fully exclusive observables are measured in the LAB system. The kinematic configuration of three-body final system is characterized by the polar and azimuthal angles $\Omega_i = (\theta_i, \phi_i)$ of the two detected particles which we take to be the valence/knockout nucleon and the proton target $p$. The kinematically fully exclusive cross section is then

$$\frac{d^3\sigma}{dK_N dK_p} = \sum_i \left| T^{0\alpha_i} \right|^2 \rho_i ,$$

(5)

where $\rho_i$ is a phase-space factor given by

$$\rho_i = (2\pi)^4 \left( \frac{(m_N + m_p)m_C m_N}{K_{LAB}} \right)^2 \frac{K_n^2}{\left| (m_N + m_C)K_N - m_N (K_{LAB} - K_p) \cdot \hat{K}_N \right|} ,$$

(6)

and the sum on $i$ involves the momenta $K_N$ given by the zero’s of the argument of the energy conserving $\delta$-function

$$E_{LAB} - \epsilon - \frac{(K_{LAB} - K_N - K_p)^2}{2m_C} - \frac{K_n^2}{2m_N} - \frac{K_p^2}{2m_p} = 0 ,$$

(7)

where $\epsilon = E^* - Q_0$ being $E^*$ the excitation energy of the heavy fragment, $Q_0$ the mass excess, $K_{LAB}$ is the beam momentum in the LAB frame and $E_{LAB}$ the corresponding energy.

In the standard Faddeev/AGS reaction framework, the single scattering amplitude, includes the term where the proton target scatters from the knockout nucleon ($p-N$) and from the heavy fragment ($p-C$). As one can see from the Fig. 1, the single scattering contribution where the proton target scatters from the heavy fragment (dashed-dotted line) is very significant. The case where one includes only the single scattering of the knockout nucleon by the proton target is standardly referred as the Plane Wave Impulse Approximation (PWIA). Within the Faddeev/AGS reaction framework, distortion effects with respect to the PWIA approximation result from the simultaneous combination of the $p-C$ single scattering term and of higher order multiple scattering contributions. From Eq. 5, it can be seen that formally the distortion effects do not factorize into a renormalization factor to the PWIA term. Nevertheless, a systematic study made in [3] has shown that, at high energies, there are subtle cancellations between the single scattering ($p-C$) and higher order multiple scattering terms such that (under the assumption of a single core state) a distortion parameter (defined as the relative difference between the PWIA and the full Faddeev/AGS total cross section) has a nearly logarithmic dependence on the separation energy of the knockout nucleon, for all considered values of the angular momentum of the knockout particle. A timely result was also given in Ref. [3]: the calculated ratio of the full to the plane wave total cross section was found to have a roughly linear trend when expressed as a function of the asymmetry parameter with a smaller slope than the ratio of the theoretical to the experimental total cross section for nucleon knockout found in the work of [15]. Further insight on the dependence of the distortion with respect to the angle of the light particles will be of interest for future experiments at Radioactive Ion Beam Facilities.

One key aspect that needs to be addressed is the validity of the choice of the truncated Hilbert space at a given energy regime. Standard developments of the Faddeev/AGS reaction approach assume a truncated Hilbert space for the three particle system, in particular it presupposes an inert core configuration. Recently, the Faddeev/AGS momentum-space framework for nuclear reactions has been extended to include also excitations of the core nucleus. In this case, the contribution of the transition amplitude for the scattering between the Core and the proton...
target induces the core excitation mechanism. Important core excitation effects were found at intermediate energies for the breakup of $^{11}$Be on a proton target at 64 MeV/u [4] and at low energies for the case of the transfer reaction $^{10}$Be$(d,p)^{11}$Be at $E_d = 15$ and 21.4 MeV leading to the ground state $1/2^+$ of $^{11}$Be [5].

3. The DWIA

The distorted-wave impulse-approximation (DWIA) reaction framework [6], can be viewed as an incomplete and truncated multiple scattering series [7], commonly used to analyze the knockout reaction $A(a,ab)B$ where an incident particle $a$ knocks out a nucleon or a bound cluster $b$ in the target nucleus $A$ resulting in three particles $(a,b,B)$ in the final state.

This reaction framework relies mainly in two basic assumptions: (i) that the projectile struck the ejected particle freely and (ii) that the 3-body wave function for the final state can be written as a factorized product of two wave functions that describe the $a + B$ and $b + B$ scattering $\eta_{Bab} \sim \eta_{aB}^{(+)} \eta_{bB}^{(+)}$. The transition amplitude is written as

$$T_{AB} = \langle \eta_{aB}^{(-)} \eta_{bB}^{(-)} | t_{ab} | \phi_{Bb}^{(+)} \eta_{aA}^{(+)} \rangle,$$

where the wave function for the relative motion of the particles in the entrance channel satisfy

$$(T_{aA} + V_{aA} - \epsilon_{aA}) \eta_{aA}^{(+)} = 0,$$

where $\epsilon_{aA}$ is the relative kinetic energy and $T_{aA}$ the relative kinetic energy operator. For the exit channel, the relative wave functions satisfy:

$$(T_{aB} + V_{aB} - \epsilon_{aB}) \eta_{aB}^{(+)} = 0,$$

and

$$(T_{bB} + V_{bB} - \epsilon_{bB}) \eta_{bB}^{(+)} = 0.$$

The potentials $V_{aB}$ and $V_{bB}$ are taken to be the optical potentials which describe the $a + B$ and $b + B$ scattering at energies $\epsilon_{aB}$ and $\epsilon_{bB}$ respectively. Further approximations are usually made in standard applications of the DWIA when evaluating the transition amplitude, namely (i) the neglect of the spin-orbit interactions in the distorting potentials (ii) the factorization approximation, which is only exact in PWIA (iii) the on-shell approximation of the transition amplitude. For $(p,pN)$ reactions this leads to the result

$$\frac{d^2\sigma}{dK_N dK_p} = F \sum_{\lambda} |P^\lambda(K_C,E)|^2 \frac{d\sigma}{d\Omega_{pN}}(E_{LAB},K_p,K_N)$$

where $F$ is a kinematic factor, $\lambda$ the relevant quantum numbers and the quantity $\sum_{\alpha} |P^\lambda(K_C,E)|^2$ represents the distorted momentum distribution of the knockout particle in the composite system. We note that extracting a spectroscopic factor from comparing with the data the calculated cross section using the DWIA framework relies on the validity of writing the transition amplitude as a product of a structure factor with a dynamical cross section (for the scattering between the proton and the knockout particle). This is an approximation, that needs to be tested and validated as discussed in detail in the previous section. In addition, one should also note that the distortion momentum distribution can only be uniquely determined from Eq. 12 if the ratio of the knockout cross section to the $p-N$ cross section is independent of the $p-N$ angle pair. Recent applications of the DWIA approach make further use of eikonal-forward approximation in the evaluation of the distorted momentum distributions [8, 9]. The eikonal approximation produces an additional truncation of the incomplete multiple scattering expansion to second order.
4. The continuum discretization reaction approach
The continuum discretized reaction method consists of an approximation to the Faddeev formalism. It makes use of a truncated model space where the three-body wave function is expanded in terms of all states of a given interacting pair.

In this reaction formalism, assuming that the projectile is well described by an inert core $C$ and a valence particle $v$, the Schrödinger equation is commonly solved in a model space in which, the three-body wave function is expanded in the internal states (bound and continuum resonant and nonresonant states) of the two-body projectile [10]. The exact three-body wave function satisfies the Schrödinger equation

\[(H - E)\Psi_{K_0}^{+}(R, r) = 0 ,\] (13)

where $H$ is the three-body Hamiltonian, $E$ the total c.m. energy of the system, $R$ the relative distance between the c.m. of the projectile and the target and $r$ the relative distance between the valence particle and the core. Finally $K_0$ is the incident wave number of the projectile in the c.m. frame.

The Hamiltonian $H$ for this system can be written as a sum of the relative motion between the projectile and target and the internal Hamiltonian of the projectile $H_{int}$,

\[H = T_R + \sum_{j=C,v} V_{jt}(R, r) + H_{int} \] where $T_R$ is the c.m. kinetic energy. The exact wave function is then expressed as an expansion in terms of the states (bound and bin) $\phi_\alpha(r)$ of the two-body Hamiltonian $H_{int}$

\[\Psi_{K_0}^{CDCC}(R, r) = \sum_{\alpha=0}^{N} \phi_\alpha(r) \omega_\alpha(R) \] (14)

where $\alpha=0$ refers to the projectile ground state. The bin states include both the resonant and the nonresonant part of the continuum. The wave functions $\omega_\alpha(R)$ of the projectile-target relative motion are solutions of the coupled-channels equations

\[(E_\alpha - T_R - V_{\alpha\alpha}(R)) \omega_\alpha(R) = \sum_{\beta \neq \alpha} V_{\alpha\beta}(R) \omega_\beta(R) ,\] (15)

where $E_\alpha = E - \varepsilon_\alpha$ and $V_{\alpha\beta}(R) = \langle \phi_\alpha | \sum_{j=C,v} V_{jt}(R, r) | \phi_\beta \rangle$ the coupling potentials. Alternatively for breakup/nucleon knockout the three-body wave function can be expanded in the p-N states [11, 12].

This formalism was extended recently to include core dynamical excitations [13, 14].

5. The adiabatic-eikonal Glauber formalism
The adiabatic-eikonal Glauber approach can be understood in terms of the multiple scattering Watson series [7] and makes use of two key approximations (i) the eikonal forward scattering approximation leading to a second order truncation of the series (ii) the adiabatic or sudden approximation which consists in replacing the internal hamiltonian of the few-body system by a constant $\bar{H}$. In this framework, the projectile resolvent $\hat{G}(z) = (z - H_0 - \sum V_{I,J})^{-1}$ where $V_{I,J}$ is the interaction between subsystems $I$ and $J$ of the composite projectile is replaced by

\[\hat{G}(z) \rightarrow G^{eik} = (z - K_P - \hat{H})^{-1} \] (16)

in the transition amplitude for the composite projectile cluster - target pair

\[\tau_{IJ}^{eik} = v_I \hat{G}^{eik} \tau_{IJ}^{eik} .\] (17)

This means that rescattering terms between the subsystem of the composite projectile are taken into account in an approximate way and the validity of this approximation needs to be assessed if one wants to extract accurate structure information from the data [7, 19].
Applications of this framework have been done for the analysis of inclusive target- and core- inclusive knockout data for light nuclei [15] (and references therein). In these analysis completeness of the bound and continuum two-body eigenstates of the composite projectile is used and it is further assumed that contributions from single particle excitations can be neglected [16].

6. Reaction formalisms validity barometer
To have a reliable interpretation of reaction data, as a first step one needs to include in the reaction formalism all the relevant excitation mechanisms, that is one has to include all the relevant degrees of freedom in the Hilbert space for the projectile-target system.

In addition to this, one also needs to include accurately all the dynamics of the reaction. Reaction methods often include approximations to the treatment of the dynamics. The validity of these approximations needs to be estimated.

Benchmark calculations for elastic, breakup, and transfer observables using the continuum discretized and the Faddeev/AGS standard reaction frameworks [11, 17] exhibit some discrepancies. The results indicate that for the case of elastic scattering and breakup of deuterons on $^{12}$C at $E_d=56$ MeV the continuum discretized is in agreement with the AGS/Faddeev three-body results. However, for deuteron breakup a large disagreement is found at the lower energy of $E_d=12$ MeV. For the p-$^{11}$Be test case, while for elastic scattering at 39 MeV/u the two methods are in good agreement, the calculated transfer cross section at 38.4 MeV/u using the continuum discretized reaction framework underestimates the result obtained from solving the Faddeev/AGS equations. For the breakup observables, good agreement between the two reaction approaches can be obtained in certain regions of the phase space, depending on the choice of basis used for the expansion of the wave function. It was also found that in the case of the transfer cross section for deuterons on $^{10}$Be the disagreement between the two reaction frameworks increases with energy.

In the work of Ref. [7] an attempt was made to bridge the exact standard Faddeev/AGS reaction framework with the approximate Glauber formalism for the case of elastic scattering. It was shown in this work that third order rescattering terms absent from the Glauber reaction formalism due to the eikonal approximation are important to describe elastic scattering. Further insight on the validity of the adiabatic approximation used in Glauber calculations is necessary, therefore benchmark calculations for other reaction channels need to be performed in the future.

In the work of Refs. [18, 19] an attempt was also made to bridge the exact standard Faddeev/AGS reaction framework with the approximate DWIA formalism. In that work it was shown that the DWIA transition amplitude can be approximately expressed in terms of a multiple scattering (MS) expansion series referred as DWIA-MS, and shown therein to be incomplete. The DWIA-MS containing multiple scattering terms up to second order, underestimate the corresponding second order Faddeev/AGS result by about 20%. The agreement found between the 2nd order DWIA-MS results and the converged Faddeev/AGS result, is unclear. Further work is needed to understand if there are cancellations between higher order multiple scattering terms or if this agreement is furtuitous.

Benchmark calculations between the two reaction frameworks are also needed in the future.

7. Conclusions and outlook
We have reviewed the theory of the standard Faddeev/AGS few-body reaction framework along with other standard formalisms (DWIA, CDCC and Glauber). When viewed as a multiple scattering expansion in terms of transition operators for the interacting pairs, these approximate reaction frameworks are described by truncated and incomplete multiple scattering series. In the Faddeev/AGS framework the sum of the series can be included. Benchmark calculations are needed to estimate the validity of approximate treatments of the dynamics of the reaction.
In addition, for the energy regime under consideration, the contribution of other excitation mechanisms such as core dynamical excitations during the collision process, highly excited states above evaporation or multiple particle knockout (p,pNN’..) should also be estimated.

Acknowledgments
The author would like to thank E. Cravo and A. Deltuva for reading this manuscript.

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