Sensor Networks TDOA Self-Calibration: 2D Complexity Analysis and Solutions

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Abstract—Given a network of receivers and transmitters, the process of determining their positions from measured pseudoranges is known as network self-calibration. In this paper we consider 2D networks with synchronized receivers but unsynchronized transmitters and the corresponding calibration techniques, known as TDOA techniques. Despite previous work, TDOA self-calibration is computationally challenging. Iterative algorithms are very sensitive to the initialization, causing convergence issues. In this paper, we present a novel approach, which gives an algebraic solution to three previously unsolved scenarios. Our solvers can lead to a position error < 1.2% and are robust to noise.

I. INTRODUCTION

Wireless Sensor Networks have been widely studied [19] and have been successfully applied to several domains, such as positioning [16], mapping [18], microphone array calibration [13] and beamforming [12]. In order to be properly used, the network must first be calibrated, i.e. the positions of its nodes need to be determined. This can be done e.g. using transmitters at a known position and trilaterating the network nodes. However, several applications require simultaneous localization of both receivers and transmitters [11]. This is known as network self-calibration [10] and it is the main theme of this paper.

Let us consider a network with $m$ receivers and $n$ transmitters, shortly denoted as $mn$. From the measurements $mn$ equations will be available to solve the self-calibration problem. Now, three calibration scenarios can be identified:

- **Synchronized RXs and TXs**: In this situation, the time instants at which the signal is transmitted and measured are known and hence the distances between RX and TX are measured. In total we will have $K(m+n)−G$ degrees of freedom (DoF), where $K$ denotes the spatial dimension ($K = 2, 3$) and $G$ is the Gauge freedom ($G = 3$ in 2D and $G = 6$ in 3D). Since we measure only distances, the positions can be recovered only up to a Euclidean transformation. Practically, this means that the coordinate systems in which we solve the coordinates can be chosen freely, reducing the degrees of freedom. This formulation is known as Time-Of-Arrival (TOA).

- **Synchronized RXs and unsynchronized TXs**: In this situation, all receivers will measure the time of arrival in the same clock frame. However, for each transmitter we will have one extra unknown, the time offset between the transmitter local clock and the receivers clock. The total number of degrees of freedom will thus be $K(m+n)+n−G$. This approach is known as Time-Difference-Of-Arrival (TDOA).

- **All RXs and TXs unsynchronized**: In this situation we can choose one clock as reference (say the first receiver) and all other devices will have one unknown clock offset, leading to $(K+1)(m+n)−G−1$ degrees of freedom. This is known as Unsynchronized Time Difference of Arrival (UTDOA).

This paper focuses on TDOA, particularly to the 2D case ($K = 2, G = 3$). The total number of DoF will thus be $2m+3n−3$. The configurations for which the number of DoF equals the number of equations are referred to as minimal configurations and the problems of network self-calibration with the minimum amount of receivers and transmitters are called minimal problems. The minimal configurations for 2D TDOA are 6r/3s and 4r/5s. Previous work developed algebraic solvers for non-minimal cases, such as 7r/6s, 5r/6s [7] and 8r/4s [14], however the minimal problems cannot be solved algebraically yet.

In this paper, we fill the gap towards minimal problems, proposing a new approach, able to solve three previously unsolved configurations, as summarized in Table I. Opposed to previous methods, where TDOA was tackled by first solving for the offsets and then solving the remaining TOA problem, our approach combines TOA and TDOA ideas and jointly solves both offsets and positions, reducing the overall computation load of the pipeline. Furthermore, we provide a quantitative estimate of the computational complexity of several unsolved situations. This paper focuses only on the 2D case. For 3D, the gap between the state-of-the-art and solving...
minimal solutions is even bigger. However, we also present a simplistic yet realistic geometrical configuration where 2D solvers can be used for 3D cases, allowing a significant reduction of computational complexity.

The paper is structured as follows: in Section II the current state of the art of TOA and TDOA solving techniques is reviewed, giving also an overview of the algebraic methods used to efficiently solve systems of polynomial equations. In Section III our proposed method is explained, and our solvers are benchmarked in Section IV. Finally, conclusions are drawn in Section V.

II. BACKGROUND

In this section, we review the algebraic formulations exploited in sensor networks self-calibration, focusing particularly on planar configurations. Given $m$ receivers and $n$ transmitters at unknown positions, the previously mentioned self-calibration scenarios can be mathematically formulated as follows:

- Time-of-Arrival (TOA): $\|r_i - s_j\| = f_{ij}$
- Time-Difference-of-Arrival (TDOA): $\|r_i - s_j\| + \alpha_i = f_{ij}$
- Unsynchronized-Time-Difference-of-Arrival (UTDOA): $\|r_i - s_j\| + \alpha_i + \alpha_j = f_{ij}$

For TOA problems, both receivers and transmitters are assumed to be synchronized and the measured pseudorange $f_{ij}$ between a receiver $r_i$ and a transmitter $s_j$ will correspond to the distance between them. For TDOA, the receivers are assumed to be synchronized but the transmitters are not (or vice versa), introducing an extra unknown bias term $\alpha_i$ for each transmitter. For UTDOA, neither transmitters nor receivers are synchronized, introducing offset terms $\alpha_i$ and $\alpha_j$ both for receivers and transmitters, respectively. In this section, we review how TOA and TDOA problems have been tackled so far. Further information regarding UTDOA can be found e.g. in [3].

The above mentioned equations could be solved by numerical iterative methods [13, 2]. These approaches, however, can suffer from several issues such as getting stuck in local minima, slow convergence and sensitivity to outliers. It has been showed that algebraic non-iterative approaches can achieve higher accuracies [6], [7], [1], [5]. Furthermore, using the algebraic solution as initial value for iterative methods allows a faster and more accurate convergence.

A. TOA solving techniques

From the measured distances $d_{ij}$ we can define the compaction matrix $\tilde{D} \in \mathbb{R}^{(m-1) \times (n-1)}$ such that

$$\tilde{D}_{ij} = d_{i+1,j+1}^2 - d_{i,j+1}^2 - d_{i+1,j}^2 + d_{i+1,j+1}^2.$$  (1)

With algebraic manipulation, it can be shown that the following factorization holds

$$\tilde{D} = -2R^T S,$$  (2)

where $R_i = [r_i+1 - r_i]$ for $i = 1 \ldots m - 1$ and similarly $S_j = [s_j+1 - s_j]$ for $j = 1 \ldots n - 1$.

Using SVD, a factorization $\tilde{D} = \tilde{R}\tilde{S}$ can be computed. Clearly, for each full-rank matrix $L$, we will have $\tilde{D} = \tilde{R}^T L^{-1}\tilde{L} S = \tilde{R}^T S$. The problem is now reduced to determine the matrix $L$ so that $\tilde{R} = L^{-T}\tilde{R}$ and $S = \tilde{L} S$. As the positions can only be determined up to a Euclidean transformation, we can fix the origin of the coordinate system imposing $r_1 = 0$, leading to the following parametrizations

$$s_i = Lb,$$

$$r_i = L^T R_{i-1}, \quad i = 2 \ldots m,$$

$$s_j = L \left( -\frac{1}{2}S_{j-1} + b \right), \quad j = 2 \ldots n,$$  (3)

where $b$ is a vector to be determined. Finally, defining $H = (L^T L)^{-1}$, the following equations can be derived

$$d_{i1}^2 = b^T H^{-1} b,$$

$$d_{ij}^2 - d_{i1}^2 = \frac{1}{4}S_{j-1}^T H^{-1} S_{j-1} - b^T H^{-1} S_{j-1},$$

$$d_{i1}^2 - d_{i1}^2 = \tilde{R}_{i-1}^T H \tilde{R}_{i-1} - 2b^T \tilde{R}_{i-1},$$  (4)

with $i = 2 \ldots m$ and $j = 2 \ldots n$. Since the matrix $H$ is symmetric, for the 2D case we will have five unknowns, three for $H$ and two for $b$, respectively. The equations in (4) will lead to polynomials of degree 3, 2 and 1 respectively. Given $m$ receivers and $n$ transmitters, one equation of type (A), $n-1$ equations of type (B) and $m-1$ equations of type (C) will be obtained. The minimal cases for this TOA formulation were studied and solved in [6], and for the 2D case a solution using a different parametrization was given in [17].

B. TDOA solving techniques

Similarly to TOA, the matrix $\tilde{D}$ can be constructed, observing that $d_{ij}^2 = (f_{ij} - \alpha_i)^2$. Now, however, $\tilde{D}$ will depend on the offset and hence it cannot be factorized numerically. The matrix has to be rank 2 in the 2D case. If $m > 3$ and $n > 3$, this means that $\tilde{D}$ is rank deficient. For a matrix larger than $2 \times 2$, this implies that all $3 \times 3$ subdeterminants should be equal to zero. Despite having in general $\binom{m-1}{3} \cdot \binom{n-1}{3}$ subdeterminants, the following theorem holds.

**Theorem 1**: Given a rank 2 matrix $A \in \mathbb{R}^{(m \times n)}$, $m, n > 2$ then $(m-2)(n-2)$ independent rank constraints can be obtained.

Since our compaction matrix is $\tilde{D} \in \mathbb{R}^{(m-1) \times (n-1)}$, given $m$ receivers and $n$ transmitters, $(m-3)(n-3)$ independent constraints can be obtained. This approach was used to solve some TDOA configurations in [7]. It is good to notice, however, that rank constraints alone cannot be used to solve the minimal cases of TDOA, not even in the two dimensional case. For the minimal problem 6r/3s, the compaction matrix will already have two columns, and hence no rank constraints can be derived. For the other minimal case, 4r/5s, only two independent constraints can be obtained, which is not enough to solve for the five unknown offsets.
C. Generating efficient solvers for polynomial systems

The formulations described above lead to systems of polynomial equations. Furthermore, once the problem we want to solve is fixed, the structure of the polynomial system will also be fixed, with only its coefficients changing from one instance to another. It is thus appealing to exploit algebraic geometry techniques to produce optimized solvers, instead of employing general numerical techniques. The technique used here involves computing the action matrix \( \mathbf{M} \) from the original system. The solutions of the system can then be extracted from the eigenvectors of the matrix \( \mathbf{M} \). The size of \( \mathbf{M} \), and hence the number of solutions, can be predicted computing the standard monomial basis \( \mathcal{B} \) of the ideal generated by the polynomials \([4]\). If the size of the standard monomial basis is \( n \) than \( \mathbf{M} \) will be an \( n \times n \) matrix. This is an important metric to roughly estimate the numerical stability of the problem, the bigger the size of \( \mathcal{B} \) the more sensitive to numerical issues the problem will be.

To obtain the action matrix from the original system, new polynomials need to be computed from the original ones. Determining the needed polynomials can be extremely time consuming, as it involves heavy symbolical computations. Luckily, the structure of needed polynomials depends only on the structure of the original problem, but not on the specific values of the coefficients. For this reason, we follow the strategy proposed in \([8]\), where a specific solver is automatically generated. This method consists of two phases:

- In an offline phase, the polynomials needed to obtain the action matrix are computed. These polynomials are stored as an elimination template, a matrix where each row corresponds to a polynomial and each column to a monomial. At the end of this phase, a program to compute the elimination template from the original data is generated. The template size, similarly to the standard monomial basis, can also be used to roughly predict the performance of the final solver. A bigger template will imply numerical manipulation of bigger matrices, making the solver more prone to numerical error propagation.

- In the online phase, the previously generated elimination template is used to quickly compute the action matrix. Finally, the solution of the system is extracted.

III. PROPOSED METHOD

In this section we describe the numerical techniques used to solve the problems arising in TDOA. The core idea is to use the factorization in \([2]\) to produce new equations depending on both offsets and coordinates. Opposed to previous TDOA approaches, we aim at solving all unknowns in one step. First, we show how a trivial factorization can be obtained when only three transmitters are present. Next, we show how this can be generalized to more transmitters. In all our formulations, we fix the Gauge freedom by imposing \( r_1 = 0 \) and \( r_2 = [r_{2x}, 0]^T \).

A. Three transmitters

If only three transmitters are available, no rank constraints can be imposed. However, it can be noticed that the following holds

\[
\mathbf{D} = (\mathbf{D}^T)^T \mathbf{I},
\]

where \( \mathbf{I} \) is the identity matrix. Hence, we can formulate the equations in \([3]\) imposing \( \mathbf{R} = \mathbf{D}^T \) and \( \mathbf{S} = \mathbf{I} \), obtaining \( m + 2 \) equations in \( 8 \) unknowns (for \( \mathbf{H} \) and \( \mathbf{b} \) and 3 for the offsets). The minimal case is, as previously shown, \( 6t/3s \). It is good to notice that now equations of type (C), despite being linear in \( \mathbf{H} \) and \( \mathbf{b} \), are overall of degree 3, as \( \mathbf{R} \) depends on the unknown offsets. For the subminimal cases \( (m > 6) \), we have more constraints than unknowns. This raises the question how should we pick the eight equations from the \( m + 2 \) available? To answer this, we compute the standard monomial bases of different formulations. We introduce the notation \( abc \) to denote the formulation using \( a \) equations of type A, \( b \) equations of type B and \( c \) equations of type C. The results of the simulation is shown in Table \([II]\).

TABLE II: Standard monomial bases of different formulations using 3 receivers.

| A | B | C | \( [s] \) |
|---|---|---|---|
| 0 | 0 | 8 | 75 |
| 0 | 1 | 7 | 116 |
| 1 | 0 | 7 | 160 |
| 1 | 1 | 6 | 198 |
| 0 | 2 | 6 | 144 |
| 1 | 2 | 5 | 181 |

As can be noticed from Table \([II]\) equations of type C lead to the lowest computational complexity and equations of type A to the highest computational complexity. Particularly, for the \( 9t/3s \) case, the problem can be solved using only equations of type C. The last row of the table corresponds to the minimal problem \( 6t/3s \). Based on this, the formulation \( 008 \) should be chosen to solve the \( 9t/3s \) case. Observing Table \([II]\) this new solver allows to calibrate a network using as little as three transmitters, whereas previous state-of-the-art solvers required at least four.

B. More than three transmitters

If we have more than three transmitters, than \( \hat{\mathbf{S}} \) will not be square and it cannot be directly replaced by the identity matrix. Thus, in order to use the factorization in \([5]\), we need to discard some transmitters. Particularly, we consider two cases:

- \( n = 4 \): the last transmitter can be discarded and receivers, offsets and the first two transmitters can be solved using the formulation of the previous section together with the rank constraints. Finally, the last transmitter can be solved by trilateration.

- \( n > 4 \): The transmitters can be grouped in triplets \( \{s_1, s_8, s_{i+1}\} \) for \( i = 2, 4, \ldots \). For each triplet, the factorization in \([5]\) can now be formed. It is important to note that now each triplet will lead to its own unknowns \( \mathbf{H}_i, \mathbf{b}_i \), i.e. five unknowns per triplet. Once \( \mathbf{H}_i \) and \( \mathbf{b}_i \) are solved, receivers and transmitters can be extracted.
with \( \mathbf{B} \). Next each triplet needs to be normalized by fixing the Gauge as described above. If \( n \) is even, then the last transmitter will be left out and solved at the end by trilateration.

In this paper two previously unsolved cases were solved:

- **6r/4s**: For this problem we will have five equations of type (C), two of type (B) and one of type (A). Furthermore, from the rank constraints we will obtain 10 equations, out of which only three are independent. The extra rank constraints do not add any additional information, but they do not contradict the previous equations either. Adding the redundant equations, however, can help reduce the computational complexity. Our final solver, containing all 10 rank constraints, all equations of type (C) and one equation of type (B) has \( |B| = 22 \). Using only three rank constraints would lead to \( |B| = 66 \).

- **6r/5s**: For this problem we will have 40 rank equations, out of which six are independent. As we have only five offsets, we can first solve for the offsets using the rank equations and next for the receivers and transmitters solving two linear systems from the five equations of type (C). For this formulation we obtained \(|B| = 6\).

### IV. Results

In this section, the experiments to quantitatively evaluate our solvers are presented. The solvers are benchmarked against synthetic data. The positions of the receivers and transmitters are sampled from a zero-mean normal distribution with standard deviation 10. The time offsets are generated from a standard normal distribution. Table III gives some technical details of the solvers. The size of the standard monomial basis directly defines the size of the action matrix. The bigger the action matrix, the more the numerical error will propagate when computing the eigenvectors. The elimination template is used to compute the action matrix. Practically, this step involves computing Gauss-Jordan elimination on the elimination template. Clearly, the bigger the elimination template, the more numerically challenging this step will be. As a rule of thumb [9], templates much smaller than \( \sim 200 \text{ms} \) can lead to very fast and stable solvers. Despite our solvers are at the edge of the feasible zone, our results show that the generated Matlab solver can still lead to accurate solutions and runs in \( \sim 200 \text{ms} \) on an Intel i7-8565U processor, thus being suitable for near-real-time applications.

In addition to the three proposed solvers, we also computed the standard monomial basis for configurations between the previously solved and the minimal ones, using the approach described in this paper. The results, depicted in Table IV, can be used to roughly assess the computational complexity of the unsolved cases, giving hints on their feasibility.

#### A. Clean data

To evaluate our solver, we randomly generate input data and solve the self-calibration problem using our proposed solvers. At this step, we consider noiseless data. The relative error distributions, obtained running the solvers 5000 times with different random data, are shown in Fig. 1 and the median relative errors in Table V. As can be observed, the solver r6/s5 alone itself is already stable, with the relative error being on average \( 10^{-11} \). The r6/s4 and r9/s3 solvers, due to their huge template size, appear to be more prone to numerical stability issues. This issue can be however easily removed by refining the initial estimate with nonlinear optimization (NO). As Table V and Fig. 1 show, our solver+NO can lead to a very accurate estimate. Furthermore, using the solution of our solver as a starting point, the NO algorithm converges already after very few iterations.

### IV. Results

#### B. Noisy data

We also investigate how our solvers perform with noisy data, adding zero-mean gaussian noise with varying standard deviation \( \sigma \) to the measurements \( f_{ij} \). To solve the problem, we first determine an initial estimate using our solver and then perform nonlinear optimization using Levenberg-Marquardt algorithm. The results of the simulation are shown in Fig. 2. TDOA problems are sensitive to the choice of the initial value. Indeed, with random initialization the nonlinear optimization algorithm fails to converge even for small noise levels. Using our solver removes this problem, allowing fast and accurate convergence, even at higher noise levels.

#### C. Degenerate configurations

The geometry of receivers and transmitters can also affect the stability of the solver. Particularly, some configurations, referred to as degenerate configurations can cause the solver to fail. For the 2D case studied in this paper, a trivial degenerate configuration is the case where all receivers and transmitters are collinear.

| Solver       | Position error | Offset Error |
|--------------|----------------|--------------|
| 9r/3s+NO     | 1.2%           | 6.3%         |
| 9r/3s        | 0.25%          | 0.13%        |
| 6r/4s+NO     | 10−15          | 10−14        |
| 6r/4s        | 10−15          | 10−14        |
| 6r/5s        | 10−11          | 10−10        |

### Table III: Technical details of the generated solvers.

| Solver | Template Size | | |
|--------|---------------|---|---|
| 9r/3s  | 2744x2819     | 75|   |
| 6r/4s  | 1005x1027     | 22|   |
| 6r/5s  | 39x45         | 6 |   |

### Table IV: Complexity estimate of different TDOA configurations. X: solved in 141. -: unsolved.

| m/n   | 3 | 4 | 5 | 6 |
|-------|---|---|---|---|
| 4     |   |   |   |   |
| 5     |   |   |   |   |
| 6     |   |   |   |   |
| 7     |   |   |   |   |
| 8     |   |   |   |   |
| 9     |   |   |   |   |
lie on a line. A more interesting, non-trivial degenerate configuration is obtained when all receivers and transmitters lie on a conic, such as a circle, ellipse, hyperbola or parabola. As an example, the error distribution when the points lie on a parabola is shown in Fig. 3. Comparing this to the histograms in Fig. 1 the significant loss in numerical stability can be noticed.

D. 2D Solvers for 3D Geometry

The cases considered so far were limited to a 2D case. In this section, we present a simplistic yet realistic 3D geometry, which can be reduced to 2D, significantly decreasing the computational complexity of the problem. We consider the case where all receivers lie on a plane Π₁ and all transmitters on a plane Π₂ parallel to Π₁, so that the distance between the planes is $h > 0$. We fix the coordinate system so that all receivers have $z = 0$ and all transmitters have $z = h$. In this situation, the distance between a receiver $r_i$ and a transmitter $s_j$ can be rewritten as

$$d_{ij} = \left( f_{ij} - o_j \right)^2 = d_{ij}^2 + h^2,$$

where $d_{ij}'$ is the projection of $d_{ij}$ to the plane $\Pi_1$. Substituting this into equations of type (B) and (C) in (5), it can be noticed that the resulting equations do not depend on $h$. Thus, the offsets and the $x$ and $y$ coordinates can be solved using the 2D solvers described above. Finally, the missing unknown $h$ can be solved from the original equations. The performances of our solvers for 3D data are shown in Fig. 4

V. CONCLUSIONS AND FUTURE WORK

In this paper we considered the sensor network self-calibration problem from TDOA measurements. Focusing on the 2D case, we proposed a novel algorithm which led to new robust and efficient polynomial algebraic solvers. We showed that the solutions obtained with our approach are stable both for clean and noisy data. Moreover, we showed how our 2D solvers can also be applied in some 3D configurations. Despite the improvement in this paper, the minimal cases remain unsolved. Future work will focus on investigating alternative...
formulations and algebraic techniques to generate solvers for the still unsolved cases.

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