Research Article

Evaluation Approach of Rock Brittleness Index for Fracturing Acidizing Based on Energy Evolution Theory and Damage Constitutive Relation

Shaobo Jin,1 Xin Wang,2,3 Zhen Wang,2,3 Shaoyuan Mo,2,3 Fengshou Zhang,4 and Jizhou Tang1

1State Key Laboratory of Marine Geology, Tongji University, Shanghai, China
2Research Institute of Petroleum Exploration & Development, PetroChina, Beijing, China
3The Key Laboratory of Reservoir Stimulation, PetroChina, Langfang, China
4Department of Geotechnical Engineering, College of Civil Engineering, Tongji University, Shanghai, China

Correspondence should be addressed to Jizhou Tang; jeremytang@tongji.edu.cn

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Acidizing, as an essential approach for well stimulation of sandstone or carbonate reservoirs, greatly affects the brittleness of the rock mass. Therefore, it is of great significance to develop a scientific brittleness evaluation methodology for the acid-corroded rock. In this paper, firstly, a damage constitutive model considering the compression hardening process of the acid-corroded sandstone under uniaxial loading is established and verified. Then, the evolution formulae of the relevant mechanical and fitting parameters are derived, and the stress-strain curve of the sandstone subjected to acid corrosion with soaking time is predicted. Finally, a theoretical model for evaluating the brittleness index (BI) of the acid-corroded sandstone based on energy evolution theory and damage constitutive relation is proposed. Based on this model, the BI of the sandstone subjected to acid corrosion is calculated and analyzed, and the BI of the acid-corroded sandstone with the soaking time is predicted. Results show that the BI of the sandstone is negatively correlated with the soaking time, and the rate of descent of the BI decreases with the increment of the soaking time. In addition, the decline degree of the BI has a negative correlation with the pH value. On the other hand, the temperature (25°C, 50°C, and 75°C) has greater weakening effects on the BI compared with the impact of the pressure (5 MPa, 10 MPa, and 15 MPa). Besides, days 50 and 120 are two turning points where the decreasing rates of the BI change from rapid to slow and slow to almost constant, respectively. Furthermore, a coefficient (θ) is proposed to quantify the effect of the acid corrosion, and some suggestions are provided for the application of the acidizing treatment.

1. Introduction

As a crucial parameter in evaluating the fracability of the target formation [1], rock brittleness becomes decisive in detecting engineering sweet spots, which enables to guide fracturing design. In the aspect of well stimulation for sandstone or carbonate reservoirs, matrix acidizing is regarded as an essential approach. This technique is to inject acid into the formation and then dissolve the matrix [2], which further improves the formation permeability and reduces the breakdown pressure during the fracturing treatment [3–5]. Acid corrosion causes the change of the microstructure and mineral compositions, which has weakening effects on the physical and mechanical properties of rocks [3]. Rock brittleness, as a parameter determined by the aforementioned rock properties, would be probably affected by the acidizing, and thus, it is highly worthwhile to develop a scientific BI evaluation methodology for acid-corroded rocks.

In recent years, numerous studies have been carried out to analyze the influence of acid corrosion on rocks. For example, Gou et al. [6] conduct true triaxial fracturing experiments on carbonate rocks and observe that viscoelastic surfactant acid makes the hydraulic fracture more conducive to connecting the nature fractures. Guo et al. [7] further
carry out a series of hydraulic fracturing experiments and find that etching wormholes induced by the rock-acid reaction would change the pore structure of carbonate rocks, and gelled acid enables to increase both the width and the roughness of main fracture surfaces. Based on freeze-thaw cycles tests on sandstone immersed in different chemical solutions, Li et al. [8] define a pore evolution law of sandstone and find that the porosity alteration is the most obvious in the first ten freeze-thaw cycles. Huang et al. [9] state that iron ions and magnesium ions provided by the dissolution of ankerite in the acid fluid can promote the formation of chlorite, which is likely to block the pore throats and thus decrease the rock permeability. Besides, Yuan et al. [10] carry out dry-wet cycles on sandstone and claim that the deterioration of the shear strength of sandstone in acid environment is more obvious than that in alkaline environment. Yao and Zhang [11] perform triaxial compression tests on limestone soaked in acidic solutions with different pH values and conclude that the deterioration level of the limestone strength increases with the decrement of pH value with the same saturated time and confining pressure. Jiang et al. [12] find that the acid mine drainage can change the interior molecule structure and the framework composition, which degrades the cohesion and the angle of internal friction of the sandstone. Additionally, alterations of mortar mass [13] and longitudinal wave velocity [7, 14] of rocks are also studied. From aforementioned studies, it is universally accepted that acid corrosion has significant weakening effects on the physical and mechanical properties of rocks. Thus, rock brittleness would be seriously affected after the acid corrosion.

So far, more than 20 definitions of rock BI have been proposed. However, there is no uniformly accepted definition. The existing definitions can be classified into five categories: (a) mineral compositions [Jarvie et al. [15], Jin et al. [16], Rybacki et al. [17]), (b) elastic parameters (Ai et al. [18], Rickman et al. [19], Rybacki et al. [20]), (c) strength parameters (Zhou et al. [21], Xia et al. [22], Wang et al. [23]), (d) strain parameters (Hucka and Das [24], Rahimzadeh Kivi et al. [25], Munoz et al. [26], Geng et al. [27]), and (e) energy (Tarasov and Potvin [28], Zhang et al. [29], Li et al. [30]). Each definition has the limitation due to the fact that some influential factors are not considered. For instance, in the group (a), Jarvie et al. [15] propose a BI based on mineral compositions, and the influence of diagenesis on BI is not taken into consideration; in the group (b), Rickman et al. [19] believe that the BI increases with the increment of elastic modulus and the decrement of Poisson’s ratio, only within the prepeak stress stage; in the group (c), Wang et al. [23] reestablish a BI based on the initial rupture stress, and the influence of rock deformation characteristics is not considered; in the group (d), Hucka and Das [24] define a BI as the ratio of the elastic strain to the total strain, only within the prepeak stress stage; in the group (e), Tarasov and Potvin [28] only take into account the energy behavior of rocks at the postpeak stress stage. Researchers have made great efforts to define the BI of rocks, while by far the investigation regarding acid corrosion effects on the BI of sandstone is relatively rare.

Compared to the BI based on mineral compositions and mechanical parameters, the energy-based BI is reliable to the evaluation of rock brittleness because the deformation and failure of the rock are accompanied by a complex process of energy absorption and release, and the fracture is essentially the structural instability induced by the released energy. The energy evolution of the rock deformation and failure can be characterized by the stress-strain curve. In this paper, a damage constitutive model is established and verified, which shows a good agreement with the stress-strain curve of the acid-corroded sandstone. This model can accurately present the energy evolution of the acid-corroded sandstone during the uniaxial loading. Therefore, a theoretical model for evaluating the BI of acid-corroded sandstone based on energy evolution theory and damage constitutive relation is proposed. The BI of the sandstone subjected to acid corrosion is calculated and analyzed by using this model. Additionally, the BI of the sandstone subjected to acid corrosion with the soaking time is predicted. Finally, both the effect of the acid corrosion related to the decreasing rate of BI and the strength of the sandstone are discussed. Our research can provide a guideline for the application of matrix acidizing treatment in sandstone reservoirs.

2. Damage Constitutive Model

2.1. Model Establishment. The rock damage can be categorized into the acid-corroded damage and the loading damage. The acid-corroded damage is reflected in the alteration of the static elastic modulus, and the loading damage can be expressed based on the statistical theory of the micounit strength. In this section, firstly, a classical damage constitutive equation under uniaxial loading is derived by assuming that the micounit strength of the rock satisfies the Weibull distribution. Then, the damaged static elastic modulus is substituted into the classical damage constitutive equation to replace the initial static elastic modulus of rocks. Besides, a compaction hardening coefficient is proposed and introduced into the classical damage constitutive equation. Finally, the damage constitutive equation of acid-corroded rock under uniaxial loading is established.

Before modelling, the rock is assumed to be a homogeneous and isotropic material on a macro scale, and the micounit is assumed to have the following properties: (1) the first one is failure or intact in which the micounit has the property of 0-1 distribution; (2) the micounit exhibits linear elasticity before failure; and (3) the strength of each micounit is different, but it obeys some distributions. Assuming that the micounit strength of the rock satisfies the Weibull distribution, then the corresponding probability density formula can be written as

\[ P(W) = \frac{m}{a} \left( \frac{W}{a} \right)^{m-1} \exp \left[ - \left( \frac{W}{a} \right)^{m} \right], \]

where \( W \) denotes the micounit strength, and \( m \) and \( a \) represent the shape parameter and the scale parameter of the Weibull distribution, respectively.
According to the maximum-tensile strain yield criterion, the microunit strength variable \( W \) can be replaced by \( \varepsilon \).

The probability density formula can be written as

\[
P(\varepsilon) = \frac{m}{a} \left( \frac{\varepsilon}{a} \right)^{m-1} \exp \left[ -\left( \frac{\varepsilon}{a} \right)^{m} \right],
\]

where \( \varepsilon \) is the strain of the rock.

The damage variable of the rock \( D_L \) under a certain loading can be defined as the ratio of the number of failed units, \( N_F \), to the total number of microunits, \( N \).

\[
D_L = \frac{N_F}{N}.
\]

Substituting Equation (4) into Equation (3), then the evolution equation of the loading damage variable can be written as

\[
D_L = 1 - \exp \left[ -\left( \frac{\varepsilon}{a} \right)^{m} \right],
\]

where \( D_L \) is the loading damage variable.

Assuming that the microunit strength obeys the generalized Hooke’s law, then we have

\[
\sigma = (1 - D_L)E_0\varepsilon,
\]

where \( E_0 \) is the static elastic modulus of rock (has not been treated with acid corrosion).

**Figure 1:** Comparison between the fitting curves and the measured data [31] of the sandstone soaked in acid solutions with different pH values and soaking time. (a) pH = 1 H\(_2\)SO\(_4\). (b) pH = 3 H\(_2\)SO\(_4\). (c) pH = 5 H\(_2\)SO\(_4\). (d) pH = 7 H\(_2\)O.
Substituting Equation (5) into Equation (6), the classical damage constitutive equation of fresh rock under uniaxial loading can be written as

\[\sigma = E_0 \varepsilon \cdot \exp \left( -\frac{\varepsilon}{\varepsilon_e} \right)^m.\]  

(7)

The acid-corroded damage variable of rock can be characterized by the static elastic modulus, and it can be written as

\[D_T = 1 - \frac{E_T}{E_0},\]  

(8)

where \(D_T\) is the acid-corroded damage variable and \(E_T\), as the static elastic modulus of acid-corroded rocks, denotes the slope of the stress-strain curve between 40% and 60% of the rock strength.

Considering the acid-corroded damage, the constitutive equation of acid-corroded rock under uniaxial loading can be obtained by replacing \(E_0\) with \(E_T\):

\[\sigma = E_T \varepsilon \cdot \exp \left( -\frac{\varepsilon}{\varepsilon_e} \right)^m.\]  

(9)

Nevertheless, the compaction hardening process at the initial stage under uniaxial compression loading is not considered in the above constitutive equation (the fitting curve of the classical damage constitutive increases linearly at the initial compression hardening stage). It is known to us that both the pore structure and microcracks within the rock are changed after acid corrosion, and an obvious concave increment becomes obvious at the initial compression hardening stage. Thus, a compaction hardening coefficient defined as the ratio of the slope of the stress-strain curve to the elastic modulus is proposed:

\[\beta = \log \left( \frac{(n-1)\varepsilon_e}{\varepsilon_e} + 1 \right),\]  

(10)

where \(\beta\) is the compaction hardening coefficient, \(n\) is the constant obtained by fitting the experiment data (related to the density of rock), and \(\varepsilon_e\) is the strain of rock corresponding to the stress at the endpoint of the compaction hardening stage.

By introducing \(\beta\) into Equation (9), the damage constitutive model is derived as

\[\sigma = \log \left( \frac{(n-1)\varepsilon_e}{\varepsilon_e} + 1 \right) \cdot E_T \varepsilon \cdot \exp \left( -\frac{\varepsilon}{\varepsilon_e} \right)^m.\]  

(11)

### 2.2. Model Verification

The experimental data of the sandstone from three references are cited and discussed to verify the feasibility and accuracy of the above proposed damage constitutive model. The details are listed as follows.

#### 2.2.1. Li et al. [31] Uniaxial Compression Test Data.

Li et al. [31] carry out a series of uniaxial compression tests on acid-corroded sandstone specimens. Before conducting the uniaxial compression tests, the sandstone specimens are immersed in sulfuric acid solutions with different pH values (i.e., pH1, pH3, pH5, and pH7) and soaking time (i.e., 30 days, 90 days, and 180 days). The damage constitutive model is used to fit the measured data from experiments, and the comparisons between the fitting curves and measured data are shown in Figure 1. In this figure, the compression hardening process and the strain at the stress peak of the sandstone are positively correlated with the soaking time, while the compression strength decreases with the increment of the soaking time.

| Soaking solution | Soaking time (T) (days) | Mechanical parameters | Fitting parameters | R² |
|------------------|-------------------------|-----------------------|-------------------|----|
|                  |                         | \(E_T\) (GPa) | \(\varepsilon_e\) \times 10^{-3} | \(n\) | \(a\) \times 10^{-3} | \(m\) |
| \(\text{H}_2\text{SO}_4\) (pH = 1) | 30 | 13.896 | 2.041 | 67.525 | 5.880 | 34.868 | 0.997 |
|                  | 90 | 5.499 | 4.312 | 28.490 | 13.250 | 27.617 | 0.997 |
|                  | 180 | 2.924 | 7.33 | 10.472 | 22.440 | 18.267 | 0.999 |
| \(\text{H}_2\text{SO}_4\) (pH = 3) | 30 | 15.966 | 1.889 | 77.258 | 5.610 | 32.419 | 0.999 |
|                  | 90 | 6.855 | 4.001 | 32.353 | 12.030 | 25.503 | 0.996 |
|                  | 180 | 5.213 | 5.387 | 11.790 | 15.850 | 14.780 | 0.999 |
| \(\text{H}_2\text{SO}_4\) (pH = 5) | 30 | 23.026 | 1.425 | 85.395 | 5.070 | 29.358 | 0.999 |
|                  | 90 | 8.585 | 3.070 | 37.410 | 10.550 | 27.354 | 0.999 |
|                  | 180 | 6.622 | 3.871 | 19.083 | 13.730 | 14.831 | 0.998 |
| \(\text{H}_2\text{O}\) (pH = 7) | 30 | 22.036 | 1.426 | 85.395 | 5.070 | 29.358 | 0.999 |
|                  | 90 | 14.737 | 2.432 | 43.396 | 7.630 | 26.301 | 0.998 |
|                  | 180 | 10.409 | 3.140 | 25.079 | 10.760 | 11.719 | 0.999 |
| Nature | 0 | 27.812 | 0.594 | 179.728 | 4.270 | 40.424 | 0.997 |
The fitting curves agree well with the experimental data, and the compaction hardening processes are accurately exhibited, which illustrates that the damage constitutive model can accurately describe the mechanical behavior of the sandstone under uniaxial compression loading. The related mechanical parameters and fitting parameters for the damage constitutive model of sandstone soaked in sulfuric acid solutions with different pH values and soaking time are listed in Table 1.

2.2.2. Huo et al. [32] Uniaxial Compression Test Data. Huo et al. [32] conduct uniaxial compression tests on sandstone specimens subjected to acid corrosion with different concentrations of sulfuric acid solutions (i.e., pH1, pH3, pH5, and pH7) and soaking times (i.e., 30 days, 60 days, 90 days, and 180 days). The established damage constitutive model is used to fit the measured data from experiments. The fitting results of the measured data from Huo et al. [32] are similar to those from Li et al. [31]. The comparisons between the fitting curves and measured data are shown in Figure 2, and the related mechanical parameters and fitting parameters for the damage constitutive model of sandstone soaked in sulfuric acid solutions with different pH values and soaking time are listed in Table 2.

2.2.3. Li et al. [33] Uniaxial Compression Test Data. According to the uniaxial compression tests on sandstone specimens subjected to acid corrosion (pH3 H2SO4) with different temperatures (i.e., 25°C, 50°C, and 75°C) and pressures (i.e., 5 MPa, 10 MPa, and 15 MPa), the damage constitutive model is adopted to fit the measured data from experiments. Figure 3 shows the fitting curves and the measured data. As the pressure of the H2SO4 solution is 5 MPa, the compression hardening process and the strain at the
the experimental data, and the constitutive model established above, there are two mechanical parameters, compression loading. The related mechanical parameters respond of the acid-corroded sandstone under uniaxial of the damage constitutive model enables to accurately describe the mechanical experimental data, which indicates that the proposed damage constitutive model based on the experimental fit parameters can be obtained from the experimental data, and the fitting parameters can be obtained by fitting the experimental data. To investigate variations of the mechanical parameters and the fitting parameters with the soaking time, the five fitting parameters of the damage constitutive model based on the experimental data from Li et al. [31] are taken as an example for parametric analyses. The detailed steps are as follows.

2.3. Parametric Analyses. In terms of the damage constitutive model established above, there are two mechanical parameters, \( E_T \) and \( \varepsilon_r \), and three fitting parameters, \( n \), \( a \), and \( m \). The mechanical parameters can be obtained from the experimental data, and the fitting parameters can be obtained by fitting the experimental data. To investigate variations of the mechanical parameters and the fitting parameters with the soaking time, the five fitting parameters of the damage constitutive model based on the experimental data from Li et al. [31] are taken as an example for parametric analyses. The detailed steps are as follows.

### Table 2: Fitting and mechanical parameters for the damage constitutive model [32].

| Soaking solution | Soaking time (T) (days) | Mechanical parameters \( E_T \) (GPa) | \( \varepsilon_r \) \( (10^{-3}) \) | Fitting parameters \( a \) \( (10^{-3}) \) | \( m \) | \( R^2 \) |
|------------------|------------------------|----------------------------------------|--------------------------------|--------------------------------|------|------|
| \( \text{H}_2\text{SO}_4 \) (pH = 1) | 30 | 14.790 | 2.413 | 20.084 | 6.310 | 27.036 | 0.998 |
|                  | 60 | 7.360 | 3.624 | 15.421 | 10.730 | 25.421 | 0.997 |
|                  | 90 | 5.710 | 4.511 | 12.916 | 12.710 | 23.331 | 0.997 |
|                  | 180 | 3.500 | 5.373 | 8.488 | 21.530 | 16.876 | 0.996 |
| \( \text{H}_2\text{SO}_4 \) (pH = 3) | 30 | 15.630 | 2.224 | 24.173 | 5.220 | 25.772 | 0.996 |
|                  | 60 | 9.110 | 3.390 | 17.682 | 8.530 | 23.094 | 0.996 |
|                  | 90 | 7.030 | 3.791 | 14.630 | 11.910 | 20.128 | 0.998 |
|                  | 180 | 6.010 | 4.587 | 10.958 | 14.53 | 13.967 | 0.997 |
| \( \text{H}_2\text{SO}_4 \) (pH = 5) | 30 | 19.130 | 1.932 | 30.050 | 5.050 | 21.9137 | 0.995 |
|                  | 60 | 11.510 | 2.644 | 20.425 | 8.030 | 24.391 | 0.996 |
|                  | 90 | 8.130 | 3.311 | 16.855 | 10.150 | 15.394 | 0.999 |
|                  | 180 | 7.240 | 3.801 | 13.956 | 12.470 | 10.113 | 0.999 |
| \( \text{H}_2\text{O} \) (pH = 7) | 30 | 20.450 | 1.731 | 37.801 | 4.950 | 23.29 | 0.999 |
|                  | 60 | 16.180 | 2.249 | 27.459 | 5.980 | 17.581 | 0.995 |
|                  | 90 | 12.820 | 2.723 | 22.884 | 7.560 | 13.889 | 0.999 |
|                  | 180 | 8.418 | 3.114 | 16.920 | 9.420 | 9.341 | 0.996 |
| Nature | 0 | 24.150 | 1.160 | 52.556 | 4.410 | 41.561 | 0.990 |

2.3.1. Evolution Formulae of Mechanical Parameters. The variation of \( E_T \) of the acid-corroded sandstone with soaking time is shown in Figure 4. It is indicated that the mechanical parameter, \( E_T \), decreases in the form of a negative exponential function with soaking time at each pH value. The relationships between static elastic modulus (\( E_T \)) and the soaking time (\( T \)) are fitted by

\[
\begin{align*}
E_T(\text{pH} = 1) &= 24.837e^{-0.0267T} + 2.924 \quad (R^2 = 0.999), \\
E_T(\text{pH} = 3) &= 23.152e^{-0.0247T} + 4.727 \quad (R^2 = 0.998), \\
E_T(\text{pH} = 5) &= 23.487e^{-0.0167T} + 4.807 \quad (R^2 = 0.957), \\
E_T(\text{pH} = 7) &= 21.088e^{-0.0104T} + 6.991 \quad (R^2 = 0.988). 
\end{align*}
\]

Figure 5 shows the fitting curves of the rock strain (\( \varepsilon_r \)) for cases with different pH values. As the pH value of the soaking solution is unchanged, it can be seen that \( \varepsilon_r \) has a positive exponential relationship with the soaking time (\( T \)). Then, the empirical formulae are concluded as follows:

\[
\begin{align*}
\varepsilon_r(\text{pH} = 1) &= 1.453 \times 10^{-2} \ln \left[ 0.157(T + 307.721) \right] - 5.570 \times 10^{-2} \quad (R^2 = 0.995), \\
\varepsilon_r(\text{pH} = 3) &= 0.332 \times 10^{-2} \ln \left[ 0.756(T + 53.177) \right] - 1.172 \times 10^{-2} \quad (R^2 = 0.980), \\
\varepsilon_r(\text{pH} = 5) &= 0.157 \times 10^{-2} \ln \left[ 0.998(T + 24.799) \right] - 0.447 \times 10^{-2} \quad (R^2 = 0.991), \\
\varepsilon_r(\text{pH} = 7) &= 0.142 \times 10^{-2} \ln \left[ 1.013(T + 35.205) \right] - 0.449 \times 10^{-2} \quad (R^2 = 0.989). 
\end{align*}
\]
2.3.2. Evolution Formulae of Fitting Parameters. Similar to the mechanical parameter $E_T$, the fitting parameter, $n$, decreases in the form of a negative exponential function with the soaking time at each pH value (Figure 6). The regression equations between $n$ and the soaking time ($T$) are fitted by

\[
\begin{align*}
  n(&\text{pH} = 1) = 163.692e^{-0.0369T} + 15.564 \quad (R^2 = 0.986), \\
  n(&\text{pH} = 3) = 163.295e^{-0.0310T} + 15.704 \quad (R^2 = 0.987), \\
  n(&\text{pH} = 5) = 156.226e^{-0.0325T} + 22.908 \quad (R^2 = 0.988), \\
  n(&\text{pH} = 7) = 150.676e^{-0.0310T} + 28.423 \quad (R^2 = 0.989).
\end{align*}
\]  

(14)

The fitting curves of the parameter $a$ and the soaking time ($T$) are shown in Figure 7. It is observed that the

\[
\begin{align*}
  a(&\text{pH} = 1) = 163.692e^{-0.0369T} + 15.564 \quad \text{(Fitted)}
\end{align*}
\]

Table 3: Fitting and mechanical parameters for the damage constitutive model [33].

| Test condition | Mechanical parameters | Fitting parameters | $R^2$ |
|---------------|-----------------------|--------------------|-------|
|               | $E_T$ (GPa)           | $\varepsilon_e$ $(10^{-3})$ | $n$ | $a$ $(10^{-3})$ | $m$ | |
| Untreated     | 32.250                | 0.411              | 106.280 | 7.512 | 120.891 | 0.997 |
| 25°C, 5 MPa   | 28.050                | 1.202              | 38.164 | 8.821 | 97.476 | 0.999 |
| 50°C, 5 MPa   | 23.390                | 1.577              | 14.885 | 9.650 | 53.759 | 0.986 |
| 75°C, 5 MPa   | 17.870                | 1.661              | 5.369  | 10.863 | 22.765 | 0.997 |
| 25°C, 10 MPa  | 24.730                | 1.444              | 17.262 | 9.524 | 55.098 | 0.999 |
| 25°C, 15 MPa  | 25.060                | 1.491              | 9.983  | 10.640 | 15.787 | 0.999 |

Figure 3: Comparison between the measured data [33] and the fitting curves by using the damage constitutive model. (a) Temperatures. (b) Pressures.

Figure 4: Variations of $E_T$ with the soaking time in solutions with different pH values.
parameter, $a$, increases linearly with the soaking time, and the empirical formulae can be written as

\[
\begin{align*}
  a_{pH = 1} &= -1.042 \times 10^{-4} T + 3.64 \times 10^{-3} \quad (R^2 = 0.991), \\
  a_{pH = 3} &= -6.730 \times 10^{-5} T + 4.39 \times 10^{-3} \quad (R^2 = 0.940), \\
  a_{pH = 5} &= -5.480 \times 10^{-5} T + 4.38 \times 10^{-3} \quad (R^2 = 0.945), \\
  a_{pH = 7} &= -3.682 \times 10^{-5} T + 4.17 \times 10^{-3} \quad (R^2 = 0.996). 
\end{align*}
\]

Figure 8 shows variations of the fitting parameter $m$ as a function of the soaking time in soaking solutions with different pH values. The fitting regression equations between $m$ and the soaking time ($T$) are obtained as

\[
\begin{align*}
  m_{pH = 1} &= -0.120 T + 39.296 \quad (R^2 = 0.983), \\
  m_{pH = 3} &= -0.135 T + 38.425 \quad (R^2 = 0.963), \\
  m_{pH = 5} &= -0.129 T + 38.008 \quad (R^2 = 0.915), \\
  m_{pH = 7} &= -0.144 T + 37.775 \quad (R^2 = 0.907). 
\end{align*}
\]
2.4. Predicting Stress-Strain Curve of Acid-Corroded Sandstones. To obtain the effect of acid corrosion on mechanical properties of the sandstone more accurately, the sandstone immersed in H₂SO₄ solution (pH = 1) is selected as an example (Li et al. [31]), and the stress-strain curves of the sandstone subjected to acid corrosion for different soaking time (e.g., 10 days, 20 days, 50 days, 70 days, 120 days, and 150 days) are predicted based on the damage constitutive model established above.

In terms of the predicted stress-strain curves with different soaking times, the corresponding mechanical parameters (\(E_T\) and \(\varepsilon_e\)) and the three fitting parameters (\(n\), \(a\), and \(m\)) can be derived by Equations (12)–(16). The calculated results are shown in Table 4. By substituting these parameters into Equation (11), the stress-strain curves of the sandstone subjected to acid corrosion for different soaking time before the peak stress can be obtained (Figure 9). It shows that both the compaction hardening process and the strain at peak point increase with the increment of the soaking time, while the uniaxial compression strength shows a decreasing trend with the increment of the soaking time. The variation trends show good agreements with the experimental results (sandstones subjected to the acid corrosion for 30 days, 90 days, and 180 days) from Li et al. [31], which indirectly indicates that the damage constitutive model and the fitting process of the mechanical and fitting parameters are reliable.

### Table 4: Mechanical and fitting parameters for the predicted stress-strain curves.

| Soaking solution | Soaking time (\(T\)) (days) | \(E_T\) (GPa) | \(\varepsilon_e\) (10⁻³) | \(n\) | \(m\) | \(a\) (10⁻³) |
|------------------|---------------------------|--------------|-----------------|-----|-----|--------|
| pH = 1           | 10                        | 21.941       | 1.075           | 128.763 | 38.096 | 4.682   |
|                  | 20                        | 17.484       | 1.525           | 93.845 | 36.896 | 5.725   |
|                  | 50                        | 9.460        | 2.798           | 41.452 | 33.295 | 8.851   |
|                  | 70                        | 6.756        | 3.588           | 27.944 | 30.894 | 10.936  |
|                  | 120                       | 3.932        | 5.395           | 17.522 | 24.893 | 16.147  |
|                  | 150                       | 3.377        | 6.380           | 16.211 | 21.292 | 19.274  |

3. BI of Sandstone Subjected to Acid Corrosion

3.1. BI of Rock Based on Energy Evolution Theory and Damage Constitutive Relation. The entire process of a rock from being loaded to generating fractures is essentially a dynamic equilibrium process of the energy absorption and release. In the prepeak stage, energy is continuously accumulated in the rock and some energy is dissipated due to the plastic deformation during the loading process. For a rock with relatively high brittleness (similar to the brittleness of the sandstone selected in this paper), the energy required for postpeak cracking is mainly supplied by the elastic strain energy stored in rock before the stress arrives the peak value. Hence, when the stress within the rock reaches its peak value, the more elastic the strain energy stored in rock, the more severe the rock failure at the postpeak stage, and the more brittle the rock.

The stress-strain curve of a rock is often used to analyze the energy evolution process of the rock during the loading process [34, 35]. Similarly, this method is also adopted in our studies. Figure 10 shows a schematic diagram of the energy evolution process for a relatively brittle rock from being loaded to reaching the peak stress. As can be seen from Figure 10, the total strain energy, defined as \(S\), is the area integral under the stress-strain curve (OBC), the dissipated strain energy, defined as \(S_1\), is the shaded part encircled by OAC, and the elastic strain energy, defined as \(S_2\), is the shaded part encircled by ABC.

The mathematical relationship among the total strain energy, the dissipative strain energy, and the elastic strain energy can be written as

\[
S = S_1 + S_2, \quad (17)
\]

According to analyses above, the BI of the rock can be characterized by the ratio of the elastic energy to the total strain energy, and it can be formulated in the form of

\[
B_E = \frac{S_2}{S}. \quad (18)
\]
The elastic strain energy stored in the rock before arriving the stress peak can be calculated as

\[ S_2 = \frac{\sigma_p^2}{2E_T}, \]  \hspace{1cm} (19)

where \( \sigma_p \) and \( E_T \) denote the peak stress and the static elastic modulus, respectively, and these two parameters can be obtained from the fitting curve and the measured data from experiments, respectively.

The investigation in Section 2.2 illustrates that the fitting curves have a good agreement with the experimental data, and the damage constitutive model can accurately describe the mechanical response of the rock. Therefore, the total strain energy can be obtained by calculating the area integral between the fitting curve before the stress peak and the x-axis \[ (20), \] it can be written as

\[ S = \int_0^{\epsilon_p} \sigma \, d\epsilon, \]  \hspace{1cm} (20)

where \( \epsilon_p \) is the rock strain at the peak stress (\( \sigma_p \)) and it can be obtained from the fitting curve.

Substituting Equations (19) and (20) into Equation (18), the BI of the acid-corroded rock based on energy evolution theory and damage constitutive relation can be written as

\[ B_E = \frac{\sigma_p^2}{2E_T \int_0^{\epsilon_p} \epsilon \, d\epsilon}. \]  \hspace{1cm} (21)

\( B_E \) can characterize the ability of a rock to maintain self-fracture at the postpeak failure stage, and it increases with the increment of the proportion of the elastic strain energy in the total strain energy. When the stress within the rock reaches its peak value, the brittleness index \( B_E \) has a positive correlation with the elastic strain energy storage rate and the supplied energy for the postpeak fracture as well. \( B_E \) ranges from 0 to 1, when the rock is an ideal plastic material, the total strain energy \( S \) is completely converted into dissipative strain energy \( S_1 \), i.e., \( B_E = 0 \), and when the rock is an ideal brittle material, all the total strain energy is converted into the elastic strain energy \( S_2 \), i.e., \( B_E = 1 \); for example, if \( S_1 \) gets 0, a sudden stress drop and dynamic loss of stability will happen in the rock after peak stress, which is the evident indicator of ideal brittle.

3.2. BI of the Sandstone after Acid Corrosion. The BI of the acid-corroded sandstone of three references is directly calculated by using the theoretical model proposed in Section 3.1. The BI evaluations of acid-corroded sandstone of Li et al. \cite{31} and Huo et al. \cite{32} will be discussed together, and those of the acid-corroded sandstone of Li et al. \cite{31} will be discussed individually. More details are claimed as follows.

3.2.1. BI of the Acid-Corroded Sandstone of Li et al. \cite{31} and Huo et al. \cite{32}. Table 5 shows the calculation results of the BI of the acid-corroded sandstone of three references is directly calculated by using the theoretical model proposed in Section 3.1.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Figure10.png}
\caption{Energy evolution of the rock at the prepeak stage of the stress-strain curve.}
\end{figure}

The BI evaluations of acid-corroded sandstone from Li et al. \cite{31} and Huo et al. \cite{32} are similar, here, the calculation of BI from the case of Huo et al. \cite{32} is...
taken as an example for the analyses. As can be seen from Figure 12, the BI of sandstones immersed in acid solutions with different pH values is negatively correlated with the soaking time, and the decline rate (slope of the line segment) decreases with the increasing of the soaking time. For instance, the BI of the nature sandstone is 0.973 as the rock immersed in H₂SO₄ solution with the pH value equivalent to 1; after soaking for 180 days, the BI decreases to 0.843. And the decline rate of the BI in the first 30 days is $2.467e^{-3/T}$ (days), while the rate of descent decreases to $0.144e^{-3/T}$ (days) from day 90 to day 180. It shows a sharp decrease in the decline rate of the BI. In addition, the decline degree of the BI decreases with the increment of the PH value. For example, after 180 days of the acid corrosion, the BI of sandstones immersed in H₂SO₄ solution (pH = 1) decreases by 13.361%, while that of the sandstone immersed in H₂SO₄ solutions (pH = 5) only decreases by 10.380%. The reason for the above variations of the BI is probably due to the fact that the BI largely depends on physical and mechanical properties of the sandstone, and the acid solutions with different pH values have different degrees of weakening effects on physical and mechanical properties. For example, in terms of the acid-corroded sandstone from Huo et al. [32], the mass of the sandstone is negatively correlated with the soaking time. The rate of mass loss decreases with both the increment of the soaking time and the pH value of the acid solution. Besides, the same behaviors occur in other mechanical parameters, such as the elastic modulus and the strength.

### 3.2.2. BI of the Acid-Corroded Sandstone of Li Et al. [33]

The calculated BI of acid-corroded sandstone are listed in Table 7, and the variations of the BI of sandstone immersed in acid solutions with different temperatures and pressures are shown in Figure 13. As can be seen from Figure 13, the BI of acid-corroded sandstone at different temperatures and pressures rapidly decreases in the early stages and shows a weak decreasing trend in the later stages with the increment of the temperature and pressure. But the decline rate of the BI shows a remarkable difference under the two curing conditions. For example, when the sandstone is immersed in the acid solution at the pressure of 5 MPa, the BI decreases by 0.037 with the temperature increases from 25°C to 75°C. However, when the sandstone is immersed in acid solution at the temperature of 25°C, the BI only decreases by 0.011 with the pressure increases from 5 MPa to 15 MPa. This is because temperature and pressure have different extents of weakening effects on the physical and mechanical properties of sandstone. For instance, in the original reference, the authors state that the mass, the microstructure, the strength, and the elastic modulus of acid-corroded sandstone show noticeable deterioration with the increment of the temperature, while these properties show inconspicuous alteration with the increment of the pressure. Thus, when the temperature and pressure of the sandstone reservoir are close to those of the experiment, it can be

**Table 5: BI of sandstone [31] under different soaking conditions.**

| Soaking solution | Soaking time (T) (days) | $\sigma_p$ (MPa) | $\varepsilon_p$ ($10^{-2}$) | $E_T$ (GPa) | $S_2$ | $S$ | $B_E$ |
|------------------|------------------------|-----------------|--------------------------|-----------|------|---|------|
| H₂SO₄ (pH = 1)   | 30                     | 58.092          | 0.535                    | 13.896    | 0.121| 0.135| 0.899 |
|                  | 90                     | 48.842          | 1.187                    | 5.499     | 0.217| 0.254| 0.856 |
|                  | 180                    | 45.124          | 1.951                    | 2.924     | 0.348| 0.413| 0.843 |
| H₂SO₄ (pH = 3)   | 30                     | 64.164          | 0.497                    | 15.966    | 0.129| 0.142| 0.905 |
|                  | 90                     | 57.369          | 1.044                    | 6.855     | 0.240| 0.277| 0.866 |
|                  | 180                    | 54.126          | 1.290                    | 5.213     | 0.281| 0.330| 0.851 |
| H₂SO₄ (pH = 5)   | 30                     | 70.742          | 0.489                    | 20.307    | 0.123| 0.135| 0.911 |
|                  | 90                     | 62.287          | 0.950                    | 8.585     | 0.226| 0.256| 0.883 |
|                  | 180                    | 59.597          | 1.158                    | 6.622     | 0.268| 0.308| 0.872 |
| H₂O (pH = 7)     | 30                     | 77.226          | 0.456                    | 23.026    | 0.130| 0.140| 0.924 |
|                  | 90                     | 68.602          | 0.678                    | 14.737    | 0.160| 0.175| 0.915 |
|                  | 180                    | 64.988          | 0.891                    | 10.409    | 0.203| 0.223| 0.910 |
| Nature           | 0                      | 88.980          | 0.392                    | 27.812    | 0.142| 0.146| 0.973 |

**Figure 11: BI of sandstones [31] immersed in acid solutions with different pH values and soaking time.**
concluded that the temperature has greater weakening effects on the BI of sandstone reservoirs than the pressure in practical acidizing treatment.

3.3. Predicting the BI of the Acid-Corroded Sandstone. To obtain the variation of the BI of the acid-corroded sandstone [31] with the soaking time more accurately, the predicted BI of the acid-corroded sandstone is calculated based on the stress-strain curve predicted in Section 2.4. The calculated results are shown in Table 8. Figure 14 graphically illustrates the variation tendency for the predicted BI of the sandstone immersed in H₂SO₄ solution (pH = 1) with the soaking time.

As can be seen from the figure, the predicted BI (the red points) decreases with the increment of the soaking time, and it is consistent with the evolution trend of the existing BI (the blue points, BI of the sandstone immersed in acid solution for 0, 30, 90, and 180 days). In addition, it can also be found from the figure that the BI decreases fastest in the first 50 days, slowly from day 50 to day 120, and approaches an almost constant value from day 120 to day 180. It is worth noting that day 50 is a turning point where the decreasing rate of the BI changes from rapid to slow, and day 120 is a turning point where the decreasing rate of the BI changes from slow to almost constant.

4. Discussion

A theoretical model is proposed in this paper to evaluate the BI of the acid-corroded sandstone. As is known to us, the rock mass with higher brittleness would spread over the formation with complex fracture networks during the fracturing treatment [37]. However, the above studies show that the acid corrosion has weakening effects on the BI of the sandstone. One of the main purposes of the acidizing projects is to reduce the breakdown pressure of the sandstone or carbonate reservoirs. In other related studies, it is suggested that the breakdown pressure is positively correlated with the rock strength [38]. Thus, the effect of the acid corrosion on the breakdown pressure of rocks can be characterized by the variation of the rock strength. The aforementioned reference illustrates that the acid corrosion has weakening effects on the sandstone [32] strength. For instance, as shown in Figure 15, the strength decreases with the soaking time as the sandstone immersed in acid solution (pH = 1), and the rock strength decreases in the form of a negative exponential function with the soaking time.

| Soaking solution | Soaking time (T) (days) | σₚ (MPa) | εₚ (10⁻²) | Eₚ (GPa) | S₂ | S | Bₑ |
|------------------|------------------------|---------|-----------|---------|----|---|----|
| H₂SO₄ (pH = 1)   | 30                     | 57.893  | 0.552     | 14.79   | 0.114 | 0.126 | 0.894 |
|                  | 60                     | 52.580  | 1.032     | 7.36    | 0.174 | 0.218 | 0.861 |
|                  | 90                     | 50.654  | 1.197     | 5.71    | 0.210 | 0.267 | 0.844 |
|                  | 180                    | 45.181  | 1.862     | 3.5     | 0.295 | 0.349 | 0.836 |
| H₂SO₄ (pH = 3)   | 30                     | 63.471  | 0.512     | 15.63   | 0.139 | 0.143 | 0.901 |
|                  | 60                     | 59.692  | 0.841     | 9.11    | 0.215 | 0.226 | 0.868 |
|                  | 90                     | 58.334  | 1.043     | 7.03    | 0.253 | 0.283 | 0.856 |
|                  | 180                    | 55.926  | 1.291     | 6.01    | 0.263 | 0.307 | 0.847 |
| H₂SO₄ (pH = 5)   | 30                     | 69.030  | 0.497     | 19.13   | 0.128 | 0.138 | 0.909 |
|                  | 60                     | 65.902  | 0.761     | 11.51   | 0.189 | 0.213 | 0.887 |
|                  | 90                     | 62.055  | 0.940     | 8.13    | 0.237 | 0.270 | 0.879 |
|                  | 180                    | 61.662  | 1.164     | 7.24    | 0.262 | 0.303 | 0.869 |
| H₂O (pH = 7)     | 30                     | 75.479  | 0.470     | 20.450  | 0.192 | 0.151 | 0.922 |
|                  | 60                     | 70.285  | 0.587     | 16.180  | 0.361 | 0.168 | 0.912 |
|                  | 90                     | 67.305  | 0.687     | 12.820  | 0.453 | 0.195 | 0.907 |
|                  | 180                    | 65.627  | 0.899     | 8.418   | 0.834 | 0.284 | 0.902 |
| Nature           | 0                      | 83.313  | 0.399     | 24.15   | 0.150 | 0.148 | 0.971 |

Figure 12: BI of sandstones [32] with the soaking time in solutions with different pH values.

As can be seen from the figure, the predicted BI (the red points) decreases with the increment of the soaking time, and it is consistent with the evolution trend of the existing BI (the blue points, BI of the sandstone immersed in acid solution for 0, 30, 90, and 180 days). In addition, it can also be found from the figure that the BI decreases fastest in the first 50 days, slowly from day 50 to day 120, and approaches an almost constant value from day 120 to day 180. It is worth noting that day 50 is a turning point where the decreasing rate of the BI changes from rapid to slow, and day 120 is a turning point where the decreasing rate of the BI changes from slow to almost constant.

4. Discussion

A theoretical model is proposed in this paper to evaluate the BI of the acid-corroded sandstone. As is known to us, the rock mass with higher brittleness would spread over the formation with complex fracture networks during the fracturing treatment [37]. However, the above studies show that the acid corrosion has weakening effects on the BI of the sandstone. One of the main purposes of the acidizing projects is to reduce the breakdown pressure of the sandstone or carbonate reservoirs. In other related studies, it is suggested that the breakdown pressure is positively correlated with the rock strength [38]. Thus, the effect of the acid corrosion on the breakdown pressure of rocks can be characterized by the variation of the rock strength. The aforementioned reference illustrates that the acid corrosion has weakening effects on the sandstone [32] strength. For instance, as shown in Figure 15, the strength decreases with the soaking time as the sandstone immersed in acid solution (pH = 1), and the rock strength decreases in the form of a negative exponential function with the soaking time.
Besides, the decreasing rate of rock strength declines with the increment of the pH value with the same soaking time. On the other hand, as the sandstone is immersed in acid solutions, the temperature (25°C, 50°C, and 75°C) has greater weakening effects on the rock strength compared with the influence caused by pressure (5 MPa, 10 MPa, and 15 MPa).

The weakening effects of the acid corrosion on sandstones can be characterized by the decreasing ratio of the BI:

\[ \Delta B = \frac{B_{EO} - B_{EC}}{B_{EO}}, \]  

where \( B_{EO} \) is the BI of the nature sandstone and \( B_{EC} \) is the BI of the sandstone after the acid corrosion.

Table 7: BI of sandstone [33] at different temperatures and pressures.

| Test conditions | \( \sigma_p \) (MPa) | \( \varepsilon_p \) (10^{-2}) | \( E_T \) (GPa) | \( S_2 \) | \( S \) | \( B_E \) |
|-----------------|----------------------|-------------------------------|----------------|--------|------|--------|
| Untreated       | 195.450              | 0.613                         | 32.25          | 0.592  | 0.602| 0.983  |
| 75°C, 5 MPa     | 143.903              | 0.974                         | 17.87          | 0.579  | 0.648| 0.894  |
| 50°C, 5 MPa     | 164.164              | 0.894                         | 23.39          | 0.576  | 0.639| 0.902  |
| 25°C, 5 MPa     | 187.457              | 0.837                         | 28.05          | 0.626  | 0.672| 0.931  |
| 25°C, 10 MPa    | 184.381              | 0.891                         | 24.73          | 0.687  | 0.746| 0.921  |
| 25°C, 15 MPa    | 181.758              | 0.927                         | 25.06          | 0.659  | 0.716| 0.920  |

Figure 13: BI of sandstone [33] at different (a) temperatures and (b) pressures.

Table 8: Predicted BI for the acid-corroded sandstone [31] with different soaking time.

| Soaking time (\( T \)) (days) | \( \sigma_p \) (MPa) | \( E_T \) (GPa) | \( S_2 \) | \( S \) | \( B_E \) |
|------------------------------|----------------------|----------------|--------|------|--------|
| 10                           | 73.945               | 21.941         | 0.125  | 0.131| 0.954  |
| 20                           | 64.304               | 17.484         | 0.118  | 0.128| 0.927  |
| 50                           | 51.792               | 9.460          | 0.142  | 0.163| 0.871  |
| 70                           | 48.936               | 6.756          | 0.177  | 0.205| 0.864  |
| 120                          | 47.047               | 3.932          | 0.281  | 0.332| 0.847  |
| 150                          | 46.860               | 3.377          | 0.325  | 0.384| 0.845  |

Figure 14: The calculated BI of the sandstone [31] immersed in acid solution with different soaking time.

Besides, the decreasing rate of rock strength declines with the increment of the pH value with the same soaking time. On the other hand, as the sandstone [33] is immersed in acid solutions, the temperature (25°C, 50°C, and 75°C) has greater weakening effects on the rock strength compared with the influence caused by pressure (5 MPa, 10 MPa, and 15 MPa).

The weakening effects of the acid corrosion on sandstones can be characterized by the decreasing ratio of the BI:

\[ \Delta B = \frac{B_{EO} - B_{EC}}{B_{EO}}, \]  

where \( B_{EO} \) is the BI of the nature sandstone and \( B_{EC} \) is the BI of the sandstone after the acid corrosion.
Alternatively, the weakening effects of the acid corrosion on sandstones can be also characterized by the decreasing ratio of the strength:

\[ \Delta K = \frac{R_0 - R_C}{R_0}, \]

where \( R_0 \) is the strength of the nature sandstone and \( R_C \) is the strength of the sandstone after the acid corrosion.

**Table 9:** Calculated results of \( \theta \) of the acid-corroded sandstone [32] under different soaking conditions.

| Soaking solution | Soaking time \((T)\) (days) | \( \Delta B \) | \( \Delta K \) | \( \theta \) |
|------------------|-----------------------------|----------------|----------------|----------|
| \( \text{H}_2\text{SO}_4 \) (pH = 1) | 30 | 0.079 | 0.341 | 4.303 |
| | 60 | 0.113 | 0.407 | 3.589 |
| | 90 | 0.131 | 0.444 | 3.395 |
| | 180 | 0.139 | 0.482 | 3.467 |
| \( \text{H}_2\text{SO}_4 \) (pH = 3) | 30 | 0.072 | 0.274 | 3.805 |
| | 60 | 0.106 | 0.324 | 3.053 |
| | 90 | 0.118 | 0.347 | 2.926 |
| | 180 | 0.128 | 0.388 | 3.034 |
| \( \text{H}_2\text{SO}_4 \) (pH = 5) | 30 | 0.064 | 0.210 | 3.292 |
| | 60 | 0.087 | 0.263 | 3.040 |
| | 90 | 0.095 | 0.273 | 2.872 |
| | 180 | 0.105 | 0.318 | 3.026 |

**Table 10:** Calculated results of \( \theta \) of the acid-corroded sandstone [33] at different temperatures and pressures.

| Test condition | \( \Delta B \) | \( \Delta K \) | \( \theta \) |
|----------------|----------------|----------------|----------|
| 75°C, 5 MPa | 0.096 | 0.287 | 3.175 |
| 50°C, 5 MPa | 0.082 | 0.194 | 2.357 |
| 25°C, 5 MPa | 0.053 | 0.083 | 1.563 |
| 25°C, 10 MPa | 0.063 | 0.121 | 1.912 |
| 25°C, 15 MPa | 0.064 | 0.120 | 1.870 |

Alternatively, the weakening effects of the acid corrosion on sandstones can be also characterized by the decreasing ratio of the strength:

\[ \Delta K = \frac{R_0 - R_C}{R_0}, \]

where \( R_0 \) is the strength of the nature sandstone and \( R_C \) is the strength of the sandstone after the acid corrosion.
The weakening effects of the acid corrosion on the BI are similar to those of the strength. The BI and the rock strength are two equally important factors, which reflect the effect of the acid corrosion. Therefore, it is necessary to find an appropriate relation between the BI and the strength. Here, a coefficient, $\theta$, is proposed to quantify the effect of the acid corrosion.

$$\theta = \frac{\Delta K}{\Delta B}$$ \quad (24)

The coefficient is defined as the ratio of the decreasing rate of the strength to that of the BI. The larger the value of $\theta$, the better the effect of the acid corrosion.

The calculated results of $\theta$ of the sandstone after the acid corrosion in different soaking conditions are listed in Tables 9 and 10, respectively. The variations of $\theta$ under different soaking conditions are shown in Figure 16. As can be seen from Figure 16(a), the value of $\theta$ firstly decreases and then increases with the soaking time, but the increase rate shows a slower increasing trend. The lower the pH value, the better the effect of the acid corrosion. In addition, the value of $\theta$ shows an increasing trend with the increment of the temperature (Figure 16(b)), and that value firstly increases and then decreases with the increment of the pressure (Figure 16(c)).

From above analyses, our research can provide guidelines for the acidizing projects. For example, after comprehensive considerations of all kinds of factors, the acid solution with higher concentration is required. To achieve a better acidizing performance, the acidizing process is suggested to be terminated in a short time window. Moreover, the effect of the acidizing treatment can be improved by increasing the temperature (range from 25°C to 75°C) of the pumping acid solution, and the appropriately increased pumping pressure is recommended.

5. Conclusion

In this study, a theoretical model for evaluating the BI of the acid-corroded sandstone is proposed based on energy evolution theory and damage constitutive relation. A damage constitutive equation considering the compression hardening process is established and verified and then implanted into
the theoretical model. The BI of the sandstone immersed in H₂SO₄ corrosion with different pH values and soaking time (and different temperatures and pressures) is calculated and analyzed. The BI of sandstone subjected to acid corrosion with soaking time is predicted. Moreover, a coefficient (θ) defined as the ratio of the decreasing rate of strength to that of the BI is proposed to quantify the effect of the acid corrosion. The major conclusions can be drawn as follows:

(1) By introducing a compaction hardening coefficient (β) to the classical damage constitutive model and replacing the elastic modulus of the sandstone (has not been treated with acid corrosion) with that of the acid-corroded sandstone, a damage constitutive model for the acid-corroded sandstone is established. The damage constitutive model is verified and has a good agreement with the experimental data from three references.

(2) The mechanics and fitting parameters are derived by five evolution formulae. By substituting these parameters into the damage constitutive model, the stress-strain curves of sandstone subjected to acid corrosion after different soaking time can be predicted.

(3) The BI of the sandstone subjected to the acid corrosion is negatively correlated with the soaking time, and the decline rate of the BI decreases with the increment of soaking time. In addition, the decline degree of the BI decreases with the increment of pH value. Besides, the temperature (25°C, 50°C, and 75°C) has greater weakening effects on the BI compared with the impact of the pressure (5 MPa, 10 MPa, and 15 MPa).

(4) From the predicted BI of the acid-corroded sandstone, it can be noted that day 50 is a turning point where the decreasing rate of the BI changes from rapid to slow, and day 120 is a turning point where the decreasing rate of the BI changes from slow to almost constant.

(5) Our research can provide some guidelines for the acidizing projects, including the requirement of the acid solution with higher concentration, a short time window of the acidizing treatment, and the appropriately increased pumping pressure.

**Nomenclature**

- P(W): Weibull distribution function of rock microunit
- W: The variable of microunit strength
- m: A fitting constant
- a: A fitting constant
- n: A fitting constant
- ε: The strain of rock
- NF: The number of failed units
- N: The total number of microunits
- D_L: The loading damage of rock
- D_C: The acid-corroded damage of rock
- E: The static elastic modulus of fresh rock (GPa)
- E_T: The static elastic modulus of acid-corroded rock (GPa)
- β: The compaction hardening coefficient
- ε: The strain of rock corresponding to the stress at the endpoint of the compaction hardening stage
- σ: The peak stress of rock (MPa)
- ε: The rock strain at peak stress
- B_C: The BI of acid-corroded rock
- ΔB: The decreasing ratio of the BI
- B_Bo: The BI of the nature sandstone
- B_Bc: The BI of the sandstone after acid corrosion
- ΔK: The decreasing ratio of the rock strength
- R_C: The strength of the sandstone after acid corrosion (MPa)
- θ: The coefficient to quantify the effect of the acid corrosion.

**Data Availability**

All data in this study are cited from three original references.

**Conflicts of Interest**

The authors declare that they have no conflict of interest.

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