New Method to Predict the Viscosity of Bitumen Diluted with Light Oil Using a Modified Van Der Wijk Model under Reservoir Temperature and Pressure

Ronald Ssebadduka, Kyuro Sasaki, Ronald Nguele, Tumelo Kgetse Dintwe, Tiago Novaes, and Yuichi Sugai

ABSTRACT: In this study, we introduce a new method for the prediction of the viscosity of bitumen diluted with light oil under reservoir temperature and pressure. This two-step method works as follows: first, predicting the bitumen viscosity under reservoir temperature and pressure using the classical Mehrotra and Svrcek model, and then subsequently using it in the modified Van Der Wijk (MVDM) model. This model formed from the modification of the original Van Der Wijk model was developed from the consideration of the interactions between like molecules in different binary components of the mixture. In this study, the bitumen viscosity was predicted with an average absolute deviation percentage (AAD%) of 3.86. The accuracy of the MVDM was investigated from the experimental results obtained from the rheological studies of three binary mixtures of light oil (API 32°) and bitumen (API 7.39°). Dead oils were mixed on a mass fraction basis. The viscosity was measured at a temperature range of 45−110 °C and a pressure range of 0.1−6 MPa. For comparison purposes, a reworked Van Der Wijk model (RVDM) was used in the same method and compared to the MVDM. The latter was more accurate than the RVDM with AAD% values of 8.88, 8.02, and 5.07 in predicting the viscosity of the three mixtures of 25, 32.5, and 50% bitumen with light oil. On the other hand, the RVDM had AAD% values of 12.42, 11.43, and 7.87 for the same mixtures, respectively. The applicability of this method was further verified by comparing its accuracy to another reported method using published data and it was found that the MVDM had AAD% values of 1.86, 6.55, and 2.82 when predicting the viscosities of the three mixtures under reservoir temperature and pressure conditions.

1. INTRODUCTION

Enhanced oil recovery is a technique that is popular in the field of oil production because most fields have now matured and have a lot of bitumen (most viscous) or heavy oil (less viscous) that can no longer easily flow out of reservoirs. Bitumen and heavy oil are alternative energy sources because they make up some of the largest reserves of previously unexploited fossil fuels on earth. Heavy oil reservoirs tend to have low pressure and a low gas−oil ratio, resulting generally in lower recovery factors in comparison to light oil reservoirs.

The dilution of heavier crude with light crude to give a mixture with flow properties in between those of the initial components has been widely used to facilitate the transportation of bitumen. However, in countries like Venezuela, the dilution of bitumen with light oils is one such method used to increase the recovery factor, especially if coupled with either artificial lift or gas lift.

For bitumen, the fraction of light oils in the mixture can go up to 50%. Therefore, once the compatibility testing has been done between the oils, the viscosity is expected to reduce in part due to the polarity of lighter oils.

Although much efforts have been directed toward the prediction of the viscosity of bitumen with solvent mixtures, in some cases under different temperatures and pressures as shown by several researchers, there is still limited research to accurately make the prediction with light oils used in the mixtures. Chen et al. proposed a way to accurately predict the viscosity for light solvents (i.e., methane, ethane, propane, n-butane, n-pentane, N2, and CO2)—heavy oil/bitumen/water systems as a function of pressure in a temperature range of 287.9−463.4 K. The LV and ALV (L is the oleic phase, V is the vapor phase, and A is the aqueous phase) phase equilibria of the aforementioned systems were calculated using the Peng−Robinson equation of state (PR EOS) with modified a functions and binary interaction parameters (BIPs). The six widely used mixing rules for

10085

© 2021 The Authors. Published by American Chemical Society
https://doi.org/10.1021/acsomega.1c00079
ACS Omega 2021, 6, 10085−10094
predicting the viscosity of solvents-heavy oil/bitumen systems pertaining to vapor-liquid equilibria were compared and evaluated, while the linear mixing rule was used for hydrocarbons-water mixtures. Plus, effective density was successfully introduced into the volume-based mixing rules. The volume-based power law, weight-based power law, and weight-based Crague’s mixing rules were found to well reproduce the viscosity for the aforementioned systems with average absolute relative deviations (AARDs) of 15.5, 19.0, and 32.6%, respectively. Zirrahi and Hassanzadeh proposed a semitheoretical viscosity model based on the Arrhenius mixing rule and considering the effect of association between the molecules of the solvent and bitumen. They employed the thermodynamic perturbation theory to calculate the fraction of bonding solvent molecules and used the modified Enskog theory to calculate the viscosity of the solvent in wide temperature and pressure ranges. The results showed the predictions by the model to be close to the experimental data with deviations (AARDs) of 15.5, 19.0, and 32.6%, respectively.

Mishra et al. came up with a modified Guo viscosity model for heavier hydrocarbon components and their mixture. Their model could potentially be applied to determine the viscosity of mixed crude oil under pressure and temperature conditions but required bitumen, light oil, and oil mixtures to be first characterized.

Herein, we intend to use a classical Mehrotra and Svreck bitumen viscosity prediction model that contains a temperature and pressure term in the modification of the traditional Van Der Wijk binary fluid viscosity prediction model, which unlike the other previously mentioned models in this study does not use mixing rules that involve many calculations that increase the error. In studies for bitumen viscosity, Mehrotra and Svreck (1984) observed that the viscosity of Marguerite lake bitumen decreased with an increase in temperature, but the reduction in viscosity was less steep in the high-temperature region and this observation can only be attributed to increased fluidity at higher temperatures. Similar behavior was observed for Athabasca bitumen. Mehrotra and Svreck subsequently presented new data for the effect of pressure and temperature on gas-free Athabasca bitumen viscosity, covering a temperature range of 43–120 °C and pressure up to 10 MPa (1450 psi). The compression of bitumen was found to result in a significant increase in viscosity but only a small increase in density. They developed two correlations to describe the effect of pressure and temperature on the viscosity of bitumen

\[
\mu = \exp[a_1 + a_2 \ln T] + a_3P
\]

and modifying the original Van Der Wijk equation. The conceptual approach in this work is to consider the effect of like intermolecular interactions between the two components of the mixture and to use nitrogen gas, which is relatively inert and has low solubility, in crude oil to simulate reservoir pressure conditions. This is done so as to assess the effect on the viscosity by pressure without the interference of solubility. Additionally, the temperature range chosen is to mimic the reservoir temperature conditions.

2. EXPERIMENTAL SECTION

2.1. Preparation of Diluted Oils. In this study, light oil (API 32°, specific density 0.866) was mixed with bitumen (API 7.39°, specific density 1.0188) in different ratios, as shown in Table 1.

| Table 1. Mixtures in This Study Based on Weight Fractions |
|---------------------------------|----------------|----------------|
| code | bitumen (wt %) | light oil (wt %) |
| M1  | 25            | 75             |
| M2  | 33            | 67             |
| M3  | 50            | 50             |

For each respective mixture, the corresponding masses of light oil and bitumen were added into a glass flask. The mixture was then homogenized using a homogenizer (model AHG 160A) for a minimum of 2 h. After that, the mixture was left to equilibrate for 2 days. After allowing the mixture to age, a funnel filter assembly was connected to allow for the separation of any deposited materials from the oil mixture.

For this purpose, a preweighed 0.32 μm filter membrane was used. The mixture in the flask was then poured into the filter membrane-lined funnel cup sealed with an aluminum foil to prevent evaporation during filtration. The side arm of the filtration flask was then connected to a vacuum pump. At the end of the vacuum filtration, the filter paper was rinsed with hexane and dried at 65 °C for 20 min. The weight of the filter paper was then recorded. This was used to determine if any solids had been deposited from the mixture but the mixtures were found to be compatible.

The properties of the obtained diluted crude oils are outlined in Table 2.

| Table 2. Measured Physical Properties of the Crude Oils and Their Mixtures as Classified and Used in This Study |
|---------------------------------|----------------|----------------|----------------|----------------|----------------|
| code | M1 | M2 | M3 | bitumen | light oil |
| API gravity (deg) at 16 °C | 10.94 | 10.57 | 10.10 | 7.39 | 32 |
| specific gravity at 16 °C | 0.9934 | 0.996 | 0.9993 | 1.0188 | 0.866 |
| density (g/cm³) at 16 °C | 0.9923 | 0.9949 | 0.9982 | 1.0177 | 0.865 |

2.2. Classification of Crude Oils. Using the guide shown in Table 3 and the results shown in Table 2, the modeled crude oils and their components as used in this study are classified according to their API gravity and found to fall into these categories as described.

2.3. Viscosity Measurements. The rheological properties of bitumen, light oil, and the diluted oils were measured using an equipment schematized in Figure 1.

2.3.1. Methodology. A Cambridge high-pressure and temperature viscometer (model BCC-323) was connected
perpendicularly to a high-pressure cell so as to be able to measure viscosity under N₂ gas pressure. This setup was placed in an air bath with connections to a gas cylinder and a gas holding cell, as shown in the schematic diagram above. Serial communication was then established between the computer and the viscometer. The viscosity of bitumen and the light oil solvent along with the three blends was measured over a temperature range of 45–110 °C and a pressure range of 0.1–6 MPa.

Figure 2 summarizes the sequential algorithm, followed in the present work, in the development of the modified Van Der Wijk (MVDM) model and how it is used.

3. RESULTS AND DISCUSSION

The results of this work are discussed in three major sections, with the rheology of the crude oil samples being the first one. Subsequently, the viscosity modeling for the prediction of the bitumen viscosity and the study of the effects of pressure, temperature, and mass fraction of the components of the mixture are presented. The procedure followed in the development of a new method to predict the viscosity of bitumen diluted with light oil is then explained. The validation and stability analysis of the MVDM concludes this section in which the new model’s accuracy is compared to another model.

3.1. Rheology of the Crude Oils and Their Mixtures.

The rheological property of complex fluids such as heavy oils and their mixtures with light oils is extremely relevant in production and transportation. The rheological properties of crude oils are especially important in process design to handle the fluid, numerical modeling, simulation, and optimization.\(^{19-21}\) The five crude oil samples’ viscosities were measured at 110 °C and atmospheric pressure using a Brookfield viscometer model LVDV-1 prime and spindle number 18. The shear rate was calculated by multiplying a shear rate constant of

|      | Viscosity (mPa·s) | Density (kg/m³) | °API |
|------|------------------|----------------|------|
| light oil | <10¹       | <934           | >20  |
| heavy oil | 10²–10³   | 934–1000       | 10–20|
| bitumen | >10⁵       | >1000          | <10  |

“Adapted with permission from [Gray, M. Upgrading Petroleum Residues and Heavy Oils, First Edit.; London: CRC Press, 1994.]. Copyright [1994] [Taylor and Francis Group].

Figure 1. Schematic diagram of the experimental setup for viscosity measurement.

Figure 2. Flow diagram of the procedure followed in developing and using the MVDM.
1.32 by the corresponding RPM (revolutions per minute). In Figure 3, all of the plots show decreasing apparent viscosity with an increasing shear rate, a trend consistent with pseudoplastics. This type of non-Newtonian behavior is more evident in the low shearing region ($3^{-7} s^{-1}$), and beyond $7 s^{-1}$, it tends to be less sensitive to shearing force, thus becoming more Newtonian. This could be because of the increased homogeneity compared to the higher heterogeneity at lower shearing rates.21,22

3.2. Viscosity Modeling. 3.2.1. Bitumen Viscosity Modeling. The very popular three-parameter empirical correlations for the viscosity–temperature–pressure relationship originally developed for Athabasca and Cold Lake extr-a-

Figure 3. Rheology of the crude oils.

Figure 4. Plot of the experimental against the predicted bitumen viscosity using eq 3.

Figure 5. Effect of temperature on the viscosities of the crude oils and their mixtures.

Figure 6. Effect of pressure on the viscosities of bitumen and light oil.

Figure 7. Effect of pressure on the viscosity of mixture M1.

Figure 8. Effect of pressure on the viscosity of mixture M2.

Figure 9. Effect of pressure on the viscosity of mixture M3.
heavy oil (bitumen), which were proposed by Mehrotra and Svrcek, are used in this work to predict the viscosity of the bitumen samples being used under temperature and N₂ gas pressure.

These correlations are shown in eqs 1 and 2

\[
\ln(\mu) = \exp(c_1 + c_2 \ln T) + c_3 P
\]

(1)

\[
\ln \ln(\mu) = \exp(c_1 + c_2 \ln T) + c_3 P
\]

(2)

where \(c_1, c_2, \) and \(c_3\) are system-specific empirical parameters and \(\mu\) is the viscosity in centipoise (cP).

Equation 1 was rearranged using a double natural log to give eq 3

\[
\ln \ln(\mu) = c_1 + c_2 \ln T + c_3 \ln P
\]

(3)

This was applied to the experimental bitumen data and the constants \(c_1, c_2, \) and \(c_3\) were determined by regression analysis. In Figure 4, a comparison is shown by plotting the predicted values against the experimental bitumen viscosity values to highlight the accuracy of eq 3. AAD% when using this equation in the current study was obtained as 3.86%, which implies that the predicted bitumen values were close enough to the measured ones (AAD% < 5). The values of the coefficients, \(c_1, c_2, \) and \(c_3\), obtained by regression analysis were 4.9362, 0.0078, and −0.6693, respectively.

3.2.2. Prediction of the Viscosity of Crude Oil Mixtures.

3.2.2.1. Theoretical Considerations.

Numerous attempts have been made to represent the viscosity of binary mixtures as a function of the concentrations of the original two components and their respective viscosities. Many authors have expressed \(f\) differently, for example, \(f = \frac{\mu_1}{3}\) (Kendall and Monroe), \(f = 1/\mu\) (Bingham), and \(f = 2.303 \ln \mu\) (Arrhenius).

For bitumen diluent mixtures, however, only a limited number of correlations have been developed. Conventional mixing rules for the mixing of pure components are not generally applicable in the case of bitumen. This was attributed to the likely interaction of diluent molecules with the complex mixtures of hydrocarbons, which are found in bitumen. The equations presented by Kendall and Monroe and Chirinos, respectively, are highly limited in their range of applicability.

\[
\frac{V_m^{1/2} = Y_D V_D^{1/2} + (1 - Y_D)V_B^{1/2}}
\]

(4)

\[
\ln \ln(V_m + 0.7) = X_D(\ln(V_D + 0.7) - \ln(V_B + 0.7)) + \ln(V_B + 0.7)
\]

(5)

Of these two equations, eq 5 has been shown to have a more practical range of applicability but less accuracy. The equations by Cragoe and Shu have also been used over a wider range of data and gave good viscosity estimations. In addition, it has been reported that the model by Nissan and Grunberg is popular among many oil and gas companies for determining the viscosity of binary crude oil mixtures. This was, however, found to be inapplicable to the data sets used in this study. The classical Van Der Wyk model was shown to be applicable to many fluids but was found difficult to use in its original version.
Some authors\textsuperscript{12,32,33} have presented methods that can predict the viscosity of crude oil binary mixtures while considering the effect of temperature and pressure but we believe that there are limited data and research on this kind of approach and therefore the new model is compared to some of the reported models\textsuperscript{33} as shown later in this paper to indicate how accurate our approach is.

3.2.3. Effect of Temperature and Pressure on the Viscosity.

In this study, it was shown that for temperature, the effect is an exponential decrease. The plots in Figure 5 show specifically the effect of the temperature on the viscosity of the crude oils and their mixtures. This effect is more pronounced in heavier or more viscous oils as can be seen in the graph with heavy oil viscosity changing from 40761.8 cP at 45.5 °C to 374.1 cP at 110.3 °C, whereas it was about 8.1003 cP at 45 °C for light oil, it only changed from 2.6 cP at 110.4 °C to 374.1 cP at 110.3 °C, whereas it was about 38.2% at 45 °C for a pressure increase from 0.1 to 6 MPa.

In this study, for the most dilute mixture M1 (Figure 7), the viscosity increased by 38.2% at 45 °C, while for light oil, it only changed from 8.1003 cP at 45 °C to 2.6 cP at 110.4 °C. As for the pressure, its effect on the viscosity has been expressed as \( \mu_p/\mu_{\text{ref}} \), where \( \mu_{\text{ref}} \) is the viscosity at atmospheric pressure and \( \mu_p \) is the viscosity under pressure. This effect is more pronounced in the more viscous oil as shown in Figures 6, 7, and 8, and the trend is less observed in lighter oils as shown in Figure 9. In addition, it is seen much more at lower temperatures. For this study, the viscosity for bitumen was shown to have increased by 43% at 45 °C, whereas it was about 10% at 110 °C for a pressure increase from 0.1 to 6 MPa. Similarly, for the most dilute mixture M1 (Figure 7), the viscosity increased by 38.2% at 45 °C and 2.13% at 110 °C for a pressure increase from 0.1 to 6 MPa (Figure 8).

3.3. Development of a New Method to Measure the Viscosity of Diluted Bitumen under Reservoir Temperature and Pressure.

The original Van Der Wijk model, which was used as the basis for developing our new method, is based on the molecular interaction between the species of the two components in a mixture; therefore, making the assumption that the internal friction between the two different layers of the liquid components is actually a hindrance to the flow properties of the mixture. This he termed as the viscosity of this mixture.

Herein, these interaction parameters are expressed as the viscosity in logarithmic terms and the mole fraction changed to the mass fractions on the assumption that the number of moles of each component is proportional to its mass fraction in the mixture. To introduce the pressure and temperature terms, the co-efficient of the internal friction between molecules from the two components.

\[ \ln \mu = X_b \ln \left( \frac{\mu_b/\mu_L}{k} \right) + 2X_b \ln \left( \frac{k}{\mu_L} \right) + \ln \mu_L \]

(6)

Van Der Wijk model\textsuperscript{34} has been modified by reworking it to be linear, then adding a new term for the like molecule interactions in different components, and finally using eq 3 to substitute for the bitumen term.

The original Van Der Wijk correlation is

\[ \ln \mu = a_1 + a_2X_b^2 \ln \mu_L + a_3X_b^2 \ln \mu_L + a_4X_b(\mu_L + \mu_b) + a_5 \ln \mu_L^{X_bL-1} \]

(7)

Equation 6 was simplified to give the RVDM

\[ \ln \mu = a_1 + a_2X_b^2 \ln \mu_L + a_3X_b^2 \ln \mu_L + a_4X_b(\mu_L + \mu_b) + a_5 \ln \mu_L^{X_bL-1} \]

(7)

Equation 6, which is the old version of the Van Der Wijk model was developed based on the assumption that there is only one coefficient of the internal friction between the unlike molecules from the two compounds in the mixture. However, bitumen always has some molecules that are similar to the molecules in light oil. Therefore, we propose that for two crude oil mixtures, there is also the consideration of the interaction of like molecules in the different crudes, which also affects the overall

### Table 4. Correlation Constants for the Viscosity Equations of the Different Crude Oil Mixtures for the MVDM (Equation 8)

| mixture | \( a_1 \) | \( a_2 \) | \( a_3 \) | \( a_4 \) | \( a_5 \) |
|---------|---------|---------|---------|---------|---------|
| M1      | -21.8072 | 81.11093 | 114.5818 | -0.000051 | -16.1328 |
| M2      | 4.16898  | 9.89299 | 44.7571  | -0.000051 | -14.08551 |
| M3      | -4.47357 | 3.77817 | 4.39283  | 0        | -0.000032 |

### Table 5. Statistical Results for the Mixtures in Table 1 Using the MVDM

| mixture | AAD% | RMSE (mPa s) | SD (mPa s) |
|---------|------|-------------|-----------|
| M1      | 5.07 | 62.73       | 490.63    |
| M2      | 12.42| 30.94       | 88.31     |
| M3      | 11.43| 31.20       | 38.97     |

### Table 6. Correlation Constants for the Viscosity Equations of the Different Crude Oil Mixtures for the RVDM (Equation 7)

| mixture | \( a_1 \) | \( a_2 \) | \( a_3 \) | \( a_4 \) | \( a_5 \) |
|---------|---------|---------|---------|---------|---------|
| M1      | -21.8072 | 81.11093 | 114.5818 | -0.000051 | -16.1328 |
| M2      | 4.16898  | 9.89299 | 44.7571  | -0.000051 | -14.08551 |
| M3      | -4.47357 | 3.77817 | 4.39283  | 0        | -0.000032 |

### Table 7. Statistical Results for the Mixtures in Table 1 Using the RVDM

| mixture code | AAD% | RMSE (mPa s) | SD (mPa s) |
|--------------|------|-------------|-----------|
| M1           | 12.42| 23.00       | 138.77    |
| M2           | 11.43| 31.20       | 38.97     |
| M3           | 7.871| 80.46       | 497.67    |

### Table 8. Mass Fraction Composition of the Crude Oil Mixtures in This Study

| mixture | heavy oil (%) | light oil (%) |
|---------|---------------|---------------|
| M11     | 10            | 90            |
| M12     | 60            | 40            |
| M13     | 90            | 10            |

Van Der Wijk model\textsuperscript{34} has been modified by reworking it to be linear, then adding a new term for the like molecule interactions in different components, and finally using eq 3 to substitute for the bitumen term.

### Table 9. Statistical Results for the Mixtures in Table 1 Using the RVDM

| mixture code | AAD% | RMSE (mPa s) | SD (mPa s) |
|--------------|------|-------------|-----------|
| M1           | 12.42| 23.00       | 138.77    |
| M2           | 11.43| 31.20       | 38.97     |
| M3           | 7.871| 80.46       | 497.67    |
Due to this consideration, the term $X_B \ln \left( \frac{\mu_B + X_L \mu_L}{\mu_B} \right)$ to represent this interaction was added to eq 7 and also found to be a good fitting parameter to make eq 8, which represents the MVDM:

$$\mu = a_1 + a_2 X_B^2 \ln \mu_B + a_3 X_B \ln \mu_L + a_4 X_B (\mu_L + \mu_B) + a_5 \ln \mu_L^{2X_L^{-1}} + a_6 X_B X_L \ln (X_B \mu_B + X_L \mu_L) \quad (8)$$

Equation 3 is used to obtain the bitumen viscosity, which is substituted into eq 8 for the $\mu_B$ term to predict the viscosity of bitumen diluted with light crude oils under reservoir temperature and pressure. This is applicable when light oil of known viscosity $\mu_B$ is present.

Table 9. Summary of the Viscosities of the Crude Oils Presented by Al-Besharah" under Temperature and Pressure

| mixture     | temp. (°C) | pressure (MPa) | viscosity (mPa·s) | viscosity (mPa·s) |
|-------------|------------|----------------|-------------------|-------------------|
| light oil   | 25         | 0.101          | 11.42             | 28.15             |
| light oil   | 30         | 13.79          | 13.38             | 35.75             |
| light oil   | 40         | 27.58          | 16.33             | 45.32             |
| light oil   | 50         | 41.37          | 20.21             | 55.62             |
| heavy oil   | 25         | 55.16          | 24.44             | 71.81             |
| heavy oil   | 30         | 27.58          | 15.28             | 42.89             |
| heavy oil   | 40         | 41.37          | 18.52             | 52.9              |
| heavy oil   | 50         | 55.16          | 22.22             | 65.91             |

Table 10. Viscosities of Binary Crude Oil Mixtures Extracted from Al-Besharah’

| mixture code | temp. (°C) | pressure (MPa) | viscosity (mPa·s) |
|--------------|------------|----------------|-------------------|
| M11          | 25         | 0.101          | 11.42             |
| M12          | 25         | 13.79          | 13.38             |
| M13          | 25         | 27.58          | 16.33             |
|               | 25         | 41.37          | 20.21             |
|               | 25         | 55.16          | 24.44             |
|               | 30         | 0.101          | 10.55             |
|               | 30         | 13.79          | 12.9              |
|               | 30         | 27.58          | 15.28             |
|               | 30         | 41.37          | 18.52             |
|               | 30         | 55.16          | 22.22             |
|               | 40         | 0.101          | 15.43             |
|               | 40         | 13.79          | 20.22             |
|               | 40         | 27.58          | 24.06             |
|               | 40         | 41.37          | 30.95             |
|               | 40         | 55.16          | 37.72             |
|               | 50         | 0.101          | 5.909             |
|               | 50         | 13.79          | 7.146             |
|               | 50         | 27.58          | 8.622             |
|               | 50         | 41.37          | 10.48             |
|               | 50         | 55.16          | 12.61             |

Table 11. AAD% Comparison of the MVDM, the RVDM, and the Al-Besharah Model

| mixture code | Al-Besharah | MVDM | RVDM |
|--------------|-------------|------|------|
| M11          | 3.233       | 1.86 | 3.12 |
| M12          | 3.62        | 6.55 | 5.89 |
| M13          | 3.65        | 2.823| 3.44 |

Adapted with permission from [Al-Besharah, J. M.; Akashah, S. A.; Mumford, C. J. The Effect of Temperature and Pressure on the Viscosities of Crude Oils and Their Mixtures. *Ind. Eng. Chem. Res.* 1989, 28 (2), 213–221.]. Copyright [1989] [ACS Publications].
Equation 8 shows that there is a direct proportionality between the mixture viscosity and the mass fraction of the components, especially the bitumen content in this case. To highlight this relationship, the viscosity of the crude oil mixture was plotted against the mass fraction as shown in Figure 10. The mixture viscosity was measured at atmospheric pressure and a temperature of 45.5 °C.

3.3.1. Statistical Error Analysis. To determine how accurate our methods are, we used statistical methods like the percentage average absolute deviation (AAD%), the root-mean-square error (RMSE), and the standard deviation (SD) as shown in eqs 9–11, respectively. The arithmetic average of the absolute values of the relative errors (AAD%) is an indication of the accuracy of the model. A low value of AAD% shows a better correlation and a lower error for the predicted values of viscosity. The RMSE is a measure of similar performance as indicated by the AAD%, and the SD indicates how far the predicted values deviate from the actual result.

\[
\text{AAD} = \left( \frac{\mu_{\text{exp}} - \mu_{\text{calcd}}}{\mu_{\text{exp}}} \right) \times 100
\]

\[
\text{RMSE} = \sqrt{\frac{1}{n} \sum_{k=1}^{n} (\mu_{\text{exp}} - \mu_{\text{calcd}})^2}
\]

\[
\text{SD} = \sqrt{\sum_{k=1}^{n} (X_n - \bar{X})^2} / n
\]

where \(\mu_{\text{exp}}\) is the experimentally measured viscosity, \(\mu_{\text{calcd}}\) is the predicted viscosity, \(n\) is the number of data points, \(\bar{X}\) is the mean of the data set, and \(X_n\) is the value of a single data point.

3.4. Validation and Stability of the MVDM. 3.4.1. Comparison of the RVDM with the MVDM. To determine whether the MVDM is an upgrade on the RVDM in terms of accuracy in predicting the viscosity of binary crude oil mixtures, the plots in Figure 11(a) for M1, (b) for M2, and (c) for M3 were made. All plots showed the MVDM line to be closer to the experimental viscosity than the RVDM at predicting the viscosity of a mixture of bitumen and light oil.

To use the method of recovery of bitumen by dilution with lighter oil and with gas lift assistance, the viscosity of the diluent under different temperatures and pressures can be readily obtained but not that of the resultant mixture. Using the Mehrotra/Svrcek model combined with the new Van Der Wijk model, we can determine the viscosity of the diluted bitumen mixture for a reservoir of known temperature and pressure without necessarily knowing the bitumen viscosity. When this new method was used, the AAD% for M1 was only 8.88, while it was around 12.42 when the RVDM was used. A summary of the statistical results of this work is shown in Tables 4 and 5 shows the correlation coefficients of the MVDM. For comparison purposes, the results showing the correlation coefficients for the RVDM are also shown in Table 6 and the error analysis in Table 7.

3.5. Comparison of the MVDM with Literature Data (Al-Besharah). We compared the RVDM and the proposed model (MVDM) to the data available in the literature. Al-Besharah measured the viscosities of crude oils and their mixtures among which some were binary. Al-Besharah also reported some of his experimental data in his paper and that is why we use it to check the accuracy of our approach (Table 8).
An excerpt from the measurements reported by Al-Besharah et al.\textsuperscript{33} as shown in Tables 9 and 10 was used to compare their approach to the method we propose herein for determining the viscosity of the binary mixtures of crude oils under temperature and pressure. A description of the compositions of three mixtures on a weight percentage basis from his study is shown in Table 8 below.

A comparison between the MVDM, the RVDM, and the Al-Besharah model based on the data in Table 10 was also made, as shown in Table 11, to determine the most accurate way of predicting the binary mixture viscosity under temperature and pressure. The MVDM with an average AAD\% of about 3.744\% compared to the 4.15\% of the RVDM is shown as a highly accurate model.

Furthermore, the same set of data summarized in Table 10 from a report by Al-Besharah\textsuperscript{35} was also used to compare more statistical errors between the two versions of the Van Der Wijk model. The results of this comparison are shown in Table 12.

### 3.6. Analysis of MVDM Stability

To determine how many initial measurements are needed to have stable values of the coefficients in the new method, stability analysis was carried out using different data points for the NVDM and the MVDM. This was done for the selected mixture M1, and comparisons between the stability of the MVDM and the RVDM are shown in Figures 12 and 13 for M1. In all cases, the MVDM shows better stability than the RVDM after 15 initial measurements.

Furthermore, the errors in the models are compared using the SD for the AAD\% in 15–25 observations so as to show which model requires the least amount of observations to obtain a near accurate prediction of the mixture viscosity. It is shown in Figure 14 that the MVDM has better accuracy in this regard as well.

In a further exploratory data analysis, for a total of five experiments that were done to measure the viscosity of crude oils under temperature and pressure, three were for the binary mixed crude oils of bitumen and light oil. A total of 25 data points were obtained for each of the five experiments done. In Table 13, a summary of the data analysis is presented based on the AAD for 75 data points.

Similarly, the histogram in Figure 15 is used to compare the distribution of the AAD for the RVDM to that for the MVDM across 75 data points during the prediction of the M1, M2, and M3 viscosity. From this, we can clearly see that the MVDM outdoes the RVDM over the data range used in testing this model (Figure 16).

The plots of RMSE against the number of data points in Figure 16 are used to determine the degree of prediction bias of the new method. In this report, the plots for the three mixtures (M1, M2, and M3) took on a sigmoid shape, which implies that the degree of error reduces as the number of data points increases until convergence is obtained at a sample size of 20 observations.

### 4. CONCLUSIONS

In this work, focused on developing a new method to predict the viscosity of bitumen diluted with light oil using a modified Van Der Wijk model, we used the Mehrorat and Svcek model to predict the viscosity of bitumen under reservoir temperature and pressure. The obtained bitumen viscosity was then combined with a reworked Van Der Wijk correlation to predict the viscosity of a binary mixture of light oil and bitumen. Furthermore, the same two-step process was repeated using a modified Van Der Wijk correlation in step two to predict the viscosity of the same binary component mixture under reservoir temperature and pressure conditions. The key findings in the development of the new method, which is limited to oil blends whose API and viscosity are above those of light oil (0% bitumen mass fraction), are summarized as follows:

1. The two models were compared in terms of accuracy with the latter having better AAD\% values of 8.88, 8.02, and 5.07 for M1, M2, and M3, respectively, while the former had AAD\% values of 12.42, 11.43, and 7.871, for M1, M2, and M3, respectively.

2. The model showed a good accuracy as well when compared to some literature data by Al-Besharah. The AAD\% values for MVDM were 1.86, 6.55, and 2.83 for M11, M12, and M13, respectively, against 233, 3.62, and 3.65 when the Al-Besharah model was used.

### REFERENCES

(1) Alvarado, V.; Manrique, E. Enhanced Oil Recovery: An Update Review. Energies 2010, 3, 1529–1575.
(2) Denney, D. Heavy-Oil Production in Venezuela. J. Pet. Technol. 1999, 51, 110–114.
(3) Devold, H. Oil and Gas Production Handbook: An Introduction to Oil and Gas Production, Transport, Refining and Petrochemical Industry; ABB Oil and Gas, 2013; Vol. 53.
(4) Guevara, E.; Nínez, G.; Gonzalez, J. Highly Viscous Oil Transportation Methods in the Venezuelan Oil Industry. In Exploration, Production Downstream (Refining and Petrochemicals), 15th ed.; Wiley: 1997; Vol. 2, pp 495–502.
(5) Hart, A. A Review of Technologies for Transporting Heavy Crude Oil and Bitumen via Pipelines. J. Pet. Explor. Prod. Technol. 2014, 4, 327–336.
(6) Zhong, H. Q.; Zhu, S.; Zeng, W. G.; Wang, X. L.; Zhang, F. Research on Heavy Oil Gas Lift Assisted with Light Oil Injected from the Annulus. J. Pet. Explor. Prod. Technol. 2018, 8, 1465–1471.
(7) Brito, F.; García, L.; Brown, J. In Use of Natural Gas as a Driving Force in a Diluent-Gas Artificial-Lift System Applied to Heavy Oils, SPE Latin American and Caribbean Petroleum Engineering Conference; Society of Petroleum Engineers, 2010; pp 999–1011.
(8) Hou, H.; Chang, Y.; Hu, D.; Cai, W.; Zhao, G. Application of Gas Lift Technology to a High-Water-Cut Heavy Oil Reservoir in Intercampo Oilfield, Venezuela. SPE Prod. Oper. 2007, 22, 46–49.
(9) Hoffmann, A.; Astutik, W.; Rasmussen, F.; Whitson, C. H. In Diluent Injection Optimization for a Heavy Oil Field, SPE Heavy Oil Conference and Exhibition; Society of Petroleum Engineers, 2016; pp 6–8.
(10) Gateau, P.; Héaut, I.; Barré, L.; Argüller, J. F. Heavy Oil Dilution. Oil Gas Sci. Technol. 2004, 59, 503–509.
(11) Chen, Z.; Li, X.; Yang, D. Quantification of Viscosity for Solvents-Heavy Oil/Bitumen Systems in the Presence of Water at High Pressures and Elevated Temperatures. Ind. Eng. Chem. Res. 2019, 58, 1044–1054.
(12) Zirrah, M.; Hassanzadeh, H.; Abidi, J. Modelling of Bitumen-and-Solvent-Mixture Viscosity Data Using Thermodynamic Perturbation Theory. J. Can. Pet. Technol. 2014, 53, 48–54.
(13) Mishra, A. K.; Kumar, A. Modified Guo Viscosity Model for Heavier Hydrocarbon Components and Their Mixtures. J. Pet. Sci. Eng. 2019, 182, No. 106248.
(14) Mehratra, A. K.; Svrcek, W. Y. Viscosity, Density and Gas Solubility Data for Oil Sand Bitumens. Part I: Athabasca Bitumen Saturated with CO and C2H6, AOSTRA J. Res. 1985, 1, 263–268.
(15) Mehratra, A. K.; Svrcek, W. Y. Viscosity of Compressed Athabasca Bitumen. Can. J. Chem. Eng. 1986, 64, 844–847.
(16) Comberati, J. R.; Zammerilli, A. M. Effects of Petroleum-Reservoir Conditions on Oil Recovery by Carbon Dioxide Injection (No. DOE/METC/TPR-83-4); Department of Energy, Morgantown Energy Technology Center: Morgantown, WV, USA 1982.
(17) Gray, R. M. Upgrading Petroleum Residues and Heavy Oils, 1st ed.; CRC Press: London, 1994.
(18) Ulkuwuoma, O.; Ademodi, B. The Effects of Temperature and Shear Rate on the Apparent Viscosity of Nigerian Oil Sand Bitumen. Fuel Process. Technol. 1999, 60, 95–101.
(19) Ghanam, M. T.; Hasan, S. W.; Abu-Jdayil, B.; Esmail, N. Rheological Properties of Heavy & Light Crude Oil Mixtures for Improving Flowability. J. Pet. Sci. Eng. 2012, 81, 122–128.
(20) Mortazavi-Manesh, S.; Shaw, J. M. Effect of Diluents on the Rheological Properties of Maya Crude Oil. Energy Fuels 2016, 30, 766–772.
(21) Alada, O. S.; Ademodi, B.; Sasaki, K.; Sugai, Y.; Kumazaka, J.; Ogunlaja, A. S. Development of Models to Predict the Viscosity of a Compressed Nigerian Bitumen and Rheological Property of Its Emulsions. J. Pet. Sci. Eng. 2016, 145, 711–722.
(22) Behzadfar, E.; Hatzikiriakos, S. G. Rheology of Bitumen: Effects of Temperature, Pressure, CO2 Concentration and Shear Rate. Fuel 2014, 116, 578–587.
(23) Shu, W. R. A Viscosity Correlation for Mixtures Of Heavy Oil, Bitumen, and Petroleum Fractions. Soc. Pet. Eng. J. 1984, 24, 115–277.
(24) Anhorn, J. L.; Badakshan, A. A Carrier for Heavy Oil Transportation and Viscosity Mixing Rule Applicability. Can. Pet. Tech. 1994, 33, 17–21.
(25) Miadonye, A.; Latour, N.; Puttagunta, V. R. Correlation for Viscosity and Solvent Mass Fraction of Bitumen-Diluent Mixtures. Pet. Sci. Technol. 2000, 18, 1–14.
(26) Kendall, J.; Monroe, K. P. Viscosity of Liquids II. The Viscosity Composition Curve for Ideal Liquid Mixtures. J. Am. Chem. Soc. 1917, 39, 1787–1807.
(27) Grunberg, L.; Nissan, A. H. The energies of vapourisation, viscosity and cohesion and the structure of liquids. Trans. Faraday Soc. 1948, 45, 125–137.
(28) Zhmud, B. Viscosity Blending Equations. Lube Mag 2014, 121, 2–5.
(29) Mehratra, A. K. Viscosity of Cold Lake Bitumen and Its Fractions. Can. J. Chem. Eng. 1990, 68, 348.
(30) Chirinos, M. L.; Gonzalez, J.; Layrisse, I. Rheological Properties of Crude Oils From The Orinoco Oil Belt and the Mixtures with Diluents. Rev. Tec. INTEVEP 1983, 3, 103–115.
(31) Cragoe, C. S. In Changes in the Viscosity of Liquids with Temperatures Pressure and Composition, 1st World Petroleum Congress; World Petroleum Congress, 1933; pp 529–541.
(32) Zirrah, M.; Hassanzadeh, H.; Abidi, J. In Prediction of Bitumen and Solvent Mixture Viscosity Using Cubic-Plus-Association Equation of State, SPE Heavy Oil Conference Canada; Society of Petroleum Engineers, 2012; pp 1523–1530.
(33) Al-Besharah, J. M.; Akashah, S. A.; Mumford, C. J. The Effect of Temperature and Pressure on the Water (1 mol % Water)Viscosity of Crude Oils and Their Mixtures. Ind. Eng. Chem. Res. 1989, 28, 213–221.
(34) Wyk, V. Der. Viscosity of binary mixtures. Nature 1936, 138, 845–846.
(35) Hind, R. K.; McLaughlin, E.; Ubbelohde, A. R. Structure and Viscosity of Liquids Camphor+pyrene Mixtures. Trans. Faraday Soc. 1960, 56, 328–330.
(36) Martin, R. L.; Winters, J. C. Composition of Crude Oil through Seven Carbons as Determined by Gas Chromatography. Anal. Chem. 1959, 31, 1954–1960.
(37) Subramanian, M.; Deo, M. D.; Hanson, F. V. Compositional Analysis of Bitumen and Bitumen-Derived Products. J. Chromatogr. Sci. 1996, 34, 20–26.
(38) Rahimi, P. M.; Gentzis, T. The Chemistry of Bitumen and Heavy Processing. In The Chemistry of Bitumen and Heavy Oil Processing; Springer: New York, NY, 2006; Chapter 19, pp 149–186.