Consequences of dark matter-dark energy interaction on cosmological parameters derived from type Ia supernova data

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Models where the dark matter component of the Universe interacts with the dark energy field have been proposed as a solution to the cosmic coincidence problem, since in the attractor regime both dark energy and dark matter scale in the same way. In these models the mass of the cold dark matter particles is a function of the dark energy field responsible for the present acceleration of the Universe, and different scenarios can be parametrized by how the mass of the cold dark matter particles evolves with time. In this article we study the impact of a constant coupling $\delta$ between dark energy and dark matter on the determination of a redshift dependent dark energy equation of state $w_{DE}(z)$ and on the dark matter density today from SNIa data. We derive an analytical expression for the luminosity distance in this case. In particular, we show that the presence of such a coupling increases the tension between the cosmic microwave background data from the analysis of the shift parameter in models with constant $w_{DE}$ and SNIa data for realistic values of the present dark matter density fraction. Thus, an independent measurement of the present dark matter density can place constraints on models with interacting dark energy.

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I. INTRODUCTION

Supernovae of type Ia (SNIa) chronicle the recent expansion history of the Universe. The accumulated data encrypts information about the composition of the Universe and the physical properties of its main components, in particular, of the dark energy (DE) [1] that drives the accelerated expansion today.

Data from SNIa [2,3], the cosmic microwave background (CMB) [4] and large scale structure [5] converged to a concordance $\Lambda$CDM model [6], with a nearly flat Universe where a cosmological constant $\Lambda$ is the current dominant energy component, accounting for approximately 74% of the critical density, the remaining being nonrelativistic nonbaryonic dark matter (DM, 22%) and baryonic matter (4%).

This simple “vanilla” $\Lambda$CDM model is still compatible with data but is not satisfactory mainly because it requires a large amount of fine tuning in order to make the cosmological constant energy density dominant at recent epochs. DE can also be modeled by a scalar field, the so-called quintessence models, either slowly rolling towards the minimum of the potential or already trapped in this minimum [1,7–9]. In this case, the equation of state of DE may vary with cosmological time.

In spite of the success of the concordance model, one should keep in mind that there exists some tension between SNIa and CMB data when the DE equation of state is allowed to be different from a cosmological constant. Best-fit models for one set of data alone is usually ruled out by the other set at a large confidence limit [10]. SNIa data typically favors large values of nonrelativistic dark matter abundance $\Omega_{DM}$ and a phantomlike DE equation of state $\omega < -1$. Of course these conclusions are valid only in standard models of DE and DM, that is, models where DE and DM are decoupled. This tension has been ameliorated with the new data from the Supernova Legacy Survey (SNLS) [3], as shown in [11].

An intriguing possibility is that DM particles could interact with the DE field, resulting in a time-dependent mass for the DM particles and a modification in its equation of state. In this scenario, sometimes called VAMPs (VVariable-Mass Particles) [12], the mass of the DM particles evolves according to some function of the dark energy field $\phi$ such as, for example, a linear function of the field [12–14] with a inverse power law dark energy potential or an exponential function [15–19] with an exponential dark energy potential. Some of these models have a tracker solution, that is, there is a stable attractor regime where the effective equation of state of DE mimics the effective equation of state of DM [15,19].

The tracker behavior is interesting because once the attractor is reached, the ratio between DM energy density $\rho_{DM}$ and DE energy density $\rho_{DE}$ remains constant afterwards. This behavior could solve the “cosmic coincidence problem”, that is, why are the DE and DM energy densities similar today. However, even in these cases a large amount of fine-tuning is required for the energy density scale of the scalar potential [20].

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In this paper we investigate the effect of a DE-DM coupling in deriving bounds on the DE equation of state and on $\Omega_{\text{DE}}$ from SNIa data. In particular, we show that analyzing the SNIa data with a large positive coupling results in a larger value of $\Omega_{\text{DE}}$, thus potentially increasing the tension with CMB data. This is confirmed by an analysis of the CMB shift parameter [21] for interacting models with constant equation of state.

II. A PHENOMENOLOGICAL MODEL

Variable-mass particles generically arise in models where the quintessence field is coupled to the nonbaryonic dark matter field (coupling to baryonic matter is severely restricted [22]). Such a coupling represents a particularly simple and relatively general form of modified gravity: they appear in fact in scalar-tensor models (in the Einstein frame) and in simple versions of higher-order gravity theories in which the action is a function of the Ricci scalar. From a Lagrangian point of view, these couplings could be of the form $g(\Phi)m_0\phi^4\psi + h(\Phi)\phi^2\phi^2$ for a fermionic or bosonic dark matter represented by $\psi$ and $\phi$ respectively, where the functions $g$ and $h$ of the quintessence field $\Phi$ can in principle be arbitrary.

Instead of postulating a definite model by choosing two functions defining a DE self-coupling potential and DE-DM coupling [23], one could alternatively follow an approach that is more model-independent and closer to observations by introducing a parametrization for the DE equation of state $w_{\text{DE}}(a)$ and for the coupling function $\delta(a)$, where $a(t)$ is the scale factor of the Universe.

The dynamics of the quintessence field, governed by its potential, induces a time dependence in the mass of dark matter particles. Therefore one would have $m = m(\Phi(a))$ that we will parametrize in terms of a function of the scale factor $\delta(a)$ in the following way:

$$m(a) = m_0 e^{\int_0^a \delta(\alpha)/d\ln a}$$

(1)

where $m_0$ is the particle mass today. In other words, in addition to the usual parametrizations of the DE variable equation of state we introduce a parametrization for the DM variable mass. Just as for the equation of state $w_{\text{DE}}(a)$, this parametrization allows to study the observational data in a systematic fashion in search of new physical phenomena. The physical interpretation of the coupling $\delta(a)$ is therefore straightforward: it represents the rate of change of the DM particle mass, $\delta = d \ln m / d \ln a$. In this paper we will focus on the simplest case, a constant coupling $\delta$.

This variable-mass results in the following equation for the evolution of the DM energy density $\rho_{\text{DM}}$:

$$\dot{\rho}_{\text{DM}} + 3H \rho_{\text{DM}} - \delta(a)H \rho_{\text{DM}} = 0$$

(2)

where $H = \dot{a}/a$ is the Hubble parameter. Conservation of the total stress-energy tensor them implies that the dark energy density should obey

$$\dot{\rho}_{\text{DE}} + 3H \rho_{\text{DE}}(1 + w_{\text{DE}}) + \delta(a)H \rho_{\text{DM}} = 0$$

(3)

Recently, Majerotto et al. [24] (see also [25]) considered the case of constant $w_{\text{DE}}$ and assumed a tracking behavior of the DM and DE densities ($\rho_{\text{DE}} / \rho_{\text{DM}} \propto a^{3\delta}$) in their analysis of SNIa data. In this class of models the modifications of the abundance of mass-varying DM particles were studied in [26] and consequences for the evolution of the Universe were analyzed in [27].

However, in quintessence models the equation of state is generally time-dependent. Therefore in this work we will study the impact of a constant DM-DE coupling $\delta$ on the determination of a redshift dependent DE equation of state $w_{\text{DE}}(z)$ and on the best-fit value of the dark matter abundance today from SNIa data.

In the case of a constant interaction, Eq. (2) can be easily solved:

$$\rho_{\text{DM}}(a) = \rho_{\text{DM}}^{(0)} a^{-3 + \delta},$$

(4)

where $\rho_{\text{DM}}^{(0)}$ is the nonbaryonic DM energy density today. Substituting this solution in Eq. (3) we obtain a differential equation in the scale factor $a$

$$\frac{d\rho_{\text{DE}}}{da} + \frac{3}{a} \rho_{\text{DE}}(1 + w_{\text{DE}}) + \frac{\delta}{\delta + 3w} \rho_{\text{DM}}^{(0)} a^{-4 + \delta} = 0.$$  

(5)

Before proceeding to an evolving equation of state, it is instructive to study the particular case of a constant $w_{\text{DE}} = w$, where the solution to Eq. (5) is given by

$$\rho_{\text{DE}}(a) = \rho_{\text{DE}}^{(0)} a^{-3(1 + w)} + \frac{\delta}{\delta + 3w} \rho_{\text{DM}}^{(0)} a^{-3(1 + w)} - a^{-3 + \delta}.$$  

(6)

where $\rho_{\text{DE}}^{(0)}$ is the DE energy density today. The first term of the solution is the usual evolution of DE without the coupling to DM. From this solution it is easy to see that one must require a positive value of the coupling $\delta > 0$ in order to have a consistent positive value of $\rho_{\text{DE}}$ for earlier epochs of the Universe (it is to be noted however that negative values of $\rho_{\text{DE}}$ could be allowed if the dark energy is in fact a manifestation of modified gravity, see e.g. [28]). This feature remains in the case of varying $w$ and we will consider only positive values of $\delta$ throughout the paper.

Furthermore, if $\delta < -3w$ one has a tracking of DE and DM densities at earlier epochs

$$\frac{\rho_{\text{DM}}(a)}{\rho_{\text{DE}}(a)} \rightarrow -\frac{\delta + 3w}{\delta},$$

(7)

resulting in

$$\Omega_{\text{DE}}(a) = \frac{\rho_{\text{DE}}(a)}{\rho_{\text{DE}}(a) + \rho_{\text{DM}}(a)} \rightarrow -\frac{\delta}{3w}.$$  

(8)

Therefore, requiring $\Omega_{\text{DE}} < 0.1$ in the past fixes $\delta < -0.3w$ in this simple case of constant $w$. It is also interesting that one can analytically compute the scale factor $a_{tr}$
where the transition from DM to DE occurs in this simple case:
\[
a_t = \left( -1 - \frac{\delta + 3w}{3} \frac{\rho_{\text{DE}}^{(0)}(z)}{\rho_{\text{DE}}^{(0)}} \right)^{1/(\delta + 3w)}.
\] (9)

For instance, for \( w = -1 \) and \( \delta = 1.6 \) we find \( a_t = 0.665 \) (corresponding to \( z_t = 0.5 \)).

For the remainder of this work, we will study an equation of state with the commonly used parametrization
\[
\rho_{\text{DE}}(z) = \rho_{\text{DE}}^{(0)}(1 + \Theta(z, w_0, w_1, \delta))
\] (11)

where
\[
\rho_{\text{DE}}^{(0)}(z) = \rho_{\text{DE}}^{(0)} e^{-3w_1(z)}(1 + z)^3(1 + w_0 - w_1)
\] (12)
is the usual evolution of noninteracting (NI) DE density for this parametrization of the equation of state and the correction \( \Theta \) is given by
\[
\Theta(z, w_0, w_1, \delta) = \delta e^{-3w_1(z)}(1 + z)^3(1 + w_0 - w_1) \times \Gamma(-(3w_0 - w_1) - \delta, 3w_1, 3w_1(1 + z))
\] (13)

where \( \Gamma(a, x_0, x_1) \) is the generalized incomplete gamma function
\[
\Gamma(a, x_0, x_1) = \int_{x_0}^{x_1} t^{a-1} e^{-t} dt.
\] (14)

Notice that the correction \( \Theta \) vanishes in the case of no interaction, \( \delta = 0 \), as it should be. We have also found solutions for different parametrizations of the dark energy equation of state but will not use them in this paper.

We use the Hubble-free dimensionless luminosity distance \( d_h \) defined in terms of the usual luminosity distance \( d_L \) as
\[
d_h = \frac{H_0}{c} d_L.
\] (15)

The Hubble-free luminosity distance in our model for the parametrization used is given by (we are assuming a flat Universe throughout the paper)
\[
d_h(z, \Omega_{\text{DE}}, w_0, w_1, \delta) = (1 + z) \int_0^z d\zeta [\Omega_{\text{DM}}^{(0)}(1 + \zeta^3) + \Omega_{\text{DE}}^{(0)}(1 + \zeta^3)3 - 3 - (1 - \Omega_{\text{DE}}^{(0)})]^{-1/2}
\] (16)

where \( \Omega_{\text{DM}}^{(0)} \) and \( \Omega_{\text{DE}}^{(0)} \) are the dark matter and the baryonic density fractions today, respectively. In the next section we will compare \( d_h \) (or \( d_L \) for a given value of the Hubble parameter) obtained from our model to observations in order to study the consequences of the coupling between DE and DM.

### III. RESULTS

We will work with two data sets, the so-called gold data set consisting of 157 SNIa [2] and the recent SNLS data set of 71 new SNIa with high redshift [3]. Since these two data sets were obtained with different detection techniques and use different methods to extract the distance moduli of the supernovae, we will analyze them separately. Other recent analysis can be found in [11,29–31].

In both cases we use the maximum likelihood method for estimating the cosmological parameters. We find the best-fit values by numerically minimizing the likelihood function and the 68% confidence level contour plots were obtained by direct integration of the likelihood function. We fixed \( \Omega_{b}^{(0)} = 0.05 \) throughout our analysis. Since the impact of \( \Omega_{b}^{(0)} \) on \( d_h \) is very weak, the exact value of \( \Omega_{b}^{(0)} \) is not important.

The apparent luminosity \( m \) of a supernova in terms of its absolute magnitude \( M \) is usually written as
\[
m = M + 5 \log_{10} d_h + 5 \log_{10} c H_0^{-1} + 25.
\] (17)

The absolute magnitude of each SNIa has a spread around a standard value
\[
M = M_0 + \Delta
\] (18)

where \( M_0 \) is the absolute magnitude of a standard SNIa and \( \Delta \) represents the corrections arising from fits to the color and light curves of each supernova. Hence one can write the apparent magnitude as
\[
m_{\text{corr}} = m - \Delta = 5 \log_{10} d_h + \bar{M}
\] (19)

where the so-called nuisance parameter \( \bar{M} \) is given by
\[
\bar{M}(H_0, M_0) = M_0 + 5 \log_{10} c H_0^{-1} + 25.
\] (20)

We will use the values of the distance modulus \( \mu \) given by
\[ \mu = m - \Delta - \bar{M} = 5 \log_{10} d_h \] (21)
in order to derive bounds in the parameters in our model.

For the analysis of the SNIa gold data set, we follow Ref. [32–34] and estimate the likelihood function already marginalized over the nuisance parameter \( \bar{M} \), which includes \( H_0 \), for different values of the coupling \( \delta \).

\[ L(w_0, w_1, \Omega_{\text{DM}}) = \exp\left[-\frac{1}{2} \sum_{i=1}^{71} \frac{(\mu_i - 5 \log_{10} d_L(h = 0.7, w_0, w_1, \Omega_{\text{DM}}; z_i) - 25)^2}{\sigma_{\mu_i}^2 + \sigma_v^2 + \sigma_{\text{int}}^2}\right] \] (22)

where \( \sigma_{\mu_i} \) is the uncertainty associated with the observational techniques in determining the magnitudes, \( \sigma_v \) is associated with the peculiar velocities (and hence negligible for large redshifts) and \( \sigma_{\text{int}} \) is due to the intrinsic dispersion of the absolute magnitudes.

In the case where there is no interaction between DE and DM, we find that the best-fit values for \( \Omega_{\text{DM}}^0 \) in a \( \Lambda \)CDM model (\( w_1 = 0 \) and \( w_0 = -1 \)) are \( \Omega_{\text{DM}}^0 = 0.26 \pm 0.04 \) and \( \Omega_{\text{DM}}^0 = 0.19 \pm 0.02 \) for the gold set and SNLS data, respectively. The best fit from the SNLS data is in remarkable agreement with the recent analysis of the 3-year data from the Wilkinson Microwave Anisotropy Probe (WMAP3y) [4] alone, which results in \( \Omega_{\text{DM}}^0 = 0.18 \pm 0.04 \), whereas the best fit from the gold data set has a somewhat poorer agreement.

Many parametrizations for the DE equation of state were tested with SNIa data but frequently fixing a particular value of \( \Omega^0_\Lambda \) or marginalizing over a flat or gaussian prior around \( \Omega^0_\Lambda = 0.27 \) [2,10,32–34]. However, the agreement between SNIa and CMB gets particularly worse for the gold data set when we allow for \( w_{\text{DE}}(z) \) and \( \Omega_{\text{DM}}^0 \) to vary simultaneously without any priors. For instance, fixing \( w_1 = 0 \), that is, a constant DE equation of state, we find the best-fit values are \( w_0 = -2.4 \) and \( \Omega_{\text{DM}}^0 = 0.44 \) for the gold set and \( w_0 = -1.0 \) and \( \Omega_{\text{DM}}^0 = 0.21 \) for the SNLS data. Allowing for an evolving equation of state does not significantly alter the fits. A possible tension between SNIa and CMB data which was present in the gold data set when one considers models other than the \( \Lambda \)CDM model practically disappeared in the SNLS data [11].

We want in this article the effects of adding a coupling between DE and DM on these fits to SNIa data. The coupling will be characterized by the constant \( \delta \). We analyze first the case of constant \( w \) (i.e. \( w_1 = 0 \)) and then generalize to a variable equation-of-state.

**A. Constant equation of state**

We show in Fig. 1 the 68% confidence level contour plots in the \( w_0 \times \Omega_{\text{DM}}^0 \) plane for different values of the coupling \( \delta \), keeping \( w_1 = 0 \) (constant equation of state). Notice that the SNLS data results in a better agreement with CMB measurements. One can see the existence of a correlation between \( w_0 \) and \( \Omega_{\text{DM}}^0 \) and that turning on the interaction results in a marked tendency towards increasing the best value for \( \Omega_{\text{DM}}^0 \) against the CMB results.

However, a direct comparison of our result for \( \Omega_{\text{DM}}^0 \) with the CMB results is not possible, since the latter has been obtained in the context of uncoupled models. It is therefore important to see whether there are upper limits to \( \Omega_{\text{DM}}^0 \) which are independent of the cosmological model. An upper limit to \( \Omega_{\text{DM}}^0 \) which does not depend on the background cosmology can be obtained from the galaxy cluster dynamics. However the current data yield very weak constraints: Ref. [35] gives \( \Omega^0_\Lambda = 0.30^{+0.17}_{-0.07} \) so that even \( \Omega_{\text{DM}}^0 = 0.6 \) is not excluded at more than 95% c.l. and actually the strong degeneracy with \( \sigma_8 \) allows for even higher values.

On the other hand, for a constant equation of state, we can use the CMB shift parameter in order to study the effects of interaction. The shift parameter encapsulates information contained in the detailed CMB power spectrum and, more importantly, its measured values is weakly dependent of assumptions made about dark energy. It has been used recently to put constraints on models with braneworld cosmologies [36].

![Contour plots in the \( w_0 - \Omega_{\text{DM}} \) plane](image-url)
The shift parameter $R$ is defined by

$$R = \left( \Omega_M^{1/2} \right) \int_{z_{dec}}^{z_{dec}} \frac{d\zeta'}{E(\zeta')}$$

(23)

where $z_{dec} = 1088$ is the redshift of matter-radiation de-

FIG. 2. Contour plots in the $w_0 - \Omega_{DM}$ plane for a constant equation of state with $3\sigma$ confidence level for different values of the DM-DE coupling $\delta$. We compare constraints from the gold SNIa (solid line) and CMB (dashed line) data.

FIG. 3. Contour plots in the $w_0 - \Omega_{DM}$ plane for a constant equation of state with $3\sigma$ confidence level for different values of the DM-DE coupling $\delta$. We compare constraints from the SNLS SNIa (solid line) and CMB (dashed line) data.

coupling and $E(z) = H(z)/H_0$. We will use the value $R = 1.70 \pm 0.03$, obtained from WMAP3y data [21]. Clearly for such calculations a radiation component with standard conservation equation was appropriately included.

In Figs. 2 and 3 we compare the resulting $3\sigma$ bounds obtained from $R$ and from SNIa data. Figure 2 clearly shows the tension between CMB and the gold data set referred to earlier. It is clear that the introduction of the coupling makes the CMB and the gold data set more incompatible with each other. The coupling favors smaller values of $\Omega_{DM}$ from CMB at the same time favoring larger values of the same quantity from SNIa data. The same qualitative behavior is seen for the SNLS data, although since there is already a good agreement with $\delta = 0$, the situation is less drastic in this case. It is interesting to notice that assuming a constant $w_{DE} = -1$ and requiring $\Omega_{DM} \approx 0.1$ seems to rule out a coupling $\delta \approx 0.5$ from the bounds arising from the shift parameter.

B. Variable equation of state

Allowing $w_1 \neq 0$ does not change the qualitative features of Fig. 1. We show in Fig. 4 the $68\%$ confidence level contour plots in the $w_0 \times \Omega_{DM}$ plane obtained with a marginalization over $w_1$ and as expected the allowed re-

FIG. 4. Contour plots in the $w_0 - \Omega_{DM}$ plane marginalized over values of $w_1$ with $68\%$ confidence level for different values of the DM-DE coupling $\delta$ ($\delta = 0$ (dotted line), $\delta = 0.2$ (dashed line), and $\delta = 0.6$ (solid line)). The best-fit values are marked by an “*” ($\delta = 0$), “+” ($\delta = 0.2$) and an “X” ($\delta = 0.6$), using the gold SNIa (left panel) and SNLS (right panel) data.

FIG. 5. Plot of the likelihood function for $\Omega_{DM}$ with $w_0$ and $w_1$ marginalized for different values of the DM-DE coupling $\delta$ ($\delta = 0$ (dotted line), $\delta = 0.2$ (dashed line), and $\delta = 0.6$ (solid line)) using the gold SNIa (left panel) and SNLS (right panel) data.
TABLE I. Best fits obtained for \( w_0 \) and \( \Omega_{DM} \) for both sets of SNIa data.

| \( \delta \) | Gold \( w_0 \) | Gold \( \Omega_{DM} \) | SNLS \( w_0 \) | SNLS \( \Omega_{DM} \) |
|----------|--------------|---------------------|--------------|---------------------|
| 0.0      | \(-2.6^{+1.4}_{-1.1}\) | \(-0.95^{+0.32}_{-0.42}\) | \(0.48^{+0.03}_{-0.04}\) | \(0.37^{+0.06}_{-0.08}\) |
| 0.2      | \(-3.2^{+1.5}_{-2.8}\) | \(-1.00^{+0.35}_{-0.52}\) | \(0.51^{+0.04}_{-0.05}\) | \(0.40^{+0.07}_{-0.08}\) |
| 0.6      | \(-6.0^{+2.4}_{-3.7}\) | \(-1.15^{+0.42}_{-0.62}\) | \(0.58^{+0.03}_{-0.04}\) | \(0.48^{+0.08}_{-0.09}\) |

FIG. 6. Contour plots in the \( w_0 \times w_1 \) plane obtained by marginalizing the likelihood function over \( \Omega_{DM} \), using a gaussian prior \( \Omega_{DM} = 0.18 \pm 0.04 \), obtained from the WMAP3y data. The effect of the coupling is very small when the prior on the DM density is taken into account.

IV. CONCLUSIONS

There is a vast amount of work studying the possibility of having an interaction between the dark energy and the dark matter components of our Universe. In this paper we analyzed a simple model for the interaction between these two fluids, in which the mass of the dark matter particles increases at a constant rate. We have shown that introducing a coupling between the dark energy component of the Universe with dark matter particles has the effect of increasing the best-fit values of the DM density today obtained from current SNIa data.

We performed a comparison with CMB data for the case of constant equation of state, using the shift parameter. We found that the introduction of the coupling results in a poorer compatibility between CMB and SNIa data. Our results showed that assuming a constant \( w_{DE} \approx -1 \) and requiring \( \Omega_{DM} \geq 0.1 \), a coupling \( \delta \geq 0.5 \) seems to be ruled out. This must be checked by model-independent measurements of \( \Omega_{DM} \) from large scale structure and also with a more careful analysis of the CMB data including more observables in addition to the shift parameter. We also found that introducing the coupling does not change significantly the determination of the DE equation of state when a prior on \( \Omega_{DM} \) is adopted.

We worked with a simple parametrization of the dark energy equation of state and for the DM-DE coupling but we believe that our results are fairly general for the type of interaction that we introduced. It would be interesting to extend our analysis to more general parametrizations for both the interaction and the equation of state available in the literature. However, it should be stressed that SNIa data is not currently very sensitive to a time-varying \( w_{DE} \).

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For reviews on dark energy, see e.g. P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. 75, 559 (2003); T. Padmanabhan, Phys. Rep. 380, 235 (2003); E. J. Copeland, M. Sami, and S. Tsujikawa Int. J. Mod. Phys. D 15, 1753 (2006).

A. G. Riess et al., Astrophys. J. 607, 665 (2004).

P. Astier et al., Astron. Astrophys. 447, 31 (2006).

D. N. Spergel et al., Astrophys. J. Suppl. Ser. 148, 175 (2003).

See e.g. K. Abazajian et al., Astron. J. 126, 2081 (2003).

S. L. Bridle, O. Lahav, J. P. Ostriker, and P. J. Steinhardt, Science 299, 1532 (2003); M. Tegmark et al., Phys. Rev. D 69, 103501 (2004).

R. R. Caldwell, R. Dave, and P. J. Steinhardt, Phys. Rev. Lett. 80, 1582 (1998).

B. Ratra and P. J. E. Peebles, Phys. Rev. D 37, 3406 (1988); P. J. E. Peebles and B. Ratra, Astrophys. J. Lett. 325, L17 (1988).

C. Wetterich, Nucl. Phys. B302, 668 (1988).

H. K. Jassal, J. S. Bagla, and T. Padmanabhan, Phys. Rev. D 72, 103503 (2005).

H. K. Jassal, J. S. Bagla, and T. Padmanabhan, astro-ph/0601389.

G. W. Anderson and S. M. Carrol, astro-ph/9711288.

J. A. Casas, J. García-Bellido, and M. Quiro ´s, Class. Quant. Grav. 9, 1371 (1992).

G. R. Farrar and P. J. E. Peebles, Astrophys. J. 604, 1 (2004).

L. Amendola, Phys. Rev. D 62, 043511 (2000).

L. Amendola and D. Tocchini-Valentini, Phys. Rev. D 64, 043509 (2001).

L. Amendola, Mon. Not. R. Astron. Soc. 342, 221 (2003).

M. Pietroni, Phys. Rev. D 67, 103523 (2003).