Excited isovector mesons using the stochastic LapH method

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Outline

- project goals:
  - comprehensive survey of QCD stationary states in finite volume
  - hadron scattering phase shifts, decay widths, matrix elements
  - focus: large $32^3$ anisotropic lattices, $m_\pi \sim 240$ MeV

- extracting excited-state energies

- single-hadron and multi-hadron operators

- the stochastic LapH method

- level identification issues

- preliminary results for 20 channels $I = 1, \ S = 0$
  - correlator matrices of size $100 \times 100$
  - large number of extended single-hadron operators
  - attempt to include all needed 2-hadron operators

- preliminary results for $I = \frac{1}{2}, \ S = 1, T_{1u}$

- future work
Dramatis Personae

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Excited Isovectors
Building blocks for single-hadron operators

- building blocks: covariantly-displaced LapH-smeared quark fields
- stout links $\tilde{U}_j(x)$
- Laplacian-Heaviside (LapH) smeared quark fields

$$\tilde{\psi}_{a\alpha}(x) = S_{ab}(x,y) \psi_{b\alpha}(y), \quad S = \Theta \left( \sigma_s^2 + \tilde{\Delta} \right)$$

- 3d gauge-covariant Laplacian $\tilde{\Delta}$ in terms of $\tilde{U}$
- displaced quark fields:

$$q^A_{a\alpha j} = D^{(j)} \tilde{\psi}^{(A)}_{a\alpha}, \quad \overline{q}^A_{a\alpha j} = \overline{\tilde{\psi}}^{(A)}_{a\alpha} \gamma_4 D^{(j)\dagger}$$

- displacement $D^{(j)}$ is product of smeared links:

$$D^{(j)}(x,x') = \tilde{U}_{j_1}(x) \tilde{U}_{j_2}(x+d_2) \tilde{U}_{j_3}(x+d_3) \ldots \tilde{U}_{j_p}(x+d_p) \delta_{x'} x + d_{p+1}$$

- to good approximation, LapH smearing operator is

$$S = V_s V_s^\dagger$$

- columns of matrix $V_s$ are eigenvectors of $\tilde{\Delta}$
Extended operators for single hadrons

- quark displacements build up orbital, radial structure

Meson configurations

Baryon configurations

\[
\Phi_{AB}^{\alpha\beta}(p, t) = \sum_x e^{ip \cdot (x + \frac{1}{2} (d_\alpha + d_\beta))} \delta_{ab} \overline{q}_b^B(x, t) \overline{q}_a^A(x, t)
\]

\[
\Phi_{ABC}^{\alpha\beta\gamma}(p, t) = \sum_x e^{ip \cdot x} \varepsilon_{abc} \overline{q}_c^C(x, t) \overline{q}_b^B(x, t) \overline{q}_a^A(x, t)
\]

- group-theory projections onto irreps of lattice symmetry group

\[
\overline{M}_l(t) = c^{(l)*}_{\alpha\beta} \Phi_{AB}^{\alpha\beta}(t) \quad \overline{B}_l(t) = c^{(l)*}_{\alpha\beta\gamma} \Phi_{ABC}^{\alpha\beta\gamma}(t)
\]

- definite momentum \( p \), irreps of little group of \( p \)
Small $-a$ expansion of probes

- link variables in terms of continuum gluon field

$$U_\mu(x) = \mathcal{P} \exp \left\{ ig \int_x^{x+\hat{\mu}} d\eta \cdot A(\eta) \right\},$$

- classical small $-a$ expansion of displaced quark field:

$$U_j(x) U_k(x + \hat{j}) \psi_\alpha(x + \hat{j} + \hat{k}) = \exp(aD_j) \exp(aD_k) \psi_\alpha(x).$$

- where $D_j = \partial_j + igA_j$ is covariant derivative
- must take smearing of fields into account
- radiative corrections of expansion coefficients (hopefully small due to smearing)
iso-vector meson continuum probe operators

$$M_{\mu j_1 j_2 \ldots} = \chi^d \Gamma_{\mu} D_{j_1} D_{j_2} \cdots \psi^u,$$

$$\chi = \bar{\psi} \gamma_4$$

where $$\Gamma_0 = 1$$ and $$\Gamma_k = \gamma_k$$ (analogous table inserting $$\gamma_4, \gamma_5, \gamma_4 \gamma_5$$)

| $$J^P G$$ | $$O^G_{h}$$ irrep | Basis operator |
|----------|-----------------|----------------|
| 0++      | $$A^+_{1g}$$    | $$M_0$$        |
| 1−+      | $$T^+_{1u}$$    | $$M_1$$        |
| 1−−      | $$T^-_{1u}$$    | $$M_{01}$$     |
| 0+-      | $$A^-_{1g}$$    | $$M_{11} + M_{22} + M_{33}$$ |
| 1+-      | $$T^-_{1g}$$    | $$M_{23} - M_{32}$$ |
| 2++      | $$E^-_{g}$$     | $$M_{11} - M_{22}$$ |
|          | $$T^-_{2g}$$    | $$M_{23} + M_{32}$$ |
| 0++      | $$A^+_{1g}$$    | $$M_{011} + M_{022} + M_{033}$$ |
| 1+-      | $$T^-_{1g}$$    | $$M_{023} - M_{032}$$ |
| 2++      | $$E^+_{g}$$     | $$M_{011} - M_{022}$$ |
|          | $$T^+_{2g}$$    | $$M_{023} + M_{032}$$ |
isovector meson continuum probe operators

\[ M_{\mu j_1 j_2 \cdots} = \chi^d \Gamma_\mu \mathcal{D}_{j_1} \mathcal{D}_{j_2} \cdots \psi^u, \quad \chi = \overline{\psi} \gamma_4 \]

where \( \Gamma_0 = 1 \) and \( \Gamma_k = \gamma_k \) (analogous table inserting \( \gamma_4, \gamma_5, \gamma_4 \gamma_5 \))

| \( J^P \) | \( O_h^G \) irrep | Basis operator |
|---|---|---|
| 0^{--} | \( A_{1u}^- \) | \( M_{123} + M_{231} + M_{312} - M_{321} - M_{213} - M_{132} \) |
| 1^{--} | \( T_{1u}^+ \) | \( M_{111} + M_{122} + M_{133} \) |
| 1^{--} | \( T_{1u}^+ \) | \( 2M_{111} + M_{221} + M_{331} + M_{212} + M_{313} \) |
| 1^{--} | \( T_{1u}^- \) | \( M_{221} + M_{331} - M_{212} - M_{313} \) |
| 2^{--} | \( E_u^- \) | \( M_{123} + M_{213} - M_{231} - M_{132} \) |
| \( T_{2u}^- \) | \( M_{221} - M_{331} + M_{313} - M_{212} \) |
| 2^{--} | \( E_u^+ \) | \( M_{123} + M_{213} - 2M_{321} - 2M_{312} + M_{231} + M_{132} \) |
| \( T_{2u}^+ \) | \( M_{221} - M_{331} - 2M_{122} + 2M_{133} - M_{313} + M_{212} \) |
| 3^{--} | \( A_{2u}^+ \) | \( M_{123} + M_{231} + M_{312} + M_{213} + M_{321} + M_{132} \) |
| \( T_{1u}^+ \) | \( 2M_{111} - M_{221} - M_{331} - M_{212} - M_{313} - M_{122} - M_{133} \) |
| \( T_{2u}^+ \) | \( M_{331} - M_{212} + M_{313} - M_{122} + M_{133} - M_{221} \) |
Two-hadron operators

- our approach: superposition of products of single-hadron operators of definite momenta
  \[ c_{p_a I_3 b} I_{3 a I_3 b} p_b \lambda_b B_{p_a I_3 a} I_{3 a I_3 a} S_a \lambda_a \Lambda_a i_a R^p_{p_a I_3 b} I_{3 b} \lambda_b i_b \]
- fixed total momentum \( p = p_a + p_b \), fixed \( \Lambda_a, i_a, \Lambda_b, i_b \)
- group-theory projections onto little group of \( p \) and isospin irreps
- restrict attention to certain classes of momentum directions
  - on axis \( \pm \hat{x}, \pm \hat{y}, \pm \hat{z} \)
  - planar diagonal \( \pm \hat{x} \pm \hat{y}, \pm \hat{x} \pm \hat{z}, \pm \hat{y} \pm \hat{z} \)
  - cubic diagonal \( \pm \hat{x} \pm \hat{y} \pm \hat{z} \)
- crucial to know and fix all phases of single-hadron operators for all momenta
  - each class, choose reference direction \( p_{\text{ref}} \)
  - each \( p \), select one reference rotation \( R^p_{p_{\text{ref}}} \) that transforms \( p_{\text{ref}} \) into \( p \)
- efficient creating large numbers of two-hadron operators
- generalizes to three, four, … hadron operators
Quark line diagrams

- temporal correlations involving our two-hadron operators need
  - slice-to-slice quark lines (from all spatial sites on a time slice to all spatial sites on another time slice)
  - sink-to-sink quark lines

- isoscalar mesons also require sink-to-sink quark lines

- solution: the stochastic LapH method!
Stochastic estimation of quark propagators

- do not need exact inverse of Dirac matrix $K[U]
- use noise vectors $\eta$ satisfying $E(\eta_i) = 0$ and $E(\eta_i \eta_j^*) = \delta_{ij}$
- $Z_4$ noise is used \{1, $i$, $-1$, $-i$\}
- solve $K[U]X^{(r)} = \eta^{(r)}$ for each of $N_R$ noise vectors $\eta^{(r)}$, then obtain a Monte Carlo estimate of all elements of $K^{-1}$

$$K^{-1}_{ij} \approx \frac{1}{N_R} \sum_{r=1}^{N_R} X^{(r)}_i \eta^{(r)*}_j$$

- variance reduction using noise dilution
- dilution introduces projectors

$$P^{(a)}P^{(b)} = \delta^{ab}P^{(a)}, \quad \sum_a P^{(a)} = 1, \quad P^{(a)}\dagger = P^{(a)}$$

- define

$$\eta^{[a]} = P^{(a)}\eta, \quad X^{[a]} = K^{-1}\eta^{[a]}$$

to obtain Monte Carlo estimate with drastically reduced variance

$$K^{-1}_{ij} \approx \frac{1}{N_R} \sum_{r=1}^{N_R} \sum_a X^{(r)[a]}_i \eta^{(r)[a]}_j$$

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Stochastic LapH method

- introduce $Z_N$ noise in the LapH subspace
  \[ \rho_{\alpha k}(t), \quad t = \text{time}, \alpha = \text{spin}, \ k = \text{eigenvector number} \]
- four dilution schemes:

\[
\begin{align*}
P_{ij}^{(a)} &= \delta_{ij} \quad a = 0 \quad \text{(none)} \\
P_{ij}^{(a)} &= \delta_{ij}\delta_{ai} \quad a = 0, 1, \ldots, N-1 \quad \text{(full)} \\
P_{ij}^{(a)} &= \delta_{ij}\delta_{ai,Ki/N} \quad a = 0, 1, \ldots, K-1 \quad \text{(interlace-}K) \\
P_{ij}^{(a)} &= \delta_{ij}\delta_{ai,i \mod k} \quad a = 0, 1, \ldots, K-1 \quad \text{(block-}K)
\end{align*}
\]
- apply dilutions to
  - time indices (full for fixed src, interlace-16 for relative src)
  - spin indices (full)
  - LapH eigenvector indices (interlace-8 mesons, interlace-4 baryons)
The effectiveness of stochastic LapH

- comparing use of lattice noise vs noise in LapH subspace
- $N_D$ is number of solutions to $Kx = y$

![Graph](image)
Quark line estimates in stochastic LapH

- each of our quark lines is the product of matrices
  \[ Q = D^{(j)} SK^{-1} \gamma_4 SD^{(k)^\dagger} \]
- displaced-smeared-diluted quark source and quark sink vectors:
  \[ \varrho^{[b]}(\rho) = D^{(j)} V_s P^{(b)} \rho \]
  \[ \varphi^{[b]}(\rho) = D^{(j)} SK^{-1} \gamma_4 V_s P^{(b)} \rho \]
- estimate in stochastic LapH by \((A, B, \text{flavor, } u, v, \text{ compound: space, time, color, spin, displacement type)}\)
  \[ Q_{uv}^{(AB)} \approx \frac{1}{N_R} \delta_{AB} \sum_{r=1}^{N_R} \sum_{b} \varphi^{[b]}(\rho^r) \varrho^{[b]}(\rho^r)^* \]
- occasionally use \(\gamma_5\)-Hermiticity to switch source and sink
  \[ Q_{uv}^{(AB)} \approx \frac{1}{N_R} \delta_{AB} \sum_{r=1}^{N_R} \sum_{b} \overline{\varphi^{[b]}(\rho^r)} \overline{\varrho^{[b]}(\rho^r)^*} \]
- defining \(\overline{\varrho}(\rho) = -\gamma_5 \gamma_4 \varrho(\rho)\) and \(\overline{\varphi}(\rho) = \gamma_5 \gamma_4 \varphi(\rho)\)
Source-sink factorization in stochastic LapH

- baryon correlator has form
  \[ C_{\bar{l}l} = c_{ijk} c_{\bar{i}\bar{j}\bar{k}}^{(l)} Q_{\bar{i}i}^A Q_{\bar{j}j}^B Q_{\bar{k}k}^C \]

- stochastic estimate with dilution
  \[ C_{\bar{l}l} \approx \frac{1}{N_R} \sum_r \sum_{d_A d_B d_C} c_{ijk} c_{\bar{i}\bar{j}\bar{k}}^{(l)} \left( \varphi_i^{(Ar)} [d_A] \varphi_j^{(Ar)} [d_B] \varphi_k^{(Ar)} [d_A]^* \right) \times \left( \varphi_j^{(Br)} [d_B] \varphi_j^{(Br)} [d_B]^* \right) \left( \varphi_k^{(Cr)} [d_C] \varphi_k^{(Cr)} [d_C]^* \right) \]

- define baryon source and sink
  \[ B_l^{(r)[d_A d_B d_C]} (\varphi^A, \varphi^B, \varphi^C) = c_{ijk} \varphi_i^{(Ar)} [d_A] \varphi_j^{(Br)} [d_B] \varphi_k^{(Cr)} [d_C] \]
  \[ B_l^{(r)[d_A d_B d_C]} (\varphi^A, \varphi^B, \varphi^C) = c_{ijk} \varphi_i^{(Ar)} [d_A] \varphi_j^{(Br)} [d_B] \varphi_k^{(Cr)} [d_C] \]

- correlator is dot product of source vector with sink vector
  \[ C_{\bar{l}l} \approx \frac{1}{N_R} \sum_r \sum_{d_A d_B d_C} B_l^{(r)[d_A d_B d_C]} (\varphi^A, \varphi^B, \varphi^C) B_{\bar{l}}^{(r)[d_A d_B d_C]} (\varphi^A, \varphi^B, \varphi^C)^* \]
Correlators and quark line diagrams

- baryon correlator
  \[ C_{\bar{l}l} \approx \frac{1}{N_R} \sum_r \sum_{d_A d_B d_C} B_l^{(r)[d_A d_B d_C]} (\phi^A, \phi^B, \phi^C) B_{\bar{l}}^{(r)[d_A d_B d_C]} (\bar{\phi}^A, \bar{\phi}^B, \bar{\phi}^C) \]

- express diagrammatically

- meson correlator
More complicated correlators

- two-meson to two-meson correlators (non isoscalar mesons)
Quantum numbers in toroidal box

- periodic boundary conditions in cubic box
  - not all directions equivalent ⇒ using $J^{PC}$ is wrong!!

- label stationary states of QCD in a periodic box using irreps of cubic space group even in continuum limit
  - zero momentum states: little group $O_h$
    \[ A_{1a}, A_{2a}, E_a, T_{1a}, T_{2a}, G_{1a}, G_{2a}, H_a, \quad a = g, u \]
  - on-axis momenta: little group $C_{4v}$
    \[ A_1, A_2, B_1, B_2, E, \quad G_1, G_2 \]
  - planar-diagonal momenta: little group $C_{2v}$
    \[ A_1, A_2, B_1, B_2, \quad G_1, G_2 \]
  - cubic-diagonal momenta: little group $C_{3v}$
    \[ A_1, A_2, E, \quad F_1, F_2, G \]

- include $G$ parity in some meson sectors (superscript $+$ or $-$)
Spin content of cubic box irreps

- numbers of occurrences of $\Lambda$ irreps in $J$ subduced

| $J$ | $A_1$ | $A_2$ | $E$ | $T_1$ | $T_2$ |
|-----|-------|-------|-----|-------|-------|
| 0   | 1     | 0     | 0   | 0     | 0     |
| 1   | 0     | 0     | 0   | 1     | 0     |
| 2   | 0     | 0     | 1   | 0     | 1     |
| 3   | 0     | 1     | 0   | 1     | 1     |
| 4   | 1     | 0     | 1   | 1     | 1     |
| 5   | 0     | 0     | 1   | 2     | 1     |
| 6   | 1     | 1     | 1   | 1     | 2     |
| 7   | 0     | 1     | 1   | 2     | 2     |

| $J$ | $G_1$ | $G_2$ | $H$ | $J$ | $G_1$ | $G_2$ | $H$ |
|-----|-------|-------|-----|-----|-------|-------|-----|
| $\frac{1}{2}$ | 1    | 0     | 0   | $\frac{9}{2}$ | 1     | 0     | 2   |
| $\frac{3}{2}$ | 0    | 0     | 1   | $\frac{11}{2}$ | 1     | 1     | 2   |
| $\frac{5}{2}$ | 0    | 1     | 1   | $\frac{13}{2}$ | 1     | 2     | 2   |
| $\frac{7}{2}$ | 1    | 1     | 1   | $\frac{15}{2}$ | 1     | 1     | 3   |
Common hadrons

Irreps of commonly-known hadrons at rest

| Hadron | Irrep   | Hadron | Irrep   | Hadron | Irrep   |
|--------|---------|--------|---------|--------|---------|
| $\pi$  | $A_{1u}^-$ | $K$    | $A_{1u}$ | $\eta, \eta'$ | $A_{1u}^+$ |
| $\rho$ | $T_{1u}^+$ | $\omega, \phi$ | $T_{1u}^-$ | $K^*$ | $T_{1u}$ |
| $a_0$  | $A_{1g}^+$ | $f_0$ | $A_{1g}^+$ | $h_1$ | $T_{1g}$ |
| $b_1$  | $T_{1g}^+$ | $K_1$ | $T_{1g}$ | $\pi_1$ | $T_{1u}^-$ |
| $N, \Sigma$ | $G_{1g}$ | $\Lambda, \Xi$ | $G_{1g}$ | $\Delta, \Omega$ | $H_g$ |
Ensembles and run parameters

- plan to use three Monte Carlo ensembles
  - \((32^3|240)\): 412 configs \(32^3 \times 256\), \(m_\pi \approx 240\) MeV, \(m_\pi L \sim 4.4\)
  - \((24^3|240)\): 584 configs \(24^3 \times 128\), \(m_\pi \approx 240\) MeV, \(m_\pi L \sim 3.3\)
  - \((24^3|390)\): 551 configs \(24^3 \times 128\), \(m_\pi \approx 390\) MeV, \(m_\pi L \sim 5.7\)

- anisotropic improved gluon action, clover quarks (stout links)
- QCD coupling \(\beta = 1.5\) such that \(a_s \sim 0.12\) fm, \(a_t \sim 0.035\) fm
- strange quark mass \(m_s = -0.0743\) nearly physical (using kaon)
- work in \(m_u = m_d\) limit so \(SU(2)\) isospin exact
- generated using RHMC, configs separated by 20 trajectories

- stout-link smearing in operators \(\xi = 0.10\) and \(n_\xi = 10\)
- LapH smearing cutoff \(\sigma_s^2 = 0.33\) such that
  - \(N_v = 112\) for \(24^3\) lattices
  - \(N_v = 264\) for \(32^3\) lattices

- source times:
  - 4 widely-separated \(t_0\) values on \(24^3\)
  - 8 \(t_0\) values used on \(32^3\) lattice
Use of XSEDE resources

- use of XSEDE resources crucial
- Monte Carlo generation of gauge-field configurations: $\sim 200$ million core hours
- quark propagators: $\sim 100$ million core hours
- hadrons + correlators: $\sim 40$ million core hours
- storage: $\sim 300$ TB

Kraken at NICS
Stampede at TACC
correlator software *last_laph* completed summer 2013
  - testing of all flavor channels for single and two-mesons completed
  - testing of all flavor channels for single baryon and meson-baryons ongoing

small-\(a\) expansions of all operators completed

inclusion of all possible 2-meson operators

3-meson operators currently neglected

still finalizing analysis code

initial focus: the 20 bosonic channels with \(I = 1, \; S = 0\)
Operator accounting

- numbers of operators for $I = 1$, $S = 0$, $P = (0,0,0)$ on $32^3$ lattice

| $(32^2|240)$ | $A_{1g}^+$ | $A_{1u}^+$ | $A_{2g}^+$ | $A_{2u}^+$ | $E_g^+$ | $E_u^+$ | $T_{1g}^+$ | $T_{1u}^+$ | $T_{2g}^+$ | $T_{2u}^+$ |
|-----------|-----------|-----------|-----------|-----------|--------|--------|-----------|-----------|-----------|-----------|
| SH        | 9         | 7         | 13        | 13        | 9      | 9      | 14        | 23        | 15        | 16        |
| “$\pi\pi$”| 10        | 17        | 8         | 11        | 8      | 17     | 23        | 30        | 17        | 27        |
| “$\eta\pi$”| 6         | 15        | 10        | 7         | 11     | 18     | 31        | 20        | 21        | 23        |
| “$\phi\pi$”| 6         | 15        | 9         | 7         | 12     | 19     | 37        | 11        | 23        | 23        |
| “$K\bar{K}$”| 0         | 5         | 3         | 5         | 3      | 6      | 9         | 12        | 5         | 10        |
| Total     | 31        | 59        | 43        | 43        | 43     | 69     | 114       | 96        | 81        | 99        |

| $(32^2|240)$ | $A_{1g}^-$ | $A_{1u}^-$ | $A_{2g}^-$ | $A_{2u}^-$ | $E_g^-$ | $E_u^-$ | $T_{1g}^-$ | $T_{1u}^-$ | $T_{2g}^-$ | $T_{2u}^-$ |
|-----------|-----------|-----------|-----------|-----------|--------|--------|-----------|-----------|-----------|-----------|
| SH        | 10        | 8         | 11        | 10        | 12     | 9      | 21        | 15        | 19        | 16        |
| “$\pi\pi$”| 3         | 7         | 7         | 3         | 8      | 11     | 22        | 12        | 12        | 15        |
| “$\eta\pi$”| 26        | 15        | 10        | 12        | 24     | 21     | 25        | 33        | 28        | 30        |
| “$\phi\pi$”| 26        | 15        | 10        | 12        | 27     | 22     | 26        | 38        | 30        | 32        |
| “$K\bar{K}$”| 11        | 3         | 4         | 2         | 11     | 5      | 12        | 5         | 12        | 6         |
| Total     | 76        | 48        | 42        | 39        | 82     | 68     | 106       | 103       | 101       | 99        |
Operator accounting

- numbers of operators for $I = 1$, $S = 0$, $P = (0, 0, 0)$ on $24^3$ lattice

| (24²|390) | $A_{1g}^+$ | $A_{1u}^+$ | $A_{2g}^+$ | $A_{2u}^+$ | $E_g^+$ | $E_u^+$ | $T_{1g}^+$ | $T_{1u}^+$ | $T_{2g}^+$ | $T_{2u}^+$ |
|----------------|----------|----------|----------|----------|--------|--------|--------|--------|--------|--------|
| SH            | 9        | 7        | 13       | 13       | 9      | 9      | 14      | 23      | 15      | 16      |
| “ππ”          | 6        | 12       | 2        | 6        | 8      | 9      | 15      | 17      | 10      | 12      |
| “ηπ”          | 2        | 10       | 8        | 4        | 8      | 11     | 21      | 14      | 14      | 13      |
| “φπ”          | 2        | 10       | 8        | 4        | 8      | 11     | 23      | 3       | 14      | 13      |
| “K−K”         | 0        | 4        | 1        | 4        | 1      | 4      | 8       | 10      | 4       | 6       |
| Total         | 19       | 43       | 32       | 31       | 34     | 44     | 81      | 67      | 57      | 60      |

| (24²|390) | $A_{1g}^-$ | $A_{1u}^-$ | $A_{2g}^-$ | $A_{2u}^-$ | $E_g^-$ | $E_u^-$ | $T_{1g}^-$ | $T_{1u}^-$ | $T_{2g}^-$ | $T_{2u}^-$ |
|----------------|----------|----------|----------|----------|--------|--------|--------|--------|--------|--------|
| SH            | 10       | 8        | 11       | 10       | 12     | 9      | 20      | 15      | 19      | 16      |
| “ππ”          | 1        | 5        | 6        | 2        | 3      | 7      | 18      | 8       | 10      | 9       |
| “ηπ”          | 19       | 9        | 4        | 6        | 13     | 12     | 11      | 18      | 15      | 14      |
| “φπ”          | 18       | 9        | 4        | 6        | 14     | 12     | 11      | 19      | 15      | 15      |
| “K−K”         | 7        | 2        | 2        | 2        | 6      | 4      | 9       | 4       | 8       | 4       |
| Total         | 55       | 33       | 27       | 26       | 48     | 44     | 69      | 64      | 67      | 58      |
Excited states from correlation matrices

- in finite volume, energies are discrete (neglect wrap-around)

\[ C_{ij}(t) = \sum_n Z_i^{(n)} Z_j^{(n)*} e^{-E_n t}, \quad Z_j^{(n)} = \langle 0 | O_j | n \rangle \]

- not practical to do fits using above form
- define new correlation matrix \( \tilde{C}(t) \) using a single rotation

\[ \tilde{C}(t) = U^\dagger C(\tau_0)^{-1/2} C(t) C(\tau_0)^{-1/2} U \]

- columns of \( U \) are eigenvectors of \( C(\tau_0)^{-1/2} C(\tau_D) C(\tau_0)^{-1/2} \)
- choose \( \tau_0 \) and \( \tau_D \) large enough so \( \tilde{C}(t) \) diagonal for \( t > \tau_D \)
- effective masses

\[ \tilde{m}_{\alpha}^{\text{eff}}(t) = \frac{1}{\Delta t} \ln \left( \frac{\tilde{C}_{\alpha\alpha}(t)}{\tilde{C}_{\alpha\alpha}(t + \Delta t)} \right) \]

- tend to \( N \) lowest-lying stationary state energies in a channel
- 2-exponential fits to \( \tilde{C}_{\alpha\alpha}(t) \) yield energies \( E_\alpha \) and overlaps \( Z_j^{(n)} \)
$I = 1, \ S = 0, \ T_{1u}^+ \ channel$

- effective masses $\tilde{m}^{\text{eff}}(t)$ for levels 0 to 15
- results for $32^3 \times 256$ lattice for $m_\pi \sim 240$ MeV
- two-exponential fits
$I = 1$, $S = 0$, $T^+_1$ energy extraction, continued

- Effective masses $\tilde{m}^{\text{eff}}(t)$ for levels 16 to 31
- $32^3 \times 256$ lattice for $m_\pi \sim 240$ MeV
effective masses $\tilde{m}^{\text{eff}}(t)$ for levels 32 to 47

- 32$^3 \times 256$ lattice for $m_\pi \sim 240$ MeV
Level identification

- level identification inferred from $Z$ overlaps with probe operators
- analogous to experiment: infer resonances from scattering cross sections
- keep in mind:
  - probe operators $\overline{O}_j$ act on vacuum, create a “probe state” $|\Phi_j\rangle$,
    $Z$’s are overlaps of probe state with each eigenstate
    $$|\Phi_j\rangle \equiv \overline{O}_i|0\rangle,$$
    $$Z^{(n)}_j = \langle \Phi_j | n \rangle$$
  - have limited control of “probe states” produced by probe operators
    - ideal to be $\rho$, single $\pi\pi$, and so on
    - use of small $-a$ expansions to characterize probe operators
    - use of smeared quark, gluon fields
    - field renormalizations
  - mixing is prevalent
  - identify by dominant probe state(s) whenever possible
Level identification

- overlaps for various operators

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Excited Isovectors
Identifying resonances

- resonances: finite-volume “precursor states”
- probes: *optimized* single-hadron operators
  - analyze matrix of just single-hadron operators $O^{[SH]}_i$
  - rotation to build probe operators $O^{[SH]}_m = \sum_i v_i^m O_i^{[SH]}$
- obtain $Z'$ factors of these probe operators $Z_m^{(n)} = \langle 0 | O^{[SH]}_m | n \rangle$

VERY PRELIMINARY
Bosonic $I = 1$, $S = 0$, $T_{1u}^+$ channel

- finite-volume stationary-state energies: “staircase” plot
- $32^3 \times 256$ lattice for $m_\pi \sim 240$ MeV
- use of single- and two-meson operators only
- blue: levels of max overlaps with SH optimized operators

![Graph showing levels of energy versus levels](image)

- blue: levels of max overlaps with SH optimized operators

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Excited Isovectors
Bosonic $I = 1, \ S = 0, \ A_{1u}^{-}$ channel

- finite-volume stationary-state energies: “staircase” plot
- $32^3 \times 256$ lattice for $m_\pi \sim 240$ MeV
- use of single- and two-meson operators only
- blue: levels of max overlaps with SH optimized operators

\[
B(3u)_{1u}^{-}
\]
**Bosonic $I = 1$, $S = 0$, $E_u^+$ channel**

- finite-volume stationary-state energies: “staircase” plot
- $32^3 \times 256$ lattice for $m_\pi \sim 240$ MeV
- use of single- and two-meson operators only
- blue: levels of max overlaps with SH optimized operators

![Graph showing energy levels](image.png)
Bosonic $I = 1$, $S = 0$, $T_{1g}^-$ channel

- finite-volume stationary-state energies: “staircase” plot
- $32^3 \times 256$ lattice for $m_\pi \sim 240$ MeV
- use of single- and two-meson operators only
- blue: levels of max overlaps with SH optimized operators
Bosonic $I = 1, \ S = 0, \ T_{1u}^-$ channel

- finite-volume stationary-state energies: “staircase” plot
- $32^3 \times 256$ lattice for $m_\pi \sim 240$ MeV
- use of single- and two-meson operators only
- blue: levels of max overlaps with SH optimized operators

![Graph showing energy levels vs. levels]

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Excited Isovectors
Bosonic $I = \frac{1}{2}$, $S = 1$, $T_{1u}$ channel

- kaon channel: effective masses $\tilde{m}^{\text{eff}}(t)$ for levels 0 to 8
- results for $32^3 \times 256$ lattice for $m_\pi \sim 240$ MeV
- two-exponential fits
Bosonic $I = \frac{1}{2}$, $S = 1$, $T_{1u}$ channel

- effective masses $\tilde{m}^{\text{eff}}(t)$ for levels 9 to 17
- results for $32^3 \times 256$ lattice for $m_\pi \sim 240$ MeV
- two-exponential fits
Bosonic $I = \frac{1}{2}, S = 1, T_{1u}$ channel

- effective masses $\tilde{m}^{\text{eff}}(t)$ for levels 18 to 23
- dashed lines show energies from single exponential fits

C. Morningstar Excited Isovectors
Bosonic $I = \frac{1}{2}, \ S = 1, \ T_{1u}$ channel

- finite-volume stationary-state energies: “staircase” plot
- $32^3 \times 256$ lattice for $m_\pi \sim 240$ MeV
- use of single- and two-meson operators only
- blue: levels of max overlaps with SH optimized operators
Issues

- address presence of 3 and 4 meson states
- in other channels, address scalar particles in spectrum
  - scalar probe states need vacuum subtractions
  - hopefully can neglect due to OZI suppression
- infinite-volume resonance parameters from finite-volume energies
  - Luscher method too cumbersome, restrictive in applicability
  - need for new hadron effective field theory techniques
$I = 1$ $\pi \pi$ scattering phase shift

- various channels, various total momenta, $32^3 \times 256$, $m_\pi \approx 240$ MeV
- Brendan Fahy talk (Monday), collaborator Ben Hoerz (Dublin)
- results below very preliminary
References

S. Basak et al., *Group-theoretical construction of extended baryon operators in lattice QCD*, Phys. Rev. D *72*, 094506 (2005).

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C. Morningstar et al., *Improved stochastic estimation of quark propagation with Laplacian Heaviside smearing in lattice QCD*, Phys. Rev. D *83*, 114505 (2011).

C. Morningstar et al., *Extended hadron and two-hadron operators of definite momentum for spectrum calculations in lattice QCD*, Phys. Rev. D *88*, 014511 (2013).
Conclusion and future work

- goal: comprehensive survey of energy spectrum of QCD stationary states in a finite volume
- stochastic LapH method works very well
  - allows evaluation of all needed quark-line diagrams
  - source-sink factorization facilitates large number of operators
  - last_laph software completed for evaluating correlators
- analysis of 20 channels $I = 1$, $S = 0$ for $(24^3|390)$ and $(32^3|240)$ ensembles nearing completion
- can evaluate and analyze correlator matrices of unprecedented size $100 \times 100$ due to XSEDE resources
- study various scattering phase shifts also planned
- infinite-volume resonance parameters from finite-volume energies $\rightarrow$ need new effective field theory techniques