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Temperature-Dependent Third Cumulant of Tunneling Noise

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Poisson statistics predicts that the shot noise in a tunnel junction has a temperature independent third cumulant $e^2I$, determined solely by the mean current $I$. Experimental data, however, show a puzzling temperature dependence. We demonstrate theoretically that the third cumulant becomes strongly temperature dependent and may even change sign as a result of feedback from the electromagnetic environment. In the limit of a noninvasive (zero impedance) measurement circuit in thermal equilibrium with the junction, we find that the third cumulant crosses over from $e^2I$ at low temperatures to $-e^2I$ at high temperatures.

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Shot noise of the electrical current was studied a century ago as a way to measure the fundamental unit of charge [1]. Today shot noise is used for this purpose in a wide range of contexts, including superconductivity and the fractional quantum Hall effect [2]. Already in the earliest work on vacuum tubes it was realized that thermal fluctuations of the current can mask the fluctuations due to the discreteness of the charge. In semiconductors, in particular, accurate measurements of shot noise are notoriously difficult because of the requirement to maintain a low temperature at a high applied voltage.

Until very recently, only the second cumulant of the fluctuating current was ever measured. The distribution of transferred charge is nearly Gaussian, because of the law of large numbers, so it is quite nontrivial to extract cumulants higher than the second. Much of the experimental effort was motivated by the prediction of Levitov and Reznikov [3] that odd cumulants of the current through a tunnel junction should not be affected by the thermal noise that contaminates the even cumulants. This is a direct consequence of the Poisson statistics of tunneling events. The third cumulant should thus have the linear dependence on the applied voltage characteristic of shot noise, regardless of the ratio of voltage and temperature. In contrast, the second cumulant levels off at the thermal noise for low voltages.

The first experiments on the voltage dependence of the third cumulant of tunnel noise have now been reported [4]. The pictures are strikingly different from what was expected theoretically. The slope varies by an order of magnitude between low and high voltages, and for certain samples even changes sign. Such a behavior is expected for a diffusive conductor [5], but not for a tunnel junction. Although the data are still preliminary, it seems clear that an input of new physics is required for an understanding.

It is the purpose of this paper to provide such input.

We will show that the third cumulant of the measured noise (unlike the second cumulant [6]) is affected by the measurement circuit in a nonlinear way. The effect can be seen as a backaction of the electromagnetic environment.

FIG 1 Two resistors in series with a voltage source. The fluctuating current and voltage are indicated.
The result is
\[ \text{can represent 7 or} \]
where \( X \) and \( \Upsilon \) power is a factor of 2 larger if positive and negative
mal noise and shot noise coexist, according to [2]

\[ f \text{ depends on the model for the} \]

\[ \text{independent of the voltage.} \]

\[ \text{The noise \( t \),} \]

\[ \text{tunnel junction both ther-} \]

\[ \text{at temperature} \]

\[ \text{independent in the frequency} \]

\[ \text{range of the measurement.} \]

\[ \text{where} \]

\[ \text{all averages} \]

\[ \text{cumulant of} \]

\[ \text{p} \]

\[ \text{first consider} \]

\[ \text{we first consider} \]

\[ \text{Before addressing the case} \]

\[ \text{When all averages} \]

\[ \text{the resistors} \]

\[ \text{macroscopic resistor.} \]

\[ \text{feedback into the Langevin equation} \]

\[ \text{Equation (5) applies to a time independent mean volt-} \]

\[ \text{For a time dependent perturbation} \]

\[ \text{order,} \]

\[ \text{where} \]

\[ \text{we will use this equation, with} \]

\[ \text{to describe the} \]

\[ \text{This assumes a separation of time scales between} \]

\[ \text{Turning now to the third cumulant, we first note that at} \]

\[ \text{fixed voltage the intrinsic current fluctuations} \]

\[ \text{we introduce the nonlinear feedback from the voltage} \]

\[ \text{fluctuations through the relation} \]

\[ \text{The variable} \]

\[ \text{the sum is over the} \]

\[ \text{cyclic permutations} \]

\[ \text{Equation (10) has the same form} \]

\[ \text{as the “cascaded} \]

\[ \text{Equation (10) determines the current and voltage cor-} \]

\[ \text{We introduce the nonlinear feedback from the voltage} \]

\[ \text{The spectral density} \]

\[ \text{with} \]

\[ \text{The nonlinear feedback from the voltage} \]

\[ \text{Equation (10) has the same form as the “cascaded} \]

\[ \text{The variable} \]

\[ \text{Equation (10) determines the current and voltage cor-} \]
We find

\[ C_{III} = Z^{-3}[Z_1^3C_{C1}^{(3)}(V) + Z_2^3C_{C2}^{(3)}(V_0 - V)] + 3S_{IV} \frac{d}{dV} S_{II}. \]

\[ C_{VVV} = Z^{-3}(Z_1Z_2)^3[C_{C1}^{(3)}(V_0 - V) - C_{C1}^{(3)}(V)] + 3S_{VVV} \frac{d}{dV} S_{VVV}. \]

\[ C_{VVI} = Z^{-3}(Z_1Z_2)^2[Z_1C_{C1}^{(3)}(V) + Z_2C_{C2}^{(3)}(V_0 - V)] + 2S_{VIV} \frac{d}{dV} S_{IV} + S_{IV} \frac{d}{dV} S_{VV}. \]

\[ C_{IV} = Z^{-3}Z_1Z_2[Z_2^3C_{C2}^{(3)}(V_0 - V) - Z_1C_{C1}^{(3)}(V)] + 2S_{IV} \frac{d}{dV} S_{IV} + S_{VV} \frac{d}{dV} S_{II}. \]

We apply the general result (12) to a tunnel barrier (resistor number 1) in series with a macroscopic resistor (number 2). The spectral densities \( C_{C1}^{(3)} \) and \( C_{C2}^{(3)} \) are given by Eqs (4) and (9), respectively. For \( C_{C2}^{(3)} \) we use Eq (3), while \( C_{C1}^{(3)} = 0 \). From this point on we assume \( V \)-independent \( Z \)'s.

We compare \( C_I = C_{III} \) with \( C_V = -C_{VVV}/Z_2^3 \). The choice of \( C_V \) is motivated by the typical experimental situation in which one measures the current fluctuations indirectly through the voltage over a macroscopic series resistor. From Eq (12) we find

\[ C_\alpha = \frac{e^2I}{(1 + Z_2/Z_1)^3} \times \left( \frac{3(\sinh \epsilon \epsilon_0 u - u)(T_2 \epsilon_0 - \epsilon_0)}{1 + Z_1/Z_2}\sinh \epsilon \epsilon_0 u - \cosh \epsilon \epsilon_0 u \right). \]

with \( g_I = 1 \), \( g_V = -Z_1/Z_2 \), and \( u = eV/2kT_1 \).

In the shot noise limit \( (eV \gg kT_1) \) we recover the third cumulant obtained in Ref [7] by the Keldysh technique

\[ C_I = C_V = e^2I \left( 1 - 2Z_2/Z_1 \right) \left( 1 + Z_2/Z_1 \right)^4. \]

In the opposite limit of small voltages \( (eV \ll kT_1) \) we obtain

\[ C_I = e^2I \frac{1 + (Z_2/Z_1)(2T_2/T_1 - 1)}{(1 + Z_2/Z_1)^4}, \]

\[ C_V = e^2I \frac{1 - Z_2/Z_1 - 2T_2/T_1}{(1 + Z_2/Z_1)^4}. \]

We conclude that there is a change in the slope \( dC_I/dI \) from low to high voltages. If the entire system is in thermal equilibrium \( (T_2 = T_1) \), then the change in slope is a factor \( \pm(1 - 2Z_2)(Z_1 + Z_2)^{-1} \), where the + sign is for \( C_I \) and the - sign for \( C_V \). In Fig 2 we plot the entire voltage dependence of the third cumulants.

The limit \( Z_2/Z_1 \to 0 \) of a noninvasive measurement is of particular interest. Then \( C_I = e^2I \) has the expected result for an isolated tunnel junction [3], but \( C_V \) remains affected by the measurement circuit.

\[ \lim_{Z_2/Z_1 \to 0} C_V = e^2I \left( 1 - \frac{T_2}{T_1} \frac{3(\sinh \epsilon \epsilon_0 u - u)}{usinh \epsilon \epsilon_0 u} \right). \]

This limit is also plotted in Fig 2, for the case \( T_2 = T_1 = T \) of thermal equilibrium between the tunnel junction and the macroscopic series resistor. The slope then changes from \( dC_V/dI = -e^2 \) at low voltages to \( dC_V/dI = e^2 \) at high voltages. The minimum \( C_V = -0.69 \) is reached at \( eV = 2.7kT \).

In conclusion, we have demonstrated that feedback from the measurement circuit introduces a temperature dependence of the third cumulant of tunneling noise. The graph shows the voltage dependence of the third cumulants \( C_I \) and \( C_V \) of current and voltage for a tunnel junction (resistance \( Z_1 \)) in series with a macroscopic resistor \( Z_2 \). The two solid curves are for \( Z_2/Z_1 \to 0 \) and the dashed curves for \( Z_2/Z_1 = 1 \). The curves are computed from Eq (13) for \( T_1 = T_2 = T \). The high voltage slopes are the same for \( C_I \) and \( C_V \), while the low voltage slopes have the opposite sign.

FIG 2 Voltage dependence of the third cumulants \( C_I \) and \( C_V \) of current and voltage for a tunnel junction (resistance \( Z_1 \)) in series with a macroscopic resistor \( Z_2 \). The two solid curves are for \( Z_2/Z_1 \to 0 \) and the dashed curves for \( Z_2/Z_1 = 1 \). The curves are computed from Eq (13) for \( T_1 = T_2 = T \). The high voltage slopes are the same for \( C_I \) and \( C_V \), while the low voltage slopes have the opposite sign.
temperature independent result $\varepsilon e^2$ of an isolated tunnel junction [3] acquires a striking temperature dependence in an electromagnetic environment, to the extent that the third cumulant may even change its sign. Precise predictions have been made for the dependence of the noise on the environmental impedance and temperature, which can be tested in ongoing experiments [4].

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Note added—For a comparison of our theory with experimental data, see Reulet, Senzier, and Prober [15].

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