Phantom Energy Accretion by a Class of Black Holes *

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Abstract

This paper deals with phantom energy accretion by a class of black holes. We give the general formulation for the accretion process by using general static spherically symmetric spacetime. This is then applied to SdS and global charged monopole black holes. We find that the mass of black holes decreases due to phantom accretion. The conditions for critical accretion are also explored which imply a unique critical point in each case.

Keywords: Phantom Energy; Black Holes; Accretion; Critical Accretion.

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1 Introduction

Recently, the concept of dark energy (DE) has been recognized by observing that our universe is in accelerating phase. It has been discussed in many cosmological scenarios that the existence of DE with negative pressure leads to a new exotic model of the universe in which it dies as a result of Big Rip [1]-[4]. Also, the anisotropies in cosmic microwave background radiations as

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observed by WMAP [5]-[7] favor the accelerating behavior of universe. The substance responsible for this behavior is speculated as DE. Due to observational facts, this is a well motivated problem for the theoretical physicists.

Different models like quintessence [8], phantom [9], tachyon field [10], holographic [11] and brane-world [12] help to understand the nature of DE. The simplest form is the cosmological constant for which equation of state (EoS) parameter is \( \omega = -1 \). The quintessence and phantom are in a hypothetical form of DE for which \( \omega > -1 \) and \( \omega < -1 \), respectively [13]-[15]. If \( \omega \geq -1 \), then the DE density is not increasing or decreasing as the universe expands. On the other hand, if we allow \( \omega < -1 \), then the DE density grows and becomes infinite in a finite time. The expansion of the universe is dominated by the phantom energy which diverges to future singularity (Big Rip).

A massive object surrounded by a matter can capture particles of the matter that passes within certain distance from the massive object. This phenomena is termed as accretion of matter by the massive objects. Bondi [16] originally formulated the problem of matter accretion by the compact objects in Newtonian theory. Michel [17] was the pioneer who studied accretion of gas onto the Schwarzschild black hole (BH) in the relativistic physics. Babichev et al. [18] have shown that BH mass diminishes due to phantom accretion. Jamil et al. [19] have explored the effects of phantom energy accretion onto the charged BH. They pointed out that if the mass of BH becomes smaller (due to accretion of phantom energy) than its charge, then cosmic censorship hypothesis is violated. The same conclusion was deduced by Babichev et al. [20] in a study of the phantom energy accretion onto charged BH with generalized linear EoS.

In the papers [21]-[23], the fate of different BHs in the phantom cosmology has been discussed. In recent papers [24], [25], we have investigated some work about phantom energy accretion. This piece of work is an extension of the previous one for another interesting BH, known as global charged monopole BH by using the procedure of Michel [17]. We also discuss locations of the critical points of accretion. The format of the paper is as follows: In the next section, general formulation is given for spherically symmetric spacetimes. Sections 2 and 3 provide applications to two BHs.Finally, the last section gives the conclusion.
2 General Formalism

A static spherically symmetric BH solution is given by

$$ds^2 = D(r)dt^2 - \frac{1}{D(r)}dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2).$$

(1)

The horizons of BH can be found by the zero of the lapse function \(D(r)\). The energy-momentum tensor for perfect fluid is

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu},$$

(2)

where \(\rho\) is the energy density and \(p\) is the pressure which violates the null energy condition i.e., \(\rho + p < 0\). Also, the only non-zero components of the 4-vector are \(u^0 = \frac{dt}{ds}\) and \(u^r = \frac{dr}{ds}\) which satisfies the normalization condition, i.e., \(u^\mu u_\mu = 1\).

The conservation equation gives

$$r^2 u(\rho + p)\sqrt{D(r)} + u^2 = A_0,$$

(3)

where \(A_0\) is an integration constant with no dimension and \(u^r = u < 0\) for inward flow. Also, by projecting the conservation equation on four-velocity, i.e., \(u_\mu \nabla_\nu T^{\mu\nu} = 0\), it follows that

$$r^2 u \exp(n) = -A_1,$$

(4)

where \(n = \int_{\rho_\infty}^{\rho} \frac{dp}{\rho + p(\rho)}\) and \(A_1 > 0\) is another integration constant (related to the energy flux) implying the dimension of quadratic in length. Here \(\rho\) and \(\rho_\infty\) are densities of the phantom energy at finite and infinite \(r\). From Eqs.(3) and (4), we get

$$(\rho + p)\left(D(r) + u^2\right)\frac{1}{2} \exp(-n) = A_2,$$

(5)

where \(A_2 = -\frac{A_0}{A_1} = \rho_\infty + p(\rho_\infty)\). The rate of change of mass due to phantom energy accretion onto 4D BH is given by [18]

$$\dot{m} = -4\pi r^2 T_{00}.$$

(6)

Using Eqs.(3)-(5) in the above equation, it follows that

$$\dot{m} = 4\pi A_1 [\rho_\infty + p_\infty].$$

(7)
This implies that the mass of BH increases when realistic matter with \( p > 0 \) accretes onto the BH. For phantom energy which violates null energy condition \( \rho_\infty + p_\infty < 0 \), Eq. (7) shows that the accretion of phantom energy onto a BH decreases its mass. The physical reason for the decrease of BH mass lies in the fact that the energy flux associated with the falling phantom energy is always directed away from the BH. The decrease of BH mass is independent of EoS rather than depends only on the violation of null energy condition. It is mentioned here that one can solve Eq. (7) for \( m \) by using EoS \( p = k \rho \). Since all \( p \) and \( \rho \) violating null energy condition must satisfy this equation, thus it holds in general.

Now we find such points at which flow speed is equal to the speed of sound. For the discussion of critical points of accretion, we follow the procedure introduced by Michel [17]. The conservation of mass flux, \( \nabla_\mu J^\mu = 0 \), yields

\[
\rho u r^2 = b, \tag{8}
\]

where \( b \) is an integration constant. Equations (4) and (8) lead to

\[
\left( \frac{\rho + p}{\rho} \right)^2 \left( D(r) + u^2 \right) = b_1, \tag{9}
\]

where \( b_1 = (\frac{db}{b})^2 \). Further, Eqs. (8) and (9) yield

\[
\frac{dr}{r} \left[ 2V^2 - \frac{D'(r)}{D(r) + u^2} \right] + \frac{du}{u} \left[ V^2 - \frac{u^2}{D(r) + u^2} \right] = 0, \tag{10}
\]

where \( V^2 = \frac{d\ln(\rho + p)}{d\ln \rho} - 1 \) and prime denotes differentiation with respect to \( r \). This equation shows that critical points occur where both the square brackets vanish. The physically acceptable solutions of the above equation are obtained if \( u_c^2 > 0 \) and \( V_c^2 > 0 \).

For the sake of application, we apply the general formulation to SdS as well as global charged monopole BHs.

### 3 Phantom Energy Accretion by SdS BH

The SdS BH is obtained by taking \( D(r) = (1 - \frac{2m}{r} - \frac{a^2}{r^2}) \) in Eq. (1), where \( a = \sqrt{\frac{3}{\Lambda}} \), \( m \) and \( \Lambda \) are constants. This metric has essential singularity at
\( r = 0 \) covered by the black hole horizons. Such horizons can be found by solving \( g_{00} = 1 - \frac{2m}{r} - \frac{r^2}{a^2} = 0 \) for \( r \) whose positive real roots will give horizons. The corresponding steady state equation of motion (Eq.(10)) becomes
\[
\frac{dr}{r} [2V^2 - \frac{m}{r} + \frac{r^2}{a^2} + u^2] + \frac{du}{u} [V^2 - \frac{u^2}{1 - \frac{2m}{r} - \frac{r^2}{a^2} + u^2}] = 0. \tag{11}
\]

For the turn-around points, it follows that
\[
u_c^2 = \frac{ma^2 + r_c^3}{2a^2r_c}, \quad V_c^2 = \frac{ma^2 + r_c^3}{2a^2r_c - 3ma^2 - r_c^3}. \tag{12}\]

The solutions are found when \( u_c^2 > 0 \) and \( V_c^2 > 0 \) implying that
\[
2a^2r_c - 3ma^2 - r_c^3 > 0, \quad ma^2 + r_c^3 > 0. \tag{13}\]

The cubic equation (13) can be solved (using the technique discussed in [26]) as follows. For \( \frac{m}{a} < \frac{4\sqrt{2}}{9\sqrt{3}} \), there are three real roots out of which two are positive and one is negative (neglected). The positive roots are given by
\[
r_{c1} = \frac{2a\sqrt{2}}{\sqrt{3}} \sin \chi, \quad r_{c2} = \sqrt{2a}(\cos \chi - \frac{1}{\sqrt{3}} \sin \chi), \tag{14}\]

where \( \chi = \frac{1}{3} \sin^{-1}(\frac{m(\frac{3}{2})^5}{a}) \). The critical points are found by using Eq.(14) in the 2nd equation of (13) so that
\[
\frac{m}{a} = \frac{16}{3} \sqrt{\frac{2}{3}} \sin \chi (\cos^2 \chi - 1), \tag{15}\]

and
\[
\frac{m}{a} = \frac{2}{27} \sqrt{2}(\sqrt{3} \sin \chi - 3 \cos \chi). \tag{16}\]

Since \( |\cos^2 \chi| \leq 1 \), so Eq.(15) implies that \( \frac{m}{a} \leq 0 \) - a contradiction, hence \( r_{c1} \) is not a critical point. On the other hand \( r_{c2} \) is a critical solution of flow if \( \chi > \frac{\pi}{3} \). When \( \frac{m}{a} \geq \frac{4\sqrt{2}}{9\sqrt{3}} \), there is no physical solution and hence no critical point.
4 Phantom Energy Accretion by Global Charged Monopole BH

The influence of cosmological phase transitions upon the cosmological defects such as domain wall, strings and monopoles has attracted the attention of the researchers. The monopoles are formed as a result of the gauge symmetry breaking. The simplest model that describes the monopoles is given by the Lagrangian [27]

\[ L = \frac{1}{2} \partial_\mu \chi^a \partial^\mu \chi^a - \frac{\lambda}{4} (\chi^a \chi^a - \eta^2)^2, \]  
(17)

where \( \chi^a = \frac{w^a}{8\pi} \) is isoscalar triplet and \( a = 1, 2, 3 \). The monopole charge parameter \( \eta \) causes to break the asymptotically flatness of the Schwarzschild solution. A spherically symmetric gravitational collapse of the matter around the monopole forms a BH.

The metric for global charged monopole BH is obtained [28] by replacing \( D(r) = (1 - \eta^2 - \frac{2m}{r}) \) in Eq.(1), where \( m \) is the mass of BH and \( \eta \) is global monopole charge parameter such that \( \eta^2 << 1 \). This metric has essential singularity at \( r = 0 \) covered by the BH horizon at \( r_h = \frac{2m}{1-\eta^2} \). The equation of motion (10) gives

\[ \frac{du}{u} \left[ V^2 - \frac{m}{r(1-\eta^2 - \frac{2m}{r} + u^2)} \right] + \frac{dr}{r} \left[ 2V^2 - \frac{u^2}{(1-\eta^2 - \frac{2m}{r} + u^2)} \right] = 0. \]  
(18)

This shows that turn-around points (critical points) are located at

\[ u_c^2 = \frac{m}{2r_c}, \quad V_c^2 = \frac{m}{2r_c(1-\eta^2) - 3m}. \]  
(19)

The physical solutions exist for \( u_c^2 \geq 0 \) and \( V_c^2 \geq 0 \) implying that

\[ 2r_c(1-\eta^2) - 3m \geq 0, \quad m \geq 0. \]  
(20)

It is worthwhile to mention here that for \( \eta \to 0 \), the above equations reduce to the results of accretion onto Schwarzschild BH. Further, Eq.(20) implies that

\[ r_c = \frac{3m}{2(1-\eta^2)} > 0. \]  
(21)

The relation between horizon radius and critical point is \( r_c = \frac{3}{4} r_h \) which implies that the critical point lies inside the horizons.
5 Outlook

We have formulated the equation of motions for steady state spherically symmetric fluid flow near BH. It has been assumed that infalling fluid does not disturb the generic properties of the BH. Following the procedure introduced by Michel [17], we have derived the relations for the accretion and critical accretion by such a BH for which $g_{11} = (g_{00})^{-1}$. We have applied this formalism to SdS and global charged monopole. For both BHs, we have found that the mass of BHs decreases due to phantom energy accretion on to the BHs. For SdS BH, the critical point can exist only if $\chi > \frac{\pi}{3}$. The critical accretion on the global monopole charged BH implies that $\frac{r_c}{r_h} = \frac{3}{4} < 1$ which indicates that $r_c < r_h$. We conclude that critical point lies inside the horizon of the BH.

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