Research Article

Analytical Fuzzy Analysis of a Fractional-Order Newell-Whitehead-Segel Model with Mittag-Leffler Kernel

Yousuf Alkhezi, Nehad Ali Shah, and Davis Bundi Ntwiga

1Public Authority for Applied Education and Training, College of Basic Education, Mathematics Department, Kuwait
2Department of Mechanical Engineering, Sejong University, Seoul 05006, Republic of Korea
3Department of Mathematics, University of Nairobi, Kenya

Correspondence should be addressed to Davis Bundi Ntwiga; dbundi@uonbi.ac.ke

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In this paper, the method for evaluating an analytical solution of fuzzy Newell-Whitehead-Segel equation with certain affecting terms of force has been given. The notions of an Atangana-Baleanu-Caputo derivative in the vague or uncertainty form are used to reach this type of result for the solution as mentioned earlier. The fuzzy Laplace transformation is implemented at the first attempt to achieve the series form result. Secondly, the iterative method is applied to investigate the suggested solution by inverse Laplace transform. Some new solutions on the Laplace transform of an arbitrary derivative under uncertainty are presented. The solution has been provided in terms of infinite series for the research, which reduces the problem to a few equations. The required results are then calculated in a series solution form that quickly leads to the analytical answer. The solution is divided into two sections, or fuzzy branches, the lower and upper branches. We proved certain test problems to demonstrate the effectiveness of the recommended approach.

1. Introduction

Fuzzy set theory is a valuable method for modeling uncertain issues. As a result, fuzzy concepts have been used to simulate a variety of natural phenomena. In particular, fuzzy partial differential equations (PDEs) have been used in engineering, pattern formation theory, control systems, population dynamics, knowledge-based systems, power engineering, image processing, trial automation, consumer electronics, robotics, management, operations research, and artificial expert/intelligence. Due to its importance in a variety of scientific areas, fuzzy set theory has a close link with fractional calculus [1–4]. Byatt and Kandel [5] proposed fuzzy DEs in 1978, while Agarwal et al. [6] were the first to address the Riemann-Liouville differentiability and fuzziness concept under the Hukuhara differentiability. FC and fuzzy set theory integrates several numerical methodologies that allow for more in-depth knowledge of complex systems while also lowering the computing cost of solving them.

Physical models of real-world phenomena usually contain considerable uncertainty for many reasons. Fuzzy sets appear to be a good tool for modeling the uncertainty that ambiguity and impreciseness bring up. We use it in disciplines including medical, environmental, economic, medical, social, economic, physical, and social science, where data contains uncertainty. To investigate these issues, Zadeh introduced fuzziness to set theory in 1965. Fractional calculus has risen in prominence over the last forty years due to its various applications in engineering and physics [7–11]. In the behavior of defined system processes, many examples exhibit fuzzy instead of stochastic uncertainty. Several writers have been interested in studying the theoretical underpinnings of fuzzy issues in recent years. Fractional fuzziness differential equations (FFDEs) are particularly valuable in modeling scientific and technical problems, like population models, civil engineering, electrohydraulics modeling, and weapon systems evaluation. In mathematics, fractional calculus, along with the help of fuzzy theory, is a key instrument for dealing with ambiguity and determining
subjective or ambiguous status and providing more general solutions. This has been addressed in a variety of real-world situations, including the golden mean [12], practical systems [13], medicine [14], gravity [15], engineering phenomena, and quantum physics. For the very first time, Zadeh [16] became acquainted with fuzzy sets. Then, there was work on the concept of a fuzzy number and its use in fuzzy control [17].

The necessity to model some real-world challenges while accounting for data uncertainty led to fuzzy partial differential equations. As we will see subsequently, partial differential equations are important in many domains of engineering and science. Heat transfer is an important part of mechanical and aerospace engineering study because many equipment and systems in both aerospace and mechanical engineering disciplines are subject to heat [18, 19]. The governing differential equation for the above-mentioned physical setting may be derived using heat conduction equations. An engineer can foresee possible forms of changes of the plate in vibrations based on the equations mentioned earlier in simulation findings. Numerous engineering problems fall into this category, and engineers should address them using numerical approaches; see for more details [20, 21].

The classic Newell-Whitehead-Segel model (NWS) is among the most widely used amplitude equations for presenting the occurrence of stationary spatial stripe patterns in a two-dimensional system, as well as the dynamic behavior near the Rayleigh-Bénard convection bifurcation point of binary fluid mixtures [22]. The rolling pattern, in which the cylinders are made using fluid streamlines that may be twisted to form the hexagonal, and spiral-like pattern, in which the striped cells and honeycomb are formed by splitting the flow of the fluid, are both visible. For example, stripe patterns can be observed in the human fingerprints, visual cortex, and zebra skin. It is important to note that hexagonal patterns may be created using laser beam propagation across the nonlinear optical medium in a diffusion and chemical reaction model [23].

Nonlinear fractional partial differential equations (FPDEs) are seen in many mathematical structures. Solving these equations, on the other hand, is frequently challenging. Effective and established techniques are required to find approximate or analytical solutions to these equations. There are many standard numeric-analytic strategies, like Sumudu transform method [24], homotopy analysis method [25], variational iteration method [26], fractional complex transform method [27], and homotopy perturbation algorithm [28].

2. Basic Definitions

Definition 1. Let us consider fuzzy continuous term of $\tilde{\Theta}(\eta)$ on $[0,\eta] \subset R$ in sense of Atangana-Baleanu-Caputo (ABC) operator with respect to $\eta$ as [29].

The ABC derivative of $\tilde{\Theta}(\eta)$ in $H^1[0,\eta]$ expressed as

$$D_\eta^\delta \tilde{\Theta}(\eta) = \frac{ABC(\gamma)}{1 - \gamma} \int_0^\eta \frac{d}{de} \tilde{\Theta}(e) M_\gamma \left[ \frac{-\gamma}{1 - \gamma} (\eta - e)^\gamma \right] de.$$  (1)

Replacing $E_\mu\left[ (\gamma - \gamma)/(1 - \gamma) \right] \gamma$ by $E_\mu\left[ (\gamma - \gamma)/(1 - \gamma) \right] \gamma$, we have “differential Caputo-Fabrizio derivative.” Further, if $\tilde{\Theta}(\eta) \in C^\gamma[0,\eta] \cap L^\gamma[0,\eta]$, such that $\tilde{\Theta}(\eta) = [\Theta_\gamma, \Theta_\gamma(\eta)]$, $\gamma \in [0, 1]$ and $\eta_0 \in (0, \rho)$, then the fractional fuzzy ABC derivative is defined as

$$D_\eta^\delta \tilde{\Theta}(\eta) = \left[ D_\eta^\delta \tilde{\Theta}_\gamma(\eta_0), D_\eta^\delta \tilde{\Theta}_\gamma(\eta_1) \right], \quad 0 \leq \delta \leq 1,$$  (2)

such that

$$D_\eta^\delta \tilde{\Theta}_\gamma(\eta) = \frac{ABC(\gamma)}{1 - \gamma} \int_0^\eta \frac{d}{de} \tilde{\Theta}(e) E_\gamma \left[ \frac{-\gamma}{1 - \gamma} (\eta - e)^\gamma \right] de,$$  (3)

$$D_\eta^\delta \tilde{\Theta}_\gamma(\eta) = \frac{ABC(\gamma)}{1 - \gamma} \int_0^\eta \frac{d}{de} \tilde{\Theta}(e) E_\gamma \left[ \frac{-\gamma}{1 - \gamma} (\eta - e)^\gamma \right] de.$$  (4)

Further, if $\tilde{\Theta}(\eta) \in C^\gamma[0,\eta] \cap L^\gamma[0,\eta]$, where $L^\gamma[0,\eta]$ and $C^\gamma[0,\eta]$ define the “space continues fuzzy term 1” is the space of “fuzzy integrable Lebesgue terms,” respectively. Then, fuzzy fractional ABC integral is defined as

$$\int_0^\delta \tilde{\Theta}(\eta) = \left[ \int_0^\delta \tilde{\Theta}_\gamma(\eta), \int_0^\delta \tilde{\Theta}_\gamma(\eta) \right], \quad 0 \leq \delta \leq 1,$$  (5)

such that

$$\int_0^\delta \tilde{\Theta}_\gamma(\eta) = \frac{1 - \gamma}{ABC(\gamma)} \tilde{\Theta}(\eta) + \frac{\gamma}{ABC(\gamma)} \Gamma(\gamma) \int_0^\eta \frac{(\eta - e)^{\gamma - 1}}{\Gamma(\gamma)} \tilde{\Theta}(e) de,$$  (6)

$$\int_0^\delta \tilde{\Theta}_\gamma(\eta) = \frac{1 - \gamma}{ABC(\gamma)} \tilde{\Theta}(\eta) + \frac{\gamma}{ABC(\gamma)} \Gamma(\gamma) \int_0^\eta \frac{(\eta - e)^{\gamma - 1}}{\Gamma(\gamma)} \tilde{\Theta}(e) de.$$  (7)

Definition 3. The “Fuzzy Laplace transform” of ABC derivative of $\tilde{\Theta}(\eta)$ is given as [29]

$$\mathcal{L}\left[ D_\eta^\delta \tilde{\Theta}(\eta) \right] = \frac{ABC(\gamma)}{[\gamma(1 - \gamma) + \gamma]} \left[ \gamma \mathcal{L}\left[ \tilde{\Theta}(\eta) - \frac{\gamma - 1}{\gamma - 1} \tilde{\Theta}(0) \right] \right].$$  (8)

Definition 4. The function of “Mittag-Leffler” $E_\beta(\eta)$ is
defined as [29]

\[ E_\beta(\eta) = \sum_{n=0}^{\infty} \frac{\eta^n}{\Gamma(n\beta + 1)}, \quad \beta > 0. \tag{8} \]

**Definition 5.** A mapping \( \kappa : R \rightarrow [0, 1] \). If it holds, it is considered to be a fuzzy number [29]. (i) \( \kappa \) is upper semicontinuous;

(ii) \( \left\{ \kappa(\mu_1) + \kappa(\mu_2) \right\} \geq \min \left\{ \kappa(\mu_1), \kappa(\mu_2) \right\} \)

(iii) \( \exists \epsilon_0 \in R \) such that \( \kappa(\epsilon_0) = 1 \)

(iv) \( \text{cl}\{r \in R, \kappa(r) > 0\} \) is compact

**Definition 6.** The parametric form of a fuzzy number \((\kappa(\delta), \bar{\kappa}(\delta))\) such that \( 0 \leq \delta \leq 1 \) and conditions [29]

(i) \( \bar{k}(\delta) \) increasing, left-continuous over \((0, 1]\) and right continues at 0

(ii) \( \bar{k}(\delta) \) decreasing, left-continuous over \((0, 1]\) and right continues at 0

(iii) \( \kappa(\delta) \leq \bar{k}(\delta) \)

### 3. Methodology

In this section, we apply Laplace transform to analyze the general solution of PDE. On both sides implementing Laplace transform, we get

\[
\mathcal{L}\left[ p'_h(\hat{\Theta}(\psi, \eta)) \right] = \mathcal{L} \left[ A \frac{\partial^2}{\partial \psi^2} (\hat{\Theta}(\psi, \eta)) + \frac{\partial}{\partial x} (h(\psi)\hat{\Theta}(\psi, \eta)) \right].
\tag{9}
\]

Evaluating the Laplace transformation, Equation (9) implies that

\[
\frac{ABC(\gamma)}{[\gamma'(1 - \gamma) + \gamma]} [s^2 \mathcal{L}[\hat{\Theta}(\psi, \eta)] - s^{n-1} \hat{\Theta}(\psi, 0)] = \mathcal{L} \left[ A \frac{\partial^2}{\partial \psi^2} (\hat{\Theta}(\psi, \eta)) + \frac{\partial}{\partial x} (h(\psi)\hat{\Theta}(\psi, \eta)) \right].
\tag{10}
\]

By applying the initial condition, we have

\[
s^2 \mathcal{L}[\hat{\Theta}(\psi, \eta)] = s^{n-1} \hat{\Theta}(\psi, 0) + \frac{[\gamma'(1 - \gamma) + \gamma]}{ABC(\gamma)} \mathcal{L} \left[ A \frac{\partial^2}{\partial \psi^2} (\hat{\Theta}(\psi, \eta)) + \frac{\partial}{\partial x} (h(\psi)\hat{\Theta}(\psi, \eta)) \right].
\tag{11}
\]

or

\[
\mathcal{L} \left[ \hat{\Theta}(\psi, \eta) \right] = \frac{1}{s} g(\psi, \eta) + \frac{[(\gamma' + \gamma)}{ABC(\gamma)} \mathcal{L} \left[ A \frac{\partial^2}{\partial \psi^2} (\hat{\Theta}(\psi, \eta)) + \frac{\partial}{\partial x} (h(\psi)\hat{\Theta}(\psi, \eta)) \right].
\tag{12}
\]

We can write the unknown functions as to investigate the series-type solution \( \hat{\Theta}(\psi, \eta) = \sum_{n=0}^{\infty} \hat{\Theta}_n(\psi, \eta) \). We can write of Equation (9), and we get

\[
\mathcal{L} \left[ \sum_{n=0}^{\infty} \hat{\Theta}_n(\psi, \eta) \right] = \frac{1}{s} g(\psi, \eta) + \frac{[(\gamma' + \gamma)}{ABC(\gamma)} \mathcal{L} \left[ A \frac{\partial^2}{\partial \psi^2} (\hat{\Theta}_n(\psi, \eta)) + \frac{\partial}{\partial x} (h(\psi)\hat{\Theta}_n(\psi, \eta)) \right].
\tag{13}
\]

Comparisons of term by term of Equation (13), we have

\[
\mathcal{L} \left[ \hat{\Theta}_n(\psi, \eta) \right] = \frac{1}{s} g(\psi, \eta),
\]

\[
\mathcal{L} \left[ \hat{\Theta}_1(\psi, \eta) \right] = \frac{[(\gamma' + \gamma)}{ABC(\gamma)} \mathcal{L} \left[ A \frac{\partial^2}{\partial \psi^2} (\hat{\Theta}_1(\psi, \eta)) + \frac{\partial}{\partial x} (h(\psi)\hat{\Theta}_1(\psi, \eta)) \right].
\]

\[
\mathcal{L} \left[ \hat{\Theta}_2(\psi, \eta) \right] = \frac{[(\gamma' + \gamma)}{ABC(\gamma)} \mathcal{L} \left[ A \frac{\partial^2}{\partial \psi^2} (\hat{\Theta}_2(\psi, \eta)) + \frac{\partial}{\partial x} (h(\psi)\hat{\Theta}_2(\psi, \eta)) \right].
\]

\[
\vdots
\]

\[
\mathcal{L} \left[ \hat{\Theta}_n(\psi, \eta) \right] = \frac{[(\gamma' + \gamma)}{ABC(\gamma)} \mathcal{L} \left[ A \frac{\partial^2}{\partial \psi^2} (\hat{\Theta}_n(\psi, \eta)) + \frac{\partial}{\partial x} (h(\psi)\hat{\Theta}_n(\psi, \eta)) \right]. \quad n \geq 0.
\tag{14}
\]

Using inverse Laplace transformation in Equation (14), we get

\[
\hat{\Theta}_0(\psi, \eta) = \mathcal{L}^{-1} \frac{1}{s} g(\psi, \eta),
\]

\[
\hat{\Theta}_1(\psi, \eta) = \mathcal{L}^{-1} \frac{[(\gamma' + \gamma)}{ABC(\gamma)} \mathcal{L} \left[ A \frac{\partial^2}{\partial \psi^2} (\hat{\Theta}_1(\psi, \eta)) + \frac{\partial}{\partial x} (h(\psi)\hat{\Theta}_1(\psi, \eta)) \right].
\]

\[
\hat{\Theta}_2(\psi, \eta) = \mathcal{L}^{-1} \frac{[(\gamma' + \gamma)}{ABC(\gamma)} \mathcal{L} \left[ A \frac{\partial^2}{\partial \psi^2} (\hat{\Theta}_2(\psi, \eta)) + \frac{\partial}{\partial x} (h(\psi)\hat{\Theta}_2(\psi, \eta)) \right].
\]

\[
\vdots
\]

\[
\hat{\Theta}_n(\psi, \eta) = \mathcal{L}^{-1} \frac{[(\gamma' + \gamma)}{ABC(\gamma)} \mathcal{L} \left[ A \frac{\partial^2}{\partial \psi^2} (\hat{\Theta}_n(\psi, \eta)) + \frac{\partial}{\partial x} (h(\psi)\hat{\Theta}_n(\psi, \eta)) \right]. \quad n \geq 0.
\tag{15}
\]

Thus, the fuzzy solutions are obtained as

\[
\hat{\Theta}(\psi, \eta) = \sum_{n=0}^{\infty} \hat{\Theta}_n(\psi, \eta),
\]

\[
\Theta(\psi, \eta) = \sum_{n=0}^{\infty} \Theta_n(\psi, \eta).
\tag{16}
\]
4. Numerical Results

4.1. Case I. Consider the fractional fuzzy Newell-Whitehead-Segel equation:

\[
\text{ABC} D^\alpha \Theta - \Theta_{xy} - 2\Theta + 3\Theta^2 = 0, \quad \eta > 0, \quad \psi \in R, \quad 0 < \gamma \leq 1, \tag{17}
\]

with the initial condition

\[
\Theta(\psi, 0) = \lambda, \quad \bar{k} = (\kappa(\delta) k(\delta)) = (\delta - 1, 1 - \delta). \tag{18}
\]

Applying the suggested technique, we get

\[
\Theta_0(\psi, \eta) = \frac{\kappa(\delta) \lambda(2 - 3\lambda)}{\text{ABC}(\gamma)} \left[ 1 - \gamma + \frac{\eta \gamma}{\Gamma(\gamma + 1)} \right],
\]

\[
\Theta_1(\psi, \eta) = \frac{\kappa(\delta) \lambda(2 - 3\lambda)}{\text{ABC}(\gamma)^2} \left[ 1 - \gamma + \frac{\eta \gamma}{\Gamma(\gamma + 1)} \right],
\]

\[
\Theta_2(\psi, \eta) = \frac{\kappa(\delta) \lambda(2 - 3\lambda)(1 - 3\lambda)}{\text{ABC}(\gamma)^3} \left[ (1 - \gamma)^2 + \frac{2\gamma(1 - \gamma)\eta \gamma}{\Gamma(\gamma + 1)} + \frac{\gamma^2 \eta^2 \gamma}{\Gamma(2\gamma + 1)} \right],
\]

\[
\Theta_3(\psi, \eta) = \frac{\kappa(\delta) \lambda(2 - 3\lambda)(1 - 3\lambda)}{\text{ABC}(\gamma)^4} \left[ (1 - \gamma)^2 + \frac{2\gamma(1 - \gamma)\eta \gamma}{\Gamma(\gamma + 1)} + \frac{\gamma^2 \eta^2 \gamma}{\Gamma(2\gamma + 1)} \right]. \tag{19}
\]

The higher terms can be obtained in a similar way. We used to find the series solution Equation (17); therefore, we write

\[
\Theta(\psi, \eta) = \Theta_0(\psi, \eta) + \Theta_1(\psi, \eta) + \Theta_2(\psi, \eta) + \Theta_3(\psi, \eta) + \Theta_4(\psi, \eta) + \cdots. \tag{20}
\]

The upper and lower fuzzy portion type can be written as

\[
\Theta(\psi, \eta) = \Theta_0(\psi, \eta) + \Theta_1(\psi, \eta) + \Theta_2(\psi, \eta) + \Theta_3(\psi, \eta) + \Theta_4(\psi, \eta) + \cdots,
\]

\[
\Theta(\psi, \eta) = \Theta_0(\psi, \eta) + \Theta_1(\psi, \eta) + \Theta_2(\psi, \eta) + \Theta_3(\psi, \eta) + \Theta_4(\psi, \eta) + \cdots, \tag{21}
\]

The exact solution is

\[
\Theta(\psi, \eta) = \bar{k} \left( -\frac{2}{3} \right) \lambda \exp^{2\eta} \left( -\frac{2}{3} \right) + \left( -\lambda - \lambda \exp^{2\eta} \right). \tag{22}
\]

In Figure 1, the first graph shows the 3D fuzzy upper and lower figure of an analytical solution at \( \gamma = 1 \) and second the fractional graph of \( \gamma = 0.8 \). Figure 2 shows the 3D fuzzy figure of upper and lower of an analytical solution at \( \gamma = 0.6 \) and the second figure at fractional order of \( \gamma = 0.4 \). In Figure 3, the figure shows the 3D fuzzy figure of upper and lower of various fractional orders of \( \gamma \). Figure 4 shows the 2D fuzzy figure of the upper and lower of various fractional orders of \( \gamma \).
4.2. Case II. Consider the fractional fuzzy Newell-Whitehead-Segel equation:

\[
\begin{align*}
ABC \frac{D_{\gamma}^{\nu} \tilde{\Theta} - \tilde{\Theta}_{\psi\psi} - \tilde{\Theta}(1 - \tilde{\Theta})}{2} &= 0, \quad \eta > 0, \quad \psi \in \mathbb{R}, \quad 0 < \gamma \leq 1. \\
\end{align*}
\] (23)

The initial condition is

\[
\begin{align*}
\tilde{\Theta}(\psi, 0) &= \frac{1}{(1 + \exp^{\psi/\sqrt{\gamma}})^2} \quad \text{(24)} \\
\tilde{\kappa} &= (\kappa(\delta) / \kappa(\delta)) = (\delta - 1, 1 - \delta). \\
\end{align*}
\]

Using the proposed method, we get

\[
\begin{align*}
\tilde{\Theta}_0(\psi, \eta) &= \kappa(\delta) \frac{1}{(1 + \exp^{\psi/\sqrt{\gamma}})^2}, \\
\tilde{\Theta}_0(\psi, \eta) &= \tilde{\kappa}(\delta) \frac{1}{(1 + \exp^{\psi/\sqrt{\gamma}})^2}, \\
\tilde{\Theta}_1(\psi, \eta) &= \frac{\kappa(\delta)}{ABC(\gamma) \frac{3}{2} \frac{\exp^{\psi/\sqrt{\gamma}}}{(1 + \exp^{\psi/\sqrt{\gamma}})^2}} \left[ 1 - \frac{\gamma}{2} \right], \\
\tilde{\Theta}_1(\psi, \eta) &= \frac{\tilde{\kappa}(\delta)}{ABC(\gamma) \frac{3}{2} \frac{\exp^{\psi/\sqrt{\gamma}}}{(1 + \exp^{\psi/\sqrt{\gamma}})^2}} \left[ 1 - \frac{\gamma}{2} \right], \\
\tilde{\Theta}_2(\psi, \eta) &= \frac{\kappa(\delta)}{ABC(\gamma)^2 \frac{25}{18} \left( \frac{\exp^{\psi/\sqrt{\gamma}}}{1 + \exp^{\psi/\sqrt{\gamma}}} \right)^4} \left[ 1 - \frac{\gamma}{2} \right], \\
\tilde{\Theta}_2(\psi, \eta) &= \frac{\tilde{\kappa}(\delta)}{ABC(\gamma)^2 \frac{25}{18} \left( \frac{\exp^{\psi/\sqrt{\gamma}}}{1 + \exp^{\psi/\sqrt{\gamma}}} \right)^4} \left[ 1 - \frac{\gamma}{2} \right].
\end{align*}
\] (25)

The higher terms can be obtained in a similar way. We used to find the series solution Equation (23); therefore, we write

\[
\tilde{\Theta}(\psi, \eta) = \tilde{\Theta}_0(\psi, \eta) + \tilde{\Theta}_1(\psi, \eta) + \tilde{\Theta}_2(\psi, \eta) + \tilde{\Theta}_3(\psi, \eta) + \tilde{\Theta}_4(\psi, \eta) + \cdots.
\] (26)

The upper and lower fuzzy portion type can be written
Figure 4: The first graph shows the fuzzy 2D upper and lower figure of the various fractional orders of $\gamma$.

Figure 5: The first graph shows the fuzzy 3D upper and lower figure of analytical solution at $\gamma = 1$ and the second graph at fractional of $\gamma = 0.8$.

Figure 6: The first graph shows the fuzzy 3D upper and lower figure of analytical solution at $\gamma = 0.6$ and the second graph at fractional of $\gamma = 0.4$. 
as

\[
\Theta(\psi, \eta) = \Theta_0(\psi, \eta) + \Theta_1(\psi, \eta) + \Theta_2(\psi, \eta) + \Theta_3(\psi, \eta) + \Theta_4(\psi, \eta) + \cdots,
\]

\[
\Theta(\psi, \eta) = \Theta_0(\psi, \eta) + \Theta_1(\psi, \eta) + \Theta_2(\psi, \eta) + \Theta_3(\psi, \eta) + \Theta_4(\psi, \eta) + \cdots.
\]

\[
\Theta(\psi, \eta) = \Theta_0(\psi, \eta) \cdot \frac{1}{(1 + \exp^{\psi/\gamma})^2} + \chi(\delta) \frac{5}{(1 + \exp^{\psi/\gamma})^3} \begin{pmatrix}
1 - \gamma + \frac{\eta \gamma}{\Gamma(\gamma + 1)} & 1 + \frac{\kappa(\delta) 25}{(1 + \exp^{\psi/\gamma})^2} \\
\exp^{\psi/\gamma}(1 - 2 \exp^{\psi/\gamma}) & \exp^{\psi/\gamma}(1 - 2 \exp^{\psi/\gamma})
\end{pmatrix} \begin{pmatrix}
1 - \gamma + \frac{\eta \gamma}{\Gamma(\gamma + 1)} & 1 + \frac{\kappa(\delta) 25}{(1 + \exp^{\psi/\gamma})^2} \\
\exp^{\psi/\gamma}(1 - 2 \exp^{\psi/\gamma}) & \exp^{\psi/\gamma}(1 - 2 \exp^{\psi/\gamma})
\end{pmatrix} + \cdots.
\]

(27)

The exact solution is

\[
\tilde{\Theta}(\psi, \eta) = \kappa \left( \frac{1}{1 + \exp^{\psi/\gamma}(5/6) \eta} \right)^2.
\]

(28)

In Figure 5, the first graph shows the 3D fuzzy upper and lower figure of an analytical solution at \( \gamma = 1 \) and the second the fractional graph of \( \gamma = 0.8 \). Figure 6 shows the 3D fuzzy figure of upper and lower of an analytical solution at \( \gamma = 0.6 \) and the second figure at fractional order of \( \gamma = 0.4 \). In Figure 7, the figure shows the 3D fuzzy figure of upper and lower of various fractional orders of \( \gamma \). Figure 8 shows the 2D fuzzy figure of the upper and lower of various fractional orders of \( \gamma \).

4.3. Case III. Consider the fractional fuzzy Newell-Whitehead-Segel equation:

\[
ABC D_\eta^\gamma \tilde{\Theta} - \tilde{\Theta}_{\psi \psi} - \tilde{\Theta} + \tilde{\Theta}^4 = 0, \quad \eta > 0, \quad \psi \in R, \quad 0 < \gamma \leq 1.
\]

(29)

The initial condition is

\[
\tilde{\Theta}(\psi, 0) = \frac{1}{(1 + \exp^{\psi/\gamma})^{2/3}},
\]

\[
\kappa = (\kappa(\delta) \kappa(\delta)) = (\delta - 1, 1 - \delta).
\]

(30)

The higher terms can be obtained in a similar way. We used to find the series solution Equation (29); therefore, we write

\[
\Theta(\psi, \eta) = \Theta_0(\psi, \eta) + \Theta_1(\psi, \eta) + \Theta_2(\psi, \eta) + \Theta_3(\psi, \eta) + \Theta_4(\psi, \eta) + \cdots.
\]

(31)
Figure 8: The first graph shows the fuzzy 3D upper and lower figure of the various fractional orders of $\gamma$.

Figure 9: The first graph shows the fuzzy 3D upper and lower figure of analytical solution at $\gamma = 1$ and the second graph at fractional of $\gamma = 0.8$.

Figure 10: The first graph shows the fuzzy 3D upper and lower figure of analytical solution at $\gamma = 0.6$ and the second graph at fractional of $\gamma = 0.4$. 
Using the proposed method, we get

\[
\begin{align*}
\hat{\Theta}_i(\psi, \eta) &= \frac{k(\delta)}{1 + \exp(\eta \psi)} \left[ 1 - \frac{\gamma \eta}{T(y + 1)} + \frac{\gamma^2 \eta^2}{T(2y + 1)} \right] + \cdots, \\
\hat{\Theta}_2(\psi, \eta) &= \frac{k(\delta)}{1 + \exp(\eta \psi)} \left[ 1 - \frac{\gamma \eta}{T(y + 1)} + \frac{\gamma^2 \eta^2}{T(2y + 1)} \right] + \cdots.
\end{align*}
\]

The exact solution is

\[
\hat{\Theta}(\psi, \eta) = \frac{1}{2} \tanh \left( \frac{3}{2\sqrt{10}} \left( \psi - \frac{7}{\sqrt{10}} \eta \right) \right).
\]
Figure 12: The first graph shows the fuzzy 3D upper and lower figure of analytical solution at $\gamma = 1$ and the second graph at fractional of $\gamma = 0.8$.

Figure 13: The first graph shows the fuzzy 3D upper and lower figure of analytical solution at $\gamma = 0.6$ and the second graph at fractional of $\gamma = 0.4$.

Figure 14: The first graph shows the fuzzy 3D upper and lower figure of the various fractional orders of $\gamma$. 

\[
\begin{align*}
\Theta(\psi, \eta) &= \Theta_1(\psi, \eta) + \Theta_2(\psi, \eta) + \Theta_3(\psi, \eta) + \Theta_4(\psi, \eta) + \Theta_5(\psi, \eta) + \cdots, \\
\Theta(\psi, \eta) &= \Theta_2(\psi, \eta) + \Theta_3(\psi, \eta) + \Theta_4(\psi, \eta) + \Theta_5(\psi, \eta) + \Theta_6(\psi, \eta) + \cdots, \\
\Theta(\psi, \eta) &= \kappa(\delta) \sqrt{\frac{3}{4 \exp \gamma^{\psi}} + \frac{\kappa(\delta)}{9 \text{ARC} \gamma^2}} \\
&\quad \cdot \left( \frac{\Delta \exp \gamma^{\psi} \exp \{\gamma(\psi)\} + \exp \{\gamma(\psi)\}}{\exp \gamma^{\psi} \exp \{\gamma(\psi)\}} \right) \\
&\quad \cdot \left( \frac{(1 - \gamma)^2 + 2y(1 - \gamma)\eta + \eta^2}{\Gamma(2 + 1) + \Gamma(2 + 1)} \right) + \cdots, \\
\Theta(\psi, \eta) &= \kappa(\delta) \sqrt{\frac{3}{4 \exp \gamma^{\psi}} + \frac{\kappa(\delta)}{9 \text{ARC} \gamma^2}} \\
&\quad \cdot \left( \frac{\Delta \exp \gamma^{\psi} \exp \{\gamma(\psi)\} + \exp \{\gamma(\psi)\}}{\exp \gamma^{\psi} \exp \{\gamma(\psi)\}} \right) \\
&\quad \cdot \left( \frac{(1 - \gamma)^2 + 2y(1 - \gamma)\eta + \eta^2}{\Gamma(2 + 1) + \Gamma(2 + 1)} \right) + \cdots.
\end{align*}
\]
The exact solution is

$$\tilde{\Theta}(\psi, \eta) = \tilde{k} \sqrt{\frac{3}{4}} \frac{\exp(\sqrt{\psi})}{\exp(\psi + \exp((-\psi/2)^2 - (9/2)(\eta))}. \tag{40}$$

In Figure 12, the first graph shows the 3D fuzzy upper and lower figure of an analytical solution at $\gamma = 1$ and the second the fractional graph of $\gamma = 0.8$. Figure 13 shows the 3D fuzzy figure of upper and lower of an analytical solution at $\gamma = 0.6$ and the second figure at fractional order of $\gamma = 0.4$. In Figure 14, the figure shows the 3D fuzzy figure of upper and lower of various fractional orders of $\gamma$.

5. Conclusion

In this paper, investigating the fractional Newell-Whitehead-Segel equation with uncertainty is not an easy problem to analyze, especially in advanced differentiability such as Atangana-Baleanu-Caputo on the fuzzy valued functions. Because the generated models are more complex to solve as parametric coupled systems than standard fuzzy differential equations, we must use the meaning of the uncertain Atangana-Baleanu-Caputo derivative to identify the signiﬁcance of these solutions to the uncertain models. It can be seen that the solution for each level is an interval at each point, implying that our solutions are fuzzy number functions at each point in the domain.

Data Availability

The numerical data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conﬂicts of interest regarding the publication of this article.

Authors’ Contributions

Yousuf Alkhezi and Nehad Ali Shah contributed equally to this work and are the co-first authors.

References

[1] A. Lachouri and A. Ardjouni, “The existence and Ulam-Hyers stability results for generalized Hilfer fractional integro-differential equations with nonlocal integral boundary conditions,” Advances in the Theory of Nonlinear Analysis and its Applications, vol. 6, no. 1, pp. 101–117, 2022.
[2] M. Naeem, A. M. Zidan, K. Nolaono, M. I. Syam, Z. Al-Zhour, and R. Shah, “A new analysis of fractional-order equal-width equations via novel techniques,” Symmetry, vol. 13, no. 5, p. 886, 2021.
[3] P. Sunthrayuth, A. M. Zidan, S. W. Yao, R. Shah, and M. Inc, “The comparative study for solving fractional-order Fornberg-Whitham equation via $\rho$-Laplace transform,” Symmetry, vol. 13, no. 5, p. 784, 2021.
[4] K. Nolaono, A. M. Alsharif, A. M. Zidan, A. Khan, Y. S. Hamed, and R. Shah, “Numerical investigation of fractional-order Swift-Hohenberg equations via a novel transform,” Symmetry, vol. 13, no. 7, p. 1263, 2021.
[5] A. Khastan, F. Bahrami, and K. Ivaz, “New results on multiple solutions for nth-order fuzzy differential equations under generalized differentiability,” Boundary value problems, vol. 2009, Article ID 395714, 13 pages, 2009.
[6] R. P. Agarwal, V. Lakshmikantham, and J. J. Nieto, “On the concept of solution for fractional differential equations with uncertainty,” Nonlinear Analysis: Theory, Methods & Applications, vol. 72, no. 6, pp. 2859–2862, 2010.
[7] R. Shah, H. Khan, D. Baleanu, P. Kumam, and M. Arif, “The analytical investigation of time-fractional multi-dimensional Navier–Stokes equation,” Alexandria Engineering Journal, vol. 59, no. 5, pp. 2941–2956, 2020.
[8] M. Alesemi, N. Iqbal, and A. A. Hamoud, “The analysis of fractional-order proportional delay physical models via a novel transform,” Complexity, vol. 2022, Article ID 2431533, 13 pages, 2022.
[9] R. Shah, U. Farooq, H. Khan, D. Baleanu, P. Kumam, and M. Arif, “Fractional view analysis of third order Korteweg-De Vries equations, using a new analytical technique,” Frontiers in Physics, vol. 7, p. 244, 2020.
[10] N. Iqbal, H. Yasin, A. Rezaigia, J. Kafie, A. O. Almatrood, and T. S. Hassan, “Analysis of the fractional-order Kaup–Kupershmidt equation via novel transforms,” Journal of Mathematics, vol. 2021, Article ID 2567927, 13 pages, 2021.
[11] R. Shah, H. Khan, and D. Baleanu, “Fractional Whitham-Broer-Kauf equations within modified analytical approaches,” Axioms, vol. 8, no. 4, p. 125, 2019.
[12] R. P. Agarwal, F. Mofarreh, R. Shah, W. Luangboon, and K. Nolaono, “An analytical technique, based on natural transform to solve fractional-order parabolic equations,” Entropy, vol. 23, no. 8, p. 1086, 2021.
[13] A. Kilbas, H. M. Srivastava, and J. J. Trujillo, Theory and Application of Fractional Differential Equations, vol. 204, Elsevier, Amsterdam, The Netherlands, 2006.
[14] I. Podlubny, Fractional Differential Equations, Academic Press, San Diego, CA, USA, 1999.
[15] N. H. Aljahdaly, R. P. Agarwal, R. Shah, and T. Botmart, “Analysis of the time fractional-order coupled burgers equations with non-singular kernel operators,” Mathematics, vol. 9, no. 18, p. 2326, 2021.
[16] S. G. Samko, A. A. Kilbas, and O. I. Marichev, Fractional Integrals and Derivatives: Theory and Applications, Gordon and Breach Science Publishers, Philadelphia, PA, USA, 1993.
[17] A. Jajarmi and D. Baleanu, “On the fractional optimal control problems with a general derivative operator,” Asian Journal of Control, vol. 23, no. 2, pp. 1062–1071, 2021.
[18] J. Reunsumrith, M. Sher, K. Shah, N. A. Alsheshidhi, and M. Shatatwi, “On fuzzy partial fractional order equations under fuzzified conditions,” Fractals, vol. 30, no. article 2240025, 2021.
[19] S. Ahmad, A. Ullah, A. Akful, and T. Abdeljawad, “Numerical analysis of fractional human liver model in fuzzy environment,” Journal of Taibah University for Science, vol. 15, no. 1, pp. 840–851, 2021.
[20] Z. Ullah, S. Ahmad, A. Ullah, and A. Akful, “On solution of fuzzy Volterra integro-differential equations,” Arab Journal of Basic and Applied Sciences, vol. 28, no. 1, pp. 330–339, 2021.
[21] A. U. K. Niazi, N. Iqbal, R. Shah, F. Wannalookkhee, and K. Nolaono, “Controllability for fuzzy fractional evolution
equations in credibility space,” *Fractal and Fractional*, vol. 5, no. 3, p. 112, 2021.

[22] R. Saadeh, M. Alaroud, M. Al-Smadi, R. R. Ahmad, and U. K. Salma Din, “Application of fractional residual power series algorithm to solve Newell-Whitehead-Segel equation of fractional order,” *Symmetry*, vol. 11, no. 12, article 1431, 2019.

[23] S. A. Ahmed and M. Elbadri, “Solution of Newell-Whitehead-Segel equation of fractional order by using Sumudu decomposition method,” *Order*, vol. 8, no. 6, pp. 631–636, 2020.

[24] N. H. Tuan, R. M. Ganji, and H. Jafari, “A numerical study of fractional rheological models and fractional Newell-Whitehead-Segel equation with non-local and non-singular kernel,” *Chinese Journal of Physics*, vol. 68, pp. 308–320, 2020.

[25] H. Kheiri, N. Alipour, and R. Dehghani, “Homotopy analysis and homotopy pade methods for the modified Burgers-Korteweg-de Vries and the Newell-Whitehead equations,” *Mathematical Sciences*, vol. 5, pp. 33–50, 2011.

[26] A. Prakash, M. Goyal, and S. Gupta, “Fractional variational iteration method for solving time-fractional Newell-Whitehead-Segel equation,” *Nonlinear Engineering*, vol. 8, no. 1, pp. 164–171, 2019.

[27] S. M. Mohamed, F. Al-Malki, R. Talib, and S. A. Taif, “Approximate analytical and numerical solutions to fractional Newell-Whitehead equation by fractional complex transform,” *International Journal of Applied Mathematics*, vol. 26, no. 6, pp. 657–669, 2014.

[28] H. K. Jassim, “Homotopy perturbation algorithm using Laplace transform for Newell-Whitehead-Segel equation,” *International Journal of Advances in Applied Mathematics and Mechanics*, vol. 2, no. 4, pp. 8–12, 2015.

[29] M. Arfan, K. Shah, A. Ullah, and T. Abdeljawad, “Study of fuzzy fractional order diffusion problem under the Mittag-Leffler kernel law,” *Physica Scripta*, vol. 96, no. 7, article 074002, 2021.