Theoretical and practical solutions to reduce the vibration level of aggregates with two rotors coupled to an elastic shaft

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Abstract. In the first part of the paper we present the results of a theoretical study about of the vibrations of a rotating assembly consisting of a turbine rotor coupled by an elastic shaft to the rotor of an electric generator, to which vibrations come from the deformation of the shaft during operation and the static imbalance. It has been presented the differential equations for the movement for rotor of the generator, and of the turbine. There have been analysed some approximate solutions for the equations and we draw some conclusions about obtaining a stable functioning of the machine. In the second part of the paper there is presented an experimental study on a hydro-aggregate who had a high vibration level in all operating modes.

1. Introduction
When designing aggregates comprising two rotors coupled by an elastic shaft, it must be considered that, following operations at high revolutions, the coupling shaft can deform. Due to this deformation, the centre of gravity moves towards the axis of the shaft and an eccentricity appears. Thus, disturbing forces arise which cause transverse vibrations of the machine. To avoid their negative manifestations, they must be studied. This avoids both excessive deformation of the shaft and the resonance phenomenon. In addition, information can be obtained on the correlation between kinematic parameters and geometric elements.

For aggregates already in operation and which exhibits a significant level of vibrations, it is important to carry out prior expertise, and then to conclude on the causes of these vibrations. If vibrations come from imbalances this must be considered and to act accordingly.

The material has been organized in this article in two parts. In the first part, it is proposed to determine the dynamic model of the rotating assembly and to obtain the differential equations of the movement. The start has been made from the Lagrange equation of second degree and use using a simplifying hypothesis that the angular velocity of the turbine is equal to the nominal mode speed.

In the second part of the paper is presented a case study, made on a Kaplan turbine hydro unit where a high vibration level was found in all operating modes. The dynamic rebalancing operation of the rotor of the generator is not always the necessary or sufficient solution to eliminate vibrations. But, in the case presented here, has proven to be sufficient.
2. Determining the dynamic model and obtaining the differential equations of the movement

To determine the dynamic model, there has been considered one of the rotors, for example the turbine rotor, mounted together with the second rotor, that of the electric generator, on an elastic shaft of diameter d; the distance between the two rotors is l (Figure 1).

![Figure 1. The rotating assembly.](image1)

Also it is known: the turbine rotor mass m, the turbine rotor eccentricity e (Figure 2.a), the moment of inertia in relation to an axis passing through the centre of gravity C parallel to the bearing line J, the elastic bending constant of the shaft right next to the turbine k, the transverse elastic modulus of the shaft G, the polar inertia moment of the transverse section of the shaft Ip, the moment of inertia of the generator rotor relative to the shaft axis J1, the moment with which the working agent acts on the turbine M1, and the moment with which the generator stator acts on the rotor M2.

![Figure 2. The dynamic model of the rotating assembly: a) the generalized coordinates; b) the distribution of forces and moments.](image2)

It is to notice that the generator rotor movement has a parallel-plane motion, and, for the study, there has been chosen a reference system Oxy, displayed in the plan of transverse symmetry of the rotor (Figure 2.b). Point C is the centre of gravity, point A is the rotor attachment point, and point O is the intersection point of the plane with the line of bearings.

The differential equations of movement are obtained starting from the Lagrange equations of second degree [1]:

\[
\frac{d}{dt} \left( \frac{\partial E_c}{\partial \dot{q}_i} \right) - \frac{\partial E_c}{\partial q_i} = Q_i
\]

(1)

where: \(E_c\) - kinematic energy; \(q_i\) - generalized coordinate; \(\dot{q}_i\) - generalized speed; \(Q_i\) - generalized force.
As generalized coordinates were chosen the parameters: \( x_1, y_1 \) - the coordinates of point A; \( \varphi \) - the angle of rotation of the rotor (Figure 2.a). With the notations \( q_1 = x_1, q_2 = y_1, q_3 = \varphi \), the differential equations of movement, obtained from Lagrange’s equation, are:

\[
m\left[\ddot{x}_1 - e\left(\dot{\varphi}\sin\varphi + \varphi^2 \cos\varphi\right)\right] = -kx_1
\]

\[
m\left[\ddot{y}_1 + e\left(\dot{\varphi}\cos\varphi - \varphi^2 \sin\varphi\right)\right] = -ky_1 - mg
\]

\[
(J + me^2)\ddot{\varphi} - e\left(\ddot{x}_1 \sin\varphi - \ddot{y}_1 \cos\varphi\right) = -mge\cos\varphi + M_1 - \frac{\varphi - \theta}{l}GL_p
\]

The differential equation of movement of the rotor is established using the kinetic momentum theorem:

\[
J_1\ddot{\varphi} = -\frac{\theta - \varphi}{l}GL_p - M_2
\]

### 2.1. The determination of approximate solutions of motion equations

The equations (2), (3), (4) and (5) form a system of unknowns \( x_1, y_1, \theta, \varphi \). Since this system cannot be integrated, approximate solutions are sought.

It has been assumed that the angular velocity of rotation of the turbine \( \dot{\varphi} \) is approximately equal to the angular velocity of the regime \( \omega_0 \) (constant). Thus, it results that the angular acceleration is null, so it can be replaced \( \ddot{\varphi} = 0 \) and the differential equations (2) and (3) become:

\[
m\left[\ddot{x}_1 - e\omega_0^2 \cos\omega_0 t\right] = -kx_1
\]

\[
m\left[\ddot{y}_1 - e\omega_0^2 \sin\omega_0 t\right] = -ky_1 - mg
\]

Equations (6) and (7) are linear differential equations of second degree and have the general solutions [2]:

\[
x_1 = \frac{e\omega_0^2 m}{k - \omega_0^2 m} \cos\omega_0 t + A \cos\omega_n t + B \sin\omega_n t
\]

\[
y_1 = -\frac{mg}{k - \omega_0^2 m} \cos\omega_0 t + C \cos\omega_n t + D \sin\omega_n t
\]

where: it was noted \( \omega_n = (k/m)^{1/2} \) and \( A, B, C, D \) are integration constants determined by the initial conditions. After replacing the solutions (8) and (9) in equation 4 and also using the notations:

- \( F(t) = M_1 - mge \cos\omega_0 t + e\left[\cos\omega_0 t \left(C \cos\omega_n t - A \sin\omega_n t\right) + \sin\omega_0 t \left(D \cos\omega_0 t - B \sin\omega_n t\right)\right]\)

- \( J_0 = J + me^2 \) (Steiner’s theorem)

it gets:

\[
J_0 J_1 (\ddot{\varphi} - \ddot{\theta}) + \frac{GI}{l}(J_0 + J_1)(\varphi - \theta) = M_1 J_1 + M_2 J_0 + J_1 F(t)
\]

This is the differential equation of the torsional deformations of the shaft, where the term \( F(t) \) has the role of disturbance, and has a solution in the form of a natural oscillation:

\[
\varphi - \theta = A \cos\omega t + B \sin\omega t
\]
2.2. The interpretation of approximate solutions of motion equations

By analysing the solutions (8) and (9) it was concluded that the bending vibrations of the rotor are periodic, and their amplitude can be reduced if the eccentricity \( e \) is diminishing. Also, the beating or resonance phenomena can be prevented if the pulse \( \omega_n \) is different from \( \omega_0 \), i.e. the angular rotation speed of the turbine does not have to coincide with the natural pulsation of the bending oscillations of the shaft. By analysing the solution (11) it is to note that, in case of shaft torsion, it is a harmonic vibration, and the own pulsation of the relative oscillations is:

\[
\alpha = \left[ \frac{GI_p (J_0 + J_1)}{IJ_0 J_1} \right]^{\frac{1}{2}}
\]

(12)

So, since the disturbance \( F(t) \) has a term in \( \cos \omega_0 t \), a condition to avoid resonance would be that the pulse of the swing oscillations \( \alpha \) is different from the pulse of the disturbance \( \omega_0 \). Also, \( F(t) \) has components in sinus products and differences of sinus and cosine functions with the arguments \((\omega_0 - \omega_n) t\) and \((\omega_0 + \omega_n) t\), so to avoid the phenomenon of resonance the mandatory condition is \( \alpha \neq \omega_0 \pm \omega_n \). The mathematical resonance avoidance conditions are:

\[
\omega_0 \neq \omega_n = (k/m)^{1/2}
\]

(13)

\[
\omega_0 \neq \left[ \frac{GI_p (J_0 + J_1)}{IJ_0 J_1} \right]^{\frac{1}{2}}
\]

(14)

\[
\omega_0 \pm \omega_n = \left[ \frac{GI_p (J_0 + J_1)}{IJ_0 J_1} \right]^{\frac{1}{2}}
\]

(15)

If considered the case when \( F(t) = 0 \) (the perturbation is null) and the turbine rotor rotates evenly, from the equation (10) is obtained:

\[
\varphi - \theta = \frac{M_1 l}{GI_p}
\]

(16)

So, the relative deformation between the turbine and generator rotors would be constant. It must be mentioned that, in part, this paragraph was the subject of a previous study [3]. Mainly, there were made no changes in the part related to the calculations; the explanations were changed, for a clearer presentation of ideas.

3. A case study

Below are presented the results of the measurements made on a Kaplan turbine hydro unit having a nominal power of the generator of 7.5 MW and a nominal revolution speed of 166 rpm, where a high vibration level was found in all operating modes [4].

3.1. Working mode and equipment used

In order to establish the causes leading to this situation the unexcited idle mode was analysed, by recording the following characteristic parameters: \( n \) rotor speed, the vibrations in the directions +x, +y in the axial radial bearing \( (LRA_{x\_rad}, LRA_{y\_rad}) \) and the vibrations in radial direction +x in the lower radial bearing \( (LRI_{x\_rad}) \). The coordinate axes \( x, y \) are located horizontally and perpendicular to the \( z \) direction, which is given by the coupled turbine and generator shafts. As the \(+y\)-axis we take the water inlet direction in the spiral chamber.
The vibration level was measured before and after the dynamic balancing of the rotor of the generator. For measurements, it was used an electrical and process parameter analyser of type VPA323 with a laptop for data processing, as in Figure 3.

![Parameter analyser of the VPA323 type.](image)

Figure 3. Parameter analyser of the VPA323 type.

The revolution speed was measured by a laser speed, the type QS30LDQ (Figure 4), and the vibration in the bearing was measured three vibration sensors mounted on the top bearing of the generator as in Figure 5, with the sensitivity 533.3 mV/g, 546.1 mV/g or 497.5 mV/g.

![QS30LDQ laser speed.](image)  ![Vibrations sensor.](image)

Figure 4. QS30LDQ laser speed.  Figure 5. Vibrations sensor.

The vibration level was measured before and after the dynamic balancing of the rotor of the generator. Subsequently, the signals were analysed using LabVIEW software. To view, it was performed a Fourier transform to highlight the range of variation of speed and amplitude of harmonic frequency (in that order).

### 3.2. Measurements before dynamic balancing

The variation of the measured parameters in the unexcited transient start-up regime and in stabilized idle unexcited mode is presented in Figure 6. The values of the measured parameters in stabilized idle mode before balancing, read at a time interval of 1 second, are listed in table 1.

Figure 6 shows that, during the transient start-up of the hydro-unit, the vibrations in the radial direction +x in the axial radial bearing have a significant variation and size, indicating instability in the rotor dynamics. Then it is to note that the vibration level stabilizes, but at a very high level of 1.8 mm/s. This indicates a significant mechanical imbalance.

The waveform in unexcited idle mode is presented in Figure 7. In Figure 8 is presented the frequency spectrum of vibrations in stabilized idle unexcited mode.
Figure 6. Variation of the measured parameters in the unexcited transient start-up regime.

Table 1. Measured parameters in unexcited idle mode before balancing.

| Time (s) | Revolution (rpm) | LRA_x_rad (mm/s) | LRA_y_rad (mm/s) | LRL_x_rad (mm/s) |
|----------|------------------|------------------|------------------|------------------|
| 0        | 166.7            | 1.87             | 0.39             | 0.11             |
| 1        | 166.61           | 2.02             | 0.28             | 0.15             |
| 2        | 166.69           | 1.67             | 0.39             | 0.12             |
| 3        | 166.53           | 2.08             | 0.41             | 0.16             |
| 4        | 166.53           | 1.93             | 0.40             | 0.13             |
| 5        | 166.36           | 1.94             | 0.40             | 0.14             |
| 6        | 166.61           | 1.70             | 0.33             | 0.21             |
| 7        | 166.53           | 1.84             | 0.38             | 0.13             |
| 8        | 166.69           | 1.74             | 0.34             | 0.14             |
| 9        | 166.53           | 1.78             | 0.33             | 0.13             |
| 10       | 166.36           | 1.67             | 0.34             | 0.13             |

Figure 7. Waveform in unexcited idle mode.
From the spectrum of vibrations measured on the upper axial radial bearing in the +x direction, it results that the amplitude of the fundamental frequency is 1.946 mm/s, a value too high for a proper operation of the hydro-unit in this mode.

In general, it is known that a high amplitude of the fundamental frequency indicates the presence of a mechanical imbalance [5-8]. Due to this fact, the hydro-aggregate rehabilitation program considered the dynamic balancing of the rotor assembly as a mandatory measure.

3.3. Measurements after dynamic balancing

After completing the dynamic rotor balancing operation, the vibration level in the idle mode has been verified. The values obtained of the parameters in the unexcited idle mode, read at a time interval of 1 second, are given in table 2, and in Figure 9 is displayed the form of variation in time of the measured parameters.

| Time (s) | U_line (V) | V_LRI_rad_+x (mm/s) | V_LRA_rad_+x (mm/s) | V_LRA_ax_+x (mm/s) |
|----------|------------|---------------------|---------------------|---------------------|
| 1        | 100.13     | 0.17                | 0.31                | 0.51                |
| 2        | 100.11     | 0.16                | 0.47                | 0.56                |
| 3        | 100.19     | 0.12                | 0.38                | 0.54                |
| 4        | 100.25     | 0.14                | 0.31                | 0.70                |
| 5        | 100.19     | 0.19                | 0.34                | 0.72                |
| 6        | 100.11     | 0.21                | 0.40                | 0.52                |
| 7        | 100.08     | 0.16                | 0.26                | 0.51                |
| 8        | 100.02     | 0.21                | 0.63                | 0.57                |
| 9        | 100.02     | 0.19                | 0.39                | 0.50                |
| 10       | 99.96      | 0.17                | 0.33                | 0.56                |
| 11       | 99.87      | 0.18                | 0.38                | 0.67                |
| 12       | 99.78      | 0.22                | 0.37                | 0.74                |
| 13       | 99.81      | 0.12                | 0.42                | 0.59                |
| 14       | 99.88      | 0.14                | 0.37                | 0.58                |
| 15       | 99.91      | 0.20                | 0.36                | 0.60                |
Table 2 shows that the vibration level dropped significantly after dynamic balancing: from 1.83 mm/s to 0.58 mm/s, demonstrating the effectiveness of dynamic balancing operations. By the mechanical balancing operation, there was also achieved a stabilization of the dynamic of the rotating parts, also eliminating any kind of rotor oscillation in the other hydro-aggregate operating modes.

4. Conclusions

On the operation of aggregates having two rotors on the same shaft can occur different types of vibrations, having as main causes: mechanical imbalance, beating phenomena, misalignment of the rotors, resonance phenomenon.

In the first part of the paper are proposed a dynamic model and a form of the differential equations of the movement. It starts from the general Lagrange equation of second degree and use the simplifying hypothesis that the angular velocity of the turbine is equal to the nominal mode speed. There were obtained the differential equation of the torsional deformations of the shaft, having a solution in the form of a natural oscillation. Then, from the interpretation of the approximate solutions of motion equations, were obtained the mathematical conditions for the avoidance of resonance. The model obtained used a simplifying hypothesis, but opens a good path for further approaches based on computational methods. These theoretical solutions must be considered in the design phase.

In the second part of the paper is presented a case study for a practical solution to these issues. After a certain amount of operating time, if vibrations occur, it must be analysed if there are any imbalances and a dynamic rebalancing is needed. In our example, at the hydro energetic complex having a Kaplan turbine with a nominal power of the generator of 7.5 MW and a nominal revolution speed of 166 rpm, there was a high vibration level in idle mode (with a maximum of 2.08 mm/s on the axial radial bearing, in the direction +x). After performing the repair work that included the dynamic balancing operation of the rotor of the generator, the vibration level has dropped substantially, almost 3 times (with a maximum of 0.74 mm/s on the axial radial bearing, in the direction +x).

It is also very important to note that the dynamic balancing is not always the solution needed to eliminate vibrations and must be done only in the situations that require it.
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