Modal parameter identification of rotating machinery based on time domain synchronous averaging

Zhuzhu Zhang1, Ting Wang, Congying Deng, Yang Zhao, Haiyan Deng
School of Advanced and Manufacturing Engineering, Chongqing University of Posts and Telecommunications, Chonqing, 400065, China
1Email: S182101009@stu.cqupt.edu.cn

Abstract: Operational modal analysis (OMA) is an important method of structural dynamic design and mechanical fault diagnosis. It is the application of parameter identification in the field of engineering vibration. Traditional operational modal analysis is based on the random response of white noise. However, when rotating machinery structure is running at high speed, it will produce a lot of harmonic interference, which will affect the identification effect. In this paper, the time domain synchronous averaging technique is used to remove the periodic harmonics in the response. For the random response part, the method of combining NExT and ITD is used to identify the modal parameters.

1. Introduction
Structural working modal parameters can be used to characterize the operating conditions of the structure, reflecting the dynamic characteristics that match the operating state, and are an important dynamic index for judging the operational safety of the structure [1]. For operating structures equipped with rotating equipment, there is a certain difference between the structural working mode and its inherent mode [2]. Therefore, the correct understanding of the structural working modal characteristics can help improve the accuracy of the judgment of the excitation force and the resonance between the structure, and thus avoid the hidden danger of structural operation safety.

Rotating mechanical structures are subject to obvious impeller and bearing rotation frequency and frequency-doubling harmonic excitation during high-speed operation. Therefore, there will be a large number of harmonic interferences in the acquisition of structural vibration signals. However, the operation mode analysis is based on the premise that the excitation of the measured object is the random excitation of white noise. At present, the processing of harmonic interference mainly includes filtering the harmonic components in the signal [3][4], using the statistical characteristics difference of random signal and harmonic signal, and improving the traditional mode identification method [5][6]. This paper uses time-domain synchronous averaging technology to separate the harmonic components from the random fluctuation components, and then performs parameter identification based on random vibration signals; for random vibration signals, this paper combines NExT and ITD methods to identify system modal parameters.

2. The Theoretical Background

2.1 The principle of time domain synchronous averaging
Assuming that the vibration signal of a rotating machine is $x(t)$, the rotation frequency is $f_0$, and the sampling interval is $\Delta$, the discrete signal corresponding to the vibration signal is $x_n = x(n\Delta)$. The
vibration signal is extracted according to the rotation frequency \( f_0 \), and \( x_n \) is intercepted as P segment, Then the period of each segment is \( T = \frac{1}{f_0} \), assuming that the number of sampling points of each segment is equal and the value is N, so the time-domain synchronous average signal \( \bar{x}_n \) can be expressed by the following formula:

\[
\bar{x}_n = \frac{1}{P} \sum_{P=0}^{P-1} x_{n+PN}
\]  

(1)

After several averaging, the random component will tend to zero.

Do Z transformation to equation (1), we get:

\[
\bar{X}(Z) = \frac{1}{P} \sum_{P=0}^{P-1} Z[x_{n+PN}]
\]

(2)

According to the time shift of Z transformation, formula (2) can be reduced to

\[
\bar{X}(Z) = \frac{1}{P} X(Z) \sum_{P=0}^{P-1} Z^{PN}
\]

(3)

Let \( Z = e^{j2\pi f\Delta N} \), the frequency response function averaged in the time domain can be obtained:

\[
H(f) = \frac{1 - e^{j2\pi f \Delta N}}{P(1 - e^{j2\pi f \Delta N})}
\]

(4)

\( \Delta N = T = \frac{1}{f_0} \), so, the above formula can be expressed as

\[
H(f) = \frac{e^{j2\pi f f_0} (e^{-j2\pi f f_0} - e^{j2\pi f f_0})}{Pe^{j\pi f f_0} (e^{-j\pi f f_0} - e^{j\pi f f_0})}
\]

(5)

The vibration signal is processed by TSA, which is equivalent to passing through the above system. The amplitude-frequency and phase-frequency characteristics of the system are shown in the following equation:

\[
|H(f)| = \frac{1}{P} \left| \frac{\sin P\pi \frac{f}{f_0}}{\sin \pi \frac{f}{f_0}} \right|
\]

(6)

\[
\Psi(f) = \frac{\pi(P-1) f}{f_0}
\]

(7)

Figure 1. Amplitude-frequency curve
The amplitude-frequency characteristic curve is shown in Figure 1. As shown in Figure 1, the vibration signal passing through the time-domain synchronous averaging is similar to passing a comb filter. The bandpass center frequency of the filter is an integer multiple of the rotation frequency \( f_0 \), that is, \( k f_0 \) (\( k=0,1,2,3 \ldots \)). It can be seen that the time-domain synchronous averaging can extract the periodic signal related to frequency \( f_0 \), and at the same time, the periodic signal can be separated from the random vibration signal by subtracting the periodic signal from the original vibration signal.

2.2 Modal parameter identification

Based on the random vibration signal separated from TSA, the cross-correlation function of two-point response is calculated by next method. According to the paper [7], the cross-correlation function and impulse response function of the response between two points of the structure under white noise excitation have similar expressions. Therefore, after obtaining the cross-correlation function between the two-point response, this paper adopts the ITD method for identification. ITD theory can refer to literature [7].

3. Simulation example

This section is mainly to verify whether the TSA method separates the periodic part from the random vibration. The picture 3 shows a four-degree-of-freedom spring damping system, whose stiffness and mass are \( m_1 = m_2 = m_3 = m_4 = 0.1 \text{Kg} \), \( k_1 = 5000 \text{N/m} \), \( k_2 = 10000 \text{N/m} \), \( k_3 = 15000 \text{N/m} \), \( k_4 = 20000 \text{N/m} \), \( k_5 = 5000 \text{N/m} \), respectively. Its damping adopts proportional damping, expressed as

\[
[C] = 0.680[M] + 1.743 \times 10^{-4}[K]
\]

(8)

Where \([C] \), \([M] \), \([K] \) are the damping matrix, mass matrix and stiffness matrix of the system, respectively. From the knowledge of mechanical vibration, the damping natural frequencies of the system are 33.86 Hz, 65.52 Hz, 94.52 Hz and 126.53 Hz, respectively.

In this paper, random white noise signals and sine signals with frequencies of 30 Hz, 60 Hz, 90 Hz, 120 Hz are selected to simulate the environmental excitation of rotating mechanical structures. Among them, 30Hz signal is used to simulate mechanical rotation frequency, 60Hz, 90Hz, 120Hz signal is used to simulate higher harmonics.

The original vibration response of the fourth degree of freedom is shown in Figure 4, and the frequency spectrum is shown in Figure 5. From the spectrogram of the original vibration signal, it can
be seen that the peak value is not only at the natural frequency but also at the harmonic frequency. At the same time, due to the influence of noise, the image has obvious burrs, so the modal parameters of the system cannot be directly derived from the original vibration.

![The original vibration diagram](image)

**Figure 4. The original vibration diagram**

The original vibration is processed by time-synchronized average (TSA) to obtain the periodic vibration signal, which can be removed from the original vibration to get the random vibration. After the random vibration is obtained, this paper combines the NExT method and the ITD method to identify the modal parameters. Curve 1 and curve 2 in Figure 6 are the structure-like pulse curve obtained by the NExT method and the curve fitted by the ITD method, respectively. It can be seen from the figure that the fitted curve is very similar to the system impulse response. The theoretical parameters of the system and the modal parameters obtained by the NExT method combined with the ITD method are shown in Table 1. According to the data in the table, the error between the natural frequency obtained by the method in this paper and the theoretical natural frequency of the system is less than 5%.

![Spectrum of original vibration](image)

**Figure 5. Spectrum of original vibration**
Figure 6. Impulse response diagram of quasi structure

| Order | Frequency (Hz) | Theoretical value (Hz) | Error rate |
|-------|----------------|------------------------|------------|
| 1     | 35.18          | 33.86                  | 3.90%      |
| 2     | 67.99          | 65.52                  | 3.77%      |
| 3     | 94.69          | 94.59                  | 0.11%      |
| 4     | 118.63         | 126.53                 | -6.24%     |

4. Conclusion
The experimental modal algorithm needs to measure the input signal in actual situations, so the working modal algorithm has advantages under the condition that the input is not easy to measure. The method based on time synchronization averaging proposed in this paper not only does not need to measure the input signal, but also can deal with the situation that the input signal contains harmonic components, which can be well applied in the parameter identification of rotating machinery.

Acknowledgments
This work is supported by the National Natural Science Foundation of China (51705058, 51807019), the Science and Technology Research Program of Chongqing Municipal Education Commission (KJZD-K201900604), the China Postdoctoral Science Foundation Funded Project under Grant 2018M633314, and the Chongqing Special Postdoctoral Science Foundation under Grant XmT2018040.

References
[1] Jianping Han, Peijuan Zheng, Hongtao Wang. Structural modal parameter identification and damage diagnosis based on Hilbert-Huang transform[J]. Earthquake Engineering and Engineering Vibration, 2014, 13(1).
[2] Xue-Mei, Niu, Wen-Jun, Meng, Hang, Cheng. Research in the Operational Modal Analysis of Rotating Machinery Reducer[J]. Sensors & Transducers, 2013, 155(8).
[3] Jun Chen Xu, Ming Hong, Hong Yu Cui. The Contrast Experimental Study on Operational Modal Analysis of Ship Structural Model[J]. Applied Mechanics and Materials, 2012, 2031.
[4] Fushun Liu, Shujian Gao, Huawei Han, et al. Interference reduction of high-energy noise for modal parameter identification of offshore wind turbines based on iterative signal extraction[J]. Ocean Engineering, 2019, 183:372-383.
[5] Prasenjit Mohanty, Daniel J. Rixen. Modified ERA method for operational modal analysis in the presence of harmonic excitations[J]. Mechanical Systems and Signal Processing. 2004 (1).

[6] Prasenjit Mohanty, Daniel J. Rixen. Modified SSTD method to account for harmonic excitations during operational modal analysis[J]. Mechanism and Machine Theory. 2004 (12)

[7] Rune Brincker, Peter Olsen, Sandro Amador, et al. Modal participation in multiple input Ibrahim time domain identification. 2019, 24(1):168-180.