A ROBUST method for measuring the Hubble parameter

M. A. Hendry and S. Rauzy

Department of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, UK

Abstract. We obtain a robust, non-parametric, estimate of the Hubble constant from galaxy linear diameters calibrated using HST Cepheid distances. Our method is independent of the parametric form of the diameter function and the spatial distribution of galaxies and is insensitive to Malmquist bias. We include information on the galaxy rotation velocities; unlike Tully-Fisher, however, we retain a fully non-parametric treatment. We find $H_0 = 66 \pm 6 \text{ km s}^{-1} \text{ Mpc}^{-1}$, somewhat larger than previous results using galaxy diameters.

1. Introduction

Despite the recent emergence of a broad consensus in estimates of $H_0$, the issue of observational selection effects remains an important one for studies of the distance scale and peculiar velocity field. Current recipes for eliminating Malmquist bias make parametric model assumptions about the distribution function of the distance indicator and the selection effects, and the spatial distribution of the observed galaxies. There is clearly an advantage in developing more robust techniques in which only minimal model assumptions are required. In this paper we present such a robust method.

2. Method and Application Using Galaxy Diameters

Our technique is based on the $C^-$ method of Lynden-Bell (1971), and provides an estimate of the cumulative distribution function (CDF) of galaxy diameter independent of any model assumption about its parametric form. Moreover, the method may be applied to data of arbitrary spatial distribution and thus requires no correction for Malmquist bias. The method is, however, applicable only to samples which are strictly complete to a given apparent magnitude or diameter. We have developed an objective test of the validity of this assumption, based on the approach of Efron & Petrosian (1992). (See Rauzy & Hendry in prep. for more details).

We applied the method to reconstruct the CDF of linear isophotal diameter, $D$, from a sample of 4005 galaxies – complete to an angular diameter limit of $D \geq 1.5'$ – from the LEDA database (Paturel et al. 1997). We carried out our analysis using the variables, $m$ and $M$, analogous to apparent magnitude and
absolute magnitude, given by

\[ m = 20 - 5 \log_{10} D \]  

\[ M = m - Z = 20 - 5 \log_{10} D - 5 \log_{10} \frac{cz}{H_0} - 25 \]  

We compared the CDF of \( M \) from the LEDA galaxies with that obtained from a set of 14 local calibrators, with HST Cepheid distances, from Theureau et al. (1997). We then varied \( H_0 \) in eq. (2) and, for each value of \( H_0 \), determined the Kolmogorov-Smirnoff (KS) distance between the two CDFs as a function of \( M \). We took as our ‘best-fit’ estimate of \( H_0 \) the value which gave the minimum KS distance – obtaining \( H_0 = 42 \) kms\(^{-1}\) Mpc\(^{-1}\). This value agrees with Sandage (1993a,b), who used M31 and M101 as standard rulers, and is consistent with the analysis of Goodwin, Gribbin & Hendry (1997), who obtained \( H_0 = 52 \pm 6 \pm 8 \) kms\(^{-1}\) Mpc\(^{-1}\) using galaxy linear diameters and a similar calibrating sample. However, their second uncertainty was an estimate of the difference in the mean intrinsic diameter of the local calibrating galaxies compared with the distant sample (even after correction for Malmquist bias) – a difference which might be systematically negative given the strategy of the HST Key Project to observe ‘Grand Design’ spirals (Kennicutt et al. 1995). We find evidence for a similar negative bias in our calibrating sample: galaxies of small diameter are relatively under-represented in the CDF of the local calibrators. This would lead to a systematic underestimate in the value of \( H_0 \).

3. Including Tully-Fisher information

We can improve our analysis by introducing galaxy rotation velocity to reduce the dispersion of the distance indicator. This is analogous to the conventional Tully-Fisher relation but – crucially – retains completely the robustness of our previous analysis. In a similar manner to the ‘Sosie’ method (c.f. Paturel et al. 1998), we select, for each local calibrator, the subset of galaxies from the distant sample with similar log rotation velocity and morphological type. For each subset we then reconstruct the diameter function, assuming initially a fiducial value of \( H_0 \). Finally we determine the value of \( H_0 \) required to match the median of the CDF to the observed diameter for that calibrator.

We applied this technique to the KLUN sample of spiral galaxies (c.f. Theureau et al. 1997). Fig. 1 shows the CDFs reconstructed from the subsets corresponding to each local calibrator. The median value of the reconstructed distribution is indicated on 9 of the panels. The remaining 3 panels correspond to the 4 local calibrators with the smallest rotation velocities: N300, N598, N925 and N496A. We exclude these calibrators since it is not clear from Fig. 1 whether their corresponding reconstructed CDFs are completely sampled – due to the presence of the lower diameter limit. Matching the median values from the 9 remaining panels to the linear diameters of the 10 remaining calibrators, and taking the logarithmic mean of these individual values gives

\[ H_0 = 66 \pm 6 \text{ kms}^{-1}\text{Mpc}^{-1} \]  

(3)
Figure 1. Reconstructed CDFs for subsets of galaxies selected from the KLUN sample (c.f. Theureau et al. 1997) with similar rotation velocities and morphological types to the indicated local calibrators.

$M \ (H_0 = 100 \ km \ s^{-1} Mpc^{-1})$

Figure 1. Reconstructed CDFs for subsets of galaxies selected from the KLUN sample (c.f. Theureau et al. 1997) with similar rotation velocities and morphological types to the indicated local calibrators.
4. Discussion

When we incorporate information on the rotation velocities of the local calibrators our results are in excellent agreement with recent determinations of $H_0$ from the conventional Tully-Fisher relation (c.f. Giovanelli et al 1997). Note, however, that our analysis is completely free of assumptions about the form of the galaxy diameter and luminosity function and the conditional distribution function of diameter at a given value of log $V_m$. We do not, for example, require to assume that this conditional distribution is Gaussian, nor indeed even that it has zero mean or constant dispersion – as is often assumed in calibrating the Tully-Fisher relation. In particular, therefore, we do not require that the Tully-Fisher relation is a straight line, nor that the distribution of residuals is symmetrical. Our method would remain applicable if, for example, the distribution of Tully-Fisher residuals displayed a long ‘tail’ for galaxies of small rotation velocity. Our method is also completely independent of Malmquist bias corrections, so that our results are unaffected by the precise ‘recipe’ adopted to correct for Malmquist bias.

In summary, the robustness and assumption-free nature of this method has important ramifications for any current debate on the cosmic distance scale. There appears to be little possibility of any remaining systematic error in $H_0$ which is introduced at the stage of linking the primary and secondary distance scales. Improving the calibration of the Cepheid distance scale, via the lowest rungs of the Cosmic Distance Ladder, must now be the main priority for the Distance Scale community.

References

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