EVOLUTION, NUCLEOSYNTHESIS, AND YIELDS OF LOW-MASS ASYMPTOTIC GIANT BRANCH STARS AT DIFFERENT METALLICITIES

S. Cristallo\textsuperscript{1}, O. Straniero\textsuperscript{1}, R. Gallino\textsuperscript{2,3}, L. Pierantoni\textsuperscript{4}, I. Domínguez\textsuperscript{4}, and M. T. Lederer\textsuperscript{5}

\textsuperscript{1} INAF-Osservatorio Astronomico di Collurania, 64100 Teramo, Italy
\textsuperscript{2} Dipartimento di Fisica Generale, Università di Torino, 10125 Torino, Italy
\textsuperscript{3} Center for Stellar and Planetary Astrophysics, School of Mathematical Sciences, Monash University, PO. Box 28, Victoria 3800, Australia
\textsuperscript{4} Departamento de Física Teórica y del Cosmos, Universidad de Granada, 18071 Granada, Spain
\textsuperscript{5} Institut für Astronomie, Türkenschanzstraße 17, A-1180 Wien, Austria

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ABSTRACT

The envelope of thermally pulsing asymptotic giant branch (TP-AGB) stars undergoing periodic third dredge-up (TDU) episodes is enriched in both light and heavy elements, the ashes of a complex internal nucleosynthesis involving p, α, and n captures over hundreds of stable and unstable isotopes. In this paper, new models of low-mass AGB stars (2 M\textsubscript{☉}), with metallicity ranging between Z = 0.0138 (the solar one) and Z = 0.0001, are presented. Main features are (1) a full nuclear network (from H to Bi) coupled to the stellar evolution code, (2) a mass loss–period–luminosity relation, based on available data for long-period variables, and (3) molecular and atomic opacities for C- and/or N-enhanced mixtures, appropriate for the chemical modifications of the envelope caused by the TDU. For each model, a detailed description of the physical and chemical evolutions is presented; moreover, we present a uniform set of yields, comprehensive of all chemical species (from hydrogen to bismuth).

The main nucleosynthesis site is the thin 13C pocket, which forms in the core–envelope transition region after each TDU episode. The formation of this 13C pocket is the principal by-product of the introduction of a new algorithm, which shapes the velocity profile of convective elements at the inner border of the convective envelope: both the physical grounds and the calibration of the algorithm are discussed in detail. We find that the pockets shrink (in mass) as the star climbs the AGB, so that the first pockets, the largest ones, leave the major imprint on the overall nucleosynthesis. Neutrons are released by the 13C(α, n)16O reaction during the interpulse phase in radiative conditions, when temperatures within the pockets attain T ∼ 10^8 K, with typical densities of (10^6–10^7) neutrons cm\textsuperscript{-3}. Exceptions are found, as in the case of the first pocket of the metal-rich models (Z = 0.0138, Z = 0.006 and Z = 0.003), where the 13C is only partially burned during the interpulse: the surviving part is ingested in the convective zone generated by the subsequent thermal pulse (TP) and then burned at T ∼ 10^8 K, thus producing larger neutron densities (up to 10^11 neutrons cm\textsuperscript{-3}).

Another neutron exposure, caused by the 22Ne(α, n)25Mg during the TPs, is marginally activated at large Z, but becomes an important nucleosynthesis source at low Z, when most of the 22Ne is primary. The final surface compositions of the various models reflect the differences in the initial iron-seed content and in the physical structure of AGB stars belonging to different stellar populations. Thus, at large metallicities the nucleosynthesis of light s-elements (Sr, Y, Zr) is favored, whilst, decreasing the iron content, the overproduction of heavy s-elements (Ba, La, Ce, Nd, Sm) and lead becomes progressively more important. At low metallicities (Z = 0.0001) the main product is lead. The agreement with the observed [hs/ls] index observed in intrinsic C stars at different [Fe/H] is generally good. For the solar metallicity model, we found an interesting overproduction of some radioactive isotopes, like 60Fe, as a consequence of the anomalous first 13C pocket. Finally, light elements (C, F, Ne, and Na) are enhanced at any metallicity.

Key words: nuclear reactions, nucleosynthesis, abundances – stars: AGB and post-AGB

Online-only material: color figures, machine-readable tables

1. INTRODUCTION

The fundamental role played by thermally pulsing asymptotic giant branch (TP-AGB) stars in the chemical evolution has been early recognized (Ulrich 1973). These stars are responsible for the nucleosynthesis of the main and the strong components of the s-process (Gallino et al. 1998; Travaglio et al. 1999) and contribute to the synthesis of various light elements, such as Li, C, N, and F. These stars are bright red giants (M\textsubscript{bol} ranging from −4 to −7). Their compact CO cores, whose mass ranges between 0.5 and 1 M\textsubscript{☉}, are progressively cooled by plasma neutrino emission. Owing to the efficient heat conduction provided by degenerate electrons, the cores are maintained in a quasi-isothermal state. Outside, there is an extended H-rich envelope, largely convective and progressively eroded by an intense mass loss (from 10\textsuperscript{−8} up to 10\textsuperscript{−3}M\textsubscript{☉} yr\textsuperscript{−1}). The most interesting nucleosynthesis processes occur in a thin He-rich zone (10\textsuperscript{−2}–10\textsuperscript{−3}M\textsubscript{☉}), located between the H and the He burning shells (hereafter He intershell).

The energy irradiated by these stars is mainly provided by the hydrogen burning. He burning is off during most of the AGB lifetime, so that the mass of the He intershell increases as a consequence of the advancing H burning shell. Then, periodic thermonuclear runaways (thermal pulses (TPs)) are driven by violent He ignitions occurring when the He buffer attains a critical value (Schwarzchild & Härn 1965; Weigert 1966). Owing to the sudden release of energy by the 3α reactions, the He intershell becomes dynamically unstable against convection, while the external layers expand and cool down. As a consequence of the convective mixing, the whole He intershell becomes enriched in C (the main product of the partial He burning) and heavy elements, as produced by the
s-process (slow neutron captures). After a short period (10–200 yr), the thermonuclear runaway ceases, a stationary He burning takes place, and convection disappears within the He intershell. Meanwhile, at the base of the H-rich envelope, the outgoing nuclear energy flux and the dropping temperature cause the radiative gradient to exceed the adiabatic gradient at positions more and more close to the He intershell. Finally, after a few hundred years, the convective envelope penetrates the He intershell zone (third dredge-up (TDU)). Straniero et al. (2003) showed that all TP-AGB stars may undergo recurrent TDU episodes, provided their envelope mass is sufficiently large. The minimum envelope mass for the occurrence of the TDU is of a few tenths of $M_\odot$, but the precise value depends on the core mass and on the envelope composition.

The occurrence of many TDU episodes in thermally pulsing low-mass AGB stars has two important consequences. First of all it is responsible for the chemical modification of the envelope, which becomes progressively enriched in primary C and in the products of the s-process. This provides a simple explanation for the observed spectroscopic sequence of AGB stars, from the M to the C type, through the MS and S stars (Smith & Lambert 1986; Malaney & Lambert 1988; Busso et al. 1995, 2001; Abia et al. 2002). The carbon excess, mixed within the envelope by previous TDU episodes, causes a significant increase of the radiative opacity in the envelope, so that the effective temperature sinks, the stellar radius increases, and the mass loss rate rises. The second important consequence of the penetration of the convective envelope into the He- and C-rich He intershell concerns the formation of the so-called $^{13}$C pocket, where the main and the strong components of the classical s-process are built up (see Gallino et al. 1998). Indeed, when the convective envelope recedes after a TDU episode, it leaves a small region (of the order of a few $10^{-4} M_\odot$) characterized by an increasing proton profile embedded in a bath of C and He (about 20% and 80%, by mass fractions, respectively). During the relatively long period that elapses between two subsequent TPs (interpulse), this region heats up; then, $^{13}$C is first produced through the $^{12}$C($p, \gamma$)$^{13}$N$(\beta^- \nu)^{13}$C and, later on (at $T \sim 0.9 \times 10^6$ K), the $^{13}$C$(\alpha, n)^{16}$O, provides the slow neutron flux required to activate the s-process nucleosynthesis (Straniero et al. 1995). This pocket, strongly enriched in heavy elements, is engulfed into the convective zone generated by the subsequent TP. When the temperature at the base of this convective zone exceeds $2.7 \times 10^8$ K, a small neutron burst is powered by the $^{22}$Ne$(\alpha, n)^{25}$Mg reaction. In low-mass AGB stars of nearly solar metallicity, this second neutron source is marginally activated (Straniero et al. 1997). Several observations confirm such a theoretical expectation (see, e.g., Lambert et al. 1995; Abia et al. 2001). However, this second neutron source plays a more relevant role at low metallicity, even in low-mass AGB stars.

Our knowledge of the physical properties of AGB stars and of their nucleosynthesis is mainly based on observations and models for intermediate age stellar populations of the Milky Way. The recent availability of optical and near IR spectroscopy of AGB stars belonging to nearby galaxies, dwarf spheroidals as well as the Magellanic Clouds, allows us to extend our investigation to metal-poor stellar populations with age of the order 1–2 Gyr (Dominguez et al. 2004; de Laverny et al. 2006; Reyniers et al. 2007; Lebzelter et al. 2008; Abia et al. 2008). In addition to that, the growing number of studies on C-enhanced metal-poor stars enriched in s-elements (CEMPs), as stimulated by dedicated surveys (Beers et al. 1999; Christlieb 2003), provides new hints on the nucleosynthesis occurred in the now extinct AGB–halo population, as due to stars whose initial masses were in the range 1–7 $M_\odot$. The aim of this paper is to investigate how the theoretical scenario changes with decreasing metallicity, from the solar value to that of the metal-poor stars belonging to the Galactic halo or to the nearby galaxies. Extant stellar models have been computed by adopting limited networks. For example, Karakas & Lattanzio (2007) include all the relevant processes and isotopes from H to Si, plus a few Fe-group isotopes and an artificial neutron sink to account for heavy-element neutron captures. In this way, they can provide a reasonable estimate of the light-element yields. Our approach is different. We compute evolutionary models, followed from the pre-main sequence up to the AGB tip, of stars with initial mass 2 $M_\odot$ and different metallicities (namely, $Z = 0.0138$, 0.006, 0.003, 0.001, and 0.0001), by adopting a full network that includes all the stable and the relevant unstable isotopes from hydrogen to bismuth. In our previous works (Gallino et al. 1998; Bisterzo et al. 2009), full nucleosynthesis calculations were obtained by means of a postprocess code: in such a case, the main physical parameters were obtained by means of stellar models calculated with a restricted nuclear network involving key isotopes and reactions (Straniero et al. 1997). Then, an ad hoc (average) $^{13}$C pocket was imposed. In the present work, stellar evolution and nucleosynthesis are coupled. If the faster postprocess calculation allows us to investigate a large area in the parameter space, the present, fully coupled, calculations shed light on the possible feedbacks between chemical and physical evolutions. These new models include several improvements in the input physics and in the computational algorithms (see Straniero et al. 2006, thereafter SGC06): the most important for the AGB evolution are illustrated in Section 2. The calibration of some free parameters used by the mixing algorithm is discussed in Section 3. Results are presented in Sections 4, 5, and 6. A final discussion follows in Section 7.

2. THE THEORETICAL RECIPE: RADIATIVE OPACITY, NUCLEAR DATA, MIXING SCHEME, AND MASS LOSS

The stellar models here presented have been computed by means of the FRANEC code (Chieffi et al. 1998, and references therein). An updated description of the various input physics adopted in the AGB computations can be found in SGC06. Owing to the particular relevance for the present paper, let us describe the radiative opacity, some key nuclear reaction rates, the mixing algorithm, and the mass loss rate.

As discussed by Marigo (2002), the increase of the radiative opacity caused by the C dredged-up induces important changes in the physical structure of the outermost layers of an AGB star, with important consequences on the effective temperature, the radius, and the mass loss rate. In addition to that, when the CN cycle takes place in the innermost part of the convective envelope or immediately below it, an important fraction of the primary C that is dredged up to the surface may be converted into N. This is a common phenomenon in massive AGBs (the so-called hot bottom burning (HBB); Sugimoto 1971; Iben 1973), even if there are several observational evidence proving that deep-mixing processes can also occur in low-mass AGBs (cool bottom process (CBP); see Nollett et al. 2003). As a consequence of the carbon dredged-up, new molecular species may form in the cool atmosphere of an AGB star, such as CN, HCN, or C$_2$. Note that since the amount of primary C synthesized by the $3\alpha$ reaction and mixed by the TDU is of the same order
of magnitude at any metallicity, the resulting overabundance of this element (and eventually N) is very large at low metallicity. Whilst at solar metallicity the final C-enhancement in the envelope may be a factor of 2–4, at \( Z = 0.0001 \) it rapidly attains a factor of 1000. For this reason, if the adoption of a proper radiative opacity (including enhancements of C and N) may only introduce limited quantitative changes in the overall picture of AGB stars with solar metallicity, its use at low metallicity is absolutely mandatory, because it leads to substantial modifications of the whole theoretical scenario (see Cristallo et al. 2007). Therefore, below \( T = 10^4 \) K we use new opacity tables derived by means of the COMA code (Aringer 2000), which include all the molecular and atomic species relevant for AGB atmospheres. At higher temperatures, the opacity tables have been calculated by means of the OPAL Web facility (http://www-phys.llnl.gov/Research/OPAL/opal.html). Table 1 lists the C- and the N-enhancement factors (with respect to the solar scaled values) adopted for the opacity tables at different metallicities. All the elements (except H, He, C, and N) are assumed to be solar scaled (namely \([\text{M}/\text{Fe}] = 0\)). The enhancement of the \( \alpha \) elements (O, Ne, Mg, Si, S, Ca), more appropriate for low-metallicity halo stars, is not considered here, given that it only adds minor effects on the Rosseland mean opacity. The full database providing low-temperature Rosseland opacities for a wide range of metallicities with varying carbon and nitrogen abundances is described in Lederer & Aringer (2009).

The nuclear network is essentially the same already described in Section 6 of SGC06: it includes about 500 isotopes (from H to Bi) and more than 700 nuclear reactions (charged particle reactions, neutron captures, and \( \beta \)-decays). However, few reaction rates have been changed or updated. In particular, concerning reactions among charged nuclei, the \(^{14}\text{N}(\alpha,\gamma)^{18}\text{F} \) rate is now taken from Görres et al. (2000), whilst for the \(^{22}\text{Ne}(\alpha, n)^{25}\text{Mg} \) rate we follow Jaeger et al. (2001). Then, we have updated the \(^{151}\text{Sm}(n,\gamma)^{152}\text{Sm} \) reaction rate, following the prescriptions reported by Abbondanno et al. (2004), and the cesium isotopes’ neutron-capture rates, as suggested by Patronis et al. (2004). Concerning the \(^{175}\text{Lu} \) unthermalized isomeric state (see Section 7.5 of SGC06), we adopt an isomeric ratio \( \text{IR} = 0.2 \) for temperatures lower than \( 2.0 \times 10^8 \) K and \( \text{IR} = 0.25 \) for higher temperatures. For a more detailed analysis of this important branching see Heil et al. (2008).

Mixing caused by convection is obtained by means of a time-dependent mixing scheme in which the degree of mixing between two mesh points, whose separation is \( \Delta R \), is linearly dependent on the ratio between the time step and the mixing timescale.\(^6\) An exhaustive description of the mixing algorithm can be found in Section 8 of SGC06 (see also Chieffi et al. 2001). Let us here recall the main concepts. The average convective velocity (\( v_c \)) is calculated according to the mixing-length theory (Cox & Giuli 1968), whereas the boundaries of the convective regions are located as given by the Schwarzschild criterion. In order to handle the instability of the convective boundary taking place during the TDU episodes, when the H-rich envelope penetrates the H-depleted core, we assume that the convective velocity drops to 0 following an exponential decline, namely,

\[
v = v_{bce} \exp \left(-\frac{d}{\beta H_P}\right), \tag{1}\]

where \( d \) is the distance from the formal convective boundary (as defined by the Schwarzschild criterion), \( v_{bce} \) is the velocity at the formal convective boundary, \( H_P \) is the pressure scale height, and \( \beta \) is a free parameter. All the evolutionary sequences presented in Section 4 and 5 of the present paper have been obtained assuming \( \beta = 0.1 \). In the next section, we will describe the procedure followed to fix the value of the \( \beta \) parameter. A similar mathematical profile has been already applied to the calculation of AGB models by Herwig et al. (1997; see also Herwig 2000). However, Herwig et al. (1997) used an exponential decline equation to calculate the diffusion coefficient, instead of the convective velocity, and applied this prescription to all the convective boundaries. In this way, they always obtain overshoot at the top and at the bottom of any convective zone, while in our models Equation (1) works only when \( v_{bce} > 0 \), as it happens in the case of the bottom of the convective envelope at the time of the TDU. A second important difference concerns the mixing scheme, which determines the chemical coupling among mesh points within the same convective zone. In the diffusive mixing adopted by Herwig et al. (1997), the variation of the composition during a given time step depends on the second derivative of the composition with \( r \) (the radial coordinate), whilst in our algorithm this relationship is linear. These differences directly affect the profile of protons in the transition zone between the fully convective envelope and the radiative core at the time of the TDU and the temperature attained in the He-rich intershell during a TP. We will discuss the consequences on the nucleosynthesis in the next section.

Concerning mass loss, we adopt a Reimers’ formula (\( \eta = 0.4 \)) for the pre-AGB evolution, whilst for the AGB we follow a procedure similar to that described by Vassiliadis & Wood (1993), but revising the mass loss–period and the period–luminosity relations, taking into account more recent infrared observations of AGB stars (see SGC06 for the references). A comparison among our mass loss rate and those obtained by means of the Reimers’ formula and the Vassiliadis and Wood prescriptions can be found in Section 5 of SGC06. An important remark concerns the dependence of the mass loss rate on the metallicity. The mass loss–period relation we use is based on data relative to Galactic AGB stars having nearly solar iron. In a recent paper, Groenewegen et al. (2007) showed that the same relation also applies to carbon stars in the Magellanic Clouds, whose metallicity is lower, on average, than that of the Galactic C-stars. Similar indications come from Spitzer observations of mass loss of carbon stars belonging to dwarf spheroidal galaxies (see Lagadec & Zijlstra 2008).

\(^6\) Here the mixing timescale is defined as \( \tau_{max} = \Delta R/v_c \).
Figure 1. Each panel shows the composition of the core–envelope transition zone of the solar metallicity model ($M = 2 M_\odot$) at the moment of the 3rd TDU, as obtained by adopting low values of the $\beta$ parameter. We report the mass fractions of $^{12}\text{C}$ (dotted line), $^{13}\text{C}$ (solid line), $^{14}\text{N}$ (short-dashed line), and H (long-dashed line) in the transition zone. The left panels refer to the epoch of the maximum penetration of the convective envelope into the He intershell during the TDU, whilst the right panels show the same region after the formation of the $^{13}\text{C}$ pocket. Note that in the $\beta = 0$ case, the convective envelope does not penetrate the H-exhausted core, so that the showed H profile is due to the previous shell-H burning, rather than to the operation of the exponential decline of the convective velocity described by Equation (1).

(A color version of this figure is available in the online journal.)

and 6.5, we discuss the effect of different mass loss prescriptions on the evolution and nucleosynthesis of the $Z = 0.0001$ model.

3. INSTABILITY OF THE CONVECTIVE BOUNDARY, DREDGE-UP, AND THE FORMATION OF THE $^{13}\text{C}$ POCKET

In this section, we describe the models obtained by changing the value of the $\beta$ parameter introduced in the previous section to treat the instability of the convective boundary layer, arising when the H-rich convective envelope penetrates the He-rich zone. During a TDU episode, a chemical discontinuity forms at the convective boundary, causing an abrupt change of the radiative opacity. If a bare Schwarzschild criterion is adopted to limit the mixing, the average convective velocity drops from about $10^4$ cm s$^{-1}$ to 0 in two adjacent mesh points. In this condition, any perturbation causing mixing of material across the boundary layer leads to an increase of the opacity in the underlying stable zone, which immediately becomes unstable against convection. As is well known, a similar condition occurs at the external border of the convective core during the central He-burning phase (Castellani et al. 1971). The most important consequence of this instability is the propagation of the convective instability; if in the case of the central-He burning such an occurrence implies the growth of the convective core, in the case of the TDU a deeper penetration of the convective envelope into the H-exhausted core is expected. Actually, there should exist a transition region, between the fully convective envelope and the radiatively stable H-exhausted core, where the convective velocity smoothly decreases to 0 and, therefore, a partial mixing takes place. In such a way, after any dredge-up episode, a zone with a smoothed H profile is left behind. Since the amount of $^{12}\text{C}$ is rather large (about 20% by mass fraction) in this transition zone, at H reignition those few protons are captured by the abundant $^{12}\text{C}$ and form a $^{13}\text{C}$ pocket. Later on, the $s$-process nucleosynthesis is activated, through the $^{13}\text{C}(\alpha, n)^{16}\text{O}$ reaction. Note that a $^{14}\text{N}$ pocket and a $^{23}\text{Na}$ pocket form, partially overlapped to the $^{13}\text{C}$ pocket (see the next section).

The mixing algorithm described in the previous section mimics the formation of such a transition zone. The free parameter ($\beta$), used to tune the decline of the average convective velocity at the base of the convective envelope, determines the extension of such a transition zone and, in turn, the amount of $^{13}\text{C}$ available for the neutron-capture nucleosynthesis. To calibrate such a parameter, we have repeated the calculation of the same sequence TP–interpulse–TP for $0 \leq \beta \leq 0.2$. This test has been performed at two metallicities ($Z = 0.0138$ and $Z = 0.0001$) in order to test the sensitivity of our calibration procedure when changing the metal content. Results are illustrated in Figures 1–4. The left panels refer to the epoch of the maximum penetration of the convective envelope during the TDU. In the

7 The heavy vertical dashed line indicates the location of the formal boundary of the convective envelope, as defined by means of the Schwarzschild criterior.
Figure 2. Same as Figure 1, but for larger values of the $\beta$ parameter.
(A color version of this figure is available in the online journal.)

Figure 3. Same as Figure 1, but relative to the 2nd TDU of the model with $Z = 0.0001$ and $M = 2 M_\odot$.
(A color version of this figure is available in the online journal.)
right panels, we show the same region, but after the H reignition, once the convective envelope recedes and the $^{13}$C pocket is fully developed. The results of these test models are quantitatively summarized in Table 2, where in Columns 1–7 we report the value of $\beta$, the mass $\Delta M_{\text{TDU}}$ of the H-depleted material that is dredged up, the ratio $\lambda = \Delta M_{\text{TDU}}/\Delta M_{\text{H}}$ ($\Delta M_{\text{H}}$ being the mass of the material that has been burned by the H-shell during the previous interpulse period), the product $\beta \times H_p$ at the epoch of the maximum penetration of the convective envelope, the total mass $\Sigma H$ of hydrogen left below the formal convective boundary at the same epoch, and the

| $\beta$ | $\Delta M_{\text{TDU}}$ ($M_\odot$) | $\lambda$ | $\Delta M_{\text{pocket}}$ ($M_\odot$) | $\beta \times H_p$ (cm) | $\Sigma H$ ($M_\odot$) | $\Sigma^{13}\text{C}_{\text{eff}}$ ($M_\odot$) |
|--------|---------------------------------|---------|------------------------------------|--------------------------|-------------------------|-----------------------------|
| 0      | 0.000E+00                       | …       | 0.00E+00                           | …                        | …                       | …                           |
| 0.01   | 0.000E+00                       | …       | 0.00E+00                           | …                        | …                       | …                           |
| 0.05   | 3.00E–04                        | 0.04    | 9.40E–05                           | 5.62E+08                 | 4.37E–05               | 8.32E–07                    |
| 0.075  | 1.5E–03                         | 0.14    | 3.47E–04                           | 8.82E+08                 | 8.49E–05               | 3.12E–06                    |
| 0.09   | 1.70E–03                        | 0.20    | 7.66E–04                           | 1.06E+09                 | 9.56E–05               | 6.41E–06                    |
| 0.1    | 2.50E–03                        | 0.28    | 5.54E–04                           | 1.22E+09                 | 1.13E–04               | 7.38E–06                    |
| 0.11   | 2.90E–03                        | 0.32    | 2.54E–04                           | 1.36E+09                 | 1.29E–04               | 4.40E–06                    |
| 0.125  | 4.10E–03                        | 0.43    | 2.00E–05                           | 1.61E+09                 | 1.76E–04               | 8.80E–07                    |
| 0.15   | 5.80E–03                        | 0.58    | 1.00E–05                           | 2.00E+09                 | 2.04E–04               | 1.42E–07                    |
| 0.2    | 7.90E–03                        | 0.75    | 0.00E+00                           | 2.73E+09                 | 1.96E–04               | 0.00E+00                    |
| $Z = 0.0138$ (3rd Pulse followed by TDU) | $Z = 0.0001$ (2nd Pulse followed by TDU) |
Figure 5. Upper panel: the evolutions of the transition region between the convective envelope and the H-exhausted core in between the 4th and the 5th TP of the $2 M_\odot$ model with solar metallicity and $\beta = 0.1$. The more external line represents the inner border of the convective envelope, while the lower line shows the location of the layer where the nuclear energy production is maximum in the H-burning shell. The 2nd and 3rd TDU episodes are easily recognized. The quantities $\Delta M_H$ and $\Delta M_{TDU}$ are also graphically illustrated. Central panel: H profile in the top layer of the H-exhausted core at the epoch of the maximum penetration of the convective envelope during the 3rd TDU episode. The shaded area represents the quantity $\Sigma H$, namely the total mass of H left below the formal border of the convective envelope. Bottom panel: the same region, but after the development of the $^{13}$C pocket. The solid and dotted lines represent the mass fractions of $^{13}$Ca and $^{14}$N, respectively, while the shaded area is $\Sigma^{13}\text{C}_{\text{eff}}$, namely the mass of the effective $^{13}$C in the pocket (see Footnote 8 for the definition of effective $^{13}$C). The arrow shows the parameter $\Delta M_{\text{pocket}}$, corresponding to the extension (in mass) of the region where the effective $^{13}$C is larger than $10^{-3}$.

Figure 6. Mass portion of the He intershell that is dredged up vs. the $\beta$ parameter after the 3rd pulse followed by TDU of the solar metallicity model and after the 2nd pulse followed by TDU of the $Z = 0.0001$ model. See the text for details.

Figure 7. Variation with $\beta$ of the effective $^{13}$C mass in the pocket after the 3rd pulse followed by TDU of the solar metallicity model and after the 2nd pulse followed by TDU of the $Z = 0.0001$ model. See the text for details.

The mass fraction of effective $^{13}$C in a given mesh point is defined as $X_{^{13}\text{C}} = X_{^{13}} - X_{^{14}}$, where $X_{^{13}}$ and $X_{^{14}}$ are the mass fractions of $^{13}$C and $^{14}$N, respectively. Owing to its large neutron-capture cross section, $^{14}$N is the most efficient neutron poison, so that the s-process nucleosynthesis is significantly depressed where it becomes comparable to the $^{13}$C. For this reason, we define the mass of the $^{13}$C pocket as the mass of the zone where $X_{^{13}\text{C}} > 10^{-3}$ (see Figure 5).

The effective $^{13}$C masses are $7.4 \times 10^{-6}$ and $5.5 \times 10^{-6} M_\odot$, at $Z = 0.0138$ and $Z = 0.0001$, respectively. These values are close to the standard one adopted by Gallino et al. (1998), namely $4 \times 10^{-6} M_\odot$. Gallino et al. (1998) showed how this choice of the $^{13}$C pocket allows the build-up of the main and the strong components, i.e., the distribution of s-elements from the Sr–Y–Zr peak up to the Pb–Bi peak in the solar system, whilst other works (Busso et al. 2001; Abia et al. 2001, 2002) argue that a certain spread of the effective $^{13}$C mass is required in order to reproduce the heavy element overabundances in a large sample of Galactic C(N type) stars. More recently, Bisterzo et al. (2009) confirm the need for such a spread, extending their analysis to CEMP stars. It is worth to note that whilst Gallino et al. (1998) assume a constant effective $^{13}$C mass, in our models we find a characteristic evolution of the $^{13}$C pocket (see Figure 8). In particular, a maximum mass of $^{13}$C is attained.

8 This amount of $^{13}$C is often referred as the ST (standard) case for low-mass AGB nucleosynthesis models.
after very few TPs (2 or 3); later on, the pocket progressively shrinks, in mass, until it disappears during the late part of the AGB evolution (see the next section). Nevertheless, the success of the postprocess calculations in reproducing a large amount of observational data concerning AGB stars (Lambert et al. 1995; Busso et al. 1995; Abia et al. 2001, 2002), their progeny on the post-AGB (Reyniers et al. 2007), Galactic evolution of s-process elements (Travaglio et al. 1999, 2001, 2004), and the isotopic composition of C-rich presolar grains (SiC; Zinner et al. 2006a; Lugaro et al. 1999) suggests to adopt $\beta$ values allowing the formation of the $^{13}$C pocket whose average size is similar to the ST case. On the other hand, as shown in Figure 7, only values of $\beta$ close to 0.1 provide enough effective $^{13}$C. For this reason, in all the computation presented here we have adopted $\beta = 0.1$ (but see Section 7 for further remarks).

Let us come back to the differences between our mixing scheme and that adopted by Herwig et al. (1997). Based on a few additional models computed by using a diffusion algorithm instead of our linear mixing, we have verified that in this case the extension in the mass of the zone where the proton abundance left by the TDU is $10^{-3} < X < 10^{-2}$ cannot be larger than about $10^{-5} M_\odot$ (for any value of the $\beta$ parameter).10 Instead, as shown in Figure 5, when our mixing algorithm is adopted, the same region extends for a few $10^{-4} M_\odot$. Such a difference affects the total mass of the effective $^{13}$C within the pocket, which may be about 20 times larger in the case of a linear mixing scheme (for comparisons, see Figure 4 in Herwig 2000). As a consequence, the resulting s-process yields are significantly reduced if a diffusive mixing is adopted. A further difference concerns the overshoot at the bottom of the convective zone generated by a TP. In our models, the exponential decline described by Equation (1) switches off when, as in this case, the radiative gradient is equal to the adiabatic gradient at the convective border, so that $v_{\text{bce}} = 0$. In contrast, Herwig et al. (1997) force a certain overshoot below the base of this convective zone, in spite of the strong negative difference between the radiative gradient and the huge entropy barrier generated by the shell-He burning. This implies stronger TPs and larger temperatures at the base of the He-rich intershell, thus increasing the efficiency of the s-process nucleosynthesis powered by the $^{22}$Ne($\alpha$, n)$^{25}$Mg reaction (see Lugaro et al. 2003).

A reliable (hydrodynamical) model of convection capable to describe the mixing across the boundaries of the major convective zones should provide a more realistic description of the AGB evolution and nucleosynthesis (for a recent attempt, see Meakin & Arnett 2007). On the other hand, comparisons between the predicted AGB nucleosynthesis and the abundances observed in AGB stars may be used to constrain the efficiency of convection. This is the approach we follow in the present paper.

4. REFERENCE MODELS AT DIFFERENT METALLICITIES

In this section, we present and compare five evolutionary sequences of AGB models having the same initial mass (2 $M_\odot$), but different initial compositions, namely, $(Z, Y) = (0.0138; 0.269), (0.006; 0.260), (0.003; 0.260), (0.001; 0.245), (0.0001; 0.245)$. The mass is representative of low-mass AGB stars and the five metallicities almost span the entire metal distribution of our Galaxy and of the extragalactic resolved stellar populations, such as the Magellanic Clouds, M31, and several dwarf spheroidal galaxies. The most metal-rich model corresponds to the composition of the presolar nebula, as derived by means of an up-to-date standard solar model obtained by adopting the latest compilation of solar abundance ratios (Lodders 2003; see Piersanti et al. 2007 for more details). The five evolutionary tracks, from the pre-main sequence up to the end of the AGB, are reported in Figure 9. The TP-AGB phase is characterized by large oscillations in luminosity, associated with the expansions and contractions powered by TPs, and an evident red excursion that is the direct consequence of the formation of new molecular species taking place after the transition from O- to C-rich atmosphere.

In Figure 10, we show the evolution, during the TP-AGB phase, of the positions, in mass coordinates, of the inner border of the convective envelope, of the location of the maximum energy production within the H-burning shell, and of the location

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10 This is the range of hydrogen mass fraction required to produce enough effective $^{13}$C. If $X > 10^{-2}$, $^{14}$N rather than $^{13}$C would be produced, while for $X < 10^{-3}$ too few $^{13}$C is synthesized.
of the maximum energy production available in the online journal.)

Table 3 reports some properties characterizing the five evolutionary sequences, namely (from left to right), the progressive number of the TP \( n_{\text{TP}} \), the total mass at the time of the onset of the TP \( M_{\text{Tot}} \), the corresponding mass of the H-exhausted core \( M_{\text{H}} \), the mass of the H-depleted material dredged-up \( \Delta M_{\text{TDU}} \), the maximum mass of the convective zone generated by the TP \( \Delta M_{\text{CZ}} \), the mass burnt by the H-shell during the previous interpulse period \( \Delta M_{\text{H}} \), the overlap factor \( r \) (defined as the fraction of \( \Delta M_{\text{CZ}} \) already included in the convective zone generated by the previous TP), the \( \lambda \) factor (defined as the ratio between \( \Delta M_{\text{TDU}} \) and \( \Delta M_{\text{H}} \)), the average number \( n_{\text{c}} \) of neutrons captured per initial iron-seed nucleus during the radiative \( ^{13}\text{C} \) pocket burning, the duration of the interpulse period preceding the current TP \( \Delta t_{\text{ip}} \), the maximum temperature attained at the bottom of the convective zone generated by the TP \( T_{\text{MAX}} \), and the surface metallicity after the dredge-up \( Z_{\text{surf}} \) and the corresponding C/O ratio. Finally, the total mass of the material cumulatively dredged up during the whole evolutionary sequence is reported in the last row \( M_{\text{tot}}^{\text{TDU}} \).

As already discussed in Straniero et al. (2003), the TDU starts when the mass of the H-exhausted core exceeds a critical value, whereas it ceases when the envelope mass, eroded by the mass loss, is reduced down to a critical value. In the five evolutionary sequences presented here, the first TDU episode takes place when (from \( Z = 0.0138 \) to \( Z = 0.0001 \)) \( M_{\text{H}} \) is 0.560, 0.569, 0.576, 0.604, 0.641 \( M_{\odot} \). Note that at the lowest metallicities (\( Z = 0.001 \) and 0.0001) a 2 \( M_{\odot} \) star already develops rather large core mass during the early-AGB, so that the TDU occurs almost soon after the beginning of the TP-AGB phase. At those metallicities, stars with smaller mass may experience TDU episodes at smaller \( M_{\text{H}} \). The last TDU...
Table 3

| n_Typ | M_{tot} | M_H | ΔM_{TUD} | ΔM_{CZ} | ΔM_{fit} | r | λ | n_{i} | Δ_{ij} | T_{MAX} | Z_{surf} | C/O |
|-------|--------|-----|----------|--------|---------|---|---|-----|------|--------|---------|-----|
|       |        |     |          |        |         |   |   |     |      |        |          |     |
| −2    | 1.91E+00 | 5.48E−01 | 0.00E+00 | 3.25E−02 | 4.20E−03 | 8.99E−01 | 0.00E+00 | ... | 1.40 | 2.20 | 6.67E−03 | 0.29 |
| −1    | 1.90E+00 | 5.53E−01 | 0.00E+00 | 3.07E−02 | 4.80E−03 | 8.66E−01 | 0.00E+00 | ... | 1.65 | 2.27 | 3.92E−03 | 0.31 |
| 1     | 1.89E+00 | 5.60E−01 | 0.00E+00 | 3.12E−02 | 7.00E−03 | 7.90E−01 | 2.90E−02 | 4.2×104 | 18.6 | 2.46 | 3.92E−03 | 0.31 |
| 2     | 1.88E+00 | 5.68E−01 | 0.00E+00 | 2.94E−02 | 7.90E−03 | 7.48E−01 | 1.52E−01 | 4.6×104 | 18.6 | 2.53 | 4.12E−03 | 0.34 |
| 3     | 1.86E+00 | 5.76E−01 | 0.00E+00 | 2.81E−02 | 9.00E−03 | 6.27E−01 | 2.78E−01 | 4.3×104 | 17.8 | 2.62 | 4.46E−03 | 0.43 |
| 4     | 1.84E+00 | 5.83E−01 | 0.00E+00 | 2.69E−02 | 9.00E−03 | 6.46E−01 | 3.74E−01 | 4.3×104 | 17.0 | 2.69 | 5.32E−03 | 0.57 |
| 5     | 1.82E+00 | 5.90E−01 | 0.00E+00 | 2.58E−02 | 1.07E−02 | 6.00E−01 | 4.21E−01 | 3.9×104 | 16.2 | 2.75 | 6.22E−03 | 0.75 |
| 6     | 1.78E+00 | 5.96E−01 | 0.00E+00 | 2.48E−02 | 1.10E−02 | 5.67E−01 | 4.45E−01 | 3.6×104 | 15.3 | 2.80 | 7.22E−03 | 0.95 |
| 7     | 1.72E+00 | 6.03E−01 | 0.00E+00 | 2.37E−02 | 1.11E−02 | 5.40E−01 | 4.41E−01 | 3.2×104 | 14.3 | 2.84 | 8.22E−03 | 1.15 |
| 8     | 1.63E+00 | 6.09E−01 | 0.00E+00 | 2.25E−02 | 1.10E−02 | 5.23E−01 | 4.36E−01 | 3.2×104 | 13.2 | 2.87 | 9.22E−03 | 1.35 |
| 9     | 1.51E+00 | 6.15E−01 | 0.00E+00 | 2.14E−02 | 1.07E−02 | 5.09E−01 | 4.11E−01 | 3.0×104 | 12.1 | 2.90 | 1.04E−03 | 1.57 |
| 10    | 1.35E+01 | 6.21E−01 | 0.00E+00 | 2.03E−02 | 1.03E−02 | 5.02E−01 | 2.96E−01 | 2.9×104 | 11.0 | 2.91 | 2.14E−03 | 1.78 |
| 11    | 1.16E+01 | 6.26E−01 | 0.00E+00 | 2.12E−02 | 1.28E−02 | 5.19E−01 | 5.70E−01 | 8.3×103 | 17.7 | 2.87 | 1.04E−03 | 1.85 |
| 12    | 9.80E−01 | 6.33E−01 | 0.00E+00 | 2.05E−02 | 8.05E−03 | 5.31E−01 | 0.00E+00 | 2.2×103 | 8.30 | 2.87 | 2.19E−02 | 2.19 |

\[ Z = 0.0138, \ Y = 0.269 \]

\[ Z = 0.0060, \ Y = 0.260 \]

\[ Z = 0.0030, \ Y = 0.260 \]
episode takes place when \( M_{\text{ens}} \) is 0.534, 0.183, 0.207, 0.292, 0.242 \( M_\odot \), for \( Z = 0.0138, 0.006, 0.003, 0.001 \) and 0.0001, respectively.

As discussed in Section 3, the introduction of a smoothed profile of the convective velocities at the bottom of the envelope favors the occurrence of TDU at smaller core masses and enhances its efficiency, with respect to models where the bare Schwarzschild criterion is adopted (Straniero et al. 1997). As an example, in the 2 \( M_\odot \) model with \( Z = 0.02 \) of Straniero et al. (1997), the first TDU takes place when the core mass is 0.61 \( M_\odot \) and has a reduced efficiency with respect to the model presented here. A second reason at the base of these differences is the rate of the \(^{14}\text{N}(p, t)^{15}\text{O}\) reaction: we are currently using the most recent-low-energy laboratory measurement of this rate by Imbriani et al. (2006), which is a factor of 2 lower with respect to the rate proposed by Caughlan & Fowler (1988) and Angulo et al. (1999). Being the \(^{14}\text{N}(p, t)^{15}\text{O}\) reaction the bottleneck of the CNO cycle, a lower rate implies a reduced H-burning efficiency, stronger TPs, and larger TDUs, as already verified by Straniero et al. (2000) and confirmed by Herwig & Austin (2004).

An important quantity affecting the nucleosynthesis is the maximum temperature attained at the base of the convective zone generated by a TP. During the AGB evolution, this value progressively increases, reaches a maximum, and then slightly decreases toward the last TP (Figure 11, upper panel). In the most metal-rich model, \( T_{\text{MAX}} \) remains always below 3.0 \times 10^8 K: this is due to the fact the H-burning shell is efficient and, therefore, the TPs are weaker with respect to lower metallicities (see Straniero et al. 2003). For that reason, in these models the \(^{22}\text{Ne}(\alpha, n)^{25}\text{Mg}\) reaction is only marginally activated. In contrast, in the two most metal-poor models, \( T_{\text{MAX}} \) attains higher values (up to 3.2 \times 10^8 K), so that the \(^{22}\text{Ne}(\alpha, n)^{25}\text{Mg}\) adds a significant contribution to the \( s\)-process nucleosynthesis, in particular for the treatment of the branchings along the \( s\)-path. We will come back on this issue in the next section.

Concerning \( \Delta M_{\text{TDU}} \), it depends nonlinearly on the metallicity, the envelope mass, and the core mass (see Formula (3) in Straniero et al. 2003). Fixing the values of the other two parameters, it is generally larger at lower metallicity. It is also larger at larger envelope mass or/and core mass. During the AGB evolution, the core mass increases, while the envelope mass decreases, so that \( \Delta M_{\text{TDU}} \) initially increases, following the growth of \( M_H \), but then decreases, when the envelope erosion by mass loss is stronger. This is clearly showed in Figure 11 (lower panel). Looking at the total mass of the material that is cumulatively dredged up, it has a minimum in the case of the most metal-rich evolutionary sequence

| \( n \) | \( \lambda \) | \( \lambda_g \) | \( T_{\text{MAX}} \) | \( Z_{\text{out}} \) | C/O |
|---|---|---|---|---|---|
| 0.25 | 0.35 | 0.45 | 0.55 | 0.65 | 0.75 |
| 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 |
| 0.15 | 0.25 | 0.35 | 0.45 | 0.55 | 0.65 |
| 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 |
| 0.05 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |

Notes.

\( ^a \) Pulse number (negative values correspond to TP not followed by TDU).

\( ^b \) \( M_{\odot} \).

\( ^c \) \( 10^3 \) yr.

\( ^d \) \( 10^8 \) K.

Table 3 (Continued)

| \( n \) | \( \lambda \) | \( \lambda_g \) | \( T_{\text{MAX}} \) | \( Z_{\text{out}} \) | C/O |
|---|---|---|---|---|---|
| 0.25 | 0.35 | 0.45 | 0.55 | 0.65 | 0.75 |
| 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 |
| 0.15 | 0.25 | 0.35 | 0.45 | 0.55 | 0.65 |
| 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 |
| 0.05 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |

Notes.

\( ^a \) Pulse number (negative values correspond to TP not followed by TDU).

\( ^b \) \( M_{\odot} \).

\( ^c \) \( 10^3 \) yr.

\( ^d \) \( 10^8 \) K.
2.15, as required by the standard solar model (see Piersanti et al. 2007), to 1.8. The evolutions of $T_{\text{MAX}}$ and $\Delta M_{\text{TDU}}$ for these two additional models are compared with the reference model in Figure 12. Either the mass loss rate and the mixing-length parameter are usually calibrated on stars with solar or nearly solar composition and little is known about the validity of these calibrations at low metallicities; for a discussion on the possible calibration of the mixing length at different $Z$ see Chieffi et al. (1995) and Ferraro et al. (2006). The early TP-AGB evolution of the Reimers’ model, which is mainly controlled by the growth of the H-exhausted core, is similar to that obtained in the case of the reference model. However, when the envelope erosion becomes important, the two sequences depart from each other. In particular, as a consequence of the higher mass loss rate, the reference model terminates sooner. As a result, the Reimers’ model experiences a larger number of TPs and TDU episodes. The total mass cumulatively dredged up is $1.6 \times 10^{-2} M_{\odot}$ in the reference model (see Table 4).

The main effect of reducing the mixing-length parameter is a smaller extension of the convective regions. As a consequence, the TDU is weaker and the temperature at the bottom of the convective zone powered by TPs is lower (see Figure 12). In this case, the total mass cumulatively dredged up is just $6.13 \times 10^{-2} M_{\odot}$ (see Table 5).

The corresponding effects on the nucleosynthesis are discussed in Section 6.5.

5. CHANGING THE MASS LOSS AND THE MIXING-LENGTH EFFICIENCY

In this section, we will show how the present theoretical scenario is affected by the assumed treatment of hydrodynamical phenomena, not explicitly included in the hydrostatic equations, such as mass loss and convection, whose efficiency in the models relies on some free parameters. For this purpose, we have calculated two additional evolutionary sequences, both at $Z = 0.0001$. In the first one we have adopted a classical Reimers’ formula ($\eta = 0.4$), up to the tip of the AGB, instead of that based on the calibrated mass loss–period–luminosity relation (see Section 2). The second evolutionary sequence has been obtained by changing the value of the mixing-length parameter ($\alpha = \Lambda/H_F$, where $\Lambda$ is the mixing length) from 2.15, as required by the standard solar model (see Piersanti et al. 2007), to 1.8. The evolutions of $T_{\text{MAX}}$ and $\Delta M_{\text{TDU}}$ for these two additional models are compared with the reference model in Figure 12. Either the mass loss rate and the mixing-length parameter are usually calibrated on stars with solar or nearly solar composition and little is known about the validity of these calibrations at low metallicities; for a discussion on the possible calibration of the mixing length at different $Z$ see Chieffi et al. (1995) and Ferraro et al. (2006). The early TP-AGB evolution of the Reimers’ model, which is mainly controlled by the growth of the H-exhausted core, is similar to that obtained in the case of the reference model. However, when the envelope erosion becomes important, the two sequences depart from each other. In particular, as a consequence of the higher mass loss rate, the reference model terminates sooner. As a result, the Reimers’ model experiences a larger number of TPs and TDU episodes. The total mass cumulatively dredged up is $1.6 \times 10^{-2} M_{\odot}$ in the reference model (see Table 4).

The main effect of reducing the mixing-length parameter is a smaller extension of the convective regions. As a consequence, the TDU is weaker and the temperature at the bottom of the convective zone powered by TPs is lower (see Figure 12). In this case, the total mass cumulatively dredged up is just $6.13 \times 10^{-2} M_{\odot}$ (see Table 5).

The corresponding effects on the nucleosynthesis are discussed in Section 6.5.

6. NUCLEOSYNTHESIS IN THE He INTERSHELL

Let us start with the solar metallicity model. As noted in Section 4, owing to the low temperature within the convective zones generated by the various TPs, the $^{22}\text{Ne}(\alpha, n)^{25}\text{Mg}$ reaction plays a marginal role as a neutron source at high metallicities, so that the most important nucleosynthesis site is the transition region between the core and the envelope, where the $^{13}\text{C}$ pocket forms. In Figure 13 we report the mass fractions of selected isotopes in this zone, after the formation of the third pocket.
Actually, three different “pockets” can be distinguished. The most internal one is the $^{13}$C pocket (solid line), which is partially overlapped to a $^{14}$N pocket (long dashed line). Note that the relevant $s$-process nucleosynthesis occurs in the left (more internal) tail of the $^{13}$C pocket, where the amount of $^{14}$N, the major neutron poison, is low. The third and smallest pocket is made of $^{23}$Na (short-long-dashed line). The latter forms where the abundance of $^{22}$Ne (short-dashed line) is comparable to the $^{12}$C abundance (dotted line), so that the $^{22}$Ne($p, \gamma$)$^{23}$Na reaction competes with the $^{12}$C($p, \gamma$)$^{13}$N in the proton capture game. As firstly suggested by Goriely & Mowlavi (2000), this $^{23}$Na pocket may provide a significant contribution to the synthesis of sodium in AGB stars.

The most interesting result obtained by coupling stellar structure evolution and nucleosynthesis concerns the variations of the size of the three subpockets when the star climbs the asymptotic giant branch. Figures 14 and 15 show, for the solar metallicity model, the physical conditions in the core–envelope transition region at the time of the maximum penetration of the 3rd and 11th (i.e., the last) TDUs, respectively. The solid and dashed lines represent the hydrogen and the $^{12}$C profiles, respectively. The abundance curves in both figures have been shifted upward in order to match the pressure scale axis. The slanting dashed area shows the region of the fully convective envelope (as defined by the Schwarzschild criterion), whilst the horizontal and vertical dashed areas mark the regions where the $^{12}$C and $^{23}$Na pockets will form later (the $^{14}$N pocket, which overlaps with both the aforementioned pockets, is not reported in the plot for graphical reasons). Pressure and pressure scale height are also shown. Note that the pressure jump between the dense core and the loose envelope is definitely steeper in the plot for graphical reasons. Pressure and pressure scale height are also shown. Note that the pressure jump between the dense core and the loose envelope.
a pressure difference of about 5 orders of magnitude. In both cases, external convection penetrates down to a layer where the pressure is about \(10^{11}\) dyne and \(H_P\) is about \(10^{10}\) cm, but due to the steeper pressure gradient the extension of the zone with a smoothed H profile is significantly reduced in the last TDU episode. In this case, the resulting pocket is definitely smaller. Table 6 lists the effective mass of \(^{13}\)C for all the pockets produced in the \(Z = 0.0138\) and \(Z = 0.0001\) models. In the table, we report (left to right) the mass of the H-exhausted core \((M_H)\), the mass of effective \(^{13}\)C in the pocket \((\Sigma^{13}\text{C}_{\text{eff}})\), and the mass extension of the pocket \((\Delta M)\). Note how the last pocket is more than 1 order of magnitude smaller than the first one.

The shrinkage of the \(^{13}\)C pocket implies a progressive decrease of the \(s\)-process efficiency as the star evolves along the AGB. For a long time, it has been assumed that the synthesis of the main component of the \(s\)-process would require the partial superposition of different neutron exposures: Clayton (1961) showed that an exponential distribution of these exposures could reproduce the observed \(\sigma_N\) curve (see also Seeger et al. 1965). Ulrich (1973) noted that if the \(s\)-process takes place at the base of the convective zone generated by a TP, the partial overlap of the recurrent convective zones provides, in a natural way, an exponential distribution of neutron exposures, thus reinforcing Clayton’s original suggestion. However, as argued by Gallino et al. (1998; see also Straniero et al. 1995; Arlandini et al. 1999), when the \(s\)-process occurs during the interpulse in the...
Note that the mass of the largest pocket is only $6 \times 10^{-4} M_\odot$. Within the 13C pocket, the scenario becomes more complex and cannot be described by a simple analytical function. Beside its relatively small thickness, the stratification of the 13C (and of the neutron poisons, as 14N) within the pocket leads to stratified neutron irradiations with different intensities. In addition to that, owing to the variation of the overall amount of 13C available in the pocket, as we find in self-consistent evolutionary models of AGB stars, this stratification changes pulse by pulse. In particular, the first pockets, the largest ones, give a major contribution to the overall AGB nucleosynthesis compared with that of the smallest late pockets. This is illustrated in Figure 16, where we report, for the solar metallicity model, the production factors, within the He intershell at the time of the 1st, 3rd, 6th, and 11th TDU episodes, of nuclei whose synthesis is mainly ascribed to the s-process (namely, nuclei whose s-process contribution is larger than 80%).

The s-process nucleosynthesis occurs in the radiative 13C pocket and leaves a thin layer highly enriched with heavy elements that is located in the middle of the He intershell. When the convection powered by the TP takes place, this material is spread out in the whole He intershell. In spite of the large dilution caused by the convective mixing, the s-element overabundances in the He intershell rapidly increase during the first TPs. A maximum is attained at the 6th TP. At that time, in the He intershell the abundances of the s-only nuclei are on the average a factor of 500 larger than the initial ones (700 for the ls element and 300 for the hs elements). Later on, the production factors of the various s-elements in the He intershell decrease. This is due to the shrinkage of the late pockets, which are no more able to compensate the convective dilution. Nevertheless, quite large overabundances are found in the He intershell up to the AGB tip, because of the partial overlap of the recurrent convective zones generated by various TPs.

6.1. Evolution of the Surface Composition: the Main and the Strong s-Process Components

The variations of the surface composition after selected TDU episodes of the solar metallicity model are reported in Figure 17. Looking at the elements beyond Fe ($Z = 26$), the most abundant species are those corresponding to isotopes with particularly small neutron-capture cross sections. In particular, the peaks corresponding to magic neutron numbers ($N = 50, 82$, and 126) clearly emerge from the bulk production of the s-process. In general, magic neutron nuclides act as bottlenecks of the s-process. At solar metallicity, owing to the relatively large amount of iron seed, compared with the number of neutrons released by the radiative 13C burning, the productions of the light, ls (Sr–Y–Zr), elements and the heavy, hs (Ba–La–Ce–Nd–Sm), elements, namely the first and second s-peaks, are favored with respect to the lead production (the third s-peak). At the end of the AGB evolution, the surface ratios [ls/ls] and [Pb/ls] are $-0.21$ and $-0.33$, respectively. These values are attained after a few TDU episodes, those following the first and largest 13C pockets. Later on, the overabundances continue to grow, as a consequence of the TDU, but these ratios remain constant.

The final surface composition of the five models with different metallicities is shown in Figure 18. Surface abundances after selected TDUs are shown in Tables 7–11. Only key elements are tabulated. Moreover, a selection of final elemental abundances, isotopic ratios, and spectroscopic indices are listed in Table 12. The 13C($\alpha$, n)16O reaction is primary-like (i.e., not directly affected by the metallicity of the pristine material), whilst the iron seeds scale with the metallicity. Thus, by lowering the metallicity, the number of neutron per seed nuclei progressively increases: this trend clearly results from Column 9 of Table 3. Within the 13C pockets, in fact, the average number $n_c$ of neutrons captured per initial iron-seed nucleus increases with the metallicity, starting from about 40 at solar metallicity up

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12 The production factor is defined as $N_j^*/N_j^\odot$, where $N_j^*$ is the abundance by number and $N_j^\odot$ is the corresponding solar scaled abundance.

13 Note that the mass of the largest pocket is only $6 \times 10^{-4} M_\odot$, while the convective zones attain about $3 \times 10^{-2} M_\odot$.

14 The complete set of these tables is available in the electronic version of this paper.
to 4000 at \( Z = 0.0001 \).\(^{15}\) Therefore, the bulk of the \( s \)-process nucleosynthesis first moves from light elements to heavy elements and, at low metallicities it shifts directly to \(^{208}\)Pb, at the termination point of the \( s \) path. This behavior is well reproduced by our models (see Figure 18). A second important change occurring at low \( Z \) concerns the second neutron source, the \(^{22}\)Ne(\( \alpha, n \))\(^{25}\)Mg. In low-metallicity models, owing to the larger temperature attained at the inner border of the convective zone generated by the TP, a second neutron burst takes places, causing interesting

\(^{15}\) Note that \( n_c \) involves neutron captures on both light and heavy elements. Neutron captures on light elements act as neutron poisons, thus decreasing the number of neutrons available for the \( s \)-process nucleosynthesis. The major neutron poison, represented by the \(^{14}\)N(\( n, \alpha \))\(^{11}\)C reaction, has already been considered in the estimate of \( n_c \).
The nucleosynthesis in the 13C pocket, since the maximum temperature in the convective shell generated by a TP increases pulse by pulse (see Figure 11), it is in the late part of the AGB that the major effects induced by the second neutron burst become important. Among them, we recall the overproduction of some neutron-rich isotopes. Owing to the larger neutron density ($10^{11}$ cm$^{-3}$ instead of $10^7$ obtained in the case of the radiative 13C pocket), several interesting branchings along the $s$-process path are activated. Thus, neutron-rich isotopes, whose production is otherwise prevented by $\beta$ decays of lighter isotopes, can be produced. Table 13 reports some characteristic isotopic ratios sensitive to the neutron density. The tabulated ratios relate to the branchings occurring at $^{85}$Kr, $^{86}$Rb, $^{95}$Zr, $^{133}$Xe, and $^{141}$Ce, respectively. As a comparison, solar ratios are also reported. Note how they increase when the metallicity decreases. In particular, the $^{87}$Rb is underproduced at large metallicity with respect to the lighter $^{85}$Rb, whereas it is overproduced at low Z. This is a direct consequence of the opening of the $^{85}$Kr and $^{86}$Rb branchings, taking place when the neutron density exceeds $10^8$-$10^9$ neutron cm$^{-3}$ (see Malaney & Lambert 1988). Note that, since $^{87}$Rb is a magic neutron nucleus, the overall production of Rb is significantly enhanced at low Z. As a result, the ratio of Rb and Is increases at low Z. For example, log(Rb/Sr) is 0.33, 0.42, 0.53, 0.77, and 0.85 at $Z = 0.0138$, 0.006, 0.003, 0.001, and 0.0001, respectively. In Figure 19, we report the maximum neutron densities attained in the models at various metallicities. As stressed before, the lower the metallicity is, the higher the neutron density attained during the TP is.

### 6.2. Evolution of the Surface Composition: from C to Fe

The most striking consequence of the TDU is the surface carbon enhancement. By reducing the metallicity, the maximum enhancement with respect to iron increases from [C/Fe] = 0.6 ($Z = 0.0138$ model) to [C/Fe] = 3 ($Z = 0.0001$ model). Obviously, this implies that the C/O ratio increases with decreasing the metallicity, passing from 1.87 at solar metallicity to 53 at $Z = 0.0001$ (see Table 12). Our models do not show evidences for HBB, therefore at low metallicities we obtain large C/N ratios and large $^{12}$C/$^{13}$C isotopic ratios. Note that possible reduction of these ratios due to deepmixing processes (as the CBP) is not explicitly considered in our models. Small enhancements of nitrogen are found, due to the dredge-up of the thin region of incomplete H burning. Almost all the $^{14}$N left by the H burning in the He intershell is converted into $^{22}$Ne during the TP phase, through the chain $^{14}$N($\alpha, \gamma$)$^{18}$F($\beta^+$)$^{18}$O($\alpha, \gamma$)$^{22}$Ne. The $^{22}$Ne plays a fundamental role, particularly in metal-poor stars. It acts as a
neutron source, as a poison and, at very low Z, as a seed capable to counterbalance the scarcity of iron (see Gallino et al. 2006). Note that at low Z most of the $^{22}$Ne is primary at $Z = 0.0001$; in fact, the amount of carbon in the envelope largely exceeds the pristine C+N+O since the first TDU episode. As a result, at the end of the AGB the amount of $^{22}$Ne in the He intershell is comparable with that found in the same region of the solar metallicity model. Correspondingly, the surface abundance of neon is significantly enhanced (see Table 12). Despite the very low neutron-capture cross section, the large abundance of $^{22}$Ne allows the production of light isotopes such as Na, Mg, and Al. Sodium is further enhanced, because of the already mentioned presence of $^{18}$O and protons (see, e.g., Forestini et al. 1992). In the convective zone, protons are available in the radiative $^{13}$C pocket and during the TP. The $^{15}$N production is due to the $^{18}$O($p$, $\gamma$)$^{15}$N reaction; therefore, the $^{19}$F production requires the simultaneous presence of $^{18}$O and protons (see, e.g., Forestini et al. 1992). In the He intershell, protons are available in the radiative $^{13}$C pocket and during the TP (see, e.g., Lugaro et al. 2004), released by the $^{14}$N$(n, p)^{14}$C reaction: neutrons are therefore required for a consistent production of $^{15}$N. In the radiative $^{13}$C pocket neutrons are produced by the $^{15}$C$(\alpha, n)^{18}$O reaction, while $^{19}$O is synthesized by means of the $^{14}$C$(\alpha, \gamma)^{18}$F$(\beta^−\nu)^{19}$O chain.

Few neutrons are also available at the beginning of a TP: they come from the burning of the $^{13}$C left by the H-burning shell in the upper zone of the He intershell. This $^{13}$C remains unburned during the interpulse and is engulfed into the convective zone generated by the TP. The $^{15}$N produced by the $^{14}$N$(n, p)^{14}$C reaction: neutrons are therefore required for a consistent production of $^{15}$N. In the radiative $^{13}$C pocket neutrons are produced by the $^{15}$C$(\alpha, n)^{18}$O reaction, while $^{19}$O is synthesized by means of the $^{14}$C$(\alpha, \gamma)^{18}$F$(\beta^−\nu)^{19}$O chain.

Concerning O, it is only marginally affected by the internal nucleosynthesis at large Z. In low-metallicity models, the oxygen enhancements result larger ([O/Fe] = 1 at $Z = 0.0001$) because of the reduced initial $^{16}$O abundance. We recall that we have assumed a solar scaled initial composition, so that [O/Fe] is nearly 0 at the beginning of the AGB for all the models presented here. Thus, in order to compare theoretical O overabundances to those measured in C-enhanced stars belonging to the Galactic halo (CEMP), one has to add an initial overabundance of, at least, 0.4–0.5 dex.

Among the light elements, $^{19}$F is produced at all metallicities, with a significant production at the lowest one. Fluorine is synthesized by the $^{15}$N$(\alpha, \gamma)^{16}$O reaction in the convective zone generated by a TP. The $^{15}$N production is due to the $^{18}$O$(\alpha, \gamma)^{15}$N reaction; therefore, the $^{19}$F production requires the simultaneous presence of $^{18}$O and protons (see, e.g., Forestini et al. 1992). In the He intershell, protons are available in the radiative $^{13}$C pocket and during the TP (see, e.g., Lugaro et al. 2004), released by the $^{14}$N$(n, p)^{14}$C reaction: neutrons are therefore required for a consistent production of $^{15}$N. In the radiative $^{13}$C pocket neutrons are produced by the $^{15}$C$(\alpha, n)^{18}$O reaction, while $^{18}$O is synthesized by means of the $^{14}$C$(\alpha, \gamma)^{18}$F$(\beta^−\nu)^{19}$O chain.

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the convective shell. In summary, we have two main channels for the $^{15}$N production in the He intershell. At solar metallicity, the two sources equally contribute to the fluorine production; at low metallicities, the $^{15}$N accumulated in the radiative $^{13}$C pocket is the main source of fluorine, because the $^{13}$C left by the full CNO burning is rather low for the major part of the AGB lifetime (but see Section 6.5).

6.3. Yields

One of the aims of this paper is to provide a complete and uniform set of AGB yields, comprehensive of all chemical species, starting from hydrogen up to the Pb–Bi s-process ending point. Yields of selected isotopes at different metallicities are reported in Table 14: being the number of considered isotopes too large to provide it in a paper format, a constantly updated database of AGB yields is available on the Web\textsuperscript{16} and in the electronic version of this paper. According to Tinsley (1980), the yield is

$$M_r(k) = \int_0^{t(M)} [X(k) - X^0(k)] \frac{dM}{dt} dt,$$

where $dM/dt$ is the mass loss rate, while $X(k)$ and $X^0(k)$ stand for the current and the initial mass fraction of the isotope $k$, respectively. Yields are given in solar mass units. We recall that our calculation have been stopped after the last TDU episode, when the residual envelope mass is of the order of (0.2–0.5) $M_\odot$. Except for the natural decay of the eventually surviving unstable isotopes, the envelope composition is frozen after this moment. Then, in computing the yields, we assume that the star loses the whole residual envelope through a single mass loss episode.

6.4. Short-Lived Radioactive Isotopes

In this section, we discuss the synthesis of a few short-lived radioactive isotopes, namely, $^{26}$Al, $^{36}$Cl, $^{41}$Ca, $^{60}$Fe, $^{107}$Pd, and $^{205}$Pb. The present theoretical predictions for low-mass AGB stars may be used to interpret the evidence for the presence of these radioactive isotopes in the early solar system (ESS; see Wasserburg et al. 2006 for a review on short-lived radioactive from AGB stars) and in presolar dust grains (Zinner et al. 2006b). Moreover, $^{26}$Al and $^{60}$Fe are particularly important in the field of $\gamma$-ray astronomy (Diehl 2006): their detection from selected AGB sources could be possible in the next future by means of $\gamma$-rays instruments mounted on high-energy astronomical satellites. Let us start from these two short-lived isotopes.

The ground state of $^{26}$Al has a terrestrial half-life of 7.16 × 10$^5$ yr, which is comparable to the duration of the whole TP-AGB phase of a low-mass star. $^{26}$Al is produced in the H-burning shell by proton captures on $^{25}$Mg. Within the H-depleted (and $^{26}$Al-rich) region, we need to distinguish two zones: the upper one, which extends from the tip of the convective shell generated by a TP up to the H-shell, and the lower one, which is engulfed in the convective shell. In the former region $^{26}$Al survives and is dredged up to the surface, whilst in the latter one, owing to its large neutron-capture cross section, it is easily destroyed by neutron captures. This happens in radiative conditions within the

\textsuperscript{16} http://www.oa-teramo.inaf.it/osservatorio/personale/cristallo/data_online.html.
\[ Z_{1011} \text{ cm} \] generated by the TP and is rapidly burned at higher temperature. In the upper part of the He intershell (not engulfed in the \[ 13C \] pockets and, more importantly, within the convective TPs. The temperature developed in the pocket during the first TP \((Z_{107} \times 10^7 \text{ K})\) is about 10 times the value reported by Wasserburg et al. (2006), as obtained by means of a postprocess calculation, where the effect of the peculiar first \[ 13C \] pocket was not considered. Let us finally discuss the production of the short-lived radioactive isotopes \([^{36}\text{Cl},^{41}\text{Ca},^{107}\text{Pd},\text{and}^{205}\text{Pb}]\), whose traces have been found in the ESS and whose origin can be ascribed to an AGB star. Concerning \([^{41}\text{Ca}]\), an equilibrium value \(\left({^{41}\text{Ca}} / {^{40}\text{Ca}}\right) = 1.16E - 03\) is rapidly attained whenever a neutron source is activated: this value is inversely proportional to the ratio of the corresponding neutron-capture cross sections. \(107\text{Pd}\) and \(205\text{Pb}\) are mainly synthesized during the standard radiative \[ 13C \] burning at any metallicity. The variations of the total yields of these two short-lived nuclei with the metallicity mainly reflect the increase of the neutron exposure when the metallicity decreases (because the number of neutrons per iron seed increases). This is only partially counterbalanced by the larger dredged-up mass.

As a general remark, since the decay rates depend on the environmental conditions (temperature and electron density), the use of stellar models, which follow the temporal evolution of these conditions in detail and in all the layers where radioactive isotopes are stored after their production, is mandatory. Such a peak corresponding to the ingestion of the first \[ 13C \] pocket into the convective shell generated by the following TP (this fact occurring for the metal-rich models only). Although the effect of this anomalous convective \(13C\) burning on the overall s-process nucleosynthesis is negligible, some neutron-rich isotopes as, for example, \(60\text{Fe}\) preserve the signature of such a peculiar event. Note that the surface isotopic ratio \(60\text{Fe} / 56\text{Fe}\) we obtain in the solar metallicity model (namely \(4.0 \times 10^{-2}\)) is about 10 times higher than the value reported by Wasserburg et al. (2006), as obtained by means of a postprocess calculation, where the effect of the peculiar first \(13C\) pocket was not considered.

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As a general remark, since the decay rates depend on the environmental conditions (temperature and electron density), the use of stellar models, which follow the temporal evolution of these conditions in detail and in all the layers where radioactive isotopes are stored after their production, is mandatory. Such a
warning should be seriously considered, particularly, when these isotopes are engulfed in convective zones where the temperature and the density vary considerably from the bottom to the top. As an example, $^{205}$Pb has a rather long terrestrial half-life (1.5 × 10$^7$ yr), but at the temperature of the He intershell of an AGB star its half-life is several orders of magnitude shorter (Takahashi & Yokoi 1987). The faster decays occurring in radiative conditions during the time elapsed between the TP and the TDU (about 10$^3$ yr) significantly affect the resulting surface abundances of $^{205}$Pb (see also Mowlavi et al. 1998).

In Table 15, we list the final surface abundances (by mass fraction) of the aforementioned shot-lived radioactive isotopes (with the corresponding stable isotopes) at different metallicities.

### Table 15
Final Isotopic Surface Abundances (by Mass Fraction) of Selected Short-Lived Radioactive Isotopes (with the Corresponding Stable Isotopes) at Different $Z$

| Isotope | $Z = 1.38 \times 10^{-2}$ | $Z = 6.0 \times 10^{-3}$ | $Z = 3.0 \times 10^{-3}$ | $Z = 1.0 \times 10^{-3}$ | $Z = 1.0 \times 10^{-4}$ |
|---------|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|
| $^{205}$Al | 3.07E−07 | 1.41E−07 | 7.80E−08 | 7.37E−08 | 5.33E−08 |
| $^{32}$Al | 6.14E−05 | 2.77E−05 | 1.43E−05 | 6.43E−06 | 1.98E−06 |
| $^{35}$Cl | 3.72E−06 | 1.61E−06 | 7.97E−07 | 2.59E−07 | 2.78E−08 |
| $^{56}$Cl | 1.70E−09 | 1.51E−09 | 9.29E−10 | 5.35E−10 | 3.50E−11 |
| $^{40}$Ca | 6.26E−05 | 2.70E−05 | 1.34E−05 | 4.35E−06 | 4.78E−07 |
| $^{41}$Ca | 5.13E−09 | 3.52E−09 | 1.90E−09 | 1.11E−09 | 7.11E−11 |
| $^{56}$Fe | 1.20E−03 | 5.17E−04 | 2.56E−04 | 8.26E−05 | 8.35E−06 |
| $^{60}$Fe | 4.78E−08 | 3.93E−08 | 5.70E−08 | 1.38E−07 | 2.04E−08 |
| $^{204}$Pb | 2.42E−09 | 1.24E−08 | 1.58E−08 | 4.80E−09 | 1.50E−10 |
| $^{205}$Pb | 1.58E−10 | 1.04E−09 | 1.55E−09 | 4.61E−10 | 1.31E−11 |

### Table 16
Final Composition of the Three Models at $Z = 10^{-4}$ (Reference, Reimers, and Low Mixing Length)

| Element | Reference | Reimers | $\alpha = 1.8$ |
|---------|-----------|---------|---------------|
| [C/Fe]  | 3.00      | 3.20    | 2.81          |
| [N/Fe]  | 0.86      | 1.15    | 0.68          |
| [O/Fe]  | 1.01      | 1.17    | 0.88          |
| [F/Fe]  | 2.90      | 3.27    | 2.66          |
| [Ne/Fe] | 2.27      | 2.68    | 1.90          |
| [Na/Fe] | 1.77      | 2.26    | 1.36          |
| [Mg/Fe] | 1.30      | 1.82    | 0.84          |
| [Al/Fe] | 0.64      | 0.97    | 0.47          |
| [Si/Fe] | 1.10      | 1.30    | 0.85          |
| [S/Fe]  | 1.56      | 1.85    | 1.33          |
| [Pb/Fe] | 2.88      | 3.00    | 2.76          |
| [ls/ls] | 0.46      | 0.55    | 0.48          |
| C/O    | 53        | 52      | 46            |
| C/N    | 406       | 325     | 388           |
| $^{12}$C/$^{13}$C | $1.72 \times 10^4$ | $1.35 \times 10^4$ | $1.72 \times 10^4$ |

### Figure 20
Final elemental surface composition of the $Z = 0.0001$ reference model is compared with those of the two additional models obtained by adopting a different mass loss rate (Reimers $\eta = 0.4$) or by decreasing the mixing-length parameter ($\alpha_{\text{mix}} = 1.8$). See the text for details.

(A color version of this figure is available in the online journal.)

### 6.5. Changing the Mass Loss and the Mixing-Length Efficiency: Effects on the Nucleosynthesis

In Section 5, we have shown how the physical parameters of the AGB evolutions depend on the choices of the mass loss rate and the mixing-length efficiency. In this section, we discuss the corresponding effects on the nucleosynthesis. The results are illustrated in Figure 20 and Table 16.

The final overabundances (with respect to iron) of the Reimers model are generally larger than those found in the reference model. This is a consequence of the larger duration of the AGB, as obtained when the mass loss rate is lower, so that the total dredged-up mass is larger. On average, the abundances of the s-elements increase by a factor of 2, while for the light elements we found a variation in the range between +0.2 dex (carbon) and +0.5 dex (magnesium). Concerning the fluorine production, we note that the delayed end of the AGB phase favors the second fluorine source even in low-Z models (see Section 6.2). The primary $^{13}$C in the ashes of the H burning becomes, in the late TP-AGB phase of low-Z models, as large as the $^{13}$C found in the reference model with solar metallicity. As explained before, neutrons released by the burning of this $^{13}$C at the beginning of each TP provide an additional channel for the production of $^{15}$N. As a result, a longer AGB phase, as obtained by reducing the mass loss rate, favors this fluorine source, at any Z. Although the absolute abundances depend on the mass loss rate, the abundance ratios are less sensitive to the AGB duration. It occurs because the bulk of the s-process nucleosynthesis has to be ascribed to the first $^{15}$C pockets, the largest ones, so that a freezeout of the abundance ratios takes place after very few TPs in the He intershell material (see Section 6.1). For this reason, the [ls/ls] and [Pb/ls] obtained in the case of the Reimers model are very similar to those of the reference model. Interestingly, the final C/N and $^{12}$C/$^{13}$C isotopic ratios of the Reimers model decrease with respect to the reference one (where monotonic trends are found), even if HBB is not at work. This different behavior, which appears in the late AGB phase, is due to an increase with the core mass of the temperature at the bottom...
of the convective envelope during TDU episodes, which leads to a partial H-burning. During these phases, therefore, mixing and burning simultaneously occur. This phenomenon, already found by Goriely & Siess (2004) in a model with initial mass $M = 3M_\odot$ and $Z = 0.0001$, enhances the $^{13}$C and the $^{14}$N in the envelope more rapidly than $^{12}$C (which, in any case, increases after each TDU episode). When applying a velocity profile at the base of the convective envelope, in fact, protons are mixed to higher temperatures with respect to those attained when using the bare Schwarzschild criterion. A more detailed analysis of this phenomenon, based on models with different initial masses, will be presented in a forthcoming paper.

Finally, we evaluated the effects of varying the mixing efficiency by reducing the mixing-length parameter (case $\alpha = 1.8$). In that case, the lower cumulative dredged-up mass leads to smaller overabundances. As in the Reimers model, the elemental ratios, which are sensitive to the metallicity and to the $^{13}$C mass in the pocket, are less affected by the structural model change.

### 7. CONCLUSION

This paper reports the first systematic calculation of low-mass AGB models at different metallicities, in which the physical evolution of the star is coupled to a full nuclear network, from H to Bi. The major input physics, such as nuclear reaction rates, radiative opacity, and mass loss rate, have been revised, in order to provide a reliable set of theoretical stellar yields.

The [hs/ls] and the [Pb/hs] ratios at different Z provide important hints about the nucleosynthesis occurring at different metallicities and represent useful tools to verify the goodness of our theoretical models, when compared with observational data. As shown in Table 12, the [hs/ls] is rather low at large Z, because the low number of neutron per seed limits the production of heavy s-elements. When decreasing the metallicity, this ratio increases, achieving a sort of saturation for $Z < 0.001$. Below this threshold, the scarcity of seeds (essentially Fe) is partially compensated by a reduction of the mass of the $^{13}$C pocket (see below). [Pb/hs] provides a more sensitive spectroscopic index to test low-metallicity models. It is expected to monotonically increase from high to low Z, indeed. Let us stress that, since the [hs/ls] and [Pb/Fe] are practically frozen after a few dredge-up episodes, these indices are almost independent of the assumed mass loss rate and the efficiency of the TDU, which are two of the most uncertain quantities in AGB modeling. In practice, as noted by Busso et al. (2001); see also Gallino et al. 2008; Bisterzo et al. 2009), they essentially depend on the effective $^{13}$C mass within the $^{13}$C pockets. In this context, by comparing the theoretical predictions of these indices with spectroscopic ratios at different Z, we may have a direct check of the validity of our choice of the $\beta$ parameter.

In Figure 21 we have reported the observed [hs/ls] indices, as measured in a sample of Galactic and extragalactic C-stars with different metallicities (Abia et al. 2002, 2008; de Laverny et al. 2006). Spectroscopic and photometric studies indicate that these stars are intrinsic (N type) C-stars, i.e., low-mass AGB stars undergoing the TDU. The agreement with our theoretical predictions is comfortable. In particular, data confirm the increase of [hs/ls], when the metallicity is reduced below the solar value, up to a plateau attained at intermediate Z. Only the most metal-poor C(N) star of the sample, namely ALW-C7, a carbon star of the Carina dwarf galaxy, shows a too large [hs/ls] compared with the low-Z plateau. However, as suggested by Abia et al. (2008), this star shows a particularly low abundance of Zr compared with other light s-elements.

Moreover, the fitting process in determining the ls abundances in ALW-C7 resulted more difficult with respect to ALW-C6, making the determination of the [hs/ls] even more uncertain (C. Abia 2008, private communication).

A second interesting check for the new theoretical scenario concerns the comparison with nucleosynthesis models based on postprocesses calculations (Gallino et al. 2008; Bisterzo et al. 2009). In the postprocess calculations, the relevant stellar parameters are derived according to older stellar evolutionary models (Straniero et al. 1997) and, where models were not available, by using the interpolation formulae provided by Straniero et al. (2003). Since these models have been computed by assuming a Reimers mass loss rate, the duration of the AGB, and, in turn, the total mass of H-depleted and s-enriched material dredged-up, is larger than that found in the present computations. Although this difference affects the predicted overabundance of a single element, it has negligible effects on the [hs/ls] or the [Pb/hs] indices. A second important difference of the postprocess calculations concerns the mass of the $^{13}$C pocket, which is fixed to a constant value for the full AGB evolution; according to Gallino et al. (1998), the value of the $^{13}$C mass is a free parameter of the nucleosynthesis model and the ST (standard) case corresponds to $\Sigma^{13}\text{C}_{\text{eff}} = 4 \times 10^{-6}M_\odot$ (see the discussion in Section 3). The third important difference concerns the chemical profiles of $^{13}$C and $^{14}$N within the pocket (for the postprocess calculation see Figure 1 of Gallino et al. 1998). In Figure 22, the various lines represent the results of the postprocess calculations (Gallino et al. 2008; Bisterzo et al. 2009), whilst the filled squares are our new predictions. In the plot, ST refers to the standard case, whilst the cases labeled with $ST \times k$ or $ST/k$ correspond to $^{13}$C pockets whose mass is $k$ and $1/k$ times the standard one, respectively. At large metallicities, our predictions for [hs/ls] and [Pb/hs] are in good agreement with those of the ST case (dotted lines). When the metallicity is reduced, however, the two spectroscopic indices resulting from the new calculation move progressively toward the lines corresponding to smaller $^{13}$C masses. This behavior is likely due to the smaller $^{13}$C pockets we found at low Z (see Figure 8 and Table 6), whose effect is to limit the maximum neutron exposure.

Let us conclude with a comment on the question of the spread of the $^{13}$C pocket. As discussed by Bisterzo et al. (2009), a certain spread is required to explain the variety of [hs/ls] observed in the available sample of carbon-enhanced metal-poor and s-rich stars (CEMPs), whose overabundance...
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Figure 22. Spectroscopic indices predicted by our new models (filled squares) compared with postprocess calculations (Gallino et al. 2008; Bisterzo et al. 2009). In panel (a) we report the [hs/ls] index, whilst in panel (b) we plot the [Pb/ls] index. See the text for details.
