Canonical quantization of Chern-Simons on the light-front

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Abstract

By performing the canonical quantization of the Abelian Chern-Simons model on the light-front (as suggested by Dirac), we clarify some controversies appearing in recent papers that discuss the relation between the existence of excitations carrying fractional spin and statistics (anyons) and this model. Properties of the Chern-Simons model on the light-front are investigated in detail, following the Dirac method for constrained dynamical systems, both for a coupled complex scalar field as well as for a spinor field.

1 Introduction

Dirac [1], in 1949, noticed that it is possible to quantize dynamical systems on any space-like surface, in particular a plane wave front moving with the speed of light, defined by the condition $x^0 + x^3 = \text{const}$. For simplicity we shall call that method light-front quantization (LF), in contrast to the usual equal-time quantization (also referred to as instant form).

LF has some advantages in that seven out of the ten Poincaré generators (in 4 dimensions) are kinematical, while in the instant form only six have this property. It should also be noticed that non-locality with respect to the longitudinal coordinate $x^-$ is expected in the LF, because it happens that the commutators are non-vanishing everywhere on
the light-like lines of the light-front hyperplane \[ \mathcal{H} \]. In addition, LF always gives rise to constrained Lagrangians, demanding the use of the well-known Dirac procedure in order to construct the Hamiltonian.

In 1966, Weinberg [3] obtained Feynman rules in the infinite momentum frame. In 1970, Kogut and Soper [4] proved that those rules correspond to LF quantization. Even before \[ \mathcal{H} \], making \( p \to \infty \) was used to derive current algebra sum rules, and it was noticed that this is the same as using some LF commutators of the currents.

Recently, interest in LF quantization has been renewed \[ \mathcal{H} \] due to the difficulties in the calculation of non-perturbative effects in the instant form QCD. It occurs that, due to infrared slavery, the QCD vacuum contains gluonic and fermionic condensates. On the LF, one obtains a simpler vacuum, often coincident with the perturbative vacuum. This is so because, for a massive particle on the mass shell, its LF momentum components, \( k^\pm \), are positive definite, thus not allowing the excitation of these degrees of freedom on the LF vacuum, in view of conservation of total longitudinal momentum.

In the context of string theory LF has been used, for example, in the case of heterotic string \[ \mathcal{H} \]. Recently, LF has also been used to treat the chiral Schwinger model \[ \mathcal{H} \].

On the other hand, Chern-Simons models \[ \mathcal{H}, \mathcal{H} \] have been used to treat planar condensed matter physics systems, for instance, low temperature superconductivity and quantum Hall effect \[ \mathcal{H} \].

In this paper, we apply LF quantization to Abelian Chern-Simons model in 2+1 dimensions \[ \mathcal{H} \], coupled to scalar or spinor fields. As already pointed out, we make use of Dirac procedure to treat the resulting constrained systems. Our main goal is to clarify the relation between Chern-Simons systems and the existence, due to commutation of rotations and no \textit{a priori} quantization of angular momentum, of excitations carrying fractional spin and statistics (anyons) \[ \mathcal{H}, \mathcal{H} \]. In section 2, we treat Chern-Simons coupled to a complex scalar field. In section 3, we treat coupling to a fermionic field. In section 4, we compare both and discuss our results.

## 2 Chern-Simons coupled to scalar fields

We begin by defining the LF coordinates (in 2+1 dimensions)

\[
x^\pm = \frac{x^0 \pm x^2}{\sqrt{2}} = x_\pm.
\]  

We shall adopt \( x^+ \equiv \tau \) as our \textit{time} coordinate, while \( x^- \) is our spatial longitudinal component. The remaining component \( x^1 \) is the spatial transverse component also referred to as \( x^\perp \). Therefore the greek indices run through \(+, -, 1\) while the latin indices take the
values $-1$. The LF coordinates are clearly not related to the usual ones by any Lorentz transformation.

The theory to be treated here is given by the following Lagrangian density:

$$\mathcal{L} = (\mathcal{D}^\mu \phi)(\overline{\mathcal{D}}^\mu \phi^*) + ae^{\mu \nu \rho} A_\mu \partial_\nu A_\rho, \quad (2)$$

where $a$ is a constant, $\phi$ is a complex scalar minimally coupled to the Abelian gauge field $A_\mu$ by means of the covariant derivative

$$\mathcal{D}_\mu = (\partial_\mu + iA_\mu) \quad (3)$$

$$\overline{\mathcal{D}}_\mu = (\partial_\mu - iA_\mu). \quad (4)$$

The gauge-invariant Noether current

$$j^\mu = ie(\phi^* \mathcal{D}_\mu \phi - \phi \overline{\mathcal{D}}^\mu \phi^*) \quad (5)$$

is conserved.

The field equations are easily obtained and read as below

$$\partial_+ \partial_- \phi = \frac{1}{2} \mathcal{D}_1 \mathcal{D}_1 \phi - iA_+ \partial_- \phi - \frac{i}{2}(\partial_- A_+)\phi \quad (6)$$

$$\partial_+ \partial_- \phi^* = \frac{1}{2} \overline{\mathcal{D}}_1 \overline{\mathcal{D}}_1 \phi^* + iA_+ \partial_- \phi^* + \frac{i}{2}(\partial_- A_+)\phi^* \quad (7)$$

$$2a \partial_- A_1 = j^+ = i(\phi^* \partial_- \phi - \phi \partial_- \phi^*) \quad (8)$$

$$2a(\partial_1 A_+ - \partial_+ A_1) = j^- = i(\phi^* \mathcal{D}_1 \phi - \phi \overline{\mathcal{D}}_1 \phi^*) \quad (9)$$

$$-2a \partial_- A_+ = j^1 = -i(\phi^* \mathcal{D}_1 \phi - \phi \overline{\mathcal{D}}_1 \phi^*), \quad (10)$$

after we choose the gauge $A_- \approx 0$.

The $\phi$-field satisfy null boundary conditions at spatial infinity $(x^-, x^1)$, while the gauge field satisfy anti-periodic boundary conditions at infinite $x^-$ and null at infinite $x^1$. The anti-periodic boundary conditions are in order to allow non-zero electric charge:

$$Q = \int d^2x \, j^+ = 2a \int dx^1 [A_1(x^- = \infty, x^1) - A_1(x^- = -\infty, x^1)]. \quad (11)$$

By calculating the canonically conjugate momenta, $\pi$, $\pi^*$ and $\pi^\mu$, we conclude that this is a constrained system, and the constraints are given by

$$\pi^+ \approx 0 \quad (12)$$

$$\chi^i \equiv \pi^i - ae^+ i j A_j \approx 0 \quad (i, j = -1) \quad (13)$$

$$\chi \equiv \pi - \overline{\mathcal{D}}_+ \phi^* \approx 0 \quad (14)$$

$$\chi^* \equiv \pi^* - \mathcal{D}_- \phi \approx 0. \quad (15)$$

3
The canonical Hamiltonian is obtained in the usual way (after integrating by parts):

\[
H_c = \int d^2x \left[ (\mathcal{D}_1\phi)(\bar{\mathcal{D}}_1\phi^*) - A_+\Omega \right],
\]

where

\[
\Omega = i(\pi\phi - \pi^*\phi^*) + a\epsilon^{ij}\partial_iA_j + \partial_i\pi^i.
\]

Now, we apply the Dirac procedure. Demanding the persistence in time of the constraints, we obtain \(\Omega \approx 0\). So, we end up with two first class constraints, \(\pi^+ \approx 0\) and \(\Omega \approx 0\), which generate the gauge transformations. The others are second class. The pair \(A_+, \pi^+\) decouples, so we only have to deal with \(\Omega\). We consistently add the constraint \(A_- \approx 0\) (thus justifying our choice on the field equations).

We are ready to define the Dirac brackets,

\[
\{f, g\}_D = \{f, g\} - \int d^2u \, d^2v \, \{f, T_m(u)\} \, C^{-1}_{mn}(u, v) \, \{T_n(v), g\},
\]

where \(T_m\) stands for one of the constraints \(\chi^i, \chi, \chi^*, A_-\) and \(\Omega\), and \(C_{mn}(x, y) = \{T_m(x), T_n(y)\}\). The constraint matrix is inverted and we obtain for \(C^{-1}_{mn}(x, y)\)

\[
\begin{pmatrix}
0 & -4a\partial_x^- & 0 & 0 & 0 \\
4a\partial_x^- & [\phi^*(x)\phi(y) + \phi(x)\phi^*(y)] & 2ai\phi(x) & -2ai\phi^*(x) & 0 & -4a \\
0 & 2ai\phi(y) & 0 & (2a)^2 & 0 & 0 \\
0 & -2ai\phi^*(y) & (2a)^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2(2a)^2 & 0 \\
0 & -4a & 0 & 0 & 2(2a)^2 & 0
\end{pmatrix}
K(x - y) \frac{1}{(2a)^2},
\]

where

\[
K(x - y) = \frac{1}{4} \epsilon(x^- - y^-)\delta(x^1 - y^1),
\]

being \(\epsilon(x)\) the signal function. This function is antiperiodic at infinite \(x^-\) as long as the above discussed boundary conditions are concerned. The constraint \(\Omega \approx 0\) gives, in accordance with the field equations,

\[
A_1 = \frac{1}{4a} \int d^2y \epsilon(x^- - y^-)\delta(x^1 - y^1)j^+(y),
\]

and the same boundary condition considerations apply. With \(A_-\) and \(A_+\) eliminated in view of the constraints, this leaves only \(\phi\) and \(\phi^*\) as independent fields.

By eliminating the constraints, we obtain self-consistently the LF Hamiltonian

\[
H^{l.f.} = \int d^2x (\mathcal{D}_1\phi)(\bar{\mathcal{D}}_1\phi^*).
\]
This Hamiltonian has non-local features as discussed in the Introduction. We also obtain easily:

\[
\{\phi, \phi^*\}_D = \{\phi^*, \phi\}_D = 0
\]

(23)

\[
\{\phi, \phi^*\}_D = \{\phi^*, \phi\}_D = K(x - y)
\]

(24)

\[
\{\pi, \phi\}_D = \{\pi^*, \phi^*\}_D = -\frac{1}{2} \delta^2(x - y)
\]

(25)

\[
\{\pi, A_1\}_D = -\frac{i}{4a} \left[ -4\pi(x)K(x - y) + \phi^* \delta^2(x - y) \right]
\]

(26)

\[
\{\phi, A_1\}_D = \frac{i}{2a} \left[ \phi(y) - 2\phi(x) \right] K(x - y)
\]

(27)

\[
\{A_1, A_1\}_D = \frac{1}{(2a)^2} \left[ \phi(x) \phi^*(y) + \phi^*(x) \phi(y) \right] K(x - y).
\]

(28)

When checking the self-consistency, we try to recover the field equations using this brackets. So, we are led to define

\[
A_+ \equiv -\frac{1}{4a} \int d^2 y \, \epsilon(x^- - y^-) \delta(x^1 - y^1) j^1(y),
\]

(29)

that is identical to \( A_+ \). This means that, although decoupled throughout the Dirac procedure, this component must obey the above definition. At the end, we recover the field equations, having done \( A_- = 0 \) as a gauge condition, either in Lagrangian or Hamiltonian formalism.

We are ready to outline the construction of the Fock states. The Dirac bracket are promoted to commutators,

\[
[\phi, \phi^*] = iK(x - y)
\]

(30)

\[
[\phi, \phi] = 0.
\]

(31)

In order to satisfy them, we use the momentum space expansions

\[
\phi = \frac{1}{2\pi} \int d^2 k \frac{\theta(k^+)}{\sqrt{2k^+}} \left[ a(k^+, k^1; \tau) e^{-i(k^+ x^- - k^1 x^1)} + b^\dagger(k^+, k^1; \tau) e^{i(k^+ x^- - k^1 x^1)} \right]
\]

(32)

\[
\phi^* = \frac{1}{2\pi} \int d^2 k \frac{\theta(k^+)}{\sqrt{2k^+}} \left[ a^\dagger(k^+, k^1; \tau) e^{i(k^+ x^- - k^1 x^1)} + b(k^+, k^1; \tau) e^{-i(k^+ x^- - k^1 x^1)} \right],
\]

(33)

where \( d^2 k = dk^+ dk^- \) and \( \theta \) is the Heaviside step function, and with

\[
[a(k), a^\dagger(k')]_{\tau = \tau'} = [b(k), b^\dagger(k')]_{\tau = \tau'} = \delta^2(k - k') = \delta(k^+ - k'^+) \delta(k^1 - k^1),
\]

(34)
If we notice that $A_1$ is a pure gauge,

$$A_1 = \partial_1 \Lambda$$

$$\Lambda(x) = \frac{1}{8a} \int d^2y \, \epsilon(x^- - y^-) \epsilon(x^1 - y^1) j^+(y),$$

then we may define

$$\hat{\phi} = e^{i\Lambda} \phi,$$

and rewrite the Hamiltonian as a free one in the field $\hat{\phi}$

$$H = \int d^2x (\partial_1 \hat{\phi})(\partial_1 \hat{\phi}^*).$$

The $\hat{\phi}$-field does not have a vanishing Dirac bracket with itself,

$$[\hat{\phi}(x), \hat{\phi}(y)] = e^{\alpha(x,y)} [e^{i\Lambda(x)}, e^{i\Lambda(y)}] \phi(y) \phi(x) +$$

$$+ \frac{1}{2} \left[ e^{i\Lambda(x)} e^{i\Lambda(y)} \phi(y^-, x^1) \phi(y)(1 - e^{\alpha(x,y)}) - (x \leftrightarrow y) \right]$$

where

$$\alpha(x, y) = -\frac{i}{8a} \epsilon(x^- - y^-) \epsilon(x^1 - y^1).$$

This is the equivalent on the LF of the equal-time graded commutation relations, exhibiting manifestly fractional statistics. Due to the boundary conditions, though, the phase factor $\Lambda$ is single-valued on the LF, while its equal-time equivalent’s expression includes an angle and therefore is multi-valued [12].

It has been shown [14] that anyonicity seems not to be related to rotational anomaly (the latter being a gauge artifact), but rather to the dynamics of the CS system, as given by the Hamiltonian and the Dirac brackets.

### 3 Chern-Simons coupled to spinor fields

We shall use the following representation of the gamma-matrices in 2+1 dimensions:

$$\gamma^0 = \sigma_1$$

$$\gamma^1 = i\sigma_3$$

$$\gamma^2 = i\sigma_2.$$

It will be useful to define the LF components of the gamma-matrices

$$\gamma^\pm \equiv \frac{1}{\sqrt{2}}(\gamma^0 \pm \gamma^2)$$
and the projectors
\[ \Lambda^\pm \equiv \frac{1}{\sqrt{2}} \gamma^0 \gamma^\pm. \] (45)

Our theory is now defined by (writing the spinorial indices explicitly for later convenience):
\[ \mathcal{L} = \frac{i}{2} \left[ \psi_\alpha^* (\gamma^0 \gamma^\mu)_{\alpha\beta} D_\mu \psi_\beta - \bar{D}_\mu \psi_\alpha^* (\gamma^0 \gamma^\mu)_{\alpha\beta} \psi_\beta \right] + a \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho, \] (46)
where the definition of the covariant derivative still holds.

The gauge-invariant conserved Noether current now reads
\[ j_\mu = \psi_\alpha^* (\gamma^0 \gamma^\mu)_{\alpha\beta} \psi_\beta. \] (47)

The field equations in the gauge \( A_- \approx 0 \) are
\[ \sqrt{2} \partial_- \psi_1 - i D_1 \psi_2 = 0 \] (48)
\[ \sqrt{2} D_+ \psi_2 + i D_1 \psi_1 = 0 \] (49)
\[ 2a(\partial_- A_1) = j^+ = \sqrt{2} \psi_2^* \psi_2 \] (50)
\[ 2a(\partial_1 A_+ - \partial_+ A_1) = j^- = \sqrt{2} \psi_1^* \psi_1 \] (51)
\[ -2a(\partial_- A_+) = j^1 = i(\psi_1 \psi_2^* - \psi_1^* \psi_2). \] (52)

Notice that the first field equation has no time derivatives, turning out to be a constraint in the Hamiltonian formalism. The fields obey boundary conditions similar to the former case.

We calculate the conjugate momenta, \( \pi_\alpha \) and \( \pi^\mu \), obtaining the constraints
\[ \chi_\alpha \equiv \pi_\alpha - \frac{i}{2} \psi_\beta^* (\gamma^0 \gamma^+)_{\beta\alpha} \approx 0 \] (53)
\[ \pi^+ \approx 0 \] (54)
\[ \chi^i \equiv \pi^i - a \epsilon^{+ij} A_j \approx 0. \] (55)
(Here \( \pi_1 \) is conjugate to \( \psi_1 \) and \( \pi^1 \) is conjugate to \( A_1 \)). The canonical Hamiltonian is
\[ H_c = \int d^2 x \left\{ \frac{i}{2} \left[ (\bar{D}_i \psi_\alpha^*) (\gamma^0 \gamma^i)_{\alpha\beta} \psi_\beta - \psi_\alpha^* (\gamma^0 \gamma^i)_{\alpha\beta} (D_i \psi_\beta) \right] - A_+ \Omega \right\}, \] (56)
where
\[ \Omega = i(\pi_\alpha \psi_\alpha - \pi^*_\alpha \psi^*_\alpha) + a \epsilon^{+ij} \partial_i A_j + \partial_i \pi^i. \] (57)

As before, \( \Omega \approx 0 \) is a secondary constraint. Further, the persistence in time of \( \chi^*_\alpha \) gives another secondary constraint, that we shall call \( \Sigma \approx 0. \Sigma \), as we expect, is exactly
the first of the field equations, as long as we set $A_- \approx 0$. This last choice is necessary, again, in order to render the matrix of the Poisson brackets of the constraints invertible.

So, we define the Dirac brackets, now with $T_m(x)$ being $\chi_1$, $\chi_2$, $\chi_1^*$, $\chi_2^*$, $\chi^+$, $\chi^1$, $\Sigma$, $\Sigma^*$, $A_-$ and $\Omega$. The resulting matrix is then inverted. We have developed a technique to simplify the inversion of the symmetric $10 \times 10$ matrix. It occurs that the matrix is sparse, so we can take advantage of this fact, by first determining which elements of the inverse are necessarily zero. By doing this, we lower from $55 \times 55$ to $10 \times 10$ the system to effectively determine, with the latter being already block diagonal.

As before, the pair $\pi^+$ and $A_+$ decouples. The constraint $\Omega = 0$ gives

$$A_1(x) = -\sqrt{2} a \int d^2 y K(x-y) \psi_2(y) \psi_2^*(y),$$

and now, in addition, $\Sigma = 0$ gives

$$\psi_1 = -i \sqrt{2} \int d^2 y K(x-y) D_1^y \psi_2(y),$$

leaving $\psi_2$ and $\psi_2^*$ as the only independent degrees of freedom.

The LF Hamiltonian ends up being

$$H^{l.f.} = \frac{1}{2} \int d^2 x [\psi_2^*(\partial_1 \psi_1) + (\partial_1 \psi_2^*) \psi_2 - 2a A_1 \partial_- A_+] ,$$

and we now calculate the following Dirac brackets:

$$\{ \psi_2, \psi_2 \}_D = 0$$

$$\{ \psi_2, \psi_2^* \}_D = -\frac{i}{\sqrt{2}} \delta^2(x-y)$$

$$\{ \psi_2, \pi_2 \}_D = \frac{1}{2} \delta^2(x-y)$$

$$\{ \psi_2, j^+ \}_D = -i \psi_2 \delta^2(x-y)$$

$$\{ \psi_2, A_1 \}_D = -\frac{i}{a} \psi_2(x) K(x-y)$$

$$\{ j^+, j^+ \}_D = 0$$

$$\{ A_1, A_1 \}_D = 0.$$

Now, the charge density commutes with itself, and eq. (62) yields a local commutator, in contrast with the case of the scalar field. We consistently recover the field equations in the Hamiltonian formalism.
We can also build up the Fock states here, satisfying
\[ [\psi_2, \psi_2^\dagger]_+ = \frac{1}{\sqrt{2}} \delta^2(x - y), \tag{68} \]
by using
\[ \psi_2 = \frac{1}{2^{2+2\pi}} \int d^2k \left[ a(k^+, k^1; \tau)e^{-i(k^+x^- - k^1x^1)} + b^\dagger(k^+, k^1; \tau)e^{i(k^+x^- - k^1x^1)} \right], \tag{69} \]
with
\[ [a(k), a^\dagger(k')]_{\tau = \tau'} = [b(k), b^\dagger(k')]_{\tau = \tau'} = \delta^2(k - k') = \delta(k^+ - k'^+)\delta(k^1 - k^1'). \tag{70} \]
The remainder of the construction follows as usual.

We can define a \( \hat{\psi}_2 \),
\[
\hat{\psi}_2 = e^{i\Lambda} \psi_2,
\]
\[
\Lambda(x) = \frac{1}{8\alpha} \int d^2y \epsilon(x^- - y^-)\epsilon(x^1 - y^1)j^+(y). \tag{72} \]

Now, if we use the brackets above, the equivalent of the graded anti-commutation relation of the equal time formulation reads in the LF
\[ [\hat{\psi}_2, \hat{\psi}_2^\dagger]_+ = 0. \tag{73} \]
This is also in contrast with the case of the scalar field.

4 Discussion and final remarks

We have treated an Abelian Chern-Simons model minimally coupled to a complex scalar field and a spinor field separately, using a non-covariant gauge, on the LF. Using the canonical approach and the Dirac procedure for the resulting constrained system, we have obtained a non-local Hamiltonian with sixth-order interactions, and showed the self-consistency of our treatment. The gauge field has been eliminated, leaving just the 2 matter degrees of freedom. We outlined the construction of the Fock states, in terms of creation and annihilation operators.

We have defined, in the terms of the equal-time formalism, a dual description, in which the matter fields are multivalued and the Hamiltonian has been claimed to be a free one. It happens that in the equal-time formalism this can not be so, because
having a multivalued field inside an integral would force us to choose a branch, in order to integrate, thus introducing a discontinuity and a delta-like interaction. On the other hand, on the LF, we do not have multivalued fields, in view of our boundary conditions. So, our Hamiltonian written in terms of the dual description field is indeed a free one. This dual description field is single-valued, but does not have usual statistics, at least in the case of the scalar field.

The spinor field has a local Dirac bracket, in spite of the fact that this was not necessary in view of the LF coordinates, and it also has a vanishing anti-commutator with itself. What remains to be interpreted is the meaning of the null anti-commutator on the LF.

The next natural step should be to analyse a model with a self-interaction potential, leading to vortex-like configurations. The above mentioned boundary conditions should play a central role in such a case.

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