The photometric method of mode identification for rapidly rotating SPB stars. 
An application to $\mu$ Eridani

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Abstract. We present an outline of a method for mode identification, which relies on a nonadiabatic version of the traditional approximation and involves instability and visibility considerations. Determination of the angular degrees, $(\ell, m)$, is done simultaneously with the rotation velocity and inclination angle. The method is applied to oscillation frequencies detected in the rapidly rotating SPB star $\mu$ Eridani.

1. Introduction

Slowly Pulsating B-type stars are main sequence variables of B3-B9 spectral types. In these stars high-order g modes are excited by the $\kappa$ mechanism operating in the metal opacity bump layer. In recent years these stars attracted a considerably attention among observers. Lots of photometry and spectroscopy data on modes detected in individual objects has been collected. These data, however, are still awaiting exploitation in the context stellar modeling.

Asteroseismic data on SPB stars are much harder to use than similar data on their neighbors in the H-R diagram, $\beta$ Cephei stars. In fact, the prospect for construction of a seismic model for any SPB star seems remote. Firstly, the frequency spectra of these pulsators are so dense that a unique identification of the radial order, $n$, of the individual modes is impossible. Secondly, the oscillation frequencies are often of the order of the angular rotation rate and simple equidistant pattern, that facilitate identification of angular numbers $(\ell, m)$, is not expected.

When oscillation and rotation frequencies are comparable, a more sophisticated formalism, than the perturbation theory, is needed to calculate frequencies and other observable mode characteristics, like relative light and radial velocity amplitudes. The easiest method to describe the low frequency modes in rotating stars employs the traditional approximation, which allows to express the angular dependence of surface distortion in terms of the Hough functions (e.g. Lee & Saio 1997, Townsend 2003a). The semi-analytical formula for the light variation due to low frequency g modes was derived for by Townsend (2003b).

Using nonadiabatic generalization of the traditional approximation, Daszyńska-Daszkiewicz et al. (2007) studied how rotation affects observable amplitudes of high-order g modes and mixed
gravity-Rossby modes. We showed that determination of angular numbers from diagnostic diagrams employing multicolour photometry and radial velocity data is possible. However, unlike in $\beta$ Cep stars, it must be done in conjunction with determination of the rotation rate and inclination angle. Constraints on these parameters are valuable.

If we have single passband data, certain constraints on mode identification come from instability and visibility conditions. We used this approach in our interpretation of the rich oscillation spectrum of rapidly rotating B type star HD163868, obtained from the MOST satellite (Dziembowski et al. 2007).

In this paper, we apply the method of mode identification described by Daszyńska-Daszkiewicz et al (2007) to the rapidly rotating SPB star $\mu$ Eridani, for which six oscillation modes were found from uvby Strömgren photometry data. Here, we present only our initial results. The full analysis will be published elsewhere.

In Section 2, we introduce briefly the hero of this paper, $\mu$ Eri. Section 3 presents an outline of the traditional approximation. In Section 4, we recall our expression for pulsational light variation in rotating star. The procedure for mode identification based on multicolor photometry is described in Section 5. Some numerical results and conclusions are given in Sections 6 and 7, respectively.

2. The star $\mu$ Eridani
$\mu$ Eri (HD 30211, $V = 4.0$ mag) is a fast rotating star of B5IV spectral type. Two values of $V_{\text{rot}}\sin i$ can be found in the literature: 190 km/s derived by Bernacca & Perinotto (1970) and, more recent value of 150 km/s, obtained by Abt, Levato & Grosso (2002). The star is the primary of a spectroscopic binary of the SB1 type (Frost et al 1926, Hill 1969). This is a well detached system with the orbital period $P_{\text{orb}} = 7.36$ d and eccentricity $e = 0.26$. Recently, Jerzykiewicz et al. (2005, hereafter J2005), found from photometric observations that the star is also an eclipsing binary. From the eclipse, which seems total, they deduced that the secondary is fainter than the primary by several magnitudes. From the same data, J2005 extracted six oscillation frequencies and classified $\mu$ Eri as a Slowly Pulsating B-type star.

2.1. Stellar parameters
Combining the mean Geneva photometric indexes (Rufener 1976) and the Hipparcos parallax, $\pi = 6.13 \pm 1.03$ mas, we obtained the following stellar parameters: $\log T_{\text{eff}} = 4.195 \pm 0.010$ and $\log L = 3.46 \pm 0.15$. These parameters were determined using Kurucz atmosphere models for the standard solar metallicity, $[\text{m/H}]=0.0$, and microturbulence velocity, $\xi_t = 2$ km/s. In luminosity, we took into account the Lutz-Kelker correction. In Fig. 1, we plot the location of $\mu$ Eri in the HR diagram. As one can see, the observational error box is mostly outside the main sequence. Nevertheless, we assume that the star is on the main sequence, because after hydrogen exhaustion, the evolution is very fast and, moreover, g mode oscillations are strongly damped. There are several effects which shift the TAMS to lower effective temperatures. One is the mean centrifugal force, which we include in our evolutionary calculations. The left panel of Fig. 1 shows the effect of the rotation velocity on the evolutionary tracks. The other is the metallicity increase. The effect of $Z$ value on evolutionary tracks is shown in the right panel of Fig. 1. The rightward shift of TAMS can be achieved also by including the convective core overshooting, but here we do not take this effect into account.

2.2. $\mu$ Eri oscillations
$\mu$ Eri was discovered as a Slowly Pulsating B-type star by J2005, who analyzed data from two multi-site photometric campaigns. The campaigns, organized in 2002-2004, were aimed at the $\beta$ Cep star $\nu$ Eri and $\mu$ Eri, which was one of two comparison stars, turned out variable.
Figure 1. The location of $\mu$ Eri in the HR diagram. The effects of rotation velocity and the metallicity on the evolutionary tracks are shown in the left and right panel, respectively.

J2005 extracted from $uvy$ Strömgren photometry six oscillation frequencies for $\mu$ Eri. The dominant frequency, $f_1 = 0.616$ c/d, was already found by Handler et al. (2004), who analyzed observations only from the first campaign, but authors were not convinced whether this was due to pulsation or rotational modulation. The values of six frequencies and corresponding periods, obtained by J2005, are: $\nu_1 = 0.61574$ c/d ($P_1 = 1.62406$ d), $\nu_2 = 0.70075$ c/d ($P_2 = 1.42703$ d), $\nu_3 = 0.81319$ c/d ($P_3 = 1.22972$ d), $\nu_4 = 1.20554$ c/d ($P_4 = 0.82951$ d), $\nu_5 = 0.65882$ c/d ($P_5 = 1.51786$ d), $\nu_6 = 0.56799$ c/d ($P_6 = 1.76059$ d).

The rotation frequency of $\mu$ Eri is of the order of oscillation frequencies, for example, taking $V_{\text{rot}} = 200$ km/s and $R = 6R_\odot$, we get value of about 0.65 c/d.

As one can notice, first three peaks are equidistant in period. However, they cannot correspond to consecutive overtones, as suggested by J2005, because the separation between peaks is too large. Thus, from the oscillation spectrum itself we cannot make any guess for mode identification.

3. Slow modes in the traditional approximation

This is a very convenient approximation to describe g modes in rotating stars, which demands the following assumptions:

- both pulsation and rotation frequencies are much smaller than the Brunt-Väisälä frequency, $(\omega \sim \Omega \ll N)$, which allows to drop out certain terms in pulsational equations arising from the Coriolis force,
- stellar rotation is much below the critical value, $\Omega/\Omega_{\text{crit}} \ll 1$, which implies that effects of centrifugal distortions may be neglected.

In computation, we use the Cowling approximation, which is very adequate because of short radial wavelength for g mode oscillations. In the framework of the traditional approximation, it is possible to separate the radial and the angular dependence of eigenfunctions. The latter dependence is given by the Hough functions, which are obtained from the Laplace tidal equation.
The most important property of the Hough functions is the equatorial confinement of the amplitudes growing with the increasing rotation velocity (see e.g. Bildsten et al. 1996, Lee & Saio 1997, Townsend 2003a).

The pulsation equations look very similar to those for nonrotating stars except that \( \ell(\ell+1) \) is replaced with the separation parameter \( \lambda \), which for specified mode angular degree and azimuthal order, \( m \), depends on the spin parameter \( s = 2\Omega/\omega \). The frequency measured in the inertial system is given by \( \omega_{\text{obs}} = \omega + m\Omega \). The mode degree, \( \ell \), for \( g \) modes is defined by the asymptotic relation \( \lambda \to \ell(\ell+1) \) at \( s \to 0 \). The mixed gravity-Rossby modes (\( r \) modes), which do not exist in non-rotating stars, occur only at \( m < 0 \). In radiative zones, they are propagatory (\( \lambda > 0 \)) at \( s > |m| + 1 \).

4. Light variations

To calculate the light variations, we have to consider changes in various atmospheric parameters. The radial displacement can be expressed as

\[ \xi_r = \varepsilon R \Theta^m_{\lambda}(\theta)Z, \]  

where \( \Theta^m_{\lambda}(\theta) \) is the Hough function describing the latitudinal dependence, \( Z = \exp[i(m\varphi - \omega t)] \) gives the azimuthal and temporal dependencies, and \( \varepsilon \) is the intrinsic mode amplitude.

Then, for the change of the normal to the stellar surface, we have

\[ \delta n_s = -\varepsilon \nabla_H(\Theta^m_{\lambda}(\theta)Z) = -\varepsilon \left( 0, \frac{\partial \Theta^m_{\lambda}(\theta)}{\partial \theta}, \frac{\partial \Theta^m_{\lambda}(\theta)}{\sin \theta} \right) Z, \]

and for the change of the directed element of the surface

\[ \frac{\delta dS}{dS} = \varepsilon \left( 2\Theta^m_{\lambda}(\theta), -\frac{\partial \Theta^m_{\lambda}(\theta)}{\partial \theta}, -\frac{im\Theta^m_{\lambda}(\theta)}{\sin \theta} \right) Z, \]

where \( dS = R^2 \mu d\varphi \). The perturbation of the bolometric flux is given by

\[ \frac{\delta F_{\text{bol}}}{F_{\text{bol}}} = \varepsilon f \Theta^m_{\lambda}(\theta)Z, \]

where \( f \) is the ratio of the flux perturbation to the radial displacement and it is a function of model parameters as well as of \( \omega \) and \( \lambda/\omega^2 \). Finally, the gravity variations, which are small, we write as

\[ \frac{\delta g}{g} = -\varepsilon \left( \omega^2 R^3/2GM + 2 \right) \Theta^m_{\lambda}(\theta)Z. \]

With the above expressions, the complex amplitude of the light variation in the \( k \) passband can be expressed as follows

\[ A_k(i) = D^k(i)f \tilde{\varepsilon} + \varepsilon^k(i) \tilde{\varepsilon}, \]

where

\[ D^k(i) = -1.086 \left( \frac{\alpha_k}{4} B_1 + B_3 \right), \]

\[ \varepsilon^k(i) = -1.086[2B_1 + B_2 - (2 + w^{-1})(\alpha_k B_1 + B_4)], \]

and \( \tilde{\varepsilon} = \varepsilon \exp(i m \varphi_0) \). The \( B_i \) coefficients are two-dimensional integrals over stellar disc, defined in Daszyńska-Daszkiewicz et al. (2007), which, for a given oscillation mode, depend on the inclination angle, rotation and the limb darkening.
The normalized growth rate, $\eta$, as a function of the frequency in the observer system, for the star model with mean parameters $M = 6.5 M_\odot$, log $T_{\text{eff}} = 4.199$, log $L/L_\odot = 3.369$ and metallicity $Z = 0.02$. Three values of a rotational velocity were considered: 150 (a), 200 (b) and 250 km/s (c). Individual peaks correspond to the observed frequencies of $\mu$ Eri and their height is related to the amplitude in the $y$ filter (the right linked Y axes).

**Figure 2.** The normalized growth rate, $\eta$, as a function of the frequency in the observer system, for the star model with mean parameters $M = 6.5 M_\odot$, log $T_{\text{eff}} = 4.199$, log $L/L_\odot = 3.369$ and metallicity $Z = 0.02$. Three values of a rotational velocity were considered: 150 (a), 200 (b) and 250 km/s (c). Individual peaks correspond to the observed frequencies of $\mu$ Eri and their height is related to the amplitude in the $y$ filter (the right linked Y axes).
5. Procedure for mode identification
5.1. Selection of pulsation modes
We measure the robustness of the mode excitation by the normalized growth rate (Stellingwerf 1978)
\[ \eta = \frac{W}{\int_0^R \left| \frac{dW}{dr} \right| dr}, \]
which changes from \(-1\) (full damping) to \(+1\) (full driving). We consider g modes with \(\ell \leq 4\) and mixed gravity-Rossby modes with \(m = -1, -2\). Those modes with \(\eta > 0\) and frequencies close to the observed values, \(\nu_{\text{cal}} \approx \nu_{\text{obs}}\), we regard as possible candidates for mode identification.

As an example, in Fig. 2 we depict the instability parameter, \(\eta\), as a function of the pulsation frequency in the observer system, \(\nu_{\text{obs}}\), for g modes with \(\ell = 1\) and Rossby r modes with \(m = -1\). We chose the star model with the following stellar parameters: \(M = 6.5 M_\odot\), \(\log T_{\text{eff}} = 4.199\), \(\log L/L_\odot = 3.369\) and metallicity \(Z = 0.02\). The instability domains are shown for three values of the rotation rate: 150, 200 and 250 km/s. The right linked Y axes correspond to the \(y\) amplitude of the observed frequencies.

5.2. Mode discriminant
The most popular tools of mode identification from multicolour photometry are amplitude ratios and phase differences. However, in this approach one filter is favored, which is chosen for normalization. Instead of using these observables, we propose an alternative discriminant which includes amplitudes and phases themselves.

For specified stellar model and pulsation mode, we find an optimal value of \(\varepsilon\) to fit photometric amplitudes and phases in all passbands simultaneously
\[ \frac{\partial \chi^2}{\partial \varepsilon} = 0. \]

With the above condition, after some algebra, we get
\[ \chi^2 = \frac{1}{2N - M} \left( \sum_{k=1}^{N} w_k |A_k|^2 - \frac{\sum_{k=1}^{N} w_k |A_k F_k|^2}{\sum_{k=1}^{N} w_k |F_k|^2} \right), \]
where \(A_k\) is the observed complex amplitude in the \(k\) passband and
\[ F_k(i) = D^k(i) f + E^k(i) \]
is calculated, for a given oscillation mode, with the input from linear nonadiabatic theory of stellar pulsation and models of stellar atmospheres. \(w_k\)'s are statistical weights given by the observational errors. The indexes \(N\) and \(M\) are the number of passbands and the number of unknowns to be determined, respectively. In our case \(N = 3\) (\(uvy\) photometry) and \(M = 2\) (complex \(\varepsilon\)).

The \(\chi^2\) minimization yields also a value of \(|\varepsilon|\), which can be used as condition for the mode visibility. In the case of SPB stars the observed photometric amplitudes do not exceed 0.1 mag and the theoretical values of \(F_k(i)\) are not larger than 10. Thus we assume the upper limit for the value of the intrinsic amplitude, \(|\varepsilon|\), as 0.01.

6. Results
Having amplitudes and phases in three Strömgren passbands for six pulsation frequencies of \(\mu\) Eri, we can make an attempt of mode identification, following the above described strategy. As an example, we illustrate results for the dominant frequency obtained for the star model.
Figure 3. The $\chi^2$ value as a function of inclination angle for the dominant oscillation frequency of $\mu$ Eri. The same stellar model as in Fig.3 was considered and a rotational velocity of 200 km/s was assumed.

Figure 4. A similar plot as Fig. 3 but for determined values of the intrinsic amplitude, $|\varepsilon|$, for the dominant frequency.

Table 1. Mode identification for the six observed frequencies in the SPB star $\mu$ Eri. The adopted stellar parameters were: $M = 6.5M_\odot$, $\log T_{\text{eff}} = 4.199$, $\log L/L_\odot = 3.369$, $Z = 0.02$ and a rotational velocity of $V_{\text{rot}} = 200$ km/s. In the fourth column we give determined values of the intrinsic amplitude. Quantities in the last column are explained in the text.

| frequency [c/d] | ($\ell, m$) | min$\chi^2$ | $|\varepsilon|$ | $A_{\text{rad}}^\text{cal}$ [km/s] |
|-----------------|------------|-------------|---------------|-----------------|
| $\nu_1 = 0.61574$ | (1, 0) | 0.78 | 0.0037 | 5.9 |
| | (3, −1) | 1.72 | 0.0113 | 6.3 |
| $\nu_2 = 0.70075$ | (2, 0) | 0.43 | 0.0026 | 3.0 |
| | (3, 0) | 1.28 | 0.0028 | 4.6 |
| | (3, −1) | 1.03 | 0.0081 | 3.1 |
| $\nu_3 = 0.81319$ | (4, 0) | 1.54 | 0.0085 | 13.1 |
| $\nu_4 = 1.20554$ | (2, +1) | 2.76 | 0.0010 | 2.1 |
| $\nu_5 = 0.65882$ | (1, 0) | 1.27 | 0.0018 | 2.5 |
| | (3, −1) | 1.45 | 0.0048 | 2.6 |
| $\nu_6 = 0.56799$ | (2, −1) | 1.05 | 0.0011 | 0.6 |
| | (4, −1) | 0.86 | 0.0013 | 1.0 |

with $M = 6.5M_\odot$, $\log T_{\text{eff}} = 4.199$, $\log L/L_\odot = 3.369$, $Z = 0.02$ and the rotational velocity of $V_{\text{rot}} = 200$ km/s. In Fig.3, the values of $\chi^2$ as a function of inclination angle are plotted for considered modes. Fig.4 shows corresponding intrinsic amplitudes which were determined simultaneously. The range of the inclination angles, which gives observed values of $V_{\text{rot}} \sin i = 150 − 190$ km/s, is $i \in (48^\circ, 72^\circ)$.

In Table 1 we present mode identification obtained for all six modes at $V_{\text{rot}} = 200$ km/s. In the third column we give min$\chi^2$ in the aspect range $i \in (48^\circ, 72^\circ)$. The last column contains the expected amplitude of radial velocity variations adopting determined values of $\varepsilon$ (listed in the
fourth column). For four frequencies there are more than one candidate for mode identification. 
In the case of $\nu_1$, only the $(\ell = 1, m = 0)$ mode is possible because the $(3, -1)$ pair gives large intrinsic amplitude ($|\varepsilon| > 0.01$).

7. Conclusions
Identification of angular numbers of pulsation modes in rapidly rotating stars is possible. It has to be done simultaneously with determination of the inclination angle and rotation velocity. In this procedure, we take into account instability and visibility considerations. The latter is given by the intrinsic amplitude obtained from the $\chi^2$ minimization.

There is some ambiguity in mode identification. Nevertheless, at least four frequencies detected in photometric variations of $\mu$ Eri can be explained by unstable low degree g modes. For better constraints, spectroscopic observations would be desirable.

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