Do Gravitational and Electromagnetic Fields Have Rest Masses in the Fractal Universe?

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As well known rest masses of elementary particles and physical fields appear when temperature of Universe become low enough and symmetry broke. Are there another sources of rest masses that are consequences of other nature of rest masses or it is the only methods for generating rest masses? What will be with massless fields when the laws of symmetry are not exact laws but only are very good approximation and the time is not homogeneous, as it is in the fractal world? In this paper based on the fractal theory of time and space (developed by author earlier) a possible source of rest masses caused by the fractional dimensions (FD) of the time is considered. It gives rest masses (very small) for all particles and fields including gravitational and electromagnetic fields. The estimation of the values of the rest masses gives $m_g = \sqrt{\varepsilon(t_0)}$, $m_f = \sqrt{(t_0t_1)}$ where $t_i$ is the necessary time for photons and gravitons with rest masses $m_{ph}$ and $m_g$ to get the velocity equal the speed of light (in the fractal Universe the moving with any velocities is possible), $t_0$ is the time of existence of Universe. Under some assumption preliminary values of rest masses photons and gravitons are obtained: for a rest mass of photons $m_f \sim 10^{-39} g$, for a rest mass of gravitons $m_g \sim 10^{-42} g$.

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I. INTRODUCTION

The problems of existence rest masses of electromagnetic, gravitational and neutrino fields are open till now. From one side the experiments can not discover rest masses of these fields, from other side there is unexplained question why some physical fields have rest masses, the other fields not have it. In early years it was wide-spread the opinion that the fields of the interactions (electromagnetic, gravitational an so on) between particles are massless and must not take a charges, but example of the electro-weak interactions by means of vector mesons broke this opinion and interactions of quarks broke the opinions about chargeless of interactions fields. Thus the situation is: there are the physical fields with rest masses and there are the physical fields without rest masses; there are the interactions fields with charges and without charges. There are the theorems (Goldstone and analogies) about massless boson fields as consequence of symmetry laws. In the space with the fractal time (see \[1\], \[3\]) not all the symmetry laws are fulfilled and these theorems work only as a good approach. Only in the case when the fractal dimension of time almost integer the symmetry laws are almost fulfill, not exactly, but as a good approach. So the existence in the fractal world rest masses of bosons field such as electromagnetic and gravitational do not contradicts any theorems true for world with exact symmetry laws and homogeneous time. In this paper shown that in the space with fractal time (dimensions of time $d_t = 1 + \varepsilon (r,t) \neq 1$) in the frame-work of the fractal theory of time and space \[1\], \[3\] all physical fields have rest masses originated by fractional additives $\varepsilon$ to integer time dimension. For the fields with rest masses in the integer dimensions time these additional masses are non-essential, but it gives the main masses for the massless early fields such as electromagnetic and gravitational fields. So the symmetry for all physical fields in respect to rest masses is restored: in fractal Universe all physical fields have rest masses: some fields have rest masses originated when temperature of Universe become low enough and symmetry laws is broken, other fields got rest masses by means of fractional dimensions of time (or space) from the time of beginning of the "big bang", when the time and space with fractal dimensions were originated.

II. HOW TO DESCRIBE THE FUNCTIONS WITH DEPENDENCIES AT TIME AND COORDINATES IN THE FRACTAL WORLD?

As the first step of describing the physical functions in the fractal Universe (the world with fractional dimensions of time and space) it is necessary to introduce the definitions of fractal time and fractal space (for the purpose of this paper it is enough to introduce only the fractal time) and mathematical methods its describing. Following \[1\], \[2\], \[4\], we will consider both time and space as the initial real material fields existing in the world and generating all other physical fields. Therefore, we introduce the integral functionals (both left-sided and right-sided) which are suitable to describe the dynamics of functions defined on multifractal sets of time and space (generalized fractional derivatives (GFD), see \[1\], \[2\], \[4\]) and replace by GFD the usual derivatives and integral respect to time.
and space coordinates in the fractional dimensions functional. These functionals GFD are simple and natural generalization of the Riemann-Liouville fractional derivatives and integrals:

\[ D^d_{\pm,t} f(t) = \left( \frac{d}{dt} \right)^n \int_a^t \frac{f(t')dt'}{\Gamma(n-d(t'))(t-t')^{d(t')-n+1}} \]  
(1)

\[ D^d_{\pm,t} f(t) = (-1)^n \left( \frac{d}{dt} \right)^n \int_t^b \frac{f(t')dt'}{\Gamma(n-d(t'))(t-t')^{d(t')-n+1}} \]  
(2)

where \( \Gamma(x) \) is Euler’s gamma function, and \( a \) and \( b \) are some constants from \([0, \infty)\). In these definitions, as usually, \( n = \{d\} + 1 \) where \( \{d\} \) is the integer part of \( d \) if \( d \geq 0 \) (i.e. \( n - 1 \leq d < n \)) and \( n = 0 \) for \( d < 0 \). If \( d = const \), the generalized fractional derivatives (GFD) \([\delta]-[\beta] \) coincide with the Riemann - Liouville fractional derivatives \((d \geq 0)\) or fractional integrals \((d < 0)\). When \( d = n + \varepsilon(t), \varepsilon(t) \to 0 \), GFD can be represented by means of integer derivatives and integrals. We pay attention that there are relations between GFD and ordinary derivatives for \( d_n \) near integer values. If \( d_n \to n \) where \( n \) is an integer number, for example \( d_n = 1 + \varepsilon(q(t), t), \alpha = r, t \), in that case it is possible represent GFD by approximate relations (see \([\delta]-[\beta] \))

\[ D^{1+\alpha}_{\pm,t} f(\varepsilon(t), t) = \frac{\partial}{\partial x_\alpha} f(\varepsilon(t), t) + \frac{\partial}{\partial x_\alpha} [\varepsilon(\varepsilon(t), t)f(\varepsilon(t), t)] \]  
(3)

For \( n = 1 \), that is, \( d = 1 + \varepsilon, |\varepsilon| << 1 \) it is possible to obtain:

\[ D^{1+\varepsilon}_{\pm,t} f(t) \approx \frac{\partial}{\partial t} f(t) + a \frac{\partial}{\partial t} [\varepsilon(\varepsilon(t), t)f(t)] \]  
(4)

where \( a \) is constant and defined by the choice of the rules of regularization of integrals \([\delta]-[\beta] \) (for more detailed see \([\delta]-[\beta] \)). The selection of the rule of regularization that gives a real additives for usual derivative in \([\delta]-[\beta] \) yield \( a = 0.5 \) for \( d < 1 \) \([\delta]-[\beta] \). The functions under integral sign in \([\delta]-[\beta] \) we consider as the generalized functions defined on the set of the finite functions \([\delta]-[\beta] \). The notions of GFD, similar to \([\delta]-[\beta] \), can also be defined and for the space variables \( r \). In the definitions of GFD \([\delta]-[\beta] \) the connections between fractal dimensions of time \( d_{\pm}(r(t), t) \) and main characteristics of physical fields (say, potentials \( \Phi_i(r(t), t), i = 1, 2, \ldots \) or densities of Lagrangians \( L_i \)) are determined. Following \([\delta]-[\beta] \), we define this connection by the relation

\[ d_i(r(t), t) = 1 + \sum_i \beta_i L_i(\Phi_i(r(t), t)) \]  
(5)

where \( L_i \) are densities of energy of physical fields, \( \beta_i \) are dimensional constants with physical dimension of \([L_i]^{-1} \) (it is worth to choose \( \beta_i' \) in the form \( \beta_i' = a^{-1} \beta_i \) for the sake of independence from regularization constant). The definition of time as the system of subsets and definition the FD \( d \) (see \([\delta]-[\beta] \) connects the value of fractional (fractal) dimension \( d_i(r(t), t) \) with each time instant \( t \). The latter depends both on time \( t \) and coordinates \( r \). If \( d_i = 1 \) (an absence of physical fields) the set of time has topological dimension equal to unity.

Thus, in the fractal Universe for changes of any physical functions describing it is necessary to use the GFD instead of ordinary derivatives and integrals. Only in the case when corrections to integer dimensions is small (it is the wide-spread case in our world) it is possible replace the GFD by ordinary derivatives using the relations \([\delta]-[\beta] \).

We stress now that GFD \( D_{\pm,t}^d \) take into account the influences of the past of the phenomena on the present time state of the phenomena. The GFD \( D_{\pm,t}^d \) take into account the influences of future times on the present time state of the phenomena. So the mathematical methods of GFD defined on the multifractal sets allow to take into account the influences of past and future on the state of system at the present time. In the fractal world describing by GFD there are no stable phenomena because the differentiation by means of GFD of constant value do not gives zero.

### III. THE FRACTAL UNIVERSE IS IN CONTINUAL CHANGING AND EXPANDING

The hypotheses of the ”big bang ” as the originate of our Universe gives continual changing and expanding of time and space, but the mathematical methods of ordinary differential and integral calculus based on usual mathematical analysis do not give possibility to describe this changing correctly. In particular, thou the time and space are continually expanding (in the frame of GR) from point of view of ordinary analysis in this expanding world exist constant physical values (for example a volume, an energy and so on). The fractal Universe is an open system (about open systems see \([\beta]-[\gamma] \) defined on the measure carrier (see \([\delta]-[\beta] \)). It is expanding, so all physical values in our world must not be constant. Thou its the changing is very small it has a principle role. The GFD gives such changing of all physical phenomena and values. That changes originate by fractional dimensions of time ( and the space) and allow to describe the changing concerned with Universe expanding. Let us see now how to calculate the GRD \( [\beta]-[\gamma] \) with respect to \( t \) at constant value \( a_0 = const \). In the fractal world there are no constant values because GFD of constant values do not equal zero

\[ D^{1+\varepsilon}_{\pm,t} a_0 = \frac{a_0 \varepsilon}{t^{1+\varepsilon} \Gamma(1+\varepsilon)} \approx \frac{a_0 \varepsilon}{t^{1+\varepsilon}} \]  
(6)

for large \( t \). So if our world has the fractal dimensions any values in this world including the constant values of our world may be expanded in infinite power series in \( t \). The
GFD $D_{d_t}^a$ of constant value $a_0$ ($d_t = 1 + \varepsilon (r, t), |\varepsilon| < < 1$) is equal

$$D_{d_t}^a a_0 \approx a_0 \frac{\varepsilon}{B - t}$$

(7)

In (5) the $B$ is the upper limit of integral in GFD. The time $t$ in (1) - (3) is the time of existence of Universe. It is useful for symmetry of (6) and (7) to choose $B$ to equal the time of existence of Universe.

In (8) the factor $\exp (ct - \varepsilon)$ describes the corrections given by the fractal dimensions of time. The ordinary differentiation of (8) with respect to $t$ coincide with result of GFD of $a$ with respect to $t$. If $d_a = 1$ the time dependence in (3) disappears ($\varepsilon = 0$). The (3) may be used for calculation of the ordinary derivatives in the theory of the fractal time on base of relation (4) if $d_t = 1 + \varepsilon (r, t)$ and $|\varepsilon| << 1$. In the fractal time and space the equations of all physical fields must be wrote by means of GFD. For case when FD of time has small fractal corrections to unit these equations may be wrote by means of ordinary derivatives but using (3). Than in the equations with GFD appear the members proportional to fields which usually describe the rest masses. These members are result of existence of fractal dimensions of time in fractal Universe where the time treats as the real field and the field of time originate all physical fields (see (1) - (3)) including the rest masses of massless fields. As the examples of originating the last masses by GFD the demonstration of appealing rest masses for electromagnetic and gravitational fields will be considered in next paragraph.

IV. HOW THE FRactal DIMENSIONS OF TIME ORIGINATE REST MASSES IN THE MASSLESS FIELD THEORIES?

The corrections given by GFD in case of using it for differentiation of constant value were considered early. Let us now represent (2) in the form when its main characteristics conserves but the derivatives are the ordinary. Thus an any constant value $a_0$ in fractal world may be represented as the value with dependence of time $t$ (or coordinates $r$)

$$a \approx a_0 \exp \frac{\varepsilon}{t}$$

(8)

In (6) the factor $\exp (ct - \varepsilon)$ describes the corrections given by the fractal dimensions of time. The ordinary differentiation of (6) with respect to $t$ coincide with result of GFD of $a$ with respect to $t$. If $d_t = 1$ the time dependence in (3) disappears ($\varepsilon = 0$). The (3) may be used for calculation of the ordinary derivatives in the theory of the fractal time on base of relation (4) if $d_t = 1 + \varepsilon (r, t)$ and $|\varepsilon| << 1$. In the fractal time and space the equations of all physical fields must be wrote by means of GFD. For case when FD of time has small fractal corrections to unit these equations may be wrote by means of ordinary derivatives but using (3). Than in the equations with GFD appear the members proportional to fields which usually describe the rest masses. These members are result of existence of fractal dimensions of time in fractal Universe where the time treats as the real field and the field of time originate all physical fields (see (1) - (3)) including the rest masses of massless fields. As the examples of originating the rest masses by GFD the demonstration of appealing rest masses for electromagnetic and gravitational fields will be considered in next paragraph.

V. THE REST MASSES OF PHOTONS AND GRAVITONS IN THE UNIVERSE WITH FRACTAL TIME

The equations of electromagnetic and gravitational fields in the fractal Universe if take into account both the corrections giving by FD and modifying SR (see also (1), (10), (14), (15)) reads:

a) electromagnetic fields equations

$$[D_{d_t}^e D_{d_t}^x - \frac{1}{c^2} D_{d_t}^a D_{d_t}^x + m^2] I A_\mu (x) = [\alpha_1 \frac{4\pi}{\epsilon} i j_\mu (x) +$$

$$+ 2 \alpha_0 D_{d_t}^a D_{d_t}^x \alpha_2] A_\mu (x), \mu = 0, 1, 2, 3$$

$$j_i = e D_{d_t}^a r_i, \quad i = 1, 2, 3$$

(9)

b) gravitational fields equations (the measure carrier is Minkowski time-space)

$$\gamma^{\alpha \beta} D_{-d_t}^a D_{d_t}^i D_{d_t}^j I \tilde{\Phi}^{\mu \nu} = \alpha_1 b^2 \tilde{\Phi}^{\mu \nu} + \lambda \tilde{\Phi}^{\mu \nu} (\gamma^{\mu \nu}, \Phi_A) +$$

$$+ 2 \alpha_0 \gamma^{44} D_{-d_t}^a D_{d_t}^i \alpha_2 \tilde{\Phi}^{\mu \nu}$$

(10)

In (10) and (11) $\alpha_1, \alpha_2$ are Dirac type matrices, $I$ is unit four column matrix, the function $a_0$ has the form

$$a_0 = \varepsilon (r_t, t) = a_g + a_e + a_n =$$

$$= \beta_L g (r, t) + \beta_n L_n (r, t)$$

(11)

where $L_g, \beta_n, L_n$ are Lagrangians density of energies for gravitational, electro-weak and strong fields, $\beta_i$ is the full energies of the fields that constructed the $L_i$.

We consider now the cases of free electromagnetic and gravitational fields and neglect by influences of the fields with imaginary energies (this influences gives non-essential corrections see (13)) and omit the members with $m^2$ and $b^2$ (which were formal introduced for fulfillment of the conditions of existence the GFD). Thus we have

$$[D_{d_t}^e D_{d_t}^x - \frac{1}{c^2} D_{d_t}^a D_{d_t}^x] I A_\mu (x) = 0$$

(13)

$$\gamma^{\alpha \beta} D_{-d_t}^a D_{d_t}^i D_{d_t}^j I \tilde{\Phi}^{\mu \nu} = 0$$

(14)

Consider the case when electromagnetic or gravitational waves propagate in the space with $a_0 \approx constant$, i.e in the domain of space that far away from the sources of these fields. If neglect by the members describing the interactions with the fields with imaginary masses and replace the functions $A_\mu$ and $\Phi^{\mu \nu}$ by the plane waves with frequencies $\omega$ we obtain (using (6))

$$F = (1 + a_0) \frac{\partial}{\partial t} A_\mu + \frac{\partial a_0}{\partial t} A_\mu$$

or

$$F = (1 + a_0) \frac{\partial}{\partial t} A_\mu + \frac{\partial a_0}{\partial t} A_\mu$$

(15)
\[ \omega^2 - 2\frac{\partial a_0}{\partial t} \omega - \left( \frac{\partial^2 a_0}{\partial t^2} + (\frac{\partial a_0}{\partial t})^2 \right) - k^2 c^2 = 0, \quad (16) \]

For \( \omega \) obtain

\[ w = i\gamma \pm \sqrt{\frac{\partial^2 a_0}{\partial t^2} + k^2 c^2} \]
\[ \gamma = \frac{\partial a_0}{\partial t} \quad (17) \]

So for \( |k| = 0 \) or \( k^2 c^2 << \frac{\partial^2 a_0}{\partial t^2} \) we read

\[ \omega_0 = \sqrt{\frac{\partial^2 a_0}{\partial t^2} + i\gamma} \quad (18) \]

and for \( \frac{\partial^2 a_0}{\partial t^2} < k^2 c^2 \) obtain

\[ \omega = |kc| + 0.5 \frac{\omega_0^2}{|kc|} + i\gamma \quad (19) \]

Thus it was shown that the massless fields got the rest masses and these masses defined by the fractal dimensions of the time. If the fractal corrections to the fractal dimensions of time are equal zero the rest masses are also equal zero. The value of rest masses depends of the FD of time \( (d = 1 + \varepsilon(t, t)) \). So the rest masses are functions of time and coordinates included by means of the densities of energies \( \beta_i L_i(\mathbf{r}(t, t)) \) and differs for different time and in different coordinates. This value and its derivates with respect to \( t \) gives the rest masses for case when its velocity is equal zero. In fractal theory of time and space a masses may have any velocities. Now we must take into account this possibility and use the relation for moving particles (see [8]) in modified SR for multifractal time

\[ E = \beta^{s-1} E_0 = \frac{E_0}{\sqrt{1 - \frac{v^2}{c^2}} + 4a^2} \quad (20) \]

where \( a^2 = (\frac{\partial a}{\partial t})^2 r^2 \). Thus for the rest masses originated by the fractal dimensions of time for the case when its velocity \( v \) equal speed of light \( v = c \) obtain

\[ \omega_c = \frac{\omega_0}{\beta^s} = \sqrt{\frac{\omega_0^2}{2a}} \quad (21) \]

The considered theory do not define the values of parameters \( t \) and \( t_i \) included in relations (16)-(22), so any estimations of the values of rest masses of gravitons and photons are preliminary and depends of hypothesis laying in the methods of selection \( t \) and \( t_i \). Nevertheless, as example, consider the order of the rest masses of gravitons and photons (neutrinos also have the rest masses originated by FD of time, but it seems that the main part of the rest masses of neutrinos may be explained by ordinary theories).

a) gravitons masses

Let \( a_0 = \varepsilon \) and \( \varepsilon \) with grate accuracy may be treated as constant value. Than

\[ \frac{\partial \varepsilon}{\partial t} \approx \frac{\varepsilon^2}{t} \quad (22) \]

and for large \( t \) \( (t \) is the time of existence of Universe) \( \frac{\partial \varepsilon}{\partial t} \) also may be treated as constant value. In this case

\[ \frac{\partial^2 \varepsilon}{\partial t^2} \approx \frac{\varepsilon^3}{t^2} \quad (23) \]

and for the rest mass of gravitons \( m_g \) we obtain

\[ m_g \approx \sqrt{\frac{\varepsilon}{t_1 \hbar c^{-2}}} \quad (24) \]

For \( \varepsilon \approx \frac{\varepsilon}{r_0} \) where \( r_0 \) and \( r \) are the gravitational radius of Earth and its radius, \( t \sim 10^{16} \text{sec} \) and \( t_i \sim 10^{-34} \text{sec} \) \( (t_i \) is the time needs for graviton to receive the velocity of light) is the time of ”Big Bang” origin, we obtain \( m_g \approx 10^{-43} \text{g} \). If differentiate \( a_0 \) with respect to \( t \) then for estimation of \( m_g \) we receive

\[ m_g \approx \sqrt{\frac{\varepsilon}{t \hbar c^{-2}}} \quad (25) \]

and \( m_g \approx 10^{-39} \text{g} \).

b) The rest masses of photons

We may use the same assumptions for estimation the rest masses of photons that have been used for the estimation of the rest masses of gravitons. So we receive the same formula. The estimation of \( t_i \) will be the same that for gravitons \( (m_{ph} \sim 10^{-39} \text{g}) \). The another selection of \( t_i \) as the time of emitting of quanta of light \( t_i \sim 10^{-12} \text{sec} \) gives \( m_{ph} \sim 10^{-59} \text{g} \). We pay attention, as \( \varepsilon \) is in general not constant value, on not necessity the using of \( \bar{m} \) or \( \bar{m} \) for calculation the derivatives of \( \varepsilon \) with respect to \( t \) if is known the dependence of \( \varepsilon \) at \( t \).

VI. THE PROBLEMS OF HIDDEN MASSES OR BLACK MATTER IN THE FRACTAL UNIVERSE

As well known, the theories of galaxies moving needs for the value of mass of Universe the value almost in nine times large than the masses observed by astronomers. So, where is the deficit of matter hidden? Is it the masses of interstellar gas substance pressed by gravitational fields, or is it the black matter of unknown nature? These questions do not have now unambiguously answer. The existence of the rest masses of photons and gravitons allows to include these masses in the possible scenario of the models of hidden and black matter. Really, If the masses of photons and gravitons have the value of order \( 10^{-39} \text{g} \) it is quite enough for explaining the necessary deficit of masses in the Universe.
VII. CAN THE BOSE-CONDENSATES OF PHOTONS OR GRAVITONS EXIST IN THE FRACTAL UNIVERSE?

Is it possible to imagine the flow of photons or gravitons with very small wave numbers and very low temperature that satisfy the conditions that are necessary for Bose-condensation? It seems (if its moving take place in the very strong gravitational or electromagnetic fields that makes its velocities (and kinetic energy) very small) because of existence of the rest masses such condensation is not contradicts the physical laws. The really answer now is unknown.

VIII. CONCLUSION

The main results obtained in this paper are:

1. All physical fields have rest masses originated by the fractal dimensions of time;
2. The rest masses originated by the FD of time have similar characteristics (in our approach) for any massless fields;
3. The rude estimation of values of rest masses gives satisfactory values for explaining the problem of black matter and hidden masses;
4. The really existence of the rest masses of photons and gravitons makes unnecessary aspiration of the formal introduced masses in theories \[1] - \[3] to zero;
5. Some questions arise: have rest masses of photons and gravitons unknown charges?; can a charge photons (or a gravitons) interact with each other? What may be answers on these questions and may the theory of open systems \[19\] - \[20\] (quantum variants) give these answers?
6. The fractal theory of time and space restore the symmetry respect rest masses for all physical fields;

[1] Kobelev L.Ya. Fractal Theory Time and Space, Ekaterinburg: Konross, 1999, 136p.(in Russian); Fractal Theory Time and Space /Kobelev L.Ya./ // Ural State Univ., Ekaterinburg, 1998.-158p.-Bibliogr.51Nam.-Rus.-Dep.v VINITY 22.01.99,189-B99 (in Russian);
[2] Kobelev L.Ya., What Dimensions Do the Time and Space Have: Integer or Fractional? \[arXiv:physics/0001035\]
[3] Kobelev L.Ya., Generalized Riemann -Liouville Fractional Derivatives for Multifractal Sets, \[arXiv:math.CA/0002008\];
[4] Kobelev L.Ya. Multifractality of Time and Special Theory of Relativity / Kobelev L.Ya.// Ural State Univ., Ekaterinburg, 1999.-21p.-Bibliogr.1Ref.-Rus.-Dep.v VINITY 19.08.99, 2677-B99.01.99,(in Rus.); Kobelev L.Ya./ Dep. v VINITI, Ekaterinburg, 20.10.99.3128-B99;
[5] Kobelev L.Ya., Can a Particle’s Velocity Exceeds the Speed of Light in the Empty Space? \[arXiv:gr-qc/0001042\]
[6] Kobelev L.Ya., Physical Consequences of Moving Faster than Light in Empty Space, \[arXiv:gr-qc /0001043;\]
[7] Kobelev L.Ya. , Multifractality of Time and Space. Covariant Derivatives and Gauge Invariance, \[arXiv:hep-th/0002003;\]
[8] Kobelev L.Ya., The Multifractal Time and Irreversibility in Dynamic Systems, \[arXiv:physics/0002002;\]
[9] Kobelev L.Ya., Is it Possible to Transfer an Information with the Velocities Exceeding Speed of Light in Empty Space?, \[arXiv:physics/0002003;\]
[10] Kobelev L.Ya., Maxwell Equation, Shroedinger Equation, Dirac Equation, Einstein Equation Defined on the Multifractal Sets of the Time and the Space, \[arXiv:gr-qc/0002003;\]
[11] Does Special Relativity Have Limits of Applicability in the Domain of Very Large Energies?, \[arXiv:physics/0005060;\]
[12] Kobelev L.Ya. Is the Time a Dimension of an Alien Universe? xxx.arXiv:physycs/0005070
[13] Kobelev L.Ya. Are the Laws of Thermodynamics Consequences of a fractal Properties of Universe? xxx \[arXiv:physics/0003039;\]
[14] The Theory of Fractal Time: Field Equations the Theory of Almost Inertial Systems and Modified Lorents Transformations), \[arXiv:physics/0005060;\]
[15] Kobelev L.Ya., The Theory of Gravitation in the Space - Time with Fractal Dimensions and Modified Lorents transformations, \[arXiv:gr-qc/00060xx\] 
[16] S.G.Samko , A.A.Kilbas , O.I.Marichev, Fractional integrals and derivatives - theory and applications (Gordon and Breach, New York, 1993)
[17] I.M.Gelfand, G.E.Shilov, Generalized functions (Academic Press, New York, 1964)
[18] Logunov A.A., Mestvirichvili M.A.,Theoretical and Mathematical Physics,1997, v.110, p.1-20 (in Russian)
[19] Klimontovich Yu.L. Statistical theory of open systems (Kluwer, Dordrecht, 1995);
[20] Klimontovich Yu.L.,Statistical theory of open systems 2,( Moscow: Yanus,1999, 439p)(in Russian)