Gravity as the affine Goldstone phenomenon and beyond

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Abstract

The two-phase structure is imposed on the world continuum, with the graviton emerging as the tensor Goldstone boson during the spontaneous transition from the affinely connected phase to the metric one. The physics principle of metarelativity, extending the respective principle of special relativity, is postulated. The theory of metagravitation as the general nonlinear model $GL(4, R)/SO(1, 3)$ in the arbitrary background continuum is built. The concept of the Metauniverse as the ensemble of the regions of the metric phase inside the affinely connected phase is introduced, and the possible bearing of the emerging multiple universes to the fine tuning of our Universe is conjectured.

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1 Introduction

The General Relativity (GR) is the well-stated theory attributing gravity to the Riemannian geometry of the space-time. Nevertheless, the ultimate nature of gravity awaits, conceivably, its future explanation. In this respect, it is of great interest the approach to gravity as the Goldstone phenomenon corresponding to the broken global affine symmetry \[ GL(4, R) \rightarrow SO(1, 3) \] in the Minkowski background space-time, as distinct from the geometrical framework of the GR.

In the present paper, we adhere to the viewpoint that the above construction is more than just the mathematical one, but has a deeper physics foundation underlying it. In this respect, the new insights motivating and extending the Goldstone approach to gravity are put forward. Of principle, we go beyond the framework of the Riemannian geometry. Namely, we start with the world continuum considered as the affinely connected manifold without metric and end up in the space-time with the effective Riemannian geometry.

Our main results are threefold:

(i) The physics principle of extended relativity is introduced as a substitution for that of special relativity. It states the physics invariance, at an underlying level, relative to the choice within the extended set of the local coordinates, including the inertial ones. The principle justifies the pattern of the affine symmetry breaking \[ GL(4, R) \rightarrow SO(1, 3) \] required for the Goldstone approach to gravity.

(ii) The extended theory of gravity, with the GR as the lowest approximation, is built as the proper nonlinear model in an arbitrary background continuum. The natural hierarchy of the possible GR extensions, according to the accuracy of the affine symmetry realization, is put forward.

(iii) The extended Universe, as the ensemble of the Riemannian metric universes inside the affinely connected world continuum, is considered. It is conjectured that the multiple universes may clarify the fine tuning problem of our Universe.

The content of the paper is as follows. In sec. 2 the principle of extended relativity is introduced. The spontaneous breaking of the ensuing global symmetry, the affine one, with the residual Poincare symmetry and the emerging tensor Goldstone boson is then considered. In sec. 3 the nonlinear realization of the broken affine symmetry is studied. In sec. 4 the respective nonlinear model in the tangent space is developed. Its prolongation to the space-time, the extended gravitation, is presented in sec. 5. Finally, the concept of the extended Universe is discussed in sec. 6 with some remarks in conclusion.

2 Metarelativity

2.1 Affine symmetry

Conventionally, the GR starts by postulating that the world continuum, i.e. the set of the world events (points), is the Riemannian manifold. In other words, a metric is imposed on the world ab initio. The metric specifies all the fine properties of the continuum converting the latter into the space-time. Nevertheless, not all of the properties of the space-time
depend crucially on the metric.\textsuperscript{1} To appreciate the deeper meaning of the gravity and the very space-time, one needs possibly go beyond the Riemannian geometry.

To this end, consider the space-time not as a priori existing but as emerging in the processes of the world structure formation. Namely, suppose that at an underlying level the continuum is endowed only with the topological structure (without metric, yet). More particularly, it is the affinely connected manifold. The affine connection supports the detailed continuity properties, such as the parallel transport of the vector fields, their covariant derivatives, etc. In particular, the connection produces the curvature tensor as a result of the parallel transport of a vector around the infinitesimal closed contour. But there is yet no geometrical structures which would be inherent in the metric, such as the interval, distances, angles, etc.

Let $x^\mu, \mu = 0, \ldots, 3$ be the world coordinates, generally, in the patches. There being, in absence of the metric, no partition of the continuum onto the space and time, the index 0 has yet no particular meaning and is just the notational one. In ignorance of the underlying “dynamics”, consider all the structures related to the underlying level of the world continuum as the background ones. Let $\bar{\phi}_\lambda^{\mu\nu}(\bar{x})$ be the background affine connection and let $\bar{\xi}^\alpha, \alpha = 0, \ldots, 3$ be the background affiliated coordinates where the connection have a particular, to be defined, form $\bar{\phi}^\gamma_{\alpha\beta}(\bar{\xi}).$\textsuperscript{2} The connections are related as usually:

$$\bar{\phi}^\gamma_{\alpha\beta}(\bar{\xi}) = \frac{\partial x^\mu}{\partial \bar{\xi}^\alpha} \frac{\partial x^\nu}{\partial \bar{\xi}^\beta} \left( \frac{\partial \bar{\xi}^\gamma}{\partial \bar{x}^\lambda} \bar{\phi}^\lambda_{\mu\nu}(x) - \frac{\partial^2 \bar{\xi}^\gamma}{\partial x^\mu \partial x^\nu} \right).$$  \hspace{1cm} (1)

It follows thereof, that the parts antisymmetric and symmetric in the lower indices transform independently, respectively, homogeneously and inhomogeneously. In particular, being zero in a point in some coordinates, the antisymmetric part (the torsion) remains to be zero independent of the coordinates. Thus one can adopt the background torsion to be absent identically. As for the symmetric part, one is free to choose the special coordinates to make the physics description as transparent as possible.

So, let $P$ be a fixed but otherwise arbitrary point (the reference point) with the world coordinates $X^\mu$. Adjust to this point the local coordinates as follows:

$$\bar{\xi}^\alpha = \bar{\Xi}^\alpha + \bar{e}_\lambda^\alpha(X) \left( (x - X)^\lambda + \frac{1}{2} \bar{\phi}^\lambda_{\mu\nu}(X) (x - X)^\mu (x - X)^\nu \right) + \mathcal{O}((x - X)^3).$$  \hspace{1cm} (2)

Here $\bar{\Xi}^\alpha \equiv \bar{\xi}^\alpha(X)$ and $\bar{e}_\lambda^\alpha(X) \equiv \partial \bar{\xi}^\alpha / \partial x^\lambda \big|_{x=X}$ is the tetrad, with $\bar{e}_\alpha(X)$ being the inverse one. These parameters are still arbitrary and liable to further specification. In the vicinity of $P$, the affine connection now looks like:

$$\bar{\phi}^\gamma_{\alpha\beta}(\bar{\xi}) = \frac{1}{2} \bar{\rho}^\gamma_{\alpha\delta\beta}(\bar{\Xi}) (\bar{\xi} - \bar{\Xi})^\delta + \mathcal{O}((\bar{\xi} - \bar{\Xi})^2),$$  \hspace{1cm} (3)

with $\bar{\rho}^\gamma_{\alpha\delta\beta}(\bar{\Xi})$ being the background curvature tensor in the reference point. In the coordinates chosen, the affine connection vanishes in the reference point: $\bar{\phi}^\gamma_{\alpha\beta}(\bar{\Xi}) = 0$.

Now, consider the whole set of the local coordinates nullifying the affine connection in the reference point $P$. Under the world coordinates $x^\mu$ being fixed, one can choose

\textsuperscript{1}Cf. in this respect the reflections on the space-time structure due to E. Schrödinger \textsuperscript{3}.

\textsuperscript{2}The bar sign refers in what follows to the background. The indices $\alpha, \beta$, etc., are those of the special coordinates, while the indices $\lambda, \mu$, etc., are the arbitrary world ones.
a priori any one of the local coordinates $\bar{\xi}^\alpha$. The group of replacements $\bar{\xi}^\alpha \rightarrow \xi'^{\alpha}$ among the latter ones is given by transformations:

$$
(A, a) : \bar{\Xi}^\alpha \rightarrow \bar{\Xi}'^\alpha = A^{\alpha\beta} \bar{\Xi}^\beta + a^\alpha, \quad \bar{\epsilon}_\mu^\alpha \rightarrow \bar{\epsilon}'_\mu^\alpha = A^{\alpha\beta} \bar{\epsilon}_\mu^\beta,
$$

with $A^{\alpha\beta}$ being an arbitrary nondegenerate matrix and $a^\alpha$ being an arbitrary vector. The transformations $(A, a)$ being independent of $\bar{\xi}$, the group is the global one. This is the inhomogeneous general linear group $IGL(4, R) = T_4 \odot GL(4, R)$ (the affine one). Under these and only under these transformations, the affine connection remains still to be zero in the reference point. The respective coordinates will be called the local affine ones.\(^3\) In these coordinates, the world continuum is approximated by the affinely flat manifold in a neighbourhood of the reference point $P$. Particularly, the covariant derivative in the affine coordinates in the point $P$ coincides with the ordinary one. Being changeable under $A$, the nonzero background torsion in the point $P$ would violate explicitly the affine symmetry. Just to abandon this, the torsion is adopted to be zero identically.

The affine group $IGL(4, R)$ is 20-parametric and extends the 10-parameter Poincare group $ISO(1, 3) = T_4 \odot SO(1, 3)$ by the transformations varying the scales, the affine dilatations,\(^4\) the rest being the scale preserving, unimodular, affine transformations. The dilatations $A = \Delta$ belong to the one-parametric multiplicative group of the positive real numbers, $\Delta^{\alpha\beta} = e^{-\lambda \delta^{\alpha\beta}}$, with $\lambda$ any real. The unimodular affine transformations are the 9-parametric part of the special linear group $SL(4, R) \ni A_0$, with $\det A_0 = 1$ (times reflections).

According to the well-known principle of special relativity, the present-day local physics laws are unchanged under the choice of the inertial coordinates, the Poincare group being the physics invariance symmetry. Now, introduce the principle of extended relativity, or the metarelativity, stating that the local physics laws at the underlying level are unchanged relative to the choice of the affine coordinates. This extends the physics invariance symmetry from the Poincare symmetry to the affine one.

### 2.2 Spontaneous symmetry breaking

Presently, there is known no exact affine symmetry. Thus, the latter should be broken in transition from the underlying level to the effective one. Postulate that this is achieved due to the spontaneous emergence of the background metric $\bar{\phi}_{\mu\nu}$ in the world continuum. The metric, with the Minkowskian signature, is assumed to be correlated with the background affine connection so that to look in the affine coordinates as

$$
\bar{\phi}_{\alpha\beta}^\gamma(\bar{\xi}) = \bar{\eta}_{\alpha\beta}^\gamma - \frac{1}{2} \bar{\rho}_{\gamma\alpha\delta\beta}(\bar{\Xi}) (\bar{\xi} - \bar{\Xi})^\gamma (\bar{\xi} - \bar{\Xi})^\delta + O((\bar{\xi} - \bar{\Xi})^3).
$$

Here one puts $\bar{\eta}_{\alpha\beta} \equiv \bar{\phi}_{\alpha\beta}(\bar{\Xi})$ and $\bar{\rho}_{\gamma\alpha\delta\beta}(\bar{\Xi}) = \bar{\eta}_{\gamma\gamma'} \bar{\rho}_{\alpha\delta\beta}'(\bar{\Xi})$. The metric eq. (5) is such that the Christoffel connection $\bar{\chi}_\gamma^{\alpha\beta}(\phi)$, determined by the metric, matches with the

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\(^3\) Being understood, the term “local” will be omitted in what follows.

\(^4\) The world coordinates being fixed, the affine dilatations are to be distinguished from the conventional ones in the space-time \([4]\).
affine connection \( \tilde{\gamma}^{\alpha\beta} \) in the sense, that the connections coincide locally, up to the first derivative: 
\[
\tilde{\chi}^{\alpha\beta} = \tilde{\gamma}^{\alpha\beta} + O((\tilde{\varphi} - \tilde{\xi})^2).
\]
This is quite reminiscent of the well-known fact that the metric in the Riemannian manifold may be approximated locally, up to the first derivative, by the Euclidean metric. In the wake of the emerging background metric, there appears the (yet primordial) partition of the world continuum onto the space and time.

Under the linearly realized affine symmetry, the background metric ceases to be invariant. But it still possesses an invariance subgroup. To find it note that, without any loss of generality, one can choose among the affine coordinates the particular ones with \( \tilde{\eta}_{\alpha\beta} \) being the Minkowski tensor \( \eta = \text{diag}(1, -1, -1, -1) \). The respective coordinates will be called the background inertial ones.\(^5\)

Under the affine transformations, one has
\[
(A, a) : \eta \rightarrow \eta' = A^{-1}T \eta A^{-1} \neq \eta, \quad (6)
\]
whereas the Poincare transformations still leave \( \eta \) invariant:
\[
(\Lambda, a) : \eta \rightarrow \eta' = \Lambda^{-1}T \eta \Lambda^{-1} = \eta. \quad (7)
\]

It follows that the group of invariance is isomorphous to the Poincare group \( ISO(1, 3) \subset IGL(4, R) \) for any fixed \( \tilde{\eta}_{\alpha\beta} \). Physically, the spontaneous symmetry breaking corresponds to fixing, modulo the Poincare transformations, the class of the distinguished coordinates among the affine ones. These coordinates correspond to the particular choice for \( \tilde{\eta}_{\alpha\beta} \). Of course, the fact that the distinguished coordinates are precisely those with the Minkowskian \( \eta_{\alpha\beta} \) is no more than the matter of convention, corresponding to the proper inner automorphism of the affine group.

Thus under the appearance of the metric, the affine symmetry is broken spontaneously to the residual Poincare one:
\[
IGL(4, R) \xrightarrow{M_A} ISO(1, 3). \quad (8)
\]

For the symmetry breaking scale \( M_A \), one expects a priori \( M_A \sim M_{Pl} \), with \( M_{Pl} \) being the Planck mass. More particularly the relation between the scales is discussed in sec. 5. Due to the spontaneous breaking, the affine symmetry should be realized in the nonlinear manner \([5]\), with the nonlinearity scale \( M_A \), the residual Poincare symmetry being still realized linearly. The unitary linear representations of the latter correspond to the matter particles, as usually. The translation subgroup being intact, the broken part coincides with \( GL(4, R)/SO(1, 3) \). The latter should be realized in the Nambu-Goldstone mode.

Accompanying the spontaneous emergence of the metric, there should appear the 10-component affine Goldstone boson which corresponds to the ten generators of the broken affine transformations. The effective field theory of the Goldstone boson is given by the relevant nonlinear model, to be studied in what follows.

### 2.3 Lorentz symmetry

The group \( GL(4, R) \) possesses the 16 generators \( \Sigma^{\alpha\beta} \). By means of \( \eta_{\alpha\beta} \), one can redefine the generators as \( \Sigma^{\alpha\beta} \equiv \Sigma^{\alpha\gamma} \eta_{\gamma\beta} \) and substitute the later ones by the symmetric and antisymmetric combinations \( \Sigma^{\alpha\beta}_{\pm} = \Sigma^{\alpha\beta} \pm \Sigma^{\beta\alpha} \). Clearly, this partition is affine noncovariant.

\(^5\)The latter ones are to be distinguished from the effective inertial coordinates, to be introduced.
The respective commutation relations read as follows:

\[
1 \frac{1}{i} [\Sigma^\alpha_-, \Sigma^\gamma_+] = \eta^{\alpha\gamma} \Sigma^\beta_+ \pm \eta^{\alpha\delta} \Sigma^\beta_- \pm (\alpha \leftrightarrow \beta),
\]

\[
1 \frac{1}{i} [\Sigma^\alpha_+, \Sigma^\gamma_-] = \eta^{\alpha\gamma} \Sigma^\beta_+ + \eta^{\alpha\delta} \Sigma^\beta_- - (\alpha \leftrightarrow \beta).
\]  

(9)

The generators \( \Sigma^\alpha_- \) correspond to the residual Lorentz symmetry, whereas \( \Sigma^\alpha_+ \) to the broken part of the affine symmetry. The broken generators contain, in turn, the dilatation one \( i\eta_{\alpha\beta}\Sigma^\alpha_+ \). The latter commutes with all the generators and is thus proportional to unity in any irreducible representation. For the generators \( \sigma^{\alpha\beta} \) in the adjoint representation one has \( (\sigma^{\alpha\beta})^\gamma_\delta = 1/i \delta^\alpha_\delta \delta^\beta_\gamma \), so that the respective generators \( \sigma^{\alpha\beta}_\pm \) are as follows:

\[
(\sigma^{\alpha\beta}_\pm)^\gamma_\delta = \frac{1}{i}(\delta^\alpha_\delta \eta^{\beta\gamma} \pm \delta^\beta_\delta \eta^{\alpha\gamma}).
\]  

(10)

The above partition of generators is used in what follows in constructing the nonlinear model. First, we study the three kinds of the substance, i.e., the affine Goldstone boson, matter and radiation, which are characterized by the three distinct types of the nonlinear realization. With these building blocks, we then construct the nonlinear model itself.

## 3 Nonlinear realization

### 3.1 Affine Goldstone boson

Let \( \bar{\xi}^\alpha \) be the background inertial coordinates adjusted to the space-time point \( P \). Attach to this point the auxiliary linear space \( T \), the tangent space in the point. By definition, \( T \) is isomorphous to the Minkowski space-time. The tangent space is the structure space of the theory, whereupon the realizations of the physics space-time symmetries, the affine and the Poincare ones, are defined. Introduce in \( T \) the coordinates \( \xi^\alpha \), the counterpart of the background inertial coordinates \( \bar{\xi}^\alpha \) in the space-time. By construction, the connection in the tangent space is zero identically. For the connection in the space-time point \( P \) in the coordinates \( \bar{\xi}^\alpha \) to be zero, too, the coordinates in the vicinity of the reference point are to be related as \( \xi^\alpha = \bar{\xi}^\alpha + \mathcal{O}((\bar{\xi} - \bar{\Xi})^3) \). The coordinates \( \xi^\alpha \) are the ones, wherein all the constructions in \( T \) are originally built. The latter ones being done, one can use in \( T \) the arbitrary coordinates.

According to ref. [5], the nonlinear realization of the symmetry \( G \) spontaneously broken to the symmetry \( H \subset G \) can be built on the quotient space \( K = G/H \), the residual subgroup \( H \) serving as the classification group. In the present case, one is interested in the pattern \( GL(4, R)/SO(1, 3) \), with the quotient space consisting of all the broken affine transformations. Let \( \alpha(\xi) \in K \) be the coset function on the tangent space. It can be represented by a group element \( k(\xi) \in G \). Under the affine transformations \( (A, a) \), the representative group element is to transform in the vicinity of the reference point as

\[
(A, a) : k(\xi) \rightarrow k'(\xi') = Ak(\xi)A^{-1},
\]  

(11)
where $\Lambda$ is the appropriate element of the residual group, here the Lorentz one $SO(1,3)$. One has similarly: $k^{-1} \rightarrow \Lambda k^{-1} A^{-1}$. In the same time, by the very construction, the Minkowskian $\eta$ stays invariant under the nonlinearly realized affine symmetry:

$$(A, a) : \eta \rightarrow \eta' = \Lambda^{-1T} \eta \Lambda^{-1} = \eta,$$ (12)

in distinction with the linear realization, eq. (6). Accounting for eq. (12), one gets in the other terms:

$$(A, a) : k(\xi) \eta \rightarrow k'(\xi') \eta = Ak(\xi) \eta A^T. \quad (13)$$

To represent unambiguously the coset by the group element $k$, one should impose on the latter some auxiliary condition. E.g., require $k$ to be pseudosymmetric in the sense that $k\eta = (k\eta)^T$ (and similarly for $k^{-1}$). This ensures that $k$ has ten independent components, indeed, in accord with the ten broken generators. Under the affine transformations, this results in the restriction $Ak\eta A^T = \Lambda k^T A^T$. This entails implicitly the dependence of the Lorentz transformation on the Goldstone boson: $\Lambda = \Lambda(A, k)$. Hereof, the term “nonlinear” follows. This construction implements the nonlinear realization of the whole broken group $GL(4, R)$, the residual Lorentz subgroup $SO(1,3)$ being still realized linearly, i.e., $\Lambda(A, k)|_{A=\Lambda} = \Lambda$. And what is more, the dilatations $\Delta$ being Abelian, one gets $\Lambda(\Delta, k) = 1$, so that $\Lambda(A, k) = \Lambda(A_0, k)$, with $A \equiv \pm A_0 \Delta$ and $A_0 \in SL(4, R)$.

By doing as above, one retains only the independent Goldstone components but looses the local Lorentz symmetry.$^6$ For this reason, we will not impose any auxiliary condition. Instead, we extend the affine symmetry by the hidden local symmetry$^7$ $\hat{H} \simeq H = SO(1,3)$, with the symmetry breaking pattern $G \times \hat{H} \rightarrow H$. For quantities in the tangent space, one should distinguish now two types of indices: the affine ones, acted on by the global affine transformations $A \in G$, and the Lorentz ones, acted on by the local Lorentz transformations $\Lambda(\xi) \in \hat{H}$. To make this difference explicit, designate the affine indices in the tangent space as before $\alpha, \beta$, etc, while the Lorentz ones as $a, b$, etc. The Goldstone field is represented in this case by the arbitrary $4 \times 4$ matrix $\kappa_{\alpha a}(\xi)$ (respectively, $\kappa^{-1}_{a\alpha}$), which transforms similar to $k$ by eq. (13) but with arbitrary $\Lambda(\xi)$. In what follows, it is understood that the Lorentz indices are manipulated by means of the Minkowskian $\eta_{ab}$ (respectively, $\eta^{ab}$). So, in the component notation, $k\eta$ looks like $\kappa^{\alpha a}$ (similarly, $\eta k^{-1}$ is $\kappa^{-1}_{a\alpha}$). This is the linearization of the nonlinear model, with the extra Goldstone degrees of freedom being unphysical due to the gauge Lorentz transformations $\Lambda(\xi)$. The auxiliary gauge boson, corresponding to the generators $\Sigma_{ab}^\alpha$ of the local Lorentz symmetry, is expressed due to the equation of motion through $\kappa$ and its derivative. With this in mind, the abrupt expressions entirely in terms of $\kappa$ and its derivative are used in what follows. The versions differ by the higher order corrections.

3.2 Matter

The affine symmetry contains the Abelian, though broken, subgroup of the affine dilatations. For this reason, the generic matter fields $\phi$ may additionally be classified by their

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$^6$Such a procedure was adopted in refs. [1, 2].

$^7$The hat sign refers in what follows to the local Lorentz symmetry.
affine scale dimension $l_\phi$,\(^8\) so that:

$$(A, a) : \phi(\xi) \rightarrow \phi'(\xi') = e^{\phi\hat{\rho}(\Lambda)}\phi(\xi),$$

with $\hat{\rho}(\Lambda)$ taken in the proper Lorentz representation. According to eq. (11), the scale dimension of $\kappa$ is $l_\kappa = -1$ (respectively, $l_{\kappa^{-1}} = 1$). Thus, with account for the transformation $\det \kappa \rightarrow e^{-4\lambda}\det \kappa$ under dilatations, one can rescale the matter fields to the effective ones $\hat{\phi} = (\det \kappa)^{\phi/4}$. The new fields are affine scale invariant, i.e. correspond to $l_\phi = 0$, and transform simply as the local Lorentz representations. They are to be used in constructing the nonlinear model. If the affine symmetry is not explicitly violated, only the rescaled matter fields enter the action. In any case, one can choose $\hat{\phi}$ and $\det \kappa$ as the independent field variables. Thus, instead of $\hat{\phi}$, the matter fields will be designated in what follows simply as $\phi$ with $l_\phi = 0$.

3.3 Radiation

From the point of view of the nonlinear realization, the gauge bosons of the internal symmetries constitute one more separate kind of the substance, the radiation. By definition, the gauge boson fields $V_\alpha$ transform under the affine transformations linearly as the derivative $\partial_\alpha \equiv \partial/\partial \xi^\alpha$:

$$(A, a) : V(\xi) \rightarrow V'(\xi') = A^{-1T}V(\xi),$$

corresponding thus to the scale dimension $l_V = 1$. For this reason, redefine the gauge fields as $\hat{V}_a = \kappa^a_\alpha V_\alpha$. The new fields transform as the local Lorentz vectors

$$\hat{\dot{V}}(\xi) \rightarrow \hat{\dot{V}}'(\xi') = \Lambda^{-1T}\hat{\dot{V}}(\xi)$$

and correspond to $l_{\dot{V}} = 0$. These gauge fields are to be used in the model building. Altogether, this exhausts the description of all the three kinds of the substance: the affine Goldstone boson, matter and radiation.

4 Nonlinear model

4.1 Nonlinear connection

To explicitly account for both the affine symmetry and the local Lorentz one, it is convenient to start with the composite objects transforming only under the latter symmetry. Clearly, any nontrivial combinations of $\kappa$ and $\kappa^{-1}$ alone transform explicitly under $A$. Thus the derivative terms are inevitable. To describe the latter ones, introduce the Maurer-Cartan one-form:

$$\hat{\Omega} = \eta \kappa^{-1}d\kappa,$$

with $d\kappa$ being the ordinary differential of $\kappa$. Under the affine transformations $\kappa \rightarrow \kappa' = Ak\Lambda^{-1}$, the one-form transforms as the local Lorentz representation:

$$\hat{\Omega}(\xi) \rightarrow \hat{\Omega}'(\xi') = \Lambda^{-1T}\hat{\Omega}(\xi)\Lambda^{-1} + \Lambda^{-1T}\eta d\Lambda^{-1},$$

\(^8\)The latter is to be distinguished from the conventional scale dimension.
with \(d\Lambda\) being the ordinary differential of \(\Lambda(\xi)\). Here use is made of the relation \(\eta \Lambda \eta = \Lambda^{-1T}\) for the Lorentz transformations.

In the component notation, the so defined one-form looks like \(\hat{\Omega}_{ab}\). Decompose it as

\[
\hat{\Omega}_{ab} \equiv \sum_{\pm} \hat{\Omega}^{\pm}_{ab} = \sum_{\pm} [\eta \kappa^{-1} d\kappa]^{\pm}_{ab},
\]

(19)

where \([\ldots]^{\pm}\) means the symmetric and antisymmetric parts, respectively. One sees that \(\hat{\Omega}^{\pm}_{ab}\) transform independently of each other:

\[
\hat{\Omega}^{\pm}(\xi) \rightarrow \hat{\Omega}'^{\pm}(\xi') = \Lambda^{-1T} \hat{\Omega}^{\pm}(\xi) \Lambda^{-1} + \delta^{\pm},
\]

(20)

where

\[
\delta^{-} = \Lambda^{-1T} \eta d\Lambda^{-1},
\]

\[
\delta^{+} = 0.
\]

(21)

Transforming homogeneously, the symmetric part \(\hat{\Omega}^{+}\) can naturally be associated with the nonlinear covariant differential of the Goldstone field. At the same time, the antisymmetric part \(\hat{\Omega}^{-}\) transforms inhomogeneously and allows one to define the nonlinear covariant differential of the matter fields: \(D\phi = (d + i/2 \hat{\Omega}^{-}_{ab} \hat{\Sigma}^{ab}_{\phi})\phi\), with \(\hat{\Sigma}^{ab}_{\phi}\) being the Lorentz generators in the representation \(\hat{\rho}_{\phi}\). The so defined \(D\phi\) transform homogeneously, like \(\phi\) themselves.

The generic nonlinear covariant derivative \(D_{\alpha} \equiv D/d\xi^{\alpha}\) transforms as the affine vector. The effective covariant derivative, which transforms as the local Lorentz vector, can be constructed as follows:

\[
\hat{D}_{a} \equiv \kappa^{\alpha}_{a} D_{\alpha} = \kappa^{\alpha}_{a} D/d\xi^{\alpha}.
\]

(22)

Thus one gets for the covariant derivative of the one-form:

\[
\hat{\Omega}^{\pm}_{abc} = \kappa^{\gamma}_{c} \hat{\Omega}^{\pm}_{ab}/d\xi^{\gamma} = [\eta \kappa^{-1} \hat{\partial}_{c} \kappa]^{\pm}_{ab},
\]

(23)

where

\[
\hat{\partial}_{c} \equiv \kappa^{\gamma}_{c} \partial_{\gamma} = \kappa^{\gamma}_{c} \partial/\partial\xi^{\gamma}
\]

(24)

is the effective, Goldstone boson dependent, partial derivative. It follows that \(\hat{\Omega}^{\pm}_{abc}\) could be used as the connection for the nonlinear realization. Note, that this expression precisely corresponds to the case of the nonlinear realization of the spontaneously broken internal symmetry, where this connection is determined uniquely. But in the present case of the space-time symmetry, the coordinates transform under the same group as the fields. This results in the possible ambiguity of the nonlinear connection.

Namely, the transformation properties of the covariant derivative do not change if one adds to the above minimal connection the properly modified terms \(\hat{\Omega}^{\pm}_{abc}\), the latter ones transforming homogeneously. For reason justified later on in this section, we choose for the nonminimal connection the following special combination:

\[
\hat{\omega}_{abc} = \hat{\Omega}^{-}_{abc} + \hat{\Omega}^{+}_{cab} - \hat{\Omega}^{+}_{cda} = [\eta \kappa^{-1} \hat{\partial}_{c} \kappa]^{\pm}_{ab} + [\eta \kappa^{-1} \hat{\partial}_{b} \kappa]^{+}_{ca} - [\eta \kappa^{-1} \hat{\partial}_{a} \kappa]^{+}_{cb}.
\]

(25)

8
The nonlinear covariant derivative of the matter fields now becomes

\[ \hat{D}_c \phi = \left( \partial_c + i \frac{1}{2} \hat{\omega}_{abc} \hat{\Sigma}^{ab} \right) \phi. \]  

(26)

\( \hat{D}_c \phi \) transforms homogeneously and can be used in model building.

### 4.2 Gauge interactions

**Internal symmetry**  Let \( \hat{V}_a \) be the generator valued gauge fields for the internal gauge symmetry. The gauge fields are supposed to be coupled universally via the nonlinear connection, eq. (25). With account for the Lorentz generators \( -(\hat{\sigma}^{ab})^T \) in the contravariant adjoint representation (being given by eq. (10) with the obvious substitution for the indices), one gets for the nonlinear derivative of the fields:

\[ \hat{D}_a \hat{V}_b = \left( \delta^c_b \partial_a + \hat{\omega}_{cba} \right) \hat{V}_c. \]  

(27)

It follows thereof, in particular, that \( \hat{D}_c \eta_{ab} = 0 \). Define the gauge strength as usually:

\[ \hat{F}_{ab} = \left( \hat{D}_a + i \hat{V}_a \right) \hat{V}_b - (a \leftrightarrow b). \]  

(28)

It follows that the so defined gauge strength takes the form \( \hat{F}_{ab} = \kappa^a_a \kappa^b_b F_{\alpha\beta} \), with

\[ F_{\alpha\beta} = \left( \partial_\alpha + i V_\alpha \right) V_\beta - (\alpha \leftrightarrow \beta). \]  

(29)

Thus \( \hat{F}_{ab} \) does possess the correct transformation properties with respect to both the affine symmetry and the internal gauge symmetry.

**Lorentz symmetry**  Further, consider the local Lorentz symmetry as the gauge one with the connection \( \hat{\omega}_c \equiv 1/2 \hat{\omega}_{abc} \hat{\Sigma}^{ab} \), where \( \hat{\Sigma}^{ab} \) are some generic Lorentz generators. Define the corresponding gauge strength for the affine Goldstone boson as

\[ \hat{G}_{cd} = (\hat{\partial}_c + i \hat{\omega}_c) \hat{\omega}_d - (c \leftrightarrow d) \equiv \frac{1}{2} \hat{R}_{abcd} \hat{\Sigma}^{ab}. \]  

(30)

This gives

\[ \hat{R}_{abcd} = \hat{\partial}_c \hat{\omega}_{abd} - \hat{\omega}_f^{ac} \hat{\omega}_{fbd} - (c \leftrightarrow d). \]  

(31)

This quantity transforms homogeneously as the local Lorentz tensor (and similarly for its partial contraction \( \hat{R}_{bd} \equiv \hat{R}^{a}_{bad} \)). The total contraction

\[ \hat{R} \equiv \hat{R}^{ab}_{ab} = 2\hat{\partial}_a \hat{\omega}_b^{ab} - \hat{\omega}_f^{a} \hat{\omega}_f^{b} + \hat{\omega}_f^{a} \hat{\omega}_f^{b} \]  

(32)

is the local Lorentz scalar and can be used in building the Lagrangian for the Goldstone boson.
4.3 Lagrangian

Lorentz invariant form The constructed objects can serve as the building blocks for the nonlinear model $GL(4, R)/SO(1, 3)$ in the tangent space. Postulate the equivalence principle in the sense that the tangent space Lagrangian should not depend explicitly on the tangent space counterpart of the background curvature $\bar{\rho}_{\gamma\alpha\delta\beta}$, eq. (5). Thus, the Lagrangian may be written as the general Lorentz invariant function built of $\hat{R}$, $\hat{F}_{ab}$, $\hat{D}_a \phi$ and $\phi$ themselves. As usually, we restrict ourselves by the terms containing two derivatives at the most.

The generic Lorentz (and thus affine) invariant Lagrangian in the tangent space is

$$L = L_g(\hat{R}) + L_r(\hat{F}_{ab}) + L_m(\hat{D}_a \phi, \phi).$$

(33)

In the above, the basic Goldstone Lagrangian $L_g$ is as follows:

$$L_g = c_g M_A^2 \left( -\frac{1}{2} \hat{R}(\hat{\omega}_{abc}) + \Lambda \right),$$

(34)

with $c_g$ being a dimensionless constant, to be chosen, and $\Lambda$ proving to be in what follows the cosmological constant. The radiation Lagrangian $L_r$ is as usually

$$L_r = -\frac{1}{4} \text{tr}(\hat{F}^{ab} \hat{F}_{ab}).$$

(35)

Finally, $L_m$ is the proper matter Lagrangian. As for the radiation and matter, their Lagrangian could well be the affine invariant Lagrangian of the Standard Model or of any its extension. In fact, the given nonlinear model can accommodate any field theory.

Affine invariant form The Lagrangian above gives the basic dynamical description of the affine Goldstone boson, radiation and matter. The local Lorentz representations are necessary to construct the Lagrangian. The latter being built, one can rewrite it in terms of the affine representations. This allows one to make explicit the geometrical structure of the theory and to relate it with the gravity. This is achieved by the proper regrouping the factors $\kappa^a_\alpha$ and $\kappa^{-1a}_\alpha$, so that to make the affine indices to be explicit. The Lagrangian now becomes

$$L = c_g M_A^2 \left( -\frac{1}{2} R(\gamma_{\alpha\beta}) + \Lambda \right) + L_r(F_{\alpha\beta}) + L_m(D_a \phi, \phi).$$

(36)

Here

$$\gamma_{\alpha\beta} = \kappa^{-1a}_\alpha \eta_{ab} \kappa^{-1b}_{-\beta}$$

(37)

transforms as the affine tensor

$$(A, a) : \gamma_{\alpha\beta} \rightarrow \gamma'_{\alpha\beta} = A^{-1a}_\alpha \gamma_{\gamma\delta} A^{-1b}_{-\beta}.$$
Riemann-Christoffel curvature tensor with the metric. In this, all the contractions of the affine indices are understood with $\gamma_{\alpha\beta}$ (respectively, $\gamma^{\alpha\beta}$).

Similarly, $D_\alpha \phi$ looks like the generally covariant derivative for the matter fields:

$$D_\gamma \phi = \left( \partial_\gamma + \frac{i}{2} \omega_{ab\gamma} \hat{\Sigma}^{ab} \right) \phi,$$

with the spin-connection

$$\omega_{ab\gamma} \equiv \kappa^{-1\gamma}_{a} \hat{\omega}_{abc} = \kappa^\beta_{a} \nabla_\gamma \kappa^{-1}_{b\beta} - (a \leftrightarrow b).$$

(40)

In the above, $\nabla_\gamma \kappa^{-1}_{b\beta} \equiv (\delta^a_\beta \partial_\gamma - \Gamma^a_{\beta\gamma}) \kappa^{-1}_{b\alpha}$ is the covariant derivative calculated with the Christoffel connection

$$\Gamma^\alpha_{\beta\gamma} = \kappa^\alpha_{a} \kappa^{-1b}_{\beta} \kappa^{-1\gamma}_{c} \hat{\omega}_{abc} + \kappa^a_{a} \partial_\gamma \kappa^{-1b}_{\beta}$$

$$= \frac{1}{2} \gamma^{\alpha\delta} \left( \partial_\beta \gamma_{\delta\gamma} + \partial_\gamma \gamma_{\delta\beta} - \partial_\delta \gamma_{\beta\gamma} \right).$$

(41)

In particular, one gets $\nabla_\gamma \gamma_{\alpha\beta} = 0$ as the affine counterpart of the Lorentz relation $\hat{D}_c \eta_{ab} = 0$. For the radiation Lagrangian, one has the usual expression

$$L_r = -\frac{1}{4} \text{tr}(F_{\alpha\beta} F^{\alpha\beta}),$$

(42)

with $F_{\alpha\beta}$ given by eq. (29). Finally, the matter Lagrangian is obtained straightforwardly from $L_m$, eq. (33), with account for eq. (37) and the relation $\hat{D}_a = \kappa^a_{a} D_\alpha$, eq. (22).

Clearly, $L_g$ looks like the GR Lagrangian in the tangent space considered as the effective Riemannian manifold with the metric $\gamma_{\alpha\beta}$, the Christoffel connection $\Gamma^\alpha_{\beta\gamma}$, the Riemann-Christoffel curvature tensor $R^\gamma_{\alpha\beta\gamma}$, the Ricci tensor $R_{\alpha\beta}$, the Ricci scalar $R$ and the tetrad $\kappa^{-1a}_{\alpha}$ (the inverse one $\kappa^a_{\alpha}$). This is in no way accidental. Namely, as it is shown in ref. [2], under the special choice of the nonlinear connection eq. (25), the Lagrangian becomes conformally invariant, too. In this, the dilaton of the conformal symmetry coincides with the affine dilaton, while the vector Goldstone boson of the conformal symmetry, proving to be the derivative of the dilaton, is auxiliary. Further, according to the theorem due to Ogievetsky [3], it follows that the theory which is invariant both under the conformal symmetry and the global affine one is generally covariant, as well. After the proper choice of the metric, this imposes the effective Riemannian structure onto the tangent space. In the world coordinates, this will result in the generally covariant theory (the GR, in particular). Precisely this property justifies the special choice eq. (25) for the nonlinear connection, with the Goldstone boson being the graviton in disguise.

5 Metagravitation

5.1 General covariance

Accept the tangent space Lagrangian as that for the space-time, being valid in the background inertial coordinates in the infinitesimal neighbourhood of the reference point $P$.\footnote{For short, the term “effective” will be omitted, while that “background” will, in contrast, be retained.}
After the subsequent multiplication of the Lagrangian by the covariant volume element \((-\gamma)^{1/2} \text{d}^4\bar{\Xi}\), with \(\gamma \equiv \det\gamma_{\alpha\beta}\), one gets the contribution into the action of the infinitesimal neighbourhood of the point \(P\). Now one has to convert this contribution into the arbitrary world coordinates and to sum over the whole space-time.

The relation between the background inertial and world coordinates is achieved by means of the background tetrad \(e_a^\mu(X)\), eq. \((2)\), with the world metric being as follows:

\[
g_{\mu\nu}(X) = e_\mu^a(X)\gamma_{\alpha\beta}(\bar{\Xi})e_\nu^\beta(X). \tag{43}\]

With account for eqs. \((4)\), \((38)\), this metric is invariant under the affine transformations

\[(A, a) : g_{\mu\nu} \rightarrow g_{\mu\nu}. \tag{44}\]

By the very construction, the world coordinates are unchanged as well:

\[(A, a) : X^\mu \rightarrow X^\mu. \tag{45}\]

As a result, the effective interval \(ds^2 = g_{\mu\nu} dX^\mu dX^\nu\) remains invariant, too.

Now, introduce the effective tetrad related with the background one as

\[e_\mu^a(X) = \kappa^{-1\alpha}(\Xi) e_\mu^\alpha(X). \tag{46}\]

The effective tetrad transforms as the local Lorentz vector:

\[e_\mu(X) \rightarrow e'_\mu(X) = \Lambda(X) e_\mu(X). \tag{47}\]

Due to the local Lorentz transformations \(\Lambda(X)\), one can eliminate six components out of \(e^a_\mu\), the latter having thus ten independent components. In this terms, the world metric is

\[g_{\mu\nu}(X) = e_\mu^a(X)\eta_{ab} e_\nu^b(X). \tag{48}\]

In other words, the tetrad \(e_\mu^a\) defines the effective inertial coordinates. Physically, eq. \((46)\) describes the disorientation of the effective inertial and background inertial frames depending on the distribution of the affine Goldstone boson (and thus the gravity).

With account for the relation \(d\bar{\Xi}^a = e^a_\mu(X) dX^\mu\) between the displacements of the point \(P\) in the background inertial and world coordinates, and thus \(\partial \bar{\Xi}^a /\partial X^\mu = e^a_\mu\), one has

\[\Gamma^\lambda_{\mu\nu} = e^\lambda_\alpha e^\beta_\mu e^\gamma_\nu \Gamma^\alpha_{\beta\gamma} + \partial_\mu e^\alpha_\nu e^\gamma_\nu. \tag{49}\]

where \(\partial_\mu = \partial /\partial X^\mu\). This can be rewritten as usually:

\[\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\rho}(\partial_\mu g_{\rho\nu} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu}). \tag{50}\]

By construction, the world indices are manipulated via \(g_{\mu\nu}\) and \(g^{\mu\nu}\). The spin-connection looks in the world coordinates like

\[\omega_{ab\mu} = \omega_{ab\gamma} e^\gamma_\mu = e^\nu_\mu \nabla_\mu e_{ab} - (a \leftrightarrow b), \tag{51}\]
with the generally covariant derivative $\nabla_\mu$ defined via the Christoffel connection $\Gamma^\lambda_{\mu\nu}$, as usually. Respectively, the covariant derivative of the matter fields looks like

$$D_\mu \phi = \left( \partial_\mu + \frac{i}{2} \omega_{ab\mu} \tilde{\Sigma}^{ab} \right) \phi.$$ \hfill (52)

In the similar way, one finds the usual expressions for the Riemann-Christoffel tensor $R^\lambda_{\mu\rho\nu}(g)$, the Ricci tensor $R_{\mu\nu} = R^\lambda_{\mu\lambda\nu}$ and the Ricci scalar $R = g^{\mu\nu} R_{\mu\nu}$. The same is true for the gauge strength:

$$F_{\mu\nu} = (\partial_\mu + iV_\mu)V_\nu - (\mu \leftrightarrow \nu).$$ \hfill (53)

Plugging the above modified objects into the Lagrangians for the affine Goldstone boson, radiation and matter and integrating with the invariant volume element one gets the total action, the Einstein-Hilbert one including:

$$S = \int \left[ M_{Pl}^2 \left( -\frac{1}{2} R(g_{\mu\nu}) + \Lambda \right) + L_r(F_{\mu\nu}) + L_m(D_\mu \phi, \phi) \right] (-g)^{1/2} d^4X,$$ \hfill (54)

with $g \equiv \det g_{\mu\nu}$. In the above, the constant $c_g$ in eq. (36) is chosen so that $c_g M_A^2 = 1/(8\pi G_N) \equiv M_{Pl}^2$, with $G_N$ being the Newton’s constant and $M_{Pl}$ being the Planck mass. Varying the action with respect to the metric $g^{\mu\nu}$ one arrives at the well-known equation of motion for gravity:

$$G_{\mu\nu} = M_{Pl}^{-2} T_{\mu\nu}.$$ \hfill (55)

In the above, $G_{\mu\nu}$ is the gravity tensor:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu}$$ \hfill (56)

and $T_{\mu\nu} = T_{\mu\nu}^r + T_{\mu\nu}^m$ is the conventional energy-momentum tensor of the radiation and matter, produced by $L_r$ and $L_m$.

### 5.2 General covariance violation

By choosing the generally covariant Lagrangian in the tangent space one arrives at the generally covariant theory in the space-time. Modulo the choice of the Lagrangian, such a theory is unique, independent of the choice of the coordinates. In particular, one manages to express everything exclusively in the internal dynamical terms (but for the numerical parameters). Under extension of the tangent space Lagrangian beyond the generally covariant one, the theory in the space-time ceases to be generally covariant and thus unique. It depends not only on the Lagrangian but on the choice of the coordinates, too. Relative to the general coordinate transformations, the variety of theories divides into the observationally inequivalent classes, each of which being characterised by the particular set of the background parameter-functions. Precisely the latter ones make the coordinates distinguishable. A priori, no one of the sets of parameter-functions is preferable. Which one is suitable (if any), should be determined by observations. Each class of theories consists of the equivalent theories related by the residual covariance group. The latter consists of the coordinate transformations leaving the background parameter-functions...
invariant. On the contrary, one class can be obtained from another by the coordinate transformations changing these parameter-functions. Weakening the requirements on the bundling of the tangent spaces, one extends the set of the admissible theories, but arrives, instead, at the dependence of the theory in the space-time on the more elaborate properties of the background.

To clarify the corresponding parameter-functions, construct the background metric

$$\bar{g}_{\mu\nu}(X) = \bar{e}_A^\mu(X)\eta^{AB}\bar{e}_B^\nu(X),$$

with the inverse one

$$\bar{g}^{-1\mu\nu}(X) = \bar{e}_A^\mu(X)\eta^{AB}\bar{e}_B^\nu(X).$$

Here the generic index $A$ means $a$ or $\alpha$, as appropriate (and similarly for $B$, $b$, $\beta$, etc).

This metric transforms intricately under the arbitrary affine transformations:

$$(A, a) : \bar{g}_{\mu\nu} \rightarrow \bar{g}'_{\mu\nu} = \bar{e}^{T\mu A}T^\eta A^{\mu\nu} = \bar{g}_{\mu\nu},$$

though being invariant under the Poincare transformations ($\Lambda, a$). The metric $\bar{g}_{\mu\nu}$ is the next-of-kin to the primordial one $\bar{\varphi}_{\mu\nu}$, eq. (5). The former approximates the latter as closely as possible in the lack of the knowledge of the primordial background curvature $\bar{\rho}^\gamma_{\alpha\delta\beta}$, eq. (5). According to the equivalence principle, this curvature does not enter the tangent space Lagrangian and thus is inessential. Respectively, the Christoffel connection $\bar{\Gamma}^\lambda_{\mu\nu}(\bar{g})$ approximates with the same accuracy the Christoffel connection $\bar{\chi}^\lambda_{\mu\nu}(\bar{\varphi})$ and thus the primordial affine connection $\bar{\phi}^\lambda_{\mu\nu}$, i.e., $\bar{\Gamma}^\lambda_{\mu\nu} \simeq \bar{\chi}^\lambda_{\mu\nu} \simeq \bar{\phi}^\lambda_{\mu\nu}$. So, in the reasonable assumptions, it suffices to know only the background metric $\bar{g}_{\mu\nu}$.

**Affine symmetry preservation** To be more specific, consider the extension of the tangent space Lagrangian for the Goldstone boson by means of the terms depending explicitly on $\hat{\Omega}^{a b c}$, eq. (23). E.g., one can add to the basic Goldstone Lagrangian eq. (34) the quadratic piece:

$$\Delta L_g^{(0)} = \frac{1}{2} \varepsilon_0 M^2_{Pl} \hat{\Omega}^{a b c} \gamma^a \gamma^b \gamma^c$$

(60)

where $\varepsilon_0$ is a dimensionless constant. With account for eqs. (23) and (41), one gets for $\sigma_\alpha \equiv -\kappa^{-1} \hat{\Omega}^{+ b a}$ the relation $\sigma_\alpha = \Gamma^\beta_{\beta\alpha}(\gamma) = \partial_\alpha \sigma$, where $\sigma \equiv 1/2 \ln(-\gamma)$ and $\gamma \equiv \det \gamma_{\alpha\beta} = -(\det \kappa^{\alpha}_a)^{-2}$. In the affine terms, one has

$$\Delta L_g^{(0)} = \frac{1}{2} \varepsilon_0 M^2_{Pl} \gamma^{\alpha\beta} \partial_\alpha \sigma \partial_\beta \sigma.$$  

(61)

This Lagrangian violates the conformal symmetry in the tangent space (more particularly, the local dilatation), as well as the general covariance, though not violating the global affine symmetry.

It follows from eq. (43) that $\gamma = -g/\bar{g}$, where $\bar{g} = \det \bar{g}_{\mu\nu} = -(\det \bar{e}_\mu^\nu)^2$. In the world coordinates, the Lagrangian $\Delta L_g^{(0)}$ becomes

$$\Delta L_g^{(0)} = \frac{1}{2} \varepsilon_0 M^2_{Pl} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma,$$

(62)
with
\[ \sigma \equiv 1/2 \ln(g/\bar{g}) \]  
(63)
and
\[ \partial_\mu \sigma = \Gamma^\lambda_\lambda_\mu (g) - \bar{\Gamma}^\lambda_\lambda_\mu (\bar{g}). \]  
(64)
Thus, all the background dependence in the given case is determined only by the scalar density \( \bar{g} \). Note that \( \partial_\mu \sigma \) transforms homogeneously and thus can not be eliminated by the coordinate transformations, though each one of the contributions could separately be nullified by the choice of coordinates.\(^{11}\)

Varying the total action (the Lagrangian \( \Delta L_{(0)}^g \) included), one arrives at the modification of the equation of motion for gravity, eq. [55], with the extra piece in the gravity tensor \( \Delta G_{\mu\nu}^{(0)} \). Introducing the derivative couplings of \( \sigma \) with matter, not violating explicitly the affine symmetry, one would get the extra piece \( \Delta T_{\mu\nu}^{m} \) in the energy-momentum tensor for matter. Clearly, the modified theory, though not being generally covariant, is consistent with the unimodular covariance, i.e., that leaving \( \bar{g} \) (as well as \( g \)) invariant. The unimodular covariance is next-of-kin to the general one. Due to this residual covariance, the given GR extension describes only three physical degrees of freedom corresponding to the “scalar” and massless tensor gravitons. In the case \( \varepsilon_0 = 0 \), the general covariance is restored eliminating thus one more degree of freedom. This leaves just two of them with helicities \( \lambda = \pm 2 \), as it should be for the massless spin-2 particle.

The extra terms in the Goldstone boson Lagrangian would make physical the other latent degrees of freedom of the gravity field, but by the cost of further violating the general covariance. E.g., one could supplement the Goldstone Lagrangian by the other independent quadratic pieces:
\[
\begin{align*}
\Delta L_{(1)}^g & = \varepsilon_1 M^2_P \hat{\Omega}^+_{ab} \hat{\Omega}^{+ac}, \\
\Delta L_{(2)}^g & = \varepsilon'_2 M^2_P \hat{\Omega}_{abc}^{+} \hat{\Omega}^{+abc} + \varepsilon''_2 M^2_P \hat{\Omega}_{abc}^{+} \hat{\Omega}^{+abc}.
\end{align*}
\]  
(65)
This would, in particular, violate causality for the “vector” graviton, as well as modify interactions for the tensor graviton. Phenomenologically, these and similar modifications could be done as small as necessary by the choice of the numerical parameters \( \varepsilon \). This is insured by the fact that in the limit when these parameters vanish, the symmetry of the theory increases up to the general covariance.

**Affine symmetry violation** The derivative couplings above preserve the affine symmetry, though violating the general covariance. It is conceivable another way of the general covariance violation by introducing into the tangent space Lagrangian the potential \( U_g(\kappa) \), which contains only the derivativeless couplings of the Goldstone boson. Of necessity, this would explicitly violate the affine symmetry, too. To preserve the local Lorentz symmetry, the potential should depend only on \( \gamma_{\alpha\beta} \) (and/or \( \gamma^{\alpha\beta} \)). Not to violate the global Lorentz symmetry, too, the potential is to be chosen as a Lorentz invariant function as follows:
\[ U_g = U_g \left( \det \gamma, \text{tr} (\gamma \eta)^n \right), \]  
(66)
\(^{11}\)Under \( \bar{g} = -1 \), the given GR extension reduces to that of ref. [7].
with any degree \( n \). In the above, one puts \((\gamma \eta)^A_B \equiv \gamma_{AA'} \eta^{A'B} \), were as before \( A = a \) or \( \alpha \), etc, as appropriate. At \( n < 0 \), one uses the relation \((\gamma \eta)^n = (\eta \gamma^{-1})^{|n|} \), with \( \gamma^{-1 \alpha \beta} \equiv \gamma^{\alpha \beta} \).

It follows thereof that in the world terms the potential should depend on \( g \bar{g}^{-1} \):

\[
U_g = U_g \left( \det (g \bar{g}^{-1}), \text{tr} (g \bar{g}^{-1})^n \right),
\]

(67)

with the background metric given by eqs. (57), (58). Generally, one has \( \bar{g}^{-1 \mu \nu} \neq \bar{g}^{\mu \nu} \equiv g^{\mu \nu} \bar{g}^{\nu \mu} \) (and similarly, \( \bar{g}^{-1}_{\mu \nu} \equiv g_{\mu \nu} g^{\nu \rho} \bar{g}^{-1}_{\rho \mu} \neq \bar{g}_{\mu \nu} \)). At the negative \( n \), one puts \((g \bar{g}^{-1})^n = (\bar{g} g^{-1})^{|n|} \), with \( g^{-1 \mu \nu} \equiv g^{\mu \nu} \). In this, the terms depending only on \( \det (g \bar{g}^{-1}) = e^{2 \sigma} \) are unimodular covariant. The potential above corresponds to the case of the most general graviton mass with the Lorentz symmetry preservation.\(^{12}\)

With advent of the potential, the only modification of the gravity equation of motion is the appearance of the extra piece \( \Delta G_{\mu \nu}^{(U)} \) in the l.h.s. of eq. (55). The Bianchi identity states the covariant divergenceless of the gravity tensor \( G_{\mu \nu} \), eq. (56). Due to this identity, there appear four constraints on the metric field and its first derivative. These constraints substitute the Lorentz-Hilbert gauge condition. Thus at the level of the equation of motion, the theory describes six physical degrees of freedom, the massive scalar and tensor gravitons. Choosing different contributions to \( U_g \), one can vary the relation between the respective masses. In the limit of vanishing potential, the general covariance is restored and one recovers smoothly the GR with the massless two-component tensor graviton.

One more similar source of the general covariance violation could be due to the derivativeless couplings of the affine Goldstone boson with matter. Violating the affine symmetry, all the derivativeless couplings are expected naturally to be suppressed (if any). This is in distinction with the extra terms depending on the derivatives of the Goldstone boson. The latter terms also result in the general covariance violation. Nevertheless, being affine invariant, they are not expected a priori to be small.

This exhausts the foundations of the effective field theory of the gravity, radiation and matter. The above theory, embodying the GR and its extensions in the framework of the affine symmetry and the general relativity, may be called the metagravitation.

\section{Metauniverse}

\subsection{World continuum}

The ultimate goal of the Goldstone approach to gravity is to go beyond the effective metric theory and to build the underlying premetric one. In what follows, we present some hints of the respective scenario. Of necessity, we will be very concise, just to indicate the idea.

The forebear of the space-time is supposed to be the world continuum. At the very least, the latter is to be endowed with the defining structure, the continuity in the topological sense. Being covered additionally with the patches of the smooth real coordinates \( x^\mu \), \( \mu = 0, 1, \ldots, d - 1 \) (index 0 having yet no particular meaning), the continuum acquires the structure of the differentiable manifold of the dimension \( d \) (4, for definiteness). There exist in the continuum the tensor densities, in particular, the volume element. Thus, the

\footnote{For the theory of the massive tensor field in the Minkowski background space-time see, e.g., ref. \cite{8}. For the phenomenology of the graviton mass and for further references on the subject cf., e.g., ref. \cite{9}.}
integration over the manifold is allowed. But this does not suffice to define the covariant derivative and thus to get the covariant differential equations, etc. Suppose now, that the continuum can exist in two phases with the following affinity properties.

**Affine connection**  Being endowed with the primordial affine connection $\tilde{\phi}^\lambda_{\mu\nu}$, the continuum becomes the affinely connected manifold. Generally, the connection is the 64-parametric structure. It defines the parallel transport of the world vectors, as well as their covariant derivatives. The parallel transport along the infinitesimal closed contour defines, in turn, the background curvature tensor $\tilde{\rho}^\lambda_{\mu\nu}$ and thus its contraction $\tilde{\rho}_{\mu\nu} = \tilde{\rho}^\lambda_{\mu\lambda\nu}$ (but not yet the scalar $\tilde{\rho}$). To every point $P$, there can be attached the coordinates $\tilde{\xi}^\alpha$, where the symmetric part of the connection locally nullifies, the manifold becoming thus locally affinely flat. This defines the global affine symmetry. For the symmetry to be exact, the antisymmetric part of the connection, the torsion, should be trivial, with the connection being just 40-parametric. In this phase, there is yet no metric and thus no space and time directions, even no definite space-time signature, no lengths and angles, no preferred Lorentz group and thus no finite dimensional spinors, no preferred Poincare group and thus no conventional particles, no invariant intervals, no quadratic invariants, no causality, etc. Though there can be implemented the principle of the least action with the primitive Lagrangians, the world structure is still rather dull. Nevertheless, it should ultimately lead to the spontaneous transition from the given phase to the metric one.

**Metric**  Further, being endowed spontaneously with the metric $\tilde{\phi}^{\mu\nu}$ having the Minkowskian signature, the continuum becomes the metric space, i.e., the space-time. The metric is much more restrictive 10-parametric structure. It defines the background Riemannian geometry. Accompanying the emergence of the metric and the spontaneous breaking of the affine symmetry, there appears the affine Goldstone boson serving as the graviton in disguise. This results in the effective Riemannian geometry with the effective metric $g^{\mu\nu}$, etc. Now there appear the preferred time and space directions, the lengths and angles, the definite Lorentz group and thus the finite dimensional spinors, the definite Poincare group and thus the particles, the invariant intervals, the quadratic invariants, the causality, etc. The world structure becomes now very flourishing. In the wake of the gravity, there appear the conventional matter. The spontaneous breaking of the affine symmetry to the Poincare one reflects the appearance of the coherent particle structure, among a lot of a priori possible ones corresponding to the various choices of the Poincare subgroup. Formally, the effective Riemannian geometry is to be valid at all the space-time intervals. Nevertheless, its accuracy worsen when diminishing the intervals, requiring more and more terms in the decomposition over the ratio of the energy to the symmetry breaking scale $M_A$, as it should be for the effective theory. Thus, the scale $M_A$ (or, rather, the Planck mass $M_{Pl}$) is a kind of the inverse minimal length in the nature.

### 6.2 The Universe

Conceivably, the formation of the Universe is the result of the actual transition between the two phases of the continuum. This transition is thus the “Grand Bang”, the origin not only of the Universe but of the very space-time. At this stage, there appears the world “arrow
of time” as the reflection of the spontaneous synchronization of the chaotic local times. The residual dependence of the structure of the Universe on the background parameter-functions could result in the variety of the primordial effects, such as the anisotropy, inhomogeneity, etc. And what is more, there is conceivable the appearance (as well as disappearance and coalescence) of the various regions of the metric phase inside the affinely connected one (and v.v.). These regions are to be associated with the multiple universes. One of the latter ones happens to be ours. Call the ensemble of the universes the Metauniverse. Within the concept of the Metauniverse, there becomes sensible the notion of the wave function of the Universe. Hopefully, this may clarify the long-standing problem of the fine tuning of our Universe.\footnote{13}{Cf., e.g., ref. \cite{10}.}

7 Conclusion

To conclude, the theory proposed realizes consistently the approach to gravity as the Goldstone phenomenon. It proceeds, in essence, from the two basic symmetries: the global affine one and the general covariance. The affine symmetry is the structure symmetry which defines the theory in the small. The general covariance is the bundling symmetry which terminates the a priori admissible local theories according to their ability to be prolonged onto the space-time. The theory embodies the GR as the lowest approximation. Its distinction with the GR are twofold. At the effective level, the given theory predicts the natural hierarchy of the conceivable GR extensions, according to the accuracy of the affine symmetry realization. At the underlying level, the theory presents the new look at the gravitation, the Universe and the very space-time.

The author is grateful to V. V. Kabachenko for useful discussions.

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