Macroscopic Estimate of the Average Speed for New Particles Created in Collision

A. Kwang-Hua Chu

P.O. Box 39, Tou-Di-Ban, Road XiHong, Urumqi 830000

Abstract

A heuristic approach is proposed to estimate the average speed of particles during binary encounters by using the macroscopic variables with their extended gradient-type which are the fundamental independent variables in extended thermodynamics theory. We also address the missing contribution (say, due to creation of new particles: Acousticons) in conventional Bremsstrahlung.

Key Words: Dynamic Casimir effect; Bremsstrahlung; stationary conservation form; hard-sphere.

PACS Codes: 02.30.Jr; 05.60.-k; 34.10.+x; 12.90.+b

1 Introduction

The rough approximations to the velocity of particles for a specific distribution have been a fundamental issue in the kinetic theory of gas [1]. Normally the molecular speed varies with the molecular weight and absolute temperature if we only consider the translation part of the kinetic energy [2] for a gas in a uniform steady state.

We shall discuss the approximate estimate of average particle speed (in 1-D. sense, \(c_x\)) from the stationary equations of wave-breaking-like conservation laws [3-4]. The flow is assumed to be uniformly bounded and avoid the vacuum state. Since a conservation law is an integral relation, it may be satisfied by functions which are not differentiable (like the discrete particle-or molecule-based flow using Boltzmann approach for dilute gas), not even continuous, merely measurable and bounded.

We noticed that steady shocks can occur in an ideal case [5-6] or in a microscopic way [7-9]. In this short paper, we will investigate this kind of 1D flow in a heuristic way. The flow field (if in terms of the flow velocity) depends on the pressure gradient and density gradient only.
2 Formulation

2.1 Stationary Weak Shock

Starting from the integral form of the balanced equations for the one-dimensional flow allowing discontinuity in x-direction velocity \( u \) (cf. Fig. 1):

\[
\frac{d}{dt} \int_{x_l}^{x_r} f \, dx + [g]_{x_l}^{x_r} = 0,
\]

(1)

here, \([\cdot]\) relates to the jump [10]; \(f, g\) can be the density and flux of mass, or the density and flux of momentum, and we neglect the source-term effects, e.g., body force in the momentum-balance analogy. We assume that \( u \) has continuous first derivatives and \( f, g \) are functions of \( x, t, u \). Thus, together with jump condition [10] and entropy condition for a weak solution [3,4], (1) becomes

\[
\frac{\partial f(x, t, u)}{\partial t} + \frac{\partial g(x, t, u)}{\partial x} = 0.
\]

(2)

Let \( f \) be the density and flux of mass, \( g \) be the flux of mass and momentum, then we have

\[
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} = 0,
\]

(3)

\[
\frac{\partial (\rho u)}{\partial t} + \frac{\partial (\rho u^2 + p)}{\partial x} = 0.
\]

(4)

Here,

\[
p = -\frac{1}{3} p_{ii} = \frac{1}{3} \int_{-\infty}^{\infty} m c_i^2 F \, dc,
\]

\( F \) is the velocity distribution function of molecules from the kinetic theory of gases, \( m \) is the mass of a molecule, \( c \) (or \( c_i \)) measures the difference of the molecular velocity from the mean value or macroscopic velocity \( \int_{-\infty}^{\infty} m v f \, dv \); \( v \) is the absolute molecular velocity.

The stationary solution \( u \) from Eqns. (3) and (4) then is

\[
u = (\frac{\partial p}{\partial x})^{1/2}.
\]

(5)

For the cases of weak shocks, Eqn. (5) tends to the characteristic velocity

\[
U = (\frac{\partial p}{\partial \rho})^{1/2},
\]

(6)
in the limit as the shock strength approaches zero, which is just the generalization of 1D sound speed. Up to now, the internal energy $e$ or the enthalpy $h = e + p/\rho$, which for the ideal gas is a function of temperature alone, are still not specified yet. Besides, we neglect the viscous and heat-conducting effects in general. Thus, this kind of stationary shock can only exist either in discrete sense [11-12] or in microscopic way (e.g., induced by molecular collisions) [9,13]. Considering the time scale of collisions, e.g., the mean collision time, since it is much shorter than the relaxation time, so, once we neglect the high-frequency behavior or relaxation effects and only take the low-frequency limit into account, then the stationary shock concept is valid.

2.2 Application to Bremsstrahlung and New Particles Creation

The conservation equations obtained after we impose weak formulations from the integral forms which are similar to the treatments of weak-shock problems [3] are constructed from the collision diagram as shown in Fig. 1 with respect to the axes $x$ and $y$. The one-dimensional velocity $c$ (in average) for particles during a binary encounter is in the $x$-direction for simplicity. The stationary equations, if we neglect the time-dependent effects, are

\begin{align}
(p c)_x &= 0, \\
(p c^2 + p)_x &= 0.
\end{align}

(7)

c has been spatially and locally homogenized [14] with $c \in C^1$. Thus, similar to the derivation of $u$ (cf. the equation (5)) we can get the average estimate of $c$ as

$$c = (\frac{p_x}{\rho_x})^{1/2}.$$  

Note that this velocity could be linked to the sound speed by the extended thermodynamics theory [15-16] and might be related to the neglected (energy) contributions during Bremsstrahlung (or collisions of particles) since it is rather weak (however, it should not be neglected considering the strong and weak interactions of particles) compared to the photon emission or others! To be precise, the approach we used above could be applied to the dynamic Casimir effect and the more interesting manifestation of the dynamic behavior is the creation of particles from vacuum by a moving boundary (here, the moving boundary is related to the non-flat shape).
The vacuum fluctuations, according to the Casimir effect, can generate the pressure field and thus the acoustic field as mentioned above! The effect of creation of particles from vacuum by nonstationary electric and gravitational fields is well known (see, e.g., [20,21]). As noted above, boundary conditions are idealizations of concentrated external fields. It is not surprising, then, that moving boundaries act in the same way as a nonstationary external field. As to the possibility of experimental observation of the photons created by the moving mirrors. Additional factors such as imperfectness of the boundary mirrors, back reaction of the radiated photons upon the mirror, etc., could be traced in [22].

3 Results & Discussions

From Fig. 1 we know that, if we transform the coordinate system into the one based on the mass-center of these two colliding particles or molecules, as the particles are assumed to be hard-sphere ones and have equal mass, so, the mass-center (located at the contact point or the cross of $x$ & $y$-axis) will move with the speed ($c_{av}$) of total momentum divided by the total mass [23]. The collisions are assumed to be elastic. In fact, $c_{av}$ is equivalent to the speed of one-dimensional shock front. The energy associated to this velocity is rather weak (in intermediate regime) and thus was neglected in conventional Bremsstrahlung (emission of photons or other radiations) but, as mentioned above, it should be considered in the weak and strong interactions. The remaining question is how to detect this kind of energy in the test section for creation of new particles (say, if we termed these particles as Acoustons) subjected to collisions.
Fig. 1  A head-on collision; after homogenization.

Acknowledgements

This work was the extension part of the author’s PhD thesis (dated 1997-Dec.) [16].

References

[1] J.H. Jeans, The Dynamic Theory of Gases (Cambridge University Press, 4th. ed., 1925) p. 25, 118.

[2] S. Chapman and T.G. Cowling, The Mathematical Theory of Non-Uniform Gases, Cambridge University Press, 3rd. ed., 1970, p. 36.

[3] P.D. Lax, Hyperbolic Systems of Conservation Laws and the Mathematical Theory of Shock Waves, SIAM, 1973, p. 4.

[4] R.J. DiPerna, in Nonlinear Partial Differential Equations in Applied Science; Proc. of the US-Japan Seminar, Tokyo, 1982 (eds. H. Fugita, P.D. Lax, and G. Strang, Lecture Notes in Num. Appl. Anal. 5, North-Holland Publ. Co., 1983) p. 1.

[5] Z.-Y. Han and X.-Z. Yin, Shock Dynamics, Kluwer Academic Pub., New York, 1993, pp. 76 & 238.
[6] I.I. Glass and J.P. Sislian, Nonstationary Flows and Shock Waves, Clarendon Press, Oxford Engg. Sci. Series, 39; London, 1994, p. 37.

[7] K.G. Gureev and V.O. Zolotarev, Zhurnal Tekhnicheskoi Fiziki 60 (Feb. 1990) 22.

[8] B.-C. Eu, Kinetic Theory and Irreversible Thermodynamics, John Wiley & Sons, Inc., New York, 1992, p. 404.

[9] A.A. Vlasov, Many-Particle Theory and its Application to Plasma, Gordon and Breach, Science Publ., New York, 1961, p. 266.

[10] G.B. Whitham, Linear & Nonlinear Waves, John Wiley & Sons, Singapore, 1974, pp. 39, 138, 170 & 208.

[11] G. Jennings, Comm. Pure Appl. Math. 27 (1974) 25.

[12] H.-L. Liu and J.-H. Wang, Math. Comp. 65 (1997) 1137.

[13] M.S. Ivanov, S.F. Gimelshein, and A.E. Beylich, Phys. Fluids 7 (1995) 685.

[14] G. Allaire, in Homogenization and Porous Media (U. Hornung ed., Springer, 1997) p. 225.

[15] I. Müller and T. Ruggeri, Extended Thermodynamics, Springer-Verlag, Berlin 1993.

[16] K.-H. Chu, PhD. Thesis. Hong Kong University of Science and Technology, Hong Kong (PR China), Jan. 1998.

[17] G.T. Moore, J. Math. Phys. 11 (1970) 2679.

[18] S.A. Fulling and P.C.W. Davies, Proc. Roy. Soc. London A 348 (1976) 393.

[19] V.V. Dodonov and A.B. Klimov, Phys. Rev. A 53 (1996) 2664.

[20] A.A. Grib, S.G. Mamayev, V.M. Mostepanenko, Vacuum Quantum Effects in Strong Fields, Friedmann Laboratory Publishing, St. Petersburg, 1994.
[21] N.D. Birrell, P.C.W. Davies, Quantum Fields in Curved Space, Cambridge University Press, Cambridge, 1982.

[22] M. Bordag, U. Mohideen, V.M. Mostepanenkoc, Phys. Rep. 353 (2001) 1.

[23] F. Reif, Fundamentals of Statistical and Thermal Physics, McGraw-Hill, Inc., 1965, p. 516.