Non-isothermal analysis of die corner gap formation for materials deformed by multi-pass ECAP

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Abstract. In this paper, the upper-bound solutions proposed by Eivani and Karimi Taheri [Comp. Mater. Sci. 42 (2008) 14] to calculate processing force and evaluate die corner angle $\Psi$ formation in terms of tribology and die configurations during cold single pass equal channels angular pressed metals with constant flow stress were extended to work-hardening metals processed by two passes according to route A by using the Swift model combined to von Mises isotropic plasticity criterion. Also, adiabatic heat equation was coupled to solutions to express the final temperature of the workpiece. For that, thermomechanical properties of a hot-dip galvanized interstitial-free (IF) were considered and its behavior under pressing was evaluated to non-hardening and work-hardening conditions in all performed analyses. By including work-hardening in the models and for the critical friction factor of 0.4, theoretical predictions after single pass showed a decreasing of die corner angle and pressing force predictions and increasing of effective plastic strain and end temperature for all friction conditions and tooling geometries evaluated. In addition, after second pass, these responses showed higher values. Finally, with the proposed upper-bound models it was possible to analyze the dependency of angle $\Psi$, effective plastic strain, pressing load and sample temperature with the instantaneous workpiece height at the entry surface of deformation zone.

1. Introduction

The equal channel angular pressing technique, henceforward, ECAP, became a very promising severe plastic deformation (SPD) process to understand the formation of bulk ultra-fine grained materials which allow to obtain improved mechanical properties [1,2]. This process is performed with a well lubricated billet, usually with a constant squared cross-section, which is forced to pass through a two-channel die designed with an intersection angle $\Phi$ owing to the action of a ram plunger.

As pointed out by Kim et al. [3], in the case of ECAP die design with an outer corner radius one must adopt the angle $\Psi$ as the arc of curvature of the workpiece instead of the die corner angle due to the formation of a corner gap between the workpiece and the die. Otherwise, the equivalent strain could be overestimated unless the workpiece fully fills the die as would be the case of the ECAP technique employing a back-pressure.

Altan et al. [4] obtained the conditions for which the deformation zone can be formed in the ECAP with an intersection die angle $\Phi = 90^\circ$ for a non-hardening billet material considering both the inner die radius and the friction between the workpiece and die walls channels. Eivani and Taheri [5] also analyzed the dead metal zone formation in the ECAP process, after single pass at room temperature, for a die with sharp corners taken into account the angle $\Phi$, friction conditions and material strain-hardening through an average shear flow stress defined from the Hollomon law along with the von Mises isotropic yield criterion.

From these previous works mentioned related to solutions for the die corner angle of curvature $\Psi$, which defines the extension of the deformation zone and plays a major role upon the resulting strain homogeneity imposed to the billet material and, therefore, is an important parameter to optimize the ECAP die design, and the lack of a general solution for work-hardening materials processed by multipasses, an extension of the original upper bound solutions of Eivani and Taheri [5] is proposed in the present work to account for both strain hardening, friction conditions and tooling geometry. Also, after each processing cycle, the workpiece final temperature can be estimated by assuming adiabatic heating condition.
2. Material and methods

2.1. Material
In this work, it was considered the mechanical behavior of a hot-dip galvanized interstitial-free (IF) steel obtained by Freitas et al. [6] from uniaxial tensile tests listed in Table 1 by fitting the experimental true stress-strain curve with Swift hardening power law.

| Density (kg/m³) | Specific heat (J/kg K) | K (MPa)       | ε₀       | n       | σy (0.2%) |
|----------------|------------------------|---------------|----------|---------|-----------|
| 7861.09        | 481                     | 576.53        | 0.0164   | 0.309   | 170.33    |

2.2. Methods
According to analytical modeling strategy adopted in the present work, the material presents an initial yield stress, σy, before its crossing at the die channels intersection region, called zone II in Fig. 1 and inside zone II it undergoes the mean effective stress, $\overline{\sigma}_{DZ}$, that is:

$$\overline{\sigma}_{DZ} = \frac{1}{\varepsilon} \int_{\varepsilon_0}^{\varepsilon} K (\varepsilon_0 + \varepsilon)^n \, d\varepsilon = \frac{K}{\varepsilon (n + 1)} \left[ (\varepsilon_0 + \varepsilon)^{n+1} - \varepsilon_0^{n+1} \right]$$

(1)

where $\overline{\varepsilon}$ denotes the accumulated effective plastic strain during the material processing.

Figure 1. Deformation model adopted in the present work. Adapted from [5]
At the end of zone II and at the zone III, it is assumed that the material mechanical behavior is described by the final effective stress, $\bar{\sigma}_F$, which is given by:

$$\bar{\sigma}_F = K(\varepsilon_0 + \varepsilon)^n$$

(2)

Based on the work of Eivani and Karimi Taheri [5] and taking into account the deformed model presented in Fig. 1, in the sense of von Mises criterion, the proposed extended solution to express the pressing force $F$ is given by:

$$F = \frac{L^2}{\sqrt{3}} \sigma_{DZ} - \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Phi + \Psi}{2}}{\sigma_f \cotan \frac{\Phi + \Psi}{2}} + \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Phi + \Psi}{2}}{\sigma_f \cotan \frac{\Psi}{2}} + \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Psi}{2}}{\sigma_f \cotan \frac{\Psi}{2}} + \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Psi}{2}}{\sigma_f \cotan \frac{\Psi}{2}} +$$

$$+ \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Psi}{2}}{\sigma_f \cotan \frac{\Psi}{2}} + \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Psi}{2}}{\sigma_f \cotan \frac{\Psi}{2}} + \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Psi}{2}}{\sigma_f \cotan \frac{\Psi}{2}} + \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Psi}{2}}{\sigma_f \cotan \frac{\Psi}{2}} +$$

$$+ \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Psi}{2}}{\sigma_f \cotan \frac{\Psi}{2}} + \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Psi}{2}}{\sigma_f \cotan \frac{\Psi}{2}} + \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Psi}{2}}{\sigma_f \cotan \frac{\Psi}{2}} + \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Psi}{2}}{\sigma_f \cotan \frac{\Psi}{2}} +$$

$$+ \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Psi}{2}}{\sigma_f \cotan \frac{\Psi}{2}} + \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Psi}{2}}{\sigma_f \cotan \frac{\Psi}{2}} + \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Psi}{2}}{\sigma_f \cotan \frac{\Psi}{2}} + \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Psi}{2}}{\sigma_f \cotan \frac{\Psi}{2}} +$$

$$+ \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Psi}{2}}{\sigma_f \cotan \frac{\Psi}{2}} + \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Psi}{2}}{\sigma_f \cotan \frac{\Psi}{2}} + \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Psi}{2}}{\sigma_f \cotan \frac{\Psi}{2}} + \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Psi}{2}}{\sigma_f \cotan \frac{\Psi}{2}} +$$

(3)

where $L$, $H_{IN}$ and $H_{OUT}$ denote the die channels width and instantaneous billet height at the entry and exit deformation zone surfaces, see Fig. 1, respectively. It was considered $L$ equals to 10 mm. In addition, $\Phi$ is the die channels intersection angle and $m$ defines the Tresca's friction factor.

Regarding volumetric incompressibility it is possible to obtain a relationship between the instantaneous heights $H_{IN}$ and $H_{OUT}$, i.e.,

$$H_{OUT} = H_{INITIAL} - L \cotan \frac{\Phi}{2} - H_{IN}$$

(4)

where $H_{INITIAL}$ denotes billet total height that was assumed to be 50 mm. Also, according to reference [5], it is possible to find that, $H_{IN} = H_{INITIAL} - L = 40$ mm. For materials with constant flow stress ($\sigma_D$ and $\sigma_{DZ}$ equal to $\sigma_f$) and by introducing the parameter $H = H_{IN} + H_{OUT}$, the Eq. (3) returns that proposed by Eivani and Karimi Taheri [5].

To calculate the effective plastic strain and also the flow stresses defined by Eqs. (1-2) it was used the solution proposed by Iwahashi et al. [7] that is given by:

$$\frac{\partial F}{\partial \Psi} = 0$$

(6)

And by applying (6) into (3) one can obtain a nonlinear function in terms of $\Psi$ that clearly exhibits its dependence with tribology, tooling geometry and material properties. Thus:

$$\frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Phi + \Psi}{2}}{\sigma_f \cotan \frac{\Phi + \Psi}{2}} + \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Phi + \Psi}{2}}{\sigma_f \cotan \frac{\Phi + \Psi}{2}} + \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Phi + \Psi}{2}}{\sigma_f \cotan \frac{\Phi + \Psi}{2}} + \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Phi + \Psi}{2}}{\sigma_f \cotan \frac{\Phi + \Psi}{2}} +$$

$$+ \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Phi + \Psi}{2}}{\sigma_f \cotan \frac{\Phi + \Psi}{2}} + \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Phi + \Psi}{2}}{\sigma_f \cotan \frac{\Phi + \Psi}{2}} + \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Phi + \Psi}{2}}{\sigma_f \cotan \frac{\Phi + \Psi}{2}} + \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Phi + \Psi}{2}}{\sigma_f \cotan \frac{\Phi + \Psi}{2}} +$$

$$+ \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Phi + \Psi}{2}}{\sigma_f \cotan \frac{\Phi + \Psi}{2}} + \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Phi + \Psi}{2}}{\sigma_f \cotan \frac{\Phi + \Psi}{2}} + \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Phi + \Psi}{2}}{\sigma_f \cotan \frac{\Phi + \Psi}{2}} + \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Phi + \Psi}{2}}{\sigma_f \cotan \frac{\Phi + \Psi}{2}} +$$

$$+ \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Phi + \Psi}{2}}{\sigma_f \cotan \frac{\Phi + \Psi}{2}} + \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Phi + \Psi}{2}}{\sigma_f \cotan \frac{\Phi + \Psi}{2}} + \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Phi + \Psi}{2}}{\sigma_f \cotan \frac{\Phi + \Psi}{2}} + \frac{L^2}{\sqrt{3}} \frac{\sigma_y - \sigma_f \cotan \frac{\Phi + \Psi}{2}}{\sigma_f \cotan \frac{\Phi + \Psi}{2}} +$$

(7)
The first derivatives in the Eq. (7) are given by:

\[
\frac{\partial \sigma_{\text{ZD}}}{\partial \Psi} = \frac{K}{(n+1)} \left[ \frac{(n+1)}{\epsilon} (\epsilon_o + \epsilon)^n \frac{\partial \epsilon}{\partial \Psi} \frac{1}{\epsilon^2} (\epsilon_o + \epsilon)^{n+1} \frac{\partial \epsilon}{\partial \Psi} \right]
\]

\[
\frac{\partial \epsilon}{\partial \Psi} = nK(\epsilon_o + \epsilon)^{n-1} \frac{\partial \epsilon}{\partial \Psi}
\]

\[
\frac{\partial \epsilon}{\partial \Psi} = \frac{NP}{\sqrt{3}} \left\{ \csc \left( \frac{\Phi + \Psi}{2} \right) - \Psi \cot \left( \frac{\Phi + \Psi}{2} \right) \cot \left( \frac{\Phi + \Psi}{2} \right) \right\} - \left[ 1 + \cot^2 \left( \frac{\Phi + \Psi}{2} \right) \right]
\]

To describe the thermo-plastic coupling between deformation work and temperature increasing during ECAP, the adiabatic heat equation was employed here, to calculate the temperature increment after each pressing cycle, that is,

\[
\Delta T = \frac{\eta}{\rho c_p} \epsilon
\]

where \( \eta \), \( \rho \) and \( c_p \) denote the Taylor-Quinney factor (assumed to be 0.9), the material density and constant pressure specific heat, respectively. Thus, the workpiece final temperature, \( T_f \), is given by:

\[
T_f = T_{\text{ini}} + \Delta T
\]

where \( T_{\text{ini}} \) denotes the sample temperature before ECAP. As we are considering cold pressing, it was assumed \( T_{\text{ini}} = 298.15 \) Kelvin (or 25ºC).

It is important to point out that a Fortran 90 - based algorithm was developed to computationally implement Eqs. (1-10). In particular, Eq. (7) was numerically solved using bisection method to predict the angle \( \Psi \).

Table 2 lists the conditions assumed in the theoretical analyses performed in the present work. Also, to non-hardening case we have \( \sigma_y = \sigma_{\text{ZD}} = \sigma_F = K \).

| Analysis type | Parameters | Evaluated response |
|---------------|------------|-------------------|
|               | m | \( \Phi \) (degrees) | \( H_{\text{IN}} \) (mm) | \( \Psi \) | \( \bar{\epsilon} \) | T | F |
| Non-hardening | 0.0 | 1.0 | 60 | 135 | - | - | x | x | x | x |
| Hardening     | 0.4 | 1.0 | 60 | 135 | 0 | 50 | x | x | x | x |

Finally, to calculate the responses associated to second pass, it was assumed \( (\sigma_y)_{\text{pass 2}} = (\sigma_F)_{\text{pass 1}} \) and the other material parameters listed in Tab.1 were fixed.
3. Results and discussion

3.1. Effect of tribology and die geometry

Figure 2 shows the analytical predictions of angle $\Psi$ for distinct friction conditions and die channels angle $\Phi$ to either non-hardening or work-hardening IF-steel. After one pass, by adopting a hardening exponent higher than zero, see Fig. 2b, it was observed the decreasing of $\Psi$ calculated with Eq. (7), for a fixed friction factor $m$, once the material mechanical strength decreases in comparison to non-hardening assumption depicted in Fig. 2a. On the other hand, for a constant $\Phi$, higher values of $m$ are associated to greater resistance to the processing of either perfectly plastic or hardenable materials. Also, one can observe that for second pass with work-hardening assumption, see Fig. 2c, the angle $\Psi$ shows maximum predictions that violate the geometrical limit $\pi/2 \leq \Phi + \Psi \leq \pi$ observed by Eivani and Karimi Taheri [5]. It is due to constant thermomechanical properties adopted for all ECAP passes.

Figure 2. Predictions of angle $\Psi$ after one and two passes of ECAP: (a) non-hardening assumption; (b) and (c) work-hardened IF-steel.
Figure 3 presents the theoretical predictions of effective plastic strain associated to same conditions used in Fig. 2. It is clear that, according to Eq. (5), as the effective plastic strain is dependent of angle $\Psi$, it is affected by material plastic behavior. Thus, by the trigonometric characteristic of Eq. (5) one can expect a decreasing of results for work-hardening materials in comparison to non-hardening ones, as showed in Figs. 3a and 3b, for each individual $\Phi$ and $m$. However, as the predictions of angle $\Psi$ for work-hardened material were lesser than for non-hardening assumption due to critical friction factor of 0.4, the results of effective plastic strain increased for hardened one. After second pass, the increasing of angle $\Psi$ for $n>0$, see Fig. 3d, caused higher predictions for effective plastic strain in comparison to non-hardened material, see Fig. 3e.

![Figure 3](image-url)
Figure 4 shows the predictions of workpiece final temperature after multipasses of ECAP in relation to die angle $\Phi$ and friction conditions for either perfectly plastic or hardenable IF-steel. One can observe that in absence of hardening, the heating is more considerable in comparison with work-hardened material, independently of the pressing cycle. In fact, as the temperature is affected by angle $\Psi$ by means of effective plastic strain, it is verified that lesser values of this parameters leads to decreasing of sample heating.

Figure 4. Estimative of workpiece final temperature after one and two passes of ECAP: (a) and (c) non-hardening assumption; (b) and (d) work-hardened IF-steel.
Figure 5 depicts the predictions of pressing force associated to non-hardening and hardening behaviors of IF-steel, obtained from alterations of angle $\Phi$ and friction conditions. In general words, after one pass, the work-hardening consideration dropped down the necessary load to deform the material in comparison with non-hardening condition and it was consistent with the associated falling in the mechanical strength to plastic yielding. Also, the effects of angle $\Phi$ and tribology are analogous those already discussed for $\Psi$ and, as presented in Figs. 5a and 5b. For these same reasons, the predictions of pressing force after the second pass were higher for work-hardened sample those observed for non-hardened one.

(a) (b) (c)

Figure 5. Predictions of angle pressing force after one and two passes of ECAP: (a) non-hardening assumption; (b) and (c) work-hardened IF-steel.
3.2. Influence of instantaneous height $H_{IN}$

Figure 6 presents the relationship between angle $\Psi$, effective plastic strain, billet final temperature and pressing force and sample instantaneous height $H_{IN}$ for hardenable IF-steel for $\Phi = 90^\circ$ and constant friction factor. It was observed, after first pass, consistent decreasing of angle $\Psi$ and increasing of effective plastic strain. Also, higher initial values of $H_{IN}$ lead to increasing of billet temperature and a falling of pressing force. These aspects are observed by Pérez et al. [7] and Reihanian et al. [8]. After second pass, the obtained responses are associated with fixed material thermomechanical properties.

![Figure 6](image)

(a) (b) (c) (d)

Figure 6. Responses dependency with instantaneous billet height $H_{IN}$ for hardened IF-steel: (a) angle $\Psi$; (b) effective plastic strain; (c) workpiece final temperature; (d) pressing force.

4. Conclusion

Analytical solutions based on the upper-bond method reported in the literature [5] for materials with constant flow stress were extended in the present work by including material nonlinear work-hardening and evaluated in terms of variations in tooling geometry and processing conditions to estimate the die corner angle $\Psi$, effective plastic strain and pressing force for an IF-steel during after two passes of ECAP at room temperature. Therefore, the following conclusion can be outlined:
1) Theoretical results showed the consistent effect of work hardening on angle $\Psi$, effective plastic strain, workpiece final temperature and pressing force in comparison with non-hardening assumption distinct tooling geometry and friction conditions. Also, the critical friction conditions could be evaluated for analytical pressing force results. In addition, by considering the volumetric incompressibility, it was also possible to analyze the effect of billet entry deformation zone instantaneous height, $H_{IN}$ on these responses for strain-hardenable material.

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References
[1] Segal VM 1995 Mater. Sci. Eng. A 197 157.
[2] Segal VM 2002 Mater. Sci. Eng. A 338 331.
[3] Kim HS, Seo MH and Hong SI 2000 Mater. Sci. Eng. A 291 86.
[4] Altan BS, Purcek G and Miskioglu I 2005 J. Mat. Proc. Technol. 168 137.
[5] Eivani AR and Taheri AK 2008 Comp. Mat. Sci. 42 14.
[6] Freitas MCS, Moreira LP and Velloso RG 2013 Mater. Res. 16 351.
[7] Pérez CJL and Luri R 2008 Mech. Mater. 40 617.
[8] Reihanian M, Ebrahimi R and Moshksar MM 2009 Mater. Design 30 28.