We study stationary axisymmetric self-gravitating non-linear $\sigma$-model in five-dimensional spacetime admitting three commutating Killing vector fields. We show that the only asymptotically flat black ring solution with a regular rotating event horizon is the black ring characterized by mass and two angular momenta with constant mapping.

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I. INTRODUCTION

It has been renewed of interests in higher dimensional black hole solutions triggered by the unification attempts, both in the context of M/string theory as well as the brane world scenario. The unification attempts such as M/string theory described our Universe as a brane or defect emerged in higher dimensional geometry. $E_8 \times E_8$ heterotic string theory at strong coupling may be described in terms of M-theory acting in eleven-dimensional spacetime with boundaries where ten-dimensional Yang-Mills gauge theories reside on two boundaries [1]. On the other hand, the so-called TeV gravity acquires much attention to higher dimensional black hole because of the idea that such kind of objects may be produced in the near future, in high energy experiments [2]. This kind of black holes (the so-called mini black holes) are classical solutions of higher dimensional Einstein’s equations. The radius of their event horizon is much smaller that the scale of extra dimensions.

In four-dimensional spacetime the classification of non-singular black hole solutions began with the Israel’s work [3], then Müller zum Hagen et al. [4] and Robinson [5], provided other contributions to the problem [6, 7, 8, 9, 10]. Both for vacuum and Einstein-Maxwell (EM) black holes the condition of non-degeneracy of the event horizon was removed Refs.[11, 12]. It was proved that for the static electro-vacuum black holes all degenerate components of the event horizon should have charges of the same signs. However, recently this last restriction was removed from the theorem [13, 14].

The problem of uniqueness theorem for stationary axisymmetric black hole was considered in Refs.[15, 16] and the complete proof was provided by Mazur [17] and Bunting [18] (see also for a review of the uniqueness of black hole solutions story see [19] and references therein).

Studies of the low-energy string theory also renew the resurgence of works concerning the mathematical aspects of the black holes emerging in it. Namely, the staticity theorem for Einstein-Maxwell axion dilaton (EMAD) gravity was studied in Ref.[20] and uniqueness of the black hole solutions in dilaton gravity was proved in works [21, 22], while the uniqueness of the static dilaton $U(1)^2$ black holes being the solution of $N = 4, d = 4$ supergravity was provided in [23]. The extension of the proof to $U(1)^N$ static dilaton black holes was established in Ref.[24].

The possibility of production of higher dimensional black holes in accelerators caused the considerable interests in $n$-dimensional black hole uniqueness theorem, both in vacuum and charged case [25, 26, 27]. The uniqueness theorem for self-gravitating nonlinear $\sigma$-models in higher dimensional spacetime was obtained in [28].

The complete classification of $n$-dimensional charged black holes having both degenerate and non-degenerate components of event horizon was initiated in Ref.[29]. Studies of the near-horizon geometry of degenerate horizons enable to eliminate the previous restriction bounded with the inequality satisfied by the charges of the adequate components of the aforementioned black hole horizons [30]. Uniqueness theorem for $n$-dimensional static black hole carrying electric and magnetic components of $(n - 2)$-gauge containing an asymptotically flat hypersurface with compact interior and nondegenerate components of the event horizon was presented in Ref.[31]. The staticity theorem for generalized Einstein-Maxwell (EM) system in $n$-dimensional spacetime was presented in [32].

It was argued that future experiments of colliding high energy particles held great promise for illuminating the nature of mini black holes. One of the first signature of appearing black holes in future accelerator experiments will be a Hawking emission and generically the black holes will have angular momentum. This fact causes that the uniqueness of rotating black holes in higher dimensions is of a great importance in studies mathematical properties
of them. A salient feature of the axisymmetric stationary solutions in higher dimensions is the fact that they can admit event horizon with non-spherical topology in contrast to four-dimensional case. The topology of the event horizon can not be uniquely determined. For example in five-dimensional case one has the topology of $S^3$ sphere or $S^1 \times S^2$ [33], while in higher dimensions the topology is far more complicated [34, 35]. It was advocated that generalization of the Kerr metric to arbitrary $n$-dimensions proposed by Myers and Perry [36] is not unique. The counterexample showing that a five-dimensional rotating black hole ring solution has the same angular momentum and mass as five-dimensional axisymmetric stationary black hole. But as far as topology was concerned, its event horizon was homeomorphic to $S^1 \times S^2$ [37, 38]. In Ref. [39] it was shown that Myers-Perry solution is the unique black hole in five-dimensions in the class of spherical topology with three commuting Killing vectors. On the other hand, the uniqueness for stationary self-gravitating nonlinear $\sigma$-models in five-dimensional spacetime was obtained in [40], where it was shown that vacuum Myers-Perry Kerr solution is the only maximally extended stationary axisymmetric asymptotic flat solution having the regular event horizon with constant mapping.

It was proved in Ref. [41] that a higher dimensional stationary rotating black hole must be axisymmetric with no assumptions concerning the topology of the horizon cross-section other than compactness of it. The also assume that the horizon is non-degenerate and spacetime is analytic. In Ref. [42] the authors showed that two asymptotically flat five-dimensional black hole solutions of vacuum Einstein Eqs. with the same topology, mass and angular momentum and moreover with the same rod structure [43] are isometric to each other. Then, the proof was generalized to include Maxwell fields [44].

Recently, it was shown in Ref. [45] that assuming the existence of two additional commuting axial Killing vector fields and the horizon topology of black ring $S^1 \times S^2$, the only asymptotically flat black ring solution with a regular horizon is the Pomeransky-Sen’kov (PS) black ring [46].

Finding the black ring solutions triggered interests in these blossoming subject. Many other black ring, or one should say black object solutions were found e.g., black Saturn (an object consisting of rotating black rings with a spinning black hole as its center), the di-ring, the bi-ring etc. The recent summary and review of our understanding of the aforementioned problem is presented in Ref. [47]. Also thermodynamics of these black objects were intensively studied (see e.g., Refs. [48]).

In our paper we shall treat the problem of uniqueness of five-dimensional axisymmetric, stationary black ring solution for self-gravitating non-linear $\sigma$-models. In Sec. II we present general self-gravitating $\sigma$-model and establish the main result of our work that the only regular black ring solution with regular rotating event horizon is the five-dimensional vacuum PS black ring with constant mapping.

## II. FIVE-DIMENSIONAL ROTATING $\sigma$-MODELS

In this section we shall derive equations of motion for $n$-dimensional self-gravitating $\sigma$-model being subject to the following action:

$$I = \int d^n x \sqrt{-(n)} g \left[ \frac{1}{2} G_{AB}(\varphi(x)) \varphi^A_{,\mu} \varphi^B_{,\mu} \right].$$  \hspace{1cm} (1)

The equations of motion for our non-linear $\sigma$-model can be derived from the variational principle. They yield

$$\nabla_\gamma \nabla_\gamma \varphi^A + \Gamma^A_{BC} \varphi^B_{,\mu} \varphi^C_{,\mu} = 0,$$

$$\left(\frac{1}{2}G_{\mu\nu} = T_{\mu\nu}(\varphi), \right)$$  \hspace{1cm} (2)

where the energy momentum for the underlying model has the form

$$T_{\mu\nu}(\varphi) = G_{AB}(\varphi(x)) \varphi^A_{,\mu} \varphi^B_{,\nu} - \frac{1}{2} G_{AB}(\varphi(x)) \varphi^A_{,\gamma} \varphi^B_{,\gamma} g_{\mu\nu}.$$  \hspace{1cm} (3)

In what follows we shall take into account the asymptotically, five-dimensional flat spacetime, i.e., the spacetime will contain a data set $(\Sigma_{end}, g_{ij}, K_{ij})$ with scalar fields of $\varphi$ such that a spacelike hypersurface $\Sigma_{end}$ is diffeomorphic to $\mathbb{R}^4$ minus a ball. The asymptotical conditions of the following forms should also be satisfied:

$$|g_{ij} - \delta_{ij}| + r |\partial_\alpha g_{ij}| + \ldots + r^m |\partial_{a_1 \ldots a_m} g_{ij}| + r |K_{ij}| + \ldots + r^m |\partial_{a_1 \ldots a_m - 1} K_{ij}| \leq \mathcal{O}\left(\frac{1}{r}\right),$$  \hspace{1cm} (5)

where $g_{ij}$ and $K_{ij}$ are induced on $\Sigma_{end}$. $K_{ij}$ is the extrinsic curvature tensor of the hypersurface $\Sigma_{end}$. It is required that in the local coordinates on $\Sigma_{end}$ the scalar field satisfies the following fall-off condition:

$$\varphi^A = \varphi^A_{\infty} + \mathcal{O}\left(\frac{1}{r^{3/2}}\right).$$  \hspace{1cm} (6)
As we shall consider stationary axisymmetric five-dimensional spacetime, thus it will admit three commutating Killing vector fields \( k_\mu, \phi_\mu, \psi_\mu \)

\[
[k, \phi] = [k, \psi] = [\phi, \psi] = 0. \tag{7}
\]

\( k_\mu \) is an asymptotically timelike Killing vector field for which \( V = -k_\mu k^\mu \), while \( \phi_\mu \) and \( \psi_\mu \) are spacelike Killing vector fields. They have all closed orbits. Then, denoting by \( \mathcal{L} \) the Lie derivative with respect to the adequate Killing vector fields one obtains the following:

\[
\mathcal{L}_k g_{\mu\nu} = \mathcal{L}_\phi g_{\mu\nu} = \mathcal{L}_\psi g_{\mu\nu} = 0, \tag{8}
\]

The scalar field \( \varphi \) will be also invariant due to the action of Killing vector fields. Namely, we have

\[
\mathcal{L}_k \varphi = \mathcal{L}_\phi \varphi = \mathcal{L}_\psi \varphi = 0. \tag{9}
\]

The metric of the general black ring solution is given by \([15]\)

\[
ds^2 = \frac{H(y, x)}{H(x, y)} \left( dt + \Omega \right)^2 - \frac{F(x, y)}{H(y, x)} d\phi^2 - 2 J(x, y) \frac{F(x, y)}{H(y, x)} d\psi + \frac{F(x, y)}{H(y, x)} dy^2 + \frac{2k^2 H(x, y)}{(x - y)^2 (1 - \nu)^2} \left( \frac{dx^2}{G(x)} - \frac{dy^2}{G(y)} \right), \tag{10}
\]

where the range of \( x, y \) coordinates is \(-1 \leq x \leq 1 \) and \((-\lambda + \sqrt{\Delta - 4\nu})/2 \leq y < \infty \) or \(-\infty < y \leq -1 \). The considered solution has four independent parameters which are subject to the inequalities. Namely, \( 0 \leq \nu < 1, 2\sqrt{\nu} \leq \lambda < 1 + \nu, k > 0 \), and \( c \leq b < 1 \) with the additional condition of \( c \) being equal to \( \sqrt{\Delta - 4\nu}/(1 - \nu) \). It happened that constant \( c \) has the geometrical meaning as being the ratio of the radius of \( S^2 \) to the radius \( S^1 \). One can introduce the canonical coordinates \((\rho, z)\) (which explicit form is given in Ref.\([15]\)). This enables us to write the underlying metric in the Weyl-Papapetrou form as

\[
ds^2 = -\rho^2 dt^2 + f_{ab} \left( dx^a + \omega^a dt \right) \left( dx^b + \omega^b dt \right) + \frac{e^{2\sigma}}{f} \left( d\rho^2 + dz^2 \right), \tag{11}
\]

where all functions appearing in the above line element have the only \( \rho \) and \( z \) dependence. Furthermore, the metric \([11]\) can be rearrange in the form which implies

\[
ds^2 = \sigma_{ab} dx^a dx^b + \gamma_{ij} dx^i dx^j, \tag{12}
\]

where \( a, b = t, \phi, \psi \) and comprises the first two components in expression \([11]\) while \( i, j = \rho, z \) and describes \( \gamma_{ij} = X g_{ij} \). \( g_{ij} \) stands for the metric of a flat spacetime written in \((\rho, z)\) coordinates. The conformal factor is equal to \( X = e^{2\sigma}/f \). Using rules of conformal transformation, after some algebra, we find expressions for the Ricci tensor components:

\[
R_{ij} = (\gamma) R_{ij} + \frac{1}{2} \gamma_{ij} \nabla^2 \ln X - \frac{1}{2\rho X} \left( (\gamma) \nabla_i \rho (\gamma) \nabla_j X + (\gamma) \nabla_i X (\gamma) \nabla_j \rho - \gamma_{ij} (\gamma) \nabla k \rho (\gamma) \nabla k X \right) \tag{13}
\]

Consequently equations of motion may be written as

\[
R_{pp} - R_{zz} + \frac{1}{\rho X} (g) \nabla_\rho X - \frac{1}{\rho^2} + \frac{1}{4} (g) \nabla_\rho \sigma_{\rho\sigma} - (g) \nabla_\rho \sigma_{\rho\sigma} (g) \nabla_\rho \sigma_{\rho\sigma} = \frac{2}{\rho} (g) \nabla_\rho \sigma, \tag{14}
\]

\[
2R_{\rho\rho} + \frac{1}{\rho X} (g) \nabla_\rho X - (g) \nabla_\rho \sigma_{\rho\sigma} = \frac{2}{\rho} (g) \nabla_\rho \sigma, \tag{15}
\]

\[
R_{zz} + R_{pp} - (g) \nabla_\rho (g) \nabla^\rho \ln X + \frac{1}{\rho X} (g) \nabla_\rho X - \frac{1}{\rho^2} (g) \nabla^\rho \rho (g) \nabla_\rho X - \frac{1}{\rho^2} + \frac{1}{4} (g) \nabla_\rho \sigma_{\rho\sigma} (g) \nabla_\rho \sigma_{\rho\sigma} = -\frac{2}{\rho} (g) \nabla^\rho (g) \nabla_\rho \sigma, \tag{16}
\]

where \((g) \nabla \) is the derivative with respect to \( g_{ij} \) metric. The linearity of the above equations implies directly that \( \sigma(\rho, z) \) reduces to the sum of two components as follows:

\[
\sigma = \sigma(\text{vac}) + \sigma(\varphi), \tag{17}
\]
where \( \sigma(vac) \) is the solution of five dimensional vacuum equations of motion while \( \sigma(\varphi) \) is connected with the solution of matter equations.

On the other hand, equations of motion for self-gravitating non-linear \( \sigma \)-model provide the following:

\[
\frac{1}{\rho} \nabla_z \sigma(\varphi) = \frac{1}{2} G_{AB}(\varphi(x)) \left( (g) \nabla_\rho \varphi^A (g) \nabla_\varphi^B + (g) \nabla_\varphi \varphi^A (g) \nabla_\rho \varphi^B \right),
\]

\[
\frac{1}{\rho} (g) \nabla_\rho \sigma(\varphi) = \frac{1}{2} G_{AB}(\varphi(x)) \left( (g) \nabla_\rho \varphi^A (g) \nabla_\rho \varphi^B - (g) \nabla_\varphi \varphi^A (g) \nabla_\rho \varphi^B \right),
\]

\[
(g) \nabla_m (g) \nabla_m \sigma(\varphi) = -\frac{1}{2} G_{AB}(\varphi(x)) \left( (g) \nabla_\rho \varphi^A (g) \nabla_\rho \varphi^B + (g) \nabla_\varphi \varphi^A (g) \nabla_\rho \varphi^B \right).
\]

In order to prove the uniqueness theorem for five-dimensional axisymmetric stationar non-linear \( \sigma \)-model we shall use the idea presented in Ref. [49]. First, one chooses a two-dimensional vector in the form as

\[
\Pi_j = \rho \ (g) \nabla_j e^{-\sigma(\varphi)}.
\]

Then, by virtue of Stokes’ theorem for \( \Pi_j \) vector and integration over the region \( \Sigma = \{(\rho, z) \mid \rho \geq 0, -\infty < z < \infty\} \) we get

\[
D_{\partial \Sigma} = \int_{\partial \Sigma} \rho e^{-\sigma(\varphi)} \left( (g) \nabla_z \sigma(\varphi) \, d\rho - (g) \nabla_\rho \sigma(\varphi) \, dz \right)
\]

\[
= \int_{\Sigma} d\rho dz \rho e^{-\sigma(\varphi)} \left[ (g) \nabla^i \sigma(\varphi) \nabla_i \sigma(\varphi) - \frac{1}{\rho} (g) \nabla_\rho \sigma(\varphi) + (g) \nabla_i (g) \nabla_i \sigma(\varphi) \right].
\]

From Eqs. (18), (19) and (20) it follows in particular that the second term on the right-hand side of (22) is greater or equal to zero. It implies that the right-hand side is the sum of two non-negative terms.

Now, let us calculate the left-hand side of expression (22). In order to do so we have to decompose the integral over the segments of the rod and the integral over infinity. So let us give a brief account of the rod structure of the underlying black ring [42, 43]. Namely, the rod structure we should take into account is as follows:

1. the semi-infinite spacelike rod \([-\infty, -ck^2]\) and the finite rod \([ck^2, k^2]\) have the direction \( v = (0, 0, 1) \). It means that for \( \rho = 0, z \in [-\infty, -ck^2] \) and for \( \rho = 0 \) and \( z \in [ck^2, k^2] \) the following relation is fulfilled \( g_{ij} v^j = 0 \).

   Because of the fact that \( g_{\psi\psi} = 0 \) the conditions \( \rho = 0, z \in [-\infty, -ck^2] \) together with \( \rho = 0 \) and \( z \in [ck^2, k^2] \) denote \( \psi \)-axis,

2. the finite timelike rod for the coordinates range \( \rho = 0 \) and \( z \in [-ck^2, ck^2] \) with \( g_{ij} v^j = 0 \). It corresponds to the event horizon with topology \( S^1 \times S^2 \). The Killing vector field \( \psi_\mu \) vanishes on both sides of this rod, while vector \( v_\mu \) has values \((1, \Omega_\phi, \Omega_\psi)\), where \( \Omega_\phi \) denotes the angular velocity along the direction of \( \phi_\mu \) Killing vector field and \( \Omega_\psi \) is the angular velocity along the direction of Killing vector \( \psi_\mu \).

3. The semi-infinite spacelike rod for the range of coordinates \( \rho = 0, z \in [k^2, \infty] \), with the direction vector \( v = (0, 1, 0) \). Since one has that \( g_{\phi\phi} = 0 \) than it denotes \( \phi \)-axis.

Having in mind the rod structure of the black ring solution, as we have mentioned, one has to decompose the boundary integral on the left-hand side of Eq. (22) over the segments of the rod and the integral over infinity. Consequently with this remark it leads to the following:

\[
\partial \Sigma = \partial \Sigma_1 + \partial \Sigma_2 + \partial \Sigma_3 + \partial \Sigma_4 + \partial \Sigma_\infty,
\]

we have denoted

\[
\begin{align*}
\partial \Sigma_1 &= \{ \rho = 0, \ z \in [-\infty, -ck^2] \}, \\
\partial \Sigma_2 &= \{ \rho = 0, \ z \in [-ck^2, ck^2] \}, \\
\partial \Sigma_3 &= \{ \rho = 0, \ z \in [ck^2, k^2] \}, \\
\partial \Sigma_4 &= \{ \rho = 0, \ z \in [k^2, \infty] \}, \\
\partial \Sigma_\infty &= \{ (\rho, \ z) \mid \sqrt{\rho^2 + z^2} \rightarrow \infty \ with \ z/\sqrt{\rho^2 + z^2} \ finite \}. 
\end{align*}
\]
Having in mind relations (25) and (26) we can establish that \( \sigma(\phi) \), \( \sigma(\phi) \mu \) and \( e^{-\sigma(\phi)} \) remain finite along the boundaries \( \partial \Sigma^{(i)} \) for \( i = 1, 2, 3, 4 \) and they all vanish along these parts of the boundary. Just, it remains to consider the last part of the boundary \( \partial \Sigma_{\infty} \). In order to do so we introduce the coordinate \((r, \theta)\) defined by \( \rho = \frac{r^2}{2} \sin 2\theta \) and \( z = \frac{r}{2} \cos 2\theta \). Using Eqs. (18) and (19) in \((r, \theta)\) coordinates one reaches to the following expressions for \( \sigma(\phi),r \) and \( \sigma(\phi),\theta \):

\[
\sigma(\phi),r = \frac{G_{AB}}{2} \left[ \frac{r}{\sin^2 2\theta} \varphi^A_r \varphi^B_r + \frac{ctg4\theta}{2r} \varphi^A_{\theta} \varphi^B_{\theta} - \frac{ctg4\theta}{2\cos^2 2\theta} \varphi^A_{r} \varphi^B_{r} \right],
\]

\[
\sigma(\phi),\theta = \frac{G_{AB}}{4} \left[ -r^2 ctg2\theta \varphi^A_r \varphi^B_r + \frac{1}{\sin 2\theta} \varphi^A_{\theta} \varphi^B_{\theta} + 4r^2 \cos^2 2\theta \varphi^A_{r} \varphi^B_{r} \right].
\]

Hence, in terms of Eqs. (25) and (26) we arrive at the following relation:

\[
D_{\partial \Sigma_{\infty}} = \lim_{r \to \infty} \int d\theta \left( e^{-\sigma(\phi)} r^3 \sigma, r \sin 2\theta - e^{-\sigma(\phi)} r^2 \sigma, \theta \sin 2\theta ctg4\theta + O\left( \frac{1}{r^n} \right) \right),
\]

(27)

where \( n \geq 2 \). Having in mind the asymptotical properties of the derivatives of scalar field \( \phi \)

\[
\varphi^A_r = O\left( \frac{1}{r^{3/2}} \right), \quad \varphi^A_{r,\theta} = O\left( \frac{1}{r^{5/2}} \right),
\]

(28)

we conclude that the above entire integral vanishes to the fact that \( \lim_{r \to \infty} r^3 \sigma, r = 0 \) and \( \lim_{r \to \infty} r^2 \sigma, \theta = 0 \). Hence \( \partial_{\rho} \nabla^a \sigma(\phi) \) and \( \frac{1}{\rho} \nabla_{\rho} \sigma(\phi) + (\sigma/\nabla^a \nabla(\phi)) \) are equal to zero. It occurs that \( \sigma(\phi) \) is constant in the considered domain \( \Sigma \), but using the fact that \( \sigma(\phi) \) tends to zero as \( r \to \infty \) we get that \( \sigma(\phi) = 0 \) which in turn implies that \( \phi \) is constant in the entire domain \( \Sigma \). Just from Eq. (17) one can deduced that \( \sigma_{(\text{vac})} \) is the only solution of equations of motion. In Ref. [15] uniqueness of the asymptotically flat, stationary five-dimensional black ring solution being the solution of Einstein vacuum equations with regular event horizon homeomorphic to \( S^1 \times S^2 \) and admitting three commutating Killing vector fields (two spacelike and one timelike), specified by mass and two angular momenta and the ratio of the radius of \( S^2 \) to \( S^1 \) was shown. Thus, we can assert the main conclusion of our work: Theorem:

Let us consider a stationary axisymmetric solution to five-dimensional self-gravitating non-linear \( \sigma \)-models with an asymptotically timelike Killing vector field \( k_\mu \) and two spacelike Killing vector fields \( \phi_\mu \) and \( \psi_\mu \). The scalar field is invariant under the action of the Killing vector fields. Then, the only asymptotically flat black ring solution with regular rotating event horizon is the five-dimensional PS vacuum black ring solution with a constant mapping \( \phi \).

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