CONNECTION BETWEEN THE LARGEST LYAPUNOV EXPONENT, DENSITY FLUCTUATION AND MULTIFRAGMENTATION IN EXCITED NUCLEAR SYSTEMS

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Abstract

Within a quantum molecular dynamics model we calculate the largest Lyapunov exponent (LLE), density fluctuation and mass distribution of fragments for a series of nuclear systems at different initial temperatures. It is found that the LLE peaks at the temperature ("critical temperature") where the density fluctuation reaches a maximal value and the mass distribution of fragments is best fitted by the Fisher’s power law from which the critical exponents for mass and charge distribution are obtained. The time-dependent behavior of the LLE and density fluctuation is studied. We find that the time scale of the density fluctuation is much longer than the inverse LLE, which indicates

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that the chaotic motion can be well developed during the process of fragment formation. The finite-size effect on "critical temperature" for nuclear systems ranging from Calcium to superheavy nuclei is also studied.

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1. INTRODUCTION

Recent interest in the phase transition in finite nuclear systems [1,2] and the study of the related dynamical feature has stimulated the investigation of the, so far, obscure relation between an anomalous increase of fluctuations at a phase transition and a rapid increase of chaoticity at the microscopic level. In this pursuit several papers have been published [3–11]. In [3–9] the studies were carried out for the excited drops made up of particles interacting via Lennard-Jones (LJ) potential, whereas the authors in [10] and [11] used the microcanonical lattice gas model and Vlasov model, respectively.

A way to characterize the dynamics in the phase transition is to calculate the largest Lyapunov exponent (LLE), which is a measure of the sensitivity of the system to initial conditions and also gives an idea of the velocity at which the system explores the available phase space. In [12] the LLE has been used to study the solid-like to liquid-like phase transition in LJ clusters. In this case the LLE can be understood as an average of the behavior of the system along an infinite trajectory in phase space. However in the case of nuclear fragmentation under study, a nucleus evolves from a highly excited state into a set of nucleons and clusters, it means that a given trajectory in the phase space will never come back close to initial state of the system. Hence such the average over the infinite trajectory will erase the relevant information of the critical behavior. In order to avoid this feature we have to calculate the local-in-time LLE over an ensemble of trajectory whose initial condition is consistent with the nucleus at a given excitation energy. Because of the absence of boundary conditions, the ergodic theorem could not be applied. Therefore, we
pay a great attention to study the time scales of the inverse LLE and density fluctuation. We find that the time scale of the inverse LLE is much less than that of density fluctuation. This means that the dynamics during multifragmentation is chaotic enough, as a result the different events can sample the whole phase space and the ensemble of trajectories becomes equivalent to the infinitely long trajectory of the system. In this way the LLE calculated over an ensemble of trajectories can carry the full information of multifragmentation.

In this paper we employ the quantum molecular dynamics (QMD) model to study the critical behavior of real nuclear systems where the effective nuclear force (Skryme force) and Coulomb force are included. With the QMD model we first study the time evolution of both the LLE and density fluctuation to find their time scales which are very important in this study. Then we study the chaoticity characterized by the LLE, the density fluctuation, and the mass (and charge) distribution of fragments at various initial temperatures, so that we can obtain the correlation between the characteristic temperatures for the system reaching a maximal chaoticity, maximal density fluctuation, and attaining a power law mass spectra. In order to study finite size effects on ”critical temperature”, numerical studies are carried out for nuclear systems ranging from $^{40}\text{Ca}$ to the superheavy nucleus $^{298}\text{Hg}$. We expect that, through our study, a better understanding of nuclear multifragmentation and its relation to the Lyapunov instability are obtained.

This paper is organized as follows. In Sec.2 we briefly introduce the quantum molecular dynamics model (QMD) used in our numerical calculations. Then the calculated results are shown in section 3. In Sec.4, a possible connection between the LLE and the fluctuation of nuclear density is discussed preliminary. Finally, a brief summary is given in Sec.5.

2. MODEL

The QMD model is employed in describing the dynamic evolution of an excited nuclear system, which contains not only some quantum features but also many-body correlations. Therefore, it has been widely used [13–15] in modelling intermediate and high energy heavy-ion collisions, and it successfully provides much dynamic information about nuclear reactions
and multifragmentation due to its practical approach to studying heavy nuclear systems. Compared with Anti-symmetrized Molecular Dynamics [16] and Fermionic Molecular Dynamics [17], our model treats the effect of the Pauli principle approximately [18]. The phase space constraint proposed by Papa et. al. [19] is introduced, that is, the one-body occupation number in a volume $h^3$ of phase space centered at $(\vec{r}_i, \vec{p}_i)$, corresponding to the centroid of the wave pocket of particle $i$, should always be not larger than 1. For reader convenience, in this section we briefly introduce the model. In the model, each nucleon is represented by a coherent state of a Gaussian wave packet

$$\psi(\vec{r}, t) = \frac{1}{(2\pi\sigma^2)^{3/4}} \exp\left[-(\vec{r} - \vec{r}_i)^2/4\sigma^2_r\right] \exp[i\vec{p}_i \cdot \vec{r}/\hbar].$$  \hspace{1cm} (1)

Where, $\vec{r}_i$ and $\vec{p}_i$ are the centers of the wave packet of particle $i$ in the coordinate and momentum space, respectively. $\sigma_r$ represents the spatial spread of the wave packet. Through a Wigner transformation of the wave function, the one-body phase space distribution function for N-distinctable particles is given by

$$f(\vec{r}, \vec{p}) = \frac{1}{(\pi\hbar)^3} \sum_{i=1}^{N} \exp\left[-(\vec{r} - \vec{r}_i)^2/2\sigma^2_r - (\vec{p} - \vec{p}_i)^2/2\sigma^2_p/\hbar^2\right].$$  \hspace{1cm} (2)

The density and momentum distribution functions of a system read

$$\rho(\vec{r}) = \int f(\vec{r}, \vec{p}) d\vec{p} = \sum_{i=1}^{N} \rho_i(\vec{r}),$$  \hspace{1cm} (3)

and

$$g(\vec{p}) = \int f(\vec{r}, \vec{p}) d\vec{r} = \sum_{i=1}^{N} g_i(\vec{p}),$$  \hspace{1cm} (4)

respectively, where the sum runs over all particles in the system. $\rho_i(\vec{r})$ and $g_i(\vec{p})$ are the density and momentum distribution functions of nucleon $i$:

$$\rho_i(\vec{r}) = \frac{1}{(2\pi\sigma^2)^{3/2}} \exp\left(-\frac{(\vec{r} - \vec{r}_i)^2}{2\sigma^2_r}\right),$$  \hspace{1cm} (5)

$$g_i(\vec{p}) = \frac{1}{(2\pi\sigma^2_p)^{3/2}} \exp\left(-\frac{(\vec{p} - \vec{p}_i)^2}{2\sigma^2_p}\right).$$  \hspace{1cm} (6)
where $\sigma_r$ and $\sigma_p$ are the widths of wave packets in the coordinate and momentum space, respectively, and they satisfy the minimum uncertainty relation: $\sigma_r \cdot \sigma_p = \frac{\hbar}{2}$. The time evolution of $\vec{r}_i$ and $\vec{p}_i$ is governed by Hamiltonian equations of motion

$$
\dot{\vec{r}}_i = \frac{\partial H}{\partial \vec{p}_i}, \quad \dot{\vec{p}}_i = -\frac{\partial H}{\partial \vec{r}_i}.
$$

The Hamiltonian $H$ is made up of the kinetic energy and the effective interaction potential energy:

$$
H = E_k + U,
$$

where

$$
E_k = \sum_{i=1}^{N} \frac{\vec{p}_i^2}{2m}.
$$

The effective interaction potential energy reads

$$
U = \frac{\alpha}{2} \sum_{i=1}^{N} < \rho \rho_0 >_i + \frac{\beta}{3} \sum_{i=1}^{N} < \rho^2 \rho_0 >_i + \frac{C_s}{2} \int \frac{(\rho_p - \rho_n)^2}{\rho_0} d\vec{r}
\quad + \frac{C_y}{2} \sum_{i=1}^{N} \int \rho_i(\vec{r}) \frac{\exp[-\gamma |\vec{r} - \vec{r}_j|]}{|\vec{r} - \vec{r}_j|} \rho_j(\vec{r}) d\vec{r} d\vec{r}'
\quad + \frac{1}{2} \sum_{i,j \neq (i,j) \text{for protons}} \int \rho_i(\vec{r}) \frac{e^2}{|\vec{r} - \vec{r}_j|} \rho_j(\vec{r}) d\vec{r} d\vec{r}'.
$$

The parameters $\alpha, \beta, C_s$ and $C_y$ in the model are taken to be the same as in Ref [20]. In order to prepare a ground state nucleus, we first calculate the neutron and proton density distribution, binding energy and nuclear radius of the ground state by the relativistic mean field theory (RMF) [21]. Then the position of each nucleon in the nucleus is sampled according to the density distribution obtained. The momentum of each wave packet is assigned randomly between zero and local Fermi momentum $P_f(r)$ obtained by the local density approximation. Each created nucleus is examined according to the properties of the ground state (i.e., the binding energy and the nuclear radius) and the time evolution of the binding energy and root-mean-square radius. Only those whose properties are in consistent with those of the ground state are accepted as initial ground state nuclei (see,
Ref. [18] for detail). The initial excited nuclei are obtained by the following procedures: the position of each nucleon is taken to be the same as its ground state, but the momentum of each nucleon is re-sampled according to the Fermi-Dirac distribution at a certain value of chemical potential and temperature $T$. By varying the temperature, we put different initial kinetic energies (initial excitation energies) into nuclear systems. With this procedure, we prepare a series of initial excited nuclei, such as $^{40}Ca$, $^{58}Ni$, $^{90}Zr$, $^{124}Sn$, $^{144}Nd$, $^{197}Au$, $^{208}Pb$, $^{266}Ra$, $^{238}U$, and $^{298}114$ for dynamic studies.

3. NUMERICAL STUDY OF MULTIFRAGMENTATION

3.1 LARGEST LYAPUNOV EXPONENT

The largest Lyapunov exponent is defined as [12,1,11]

$$
\lambda = \lim_{n \to \infty} \frac{1}{n\tau} \ln \frac{\|d\vec{X}_n\|}{\|d\vec{X}_0\|}.
$$

(11)

The quantity $\|d\vec{X}_n\|$ is the phase space distance between two trajectories corresponding to two concerned events at time $t = n\tau$ and the phase space distance $\|d\vec{X}\|$ reads

$$
\|d\vec{X}\| = \sqrt{\sum_{i=1}^{N}[(\vec{r}_1^{i}/rms - \vec{r}_2^{i}/rms)^2 + (\vec{p}_1^{i}/avp - \vec{p}_2^{i}/avp)^2]_i}.
$$

(12)

Where the sum runs over all $N$ nucleons of the system, the subscripts 1 and 2 refer to two events which differ of an infinitesimal quantity of $\|d\vec{X}_0\| = 10^{-7}$ or less at initial time. In Eq.(12), the dimensionless coordinate and momentum, scaled by the root-mean-square radius ($rms$) and the average momentum ($avp$), respectively, are used. In our model, the trajectory $\vec{X}(t)$ is a function of a set of $\{\vec{r}_i, \vec{p}_i\}$ which defines the states of a nucleus.

In numerical calculations of the LLE, the initial excited nuclei are created by the method mentioned in section 2. For each temperature $T$, 50 test events are generated, and for each test event 40 other different events are generated, each of them differing from the corresponding test event by $\|d\vec{X}_0\|$ at the initial time. The LLE is obtained by averaging over trajectories of all events evolving according to a set of equations of motion (Eq.(7)). To
show the feature of time evolution of the LLE at different temperatures, as an example, we plot \( \lambda(t) \) for \(^{208}\text{Pb}\) at temperatures of \( T = 2, 11, 30\text{MeV} \) in Fig.1. From this figure one can see that the behavior of \( \lambda(t) \) for \( T=11 \text{ MeV} \) is quite different from the cases of \( T=2 \) and 30 MeV. For the case of \( T = 2 \text{ MeV} \) the \( \lambda(t) \) reaches a constant value after \( T = 110 \text{ fm/c} \), and for \( T=30 \text{MeV} \) the \( \lambda(t) \) firstly decreases with time and finally approaches to an asymptotic value. Whereas for the case of \( T=11 \text{ MeV} \) there appears a plateau in the LLE from time 130 fm/c to 175 fm/c and after then the LLE decreases again. The saturation behavior in \( T=2 \) and 30 MeV is simply due to the fact that the available phase space is limited. Whereas for the case of \( T=11 \text{ MeV} \) the LLE maintains at a constant value only during the finite time in which multifragmentation takes place. After fragment formation the collective expansion motion of the fragmenting system may play a major role, this ordered motion results in a reduction of the LLE. Considering this feature, we take the \( \lambda(t) \) value at the plateau as the LLE of the fragmenting system. In order to find a "critical temperature" we calculate the LLE of selected systems at different temperatures in step of 1 MeV, and as examples, we show the results for \(^{124}\text{Sn}\) and \(^{208}\text{Pb}\) in Fig. 2. One can find that, with the increase of temperature \( T \), the LLE increases until it reaches a maximal value at a certain temperature, and afterwards the LLE decreases as temperature further increases. We call the temperature corresponding to the maximal LLE as the "critical temperature". This temperature is just the one at which a plateau appears in the time evolution of \( \lambda(t) \). The behavior of the LLE shown in Fig. 2 is also found in all nuclear systems studied (for example, \(^{144}\text{Nd}\), \(^{197}\text{Au}\), \(^{226}\text{Ra}\), \(^{238}\text{U}\), \(^{208}\text{114}\), \(^{40}\text{Ca}\), and \(^{58}\text{Ni}\)) and was observed in Ref. [5]. Here we notice that the numerical evaluation about the "critical temperature" is a bit uncertain because the saddle-point has to be found out within a fluctuating signal. This uncertainty is even a bit more for light nuclear systems, due to stronger finite size effects. However, for heavy nuclear systems, since the observed change of the signature is strong enough, the peak in the LLE is well pronounced. The behavior of the LLE as a function of temperature can be easily understood as follows: The raising branch is obviously due to the increase of fluctuation with temperature, and the presence of the maximum in the LLE, which signals a transition
from a chaotic to a more ordered motion, results form multifragmentation; The behavior of the
descent branch of $\lambda \sim T$ can be traced to the fact that in this temperature region the system
breaks up very soon and the ordered expansion collective motion dominates the evolution
of the system.

The mass dependence of the "critical temperature" for nuclei ranging from $^{40}Ca$ to
$^{298}114$ is shown by the line with solid squares in Fig.3. One can see from the curve that: as
the system size increases, the "critical temperature" increases slightly, and for the systems
heavier than $^{197}Au$, the "critical temperature" approximately approaches to a constant value
of about $T_c = 11MeV$. The trend of the dependence of the "critical temperature" for finite
systems on the nuclear mass is consistent with the other model calculations [22–24].

3.2 DENSITY FLUCTUATION

In the QMD model, the many body correlation can be taken into account, which allows
us to study the density fluctuation defined as

$$\sigma^2_\rho = \frac{<\rho^2(t)> - <\rho(t)>^2}{<\rho(t)>^2}.\quad (13)$$

Here,

$$<\rho(t)> = \int \rho(t)\rho(t)d\vec{r},\quad (14)$$

and

$$<\rho(t)^2> = \int \rho(t)^2 \rho(t)d\vec{r}.\quad (15)$$

The integration is over the whole space. Since the system studied is an isolated one, the
density fluctuation should asymptotically approach to a saturation value under the influence
of the mean field and the kinetic energy term. Fig. 4 shows the time evolution of the density
fluctuation at temperatures of $T = 3, 11$ and $20$ MeV for $^{208}Pb$. From the figure one sees
that for the case of $T=3$ MeV, the $\sigma^2_\rho$ is very small and there is a very slow increase in the
density fluctuation. This is because of the effect of neutron evaporation. For $T=20$ MeV, the
density fluctuation sooner reaches a saturation value since the system breaks up very soon.
While for the case of T=11 MeV, the density fluctuation grows abnormally till t=175 fm/c and after then there appear small jumps. The abnormal growth rate and jumps seen in the curve signal the process of fragmentation. In order to see the saturation behavior the effect resulting from evaporation process should be subtracted. We show the density fluctuation with the neutron evaporation subtracted for T=3 MeV case and the secondary evaporation subtracted for T=11 MeV case in the small figure inserted in Fig.4. For T=20 MeV case the system breaks up very fast and the evaporation almost has no effect on the density fluctuation of the system. From the inserted figure, one sees the saturation behavior of $\sigma^2/\rho$ for T=3 MeV case and the asymptotically saturation behavior of $\sigma^2/\rho$ for T=11 MeV case, respectively. By comparing the time dependent behavior of the density fluctuation with that of the LLE, we find quite different time scales: the time scale for density fluctuation growth ($\sim 150$ fm/c) is much longer than the inverse largest Lyapunov exponent ($\sim 40$ fm/c). This finding indicates the fact that the dynamics during fragmentation of the nuclear system is chaotic enough. Based on this finding we are allowed to use the Lyapunov exponent to characterize the dynamics of fragmentation in finite nuclear systems (see introduction).

Then we study the evolution of saturation values of $\sigma^2/\rho$ with temperatures for the systems studied in Sec. 3.1. We find that the behavior of $\sigma^2/\rho \sim T$ is quite similar with that of the LLE$\sim T$ (shown in Fig. 2). Similarly, we can extract the "critical temperature" from the maximal value of $\sigma^2/\rho \sim T$. The mass dependence of the "critical temperature" extracted by the density fluctuation is also shown in Fig. 3. We find that the "critical temperatures" obtained from both the density fluctuation and the largest Lyapunov exponent are in well coincidence. It implies that the abnormal fluctuation in the density emerges from the deterministic chaos and thus a small uncertainty in the initial condition can produce a large dynamical fluctuation in final observables.

3.3 MULTIFRAGMENTATION

In this section we study the mass and charge distributions of fragments for the selected systems mentioned in section 3.1 at different temperatures. In our calculation, the fragment
recognition is in terms of the conventional coalescence model [25], in which particles with relative momenta smaller than $P_0$ and relative distance smaller than $R_0$ are considered to belong to one cluster. Here $R_0$ and $P_0$ are taken to be 3.5 fm and 300 MeV/c, respectively [26,27]. In Fig. 5 we show the calculated mass distributions at different temperatures for systems of $^{124}Sn$ and $^{208}Pb$. From this figure we can see that at low temperatures (for example, $T=4$ MeV) only a few nucleons are vaporized, and the mass of residues is peaked near the original nucleus (U shaped mass spectra). However, for high temperatures (for instance, $T=25$ MeV) the system breaks up into nucleons and light fragments, thus the mass distribution of fragments peaks at the very small mass number, and the system is considered to be at a "vapor" phase. If the initial temperature is in between, for an example, at $T=8$ MeV the system starts to fragment and the coexistence of "liquid" and "vapor" may appear. At $T=10$ MeV for $^{124}Sn$ and 11 MeV for $^{208}Pb$, the mass of fragments is distributed over a wide range from free nucleon to about a half of the total mass of the system, and the maximal fragmentation seems to appear. The sequence of shapes of above mass spectra is the same as the one predicted by Fisher’s model [28] of liquid-gas phase transition. In the latter model the probability of having a drop of size $A$ in the vapor is given by

$$P(A) = Y_0 A^{-\tau} exp[-(\mu_l - \mu_g)A + 4\pi \ast r_0^2 \sigma(T)A^{2/3}].$$  \hspace{1cm} (16)

Here, $\mu_l$ and $\mu_g$ are the chemical potential of the liquid and vapor phases, and $\sigma$ is the surface tension. In the critical point, $\mu_l$ equals $\mu_g$ and $\sigma(T_c)$ equals zero, then the power law is obtained. In order to fit the power law, we give a double-logarithmic plot of the mass (and charge) spectra at $T=11$ MeV for $^{208}Pb$ in Fig. 6. The solid squares (open circles) denote the numerical results for mass (charge) spectra, and the dashed lines denote their best fits to mass and charge spectra with $\chi^2$ of 0.678 and 1.140, respectively. From the best fits to mass and charge spectra for all selected systems we can obtain the critical temperature $T_c$ and the critical exponents $\tau_m$ (for mass) and $\tau_z$ (for charge). Table 1 lists those results with the values of $\chi^2$. We note that all extracted exponents are larger than 2.0, and they are in agreement with what is expected from the Fisher’s power law for the nuclear
liquid-gas phase transition. The calculated critical exponents for charge distributions are quite close to the recently obtained experiment value of $\tau_z \sim 2.35 \pm 0.05$ [29]. We find that the critical temperatures obtained by the best fit to power law are the same as those obtained by means of the LLE and density fluctuation. This means that the power law behavior of mass spectra is closely related to the pronounced peak in the LLE and density fluctuation. As is shown in section 3.1 that the maximum in the LLE corresponds to the largest local rate of the divergence of trajectories, thus to the maximum in the available phase space of trajectories. In fact, if the dynamics is chaotic, strong fluctuation is expected from one microscopic (collision) event to another, each event ending in a different region of the available phase space. On the average, therefore, if chaoticity is strong enough, the population of the final channels will be dominated by the available phase space. From this point of view, Fig. 5 may tell us that a very large population of the decay channels can be obtained even at the "critical temperature" $T_c$ at which the possible configurations show the maximal variation. Thus, the correspondence between multifragmentation and maximum chaoticity in microscopic level can been established.

4. BRIEF DISCUSSION OF CONNECTION BETWEEN THE LLE AND DENSITY FLUCTUATION

In this part, we further discuss the connection between the Lyapunov exponent of trajectories around an ensemble of initial states consistent with given initial heated nuclei and the fluctuation of nuclear density related to the decomposition of a nucleus into fragments. As is described in section 3.1 that the trajectory $\vec{X}(t)$ is a function of a set of $\{\vec{r}_i, \vec{p}_i\}$ which defines the states of a nucleus. The phase density $\varrho(\vec{X})$ is, therefore, also a function of the set of $\{\vec{r}_i, \vec{p}_i\}$.

The general connection between the LLE and the phase density $\varrho(\vec{X})$ is given in Refs. [30,31] as

$$\lim_{t \to \infty} \frac{1}{2t} \ln h(\varrho) = \max_{\vec{X} \in \Lambda} \lambda(\vec{X}),$$

(17)
where the heterogeneity of the phase density is defined as

$$h(\rho) = \int \left| \frac{\partial \rho(\vec{X})}{\partial X} \right|^2 d\vec{X} / \int |\rho(\vec{X})|^2 d\vec{X},$$

(18)

where the $\lambda(\vec{X})$ is the LLE of the trajectory $X(t)$ (defined in Eq.(11)), the $\Lambda$ is the non-zero domain of $|\nabla \rho^0(\vec{X})|$ in phase space and the $\rho^0(\vec{X})$ denotes the phase density $\rho(\vec{X})$ at time $t=0$. Here the maximum has to be understood in measure theoretical sense. The expression (17) is known to be a quite general formula. We will make a qualitative discussion about the connection between the LLE and density fluctuation by means of expressions of (17) and (18), although use of expression (17) in the phase transition regime might be questionable. According to the definition of $\rho(\vec{X}(t))$ and $\vec{X}$, the phase density $\rho$, in a certain sense, can be roughly considered as a function of nuclear density $\rho$ (see, Eq.(3)) and momentum distribution $g$ (see, Eq.(4)), thus one can easily deduce that the heterogeneity of the phase density $h(\rho)$ increases with $\langle \rho^2 \rangle - \langle \rho \rangle^2$, i.e., increases with the density fluctuation $\sigma_\rho^2$. From expression (17) the LLE of trajectory should increase with the density fluctuation. Of course, this is only a very qualitative discussion. However, based on our numerical results the correspondence of the LLE and the density fluctuation for different systems can be illustrated. Fig. 7 shows the relation between the LLE $\lambda(\vec{X})$ and the density fluctuation $\sigma_\rho^2$ at different temperatures from 3 MeV to 19 MeV for systems of $^{124}Sn$ (Fig. 7(a)) and $^{208}Pb$ (Fig. 7(b)). One can see that the maximum values of both the LLE and density fluctuation are located at the same temperature, i.e. the "critical temperature". There are two branches in $\lambda(\vec{X}) \sim \sigma_\rho^2$, one corresponding to the temperature lower than the "critical temperature" and another corresponding to the temperature higher than the "critical temperature". For the low temperature branch, both of the $\lambda(\vec{X})$ and $\sigma_\rho^2$ increase as the temperature increases, whereas for the high temperature branch they increase as the temperature decreases. Both branches show that the $\lambda(\vec{X})$ roughly linearly increases with $\sigma_\rho^2$. This correspondence between the $\lambda(\vec{X})$ and $\sigma_\rho^2$ is qualitatively in consistence with the discussion based on the expressions of (17) and (18).

5. SUMMARY

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In this work we have systematically studied the fragmentation process of hot nuclei in terms of the LLE, the density fluctuation and the mass(charge) spectrum. The character of the LLE at the "critical temperature" is that not only its value reaches the largest one, but also there appears a plateau in its time evolution, which represents the period of formation process of fragments. Simultaneously, at the "critical temperature" in the time evolution of the density fluctuation there appear an abnormal growth rate and jumps which indicate the process of decomposition of a nucleus into fragments. Our study further demonstrates that the time scale of the density fluctuation is much longer than the inverse largest Lyapunov exponent, which means that the chaotic motion can be well developed during the process of fragment formation. Therefore, the deterministic chaotic mechanism is allowed to describe the fragmentation in finite nuclear systems, and it seems to be of a crucial importance for the phase transition.

Our study shows that the LLE peaks at the temperature, at which the density fluctuation grows abnormally and the mass distribution of fragments fits best to the Fisher's power law, for all selected nuclear systems ranging from $^{40}\text{Ca}$ to the superheavy nucleus $^{298}\text{114}$. This means that all three signatures are in coincidence at the "critical temperature". As can be seen that the observed changes in the key signatures seem to be strong enough and therefore they could survive even when associating each point with reasonable error bars caused by numerical uncertainties and the approximate treatment of the Pauli principle.

We have further investigated finite size effects on "critical temperature", and it is observed that for systems lighter than $^{197}\text{Au}$ the "critical temperature" increases with mass and for systems heavier than $^{197}\text{Au}$ a saturation value of about $T_c = 11\text{MeV}$ seems to be reached.
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TABLES

TABLE I. The power law of mass and charge distributions for systems $^{124}\text{Sn}$, $^{144}\text{Nd}$, $^{197}\text{Au}$, $^{208}\text{Pb}$, $^{226}\text{Ra}$, $^{238}\text{U}$, and $^{298}\text{I}$ at their ”critical temperatures”.

|       | $^{124}\text{Sn}$ | $^{144}\text{Nd}$ | $^{197}\text{Au}$ | $^{208}\text{Pb}$ | $^{226}\text{Ra}$ | $^{238}\text{U}$ | $^{298}\text{I}$ |
|-------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| $T_c$ | 10 MeV            | 10 MeV            | 11 MeV            | 11 MeV            | 11 MeV            | 11 MeV            | 11 MeV            |
| $\tau_m$ | 2.679            | 2.672             | 2.696             | 2.676             | 2.642             | 2.700             | 2.660             |
| $(\chi^2)$ | 0.510             | 0.611             | 0.594             | 0.678             | 1.353             | 0.792             | 0.531             |
| $\tau_z$ | 2.514             | 2.496             | 2.477             | 2.453             | 2.406             | 2.453             | 2.432             |
| $(\chi^2)$ | 0.146             | 0.256             | 1.067             | 1.140             | 1.184             | 1.497             | 1.606             |


FIGURES

FIG. 1. The time evolution of $\frac{1}{\tau} \ln \frac{||d\vec{X}(\tau)||}{||d\vec{X}_0||}$ for the system $^{208}Pb$ at temperatures of $T = 2, 11,$ and $30 MeV$.

FIG. 2. The largest Lyapunov exponent as a function of temperature for systems of $^{124}Sn$ and $^{208}Pb$.

FIG. 3. The "critical temperature" obtained from the $LLE$ (line with solid squares) and density fluctuation (line with open circles) for various nuclear systems.

FIG. 4. The time evolution of the density fluctuation for $^{208}Pb$ at different initial temperatures of $T = 3, 11,$ and $20 MeV$.

FIG. 5. The mass distribution of fragments at various temperatures for systems of $^{124}Sn$ and $^{208}Pb$.

FIG. 6. The double-logarithmic plot of the mass and charge distribution of fragments at the "critical temperature" for $^{208}Pb$.

FIG. 7. The relation between the LLE and the density fluctuation at temperatures from 3 MeV to 19 MeV for systems of $^{124}Sn$ (Fig. 7(a)) and $^{208}Pb$ (Fig. 7(b)).
Yield (arb. units) vs. Mass (Charge)

- Mass
- Charge

$^{208}$Pb
Figure (a) shows the density fluctuation $\lambda$ (c/fm) as a function of energy for $^{124}$Sn. The graph plots points for energies ranging from 3 MeV to 19 MeV, with each energy level marked on the graph. The x-axis represents the density fluctuation, while the y-axis represents $\lambda$ (c/fm).
