Supermembranes and Super Matrix Theory

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Abstract: We review recent developments in the theory of supermembranes and their relation to matrix models.
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1 Supersymmetric quantum mechanics

Consider the class of supersymmetric Hamiltonians of the form

\[ H = \frac{1}{g} \text{Tr} \left[ \frac{1}{2} P^2 - \frac{1}{4} [X^a, X^b]^2 + \frac{1}{2} g \theta^T \gamma_a [X^a, \theta] \right], \tag{1} \]

depending on a number of \( d \)-dimensional coordinates \( X = (X^1, \ldots, X^d) \), corresponding momenta \( P \), as well as real spinorial anticommuting coordinates \( \theta \), all taking values in the matrix representation of some Lie algebra. The phase space is restricted to the subspace invariant under the corresponding (compact) Lie group and is therefore subject to Gauss-type constraints. The above Hamiltonians arise in the zero-volume limit of supersymmetric Yang-Mills theories, which explains the presence of these constraints.

The theories based on (1) were proposed long ago as extended models of supersymmetric quantum mechanics with more than four supersymmetries (1). The spatial dimension \( d \) and the corresponding spinor dimension are severely restricted. The models exist for \( d = 2, 3, 5 \), or 9 dimensions; the (real) spinor dimension equals 2, 4, 8, or 16, respectively. Naturally this is also the number of independent supercharges. In what follows we restrict ourselves to the highest-dimensional case, where the model contains 16 supercharges. However, additional charges can be obtained by
splitting off an abelian factor of the gauge group (we will mainly consider the gauge group $U(N)$),

$$Q^+ = \text{Tr} \left[ (P^a \gamma_a + \frac{1}{2} i [X^a, X^b] \gamma_{ab}) \theta \right], \quad Q^- = g \text{Tr} \left[ \theta \right].$$

(2)

The $Q^+$ generate the familiar supersymmetry algebra (in the group-invariant subspace),

$$\{Q^+_\alpha, Q^+_\beta\} \approx H \delta_{\alpha\beta}.$$

(3)

A central theme of this lecture is that the supermembrane in the light-cone formulation is described by a quantum-mechanical model of the type above with an infinite-dimensional gauge group corresponding to the area-preserving diffeomorphisms of the membrane spacesheet \([\mathbb{R}^N]/\mathbb{S}_N\); the coupling constant $g$ is then equal to the total light-cone momentum $(P^-)_0$, which in a flat target space equals $P^+_0$. In 11 spacetime dimensions the supermembrane is subject to 32 supercharges. The 16 charges $Q^-$ given in (2) are then associated with the center-of-mass superalgebra. The connection with the supermembrane shows that the manifest $SO(9)$ symmetry, which from the viewpoint of the supermembrane is simply the exact transverse rotational invariance of the lightcone formulation, extends to the 11-dimensional Lorentz group in the limit of an appropriate infinite-dimensional gauge group \([3, 4]\).

Classical zero-energy configurations require all commutators $[X^a, X^b]$ to vanish. Dividing out the gauge group implies that zero-energy configurations are thus parametrized by $\mathbb{R}^{9N}/\mathbb{S}_N$. The zero-energy valleys in the potential extend all the way to infinity where they become increasingly narrow. Their existence raises questions about the nature of the spectrum of the Hamiltonian \([\mathbb{I}]\). In the bosonic versions of these models the wave function cannot freely extend to infinity, because at large distances it becomes more and more squeezed in the valley. By the uncertainty principle, this gives rise to kinetic-energy contributions which increase monotonically along the valley. Another way to see this effect is by noting that oscillations perpendicular to the valleys give rise to a zero-point energy, which induces an effective potential barrier that confines the wave function. This confinement causes the spectrum to be discrete. However, for the supersymmetric models defined by \([\mathbb{I}]\) the situation is different. Supersymmetry can cause a cancelation of the transverse zero-point energy. Then the wave function is no longer confined, indicating that the supersymmetric models have a continuous spectrum. The latter was rigourously proven for the gauge group $SU(N)$ \([\mathbb{I}]\).

For the supermembrane, the classical zero-mass configurations correspond to zero-area stringlike configurations of arbitrary length. As the supermembrane mass is described by a Hamiltonian of the type \([\mathbb{I}]\), the mass spectrum of the supermembrane is continuous for the same reasons as given above. For a supermembrane moving in a target space with compact dimensions, winding may raise the mass of the membrane state. This is so because winding in more than one direction gives rise to a nonzero central charge in the supersymmetry algebra, which sets a lower limit on the membrane mass. This fact should not be interpreted as an indication that the spectrum becomes discrete. The possible continuity of the spectrum hinges on the
two features mentioned above. First the system should possess continuous valleys of classically degenerate states. Qualitatively one recognizes immediately that this feature is not directly affected by winding. A classical membrane with winding can still have stringlike configurations of arbitrary length, without increasing its area. Hence the classical instability persists. The second feature is supersymmetry. Without winding it is clear that the valley configurations are supersymmetric, so that one concludes that the spectrum is continuous. With winding the latter aspect is more subtle. However, we note that, when the winding density is concentrated in one part of the spacesheet, then valleys can emerge elsewhere corresponding to stringlike configurations with supersymmetry. Hence, as a space-sheet local field theory, supersymmetry can be broken in one region where the winding is concentrated and unbroken in another. In the latter region stringlike configurations can form, which, at least semiclassically, will not be suppressed by quantum corrections [6]. However, in this case we can only describe the generic features of the spectrum. Our arguments do not preclude the existence of mass gaps.

Finally, whether or not the Hamiltonian (1) allows normalizable or localizable zero-energy states, superimposed on the continuous spectrum, is a subtle question. Early discussion on the existence of such zero-energy states can be found in [2, 7]; more recent discussions can be found in [3, 8]. According to [4] such states do indeed exist in $d = 9$. There is an important difference between states whose energy is exactly equal to zero and states of positive energy. The supersymmetry algebra implies that zero-energy states are annihilated by the supercharges. Hence, they are supersinglets. The positive-energy states, on the other hand, must constitute full supermultiplets. So they are multiplets consisting of multiples of $1 + 1, 2 + 2, 8 + 8,$ or $128 + 128$ bosonic + fermionic states, corresponding to $d = 2, 3, 5$ or 9, respectively.

To prove or disprove the existence of discrete states with winding is even more difficult. While the contribution of the bosonic part of the Hamiltonian increases by concentrating the winding density on part of the spacesheet, the matrix elements in the fermionic directions will also grow large, making it difficult to estimate the eigenvalues. At this moment the only rigorous result is the BPS bound that follows from the supersymmetry algebra. Obviously, the state of lowest mass for given winding numbers is always a BPS state, which is invariant under some residual supersymmetry. The counting of states proceeds in a way that is rather similar to the case of no winding.

2 Supermembranes

Fundamental supermembranes can be described in terms of actions of the Green-Schwarz type, possibly in a nontrivial but restricted (super)spacetime background [10]. Such actions exist for supersymmetric $p$-branes, where $p = 0, 1, \ldots$ defines the spatial dimension of the brane. Thus for $p = 0$ we have a superparticle, for $p = 1$ a superstring, for $p = 2$ a supermembrane, and so on. The dimension of spacetime in which the superbrane can live is very restricted. These restrictions arise from the
fact that the action contains a Wess-Zumino-Witten term, whose supersymmetry depends sensitively on the spacetime dimension. If the coefficient of this term takes a particular value then the action possesses an additional fermionic gauge symmetry, the so-called \( \kappa \)-symmetry. This symmetry is necessary to ensure the matching of (physical) bosonic and fermionic degrees of freedom. In the following we restrict ourselves to supermembranes (i.e., \( p = 2 \)) in 11 dimensions.

The supermembrane action \([10]\) is written in terms of superspace embedding coordinates \( Z^M(\zeta) = (X^\mu(\zeta), \theta(\zeta)) \), which are functions of the three world-volume coordinates \( \zeta^i \) (\( i = 0, 1, 2 \)). It takes the following form,

\[
S[Z(\zeta)] = \int d^3\zeta \left[ -\sqrt{-g(Z(\zeta))} - \frac{1}{3} \varepsilon^{ijk} \Pi_i^A \Pi_j^B \Pi_k^C B_{CBA}(Z(\zeta)) \right],
\]

where \( \Pi_i^A = \partial Z^M / \partial \zeta^i E_M^A \) and the induced metric equals \( g_{ij} = \Pi_i^A \Pi_j^B \eta_{rs} \), with \( \eta_{rs} \) the constant Lorentz-invariant metric. Flat superspace is characterized by

\[
\begin{align*}
E_\mu^r &= \delta_\mu^r, & E_\mu^a &= 0, \\
E_\alpha^a &= \delta_\alpha^a, & E_\alpha^r &= -(\bar{\theta} \Gamma^r)_\alpha, \\
B_{\mu\nu\alpha} &= (\bar{\theta} \Gamma_{\mu\nu})_\alpha, & B_{\mu\alpha\beta} &= (\bar{\theta} \Gamma_{\mu\nu})_\alpha (\bar{\theta} \Gamma^\nu)_\beta, \\
B_{\alpha\beta\gamma} &= (\bar{\theta} \Gamma_{\mu\nu})_\alpha (\bar{\theta} \Gamma^\mu)_\beta (\bar{\theta} \Gamma^\nu)_\gamma, & B_{\mu\nu\rho} &= 0.
\end{align*}
\]

The gamma matrices are denoted by \( \Gamma^r \); gamma matrices with more than one index denote antisymmetrized products of gamma matrices with unit weight. In flat superspace the supermembrane Lagrangian, written in components, reads (in the notation and conventions of \([2]\))

\[
\mathcal{L} = -\sqrt{-g(X, \theta)} - \varepsilon^{ijk} \bar{\theta} \Gamma_{\mu\nu} \partial_k \theta \left[ \frac{1}{2} \partial_\mu X^\mu (\partial_j X^\nu + \bar{\theta} \Gamma^\nu \partial_j \theta) + \frac{1}{6} \bar{\theta} \Gamma^\mu \partial_\theta \bar{\theta} \Gamma^\nu \partial_j \theta \right],
\]

The target space can have compact dimensions which permit winding membrane states \([3]\). In flat superspace the induced metric,

\[
g_{ij} = (\partial_i X^\mu + \bar{\theta} \Gamma^\mu \partial_i \theta)(\partial_j X^\nu + \bar{\theta} \Gamma^\nu \partial_j \theta) \eta_{\mu\nu},
\]

is supersymmetric. Therefore the first term in \([3]\) is trivially invariant under spacetime supersymmetry. In 4, 5, 7, or 11 spacetime dimensions the second term proportional to \( \varepsilon^{ijk} \) is also supersymmetric (up to a total divergence) and the full action is invariant under \( \kappa \)-symmetry.

In the case of the open supermembrane, \( \kappa \)-symmetry imposes boundary conditions on the fields \([11]\). They must ensure that the following integral over the boundary of the membrane world volume vanishes,

\[
\int_{\partial M} \left[ \frac{1}{2} dX^\mu \wedge (dX^\nu + \bar{\theta} \Gamma^\nu d\theta) \bar{\theta} \Gamma_{\mu\nu} \delta_\kappa \theta + \frac{1}{6} \bar{\theta} \Gamma^\mu d\theta \wedge \bar{\theta} \Gamma^\nu d\theta \bar{\theta} \Gamma_{\mu\nu} \delta_\kappa \theta \\
+ \frac{1}{2} (dX^\mu - \frac{1}{3} \bar{\theta} \Gamma^\mu d\theta) \wedge \bar{\theta} \Gamma_{\mu\nu} d\theta \bar{\theta} \Gamma^\nu \delta_\kappa \theta \right] = 0.
\]

This can be achieved by having a “membrane D-\( p \)-brane” at the boundary with \( p = 1, 5, \) or 9, which is defined in terms of \((p + 1)\) Neumann and \((10 - p)\) Dirichlet
boundary conditions for the $X^\mu$, together with corresponding boundary conditions on the fermionic coordinates. More explicitly, we define projection operators

$$P_\pm = \frac{1}{2} \left( 1 \pm \Gamma^{p+1} \Gamma^{p+2} \cdots \Gamma^{10} \right), \quad (9)$$

and impose the Dirichlet boundary conditions

$$\partial_\parallel X^M | = 0, \quad M = p + 1, \ldots, 10,$$

$$P_- \theta | = 0, \quad (10)$$

where $\partial_\perp$ and $\partial_\parallel$ define the world-volume derivatives perpendicular or tangential to the surface swept out by the membrane boundary in the target space. Note that the fermionic boundary condition implies that $P_- \partial_\parallel \theta | = 0$. Furthermore, it implies that spacetime supersymmetry is reduced to only 16 supercharges associated with spinor parameters $P_+ \epsilon$, which is chiral with respect to the $(p+1)$-dimensional world volume of the D-$p$-brane at the boundary. With respect to this reduced supersymmetry, the superspace coordinates decompose into two parts, one corresponding to $(X^M, P_- \theta)$ and the other corresponding to $(X^m, P_+ \theta)$ where $m = 0, 1, \ldots, p$. While for the five-brane these superspaces exhibit a somewhat balanced decomposition in terms of an equal number of bosonic and fermionic coordinates, the situation for $p = 1, 9$ shows heterotic features in that one space has an excess of fermionic and the other an excess of bosonic coordinates. Moreover, we note that supersymmetry may be further broken, e.g. by choosing different Dirichlet conditions on nonconnected segments of the supermembrane boundary.

The Dirichlet boundary conditions can be supplemented by the following Neumann boundary conditions,

$$\partial_\perp X^m | = 0 \quad m = 0, 1, \ldots, p,$$

$$P_+ \partial_\perp \theta | = 0. \quad (11)$$

These do not lead to a further breakdown of the rigid spacetime symmetries.

We now continue and follow the light-cone quantization described in [2]. The supermembrane Hamiltonian takes the form

$$H = \frac{1}{P_0^+} \int d^2 \sigma \sqrt{w} \left[ \frac{P^a P_a}{2w} + \frac{1}{4} \{ X^a, X^b \}^2 - P_0^+ \bar{\theta} \gamma_- \gamma_a \{ X^a, \theta \} \right]. \quad (12)$$

Here the integral runs over the spatial components of the world volume denoted by $\sigma^1$ and $\sigma^2$, while $P^a(\sigma)$ ($a = 2, \ldots, 9$) are the momenta conjugate to the transverse coordinates $X^a$. In this gauge the light-cone coordinate $X^+ = (X^1 + X^0)/\sqrt{2}$ is linearly related to the world-volume time denoted by $\tau$. The momentum $P_-$ is time independent and proportional to the center-of-mass value $P_0^+ = (P_-)_0$ times some density $\sqrt{w(\sigma)}$ of the spacesheet, whose spacesheet integral is normalized to unity. The center-of-mass momentum $P_0^+$ is equal to minus the Hamiltonian (12) subject to the gauge condition $\gamma_+ \theta = 0$. And finally we made use of the Poisson bracket
\{A, B\} defined by
\[
\{A(\sigma), B(\sigma)\} = \frac{1}{\sqrt{w(\sigma)}} \varepsilon^{rs} \partial_r A(\sigma) \partial_s B(\sigma).
\] (13)

Note that the coordinate \(X^- = (X^1 - X^0)/\sqrt{2}\) itself does not appear in the Hamiltonian [12]. It is defined via
\[
P_0^+ \partial_r X^- = -\frac{P \cdot \partial_r X}{\sqrt{w}} - P_0^+ \bar{\theta} \gamma_\partial \partial, \quad (14)
\]
and implies a number of constraints that will be important in the following. Obviously, the right-hand side of (14) must be closed; without winding in \(X^-\), it must be exact.

The equivalence of the large-\(N\) limit of SU(\(N\)) quantum mechanics with the closed supermembrane model is based on the residual invariance of the supermembrane action in the light-cone gauge. This invariance corresponds to the area-preserving diffeomorphisms of the membrane surface. These are defined by transformations of the worldsheet coordinates
\[
\sigma^r \rightarrow \sigma^r + \xi^r(\sigma), \quad (15)
\]
with
\[
\partial_r (\sqrt{w(\sigma)} \xi^r(\sigma)) = 0. \quad (16)
\]
It is convenient to rewrite this condition in terms of dual spacesheet vectors by
\[
\sqrt{w(\sigma)} \xi^r(\sigma) = \varepsilon^{rs} F_s(\sigma). \quad (17)
\]
In the language of differential forms the condition (13) may then be simply recast as \(dF = 0\). The trivial solutions are the exact forms \(F = d\xi\), or in components,
\[
F_s = \partial_s \xi(\sigma), \quad (18)
\]
for any globally defined function \(\xi(\sigma)\). The nontrivial solutions are the closed forms which are not exact. On a Riemann surface of genus \(g\) there are precisely \(2g\) linearly independent non-exact closed forms, whose integrals along the homology cycles are normalized to unity\(^1\). In components we write
\[
F_s = \phi(\lambda)\xi(\sigma), \quad \lambda = 1, \ldots, 2g. \quad (19)
\]

The commutator of two infinitesimal area-preserving diffeomorphisms is determined by the product rule
\[
\xi^{(3)}_r = \partial_r \left( \frac{\varepsilon^{st} \xi^{(2)}_s \xi^{(1)}_t}{\sqrt{w}} \right), \quad (20)
\]
\(^1\)In the mathematical literature the globally defined exact forms are called “hamiltonian vector fields”, whereas the closed but not exact forms which are not globally defined go under the name “locally hamiltonian vector fields”.

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where both $\xi^{(1,2)}$ are closed vectors. Because $\xi^{(3)}$ is exact, the exact vectors thus generate an invariant subgroup of the area-preserving diffeomorphisms. As we shall discuss in the next section this subgroup can be approximated by $SU(N)$ in the large-$N$ limit, at least for closed membranes. For open membranes the boundary conditions on the fields (14) lead to a smaller group, such as $SO(N)$.

The presence of the closed but non-exact forms is crucial for the winding of the embedding coordinates. More precisely, while the momenta $P(\sigma)$ and the fermionic coordinates $\theta(\sigma)$ remain single valued on the spacesheet, the embedding coordinates, written as one-forms with components $\partial_r X(\sigma)$ and $\partial_r X^-(\sigma)$, are decomposed into closed one-forms. Their non-exact contributions are multiplied by an integer times the length of the compact direction. The constraint alluded to above amounts to the condition that the right-hand side of (14) is closed.

Under the full group of area-preserving diffeomorphisms the fields $X^a$, $X^-$ and $\theta$ transform according to

$$\delta X^a = \varepsilon^{rs} \frac{\xi_r}{\sqrt{w}} \partial_s X^a, \quad \delta X^- = \varepsilon^{rs} \frac{\xi_r}{\sqrt{w}} \partial_s X^-, \quad \delta \theta^a = \varepsilon^{rs} \frac{\xi_r}{\sqrt{w}} \partial_s \theta,$$

(21)

where the time-dependent reparametrization $\xi_r$ consists of closed exact and non-exact parts. Accordingly there is a gauge field $\omega_r$, which is therefore closed as well and transforming as

$$\delta \omega_r = \partial_0 \xi_r + \partial_r \left( \frac{\varepsilon^{st}}{\sqrt{w}} \xi_s \omega_t \right).$$

(22)

Corresponding covariant derivatives are

$$D_0 X^a = \partial_0 X^a - \varepsilon^{rs} \frac{\omega_r}{\sqrt{w}} \partial_s X^a, \quad D_0 \theta = \partial_0 \theta - \varepsilon^{rs} \frac{\omega_r}{\sqrt{w}} \partial_s \theta,$$

(23)

and likewise for $D_0 X^-$. The action corresponding to the following Lagrangian density is then gauge invariant under the transformations (21) and (22),

$$L = P_0^+ \sqrt{w} \left[ \frac{1}{2} (D_0 X)^2 + \bar{\theta} \gamma_+ D_0 \theta - \frac{1}{4} (P_0^+)^{-2} \{ X^a, X^b \}^2 + (P_0^+)^{-1} \bar{\theta} \gamma_- \gamma_a \{ X^a, \theta \} + D_0 X^- \right],$$

(24)

where we draw attention to the last term proportional to $X^-$, which can be dropped in the absence of winding. Moreover, we note that for open supermembranes, (24) is invariant under the transformations (21) and (22) only if $\xi_{\parallel} = 0$ holds on the boundary. This condition defines a subgroup of the group of area-preserving transformations, which is consistent with the Dirichlet conditions (10). Observe that here $\partial_{\parallel}$ and $\partial_{\perp}$ refer to the spacesheet derivatives tangential and perpendicular to the membrane boundary.

\[2\] Consistency of the Neumann boundary conditions (11) with the area-preserving diffeomorphisms (21) further imposes $\partial_{\perp} \xi_{\parallel} = 0$ on the boundary, where indices are raised according to (17).
The action corresponding to (24) is also invariant under the supersymmetry transformations
\[
\delta X^a = -2 \bar{\epsilon} \gamma^a \theta,
\]
\[
\delta \theta = \frac{1}{2} \gamma_+ (D_0 X^a \gamma_a + \gamma_-) \epsilon + \frac{1}{4} (P_0^+)^{-1} \{X^a, X^b\} \gamma_+ \gamma_{ab} \epsilon,
\]
\[
\delta \omega_r = -2 (P_0^+)^{-1} \bar{\epsilon} \partial_r \theta.
\]
(25)

The supersymmetry variation of $X^-$ is not relevant and may be set to zero. For the open case one finds that the boundary conditions $\omega_\parallel = 0$ and $\epsilon = P_+ \epsilon$ must be fulfilled in order for (25) to be a symmetry of the action. In that case the theory takes the form of a gauge theory coupled to matter. The pure gauge theory is associated with the Dirichlet and the matter with the Neumann (bosonic and fermionic) coordinates.

In the case of a ‘membrane D-9-brane’ one now sees that the degrees of freedom on the ‘end-of-the world’ 9-brane precisely match those of 10-dimensional heterotic strings. On the boundary we are left with eight propagating bosons $X^m$ (with $m = 2, \ldots, 9$), as $X^{10}$ is constant on the boundary due to (10), paired with the 8-dimensional chiral spinors $\theta$ (subject to $\gamma_+ \theta = P_- \theta = 0$), i.e., the scenario of Hořava-Witten [12].

The full equivalence with the membrane Hamiltonian is now established by choosing the $\omega_r = 0$ gauge and passing to the Hamiltonian formalism. The field equations for $\omega_r$ then lead to the membrane constraint (14) (up to exact contributions), partially defining $X^-$. Moreover the Hamiltonian corresponding to the gauge theory Lagrangian of (24) is nothing but the light-cone supermembrane Hamiltonian (12). Observe that in the above gauge theoretical construction the space-sheet metric $w_{rs}$ enters only through its density $\sqrt{w}$ and hence vanishing or singular metric components do not pose problems.

We are now in a position to study the full 11-dimensional supersymmetry algebra of the winding supermembrane. For this we decompose the supersymmetry charge $Q$ associated with the transformations (23), into two 16-component spinors,
\[
Q = Q^+ + Q^-,
\]
where $Q^\pm = \frac{1}{2} \gamma^\pm \gamma \mp Q$,
(26)
to obtain
\[
Q^+ = \int d^2 \sigma \left( 2 P^a \gamma_a + \sqrt{w} \{X^a, X^b\} \gamma_{ab} \right) \theta,
\]
\[
Q^- = 2 P_0^+ \int d^2 \sigma \sqrt{w} \gamma_- \theta.
\]
(27)

In the presence of winding the supersymmetry algebra takes the form [6]
\[
(Q^+_{\alpha}, Q^+_{\beta})_{DB} = 2 (\gamma^+)_{\alpha \beta} H - 2 (\gamma_a \gamma^+)_{\alpha \beta} \int d^2 \sigma \sqrt{w} \{X^a, X^\pm\},
\]
\[
(Q^+_{\alpha}, Q^-_{\beta})_{DB} = -(\gamma_a \gamma^+ \gamma^-)_{\alpha \beta} P_0^a - \frac{1}{2} (\gamma_{ab} \gamma^+ \gamma^-)_{\alpha \beta} \int d^2 \sigma \sqrt{w} \{X^a, X^b\},
\]
\[
(Q^-_{\alpha}, Q^-_{\beta})_{DB} = -2 (\gamma^-)_{\alpha \beta} P_0^+,\n\]
(28)
where use has been made of the Dirac brackets of the phase-space variables and the defining equation (14) for $\partial_r X^-$. The new feature of this supersymmetry algebra is the emergence of the central charges in the first two anticommutators, which are generated through the winding contributions. They represent topological quantities obtained by integrating the winding densities

$$z^a(\sigma) = \varepsilon^{rs} \partial_r X^a \partial_s X^-$$

and

$$z^{ab}(\sigma) = \varepsilon^{rs} \partial_r X^a \partial_s X^b$$

over the space-sheet. It is gratifying to observe the manifest Lorentz invariance of (28). Here we should point out that, in adopting the light-cone gauge, we assumed that there was no winding for the coordinate $X^+$. In [13] the corresponding algebra for the matrix regularization was studied. The result coincides with ours in the large-$N$ limit, in which an additional longitudinal five-brane charge vanishes, provided that one identifies the longitudinal two-brane charge with the central charge in the first line of (28). This identification requires the definition of $X^-$ in the matrix regularization, a topic that we return to in the next section. The form of the algebra is another indication of the consistency of the supermembrane-supergravity system.

Until now we discussed the general case of a flat target space with possible winding states. To make the identification with the matrix models more explicit, let us ignore the winding and split off the center-of-mass (CM) variables. First of all, the constant $P^+_0$ represents the membrane CM momentum in the direction associated with the coordinate $X^-$,

$$P^+_0 = \int d^2 \sigma P^+.$$

The other CM coordinates and momenta are

$$P_0 = \int d^2 \sigma P, \quad X_0 = \int d^2 \sqrt{w(\sigma)} X(\sigma), \quad \theta_0 = \int d^2 \sqrt{w(\sigma)} \theta(\sigma).$$

In the light-cone gauge we are left with the transverse coordinates $X$ and corresponding momenta $P$, which transform as vectors under the SO(9) group of transverse rotations. Only sixteen fermionic components $\theta$ remain, which transform as SO(9) spinors. Furthermore we have the CM momentum $P^+_0$ and the CM coordinate $X^-_0$ (the remaining modes in $X^-$ are dependent), while the CM momentum $P^-_0$ is equal to minus the supermembrane Hamiltonian and takes the following form

$$H = \frac{\bar{P}^2_0}{2P_0^+} + \frac{\mathcal{M}^2}{2P_0^+}.$$

Here $\mathcal{M}$ is the supermembrane mass operator, which does not depend on any of the CM coordinates or momenta. The explicit expression for $\mathcal{M}^2$ is

$$\mathcal{M}^2 = \int d^2 \sigma \sqrt{w(\sigma)} \left[ \frac{\bar{P}^2(\sigma)}{w(\sigma)} + \frac{1}{2} \left\{ X^a, X^b \right\}^2 - 2P^+_0 \bar{\theta} \gamma_a \left\{ X^a, \theta \right\} \right],$$

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where $[\mathbf{P}^2]'$ indicates that the contribution of the CM momentum $\mathbf{P}_0$ is suppressed.

The structure of the Hamiltonian (33) shows that the wave functions for the supermembrane now factorize into a wave function pertaining to the CM modes and a wave function of the supersymmetric quantum-mechanical system that describes the other modes. For the latter the mass operator plays the role of the Hamiltonian. When the mass operator vanishes on the state, then the 32 supercharges act exclusively on the CM coordinates and generate a massless supermultiplet of eleven-dimensional supersymmetry. In case there is no other degeneracy beyond that caused by supersymmetry, the resulting supermultiplet is the one of supergravity, describing the graviton, the antisymmetric tensor and the gravitino. In terms of the $SO(9)$ helicity representations, it consists of $44 \oplus 84$ bosonic and $128$ fermionic states. For an explicit construction of these states, see [14]. When the mass operator does not vanish on the states, we are dealing with huge supermultiplets consisting of multiples of $2^{15} + 2^{15}$ states.

3 The matrix approximation

The expressions for the Hamiltonian (12), the supercharges (27) and the constraints associated with (14) are clearly in direct correspondence with the Hamiltonian, supersymmetry charges and the Gauss constraints for the matrix models introduced in section 1. This correspondence between de supermembrane and supersymmetric quantum mechanics becomes exact after one replaces $P_0^+$ by the coupling constant $g$ and rewrites the spinor coordinates in terms of a real $SO(9)$ spinor basis. In order to make the relation more explicit one may expand functions on the spacesheet in a complete set of functions $Y^A$ with $A = 0, 1, 2, \ldots, \infty$. It is convenient to choose $Y^0 = 1$. Furthermore we choose a basis of the closed one-forms, consisting of the exact ones, $\partial_r Y^A$, and a set of closed nonexact forms denoted by $\phi^{(\lambda)}_r$. Completeness implies the following decompositions,

$$\{Y^A, Y^B\} = f^{ABC} Y^C,$$

$$\varepsilon^{rs}_{\lambda} \phi^{(\lambda)}_r \partial_s Y^A = f_{\lambda A}^B Y^B,$$

$$\varepsilon^{rs}_{\lambda \lambda'} \phi^{(\lambda)}_r \phi^{(\lambda')}_s = f_{\lambda \lambda'}^A Y^A,$$  (35)

so that the constants $f^{ABC}, f_{\lambda A}^B$ and $f_{\lambda \lambda'}^A$ represent the structure constants of the infinite-dimensional group of area-preserving diffeomorphisms. Lowering of indices can be done with the help of the invariant metric

$$\eta_{AB} = \int d^2 \sigma \sqrt{w(\sigma)} Y^A(\sigma) Y^B(\sigma).$$  (36)

There is no need to introduce a metric for the $\lambda$ indices. Observe that we have $\eta_{00} = 1$. Furthermore it is convenient to choose the functions $Y^A$ with $A \geq 1$ such
that $\eta_{0A} = 0$. Completeness implies

$$
\eta^{AB} Y_A(\sigma) Y_B(\rho) = \frac{1}{\sqrt{w(\sigma)}} \delta^{(2)}(\sigma, \rho). 
$$

(37)

After lowering of upper indices, the structure constants are defined as follows

\begin{align*}
    f_{ABC} &= \int d^2 \sigma \varepsilon^{rs} \partial_r Y_A(\sigma) \partial_s Y_B(\sigma) Y_C(\sigma), \\
    f_{\lambda BC} &= \int d^2 \sigma \varepsilon^{rs} \phi_{(\lambda)r}(\sigma) \partial_s Y_B(\sigma) Y_C(\sigma), \\
    f_{\lambda \lambda' C} &= \int d^2 \sigma \varepsilon^{rs} \phi_{(\lambda)r}(\sigma) \phi_{(\lambda')s}(\sigma) Y_C(\sigma). 
\end{align*}

(38)

Note that we have $f_{AB0} = f_{\lambda B0} = 0$.

Using the above basis one may write down the following mode expansions for the phase-space variables of the supermembrane,

\begin{align*}
    \partial_r X(\sigma) &= \sum_{\lambda} X^\lambda \phi_{(\lambda)r}(\sigma) + \sum_A X^A \partial_r Y_A(\sigma), \\
    P(\sigma) &= \sum_A \sqrt{w(\sigma)} P^A Y_A(\sigma), \\
    \theta(\sigma) &= \sum_A \theta^A Y_A(\sigma), 
\end{align*}

(39)

introducing winding modes for the transverse coordinates $X$. A similar expansion exists for $X^-$.

Other tensors are needed, for instance, to write down the Lorentz algebra generators [3]. An obvious tensor is given by

$$
    d_{ABC} = \int d^2 \sigma \sqrt{w(\sigma)} Y_A(\sigma) Y_B(\sigma) Y_C(\sigma), 
$$

(40)

which is symmetric in all three indices and satisfies $d_{AB0} = \eta_{AB}$. Another tensor, whose definition is more subtle, arises when expressing $X^-$ in terms of the other coordinates and momenta. We recall that $X^-$ is restricted by (14), which implies the following Gauss-type constraint,

$$
    \varphi^A = f_{BC}^A \left[ P^B \cdot X^C + P_0^+ \bar{\theta}^B \gamma_- \theta^C \right] + f_{BA}^A P^B \cdot X^A \approx 0. 
$$

(41)

The coordinate $X^-$ receives contributions proportional to $Y_A(\sigma)$, which can be parametrized by ($A \neq 0$)

$$
    X^-_A \approx \frac{1}{2P_0^+} c_{BC}^A \left[ P^B \cdot X^C + P_0^+ \bar{\theta}^B \gamma_- \theta^C \right] + \frac{1}{2P_0^+} c_{B\lambda}^A P^B \cdot X^\lambda. 
$$

(42)

In addition $X^-$ has CM and winding modes. Observe that the tensors $c_{ABC}^A$ and $c_{B\lambda}^A$ are somewhat ambiguous, as (12) is only defined up to the constraints (11). The
symmetric component of $c^A_{BC}$ is, however, fixed and given by $c^A_{BC}+c^A_{CB}=-2d_{ABC}$. Note that $c^A_{B0}=0$. There are many other identities between the various tensors, such as

$$f_{[AB}^E f_{C]E}^D = d_{(AB}^E f_{C)E}^D = d_{ABC} f_{[DE}^B f_{FG]}^C = c_{DE}^A f_{BC]E} = d_{EA]BD} c_{D]E} = 0.$$ (43)

If we replace the group of the area-preserving diffeomorphisms by a finite group, then (34) defines the Hamiltonian of a supersymmetric quantum-mechanical system based on a finite number of degrees of freedom [15]. In the limit to the infinite-dimensional group we thus recover the supermembrane. This observation enables one to regularize the supermembrane in a supersymmetric way by considering a limiting procedure based on a sequence of groups whose limit yields the area-preserving diffeomorphisms. For closed membranes of certain topology it is known how to approximate a (sub)group of the area-preserving diffeomorphisms as a particular $N \to \infty$ limit of SU($N$). To be precise, it can be shown that the structure constants of SU($N$) tend to those of the diffeomorphism subgroup associated with the Hamiltonian vectors, up to corrections of order $1/N^2$. While some of the identities (43) remain valid at finite $N$, others receive corrections of order $1/N^2$. Furthermore, the tensors $c^A_{BC}$ and $c^A_{B\lambda}$ are intrinsically undefined at finite $N$. Therefore, the expression for $X^-$ is ambiguous for the matrix model and Lorentz invariance holds only in the large-$N$ limit [3, 4].

The nature of the large-$N$ limit itself is subtle and depends on the membrane topology. As long as $N$ is finite, no distinction can be made with regard to the topology. In some sense, all topologies are thus included at the level of finite $N$. However, the diffeomorphisms associated with the harmonic vectors are problematic, because they cannot be incorporated for finite $N$, at least not at the level of the Lie algebra. This was shown in [3], where it was established that the finite-$N$ approximation to the structure constants $f_{ABC}$ violates the Jacobi identities for a toroidal membrane. Therefore it seems impossible to present a matrix model regularization of the supermembrane with winding contributions. There exists a standard prescription for dealing with matrix models with winding [16], however, which is therefore conceptually different. The consequences of this difference are not well understood. The prescription amounts to adopting the gauge group $[U(N)]^M$, for winding in one dimension, which in the limit $M \to \infty$ leads to supersymmetric Yang-Mills theories in $1+1$ dimensions [10]. Hence, in this way it is possible to extract extra dimensions from a suitably chosen infinite-dimensional gauge group. Obviously this approach can be generalized to a hypertorus.

Finally we add that the matrix regularization works also for the case of open supermembranes. In that case one deals with certain subgroups of SU($N$). We refer to [11] for further details.
4 Membranes and matrix models in curved space

So far we considered a supermembrane moving in a flat target superspace. To that order we substituted the flat superspace expressions (3) into the supermembrane action (4). However, these expressions can in principle be evaluated for nontrivial backgrounds, such as those induced by a nontrivial target-space metric, a target-space tensor field and a target-space gravitino field, corresponding to the fields of (on-shell) 11-dimensional supergravity. This background can in principle be incorporated into superspace by a procedure known as ‘gauge completion’ [17]. For 11-dimensional supergravity, the first steps of this procedure have been carried out long ago [18], but unfortunately only to first order in fermionic coordinates $\theta$.

For brevity of the presentation, let us just confine ourselves to the purely bosonic case and present the light-cone formulation of the membrane in a background consisting of the metric $G_{\mu\nu}$ and the tensor gauge field $C_{\mu\nu\rho}$ [19]. The Lagrangian density for the bosonic membrane follows directly from (4),

$$L = -\sqrt{-g} + \frac{1}{6}\varepsilon^{ijk}\partial_i X^\mu \partial_j X^\nu \partial_k X^\rho C_{\mu\nu\rho},$$

(44)

where $g_{ij} = \partial_i X^\mu \partial_j X^\nu \eta_{\mu\nu}$. For the light-cone formulation, the coordinates are treated in the usual fashion in terms of light-cone coordinates $X^\pm$ and transverse coordinates $X$. Furthermore we use the diffeomorphisms in the target space to bring the metric in a convenient form [20],

$$G_{--} = G_{a-} = 0.$$

(45)

Following the same steps as for the membrane in flat space, discussed in section 2, one again derives a Hamiltonian formulation. Interestingly enough, the constraint takes the same form as (14). Of course, the definition of the momenta in terms of the coordinates and their derivatives does involve the background fields, but at the end all explicit dependence on the background cancels out.

The Hamiltonian now follows straightforwardly. After additional gauge choices,

$$C_{++} = 0, \quad C_{-a} = 0, \quad G_{--} = 1,$$

(46)

it takes the form

$$H = \int d^2 \sigma \left\{ \frac{1}{P_-} \left[ \frac{1}{2}(P_a - G_{a+} - P_- G_{a+})^2 + \frac{1}{4}(\varepsilon^{rs} \partial_r X^a \partial_s X^b)^2 \right] \\
- \frac{1}{2} P_- G_{++} - \frac{1}{2} \varepsilon^{rs} \partial_r X^a \partial_s X^b C_{+ab} \right\},$$

(47)

We want to avoid explicit time dependence of the background fields, so we assume the metric and the tensor field to be independent of $X^+$. If we assume, in addition, that they are independent of $X^-$, it turns out that $P_-$ becomes $\tau$-independent. This allows us to set $P_-(\sigma) = (P_-)_0 \sqrt{w(\sigma)}$, exactly as in flat space. With these restrictions, it is possible to write down a gauge theory of area-preserving diffeomorphisms for the membrane in the presence of background fields. Its Lagrangian density equals

$$w^{-1/2} \mathcal{L} = \frac{1}{2}(D_0 X^a)^2 + D_0 X^a \left( \frac{1}{2} C_{abc} \{X^b, X^c\} + G_{a+} \right) - \frac{1}{4} \{X^a, X^b\}^2 + \frac{1}{2} G_{++} + \frac{1}{2} C_{+ab} \{X^a, X^b\},$$

(48)
where we used the metric $G_{ab}$ to contract transverse indices; the Poisson bracket and the covariant derivatives were already introduced in section 2. For convenience we have set $(P_0)_0 = 1$.

The action corresponding to (48) is manifestly invariant under area-preserving diffeomorphisms in the presence of the background fields. It is now straightforward to write it in terms of a matrix model, by truncating the mode expansion for coordinates and momenta as explained in the previous section. Matrix models in curved space have been discussed before [21]; for more recent papers dealing with matrix models in the presence of certain backgrounds, see [22]. A more explicit derivation of the results of this section and their supersymmetric extension will appear in a forthcoming publication [23].

5 The continuous supermembrane mass spectrum

The continuous mass spectrum of the supermembrane forms an obstacle in interpreting the membrane states as elementary particles, in analogy to what is done in string theory. Instead the continuity of the spectrum should be viewed as a result of the fact that supermembrane states do not really exist as asymptotic states. The membrane collapses into stringlike configurations and is to be interpreted as a multimembrane state. Obviously such states exhibit a continuous mass spectrum. As we alluded to earlier, there is evidence that massless ground states exist, probably associated with the states of 11-dimensional supergravity [9]. In the winding sector there may exist massive BPS states, which are the lowest-mass states for given winding number. Whether additional non-BPS bound states exist is not known. It could be that beyond the massless and BPS winding states, there is nothing than a continuum of multimembrane states.

Qualitatively, the situation is the same for the matrix models (1) based on a finite number of degrees of freedom. Among the zero-energy states there are those where the matrices take a block-diagonal form, which can be regarded as a direct product of states belonging to lower-rank matrix models [24]. The fact that the moduli space of ground states, whose nature is protected by supersymmetry at the quantum-mechanical level, is isomorphic to $R^{9N}/S_N$, is already indicative of a corresponding description in terms of an $N$-particle Fock space. The finite-$N$ matrix models have an independent interpretation in string theory. Strings can end on certain defects by means of Dirichlet boundary conditions. These defects are called D-branes (for further references, see [25]). They can have a $p$-dimensional spatial extension and carry Ramond-Ramond charges [26]. D-Branes play an important role in the non-perturbative behaviour of string theory. The models of section 1 are relevant for D0-branes (Dirichlet particles), but we note in passing that there are similar models relevant for higher-dimensional D-branes, which emerge in the zero-volume limit of supersymmetric gauge theories coupled to matter.

The effective short-distance description for D-branes can be derived from simple arguments [27]. As the strings must be attached to the $p$-dimensional branes, we are dealing with open strings whose endpoints are attached to a $p$-dimensional
subspace. At short distances, the interactions caused by these open strings are
determined by the massless states of the open string, which constitute the ten-
dimensional Yang-Mills supermultiplet, propagating in a reduced \((p+1)\)-dimensional
spacetime. Because the endpoints of open strings carry Chan-Paton factors the ef-
efective short-distance behaviour of \(N\) D-branes can be described in terms of a U(\(N\)
) ten-dimensional supersymmetric gauge theory reduced to the \((p+1)\)-dimensional
world volume of the D-brane. The U(1) subgroup is associated with the center-of-
mass motion of the \(N\) D-branes.

In the type-IIA superstring one has Dirichlet particles moving in a 9-dimensional
space. As the world volume of the particles is one-dimensional \((p = 0)\), the short-
distance interactions between these particles is thus described by the model of sec-
tion 1 with gauge group U(\(N\)) and \(d = 9\). The continuous spectrum without gap is
natural here, as it is known that, for static D-branes, the Ramond-Ramond repulsion
cancels against the gravitational and dilaton attraction, a similar phenomenon as for
BPS monopoles. With this gauge group the coordinates can be described in terms
of \(N \times N\) hermitean matrices. The valley configurations correspond to the situation
where all these matrices can be diagonalized simultaneously. The eigenvalues then
define the positions of \(N\) D-particles in the 9-dimensional space. As soon as one or
several of these particles coincide then the \([U(1)]^N\) symmetry that is left invariant
in the valley, will be enhanced to a nonabelian subgroup of U(\(N\)). Clearly there
are more degrees of freedom than those corresponding to the D-particles, which are
associated with the strings stretching between the D-particles. As we alluded to
above the model naturally incorporates configurations corresponding to widely sep-
parated clusters of D-particles, each of which can be described by a supersymmetric
quantum-mechanics model based on the product of a number of U(\(k\)) subgroups
forming a maximal commuting subgroup of U(\(N\)). When all the D-particles move
further apart this corresponds to configurations deeper and deeper into the potential
valleys. These D-particles thus define an independent perspective on the models
introduced in section 1, which can be used to study their dynamics. We refer to [28]
for work along these lines.

The study of D-branes was further motivated by a conjecture according to which
the degrees of freedom of M-theory are fully captured by the U(\(N\)) super-matrix
models in the \(N \rightarrow \infty\) limit [24]. The elusive M-theory is defined as the strong-
coupling limit of type-IIA string theory and is supposed to capture all the relevant
degrees of freedom of all known string theories, both at the perturbative and the
nonperturbative level [29, 30]. In this description the various string-string dualities
are fully incorporated. At large distances M-theory is described by 11-dimensional
supergravity. A direct relation between supermembranes and type-IIA string theory
was emphasized in [25], based on the relation between extremal black holes in 10-
dimensional supergravity [31] and the Kaluza-Klein states of 11-dimensional super-
gravity in an \(S^1\) compactification. In this compactification the Kaluza-Klein photon
coincides with the Ramond-Ramond vector field of type-IIA string theory. Therefore
Kaluza-Klein states are BPS states whose Ramond-Ramond charge is proportional
to their mass. Hence they have the same characteristics as the Dirichlet particles.
On the other hand, the effective interaction between infinitely many Dirichlet particles leads to a theory that is identical to that of an elementary supermembrane. There are alternative compactifications of M-theory which make contact with other string theories. Supermembranes have been used to provide evidence for the duality of M-theory on $\mathbb{R}^{10} \times S^1/\mathbb{Z}_2$ and 10-dimensional $E_8 \times E_8$ heterotic strings [12]. Finally the so-called double-dimensional reduction of membranes leads to fundamental string states [32].

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