Effect of first-forbidden decays on the shape of neutrino spectra

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(Received 25 September 2014; revised manuscript received 5 January 2015; published 19 February 2015)

We examine the effect of first forbidden (FF) decays on $\beta$-decay neutrino spectra by performing microscopic nuclear structure calculations. By analyzing the FF decay branches of even-even nuclei, we conclude that FF decays may be responsible for part of the missing neutrinos in the so-called reactor neutrino anomaly. Further calculations and more experimental data are needed for a firm conclusion.

DOI: 10.1103/PhysRevC.91.025503

PACS number(s): 23.40.Bw, 14.60.Lm, 21.60.Jz

I. INTRODUCTION

The “reactor antineutrino anomaly” is the observation that the average of the experimentally determined reactor antineutrino flux at reactor-detector distances less than 100 m accounts for only $0.946 \pm 0.023$ of the theoretical expectation [1–3]. One of the explanations for this anomaly is that standard antineutrino flux at reactor-detector distances less than 100 m is correct. For 0 electron mass, $\omega$ is the $\beta$-decay energy in the units of electron mass, and $p = \sqrt{\omega^2 - 1}$ is the momentum of the electron. $F(Z,\omega)$ is the Fermi factor, which takes into account the nuclear charge on the shape of the spectra for the emitted electron.

The nuclear structure dependence on the shape of the emitted leptons is contained in $C(\omega)$. It has different $\omega$ dependencies for different kinds of decays that lead to different spectra for emitted electrons and neutrons. For allowed decay, $C(\omega)$ is independent of $\omega$. For the FF decay, the dependence can be written in the following form [7]:

$$C(\omega) = K_0 + K_1 \omega + K_{-1}/\omega + K_2 \omega^2.$$  \hfill (3)

For FF decays one has three different types of transitions associated with the change of spins, $\Delta J^z = 0^-, 1^-, 2^-$, and they have different matrix elements and $\omega$ dependencies:

$$C_{\Delta J^z=0}(\omega) = K_0 + K_{-1}/\omega,$$

$$C_{\Delta J^z=1}(\omega) = K_0 + K_1 \omega + K_{-1}/\omega + K_2 \omega^2,$$

$$C_{\Delta J^z=2}(\omega) = K_0 + K_1 \omega + K_2 \omega^2.$$  \hfill (4)

The detailed expressions for the $K$’s can be obtained from Refs. [7,8]. For $0^-$, there are three matrix elements, $M_0^u$, $M_0^d$, and $M_0^s$; for $1^-$, one has five matrix elements involving $u$, $u'$, $x$, $x'$, and $y$; and for $2^-$, just one matrix element $z$ is involved. The expressions for these matrix elements are given in Ref. [7]. In Ref. [5] only the $M_{\Delta J^z}^u$, $u$, $x$, and $z$ terms were used for FF branches. Our additional terms result in some differences between our results and those of Ref. [5].

II. THEORY OF $\beta$ DECAY

The decay rate for $\beta$ decay can be written generally as [7,8]

$$\lambda = \ln 2/t_{1/2} = \sum_i \lambda_i.$$  \hfill (1)

With the conventions and numerical constants used in Refs. [7,8] one obtains

$$f = 8896 \text{ s}^{-1} \lambda = \sum_i \int_0^{\infty} C(\omega)F(Z,\omega)p\omega(\omega_0 - \omega)^2 d\omega.$$  \hfill (2)

Here $\omega \equiv E_e/m_e$ is the energy of the emitted electron in the units of electron mass, $\omega_0$ is the $\beta$-decay energy in the units of electron mass, and $p = \sqrt{\omega^2 - 1}$ is the momentum of the electron. $F(Z,\omega)$ is the Fermi factor, which takes into account the nuclear charge on the shape of the spectra for the emitted electron.

The decay rate for $\beta$ decay on the shape of the neutrino spectra by performing microscopic nuclear structure calculations. By analyzing the FF decay branches of even-even nuclei, we conclude that FF decays may be responsible for part of the missing neutrinos in the so-called reactor neutrino anomaly. Further calculations and more experimental data are needed for a firm conclusion.

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To get the electron or neutrino spectra, we take derivatives over the respective energies:

\[
\frac{dN_e}{d\omega} = N \frac{d\lambda_e}{d\omega} = C(\omega) F(Z, \omega) p(\omega_0 - \omega)^2,
\]

\[
\frac{dN_\nu}{d\omega} = N \frac{d\lambda_\nu}{d\omega} = C(\omega_0 - \omega_\nu) F(Z, \omega_0 - \omega_\nu)
\times \omega_\nu^2 \sqrt{(\omega_0 - \omega_\nu)^2 - 1}.
\]

The spectra for FF decays are different from that of allowed GT, and their shape depends on the decay modes \((J^\pi)^f\). In this work we generally follow the previous work where we used \(g_A(0^+) = 0.5 g_{A\nu(0^+)}\) for all types of transitions but slightly change some of the parameters to better reproduce the log \(ft\) values in the Xe region: \(g_A(1^+) = 0.4 g_A\) and \(g_V(0^+) = 0.6 g_V\). The same quenching values are used for the SM calculations.

In Table I we present the comparisons of experimental results with the SM and QRPA methods for the two even-even nuclei. For \(^{136}\text{Te}\), where the experimental data and both calculations are possible, we see good agreement among them. A one-to-one correspondence of most decay branches can be found between the SM calculations and the experimental results, and the difference of the log \(ft\) values are within 0.2, which means a factor of 1.5 in the transition rates. The QRPA calculations agree with the shell model with differences for log \(ft\) values around 0.1–0.2. Another even-even nucleus which has been measured is \(^{140}\text{Xe}\). However, it is beyond the

| \(J^\pi\) | \(r(s)\) | \(\text{Exp. \cite{12}}\) | \(\text{ShM}\) | \(\text{QRPA}\) |
|---|---|---|---|---|
| \(J^\pi\) | \(E_{ex}\) | \(\log ft\) | \(J^\pi\) | \(E_{ex}\) | \(\log ft\) | \(J^\pi\) | \(E_{ex}\) | \(\log ft\) |
| \(^{136}\text{Te}\) | 0\(^+\) | 17.63 | 0 | >6.7 | 1\(^-\) | 0 | 6.85 | 0\(^-\) | 0 | 6.37 |
| | (0\(^+\)) | 0.222 | 7.23 | 0.095 | 7.37 | 1\(^-\) | 0.171 | 6.95 |
| | (0\(^-\)) | 0.334 | 6.27 | 0.133 | 6.41 | 2\(^-\) | 0.194 | 7.89 |
| | (0\(^-\)) | 0.631 | 6.28 | 0.426 | 6.26 | 2\(^-\) | 0.541 | 6.99 |
| | (0\(^-\)) | 0.738 | 7.57 | 0.507 | 6.71 | 1\(^-\) | 0.747 | 6.13 |
| | 13.6 | 0.080 | 6.14 | 0 | 0 | 6.15 |
| \(^{140}\text{Xe}\) | 0\(^+\) | 0.515 | 6.82 | 1\(^-\) | 0.127 | 6.77 |
| | (0\(^-\)) | 0.653 | 5.98 | 2\(^-\) | 0.365 | 7.01 |
| | (1\(^\pm\)) | 0.800 | 7.21 | 2\(^-\) | 0.586 | 6.05 |
| | 1\(^-\) | 0.966 | 6.77 | 1\(^-\) | 1.353 | 6.75 |
reach of our current SM computational capacity, so only QRPA results are shown. One finds that for this nucleus, the QRPA calculations are in good agreement with the measurement.

As we have stated above, different decay channels may have different shapes due to different dependencies over energy $\omega$, so we need to investigate the effects of these decay channels on the neutrino spectra shape. For the odd-odd or odd-A nuclei there is usually mixing between different decay channels as $|J_i - J_f| \leq \Delta J \leq J_i + J_f$, but for even-even nuclei, because the ground states of the parent nuclei has always $J_i = 0$, $\Delta J$ is unique for specific final state of daughter nuclei, there will be no mixing among different channels, and it is easy to isolate different shape changes in different decay channels.

In Fig. 1, we compare the neutrino spectra shape changes relative to the allowed shape for different channels with different methods for two even-even nuclei ($^{136}$Te and $^{140}$Xe). For each nucleus we show the $0^-$, $1^-$, and $2^-$ decay branches. The SM and QRPA methods agree well with each other. For $0^-$ decays, the change of the spectra is small and it is a good approximation to treat the $0^-$ decay as allowed decay. For $1^-$ decay the change is large with the peak of the neutrino spectra shifted downwards. This means that more neutrinos have less energy than expected from the previous simulation [2] using the allowed type of phase space. For $2^-$ decay the behavior of the change to the shape is a bit different from that of $1^-$ as seen from Fig. 1, where the shape of the neutrino spectra for this decay branch is broadened.

We also make a comparison of the full microscopic calculations to the approximations made in Ref. [5], where 4 out of 9 matrix elements are used (affecting the $0^-$ and $1^-$ decays). For the $0^-$ decay, the approximation used in Ref. [5] gives a result that is opposite to the full microscopic calculations, slightly shifting the neutrino spectra to lower energy. For $1^-$ decay, the approximation completely changes the behavior of the neutrino spectra. Due to the oversimplified forms in Ref. [5], the behavior of an overall shift of spectra to low energies disappears now. This comes from the fact that for simplified $1^-$ decay in Table I of [5] one of its matrix elements ($[\Sigma, r]^{1-}$ or $u$ in this work) has the same form as that for $2^-$ decay ($[\Sigma, r]^{2-}$ or $z$ in this work).

There is similar behavior between $^{136}$Te and $^{140}$Xe. We would also expect the same behaviors of these FF decay channels in odd-mass or odd-odd nuclei since they have the same transition operators as the even-even nuclei. From these results, we conclude that the inclusion of FF decays could eliminate the “reactor antineutrino anomaly” if there are enough $\beta$ branches containing $1, 2^-$ transitions with suitable endpoint energies, especially $1^-$. However, if we examine the nuclear chart for the decay branching ratios, we find that $1, 2^-$ are usually accompanied with $0^-$ decays, which usually have much smaller log $\beta$ values (a stronger transition probability). This would reduce the overall changes to the spectra.

To quantify the change in the neutrino spectrum due to the change of phase space, we integrate over the spectra with the two phase spaces as follows:

$$\delta = \frac{1 - n_{FF}(E < E_i)}{1 - n_{GT}(E < E_i)}$$

$$n_I(E < E_i) = \int_0^{E_i} \frac{dN}{dE_\nu}(E_\nu)dE_\nu,$$

where $n_{FF}$ and $n_{GT}$ denote the number of transitions for the full and allowed phase spaces, respectively. $\delta$ measures the relative change in the number of transitions for a given endpoint energy $E_i$.
with \( \int_0^{E_{\text{end}}} dN/dE_\nu(E_\nu) dE_\nu = 1 \). \( E_t \) is the energy needed to trigger the interaction \( \bar{\nu}_e + p \rightarrow e^{-} + n \), and \( E_{\text{end}} \) is the maximum energy of emitted neutrinos. The reduction in the number of low-energy neutrinos is given by \( \Delta = 1 - \delta \). The change depends on the endpoint energy \( E_{\text{end}} \), which can be expressed as \( Q_\beta - m_e - E_{\text{ex}} \). So we need precise excitation energies for the determination of neutrino spectra. This result can then be compared with the value of the reactor neutrino anomaly to see if the lack of FF phase space factor in the simulation can explain the missing neutrinos. The results for single decay branches are listed in Table II. A comparison between QRPA and shell model shows similarities for the ratio \( \delta \), which agrees with Fig. 1. For the detailed values, the change \( \Delta \) of the \( 0^- \) decay is negligible, for \( 1^- \), \( \Delta \) goes up to 10\%, and for \( 2^- \), \( \Delta \) is only 2–3\%.

To obtain quantitative results on the dependence of the detailed changes on the endpoint energies of the decay branches, we vary the \( Q \) values in the calculations for the two nuclei \(^{136}\text{Te}\) and \(^{140}\text{Xe}\). The results are plotted in Fig. 2, where one observes that to a large extent this relation is nucleus independent. \( \Delta \) for the \( 0^- \) branches are near zero except below endpoint energies of 3 MeV. For small endpoint energies, \( \Delta \) is large due to the shape changes at the spectra tail, but these are not important since contributions of these branches to the total spectra are small; see Fig. 3 of Ref. [5]. For \( 2^- \) decay the dependence of \( \delta \) on the endpoint energies is independent of \( \log f t \) values since it has only one component. For endpoint energies from 4 to 6 MeV, \( \Delta \) is around 3–4\%.

However, for \( 1^- \) decays \( \delta \) depends on both \( E_{\text{end}} \) and \( \log f t \). To see this we also plot the \( 1^- \) decay branches for the two nuclei in Fig. 2. Compared to \( 1^- \) the FF decays to the \( 1^- \) states have smaller \( \log f t \) values (Table I) (i.e., they are stronger) and have smaller \( \Delta \) values (Fig. 2). The reason of this comes from the fact that the transition rates of \( 1^- \) are determined by five different components. They are combined to give the final decay rates, and their different combinations have different energy dependencies. At \( E_{\text{end}} \sim 4–6 \) MeV, \( \Delta \) is 5–15\%. It was estimated in Ref. [5] that 30\% of the decay branches of the fission products are FF. Thus, in the most extreme case where the FF is dominated by \( \Delta J^{\pi} = 1^- \) the change of the neutrino spectrum could be as large as \( \Delta = 4.5\% \).

**IV. CONCLUSION**

In this work, explicit analysis of \( \beta \)-decay neutrino spectra with inclusion of the first forbidden part has been performed. One finds that use of the allowed decay phase space factor results in a correction of up to about \( \Delta \sim 4.5\% \) due to \( \Delta J^{\pi} = 1^- \) FF transitions. An average over all types of FF transitions, endpoint energies, and \( \log f t \) values would result in a smaller value of \( \Delta \sim 1^- \) 2–3\%. The finite size effects and the weak magnetism corrections obtained in Ref. [4] for the allowed (GT) decays are estimated to be \( \Delta \sim 2–3\% \). If the average branching ratios for all types of FF are estimated, they can be combined with our results to obtain an improved correction for the shape of the neutrino spectra.

**ACKNOWLEDGMENTS**

We thank Prof. A. Hayes for useful discussions and helpful data. This work was supported by the US NSF Grants No. PHY-0822648 and No. PHY-1404442.

TABLE II. The percentage of the numbers of neutrinos of the actual decay compared with the allowed shapes used in the simulation for single-decay branches of \(^{136}\text{Te}\) and \(^{140}\text{Xe}\), denoted by \( \delta \) defined in text. The superscripts here are \( Q \) for QRPA and \( S \) for shell model; the subscript “simp” means that we used the simplified FF matrix elements used in Ref. [5].

| \( E_{\text{ex}} \) | \( \delta^Q \) | \( \delta^Q_{\text{simp}} \) | \( E^S_\text{ex} \) | \( \delta^S \) | \( E^Q_\text{ex} \) | \( \delta^Q \) | \( \delta^Q_{\text{simp}} \) |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 0^- | 0.0 | 1.000 | 0.995 | 0.133 | 1.000 | 0.0 | 1.003 | 0.990 |
| 1^- | 0.171 | 0.899 | 0.929 | 0.0 | 0.902 | 0.127 | 0.875 | 0.949 |
| 1^- | 0.747 | 0.938 | 0.971 | 0.426 | 0.933 | 0.586 | 0.919 | 0.981 |
| 2^- | 0.194 | 0.968 | 0.065 | 0.970 | 0.060 | 0.960 | 0.971 |
| 2^- | 0.541 | 0.968 | 0.507 | 0.982 | 0.365 | 0.976 |

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