Complete Wetting of Gluons and Gluinos *

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Abstract

Complete wetting is a universal phenomenon associated with interfaces separating coexisting phases. For example, in the pure gluon theory, at $T_c$ an interface separating two distinct high-temperature deconfined phases splits into two confined-deconfined interfaces with a complete wetting layer of confined phase between them. In supersymmetric Yang-Mills theory, distinct confined phases may coexist with a Coulomb phase at zero temperature. In that case, the Coulomb phase may completely wet a confined-confined interface. Finally, at the high-temperature phase transition of gluons and gluinos, confined-confined interfaces are completely wet by the deconfined phase, and similarly, deconfined-deconfined interfaces are completely wet by the confined phase. For these various cases, we determine the interface profiles and the corresponding complete wetting critical exponents. The exponents depend on the range of the interface interactions and agree with those of corresponding condensed matter systems.

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1 Introduction

Supersymmetric gauge theories have attracted a lot of attention because, for example, some aspects of their dynamics are accessible to analytic methods. In particular, techniques from string theory can be applied to these systems. The $SU(N)$ Yang-Mills theory with $\mathcal{N} = 1$ supersymmetry describes the interactions between gluons and gluinos. Here we investigate this model both at zero and at non-zero temperatures. Of course, at non-zero temperatures supersymmetry is explicitly broken due to different boundary conditions for gluons and gluinos in the Euclidean time direction. This is no problem because we will use methods of effective field theory that do not rely on supersymmetry. At low temperatures, the theory is in a confined phase with a spontaneously broken $Z(N)_\chi$ chiral symmetry, while at high temperatures one expects chiral symmetry to be restored and gluons and gluinos to be deconfined. In the deconfined phase, the $Z(N)_c$ center symmetry of the $SU(N)$ gauge group is spontaneously broken. In addition, at zero temperature gluons and gluinos may exist in a non-Abelian Coulomb phase without confinement or chiral symmetry breaking. The existence of the Coulomb phase is a controversial issue. For the rest of this paper, we assume that it exists. The rich phase structure gives rise to interesting effects related to the interfaces separating various bulk phases. Due to spontaneous $Z(N)_\chi$ breaking, there are $N$ distinct confined phases with different values of the gluino condensate $\chi^{(n)} = \chi_0 \exp(2\pi in/N)$, $n \in \{1, 2, ..., N\}$. These phases are separated by confined-confined interfaces. Similarly, at high temperatures, $Z(N)_c$ breaking gives rise to $N$ deconfined phases with different values of the Polyakov loop $\Phi^{(n)} = \Phi_0 \exp(2\pi in/N)$, $n \in \{1, 2, ..., N\}$, which are separated by deconfined-deconfined interfaces. Assuming a first order phase transition, at $T_c$ there are, in addition, confined-deconfined interfaces. Finally, at zero temperature, the confined phases can coexist with the Coulomb phase, which has a vanishing gluino condensate $\chi^{(0)} = 0$. Hence, there are also confined-Coulomb interfaces.

The properties of confined-Coulomb and confined-confined interfaces at zero temperature have been studied in detail by Shifman and collaborators [1, 2]. In particular, the tension of an interface separating two bulk phases with gluino condensates $\chi^{(m)}$ and $\chi^{(n)}$ is constrained by the inequality

$$\alpha_{mn} \geq \frac{N}{8\pi^2} |\chi^{(m)} - \chi^{(n)}| = \frac{N\chi_0}{8\pi^2} |\exp(2\pi im/N) - \exp(2\pi in/N)|.$$

If the interface represents a BPS saturated state, this inequality is satisfied as an equality. Indeed, confined-Coulomb interfaces are BPS saturated [2], such that

$$\alpha_{0n} = \frac{N\chi_0}{8\pi^2}.$$

The interface tensions determine the shape of a droplet of Coulomb phase that wets a confined-confined domain wall. As shown in figure 1a, such a droplet forms a lens
with opening angle \( \theta \), where

\[
\alpha_{mn} = 2\alpha_0 \cos \frac{\theta}{2}.
\]

This equation for \( \theta \) follows from the forces at the corner of the lens being in equilib-

![Diagram](image_url)

**Figure 1:** *Incomplete versus complete wetting.* (a) For \( \alpha_{mn} < 2\alpha_0 \) one has incomplete wetting with \( \theta \neq 0 \). Then the Coulomb phase forms a lens shaped droplet at the confined-confined interface. (b) Complete wetting corresponds to \( \alpha_{mn} = 2\alpha_0 \). Then \( \theta = 0 \), and the Coulomb phase forms a film that splits the confined-confined domain wall into two confined-Coulomb interfaces.

In condensed matter physics the inequality

\[
\alpha_{mn} \leq 2\alpha_0,
\]

was derived by Widom and is also known as Antonoff’s rule. It follows from thermodynamic stability because a hypothetical confined-confined domain wall with \( \alpha_{mn} > 2\alpha_0 \) would simply split into two confined-Coulomb interfaces. Combining Antonoff’s rule with the BPS inequality, one finds

\[
\frac{N\chi_0}{8\pi^2} |\exp(2\pi im/N) - \exp(2\pi in/N)| \leq \alpha_{mn} \leq \frac{N\chi_0}{4\pi^2}.
\]

Assuming BPS saturation for confined-confined interfaces, the values of the interface tensions from above imply

\[
\theta = \pi - 2\pi \left( \frac{m - n}{N} \right), \text{ for } 1 \leq m - n \leq \frac{N}{2}.
\]
Then, for odd $N$, $\theta \neq 0$, such that the Coulomb phase forms droplets at a confined-confined interface. In condensed matter physics this phenomenon is known as incomplete wetting. For even $N$ and for $m - n = N/2$, on the other hand, $\alpha_{mn} = 2\alpha_{0n}$ and $\theta = 0$. In that case, the lens shaped droplet degenerates to an infinite film, as shown in figure 1b. When such a film is formed, this is called complete wetting. For example, the human eye is completely wet by a film of tears that splits the eye-air solid-gas interface into a pair of solid-liquid and liquid-gas interfaces. Complete wetting may occur when several bulk phases coexist with each other. For condensed matter systems, achieving phase coexistence usually requires fine-tuning of, for example, the temperature, to a first order phase transition. In supersymmetric theories, on the other hand, drastically different phases, like confined and Coulomb, may coexist at zero temperature, since their bulk free energies are identical due to supersymmetry. This implies that complete wetting appears naturally, i.e. without fine-tuning, in supersymmetric theories.

Complete wetting is a universal phenomenon of interfaces characterized by several critical exponents. For example, the width $r$ of the complete wetting layer diverges as

$$r \propto (T - T_c)^{-\psi},$$

where $T - T_c$ measures the deviation from the point of phase coexistence and $\psi$ is a critical exponent. The value of $\psi$ depends on the range of the interactions between the two interfaces that enclose the complete wetting layer. For condensed matter systems, the interaction energy per unit area is often due to repulsive van der Waals forces and is given by $c/r^p$. In addition, away from $T_c$ the phase that forms the wetting layer has a slightly larger bulk free energy than the other phases. Close to $T_c$, the additional bulk free energy per unit area for a wetting layer of width $r$ is given by $\mu(T - T_c)r$. Hence, the total free energy per unit area of the two interface system relative to the free energy of two infinitely separated interfaces at $T_c$ is given by

$$\alpha_{mn}(T) - 2\alpha_{0n}(T_c) = \frac{c}{r^p} + \mu(T - T_c)r.$$  

Minimizing the free energy with respect to $r$, one finds the equilibrium width

$$r \propto (T - T_c)^{-1/(p+1)},$$

such that $\psi = 1/(p + 1)$. Inserting the equilibrium value of $r$ one finds

$$\alpha_{mn}(T) - 2\alpha_{0n}(T_c) \propto (T - T_c)^{p/(p+1)} \propto (T - T_c)^{1-\psi}.$$  

Some condensed matter systems with interfaces interact via retarded van der Waals forces with a potential $c/r^3$, and thus have $\psi = 1/4$. As we will see, the confined-confined interfaces of the $\mathcal{N} = 1$ supersymmetric $SU(N)$ gauge theory, which are completely wet by the non-Abelian Coulomb phase, have the same critical exponent.

Complete wetting does not only occur in supersymmetric gauge theories, although only there it occurs naturally without fine-tuning. In fact, Frei and Patkós
conjectured that complete wetting occurs in the non-supersymmetric $SU(3)$ pure
gauge theory at the high-temperature deconfinement phase transition \cite{5}. In the
deconfined phase the $Z(3)_c$ center symmetry is spontaneously broken. Hence, there
are three distinct high-temperature phases separated by deconfined-deconfined inter-
faces. When a deconfined-deconfined interface is cooled down to the phase tran-
sition, the low-temperature confined phase forms a complete wetting layer. In that
case, the interactions between the interfaces are short-ranged, such that
\begin{equation}
\alpha_{mn}(T) - 2\alpha_{0n}(T_c) = a \exp(-br) + \mu(T - T_c)r.
\end{equation}
Minimizing the free energy with respect to $r$ now gives
\begin{equation}
r \propto \log(T - T_c),
\end{equation}
such that $\psi = 0$. Substituting the value of $r$, one obtains
\begin{equation}
\alpha_{mn}(T) - 2\alpha_{0n}(T_c) \propto (T - T_c) \log(T - T_c).
\end{equation}
This is the critical behavior that was observed in ref.\cite{6}. As we will see, the same
exponents follow for the high-temperature deconfinement transition in the supers-
symmetric case.

As argued before, for $N = 1$ supersymmetric $SU(3)$ gauge theory (and generally
for odd $N$) complete wetting may occur at zero temperature only if confined-confined
domain walls are not BPS saturated. Recently, we have shown that complete wetting
occurs at the high-temperature deconfinement phase transition \cite{7}. In this case, a
confined-confined interface is completely wet by one of the three deconfined phases
when it is heated up to the phase transition. The occurrence of deconfined phase at
the center of a confined-confined interface has interesting dynamical consequences.
It implies that a static quark has a finite free energy close to such a domain wall
and the string emanating from it can end at the wall. This effect was first derived
by Witten in the framework of M-theory \cite{8}. Complete wetting provides a field
theoretic explanation of the same effect without reference to string theory. Here
we concentrate on the derivation of the corresponding complete wetting critical
exponents. As in the non-supersymmetric case, we find $\psi = 0$ due to short-range
interactions between interfaces.

\section{Complete Wetting at Zero Temperature}

In $\mathcal{N} = 1$ supersymmetric $SU(N)$ gauge theory with even $N$ a confined-confined
interface separating phases with gluino condensates $\chi^{(m)}$ and $\chi^{(n)}$ is completely wet
by the non-Abelian Coulomb phase if $m - n = N/2$. In this case, Antonoff’s rule
implies BPS saturation of confined-confined domain walls. The simplest case we can
consider is $SU(2)$, where $\chi^{(1)} = \chi_0$ and $\chi^{(2)} = -\chi_0$. The universal aspects of the interface dynamics can be derived from an effective action

$$ S[\chi] = \int d^4x \left[ \frac{1}{2} \partial_{\mu} \chi \partial_{\mu} \chi + V(\chi) \right], $$

for the gluino condensate $\chi$. Under the $\mathbb{Z}(2)$ chiral symmetry $\chi$ changes sign and $V(-\chi) = V(\chi)$. When the two confined phases coexist with the Coulomb phase, $V(\chi)$ has three degenerate minima, two at $\chi = \pm \chi_0$ and one at $\chi = 0$. The confined phase is massive, and thus $V(\chi)$ is quadratic around $\chi = \pm \chi_0$. The Coulomb phase, on the other hand, is massless and $V(\chi)$ turns out to be quartic around $\chi = 0$ [2]. A simple potential with these properties can be written as

$$ V(\chi) = \chi^4(a + b\chi^2 + c\chi^4). $$

For our purposes it is not essential to use the actual potential of the supersymmetric theory. We are interested only in the universal complete wetting aspects of the interface dynamics. These are the same for the simple potential from above. For $a > 0$ there is a quartic minimum at $\chi = 0$ corresponding to the Coulomb phase. For $0 < a < 9b^2/32c$ and $b < 0$ there are two other minima at

$$ \chi_0^2 = \frac{-3b + \sqrt{9b^2 - 32ac}}{8c}, $$

corresponding to the two confined phases. Phase coexistence corresponds to $b^2 = 4ac$, because then all three minima are degenerate.

We now look for solutions of the classical equation of motion representing static planar domain walls, i.e. $\chi(x, y, z, t) = \chi(z)$, where $z$ is the coordinate perpendicular to the wall. The equation of motion then takes the form

$$ \frac{d^2 \chi}{dz^2} = \frac{\partial V}{\partial \chi} = 2\chi^3(2a + 3b\chi^2 + 4c\chi^4). $$

The interface action per unit area and unit time gives the interface tension $\alpha_{mn}$. Figure 2 shows a numerical solution of the equation of motion for a domain wall separating the two confined phases with boundary conditions $\chi(\pm \infty) = \pm \chi_0$. Figure 2a corresponds to a high temperature where only the confined phase is thermodynamically stable. Then the interface profile interpolates directly between the two confined phases. Figure 2b corresponds to an almost zero temperature where the Coulomb phase is metastable. Then the confined-confined domain wall splits into two confined-Coulomb interfaces and the Coulomb phase forms a complete wetting layer between them. Figure 3 shows the width $r$ of the complete wetting layer as a function of the deviation $\Delta = 1 - 4ac/b^2$ from phase coexistence. One finds $r \propto \Delta^{-1/4}$, such that the critical exponent is $\psi = 1/4$. This is the same value that one finds for condensed matter interfaces that interact via long-range retarded van der Waals forces with a potential $c/r^3$ [3]. We have verified numerically that
Figure 2: Profile $\chi(z)$ of a confined-confined domain wall. (a) At high temperatures the Coulomb phase is unstable, and one goes directly from one confined phase to the other. (b) At almost zero temperature the Coulomb phase is metastable and forms a complete wetting layer that splits the confined-confined domain wall into a pair of confined-Coulomb interfaces.
Figure 3: Determination of the critical exponent $\psi$. The straight line in the double-logarithmic plot shows the width of the wetting layer as $r \propto \Delta^{-1/4}$, such that $\psi = 1/4$.

$\alpha_{mn}(T) - 2\alpha_{0n}(T_c) \propto \Delta^{3/4}$ in agreement with the arguments presented in the introduction. The order parameter at the center of the wetting layer behaves as

$$\frac{d\chi}{dz}(0) \propto \Delta^{1/2},$$

which defines another critical exponent.

Let us now turn to $SU(3)$ supersymmetric Yang-Mills theory. In that case, complete wetting can occur at zero temperature only if confined-confined domain walls are not BPS saturated. For the following discussion we will assume that they are not. The universal aspects of the interface dynamics are again captured by an effective action

$$S[\chi] = \int d^4 x \left[ \frac{1}{2} \partial_\mu \chi^* \partial_\mu \chi + V(\chi) \right],$$

but the gluino condensate $\chi = \chi_1 + i\chi_2$ is now a complex field. The effective potential $V(\chi)$ is restricted by $\mathbb{Z}(3)_\chi$ and charge conjugation symmetry. Under chiral transformations $z \in \mathbb{Z}(3)_\chi$, the gluino condensate transforms into $\chi' = \chi z$ and under charge conjugation it gets replaced by its complex conjugate. This implies

$$V(\chi z) = V(\chi), \quad V(\chi^*) = V(\chi).$$
Again, one needs the potential to be quadratic around the confined phase minima and quartic around the Coulomb phase minimum. A simple potential with these properties is
\[
V(\chi) = |\chi|^2(a|\chi|^2 + b\chi_1(\chi_1^2 - 3\chi_2^2) + c|\chi|^4). \tag{8}
\]
At zero temperature (corresponding to \(b^2 = 4ac\)), the above potential has four degenerate minima at \(\chi = \chi^{(n)}, n \in \{1, 2, 3\}\), representing the three confined phases and at \(\chi = \chi^{(0)} = 0\) representing the Coulomb phase.

Again, we look for solutions of the classical equations of motion representing planar domain walls. Figure 4 shows a numerical solution of these equations for a domain wall separating two confined phases with boundary conditions \(\chi(\infty) = \chi^{(1)}, \chi(-\infty) = \chi^{(2)}\). Figure 4a corresponds to a temperature deep in the confined phase. In this case, the Coulomb phase is unstable and the domain wall interpolates directly from one confined phase to the other. Figure 4b shows the situation at almost zero temperature. Then the confined-confined domain wall splits into two confined-Coulomb interfaces and the Coulomb phase forms a complete wetting layer between them.

As in the \(SU(2)\) case, we have determined the width of the complete wetting layer numerically. Again, one finds \(r \propto \Delta^{-1/4}\), such that \(\psi = 1/4\). In addition, \(\alpha_{mn}(T) - 2\alpha_{nn}(T_c) \propto \Delta^{3/4}\), as it should be. The order parameter at the center of the wetting layer now behaves as
\[
\chi_1(0) \propto \Delta^{1/4}, \quad \frac{d\chi_2}{dz}(0) \propto \Delta^{1/2}, \tag{9}
\]
again in agreement with the \(SU(2)\) critical exponent. We conclude that the critical exponents are independent of \(N\) and depend only on the range of the interface interactions. To summarize, at zero temperature complete wetting occurs for even \(N\) and \(m - n = N/2\). For odd \(N\) or \(m - n \neq N/2\) complete wetting can occur only if confined-confined domain walls are not BPS saturated. When a confined-confined domain wall is completely wet by the Coulomb phase, the complete wetting layer grows with the fourth root of the deviation from the critical temperature. This is due to long-range interactions in the Coulomb phase mediated by massless gluons and gluinos (rather than massive glueballs), which lead to \(\psi = 1/4\).

3 Complete Wetting at High Temperature

Let us now discuss complete wetting at high temperatures where a phase transition separates the confined phase from a high-temperature deconfined phase. We assume that this phase transition is first order, such that confined and deconfined phases coexist at \(T_c\). In the case of \(\mathcal{N} = 0\) non-supersymmetric \(SU(3)\) Yang-Mills theory at high temperatures the universal aspects of the interface dynamics are captured
Figure 4: Shape of a confined-confined domain wall. (a) Deep in the confined phase the domain wall profile interpolates directly from one confined phase to the other. (b) At almost zero temperature the wall splits into two confined-Coulomb interfaces with a complete wetting layer of Coulomb phase between them.
by a 3-d effective action \[ S[\Phi] = \int \! d^3 x \left[ \frac{1}{2} \partial_i \Phi^* \partial_i \Phi + V(\Phi) \right], \] for the Polyakov loop \( \Phi \), which is a gauge invariant complex scalar field. Its expectation value \( \langle \Phi \rangle \propto \exp(-F/T) \), measures the free energy \( F \) of a static quark. In the confined phase, \( F \) diverges and \( \langle \Phi \rangle \) vanishes, while in the deconfined phase, \( F \) is finite and \( \langle \Phi \rangle \) is non-zero. Under topologically non-trivial gauge transformations, which are periodic in Euclidean time up to a center element \( z \in \mathbb{Z} \), the Polyakov loop changes into \( \Phi' = \Phi z \). Hence, the \( \mathbb{Z} \) symmetry is spontaneously broken in the deconfined phase. Under charge conjugation, the Polyakov loop is replaced by its complex conjugate. The effective potential \( V(\Phi) \) is restricted by \( \mathbb{Z} \) and charge conjugation symmetry, i.e.

\[
V(\Phi z) = V(\Phi), \quad V(\Phi^*) = V(\Phi).
\]

In this case, all phases are massive (note that the deconfined phase has a finite screening length), such that the potential is quadratic around all minima. A simple potential with these properties takes the form

\[
V(\Phi) = a|\Phi|^2 + b\Phi_1(\Phi_1^2 - 3\Phi_2^2) + c|\Phi|^4,
\]

where \( \Phi = \Phi_1 + i\Phi_2 \). One can restrict oneself to quartic potentials because they are sufficient to explore the universal features of the interface dynamics. At the deconfinement phase transition temperature (corresponding to \( b^2 = 4ac \)), the above potential has four degenerate minima at \( \Phi = \Phi^{(n)} \), \( n \in \{1, 2, 3\} \) representing the three deconfined phases and at \( \Phi = 0 \) representing the confined phase. In ref. it was shown that a deconfined-deconfined domain wall is completely wet by the confined phase and the corresponding critical exponents have been determined analytically. As expected for short-range forces, one finds \( \psi = 0 \), i.e. the width of the complete wetting layer diverges logarithmically. In addition, for the order parameter at the center of the wetting layer one obtains

\[
\Phi_1(0) \propto \Delta^{1/2} \quad \text{and} \quad d\Phi_2/dz(0) \propto \Delta^{1/2}.
\]

In \( \mathcal{N} = 1 \) supersymmetric \( SU(3) \) Yang-Mills theory the \( \mathbb{Z}(3)_c \) chiral symmetry is spontaneously broken in the confined phase. At high temperatures, one expects chiral symmetry to be restored and — as in the non-supersymmetric theory — the \( \mathbb{Z}(3)_c \) center symmetry to be spontaneously broken due to deconfinement. Consequently, the effective action describing the interface dynamics now depends on both order parameters \( \Phi \) and \( \chi \), such that

\[
S[\Phi, \chi] = \int \! d^3 x \left[ \frac{1}{2} \partial_i \Phi^* \partial_i \Phi + \frac{1}{2} \partial_i \chi^* \partial_i \chi + V(\Phi, \chi) \right].
\]

The most general quartic potential consistent with \( \mathbb{Z}(3)_c, \mathbb{Z}(3)_\chi \) and charge conjugation now takes the form

\[
V(\Phi, \chi) = a|\Phi|^2 + b\Phi_1(\Phi_1^2 - 3\Phi_2^2) + c|\Phi|^4 + d|\chi|^2 + e\chi_1(\chi_1^2 - 3\chi_2^2) + f|\chi|^4 + g|\Phi|^2|\chi|^2.
\]
We assume that deconfinement and chiral symmetry restoration occur at the same temperature and that the phase transition is first order. Then three chirally broken confined phases coexist with three distinct chirally symmetric deconfined phases. The three deconfined phases have $\Phi = \Phi^{(n)}$, $n \in \{1, 2, 3\}$, and $\chi = 0$, while the three confined phases are characterized by $\Phi = 0$ and $\chi = \chi^{(n)}$, $n \in \{1, 2, 3\}$. The phase transition temperature corresponds to a choice of parameters $a, b, \ldots, g$ such that all six phases represent degenerate absolute minima of $V(\Phi, \chi)$.

Again, we look for solutions of the classical equations of motion representing planar domain walls. Figure 5 shows a numerical solution of these equations for a domain wall separating two confined phases with boundary conditions $\chi^{(\infty)} = \chi^{(1)}$, $\Phi^{(\infty)} = 0$ and $\chi^{(-\infty)} = \chi^{(2)}$, $\Phi^{(-\infty)} = 0$. Figure 5a corresponds to a temperature deep in the confined phase. Still, at the domain wall the Polyakov loop is non-zero, i.e. the center of the domain wall shows characteristic features of the deconfined phase. As a consequence, close to the wall the free energy of a static quark is finite and its string can end there. Thus, as discussed in detail in ref. [7], wetting provides a field theoretic explanation for why QCD strings can end on domain walls. This effect was first described by Witten in the framework of M-theory [8]. Figure 5b corresponds to a temperature very close to the phase transition. Then the confined-confined domain wall splits into two confined-deconfined interfaces and the deconfined phase forms a complete wetting layer between them.

For the special values $d = a = 0$, $e = b$, $f = c$, $g = 2c$ one can find an analytic solution for a confined-deconfined interface. Combining two of these solutions to a confined-confined interface, one obtains

\begin{align}
\Phi_1(z) &= -\frac{1}{2} \Phi_0 \left[ \tanh \alpha(z - z_0) - \tanh \alpha(z + z_0) \right], \quad \Phi_2(z) = 0,
\chi_1(z) &= -\frac{1}{4} \chi_0 \left[ 2 + \tanh \alpha(z - z_0) - \tanh \alpha(z + z_0) \right],
\chi_2(z) &= \frac{\sqrt{3}}{4} \chi_0 \left[ \tanh \alpha(z - z_0) + \tanh \alpha(z + z_0) \right],
\end{align}

where $\Phi_0 = \chi_0 = -3b/4c$ and $\alpha = -3b/4\sqrt{c}$. The critical temperature corresponds to $e^4/f^3 = b^4/c^3$. Near criticality, where $\Delta = e^4/f^3 - b^4/c^3$ is small, the above solution is valid up to order $\Delta^{1/2}$, while now $e = b$ and $f = c$ are satisfied to order $\Delta$. The width of the deconfined complete wetting layer,

$$r = 2z_0 \propto \log \Delta,$$

grows logarithmically as we approach the phase transition temperature. This is the expected critical behavior for interfaces with short-range interactions, which have $\psi = 0$. We have also checked that $\alpha_{mn}(T) - 2\alpha_{0n}(T_c) \propto \Delta \log \Delta$, as expected. Again, the order parameter at the center of the wetting layer behaves as

$$\chi_1(0) \propto \Delta^{1/2}, \quad \frac{d\chi_2}{dz}(0) \propto \Delta^{1/2}.$$
Figure 5: Shape of a confined-confined domain wall. (a) Deep in the confined phase $\Phi_1(0) \neq 0$, i.e. the center of the wall has properties of the deconfined phase. (b) Close to the phase transition the wall splits into two confined-deconfined interfaces with a complete wetting layer of deconfined phase between them.
Complete wetting also occurs on the other side of the phase transition. Then the confined phase completely wets a deconfined-deconfined interface. The treatment of this case is completely analogous, and the resulting critical exponents are again those for interfaces with short-range interactions. We expect the same high-temperature interface critical behavior for other values of $N$.

4 Conclusions

We have investigated universality classes for complete wetting in $\mathcal{N} = 1$ supersymmetric $SU(N)$ Yang-Mills theory. At zero temperature, the Coulomb phase may completely wet a confined-confined interface. In that case the interfaces interact via long-range forces. The corresponding critical exponents are the same as those for condensed matter systems with retarded van der Waals forces with a $c/r^3$ potential. One should keep in mind that we have assumed that the Coulomb phase indeed exists in supersymmetric Yang-Mills theory. Should this not to be the case, our calculation, of course, does not apply to this theory. Even then, it still correctly describes the effective theories discussed in this paper. At high temperatures the massive deconfined phase completely wets a confined-confined interface, and the interactions between interfaces are short-ranged. Then the width of the complete wetting layer grows logarithmically as one approaches $T_c$. Above $T_c$, the confined phase can wet a deconfined-deconfined domain wall, with the same critical exponents as before. In general, the critical exponents are independent of $N$ and depend only on the range of the interface interactions.

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