Axisymmetric turbulent methane jet propagation in a wake air flow under combustion at a finite velocity

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Abstract. This article proposes a numerical method for solving the problem of an axisymmetric methane jet propagation in an infinite wake air flow. The dimensionless equations of the turbulent boundary layer of reacting gases in von Mises coordinates and the $k-e$ turbulence model were used in modeling. The equations for $N$ components of the gas mixture were reduced to two equations by introducing the Schwab-Zeldovich functions. To solve the problem in von Mises coordinates, a two-layer, six-point implicit finite-difference scheme was used, which provided the second order of accuracy of the approximation in coordinates. An iterative process was realized due to the nonlinearity of the equations for the conservation and transfer of substances. The effect of the radius of fuel nozzle on the indices of turbulent jet and flame was investigated. It was found that in an infinite wake flow of fuel with a decrease in the nozzle radius, the rate of chemical reaction and the highest temperature in the calculation area decrease, and the amount of unburnt fuel increases.

1. Introduction
Combustion is a complex physicochemical process of converting initial (source) substances into combustion products in the course of exothermic reactions, accompanied by intense heat release.

The mastery of fire played a key role in the development of civilization. Fire opened to people the possibility of heat treatment of food and heating of homes, and subsequently - the development of metallurgy, energy and the creation of new, more advanced tools and technologies, where solid, liquid and gaseous substances are used as fuel. Combustion control underlies the creation of modern engines for cars, airplanes, ships and rockets, where the combustion of pre-mixed mutually reacting substances is used.

However, the efficiency of devices used in the Republic of Uzbekistan for burning natural and liquefied gases is low. In this regard, a thorough study of the processes of flame formation and propagation, the methods of effective control of processes, the modernization and development of existing mathematical models based on new achievements in the fields of turbulence, jet flows and the theory of combustion are required.
Combustion problems are formed using the systems of complex nonlinear partial differential and closing relations [1-3]. An expedient way to solve them is the methods of mathematical modeling, including the development of numerical algorithms and conducting a computational experiment using the capabilities of modern info-communication technologies.

The development of numerical methods to calculate the parameters of reacting gas flow was a result of the search for the methods, alternative to full-scale experiments, to obtain the characteristics of a flow with chemical reactions [4]. For practical applications, the models of the flow of reacting gases in free or wake jets were used. The paper presents the equations of the boundary layer of a multicomponent gas in plane and axisymmetric statements. The equations, parabolized in von Mises coordinates, were solved numerically. The model includes equations for the concentrations of toxic nitrogen oxides. The results of a computational experiment are presented for various problems with the “frozen flow” approach and the Arrhenius law for 15 reactions with 11 components in the calculation area. It is shown that the flow swirling leads to a sharp shortening of the flame length and to a decrease in the formation of thermal nitrogen oxides.

In [5], a numerical study of various approaches to the modeling of laminar diffusion combustion was conducted. The focus was on the “flamelet” model. The energy loss due to radiation was taken into account. The models were evaluated for the problems with counter and wake modes of fuel and oxidizer supply. The distribution profiles of temperature and component concentrations were obtained. The calculated values were compared with experimental data.

The results of numerical studies of the processes of mixing, ignition and combustion during the outflow of a propane jet into a wake air flow in an axisymmetric channel were presented in [6]. Numerical modeling of turbulent subsonic nonequilibrium flow was performed within the framework of two-dimensional stationary equations of a “narrow channel” taking into account the finite velocities of chemical kinetics in a global mechanism. The effect of ignition on the delay of the mixing process in the flow of unmixed components was shown. Proposals are formulated for the choice of rational temperature of the wake flow and the location of the ignition cross section, allowing to intensify the development of the combustion process.

The studies by V.T. Zhukov, N. D. Novikova and O.B. Feodoritova [7] present a methodology for the numerical simulation of flows in a high-speed combustion chamber, based on the solution of the Navier-Stokes equations for a reacting multicomponent medium. The dynamics of combustion processes was investigated depending on the oxidizer excess coefficient; numerical methods technology was developed.

In [8], the problem of kinetic combustion of a propane jet flowing into a wake subsonic air flow in an axisymmetric channel was considered. To describe the flow, a “narrow channel” model and a differential one-parameter turbulence model were used. The kinetics of propane combustion in air is described by a one-stage stoichiometric gross reaction. The influence of the initial turbulent viscosity and the location of the artificial ignition cross-section on the development of the combustion process was investigated.

The modeling of turbulent combustion of methane in areas of complex geometry was conducted in [9]. Parameters were selected for one-stage and two-stage models of chemical kinetics. The parameters of chemical kinetics significantly affect the correctness of mathematical model of combustion used to calculate the explosion characteristics of a methane-air mixture in a residential building. A comparison with the results of experimental measurements was conducted.

The study in [10] presents a model of combustion of a carbon particle of high porosity in oxygen, which takes into account the occurrence of heterogeneous and homogeneous chemical reactions inside the particle and heat exchange by radiation. The boundaries of the area to which the dependence of combustion on particle temperature belongs were determined. The possibility of the existence of two combustion modes was shown: a high-velocity mode, in which the reaction of carbon with oxygen occurs in a layer near the particle surface, and a low-velocity mode, in which the reaction occurs throughout the entire volume of the particle.
A.G. Demenkov et al. [11] used a semi-empirical model of second-order turbulence, including differential transfer equations of normal Reynolds stresses. Flows in axisymmetric turbulent jets were considered.

The mechanisms of flame acceleration and transition from combustion to detonation in a coal mine were considered in [12] by Yu.A. Galenko, E.V. Sypin, A.N. Pavlov. Approaches to the study of flame acceleration and combustion-to-detonation transition, and to the simulation of turbulent flows under unsteady combustion were analyzed.

The simulation was supposed to be performed for a pipe with one closed end and one open end and the obstacles inside the pipe; the ignition was executed at the closed end. The obstacles that turbulize combustion presented the annular partitions with geometrical parameters corresponding to the dimensions of the lining in coalmines.

In some cases, combustion occurs in an enclosed space or in layers. For example, diffusion of a reactive gas in pores was first considered theoretically by Ya.B. Zeldovich in [13], where the existence of three modes of the reaction on a porous catalyst: kinetic, inside diffusion and outside diffusion, was shown. The kinetic mode of the heterogeneous reaction was characterized by a constant concentration of the gas reagent throughout the entire volume of the catalyst. As the temperature rises, the rate constant of chemical conversion increases according to the Arrhenius law, while the diffusion rate increases according to the power law. Therefore, starting from a certain temperature, the rate of the chemical reaction outruns the rate of established diffusion equilibrium in the entire volume of a rigid body. The reagent concentration in the bulk of the particle becomes less than at the surface, and a diffusion regime occurs inside.

Analysis of the references [7, 9 - 13] shows that the processes of homogeneous combustion of fuel gases, the main component of which is methane, are studied insufficiently. Due to the shortcomings of theoretical foundations of chemical exothermic reactions under turbulent flow, theoretical work in this area continues. Taking these aspects into account, the article discusses the distribution of methane in the wake air flow and the problem of flame control.

2. Method

A jet of combustible gas is considered, which flows out of a circular nozzle of diameter $2a$ and propagates in a wake flow of an oxidizer at a finite velocity of chemical reaction. The velocity distribution in the nozzle outlet and in the wake flow, as well as the initial (at $x = 0$) distributions of temperature and the fuel and oxidizer concentration, are considered as given, uniform, and homogeneous ones. There is a tangential discontinuity at the nozzle boundary.

In parallel with the turbulent mixing of two flows, a chemical reaction takes place between the interacting components - fuel and oxygen from the air. The mixing area expands gradually.

The aim of this study is to develop a numerical method for calculating the mixing, combustion, and propagation of different compositions of combustible mixtures in a wake flow, which makes it possible to carry out a computational experiment to investigate the processes of heat and mass transfer. The first step is to select a suitable turbulence model to describe a jet flow with intense chemical transformation. On the other hand, disregarding the equations for turbulence results in formation of $N + 3$ nonlinear partial differential equations. It is necessary to reduce the number of $N$ equations of mass components conservation. Otherwise, for each chemical composition of mixtures introduced by the main or wake flows, it is required to solve a system with different number of equations.

In the approximation of the theory of a turbulent boundary layer, taking into account the $k - \varepsilon$ model [9,15], the complete system of Navier-Stokes equations at the Lewis number equal to unity for the components ($Le_i = 1$, i.e. $Pr = Sc = 0.72$) can be written in dimensionless von Mises variables [22 - 26]:
\[
\begin{align*}
\n\frac{\partial u}{\partial \xi} &= \frac{1}{\psi} \frac{\partial u}{\partial \psi} \left[ \rho^2 u \frac{r^2}{\psi} \left( v_i + v_j \right) \frac{\partial u}{\partial \psi} \right], \\
\n\frac{\partial \tilde{H}}{\partial \xi} &= \frac{1}{Pr \psi} \frac{\partial}{\partial \psi} \left[ \rho^2 u \frac{r^2}{\psi} \left( v_i + v_j \right) \frac{\partial \tilde{H}}{\partial \psi} \right], \\
\n\frac{\partial c_2}{\partial \xi} &= \frac{1}{Sc \psi} \frac{\partial}{\partial \psi} \left[ \rho^2 u \frac{r^2}{\psi} \left( v_i + v_j \right) \frac{\partial c_2}{\partial \psi} \right] - A_1 \frac{c_2^2}{u} \exp(-A_2 / T), \\
\n\frac{\partial k}{\partial \xi} &= \frac{1}{\delta_k \psi} \frac{\partial}{\partial \psi} \left( \rho^2 u r^2 \frac{\partial k}{\partial \psi} \right) + 4 \rho^2 u r^2 \frac{\partial^2 k}{\partial \psi^2} \frac{\partial v_i}{\partial \psi} \left( \frac{\partial u}{\partial \psi} \right)^2 - \frac{\varepsilon}{u}, \\
\n\frac{\partial \varepsilon}{\partial \xi} &= \frac{1}{\delta_\varepsilon \psi} \frac{\partial}{\partial \psi} \left( \rho^2 u r^2 \frac{\partial \varepsilon}{\partial \psi} \right) + 4 \hat{c}_1 \frac{\varepsilon}{k} \rho^2 u r^2 \frac{\partial v_i}{\partial \psi} \left( \frac{\partial u}{\partial \psi} \right)^2 - \frac{\varepsilon^2}{ku}.
\end{align*}
\]

This system is closed by the following dependencies:

\[
H = c_p T + c_z h_z^*, \quad c_p = \sum_{n=1}^{N} c_{pm} c_n, \quad p = \rho \frac{R_0}{m} T = \text{const}, \quad m = \left( \sum_{n=1}^{N} m_n / c_n \right)^{-1},
\]

\[
\varepsilon = \frac{c_{\mu} k^2}{\varepsilon}, \quad G = 4 \rho v_i \left( \frac{\partial u}{\partial r} \right)^2,
\]

\[
\hat{c}_1 = 1.44, \quad \hat{c}_2 = 1.92, \quad c_{\mu} = 0.09, \quad \delta_k = 1, \quad \delta_\varepsilon = 1.3, \quad A_1 = 0.05, \quad A_\mu = 0.0165.
\]

Hereinafter, \( u, v \) are the averaged longitudinal and transverse (radial) components of the velocity vector in cylindrical coordinates; \( \xi, \psi \) are the von Mises coordinates (\( \text{m} \cdot \text{kg}^{1/2} / \text{s}^{1/2} \)); \( \rho, T \) are the density \( \text{kg} / \text{m}^3 \) and absolute temperature (\( \text{K} \)) of the gas mixture; \( p \) is the hydrostatic pressure (\( \text{Pa} \)); \( \text{Pr}, \text{Sc}_n \) are the turbulent analogs of Prandtl and Schmidt numbers; \( c_n \) is the mass concentration of the \( n \)-th component of gas in the mixture (\( \text{kg} / \text{kg} \)); \( \omega_n \) is the mass rate of formation or disappearance of the \( n \)-th component of gas \( \text{kg} / \text{m}^3 / \text{c} \); \( c_p = \sum_{n=1}^{N} c_{pm} c_n \) and \( c_{pm} \) is the heat capacity of the gas mixture and the \( n \)-th component under constant pressure \( J / \text{kg} / \text{K} \); \( h_n^* \) – fuel (heat) capacity of the \( n \)-th component \( J / \text{kg} / \text{K} \); \( V, V_i \) are the kinematic coefficients of laminar and turbulent viscosity \( \text{m}^2 / \text{s} \); \( H \) is the total gas enthalpy \( J / \text{kg} \); \( k \) is the kinetic energy of pulsating motion of a unit mass of gas \( \text{m}^2 / \text{s}^2 \); \( \varepsilon \) is the dissipation rate of turbulence energy of a unit mass of gas \( \text{m}^2 / \text{s}^3 \).

The gas mixture is assumed to be ideal, therefore its state satisfies the Mendeleev-Clapeyron equation:

\[
p = \rho R_0 T / m, \quad \text{where} \quad m = \left( \sum_{n=1}^{N} c_n / m_n \right)^{-1} \text{and} \quad m_n \text{ are the molecular weights of the gas mixture and the} \ n \text{-th gas component} \text{ kg / mole; } R_0 \text{ is a universal gas constant } (8.34141 J / \text{mole} / \text{K}).
\]

Since we are considering direct-flow free extended jets, based on data known from theory and practice in the flow area, the static pressure value is assumed to be constant \( p = \text{const} \) (according to Bai Shi-I
data for jet flows without combustion, the change in static pressure is no more than 0.5% compared to
dynamic pressure; and for the jets with combustion this fact is confirmed by experimental and
theoretical data obtained by Spalding).

Methane is taken as a combustible gas, the one-
stage kinetics of methane combustion in the air is
specified through the stoichiometric equation

\[ \text{CH}_4 + 2\text{O}_2 \rightarrow \text{CO}_2 + 2\text{H}_2\text{O} + h^*_2. \]

The indices 1, 2, 3, 4 and 5 mark the parameters of the components - oxygen, methane, carbon
dioxide, water vapor and nitrogen, respectively. The reaction rate of methane combustion with oxygen
in the equation of fuel mass conservation, proposed in [16, 18], in dimensionless von Mises
coordinates has the form:

\[ \tilde{\omega}_2 = -A_1 \frac{c_2 \tilde{p}^2}{\bar{u}} \exp(-A_r / T) \]

where \( A_1 = 1.35 \times 10^{20} \), \( \frac{E_\nu}{R} = 15.05 K = A_2 K. \)

The method of reducing \( N \) differential equations of components conservation to two equations
(with respect to the combustible component and the Schwab-Zeldovich function) at \( n = 1..5 \) and the
formulas for the reverse transition to the mass concentration are given in [22].

The combustion of gas fuel in the combustion process is carried out at sufficiently low
velocity relative to the sound velocity. Therefore, in the simulation, the kinetic energy of gas can be neglected.
Then a substitution similar to the one introduced for the concentration can be used in relation to the
total enthalpy, which leads to the normalization of the total enthalpy

\[ \tilde{H} = \frac{H - H_1}{H_2 - H_1} = \tilde{C} \]

Here \( H_1 = \left( c_p \right)_1 T_1 + \left( c_2 \right)_1 h^*_2 \), \( H_2 = \left( c_p \right)_2 T_2 + \left( c_2 \right)_2 h^*_2 \).

The value of the mixture temperature is calculated by the formula

\[ T = \left( H - c_p h^*_2 \right) / c_p = \left[ H_1 + \left( H_2 - H_1 \right) \tilde{H} - c_2 h^*_2 \right] / \sum_{n=1}^{N} c_{p_n} c_n. \]

The value of the gas mixture density required for further calculations is found by the formula

\[ \rho = \frac{p}{R \tilde{\theta} \sum_{n=1}^{N} c_n / m_n}. \]

At the entrance \( \xi = 0 \) to the computational domain, the following conditions are imposed:

at \( 0 \leq \psi < 1 \): \( u = 1 \), \( \tilde{H} = 1 \), \( c_2 = \left( c_2 \right)_2 \), \( k = 1 \), \( \varepsilon = 1 \).

at \( 1 \leq \psi < \psi_c \): \( u = \left( u \right)_1 \), \( \tilde{H} = 0 \), \( c_2 = \left( c_2 \right)_1 \), \( k = \left( k \right)_1 \), \( \varepsilon = \left( \varepsilon \right)_1 \).

At \( \xi > 0 \) we have

at \( \psi = 0 \): \( \frac{\partial u}{\partial \psi} = 0 \), \( \frac{\partial \tilde{H}}{\partial \psi} = 0 \), \( \frac{\partial c_2}{\partial \psi} = 0 \), \( \frac{\partial k}{\partial \psi} = 0 \), \( \frac{\partial \varepsilon}{\partial \psi} = 0 \).

at \( \psi \to \psi_c \): \( u = \left( u \right)_1 \), \( \tilde{H} = 0 \), \( c_2 = \left( c_2 \right)_1 \), \( k = \left( k \right)_1 \), \( \varepsilon = \left( \varepsilon \right)_1 \).

The problem was solved by the finite difference method. Integration steps \( h_\xi \) and \( h_\psi \) had constant
values. The jet boundary corresponded to a discrete coordinate \( N \).

The first three equations are written in a unified form:
\[ \frac{\partial U}{\partial \xi} = \frac{1}{\delta_U \psi} \frac{\partial}{\partial \psi} \left( \rho^2 u \frac{r^2}{\psi} (v_i + v_r) \frac{\partial U}{\partial \psi} \right) + f_U. \]

Here

\[ U = u; \quad \delta_U = 1, \quad f_U = 0; \]

\[ U = \bar{H}; \quad \delta_H = \Pr \left( = Sc \right), \quad f_H = 0; \]

\[ U = c_2; \quad \delta_{c_2} = Sc, \quad f_{c_2} = -A_{r1} \frac{c_1^2 c_2 \rho^2}{\bar{u}} \exp(-A_{r2} I/T). \]

The right-hand side of the equations contains the square of dimensionless radial coordinate \( r^2 \). It is calculated with the second order of accuracy

\[ \left( r_{i,j}^s \right)^2 = \left( r_{i,j-1}^s \right)^2 + 2 \frac{\psi_j^2 - \psi_{j-1}^2}{\rho_{i,j-1}^s u_{i,j-1}^s + \rho_{i,j}^s u_{i,j}^s}. \]

This is the formula for the \( i \)-th section; \( S \) is the ordinal number of the iteration for the \( i \)-th section. For the \( i-1 \)-th section, \( \bar{r}_i^2 \) is calculated according to the data of the previous section.

To reduce the amount of records and computations, first calculate the following

\[ K_{i,j} = \frac{\left( \rho_{i,j} \right)^2}{2} u_{i,j} \left( \bar{r}_{i,j} \right)^2 (v_i + (v_r)_{i,j}), \]

where \( \bar{r}_{i,j} = \left( r_{i,j} \right)^2 / h_{\psi}^2 \).

At \( j = 0 \) we take \( K_{i,0} = 0 \) and \( \bar{r}_{i,0}^2 = 0 \).

Let us approximate the equations for a fictitious node \( (i+0.5, j) \) with the second order of accuracy in both coordinates:

\[ \frac{U_{i+1,j}^{s+1} - U_{i,j}^{s+1}}{h_{\psi}} = \frac{1}{4 \delta_U j h_{\psi}^2} \left[ \left( K_{i,j+1}^s + K_{i,j}^s \right) \left( U_{i,j+1}^{s+1} - U_{i,j}^{s+1} \right) - \left( K_{i,j}^s + K_{i-1,j}^s \right) \left( U_{i,j}^{s+1} - U_{i,j-1}^{s+1} \right) \right] + \left( f_{i,j} \right)_{i,j} + \left( f_{i-1,j} \right)_{i,j} - \frac{1}{2} \left( f_{i,j} \right)_{i,j} - \frac{1}{2} \left( f_{i-1,j} \right)_{i,j}. \]

We multiply both sides of the equation by \( 4 \delta_U j h_{\psi}^2 \) and give similar terms selecting the terms with the elements of the previous section:

\[ \left( K_{i,j+1}^s + K_{i,j}^s \right) U_{i,j+1}^{s+1} - \left( K_{i,j+1}^s + 2 K_{i,j}^s + K_{i,j-1}^s + 4 \delta_U j / \sigma \right) U_{i,j}^{s+1} + \left( K_{i,j}^s + K_{i-1,j}^s \right) U_{i-1,j}^{s+1} = -2 \delta_U j h_{\psi}^2 \left( f_{i,j} \right)_{i,j} - \left( K_{i+1,j}^s + K_{i,j+1}^s \right) U_{i,j+1}^{s+1} + \left( K_{i+1,j}^s + 2 K_{i+1,j}^s + K_{i+1,j-1}^s - 4 \delta_U j / \sigma \right) U_{i-1,j}^{s+1} - \left( K_{i,j}^s + K_{i-1,j}^s \right) U_{i,j-1}^{s+1} - 2 \delta_U j h_{\psi}^2 \left( f_{i,j} \right)_{i-1,j}, \]
where \( \sigma = h_s / h_w^2 \).

Introduce notation:

\[
\alpha^{(U)}_j = K_{i,j+1}^s + K_{i,j}^s, \quad c^{(U)}_j = K_{i,j}^s + K_{i,j-1}^s, \\
b^{(U)}_j = a^{(U)}_j + c^{(U)}_j + 4\delta_{ij}/\sigma, \quad d^{(U)}_j = -2\delta_{ij} h^2_j \left( f_j \right)^2, \\
g^{(U)}_j = -\left( K_{i,j+1} + K_{i,j} \right) U_{i,j+1,1} + \left( K_{i,j} + K_{i,j-1} \right) U_{i,j-1,1} + \\
\left( K_{i,j},j+1 + 2K_{i,j} - 4\delta{ij} / \sigma \right) U_{i,j-1,1} - 2\delta_{ij} h^2_j \left( f_j \right)^2.
\]

As a result, the finite difference equation takes the form:

\[
a^{(U)}_{j, i,j+1} U_{i,j+1,1}^{s+1} - b^{(U)}_{j, i,j} U_{i,j,1}^{s+1} + c^{(U)}_{j, i,j-1} U_{i,j-1,1}^{s+1} = d^{(U)}_{j, i,j} + g^{(U)}_{j, i,j}.
\]

If we assume that the values of the coefficients \( \alpha^{(U)}_{j-i} \) and \( \beta^{(U)}_{j-i} \) in \( U_{i,j+1}^{s+1} = \alpha^{(U)}_{j-i} U_{i,j}^{s+1} + \beta^{(U)}_{j-i} \), are known, then the values of the sweep coefficients are calculated by the formulas:

\[
\alpha^{(U)}_{j} = \frac{a^{(U)}_{j}}{b^{(U)}_{j} - \alpha^{(U)}_{j-i} c^{(U)}_{j}}, \quad \beta^{(U)}_{j} = \frac{c^{(U)}_{j} \beta^{(U)}_{j-i} - a^{(U)}_{j} + g^{(U)}_{j}}{b^{(U)}_{j} - \alpha^{(U)}_{j-i} c^{(U)}_{j}}.
\]

Selection of \( g^{(U)}_{j} \) requires additional computer memory, but at the same time, in the process of successive approximation, it significantly reduces the amount (time) of calculations and allows using a greater step in \( \xi \).

The boundary conditions are implemented in the usual mode [21].

The second group of equations includes the two remaining equations:

\[
\frac{\partial V}{\partial \xi} = \frac{1}{\delta_{ij}} \frac{\partial}{\partial \psi} \left( \frac{\rho^2 u r^2}{\psi} \frac{\partial V}{\partial \psi} \right) + f_v.
\]

Here \( V = k \): \( \delta_{\xi} = 1 \), \( f_k = 4 \frac{\rho^2 u r^2}{\psi} v_1 \left( \frac{\partial u}{\partial \psi} \right)^2 - \frac{v}{u}, \)

\[
V = \varepsilon : \delta_{\xi} = 1.3, \quad f_{\varepsilon} = 4 \frac{\rho^2 u r^2}{\psi} v_1 \left( \frac{\partial u}{\partial \psi} \right)^2 - \varepsilon \frac{\partial^2 \varepsilon}{k u}.
\]

The approximation of the equations of this group is carried out similarly to the above.

The calculation for a fixed cross section of the jet consisted of three stages. In the first stage, the components of the finite-difference equations were calculated with the indices of the previous section: \( \tilde{r}_{i,j}, \ K_{i,j}, \ g^{(c)}_{i,j} \) and a zero approximation was organized. The other two stages presented iterative processes in terms of velocity and other criteria. The stage of iterative process in terms of velocity includes computations of \( \tilde{r}_{i,j}, \ K_{i,j}^s, \ \tilde{K}_{i,j}^s, \ g^{(c)}_{i,j} \), a running process, and checking the fulfillment of the approximation condition \( \max_{i=0,1,..,N} \left| u_{i,j}^m - u_{i,j}^{m-1} \right| < \varepsilon \) for the iterative process by \( u \) (\( m \) – is the iteration number). If the condition is not met, then the iterative process continues in terms of velocity.

If the condition is met, then the transition to the third stage is carried out, where the turbulence, fuel concentration and total enthalpy indices for the next approximation are determined by the sweep method, and the approximation in total enthalpy and density is checked. If the conditions for the total
enthalpy and density are not met, then the velocity calculations (the second part) are resumed with verification and the third stage is repeated. If the conditions are met, then the density and temperature of gas in the computational nodes are calculated in a new approximation.

The calculation continued until the condition $\bar{H}(\bar{x}, 0) \leq 0.02$ was met, and the distance to this section was taken as the conditional flame length.

In the calculations, the path changes of added mass, excess momentum, enthalpy, and fuel mass were calculated.

3. Results and Discussions

Based on the presented material, a program was drawn up and calculations were carried out. The air mass composition was set in the form $(c_1)_1 = 0.232, (c_2)_1 = 0.768$. Methane without impurities was considered as a fuel. The velocity of the main stream was 61 m/s, and the wake flow velocity was 18.3 m/s. Temperature of air and combustible mixture was $T_1 = T_2 = 293.15$ K. The thermo physical parameters of the components and the fuel lower heat value of methane (50.2 MJ/kg) were taken from [16, 17].

Numerical results were obtained for the total enthalpy, longitudinal velocity, concentration of components, and temperature along the flame length for the values of the nozzle radius $a$: 0.05, 0.01, 0.005, 0.002, 0.001, 0.0005, 0.0002, 0.0001, 0.00005, 0.00002 m.

Some calculation results are given in the form of graphs.

![Figure 1](image)

Figure 1. Isotherms in the field of intense heat and mass transfer at $a = 0.05 m$.

Near the nozzle, the highest temperature was reached at a certain distance from the axis. Nonmonotonic isotherms up to $x/a \approx 78$ show that the line of the highest temperature in the cross section first moves away from the jet axis. Having reached the line $r/a \approx 23$, it moves towards the axis and after $x/a \approx 78$ the maximum temperature in the section is represented by the axial temperature. Further, the axial temperature is calculated and at $x/a \approx 146$, it reaches $2052 K$. Hereafter, the highest temperature in the section, i.e. the axial temperature decreases. At the end of the conditional section $\bar{H}_{axis} = 0.02$ on the axis, the temperature is $761 K$. 
Figure 2. Isotherms in the field of heat and mass transfer at $a = 0.0005 m$.

Similar patterns were plotted for other values of the radius of the fuel nozzle. As the radius of the fuel nozzle decreased, the section where the highest temperature was reached outside the jet axis gradually decreased, and the boundary of disturbances in the dimensionless radial coordinate increased (Fig. 2). At $a = 0.0005 m$, the highest temperature in the calculation area decreased to 970 K. At $0.0005 m \leq a \leq 0.05 m$, the conditional flame length in dimensionless coordinates oscillated about $l^* / a = 700$, and at a further decrease in the nozzle radius, the computational domain reduced. At $a \approx 0.00005 m$ the highest temperature in the computational domain was 553 K, and the conditional length was $x / a \approx 590$.

The change in the highest temperature value in the area of heat and mass transfer depending on the radius $a$, given in logarithms, is shown in Fig. 3.
Figure 3. Change of the highest temperature value in the area of heat and mass transfer depending on the radius of the fuel nozzle.

The summary data of the graph of changes in the conditional flame length are shown in Fig. 4.

Figure 4. Change in conditional flame length $\tilde{H}_{axis} = 0.02$ depending on the radius of the fuel nozzle.

The conditional length of the intense area of heat and mass transfer is practically directly proportional to the radius of the fuel nozzle.
Figure 5. Change in the axial temperature, reduced by the temperature of the wake flow at \( a = 0.01 \) m: 1 - calculation; 2 - experiment [3].

4. Conclusions
The methods of introducing the Schwab-Zeldovich functions and relative-excess enthalpy to the case of methane combustion in a wake air flow of a finite velocity were demonstrated as a part of the study. A numerical method was developed for solving the problem of an axisymmetric fuel jet propagation in a wake air flow in von Mises variables using a modified \( k-e \) model for the turbulence coefficient.

For the numerical solution to the problem, an approximation scheme of the second order of accuracy was used in both coordinates; it gave a more accurate picture of the object. The calculation algorithm and program with internal and external iteration for the longitudinal velocity and other indices of heat and mass transfer processes were developed.

The influence of the radius of the fuel nozzle on the parameters of the jet and flame was investigated. The conditional flame length was taken as \( \frac{H_{\text{ad}} - H_f}{H_2 - H_f} = 0.02 \). It was found that in an infinite wake flow of fuel at a decrease in the nozzle radius, the rate of the chemical reaction and the highest temperature in the calculation area decreased, and the amount of unburnt fuel increased.

The calculation results showed that the amount of unburnt fuel at the end of conditional flame length decreased depending on the nozzle radius. This explains the decrease in the highest temperature value in the computational domain at a decrease in the nozzle radius.

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