SUSY Dark Matter in Scalar-Tensor Cosmologies

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Abstract. Given that nucleosynthesis (BBN) provides the oldest available information we have on the evolution of the Universe, it is worthwhile exploring possible deviations from the standard Big Bang cosmology based on General Relativity in pre-BBN epochs. Non-GR pre-BBN cosmologies affect the evolution of the WIMP DM component, in particular determining relevant shifts on its freeze-out temperature. The impact of this new "degree of freedom" in the discussion of naturalness of WIMPs in playing the role of DM is presented in the context of Scalar Tensor (ST) theories of gravity.

1. Introduction
So far the two firmest “observational” pieces of evidence we have to assess the presence of new physics (NP) beyond the SM are mass of neutrinos and the presence of a large amount of non-baryonic dark matter (DM). As for the latter one, given the difficulty to substitute the presence of DM with modifications of newtonian gravity at different scales, we can say that not only at a quantitative level, but also at a qualitative one, it represents the most formidable indication for the existence of new particles which are completely ignored in the SM particle spectrum. In the case of the mass of neutrinos, one can account for it by an enlargement of the SM spectrum with the inclusion of a right-handed counterpart of the familiar two-component left-handed neutrino. On the other hand, in the case of DM one has really to introduce an entirely new particle in addition to the massive neutrinos who cannot constitute the main bulk of the DM.

Unfortunately, both puzzles, the neutrino mass and the DM, do not yield any clear indication about the energy scale at which their related NP should set in. In fact, to be true, the neutrino mass problem seems to point to a large scale (possibly much larger than the electroweak scale) in its see-saw resolution. As for DM, it is true that one can exhibit NP candidates at scales which have nothing to do with the electroweak scale (for instance, in the case of the axion or of superheavy DM, like wimpzillas), however, fortunately for us, there exists a strong hint that the DM issue may be connected with NP at the TeV scale: this hint goes under the name of “WIMP miracle”.

Remarkably, fundamental parameters entering the two Standard Models of particle physics and cosmology conspire together to individuate a weakly interacting massive particle (WIMP), with a mass at the electroweak scale, as a correct cold DM candidate. The word “conspire” is here used in connection with the issue of naturalness. Every time one has some relation among different parameters giving rise to a certain value for some physical observable, one can ask how “stable” such a result is against small changes in the parameters: in the WIMP “miracle”, it turns out that for WIMP masses in the 100-1000 GeV range and annihilation cross sections...
which are typical of a weakly interacting particle, the WIMP provides the correct amount of DM.

Obviously, to proceed beyond the general model-independent statement, one has to consider specific NP models where such WIMPs arise as stable particles. Being the WIMP mass at the electroweak scale, one is lead to explore NP at such scale. Practically all relevant TeV NP models, at least if supplemented with convenient discrete symmetries, yield interesting WIMP candidates. For instance, in the case of models with extra dimensions, the lightest Kaluza-Klein mode can be stable and represent a valid WIMP candidate. Among the WIMPs related to TeV NP, the lightest SUSY particle (LSP) certainly plays a major role. The point is that, differently from other TeV NP cases, in the SUSY case the imposition of the discrete symmetry known as R parity was not dictated by the request of having a valid WIMP candidate, but, rather, by the completely different motivation of preserving matter stability. Indeed, it was the proton decay issue which induced SUSY model builders to make use of R parity, consequently ensuring the stability of the LSP. In the supergravity schemes the most likely LSP is the lightest of the four neutralinos. That’s why the neutralino has become the prototype of the "WIMP miracle" establishing a direct link between the existence of a NP stabilizing the electroweak breaking scale and the presence of a stable particle with all the features of a WIMP.

However, when going to specific models, like some version of the minimal SUSY SM (MSSM), it turns out that only very special choices of the SUSY parameter space allow for the correct DM density. In fact, what is even more worrying, is that the vast regions of SUSY parameter space not allowing for the right amount of DM generally lead to a too large matter density exceeding the critical energy density. Hence, all these points are excluded; indeed, not only the DM requirement, but even the mere requirement of not having too much matter constitutes the toughest test to pass for a point of the SUSY parameter space.

On the other hand, the narrow regions of the SUSY parameter space allowing for the LSP to be a good DM WIMP depend not only on the particle physics aspect (namely, the specific SUSY variant one is considering), but also on the conditions present at the moment of the WIMP decoupling. In turn, these conditions are determined by the cosmological evolution of the Universe in the pre-BBN epoch (the WIMP freezeout occurs at temperatures of O(5-100) GeV, i.e. much higher than the MeV temperatures at which BBN takes place). We know that at BBN the FRW universe described by General Relativity (GR) has to be reproduced; however, very little is known about the cosmological picture before BBN.

Here I wish to point out that non-GR cosmologies are possible to occur with remarkable differences with respect to the canonical GR FRW universe in pre-BBN epochs (in particular, at the WIMP freeze-out), while fully reproducing the GR cosmology at and after BBN. I will consider the specific example of Scalar Tensor (ST) theories of gravity showing that in such a framework it is possible to realize both an enhancement or a reduction of the rate at which the universe expands w.r.t. the case of the standard FRW GR cosmology. The ST cosmologies lead to an enhanced universe expansion rate has been known in the literature for quite some time; the novelty here is the fact that it is possible to obtain a sizeable reduction (up to about 2 orders of magnitude) of the Hubble rate prior to BBN. I will discuss the impact of these cosmological models on the relic abundance of DM in minimal Supergravity models and show that the cosmologically allowed regions in parameter space are significantly enlarged. The consequent change in the potential reach of LHC on the neutralino phenomenology will be briefly analysed.

This talk complements the presentation made by Graciela Gelmini at this meeting. I refer the interested reader to her talk for a general discussion on non-standard cosmologies and DM and the related references [1].
2. Changing the expansion rate in the past

In a standard flat FRW universe described by GR, the expansion rate of the universe, \( H_{GR} \equiv \dot{a}/a \), is set by the total energy density, \( \tilde{\rho}_{tot} \), according to the Friedmann law,

\[
H_{GR}^2 = \frac{1}{3M_p^2} \tilde{\rho}_{tot},
\]

(1)

where \( M_p \) is the Planck mass, related to the Newton constant by \( M_p = (8\pi G)^{-1/2} \). If the total energy density is dominated by relativistic degrees of freedom, the expansion rate is related to the temperature through the relation

\[
H_{GR} \simeq 1.66 \ g_*^{1/2} \frac{T^2}{M_p},
\]

(2)

with \( g_* \) the effective number of relativistic degrees of freedom.

We will modify the above \( H-T \) relation by considering a modification of GR in which an effective Planck mass, different from \( M_p \), appears in (2). This can be realized in a fully covariant way in ST theories [2]. We will consider the class of ST theories which can be defined by the following action [3],

\[
S = S_g + \sum_i S_i,
\]

(3)

where \( S_g \) is the gravitational part, given by the sum of the Einstein-Hilbert and the scalar field actions,

\[
S_g = \frac{M_p^2}{2} \int d^4x \sqrt{-g} \left[ R + g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{2}{M_*^2} V(\varphi) \right],
\]

(4)

where \( V(\varphi) \) can be either a true potential or a (Einstein frame) cosmological constant, \( V(\varphi) = V_0 \). The \( S_i \)’s are the actions for separate “matter” sectors

\[
S_i = S_i[\Psi_i, A_\varphi^i(\varphi)g_{\mu\nu}],
\]

(5)

with \( \Psi_i \) indicating a generic field of the \( i \)-th matter sector, coupled to the metric \( A_\varphi^i(\varphi)g_{\mu\nu} \). The actions \( S_i \) are constructed starting from the Minkowski actions of Quantum Field Theory, for instance the SM or the MSSM ones, by substituting the flat metric \( \eta_{\mu\nu} \) everywhere with \( A_\varphi^i(\varphi)g_{\mu\nu} \).

The emergence of such a structure, with different conformal factors \( A_\varphi^i \) for the various sectors can be motivated in extra-dimensional models, assuming that the two sectors live in different portions of the extra-dimensional space.

We consider a flat FRW space-time

\[
\text{d}s^2 = \text{d}t^2 - a^2(t) \text{d}^3x,
\]

where the matter energy-momentum tensors, \( T_{\mu\nu}^i \equiv (\rho_i + p_i) u_\mu u_\nu - p_i g_{\mu\nu} \), admit the perfect-fluid representation

\[
T_{\mu\nu}^i = \left( \rho_i + p_i \right) u_\mu u_\nu - p_i g_{\mu\nu},
\]

(6)

with \( u_{\mu} u^{\mu} = 1 \).

The cosmological equations then take the form

\[
\frac{\ddot{a}}{a} = -\frac{1}{6M_*^2} \left[ \sum_i (\rho_i + 3p_i) + 2M_*^2 \dot{\varphi}^2 - 2V \right],
\]

(7)

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{1}{3M_*^2} \left[ \sum_i \rho_i + \frac{M_*^2}{2} \dot{\varphi}^2 + V \right],
\]

(8)

\[
\ddot{\varphi} + 3\frac{\dot{a}}{a} \dot{\varphi} = -\frac{1}{M_*^2} \left[ \sum_i \alpha_i (\rho_i - 3p_i) + \frac{\partial V}{\partial \varphi} \right],
\]

(9)
where the coupling functions $\alpha_i$ are given by

$$\alpha_i \equiv \frac{d \log A_i}{d \varphi}.$$  \hspace{1cm} (10)

The Bianchi identity holds for each matter sector separately, and reads,

$$d(\rho_i a^3) + p_i da^3 = (\rho_i - 3 p_i) a^3 d \log A_i(\varphi),$$  \hspace{1cm} (11)

implying that the energy densities scale as

$$\rho_i \sim A_i(\varphi)^{1-3w_i} a^{-3(1+w_i)},$$  \hspace{1cm} (12)

with $w_i \equiv p_i/\rho_i$ the equation of state associated to the $i$-th energy density (assuming $w_i$ is constant).

### 2.1. GR as a fixed point

To start, consider the case of a single matter sector, $S_M$. In order to compare the ST case with the GR one of eqs. (1, 2), it is convenient to Weyl–transform to the so-called Jordan Frame (JF), where the energy-momentum tensor is covariantly conserved. The transformation amounts to a rescaling of the metric according to

$$\tilde{g}_{\mu\nu} = A^2_M(\varphi) g_{\mu\nu},$$  \hspace{1cm} (13)

keeping the comoving spatial coordinates and the conformal time $d\tilde{t} = dt/a$ fixed [4]. The JF matter energy-momentum tensor, $\tilde{T}^M_{\mu\nu} = 2(-\tilde{g})^{-1/2} \delta S_M/\delta \tilde{g}^{\mu\nu}$, is related to that in eq. (6) by

$$\tilde{T}^M_{\mu\nu} = A^{-2}_M T^M_{\mu\nu},$$

so that energy density and pressure transform as

$$\tilde{\rho}_M = A^{-4}_M \rho_M, \quad \tilde{p}_M = A^{-4}_M p_M,$$  \hspace{1cm} (14)

while the cosmic time transforms as $d\tilde{t} = A_M dt$. One can easily verify that the above defined quantities satisfy the usual Bianchi identity, that is eq. (11) with vanishing RHS, and that, as a consequence, $\tilde{\rho}_M \sim \tilde{a}^{-3(1+w_M)}$. The expansion rate, $H_{ST} \equiv d \log \tilde{a}/d\tilde{t}$, is given by

$$H_{ST} = \frac{1 + \alpha_M(\varphi) \varphi'}{A_M(\varphi) \alpha},$$  \hspace{1cm} (15)

where we have defined $\alpha_M$ according to eq. (10), and $\varphi' \equiv d(\varphi)/d \log a$. Using (15) and (14) in (8), we obtain the Friedmann equation in the ST theory,

$$H^2_{ST} = \frac{A^2_M(\varphi) (1 + \alpha_M(\varphi) \varphi')^2}{3M^2_2} \left[ \tilde{\rho}_M + \tilde{V} \right],$$  \hspace{1cm} (16)

where $\tilde{V} \equiv A^{-4}_M V$. Comparing to eq.(1), we see that apart from the extra contribution to $\tilde{\rho}_{tot}$ from the scalar field potential, the ST Friedmann equation differs from the standard one of GR by the presence of an effective, field-dependent Planck mass,

$$\frac{1}{3M^2_p} \rightarrow \frac{A^2_M(\varphi) (1 + \alpha_M(\varphi) \varphi')^2}{3M^2_2} \frac{1}{1 - (\varphi')^2/6},$$  \hspace{1cm} (17)

where the last equality holds with very good approximations for all the choices of $A_i$ functions considered in the present paper.
If the conformal factor $A_M^2(\varphi)$ is constant, then the full action $S_g + S_M$ is just that of GR (with $M_p = M_s/A_M$) plus a minimally coupled scalar field. Therefore, the coupling function $\alpha_M$, defined according to eq. (10), measures the “distance” from GR of the ST theory, $\alpha_M = 0$ being the GR limit. Changing $A_M$, and, therefore, changing the effective Planck mass, opens the way to a modification of the standard relation between $H$ and $\tilde{\rho}$, or $T$. In order to study the evolution of $A_M(\varphi)$, one should come back to eq. (9). Considering an initial epoch deeply inside radiation domination, we can neglect the contribution from the potential on the RHS. The other contribution, the trace of the energy–momentum tensor ($\rho_M - 3p_M$) is zero for fully relativistic components but turns on to positive values each time the temperature drops below the mass threshold of a particle in the thermal bath. Assuming a mass spectrum – e.g. that of the SM, or of the MSSM – one finds that this effect is effective enough to drive the scalar field evolution even in the radiation domination era [5].

The key point to notice is that if there is a field value, $\varphi_0$, such that $\alpha_M(\varphi_0) = 0$, this is a fixed point of the field evolution [6, 7]. Moreover, if $\alpha_M'$ is positive (negative) the fixed point is attractive (repulsive). Since $\alpha_M = 0$ corresponds to the GR limit, we see that GR is a —possibly attractive— fixed point configuration.

The impact on the DM relic abundance of a scenario based on this mechanism of attraction towards GR was considered in [5, 8]. It was shown there that, regardless of the particular form of the $A_M(\varphi)$ function, the requirement that an attractive fixed point towards GR exists implies that the effective Planck mass in the past was not smaller than today, that is to say that, at a certain temperature $T$, for instance at DM freeze out, the universe was expanding not more slowly than in the standard GR case.

### 2.2. Lowering $H$ in the past

In this subsection, we will show that adding more matter sectors, with different conformal factors, allows us to keep the desirable property of late time convergence to GR and, at the same time, to have a lower expansion rate in the past. To illustrate this point, we will consider just two matter sectors, a “visible” one, containing the SM or one of its extensions, and a “hidden” one. The full action is then given by

$$S = S_g + S_v + S_h,$$

where the two matter actions $S_v$ and $S_h$ have two different conformal functions $A_v(\varphi)$ and $A_h(\varphi)$. The discussion follows quite closely that of the previous subsection. The only subtle point is to notice that, if $A_v(\varphi) \neq A_h(\varphi)$ there is no Weyl transformation that gives covariantly conserved energy–momentum tensors both for the visible and for the hidden sector. Since particle masses, reaction rates and so on, are computed in terms of parameters of the “visible” action, the transformation to perform in order to compare with the standard GR case is the one leading to a conserved $T^\mu_\nu$, that is [3]

$$\tilde{g}_{\mu\nu} = A_v^2(\varphi)g_{\mu\nu},$$

and so on. As a consequence, the expansion rate in this case is given by

$$H^2_{ST} = \frac{A_v^2(\varphi)}{3M^2_v} \frac{(1 + \alpha_v(\varphi)\varphi')^2}{1 - (\varphi')^2/6} \left[ \tilde{\rho}_v + \tilde{\rho}_h + \tilde{V} \right],$$

where

$$\tilde{\rho}_v \sim \tilde{a}^{-3(1+w_v)},$$

while

$$\tilde{\rho}_h \sim \tilde{a}^{-3(1+w_h)} \left( \frac{A_h}{A_v} \right)^{1-3w_h}. $$
In order to study the existence of a fixed point, it is still convenient to revert to the Einstein Frame field equation, eq. (9). The RHS, is now given by the field derivative of the effective potential

$$V_{eff} = \rho_v + \rho_h + V,$$

(21)

with the field-dependence of $\rho_{v,h}$ given by eq. (12). The condition to have a fixed point is then

$$\sum_{i=v,h} \alpha_i (1 - 3w_i) \rho_i + V' = 0,$$

(22)

while, asking that the fixed point is stable implies

$$\sum_{i=v,h} \left( \alpha'_i (1 - 3w_i) \rho_i + \alpha_i^2 (1 - 3w_i)^2 \rho_i + V'' \right) \geq 0.$$

(23)

From eq. (20) we see that, away from the fixed point, $H_{ST}$ is lower than the one obtained in GR with the same matter content but frozen scalar fields if

$$\frac{d^2}{d\varphi^2} \left( A_v^2(\varphi) \frac{1 + \hat{\rho}_h / \hat{\rho}_v |_{ST}}{1 + \hat{\rho}_h / \hat{\rho}_v |_{GR}} \right) < 0,$$

(24)

where, again, we have assumed that the second fraction in eq. (20) is approximately one, and we have neglected the contribution from the scalar potential.

As an example, we now consider $A_i$ functions of the form

$$A_{v,h}(\varphi) = 1 + b_{v,h} \varphi^2.$$

(25)

Neglecting again the potential, we see that the fixed point condition, eq. (22), is solved by the symmetric point $\varphi = 0$. The stability condition, eq. (23), translates into

$$\sum_{i=v,h} b_i (1 - 3w_i) \rho_i \geq 0,$$

(26)

which, according to eq. (24), is compatible with a lower $H_{ST}/H_{GR}$ outside the fixed point (i.e. in the past), if

$$b_v < 0,$$

(27)

where we have assumed $\rho_h \ll \rho_v$ close to the fixed point, since we are interested in a physical situation in which most of the dark matter lives in the “visible” sector (as in the MSSM).

### 2.3. Numerical examples

To be implemented in a sensible cosmological model, the previously discussed mechanism for lowering the expansion rate in the past has to respect the severe bound coming from BBN, namely [9]

$$\left| \frac{H_{ST} - H_{GR}}{H_{ST}} \right| < 10\% \quad \text{at BBN}. $$

(28)

We now show in a few examples that indeed the bound (28) can be satisfied even by points of the parameters space $(b_v, b_h, \varphi_{in})$ giving rise to a pre–BBN value of the speedup ratio as low as $10^{-3}$. Such important deviations from standard cosmology are allowed in the present scenario by the effectiveness of the GR fixed point.

In order to numerically solve eqs. (7–9) we need the equation of state parameters

$$1 - 3w_i(y) = \frac{I_i(y)}{1 + e^{y-y_{eq}}} + \frac{1}{e^{y_{eq}-y} + 1}, \quad i = v, h$$

(29)
where \( y = \log \tilde{a} \) and \( y_{eq}^i \) refers to the equivalence in the visible (\( i = v \)) and hidden (\( i = h \)) sectors respectively. The functions \( I_i(y) \) are given by [5]

\[
I_i(y) = \sum_{A_i} \frac{15 g_{A_i}}{\pi^2} \frac{g_i}{g_i^{eq}} e^{2(y-y_{A_i})} \times \int_{0}^{+\infty} \frac{z_{A_i}^2 d\tau_{A_i}}{\sqrt{e^{2(y-y_{A_i})} + z_{A_i}^2}} \pm 1
\]

(30)

where \( y_{A_i} = -\log m_{A_i}/T_0 \), \( m_{A_i} \) and \( g_{A_i} \) are the masses and the relativistic degrees of freedom of the particles \( A_i \) and \( T_0 = 2.73 \text{ K} \approx 2.35 \times 10^{-13} \text{ GeV} \) is the current temperature of the Universe.

As mass thresholds for the visible sector, namely \( y_{A_v} \), we use the one given by a MSSM–like mass spectrum \(^1\) while the equivalence time \( y_{eq}^v \) has been computed according to [10]. The analogous quantities for the hidden sector are free parameters of the theory that nevertheless do not have any drastic impact on the final result. The only significant assumption we do is that \( \lim_{y \to \bar{y}} (1 - 3w_h(y)) \approx 1 \) for \( \bar{y} \ll y_{BBN} \). In such a way the contribution of the hidden sector to the RHS of the scalar field equation before BBN can be dominant w.r.t. the one of the visible sector.

We can now integrate the equation of motion for the scalar field. Fixing as the initial condition \( \varphi_{in} = 1 \) (in Planck units), we plot in figures (1) the evolution of the ratio \( H_{ST}^2/H_{GR}^2 \).

As anticipated, an agreement with the BBN bound is achieved even by solutions with a pre–BBN value of the ratio \( H_{ST}^2/H_{GR}^2 \) of order \( 10^{-3} \).

3. Implications for dark matter in the CMSSM

A modification of the Hubble rate at early times has impact on the formation of dark matter as a thermal relic, if the particle freeze–out occurs during the period of modification of the expansion rate. ST cosmologies with a Hubble rate increased with respect to the GR case have been discussed in Refs. [5, 8, 11], where the effect on the decoupling of a cold relic was discussed

\(^1\) Only particles with a mass smaller than the temperature of the phase transition by means of which they become massive have to be considered (see also [5] and references therein).
and bounds on the amount of increase of the Hubble rate prior to Big Bang Nucleosynthesis have been derived from the indirect detection signals of dark matter in our Galaxy. For cosmological models with an enhanced Hubble rate, the decoupling is anticipated, and the required amount of cold dark matter is obtained for larger annihilation cross–sections: this, in turn, translates into larger indirect detection rates, which depend directly on the annihilation process. In Refs. [8, 11] we discussed how low–energy antiprotons and gamma–rays fluxes from the galactic center can pose limits on the admissible enhancement of the pre–BBN Hubble rate. We showed that these limits may be severe: for dark matter particles lighter than about a few hundred GeV antiprotons set the most important limits, which are quite strong for dark matter masses below 100 GeV. For heavier particles, gamma–rays are more instrumental in determining significant bounds. Further recent considerations on the effect of cosmologies with modified Hubble rate are discussed in Refs. [12, 13, 14, 15, 16, 17].

In the case of the cosmological models which predict a reduced Hubble rate, the situation is opposite: a smaller expansion rate implies that the cold relic particle remains in equilibrium for a longer time in the early Universe, and, as a consequence, its relic abundance turns out to be smaller than the one obtained in GR. In this case, the required amount of dark matter is obtained for smaller annihilation cross sections, and therefore indirect detection signals are depressed as compared to the standard GR case: as a consequence, no relevant bounds on the pre–BBN expansion rate can be set. On the other hand, for those particle physics models which typically predict large values for the relic abundance of the dark matter candidate, this class of ST cosmologies may have an important impact in the selection of the regions in parameter space which are cosmologically allowed.

A typical and noticeable case where the relic abundance constraint is very strong is offered by minimal SUGRA models, where the neutralino is the dark matter candidate and its relic abundance easily turns out to be very large, in excess of the cosmological bound provided by WMAP [10]:

\[ 0.092 \leq \Omega_{\text{CDM}}h^2 \leq 0.124 \]  

(31)

Large sectors of the supersymmetric parameter space are excluded by this bound. A reduction of the expansion rate will therefore have a crucial impact on the allowed regions in parameter space, which are therefore enlarged. The potential reach of accelerators like the Large Hadron Collider (LHC) or the International Linear Collider (ILC) on the search of supersymmetry may therefore be affected by this broadening of the allowed parameter space, especially for the interesting situation of looking for supersymmetric configurations able to fully explain the dark matter problem.

We have therefore studied how the allowed parameter space of minimal SUGRA changes in ST cosmologies with a reduced Hubble rate. We have used a cosmological model of the type of Model 3 discussed in the previous Section and depicted in figure 1. For the calculations of the neutralino relic density we have used the DarkSUSY package [18], implemented by the ISAJET package [19] for the minimal SUGRA parameter space determination, with two major modifications. First, the relic density is obtained by a numerical solution of a modified Boltzmann equation (similar to the implementaion used in Refs. [5, 8, 11] for the enhanced case). Second, the NNLO contributions to the Standard Model branching ratio of the \( \text{BR}[\bar{B} \rightarrow X_s\gamma] \), which have been recently determined [20] to be \( \text{BR}[\bar{B} \rightarrow X_s\gamma]_{\text{SM}} = (3.15 \pm 0.23) \times 10^{-4} \) \( (E_\gamma > 1.6 \text{ GeV}) \), have been included.

3.1. Low \( \tan \beta \)

As a first example, we scan the universal gaugino mass \( m_{1/2} \) and soft scalar–mass \( m_0 \) parameters of minimal SUGRA for a low value of the \( \tan \beta \) parameter (\( \tan \beta \) is defined, as usual, as the ratio of the two Higgs vacuum expectation values \( v_2 \) and \( v_1 \), where \( v_2 \) (\( v_1 \)) gives mass to the top(down)–type fermions) and a vanishing universal trilinear coupling \( A_0 \). The higgs–mixing
parameter $\mu$, derived by renormalization group equation (RGE) evolution and electro–weak symmetry breaking (EWSB) conditions, is taken as positive. Our choice of parameters is here:

$$\tan \beta = 10 \quad \text{sgn}(\mu) = + \quad A_0 = 0 \quad (32)$$

Figure 2 shows the result in the plane $(m_{1/2}, m_0)$, both for the standard GR case and for the ST case of Model 3. Shaded areas denote regions which are excluded either by theoretical arguments or by experimental constraints on higgs and supersymmetry searches, as well as supersymmetric contributions to rare processes, namely to the $\text{BR}[B \to X_s \gamma]$ and to the muon anomalous magnetic moment $(g-2)_\mu$. More specifically, the upper wedge refers to the non–occurrence of the radiative EWSB and the lower–right area to the occurrence of a stau LSP. The low–$m_{1/2}$ vertical band is excluded by the quoted experimental bounds.

The sector of the supersymmetric parameter space which provides LSP neutralinos with a relic abundance in the cosmological range of eq. (31) are denoted by the open circles: in the so–called “bulk region” (low values of both $m_{1/2}$ and $m_0$), the lower [black] points fulfill the density constraint in the standard GR cosmology, while the upper [red] points in the modified ST cosmology with reduced Hubble rate. In the region above the points, the neutralino relic abundance exceeds the cosmological bounds, and therefore refers to supersymmetric configurations which are excluded by cosmology. Figure 2 shows that in our modified cosmological scenario, the allowed regions in parameter space are enlarged (the relic density has been decreased 1.4 times to 4.4 times compared to the standard case) and those which refer to dominant neutralino dark matter are shifted towards larger values of the supersymmetric parameters $m_{1/2}$ and $m_0$. The bulk region now occurs for values of $m_0$ larger by a factor of 2 (while the bulk region for the GR case now refers to cosmologically subdominant neutralinos). Nevertheless, this sector of the parameter space is already mostly excluded by accelerator searches. In the coannihilation channel, which extends for low values of the ratio $m_0/m_{1/2}$, along the boundary of the stau excluded region, the change is more dramatic: this coannihilation region, which appears to be fully explorable at the LHC, now extends towards larger values of $m_{1/2}$, beyond the estimated LHC reach.

In the cosmologically allowed region of large $m_0$, where a gaugino–mixing becomes possible and therefore the neutralino can efficiently annihilate and provide an acceptable relic abundance (a mechanism discussed in Ref. [21] and lately dubbed as “focus point region” in Ref. [22]), the effect of reducing the Hubble rate translates into a slight lowering of the cosmologically relevant region, with no drastic phenomenological effect in this case.

3.2. Large $\tan \beta$

As a second example, we consider the case of large values of $\tan \beta$. We show two cases, one which refers to a negative $\mu$ parameter:

$$\tan \beta = 45 \quad \text{sgn}(\mu) = - \quad A_0 = 0 \quad (33)$$

and one to a positive value of $\mu$:

$$\tan \beta = 53 \quad \text{sgn}(\mu) = + \quad A_0 = 0 \quad (34)$$

The results are shown in figure 3. In this case the change in the cosmological scenario is more relevant, not only in the coannihilation channel, but also in the “funnel” region [23] which occurs for intermediate values of the ratio $m_0/m_{1/2}$. In the GR case, almost the full cosmologically allowed parameter space may be explored by the LHC. When the ST cosmology is considered, the funnel region dramatically extends towards large values of $m_0$ and $m_{1/2}$ and goes well beyond the accelerators reach. Also the coannihilation region now extends to values of $m_{1/2}$.
Figure 2. Regions in the \((m_{1/2}, m_0)\) parameter space where the neutralino relic abundance falls in the cosmological interval for cold dark matter obtained by WMAP, for \(\tan \beta = 10\), \(A_0 = 0\) and positive \(\mu\). In the bulk region, the lower [black] points refer to GR cosmology, while the upper [red] points stand for a ST cosmology with a reduced Hubble rate. The shaded areas are forbidden by theoretical arguments and experimental bounds. The two curves are indicative of the reach for 100 fb\(^{-1}\) of the LHC and of the ILC at \(\sqrt{s} = 1000\) GeV energy.

well in excess of 2 TeV. In the case of the positive \(\mu\) reported in the right panel of figure 3, also the focus–point region shows a deviation from the GR case, and is shifted towards lower values of the \(m_0\) parameter. In summary, for these large values of \(\tan \beta\) the reach of LHC on the cosmologically relevant configurations is less strong than in the case of GR: a discover of supersymmetry will be more likely related to a subdominant neutralino dark matter.

4. Conclusions
We have discussed Scalar Tensor cosmologies by determining under what conditions these theories can predict an expansion rate which is reduced as compared to the standard General Relativity case. We showed that in the case of ST theories with a single matter sector, the typical behaviour is an enhancement of the Hubble rate in the past: this arises as a consequence of the requirement of an attractive fixed point towards GR at late times. Instead, when additional matter sectors, with different conformal factors, are added, we can maintain the desirable property of late time convergence to GR and, at the same time, obtain a reduced expansion rate in the past. We showed that, for suitable choices of the parameters which govern the scalar field evolution, a sizeable reduction (up to about 2 orders of magnitude) of the Hubble rate prior to Big Bag Nucleosynthesis can be obtained. Large reductions come along with some fine–tuning on the scalar field parameters, while a milder decrease occurs without tuning problems.

We have then applied the results obtained on the reduction of the early–time Hubble rate to the formation of dark matter and the determination of its relic abundance. If the dark matter decouples during the period of Hubble–rate reduction, the relic abundance turns out to be
Figure 3. Regions in the \((m_{1/2}, m_0)\) parameter space where the neutralino relic abundance falls in the cosmological interval for cold dark matter obtained by WMAP. The left panel refers to \(\tan \beta = 45\), \(A_0 = 0\) and negative \(\mu\). The right panel is obtained for \(\tan \beta = 53\), \(A_0 = 0\) and positive \(\mu\). Notations are as in figure 2.

reduced as compared to the standard GR case. This has therefore impact on the determination of the cosmologically allowed parameter space in minimal SUGRA models, where, in large portions of the parameter space and for the GR case, the neutralino relic abundance is large and in excess of the WMAP bound. We have therefore explicitly shown what are the modifications to the minimal SUGRA allowed parameter space when ST cosmologies with a reduced Hubble rate are considered and we have quantified the effect in view of the reach of LHC and ILC on the searches for supersymmetry at future accelerators. These modifications move the cosmologically relevant regions up to a few TeV for the \(m_{1/2}\) parameters, since they significantly extend the coannihilation corridor and the funnel region which occurs at large values of \(\tan \beta\).

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