On electron–positron annihilation into nucleon–antinucleon pairs

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Abstract. We discuss the puzzling experimental results on baryon–antibaryon production in $e^+e^-$ annihilation close to the threshold, in particular the fact that $\sigma(e^+e^- \rightarrow \bar{nn}) \gtrsim \sigma(e^+e^- \rightarrow \bar{pp})$. We discuss an interpretation in terms of a two-step process, via an intermediate coherent isovector state serving as an intermediary between $e^+e^-$ and the baryon–antibaryon system. We provide evidence that the isovector channel dominates both $e^+e^- \rightarrow$ pions and from $\bar{NN}$ annihilation at rest, and show that the observed ratio of $\sigma(e^+e^- \rightarrow \bar{nn})$ to $\sigma(e^+e^- \rightarrow \bar{pp})$ can be understood quantitatively in this picture.

Experimental data from the FENICE collaboration [1] indicate that $\sigma(e^+e^- \rightarrow \bar{nn})$ is relatively large close to threshold. Their data may be compared with earlier data on $e^+e^- \rightarrow \bar{pp}$ [2, 3] and also with data on the time-reversed reaction $\bar{pp} \rightarrow e^+e^-$, for which more precise data are available [4]. As seen in figure 1, the combined data indicate that $\sigma(e^+e^- \rightarrow \bar{nn})/\sigma(e^+e^- \rightarrow \bar{pp}) \gtrsim 1$ when $E_{CM} \sim 2$ GeV. Averaging over the available data on both the direct and time-reversed reactions, which are very consistent, and ignoring any possible variation with energy, we find

$$\frac{\sigma(e^+e^- \rightarrow \bar{pp})}{\sigma(e^+e^- \rightarrow \bar{nn})} = 0.66^{+0.16}_{-0.11}. \quad (1)$$

The fact that this ratio is less than unity requires confirmation, but even equal cross-sections for $e^+e^- \rightarrow \bar{pp}$ and $e^+e^- \rightarrow \bar{nn}$ would be quite surprising.

We recall that the ratio of the cross-sections for the corresponding $t$-channel processes $ep(n) \rightarrow ep(n)$ should be infinite at zero momentum transfer, where the form factor simply measures the total proton and neutron charges, corresponding to a coherent sum over the electromagnetic charges of their constituent quarks. It is also believed that the ratio (1) should

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Graph showing the combined data indicating the ratio of $\sigma(e^+e^- \rightarrow \bar{nn})$ to $\sigma(e^+e^- \rightarrow \bar{pp})$.}
\end{figure}

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Figure 1. Comparison of the cross-sections for $e^+e^- \rightarrow \bar{nn}$ and $\bar{pp}$ in the threshold region $E_{CM} \sim 2$ GeV. In the case of $e^+e^- \rightarrow \bar{pp}$, the direct-channel data are combined with the data for the time-reversed reaction $\bar{pp} \rightarrow e^+e^-$ (marked by $\times$). The dash–dotted and dotted lines denote the average and 1σ error bars, respectively, for the $\bar{pp}$ and $\bar{nn}$ data sets.

be large at high momentum transfers. In a naive perturbative description of $e^+e^-$ annihilation into baryons, the virtual timelike photon first makes a ‘primary’ $\bar{q}q$ pair, which is then dressed by two additional quark–antiquark pairs that pop out of the vacuum. This dressing is thought to be a perturbative QCD process at high momentum transfers, which does not distinguish between the $u$ and $d$ quarks, since gluon couplings are flavour-blind. Thus, in this conventional perturbative picture, the only difference between the production rates of protons and neutrons is through the different electric charges of the primary $\bar{q}q$ pairs. The total perturbative cross-section is obtained by superposing the amplitudes with different primary $\bar{q}q$ pairs and squaring the result:

$$\sigma(e^+e^- \rightarrow \bar{NN}) \propto \left| \sum_{q \in N} Q_q a_q^N(s) \right|^2,$$

where $a_q^N(s)$ denotes the amplitude at $E_{CM}^2 = s$ for making the baryon N with a given primary flavour $q$, which is determined by the baryon wavefunctions.

Since the wavefunctions of the baryon octet have a mixed symmetry, the amplitudes $a_q^N(s)$ tend to be highly asymmetric in specific models. For example, in the Chernyak–Zhitnitsky proton wavefunction [5], the $u$ quark dominates, i.e., $a_u^p = \mathcal{O}(1)$, $a_d^p \ll 1$ and similarly $a_u^n = \mathcal{O}(1)$, $a_d^n \ll 1$. In such a limiting case we have

$$\frac{\sigma(e^+e^- \rightarrow \bar{pp})}{\sigma(e^+e^- \rightarrow \bar{nn})} \rightarrow \frac{Q_u^2}{Q_d^2} = 4.$$
While this is an extreme case, on general grounds we expect that the u contribution dominates in the proton and the d in the neutron, so \( \sigma(e^+e^- \rightarrow \bar{p}p)/\sigma(e^+e^- \rightarrow \bar{n}n) \gg 1 \) at large momentum transfers.

We find it puzzling that the experimental ratio (1) is apparently below unity when \( E^2_{CM} = s \sim 4 \text{ GeV}^2 \), whereas the ratio should be much larger than unity at both larger (timelike) and smaller (spacelike) momentum transfers. Clearly, the mechanism at work here is qualitatively different from those providing the above intuition.

The lack of a conventional theoretical explanation is part of the motivation for the developing proposed new asymmetrical \( e^-e^+ \) high-statistics collider at SLAC for the regime \( 1.4 < \sqrt{s} < 2.5 \text{ GeV} \) [6]. This machine will yield high-precision data on baryon production in \( e^-e^+ \) annihilation at threshold, providing a check on the FENICE data and an accurate benchmark for testing possible theoretical explanations.

The first thing one must realize is that even though \( q^2 \gtrsim 4m_N^2 \gg \Lambda_{QCD}^2 \), the process is highly nonperturbative. This is because the ‘extra’ kinetic energy available to the quarks is very small.

Our approach [7] to this puzzle is based on thinking about the time-reversed processes: \( \bar{NN} \rightarrow e^+e^- \). These may be viewed as two-step processes, with a coherent meson state serving as an intermediate state. One possible motivation for this picture might be provided by the Skyrme model [8, 9], according to which baryons appear as solitons in a purely bosonic chiral Lagrangian. This model is formally justified as a low-energy approximation to large-\( N_c \) QCD [10, 11], and is known to provide a good description of many low-energy properties of baryons; see [12, 13] for reviews. Skyrmion–anti-Skyrmion annihilation provides [14]–[17] a fairly accurate description of low-energy baryon–antibaryon annihilation. Just after the Skyrmion and anti-Skyrmion touch, they ‘unravel’ each other, and a coherent classical pion wave emerges as a burst that takes away energy and baryon number as quickly as causality permits. A specific parametrization of the initial pion configuration is [16]

\[
F(r, t = 0) = h \frac{r}{r^2 + a^2} e^{-r/a},
\]

where \( F \) is the profile of the chiral field, \( U = \exp[i\tau \hat{F}(r, t)] \), \( a \) is a range parameter, \( h \) is chosen such that the total energy is that of the \( \bar{NN} \) pair, and the form of \( F \) guarantees that the pion configuration has zero net baryon number. This crude model has been shown [16] to reproduce satisfactorily the inclusive single-pion spectrum in \( \bar{p}p \) annihilation at rest and the branching ratios for multi-pion final states.

The details of this specific configuration are unimportant for our purposes; what is important is that the data are not inconsistent with such a model. Indeed, although the Skyrme model provides some motivation for our approach, even it is not essential for our purpose. What is important is that a single intermediate state should dominate the two-step \( \bar{NN} \rightarrow e^+e^- \) process. This could, for example, equally well be a single intermediate \( J^{PC} = 1^- \) resonant meson state.

To be more precise, since \( \bar{NN} \) annihilation is a strong-interaction process, one must consider separately the \( I = 1 \) and 0 channels. Accurately stated, our key assumption is that both of these channels are dominated by single states. These might be some excited \( \rho^* \)-and \( \omega^* \)-mesons, for example, as well as coherent pion configurations (4) with \( I = 1 \) and 0.

With this picture in mind, we write the \( I = 1, 0 \bar{NN} \rightarrow e^+e^- \) annihilation amplitudes as \( A_1, e^{i\alpha}A_0 \), where the overall phase is irrelevant, \( A_1 \) and \( A_0 \) are relatively real, and \( \alpha \) is the relative
Figure 2. Cross-sections for $e^+e^- \rightarrow$ multi-pion final states, for $E_{CM} \sim 2$ GeV [19]. We note the dominance of $I = 1$ final states with even numbers of pions by about an order of magnitude over $I = 0$ states with odd numbers of pions.

phase between the $I = 1$ and 0 amplitudes. We then have

$$f \equiv \frac{\sigma(e^+e^- \rightarrow \bar{p}p)}{\sigma(e^+e^- \rightarrow \bar{nn})} = \left| \frac{A_1 + e^{i\alpha}A_0}{A_1 - e^{i\alpha}A_0} \right|^2. \quad (5)$$

It is apparent from (5) that $\sigma(e^+e^- \rightarrow \bar{nn})/\sigma(e^+e^- \rightarrow \bar{p}p) \sim 1$ if either $A_1 \gg A_0$ or vice versa.

Remarkably, there is evidence from both $e^+e^-$ and $\bar{NN}$ annihilations that $I = 1$ final states dominate by large factors.

The clearest evidence comes from $e^+e^- \rightarrow n\pi$, where it is found by measuring final states with even and odd numbers of pions respectively that

$$\frac{\sigma(e^+e^- \rightarrow (2m)\pi)}{\sigma(e^+e^- \rightarrow (2m+1)\pi)} \sim 9 \quad \text{for } E_{CM} \sim 2 \text{ GeV}, \quad (6)$$

as seen in figure 2†. At these energies, we expect most final states created by $e^+e^- \rightarrow \bar{s}s$ to contain $K\bar{K}$ pairs, so (6) corresponds to the non-$\bar{s}s$ initial states that we expect to dominate in $\bar{NN}$ annihilation. The value (6) is similar to that found at lower energies, where $\Gamma(\rho \rightarrow e^+e^-) \sim 9\Gamma(\omega \rightarrow e^+e^-)$, in agreement with naive quark models. The fact that the ratio $\sigma(I = 1)/\sigma(I = 0)$ continues to be large at higher energies is consistent with ideas of generalized vector meson

† The five-pion final state is predominantly $\omega\pi\pi$. The cross-section in figure 2 corresponds to the final state $\omega\pi^+\pi^-$; for the total $\omega\pi\pi$ one should multiply it by 1.5 [19].
vacuum. The ratio (7) would be small if primary various P waves, but we are interested only in the generalized vector meson dominance in each isospin channel by a single state (either a coherent multi-pion state (4) or a comparable analysis of multi-pion final states. Because the initial state is a mixture with different liquid, as has been done in the analysis of OZI-violating final states, but we are unaware of a principle possible to distinguish different initial states by comparing annihilations in gas and liquid, as has been done in the analysis of OZI-violating final states, but we are unaware of a comparable analysis of multi-pion final states. Because the initial state is a mixture with different \( G = \pm 1 \), it is not possible to separate \( I = 1 \) from 0 simply by counting pions, as was the case in \( e^+e^- \) annihilation.

The most convincing experimental information known to us comes from an analysis of \( \bar{NN} \rightarrow \bar{KK} \). By comparing the rates for \( \bar{pp} \rightarrow K^+K^- \) and \( \bar{pp} \rightarrow K^0\bar{K}^0 \), it has been possible to extract [18]

\[
\frac{|A(3S/D_1 \rightarrow \bar{KK})_{I=1}|^2}{|A(3S/D_1 \rightarrow \bar{KK})_{I=0}|^2} \sim 5-10,
\]

which is comparable to the corresponding ratio (6) in \( e^+e^- \) annihilation. The fact that \( I = 1 \) dominates over \( I = 0 \) in the ratio (7) is consistent with the hypothesis that most of the \( \bar{KK} \) final states are created by \( \bar{uu} \) and \( \bar{dd} \) pairs in the initial \( NN \) state, with a \( \bar{ss} \) pair popping out of the vacuum. The ratio (7) would be small if primary \( \bar{ss} \) pairs dominated.

We now use the experimental information on the dominance exerted by the \( I = 1 \) channel in both \( e^+e^- \) (6) and \( 3S/D_1 \) annihilation (7) in a quantitative analysis of the \( e^+e^- \rightarrow NN \) production \( f \)-ratio (5). Defining \( \epsilon \equiv A_0/A_1 \), we can rewrite (5) as

\[
f = \left| \frac{1 + e^{i\alpha}\epsilon}{1 - e^{i\alpha}\epsilon} \right|^2.
\]

It is clear that the zero-momentum-transfer limit \( \sigma(e^+e^- \rightarrow \bar{pp}) \gg (e^+e^- \rightarrow \bar{nn}) \) is obtained in the limit \( \epsilon \rightarrow 1, \alpha \rightarrow 0 \), and that the high-momentum-transfer limit \( \sigma(e^+e^- \rightarrow \bar{pp}) \sim 4(e^+e^- \rightarrow \bar{nn}) \) is obtained in the limit \( \epsilon \rightarrow 1/3, \alpha \rightarrow 0 \). In order to estimate \( \epsilon = A_1/A_0 \), our assumption of dominance in each isospin channel by a single state (either a coherent multi-pion state (4) or a generalized vector meson \( V^* \)) tells us that

\[
\epsilon = \sqrt{\frac{\sigma(e^+e^- \rightarrow (I = 0))}{\sigma(e^+e^- \rightarrow (I = 1))}} \sqrt{\frac{\sigma(\bar{NN} \rightarrow (I = 0))}{\sigma(\bar{NN} \rightarrow (I = 1))}}.
\]

Inserting the experimental indications (6), (7) into (9), we estimate that

\[
\epsilon \sim \frac{1}{3}.
\]

The top end of this range seems to us quite conservative, whereas the lower end certainly requires more justification from \( \bar{NN} \) annihilation data. In the following numerical analysis, we keep \( \epsilon \) general, but focus extra attention on the limits \( \epsilon = 1/3 \) and \( 1/10 \).

It is apparent from (8) that \( f \sim 1 \) is possible for any value of \( \epsilon \), for a restricted range of the relative phase \( \alpha \sim \pi/2 \). However, the allowed range of \( \alpha \) is extended if \( \epsilon \) is small. It is easy to see that \( f \) lies in a narrow range \( \Delta f \) around unity if \( \alpha \) falls within the following range:

\[
\Delta \alpha \simeq \frac{\Delta f}{4\epsilon}.
\]

It is apparent that \( f \sim 1 \) for all values of \( \alpha \) if \( \epsilon \) is small, as suggested (10) by the available data on \( e^+e^- \) and \( \bar{NN} \) annihilation.

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Figure 3. A three-dimensional plot of the cross-section ratio \( f \equiv \sigma(e^+e^- \to \bar{p}p)/\sigma(e^+e^- \to \bar{n}n) \) as a function of \( \epsilon \) and \( \alpha \), indicating the region where \( f \) falls within the range (1).

The quantitative behaviour of \( f \) as a function of \( 0 < \epsilon < 1/2 \) and \(-\pi < \alpha < \pi \) is shown in figure 3. Displayed explicitly is the region of the \((\epsilon, \alpha)\) plane where \( f \) falls within the experimental range (1). We see that this range favours \(|\alpha| > \pi/2\), whatever the value of \( \epsilon \). Figure 4 displays projections of figure 3 for the two limiting values \( \epsilon = 1/3 \) and \( 1/10 \). The allowed range (1) of \( f \) and the corresponding ranges of \( \alpha \) are also shown.

We conclude that the \textit{a priori} puzzling large experimental value of the ratio \( \sigma(e^+e^- \to \bar{n}n)/\sigma(e^+e^- \to \bar{p}p) \) can be understood qualitatively. This is relatively easy if the \( I = 1 \) amplitude dominates over the \( I = 0 \) amplitude, as suggested by the available data on \( e^+e^- \) and \( NN \) annihilation and our assumption of dominance by a single coherent state in each isospin channel. The specific range (1) can be understood quantitatively if the \( I = 1 \) and 0 amplitudes have a large relative phase \( \alpha \). We are not in a position to judge the plausibility of such a large value of \( \alpha \), from either an experimental or a theoretical point of view. It would be interesting to make tests of this possibility.

We comment finally on the surprisingly large value of the ratio \( \sigma(\gamma\gamma \to \bar{\Lambda}\Lambda)/\sigma(\gamma\gamma \to \bar{p}p) \) observed by the CLEO Collaboration [20] (see also [21]–[23] for related experimental work). CLEO find that \( \sigma(\gamma\gamma \to p\bar{p}) \approx \sigma(\gamma\gamma \to \Lambda\bar{\Lambda}) \) close to threshold, which seems analogous to the FENICE result for the \( \bar{n}n/\bar{p}p \) ratio. For a given quark flavour, the perturbative amplitude for baryon–antibaryon production in the photon–photon reaction scales like the quark charge squared, compared with the linear dependence of the amplitudes on the quark charge in the \( e^+e^- \) case discussed earlier. Thus one might naively expect the ratio \( \sigma(\overline{\Lambda}\Lambda)/\sigma(\overline{p}p) \) to be even smaller than the corresponding perturbative prediction for \( \sigma(\bar{n}n)/\sigma(\bar{p}p) \) in \( e^+e^- \) annihilation. It would be interesting to approach this puzzle from a point of view similar to that adopted in this paper. However, the situation in \( \gamma\gamma \) collisions is more complicated, because of the wider
range of possible spin and isospin states. Also, the information available on the isospin and spin decomposition is sparse compared with that on $e^+e^-$ annihilation, which we used above. Data for $\gamma\gamma \rightarrow \bar{n}n$ close to threshold might cast light on the $\sigma(\gamma\gamma \rightarrow \Lambda\Lambda)/\sigma(\gamma\gamma \rightarrow \bar{p}p)$ puzzle, but are not yet available.

We have proposed in this paper a simple model that is able to accommodate the surprisingly large observed value of the ratio $\sigma(e^+e^- \rightarrow \bar{n}n)/\sigma(e^+e^- \rightarrow \bar{p}p)$. Our suggestion is based on a simple two-step approach, in which a single intermediate state with $I = 1$ dominates over $I = 0$. This dominant intermediate state could be motivated by a Skyrmion–anti-Skyrmion picture, or could be some excited $\rho^*$-resonance. Our model could be tested by further measurements of the ratios of different isospin amplitudes in $e^+e^-$ and $\bar{N}N$ annihilation, and suggests a relatively large phase difference between $I = 1$ and 0 amplitudes. We look forward to seeing more experimental data bearing on these issues, for example from a new low-energy $e^+e^-$ collider [6].

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