Two particle correlation effects and Differential HBT for rotation in heavy ion collisions.

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Abstract

Peripheral heavy ion reactions at ultra relativistic energies have large angular momentum that can be studied via two particle correlations using the Differential Hanbury Brown and Twiss method. We analyze the possibilities and sensitivity of the method in a rotating system. We also study an expanding solution of the fluid dynamical model of heavy ion reactions.

1 Introduction

Collective flow is one of the most dominant observable features in heavy ion reactions up to the highest available energies, and its global symmetries as well as its fluctuations are extensively studied. Especially at the highest energies for peripheral reaction the angular momentum of the initial state is substantial, which leads to observable rotation according to fluid dynamical estimates [1]. Furthermore the low viscosity quark-gluon fluid may lead to to initial turbulent instabilities, like the Kelvin Helmholtz Instability (KHI), according to numerical fluid dynamical estimates [2], which is also confirmed in a simplified analytic model [3]. These
turbulent phenomena further increase the rotation of the system, which also leads to a large vorticity and circulation of the participant zone one order of magnitude larger than from random fluctuations in the transverse plane [4, 5, 6].

The Differential Hanbury Brown and Twiss (DHBT) method has been introduced in [9]. The method has been applied to a high resolution Particle in Cell Relativistic (PICR) fluid dynamical model [10].

2 The two particle correlation

The pion correlation function is defined as the inclusive two-particle distribution divided by the product of the inclusive one-particle distributions, such that [12]:

\[ C(p_1, p_2) = \frac{P_2(p_1, p_2)}{P_1(p_1)P_1(p_2)}, \] (1)

where \( p_1 \) and \( p_2 \) are the 4-momenta of the pions and \( k \) and \( q \) are the average and relative momentum respectively.

We use a method for moving sources presented in Ref. [14]. In the formulae the \( \bar{h} = 1 \) convention is used and \( k \) and \( q \) are considered as the wavenumber vectors. The correlation function is:

\[ C(k, q) = 1 + \frac{R(k, q)}{\left| \int d^4x S(x, k) \right|^2}, \] (2)

where

\[ R(k, q) = \int d^4x_1 d^4x_2 \cos[q(x_1 - x_2)]S(x_1, k + q/2) S(x_2, k - q/2). \] (3)

Here \( R(k, q) \) can be calculated [14] via the function and we obtain the \( R(k, q) \) function as

\[ R(k, q) = \text{Re} \left[ J(k, q) J(k, -q) \right] \] (4)

The corresponding \( J(k, q) \) function will become

\[ J(k, q) = \int d^4x S(x, k) \exp \left[ -\frac{q \cdot u(x)}{2T(x)} \right] \exp(iqx). \] (5)

For the phase space distribution we frequently use the Jüttner (relativistic Boltzmann) distribution, in terms of the local invariant scalar particle density the Jüttner distribution is [15]

\[ f^J(x, p) = \frac{n(x)}{C_n} \exp \left( -\frac{p^\mu u_\mu(x)}{T(x)} \right), \] (6)

where \( C_n = 4\pi m^2 TK_2(m/T) \). We assume a spatial distribution:

\[ G(x) = \gamma n(x) = \gamma n_s \exp \left( -\frac{x^2 + y^2 + z^2}{2R^2} \right). \] (7)
Here \( n_s \) is the average density of the Gaussian source, \( s \), (or fluid cell) of mean radius \( R \).

**Asymmetric Sources:** we have seen in few source model examples \[9\] that a highly symmetric source may result in correlation functions that are sensitive to rotation, however, these results were not sensitive to the direction of the rotation, which seems to be unrealistic. We saw that this result is a consequence of the assumption that both of the members of a symmetric pair contribute equally to the correlation function even if one is at the side of the system facing the detector and the other is on the opposite side. The expansion velocities are also opposite at the opposite sides. The dense and hot nuclear matter or the Quark-gluon Plasma are strongly interacting, and for the most of the observed particle types the detection of a particle from the side of the system, – which is not facing the detector but points to the opposite direction, – is significantly less probable. The reason is partly in the diverging velocities during the expansion and partly to the lower emission probability from earlier (deeper) layers of the source from the external edge of the timelike (or spacelike) FO layer.

For the study of realistic systems where the emission is dominated by the side of the system, which is facing the detector, we cannot use the assumption of the symmetry among pairs or groups of the sources from opposite sides of the system. Even if the FO layer has a time-like normal direction, \( \hat{\sigma}^\mu \) the \((k^\mu \hat{\sigma}^\mu)\) factor yields a substantial emission difference between the opposite sides of the system.

The correlation function, \( C(k, q) \) is always measured in a given direction of the detector, \( \vec{k} \). Obviously only those particles can reach the detector, which satisfy \( k^\mu \hat{\sigma}_\mu > 0 \). Thus in the calculation of \( C(k, q) \) (see Fig. 1) for a given \( \vec{k} \)- direction we can exclude the parts of the freeze out layer where \( k^\mu \hat{\sigma}_\mu < 0 \) (see Eq. (10) of Ref.[16] or Ref.[18]. For time-like FO a simplest approximation for the emission possibility is \( P_{esc}(x) \propto k^\mu u_\mu(x) \) [17].

### 3 The DHBT method and fluid dynamical results

Based on the few source model results the Differential HBT method \[9\] was introduced by evaluating the difference of two correlation functions measured at two symmetric angles, forward and backward shifted in the reaction plane in the participant c.m. frame by the same angle, i.e. at \( \eta = \pm \text{const.} \), so that

\[
\Delta C(k, q) \equiv C(k_+, q_{out}) - C(k_-, q_{out}).
\]  

For the exactly \( \pm x \)-symmetric spatial configurations (i.e. \( k_{+x} = k_{-x} \) and \( k_{+z} = -k_{-z} \)), e.g. central collisions or spherical expansion, \( \Delta C(k, q) \) would vanish! It would become finite if the rotation introduces an asymmetry.

The sensitivity of the standard correlation function on the fluid cell velocities decreases with decreasing distances among the cells. So, with a large number of densely placed fluid cells where all fluid cells contribute equally to the correlation function, the sensitivity on the flow velocity becomes negligibly weak.
Figure 1: (color online) The dependence of the standard correlation function in the $\hat{k}_+$ direction from the collective flow, at the final time. From ref. [10].

Thus, the emission probability from different ST regions of the system is essential in the evaluation. This emission asymmetry due to the local flow velocity occurs also when the FO surface or layer is isochronous or if it happens at constant proper time.

Figure 2: (color online) The differential correlation function $\Delta C(k, q)$ at the final time with and without rotation. From ref. [10].

We studied the fluid dynamical patterns of the calculations published in Ref. [2], where the appearance of the KHI is discussed under different conditions. We chose the configuration, where both the rotation [1], and the KHI occurred, at $b = 0.7b_{max}$ with high cell resolution and low numerical viscosity at LHC energies, where the angular momentum is large, $L \approx 10^6 \hbar$ [13]. Fig. 2 shows the DHBT for the FD model.
The standard correlation function is both influenced by the ST shape of the emitting source as well as its velocity distribution. The correlation function becomes narrower in $q$ with increasing time primarily due to the rapid expansion of the system. At the initial configuration the increase of $|\vec{k}|$ leads to a small increase of the width of the correlation function.

Nevertheless, in theoretical models we can switch off the rotation component of the flow, and analyse how the rotation influences the correlation function and especially the DCF, $\Delta C(k, q)$. 

![Figure 3](image)

**Figure 3**: (color online) The Differential Correlation Function (DCF) at average pion wavenumber, $k = 5/\text{fm}$ and fluid dynamical evolution time, $t = 3.56 \text{fm/c}$, as a function of the functions of momentum difference in the "out" direction $q$ (in units of $1/\text{fm}$). The DCF is evaluated in a frame rotated in the reaction plane, in the c.m. system by angle $\alpha$. From ref. [10].

Fig. [1] compares the standard correlation functions with and without the rotation component of the flow at the final time moment. Here we see that the rotation leads to a small increase of the width in $q$ for the distribution at high values of $|\vec{k}|$, while at low momentum there is no visible difference.

In Fig. [2] $\Delta C(k, q)$ is shown for the configuration with and without rotation. For $k = 5/\text{fm}$ the rotation increases both the amplitude and the width of $\Delta C$. The dependence on $|\vec{k}|$ is especially large at the final time.

Fig. [3] shows the result where the rotation component of the velocity field is removed. The DCF shows a minimum in its integrated value over $q$, for $\alpha = -11$ degrees. The shape of the DCF changes characteristically with the angle $\alpha$. Unfortunately this is not possible experimentally, so the direction of the symmetry axes should be found with other methods, like global flow analysis and/or azimuthal HBT analysis.

Finally we separated the effect of the rotation by finding the symmetry angle where the rotation-less configuration yields vanishing or minimal DCF for a given transverse momentum $k$. This could be done in the theoretical model. We did this for two different energies, Pb+Pb / Au+Au at $\sqrt{s_{NN}} = 2.36/0.2 \text{ TeV}$ respectively,
while all other parameters of the collision were the same. The deflection angle of the symmetry axis was \( \alpha = -11/ -8 \) degrees\(^1\) respectively. In these deflected frames we evaluated the DCF for the original, rotating configurations, which are shown in Fig. 4. This provides an excellent measure of the rotation.

4 Summary

We show that two particle correlation measurements can be sensitive to the rotation of the emitting system. The analysed model calculations show that the Differential HBT analysis can give a good quantitative measure of the rotation in the reaction plane of a heavy ion collision.

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