Sensitive Dependence of Ultrarelativistic Electron Precipitation on EMIC Wave Frequency

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Abstract Electromagnetic ion cyclotron (EMIC) waves have long been considered to cause atmospheric precipitation loss of relativistic electrons, the quantitative evaluation of which is critical for understanding the radiation belt electron flux dropouts. In this study, we report test particle calculation results on precipitation loss rates of 5 MeV electrons with initial pitch angles of 4–10° at the equator by He⁺ band EMIC waves. We find a sensitive dependence of the precipitation loss rates on the wave frequency in that a preferred wave frequency (maximum loss frequency) exists at which the loss rate peaks predominantly and away from which it declines substantially. According to a cold plasma dispersion relation, a small difference in wave frequency just below the He⁺ gyrofrequency corresponds to a significant difference in wavenumber. Different frequencies lead to phase trapping of different extents, which determines the significance of loss cone scattering at the exit latitude from the wave zone. At frequencies below the maximum loss frequency, phase trapping causes large advective pitch angle changes, prohibiting loss cone scattering. At the maximum loss frequency, the phase-trapping effect is optimized such that diffusion is dominant over advection and large enough to allow effective loss cone scattering. At frequencies above the maximum loss frequency, the phase-trapping effect disappears, and the resulting (both diffusive and advective) scattering is too weak to drive loss cone scattering unless the initial pitch angle is close enough to the loss cone angle.

Plain Language Summary The Earth’s outer radiation belts are filled with electrons of very high energy (on the order of 10⁶ electron volts). The electron population of the belts often experiences dynamic changes, ranging from decreases in the background noise level to increases of a few orders of magnitude. Electromagnetic ion cyclotron (EMIC) waves have long been considered a cause of the electron flux decreases through loss into the atmosphere (precipitation loss) due to the scattering of electron motion. EMIC waves in the inner magnetosphere are typically observed at a narrow frequency band around 1 Hz. A quantitatively realistic evaluation of the EMIC wave contribution to these electron flux dropouts remains a major research topic partly due to the lack of understanding of the related physics. In our new study, we take a step forward by demonstrating the sensitive dependence of electron precipitation loss on EMIC wave frequency. This finding implies the existence of a well-defined frequency of EMIC waves that the Earth’s magnetosphere prefers for radiation belt electron precipitation. This conclusion is applicable to related problems in other planetary magnetospheres and astrophysical situations.

1. Introduction

Electromagnetic ion cyclotron (EMIC) waves have long been known to be amplified by energetic (a few to a few tens of keV) ions with $T_\perp > T_\parallel$, where $T_\perp$ and $T_\parallel$ refer to the ion perpendicular and parallel temperatures relative to the background magnetic field, respectively (Cornwall 1965; C. F. Kennel & Petschek 1966). Recent studies (e.g., J. H. Cho et al., 2017, 2016; Noh et al., 2018; Remya et al., 2018; Sigsbee et al., 2020) have confirmed that substorm injections and solar wind dynamic pressure enhancements can provide an anisotropic ion distribution that is suitable for triggering EMIC waves in the inner magnetosphere.

Many previous works suggested that EMIC waves can cause atmospheric precipitation of relativistic electrons and energetic ions in the inner magnetosphere by pitch angle scattering. Indeed, ever since the earliest suggestions by C. F. Kennel & Petschek (1966), Cornwall et al. (1970), and Thorne and Kennel (1971), a continuous accumulation of observational support for particle precipitation by EMIC waves has occurred.
Here, we consider the interaction between EMIC waves and ultrarelativistic electrons, the detailed physics of which remains unclarified despite many previous works. This evaluation is critical for understanding the flux dropouts that affect the radiation belt dynamics and evolution (e.g., Y. Y. Shprits et al., 2013). Following the early work by Lyons and Thorne (1972), who formulated specifically the bounce-averaged diffusion coefficients based on quasilinear resonant diffusion theory (C. F. Kennel & Engelmann, 1966), efforts in theory and simulations have continued until recently, yielding more extensive and realistic estimations of the quasilinear resonance diffusion coefficients (Albert, 2003; Kang et al., 2015, 2016; Kersten et al., 2014; Jordanova et al., 2015, 2016; W. Li et al., 2007; Ni et al., 2015, 2018; Shprits et al., 2009, 2013; Summers & Thorne, 2003). The EMIC wave-electron interaction has also been examined via the nonlinear test particle approach (Albert & Bortnik, 2009; Lee et al., 2020, 2018; Lemons et al., 2009; Liu et al., 2012, 2010; Zhu et al., 2020), and the validity of the quasilinear diffusion theory has been examined with test particle simulations (Fu, Ni, Tao, et al., 2019; Su et al., 2012, 2013). Other aspects of the EMIC wave-electron interaction that have been examined previously include oblique wave effects on relativistic electron scattering (Lee et al., 2018, 2020; G. Wang et al., 2017), precipitation by EMIC rising tone emission (Grach & Demekhov, 2020; Kubota et al., 2015; Kubota & Omura, 2016; Omura & Zhao, 2012, 2013), and nonresonant scattering (Chen et al., 2016). The typical electron resonance energy with EMIC waves is ∼MeV (Lorentzen et al., 2000), but lower resonance energy may be possible (Capannolo et al., 2018; Denton et al., 2019; Ukhorskiy et al., 2010; X. Zhang et al., 2019).

In the present work, we focus on the interaction between parallel propagating EMIC waves at the He+ band frequency and ultrarelativistic (5 MeV) electrons with nearly field-aligned initial pitch angles at the equator (≤10° at the equator) under a dipole field. While we attempt to quantify the loss rate of the electrons due to precipitation into the loss cone, we aim mainly to demonstrate the extent to which the loss rate depends on the EMIC wave frequency, specifically over the He+ band frequency, and to clarify the physics behind the frequency dependence of the loss rates. For this purpose, we perform test particle calculations and identify nonlinear scattering aspects that are not addressed by the usual quasilinear diffusion theory. In Section 2, we provide a description of the mathematical models taken for this study, and in Section 3, we present the main results. We discuss some main issues in Section 4 and give conclusions in Section 5.

2. Mathematical Formulation

We start from the relativistic equation of motion for electrons interacting with parallel propagating monochromatic EMIC waves:

$$\frac{dp}{dt} = -e\left(E + \frac{1}{\gamma m} p \times B\right) = -e\left(E_w + \frac{1}{\gamma m} p \times B_w\right) = -\frac{e}{\gamma m} p \times B_0$$

(1)

where $\gamma$ is the usual relativistic factor.

For the background magnetic field $B_0$ in Equation 1, we take a simplified dipole model given by:

$$B_0 = \left(\frac{x}{2}, \frac{y}{2}, B_{z0}\right)$$

with $B_{z0}(\lambda) = B_{z0}(\lambda = 0)\sqrt{1 + \frac{3}{4}\sin^2\lambda / \cos\lambda}$ in local (x, y, z) coordinates. This model has often been used in previous works (e.g., Su et al., 2012; G. Wang et al., 2017).

For the wave fields $E_w$ and $B_w$ in Equation 1, we use the specific expressions below to represent the parallel propagating L-mode wave (e.g., G. Wang et al., 2017):

$$E_w = -\hat{\lambda}E_{\phi0} \sin(\phi) - \hat{\phi}E_{\phi0} \cos(\phi)$$

(2)

$$B_w = \hat{\lambda}B_{\phi0} \cos(\phi) - \hat{\phi}B_{\phi0} \sin(\phi)$$

(3)
here, $\Phi$ is the wave phase, the time evolution of which is given by the following equation:

$$\frac{d\Phi}{dt} = \omega - k_z v_z \tag{4}$$

We assume that this L-mode wave with a constant frequency $\omega$ exists in the latitudinal region within $20^\circ$ from the equator. The wavenumber vector $k$ is directed away from the equator.

The relativistic equation of motion for the electrons interacting with this wave can be cast into the following forms in local coordinates defined by the set $\{p, p_{\perp}, \eta \}$, which refer to, respectively, the $z$-component of the electron momentum vector, the magnitude of the perpendicular momentum, and the phase difference in the plane perpendicular to the local background magnetic field between the wave magnetic field vector and electron momentum vector (Bell, 1984; D. Lee et al., 2020) (see the supporting information in D. Lee et al. (2020) for details).

$$\frac{dp_z}{dt} = -\frac{\Omega_e p_{\perp}}{\gamma} \sin(\eta) - \frac{e}{\gamma m} (p_{\perp} \times B_{B_0}) \hat{z} \tag{5}$$

$$\frac{dp_{\perp}}{dt} = \left( \frac{\Omega_e p_{\perp}}{\gamma} - a_E \right) \sin(\eta) + \frac{e p_z}{\gamma m p_{\perp}} (p_{\perp} \times B_{B_0}) \hat{z} \tag{6}$$

$$\frac{d\eta}{dt} = \omega - k_z p_{\perp} \frac{\Omega_e}{\gamma m} + \frac{1}{p_{\perp}} \left( \frac{\Omega_e p_{\perp}}{\gamma} - a_E \right) \cos(\eta) - \frac{e p_z}{\gamma m} \frac{p_{\perp} B_{B_0}}{p_{\perp}^2} \tag{7}$$

where $\Omega_e = -\frac{e}{2m} (B_{ax} + B_{ay})$ and $a_E = -\frac{e}{2} (E_{nx} + E_{ny})$.

The set of Equations 5–7 is similar to that in Bell (1984) derived for the interaction between electrons and R-mode whistler waves. In 7, while the leading term $\frac{\Omega_e}{\gamma}$ corresponds to the gyrofrequency under the background magnetic field, the fourth term on the right-hand side reflects the finite wave field effect on the electron gyro-phase evolution, which becomes significant at low pitch angles due to the $1/p_{\perp}$ dependence.

The last terms in Equation (5–7) are due to the nonuniform background magnetic field $B_0$ and $B_{B_0}$ is its component in the local $x$-$y$ plane which is responsible for the mirror force (Bell, 1984).

For a chosen wave frequency, we determine the wavenumber $k$ from a cold plasma dispersion relation. Figure 1 shows the dispersion curves (L mode in red (He$^+$ band) and blue (H$^+$ band), R mode in black) and the minimum kinetic energy of electrons for resonant interaction with L-mode waves. The solid lines in Figure 1 are obtained by assuming a plasma consisting of cold ions (80% protons +20% He$^+$) and cold electrons with $\Omega_e^2 / \omega_{pe}^2 = 0.0125$, where $\Omega_e$ and $\omega_{pe}$ are the electron gyrofrequency and the electron plasma frequency, respectively (see Equation 1 in Summers & Thorne, 2003). In reality, all the background plasma parameters that determine the dispersion relation are unlikely to be constant along the field lines in the inhomogeneous plasma. To demonstrate this, Figure 1 also shows two additional cases in He$^+$ band (the dotted and dashed red curves) for the conditions at the latitude $\lambda = 10^\circ$ and $20^\circ$. These were obtained by assuming the plasma density model that varies as $\cos^{-4} \lambda$ (Bortnik et al., 2008; Chen et al., 2016) and using the simple dipole field model described above. With the conditions at the higher latitudes, the dispersion relation gives a smaller wavenumber and accordingly a larger resonance energy for a fixed wave frequency. A realistic work should adopt suitable models for all the parameters. However, these models may create additional effects due to the inhomogeneity beyond the main physics that we want to focus on in the present work. Therefore, to avoid any compounding effects, we apply this simple plasma model to the entire region in space.

For the calculations presented in Section 3, we consider the wave frequency domain just below the He$^+$ gyrofrequency. Specifically, we focus on a narrow frequency domain, $0.22\Omega_p < \omega < 0.25\Omega_p$, over which the wavenumber is sensitive to a small difference in the wave frequency (Figure 1a). Note that this sensitivity
holds true similarly for three red curves of L mode representing different latitudes. It also remains the case even if one includes cold oxygen ions in the dispersion relation. Additionally, the minimum resonance energy over this narrow frequency range is below 5 MeV again for all three latitudes (Figure 1b).

For a chosen set of wave frequencies and wavenumbers, the polarization relations of the wave electric and magnetic field components are determined self-consistently from the cold plasma dispersion relation and Maxwell equations (see the supporting information in D. Lee et al. (2020) for details). In practice, the parameter \( w_xB \) is the only free parameter that we need to specify the wave amplitude and polarization. For all the results in Section 3, we take \( w_xB = 1\% \) of the background magnetic field, which corresponds to 2.5 nT at \( r = 5R_E \) for a dipole field. (In Section 4, we will discuss a larger wave amplitude case by taking \( w_xB = 5\% \), corresponding to 12.5 nT at \( r = 5R_E \) for a dipole field).

Furthermore, we focus on mostly field-aligned pitch angle electrons with energy of 5 MeV, specifically electrons with initial pitch angles of 4°–10° at the equator of the L = 5 field line, where they are launched upward in the same direction as the wavenumber vector. These field-aligned pitch angle electrons may naturally be expected to be the ones that most likely suffer from scattering into the loss cone. We test these electrons with various initial phases \( \eta \) at the equator and aim to see how these different phase electrons are scattered by the EMIC waves. Specifically, in the results presented in the next section, we consider 48 electrons with different initial phases, separated evenly for the same given initial pitch angle. We define “loss” of the electrons when the equatorially mapped pitch angle of their local pitch angle at the exit latitude of 20° is smaller than the equatorial loss cone angle: the loss cone angle at \( L = 5 \) is \( \sim 3.8° \) at the equator for a dipole field.

3. Results and Interpretation

Figure 2 shows the loss rate of electrons as a function of the wave frequency for the wave magnetic field intensity of \( B_{w,B} = 1\% \). Here, the loss rate refers to the percentage of electrons precipitating into the loss cone angle at the exit latitude \( \lambda = 20° \) out of the total 336 electrons (7 different
initial pitch angles separated by 1° from 4° to 10°, and 48 different initial phases for each pitch angle) for a
given wave frequency. The loss rate peaks (15.5%) at the wave frequency \( \omega = 0.23\Omega_p \). The loss rate rapidly
decreases at lower frequencies, becoming zero at \( \omega = 0.225\Omega_p \). It also declines, though less rapidly, over a
higher frequency range down to a few percent just below the \( \text{He}^+ \) gyrofrequency (2.7% at \( \omega = 0.248\Omega_p \)).

To demonstrate the sensitive dependence of the loss rate on the wave frequency in Figure 2, we compare
the electron trajectories in detail among three wave frequency cases: \( \omega = 0.225\Omega_p \) (zero-loss rate case),
\( 0.23\Omega_p \) (peak loss rate case), and \( 0.24\Omega_p \) (reduced loss rate case). First, Figure 3 shows the information on
electron trajectories for \( \omega = 0.225\Omega_p \). For demonstration purposes, we select eight electrons with the same
initial pitch angle of 4° but with different initial phase \( \eta \), values, as indicated in the figure legend. Figure 3a
shows the changes in the equatorially mapped pitch angles versus latitude as the electrons travel along the
field line from the equator to higher latitudes. All eight electrons regularly oscillate near the loss cone angle
(3.8°, as indicated) until they arrive at the latitude marked by the red horizontal line with an arrow. Then, the
electrons go through much larger pitch angle changes during their traversal over this region such that
the pitch angles change collectively toward higher values (corresponding to a positive advective change).
Consequently, they exit the wave zone with these large pitch angles (far larger than the loss cone angle) at
\( \lambda = 20° \). Therefore, none of the electrons suffers from scattering into the loss cone angle, explaining the
zero-loss rate for this frequency shown in Figure 2.

Figure 3b shows how the local phase \( \eta \) changes in association with the pitch angle changes. The eight elec-
trons leave the equator with evenly separated initial phases. We pay attention to the latitudinal domain indi-
cated by the red horizontal line with an arrow, where these electrons become phase-trapped within a limit-
ed phase angle range. The red horizontal line with an arrow is only a rough indication of the phase-trapping
region, while the precise phase-trapping latitude differs among the electrons to some modest extent (see the
further discussion in Figure 5 below). Note that this is the same latitudinal zone as that in Figure 3a. From a
close comparison between the pitch angle changes in Figure 3a and the phase-trapping feature in Figure 3b
for each electron, we identify that for all the electrons, the pitch angle increases farther away from the loss
cone once phase trapping begins. That is, the large positive advective change in the pitch angles near the
exit altitude shown in Figure 3a is due to phase trapping, which consequently prohibits precipitation loss.

Incidentally, we recognize a few additional features in Figure 3. First, in the later part of the phase-trapping
zone, the phase bunching effect occurs to a substantial degree (blue horizontal line with an arrow in Fig-
ure 3b). Second, we identify three electrons (blue, magenta, purple) in Figure 3a that show pitch angles very
close to zero at the positions marked by the thick black arrows before they scatter back toward higher pitch
angles. This locally very-low pitch angle leads to large phase variation, as shown in Figure 3c, known as the
loss cone reflection effect (Inan et al., 1978), which is due to the fourth term being proportional to \( 1/p_\perp \)
on the right-hand side of phase Equation 7. Last, Figure 3c indicates that \( \text{d}\eta/\text{d}t = 0 \) at some points during the
phase-trapping period, implying the occurrence of resonance. Considering that all the electrons do not
precipitate, this observation implies that one should not take it for granted that the occurrence of resonance
leads to loss cone scattering for precipitation.

Similarly, Figure 4 shows the calculation results for the case of \( \omega = 0.23\Omega_p \) at which the loss rate peaks, as
shown in Figure 2. Figure 4a shows the equatorially mapped pitch angles for the eight electrons with differ-
ent initial phases but the same initial pitch angle of 4°. It indicates that the large pitch angle changes occur
primarily at latitudes quite close to the exit latitude (as indicated by the red horizontal line with an arrow); note
that this large scattering latitude is “closer” to the exit latitude compared to the case of \( \omega = 0.225\Omega_p \) in
Figure 3. More importantly, the pitch changes at the exit latitude are now somewhat diffusive among the
eight electrons, which is in contrast to the dominantly advective behavior in the \( \omega = 0.225\Omega_p \) case in Fig-
ure 3. Consequently, some of the electrons (blue, magenta) exit the wave zone with pitch angles less than
the loss cone angle, thus allowing precipitation loss for these electrons. The loss due to this diffusive pitch
angle change also occurs for electrons at other initial pitch angles. While we further discuss the diffusion
versus advection later in Figure 8, here in Figure 4c, we show the loss rates calculated for other initial pitch
angles. Specifically, the loss rates in Figure 4c are estimated separately for each of the 7 initial pitch angles
from 4° to 10° using 48 different initial phases for each pitch angle. The figure indicates that loss occurs for
electrons with an initial pitch angle of up to 8°.
Figure 3. Test particle calculation results for eight selected electrons with different initial phases $\eta_0$ but with the same initial pitch angle of 4° interacting with EMIC waves of frequency $\omega = 0.225\Omega_p$ and amplitude $B_{wx} = 1\%$. (a) Equatorially mapped pitch angle (PA), meaning the pitch angle mapped adiabatically to the equator from the local electron position, versus latitude. (b and c) Local phase $\eta$ and its time derivatives corresponding to the eight electrons in (a). The loss cone angle of 3.8° is marked in (a). EMIC, Electromagnetic ion cyclotron.
Figure 4. Test particle calculation results for electrons interacting with EMIC waves of frequency $\omega = 0.23\Omega_p$ and amplitude $B_{wx} = 1\%$. (a) Equatorially mapped pitch angles versus latitude for eight selected electrons with different initial phases $\eta_o$ but with the same initial pitch angle $= 4^\circ$ (note the different vertical axis scales used in Figures 3a and 4a). (b) Local phase $\eta$ corresponding to the eight electrons in (a). (c) Loss rate of electrons for 7 different initial pitch angles. The percentage rate is calculated from 48 different initial phases for each pitch angle. EMIC, Electromagnetic ion cyclotron.
What is the role of phase trapping at this higher frequency ($\omega = 0.23\Omega_p$) compared to the $\omega = 0.225\Omega_p$ case in Figure 3? Figure 4b shows the phase evolution of the eight electrons with the same initial pitch angle of 4° presented in Figure 4a. As indicated by the red horizontal lines with arrows, the large pitch angle changes in Figure 4a are associated with the occurrence of phase trapping within a narrow phase angle range in Figure 4b. This situation may seem similar to that in Figure 3. However, we emphasize that the latitude where this phase trapping begins is somewhat "closer" to the exit latitude $\lambda = 20^\circ$, and consequently, the latitudinal zone of this phase trapping is narrower compared to the $\omega = 0.225\Omega_p$ case in Figure 3. This narrow zone leaves much less room for the phase trapping to cause a large advective scattering, unlike the $\omega = 0.225\Omega_p$ case in Figure 3.

To demonstrate the phase-trapping effect further, we use the phase-trapping condition given by the following expression (Omura & Zhao, 2012):

$$v_R - \frac{2\omega_e}{k_z} < v_z < v_R + \frac{2\omega_e}{k_z}$$

where $v_R = (\omega + \Omega_e / \gamma) / k_z$ and $\omega_e = (k_e e B_{ext} / \gamma^2 m_e^2) p_z$.

This is an approximate expression derived by neglecting the term proportional to the $\cos \eta / p_z$ contribution and the inhomogeneity term in the phase evolution Equation 7. The numerical evaluation results of this simple expression are shown for two frequencies, that is, $\omega = 0.225\Omega_p$ and $\omega = 0.23\Omega_p$ in Figure 5. The calculations are performed for 48 electrons with different initial phases but the same initial pitch angle of 4°. The 48 electrons are identified by sequential numbers along the horizontal axis, and the vertical bars refer to the latitudinal zones where each electron satisfies the trapping condition (8). Clearly, the phase trapping occurs closer to the exit latitude $\lambda = 20^\circ$ for $\omega = 0.23\Omega_p$ (red) than for $\omega = 0.225\Omega_p$ (blue). This result is consistent with the specific trajectory calculations shown in Figures 3 and 4. Recall that for $\omega = 0.225\Omega_p$ in Figure 3, the phase-trapping effect begins well prior to the exit latitude, which causes the large pitch angle advection away from the loss cone. In contrast, for $\omega = 0.23\Omega_p$, the latitudinal zone where the phase trapping can play a significant role in causing a large advection before the electrons exit the wave zone is more limited. This situation makes the diffusive nature of the pitch changes dominant to a larger extent, causing some of the electrons to enter the loss cone.

The above results in Figures 3–5 indicate that the small difference between the two wave frequencies, $\omega = 0.225\Omega_p$ and $\omega = 0.23\Omega_p$, causes the drastic difference in the precipitation loss rate. This small difference in wave frequency corresponds to a more significant difference in wavenumber (equivalently, wavelength) that the electrons see while traveling. As shown in the dispersion relation curve in Figure 1, the corresponding wavenumbers are 0.14115 and 0.15951, in units of $\Omega_e/c$. In terms of the wavelength, the electrons see shorter wavelengths (by $\sim 11.6\%$) for $\omega = 0.23\Omega_p$ than for $\omega = 0.225\Omega_p$, while the frequency difference is only $\sim 2\%$. This difference in wavelength is large enough to yield a notable distinction in the extent to which phase trapping plays a critical role in determining loss cone scattering at the exit latitude.

As the last example, Figure 6 considers the case of $\omega = 0.24\Omega_p$. Figure 6a shows the eight electrons with an initial pitch angle of 4°, where the pitch angle changes are overall much less significant than in the two frequency cases above. The corresponding phase variations in Figure 6b indicate no meaningful feature. For this frequency, the phase trapping that could lead to large scattering does not occur within the wave zone, unlike at the previous two lower frequencies. In fact, we find that this small scattering is the case for all pitch angles (4°–10°) at this frequency. Two examples of higher pitch angles (initial pitch angles = 5 and 8°) are shown in Figure 6c, which indicates that the exit pitch angles are well outside the loss cone angle due to the very-small scattering effect. Although some of the 4° electrons (Figure 6a) undergo loss cone scattering,
higher pitch angle electrons do not. This situation leads to the reduced total loss rate at $\omega = 0.24\Omega_p$ compared to that in the $\omega = 0.23\Omega_p$ case in Figure 2.

To further demonstrate the reduced loss rates for a frequency range higher than $\omega = 0.23\Omega_p$, Figure 7 shows the loss rates for selected frequencies from the range of $\omega \geq 0.23\Omega_p$. They were computed using 48 different phases separately for each of 7 different initial pitch angles. The curve for $\omega = 0.23\Omega_p$ is the same as that in

**Figure 6.** Test particle calculation results for electrons interacting with EMIC waves of frequency $\omega = 0.24\Omega_p$ and amplitude $B_{wx} = 1\%$. (a) Equatorially mapped pitch angles versus latitude for eight selected electrons with different initial phases $\eta_0$ but with the same initial pitch angle = 4°. (b) Local phase $\eta$ corresponding to the eight electrons in (a). (c) Same as (a) but for two initial pitch angles = 5° and 8°. EMIC, Electromagnetic ion cyclotron.
Figure 4c. Clearly, as the frequency increases, the net loss comes preferentially from the initial pitch angles that are closer to the loss cone angle. Consequently, this leads to the reduced total loss rates for $\omega > 0.23\Omega_p$ in Figure 2.

We take another way to evaluate the significance of pitch angle scattering at the exit latitude and its dependence on the wave frequency by expressing the degree of pitch angle changes in terms of advection and diffusion. For a diffusive pitch angle change, we take the root mean square average:

$$\Delta \alpha_{eq} = \sqrt{\langle (\alpha_{eq} - \langle \alpha_{eq} \rangle)^2 \rangle}$$  \hspace{1cm} (9)

where $\langle \rangle$ indicates the average over different values of the initial phase. Here, $\alpha_{eq}$ is the equatorial pitch angle mapped from a given local pitch angle via the first adiabatic invariant conservation. This definition of the diffusive pitch angle change represents the average extent to which pitch angles with different initial phases deviate from the phase-averaged pitch angle. Next, as a simple measure of the nondiffusive pitch angle change, we use advection as defined below:

$$\Delta \alpha_{adv} = \alpha_{eq} - \alpha_0$$ \hspace{1cm} (10)

where $\alpha_0$ is the initial pitch angle.

The definitions (9) and (10) are basically equivalent to those used for calculations of the diffusion and advection coefficients in the test particle simulations of combined Landau and bounce resonant interactions of electrons with magnetosonic waves by Fu, Ni, Zhou, et al. (2019). Similar definitions were used in da Silva et al. (2018), but for the test particle simulations of electron interactions with whistler-mode waves. One difference is that unlike in the present work, the diffusion and advection coefficients in Fu, Ni, Zhou, et al. (2019) were calculated for magnetosonic waves having a Gaussian frequency distribution and those in da Silva et al. (2018) were based on whistler mode wave packets generated from a self-consistent simulation, which accordingly led them to consider comparison with the quasilinear theory calculations.

The quasilinear diffusion equation includes an inherent advection effect (Liu et al., 2012; Tao et al., 2008). Liu et al. (2012) demonstrate that in the early stage of the wave-particle interaction when the mean pitch angle change grows approximately linear in time, the advection rate inferred from the quasilinear diffusion theory is consistent with that from test particle simulations using Equation 10 above but begins to differ for large amplitude waves. We use the definition (10) to include nonlinear advection that is additional to the inherent advection in the quasilinear diffusion equation, although the definition (10) does not distinguish between the phase bunching and trapping effects which is the cause of nonlinear advection (Albert et al., 2013; Artemyev et al., 2016).

Figure 8 shows the computed diffusive and advective pitch angle changes for selected cases. Figure 8a compares the results among three selected wave frequencies over the entire pitch angle range (4°–10°). For $\omega = 0.225\Omega_p$ (orange), the advection is positive and very large such that it overwhelms the diffusion for all (but 10°) pitch angles. This situation makes loss cone scattering impossible, explaining the zero-loss rate at this frequency, as shown in Figure 2. For $\omega = 0.23\Omega_p$ (olive) at which the loss rate in Figure 2 peaks, the advection effect decreases substantially such that the diffusion now overwhelms the advection for all (but 4°) pitch angles. The diffusion itself is significant enough to allow many electrons to be scattered into the loss cone. For $\omega = 0.24\Omega_p$ (red), as the advection effect becomes even further reduced, the diffusion itself also becomes quite small (<1°). This holds over all the initial pitch angles. Consequently, the possibility of electrons undergoing loss cone scattering is low unless the initial pitch angle is very close to the loss cone angle.

To complement the discussion above, Figure 8b shows the diffusion (blue) and advection (red) over the entire frequency range for two initial pitch angles: 4° (solid) and 10° (dotted). For an initial pitch angle of 4°, the positive advection is large and dominant over diffusion for $\omega$ values well less than 0.23$\Omega_p$, which...
explains the rapid drop in the loss rate in this lower frequency regime. In contrast, the trend reverses in the higher frequency regime, $\omega > 0.23\Omega_p$, which may allow some electrons to be scattered into the loss cone. However, for $\omega > 0.23\Omega_p$, the amount of both types of pitch angle changes is reduced substantially. Although details differ somewhat, the overall trend for the initial pitch angle of 10° is not much different from the initial pitch angle of 4°. As explained above, the reduced amount of diffusion for the higher frequency implies that electrons are increasingly less likely to be scattered into the loss cone unless the initial pitch angles are close enough to the loss cone angle. This explains the reduced total loss rates over this higher frequency shown in Figure 2.

Incidentally, according to some previous works (Albert & Bortnik, 2009; Omura et al., 2012, 2008; Su et al., 2012), the competition between the adiabatic motion and wave-induced motion is characterized by a dimensionless parameter, called "inhomogeneity parameter." Based on this parameter, it has been suggested that wave-electron interactions are expected to be linear (that is, diffusive) for near-loss cone pitch angles but otherwise nonlinear for a wide pitch angle range. In contrast, the present work suggests that the nonlinear nature of the interaction depends sensitively on specific wave frequency such that it can occur even for near-loss cone pitch angles as shown in Figures 3–6 and 8.

4. Summary and Discussion

4.1. Summary of Key Results

In summary, the present work addresses one key point: a sensitive dependence of the precipitation loss rate on the EMIC wave frequency exists in the He$^+$ band domain. We find that a well-defined wave frequency (maximum loss frequency) exists at which the loss rate peaks predominantly and that the loss rate declines substantially away from that preferred frequency. Despite a small difference in wave frequency, electrons can see a more significant difference in the wavenumber, as predicted from the cold plasma dispersion relation. This difference causes substantially different behavior of the electron pitch angle changes near the exit point from the wave zone. This scenario can be interpreted in terms of the combination of diffusive and advective pitch angle scatterings at the exit point, which is determined by the extent to which the phase trapping plays a role. Figure 9 is a reproduction of Figure 1 with comments that summarize our conclusions. At frequencies below the maximum loss frequency, the occurrence of phase trapping over a sufficiently wide latitudinal zone prior to the exit latitude greatly contributes to causing the substantial nonlinear advective pitch angle changes. This contribution limits the occurrence of loss cone scattering. At the maximum loss frequency, the phase-trapping effect is optimized such that the advective pitch angle changes become much.
smaller, whereas the diffusive pitch angle changes remain large enough to allow effective loss cone scattering. At frequencies higher than the maximum loss frequency, phase trapping does not occur within the wave zone, leading to only a weak scattering effect. Consequently, while the pitch angle scattering is more diffusive than advective, the diffusion itself is too weak to cause loss cone scattering unless the initial pitch angles are sufficiently close to the loss cone angle. Accordingly, the net loss rate decreases for the higher frequency range.

4.2. Discussion
4.2.1. Latitudinal Distribution of Waves
In Section 3, we took a simple model for a latitudinal distribution of EMIC waves where a constant amplitude wave is present only within the latitude of 20° from the equator. Rigorously, the observational justification of this simple model is not necessarily guaranteed. Previous simulations employed somewhat different models for a latitudinal distribution of waves along the field line. For example, Chen et al. (2016) in their test particle simulations of nonresonant electron interaction with EMIC waves adopted a model of the wave zone within 20° above the equator but with varying latitudinal width of wave edges. G. Wang et al. (2017) assumed the EMIC wave existence within 24° from the equator for their test particle simulation of interaction between ultrarelativistic electrons and oblique EMIC waves. Su et al. (2012) in their comparative study between the test particle and quasilinear simulations assumed that the EMIC waves were confined within 28° from the equator. In the test particle simulations by Kubota and Omura (2016) on radiation belt electron precipitation by EMIC rising tone emissions, the waves were mostly confined within the latitude of ~20–30°. Although it is observationally true that the EMIC waves can exist over a high latitudinal zone (e.g., Matsuda et al., 2018) and the wave zone boundary in latitude may not be sharp, a well-established observation-based model of specific field-aligned distribution of the EMIC wave properties is presently limited. The loss rates in Section 3 will change quantitatively if one adopts a different (more realistic) model distribution of the waves along the field line.

Nevertheless, the key point in Section 3, that is, the sensitive dependence of the loss rates on wave frequency, remains true even if we take a wider latitudinal zone of the waves. To demonstrate this, Figure 10a (black line) shows the loss rate when the latitudinal boundary of the wave zone is taken to be 25°. For this result, all the other conditions were taken the same as for the result in Section 3. The maximum loss frequency shifts toward a higher frequency, 0.237Ωp, compared to the case of the latitudinal boundary of the wave zone = 20° (red line in Figure 10a, the same curve as in Figure 2 in Section 3). For this broader latitudinal zone of the wave, the excessive phase-trapping effect, which drives a large advective pitch angle change, simply extends to the higher frequency domain. Accordingly, the phase-trapping effect becomes more optimistic at the higher frequency, 0.237Ωp, in the sense that at this frequency the advective pitch angle change is sufficiently reduced and the diffusive loss cone scattering is most effective. Therefore, the basic explanation in Section 4.1 with Figure 9 remains the same for this wider latitudinal boundary for the wave zone.

Additionally, the calculated loss rates in Section 3 remain valid for the condition that the wave zone boundary in latitude is sufficiently thin: that is, the wave amplitude drops fast enough over a narrow latitudinal layer. To demonstrate this, the same calculation of the loss rates was repeated by taking a decaying wave boundary model where the wave amplitude decays rather rapidly from latitude λ = 20° at a rate of Bw = 1/10 (for λ = 20–22°), 1/100 (for λ = 22–24°), 1/1,000 (for λ = 24–26°) of the main wave amplitude within λ = 20°, and Bw = 0 beyond λ = 26°. The loss rate for this simple model case is shown in Figure 10a (green line). It is clear that the loss rate curve does not differ much from the original sharp boundary case (red line), implying that such a decaying wave boundary model is sharp enough to keep the main point of the present work valid. However, the methodology in Section 3 does not apply to the situation where the wave amplitude decays “too gradually” in latitude in the sense that a clear identification of the wave zone

Figure 9. Summary of the main conclusions, with the loss rate curve (Bw = 1% case) taken from Figure 1.
boundary is not practical. Such a problem should be treated by a different approach. To be realistic, reliable models of wave zone boundary edges that are supported by observations should be used, which we leave as another project.

4.2.2. Wave Amplitude Effect

In Section 3, we have taken \( \omega x B \) = 1% of the background magnetic field. Observations have indicated that stronger EMIC waves are possible. Nakamura et al. (2019) reported EMIC wave amplitudes as large as 7% of the background magnetic field. Engebretson et al. (2015) reported an EMIC wave event with peak amplitudes often >12 nT (up to 25 nT) at \( r \sim 5.5 \text{RE} \), corresponding to >6% (up to \( \sim 13% \)) of the background magnetic field.

With a larger wave amplitude, one may expect more significant nonlinear effects. As an example, Figure 10a (blue line) shows the loss rate for \( \omega x B = 5% \), corresponding to 12.5 nT at \( r = 5 \text{RE} \) for a dipole field, with all the other conditions taken to be the same as the case of \( \omega x B = 1% \). The overall trend of the frequency dependence is quite similar to \( \omega x B = 1% \), except that the loss rate curve shifts toward a higher frequency regime and the peak loss rate is somewhat higher (20.2%) and occurs at a higher frequency (\( \omega = 0.237\Omega_p \)). The reason for the shift of the maximum loss frequency toward the higher frequency regime is due to the larger nonlinear advection effect which extends to the higher frequency regime and the peak loss rate is somewhat higher (20.2%) and occurs at a higher frequency (\( \omega = 0.237\Omega_p \)). The reason for the shift of the maximum loss frequency toward the higher frequency regime is due to the larger nonlinear advection effect which extends to the higher frequency regime and the peak loss rate is somewhat higher (20.2%) and occurs at a higher frequency (\( \omega = 0.237\Omega_p \)). The reason for the shift of the maximum loss frequency toward the higher frequency regime is due to the larger nonlinear advection effect which extends to the higher frequency regime and the peak loss rate is somewhat higher (20.2%) and occurs at a higher frequency (\( \omega = 0.237\Omega_p \)).

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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure10.png}
\caption{(a) Dependence of the loss rates on the wave zone latitude \( \lambda_{\text{wave}} \) and the wave amplitude \( B_{\text{wave}} \). The red and blue lines refer to the cases where the wave exists up to \( \lambda_{\text{wave}} = 20\degree \) but with different wave amplitudes (note that the red line is the same one as in Figure 2, repeated here for comparison). The black line is the result for the wave existence up to the higher latitude, \( \lambda_{\text{wave}} = 25\degree \). The green line is a model case representing a finite width boundary where the wave amplitude decays rapidly over a narrow latitudinal layer from \( \lambda_{\text{wave}} = 20\degree \). (b) Comparison between advection and diffusion for two selected initial pitch angles as a function of the wave frequency for the case, \( B_{\text{wave}} = 5\% \) and \( \lambda_{\text{wave}} = 20\degree \), shown in the same format as in Figure 8b.}
\end{figure}

4.2.3. Higher Pitch Angle Electrons

The loss rate presented in Section 3 was computed for electrons with initial pitch angles of 4\degree–10\degree at the equator. Clearly, higher pitch angles need to be incorporated for a more realistic estimation of the loss rate. In fact, EMIC waves can scatter electrons with any pitch angle, which induces diffusion in the pitch angle.
space. The quasilinear diffusion simulations by Kersten et al. (2014) and Usanova et al. (2014) indicated that the resonant diffusion effect is significant up to pitch angles of $\sim 60^\circ$ and $\sim 45^\circ$, respectively. Further studies suggested that even high-pitch-angle (near-90°) electrons can be scattered by EMIC waves. Landau resonance with oblique EMIC waves is one possible method (Fu et al., 2018; B. Wang et al., 2016). Additionally, Cao et al. (2017) considered the bounce resonant scattering of near-equatorially mirroring electrons by H⁺-band EMIC waves (Roberts & Schulz, 1968) to more field-aligned pitch angles, where the cyclotron resonance can further scatter electrons into the loss cone. Most recently, D. Lee et al. (2020) demonstrated that even electrons with a pitch angle of exactly 90° can be directly scattered by large-amplitude EMIC waves, which can eventually result in cyclotron resonant interactions leading to significant pitch angle changes. This process can take place without Landau or bounce resonance. Additionally, combined EMIC and whistler mode wave scattering has been suggested to affect high-pitch-angle electrons with reduced lifetimes (Mourenas et al., 2016; X. Zhang et al., 2017). Therefore, a more realistic evaluation of the loss rate requires taking account of all these possibilities, which can scatter electrons with pitch angles farther away from the loss cone angle. One aspect to consider when incorporating the high pitch angle electrons is that the loss time scales on which the electrons are scattered into the loss cone differ from those for the low pitch angle electrons. In the present work, we followed the low pitch angle electron trajectories only until the time when the electrons exit the wave zone in the upper-half plane. We expect that multiple traversals over the wave zone may cause more electron loss but details of this process depend on pitch angle. Accordingly, one needs to define an effective loss rate that suitably account for all pitch angles in the test particle calculations. Our loss rate estimation results are thus subject to change quantitatively by the inclusion of all these aspects.

4.2.4. H⁺ Band Frequency Range

In the present work, we considered the wave frequency domain just below the He⁺ gyrofrequency. As discussed in Section 3 and shown in Figure 1, the wavenumber is sensitive to a small difference in the wave frequency in this He⁺ band domain, particularly $\omega \sim 0.22\Omega_p - 0.25\Omega_p$, that is, the frequency range that we have chosen to focus on in the present work. In this same sense, the dependence of the wavenumber on the wave frequency is overall less sensitive for the H⁺ band frequency, as shown in Figure 1a. In particular, for $\omega > \sim 0.5\Omega_p$, a large wavenumber difference would require a far larger difference in wave frequency (blue in Figure 1a) than that in the He⁺ band domain (red in Figure 1a). Additionally, note that the minimum resonant energy increases rapidly with decreasing frequency away from each ion gyrofrequency, becoming much greater than 5 MeV at $\omega < \sim 0.2\Omega_p$ for the He⁺ band waves (red in Figure 1b) and $\omega < \sim 0.5\Omega_p$ for the H⁺ band waves (blue in Figure 1b). We leave a more comprehensive study on the dependence of the precipitation loss rate on frequency over a wider range as a future work.

4.2.5. Lower Energies

While the present paper focused on electrons of 5 MeV representing ultrarelativistic energies, the nonlinear advection effect, which is the key factor in determining the loss cone scattering effect, depends on particle energy as well as wave frequency (Albert & Bortnik, 2009; D. Lee et al., 2018; Su et al., 2012). For example, the nonlinear advection effect can be significant for electrons of 1–2 MeV relativistic energies, which is usually the largest population of the outer radiation belts. However, although the results are not presented here, its dependence on the wave frequency for the lower energies is quite different from that for 5 MeV. In addition, the lower energy electrons meet the resonance condition closer to the equator while the wave distribution may extend to a high latitude. These differences together result in the loss rate dependence on wave frequency in a different way from that for 5 MeV. A comprehensive description on energy dependence requires another paper.

4.2.6. Time-Varying Frequency and Subpacket Wave Structure

Previous works report on EMIC rising tone emissions which have subpacket structure (e.g., Nakamura et al., 2019, 2015; Shoji & Omura, 2013). The simulations of Kubota and Omura (2016) and Omura and Zhao (2012) predicted that the EMIC rising tones are quite effective for rapid precipitation of electrons. Observationally Nakamura et al. (2019) is the first report on rapid precipitation of electrons by EMIC rising tone emissions which have subpacket structure. Omura and Zhao (2012) suggested that there is a threshold wave amplitude above which the nonlinear electron trapping by EMIC rising tones can take place. This
effect depends on the frequency sweep rate. In contrast, the nature and intention of the present work are different from the rising tone emission problems. The present work was designed to emphasize the key point in a straightforward way that there is a narrow wave frequency domain just below the He\(^+\) gyrofrequency over which the corresponding wavenumber range is much broader and accordingly the loss rates differ drastically. To demonstrate it most efficiently, the present work adopted a monochromatic constant frequency EMIC wave, which thus does not apply directly to the rising tone emission situations.

### 4.2.7. Wavenumber From Warm Plasma Model

In this study, we adopted a specific cold plasma model from which we obtained the wavenumber for a given wave frequency on which the loss rate depends sensitively. Clearly, cold plasma models with different parameter values and, more realistically, with a density variation along the field line will lead to quantitatively different results. In addition, warm plasma effects may be significant in reality. The use of a kinetic dispersion relation for a finite temperature plasma can give a different wavenumber from those predicted by cold plasma models (Chen et al., 2016; Gary et al., 2012; D. Y. Lee et al., 2017; Silin et al., 2011). Thus, it is expected that our present results in Section 3 are subject to change if one includes kinetic effects in determining the wavenumber.

### 5. Conclusions

In conclusion, there is a well-defined frequency within a narrow frequency range just below the He\(^+\) gyrofrequency at which the ultrarelativistic electron loss rate is maximized. This maximum loss frequency occurs when an optimistic condition is met such that the nonlinear advection effect is sufficiently weak and the diffusive loss cone scattering effect is strong enough just before the electrons exit the main wave zone. At frequencies slightly away from this maximum loss frequency, such optimistic condition is not satisfied and accordingly the loss rates drop substantially. Quantitatively, the specific values of the maximum loss frequency depend on specific conditions regarding the wave zone latitude (and its boundary structure) and wave amplitude.

### Data Availability Statement

No observational data were used in this research. All figures were constructed from the numerical data obtained by solving the set of equations described in Section 2, which can be obtained in a Zenodo data repository at http://doi.org/10.5281/zenodo.4444252.

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