Complementarity and Chiral Fermions in SU(2) Gauge Theories

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Abstract

Complementarity - the absence of a phase boundary separating the Higgs and confinement phases of a gauge theory - can be violated by the addition of chiral fermions. We utilize chiral symmetry violating fermion correlators such as \( \langle \bar{\psi} \psi \rangle \) as order parameters to investigate this issue. Using inequalities similar to those of Vafa-Witten and Weingarten, we show that SU(2) gauge theories with Higgs and fermion fields in the fundamental representation exhibit chiral symmetry breaking in the confined phase and therefore do not lead to massless composite fermions. We discuss the implications for the Abbott-Farhi strongly interacting standard model.

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1 Complementarity and all that

Certain gauge theories with scalars in the fundamental representation can be shown to exhibit a remarkable property known as complementarity \([1, 2, 3]\). Complementarity means that the Higgs phase (large vacuum expectation value \(v\), small gauge coupling \(g\)) and confinement phase (small \(v\), large \(g\)) are not separated by a phase boundary. (Here both \(g = g(\Lambda)\) and \(v\) are defined in terms of some lattice spacing \(\Lambda^{-1}\).) The result, proved by Fradkin and Shenker \([2]\) using results of Osterwalder and Seiler \([1]\) (see also Banks and Rabinovici \([4]\) for a similar result for \(U(1)\) theories), consists of demonstrating that in a lattice formulation of the theory all correlators (ie free energy, n-point Greens functions) are analytic functions of \(g\) and \(v\) in a connected region which contains both the Higgs and confinement phases. (See figure 1 for a typical phase diagram.) Therefore, quantities such as the free energy of the theory vary smoothly without discontinuity as we interpolate between the two regions.

The rigorous demonstration of complementarity coincided with observations by ‘t Hooft \([5]\) and Susskind (unpublished) that there exists a strong similarity between the spectrum of states in the standard Higgs picture and the confined picture of a gauge theory with scalars in the fundamental. This led ‘t Hooft to remark that the question of confinement in this class of models could only be answered dynamically - there being no fundamental difference between the confining and spontaneously broken phases. (Indeed, these remarks apply equally to QCD, despite its lack of fundamental colored scalars, because of composite fields which can be formed out of glue and fermions.)

In this letter we wish to examine complementarity in gauge-Higgs models when chiral fermions are included. We will demonstrate in the \(SU(2)\) case that the addition of chiral fermions is capable of drastically altering the phase diagram of the theory. The models we study exhibit a phase transition associated with chiral symmetry breaking (\(\chi_{SB}\)) as we move from the broken to confined phase. The above result was established previously by I.-H. Lee and R. E. Shrock \([6]\) using analytical and numerical techniques on the lattice. Our analysis will be in the continuum, which makes it less rigorous from the viewpoint of constructive quantum field theory but perhaps easier to understand to theorists who work in the continuum.

It is straightforward to argue that addition of chiral fermions to a purely bosonic theory can lead to a violation of complementarity. One has merely to consider the ‘t Hooft anomaly matching conditions \([3]\), which are necessary but not sufficient conditions for the existence of massless composite fermions. If the anomalies resulting from the fundamental fermion triangle graphs do not match those of the (putative) massless composite fermions, one can immediately conclude that there are no massless composites and chiral symmetries are broken. The effects of the anomaly are then reproduced by the Goldstone modes via the Wess-Zumino term \([7]\) in the chiral lagrangian. Here we will examine a more interesting class
of models, where the 't Hooft matching conditions are satisfied by the composite fermions, and therefore dynamical information is necessary to determine the status of the chiral symmetries. There has been much interest in models of this type in which the known fermions of the standard model are massless bound states of more fundamental preons. Dimopoulos, Raby and Susskind gave a physically motivated construction involving tumbling (via fermionic condensates) of chiral gauge theories which always yields solutions to the 't Hooft conditions.

More information is required to conclude that models of the above sort actually yield massless composites. The only cases this author is aware of where the existence of massless composites can be demonstrated is in models where the large-N approximation (see Eichten et. al.) can be used to show the absence of goldstone bosons at leading order in $1/N$. Here we will prove that in a certain class of SU(2) gauge models, which satisfy both the bosonic complementarity conditions and 't Hooft’s matching conditions, chiral symmetry is indeed broken and no massless composites are formed. The essence of the argument is that since representations of SU(2) are real, the model can be rewritten as a vectorlike gauge theory. Weingarten and Vafa and Witten have shown that in vectorlike theories rigorous inequalities apply to the correlators of certain conserved currents. Using a similar analysis, we show that the mass of a composite fermion formed of a scalar and a fundamental fermion is nonzero in the chiral limit of the model. This is sufficient to rule out the massless composite realization of 't Hooft’s conditions and therefore implies the existence of Goldstone bosons and $\chi_{SB}$. The above result stands in contradiction to one of the key dynamical assumptions of the Abbott-Farhi strongly interacting standard model, which relies on the existence of (nearly) massless composites. In the following section we will present our argument for $\chi_{SB}$ in certain SU(2) theories, and discuss the implications for the phase diagrams of those theories. In the final section we will discuss the implications for the Abbott-Farhi model and preonic models.

2 Fermions and chiral symmetries

Consider an SU(N) gauge theory with (N-1) Higgs bosons in the fundamental representation (sufficient to completely break the gauge symmetry). It can be rigorously shown using lattice methods that this theory exhibits complementarity and exhibits a phase diagram similar to the one in figure 1. Now consider adding $N_f$ massless, chiral fermions $\psi_i$ to the theory. At the classical level there is an exact $SU(N_f)$ chiral symmetry associated with...
the $N_f$ fermions. In what follows we will use chiral symmetry violating correlators such as 
$\langle \bar{\psi}\psi \rangle \equiv \langle \bar{\psi}^\alpha_i \psi_{i\alpha} \rangle$ as order parameters to investigate the phase diagram for this theory. Here 
$\alpha$ is an SU(N) index and the flavor index $i$ is left arbitrary. A nonzero value of $\langle \bar{\psi}\psi \rangle$ for any 
value of $i$ will be sufficient to show a violation of complementarity.

In the perturbative Higgs regime ($g$ small, $v$ large) these symmetries remain unbroken by 
quantum effects. We can argue this result as follows: suppose the weak coupling effects are 
sufficient to break some of the chiral symmetries. Then by the Goldstone theorem there must 
exist massless composite Goldstone bosons, formed from the massless fermions. However the 
binding energy of the composite must be sufficient to cancel the positive kinetic energy of 
the two fermions confined to a region of the size of the Goldstone boson. Since in weak 
coupling one expects the binding energy to be proportional to the fermion mass $m_f$, this 
is impossible if the size of the Goldstone boson is to remain finite in the zero mass limit. 
(A more rigorous, lattice argument for the absence of spontaneous symmetry breaking at 
arbitrarily weak coupling has been given by Lee and Shrock. See the early papers in [6].)

The above argument holds for sufficiently small coupling $g$. Therefore there must exist a 
small patch in the upper left hand corner of figure 1, in which there is no $\chi_{SB}$ and the order 
parameter $\langle \bar{\psi}\psi \rangle$ is exactly zero. However, by analyticity\footnote{Technically, to apply analyticity we must verify that the correlator still vanishes when the parameters $g$ and $v$ are given infinitesimal imaginary parts. We will assume this is the case. The lattice proofs of analyticity of course still apply when $g$ and $v$ have small imaginary parts.}, the vanishing of this correlator 
can be extended throughout the entire region where complementarity applies. In particular, 
if figure 1 truly represents the phase diagram of the theory we can conclude that $\chi_{SB}$ does 
not occur in the confined phase. If, on the other hand, we can demonstrate that chiral 
symmetries are broken in the confined phase it will imply the existence of a phase boundary 
between the Higgs and confinement regions, and a violation of complementarity.

We will now proceed to show that chiral symmetries are indeed broken in the confined 
phase of the above theory when $N = 2$. (We continue to assume as above that there are 
no Yukawa couplings between the scalar and fermions and no explicit $\chi_{SB}$.) This is a very 
plausible result for SU(2) gauge theories for the following reason: because representations of 
SU(2) are real, it is always possible to rewrite a $N_f$ flavor SU(2) theory as a vectorlike $N_f/2$ 
flavor theory. ($N_f$ must be even to guarantee vanishing of Witten’s global SU(2) anomaly \[11\]. ) The latter theory, in the absence of fundamental scalars, is merely two color QCD 
which certainly breaks its chiral symmetries. Therefore, unless the presence of fundamental 
scalars somehow dramatically alters the dynamics of the theory, we expect the same to 
be true here. In particular, it is clear that if the scalar mass is taken to infinity, thereby 
decoupling it from the low energy dynamics, $\chi_{SB}$ must occur.

We will show that $\chi_{SB}$ occurs for a large range of values of the scalar mass. Our strategy
is to prove that composites with interpolating fields given by

\[ Q^i \equiv \phi^{*\alpha}\psi^i_{\alpha} \]
\[ \tilde{Q}^i \equiv \phi_{\alpha}\epsilon^{\alpha\beta}\psi^i_{\beta} \]

are not massless as long as the scalar mass is nonzero. In the confined phase of the theory there are only two candidates for matching the anomalies of the fundamental fermions \( \psi^i \): the SU(2) singlet composites \( Q^i, \tilde{Q}^i \) and the Goldstone bosons \( \pi^{ij} \sim \bar{\psi}^i\psi^j \). If \( Q^i, \tilde{Q}^i \) are not massless, the matching conditions must be satisfied by the Goldstone bosons and therefore \( \chi_{SB} \) must occur.

We first rewrite the model in a vectorlike manner. Define

\[ \chi^i = (\psi^{i+N_f/2})^c = i\gamma_2(\psi^{i+N_f/2})^* \]

Note that the \( \chi \) fields have the opposite chirality of the \( \psi \) fields, but are still doublets. The theory is clearly vectorlike as we can now add gauge invariant mass terms to the Lagrangian, pairing \( \chi^i \) with \( \psi^i \).

Now let \( \Psi^i = \psi^i + \chi^i \). With the addition to the Lagrangian of mass terms \( m_i\bar{\Psi}^i\Psi^i \) our theory is now simply \( N_f/2 \) flavor, two color QCD with massive dirac fermions and an extra colored scalar. The fermion masses and the “vectorization” of the model are necessary for technical reasons in order to apply certain rigorous results similar to those first derived by Vafa and Witten [12]. At the end of the calculation we will take \( m_i \to 0 \) to reduce it to the original, with classical chiral symmetries intact.

Let us temporarily redefine the interpolating fields \( Q^i, \tilde{Q}^i \) so that they each contain a Dirac fermion \( \Psi^i \) rather than a Weyl fermion as previously defined. The index \( i \) now runs from 1 to \( N_f/2 \). If massless composite fermions are to exist in the limit \( m_i \to 0 \) corresponding to the old \( Q^i, \tilde{Q}^i \), then the new \( Q^i, \tilde{Q}^i \) must also be massless in that limit. We will now demonstrate that this is not the case.

Consider the Euclidean propagator for the Q field: (From here on we selectively suppress flavor and color indices for simplicity. Q refers to either of \( Q^i, \tilde{Q}^i \).)

\[ \langle T(\bar{Q}(x)Q(y)) \rangle = Z^{-1}\int DA D\phi D\bar{\Psi} D\Psi \exp(-S_E[A,\phi,\Psi]) \bar{Q}(x)Q(y), \]

where \( Z^{-1} \) is the standard normalization factor and the Euclidean action is

\[ S_E[A,\phi,\Psi] = \int d^4x \frac{1}{2g^2}Tr FF + |D\phi|^2 + M^2\phi^2 + \sum_i \bar{\Psi}(\bar{\Psi} + m_i)\Psi. \]

Figure 2 gives a pictorial description of the expectation value in equation 4. The normalization factor \( Z^{-1} \) divides out all vacuum bubbles, so we are left with \( \phi \) and \( \Psi \) propagators.

\[ \section{In other words, we will prove that the N = 2 theories satisfy a “persistence of mass” condition [10].} \]
summed over all possible gauge backgrounds, with scalar and fermion loops included. Conservation of flavor and scalar number (for the moment we neglect scalar self-interactions) prevents either the $\Psi$ or $\phi$ lines from terminating except on an insertion of $Q$.

We can rewrite (4) in the following manner by integrating out the scalar and fermion fields:

$$\langle T(Q(x)Q(y)) \rangle = Z^{-1} \int D\mu \ (D^{2} + M^{2})^{-1}_{A,xy} \ (\bar{\phi} + m)^{-1}_{A,xy}$$

$$D\mu \equiv DA \exp(- \int d^{4}x \frac{1}{2g^{2}} Tr FF) \ det^{-1/2}(D^{2} + M^{2}) \prod_{i} \ det(\bar{\phi} + m_{i}).$$

Here the determinants and propagators are evaluated in an arbitrary gauge background $A_{\mu}$, which is then integrated over. The key point is that the measure of integration $D\mu$ can be shown to be positive definite. This is because both the scalar and gauge field Euclidean actions are real and the fermion determinant is always real and positive in a vectorlike theory $[12]$. (It is also necessary to choose the topological $\theta$ term to be zero.)

Since the measure is positive definite, any $A_{\mu}$ independent bound that can be placed on the integrand will yield a bound on the $Q$ propagator. The above integrand consists of the product of the scalar and fermion propagators in arbitrary gauge background. A great deal is known about the behavior of such propagators at large separations $|x - y|$. For example, Kato’s inequality $[15]$ asserts that $|(D^{2} + M^{2})^{-1}_{A,xy}| \leq |(D^{2} + M^{2})^{-1}_{A=0,xy}|$. That is, the scalar propagator in an arbitrary gauge background falls off faster than its free counterpart (ie in zero gauge field background). A similar result, involving for technical reasons a smeared fermion propagator also applies. Here we will sketch the arguments from $[12]$ which apply to a smeared propagator of either scalar or fermionic type. Consider the smeared propagator

$$(\bar{\phi} + m)^{-1}_{A,\alpha\beta} \equiv \langle \alpha |(\bar{\phi} + m)^{-1}_{A} |\beta \rangle,$$

where $|\alpha\rangle, |\beta\rangle$ are localized wave packet states, rather than position eigenstates $|x\rangle, |y\rangle$. By a wave packet state we mean that $\Psi(x)|\alpha\rangle = \phi(x)|\alpha\rangle = 0$ outside a compact region $\alpha$ centered at $x$ with size much smaller than $|x - y|$. 

We can bound the smeared propagator by the following trick:

$$\langle \alpha |(\bar{\phi} + m)^{-1}_{A} |\beta \rangle = \int_{0}^{\infty} dt \ \langle \alpha |\exp[-(\bar{\phi} + m)_{A}t] |\beta \rangle$$

$$= \int_{0}^{\infty} dt \ e^{-mt} \ \langle \alpha |\exp[-i(\bar{\phi}h)t] |\beta \rangle.$$  

The last expression has the form of a quantum mechanical transition amplitude in a (4+1) dimensional theory with Dirac Hamiltonian $H = -i\bar{\phi}$. By causality, we have $\langle \alpha |e^{-iHt}|\beta \rangle = 0$ for $0 \leq t < |x - y|$. Therefore

$$\langle \alpha |(\bar{\phi} + m)^{-1}_{A} |\beta \rangle = \int_{t=|x-y|}^{\infty} dt \ e^{-mt} \ \langle \alpha |e^{-iHt}|\beta \rangle,$$

$$|\langle \alpha |(\bar{\phi} + m)^{-1}_{A} |\beta \rangle| \leq \frac{1}{m} e^{-m|x-y|} \ |\alpha||\beta|.$$
where we have used (4+1) unitarity and the Schwarz inequality to obtain the last expression, and $|\alpha|, |\beta|$ are the norms of the states $|\alpha\rangle, |\beta\rangle$. A similar result applies in the scalar case. Note that for position eigenstates the corresponding norms are infinite and hence do not yield a useful bound.

Putting the above results together, we have:

$$|\langle T(\bar{Q}(x)Q(y))_s \rangle| \leq \frac{C}{m} \exp\left(- (M + m) |x - y| \right), \quad (13)$$

where the subscript “s” means smeared and $C$ is a numerical constant. This bound precludes the existence of a massless bound state $Q$ in the limit of zero fermion mass and massive scalar ($m \to 0, M$ fixed). Note that this type of bound does not preclude the existence of a massless bound state consisting of two massless fermions, as in that case as $m \to 0$ the bound disappears. This is crucial, as we expect to find massless composite Goldstone bosons $\pi^{ij} \sim \bar{\psi}^i \psi^j$ in the chiral limit.

An important technical point is that the above arguments require a cutoff, $\Lambda$, for the theory because the masses $M, m$ appearing in the various inequalities are actually bare masses, $M_0, m_0$. We can either imagine that this entire analysis has been conducted on the lattice (see [13]), or that a suitable, gauge invariant regularization such as Pauli-Villars has been carried out. Because of this technical requirement, there is a problem with the inequality (13) which stems from the unnaturalness of scalar models. The problem is that as the cutoff $\Lambda$ is taken to $\infty$, the bare scalar mass required to yield a fixed physical scalar mass becomes negative. This is easy to see in perturbation theory, as the one loop correction to the bare mass has the form

$$M^2_{\text{physical}} = M^2_0 + \frac{\lambda}{32\pi^2} \Lambda^2 + \frac{y^2}{32\pi^2} \Lambda^2, \quad (14)$$

where $\lambda, g$ are respectively the $\phi^4$ and gauge couplings and we have computed the latter in Landau gauge. We have suppressed subleading logarithmic corrections. Note that if one wishes to take the cutoff arbitrarily large with respect to the physical scalar mass, an arbitrarily large and negative bare mass is required. Therefore, as $\Lambda \to \infty$ our bound (13) becomes useless.

It is possible to choose bare parameters such that (13) holds with positive $M^2_0$ if the cutoff is not chosen too large. (We also require $\lambda(\Lambda) = 0$ in order to perform the scalar functional integration in Eq. (4).) One would like to keep the cutoff large compared to the scale $\Lambda_2$ at which $SU(2)$ becomes strongly interacting, while keeping $M_{\text{physical}} \simeq \Lambda_2$.

£This choice of $\lambda(\Lambda)$ does not imply the theory is unbounded from below. If one computes the renormalization group improved effective potential, it is easy to see that a positive renormalized mass squared compensates for $\lambda(\mu)$ being driven (logarithmically) negative in the infrared. There is also no vacuum expectation induced for the scalar unless the ratio $\Lambda/M_{\text{physical}}$ is taken very large.
Whether this is possible depends on the evolution of the gauge coupling constant between $\Lambda$ and $\Lambda_2$. This in turn depends on the number of fermion flavors in the model. If we fix $M_{\text{physical}} = \Lambda_2$, for $N_f = 2$ we get $\Lambda/\Lambda_2 \simeq 3.5$ while for $N_f = 12$ (the Abbott-Farhi case) we get $\Lambda/\Lambda_2 \simeq 2$. (Note that for $N_f = 2$ the axial chiral symmetry is anomalous, and therefore already explicitly violated by quantum effects.) Larger ratios of $\Lambda/\Lambda_2$ are possible if we allow $M_{\text{physical}} > \Lambda_2$. This verifies our intuition that very heavy scalars should decouple from the strong $SU(2)$ dynamics, leaving behind a theory with broken chiral symmetries. However, the Abbott-Farhi model assumes a phase transition between $\chi_{SB}$ and no $\chi_{SB}$ as the mass of the scalar is lowered, and hence we are more interested in what happens when $M_{\text{physical}} \simeq \Lambda_2$.

Should it concern us that the cutoff must be taken so low? If we think of the renormalization group in Wilson’s language we know that we can start with a continuum theory (or one with arbitrarily large cutoff, momentarily ignoring triviality problems of scalar theories) and relate it to an effective theory at scale $\mu$ by systematically integrating out degrees of freedom. The information from the high momentum modes will be contained in the running coupling constants $g_i(\mu)$ and the higher dimension operators $O_i(\mu)$ induced by this procedure. The theory defined with cutoff $\Lambda$, “bare” couplings equal to $g_i(\Lambda)$ and additional non-renormalizable interactions given by $O_i(\Lambda)$ is then completely equivalent to the original continuum theory.

In deriving our inequalities we assumed that there were no non-renormalizable operators in the bare theory. Perhaps this is justified because we are attempting to determine whether an exactly massless state exists in a certain channel. We are therefore interested in physics at very low momentum scales, and at arbitrarily long distances, where higher-dimension operators should be irrelevant. One might expect that the long distance behavior of two theories differing by some set of such irrelevant operators $O_i$ should be the same. (By assumption the operators $O_i$ respect the fermion chiral symmetries, otherwise complementarity is already violated.) However, a loophole in this line of reasoning is that the higher-dimension operators may actually shift the ground state of the theory from chiral symmetry preserving to breaking. This is possible in principle if spontaneous $\chi_{SB}$ is due only to gauge dynamics of momentum scale $k \simeq \Lambda_2$ rather than $k << \Lambda_2$. In that case the operators $O_i$ are only suppressed by powers of $\Lambda_2/\Lambda$ to some power, and may be important if this ratio is not large. Because of this possibility, our results can probably only be rigorously applied for $M_{\text{physical}} > (\text{few}) \Lambda_2$. This leaves a small window for the Abbott-Farhi model with a very light Higgs scalar. However, the lattice work of Lee and Shrock [6] is valid for all values of the scalar mass, and hence is sufficient to close this window.
3 Further implications: the Abbott-Farhi model

The existence of a chiral phase boundary in the above class of models has strong implications for the Abbott-Farhi model. Abbott and Farhi [16] formulated a strongly coupled version of the electroweak theory, in which the the Higgs vacuum expectation value is small and the SU(2) coupling large. This model is successful in roughly reproducing the observed spectrum of the electroweak theory, generating W and Z bosons as bound states of the scalar doublet field, although its current phenomenological viability is subject to certain unproven dynamical assumptions (see last reference in [16]).

One of the key dynamical assumptions is that chiral symmetries remain unbroken in the confined phase and that in the limit of zero fermion-Higgs Yukawa couplings there are massless, left handed fermionic bound states \( Q^i_L \). Nonzero Yukawa couplings \( \lambda_i \) provide mass terms which marry the \( Q^i_L \) to SU(2) singlet right handed fermions \( \psi^i_R \) yielding Dirac fermions of mass \( \sim \lambda_i \Lambda_{SU(2)} \), where \( \Lambda_{SU(2)} \approx 256 \text{ GeV} \) is approximately the weak scale.

Since the electroweak theory is intrinsically chiral, one might wonder how the results of the previous section can be applicable. The answer is that, perhaps surprisingly, the electroweak theory is actually vectorlike in the limit where we ignore hypercharge and color. In the absence of hypercharge, color and Yukawa couplings the electroweak theory belongs to the class of models studied in section 2. (It is easy to show that the electroweak theory has no physical \( \theta \) angle.) Therefore, to the extent to which those couplings can be treated as perturbations the results of the previous section should apply to the Abbott-Farhi model. Indeed, all of the relevant coupling constants are small at scale \( \Lambda_2 \), with the possible exception of the top Yukawa coupling. This suggests that the masses of the fermions in that theory do not resemble those of their perturbative electroweak counterparts. We expect the right handed fermions to remain nearly massless, with masses of order \( (\lambda v)^2/M_L \) induced by their interactions with the \( Q^i_L \). We also expect a plethora of relatively light pseudo-Goldstone bosons which are bound states of left handed leptons and quarks. Finally, the condensates which form will spontaneously break color and hypercharge.

Because our results are specific to gauge groups which have (pseudo)real representations, it is not always possible to apply them to more complicated composite models [8]. However, it seems possible to this author that many models of the sort first constructed by Dimopoulos et. al. [3] may indeed undergo \( \chi_{SB} \) rather than produce massless composite fermions in the confined phase. Additional dynamical information is required to decide the issue.
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**Figure 1:** Typical phase diagram for theories exhibiting complementarity. All physical quantities are analytic functions of $\nu$ and $g$ in the shaded region.

**Figure 2:** Q field propagator in arbitrary gauge field background.