Deeply bound $\Xi$ tribaryon

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Abstract

We have used realistic local interactions based on the recent update of the strangeness $-2$ Nijmegen ESC08c potential to calculate the bound state problem of the $\Xi NN$ system in the $(I)JP = (\frac{1}{2})^{3+}$ state. We found that this system presents a deeply bound state lying 13.5 MeV below the $\Xi d$ threshold. Since in lowest order, pure $S$–wave configuration, this system can not decay into the open $\Lambda NN$ channel, its decay width is expected to be very small. We have also recalculated the $(I)JP = (\frac{3}{2})^{1+}$ state and we have compared with results of quark-model based potentials.

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I. INTRODUCTION

The interaction between baryons in the strangeness $-2$ sector has been in the focus of interest for many years. The new hybrid experiment $E07$ recently approved at J–PARC is expected to record of the order of $10^4 \Xi^-$ stopping events [1], one order of magnitude larger than the previous $E373$ experiment. Under the development of an overall scanning method a first output of this ambitious project was already obtained, the so–called KISO event, the first clear evidence of a deeply bound state of $\Xi^- - ^{14}\text{N}$ [2]. Together with other indications of certain emulsion data, these findings suggest that the average $\Xi N$ interaction may be attractive [3–5]. In particular, the ESC08c Nijmegen potential for baryon–baryon channels with total strangeness $-2$ predicted an important attraction in the isospin 1 $\Xi N$ interaction, with a bound state of 8.3 MeV in the $\Xi N$ channel with isospin-spin quantum numbers, $(i, j) = (1, 1)$ [3]. The recent update of the ESC08c Nijmegen potential to take into account the new experimental information of Ref. [2] concludes the existence of a bound state in the $(i, j) = (1, 1) \Xi N$ channel with a binding energy of 1.56 MeV [4]. It is worth to mention that the latest, although still preliminary, results from lattice QCD simulations of the strangeness $-2$ baryon–baryon interactions also suggest an overall attractive $\Xi N$ interaction [6].

In a recent work [7] we have used the results of the new ESC08c Nijmegen strangeness $-2$ baryon–baryon interaction [4] to analyze the possible existence of $\Xi NN$ bound states in isospin $3/2$ channels. Our main motivation was the decoupling of $I = 3/2 \Xi NN$ channels from the lowest $\Lambda \Lambda N$ channel, due to isospin conservation, what would make a possible bound state stable. In particular, for the case of the $J^P = \frac{1}{2}^+$ state we showed that the existence of the deuteron–like $\Xi N (i, j^p) = (1, 1^+) D^*$ bound state predicted by the ESC08c Nijmegen model is sufficient to guarantee the existence of a $\Xi NN J^P = \frac{1}{2}^+$ bound state with a binding energy of about 2.5 MeV [7]. Besides, for the $J^P = \frac{3}{2}^+$ channel we pointed out that a bound state can not exist as a consequence of the Pauli principle, since this would require two nucleons in a state with total spin 1 and total isospin 1, which is forbidden in S–wave. The $(I)J^P = (\frac{1}{2})\frac{1}{2}^+$ state has been recently analyzed making use of the new ESC08 Nijmegen baryon–baryon interactions for the systems with strangeness 0, −1, and −2, concluding the existence of a tribaryon with mass of 3194 MeV, just below the $\Xi d$ threshold [8].

In this paper we study the existence of deeply bound states in other partial waves, as a
possible guide to future experiments at J–PARC. For this purpose we will construct realistic local potentials of the ESC08c Nijmegen S–wave ΞN (i, j) = (0, 1) and (1, 1) interactions, which contribute to the ΞNN (I)JPG = (1/2)3+ state. We will also construct a realistic local potential of the ΞN (i, j) = (1, 0) channel in order to compare with the (I)JP = (3/2)1+ ΞNN state which was already studied in our previous work [7], but however considering only the ΞN (i, j) = (1, 1) channel.

In addition to the ESC08c Nijmegen model of the ΞN interaction we will also consider the chiral constituent quark model of Ref. [9]. The reason behind this choice is that these two models present at least one ΞN bound state. In the case of the ESC08c model they have incorporated in their analysis the Nagara event [10] and the KISO event [2]. As mentioned above, preliminary results from lattice QCD also suggest an overall attractive ΞN interaction [6]. There are other models for the ΞN interaction, like the hybrid quark–model based analysis of Ref. [11], the effective field theory approach of Ref. [12], or even some of the earlier models of the Nijmegen group [13] that do not present ΞN bound states and, in general, the interactions are weakly attractive or repulsive. Thus, one does expect that these models will give rise to ΞNN bound states.

The paper is organized as follows. We will use Sec. II for describing all technical details to solve the ΞNN bound–state Faddeev equations. In Sec. III we will construct the two–body amplitudes needed for the solution of the bound state three–body problem. Our results will be presented and discussed in Sec. IV. We will also compare with results from quark-model based potentials. Finally, in Sec. V we summarize our main conclusions.

II. THE ΞNN BOUND-STATE FADDEEV EQUATIONS

We will restrict ourselves to the configurations where all three particles are in S–wave states and assume that Ξ is particle 1 and the two nucleons are particles 2 and 3, so that the Faddeev equations for the bound–state problem in the case of three baryons with total
isospin $I$ and total spin $J$ are,

$$T^{i;j}_{i;I} (p_i q_i) = \sum_{j \neq i} \sum_{i,j} h^{i;i;ij}_{ij;1J} \left( \frac{1}{2} \int_{0}^{\infty} q_j^2 dq_j \right) \times \int_{-1}^{1} d\cos \theta t_{i;i;j} (p_i, q_i; E - q_i^2/2\nu_i) \times \frac{1}{E - p_j^2/2\mu_j - q_j^2/2\nu_j} T^{i;j}_{j;1J} (p_j q_j),$$  \hspace{1cm} (1)

where $t_{1;i;1j}$ stands for the two–body $NN$ amplitudes with isospin $i_1$ and spin $j_1$, and $t_{2;i;2j}$ ($t_{3;i;3j}$) for the $\Xi N$ amplitudes with isospin $i_2$ ($i_3$) and spin $j_2$ ($j_3$). $p_i$ is the momentum of the pair $jk$ (with $ijk$ an even permutation of 123) and $q_i$ the momentum of particle $i$ with respect to the pair $jk$. $\mu_i$ and $\nu_i$ are the corresponding reduced masses,

$$\mu_i = \frac{m_j m_k}{m_j + m_k},$$
$$\nu_i = \frac{m_i (m_j + m_k)}{m_i + m_j + m_k},$$  \hspace{1cm} (2)

and the momenta $p'_i$ and $p_j$ in Eq. (1) are given by,

$$p'_i = \sqrt{q_j^2 + \frac{\mu_i^2}{m_k} q_i^2 + 2 \frac{H_i}{m_k} q_i q_j \cos \theta},$$
$$p_j = \sqrt{q_i^2 + \frac{\mu_j^2}{m_k} q_j^2 + 2 \frac{H_j}{m_k} q_i q_j \cos \theta}.$$  \hspace{1cm} (3)

$h^{i;i;ij}_{ij;1J}$ are the spin–isospin coefficients,

$$h^{i;i;ij}_{ij;1J} = (-)^{i_j + \tau_j - I} \sqrt{(2i_i + 1)(2i_j + 1)} W(\tau_j \tau_k I \tau_i; i_i i_j) \times (-)^{j_j + \sigma_j - J} \sqrt{(2j_i + 1)(2j_j + 1)} W(\sigma_j \sigma_k J \sigma_i; j_i j_j),$$  \hspace{1cm} (4)

where $W$ is the Racah coefficient and $\tau_i$, $\sigma_i$, and $I$ ($\sigma_i$, $j_i$, and $J$) are the isospins (spins) of particle $i$, of the pair $jk$, and of the three–body system.

Since the variable $p_i$ in Eq. (1) runs from 0 to $\infty$, it is convenient to make the transformation

$$x_i = \frac{p_i - b}{p_i + b},$$  \hspace{1cm} (5)

where the new variable $x_i$ runs from $-1$ to 1 and $b$ is a scale parameter that has no effect.
on the solution. With this transformation Eq. (11) takes the form,

$$T_{i;IJ}^{i;j}(x_i q_i) = \sum_{j \neq i} \sum_{i,j} h_{ij;ij}^{i;j} \frac{1}{2} \int_0^\infty q_j^2 dq_j \times \int_1^1 d\cos \theta \ t_{i;i,j}(x_i, x'_i; E - q_i^2/2\nu_i) \times \frac{1}{E - p_j^2/2\mu_j - q_j^2/2\nu_j} \ T_{j;/IJ}^{i;j}(x_j q_j).$$

(6)

Since in the amplitude $t_{i;i,j}(x_i, x'_i; e)$ the variables $x_i$ and $x'_i$ run from $-1$ to $1$, one can expand this amplitude in terms of Legendre polynomials as,

$$t_{i;i,j}(x_i, x'_i; e) = \sum_{nr} P_n(x_i) \tau_{i;i,j}^{nr}(e) P_r(x'_i),$$

(7)

where the expansion coefficients are given by,

$$\tau_{i;i,j}^{nr}(e) = \frac{2n + 1}{2} \frac{2r + 1}{2} \int_{-1}^1 dx_i \int_{-1}^1 dx'_i \ P_n(x_i) t_{i;i,j}(x_i, x'_i; e) P_r(x'_i).$$

(8)

Applying expansion (7) in Eq. (6) one gets,

$$T_{i;IJ}^{i;j}(x_i q_i) = \sum_n P_n(x_i) T_{i;IJ}^{ni;ji}(q_i),$$

(9)

where $T_{i;IJ}^{ni;ji}(q_i)$ satisfies the one–dimensional integral equation,

$$T_{i;IJ}^{ni;ji}(q_i) = \sum_{j \neq i} \sum_{m;i,j} \int_0^\infty dq_j A_{ij;IJ}^{ni;ji;mj;j}(q_i, q_j; E) T_{j;IJ}^{mj;j}(q_j),$$

(10)

with

$$A_{ij;IJ}^{ni;ji;mj;j}(q_i, q_j; E) = h_{ij;IJ}^{i;j} \sum_r \tau_{i;i,j}^{nr}(e - q_i^2/2\nu_i) q_j^2/2 \times \int_{-1}^1 d\cos \theta \ P_r(x'_i) P_m(x_j) \frac{P_r(x'_i) P_m(x_j)}{E - p_j^2/2\mu_j - q_j^2/2\nu_j}.$$  

(11)

The three amplitudes $T_{1;IJ}^{ri;ji}(q_1)$, $T_{2;IJ}^{mj;j2}(q_2)$, and $T_{3;IJ}^{mi;ji}(q_3)$ in Eq. (10) are coupled together. The number of coupled equations can be reduced, however, since two of the particles are identical. The reduction procedure for the case where one has two identical fermions has been described before [14, 15] and will not be repeated here. With the assumption that particle 1 is the Ξ and particles 2 and 3 are the nucleons, only the amplitudes $T_{1;IJ}^{ri;ji}(q_1)$ and $T_{2;IJ}^{mj;j2}(q_2)$ are independent from each other and they satisfy the coupled integral equations,

$$T_{1;IJ}^{ri;ji}(q_1) = 2 \sum_{m;i,j} \int_0^\infty dq_3 A_{13;IJ}^{ri;ji;mi;j2}(q_1, q_3; E) T_{2;IJ}^{mi;j2}(q_3),$$

(12)
\[ T_{2,ij}^{n_1j_2}(q_2) = \sum_{m_{i3j_3}} g \int_0^{\infty} dq_3 A_{23,ij}^{n_1j_2;m_{i3j_3}}(q_2, q_3; E) T_{2,ij}^{m_{i3j_3}}(q_3) + \sum_{r_{i1j_1}} \int_0^{\infty} dq_1 A_{31,ij}^{n_1j_2;r_{i1j_1}}(q_2, q_1; E) T_{1,ij}^{r_{i1j_1}}(q_1), \] 

with the identical–particle factor

\[ g = (-1)^{1+\sigma_1+\sigma_3-j_2+\tau_3-i_2}, \] 

where \( \sigma_1 \) (\( \tau_1 \)) and \( \sigma_3 \) (\( \tau_3 \)) stand for the spin (isospin) of the \( \Xi \) and the \( N \), respectively.

Substitution of Eq. (12) into Eq. (13) yields an equation with only the amplitude \( T_2 \),

\[ T_{2,ij}^{n_1j_2}(q_2) = \sum_{m_{i3j_3}} \int_0^{\infty} dq_3 K_{ij}^{n_1j_2;m_{i3j_3}}(q_2, q_3; E) T_{2,ij}^{m_{i3j_3}}(q_3), \] 

where

\[ K_{ij}^{n_1j_2;m_{i3j_3}}(q_2, q_3; E) = g A_{23,ij}^{n_1j_2;m_{i3j_3}}(q_2, q_3; E) + 2 \sum_{r_{i1j_1}} \int_0^{\infty} dq_1 A_{31,ij}^{n_1j_2;r_{i1j_1}}(q_2, q_1; E) \times A_{13,ij}^{r_{i1j_1};m_{i3j_3}}(q_1, q_3; E). \] 

III. TWO–BODY AMPLITUDES

We have constructed the two–body amplitudes by solving the Lippmann–Schwinger equation of each \( (i, j) \) channel,

\[ t_{ij}(p, p'; e) = V_{ij}(p, p') + \int_0^{\infty} p''^2 dp'' V_{ij}(p, p'') \times \frac{1}{e - p''^2/2\mu} t_{ij}(p'', p'; e), \] 

where

\[ V_{ij}(p, p') = \frac{2}{\pi} \int_0^{\infty} r^2 dr \ j_0(pr) V_{ij}(r) j_0(p'r), \] 

and the two–body potentials consist of an attractive and a repulsive Yukawa term, i.e.,

\[ V_{ij}(r) = -A e^{-\mu r_p} r + B e^{-\mu r_p} r, \] 

where the parameters of the three \( \Xi N \) channels were obtained by fitting the low–energy data of each channel of Ref. [4]. The parameters of these models are given in Table I. In the case of the \( NN \) (0, 1) and (1, 0) channels we used the Malfliet–Tjon models with the parameters given in Ref. [16].
TABLE I: Low–energy parameters of the ΞN channels of the ESC08c Nijmegen interactions of Ref. [4] and the parameters of the corresponding local potentials given by Eq. (19).

| (i, j) | a(fm)  | r₀(fm) | A(MeV fm) | µ_A(fm⁻¹) | B(MeV fm) | µ_B(fm⁻¹) |
|--------|--------|--------|-----------|-----------|-----------|-----------|
| (0, 1) | -5.357 | 1.434  | 377       | 2.68      | 980       | 6.61      |
| (1, 0) | 0.579  | -2.521 | 290       | 3.05      | 155       | 1.60      |
| (1, 1) | 4.911  | 0.527  | 568       | 4.56      | 425       | 6.73      |

IV. RESULTS AND DISCUSSION

We show in Table II the results for both the (I)J^P = (1/2)3^+ and (I)J^P = (3/2)1^+ ΞNN states calculated with the ESC08c Nijmegen interactions of Table I as well as the results obtained with a quark-model based potential, the chiral constituent quark model (CCQM) of Ref. [9]. These binding energies are measured with respect to the lowest threshold which in the case of the (1/2)3^+ state is the Ξd threshold and in the case of the (3/2)1^+ state are the ND^* threshold for the ESC08c Nijmegen model (1.56 MeV below the ΞNN mass) and the N − ΞN(1,0) threshold for the CCQM model (4.8 MeV below the ΞNN mass).

As one can check, the binding energy of the (3/2)1^+ state of the ESC08c Nijmegen model is smaller than that obtained in Ref. [7] (2.50 MeV) since we have now included in addition to the ΞN (1, 1) channel also the ΞN (1, 0) channel, which is mainly repulsive. In the case of the CCQM, in Ref. [9] we had performed the calculation considering both ΞN channels, thus in perfect agreement with the present results. Let us note again that the binding energies obtained from the CCQM are much smaller than those obtained from the ESC08c Nijmegen model, since for the (I)J^P = (3/2)1^+ state the dominant channel is the (i, j) = (1, 1) NΣ subsystem, that it is almost bound with the CCQM model while it has a binding energy of 1.56 MeV with the ESC08c Nijmegen model [4].

The most interesting result of Table II is the very large binding energy of the (1/2)3^+ state predicted by the ESC08c Nijmegen potential model, which would make it easy to identify experimentally as a sharp resonance lying some 15.7 MeV below the ΞNN threshold. The ΛΛ − ΞN (i, j) = (0, 0) transition channel, which is responsible for the decay ΞNN → ΛΛN, does not contribute to the (I)J^P = (1/2)3^+ state in a pure S–wave configuration. One would need at least the spectator nucleon to be in a D wave or that the ΛΛ − ΞN transition
TABLE II: Binding energies of the $\Xi NN$ $(I)J^P = (\frac{1}{2})\frac{3}{2}^+$ and $(I)J^P = (\frac{3}{2})\frac{1}{2}^+$ states (in MeV) calculated with the interactions based in the ESC08c Nijmegen model \[4\] and with those of a chiral constituent quark model \[9\].

| Model   | $(\frac{1}{2})\frac{3}{2}^+$ | $(\frac{3}{2})\frac{1}{2}^+$ |
|---------|-------------------------------|-------------------------------|
| ESC08c  | 13.54                         | 1.33                          |
| CCQM    | 1.15                          | 0.43                          |

channel be in one of the negative parity $P$-wave channels with the nucleon spectator also in a $P$-wave, so that due to the angular momentum barriers the resulting decay width is expected to be very small. We finally note that the binding energy obtained from the CCQM is larger than in the $I = 3/2$ channel, but once again much smaller than the prediction of the recent update of the ESC08c Nijmegen potential \[4, 5\] incorporating the recent experimental results of Ref. \[1\].

V. SUMMARY

Recent results in the strangeness $-2$ sector, the so-called KISO event, reported clear evidence of a deeply bound state of $\Xi^- - ^{14}N$ what could point out that the average $\Xi N$ interaction might be attractive. We have made use of the recent update of the ESC08c Nijmegen potential taking into account the recent experimental information, to study the bound state problem of the $\Xi NN$ system in the $(I)J^P = (\frac{1}{2})\frac{3}{2}^+$ state. We have found that this system has a deeply bound state that lies 13.5 MeV below the $\Xi d$ threshold. Since at lowest order, pure $S$-wave configuration, this system can not decay into the open $\Lambda \Lambda N$ channel, its decay width is expected to be very small, what would make it very easy to be identified experimentally.

The huge amount of $\Xi^-$ stopping events that will be recorded at the recently approved hybrid experiment $E07$ at J-PARC, is expected to shed light on the uncertainties of our knowledge of the baryon-baryon interaction in the strangeness $-2$ sector. Meanwhile the scarce experimental information gives rise to an ample room for speculation. The present detailed theoretical investigation of the possible existence of deeply bound states based on realistic models are basic tools to advance in the knowledge of the details of the $\Xi N$ interac-
tion. First, it could help to raise the awareness of the experimentalist that it is worthwhile to investigate few-baryon systems, specifically because for some quantum numbers such states could be stable. Secondly, it makes clear that strong and attractive \( \Xi N \) interactions, like those suggested by the ESC08c Nijmegen model, have consequences for the few-body sector and can be easily tested against future data. Observations like the ones reported in Ref. [2] are interesting. However, in this case microscopic calculations are impossible and, consequently, their interpretation will be always afflicted by large uncertainties. The identification of strangeness \(-2\) hypernuclei in coming experiments at J–PARC would contribute significantly to understand nuclear structure and baryon–baryon interactions in the strangeness \(-2\) sector.

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