Electron-polaron–electron-polaron bound states in mass-gap graphene-like planar quantum electrodynamics: s-wave bipolarons

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A Lorentz invariant version of a mass-gap graphene-like planar quantum electrodynamics, the parity-preserving $U(1) \times U(1)$ massive QED$_3$, exhibits attractive interaction in low-energy electron-polaron–electron-polaron s-wave scattering, favoring quasiparticles bound states, the s-wave bipolarons.

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I. INTRODUCTION

The seminal works by Deser, Jackiw, Templeton and Schonfeld [1] have attracted attention to the quantum electrodynamics in three space-time dimensions (QED$_3$) in view of its potentiality as theoretical foundation for quasi-planar condensed matter phenomena, such as high-$T_c$ superconductors [2], quantum Hall effect [3], topological insulators [4], topological superconductors [5] and graphene [6]. Since then, the planar quantum electrodynamics has been widely studied in many physical configurations, namely, small (perturbative) and large (non perturbative) gauge transformations, Abelian and non-Abelian gauge groups, fermions families, odd and even under parity, compact space-times, space-times with boundaries, curved space-times, discrete (lattice) space-times, external fields and finite temperatures.

The proposed issue in this work about the possibility of s-wave bipolarons emerging from the parity-preserving $U(1) \times U(1)$ massive QED$_3$ – a mass-gap graphene-like [7] planar quantum electrodynamics – is presented as follows. Initially, the model defined by its discrete and continuous symmetries is introduced and, since the interactions are nonconfining – the vector mesons, the photon and the Néel quasiparticle, are massive – the asymptotic states for the fermions (electron polarons) are established. Hereafter, in the low-energy limit, the s- and p-wave Møller (e$^-$-polaron–e$^-$-polaron) scattering amplitudes are computed and their respective interaction potentials obtained and analysed. However, from this analysis, it was found conditions on the parameters which, in spite of the p-wave scattering potential still remains repulsive, the s-wave interaction potential becomes attractive. The latter shall favour e$^-$–polaron–e$^-$–polaron bound states – provided the attractive s-wave scattering potential satisfies necessary conditions [8–11] – giving rise to the s-wave bipolarons condensates [12].

II. THE MODEL

The Lorentz invariant version of mass-gap graphene-like planar quantum electrodynamics, the parity-even $U(1) \times U(1)$ massive QED$_3$, is defined by the action:

\[ S = \int d^3x \left\{ -i \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - i \frac{1}{4} f^{\mu\nu} f_{\mu\nu} + \frac{1}{2} \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho + + \partial_\mu \bar{\psi}_\pm \sigma_\mu \psi_\pm - m(\bar{\psi}_+ \psi_- - \bar{\psi}_- \psi_+) + \right. \]
\[ - \frac{1}{2} \partial^\mu (\partial^\mu A_\mu)^2 - \frac{1}{2} \partial^\mu (\partial^\mu a_\mu)^2 \right\}, \tag{1} \]

where $\partial_\mu = (\partial_0 \pm ieA_0 \pm igf_0)\psi_\pm$, and any object $X = X^\mu \gamma^\mu$. The coupling constants $e$ and $g$ are dimensional, with mass dimension $\frac{1}{2}$, and, $m$ and $\mu$ are mass parameters with mass dimension 1. Also, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$ are the field strengths associated to the electromagnetic field ($A_\mu$) and the Néel gauge field ($a_\mu$), respectively, $\psi_+$ and $\psi_-$ are two kinds of fermions – each of them describing electron polarons (electron-phonon) and hole polarons (hole-phonon) quasiparticles – where the ± subscripts are related to their spin sign [13], and the gamma matrices are $\gamma^\mu = (\sigma_2, -i\sigma_x, i\sigma_y)$.

A. The symmetries: parity and $U(1) \times U(1)$

The CPT-even action (1) is invariant under:

1. parity symmetry $(P)$:

\[ x_\mu \rightarrow x'_\mu = (x_0, -x_1, x_2), \]
\[ \psi_\pm \rightarrow \psi_\pm' = -i\gamma^1 \psi_\pm, \quad \bar{\psi}_\pm \rightarrow \bar{\psi}_\pm' = i\bar{\psi}_\pm \gamma^1, \]
\[ A_\mu' = A_\mu, \quad a_\mu' = (A_0, -A_1, A_2), \]
\[ a_\mu'^P = a_\mu, \quad (a_0, a_1, a_2). \tag{2} \]

2. gauge $U(1)_A \times U(1)_a$ symmetry $(\delta_\xi)$:

\[ \delta_\xi \psi_\pm (x) = i(\theta(x) \pm \omega(x))\psi_\pm (x), \]

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\[
\delta_{k} \overline{\psi}_{\pm}(x) = -i[\theta(x) \pm \omega(x)]\overline{\psi}_{\pm}(x), \\
\delta_{k} A_{\mu}(x) = -\frac{1}{e} \partial_{\mu} \theta(x), \\
\delta_{k} a_{\mu}(x) = -\frac{1}{g} \partial_{\mu} \omega(x).
\]

(3)

B. The spectrum: degrees of freedom, spin, masses and charges

The free Dirac equations associated to \(\psi_{+}\) and \(\psi_{-}\), which stem from the action (1), read:

\[
(i\partial - m)\psi_{+} = 0 \quad \text{and} \quad (i\partial + m)\psi_{-} = 0,
\]

(4)

So, by expanding the operators \(\psi_{+}\) and \(\psi_{-}\) in terms of the \(c\)-number plane wave solutions of the Dirac equations, with operator-valued amplitudes, \(a_{+}, b_{+}, a_{-}\) and \(b_{-}\) (annihilation operators), and \(a_{+}^{\dagger}, b_{+}^{\dagger}, a_{-}^{\dagger}\) and \(b_{-}^{\dagger}\) (creation operators):

\[
\psi_{+}(x) = \int \frac{d^{2}k}{(2\pi)^{2}} \frac{m}{k^{0}} \{a(k)u(k)e^{-ikx} + b_{+}^{\dagger}(k)v_{+}(k)e^{ikx}\},
\]

(5)

\[
\psi_{-}(x) = \int \frac{d^{2}k}{(2\pi)^{2}} \frac{m}{k^{0}} \{a_{-}(k)u_{-}(k)e^{-ikx} + b_{-}^{\dagger}(k)v_{-}(k)e^{ikx}\},
\]

(6)

where \(\overline{\psi}_{\pm} = \psi_{\pm}^{\dagger}\gamma^{0}\). Consequently, from (4) and (5)-(6), by assuming \(p^{\mu} = (E, p_{x}, p_{y})\), the wave functions, \(u_{+}, v_{+}, u_{-}\) and \(v_{-}\), are given by:

\[
u_{+}(p) = \frac{\hat{g} + m}{\sqrt{2m(E + m)}} u_{+}(m, \bar{0}),
\]

(7a)

\[
u_{-}(p) = \frac{\hat{g} - m}{\sqrt{2m(E + m)}} u_{-}(m, \bar{0}),
\]

(7b)

satisfying the following conditions:

\[
u_{+}(p)u_{+}(p) = 1 \quad \text{and} \quad \nu_{+}(p)v_{+}(p) = -1,
\]

(9a)

\[
u_{-}(p)u_{-}(p) = -1 \quad \text{and} \quad \nu_{-}(p)v_{-}(p) = 1,
\]

(9b)

where

\[
u_{+}(m, \bar{0}) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad v_{+}(m, \bar{0}) = \begin{pmatrix} 0 \\ 1 \end{pmatrix},
\]

(11a)

\[
u_{-}(m, \bar{0}) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \text{and} \quad v_{-}(m, \bar{0}) = \begin{pmatrix} 1 \\ 0 \end{pmatrix},
\]

(11b)

are the momenta space solutions of the Dirac equations at the particle rest-frame, \(p^{\mu} = (m, \bar{0})\). The microcausality conditions for \(\psi_{+}\) and \(\psi_{-}\):

\[
\{\psi_{\pm}(x), \psi_{\pm}^{\dagger}(y)\}_{x^{0} = y^{0}} = \delta^{2}(\vec{x} - \vec{y}),
\]

(13)

together with the Dirac equations (4) and the normalization conditions (9)-(10), implies that:

\[
\{a_{\pm}(k), a_{\pm}^{\dagger}(p)\} = (2\pi)^{2} \frac{k^{0}}{m} \delta^{2}(\vec{k} - \vec{p}),
\]

(14)

\[
\{b_{\pm}(k), b_{\pm}^{\dagger}(p)\} = (2\pi)^{2} \frac{k^{0}}{m} \delta^{2}(\vec{k} - \vec{p}),
\]

(15)

where all other anticommutators vanish and, for the vacuum state \(|0\rangle\), \(a_{\pm}(k)|0\rangle = b_{\pm}(k)|0\rangle = 0\).

The quantum operators associated to space-time \(SO(1,2)\) symmetry and internal \((U(1)_{A} \times U(1)_{Q})\) symmetry, spin \((S)\), electric charge \((Q_{\pm})\) and Néel (chiral) charge \((q_{\pm})\), are

\[
S = \frac{1}{2} \sigma_{z},
\]

(16)

\[
Q_{\pm} = -e \int d^{2}x : \psi_{\pm}^{\dagger}(x)\psi_{\pm}(x):,
\]

(17)

\[
= -e \int d^{2}k \frac{m}{(2\pi)^{2} k^{0}} \{a_{\pm}(k)a_{\pm}(k) - b_{\pm}^{\dagger}(k)b_{\pm}(k)\},
\]

(18)

\[
q_{\pm} = \mp g \int d^{2}x : \psi_{\pm}^{\dagger}(x)\psi_{\pm}(x):
\]

(19)

\[
= \mp g \int d^{2}k \frac{m}{(2\pi)^{2} k^{0}} \{a_{\pm}(k)a_{\pm}(k) - b_{\pm}^{\dagger}(k)b_{\pm}(k)\},
\]

respectively, with their action upon the asymptotic fermion (antifermion) states with spin up and spin down, \(|f_{\uparrow}\rangle \langle f_{\uparrow}|\) and \(|f_{\downarrow}\rangle \langle f_{\downarrow}|\):

\[
S|f_{\uparrow}\rangle = +\frac{1}{2}|f_{\uparrow}\rangle, \quad S|f_{\downarrow}\rangle = -\frac{1}{2}|f_{\downarrow}\rangle,
\]

(20a)

\[
Q_{+}|f_{\uparrow}\rangle = -e|f_{\uparrow}\rangle, \quad Q_{-}|f_{\uparrow}\rangle = +e|f_{\uparrow}\rangle,
\]

(20b)

\[
q_{+}|f_{\uparrow}\rangle = -g|f_{\uparrow}\rangle, \quad q_{-}|f_{\uparrow}\rangle = +g|f_{\uparrow}\rangle,
\]

(20c)

\[
q_{+}|f_{\downarrow}\rangle = +g|f_{\downarrow}\rangle, \quad q_{-}|f_{\downarrow}\rangle = -g|f_{\downarrow}\rangle,
\]

(20d)

where

\[
|f_{\uparrow}\rangle = a_{\uparrow}^{\dagger}(k)|0\rangle, \quad |f_{\downarrow}\rangle = b_{\uparrow}^{\dagger}(k)|0\rangle,
\]

(21a)

\[
|f_{\downarrow}\rangle = a_{\downarrow}^{\dagger}(k)|0\rangle, \quad |f_{\uparrow}\rangle = b_{\downarrow}^{\dagger}(k)|0\rangle,
\]

(21b)

which means that, \(a_{\uparrow}^{\dagger}\) \((a_{\downarrow}^{\dagger})\) creates a spin-up (spin-down) fermion (electron polaron) and \(b_{\uparrow}^{\dagger}\) \((b_{\downarrow}^{\dagger})\) creates a spin-down (spin-up) antifermion (hole polaron). Moreover, from the results above, for any fermion or antifermion (spin up or down) quantum state \(|\psi\rangle\), it is verified that

\[
S|\psi\rangle = -\frac{1}{2g} q_{\pm}|\psi\rangle,
\]

(22)

which proves the correlation among spin and chiral charge (see TABLE I).

In the low-energy limit (Born approximation), the two-particle scattering potential is given by the Fourier transform of the two-particle \(t\)-channel scattering amplitude
(direct scattering) [14]. However, so as to compute the scattering amplitudes, use has been made of the propagators. Hence, switching off the coupling constants ($e$ and $g$), the tree-level propagators in momentum space, for all the fields, read:

\[
\Delta_{\pm\pm}(k) = i\frac{k^2 - m^2}{k^2 - m^2}, \quad \Delta_{\pm\mp}(k) = i\frac{k^2 + m^2}{k^2 - m^2};
\]

\[
\Delta_{A\pm}(k) = -i\left\{\frac{1}{k^2 - m^2} (\eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}) + \alpha \frac{k^\mu k^\nu}{k^2}\right\},
\]

\[
\Delta_{a\pm}(k) = -i\left\{\frac{1}{k^2 - m^2} (\eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}) + \beta \frac{k^\mu k^\nu}{k^2}\right\},
\]

\[
\Delta_{A\pm a}(k) = \Delta_{aA}(k) = \frac{i\mu}{k^2(k^2 - \mu^2)} \epsilon^{\mu\rho\sigma\nu} k_\rho.
\]

From the propagators above, $\Delta_{\pm\pm}, \Delta_{\pm\mp}, \Delta_{A\pm a}, \Delta_{a\pm}$ and $\Delta_{A\pm a}$, the spectrum and the tree-level unitarity of the model can be analyzed by coupling them to external currents, $J_{\Phi_i} = (J_{\Phi_i}, J_{\Phi_i}^\rho, J_{\Phi_i}^{\rho\sigma\nu})$, compatible with the symmetries of the model, where the current-current transition amplitudes in momentum space are written as: $A_{\Phi_i, \Phi_j} = J_{\Phi_i}^\rho(k) \langle \Phi_i(k) \Phi_j(k) \rangle J_{\Phi_j}(k)$. Then, by taking the imaginary part of the residues of the current-current amplitudes, $A_{\Phi_i, \Phi_j}$, at the poles, it can be probed the necessary conditions for unitarity – positive imaginary part of the residues of the transition amplitudes, $\Im \text{Res} A_{\Phi_i, \Phi_j} > 0$, as a consequence of the $S$-matrix be unitary – at the tree-level and the counting of the degrees of freedom described by the fields, $\Phi_i = (\psi_+, \psi_-, \pi_1, \pi_2)$. In summary, it has been concluded [15] that the two kind of fermions, $\psi_+$ and $\psi_-$, hold two massive degrees of freedom with mass $m$ – the electron-polaron $|f^e_+\rangle (u_+)$ and the hole-polaron $|f^e_-\rangle (v_+)$ associated to the spinor $\psi_+$, and the electron-polaron $|f^e_+\rangle (u_-)$ and the hole-polaron $|f^e_-\rangle (v_-)$ associated to the spinor $\psi_-$. Also, the vector fields, the electromagnetic field ($A_{\mu}$) and the Néel gauge field ($a_{\mu}$), carry each one two massive degrees of freedom with mass $\mu$, moreover, it shall be noticed that the single massless mode in model, displayed in $\Delta_{A\pm a}$, does not propagate, it decouples. From the results presented above, it can be concluded that the the parity-preserving $U(1) \times U(1)$ massive QED$_3$ is free from tachyons and ghosts at the classical level. Nevertheless, to have full control of the unitarity at tree-level, it is still necessary to study the behaviour of the scattering cross sections in the limit of high center of mass energies, by analyzing the Froissart-Martin bound [16].

### III. THE MÖLLESCATTERING

In order to calculate the scattering amplitudes, it remains the vertex Feynman rules associated to the interaction vertices $-e\bar{\psi}_\pm A\psi_\pm$ and $\mp g\bar{\psi}_\pm \gamma^\mu \psi_\pm$: $T_{\pm\pm}^\mu = ie\gamma^\mu$ and $t_{\pm\pm}^\mu = \pm ig\gamma^\mu$, respectively.

![FIG. 1. $e^−$-polaron–$e^−$-polaron (Møller) t-channel scattering mediated by electromagnetic ($A_\nu$) and Néel ($A_\nu$) quantum fields.](image-url)
A. Scattering potentials

In the low-energy (nonrelativistic) limit, the two-particle interaction potential, in the Born approximation, is nothing but the two-dimensional Fourier transform of the lowest-order two-particle $\mathcal{M}$ scattering amplitude:

$$\mathcal{V}(r) = \int \frac{d^2k}{(2\pi)^2} \mathcal{M} e^{ikr}.$$  \hspace{1cm} (36)

Accordingly to the Born approximation (36), the electron-polaron–electron-polaron $s$- and $p$-wave scattering potentials, mediated by the photon and the Néel quasiparticle, read:

$$\mathcal{V}_s(r) = \frac{1}{2\pi} \left( e^2 - g^2 \right) K_0(\mu r),$$  \hspace{1cm} (37)

$$\mathcal{V}_p(r) = \frac{1}{2\pi} \left( e^2 + g^2 \right) K_0(\mu r).$$  \hspace{1cm} (38)

Thereafter, it can be concluded from (38) that, regardless the values of the electromagnetic and the chiral coupling constants $-e$ and $g$, respectively – the $e^-$-polaron–$e^-$-polaron interaction in $p$-wave state ($|\uparrow\rangle + |\downarrow\rangle$ or $|\downarrow\rangle + |\uparrow\rangle$) is always repulsive. Nevertheless, from (37), it shall be stressed about the possibility of attractive $e^-$-polaron–$e^-$-polaron interaction in $s$-wave state ($|\uparrow\rangle + |\downarrow\rangle$) provided $g^2 > e^2$. In this case, where $g^2 > e^2$, the $s$-wave interaction potential $\mathcal{V}_s(r)$ is attractive,

$$\mathcal{V}_s(r) = -\frac{1}{2\pi} \left( g^2 - e^2 \right) K_0(\mu r),$$  \hspace{1cm} (39)

however, this is not a sufficient condition for the existence of bound states.

B. Bound states

Beyond the attractive nature, provided that $g^2 > e^2$, of $s$-wave interaction potential (39) it has to be weak in the sense of Kato [9],

$$\int_{0}^{\infty} d\rho \rho [ \ln(\rho) ] |\mathcal{V}(\rho)| < \infty, \quad \rho = \mu r,$$  \hspace{1cm} (40)

so as to satisfy the Newton-Setô and the Bargmann bounds [10, 11], which guarantee bound states and establish an upper bound for their number ($N^0$) for vanishing angular momentum ($l = 0$):

$$N^0 < 1 + \frac{\left( \frac{2\mu_r}{\hbar^2} \right)^2}{2} \int_{0}^{\infty} \int_{0}^{\infty} \rho |\ln(\rho)| |\mathcal{V}(\rho)||\mathcal{V}(\rho')| d\rho d\rho'$$

$$+ \frac{1}{2} \left( \frac{2\mu_r}{\hbar^2} \right)^2 \int_{0}^{\infty} \rho |\mathcal{V}(\rho)| d\rho,$$  \hspace{1cm} (41)

and upper bound for their number ($N^l$) for nonvanishing angular momentum ($l \neq 0$):

$$N^l < \frac{1}{2l} \left( \frac{2\mu_r}{\hbar^2} \right) \frac{1}{\mu^2} \int_{0}^{\infty} \rho |\mathcal{V}(\rho)| d\rho,$$  \hspace{1cm} (42)

respectively, where $\hbar = 1$ and $\mu_r = \frac{\mu}{2}$ is the $e^-$-polaron–$e^-$-polaron reduced mass.

It has been proved elsewhere [17] that, whenever an interaction potential of the type $\mathcal{V}(r) = C K_0(\mu r)$ is attractive ($C < 0$), it satisfies the following criteria: the weakness in the sense of Kato (40); the Newton-Setô bound (41) for $l = 0$; and the Bargmann bound (42) for all $l$ such that $l \leq l_m = \left( \frac{\mu_r |C|}{\sqrt{\pi}} \right)$ (where $[x]$ is the floor function of $x$). In the same manner, by means of the effective potential $\mathcal{V}_{\text{eff}}(r) = \frac{\hbar^2 (r^2 - \frac{l^2}{r^2})}{2\mu r^2} + \mathcal{V}(r)$ with $0 \leq l \leq l_m$, it can be figured out that bound states arise (see FIG. 2). In addition to, it shall be stressed that these fulfilled conditions, (40), (41) and (42), guarantee the existence of bound states for any kind of three-dimensional space-time model which exhibits scattering potential of the type $\mathcal{V}(r) = C K_0(\mu r)$ ($C < 0$).

IV. CONCLUSIONS

In conclusion, the parity-preserving $U(1) \times U(1)$ massive QED$_3$, a mass-gap graphene-like planar quantum electrodynamics model, yields in low-energy electron-polaron–electron-polaron $s$-wave scattering an attractive interaction, with the scattering potential given by $\mathcal{V}_s(r) = -\frac{1}{2l} \left( g^2 - e^2 \right) K_0(\mu r)$ (39), whenever the $e^-$-polaron–Néel-field coupling strength ($|g|$) be stronger than the strength of $e^-$-polaron–photon-field coupling ($|e|$), $g^2 > e^2$. Moreover, owing to the fact that the $s$-wave attractive scattering potential (39) satisfies the Kato condition [9], and the Newton-Setô and the Bargmann upper bounds [10, 11], electron-polaron–electron-polaron quasiparticles bound states may emerge, namely, the $s$-wave bipolarons.

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