Introduction. Magnetic fields pervading astrophysical fluid systems such as stars and galaxies are commonly thought to be excited and further sustained by a variety of self-amplifying magnetohydrodynamic (MHD) dynamo effects converting kinetic energy of turbulent flows of electrically-conducting fluid into magnetic energy [1–7]. In order to sustain large-scale magnetic fields via a turbulent dynamo, however, some underlying system-scale symmetry-breaking is generically required. Typically, this is provided in astrophysical systems by large-scale rotation and/or shear. In particular, by breaking the parity/mirror invariance of an otherwise isotropic, homogeneous turbulence, rotation makes the turbulence helical. This creates the conditions for statistical dynamo effects that can in principle amplify large-scale magnetic fields exponentially on a rotation timescale [8–11]. The most well-known, the $\alpha$ effect [12], is generally considered a key ingredient of magnetic-field generation in the Sun.

While a helical turbulent dynamo provides an appealing phenomenological explanation for the large-scale magnetism of rotating astrophysical bodies, there remain major open questions regarding its actual viability and efficiency in the regime of large magnetic Reynolds numbers $Rm$ and comparable flow turnover times and correlation times, an astrophysically-relevant non-perturbative limit for which no analytical theory is available [13, 14] ($Rm = UL/\eta$ is larger than $10^6$ in the Sun, and $10^{20}$ in galaxies; $U$ denotes a typical velocity field amplitude, $L$ is the typical scale of the turbulence, and $\eta$ is the magnetic diffusivity). Numerical studies have shown that large-scale exponential dynamo growth driven by helical turbulence, such as rotating convection, is possible at mild $Rm < O(100)$ ([15–19], see also [20–22] for reviews in various astrophysical contexts), however this regime is still far from asymptotic in practice.

First of all, a distinct small-scale “fluctuation” dynamo mechanism is activated beyond $Rm = O(100)$, that amplifies magnetic fields on fast time and spatial scales comparable to the flow turnover scales [18–22]. This dynamo populates a large-$Rm$ turbulent MHD fluid with dynamical small-scale fields affecting the structure of the flow much faster than the helical dynamo can grow a large-scale field [23–25]. Besides, independently of a small-scale dynamo, turbulent tangling of a growing large-scale field produces dynamical magnetic fluctuations at increasingly smaller-scales as $Rm$ increases (typically $\propto Rm^{-1/2}$), resulting in small-scale dynamical feedback on the flow well before the large-scale field has itself saturated [26]. In the case of homogeneous helical turbulence producing an $\alpha$ effect, this problem takes a particular pathological form: the large-scale field does ultimately reach super-equipartition levels, but it can only do so on a long, system-scale resistive time, a consequence of a resistive bottleneck in the dissipation of small-scale magnetic twists also responsible for the dynamical reduction of the $\alpha$ effect [16, 27–29]. This is usually referred to as the catastrophic $\alpha$-quenching problem. Finally, the resistive-scale dynamics of saturated MHD dynamos may undergo a fast-reconnection transition at $Rm = O(S_c)$, where $S_c = O(10^5)$ is the critical value of the Lundquist number $S = LV/\eta$ at which MHD reconnection becomes fast [30–32] (assuming an Alfvén speed $V_A \sim U$ in the saturated regime). Its implications for the dynamics of helical fields, such as produced by the $\alpha$ effect, have so far barely been touched on [7, 33–38].

We aim to further explore the nonlinear helical dynamo problem at large $Rm$. A long-envisioned possible solution to catastrophic quenching is via removal or spatial redistribution of small-scale magnetic helicity by helicity fluxes [39–47]. The most studied case [48, 50] involves...
expulsion of magnetic helicity through system boundary winds. Simulations up to $Rm \simeq 10^3$ suggest that a regime with subdominant resistive effects is achieved at large $Rm$ \cite{50}. Alternatively, a similar state may be achieved via magnetic helicity fluxes driven through an equator by a hemispheric distribution of kinetic helicity, a simple configuration typical of rotating astrophysical systems \cite{51}. This case has so far only been studied at low $Rm$ where resistive effects dominate over helicity fluxes \cite{52,53}. The numerical identification, up to $Rm \simeq 3 \times 10^3$, of a nonlinear helical state with subdominant resistive effects is the main result of this work.

**Model.** We address the problem from a standard perspective of magnetic helicity $\mathbf{A} \cdot \mathbf{B}$ dynamics ($\mathbf{A}(r,t)$ is the magnetic vector potential, $\mathbf{B} = \nabla \times \mathbf{A}$ the magnetic field), a local evolution equation of which can be derived from the induction equation,

$$\frac{\partial}{\partial t}(\mathbf{A} \cdot \mathbf{B}) + \nabla \cdot \mathbf{F}_{Hm} = -2\eta (\nabla \times \mathbf{B}) \cdot \mathbf{B},$$

where $\mathbf{F}_{Hm} = c(\varphi \mathbf{B} + \mathbf{E} \times \mathbf{A})$ is a magnetic-helicity flux, $c$ is the speed of light, $\mathbf{E}$ is the electric field, and $\varphi$ is the electrostatic potential. We split each field into a mean, large-scale part, defined below as an average over the $(x, y)$ plane and denoted by an overline, and a fluctuating, small-scale part, denoted by lower case letters, $\mathbf{B}(r,t) = \overline{\mathbf{B}}(z,t) + \mathbf{b}(r,t)$. Manipulating the small- and large-scale components of the induction equation, using $\mathbf{E} \cdot \mathbf{B} = 0$, where $\mathbf{E} = \mathbf{u} \times \mathbf{B}$ is the electromotive force (EMF) for a flow $\mathbf{u}$, one obtains helicity budget equations

$$\frac{\partial}{\partial t}(\overline{\mathbf{a}} \cdot \mathbf{b}) + \nabla \cdot \mathbf{F}_{Hm,ss} = -2\mathbf{E} \cdot \mathbf{B} - 2\eta (\nabla \times \mathbf{b}) \cdot \mathbf{B},$$
$$\frac{\partial}{\partial t}(\mathbf{a} \cdot \mathbf{B}) + \nabla \cdot \mathbf{F}_{Hm,lss} = 2\mathbf{E} \cdot \mathbf{B} - 2\eta (\nabla \times \mathbf{B}) \cdot \mathbf{B}.$$

In these equations, describe the first r.h.s. terms describe the production of magnetic helicity, the second r.h.s. terms its destruction by resistivity, and the second l.h.s. terms describe the transport of magnetic helicity through the divergence of mean fluxes of fluctuating/mean helicities

$$\mathbf{F}_{Hm,ss} = c(\varphi \mathbf{B} + \mathbf{e} \cdot \mathbf{a}), \quad \mathbf{F}_{Hm,lss} = c(\nabla \mathbf{B} + \mathbf{E} \times \mathbf{A}) = \mathbf{F}_{Hm} - \mathbf{F}_{Hm,ss},$$

where $\mathbf{a}$ and $\mathbf{e}$ denote fluctuations of the vector potential and electric field.

We compute these budgets in the Coulomb gauge $\mathbf{∇} \cdot \mathbf{A} = 0$ \cite{51} for three-dimensional, spatially-periodic cartesian simulations of nonlinear, helical, incompressible, viscous, resistive MHD, carried out with the SNOOPY spectral code with 2/3 dealiasing \cite{55}. An inhomogeneous body force inspired by the Galloway-Proctor flow \cite{56,57} is implemented in the momentum equation,

$$\mathbf{f}(r,t) = k_f A_f \times \left(\begin{array}{c}
-2\sin \left(\frac{2\pi y}{L_f} + \sin \omega ft\right) \sin \left(\frac{2\pi z}{L_z}\right) \\
-2\cos \left(\frac{2\pi x}{L_f} + \cos \omega ft\right) \sin \left(\frac{2\pi z}{L_z}\right) \\
\sin \left(\frac{2\pi x}{L_f} + \cos \omega ft\right) + \cos \left(\frac{2\pi y}{L_f} + \sin \omega ft\right)
\end{array}\right),$$

where $\omega_f$ and $A_f$ are a forcing frequency and amplitude, $L_x = L_y = L_f$ and $k_f = 2\pi / L_f$ is the forcing wavenumber. This forcing drives a flow with a statistically-steady sinusoidal kinetic helicity profile in $z$. Fig. 1 shows kinetic and current helicity profiles at saturation. Both are positive (resp. negative) for $z < L_z/2$ (resp. $z > L_z/2$), and change sign at “equators” $z = L_z/2$ and $z = 0$ (replicated at $z = 4$ in a periodic set-up). The mean, domain-averaged helicities are zero. Hence, this configuration i) mimicks a hemispheric distribution of kinetic helicity; ii) can potentially bypass catastrophic resistive quenching present in the standard homogeneous case by enabling equatorial turbulent magnetic helicity fluxes; iii) keeps the system complexity minimal so as to maximise $Rm$.

We performed a parametric study for different Reynolds $Re = u_{ rms}/(k_f \nu)$ and magnetic Reynolds numbers $Rm = u_{ rms}/(k_f \eta)$, where $u_{ rms}$ is the r.m.s. flow amplitude (over time and space). We set $\omega_f = 1$, $A_f = 0.1$, $L_f = 1$, and $L_z = 4L_f$ in all simulations to ensure a minimal scale separation between the turbulence forcing scale and the scales of the helical inhomogeneity and emergent large-scale statistical dynamics. Each run (Tab. 1) was integrated for at least 50 forcing times $2\pi/\omega_f$ (175–200 flow turnover times $L_f/u_{ rms}$). To isolate the weaker slow, large-scale signal from the fast turbulent noise, the magnitude of each term in equations (2)-(3) was estimated by Fourier-filtering them on $z$-scales larger than the forcing scale ($k_z < k_f$), then taking the r.m.s. values (over $z$) of their time-averages after initial growth of the dynamo.

![Fig. 1: Time- and (x, y)-averaged kinetic and current helicity z-profiles in a typical simulation (run T06, $Rm \simeq 2800$, $Re \simeq 700$, $L_f = 1$, $L_z = 4$).](image-url)
TABLE I: Run index. $L_x, L_y = L_z/4$ for all runs.

| Run | $N_x, N_z$ | $N_z$ | $\nu^{-1}$ | $Re$ | $Rm$ | $\eta_{rms}$ | $B_{rms}$ | $\overline{B}_{rms}$ |
|-----|------------|-------|------------|------|------|-------------|----------|----------------|
| V01 | 64$^2$ | 256 | 500 | 125 | 46.7 | 11.7 | 0.59 | 0.49 | 0.39 |
| V02 | 64$^2$ | 256 | 500 | 500 | 46.2 | 46.2 | 0.58 | 0.57 | 0.42 |
| V03 | 128$^2$ | 512 | 500 | 2000 | 39.9 | 159.7 | 0.50 | 0.54 | 0.21 |
| V04 | 128$^2$ | 512 | 500 | 8000 | 35.6 | 570.0 | 0.44 | 0.58 | 0.14 |
| V05 | 256$^2$ | 1024 | 500 | 16000 | 32.9 | 1054.0 | 0.41 | 0.59 | 0.12 |
| M01 | 64$^2$ | 256 | 2000 | 125 | 201.2 | 12.6 | 0.63 | 0.51 | 0.42 |
| M02 | 64$^2$ | 256 | 2000 | 500 | 197.6 | 49.4 | 0.62 | 0.56 | 0.38 |
| M03 | 128$^2$ | 512 | 2000 | 2000 | 177.0 | 177.0 | 0.56 | 0.59 | 0.32 |
| M04 | 128$^2$ | 512 | 2000 | 8000 | 163.3 | 653.2 | 0.51 | 0.60 | 0.18 |
| M05 | 256$^2$ | 1024 | 2000 | 16000 | 165.9 | 1327.5 | 0.52 | 0.60 | 0.12 |
| T01 | 128$^2$ | 512 | 8000 | 125 | 804.4 | 12.6 | 0.63 | 0.48 | 0.37 |
| T02 | 128$^2$ | 512 | 8000 | 500 | 846.6 | 52.9 | 0.66 | 0.58 | 0.42 |
| T03 | 128$^2$ | 512 | 8000 | 2000 | 749.3 | 187.3 | 0.59 | 0.58 | 0.25 |
| T04 | 128$^2$ | 512 | 8000 | 8000 | 748.1 | 748.1 | 0.58 | 0.60 | 0.16 |
| T05 | 256$^2$ | 1024 | 8000 | 16000 | 683.4 | 1366.8 | 0.54 | 0.63 | 0.17 |
| T06 | 512$^2$ | 2048 | 8000 | 32000 | 694.5 | 2778.0 | 0.55 | 0.62 | 0.12 |

FIG. 2: Small- and large-scale helicity budgets on large scales in $z$ ($k_z < k_f$) as a function of $Rm$, for different $Re$.

Results. Helicity budgets as a function of $Rm$ are shown in Fig. 2. Results are only weakly dependent on $Re$. At low $Rm$, both budgets are characterized by a balance between resistive and helical EMF terms. As $Rm$ reaches 50 – 100, the large-scale budget transitions to a regime characterized by a dominant balance between the large-scale $z$-flux of large-scale helicity and EMF. However, for $Rm < 300 – 500$, the dominant balance in the small-scale helicity budget remains between the resistive dissipation of small-scale helicity and EMF. Hence, the large-scale dynamics is still affected by resistive effects in this $Rm$ range. This regime is nevertheless interesting in that it can only be realized for non-uniform flow helicity. It really takes $Rm > 1000$ to reach a regime characterized by a dominant non-resistive balance in both large- and small-scale helicity budgets, the latter now being between the large-scale $z$–flux of small-scale helicity and the EMF term. A weak residual dependence of both terms on $Rm$ remains in the range of $Rm$ probed.

A detailed comparison, between low and large-$Rm$ runs with identical viscosity, of the time-averaged $z$-dependent quantities in equations (2)-(3) again filtered on $z$-scales larger than the flow forcing scale, is shown...
in Fig. 3 to make more explicit the transition between the resistively-dominated and asymptotic regimes. Estimates of the flux divergences carried out in the Coulomb gauge were always found to be in good agreement with the calculation of the (gauge-independent) r.h.s. of equations (2)-(3) at large $Rm$, as expected in a statistically steady state [54]. Hence, we are confident that the main trends reported here do not depend on our gauge choice.

Fig. 4 shows the energy of $\mathbf{B}$ as a function of $Rm$. The results at intermediate $Rm$ are consistent with a $Rm^{-1}$ scaling, in line with theory expectations and earlier simulations [52, 53]. There is as yet no clear-cut evidence for an asymptotic regime entirely independent of $Rm$ (maybe because the small-scale helicity dissipation term only seemingly decreases slowly as $Rm^{-1/2}$ at large $Rm$), however we observe a clear deviation away of the $Rm^{-1}$ scaling for the energy of the mean-field at the largest $Re$ and $Rm$ probed. Mean-field models assuming turbulent diffusive expressions for the helicity fluxes also suggest that convergence of $B_{\text{rms}}^2/B_{\text{rms}}^2$ towards an $Rm$-independent value should be slow at large $Rm$ [53, 55].

For our parameters, convincing access to a regime with subdominant resistive contributions required a (spectral) resolution of 512 per $L_f$ (run T06, $Rm \simeq 2800$, $Re \simeq 700$). The evolution of energy densities, and time-averaged energy spectra in the statistically steady state of T06 are shown in Fig. 5. The evolutions of the large-scale field component $\mathbf{B}_x(z,t)$ and large-scale magnetic energy densities are shown in Fig. 6. $\mathbf{B}$ displays bursty oscillations, on a timescale $\sim 50-60 \omega_f^{-1}$ ($\sim 30$ turnover times), that appear to propagate spatially towards the equator associated with the node of kinetic helicity at $z = 2$, a likely consequence of the symmetry-breaking flow helicity profile in $z$ [58]. Snapshots at the end of the run (Fig. 7) show complex, turbulent magnetic structures with tentative nascent plasmoids typical of reconnection in nonlinear tangled magnetic fields at large $Rm$ [7, 38], e.g. at $(x,z) \simeq (0.5, 0.75); (0.75, 1.3); (0.8, 3.2); (0.75, 3.5)$. The “small-scale” horizontal structure visible in the bottom plot is the direct imprint of the forcing at $L_f$ and is distinct from the weaker, but larger-scale emergent statisti-
FIG. 6: Evolution of (top) $B_x(t, z)$ and (bottom) energy density of mean field (T06: $Rm \simeq 2800$, $Re \simeq 700$).

FIG. 7: $(x, z)$ (top) and $(x, y)$ (bottom) out-of-plane magnetic-field snapshots (T06: $Rm \simeq 2800$, $Rm = 700$).

turbulent diffusive fluxes, but with different symmetries). In all cases, the strong dependence of the saturated state on $\eta$ is circumvented at large $Rm$ by non-resistive helicity fluxes. The nonlinear state achieved here, anticipated in [52 [53 [58, is particularly appealing in that it stems from a very simple inhomogeneous, hemispheric distribution of kinetic helicity also typical of rotating astrophysical systems. While the solution is dominated by small-scale fields, a clear magnetic activity pattern migrating towards the equator is present on scales larger than the flow forcing scale. Further investigations are needed to determine whether a mean shear may boost the generation of a streamwise “azimuthal” large-scale field, and how such a shear, or the transition to the low $Pm$, large $Rm$ regime typical of stellar dynamos may affect the properties of the identified travelling wave pattern.

Modelling helicity fluxes as turbulent diffusive fluxes also suggests that the transition $Rm$ (to the regime with a subdominant resistive term) scales as $(k_f/k)^2$, where $k$, the scale of $\mathbf{B}$, should be comparable to the helicity modulation scale [53]. If this scaling applies, something we could unfortunately not test due to limited computing resources, the asymptotic regime of large-scale astrophysical dynamos typically involving large scale-separations may be at significantly higher $Rm$ than that determined here for $L_z/L_f = 4$. As global simulations are currently limited to $Rm$ of a few hundreds (also uncomfortably close to the small-scale dynamo threshold), this raises the question of their lack of asymptoticity for the foreseeable future. Our results may provide a useful reference point to assess such future simulations in this respect.

An in-depth understanding of this large-$Rm$ MHD state remains to be developed. One may be tempted to interpret it “classically” as the nonlinear outcome of an $\alpha$ effect dynamo [1 [8 [12, see also [59 for theoretical work involving helicity fluxes]. Oscillations of a weak large-scale field on top of helical turbulent MHD background also suggest a (possibly connected) phenomenological interpretation in terms of simple large-scale magnetoelastic waves in small-scale tangled fields [59]. Finally, while we may have tentatively observed reconnection plasmoids in these simulations, providing a new independent estimate of the minimal (spectral) resolution required to accommodate fast reconnection in turbulent MHD, much more numerical work will be required in the future at even higher resolution to fully characterize it, and its so far poorly-understood possible effects on large-scale magnetic field generation at asymptotically large $Rm$.

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