An efficient method to identify thermal conductivity of orthotropic material based on BP neural network algorithm

Xiang LI, Rui-feng MA, Zhi-yin TANG
Naval University of Engineering, Wuhan 430033, China
*Corresponding author’s e-mail: suplixiang@163.com

Abstract. BP neural network algorithm is used to identify the three-dimensional orthotropic thermal conductivity. The finite volume method is used to solve the forward heat transfer problem, and the training samples for neural network inversion identification are obtained. According to the identification results, the temperature field is reconstructed. Through the analysis of the calculation results, it is found that the accurate identification of three-dimensional orthotropic thermal conductivity can be realized by using BP neural network algorithm. When the temperature measurement error is less than 0.5 ℃, the maximum identification error of thermal conductivity is only 3.5%.

1. Introduction
The thermophysical properties of modern composite materials are almost anisotropic, and the traditional methods to determine the thermal properties have become increasingly unable to meet the requirements, and modern industry increasingly requires accurate determination of the thermal properties of materials under actual working conditions. Due to the limitation of experimental methods, the method of parameter identification based on inverse heat conduction problem has been paid great attention.

The solution of the inverse problem of anisotropic thermal conductivity generally integrates three aspects: experimental measurement technology, numerical or analytical calculation method and optimization technology. The idea is to start from an initial model, searching around the initial model according to some algorithm, get the correction value of the function, and then modify the value of the function according to the correction value to get a new function value. The calculation is repeated until the convergence conditions given in mathematics are met. The inverse problem of anisotropic thermal conductivity is a complex nonlinear problem. Neural network algorithm is a nonlinear and adaptive information processing system composed of a large number of processing units interconnected, which has strong nonlinear fitting ability and can map any complex nonlinear relationship. At present, the most widely used and mature method is forward feedback (BP).

In order to solve the above problems, the finite element method is used to solve the three-dimensional heat transfer forward problem. The corresponding temperature values of temperature measurement points on the research object are obtained by setting different thermal conductivity values under specific boundary conditions and loads. These data are used as sample data to train BP neural network model, and the temperature values of temperature measurement points obtained from experiments are taken as the input of the model, Then the physical parameters to be determined are identified in reverse.

2. Positive problem of heat conduction
A typical three-dimensional steady-state anisotropic heat conduction model can be described in figure 1. The governing equations, initial conditions and boundary conditions are as follow.
\[
\begin{align*}
    k_x \frac{\partial^2 T}{\partial x^2} + k_y \frac{\partial^2 T}{\partial y^2} + k_z \frac{\partial^2 T}{\partial z^2} &= 0 \\
    k_z \frac{\partial T}{\partial z} &= q \quad (x, y, z) \in \Omega_i \\
    T(x, y, z) &= T_0 \quad (x, y, z) \in \Omega_2 
\end{align*}
\]

The other boundary conditions are adiabatic, meeting

\[
    k_i \frac{\partial T}{\partial i} = 0 \quad i = x, y, z \quad (x, y, z) \in \{\Omega - \Omega_i \cup \Omega_2\}
\]

Among them, \(k_x\), \(k_y\), and \(k_z\) represent thermal conductivity in \(x\), \(y\), and \(z\) directions respectively. \(\Omega\) represents for all boundary of research object. \(\Omega_i\) represents temperature boundary and \(\Omega_2\) represents heat flow boundary. \(a\), \(b\), and \(c\) represent dimension of research object in \(x\), \(y\), and \(z\) directions.

3. BP neural network algorithm inversion identification

In this paper, the BP neural network is used to identify the anisotropic thermal conductivity. The idea is to use the method of solving the three-dimensional anisotropic heat conduction forward problem to generate the training samples of the BP network. Under the specific boundary conditions, different thermal conductivity parameters are set for the research object, and the finite element method is used to solve the thermal conductivity forward problem to obtain the temperature field under each group of thermal conductivity. Then, the temperature values \(T_m (T_1, T_2, T_n)\) at specific positions (monitoring points) in these temperature fields are taken as the input of BP network, and the corresponding thermal conductivity \((k_x, k_y, k_z)\) is output to train BP network for training. Finally, the measured temperature values of monitoring points of the research object are input into the trained neural network model, and the output is the thermal conductivity of the research object. The thermal conductivity identification model of BP neural network is shown in figure 2.

4. Example and analysis

4.1 Data acquisition of orthotropic thermal conductivity inversion

As shown in Figure 3, for an orthotropic cuboid in engineering practice, its length, width and height are setting as \(a = 0.3m\), \(b = 0.1m\), \(c = 0.4m\). It is difficult to set the boundary conditions and loads of large
area accurately under the actual conditions. Therefore, heat flow load $q = \frac{50000}{m^2}$ and temperature boundary $T_0 = 20^\circ C$ are set respectively for small area $\Omega_1$ and small area $\Omega_2$ to make the research object reach steady state. Assuming that four temperature sensors are set on one side of the surface, and the steady-state temperature of the four point is $[T_1, T_2, T_3, T_4] = [623^\circ C, 204^\circ C, 277^\circ C, 328^\circ C]$.

![Figure 3. Experimental model of thermal conductivity inversion](image)

4.2 **BP neural network identification of thermal conductivity inversion**

Under the same boundary conditions and loads as section 2, the temperature values of monitoring points are solved with finite element method, and these data are used as real values to train BP neural network model. Some sample data are shown in Table 1.

| Serial number | Thermal conductivity $k_x$ | Monitoring point temperature $T_1$ | $T_2$ | $T_3$ | $T_4$ |
|---------------|-----------------|-----------------|-------|-------|-------|
| 1             | 212             | 1000            | 327   | 541   | 704   |
| 2             | 233             | 714             | 247   | 380   | 480   |
| 3             | 170             | 555             | 194   | 266   | 317   |
| ...           | ...             | ...             | ...   | ...   | ...   |
| 23            | 128             | 641             | 209   | 287   | 342   |
| 24            | 117             | 686             | 217   | 302   | 361   |

The training process is shown in Figure 4. When the training reaches 42709 generation, the mean square error is less than 0.001, and the model has good convergence.

![Figure 4. BP neural network training process](image)

4.3 **BP model accuracy verification**

In order to verify the accuracy of the established BP neural network model, the BP neural network model trained in the previous section is adopted, the temperature data of monitoring points in Table 1 is taken
as the input of the model, and the thermal conductivity data in Table 1 are inversely identified, and the comparison results are shown in Fig. 5. From the inversion results, the maximum error between the thermal conductivity obtained by inversion identification and the real value is less than 1%. Therefore, it can be considered that the BP neural network is accurate and effective, and can be used to identify the orthotropic thermal conductivity.

4.4 Results and analysis

Taking the temperature sensor test data \([T_{n1}, T_{n2}, T_{n3}, T_{n4}] = [623\, ^\circ C, 204\, ^\circ C, 277\, ^\circ C, 328\, ^\circ C]\) as the input of BP neural network, the thermal conductivity inversion identification result of the research object are \(x_k = 123.7, y_k = 157.3, z_k = 284.3\). In order to verify the correctness of the inversion identification result, the thermal conductivity value obtained by inversion identification is brought into the solution model of heat conduction forward problem in Section 1.1, and the temperature of monitoring point is calculated as \([T'_{n1}, T'_{n2}, T'_{n3}, T'_{n4}] = [627.9\, ^\circ C, 205\, ^\circ C, 278.8\, ^\circ C, 329\, ^\circ C]\). Compared with the test data of temperature sensor \([T_{n1}, T_{n2}, T_{n3}, T_{n4}]\), the maximum error is less than 0.64%, It shows that the inversion result is accurate and reliable.

In order to consider the influence of data noise on the identification results, the temperature measurement error is assumed to be normal distribution. It is assumed that \(T_{p} = (1 + \sigma \cdot \xi) \cdot T_{e}\) is the temperature information of measuring point with measurement error. \(T_{e}\) is the accurate value of the temperature information of the measuring point, which is given by the numerical solution of the forward heat transfer problem. \(\sigma\) is the error level and \(\xi\) is random variable obeying normal distribution \([5]\). Table 2 shows the quantitative identification results of thermal conductivity under different temperature measurement errors. When the temperature measurement error is less than 0.5 \(^\circ C\), the identification error of thermal conductivity is less than 3.5%.

| \(\sigma\) (\(^\circ C\)) | Result \((W/(m \cdot K))\): \(x_k, y_k, z_k\) | Relative error (%): \(E_{r_x}, E_{r_y}, E_{r_z}\) |
|---------------------|-----------------|---------------------------------|
| 0.1                 | 124.4, 158.7, 284.2 | 0.6, 0.8, 0.03                  |
| 0.2                 | 126.6, 157.4, 288.0 | 2.4, 0.08, 1.3                  |
| 0.5                 | 128.1, 158.9, 283.9 | 3.5, 1.0, 0.1                   |

Figure 5. Comparison between BP neural network recognition results and real values
5. Conclusion
In this paper, in order to accurately measure the thermophysical properties of materials under the actual working conditions in industry, a set of three-dimensional anisotropic thermal conductivity inversion identification method based on BP neural network is proposed. Numerical experiments show that the proposed algorithm has high accuracy and good noise immunity, and can be used as a powerful tool for solving inverse heat conduction problems with multiple variables.

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