On the consequences of draw restrictions in knockout tournaments

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Abstract

The paper analyses how draw constraints influence the outcome of a knockout tournament. The research question is inspired by the rules of European club football competitions: in order to maintain their international character, the organiser usually imposes an association constraint both in the group stage and the first round of the subsequent knockout phase, that is, teams from the same country cannot be drawn against each other. The effects of similar restrictions are explored in both theoretical and simulation models. Using an association constraint in the first round(s) is verified to increase the likelihood of a same nation matchup to approximately the same extent in each subsequent round. Furthermore, if the favourite teams are concentrated in some associations, they have a higher probability to win the tournament in the presence of association constraints. Our results essentially justify a recent decision of the Union of European Football Associations (UEFA).

Keywords: OR in sports; draw procedure; simulation; tournament design; UEFA Europe League

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JEL classification number: C44, C63, Z20

Wir haben diese einfachen Vorstellungen an die Wirklichkeit angeknüpft und so den Weg gezeigt, wie man aus der Wirklichkeit zu jenen einfachen Vorstellungen wieder zurückgelangen und also festen Grund gewinnen kann, damit man nicht genötigt sei, im Räsonnement zu Stützpunkten seine Zuflucht zu nehmen, die selbst in der Luft schweben.1

(Carl von Clausewitz: Vom Kriege)

1 “We have connected these simple ideas with reality, and therefore shown the way by which we may return again from the reality to those simple ideas, and obtain firm ground, and not be forced in reasoning to take refuge on points of support which themselves vanish in the air.” (Source: Carl von Clausewitz: On War, Book 6, Chapter 8 [Methods of Resistance]. Translated by Colonel James John Graham, London, N. Trübner, 1873. http://clausewitz.com/readings/OnWar1873/TOC.htm)
1 Introduction

The Operations Research community has recently made a substantial effort to understand the implications of various sports rules (Wright, 2014; Kendall and Lenten, 2017; Csató, 2021e). The present paper examines how draw constraints affect the outcome of a knockout tournament. Our study is inspired by the football club competitions organised by the Union of European Football Associations (UEFA). Since more than one team can qualify for these tournaments from certain countries, UEFA usually imposes an association constraint both in the group stage and the first round of the subsequent knockout phase, that is, teams from the same country cannot be drawn against each other. The decision makers likely have an underlying preference to maintain the international character of the tournaments. However, this restriction may have some unforeseen consequences and it is far from clear why it is not applied in later rounds. We want to address these issues in the following.

The design of knockout tournaments has extensive literature, especially on finding an optimal seeding. Hwang (1982) defines a bracket to be monotone if the winning probability grows with team skill. This axiom may be violated by the traditional seeding regime but reseeding after each round guarantees monotonicity. Horen and Riezman (1985) investigate four-team tournaments with respect to various objective functions such as maximising the winning probability for the top player or the expected strength of the winner. Schwenk (2000) provides a randomisation procedure, which is free of perverse incentives and satisfies two further favourable properties. Vu and Shoham (2011) introduce alternative criteria of fairness (envy-freeness and order preservation) to derive some impossibility results. Groh et al. (2012) analyse optimal seedings in a model where the agents should exert effort in order to win a match and advance to the next stage, thus the winning probabilities are endogenous and depend on strategic choices. Karpov (2016) analyses equal gap seeding, the only one maximising the probability that the strongest competitor is the winner, the two strongest competitors are the finalists, etc., and characterises it by a set of theoretical requirements. Karpov (2018) extends this model to tournaments with more than two participants in one match. Dagaev and Suzdal’stev (2018) solve a discrete optimization problem to maximise the number of matches with high competitive intensity and high quality. Della Croce et al. (2020) analyse the allocation of unseeded players in tennis tournaments to ensure diversification and fairness. Arlegi and Dimitrov (2020) specify the class of seeding rules that satisfy equal treatment and monotonicity in strengths.

Further papers address the likelihood of winning in knockout tournaments. Marchand (2002) compares the winning probability of the top-seeded player for standard and random versions. Adler et al. (2017) provide upper and lower bounds for the winning probabilities in random designs. Kulhanek and Ponomarenko (2020) explore the cases when the second-best player is more likely to win than to finish second in both the random and seeded formats. Arlegi (2021) presents a procedure that, for each possible seeding of the players, obtains a matrix of pairwise winning probabilities, which favours a weaker player over a stronger player and identifies those players.

Academic researchers have also widely discussed the implications of draw constraints in the group stage of sports tournaments. The FIFA World Cup draw has been demonstrated to be unevenly distributed due to geographical restrictions (Jones, 1990; Rathgeber and Rathgeber, 2007; Guyon, 2015). The draw of the European Qualifiers to the 2022 FIFA World Cup has suffered from the same shortcoming (Csató, 2021a). Several papers have made suggestions to create more balanced groups in the FIFA World Cup (Guyon, 2015; Laliena and López, 2019; Cea et al., 2020). Draw constraints present a powerful tool
to avoid matches where a team has misaligned incentives (Csató, 2021b). Guyon (2021) proposes a novel format for hybrid tournaments consisting of a preliminary group stage followed by a knockout phase and adapts it to the constraints put by the UEFA on the draw.

On the other hand, draw restrictions in knockout tournaments have received far less attention. According to Klößner and Becker (2013), the mechanism used in the UEFA Champions League Round of 16 draw implies that every result of the draw could not have the same probability. Nonetheless, the UEFA draw procedure remains close to a constrained-best in terms of fairness (Boczoń and Wilson, 2018).

However, none of the above papers deal with the potential implications of draw constraints for the outcome of the tournament. The only exception is Boczoń and Wilson (2018), where a figure presents that a same nation pairing allowed in the UEFA Champions League Round of 16 draw decreases the number of same nation match-ups in the subsequent rounds by about 10%—but the issue is discussed in a single paragraph without any theoretical background and the effect of alternative policies is not analysed.

Our main contributions to the topic can be summarised as follows:

- First in the literature, the effects of draw constraints in a knockout tournament are extensively explored in both theoretical and simulation models.
- Introducing an association constraint in the first round(s) is documented to increase the likelihood of a same nation matchup to approximately the same extent in each subsequent round.
- We show that, if the favourite teams are concentrated in some national associations, they will have a higher probability to win the tournament in the presence of an association constraint. This is especially important for international tournaments where higher-ranked countries can delegate more teams such as in the club football competitions of the UEFA.
- According to our simulations, the UEFA policy of banning the same nation matchups in the first round of the knockout stage turns out a strange compromise between maintaining the international character of the tournament and avoiding the dominance of particular nations. Imposing association constraints in later rounds is an option worth further consideration.

The simulation model is based on the structure of the UEFA Europa League used between the 2009/10 and 2020/21 seasons when the knockout phase started with the Round of 32. Currently, the Europa League consists of knockout round play-offs with 16 teams, followed by the Round of 16, quarterfinals, semifinals, and the final. Since there is an association constraint in the draw of both the knockout round play-offs and the Round of 16, our results essentially justify a recent decision of the UEFA.

The remainder of the paper is structured as follows. Section 2 motivates the research topic through the example of the UEFA Europa League. A basic mathematical model is presented in Section 3. Section 4 investigates the implications of the association constraint in 12 recent Europa League seasons via simulations. In particular, the methodology is detailed in Section 4.1 and the results are discussed in Section 4.2. Section 5 provides concluding remarks and some directions for future research.
Table 1: Same nation matchups in the UEFA Europa League knockout stage between the 2009/10 and 2020/21 seasons

| Season       | Round         | Association |
|--------------|---------------|-------------|
| 2009/10      | quarterfinals | Spain       |
| 2010/11      | semifinal     | Portugal    |
| 2010/11      | final         | Portugal    |
| 2011/12      | semifinal     | Spain       |
| 2011/12      | final         | Spain       |
| 2012/13      | —             | —           |
| 2013/14      | Round of 16   | Italy       |
| 2013/14      | Round of 16   | Spain       |
| 2013/14      | semifinals    | Spain       |
| 2014/15      | Round of 16   | Italy       |
| 2014/15      | Round of 16   | Spain       |
| 2015/16      | Round of 16   | England     |
| 2015/16      | Round of 16   | Spain       |
| 2015/16      | quarterfinals | Spain       |
| 2016/17      | Round of 16   | Belgium     |
| 2016/17      | Round of 16   | Germany     |
| 2017/18      | —             | —           |
| 2018/19      | quarterfinals | Spain       |
| 2018/19      | final         | England     |
| 2019/20      | —             | —           |
| 2020/21      | —             | —           |

2 Motivation: tournament rules and outcomes in the UEFA Europa League

The UEFA Europa League is the secondary club football tournament of Europe. Between the 2009/10 and 2021/21 seasons, its group stage has consisted of 12 groups with four teams each. The top two from each group—altogether 24 teams—have qualified for the knockout stage, where eight third-placed teams from the UEFA Champions League (the most prestigious club football tournament of the continent) group stage have joined them. In this period, the rules of the knockout phase have not changed. The Round of 32 pairings have been determined by a draw according to some constraints (UEFA, 2020, Article 17):

- The 12 Europa League group winners and the four best third-placed teams from the Champions League group stage are drawn against the 12 Europa League group runners-up and the remaining four third-placed teams from the Champions League group stage;

- Clubs from the same national association cannot play against each other;

- The winners and runners-up of the same Europa League group cannot play against each other.

However, there have not been any restrictions in the draw of the subsequent rounds (Round of 16, quarterfinals, semifinals), hence some clashes have taken place between teams from
Table 2: The national associations of the UEFA Europa League finalists between the 2009/10 and 2020/21 seasons

| Season   | Association of the winner | Association of the runner-up |
|----------|---------------------------|-----------------------------|
| 2009/10  | Spain                     | England                     |
| 2010/11  | Portugal                  | Portugal                    |
| 2011/12  | Spain                     | Spain                       |
| 2012/13  | England                   | Portugal                    |
| 2013/14  | Spain                     | Portugal                    |
| 2014/15  | Spain                     | Ukraine                     |
| 2015/16  | Spain                     | England                     |
| 2016/17  | England                   | Netherlands                 |
| 2017/18  | Spain                     | France                      |
| 2018/19  | England                   | England                     |
| 2019/20  | Spain                     | Italy                       |
| 2020/21  | Spain                     | England                     |

the same country as Table 1 reveals. This implies the first question to be addressed here: What is the effect of the association constraint in the Round of 32 on the likelihood of a same nation matchup in the subsequent rounds?

According to Table 2, the Europa League has recently been dominated by English and Spanish teams. Among the 12 winners, three have come from England and eight from Spain (more than 90%), while there have been seven English and nine Spanish finalists out of 24 (two-third). Consequently, the organiser should be careful not to use a rule that significantly favours these teams.

In the Round of 32 draw, the group constraint can hardly be debated; repeated matchups are worth avoiding as they are probably less interesting for the spectators. The association constraint might also be explained by a similar argument because these teams play against each other in their domestic leagues, too. But the knockout phase of the parallel UEFA Champions League starts with the Round of 16, where both the group and association constraints are valid. Hence the association constraint would be reasonable to require in the Europa League Round of 16 draw.

To better understand the effects of the association constraint, we compare three opportunities to draw a knockout tournament with 32 teams:

- **Method 1**: there are no restrictions in the draw due to the associations of the teams.

- **Method 2**: two teams from the same association cannot be drawn against each other in the Round of 32.

- **Method 3**: two teams from the same association can be drawn against each other neither in the Round of 32 nor in the Round of 16.

The three options will be evaluated first in a simple mathematical model, followed by a simulation based on the historical results of the UEFA Europa League.
3 Analytical results

Some consequences of the draw constraints can be uncovered in a basic mathematical model. Assume that there are two teams from the same country \( W \) in a knockout tournament with 32 teams, each of them having a probability \( w \) of advancing against any of the remaining 30 teams and 0.5 against each other. All other teams are equally skilled, they win with a 50% chance against each other.

Inspired by Section 2, two measures will be analysed:

- The probability that the two teams from country \( W \) play against each other;
- The probability that the winner comes from country \( W \).

Consider Method 1. The probability of a same nation matchup in the Round of 32 is:

\[
P_{32}^1 = \frac{1}{31}.
\]

In the Round of 16, it is:

\[
P_{16}^1 = \left(1 - P_{32}^1\right) \cdot \frac{1}{15} \cdot w^2 = \frac{30}{31} \cdot \frac{1}{15} \cdot w^2.
\]

In the quarterfinals, it is:

\[
P_8^1 = \frac{30}{31} \cdot \frac{14}{15} \cdot \frac{1}{7} \cdot w^4.
\]

In the semifinals, it is:

\[
P_4^1 = \frac{30}{31} \cdot \frac{14}{15} \cdot \frac{6}{7} \cdot \frac{1}{3} \cdot w^6.
\]

In the final, it is:

\[
P_2^1 = \frac{30}{31} \cdot \frac{14}{15} \cdot \frac{6}{7} \cdot \frac{2}{3} \cdot w^8.
\]

Consider Method 2. The probability of a same nation matchup in the Round of 32 is \( P_{32}^2 = 0 \). In the Round of 16, it is:

\[
P_{16}^2 = \frac{1}{15} \cdot w^2.
\]

In the quarterfinals, it is:

\[
P_8^2 = \frac{14}{15} \cdot \frac{1}{7} \cdot w^4.
\]

In the semifinals, it is:

\[
P_4^2 = \frac{14}{15} \cdot \frac{6}{7} \cdot \frac{1}{3} \cdot w^6.
\]

In the final, it is:

\[
P_2^2 = \frac{14}{15} \cdot \frac{6}{7} \cdot \frac{2}{3} \cdot w^8.
\]

Consider Method 3. The probability of a same nation matchup in the Round of 32 is \( P_{32}^3 = 0 \). In the Round of 16, it is \( P_{16}^3 = 0 \), too. In the quarterfinals, it is:

\[
P_8^3 = \frac{1}{7} \cdot w^4.
\]
In the semifinals, it is:

\[ P_3^3 = \frac{6}{7} \cdot \frac{1}{3} \cdot w^6. \]

In the final, it is:

\[ P_2^2 = \frac{6}{7} \cdot \frac{2}{3} \cdot w^8. \]

Now we turn to the second issue. Consider Method 1. Any team from country \( W \) can win the tournament in two ways. First, the two teams from country \( W \) might play a match against each other, hence the probability of being the final winner is:

\[ P_1^{32} \cdot w^4 + P_1^{16} \cdot w^3 + P_1^8 \cdot w^2 + P_1^4 \cdot w + P_1^2. \]

Second, the other team from country \( W \) might be eliminated before it plays against the winner, which gives the probability:

\[ 2w^5 \cdot \left[ \frac{30}{31} \cdot (1 - w) + \frac{30}{15} \cdot \frac{14}{15} \cdot w \cdot (1 - w) + \frac{30}{31} \cdot \frac{14}{15} \cdot \frac{6}{7} \cdot w^2 \cdot (1 - w) + \frac{30}{31} \cdot \frac{14}{15} \cdot \frac{6}{7} \cdot \frac{2}{3} \cdot w^3 \cdot (1 - w) \right]. \]

The sum of the two terms provides the chance that the winner comes from association \( W \).

Consider Method 2. The following formula gives the probability that the winner is from association \( W \) and plays against the other team from its association:

\[ P_2^{32} \cdot w^4 + P_2^{16} \cdot w^3 + P_2^8 \cdot w^2 + P_2^4 \cdot w + P_2^2. \]

Analogously, the probability that the winner comes from association \( W \) and does not play against the other team from its association in the tournament is:

\[ 2w^5 \cdot \left[ (1 - w) + \frac{14}{15} \cdot w \cdot (1 - w) + \frac{14}{15} \cdot \frac{6}{7} \cdot w^2 \cdot (1 - w) + \frac{14}{15} \cdot \frac{6}{7} \cdot \frac{2}{3} \cdot w^3 \cdot (1 - w) \right]. \]

Consider Method 3. The following formula gives the probability that the winner is from association \( W \) and plays against the other team from its association:

\[ P_3^{32} \cdot w^4 + P_3^{16} \cdot w^3 + P_3^8 \cdot w^2 + P_3^4 \cdot w + P_3^2. \]

Similarly, the probability that the winner comes from association \( W \) and does not play against the other team from its association in the tournament is:

\[ 2w^5 \cdot \left[ (1 - w) + w \cdot (1 - w) + \frac{6}{7} \cdot w^2 \cdot (1 - w) + \frac{6}{7} \cdot \frac{2}{3} \cdot w^3 \cdot (1 - w) \right]. \]

These calculations can be easily generalised to a knockout tournament with \( 2^k \) teams.

Figure 1 shows how the introduction of a draw restriction affects the probability that the two teams from country \( W \) meet in the tournament. If the association constraint holds only in the Round of 32 (Method 2), the probability of such a clash is multiplied by the same factor in any subsequent rounds since \( P_2^{32} \cdot P_2^{16} = P_2^8 \cdot P_2^4 = P_2^4 \cdot P_2^1 = P_2^1 \cdot P_2^1 = \frac{31}{30} \).

Analogously, if the association constraint holds in both the Round of 32 and the Round of 16, the probability of such a clash is multiplied by the same factor in any subsequent rounds since \( P_2^{32} \cdot P_2^{16} = P_2^8 \cdot P_2^4 = P_2^4 \cdot P_2^1 = P_2^1 \cdot P_2^1 = \frac{31}{30} \cdot \frac{15}{14} \).

However, the sums of these probabilities are naturally not linear functions of each other because they depend on the winning probability \( w \). According to the left panel of Figure 1, Method 2 is able to reduce the probability of a same nation matchup by at most \( \frac{1}{31} \), which is the chance
that the two teams from country $W$ are paired in the Round of 32 in the absence of the draw constraint (Method 1). On the other hand, Method 3 becomes the most effective when the winning probability is around 0.7, that is, association $W$ is relatively strong. The explanation is obvious: prohibiting a particular match in the Round of 16 can be useful if the two teams reach this stage with a substantial probability. The chart suggests a theoretical result, too.

**Proposition 3.1.** The probability that the two teams from country $W$ play against each other is not increased by the draw restrictions.

**Proof.** For Method 2, $P_{32}^1 + P_{16}^1 + P_{8}^1 + P_{4}^1 + P_{2}^1 = \frac{1}{31} + \frac{30}{31} \cdot (P_{16}^2 + P_{8}^2 + P_{4}^2 + P_{2}^2) \geq P_{16}^2 + P_{8}^2 + P_{4}^2 + P_{2}^2$ because the latter sum is a probability, hence it cannot be greater than one.

The calculation for Method 3 is also elementary and left to the reader. \qed

The cost of a match between the two teams from the same country $W$ is not necessarily uniform, it might be more embarrassing in a later round of the tournament. For this purpose, a weighted probability is computed with the weights derived from the 2019/20 UEFA club competitions revenue distribution system (UEFA, 2019). Qualification for the knockout stage of the Europa League is awarded by the following amounts: (a) 0.5 million Euros for the Round of 32; (b) 1.1 million Euros for the Round of 16; (c) 1.5 million Euros for the quarterfinals; (d) 2.4 million Euros for the semifinals; and (e) 4.5 million Euros for the final. Since the prize money provides the weights, the weighted probability of a same nation matchup under Method 1 is:

$$\frac{0.5P_{32}^1 + 1.1P_{16}^1 + 1.5P_{8}^1 + 2.4P_{4}^1 + 4.5P_{2}^1}{4.5}.$$
However, this value can serve only for comparative purposes; its maximum is less than one as the two dominating teams do not necessarily meet in the final even if $w = 1$.

The right panel of Figure 1 shows the effects of restrictions on the weighted probability of a match between the two teams from country $W$. While Methods 2 and 3 are better than Method 1 if $w$ does not exceed 0.75, the prohibition of such a clash in the first round(s) becomes detrimental in the case of strong teams.

Turning to the second question of our study, Figure 2 reveals how the impact of the draw constraint depends on the winning probability $w$ with respect to the likelihood that the winner of the tournament comes from country $W$. Obviously, if the teams of this association are weak ($w < 0.5$), both Methods 2 and 3 are unfavourable for them as they cannot play against each other at the beginning of the tournament. On the other hand, the consequences are more serious when they often defeat any other team. Again, the result is intuitive; banning a match between the strong teams helps them to win the tournament.

To summarise, the abstract mathematical model conveys two important messages:

- If draw constraints apply in the first round(s), the probability of a clash between the two teams from the same association is multiplied by the same factor in each subsequent round.

- If the association with two teams is relatively strong, imposing an association constraint favours its teams.

The second observation is especially relevant because the winning probability $w$ is more likely to be above half in the real world. The reason is that the teams qualifying for similar knockout tournaments are not chosen randomly, countries with better teams usually get more places in the group stage and their clubs also have a higher probability to qualify for the knockout stage.
4 Simulation results

In the following, the effects of the association constraint in the UEFA Europa League will be quantified. As Section 3 demonstrated, an exact mathematical computation remains impossible due to the complex interactions between the constraints for different national associations. Therefore, the research questions will be addressed via Monte Carlo simulations, a standard approach in the analysis of tournament designs (Scarf et al., 2009; Goossens et al., 2012; Lasek and Gagolewski, 2018; Csató, 2021d). To that end, the draw procedure should be replicated and it needs to be determined which team advances to the next stage from a given match. We will also try to connect the results of the simulations to the analytical findings.

4.1 Methodology

If there are some draw constraints, it is a non-trivial problem to pair the competitors since a valid matching should be obtained (Csató, 2021c). UEFA has followed its usual procedure (Klößner and Becker, 2013; Boczoń and Wilson, 2018) in the Europa League Round of 32 draw between the 2009/10 and the 2020/21 seasons. In particular, the 16 unseeded teams (12 Europa League group runners-up and four lower-ranked third-placed teams from the Champions League) are drawn randomly from an urn. For each team, the set of possible opponents is established by a computer program in order to avoid any dead end, a situation when the remaining teams cannot be paired. Then a club is drawn randomly from this set of possible opponents, and the pair of the two drawn teams is added to the matching. The video of the 2020/21 UEFA Europa League Round of 32 draw is available at https://www.uefa.com/uefaeuropaleague/draws/2021/2001241/.

Example 1. Assume that there are four seeded teams $T_1$–$T_4$ and four unseeded teams $T_5$–$T_8$ still to be drawn under the following constraints:

- Certain pairs of clubs have played in the same group, hence they cannot be drawn against each other ($T_1$–$T_5$, $T_2$–$T_6$, $T_3$–$T_7$, $T_4$–$T_8$);

- Some clubs are from the same national association: teams $\{T_2, T_5\}$ from country $A$ and teams $\{T_1, T_4, T_6\}$ from country $B$.

First, $T_5$ is drawn. It has two possible opponents, $T_3$ and $T_4$, because $T_1$ is excluded by the group constraint and $T_2$ is excluded by the association constraint. However, if $T_5$ plays against $T_3$, there remains no feasible assignment for $T_6$ ($T_2$ is prohibited by the group constraint, whereas $T_1$ and $T_4$ are excluded by the association constraint). Hence, $T_5$ should be drawn against $T_4$ since a draw condition is anticipated to apply. Second, $T_6$ is drawn and paired with $T_3$. Finally, there are two group winners and two group runners-up without any restriction, thus the opponent of $T_7$ will be either $T_1$ (implying a match between $T_8$ and $T_2$) or $T_2$ (implying a match between $T_8$ and $T_1$).

According to Example 1, the draw in the Round of 32 is more complicated than it might seem at first glance. For instance, the conditions affecting the clubs still to be drawn should be taken into account, and the cardinality of the set of teams against which an unseeded team is allowed to play is not necessarily decreasing.

Our simulation is based on the UEFA mechanism above if there exists any draw restriction, that is, in the Round of 32 under Method 2, as well as in the Round of 32 and
Table 3: Winning probabilities in the UEFA Europa League knockout stage between the 2009/10 and 2020/21 seasons

(a) By type of the qualification

| Team 1               | Team 2               | Winning probability of Team 1 |
|----------------------|----------------------|------------------------------|
| EL group winner      | EL group runner-up   | 85/139 \approx 0.612         |
| EL group winner      | CL seeded            | 22/43 \approx 0.512          |
| EL group winner      | CL unseeded          | 30/59 \approx 0.508          |
| EL group runner-up   | CL seeded            | 14/53 \approx 0.264          |
| EL group runner-up   | CL unseeded          | 5/11 \approx 0.455           |
| CL seeded            | CL unseeded          | 11/16 = 0.6875               |

Abbreviations: EL group winner = UEFA Europa League group winner; EL group runner-up = UEFA Europa League group runner-up; CL seeded = higher-ranked third-placed team from the UEFA Champions League group stage; CL unseeded = lower-ranked third-placed team from the UEFA Champions League group stage.

(b) By national association

| Team 1 | Team 2 | Winning probability of Team 1 |
|--------|--------|------------------------------|
| English| Spanish| 6/16 \approx 0.375            |
| English| Other  | 50/67 \approx 0.746           |
| Spanish| Other  | 58/73 \approx 0.795           |

in the Round of 16 under Method 3. In all other rounds, a random matching is picked up due to the absence of draw constraints.

The UEFA emergency panel ruled on 17 July 2014 that Ukrainian and Russian clubs could not be drawn against each other due to the political unrest between the countries. This constraint is never taken into account in our study.

Regarding the likelihood of advancing, we use separate assumptions for the two issues addressed:

- Same nation matchup: the probability of winning depends on the past performance of the club. Four cases are distinguished according to the rules of the Round of 32 draw (Europa League group winner, Europa League runner-up, seeded Champions League third-placed team, unseeded Champions League third-placed team).

- The winner comes from a particular country: the probability of winning depends on the national association of the club. Three cases are distinguished as suggested by Table 2 (English clubs, Spanish clubs, clubs from all other countries).

In both cases, the winning probability is determined by historical data from the 12 Europa League seasons organised between 2009/10 and 2020/21. They are presented in Table 3. For example, Europa League group winners have played 139 matches against Europa League group runners-up, and the former team has won 85 times (Table 3.a). Analogously, there have been 67 clashes between clubs from England and all other nations except for Spain, among which 50 have been won by the English club (Table 3.b). If the two clubs have the same type, the probability of winning is assumed to be 0.5.

The effect of the association constraint depends on the identity of the participants. Table 4 details the association constraints in each season. Note that at least two teams from
Table 4: National associations with more than one club in the UEFA Europa League knockout stage by season

|        | 2009/10 | 2010/11 | 2011/12 | 2012/13 | 2013/14 | 2014/15 | 2015/16 | 2016/17 | 2017/18 | 2018/19 | 2019/20 | 2020/21 |
|--------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| AUT    | 110| 110| 010| 010| 010| 100| 100| 100| 100| 100| 100| 100|
| BEL    | 200| 300| 010| 010| 110| 120| 120| 120| 120| 120| 120| 120|
| CZE    | 100| 100| 010| 010| 110| 120| 120| 120| 120| 120| 120| 120|
| ENG    | 020| 020| 120| 120| 120| 130| 130| 130| 130| 130| 130| 130|
| FRA    | 010| 200| 110| 120| 010| 120| 120| 120| 120| 120| 120| 120|
| GER    | 120| 200| 110| 120| 110| 130| 130| 130| 130| 130| 130| 130|
| GRE    | 020| 010| 110| 110| 020| 010| 110| 110| 010| 110| 110| 110|
| ITA    | 100| 020| 020| 120| 110| 310| 210| 200| 300| 120| 110| 300|
| NED    | 120| 020| 210| 120| 110| 130| 130| 130| 130| 130| 130| 130|
| POL    | 020| 020| 110| 110| 020| 020| 110| 110| 010| 110| 110| 110|
| POR    | 200| 200| 110| 110| 010| 110| 110| 110| 010| 110| 110| 110|
| ROU    | 010| 010| 010| 010| 110| 110| 110| 110| 010| 110| 110| 110|
| RUS    | 200| 200| 110| 110| 110| 010| 110| 110| 110| 110| 110| 110|
| SCO    | 020| 020| 020| 020| 020| 020| 020| 020| 020| 020| 020| 020|
| SRB    | 120| 110| 110| 110| 110| 210| 110| 110| 110| 110| 110| 110|
| ESP    | 120| 120| 020| 020| 210| 020| 110| 110| 300| 210| 300| 200|
| SUI    | 010| 010| 010| 010| 010| 010| 010| 010| 010| 010| 010| 010|
| TUR    | 200| 010| 010| 010| 010| 200| 100| 100| 010| 010| 010| 010|
| UKR    | 100| 200| 030| 110| 110| 110| 110| 110| 110| 110| 110| 110|

** Abbreviations:** AUT = Austria; BEL = Belgium; CZE = Czech Republic; ENG = England; FRA = France; GER = Germany; GRE = Greece; ITA = Italy; NED = Netherlands; POL = Poland; POR = Portugal; ROU = Romania; RUS = Russia; SCO = Scotland; SRB = Serbia; ESP = Spain; SUI = Switzerland; TUR = Turkey; UKR = Ukraine.

**Note:** In any tuple $i j k \ell$, $i$ indicates the number of Europa League group winners, $j$ the number of Europa League runners-up, $k$ the number of seeded third-placed teams from the Champions League group stage, and $\ell$ the number of unseeded third-placed teams from the Champions League group stage. The total number of exclusions generated by an association in the Round of 32 draw equals $(i + k) \times (j + \ell)$. 
Spain have qualified for the Round of 32 every year, while the association constraint for England and Germany have not influenced the tournament in one season only, respectively. Therefore, all simulations are run separately for the 12 seasons with 10 million iterations.

4.2 Assessing the implications of the association constraint

Figure 3 shows how the association constraint influences the frequency of a match played by clubs from the same country. The UEFA policy has increased the probability of such a clash in each round of the Europa League between 1% and 5% in the period considered. Remarkably, the changes are mostly driven by the composition of the competitors, that is, they almost coincide for the Round of 16, quarterfinals, semifinals, and the final, which reinforces our finding from the basic mathematical model. This observation remains valid if the association constraint is introduced in the Round of 16, too, when the probability of a same nation matchup grows by at least 6% but not more than 12% in every subsequent round.

Imposing the association constraint raises the danger of a game played by two teams from the same country in all subsequent rounds. Thus Figure 4 plots the cumulated impact both in the unweighted and weighted settings; in the latter case, a match at the end of the tournament is judged to be more costly, similarly to Section 3. Although UEFA has reduced the probability of such an unwanted matchup by at most 60%, the gain remains below 30% if the rounds of these clashes are taken into account. On the other hand, extending the association constraint to the Round of 16 would have cut the likelihood of a same nation matchup by at least 60% in the unweighted scenario, and by more than 30% even in the weighted case. Shortly, the effectiveness of Method 3 in the worst case almost coincides with the effectiveness of Method 2 in the best case.

The introduction of the association constraint may contribute to the dominance of
countries with strong teams. According to Table 3, Spanish clubs progress against any non-English team with a probability of 80%, which explains their excellent performance in the Europa League (see Table 2). As Figure 5 presents, the UEFA rule has only marginally increased the already high winning probability of Spanish teams that has been above 40% in each season. Nonetheless, this country has benefited each year from prohibiting clashes
between its teams in the Round of 32. Imposing the association constraint in the Round of 16 (Method 3), too, would have decreased further the chances of teams outside England and Spain, but the effects seem to remain at a tolerable level. The simulation reinforces our result obtained from the mathematical model, namely, the association constraint favours the teams of relatively strong countries with respect to their winning probabilities.

Consequently, the rules of the draw in the UEFA Europa League knockout stage can hardly be justified: requiring the association constraint only in the Round of 32 is a strange compromise between avoiding same nation matchups and the dominance of English and Spanish clubs. Banning such clashes in the Round of 16 would have significantly reduced the likelihood of their occurrence at a moderated price. UEFA is encouraged to investigate the issue more deeply and consider applying the association constraint in the later rounds of international tournaments.

With the start of a new competition called UEFA Europa Conference League—the third tier of European club football—from the 2021/22 season, the Europa League has also been reformed to contain knockout round play-offs with 16 teams (contested by the eight group runners-up and the eight third-placed teams from the Champions League group stage) and a knockout stage starting with the Round of 16 (contested by the eight group winners and the eight winners of the play-offs). Analogously, the Europa Conference League group stage is followed by knockout round play-offs with 16 teams (contested by the eight group runners-up and the eight third-placed teams from the Europa League group stage) and a knockout stage starting with the Round of 16 (contested by the eight group winners and the eight winners of the play-offs). Since there exists an association constraint in both the Europa League (UEFA, 2021b, Articles 17 and 18) and the Europe Conference League (UEFA, 2021a, Articles 17 and 18) draws of the knockout round play-offs and the Round of 16, UEFA has essentially implemented our recommendation above.

5 Conclusions

Inspired by the recent seasons of the UEFA Europa League, the present paper has attempted to uncover some consequences of applying draw constraints in a knockout tournament. First, we have formulated a simple mathematical model to understand how a restriction can affect the probability of a match played by two teams from the same country, as well as the likelihood that the winner comes from a particular country. After that, the role of the association constraint has been analysed via Monte Carlo simulations based on historical results of the Europa League. Imposing the association constraint in the Round of 32 draw has increased the chance of a same nation matchup by less than 5% in any subsequent round and has not favoured England and Spain to a great extent. Nonetheless, the international character of the Europa League could have been improved further by prohibiting matches between teams from the same country even in the Round of 16 draw.

There are several ways to continue our research. Even though the complex interactions between competitors from many countries are difficult to handle by analytical tools, mathematical results can be developed. The simulation model can be refined with respect to the winning probabilities. Finally, this study has ignored the problem that the restrictions make the draw unevenly distributed, which is unfair (Klößner and Becker, 2013). Unfortunately, these distortions can be mitigated only by slacking the constraints (Boczoń and Wilson, 2018).
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