Decay coupling constant sum rules for tetraquarks \( T[(\bar{Q}q)(Q\bar{q})] \)
with broken SU(3) symmetry

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Abstract

For tetraquarks of the form \( T[(\bar{Q}q)(Q\bar{q})] \) we give sum rules for their decay coupling constants, taking into account the SU(3) symmetry breaking interactions to first order.

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I. INTRODUCTION

Two hidden-bottom charged meson resonances, $Z_b(10610)$ and $Z_b(10650)$ with $J^P = 1^+$, were observed recently by the Belle Collaboration [1]. Since these decay into $\pi^\pm \Upsilon(nS)$ ($n = 1, 2, 3$) and $\pi^\pm h_b(mP)$ ($m = 1, 2$) they have been interpreted [2–4] as tetraquarks of the form $T(Q\bar{Q} q\bar{q})$. A natural decay mode for such a tetraquark would be

$$T(Q\bar{Q} q\bar{q}) \rightarrow H(Q\bar{Q}) + M(q\bar{q}),$$

(1)

where $H(Q\bar{Q})$ is a heavy meson with $Q = b$ or $c$ and $M(q\bar{q})$ could be the pseudoscalar or vector meson SU(3) flavour octet. For such tetraquarks we recently [5] gave sum rules for tetraquark decay coupling constants taking into account the SU(3) breaking interactions to first order.

In this paper we consider tetraquarks of the form $T[(\bar{Q}q)_k (Q\bar{q})_i]$ where the heavy quark $Q = b$ or $c$ and the light quarks $q = u, d, s$. For a given $Q$ there will be nine such tetraquarks. The natural decay mode will be into two mesons $(\bar{Q}q)_k$ and $(Q\bar{q})_i$ subject to angular momentum and parity selection rules. For example, a $J^P = 0^+$ tetraquark would decay into two pseudoscalar mesons.

We consider the decays

$$T[(\bar{Q}q) (Q\bar{q})] \rightarrow M(\bar{Q}q) + M(Q\bar{q}),$$

(2)

in Sec. III below. We give sum rules for the nine decay couplings constants taking into account the SU(3) breaking interactions to first order.

An important difference between the tetraquarks $T(Q\bar{Q} q\bar{q})$ considered earlier and the tetraquarks $T[(\bar{Q}q) (Q\bar{q})]$ considered here is that the latter can have OZI [6–8] suppressed decays through annihilation of $Q^a Q_\beta$ through gluons into lighter quarks $\bar{q}^a q_\beta$ (greek letters represent the color indices). These modes are briefly discussed in the concluding remarks.

II. NOTATION

We denote the tetraquark made of a heavy meson and its antiparticle as

$$T^i_k = T[(\bar{Q}q_k) (Q\bar{q})],$$

(3)

with $i, k = 1, 2, 3$ (latin letters representing flavour indices). The heavy quark $Q = b$ or $c$ is SU(3) flavour singlet. The light quarks $q_k$ ($k = 1, 2, 3$) transform as SU(3) 3 while the antiquarks $\bar{q}^i$ ($i = 1, 2, 3$) transform as $\bar{3}$. In color space, $M_k \equiv (\bar{Q}q_k)$ and $M^i \equiv (Q\bar{q})^i$ form color singlets and transform as 3 and $\bar{3}$ of SU(3) flavour, respectively.

The natural decay mode for the tetraquark would be

$$T^i_k \rightarrow M_k + M^i.$$  

(4)

The tetraquark states can be represented as a $3 \times 3$ matrix $T$ with $T^i_k$ as matrix elements, so that,

$$T = \begin{pmatrix} T^1_1 & T^1_2 & T^1_3 \\ T^2_1 & T^2_2 & T^2_3 \\ T^3_1 & T^3_2 & T^3_3 \end{pmatrix}. $$  

(5)
In SU(3) flavour space this transforms as $1 \oplus 8$. The mesons $M_k$ and $\bar{M}^i$ transform as SU(3) $3$ and $\bar{3}$, respectively. The $3 \times 3$ matrix $\mathcal{M}$ representing the final states is

$$
\mathcal{M} = \begin{pmatrix}
    M^1_1 & M^2_1 & M^3_1 \\
    M^1_2 & M^2_2 & M^3_2 \\
    M^1_3 & M^2_3 & M^3_3
\end{pmatrix},
$$

(6)

where $\mathcal{M}^i_k$ represents $M_k + \bar{M}^i$ in the final state. In flavour space it also transforms as $1 \oplus 8$, this is clear since $(\bar{Q}q) \otimes (Q\bar{q})$ transform as $3 \otimes \bar{3} = 1 \oplus 8$.

### III. SUM RULES

In unbroken SU(3) there will be a single decay coupling constant $G_0$; with $\lambda_8$ breaking the tetraquark octet can form two octets, symmetric (coupling constant $G_D$) and antisymmetric (coupling constant $G_F$). In matrix form we can write the decay coupling constant as

$$
G \left[ T^i_k (8) \rightarrow M_k + \bar{M}^i \right] = G_0 \text{Tr}[\bar{T}\mathcal{M}] + G_D \text{Tr}[\bar{T}(\lambda_8\mathcal{M} + \mathcal{M}\lambda_8)] + G_F \text{Tr}[\bar{T}(\lambda_8\mathcal{M} - \mathcal{M}\lambda_8)].
$$

(7)

Explicitly, for the nine tetraquark decays

$$
G(T^1_1 \rightarrow M_1 + \bar{M}^1) = G_0 + 2G_D,
$$

(8)

$$
G(T^2_2 \rightarrow M_2 + \bar{M}^2) = G_0 + 2G_D,
$$

(9)

$$
G(T^3_3 \rightarrow M_3 + \bar{M}^3) = G_0 - 4G_D,
$$

(10)

$$
G(T^2_1 \rightarrow M_1 + \bar{M}^2) = G_0 + 2G_D,
$$

(11)

$$
G(T^1_2 \rightarrow M_2 + \bar{M}^1) = G_0 + 2G_D,
$$

(12)

$$
G(T^3_1 \rightarrow M_1 + \bar{M}^3) = G_0 - G_D + 3G_F,
$$

(13)

$$
G(T^3_2 \rightarrow M_2 + \bar{M}^3) = G_0 - G_D + 3G_F,
$$

(14)

$$
G(T^1_3 \rightarrow M_3 + \bar{M}^1) = G_0 - G_D - 3G_F,
$$

(15)

$$
G(T^2_3 \rightarrow M_3 + \bar{M}^2) = G_0 - G_D - 3G_F.
$$

(16)

Nine decays and three coupling constants. So, six sum rules or relations,

$$
G(T^1_1 \rightarrow M_1 + \bar{M}^1) = G(T^2_2 \rightarrow M_2 + \bar{M}^2) = G(T^3_1 \rightarrow M_1 + \bar{M}^3) = G(T^3_2 \rightarrow M_2 + \bar{M}^3); \ (17)
$$

3
\[ G(T_1^3 \rightarrow M_1 + \bar{M}^3) = G(T_2^3 \rightarrow M_2 + \bar{M}^3), \quad G(T_3^1 \rightarrow M_3 + \bar{M}^1) = G(T_3^2 \rightarrow M_3 + \bar{M}^2); \quad (18) \]

\[ G(T_2^2 \rightarrow M_2 + \bar{M}^2) + G(T_3^3 \rightarrow M_3 + \bar{M}^3) = G(T_1^3 \rightarrow M_1 + \bar{M}^3) + G(T_1^1 \rightarrow M_3 + \bar{M}^1). \quad (19) \]

If and when such tetraquarks are observed one can extract the coupling constants from the observed decay rates. For a $J^P = 0^+$ tetraquark decaying into two pseudoscalar mesons the decay width is

\[ \Gamma = \frac{1}{8\pi} \frac{k}{M^2} |A|^2, \quad (20) \]

where $M$ is the tetraquark mass, $k$ is the momentum of a decay particle and $A$ is the transition amplitude. For $s$-wave decay $(0^+ \rightarrow 0^- + 0^-)$ $A = GM$ with $G$ the coupling constant.

IV. CONCLUDING REMARKS

To date the tetraquarks $T[(\bar{Q}q)(Q\bar{q})]$ have not been observed experimentally. However, their decays products, the heavy mesons (for example $D_S^+ = c\bar{s}$, $B^+ = u\bar{b}$, etc.) have been observed [3]. It is conceivable that this type of tetraquark may be seen as a resonance in the heavy quark particle-antiparticle mass spectra, for example, $D_S^+ + D_S^-, D^0 + \bar{D}^0$, etc. Another possibility is that they may be observed through their decay into light meson plus anti meson pairs, for example, $q_k\bar{q}^l + q_l\bar{q}^i$ (see Fig. 1). These are possible through annihilation of $QQ$ through gluons into $\bar{q}q$. However, such decays are likely to be OZI [6–8] suppressed.

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[1] Belle Collab. (A. Bondar et al.), Phys. Rev. Lett. 108, 122001 (2012).
[2] T. Guo, L. Cao, M. Z. Zhou and H. Chen, arXiv: 1106.2284 [hep-ph].
[3] F. S. Navarra, M. Nielsen and J.-M. Richard, arXiv: 1108.1230 [hep-ph].
[4] For a discussion of all tetraquarks containing only $c$ and $s$ quarks see: V. Gupta, Int. J. Mod. Phys. A. 20, 5891 (2005).
[5] V. Gupta, G. Sánchez-Colón, and S. Rajpoot, Mod. Phys. Lett. A 27, 1250165 (2012).
[6] S. Okubo, Phys. Lett. 5, 165 (1963).
[7] G. Zweig, CERN Report. TH 401 and 412 (1964).
[8] J. Izuka, Prog. Theor. Phys. Suppl. 37 and 38, 21 (1966).
[9] Particle Data Group (J. Beringer et al.), Phys. Rev. D 86, 010001 (2012).
FIG. 1: Tetraquarks $T[(Qq)(Qq)]$ observed through their OZI suppressed decay into light meson plus anti meson pairs, $q_k \bar{q}_l + q_l \bar{q}_k$. Greek letters represent the color indices and latin letters represent flavor indices.