NON-PERTURBATIVE STRUCTURE OF THE NUCLEON

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ABSTRACT

Chiral perturbation theory is the effective field theory of the standard model. In this talk, I discuss some applications of this framework to the pion–nucleon system. These are chiral corrections to the S-wave pion–nucleon scattering lengths, the reaction $\pi N \rightarrow \pi \pi N$ at threshold and low–energy theorems in $\pi^0$ photoproduction.

1. Effective Field Theory of QCD

In the sector of the three light quarks ($u, d, s$), one can write the QCD Lagrangian as

$$
\mathcal{L}_{\text{QCD}} = \mathcal{L}^0_{\text{QCD}} - \overline{q} \mathcal{M} q,
$$

with $q^T = (u, d, s)$ and $\mathcal{M} = \text{diag}(m_u, m_d, m_s)$ the current quark mass matrix. The current quark masses are believed to be small compared to the typical hadronic scale, $\Lambda \chi \approx 1 \text{ GeV}$. $\mathcal{L}^0_{\text{QCD}}$ admits a global chiral symmetry, i.e. one can independently rotate the left– and right–handed components of the quark fields. This symmetry is spontaneously broken down to its vectorial subgroup, $\text{SU}(3)_{L+R}$, with the appearance of eight massless Goldstone bosons. The explicit chiral symmetry breaking due to the quark mass term gives these particles, identified with the pions, kaons and eta, a small mass. The consequences of the spontaneous and the explicit chiral symmetry breaking can be calculated by means of an effective field theory (EFT), called chiral perturbation theory. $\mathcal{L}_{\text{QCD}}$ is mapped onto an effective Lagrangian with hadronic degrees of freedom,

$$
\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{eff}}[U, \partial U, \ldots, \mathcal{M}, N],
$$

where the matrix–valued field $U(x)$ parametrizes the Goldstones, $\mathcal{M}$ keeps track of the explicit symmetry violation and $N$ denotes matter fields (like e.g. the nucleon). While the latter are not directly related to the symmetry breakdown, their interactions are severely constrained by the non–linearly realized chiral symmetry and one can thus incorporate them unambiguously. $\mathcal{L}_{\text{eff}}$ admits an energy expansion,

$$
\mathcal{L}_{\text{eff}} = \mathcal{L}^{(2)}_{\pi\pi} + \mathcal{L}^{(4)}_{\pi\pi} + \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(3)}_{\pi N} + \mathcal{L}^{(4)}_{\pi N} + \ldots,
$$

where the superscript $(i)$ refers to the number of derivatives and/or meson mass insertions. I restrict myself to the two–flavor sector. The first two terms in Eq. comprise the meson sector whereas the next four are relevant for processes involving one single nucleon. The ellipsis stands for terms with more nucleon fields and/or more derivatives. The various terms
contributing to a certain process are organized by their *chiral* dimension $D$ (which differs in general from the physical dimension) as follows:\(^2\)

$$D = 2L + 1 \sum_d (d-2) N^\pi\pi_d + \sum_d (d-1) N^\pi\pi_d,$$

with $L$ the number of (pion) loops and $d$ the vertex dimension (derivatives or factors of the pion mass). Lorentz invariance and chiral symmetry demand that $d \geq 2$ ($\geq 1$) for mesonic (pion–nucleon) interactions. So to lowest order, one has to deal with tree diagrams ($L = 0$) which is equivalent to the time–honored current algebra (CA). However, we are now in the position of *systematically* calculating the corrections to the CA results. It is also important to point out that $\mathcal{L}^{(4)}_{\pi\pi}$ and $\mathcal{L}^{(2,3,4)}_{\pi\pi}$ contain parameters not fixed by symmetry, the so–called low–energy constants (LECs). These have to be determined from data or can be estimated from resonance exchange.\(^5\) The whole machinery is well documented, see e.g. Ref.\(^6\).

2. Structure of the Nucleon

Here, I wish to list some processes which have been studied in detail to give a flavor about where the CHPT machinery does apply in the (single) nucleon sector. References can be traced back from.\(^7\) It is worth to stress that tests of chiral dynamics heavily rely on very precise data at low energies. Fortunately, over the last few years, such accurate data (for very different processes) have become available and much more are coming.

- $\pi N \to \pi N$: Of particular interest are the S–wave scattering lengths and the so–called pion–nucleon $\sigma$–term (strangeness in the nucleon). In section 3, I will consider the chiral corrections to the threshold $\pi N$ amplitudes.

- $\pi N \to \pi \pi N$: This reaction has attracted particular interest since it supposedly allows to pin down the S-wave $\pi \pi$ scattering lengths. The corresponding accurate CHPT predictions\(^2\) are one of the premier tests of chiral (Goldstone boson) dynamics. These issues are discussed in section 4.

- $\gamma N \to \gamma N$: Low energy Compton scattering has been investigated in detail experimentally as well as theoretically over the last years. The empirical facts concerning the nucleons’ electromagnetic polarizabilities find a natural explanation in CHPT. Furthermore, predictions for the spin–dependent amplitude have been made (spin–polarizability, slope of the generalized DHG sum rule). These predictions will be tested at BNL and CEBAF. Extensions to the three flavor sector have also been performed and measurements using the hyperon beams at CERN and FNAL are eagerly awaited for.

- $\gamma N \to \pi^0 N$: Here, it was shown that an existing low–energy theorem for the S–wave multipole $E_{0+}$ was incorrect and that the chiral expansion for the electric dipole amplitude is slowly converging. New data from MAMI and SAL (for $\gamma p \to \pi^0 p$) seem to indicate a smaller value (in magnitude) of $E_{0+}$ in agreement with CHPT predictions. Also, novel P–wave LETs have been given and agree with indirect determinations. An accepted experiment at MAMI involving polarization will give a direct test. Also, the reaction $\gamma n \to \pi^0 n$ should be measured. A brief discussion is given in Section 5.

- $\gamma^* N \to \pi^* N$: Charged pion electroproduction is of particular interest since it allows to determine the axial form factor of the nucleon at small momentum transfer. A venerable LET due to Nambu et al. was modified and previously existing discrepancies
to determinations of $G_A(t)$ from neutrino–nucleon reactions could be explained. Furthermore, the new NIKHEF and MAMI data on neutral pion electroproduction show some puzzling features which have yet to be understood.

- $\mu p \rightarrow n\nu\mu$: Ordinary muon capture at rest allows to measure the induced pseudoscalar coupling constant $g_P$. In CHPT, a very accurate prediction can be made, $g_P = 8.44 \pm 0.23$. Presently available determinations are not yet accurate enough to disentangle this from the simple pion pole (CA) prediction.

To end this short survey, I would like to stress again that all these processes are to be considered in the threshold region, i.e. at small energy and momentum transfer. Only there the CHPT machinery applies.

### 3. The Isovector Pion–Nucleon S–Wave Scattering Length

One of the most splendid successes of current algebra was Weinberg’s prediction for the S–wave pion–nucleon scattering lengths:

$$a_{CA}^+ = 0, \quad a_{CA}^- = \frac{M_\pi}{8\pi F_\pi^2} \left( \frac{1}{1 + M_\pi/m_p} \right) = 0.079 M_\pi^{-1},$$

with $M_\pi = 139.57$ MeV the charged pion mass, $m_p = 938.27$ MeV the proton mass, $F_\pi = 92.5$ MeV the pion decay constant and the superscripts $+/−$ refer to the isoscalar and isovector $\pi N$ amplitude, respectively. The Karlsruhe–Helsinki phase shift analysis of $\pi N$ scattering leads to $a^- = 0.092 \pm 0.002 M_\pi^{-1}$ and $a^+ = −0.008 \pm 0.004 M_\pi^{-1}$, impressively close to the CA prediction, Eq.5. However, over the last few years there has been some controversy about the low–energy $\pi N$ data which has not yet been settled. Consequently, the uncertainties in $a^\pm$ are presumably larger and even the sign of $a^+$ could be positive. A more direct way to get a handle at these zero momentum (i.e. threshold) quantities is the measurement of the strong interaction shift ($\epsilon_{1S}$) and the decay width ($\Gamma_{1S}$) in pionic atoms. The PSI-ETH group has recently presented first results of their impressive measurements in pionic deuterium and pionic hydrogen. The consequent analysis of the data leads to $a^- = 0.096 \pm 0.007 M_\pi^{-1}$ and $a^+ = −0.0077 \pm 0.0071 M_\pi^{-1}$. If one combines the pionic hydrogen shift measurement with the one from the pionic deuterium, one has $a^- = 0.086 \pm 0.002 M_\pi^{-1}$ and $a^+ = 0.002 \pm 0.001 M_\pi^{-1}$. The largest uncertainty comes from the width measurement of pionic hydrogen. Both determinations are consistent within one standard deviation. We conclude that $a^-$ is larger than the CA value and that $a^+$ is consistent with zero.

Within CHPT, the chiral corrections to Eq.5 have been calculated in Ref. There it was shown that the isoscalar scattering length is very sensitive to some LECs which are not known to such an accuracy. In contrast, to order $M_\pi^3$ the only sizeable corrections to $a^-$ come from the one loop diagrams, the counter term contribution is small (as estimated from $\Delta(1232)$ and $N^*(1440)$ exchange). In Ref. it was furthermore shown that the one loop graphs with exactly one insertion from $\mathcal{L}_{\pi N}^{(2)}$ sum up to zero. Contact terms from $\mathcal{L}_{\pi N}^{(4)}$ can not contribute to $T^-\bar{u}d$ due to crossing. Estimating the uncertainties conservatively, one therefore arrives at a band for $a^-$,

$$0.088 M_\pi^{-1} \leq a^- \leq 0.096 M_\pi^{-1},$$

which is consistent with the various empirical values discussed before and 10...20% larger than the CA prediction. As already stressed in Ref., it is the chiral loop correction at
order $M^3_\pi$ which closes the gap between the lowest order (CA) prediction and the empirical value. An indication of the size of the next corrections can be obtained by writing the one-loop result as $a^- = a^-_{\text{CA}}(1 + \delta_1) \simeq a^-_{\text{CA}} \exp(\delta_1)$. The next correction follows to be $\delta_2^2/2$ which is of the order of $1 \ldots 2\%$ of $a^-_{\text{CA}}$.

4. $\pi N \rightarrow \pi \pi N$ at Threshold

The reaction $\pi N \rightarrow \pi \pi N$ is of particular interest since it contains, besides many other contributions, the four-pion vertex. This offers the possibility to extract the S-wave $\pi\pi$ scattering lengths which are of fundamental importance to our understanding of the chiral QCD dynamics. Over the last years, many accurate threshold data have been compiled, but their theoretical interpretation rested on the ancient Olsson–Turner model (which is the same as tree level CHPT when one sets the pre-QCD chiral symmetry breaking parameter $\xi = 0$). In Ref.\textsuperscript{14} the first corrections to the threshold amplitudes $D_1$ and $D_2$ were calculated and some novel LETs were formulated. $D_{1,2}$ are related to the more commonly used $A_{2I=0,I=0}$ via

$$A_{32} = \sqrt{10} D_1, \quad A_{10} = -2 D_1 - 3 D_2,$$

with $I_{\pi N}$ the total isospin of the initial pion–nucleon system and $I_{\pi\pi}$ the isospin of the final two–pion system. These LETs show the expected pattern of deviation from the empirical values, namely small and sizeable for $A_{32}$ and $A_{10}$, respectively. To that order, however, nothing can be said about the $\pi\pi$ scattering amplitude. The task of calculating the second corrections has been taken up in Ref.\textsuperscript{15} $D_{1,2}$ admit an expansion of the form

$$D_i = d_i^0 + d_i^1 \mu + d_i^2 \mu^2 + \mathcal{O}(\mu^3), \quad i = 1, 2,$$

modulo logs and $\mu = M_\pi/m$. The upshot of the lengthy calculation in Ref.\textsuperscript{15} is the following (the numbers given here should be considered preliminary). The chiral expansion for $D_1$ converges nicely and one thus is able to extract the isospin two S-wave scattering length $a^2_0$,

$$a^2_0 = -0.052 \pm 0.013,$$

compatible with the one loop CHPT prediction of $a^2_0 = -0.042 \pm 0.008$\textsuperscript{12}. In contrast, in case of $D_2$ the terms of order $\mu^2$ are large and have a sizeable uncertainty. Therefore, the small contribution from the $\pi\pi$ interaction can only be isolated with poor precision. In particular, the excitation of the Roper resonance and its subsequent decay into the nucleon and two pions in the S-wave has to be understood much better and also certain LECs related to the $\pi N$ scattering amplitude. The corresponding extracted value for $a^0_0$ is

$$a^0_0 = 0.16 \pm 0.05,$$

which is compatible with the CA value of 0.16 and the one–loop CHPT prediction of $0.20 \pm 0.01$\textsuperscript{12}. As stated before, the theoretical uncertainty is much smaller than the one deduced from the $\pi\pi N$ threshold amplitude $A_{10}$. These are good and bad news. Of course, $a^0_0$ can be determined e.g. from $K_{l4}$ decays or pionic molecules, so the reaction $\pi N \rightarrow \pi \pi N$ offers the complementary information on $a^2_0$. A calculation of the next corrections to $D_1$ would be very welcome to be able to further sharpen the extraction of $a^2_0$. For all the details, see\textsuperscript{15}. 
5. Low-Energy Theorems in $\pi^0$ Photoproduction

Space forbids to discuss in detail the interesting story of the theoretical and experimental determinations of the S–wave multipole $E_{0+}$ in neutral pion photoproduction off protons. Already in 1991 it was shown that in the derivation of the old "LET" some terms at order $M^2_\pi$ had been overlooked. Unfortunately, the reexamination of the Saclay and Mainz data gave a value seemingly supporting this incomplete expression. This has led many to try to resurrect or reinterpret the old "LET". An educational discussion is given in Ref.\textsuperscript{16} Well, there is yet another twist to the story. The new data from SAL and MAMI are in the process of being analyzed and seem to indicate a value of $E_{0+}$ much smaller (in magnitude) than the old "LET" value and a less pronounced energy dependence in the first 15 MeV above threshold.\textsuperscript{17,18} If confirmed, this would be a nice support of the CHPT calculations. Although the expansion of the electric dipole amplitude in powers of the pion mass is slowly converging, the range of values predicted is definitively smaller in magnitude than the presently believed empirical value of $(-2.0\pm0.2)\times10^{-3}M^{-1}_\pi$. In fact, the preliminary analysis shown by Bernstein at this symposium points toward a great success of CHPT - only with the loop contribution one is able to understand the result for $E_{0+}(\omega)$.

Almost as a by–product came the formulation of LETs for the slopes of the P–wave multipole $P_{1,2}$ at threshold.\textsuperscript{19} These are related to the more commonly used electric and magnetic multipoles via

$$P_1 = 3E_{1+} + M_{1+} - M_{1-}, \quad P_2 = 3E_{1+} - M_{1+} + M_{1-}. \quad (11)$$

The expansion of $P_{1,2}$ in powers of $M_\pi$ is quickly converging and thus these quantities are a good testing ground for the chiral dynamics. The third P–wave multipole $P_3 = 2M_{1+} + M_{1-}$ is dominated by the $\Delta(1232)$. These statements are counterintuitive to many practitioners in the field. It is therefore important to stress that only with the new CV machines and improved detector technology one can access this narrow window of chiral physics. It is a sociologically interesting phenomenon how these facts are ignored by some, who believe that nothing can’t be learned any more from such ”old” physics. Quite the contrary is true. One can indirectly infer these P–wave combinations from the unpolarized data and finds satisfactory agreement with the predictions. However, some assumptions have to be made since the system is underdetermined, so such results can only be considered indicative. A direct test of these P–wave LETs will come once the reaction $\bar{\gamma}p \to \pi^0p$ has been measured and analyzed at MAMI. The extension to electroproduction can be found in Ref.\textsuperscript{20}

Finally, a remark on the resonance contributions is at order since that still seems to be a red herring to some. In the $q^4$ calculation\textsuperscript{19} of $\gamma p \to \pi^0p$, three LECs appear, one related to $P_3(\omega)$ and two to $E_{0+}(\omega)$. The numerical value of the P–wave LEC can be quite accurately understood from $\Delta(1232)$ and $\rho$–meson exchange. The old Mainz data, however, led to a puzzle for the two S–wave LECs.\textsuperscript{13} To the contrary, analyzing the new MAMI data, one finds that the numerical values of these two LECs obtained from a best fit to the total and differential cross sections in the threshold region can be reproduced by resonance exchange to a good accuracy. Thus, in the threshold region, resonances do not pose a problem to CHPT, even not the close–by $\Delta$. This important lesson has not yet penetrated the prejudices of most people.
6. Outlook

More accurate data are necessary to systematically explore the strictures of the explicit and spontaneous chiral symmetry breaking in QCD. As has become clear from the many interesting talks at this symposium, we will soon be in a much better situation to pin down certain LECs and make more detailed predictions. From the theoretical side, two major issues need clarification, these are the extension to the three–flavor case and the consistent implementation of isospin–breaking (via the quark mass differences and virtual photons).

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