Early thermalization at RHIC*

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It is shown that recent RHIC data on hadron spectra and elliptic flow can be excellently reproduced within a hydrodynamic description of the collision dynamics, and that this provides strong evidence for rapid thermalization while the system is still in the quark-gluon plasma phase. But even though the hydrodynamic approach provides an impressive description of the single-particle momentum distributions, it fails to describe the two-particle momentum correlation (HBT) data for central Au+Au collisions at RHIC. We suggest that this is not likely to be repaired by further improvements in our understanding of the early collision stages, but probably requires a better modelling of the freeze-out process. We close with a prediction of the phases of the azimuthal oscillations of the HBT radii in noncentral collisions at RHIC.

1. ELLIPTIC FLOW AS AN EARLY QGP SIGNATURE

The quark-gluon plasma (QGP) is a thermalized system and, as such, has thermal pressure. If the QGP is created in a heavy-ion collision, this pressure acts against the surrounding vacuum and causes a rapid collective expansion ("flow") of the reaction zone, the "Little Bang". Collective flow is an unavoidable consequence of QGP formation in heavy-ion collisions, and its absence could be taken as proof that no such plasma was ever formed. Its presence, on the other hand, does not automatically signal QGP formation. Detailed studies of the observed final state flow pattern are necessary to convince oneself that the reflected time-integrated pressure history of the collision region indeed requires a thermalized state in the early collision stage whose pressure and energy density are so high that it can no longer be mistaken as consisting of conventional hadronic matter.

Much progress in this direction was recently achieved by studying elliptic flow [1] in non-central (non-zero impact parameter) heavy-ion collisions. It characterizes the azimuthal anisotropy in the transverse plane of the final momentum distribution near midrapidity (y = 0) and is quantified by the second harmonic coefficient $v_2(y, p_\perp; b)$ of a Fourier expansion in $\phi_p$ of the measured hadron spectrum $dN/(dy \ p_\perp \ dp_\perp \ d\phi_p)$ [2]. (The first harmonic coefficient $v_1$ measures the directed flow or “bounce-off” of the colliding nuclei at forward and backward rapidities [3]; it vanishes at midrapidity by symmetry.)

Since individual nucleon-nucleon collisions produce azimuthally symmetric spectra, such final state momentum anisotropies must be generated dynamically during the nuclear reaction. They require the existence of an initial spatial anisotropy of the reaction zone,

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either by colliding deformed nuclei [4–6] or by colliding spherical nuclei at non-zero impact parameter \( b \neq 0 \) (the practical method of choice so far). The transfer of the initial spatial anisotropy onto a final momentum anisotropy requires final state interactions (rescattering) within the produced matter; without them the primordial azimuthally symmetric momentum distribution survives. Microscopic transport calculations [7,8] show a monotonic dependence of \( v_2 \) on the opacity (density times scattering cross section) of the produced matter which is inversely related to its thermalization time. These studies strongly suggest that, for a given initial spatial anisotropy \( \epsilon_x \), the maximum momentum-space response \( v_2 \) is obtained in the hydrodynamic limit which assumes perfect local thermal equilibrium at every space-time point (i.e. a thermalization time which is much shorter than any macroscopic time scale in the system). Any significant delay of thermalization (modelled, for example, as an initial free-streaming stage) causes a decrease of the initial spatial anisotropy without concurrent build-up of momentum anisotropies, thereby reducing the finally observed elliptic flow signal [3].

This specific sensitivity of the elliptic flow to rescattering and pressure build-up in the early collision stages [1,10] (before the spatial deformation and the resulting anisotropies of the pressure gradients have disappeared [3]) puts \( v_2 \) on the list of “early signatures” of the collision dynamics. In contrast to other early probes (which use rare signals such as hard photons and dileptons, heavy quarkonia and jets), \( v_2 \) can be extracted from the bulk of the measured hadrons which are very abundant and thus easily accessible. In fact, the elliptic flow measurement in Au+Au collisions at \( \sqrt{s} = 130 \text{ AGeV} \) [11] became the second publication of RHIC data and appeared within days of the end of the first RHIC run.

In this talk I will present results from hydrodynamic simulations of hadronic spectra and elliptic flow at RHIC energies. We will see that the hydrodynamic approach provides an excellent quantitative description of the bulk of the data and fails only for very peripheral Au+Au collisions and/or at high \( p_{\perp} > 1.5 - 2 \text{ GeV/c} \). That the hydrodynamic approach fails if the initial nuclear overlap region becomes too small or the transverse momentum of the measured hadrons becomes too large is not unexpected. However, where exactly hydrodynamics begins to break down gives important information about the microscopic rescattering dynamics. What is really surprising is that the hydrodynamic approach works so well in semi-central collisions where it quantitatively reproduces the momenta of more than 99% of the particles: below \( p_{\perp} = 1.5 \text{ GeV/c} \) the elliptic flow data [11,13] actually exhaust the hydrodynamically predicted [6,14–17] upper limit. The significance of this agreement can hardly be overstressed (see also E. Shuryak’s concluding talk), and it poses significant challenges for microscopic descriptions of the early collision dynamics.

However, not all is well with hydrodynamics: in its present form, it fails to reproduce the HBT measurements [15,19] which constrain the freeze-out distribution in space-time. This problem, which affects more our understanding of the late freeze-out stage than of the early thermalization stage, will be discussed in Sec. 4.

2. HYDRODYNAMIC FIREBALL EXPANSION

The natural language for describing collective flow phenomena is hydrodynamics. Its equations control the space-time evolution of the pressure, energy and particle densities and of the local fluid velocity. The system of hydrodynamic equations is closed by speci-
fying an equation of state which gives the pressure as a function of the energy and particle densities. In the ideal fluid (non-viscous) limit, the approach assumes that the microscopic momentum distribution is thermal at every point in space and time (note that this does not require chemical equilibrium – chemically non-equilibrated situations can be treated by solving separate and coupled conservation equations for the particle currents of individual particle species). Small deviations from local thermal equilibrium can in principle be dealt with by including viscosity, heat conduction and diffusion effects, but such a program is made difficult in practice by a number of technical and conceptual questions [20] and has so far not been very successful for relativistic fluids. Stronger deviations from local thermal equilibrium require a microscopic phase-space approach (kinetic transport theory), but in this case the concepts of an equation of state and of a local fluid velocity field themselves become ambiguous, and the direct connection between flow observables and the equation of state of the expanding matter is lost.

The assumption of local thermal equilibrium in hydrodynamics is an external input, and hydrodynamics offers no insights about the equilibration mechanisms. It is clearly invalid during the initial particle production and early recattering stage, and it again breaks down towards the end when the matter has become so dilute that rescattering ceases and the hadrons “freeze out”. The hydrodynamic approach thus requires a set of initial conditions for the hydrodynamic variables at the earliest time at which the assumption of local thermal equilibrium is applicable, and a “freeze-out prescription” at the end. For the latter we use the Cooper-Frye algorithm [21] which implements an idealized sudden transition from perfect local thermal equilibrium to free-streaming. This is not unreasonable because freeze-out (of particle species $i$) is controlled by a competition between the local expansion rate $\partial \cdot u(x)$ (where $u^\mu(x)$ is the fluid velocity field) and the local scattering rate $\sum_j \langle \sigma_{ij} v_{ij} \rangle \rho_j(x)$ (where the sum goes over all particle species with densities $\rho_j(x)$ and $\langle \sigma_{ij} v_{ij} \rangle$ is the momentum-averaged transport cross section for scattering between particle species $i$ and $j$, weighted with their relative velocity); while the local expansion rate turns out to have a rather weak time-dependence, the scattering rate drops very steeply as a function of time, due to the rapid dilution of the particle densities $\rho_j$ [22], causing a rapid transition to free-streaming. – A better algorithm [23,15] switches from a hydrodynamic description to a microscopic hadron cascade at or shortly after the quark-hadron transition, before the matter becomes too dilute, and lets the cascade handle the freeze-out kinetics. The resulting flow patterns [13] from such an improved freeze-out algorithm don’t differ much from our simpler Cooper-Frye based approach.

The main advantage of the microscopic freeze-out algorithm [23,15] is that it also correctly reproduces the final chemical composition of the fireball, since the particle abundances already freeze out at hadronization, due to a lack of particle-number changing inelastic rescattering processes in the hadronic phase [24]. Our version of the hydrodynamic approach uses an equation of state which assumes local chemical equilibrium all the way down to kinetic freeze-out at $T_f \approx 125$ MeV and thus is unable to reproduce the correct hadron yield ratios. We therefore adjust the normalization of the momentum spectra for the rarer particle species (kaons, protons, antiprotons) by hand to reproduce the chemical equilibrium ratios at a chemical freeze-out temperature $T_{\text{chem}} = 165$ MeV. The absolute normalization of the pion spectra is adjusted through the initial energy density in central collisions; for non-central collisions no new parameters enter since the centrality
dependence of the initial conditions is completely controlled by the collision geometry (see below).

3. RADIAL AND ELLIPTIC FLOW FROM HYDRODYNAMICS

We solve the relativistic equations for ideal hydrodynamics (for details see [14,6]). To simplify the numerical task, we analytically impose boost invariant longitudinal expansion [25,1], without giving up any essential physics as long as we focus on the transverse expansion dynamics near midrapidity (the region which most RHIC experiments cover best).

Figure 1. Charged pion and antiproton spectra from central (left panel) and semi-central to peripheral (middle and right panel) Au+Au collisions at $\sqrt{s} = 130$ $A$ GeV. The data were taken by the PHENIX [29] and STAR [30] collaborations, the curves show hydrodynamical calculations (for details see text).

The hydrodynamic expansion starts at time $\tau_{eq}$ which we fixed by a fit to hadron spectra from central Pb+Pb collisions at the SPS and then extrapolated to RHIC initial conditions, taking into account the higher initial parton density [6]. For each impact parameter the initial energy density profile in the transverse plane is calculated from a Glauber parametrization using a realistic nuclear thickness function [6,26]. We have explored a number of different options involving various combinations of “hard” and “soft” production mechanisms for the initial particle production [26]. While all were found to give almost indistinguishable results for the charged particle elliptic flow, the radial flow (i.e. the slopes of the hadron transverse mass spectra) and the centrality dependence of the charged particle rapidity density at midrapidity, $(dN_{ch}/dy)(y = 0)$, exhibit sensitivity to the initial transverse density profile [26]. We here present results for initial conditions at $\tau_{eq} = 0.6$ fm/c calculated from a mixture of 25% “hard” (binary collision) and 75% “soft” (wounded nucleon) contributions [26] to the initial entropy density (or parton density), with a maximal entropy density $s_{\text{max}} = 85$ fm$^{-3}$ at the fireball center in central collisions (corresponding to a maximal energy density $e_{\text{max}} = 21.4$ GeV/fm$^3$ and a maximal temperature $T_{\text{max}} = 328$ MeV). At the standard time $\tau = 1$ fm/c for energy density estimates from the measured multiplicity density using Bjorken’s formula [25], this corresponds to an average energy density $\langle e \rangle (1 \text{ fm/c}) = 5.4$ GeV/fm$^3$ which is about 70% higher than the value reported from 158 A GeV Pb+Pb collisions at the SPS. (Note that $\langle e \rangle$ at $\tau_{eq} = 0.6$ fm/c is nearly twice as large!) The corresponding central values and profiles for
peripheral collisions are then given by the Glauber model [6,26]. For the kinetic freeze-out temperature we took $T_f = 128$ MeV, independent of centrality.

In Fig. 1 we show the (absolutely normalized) single particle $p_\perp$-spectra for charged pions and antiprotons measured in Au+Au collisions at RHIC together with the hydrodynamical results. The latter were normalized in central collisions as described at the end of Sec. 2, but their centrality dependence and shapes are then completely fixed by the model. The agreement with the data is impressive; for antiprotons the data go out to $p_\perp \leq 3$ GeV/$c$, and the hydrodynamic model still works within error bars! Only for very peripheral collisions (impact parameter $b > 10$ fm) the data show a significant excess of high-$p_\perp$ particles at $p_\perp > 1.5$ GeV/$c$. Teaney et al. [15] showed that this excellent agreement requires a phase transition (soft region) in the equation of state; without the transition, the agreement is lost, especially when the constraints from SPS data and from the elliptic flow measurements below are taken into account.

It should be stressed that in the hydrodynamic picture the fact that for $p_\perp > 2$ GeV/$c$ antiprotons become more abundant than pions (left panel of Fig. 1) is not surprising, but a simple consequence of the strong radial flow at RHIC. For a hydrodynamically expanding thermalized fireball, at relativistic transverse momenta $p_\perp \gg m_0$ all hadron spectra have the same slope [31], and at fixed $m_\perp \gg m_0$ their relative normalization is given by $(g_i\lambda_i)/(g_j\lambda_j)$ (where $g_{i,j}$ is the spin-isospin degeneracy factor and $\lambda_{i,j} = e^{\mu_{i,j}/T}$ is the fugacity of hadron species $i,j$). At RHIC the baryon chemical potential at chemical freeze-out is small, $\mu_B/T_{\text{chem}} \approx 0.26$ [32], and $\mu_\pi = 0$; the $\bar{p}/\pi^-$ ratio at fixed and sufficiently large $m_\perp$ is thus predicted to be larger than 1: $(\bar{p}/\pi^-)_{m_\perp} = 2 \exp[-(\mu_B + \mu_\pi)/T_{\text{chem}}] \approx 1.5$ (where the factor 2 arises from the spin degeneracy of the $\bar{p}$).

Figure 2 shows the elliptic flow coefficient $v_2$ from Au+Au collisions at RHIC [11,33,34] and for identified pions and protons (right panel [13]) from 130 A GeV Au+Au collisions. The left panel shows the $p_\perp$-averaged elliptic flow as a function of collision centrality, parametrized by the charged multiplicity density $n_{ch}$ at midrapidity ($n_{\text{max}}$ corresponds to the value in central collisions). The right two panels show the differential elliptic flow $v_2(p_\perp)$ for minimum bias collisions. The data were collected by the STAR collaboration [11,33,34]. The curves in the left two panels are hydrodynamic calculations corresponding to different choices for the initial energy density profile (see [26] for details). The curves in the right panel were published in [17].
compared with hydrodynamic calculations. For impact parameters $b \leq 7$ fm (corresponding to $n_{ch}/n_{max} \geq 0.5$) and transverse momenta $p_{\perp} \leq 1.5 - 2$ GeV/c the data are seen to exhaust the upper limit for $v_2$ obtained from the hydrodynamic calculations. For larger impact parameters $b > 7$ fm the $p_{\perp}$-averaged elliptic flow $v_2$ increasingly lags behind the hydrodynamic prediction, indicating a lack of early thermalization when the initial overlap region becomes too small. The $p_{\perp}$-differential elliptic flow stops following the hydrodynamic curves for $p_{\perp} > 2$ GeV/c [33] (not shown in Fig. 2), indicating incomplete thermalization of high-$p_{\perp}$ particles. Both these effects are expected; what is surprising is the excellent agreement otherwise, including the hydrodynamically predicted mass-dependence of $v_2$ [17] as seen in the right panel of Fig. 2.

The high level of agreement with hydrodynamics becomes even more impressive after you begin to realize how easily it is destroyed: As stressed in Sec. 1, it requires the build-up of momentum anisotropies during the very early collision stages when the spatial anisotropy of the reaction zone is still appreciable, causing significant anisotropies of the pressure gradients. A delay in thermalization by more than about 1 fm/c (2 fm/c) dilutes the spatial anisotropy and the hydrodynamically predicted elliptic flow coefficient by 10% (25%) [3] which is more than is allowed by the data. Parton cascade simulations with standard HIJING input generate almost no elliptic flow and require an artificial increase of the opacity of the partonic matter by a factor 80 to reproduce the RHIC data [8]. Hadronic cascades of the RQMD and URQMD type (in which the high-density initial state is parametrized by non-interacting, pressureless QCD strings) predict [35] too little elliptic flow and a decrease of $v_2$ from SPS to RHIC, contrary to the data.

The elliptic flow is self-quenching [3]: it makes the reaction zone grow faster along its initially short direction and thus eventually eliminates its own cause. As the spatial deformation of the fireball goes to zero, the elliptic flow saturates [3]. The saturation time scale times $c$ is of the order of the transverse size of the initial overlap region (at lower energies it is a bit longer, see Figs. 7, 9 in [4]). At RHIC energies and above, the time it takes the collision zone to dilute from the high initial energy density to the critical value for hadronization is equal to or longer than this saturation time: most or all of the elliptic flow is generated before any hadrons even appear! It thus seems that the only possible conclusion from the successful hydrodynamic description of the observed radial and elliptic flow patterns is that the thermal pressure driving the elliptic flow is partonic pressure, and that the early stage of the collision must have been a thermalized quark-gluon plasma.

4. THE RHIC HBT PUZZLE

Hydrodynamics not only predicts the momentum-space structure of the hadron emitting source at freeze-out, but also its spatial structure. Bose-Einstein (a.k.a. Hanbury Brown-Twiss (HBT)) two-particle intensity interferometry allows to access the r.m.s. widths of the space-time distribution of hadrons with a given momentum $p$ [33]. One of the interesting questions one can try to address with this tool is whether at RHIC the reaction zone really flips the sign of its spatial deformation between initial impact and final freeze-out, as predicted by hydrodynamics [3] where the reaction zone changes from a significant initial elongation perpendicular to the reaction plane to a smaller final elongation into the
reaction plane. The answer to this question turns out to be non-trivial, on two different levels: first, hydrodynamics, at least with the presently implemented initial conditions and freeze-out algorithm, fails to reproduce even for central Au+Au collisions the measured HBT radii extracted from two-pion correlations \cite{18,19}. I’ll show how and explain why. Second, for expanding systems the HBT radii don’t measure the entire freeze-out region, but only the effective emission regions (“regions of homogeneity”) for particles of given momentum \cite{36}. For non-central collisions, due to the anisotropic transverse flow these turn out to have a different spatial deformation than the entire (momentum-integrated) freeze-out region, giving rise to a different behaviour of the HBT radii observed at different angles relative to the reaction plane than perhaps naively expected.

![Density contours for the effective emission regions of $Y=0$ pion pairs with transverse momentum $K_{\perp}=0$ (left panels) and $K_{\perp}=0.5$ GeV/c in $x$-direction (right panels). The upper and lower panels show projections on the transverse $x$-$y$ plane and on the $x$-$t$ plane, respectively.](image)

Figure 3. Density contours for the effective emission regions of $Y=0$ pion pairs with transverse momentum $K_{\perp}=0$ (left panels) and $K_{\perp}=0.5$ GeV/c in $x$-direction (right panels). The upper and lower panels show projections on the transverse $x$-$y$ plane and on the $x$-$t$ plane, respectively.

Figure 3 shows the effective emission regions for pion pairs with vanishing and non-vanishing transverse momentum $K_{\perp}$. One sees that the emission region for pions with $K_{\perp}=0$ is spherically symmetric around the fireball center, whereas pions with non-zero $K_{\perp}$ are emitted from a relatively thin crescent-shaped region near the edge of the fireball. This apparent “opacity” of the source \cite{37} is a result of the sharp Cooper-Frye freeze-out combined with the strong radial flow. It correctly reproduces the steeper decrease for increasing $K_{\perp}=0$ of the outward radius $R_{\text{out}}$ compared with $R_{\text{side}}$, which is seen in the RHIC data \cite{18,19} (cf. Fig. 4) and which was already observed (albeit more weakly) at the SPS \cite{38,39}.

Unfortunately, Fig. 4 also shows that the absolute values of $R_{\text{side}}$, $R_{\text{out}}$, and $R_{\text{long}}$ come out quite wrong: $R_{\text{side}}$ is too small whereas $R_{\text{out}}$ and $R_{\text{long}}$ are both too large when compared with the data. The problem with $R_{\text{side}}$ from hydrodynamics being too small
is well-known from the SPS [40]; presumably it is mostly due to the sharp Cooper-Frye freeze-out and seems to be at least partially resolved if the freeze-out kinetics is handled microscopically within a hadronic cascade [41]. The latter gives a more “fuzzy” spatial freeze-out distribution with larger r.m.s. width in the sideward direction.

![Graph](image)

**Figure 4.** HBT radii $R_{\text{out}}$, $R_{\text{side}}$, $R_{\text{long}}$ for central Au+Au collisions at $\sqrt{s} = 130$ A GeV [18], compared with hydrodynamic predictions from the same simulations which provide an excellent fit to the spectra and elliptic flow (solid lines). See text for the other lines.

On the other hand, this “fuzziness” only exacerbates the problems with $R_{\text{long}}$ and $R_{\text{out}}$. It is well known [36] that in high energy heavy-ion collisions the longitudinal HBT radius is controlled by the dynamics of the expanding source via the longitudinal velocity gradient at freeze-out. For a boost invariant longitudinal flow profile this gradient decreases with time as $1/\tau$, leading to rather weak gradients (and correspondingly large values for $R_{\text{long}}$) at the typical hydrodynamic freeze-out time of $\sim 15$ fm/c. The left lower panel of Fig. 3 shows also a long emission time duration. This is actually a consequence of the large $R_{\text{long}}$ because it causes substantial time variations along the constant-$\tau$ freeze-out hypersurface within the longitudinal homogeneity length. The resulting large value $(\delta t)^2 \equiv \langle (t-\bar{t})^2 \rangle$ for the emission duration adds to $R_{\text{out}}^2 = \langle (x-\bar{x})^2 \rangle + \beta_{\perp}^2 (\delta t)^2 - 2 \beta_{\perp} \langle (x-\bar{x})(t-\bar{t}) \rangle$ [38] (where $\beta_{\perp} = K_{\perp}/K^0$ is the transverse velocity of the pion pair) and makes $R_{\text{out}}$ come out significantly larger than $R_{\text{side}}$, contrary to the data (see Fig. 4). These problems are, if anything, worse when freeze-out is handled microscopically [11].

One possibility to make both $R_{\text{long}}$ and $R_{\text{out}}$ smaller would be to force the system to decouple earlier. If there were already some initial transverse collective motion at the thermalization time when the hydrodynamic simulation is started, this might help to build up transverse flow more quickly, leading to an earlier decoupling. We have tested this idea [42] with two extreme assumptions about the transverse expansion prior to thermalization: in one simulation, shown as the dot-dashed curve in Fig. 4, we started the hydrodynamic evolution directly at the parton formation time (for which we took the somewhat arbitrary value $\tau_{\text{form}} = 0.2$ fm/c). In another limit (dashed lines in Fig. 4), we let the partons stream freely from time $\tau_{\text{form}}$ to $\tau_{\text{eq}}$ and matched at $\tau_{\text{eq}}$ the first row of the energy momentum tensor to an ideal fluid form, thereby extracting an initial transverse flow velocity at $\tau_{\text{eq}}$. In both cases the resulting transverse flow “seed” at $\tau_{\text{eq}} = 0.6$ fm/c caused the system to expand more rapidly and farther out into the transverse direction, freezing out 10-20% earlier. Fig. 4 shows that this helps with both $R_{\text{long}}$ and $R_{\text{out}}$, but not as much as required.
by the data. And even though the system expanded to larger values of \( r \), \( R_{\text{side}} \) wouldn’t grow (see Fig. 4) because the homogeneity region only moved farther out but its size did not increase. We don’t see a way to move closer to the data by further modifying the initial conditions, and even the alterations we made to obtain Fig. 4 may turn out to be excluded by the singles spectra and the elliptic flow data (which we haven’t tested yet). We therefore believe that a resolution to the HBT puzzle must lie in the handling of the freeze-out process (although we do not know yet how).

Let me close with a quick preview of results for the HBT radii for non-central collisions, in particular their dependence on the angle \( \Phi \) of the transverse pair momentum \( \vec{K}_\perp \) relative to the reaction plane [36]. Fig. 5 shows hydrodynamic results for \( R_{\text{side}}^2 \), \( R_{\text{out}}^2 \), \( R_{\text{os}}^2 \), and \( R_{\text{long}}^2 \), plotted as functions of \( \Phi \) for a number of different values of \( K_\perp \). While \( R_{\text{long}}^2 \) is almost independent of \( \Phi \), the three other radius parameters show marked azimuthal dependences of the generic form (with all coefficients being positive)

\[
R_{\text{side}}^2(\Phi) = R_{s,0}^2 + R_{s,2}^2 \cos(2\Phi), \quad R_{\text{out}}^2(\Phi) = R_{o,0}^2 - R_{o,2}^2 \cos(2\Phi), \quad R_{\text{os}}^2(\Phi) = R_{\text{os},2}^2 \sin(2\Phi).
\]

Although the magnitudes of the coefficients \( R_{\alpha}^2 \) in Fig. 5 are quite different (and presumably not too trustworthy, given the disagreement with the data for central collisions in Fig. 4), it is surprising that the signs and phases of the oscillations are identical to those calculated and measured at the AGS [44]! At the AGS radial flow effects are thought to be sufficiently weak that the oscillations can be interpreted purely geometrically [44], reflecting a spatially deformed source which is elongated perpendicular to the reaction plane.

Figure 5. HBT radius parameters for 130 A GeV Au+Au collisions at \( b=6 \, \text{fm} \) as a function of the angle \( \Phi \) of \( \vec{K}_\perp \) with respect to the reaction plane, from hydrodynamics.
(as is the case for the initial overlap region). But hydrodynamics predicts a freeze-out configuration which is slightly longer in the reaction plane, as a result of a much stronger expansion in this direction than in the perpendicular one. Shouldn’t the oscillations then have different phases?

The answer is: not necessarily. Due to the strong radial flow at RHIC, the effective emission regions for pairs of given momentum $K_{\perp}$ (as measured by the HBT correlations) cover only a fraction of the transverse source area. Fig. 3 shows that for sufficiently large $K_{\perp}$, these effective emission regions are pushed out of the fireball center and “squashed” towards the transverse edges of the source where one has the largest transverse flow velocities. This effect increases with the transverse flow and with $K_{\perp}$. Flow gradients reduce the regions of homogeneity [36], leading to the shortest correlation radii in the direction with the fastest expansion.

![Schematic plot of the effective emission regions in the fireball created in peripheral Au+Au collisions at RHIC, as predicted by hydrodynamics.](image)

Fig. 6 shows that if the transverse flow is stronger in the reaction plane than perpendicular to it ($v_x > v_y$), then for pairs emitted in $x$-direction the “outward” width $\Delta x_{\text{out}}$ is smaller than for pairs emitted in the $y$-direction. Conversely, the “sideward” width $\Delta x_{\text{side}}$ is larger for pairs emitted in $x$-direction, since for them the flow gradients in the $x_{\text{side}}$-direction are weaker than for pairs emitted in the $y$-direction. Hence we have (at least for large $K_{\perp}$) $\Delta x_{\text{out}}(\Phi=0^\circ) < \Delta x_{\text{out}}(\Phi=90^\circ)$ and $\Delta x_{\text{side}}(\Phi=0^\circ) > \Delta x_{\text{side}}(\Phi=90^\circ)$. For small $K_{\perp} \approx 0$ the effective emission region is centered at $x=y=0$, but the stronger flow gradients in $x$-direction lead in this case to $\Delta x < \Delta y$ which, after translation into $\Delta x_{\text{side}}$ and $\Delta x_{\text{out}}$ at $\Phi=0$ and $90^\circ$, gives again rise to the same ordering. Assuming that the contributions to $R^2_{\text{out}}$ involving the emission time don’t depend strongly on $\Phi$, this explains the sign of the oscillations in the upper row of Fig. 5. In the cross term $R^2_{\text{os}}$, the positive sign of the coefficient multiplying $\sin(2\Phi)$ indicates that the major axes $(x_1, x_2)$ of the emission ellipsoid are tilted clockwise relative to the $(x_{\text{out}}, x_{\text{side}})$ axes in the 1$^{\text{st}}$ and 3$^{\text{rd}}$ quadrant and counterclockwise in the 2$^{\text{nd}}$ and 4$^{\text{th}}$ quadrant. This is again consistent with the flow picture, since the stronger flow in $x$-direction compresses the emission region more strongly in the $x$- than in the $y$-direction, leading to exactly such a tilt.
Hydrodynamics thus makes a clear prediction: at RHIC energies, the phases of the oscillations of the HBT radii as functions of the azimuthal angle $\Phi$ are completely dominated by the anisotropic flow pattern; dynamics rules over geometry. Even if the magnitudes of the radii so far do not fully agree with the data, this qualitative prediction seems quite robust. It will be interesting to see it confirmed (or contradicted) by the experiments.

5. EPILOGUE

I am deeply grateful to Helmut Satz and Frithjof Karsch for the invitation to speak at this conference. When Helmut held his first meeting on Statistical QCD in Bielefeld in 1980 I was still too young to attend: I had just obtained my Ph.D. with a thesis on heavy-ion collisions at the Coulomb barrier, and I was on my way to my first postdoctoral period in the U.S., eager to learn quantum field theory and to work my way up in energy. But I came here two years later, to Helmut’s second Bielefeld meeting, and gave my first talk on quark-gluon transport theory for relativistic heavy-ion collisions. Although Helmut and I never published a paper together, our interactions have always been strong: he influenced and stimulated me, and he supported and challenged me. Sometimes I had to work hard until my arguments were sharp enough to convince him; and sometimes this never happened. My choice of physics problems was affected by my interactions with him, personally and through the literature, and without him my physics career would very likely have evolved quite differently. His famous crystal clear (sometimes I felt: too clear!) presentations of many issues in heavy-ion physics inspired and challenged me. I am sure that I am not the only one who can say this of Helmut, and to express this loudly and clearly is perhaps the greatest compliment we can pay him. Thanks, Helmut, and all my best wishes for a very active career as professor emeritus!

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