Mathematical learning trajectory in primary school

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Abstract. Piaget's theory of cognitive development states that every child has their own way / strategy in interpreting and adapting to their environment. The strategy or method used by students towards learning situation is one of the stages of the learning trajectory. Learning trajectory is a learning line or route that provides an overview of the prerequisite knowledge students already have (as a starting point) and each step from one point to the next that describes the thinking process and methods used by students. This means that students' factors are the main case and must be considered in creating a learning design. By understanding the learning flow, students can provide various alternative strategies or scaffolding to help and overcome students who have difficulty in understanding mathematical concepts in elementary schools.

1. Introduction

Children who are in the early grades of elementary schools are children who are in the early childhood range. This early childhood is a short period, and it is a critical period in one's life. Therefore, all the potential that the child has needs to be encouraged so that it will develop optimally.

Piaget stated that every child has their way of interpreting and adapting to their environment (cognitive development theory). Piaget added that every child has a cognitive structure called schemata, which is a system of concepts that exist in mind as a result of understanding the objects in their environment. These objects occur through assimilation (connecting objects with existing concepts in mind) and accommodation (utilizing the concepts in mind to interpret objects). The two processes, if they occur continuously, will balance the old knowledge and new knowledge. In this way, children can gradually build knowledge through interaction with their environment. Based on this, children's learning behavior is greatly influenced by aspects inside themselves and their environment. The two things cannot be separated because the learning process occurs in the child's interaction with their environment.

Strategies or ways of children towards learning situations and reasoning, and mental development of numeracy vary widely \[1\]. The students' strategies or methods are one of the stages of the learning trajectory. According to Simon \[2\], the learning trajectory is a route that provides an overview of the knowledge or prerequisites that students already have as a starting point, and each step from one point to the next describes the thinking process and methods used by students and the level of thinking shown by the student.

Activities that do not limit the student's path to one route are the key to planning learning experiences that stimulate students' thinking patterns \[3\]. Sarama and Clements \[4\] asserted that understanding children's level of thinking in the classroom is vital in serving all children's needs. Effective learning requires teachers to meet student needs and help build knowledge that students know. So, teachers
should understand learning trajectories, which are how children think and learn mathematics and how to help children learn.

Given the importance of sustainable education in terms of content and process, mathematical learning trajectory becomes an essential issue in learning mathematics. For this reason, as a practitioner of primary education, the author is interested in discussing the mathematical learning trajectory.

2. Methods
The method used is library research [5] by reviewing journals and books that discuss mathematical learning trajectories with the following procedure in figure 1.

![Figure 1. Research procedure.](image)

3. Results and discussion

3.1. Learning trajectory
Simon [2] first used the term learning trajectory, who proposed the hypothetical concept as follows:

A hypothetical learning trajectory provides the teacher with a rationale for choosing a particular instructional design; thus, I (as a teacher) make my design decisions based on my best guess of how learning might proceed. This can be seen in the thinking and planning that preceded my instructional interventions ... as well as the spontaneous decisions that I make in response to students' thinking. The hypothetical learning trajectory is formulated based on three components, namely: the learning goal that defines the direction, the learning activities, and the hypothetical learning process - a prediction of how the students' thinking and understanding will evolve in the context of the learning activities.

Learning trajectories are how children think when they learn to achieve specific goals in mathematical concepts through a series of instructional tasks designed to generate mental processes or actions hypothesized to transfer children's cognitive development by developing children's thinking. Learning trajectory is used to describe the learning transformation resulting from participation in mathematics learning activities.

There are three main components of the learning trajectory: learning goals, learning activities, and hypothetical learning processes. Learning objectives are the first component indicating the need to formulate learning objectives to be achieved after the learning process. The determination of learning objectives is beneficial in determining the direction and learning strategies to be used. Based on the learning objectives that have been formulated, learning activities to achieve learning objectives can be designed. Learning activities are arranged into several sub-activities with sub-learning objectives. The last component is the student learning process hypothesis, which is useful for designing action or alternative strategies to overcome various problems students may face in the learning process.

The following is a learning cycle (Figure 2) that contains a learning flow that is constructed by the teacher for learning planning that refers to: (a) learning objectives, (b) activity and learning arrangements, and (c) learning processes that are possible to involve learners actively.
Simon gives an illustration of the relationship between the learning trajectory and the hypothetical learning trajectory as follows: "In the beginning, you may plan a trip from one place to another as a whole or in part. You will introduce the trip according to your plan. On the way, you must constantly adjust the conditions you encounter. Therefore, you keep trying to gain knowledge about the trip, about the conditions that exist, and about the areas you want to visit. Based on this, you may change your plans related to your previous travel destinations. You modify your route or travel route as a result of interactions with people you meet along the way. You added new goals that you were not familiar with before." The flow that is traversed is called the flow of the trajectory. Meanwhile, the path that is changed at a certain point in the journey is called a hypothetical trajectory.

Based on the illustration above, the learning flow provides a complete picture of what happened or encountered, and the areas stopped by along the way. Thus, a learning flow provides an overview of the prerequisite knowledge that students have as a starting point, and each step from one point to the next describes the thinking process students use, the methods students use, or the levels of thinking that students demonstrate.

By knowing the level and flow of thinking that the child has, the teacher can determine which ones should come first in the development process. The learning flow provides a framework for teachers to develop knowledge of student thinking and learning. Furthermore, knowledge about students' thinking and learning can be used to plan lessons.

In general, the development of children's cognitive abilities begins with concrete things and gradually leads to abstract things. Piaget stated that every child has a cognitive structure called schemata, a system of concepts in mind due to understanding the objects in their environment. Understanding these objects takes place through assimilation (connecting objects with existing concepts in mind) and accommodation (utilizing the concepts in mind to interpret objects). The two processes, if they occur continuously, will balance the old knowledge and new knowledge. In this way, children can gradually build knowledge through interaction with their environment. Based on this, children's learning behavior is greatly influenced by aspects inside themselves and their environment. The two things cannot be separated because the learning process occurs in the child's interaction with their environment.

According to Piaget theory, elementary school-age children are at a concrete operational stage. At this age range, children begin to show learning behaviors as follows:

- Starting to look at the world objectively, shifting from one aspect of the situation to another in a reflective way and seeing the elements simultaneously,
Starting to think operationally,
Using operational thinking to classify things,
Forming and making use of connected rules, simple scientific principles, and employing causal relationships,
Understanding the concept of substance, volume of liquid, length, width, area, and weight, and
Noting the stages of thinking development, the learning tendency of elementary school-age children has three characteristics, namely:
  o Concrete. Concrete contains the meaning of the learning process that departs from things such as being seen, heard, smelled, touched, and manipulated, emphasizing the use of the environment as a learning resource. The use of the environment will produce more meaningful and valuable learning processes and results because students are exposed to actual events and circumstances so that their experiences are more real, factual, meaningful, and accountable.
  o Integrative. At the stage of elementary school-age, children see what is learned. They have not been able to sort out the concepts from various disciplines. This illustrates the deductive way of thinking of children, namely from general things to sectional things.
  o Hierarchical. At the elementary school-age, the way children learn develops gradually from simple things to complex things. This needs to be considered according to the logical order, the relationship between the materials, and the material's breadth and depth.

For every child, the journey from concrete to abstract can be different. Some are fast and some are slow. It may not require many stages for those who are fast, but it needs to go through many stages for those who are slow. Thus, each child may require a different learning trajectory or learning path.

3.2. Benefits of learning trajectory
A learning trajectory provides instructions for the teacher to determine and formulate learning objectives to be achieved. Furthermore, the teacher can make decisions about the strategies that will be used to achieve these goals. Before determining the steps to be taken in learning or problem solving, the teacher should have information in advance about the prerequisite knowledge, the thinking strategies children use, the level of thinking they demonstrate, and how various activities can help them develop the thinking needed for their goals. All of these can be seen in the hypothetical learning trajectory. The learning trajectory serves as a compass that guides learning. The learning trajectory can be structured based on past teaching experiences through trial results, conjectures built on theory or personal experience, and relevant research results [8].

When designing a learning activity, teachers should formulate hypotheses from students' reactions at each learning stage. In the early stages of lesson planning, the hypothesis is based on estimating the prerequisite knowledge that students already have and based on previous learning experiences or practices [9]. Some of the benefits of the hypothetical learning trajectory are as follows:

- Hypothetical learning trajectory can provide an understanding to the teacher about the importance of paying attention to students' initial knowledge and the differences in students' abilities in compiling learning designs.
- Hypothetical learning trajectory can be used as a teacher's guide in dividing the learning stages by making several sub-learning objectives to achieve the main learning objectives.
- Hypothetical learning trajectory is useful as a guide for implementing learning activity while providing various alternative strategies or scaffolding to help students overcome difficulties in understanding the learned concepts.

4. Conclusion
Piaget's theory has had a lot of influence on learning design. Teacher-centered learning has changed to become student-centered. This means that student factors are the main thing and must be considered in making a learning design. Hypothetical learning trajectory can be arranged based on the empirical
learning trajectory pattern so that the teacher can use it as a guide in dividing the learning stages and can provide various alternative strategies or scaffolding to help and overcome students who have difficulty in understanding mathematical concepts in elementary schools.

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