An Encoding Strategy of Swarm Intelligence Algorithms for Solving the Set-covering Location-allocation Problem with Collaborative Inventory

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Abstract. This paper proposed a novel encoding strategy with multiple segments of swarm intelligence optimization algorithms to solve the set-covering location-allocation problem with collaborative inventory. It divided an individual expression into several segments and then decoded the location and allocation information. By this encoding strategy, the discrete and continuous problems can be analysed simultaneously. It could improve the search efficiency of the swarm intelligence optimization algorithms to solve the problem. The experiment results showed that the strategy was viable and could be applied for the selected algorithms to optimize the set-covering location-allocation problem with collaborative inventory.

1. Introduction
The location-allocation problem based on set covering is to select distribution centers and allocate the resources of distribution centers to make the total cost lowest under the premise of covering all demand points. Many researchers have done on it. Gokbayrak et al. [1] introduced a new continuous location-allocation problem where the facilities have both a fixed opening cost and a coverage distance limitation and presented a three-stage heuristic algorithm for its solution. Venkateshan et al. [2] consider the problem of locating up to a given number of facilities in continuous Euclidean space. Rodriguez-Verjan et al. [3] presented a facility location-allocation (HAH) problem, and solved it by using the branch and bound method. Mohamed et al. [4] provide a facility location-allocation problem in the strategic stage of the supply chain planning.

In the traditional set-covering location-allocation problem, each distribution center is operated independently without collaboration. Obviously, the relationship between distribution center and demand point is 1-to-1 or 1-to-N. Now, with the development of collaboration, some researchers propose a new model which introduces a collaborative model. It makes the distributions as an alliance which provides logistics service for all the customers. That means one demand point would be served by more than one distribution center. This problem needs to solve two problems simultaneously: distribution center location and demand coverage ratio. It is much more complex than the traditional model and needs powerful tools. And Swarm intelligence algorithms have shown a powerful ability in solving complex problems. It also can be applied to solve this problem. But many decoding strategies of swarm intelligence algorithms are not suitable for this problem because of the difficulty in coding and low efficiency. In order to improve the efficiency of swarm intelligence optimization algorithms applied in the problem, this paper presents a different encoding strategy considering the encoding and decoding...
need of SI algorithms. It can decode the discrete location information and the continuous proportional coverage information simultaneously.

2. Assumption and Model

2.1. Assumption

- \( I \) is demand point set; \( J \) is candidate distribution center set;
- \( Q_i \) Requirement of Demand point \( i \), \( \forall i \in I \);
- \( H_j \) Cost of construction of distribution center \( j \), \( \forall j \in J \);
- \( p' \) Unit price per unit distance;
- \( p^* \) Unit price unit purchase;
- \( p^t \) Unit commodity storage cost;
- \( d_{ij} \) Distance from point \( i \) to distribution center \( j \), \( \forall i \in I, \forall j \in J \);
- \( X_j \) 0, 1 \( \forall j \in J \), when distribution center \( j \) is selected, \( X_j = 1 \); otherwise, \( X_j = 0 \);
- \( Z_{ij} \) Coverage ratio of distribution center \( j \) to demand point \( i \), \( \forall i \in I, \forall j \in J \);
- \( a_j \) Cost coefficient of station construction in distribution center \( j \), \( \forall j \in J \);

2.2. Model objective

The objective of the model is to minimize the total cost of the system, including location cost and assignment cost. Therefore, a two-layer location-allocation model is introduced\(^{[5]}\). The upper model \( F^w \) solves the location problem, and the lower model \( F^d \) aims at the demand assignment problem. The model is shown as follows.

\[
\begin{align*}
\min F^w &= \sum_j H_j X_j \quad \forall j \in J \\
\min F^d &= p' \sum_i \sum_j d_{ij} Q_i Z_{ij} X_j + p^* \sum_i Q_i + p^t \sum_j \sqrt{Q_j} \quad \forall i \in I, \forall j \in J \\
\text{s.t.} & \quad H_j = a_j \sqrt{\sum_i Q_i Z_{ij}} \\
& \quad X_j = 0, 1 \quad \forall j \in J \\
& \quad 0 \leq Z_{ij} \leq 1 \quad \forall i \in I, \forall j \in J \\
& \quad \sum_j Z_{ij} X_j = 1 \quad \forall i \in I \\
& \quad \sum_j X_j \geq 1
\end{align*}
\]

3. Multiple segments encoding strategy

3.1. Individual representation design

Assuming that an appropriate number of distribution centers are to be selected from \( M \) candidate points to meet the requirements of \( N \) demand points. Then an individual \( P \) is divided into \( N \) segments with a length of \((M+1)\) and the total coding length is \( N \times (M+1) \). Each dimensional value is a real number.

As shown in Figure 1, each segment of the individual consists of two parts, one of which identifies the decision position of the distribution center and another represents the decision position of the demand point allocation ratio. The first segment’s length is 1, and its position is \( P_{00} \). In this section, the number
information of distribution center is mainly represented, and the range of values is any real number in \([I, M]\). The second segment’s length is \(M\) and represented as \((P_{i1}, P_{i2}, P_{i3}, \ldots, P_{iM})\). In this section, the information about the coverage ratio of different distribution to different demand points can be realized.

![Figure 1. Individual segment](image1)

According to the above description, the complete representation of an individual in the swarm intelligence algorithm is shown in Figure 2.

![Figure 2. Representation of individual encoding](image2)

3.2. Decoding process

1) Distribution center selection decoding

Each segment of the individual \(P\) can be extracted into a matrix \(S\) composed of \(N\) rows and \((M+1)\) columns, as shown in Figure 3. Firstly, we can extract the first column element in matrix \(S\) to get the array \(L_1=(P_{10}, P_{20}, \ldots, P_{N0})\). Then we round the elements to get a new integer array \(H\). The integers in \(H\) are between \([1, M]\) which can be used as the selected distribution center.

![Figure 3. Matrix S and array L1](image3)

Take an individual as an example (Figure 4), the extraction array is \(L_1=(3.12, 1.52, 1.13, 4.32)\). After being rounded, the array is \(H=(3, 1, 1, 4)\). That is, the distribution center 1, 3 and 4 are selected.

![Figure 4. Decoding process](image4)

2) Demand point allocation ratio decoding

Extract the second column to the \((M+1)\) column from \(S\) and construct a new matrix \(L_2\). Now \(L_2=[P_{11} \; P_{12} \; P_{13} \; \ldots \; P_{1M}; \; P_{21} \; P_{22} \; P_{23} \; \ldots \; P_{2M}; \; \ldots; \; P_{N1} \; P_{N2} \; P_{N3} \; \ldots \; P_{NM}]\).
Once the distribution center has been determined, the coverage ratio $r_{ij}$ can be calculated according to the value of $P_{ij}$, where $i$ and $j$ are the number of demand point and the service-provided distribution center separately. Assume that the numbers of one demand point is $i \in I$, and the set of distribution centers responsible for the demand is $O$. Then the ratio $r_i$ that each distribution center $j$ in set $O$ should be responsible for demand point $i$ can be calculated by the following formula (8):

$$ r_j = \frac{P_j}{\sum_{j \in O} P_j} $$

Taking the case in Figure 4 as an example, the selected distribution centers are 1, 3 and 4. The extraction matrix $L_2$ is shown in Figure 6. For demand point 1, the ratio of demand allocation is calculated according to formula (8) as follows.

$$ r_{11} = \frac{1.26}{1.26+1.21+4.23} = 0.188, \quad r_{13} = \frac{1.21}{1.26+1.21+4.23} = 0.181, \quad r_{14} = \frac{4.23}{1.26+1.21+4.23} = 0.631 $$

The values of each demand points are calculated in the same way.

4. Case study

In order to test the efficiency of the algorithms for solving the problem, three groups of cases with different scales are designed in this experiment. The test data of each group of cases are randomly generated, where coordinate of the candidate distribution centers are randomly generated within (1, 100), and the coordinate of the demand point is randomly generated within (1, 100). The cost coefficient of the candidate distribution center is generated according to normal distribution $N(50,602)$. The demands are generated randomly within (100, 400); the unit price of unit transportation is 1; the price of single commodity is 200. Also, the order fee is 3,000; the unit commodity storage fee is 4.

4.1. Parameter setting

In order to verify the effectiveness of the coding strategy, this paper selects some similar swarm intelligence algorithms applied widely to make a comparison, including: CLPSO、BBO、FA、ABC、BA、GA、PSODT-SNE and SNSO. And the same individual encoding strategy and fitness calculation method are applied by each algorithm. For all the case, each algorithm will run 20 times independently.

4.2. Experiments and Results

We set up three groups of random cases in this experiment, the scales of which are (5,10), (8,15), (15,50). Then the number of alternative distribution center stations is 5, 8 and 15, and the number of demand points are 10, 15 and 50 respectively. In the test of the small-scale case (5, 10), result in Figure 7a shows that all the selected algorithms can get a quite close result based on the encoding strategy. However different algorithms show different searching performance. The SNSO algorithm is superior to the other seven algorithms in terms of accuracy from the average convergence curve (Figure 7a), it can be seen that SNSO shows better characteristics in search speed and convergence.
From the test results (Figure 7b) of medium case (8, 15), it can be seen that the increasing size of the problem makes the search speed of most algorithms a bit slower than that in small scale cases. But most of them still can continue to search in the middle and late period to find better solutions.

Finally, for the larger scale case (Figure 7c), the search speed of several algorithms is slowed down, but the SNSO, PSODT-SNE, BBO and GA show a better stability the search speed of which remain better. After an early slow search, they can accelerate searching in mid-late period. It shows that these algorithms are not trapped in the local optimum and can be searched effectively.

5. Conclusion
This paper presents a novel encoding method of swarm intelligence algorithms to solve the set-covering location-allocation problem with collaborative inventory. The discrete location problem and the continuous proportional coverage problem can be decoded simultaneously for each individual in a swarm based on the proposed strategy. The experiment results indicate that the new encoding strategy is applied to solve the set-covering location-allocation problem with collaborative inventory effectively.

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