The determining of stressed state in the elastic body with a broken line shaped inclusion when the harmonic oscillations of the longitudinal shear

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Abstract. The problem the determining of stressed state in the vicinity of the tunnel rigid inclusion when it’s cross-section a broken is considered. The inclusion is located in an infinite elastic medium and harmonic shear force impacts on it. It is supposed the inclusion is fully coupled with the medium. The problem is reduced to the solution of at system singular integral equations with fixed singularities. A numerical method of the solving of this system with regard to true asymptotic of the unknown functions is developed.

1. Introduction
The problem of determining the dynamic stress state in bodies with thin defects in the form of a segment straight line or arc of a smooth curve can now be considered solved. Haw examples of such works can be called the next [1–8]. But, as you know, real defects can be more complicated configuration: piecewise smooth, intersecting or branching. The problems of determining the stress state in bodies with like defects was solved significantly less. This is due to mathematical difficulties in solving such problems by the method of integral equations. In particular, they consist in the fact that the resulting singular integral equations contain fixed singularities in the kernels. The problems of equilibrium of bodies with cracks are the most studied, among which, first of all, it is necessary to indicate [9–13]. Exact solutions are obtained using the Wiener-Hopf method in these works and the stress intensity factor (SIF) is exactly determined. These solutions and results of work [13] indicate that the presence of kernels with fixed singularities influences the singularity of solutions in neighborhoods of the ends of integration intervals. The authors of the articles [13–19] studied the stress state near branching broken and edges cracks by a numerical method. The method of boundary integral equations has been used to study the stress state in bodies with inclusions in the form of broken lines or with branching inclusions in [20, 21]. What unites these works is that in constructing a numerical solving of the obtained integral equations, the real asymptotic of the solutions is not taken into account.

Solutions of similar dynamic problems are practically absent even for the case of harmonic oscillations. Among the few in existence works, it should be noted [22–25] where the interaction of plane longitudinal shear harmonic waves with two defects extending from one point and defect in the shape of a three-link broken line was investigated. Original numerical method for solving the systems of singular integral and integro-differential equations with fixed singularities are obtained in this case was proposed. This method takes into account the real singularity of
2. Statement of the problem and its reduction to a system of singular integral equations

Let an isotropic elastic body be in a condition of deformation of longitudinal shear and contain a thin absolutely rigid tunnel inclusion, which in cross section is shape of broken line from $N$ links (figure 1).

Links of this broken line occupy segments of length $2d_l$ in the plane $Oxy$. The inclusion is affected by the harmonic shear force $P e^{-i\omega t}$ along the axis $Oz$. The time dependence is given by the factor $\exp(-i\omega t)$ that is omitted in what follows. Let the vector of displacements are caused wave is reflected from defect have the single component $W(x,y)$ which is nonzero at deformation of longitudinal shear. In the coordinate system $Oxy$ it satisfies the Helmholtz equation

$$\Delta W + \kappa_2^2 W = 0, \quad \kappa_2^2 = \frac{\rho \omega^2}{G}. \quad (1)$$

The symbols $G$ and $\rho$ stand for the shear modulus and the medium density.

For the formulation of the boundary conditions on the inclusion of each of its links connect local coordinate system $O_l x_l y_l, l = 1, 2, \ldots, N$ (figure 1). The axis $O_l x_l$ of the local coordinate system is directed along the corresponding inclusion link and forms an angle $\beta_l, l = 1, 2, \ldots, N$ with the axis $Ox$. The center $O_l(a_l, b_l)$ of the local coordinate system coincides with the middle of this link. The relationship between the coordinate systems is given by the formulas

$$\begin{align*}
x_k &= A_{lk} x_l + x_l \cos \beta_{lk} - y_l \sin \beta_{lk}, \\
y_k &= B_{lk} x_l + x_l \sin \beta_{lk} + y_l \cos \beta_{lk}, \\
A_{lk} &= (a_l - a_k) \cos \beta_k + (b_l - b_k) \sin \beta_k, \\
B_{lk} &= -(a_l - a_k) \sin \beta_k + (b_l - b_k) \cos \beta_k, \quad k, l = 1, \ldots, N. \quad (2)
\end{align*}$$

Assume that the condition of perfect contact is realized between the body and inclusion, therefor

$$W_l(x_l, 0) = c, \quad l = 1, \ldots, N, \quad (3)$$

where

$$W_l(x_l, y_l) = W(a_l + x_l \cos \beta_l - y_l \sin \beta_l, b_l + x_l \sin \beta_l + y_l \cos \beta_l).$$
On the surface of the inclusion, the tangential stresses are discontinues and their jumps are denoted
\[ \tau_{zy}(x_l, +0) - \tau_{zy}(x_l, -0) = \chi_l(x_l), \quad -d_l < x_l < d_l, \quad l = 1, \ldots, N, \quad (4) \]

In equality (3) \( c \) is the amplitude of longitudinal oscillations of the inclusion from the equation of its motion as a solid body. In the case of harmonic oscillations, this equation takes the form
\[ -\omega^2 c \sum_{l=1}^{N} m_l = P + \sum_{l=1}^{N} \int_{-d_l}^{d_l} \chi_l(\eta) d\eta, \quad m_l = 2\rho_0 d_l h, \quad (5) \]

where \( h \) is the thickness of inclusion, \( \rho_0 \) is density of the inclusion.

We start the solution of the posed problem (1)–(5) from the construction of a discontinuous solution of the Helmholtz equation with jumps (4) in the coordinate system \( O_kx_ky_k \) for each link of the inclusion [6, 7]
\[ W_k^d(x_k, y_k) = \int_{-d_k}^{d_k} \frac{\chi_k(\eta)}{G} r_2(\eta - x_k, y_k) d\eta, \quad r_2(\eta - x_k, y_k) = \frac{i}{4} H_0^1(\kappa_0 \sqrt{\eta^2 + y_k^2}) \quad (6) \]

where \( H_0^1(z) \) is Chancels function.

Then the displacements can be presented in the form
\[ W(x, y) = \sum_{k=1}^{N} W_k^d(x, y), \quad (7) \]

where
\[ W_k^d(x, y) = W_k^d((x - a_k) \cos \beta_k + (y - b_k) \sin \beta_k, -(x - a_k) \sin \beta_k + (y - b_k) \cos \beta_k), \]

obtained from (7) due to the transformation of coordinates by the formulas (2) In order to finally determine the displacements by the formulas (7), we should find the unknown jumps (4). To determine these jumps from the boundary conditions (3) we can obtain a system of singular integral equations. In order that the singular component of their kernels contains a singularity in the form of the Cauchy kernel, instead of (3), we should use [7] the following conditions:
\[ \frac{\partial W_l(x_l, 0)}{\partial x_l} = 0, \quad W_l(-d_l, 0) = c, \quad l = 1, \ldots, N. \quad (8) \]

The first equality (8) is the result of applying to (3) the operation of differentiation, and the second equality is equivalence of differentiated and initial equalities. The result of substitution of (7), (6) into (8) is the singular integral equation with an auxiliary conditions. After the separation singular components of the kernels and some transformations, this system takes the form
\[ \int_{-1}^{1} \left[ -\frac{E}{\tau - \xi} + Q(\tau, \xi) + R(\tau, \xi) \right] \Phi(\tau) d\tau = 0, \quad -1 < \xi < 1, \quad (9) \]

The follow notations are introduced in system (9):
\[ \Phi(\tau) = \begin{pmatrix} \varphi_1(\tau) \\ \varphi_2(\tau) \\ \vdots \\ \varphi_N(\tau) \end{pmatrix}, \quad \varphi_k(\tau) = \frac{\chi_k(d_k\tau)}{G}, \quad \eta = d_k\tau, \quad x_l = d_l\xi, \quad (10) \]
\[ \gamma_l = \frac{d_l}{d}, \quad c_0 = \frac{c}{d}, \quad \kappa_0 = \kappa_2d, \quad d = \max(d_1, d_2, \ldots, d_N), \]
$E$ is unit $N \times N$ matrix, $U$ is diagonal matrix with elements $\gamma_1, \ldots, \gamma_N$, on the main diagonal. The matrices $R(\tau, \xi), D(\tau)$ are composed of continuous functions when $-1 \leq \tau, \xi \leq 1$. Matrix $Q(\tau, \xi)$ has the form

$$Q(\tau, \xi) = \begin{pmatrix}
0 & q_{12} & 0 & 0 & \cdots & 0 & 0 & 0 \\
q_{21} & 0 & q_{23} & 0 & \cdots & 0 & 0 & 0 \\
0 & q_{32} & 0 & q_{34} & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & q_{N-1N-2} & 0 & q_{N-1N} \\
0 & 0 & 0 & 0 & \cdots & 0 & q_{NN-1} & 0 
\end{pmatrix}.$$  

The nonzero elements of this matrix are the functions

$$q_{ll+1}(\tau, \xi) = \frac{\gamma_{l+1}(\gamma_{l+1}(\xi \pm 1) - \gamma_{l+1}(\tau \mp 1) \cos \beta_{l+1})}{s_{l+1}(\tau, \xi)},$$

$$s_{l+1} = \gamma_{l+1}(\tau \mp 1)^2 - 2\gamma_{l+1}(\tau \mp 1)(\xi \pm 1) \cos \beta_{l+1} + \gamma_l^2(\xi \pm 1)^2$$

To the system (9) we must also add the equality (5) to determine the unknown amplitudes of oscillation of inclusions in the notation (10) take the form

$$c_0 = -\frac{1}{2l_0} \left[ P_0 + \sum_{l=1}^{N} \gamma_l \int_{-1}^{1} \varphi_l(\tau) d\tau \right], \quad l_0 = \frac{P}{Gd}, \quad P_0 = \frac{P}{Gd}, \quad \bar{\rho} = \rho_0 \rho, \quad \varepsilon = \frac{h}{d}.$$  

3. Numerical solution of the system of integro-differential equations

As is seen from (11), functions $q_{l+1}(\tau, \xi)$ have singularities at $\tau = 1, \xi = -1$ and $\tau = -1, \xi = 1$. The presence of fixed singularities in singular components of system (12) influences the behavior of solutions in neighborhoods of the points $\tau = \pm 1$. Their asymptotic in neighborhoods of these points are determined in the same way as in works [24, 25]. As a result, we found unknown functions should be sought in the form

$$\varphi_l(\tau) = (1 - \tau)^{-\sigma_{l+1}}(1 + \tau)^{-\sigma_l}\psi_l(\tau).$$  

There the exponents are as follows

$$\sigma_1 = \sigma_{N+1} = \frac{1}{2}, \quad \sigma_l = \begin{cases} 
\alpha_{l-1} - \pi, & -\pi \leq \alpha_{l-1} < 0 \\
\alpha_{l-1}, & 0 \leq \alpha_{l-1} < \pi,
\end{cases} \quad \alpha_{l-1} = \beta_l - \beta_{l-1}, \quad l = 2, \ldots, N.$$  

Now, if we consider the function

$$\psi_{2l}(\tau) = \psi_l(\tau) - \frac{\psi_l(-1)}{2}(1 - \tau) - \frac{\psi_l(1)}{2}(1 + \tau)$$

we can see $\psi_{2l}(\pm 1) = 0$. This is why we can assume that

$$\psi_{2l}(\tau) = (1 - \tau)^2g_l(\tau),$$

where $g_l(\tau)$ are new unknown functions. Substituting (15) and (14) into (13), we get the following representation of the solutions system of the integrals equations (9):

$$\varphi_l(\tau) = (1 - \tau)^{\sigma_{l+1}}(1 + \tau)^{1-\sigma_l}g_l(\tau) + (1 - \tau)^{1-\sigma_{l+1}}(1 + \tau)^{-\sigma_l}\psi_l(-1) + (1 - \tau)^{-\sigma_{l+1}}(1 + \tau)^{1-\sigma_l}\psi_l(1).$$
Further, the approximate method of solution is based on the approximation of the functions \( g_l(\tau) \).

Formatting the title by the interpolation polynomials of the \((n-1)\)st power:

\[
g_l(\tau) \approx g_{l,n-1}(\tau) = \sum_{m=1}^{n} g_{ml}(\tau) \frac{Q_{nl}(\tau)}{[Q_{nl}(\tau_m)]'}, \quad g_{ml} = g_l(\tau_{ml}),
\]

\( Q_{nl}(\tau) = P_n^{\alpha_{l+1},\alpha_{l}}(\tau) \) are the Jacobi polynomials and \( \tau_{ml} \) are the roots of these polynomials.

Then, for the singular integral with the Cauchy kernel the following quadrature formula holds [25, 26]

\[
\int_{-1}^{1} \frac{\varphi_l(\tau)}{\tau - \xi_{jl}} d\tau = \sum_{m=1}^{n} g_{ml} \frac{A_{ml}}{\tau_{ml} - \xi_{jl}} + \frac{\psi_l(-1)}{2} b_j^- + \frac{\psi_l(1)}{2} b_j^+,
\]

where \( \xi_{jl}, j = 1, \ldots, n + 1 \) are the roots of the Jacobi functions of the second kind \( J_n^{\alpha_{l+1},\alpha_{l}}(\zeta) \) and \( A_{ml} \) are the coefficients of the corresponding Gauss–Jacobi quadrature formula [27]. When obtaining formulas (18), the integrals

\[
b_j^- = \int_{-1}^{1} \frac{(1 - \tau) - \alpha_{l+1}(1 + \tau) - \alpha_{l}}{\tau - \zeta_{jl}} d\tau, \quad b_j^+ = \int_{-1}^{1} \frac{(1 - \tau) - \alpha_{l+1}(1 + \tau) - \alpha_{l}}{\tau - \zeta_{jl}} d\tau
\]

were found by the method described in [28] and based on their conversion to the Mellin convolution. Further, the use of the convolution theorem makes it possible to represent these integrals as the sum of residues at the poles.

Quadrature formulas (18) for Cauchy integrals make possible to construct approximate solution of the system (10) using the roots of Jacobi functions as points of collocation. But analogous quadrature formulas for the integrals with fixed singularities are necessary for this.

Below, we obtain formulas for the integrals

\[
E_{l}^{\pm} = \int_{-1}^{1} q_{ll+1}(\tau, \xi_{jl}) \varphi_{l+1}(\tau) d\tau, \quad l + 1 = 2, \ldots, N, \quad l - 1 = 1, \ldots, N - 1.
\]

Let \( 0 < r_1 < 1 \) be some positive number. In the case where \( 1 \pm \xi_{jl} > r_1 \) the integral (20) is not singular, and it can be calculated with the help of the Gauss–Jacobi quadrature formulas with the corresponding weight function [27]. The principal difficulty is related to the calculation of these integrals as \( 1 \pm \xi_{jl} \to +0 \). To this end, we use representation (16), (17) and use the method based on using Mellin integral transformation [28]. As result, we represent integrals (20) in terms of the following power series convergent for \( 0 \leq 1 \pm \xi_{jl} < r_1 < 1 \)

\[
E_{l}^{\pm} = \sum_{m=1}^{n} g_{ml} H_{m}^{\pm 1}(\zeta) + \frac{\psi_l(-1)}{2} s_{l+1}^- + \frac{\psi_l(1)}{2} s_{l+1}^+.
\]
We introduced new notation in formulas (21)

\[ H_{jm}^{ll\pm} = \frac{1}{Q_n'(\tau_{ml\pm})} \left[ q_{l\pm}(\tau_{ml\pm}, \xi_{jl\pm})Q_n'(\tau_{ml\pm})A_{ml\pm} \right. \\
\pm \frac{\gamma(l\pm1, \xi_{jl\pm} \mp 1)}{Q_n(\tau_{ml\pm}, \xi_{jl\pm})} B_n^{(1)}(\sigma_l, \sigma_{l+1}, \beta_{ll\pm}, 1 \mp \xi_l) \\
\left. \mp \frac{\gamma^2(l\pm1, \xi_{jl\pm} \mp 1)}{Q_n(\tau_{ml\pm}, \xi_{jl\pm})} B_n^{(2)}(\sigma_l, \sigma_{l+1}, \beta_{ll\pm}, 1 \mp \xi_l) \right]. \]

\[ B_n^{(\nu)}(\alpha, \beta, \theta, Y) = \frac{2^{2-\alpha-\beta} \Gamma(2+n-\beta)}{n!} \]

\[ \times \left\{ -y^{1-\alpha}\sin(\pi \beta) \sum_{p=0}^{\infty} pC_{pn} \cos[\theta(p+3-v-\alpha)]Y^p \\
+ \sin[\pi(\alpha+\beta)] \sum_{p=0}^{\infty} D_{pn} \cos[\theta(p+2-v)]Y^p \right\}, \]

\[ C_{pn} = \frac{\Gamma(p+n+2-\alpha)\Gamma(p+n+\beta)(-1)^p}{(p-n-1+\beta)\Gamma(p+2-\beta)p!}, \]

\[ D_{pn} = \frac{\Gamma(p+n+1)\Gamma(p+n-1+\alpha+\beta)(-1)^p}{(p-n+\nu-3+\alpha+\beta)\Gamma(p+2-\alpha)p!}. \]

The integrals

\[ s_{jl\pm}^{\pm} = \int_{-1}^{1} q_{l\pm}(\tau, \xi_{jl})(1-\tau)^{\nu_{\pm}-\sigma_{l+1}}(1+\tau)^{\nu_{\pm}-\sigma_l}d\tau \]

included in the formula are calculated analogously for \( 1 \pm \xi_{jk} \to +0. \)

\[ s_{jl+1}^{\pm} = \frac{\Gamma(2-\nu_{\pm}-\sigma_{l+1})}{2\sigma_l+\sigma_{l+1}-1} \]

\[ \times \left[ \sin(\pi \sigma_{l+1}) \sum_{p=0}^{\infty} C_{p}^{\pm} \cos[(\sigma_l-p-1-\nu_{\pm})\beta_{ll+1}](\frac{y}{\gamma_{ll+1}})^p \right. \\
\pm \left. \frac{\sin(\pi(\sigma_l+\sigma_{l+1}))}{\sin(\pi \sigma_l)} \sum_{p=0}^{\infty} D_{p}^{\pm} \cos[(p+1)\beta_{ll+1}](\frac{y}{\gamma_{ll+1}})^p \right], s_{jl-1}^{\pm} = -s_{jl+1}^{\pm}. \]

\[ C_{p}^{\pm} = \frac{\Gamma(p+\sigma_{l+1}+\nu_{\pm})(-1)^p}{(p+\sigma_l+\nu_{\pm})p!}, D_{p}^{\pm} = \frac{\Gamma(p+\sigma_l+\sigma_{l+1})(-1)^p}{(p+\sigma_l+\sigma_{l+1}-1)\Gamma(p+\sigma_l+\nu_{\pm})}. \]

Let us consider the integrals with logarithmic singularity. As a result use of representations (16) and (17) the next quadrature formulas are obtained

\[ \int_{-1}^{1} \varphi_l(\tau) \ln|\tau \pm 1|d\tau = \sum_{m=1}^{n} A_{ml}g_{ml}\theta_{ml} + \frac{\psi_l(-1)}{2} E_{l}^{-} + \frac{\psi_l(1)}{2} E_{l}^{+}. \]  

Next notations are introduced in formulas (22)

\[ \theta_{ml} = \sum_{s=0}^{n} \frac{u_{ml}}{\sigma_{sl}} Q_{sl}(\tau_{ml}), \quad u_{ml} = \int_{-1}^{1} (1-\tau)^{1-\sigma_{l+1}}(1+\tau)^{1-\sigma_l} Q_{sl}(\tau) \ln|\tau \pm 1|d\tau, \]

\[ E_{l}^{\pm} = \int_{-1}^{1} \ln|\tau \pm 1|(1-\tau)^{\nu_{\pm}-\sigma_{l+1}}(1+\tau)^{\nu_{\pm}-\sigma_l}d\tau. \]
The last integrals are calculated by methods for calculating integrals with orthogonal polynomials [15] and are equal

\[ u_{0l} = \frac{2^{3-\sigma_l-\sigma_{l+1}} \Gamma(2-\sigma_l) \Gamma(2-\sigma_{l+1})}{\Gamma(4-\sigma_l-\sigma_{l+1})} \left[ \ln 2 + \Psi(2-\sigma_{l+1}) - \Psi(4-\sigma_l-\sigma_{l+1}) \right], \]

\[ u_{sl} = -\frac{2^{2-\sigma_l-\sigma_{l+1}} \Gamma(2-\sigma_l+s) \Gamma(2-\sigma_{l+1})}{s \Gamma(4-\sigma_l-\sigma_{l+1}+s)} \] \[ E_l^\pm = \frac{2^{2-\sigma_l-\sigma_{l+1}} \Gamma(1+\nu^\pm - \sigma_l) \Gamma(1+\nu^\pm - \sigma_{l+1})}{\Gamma(3-\sigma_l-\sigma_{l+1})} \left[ \ln 2 + \Psi(1+\nu^\pm - \sigma_{l+1}) - \Psi(3-\sigma_l-\sigma_{l+1}) \right]. \]

After use the quadrature formulas (18), (21), (22), system (9), (12) was reduced to a system of linear algebraic equations for

\[ g_{ml} = g(\tau_{ml}), \quad \psi_l(\pm 1) \quad (l = 1,\ldots,N), \quad c_0. \]

We are got the following formulas for the approximate values of stress intensity factors (SIF) [29], after solving this system:

\[ K^- = G \sqrt{d_1} \psi_1(-1), \quad K^+ = G \sqrt{d_N} \psi_N(1). \quad (23) \]

4. Results of numerical analysis

While performing the numerical realization, we tried, first, to study the practical convergence of the proposed method of numerical solution. For this, we considered a inclusion with links of the same length of the configuration presented in figure 2.

By formulas (23), we calculated the dimensionless values of stress intensity factors \( k = K/G\sqrt{d} \), \( K_1 = K_3 = K. \) The calculation was carried out for the angle \( \beta = 45^\circ \) and \( \bar{\rho} = p_k/\rho = 1, \) \( \varepsilon = h/d = 0.05. \) The results of calculations are given in figure 3, as plots of the absolute value of stress intensity factor as of the dimensionless wave number \( \kappa_0 = \kappa_2 d. \) The values \( n = 5, 10, 15, 20 \) correspond to the number of the interpolation nodes in formula (17). We see that it is sufficient to take at most 20 interpolation nodes in (17) in order to get the values of stress intensity factors with an error of at most 0.1%. For the loading with small frequencies it is sufficient to take 5 nodes.

We also investigated the influence of the inclusion shape on the value of stress intensity factor by the example of the inclusion shown in figure 4. The influence on the values of the SIF and the amplitudes of the oscillations of the inclusion of its shape, namely the value of the angle \( \beta \) between its links, is clarified. We took the following relation between the lengths of links:
\[ d_2 = 3d_1 = 3d_3. \] The results of calculations are given in figures 5 and 6. The curves correspond to values: 1 — $\beta = 5^\circ$, 2 — $\beta = 45^\circ$, 3 — $\beta = 90^\circ$, 4 — $\beta = 135^\circ$, 5 — $\beta = 165^\circ$, 6 — $\beta = 170^\circ$. The calculation results show that, at low oscillation frequencies $\kappa_0 \leq 0.5$, the SIF values decrease with increasing angle $\beta$. In the region of higher frequencies, the SIF dependence has a complicated form, especially at angles $\beta > 90^\circ$. We can see that there are frequencies at which the SIF value has local maximum. The values of these frequencies change with changing angle $\beta$. The largest SIF values in the frequency region $0 \leq \kappa_0 \leq 5$ considered are reached for angles of large $135^\circ$. The curves located in figure 6 show that the amplitude of the inclusion oscillations decreases with increasing frequency for all values of the angle $\beta$.

**Conclusions**

The results presented above enable us to make the following conclusions:

1. The proposed numerical method for solving singular integral equations with fixed singularities makes it possible to approximately calculate stress intensity factors in the vicinity of thin inclusions, with the broken line shaped cross section.
2. Determining and taking into account the real asymptotic behavior of the solutions of singular integral equations with fixed singularities and using special quadrature formulas for singular integrals ensure fast convergence and stable numerical results in a wide frequency range.
3. The influence of the shape and the oscillation frequency of inclusions on the dependence of the stress intensity factor on frequency is demonstrated.
4. The existence of frequencies at which the maximum values of stress intensity factors are observed is shown.
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