Influence of probability density function of the passage time in the method of non-equilibrium statistical operator on non-equilibrium properties of the system

V.V. Ryazanov
Institute for Nuclear Research, Kiev, pr.Nauki, 47 Ukraine
vryazan@kinr.kiev.ua
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Abstract

A family of non-equilibrium statistical operators (NSO) is introduced which differ by the system lifetime distribution over which the quasi-equilibrium (relevant) distribution is averaged. This changes the form of the source in the Liouville equation, as well as the expressions for the kinetic coefficients, average fluxes, and kinetic equations obtained with use of NSO. It is possible to choose a class of lifetime distributions for which thermodynamic limiting transition and to tend to infinity of average lifetime of system is reduced to the result received at exponential distribution for lifetime, used by Zubarev. However there is also other extensive class of realistic distributions of lifetime of system for which and after to approach to infinity of average lifetime of system non-equilibrium properties essentially change. For some distributions the effect of "finite memory" when only the limited interval of the past influence on behaviour of system is observed. It is shown, how it is possible to spend specification the description of effects of memory within the limits of NSO method, more detailed account of influence on evolution of system of quickly varying variables through the specified and expanded form of density of function of distribution of lifetime. The account of character of history of the system, features of its behaviour in the past, can have substantial influence on non-equilibrium conduct of the system in a present moment time.

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1 Introduction

One of the most fruitful and successful ways of development of the description of the non-equilibrium phenomena are served by a method of the non-equilibrium statistical operator (NSO) \[1, 2, 3, 4, 5\]. In work \[6\] new interpretation of a method of the NSO is given, in which operation of taking of invariant part \[1, 2, 3, 4, 5\] or use auxiliary "weight function" (in terminology \[7, 8\]) in NSO are treated as averaging of quasi-equilibrium statistical operator on distribution of past lifetime of system. In work \[2\] is chosen uniform distribution for an initial moment \(t_0\), which after change of integration passes to the exponential distribution \(p_q(u) = \exp\{-\varepsilon u\}\). Such distribution serves as the limiting distribution of lifetime \[9\], of the first passage time of level. In general case it is possible to choose a lot of functions for the obvious type of distribution \(p_q(u)\), that marked in works \[6, 10\]. This approach adjust with the operations spent in the general theory of random processes, in the renewal theory, and also with the lead Zubarev in work as \[2\] reception NSO by means of averaging on the initial moment of time.

In Kirkwood’s works \[11\] it was noticed, that the system state in time present situation depends on all previous evolution of the non-equilibrium processes developing it. For example, in real crystals it is held in remembrance their formation in various sorts "defects", reflected in structure of the crystals. Changing conditions of formation of crystals, we can change their properties and create new materials. In works \[7, 8\] it is specified, that it is possible to use many "weight functions". Any form of density of lifetime distribution gives a chance to write down a source of general view in dynamic Liouville equation which thus becomes, specified Boltzmann and Prigogine \[7, 8, 12\], and contains dissipative items.

If in Zubarev’s works \[1, 2\] the linear form of a source corresponding limiting exponential distribution for lifetime is used other expressions for density of lifetime distribution give fuller and exact analogues of "integrals of collisions".

In works \[13, 14\] sources in the Liouville equation different from sources, entered in the NSO method in works \[1, 2\] are considered. But in works \[13, 14\] at \(\varepsilon \to 0\) (and \(\Gamma \to \infty\) in interpretation paper \[6\]) this source turn into a zero. In the present work the sources which are not turn into a zero at \(\Gamma \to \infty\) are considered. Such source is caused by external influences on system. The approach of present paper differs from the methods used in \[13, 14\]. But use of distribution of a lifetime of system in the present work can be compared with noted in work \[13\] enlarged the set of macroobservables, include besides the physically necessary macroobservables additional ones, namely lifetime.

In work \[15\] irreversible transfer equations are received in assumptions of coarsening of the distributions, a certain choice of macroskopical variables and the analysis of division of time scales of the description (last circumstance was marked in \[16\]). Importance and necessity of the analysis of the time scales playing a fundamental role in the description of macroskopical dynamics of system is underlined. Evolution of slow degrees of freedom is described by Markovian equations. Thus the time scale on which observable variables evolve, should be much more time of memory on which the residual effects
brought by irrelevant degrees of freedom are considered. As marked in paper [13], it requires the condition of a maximum damping of the non-macroscopic information.

Otherwise effects of memory play an essential role. Memory time is estimated in work [15] for Boltzmann equation. In the present work the consideration subject is made by situations when it is necessary to consider effects of memory. Examples of such situations are given in [15].

In work [10] it is shown, in what consequences for non-equilibrium properties of system results change of lifetime distribution of system for systems of the limited volume with finite lifetime. In the present work are considered also infinitely greater systems with infinite average lifetime.

2 New interpretation of NSO

In [6] the Nonequilibrium Statistical Operator introduced by Zubarev [1, 2] rewritten as

\[
\ln q(t) = \int_0^\infty p_q(u) \ln q(t-u,-u) du, \quad \ln q(t,0) = -\Phi(t) - \sum_n F_n(t) P_n;
\]

\[
\ln q(t,t_1) = e^{(-t_1H/\hbar)} \ln q(t,0) e^{(t_1H/\hbar)}; \quad \Phi(t) = \ln S p \exp \left\{ \sum_n F_n(t) P_n \right\},
\]

where \( H \) is hamiltonian, \( \ln q(t) \) is the logarithm of the NSO in Zubarev’s form, \( \ln q(t,0) \) is the logarithm of the quasi-equilibrium (or relevant); the first time argument indicates the time dependence of the values of the thermodynamic parameters \( F_n \); the second time argument \( t_2 \) in \( q_q(t_1, t_2) \) denotes the time dependence through the Heizenberg representation for dynamical variables \( P_m \) from which \( q_q(t, 0) \) can depend [1, 2, 6, 7, 8]. In [6] the auxiliary weight function \( p_q(u) = \varepsilon \exp(-\varepsilon u) \) was interpreted as the density of probability distribution of lifetime \( \Gamma \) of a system. There \( \Gamma \) is random variables of lifetime from the moment \( t_0 \) of its birth till the current moment \( t; \varepsilon^{-1} = (t - t_0); \langle t - t_0 \rangle = \langle \Gamma \rangle \), where \( \langle \Gamma \rangle = \int u p_q(u) du \) is average lifetime of the system. This time period can be called the time period of getting information about system from its past. Instead of the exponential distribution \( p_q(u) \) in (1) any other sample distribution could be taken. This fact was marked in [6] and [7, 8] (where the distribution density \( p_q(u) \) is called auxiliary weight function \( w(t, t') \)). From the complete group of solutions of Liouville equation (symmetric in time) the subset of retarded “unilateral” in time solutions is selected by means of introducing a source in the Liouville equation for \( \ln g(t) \)

\[
\frac{\partial \ln g(t)}{\partial t} + iL \ln g(t) = -\varepsilon (\ln g(t) - \ln q_q(t, 0)) = J,
\]

which tends to zero (value \( \varepsilon \rightarrow 0 \)) after thermodynamic limiting transition. Here \( L \) is Liouville operator; \( iL = -\{ H, g \} = \sum_k \left\{ \frac{\partial H}{\partial p_k} \frac{\partial g}{\partial q_k} - \frac{\partial H}{\partial q_k} \frac{\partial g}{\partial p_k} \right\}; H \) is Hamilton function, \( p_k \) and \( q_k \) are momenta and coordinates of particles; \{...\} is
Poisson bracket. In [17] it was noted that the role of the form of the source term in the Liouville equation in NSO method has never been investigated. In [18] it is stated that the exponential distribution is the only one which possesses the Markovian property of the absence of contagion, that is whatever is the actual age of a system, the remaining time does not depend on the past and has the same distribution as the lifetime itself. It is known [1, 2, 6, 7, 8] that the Liouville equation for NSO contains the source

\[ J_{\text{zub}} = -\varepsilon \left[ \ln \varrho(t) - \ln \varrho_0(t,0) \right] \]

which becomes vanishingly small after taking the thermodynamic limit and setting \( \varepsilon \to 0 \), which in the spirit of the paper [1] corresponds to the infinitely large lifetime value of an infinitely large system. For a system with finite size this source is not equal to zero. In [8] this term enters the modified Liouville operator and coincides with the form of Liouville equation suggested by Prigogine [12] (the Boltzmann-Prigogine symmetry), when the irreversibility is entered in the theory on the microscopic level. We note that the form of NSO by Zubarev cast in [6] corresponds to the main idea of [12] in which one sets to the distribution function \( \varrho(q) \) (in Zubarev’s approach) which evolves according to the classical mechanics laws, the coarse distribution function \( \tilde{\varrho}(t) \) in the case of Zubarev’s NSO) whose evolution is described probabilistically since one perform an averaging with the probability density \( p_q(u) \). The same approach (but instead of the time averaging the spatial averaging was taken) was performed in [19].

Besides the Zubarev’s form of NSO [1, 2], NSO Green-Mori form [20, 21] is known, where one assumes the auxiliary weight function [7] to be equal

\[ W(t,t') = 1 - (t-t')/\tau; w(t,t') = dW(t,t')/dt' = 1/\tau; \tau = t-t_0. \]

After averaging one sets \( \tau \to \infty \). This situation at \( p_q(u = t - t_0) = w(t,t' = t_0) \) coincides with the uniform lifetime distribution. The source in the Liouville equation takes the form \( J = \ln \varrho_0(t)/\tau \). In [1] this form of NSO is compared to the Zubarev’s form.

One could name many examples of explicit defining of the function \( p_q(u) \). Every definition implies some specific form of the source term \( J \) in the Liouville equation, some specific form of the modified Liouville operator and NSO. Thus the family of NSO is defined.

3 Modifications to the nonequilibrium description

Let’s consider now, what consequences follow from such interpretation of NSO.

3.1 Families of NSO

Setting various distributions for past lifetime of the system, we receive a way of recording of families of NSO. Class of NSO from this family will be connected with a class of distributions for lifetime (taken, for example, from the stochastic theory of storage processes, the theory of queues etc.) and with relaxation properties of that class of physical systems which is investigated. The general expression for NSO with any distribution
\[ \ln q(t) = \int_{0}^{\infty} p_q(u) \ln q(t-u,-u)du = \]

\[ = \ln q(t,0) - \int_{0}^{\infty} \left( \int p_q(u)du \right) \frac{d\ln \rho(t-u,-u)}{du}du, \]

where integration by parts in time is carried out at \( \int p_q(y)dy|_{y=0} = -1; \int p_q(y)dy|_{y=-\infty} = 0; \) at \( p_q(y) = \varepsilon \exp\{-\varepsilon y\}; \varepsilon = 1/(\Gamma), \) the expression (1) passes in NSO from [11] [2]. In [18] it is shown, how from random process \( X(t), \) corresponding to evolution of quasi-equilibrium system, it is possible to construct set of new processes, introducing the randomized operational time.

It is supposed, that to each value \( t > 0 \) there corresponds a random value \( \Gamma(t) \) with the distribution \( p_{\Gamma}^t(y). \) The new stochastic kernel of distribution of a random variable \( X(\Gamma(t)) \) is defined by equality of a kind (1). Random variables \( X(\Gamma(t)) \) form new random process which, generally speaking, need not to be of Markovian type any more. Each moment of time \( t \) of "frozen" quasi-equilibrium system is considered as a random variable \( \Gamma(t) \) the termination of lifetime with distribution \( p_{\Gamma}^t(y). \) Any moment of lifetime can be with certain probability the last. That the interval \( t - t_0 = y \) was enough large (that became insignificant details of an initial condition as dependence on the initial moment \( t_0 \) is nonphysical [1, 2]), it is possible to introduce the minimal lifetime \( \Gamma_{\text{min}} = \Gamma_1 \) and to integrate in (3) on an interval \( (\Gamma_1, \infty). \) It results to the change of the normalization density of distribution \( p_q(y). \) For example, the function \( p_q(y) = \varepsilon \exp\{-\varepsilon y\} \) will be replaced by \( p_q(y) = \varepsilon \exp\{\varepsilon \Gamma_1 - \varepsilon y\}, \) \( y \geq \Gamma_1; p_q(y) = 0, \) \( y < \Gamma_1. \) The under limit of integration in (3) by \( \Gamma_1 \to 0 \) is equal 0.

It is possible to choose \( p_q(y) = Cf(y), \) \( y < t_1; p_q(y) = \varepsilon \exp\{-\varepsilon y\}, \) \( y \geq t_1; C = (1 - \exp\{-\varepsilon t_1\})/(\int_{t_1}^{\infty} f(y)dy). \) The function \( f(y) \) can be taken from models of the theory of queues, the stochastic theory of storage and other sources estimating the lifetime distribution for small times (for example [22] [23] [9] [24]). The value \( t_1 \) can be found from results of work [9]. It is possible to specify many concrete expressions for lifetime distribution of system, each of which possesses own advantages. To each of these expressions there corresponds own form of a source in Liouville equation for the nonequilibrium statistical operator. In general case any functions \( p_q(u) \) the source is:

\[ J = p_q(0) \ln q(t,0) + \int_{0}^{\infty} \frac{\partial p_q(y)}{\partial y} (\ln q(t-y,-y))dy \]

(when values \( p_q(0) \) disperse, it is necessary to choose the under limit of integration equal not to zero, and \( \Gamma_{\text{min}} \).) Such approach corresponds to the form of dynamic Liouville equation in the form of Boltzmann-Bogoliubov-Prigogine [7] [8] [12], containing dissipative items.

Thus the operations of taking of invariant part [11], averaging on initial conditions [2], temporary coarse-graining [11], choose of the direction of time [7] [8], are replaced by averaging on lifetime distribution.

It is essentially that and in exponential distribution from [11] [2] \( \varepsilon \neq 0. \) The thermodynamic limiting transition is not performed, and actually important for many physical phenomena dependence on the size of system are considered.
We assume $\varepsilon$ and $\langle \Gamma \rangle$ to be finite values. Thus the Liouville equation for $\rho(t)$ contains a finite source. The assumption about finiteness of lifetime breaks temporary symmetry. And such approach (introduction $\rho_q(y)$, averaging on it) can be considered as completing the description of works [1, 2].

In work [9] lifetimes of system are considered as the achievement moments by the random process characterizing system, certain border, for example, zero. In [9] are received approached exponential expressions for density of probability of lifetime, accuracy of these expressions is estimated. In works [25, 26] lifetimes of molecules are investigated, the affinity of real distribution for lifetime and approached exponential model is shown. It is possible to specify and other works (for example [27, 28, 29]) where physical appendices of concept of lifetime widely applied in such mathematical disciplines, as reliability theory, the theory of queues and so forth (under names non-failure operation time, the employment period, etc.) are considered. In the present section lifetime joins in a circle of the general physical values, acting in an estimation or management role (in terminology of the theory of the information [30]) for the quasi-equilibrium statistical operator that allows to receive the additional information on system.

Let’s notice, that in a case when value $d\ln \rho(t,y, -y)/dy$ (the operator of entropy production $\sigma$ [1]) in the second item of the right part (3) does not depend from $y$ and is taken out from under integral on $y$, this second item becomes $\sigma \langle \Gamma \rangle$, and expression (3) does not depend on form of function $p_q(y)$. There is it, for example, at $\rho_q(t) \sim \exp\{-\sigma t\}$. In work [31] such distribution is received from a principle of a maximum of entropy at the set of average values of fluxes.

3.2 Physical sense of distributions for past lifetime of system

As is known (for example, [22, 9, 24]), exponential distribution for lifetime

$$p_q(y) = \varepsilon \exp\{-\varepsilon y\}, \quad (5)$$

used in Zubarev’s works [1, 2], is limiting distribution for lifetime, fair for large times. It is marked in works [1, 2] where necessity of use of large times connected with damping of nonphysical initial correlations. Thus, in works [1, 2] the thermodynamic result limiting and universal is received, fair for all systems. It is true in a thermodynamic limit, for infinitely large systems. However real systems have the finite sizes. Therefore essential there is use of other, more exact distributions for lifetime. In this case the unambiguity of the description peculiar to a thermodynamic limit is lost [32].

For NSO with Zubarev’s function (4) the value enter in second item

$$-\int p_q(y)dy = \exp\{-\varepsilon y\} = 1 - \varepsilon y + (\varepsilon y)^2/2 - \ldots = 1 - y/\langle \Gamma \rangle + y^2/2\langle \Gamma \rangle^2 - \ldots \quad (6)$$

Obviously that to tend to infinity of average lifetime, $\langle \Gamma \rangle \to \infty$, correlation (6) tends to unity.
Besides exponential density of probability (5), as density of lifetime distribution Erlang distributions (special or the general), gamma distributions etc. (see [22, 23]), and also the modifications considering subsequent composed asymptotic of the decomposition [24] can be used. General Erlang distributions for \( n \) classes of ergotic states are fair for cases of phase transitions or bifurcations. For \( n = 2 \) general Erlang distribution looks like \( p_q(y) = \theta \rho_1 \exp\{-\rho_1 y\} + (1 - \theta) \rho_2 \exp\{-\rho_2 y\}, \theta < 1 \). Gamma distributions describe the systems which evolution has some stages (number of these stages coincides with gamma distribution order). Considering real-life stages in non-equilibrium systems (chaotic, kinetic, hydrodynamic, diffusive and so forth [16]), it is easy to agree, first, with necessity of use of gamma distributions of a kind

\[
p_q(y) = \varepsilon(\varepsilon y)^{k-1} \exp\{-\varepsilon y\}/\Gamma(k)
\]

(\( \Gamma(k) \) is gamma function, at \( k = 1 \) we receive distribution (5)), and, secondly, with their importance in the description of non-equilibrium properties. The piecewise-continuous distributions corresponding to the different stages of evolution of the system will be used below.

More detailed description \( p_q(u) \) in comparison with limiting exponential (5) allows to describe more in detail real stages of evolution of system (and also systems with small lifetimes). Each from lifetime distributions has certain physical sense. In the theory of queues, for example [33], to various disciplines of service there correspond various expressions for density of lifetime distribution. In the stochastic theory of storage [33], to these expressions there correspond various models of an exit and an input in system.

The value \( \varepsilon \) without taking into account of dissipative effects can be defined, for example, from results of work [9]. The value \( \varepsilon \) is defined also in work [6] through average values of operators of entropy and entropy production, flows of entropy and their combination.

How was already marked, it is possible to specify very much, no less than 1000, expressions for the distributions of past lifetime of the system. Certain physical sense is given to each of these distributions. To some class functions of distributions, apparently, some class of the physical systems corresponds, the laws of relaxation in which answer this class of functions of distributions for lifetime.

### 3.3 Influence of the past on non-equilibrium properties

In [18] by consideration of the paradox connected with a waiting time, the following result is received: let \( X_1 = S_1; X_2 = S_2 - S_1; \ldots \) are mutually independent also it is equally exponential the distributed values with average \( 1/\varepsilon \). Let \( t > 0 \) is settled, but it is any. Element \( X_k \), satisfying to condition \( S_{k-1} < t \leq S_k \), has density \( \nu_t(x) = \varepsilon^2 x \exp\{-\varepsilon x\}, 0 < x \leq t; \nu_t(x) = \varepsilon (1 + \varepsilon x) \exp\{-\varepsilon x\}, x > t \). In Zubarev’s NSO [1, 2] the value of lifetime to a present moment \( t \), belonging lifetime \( X_k \), influence of the past on the present is considered. Therefore the value \( p_q(u) \) should be chosen not in the form of exponential distribution (5), and in a form
that in the form of gamma distribution (7) at \( k = 2 \). In this case distribution (8) coincides with special Erlang distribution of order 2 [22], when refusal (in this case - the moment \( t \)) comes in the end of the second stage [22], the system past consists of two independent stages. Function of distribution is equal \( P_q(x) = 1 - \exp\{-\varepsilon x\} - \varepsilon x \exp\{-\varepsilon x\} \), \( p_q(u) = dP_q(u)/du \), unlike exponential distribution, when \( P_q(x) = 1 - \exp\{-\varepsilon x\} \). The behaviour of these two densities of distribution of a form (5) and (8) essentially differs in a zero vicinity. In case of (8) at system low probability to be lost at small values \( y \), unlike exponential distribution (5) where this probability is maximal. Any system if has arisen, exists any minimal time, and it is reflected in distribution (8).

In work [10] it is shown, in what differences from Zubarev’s distribution (3) with exponential distribution of lifetime (5) results gamma distribution (7), (8) use. Additional items in NSO, in integral of collisions of the generalized kinetic equation, in expressions for average fluxes and self-diffusions coefficient are considered. The same in [10] is done and for special Erlang distribution \( k = 2, 3, 4, \ldots, n \), \( P_q(x) = 1 - \exp\{-\varepsilon x\}\{1 + \varepsilon x/1! + \ldots + (\varepsilon x)^{k-1}/(k-1)!\}; \varepsilon = k/\langle \Gamma \rangle \).

For distributions (7), (8) is correct correlation (4), value \(-\int p_q(u)du \to 1\) by \( \langle \Gamma \rangle \to \infty \).

Thus the multi-stage model of the past of system is introduced. Non-equilibrium processes usually proceed in some stages, each of which is characterized by the time scale. In distribution (8) the account of two stages, possibly, their minimal possible number is made. Other distributions can describe any other features of the past. Corresponding additives will be included into expressions for fluxes, integral of collisions, kinetic coefficients. Besides special Erlang distributions with whole and specified values \( k = n \), that does not deduce us from set of one-parametrical distributions, the general already two-parametrical gamma distribution where the parameter \( k \) can accept any values can be used. In this case \( \langle \Gamma \rangle = k/\varepsilon \). The situation (formally), when \( k < 1 \) is possible. Then sources will tends to infinity, as \((t - t_0)^{k-1} \to \infty\) at \( k < 1 \). This divergence can be eliminated, having limited from below the value \( t - t_0 \) of minimal lifetime value \( \Gamma_{min} \), having replaced the under zero limit of integration on \( \Gamma_{min} \). Then to expression for a source (4) it is added item \([\varepsilon \Gamma_{min}]^{k-1}/\Gamma(k)\varepsilon \exp\{-\varepsilon \Gamma_{min}\} \ln g_q(t - \Gamma_{min}, -\Gamma_{min})\).

In [18] it was shown that the exponential lifetime distribution \( t_f - t_0 \) (\( t_f \), \( t_0 \) are random moments of system death and birth) at big \( t \)'s the "age" of a system \( t - t_0 \) tends to the exponential form. In Zubarev’s NSO [12] the lifetime value \( t - t_0 \) to the current time \( t \), which is a part of the total lifetime \( t_f - t_0 \), is considered, that is the influence of the past on the current moment is taken into account. The full lifetime distribution, as well as the "past" lifetime (i.e. time from the system birth \( t_0 \) till the current time \( t \)) need not be exponential. The logarithm of NSO in the case (8) has the form

\[
\ln g(t) = \int_0^\infty p_q(u) \ln g_q(t - u, -u)du = \]

\( (9) \)
\[
\int_0^\infty \varepsilon^2 u \exp\{-\varepsilon u\} \ln q(t-u) \, du = \\
\ln q(t,0) + \int_0^\infty \sigma(t-u) \exp\{-\varepsilon u\} \, du = \\
\ln q_{\text{ub}}(t) + \int_0^\infty \sigma(t-u) \varepsilon u \exp\{-\varepsilon u\} \, du; \\
\sigma(t-u) = d\ln q(t-u) / du,
\]

where \(\ln q_{\text{ub}}(t) = \ln q(t,0) + \int_0^\infty \sigma(t-u) \exp\{-\varepsilon u\} \, du\) is the Zubarev’s form of the NSO. \(\sigma(t) = \partial \ln q(t-u) / \partial u|_{u=0} = -\partial \ln q(t,0) / \partial t\) is the entropy production operator \([1]\). It is seen from \([3]\) that the logarithm of the NSO has an additional term in comparison to the Zubarev’s form. The source in the rhs of the Liouville equation (or dissipative part of the Liouville operator \([7]\)) equal \(J = -\varepsilon [\ln q(t) - \ln q_{\text{ub}}(t)]\), that is the system relaxes not towards \(\ln q(t,0)\), like it is the case of Zubarev’s NSO, but towards \(\ln q_{\text{ub}}(t)\). From the expression \([10]\) it is seen that introduced NSO contains amendments to the Zubarev’s NSO \([1,2]\). The physical results obtained with use of \([3]\) also contains additional terms in comparison to Zubarev’s NSO. The additional terms describe the influence of the lifetime finiteness on the kinetic processes. The expressions for average fluxes \([1]\) averaged over \([9]\) have the form

\[
< j^m(x) > = < j^m(x) >_{\text{ub}} + \\
\sum_n \int_{-\infty}^t \varepsilon(t-t') \exp\{\varepsilon(t'-t)\} \{j^m(x), j^n(x', t'-t)\} X_m(x', t') \, dt \, dx',
\]

where \(< j^m(x) >_{\text{ub}} = < j^m(x) >_{t} + \\
\sum_n \int_{-\infty}^t \exp\{\varepsilon(t'-t)\} \{j^m(x), j^n(x', t'-t)\} X_m(x', t') \, dt \, dx'\) are fluxes in the form obtained by Zubarev \([1]\). \(j^n\) are flux operators, \(X_m\) are corresponding thermodynamical forces; \((j^m(x), j^n(x', t)) = \beta^{-1} \int_0^\infty q^0 < j^m(x), j^n(x', t, \tau) >_{t} \, d\tau\) are quantum time correlation functions, \(j^n(x', t, \tau) = \exp\{-\beta^{-1} A\tau\} j^n(x', t) \exp\{-\beta^{-1} A\tau\}; A = \sum_m \int F_m(x, t) P_m(x) \, dx\). The collision integrals of the generalized kinetic equation \([1]\), averaged over \([3]\) have the amendments

\[
S^{(2)}_{\text{add}} = -\hbar^{-2} \int_{-\infty}^0 dt \varepsilon(t) \exp\{\varepsilon(t)\} < [H_1(t), [H_1, P_k]] + i\hbar \sum_m P_m \frac{\partial S^{(1)}_k}{\partial [P_m]} > > q
\]

to Zubarev result \([1]\): \(S^{(2)} = -\hbar^{-2} \int_{-\infty}^0 dt \exp\{\varepsilon(t)\} < [H_1(t), [H_1, P_k]] + i\hbar \sum_m P_m \frac{\partial S^{(1)}_k}{\partial [P_m]} > > q\), where the Hamiltonian of the system is \(H = H_0 + H_1\), \(H_1\) is the Hamiltonian of the interaction which contains the longtime correlations \([7]\), \(S^{(1)}_k = < [P_k, H_1] > > \). The same is valid for the generalized transport equations \([1]\), kinetic coefficients etc. Thus the selfdiffusion coefficient (or, to be exact, its Laplace transform over time and space) obtained in \([2]\) in the form
\[ D(\omega, q) = \frac{q^{-2} \Phi(\omega, q)}{1 + \Phi(\omega, q)}, \quad (12) \]

where \( \Phi(\omega, q) = \frac{\int_0^\infty dt \exp\{i(\omega - \epsilon)t\} \int_0^\infty \delta_t < n_q - n_q(\tau + i\hbar \lambda) > d\lambda}{\int_0^\infty \delta_t < n_q - n_q(\tau + i\hbar \lambda) > d\lambda}, \]
\[ n_q = \int n(x) \exp\{i(qx)\} dx = \sum_j \exp\{i(qx_j)\}, \] after use of (9) takes on the form
\[ D(\omega, q) = q^{-2} [\Phi(\omega, q) + \frac{\epsilon d\Phi(\omega, q)}{d(\omega - \epsilon)}] \]  
\[ 1 + \frac{\Phi(\omega, q)}{(i\omega - \epsilon)} + \frac{\epsilon d\Phi(\omega, q)}{d(\omega - \epsilon)}, \quad (13) \]

At \( \epsilon \to 0 \), for infinitely large system in the thermodynamic limit this expression (13) coincides with (12) at \( \epsilon \to 0 \) [2]. For finite size systems (as well as for the case \( \omega \to 0 \)) the results differ.

4 Systems with infinite lifetime

Above, as well as in work [10], additives to NSO in the Zubarev’s form are received for systems of the finite size, with finite lifetime. We will show, as for systems with infinite lifetime, for example, for systems of infinite volume, after thermodynamic limiting transition, the same effects, which essence in influence of the past of system, its histories, on its present non-equilibrium state are fair.

In work [10] it is shown, as changes in function \( p_q(u) \) influences on non-equilibrium descriptions of the system. But for those distributions \( p_q(u) \), which are considered in [10] (gamma-distributions, (7), (8)) the changes show up only for the systems of finite size with finite lifetimes. Additions to unit in equation (6) becomes vanishingly small to tend to infinity of sizes of the system and its average lifetime, as in the model distribution (5) used in Zubarev’s NSO (3).

For the systems of finite size and the exponential distribution results to nonzero additions in expression (3). Thus, these additions to NSO and proper additions to kinetic equations, kinetic coefficients and other non-equilibrium descriptions of the system, are an effect finiteness of sizes and lifetime of the system, not choice of distribution of lifetime of the system. We will find out, whether there are distributions of lifetime of system for which and for the infinitely large systems with infinitely large lifetime an additional contribution to NSO differs from Zubarev’a NSO.

We will consider a few examples of task of function \( p_q(u) \) in (2), (3). We will be limited to the examples of task of the piecewise-continuous distributions, which result in results different from works [11] [2]. There are numerous experimental confirmations of such change of distribution of lifetime of the system \( p_q(u) \) on the time domain of life of the system. This and transition to chaos and transition from the laminar mode to turbulent one is also accompanied by the change of distribution of \( p_q(u) \). In works [34] [35] is shown transition of distribution of the first passage processes from Gaussian regime to the not Levy conduct in some point of time. Besides the real systems possess finite sizes and finite lifetime. Therefore influence of surroundings on them is always present.
That a source not is equal to the zero and in a limit infinitely large systems, is
related to the openness of the system, influence on her of surroundings
4a). We will put
\[ p_q(u) = \begin{cases} \frac{ka^k}{(u+a)^{k+1}}, & u < c, \\ b \varepsilon \exp(-\varepsilon u), & u \geq c; \end{cases} \]  
(14)

First part of distribution \[14\], Pareto distribution, in work \[23\] got from
the exponential distribution with the random parameter of intensity \(\rho\), when
\[
J = \frac{k}{a} \ln \varrho(t, 0) - \int_0^c \frac{k(k+1)a^k}{(u+a)^{k+2}} Sdu - \varepsilon e^c \left(\frac{a}{a+c}\right)^k \int_0^\infty \varepsilon^2 e^{-\varepsilon u} Sdu, 
\]
(17)
where \(S = \ln \varrho(t-u, -u)\). Distribution of NSO \[3\] in the case of distribution
\[14\] equal
\[
\langle \Gamma \rangle = \frac{a}{k-1} + \left(\frac{a}{a+c}\right)^k \frac{1}{\varepsilon} \left(1 + \varepsilon c - \left(\frac{k c + a}{(k-1)}\right)^{\frac{1}{\varepsilon}}\right). 
\]
(15)

The value \[15\] \(\langle \Gamma \rangle \to \infty\) at \(\varepsilon \to 0\). From \[14\] we find
\[
- \int p_q(u) = \begin{cases} \frac{a}{u+a}^k, & u < c, \\ b \exp(-\varepsilon u), & u \geq c. \end{cases} \]  
(16)

Source in right part of Liouville equation \[2\] for distribution \[14\] in accor-
dance with expression \[4\] equal
\[
\Delta = \int_0^c \left[ \frac{a}{u+c}^k - \varepsilon e^c \left(\frac{a}{a+c}\right)^k \right] \sigma du + \int_0^\infty \varepsilon^2 e^{-\varepsilon u} \sigma du, 
\]
where \(\sigma = \frac{d \ln \varrho(t-u, -u)}{du}\); \(\ln zub(t) = \ln \varrho_q(t, 0) + \int_0^\infty e^{-\varepsilon u} \sigma du\) is distribution got
by Zubarev in \[11, 2\], and \(\Delta\) are amendments to him.
4b). We will consider distribution of kind now
\[
p_q(u) = \begin{cases} d, & u < c, \\ \varepsilon \exp(-\varepsilon u), & u \geq c; \end{cases} 
\]  
(19)

From the condition of the normalization we find a value \(d = \frac{1}{\varepsilon} (1 - e^{-\varepsilon c})\). Average lifetime
\[
\langle \Gamma \rangle = \frac{d e^2}{2} + \frac{1}{\varepsilon} e^{-\varepsilon c} (1 + \varepsilon c). 
\]
(20)
Average lifetime $\langle \Gamma \rangle \to \infty$ at $\varepsilon \to 0$. Source (4) in equation (2) in this case equal

$$J = d\ln q_0(t, 0) - \int_0^\infty \varepsilon^2 e^{-\varepsilon u} Sdu.$$  \hspace{1cm} (21)

NSO is equal

$$\ln q(t) = \ln q_{zub}(t, 0) - \int_0^c [e^{-\varepsilon u} + \frac{1}{c}(1 - e^{-\varepsilon c})u]\sigma du;$$  \hspace{1cm} (22)

$$\Delta = -\int_0^c [e^{-\varepsilon u} + \frac{1}{c}(1 - e^{-\varepsilon c})u]\sigma du.$$

We get in this case, that in an additional item memory of the system is limited by a size, is observed effect of limited memory. It is possible to consider and other examples of task distributions $p_q(u)$ which reduce to limited memory.

4c). For the exponential density of distribution with different intensities in different temporal intervals

$$p_q(u) = \begin{cases} 
\varepsilon_1 \exp\{-\varepsilon_1 u\}, & u < c, \\
\varepsilon_2 \exp\{-\varepsilon_2 u\}, & u \geq c, 
\end{cases}$$  \hspace{1cm} (23)

from the condition of the normalization finds that $b = e^{(\varepsilon_2 - \varepsilon_1)}$;

$$\langle \Gamma \rangle = \frac{1}{\varepsilon_1} [1 - e^{-\varepsilon_1 c}(1 + \varepsilon_1 c)] + \frac{1}{\varepsilon_2} e^{-\varepsilon_1 c}(1 + \varepsilon_2 c).$$  \hspace{1cm} (24)

At $\langle \Gamma \rangle \to \infty$, $\varepsilon_2 \to 0$.

$$J = \varepsilon_1 \ln q_0(t, 0) - \int_0^c \varepsilon_1^2 e^{-\varepsilon_1 u} Sdu - e^{(\varepsilon_2 - \varepsilon_1)} \int_c^\infty \varepsilon_2^2 e^{-\varepsilon_2 u} Sdu;$$

$$\ln q(t) = \ln q_{zub}(t) + \Delta;$$

$$\Delta = \int_0^c [e^{-\varepsilon_1 u} - e^{-\varepsilon_2 u}]\sigma du + \int_c^\infty [e^{(\varepsilon_2 - \varepsilon_1)} - 1]e^{-\varepsilon_2 u}\sigma du;$$

$$\Delta_{\varepsilon_2 \to 0} \to \int_0^\infty [e^{-\varepsilon_1 u} - 1]\sigma du.$$

4d). We will consider yet distribution of kind

$$p_q(u) = \begin{cases} 
\varepsilon^2 u \exp\{-\varepsilon u\}, & u < c, \\
\frac{\exp\{-\gamma u\}}{[1 + \frac{1}{q}\gamma \exp\{\gamma u\}]^{(1-q)}}, & u \geq c. 
\end{cases}$$  \hspace{1cm} (26)

The second part of distribution (26) can be received from results of works [36] and like to Tsallis distributions [37]. For (26) $b = \frac{\gamma a[1 + \frac{1}{q}\gamma]}{B(1-p, 1)(1-e^{-\varepsilon c})}$, where $B(1 - p, 1)$ is incomplete beta function [38], $p = (\varepsilon - 1)\gamma \exp\{-\gamma ac\}$.

Average lifetime

$$\langle \Gamma \rangle = \frac{2}{\varepsilon} [1 - e^{-\varepsilon c}(1 + \varepsilon c + (\varepsilon c)^2/2)] +$$
\[ +e^{-\varepsilon c}(1 + \varepsilon c)\frac{\Gamma^2\left(\frac{1}{a}\right) F_2\left(\frac{1}{a}, \frac{1}{a}, \frac{1}{a}; 1 + \frac{1}{a}, 1 + \frac{1}{a}; p\right)}{(a^2\gamma)^2 F_1\left(\frac{1}{a}, \frac{1}{a}; \frac{1}{a}; 1 + \frac{1}{a}; p\right)}, \]

where \( \Gamma(\cdot) \) is gamma function, \( _mF_n \) is hypergeometrical functions \[38\], tends to infinity at \( \gamma \to 0 \). In this case value of the normalization \( b \to 0 \). Therefore at \( \langle \Gamma \rangle \to \infty, \gamma \to 0 \) and \( b \to 0 \);

\[ \ln \varrho(t) = \ln \varrho_q(t, 0) + \int_0^u e^{-\varepsilon u}(1 + \varepsilon u)\sigma du; J = -\int_0^u e^{-\varepsilon u}e^2(1 - \varepsilon u)Sdu. \]

The effect of limited memory shows up in this case obviously.

Why the examples of this section differ from examples of section 3? In interpretation \[2\] fluctuate the random value \( t_0 \) in \( u = t - t_0 \). In \[2\] limiting transition is conducted for the parameter \( \varepsilon, \varepsilon \to 0 \) in the exponential distribution \( p_q(u) = \varepsilon \exp\{-\varepsilon u\} \) after thermodynamical limiting transition. In interpretation of work \[6\] it corresponds to that mean lifetime of the system \( \langle \Gamma \rangle = \langle t - t_0 \rangle = 1/\varepsilon \to \infty \). But middle intervals between random shoves infinitely increase, exceeding lifetime of the system. Therefore an item with a source in Liouville equation applies in a zero. If there is the change of distribution \( p_q(u) \) on the time domain of life of the system, as in examples 4a)-4d), influence of surroundings, followed this change with, remains in the time domain of life even at tendency to infinity of mean lifetime.

5 Application of distributions of section 4 to research of conductivity

On the example of determination of conductivity we will consider, in what consequences the change of type of functions \( p_q(u) \) and \( \varrho(t) \) results as compared to an exponential law for \( p_q(u) \), used in work \[5\].

Determination of conductivity by a method NSO is considered in works \[39, 40, 41, 42\]. In this section we will consider the transport of charges in the electric field, as linear reaction on mechanical perturbation, conductivity in the linear approaching, following results of work \[9\] and, as in \[5\], we will be limited to the important special case - reaction of the equilibrium system on the spatially homogeneous variable field.

\[ \vec{E}(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} \vec{E}(\omega). \]

Hamiltonian of perturbation is given by a formula

\[ H^1_{1} = -\vec{P} \vec{E}(t), \]

where \( \vec{P} \) is operator proper to the vector of polarization of the system. In coordinate representation this operator is written down as

\[ \vec{P} = \sum_i c_i \vec{r}_i, \]
where $e$ is charge of particle, and $\mathbf{r}$ is its radius vector. The operator of current is equal

$$\mathbf{J} = \dot{\mathbf{P}} = e \sum_j \dot{\mathbf{r}}_j = \frac{e}{m} \sum_j \ddot{\mathbf{p}}_j,$$

where $\ddot{\mathbf{p}}_j$ is particle momentum, $m$ is mass of a particle. We will choose a model in which coulomb interaction is taken into account as self-consistent screening of the field, i.e. we will consider that $\mathbf{E} = \mathbf{E}_0$. Most essential difference from work [5] at consideration of this problem consists of replacement of Laplace transformation used in [5], form

$$\langle A; B \rangle_{\omega + i\varepsilon} = \int_0^\infty dt e^{i(\omega + i\varepsilon)t} \langle A(t), B \rangle, (\varepsilon > 0), \quad (27)$$

where $\langle A(t), B(t') \rangle = \int_0^1 dx Tr\{\Delta A(t) \Delta B(t' + i\beta x) \rho_{eq}\}$ is temporal correlation function [5], by other integral transformation. So, for an example 4a) with distribution of form (14), (16) expression (27) it is replaced on

$$\langle A; B \rangle_{\omega + i\varepsilon} = \int_c^\infty dt e^{i(\omega + i\varepsilon)t} \langle A(t), B \rangle, (\varepsilon > 0), \quad (29)$$

We will consider a isotropic environment in which tensor of conductivity is diagonal. In work [5] for Laplace transformation of kind (27) expression is got for specific resistance $\rho(\omega)$ of kind

$$\rho(\omega) = \frac{1}{\sigma(\omega)} = \frac{3V}{\beta(\mathbf{J}, \mathbf{J})} [-i\omega + M]; \quad (30)$$

$$M = \frac{\langle \dot{\mathbf{J}}; \dot{\mathbf{J}} \rangle_{\omega + i\varepsilon}}{\langle \mathbf{J}, \dot{\mathbf{J}} \rangle + \langle \mathbf{J}, \dot{\mathbf{J}} \rangle_{\omega + i\varepsilon}}, \quad (31)$$

where $V$ is the volume of the system, $\beta$ is inverse temperature, $\sigma(\omega)$ is scalar coefficient of conductivity or simply conductivity. In the examples considered below in expression (31) a value $M$ will change. Doing the operations conducted in work [5], with replacement of expression (27) by expression (28), we will get for this case in place of correlation (31) more difficult expression of kind

$$M = \frac{\langle \dot{\mathbf{J}}; \dot{\mathbf{J}} \rangle_{\omega + i\varepsilon} + \frac{i\omega}{i(\omega + i\varepsilon)} \epsilon i\omega c \left( \frac{a}{a + c} \right)^k \langle \dot{\mathbf{J}}; \dot{\mathbf{J}} \rangle_{\omega + i\varepsilon} (c, \infty) }{K};$$

$$K = (\mathbf{J}(0), \mathbf{J}) - \frac{k}{a} \langle \dot{\mathbf{J}}; \dot{\mathbf{J}} \rangle_{\omega + i\varepsilon} - (1 - \frac{i\omega}{i(\omega + i\varepsilon)}) \epsilon i\omega c \left( \frac{a}{a + c} \right)^k (\mathbf{J}(c), \mathbf{J}) +$$

$$+ \langle \dot{\mathbf{J}}; \dot{\mathbf{J}} \rangle_{\omega + i\varepsilon} \epsilon i\omega c \left( \frac{a}{a + c} \right)^k (\mathbf{J}(c), \mathbf{J})_\infty.$$
At \( \varepsilon \to 0 \)

\[
M = \frac{\langle \dot{J}; \dot{J} \rangle_{\omega;0,k} + e^{i\omega t} \left( \frac{d}{dt} \right)^{k} \langle \dot{J}; \dot{J} \rangle_{\omega;1;\infty}}{\langle \dot{J}(0), \dot{J} \rangle - \frac{i}{\omega} \langle \dot{J}; \dot{J} \rangle_{\omega;0,k+1} + \langle \dot{J}; \dot{J} \rangle_{\omega;0,k} + \langle \dot{J}; \dot{J} \rangle_{\omega;1;\infty}}.
\]

For distribution (19) in 4b) Laplace transformation of kind (27) it is replaced on

\[
-d(A;B)_{\omega;c,t} + \langle A;B \rangle_{\omega+ic;1;\infty},
\]

where \( \langle A;B \rangle_{\omega;c,t} = \int_{0}^{c} dt e^{i\omega t} (A(t),B) \), the value \( d \) is given in (19), (20), \( \langle A;B \rangle_{\omega+ic;1;\infty} \) is given in (29). In place of expression (31) in this case we will get expression

\[
M = \frac{\langle \dot{J}; \dot{J} \rangle_{\omega+ic;\infty} - d[\langle \dot{J}; \dot{J} \rangle_{\omega;c,t} + \langle \dot{J}; \dot{J} \rangle_{\omega;1;\infty}]}{K_1},
\]

\[
K_1 = \frac{i\omega}{i(\omega + i\varepsilon)} \langle \dot{J}; \dot{J} \rangle_{\omega+ic;\infty} - d[\langle \dot{J}; \dot{J} \rangle_{\omega;c,t} + \langle \dot{J}; \dot{J} \rangle_{\omega;1;\infty}]
\]

If \( \varepsilon \to 0, (\Gamma) \to \infty, d \to 0, \) and

\[
M = \frac{\langle \dot{J}; \dot{J} \rangle_{\omega;1;\infty}}{e^{i\omega c}(\dot{J}(c),\dot{J}) + \langle \dot{J}; \dot{J} \rangle_{\omega;1;\infty}}.
\]

This expression at the small values \( c \) low differs from (31). For a case 4c) with distribution \( p_{q}(u) \) of kind (23) Laplace transformation (27) it is substituted by a value

\[
\langle A;B \rangle_{\omega+ic;0;\infty} + e^{i(\varepsilon_2 - \varepsilon_1)} \langle A;B \rangle_{\omega+ic;2;\infty},
\]

where \( \langle A;B \rangle_{\omega+ic;0;\infty} = \int_{0}^{\infty} dt e^{i(\omega + ic)t} (A(t),B), \langle A;B \rangle_{\omega+ic;2;\infty} \) is given in (29) and value \( M \) from (31) it is substituted by a value

\[
M = \frac{\langle \dot{J}; \dot{J} \rangle_{\omega+ic;1;\infty} + e^{i(\varepsilon_2 - \varepsilon_1)} \frac{i(\omega + i\varepsilon_1)}{i(\omega + i\varepsilon_2)} \langle \dot{J}; \dot{J} \rangle_{\omega+ic;2;\infty}}{K_2},
\]

\[
K_2 = (\dot{J}(0),\dot{J}) - e^{i(\omega + ic)c} (\dot{J}(c),\dot{J}) + \langle \dot{J}; \dot{J} \rangle_{\omega+ic;1;\infty} +
\]

\[
e^{i(\varepsilon_2 - \varepsilon_1)} \frac{i(\omega + i\varepsilon_1)}{i(\omega + i\varepsilon_2)} [e^{i(\omega + ic)c} (\dot{J}(c),\dot{J}) + \langle \dot{J}; \dot{J} \rangle_{\omega+ic;1;\infty}].
\]

At \( (\Gamma) \to \infty \) and \( \varepsilon_2 \to 0 \) the value \( M \) changes unessential, assuming a form

\[
M = \frac{\langle \dot{J}; \dot{J} \rangle_{\omega+ic;1;\infty} + e^{-ic} \frac{i(\omega + ic)}{i\omega} \langle \dot{J}; \dot{J} \rangle_{\omega;1;\infty}}{K_3},
\]

\[
K_3 = (\dot{J}(0),\dot{J}) - e^{i(\omega + ic)c} (\dot{J}(c),\dot{J}) + \langle \dot{J}; \dot{J} \rangle_{\omega+ic;1;\infty} +
\]

15
\[ +e^{-c\varepsilon_1} \frac{i(\omega + i\varepsilon_1)}{i\omega} [\varepsilon i\omega(\vec{J}(c), \vec{J}) + \langle \dot{\vec{J}}; \vec{J} \rangle \omega(c, \infty)]. \]

At the small values of value \( \varepsilon_1 \) this expression close to (31). From (24) it is visible that \( \lim_{\varepsilon_2 \to 0} \varepsilon_2 \langle \Gamma \rangle = e^{-c\varepsilon}. \)

Unfortunately, to compare the got results to the experiment difficultly, because parameters \( c, a, k, \varepsilon_1 \) are unknown. But, apparently, through the choice of type of distribution \( p_q(u) \) and selection of his parameters it is possible to obtain very good accordance with experimental results.

6 Comparison with the theory of transport processes by McLennan

In works [43, 44] the statistical theory of transport processes, based on introduction of external forces of unpotential behavior, describing influence of surroundings on this system, is built. In the Appendix II to work [1] this theory is compared with the method of NSO.

The function of distribution of the complete system \( f_u \) is described by Liouville equation

\[
\frac{\partial f_u}{\partial t} + \{f_u, H_u\} = 0,
\]

(32)

where \( \{,\} \) are classic Poisson brackets, \( H_u = H + H_s + U \), \( H \) is the Hamiltonian of the concerned system, \( H_s \) is Hamiltonian of surroundings, \( U \) is Hamiltonian of interaction of the system with surroundings. The functions of distributions of the concerned system \( f \) and surroundings \( g \) are accordingly equal

\[
f = \int f_u d\Gamma_s, \quad g = \int f_u d\Gamma,
\]

where \( d\Gamma_s \) and \( d\Gamma \) are elements of phase volume of surroundings and concerned system. Integrating (32) on phase space of surroundings \( d\Gamma_s \), we get equation of motion for \( f \), containing a source. At introduction of function \( X \) describing correlation of the system with surroundings

\[
f_u = f g X,
\]

(33)
equation for \( f \) is written down in a form

\[
\frac{\partial f}{\partial t} + \{f, H\} + \frac{\partial(f F_\alpha)}{\partial p_\alpha} = 0,
\]

(34)

where

\[
F_\alpha = -\int g X \frac{\partial U}{\partial q_\alpha} d\Gamma_s,
\]

(35)

\( q_\alpha \) and \( p_\alpha \) are coordinates and impulses of the system, in (34) summation up is assumed on \( \alpha \). A value \( F_\alpha \) makes sense ”force” presenting the action of surroundings on the system. In [43, 44] from the physical considering justice of expression is assumed

\[
\frac{\partial F_\alpha}{\partial p_\alpha} = -\int \vec{j}_s(x) ds,
\]

(36)
where \( \vec{j}_s(\vec{x}) \) is density of entropy flow (including the work accomplished above the system), \( d\vec{s} \) is element of surface limiting the system. For the function of distribution in [1] expression coincident with expression for NSO at \( \varepsilon \to 0 \) is got. If to accept, that correlation between the system and surroundings results in replacement of expression (36) on expression

\[
\frac{\partial F_\alpha}{\partial p_\alpha} = -\int \vec{j}_s(\vec{x})Y(\vec{x}, t)d\vec{s},
\]

and \( Y(x, t) = -\int p_\alpha(t)dt \), after integration on time, conducted at the decision of equation for the function of distribution, we will get for the function of distribution of the system expressions coincident with expressions of the this work.

Indeed, the function of distribution of time of past life \( t - t_0 \) of the system \( p_\alpha(u) \) depends and from properties of the system and from surroundings, as \( X \). A function \( X \) must depend on time. Then, for example, for expression 4a) with distribution [13], (16) function \( -\int p_\alpha(t)dt \) related to \( X(t) \), to the moment of time \( c \) decreases, as \( (\frac{a}{\varepsilon + c})^k \), and from a moment \( c \) assumes a form \( (\frac{a}{\varepsilon + c})^k e^{-\varepsilon c}e^{-\varepsilon t}, \varepsilon \to 0 \) after thermodynamic limiting transition and tendency \( \langle \Gamma \rangle \to \infty \). Time \( c \) comes forward in a role of time of establishment of stationary flows. For an example 4c) with distribution [23] a function \( Y(x, t) \) decreases as \( e^{-\varepsilon c}t \) at \( t < c \) and as \( e^{c(\varepsilon_1 - \varepsilon_2)}e^{-\varepsilon_2 t} \) at \( t \geq c \), \( \varepsilon_2 \to 0 \) at \( \langle \Gamma \rangle \to \infty \). Thus, the function of distribution \( p_\alpha(u) \) of time of past life of the system (more precisely, integral from her with a reverse sign) can be interpreted and in connection with the function of correlation \( X \) between the system and surroundings.

7 The conclusion

As it is specified in work [15], existence of time scales and a stream of the information from slow degrees of freedom to fast create irreversibility of the macroscopical description. The information continuously passes from slow to fast degrees of freedom. This stream of the information leads to irreversibility. The information thus is not lost, and passes in the form inaccessible to research on Markovian level of the description. For example, for the rarefied gas the information is transferred from one-partial observables to multipartial correlations. In work [6] values \( \varepsilon = 1/\langle \Gamma \rangle \) and \( p_\alpha(u) = \varepsilon \exp\{-\varepsilon u\} \) are expressed through the operator of entropy production and, according to results [15], through a stream of the information from relevant to irrelevant degrees of freedom. Introduction in NSO function \( p_\alpha(u) \) corresponds to specification of the description by means of the effective account of communication with irrelevant degrees of freedom. In the present work it is shown, how it is possible to spend specification the description of effects of memory within the limits of method NSO, more detailed account of influence on evolution of system of quickly varying variables through the specified and expanded kind of density of function of distribution of time the lived system of a life.

In many physical problems finiteness of lifetime can be neglected. Then \( \varepsilon \sim 1/\langle \Gamma \rangle \to 0 \). For example, for a case of evaporation of drops of a liquid
it is possible to show \cite{45}, that non-equilibrium characteristics depend from $\exp(y^2); y = \varepsilon/(2\lambda_2)^{1/2}, \lambda_2$ is the second moment of correlation function of the fluxes averaged on quasi-equilibrium distribution. Estimations show, what even at the minimum values of lifetime of drops (generally - finite size) and the maximum values size $y = \varepsilon/(2\lambda_2)^{1/2} \leq 10^{-5}$. Therefore finiteness of values $\langle \Gamma \rangle$ and $\varepsilon$ does not influence on behaviour of system and it is possible to consider $\varepsilon = 0$. However in some situations it is necessary to consider finiteness of lifetime $\langle \Gamma \rangle$ and values $\varepsilon > 0$. For example, for nanodrops already it is necessary to consider effect of finiteness of their lifetime. For lifetime of neutrons in a nuclear reactor in work \cite{6} the equation for $\varepsilon = 1/\langle \Gamma \rangle$ which decision leads to expression for average lifetime of neutrons which coincides with the so-called period of a reactor is received. In work \cite{46} account of finiteness of lifetime of neutrons result to correct distribution of neutrons energy.

Use of distributions \cite{7, 8} and several more obvious forms of lifetime distribution in quality $p_q(u)$ leads to a conclusion, that the deviation received by means of these distributions values $\ln g(t)$ from $\ln g_{zub}(t)$ is no more $\varepsilon \sim 1/\langle \Gamma \rangle$. Therefore in expression \cite{45} additives to the result received by Zubarev, are proportional $\varepsilon$. This result corresponds to mathematical results of the theory asymptotical phase integration of complex systems \cite{24} according to which distribution of lifetime looks like $p_q(u) = \exp\{-\varepsilon u\} + \lambda\varphi_1(u) + \lambda^2\varphi_2(u) + ...$, where the parameter of smallness $\lambda$ in our case corresponds to value $\varepsilon \sim 1/\langle \Gamma \rangle$. Generally the parameter $\lambda$ can be any.

For distributions of kind \cite{14, 19, 23}, having a various form for different times, additives to Zubarev’s NSO are distinct from zero and for infinitely large systems with infinitely large lifetimes. In the present work it is shown, that it is possible, for example, for distributions of lifetime of the system, having a various appearance at different stages of evolution of system. Such behaviour will be coordinated with known division of process of evolution of system into a number of stages \cite{16}. For some distributions the effect of “finite memory” when only the limited interval of the past influences on behaviour of system is observed.

Probably, similar results will appear useful, for example, in researches of small systems. All greater value is acquired by importance of description of the systems in mesoscopical scales. A number of the results following from interpretation of NSO and $p_q(u)$ as density of lifetime distribution of system \cite{6}, it is possible to receive from the stochastic theory of storage \cite{33} and theories of queues. For example, in \cite{33} the general result that the random variable of the period of employment (lifetime) has absolutely continuous distribution $p_q(u) = g(u, x) = xk(u - x, u), u > x > 0$ is resulted; $g(u, x) = 0$ in other cases, where $k(x, t)$ is absolutely continuous distribution for value $X(t) - input to system.$

The form of distribution chosen by Zubarev for lifetime represents limiting distribution. The choice of lifetime distribution in NSO is connected with the account of influence of the past of system, its physical features, for the present moment, for example, with the account only age of system, as in Zubarev’s NSO \cite{1, 2, 6} at $\varepsilon > 0$, or with more detailed characteristic of the past evolution of system. The received results are essential in cases when it is impossible to
neglect effects of memory when memory time there is not little. The analysis of time scales as it is noted in [15], it is necessary to spend in each problem.

To determination of type of distribution $p_q(u)$ it is possible to apply general principles, for example, principle of maximum of entropy, as this it was done in the work [47] for determination of type of source in kinetic equation. It was marked that exists many possibilities of choice of function $p_q(u)$. But by virtue of a number of the reasons not all functions can be used in NSO. So, in work [14] it was marked that NSO can exist only if corresponding retarded theories are memory renormalizable. In general case offered approach can be interpreted as working out in detail of history of evolution of the system, clarification of different stages of its conduct.

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