Abstract:
Nowadays, vehicles are being abandoned by their users due to their high fuel consumption which had not been studied by the user from the start. Thus, the need to study the fuel consumption of vehicles due to one of the factors which greatly affects it; drag force, so as to produce information which vehicle users can have before purchasing their vehicles. With regards to this, this work is focused on the development of a computer program able to evaluate the fuel consumption of light vehicles. To achieve this, the basic equations of consumption are used to arrive at a mathematical relation between drag force and fuel consumption. This mathematical model is further implemented on the analytical software Matlab in order to produce the various consumption curves of the vehicles case study. A simulator which takes into consideration a vehicle’s engine data in order to produce specific consumption curves and provide valid information on the fuel consumption of the vehicle is developed from this mathematical model. It can be used in automotive construction companies and also by any individual.

Keywords: Fuel; Consumption; Drag Force; Matlab.

1. Introduction

The rise in petroleum products especially fuel in automobile industry has for many years been a topic of concern to vehicle users and manufactures. Hence, there is a great need for studies to be carried out in order to determine the rate of fuel use during vehicle functioning in so as to avoid sudden car breakdown on the roads. Some faults have proven to be causes of high vehicle consumption such as; engine malfunction, electric and electronic problems in the vehicle, as well as transmission faults. This paper presents one of the factors which greatly influence a vehicle’s fuel consumption, being the vehicle’s drag force and is directly related to the vehicle’s shape and
size. As such, this paper is aimed at providing useful information to users on their vehicle’s fuel consumption as well as, educating them on how to choose a vehicle depending on its size thus inherently its drag force in order to minimize fuel consumption.

2. Materials and Methods

2.1. Materials

Aerodynamics is a part of fluid mechanics, applied specifically to air. As such, the mathematical laws that on which this model will be based are:

- The Navier-Stokes equations in which the viscous effects are non-negligible.
- Euler or perfect fluid equations, in which the viscous effects are negligible;

\[
\overrightarrow{F_a} = \overrightarrow{F_{\text{Fluid} \rightarrow \text{Geom}}} = F_X \overrightarrow{e_X} + F_Y \overrightarrow{e_Y} + F_Z \overrightarrow{e_Z} \tag{1}
\]

\[
\overrightarrow{F_a} = \overrightarrow{F_p} + \overrightarrow{F_f} = \iint_S (P - P_{\text{ref}}) \overrightarrow{n} dS + \iint_S \tau \mu \overrightarrow{t} dS \tag{2}
\]

- With \( \overrightarrow{n} \) a normal vector to the wall and \( \overrightarrow{t} \) a vector tangential to the wall, \( P \) corresponds to the local static pressure, \( P_{\text{ref}} \) the reference pressure and \( dS \) the integration surface element. The aerodynamic moment defined at point \( P \) relative to the center of gravity \( G \) is:

\[
\overrightarrow{M_P} = \overrightarrow{F_a} \times \overrightarrow{G_p} = M_X \overrightarrow{e_X} + M_Y \overrightarrow{e_Y} + M_Z \overrightarrow{e_Z} \tag{3}
\]

Figure 1: Aerodynamic forces and moments in the vehicle reference frame

The pressure field exerted on a road obstacle generally induces a set of efforts where one considers usually:

- A drag force \( F_X \), parallel to the mean flow direction.
- A drift force \( F_Y \), perpendicular to the mean flow direction, in the horizontal plane.
- A force of lift \( F_Z \), perpendicular to the mean flow direction, in the vertical plane.
The expression of force is of the general form:

\[ F = q \times S \times C \]  

(4)

With \( q = \frac{1}{2} \times \rho \times V^2 \)

Thus, “q” in equation (4) above, Force becomes the following;

\[ F = \frac{1}{2} \times \rho \times V^2 \times S \times C \]  

(5)

### 2.1.1. Aerodynamic Coefficients

The aerodynamic coefficients are dimensionless coefficients used to quantify the forces in x, y, z.

- Cx: the drag coefficient;
- Cy: The coefficient of lateral drift;
- Cz: The coefficient of drift.

As the forces are calculated or measured experimentally (in the wind tunnel), the coefficients are determined by:

\[ C = \frac{F}{q \times S} \]  

(6)

Substituting the expression for q in equation (6) above, the final expression for determining the aerodynamic coefficients is gotten:

\[ C_{x,y,z} = \frac{F_{x,y,z}}{\frac{1}{2} \times \rho \times V^2 \times S} \]  

(7)

### 2.1.2. The Drag

The drag coefficient is the ratio of drag to the product of the reference surface and the dynamic pressure. The drag force is:

\[ \vec{F}_x = \frac{1}{2} \times \rho_{air} \times \vec{V}^2 \times S \times C_x \]  

(8)

With Cx = 1. We can also determine this force by considering the inertia force defined by:

\[ \vec{F} = m \vec{a} \]  

(9)

Where,

- m: The mass of air defined by the expression \( m = \rho_{air} \times S \times V \times T \)

- a: The acceleration defined by the expression \( a = \frac{1}{2} \times \frac{V}{T} \)
Therefore, replacing these expressions of mass and air into equation (9), the inertia force becomes:

\[ F_x = \frac{1}{2} \rho_{\text{air}} \times V^2 \times S \times C_x \tag{10} \]

**2.1.3. The Lift**

The lift equation is similar to that of the drag with Cx replaced by Cz or Cy, for lateral lift [7]:

\[ F_x = \frac{1}{2} \rho_{\text{air}} \times V^2 \times S \times C_x \tag{11} \]

**2.1.4. The Drift**

The drift equation is similar to that of the drag with Cx replaced by Cy where:

\[ F_Y = \frac{1}{2} \rho_{\text{air}} \times V^2 \times S \times C_Y \tag{12} \]

**2.1.5. Aerodynamic Moments**

It is defined according to three components [8]:

The moment of roll \( M_x = L = \frac{1}{2} \rho C_l S V^2 \) \tag{13} \]

The pitch moment \( M_y = N = \frac{1}{2} C_n S V^2 \) \tag{14} \]

The yaw Moment \( M_z = M = \frac{1}{2} \rho C_m S V^2 \) \tag{15} \]

**2.1.6. The Adherence of Tires**

This part will deal with the mechanical effects between the ground and the tire. The adhesion between the tire and the ground depends on two basic parameters:
- The adhesion coefficient;
- The adhesion forces.

**2.1.7. The Adherence Coefficient**

It depends on the state of the tyre and the nature of the soil; one note it to be \( \mu \).

| The coefficient of adhesion (\( \mu \)) | Nature and condition of the roadway |
|----------------------------------------|-----------------------------------|
| New tyre | Worn tyre |
| 0,8 | 0,95 | Asphalt road or dry concrete |
| 0,6 | 0,2 | Wet road (0.2 mm) |
| 0,3 | 0,1 | Heavy rain (1mm) |
| 0,3 | 0,1 | Snowy Road |
| 0,05 | 0,05 | Icy Road |
Table 2: Annotations of the isolated wheel

| Notation | Designation                                      | Notation | Designation                      |
|----------|--------------------------------------------------|----------|----------------------------------|
| N        | ground action on wheel due to weight             | V        | direction of vehicle movement    |
| T        | Forces Retardant (dragged)                       | \( \propto \) | angle of adhesion                |
| A        | ground action on the wheel due to braking        |          |                                  |

The relationship of the following coefficient of adhesion

\[ \mu = \tan \alpha = \frac{T}{N} \]  \hspace{1cm} (16)

With: \( \mu \): The coefficient of adhesion in \([\text{N}\cdot\text{S}^{-1}\cdot\text{m}^{2}]\)

2.1.8. The Adherence Strength

The adherence strength corresponds to the action (A) of the ground on the wheel.
- Has the stop (in static) \( A = N \) (i.e. the weight of the vehicle);
- To braking (dynamic) \( A \) is equal to the resulting \( N \) and \( T \).

- There is 100% of slip when \( V_{R}=0 \)
  (the wheel is blocked) and \( V_{V}>0 \);

The following points are important for these analysis:

1) **Hourly Consumption**

Hourly consumption is the mass of fuel consumed per unit of time; it is expressed as gram/hour [11].
\[ G_T = \frac{mc}{t} \]  

(17)

2) Effective Specific Consumption

The specific consumption "\( g_e \)" is the mass of fuel (in grams) that the engine would consume to deliver a power of 1kW for one hour (i.e. work of 3600 KJ).

\[ g_e = \frac{mc(g)}{N_e(KW) \times t(hour)} \]  

(18)

Consumption Per Hundred Kilometers

This refers to the amount of fuel in liter burned to travel one hundred (100) kilometers. The fuel consumption per hundred kilometers is calculated thus:

\[ Q_S = \frac{g_e \times N_e}{10.\rho \times V} \times 100 \]  

[11] [12]  

(19)

Relationship Between Consumption in One Hundred Kilometers and The Drag Force

The consumption over one hundred kilometers is defined by equation (19) above:

But, \( N_e = F_r \times V \) Also, \( F_r = F_x + G[f \cos \alpha + \sin \alpha] \) and \( F_x = \frac{1}{2} \rho \text{air} \times S \times C_x \).

Hence, \( N_e = \left\{ \frac{1}{2} \rho \text{air} \times S \times C_x \times V^2 + G[f \cos \alpha + \sin \alpha] \right\} V \)  

(20)

Where; \( \rho = \frac{W_e}{W_c} \) 
\( \eta_g = \frac{W_e}{W_c} \)

Substituting \( \rho \), \( g_e \) and equation (20) in equation (19) we get the following equation for consumption:

\[ Q_S = \frac{mc}{W_e} \left\{ \frac{1}{2} \rho \text{air} \times S \times C_x \times V^2 + G[f \cos \alpha + \sin \alpha] \right\} V \times 10 \]  

\[ \frac{mc}{W_c} \times 100 \]  

(21)

Considering the vehicle driving on a horizontal road, Hence, we get the following equation for consumption for a hundred kilometers without to the drag force.

\[ Q_S = \frac{\frac{1}{2} \rho \text{air} \times S \times C_x \times V^2 + G[f \cos \alpha]}{10 \times \rho \text{air} \times \eta_g} \times 100 \]  

(22)

For a given vehicle, all these values represent constants in the course of its displacement; thus, only speed is the parameter which varies during the movement of the vehicle.

2.1.9. Experimental Determination of Drag Coefficient (Wind Tunnel Method)

This section deals with experimentally determining the wind drag coefficient. To achieve this, the SOLIDWORKS software was used to simulate the effects of wind on the bodywork of a CHEVROLET CAMORO vehicle, drawn on the same software.
3. Presentation of Results and Discussions

To better explain this paragraph, we will present the results of some simulation plans and we recorded the elementary pressures on the elementary surfaces.

3.1. Visualization of the Pressure Field According to the Plane of Symmetry

Figure 4: Pressure field on the plane of symmetry

This visualization allows us to visualize several pressure zones:
- The blue zones materialize the fluid particles of low pressures;
- The red areas represent the fluid particles of high pressures;
- High pressures are recorded at the front of the vehicle and low pressures at the rear.

3.2. Air Flow Around the Vehicle

Figure 5: Airflow at the front of the vehicle

Figure 6: Air flow over the entire vehicle
The figures above indicate the path occupied by the flow of air during the movement of the vehicle; the presence of several colors indicates the variation of the air pressure on each part of the surface of the automobile and where the pressure is more accentuated.

![Figure 7: Airflow at the rear of the vehicle](image)

The flow at the rear of the vehicle creates a high-speed (swirling) vacuum that generates a drag force opposing the vehicle's advancement.

**Determination of Drag Coefficients**

![Figure 8: Representation of the width and height off turn of the Chevrolet](image)

Determination of pressure

\[ p_{moy} = \frac{\sum_{i=1}^{10} p_i}{\text{number of pressure}} \quad \rightarrow p_{moy} = 405.71 \text{Pa} \]

The drag coefficient:

\[ C_x = \frac{2F_x}{\rho_{\text{air}}xv^2S} \quad \text{but} \quad \frac{F}{S} = C_x \quad \rightarrow \quad C_x = \frac{2P}{\rho_{\text{air}}y^2} \quad (3.27) \]

\[ C_x = \frac{2 \times 405.71}{48^2} \quad \rightarrow \quad C_x = 0.35 \]
Table 3: Characteristics of the vehicles traced on the software

| Marks  | Model   | Master couple (m²) | Drag coefficient | Maximum speed (m/s) | Empty load or mass (kg) | Died max (tr/min) | Maximum power (ch) | Maximum torque (N.m) | Transmission | Type of engine | Release year |
|--------|---------|--------------------|------------------|--------------------|------------------------|------------------|-------------------|----------------------|--------------|----------------|--------------|
| Mercedes | C 280  | 2.17               | 0.32             | 94.16              | 1615                   | 250              | 232               | 299                  | A            | D             | 2007         |
| Peugeot | 308     | 2.2                | 0.301            | 57.78              | 1598                   | 2000             | 141               | 340                  | M            | D HDI         | 2008         |
| Opel   | OMEGA   | 2.15               | 0.29             | 58.06              | 1536                   | 4000             | 136               | 186                  | M            | E (inj)       | 2000         |
| Lexus  | GS 300  | 2.19               | 0.27             | 69.17              | 1619                   | 3500             | 249               | 310                  | A            | E (inj)       | 2009         |

Let's introduce the characteristics of each of these vehicles in the software in order to evaluate their respective consumption per hundred kilometers according to the drag force. This being done click on draw; then export to get the analysis report and its consumption curve.

**Analysis Report**

**Vehicle 1**
- Mark: Mercedes
- Model: C 280
- Density: 1000 kg/m³
- Master couple: 2.170000e+00 m²
- Drag Coefficient: 3.200000e-01
- Speed 9.416000e+01 m/s
- Mass: 1615 kg
- Resistance Coefficient: 4.000000e-02
- Slope: 0
- Calorific Power: 43000 MJ/L
- Efficiency: 4.200000e-01

**Vehicle 2**
- Mark: Opel
- Model: OMEGA
- Density: 1000 kg/m³
- Master couple: 2.150000e+00 m²
- Drag Coefficient: 2.900000e-01
- Speed 5.806000e+01 m/s
- Mass: 1536 kg
- Resistance Coefficient: 4.000000e-02
- Slope: 0
- Calorific Power: 42500 MJ/L
- Efficiency: 3.600000e-01
The graph above shows the consumption over one hundred kilometers of Mercedes, Peugeot, Opel and Lexus vehicle brands.
4. Conclusion

This main objective of this paper was to realize a software for the evaluation of a fuel consumption with respect to the effect of its drag force. To achieve this, Matlab was used to quantify and plot the characteristic curves of some light vehicles through the programming of the existing mathematical relationship between consumption over one hundred kilometers and the drag force. The preliminary studies allowed us to find the real program capable of generating a source code necessary for the compilation and realization of the software. This software will greatly help vehicle users to evaluate how much quantity of fuel consumption their vehicle can consume. Automotive engineers will see in this software an important tool in the quest for producing better, reliable and affordable engines.

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