Study on Radial Vibration of Circular Piezoelectric Ceramic

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Abstract. The core component of the piezoelectric transducer is piezoelectric ceramic oscillator. The vibration of the piezoelectric ceramic oscillator is affected by piezoelectric ceramics. In this paper, the radial vibration is analyzed by using ANSYS. The variation of voltage and strain displacement, poisson’s ratio and resonance frequency are similar, and they are approximately linear. Moreover, the normalized resonant frequency gradually decreases with the thickness diameter ratio of \( t \geq 1 \) and \( t \leq 1 \). The accuracy of ANSYS to solve the resonance frequency of the radial vibration of piezoelectric ceramics is proved by experiment, theoretical calculation and ANSYS simulation results.

1. Introduction

Transducer is an important device for energy conversion. It can realize mutual conversion between electrical energy and mechanical energy[1]. Thus, it is able to applied to engineering practice[2]. The core component of piezoelectric transducer is piezoelectric ceramic, which has been widely used in transducer for its advantages of high accuracy and low energy consumption.

Chen[3] et al., used ANSYS software to study the resonance frequency of circular piezoelectric ceramic vibration by Admittance-Frequency screening method. Li[4] et al., studied the changed law of radial vibration of piezoelectric ceramic with different radii by using ANSYS software. Fan[5] et al., pointed that the piezoelectric ceramic model was developed for simulated investigation, and results provided theoretical guidance for improving transducer performance. Xiang[6] et al., had been actively conducting research on resonance frequency equation of piezoelectric ceramic ring. Jia[7] et al., deduced the resonance and anti-resonance frequency formula of the equivalent circuit of ultrasonic transducer. Qin[8] et al., had deduced the resonant frequency of piezoelectric ring crystals, and the correctness was verified by experiments. Liu and Mao[9-10] et al., studied the vibration performance of piezoelectric ceramic plates.

In this work, the vibration of radial piezoelectric ceramic is analyzed. By comparing theoretical and experimental results with ANSYS simulation, the correctness of ANSYS analysis results of piezoelectric ceramic resonance frequency is verified.

2. Finite element analysis theory of piezoelectric ceramic coupling field

Piezoelectric ceramic has piezoelectricity. The main description of piezoelectricity is the interaction between mechanical quantity (stress tensor \( T \) and strain tensor \( S \)) and electric quantity (electric field
strength $E$ and electric displacement vector $D$). In ANSYS, the second kinds of piezoelectric equations usually is chosen, and the equation is shown as follows:

$$
\begin{align*}
T &= e^E S - e^E E \\
D &= e^E S + e^E E
\end{align*}
$$

(1)

Where, $e^E$ is piezoelectric ceramic elastic matrix, $e^E$ is piezoelectric ceramic stress matrix, $e^S$ is the transposed matrix of $e^E$, $\varepsilon$ is the dielectric constant.

Modal analysis is used to determine the vibration characteristics of structure. The equation for ANSYS to dealt with dynamic problems as follows:

$$
M \ddot{u} + Cu + Ku = F
$$

(2)

Where, $M$, $C$ and $K$ are the mass matrix, composite damping and stiffness matrix, respectively. $F$ is external load.

For the undamped linear system, the expression can be changed to next equation:

$$
Mu + Ku = 0
$$

(3)

The solution of expression (3) is shown as following:

$$
u = \phi_i \cos \omega t
$$

(4)

Where, $\phi_i$ is the formation eigenvector corresponding to the $i$ order mode, $\omega_i$ is the natural frequency of the $i$ order mode (unit: rad/s). $t$ is time (unit: s).

Substituting expression (4) into (3), then:

$$
(K - M \omega_i^2) \phi_i = 0
$$

(5)

The characteristic equation of vibration equation is:

$$
\det[K - M \omega^2] = 0
$$

(6)

For calculating natural frequency is shown as follows:

$$
f_i = \frac{\omega_i}{2\pi}
$$

(7)

Where, the unit of $f_i$ is Hz, i.e. rpm.

3. Theoretical analysis of resonance frequency

The piezoelectric ceramic is a device for realizing electromechanical conversion, which can realize the mutual conversion between machine and electricity (positive piezoelectric effect) or electric machine (reverse piezoelectric effect). An axially polarized piezoelectric ceramic is shown in Figure 1.

![Figure 1. Geometric schematic of the piezoelectric ceramic radial vibration](image)

In Figure 1, $R$ and $h$ are radius and thickness, respectively. $F_{ra}$ and $v_{ra}$ represent the vibration velocity and radial force, respectively. The motion equation of the piezoelectric ceramic radial vibration is shown as follows:

$$
\rho \frac{\partial^2 \xi}{\partial t^2} = \frac{\partial T_r}{\partial r} + \frac{T_r - T_\theta}{r}
$$

(8)
Where, $\rho$ is the density. $r$ is the radial coordinate. $\xi = \xi_r e^{jwt}$ is the radial vibration displacement. $T_r$ is the radial stress. $T_\theta$ is the tangential stress. The piezoelectric equation is shown as follows

$$
S_r = s_{11}^E T_r + s_{12}^E T_\theta + d_{31} E_z
$$

$$
S_\theta = s_{12}^E T_r + s_{11}^E T_\theta + d_{31} E_z
$$

$$
D_z = d_{31} T_r + d_{33} T_\theta + e_{33}^E E_z
$$

(9)

Where, $d_{31}$ is piezoelectric strain constant. $s_{11}^E$ and $s_{12}^E$ are piezoelectric flexible constant, respectively. $\varepsilon_{33}^T$ is permittivity. $E_z$ is polarization direction in axial direction.

According to mechanical vibration characteristics of piezoelectric ceramic, the radial force and tangential force of piezoelectric ceramic are shown as follows

$$
T_r = \frac{1}{2} \left[ (s_{11}^E - s_{12}^E) \frac{s_{11}^E + s_{12}^E - 2d_{31} E_z}{s_{11}^E + s_{12}^E} \right]
$$

$$
T_\theta = \frac{1}{2} \left[ (s_{11}^E - s_{12}^E) \frac{s_{11}^E + s_{12}^E - 2d_{31} E_z}{s_{11}^E + s_{12}^E} \right]
$$

(10)

(11)

Substituting (10), (11) into (8) can be obtained:

$$
\rho \frac{1}{r} \frac{\partial^2 \xi}{\partial r^2} + \frac{1}{r} \frac{\partial \xi}{\partial r} - \xi = \frac{1}{r^2} \frac{\partial \xi}{\partial r} - \xi
$$

(12)

Where, $E_r$ is elastic constant, $E_r = s_{11}^E / (s_{11}^E + s_{12}^E)(s_{11}^E + s_{12}^E) = 1/ s_{11}^E (1-v_{12}^2)$. $v_{12} = s_{12}^E / s_{11}^E$ is poisson’s ratio.

After finishing formula (12) can be obtained:

$$
\frac{\partial^2 \xi}{\partial r^2} + \frac{1}{r} \frac{\partial \xi}{\partial r} - \frac{\xi}{r^2} + k^2 \xi = 0
$$

(13)

Where, $k=\omega/\nu$ is wave number. $\omega=2\pi f$ is angular frequency. $\nu$ is radial vibration velocity, $\nu_2 = \nu E_\nu / \rho (1-\sigma)$.

The expression (13) represents Bessel equation, and the solution of this equation is shown as follows:

$$
\xi = AJ_1(kr) + BY_1(kr)
$$

(14)

Where, $J_1(kr)$ and $Y_1(kr)$ represent the first-order and second-order Bessel functions. While, $A$ and $B$ represent constants. The radial vibration velocity and displacement can be obtained as follows:

$$
\nu = j \omega AJ_1(kr)
$$

(15)

$$
\nu = j \omega AJ_1(kr)
$$

(16)

According to the geometric schematic of the piezoelectric ceramic radial vibration in Figure 1.

$$
\frac{\partial \xi}{\partial r}\bigg|_{r=R} = -\nu_{ra}
$$

(17)

After finishing, the above formulas (16) and (17) can be got the constant $A$:

$$
A = \frac{j \omega \nu_{ra}}{J_1(kR)}
$$

(18)

Based on the above expressions, the radial force $T_r$ is obtained as follows:

$$
T_r = \nu_{ra} \frac{(1-v_{12})J_1(kR)}{j \omega J_1(kR) J_1^2(kR)(1-v_{12})} - \frac{d_{31} E_z}{s_{11}^E (1-v_{12})}
$$

(19)

According to the geometric schematic of the piezoelectric ceramic radial vibration in Figure 1.

$$
F_{ra} = -T_r \bigg|_{r=R} S_a
$$

(20)

Where $S_a$ represents piezoceramic superficial area, $S_a = 2 \pi ah$. After simultaneous sorting of equations (18). (19) and (20) can be obtained:
\[-F_{ra} = -j\rho U_r S_a \left(1 - \nu_{12}\right) \frac{J_0(kR)}{J_1(kR)} \nu_{ra} - \frac{2\pi d_{31} E_h}{s_{11}^p(1 - \nu_{12})}\]  

(21)

The expression (21), after simplification and sorting:

\[F_{ra} = Z_r \nu_{ra} + AU_3\]  

(22)

Where, \(U_3\) represents voltage, \(Z_r = \rho U_r S_a, \ Z_r = jZ_{ra}\{1 - \nu_{12}\}/kRJ_0(kR)/J_1(kR)\}, \(A = 2\pi d_{31}/s_{11}^p(1 - \nu_{12})\), \(n\) represents mechatronic conversion coefficient.

According to electric characteristics of piezoelectric ceramics, it can be obtained by combining expressions (10), (11) and (12), then:

\[\epsilon_{33}^T E_3 = \frac{2d_{31} E_h d_{31}}{s_{11}^p + s_{12}^p} + \pi R \left(\frac{\epsilon_{33}^T E_3}{s_{11}^p + s_{12}^p}\right)\]  

(24)

Where, \(C_0 = \varepsilon_{33}^T S\{1 - 2d_{31}/\varepsilon_{33}^T(s_{11}^p + s_{12}^p)/h, S = \pi R^2\) represents piezoelectric ceramic area.

The admittance of piezoelectric ceramic is:

\[Y = \frac{I}{U_3}\]  

(25)

The resonance frequency equation is expressed as follows:

\[kR J_0(kR) = (1 - \nu_{12})J_1(kR)\]  

(26)

The resonant frequency equation is a transcendental equation. If, the solution of the transcendental equation is \(R(n)\), the expression for piezoelectric ceramic radial vibration resonance frequency \(f_n\) is shown as follows:

\[f_n = \frac{R(n)}{2\pi R \sqrt{s_{11}^p(1 - \nu_{12}^2)}}\]  

(27)

Where, \(n\) is vibration order of piezoelectric ceramic.

4. Results and discussion

4.1 The relationship between displacement and voltage

In Figure 2, the experimental result and the simulation result are basically identical. The experimental result is slightly higher than the simulation result, but the error between them is less than 8%. Thus, it is proved that there is an approximate linear relationship between piezoelectric ceramic thickness and voltage. While, the approximate linear relationship meets the following requirements: \(y = au + b\).
Figure 2. The displacement varying with voltage

In Figure 3, with the voltage loaded on the surface of piezoelectric ceramic, the maximum displacement of piezoelectric ceramic varies linearly with the voltage, and the displacement is gradually increase with the increase of voltage. Piezoelectric ceramic at the same voltage will vibrate more violently and larger amplitude as the thickness is thinner. The amplitude of piezoelectric ceramic is symmetrical distribution about 0V voltage. Because of displacement is a vector, the difference of voltage polarity only changes the direction of piezoelectric ceramic amplitude, but it can not change the magnitude of its amplitude. The larger the voltage, the more intense vibration of piezoelectric ceramic plate. Therefore, the strain of piezoelectric ceramic can be realized by controlling the voltage loaded on the piezoelectric ceramic plate surface.

Figure 3. The relationship between voltage and displacement

4.2 The relationship between poisson’s ratio and resonant frequency

In order to investigate the relationship between poisson’s ratio and resonant frequency. The modeling process is repeated in ANSYS and piezoelectric ceramic with different poisson’s ratios are analyzed, as shown in Figure 4. In Figure 4, there is an approximate linear relationship between the poisson’s ratio and the resonant frequency in the range of poisson’s ratio $\nu=0.27~0.45$ selected for study, i.e. the resonant frequency is gradually increase. The resonance frequency at poisson’s ratio $\nu=0.45$ is 1.14 times of that at $\nu=0.27$. Therefore, the sensitivity of resonance frequency is obviously influenced by poisson’s ratio.
4.3 The relationship between thickness diameter ratio and resonant frequency

Piezoelectric ceramics are created in ANSYS. The thickness-diameter ratio $t$ is defined as $t = h/R$. Normalized resonant frequency is shown in Figure 5 and 6. Wherein, the normalized resonance frequency $f$ of the piezoelectric ceramic is the ratio of the currently resonance frequency to maximum resonance frequency.

In Figure 5, when thickness-diameter ratio ($t \leq 1$), normalized resonance frequency of numerical solution, theoretical value and experimental value decrease gradually with the increase of thickness-diameter ratio. The resonance frequency of thickness-diameter ratio $t=0.1$ is about seven times of $t=1$. It shows that the smaller $t$, the higher the resonance frequency value. It provides theoretical guidance for the selection of piezoelectric ceramic plate.

In Figure 6, the relationship between thickness diameter ratio ($t \geq 1$) and resonance frequency is the normalized resonance frequency of numerical solution, theoretical value and experimental value decrease gradually with the increase of thickness-diameter ratio. The change regulation of thickness-diameter ratio $t \geq 1$ and $t \leq 1$ is similar, but the slope of the curves is different. Moreover, the normalized resonance frequency of piezoelectric ceramic plate with thickness-diameter ratio $t=1$ is about 1.13 times of $t=10$. It shows that the increase of thickness-diameter ratio of the piezoelectric ceramic has little effect on the normalized resonant frequency.
Figure 6. The relationship between thickness diameter ratio \( t (t \geq 1) \) and normalized resonance frequency

In Table 1, the numerical solution, theoretical value and experimental value are represented by \( f_1 \), \( f_2 \) and \( f_3 \) respectively. The error between numerical solution, theoretical value and experimental value are represented by \( \Delta_1 \) and \( \Delta_2 \), i.e. \( \Delta_1 = \frac{|f_1 - f_2|}{f_2} \) and \( \Delta_2 = \frac{|f_1 - f_3|}{f_3} \). In Table 1, the \( \Delta_1 \) and \( \Delta_2 \) value are both very small and the \( \Delta_1 \) value is less than the \( \Delta_2 \). It indicates that the numerical solution is closer to the theoretical value. Although the \( \Delta_2 \) value is greater than the \( \Delta_1 \), but the \( \Delta_2 \) value is less than 2\%. It not only indicates that ANSYS simulation result can be fully meet the need of engineering applications, and but also ANSYS is used to solve the resonant frequency of the accuracy and effectiveness of piezoelectric ceramic plate.

Table 1. The error analysis of numerical solution, theoretical value and experimental value

| \( h \) | \( R \) | \( f_1/\text{KHz} \) | \( f_2/\text{KHz} \) | \( f_3/\text{KHz} \) | \( \Delta_1/% \) | \( \Delta_2/% \) |
|---|---|---|---|---|---|---|
| 2 | 15 | 76.615 | 76.507 | 77.528 | 0.14 | 1.18 |
| 3 | 30 | 38.727 | 38.253 | 39.329 | 0.98 | 1.18 |

5. Conclusions

(1) There is an appropriate linear relationship between voltage and displacement, poisson’s ratio and resonant frequency. When thickness-diameter ratio is \( t \geq 1 \) and \( t \leq 1 \), the normalized resonance frequency of piezoelectric ceramic plate is similar to the change curve of thickness-diameter ratio, and the normalized resonance frequency decreases with the increase of thickness ratio.

(2) The results of numerical solution, theoretical value and experimental value analysis indicate that the numerical solution is closer to the theoretical value. Although the value of \( \Delta_1 \) is lower than that of \( \Delta_2 \), the error value is less than 2\%. Therefore, the results obtained by ANSYS software are real, effective and conform to the actual reality.

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