The method for achievement of group interchangeability accuracy for multilink dimension chains with gaps

Sergey Shilyaev, Evgeniy Slashchev* and Yurii Mikhaylov
Izhevsk State Technical University, 7, Studencheskaya street, Izhevsk, 426069, Russia

* shiljaev@mail.ru

Abstract. The article provides recurrent dependencies for determining the upper and lower deviations of the sorting groups of the component parts of a dimensional chain calculated by the group interchangeability method for tasks when the closing link is a gap. The article also substantiates the necessity of formalizing the group interchangeability method for multilink dimension chains, which are divided into groups by the equal tolerance method. Using the sorting of parts for multilink dimension chains into groups and the assembly of parts within a group by the method for complete interchangeability, the technologist provides high accuracy, productivity and economy of production. At the same time, for multilink dimension chains, a graphical method is used to calculate the upper and lower deviations of the sorting groups. In this regard, a technique is proposed for automation, which allows formalizing the procedure for determining the parameters of a dimensional chain. At the same time, the field of application of the method expands, the requirements for the designer’s qualification reduce, and subjective errors are eliminated.

1. Introduction

The need to apply the group interchangeability method, the number of links (parts, sizes), which are divided into groups with more "strict" tolerances, and the number of groups are determined by the technologist, who assigns tolerances that are economically achievable under given conditions for the manufacture of parts included in a dimensional chain. Using the sorting of parts into groups and the assembly of parts within a group by the method for complete interchangeability, the technologist provides high accuracy, productivity and production efficiency [1-20].

Improving the group interchangeability method (selective assembly) while ensuring the accuracy of the closing link (gap) involves developing a method for calculating a dimensional chain using recurrent dependencies that allow determining the upper and lower dimensional deviations for a multilink dimension chain (the number of component links \( m \geq 3 \)), based on which the necessary conditions for sorting and assembling parts are organized. At the same time, the number of component links sorted into groups is not significant.

The proposed technique allows formalizing and automating the procedure for determining the parameters of a dimensional chain using the group interchangeability method for multilink dimension chains. At the same time, the field of application of the method expands, the requirements for the designer’s qualification reduce, and subjective errors are eliminated.
To derive the recurrent dependencies for each typical problem encountered in production practice, it is necessary to formulate and use the rules and restrictions when assigning and constructing field diagrams for dimensional tolerances included in a dimension chain, which are as follows:

**Rule 1.** If a dimensional chain with a closing link in the form of a gap has two or more increasing links, then when deriving recurrent dependencies, their tolerances in the diagram are arranged in the form of total (combined) vertical tolerance. The tolerances are combined according to the corresponding groups. In this case, the sum of the coordinates of the midpoints of the tolerance fields of the links of a dimension chain located on the zero line horizontally is equal to the coordinate of the midpoint of the total (combined) tolerance field located vertically (see figure 1).

**Rule 2.** The relation is used: the coordinate of the tolerance field midpoint of the closing link must be equal to the sum of the coordinates of the midpoints of the tolerance fields of the component links, with the appropriate gearing ratio. Graphically on the layout of the tolerance fields, this is achieved as follows:
- if the closing link is a gap, then the tolerance field of one of the reducing links is located relative to the tolerance field of the first group of the increasing link, shifted by the minimum gap, and the tolerance fields of the other reducing links are in place (under the zero line, the main deviation is $h$, see figures 2 – 5.

![Figure 1. Location of the tolerance fields (Δi are the midpoint coordinates of the tolerance fields of links)](image)

**Rule 3.** The equality of the sum of the tolerances of the increasing and reducing component links is accepted.

**Rule 4.** Sorting group numbers in the tolerance field layout are indicated from the top down. In the first place in the designation of group tolerance is the group number, in second place is the number of the link.

2. **General procedure for the implementation of the group interchangeability method**

The general procedure for implementation of the group interchangeability method during assembly, which will be used in the future for multilink dimension chains with specific features and are often found in manufacturing practice, can be explained with definite production examples:

**Task 1.** Given: a multilink dimension chain and a layout of tolerance fields (see Figures 2 and 3). The dimension chain consists of one increasing and three reducing links. In a dimension chain, the tolerance of the increasing link is equal to the sum of the tolerances of reducing links $\sum_{i=1}^{m_{red}} T_i = T_{TD}$. Besides, the reducing links have equal tolerances. The method for equal tolerances is used. The number of reducing links sorted into groups $m_{red} = 3$, the number of sorting groups $n = 2$.

Determine: the upper and lower deviations of the component links when applying the group interchangeability method and method for equal tolerances.

The scheme in the Figure 3 is typical and illustrates the second rule for the location of tolerance fields: the equality of the coordinates of the midpoints of the closing link tolerance fields and the sum of the coordinates of the tolerance fields of the component links with the corresponding gearing ratio.

![Figure 2. Multilink dimension chain](image)
Determine the upper and lower deviations of the increasing link within the group by formulas 1 and 2:

\[ ES_A(i) = TD(1-\frac{i-1}{n}), \]
\[ EI_A(i) = TD(1-\frac{i}{n}), \]

where: \( i = 1,2…n \) is the current variable characterizing the group number; TD-tolerance of the increasing link.

For the upper and lower deviations of the reducing link, analyzing the scheme (see Figure 3), we derive the recursive formulas 3 and 4 by the induction method:

\[ esA(i, j) = (Td - Td/n - S_{min}) \frac{\Pi_{k=2}^{m_{red}} (k-j)}{(m_{ym}-1)!} - \frac{Td \cdot (i-1)}{m_{ym} \cdot n}, \]
\[ eiA(i, j) = (Td - Td/n - S_{min}) \frac{\Pi_{k=2}^{m_{red}} (k-j)}{(m_{ym}-1)!} - \frac{Td \cdot (i)}{m_{ym} \cdot n}, \]

where the product is defined as follows (5):

\[ \Pi_{k=2}^{m_{ym}} (k-j) = \frac{(2-j)(3-j)(4-j)}{1 \cdot 2 \cdot (m_{ym} - 1)}. \]

Figure 3. The layout of the of tolerance fields

For the layout on the figure 3: \( m_{ym} = 3; i = 1,2; j = 1,2,3. \)

The calculation is made with the following initial data: \( n = 2, m_{red} = 3, TD = TA_4 = 12 \mu m, TA_1 = TA_2 = TA_3 = 4 \mu m, TD = \sum_{i=1}^{m_{red}} TA_i = 12 \mu m, S_{min} = 2 \mu m, S_{max} = 14 \mu m. \) The calculation results are given in the table 1.

Formulas 3 and 4 are used when the nominal dimensions of the links are in the same interval of the normal series of numbers.

Table 1. The results of the task 2 solving (\( \mu m \))

| Group (n=2) | A_4 | A_1 | A_2 | A_3 | Gap | TS |
|------------|-----|-----|-----|-----|-----|----|
| 1          | 12  | 6   | 4   | 2   | -2  | 14 | 2  | 12 |
| 2          | 6   | 0   | 2   | 0   | -2  | 14 | 2  | 12 |
Task 2. Given: a multilink dimension chain (see Figure 4) and the layout of the tolerance fields (Figure 5). The chain has two increasing links and three reducing links. The condition of equality of the amounts of tolerances of increasing and reducing links is fulfilled. The method for equal tolerances is applied (separately for increasing and reducing links). For this case, the number of groups \( n = 3 \), and the number of increasing and reducing units, sorted into groups, is respectively: \( m_{\text{inc}} = 2 \) и \( m_{\text{red}} = 3 \).

Determine: the upper and lower deviations of the component links when applying the group interchangeability method and the method for equal tolerances (separately for the increasing and reducing links).

\[
\text{Figure 4. Multilink dimension chain}
\]

\[
\text{Figure 5. The layout of the tolerance fields}
\]

When using the first rule of tolerance fields’ location for two increasing links, the recurrent dependencies for determining the upper and lower deviations for each group of increasing links are (formulas 6.7):

\[
ES A(i, k) = \frac{TD}{m_{\text{inc}}} \left(1 - \frac{i-1}{n}\right),
\]

\[
EI A(i, k) = ES A(i, k) - \frac{TD}{m_{\text{inc}}} \cdot n = 27 - 9 = 18 \, \mu m \text{ etc.},
\]

where \( k = m_{\text{red}} + j; j = 1, 2..5; \)

When using the second rule for the location of tolerance fields, the upper and lower deviations of reducing links are determined by formulas (3) and (4). However, the shift of the tolerance field of one of the reducing links by \( S_{\text{min}} \) is relative to the first group of the generalized (total) tolerance of the increasing links (if there are two increasing links, then a shift relative to the sum of their tolerances of the first group).

The calculation was made with the following initial data: nominal dimensions of the components \( A_1 = A_2 = A_3 = 14 \, \text{mm}, A_4 = A_5 = 21 \, \text{mm} \); limiting dimensions of the closing link \( S_{\text{min}} = 9 \, \mu m, S_{\text{max}} = 45 \, \mu m; n = 3; m_{\text{inc}} = 2 \) и \( m_{\text{red}} = 3 \).

The results of the calculations are given in the table 2.
Table 2. The results of the calculations (µm)

| n | Increasing links | Reducing links | Gap (S) | Clearance TS |
|---|-----------------|---------------|---------|--------------|
|   | A_4 | A_5 | A_1 | A_2 | A_3 | Max | Min |
| ES | El | ES | El | es | ei | es | ei | TS |
| 1  | 27 | 18 | 27 | 21 | 0 | -6 | 0 | 45 | 9 | 36 |
| 2  | 18 | 9 | 18 | 21 | 15 | -12 | -6 | 45 | 9 | 36 |
| 3  | 9 | 0 | 9 | 15 | 9 | -12 | -18 | 45 | 9 | 36 |

3. Conclusion

Thus, the main difference of the proposed technique from the existing ones is to develop the principles of a formalized approach and use recurrent dependencies that allow to easily define the boundaries of sorting parts (upper and lower deviations) assembled by group interchangeability for multilink dimension chains. By automating the calculation procedure, the procedure for assigning and ensuring dimensional tolerances included in a dimension chain is simplified, taking into account production conditions and economically achievable accuracy, by changing the number of sorting groups.

Acknowledgments

The publication was prepared within the framework of project №. №15.06.01/18IIICA "Development of scientific and methodological foundations for the design and creation of technological machines in order to ensure reliable and safe operation and life extension implemented on the basis of the order of the rector of the Kalashnikov Izhevsk State Technical University from December 29, 2018 № 1493 "About grant support for priority research scientists of Kalashnikov ISTU" with the financial support of the "Kalashnikov ISTU".

References

[1] Altschul R 1994 Case Study in Statistical Tolerancing Manufacturing Review of the AMSE no 7 pp 52-56
[2] Anurev V I 2001 Reference Book for Designer-Mechanical Engineer vol 2 8th ed Zhestkova I N (Moscow: Mashinostroenie) p 912
[3] Bezyazychny V F and Nepomiluyev V V Possible Ways to Improve the Quality of Manufacturing of Engineering Products (Assembly in mechanical engineering, instrument making) no 1 pp 17-20
[4] Chase K W, Gao J, Magleby S P and Sorensen C D 1996 Including Geometric Feature Variations in Tolerance Analysis of Mechanical Assemblies (Russia: Institute of Industrial Engineers/IE Transactions) 28 (10) pp 795-807
[5] Goldfarb V, Malina O and Trubachev E 2016 New Concept of the Process of Designing Gearboxes and Gear Systems. (Mechanisms and Machine Science 34) pp 405-423
[6] Henzold G 2006 Geometrical Dimensioning and Tolerancing for Design Manufacturing and Inspection (Elsevier: Oxford, UK)
[7] Kolesov I M 1999 Fundamentals of Mechanical Engineering Technology: a Textbook (Moscow: Higher School) 2nd ed
[8] Laaneots R 2004 Modified Calculation Method of Tolerance of Dimensional Chain Dependent Link. (Tallinn 4th international DAAAM Symp. of industrial engineering – innovation as competitive edge SME) pp 43-46
[9] Lebedevsky M S, Weitz V L and Fedorov A I 1985 Scientific Basis of Automatic Assembly (Moscow: Mashinostroenie) p 316
[10] Mishunin V P and Osetrov V G 2002 Optimization in Achieving the Accuracy of the Axial Clearance in Gearboxes (Russia: Assembly in mechanical engineering, instrument making) 6 pp 2-4

[11] Nepomiluev V V, Oleynikova E V, Gusarova N I 2015 Probabilistic-Statistical Model of the Process of Individual Selection (Russia: Intelligent systems in production) 1 pp 8-13

[12] Osetrov V G, Slashchev E S and Shilyaev S A 2013 Stages of Development of Engineering Technology in Udmurtia (Monograph) (Izhevsk: Computer Research Institute) p 160

[13] Osetrov V G and Slashchev E S 2014 Improving the Calculations of the Dimensional Chain when using the method of group interchangeability (Russia: Assembly in mechanical engineering, instrument making) 7 pp 24–29

[14] Osetrov V G, Slashchev E S 2014 The Calculation of the Accuracy of Compounds with Tension Using the Method of Group Interchangeability (Russia: Intelligent systems in production) 2 (24) pp 52-56

[15] Scholz F 1995 Tolerance Stack Analysis Methods Research and Technology (Boeing Information & Support Services) p 44

[16] Sorokin M N, Anurov Yu N 2011 Formalization of the Method Group Interchangeability in the Implementation of Selective Assembly of Products (Russia: Assembly in mechanical engineering, instrument making) 8 pp 16-19

[17] Sorokin M N, Anurov Yu N 2016 Rolling Bearing Assembly (Russia: Assembly in Mechanical Engineering, Instrument Making) 2 pp 18-23

[18] Srinivasan V 2008 Standardizing the Specification, Verification, and Exchange of Product Geometry Research, Status and Trends (Computer-Aided Design) 40 (7) pp 738–749

[19] Sun Y., Gupta M 2004 Optimization of a Flat Die (Mechanical Engineering – Engineering Mechanics Department Michigan Technological University Houghton ANTEC) p 3007-11

[20] Shilyaev S A 2009 Thermal Processes in Rotational Tape. Russian engineering research. (Russia: MAIK Science / Interperiodica LLC for Allerton Press, Inc.) pp 948-950