Comparison of linear accelerometers calibrated by precise centrifuges

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Abstract: The paper described comparison of linear accelerometers (LA) calibrated by precise centrifuges (PC) in China from 2015 through 2017. It covered purpose, method result and conclusion of the comparison.

1. Introduction

The comparison of the linear accelerometers (LA) calibrated by the precise centrifuges (PC) was carried out in China from 2015 through 2017[1]. Purpose of the comparison was investigation of agreement while a LA was calibrated by a few PCs belonging to several institutes. There were five labs participating with its own PC (see fig.1), and the three LA were calibrated as transfer standard (see fig.2). Table 1 listed information of the labs, PCs and LAs, while the lab A was taken as reporting lab.

| Code of LAs | LA-01 | LA-02 | LA-03 |
|-------------|-------|-------|-------|
| Range of LAs, g = 9.80665 m/s² | 25 | 100 | 100 |
| Code of Labs | A | B | C | D | E |
| Range of PCs, g | 200 | 200 | 75 | 100 | 150 |
| LAs tested | LA-01, LA-02, LA-03 | LA-02 | LA-01, LA-02, LA-03 |

The tests covered: 1) $k_{10}$, which expressed the first order of the coefficients in the linear calibration equation tested in gravity field; 2) R, which expressed the radius of rotation while the
LA was fixed in a PC turned around its turning-axial; 3) calibration equation of the LA while it is fixed in the PC, and acceleration was applied by the PC turned on.

The three LAs were tested by the labs A, B, C, D and E one by one exception of the lab C tested the LA-02 only, of which the lab A had tested the LA-02 and the LA-03 for two times during 15 days in order to know reproducibility of the tests. Based on calibration results, the authors had made calculation, statistic and analysis, which included uncertainty evaluation of the k_{10}, the R and the output x of the LA. Finally, all of normalized error E_{n} was calculated while the data obtained by the lab A at the first time testing were taken as reference value in the comparison.

2. Method of the comparison and calibration

2.1 The first order of the coefficients k_{10}

A LA was fixed in a special SHD (see fig.2), put it vertically on a horizontal plat in a gravity field while the head of the LA up and down, as well as horizontally. A digital meter was used to measure output of the LA. The k_{10} will be calculated by a linear equation as following:

\[ x_{rd} = x_{00} + k_{10} g_{loc} \]  \( (1) \)

where,  \( x_{rd} \) output reading of the LA,  \( mv \) or \( v \);  
\( g_{loc} \)-the local gravity acceleration applied on the LA,  \( m/s^2 \);  
\( x_{00} \)- output of the LA while the acceleration applied being equal to 0,which was called a zero-output of the LA,  \( mv \) or \( v \);  
\( x = x_{rd} - x_{00} \) output of the LA,  \( mv \) or \( v \).

The average of \( k_{10} \) measured was taken as final \( k_{10} \).

2.2 Radius of rotation: \( R \)

The radius were tested with three turning speeds being little higher, middle and lower, while the HSD with the LA was fixed in the PC, which would generate approx. one local gravity acceleration \( g_{loc} \):

\[ R = \frac{x_{rd} - x_{0}}{k_{10}} \cdot \frac{30^2 g_{loc}}{\pi^2 n^2} (1 + \frac{60}{\pi n} \sin \varphi) \]  \( (2) \)

Where,  \( x_{rd} \) output reading of the LA,  \( mv \) or \( v \);  
\( x_{0} \)-zero-output of the LA,  \( mv \) or \( v \);  
\( n \)-turning-speed of the PC, rpm;  
\( \omega \)-self-turning-speed of the earth being equal to \( 2\pi/(24x60x60) \), rad/s  
\( \varphi \)-latitude of the PC located on the earth. (Note: when the PC is located on the northern of the earth, and turning clockwise, the \( \sin \varphi \) will be taken plus, otherwise minus.)

The average of the three radius measured with the three speeds was taken as the final radius.

2.3 Calibration equation

After the radius was measured, turning the PC, step by step, from rest, a \( g_{loc} \) 3\( g_{loc} \) 5\( g_{loc} \) ---, up to 25\( g_{loc} \) for the LA-01, and going down for hysteresis tested; from rest, a \( g_{loc} \) 5\( g_{loc} \) 10\( g_{loc} \) 20\( g_{loc} \) ---, up to max. acceleration for the LA-02, and LA-03, and going down. Five circles above had been done in order to calculate repeatability of the output.

Based on the the acceleration applied and the output measured of the LA, the following
calibration equation was fitted by the least-square method,
\[ x = k_0 + k_1a + k_2a^2 + k_3a^3 \]  
(3)
where, \( x \) - the output of the LA, mv, or v, or mA;
\( k_0, k_1, k_2, k_3 \) - zero order, first order, second and third order of coefficients separately;
\( a \) - acceleration applied, \( \text{m/s}^2 \), or \( g_{45.0} \)

3. Result of the comparison

3.1 Calibration results of \( k_{10} \)
Table 2 summarized all of \( k_{10} \) measured, its relative expanded uncertainty \([2][3][4][5]\) \( W_{k_{10}} \), \( k=2 \), as well as relative deviation \( \delta k_{10A} \) between \( k_{10} \) measured by \( i^{th} \) lab, and \( k_{10A} \) measured by the lab A, which were less than 5.5E-4 for the lab B, C and D, and 3.6E-3 for the lab E. There were four main budgets in the combined uncertainty of \( k_{10} \), which covered the repeatability and the reproducibility of the output indicated, as well as the zero output, and the local gravity acceleration (relative deviation less than 1.5E-5). Normalized error \([6]\) \( E_{n,i,A} \) of the \( k_{10i} \) vs \( k_{10A} \) were evaluated, all of which \( |E_{n}| \) less than 1, exception of the LA-02 measured by the lab B being 1.6.

Table 2 Summery of \( k_{10} \), its relative expanded uncertainty \( W_{k_{10}} \), \( k=2 \), \( \delta k_{10A} \)

| Lab Code | \( k_{10}-LA-01 \) | \( k_{10}-LA-02 \) | \( k_{10}-LA-03 \) | \( W_{k_{10}} \)-LA-01 | \( W_{k_{10}} \)-LA-02 | \( W_{k_{10}} \)-LA-03 | \( \delta k_{10A} \) |
|----------|-----------------|-----------------|-----------------|----------------|----------------|----------------|----------------|
| A        | 1.21868         | 0.83809         | 0.90919         | 1.1E-04        | 2.2E-04        | 2.7E-05        |                |
| B        | 1.21868         | 0.83763         | 0.90921         | 3.0E-03        | 8.0E-05        | 2.5E-04        | -4.0E-07       |
| C        | 0.83808         |                 |                  | 1.9E-03        |                |                | -1.6E-05       |
| D        | 1.21850         | 0.83801         | 0.90908         | 1.2E-04        | 1.8E-04        | 2.0E-04        | -1.5E-04       |
| E        | 1.21655         | 0.84107         | 0.91051         | 6.0E-03*       |                |                | -1.7E-03       |

Remarks: *it was given by the lab E

3.2 Calibration result of the radius, \( R \)
Table 3 summarized the radius measured, and the reproducibility \( \delta A_{12} \) measured by the lab A.

Table 3 Summery of the radius, m

| No. | LA code | LA-01   | LA-02   | LA-03   |
|-----|---------|---------|---------|---------|
| 1   |         | R       | R       | R       |
| 2   | A_1     | 0.60931 | -0.69485| 0.61252 | -0.69622 | 0.61239 | -0.69599 |
| 3   | B       | 0.55018 | -0.52013*| 0.53524*| -0.53507* |
| 4   | C       |         | 0.92868"| -0.92149"|         |         |
| 5   | D       | 1.16459 | -1.15550| 1.15561 | -1.15395 | 1.15852 | -1.15678 |
| 6   | E       | 0.69989 | -0.69854| 0.66987 | -0.69910 | 0.67742 | -0.70562 |
| 7   | A_2     |         | 0.60949 | -0.69285| 0.60939 | -0.69271 |
### Table 1

| Condition | \( \delta A_{12} \) | 0.50% | 0.49% | 0.49% | 0.47% |
|-----------|----------------|-------|-------|-------|-------|
| E         |                | 0.70005*** |
| \( \delta E_{\text{norm-meas}} \) | 0.02% | 0.22% | 4.31% | 0.14% | 3.23% | -0.80% |

Remarks: 1) * given by the Lab B; ** given by the Lab C; *** given by the lab E:

2) \( \delta E_{\text{norm-meas}} \) was the relative deviation between the radius measured and the normal for the lab E, being from -0.80% through 4.31%. 3) R,-which means the head of the LA facing the turning-axial of the PC, otherwise R, as same as followings.

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**Fig.3** showed summary of relative combined uncertainty of the radius measured.

There were four budgets in the relative combined uncertainty, which covered the turning-speed \( n \), the local gravity acceleration \( g_{\infty} \) the \( k_{10} \) measured and the output \( x \) of the LA. Its main effect was the \( k_{10} \). Besides that, the maximum deviation among the three combined uncertainties of \( w_{i} \) responsible to the three \( n \) was taken as its budget, which was the main effect.

It would be pointed out that 1) the radius of a LA would be tested, and used in the calibration equation, instead of the normal value; 2) after the LA re-fixed on a PC, its radius could be changed.

The relative combined uncertainty of the radius given by the lab B and C was less than that evaluated by the lab A, which had effect seriously in evaluation of uncertainty of the output \( x \) during the calibration equation tested (see the next paragraph).

#### 3.3 Result of the calibration equation

##### 3.3.1 Result of the LA-01

The fig.4 showed relative deviation between the output of the LA-01 tested by the lab A and other 3 labs (B, D, E). The fig.5 showed relative expanded uncertainty, \( k=2 \), of the output tested by the 4 labs. There were 6 budgets in the combined uncertainty including acceleration generated by the PC, repeatability of output of the LA, as well as its hysteresis, indication resolution, zero recovery, interpolation error, of which main effects were radius responsible to acceleration applied, and interpolation while the calibration range of acceleration being over larger.

It would be pointed out in the fig.4 and the fig.5 that: 1) the \( \delta_{A-E} \), had been corrected based on the relative deviation between R, measured and normal used by the lab E, as well as \( \delta_{A-E} \).
there were similar correction for LA-02 and LA-03). Besides that, the relative expanded uncertainty, $k=2$, $W_B=0.1\%$ was given by the lab B, as well as the $W_D$, given by the lab D; $W_E=W_F=0.3\%$ was given by the lab E. The data mentioned above had been used in the evaluation of $E_n$. After considering all the corrections, almost $|E_n|$ values, $k=3$, were less than 1 exception acceleration of $a_{g45.0}$ and $3g_{45.5}$ tested by the lab B, of which $E_n$ values were -36 and -1.2 separately. It means that the output tested by the 3 labs of B, D, E with the LA-01 were agree with the lab A.

3.3.2 Result of the LA-02
Fig.6 showed the relative deviation of output between $A_1$ and $A_2$ of lab A, as well as $A_1$ with other 4 lab, of which the $\delta_{A1-A2}$, $\delta_{A1-A2}$ represented the reproducibility of the output tested by the lab A, being from 1.3E-04 through -5.6E-05. The fig.7 showed relative expanded uncertainty, $k=2$, of output tested by $A_1$ and $A_2$, as well as other 4 Labs.

The relative expanded uncertainty, $k=2$, $W_B=0.03\%$ was given by the lab B, as well as the $W_C=0.2\%$ given by the lab C; $W_E=W_F=0.12\%$ was given by the lab E, which had been used in the evaluation of $E_n$. After considering the correction above, all of $|E_n|$ were showed less than 1.

3.3.3 Result of the LA-03
The fig.8 showed relative deviation of output between lab A and other 3. The reproducibility of the lab A for LA-03 $|\delta_{A1-A2}|$, $|\delta_{A1-A2}|$ were less than 6E-5. The fig.9 showed relative expanded uncertainty, $k=2$, of output tested by the 4 labs.
After considering the correction above, all of $|E_n|$ values were less than 1.
4. Conclusion

1) It is necessary to test the $k_{10b}$ $R$ and calibration equation. All of the normalized error $|E_n|$ for $k_{10}$ less than 1 except of the one. 2) The radius of a LA would be tested, and used in the calibration equation, instead of the normal value; after the LA re-fixed on the PC, its radius could be changed. 3) The output reproducibility of LA -03 and LA-04 tested by the lab A was 2E-4. The output tested by the lab D for 3 LAs was quite agree with the lab A, as well as the lab B. 4) After considering the correction, the lab C agreed with lab A. 5) Regarding the lab E, since the normal values of the radius 0.70005 m had been used while the calibration equations were tested, all of results on it had been made corrections of the radius based on the relative deviations. After that, the lab E agreed with the lab A.

Acknowledgments

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Reference

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