Investigating two-zero textures of inverse neutrino mass matrix under the lamp post of LMA and LMA-D solutions and symmetry realization

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Abstract In this work we have investigated the phenomenological consequences of two-zero textures of inverse neutrino mass matrix \( (M^{-1}_\nu) \) in light of the large mixing angle (LMA) and large mixing angle-dark (LMA-D) solutions, later of which originates if neutrinos exhibit non-standard interactions with matter. Out of fifteen possibilities, only seven two-zero textures of \( M^{-1}_\nu \) are found to be phenomenologically allowed under LMA and/or LMA-D descriptions. In particular, five textures are in consonance with both LMA and LMA-D solutions and are necessarily CP violating while remaining two textures are found to be consistent with LMA solution only. The textures with vanishing (1, 1) element of \( M^{-1}_\nu \) are, in general, disallowed. All the textures allowed under LMA and LMA-D solutions follow the same neutrino mass hierarchy. Furthermore, textures with vanishing (2, 3) element of \( M^{-1}_\nu \) are found to be either disallowed or are consistent with LMA description only. We have, also, obtained the implication of the model for \( 0\nu\beta\beta \) decay amplitude \( |M_{ee}| \). For most of the textures the calculated 3\( \sigma \) lower bound on \( |M_{ee}| \) is \( \mathcal{O}(10^{-2}) \), which is within the sensitivity reach of \( 0\nu\beta\beta \) decay experiments. We have, also, proposed a flavor model based on discrete non-Abelian flavor group \( A_4 \) wherein such textures of \( M^{-1}_\nu \) can be realized within Type-I seesaw setting.

1 Introduction

The experimental evidences accumulated in last two decades have convincingly established that not only neutrino have non-zero mass but its dynamical origin is beyond our current understanding of the standard model (SM). Despite the best efforts to precisely decipher the structure of neutrino mixing matrix, it still has certain unknowns, for example, \( CP \) violating phases, octant of \( \theta_{23} \) and neutrino mass hierarchy, to name a few. The theoretical frameworks to explain neutrino mass and its manifestations like neutrino oscillations have been developed assuming standard [charged (CC) and neutral current (NC)] interactions between neutrino and matter. In fact LMA has been independently confirmed as the solution to solar neutrino problem (SNP) in solar and KamLAND reactor experiments [1]. It has been shown that for positive solar mass-squared difference, \( \sin^2 \theta_{12} \) cannot be greater than 0.5. However, at the subdominant level, there still have possibilities for additional contributions to neutrino oscillations such as non-standard interactions (NSI) of neutrino with matter fields [2–4]. The future hyper-technological experiments will have the access to these unexplored regions. In presence of NSI, \( \theta_{12} \) can be in the second octant. Thus, solar neutrino problem may have another degenerate solution in which \( \sin^2 \theta_{12} \approx 0.7 \). This solution is termed as large mixing angle-dark (LMA-D) solution. In general, the degeneracy between LMA and LMA-D solutions cannot be alleviated by oscillation experiments due to generalized mass hierarchy degeneracy in presence of NSI. However, a combined measurements from neutrino oscillation and scattering experiments may have imperative implication with regard to lifting of these degeneracies [5–7]. Although non-standard interactions of neutrino with matter are severely constrained, the latest global fit, incorporating neutrino oscillation and COHERENT data, still allows LMA-D solution at 3\( \sigma \) confidence level [8]. The only difference in LMA and LMA-D is in
The octant of $\theta_{12}$, however, the solar mass-squared difference remains the same.

The progress in understanding the origin of neutrino mass centrally involves explaining the observed pattern of neutrino mixing which is encoded in the neutrino mass matrix obtained after electroweak symmetry breaking. Assuming neutrino to be Majorana particle the mass matrix contain nine free parameters viz. three neutrino mass eigenvalues, three mixing angles and three CP violating phases. The two mass-squared differences and three mixing angles have been measured in neutrino oscillation experiments with high degree of precision. Seesaw mechanism is a natural and most effective way to explain the smallness of neutrino mass. Within Type-I seesaw [9,10] paradigm, the low energy effective neutrino mass matrix ($M_D$) and heavy right-handed Majorana neutrino mass matrix ($M_R$) using the relation: $M_v \approx M_D M_R^{-1} M_D^T$. The existence of near degeneracy in LMA and LMA-D solutions have imperative implications for models of neutrino mass and associated phenomenology. Recently, the LMA and LMA-D phenomenology of Majorana neutrino mass matrix has been studied assuming (i) zero textures of the neutrino mass matrix, $M_\nu$ [11–13] (ii) in presence of one-sterile neutrino [14]. Texture-zeros in the effective low energy Majorana neutrino mass matrix may have seesaw origin in which they can be realized from the zeros in $M_D$ and $M_R$ [15]. In literature, there have been phenomenological studies with texture one-zero [16–19] and two-zeros [20–24] while three-zeros or more, in neutrino mass matrix, are ruled out by current neutrino oscillation data [25]. In Type-I seesaw, an interesting scenario may emerge if we work in $M_D$-diagonal basis. In this basis, the zero(s) in $M_v^{-1}$ is same as the the zero(s) in $M_R$ i.e. $M_v^{-1} \approx M_D^{-1} M_R M_D^{-1}$ [26]. In Ref. [13], the authors have investigated the phenomenological consequences of one-zero texture in $M_v^{-1}$ with in the context of trimaximal mixing. The LMA phenomenology of two-zero textures of $M_v^{-1}$ has been investigated in [27]. It is to be noted that the texture zeros in $M_v$ and $M_v^{-1}$ are, in general, independent and may have distinguishing phenomenology. Motivated by the capabilities of the future neutrino oscillation experiments in resolving these subdominant effects [28] and its important model building perspective, we investigate the phenomenological consequences of two-zero textures of $M_v^{-1}$. Also, we have proposed a flavor model based on non-Abelian discrete group $A_4$ and Type-I seesaw, where such zeros can be realized in the inverse neutrino mass matrix. There are fifteen possibilities to have two zeros in $M_v^{-1}$. We investigate the LMA and LMA-D phenomenology of these fifteen textures.

The paper is organized as follows. In Sect. 2, we briefly introduce the formalism of two-zero texture of $M_v^{-1}$ and the details of the numerical analysis. Section 3 is devoted to the investigation and discussion of LMA/LMA-D phenomenology of the seven allowed textures of $M_v^{-1}$. A flavor model based on non-Abelian $A_4$ symmetry is discussed in Sect. 4. Finally, we brief our conclusions in Sect. 5.

2 Formalism of two-zero textures of inverse neutrino mass matrix

In charged lepton basis, the neutrino mass matrix is given by

$$M_v = V M_v^{diag} V^T,$$

where $M_v^{diag} = diag(m_1, m_2, m_3)$ is the neutrino mass eigenvalue matrix, $V = U_P$ is complex unitary neutrino mixing matrix. The matrix $U$ is Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix which in the PDG representation can be written as [29]

$$U = \begin{pmatrix}
U_{11} & U_{12} & U_{13} \\
U_{21} & U_{22} & U_{23} \\
U_{31} & U_{32} & U_{33}
\end{pmatrix} = \begin{pmatrix}
c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i \delta} \\
-s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i \delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i \delta} & c_{23} c_{13} \\
- s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i \delta} & c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i \delta} & s_{23} c_{13}
\end{pmatrix},$$

(2)

where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$ and $\delta$ is Dirac-type $CP$ violating phase. Also, $P$ is the phase matrix given by

$$P = \begin{pmatrix}
1 & 0 & 0 \\
0 & e^{i \alpha} & 0 \\
0 & 0 & e^{i (\beta + \gamma)}
\end{pmatrix},$$

where $\alpha, \beta$ are Majorana-type $CP$ violating phases. The inverse neutrino mass matrix can be derived from Eq. (1) as

$$M_v^{-1} = (V M_v^{diag} V^T)^{-1}.$$  

(3)

Using Eqs. (1) and (2), six independent elements of ($M_v^{-1}$) can be written as
\[
(M^{-1}_\nu)_{11} = \frac{1}{m_1 m_2 m_3} \left[ c_{13} e^{-2i\alpha} m_3 (c_{12} e^{2i\alpha} m_2 + m_1 s_{12}^2) + e^{-2i\beta} s_{13} m_1 m_2 \right],
\]
\[
(M^{-1}_\nu)_{12} = \frac{1}{m_1 m_2 m_3} \left[ e^{-i(2(\alpha + \beta) + \delta)} (c_{13} e^{2i\beta} m_1 m_3 s_{12} (c_{12} c_{23} e^{i\delta} - s_{12} s_{13} s_{23}) + c_{13} e^{2i\alpha} m_2 (-c_{12} c_{23} e^{i(2\beta + \delta)} m_3 s_{12}) + (-c_{12} e^{2i\beta} m_3 + m_1) s_{13} s_{23} \right],
\]
\[
(M^{-1}_\nu)_{13} = \frac{1}{m_1 m_2 m_3} \left[ e^{-i(2(\alpha + \beta) + \delta)} (c_{13} c_{12} (e^{2i\alpha} m_1 m_2 - e^{2i\alpha} m_3 (c_{12} e^{2i\alpha} m_2 + m_1 s_{12}^2)) s_{13} - c_{12} c_{13} e^{i(2\beta + \delta)} (m_1 - e^{2i\alpha} m_2) m_3 s_{12} s_{23} \right],
\]
\[
(M^{-1}_\nu)_{22} = \frac{1}{m_1 m_2 m_3} \left[ e^{-2i(\alpha + \beta)} m_1 m_3 (c_{12} c_{23} e^{i\delta} - s_{12} s_{13} s_{23})^2 + m_2 (c_{13}^2 m_3 s_{12}^2 + 2 c_{12} c_{23} m_3 s_{12} s_{13} s_{23}
\]
\[+ e^{-2i(\beta + \delta)} (c_{13}^2 m_1 + c_{12}^2 e^{i\beta} m_3 s_{13}^2 s_{23}^2) \right],
\]
\[
(M^{-1}_\nu)_{23} = \frac{1}{m_1 m_2 m_3} \left[ e^{-2i(\alpha + \beta)} m_1 m_3 (c_{12} c_{23} e^{i\delta} + c_{12} e^{i\delta} s_{23}) (c_{12} c_{23} e^{i\delta} - s_{12} s_{13} s_{23})
\[+ e^{-2i(\beta + \delta)} m_2 (c_{13}^2 m_1 - e^{2i(\beta + \delta)} m_3 s_{12}^2) + c_{13}^2 c_{23} e^{i\beta} m_3 s_{13}^2 s_{23}
\[+ c_{12} e^{i(2\beta + \delta)} m_3 s_{12} s_{23} (c_{23}^2 - s_{23}^2) \right],
\]
\[
(M^{-1}_\nu)_{33} = \frac{1}{m_1 m_2 m_3} \left[ e^{-2i(\alpha + \beta)} c_{23}^2 (c_{13}^2 e^{2i\alpha} m_2 + e^{2i\beta} m_3 (c_{13}^2 e^{2i\alpha} m_2 + m_1 s_{12}^2)) s_{13}
\[+ e^{i(2\alpha + \beta)} c_{12} c_{23} (m_1 - e^{2i\alpha} m_2) m_3 s_{12} s_{13} s_{23} + e^{-2i\alpha} m_3 (c_{12} m_1 + e^{2i\alpha} m_2 s_{12}^2) s_{23} \right].
\]
\[
(4)
\]

There are fifteen possible two-zero textures of $M^{-1}_\nu$ which we categorize in five classes viz. class $A$, $B$, $C$, $D$ and $E$ as shown in Table 1. Symbolically, these 15 textures can be represented as

\[
A_1 = \begin{pmatrix} 0 & 0 & X \\ 0 & X & X \\ X & X & X \end{pmatrix}, \quad A_2 = \begin{pmatrix} 0 & X & 0 \\ X & X & X \\ 0 & X & X \end{pmatrix}, \quad A_3 = \begin{pmatrix} 0 & X & X \\ 0 & X & X \\ X & X & X \end{pmatrix},
\]

\[
A_4 = \begin{pmatrix} 0 & X & 0 \\ X & X & 0 \\ X & 0 & X \end{pmatrix}, \quad A_5 = \begin{pmatrix} 0 & X & 0 \\ X & X & 0 \\ 0 & X & X \end{pmatrix},
\]

\[
B_1 = \begin{pmatrix} X & 0 & 0 \\ 0 & X & X \\ 0 & X & X \end{pmatrix}, \quad B_2 = \begin{pmatrix} X & 0 & X \\ 0 & X & X \\ 0 & X & X \end{pmatrix}, \quad B_3 = \begin{pmatrix} X & 0 & X \\ 0 & X & 0 \\ 0 & X & 0 \end{pmatrix},
\]

\[
B_4 = \begin{pmatrix} X & 0 & X \\ 0 & X & 0 \\ X & 0 & 0 \end{pmatrix},
\]

\[
C_1 = \begin{pmatrix} X & 0 & X \\ 0 & X & 0 \\ 0 & X & X \end{pmatrix}, \quad C_2 = \begin{pmatrix} X & 0 & X \\ 0 & X & 0 \\ 0 & 0 & X \end{pmatrix}, \quad C_3 = \begin{pmatrix} X & 0 & X \\ X & X & 0 \\ X & 0 & 0 \end{pmatrix},
\]

\[
D_1 = \begin{pmatrix} X & X & X \\ X & 0 & 0 \\ X & 0 & X \end{pmatrix}, \quad D_2 = \begin{pmatrix} X & X & X \\ X & X & X \\ X & X & X \end{pmatrix}.
\]

where “$X$” denotes the non-zero element of inverse neutrino mass matrix. In the present work, we investigate the phenomenological consequences of these 15 possible two-zero textures in $M^{-1}_\nu$ within the paradigm of LMA and LMA-D descriptions of neutrino oscillation phenomenon.

In general, two-zero texture of $M^{-1}_\nu$ result in constraining equations

\[
(M^{-1}_\nu)_{pq} = 0; \quad (M^{-1}_\nu)_{rs} = 0
\]
\[
(5)
\]
where $p, q, r$ and $s$ can take value 1, 2, 3 such that $p \leq q$, $r \leq s$.

Using Eq. (3), the constraints described by Eq. (5) can be written as

\[
\lambda_1 U_{1p}^{-1} U_{1q}^{-1} + \lambda_2 U_{2p}^{-1} U_{2q}^{-1} + \lambda_3 U_{3p}^{-1} U_{3q}^{-1} = 0,
\]
\[
(6)
\]
and

\[
\lambda_1 U_{1r}^{-1} U_{1s}^{-1} + \lambda_2 U_{2r}^{-1} U_{2s}^{-1} + \lambda_3 U_{3r}^{-1} U_{3s}^{-1} = 0,
\]
\[
(7)
\]
Table 1 Fifteen possible two-zero texture patterns with corresponding first and second zero

| Class | Textures | $(M_{21}^{-1})_{pq} = 0$ | $(M_{23}^{-1})_{rs} = 0$ |
|-------|----------|------------------------|------------------------|
| A     | $A_1$    | $(M_{21}^{-1})_{11}$   | $(M_{23}^{-1})_{12}$   |
|       | $A_2$    | $(M_{21}^{-1})_{11}$   | $(M_{23}^{-1})_{13}$   |
|       | $A_3$    | $(M_{21}^{-1})_{11}$   | $(M_{23}^{-1})_{22}$   |
|       | $A_4$    | $(M_{21}^{-1})_{11}$   | $(M_{23}^{-1})_{23}$   |
|       | $A_5$    | $(M_{21}^{-1})_{11}$   | $(M_{23}^{-1})_{33}$   |
| B     | $B_1$    | $(M_{21}^{-1})_{12}$   | $(M_{23}^{-1})_{13}$   |
|       | $B_2$    | $(M_{21}^{-1})_{12}$   | $(M_{23}^{-1})_{22}$   |
|       | $B_3$    | $(M_{21}^{-1})_{12}$   | $(M_{23}^{-1})_{23}$   |
|       | $B_4$    | $(M_{21}^{-1})_{12}$   | $(M_{23}^{-1})_{33}$   |
| C     | $C_1$    | $(M_{21}^{-1})_{13}$   | $(M_{23}^{-1})_{22}$   |
|       | $C_2$    | $(M_{21}^{-1})_{13}$   | $(M_{23}^{-1})_{23}$   |
|       | $C_3$    | $(M_{21}^{-1})_{13}$   | $(M_{23}^{-1})_{33}$   |
| D     | $D_1$    | $(M_{21}^{-1})_{22}$   | $(M_{23}^{-1})_{23}$   |
|       | $D_2$    | $(M_{21}^{-1})_{22}$   | $(M_{23}^{-1})_{33}$   |
| E     | $E_1$    | $(M_{21}^{-1})_{23}$   | $(M_{23}^{-1})_{33}$   |

where

$$\lambda_1 = \frac{1}{m_1}, \quad \lambda_2 = \frac{e^{-2i\alpha}}{m_2}, \quad \lambda_3 = \frac{e^{-2i(\beta+\delta)}}{m_3}.$$  

We solve Eqs. (6) and (7) for two mass ratios

$$\begin{align*}
\frac{m_1}{m_2} e^{-2i\alpha} &= U^{-1} v_{e} U^{-1} p U^{-1} q U^{-1} 1 \equiv U^{-1} v_{e} U^{-1} p U^{-1} q U^{-1} 1 s, \\
\frac{m_1}{m_3} e^{-2i(\beta+\delta)} &= U^{-1} v_{e} U^{-1} p U^{-1} q U^{-1} 1 \equiv U^{-1} v_{e} U^{-1} p U^{-1} q U^{-1} 1 s, \\
\end{align*}$$

(8)

The absolute values of mass ratios are given by

$$\begin{align*}
\frac{m_1}{m_2} &= \left| U^{-1} v_{e} U^{-1} p U^{-1} q U^{-1} 1 \right| = \left| U^{-1} v_{e} U^{-1} p U^{-1} q U^{-1} 1 s \right|, \\
\frac{m_1}{m_3} &= \left| U^{-1} v_{e} U^{-1} p U^{-1} q U^{-1} 1 \right| = \left| U^{-1} v_{e} U^{-1} p U^{-1} q U^{-1} 1 s \right|. \\
\end{align*}$$

(10)

and two Majorana phases are obtained as

$$\alpha = -\frac{1}{2} \text{Arg}$$

It is to be noted that the mass ratios \(\frac{m_1}{m_2} e^{-2i\alpha}, \frac{m_1}{m_3} e^{-2i(\beta+\delta)}\) are different for each texture as they depend on the position of zero in \(M_{21}^{-1}\). For example, in case of \(A_1\) texture

$$p = 1, \quad q = 1, \quad r = 1, \quad s = 2,$$

therefore, Eqs. (8) and (9) become

$$\begin{align*}
\frac{m_1}{m_2} e^{-2i\alpha} &= U^{-1} v_{e} U^{-1} p U^{-1} q U^{-1} 1 \equiv U^{-1} v_{e} U^{-1} p U^{-1} q U^{-1} 1 s, \\
\frac{m_1}{m_3} e^{-2i(\beta+\delta)} &= U^{-1} v_{e} U^{-1} p U^{-1} q U^{-1} 1 \equiv U^{-1} v_{e} U^{-1} p U^{-1} q U^{-1} 1 s, \\
\end{align*}$$

(12)

while for \(B_2\) texture

$$p = 1, \quad q = 2, \quad r = 2, \quad s = 2,$$

resulting in mass ratios

$$\begin{align*}
\frac{m_1}{m_2} e^{-2i\alpha} &= U^{-1} v_{e} U^{-1} p U^{-1} q U^{-1} 1 \equiv U^{-1} v_{e} U^{-1} p U^{-1} q U^{-1} 1 s, \\
\frac{m_1}{m_3} e^{-2i(\beta+\delta)} &= U^{-1} v_{e} U^{-1} p U^{-1} q U^{-1} 1 \equiv U^{-1} v_{e} U^{-1} p U^{-1} q U^{-1} 1 s, \\
\end{align*}$$

(13)

We have analytically solved the mass ratios for all possible two-zero textures which are shown in the Tables 2, 3 and 4.

The two mass-squared differences \(\Delta m_{21}^2 = m_2^2 - m_3^2\) and \(\Delta m_{32}^2 = m_3^2 - m_1^2\) along with mass ratios \(\frac{m_1}{m_2} e^{-2i\alpha} \equiv R_{12}\), \(\frac{m_1}{m_3} e^{-i(\beta+\delta)} \equiv R_{13}\) yield two values of neutrino mass \(m_1\) given by

$$m_1^a = |R_{12}| \sqrt{\frac{\Delta m_{21}^2}{1 - |R_{12}|^2}}, \quad m_1^b = |R_{13}| \sqrt{\frac{\Delta m_{32}^2}{1 - |R_{13}|^2}},$$

(14)

respectively. In order to ensure the consistency of the formalism the two values \((m_1^a, m_1^b)\) must be equal which results in mass ratio parameter

$$\frac{\Delta m_{21}^2}{\Delta m_{32}^2} = \frac{|R_{13}|^2 (1 - |R_{12}|^2)}{|R_{12}|^2 - |R_{13}|^2} \equiv R_v.$$

(15)
The $3\sigma$ experimental range of parameter $R_{\nu}$ defined in Eq. (15) is $0.02590 < R_{\nu} < 0.03656$. The allowed phenomenology of the model is obtained by restricting $R_{\nu}$ in the $3\sigma$ experimental range. The neutrino mass eigenvalues $m_{2}^{2}$ and $m_{3}^{2}$ can be obtained using mass square differences ($\Delta m_{21}^{2}$, $\Delta m_{32}^{2}$) as

\[ m_{2}^{2} = \sqrt{m_{1}^{2} + \Delta m_{21}^{2}}; \]
\[ m_{3}^{2} = \sqrt{m_{2}^{2} + \Delta m_{32}^{2}} \text{ for normal hierarchy (NH)} \]
\[ (m_{1} < m_{2} < m_{3}), \] (16)

and

\[ m_{2}^{2} = \sqrt{m_{1}^{2} + \Delta m_{21}^{2}}; \]
\[ m_{1} = \sqrt{m_{2}^{2} + \Delta m_{32}^{2} - \Delta m_{21}^{2}} \text{ for inverted hierarchy (IH)} \]
\[ (m_{3} < m_{1} < m_{2}). \] (17)

Also, the effective Majorana mass which governs the neutrinoless double beta($0\nu\beta\beta$) decay process is given by

\[ |M_{ee}| = \left| \sum_{i} V_{ei}^{*} m_{i} \right| = m_{1} c_{12}^{2} c_{13}^{2} e^{-2\alpha} + m_{2} s_{12}^{2} c_{13}^{2} e^{2\beta} + m_{3} s_{12}^{2} s_{13}^{2} e^{2\beta}. \] (18)

The Jarlskog $CP$ invariant is defined as $[30,31]$

\[ J_{CP} = s_{23} c_{23} s_{12} c_{12} s_{13} c_{13}^{2} \sin \delta. \]

In the numerical analysis, to study LMA phenomenology of the model, we have randomly generated the known neutrino oscillation parameters such as $\theta_{ij}$ ($i, j = 1, 2, 3; i < j$) and $\Delta m_{ij}^{2}$ ($i > j$) using Gaussian distribution within allowed experimental range shown in Table 5. However, in order to study the viability of the model under LMA-D solution, $\theta_{12}$ is randomly generated using the uniform distribution within the range $\{53.71^\circ, 58.37^\circ\}$ $[32-35]$.

### 3 LMA and LMA-D phenomenology

In this section, we have investigated the phenomenology of all possible two-zero textures of $M_{\nu}^{-1}$ under the paradigm of LMA and LMA-D solutions to the neutrino oscillation phenomenon.

#### 3.1 Class A

Class A is disallowed for both LMA and LMA-D solutions. As a representative case, we have discussed the viability of $A_{1}$ texture in the following. The mass ratios ($|R_{12}|$, $|R_{13}|$) for $A_{1}$ texture up-to first order in $s_{13}$ can be written as

\[ |R_{12}| \equiv \frac{m_{1}}{m_{2}} \approx \frac{c_{12}^{2}}{s_{12}^{2}} + \frac{c_{12} c_{23} \cos \delta}{s_{12} s_{23}} s_{13}, \] (19)
Table 3 Mass ratios for class \( B \)

| Class | Texture | Mass ratios \((R_{12}, R_{13})\) |
|-------|---------|---------------------------------|
| \( B \) | \( B_1 \) | \[
\begin{align*}
\frac{m_1}{m_2} e^{-2\alpha} &= 1 \\
\frac{m_1}{m_3} e^{-i(2\beta+\delta)} &= 1 \\
\end{align*}
\]

| \( B_2 \) | \[
\begin{align*}
\frac{m_1}{m_2} e^{-2\alpha} &= \frac{c_{21}^2 e^{i\delta} s_{12} + c_{12} s_{13} s_{23}}{c_{12} e^{i\delta} s_{12} + c_{12} e^{i\delta} s_{13} s_{23} - s_{12} s_{23}} \\
\frac{m_1}{m_3} e^{-i(2\beta+\delta)} &= c_{23} \left( c_{23} e^{i\delta} s_{12} + c_{12} e^{i\delta} s_{13} s_{23} - s_{12} s_{23} \right) \\
\end{align*}
\]

| \( B_3 \) | \[
\begin{align*}
\frac{m_1}{m_2} e^{-2\alpha} &= \frac{c_{23} e^{i\delta} s_{12} + c_{12} e^{i\delta} s_{13} s_{23}}{c_{23} s_{12} + c_{12} e^{i\delta} s_{13} s_{23} - s_{12} s_{23}} \\
\frac{m_1}{m_3} e^{-i(2\beta+\delta)} &= c_{23} e^{i\delta} s_{12} + c_{12} e^{i\delta} s_{13} s_{23} \\
\end{align*}
\]

| \( B_4 \) | \[
\begin{align*}
\frac{m_1}{m_2} e^{-2\alpha} &= \frac{c_{12} c_{23} e^{i\delta} s_{12} s_{23} + c_{23} e^{2i\delta} s_{12} s_{23}}{c_{23} s_{12} s_{23} + c_{12} e^{i\delta} s_{12} s_{23} - c_{12} e^{i\delta} s_{12} s_{23}} \\
\frac{m_1}{m_3} e^{-i(2\beta+\delta)} &= \frac{c_{12} c_{23} e^{2i\delta} s_{12} s_{23} + c_{23} e^{i\delta} s_{12} s_{23}}{c_{23} s_{12} s_{23} + c_{12} e^{i\delta} s_{12} s_{23} - c_{12} e^{i\delta} s_{12} s_{23}} \\
\end{align*}
\]

Table 4 Mass ratios for class \( C, D \) and \( E \)

| Class | Texture | Mass ratios \((R_{12}, R_{13})\) |
|-------|---------|---------------------------------|
| \( C \) | \( C_1 \) | \[
\begin{align*}
\frac{m_1}{m_2} e^{-2\alpha} &= \frac{c_{21}^2 e^{i\delta} s_{12} + c_{12} e^{i\delta} s_{13} s_{23} \left( 2c_{23}^2 + s_{12}^2 - s_{13}^2 s_{23} \right)}{c_{23}^2 s_{12} s_{23} + c_{12} e^{i\delta} s_{13} s_{23} - s_{12} s_{23}} \\
\frac{m_1}{m_3} e^{-i(2\beta+\delta)} &= \frac{c_{23} e^{i\delta} s_{12} + c_{12} e^{i\delta} s_{13} s_{23}}{c_{23} s_{12} + c_{12} e^{i\delta} s_{13} s_{23} - s_{12} s_{23}} \\
\end{align*}
\]

| \( C_2 \) | \[
\begin{align*}
\frac{m_1}{m_2} e^{-2\alpha} &= \frac{c_{23}^2 e^{i\delta} s_{12} + c_{12} e^{i\delta} s_{13} s_{23} \left( 2c_{23}^2 + s_{12}^2 - s_{13}^2 s_{23} \right)}{c_{23} s_{12} s_{23} + c_{12} e^{i\delta} s_{13} s_{23} - s_{12} s_{23}} \\
\frac{m_1}{m_3} e^{-i(2\beta+\delta)} &= \frac{c_{23} e^{i\delta} s_{12} + c_{12} e^{i\delta} s_{13} s_{23}}{c_{23} s_{12} + c_{12} e^{i\delta} s_{13} s_{23} - s_{12} s_{23}} \\
\end{align*}
\]

| \( C_3 \) | \[
\begin{align*}
\frac{m_1}{m_2} e^{-2\alpha} &= \frac{c_{23} e^{i\delta} s_{12}}{c_{23} s_{12} + c_{12} e^{i\delta} s_{13} s_{23} - s_{12} s_{23}} \\
\frac{m_1}{m_3} e^{-i(2\beta+\delta)} &= \frac{c_{23} e^{i\delta} s_{12}}{c_{23} s_{12} + c_{12} e^{i\delta} s_{13} s_{23} - s_{12} s_{23}} \\
\end{align*}
\]

| \( D \) | \( D_1 \) | \[
\begin{align*}
\frac{m_1}{m_2} e^{-2\alpha} &= -\frac{s_{12} \left( c_{23} e^{i\delta} s_{12} + c_{12} s_{13} s_{23} \right)}{c_{12} \left( c_{23} e^{i\delta} s_{12} - s_{12} s_{23} \right)} \\
\frac{m_1}{m_3} e^{-i(2\beta+\delta)} &= \frac{s_{13} \left( c_{23} e^{i\delta} s_{12} + c_{12} s_{13} s_{23} \right)}{c_{12} c_{23} e^{i\delta} s_{23}} \\
\end{align*}
\]

| \( D_2 \) | \[
\begin{align*}
\frac{m_1}{m_2} e^{-2\alpha} &= -\frac{s_{12} \left( 2c_{12} c_{23} s_{12} s_{23} + c_{23} e^{i\delta} s_{12} - s_{12} s_{23} \right)}{c_{12} \left( -2c_{12} c_{23} s_{12} s_{23} + c_{23} e^{i\delta} s_{12} - s_{12} s_{23} \right)} \\
\frac{m_1}{m_3} e^{-i(2\beta+\delta)} &= \frac{s_{13} \left( 2c_{23} c_{12} e^{i\delta} s_{12} - s_{12} s_{23} \right) - 2c_{12} c_{23} s_{12} s_{23} \left( c_{23} e^{i\delta} s_{12} - s_{12} s_{23} \right) + s_{12} s_{23} (s_{12}^2 - c_{12}^2)}{c_{12} c_{23} e^{i\delta} s_{12} - c_{23} s_{12} s_{23} s_{23} - c_{12} e^{i\delta} s_{23}} \\
\end{align*}
\]

| \( E \) | \( E_1 \) | \[
\begin{align*}
\frac{m_1}{m_2} e^{-2\alpha} &= \frac{s_{12} \left( c_{12} c_{23} s_{13} - s_{12} s_{23} \right)}{c_{12} e^{i\delta} s_{23} + c_{12} e^{i\delta} s_{13} s_{23}} \\
\frac{m_1}{m_3} e^{-i(2\beta+\delta)} &= \frac{s_{13} \left( -e^{-i\delta} c_{12} c_{23} s_{13} + s_{12} s_{23} \right)}{c_{12} c_{23} e^{i\delta} s_{13}} \\
\end{align*}
\]
Table 5: Global fit values of neutrino oscillation parameters [36]

| Parameters                        | Best fit ±1σ range (NH) | Best fit ±1σ range (IH) |
|-----------------------------------|--------------------------|--------------------------|
| $\Delta m^2_{31}[10^{-5}$ eV$^2]$ | $7.50^{+0.22}_{-0.20}$  | $7.50^{+0.22}_{-0.20}$  |
| $|\Delta m^2_{31}|[10^{-3}$ eV$^2]$ | $2.55^{+0.02}_{-0.03}$  | $2.45^{+0.02}_{-0.03}$  |
| $\theta^{0}_{12}$                 | $34.3 \pm 1.0$          | $34.3 \pm 1.0$          |
| $\theta^{0}_{23}$                 | $49.26 \pm 0.79$        | $49.46^{+0.60}_{-0.97}$ |
| $\theta^{0}_{13}$                 | $8.53^{+0.13}_{-0.14}$  | $8.53^{+0.12}_{-0.14}$  |

![Diagram showing $R_\nu$ vs $|R_{12}|$ for texture $A_1$]

Fig. 1: Correlation between $|R_{12}|$ and $R_\nu$ for texture $A_1$

$$|R_{13}| \equiv \frac{m_1}{m_3} \approx \frac{c^2_{12}c^2_{23}}{s^2_{12}s^2_{23}} \cos \delta + c_{12}c_{23}(c^2_{12}c^2_{23} - 4s^2_{12}s^2_{23} + 3c^2_{12}c^2_{23} \cos 2\delta)$$

(20)

**LMA Scenario:** For $\delta$ in the range $0^\circ \leq \delta \leq 90^\circ$ or $270^\circ \leq \delta \leq 360^\circ$, Eq. (19) results in $m_1/m_2 > 1$. Furthermore, if $\delta$ lies in the range $90^\circ < \delta < 270^\circ$ and $\sin \theta_{23} \approx \cos \theta_{23}$, solar mass hierarchy requires $\sin 2\theta_{12} < 4 \sin \theta_{13}$, however, from neutrino oscillation data (Table 5) $\sin 2\theta_{12} > 4 \sin \theta_{13}$ implying that $A_1$ texture is disallowed.

**LMA-D Scenario:** The parameter $(R_\nu)$ up-to-first order in $s_{13}$ can be written as

$$R_\nu \approx \left(-1 + \frac{c^4_{12}}{s^4_{12}}\right) + \frac{2c^3_{12}c_{23} \cos \delta}{s^3_{12}s^3_{23}} s_{13}.$$  

(21)

Using $\theta_{12} = 55^\circ$ and $\delta= 80^\circ (190^\circ)$, numerical value of $R_\nu$ is found to be 0.74 (0.89) which lies outside the 3σ range of $R_\nu$. The above observations are, also, evident from Fig. 1.

Similar analysis can be done for all remaining textures in class A. Therefore, neutrino mass model, with two-zero textures in $M^{-1}_{\nu}$, wherein one of the texture zero is at $(1,1)$ place in $M^{-1}_{\nu}$ is disallowed as shown in Table 6.

3.2 Class B

In class B, mass ratios for textures $B_1$ are given by

$$|R_{12}| \equiv \frac{m_1}{m_2} = 1, \quad |R_{13}| \equiv \frac{m_1}{m_3} = 1,$$

Table 6: Allowed/disallowed two-zero textures of $M^{-1}_{\nu}$ under LMA and LMA-D solutions. The ✓ (✓) mark is used to denote allowed (disallowed) texture

| Class | Texture  | LMA | LMA-D |
|-------|----------|-----|-------|
| A     | $A_1$ (NH/IH) | ✓/x | ✓/x   |
|       | $A_2$ (NH/IH) | ✓/x | ✓/x   |
|       | $A_3$ (NH/IH) | ✓/x | ✓/x   |
|       | $A_4$ (NH/IH) | ✓/x | ✓/x   |
|       | $A_5$ (NH/IH) | ✓/x | ✓/x   |
| B     | $B_1$ (NH/IH) | ✓/x | ✓/x   |
|       | $B_2$ (NH/IH) | ✓/x | ✓/x   |
|       | $B_3$ (NH/IH) | ✓/x | ✓/x   |
|       | $B_4$ (NH/IH) | ✓/x | ✓/x   |
| C     | $C_1$ (NH/IH) | ✓/x | ✓/x   |
|       | $C_2$ (NH/IH) | ✓/x | ✓/x   |
|       | $C_3$ (NH/IH) | ✓/x | ✓/x   |
| D     | $D_1$ (NH/IH) | ✓/x | ✓/x   |
|       | $D_2$ (NH/IH) | ✓/x | ✓/x   |
| E     | $E_1$ (NH/IH) | ✓/x | ✓/x   |
Fig. 2 Correlation plots for textures $B_2$-NH (left panel) and $B_4$-IH (right panel) under LMA scenario.
which result in degenerate neutrino masses (i.e. \( m_1 = m_2 = m_3 \)) and are inconsistent with neutrino oscillation data. Also, for texture \( B_3 \), mass ratios are given by

\[
|R_{12}| = \left| \frac{c_{23} e^{i \delta} s_{12} + c_{12} s_{13} s_{23}}{c_{23} s_{12} + c_{12} e^{i \delta} s_{13} s_{23}} \right| = 1,
\]

\[
|R_{13}| = \left| \frac{c_{23} e^{i \delta} s_{12} + c_{12} s_{13} s_{23}}{c_{23} s_{12} + c_{12} e^{i \delta} s_{13} s_{23}} \right| = 1,
\]

resulting in degenerate neutrino masses. Therefore, textures \( B_1 \) and \( B_3 \) are disallowed. In the following we have investigated the phenomenological consequences of \( B_2 \) and \( B_4 \) textures under LMA and LMA-D solutions.

The mass ratios (\(|R_{12}|, |R_{13}|\)) for texture \( B_2 \), up-to first order in \( s_{13} \), can be written as

\[
|R_{12}| \equiv \frac{m_1}{m_2} \approx 1 + \frac{2 \cos \delta}{c_{12} c_{23} s_{12} s_{23}} s_{13},
\]

\[
|R_{13}| \equiv \frac{m_1}{m_3} \approx \frac{c_{23}}{s_{23}} + \frac{c_{12} e^{i \delta} \cos \delta}{s_{12} s_{23}} s_{13}.
\]

From Eq. (24), in order to satisfy the solar mass hierarchy i.e. \(|R_{12}| = \frac{m_1}{m_2} < 1\), \( \delta \) should be in the range \( 90^\circ < \delta < 270^\circ \). Also, texture \( B_2 \) predicts the normal hierarchical neutrino masses for \( \theta_{23} \) above maximality (\( \theta_{23} > 45^\circ \)) and \( \cos \delta \) negative such that \(|R_{13}| \equiv \frac{m_1}{m_3} < 1\). The above observations are, also, depicted in the Fig. 2. Similar analysis can also be done for texture \( B_4 \).

The texture \( B_2 (B_4) \) is allowed for both LMA and LMA-D solutions with normal (inverted) hierarchy. The allowed parameter space for these textures are shown in Fig. 2 as correlation plots amongst different parameters. Both these textures are found to have identical phenomenology under LMA and LMA-D solutions. In Fig. 2 we have shown the LMA scenario for \( B_2 \) (NH) and \( B_4 \) (IH) textures. In the left (right) panel we have depicted the correlation plots for \( B_2 \) (\( B_4 \)) texture. The atmospheric mixing angle \( \theta_{23} \) is found to be above maximality for both the textures. The \( CP \) violating phases \( \alpha, \beta \) and \( \delta \) are found to be sharply constrained. The Dirac-type \( CP \) violating phase \( \delta \) is found to be maximal (around \( 90^\circ \) and \( 270^\circ \)) and the Jarlskog rephasing invariant \( J_{CP} \neq 0 \), thus, these textures are necessarily \( CP \) violating. We can, also, appreciate the \( CP \) violating nature of these textures analytically. For example, for texture \( B_2 \) with \( \delta = 0^\circ \) \( (CP \) conserving scenario), we obtain \( R_\nu \) to the first order in \( s_{13} \) using Eq. (15) and values of mass ratios given in Table 3.

\[
R_\nu \approx \frac{2 c_{23}}{c_{12} s_{12} s_{23}} \left( s_{23}^2 - s_{13}^2 \right) s_{13} - \frac{2 c_{23} s_{23}}{c_{12} s_{12} \left( c_{23}^2 - s_{23}^2 \right)} s_{13}.
\]

Using the best-fit values given in Table 5, \(|R_\nu| \approx 1.7\) which is outside \( 3\sigma \) range, thus, \( \delta = 0^\circ \) is disallowed implying \( B_2 \) texture is necessarily \( CP \) violating. Similarly, for texture \( B_4 \), taking \( \delta = 0^\circ \) \( R_\nu \) can, approximately, be written as

\[
R_\nu \approx \frac{s_{23}^2 (2 c_{23}^2 - 2 c_{23}^2 s_{23}^2 + 2 c_{23}^2 s_{12}^2 - s_{23}^2)}{c_{12} s_{12} s_{23} (c_{23}^2 - c_{23}^2)} s_{13},
\]

which again result in \(|R_\nu| > 1\) for best-fit values of the mixing angles. Similar analysis can be done for \( CP \)-violating textures of class C and D discussed in the following sections.

We have, also, obtained the implication of the model for neutrinoless double beta \((0\nu\beta\beta)\) decay amplitude \(|M_{ee}|\). It is evident from Fig. 2 that there exist a lower bound on \(|M_{ee}|\) in both the textures. For texture \( B_2 (B_4) \), \(|M_{ee}| < 0.03\) eV (0.06 eV). The prediction for \(|M_{ee}|\) has, also, been tabulated in Table 7.

### 3.3 Class C

For texture \( C_2 \), the mass ratios \(|R_{12}| \) and \(|R_{13}| \) are equal to 1 resulting in degenerate neutrino masses which is in contradiction with neutrino oscillation data. Therefore, texture \( C_2 \) is disallowed. For textures \( C_1 \), the mass ratios \(|R_{12}| \) and \(|R_{13}| \), up-to first order in \( s_{13} \), are given by

\[
|R_{12}| \equiv \frac{m_1}{m_2} \approx 1 - \frac{c_{23} s_{13} \cos \delta}{c_{12} s_{12} s_{23}^3},
\]
\[ |R_{13}| \equiv \frac{m_1}{m_3} \approx \frac{c_{23}^2}{s_{23}} - \frac{c_{12} c_{23} s_{13} \cos \delta}{s_{12} s_{23}}. \]  

(27)

It can be seen from Eq. (26) that \( \delta \) should lie in first and fourth quadrant to have \(|R_{12}| \equiv \frac{m_1}{m_2} < 1 \). Also, for \( \theta_{23} \) above maximality \(( \theta_{23} > 45^\circ )\) and \( \cos \delta \) positive the model predicts normal hierarchical neutrino masses. The above observations are, also, supplemented by Fig. 3. Similar analysis can, also, be done for texture \( C_3 \). In fact, it is evident from Fig. 3 that \( C_3 \) admit inverted hierarchical neutrino masses. Also, it is to be noted that the phenomenology of both the textures are found to be similar under LMA and LMA-D solutions. In Fig. 3, we have shown the allowed parameter space considering LMA solution. The Majorana phases are sharply correlated and constrained to very narrow ranges giving a lower bound on \( 0\nu\beta\beta \) decay amplitude \( |M_{ee}| \). For texture \( C_1 \) \(( |M_{ee}| > 0.02 \) (0.06) eV (Table 7). The Dirac-type \( CP \) violating phase \( \delta \) is sharply constrained around \( 90^\circ \) and \( 270^\circ \), thus, allowing a maximal \( CP \) violation. The Jarlskog rephasing invariant \( J_{CP} \) is non-zero which is, also, shown in Fig. 3.

### 3.4 Class D

For textures \( D_1 \), the mass ratios, up-to first order in \( s_{13} \), are given by

\[ |R_{12}| \equiv \frac{m_1}{m_2} \approx \frac{s_{12}^2}{c_{12}^2} + \frac{c_{12} s_{23} s_{13} \cos \delta}{c_{12} s_{23}}, \]  

(28)

\[ |R_{13}| \equiv \frac{m_1}{m_3} \approx \tan \theta_{12} \tan \theta_{23}. \]  

(29)

For texture \( D_1 \), LMA-D solution is disallowed as \(|R_{12}| > 1 \) (Eq. (28)). However, for LMA solution, it is evident that \(|R_{13}| < 1 \) (Eq. (29)) implying normal hierarchical neutrino masses. The above analytical observations are, also, supplemented by the correlation plots in Figs. 4 and 5. In addition, the Majorana phases are sharply constrained and correlated in such a way giving vanishing value of \( 0\nu\beta\beta \) decay amplitude \(|M_{ee}| \) (Table 7).

Texture \( D_2 \) is found to be consistent with both LMA and LMA-D descriptions with normal hierarchical neutrino masses, as shown in Figs. 6 and 7. It is evident from \(( \theta_{23} - |M_{ee}|) \) correlation plot in Fig. 7 that there exist a 3\( \sigma \) lower bound on \( 0\nu\beta\beta \) decay amplitude \(|M_{ee}| > 0.02 \) eV (see Table 7). Furthermore, in contrast to \( D_1 \), texture \( D_2 \) is found to be necessarily \( CP \) violating as depicted in \(( \theta_{13}, J_{CP}) \) (Fig. 7) and \((\theta_{23}, \delta) \) (Fig. 8) correlation plots.

### 3.5 Class E

The mass ratios for texture \( E_1 \) can be obtained from Eqs. (28) and (29) by using the transformation \( c_{23} \rightarrow s_{23}; s_{23} \rightarrow c_{23} \) viz.,

\[ |R_{12}| \equiv \frac{m_1}{m_2} \approx \frac{s_{12}^2}{c_{12}^2} + \frac{s_{12} c_{23} s_{13} \cos \delta}{c_{12} s_{23}}, \]  

(30)

\[ |R_{13}| \equiv \frac{m_1}{m_3} \approx \tan \theta_{12} \tan \theta_{23} s_{13}. \]  

(31)

It can be seen from Eq. (30) that \( E_1 \) is not consistent with LMA-D solution as \(|R_{12}| > 1 \). The allowed parameter space for LMA with NH is shown in Figs. 9 and 10. Also, the Majorana phases are correlated in such a way that \( 0\nu\beta\beta \) decay amplitude \(|M_{ee}| \) is found to be vanishing in this case. Furthermore, the texture allows for both \( CP \) conserving and violating solutions as is evident from Fig. 10.

### 4 Symmetry realization

In this section, we discuss the minimal realization of two-zero texture of \( M^{-1}_\nu \). Motivated by the pivotal character played by the discrete flavor symmetry groups in explaining the observed neutrino oscillation data [37,38], we obtain the \( A_4 \) flavor group based symmetry realisation of inverse neutrino mass matrix \((M^{-1}_\nu)\) by extending the standard model particle content in the lepton sector. \( A_4 \) is a non-Abelian discrete group of even permutations. \( A_4 \) group is a orientation-preserving symmetry of a regular tetrahedron. It has four irreducible representations \( 1, 1', 1'' \) and \( 3 \) and can be generated using \( S \) and \( T \) generators satisfying the relations

\[ S^2 = T^3 = (ST)^3 = 1. \]

Here, we choose basis for \( A_4 \) group in which \( T \) generator takes the diagonal form. The reason behind choosing this particular representation is that it facilitates the diagonal mass matrix for charged leptons. In T-diagonal basis, one dimensional unitary representation \( 1, 1' \) and \( 1'' \) with generator \( S \) and \( T \) can be written as

\[ 1: \quad S = 1, \quad T = 1, \]

\[ 1': \quad S = 1, \quad T = \omega, \]

\[ 1'': \quad S = 1, \quad T = \omega^2, \]

such that \( \omega = e^{2\pi i/3} \) whereas three-dimensional unitary representation is given by

\[ T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad S = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}. \]

The multiplication rules for the representations of \( A_4 \) are as follows

\[ 1 \otimes 1' = 1'', \quad 1'' \otimes 1'' = 1', \]

\[ 1' \otimes 1'' = 1, \quad 1'' \otimes 1 = 1'', \]

\[ 1 \otimes 1' = 1', \quad 3 \otimes 1' = 3, \quad 3 \otimes 1'' = 3, \]
Fig. 3 Correlation plots for textures C$_1$-NH (left panel) and C$_3$-IH (right panel) under LMA scenario.
Fig. 4 Correlation plots between $(\theta_{23} - \delta)$ (left) and $(|R_{12}| - R_{\nu})$ (right) for $D_1$ texture under LMA solution with NH

Fig. 5 Correlation plots for $D_1$ texture under LMA solution with NH

Fig. 6 Correlation between $(|R_{12}| - R_{\nu})$ for $D_2$ texture

$$3 \otimes 3 = 1 \oplus 1' \oplus 1'' \oplus 3_s \oplus 3_a.$$ 

In the $T$-diagonal basis, the Clebsch–Gordan decomposition of two triplets, $a = (a_1, a_2, a_3)$ and $b = (b_1, b_2, b_3)$ is given as

$$(a \otimes b)_1 = a_1 b_1 + a_2 b_3 + a_3 b_2,$$

$$(a \otimes b)_1' = a_3 b_3 + a_1 b_2 + a_2 b_1,$$

$$(a \otimes b)_1'' = a_2 b_2 + a_1 b_3 + a_3 b_1,$$

$$(a \otimes b)_3_s = \frac{1}{3} (2a_1 b_1 - a_2 b_3 - a_3 b_2, 2a_3 b_3 - a_1 b_2 - a_2 b_1, -a_2 b_1, 2a_3 b_2 - a_1 b_3 - a_3 b_1),$$

$$(a \otimes b)_3_a = \frac{1}{2} (a_2 b_3 - a_3 b_2, a_1 b_2 - a_2 b_1, a_1 b_3 - a_3 b_1).$$

Here, we have worked in the framework of Type-I seesaw. In addition, we have minimally extended the standard model by adding three right-handed neutrino fields ($\nu_iR; i = 1, 2, 3$) and one scalar field ($\chi$), having singlet representation under $A_4$ symmetry as shown in the Table 8. In general, for any Yukawa coupling to be non-zero, its Yukawa Lagrangian term must be in singlet-invariant representation of $A_4$ with mass dimension four at tree level.

**Texture $B_3$:**

Using the tensor products in Eq. (32), the invariant Yukawa Lagrangian is given by

$$-\mathcal{L} = \cdots + y_\nu D_{\nu L} \phi e_R + y_\mu D_{\mu L} \phi \mu R + y_\tau D_{\tau L} \phi \tau R + y_1 D_{\nu L} \phi e_R + y_2 D_{\mu L} \phi \mu R + y_3 D_{\tau L} \phi \tau R + \frac{1}{2} \left[ M_1 (v_{1R}^T C^{-1} v_{1R}) + M_2 (v_{2R}^T C^{-1} v_{3R} + v_{3R}^T C^{-1} v_{2R}) \right]$$
Fig. 7 Correlation plots for $D_2$ texture under LMA solution with NH

Fig. 8 Correlation between $(\theta_{23} - \delta)$ for $D_2$ texture with NH under LMA (left) and LMA-D (right)

Fig. 9 Correlation between $|R_{12}|$ for texture $E_1$ under LMA solution with NH

\[ + \frac{1}{2} \left[ \left( y_{\chi_1} (v_{e R}^T C^{-1} v_{2 R}) + y_{\chi_2} (v_{\mu R}^T C^{-1} v_{3 R} + v_{\tau R}^T C^{-1} v_{1 R}) \right) \right] \chi \],

(33)

where $y_{k}, y_{i}$ ($k = e, \mu, \tau; i = 1, 2, 3$) are Yukawa coupling constants, $M_{1,2}$ are bare mass terms for right-handed Majorana neutrinos, $y_{\chi_{1,2}}$ denotes Yukawa coupling constant for interaction terms with scalar field $\chi$ and $\phi = i \tau_2 \phi^*$; $\tau_2$ being Pauli matrix.

The dots in the Lagrangian represents the other kinetic and scalar potential terms. We have restricted up to Yukawa interactions pertaining to mass terms. Spontaneous symmetry breaking (SSB) occurred with vacuum expectation values ($v_1$,$v_2$) $v$ and $w$ for the Higgs doublet and scalar singlet field, respectively. The Yukawa Lagrangian (Eq. (33)) leads to the mass matrices as

\[ M_L = \begin{pmatrix} y_e v & 0 & 0 \\ 0 & y_\mu v & 0 \\ 0 & 0 & y_\tau v \end{pmatrix}, \quad M_D = \begin{pmatrix} y_{11} v & 0 & 0 \\ 0 & y_{22} v & 0 \\ 0 & 0 & y_{33} v \end{pmatrix} \]

(34)

and

\[ M_R = \begin{pmatrix} M_1 & 0 & y_{12} w \\ 0 & y_{21} w & M_2 \end{pmatrix}, \quad M_T \]

(35)

where $M_L, M_D, M_R$ corresponds to charged lepton mass matrix, Dirac mass matrix and right-handed Majorana mass matrix, respectively (Table 8).

Type-I seesaw contribution to effective Majorana neutrino mass matrix is given by

\[ M_\nu = M_D M_R^{-1} M_D^T. \]

(36)

Also, the inverse neutrino mass matrix can be written as

\[ M_\nu^{-1} = M_D^{-T} M_R M_D^{-1}. \]

(37)
Correlation plots for texture $E_1$ under LMA solution with NH

Table 8  Field content and charge assignments in the model under $SU(2)_L$ and $A_4$ symmetries

| Symmetry | $D_{eL}$ | $D_{3eL}$ | $D_{1L}$ | $e_R$ | $\mu_R$ | $\tau_R$ | $v_{1R}$ | $v_{2R}$ | $v_{3R}$ | $\phi$ | $\chi$ | Obtained two-zero textures |
|----------|---------|----------|---------|-----|------|------|-------|-------|-------|-------|------|--------------------------|
| $SU(2)_L$ | 2       | 2        | 2       | 1   | 1    | 1    | 1     | 1     | 1     | 2     | 1   |             |
| $A_4$    | 1       | 1"     | 1'     | 1'  | 1"  | 1'  | 1"   | 1'   | 1"   |       |    |             |

In $M_D$-diagonal basis, the peculiar feature of implementation of type-I seesaw for $M_v^{-1}$ is that the zero(s) in $M_R$ corresponds to zero(s) in $M_v^{-1}$. Using the Eqs. (34) and (35), the $M_v^{-1}$ is given by

$$M_v^{-1} = \begin{pmatrix} M_1 & 0 & y_{\chi_1} w \\ 0 & M_2 & y_{\chi_2} w \\ y_{\chi_1} w & y_{\chi_2} w & 0 \end{pmatrix}$$

(38)

which symbolically can be written as

$$M_v^{-1} = \begin{pmatrix} X & 0 & X \\ 0 & X & X \\ X & X & 0 \end{pmatrix}$$

(39)

corresponding to texture $B_4$.

**Texture $C_1$:**

For the realization of texture $C_1$, we change the irreducible representation of scalar field $\chi$ to be $1"$.

The relevant Yukawa Lagrangian is

$$-\mathcal{L} = \ldots + y_e \bar{D}_{eL} \phi e_R + y_{\mu} \bar{D}_{\mu L} \phi \mu_R + y_{\tau} \bar{D}_{\tau L} \phi \tau_R + y_1 \bar{D}_{eL} \phi v_{eR} + y_2 \bar{D}_{\mu L} \phi v_{\mu R} + y_3 \bar{D}_{\tau L} \phi v_{\tau R}$$

where $y_{k}, y_{i}(k = e, \mu, \tau; i = 1, 2, 3)$ are Yukawa coupling constants, $M_{1,2}$ are bare mass terms for right-handed Majorana neutrinos, $y_{\chi 1,2}$ denotes Yukawa coupling constant for interaction terms with scalar field $\chi$ and $\hat{\phi} = i \tau_2 \phi^*$; $\tau_2$ being Pauli matrix.

After SSB, charged lepton mass matrix and Dirac mass matrix remains diagonal as shown in Eq. (34). But Majorana mass matrix gets modified and takes the form

$$M_R = \begin{pmatrix} M_1 & y_{\chi_2} w & 0 \\ y_{\chi_2} w & M_2 & 0 \\ 0 & 0 & M_2 \end{pmatrix}$$

(41)
In $M_D$-diagonal basis, the inverse neutrino mass matrix is given by

$$M_{\nu}^{-1} = \begin{pmatrix}
\frac{M_D}{v^2} & \frac{y_{\chi_2} y_{\nu_1}}{v^2 y_{\nu_1} y_{\nu_2}} & 0 \\
\frac{y_{\chi_2} y_{\nu_1}}{v^2 y_{\nu_1} y_{\nu_2}} & \frac{M_D}{v^2 y_{\nu_1} y_{\nu_2} y_{\nu_3}} & 0 \\
0 & \frac{M_D}{v^2 y_{\nu_1} y_{\nu_2} y_{\nu_3}} & \frac{M_D}{v^2 y_{\nu_2}^2 y_{\nu_3}}
\end{pmatrix}, \quad (42)$$

which symbolically can be written as

$$M_{\nu}^{-1} = \begin{pmatrix}
X & X & 0 \\
X & 0 & X \\
0 & X & X
\end{pmatrix}, \quad (43)$$

corresponding to texture $C_1$.

5 Conclusions

In conclusion, we have investigated the phenomenological implications of two-zero textures in inverse neutrino mass matrix $M_{\nu}^{-1}$. In the basis where Dirac neutrino mass matrix $M_D$ is diagonal, zeros in right-handed Majorana neutrino mass matrix $M_R$ correspond to zeros of $M_{\nu}^{-1}$. We have, also, proposed symmetry realization, based on discrete flavor group $A_4$, wherein such texture zeros can emerge in $M_{\nu}^{-1}$. Further, we investigate the viability of all two-zero textures in $M_{\nu}^{-1}$ under LMA and LMA-D solutions. We have categorized the possible textures in five classes viz. class A, B, C, D and E. Out of fifteen possible two-zero textures of $M_{\nu}^{-1}$, seven are found to be in consonance with LMA and/or LMA-D scenario. The general remarks about the obtained phenomenology are as under:

- The textures in class A are all disallowed as they do not reproduce the correct neutrino phenomenology. Thus, texture with $(M_{\nu}^{-1})_{11} = 0$ is disallowed, in general.
- In class B, $B_2$ and $B_4$ textures are consistent with both LMA and LMA-D solutions. $B_2$ ($B_4$) predicts normal (inverted) hierarchical neutrino masses. For texture $B_2$, the $3\sigma$ lower bound on $0\nu\beta\beta$ decay amplitude $|M_{ee}|$ is found to be 0.02 eV (0.04 eV) under LMA and LMA-D, respectively. For $B_4$ texture, it is about 0.06 eV for both LMA and LMA-D solutions.
- In class C, $C_2$ is disallowed. $C_1$ and $C_3$ textures are consistent with both LMA and LMA-D solutions. $C_1$ ($C_3$) predicts normal (inverted) hierarchical neutrino masses. For texture $C_1$ ($C_3$), the $3\sigma$ lower bound on $|M_{ee}|$ is 0.02 eV (0.06) eV under both LMA and LMA-D solutions.
- For textures $B_2$, $B_4$, $C_1$ and $C_3$, the Dirac and Majorana-type $CP$ violating phases are sharply constrained and these textures are found to be necessarily $CP$ violating.

- Textures $D_1$ and $E_1$ predict normal hierarchical neutrino masses and are found to be consistent with LMA solution. LMA-D solution is disallowed by these textures. Also, $|M_{ee}|$ is vanishing in these textures. In general, we conclude that the textures for which LMA-D is disallowed, $|M_{ee}|$ is vanishing. Similar inference is observed in Ref. [11] wherein the authors analyzed phenomenology of Majorana neutrino textures in the light of LMA-D solution.
- Texture $D_2$ predicts normal hierarchical neutrino masses and is consistent with both LMA and LMA-D phenomenology. In LMA (LMA-D) scenario, there exist a $3\sigma$ lower bound on $|M_{ee}| > 0.01$ eV (0.02 eV).
- The generic feature of the class of model, discussed in the present work, is the existence of neutrino mass hierarchy degeneracy in a particular texture. For example, if a texture is allowed by LMA solution with “X” neutrino mass hierarchy then, if LMA-D is allowed, it is allowed with the same hierarchy “X”.

The allowed two-zero texture of $M_{\nu}^{-1}$ viz. $B_2$, $B_4$, $C_1$, $C_3$, $D_1$, $D_2$ and $E_1$ has imperative predictions for $|M_{ee}|$ [33]. Except for $D_1$ and $E_1$ textures, the predicted $3\sigma$ lower bound on $0\nu\beta\beta$ decay amplitude $|M_{ee}|$ is $O(10^{-2})$ which is within the sensitivity reach of $0\nu\beta\beta$ decay experiments like SuperNEMO [39], KamLAND-Zen [40], NEXT [41,42], and nEXO [43]. For example, the non-observation of $0\nu\beta\beta$ decay down to these high sensitivities will refute all the textures except $D_1$ and $E_1$. Also, we have shown that the allowed $M_{\nu}^{-1}$ textures can be accommodated in an extension of the SM with three right-handed neutrinos and one scalar singlet field. As representative realizations, we have obtained two such textures $B_4$ and $C_1$ within Type-I seesaw scenario using $A_4$ discrete flavor symmetry.

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