Quintessence in a quandary: On prior dependence in dark energy models

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The archetypal theory of dark energy is quintessence: a minimally coupled scalar field with a canonical kinetic energy and potential. By studying random potentials we show that quintessence imposes a restricted set of priors on the equation of state of dark energy. Focusing on the commonly-used parametrisation, \( w(a) \approx w_0 + w_a (1 - a) \), we show that there is a natural scale and direction on the \((w_0, w_a)\) plane that distinguishes quintessence as a general framework. We calculate the expected information gain for a given survey and show that, because of the non-trivial prior information, it is a function of more than just the figure of merit. This allows us to make a quantitative case for novel survey strategies.

What drives the accelerated expansion of the Universe? Anything with a sufficiently negative equation of state will do. Consequently, there are a vast number of possible mechanisms that fall under the umbrella term ‘dark energy’ (DE). The equation of state can depend on the scale factor, \(a\), and, as such, is used to parametrise a wide range of these theories. The problem is that this doesn’t give us a natural scale to aim for in astronomical surveys, and so we are left without a clear idea of how accurate observations must be to actually constrain DE.

For simplicity, consider the commonly-used series expansion of the equation of state, \( w \approx w_0 + w_a (1 - a) \) [1, 2], an excellent approximation for \( z \lesssim 3 \) [3]. Given finite resources, what is the optimal precision to which we should measure \( w_0 \) and \( w_a \)? To tackle this question we need some theoretical input to identify the regions within the \((w_0, w_a)\) plane that would allow us to say something meaningful about DE.

The archetypal physical model of DE is quintessence [4–6]: a scalar field with a potential energy that dominates at late times. If one assumes the well-motivated case of a canonical kinetic energy term, different models consist solely of particular choices of potentials. What we then have is a model-dependent way of parametrising DE that can be directly related to physical principles. If the scalar field of quintessence is subject to the rules of effective field theory (EFT), for example, the potential is restricted to a particular functional form, with coupling constants of a particular amplitude (modulo the cosmological constant, \( \Lambda \), problem). Similar restrictions arise in specific models within particle physics and string theory, such as pseudo-Nambu-Goldstone Bosons (PNGBs) [7, 8] or axions (e.g. [9]), moduli of extra dimensional theories (e.g. [10–12]), and monodromy [13–15]. Models with generic and/or random quintessence potentials have been studied in, e.g., Refs. [3, 16, 17], although under different circumstances.

In this Letter we show that quintessence \emph{a priori} defines a natural scale and degeneracy direction on the \((w_0, w_a)\) plane when various physical guiding principles are taken into account. This is demonstrated graphically in Fig. 1, which is a new result of this work. A typical error ellipse for a future galaxy survey with figure of merit (FOM) \( \sim 600 \) is shown by the filled contours (1 and 2\( \sigma \) regions), and 95\% CL regions for the physical quintessence priors are shown by the unfilled contours. The way that these two areas overlap allows us to quantify the information that can actually be gained about DE by undertaking a given survey. The rest of this Letter describes how we reach these theoretical priors, and suggests how to compare different surveys in light of them.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{quintessence_priors.png}
\caption{Quintessence priors in the \((w_0, w_a)\) plane (out to 95\% CL).} \label{fig:quintessence}
\end{figure}

\begin{equation}
3 \left( \frac{\ddot{a}}{a} \right)^2 = \rho_c \phi^4 - 4 \left[ 1 + a (1 + z_{eq}) \right] + \frac{1}{2} \phi^2 + A \mathcal{P}(\phi),
\end{equation}

\begin{equation}
-6 \left( \frac{\dot{a}}{a} \right)^2 = \rho_c \phi^4 - 2 \left[ 2 + a (1 + z_{eq}) \right] + \frac{1}{2} \phi^2 + A \mathcal{P}(\phi),
\end{equation}

\begin{equation}
\ddot{\phi} = -3 \phi \dot{a}/a - A \mathcal{P}_\phi, \nonumber
\end{equation}

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where \( \mathcal{P}(\phi) \) is the dimensionless functional form of the quintessence potential and \( A \) is its overall scale, \( V(\phi) = AM_{p}^{2}M_{A}^{2}\mathcal{P}(\phi) \). We will work in units of the reduced Planck mass (energy scale), \( M_{p} = 1/\sqrt{8\pi G} = 2.435 \times 10^{22} \text{eV} \), and the Hubble rate (time scale), \( M_{H} = 100 \text{ km s}^{-1} \text{Mpc}^{-1} = 2.13 \times 10^{-33} \text{eV} \), so that the dimensionless evolution variables are \( \phi = \phi / M_{p} \) and \( \tau = M_{H} t \). In the above, we have used the redshift of matter-radiation equality, \( \xi_{eq} \) (which we later marginalise over), and the CMB temperature today \( (T_{CMB} = 2.725 \text{ K}) \) to fix the relative matter and radiation densities, \( \rho_{r,0} = 1.681 \times 10^{43} \text{cm}^{-3} \), where the leading numerical factor accounts for photons and three generations of neutrinos with negligible mass.

The DE equation of state is \( w(a) = P_{\phi}/\rho_{\phi} \), which we parametrise using a series expansion, \( w(a) \approx w_{0} + (1-a)w_{a} \). The coefficients can be evaluated directly at \( a = 1 \):

\[
w_{0} = \frac{\phi^{2}}{\phi^{2} + 2A\mathcal{P}(\phi)\phi^{2} + \mathcal{P}(\phi)\phi^{2}H^2 + \mathcal{P}(\phi)\phi H\dot{\phi}}.
\]

Alternatively, one could find \( w_{0} \) and \( w_{a} \) by fitting to \( w(a) \), but over the range of redshifts where such an expansion is valid, the series expansion agrees well [3]. In our units, the fractional density in a given component is \( \Delta \xi (a) = \rho_{X}(a)/3H^{2}(a) \). Where relevant, we include the cosmological constant (c.c.) within \( V(\phi) \) and hence \( w \).

We proceed by Monte Carlo sampling (a) various random functional forms for the potential, (b) the parameters of these functional forms, and (c) the initial conditions of the field. The resulting cosmologies are subjected to loose observations to ensure broad consistency with the real Universe.

**Functional forms** — We consider a number of general quintessence potentials with functional forms \( \mathcal{P}(\phi) = c_{1}\xi c_{2} + f(\phi) + \sum_{n=0}^{n_{\text{max}}} c_{n}b_{n}(\phi) \), where \( c_{n} \) is a deterministic constant, \( \xi \) is a random variable, \( b_{n}(\phi) \) is a basis function and \( f(\phi) \) is a leading contribution to the potential [18]. The term \( c_{1}\xi c_{2} \) takes account of the c.c., with \( c_{1} = 0.1 \) switching it off/on. The random coefficients are drawn from a unit Gaussian distribution, \( \xi \equiv \xi \in \mathcal{N}(0,1) \). All potentials are truncated at finite order \( n_{\text{max}} \), taken with some distribution, while \( n_{\text{min}} \) is model-specific. In this paper, we consider the following types of potential (summarised in Table I):

**Kac/Weyl:** Simple random polynomial functions [19]. These will serve as baseline random potentials, but have no physical motivation.

**Monomial:** An integer power law, with only a leading order part, \( f(\phi) = \phi^{c} \). These are included as simple examples, and in some cases yield analytic results. Although possible physical motivations include possible relation to chaotic inflation [20], or as large-field limits of certain monodromy models, our chief reason for including these potentials is simplicity.

**EFT:** Consists of a leading ‘classical contribution’ [21], \( f(\phi) = \phi^{4}\xi^{2} + \xi^{2} + \phi^{2} \), plus a random polynomial of ‘quantum corrections’ expanded in an energy scale parameter, \( \xi_{\phi} \). To allow quintessence-like masses and energy densities, one requires \( |\phi| > 1 \), and therefore the EFT must be controlled by a super-Planckian shift symmetry, \( F > M_{p} \) [22]. For $\xi_{\phi} = M_{p}/F < 1$, this fixes \( c_{n} = \xi_{\phi}^{n} \). In order to have the expansion begin at some leading order beyond the classical contribution, \( n_{\text{min}} = n_{\pi F} > 4 \). The number of quantum correction terms is \( n_{\phi} = n_{\text{max}} - n_{\pi F} + 1 \).

**Axion/PNGB:** A sum of cosines, with the leading term fixed positive with a minimum at zero, and higher-order non-perturbative corrections suppressed by \( \epsilon_{\pi F} < 1 \). We choose \( f(\phi) = 0 \) such that the leading term contributes no c.c. in the vacuum, as is conventional for axions. As with EFT, the shift symmetry is controlled by the scale \( F > M_{p} \), giving \( \epsilon_{\pi F} < 1 \).

**Modulus:** Moduli of extra dimensional theories generically acquire exponential potentials when stabilised and canonically normalised (e.g. Ref. [23]). There can be leading positive exponentials, with higher-order negative exponentials suppressed by the compactification scale, \( \epsilon_{s} = (\tilde{M})^{-2} \), where \( l < 10^{-6} \text{ m} \) is a length scale and \( M < M_{p} \) a mass scale. This gives \( f(\phi) = 0 \) for each of our models is given in Table I. For the Kac and Weyl polynomials, the natural range of field values runs from Planck scale, \( \phi \in [-1,1] \). Outside this range, the potentials become steep or tend to have zeros [19], and are unsuitable quintessence candidates. For the monomial potentials, super-Planckian field excursions are necessary to have large DE energy density, and symmetry/positivity allow us to restrict \( \phi > 0 \), so we take \( \phi \in [0,4] \). For EFT controlled by a super-Planckian shift symmetry, the natural range is \( [-\epsilon_{\pi F}^{-1}, \epsilon_{\pi F}^{-1}] \); for PNGB/axions it is \( [-\pi, \pi] \); and for moduli it is \( [-1,1] \), emerging from \( \epsilon_{D} < 1 \) and \( 1/l < M < M_{p} \) for sub-Planckian compactification.

We consider two types of initial condition on the

| Model   | $b_{n}(\phi)$ | $c_{n}$ | $n_{\text{min}}$ | $f(\phi)$ | $\phi$ |
|---------|----------------|---------|------------------|-------------|--------|
| Kac     | $\phi^{c}$     | 1       | 1                | 0           | $[-1,1]$ |
| Weyl    | $\phi^{c}$     | 1       | 1                | 0           | $[-1,1]$ |
| Mono.   | 0              | -       | -                | $\phi^{N}$  | [0,4]  |
| EFT     | $\phi^{n}$     | $\epsilon_{F}^{n}$ | $\rho_{E}$ | $\xi_{\phi}^{2} + \phi^{2}$ | $[-\epsilon_{F}^{-1}, \epsilon_{F}^{-1}]$ |
| Axion   | $\cos(n\epsilon_{F})\phi$ | $\epsilon_{\pi F}^{n-1}$ | 2 | 1 + $\cos \epsilon_{F}\phi$ | $[-\frac{\pi}{\epsilon_{F}}, \frac{\pi}{\epsilon_{F}}]$ |
| Modulus | $\epsilon^{n}(p_{\phi}-n)\phi$ | $\epsilon_{D}^{n}$ | 0 | 0 | $[-1,1]$ |

**TABLE I.** Model specifications for the functional form \( \mathcal{P}(\phi) \).

$\epsilon_{F} = M_{p}/F < 1 \), this fixes $c_{n} = \xi_{\phi}^{n}$. In order to have the expansion begin at some leading order beyond the classical contribution, $n_{\text{min}} = n_{\pi F} > 4 \). The number of quantum correction terms is $n_{\phi} = n_{\text{max}} - n_{\pi F} + 1 \).

**Axion/PNGB:** A sum of cosines, with the leading term fixed positive with a minimum at zero, and higher-order non-perturbative corrections suppressed by $\epsilon_{\pi F} < 1 \). We choose $f(\phi) = 0$ such that the leading term contributes no c.c. in the vacuum, as is conventional for axions. As with EFT, the shift symmetry is controlled by the scale $F > M_{p} \), giving $\epsilon_{\pi F} < 1 \).

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FIG. 2. The (log-scaled) density of prior samples in the \((w_0,w_a)\) plane before (top) and after (middle) the observational cuts, for models with a cosmological constant and \(\phi_i = 0\). Models with \(\Lambda = 0\) are shown at the bottom, also after observational cuts. Sharp edges in many of the distributions emerge from the fact that \(|w(z)| \leq 1\) when \(\Omega_{\text{DE}} > 0\) in quintessence models [3].

| Parameter | Model | Dist. |
|-----------|-------|-------|
| \(\log_{10} A\) | All | \(U(-1,1)\) |
| \(N\) | Monomial | \(U_{2}(1.7)\) |
| \(n_{\text{max}}\) | Kac, Weyl, Ax., Mod. | \(U_{2}(10,20)\) |
| \(n_{\Omega, PE}\) | EFT | \(U_{2}(5,10)\) |
| \(\log_{10} \varepsilon_{\text{NP, D}}\) | EFT, Ax., Mod. | \(U(-3, -1)\) |
| \(\alpha\) | Modulus | \(U_{2}(1,5)\) |

TABLE II. Parameter distributions for the models in Table I. U is the uniform distribution, and subscript ‘Z’ indicates that the distribution is over the integers.

field velocity: static, \(\phi_i = 0\), and tracking, \(\phi_i \sim U[-\sqrt{2V(\phi)}, + \sqrt{2V(\phi)}]\), the latter giving \(w(a_i) \leq \frac{1}{3}\) and tracking radiation when this is saturated. We observed little difference in the resulting \((w_0,w_a)\) priors between the static and tracking cases: Hubble friction damps the field motion at high-\(z\). We have also tested our models with log-flat priors on the initial conditions for the field and field velocity, and found that this also had little effect on the \((w_0,w_a)\) priors.

Observational cuts — Models with excessive amounts of early DE are discarded by requiring \(\Omega_{\text{DE}}(z_{SS} \approx 1090) < 0.042\) [24], we require that the present Hubble rate, \(h = H_0/M_H\), is between 0.6 < \(h < 0.8\), and the fractional DE density is between 0.6 < \(\Omega_{\text{DE}} < 0.8\). We also put a weak prior on the present-day matter density by sampling \(z_{eq} \sim U(2000,4000)\). (The radiation density is extremely well-measured, so \(\Omega_{\text{CMB}}\) is fixed; we do not vary the neutrino density.) Many of our random potentials can become negative

and lead the universe to collapse, so we reject any cosmologies that do not reach \(a = 1\) as well.

These cuts are substantially broader than current observational constraints allow; their purpose is only to act as priors to ensure that we are considering somewhat realistic cosmologies. Typically 1 – 10\% of the samples remain after the various cuts are applied, so we draw \(\sim 10^6\) samples for each model to ensure sufficient statistics. Fig. 2 shows the Monte Carlo prior samples before and after cuts for each model.

Results — Not all combinations of cosmological parameters correspond to physically reasonable cosmological models. There is a strong correlation between the equation of state values at different redshifts in quintessence models [3], which is observed as restrictive joint priors on \((w_0,w_a)\) once our broad priors on other cosmological parameters are imposed, as shown in Figs. 1 and 2. The more typical assumption of independent uniform priors on all of \(\{H_0, \Omega_{\text{M}}h^2, w_0, w_a\}\) is not valid for generic physical quintessence models.

The marginal 95\% confidence intervals for the priors, after cuts and with \(\Lambda = 0\), were already shown in Fig. 1. The quintessence models define a narrow strip in the \((w_0,w_a)\) plane, with which certain fiducial models (such as the reference point \((-0.95,0)\) [25]) are inconsistent. Most of the allowed prior region has \(w_a < 0\) because acceptable potentials are shallow and typically do not have runaway behaviour as long as the universe is expanding [26]. In such a potential, sub-dominance of DE at early times and Hubble friction send \(w \rightarrow -1\) at high-\(z\), while friction lessens at low-\(z\), allowing \(w\) to become larger. Thus the prior lies near (but not exactly on) the ‘thawing’ region of Ref. [27]. The random quintessence models studied by Ref. [3] constructed using priors in the flow equations were found to be almost entirely ‘freezing’. This shows the difference between priors based on random mod-
The likelihoods are fixed at given fiducial values of $(w_0, w_a)$, while the black lines are for $\Delta S$ marginalised over all fiducial values, $\langle \Delta S \rangle$. The thick blue line shows $\Delta S$ for a uniform prior over $(w_0, w_a)$-space, and does not depend on the fiducial point.

**Fig. 3.** Relative entropy as function of figure of merit for a typical future galaxy survey (solid lines), and for the same but with error ellipse rotated by $90^\circ$ (dashed lines). The red and grey lines are for $\Delta S$ defined as $\langle Q(x,y)\Delta S(x,y)dx\,dy \rangle / \langle Q(x,y)dx\,dy \rangle$, where $(x,y)$ run over all fiducial values of $(w_0, w_a)$. We rescale the covariance matrix by the figure of merit, $\text{FOM} = 1/\sqrt{\det S_{00}^{-1}}$, which (loosely) increases with the increasing accuracy of distance measurements from a survey. We also consider the possibility of having an error ellipse that is orthogonal to that of a galaxy survey, which could be achieved in practise by (e.g.) cosmic shear [33] or redshift drift measurements [34]. It is also possible to partially rotate the error ellipse of a redshift survey by making an appropriate choice of target redshift and binning.

In Fig. 3 we show $\Delta S$ as a function of FOM for physical quintessence as a whole (i.e. combining, with equal weights, the normalised prior distributions for all but the unmotivated Kac, Weyl, and Monomial models), and compare this to the $\Delta S$ that would be obtained if uniform priors on $(w_0, w_a)$ were assumed. The value of $\Delta S$ is larger for the uniform prior – since quintessence disfavours large regions of the $(w_0, w_a)$ plane a priori, there is less information to be gained from a given survey. The orientation of the error ellipse, though unimportant in the uniform case, has a substantial effect for the quintessence prior; with the orthogonal ellipse, one always finds a greater information gain. For example, a survey with an orthogonal ellipse and FOM $\sim 100$ offers an equivalent $\Delta S$ to a standard galaxy redshift survey with a much higher FOM of $\sim 250$. This is due to the near-alignment of the quintessence prior with the typical degeneracy direction of the galaxy survey error ellipse (Fig. 1); an orthogonal ellipse cuts through the prior more effectively. The alignment is not exact, however; rotating $20^\circ$ towards the $w_a = 0$ axis matches the ellipse to the prior direction, minimising $\Delta S$ to 0.95 for a survey with FOM of 600 (down from 1.2 for the unrotated case). The same rotation in the opposite direction increases $\Delta S$ to 1.38.

With quintessence priors, one can also observe features in $\Delta S$ as a function of FOM as the observational error shrinks inside the prior region about a fixed fiducial point (c.f. the results for the point $(-0.95, -0.07)$ in Fig. 3). The function marginalised over all fiducial points, $\langle \Delta S \rangle$, does not show such a feature however; this is because the prior is still dominated by the $\Lambda$-like peak at $(w_0, w_a) = (-1.0)$. The implication is that if one suspects that DE is described by quintessence with a particular $(w_0, w_a) \neq (-1.0)$, then there is potentially a scale of FOM to aim for, but in the general case where we are ignorant about DE there is no preferred scale.

In this Letter we considered random, physically-motivated models of quintessence, which were found to impose a specific structure on the DE equation of state. The resulting prior on $(w_0, w_a)$ is only weakly sensitive to the details of how the models are constructed, and is therefore suitable as a guide to regions of observational interest. The theory priors cluster around $\Lambda$-like behaviour and therefore give no “target” FOM. Nonetheless our results quantify how surveys at fixed FOM are not equivalent in the amount of information on DE they bring to bear. This should be accounted for in survey design.
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[1] M. Chevallier and D. Polarski, Int. J. Mod. Phys. D10, 213 (2001), arXiv:gr-qc/0009008 [gr-qc].
[2] E. V. Linder, Phys. Rev. Lett. 90, 091301 (2003), arXiv:astro-ph/0208512 [astro-ph].
[3] D. Huterer and H. V. Peiris, Phys. Rev. D75, 083503 (2007), arXiv:astro-ph/0610427 [astro-ph].
[4] C. Wetterich, Nuclear Physics B 302, 608 (1988).
[5] P. J. E. Peebles and B. Ratra, Astrophys. J. 325, 1582 (1988).
[6] R. Caldwell, R. Dave, and P. J. Steinhardt, Phys. Rev. Lett. 80, 1582 (1998), arXiv:astro-ph/9708089 [astro-ph].
[7] C. Hill and G. Ross, Nuclear Physics B 311, 253 (1998).
[8] J. A. Frieman, C. T. Hill, A. Stebbins, and I. Waga, Phys. Rev. Lett. 75, 2077 (1995).
[9] N. Kaloper and L. Sorbo, Phys. Rev. D79, 043528+ (2009).
[10] D. J. E. Marsh, Phys. Rev. D83, 123526+ (2011), arXiv:1102.4851.
[11] M. Cicoli, F. G. Pedro, and G. Tasinato, JCAP 1007, 044 (2012), arXiv:1203.6655 [hep-th].
[12] D. J. E. Marsh, E. R. M. Tarrant, E. J. Copeland, and P. G. Ferreira, Phys. Rev. D86, 023508+ (2012).
[13] E. Silverstein and A. Westphal, Phys. Rev. D78, 106003+ (2008).
[14] L. McAllister, E. Silverstein, and A. Westphal, Phys. Rev. D82, 046003 (2010), arXiv:0808.0706 [hep-th].
[15] S. Panda, Y. Sumitomo, and S. P. Trivedi, Phys. Rev. D83, 083506 (2011), arXiv:1011.5877 [hep-th].
[16] S. Chongchitnan and G. Efstathiou, Phys. Rev. D76, 043508 (2007), arXiv:0705.1955 [astro-ph].
[17] E. V. Linder, Gen. Rel. Grav. 40, 329 (2008), arXiv:0704.2064 [astro-ph].
[18] All models here could be considered with $f(\phi) = 0$ if $c_n$ are individually specified, but this notation is more compact. Allowing $f(\phi) \neq 0$ and product basis functions can model more general potentials within our scheme, such as monodromy and Albrecht-Skordis/SUGRA [35].
[19] http://terrytao.wordpress.com/tag/kac-polynomials/.
[20] A. D. Linde, Phys.Lett. B129, 177 (1983).
[21] Note that the classical contribution is also suppressed by powers of $e_F$; this is formally a tuning, and reflects the unknown solution to the ‘old’ c.c. problem [36].
[22] This may be problematic to realise in UV models including quantum gravity [37].
[23] P. G. Freund and M. A. Rubin, Phys.Lett. B97, 233 (1980).
[24] S. Joudaki, Phys. Rev. D87, 083523 (2013), arXiv:1202.0005 [astro-ph.CO].
[25] L. Amendola et al. (Euclid Theory Working Group), Living Rev.Rel. 16, 6 (2013), arXiv:1206.1225 [astro-ph.CO].
[26] The moduli models, having exponential potentials, can more easily support tracking solutions [38] and positive runaways, and so have more prior weight with $w_\alpha > 0$.
[27] R. Caldwell and E. V. Linder, Phys. Rev. Lett. 95, 141301 (2005), arXiv:astro-ph/0505494 [astro-ph].
[28] R. Kallosh, A. Linde, S. Prokushkin, and M. Shmakova, Phys. Rev. D66, 123503+ (2002).
[29] R. Kallosh, J. Kratochvil, A. Linde, E. V. Linder, and M. Shmakova, JCAP 2003, 015+ (2003).
[30] D. H. Lyth, Phys. Rev. Lett. 78, 1861 (1997).
[31] S. Kullback and R. A. Leibler, Ann. Math. Statist. 22, 79 (1951).
[32] For our illustration we approximate the Fisher matrix as being constant as a function of fiducial point, but this would not be the case for a real survey.
[33] E. V. Linder, Phys. Rev. D68, 083503 (2003), arXiv:astro-ph/0212301 [astro-ph].
[34] A. G. Kim, E. V. Linder, J. Edelstein, and D. Erskine, (2014), arXiv:1402.6614 [astro-ph].
[35] A. Albrecht and C. Skordis, Phys. Rev. Lett. 93, 023503 (1998), arXiv:astro-ph/9711102 [astro-ph].