Black Holes, Entropy Bound and Causality Violation

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The gravity/gauge theory duality has provided us a way of studying QCD at short distances from straightforward calculations in classical general relativity. Among numerous results obtained so far, one of the most striking is the universality of the ratio of the shear viscosity to the entropy density. For all gauge theories with Einstein gravity dual, this ratio is $\eta/s = 1/4\pi$. However, in general higher-curvature gravity theories, including two concrete models under discussion - the Gauss-Bonnet gravity and the (Riemann)$^2$ gravity - the ratio $\eta/s$ can be smaller than $1/4\pi$ (thus violating the conjecture bound), equal to $1/4\pi$ or even larger than $1/4\pi$. As we probe spacetime at shorter distances, there arises an internal inconsistency in the theory, such as a violation of microcausality, which is correlated with a classical limit on black hole entropy.

Keywords: AdS/CFT, higher-derivative black holes, entropy bound, causality violation

1. Introduction

According to the famous dictum of gravity/gauge theory duality or Maldacena’s Anti de Sitter (AdS) conformal field theory (CFT) correspondence, an AdS space with a black hole is dual to a field theory at a finite temperature. AdS/CFT has been an excellent tool to study not only strongly coupled gauge theories at a large ’t Hooft coupling limit but also to study hydrodynamic properties of a certain class of boundary CFTs at a finite temperature. One of the highly celebrated results over the last decade is that, for a large class of four-dimensional CFTs, the ratio of the shear viscosity $\eta$ to the entropy density $s$ is (in units $\hbar = k_B = 1$) given by

$$\frac{\eta}{s} = \frac{1}{4\pi}. \quad (1)$$

This result has been known to hold for all gauge theories with Einstein gravity dual. This is due to the fact that in pure Einstein gravity all black holes satisfy the Bekenstein-Hawking entropy law or the area formula of black hole entropy

$$S = \frac{k_B c^3 A}{\hbar 4G_N}, \quad (2)$$

where $\hbar$ is the Planck constant, $k_B$ is the Boltzmann constant, $A$ is the area of the (black hole) horizon corresponding to the surface at $r = r_+$ and $G_N$ is the Newton’s constant $G_N$. In general gravity theories with higher derivative or higher-curvature
corrections, however, the ratio \( \eta/s \) can be different from \( 1/4\pi \). In the same context, a violation of causality might occur in the boundary CFT when \( \eta/s \) is too low (\( \ll 1/4\pi \)). Here we shall argue that any such a violation of micro-causality can be related to a violation of black hole entropy bounds or a violation of certain laws of black hole thermodynamics.

To show that this might be the case, and also to establish better contact with QCD via AdS holography, in a holographic context (see \(^3\) and references therein), one might like to consider the following gravitational action

\[
I_g = \frac{1}{16\pi G_N} \int d^d x \sqrt{-g} \left[ R - 2\Lambda + \alpha' L^2 \left( a R^2 + b R_{\mu\nu} R^{\mu\nu} + c R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho} \right) \right],
\]

(3)

where \( \alpha' \) is a dimensionless coupling and \( \Lambda \equiv -(d-1)(d-2)/2L^2 \) is a bulk cosmological term. In fact, the Regge slope or the coefficient \( \alpha' \) appearing in the effective action (3) can be a complicated function of some microscopic parameters of the quantum gravity theory, which may even depend on the details of compactification and on dilaton couplings. Nevertheless, in the simplest scenario to be considered in this paper, one would assume that \( \alpha' = \ell_s^2 / L^2 \lesssim \mathcal{O}(1/10) \). Here one might also note that, especially, in an AdS\(_5\) space, the coefficients of \( R^2 \) terms in a dual supergravity action are determined by the central charges of a CFT in four dimensions.

In most of our discussions we will focus to the gravity sector in a five-dimensional AdS space, for which we have from the AdS/CFT correspondence, \( G_N \equiv (\pi L^3/4N_c^2) \) and \( L = (4\pi g_s N_c)^{1/4} \ell_s \), where \( \ell_s \) is the string scale and \( g_s \) the string coupling and \( N_c \) is the number of color charges or rank of the gauge group. In the dual supergravity description, a small \( \alpha' \) corresponds to the strong coupling limit, i.e., \( \lambda \equiv g_s^2 N_c \gg 1 \), since \( \alpha' \sim 1/\sqrt{\lambda} \). The Gauss-Bonnet term is obtained by setting \( a = c = 1 \) and \( b = -4 \) in (3), for which there would be no need to treat \( \alpha' \) as small, at least, for the purpose of obtaining exact (black hole) solutions.\(^4,5\)

Recently, Kats and Petrov\(^6\) and Brigante et al.\(^7\) have shown that, for a class of CFTs in flat space with Gauss-Bonnet gravity dual, the ratio \( \eta/s \), which reads

\[
\frac{\eta}{s} = \frac{1}{4\pi} \left( 1 - \frac{2(d-1)\lambda_{GB}}{(d-3)} \right),
\]

(4)

where \( \lambda_{GB} \equiv (d-3)(d-4)\alpha' \), can be smaller than \( 1/4\pi \) for \( \lambda_{GB} > 0 \) and \( d \geq 5 \). Based on the bulk causal structure of an AdS\(_5\) black brane solution, one may actually find an even more stronger bound for \( \lambda_{GB} \), that is,

\[
\lambda_{GB} < \frac{9}{100} \quad \text{or, equivalently,} \quad \frac{\eta}{s} > \frac{16}{25} \left( \frac{1}{4\pi} \right),
\]

(5)

which otherwise violates a microcausality in the dual CFT defined on a flat space. Here we show that the critical value of \( \lambda_{GB} \) beyond which the theory becomes inconsistent is related to the entropy bound for a large class of AdS black holes. In fact, in the holographic context, AdS black hole solutions with spherical and hyperbolic event horizons allow much wider possibilities for \( \eta/s \).\(^8\)
It is quite plausible that in the presence of curvature terms like $R^n$ with $n \geq 3$, which generate six and higher derivatives in the metric, or the (Weyl)$^4$ terms arising as $\alpha'^3 R^4$ corrections to low energy effective action of type IIB string theory, the ratio $\eta/s$ takes a value slightly larger than $1/4\pi$ (see\textsuperscript{10} for a review). However, in any consistent gravity models, interaction terms like $\lambda L^4 R_{abcd} R^{abcd}$, where $\lambda \sim \ell_p^4/L^4 \ll \alpha'$, must be suppressed in a sensible derivative expansion, implying that such corrections could arise only as subleading or next-to-subleading terms. Here we show that when an underlying theory admits a large violation of the conjectured KSS bound $\eta/s \geq 1/4\pi$ or the ratio $\eta/s$ becomes too small, then certain laws of thermodynamics can be violated in the same limit. This effect can be seen also in terms of a violation of causality in the boundary field theory. The bound $\eta/s \geq 1/4\pi$ may be restored only at weak couplings or in the limit of large $N_c$, since in such cases the shear viscosity can grow faster as compared to the entropy density.

2. Gauss-Bonnet Gravity and Causality Violation

For an AdS GB black hole, the entropy and Hawking temperature are given by\textsuperscript{5}

$$S = \frac{A}{4G_N} \left(1 + \frac{2(d-2)k\lambda_{GB}}{(d-4)x^2}\right), \quad T = \frac{(d-1)x^4 + k(d-3)x^2 + (d-5)k^2\lambda_{GB}}{4\pi Lx(x^2 + 2k\lambda_{GB})}$$

(6)

(in units $c = \hbar = k_B = 1$) where $x \equiv r_+/L$, $A \equiv V_{d-2}r_+^{d-2}$, with $V_{d-2}$ being the unit volume of the base manifold or the hypersurface $\mathcal{M}$. The entropy of a GB black hole depends on the curvature constant $k$, whose value determines the geometry of the event horizon $\mathcal{M} = S^{d-2}, R^{d-2}$ and $H^{d-2}$, respectively, for $k = +1, k = 0$ and $k = -1$. Especially, at the $k = -1$ extremal state with zero temperature,

$$S|_{T\to 0} = \frac{V_3}{G_N} \frac{L^3}{2^{7/2}}(1 - 12\lambda_{GB}), \quad E|_{T\to 0} = 0.$$

(7)

Thus, beyond a critical coupling $\lambda_{GB} > \lambda_{crit}$, the entropy $S$ becomes negative, which indicates a violation of cosmic censorship or the second law of the thermodynamics. In the AdS$_5$ case, $\lambda_{crit} = 1/12$. This critical value of $\lambda_{GB}$ above which the theory is inconsistent nearly coincides with the bound $\lambda_{GB} < 9/100$ required for a consistent formulation of a class of CFTs in a flat space with Gauss-Bonnet gravity dual.

To be more specific, let us consider small metric fluctuations $\phi = h^2$ around an AdS GB black hole metric.\textsuperscript{4}

\textsuperscript{4}The GB term is topological in $d = 4$, especially, with a constant coupling, $\alpha' L^2 = \text{const}$, so we take $d \geq 5$.

\textsuperscript{5}There can be three different modes of metric fluctuations, each of which may be decoupled from others: the scalar, vector and tensor fluctuations. For simplicity, and especially for the purpose of calculating shear viscosity, one may study only the tensor fluctuations $h_{xy}$, using $\phi$ to denote this perturbation with one index raised $\phi = h_X^Y$ and writing $\phi$ in the basis $\phi(t, x_3, z) = \phi(z)e^{-iwt+iqx_3}$.
where $k = 0, \pm 1$ and $N_* \equiv a = [(1 + \sqrt{1 - 4\lambda_{GB}})/2]^{1/2}$ and $L$ is the curvature of AdS$_5$ space. The scalar metric fluctuation (along the $x_3$ direction)

$$\phi(t, x_3, z) \equiv \int \frac{dw dq}{(2\pi)^3} \phi(z; \hat{k}) e^{-i\omega t + iqx_3}, \quad \phi(z; -\hat{k}) = \phi^*(z, \hat{k}),$$

(9)

(where $\hat{k} = (w, 0, 0, q)$) satisfies the following (linearized) equation of motion

$$K \partial^2_k \phi + \partial_z K \partial_z \phi + K_2 \phi = 0.$$  

(10)

This structure is not affected by Maxwell type charges but it may well be affected by dilatonic scalar charges and rotation parameters. In the simplest scenario one may neglect both the scalar charge and rotation parameters. In pure GB gravity defined on an AdS$_5$ spacetime, we find $K = z^2f(z - \lambda_{GB}\partial_z f), \ K_2 = (z^2\hat{w}^2/N_*^2 f)(z - \lambda_{GB}\partial_z f) - z(1 - \lambda_{GB}\partial_z^2 f) (\hat{q}^2 + 2\hat{k})$ and

$$f(z) = \frac{\hat{k}}{2\lambda_{GB}} \left[1 \pm \sqrt{1 - 4\lambda_{GB} + \frac{4\lambda_{GB}}{z^4} \left(1 + \frac{k}{x^2} + \frac{\lambda_{GB}k^2}{x^4}\right)}\right],$$

(11)

where $z \equiv r/r_+, \ x \equiv r_+/L, \ \hat{k} = k/x^2, \ \hat{w} \equiv wL/x$ and $\hat{q} \equiv qL/x$. Eq. (10) may be solved with the following incoming boundary conditions at the horizon

$$\phi(z; \hat{k}) = a_{in}(\hat{k})\phi_{in}(z; \hat{k}) + a_{out}(\hat{k})\phi_{out}(z; \hat{k}), \quad a_{out} \equiv 0, \quad a_{in} \equiv J(\hat{k}),$$

(12)

where $J(\hat{k})$ is an infinitesimal boundary source for the fluctuating field $\phi$. To the leading order in $\hat{w}$, and in the limit $\hat{q} \rightarrow 0$, the solution is given by

$$\phi(z; \hat{k}) = J(\hat{k}) \left[1 + \frac{i\hat{w}}{4N_*} a^2 \sqrt{1 - 4\lambda_{GB}} \left(\frac{1}{z^4} - \frac{4\lambda_{GB}\hat{k}}{3(1 + \sqrt{1 - 4\lambda_{GB}})} \frac{1}{z^6} + \mathcal{O}(z^{-8})\right)\right].$$

(13)

By identifying $\phi \sim T^3$, where $T^3$ is a component of the energy-momentum tensor on the field theory side, we may obtain the retarded two-point Green’s function. Indeed, GB black holes admit a stable five-dimensional anti-de Sitter vacuum only if $\lambda_{GB} < 1/4$. This result is independent of the types of interactions the Gauss-Bonnet term can have with matter fields (including gauge and scalar fields).

From the above result, we can easily see that the curvature on a boundary may not affect the shear viscosity

$$\eta = \frac{1}{16\pi G_N} \left(\frac{\hat{r}_s^2}{L^5}\right)(1 - 4\lambda_{GB})$$

(14)

obtained using the Kubo formula

$$\eta = \lim_{w \rightarrow 0} \frac{1}{2iw} \left[G^A_{12,12}(w, 0) - G^R_{12,12}(w, 0)\right] \equiv \lim_{w \rightarrow 0} \frac{1}{w} \text{Im} G^R_{12,12}(w, 0).$$

(15)
As is evident, this formula relates $\eta$ to zero spatial momentum ($q = 0$), low frequency limit of the retarded two-point Green’s function

$$G_{12,12}^R(w, 0) = -i \int d^4x e^{iwt} \theta(t) \langle [T_{12}(t, \vec{x}), T_{12}(0, 0)] \rangle,$$  

which satisfies $G^A(w, \vec{q}) \equiv G^R(w, \vec{q})^*$. The ratio $\eta/s$ is now modified as

$$\frac{\eta}{s} = \frac{1}{4\pi} \frac{(1 - 4\lambda_{GB})}{(1 + 6k\lambda_{GB})}.$$  

The standard result in Einstein gravity, i.e. $\eta/s = 1/4\pi$, is obtained only at a fixed $\tilde{k}$, i.e. when $k = -1$ and $x = r_+/L = \sqrt{3}/2$. The minimum of entropy density actually occurs at $x = \sqrt{1/2}$, implying that

$$s = \frac{1}{G_N} \frac{1}{27/2} (1 - 12\lambda_{GB}), \quad \eta = \frac{1}{4\pi G_N} \frac{1}{27/2} (1 - 4\lambda_{GB}).$$  

At this extremal state the shear viscosity is given by

$$\eta = \frac{1}{4\pi G_N} \frac{1}{27/2} (1 - 4\lambda_{GB}).$$  

The positivity of extremal entropy density implies that $\lambda_{GB} \leq 1/12$ and hence $\eta/s \geq 1/6\pi$ for the $k = 0$ and $\eta/s < 5/12\pi$ for the $k = -1$ solutions. It is quite remarkable that the lower bound found above, i.e. $\eta/s \approx 0.66/4\pi$, is similar to a lower value of $\eta/s$ found at some relativistic heavy ion collision experiments.\(^{11}\) The bound $\eta/s > 0.09$ was found in\(^{7}\) by considering a Ricci flat horizon or assuming that the boundary theory is defined on a flat space. Here we note that, in the $k = +1$ case, the lower value of $\eta/s$ can be slightly stronger than that in flat space. Especially, in a flat space, a non-violation of causality requires the square of local speed of graviton on a constant $z$-hypersurface to be less than unity, i.e.

$$c_g^2 = 1 - \left( \frac{5}{2} - \frac{2}{1 - 4\lambda_{GB}} + \frac{1}{2\sqrt{1 - 4\lambda_{GB}}} \right) \frac{1}{z^4} + O(z^{-8}) < 1$$  

or $\lambda_{GB} < 0.09$. This limit can be slightly altered by the Maxwell term $F_{\mu \nu} F^{\mu \nu}$ and also by $F^4$ type corrections to Maxwell fields\(^{12,14}\). Specifically, in the presence of a Maxwell charge $q$, the ratio $\eta/s$ is given by $\eta/s = \frac{1}{4\pi} (1 - 4\lambda_{GB} + 2Q\lambda_{GB})$, where $Q \equiv \frac{q^2 L^2}{r_+^3}$. In the standard case of an non-extremal black hole one may be required to satisfy two separate conditions $Q < 2$ and $\lambda_{GB} < 1/24$, leading to the bound $\eta/s \geq 0.09 \times \frac{1}{z^4} + \frac{5}{z^8}$ as first reported in\(^{12}\).

For the consistency of a dual Gauss-Bonnet gravity action, and also from a viewpoint of classical stability of GB black holes, one is required to take

$$\lambda_{GB}^{d=6} \lesssim 0.1380 \quad \text{and} \quad \lambda_{GB}^{d=7} \lesssim 0.1905.$$  

\(^{11}\)When matter fields (including gauge field and scalar fields) are coupled to higher-curvature terms, and as long as Einstein’s field equations contains at most second derivatives of the metric component $g_{xy}$ as a function of $t, z, x_i$, which is the case only with the Gauss-Bonnet gravity, then the perturbation of the transverse gravitons can get decoupled from the fluctuations of matter fields.
respectively, in the AdS$_6$ and the AdS$_7$ cases.

Next let us briefly discuss the results in the (Riemann)$^2$ gravity, which is obtained by setting $a = 0 = b$ and $d = 5$ in Eq. (3). In this theory, we find

$$\frac{\eta}{s} = \frac{1}{4\pi} \left( 1 - \frac{4\lambda_{\text{Riem}}}{1 + 4\lambda_{\text{Riem}}} \right) \approx \frac{1}{4\pi} \left( 1 - 8\lambda_{\text{Riem}} + \cdots \right) = \frac{1}{4\pi} \left( 1 - \frac{1}{N_c} + \cdots \right). \quad (22)$$

This result shows that the KSS bound $\eta/s \geq 1/(4\pi)$ can be violated for a finite $N$, unless that the contribution to $\eta/s$ coming from next-to-leading order terms, such as (Weyl)$^4$ terms, are of a comparable magnitude to that of the $R^2$ terms. This scenario is, however, almost unlikely since the couplings associated with the cubic and higher powers in Riemann tensors, such as $\lambda L^4 R_{abcd}R^{abcd}$ with $\lambda \sim \ell_P^4/L^4 \ll \alpha'$, must be suppressed in a sensible derivative expansion.

Notice that the limit $\lambda_{\text{Riem}} < 1/8$, as implied by Eq. (22) and required for positivity of $\eta/s$, is the same as implied by the positivity of extremal entropy of a (Riemann)$^2$-corrected AdS black hole. This result is consistent with the following observation. The conformal anomaly of a four-dimensional CFT can be identified by considering the theory in a curved spacetime and writing

$$16\pi^2 \langle T_{\mu\nu} \rangle_{\text{CFT}} = -c_1 E_4 - c_2 W_4, \quad (23)$$

where $E_4 \equiv R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$ and $W_4 \equiv R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma} - 2R_{\mu\nu}R^{\mu\nu} + R^2/3$ are, respectively, the four-dimensional Euler density and the square of the Weyl curvature, and $c_1$, $c_2$ are the two central charges of a dual CFT. As explained by Hofman and Maldacena in$^{13}$ (see also$^{10}$), for the consistency of a low energy action with sensible derivative expansion, one may be required to consider CFTs for which

$$|1 - \frac{c_1}{c_2}| \lesssim O(1/10). \quad (24)$$

In fact, the gravity dual of a class of CFTs for which $\lambda_{\text{Riem}} \simeq \frac{1}{8} \left( 1 - \frac{c_2}{c_1} \right) \neq 0$, must be defined in a curved background, so the choice $k = 0$ may not be very physical, at least, in the presence of (Riemann)$^2$ type corrections.

Taking into account all three possibilities for the boundary topology that $k = 0$ or $k = \pm 1$, and demanding that $\lambda_{\text{Riem}} \simeq \frac{1}{8\pi^2} (c_2 - c_1) < 1$, we find

$$0 < \frac{\eta}{s} \leq \frac{3}{2} \left( \frac{1}{4\pi} \right). \quad (25)$$

It is not known yet whether either of these limits applies to nuclear matters at extreme densities and temperatures, or heavy ion collision experiments, but it would be interesting to know anything specific to the universality of the result $\eta/s \approx 1/4\pi$ through numerical hydrodynamic simulations of data from high energy experiments, including the relativistic heavy ion collision (RHIC) and large hadron collider.

$^4$We believe the result in$^7$ i.e. $\eta/s = \frac{1}{16\pi} \left( 1 - 4\lambda_{\text{Riem}} + \cdots \right)$ should be corrected by a factor of 2: the source of this difference is that in (Riemann)$^2$ gravity the entropy function is increased by a factor of $(1 + 4\lambda_{\text{Riem}})$, in the case of a Ricci-flat horizon, which was however not considered there.
3. Conclusion

Recent developments in gravity/gauge theory duality, supersymmetric field theories and black hole mechanics have shown that AdS black holes are excellent objects to study the properties of strongly coupled gauge theories. In this note we have studied the limits on black hole entropy and shear viscosity for the simplest class of higher curvature-corrected black hole solutions defined on AdS spaces. We also considered the causal problem on the boundary field theory to see what kind of constraints we can get on the ratio \( \frac{\eta}{s} \) (where \( \eta \) is the shear viscosity and \( s \) is the entropy density) so as to keep the theory phenomenologically viable and internally consistent.

It has been known for quite sometimes that the limit \( \lambda_{GB} < \frac{1}{4} \) on the Gauss-Bonnet coupling can be viewed as a classical limit of a consistent theory of quantum gravity.\(^{4,5}\) Recent results coming from the studies of viscosity bound violation in higher curvature gravity\(^7,8\) have shown that the bound on the shear viscosity of any fluid in terms of its entropy density may be saturated, i.e. \( \eta/s = \frac{1}{4\pi} \), by all gauge theories but only at large \('t\) Hooft coupling, as this limit correspond to the cases where all higher-order curvature contributions are absent. Nevertheless, this bound is naturally in immediate threat of being violated in the presence of generic higher derivative and higher-order curvature corrections to the Einstein-Hilbert action.

It is important to note that by tuning of the Gauss-Bonnet coupling or the generic higher derivative couplings, the ratio \( \frac{\eta}{s} \) can be adjusted to a small positive value. Causality violation might take place when \( \eta/s \) is too low, i.e. \( \eta/s \ll \frac{1}{4\pi} \).

We have shown that limits on curvature coupling can be imposed by demanding the positivity of extremal black hole entropy or by keeping boundary causality intact, or both. We have not found any obvious explicit bound on \( \lambda_{GB} \) from the thermodynamics of spherically symmetric AdS Gauss-Bonnet black holes. This could however arise as a consequence of causality violation of a boundary CFT. The critical value of \( \lambda_{GB} \) beyond which the theory becomes inconsistent is found to be related to the entropy bound for an AdS GB black hole with a hyperbolic or Euclidean anti-de Sitter event horizon. This remark applies also to the (Riemann)\(^2\) gravity. Some other inconsistencies of higher-derivative gravity, such as an appearance of tachyonic mode, or semi-classical instability at short distances, can also be related to classical limits on black hole entropy and viscosity bounds.

We conclude with a couple of remarks. In the presence of a Maxwell type charge \( q \), the ratio \( \frac{\eta}{s} \) is generally modified, which is given by \( 4\pi (\eta/s) = 1 - 4\lambda_{GB} (1 - Q/2) \), where \( Q = q^2 L^6 / r^2 \). In the extremal limit \( (Q \rightarrow 2) \), we put a restriction on the coupling \( \lambda_{GB} \) such that \( \lambda_{GB} \leq 1/24 \), which guarantees that the gravitational potential of a black brane is positive and bounded. The KSS bound \( \eta/s \geq 1/4\pi \) is saturated only in the extremal limit, while in general it is violated also by charged Gauss-Bonnet black brane solutions. The bound \( 4\pi (\eta/s) \geq 5/6 \) found in\(^{12}\) with a nonzero charge is stronger than for pure Einstein-Gauss-Bonnet gravity in flat space, namely \( 4\pi (\eta/s) \geq 2/3 \). Our analysis showed that in the AdS\(_5\) case, the entropy of a GB black hole could be negative for \( 1/12 < \lambda_{GB} < 1/4 \), leading...
to a possible violation of unitarity in this range. As shown recently in, a scalar dilaton field coupled to the Gauss-Bonnet can have an additional non-trivial contribution to the ratio $\eta/s$: a scalar charge generally lowers the KSS bound. In some more recent papers, there have appeared new examples, including string theory constructions, where the standard result of Einstein gravity, namely $\eta/s = 1/4\pi$, is modified in a modest way.

In conclusion, we have explored several interesting connections between the bulk causality and limits on black hole entropy, specific to the modified Gauss-Bonnet and the (Riemann)$^2$ gravity models.

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