Estimation of thermal contact parameters at the interface of two sliding bodies

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Abstract. The knowledge of heat transfer between sliding solids has determinant applications in many industrial sectors (motor, railway, space…). The heat flux generated by friction increases the temperatures, which can create an excessive deformation of these solids, leading to the damaging of surfaces. The thermal coupling at the interface of two solids can be described either by a model of ‘perfect contact’ which assumes the equality of surface temperatures of solids, or by a model of ‘imperfect contact’ where a difference of temperatures is introduced at the interface due to surfaces irregularities. The modelling of heat transfer between two sliding solids introduces three macroscopic parameters: $h_c$ (the thermal conductance), $\alpha$ (the local heat partition coefficient) and $\phi_g$ (the generated flux). Some numerical approaches of the simultaneous identification of these parameters have been developed with an assumption of constant parameters. This article presents the first estimation of the three parameters for the dry real sliding contact.

1. Introduction
The phenomenon of heat transfer between two sliding solids limits many engineering applications. Some examples where friction plays an important role can be given: ball and roll bearings, gears, braking systems, etc. The heat flux generated by friction increases the temperatures and therefore leads to stress and deformation in solids. An intensification of this phenomenon can lead to damages of surfaces. Several investigations have been developed in tribological and mechanical journals (e.g. ASME Journal of Tribology, Wear, International Tribology, Thermal Stress, ...) in order to study this problem. In a first approach, the assumption of perfect contact has been used by many authors to determine the surface temperature and the partition of heat between two sliding solids. In a second improved approach, the concept of an imperfect contact, which is well known for static solids, has been extended to sliding contacts. Analyzing the thermal constriction phenomenon at the vicinity of the interface between two sliding solids, Bardon [1] proposed an expression to describe the interfacial heat exchange. This approach introduces two contact parameters: (i) the thermal contact conductance $h_c$ that is the inverse of the thermal contact resistance ($R_c$), and (ii) the intrinsic heat partition coefficient, both are dependent on the thermal constriction resistances. An equivalent expression has been proposed by Tseng [2] to study heat transfer for rolling systems. The authors assume that the heat

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flux generated by friction is equally partitioned between the two solids (i.e. the intrinsic heat partition coefficient is equal to 0.5). Laraqi [3] presents another formulation based on a Gaussian distribution of the heat flux generated by friction around the sliding contact. Some analytical and numerical models have been performed in order to study the thermal constriction phenomenon in sliding configuration [4][5]. In the literature, the identification of TCR for non-static contacts has been conducted. Orlande and Ozişik [6] provided an inverse technique based on the conjugate gradient methods with an adjoint equation to study the periodic contact problem. Chen and Tuan [7] proposed an other method, using the Kalman filter and a recursive least-square algorithm, to estimate the thermal conductance for periodic contact. The TCR between two rotating discs has been studied by Chantrenne [8] using an inverse heat conduction method. LeMeur et al. [9] conducted a numerical study, based on the least-square method, in order to estimate the contact parameters for a thermo-electrical imperfect contact which is a similar problem. Then, a numerical study [10] has been performed to estimate simultaneously the heat flux generated by friction, the thermal contact conductance and the intrinsic heat partition coefficient for the problem of sliding contacts. A sensitivity analysis has been also performed in order to define the appropriate conditions for an accurate estimation of the thermal parameters. In this paper, after a brief description of the physical problem and the experimental device, we present the first experimental estimation of the thermal contact parameters in a real sliding contact.

2. Description of the physical problem.

The thermal behavior of solids in dry contact is closely related to heat exchanges, which occurred at the interface. This problem is complex and has not been widely investigated. This difficulty is due to the coexistence of several physical phenomena concerning different disciplines: Tribology, Mechanics, physical chemistry, heat transfer, physics of solids, etc. At present, a physical model has been developed [1] in order to describe the thermal coupling at the interface of solids. The model is represented in figure 1. The heat flux generated by friction $\phi_g$ is split between solids across thermal constriction resistances.

Assuming a resistive scheme, we show that temperatures at the boundaries of the constriction zone could be connected by a thermal contact resistance crossed by a heat flux including the conductive one (in solid 1) and a part of the frictional heat generation. This physical model introduces two macroscopic parameters: $R_{cg}$ (the thermal contact resistance: $R_{cg} = R_{c1} + R_{c2} + R_{a2}$) and $\alpha$ (the local heat partition coefficient). In imperfect contact problems, the former used to define the temperature thermal discontinuity at the interface is well known. The latter, less known, represents the local heat partition of generated flux. The resulting equation of heat transfer at the interface is as follows:
The first term on the r.h.s. of equation (1) represents the part of generated flux entering the solid 1. The second term concerns the heat flux from solid 2 through solid 1 across the thermal contact resistance $R_{cg}$. The entering flux into solid (2) is the complementary part of $\phi_g$ relative to $\phi_f$. The problem consists of estimating $\alpha$ and $R_{cg}$ (that could vary as a function of time).

3. Experimental device

The experimental device (figure 3), which simplified schema is presented in figure 2, is composed of two hollow cylinders in axial friction. The depth of the cylinders is small compared to their radii and the lateral surfaces are insulated in order to obtain an unidirectional heat transfer (on axial direction). This configuration is favourable for the identification of the thermal coupling parameters. The measurement of the contact resistance requires the existence of a heat gradient at the interface. In order to generate this gradient, a hot element is linked with the upper end of the rotating sample. An electric motor, whose rotating velocity is assumed to be known, drives the ensemble. The non-rotating cylinder is fixed on a caradan joint in order to ensure the uniformity of the contact between the two solids. The axial loading is transmitted to the hollow cylinder via this caradan. The temperatures are measured on different heights of the cylinder by 100 µm diameter K-type thermocouples (4 by sample). The thermocouples of the revolving hollow cylinder are connected to a transmitter with mercury.

![Figure 2. Scheme of the experimental device.](image1)

![Figure 3. Experimental setup](image2)

4. Experimental results on real dry sliding surfaces

4.1. Global Method

The estimation of $\alpha$ and $R_{cg}$ and $\phi_g$ is performed by minimizing the square of the difference between measured and calculated temperatures [11]. The functional of the least-square method is given by equation 2.
\[ F_n = \sum_{i=1}^{i=N} \sum_{j=1}^{j=M} \left( T_j^i - T_j^\mu \right)^2 \]  

(2)

where \( T_j^i \) is the measured temperature at time \( t_j \) and abscissa \( x_i \) and \( T_j^\mu \) is the calculated temperature at the same time and abscissa.

4.2. Experimental data

The experimental results presented in this study correspond to the dry friction between a steel cylinder and one made of bronze [10]. Both cylinders are smooth. This is the first time that some results of identification of the thermal contact parameters in a real dry sliding contact are obtained. The experimental results for the configuration of macro constriction were already presented [12]. In addition, the results of an experimental estimation procedure for real sliding surfaces are given. The thermal properties of materials are listed in table 1.

| Table 1. Thermal properties |
|----------------------------|
| Steel | Bronze |
|----------------------------|
| Thermal conductivity [W/m K] 50 | 110 |
| Thermal diffusivity [m²/s] 1.47 x 10⁻⁵ | 2.98 x 10⁻⁵ |
| Thermal effusivity [W s⁻¹/²/m² K] 13055 | 20163 |

The experiment take into account a contact pressure of 49.6 x 10³ Pa and a linear velocity of 0.495 m/s. Measures are recorded for different depths of the sliding contact in both cylinders [10]. The measurements are represented in figure 4 (two thermocouples on each cylinder and the calculated surface temperature). Two sliding periods are created during this experiment (Approximately 500-600 seconds and 800-950 seconds). Figure 5 presents the results for the surface heat flux calculated by successive time integrations [13] and the generated heat flux. We can notice that the phenomenon is reproducible. The value of the generated heat flux is similar for both sliding periods, about 7x10⁴ [W/m²].

![Figure 4](image1.png)

**Figure 4.** Temperatures of the cylinders and calculated surface temperatures.

![Figure 5](image2.png)

**Figure 5.** Calculated heat flux.
4.3. Sensitivity analysis
As it is shown through the numerical sensitivity analysis [10], the problem is ill-posed. $\alpha$ and $h_{cg}$ (the thermal contact conductance) have an important correlation. But we can notice on figure 6 that the ratio between the sensitivity vectors of $\alpha$ and $h_{cg}$ is not constant.

![Figure 6. Ratio of the sensitivity vectors of $\alpha$ and $h_{cg}$.](image)

$X_{\alpha}$ is the sensitivity vector of the parameter $\alpha$ and $X_{h_{cg}}$ the sensitivity vector of the parameter $h_{cg}$. They are calculated for two thermocouples, one for each cylinder and for 100 future time steps. The sensitivity vector of $\varphi_g$ is not correlated to the others. Although the rate of correlation between $\alpha$ and $h_{cg}$ is elevated, the thermal contact parameters can be estimated with these data.

4.4. Experimental results
The difference of temperatures in the contact increase, in dynamic configuration, about 0.5 K (20% more than the static contact). Experimental estimations of $\alpha$ and $R_{cg}$ are given in table 2. The minimisation is carried out assuming constant parameters. It takes into account 100 time steps with the time step equal to 0.65 s.

| Parameters   | $\alpha$ | $R_{cg}$ [m² K/W] | $\varphi_g$ [W/m²] |
|--------------|----------|-------------------|-------------------|
| Identification | 0.52     | $3.59 \times 10^{-4}$ | 6954              |

The choice of the period of identification changes the value of the estimate (cf. 4.5. ). The variation of the estimate is about 10% for the local heat partition coefficient and 5% for the sliding contact resistance. The estimation of the thermal sliding contact resistance must be compared with the thermal static contact resistance before and after the friction. The mean value before sliding is about $3,139 \times 10^{-4}$ [m² K/W] and after sliding about $3,125 \times 10^{-4}$ [m² K/W]. We notice an increase of about 20% of the thermal sliding contact resistance with regard to the static one.
4.5. Study of residuals
The quality of the identification parameters can be analyzed from the temperature residuals. This study brings more information on the physical phenomenon. Certainly, the assumption of constant parameters used to perform the estimation in the direct model can be validated if there is not a deviation of the temperature residuals. But, we can observe in figure 7 that during the first 20 seconds of the sliding contact, residuals move away from their mean values.

![Figure 7. Ratio of the sensitivity vectors of α and $h_{cg}$.](image)

Physically, we can explain this evolution by the fact that the contact configuration doesn’t change immediately from static to dynamic. A transient period where parameters are not constant appears at the beginning of the sliding contact (the generated heat flux doesn’t take the maximal value immediately. It is the same for $\alpha$ and $h_{cg}$). The appearance of the experimental residuals is similar for a numerical result calculated with exponential expression for the three parameters [14]. The study of residuals shows that the coefficients can be assumed to be constant along the estimation time. As it can be seen, there is no divergence in the residuals after 10-15 seconds (period of the evolution of the thermal contact parameters). As expected, the transient period between static and sliding contact clearly appears on the residuals of all experiences. During this time, it is obvious that the thermal contact parameters are not constant. Then, we take into account one hundred time steps (about 65 seconds) which is more than the transient period of the parameters. In this case, the error of the estimated parameters due to the assumption of constant parameters is satisfactory. The identification of the local heat partition coefficient is sensitive (error of 10%) to this variation of the parameters in the first instants. On the other hand, the estimation of the thermal contact resistance and the generated flux will be little affected by a variation of the parameters at the beginning of the sliding contact.

5. Error estimation according to the known parameters
The results obtained by inverse method must be assessed according to the errors of the "known" parameters, entries of the direct model [1]. Indeed, location of the detectors, thermal conductivity, measures of thermocouples and convection coefficient of the model cannot be known with an infinite precision.

5.1. Noise of measurement
A stochastic approach of the problem allows estimating the error of identification according to the noise of measurement. It allows giving a reliable interval to the presented results. The noise of
measurement is about $1.17 \times 10^{-2}$ [K]. Table 3 present the standard deviation ($\sigma_{\text{identification}}$ ) and the error calculated for each parameters by the stochastic approach. $X$ is the sensitivity matrix and $p$ the parameter: $p = \alpha$, $p = h_{cg}$, $p = \varphi_g$. We notice that the influence of the noise of measurement on the estimation of the parameters is not too important. The local heat partition coefficient is the most sensitive parameter to this noise of measurement.

| Table 3. Standard deviation of estimated parameters for a noise of measurement about $1.17 \times 10^{-2}$ [K]. |
|--------------------------------------------------|
| Parameter | $\alpha$ | $h_{cg}$ | $R_{cg}$ | $\varphi_g$ |
| Estimated value | 0.52 | 2787 W/m² K | $3.59 \times 10^{-4}$ m² K/W | 6954 W/m² |
| $(X^T X)^{-1}$ | 5,50 | 1,21 $10^4$ | - | 1,99 $10^3$ |
| $\sigma_{\text{identification}}$ | 0.064 | 141.9 [W/m² K] | 1,74 $10^{-5}$ [m² K/W] | 23.30 [W/m²] |
| Percentage of error | 12,35% | 5,09% | 4,84% | 0,34% |

5.2. Location of thermocouples

It is possible to give a margin of error according to the location of thermocouples. Supposing that the abscissa of the thermocouple is known with about two-tenths of a millimetre, the maximum error of estimation for each parameter is given in table 4.

| Table 4. Maximum error for an error of $+/-0.2\ mm$ on the location of the thermocouple |
|-----------------------------------------------|
| Parameter | $\alpha$ | $R_{cg}$ | $\varphi_g$ |
| Error on $x_{T3}$ (steel) | 5,85% | 0,71% | 3,21% |
| Error on $x_{T7}$ (bronze) | 13,44% | 2,26% | 5,87% |

The higher the conductivity of the material the more important the incidence of the location error on the estimation of parameters. Furthermore, the estimation of $\alpha$ is more sensitive to the location errors than the resistance.

5.3. Conductivity of the material

Once again, the most important error of estimation will be made on the coefficient of $\alpha$. If the maximum error of thermal characterization of materials is about 2 %, the table 5 presents the error of the identification made on the parameters of sliding contact.

| Table 5. Maximum error for an error of $+/-2\%$ on the conductivity of the materials |
|-----------------------------------------------|
| Parameter | $\alpha$ | $R_{cg}$ | $\varphi_g$ |
| Error on $\lambda_{\text{bronze}}$ | 5,47% | 1,00% | 3,15% |
| Error on $\lambda_{\text{acier}}$ | 2,00% | 0,05% | 1,84% |

5.4. Lateral convection coefficient

The lateral surfaces are insulated with Teflon. The conductance on theses lateral surfaces is at most about 0.5 [W/m² K]. In that case the errors of estimation of the parameters are given in table 6. This time, the most important error is made on the generated heat flux. It is the estimation of $\alpha$ that is the least affected by a bad estimation of the lateral convection coefficient.
Table 6. Maximum parameters’ errors when $h_{lateral}=0.5 \text{ [W/M}^2 \text{K]}$.

| $\alpha$ | $R_{\varphi}$ | $\varphi_g$ |
|---------|-------------|-------------|
| 1.47%   | 2.65%       | 6.67%       |

### 6. Conclusion

The study presented in this paper exposes the first results obtained in the determination of thermal contact parameters for a real sliding contact from experimental data. The model of three sliding contact parameters is experimentally validated. The feasibility of parameters identification at the interface of two sliding solids is proved. The previous experimental studies reported some values of resistances but the local heat partition coefficient didn’t appear. Indeed, there are numerous theoretical studies that consider this coefficient but none experimental work has highlighted it. A detailed sensitivity study has been performed in order to define the standard deviation of the estimated parameters. The numerical values obtained from experimental data have a physical meaning. Then, the goal of future experiments will be, on the one hand, to identify parameters on several experiments and, on the other hand, to correlate the values of the thermal contact parameters to the mechanical parameters of the sliding contact: roughness, velocity, pressure, etc.

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