Role of symmetry in the interplay of $T = 0$ quantum-phase transitions with unconventional $T > 0$ transport properties in integrable quantum lattice systems

J. M. P. Carmelo$^{1,2}$ (a), S.-J. Gu$^3$ and N. M. R. Peres$^1$

$^1$ GCEP-Center of Physics, U. Minho, Campus Gualtar, P-4710-057 Braga, Portugal
$^2$ Massachusetts Institute of Technology, Cambridge, MA 02139-4307, USA
$^3$ Department of Physics and ITP, The Chinese University of Hong Kong, Hong Kong, China

PACS 72.10.-d – Theory of electronic transport; scattering mechanisms
PACS 71.27.+a – Strongly correlated electron systems
PACS 05.60.Gg – Quantum transport

Abstract. - We show that a generalized charge SU(2) symmetry of the one-dimensional (1D) Hubbard model in an infinitesimal flux $\phi$ generates half-filling states from metallic states which lead to a finite charge stiffness $D(T)$ at finite temperature $T$, whose $T$ dependence we study. Our results are of general nature for many integrable quantum lattice systems, reveal the microscopic mechanisms behind their exotic $T > 0$ transport properties and the interplay with $T = 0$ quantum-phase transitions, and contribute to the further understanding of the transport of charge in systems of interacting ultracold fermionic atoms in 1D optical lattices, quasi-1D compounds, and 1D nanostuctures.

There has been a recent increasing interest in the unusual transport and spectral properties of systems of interacting ultracold fermionic atoms in one-dimensional (1D) optical lattices [1] and a renewed interest in those of quasi-1D carbon nanotubes, ballistic wires, and quasi-1D compounds [2, 3]. Quantum effects are strongest at low dimensionality leading to unusual phenomena such as charge-spin separation at all energies [3] and persistent currents in mesoscopic rings [4]. The ultracold atom technology can realize a 1D closed optical lattice experimentally with a tunable boundary phase twist capable of inducing atomic ”persistent currents” [1]. Thus, the further understanding of the microscopic mechanisms behind the transport of charge in low-dimensional correlated systems and materials is a topic of high scientific and technological interest. Recently there has been an enormous grow in the literature on the quantum phase transitions in such systems [5]. For instance, the relation of the unusual $T > 0$ charge-transport properties observed in low-dimensional correlated systems to their $T = 0$ quantum phase transitions is also a problem which is far of being fully understood.

For the 1D Hubbard model [6] or any other interacting electronic quantum system defined in a 1D lattice of $N_a$ sites of length $L = aN_a = N_a$ ($k_B = \hbar = a = -e = 1$) the $T > 0$

(a)E-mail: carmelo@fisica.uminho.pt
ideal conducting and insulating behaviors are defined in terms of the real-part of the optical conductivity for finite temperatures \( T > 0 \), which reads \( \sigma (\omega ) = 2 \pi D \delta (\omega ) + \sigma _{\text{reg}} (\omega ) \). Here the charge stiffness is given by \( D = D(N, T) = d/Z \) where \( Z = Z(N, T) = \sum _{m} p_m \) is the partition function of the N-electron system, the \( m \) summation runs over the eigenstates \( |m\rangle \) of energy \( E_m \), and \( p_m = e^{-E_m/T} \) are the corresponding Boltzmann weights. At \( T = 0 \) the charge stiffness is related to the boundary energy of the lattice model under twisted boundary conditions [7] and for \( T \geq 0 \) \( d = d(N, T) \) can be related to the thermal average of curvatures of energy levels subject to an infinitesimal flux \( \phi \) (in units of \( 2\pi /L \) and \( L \to \infty \)),

\[
d = \frac{1}{L} \sum _{m} p_m \left. \frac{1}{2} \frac{\partial ^2 E_m (\phi )}{\partial \phi ^2} \right| _{\phi \to 0} .
\]

At \( T > 0 \) a normal conductor is such that \( D(N, T) = 0 \) and \( \sigma (\omega \to 0) > 0 \), whereas an ideal conductor is characterized by \( D(N, T) > 0 \). Furthermore, an insulator can develop into a normal conductor, become an ideal conductor, or display ideal insulating behavior such that \( D(T) = 0 \) and \( \sigma (\omega \to 0) = 0 \). 1D integrable systems whose Hamiltonians commute with the current operator behave as ideal conductors and ideal insulators for the densities corresponding to the \( T = 0 \) metallic and insulating quantum phases, respectively. There is strong numerical evidence that the same occurs concerning the \( T > 0 \) ideal conducting behavior for densities referring to \( T = 0 \) metallic phases of 1D integrable quantum systems whose Hamiltonian does not commute with that operator, such as the 1D Hubbard model [8, 9]. However, whether for the densities corresponding to the \( T = 0 \) insulating quantum phases such systems behave as ideal insulators remains an open question. In contrast, the non-integrable 1D interacting systems are generic conductors and activated ones in the metallic and insulating phases, respectively. 

In this Letter we clarify such an issue by showing that a generalized charge \( SU(2) \) symmetry of the 1D Hubbard model in an infinitesimal flux \( \phi \) generates half-filling states from metallic states which lead to a finite half-filling charge stiffness \( D = D(N_a, T) \) at finite temperature \( T \). We study its \( T \) dependence by characterizing its different behaviors in three well-defined temperature regimens. Interestingly, the above half-filling states do not belong to the integrability Hilbert subspace where the model is solvable by the Bethe ansatz [6]. The latter subspace is spanned by the lowest-weight states (LWSs) of both the \( \eta \)-spin and spin \( SU(2) \) algebras [10]. When defined in such a subspace, the quantum problem has an infinite number of conservation laws [11]: as in the corresponding classical systems, the occurrence of an infinite number of such laws is behind the integrability of the quantum models. Moreover, we present numerical evidence that when acting onto the integrability subspace the 1D Hubbard model is an ideal insulator. Indeed, we find that in the thermodynamic limit the half-filling states that span such a subspace do not carry charge current, consistently with the general predictions of Ref. [8]. Importantly, our results provide useful information about the microscopic mechanisms behind the interplay of the \( T = 0 \) Mott-Hubbard insulator - metal quantum-phase transition with the \( T > 0 \) thermal properties in 1D correlated systems and new insights into the exotic microscopic processes of charge transport in systems of interacting ultracold fermionic atoms in 1D optical lattices, carbon nanotubes, ballistic wires, and quasi-1D compounds [1–3]. 

The 1D Hubbard model in an infinitesimal flux \( \phi \) reads,

\[
\hat{H}(\phi ) = -t \sum _{j, \sigma } \left[ e^{i\phi } c_{j, \sigma }^\dagger c_{j+1, \sigma } + \text{h.c.} \right] + U \hat{D}_- , 
\]

where \( \hat{D}_- = \sum _{j} \{[\hat{n}_{j, \uparrow } - 1/2][\hat{n}_{j, \downarrow } - 1/2] + 1/4 \} \), \( c_{j, \sigma }^\dagger \) and \( c_{j, \sigma } \) (and below \( c_{k, \sigma }^\dagger \) and \( c_{k, \sigma } \)) are spin-projection \( \sigma = \uparrow, \downarrow \) electron operators at site \( j = 1, 2, ..., N_a \) (of momentum \( k \)), the number \( N_a \) is even and very large, and \( \hat{n}_{j, \sigma } = c_{j, \sigma }^\dagger c_{j, \sigma } \). We use periodic boundary conditions and consider zero spin density \( [n_{\uparrow } - n_{\downarrow }] = 0 \) and electronic densities \( n = [n_{\uparrow } + n_{\downarrow }] \).
The Bethe-ansatz equations given in Eq. (10)- (12) of Ref. [9] for the off-diagonal generators of the generalized algebra become those of the \( \eta \) spin algebra \[10\]. Since here we are mostly interested in the transport properties in integrable half-filling \( \eta \)-spin-triplet states (left current spectrum) and corresponding singlet states (right current spectrum) for \( U/4t = 1 \) and \( N = N_{\sigma} = 66 \).

and corresponding hole concentrations \( \delta = 1 - n \) in the range \( 0 \leq \delta \leq 1 \), where \( n_{\sigma} = N_{\sigma}/L = N_{\sigma}/N_{\sigma} \) and \( N_{\sigma} \) such that \( N = [ N_{\uparrow} + N_{\downarrow} ] \) is the number of \( \sigma \) electrons. The following \( \phi = 0 \) properties of the Hamiltonian (2) play an important role in our study: (i) Its zero-energy level refers to the \( \delta = 0 \) insulating ground state, \( \varepsilon_0(N_\sigma) = 0 \), and is located in the middle of the Mott-Hubbard gap \[6\], \( \Delta_{MH} = [(4t)^2/|U|] \int_0^\infty d\omega \sqrt{\omega^2 - 1/4} \sin(2\pi\omega/U) \), such that \( \Delta_{MH} \approx [8\sqrt{t/t}/\pi] e^{-2\pi t/U} \) for \( U/t << 1 \) and \( \Delta_{MH} \approx U - 4t \) for \( U/t >> 1 \); (ii) The ground-state energies associated with even values of \( N \leq N_\sigma \) are such that \( \varepsilon_0(N-2) > \varepsilon_0(N) \) where \( \varepsilon_0(N) \) changes from \( [4N_\sigma /\pi][1 - \cos(\pi\delta)/2] \) for \( U/t << 1 \) to \( [\delta N_\sigma U/2] [1 - 4\sin(\pi\delta)/\pi\delta U] \) for \( U/t >> 1 \); (iii) The \( T = 0 \) chemical potential \( \varepsilon_\mu(N,0) \) is an increasing function of \( \delta \), changing from \( 4t\sin(\pi\delta/2) \) for \( U/t << 1 \) to \( U - 4t\cos(\pi\delta) \) for \( U/t >> 1 \) and is given by \( \varepsilon_\mu(N,0) \approx \Delta_{MH} + \delta^2/\pi^2/m^*_{\sigma} \) and \( \varepsilon_\mu(0,0) = U + 4t > \Delta_{MH} \) for \( \delta \) small and \( \delta = 1 \), respectively, and all values of \( U/t \); (iv) \( m^*_{\sigma} \) is a charge mass given in Eq. (48) of Ref. [12] whose limiting values are \( m^*_{\sigma} \to 0 \) as \( U/t \to 0 \) and \( m^*_{\sigma} \to 1/2t \) as \( U/t \to \infty \).

For \( \phi = 0 \) the Hamiltonian (2) commutes with the three generators of the \( \eta \)-spin \( SU(2) \) algebra \[10\]. While for \( \phi > 0 \) it does not commute with the two off-diagonal generators of that algebra, a result which plays a central role in our study is that for \( \phi > 0 \) it commutes with the three generators \( \hat{\eta}_0 = -\frac{1}{2} \sum_j [\hat{n}_{\uparrow, j} - \hat{n}_{\downarrow, j}], \quad \hat{\eta}_\uparrow = \sum_k \hat{c}^\dagger_{\uparrow \phi \rightarrow \downarrow, k} \hat{c}_{\uparrow \downarrow, k}, \quad \hat{\eta}_\downarrow = \sum_k \hat{c}^\dagger_{\uparrow \phi \rightarrow \downarrow, k} \hat{c}_{\uparrow \downarrow, k}, \) and \( \hat{\eta}_\phi = \sum_k c^\dagger_{\uparrow \phi \rightarrow \downarrow, k} c_{\uparrow \downarrow, k} \) of a generalized \( SU(2) \) algebra. Since for \( \phi > 0 \) the \( \eta \)-spin states and \( \eta \)-spin algebra remain good quantum numbers, the diagonal generator \( \hat{\eta}_0 \) is independent of \( \phi \). Furthermore, note that as \( \phi \to 0 \) the above off-diagonal generators of the generalized algebra become those of the \( \eta \)-spin algebra \[10\]. For \( \phi > 0 \) the Hamiltonian (2) remains integrable. However, as for \( \phi = 0 \), the generalized Bethe-ansatz equations given in Eq. (10)-(12) of Ref. [9] for \( \phi > 0 \) refer to the Hilbert subspace spanned by the LWSs of both the \( \eta \)-spin and spin algebras such that \( 2\eta = [N_\sigma - N] \) and \( 2S = [N_\uparrow - N_\downarrow] \), where \( S \) is the spin. Thus, for half filling such an integrability subspace is spanned by the \( \eta = 0 \), \( \eta \)-spin singlet states. The full Hilbert space contains another subspace which is spanned by energy eigenstates generated from application onto the LWSs of the off-diagonal generators of the generalized algebras. Since here we are mostly interested in the transport of charge, for simplicity we limit our study to the charge and \( \eta \)-spin degrees of freedom, yet it is straightforward to extend our analysis to the spin degrees of freedom. Let \( |m, N, \phi \rangle \) be a \( N \)-electron energy eigenstate of the Hamiltonian (2) for \( \phi > 0 \), such that \( 2\eta > [N_\sigma - N] \). This state does not belong to the subspace where the model is integrable but can be generated from the corresponding \( N' \)-electron LWS such that \( N' = [N_\sigma - 2\eta] < N \) as follows,

\[
|m, N, \phi \rangle = \frac{1}{C} \left[ \hat{\eta}_\phi \right]^{N_\sigma - N} |m', N', \phi \rangle ,
\]

where the normalization constant reads \( C = \delta_{\phi, \phi} \left( N_\sigma - N \right) + \prod_{l=1}^{N_\sigma - |N_\sigma - N|} l [2\eta + 1 - l] \). Since the
Thus, \( N \) and \( \min \) being such that \( \Delta T > \eta \). M. J. P. Carmelo

integrability, for finite values of \( \eta \) of the above states \( |m, N, \phi \rangle \) and \( |m', N', \phi \rangle \) are identical.

The microscopic mechanism described in the following implies that, in spite of the model integrability, for finite values of \( U/t \) the \( \delta = 0 \) charge stiffness is finite for finite \( T \) values and vanishes both as \( T \to 0 \) and \( T \to \infty \). For each energy eigenstate \( |m, N, \phi \rangle \) of electronic-number \( N \) and \( 2\eta > |N_a - N| \) there is one and only one \( \eta \)-spin LWS \( |m', N', \phi \rangle \) with the same \( \eta \) value, \( N' < N \), and \( 2\eta = |N_a - N'| \) whose energy is such that \( E_m(\phi) = E_{m'}(\phi) \), where \( E_m(\phi) \) is that of \( |m, N, \phi \rangle \). In turn, there are \( 2\eta \) states \( |m, N, \phi \rangle \) such that \( N = N_a - 2\eta + 2j \) with \( j = 1, 2, \ldots, 2\eta \) corresponding to the same \( \eta \)-spin LWS \( |m', N', \phi \rangle \).

Thus, \( Z(N, T) = \delta_{N,N_a} + Z_{LWS}(N, T) + Z_{R}(N, T) = \delta_{N,N_a} + \sum_{N'=0}^{N} Z_{LWS}(N', T) \) (and \( d(N, T) = d_{LWS}(N, T) + d_{R}(N, T) = \sum_{N'=0}^{N} d_{LWS}(N', T) \) where \( \delta_{N,N_a} \) and \( Z_{LWS}(N, T) \) correspond to the contributions from all the \( \eta \)-spin LWSs of electronic number \( N \) and \( Z_{R}(N, T) = \sum_{N'=0}^{N-2} Z_{LWS}(N', T) \) (and \( d_{R}(N, T) = \sum_{N'=0}^{N-2} d_{LWS}(N', T) \) is such that the \( N' \) summation refers to all LWSs with even electronic numbers \( N' = 0, 2, 4, \ldots, N - 2 \) from which one can generate all non-LWSs with \( N \) electrons. (The above contribution \( \delta_{N,N_a} \) refers to the insulating ground state.) Such results imply that \( \max d(N, T) = d(N_a, T) \) and thus if \( d(N, T) > 0 \) for \( N \) even, \( N < N_a \), and \( T > 0 \) then \( d(N, T) > 0 \) for \( N = N_a \) and \( T > 0 \).

Without the construction of the states (3) for \( \phi > 0 \) the above results could not be reached. Their use reveals that a parameter which plays key role in our study is,

\[
\alpha(\Delta N, T) = \tilde{Z}(\Delta N, T)/Z(N_a - \Delta N, T); \quad \tilde{Z}(\Delta N, T) = \sum_{N' = N_a - \Delta N - 2}^{N_a} Z_{LWS}(N', T),
\]

where \( Z(N_a - \Delta N, T) \) is the partition function for \( N = N_a - \Delta N \) and the \( N' \) sum runs over even values only. For finite values of \( T \) and \( U/t \) one has that \( \alpha(\Delta N, T) < 1 \) for \( \Delta N \leq \Delta N_0 = \Delta N_0(T) \), where the critical value \( \Delta N_0(T) = 2, 4, 6, \ldots \) increases with \( T \), being such that \( \Delta N_0(T) \to 2 \) as \( T \to 0 \) and \( \Delta N_0(T) \to N_a \) as \( T \to \infty \). Moreover, \( \min\{\alpha(\Delta N, T)\} = \alpha(2, T), \max\{\alpha(2, T)\} = \alpha(2, 0) = 1, \) and \( \alpha(2, T) < 1 \) for \( T > 0 \).

Three typical regimes correspond to (a) \( 2\mu(N_a, T)/T \to \infty \) when \( T << 2\mu(N_a, 0) = \Delta_{MH} \), (b) \( 2\mu(N_a, T)/T \approx 0 \) for intermediate temperatures \( T \approx 2\mu(N_a, 0) = \Delta_{MH} \), and (c) \( 2\mu(0, T)/T \to -\infty \) when \( T \to \infty \). We find from the above results that for \( U/t > 0 \) the half-filling charge stiffness is of the general form,

\[
D(N_a, T) = \frac{D(N_a - \Delta N_0, T)}{1 + F(\Delta N_0, T)} + \gamma(\Delta N_0, T),
\]

where \( F(\Delta N, T) = \alpha(\Delta N, T) + 1/Z(N_a - \Delta N, T) \) and

\[
\gamma(\Delta N, T) = \frac{[d(N_a, T) - d(N_a - \Delta N, T)]/Z(N_a, T)}{Z(N_a, T)},
\]
is such that \( \gamma(\Delta N, T) \geq 0 \) for all \( \Delta N = 2, 4, ..., N_a \). For the regimen (a), \( e^{-\Delta_{MH}/T} \approx e^{-\infty} \) and thus also \( e^{-2\mu(N',0)/T} \approx e^{-\infty} \), because \( 2\mu(N',0) > \Delta_{MH} \) for \( 0 \leq N' < N_a \). Thus, one has both that,

\[
\frac{Z_{LWS}(N', T)}{Z_{LWS}(N' - 2, T)} \approx e^{-2\mu(N',0)/T} \approx e^{-\infty} \quad \text{and} \quad \frac{d_{LWS}(N', T)}{d_{LWS}(N' - 2, T)} \approx e^{-2\mu(N',0)/T} \approx e^{-\infty},
\]

and only the term \( d_{LWS}(N_a - 2, T) \) contributes to \( d_R(N_a, T) = \sum_{N' = 0}^{N_a - 2} d_{LWS}(N', T) \). We then find from use of Eq. (5) that,

\[
D(N_a, T) \propto e^{-\Delta_{MH}/T} \sqrt{T/m_h^*}; \quad T \ll \Delta_{MH}.
\]

In turn, as the temperature increases and reaches the regimen (b) such that \( T \approx \Delta_{MH} \), one finds that \( e^{2\mu(N',T)/T} \approx 1 \) for an increasing number of \( Z_{LWS}(N', T) \) and \( d_{LWS}(N', T) \) terms of the expressions \( Z_R(N_a, T) = \sum_{N' = 0}^{N_a - 2} Z_{LWS}(N', T) \) and \( d_R(N_a, T) = \sum_{N' = 0}^{N_a - 2} d_{LWS}(N', T) \) respectively, so that \( Z_R(N_a, T) \) and \( d_R(N_a, T) \) have significant contributions from a maximum number of such terms. Moreover, \( \delta_0 = \Delta N_0(T)/N_a \) is now finite and \( e^{2\mu(N_a-\Delta N_0,T)/T} \approx 1 \) implies that \( Z(N_a-\Delta N_0, 0, T) \gg 1 \) and thus \( F(\Delta N_0, T) < 1 \) or \( F(\Delta N_0, T) \approx 1 \). Therefore, since for \( \delta_0 \) finite the system is an ideal conductor for \( N = N_a - \Delta N_0 \) [8, 9], \( D(N_a, T) \), Eq. (5), is finite for the regimen (b). Furthermore, the achieved maximum contribution from the whole set of above terms leads to a maximum value of \( D(N_a, T) \) for \( T \approx \Delta_{MH} \). For increasing values of \( T \), \( D(N_a, T) \) becomes a decreasing function of \( T \) and finally when \( T \to \infty \) the regimen (c) such that the two ratios of Eq. (7) reach the values \( e^{-2\mu(N',T)/T} \approx e^{-\infty} \) and \( e^{2\mu(N',T)/T} \approx e^{\infty} \), respectively. We then find from use of Eq. (5) that,

\[
D(N_a, T) \propto 1/T; \quad T \to \infty,
\]

which is the expected behavior of systems on a lattice. The overall charge-stiffness \( T \) dependence is thus similar to that of the \( \delta = 0 \) dimerized model plotted in Fig. 6 of Ref. [9], with the dimerized gap \( \Delta \) replaced by \( \Delta_{MH} \). The temperature \( T \approx \Delta_{MH} \) associated with the maximum value of \( D(N_a, T) \) is an increasing function of \( U/t \). For \( U/t \to \infty \) one has that \( T \approx \Delta_{MH} \approx U - 4t \to \infty \) and thus \( D(N_a, T) \) is given by Eq. (8) for all ranges of finite \( T \) values. Thus, for \( U/t \to \infty \) the regimens (b) and (c) are never reached for finite \( T \) values. In the opposite limit, when upon decreasing the value of \( U/t \) one reaches \( T \approx \Delta_{MH} \approx \sqrt{8U/t/\pi}e^{-2\pi/|U|}; \) the temperature range above \( T = 0 \) where \( D(N_a, T) \) is given by Eq. (8) becomes smaller and smaller, until it shrinks as both \( \Delta_{MH} \to 0 \) and \( m_h^* \to 0 \) and the limit \( U/t = 0 \) is reached. In that limit a maximum value \( D(N_a, 0) = 2t/\pi \) occurs for \( T = \Delta_{MH} \to 0 \) and thus \( D(N_a, T) \) becomes finite and largest at \( T = 0 \). Thus, the regimen (a) is absent for \( U/t \to 0 \), which implies a metallic ground state for \( U/t = 0 \) and \( \delta = 0 \).
and ideal conducting behavior associated with the regimen \( \text{b} \) for finite temperatures. Our above results then reveal the microscopic processes behind the \( T = 0 \) half-filling \( U/t \to 0 \) Mott-Hubbard insulator - metal phase transition. Indeed, for finite \( U/t \) values \( D(N_a, T) \) displays a maximum value at \( T \approx \Delta_{M_H} \) which corresponds to the merging of the regimen (b) and vanishes both for \( T \to 0 \) and \( T \to \infty \) as given in Eqs. (8) and (9), respectively. However, for \( U/t \to 0 \) the maximum value is shifted to \( T = 0 \) as \( \Delta_{M_H} \to 0 \), which is behind the \( T = 0 \) quantum phase transition. The above mechanism reveals that the finite values \( D(N_a, T) > 0 \) for \( T > 0 \) and \( U/t > 0 \) result from half-filling states generated by the generalized charge \( SU(2) \) algebra from metallic states.

For \( L \to \infty \) the charge stiffness can be expressed as \( D = [1/2T Z L] \sum_m p_m \langle j_m \rangle^2 \) in terms of the expectation values \( j_m = \langle m | j | m \rangle = -\partial E_{m}(\phi)/\partial \phi |_{\phi=0} \) of the current operator \( j \) and thus \( D = D(N, T) \) vanishes if \( j_m = 0 \) for all energy eigenstates \([9]\). In order to confirm that the \( \delta = 0 \) finite charge currents \( j_m \), are carried by states generated from metallic states, we provide numerical evidence that the \( \delta = 0 \eta \)-spin singlet energy eigenstates which span the integrability subspace carry no charge current and thus that \( d_{L W S}(N_a, T) = 0 \). Our numerical results also confirm that the \( \delta = 0, \eta > 0 \) states carry finite current, in agreement with our above analysis. We have used the \( L \gg 1 \) Bethe-ansatz equations of Ref. \([9]\) with \( L \) finite and checked that our finite-size results are in excellent numerical agreement with all known quantities. In Figs. 1 and 2 we plot the \( \delta = 0 \) energy spectrum as a function of the momentum of the \( \eta = 1, \eta \)-spin-triplet energy eigenstates and of the corresponding \( \eta = 0, \eta \)-spin-singlet states whose energy spectrum is degenerated with that of the former states for two values of \( U/4t \). Such \( \eta = 0 \) states have one charge string of length one \([9]\). These figures also display the charge-current spectrum of these two types of states. Note that the current value is finite for the \( \eta \)-spin triplet states, in agreement with our above analysis, whereas for the \( \eta = 0 \) states it is very small for the finite system and vanishes as \( L \to \infty \). In figure 3 we show the current spectrum of the same \( \eta = 0 \) states for \( U/4t = 1 \) and different values of \( N_a \). The data of the figure confirm that the current values decrease for increasing values of \( L \) and our extrapolation for \( L \to \infty \) reveals that they vanish in that limit. Finally, we study the current expectation values of all \( \delta = 0, \eta = 2 \) states and of the corresponding \( \eta \)-spin-singlet states whose energy spectrum is degenerated with that of these states. There are two types of such \( \eta = 0 \) states, which have one charge string of length two and two charge strings of length one, respectively \([9]\). We consider the square root of the summation of the currents of these two classes of states. The results are plotted in Fig. 4 for \( U/4t = 10 \) as a function of \( 1/L \) and indicate that the currents of these states also vanish as \( 1/L \to 0 \). A similar analysis for other values of \( U/4t \) and random samples of \( \eta \)-spin-singlet states whose energy is degenerated with that of \( \delta = 0, \eta > 2 \) states also led to vanishing currents as \( L \to \infty \).
In this Letter we have shown that at $N = N_a$ the 1D Hubbard model is not an ideal insulator because a generalized charge $SU(2)$ symmetry of the model in an infinitesimal flux generates half-filling states from metallic states that carry finite current. Moreover, we found evidence that when acting onto the Hilbert subspace spanned by the energy eigenstates associated with integrability the 1D Hubbard model is at half filling an ideal insulator. Our results also reveal the microscopic mechanisms behind the interplay of the half-filling $T = 0$ Mott-Hubbard insulator - metal quantum-phase transition with the model unusual $T > 0$ properties and contribute to the further understanding of the transport of charge in systems of interacting ultracold fermionic atoms in 1D optical lattices, quasi-1D compounds, and 1D nanostructures.

We thank P. A. Lee for discussions and the support of the Gulbenkian Foundation, Fulbright Commission, ESF Science Program INSTANS, and grants POCTI/FIS/58133/2004 and RGC CUHK 401504.

REFERENCES

[1] D. Jaksch and P. Zoller, Ann. of Phys., 315 (2005) 52; L. Amico, A. Osterloh and F. Cataliotti, Phys. Rev. Lett., 95 (2005) 063201.
[2] I. Hiroyoshi et al., Nature, 426 (2003) 540; O. M. Auslaender et al., Science, 308 (2005) 88; T. Lorenz et al., Nature, 418 (2002) 614.
[3] R. Claessens et al., Phys. Rev. Lett., 88 (2002) 096402.
[4] U. Eckern and P. Schwab, J. Low Temp. Phys., 126 (2002) 1291.
[5] D. Belitz, T. R. Kirkpatrick and T. Vojta, Rev. Mod. Phys., 77 (2005) 579.
[6] E. H. Lieb and F. Y. Wu, Phys. Rev. Lett., 20 (1968) 1445.
[7] B. S. Shastry and B. Sutherland, Phys. Rev. Lett., 65 (1990) 243.
[8] P. Prelovšek, S. El Shawish, X. Zotos and M. Long, Phys. Rev. B, 70 (2004) 205129.
[9] N. M. R. Peres, R. G. Dias, P. D. Sacramento and J. M. P. Carmelo, Phys. Rev. B, 61 (2000) 5169.
[10] O. J. Heilmann and E. H. Lieb, Ann. NY Acad. Sci., 172 (1971) 583.
[11] X. Zotos, F. Naef and P. Prelovšek, Phys. Rev. B, 55 (1997) 11029.
[12] J. Carmelo, P. Horsch, P. A. Bares and A. A. Ovchinnikov, Phys. Rev. B, 44 (1991) 9967.