One-time measurement to characterize complex amplitude single component in symmetry superposed optical vortices

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Abstract: Complex amplitude measurement is an essential prerequisite for light field characterization. In this work, we propose a one-time measurement method to characterize the complex amplitude of symmetry superposed optical vortices (SSOVs) with only one picture registered by CCD. This method includes two strategies we proposed. One is the ring extraction strategy for amplitude measurement, and another is the rotational measurement strategy for phase measurement. In the proof-of-concept experiment, the complex amplitude is characterized, and the mode purity is well measured. This method has excellent flexibility, rapidity, and robustness, which can be applied to various occasions and harsh conditions. Careful alignment and optimized error analysis allow us to generate and measure a single component with mode purity as high as 99.99%. © 2020 Optical Society of America

1. INTRODUCTION

In 1992, Allen et al. proved that an optical vortex (OV) whose complex amplitude comprises the helical term \( \exp(i m \theta) \) carries an orbital angular momentum (OAM) per photon of \( m \hbar \) [1], which has aroused widespread concern among researchers [2-17], where \( m \) denotes the topological charge (TC), \( \theta \) represents the azimuthal angle and \( \hbar \) is the reduced Planck constant. Since then, OVs have been utilized in a plethora of applications in the field of optical micro-manipulation [18, 19], plasmonics [20, 21], rotation probing [22-28], optical communication [29-31], gravitational wave detection [32, 33] and so on. The symmetry superposed OVs (SSOVs) are often used here instead of a single component. Before practical application, we need to characterize the complex amplitude of a single component in SSOVs and measure its mode purity. It is an essential quality reference for an OV, which determines OV’s performance in various applications. In the rotating Doppler effect [34, 35], low mode purity means frequency spectrum expansion, significantly increasing signal detection difficulty. In the field of micro-manipulation, high mode purity is more conducive to the use of superposed OVs to achieve static capture of particles [36].

Many demonstrated methods characterize and measure the mode purity of SSOVs [37-45]. But the most versatile approach is to resolve a field into a coherent sum of modes, each with a particular amplitude weighting and phase: a so-called modal decomposition [46]. However, we need to use an SLM to scan multiple holograms one by one to complete the measurement. Is there a way to characterize the complex amplitude of SSOVs with only one-time measurement to improve measurement efficiency? The answer is yes. Recent work showed that the phase-shifting holography in an interferometer could be used to measure the mode purity of OVs [47]. This work required a separate reference path that increased the setup’s complexity and
introduced additional phase noise. In 2019, Andersen et al. modified the work mentioned above to achieve the same goal with only one path[48]. However, these two works need multiple measurements to characterize the complex amplitude of the aimed optical field.

In this work, we propose a one-time measurement method to characterize the complex amplitude of SSOVs with only one picture registered by CCD based on phase-shifting technology, which is reported for the first time. We can use this method to evaluate the quality of the generated SSOVs. This method includes two strategies we proposed. One is the ring extraction strategy for amplitude measurement, and another is the rotational measurement strategy for phase measurement. This method takes a short time, and we can characterize the complex amplitude of the SSOVs and achieve the measurement of mode purity in only 0.24 seconds. Since we use part of the SSOVs to be measured as the reference light, though we use the phase-shifting technology, no additional reference beams are required for interference anymore, which has not been reported before.

2. THEORY AND METHODS

A PHASE-SHIFTING THEORY OF SSOVs

The self-interference method requires four intensities \( I(x,y;\phi) \) of SSOVs. \( \phi \) denotes the shifting phase of one single component. We only consider that the SSOVs only have two symmetry components for simplicity without loss of generality.

May the two components of SSOVs be

\[
E_1 = A_1 \exp(i\phi)\exp(i\phi_1),
E_2 = A_2 \exp(-i\phi)
\]

where \( \phi = 0, \pi / 2, \pi, 3\pi / 2 \). The four intensity distributions of SSOVs are:

\[
I_0(x,y;0) = I(x,y) + I'(x,y)\cos(2\phi);
I_0(x,y;\pi / 2) = I(x,y) - I'(x,y)\sin(2\phi);
I_0(x,y;\pi) = I(x,y) - I'(x,y)\sin(2\phi);
I_0(x,y;3\pi / 2) = I(x,y) + I'(x,y)\sin(2\phi)
\]

where \( I(x,y) = A_1^2 + A_2^2 \), \( I'(x,y) = 2A_1A_2 \). Consequently, we can calculate the phase \( \phi \):

\[
\phi = \frac{1}{2} \arctan\left[ \frac{I_0(x,y;3\pi / 2) - I_0(x,y;\pi / 2)}{I_0(x,y;\pi) - I_0(x,y;0)} \right]
\]

Although the intensity of the two components does not affect the final result, according to interference contrast:

\[
\gamma = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}
\]

we set the two equals to get the best interference pattern. Where

\[
I_{\text{max}} = A_1 + A_2; I_{\text{min}} = A_1 - A_2
\]

B ENCODING HOLOGRAM

In this part, we briefly introduce the process of encoding the hologram to generate SSOVs. We use the technique demonstrated in ref.[49, 50] to encode the hologram. The hologram can be used to achieve complex amplitude modulation of incident light on a phase-only SLM. The process of encoding the hologram is shown in Fig. 1. The expression of the hologram is:

\[
\phi_{\text{hol}} = F(I_n)\sin(\phi + \phi_{\text{gra}})
\]

where \( I_n \) denotes the normalized amplitude of SSOVs, \( F(I_n) \) is the numerical inversion result of the first-order Bessel function of \( I_n \) in [0, 1.84], \( \phi \) represents the phase of SSOVs, and \( \phi_{\text{gra}} \) is the blazed grating phase. The blazed grating prevents part of the unmodulated light from mixing into the required SSOVs, diffracting the required light beam to the first order, and
leaving the unmodulated light in the zero-order. The unmodulated light is due to the gaps in the SLM liquid crystal arrangement. Although using an SLM for complex amplitude modulation sacrifices phase depth, it allows us to radially modulate the incident light field to generate an eigenmode of SSOVs instead of hypergeometric mode[51]. Load the encoded hologram onto the SLM, and we can get the intensity $I_{0}(x, y; 0)$ of SSOVs.

![Fig. 1 Schematic representation of encoding the hologram. The TCs of SSOVs here are $\pm 1$. (a) The normalized amplitude of the SSOVs. (b1) The phase of a blazed grating. (b2) The phase of the SSOVs. (c) Encoded hologram. (d) The experimental intensity $I_{0}(x, y; 0)$ of the SSOVs generated by (c). The color bar 1 indicates the phase range. The color bar 2 shows the range of (a) and (d).]

**C RING EXTRACTION AND ROTATIONAL MEASUREMENT STRATEGY**

To obtain the complex amplitude of SSOVs through one measurement, we propose two strategies. One is the ring extraction strategy for amplitude measurement, and another is the rotational measurement strategy for phase measurement.

In the ring extraction strategy, we consider a typical OV called the Laguerre-Gaussian (LG) beam. In this case, the amplitudes in Eqs.(1) are:

$$A(x, y) = A_0 \exp \left( \frac{-r^2}{\omega^2} \right) L \left( \frac{2r^2}{\omega^2} \right)$$  \hspace{1cm} (7)

where $r = \sqrt{x^2 + y^2}$, $p$ represents the radial node, $\omega$ is the waist radius, and $L$ denotes the Laguerre polynomial. The initial interference intensity in Eqs.(2) turns to:

$$I_{0}(x, y; 0) = 2A^2(x, y) + 2A^2(x, y) \cos(2\phi)$$  \hspace{1cm} (8)

where

$$I_{0_{\text{max}}} = 4A^2 \quad (\phi = \pi n, n \in \mathbb{Z})$$  \hspace{1cm} (9)

It can be seen from the Eqs.(9) that interference affects the angular distribution of SSOVs, but the light intensity before interference can always be restored in the radial direction (divide $I_{0_{\text{max}}}$ by four), which makes it possible to restore the original single-component intensity through interference intensity by extracting the maximum intensity in rings in the radial direction. In the actual extraction, we do not need to divide $I_{0_{\text{max}}}$ by four because we will also normalize the entire intensity value when all extractions are over. Schematic representation of the ring extraction strategy is shown in Fig. 2 (a1-a4). In the interference intensity registered by CCD, we take multiple rings at equal intervals $d$ in the radial direction. In each ring, we take the maximum intensity in the ring and assign this value to the ring’s corresponding spatial position.
As the interval gradually decreases, the restored intensity distribution is closer to the original single component in SSOVs, as depicted in Fig. 2 (a2-a4).

In the rotational measurement strategy, we replace the actual phase-shifting operation by rotating the picture shown in Fig. 2 (b1). A schematic representation of the rotational measurement strategy is shown in Fig. 2 (a1-a4). Through a simple rotation operation, we can quickly obtain the four interference intensities required by Eqs.(2). It should be noted that the corresponding rotational angle of high-order SSOVs should be 45 / m degrees, 90 / m degrees, and 135 / m degrees, where m denotes TCs of a single component in high-order SSOVs.

![Schematic representation of the rotational measurement strategy.](image)

**Fig. 2 Schematic representation of (a1-a4) the ring extraction strategy and (b1-b4) the rotational measurement strategy.** The TCs of SSOVs here are ±1. (a1) SSOVs' intensity registered by CCD. White dotted lines denote the extracting and filling position. (a2-a4) Extracted intensities with d = 0.5mm, d = 0.1mm, and d = 0.005mm, respectively. (b1) SSOVs' intensity registered by CCD. (b2-b4) Rotated intensities at 45 degrees, 90 degrees, and 135 degrees, respectively. White dotted lines denote the axis of symmetry.

**D CALCULATING THE PHASE OF SINGLE COMPONENT IN SSOVS**

Theoretically speaking, after obtaining four intensity distribution results, the desired phase can be deduced inversely. However, due to trigonometric functions’ introduction, the desired phase is wrapped into $2\pi$ in the actual calculation. We cannot get the desired result by simply dividing by 2, as shown in Eqs.(3). Consequently, the phase unwrapping technology is required[52]. Firstly, we need to trim the phase to achieve maximum phase unwrapping efficiency, as depicted in Fig. 3 (b) and Fig. 3 (c). Finally, we can get the unwrapping phase by employed the phase unwrapping technology shown in ref.[52], as depicted in Fig. 3 (e1) and Fig. 3 (e2). What is implemented here is not phase unwrapping in the traditional sense: the phase is restored to a linear increase. The purpose of phase unwrapping here is to restore the phase to one-half of the calculated phase after Fig. 3 (b). We can directly perform phase unwrapping or perform windowed Fourier transform[53] (WFT) first and then perform phase unwrapping. The WFT is adopted here to filter the trimmed phase to reduce noise further and smooth the phase. The unwrapped phase obtained directly in the laboratory environment is ideal, but it does not mean that good results are maintained under complex conditions such as outdoors. Therefore, The WFT is optional according to the actual situation.
Fig. 3 Schematic representation of calculating required phases. The TCs of SSOVs here are $\pm 1$. (a1-a4) four rotated intensities SSOVs. (b) Calculated phase. The red dotted lines indicate the trimming area. (c) Trimmed phase. (d) Trimmed phase filtered by WFT (optional). (e1-e2) Unwrapped phase. The color bar 1 indicates the phase range. The color bar 2 indicates the range of intensities.

3. EXPERIMENTAL RESULTS

The experimental setup employed to generate a PPOV is shown in Fig. 4. The laser (NEWPORT N-LHP-151) delivers a collimated Gaussian beam with a wavelength of 632.8nm after a linear polarizer (LP), a half-wave plate (HWP), and a telescope consisting of two lenses (L1, L2) are used for collimation. The combination of the LP and the HWP is served to rotate the laser polarization state along the SLM’s long display axis and adjust the incident light’s power on SLM. The SLM (UPOLABS HDSLM80R) precisely modulates the incident light via loading a hologram mentioned above. The aperture (AP) is then used to select the beam’s first diffraction order to avoid other stray light. A CCD camera (NEWPORT LBP2) registers the intensity pattern after L4.
Experimental results of one-time measurement are shown in Fig. 5. In the proof-of-concept experiment, we select four typical SSOVs (one order, low order, high order, and radial node) to characterize the complex amplitude with only one picture registered by CCD shown in Fig. 5 (a1-d1). We obtain the intensity distributions of a single component in SSOVs through the ring extraction strategy, as shown in Fig. 5 (a3-d3). We get the phase distributions of a single component in SSOVs by rotational measurement strategy after unwrapping.

![Experimental setup for measuring mode purity of single component in superposed OVs.](image)

Fig. 4 Experimental setup for measuring mode purity of single component in superposed OVs. LP: linear polarizer. HWP: half-wave plate. L: lens. SLM: spatial light modulator. AP: aperture. CCD: charge-coupled device.

To prove the accuracy of the detection method, we selected two SSOVs to calculate the mode purity. The mode purity for an arbitrary field $\Psi$ can be calculated with\[54, 55\]:

$$\Gamma_p = \frac{\sum_{m} \sum_{n} C_{m}^{p}}{\sum_{p=0}^{+\infty} \sum_{n=-\infty}^{+\infty} C_{m}^{p}}$$

$$\Gamma_n = \frac{\sum_{m} \sum_{n} C_{m}^{n}}{\sum_{p=0}^{+\infty} \sum_{n=-\infty}^{+\infty} C_{m}^{p}}$$

(10)
where

\[ C_m^p = \int_0^\infty (a_m^p(r, \phi, z) \bar{a}_m^p(r, \phi, z))^2 \, dr \]  \hspace{1cm} (11)

\[ a_m^p(r, z) = 1/(2\pi)^{1/2} \int_0^{2\pi} \Psi(r, \phi, z) \text{LG}_m^p d\phi \]  \hspace{1cm} (12)

where \( \text{LG}_m^p \) denotes the LG beam. The calculated mode purity of a single component is shown in Fig. 6. Whether it is radial decomposition or angular decomposition, the model purity can be calculated by the one-time measurement method. Experimental results show that the angular mode’s purity (OAM spectrums) is higher than that of the radial mode (radial distributions), consistent with the more robustness of the angular distribution of an OV.

\[ \text{Fig. 6} \text{ Experimental distributions of single component mode purity. (a) Radial distributions and (b) OAM spectrums of } m = 3 \ (p = 0) \text{ and } m = 1 \ (p = 1) \text{ from SSOVs } m = \pm 3 \ (p = 0) \text{ and } m = \pm 1 \ (p = 1), \text{ respectively.} \]

4. DISCUSSION

A ANGULAR ERROR

Firstly, we demonstrate the rotational measurement strategy’s angular errors (new strategy) compared with the phase-shifting method (old method using four interferograms). The experimental results are depicted in Fig. 7. In the phase measurement of a single component in SSOVs, the low-order OV phase restored using the new strategy has a significant offset compared with the old method, as shown in Fig. 7 (a). However, the measured model purity is not much different. In the phase measurement of high-order OV, excellent performance in both phase distribution and measurement results are shown in Fig. 7 (b). The experimental results show the accuracy and robustness of the rotational measurement strategy.

\[ \text{Fig. 7} \text{ Comparison of the rotational measurement strategy (new strategy) with the phase-shifting method (old method) on the measurement of OAM spectrums. (a) OAM spectrums of } m = 1 \ (p = 1). \text{ (b) OAM spectrums of } m = 3 \ (p = 0). \]

B RADIAL ERROR
Secondly, we’d like to show the radial errors introduced by interval $d$ in the ring extraction strategy. The interval $d$ determines the restored degree of the intensity distribution. As the interval $d$ decreases, the intensity distribution is closer to the ideal situation (red line), as shown in Fig. 8 (a). Interestingly, although the interval affects the restored intensity distribution, it has little effect on the measurement results of model purity, which fully demonstrates this method’s robustness, as shown in Fig. 8 (b) and (c).

![Fig. 8](image)

**Fig. 8** The radial errors introduced by interval $d$ in the ring extraction strategy. The TC is 1 and the radial node is 0 here. (a) Intensity distribution curves intercepted at the white dotted lines shown in (d1-d3). The number identified in the upper right corner identifies the correlation coefficient between the current intensity distribution and the ideal intensity distribution of the single component in SSOVs. (b) Radial distributions. (c) OAM spectrums. (d1) Ideal intensity distribution. (d2) Restored intensity distribution with $d = 0.005\text{mm}$. (d3) Restored intensity distribution with $d = 0.1\text{mm}$.

**C EXPERIMENTAL ERROR**

Finally, we demonstrate the effect of experimental errors on the measurement. One is the effect of different exposure times on the measurement. The results show that as the exposure time increases, the measured mode purity decreases slightly, as shown in Fig. 9 (a). This can be explained that without affecting the sampling, the increase in exposure time causes the image’s noise to increase, so the mode purity is slightly reduced. Another is the effect of CCD tilt on the measurement. The results show that as the CCD tilt increases, the measured mode purity decreases somewhat, as shown in Fig. 9 (a). This can be explained that the tilt of CCD affects the symmetry distribution of SSOVs, which affects the accuracy of the phase calculation, as shown in Fig. 9 (b). The third one is the effect of the CCD shift on the measurement. The results show that as the CCD shift increases, the measured mode purity decreases slightly, as shown in Fig. 9 (c). Although the CCD shift does not change the SSOVs’ distribution, if we still perform mode decomposition at the position shown in 0 pixels, the mode decomposition’s accuracy will be reduced. The above three experimental errors cause 0.1% to 0.01% deviation of the measured mode purity. But overall, the experimental results show that the self-interference method has excellent robustness. Careful alignment and optimized error analysis allow us to generate and measure a single component with mode purity as high as 99.99%.

![Fig. 9](image)

**Fig. 9** The effect of the experimental error on the results. The effect of (a) exposure times, (b) CCD tilt, and (c) CCD shift on the measurement of the mode purity.
5. CONCLUSION

In summary, we propose a one-time measurement method to characterize the complex amplitude of SSOVs with only one picture registered by CCD based on phase-shifting technology, which is reported for the first time. Firstly, we introduce the phase-shifting theory of SSOVs. Secondly, we demonstrate the process of encoding the hologram. Thirdly, we demonstrate two strategies we proposed. One is the ring extraction strategy for amplitude measurement, and another is the rotational measurement strategy for phase measurement. Fourthly, we show the process of calculating the phase of a single component in SSOVs. In the proof-of-concept experiment, the complex amplitude is characterized, and the mode purity is well measured. Finally, we demonstrate the angular and radial errors on the experimental results and discuss the effect of experimental error of exposure time, CCD tilt, and CCD shift. The experimental results indicate that the one-time measurement has excellent flexibility, rapidity, and robustness, which can be applied to various occasions and harsh conditions.

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Disclosures

The authors declare no conflicts of interest.

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