Direct Spherical Azimuth Angles Determination

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Abstract: In this paper, we will see that the determination of direct bearing angles. As it is known, in bearing angles are often computed used formulas with arctan function. The arctan function gives an angle values between -90° and + 90°. However, the bearing angle is by definition 0° to 360°. Consequently, it is inevitable to examine the process of obtaining the azimuth angle.

Classic formulas only work correctly if the edge is in the 1st quarter. If the edge is located in the other quarters, the angles of the bearing should be examined. In this work we proposed a new formulas for direct bearing angles on globe (sphere). Using the formula that we propose will save execution time in codes with intensive geodesic calculations.

Keywords: Azimuth Angles, Sphere, Geographical coordinates

1. INTRODUCTION

For example, First Geodetic Basic problem; P1(φ1, λ1) the geographic coordinates of a point P1 are given in latitude longitude values , S₁₂ the geodetic curve length from point P1 to point P2 , A₁₂ the azimuth angle (bearingangle) of the length and desired P2(φ₂, λ₂) the geographic coordinates of a point P2. The azimuth A₂₁ is desirable which is corresponding A₁₂ azimuth angle. Because there is approximately 180 degrees difference between A₁₂ and A₂₁. Thus, the region of A₂₁ is easily predicted. If the two points are on the same meridian or the same parallel circle the difference between A₁₂ and A₂₁ is exactly 180° degrees [3],[6].

However, in the 2nd Geodetic basic problem; the geographic coordinates latitude and longitude values of the two points are given ; P₁(φ₁, λ₁), P₂(φ₂, λ₂)and required the geodesic curve length between the two points is S₁₂ and the corresponding azimuths A₁₂ and A₂₁ between the two points. The azimuth calculation is not as easy as in the 1st Geodetic basic problem assignment. If the A₁₂ azimuth is calculated incorrectly, the A₂₁ azimuth will also be incorrect by itself.

In this proposed method, formulas are given for how to obtain the azimuth angle directly without any examination. The given method can calculate azimuth without reducing the sphere and ellipsoid surface.

2. MATERIAL AND METHODS

Calculation of between the two points S₁₂ and the corresponding azimuths A₁₂ and A₂₁ from known P1,P2 points geographical coordinates is also called as geodetic 2nd basic problem solution (see Fig.1). Problem is solved classically with below formulas[1], [2], [4]
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Figure 1.

Classic Method:

\[ \sigma = \arccos(\sin \varphi_1 \sin \varphi_2 + \cos \varphi_1 \cos \varphi_2 \cos \Delta \lambda) \]

\[ A_{12} = \arctan \left( \frac{\sin \Delta \lambda}{\tan \varphi_2 \cos \varphi_1 - \sin \varphi_1 \cos \Delta \lambda} \right) \]

\[ A_{21} = \arctan \left( \frac{\sin \Delta \lambda}{\cos \Delta \lambda \sin \varphi_2 - \cos \varphi_2 \tan \varphi_1} \right) + \pi \]

Here \( \sigma \) is the angular equivalent of the edge. If you want to find the metric of the edge

\[ S = \sigma / \rho \cdot R \]

\( \rho = 180^\circ / \Pi \)

\( R \): radius of the earth

It is important to remember that these classic formulas only work correctly if the edge is in the 1st quarter. If the edge is located in the other quarters, the angles of the bearing angle should be examined. For correct angles, the necessary additions should be made according to the Table-1 below.

**Table 1. Fixed value to add for bearing angles**

| Quadrant     | fixed value to add for \( A_{12} \) | fixed value to add for \( A_{21} \) |
|--------------|-------------------------------------|-------------------------------------|
| 1.Quadrant   | -                                   | -                                   |
| 2.Quadrant   | +180°                               | +180°                               |
| 3.Quadrant   | +180°                               | -180°                               |
| 4.Quadrant   | +360°                               | -                                   |

Proposed Method

For direct determination of azimuth by geographic coordinates, we give below formulas. In this proposed method, formulas are given how to obtain the azimuth angle directly without any examination. The proposed method can calculate direct azimuth angles on the sphere and ellipsoid surface [3].

\[ I = \sin \Delta \lambda \]

\[ II = \tan \varphi_2 \cos \varphi_1 - \sin \varphi_1 \cos \Delta \lambda \]

\[ A_{12} = 2 \cdot \arctan \left( \frac{I}{I - \sqrt{I^2 + II^2}} \right) + 180^\circ \]

\[ III = \tan \varphi_1 \cos \varphi_2 - \sin \varphi_2 \cos \Delta \lambda \]

\[ A_{21} = 2 \cdot \arctan \left( \frac{I}{-III + \sqrt{I^2 + III^2}} \right) + 180^\circ \]
Numerical Example

To compare direct formula and classical formula results, from the point P1 to the point P2 which is located in different quarters each time, the second basic problem solutions were made and the bearing angles calculations were made.

\[ P_2(\varphi_1, \lambda_1) \] the geographic coordinates of a point P1 are given in latitude longitude values:

\[ \varphi_1 = 30^\circ, \lambda_1 = 30^\circ \quad R=6370000m \]

Required: \( s, A_{12}, A_{21} \)

If we use the above classic method formula and proposed method formula for the solution, for results please see Table-2

| Table 2. Direct formula and classical formula results |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|
| Quadrant | \( \varphi_2 \) | \( \lambda_2 \) | S \( A_{12} \) | \( A_{21} \) |
| 1 | 32 | 31 | 241911.948 | 22.94320 | 203.45833 | 22.94320 | 203.45833 |
| 2 | 29 | 32 | 233183.087 | -60.61890 | 120.36606 | 119.38110 | 300.36606 |
| 3 | 28 | 29 | 242683.026 | 23.86428 | 203.37939 | 203.86428 | 23.37939 |
| 4 | 32 | 29 | 241911.948 | -22.94320 | 156.54167 | 337.05680 | 156.54167 |

3. Result and Discussion

In this proposed method, formulas are given for how to obtain the azimuth angle directly without any examination. The given method can calculate azimuth without reducing the sphere and ellipsoid surface.

The numerical example that we have given shows the accuracy of the method we propose. The advantage of the method is that no examination is required. In computer calculations, if..end blocks are not used when direct formulas are used. The if..end blocks reduce the execution speed in computer calculations. For future studies, researchers are advised to try to find more simple direct formulas.

4. Conclusion

In this proposed method, formulas are given for how to obtain the azimuth angle directly without any examination. The given method can calculate azimuth without reducing the sphere and ellipsoid surface. Using the formula that we propose will save execution time in codes with intensive geodesic calculations.

References

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