Strong Correlation Effects on Surfaces of Topological Insulators via Holography

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We investigate effects of strong correlation on the surface state of topological insulator (TI). We argue that electrons in the regime of crossover from weak anti-localization to weak localization, are strongly correlated and calculate magneto-transport coefficients of TI using gauge gravity principle. Then, we examine, magneto-conductivity (MC) formula and find excellent agreement with the data of chrome doped Bi$_2$Te$_3$ in the crossover regime. We also find that cusp-like peak in MC at low doping is absent, which is natural since quasi-particles disappear due to the strong correlation.

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Introduction: Understanding strongly correlated electron systems has been a theoretical challenge for several decades. Typically, such systems lose quasi-particles and show mysteriously rapid thermalization [11–14], which provide the hydrodynamic description [5, 6] of them near quantum critical point (QCP). Recently, the principle of gauge-gravity duality [7, 8] attracted much interest as a possibility of the paradigm for strongly interacting systems, where the system near QCP is mapped to a black hole. More recently, large violation of Widermann-Frantz law was observed in graphene near charge neutral point, indicating that it is a strongly interacting system [10] in a window of temperature, and the gauge gravity principle applied to it exhibited remarkable agreement with the experimental data [11].

The fundamental reason for the appearance of the strong interaction in graphene is the smallness of the fermi sea: in the presence of the Dirac cone, when fermi surface is near the tip of the cone, electron hole pair creation from such a small fermi sea is insufficient to screen the Coulomb interaction. Because this is so simple and universal, one can expects that for any Dirac material, there should be a regime of parameters where electrons are strongly correlated. Dirac cone also provides the reason why it is a quantum critical system with Lorentz invariance. The most well known Dirac material other than the graphene is the surface of a topological insulator (TI) [12, 13]. The latter has an unpaired Dirac cone and strong spin-orbit coupling, and as a consequence, it has a variety of interesting physics [14, 16] including weak anti-localization (WAL) [17].

Magnetic doping in TI can open a gap in the surface state by breaking the time reversal symmetry [18–20]. and it is responsible for the transition from WAL to weak localization (WL). For extreme low doping, the sharp horn of the magneto-conductivity curve near zero magnetic field can be described by Hikami-Larkin-Nagaoka (HLN) function [21]. However, for intermediate doping where the tendency of WAL and weak localization (WL) compete, a satisfactory theory is still wanted [18, 20, 22] although there is a phenomenological description [23]. Even in the case the fermi surface is large at low doping so that the system is a fermi liquid, increasing the surface gap pushes up the dispersion curve, which makes the fermi sea small. Then, the logic for strong correlation in graphene works for transition region in surface of TI. Therefore electron system near the transition region should be strongly correlated.

In this paper, we investigate magneto-conductivity (MC) for the surface of a topological insulator with correlated electrons using gauge gravity principle. We will give analytic formulae of all the magneto-transport on the surface of TI with strong correlation as a function of magnetic field, temperature and impurity density and compare the result with Bi$_2$Te$_3$ data of [20]. Most interestingly, in the doping regime with crossover from WAL to WL, our theory agrees with experimental data nicely in a window of temperature justifying our suggestion that electrons in the experimented material are strongly correlated in this regime. Our results also show that the cusp-like peak in MC curve at fixed temperature, which is the hallmark of WAL in the weakly interacting system, is absent, which can be argued to be a consequence of strong correlation.

Idea of the model: Our system is the surface of topological insulator which is a 2+1 dimensional system with odd number of Dirac cones. Our question is what happens if such system has strong correlation as well and the recipe for strong electron-electron interaction is to use gauge gravity principle or holography. For TI, special care is necessary to encode strong spin-orbit coupling (SOC). Our holographic model is defined on a manifold $\mathcal{M}$ which is asymptotically AdS$_4$. With these setup, our model is defined by the action,

$$2\kappa^2 S = \int_{\mathcal{M}} d^4x \sqrt{-g} \left[ R + \frac{6}{L^2} - \frac{1}{4} F^2 - \sum_{I,a=1,2} \frac{1}{2} (\partial \chi_I^{(a)})^2 \right] - \frac{q_\Lambda}{16} \int_{\mathcal{M}} \sum_{I=1,2} (\partial \chi_I^{(2)})^2 F \wedge F$$

(1)

where $q_\Lambda$ is the coupling and $\kappa^2 = 8\pi G$ and $L$ is the AdS radius. From now on, we set $2\kappa^2 = L = 1$. The action contains two pairs of bosons, one for the magnetic impurities and the other for the non-magnetic ones. To encode the effect of SOC in the presence of the magnetic doping, we introduced the last term which is a coupling between the impurity density and the instanton density.
Such an interaction term was first introduced in [21] by us to discuss the SOC. The strong SOC provides the band inversion that induces massless chiral fermions at the boundary, which in turn induces the chiral anomaly as a nontrivial divergence of the chiral current. In fact, our interaction term is unique in that it is the leading order term that can take care of anomaly and its coupling to impurity in a manner with time reversal symmetry broken.

The solution of equation of motion is given by

\[ A = a(r)dt + \frac{1}{2}H(xdy - ydx), \]

\[ \chi_1 = a\left(\frac{x}{y}\right), \quad \chi_2 = \lambda\left(\frac{x}{y}\right), \]

\[ ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + r^2(dx^2 + dy^2), \]

with \( U(r) = r^2 - \frac{\alpha^2 + \lambda^2 - m_0^2}{2r} + \frac{q^2 + H^2}{4r^2} + \frac{\lambda^4 H^2 q_0^2}{20r^6} - \frac{\lambda^2 H q q_0}{6r^4}, \]

\[ a(r) = \mu - \frac{q}{r} + \frac{\lambda^2 H q_0}{3r^3}, \]

where \( \mu \) is the chemical potential, \( q \) is charge carrier density. \( q \) and \( m_0 \) is determined by the regularity condition at the black hole horizon, \( A_1(r_0) = U(r_0) = 0 \).

\[ q = \mu r_0 + \frac{1}{3} \theta H \quad \text{with} \quad \theta = \frac{\lambda^2 q_0}{r_0^2} \]

\[ m_0 = r_0^3 \left(1 + \frac{r_0^2 \mu^2 + H^2}{4r_0^2} - \frac{\alpha^2 + \lambda^2}{2r_0^2} + \frac{\theta^2 H^2}{45r_0^2}\right). \]

Usually the chemical potential is proportional to the charge density. However, in our model, there is extra term \( \sim \theta H \), which represents the Witten effect, the magnetic field generation by electric charge and vice versa. It comes from the last term of the action whose microscopic origin is the spin-orbit interaction [25, 26].

The temperature of the physical system is identified by the Hawking temperature of the black hole given by

\[ T = \frac{12 r_0^3}{16 \pi r_0^3} \left[ H^2 + 2 r_0^2 (\alpha^2 + \lambda^2) + (q - H \theta)^2 \right] \]

and the entropy and energy densities are given by \( s = 4 \pi r_0^2 \), \( \epsilon = 2 m_0 \) respectively. Since the boundary on-shell action is related with pressure by \( S_{\text{onshell}} = -P \), we get \( \varepsilon + P = s T + \mu q \). Then, the magnetization can be obtained from \( M = -\frac{\partial \mu}{\partial H} \).

**DC transport coefficients** can be calculated by turning on small fluctuations around background [3, 22];

\[ \delta G_{ii} = -tU(r)\zeta_i + \delta g_{ii}(r), \quad \delta G_{ri} = r^2 \delta g_{ri} \]

\[ \delta A_i = t(-E_i + \zeta_i a(r)) + \delta a_i(r), \]

where \( i = x, y \). Notice that equations of motion for fluctuation are time-independent, although there is explicit time dependence in above ansatz. Here \( E_i \) corresponds to the external electric field and \( \zeta_i = -\partial_i T/T \). We define bulk currents by

\[ J^i = \sqrt{-g} F^{ir}, \quad Q^i = U(r)^{2} \partial_r \left( \frac{\delta g_{ii}(r)}{U(r)} \right) - a_i(r) J^i. \]

which become the electric and the heat current \( J^i, Q^i = (T^{ii}) - \mu J^i \) respectively at the boundary \( (r \rightarrow \infty) \). Using the equations of motion of the fluctuation fields together with the horizon regularity condition, we can get electric and heat current at the boundary in terms of the external sources;

\[ J^i = \frac{\mathcal{F} + G^2}{\mathcal{F}^2 + H^2 G^2} E_i \]

\[ \left( \frac{\theta + H \mathcal{G}(2 \mathcal{F} + G^2 - H^2)}{\mathcal{F}^2 + H^2 G^2} \right) \epsilon_{ij} E_j + \frac{s T \mathcal{G}(F - H^2)}{\mathcal{F}^2 + H^2 G^2} \zeta_i + \frac{s T \mathcal{H}(F + G^2)}{\mathcal{F}^2 + H^2 G^2} \epsilon_{ij} \zeta_j \]

\[ Q^i = \frac{s T \mathcal{G}(F - H^2)}{\mathcal{F}^2 + H^2 G^2} E_i + \frac{s T \mathcal{H}(F + G^2)}{\mathcal{F}^2 + H^2 G^2} \epsilon_{ij} E_j \]

\[ + \frac{s T^2 T^2 \mathcal{F}}{\mathcal{F}^2 + H^2 G^2} \zeta_i + \frac{s T^2 T^2 \mathcal{H}^2 \mathcal{G}}{\mathcal{F}^2 + H^2 G^2} \epsilon_{ij} \zeta_j, \]

where \( \zeta_i = -(\nabla_i T)/T \) as before and

\[ \mathcal{F} = r_0^2 (\alpha^2 + \lambda^2) + (1 + \theta^2) H^2 - q \theta H \]

\[ \mathcal{G} = q - \theta H. \]

Now, the transport coefficients can be read off from

\[ \left( \begin{array}{c} J^i \\ Q^i \end{array} \right) = \left( \begin{array}{cc} \sigma_{ij} & \alpha_{ij} T \\ \tilde{\alpha}_{ij} T & \tilde{\kappa}_{ij} \end{array} \right) \left( \begin{array}{c} E_j \\ \zeta_j \end{array} \right). \]

In \( q_\lambda \rightarrow 0 \) limit, Eq. (6) are reduced to those of dyonic black hole [25, 31]. There are two important symmetries of the DC conductivities: one is the anti-symmetry of the off-diagonal components, i.e., \( X_{ij} = -X_{ji} \) for all \( X = \sigma, \alpha, \kappa \) and the other is \( \alpha_{ij} = \tilde{\alpha}_{ij} \), which is Onsager’s relation. If we further take \( H \rightarrow 0 \) limit,

\[ \sigma_{xx} \rightarrow 1 + \frac{q^2}{r_0^2 (\alpha^2 + \lambda^2)}. \]

Notice that if we define \( \beta^2 = \alpha^2 + \lambda^2, \gamma = \frac{\lambda^2}{\alpha^2 + \lambda^2} \), then \( \beta^2 \) plays the role of the total impurity density used in [24], and \( \lambda^2 \) and \( \alpha^2 \) can be interpreted as the magnetic and non-magnetic impurity density respectively. Therefore \( \gamma \) corresponds to the magnetic doping parameter, which is usually denoted by \( x \) in the literature.

**Magnetono-conductance:**

To compare our results with the data for the non-ferro magnetic material, we take \( \mu = 0 \) to set the ferromagnetic
magnetization zero. The longitudinal conductivity in this limit is

$$\sigma_{xx} = \frac{(F + G^2)(F - H^2)}{F^2 + H^2G^2}. \quad (13)$$

The MC is defined by $\Delta\sigma = \sigma_{xx}(H) - \sigma_{xx}(0)$.

Consider the evolution of the system with the doping. As the surface gap increases, the size of the fermi surface decreases. See figure 1(a). At $x=0.08$ gap is large enough to see transition from WAL to WL for some temperature, but fermi surface is still large so that particle character remains. At $x=0.1$, gap is large enough and fermi surface is small enough to show strong coupling behavior, so that our theory is well applicable. Figure 1(b) shows the evolution of MC curve as we raise the doping rate assuming that entire regime can be described holographically. However the real system is strongly correlated only when fermi surface is small enough. Therefore we expect that our theory is valid only in a window of doping rate as well as that of temperature. This is indeed what happens. In figure 1(b), the green color indicates the validity island in parameter space of $(\gamma, H)$, where our theory agrees with experimental result of ref.[20].

FIG. 1. Evolution of (a) density of state and (b) MC as we vary the doping. Again, our theory fit data only in an island of parameter space $(H, \gamma)$, where $\gamma = x$.

As we discussed earlier, the problematic part of the data fitting in weakly interacting picture is the medium doping regime $x \sim 0.1$ where the transition between the WAL to WL is smooth. Does our theory fit data in such region? The answer is given in figure 2, where we took the data for $x = 0.1$. Here again, our theory is valid only in an island of parameter space $(H, T)$. There are only 4 adjustable parameters: $\gamma, \beta, \xi, v_F$. Others $(T, H, \mu)$ are plot variables. From the data fitting point of view, the 1.9 K data is difficult to fit because it has too steep curvature near zero magnetic field $H = 0$. If we fit it for small field region, medium and large field regions are not fit at all. We believe that at $T = 1.9K$ the fermi surface is still not small enough and our theory can not fit such weakly interacting regime by tuning all 4 parameters.

Another important question is whether our result is universal, namely, independent of details of the matter. To answer this question at least partially, we worked out two materials in the validity islands which is shown in figure 3(b). Figure 3(a) shows a remarkable similarity in MC curves for different TI material. The transition behavior seems be universal and well described by our theory.

FIG. 2. (a) Theory v.s data (circle) for $x=0.1$. $T=1.9K$ is in fermi liquid regime where our theory does not work. (b) $\Delta\sigma$ as function of $H$ and $T$. Our theory works in the green colored island of $(H, T)$ space, where the system is strongly correlated. We used $\beta^2 = \frac{2\gamma_{\text{eff}}}{(\mu m)^2}, v_F = 7.5 \times 10^4 m/s, q_x = 7.12$.

FIG. 3. Universality of transition behavior: two different materials are described by the same analytic expression with different parameter values. (a) MC for Cr doped $Bi_2Te_3$ (left) and for Mn doped $Bi_2Se_3$ (right). The data are from ref. [20] and [19] respectively. (b) strong correlation islands for the two. $Bi_2Se_3$ has bigger island due to the bigger bulk gap.

In weakly interacting picture, the non-trivial behavior of magneto-conductivity in crossover regime is understood by the competition between anti-localization induced by spin-orbit coupling and the localization by surface gap. In holographic picture, the enhancement in conductivity can be understood as magneto-electric effect or Witten effect. The interaction term dictates that external magnetic field generates extra charge carriers $\delta q \sim \theta H$ to increase the conductivity. The result of the competition is the sign change in the curvature of MC curve near $H = 0$, where

$$\Delta\sigma \sim -\frac{2(1 - 4\theta^2/\theta_0^2)}{r_0^2 \beta^2} H^2 + O(H^4). \quad (14)$$

and $\theta = q_x \beta^2/r_0^2$. It also explains why crossover from WAL to WL appears only in relatively low but not very low temperature region, because $r_0 \sim T$ for high temperature and $\theta$ becomes small so that $1 - 2\theta/3$ cannot
change the sign. This can be more precisely stated in terms of the phase diagram which is drawn in Fig2(b). Notice that there are only 2 phases. If $\gamma_\chi > 1/4$, there is always a phase transition from WAL to WL.

Predictions: Finally we give a list of prediction coming from our theory that can be testable by experiments.

- Near Dirac point of small doping, we will find transport anomaly, large violation of Wiedemann-Franz Law just like graphene.

- For undoped or weakly doped TI, where one normally see a sharp peak, the characteristic of weak anti-localization. We predict that if one looks at near Dirac point by adjusting fermi surface by gating for example, one will see the disappearance of the sharp peak as we move down the fermi surface.

- We claim that the transition behavior from WAL $\rightarrow$ WL in the medium doping is universal: namely, magnetic conductivity of all two dimensional Dirac material with broken TRS, can be described by our formula, independent of the detail of the system. Here, we gave only two examples: the Mn doped $Bi_2Se_3$ in figure 2(a).

- For $Cr_xBi_{2-x}Te_3$ with $x = 0.1$ where the system in our picture is strongly interacting for $T \geq 2K$, we expect that ARPES data will show fuzzy density of state(DOS). This means that DOS will be non-zero in the region between dispersion curves, where quasi-particle case would show empty DOS leading to the gap.

- All magneto-transport coefficients other than magneto-conductivity are predictions: That is we calculated all the transport coefficients: heat transports thermo-electric power as well as magneto-conductance. Once we determine all the coefficients using MC data, all other transport results are predictions. It is prediction for several observables as functions of multi-variables $(B, T, \gamma)$, containing a huge set of data.

Future directions: In this letter, we examined the zero charge sector only. Nonzero charge parameter $q$ will be discussed in followup paper. Other transport coefficients like thermal conductivities and Seebeck coefficients with or without magnetic fields are also important aspects that request future investigations. The graphene has even number of Dirac cones, weak spin-orbit interaction and different mechanism for WL/WAL. Because of such differences, we need to find other interaction term in holographic model for graphene. It is also interesting to classify all possible pattern of interaction that provides the fermion surface gap in the presence of strong e-e correlation in our context.

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