Prospects of detecting the nonlinear gravitational wave memory

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In GW150914, approximately 3\(M_{\odot}\) were radiated away as gravitational waves from the binary black hole system as it merged. The stress energy of the gravitational wave itself causes a nonlinear memory effect in the detectors here on Earth called the Christodoulou memory. We use an approximation that can be applied to numerical relativity waveforms to give an estimate of the displacement magnitude and the profile of the nonlinear memory. We give a signal to noise ratio for a single GW150914-like detection event, and by varying the total mass and distance parameters of the event, we find distances and source masses for which the memory of an optimally oriented GW150914-like event would be detectable in aLIGO and future detectors.

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I. INTRODUCTION

Recently reported observations [1–7] of gravitational waves in aLIGO consist of black hole binaries and one neutron star binary system. Binary systems which lose components are known to produce a memory [8–10]. Gravitational bremsstrahlung results in a linear memory in a detector far from the source: a permanent displacement between freely falling test masses that grows as the wave passes and persists even after the wave has passed. Christodoulou (by using the full nonlinear theory of relativity) [11] and Blanchet and Damour (by using a post-Minkowskian scheme) [12] independently discovered that the difference in relative position between ideal (freely falling) test masses long before and long after any gravitational wave has passed has a detector nonzero. This memory effect is known as the Christodoulou or nonlinear memory.

This is contrary to the standard picture of a gravitational wave that one usually imagines: a ring of particles subject to a plane gravitational wave will oscillate either in a plus or cross polarization pattern and then come back to its original orientation as a ring. In reality, the ring does not return to its original position but is instead left in a residual polarization state as in Figure 1. Two particles on the ring will either be closer or further apart depending on the sign of the memory. Knowledge of this sign clearly requires knowledge of the polarization state of the oscillatory part of the gravitational wave.

Thorne found that the oscillatory part of the gravitational wave causes the nonlinear memory by considering the wave to be made of gravitons causing a linear memory as they escape from the system [13]. Indeed, a compact binary system loses components in the form of gravitational radiation (about three solar masses in GW150914) thereby causing a nonlinear memory. Since the stress energy of the oscillating gravitational wave as it escapes to infinity causes the nonlinear memory, it can be thought of as the “wave of the wave” [14].

One may worry that LIGO is insensitive to permanent or DC changes in its arms. How then might we see such an effect? LIGO contains stationary masses held in place, but the detectors will still be able to detect the changing strain caused by the buildup of the memory [15]. Provided the compact binary system is close enough and the inclination angle is optimal, the nonlinear memory could be directly detectable in ground based detectors if enough of the change associated with the memory occurs on a timescale \(\tau \approx 1/f_{\text{opt}}\) where \(f_{\text{opt}}\) is the frequency of the detector’s peak sensitivity [13].

Prior to the first detection, stellar mass black holes were thought to have masses of at most 10\(M_{\odot}\). With each subsequent detection, we have found that these tens of solar mass systems are more common than previously thought. Because the detected systems have been so large, the question has arisen whether the nonlinear memory could be detectable with current sensitivities in ground based detectors [15].

After GW150914, Lasky, Thrane, Levin, Blackman, and Chen (LTLBC) [15] used Favata’s minimal waveform model (MWM) [16] to find a nonlinear memory waveform. They found a signal to noise ratio (SNR) of 0.42 for an optimally oriented source modeled by the MWM in aLIGO at design sensitivity. A signal with this SNR is not detectable without a clever scheme of adding sub-

FIG. 1: Two particles from a ring of particles as a plane, plus polarized gravitational wave passes perpendicular to the page. The nonlinear memory is shown here as a residual plus polarization after the wave has passed.
threshold signals as discussed in their paper.

Computation of the nonlinear memory from numerical waveforms has proved difficult, but some calculations have been done for equal mass binary systems [17]. Extraction of the memory waveform from numerical data to leading order requires two numerical integrations of the $l = 2$, $m = 0$ part of the Weyl scalar $\psi_4$, the typical output of a numerical simulation. Each integration increases the amplitude of the numerical error until it swamps small, low frequency effects in the signal such as the nonlinear memory [18]. Current attempts use Cauchy characteristic matching (see Sec 6.2 in [19]) to attempt to get more accurate modes containing the memory. However, there is always a piece that is missing from these calculations due to the computation extending only to finite times in the past (we miss the entire inspiral phase before our simulation starts).

While not likely to be detectable by aLIGO, advanced Virgo (AdV), or KAGRA (a Japanese, cryogenic, underground detector which will be operational around 2018 [20]), one may wonder what the odds of detection are with future detectors. Might we be able to detect the memory as strain sensitivity increases in ground based detectors? Third generation, ground based detectors including the Einstein Telescope (ET) reduce seismic noise by being set up deep underground. The arms of ET are planned to be 10 km long in a triangular geometry with three detectors each comprised of a low frequency, cooled detector and a high frequency detector [21]. On the same timescale is the Cosmic Explorer (CE) which has an “L” shape like aLIGO and has 40 km long arms [22].

Current ground based gravitational wave detectors hit a wall of seismic noise at about 10 Hz. Given that the memory is primarily a low frequency effect, perhaps space based detectors sensitive in the decihertz frequency regime could detect the memory. A Japanese, space based detector, DECIGO, is proposed to launch on a timescale similar to LISA [23]. This gravitational wave detector has three 1000 km arms set up in a triangular pattern and uses differential Fabry-Perot interferometry. However, this timescale includes launching DECIGO pathfinder in 2015, which did not happen. It is sensitive to signals around the decihertz frequency range, filling the frequency gap between the LISA and LIGO detectors. Space based transponder type detectors such as the planned LISA mission are sensitive in the mHz regime and will detect memories of larger binary systems such as extreme mass ratio inspirals (EMRIs) [16]. Many decades from now, we may see the Big Bang Observer (BBO) launched. BBO consists of smaller LISA type detectors situated in specific “constellations” around the sun [24]. DECIGO and BBO have strong sensitivity in regions that make them suitable for detecting the memory from a GW150914-like event.

Pulsar Timing Arrays (PTA) are sensitive to even lower frequencies than space based interferometers and are checked for memory signals from supermassive black hole binaries [25]. Among the currently operating PTA groups are the Parkes Pulsar Timing Array (PPTA), the European Pulsar Timing Array (EPTA), and the North American Nanohertz Observatory for Gravitational Waves (NANOGrav). For a recent review of these collaborations, see [26]. The International Pulsar Timing Array Group aims to combine the observational data from each group to get even better sensitivity and a greater number of pulsars [27]. In the future the Square Kilometer Array could detect many more pulsars to be used for data analysis and push sensitivity curves even deeper into the noise [28].

The nonlinear memory is interesting as a purely strong-field gravitational effect. As such, its effects are dependent on the form of Einstein’s equations and therefore are useful in theory testing. For example, theories which include scalar fields also contain extra memory modes [29]. Further, since the rise time of the memory is related to the radii of the compact binary constituents, detection of neutron star binary memory could give independent insight into the equation of state by picking a mass-radius relation and calculating the memory [30].

In this paper, we aim to give an approximation of the memory and its profile. We apply Thorne’s formula (Equation (3) from [13]) to a numerical relativity waveform (Section II). Using this model results in a calculation that is easy to use and computationally cheap. We calculate the memory for GW150914 (Section II). Next, the memory obtained from this calculation can be used to give a signal to noise ratio for a given detector (Section III). Finally, we vary the mass and distance parameters on a GW150914-like event to find the distance and total source mass for which an event would be detectable in several current and future detectors (Section IV). Throughout this paper we use geometric units ($G = c = 1$).

II. THE NONLINEAR MEMORY OF GW150914

Thorne gives a formula for “practical computations” of the memory in [13],

$$h_{\text{mem}}(t) = \frac{2}{r} \int_{-\infty}^{t} dt' \int d\Omega' \frac{d^2E}{dt'd\Omega'} (1 + \cos \theta') e^{2i\phi'},$$  \hspace{1cm} (1)

where $r$ is the distance from source to detector, and $d\Omega'$ is the solid angle. By expanding the energy flux in terms of spherical harmonics and integrating out the angular dependence, we can approximate the flux as a prefactor multiplied by the Isaacson stress energy. The full calculation is given in Appendix A and gives an approximation for the memory in optimal orientation,

$$h_{\text{max}}(t) = \frac{r}{4\pi} \int_{-\infty}^{t} dt' h_{+}^2,$$ \hspace{1cm} (2)

where $h_{+}$ is the plus polarization of the oscillatory gravitational waveform from a numerical relativity simulation.
The memory amplitude scales depending on inclination \( \Phi(\Phi) \) as

\[
\Phi(\Phi) = \sin^2 \Phi \left( 1 - \frac{\sin^2 \Phi}{18} \right).
\]

(3)

This prefactor shows larger memory effects for edge-on binary systems in contrast to the primary oscillatory wave which is strongest from face-on binaries. In systems that exhibit maximum memory \( \Phi(\Phi) = \pi/2 \), we will see only half of the maximum \( h_+ \) polarization and none of the \( h_\times \) polarization from the oscillatory part of the gravitational wave. The memory effect is not present in face-on systems \( \Phi(\Phi) = 0 \). For GW150914, \( \Phi(\Phi) = 2.856 \) rad \[31\]. Using \( \Phi(\Phi) \), we can find the memory amplitude at any inclination:

\[
h_{\text{mem}}(t) = \Phi(\Phi) \frac{r}{4 \pi} \int_{-\infty}^{t} dt' h_+^2.
\]

(4)

By using the method summarized above on the numerical relativity data given by a waveform generated with SEOBNRv4 \[32\] using the PyCBC python package \[33, 34\] and the LIGO Algorithm Library (LAL), the non-linear memory can be calculated for GW150914 as shown in Figure 2. This waveform uses the averaged parameters given in the spin precessing parameter estimation paper \[31\].

![FIG. 2: Numerical relativity waveform for GW150914 generated with PyCBC is shown in the upper plot. The memory is shown with both optimal inclination and with inclination estimated in [31] in the lower plot with maximum amplitude even after the gravitational wave has passed. The “wiggliness” of the waveform has been discussed in Appendix C of [35] and is caused by disregarding the average after taking the time integral (see Appendix A).](image)

III. FINDING THE SIGNAL TO NOISE RATIO

The signal to noise ratio (SNR) denoted \( \rho \) can be found by using a Fast Fourier Transform (FFT) routine on the memory waveform. Assuming our template is accurate,

\[
\rho^2 = 4 \int_{0}^{\infty} \left| \tilde{h}(f) \right|^2 \frac{1}{S_n(f)} df,
\]

(5)

where \( \tilde{h}(f) \) is the memory signal in the frequency domain, and \( S_n(f) \) is the one sided noise power spectral density (PSD) for the detector used to detect the memory signal. These noise curves can be found in the web application listed in the abstract of \[37\] with the exception of NANOGrav, which may be found in the 11 year data set \[38\]. Outside of the frequency range for which any gravitational wave detector noise curve is given, we take the SNR to be zero. In this paper, we take a SNR of 3 to be the threshold for detection, though if the memory was detected in coincidence with the associated waveform, a SNR of 2 might give sufficient confidence to claim detection.

A. SNR for GW150914-like events in current and future detectors

As has been discussed in the paper by LTLBC \[15\], the memory is not likely to be detectable in aLIGO without adding several detections together. We agree with their results with a calculated SNR from the of 0.45 for an optimally oriented memory source (compared with an SNR of 0.42 found in \[15\]). Therefore, we need a closer event
to detect the memory with aLIGO’s current sensitivity. However, as can be seen in Table I, both third generation ground based detectors will be able to detect the nonlinear memory effect.

From Table I, it is apparent that eLISA and LISA will do no better than ground detectors in detecting an event like GW150914. Both space based decihertz detectors, DECIGO and BBO, will certainly be able to see the memory. This is probably due to the frequency band being optimal for this signal. Additionally, these detectors have impressive projected sensitivities. Even with sensitivity reduced by a factor of 10, DECIGO would still see the memory from a GW150914-like event. Neither decihertz detector has a production timeline set, but DECIGO has an optimistic launch date as early as 2027 [23]. This launch date expected DECIGO pathfinder to launch in mid 2015 as a precursor to DECIGO. Given that this did not take place, the timeline should be adjusted accordingly.

### IV. GW150914-LIKE EVENTS WITH VARYING MASS AND DISTANCE

Since current detectors have little chance of detecting any single GW150914-like event memory, we now vary parameters on the event to find what distance and total mass $M$ would be needed for a given detector to achieve a detection. All points on the grid use the same template with varied total mass and distance yielding a similar system in a different frequency regime or reducing the SNR as distance increases. Figures 5, 6, and 7 show the SNR for a given total mass, distance combination. In the cases where it is relevant, the plots show a nearby plot extending to 2000 Mpc in distance on the left, and a plot that extends to the edge of the observable universe on the right. There are contours for $\rho = 3$, $\rho = 5$, and $\rho = 8$ on these plots to show the threshold for detection in each detector. Reported events (and one trigger) are shown along with grid lines connecting events to their estimated distance and total mass parameters. However, GW170817 is a neutron star binary while the SNR values are given based on a binary black hole system. Therefore, we expect the actual memory SNR to be less than projected on the plot and moved further to the right. All SNR values here are calculated assuming a comparable mass binary system. As the frequency band gets lower, the mass ratio of the two black holes may become more extreme. However, for higher mass ratios, the memory is less prominent [16].

Figure 5 shows current and near future detectors unable to detect the nonlinear memory further away than 250 Mpc. Given that the majority of black hole related events so far are further away than that, it seems unlikely that we will see the memory with current ground based detectors. However, outlook for future ground based detection of the memory is positive. Both proposed third generation detectors improve sensitivities and visibility distance significantly.

Even if we don’t see the nonlinear memory from current ground based detectors, it should be visible in a different regime with LISA. Supermassive black hole mergers with total mass on the order of $10^9 M_\odot$ give a promising source for memory detections as can be seen in Figure 6. DECIGO and BBO will be able to see optimally oriented memory from all of the recently reported sources. These detectors also open the exciting possibility of detecting neutron star binary gravitational nonlinear memory.

Pulsar timing arrays NANOGrav, EPTA, and IPTA have not reported detection yet. There is thought to be an upper bound of supermassive black hole mass at around $10^{10} M_\odot$ [39]. The fact that no memory has been seen in such detectors supports this as shown in Figure 7 where it can be seen that comparable mass binary systems with this mass would produce a memory which should be visible to current pulsar timing arrays.

From these figures, we can track the horizontal line from the GW150914 event until it connects with the $\rho = 3$ contour. Using the distance from this procedure and event rates from [40] of $17 \text{ Gpc}^{-3} \text{yr}^{-1}$ events like GW150914, we can see how long we can expect to wait for a detectable GW150914-like memory event. This is summarized in Table II.

| Detector | Distance (Mpc) | time/event |
|----------|---------------|------------|
| aLIGO    | 65            | 409 years  |
| ET       | 1430          | 2 weeks    |
| CE       | 4070          | 14.5 hours |
| DECIGO   | 14100         | 21 minutes |
| BBO      | 26000         | 3.5 minutes|

### V. CONCLUSION

Gravitational wave astronomy is in its infancy. Now that the primary oscillatory wave has been detected, we look toward nonlinear parts of the gravitational wave.
Current ground based detectors are unlikely to see the nonlinear memory without a clever stacking scheme as in [15]. The outlook for future detectors is positive: third generation ground based detectors (see Figure 5), space based detectors (see Figure 6), and pulsar timing arrays (see Figure 7) can all detect the nonlinear memory in different frequency regimes. DECIGO in particular yields interesting prospects for detecting the nonlinear memory from a neutron star binary system. Given that the rise time of the memory could be related to the radius in some way [30], the memory yields an independent method of constraining the neutron star equation of state. Future funding applications for newer, better gravitational wave detectors should include detecting the nonlinear memory and its applications as a science goal.

VI. ACKNOWLEDGMENTS

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This work has made use of the LIGO Algorithm Library (LAL) and the PyCBC Python package.

Appendix A: Calculation of the nonlinear memory

Following the derivations for the nonlinear memory in [14, 41], we begin with the relaxed Einstein field equation in the harmonic gauge,

$$\Box h^{\mu\nu} = -16\pi \tau^{\mu\nu}. \quad (A1)$$

where $\tau^{\mu\nu}$ contains the stress-energy $T^{\mu\nu}$, the Landau-Lifshitz pseudotensor $t^{\mu\nu}_{LL}$, and some pieces of $O(h^2)$. Assuming a flat background spacetime and ignoring the other pieces of $\tau^{\mu\nu}$, we focus on this part that sources the memory,

$$T^{\mu\nu}_{jk} = \frac{1}{r^2} \frac{d^2 E}{dt' d\Omega'} \xi_j \xi_k, \quad (A2)$$

where $\xi^i$ is a unit vector pointing from the source to solid angle $d\Omega'$ and $d^2 E/dt' d\Omega'$ is the gravitational wave flux. Solving equation (A1) by using the retarded Green’s function yields the standard result

$$\tilde{h}_{jk} = 4 \int T^{jk}_{\text{mem}}(t' - |x - x'|, x') \frac{1}{|x - x'|}. \quad (A3)$$

Next, we use a method of direct integration of the relaxed Einstein equations (DIRE) [42]. This method changes the coordinates from Cartesian to spherical and the radial coordinate $r'$ to retarded time $u' = t' - r'$. We then integrate with respect to retarded time in the wave zone. This process yields

$$\tilde{h}_{jk} = \int_{-\infty}^a du' \int \frac{dE}{d\Omega' d\Omega'} \frac{\xi_j \xi_k}{1 - \cos \theta'} d\Omega' \quad (A4)$$

where $u = t - r$ and $\xi' = x'/r'$. This is Equation (4) of [14]. We now specialize to a gravitational wave burst passing a detector at fixed $r$. Using the limiting procedure in [14] and transforming to transverse traceless gauge,

$$\tilde{h}^{TT}_{jk} = 4 \int_{-\infty}^t dt' \int \frac{d^2 E}{dt' d\Omega'} \left( \frac{\xi_j \xi_k}{1 - \cos \theta'} \right) \frac{TT}{TT} d\Omega' \quad (A5)$$

From here Thorne suggested a system of coordinates for “practical computations” and found a formula for the nonlinear memory. Equation (3) from [13] modified by replacing $dE/d\Omega'$ with $\int_{-\infty}^t (d^2 E/dt' d\Omega') dt'$ as in [30] is

$$h_{\text{mem}} = \frac{2}{r} \int_{-\infty}^t dt' \int d\Omega' \frac{d^2 E}{dt' d\Omega'} (1 + \cos \theta') e^{2i\phi'} \quad (A6)$$

where $r$ is the distance from the source to the detector, $t$ is some time after the wave has passed, and $\Omega$ is the solid angle. Notice that we are integrating over the entire history of the wave until now, hence the name “memory.” An expansion in terms of the spin-weighted spherical harmonics,

$$\frac{d^2 E}{d\Omega' dt'} = \frac{r^2}{16\pi} \sum_{l, l', m, m'} \langle \tilde{h}_{lm} \tilde{h}^*_{l'm'} \rangle - 2Y_{lm} - 2Y^*_{l'm'} \quad (A7)$$

allows one to separate the angular piece from the temporal piece. Here the $-2Y_{lm}$ are the spin-weighted spherical harmonics, and the * denotes complex conjugation. Using Equation (A7) in Equation (A6), we find the selection rule $m + m' = -2$. After performing the angular integration,
Appendix B: A note on the form of the memory and “wiggliness” seen in Figure 2 as discussed in [35].

Because of the issues with directly calculating $h_{20}$ modes of the gravitational wave discussed in Section I, we aim for an approximation. We change all smaller modes to the dominant $h_{22}$ piece of the waveform. What we would like to use here is the $(l, m) = (2, 2)$ mode that exists at the source. Instead, we are given the mode in the detector’s frame. Then,

$$h_{\text{mem}}(t) = \frac{r}{8\pi} \int_{-\infty}^{t} dt' \left( \sqrt{\frac{2}{3}} \langle \tilde{h}_{22} h_{22}^* \rangle + \sqrt{\frac{1}{6}} \langle \tilde{h}_{2-2} h_{20}^* \rangle + \frac{2}{3} \langle \tilde{h}_{2-1} h_{21}^* \rangle \right).$$

(A8)

This equation can be compared to Equation (21) in [41] from which we find the prefactor off by a factor of about 2 (after the inclination has been put in terms of $\sin^2 \nu$). The origin of this factor is unknown, but it does not significantly alter the results in a side-by-side comparison. For a direct approximation from the gravitational waves given in a data release, we can replace the dominant mode with the Isaacson stress-energy,

$$h_{\text{mem}}(t) = \frac{r}{4\pi} \int_{-\infty}^{t} dt' \langle \tilde{h}_{22}^2 \rangle.$$  

(A9)

This is one of the functions used in the approximation in [30]. A step function has a Fourier transform that is proportional to $1/f$. As mass increases in the system, we expect this signal in the frequency domain to increase as it comes into our detector’s band and then decrease as it goes out. However, this is not what was found in the ground and space based detectors (Figures 5, 6).

The signal in the frequency domain is only well approximated by a step function at low frequencies. The slow step response before merger happens on the radiation reaction timescale and the fast step response after merger happens on the merger timescale. These correspond to features that show up in the frequency domain.

Here we use a characteristic strain convention [37],

$$[h_c(f)]^2 = 4f^2 |\tilde{h}(f)|^2$$

and

$$[h_n(f)]^2 = f S_n(f).$$

This is a useful convention for plotting the memory signal over the noise curve, because the area between the two curves on the plot is now proportional to the SNR. However, we must be careful to remember that now a $1/f$ curve on the plot will be a constant. Looking at 4, we can see that lower frequencies do behave as $1/f$, but higher frequencies do not! Instead there is a local minimum in the plot and then it falls off completely shortly afterward.

As the mass increases, we find that the curve moves up and to the left. The SNR will increase until the local minimum hits the lowest frequency the detector can see and then it will increase rapidly and fall to zero shortly after. This is the behavior that is seen in Figures 5, 6.

FIG. 4: Heuristic plot showing the memory signal and a noise curve using the characteristic strain convention. Arrows show the direction the memory moves when changing the total mass, M, and the distance from the source, d.
[1] B. P. Abbott et al., Phys. Rev. Lett. 116, 061102 (2016).
[2] B. P. Abbott et al., Phys. Rev. Lett. 116, 241103 (2016).
[3] B. P. Abbott et al., Phys. Rev. X 6, 041015 (2016).
[4] B. P. Abbott et al., Phys. Rev. Lett. 118, 221101 (2017).
[5] B. P. Abbott et al., Astrophys. J. 851, L2 (2017).
[6] B. P. Abbott et al., Phys. Rev. Lett. 119, 141101 (2017).
[7] B. P. Abbott et al., Phys. Rev. Lett. 119, 161101 (2017).
[8] Y. B. Zeldovich and A. G. Polnarev, Sov. Astron. 18, 17 (1974).
[9] V. B. Braginsky and L. P. Grishchuk, Sov. Phys. JETP 62, 427 (1985).
[10] V. B. Braginsky and K. S. Thorne, Nat. 327, 123 (1987).
[11] D. Christodoulou, Phys. Rev. Lett. 67, 1486 (1991).
[12] L. Blanchet and T. Damour, Phys. Rev. D 46, 4304 (1992).
[13] K. S. Thorne, Phys. Rev. D 45, 520 (1992).
[14] A. G. Wiseman and C. M. Will, Phys. Rev. D 44, 2945 (1991).
[15] P. D. Lasky, E. Thrane, Y. Levin, J. Blackman, and Y. Chen, Phys. Rev. Lett. 117, 061102 (2016).
[16] M. Favata, Astrophys. J. 696, L159 (2009).
[17] D. Pollney and C. Reisswig, Astrophys. J. 732, L13 (2010).
[18] C. Reisswig, D. Pollney, Class. Quantum Grav. 28, 195015 (2011).
[19] J. Winicour, J. Living Rev. Relativ. 15, 2 (2012).
[20] K. Somiya, Class. Quantum Grav. 29, 124007 (2012).
[21] S. Hild et al., Class. Quantum Grav. 28, 094013 (2011).
[22] B. P. Abbott et al., Class. Quantum Grav. 34, 044001 (2017).
[23] S. Kawamura et al., J. Phys.: Conf. Ser. 120, 032004 (2008).
[24] J. Crowder and N. J. Cornish, Phys. Rev. D 72, 083005 (2005).
[25] R. van Haasteren and Y. Levin, MNRAS 401, 2372 (2010).
[26] G. Hobbs and Shi Dai, Nat. Sci. Rev. 4, 5 (2017).
[27] G. Hobbs et al., Class. Quantum Grav. 27, 084013 (2010).
[28] T. J. W. Lazio, Class. Quantum Grav. 30, 224011 (2013).
[29] S. M. Du and A. Nishizawa, Phys. Rev. D 94, 104063 (2016).
[30] D. Kennefick, Phys. Rev. D 50, 3587 (1994).
[31] B. P. Abbott et al., Phys. Rev. X 6, 041014 (2016).
[32] A. Bohe et al., Phys. Rev. D 95, 044028 (2017).
[33] T. Dal Canton et al., Phys. Rev. D 90, 082004 (2014).
[34] S. Usman et al., Class. Quantum Grav. 33, 215004 (2016).
[35] M. Favata, Phys. Rev. D 84, 124013 (2011).
[36] B. P. Abbott et al., Phys. Rev. Lett. 116, 241102 (2016).
[37] C. J. Moore, R. H. Cole, and C. P. L. Berry, Class. Quantum Grav. 32, 015014 (2014).
[38] Z. Arzoumanian, et al., Astrophys. J. 859, 47 (2018).
[39] A. King, Mon. Not. R. Astron. Soc. Lett. 456, 1 (2015).
[40] B. P. Abbott et al., Astrophys. J. 833, L1 (2016).
[41] M. Favata, Class. Quantum Grav. 27, 084036 (2010).
[42] E. Poisson, C. M. Will, Gravity: Newtonian, Post-Newtonian, Relativistic (Cambridge University Press, Cambridge, 2014), 1st ed., pages 318-319.
FIG. 5: Distance, total mass (M) parameter space for ground based detectors
FIG. 6: Distance, total mass (M) parameter space for space based detectors
FIG. 7: Distance, total mass (M) parameter space for PTAs