Fermion transmutation—A renormalization effect in gauge theory

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A new category of phenomena is predicted in which fermions of different flavours can transmute into one another, for example \( e \rightarrow \mu \) or \( e \rightarrow \tau \), as a consequence of the ‘rotating’ mass matrix due to renormalization. As examples, calculations will be presented for various such processes. Some of these could be accessible to experiments in the near future.

1 Introduction

By ‘fermion transmutation’ we mean a process in which a fermion changes its generation index as a direct consequence of the rotation of the mass matrix, and not as a secondary effect such as \( e \rightarrow \mu \) conversion via FCNC. Examples of transmutation are: \( e \rightarrow \mu \), \( e \rightarrow \tau \), \( \mu \rightarrow \tau \), which can occur e.g. in a Compton-like process schematically represented in Figure 1.

That the fermion mass matrix rotates, by which we mean that it undergoes unitary transformations as the scale changes, can be seen from the renormalization group equation, both in the standard model (SM) and in the dualized standard model (DSM).

An earlier talk\(^1\) summarizes our earlier work on DSM\(^2\). In this talk I shall report on two effects, namely transmutational decays\(^3\) and phototransmutations\(^4\), in both SM and DSM, but with emphasis on DSM.

\[ \ell_\alpha \]
\[ \ell_\beta \]

Figure 1. Photo-transmutation of leptons.
2 Mass matrix rotation

The SM renormalization group equation for the charged lepton mass matrix $L$ has a term which, given that the leptonic MNS mixing matrix $U$ is nontrivial, rotates it as the scale $\mu$ changes. The linearized equation:

$$\frac{dL}{d\mu} = \frac{3}{128\pi^2} \frac{1}{246^2} (ULU^\dagger)(ULU^\dagger)^\dagger L + \cdots,$$

where $ULU^\dagger = N$ the neutrino (Dirac) mass matrix, already shows that even if $L$ is diagonal at a chosen scale it cannot remain so at all other scales. The magnitude of the off-diagonal elements will depend on poorly known or unknown quantities such as the mixing $U$ and the Dirac mass $m_3$ of the heaviest neutrino. If we take the present popular theoretical biases of $U$ bimaximal and $m_3 \sim m_t$, then (1) gives for each decade change in energy:

$$\langle \mu|\tau \rangle \text{ changes by } \sim 5.5 \times 10^{-3} \text{ GeV}$$
$$\langle e|\tau \rangle \text{ changes by } \sim 1.8 \times 10^{-7} \text{ GeV}$$
$$\langle e|\mu \rangle \text{ changes by } \sim 1.1 \times 10^{-8} \text{ GeV}$$

In the DSM, the fermion mass matrix is of the following factorized form:

$$m = m_T \begin{pmatrix} x \\ y \\ z \end{pmatrix} (x, y, z),$$

where $m_T$ is essentially the mass of the heaviest generation. Under renormalization $m$ remains factorized, but the vector $(x, y, z)$ changes as

$$\frac{d}{d\mu} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{5}{32\pi^2 \rho^2} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix},$$

where $\rho$ is a (fitted) constant and

$$x_1 = \frac{x(x^2 - y^2)}{x^2 + y^2} + \frac{x(x^2 - z^2)}{x^2 + z^2}, \quad \text{cyclic.}$$

The off-diagonal elements have been calculated explicitly, using 3 free parameters determined by fitting experimental mass and mixing parameters (giving sensible predictions for the remaining parameters). These are shown in Figure. Hence the results we report below are entirely parameter-free.
3 Definition of lepton states

To define lepton states one must diagonalize the mass matrix, but since the eigenvectors depend on scale, there is no canonical recipe for doing so.

We suggest two quite different schemes for exploration. In the fixed scale diagonalization scheme (FSD) the state vectors are fixed at a chosen scale. As a result, the mass matrix is diagonal only at that scale. Such a scheme is applicable to SM, where it is tacitly assumed in most calculations.

For the DSM we use a more sophisticated scheme which we may call step-by-step diagonalization (SSD). It is a working criterion in which the lepton state vectors are always orthogonal and the mixing matrix is always unitary. There are 3 main steps:

- run the $3 \times 3$ mass matrix to a scale equal to mass of heaviest generation;
- corresponding eigenvector is its state vector;
- repeat with the remaining $2 \times 2$ submatrix.

In contrast to FSD, the charged lepton mass matrix here is diagonal at scales of $m_\tau, m_\mu, m_e$, as seen in Figure 2.

4 Transmutational decays

With the lepton states defined as in §3 and the rotations obtained in §2 we can now study transmutational processes, the most obvious of which are decays.
Table 1. Branching ratios of transmuational decays.

| Decays | SM est.      | DSM est.  | Expt limit   |
|--------|--------------|-----------|--------------|
| $Z^0 \rightarrow \tau^- \mu^+$ | $4 \times 10^{-10}$ | $4 \times 10^{-8}$ | $1.2 \times 10^{-5}$ |
| $\pi^0 \rightarrow \mu^- e^+$ | negligible    | $3 \times 10^{-9}$ | $1.7 \times 10^{-8}$ |
| $\psi \rightarrow \mu^+ \tau^-$ | $1 \times 10^{-8}$ | $6 \times 10^{-6}$ | not given |
| $\Upsilon \rightarrow \mu^+ \tau^-$ | negligible    | $2 \times 10^{-6}$ | not given |
| $\mu^- \rightarrow e^- \gamma$ | ?            | 0          | $4.9 \times 10^{-11}$ |
| $\mu^- \rightarrow e^- e^+ e^-$ | ?            | 0          | $1.0 \times 10^{-12}$ |

By expanding the fermion propagator we get for $\ell_\alpha \rightarrow \ell_\beta$, $\alpha \neq \beta$:

$$\text{transmutation/diagonal} \sim \langle \alpha | \beta \rangle / E,$$

where $E$ is a typical energy for the decay. Estimates are in Table 1.

With significant exceptions, SM (with FSD) estimates are all far below present experimental bounds and are hence not so interesting. An exception is the process $\mu^- \rightarrow e^- e^+ e^-$, where one could get a branching ratio of $10^{-3}$ (limit $10^{-12}$), if one applied FSD naively.

The parameter-free calculations in DSM give branching ratios which are in general larger but still below present experimental limits. It is important to note that because of SSD the branching ratios of transmutational leptonic decays are automatically zero to first order (Figure 2). The $\pi^0$ decay is of particular interest as being less than one order from the experimental bound.

5 Photo-transmutation

We have studied the following process, mainly for DSM:

$$\gamma + \ell_\alpha \rightarrow \gamma + \ell_\beta, \quad \alpha \neq \beta.$$ (6)

There are two points to note. First, the kinematics is unfamiliar, with the mass eigenstates being varying linear combinations of the $\tau, \mu, e$ states. The other point is that standard formulae for summing $\gamma$ matrix traces cannot be applied directly. We calculated the individual spin and polarization amplitudes and then summed them by hand.

We calculated the cross sections for: $\gamma e \rightarrow \gamma \mu$, $\gamma e \rightarrow \gamma \tau$, $\gamma \mu \rightarrow \gamma \tau$, leaving out $\tau$-initiated reactions as experimentally unrealistic at present. A
sample of the DSM results is presented in Figure 3. We get in general:

$\gamma \mu \rightarrow \gamma \tau > \gamma e \rightarrow \gamma \tau > \gamma e \rightarrow \gamma \mu$.

However, at low energies $\gamma e \rightarrow \gamma \mu$ becomes quite sizeable, Figure 4, with the total cross section having a peak of $\sim 100$ pb at c.m. energy $\sim 200$ MeV.

Calculations in SM depend on further assumptions. If the Dirac masses of the neutrinos are hierarchical, then above $\tau$ the cross sections are approximately given by scaling those of DSM with the relevant rotation matrix elements. For example $\gamma \mu \rightarrow \gamma \tau$ at $\sqrt{s} = 17.8$ GeV is $2 - 3$ orders smaller.

Figure 3. Differential cross sections for $\gamma e \rightarrow \gamma \tau$.

Figure 4. Total cross section for $\gamma e \rightarrow \gamma \mu$. 
6 Possible experimental tests in the near future

Since only the DSM estimates and calculations are parameter-free, while the SM results presented here are subject to further assumptions and uncertainties, we shall point out tests for DSM only here.

The estimates for the transmutational decay modes: $\pi^0 \rightarrow \mu^- e^+$, $\psi \rightarrow \tau^- \mu^+$, $\Upsilon \rightarrow \tau^- \mu^+$ could be near experimental limits and sensitivities, for LEP, BEPC and B-factories.

For photo-transmutations, one may consider virtual $\gamma$ from $e^+e^-$ colliders.

Above $\tau$, $e \rightarrow \tau$ is more important, while below it is $e \rightarrow \mu$. Again, LEP and/or BEPC may provide tests.

7 Conclusions

• Fermion transmutation necessarily occur in both SM and DSM.
• The SM results are in general smaller than DSM, with uncertainties.
• The DSM calculations are entirely parameter-free. There are no violations of data in all the cases we were able to consider.
• Experimental tests of DSM predictions seem feasible in the near future: (1) decays of $\pi^0, \psi, \Upsilon$, (2) photo-transmutation of $\gamma e \rightarrow \gamma \tau$ at high and $\gamma e \rightarrow \gamma \mu$ at low energy, (3) other processes e.g. $e^+e^- \rightarrow e^+\mu^-$.
• Transmutation leads to exciting new physics that has to be explored.

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