Evolution of non-forward parton distributions in next-to-leading order: singlet sector.

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Abstract

We present a general method for the solution of the renormalization group equations for the non-forward parton distributions on the two-loop level in the flavour singlet channel based on an orthogonal polynomial reconstruction. Using this formalism we study the effects of the evolution on recently proposed model distribution functions.

Keywords: non-forward distribution, two-loop evolution equation, anomalous dimensions

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1Alexander von Humboldt Fellow.
1 Introduction.

Parton distribution functions as gained from high-energy scattering reactions provide our main information about the non-perturbative structure of hadrons. Up to now the wealth of this knowledge was obtained from deep inelastic scattering (DIS) of leptons off a hadron target which allows to access the ordinary forward structure functions. Recently much excitement was generated by new objects which could provide a new insight into the underlying dynamics of non-perturbative QCD in hard processes — the non-forward parton distributions \[1, 2, 3\]. Several processes were proposed where the latter could be measured, e.g. deeply virtual Compton scattering, diffractive production of vector meson etc. It is unnecessary to emphasize that any precise analysis of such data will require accurate predictions from strong interaction theory for the corresponding reactions, especially for the radiative corrections to the tree level amplitudes. We solved the latter problem in a series of our recent papers (see \[4\] and references cited therein) where all necessary ingredients for a next-to-leading order (NLO) analysis were derived. In this contribution we conclude by presenting a general method for the solution of the generalized evolution equations for the non-forward parton distributions in two-loop approximation in the flavour singlet channel. The technique we use, i.e. the reconstruction of a distribution function \[\mathcal{O}(x, \zeta)\], from its conformal moments, \(\mathcal{O}_j\), via an expansion in the series of the orthogonal Jacobi polynomials \(P_j^{(\alpha, \beta)}\),

\[
\mathcal{O}(x, \zeta) \propto w(x|\alpha, \beta) \sum_{j} P_j^{(\alpha, \beta)} (2x - 1) \sum_{k} E_{jk}(\zeta) \mathcal{O}_k,
\]

is close in spirit to the one developed about two decades ago by Parisi and Sourlas \[6\] (see also Ref. \[7\]) for the usual DIS. The generalization to the singlet sector in DIS was done in Ref. \[8\]. Quite recently it was independently rediscovered by us in Refs. \[4, 9\] in relation to the problem at hand. This approach is the most suitable for our purposes because the standard methods of contour integration via the inverse Mellin transformation are very hard to implement in practice. On top of this for particular values for the parameters of the Jacobi polynomials the latter become eigenfunctions of the leading order non-forward evolution equation which favours the choice we have made. Since for the time being analytical expressions in NLO exist only for the complete set of non-forward singlet anomalous dimensions \[10, 11\] but not for the kernels themselves it is the only possible method to go beyond the one-loop approximation. Though, using the generating function for the diagonal part of the non-forward evolution kernels derived and tested at leading order in Ref. \[10\] (the non-diagonal parts are known analytically) one can extend this procedure

\[2\]The functions \(\mathcal{O}(x, \zeta)\) used here vanishes outside the region \(x \in [0, 1]\). However, below we deal with non-forward parton distributions with the support \(-1 + \zeta \leq x \leq 1\) which parametrize the matrix element of a light-ray operator. The results for the former case can be found in Appendix \[3\].
to two-loop approximation and find in principle NLO exclusive kernels, \( V(t, t') \) with \(|t|, |t'| \leq 1 \), analytically. After continuation to the entire plane \(|t|, |t'| < \infty\) might allow a direct numerical integration of the two-loop evolution equations — an alternative method used in the analysis of the forward structure functions.

Note, however, that with the former approach at hand, our numerical task is more difficult contrary to method used in usual forward scattering as we cannot restrict ourselves to the first \( N_{\text{max}} \approx 10 \) polynomials in the series (1) because in DIS the shape of the curve is roughly fixed by the weight-function, \( w(x|\alpha, \beta) = \bar{x}^\alpha x^\beta \) (with \( \bar{x} \equiv 1 - x \)), provided we have made an appropriate choice for the indices of the Jacobi polynomials, namely, with \( \beta \) given by Regge theory predictions and \( \alpha \) driven by quark counting rules near the phase space boundary. The series of polynomials leads only to small perturbations around this \( \bar{x}^\alpha x^\beta \)-behaviour. For the case at hand the situation is different since the shape of the distribution is obtained from the total series over all oscillating polynomials. In practice, due to rather rapid convergence of the series the number of terms to be taken into account is rather large but still treatable.

In the present paper we pursue the goal of developing a machinery for the solution of the two-loop renormalization group equations for the non-forward parton distributions and of studying numerically the \( Q^2 \)-dependence of the latter. Our presentation is organized as follows. In the next section we give our definitions and conventions used throughout the paper. The third section is devoted to the description of our formalism for evolution of generalized distribution functions together with a set of explicit formulae required in the numerical analysis. Since there are no experimental data yet for the non-forward parton distributions we have to rely on estimations of the form of the \((x, \zeta)\)-dependence at low normalization scale from non-perturbative approaches to QCD. In section 4 we give our results for models of the singlet non-forward parton functions. We consider CTEQ-based functions which are deduced making plausible assumptions about the behaviour of the so-called double distributions in different regions of phase space. The final section is left for the conclusions. To make the presentation self-consistent as much as possible we add three appendices. The first one contains the analytically continued forward anomalous dimension matrix responsible for the evolution of the multiplicatively renormalizable conformal moments. The second one presents the non-diagonal elements of the anomalous dimension matrix of the tree-level conformal operators which define the corrections to the eigenfunctions of the NLO evolution kernels. The last appendix contains the formulae required for reconstruction of the non-forward distributions with the support \( 0 \leq x \leq 1 \).

\[^3\text{Let us remind, however, that there exists another optimal choice of } \alpha \text{ and } \beta \text{ which leads to the same high precision reconstruction of the structure function } \text{[8]. Obviously, there is no dependence on the particular values of these parameters provided a large enough number of terms, } N_{\text{max}}, \text{ is taken into account.} \]
2 Conventions.

Since we are interested in the study of the parity even flavour singlet evolution equations we face as usual the mixing problem between quarks and gluons. To treat it in a compact way let us introduce a two-dimensional vector of singlet non-forward parton distributions composed from quark and gluon functions ($\bar{\zeta} \equiv 1 - \zeta$):

$$\mathcal{O}(x, \zeta) = \begin{pmatrix} Q(x, \zeta) \\ G(x, \zeta) \end{pmatrix} \quad \text{with} \quad -\bar{\zeta} \leq x \leq 1. \quad (2)$$

The latter are defined as the light-cone Fourier transforms:

$$\langle h' | Q \mathcal{O}(\kappa_1, \kappa_2) | h \rangle = 2 \int_{-\bar{\zeta}}^{1} dx \, e^{-i\kappa_1 x - i\kappa_2 (\zeta - x)} Q(x, \zeta), \quad (3)$$
$$\langle h' | G \mathcal{O}(\kappa_1, \kappa_2) | h \rangle = \int_{-\bar{\zeta}}^{1} dx \, e^{-i\kappa_1 x - i\kappa_2 (\zeta - x)} G(x, \zeta), \quad (4)$$

of the light-ray quark and gluon string operators ($v_+ \equiv v_\mu n_\mu$)

$$Q \mathcal{O}(\kappa_1, \kappa_2) = \bar{\psi}(\kappa_2 n) \gamma_+ \psi(\kappa_1 n) - \bar{\psi}(\kappa_1 n) \gamma_+ \psi(\kappa_2 n), \quad G \mathcal{O}(\kappa_1, \kappa_2) = G_{+\mu}(\kappa_2 n) G_{\mu+}(\kappa_1 n), \quad (5)$$

where, for brevity, we omit a path-ordered link factor which ensures gauge invariance. Here we accept the conventions advocated by Radyushkin [2], namely, $x$ is the momentum fraction of an outgoing parton w.r.t. the incoming hadron momentum $p$, $k_+ = xp_+$, and $\Delta_+ \equiv p_+ - p'_+ = \zeta p_+$, where $p'$ is an outgoing hadron momentum. These quantities are related to the variables $t$ and $\eta$ used by the authors [1, 3] according to $\eta = \frac{\zeta}{2 - \zeta}$, $t = \frac{2x - \zeta}{2 - \zeta}$, where $k_+ = t \bar{P}_+, \Delta_+ = \eta \bar{P}_+$ and the averaged momentum is introduced as follows $\bar{P} = p + p'$. An advantage of the first conventions is that the variable $x$ acquires a simple partonic interpretation in contrast to $t$. However, the range of the latter does not depend on the longitudinal asymmetry parameter $\eta$ as compared to $x$.

Due to charge conjugation properties of the non-local operators (3) it immediately follows that in the $(x, \zeta)$-plane the singlet quark distribution is anti-symmetric, $Q(x, \zeta) = -Q(\zeta - x, \zeta)$, while the gluon one is symmetric, $G(x, \zeta) = G(\zeta - x, \zeta)$, w.r.t. the line $x = \frac{\zeta}{2}$. Up to an obvious redefinition the functions $Q$ and $G$ coincide with the ones introduced in Refs. [1, 3]

$$Q \left( x = \frac{t + \eta}{1 + \eta}, \zeta = \frac{2\eta}{1 + \eta} \right) \equiv (1 + \eta)q(t, \eta), \quad (6)$$

and similar for gluons, where the variables $t$ and $\eta$ vary within the limits $-1 \leq t \leq 1$, $0 \leq \eta \leq 1$. Obviously, these singlet quark and gluon distributions are, respectively, odd and even functions of $t$.

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4Below, we will repeatedly omit the integration limits. The latter can be easily restored making use of known support properties of the distributions or/and evolution kernels.
The original functions, \( Q \) and \( G \), introduced above can be decomposed into the following quark, \( Q_O \), anti-quark, \( \bar{Q}_O \), and gluon, \( G_O \), non-forward distributions defined in the range \( x \in [0, 1] \) via

\[
Q(x, \zeta) = \frac{1}{2} \left\{ [Q_O(x, \zeta) + \bar{Q}_O(x, \zeta)] \theta(x) \theta(\bar{x}) - [Q_O(\zeta - x, \zeta) + \bar{Q}_O(\zeta - x, \zeta)] \theta(\zeta - x) \theta(x + \bar{\zeta}) \right\},
\]

\( (7) \)

\[
G(x, \zeta) = \frac{1}{2} \left\{ G_O(x, \zeta) \theta(x) \theta(\bar{x}) + G_O(\zeta - x, \zeta) \theta(\zeta - x) \theta(x + \bar{\zeta}) \right\}.
\]

\( (8) \)

However, our consequent discussion deals entirely with the non-forward distributions \( Q \) and \( G \).

3 The method.

Due to ultraviolet divergences of perturbative corrections for a product of operators separated by a light-like distance as in Eq. (1) the distributions acquire a dependence on a normalization point, \( \mu^2 \), governed by a renormalization group equation, the so-called generalized Efremov-Radyushkin-Brodsky-Lepage (ER-BL) evolution equation

\[
\mu^2 \frac{d}{d \mu^2} O(x, \zeta) = \int dx' K(x, x', \zeta | \alpha_s) O(x', \zeta),
\]

\( (9) \)

where the purely perturbative kernel \( K \) is a \( 2 \times 2 \) matrix given by a series in the coupling constant, \( \alpha_s \). The LO non-forward light-cone fraction kernels\(^5\) were evaluated many years ago in Ref. [12] for even and odd parity and chirality while corresponding light-cone position counterparts were addressed in Refs. [4, 13]-[17]. The evaluation of the two-loop corrections, however, for the Gegenbauer moments of the kernel \( K \)

\[
\int dx C^{\nu(A)}_j \left( 2x, \zeta - 1 \right) AB K(x, x', \zeta | \alpha_s) C^{\nu(B)}_k \left( 2x', \zeta - 1 \right) = -\frac{1}{2} \sum_{k=0}^j AB \gamma_{jk}(\alpha_s) C^{\nu(B)}_k \left( 2x', \zeta - 1 \right),
\]

\( (10) \)

— where the numerical values of the indices \( \nu(A, B) \) depends on the channel under consideration — has been addressed by us in Refs. [17, 10, 11]. In one-loop approximation the above kernel is diagonal \( \gamma_{jk}^{(0)} = \gamma_{j}^{D(0)} \delta_{jk} \) in this basis while beyond LO non-diagonal elements, \( \gamma_{jk}^{ND} \), appear \( \gamma_{jk} = \gamma_{jk}^{D} + \gamma_{jk}^{ND} \). The diagonal anomalous dimensions coincide up to pre-factors given in Eq. (18) with the anomalous dimensions of local operators without total derivatives which appear in the operator product expansion (OPE) for DIS. The formalism we have developed there allowed us to find all entries, \( \gamma_{jk}^{ND} \), in closed analytical form in NLO and provided a simple diagonalization of the evolution equation (9). The following discussion concerns the solution of Eq. (9) within the formalism we have sketched above.

\(^5\)For recent independent calculations of these kernels related to the problem at hand see the reviews [1, 2].
Figure 1: Two-loop diagrams giving rise to a non-vanishing contribution to the $\mathcal{W}$-functions of the generalized ER-BL evolution kernel (11).

Let us add a remark about the general structure of the all-order evolution kernel $\frac{1}{\log(\eta)} \mathbf{V} \left( \frac{x}{\eta}, \frac{x}{\eta} \right) = K \left( x = \frac{t + \eta}{1 + \eta}, x' = \frac{t' + \eta}{1 + \eta}, \zeta = \frac{2\eta}{1 + \eta} \right)$. It can be deduced in a straightforward manner from the support properties and charge conjugation symmetry of the light-ray operators and reads, e.g. for the $QQ$-channel [18],

$$V(t, t') = \Theta^{0}_{11}(t - t', t - 1)\mathcal{V}(t, t') + \Theta^{0}_{11}(-t - t', -t - 1)\mathcal{W}(-t, t') + \left\{ \begin{array}{l} t \rightarrow -t \\ t' \rightarrow -t' \end{array} \right\},$$

(11)

where we have introduced the step-function $\Theta^{0}_{11}(t, t') = \frac{1}{\log(\nu)} \{ \theta(t)\theta(-t') - \theta(-t)\theta(t') \}$ [19] and $\mathcal{V}$, $\mathcal{W}$ are analytic functions of their arguments. Here the $\mathcal{W}$-part appears first in NLO and is generated by diagrams depicted in Fig. 1. These terms give rise to mixing of partons from different regions of phase space conserved by the leading order evolution and it forces us to consider the functions (7,8) which are defined on the entire interval of $x \in \left[-\bar{\zeta}, 1\right]$.

3.1 General formalism.

In order to solve the evolution equation (9) one has to find its eigenvalues and eigenfunctions. This problem has been exhaustively treated by us in general form in Ref. [11] where it was shown that its solution can be written in terms of the partial conformal wave expansion

$$\mathcal{O}(x, \zeta, Q^2) = \sum_{j=0}^{\infty} \phi_j \left( x, \zeta \left| \alpha_s(Q^2) \right. \right) \overline{\mathcal{O}}_j(\zeta, Q^2),$$

(12)

with the partial conformal waves matrix

$$\phi_j \left( x, \zeta \left| \alpha_s(Q^2) \right. \right) = \sum_{k=j}^{\infty} \zeta^{k-j} \phi_k \left( x, \zeta \right) B_{kj},$$

(13)
being the eigenstate of the all-orders equation (9). The \( B \)-matrix defines the corrections to the eigenfunctions which at tree-level diagonalize the leading order ER-BL equations. The latter read (\( w(x|\nu) = (x\bar{x})^{\nu-1/2} \))

\[
\phi_j(x, \zeta) = \frac{1}{\zeta} \begin{pmatrix}
\frac{w\left(\frac{\zeta}{N_j}\right)}{N_j(\frac{\zeta}{N_j})} C_j^3 \left(2z - 1\right) & 0 \\
0 & \frac{w\left(\frac{\zeta}{N_{j-1}}\right)}{N_{j-1}(\frac{\zeta}{N_{j-1}})} C_{j-1}^3 \left(2z - 1\right)
\end{pmatrix}.
\]

The multiplicatively renormalizable moments, \( \widetilde{O}_j(\zeta, Q^2) \), evolve as follows

\[
\widetilde{O}_j(\zeta, Q^2) = \mathcal{E}_j \left(\alpha_s(Q^2), \alpha_s(Q_0^2)\right) \widetilde{O}_j(\zeta, Q_0^2),
\]
with an evolution operator defined by the equation

\[
\mathcal{E}_j \left(\alpha_s(Q^2), \alpha_s(Q_0^2)\right) = \mathcal{T} \exp \left\{ -\frac{1}{2} \int_{Q_0^2}^{Q^2} \frac{d\tau}{\tau} \gamma^D_j \left(\alpha_s(\tau)\right) \right\}
\]

where the operator \( \mathcal{T} \) orders the matrices of the diagonal anomalous dimensions

\[
\gamma^D_j = \begin{pmatrix}
QQ_{j\gamma}^D & QG_{j\gamma}^D \\
GQ_{j\gamma}^D & GG_{j\gamma}^D
\end{pmatrix}
\]

along the integration path. They are given as an expansion in the coupling constant by \( \gamma_j(\alpha_s) = \frac{\alpha_s}{2\pi} \gamma_j^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \gamma_j^{(1)} + \ldots \). The elements of this matrix are related to the forward anomalous dimensions \( \gamma_{j}^{fw} \) via the relations

\[
QQ_{j\gamma}^D = QQ_{j\gamma}^{fw}, \quad QG_{j\gamma}^D = \frac{6}{j} QG_{j\gamma}^{fw}, \quad GG_{j\gamma}^D = \frac{j}{6} GG_{j\gamma}^{fw}, \quad GC_{j\gamma}^D = GC_{j\gamma}^{fw}.
\]

The pre-factors \( 6/j, j/6 \) in the off-diagonal matrix elements come from the conventional definition of the Gegenbauer polynomials.

The normalization condition in Eqs. (12,13) is chosen so that there are no radiative corrections at an input scale \( Q_0 \). Apart from the minimization of the higher order corrections the advantage of this choice lies in the fact that multiplicatively renormalizable \( \widetilde{O}_j(\zeta, Q_0^2) \) are defined at \( Q_0 \) by forming ordinary Gegenbauer moments with non-forward distributions via

\[
\widetilde{O}_j(\zeta, Q_0^2) = \int dx C_j(x, \zeta) O(x, \zeta, Q_0^2), \quad \text{with} \quad C_j(x, \zeta) = \begin{pmatrix}
\zeta^j C_j^3 \left(2z - 1\right) & 0 \\
0 & \zeta^{j-1} C_{j-1}^3 \left(2z - 1\right)
\end{pmatrix}.
\]

\[\begin{array}{c}
\text{Let us emphasize that here the Gegenbauer polynomials should be understood as mathematical distributions (see Ref. [2]) according to the relation } \int_{Q_0^2}^{Q^2} \frac{d\tau}{\tau} \gamma_{j}^{fw}(\zeta-y) \propto \int_{Q_0^2}^{Q^2} dy \tilde{\gamma}_{j}(\zeta-y) \text{ in order to be able to restore the correct support properties of the non-forward distributions.}
\end{array}\]
As before [4] the restoration of the support properties of the distributions is achieved via an expansion of the latter in a series with respect to the complete set of orthogonal polynomials, \( \mathcal{P}_j^{(\alpha_p)}(t) \), on the interval \(-1 \leq t \leq 1\)

\[
\mathcal{O}(x, \zeta, Q^2) = \frac{2}{2 - \zeta} \sum_{j=0}^{\infty} \mathcal{P}_j \left( \frac{2x - \zeta}{2 - \zeta} \right) \mathcal{M}_j(\zeta, Q^2),
\]

(20)

\[
\mathcal{P}_j(t) = \left( \begin{array}{cc}
w(t|\alpha_p) P_j^{(\alpha_p)}(t) & 0 \\
0 & w(t|\alpha_p') P_j^{(\alpha_p')}(t) \end{array} \right)
\]

(21)

with \( w(t|\alpha_p) \) and \( n_j(\alpha_p) \) being weight and normalization factors, respectively (see, e.g. [22]). The matrix of moments, \( \mathcal{M}_j(\zeta, Q^2) \), is given by the sum

\[
\mathcal{M}_j(\zeta, Q^2) = \sum_{k=0}^{\infty} E_{jk}(\zeta) \mathcal{O}_k(\zeta, Q^2), \quad \text{where}
\]

(22)

\[
\mathcal{O}_j(\zeta, Q^2) = \sum_{k=0}^{j} \zeta^{j-k} B_{jk} \mathcal{O}_k(\zeta, Q^2),
\]

(23)

and the upper limit in Eq. (22) will come from the constraint \( \theta \)-functions in the expansion coefficients

\[
E_{jk}(\zeta) = \left( \begin{array}{cc}
E_{jk}(\frac{3}{2}; \alpha_p|\zeta) & 0 \\
0 & E_{j-1}(\frac{3}{2}; \alpha_p'|\zeta) \end{array} \right),
\]

(24)

with elements given by the integral (where \( \theta_{jk} = \{1, \text{if } j \geq k; 0, \text{if } j < k\})

\[
E_{jk}(\nu; \alpha, \beta|\zeta) = \frac{\theta_{jk}}{2^\nu \Gamma(\frac{1}{2}) \Gamma(k + \nu + \frac{3}{2})} \int_{-1}^{1} dt \left(1 - t^2 \right)^{k + \nu - \frac{1}{2}} \frac{d^k}{dt^k} \mathcal{P}_j^{(\alpha_p)} \left( \frac{\zeta t}{2 - \zeta} \right).
\]

(25)

Taking Jacobi polynomials, \( \mathcal{P}_j^{(\alpha_p)}(t) = P_j^{(\alpha,\beta)}(t) \), we obtain the rather complicated result

\[
E_{jk}^J(\nu; \alpha, \beta|\zeta) = \frac{\theta_{jk}}{2^\nu - 1} \frac{\Gamma(\nu)}{1^\nu} \frac{\Gamma(j + \alpha + 1)}{1^j \Gamma(j + \alpha + \beta + 1)} \frac{\Gamma(k + \nu + \frac{3}{2})}{1^k \Gamma(k + 2 + 2\nu)} \times \theta_{jk} \sum_{l=0}^{j-k} (-1)^l \frac{\Gamma(j + k + \ell + \alpha + \beta + 1)}{\Gamma(\ell + 1) \Gamma(j - k - \ell + 1) \Gamma(k + \ell + \alpha + 1)} (2 - \zeta)^{-\ell-k} F_1 \left( -\ell, k + \nu + \frac{1}{2} \bigg| \zeta \right),
\]

(26)

which significantly simplifies for the particular values of the indices \( \alpha = \beta = \mu = \nu = 1 \) when Jacobi polynomials degenerate into Gegenbauer ones. Therefore, taking the latter as a set of expansion functions for our purposes, i.e. \( \mathcal{P}_j^{(\alpha_p)}(t) = C_j^{\mu}(t) \) we have

\[
E_{jk}^G(\nu; \mu|\zeta) = \frac{1}{2} \theta_{jk} \left[ 1 + (-1)^{j-k} \right] \frac{\Gamma(\nu)}{1^\nu} \frac{(-1)^{j-k} \Gamma(\mu + \frac{j+k}{2})}{\Gamma(\mu) \Gamma(\nu + k) \Gamma(1 + \frac{j-k}{2})} \times (2 - \zeta)^{k} F_1 \left( -\frac{j-k}{2}, \mu + \frac{j+k}{2} \bigg| \zeta^2 \right).
\]

(27)
The fact that only even \( j - k \)-moments contribute is obvious since the non-forward distributions are even functions of \( \eta = \frac{\zeta}{2} \) due to time-reversal invariance and hermiticity \([1]\), — as a result only even powers of \( \eta \) can appear in the expansion.

Now we are in a position to give results for the evolution operator, \( \mathcal{E} \), and \( B \)-matrix in two-loop approximation.

3.2 Explicit NLO solution.

In NLO the evolution operator satisfies the equation

\[
\frac{d}{d \ln \alpha_s(Q^2)} \mathcal{E}_j \left( \alpha_s(Q^2), \alpha_s(Q_0^2) \right) = -\frac{1}{\beta_0} \left\{ \gamma_j^{D(0)} + \frac{\alpha_s(Q^2)}{2\pi} R_j \right\} \mathcal{E}_j \left( \alpha_s(Q^2), \alpha_s(Q_0^2) \right),
\]

where

\[
R_j = \gamma_j^{D(1)} - \frac{\beta_1}{2\beta_0} \gamma_j^{D(0)},
\]

and the boundary condition \( \mathcal{E}_j \left( \alpha_s(Q_0^2), \alpha_s(Q_0^2) \right) = \mathbb{1} \) with the unit matrix \( \mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \). Here \( \beta_0 \) and \( \beta_1 \) are the first and second coefficient in the perturbative expansion of the QCD \( \beta \)-function \( \frac{\beta}{g} = \frac{\alpha_s}{4\pi} \beta_0 + \left( \frac{\alpha_s}{4\pi} \right)^2 \beta_1 + \ldots \) and read \( \beta_0 = \frac{10}{3} T_F N_f - \frac{11}{3} C_A \) and \( \beta_1 = \frac{10}{3} C_A N_f + 2 C_F N_f - \frac{34}{3} C_A \), respectively.

The solution of the above equation is \([22, 23]\)

\[
\mathcal{E}_j \left( \alpha_s(Q^2), \alpha_s(Q_0^2) \right) = \left( P_j^+ - \frac{\alpha_s(Q^2) - \alpha_s(Q_0^2)}{2\pi} \frac{1}{\beta_0} P_j^+ R_j P_j^+ \right) \left( \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right) \frac{\gamma_j^+ / \beta_0}{1 - \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \left( \gamma_j^- - \gamma_j^+ / \beta_0 \right)} \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \gamma_j^+ / \beta_0 + \left( + \leftrightarrow - \right),
\]

where we have introduced projection operators

\[
P_j^\pm = \pm \frac{1}{\gamma_j^+ - \gamma_j^-} \left( \gamma_j^{D(0)} - \gamma_j^\pm \mathbb{1} \right),
\]

and the eigenvalues of the LO anomalous dimension matrix

\[
\gamma_j^\pm = \frac{1}{2} \left( QQ_{\gamma_j}^{D(0)} + GG_{\gamma_j}^{D(0)} \pm \sqrt{ \left( QQ_{\gamma_j}^{D(0)} - GG_{\gamma_j}^{D(0)} \right)^2 + 4 GQ_{\gamma_j}^{D(0)} QG_{\gamma_j}^{D(0)} } \right).
\]

The \( B \)-matrix which fixes the corrections to the eigenfunctions of the NLO ER-BL kernels is given by

\[
B = \mathbb{1} + B^{(1)},
\]
where $B^{(1)}$ is determined by the first order differential equation

$$\frac{d}{d\ln \alpha_s(Q^2)} B^{(1)}(\alpha_s(Q^2), \alpha_s(Q_0^2)) = -\frac{1}{\beta_0} \left\{ \left[ \gamma^{D(0)}, B^{(1)}(\alpha_s(Q^2), \alpha_s(Q_0^2)) \right]_+ + \frac{\alpha_s(Q^2)}{2\pi} \gamma^{\text{ND}(1)} \right\},$$

(34)

and reads ($j > k$)

$$B^{(1)}_{jk} = -\frac{\alpha_s(Q^2)}{2\pi} \left( \frac{P^+_j \gamma^{\text{ND}(1)}_{jk} P^+_k}{\gamma^+_j - \gamma^+_k + \beta_0} \left( 1 - \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \left( \frac{\gamma^+_j - \gamma^+_k + \beta_0}{\beta_0} \right) \right) \right) + \frac{P^+_j \gamma^{\text{ND}(1)}_{jk} P^-_k}{\gamma^+_j - \gamma^-_k + \beta_0} \left( 1 - \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \left( \frac{\gamma^-_j - \gamma^-_k + \beta_0}{\beta_0} \right) \right) + (+ \leftrightarrow -).$$

(35)

The two-loop forward anomalous dimension matrix in the singlet sector has been evaluated in Refs. [24, 25]. The non-diagonal entries, $\gamma^{\text{ND}(1)}$, of the full anomalous dimension matrix $\gamma = \gamma^D + \gamma^{\text{ND}}$ of the conformal operators have become available quite recently [10, 11]. For the reader’s convenience both of the above ingredients are summarized in appendices A and B, respectively.

Finally, the coupling constant in the corresponding order is given by the following inverse-log expansion

$$\alpha_s(Q^2) = -\frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^2_{\text{MS}})} \left( 1 + \frac{\beta_1}{\beta_0^2} \ln\left( \frac{Q^2}{\Lambda^2_{\text{MS}}} \right) \right).$$

(36)

Note, however, that due to the fact that the input scale, $Q_0$, — at which the initial conditions considered below are defined — is very low it might be more reliable and accurate to obtain $\alpha_s$ in two-loop approximation by solving the exact transcendental equation

$$-\beta_0 \ln \frac{Q^2}{\Lambda^2_{\text{MS}}} = \frac{4\pi}{\alpha_s(Q^2)} - \frac{\beta_1}{\beta_0} \ln \left( \frac{4\pi}{\beta_0 \alpha_s(Q^2)} - \frac{\beta_1}{\beta_0^2} \right).$$

(37)

This leads to $\sim 15\%$ discrepancy between them for $Q^2 = 0.6 \text{ GeV}^2$ which goes down to $3\%$ for $Q^2 = 4 \text{ GeV}^2$. However, we leave aside this source of theoretical uncertainty, as being conceptually irrelevant for our study it presents only an extra source of unnecessary complication.

### 4 Evolution of the model distributions.

For practical applications we have chosen the indices of the Jacobi polynomials in the simplest way, i.e. $\alpha = \beta = 0$. In this case the former coincide with Legendre polynomials, $P^+_j(t) = P^{(0,0)}_j(t) = C^{1/2}_j(t)$ with $w(t) = 1$ and $n_j = (2j + 1)/2$ in Eqs. (20, 21). From the last equality we can read off explicit expressions for the expansion coefficients $E_{jk}$ in Eq. (27). Contrary to our previous procedure advocated in Ref. [4] where the evolution of a limited number of points
in the discrete \( \{x, \zeta\} \)-plane has been performed exactly and interpolated afterwards by a smooth functions in spline approximation, presently we evolve the expansion coefficients of the non-forward parton distribution function in the series over orthogonal polynomials and then sum them back. Therefore, no smoothing recipes are required but the result depends on the truncation parameter \( N_{\text{max}} \). To achieve rather high reconstruction accuracy, i.e. \( \sim 10^{-2} - 10^{-3} \), we have retained up to \( N_{\text{max}} = 100 \) polynomials in the series\(^7\) (20). Even better precision, \( 10^{-4} \), is feasible by doubling the number, \( N_{\text{max}} = 100 \), of polynomials in the expansion at the price of the much more time consuming procedure. Note that only the region around the crossover point \( x = \zeta \) is sensitive to \( N_{\text{max}} \) due to rather rapid change of the curve on the small interval of \( x \). Taking only \( N_{\text{max}} \sim 50 \) the reconstructions accuracy decreases down to \( 10^{-1} - 10^{-2} \).

Note that due to symmetry properties of the distribution functions only those moments survive which acquire an operator content and evolve with anomalous dimensions deduced from OPE. This is in contrast to the case of distributions, \( A^{\mathcal{O}} \), with the support \( 0 \leq x \leq 1 \) where all moments enter on equal grounds due to lack of charge conjugation symmetry in the parametrization of the hadronic matrix elements in Eqs. (3,4,7,8). They were considered in Ref. [4] and it was crucial there to use the analytically continued anomalous dimensions\(^8\) (see appendix A) since otherwise the convergence of the series will break down already for \( N_{\text{max}} \sim 10 \). This is so in spite of the fact that the relative difference between the former and the ones deduced from an OPE analysis with \( \sigma = (-1)^{j+1} \) (see appendix A) left intact is negligible (\( \leq 10^{-4} \)). The reason is the factorial growth of the expansion coefficients which weight the contribution of moments.

Below we report on our study of the \( Q^2 \)-dependence for the distribution functions proposed in Ref. [26]. We will, however, neither speculate on the physical relevance of the latter nor discuss their adequacy to the real world since this is a disputable issue in the lack of any experimental data, but we rather accept them in order to test our formalism. We have chosen for our analysis \( N_f = 4 \) and \( A_{\text{MS}}^{(4)} = 246 \text{ MeV} \).

The non-forward functions \( A^{\mathcal{O}}(x, \zeta) \) in Eqs. (3,8) for the parton species \( A = Q, \bar{Q}, G \) are defined in terms of the double distribution function \( A^{F}(y, z) \) via the following relation\(^2\)

\[
A^{\mathcal{O}}(x, \zeta, Q^2_0) = \int_0^1 dy \int_0^1 dz \, A^{F}(y, z, Q^2_0) \theta(1 - y - z) \delta(x - y - \zeta z). \quad (38)
\]

The functional dependence of \( A^{F} \) in the \( y \)-subspace is given by the shape of the forward parton density while its \( \frac{1}{z} \)-dependence has to be similar to that of the distribution amplitude. This results

---

\(^7\)The C++ code used in the calculation is available via \url{http://www.physik.uni-regensburg.de/~nil17791}.

\(^8\)In the above mentioned paper we have expanded the anomalous dimensions in the series \( \gamma_j = \sum_{l,m} c_{lm} \ln^{l}(j+1)^{m} \) with the first few terms kept in the expansion which provides a highly accurate approximation, sufficient for numerical studies.
Figure 2: Evolution of the non-forward singlet quark distributions $Q(x,\zeta)$. The input function at $Q_0 = 0.7$ GeV is shown by the short-dashed line at different $\zeta$’s. The full curves moving away from the initial function correspond to LO results for $Q^2 = 10^3, 10^5, 10^7$ GeV$^2$, respectively. The long-dashed lines give the NLO results for the same values of the momentum scale in the same order.
Figure 3: Same as Fig. 2 but for the gluon non-forward distribution $G(x, \zeta)$. The conventions are the same as in Fig. 2.
in the following model for $A_F(y, z, Q_0^2)$ \[26\]

$$A_F(y, z, Q_0^2) = A(y, Q_0^2)\pi(y, z),$$ \hspace{1cm} (39)

with the plausible profiles $A\pi(y, z)$ \[26\]

$$Q\pi(y, z) = 6\frac{z}{y^3}(\bar{y} - z), \quad G\pi(y, z) = 30\frac{z^2}{y^5}(\bar{y} - z)^2,$$ \hspace{1cm} (40)

for quarks and gluons, respectively. In Eq. (39) the function $A(y, Q_0^2) = q(y), \bar{q}(y), yg(y)$ is an ordinary forward parton density measured in deep inelastic scattering at a low normalization point. We use the CTEQ4LQ parametrization \[27\] of the parton densities at the momentum scale $Q_0 = 0.7$ GeV:

$$xu_v(x) = x(u - \bar{u})(x) = 1.315x^{0.573}(1 - x)^{3.281}(1 + 10.614x^{1.034}),$$
$$xd_v(x) = x(d - \bar{d})(x) = 0.852x^{0.573}(1 - x)^{1.060}(1 + 4.852x^{0.693}),$$
$$x(\bar{u} + \bar{d})(x) = 0.578x^{0.143}(1 - x)^{7.293}(1 + 1.858x^{1.000}),$$
$$xg(x) = 39.873x^{1.889}(1 - x)^{5.389}(1 + 0.618x^{0.474}).$$ \hspace{1cm} (41)

The results of the evolution are shown in Figs. 2, 3 for singlet quark and gluon distributions, respectively. Due to the low input momentum scale, $Q_0$, the change of the shape is very prominent already for $Q^2 = 10$ GeV$^2$. We have plotted the curves obtained using the LO (full lines) and NLO (dashed lines) formulae. The difference between them is especially sizable ($\sim 10 - 30\%$) for quarks at small $\zeta$’s and moderately large $Q^2$. The limiting lines correspond to the extremely large momentum scales, $Q^2 = 10^{14}$ GeV$^2$, when the distributions approximately reach their asymptotic forms ($Q^2 \rightarrow \infty$) \[23\]:

$$Q_{as}(x, \zeta) \propto \frac{x}{\zeta^2} \left(1 - \frac{x}{\zeta}\right) \left(2\frac{x}{\zeta} - 1\right) \theta(\zeta - x), \quad G_{as}(x, \zeta) \propto \frac{x^2}{\zeta^3} \left(1 - \frac{x^2}{\zeta^2}\right) \theta(\zeta - x).$$ \hspace{1cm} (42)

### 5 Conclusions.

In the present paper we have generalized the formalism for the solution of NLO evolution equations for the non-forward parton distributions developed by us previously for the non-singlet case \[4, 9\] to the flavour singlet channel. It is based on the use of the Jacobi polynomials to reconstruct the $(x, \zeta)$-dependence of the function from the multiplicatively renormalizable conformal moments which diagonalize the generalized ER-BL evolution equation. Due to relatively fast convergence of the series in orthogonal polynomials we limit ourselves to at most $N_{\text{max}} \leq 100$ terms. The resummation can still be handled numerically and the precision achieved is sufficiently high for any practical application. Let us note that in absence of analytic expressions for the complete
NLO non-forward kernels in the singlet channel (at least for the time being) there is no feasible alternative to the procedure described above.

We did not analyze in full, however, how the choice for the parameters $\alpha$ and $\beta$ influences the convergence properties of the series. One should note that a clever choice can lead to an increase of reconstruction accuracy by an order of magnitude for the same $N_{\text{max}}$ \cite{8}.

We have studied the general pattern of evolution obeyed by the models for the non-forward distributions proposed recently. We have found that the difference between LO and NLO does not exceed $10 - 30\%$ which suggests that the latter effects could be taken into account provided very high accuracy data points will be available which seems to be a very hard experimental task.

An application of our considerations to the evolution of the parity-odd densities is straightforward since the non-diagonal elements, $\gamma^{\text{ND}}$, of the corresponding anomalous dimensions are known from Ref. \cite{11} while the analytically continued two-loop forward entities \cite{28} were derived in \cite{29}.

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A Diagonal (forward) anomalous dimensions.

Let us add few remarks concerning the NLO anomalous dimensions for DIS. It should be noted that beyond leading order the moments of the DGLAP splitting kernels coincide with the anomalous dimensions available in the literature evaluated by OPE methods only for even (odd) moments provided we are interested in the crossing odd (even) combination of quark–anti-quark species. We give below analytically continued anomalous dimensions from even (odd) to all values of moments $j$ \cite{30,23} where the above unfortunate feature is naturally withdrawn.

The one-loop quantities are well known and read \cite{31}

\begin{align}
\QQ_{j}^{\gamma_{\text{fw}}(0)} & = -C_{F} \left( -4\psi(j+2) + 4\psi(1) + \frac{2}{(j+1)(j+2)} + 3 \right), \tag{A.1} \\
\QC_{j}^{\gamma_{\text{fw}}(0)} & = -4N_{f}T_{F} \frac{j^{2} + 3j + 4}{(j+1)(j+2)(j+3)}, \tag{A.2} \\
\GC_{j}^{\gamma_{\text{fw}}(0)} & = -2C_{F} \frac{j^{2} + 3j + 4}{j(j+1)(j+2)}, \tag{A.3} \\
\GG_{j}^{\gamma_{\text{fw}}(0)} & = -C_{A} \left( -4\psi(j+2) + 4\psi(1) + 8\frac{j^{2} + 3j + 3}{j(j+1)(j+2)(j+3)} - \frac{\beta_{0}}{C_{A}} \right). \tag{A.4}
\end{align}
The anomalous dimensions at $\mathcal{O}(\alpha_s^2)$ are given by \cite{25}:

\[
\begin{align*}
QQ_{ij}^{\text{fw NS}(1)}(\sigma) &= \left(C_F^2 - \frac{1}{2}C_FC_A\right) \left\{ \frac{4(2j+3)}{(j+1)^2(j+2)^2} S(j+1) - 2\frac{3j^3 + 10j^2 + 11j + 3}{(j+1)^3(j+2)^3} \right. \\
&\quad + 4 \left( 2S_1(j+1) - \frac{1}{(j+1)(j+2)} \right) (S_2(j+1) - S_2'(j+1)) \\
&\quad + 16S(j+1) + 6S_2(j+1) - \frac{3}{4} - 2S_2'(j+1) + 4\sigma \frac{2j^2 + 6j + 5}{(j+1)^3(j+2)^3} \\
&\quad + C_FC_A \left\{ S_1(j+1) \left( \frac{134}{9} + \frac{2(2j+3)}{(j+1)^2(j+2)^2} \right) \\
&\quad - 4S_1(j+1)S_2(j+1) + S_2(j+1) \left( -\frac{13}{3} + \frac{2}{(j+1)(j+2)} \right) \\
&\quad - \frac{43}{24} - \frac{1151j^4 + 867j^3 + 1792j^2 + 1590j + 523}{(j+1)^3(j+2)^3} \right\} \\
&\quad + C_FT_FN_f \left\{ -\frac{40}{9} S_1(j+1) + \frac{8}{3} S_2(j+1) + \frac{1}{3} + \frac{4}{9} \frac{11j^2 + 27j + 13}{(j+1)^2(j+2)^2} \right\}, \quad \text{(A.5)}
\end{align*}
\]

\[
\begin{align*}
QQ_{ij}^{\text{fw}(1)} &= QQ_{ij}^{\text{fw NS}(1)}(\sigma = 1) - 4C_FT_FN_f \frac{j^5 + 57j^4 + 227j^3 + 427j^2 + 404j + 160}{j(j+1)^3(j+2)^3(j+3)^2}, \quad \text{(A.6)}
\end{align*}
\]

\[
\begin{align*}
QQ_{ij}^{\text{fw}(1)} &= -2C_A T_FN_f \left\{ \left( -2S_2^2(j+1) + 2S_2(j+1) - 2S_2'(j+1) \right) \frac{j^2 + 3j + 4}{(j+1)(j+2)(j+3)} \\
&\quad + 2j^9 + 15j^8 + 99j^7 + 382j^6 + 963j^5 + 1711j^4 + 2273j^3 + 2252j^2 + 1488j + 480 \\
&\quad + \frac{8}{(j+2)^2(j+3)^2} S_1(j+1) \right\} \\
&\quad - 2C_FT_FN_f \left\{ \left( 2S_1^2(j+1) - 2S_2(j+1) + 5 \right) \frac{j^2 + 3j + 4}{(j+1)(j+2)(j+3)} \\
&\quad - 4S_1(j+1) \frac{11j^4 + 70j^3 + 159j^2 + 160j + 64}{(j+1)^3(j+2)^3(j+3)} \right\}, \quad \text{(A.7)}
\end{align*}
\]

\[
\begin{align*}
GG_{ij}^{\text{fw}(1)} &= -C_F^2 \left\{ \left( -2S_1^2(j+1) + 10S_1(j+1) - 2S_2(j+1) \right) \frac{j^2 + 3j + 4}{j(j+1)(j+2)} \\
&\quad - 4S_1(j+1) \frac{12j^6 + 102j^5 + 373j^4 + 740j^3 + 821j^2 + 464j + 96}{(j+1)^3(j+2)^3} \right\} \\
&\quad - 2C_A C_F \left\{ \left( S_1^2(j+1) + S_2(j+1) - S_2'(j+1) \right) \frac{j^2 + 3j + 4}{j(j+1)(j+2)} \\
&\quad - \frac{1109j^9 + 1602j^8 + 10292j^7 + 38022j^6 + 88673j^5 + 133818j^4 + 128014j^3}{9} \frac{j^2(j+1)^3(j+2)^3(j+3)^2}{j^2(j+1)^3(j+2)^3(j+2)} S_1(j+1) \right\} \\
&\quad + \frac{172582j^2 + 21384j + 2592}{9} \frac{j^2(j+1)^3(j+2)^3(j+3)^2}{j^2(j+1)^3(j+2)^3(j+2)} S_1(j+1) \\
&\quad \end{align*}
\]

\[\text{[Page 15]}\]
\[-\frac{8}{3}C_F T_F N_f \left\{ \left( S_1(j + 1) - \frac{8}{3} \right) \frac{j^2 + 3j + 4}{j(j + 1)(j + 2)} + \frac{1}{(j + 2)^2} \right\}, \quad (A.8)\]

\[
GG_{\gamma^k j}^{\text{fw}(1)} = C_A T_F N_f \left\{ -\frac{40}{9} S_1(j + 1) + \frac{8}{3} + \frac{819j^4 + 114j^3 + 275j^2 + 312j + 138}{j(j + 1)^2(j + 2)^2(j + 3)} \right\}
+ C_F T_F N_f \left\{ 2 + 4 \frac{2j^6 + 16j^5 + 51j^4 + 74j^3 + 41j^2 - 8j - 16}{j(j + 1)^3(j + 2)^3(j + 3)} \right\}
+ C_4 \left\{ \frac{134}{9} S_1(j + 1) + 16S_1(j + 1) \frac{2j^5 + 15j^4 + 48j^3 + 81j^2 + 66j + 18}{j^2(j + 1)^2(j + 2)^2(j + 3)^2} \right\}
- \frac{16}{3} + 8 S_2(j + 1) \frac{j^2 + 3j + 3}{j(j + 1)(j + 2)(j + 3)} - 4 \frac{S_1(j + 1) S_2(j + 1)}{j^2(j + 1)^2(j + 2)^2(j + 3)^2}
+ 8 \tilde{S}(j + 1) - S_3'(j + 1) - \frac{1457j^9 + 6855j^8 + 44428j^7 + 163542j^6}{j^2(j + 1)^3(j + 2)^3(j + 3)^3}
- \frac{1376129j^5 + 557883j^4 + 529962j^3 + 308808j^2 + 101088j + 15552}{j^2(j + 1)^3(j + 2)^3(j + 3)^3} \right\}. \quad (A.9)\]

Here we should use the following expressions for the analytically continued functions \[23\]

\[
S_1(j) = \gamma_E + \psi(j + 1),
\]

\[
S_2(j) = \zeta(2) - \psi'(j + 1),
\]

\[
S_3(j) = \zeta(3) + \frac{1}{2} \psi''(j + 1),
\]

\[
S_3'(j) = \frac{1}{2}(1 + \sigma) S_3 \left( \frac{j}{2} \right) + \frac{1}{2}(1 - \sigma) S_3 \left( \frac{j - 1}{2} \right),
\]

\[
\tilde{S}(j) = -\frac{5}{8} \zeta(3) + \sigma \left\{ \frac{S_1(j)}{j^2} - \frac{\zeta(2)}{2} \left( \psi \left( \frac{j + 1}{2} \right) - \psi \left( \frac{j}{2} \right) \right) + \int_0^1 dx x^{j - 1} \frac{\text{Li}_2(x)}{1 + x} \right\}. \quad (A.14)\]

where \( \psi^{(\ell - 1)}(j) = \frac{d^{(\ell - 1)} \psi(j)}{dj^{\ell - 1}} \) is the poly-gamma function and \( \gamma_E = -\psi(1) \) is the Euler-Mascheroni constant.

The integral over the dilogarithm \( \text{Li}_2(x) \) can be evaluated in terms of an analytical function of \( j \) by using the least square fit approximation \[23\]:

\[
\frac{\text{Li}_2(x)}{1 + x} \simeq 0.0030 + 1.0990x - 1.5463x^2 + 3.2860x^3 - 3.7887x^4 + 1.7646x^5. \quad (A.15)\]

The \( \sigma = (-1)^{j + 1} \) takes the following values for “non-singlet” — charge conjugation odd, \( q_v \), and flavour NS, \( q^{NS} \), — combinations:

\[
\sigma = -1 \quad \text{for} \quad q_v = q - \bar{q}, \quad \sigma = 1 \quad \text{for} \quad q^{NS} = (u + \bar{u}) - (d + \bar{d}), \quad \text{etc.,} \quad (A.16)\]

while the singlet distributions are evolved with \( \sigma = 1 \).
B Non-diagonal anomalous dimensions.

The elements of the matrix $\gamma_{jk}^{\text{ND}(1)}$ derived in Ref. [1] read

$$
QQ_{\gamma_{jk}^{\text{ND}(1)}} = \left( QQ_{\gamma_{j}^{D(0)}} - QQ_{\gamma_{k}^{D(0)}} \right) \left\{ d_{jk} \left( \beta_{0} - QQ_{\gamma_{k}^{D(0)}} \right) + QQ_{g_{jk}} \right\} - \left( QQ_{\gamma_{j}^{D(0)}} - QQ_{\gamma_{k}^{D(0)}} \right) d_{jk} \frac{QQ_{g_{jk}} + QQ_{\gamma_{j}^{D(0)}}}{QQ_{g_{jk}}},
$$

(B.1)

$$
QQ_{\gamma_{jk}^{\text{ND}(1)}} = \left( QQ_{\gamma_{j}^{D(0)}} - QQ_{\gamma_{k}^{D(0)}} \right) d_{jk} \left( \beta_{0} - QQ_{\gamma_{k}^{D(0)}} \right) - \left( QQ_{\gamma_{j}^{D(0)}} - QQ_{\gamma_{k}^{D(0)}} \right) d_{jk} \frac{QQ_{g_{jk}} + QQ_{\gamma_{j}^{D(0)}}}{QQ_{g_{jk}}},
$$

(B.2)

$$
QQ_{\gamma_{jk}^{\text{ND}(1)}} = \left( QQ_{\gamma_{j}^{D(0)}} - QQ_{\gamma_{k}^{D(0)}} \right) \left\{ d_{jk} \left( \beta_{0} - QQ_{\gamma_{k}^{D(0)}} \right) + QQ_{g_{jk}} \right\} - \left( QQ_{\gamma_{j}^{D(0)}} - QQ_{\gamma_{k}^{D(0)}} \right) d_{jk} \frac{QQ_{g_{jk}} + QQ_{\gamma_{j}^{D(0)}}}{QQ_{g_{jk}}},
$$

(B.3)

$$
QQ_{\gamma_{jk}^{\text{ND}(1)}} = \left( QQ_{\gamma_{j}^{D(0)}} - QQ_{\gamma_{k}^{D(0)}} \right) \left\{ d_{jk} \left( \beta_{0} - QQ_{\gamma_{k}^{D(0)}} \right) + QQ_{g_{jk}} \right\} - \left( QQ_{\gamma_{j}^{D(0)}} - QQ_{\gamma_{k}^{D(0)}} \right) d_{jk} \frac{QQ_{g_{jk}} + QQ_{\gamma_{j}^{D(0)}}}{QQ_{g_{jk}}},
$$

(B.4)

Here the leading order diagonal anomalous dimensions are given by Eqs. (B.8) and (A.1); the $d$ and $g$ elements are

$$
d_{jk} = -\frac{1}{2} \left[ 1 + (-1)^{j-k} \right] \frac{(2k+3)}{(j-k)(j+k+3)},
$$

(B.5)

and

$$
QQ_{g_{jk}} = -C_{F} \left[ 1 + (-1)^{j-k} \right] \frac{(3+2k)}{(j-k)(j+k+3)} \times \left\{ 2A_{jk} + (A_{jk} - \psi(j+2) + \psi(1)) \frac{(j-k)(j+k+3)}{(k+1)(k+2)} \right\},
$$

(B.6)

$$
QQ_{g_{jk}} = -C_{F} \left[ 1 + (-1)^{j-k} \right] \frac{1}{6} \frac{(3+2k)}{(k+1)(k+2)},
$$

(B.7)

$$
QQ_{g_{jk}} = -C_{A} \left[ 1 + (-1)^{j-k} \right] \frac{(3+2k)}{(j-k)(j+k+3)} \times \left\{ 2A_{jk} + (A_{jk} - \psi(j+2) + \psi(1)) \left[ \frac{\Gamma(j+4)\Gamma(k)}{\Gamma(j)\Gamma(k+4)} - 1 \right] + 2(j-k)(j+k+3) \frac{\Gamma(k)}{\Gamma(k+4)} \right\},
$$

(B.8)

respectively. We have introduced here the matrix $\hat{A}$ whose elements are defined by

$$
A_{jk} = \psi \left( \frac{j+k+4}{2} \right) - \psi \left( \frac{j-k}{2} \right) + 2\psi(j-k) - \psi(j+2) - \psi(1).
$$

(B.10)

C NFPD with support $0 \leq x \leq 1$.

Here we give a list of formulae for the expansion of non-forward parton distributions with the support $0 \leq x \leq 1$. The only difference which arises w.r.t. the results given in the body of the
paper is the argument of the Jacobi polynomials, \( P_j^{(\alpha,\beta)}(2x - 1) \), in the expansion

\[
\mathcal{O}(x, \zeta, Q^2) = \sum_{j=0}^{\infty} \tilde{P}_j(x) \mathcal{M}_j(\zeta, Q^2), \tag{C.1}
\]

with elements \( \tilde{P}_j(x) = \frac{w(x;\alpha, \beta)}{n_j(\alpha, \beta)} P_j^{(\alpha,\beta)}(2x - 1) \) where \( n_j(\alpha, \beta) = \frac{\Gamma(j+\alpha+1)\Gamma(j+\beta+1)}{(2j+\alpha+\beta+1)\Gamma(2j+\alpha+\beta+1)} \). The Jacobi moments are given by Eq. (22) provided we will substitute the elements of the coefficient matrix (24) by (cf. Ref. [4])

\[
E_{jk}^J(\nu; \alpha, \beta|\zeta) = (-1)^{j-k} \theta_{jk} 2^{2\nu-1} \frac{\Gamma(\nu)\Gamma(k+\nu+\frac{1}{2})\Gamma(j+\beta+1)}{\Gamma(\frac{1}{2})\Gamma(2k+2\nu)\Gamma(k+\beta+1)\Gamma(j-k+1)\Gamma(j+\alpha+\beta+1)} \times \exp\left(\frac{-j+k+j+k+\alpha+\beta+1,k+\nu+\frac{1}{2}}{2k+2\nu+1,k+\beta+1} | \zeta \right). \tag{C.2}
\]

The results for other classic orthogonal polynomials can be immediately derived from this equation.

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