1. Introduction

A photon echo [1, 2] is a broad class of optical phenomena where a coherence induced in a quantum system by an optical field is emitted in a form of a well-resolved intense optical signal, similar to the spin echo in nuclear magnetic resonance. Over decades, photon echo has been in use as a powerful method of coherent spectroscopy, providing unique information on transient processes in gases, liquid, and solids [3]. Photon-echo revivals in molecular rotational coherences have been shown to enable efficient quantum control of molecular alignment [4], photochemical reactions [5], as well as synthesis of ultrashort field waveforms [6]. In the era of quantum information, photon echo is viewed as a promising strategy for quantum data storage and quantum memories [7–10] (recent reviews see also in [11–14]).

Here, we show that a photon echo can be implemented by purely optical means using an array of on-chip high-finesse ring cavities whose parameters are chirped in such a way as to support equidistant spectra of cavity modes. A classical or quantum optical signal launched into such a system becomes distributed between individual cavities, giving rise to prominent coherence echo revivals at well-defined delay times, controlled by the chirp of cavity parameters. This effect enables long storage times for high-throughput broadband optical delay and quantum memory.

We consider an array of \( N \) single-mode high-finesse chirped ring cavities (CRC) with an equidistant spectrum of modes with a mode spacing \( \Delta \) (figure 1). An optical field coupled into such an array remains distributed between the cavities until all the cavity modes can re-emit in phase, giving rise to an intense photon-echo signals at the output. With appropriate coupling between the optical nanofiber and the cavities, which is possible, e.g. with a fiber tapered to a submicron diameter [15, 16], the entire field stored in the cavity array can be retrieved within the first echo signal with a time delay \( t_{\text{echo}} \approx 2\pi / \Delta \). We found the condition when this scheme can work as an efficient QM and classical time delay line then we discuss possible experimental implementations and its further development for long lived storage.
2. Physical model and equations

In the considered CRC scheme (or frequency comb all-pass filter FC-APF), the spectrum of cavity modes is equidistant and consists of narrow lines centered at cavity eigenfrequencies $\omega_n$:

$$\omega_n = \omega_0 + n\Delta - i\gamma_n,$$

where $\gamma_n < \Delta$ is the nth mode linewidth, and $n = 0,1, \ldots, N-1$. Hamiltonian of the considered CRC system is written as

$$\hat{H} = \hat{H}_c + \hat{H}_p + \hat{V},$$

where

$$\hat{H}_c = h \sum_{n} \omega_n \hat{a}_n^\dagger \hat{a}_n \quad \text{and} \quad \hat{H}_p = h \int d\omega \omega \sum_n \hat{a}_n^\dagger \hat{a}_n \hat{f}(\omega).$$

The vacuum state $|\psi_0\rangle$ is the vacuum state, $\gamma_n$ is the mode linewidth, and $\Delta / (\gamma_n \gamma_N)$ is the cavity coupling constant.

In the case of weak coupling $\gamma_n \ll \Delta$, respectively.

$$\hat{a}_n^\dagger \hat{a}_m^\dagger + \hat{a}_m \hat{a}_n = \delta_{nm}.$$ 

As it is seen in figures 2, 3 and 5, the coupling $\gamma_n$ is assumed to be in the ground state at room temperature, we arrive at the following set of equations for the field amplitude operators of nth cavity:

$$\frac{d\hat{a}_n}{dr} = -(i\omega_n + \frac{n\Delta}{2} + i\gamma_n)\hat{a}_n + \sqrt{n\gamma_n} \hat{a}_{n,\text{in}}.$$ \hspace{1cm} (1)

To solve equation (1), we take into account the condition for the input and output fields of nth cavity $\hat{a}_{n,\text{in}} = \hat{a}_{n,\text{out}} + \sqrt{n\gamma_n} \hat{b}_n$ [18] and the relation for the fiber mode before and after its interaction with nth cavity:

$$\hat{a}_{n\text{,out}}(\omega) = e^{-i\omega(t_z - t_{\text{in}})} \hat{a}_{n\text{,in}}(\omega).$$ 

Using a Fourier transform

$$\hat{a}_{n\text{,in}}(t) = \int d\omega \hat{a}_{n\text{,in}}(\omega)e^{-i\omega t},$$

we get

$$\hat{a}_{n\text{,in}}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt e^{i\omega t} \hat{a}_{n\text{,in}}(t).$$ \hspace{1cm} (2)

Since

$$\hat{a}_{n\text{,out}}(\omega, z > \Delta N) = e^{i\Delta N \omega (t-z)} U_{m} e^{i\omega t} \hat{a}_{n\text{,in}}(\omega),$$

where

$$U_{m} = (-1)^m e^{i\omega \Delta N (z-z_0)} \prod_{n=1}^{m} \frac{1}{2} \sqrt{\frac{\kappa_n - \gamma_n - i(\omega_n - \omega)}{\kappa_n + \gamma_n + i(\omega_n - \omega)}},$$ \hspace{1cm} (4)

determines the fiber mode amplitude behind nth cavity, $\hat{a}_{n\text{,out}}(\omega, z_0)$ is the Fourier transform of the input field, and $\kappa_n = \gamma_n - i(\omega_n - \omega)$ is the cavity coupling constant.

$$\hat{b}_n(t) = \int d\omega \beta_n(\omega) U_{m} e^{-i\omega t} \hat{a}_{n\text{,in}}(\omega),$$ \hspace{1cm} (5)

with

$$\beta_n(\omega) = \frac{\sqrt{n\gamma_n}}{\sqrt{2\pi}} \frac{e^{-i\omega(t_z - t_{\text{in}})}}{\sqrt{\frac{\kappa_n - \gamma_n - i(\omega_n - \omega)}{\kappa_n + \gamma_n + i(\omega_n - \omega)}}}.$$ 

For a single-photon initial state, we have $|\psi_\text{in}\rangle = \int d\omega \hat{f}_\text{in}(\omega) \hat{a}_{n\text{,in}}(\omega) |\varnothing\rangle$, where $|\varnothing\rangle$ is the vacuum state, and wave function $f_n(\omega)$ is normalized by $\int d\omega |f_n(\omega)|^2 = 1$ with a bandwidth $\delta$. Here, the field amplitude in nth cavity $B_n(t) = \langle \varnothing | \hat{b}_n(t) | \psi_\text{in}\rangle$, the probability to find a photon in nth cavity $P_n = \langle \psi_\text{in}\rangle | \hat{b}_n(t) | \psi_\text{in}\rangle$, and the output light field amplitude $A_{\text{out}}(t, z) = \langle \varnothing | \hat{a}_{\text{out},k}(t, z) | \psi_\text{in}\rangle$ are given by

$$B_n(t) = \int d\omega e^{-i\omega t} \beta_n(\omega) U_{m} e^{-i\omega t} \hat{a}_{n\text{,in}}(\omega),$$ \hspace{1cm} (6)

$$P_n(t) = B_n^*(t) B_n(t),$$ \hspace{1cm} (7)

$$A_{\text{out}}(t, z) = \int d\omega e^{-i\omega t} \beta_n(\omega) U_{m} e^{-i\omega t} \hat{a}_{n\text{,in}}(\omega).$$ \hspace{1cm} (8)

We are interested in quantum efficiency $\eta$ of an input field retrieval in the first echo pulse:

$$\eta(t) = \frac{\int d\omega A_{\text{out}}(t, z) A_{\text{in}}(t, z)}{\int d\omega A_{\text{in}}(t, z) A_{\text{in}}(t, z)},$$ \hspace{1cm} (9)

where $A_{\text{in}}(t, z) = \langle \varnothing | \hat{a}_{\text{in}}(t, z) | \psi_\text{in}\rangle$ is the input field amplitude.

In figure 2, we plot the temporal evolution of an output field amplitude $A_{\text{out}}(t)$ calculated for a three-pulse input signal for three values of the coupling constant $\kappa = \kappa_1 = \ldots = \kappa_N$, fixed $\beta = \gamma_1 = \ldots = \gamma_N$, intrinsic cavity quality factor $Q = \omega_0 / (2\gamma)$ and finesse $F = \Delta / (2\gamma)$. In the case of weak coupling $\kappa \ll \Delta$, a large fraction of input light is transmitted through the CRC array without time delay, giving rise to a signal at $t \approx 0$ in figure 2. The remainder part of the input field is distributed between the first, second, and third echo signals observed at $t_{\text{echo}}$, $t_{\text{echo}}^2 \approx 2 t_{\text{echo}} = 4 \pi / \Delta$, and $t_{\text{echo}}^3 \approx 3 t_{\text{echo}}$, respectively. Calculations presented in figure 3 for $F > 50$ show that the first echo contains input signal with an probability (efficiency) $\eta > 0.9$. As it is seen in figures 2, 3 and 5, the coupling $\kappa \approx 0.5 \Delta$ provides the highest quantum efficiency. Herein, the storage time $t_{\text{echo}}$ is slightly sensitive to the value of $\kappa$.

3. Comparison with single frequency SCISSOR

We emphasize that, while the CRC array considered here stores light—classical or quantum—in the form of a field distributed between individual cavities, its ability to provide long storage times is due to the periodic coherence revivals, occurring at the instants of time when the fields circulating in individual cavities are all emitted in phase. In this respect, it is instructive to compare the delay-line performance of the CRC scheme considered here with a delay-line architecture based on the well-known all-pass filter (APF) called also as a side coupled integrated spaced sequence of resonators (SCISSOR) [19, 20]. It is worth noting that SCISSOR together with coupled resonator optical waveguide (CROW) configuration [21] are two complementary basic standards of an on-chip integrated optical delay line (see [22, 23]). Below we show that CRC-scheme can considerably exceed the basic characteristics of SCISSOR-scheme.

A comparison of CRC- and SCISSOR- schemes is shown in figures 4, 5 and table 1). Both the schemes can provide high quantum efficiency $\eta \approx 95\%$ and superb fidelity, with the shape of the retrieved signal very accurately following the shape of the input pulse (figures 2, 4 and 5). However, for
η ≈ 95% values, the delay times provided by a CRC array are substantially longer than the delay times attainable with the SCISSOR design. But what is even more important, is that the coupling constant needed to achieve comparable delay times is an few orders of magnitude is smaller for CRC arrays. Specifically, calculations presented in figures 4 and 5 give \( \tau = 2\pi/\Delta \approx 62.8/\delta \) for an CRC array with \( \kappa = 0.05/\delta \) versus \( \tau \approx 16.8/\delta \) for SCISSOR with \( \kappa = 7.5/\delta \). Accordingly the predominant interaction of signal fields with CRC array occurs at the off-resonant condition with the ring cavity modes that should provide lower losses in comparison with SCISSOR.

4. Experimental perspectives

The proposed purely optical scheme can be experimentally implemented on whispering gallery modes of the ring cavities [15] with cavities been coupled to integrated photonic waveguide. The cavity modes in this case are characterized by high intrinsic quality factor such as \( Q = 10^{10} \) [15] and \( Q = 10^{9} \) [24].
while being efficiently coupled to the nanofiber [25]. Below we analyze the possible implementation of CRC-scheme by means of on-chip array of ring cavities [24].

Following the work [24], we use the parameters of single mode ring cavity: ω0 = 1.26 · 10^{15} rad sec^{-1} corresponding to the telecommunication wavelength λ = 1.5 μm, cavity diameter D = 90 μm, free spectral range ω_{FSR} = 4.96 · 10^{12} rad sec^{-1} and γ ≈ γ = 1/(2τ_{cm}) = 10^7 rad sec^{-1} determined by the intrinsic quality factor Q = 1.25 · 10^{8}. By taking into account that total number of the cavities can reach 100 [22], our consideration corresponds to an intermediate number N = 61 that determines a maximum intermode spacing Δ = ω_{FSR}/60 = 8.27 · 10^{10} rad sec^{-1}. The optimal coupling constant κ = 0.5Δ = 4.135 · 10^{10} rad sec^{-1} (see figure 5) can be obtained experimentally at the appropriate spatial distance between the micro cavities and nanooptical waveguide [22]. For instance nth cavity should be tuned to the spectral detuning nΔ by using the matched spatial diameter D_{n} ≈ D - nD (where εD = ΔDf/ω_{0} = 7.37 · 10^{-4}D = 63 nm). An arbitrary spectral offsets nΔ can be also operated in tunable microresonators rely on thermo-optics, or electro-optics and free-carrier dispersion control to vary the index of refraction [22, 26].

In evaluation of efficient light pulse retrieval we apply dimensionless numerical results presented in figures 4 and 5 for different intermode spacings depending on the temporal duration of pulse as Δ = (10τ_{s})^{-1}. That is, to demonstrate we only use the intermode spacing Δ = 0.1δ while a spectral width of light pulse δ = τ_{s}^{-1} can also be in the range from few Δ to several dozen of Δ. By using two possible intrinsic quality factors (Q = 10^8, Q = 10^{10}), we find the time delays t_{delay} for SCISSOR—and CRC-schemes which correspond to the same η ≈ 0.95 and F = 500. For more evident representation, these data are summarized in table 1. The time delays in table 1 demonstrate a possibility for an efficient quantum storage of pico- and nanosecond pulses where time delays in CRC-scheme can reach t_{echo} ≈ 2 ns for τ_{s} = 32 ps, Q = 10^{8} and t_{echo} ≈ 0.2 μs for τ_{s} = 3.2 ns, Q = 10^{10}. Spatial size of the described array of 61 cavities can be estimated as L = 60 · 100 μm = 6 mm where 100 μm is a distance between the centers of two nearest cavities that indicates a significant advantage of CRC for using in on-chip nanooptical schemes in comparison with an usual SCISSOR.

Storage time in CRC-scheme t_{echo} ≈ 2π/Δ can be further increased for smaller frequency spacing Δ (i.e. for lower finesse F) but at the expense of reducing the efficiency. By using the numerical data in figure 3, we find that it is possible to get the time delay t_{delay} ≈ 334.7 ns for light pulse τ_{s} = 5.33 ns with efficiency η ≈ 0.35 (where Q = 10^{8}). For higher Q = 10^{10}, the time delay can reach 33.5 μs. This time delay is shorter in one order of magnitude in comparison with the lifetime of a light in a single ring cavity.

Besides lithography, we note femtosecond laser writing (FSLW) technology [27, 28] for fabrication of the CRC-schemes. FSLW is versatile tool for creating on chip optical waveguides in various media (glasses or crystals) for controllable coupling with many single mode cavities. Due to these properties, FSLW seems to be also suitable for long-lived quantum storage based on the crystals doped by rare-earth ions.

### 5. Discussion and conclusion

CRC-scheme can be developed for controllable retrieval which is possible by fast equalizing the cavity frequencies after absorption of an input light pulse before t ≈ 2π/Δ. The aligned cavity frequencies will lead to freezing relative phases of the cavity modes and complete stopping the input light pulse in CRC-array. Further light retrieval will be possible only after recovering the cavity frequencies providing subsequent rephasing the excited cavity modes. Although an maximum storage time will be limited by the intrinsic cavity mode Q-factor and by the additional losses caused by the non ideal switching but this technique opens a way for on demand fast light pulse retrieval. The frequency controlling of the ring cavity modes can be implemented by the number of experimental methods [22, 26, 29] providing sufficiently fast switching of the microresonator frequencies up to ten and hundreds GHz bandwidth.

Comparing with free space photon echo AFC-protocol [8] which seems promising for application in optical quantum repeaters, the described CRC-scheme uses purely optical tools and can operate at room temperature. It is also important that CRC-scheme provides a light field retrieval in forward...
direction that dramatically facilitates an implementation of high quantum efficiency. Another non-obvious property of CRC-scheme is an efficient quantum storage at the optimal coupling $\kappa \approx 0.5\Delta$ that differs this scheme from the free space AFC protocol [8] and from its recent spatial-spectral version [30]. In this respect, the optimal coupling indicates that CRC-scheme is similar to the impedance matching QMs (see, for example [31]) although it does not use any common single mode resonator coupling with all the resonant atoms (or CRCs). However CRC-impedance matching has a different physical nature since CRC-scheme eliminates any reflection of the input light fields.

The described properties of all-optical photon echo on the chip ring cavity array coupled with nanofibers demonstrate a promising credit to the room temperature time delay line and optical quantum memory suitable for application in on-chip optical schemes, quantum processing and communication.

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References

[1] Kurmit N A, Abella I D and Hartmann S R 1964 Observation of a photon echo Phys. Rev. Lett. 13 567–9
[2] Allen L and Eberly J H 1975 Optical Resonance and Two-Level Atoms (New York: Dover)
[3] Shen Y R 1984 The Principles of Nonlinear Optics (New York: Wiley)
[4] Stapelfeldt H and Seideman T 2003 Colloquium: aligning molecules with strong laser pulses Rev. Mod. Phys. 75 543–57
[5] Kartashov D, Möhring J, Andriukaitis G, Pugzlys A, Zheltikov A, Motzkuš M and Baltuska A 2012 Conf. on Lasers and Electro-Optics 2012 (OSA Technical Digest) (Optical Society of America) QThE4.6
[6] Fedotov I V, Savvin A D, Fedotov A B and Zheltikov A M 2007 Controlled rotational Raman echo recurrences and modulation of high-intensity ultrashort laser pulses by molecular rotations in the gas phase Opt. Lett. 32 1275–7
[7] Moiseev S A and Noskov M I 2004 The possibilities of the quantum memory realization for short pulses of light in the photon echo technique Laser Phys. Lett. 1 303–10
[8] de Riedmatten H, Afzelius M, Staudt M U, Simon C and Gisin N 2008 A solid-state lightmatter interface at the single-photon level Nature 456 773–7
[9] Hedges M P, Longdell J J, Li Y and Sellars M J 2010 Efficient quantum memory for light Nature 465 1052–6
[10] Hosseini M, Sparkes M, Campbell G, Lam P K and Buchler B C 2011 High efficiency coherent optical memory with warm rubidium vapour Nat. Commun. 2 174
[11] Lvovsky A I, Sanders B C and Tittel W 2009 Optical quantum memory Nat. Photon. 3 706–14
[12] Simon C et al 2010 Quantum memories Eur. Phys. J. D 58 1–22
[13] Hammerer K, Sørensen A S and Polzik E S 2010 Quantum interface between light and atomic ensembles Rev. Mod. Phys. 82 1041–93
[14] Tittel W, Afzelius M, Chanière T, Cone R L, Kröll S, Moiseev S A and Sellars M 2010 Photon-echo quantum memory in solid state systems Laser Photonics Rev. 4 244–67
[15] Braginsky V B, Gorodetsky M L and Ilen’ko V S 1989 Quality-factor and nonlinear properties of optical whispering-gallery modes Phys. Lett. A 137 393–7
[16] Gorodetsky M L and Ilen’ko V S 1999 Optical microsphere resonators: optimal coupling to high—whispering-gallery modes J. Opt. Soc. Am. B 16 147–54
[17] Heebner J, Rohit R and Ibrahim T 2008 Optical Microresonators Theory Fabrication, and Applications (Springer Series in Optical Sciences) (Berlin: Springer)
[18] Walls D F and Milburn G J 2007 Quantum Optics (Berlin: Springer)
[19] Heebner J E and Boyd R W 2002 ‘Slow’ and ‘fast’ light in resonator-coupled waveguides J. Mod. Opt. 49 2629–36
[20] Xia F, Sekaric L and Vlascov Y 2007 Ultracompact optical buffers on a silicon chip Nat. Photon. 1 65–71
[21] Yariv A, Xu Y, Lee R K and Scherer A 1999 Coupled-resonator optical waveguide: a proposal and analysis Opt. Lett. 24 711–3
[22] Chremmos I, Schwelb O and Uzunoglu N 2010 Photonic Microresonators Research and Applications (Berlin: Springer)
[23] Morichetti F, Ferrari C, Canciamilla A and Pelloni A 2012 The first decade of coupled resonator optical waveguides: bringing slow light to applications Laser Photonics Rev. 6 74–96
[24] Armani D K, Kippenberg T J, Spillane S M and Vahala K J 2003 Ultra-high-Q toroid microcavity on a chip Nature 421 925–8
[25] Herr T, Brasch V, Jost J D, Wang C Y, Kondratiev N M, Gorodetsky M L and Kippenberg T J 2014 Temporal solitons in optical microresonators Nat. Photon. 8 145–53
[26] Melloni A, Canciamilla A, Ferrari C, Morichetti F, O’Faolain L, Krauss T, De La Rue R M, Samarelli A and Sorel M 2010 Tunable delay lines in silicon photonics: coupled resonators and photonic crystals, a comparison IEEE Photonics J. 2 181–94
[27] Florea C and Winick K A 2003 Fabrication and characterization of photonic devices directly written in glass using femtosecond laser pulses J. Lightwave Technol. 21 246–53
[28] Dyakonov I V, Kalinkin A A, Saygin M Y, Abroskin A G, Radchenko I V, Straupe S S and Kulik S P 2016 Low-loss single-mode integrated waveguides in soda-lime glass Appl. Phys. B 122 245
[29] Canciamilla A, Torregiani M, Ferrari C, Morichetti F, De La Rue R M, Samarelli A, Sorel M and Pelloni A 2010 Silicon coupled-ring resonator structures for slow light applications: potential, impairments and ultimate limits J. Opt. 12 104008
[30] Tian M, Vega D and Dilles J 2013 Quantum memory based on a spatiotemporal atomic comb Phys. Rev. A 87 042338
[31] Moiseev S A 2013 Off-resonant Raman-echo quantum memory for inhomogeneously broadened atoms in a cavity Phys. Rev. A 88 012304