Abstract—We consider realistic long-haul optical links, with linear and nonlinear impairments, and investigate the application of time-frequency packing with low-order constellations as a possible solution to increase the spectral efficiency. A detailed comparison with available techniques from the literature will be also performed. We will see that this technique represents a feasible solution to overcome the relevant theoretical and technological issues related to this spectral efficiency increase and could be more effective than the simple adoption of high-order modulation formats.

Index Terms—Coherent optical systems, long-haul optical communications, faster-than-Nyquist signaling, high-order modulation formats, spectral efficiency, time-frequency packing.

I. INTRODUCTION

The ever-growing bandwidth demand on internet and data networks has pushed the research in the field of optical communications towards more sophisticated transmission techniques [1]–[3]. This field has been characterized for decades by the simplest binary formats and transmission systems, since bandwidth requirements were met with both cost-effective and technologically affordable solutions. Nowadays optical communication channels have rapidly passed from 10 Gbps to 100 Gbps, whereas the next challenging step will lead to 1 Tbps. Among the more severe limitations involved in such a system upgrade, the technological and practical issues of processing high data rates on a single channel, and the optical channel impairments related to the required transmit power (i.e., the nonlinear effects) are the most prominent. Since a higher capacity will hardly be reached with single-channel transmissions, many different solutions and related implementation techniques have been devised and proposed, in order to exploit at best the optical channel capacity under current technological constraints. Basically, all the proposed solutions, irrespectively of the particular transmission technique, are based on multi-carrier transmissions or so-called “superchannels” [4], which means that the goal capacity is reached by binding up as many single-channels together as necessary, in an efficient way. The techniques investigated in this paper belong to this class and allow an effective and reduced-complexity “packing” of the channels aiming at greatly improving the spectral efficiency (SE) of the transmission.

As in most digital communication systems, orthogonal signaling is the paradigm traditionally employed also in the design of long-haul optical systems. This paradigm consists of ensuring the absence of intersymbol interference (ISI) and, in multi-carrier scenarios, also the absence of inter-carrier interference (ICI), i.e., the absence of interference between adjacent channels. As an example, in single-carrier coherent optical systems, possibly employing polarization multiplexing (PM), given a conventional transmitter, with Mach-Zehnder (MZ) modulators and return to zero (RZ) or non-return to zero (NRZ) shaping pulses, when group velocity dispersion (GVD) and polarization mode dispersion (PMD) are effectively compensated for and nonlinear effects are limited, proper filtering and sampling at the receiver ensure that even a symbol-by-symbol detector enables an almost-optimal performance since ISI and ICI are very limited [5]. Examples of multi-carrier transmission systems based on this paradigm are represented by orthogonal frequency-division multiplexing (OFDM) [6] (see also [7] and references therein) and Nyquist wavelength-division multiplexing (WDM) [8] (and similar bandwidth narrowing techniques such as [9]), where the use of proper shaping pulses allows to remove, at least in theory, both ISI and ICI without using guard bands and thus without wasting resources. On the other hand, when practical transmit or receive filters are considered and nonlinear effects come into play, orthogonality is no more guaranteed and an unwanted interference appears. In addition, in these orthogonal signaling systems, the spectral efficiency can be improved only by increasing the constellation cardinality, thus employing modulation formats that are more sensitive to nonlinear effects and crosstalk.

An alternative paradigm to increase the SE is represented by time-frequency packing (TFP) [10]–[13]. In this case, low-order modulations, such as quaternary phase shift keying (QPSK), are employed but the spacing between two adjacent pulses in the time domain (i.e., the symbol interval) is reduced well below that corresponding to the Nyquist rate. Similarly, the frequency separation between two adjacent channels can be also reduced, with the aim of maximizing the achievable spectral efficiency, which is thus used as a performance measure instead of the minimum Euclidean distance (as in more classical faster-than-Nyquist signaling schemes, see [11] and references therein). In addition, rather than the optimal receiver, a complexity-constrained detector is considered. In other words, controlled ISI and ICI are introduced and partially coped with at the receiver either through a single-channel maximum a posteriori (MAP) symbol detector, which is designed to take into account only a limited amount of ISI, or through a suboptimal multiuser detector (MUD), which enables the joint
processing of multiple sub-channels and thus coping with a limited amount of ICI (and possibly ISI).

Advanced signal processing plays a key role in TFP systems since, besides the fundamental working principles which entail decoding and soft detection techniques, significant performance improvements derive from a skillful combination of pulse shaping, coding, the adoption of a proper linear filtering (the channel shortening technique described in \[14\]), of a proper trellis-based MAP symbol detection strategy, and possibly of a proper multiuser processing. The aim is the maximization of the achievable SE, computed by resorting to the simulation-based method described in \[13\] which allows to also take nonlinear effects into account since it holds for any channel, including nonlinear and non-Gaussian.

With respect to \[10\], which considers the case of an additive white Gaussian noise (AWGN) channel and a simple symbol-by-symbol detector after matched filtering, and \[11\] which considers the optical channel in the linear regime and a more sophisticated receiver based on trellis processing, in this paper we have the following novel main contributions. (i) First, we compute the spectral efficiency and optimize the system parameters by taking into account nonlinear effects. (ii) Then, the considered receiver structures are different. We consider a trellis-based receiver enhanced by the use of the channel shortening technique \[14\]. This makes a big difference in terms of performance. We also consider here the case of use of a nonlinear compensation technique at the receiver based on digital backpropagation. (iii) Finally, we compare the performance of the TFP technique with other solutions in the literature. A similar investigation has been performed in \[13\] with reference to the nonlinear satellite channel. Satellite nonlinearities are, in nature, very different from those affecting long-haul optical transmissions. The results and the conclusions here reported are thus very different from those reported in \[13\]. In particular, we will see here that in long-haul optical links with strong nonlinear effects, an increase of SE cannot be simply obtained by increasing the modulation order.

The remainder of this paper is organized as follows. The system model is described in Section II. The framework that we use to evaluate the SE is then described in Section III whereas the adopted receivers are described in Section IV. Numerical results are reported in Section V with a detailed comparison with alternative approaches in the literature, possibly based on the paradigm of orthogonal signaling. Finally, some conclusions are drawn in Section VI.

II. PRELIMINARIES AND SYSTEM MODEL

Let us first consider a multi-carrier system over an AWGN channel, where \(N_c\) equally-spaced adjacent carriers are associated to the same linear modulation format and shaping pulse \(p(t)\). The complex envelope of the transmitted signal can be expressed as

\[
s(t) = \sum_{k=0}^{K-1} x_k(t) p(t - kT - \tau(t)) e^{j(2\pi f_1 t + \theta(t))}
\]

where \(K\) is the number of symbols transmitted over each carrier, \(T\) the symbol interval, \(x_k(t)\) the symbol transmitted over the \(k\)th carrier during the \(k\)th symbol interval, \(\tau(t)\) and \(\theta(t)\) the delay and the initial phase of the \(k\)th carrier, respectively, and \(F\) the frequency spacing between two adjacent carriers.

In such a scenario, the capacity-achieving distribution of the transmitted symbols is Gaussian. Hence, in order to maximize the system spectral efficiency, independent and uniformly distributed (i.u.d.) Gaussian symbols \(\{x_k(t)\}\) must be employed along with a shaping pulse \(p(t)\) having a rectangular spectrum (sinc pulse) with bandwidth \(B = 1/2T\) and a frequency spacing \(F = 1/T\), i.e., no guard band is employed between two adjacent carriers. No ISI or ICI occur since orthogonal signaling is employed.

Practical systems necessarily deviate from this paradigm. First of all, the transmitted symbols \(\{x_k(t)\}\) are not Gaussian but usually belong to a properly normalized zero-mean \(M\)-ary complex constellation \(\chi\). Under these conditions, instead of trying to approach as close as possible the impractical condition of having a shaping pulse with rectangular spectrum and to reduce as much as possible the guard band, as in Nyquist-WDM systems \[8\], TFP technique intentionally introduces both ISI and ICI to improve the spectral efficiency \[10\], \[11\].

In other words, for a given shaping pulse, the symbol time \(T\) and the frequency spacing \(F\) are properly optimized, as explained in Section III to maximize the spectral efficiency by intentionally violating the orthogonal signaling paradigm.

Let us now consider a realistic optical system. In this case, the possibility to generate a transmitted signal with expression (1) is strictly related to the availability of a linear modulator. In other words, let us consider the transmitted signal associated to the carrier for \(\ell = 0\). If the pulse \(p(t)\) has support larger than \(T\), this signal cannot be directly generated through an MZ modulator unless it is properly linearized. This is due to the nonlinear transfer function of the MZ modulator between the electrical signal at its input and the optical signal at its output. We could, however, use an MZ modulator to generate a linearly modulated signal with shaping pulse having support at most \(T\) and then “stretch” the transmitted pulses through an optical filter, thus obtaining an effect similar to that obtained with time packing. Hence, in this case, time-packing is not an available option but we have a viable surrogate. The degrees of freedom are thus the frequency spacing \(F\) and the bandwidth \(B\) of the optical filter used at the MZ output \[11\], which in the present analysis is always a 4th-order Gaussian filter (both at transmit and receive side). Fig. 1 shows a schematic of the considered transmission system, irrespectively of the constellation size. Blocks related to the receiver will be explained in Sections IV and V.

We consider optical channels impaired by GVD and PMD, and in particular uncompensated links where chromatic dispersion compensation is only performed with a fixed-tap equalizer in the electrical domain at the receive side. We also take...
into account nonlinear effects, as it will be explained in Section \[\text{V}\] where a description of the simulated optical links will be provided. We will compare systems based on TFP and employing quaternary constellations with other known systems, based on higher-order modulations, which show good results in terms of spectral efficiency, namely Nyquist-WDM systems \[\text{[8]}\] and the receiver-side duobinary shaping in \[\text{[9]}\]. In the latter system, an electrical two-tap filter is used to force a duobinary shape to the received signal, thus limiting the ICI, at the expense of employing a MAP sequence (or symbol) detector to cope with the introduced ISI. This is, in practice, a heuristic version of frequency packing and the processing here described for TFP systems since it allows a larger packing in frequency at the transmitter. No comparison will be performed with OFDM systems since, from an implementation point of view, they present a few drawbacks for optical links and are also less efficient than the systems considered here \[\text{[7]}\].

### III. Spectral Efficiency Computation

We now describe the framework used to evaluate the performance limits of all optical transmission systems considered in this paper and to perform the optimization of the optical filter bandwidth and frequency spacing in case of adoption of the TFP technique.

We are considering an optical channel with linear and nonlinear distortions, simulated through the split-step Fourier method (SSFM) with proper complexity \[\text{[16]}\]. Denoting by \( y \) a proper discrete-time received sequence used for detection of the information symbols \( x = \{x_k^{(\ell)}\}_k,\ell \), the information rate (IR), i.e., the average mutual information when the information symbols are independent and uniformly distributed (i.u.d.) random variables belonging to the given constellation, is defined as

\[ I(x; y) = \lim_{K \to \infty} \frac{1}{N_c K} \mathbb{E} \left\{ \log_2 \frac{p(y|x)}{\sum_{x'} p(y|x') P(x')} \right\} \quad \text{[bit]} \quad \text{[ch. use]} \]

(2)

where \( p(\cdot) \) denotes a probability density function (PDF) and \( P(\cdot) \) a probability mass function (PMF). In \[\text{[3]}\], the information symbols are, as mentioned, i.u.d. and thus \( P(x_k^{(\ell)}) = 1/M \), for all \( k, \ell \). The spectral efficiency (SE) is the IR per unit bandwidth and unit time and reads

\[ \frac{I(x; y)}{FT} \quad \text{[b/s/Hz]} \]

since \( FT \) is the time-frequency slot devoted to the transmission of symbol \( x_k^{(\ell)} \).

The computation of IR and SE requires the availability of the pdfs \( p(y|x) \) and \( p(y) = \sum_{x'} p(y|x') P(x') \). However, they are not known in closed form nor can we resort to the simulation method in \[\text{[15]}\] to compute them. In fact, this method requires that the channel at hand is finite-state and the availability of an optimal detector for it \[\text{[15]}\]. These conditions are clearly not satisfied in our scenario \[\text{[2], [17]}\]. We may thus resort to the computation of a proper lower bound on the IR (and thus on the SE) obtained by substituting \( p(y|x) \) in \[\text{[2]}\] with an arbitrary auxiliary channel law \( q(y|x) \) with the same input and output alphabets as the original channel (mismatched detection \[\text{[11], [15], [18], [19]}\]). The resulting lower bound reads

\[ L_q(x; y) = \lim_{K \to \infty} \frac{1}{N_c K} \mathbb{E} \left\{ \log_2 \frac{q(y|x)}{\sum_{x'} q(y|x') P(x')} \right\} \quad \text{[bit]} \quad \text{[ch. use]} \]

(3)

If the auxiliary channel law is representative of a finite-state channel, pdfs \( q(y|x) \) and \( \Delta_p(y) = \sum_{x'} q(y|x') P(x') \) can be computed, this time, by using the optimal MAP symbol detector for that auxiliary channel \[\text{[15]}\]. This detector, that will be clearly suboptimal for the actual channel, will have at its input the sequence \( y \) generated by simulation according to the actual channel model, and the expectation in \[\text{[3]}\] is meant with respect to the input and output sequences generated accordingly \[\text{[15]}\]. Thus, no assumption on the real statistics of the discrete-time received sequence is required for the design of the adopted detector since it is designed for the auxiliary channel. Similarly, the knowledge of the real statistics of the sequence \( y \) are not required for its generation since it can be obtained by simulation through the SSFM. If we change the adopted receiver (or, equivalently, if we change the auxiliary channel) we obtain different lower bounds on the information rate but, in any case, these bounds are achievable by those receivers, according to mismatched detection \[\text{[15], [18]}\]. We will thus say, with an abuse of terminology, that the computed lower bounds are the SE values of the considered channel when those receivers are employed. All these considerations hold for any actual channel including nonlinear and non-Gaussian ones.

This technique thus allows to take into account receivers with reduced complexity. In fact, it is sufficient to consider an auxiliary channel which is a simplified version of the actual channel in the sense that only a portion of the actual channel memory and/or a limited number of impairments are present. The considered receivers will be described in the next section. However, we may anticipate that for all of them we will assume that parallel independent detectors are
employed, one for each carrier (and each polarization in case of polarization multiplexing). In other words, ICI is not coped with at the receiver since multiuser detection is considered too computationally demanding. This corresponds to the adoption of an auxiliary channel model that can be factorized into the product

\[ q(y|x) = \prod_{\ell} q(y^{(\ell)}|x^{(\ell)}) \]

where \( y^{(\ell)} \) is a proper discrete-time received sequence used for detection of symbols \( x^{(\ell)} = \{x_k^{(\ell)}\} \) transmitted over the \( \ell \text{th} \) carrier. Under this hypothesis and assuming a system with a large number of carriers in order to neglect border effects, it simply results

\[ L_q(x;y) = \lim_{k \to \infty} \frac{1}{K} \mathbb{E}\left\{ \log_2 \frac{q(y^{(\ell)}|x^{(\ell)})}{q_p(y^{(\ell)})} \right\}, \tag{4} \]

i.e., the result can be computed by considering only one carrier and does not depend on the specific considered carrier. In a practical scenario with a finite number of carriers, we will consider the central carrier only, avoiding the computation on the border carriers which are affected by a lower amount of ICI, thus obtaining a further lower bound. Without loss of generality, we will assume that the central carrier is that with \( \ell = 0 \).

Note that, as stated, we are not able to compute the IR of the actual channel, but this is irrelevant because the optimal receiver for the actual optical channel is unavailable and thus we can in no way achieve it. The best we can do is to employ practical suboptimal receivers and for them we are indeed able to compute the relevant IR which will be called achievable IR. The corresponding achievable (lower bound on) SE is thus

\[ \eta = \frac{1}{P_T} L_q(x;y) \quad [\text{b/s/Hz}]. \tag{5} \]

The aim of the TFP technique is to find the values of \( F \) and the bandwidth \( B \) of the optical filter after the MZ modulator providing, for each value of the signal-to-noise ratio (SNR) or, equivalently, for each value of the transmitted power, the maximum value of SE achievable by that particular receiver, which is optimal for the considered specific auxiliary channel. Namely, we compute

\[ \eta_M = \max_{F, B > 0} \eta(F, B). \tag{6} \]

Typically, the dependence on the SNR value is not critical, in the sense that we can identify two or at most three SNR regions for which the optimal spacings practically have the same value.

For fair comparisons in terms of SE, we need a proper definition of the SNR. We define the SNR as the ratio \( P/N \) between the signal power and the noise power (in the considered bandwidth). Under the assumption of a large number of carriers to avoid boundary effects, \( P/N \) can be written as

\[ \frac{P}{N} = \lim_{N_c \to \infty} \frac{N_c P_c}{B_o 2 N_0} \tag{7} \]

where \( P_c \) is the power for each carrier, \( B_o \) the overall bandwidth, and \( N_0/2 \) the two-sided power spectral density of the amplified spontaneous emission (ASE) noise per polarization, as if the channel were linear. \( P_c \) is independent of the bandwidth \( B \). It is clearly \( \frac{P_c}{N_0} = (N_c - 1) F + B \). In the limit of a large number of carriers, i.e., when border effects can be neglected, or when \( B \) is comparable with \( F \), we may approximate \( B_o \simeq N_c F \) and thus

\[ \frac{P}{N_0} \simeq \frac{P_c}{2 N_0 F}. \tag{8} \]

The SNR definition as given in (8) is independent of the transmit waveform and its parameters. This definition will be adopted even when the number of carriers \( N_c \) is rather small and, if we neglect the border effects, corresponds to the SNR per carrier. This provides a common measure to compare the performance of different solutions in a fair manner and allows also to compare the SE values obtained under nonlinear propagation conditions with the Shannon limit for the bandlimited AWGN channel. This will allow to appreciate the degradation due to the nonlinear effects.

IV. CONSIDERED RECEIVERS

The system model described in Section II is representative of the considered scenario and has been employed in the information-theoretic analysis and in the simulations results. In this section, we describe two families of employed receivers and the corresponding auxiliary channels (the channels for which those receivers represent the optimal MAP symbol detectors).

The first family is composed of receivers which completely neglect nonlinear distortions. Hence, the corresponding auxiliary channels operate in the linear regime. As far as GVD and PMD are concerned, they are assumed perfectly compensated. As known, in the absence of nonlinear effects perfect compensation is possible through a proper two-dimensional equalizer [5]. We also mentioned that multiuser detection is not considered at the receiver, i.e., in the considered auxiliary channels ICI is also neglected. Under these assumptions, the independent detectors mentioned in the previous section, one for each carrier and each polarization, have to take into account only (a portion of) the ISI intentionally or accidentally introduced in the system. As an example, in the case of TFP or the adoption of the receiver-side duobinary shaping [9], the detector takes into account (a portion of) the ISI intentionally introduced. In the case of Nyquist-WDM systems, a receiver coping with the unwanted ISI deriving from the adopted practical filters and shaping pulses is considered instead.

In the second family of auxiliary channels, part of the nonlinear effects (such as signal-ASE noise interaction that cannot be compensated by digital backpropagation) are neglected. Then we proceed as in the previous case. So, in practice, after the possible digital backpropagation

\[ \text{2In each family, we have one receiver for each considered transmission system, namely TFP, Nyquist-WDM, and receiver-side duobinary shaping.} \]

\[ \text{3In Fig. this equalizer is represented as the cascade of two fixed one-dimensional equalizers, one for each polarization, aimed at compensating for the GVD, and a short adaptive two-dimensional equalizer to cope with the PMD.} \]
and the two-dimensional equalizer, the auxiliary channels are the same in both cases. The expression of $q(y^{(0)}|x^{(0)})$ (remember that we are considering the central carrier only) will be provided at the end of the section.

In practice, we are computing the SE achievable on the optical channel when two possible receiver designs are adopted. In the first one, nonlinear effects are neglected at the receiver (i.e., the receiver is designed for the linear regime). This is obviously a worst case. The second case is when we adopt the best available technique for the compensation of nonlinear effects. As shown in Section 7, the presence or absence of digital backpropagation only affects the maximum values of achievable SE but not significantly our conclusions. We expect that they will hold also when we employ other compensation techniques.

As mentioned, for the TFP technique the carrier spacing and the bandwidth of the transmit optical filter are optimized to maximize the spectral efficiency. For a fair comparison, in the case of Nyquist-WDM systems we also optimize, from an SE point of view, the transmit optical filter bandwidth and the channel spacing, with the constraint that the spacing is not smaller than the Nyquist bandwidth, i.e., $1/T$ (otherwise we fall in the domain of the TFP technique). Under these conditions the memory of the channel in the absence of nonlinear effects is usually limited to at most $L = 2$ interfering symbols (it can be simply verified by increasing the value of $L$ assumed at the receiver and noting that no improvement is attained). On the other hand, the memory at the receiver in the case of the technique described in [9] is, by definition, $L = 1$. The memory introduced by the TFP technique is, instead, potentially very large. To limit the receiver complexity with a limited performance degradation, we apply a channel shortening (CS) technique [20]. In fact, when the memory of the channel is too large to be taken into account by a full complexity detector, an excellent performance can be achieved by properly filtering the received signal before adopting a reduced-state detector [20]. A very effective CS technique for general linear channels is described in [14].

In the case of adoption of CS, we are looking for an auxiliary channel and the corresponding optimal MAP symbol detector. As mentioned, in the auxiliary channel model we consider that nonlinearities are absent or perfectly compensated whereas GVD and PMD are assumed perfectly compensated. In addition, independent receivers, one for each carrier and each polarization are considered here since, as mentioned, ICI is neglected in the auxiliary channel model. Hence, each receiver assumes that, apart from AWGN, only one carrier is present. A set of sufficient statistics $y^{(0)}$ can thus be obtained by sampling the output of a filter matched to the transmit pulse (matched filter, MF). The $k$th element of $y^{(0)}$, under the above mentioned assumption that only the carrier with $\ell = 0$ is present, reads

$$y^{(0)}_k = \sum_i x^{(0)}_{k-i} g_i + n_k$$

where

$$g_i = \int h(t) h^*(t-iT) dt ,$$

$h(t)$ being the convolution of the shaping pulse $p(t)$ after the MZ modulator and the optical transmit filter impulse response, and $n_k$ a Gaussian process with $E\{n_{k+i} n^*_k\} = 2N_0 g_i$. Vector $y^{(0)}$ can be written as

$$y^{(0)} = Gx^{(0)} + n$$

where $G$ is a Toeplitz matrix obtained from the sequence $\{g_i\}$ whereas $n$ is a vector collecting the colored noise samples. This is the so called Ungerboeck observation model for ISI channels [21]. According to the CS approach, the considered auxiliary channel is based on the following channel law [14]

$$q(y^{(0)}|x^{(0)}) \propto \exp \left(2\Re\{y^{(0)} H^* x^{(0)}\} - x^{(0)} H^* G^* y^{(0)} \right) ,$$

(10)

where $(\cdot)^H$ denotes transpose conjugate, whereas $H^*$ and $G^*$ are Toeplitz matrices obtained from proper sequences $\{h^*_i\}$ and $\{g^*_i\}$, and are known as channel shortener and target response, respectively [14]. Matrix $H^*$ represents a linear filtering of the sufficient statistics $\{y^{(0)}\}$, and $G^*$ is the ISI to be set at the detector (different from the actual ISI) [14]. In (10), the noise variance has been absorbed into the two matrices. In order to reduce the complexity, we constrain the target response used at the receiver to

$$g^*_i = 0 \ \ |i| > L_r ,$$

(11)

which implies that the memory of the detector is $L_r$ instead of the true memory $L$ of the channel. The CS technique finds a closed form of the optimal $\{h^*_i\}$ and $\{g^*_i\}$ which maximize the achievable IR [14]. If the memory $L_r$ is larger than or equal to the actual channel memory, as in the case of Nyquist-WDM or receiver-side duobinary shaping, the trivial solution is $H^* = I/2N_0$ and $G^* = G/2N_0$, where $I$ is the identity matrix. Interestingly, when $L_r = 0$ the optimal channel shortener becomes a minimum mean square error (MMSE) feedforward equalizer [14]. The optimal MAP symbol detector for the auxiliary channel with law (10), that we used for the computation of the achievable IR according to the technique in [15], is described in [21] (see also [14, 22] for further details).

V. Numerical Results

We here report the maximum achievable spectral efficiency $\eta_M$ as a function of $P/N$ for different polarization-multiplexed systems. We assume perfect synchronization as we are interested in the evaluation of achievable bounds for the spectral efficiency. Unless otherwise stated, the employed shaping pulses are those resulting from the use of RZ pulses with duty cycle 50% , an MZ modulator, and a 4th-order Gaussian optical transmit filter with 3-dB optical filter bandwidth $B$ (specified later in this section). The considered modulation formats are QPSK and 16/64/256-ary quadrature amplitude modulations (QAMs). We first considered systems with 8 carriers (subchannels) at 140 Gbps each, irrespectively of the...
employed modulation format (thus, the baud rate changes for each format). It can be noticed that, since the bandwidth of each subchannel is highly reduced by filtering and multilevel modulations are considered, the required sampling rate is always within the state-of-the-art technology, i.e., well below 80 Gsample/s. At the receiver side, after the 4th-order Gaussian optical receive filter, with 3-dB optical filter bandwidth $B_R$, opto-electric conversion through an optical hybrid, and sampling, two non-adaptive one-dimensional equalizers (one for each polarization) perform coarse GVD compensation. Then, a two-dimensional (2-D) decision-directed (DD) adaptive feedforward equalizer (FFE) with 25 taps process the signals received over two orthogonal states of polarization to compensate for residual GVD, to demultiplex polarizations, and to complete (along with the optical filter) the implementation of the MF \(^5\) \(^6\). The number of taps of the two non-adaptive one-dimensional equalizers and that of the 2-D DD adaptive feedforward equalizer have been selected in such a way they do not affect the performance. In other words, no performance improvement has been observed by increasing the number of taps. In the case of systems employing receiver-side duobinary shaping, a further digital filter is present to perform the required shaping \(^9\). The output is provided to the MAP symbol detectors (one for each carrier and each polarization) which iteratively exchange information with the decoders for a maximum of 50 iterations.

All the considered constellations can be viewed, with a proper rotation, as two independent signals transmitted over the in-phase and quadrature components, respectively. Hence, at the receiver side, we may use two identical and independent detectors, one working on the in-phase and the other one on the quadrature component. This is beneficial in case of adoption of a MAP detector. In fact, when $L$ interfering symbols are taken into account, we have two detectors (per polarization) working on a trellis with $(\sqrt{M})^L$ states instead of a single detector working on a trellis with $M^L$ states. Hence, for a given complexity, a larger memory can be taken into account.

All spectral efficiency computations have been performed, for each point of the presented curves, on pseudo-random sequences of 900,000 bits per quadrature, following a training phase of 100,000 bits. Each sequence has been split into blocks of 50,000 bits that, properly processed as described in \([15]\), allow us to compute the achievable IR. The confidence interval on the computed information rate turned out to be less than 2%.

We considered an existing link of standard single-mode fiber (SMF), whose spans have the lengths reported in Table I simulated through the SSFM. The fiber dispersion is 16.63 ps/nm/km, the attenuation is 0.23 dB/km, the nonlinear index $\gamma$ is equal to 1.3 $W^{-1}km^{-1}$, and the noise figure of all amplifiers is equal to 6 dB. For this link, Fig. 2 shows the maximum achievable spectral efficiency $\eta_M$ for a TFP system employing QPSK (TFP-QPSK), a Nyquist-WDM system using

| span # | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  | 13  | 14  | 15  |
|--------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| SMF (km) | 70.8 | 75.5 | 55.1 | 52.1 | 40.1 | 67.1 | 53.2 | 50.1 | 80.3 | 79.1 | 53.6 | 75.1 | 90.3 | 54.2 | 99.4 |

Table I: SMF LENGTHS FOR EACH SPAN IN THE SIMULATED OPTICAL LINK.
Table II
NORMALIZED BANDWIDTHS FOR TRANSMIT- AND RECEIVE-SIDE OPTICAL FILTERS; “VARIABLE” MEANS THAT THE BANDWIDTH DEPENDS ON THE CONSIDERED SNR.

| Modulation Format | B  | B_R |
|-------------------|----|-----|
| TFP-QPSK          | variable | variable |
| 16-QAM [8]        | 1.1/T | 1/T |
| 16-QAM            | 1/T | 1/T |
| 64-QAM            | 1.1/T | 0.85/T |
| 256-QAM           | 1.25/T | 0.9/T |

a 16-QAM, a system employing receiver-side duobinary shaping, still with 16-QAM, and SE-optimized 16/64/256-QAM systems. In the case of TFP, the frequency spacing $F$ and the optical 3-dB filter bandwidth $B$ and $B_R$ have been optimized for each value of $P/N$, resulting in $B_R = B$. For the Nyquist-WDM systems, we used $B = 1.1/T$, $B_R = 1/T$, and an NRZ shaping pulse before the MZM, as suggested in [8] whereas, for receiver-side duobinary shaping we used $B = B_R = 1/T$ as in [9]. For a fair comparison, we also considered Nyquist-WDM systems with RZ pulses with duty cycle 50% optimizing also, from a spectral efficiency point of view, the bandwidths $B$ and $B_R$ of the optical filters and using $F = B$ (as mentioned, we imposed the constraint $F \geq 1/T$). The optimized values corresponding to the peak of the $\eta_M$ curves are: $B = 1/T$ and $B_R = 0.7/T$ for 16-QAM, $B = 1.1/T$ and $B_R = 0.85/T$ for 64-QAM, and $B = 1.25/T$ and $B_R = 0.9/T$ for 256-QAM. Filter bandwidths for each system are summarized in Table II.

Notice that no electric filter was considered in our simulations, but its presence does not affect the results from a qualitative point of view. At the receiver, since we arbitrarily chose $L_r = 4$ for TFP-QPSK (because corresponding to detectors with 16 states, thus with reasonable complexity), for 16-QAM we used MAP symbol detectors taking into account a memory $L = 2$. In this way, the comparison is performed for the same number of states of the MAP symbol detectors. On the other hand, for 64- and 256-QAM, in order to avoid a much larger number of states, we chose $L = 1$. The Shannon limit [23] in the absence of nonlinearities is also shown for comparison. It can be observed that, in this scenario, the TFP-QPSK system outperforms the $M$-QAM systems in spite of their higher cardinality. Although, in principle, 16-QAM, 64-QAM, and 256-QAM with polarization multiplexing could achieve spectral efficiency values (with sinc pulses) of 8, 12, and 16 b/s/Hz, respectively, they would be reached, in the linear regime, only for higher values of $P/N$, whereas the optimal launch power corresponds to an SNR value which privileges TFP-QPSK.

These information-theoretic results can be approached by using proper coding schemes. As an example, we simulated the bit-error ratio (BER) of a TFP-QPSK system using $B = 0.325/T$, $F = 0.43/T$, and employing the rate-4/5 low-density parity-check (LDPC) code having codewords of 64800 bits of the 2nd generation satellite digital video broadcasting (DVB-S2) standard [24]. Assuming a reference for the BER of $10^{-7}$, the performance of this system has been reported in Fig. 2. It may be observed that, despite the lack of an optimization in the code design, we have a loss of less than 1 b/s/Hz from the theoretical results, obtaining an SE of 7.5 b/s/Hz. The observed loss is mainly due to the presence of nonlinear effects which require a careful redesign of the code (we used a good code for the linear channel). We would like to mention that on this link an experimental field trial demonstration has been also recently conducted, reaching a spectral efficiency of more than 5 b/s/Hz despite the constraint to use (poorly performing) 1st-order Gaussian filters, not flexible and penalizing frequency grids, neighboring data channels with commercial traffic, and no time available to perform the necessary system optimizations. Similar results were obtained in a previous field trial described in [25]. The relevant results will be reported in a later paper.

Notice that the use of soft decoding has become a relevant topic in coherent optical communications, and is considered to be an important resource for performance improvement in the next future [26]. Iterative detection and decoding is, at this point, a natural development of signal processing at the receive side, as soon as the complexity of MAP detectors (in our case only 16 states for TFP) becomes feasible with the present technological progress, and with the intrinsic parallel processing of block coding.

In TFP, from a conceptual point of view, the lower the bandwidth of the optical filter and/or the frequency spacing, the higher the peak-to-average power ratio (PAPR). However, we are optimizing the amount of introduced packing (it is different for each value of the launch power) so, in practice, the introduced PAPR is, in some way, optimized. If curves in Fig. 2 were plotted as a function of the transmitted power per carrier $P_c$ instead of as a function of the SNR, it could be observed that TFP with QPSK achieves the maximum value of SE, where NL effects start dominating the performance, for a values of $P_c$ lower than those related to 16-QAM systems (but higher than those related to 64- and 256-QAM systems). This suggests that TFP with optimized packing can give values of the optical field intensity higher than those related to 16-QAM. So, by optimizing the amount of packing, a trade-off is reached between the SE improvement introduced through packing and the sensitivity to NL effects.

Fig. 3 reports a comparison between the same systems as in Fig. 2 but compensating nonlinear fiber impairments at the receive side with an ideal digital backpropagation technique [27], operating on the whole transmission bandwidth at full complexity, i.e., the same complexity used to simulate the nonlinear channel through SSFM. This will allow to increase the optimal launch power and will allow higher order modulations to better exploit their potential. In other words, since the channel is now “more linear”, we expect that Nyquist-WDM with higher order modulations outperform TFP. This can be verified Fig. 3 although TFP still outperforms 16-QAM.

We also computed the SE of the same systems but by changing the number of sub-channels and their baud rates. In this case we compared the different modulation formats...
at equal baud rate, i.e., 50 Gbaud/s, and set the number of sub-channels in order to obtain approximately the same total bandwidth occupation. Thus, by keeping the same filter bandwidths as in the previous case, we set 10 sub-channels for TFP-QPSK at 20 GHz frequency spacing, 4 sub-channels for 16-QAM and 64-QAM, respectively at 50 and 55 GHz spacings, and 3 sub-channels for 256-QAM at 62.5 GHz spacing. Results are very similar to those reported in case of equal bit rate and number of sub-channels (and thus not shown for lack of space), confirming that the TFP-QPSK performance is independent of the granularity selected to reach the target data rate.

In order to verify the range of applicability of the considered techniques, in Fig. 4 we show the maximum spectral efficiency $\eta_M$ at the optimal launch power, as a function of distance on a simplified uncompensated link (which means identical spans of length 100 km, amplifier noise figure equal to 5 dB, no polarization mode dispersion), for all systems considered in Fig. 2. TFP-QPSK is seen to reach the highest spectral efficiency in the range 1000-10000 km, i.e., when link nonlinear effects significantly affect signal propagation. Fig. 5 shows the same results but plotted as normalized SE difference between the considered systems and Nyquist-WDM 16-QAM in [8], taken as a reference, i.e., we defined

$$\Delta = \frac{\eta_M - \eta_M^{REF}}{\eta_M^{REF}},$$

(12)

where $\eta_M^{REF}$ is the maximum value of SE achievable by Nyquist-WDM 16-QAM in [8]. It can be noticed that the SE gain of each modulation format becomes constant after 2000 km, and TFP-QPSK is already the most efficient after less than 1000 km. Fig. 6 shows similar results on $\eta_M$ as a function of distance for the systems used in Fig. 4 (i.e., in the case of adoption of ideal backpropagation). It can be noticed that, again, TFP-QPSK outperforms other systems at higher distances, whereas, obviously, in links where signal propagation occurs in the weak nonlinear regime, the higher information rate achieved by high-order QAM systems prevails.
Finally, Fig. 7 shows the performance of TFP-QPSK and SE-optimized 16-QAM for different transmitted pulses, on a link corresponding to 10 spans of the scenario considered in Figs. 4-6. In particular, in addition to the case of use of a 4th-order Gaussian filter, we also considered the use of a 1st-order Gaussian filter (with optimized bandwidth) and of root raised cosine (RRC) transmit pulses (obtained through the use of proper filters) with roll-off factor α equal to 0.2 and 0.5. In the case of RRC pulses, we optimized the frequency spacing \( F \) and the symbol time \( T \) (related to the frequency support \( B = (1 + \alpha)/T \)). Results suggest that the pulse deriving from the adoption of a 4th-order Gaussian filter performs almost as good as the RRC pulse with roll-off 0.2, whereas a significant penalty is observed when using a 1st-order Gaussian filter. Nevertheless, in all cases TFP-QPSK performs better than 16-QAM under this scenario.

VI. CONCLUSIONS

We compared different techniques to improve the spectral efficiency of long-haul optical systems. With the exception of short links where signal propagation occurs in the weak nonlinear regime, the most promising solution is shown to be that based on time-frequency packing which is related to the use of narrow optical filtering and a tight packing of the carriers in frequency, giving up the signal orthogonality in the time and frequency domains, and on the adoption of detectors able to cope with the interference intentionally introduced in the system. When nonlinearities start dominating the performance, this solution provides better results than other solutions proposed in the literature and based on higher-order modulations, showing that, when nonlinear effects are present, the spectral efficiency cannot be trivially increased by increasing the modulation order. This result is confirmed also for ultra-long-haul links, up to 10000 km, whereas better SE values can be achieved using modulations with very high cardinality, such as 256-QAM, but only for short range links where fiber nonlinearities have a weak effect, or resorting to compensation techniques such as digital backpropagation. We also reported simulation results on a modulation and coding format which, on a realistic optical link, reaches a spectral efficiency of 7.5 b/s/Hz with a polarization-multiplexed time-frequency-packed QPSK, with a loss of less than 1 b/s/Hz from the information-theoretic results.

ACKNOWLEDGMENT

This work was supported in part by CNIT and by the Italian Ministero dell’Istituzione, dell’Università e della Ricerca (MIUR) under the FIRB project Coherent Terabit Optical Networks (COTONE). The authors would like to thank the Editor, Prof. Erik Agrell, and the Reviewers; their comments and constructive criticism helped to improve the quality of the paper.

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