On Maurício M. Peixoto and the arrival of Structural Stability to Rio de Janeiro, 1955.

Jorge Sotomayor∗

Abstract

This essay is an inquiry about the circumstances, and subsequent mathematical consequences, of the encounter, in 1955, of Maurício M. Peixoto (1921 - 2019) with the work of Henry F. DeBaggis (1916 - 2002) on Structural Stability, Princeton 1952, developing a notion introduced by Alexander A. Andronov and Lev S. Pontrjagin, Gorkii 1937.

Mathematical Subject Classification: 01A60, 01A67, 37C20.

Key words and phrases: Andronov, DeBaggis, Lefschetz, Peixoto, Pontrjagin, structural stability, bifurcation, nonlinear differential equation, dynamical system.

1 An accidental library encounter

At the quiet Library of the Institute of Pure and Applied Mathematics (IMPA), in his after coffee afternoon browsing books routine, Mauricio M. Peixoto (Peixoto), stumbled on the article by H. F. DeBaggis (DeBaggis) entitled Dynamical Systems with Stable Structure, [5], which captured his interest.

He read attentively the following phrases:

“... For physical systems to perform certain operations they must, if they are to be useful, possess a certain degree of stability. Small perturbations should not affect the essential features of the system.”

∗The author had the partial support of CNPq, PQ-SR - 307690/2016-4.
Since the physical components of a system can never be duplicated exactly, experimental verification would be impossible unless the system remains stable under small variations.

The stability requirements of experiments provide a clue to the restrictions a mathematician should place on his nonlinear problems."

As an engineer, self-taught mathematician and full professor of Rational Mechanics, with much higher than average knowledge of the differential equations of Classical Mechanics, Peixoto immediately noticed that a clever form of stability – or continuity – was involved in the author’s text. It was Structural Stability.

He glimpsed at the references and there he found the book *Theory of Oscillations* of Andronov and Chaikin, 1949, in the English Princeton translation [3], of the Russian original [1], 1937. He had perused it in 1951 when writing his Thesis *The General Equations of Mechanics*, [9], presented in the competition for the Chair of Rational Mechanics at the University of Brazil. The book is cited in the chapter *Stability*, of the Thesis, which deals with the stability of stationary states, in the sense of Lyapunov, and not in that of the phase space depending, as a whole, on the analytic data defining the system.

On 1951, however, the notion of Structural Stability, as mentioned along the Andronov and Chaikin book, did not attract his attention. The results by Andronov and Pontrjagin, [2], stating a characterization of Structurally Stable Systems in a planar region, whose border they cross transversally, was formulated on an appendix of [3]. No proofs were provided. This omission might have contributed to diminish the potential attractiveness of Structural Stability for many mathematically inclined readers in the West. Two exceptions – DeBaggis and Lefschetz – will be discussed in Sections 2 and 3.

Complementing the phrases above, in a second reading round of [5], Peixoto also glanced at:

"... In this paper we give a complete treatment of the theory of structural stability. We have relaxed the conditions of analyticity which were imposed on the functions by [2] and merely require that they have continuous first partials."

He focused again on the concise list of references and attempted to grasp the structure of the paper examining the sections and the figures.

In the last browsing round he also looked at the short final remarks about bifurcations – the lost of structural stability – and the insinuation of a possible continuation in that direction in future work by the author.
Time had elapsed rapidly. Approaching the librarian desk for borrowing the journal, he whispered: *This seems to be good stuff.*

As he headed back home, carrying in his briefcase the article of DeBaggis [5], his thinking was accelerated. The encounter, apparently accidental, followed by the active browsing through it, had launched Peixoto’s insightful mathematical intuition.

At home, he worked on the article until late hours. Afterwards he also discussed it with his wife Marília C. Peixoto (Marília), 1921 - 1961, also an engineer, mathematician and his assistant in the Chair of Mechanics.

They decided that, in due time, he should prepare a presentation on Structural Stability

in the Seminar he directed, which had the participation of Marília, selected students and teaching assistants of the Chair of Mechanics¹.

During the preparation of the Seminar, confusing points in [5] were detected, where corrections and essential improvements were foreseen. The idea of the handy introduction of a Functional Space of Differential Equations, emerged. On it new results could be formulated in topological terms. This appears in the research note [10] published in September 1955, containing innovative results, such as the openness of structural stability.

Along the next months, the project of a visit to Princeton to confer with Solomon Lefschetz, Editor of the Series Contributions to the Theory of Nonlinear Oscillations, began to be conceived.

2 \textbf{H. F. DeBaggis}

In 1949, on leave of absence form the University of Notre Dame at Princeton, Father Henry F. DeBaggis, CSC, browsing through the recently published book by Andronov and Chaikin [3], encountered the appendix on Structural Stability. He was looking for new horizons to go beyond Axiomatic Hyperbolic Geometry, field in which he had written a M.S. Dissertation and a PhD Thesis at Notre Dame.

He decided to undertake the challenge of giving a proof of the Theorem of Andronov and Pontrjagin stated there. Though, at the beginning, he might not have had the suitable adroitness in Differential Equations and Classical Analysis, Solomon Lefschetz supported him. DeBaggis participated in Lefschetz’ Research Project for two years. See Section 3.

No additional information concerning DeBaggis’ research in mathematics was

¹Mentioned in Peixoto’s interview *Eméritos III, 2011*, https://youtu.be/PToAegfcFKA.
found. In his Obituary one can read that he taught Mathematics in several North American Universities and also in other continents. No chronological data about him after he left Princeton was found.

3 Solomon Lefschetz, 1884 - 1972.

We quote from Griffiths, Spencer and Whitehead [7]:

“In 1943 Lefschetz became a consultant for the U.S. Navy at the David Taylor Model Basin near Washington, D.C. There he met and worked with Nicholas Minorsky, who was a specialist on guidance systems and the stability of ships and who brought to Lefschetz’ attention the importance of the applications of the geometric theory of ordinary differential equations to control theory and nonlinear mechanics. From 1943 to the end of his life, Lefschetz’ main interest was centered around ordinary nonlinear differential equations and their application to controls and the structural stabilities of systems.

Lefschetz was almost sixty years old when he turned to differential equations, yet he did original work and stimulated research in this field as a gifted scientific administrator.”

We quote from Lefschetz [8], p. 1:

“During most of World War II, the undersigned, a consultant at the David Taylor Model Basin, had frequent interviews with Dr. Nicolas Minorsky, in connection with the latter’s production of his well known Introduction to Nonlinear Mechanics.

Dr. Minorsky voiced repeated regrets at the impossibility of creating in this country anything resembling the well known Institute of Oscillations of Moscow, with its large staff of highly competent mathematicians and physicists devoted to the problems of oscillations and more generally to nonlinear mechanics (equally called nonlinear differential equations).”

We quote from Lefschetz [8], p. 15:

“Father Henry DeBaggis, an Assistant Professor of Mathematics at Notre Dame and a Ph. D. under Karl Menger at Notre Dame also, joined our Project in 1949 and was a member for two years 1949 - 51.

While his thesis was on Hyperbolic Geometry, he had little taste for that subject and wished to change over to Differential Equations.
A reading of an appendix in Andronov-Chaikin [3] recently appeared, awoke his interest in structural stability.
This highly interesting concept had been launched in a Note of the Doklady [2] by Andronov and Pontrjagin. They considered a planar system defined in a closed two-cell with vectors pointing outwards along the boundary and asked under what conditions does the topology of the system of paths remain unchanged for small variations of the vector field throughout the two-cell. They stated n.a.s.c. for this to happen but gave no proofs.

DeBaggis undertook to establish a complete theory and this objective was attained. His results were developed in a paper which appear in [5] and was subsequently translated into Russian [6].

It is only fair to say that DeBaggis derived very great benefit from discussions with M. Schiffer, D. Spencer and the Director."

Lefschetz’ interest on Structural Stability was notorious. Besides renaming it from “Roughness”, he had addressed to Andronov a letter suggesting that “a theorem of such importance should have its proof published.” Unpublished correspondence exhibited in a glass display case, together with manuscripts and books of Andronov, at the Meeting in commemoration of his 100th anniversary [4].

4 The letter of Peixoto to Lefschetz, 1956.

Peixoto proposed to establish that the systems defined by the conditions of Andronov and Pontrjagin is an open and dense set inside the open set of vector fields transversal to the border of the region supporting the system.

Upon reception of the letter, Lefschetz became deeply interested in Peixoto’s project. He promptly wrote him back and suggested how to proceed to apply for a Research Associate visiting position at Princeton.

With the support of CNPq he visited Princeton during the academic year 1957 - 58, continuing further in RIAS, Baltimore, during 1958-59.

This interesting stage of Peixoto mathematical activities, The Golden Days, in which he met Stephen Smale, are recounted in his charming Speech of Acceptance of the TWAS Mathematics Award, 1987. See [11], [13].

5 A meaning for an accidental encounter

In 1943 the eminent mathematician Solomon Lefschetz: algebraic geometer and topologist, reaching his sixties, decided to change his research interests and started
to work in Differential Equations. This happened after encounters he had with the Naval Engineer Nicholas Minorsky, an “ingenieur savant” [12]. See Section 3.

From Princeton Lefschetz launched several editorial activities to spread the interest on Nonlinear Differential Equations, [8]. One of them was the Edition of the Series Contributions to Nonlinear Oscillations, which in Vol. II published the paper of DeBaggis in 1952. In 1955 it reached Peixoto in Rio de Janeiro and, after substantial elaboration, took him to Princeton to have the benefit of The Golden Days of his mathematical career.

On the afternoon of the encounter of Peixoto with DeBaggis paper, a narrow cosmic fracture allowed a considerable amount of repressed mathematical energy to flow into the library of IMPA. It was liberated by the apparently accidental and seemingly naive ceremony of browsing a mathematics article, as a dam under pressure releases its flow through a fissure. On the right side of the dam to channel the flow, there was a receptive, insightful and sensitive mind. One that was prepared for the challenge and could grasp the complexity of the inflowing mathematical message.

Thus, Structural Stability, carrying its rich heritage and mathematical background, arrived from Gorkii, now Nizhnii Novgorod, to Rio de Janeiro, then the Capital of Brazil, after an auspicious maturation in Princeton, one of the capitals of World Mathematics.

In 1987 Peixoto received the TWAS Mathematics Award for his contribution to Structural Stability in Dynamical Systems [11].
6 Contacts and influences of Peixoto around Princeton 1958, TWAS 1987 and after

Mathematical contacts and influences mentioned by Peixoto in his charming Acceptance Speech on the occasion of the TWAS Mathematics Award that he received in 1987. There he presented an outline of crucial developments in Dynamical Systems. See [11] which had a Portuguese translation included in the obituary essay [13].

Other aspects of Peixoto’s mathematical biography can be found in [14], [15] and [16].

Acknowledgement. The author is grateful to J. Derwent and to H. Weiss for their valuable help to get access to, respectively, the works of DeBaggis and the enlightening Lefschetz Report [8]. Thanks are also due to D. Schlomiuk, M. Sotomayor, L. F. Mello, A. R. da Silva and R. A. Garcia for their helpful comments to this text.
References

[1] A. A. Andronov, S. E. Chaikin, Theory of Oscillations, Moskow -
    Leningrad, ONTI, 516, 1937.
[2] A. A. Andronov, L S. Pontrjagin, Robust systems, DAN 14, No 5 (1937).
[3] A. A. Andronov, S. E. Chaikin, Theory of Oscillations, English Language
    Edition by S. Lefschetz, Princeton University Press, 1949.
[4] A. A. Andronov Progress in nonlinear science. Vol. 1. Mathematical problems
    of nonlinear dynamics. Proceedings of the International Conference dedicated
    to the 100th Anniversary of A. A. Andronov held in Nizhny Novgorod, July 2?6,
    2001. Edited by Lev M. Lerman and Leonid P. Shil’nikov. Russian Academy
    of Sciences, Institute of Applied Physics, Nizhniy Novgorod, 2002.
[5] H. F. DeBaggis, Dynamical systems with stable structure, Contributions to
    the Theory of Nonlinear Oscillations II, S. Lefschetz, ed., Princeton University
    Press, 37-59, 1952.
[6] G. F. De Baggis, Dynamical systems with stable structure, Uspekhi Mat.
    Nauk, 1955, Volume 10, Issue 4(66), 101 - 126.
[7] Griffiths, P., Spencer, D., Whitehead, G., Solomon Lefschetz, A biogra-
    phical memoir, Nac. Acad. Sciences, 1992.
[8] S. Lefschetz, Project Director, Nonlinear Differential Equations and
    Nonlinear Oscillations, US Office of Naval Research, August 1946, Final Report
    1959.
[9] M. M. Peixoto, Equações gerais da dinâmica, 110 pgs, Rio de Janeiro, 1951.
[10] M. M. Peixoto, Note on structurally stable systems, Anais Acad. Brasil. Ci.,
     27 (1955), 35.
[11] M. M. Peixoto, Acceptance Speech, TWAS Award, 1987. Proceedings of the
     Second Conference Organized by the TWAS. World Scientific, 1989.
[12] L. Petitgirard, L'ingénieur Nicolas Minorsky (1885- 1970) et les
     mathématiques pour l'ingénierie navale, la théorie du controle et les oscilla-
     tions non linéaires. Revue d’histoire des math., 21, (2015) 173-216.
[13] J. Sotomayor, R. Garcia, L. Mello, Maurício M. Peixoto (1921 - 2019).
    Revista Matemática Universitária, SBM, in print, Vol. 1, 23 pgs, 2019.
[14] J. Sotomayor, On a list of problems of ordinary differential equations,
    SPJMS, vol 13, no. 1, (2019), 177 - 194.
[15] J. Sotomayor, Mathematical Encounters, Notices ICCM, vol. 6, no. 2,
    (2018), 94 - 98.
[16] J. Sotomayor, Editor, *The Geometry of Differential Equations and Dynamical Systems*, Computational and Applied Mathematics, 20, 1-2, 2001.

Instituto de Matemática e Estatística,
Univ. de São Paulo, Rua do Matão, 1010,
CEP: 05508 - 090,
São Paulo, S.P, Brazil
E-mail address: sotp@ime.usp.br