A Parametric Analysis of Nonlinear Lower Hybrid Effects

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Abstract. Based on a new modeling of the nonlinear behaviour of lower hybrid waves launched in tokamak plasmas triggered by the noisy background of the radiofrequency power generator, a parametric analysis of the role of the plasma parameters on the onset of the nonlinearity is here proposed. As a result, the nonlinear effects could be prevented by increasing the edge electron temperature, the relative ion-electron temperature, the edge density gradient and the ion mass.

1. Introduction

Nonlinear effects occurring at the plasma edge, namely parametric instabilities (PIs) [1], play an important role in determining the power absorption of the lower hybrid (LH) waves externally launched in tokamak plasmas. Parasitic nonlinear absorption at the edge due to strong PI was invoked to prevent [2] the power penetration into the plasma core during the early LH experiments aimed at heating the plasma ions [3,4]. LH experiments aimed at current drive [5] operate successfully only at relatively low plasma density (line averaged plasma density $n \leq 3\times10^{19}\text{m}^{-3}$) [6] due to the onset of strong PI. Following modeling results [2], a new method was developed on FTU (Frascati Tokamak Upgrade) which allowed producing LH current drive effects at central plasma densities even higher that in ITER ($n_e \approx 5\times10^{20}\text{m}^{-3}$), provided that the operating edge temperature was kept relatively high to reduce the PI [7].

Based on previous works on the ponderomotive effects driving the PI phenomenon [8,9], we present here new results that provide a more complete interpretation of the nonlinear mechanism of the PI-produced spectral broadening. In a previous work [10] we proposed a new nonlinear LH modeling able to describe the nonlinear mode coupling of the LH wave (pump wave) launched in tokamak plasmas with LH waves (sideband waves) excited by the noisy background of the radiofrequency (RF) power generator. We showed that the nonlinear mode coupling is driven by low frequency ($\leq 10\text{ MHz}$) density fluctuations, produced by the beating of the pump and the sideband waves. The amplitude spectra in the frequency and $n/\rho$ domains of the low frequency driving density fluctuations were calculated in the frame of the kinetic theory, based on the self-consistent solution of the Vlasov-Poisson equations, obtained by a perturbative approach. A system of nonlinearly coupled LH wave equations was then derived in the frame of the cold plasma approximation, taking into account the nonlinear density fluctuations in evaluating the current density perturbations. Thus, based on an iterative approach, we provided a numerical solution of the system of nonlinearly coupled LH wave equations.

In the present work, we discuss the role of the plasma parameters on the onset of the LH nonlinearity in front of the LH grill, based on the new nonlinear LH model proposed in [10].
2. Nonlinear LH Modeling

We shortly review here the new nonlinear LH modeling proposed in [10]. Based on previous works on the ponderomotive effects driving the PI phenomenon [8,9], the new modeling gives a more accurate description of the LH physics near the antenna-plasma interface, by utilizing a full wave approach that avoids using the eikonal approximation. Indeed, this approximation usually fails in the plasma region near the LH antenna mouth as the radial wavelength is often larger than the characteristic radial scale lengths of the spatial amplification and plasma inhomogeneities. In addition, the modeling allows accounting for both the spectral broadening and the pump depletion. However, it is not suitable to discuss the amplification of LH sidebands emerging from the thermal background of plasma density fluctuations, as done instead in the standard PI modeling [1]. Indeed, poloidal symmetry of low frequency (LF) modes has been assumed, i.e. only LH waves with poloidal wavenumber $k_p = 0$ are considered here. This is in agreement with our choice to evaluate the LH nonlinearity triggered by a noisy background produced by the RF power generator. Therefore, the spectral noise has the same polarization and (approximate) poloidal symmetry of the pump waves, i.e. we assume $E_p \approx 0, k_p \approx 0$.

The mode coupling among different LH wave spectral components is driven by LF density fluctuations that have angular frequency much lower than the ion-cyclotron angular frequency, $\omega' \ll \Omega_{ci}$. These fluctuations are excited by the beating of the pump wave, with operating angular frequency $\omega_\omega$, and the high frequency (HF) spectral components that are intrinsic to the noise of RF power generators. The latter are shifted from the pump wave frequency by an amount of $\omega'$. In turn, under suitable plasma conditions, the beating of the pump wave and low frequency perturbation produces the growth of LH sidebands with angular frequency $\omega = \omega_\omega \pm \omega'$.

A simple slab geometry is assumed, where $\hat{x}$, $\hat{y}$ and $\hat{z}$ are, respectively, the radial (inward), poloidal and toroidal directions of the outer plasma region of a tokamak, faced to the mouth of the LH launcher. Steady state conditions are considered and symmetry of the plasma parameter in poloidal and toroidal directions is assumed, with density and temperature gradients occurring only in the radial direction. The confinement magnetic field is assumed constant and directed in the toroidal direction. These conditions allow solving the integro-differential problem by means of a spectral method, i.e. by Fourier transforms in $y$ and $z$ coordinates and in the time $t$. The low frequency density fluctuations are assumed constant in the poloidal direction, as occur for the HF modes.

The amplitude spectra in the frequency and $k$ domains of the low frequency driving modes is calculated in the frame of the kinetic theory, based on the self-consistent solution of the Vlasov-Poisson equations obtained by a perturbative approach. A new nonlinear wave equation for the LH waves is then obtained in the frame of the cold plasma approximation, taking into account the nonlinear density fluctuations in evaluating the HF current density perturbation.

Moreover, to solve the nonlinear wave equation, we consider the following spectral representation of the total toroidal electric field:

$$\tilde{E}_{z,k_{||},\omega}(x) \equiv \tilde{E}_{z,k_{||},\omega}^p(x) + \tilde{\psi}_{k_{||},\omega}(x) \quad (1)$$

assuming a spectral separation of the sideband from the pump:

$$\tilde{E}_{z,k_{||},\omega}(x) = \tilde{E}_{z,k_{||},\omega}^p(x) \quad \text{if} \quad k_{||}, \omega \in I_{k_{||},\omega} \quad (2)$$

$$\tilde{E}_{z,k_{||},\omega}(x) = \tilde{\psi}_{k_{||},\omega}(x) \quad \text{if} \quad k_{||}, \omega \notin I_{k_{||},\omega} \quad (3)$$

where $I_{k_{||},\omega}$ is the spectral range of the pump.
We assume that the pump electric field in the domain \( I_{k_o,\omega} \) is represented by a double delta function as follows:

\[
\tilde{E}_p(x) = \tilde{E}_o(x) \delta(\omega - \omega_o) \delta(k_{||} - k_{||_0}) + \tilde{E}_o^*(x) \delta(\omega + \omega_o) \delta(k_{||} + k_{||_0})
\]

(4)

and the sideband electric field in the domain complement of \( I_{k_o,\omega} \) as follows:

\[
\tilde{\psi}_{k_o,\omega}(x) \equiv \sum_{l,m} \left[ \tilde{\psi}_{l,m}(x) \delta(\omega - \omega_l) \delta(k_{||} - k_{||_m}) + \tilde{\psi}_{l,m}^*(x) \delta(\omega + \omega_l) \delta(k_{||} + k_{||_m}) \right]
\]

(5)

The governing nonlinear LH wave equation splits in two nonlinearly coupled wave equations, one for the complex pump amplitude and one for the complex sideband amplitude, for each spectral component considered:

\[
\begin{align*}
\partial_x^2 \tilde{E}_o(x) + \alpha(x, \omega_o, k_{||_0}) \cdot \partial_x \tilde{E}_o(x) + \beta_L(x, \omega_o, k_{||_0}) \cdot \tilde{E}_o(x) + \gamma(x, \omega_o, k_{||_0}, \sum_{l,m} |\tilde{\psi}_{l,m}(x)|^2) \cdot \tilde{E}_o(x) &= 0 \\
\partial_x^2 \tilde{\psi}_{l,m}(x) + \alpha(x, \omega_l, k_{||}) \cdot \partial_x \tilde{\psi}_{l,m}(x) + \beta_L(x, \omega_l, k_{||}) \cdot \tilde{\psi}_{l,m}(x) + \sigma(x, \omega_l, k_{||}, |\tilde{E}_o(x)|^2) \cdot \tilde{\psi}_{l,m}(x) &= 0
\end{align*}
\]

(6)

where:

\[
\alpha(x, \omega_o, k_{||}) = \frac{\partial_x S(x, \omega)}{S(x, \omega)} \left( \frac{k_{||}^2 - \omega_o^2}{c^2 S(x, \omega)} \right)
\]

(7)

\[
\beta_L(x, \omega_l, k_{||}) = -\frac{P(x, \omega_l)}{S(x, \omega)} \left( \frac{k_{||}^2 - \omega_l^2}{c^2 S(x, \omega)} \right)
\]

(8)

where \( S(x, \omega) \) and \( P(x, \omega) \) are the elements of the Stix tensor [11]. The \( \gamma \) and \( \sigma \) terms, taking into account the nonlinear phenomena, are defined as follow:

\[
\gamma = -\frac{q_e^2 \omega_pe(x) \left( k_{||_0}^2 - \frac{\omega_o^2}{c^2} S(x, \omega_o) \right)}{\omega_o^2 S(x, \omega_p) \sqrt{2m_e T_e^3(x)}} \sum_{l,m} \frac{\omega_o}{\omega^2_l} \left| \tilde{\psi}_{l,m}(x) \right|^2 \mathcal{F}_c^* \left[ \frac{(\omega_l - \omega_o)(k_{||_m} - k_{||_0})v_{th,e}(x)}{(k_{||_m} - k_{||_0})} \right]
\]

(9)

\[
\sigma = -\frac{q_e^2 \omega_pe(x) \left( k_{||_0}^2 - \frac{\omega_o^2}{c^2} S(x, \omega_o) \right)}{\omega_o^2 S(x, \omega_p) \sqrt{2m_e T_e^3(x)}} \frac{\tilde{E}_o(x) \left| \mathcal{F}_c \left[ \frac{(\omega - \omega_o)(k_{||} - k_{||_0})v_{th,e}(x)}{(k_{||} - k_{||_0})} \right] \right|^2}{\omega_o}
\]

(10)

Here we define a coupling function \( F_c \) as follows:

\[
F_c(u_a) = C_1(u_a) - C_2(u_a)^2 \sum_{\beta} \left( \frac{q_{\beta} m_{a} v_{th,\beta}}{k_{||}^2} \right) \left( 1 + \sum_{\gamma} \left( \frac{C_2(u_\gamma)}{k_{||}^2} \right) \right)^{-1}
\]

(11)
where \( u_a = \omega / k |v_{th,a} \), \( v_{th,a} \) is the thermal speed of the plasma species (ions and electrons), \( C_1(u_a) \equiv (\frac{2}{3} u_a + 2u_a^2 Z(u_a) - Z(u_a)) \), \( C_2(u_a) \equiv (1 + u_a Z(u_a)) \), \( \lambda_a \equiv \sqrt{\frac{T_a}{4 \pi n_{a0}^{(0)} q_a^2}} \) is the Debye length of the species and \( Z(u_a) \) is the plasma dispersion function.

The system of coupled equations (6) is numerically tractable and we solved it by an iterative method. As a specific case, we evaluated the spectral broadening and pump depletion of LH waves injected at 8 GHz in FTU. The coupled power density is 3 kW/cm\(^2\), the peak parallel refractive index \( n_{fo} = k_0 c/\omega_0 = 1.85 \). Realistic density and temperature profiles in front of the LH launcher and a white noise of the RF generator at -60 dB relative power are considered.

As a result, LH spectral broadening, both in frequency and in \( \tau \) is predicted and a pump depletion up to 30\% (of the total LH coupled power) might occur [10].

### 3. Parametric Analysis

We analyzed the intensity of the nonlinear effects at the antenna layer for:

- **i)** different ion mass species,
- **ii)** different ion-electron temperature ratio \( \tau = T_i/T_e \),
- **iii)** different electron temperatures \( T_e \),
- **iv)** different density gradients.

![Figure 1. Analysis of the effects of the ion mass (left) and the ion temperature (right) on the coupling function.](image)

In figure 1, on the left side, we compare the coupling function for a Deuterium plasma \( (m_e = m_i/m_d \approx 2) \) with single ionized Boron plasma \( (m_i \approx 10) \); on the right side, we compare two cases with different ion temperatures \( (\tau = 1, 2) \) for a Deuterium plasma. The comparison suggests that it can be very beneficial to operate with a hot heavy ion plasma in front of the LH antenna to reduce the nonlinear spectral broadening. For larger ion mass the coupling function is localized in a smaller spectral region (expressed in terms of \( u \)) thus a reduced spectral broadening is expected to occur. For larger ion/electron temperature ratio the magnitude of the coupling function is reduced. Therefore, the amplitude of the sidebands will be reduced, though the nonlinear interaction occur in the same spectral range. This result is not in contradiction with the experimental results of reduced spectral broadening.
obtained by increasing the electron temperature. It suggests that with $\tau > 1$ the amplitude of the sideband can be further reduced.

Numerical analysis suggests that the terms proportional to the gradient of the electric field do not have a leading role in determine the solution of the nonlinear system (6) in the range of plasma parameters considered here. Therefore, to evaluate the importance of nonlinear effect, we introduce a nonlinearity parameter $\rho_{NL}$ defined as follows:

$$
\rho_{NL} \equiv \max_{\omega, k} \left( \frac{\sigma(x, \omega, k)}{\beta_1(x, \omega, k)} \right) \forall \omega, k
$$

(12)

We calculate the nonlinearity parameter at the antenna layer ($x = 0$) because the nonlinear effects are found most important in such low temperature plasma.

**Figure 2.** The ratio of the nonlinearity parameters $\rho_{NL}/\rho_{NLo}$ where $\rho_{NLo}$ is calculated for a reference Deuterium plasma is plotted (left) versus the ion mass in units of Hydrogen mass ($m_i \equiv m_i/m_H$), for different values of the ion-electron temperature ratio $\tau$ and ionization charge $Z_i$; on the right, $\rho_{NL}/\rho_{NLo}$ is plotted versus the ratio $r_{Te}$ of the electron temperature with the electron temperature of the reference plasma at the antenna layer, for different density gradients.

In figure 2 we plot the ratio $\rho_{NL}/\rho_{NLo}$, where $\rho_{NLo}$ is the nonlinearity parameter for a reference Deuterium plasma, versus the ion mass (left side) and versus the electron temperature (right side), for different values of the ion-electron temperature ratio $\tau$, ionization charge $Z_i$ and density gradient. The
reference Deuterium plasma parameters are \( \tau = 1, n_e(x = 0) = 2 \times 10^{12} \text{ cm}^{-3}, \nabla n_e(x = 0) = 2 \times 10^{12} \text{ cm}^{-4}, \)
\( T_e(x = 0) = 10 \text{ eV}, V T_e(x = 0) = 6 \text{ eV cm}^{-1}. \)

These parameters correspond to typical SOL profiles in FTU [12,13]. However, in order to show the effect of a strong density gradient, we increased the typical value observed in the experiments (with characteristic e-folding of the order of 1.5-2.0 cm) by a factor 2-4.

This analysis suggests that the nonlinearity can be strongly attenuated operating with hot (\( \tau > 1 \)) heavy ions with single ionization and hot electrons along with a steeper density gradient at the antenna mouth.

The effect of higher ionization degree of impurity ions, as shown in figure 2, is to increase the nonlinear parameter. Therefore, the impurity seeding with the aim to reduce the spectral broadening is most effective at large atomic mass and low degree of ionization.

Numerical solutions of the system (6) suggest that the nonlinear effects are significant only if \( \rho_{NL} > 1. \) Therefore this condition can be used to evaluate a threshold power density for the onset of the LH nonlinear behavior. For a monochromatic pump wave, the power flux threshold necessary for nonlinear spectral broadening, when \( n_{\rho_o}^2 / \omega_0^2 \approx 2 \) and \( \tau = 1 \) is of the order of \( P_{th} \) (kW/m²) \( \approx (n_{\rho_o}^2 - 1)^{-1} \times |\Delta n| \times T_e^{1.5} \) (eV) \( \times f_o^2 \) (GHz), where \( |\Delta n| \equiv |n_i - n_{\rho_o}| \) with \( |\Delta \omega| \equiv |\omega - \omega_o| \approx (\omega_o/c) \times |\Delta n| \times v_{th,i}. \) It is worth to note that a more general expression for sigma (not reported here) is necessary to take into account the LH pump power spectrum in frequency and wavenumber. In this case, the parameter sigma involves a sum over all the spectral components. For typical LH launched spectra, numerical analysis indicates that the power flux necessary for the onset of nonlinear behavior is increased at least by a factor 2 with respect to a pure monochromatic pump. In addition, we have neglected here the effect of the density gradient (as it is marginal with respect to the other relevant parameters). Thus the above threshold should be considered as a lower bound to the actual power flux threshold for the onset of nonlinear phenomena.

**Figure 3.** Power density threshold versus the spectral distance from the launched \( n_i \) peak.
In figure 3 the predicted power density threshold is plotted, for a launched $n_0$ spectrum peaked in $n_{lo} = 1.85$, versus the $n||$ distance from the pump, for three typical LH operating line frequencies (JET 3.7 GHz, ITER 5 GHz, FTU 8 GHz) and for two different electron temperatures (10 and 50 eV) of a reference Deuterium plasma with $\tau = 1$ and plasma density such that $\omega_\text{ce}^2 / \omega_\text{ci}^2 \approx 2$.

It is worth to note that in a wide spectral range the power density threshold is lower than the typical value of 1 kW/cm$^2$ for LH wave launchers. A more accurate evaluation of the power density threshold can be obtained by the full wave solutions of the system (6). We do not discuss here the numerical solutions that we obtained, as they depend on specific experimental scenarios. The aim of the analysis proposed here is to discuss the general dependence of the LH nonlinear behavior on the plasma and RF parameters.

4. Conclusions

In [10] we derived a system of nonlinearly coupled LH wave equations describing the amplification of the LH sidebands and the depletion of the pump wave, considering the noisy background of the RF source. Full wave numerical solutions indicate that pump depletion and spectral broadening in frequency and wavenumber might significantly reduce the CD efficiency when operating at reactor grade high plasma density. In the present paper we analyzed the nonlinear LH behaviour by means of a parametric analysis. As a result, we found that the nonlinear effects could be prevented by increasing the edge electron temperature, the relative ion-electron temperature, the edge density gradient and the ion mass.

In particular, for ITER ($f_o = 5$ GHz, assuming $n_{lo} = 1.85$ and 1 kW/cm$^2$ power density) if the electron temperature is $\geq 50$ eV at the antenna mouth, the sidebands with spectral components $n|| \geq 5$, emerging from the noisy background of the RF generator, cannot be amplified (in figure 3 it is shown that in this case the power density threshold to amplify sidebands with $|\Delta n|| \geq 3$ is greater than 1 kW/cm$^2$).

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