On the Equivalent Position Workspace for a Coal Gangue Picking Robot

P Liu¹,²,*, H W Ma¹, X H Zhang¹ and X Z Qiao¹

¹ School of Mechanical Engineering, Xi'an University of Science and Technology, Xi'an, 710054, China
² Key Laboratory of Electronic Equipment Structure Design, Ministry of Education, Xi'an, 710000, China

*Corresponding author E-mail address: 200304405liupeng@163.com

Abstract. Coal gangue separation is of key important for green mining. Robotic separation of coal and gangue based on the machine vision system, which is called a coal gangue picking robot with a gangue grab driven by four cables, is developed in this study. Here, this paper focuses on the equivalent position workspace, within which a coal gangue picking robot possesses the identical stability, to find out any more information about the structural stability for a coal gangue picking robot. First, the kinematic and kinetostatic models of the coal gangue picking robot are presented for analyzing the effects of it on the structural stability for the coal gangue picking robot. And moreover, a non-iterative polynomial-based optimization algorithm with the proper optimal objective function is presented based on the convex optimization theory, in which the cable with the minimum cable tension at any pose is determined. Then, three position performance indices are proposed to show the important effects on the structural stability for the coal gangue picking robot in a specified region of the workspace. Besides, a new workspace, the Equivalent Position Workspace (EPW), is introduced. Finally, the theoretical relationship between the two performance indices and the stability is corrected based on simulation results. The research has important guiding significance and practical value for coal gangue robotic separation.

Keywords: Cable-driven Parallel Robot; Coal Gangue Picking Robot; Stability

1. Introduction

Coal is the major fossil fuel in China and continues to play a pivotal role in the energy sector. With the widely use of mechanized coal mine production and the increasingly extraction of thin seam, large number of gangue mixed raw coal has greatly increased, which affects the efficiency of coal preparation and increases the cost of preparation. Meanwhile gangues are stacked on the ground after preparation, which become the hazard source for environment. Various separation methods, in order to reduce the damage to environment of gangue, have been proposed based on the physical property differences between coal and gangue. The separation of gangue from coal can not only improve the quality of raw coal, decrease the cost of preparation, but also provide materials to the gangue filling underground and realize the real green mining [1]. But the separation of coal and gangue depends mainly on the manual gangue picking. The main motivation for moving toward the robotic separation of coal and gangue as the future coal utilizing technique is the environmental and other advantages over the conventional manual gangue picking. And furthermore, it is a promising technology as it is a...
combination of the image processing, information sensing and electromechanical integration. However, this technology is still in a developing stage in the worldwide.

Parallel mechanisms are used in industrial applications requiring high load carrying capacity, high velocity, or precise positioning. In the family of parallel mechanisms, cable-driven parallel robots allow for larger workspaces and can produce large accelerations [2]. Therefore, these advantages make the cable-driven parallel robots a promising alternative to the rigid-link mechanisms in many industrial applications, such as medical rehabilitation [3], wind tunnel experiment [4], astronomical observation [5], material transportation [6] and other fields [7]. With advantages of high speed, high payload and good flexibility, a cable-driven robot is a necessary choice in applications for driving forces and institutional quality limited. An important application of the cable-driven robots is in the manipulation task with high payload, and indeed, a coal gangue picking robot developed here is a redundantly cable-driven parallel manipulator with high load capacity to realize roboticized separation of coal and gangue.

The coal gangue picking robot is based on the machine vision system, which is an integrated technology using computer image processing technology and analysis technology to identify the target of various patterns of coal and gangue. It mainly contains machine vision and robotic separation process. It is mainly based on coal and gangue physical feature differences and image recognition to realize separation, which is suitable for preliminary separation of coal and gangue. Scholars mainly probed into a general machine vision approach for on-line estimation of coal and gangue, the robotic separation of coal and gangue has not been studied adequately. Song et al. [8] proposed an online intelligent refuse picking system based on dual-energy X-ray recognition and 3D mechanical gripper grabbing target refuse, and moreover, the working principle, design principle and specific structural design of the picking system are discussed. In order to separate coal gangue from lump coal, the paper investigated the fundamental characteristics of coal images and gangue images on the basis of digital image processing technology. The image processing results are obtained, which showed that the coal and gangue can been distinguished based on their grayscale histogram[9]. Reddy et al. [10] proposed a system identifies the gangue by analyzing the characteristics of the image, and moreover, the research results indicated that the comparison of coal images and gangue images could successfully achieve the real-time automatic separation of gangue. Zhang et al. [1] proposed an underground pneumatic separation of coal and gangue based on the machine vision system, and moreover, the key factors affecting coal gangue pneumatic separation are analyzed. But the equipment is just applicable to the gangue with large diameter (≥ 50 mm). From above, to the best of our knowledge, the researchers have done lots of research on the recognition of coal and gangue, the robotic separation of coal and gangue has not been studied adequately. Liu et al. [11] investigated the minimum cable tension distributions in the workspace for cable-based parallel robots, and furthermore, the relationship between the stability and the minimum cable tensions is explained clearly. But the effect of the position of the end-effector on the stability of cable-based parallel robots is not considered. In this work, the equipment of robotic separation of coal and gangue is presented and the equivalent position workspace for the coal gangue picking robot is discussed. The workspace with equivalent positions for the coal gangue picking robot is addressed. This study will help to motivate both applied and theoretical research work on the roboticized separation of coal and gangue.

2. Modeling of the coal gangue picking robot

2.1 Kinematic model of the coal gangue picking robot

The coal gangue picking robot proposed in this paper is made of a gangue grab attached to a cable-driven parallel robot (Fig. 1). The each pulley can be attached to the ceiling of a mast. As a result, the gangue grab moves freely in every direction because the cables can be shortened and lengthened controlled by four servo motors mounted to the fixed base. The servo motors receive control commands from the central controller. It is worth noting that the platform is assimilated to a unique point mass. As shown in Fig.2, a base reference frame, noted $OXYZ$, is attached to the fixed base,
where $O$ is the origin point. While a mobile reference frame, noted $O'X'Y'Z'$, is attached to the mobile gangue grab, where $O'$ is the reference point. The position $P=(x, y, z)^T$ of the mobile platform is given by the vector connecting $O$ to $O'$. Point $B_i$, at which the $i$th cable ($i=1, 2, 3, 4$) enters the pulley, is assumed to be fixed to the base frame. Obviously, vectors $B_i=[B_{i,x}, B_{i,y}, B_{i,z}]^T$ are constant position vectors connecting $O$ to $B_i$ in $OXYZ$.

Referring to Fig.2, the $i$th cable length $\rho_i$, compatible with a given position $P=(x, y, z)^T$, is given by

$$\rho_i = |\mathbf{B}_i - \mathbf{P}|^2 = \sqrt{(B_{i,x} - x)^2 + (B_{i,y} - y)^2 + (B_{i,z} - z)^2}$$

(1)

When expressed in the base frame, the unit vector along cable $i$ is noted $u_i$ and can be written as:

$$u_i = (\mathbf{B}_i - \mathbf{P})/\rho_i, \ i = 1, 2, 3, 4$$

(2)

![Figure 1. Schematic of a coal gangue picking robot](image1)

![Figure 2. Kinematic notation of a coal gangue picking robot](image2)

### 2.2 Kinetostatic model of the coal gangue picking robot

In order to maintain positive cable tension, the coal gangue picking robot must be actuated redundantly due to the unilateral nature of cables. Thus, for the coal gangue picking robot with 4 cables, there will be a total of 3 unknowns which are cable tension for each cable. And therefore, the kinetostatic model of the coal gangue picking robot, using Newton method, can be developed based on [12]:

$$J^T \mathbf{T} + \mathbf{W} = 0$$

(3)

where $\mathbf{W} \in \mathbb{R}^3$ is the external wrench on the gangue grab; $\mathbf{J} = [\mathbf{J}_1, \mathbf{J}_2, \mathbf{J}_3, \mathbf{J}_4] \in \mathbb{R}^{3\times n}$ is the structure matrix of the coal gangue picking robot. And the vector $\mathbf{J}_i$ is defined as

$$\mathbf{J}_i = u_i, i = 1, 2, 3, 4$$

(4)

$\mathbf{T} = [T_1, T_2, T_3, T_4]^T$ is the vector consisting of all cable tensions.

$$\mathbf{T}_{s,min} \leq \mathbf{T} \leq \mathbf{T}_{s,max}$$

(5)

where the lower bound of the cable tension $\mathbf{T}_{s,min} = [T_{1,min}, T_{2,min}, T_{3,min}, T_{4,min}]^T$ is required to keep cables taut; while upper bound of the cable tension $\mathbf{T}_{s,max} = [T_{1,max}, T_{2,max}, T_{3,max}, T_{4,max}]^T$ is limited by the output torques of the servo motors and the maximum tension the cable can withstand without breaking. Form Eq. (5), the cable tensions may exist infinite solutions because the number of cables is larger than the degree of freedom of the gangue grab. Therefore, in order to obtain a unique solution, the minimum variance is used to optimize while using (5) as a constraint. Mathematically, the determination of the cable tensions can be formulated as follows [13]:
where  \( E(T) = \frac{t_1 + t_2 + t_3 + t_4}{4} \) is the arithmetic mean value of the vector  \( T \). According to convex optimal theory, cable tensions are determined uniquely.

The minimum cable tension  \( T_{\text{min}} \) at any position can be obtained when cable tensions are determined uniquely. And moreover, the minimum cable tension  \( T_{\text{min}} \) can be expressed as:

\[
T_{\text{min}} = \min(T)
\]

where  \( \min(\bullet) \) is the smallest element in a vector  \( T \).

Here, the cable having minimum cable tension when the gangue grab is at a specified position is determined, that is to say, the weakest constraint direction, the gangue grab may be unstable, can be obtained.

3. The three position performance indices and EPW generation algorithm

3.1 The position performance indices

Based on the solutions of the kinematic and kinetostatic models of the coal gangue picking robot, three position performance indices are discussed in more detail at a later stage. In order to make clear the physical meaning of the three position performance indices, the schematic diagram relating to them is shown in Fig.3, in which the outmost box is the task space of the coal gangue picking robot. Without loss of generality, the optional position in the workspace is denoted by  \( P \); the vertical midline of the workspace is denoted by  \( a \); the horizontal plane on which the specified position is located is highlighted with green; the intersection of the vertical midline and the horizontal plane is denoted by  \( Q \); the intersection of the vertical midline and the top of the workspace is denoted by  \( M \); the intersection of the vertical midline and the top of the task space is denoted by  \( M' \); the angles at the position  \( P, Q \) and  \( M \) between the cable having the minimum cable tension and the horizontal plane are denoted by  \( \theta_p, \theta_q \) and  \( \theta_m \) respectively.

![Figure 3. Schematic diagram relating to the three position performance indices](image)

In detail, three position performance indices having an important effect on the stability of the coal gangue picking robot are proposed to show how far the specified position is away from the center and the top of the workspace. The three position performance indices,  \( \mathcal{R}_{x_1}, \mathcal{R}_{x_2}, \) and  \( \mathcal{R}_{x_3} \) can be respectively expressed as:

\[
\mathcal{R}_{x_1} = \frac{\tan \theta_m}{\tan \theta_q}
\]

\[
\mathcal{R}_{x_2} = \frac{\tan \theta_q}{\tan \theta_p}
\]
\[ s_s \times \frac{1}{s_s} \] (10)

Comprehending the definitions of the three position performance indices leads to some conclusions that the performance index \( s_s \) shows the distributions of the equivalent positions along Z-direction; the performance index \( s_s \) is a function of the distance between the present position and the center position of the horizontal plane with the same elevation along Z-direction, therefore showing distributions of the equivalent positions along horizontal direction; And furthermore, the performance index \( s_s \) is proposed to describe the comprehensive distributions of the equivalent positions over the whole workspace. It is perfectly clear that the distributions of the equivalent positions over the whole workspace can be described based on the three performance indices.

3.2 EPW generation algorithm
Furthermore, EPW is composed of the positions of the gangue grab having the equivalent effect on the stability of the coal gangue picking robot. And thus, the workflow of EPW generation algorithm can be summarized as follows:
(1) Input the real-time gangue grab position \( X_i \) in the workspace \( (i=1, 2, \ldots, N) \). \( N \) is the total number of positions; \( H \) is the height of the towers; the external wrench \( F_i(X_i) \); the gravity \( W_i(X) \) of the gangue grab.
(2) Calculate the structure matrix \( J (X_i) \) and the null space matrix \( \text{Null}(J (X_i)) \) of it.
(3) Determine the cable tension \( T(X_i) \), using convex programming theory and Eq. (6).
(4) Obtain the minimum cable tension \( T_{\text{min}}(X_i) \) using Eq. (7), and find the cable having minimum cable tension when the gangue grab is at the position \( X_i \).
(5) Calculate the the length of the cable \( \rho_{\text{min},i} \) at the position \( P, Q \) and \( M \) and the angles at the position \( P, Q \) and \( M \) between the cable having the minimum cable tension and the horizontal plane.
(6) Calculate three position performance indices using the Eqs. (8)-(10) respectively and go to the next position.
(7) Judge \( X_i \) whether the last position. If not, go to (1) and solve three position performance indices for the next position \( X_{i+1} \); if it is, record the three position performance indices and the position \( X_i \) and moreover, depict the equipotential curves and the equivalent position workspace.

4. Simulation Examples

4.1 Description of the coal gangue picking robot
An example is now presented in order to illustrate the equivalent position workspace and the relationship between the three position performance indices and the stability of the coal gangue picking robot. The mass of the gangue grab \( m_{\text{g}}=10\text{kg} \), the bound of the cable tension \( T_{\text{s, max}}=[300 300 300] \) N, \( T_{\text{s, min}}=[10 10 10] \) N. And meanwhile, the origin of the fixed base reference frame \( OXYZ \) is located at the bottom of mast1. The position vectors that the cables enter the pulleys are: \( A_1=[0 0 23] \) dm, \( A_2=[37 0 23] \) dm, \( A_3=[37 40 23] \) dm, \( A_4=[0 40 23] \) dm. Therefore, the midpoint of the horizontal surface is at \( x=18.5\text{dm} \) and \( y=20\text{dm} \).

4.2 Results and discussion
The three position performance indices above are computed using the Eq. (8), (9) and (10) in this section for the coal gangue picking robot. And the results are displayed in Fig.4. As expected, the position performance index \( s_s \) increases as \( z \) increases along the vertical midline shown in Fig.4(a). It is worth noting that the shape of the curve is a nonlinear function of the elevations. It can be seen from Fig.4(b) that the positions where gangue grab stays hold the bigger \( s_s \) are the ones being close to the center of every horizontal plane. That is because the upper and central positions possess bigger \( T_{\text{min}} \).
than others, thereby improving the stiffness and stability of the robot. In addition, the position performance index $\mathcal{R}_s$ is depicted from Fig.4(c) to Fig.4(f). With (c) and (d) together, it is evident that the curves that have the same position performance index $\mathcal{R}_s$ are at the same elevation along $Z$-direction because they are front and lateral views of $\mathcal{R}_s$ in the workspace respectively. A further view of the figure curves indicates that the interior positions where gangue grab stays retain bigger $\mathcal{R}_s$ than others because of the uniformity of the cable tensions. And moreover, a specified value of $\mathcal{R}_s$, which is a equipotential surface over the whole workspace, is displayed in Fig.4(e), and while the planform of the equipotential surface is described. Of course, it is worth noting that the positions within the surface retain position performance index $\mathcal{R}_s$ being bigger than 0.4. The colours are worthy of note that represent the elevation along $Z$-direction. Moreover, one of its features is the symmetry about $X$-direction and $Y$-direction. It should be pointed that the equipotential surface with the bigger position performance index $\mathcal{R}_s$ is the ones being higher than the surface in Fig.4(e). As expected, further insight into the integral shape of EPW can be obtained from them.

Figure 4. The three position performance indices in the workspace.
As expected, further insight into the integral shape of EPW can be obtained from them. From above it can be concluded that the combination of all the curves in Fig.4 (a)-(d) is in conformity with Fig.4(e) and (f) that displays $\tau_1 = 0.4$ in the workspace.

5. Conclusions
Robotic separation of coal and gangue, which is called a coal gangue picking robot with a gangue grab driven by four cables, is developed in this study. The equivalent position workspace of the coal gangue picking robot is discussed using the three position performance indices based on the kinematic and kinetostatic models of the robot. Solutions to the problems of the effect of the position of the gangue grab on the stability of the robot presented to a typical coal gangue picking robot. Simulation results of the three position performance indices show that as the three performance indices increase, so do the effect of the position of the gangue grab on the stability of the robot, therefore improving the stability of the robot. In the whole workspace, the positions when the gangue grab stays in the center and on top of the workspace possess bigger position performance indices than others, leading to have better stability. It is important to notice that the positions of the gangue grab have great important effect on the stability of the coal gangue picking robot.

Acknowledgments
The research is supported by Open Fund of Key Laboratory of Electronic Equipment Structure Design (Ministry of Education) in Xidian University. The authors gratefully acknowledge the financial support of Special Scientific Research Program Project of Shaanxi Provincial Education Department under Grant No. 18JK0506.

References
[1] Zheng K., Du C., and Li J., et al. Underground pneumatic separation of coal and gangue with large size ($\geq 50$ mm) in green mining based on the machine vision system[J]. Powder Technology, 2015, 278: 223-233.
[2] Barbazza L., Zanotto D., and Rosati G., et al. Design and optimal control of an underactuated cable-driven micro–macro robot[J]. IEEE Robotics and Automation Letters, 2017, 2(2): 896-903.
[3] Mao, Y., and Agrawal, S. K., Design of a Cable-Driven Arm Exoskeleton (CAREX) for Neural Rehabilitation[J]. IEEE Trans. on Robotics, 2012, 28(4): 922-931.
[4] Zheng Y. Q., and Zhao S. H., Research Survey of Technique about Wire-Driven Parallel Suspension Systems Used in Forced Oscillation Experiments in Low-Speed Wind Tunnels for Obtaining Dynamic Derivatives[J]. Informatics in Control, Automation and Robotics, 2012, 133: 131-138.
[5] Yao R., Zhu W. B., and Sun C. H. et al., Pose Planning for the Feed Support System of FAST[J]. Advances in Mechanical Engineering, Volume 2014, Article ID 209167: 1-9.
[6] Oh S. R., Ryu J. C., and Agrawal S. K. Dynamics and control of a helicopter carrying a payload using a cable-suspended robot[J]. Journal of Mechanical Design, 2006, 128(5): 1113-1121.
[7] Tang X. Q., An Overview of the Development for Cable-Driven Parallel Manipulator[J]. Advances in Mechanical Engineering, Volume 2014, Article ID 823028: 1-9.
[8] Song W. G., Tao Y. D., and Qi C. J. et al., Design and research of intelligent robot refuse picking system [J]. Coal processing and comprehensive utilization, 2018, (9): 5-10.
[9] Wang R., and Liang Z. Automatic separation system of coal gangue based on DSP and digital image processing[C]/Photonics and Optoelectronics (SOPO), 2011 Symposium on. IEEE, 2011: 1-3.
[10] Reddy K. G. R., Tripathy D. P. Separation of Gangue From Coal Based on Histogram Thresholding [J]. International Journal of Technology Enhancements and Emerging Engineering Research, 2013, 1(4): 31-34.
[11] Liu P., Qiu Y. Y., Su Y., et al. On the Minimum Cable Tensions for the Cable-based Parallel
Robots [J]. Journal of Applied Mathematics, vol. 2014, Article ID 350492, 2014.

[12] Gosselin C., Grenier M. On the determination of the force distribution in overconstrained cable-driven parallel mechanisms[J]. Meccanica, 2011, 46(1): 3-15.

[13] Liu P., Qiu Y. Y., Su Y. A new hybrid force-position measure approach on the stability for a camera robot [J]. Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science:2016, 230(14): 2508 - 2516.