Novel stellar astrophysics from extended gravity

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Abstract – Novel implications on neutron stars come from extended gravity. Specifically, the GW190814 event indicated the probability of having large mass stars in the mass gap region 2.5–5\(M_\odot\). If the secondary component of GW190814 is a neutron star, such large masses are marginally supported by General Relativity (GR), since a very stiff Equation of State (EoS) would be needed to describe such large mass neutron stars, which would be incompatible with the GW170817 event, without any modification of gravity. In view of the two groundbreaking gravitational wave observations, we critically discuss the elevated role of extensions of GR towards the successful description of the GW190814 event, and we also speculate in a quantitative way on the important issue of the largest allowed neutron star mass.

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Neutron stars are proven literally to be the superstars of all stellar structures, not unjustifiably. Since the striking observation of Jocelyn Bell back in 1967, they have been in the epicenter of many scientific disciplines, apart from astrophysics. During the last decade two striking gravitational waves observations of the LIGO-Virgo Collaboration have set the stage for a new way of thinking both in theoretical cosmology and in theoretical astrophysics. The GW170817 event [1] has proven definitive for some classes of Horndeski theories, and also had further constrained the neutron star masses and radii, while the GW190814 event [1] has proven even more interesting due to the possibilities and perspectives of the secondary component interpretation.

Extensions of GR [2] can play a key role towards describing the secondary component of the GW190814 event, if it is proven to be a neutron star. The main reason is that the candidate theories naturally yield neutron star masses in the range 2.5–3\(M_\odot\), without “stretching” general relativistic models. The main feature of extended gravity theories is that geometry plays the role of the effective energy momentum tensor, so neutron stars with masses in the range 2.5–3\(M_\odot\) can be easily described by several candidate models [3]. In GR, where obviously modified gravity effects are absent, one needs a quite stiff EoS for nuclear matter to describe neutron stars with mass 2.5–3\(M_\odot\). On the other hand, geometric effective contribution to the energy momentum tensor renders modified gravity, in general, and extended gravity in particular, very appealing to describe large mass neutron stars. Let us briefly recall the theoretical framework of one of the most important extensions of GR, namely \(f(R)\) gravity, coinciding with the Einstein theory in the case \(f(R) = R\). The Jordan frame gravitational action is

\[
A = \frac{c^4}{16\pi G} \int d^4x \sqrt{-g} \left[ f(R) + \mathcal{L}_{\text{matter}} \right],
\]

with \(g\) being the determinant of the metric tensor \(g_{\mu\nu}\) and \(\mathcal{L}_{\text{matter}}\) the perfect matter fluid Lagrangian. In the context of the metric formalism, upon variation with respect to the metric tensor, the field equations are

\[
\frac{df(R)}{dR} R_{\mu\nu} - \frac{1}{2} f(R) g_{\mu\nu} - [\nabla_\mu \nabla_\nu - g_{\mu\nu} \Box] \frac{df(R)}{dR} = \frac{8\pi G}{c^4} T_{\mu\nu},
\]
\[
\frac{dp}{dr} = -(\rho + p) \frac{d\psi}{dr},
\]
\[
\frac{d\lambda}{dr} = \frac{e^{2\lambda}[(216\pi p + f(R)) - f'(R)(r^2 R + 2)] + 2 R^2 f''(R) r^2 + 2 r f''(R)[r R_{rr} + 2 R_r] + 2 f'(R)}{2r [2f'(R) + r R_r f''(R)]},
\]
\[
\frac{d\psi}{dr} = \frac{e^{2\lambda}[(216\pi p - f(R)) + f'(R)(r^2 R + 2)] - 2(2r f'(R) R_r + f'(R))}{2r [2f'(R) + r R_r f''(R)]},
\]
\[
\frac{d^2 R}{dr^2} = R \left( \lambda_r + \frac{1}{r} \right) + \frac{f''(R)}{f'(R)} \left[ \frac{1}{r} \left( 3\psi_r - \lambda_r + \frac{2}{r} \right) - 2 \lambda \left( \frac{R}{2} + \frac{2}{r^2} \right) \right] - \frac{R^2 f'''(R)}{f'(R)}.\]

with
\[
T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta (\sqrt{-g} \mathcal{L}_m)}{\delta g^{\mu\nu}}
\]

being the energy momentum tensor of the perfect fluid matter. A spherically symmetric, static neutron star is described by the metric
\[
ds^2 = e^{2\psi} c^2 dt^2 - e^{2\lambda} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2).
\]

Inside the neutron star, the perfect fluid matter energy momentum tensor is \(T_{\mu\nu} = \text{diag}(e^{2\psi} \rho c^2, e^{2\lambda} \rho, r^2 p, r^2 \rho \sin^2 \theta)\), where \(\rho\) and \(p\) are the matter energy density and the pressure. The Tolman-Oppenheimer-Volkoff (TOV) equations for \(f(R)\) gravity are

\[
\text{see eqs. (4)–(7) above}
\]

One of the most important \(f(R)\) gravity models is
\[
f(R) = R + \alpha R^2,
\]

where \(\alpha\) denotes the coupling parameter constrained by inflationary dynamics. The numerical integration of TOV equations for the \(R^2\) model [3] is presented in fig. 1, where, for larger values of the coupling parameter \(\alpha\), higher neutron star masses result. Clearly they can naturally fill up the mass gap region. Thus, from a phenomenological point of view, extended gravity can predict neutron stars with masses in the lower edge of the mass gap region.

Coming to the predictions of extended gravity, the fundamental question is: How large can a neutron star mass be? This question is of fundamental importance, and the only consistent way to see what the maximum neutron star in the context of any theory is, is to examine the problem by choosing the stiffest possible EoS for nuclear matter. The stiffest EoS, which simultaneously respects the high density stability condition for nuclear matter (\(\frac{dP}{d\rho} > 0\)) and the subluminality condition for the speed of sound (\(\frac{dP}{d\rho} \leq c^2\)), is the causal EoS with the following form:
\[
P_{sn}(\rho) = P_u(\rho_u) + (\rho - \rho_u)c^2,
\]

with \(\rho_u\) being the maximum density for nuclear matter, and \(P_u(\rho_u)\) the corresponding pressure. By assuming this EoS, one obtains the maximum upper mass for neutron stars, which, in the context of GR for slowly rotating neutron stars, is [4,5]
\[
M_{\text{max}}^{\text{CL}} = 3M_\odot \sqrt{\frac{5 \times 10^{14} \text{g/cm}^3}{\rho_u}},
\]

thus in the context of GR, the maximum mass limit of slowly rotating neutron stars is
\[
M_{\text{max}} \leq 3M_\odot.
\]

If rotation is taken into account, the causal limit of the maximum neutron mass becomes
\[
M_{\text{max}}^{\text{CL,rot}} = 3.89M_\odot \sqrt{\frac{5 \times 10^{14} \text{g/cm}^3}{\rho_u}}.
\]

Therefore, in the context of GR, one expects to find neutron stars in, but not deeply in the mass gap region \(M \sim 2.5 - 5M_\odot\), and specifically in the region \(M \sim 2.5 - 3M_\odot\), and this under extreme conditions. With regard to any modified gravity, the question is whether the 3 solar masses upper limit of GR is respected. Remarkably for \(f(R)\) gravity, the answer is yes [6], as we now evince.
Consider that the nuclear matter has the following causal EoS:

\[ P_{sn} = P_u(p_u) + (p - p_u)v_s^2. \]  

(13)

where \( v_s \) is the sound speed, we shall assume that it varies in the range \( c^2/3 \leq v_s^2 \leq c^2 \) [7]. Also the transition density will be assumed to be that of the SLy EoS at \( \rho_u = 2\rho_0 \), where \( \rho_0 \) is the nuclear matter saturation density. The results of the numerical integration of the TOV equations for the \( R^2 \) model are presented in table 1. The results of the numerical analysis are particularly interesting. Firstly, in all studied cases, the 3 solar masses limit of GR is well respected. Let us discuss in brief the outcomes of our analysis, starting from the effect of the parameter \( \alpha \). As can be seen in table 1, for values of the sound speed lower than the speed of light, the causal maximum mass for the \( R^2 \) model is larger than that of GR, while when the sound speed is equal to the light speed, when small values of \( \alpha \) are used, the GR causal maximum mass limit is larger compared to the \( R^2 \) model. On the contrary, for large values of \( \alpha \), the \( R^2 \) model dominates over GR. We have to note here that the parameter \( \alpha \) for the \( R^2 \) model in cosmological contexts must take small values. Specifically, the parameter \( M \) must be approximately \( 1/\sqrt{\alpha} = 1.5 \times 10^{-5} (\frac{M}{M_\odot})^{-1} M_\odot \) for early time phenomenological reasons [8], with \( N \) being the \( \epsilon \)-foldings number. So basically \( \alpha \) must take small values in order for the theory to be cosmologically consistent. This issue has been addressed more thoroughly in refs. [9,10] in the Einstein frame for potentials that belong in the same class as the \( R^2 \) model when considered in the Einstein frame. So we refer the reader to refs. [9,10] for a detailed discussion on these issues.

Thus, coming to the issue on where to expect to find the maximum neutron star masses, the prediction for extended gravity is in the range \( M \sim 2.5-3 M_\odot \). The main difference with GR is the freedom in the choice of EoS for nuclear matter.

Let us note that it is remarkable that observationally-friendly increase of neutron stars maximum mass due to extended gravity is in compliance with inflationary attractors Universe, which is consistent with Planck data.

Having discovered the mass range of neutron stars even in extended gravity, the great question is: What is the lowest mass limit of an astrophysical black hole? Without being very strict, the answer is that astrophysical black holes can have masses above 3 solar masses. But in order to be sure on this, one must calculate the baryon masses for neutron stars in the context of extended gravity. This will definitely answer, in a concrete way, the above question, because, if the baryon mass is larger than the maximum baryon mass, the neutron star will collapse to a black hole. Thus automatically, one has available the lowest astrophysical black hole mass limit. Finally, let us comment that the same procedure performed in this paper can be applied in other modified gravity theories, such as \( f(T) \) gravity, in which the spherically symmetric solutions have also extended terms similar to the ones of \( f(R) \) gravity, see for example [11,12].

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Table 1: Causal maximum mass of neutron star for the \( R^2 \) model. The parameter \( \alpha \) is \( v_s^2 = 4G^2M_\odot/c^4 \) units with \( r_g \) being the gravitational radius of the Sun.

| EoS       | \( \alpha \) | \( M_{\text{max}} \) | \( R_s \) | \( \Delta M_{\text{max}} \) |
|-----------|-------------|----------------|-------|----------------|
|           |             | (\( M_\odot \)) | km    | (\( M_\odot \)) |
| SLy+(5)   | 2.50        | 2.04           | 11.54 | 0.12            |
| with \( v_s^2 = c^2/3 \) | 10          | 2.11           | 11.69 | 0.19            |
|           | 0           | 2.97           | 12.85 | 0               |
| SLy+(5)   | 2.50        | 2.98           | 11.57 | 0.02            |
| with \( v_s^2 = c^2 \) | 10          | 3.10           | 13.71 | 0.13            |

References are included in the text.