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A mathematical model for the coverage location problem with overlap control

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ABSTRACT
The Coverage Location Problem (CLP) seeks the best locations for service to minimize the total number of facilities required to meet all demands. This paper studies a new variation of this problem, called the Coverage Location Problem with Overlap Control (CLPOC). This problem models real contexts related to overloaded attendance systems, which require coverage zones with overlaps. Thus, each demand must be covered by a certain number of additional facilities to ensure that demands will be met even when the designated facility is unable to due to some facility issue. This feature is important in public and emergency services. We observe that this number of additional facilities is excessive in some demand points because overlaps among coverage zones occur naturally in CLP. The goal of the CLPOC is to control overlaps to prioritize regions with a high density population or to minimize the number of coverage zones for each demand point. In this paper, we propose a new mathematical model for the CLPOC that controls the overlap between coverage zones. We used a commercial solver to find the optimal solutions for available instances in the literature. The computational tests show that the proposed mathematical model found appropriate solutions in terms of number of demand points with minimum coverage zones and sufficient coverage zones for high demand points.

1. Introduction

The coverage location problem (CLP) (Toregas & Revelle, 1973; Toregas, Swain, ReVelle, & Bergman, 1971) locates facilities where they will constrain the operational costs involved while offering service to all demands. Optimization results from system planning avoid excessive expenses. Objectives vary from case to case. In one case, the best way to reduce costs may be to minimize the number of facilities, while in others, it may be better to reduce maintenance and support costs or distances for distribution, for example.

In most optimization studies, however, a common problem occurs in cities. In these urban centers there are facilities that operate at their limits. When failures occur, the service system does not solve this problem efficiently, because it is unable to reassign the demand to another facility. This context can worsen when it happens in emergency situations (Weissman et al., 2007).

Take, for example, the context of the COVID-19 pandemic. There are recent papers in the literature that expose the hospital burden caused by high demand for emergency services (Ai et al., 2019). Due to this excess demand, there is a need to structure temporary places to serve the population. In this context, the question arises: how to strategically locate temporary facilities in such a way as to serve the places with the greatest number of patients?

In addition to this example, countries that are in internal conflict often experience the loss of infrastructure. Facilities are targets, affecting a whole population that can lose their services and need support to meet their needs. Where can that support come from? In another example, there is also the possibility of a collapse in services when natural disasters occur. News of tornadoes, hurricanes, tsunamis, volcanic eruptions, and electrical storms frequently report the loss of food distribution centers and emergency services when people have the greatest need of them. The lack of adequate preparation of the care system for these contexts can cost the lives of many people (Barzinpour & Esmaeili, 2014).

Daskin (1982) and Hogan and ReVelle (1986) proposed a new problem: the backup coverage location problem (BCLP). In this problem, the coverage zones must have an extra cover, or backup coverage, thus, preventing demand failures. Other models have been proposed based on BCLP. After a review of the literature, we note that these models are restricted to providing incidental support, a single layer of extra coverage to places with greater demands or higher population density. These models do not provide support to all points or do not

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consider that there are places that need more than one extra layer of coverage.

One way to avoid service overload is to prepare the system for a fixed number of extra coverage zones for all demands. Whenever there is a high possibility of loss of facilities, such as occurs during natural disasters, multiple coverage layers in overlapping zones are required. The main resistance to this alternative is its cost. Ideal coverage can easily generate costs that make it impossible to implement.

The ideal is to locate the facilities so that they offer an exact number of coverage zones for all demands. However, we noted that offering an exact number of coverage zones is impractical in many contexts, as overlaps among coverage zones occur naturally, which causes an excess of coverage at different demand points. Thus, overlap control is required.

In this paper we propose a new mathematical model for the CLP to control overlaps of coverage zones. This new problem is called the Coverage Location Problem with Overlap Control (CLPOC). The main goal is to generate solutions that locate the facilities to meet different objectives. These objectives determine the strategic location of the extra coverage zones.

The main achievements obtained are the model’s capacity to:

• Generate new solutions that prioritize overlaps in regions with high demands in order to minimize the number of facilities;
• Generate new solutions that control overlaps in order to maximize the number of demands that have exactly the minimum number of coverage zones fixed a priori, a strategy that reduces the number of coverage zones, but usually increases the number of facilities per zone;
• Take results with the minimum number of coverage zones for each demand point and rerun the model to limit the number of facilities.

We organized this paper as follows. Section 2 describes related work in the literature. Section 3 describes the mathematical model proposed to the CLPOC. Section 4 describes the computational results and Section 5 concludes this paper.

2. Review of the literature

This section shows the papers related to the Coverage Location Problem with Overlap Control (CLPOC) that serve as the basis for the new model presented in this paper.

Daskin (1982) works with maximum coverage models for ambulances in both urban and rural contexts. The paper explains that previous models did not work with the problem efficiently because they underestimated the number of vehicles needed when they did not consider service time. Therefore, a real aspect was disregarded. Through a new coverage model, called CALL, which includes calculations of probabilities of using ambulances, they were able to make coverage more efficient, reducing the underestimation found. This model was later known as the first approach to the Backup Coverage Location Problem (BCLP).

However, Hogan and ReVelle (1986) were the first to introduce the term BCLP, therefore, they are the first to clearly formalize the provision of a second way of meeting demands. They provide a model for preventing service overload in high demand locations. The mathematical formulation proposed for this problem focuses on facilities that have high demand, since those are the most susceptible to infeasibility when a large number of calls occur. In this way, extra coverage is organized in places that may present service overload. The authors exemplify a case to show that the addition of extra coverage does not result in large operating costs.

Based on Hogan and ReVelle (1986), Doerner, Gutjahr, Hartl, Karall, and Reimann (2005) and Başar, Catay, and Ünlüyurt (2011), both papers present double coverage models for the location of emergency service facilities. In these models only high demand points are covered by two facilities. However, these approaches do not control the number of demand points with more than two coverage zones, which may increase operational costs to cover all demands.

Murray, Kim, Davis, Machiraju, and Parent (2007) and Jia, Ordóñez, and Dessouky (2007) show specific cases with applications of the backup model such as for ambulances, fire trucks and video security cameras. The optimal location of service locations and monitoring cameras was calculated in order to create backup coverage. Results showed substantial advantages in treating this context as a BCLP. The models improved the performance of the systems as well as optimized the allocated resources.

Farahani, Asgari, Heidari, Hosseininia, and Goh (2012) present a literature review on coverage problems, focusing on real applications. They sought to classify papers not only by the two main categories of CLP and the maximal coverage location problem. They also used features that show other extensions in coverage problems. The authors state that the growth of attention and interest in coverage problems is currently due to its application in real problems. However, the models developed have shown themselves to be far from real situations. In their conclusions, the authors also observed that several studies treated emergency facilities as service facilities, including papers about BCLP. Such papers do not consider the possibility of disasters or emergencies on a large scale, which would require the provisional allocation of facilities from other regions. For example, natural disasters such as hurricanes, floods, and tornadoes, tend to disable emergency facilities and also increase the demand for care.

Farahani, Hassan, Mousavi, and Baygi (2014) implement a hybrid artificial bee colony method to the maximum coverage location problem. The authors address the disruption of facilities service due to natural disasters. The goal is to maximize total coverage and maintain service when facility failures occur. Although the model meets the main demands, it is unable to control the extra coverage, leading to unnecessary extra coverage in low demand locations.

Bélanger, Ruiz, and Soriano (2015) study models for locating emergency services. The authors show the importance of a real-time monitoring system to respond as quickly as possible to large-scale emergencies caused by natural disasters.

Boonmee, Ariamura, and Asada (2017) present a facility location model for minimizing response time in natural disaster cases. The authors also present a review with models used to simulate adverse conditions and risk to human life scenarios. The study shows a gap of approaches for full demand coverage and efficient approaches to prepare existing facilities to cover incidents.

Pulver and Wei (2018) make a recent application of a backup model for drone services. The paper brings good results, but disregards the possibility of simultaneous loss of a set of drones. It is also limited to the region where the research was conducted, having a difficult generic application.

The review of the literature exposed the gaps that need an appropriate approach. Among the gaps are the limitations of the BCLP, which provides for a maximum of only two coverage zones in regions of greatest vulnerability. We find contexts that need a better strategy for locating facilities to offer more service alternatives and distribute demands appropriately. The review of the literature also exposed the need to prevent facility failures. Recent models that appear in the literature are unable to handle crisis situations efficiently, as happens during natural disasters, pandemics, or war.

Another problem exposed is the need to control extra coverage areas. Each new extra coverage area to meet a demand produces the need for new facilities in regions of vulnerability, but also in regions that do not need extra coverage. In a set of facilities, these unnecessary extra coverage zones can cause extra costs, depending on the application context.
3. The coverage location problem with overlap control

This paper proposes a new mathematical model for a coverage location problem in order to generate solutions that control unnecessary extra coverage. We call this variation the Coverage Location Problem with Overlap Control (CLPOC). The aim is to generate three types of solutions with the features presented in Table 1.

Fig. 1 demonstrates the advantages of our model. In Fig. 1, parts (a) and (b) represent solutions in which all demand points (represented by points) are covered by one facility (represented by “x”). In this representation, demand points can become facilities. The three facilities generate the coverage zones in blue, red and green. Part (a) shows that the overlaps between coverage zones are disorganized, occurring in regions where there are no demand points. Part (b) presents another solution, in which the intersections are concentrated in regions with a greater number of points. Note that the region with the greatest density is covered by the three coverage zones. Therefore, our purpose in this research is to find optimal solutions that strategically organize the coverage zones as in Fig. 1(b).

3.1. Definition and mathematical model

The CLPOC locates facilities to cover all demands with a fixed number of coverage zones, controlling the intensity of overlaps and the excess of coverage zones. The main objective depends on the interest of the user, who can choose two alternatives. In the first, the intensity of overlaps in regions with greater density of points is increased. Thus, the constraint of a fixed number of coverage zones is prioritized and the number of facilities is minimized. In the second, the intersections are minimized so that the largest number of demands is covered with the exact fixed number of coverage zones, which minimizes the excess of overlaps among coverage zones. The way to achieve these objectives will be presented below, as well as the mathematical model proposed in this paper.

The parameter set of the CLPOC model can be defined as:

- \( i, j \): demand points, where each point also can be a facility;
- \( d_i \): total demand of the points that are in the coverage radius of facility \( i \);
- \( S_i \): set of points that have a distance to point \( i \) less than the coverage radius;
- \( z \): a binary parameter, in which \( z = 0 \) represents the decrease in the number of overlaps in order to decrease the number of coverage zones and \( z = 1 \) represents the increase in the number of overlaps in regions with high demand points and the decrease in the number of facilities;
- \( a_{ij} \): a binary parameter that assumes the value 1 if point \( i \) has a distance less than the coverage radius to point \( j \) and 0 otherwise;
- \( b \): the minimum number of coverage zones that each point \( i \) must have;
- \( k \): the number of facilities.

The decision variables of the CLPOC model can be defined as:

- \( y_i \): a binary variable assuming value 1 if point \( i \) is a facility and assumes value 0 otherwise;
- \( u_i \): an unrestricted variable to control overlaps that defines the objective function according to the user-defined objective (\( z = 0 \) to decrease overlaps or \( z = 1 \) to increase overlaps);

Thus, the CLPOC can be modelled as follows:

\[
\max \sum_{i=1}^{n} (d_i y_i - u_i)
\]

subject to:

\[
\sum_{j=1}^{n} \left( \frac{|S_i \cap S_j|}{|S_i| + |S_j|} - z \cdot (y_i + y_j - 1) \right) \leq u_i, \forall i
\]

(2)

\[
\sum_{j=1}^{n} a_{ij} y_j \geq b, \forall i
\]

(3)

\[
\sum_{i=1}^{n} y_i = k, \quad \text{(optional constraint)}
\]

(4)

\( y_i \in [0, 1], \ u_i \in \mathbb{R} \) and unrestricted in sign

(5)

| Objective | Solution 1 | Solution 2 | Solution 3 |
|-----------|------------|------------|------------|
| Minimize number of facilities | X | X | |
| Set fixed number of facilities | X | | |
| Cover all demand points with \( b \) coverage zones | X | X | X |
| Prioritize high demands | X | | |

Table 1: Features of the different types of solutions found by the CLPOC.

Fig. 1. Example of solutions with overlap control within coverage zones.
The objective function (1) maximizes the total demand covered by a minimum of \( b \) coverage zones and penalizes the solution with the overlap control performed by the variables \( u_i \). Constraints in (2) determine the value of \( u_i \) and how to carry out this control. Some relevant components exist on which to base the calculation. Expression 6 provides the Jaccard coefficient (Jaccard, 1912).

\[
\frac{|S_i \cap S_j|}{|S_i \cup S_j|} \quad (6)
\]

Parameter \( S_i \) shows the set of points that a facility at point \( i \) can cover. Moreover, we pre-calculated distances between any two demand points. Afterwards, the formula compares these distances to a previously determined radius value. This radius value shows the maximum service range that a facility located at a point \( i \) can meet. If the distance between a pair of points \( i \) and \( j \) is less than this radius, a facility located at one of the two points may cover the service demand of the other point. The fraction of Expression 6 quantifies the proportion of overlaps between points \( i \) and \( j \). This quantification is represented by the number of points covered in common by \( i \) and \( j \). The more points in common, the closer to 1 the value of Expression 6 is.

Expression 7 adds the binary parameter \( z \) to Expression 6. When \( z = 0 \), the value of Expression 7 is directly proportional to the amount of overlaps between \( i \) and \( j \). When it assumes value 1, the value of Expression 7 is inversely proportional. Note that the value of \( z \) influences the calculation of \( u_i \) which in turn decreases the objective function. When \( z = 0 \), the goal is to minimize the number of overlaps, and for \( z = 1 \) the goal is to maximize the number of overlaps.

\[
\frac{|S_i \cap S_j|}{|S_i \cup S_j|} - z \quad (7)
\]

In both cases, the aim is to minimize the number of facilities, except when Constraint (4) is used. In addition to the summation in the objective function, there is the burden caused by the sum of Constraints (2). When \( b \) becomes a facility, this summation will result in a positive value. So, the higher the amount resulting from the calculation of Expression 8 between different points \( j \), the higher the value of \( u_i \).

\[
\frac{|S_i \cap S_j|}{|S_i \cup S_j|} - z \cdot (y_i + y_j - 1) \quad (8)
\]

In cases where \( i \) and \( j \) are not facilities, the result of \( u_i \) is negative. Therefore, in cases where the objective is to maximize overlaps and two points that can result in a large number of overlaps are not chosen as facilities, the objective function will be penalized. In cases where the objective is to minimize overlaps, the opposite occurs and the penalty is calculated if two points that cause few overlaps are not chosen as facilities. Cases where \( i \) and \( j \) represent the same points are disregarded in the calculation.

We conclude from this that when \( z = 0 \), we calculate the constraint of \( u_i \) to decrease total number of coverage zones (the sum of the coverage zones of all points) and to maximize the demand points that have a minimum number of coverage zones in order to optimize the number of coverage zones.

Constraints in (3) establish the minimum number of coverage zones that each demand point needs. Through this information, the conclusion taken from \( z = 1 \) is that we try to minimize the number of facilities and, simultaneously, to maximize the number of overlaps. Higher priorities are given to regions with greater density of demand points. As the objective is to satisfy (3) and \( z = 1 \) maximizes the overlaps, there is a reduction in the number of facilities when compared to \( z = 0 \), because \( z = 1 \) contributes to satisfy this constraint, requiring a smaller amount of facilities.

Constraint (4) is used optionally. It fixes the amount of facilities for contexts where there is a determined number of facilities. When we use it, our objective in this paper is to control overlaps to decrease the number of coverage zones, given that the greatest advantage is found in simulations of \( z = 0 \), when decreasing the number of coverage zones for a fixed number of facilities. In this paper, we present the results of the computational tests carried out with and without this constraint.

Constraints in (5) show the domain of variables \( y_i \) and \( u_i \). Variable \( y_i \) is binary whereas variable \( u_i \) can assume any real value, including negative values.

Fig. 2 exemplifies the application of the CLPOC model. In the example all demand points require two coverage zones (\( b = 2 \)). Part (a) illustrates the result of the application with \( z = 1 \), in which the increase of overlaps was prioritized and, consequently, the number of facilities was reduced. In Part (a), three facilities are used to cover all demand points and six of the 11 points are covered by exactly two coverage zones. Five points are covered by the three coverage zones. Part (b) presents a solution with \( z = 0 \), where overlaps are avoided to increase the number of points with exactly \( b \) coverage zones. In this case, four facilities are used to cover all points and ten points are covered by two coverage zones.

4. Computational results

This section presents the computational results of the CLPOC model using the IBM ILOG CPLEX 12.6 (I.I. CPLEX, 12.6) solver. The tests were carried out with max-covering instances by simulating regions of the city of São José dos Campos - Brazil (instances with 324, 402, 500, 708
and 818 points) (Arakaki & Lorena, 2001) and by simulating regions of the Bronx and Manhattan - USA (instances with 2713 and 3839 points) (Máximo, Nascimento, & Carvalho, 2017). These instances are available at https://sites.google.com/site/antoniochaves/publications/data. We performed all computational tests on a computer with an Intel Xeon E5-1620 processor with 3.70Gz and 32 GB of memory, and Linux operational system.

4.1. Definition of scenarios

Instances have demand information for each point. We performed two simulations, one that included different values of demands and another with equal demands. Thus, we solved two types of problems found in the literature: points with different priorities and points with same priorities (demands).

The demands for the instances in the city of the São José dos Campos were defined by Arakaki and Lorena (2001). In this case, the demands for the instances with 2713 and 3839 points were randomly generated with integers in the range [1, 700] with normal distribution with parameters \( \mu = 1 \) and \( \sigma^2 = 0 \). The demand values were normalized according to the maximum values of the variable \( u_i \) in tests with different demands to achieve a balance between meeting demands and controlling overlaps. For computational tests with equal demand, the value of \( d_i = 1 \) was used.

We also performed simulations varying the \( z \) value of the CLPOC model:

- \( z = 0 \Rightarrow \) decreasing the number of overlaps
- \( z = 1 \Rightarrow \) increasing the number of overlaps.

The number of coverage zones, \( b \), was analyzed also in the computational tests. We tested the CLPOC model with one, two and four extra coverage zones. Thus, it is possible to analyze how the model behaves with a greater number of coverage zones.

In the CLPOC model, we can set the number of facilities (\( k \)) by the use of Constraint (4). Thus, we performed tests applying the number of facilities obtained in the tests with \( z = 1 \). We set the parameter \( k \) of the CLPOC model with these values and set \( z = 0 \) to decrease the number of overlaps. This approach combines the decreases in the number of facilities and coverage zones.

For each combination of instances and number of coverage zones (value of \( b \)), there is a minimum radius value for which a feasible solution was found. In this study, we calculated this radius value in order to use the smallest possible value to obtain feasible solutions. Therefore, we calculated the smallest radius value that still finds a solution where all demand points are covered by \( b \) coverage zones.

Furthermore, in this study, we considered that all demand points (set of nodes) may become facilities. If we used a smaller radius value in tests of \( b = 1 \) than in tests of \( b = 2 \), some demand points would have to become facilities because they would not have another option to cover their demands. Therefore, we use the same radius values for these tests allowing all demand points to be covered by at least one other point. For tests of \( b = 4 \) we have larger radius values.

In short, the problem context simulations combine cases of equal and different demands with the variation of the values of \( z \) and \( b \). The CLPOC model is also performed with and without Constraint (4). Therefore, we have 126 tested instances with different scenarios.

4.2. Acronyms in the tables and graphs

Tables 1 show the results of the CLPOC model. The analysis of the results of the computational tests is presented by graphs in Figs. 3–8. The acronyms for the computational results presented by the CLPOC model are:

- FO: objective function value;
- \( z \): the number of facilities given by a solution;
- TCP: Average number of coverage zones for each demand point;
- PM(%): Percentage of points with minimum coverage;
- TCF: Average number of coverage zones covered by each facility;
- TC: the sum of the number of coverage zones of each point;
- \( k \): the number of facilities fixed in the CLPOC model;
- T(s): Time (seconds) for CPLEX to find an optimal solution.

The instance names consist of three numbers: the first number represents the minimum number of coverage zones required by each demand point, the second number represents the number of demand points and the third number represents the value of the radius. For example, in instance 1,324,214 the value 1 means that we performed the test with the parameter \( b = 1 \), the instance has 324 demand points and the value of the radius is 214. For the instance name, radius values were rounded to integers. Their real values are shown in the tables (R column).

4.3. Computational results

The computational results of the CLPOC model are presented in Tables 1. We grouped the results by types of solutions. Tables 2 and 3 show the results for \( z = 1 \) with equal and different demands, respectively. Tables 4 and 5 show the results for \( z = 0 \) with equal and different demands, respectively. Tables 6 and 7 show the results for a fixed number of facilities by using Constraint (4) with equal and different demands, respectively.

All computational tests performed by CPLEX with default parameters to solve the proposed CLPOC model found the optimal solutions for the tested instances in a reasonable computational time.

The results of computational time demonstrated a greater difficulty in finding results for the cases where \( z = 0 \). The reason for this occurrence was that the objective function was encumbered as coverage zones increased and Constraint (3) required a minimum amount of coverage zones. The conflict between these goals made the problem more difficult than in cases of \( z = 1 \). For this case, the objective function benefited from the addition of overlaps, because it was compatible with the purpose of the constraint.

Computational times for most instances with equal demands were greater than those with different demands. The objective of inserting various demands was to combine the objective of controlling the overlaps and meeting the high demands. So, the objective function also benefited from the high demands and not just the overlaps. The problem of meeting the high demands was easier than the problem of controlling the overlaps. When it combined them, cases that ignored the number of overlaps arose due to the benefit given by the demand, making the problem easier.

Computational times were shorter in tests with a fixed number of facilities. In the case of these tests, there was no need to calculate the number of facilities, only to control overlaps, which may have contributed to the decrease in computational time.

Note that tests with instances with 3839 points generally had the longest computational times. Therefore, this set of instances presented the contexts that CPLEX had the greatest difficulty in returning the optimal solution. The problem dimension combined with the size of the radius and specific characteristics such as differences in distances between points, may have influenced these results.

Note that the results for \( b = 2 \) present computational times longer than the results for \( b = 4 \) in instances of 3839 points, demonstrating a greater difficulty in finding the optimal solution in problems with two coverage zones than in problems with four coverage zones.

4.4. Analyses of solutions of the CLPOC

The graph in Fig. 3 shows the evolution of the facility reduction percentage in each instance. The percentage value means by how much
the number of the facilities was reduced for tests in cases with \( z = 1 \) in comparison to tests with \( z = 0 \).

In terms of the number of facilities, the results for equal demands and \( z = 1 \) were better, meeting our expectations. The second set of results with better performance is the one for different demands and \( z = 1 \), satisfying expectation since the objective was to balance the decrease of facilities with the percentage of demands met. Since the demands for service may require a considerably greater number of coverage zones for specific points, the solutions position the set of coverage zones in order to meet this goal.

We observed that the highest percentages of facility reduction occurred when the constraint on the number of coverage zones was \( b = 1 \). The problem with \( b = 1 \) is that it created a larger universe of solutions than the context of \( b = 2 \) and \( b = 4 \). Increasing the minimum number of coverage zones also increases the difficulty of preventing unnecessary overlaps, such that solutions of \( z = 0 \) and \( z = 1 \) drew closer. Similarly, increasing the value of \( b \) decreased the range of solutions and made the problem more restrictive.

We also saw a growth of this percentage in the instances with 3839 points. The reason for this occurrence was that when the coverage zones were spaced at \( z = 0 \), there was a tendency to create facilities to fill the constraint of parameter \( b \). The more facilities need to be produced the farther the points are. Thus, it was concluded that instances with 3839 points have a greater dispersion of their points than the other instances, making the difference in the number of facilities of \( z = 0 \) and \( z = 1 \) larger.

The graph in Fig. 4 shows the results of the average of coverage zones at each demand point. The results show that the tests with \( z = 1 \) produced a greater average of coverage zones at each of the demand points. These results show that these solutions prioritize offering a greater excess of coverage zones in regions with a greater density of demand points. Therefore, the prioritization of these regions shows a strategic organization of coverage zones so that these regions have a better distribution of their demands.

Note that there is an increase in the average coverage zones according to the increase in the value of the \( b \) parameter, which is expected. However, there is also an average balance even when the number of points varies. Therefore, there is a tendency to maintain the same average of coverage zones, even when the point values vary.

There are some differences between the values of equal and different demands. These values are expected, since there is a shift from coverage zones to points with higher demand values.

Fig. 5 shows the percentage of points with minimum coverage. We obtained this value by observing how many points were covered with the minimum amount of coverage required.

The results of Fig. 5 show an expected behavior, where the results

![Fig. 3. Graph with results of evolution of the facility reduction percentage.](image1)

![Fig. 4. Graph with results of average of coverage zones at each demand point.](image2)
Fig. 5. Graph with results of percentage of points with minimum number of coverage zones.

Fig. 6. Graph with results of average of demand points covered by each facility.

Fig. 7. Graph of the percentage of points with the minimum amount of coverage zones, showing the comparison between a fixed number of facilities and tests with $z = 1$. 

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for equal demands and \( z = 0 \) proved better than the others in terms of number of coverage zones. The control of overlaps at \( z = 0 \) decreased them at each point, to obtain a lower total number of coverage zones. In contrast the use of different demands led to the displacement of the coverage to places of higher demand, resulting in an increase in the total number of coverage zones to meet the greater demands. There was a control of overlaps in the results of \( z = 0 \), since values in terms of the best solutions found were above 60%.

As we increased the minimum number of coverage zones required, the percentage decreased, which demonstrates a greater difficulty in controlling overlaps. However, the proportion of overlaps remained stable, without significant variations when there was an increase in the number of points. We conclude that the problem of controlling the overlaps becomes more difficult with the required coverage increase and that the number of points does not interfere with the percentage.

Fig. 6 shows results of the average of demand points covered by each facility. This graph shows a lower average of points covered by the facilities in tests with \( z = 0 \). This result indicates a lesser amount of work spent by the facilities, since each of them has a smaller number of demands to meet. Therefore, depending on the application, this result may indicate a lesser amount of necessary resources per facility.

The results also show an increase in the average according to the increase in the parameter \( z \), which is expected. However, there is an increase in the average according to the number of points. The results demonstrate that the number of facilities is not dependent on the number of points of the problem, since instances with 2713 points needed a lower number of facilities than instances with 818 points, for example. Therefore, the number of facilities is dependent on other characteristics of the problem. We can observe that the average increases as the number of facilities is lower and the number of coverage zones is greater.

Fig. 7 shows the results for the percentage of points with minimum coverage. In this graph, we compare the values obtained in tests with the \( z = 1 \) and then by a fixed number of facilities.

This new set of tests demonstrated that there was a relevant drop in the number of coverage zones when we fixed the number of facilities,
bringing a significant gain. In the previous case, there was an objective of decreasing the number of coverage zones or the number of facilities. In the new situation, we dedicated the formulation to optimizing only the overlaps with the number of facilities already decreased. This new set of results obtained the benefit of combining the two objectives.

Fig. 8 shows results of the average of demand points covered by each facility, comparing results for initial tests with $z = 1$ and with a fixed number of facilities. Fig. 8 also shows gains, since there is a drop in the average of points covered by each facility. Therefore, in addition to a decrease in the number of facilities, these solutions also control overlaps in order to decrease the amount of work that may be required from the facilities. We observed that, for this scenario, we achieved a decrease in facilities before the reduction in the number of coverage zones, balancing the two goals.

5. Conclusion

This paper presented a new mathematical model for the Coverage Location Problem with Overlap Control (CLPOC), by adding and controlling overlaps among coverage zones to optimize the number of facilities and, simultaneously, concentrate the extra coverage zones caused by the overlaps in regions with high density of demand points. The new mathematical model also controls overlaps between coverage zones to optimize the number of coverage zones for each demand to decrease the work done by each facility. We performed another type of test, when we fixed the number of facilities given by the previous results and performed tests to optimize the number of coverage zones to combine the advantages presented by the two previous tests.

Results were given for one coverage zone (total point coverage), as well as two and four coverage zones. Results were also given for the two scenarios with equal demands and different demands. The results show the benefits of the proposed model in terms of reducing the number of facilities and controlling overlaps.

### Table 3
Results of the computational tests for $z = 1$ and different demands

| Instances | Names | R  | FO  | F   | TCP | PM(%) | TCF | TC  | T(s) |
|-----------|-------|----|-----|-----|-----|------|-----|-----|-----|
| 1,324,224 | 224.83| 812.8418 | 35  | 1.24 | 77.78% | 11.49 | 402 | 0.10 |
| 1,402,216 | 216.30| 1219.9596 | 49  | 1.23 | 78.61% | 10.08 | 494 | 0.12 |
| 1,500,239 | 239.07| 1915.5285 | 58  | 1.20 | 80.20% | 10.33 | 599 | 0.19 |
| 1,708,348 | 348.07| 43575.24  | 41  | 1.19 | 81.07% | 20.63 | 846 | 0.76 |
| 1,818,348 | 348.07| 580878.66 | 49  | 1.28 | 72.62% | 21.33 | 1045 | 3.09 |
| 1,2713,312 | 312.76| 5579124.00 | 5  | 1.38 | 62.15% | 748.00 | 1151 | 3.09 |
| 2,324,224 | 224.83| 59421.81  | 71  | 2.36 | 71.91% | 10.77 | 765 | 0.11 |
| 2,402,216 | 216.30| 8471.83   | 98  | 2.39 | 66.92% | 9.81 | 961 | 0.07 |
| 2,500,239 | 239.07| 135949.10 | 117 | 2.30 | 73.40% | 9.84 | 1151 | 0.30 |
| 2,708,348 | 348.07| 736539.48 | 82  | 2.39 | 65.68% | 20.61 | 1690 | 0.94 |
| 2,818,348 | 348.07| 506632.32 | 96  | 2.36 | 69.32% | 20.11 | 1931 | 3.06 |
| 2,2713,312 | 312.76| 5568058.29 | 10 | 2.58 | 61.26% | 701.30 | 2940 | 4.24 |
| 2,3839,157 | 157.84| 19959252.55 | 57 | 2.57 | 56.03% | 173.39 | 9883 | 862.98 |
| 4,324,280 | 280.22| 41699.75  | 99  | 4.62 | 55.86% | 15.12 | 1497 | 0.63 |
| 4,402,277 | 277.03| 65150.72  | 121 | 4.50 | 60.45% | 14.97 | 1811 | 1.01 |
| 4,500,304 | 304.31| 102751.05 | 152 | 4.51 | 60.80% | 14.84 | 2256 | 4.43 |
| 4,708,486 | 486.58| 361259.48 | 92  | 4.66 | 52.26% | 20.61 | 1690 | 0.94 |
| 4,818,436 | 436.43| 4419.00   | 132 | 4.74 | 50.73% | 29.35 | 3874 | 1.64 |
| 2,3839,157 | 157.84| 5409197.31 | 20 | 6.17 | 45.26% | 16729 | 3.61 |
| 4,3839,229 | 229.76| 1310747.72 | 48 | 5.52 | 36.23% | 411.58 | 21196 | 6.20 |
instances with the same and different demands. This paper also used the smallest radius of service that still made it possible to find a feasible solution to the problem.

We selected sets of instances with different sizes for the computational tests. The CPLEX solver was able to solve the CLPOC model for instances with up to 3839 points.

The results of the application of the CLPOC model proved to be satisfactory, returning the optimal solution for all tests. In general, computational times were low for most tests. However, tests with minimization of the number of coverage zones and equal demands presented the longest times.

The CLPOC model demonstrated efficiency by increasing overlaps, optimizing the number of facilities and, simultaneously, concentrating the extra coverage zones caused by the overlaps in regions with high density of demand points.

The CLPOC model also demonstrated efficiency by decreasing overlaps, optimizing the number of coverage zones and the percentage of points with minimum coverage.

The results for a fixed number of facilities were also efficient, which leads us to recommend the use of this alternative: optimizing the number of facilities first and then using this number to solve this problem again to optimize the number of coverage zones. This approach combines the two advantages presented by the previous tests.

Future studies will attempt to integrate the constraint of overlaps in formulations of the Probabilistic Maximal Covering Location-allocation Problem (Marianov & Serra, 1998). This problem has constraints to maintain service quality in facilities by, for example, controlling maximum service time. We plan to apply the Adaptive Biased Random-key Genetic Algorithm (A-BRKGA) metaheuristic (Chaves, Gonçalves, & Lorena, 2018) for contexts where the commercial solver cannot find results.

### Table 5

Results of the computational tests for \( z = 0 \) and different demands.

| Instances | Results |
|-----------|---------|
| Names     | R       | FO | F  | TCP PM(%) | TCF | TC | T(s) |
| 1,324,224 | 224.83  | 2180.26 | 37 | 1.09 | 91.36% | 9.51 | 352 | 0.19 |
| 1,402,216 | 216.30  | 2441.87 | 52 | 1.11 | 89.05% | 8.60 | 447 | 0.78 |
| 1,500,239 | 229.07  | 2795.76 | 59 | 1.13 | 87.40% | 9.54 | 563 | 1.57 |
| 1,708,348 | 348.07  | 9978.74 | 43 | 1.14 | 86.44% | 18.74 | 806 | 2.74 |
| 1,818,348 | 348.07  | 11477.91 | 50 | 1.12 | 88.14% | 18.32 | 916 | 20.18 |
| 1,2713,312 | 312.76 | 1768262.87 | 5 | 1.10 | 91.63% | 588.00 | 2940 | 4.14 |
| 3,369,157 | 157.84  | 563671.83 | 30 | 1.13 | 87.08% | 144.50 | 4335 | 648.03 |
| 2,324,224 | 224.83  | 1692.85  | 72 | 2.19 | 82.10% | 9.88 | 711 | 0.54 |
| 2,402,216 | 216.30  | 1822.65  | 98 | 2.21 | 81.34% | 9.06 | 888 | 0.92 |
| 2,500,239 | 239.07  | 2067.68  | 119 | 2.21 | 81.20% | 9.29 | 1106 | 1.05 |
| 2,708,348 | 348.07  | 8931.01  | 83 | 2.19 | 77.54% | 18.66 | 1549 | 45.57 |
| 2,818,348 | 348.07  | 10235.98 | 99 | 2.20 | 80.32% | 18.15 | 1797 | 1552.76 |
| 2,2713,312 | 312.76 | 1765697.77 | 10 | 2.32 | 71.51% | 629.80 | 2940 | 3.86 |

### Table 6

Results of the computational tests for fixed number of facilities and equal demands.

| Instances | Results |
|-----------|---------|
| Names     | R       | k   | FO | TCP PM(%) | TCF | TC | T(s) |
| 1,324,224 | 224.83  | 35  | 2196.33 | 1.12 | 90.03% | 10.37 | 363 | 0.19 |
| 1,402,216 | 216.30  | 49  | 2460.01 | 1.15 | 90.50% | 9.43 | 462 | 0.62 |
| 1,500,239 | 239.07  | 58  | 2818.82 | 1.14 | 88.91% | 9.83 | 570 | 0.54 |
| 1,708,348 | 348.07  | 41  | 9963.30 | 1.14 | 87.70% | 19.71 | 808 | 1.52 |
| 1,818,348 | 348.07  | 49  | 11477.91 | 1.11 | 89.00% | 18.55 | 909 | 10.43 |
| 1,2713,312 | 312.76 | 5   | 1765976.37 | 1.08 | 91.63% | 588.00 | 2940 | 2.22 |
| 2,324,224 | 224.83  | 71  | 1727.05 | 2.22 | 82.21% | 10.11 | 718 | 0.19 |
| 2,402,216 | 216.30  | 98  | 1876.34 | 2.21 | 81.59% | 9.43 | 887 | 0.69 |
| 2,500,239 | 239.07  | 117 | 2123.56 | 2.21 | 82.43% | 9.34 | 1103 | 0.33 |
| 2,708,348 | 348.07  | 82  | 8931.01 | 2.19 | 82.06% | 18.89 | 1549 | 8.76 |
| 2,818,348 | 348.07  | 96  | 10235.98 | 2.20 | 80.93% | 18.72 | 1797 | 20.98 |
| 2,2713,312 | 312.76 | 10  | 1768581.15 | 2.32 | 71.51% | 629.80 | 6298 | 81.93 |
| 2,3839,157 | 157.84 | 5   | 1765976.37 | 1.08 | 91.63% | 588.00 | 2940 | 2.22 |
| 4,324,280 | 280.22  | 99  | 1965.82 | 4.62 | 67.85% | 14.74 | 1433 | 0.38 |
| 4,402,277 | 277.03  | 119 | 1876.34 | 4.61 | 57.44% | 15.58 | 1854 | 8.76 |
| 4,500,304 | 304.31  | 151 | 2123.56 | 4.39 | 60.62% | 14.52 | 2193 | 4.57 |
| 4,708,486 | 486.58  | 92  | 15125.07 | 4.41 | 69.75% | 33.91 | 3120 | 15.44 |
| 4,818,436 | 436.43  | 132 | 13504.18 | 4.39 | 71.48% | 27.22 | 3593 | 11.92 |
| 4,2713,330 | 330.58 | 20  | 1888032.25 | 5.03 | 49.80% | 682.85 | 13657 | 2.76 |
| 4,3839,229 | 229.76 | 48  | 1435644.48 | 4.62 | 54.81% | 369.27 | 17506 | 20.67 |
Table 7
Results of the computational tests for fixed number of facilities and different demands.

| Instances | Results |
|-----------|---------|
| Names     | R       | k   | FO   | TCP | PM(%) | TCF  | TC   | T(s)  |
| 1,324,224 | 224.83  | 35  | 2177.96 | 1.13 | 91.26% | 10.46 | 366  | 0.09  |
| 1,402,216 | 216.30  | 49  | 2432.75 | 1.15 | 88.72% | 9.47  | 464  | 0.39  |
| 1,500,239 | 239.07  | 58  | 2787.63 | 1.14 | 87.15% | 9.84  | 571  | 0.56  |
| 1,708,348 | 348.07  | 41  | 9663.30 | 1.14 | 86.31% | 19.71 | 808  | 1.21  |
| 1,818,348 | 348.07  | 49  | 11477.91| 1.12 | 88.14% | 18.67 | 915  | 13.32 |
| 1,2713,312| 312.76  | 5   | 176282.87| 1.08 | 91.63% | 588.00| 2940 | 1.97  |
| 1,3639,157| 157.84  | 29  | 596600.23| 1.15 | 84.96% | 152.58| 4425 | 75.60 |
| 2,324,224 | 224.83  | 71  | 1689.34 | 2.22 | 81.93% | 10.11 | 718  | 0.24  |
| 2,402,216 | 216.30  | 98  | 1822.65 | 2.21 | 81.34% | 9.06  | 888  | 0.54  |
| 2,500,239 | 239.07  | 117 | 2056.11 | 2.24 | 80.75% | 9.58  | 1121 | 1.04  |
| 2,708,348 | 348.07  | 82  | 8994.54 | 2.21 | 77.23% | 19.07 | 1564 | 3.85  |
| 2,818,348 | 348.07  | 96  | 10169.78| 2.23 | 79.80% | 18.99 | 1823 | 21.08 |
| 2,713,312 | 312.76  | 10  | 1765697.77| 2.32 | 71.51% | 629.80| 6298 | 2.13  |
| 2,3839,157| 157.84  | 57  | 535562.07| 2.15 | 70.03% | 144.91| 8260 | 144.17|
| 4,324,384 | 320.22  | 99  | 1930.12 | 4.59 | 52.82% | 367.02| 17617| 16.15 |
| 4,402,277 | 277.03  | 119 | 2207.09 | 4.47 | 66.92% | 15.10 | 1797 | 6.09  |
| 4,500,304 | 304.31  | 151 | 2464.86 | 4.39 | 68.03% | 14.54 | 2195 | 6.09  |
| 4,708,486 | 486.58  | 92  | 15254.48| 4.38 | 67.23% | 33.71 | 3101 | 10.96 |
| 4,818,436 | 436.43  | 132 | 1502.65 | 4.39 | 68.24% | 27.23 | 3594 | 9.27  |
| 4,2713,330| 330.58  | 20  | 1898750.96| 5.02 | 56.76% | 680.75| 13615| 2.27  |
| 4,3839,229| 229.76  | 48  | 1447737.96| 4.59 | 52.82% | 367.02| 17617| 16.15 |

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