Model measurement based identification of Francis turbine vortex rope parameters for prototype part load pressure and power pulsation prediction

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Abstract. Pressure and power fluctuations of hydro-electric power plants in part-load operation are an important measure for the quality of the power which is delivered to the electrical grid. It is well known that the unsteadiness is driven by the flow patterns in the draft tube where a vortex rope is present. However, until today the equivalent vortex rope parameters for common numerical 1D-models are a major source of uncertainty.

In this work, a new optimization-based grey box method for experimental vortex rope modelling and parameter identification is presented. The combination of analytical vortex rope and test rig modelling and the usage of dynamic measurements allow the identification of the unknown vortex rope parameters. Upscaling from model to prototype size is achieved via existing non-dimensional parameters. In this work, a new experimental setup and system identification method is proposed which are suitable for the determination of the full set of part load vortex rope parameters in the lab. For the vortex rope, a symmetric model with cavity compliance, bulk viscosity and two pressure excitation sources is developed and implemented which shows the best correspondence with available measurement data. Due to the non-dimensional parameter definition, scaling is possible. This finally provides a complete method for the prediction of prototype part-load pressure and power oscillations.

Since the proposed method is based on a simple limited control domain, limited modelling effort and also small modelling uncertainties are some major advantages. Due to the generality of the approach, a future application to other operating conditions such as full load will be straightforward.

1. Introduction

Important measures for the quality of the power which is delivered to the electrical grid are pressure and power fluctuations of hydro-electric power plants in part-load operation. In order to foresee suitable means against power oscillations at the prototype plant in an early project phase, these oscillations have to be estimated long time before erection. It is well known from literature (see e.g. [1]-[3]) that direct upscaling of pressure or power time evolutions from model measurements is not possible. The reason is basically the interaction of the turbine and especially the draft tube including the part-load vortex rope with the overall waterway system. In other words, fluctuations may look differently on different test rigs even for the same runner at the same operating point.

Thus, a physical model for the draft tube including the vortex rope which is scalable from model to prototype size has to be derived. Hence, the underlying physical phenomena have to
be modelled and understood in a first step. In the second step, these parameters are applied to a numerical simulation model of the prototype power plant which finally delivers the expected pressure and power oscillations.

Clearly, the simulation results rely on the modelling quality of the underlying physical phenomena. Even if CFD analysis became a powerful widely used approach in the last few decades, the complexity and non-stationarity of the two-phase draft tube flow is still a challenge which requires experimental verification. Another drawback is the fact that low frequency power pulsations as appearing in hydro-electric power plants at part-load operation are typically coupled to the full system dynamics including waterways, drive shaft and generator dynamics. The prototype coverage of these effects would require the coupling of CFD models with some lumped piping and generator models which is subject to ongoing research.

Hence, in this work, a more direct and practical approach is followed by applying a newly developed grey box approach for draft tube parameter identification. The dynamical model structure of the overall system is obtained from first principles and from some well known considerations (see e.g. [2]). Since power fluctuations are an integral and global quantity, a spatial 1D approach is used for the hydraulic modelling of the overall system (see [4, 5]). Draft tube models for this 1D approach are widely spread in the literature [1]. A set of scalable non-dimensional parameters is presented in [6] taking into account compressibility and dissipation. Similar considerations are performed in [7, 8].

However, even if a suitable model structure has been found, the unknown parameter values still have to be identified. This is finally achieved on the basis of suitable dynamical model measurements on the test rig in the lab. Basics on the identification of general dynamic systems are given e.g. in [9]. In the present paper, model structure and multiple measurements are combined into a new and straight forward parameter identification procedure.

The paper is organized as follows. Section 2 illustrates the mathematical modelling approach of the overall system which is used for the identification. This includes the test rig itself as well as the unknown vortex rope model. The test rig behavior itself is assumed to be known. On the basis of the experimental results which are outlined in section 3, the identification scheme is treated in section 4. The results are applied to a prototype project in section 5 and summarized in section 6.

2. System modelling: Test rig and vortex rope

A rough sketch of the existing test rig at the VOITH Hydro facilities at Heidenheim is given in Fig. 1. Dynamic pressure measurements are available at upstream pipe works, at the spiral case, in the draft tube area, and at the downstream tank.

It is well known that the vortex rope in the draft tube which is present in part-load operating conditions is able to excite the overall hydraulic system leading to significant pressure and power oscillations. This typically appears with the Rheingans frequency which turns out to be roughly constant at 20% to 40% of runner rotational frequency for all part-load operating conditions [12]. These conditions are typically present in the range of 60% to 85% of best efficiency discharge (BED). Thus, forced-response type of oscillations are assumed. Hence, there is consensus in the literature (see e.g. [1]-[3]) that an appropriate modelling approach is given by a harmonic pressure source in the draft tube. In addition, the cavitation volume of the vortex rope acts as spring between the water inertia in head and tailrace. Clearly, since the size of the cavity compliance varies with the setting of the hydraulic machine, the same is true for the dynamic behaviour of the overall system.

Beside the cavity compliance, some damping is introduced by the bulk viscosity taking into account the energy dissipation in the compression and decompression of the oscillating vortex rope.

In the past, equivalent electrical schemes became a common representation for hydro-acoustic
power plant modelling. The scheme for the considered control volume of the test rig is shown in Fig. 2. There are basically two pressure taps available in the upstream section: $H_{\text{exc},1}$ at the upper boundary of the control volume and $H_{\text{m},1}$ close to the spiral case. Two pressure taps are located in the downstream section: $H_{\text{exc},0}$ at the tailwater tank which is the lower control volume boundary and $H_{\text{m},0}$ in the draft tube region. The pressure source $H_T(Q, d\gamma)$ accounts for the net head of the Francis turbine which is a nonlinear function of discharge $Q$ and gate opening $d\gamma$. Due to the fast electrical drive system, the rotational speed can be considered constant for the model test.

Remark 1: The pressure tap $H_{\text{m},0}$ in the draft tube cone is regarded as virtual sensor since physical measurements cannot be used directly. This is due to the swirling draft tube flow. Hence, in the cone and draft tube region, at least 2 sensors (with 180° phase shift) have to be installed to compute the synchronous pressure component. At the draft tube elbow local effects arise in the pressure field due to the bend and the transition to a rectangular cross section. Therefore, the measured pressure at the draft tube elbow $H_{\text{elbow}}$ is very noisy and not considered in the following.

Remark 2: The headwater tank has been completely filled with water and free surface is present in the tailwater tank during the measurement campaign. The overall hydraulic circuit has a length of roughly 50 m. By considering a wave speed in the piping system of the test rig of about 1000 m/s a first natural frequency of 10 Hz is estimated if no vortex rope cavitation is
An upper bound for the Rheingans frequency is given by 5.3 Hz if the runner is rotating at 800 rpm. Thus, interaction between vortex rope excitation and compressible modes of the waterways is not expected and the effects due to wave speed are neglected. This model order reduction leads to a reduced numerical complexity. The resulting model is shown in Fig. 2. This assumption holds only for the test rig. For prototype power plants this assumption is in general not valid. Here, compressibility effects have to be considered for investigations.

Finally, a model for the vortex rope is required within the overall system model. For the model of the vortex rope several different model structures have been proposed in literature in the past. Common approaches are shown in Fig. 3a and 3b. Similar models have been fitted to measurement data in Dörfler [1]. More detailed models can be found in Alligné [10, 11] where the author takes care of the spatial distribution of the vortex rope and its parameters. Due to the practical considerations and the complexity of parameter identification recent publications (see e.g. [7, 8]) tend to step back to the lumped modelling approach as shown in Fig. 3a and 3b. In this work a combination of both simple models yielding a symmetric model is proposed which is illustrated in Fig. 3c. It offers an additional degree of freedom in terms of magnitude and phase of two symmetrically distributed pressure sources. Thus, also phase shifts of the vortex rope internal excitation model are covered. This feature has been proven to be essential for the replication of the full system response in both head race and tail race system and a good measurement matching is finally obtained.

System identification and parameter estimation is typically carried out using system input and output measurement data. This allows the calculation of corresponding transfer functions or parametric system models. If the system is excited by the vortex rope, there are two drawbacks. Firstly, the excitation sources \( H_{VR,1} \) and \( H_{VR,0} \) are fictitious model properties which are neither physical nor measurable. Hence, one has to deal with a system with unknown inputs. Secondly, the rope excitation pattern has narrow band behaviour. Ideally, only one single data point in the frequency domain will be observed. Therefore, one is not able to estimate the vortex rope parameters \((C_c\text{ and } R_c)\) and source terms \( (H_{VR,1} \text{ and } H_{VR,0}) \) independently. To overcome this drawback additional known and controllable excitation sources are applied. In [7, 8] a controllable rotating valve is proposed which allows the injection of unsteady discharge into the upstream system. Thus, the system dynamics are excited in a broad frequency range. Drawbacks of this solution are the requirement of major test rig modifications, multiple measurements for multiple frequencies, and an additional energy source for charging a storage element. Moreover, due to the T-flange in the inlet section, the flow into the turbine might be slightly disturbed which causes undesired effects for common standard investigation. In the present paper an alternative approach is chosen. Since the wicket gates are automatically adjustable from the test rig automation system, some artificial wicket gate movements are utilized as external excitation.
source. In this case the test rig modifications are minor. Only the actual movements have to be recorded by an additional dynamic wicket gate position transducer.

For the test rig model structure as shown in Fig. 4, the system governing equations read as follows. The evaluation of the head race pressure loop yields

\[ \begin{align*}
\frac{L_{11} + L_{12}}{L_{11} + L_{12}} \dot{Q}_1 + \left( R_{11} + R_{12} + R_c + \frac{\partial H_T}{\partial Q_1} \right) |_{OP} Q_1 - R_c Q_0 + H_c = H_{exc,1} - \frac{\partial H_T}{\partial d\gamma} |_{OP} d\gamma - H_{VR,1} \end{align*} \]

(1)

and evaluation of the tailrace pressure loop leads to

\[ \begin{align*}
\frac{L_{01} + L_{02}}{L_{01} + L_{02}} \dot{Q}_0 + (R_{01} + R_{02} + R_c) Q_0 - R_c Q_1 - H_c = -H_{exc,0} - H_{VR,0}. \end{align*} \]

(2)

The cavity compliance behavior is described by

\[ C_c \dot{H}_c - Q_1 + Q_0 = 0. \]

(3)

The controlled system input is given by the wicket gate opening \( d\gamma \). \( H_{VR,1} \) and \( H_{VR,0} \) denote additional input quantities, which are unknown as mentioned before. The pressure values \( H_{exc,1} \) and \( H_{exc,0} \) at the control volume boundaries are also considered as inputs, because they are measured during the tests. Therefore, equations (1) - (3) represent a full system description in state space including the unknown vortex rope parameters \( C_c \) and \( R_c \). The state and input vector are defined by \( x^T := [Q_1 \quad Q_0 \quad H_c] \) and \( u^T := [H_{exc,1} \quad H_{exc,0} \quad d\gamma \quad H_{VR,1} \quad H_{VR,0}] \), respectively. In this context, all quantities are relative variables denoting deviations from the stationary operating point.

Under consideration of two additional pressure loops the measurable output variables

\[ H_{m,1} = H_{exc,1} - R_{11} Q_1 - L_{11} \dot{Q}_1 \]

(4)

and

\[ H_{m,0} = H_{exc,0} + R_{02} Q_0 + L_{02} \dot{Q}_0 \]

(5)

are defined. For evaluation of equations (4) and (5) the time derivatives are substituted by equations (1) - (3) in a straightforward manner.

By defining the system output vector \( y^T := [H_{m,1} \quad H_{m,0}] \) a multi-input multi-output (MIMO) descriptor representation

\[ \begin{align*}
E \dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{align*} \]

(6)

is obtained. The matrices \( E, A, B, C, D \) depend on the unknown vortex rope parameters \( C_c \) and \( R_c \). The challenge in the subsequent sections will be the estimation of the unknown parameters and inputs from available test rig measurement data.
3. Experimental investigations

In the part load operation dynamical test sequences have been carried out for different cavitation numbers \( \sigma \). A representative set of measurement quantities is depicted in Fig. 5. To obtain a possibly broad-band easy-to-design excitation pattern, a pseudo random binary sequence (PRBS) has been applied to the wicket gate position as shown in Fig. 5a. Since the test rig was initially not designed for dynamic measurements, there have to overcome some drawbacks. First of all, the wicket gate ramping speed is rather low since precision is more important than performance for common applications. Another issue is the distinct hysteresis dead zone behaviour which allows changes in moving direction only with recognizable time delay. Due to these circumstances the band width of excitation frequencies is significantly reduced and hence the dynamic performance for system identification is negatively affected.

![Figure 5. Time signals of measured input and output quantities](image)

Regarding the dynamic pressure measurements, the signal quality is influenced by some technical constraints. The head race pressure tap was installed close to a 90° bend (cf. Fig. 1) which induced some strong turbulence and hence noise in the measurement signal is observed (cf. Fig. 5b). Further, the flow patterns in the spiral case region are rather smooth. Therefore, some correlations between pressure in spiral case and wicket gate position are clearly visible, see Figures 5a and 5c. Finally, due to the rotating vortex rope absolute values are rather low in the draft tube domain, cf. Figures 5d and 5e.

Thus, in order to obtain reliable parameters numerically advanced methods will be applied in the following.

4. Vortex rope parameter identification

For the identification of vortex rope parameters from available measurements and the development of a system model different schemes are possible. In the present case, the system is characterized by three known and two unknown input quantities, two output quantities and two unknown parameters. As a matter of fact, the estimation of excitation variables and system parameters simultaneously is a challenging task. Furthermore, there is some
interaction and coupling between all of these variables. Since the vortex rope system excitation is characterized by quite limited band width and will thus influence the system response spectrum only very locally a two step approach is chosen. In the first step, the influence of the unknown inputs (excitation sources) is neglected. This is reasonable for almost all frequencies since the controlled input (wicket gate position) is performing some sufficiently large magnitudes over a broad frequency range. Compared to this the influences from vortex rope pressure sources are negligible. The vortex rope parameters are determined such a way that excitation signal, response signals and the system transfer behaviour are matching each other in an optimal way. Therewith, the transfer behaviour from all input including the unknown vortex rope excitation to all output quantities is completely known.

In the second step, this transfer behaviour is used to calculate the remaining pressure sources of the vortex rope model. In order to match the independently moving water masses at penstock side and at tailrace side both pressure sources are obviously required.

First, the parameter-dependent transfer matrix

\[ G(j\omega, p) = C(j\omega E - A)^{-1}B + D \]  

is computed. Herein, \( p = [C_c, R_c] \) is the vector of unknown parameters.

Basically, the transfer matrix determines the relationship between input and output in the time domain. If the Fourier transforms of input and output vector are denoted by \( \mathbf{u}(j\omega) \) and \( \mathbf{y}(j\omega) \), the transfer matrix fulfills \( \mathbf{y}(j\omega) = G(j\omega, p)\mathbf{u}(j\omega) \). However, since \( \mathbf{u}(j\omega) \) and \( \mathbf{y}(j\omega) \) contain noise components of the measurement signals, an advanced approach is chosen. Instead of using Fourier transforms, auto and cross power spectral density functions are applied to the measurement signals. They can easily be computed as Fourier transform of auto and cross correlation functions. Regarding the single-input single-output case, the transfer function fulfills \( S_{yu}(j\omega) = G(j\omega, p)S_{uu}(j\omega) \) and \( S_{yy}(j\omega) = G(j\omega)S_{uy}(j\omega) \), respectively. This has to be extended to the MIMO case. Since a rectangular system with three inputs and only two outputs is present, the relation

\[ S_{yu}(j\omega) = G(j\omega, p)S_{au}(j\omega) \quad \rightarrow \quad G(j\omega) = S_{yu}(j\omega)S_{uu}^{-1}(j\omega) \]  

is utilized. Herein, the cross power spectral density is given by

\[ S_{yu}(j\omega) := \begin{bmatrix} S_{yu11}(j\omega) & \cdots & S_{yu1m}(j\omega) \\ \vdots & \ddots & \vdots \\ S_{yuq1}(j\omega) & \cdots & S_{yuqm}(j\omega) \end{bmatrix}, \quad m = 3, \quad q = 2. \]  

By using equation (8) a residual matrix is computed for each frequency and set of vortex rope parameters:

\[ \mathbf{R}(\omega, p) := S_{yu}(j\omega) - G(j\omega, p)S_{au}(j\omega). \]  

For the optimization process the cost functional

\[ J(p) := \sum_{\omega_{\min}}^{\omega_{\max}} \| R(\omega, p) \|_F^2. \]  

is calculated based on the residual matrices. Herein, \( \| \cdot \|_F \) denotes the Frobenius norm. In order to obtain optimal parameters \( p_0 \) the cost functional has to be minimized:

\[ J_0 = \min_p J(p) \Rightarrow p_0. \]
Remark 3: A numerical optimization algorithm will certainly converge to some values. However, for the vortex rope parameters $C_c$ and $R_c$, only positive values are meaningful. Thus, some penalty terms have been applied to convert the constrained optimization problem to an unconstraint problem.

When the unknown vortex rope parameters have been identified, the complete MIMO transfer matrix is known. Next, the unknown pressure sources are computed in a final step. Since a harmonic excitation is assumed, the approach

$$u_{VR}(t) = \hat{U}_{VR} \sin(\omega_{VR} t)$$

is utilized for the unknown vortex rope pressure sources. The unknown magnitude $\hat{U}_{VR}$ is a complex 2-dimensional vector representing both sources and the individual phase shifts. Since the excitation is neglected during calculation of the vortex rope parameters it is assumed that the deviation is largest for $\omega_{VR}$. Therefore, the frequency $\omega_{VR}$ is obtained from $\max_\omega \| R(\omega, p_0) \|$. For the determination of the unknown magnitude the above method using spectral density functions is applied for a second time. Since the normalized input function $\sin(\omega_{VR} t)$ is known, the excitation vector $\hat{U}_{VR}$ is computed by consideration of all system inputs at $\omega = \omega_{VR}$.

5. Application to Francis prototype project

The parameter identification method has been applied to a recently developed Voith Francis runner. Following [1, 6] some non-dimensional equivalents for $C_c$ and $R_c$ are computed by $C'_c := C_c H / D_4^2$ and $R'_c := R_c D_4 / N$ using net head $H$, rated speed $N$, and reference diameter $D_4$.

The results for the cavity compliance are depicted in Fig. 6. The Thoma number dependent curves are shown for different operating points (70% and 80% BEP discharge). Even though the excitation by the wicket gates turned out to be dynamically rather poor, the curves and especially their power trendlines are in a surprisingly good accordance with references from literature, e.g. the curve according Dörfler [1]. Recent publications (see e.g. [7, 8]) have shown that even for higher Thoma numbers the cavity volume does not vanish completely. This fact is also visible from the identified curves and was visually observed at the test rig, too.

The results of the identified bulk viscosity depending on Thoma number is shown in Fig. 7.

Finally, the pressure excitation magnitudes are illustrated in Fig. 8 which again show a clear Thoma number dependency. In Fig. 8 only the absolute value of $\hat{U}_{VR}$, normalized with the averaged net head are shown.

![Figure 6. Identified cavity compliance](image-url)
However, a more detailed view on the results shows that the magnitudes of both sources are almost identical and a 180° phase shift is present. This behavior matches well-known observations and typically leads to inversely phased oscillations of head water and tail water. This phenomena is not reproducible by one single pressure source as frequently proposed in the past.

6. Summary and Conclusion
A new experimental setup and system identification method has been proposed for the determination of the full set of part load vortex rope parameters in the lab. For the hydraulic system excitation, a predefined wicket gate motion pattern has been defined. Further, a symmetric model with cavity compliance, bulk viscosity and two pressure excitation sources has been developed and implemented for the vortex rope. A good correspondence with available measurement data is observed. Due to the non-dimensional parameter definition, scaling is possible and the parameters are applicable for prediction of pressure and power pulsations at the prototype.
Since the proposed method is based on a simple limited control domain, low modelling effort and also small modelling uncertainty are major advantages. Therefore, robustness and promising results are obtained even for weak excitation sources due to slowly moving actuators.

Numerically, some well-known system identification methods are applied resulting in the solution of a simple numerical optimization problem. Due to the generality of the approach, a future application to other operating conditions such as full load will be straightforward.

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