The solitary re-entrant superconductivity in the clean four-layered superconductor/ferromagnet system

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Abstract. The superconducting critical temperature $T_c$ for asymmetric four-layered system ferromagnet/superconductor/ferromagnet/superconductor ($F'/S/F/S'$) in the clean limit using the boundary value problem for Eilenberger function is investigated. An electron-electron coupling constant in ferromagnetic metals and pair amplitude changes along the F/S interfaces are taken into account. It is shown that 0- and $\pi$ - phase superconducting states of pure thin $F'/S/F/S'$ fourlayers are controlled by the magnitude and sign of electron correlations in the F and F' layers, as well as by the competition between homogeneous Bardeen-Cooper-Schrieffer (BCS) pairing and inhomogeneous Larkin-Ovchinnikov-Fulde-Ferrell (LOFF) pairing. A solitary re-entrant superconductivity for the 00$\pi$ state is predicted. The results of numeric calculations allow to explain the absence of the suppression of three dimensional superconductivity in short period Gd/La superlattices.

1. Introduction
One of unconventional superconducting correlations of electrons different from BCS pairing is pairing through the Larkin-Ovchinnikov-Fulde-Ferrell (LOFF) mechanism with a nonzero 3D momentum of pairs $k \neq 0$ [1, 2], which can appear in ferromagnet/superconductor (F/S) nanostructures. The appearance of this states leads to the nonmonotonic dependencies of the critical temperature $T_c$, electron density of states, and the Josephson current on the thickness $d_f$ of the F layers [3, 4]. The recently revealed absence of the suppression of three dimensional (3D) superconductivity in short period Gd/La superlattices [5, 6] constitutes a challenge for the existing one dimensional (1D) theory of the proximity effect [3, 4]. Superconductivity in F/S nanostructures usually occurs only if the F layer thickness $d_f$ is much smaller than the S layer thickness $d_s$. However, 3D superconductivity in a Gd/La superlattice not only exists at $d_f > d_s$, but appears at $T_c$ equal to the critical temperature of a bulk lanthanum sample. Based on obtained equations, our theory explains this suprising effect.

Provided the solutions of the Eilenberger equations in the F and S layers are known, the critical temperature $T_c$ may be found from the self-consistency equation for the pair potential $\Delta(r)$

$$\Delta_{s(f)}(r) = 2\lambda_{s(f)}(r)\pi T \text{Re} \sum_{\omega > 0} \langle \Phi_{s(f)}(p, q_{s(f)}, z, \omega) \rangle,$$

where $\lambda_{s(f)}$ is the electron-electron interaction parameter, $\omega = \pi T(2n + 1)$ is the Matsubara frequency, the prime means a cutoff at the Debye frequency $\omega_D$, $T$ is the temperature, and $\hbar = k_B = \mu_B = 1$ is taken hereinafter.
The Eilenberger function $\Phi(p, q, z, \omega)$ can be separately defined for layers F and S using following differential equations

$$
2\omega_s(f) - v_{sz}(f)\xi_{sz}(f) \frac{\partial^2}{\partial z^2} \Phi_{s(f)}(p, q_s(f), z, \omega) = 2\Delta_s(f)(q_s(f), z),
$$

(2)

$v_{sz}(f)$ and $\xi_{s(f)}(z)$ are the $z$ components of the Fermi velocity and correlation length, respectively, which are related as

$$
\xi_{sz} = \frac{v_{sz}}{2\omega_s}; \tilde{\omega}_s = 2\omega + iq_sv_{s\perp}
$$

(3)

for superconducting layer and

$$
\xi_{fz} = \frac{v_{fz}}{2\omega_f}; \tilde{\omega}_f = 2\omega + i(2I + q_fv_{f\perp})
$$

(4)

for ferromagnet layer, respectively. The subscripts $s$ and $f$ mark the parameters and functions in the S and F layers. The boundary conditions at the planar F/S contact $z = 0$, where the 2D projection of the Fermi momentum $p$, is conserved have the form

$$
(1 - \sigma(z))\frac{\partial \Phi(p, q, z, \omega)}{\partial z} \bigg|_{z = \pm 0} = \frac{\sigma}{2} \left[ \Phi(p, q, +0, \omega) - \Phi(p, q, -0, \omega) \right],
$$

(5)

where $\sigma$ is transparency, $\xi$ is correlation length.

The results of this work develop the theory of proximity effect of recent works [7, 8, 9] for F/S/F' trilayers on the asymmetrical four-layered system. We also take into account the magnitude and sign of the electron-electron interaction in a ferromagnet and spatial variations of the Eilenberger function along F/S contact.

2. Four-layered system

Let us consider a system consisting of four layers with different thickness: two pure ferromagnets F and F' filling the regions $-d_f < z < 0$ and $d_s < z < d_s + d'_f$ and two layers of a pure BCS superconductors S and S' with areas $0 < z < d_s$ and $-d_f - d'_s < z < -d_f$, respectively. The boundary conditions at the F/S contact $z = 0$ for the case of ideal transparency have the form

$$
\Phi_s(p, q_s, +0, \omega) = \Phi_f(p, q_f, -0, \omega) ;
$$

$$
\xi_{sz} \frac{\partial \Phi_s(p, q_s, z, \omega)}{\partial z} \bigg|_{z = 0}^{+0} = \xi_{fz} \frac{\partial \Phi_f(p, q_f, z, \omega)}{\partial z} \bigg|_{z = 0}^{-0},
$$

(6)

The boundary conditions at the S/F' contact $z = d_s$ have the form

$$
\Phi_s(p, q_s, d_s - 0, \omega) = (\pm) \Phi_{f'}(p, q_f, d_s + 0, \omega) ;
$$

$$
\xi_{sz} \frac{\partial \Phi_s(p, q_s, z, \omega)}{\partial z} \bigg|_{z = d_s}^{-0} = (\pm) \xi_{fz} \frac{\partial \Phi_{f'}(p, q_f, z, \omega)}{\partial z} \bigg|_{z = d_s}^{+0},
$$

(7)

The last boundary conditions at the S'/F contact $z = -d_f$ have the form

$$
\Phi_f(p, q_f, -d_f + 0, \omega) = [\pm] \Phi_{s'}(p, q_s, -d_f - 0, \omega) ;
$$

$$
\xi_{fz} \frac{\partial \Phi_f(p, q_f, z, \omega)}{\partial z} \bigg|_{z = -d_f}^{+0} = [\pm] \xi_{sz} \frac{\partial \Phi_{s'}(p, q_s, z, \omega)}{\partial z} \bigg|_{z = -d_f}^{-0},
$$

(8)
The “+” sign in the boundary conditions corresponds to the usual 0-phase superconductivity with the coinciding phases of the order parameters $\Delta_f$ and $\Delta'_f$, which are nonzero despite the overwhelming majority of preceding theories [3, 4] for the F/S systems, in the F and F' layers or $\Delta_s$ and $\Delta'_s$ in the S and S' layers, respectively. The lower sign “-” corresponds to the nontraditional $\pi$-phase superconductivity in which the transition through the layer is accompanied with a change in the sign of the order parameter $\Delta_f$ or $\Delta_s$ [10]. Thus, we have four different states: (00), $(0\pi)$, ($\pi 0$) and $(\pi\pi)$. First symbol relates to order parameter in superconductors, the second symbol refers to the ferromagnetic layers.

We seek for the solutions of Eqs. (2) in the respective layers in the form that excludes fluxes through the outer boundaries of the contact, i.e.,

$$\Phi_s = \frac{\Delta_{s0}}{\omega_s} + A \frac{\text{ch}(z - d_s/2)/\xi_{sz}}{\text{ch}(d_s/2\xi_{sz})} + C \frac{\text{sh}(z - d_s/2)/\xi_{sz}}{\text{sh}(d_s/2\xi_{sz})}, \quad 0 < z < d_s,$$

$$\Phi_f = \frac{\Delta_{f0}}{\omega_f} + B \frac{\text{ch}(z + d_f/2)/\xi_{fz}}{\text{ch}(d_f/2\xi_{fz})} + D \frac{\text{sh}(z + d_f/2)/\xi_{fz}}{\text{sh}(d_f/2\xi_{fz})}, \quad -d_f < z < 0,$$

$$\Phi'_s = \frac{\Delta_{s0}}{\omega'_s} + A \frac{\text{ch}(z + d'_s + d_f)/\xi_{sz}}{\text{ch}(d'_s/\xi_{sz})}, \quad -d'_s + d_f < z < -d_f,$$

$$\Phi'_f = \frac{\Delta_{f0}}{\omega'_f} + B \frac{\text{ch}(z - d_s - d'_f)/\xi'_{fz}}{\text{ch}(d'_f/\xi'_{fz})}, \quad d_s < z < (d_s + d'_f).$$

Figure 1. Phase diagrams (lower panel) $T_c(d_f)$ and (upper panel) $q(d_f)$ for the 00$\pi$ states of F'/S/F/S' fourlayers. The solid lines correspond to the lines of the transition to the BCS states with $q = 0$. Other lines are the LOFF states with $q \neq 0$. In the region of the competition between BCS and LOFF states on the wings of the peaks, the realized LOFF state has a higher $T_c$ value than the BCS state. The thickness of layers are following: a) $d'_f = 0.2d_s$, $d'_s = d_s$; b) $d'_f = 0.2d_s$, $d'_s = 0.2d_s$; c) $d'_f = 0.6d_s$, $d'_s = d_s$; d) $d'_f = 0.8d_s$, $d'_s = 0.2d_s$; e) $d'_f = d_s$, $d'_s = d_s$; g) $d'_f = d_f$, $d'_s = d_s$;
The constants $A$, $B$, $B'$, $C$ can be defined from boundary conditions (6), (7) and (8). Owing to the collectivization of electron correlations and the paramagnetic effect of the exchange field, the mutual effect of $F$, $S$, $F'$ and $S'$ metals is particularly strong in the Cooper limit when their thicknesses are small, $d_{f(s)} \ll \xi_{f(s)}$, $a_f$, where $\xi_{f(s)} = v_{f(s)}/2\pi T$ is the coherence length and $a_f = v_f/2I$ is the spin stiffness length. The set of equations on these constants have following form

$$\frac{\Delta s_0}{\omega_s} + A - C = \frac{\Delta f_0}{\omega_f} + B + D,$$

$$-A \frac{d_s}{2\xi_{sz}} + C \frac{2\xi_{sz}}{d_s} = B \frac{df}{2\xi_{fz}} + D \frac{2f_{fz}}{df},$$

$$A \frac{d_s}{2\xi_{sz}} + C \frac{2\xi_{sz}}{d_s} = (\pm) \left( \frac{\Delta f_0}{\omega_{f'}} + B' \right),$$

$$A \frac{d_s}{2\xi_{sz}} + C \frac{2\xi_{sz}}{d_s} = (\pm) \left( -B' \frac{df'}{\xi_{fz}} \right),$$

$$A \frac{d_s}{2\xi_{sz}} + C \frac{2\xi_{sz}}{d_s} = (\pm) \left( \frac{\Delta f_0}{\omega_s} + A' \right),$$

$$A \frac{d_s}{2\xi_{sz}} + C \frac{2\xi_{sz}}{d_s} = \left[ \pm \right] A' \frac{d_s}{\xi_{sz}}.$$

Then, we solve the set of Eqs.(10). Due to the Eilenberger functions depend on the polar angles in velocity space, the analysis of the obtained relations for the critical temperature is very complicated. But in case of component equality of the Fermi velocity (the same electronic structure of metals in the nanocontact) equations are strongly simplified by using the summation rule with the digamma function $\Psi$.

As a result, taking into account the two possible combinations of the exchange field $I$ in ferromagnets, we obtain eight different states $\alpha, \alpha \beta$, where the meaning of the first two symbols is explained above, and $\beta$ indicates the parallel (0) or antiparallel ($\pi$) orientation of the magnetizations of the neighboring $F$ layers.

Let us analyze the case of (00$\pi$) as the most interesting for an experimental realization, because $\pi$-magnetic states have higher critical temperature then 0-magnetic states as it was shown for four-layered system $F/S/F/S$ [11] and for superlattices [12]. Simplified equation ($v_s = v_f$) has form:

$$\ln t = \frac{(c_f + c'_f)(\lambda_f - \lambda_s)}{\lambda_s((c_s + c'_s)\lambda_s + (c_f + c'_f))} + \Psi \left( \frac{1}{2} \right) - \Re \int \frac{d\Omega}{4\pi} \psi \left( \frac{1}{2} + \frac{\Gamma}{4\pi T_{cs}t} \right),$$

$$\Gamma = 2i \left( \frac{1}{2} (c_s + c'_s) q_s v_{s,\perp} + (c_f - c'_f)(I + \frac{1}{2} q_f v_{f,\perp}) \right),$$

$$t = \frac{T_c - T_{cs}}{T_{cs}}.$$

where $T_{cs}$ is the critical temperature of the isolated $S$ layer, $\Psi(x)$ is the digamma function and $c_s$ in case of similar electronic structure are the relative weights of layers, i.e $c_s = d_s/(d_f + d'_f + d_s + d'_s)$. For certainty 2D momenta of LOFF pairs $q_f$ and $q_s$ are taken to be identical ($q_f = q_s = q$) [7].

The exchange fields $I$ and $-I$ of the neighboring $F$ layers in symmetric $F/S/F/S$ fourlayers with $d_f = d'_f$, $(c_f = c'_f)$ and $d_s = d'_s$, $(c_s = c'_s)$ completely compensate each other in $\Gamma$. Then, from the condition of the maximum of $T_c$, we obtain $q = \Gamma = 0$ and the critical temperatures $T_c$ is determined only by the first terms in Eqs.(11), i.e

$$\ln t = -\frac{(c_f + c'_f)(\lambda_s - \lambda_f)}{\lambda_s(c_f + c'_f)(\lambda_s - \lambda_f)}.$$
Figure 2. Phase diagrams (lower panel) \( T_c(d_f/d_s) \) and (upper panel) \( q(d_f) \) for the 00\( \pi \) states of \( F'/S/F/S' \) at \( \lambda_s = \lambda_f = 0.2 \). The thickness of layers are following: a) \( d'_f = 0.2d_s, d'_s = d_s \); b) \( d'_f = 0.2d_s, d'_s = 0.2d_s \); c) \( d'_f = 0.6d_s, d'_s = d_s \); d) \( d'_f = 0.8d_s, d'_s = 0.2d_s \); e) \( d'_f = d_s, d'_s = d_s \); g) \( d'_f = d_f, d'_s = d_s \).

It depends on the difference between the parameters of electron-electron interactions \( \lambda_f \) and \( \lambda_s \).

The diagrams of state \( T_c(d_f/d_s) \) for \( \lambda_s = 0.3 \) and \( \lambda_s = 0.2 \), which are calculated from Eq.(11), consist of separate peaks with reentrant superconductivity. These peaks are characterized by the LOFF-BCS-LOFF competition for various \( d_f/d_s \) values are shown in the lower panel in Fig. 1. The \( g \)-line is the curve of the maxima of \( T_c \) that is calculated by the formula in Eqs.(12) and this line describes \( T_c \) behavior of the symmetric F/S/F/S four-layered system with \( d_f = d'_f \) and \( d_s = d'_s \). The upper panel in Fig. 1 shows the corresponding \( q(d'_f/d_s) \) dependences in the units of \( a_f^{-1} \).

According to Fig. 1, the LOFF superconducting state with nonzero momentum of pairs \( (q \not= 0) \) appears in the fourlayer at small \( T_c \) and the transition of the \( F'/S/F/S' \) contact to the BCS state with \( q = 0 \) and pair amplitude oscillating in the interface plane becomes favorable with an increase in \( T_c \).

A similar set of peaks of \( T_c(d_f/d_s) \) with reentrant superconductivity and the LOFF-BCS-LOFF competition at \( \lambda_s = \lambda_f = 0.2 \) calculated from the equation in Eqs.(11) is shown in the lower panel of Fig. 2. The peaks possess the same height because the line of the maxima of \( T_c(d_f/d_s) \) for the \( \pi \)-phase superconductivity, which corresponds to the symmetric F/S/F/S fourlayers, is determined from relation ln \( t = 0 \), as it follows from Eqs.(12).

The existence of such states with \( \pi \)-phase magnetism explains the absence of the suppression of 3D superconductivity in short period Gd/La superlattices in experimental works [5] and [6]. The critical temperature \( T_c \) measured for this superlattices was 5 K under cooling in zero field; this value almost coincides with the critical temperature of bulk lanthanum! This means that the 00\( \pi \) state with \( \lambda_f = \lambda_s \) occurs in the Gd/La superlattices.

Solutions obtained above for four-layered system were verified for the limiting case when
Figure 3. The limit transition into trilayer system at \( d_s' = 0 \). The thickness of layers are following: a) \( d_f' = 0.04d_s \); b) \( d_f' = 0.12d_s \); c) \( d_f' = 0.2d_s \).

\( d_s' = 0 \), (Fig. 3). We have the complete agreement with results obtained recently for trilayer F/S/F’ [9]. Here first “0” symbol indicates the sign constancy of the superconducting order parameter at the transition through the S layer, and the symbol “\( \pi \)” relates to antiparallel configuration of the exchange fields in ferromagnets. As expected, set of diagrams has the form of solitary peaks with re-entrant superconductivity and competition LOFF-BCS-LOFF. We obtained interesting result: analytical solutions (11) do not coincide with previous expressions from [9], but the numerical solutions demonstrate the similar plots.

References

[1] Larkin A I and Ovchinnikov Yu N 1964 Zh. Eksp. Teor. Fiz. 47 1136
[2] Fulde P and Ferrell R A 1964 Phys. Rev. A 135 550
[3] Izyumov Yu A, Proshin Yu N and Khusainov M G 2002 Usp. Fiz. Nauk 172 113
[4] Buzdin A I 2005 Rev. Mod. Phys. 77 935
[5] Goff J P, Deen P P, Ward R C et al. 2002 J. Magn. Magn. Mater. 240 592
[6] Deen P P, Goff J P, Ward R C et al. 2005 J. Phys.: Condens. Matter 17 3305
[7] Khusainov M G, Khusainov M M, Ivanov N M and Proshin Yu N 2009 JETP Letters 89 626
[8] Khusainov M G, Khusainov M M, Ivanov N M and Proshin Yu N 2009 JETP Letters 90 124
[9] Khusainov M G, Khusainov M M, Ivanov N M and Proshin Yu N 2009 JETP Letters 90 359
[10] Bulaevskii L N, Kuzii V V and Sohyanin A A 1977 JETP Lett. 25 290
[11] Proshin Yu N, Zimin A, Fazleev N G and Khusainov M G 2006 Phys. Rev. B 73 184514
[12] Proshin Yu N, Izyumov Yu A and Khusainov M G 2001 Phys. Rev. B. 64 064522