Precision Electroweak Parameters and the Higgs Mass

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Abstract. The status of various precisely measured electroweak parameters is reviewed. Natural relations among them are shown to constrain the Higgs mass, $m_H$, via quantum loop effects to relatively low values. A comparison with direct Higgs searches is made.

FUNDAMENTAL PARAMETERS AND PRECISION MEASUREMENTS

The SU(2)$_L \times$ U(1)$_Y$ electroweak sector of the standard model contains 17 or more fundamental parameters. They include gauge and Higgs field couplings as well as fermion masses and mixing angles. In terms of those parameters, predictions can be made with high accuracy for essentially any electroweak observable. Very precise measurements of those quantities can then be used to test the standard model, at the quantum loop level and predict the Higgs scalar mass or search for small deviations from expectations which would indicate “New Physics”.

Some fundamental electroweak parameters have been determined with extraordinary precision. Foremost in that category is the fine structure constant $\alpha$. It is best obtained by comparing the measured [1] anomalous magnetic moment of the electron, 

$$a_e = \frac{g_e - 2}{2}$$

with the calculated 4 loop QED prediction [2]

$$a_e^{\text{exp}} = 1159652188(3) \times 10^{-12}$$

1) To be published in the Proceedings of MuMu99-5th International Conference on Physics Potential and Development of $\mu^\pm$-$\mu^-$ Colliders, San Francisco, CA, Dec. (1999)

2) This manuscript has been authored under contract number DE-AC02-98CH10886 with the U.S. Department of Energy. Accordingly, the U.S. Government retains a non-exclusive, royalty-free license to publish or reproduce the published form of this contribution, or allow others to do so, for U.S. Government purposes.
\[ a_e^{th} = \frac{\alpha}{2\pi} - 0.328478444 \left( \frac{\alpha}{\pi} \right)^2 + 1.181234 \left( \frac{\alpha}{\pi} \right)^3 - 1.5098 \left( \frac{\alpha}{\pi} \right)^4 + 1.66 \times 10^{-12} \]  

where the \( 1.66 \times 10^{-12} \) comes from small hadronic and weak loop effects. Assuming no significant “new physics” contributions to \( a_e^{th} \), it can be equated with (1) to give

\[ \alpha^{-1} = 137.03599959(40) \]  

That precision is already very impressive. Improvement by a factor of 10 appears to be technically feasible [3] and should certainly be undertaken. However, at this time such improvement would not further our ability to test QED. Pure QED tests require comparable measurements of \( \alpha \) in other processes. Agreement between two distinct \( \alpha \) determinations tests QED and probes for “new physics” effects. After \( a_e \), the next best (direct) measurement of \( \alpha \) comes from the quantum Hall effect

\[ \alpha^{-1}(qH) = 137.03600370(270) \]  

which is not nearly as precise. Nevertheless, the agreement of (3) and (4) (at the 1.50 sigma level) is a major triumph for QED up to the 4 loop quantum level.

In terms of probing “new physics”, one can search for a shift in \( a_e \) by \( m_e^2/\Lambda_e^2 \) where \( \Lambda_e \) is the approximate scale of some generic new short-distance effect. Current comparison of \( a_e \to \alpha \) and \( \alpha(qH) \) explores \( \Lambda_e \lesssim 100 \) GeV. To probe the much more interesting \( \Lambda_e \sim \mathcal{O} \) (TeV) region would require an order of magnitude improvement in \( a_e \) and about two orders of magnitude error reduction in some direct precision determination of \( \alpha \) such as the quantum Hall effect. Perhaps the most likely possibility is to use the already very precisely measured Rydberg constant in conjunction with a much improved \( m_e \) determination to obtain an independent \( \alpha \).

The usual fine structure constant, \( \alpha \), is defined at zero momentum transfer as is appropriate for low energy atomic physics phenomena. However, that definition is not well suited for short-distance electroweak effects. Vacuum polarization loops screen charges such that the effective (running) electric charge increases at short-distances. One can incorporate those quantum loop contributions into a short-distance [4] \( \alpha(m_Z) \) defined at \( q^2 = m_Z^2 \). The main effect comes from lepton loops, which can be very precisely calculated, and somewhat smaller hadronic loops. The latter are not as theoretically clean and must be obtained by combining perturbative calculations with results of a dispersion relation which employs \( \mathcal{O}(e^+e^- \to \text{hadrons}) \) data. A detailed study by Davier and Höcker found [5]

\[ \alpha^{-1}(m_Z) = 128.933(21) \]  

where the uncertainty stems primarily from low energy hadronic loops. Although not nearly as precise as \( \alpha^{-1} \), the uncertainty quoted in (5) is impressively small and a tribute to the effort that has gone into reducing it. (When I first studied this issue in 1979, I crudely estimated [4] \( \alpha^{-1}(m_Z) \simeq 128.5 \pm 1.0 \).) However, the error in
(5) is still somewhat controversial, primarily because of its reliance on perturbative QCD down to very low energies. For comparison, an earlier study by Eidelman and Jegerlehner [6], which relied less on perturbative QCD and more on $e^+e^-$ data found

$$\alpha^{-1}(m_Z) = 128.896(90) \quad (E \& J \ 1995)$$  \hspace{1cm} (6)

That estimated uncertainty is often cited as more conservative and therefore sometimes employed in $m_H$ and “new physics” constraints. As we shall see, the smaller uncertainty in (5) has very important consequences for predicting the Higgs mass. I note that a more recent study [7] by Eidelman and Jegerlehner finds

$$\alpha^{-1}(m_Z) = 128.913(35) \quad (E \& J \ 1998)$$  \hspace{1cm} (7)

which is in good accord with (5) and also exhibits relatively small uncertainty. In my subsequent discussion, I employ the result in (5), but caution the reader that a more conservative approach would expand the uncertainty, perhaps even by as much as a factor of 4.

A related short-distance coupling, $\alpha(m_Z)_{MS}$, can be defined by modified minimal subtraction at scale $\mu = m_Z$. It is particularly useful for studies of coupling unification in grand unified theories (GUTS) where a uniform comparative definition ($MS$) of all couplings is called for [8]. The quantities $\alpha(m_Z)$ and $\alpha(m_Z)_{MS}$ differ by a constant, such that [9]

$$\alpha^{-1}(m_Z)_{MS} = \alpha^{-1}(m_Z) - 0.982 = 127.951(21)$$  \hspace{1cm} (8)

In weak interaction physics, the most precisely determined parameter is the Fermi constant, $G_\mu$, as obtained from the muon lifetime. One extracts that quantity by comparing the experimental value

$$\tau_\mu = 2.197035(40) \times 10^{-6} \text{s}$$  \hspace{1cm} (9)

with the theoretical prediction

$$\tau_\mu^{-1} = \Gamma(\mu \rightarrow \text{all}) = \frac{G_\mu^2 m_\mu^5}{192\pi^3} f \left( \frac{m_e^2}{m_\mu^2} \right) (1 + \text{R.C.}) \left( 1 + \frac{3}{5} \frac{m_e^2}{m_W^2} \right) f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$$  \hspace{1cm} (10)

In that expression R.C. stands for Radiative Corrections. Those terms are somewhat arbitrary in the standard model. The point being that $G_\mu$ is a renormalized parameter which is used to absorb most loop corrections to muon decay. Those corrections not absorbed into $G_\mu$ are explicitly factored out in R.C. For historical reasons and in the spirit of effective field theory approaches, R.C. has been chosen to be the QED corrections to the old V-A four fermion description of muon decay [10]. That definition is practical, since the QED corrections to muon decay in the
old V-A theory are finite to all orders in perturbation theory. In that way, one finds

\[ R.C. = \frac{\alpha}{2\pi} \left( \frac{25}{4} - \pi^2 \right) \left( 1 + \frac{\alpha}{\pi} \left( \frac{2}{3} \ln \frac{m_\mu}{m_e} - 3.7 \right) + \left( \frac{\alpha}{\pi} \right)^2 \left( \frac{4}{9} \ln^2 \frac{m_\mu}{m_e} - 2.0 \ln \frac{m_\mu}{m_e} + C \right) \cdots \right) \]  

(11)

The leading \( \mathcal{O}(\alpha) \) terms in that expression have been known for a long time from the pioneering work of Kinoshita and Sirlin [11] and Berman [12]. Coefficients of the higher order logs can be obtained from the renormalization group constraint [13]

\[ \left( m_e \frac{\partial}{\partial m_e} + \beta(\alpha) \frac{\partial}{\partial \alpha} \right) R.C. = 0 \]

\[ \beta(\alpha) = \frac{2}{3} \frac{\alpha^2}{\pi} + \frac{1}{2} \frac{\alpha^3}{\pi^2} \cdots \]  

(12)

The -3.7 two loop constant in parenthesis was recently computed by van Ritbergen and Stuart [14]. It almost exactly cancels the leading log two loop correction obtained from the renormalization group approach (or mass singularities argument) of Roos and Sirlin [13]. Hence, the original \( \mathcal{O}(\alpha) \) correction in (9) is a much better approximation than one might have guessed. Comparing (9) and (10), one finds

\[ G_\mu = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2} \]  

(13)

There have been several experimental proposals [15] to reduce the uncertainty in \( \tau_\mu \) and \( G_\mu \) by as much as a factor of 20. Such improvement appears technically feasible and, given the fundamental nature of \( G_\mu \), should certainly be undertaken. However, from the point of view of testing the standard model, the situation is similar to \( \alpha \). \( G_\mu \) is already much better known than the other parameters it can be compared with; so, significant improvement must be made in other quantities before a more precise \( G_\mu \) is required.

Let me emphasize the fact that lots of interesting loop effects have been absorbed into the renormalization of \( \frac{g_2^2}{4\sqrt{2}m_W^0} \) which we call \( G_\mu \). Included are top quark [16] and Higgs loop corrections [17] to the \( W \) boson propagator as well as potential “new physics” from SUSY loops, Technicolor etc. Even tree level effects of possible more massive gauge bosons such as excited \( W^{*\pm} \) bosons could be effectively incorporated into \( G_\mu \). To uncover those contributions requires comparison of \( G_\mu \) with other precisely measured electroweak parameters which have different quantum loop (or tree level) dependences. Of course, those quantities must be related to \( G_\mu \) in such a way that short-distance divergences cancel in the comparison.

Fortunately, due to an underlying global SU(2)\(_V\) symmetry in the standard model, there exist natural relations among various bare parameters [18]

\[ \sin^2 \theta_W^0 = \frac{e_0^2}{g_2^2} = 1 - \left( \frac{m_W^0}{m_Z^0} \right)^2 \]  

(14)
Each of those bare unrenormalized expressions contains short-distance infinities; however, because the theory is renormalizable, the divergences are the same. Therefore, those relations continue to hold for renormalized quantities, up to finite, calculable radiative corrections [18]. The residual radiative corrections contain very interesting effects such as $m_t$ and $m_H$ dependence as well as possible “new physics”. So, for example, one can relate
\[ G_\mu = \frac{\pi\alpha}{\sqrt{2}m_W^2(1 - m_W^2/m_Z^2)}(1 + \text{rad. corr.}) \]  
and test the predicted radiative corrections, if $m_Z$ and $m_W$ are also precisely known.

Gauge boson masses are not as well determined as $G_\mu$, but they have reached high levels of precision. In particular, the $Z$ mass has been measured with high statistics Breit-Wigner fits to the $Z$ resonance at LEP with the result [19]

\[ m_Z = 91.1871(21) \text{ GeV} \]  
(16)

That determination is so good that one must be very precise regarding the definition of $m_Z$. (Remember the $Z$ has a relatively large width $\sim 2.5$ GeV.) The quantity in (16) is related to the real part of the $Z$ propagator pole, $m_Z$ (pole), and full width, $\Gamma_Z$, by [20]

\[ m_Z^2 = m_Z^2(\text{pole}) + \Gamma_Z^2 \]  
(17)

The two mass definitions $m_Z$ and $m_Z$ (pole) differ by about 34 MeV, which is much larger than the uncertainty in (16). Hence, one must specify which definition is being employed in precision studies. I note, that the $m_Z$ in (16) is also more appropriate for use in low energy neutral current amplitudes.

In the case of the $W^\pm$ bosons, the renormalized mass, $m_W$, is similarly defined by

\[ m_W^2 = m_W^2(\text{pole}) + \Gamma_W^2 \]  
(18)

That quantity is obtained [21] from studies at $p\bar{p}$ colliders, $m_W = 80.448(62)$ GeV, as well as $e^+e^- \rightarrow W^+W^-$ at LEPII, $m_W = 80.401(48)$ GeV. Together they average to

\[ m_W = 80.419(38) \text{ GeV} \]  
(19)

The current level of uncertainty, $\pm 38$ MeV, is large compared to $\Delta m_Z$. It is expected that continuing efforts at LEPII and Run II at Fermilab’s Tevatron should reduce that error to about $\pm 25$ MeV. A challenging but worthwhile goal for future high energy facilities would be to push $\Delta m_W$ to $\pm 10$ MeV or better. At that level, all sorts of interesting “new physics” effects are probed. I note that the $m_W$ defined in (19) is also the appropriate quantity for low energy amplitudes such as muon decay.
Another important quantity for precision standard model tests is \( m_t \), the top quark mass. Measurements from CDF and DØ at Fermilab give \[21\]

\[
m_t(\text{pole}) = 174.3 \pm 5.1 \text{ GeV}
\] \hspace{1cm} (20)

Reducing that uncertainty further is important for quantum loop studies, as we shall subsequently see. Future Tevatron Run II efforts are expected to reduce the error in \( m_t \) to about \( \pm 2 \) GeV. LHC and NLC studies should bring it well below \( \pm 1 \) GeV.

In addition to masses, the renormalized weak mixing angle plays a central role in tests of the standard model. That parameter can be defined in a variety of ways, each of which has its own advocates. I list three popular examples \[4,22,23\]

\[
\sin^2 \theta_W(m_Z)_{\overline{MS}} \quad (\overline{MS} \text{ definition at } \mu = m_Z) \quad (a)
\]

\[
\sin^2 \theta_W^\text{eff} \quad (Z \mu \bar{\mu} \text{ vertex}) \quad (b)
\]

\[
\sin^2 \theta_W \equiv 1 - m_W^2/m_Z^2 \quad (c)
\]

They differ by finite \( \mathcal{O}(\alpha) \) loop corrections. The \( \overline{MS} \) definition is particularly simple, being defined as the ratio of two \( \overline{MS} \) couplings \( \sin^2 \theta_W(m_Z)_{\overline{MS}} = e^2(m_Z)_{\overline{MS}}/g_Z^2(m_Z)_{\overline{MS}} \). It was introduced for GUT studies \[8\], but is useful for most electroweak analyses. The effective, \( \sin^2 \theta_W^\text{eff} \), weak angle was invented for \( Z \) pole analyses. Roughly speaking, it is defined by the ratio of vector and axial-vector components (including loops) for the on-mass-shell \( Z \mu \bar{\mu} \) vertex \( \to 1-4 \sin^2 \theta_W^\text{eff} \). Although conceptually rather simple, analytic electroweak radiative corrections expressed in terms of \( \sin^2 \theta_W^\text{eff} \) are complicated and ugly. Numerically, it is close to the \( \overline{MS} \) definition \[22\]

\[
\sin^2 \theta_W^\text{eff} = \sin^2 \theta_W(m_Z)_{\overline{MS}} + 0.00028
\] \hspace{1cm} (22)

but the analytic structure of the difference is quite complicated. For those intent on employing \( \sin^2 \theta_W^\text{eff} \), a strategy might be to calculate radiative corrections in terms of \( \sin^2 \theta_W(m_Z)_{\overline{MS}} \) and then translate to \( \sin^2 \theta_W^\text{eff} \) via (22). But why not simply use \( \sin^2 \theta_W(m_Z)_{\overline{MS}} \)?

Currently, \( Z \) pole studies at LEP and SLAC give \[19\]

\[
\sin^2 \theta_W(m_Z)_{\overline{MS}} = 0.23091 \pm 0.00021
\]

\[
\sin^2 \theta_W^\text{eff} = 0.23119 \pm 0.00021
\] \hspace{1cm} (23)

That result includes measurements of the left-right asymmetry, \( A_{LR} \), at SLAC as well as the various lepton asymmetries at LEP and SLAC. The \( A_{LR} \) contribution has for some time given a relatively low value for the weak mixing angle. The latest \[19\] SLD result is

\[
\sin^2 \theta_W(m_Z)_{\overline{MS}} = 0.23073(28)
\] \hspace{1cm} (24)
Currently, the $Z \to b\bar{b}$ forward-backward asymmetry at LEP gives a higher $\sin^2 \theta_W^{\text{eff}}$ and, if included, would bring up the average. However, the $Zbb$ coupling appears to be somewhat anomalous and suggests problematic $b$ identification; so, one should be cautious when including such results in averages.

There are very good reasons to clarify and further improve $\sin^2 \theta_W (m_Z)_{\overline{MS}}$. One could imagine redoing $A_{LR}$ at a future polarized lepton-lepton ($e^+e^-$ or $\mu^+\mu^-$) collider, but with much higher statistics. In principle, one might reduce the uncertainty in $\sin^2 \theta_W^{\text{eff}}$ by a factor of 10 to $\pm 0.00002$, an incredible achievement if accomplished [23].

The so-called on-shell or mass definition [24] in (21c) also has its advocates. It can be directly obtained from $m_W$ and $m_Z$ determinations. Indeed, at hadron colliders, the ratio $m_W/m_Z$ can have reduced systematic uncertainties. One could imagine that the current uncertainty in

$$\sin^2 \theta_W = 1 - m_W^2/m_Z^2 = 0.2222 \pm 0.0007$$ (25)

might be reduced by a factor of about 4 at the LHC. Such a reduction is extremely important since the comparison of $\sin^2 \theta_W$ and $\sin^2 \theta_W (m_Z)_{\overline{MS}}$ provides a clean probe of “new physics”. It is also possible (because of a subtle cancellation of certain loop effects [25]) to measure $\sin^2 \theta_W$ more directly in deep-inelastic $\nu\mu N$ scattering. Indeed, a recent Fermilab experiment found [26]

$$\sin^2 \theta_W = 0.2255 \pm 0.0019 \pm 0.0010$$ (26)

where the first error is statistical and the second systematic. That single measurement is quite competitive with (25) and complements it nicely. One might imagine a future high statistics effort significantly reducing the error in (26), but that would require a new, intense, high energy neutrino beam.

Two other well measured electroweak parameters are the charged and neutral leptonic partial widths of the $Z$ boson [19]

$$\Gamma(Z \to \ell^+\ell^- (\gamma)) = 83.96 \pm 0.09 \text{ MeV}$$
$$\Gamma(Z \to \Sigma\nu\bar{\nu}) = 498.8 \pm 1.5 \text{ MeV}$$ (27)

The first of those, by definition, corresponds to $Z$ decay into massless charged leptons along with the possibility of inclusive bremsstrahlung. The second represents the inclusive invisible width of the $Z$.

All of the above precision measurements can be collectively used to test the standard model, predict the Higgs mass, and search for “new physics” effects. That ability stems from the natural relations in (14) and calculations [24,27] of the radiative corrections to them. Parametrizing those radiative corrections by $\Delta r$, $\Delta r(m_Z)_{\overline{MS}}$, and $\Delta r \hat{c}$, one finds [29]

$$\frac{\pi\alpha}{\sqrt{2}G_{\mu}m_W^2} = \left(1 - \frac{m_W^2}{m_Z^2}\right)(1 - \Delta r)$$ (a)
\[
\frac{\pi \alpha}{\sqrt{2} G_\mu m_W^2} = \sin^2 \theta_W(m_Z)_{\overline{MS}}(1 - \Delta r(m_Z)_{\overline{MS}}) \quad (b)
\]
\[
\frac{4\pi \alpha}{\sqrt{2} G_\mu m_Z^2} = \sin^2 2\theta_W(m_Z)_{\overline{MS}}(1 - \hat{\Delta} r)
\]

Those expressions contain all one loop corrections to \(\alpha\), muon decay, \(m_W\), \(m_Z\) and \(\sin^2 \theta_W(m_Z)_{\overline{MS}}\) and incorporate some leading two loop contributions. The quantities \(\Delta r\) and \(\Delta \hat{r}\) are particularly interesting because of their dependence on \(m_t\) and \(m_H\). In addition, all three quantities provide probes of “new physics”.

Numerically, all three radiative corrections in (28) contain a significant contribution from vacuum polarization effects [4] in \(\alpha\), about +7%. They are basically the same as the corrections that enter into the evolution of \(\alpha\) to \(\alpha(m_Z)\). Leptonic loops contribute a significant part of that effect and can be very accurately computed. Hadronic loops are less clean theoretically and lead to a common uncertainty in \(\Delta r, \Delta r(m_Z)_{\overline{MS}},\) and \(\Delta \hat{r}\) of

\[-\alpha \Delta \alpha^{-1}(m_Z)\]  

For \(\Delta \alpha^{-1}(m_Z) = 0.021\) as in (5), that amounts to a rather negligible \(\pm 0.00015\) error. However, for \(\Delta \alpha^{-1}(m_Z) = \pm 0.090\) as in (6), it increases to \(\pm 0.00066\). That large an uncertainty would impact precision tests. If one wishes to avoid that low energy hadronic loop uncertainty, dependence on \(\alpha\) can be circumvented by considering

\[
\sin^2 \theta_W(m_Z)_{\overline{MS}} = \left(1 - \frac{m_W^2}{m_Z^2}\right)(1 - \Delta r + \Delta r(m_Z)_{\overline{MS}})
\]

Currently, that comparison suggests a very light Higgs, but is not yet quite competitive with eq. (28c) in constraining \(m_H\). However, future improvements in \(m_W\) and \(m_t\) could make it very interesting.

Another useful relation that provides some sensitivity to \(m_H\) without \(\Delta \alpha\) uncertainties involves the partial \(Z\) width when written as [28]

\[
\Gamma(Z \to \ell^+ \ell^- (\gamma)) = \frac{G_\mu m_Z^3(1 - 4\sin^2 \theta_W(m_Z)_{\overline{MS}} + 8\sin^4 \theta_W(m_Z)_{\overline{MS}})}{12\sqrt{2}\pi(1 - \Delta r_Z(m_H))}
\]

Using \(m_t = 174.3 \pm 5.1\) GeV as input, one can compute the radiative corrections in (28) as functions of \(m_H\). Those results are illustrated in table 1. Note that \(\Delta r\) is most sensitive to changes in \(m_H\) but also carries the largest uncertainty from \(\Delta m_t = \pm 5.1\) GeV (\(\pm 0.0020\)). Hence, efforts to determine \(m_H\) from \(m_W\) are starting to require a better measurement of \(m_t\). On the other hand, determining \(m_H\) from \(\sin^2 \theta_W(m_Z)_{\overline{MS}}\) via \(\Delta \hat{r}\) is less sensitive to \(\Delta m_t\) but more sensitive to \(\Delta \alpha^{-1}(m_Z)\). Those dependences as well as \(\Gamma(Z \to \ell^+ \ell^- (\gamma))\) are illustrated by the following approximate relations [30] which are very accurate up to \(m_H \sim \mathcal{O}(400\) GeV)
\[ m_W = (80.385 \pm 0.032 \pm 0.003 \text{ GeV}) \left(1 - 0.00072\ln\left(\frac{m_H}{100 \text{ GeV}}\right) - 1 \times 10^{-4}n^2\left(\frac{m_H}{100 \text{ GeV}}\right)\right) \]

\[ \sin^2 \theta_W(m_Z)_{\overline{MS}} = (0.23112 \pm 0.00016 \pm 0.00006) \left(1 + 0.00226\ln\left(\frac{m_H}{100 \text{ GeV}}\right)\right) \]

\[ \Gamma(Z \to \ell^+\ell^- (\gamma)) = (84.011 \pm 0.047 \pm 0.0028 \text{ MeV})(1 - 0.00064\ln\left(\frac{m_H}{100 \text{ GeV}}\right) - 0.00026\ln\left(\frac{m_H}{100 \text{ GeV}}\right)) \]

where the errors correspond to \( \Delta m_t = \pm 5.1 \text{ GeV} \) and \( \Delta \alpha^{-1}(m_Z) = \pm 0.021 \) respectively. Note that increasing \( \Delta \alpha^{-1}(m_Z) \) to \( \pm 0.090 \) would significantly compromise the utility of \( \sin^2 \theta_W(m_Z)_{\overline{MS}} \) for determining \( m_H \) but have less of an impact on \( m_W \). Predictions for \( m_W \) and \( \sin^2 \theta_W(m_Z)_{\overline{MS}} \) are illustrated in table 2 for various \( m_H \) values [31].

**TABLE 1.** Values of \( \Delta r, \Delta r(m_Z)_{\overline{MS}}, \) and \( \Delta \hat{r} \) for various \( m_H \). A top quark mass of 174.3 \pm 5.1 \text{ GeV} \) and \( \alpha^{-1}(m_Z) = 128.933(21) \) are assumed.

| \( m_H \) (GeV) | \( \Delta r \) \( \pm 0.0020 \pm 0.0002 \) | \( \Delta r(m_Z)_{\overline{MS}} \) \( \pm 0.0001 \pm 0.0002 \) | \( \Delta \hat{r} \) \( \pm 0.0005 \pm 0.0002 \) |
|------------------|------------------------|------------------------|------------------------|
| 75               | 0.03402                | 0.06914                | 0.05897                |
| 100              | 0.03497                | 0.06937                | 0.05940                |
| 125              | 0.03575                | 0.06955                | 0.05974                |
| 150              | 0.03646                | 0.06964                | 0.06000                |
| 200              | 0.03759                | 0.06980                | 0.06042                |
| 400              | 0.04065                | 0.07005                | 0.06144                |

**TABLE 2.** Predictions for \( m_W \) and \( \sin^2 \theta_W(m_Z)_{\overline{MS}} \) for various \( m_H \) values.

| \( m_H \) (GeV) | \( m_W \) (GeV) | \( \sin^2 \theta_W(m_Z)_{\overline{MS}} \) |
|------------------|-----------------|-------------------|
| 75               | 80.401          | 0.23097            |
| 100              | 80.385          | 0.23112            |
| 125              | 80.372          | 0.23124            |
| 150              | 80.360          | 0.23133            |
| 200              | 80.341          | 0.23148            |
| 400              | 80.289          | 0.23184            |
Employing $m_W = 80.419(38)$ GeV, $\sin^2 \theta_W(m_Z)_{\overline{MS}} = 0.23091(21)$, and $\Gamma(Z \rightarrow \ell^+\ell^-(\gamma)) = 83.96(9)$ MeV one finds from eq (32)

$$m_H = 53^{+77}_{-40} \text{ GeV} \quad \text{(from } m_W \text{)} \quad (33)$$
$$m_H = 67^{+145}_{-27} \text{ GeV} \quad \text{(from } \sin^2 \theta_W(m_Z)_{\overline{MS}}\text{)} \quad (34)$$
$$m_H = 208^{+340}_{-180} \text{ GeV} \quad \text{(from } \Gamma(Z \rightarrow \ell^+\ell^-(\gamma))\text{)} \quad (35)$$

Several features of those predictions are revealing. The first is that $\sin^2 \theta_W(m_Z)_{\overline{MS}}$ currently gives the best determination of $m_H$. Note, however, the uncertainties scale as the central value; so, the relatively small value, 67 GeV, helps reduce the uncertainties. Also, a larger $\Delta \alpha^{-1}(m_Z) = \pm 0.090$ would significantly increase the overall uncertainty [32]. In the case of $m_W$, a light Higgs is also suggested. In fact, the $m_W$ and $\sin^2 \theta_W(m_Z)_{\overline{MS}}$ determinations of $m_H$ are very consistent.

Taken together, (33) and (34) appear to point to a relatively light Higgs scalar that cannot be too far from the current LEP II bound based on the non observation of $e^+e^- \rightarrow ZH$ [33]

$$m_H > 106 \text{ GeV} \quad (36)$$

In the near future, searching for the Higgs via associated $W^\pm H$ and $ZH$ at the Fermilab $p\bar{p}$ collider during Run II promises discovery up to $m_H \sim 115$–130 GeV, perhaps even higher. Higgs unveiling may soon be at hand.

**DISCUSSION**

Within the Standard Model framework, precision electroweak studies suggest a relatively light Higgs scalar. The upper bound from global fits to all data is about [34] 235 GeV. Such a bound is in accord with “no new physics" scenarios, where perturbative validity up to $O(10^{19} \text{ GeV})$ and vacuum stability imply the range [35]

$$135 \text{ GeV} \lesssim m_H \lesssim 180 \text{ GeV} \quad (37)$$

Supersymmetry, on the other hand, is starting to be squeezed by lack of a Higgs scalar discovery at LEP II. The MSSM scenario requires $m_H \lesssim 135 \text{ GeV}$, but one generally finds that considerably lower values are preferred. The current bound of $m_H \gtrsim 106 \text{ GeV}$ will be pushed to $\sim 112 \text{ GeV}$ at LEP II and somewhat further by Run II at the Fermilab Tevatron. If MSSM has some relation to Nature’s truth, a discovery should not be far off.

What about alternative theories where there is no fundamental Higgs? In some dynamical electroweak symmetry breaking scenarios, one expects an effective $m_H \sim O(1 \text{ TeV})$. Are such ideas ruled out? They face a basic difficulty which is nicely illustrated by the $S$ & $T$ parameters of Peskin and Takeuchi [36]. For $m_H \sim 1 \text{ TeV}$, one finds the following predictions
\[ m_W \simeq 80.214 \text{ GeV} \quad (1 - 0.0036S + 0.0056T) \]
\[ \sin^2 \theta_W(m_Z)_{\text{MS}} \simeq 0.23250 \quad (1 + 0.0158S - 0.0113T) \]  
(38)
\[ \Gamma(Z \to \ell^+\ell^- (\gamma)) \simeq 83.79 \text{ MeV} \quad (1 - 0.0021S + 0.0093T) \]

where \( S \) and \( T \) represent loop effects from heavy, strongly interacting, condensate fermions. Comparison of \( m_W \) and \( \sin^2 \theta_W(m_Z)_{\text{MS}} \) via [37]

\[ S \simeq 118 \left\{ 2 \left( \frac{m_W - 80.214 \text{ GeV}}{80.214 \text{ GeV}} \right) + \frac{\sin^2 \theta_W(m_Z)_{\text{MS}} - 0.23250}{0.23250} \right\} \]  
(39)

suggests from \( m_W = 80.419(38) \text{ GeV} \) and \( \sin^2 \theta_W(m_Z)_{\text{MS}} = 0.23091(21) \)

\[ S \simeq -0.20 \pm 0.15 \]  
(40)

while consistency with eq. (38) requires

\[ T \simeq 0.33 \pm 0.17 \]  
(41)

Generating \( T \simeq 0.3 \) by loop effects is actually quite easy. It requires relatively small mass splittings of new heavy fermion isodoublets. The problem is a negative \( S \) which is possible, but not particularly natural in simple dynamical models where \( S \sim O(1) \) is more likely. However, one could easily imagine small shifts in \( m_W \) and \( \sin^2 \theta_W(m_Z)_{\text{MS}} \) which could give \( S \simeq 0 \) and \( T > 0 \). So, \( m_H \simeq 1 \text{ TeV} \) could be accommodated by non-vanishing \( S \) & \( T \), but it is much easier to simply satisfy the precision measurement constraints with \( m_H \simeq 100 \text{–} 200 \text{ GeV} \) and \( S \simeq T \simeq 0 \).

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