Scaling in the two-dimensional U(1)--Higgs model

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We study the continuum limit of the 2D U(1)--Higgs model with variable scalar field length, which is qualitatively different from the fixed length case. Our simulations concentrate on the scaling behaviour of the topological susceptibility, and an instanton-induced confinement mechanism of fractional charges is numerically confirmed.

1. Introduction

The 2D abelian Higgs model shares prominent features of the SU(2)--Higgs sector of the Standard Model related to baryon number violation. Whereas detailed studies of the model with various methods are available [1,2], it is not well understood within the euclidean lattice approach, above all for variable scalar field length. We examine the continuum limit in this case and, in particular, investigate the scaling behaviour of the topological susceptibility.

2. Simulation of the lattice model

On a two-dimensional lattice \( \Lambda \) (with spacing \( a \), extensions \( L_\mu \), and unit vectors \( \hat{\mu} \), \( \mu = 1, 2 \)) the action is given by \( S = S_g + S_\phi \),

\[
S_g = \beta \sum_{x \in \Lambda} \left( 1 - \text{Re} \, U_{p;x} \right)
\]

\[
S_\phi = \sum_{x \in \Lambda} \left\{ -2\kappa \sum_{\mu=1}^{2} \text{Re} \left( \phi_{x+\hat{\mu}}^* U_{x,\mu} \phi_x \right) + |\phi_x|^2 + \lambda \left( |\phi_x|^2 - 1 \right)^2 \right\}.
\]

These are updates in the gauge sector by a global proposal of an instanton configuration

\[
A_{x,\mu} \rightarrow A_{x,\mu} \pm \Delta A_{x,\mu}
\]

with \( \Delta A_{x,\mu} \) carrying unit topological charge and being non-zero in a region of the instanton size.

We consider expectation values built up from the operators \( \rho_x \) (scalar length), the \( \varphi \)--links

\[
L_{\varphi;x,\mu}^\pm \equiv \frac{\text{Re} \left( \phi_{x+\hat{\mu}}^* U_{x,\mu} \phi_x \right)}{\text{Im} \left( \phi_{x+\hat{\mu}}^* U_{x,\mu} \phi_x \right)}
\]

and Wilson loops \( W(R,T) \) of space-time extensions \( R,T \). Particle masses in the Higgs \( (m_H) \) and vector \( (m_W) \) channels are extracted from fits of \( \rho_x^2, L_{\varphi;x,1}^- \) and \( F_x, L_{\varphi;x,1}^- \)--correlation functions, respectively.

3. Lines of constant physics

Let us mention some limiting cases of the model. For \( \kappa = 0 \) one arrives at pure gauge theory (PGT) with confinement in two dimensions. \( \lambda = \infty \) (fixed length case \( |\varphi_x| = 1 \)) and \( \beta = \infty \) is the 2D XY-model with its Kosterlitz-Thouless phase transition between a massive vortex phase \( (\kappa < \kappa_c) \) and a massless spin wave phase \( (\kappa > \kappa_c) \). At finite \( \beta \) this transition is expected to become a crossover [3]. For any fixed \( \lambda \) and \( \beta \rightarrow \infty \) the vector mass \( a m_W \) tends to zero, defining a continuum limit \( (a \rightarrow 0) \), but \( a m_H \) stays finite. Ending up with infinite \( m_H \) at \( \beta = \infty \) for all (fixed) \( \lambda \)--values reflects the freezing of the radial mode on large scales in the 2D \( \phi_{h=2}^4 \)--theory [3].

Figure [3] illustrates the typical dependence of the Higgs and vector masses on \( \kappa \). We find a change in the behaviour of the mass spectrum in addition to a rapid breakdown of the topological

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besides the fact that the $\kappa$-dependence of $am_H$ and $am_W$ is similar.

Now we set up the lines of constant physics (LCPs) by the requirements $am_H, am_W \to 0$ at fixed scalar field VEV and mass ratio:

$$v_R \equiv \sqrt{2\kappa \langle \rho \rangle} = v, \quad R_{HW} \equiv \frac{m_H}{m_W} = \bar{R}.$$  \hfill (5)

With a tuning of $\kappa$ this can be achieved by $\beta \to \infty, \lambda \to 0$, realized for large enough $\beta$ by

$$\beta \to \infty \quad \beta \lambda = \text{constant}. \hfill (6)$$

The simulated points in parameter space are collected in table 1. One has $\beta \lambda \simeq 0.1$, and $\kappa$ was adjusted until the renormalization conditions (5) were simultaneously fulfilled within errors.

It has to be emphasized that the continuum limit (6), which amounts to send $\kappa \to \frac{1}{4}$ at the same time, see figure 2, should not be confused with the gaussian limit. The crucial point is that the relation between the dimensionful bare continuum couplings $\lambda_0, e_0$ and the lattice parameters is $\lambda \propto a^2 \lambda_0$ and $\beta = 1/a^2 e_0^2$. Hence $\lambda \to 0$ at constant $\beta \lambda$ does not imply $\lambda_0 \to 0$ for $a \to 0$.

4. Topological susceptibility

We adopt the geometric definition of the topological charge, which in two dimensions reads

$$Q_{top} \equiv \frac{e_0}{4\pi} \int d^2 x \epsilon_{\mu\nu} F_{\mu\nu}(x) \to \frac{1}{2\pi} \sum_{x \in A} F_x$$  \hfill (7)

and has only integer values. The topological susceptibility is $\chi_{top} \equiv \frac{1}{\Omega} (Q_{top})$, $\Omega$: lattice volume,
Figure 2. Scaling of $\chi_{\text{top}}$ along the LCPs.

and has been measured on the LCPs leading to the results in table 1 and figure 2. Significant finite volume effects are ruled out. Within the chosen parameter sets $\chi_{\text{top}}$ varies by orders of magnitude, and a contraction of the $\kappa$–region, which is limited by a still measurable $\chi_{\text{top}}$ from above and by the line L3 from below, is seen. Note that this LCP already lies close to PGT, where the $\beta$–dependence $\chi_{\text{top}} \to 1 + \frac{1}{4\pi^2} \beta$ for $\beta \to \infty$, $\Omega \to \infty$ is known. Except for L3, the scaling of the dimensionless ratio $\chi_{\text{top}}/m_H^2$ is rather poor.

Finally we look for confinement by instantons, suggested for this model in [5]. Using $\oint_A d\mathbf{x} = \int_A d^2 x F_{12}$ we obtain a unique lattice prescription for the Wilson loop with fractional test charge $q$ in the compact formulation:

$$W_q(R,T) = e^{iq \sum_{x \in A_{R,T}} F_{12}} A_{R,T} \in \Lambda : \text{area}.$$ (8)

Since $F_{12} = e a^2 F_{12}(x)$ for $a \to 0$, one requires $F_{12} \in [-\pi, \pi]$, so $2\pi$–ambiguities for $q = \frac{1}{2}$ as for the standard form with $A_{x\mu}$ are avoided. The static potential $V_q = -\lim_{T \to \infty} \frac{1}{T} \ln W_q$ gets in the dilute instanton gas approximation a contribution $\chi_{\text{top}} \{1 - \cos(2\pi q/e_0)\} R$, which signals confinement for non-integer $q/e_0$. We take Polyakov loop correlations $P_q(R) \equiv W_q(R,T)|_{T=L_2}$ and fit $V_q = -\frac{1}{T L_2} \ln P_q$ to a continuum Yukawa ansatz

$$V_q(R) = \frac{e^2 R}{2m_s} \left(1 - e^{-m_s R}\right) + \alpha R.$$ (9)

As exemplarily displayed in table 2 for $q = \frac{1}{2}$ in L2, lying just in the Higgs regime ($\kappa > \bar{\kappa}$), the meaning of the fit parameters $ae_R$ (renormalized gauge coupling, small corrections to $ae_0 = 1/\sqrt{\beta}$ expected), $am_s$ (screening mass, $\simeq am_W$) and $\alpha$ ($= 2\chi_{\text{top}}/q^2$) is reproduced.

| set | $am_s$ | $ae_R$ | $\alpha/8 \cdot 10^4$ |
|-----|--------|--------|------------------------|
| A   | 0.436(2) | 0.3136(3) | 1.5(1)                 |
| B   | 0.209(2) | 0.1551(3) | 0.55(6)                |
| C   | 0.098(3) | 0.0769(5) | 0.20(5)                |

Table 2. Fit parameters of $V_{1/2}$ in L2.

5. Discussion and outlook

The continuum limit in the 2D U(1)–Higgs model with variable scalar field length seems to be achieved as outlined in [5]. The scaling of $\chi_{\text{top}}$ is still unclear and will be studied further. Also the systematic errors by the statistical uncertainties in the conditions (5) should be estimated. The LCPs give strong evidence for a phase transition in $\kappa = \frac{1}{4}$ at $\beta = \infty$ and for a crossover for $\beta < \infty$.

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