Large Electric Dipole Moments of Heavy Leptons

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In many models of CP violation, the electric dipole moments (EDMs) of leptons scale as the cube of the lepton mass. In these models, the EDM of a 100 GeV heavy lepton would be a billion times greater than that of the muon, and could be as large as a 0.01 e-fermi. In other models, in which the heavy leptons have different properties from the lighter generations, a similarly large EDM can be obtained. A large EDM could dominate the electromagnetic properties of heavy leptons. The angular distribution and production cross-section of both charged and neutral heavy leptons with large dipole moments is calculated and discussed. The interesting possibility that a heavy neutrino with a large EDM could leave an ionization track in a drift chamber is investigated.

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1 Introduction

The origin of the fermion masses and mixing angles remains a mystery. Many models for explaining their values exist, most involving additional symmetries, but it will require additional experimental data to distinguish between them. Much of the excitement about the discovery of neutrino masses and mixing angles is due to the hope that their values will provide clues toward solving the mystery of flavor.

Another potential source of information may come from electric and magnetic dipole moments. These moments may also reflect the flavor structure of the model. Just as the Yukawa couplings form a matrix in generation-space, the interaction of two fermions with a photon will also be a matrix in generation-space. The real and imaginary parts of the diagonal elements will lead to the magnetic and electric dipole moments; the off-diagonal elements will lead to decays such as $\mu \rightarrow e\gamma$. Measurements of these moments will give valuable clues towards understanding flavor physics, and will give hints of physics beyond the standard model (as demonstrated by the great excitement over the apparent discrepancy [1] with the muon’s magnetic dipole moment).

In this paper, I will focus on the electric dipole moments (EDM’s) of leptons. Here, there is the possibility of substantial improvement in the experimental bounds (or discovery) in the near future. A recently approved experiment [2] expects to lower the current bound [3] on the electron EDM of $4.3 \times 10^{-27}$ e-cm to at least $10^{-29}$ e-cm and possibly a couple of orders of magnitude better. The muon EDM limit [4] is now $1.1 \times 10^{-18}$ e-cm, but proposals to lower this by six orders of magnitude exist [5]. The tau limit is now listed [6] at $3 \times 10^{-16}$ e-cm, but combining limits on the weak dipole moment with $U(1)$ symmetry improves that by a factor of thirty [9].

What values might one expect for the EDM’s? In the Standard Model, the EDM’s are extremely small [10]. However, in most extensions of the Standard Model, they are
substantially larger. In multi-Higgs models, the EDM of the muon can be as large as $10^{-24}$ e-cm, and thus within reach of currently planned experiments. In leptoquark models, the muon and tau EDMs are typically $10^{-24}$ e-cm and $10^{-19}$ e-cm, respectively. In left-right models, the muon EDM is typically $10^{-24} \sin \alpha$ e-cm, where $\alpha$ is a phase angle. In the MSSM, the electron EDM is somewhat above the experimental bounds if the phases are all of order unity. Thus, we see that a wide variety of models give EDMs that can be observed in the next round of experiments.

Recently, Babu, Barr and Dorsner discussed how the EDM's of leptons scale with the lepton masses. In many models, such as the MSSM, they scale linearly with the mass. However, in a number of models, such as some multi-Higgs, leptoquark and flavor symmetry models, the EDM scales as the cube of the lepton mass. In these models the tau EDM will be 5,000 times larger than the muon EDM. (It should be noted that the electron EDM in some of these models receives a two-loop contribution which only varies linearly, and thus it need not be negligible.)

The purpose of this paper is to point out that if a fourth generation lepton doublet exists, then reasonable models exist in which one would expect an enormous EDM. For example, in models with cubic scaling, a muon EDM in the expected range, $d_\mu = 10^{-24}$ e-cm, would mean that a 100 GeV lepton would have an EDM of 0.01 e-fermi. Such an EDM will dominate the electromagnetic interactions, dramatically changing the phenomenology of such leptons. Even more interesting is the possibility that the heavy neutrino (also expected in the 100 GeV range) could have an enormous EDM, leading to the question of whether such a neutrino could leave an ionization track.

How realistic is the possibility of such a large EDM for a heavy lepton? A specific model with a cubic scaling is the model of Bernreuther, Schroder and Pham, in which CP violation occurs in the Higgs sector. In this model, the EDM of a heavy lepton (they considered the top quark, but the results are similar) is constrained by the electron EDM.
and loop diagrams, and the maximum EDM for a heavy lepton is a factor of 100 below 0.01 e-fermi. In other models, such as the model of Ref. [15] in which one has their parameter $c = 0$, the electron EDM does not pose such a constraint and a large EDM is allowed. If we do not rely on any specific model, then if one simply assumes that CP violation is due to new physics at a TeV scale, and writes an effective dimension-five Lagrangian as $\frac{1}{n}T_LG^{\mu\nu}i\gamma^5L_RF_{\mu\nu}$, then the added assumption that all particles with $O(100)$ GeV masses have couplings of $O(1)$ (which is true for the $W$, $Z$ and top quark) results is a very large EDM of the order of 0.01 e-fermi.

As an existence proof, one could assume that the fourth generation leptons form a vectorlike isodoublet. This could explain the large neutrino mass. If they couple to singlet Higgs fields with large, complex VEVs, then a large EDM could easily be generated. Such a model is not a cubic scaling model–the EDMs of the light fermions remain negligible. Our main point is that such a large EDM is certainly not excluded, and we thus investigate its consequences.

2 Heavy Lepton Production

The most dramatic effect of a large EDM of a heavy lepton will be in the production cross-section and angular distribution. As pointed out by Escribano and Masso[9], the $U(1)$ invariant effective operator giving an EDM is given by $\frac{T_L}\sqrt{2}\sigma^{\mu\nu}i\gamma^5L_RB_{\mu\nu}$, where $B_{\mu\nu}$ is the $U(1)$ field tensor. This will lead to a coupling to the photon, which we define to be the EDM, as well as a coupling to the $Z$ which is given by the EDM times $\tan \theta_W$. We will include this coupling to the $Z$ (although, in practice, it has a very small effect on the results presented). It is important to point out that this is an assumption. One could also include an operator coupling to the $SU(2)$ field tensor, leading to a very different value for the $Z–EDM$. Rather than deal with two parameters, however, we just assume
that the latter operator is smaller. If it is large, it will (barring fine-tuning) just make the cross-section even bigger. Note that the precise value of the $Z - EDM$, unless it is much larger than we have assumed, has very little effects on the results. The lepton-lepton-photon interaction is $-ie \bar{\psi}(\gamma_\mu + D_{\mu\nu}\gamma^5q^\nu)\psi A^\mu$, where $q$ is the momentum-transfer. The electric dipole moment is defined to be $eD(q^2 = 0)$. We will assume that $D(q^2)$ does not vary rapidly with $q^2$, as is the case in virtually all models.

The matrix element for heavy lepton production in $e^+e^-$ annihilation is given by

$$\mathcal{M} = \frac{ie^2}{s}\bar{u}_e\gamma^\mu u_L(\gamma_\mu + D_{\mu\nu}\gamma^5q^\nu)v_L + \frac{ie^2}{\sin^22\theta_W(s - M_Z^2 + i\Gamma M_Z)}\bar{u}_e\gamma^\mu(C_V - C_A\gamma_5)u_e$$

$$\times \bar{u}_L(\gamma_\mu(C_V - C_A\gamma_5) + D\tan\theta_W\sigma_{\mu\nu}\gamma^5q^\nu)v_L$$

(1)

where $C_V = \frac{1}{2} - 2\sin^2\theta_W$ and $C_A = \frac{1}{2}$.

The differential cross section is found to be

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\sqrt{1 - \frac{4M^2}{s}}\left(A_1 + \frac{1}{16\sin^42\theta_W}P_{ZZ}A_2 + \frac{1}{\sin^22\theta_W}P_{\gamma Z}A_3\right)$$

(2)

where

$$A_1 = 1 + \cos^2\theta + \frac{4M^2}{s}\sin^2\theta + D^2s\sin^2\theta(1 + \frac{4M^2}{s})$$

$$A_2 = 1 + \cos^2\theta - \frac{4M^2}{s}\sin^2\theta + 4D^2s\tan^2\theta_W(\sin^2\theta + \frac{4M^2}{s}(1 + \cos^2\theta))$$

$$A_3 = \sqrt{1 - \frac{4M^2}{s}}\cos\theta + C_VD^2s\tan\theta_W(\sin^2\theta + \frac{4M^2}{s}(1 + \cos^2\theta))$$

$$P_{ZZ} = \frac{s^2}{(s - M_Z^2)^2 + \Gamma^2M_Z^2}$$

$$P_{\gamma Z} = \frac{s(s - M_Z^2)}{(s - M_Z^2)^2 + \Gamma^2M_Z^2}$$

(3)

In these expressions, we have dropped terms proportional to $C_V^2$ since they are numerically negligible. This result is plotted in Figure 1, for $M = 100$ GeV and for different values of the EDM (in units of e-cm). Changing the lepton mass does not change the qualitative feature of the results. We see that for $D = 0$, the usual $1 + \cos^2\theta + C\cos\theta$
distribution for a lepton (where the $\cos \theta$ term is due to $\gamma - Z$ interference) is found. For an EDM greater than $10^{-16}$ e-cm, the distribution is completely dominated by the $\sin^2 \theta$ contribution from the electric dipole term. (It turns out that the EDM term in the $Z$ coupling is barely noticeable in the figures.) One can see that EDM’s of the size noted earlier will dramatically alter the angular distribution.

Of course, a scalar field will also give a $\sin^2 \theta$ distribution. Could one distinguish this heavy lepton from, say, a heavy scalar lepton? This can be done easily by looking at the total cross-section, shown in Figure 2 for a 100 GeV lepton. For an EDM of $10^{-16}$ e-cm, the cross section for a $\sqrt{s} = 500$ GeV collider is quite large, substantially larger than expected for a heavy fermion (and even larger than for a heavy scalar). Note the unusual scaling behavior. This is not surprising. The cross-section, for large EDM’s, must vary as $D^2$. Thus, instead of falling as $1/s$, it becomes constant. Thus, by examining the threshold behavior, one could distinguish this model from any alternatives.

Note that the cross-section varies as $D^2$, and for an EDM as large as 1.0 e-fermi would be almost a microbarn!! Of course, cross sections this large will violate unitarity. The unitarity limit can be approximately estimated by setting the relevant effective interaction
strength, $\alpha D \sqrt{s}$ equal to unity. For $\sqrt{s} = 500$ GeV, $D = 10^{-15}$ e-cm, and $\alpha \simeq 1/125$, this effective interaction strength is 0.25. Thus the unitarity limit will be near the upper right corner of Figure 2. So, the behavior discussed above will appear for a wide range of parameter space without violating the unitarity bound. Another way of saying this is to note that the larger the EDM, the smaller the scale at which the physics responsible for the effective interaction sets in, and for an EDM larger than $10^{-15}$ e-cm, that scale is less than $\sqrt{s}$.

These results are not original (although the discussion generally does not include EDMs as large as considered here). They are given in the context of a top quark EDM by Bernreuther et al.\cite{7}, and given in the context of tau-pair production in Ref. \cite{8}. This latter paper noted how one can use CP-odd angular correlations to search for a tau EDM, and this method has been used by experimentalists. However, there has not been any discussion of the possibility of a large EDM for heavy neutrinos, and here we see a
Figure 3: Differential cross section for heavy neutrino production for various EDMs, in units of e-cm, for a lepton mass of 100 GeV.

unique signature.

Since there are no more than three light active neutrinos, the neutrino associated with the charged heavy lepton must be heavy, with a mass of at least 45 GeV. The differential cross-section is given by

\[
\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} \sqrt{1 - \frac{4M^2}{s}} \left( A_1 + \frac{1}{8\sin^42\theta_W} P_{ZZ} A_2 + \frac{1 - 4\sin^2\theta_W\tan\theta_W}{\sin^22\theta_W} P_{\gamma Z} A_3 \right) \tag{4}
\]

where

\[
\begin{align*}
A_1 &= D^2s \sin^2\theta(1 + \frac{4M^2}{s}) \\
A_2 &= 1 + \cos^2\theta - \frac{4M^2}{s} \sin^2\theta + 8C_V \cos\theta + D^2s \tan^2\theta_W(\sin^2\theta + \frac{4M^2}{s}(1 + \cos^2\theta)) \\
A_3 &= 4D^2s(\sin^2\theta + \frac{4M^2}{s}(1 + \cos^2\theta)) \tag{5}
\end{align*}
\]

The differential and total cross-sections are given in Figures 3 and 4, for a heavy neutrino mass of 100 GeV. The results are similar to the charged heavy lepton case.
Since it is generally believed that these heavy neutrinos could not be detected directly, such a calculation would only be meaningful if the heavy neutrino were heavier than the charged lepton, and could thus be detected via its decay. Here, one could again look for CP-odd correlations, as discussed in Ref. [8]. However, most models have the charged lepton heavier than the neutrino, and thus the decay can only occur through mixing with the very light neutrinos. As discussed in detail in Ref. [16], this mixing could be very small, and these heavy neutrinos could be effectively stable. As we will see in the next section, however, it may be possible to detect these neutrinos directly.

### 3 Heavy Lepton Detection

With one exception discussed below, the detection of heavy leptons is not substantially affected by a large EDM. The decays of heavy charged and neutral leptons depends sensitively on their masses and mixings with the lighter leptons. If the charged lepton,
$L$, has a mass $M_L > M_W + M_N$, where $M_N$ is the neutral lepton mass, then the primary
deay would be $L \rightarrow W + N$. If the mass is less than $M_W + M_N$, but still above that
of the $N$, then its decays will either be $L \rightarrow W^* + N, W + \nu_\tau$ or $Z + \tau$, assuming the
largest mixing is with the third generation. Which of these dominates is very sensitive
to the mixing angles (note that the $Z + \tau$ decay mode, even if rare, gives a very clean
signature). A detailed analysis and review of all of the decays, including the possibility
that the $N$ is heavier than the $L$, is given in Ref. \[16\]. There is little effect on their
analysis from the EDM.

However, there is one exception. It has always been assumed that heavy ($O(100)$
GeV) neutrinos would, if stable or fairly long-lived, simply leave the detector. But we
are interested in heavy neutrinos with a very large EDM. Would such a neutrino leave
an ionization track in a calorimeter? One can derive the expression for the ionization
energy loss following Jackson\[17\], replacing the electric field from a charge with the
electric field from a dipole. Suppose a particle is traveling in the x-direction, and an
electron is at $y = b$, where $b$ is the impact parameter. The impulse given to the electron,
$\int_{-\infty}^{\infty} E_y \, dt$ depends on the orientation of the dipole moment with the motion of the
neutrino. If the orientation is in the $(x, y, z)$ direction, the impulse is $\frac{e^2 D}{\gamma v} \left(0, 2, 2\pi\right)$. Since
the orientation is arbitrary, and we are only interested in an order of magnitude estimate,
we take the impulse to be $\frac{2 e^2 D}{\gamma v}$. Note that the impulse from a charge is just $\frac{2 e^2}{\gamma v}$, and thus
this result is expected on dimensional grounds. Converting the impulse to an energy
exchange (non-relativistically), and setting the lower bound on the impact parameter
by the maximum allowed energy exchange, one can show that the minimum impact
parameter is $b_{\text{min}}^2 = \frac{e^2 D}{m \gamma v^2}$, and thus the total energy loss is given by

$$\frac{dE}{dx} = 4\pi N_A \left(\frac{e^2}{4\pi \epsilon_0}\right) D \gamma Z \frac{A}{A} \quad (6)$$

where $N_A$ is Avogadro’s number, $A$ is the atomic number in units of grams/mole. Note
that the logarithm in the usual Bethe-Block formula is no longer present (since there is an extra power of $b$ in the denominator of the impulse) and the electron mass and velocity (except in $\gamma$) drop out. Plugging in numbers, we find that $\frac{dE}{dx} = 10^{12} D \text{ MeV g}^{-1} \text{ cm}^{2}$.

We have seen that many models give a muon EDM of approximately $10^{-24}$ e-cm, and thus cubic scaling would give a heavy lepton EDM of approximately $10^{-15}$ e-cm. In addition, models with a large dimension-five operator coefficient can have a similarly large EDM. With that value, the energy loss would be $10^{-3} \text{ MeV g}^{-1} \text{ cm}^{2}$, which is roughly one one-thousandth of the usual energy loss for a charged particle. This would pose a challenge, but not an insurmountable one, for experimentalists. If the heavy neutrino is produced in the decay of a heavy charged lepton, then it would be produced in coincidence with a real or virtual $W$, and this would virtually eliminate backgrounds. However, if the heavy neutrino is produced directly through s-channel production, the backgrounds could be formidable. Note, however, that the cross section is huge. For $10^{-15}$ e-cm, it is 100 picobarns, corresponding to an event rate of 1 Hertz at the Linear Collider. One could place very sensitive detectors some distance from the vertex, back to back, and the event rate would still be quite high. Certainly, should a heavy charged lepton be discovered with an unusual angular distribution, detection of a heavy neutrino though its ionization would seem possible.

\section{Conclusions}

In a number of models of CP violation, a heavy lepton could have an enormous EDM, which could dominate the electromagnetic properties. The angular distribution and total cross-section for both charged and neutral heavy leptons have been presented, and it has been noted that the heavy neutrino, if its EDM were sufficiently large, would leave a detectable ionization track.
In order to keep the analysis general, we have neither considered a specific model of CP violation, and nor discussed the origin of the heavy leptons. Although we have done the calculation for a sequential chiral lepton doublet, the chirality of the heavy leptons does not make much difference in our results. In a more specific model, of course, other issues, such as the contribution to electroweak radiative corrections, will become relevant.

I thank Jack Kossler for several useful discussions, and Chris Carone for reading the manuscript. This paper is dedicated to the memory of Nathan Isgur. This work was supported by the National Science Foundation through Grant PHY-9900657.
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