Performance analysis of vortex acoustic wave based on uniform circular array

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Abstract: As a carrier for information and energy, acoustic waves have been applied in underwater communication widely, however, the narrow band and low transmission speed are the main problems. Whether in the field of optics or electromagnetic waves, the orbital angular momentum (OAM) represents the natural properties of the spiral phase structure. By introducing the OAM into the acoustics field, the transmission capacity and spectrum efficiency of the underwater acoustic communication system can be expanded. Based on the analysis and detection of the vortex acoustic wave generated by the circular array of transducers, we studied the array generation method of the spiral acoustic beam, and gave the characteristics of the vortex acoustic beam when propagating under the water. In the direction of the main axis, the uniform circular array was used to generate different topological acoustic vortex. To determine the relationship between the OAM topology mode and the transducer array, the spiral acoustic waves in different topology modes were generated, and the number of array elements, array radius, transmission frequency, etc. were investigated to give the effects on OAM acoustic vortex.

1. Introduction

Underwater acoustic technology is an important technology in the development of marine communication. Underwater acoustic communication is the study of how to transmit underwater information more accurately under known conditions. In the atmosphere, electromagnetic wave bandwidth resources are abundant, frequency bands are high, and the technology of using electromagnetic waves for communication is already very mature. However, in seawater, the absorption rate of seawater to electromagnetic waves is very large, and the attenuation speed of electromagnetic waves in seawater is very fast. The electromagnetic signal will have huge distortion in both amplitude and frequency, and the transmission distance in seawater is very limited[1].

In this case, underwater sound waves have become an effective and more reliable long-distance information transmission medium. The ways to generate OAM vortex sound waves are generally divided into two categories: active and passive.

Active technology belongs to the acoustic phase control technology. The principle is to form a phase control array through independent adjustment of the acoustic transducer to produce a phase distribution that can form a spiral. The active generation method of OAM sound waves in acoustics requires expensive costs and complex circuits, and it is difficult to apply in high frequency bands[2]. Marchiano et al. used piezoelectric transducer arrays to generate high-order single vortex sound beams, and
conducted experiments to study how different vortex sound fields interact[3].

Passive materials produce acoustic OAM, which is simpler than active materials. Acoustic vortices can also be generated based on metasurface material junctions. Ye et al. used a hyperplanar structure to generate acoustic spiral waves[4], Naify et al. used metamaterial slot antennas to generate acoustic spiral waves[5].

These methods provide a reference for the low-cost generation of underwater vortex sound waves. However, the structural requirements are high, some require their own spiral distribution geometric characteristics, and the transmission efficiency of sound wave energy is also limited, which makes it difficult to conduct performance analysis and research. In order to study the generation of OAM vortex sound waves and analyze the performance of vortex sound waves, we use the transducer array in active technology to simulate the generation of OAM vortex sound waves.

2. Theoretical basis of orbital angular momentum
The wave transmission of sound waves is similar to the transmission characteristics of light waves. Vortex sound waves also have a phase factor of \( e^{-i\phi} \), but unlike vortex light waves, vortex sound waves don’t have spin effects and polarization effects, and vortex sound waves don’t carry spins. The angular momentum SAM only carries the orbital angular momentum OAM[6]. OAM vortex acoustic waves generally have four characteristics: zero field strength along the propagation direction, phase distribution within \([0, \pm 2\pi] \), self-correction of the waveform during transmission, and transmission of torque to other substances to make them rotate[7].

The phase structure of the Bessel-type vortex beam generated by the antenna array has a dependent azimuth angle, and the UCA vector potential composed of \( N \) antenna elements is expressed as

\[
A_{\text{array}}(r) = -\frac{j\mu_0}{4\pi} \sum_{n=1}^{N} e^{j \phi_n} \int_{|r-r_n|}^{\infty} e^{-ikr} dV_n \approx -\frac{j\mu_0}{4\pi} \sum_{n=1}^{N} e^{-(k |r-r_n|)} = A(r)\psi(\theta, \phi),
\]

In the formula, the vector potential \( A(r) \) corresponds to the radiation of the antenna element, and \( \psi(\theta, \phi) \) is the array factor of UCA, which can be approximated by the phase angle integral

\[
\psi_1(\theta, \phi) = \sum_{n=1}^{N} e^{-ik |r-r_n|} \approx \frac{N e^{-i\phi_0}}{2\pi} \int_{0}^{2\pi} e^{-i\sin\theta \cos\phi} e^{-i\phi} d\phi = N i^{-1} e^{-i\phi} J_1(ka \sin \theta),
\]

The electric field is expressed as

\[
E(r) = \sum_{n=0}^{N-1} e^{-ki(r-r_n)} \approx e^{-ikr} \frac{N e^{-i\phi_0}}{r} N i^{-1} e^{-i\phi} J_1(ka \sin \theta),
\]

It can be concluded from formulas (1) and (2) that by designing the radius of the array and the amplitude of the excitation signal, the intensity distribution of different OAM modes can be effectively controlled, and the main lobes of the vortex beams of different topological modes can be aligned to the same elevation angle direction without changing the phase angle distribution at the same time. Increasing the diameter of the array will increase the number of side lobes when reducing the gain angle, and part of the energy will be lost. But the increased side lobe angle is much wider than the main lobe, and the side lobe gain is also much weaker than the main lobe. So the impact is not important in the actual detection process.

For the L-G type OAM beam, there is a dependent azimuth angle of \( e^{-i\phi} \), and the expression of the field distribution in cylindrical coordinates is shown in (4)

\[
u_{\rho}(r, \phi, z) = C_{\rho} \frac{w_0}{z_p} \left[ \frac{r}{w(z)} \right] \left[ \frac{2r^2}{w(z)} \right] \times \exp \left[-\frac{r^2}{w(z)^2} \right] \times \exp \left[-\frac{j(2p+|\rho|+1)}{2z_p} \right] \exp \left[-\frac{jkr^2}{2z_p(z_p + z)^2} \right] \exp(-j\phi),
\]

Where \( w(z) = \rho_0 \sqrt{1 + (z/z_p)^2} \), \( C_{\rho} = A \sqrt{p!/(p+|\rho|)!} \), \( r \) represents the radiation distance from the detection point to the radiation axis, \( \phi \) represents the azimuth angle, \( z \) represents the propagation distance, \( k \) represents the wave vector, \( k = 2\pi/\lambda \), \( l = 1,2,...,L \) represents the number of topology modes, \( p \)
represents the number of radial nodes, \( w(z) \) represents the beam width at \( z \), \( \omega_0 \) is the beam waist radius, \( Z_R = \pi \omega_0 / \lambda \) represents the Rayleigh distance, \( \lambda \) is the wavelength, \( L_p \) represents the general Laguerre polynomial, \( C_{nl} \) is the energy distribution coefficient of the topological mode, and \( A \) is the total radiant energy.

Research has proved that the vortex acoustic wave OAM can be generated by the underwater acoustic transducer. Place the \( N \) transducer units evenly on the circle of radius \( a \) at the same interval of phase angles, and connect the transducer units with signal sources with the same frequency and the same phase difference.

3. Array cell design

![Fig. 1 UCA (Uniform Circular Array) array design drawing of single-mode orbital angular momentum](image)

As shown in Figure 1, the angular distance interval on the space coordinate angle is \( \Delta \varphi = 2 \pi / N \), and the coordinate position of the \( n \)th transducer unit is \( T_n(\alpha, \varphi_{0n}, 0) \), where \( \varphi_{0n} = (n - 1) \Delta \varphi \). The vortex sound wave of topological mode \( l \) is generated, and the minimum number of transducer units required is \( N_{\text{min}} = \text{Max}(2|l| + 1, 4) \) [8]. The sound pressure generated by the \( n \)th transducer unit can be expressed as

\[
P_{n0}(a, \varphi_{0n}, 0, t) = A_0 e^{-j \omega t} e^{j \varphi_{0n}},
\]

(5)

\( A_0 \), \( \omega \), \( \varphi_{0n} \) represent the amplitude, angular frequency and initial frequency respectively. At the position above the array plane \( p(r, \varphi, z) \), the sound intensity generated by the \( n \)th transducer unit is

\[
P_p(r, \varphi, z, t) = (A_0 / R_n) e^{-j \omega t} e^{j k R_n} e^{j \varphi_{0n}}.
\]

(6)

So the field strength of the entire UCA array at position is

\[
p_{\text{total}}(r, \varphi, z, t) = e^{-j \omega t} \sum_{n=1}^{N} (A_0 / R_n) e^{j k R_n} e^{j \varphi_{0n}},
\]

(7)

\( R_n \) represents for propagation distance \( R_n = \sqrt{(r \cos \varphi - a \cos \varphi_{0n})^2 + (r \sin \varphi - a \sin \varphi_{0n})^2 + z^2} \).

According to the sound field theory of vortex acoustic waves, simulation software is used to analyze the space-time distribution of L-G type vortex beams. Through the analysis of beam amplitude and phase, the amplitude, phase and beam normalized pattern are obtained.

4. Simulation and analysis

According to the UCA array diagram in Fig. 1, the L-G type OAM beam generated by the transducer array in cylindrical coordinates is simulated, and the simulation is performed in Matlab to obtain the spatial-temporal distribution, amplitude, phase, and main lobe of the L-G type vortex acoustic wave.
OAM beam pattern. When the number of array elements is set to N=10, frequency f=30 kHz, and array radius a=3λ, we can get the space-time distribution, amplitude and phase diagrams of vortex acoustic waves with four different topological modes under the topological mode number l =1, 2, 3, 4.

![Fig. 2 Spatiotemporal distribution, amplitude and phase diagrams of vortex acoustic waves with four different modes](image)

Figure 2 shows the temporal and spatial distribution, amplitude and phase diagrams of vortex acoustic waves under four different modes. The first column is the mode number l =1, the second column is the number of mode l=2, the third column is the number of mode l=3, and the fourth column is the number of mode l=4. Figures (a)-(d) are time-space distribution diagrams. It can be seen that as the number of topology modes increases, the main lobe beam becomes wider, that is, the main lobe beam angle increases, while the main lobe peak value decreases slightly. Figures (e)-(h) are the amplitude diagrams of vortex acoustic waves. It can be seen that the amplitude of the center position is very small, that is, there is a phase singularity. Figures (i)-(m) show the phase diagrams of vortex acoustic waves. It can be seen from the phase diagram that there is a vortex in Figure 2-(i), and it can be seen that an OAM with the mode number l =1 is generated. There are two vortices in Figure 2-(j), that is, OAM with mode number l = 2 is produced, and there are three vortices in Figure 2-(k), that is, OAM with mode number l =3 is produced. And there are four vortices in Figure 2-(m), that is, OAM with mode number l =4 is produced.

Under the condition of keeping the parameters unchanged, the peak value of the main lobe of the vortex sound wave and the beam angle pattern of the main lobe are given, as shown in Figure 3.
Figure 3 shows the peak and directional patterns of vortex acoustic waves with different mode numbers. Figure 3-(a) is the peak and directional pattern of vortex acoustic waves when the mode number $l=1$. The peak value of the main lobe is about 0.6, and the beam angle of the main lobe is about 8.6°. Figure 3-(b) is the graph when the mode number $l=2$. The peak value of the main lobe of the vortex sound wave is about 0.5 and the main lobe beam angle is about 12°. Figure 3-(c) is the peak value and direction pattern of the vortex sound wave when the mode number $l=3$, the peak of the main lobe is about 0.45, and the beam angle of the main lobe is about 15.5°. Figure 3-(d) is under the mode number $l=4$, the peak of the main lobe of the vortex acoustic wave is about 0.45, and the beam angle of the main lobe is about 18.9°. It can be clearly seen from Figure 3 that as the number of topological modes increases, the main lobe beam angle of the generated vortex acoustic wave increases, and the main lobe peak value decreases.

In further research, the main lobe peak value and main lobe beam angle of the OAM sound wave under different parameter conditions are given. By analyzing the influence of the specific array radius on the maximum amplitude of the orbital angular momentum generated and the main lobe beam angle, the results are shown in Figure 4.
Under the condition of frequency $f=30$ kHz and the number of transducer array elements $N=10$, we respectively simulate OAM vortex acoustic waves with orbital angular momentum mode number $l=1, 2, 3, 4$. Then we set the array radius $a \in [0.5\lambda, 10\lambda]$, record the main lobe beam angle and the trend of the main lobe peak value with the array radius. Figure 4-(a) is the influence diagram of the main lobe beam angle. It can be seen from the figure that under the same parameter conditions, the main lobe beam angle will increase with the number of modes. And as the array radius increases, the main lobe beam angle increases. Under the mode number $l=4$, this increasing trend is even more obvious. Figure 4-(b) is the influence diagram of the main lobe peak. It can be seen from the figure that under the same parameter conditions, the peak of the main lobe will decrease with the increase of the number of modes. And as the array radius increases, the main lobe peak decreases.

5. Conclusion
This paper studies the generation of L-G type OAM vortex sound waves through an underwater acoustic transducer array, and analyzes the influence of the array radius on the performance of the generated OAM vortex sound waves. $N$ transducers are evenly placed on the circle of radius $a$ with the same phase difference, and signal sources with the same frequency and phase difference are connected to the transducer to simulate the OAM vortex sound wave. The main lobe beam angle increases as the number of modes increases, while the main lobe peak value is the opposite. The larger the array radius is, the larger the main lobe beam angle is. And it is more obvious when the number of modes is higher, while the main lobe peak value decreases as the array radius increases.

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