Twist-2 Light-Cone Pion Wave Function.

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Abstract

We present an analysis of the existing constraints for the twist-2 light-cone pion wave function. We find that existing information on the pion wave function does not exclude the possibility that the pion wave function attains its asymptotic form. New bounds on the parameters of the pion wave function are presented.

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The light-cone wave function was introduced in perturbative quantum chromodynamics (QCD) to describe hadron form factors at large $Q^2$. At the present time, the pion light-cone wave function $\varphi_\pi(u)$ is widely used in perturbative and nonperturbative (in the framework of QCD sum rule approach) descriptions of hadron properties. For example, numerous hadron amplitudes have been determined from light-cone QCD sum rules suggested in \[4\]. In this approach, the pion light-cone wave function $\varphi_\pi(u)$ is given as the following matrix element of a twist-two operator,

$$<0|\bar{u}(x)\gamma_\mu\gamma_5d(0)|\pi(q)>_{x^2=0}=i f_\pi q_\mu \int_0^1 e^{-iu(qx)}\varphi_\pi(u)du.$$  

As results depend on the model of light-cone wave functions, it is clearly important to understand which forms for the wave functions are consistent with the existing constraints. In this paper, we analyze all such constraints on $\varphi_\pi(u)$, including the new bounds determined from a recent analysis of the structure function of the pion.

There are three relevant models for $\varphi_\pi(u)$. These are the asymptotic wave function,

$$\varphi^{as.}_\pi(u) = 6u(1-u),$$  

the Chernyak-Zhitnitsky wave function $\varphi^{CZ}_\pi(u)$,

$$\varphi^{CZ}_\pi(u) = 30u(1-u)(2u-1)^2,$$

which was suggested to describe the pion form factor for $Q^2 \sim 5 - 10 GeV^2$ in perturbative QCD, and the wave function of Braun and Filyanov $\varphi^{BF}_\pi(u)$,

$$\varphi^{BF}_\pi(u) = 6u(1-u)\left\{1 + a_2(\mu)3/2[5(2u-1)^2 - 1] + a_4(\mu)15/8[21(2u-1)^4 - 14(2u-1)^2 + 1]\right\}$$

$$(a_2 \simeq 0.44, \ a_4 \simeq 0.25; \ \mu \simeq 1 GeV),$$

where $\mu$ is the normalization point. The model of Braun and Filyanov was obtained from the light-cone QCD sum rule for the coupling constant $g_{\pi NN}$ giving the following constraint on the light-cone wave function:

$$\varphi_\pi(0.5) = 1.2.$$
The value of the second moment was obtained in [1]:

\[ m_2 = \int_0^1 u^2 \varphi_\pi(u) du = 0.35; \quad \mu \simeq 1\text{GeV}. \] (6)

However it was pointed that the QCD sum rule for the second moment of the pion wave function is not accurate enough to decide in favor of the Chernyak-Zhitnitsky wave function or the asymptotic one (see for example [3]). The asymptotic value \( m_2 = 0.3 \) is not excluded. So, it means that \( m_2 \) is known with accuracy not better that 15%. This accuracy we will use in our analysis.

There is an additional constraint formulated by Radyushkin and Rustkov [10]:

\[ I = \int_0^1 \frac{\varphi_\pi(u)}{u} du = 2.4 \] (7)

This relation was obtained from the QCD sum rule for the transition form factor \( \gamma\gamma^* \rightarrow \pi^0 \), which was compared with prediction of perturbative QCD. The authors compare their result (7) with the different models,

\[ I_{as.} = \int_0^1 \frac{\varphi_{as.}(u)}{u} du = 3 \] (8)

\[ I_{CZ} = \int_0^1 \frac{\varphi_{CZ}(u)}{u} du = 5 \] (9)

\[ I_{BF} = \int_0^1 \frac{\varphi_{BF}(u)}{u} du = 5.07 \] (10)

and interpret this as an indication that the pion wave function is not very different from its asymptotic form. Unfortunately, the authors in [10] did not discuss the accuracy of their results and we can not use this result (7) in our analysis.

Finally, we use the new constraint for \( \varphi_\pi(u) \) obtained from an analysis of the light-cone QCD sum rule for the pion structure function [11 12], which gives

\[ \varphi_\pi(0.3) = 1; \quad \mu \simeq 1\text{GeV}. \] (11)

In what follows, we will consider constraints for \( m_2, \varphi_\pi(0.5) \) and \( \varphi_\pi(0.3) \). The accuracy of the constraints for \( \varphi_\pi(0.5) \) and \( \varphi_\pi(0.3) \) is about 20%.
To begin our analysis of the existing constraints for the pion wave function, we have to choose a reasonable parametrization. We use the results of [13], in which a series expansion of light-cone wave functions was suggested with the higher-order terms corresponding to operators with increasing conformal spin. In the case of the twist-2 pion wave function, this expansion is

$$\varphi_\pi(u) = 6u(1-u) \left\{ 1 + a_2 C_2^{3/2}(2u-1) + a_4 C_4^{3/2}(2u-1) + a_6 C_6^{3/2}(2u-1) + \ldots \right\}. \quad (12)$$

Here $C_n^{3/2}$ are the Gegenbauer polynomials; $C_2^{3/2}(x) = \frac{3}{2}(5x^2 - 1)$, $C_4^{3/2}(x) = 15/8(21x^4 - 14x^2 + 1)$. If we assume that the pion wave function is not very different from its asymptotic form, then we can expect that the higher terms in (12) are small. This assumption means that there should be the following relations:

$$1 \gg a_2 \gg a_4 \gg a_6 \gg \ldots \quad (13)$$

In the present analysis we take into consideration only the three leading terms in the expansion (12): 1, $a_2$, $a_4$. At the end of our analysis we demonstrate the presented constraints indicate in favor of validity of the relations (13).

The constraint (5) in the parametrization (12) has the following form,

$$m_2 = \int_0^1 u^2 \varphi_\pi(u) du = \frac{3}{70}(7 + 2a_2) = 0.35 \pm 0.05. \quad (14)$$

Note that the higher terms of expansion (12) do not contribute to the relation (14). The second constraint (6) leads to the following result,

$$\varphi_\pi(0.5) = \frac{3}{2} \left( 1 - \frac{3}{2}a_2 + \frac{15}{8}a_4 \right) = 1.25 \pm 0.25. \quad (15)$$

Using relation (7) in the parametrization (14) gives

$$I = \int_0^1 \frac{\varphi_\pi(u)}{u} du = 3(1 + a_2 + a_4) = 2.4. \quad (16)$$

And, the last constraint (11) gives us the following formula:

$$\varphi_\pi(0.3) = 1.26(1 - 0.3a_2 - 1.317a_4) = 1 \pm 0.2. \quad (17)$$
It is convenient to present all existing constraints on a plot with axis $a_2, a_4$ (see Fig.1). Note that the hatched region in Fig.1 is in agreement with our assumption on the hierarchy (13). Points corresponding to the asymptotic wave function ($a_2 = a_4 = 0$), the Chernyak-Zhitnitsky wave function ($a_2 = \frac{2}{3}, a_4 = 0$), and the Braun-Filyanov wave function ($a_2 = 0.44, a_4 = 0.25$) are also shown on this plot.

Note that due to the relatively small coefficient of $a_2$ in eq.(14), a small uncertainty in the value of $m_2$ leads to a big uncertainty for $a_2$. Assuming that $m_2 = 0.35 \pm 0.05$ we obtain

$$0 < a_2 < 1.2.$$  \hfill (18)

The relation (18) does not determine the value of $a_2$ very accurately, but it is useful, showing that $a_2 > 0$.

The constraints for $\varphi_\pi(0.5)$ and $\varphi_\pi(0.3)$ are more sensitive to the parameters $a_2$ and $a_4$. From relations (15,17) it follows that

$$a_2 = 0.25 \pm 0.25; \quad a_4 = 0.1 \pm 0.12,$$  \hfill (19)

and we can not exclude that the pion wave function attains its asymptotic form. From relation (19) we obtain the following prediction:

$$I = 4 \pm 1.$$  \hfill (20)

Note that the best representation of the quark distribution [11,12] was obtained from the light-cone QCD sum rule for the case when the pion wave function is very close to its asymptotic form. This can be used as an argument in the favor of suggestion that pion wave function is nearly asymptotic.

In summary, we have presented an analysis of the known constraints for the twist-2 pion light-cone wave function. We have accordingly found new bounds on the form of the pion wave function. We note that the light-cone QCD sum rule for the quark distribution in a pion indicates that $\varphi_\pi(u)$ is close to its asymptotic form.

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Figure Captions

Fig.1. Constraints on the first two coefficients of the twist-two pion wave function of Eq. (12). The circle corresponds to the values of the coefficients in the Braun-Filyanov wave function, and the triangle to the coefficients in the Chernyak-Zhitnitsky wave function; the asymptotic wave function sits at the origin. The dotted line is the result with Eq. (16).
Figure 1

- $\phi_\pi(0.5) = 1.25 \pm 0.25$
- $\phi_\pi(0.3) = 1.0 \pm 0.2$