Radii of weakly bound three-body systems: halo nuclei and molecules
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A renormalized three-body model with zero-range potential is used to estimate the mean-square radii of three-body halo nuclei and molecular systems. The halo nuclei (\textsuperscript{6}He, \textsuperscript{11}Li, \textsuperscript{14}Be and \textsuperscript{20}C) are described as point-like inert cores and two neutrons. The molecular systems, with two helium atoms, are of the type \textsuperscript{4}He\textsubscript{2}−X, where X=\textsuperscript{4}He, \textsuperscript{6}Li, \textsuperscript{7}Li, or \textsuperscript{23}Na. The estimations are compared with experimental data and realistic results.

Halo nuclei and weakly bound molecules are quantum systems with very large sizes in which the constituent particles have a large probability to be found much beyond the interaction range. Under this circumstances, the physical properties of such bound systems can be defined by few physical scales.

In the present contribution we report results obtained for the root-mean-square radii of three-body halo nuclei and molecular systems that are obtained from a \textit{universal scaling function} calculated within a renormalized scheme for three particles interacting through pairwise Dirac-\(\delta\) potential. In the case where we have the two-body scattering length, \(a\), much greater than the effective range of the potential, \(r_0\), \((a/r_0 \gg 1)\) our zero-range approach is expected to be a good approximation \cite{1}. In the \textit{scaling limit} \cite{2,3} all the observables of the three-body system can be represented by a function that depends only on the three- and two-body scales.

Figure 1 and Table 1 show results for halo nuclei systems, where \(C\) represents the core and \(n\) the neutron. In Fig. 1, our results \((\hbar = m_n = 1)\) are given as functions of the mass ratio \(A \equiv m_C/m_n\), where \(m_C\) and \(m_n\) are the masses of the core and the neutron, respectively. \(r_\gamma^2\) \((\gamma \equiv C, n)\) is the mean-square distance from the particle \(\gamma\) to the center-of-mass of the system. \(r_{n\gamma}^2\) is the mean-square distance between the particles \(n\) and \(\gamma\). The results given in Fig. 1 are obtained in the limit where the two-body energies are equal to zero, such that the system have just the three-body energy as a physical scale. Our formalism is presented in detail in ref. \cite{4}, where we show that Fig. 1 applies equally well to weakly bound molecular systems, as the results are given in terms of dimensionless quantities.
Figure 1. Dimensionless products $\sqrt{\langle r_{nn}^2 \gamma \rangle E_3}$ [upper (a) plots] and $\sqrt{\langle r_C^2 \gamma \rangle E_3}$ [lower (b) plots], are given as functions of $A \equiv m_C/m_n$, for $E_{nn} = E_{nC} = 0$. The results for the ground-state are shown with solid line ($\gamma = n$) and dot-dashed line ($\gamma = C$); and, for the first excited state, with dashed line ($\gamma = n$) and dotted line ($\gamma = C$).

We can see in Fig. 1 that, although the ratio between the energies of the ground and the first excited states are about few hundreds [5], we have practically no difference between our dimensionless curves, supporting the validity of our scaling limit. The experimental results for the halo nuclei $^6$He ($^4$He+n+n) and $^{14}$Be ($^{12}$Be+n+n) are quite consistent with our predictions based on the zero-range calculations. Therefore, our assumption that such nuclei could be represented by inert cores and two neutrons, interacting through short range forces, produces a reasonable description of the average interparticle distances.

In Table 1 we present the results of the neutron-neutron root-mean-square distances, for the halo nuclei C-n-n system $^6$He, $^{11}$Li, $^{14}$Be, and $^{20}$C. Our results are compared with experimental values, given in ref. [6]. Although the calculations are in reasonable agreement with data, they systematically underestimate the measured values. Analogously, we show in Table 2 our results, obtained in ref. [4] for weakly-bound three-body molecules. In this case, we present results for the root-mean-square distances between the particles and also the root-mean-square distances of each particle to the center-of-mass of the system. For the $^4$He$_3$ trimer, our calculations deviate only about 14% from the realistic results obtained in ref. [7].

In conclusion, the present model for the weakly bound halo nuclei and molecular systems gives a reasonable description of these systems, validated by the comparison with experimental data and realistic results.

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Table 1
Results of the neutron-neutron root-mean-square radii in halo nuclei. The core \((C)\) is given in the first column, the three-body ground state energy and the corresponding two-body energy are respectively given in the second and third columns. The virtual states are indicated by \((v)\), and the \(nn\) virtual state energy is taken as 143 keV. The experimental values, in the last column, come from ref. [6].

| Core  | \(E_3^{(0)}\) (MeV) | \(E_{nC}\) (MeV) | \(\sqrt{\langle r_{nn}^2 \rangle}\) (fm) | \(\sqrt{\langle r_{nn}^{2\text{exp}} \rangle}\) (fm) |
|-------|-------------------|-----------------|------------------|------------------|
| \(^4\)He | 0.97\([8]\) | 0\([9]\) | 5.1 | 5.9±1.2 |
| \(^9\)Li | 0.32\([10]\) | 0.8\([10]\)\((v)\) | 5.9 | 6.6±1.5 |
| \(^{12}\)Be | 1.34\([8]\) | 0.002\([11]\)\((v)\) | 4.4 | 5.4±1.0 |
| \(^{18}\)C | 3.51\([8]\) | 0.16\([8]\) | 3.0 | - |

Table 2
Results for different radii of the molecular systems \(\alpha\alpha\beta\), where \(\alpha \equiv ^4\)He and \(\beta\) is identified in the first column. The ground-state energies of the triatomic molecules and the corresponding energies of the diatomic subsystems, obtained from ref. [12], are given in the second, third and fourth columns. \(\langle r_{\alpha\beta}^2 \rangle\) is the corresponding mean-square distance between the particles \(\alpha\) and \(\gamma\) (= \(\alpha, \beta\)). \(\langle r_{\gamma}^2 \rangle\) is the mean-square distance of \(\gamma\) to the center-of-mass of the system.

| \(\beta\) | \(E_3^{(0)}\) (mK) | \(E_{\alpha\alpha}\) (mK) | \(E_{\alpha\beta}\) (mK) | \(\sqrt{\langle r_{\alpha\alpha}^2 \rangle}\) (\(\AA\)) | \(\sqrt{\langle r_{\alpha\beta}^2 \rangle}\) (\(\AA\)) | \(\sqrt{\langle r_{\gamma}^2 \rangle}\) (\(\AA\)) |
|-------|-----------------|-----------------|-----------------|------------------|------------------|------------------|
| \(^4\)He | 106.0 | 1.31 | 1.31 | 9.45 | 9.45 | 5.55 | 5.55 |
| \(^6\)Li | 31.4 | 1.31 | 0.12 | 16.91 | 16.38 | 10.50 | 8.14 |
| \(^7\)Li | 45.7 | 1.31 | 2.16 | 14.94 | 13.88 | 9.34 | 6.31 |
| \(^{23}\)Na | 103.1 | 1.31 | 28.98 | 11.66 | 9.54 | 8.12 | 1.94 |

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