AdS Membranes Wrapped on Surfaces of Arbitrary Genus

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Abstract

We present and analyze solutions of $D = 11$ supergravity describing the “near-horizon” (i.e., asymptotically $AdS_4 \times S^7$) geometry of M2-branes wrapped on surfaces of arbitrary genus. We study the forces experienced by test M2-branes in such backgrounds, and find evidence that extremal branes on surfaces of genera higher than the torus are unstable. Using the holographic connection between $AdS$ spaces and superconformal field theories in the large $N$ limit, we discuss the phases of the associated $2 + 1$ dimensional theories. Finally, we also study the extension of these solutions to other branes, in particular to D2-branes.
1 Introduction

The solitonic solutions of supergravity theories \[1\] have come to play a fundamental role in our current understanding of M-theory. Very recently, it has been conjectured that by studying the region near the core of certain D- and M-branes one can extract information about the worldvolume dynamics of a large number of such parallel branes, i.e., about the dynamics of superconformal field theories (SCFTs) in the large \(N\) limit \[2, 3, 4\]. A rapidly growing number of recent papers is devoted to testing and extending this correspondence. In particular, those of direct relevance to the \(2 + 1\) SCFT on the M2-brane include \[5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\].

In this paper we present new exact solutions of \(D = 11\) supergravity, which can be interpreted as describing (in a sense to be explained below) the region near the horizon of M2-branes that wrap surfaces of arbitrary topology. First, we study the geometric features of the solutions. Next, we follow \[4, 15\] in trying to obtain information about the phases of the (poorly understood) \(2 + 1\) theory that describes the worldvolume dynamics of a large number of parallel M2-branes. Finally, we analyze the generalization of these solutions to other branes.

2 Geometry of Anti-deSitter M2-branes

Consider the long range supergravity fields describing flat, non-extremal M2-branes,

\[
\begin{align*}
    ds^2 &= H^{-2/3}(-f dt^2 + dx_1^2 + dx_2^2) + H^{1/3}(f^{-1} dr^2 + r^2 d\Omega_2^2), \\
    F_{tx_1x_2r} &= \sqrt{1 + \left(\frac{r_0}{r_2}\right)^6} \partial_r \left(\frac{1}{H}\right), \\
    H &= 1 + \left(\frac{r_2}{r}\right)^6, \quad f = 1 - \left(\frac{r_0}{r}\right)^6.
\end{align*}
\]

Here \(r = r_0\) is the location of the (outer) horizon. The geometry is asymptotic to flat Minkowski space at infinity. Now, consider the case where \(r_0 \ll r_2\), and go to the region near the horizon. This amounts to setting

\[
H = \left(\frac{r_2}{r}\right)^6, \quad F_{tx_1x_2r} = \partial_r \left(\frac{1}{H}\right),
\]

while keeping \(f\) and the rest of the solution as above. The resulting geometry asymptotes to \(AdS_4 \times S^7\), the radius of \(S^7\) being \(r_2\). This geometry can also be obtained by applying a series of U-duality and coordinate transformations to \[17, 18\], which
suggests that both solutions are somehow equivalent in the full M-theory. In fact, recent developments have stressed the fact that the dynamical aspects of branes are essentially encoded in the region near the horizon.

Along the spatial worldvolume directions \((x_1, x_2)\) the horizon of this M2-brane solution is flat. But, in general, we can compactify \(x_1\) and \(x_2\) so that the brane wraps a 2-torus. Our aim now is to construct solutions where the M2-brane wraps surfaces of other genera, like 2-spheres, or surfaces with two or more handles. We will always be working in the region “near the core,” analogous to the \(AdS\) region above. As will be apparent in a moment, these M2-brane solutions take, locally, the form of one of the following three families, labelled by the parameter \(\eta = +1, 0, -1:\)

\[
\begin{align*}
\text{ds}^2 &= H^{-2/3} (-f_\eta dt^2 + dx_1^2 + S^2_\eta(x_1)dx_2^2) + H^{1/3} (f^{-1}_\eta dr^2 + r^2 d\Omega^2_2), \\
F_{tx_1x_2r} &= S_\eta(x_1)\partial_r \left( \frac{1}{H} \right), \\
H &= \left( \frac{r_2}{r} \right)^6, \quad f_\eta = 1 + \eta \left( \frac{r_2}{r} \right)^4 - \left( \frac{r_0}{r} \right)^6,
\end{align*}
\]

where

\[
S_\eta(x_1) = \begin{cases} 
\sin \left( \frac{2\pi x_1}{r_2} \right), & \eta = +1 \\
1, & \eta = 0 \\
\sinh \left( \frac{2\pi x_1}{r_2} \right), & \eta = -1.
\end{cases}
\]

It is straightforward to check that these are solutions of the equations of \(D = 11\) supergravity. Then,

- For \(\eta = 0\) we simply recover the near horizon solution of M2-branes wrapping 2-tori.

- For \(\eta = +1\), we see that \(x_1\) and \(x_2\), restricted to \(x_1 \in [0, \pi r_2/2), x_2 \in [0, \pi r_2)\), parametrize a 2-sphere.

- For \(\eta = -1\) we get the hyperbolic metric \(H_2\) on the plane \((x_1, x_2)\). A standard result from the theory of Riemann surfaces tells us that, by taking quotients of the universal covering of \(H_2\) with discrete subgroups of its isometry group that act freely and properly discontinuously, we can generate the closed Riemann surfaces of any genus higher than 1.

\(^1\)Absence of singularities requires then that \(t\) be compact too.
These solutions are, therefore, enough to describe closed, compact, orientable M2-branes. Non-orientable surfaces, for any genus, can also be obtained by taking in addition quotients by discrete involutions. We will not be considering such branes explicitly, but much of what we will say is applicable to them, too.

The geometry of the solutions in \( S^7 \) splits again into the angular sphere of constant radius, and the asymptotically \( AdS_4 \) spacetimes spanned by \( (t, x_1, x_2, r) \). In this sense, the four-form field strength has produced a spontaneous compactification of the \( D = 11 \) theory down to \( D = 4 \) spacetime with a negative cosmological constant. Interpreted this way, the four dimensional part of the solution corresponds to black holes with horizons of arbitrary topology, which have been the focus of some interest recently (see [20] for extensive references). We will come back to this connection in section 4. For the moment, let us point out that the solutions with \( r_0 = 0 \) are all locally exactly \( AdS_4 \) (and not only asymptotically), differing from each other only in identifications of points.

Let us analyze the existence of (outer) horizons in these solutions (compare [21, 22]). If present, they will correspond to the largest real roots of the equation \( f_\eta = 0 \). For toroidal membranes \( (\eta = 0) \), these occur at \( r = r_0 \). As is well known, if \( r_0 = 0 \) then the zero is a double one and we obtain an extremal, supersymmetric, flat M2-brane.

When \( \eta = +1 \), as long as \( r_0 \neq 0 \) there exists a horizon, whose size decreases with \( r_0 \). In the limit \( r_0 = 0 \) there is no horizon but the spacetime \( (AdS_4 \times S^7) \) is completely non-singular.

The situation for \( \eta = -1 \) is slightly more complicated. It turns out that negative values of \( r_0^6 \) (and therefore imaginary values of \( r_0 \)) yield sensible solutions (i.e., non-singular horizons) as long as they are bigger than a critical value,

\[
(r_0)^6 \geq (r_0c)^6 \equiv -\frac{2(r_2)^6}{3\sqrt{3}}.
\]

Then, when \( r_0 = r_0c \), the function \( f_\eta \) has a double zero at \( r = 3^{1/4}r_2 \). Although this is not interpretable as a horizon, this extremal solution will be of relevance later: it plays the role of the ground state for higher genus membranes. For larger values of \( (r_0)^6 \) we always find a nondegenerate horizon.

If we dimensionally reduce these solutions along the \( x_2 \) direction we obtain string-like solutions of Type IIA supergravity. In particular, for \( \eta = 0 \) we find the familiar solution near the core of a fundamental string. However, for \( \eta \neq 0 \) the dilaton in these solutions becomes singular where \( S_\eta(x_1) = 0 \). E.g., for \( \eta = 1 \), reduction of a spherical
M2-brane along its parallel circles would yield an “open string”, and the singularities at $x_1 = 0, \pi$ would correspond to the endpoints of the string. Notice, however, that the singularity is present at every value of $r$, and not just at the horizon. Thus, the interpretation as an “open string” should not be taken too seriously.

Further reduction along the membrane worldvolume directions, say $x_1$, is hindered by the fact that $\partial/\partial x_1$ does not generate an isometry. For the same reason, we cannot directly apply a T-duality transformation along the “open string” solution.

An important fact to notice is that, for genus different from 1, the size of the membrane at the horizon is fixed once $r_2$ and $r_0$ are chosen. Using the Gauss-Bonnet theorem, the area of the horizon for the genus $g$ membrane ($g \neq 1$) is

$$A_h = 4\pi|g - 1| \left(\frac{r_h}{r_2}\right)^4,$$

where $r_h$ is the value of $r$ at the (outer) horizon. In contrast, for genus 1, the size of the horizon of the toroidal membrane is arbitrary, since we can choose any periodicity for the coordinates $x_1$ and $x_2$.

The charge density of these branes can be readily computed by integrating $*F$ over an angular $S^7$ at constant radius. In units where the eleven dimensional Newton’s constant is equal to $(16\pi)^{-1}$, the charge density is $q_2 = 2\pi^4(r_2)^6$. One would also like to have the energy density, or tension, of these branes. This is, however, more problematic. The ADM masses, or energy densities, are usually defined by integrals on a boundary in the asymptotically flat region. Evidently, this we cannot do in the present case. There do exist definitions of mass in asymptotically anti-deSitter spaces [23], but this is not enough. The definition of mass in asymptotically AdS$_4$ space yields, for the flat M2-brane, only the energy density above the extremal state, since, in the AdS spacetime the extremal state is the natural choice for ground state.

In order to find the mass of the extremal state with respect to the (Poincaré invariant) Minkowski vacuum, we would need to connect our solutions to that state by extending them to a suitable asymptotically flat region. At present we do not have any such extension. A shortcut might be provided by use of the BPS relation between tension and charge density, $T_2 = q_2$ (in the units chosen). The latter, however, need not be valid for M2-branes with topologies different from the torus.

\footnote{This energy is, in fact the same as the thermodynamical energy that we will find in Sec. 5.}
3 Brane probes

Parallel, flat, extremal M2-branes do not exert any static force on each other. This is a consequence of their being BPS states, and it implies that we can stack an arbitrary number of these branes without any energy cost. Such systems are marginally stable. It is of interest to see what are the forces between branes of other topologies.

An easy way to study the forces between parallel similar M2-branes is by considering a light (test) M2-brane in the background of a very massive assembly of similar, parallel M2-branes. The test M2-brane is described in terms of the action

\[ I_{M2} = - T_2 \int d^3 \xi \sqrt{- \det g_{\alpha\beta}} + Q_2 \int A, \]

where \( g_{\alpha\beta} \) and \( A_{\alpha\beta\gamma} \) are the pullbacks to the worldvolume of the spacetime metric and 3-form potential. Since we want to test the forces between M2-branes, we take \( (3) \) as backgrounds, and work in static gauge, where \( \xi^\alpha = X^\alpha \). Note that, clearly, the test M2-brane has the same topology as those creating the background. The static interaction potential \( V(r) \) that the test M2-brane experiences is obtained then from

\[ I_{M2} = - \int dt V(r) \text{.} \]

One easily finds

\[ V(r) \sim H^{-1}(\sqrt{f_\eta} - 1). \]

It is a simple task to plot this potential for the different values of \( \eta \), extremal or non-extremal. Probably, the most interesting case is that of extremal M2-branes. One finds that the test M2-brane

- Is attracted by extremal spherical branes \((\eta = 1, r_0 = 0)\).
- Experiences no static force in the background of extremal flat branes \((\eta = 0 = r_0)\).
- Is repelled by extremal higher genus branes \((\eta = -1, r_0 = r_{0c})\).

This can be taken as an indication that assemblies of spherical, toroidal, and higher genus M2-branes are, respectively, stable, marginal, and unstable. The instability of the solution for M2-branes of higher genus is presumably present already at the classical level, and is most likely due to the negative curvature of the worldsheet of the brane.

If we go on to consider the force experienced by a test M2-brane in the background of non-extremal branes, we find that, at short enough distances, the force is always
attractive. As we increase \( r \) and go to the asymptotically AdS region, the behavior becomes the same as in the extremal background. In particular, the test M2-brane in the non-extremal \( \eta = -1 \) background is attracted near the horizon, but repelled at larger distances.

4 Thermodynamics and holography

In [4, 15], the thermodynamics of the \( AdS_{p+2} \) part of the branes near the horizon has been used to extract information about the phase structure of \((p + 1)\)-dimensional SCFTs in the large \( N \) limit. This correspondence is “holographic” in the sense that the SCFT is associated to the boundary of the AdS space.

Although one is ultimately interested in the thermodynamics in the infinite volume limit, it was found that there are some interesting phenomena at finite volume. In [4], branes with the topology of spheres \( S^p \), described by the Schwarzschild-Anti-deSitter solution in \( p + 2 \) dimensions, were considered. At low temperatures, the dominant phase is described in terms of the no-black-hole, \( AdS_{p+2} \) solution, which exhibits what could be called “kinematic confinement.” At higher temperatures, a phase transition takes place to the solution containing a black hole, which is interpreted as a deconfinement phase (see also [24]). High temperatures here correspond to large horizon sizes, and the spherical brane is better and better approximated by a flat brane: finite size effects become unimportant. Thus, in the infinite volume limit the SCFT is described in terms of the flat brane geometry [15].

In the present situation, we would be dealing with the large \( N \) SCFT in \( 2 + 1 \) dimensions associated to the worldvolume of a large number \( N \) of parallel M2-branes. This theory is, in fact, very poorly understood for any \( N > 1 \). It is therefore very difficult to test the results obtained from the holographic conjecture, which means that they should instead be taken as predictions.

The solutions presented in Section 4 allow us to discuss the theory on manifolds of arbitrary spatial topology. The phases associated to flat and spherical M2-branes are just like in [4]: spherical branes at low temperatures are described by the manifold with \( r_0 = 0 \), but undergo a phase transition at higher temperatures to the manifold with \( r_0 \neq 0 \). The theory on \( \mathbb{R}^2 \) (or on a large torus) is always in the high temperature phase. The entropy in this phase, however, scales as a puzzling \( N^{3/2} \), instead of the more usual “deconfinement” dependence on \( N^2 \) [24]. We discuss now the phases of the \( 2 + 1 \) theory on surfaces with more than one handle.
Since, for the purposes of this Section, we do not need the \(S^7\) part of the metric, use of the “\(D = 11\) coordinates” in (3) becomes somewhat awkward, and it may be convenient to use “\(D = 4\) coordinates” \((t, \theta, \varphi, \rho)\),

\[
\rho = \frac{r^2}{2r_2}, \quad \theta = \frac{2x_1}{r_2}, \quad \varphi = \frac{2x_2}{r_2},
\]

and parameters \(\ell = r_2/2, \mu = \frac{r_0^6}{4r_2^4}\), in terms of which the 4-metric takes the form

\[
ds^2 = -V_\eta dt^2 + V_\eta^{-1} d\rho^2 + \rho^2 (d\theta^2 + S_\eta^2(\theta) d\varphi^2),
\]

\[
V_\eta = \eta - \frac{2\mu}{\rho} + \frac{\rho^2}{\ell^2}.
\]

In this form we can easily make connection with the recent discussion on thermodynamics of \(D = 4\) topological black holes in [21, 22], which we will reinterpret in the context of M-theory. In this parametrization, the extremal solutions for \(\eta = +1, 0\) correspond to \(\mu = 0\), with horizon at \(\rho = 0\), while for \(\eta = -1\) they correspond to \(\mu = \mu_c\) and horizon at \(\rho = \rho_c\), where

\[
\mu_c = -\frac{\ell}{3\sqrt{3}}, \quad \rho_c = \frac{\ell}{\sqrt{3}}.
\]

A concept of the temperature of the horizon can be obtained from standard Euclidean arguments, from the period of Euclidean time needed to avoid conical singularities at the horizon [3]. The extremal solutions are zero temperature states (Euclidean time can be identified with arbitrary periodicity). For states above extremality we obtain inverse temperatures \(\beta\)

\[
\beta = \frac{4\pi\rho_h \ell^2}{3\rho_h^2 + \eta\ell^2} = \frac{2\pi r_h^2 r_2^3}{3r_h^4 + \eta r_2^4},
\]

where \(\rho_h\) is the value of \(\rho\) at the horizon. Notice that for large \(\rho_h\) the temperature grows to a value independent of the topology of the horizon.

The partition function and free energy of the system are computed from the classical action,

\[
-\log Z = \beta F = I_{cl} - I_{cl}^0.
\]

The action of a reference background, \(I_{cl}^0\), must be subtracted for regularization. This reference background acts as a ground state. For flat and spherical branes the
choice is clear: it is the solution with \( r_0 = 0 \), which is locally \( AdS_4 \), and can be at zero temperature. However, for higher genera (\( \eta = -1 \)) the locally \( AdS_4 \) solution \((r_0 = 0 = \mu)\) is not the same as the extremal, zero temperature one, \( \mu = \mu_c < 0 \). Now, computation of the action requires matching the Euclidean geometries of the excited and the ground state at the asymptotic boundary. It does not seem adequate to take \( AdS_4 \) as the reference background. The reason is that absence of conical singularities at the horizon in the solution with \( \mu = 0 \) requires a specific value of Euclidean time periodicity \( \beta \). Matching the boundary geometry of this solution to one with \( \mu \neq 0 \) will introduce singularities at the horizon. In contrast, in the extremal solution with \( r_0 = r_{0c} \) one can identify Euclidean time with arbitrary period without introducing singularities. This suggests that it is the correct reference state \(^4\). It is somewhat worrying, nevertheless, that, as we argued in Sec. \(^3\), this ground state appears to be an unstable one.

Having taken the \( \mu = \mu_c \) state as the reference background, the calculation of the action is straightforward (see, e.g., \([15]\), or \([22]\)), and one finds (hereafter we set \( \eta = -1 \))

\[
\beta F = -\frac{\Omega_g}{4} \rho_h^4 + \frac{\rho_h^2 \ell^2}{4} - \beta \mu_c \frac{\Omega_g}{4\pi}. \tag{14}
\]

Here \( \Omega_g = 4\pi(g - 1) \) is the volume of the unit surface of genus \( g \). We can compute now the thermodynamic energy

\[
E = \frac{\partial (\beta F)}{\partial \beta} = \frac{\Omega_g}{4\pi} (\mu - \mu_c). \tag{15}
\]

This is always positive. Usually, in terms of flat M2-branes, this energy corresponds to the mass above the mass of the extremal state. This interpretation may also be adequate for M2-branes with different topologies. The thermodynamic entropy is given by

\[
S = \beta (E - F) = \frac{\Omega_g}{4} \rho_h^2 = \frac{A_h}{4}, \tag{16}
\]

the usual Bekenstein-Hawking formula. Had we chosen the \( AdS_4 \) solution as background we would have found a different result, as well as negative values for \( E \) \([22]\).

Notice that the ground state has non vanishing entropy, in contrast with flat and spherical branes; we do not have any good explanation for this fact. Finally, we find the specific heat

\[
C = -\beta \frac{\partial S}{\partial \beta} = \frac{\Omega_g}{2} \frac{3\rho_h^2 \ell^2 - \rho_h^2}{3\rho_h^2 + \ell^2}. \tag{17}
\]

\(^4\)This is the view advocated in \([22]\). It also eliminates the notion of negative mass black holes in the \( D = 4 \) context, see eq. \([13]\) below.
This is always positive. What we learn from here is that the 2+1 SCFT defined on a surface of genus higher than 1 does not have phase transitions at finite volume. There is no “kinematic confinement” at low temperatures, and the theory is always in the “high temperature” phase. Also, it should be clear that, as we go to the infinite volume limit, the higher genus manifold becomes indistinguishable from the flat manifold.

5 Other branes

A natural question to ask is whether one can find similar solutions for other branes. The most natural case to consider is that of a D2-brane on an arbitrary genus surface \( \Sigma_g \). But one could also think of \( p \)-branes, \( p > 2 \), on, say, \( \Sigma_g \times T^{p-2} \). For simplicity we give details only for D2-branes, the generalization to other situations presenting no further novelties.

We will only consider the cases with parameter \( r_0 = 0 \). For our present purposes, there are two main differences between the supergravity solutions corresponding to the D2-brane and the M2-brane. First, the D2-brane has non-trivial dilaton. Second, the metric of the flat D2-brane near the horizon does not split into \( AdS_4 \times S^6 \) neither in Einstein frame \( ds^2_E \) nor in string frame, \( ds^2_S \). However, it does so in a conformally related frame,

\[
\begin{align*}
\frac{ds^2}{e^{2\phi/5}} &= e^{\phi/10} ds^2_E, \\
H^{-3/5}(-dt^2 + dx_1^2 + dx_2^2) + H^{2/5}(dr^2 + r^2 d\Omega_6^2),
\end{align*}
\]

That the \((t, x_1, x_2, r)\) part of this metric is locally isometric to \( AdS_4 \) can be easily seen by changing to “\( D = 4 \) coordinates,”

\[
r = \left( \frac{9r_2\rho^2}{4} \right)^{1/3}, \quad x_1 = \ell\theta, \quad x_2 = \ell\varphi, \quad \ell \equiv \frac{2r_2}{3},
\]

in terms of which the 4-metric becomes

\[
ds^2_4 = -\frac{\rho^2}{\ell^2} dt^2 + \frac{\ell^2}{\rho^2} d\rho^2 + \rho^2 (d\theta^2 + d\varphi^2),
\]

i.e., the metric \( (10) \) with \( \eta = 0 = \mu \). We know that the solutions \( (10) \) with \( \mu = 0 \) are, for all three values of \( \eta \), locally identical to \( AdS_4 \). Therefore, the solutions with
\( \eta = \pm 1, \mu = 0, \) which correspond to different topologies, must be locally related to (21) by a change of coordinates (globally one must change the identification of points). Explicitly, if in (21) we change

\[
\begin{align*}
t &\to \ell \sqrt{\rho^2 + \eta \ell^2 \sin(\sqrt{\eta t}/\ell)} / \sqrt{\eta \Delta}, \\
\theta &\to \rho \sin(\sqrt{\eta \theta}) \cos \varphi / \sqrt{\eta \Delta}, \\
\varphi &\to \rho \sin(\sqrt{\eta \theta}) \sin \varphi / \sqrt{\eta \Delta}, \\
\rho &\to \Delta, \\
\Delta &\equiv \rho \cos(\sqrt{\eta \theta}) + \sqrt{\rho^2 + \eta \ell^2 \cos(\sqrt{\eta t}/\ell)},
\end{align*}
\]

for \( \eta = \pm 1, \) then we find the desired forms of the metrics,

\[
\begin{align*}
ds^2_{(4)} &\to ds^2_{(4)} = - \left( \eta + \frac{\rho^2}{\ell^2} \right) dt^2 + \frac{d\rho^2}{\eta + \frac{\rho^2}{\ell^2}} + \rho^2 \left( d\theta^2 + S_{\eta}(\theta) d\varphi^2 \right), \\
S_{\eta}(\theta) &= \sqrt{\eta} \sin(\sqrt{\eta \theta}),
\end{align*}
\]

Now it should be clear how to proceed to find D2-branes with topologies different from the torus: use (19) to express the transformation (21) in terms of “D = 10 coordinates,” and then apply it to the solution (18). The resulting metric can be conformally rescaled to string (or Einstein) frame using the (transformed) dilaton, to find

\[
\begin{align*}
ds^2_S &= e^{\phi/2} ds^2_E = A_{\eta}^{-1/2} \left[ H^{-1/2} \left( -f_{\eta} dt^2 + dx_1^2 + S_{\eta}(x_1) dx_2^2 \right) + H^{1/2} \left( f_{\eta}^{-1} dr^2 + r^2 d\Omega_6^2 \right) \right], \\
e^{\phi} &= H^{1/4} A_{\eta}^{-5/4}, \\
H &= \left( \frac{T_2}{r} \right)^5, \\
f_{\eta} &= 1 + \eta \left( \frac{T_2}{r} \right)^3,
\end{align*}
\]

where, now

\[
S_{\eta}(x_1) = \begin{cases} 
\sin \left( \frac{3x_1}{2r_2} \right), & \eta = +1 \\
1, & \eta = 0 \\
\sinh \left( \frac{3x_1}{2r_2} \right), & \eta = -1,
\end{cases}
\]

and

\[
A_{\eta}(t, x_1, r) = \begin{cases} 
\cos \left( \frac{3x_1}{2r_2} \right) + f_1^{1/2} \cos \left( \frac{3t}{2r_2} \right), & \eta = +1 \\
1, & \eta = 0 \\
\cosh \left( \frac{3x_1}{2r_2} \right) + f_{-1}^{1/2} \cosh \left( \frac{3t}{2r_2} \right), & \eta = -1.
\end{cases}
\]
It is in the conformal factor, or equivalently in the dilaton, where the complication resides: for $\eta = 0$ the dilaton depends only on $r$, but for $\eta = \pm 1$ it acquires also a dependence on $x_1$ and, worse, on $t$, which seems an undesirable feature of these solutions. Notice that the time dependence vanishes at the null Killing surface (which is a horizon for $\eta = -1$) where $f_\eta = 0$. However, for the spherical brane ($\eta = +1$), the conformal factor becomes singular at the equator ($x_1 = \pi r_2/3$) of the horizon.

This procedure is not valid for solutions with $r_0 \neq 0$, since these are locally different from each other. However, one would expect the solutions for these cases to present a similar dependence of the dilaton on $t$ and $x_1$.

In the case of higher $p$-branes, if we start from the flat, extremal solution near the core, compactification on $T^{p-2}$ will yield a metric that, again, can be conformally rescaled to look like $AdS_4 \times S^9$. Then we can repeat the steps above to find spatial worldvolumes of the form $\Sigma_g \times T^{p-2}$.

However, the time dependence and singularities of the dilaton obscure the significance of these solutions for dilatonic branes with non-standard topologies.

6 Final remarks

Usually, one wants to view extremal $p$-branes as solitons interpolating between two different vacua of supergravity: the one near the core ($AdS_{p+2} \times S^9$), and the Minkowski vacuum, $H = 1 = f$ in (3). In the present situation, for non-toroidal M2-branes, one should note that, first, the solution near the core is not supersymmetric. Second, the fields that result from setting $H = 1 = f_\eta$ in (3), with $S_\eta(x_1) \neq 1$ do not solve the equations of $D = 11$ supergravity. It is not clear at present what, if any, is the generalization of (3) with suitable asymptotics (flat in directions transverse to the spatial directions of the brane), in a way similar to the asymptotically flat solution (1).

Closely related to this, there is one obvious shortcoming in the M2-brane solutions that we have introduced: we have no way to define the total energy density of the M2-brane (i.e., the energy measured with respect to a vacuum that is Poincaré invariant in directions transverse to the M2-brane spatial directions), only the energy above the extremal state. Indeed, the fact that extremal M2-branes of higher genera seem to repel each other may be an indication that, for them, the energy density might be much larger than for flat branes. Certainly, they are not BPS states, so we should not, presumably, expect their energy density to be fixed by the value of their charge.
Other recent works have considered modifications of the basic, flat M2-brane solution near the core that change the geometry transverse to the brane \cite{16}, i.e., substituting the angular $S^7$ with a different manifold. Here, in contrast, we have modified the geometry of the brane itself. Both sorts of modifications can be considered and applied independently.

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**References**

[1] For a recent review, see K.S. Stelle, *BPS Branes in Supergravity*, hep-th/9803116.

[2] J. M. Maldacena, *The Large N Limit of Superconformal Field Theories and Supergravity*, hep-th/9711200.

[3] S. Gubser, I. Klebanov, and A. Polyakov, *Gauge Theory Correlators from Non-Critical String Theory*, hep-th/9802109.

[4] E. Witten, *Anti-deSitter Space and Holography*, hep-th/9802150.

[5] R. Kallosh, J. Kumar, and A. Rajaraman, *Special Conformal Symmetry of Worldvolume Actions*, hep-th/9712073.

[6] P. Claus, R. Kallosh, J. Kumar, P.K. Townsend, and A. Van Proeyen, *Conformal Theory of M2, D3, M5 and D1-Branes*, hep-th/9801206.

[7] N. Itzhaki, J.M. Maldacena, J. Sonnenschein, and S. Yankielowicz, *Supergravity and the Large N Limit of Theories with Sixteen Supercharges*, hep-th/9802042.

[8] M. Gunaydin and D. Minic, *Singletons, Doubletons and M-theory*, hep-th/9802047.

[9] L. Castellani, A. Ceresole, R. D'Auria, S. Ferrara, P. Fré, and M. Trigiante, *G/H M-branes and AdS$_{p+2}$ Geometries*, hep-th/9803039.
[10] O. Aharony, Y. Oz, and Z. Yin, *M-theory on AdS*$_p$ × $S^{11-p}$ *and Superconformal Field Theories*, hep-th/9803051.

[11] S. Minwalla, *Particles on AdS*$_{4/7}$ *and Primary Operators on M*$_{2/5}$ *brane World-volumes*, hep-th/9803053.

[12] E. Halyo, *Supergravity on AdS*$_{(4/7)}$ × $S^{(7/4)}$ *and M-branes*, hep-th/9803074.

[13] S. Ferrara, A. Kehagias, H. Partouche, and A. Zaffaroni, *Membranes and Fivebranes with Lower Supersymmetry and their AdS Supergravity Duals*, hep-th/9803109.

[14] J. Gomis, *Anti-deSitter Geometry and Strongly Coupled Gauge Theories*, hep-th/9803119.

[15] E. Witten, *Anti-deSitter Space, Thermal Phase Transition, And Confinement In Gauge Theories*, hep-th/9803131.

[16] E. Halyo, *Supergravity on AdS*$_{5/4}$ × Hopf Fibrations and Conformal Field Theories, hep-th/9803193.

[17] S. Hyun, *U-duality between Three and Higher Dimensional Black Holes*, hep-th/9704005.

[18] H.J. Boonstra, B. Peeters, and K. Skenderis, Phys. Lett. B411 (1997) 59, hep-th/9706192.

K. Sfetsos and K. Skenderis, *Microscopic Derivation of the Bekenstein-Hawking entropy formula for non-extremal black holes*, hep-th/9711138.

[19] G.W. Gibbons, *Wrapping Branes in Space and Time*, hep-th/9803206.

[20] R.B. Mann, *Topological Black Holes: Outside Looking In*, gr-qc/9709039.

[21] D. Brill, J. Louko, and P. Peldán, Phys. Rev. D56 (1997) 3600, gr-qc/9705012.

[22] L. Vanzo, Phys. Rev. D56 (1997) 6475, gr-qc/9705004.

[23] L.F. Abbott and S. Deser, Nucl. Phys. B195 (1982) 76.

[24] I. Klebanov and A.A. Tseytlin, Nucl. Phys. B475 (1996) 164, hep-th/9604089.

[25] G.W. Gibbons and P.K. Townsend, Phys. Rev. Lett. 71 (1993) 3754, hep-th/9307049.