Enhancing capacity of coherent optical information storage and transfer in a Bose-Einstein condensate

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Abstract

Coherent optical information storage capacity of an atomic Bose-Einstein condensate is examined. Theory of slow light propagation in atomic clouds is generalized to short pulse regime by taking into account group velocity dispersion. It is shown that the number of stored pulses in the condensate can be optimized for a particular coupling laser power, temperature and interatomic interaction strength. Analytical results are derived for semi-ideal model of the condensate using effective uniform density zone approximation. Detailed numerical simulations are also performed. It is found that axial density profile of the condensate protects the pulse against the group velocity dispersion. Furthermore, taking into account finite radial size of the condensate, multi-mode light propagation in atomic Bose-Einstein condensate is investigated. The number of modes that can be supported by a condensate is found. Single mode condition is determined as a function of experimentally accessible parameters including trap size, temperature, condensate number density and scattering length. Quantum coherent atom-light interaction schemes are proposed for enhancing multi-mode light propagation effects.

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I. INTRODUCTION

Light can be slowed down via electromagnetically induced transparency (EIT) in an atomic medium. This effect is solely due to the steep dispersion at the EIT resonance. It has been demonstrated both for the ultracold Bose-Einstein condensate (BEC)\[1, 2\] as well as for an atomic cloud at the room temperature\[3\]. In a typical experimental set up for EIT, a probe pulse is sent to an atomic cloud, consisting of effectively three level atoms. The atomic medium would be normally opaque to the transmission of the probe pulse at resonance. Another, and more stronger laser pulse is used to establish quantum coherence in the lower level doublet so that the system is put in a dark state, in which destructive quantum interference of probability amplitudes for upper level transitions cancel each other. This allows for propagation of the probe pulse in the medium at group speeds determined by the steepness of the dispersion curve at resonance. In the case of EIT, the dispersion is so steep that the probe pulse can only propagate at very low speeds in the order of few m/s.

In the experiments, group velocity is measured operationally and indirectly from the delay time. Probe pulse is divided into two and one pulse is propagated in free space without entering into the atomic medium. Using this pulse as a reference, delay time for the probe pulse is determined. The length of the atomic cloud is measured by using an imaging laser. The ratio of the length of the medium to the delay time, which gives the group velocity of the probe pulse, is calculated at various temperatures. The temperature dependence of the group velocity exhibits a sharp drop at the critical temperature of condensation, and can be understood from the temperature dependence of the condensate density. In order to estimate the group velocity, one can assume uniform density over the width of the condensate, which would be taken as the effective length of the condensate. From the EIT susceptibility, 

\[
\chi(\vec{r}) = \rho(\vec{r}) \frac{\mu_{31}}{\varepsilon_0 \hbar} \frac{i (i \Delta + \Gamma_2 / 2)}{[(i \Delta + \Gamma_2 / 2) (i \Delta + \Gamma_3 / 2) + \Omega_c^2 / 4]},
\]

one can calculate the group velocity, defined by

\[
\frac{1}{v_g} = \frac{1}{c} - \frac{\pi}{\lambda} \frac{\partial \chi}{\partial \omega} (\omega_0),
\]

which leads to

\[
v_g = \frac{c \varepsilon_0 \hbar \Omega_c^2}{2 \omega_0 |\mu_{31}|^2 \rho}.
\]
It should be noted that this is valid for pump laser powers in the range
\[ \sqrt{\frac{4\pi c |\mu_{31}|^2 \rho}{\varepsilon_0 \hbar}} \gg \Omega_c \gg \Gamma_{2,3}. \quad (4) \]

Slow light allows for novel nonlinear optical regimes and applications. Early proposals include enhanced nonlinear optical effects\[^3\], nonlinear magneto-optics\[^4\], nonlinear optics with single photons\[^5, 6\]. Besides, slow light is thought to be useful for optical data storage, nonlinear optics with few photons, quantum entanglement of slow photons, and enhancing acousto-optical effects\[^7\]. These effects may be useful in all-optical computers, telecommunication systems, and memory chips. Most promising application of slow light is believed to be optical data storage, whose possibility is experimentally demonstrated\[^8\]. While slow light in atomic systems, particularly in BECs, could be useful in quantum information technology, for practical quantum information processing it is necessary to be able to inject more pulses into the atomic system during the storage time\[^8\].

A significant parameter to characterize the practical value of an information storage device is the bit storage capacity, which should be as high as possible for a useful dynamic memory device. Bit storage capacity can be defined as the number of bits that can be simultaneously present in the device during the storage time. In the case of EIT based slow light in atomic systems, bits correspond to optical probe pulses. In the present experiments, both the storage time and probe pulse widths are about few microseconds. Therefore, only single pulse can be present in the condensate during the storage time.

It should be noted that multiple pulses of different polarization could be injected into the atomic cloud under EIT conditions at the cost of tolerable absorption as well as increased group speeds\[^9\]. Modification of the EIT transparency window may also be exploited to inject multiple pulses\[^10\]. In order to enhance the bit storage capacity, recent experiments and proposals indicate that perhaps the most promising and straightforward direction could be just to use shorter pulses\[^11, 12, 13\].

One serious obstacle that hinder immediate application of short pulses into atomic gases to enhance their information storage and transfer capacities is that short pulses may suffer strong pulse shape distortions due to high order dispersion effects. In particular, second order dispersion yields pulse spread due to group velocity dispersion. Different frequency components of the pulse travel at different speeds resulting in broadening of the pulse over time. Using nanosecond pulses, for example, instead of microsecond ones, may not neces-
sarily increase the bit storage capacity 1000 times more. To see this quantitatively, let us consider an ideal pulse repetition rate as $1/2\tau$. The total number of pulses to be stored simultaneously in the condensate of width $L$ for the duration $t_s = L/v_g$ is found by

$$C = \frac{L}{2v_g\tau},$$  \hspace{1cm} (5)

which shows its limitation by the pulse spread. It is therefore necessary to determine the actual bit storage capacity by taking into account higher order dispersive effects in slow light propagation via EIT through atomic condensate.

II. SHORT PULSE PROPAGATION THROUGH AN ATOMIC BOSE-EINSTEIN CONDENSATE

We calculated that for the present slow light experiments, third and higher order dispersion effects are negligible. In fact, we have found the third order dispersion coefficient is about seven orders of magnitude smaller than the second order dispersion coefficient. Taking into account this group velocity dispersion, the wave equation describing slow short pulse propagation in the condensate can be written by

$$\frac{\partial E}{\partial z} + \alpha E + \frac{1}{v_g} \frac{\partial E}{\partial t} + ib_2 \frac{\partial^2 E}{\partial t^2} = 0.$$  \hspace{1cm} (6)

Here, the attenuation constant

$$\alpha = -\frac{i\pi}{\lambda} \chi(\omega_0),$$  \hspace{1cm} (7)

group velocity,

$$\frac{1}{v_g} = \frac{1}{c} - \frac{\pi}{\lambda} \frac{\partial \chi}{\partial \omega}(\omega_0),$$  \hspace{1cm} (8)

and the second order dispersion coefficient,

$$b_2 = \frac{\pi}{2\lambda^2} \frac{\partial^2 \chi}{\partial \omega^2}(\omega_0).$$  \hspace{1cm} (9)
are all calculated at the resonance. For the range \( \sqrt{4\pi c} |\mu_{31}|^2 \rho/\varepsilon_0 \hbar \gg \Omega_c \gg \Gamma_{2,3} \), they are given by

\[
\alpha = \frac{2\pi \rho |\mu_{31}|^2 \Gamma_2}{\varepsilon_0 \hbar \lambda \Omega_c^2}, \tag{10}
\]

\[
v_g = \frac{\varepsilon_0 \hbar \Omega_c^2}{2\omega_0 |\mu_{31}|^2 \rho}, \tag{11}
\]

\[
b_2 = \frac{i 8\pi \Gamma_3 |\mu_{31}|^2 \rho}{\varepsilon_0 \hbar \lambda \Omega_c^4}. \tag{12}
\]

The range of pump power can be established for \( \Omega_c \sim 5\gamma - 15\gamma \) when \( \rho \sim 10^{20} - 10^{21} \text{m}^{-3} \). It may be noted that broadening is more sensitive to pump power than group velocity. This may be exploited to optimize group delay and broadening for the purpose of maximum bit storage capacity. For that aim let us now determine pulse broadening and capacity analytically to see how they depend on pump power. Analytical solution of the wave equation is possible for a Gaussian pulse propagating through a uniform dispersive medium. We consider an effective width \( L \), given by the rms width of the condensate, over which density can be considered uniform. This treatment can be used for qualitative understanding of the pulse shape variations in the condensate. After a delay time of

\[
t_d = \frac{L}{v_g} - \frac{L}{c}, \tag{13}
\]

the pulse width will broaden into

\[
\tau(L) = \tau_0 \sqrt{1 + \left( \frac{L}{z_0} \right)^2} \tag{14}
\]

where

\[
z_0 = -\frac{\pi \tau_0^2}{b_2}. \tag{15}
\]

When \( L \gg z_0 \) pulse spread can be determined from

\[
\tau(L) \approx \frac{|b_2| L}{\pi \tau_0}. \tag{16}
\]

Using \( C = L/2v_g \tau \) we find that

\[
C = \frac{L}{2\tau_0 \sqrt{4\pi^2 \Omega_c^4/9\lambda^4 \gamma^2 \rho^2 + 4L^2 \Gamma_3^2/\pi^2 \tau_0^2 \lambda \Omega_c^4}}. \tag{17}
\]
We observe that two terms under the square root are competing with each other, depending on the increase of the pump power. At a critical Rabi frequency, given by
\[
\Omega_{c0} = \left( \frac{3\Gamma_3 \gamma^2 \lambda^2 (\rho L)}{\pi^2 r_0^2} \right)^{1/4},
\] (18)
bit storage capacity becomes a maximum with a value
\[
C_{\text{max}} = \sqrt{\frac{3\gamma^2 \lambda^2 (\rho L)}{32\Gamma_3}}.
\] (19)
Storage time in this case turns out to be
\[
t_{s0} = \frac{\tau_0 \lambda}{2} \sqrt{\frac{3\gamma (\rho L)}{\Gamma_3}}.
\] (20)
These expressions are all show dependence on density profile of the cloud and using parameters of the trapping potential as well as temperature and atomic scattering lengths, they can be tuned for an optimum bit storage capacity obtained at a critical laser power.

Let us now examine the practical control parameters to optimize bit storage capacity by taking into account spatial inhomogeneity of cloud in detail. To take it into account analytically, we consider a semi-ideal BEC[14] where thermal and condensed components overlap negligibly. This allows for separation of atomic density in the form of
\[
\rho (\vec{r}) = \rho_c (\vec{r}) + \rho_{th} (\vec{r}),
\] (21)
where
\[
\rho_c (\vec{r}) = \frac{\mu - V (\vec{r})}{u_0} \Theta (\mu - V (\vec{r})) \Theta (T_c - T)
\] (22)
represents the condensed component under Thomas-Fermi approximation and
\[
\rho_{th} (\vec{r}) = \frac{g_{3/2} \left[ z e^{-\beta V (\vec{r})} \right]}{\lambda_f^3}
\] (23)
is the density of the thermal component of the cloud. Here $V (\vec{r}) = (m/2) (\omega_r^2 r^2 + \omega_z^2 z^2)$ is the trap potential written in cylindrical coordinates, $u_0 = 4\pi \hbar^2 a_s / m$ is the atomic interaction coefficient. Fugacity is given by $z = e^{\beta \mu}$. A Bose function is defined to be
\[
g_n (x) = \sum_j x^j / j^n.
\] (24)
This model provides a good analytical fit to the experimental BEC density profile at a finite temperature. Chemical potential $\mu$ is found from $N = \int d^3 \vec{r} \rho (\vec{r})$. At low temperatures $T < T_c$ this yields

$$\mu = \mu_{TF} \left( \frac{N_0}{N} \right)^{2/5}$$  \hspace{1cm} (25)

with

$$\frac{N_0}{N} = 1 - x^3 - s \frac{\zeta (2)}{\zeta (3)} x^2 (1 - x^3)^{2/5}.$$  \hspace{1cm} (26)

Here $x \equiv T/T_c$ and $s$ is a scaling parameter. The semi-ideal model works best for $s < 0.4$. From its definition

$$s = \frac{\mu_{TF}}{k_B T_c} = \frac{1}{2} \zeta (3)^{1/3} \left( 15 N^{1/6} a_s \right)^{2/5}$$  \hspace{1cm} (27)

where $a_h = \sqrt{\hbar/m (\omega_z \omega_r)^{1/3}}$, we find that in slow light experiments semi-ideal model description can be used up to scattering lengths $a_s < \sim 8\text{nm}$. It may be noted that at high temperatures $T > T_c$ chemical potential is determined from the usual expression

$$\text{Li}_3 (z) = \frac{\zeta (3)}{x^3},$$  \hspace{1cm} (28)

where $\text{Li}_3$ is a polylogarithm. We also note that at all temperatures it is required that $\mu \gg \hbar \omega_{r,z}$.

Using the semi-ideal model, we can establish how propagation parameters, $\alpha, v_g, b_2$ gain nonlocal character for spatially nonuniform bosonic atomic cloud\cite{15}. Assuming negligible spatial inhomogeneity over the central region of width

$$L = \left[ \frac{4 \pi}{N} \int_0^\infty r dr \int_0^\infty dz z^2 \rho (r, z) \right]^{1/2}$$  \hspace{1cm} (29)

and using the peak density value, which depends on temperature and interatomic interactions as indicated by the semi-ideal model, we can calculate the broadening of the pulse analytically\cite{15}. As the broadening is determined by the product $\rho L$, and $\rho$ and $L$ exhibit competing dependencies on temperature, a maximum value emerges just before the critical temperature in the broadening\cite{15}.

Performing numerical simulations using Crank-Nicholson method to solve the wave equation with spatially dependent coefficients, we found that analytical method describes the
behavior of broadening very well qualitatively [15], though quantitatively broadening is overestimated by the analytical method. Bit storage capacity is found higher than predicted by analytical calculation. The reason is that pulse broadens as it approaches to more dispersive central region. As a result it is guarded against further spreading. This is a unique advantage of cold Bose gas in the light propagation due to its characteristic density profile.

III. MULTIPLE OPTICAL MODES WITHIN AN ATOMIC BOSE-EINSTEIN CONDENSATE

In addition to diffraction [16], effect of finite radial size of the cloud could be excitation of multiple modes within the cloud. We note that the refractive index for the radial dimensions can be expressed as

\[ n(r) = \begin{cases} n_0 \left[ 1 - 2\kappa \left( \frac{r}{R_{TF}} \right)^2 \right]^{1/2}, & r \leq R_{TF}; \\ 1, & r \geq R_{TF}. \end{cases} \] (30)

at temperatures well below \( T_C \). Here \( n_0 = \sqrt{1 + \chi(0)} \) corresponds material refractive index at the center of the cloud, \( \kappa = \chi(0)/4 (1 + \chi(0)) \), and \( R_{TF} = \sqrt{2\mu(T)/m\omega_r^2} \) is the radial Thomas-Fermi radius. It should be noted that under EIT conditions, thermal cloud and the condensed component would have same refractive index at EIT resonance. On the other hand, slightly detuned pulse may excite multiple modes in the condensate. We now establish conditions for single mode and multi-mode excitations for such a case.

Ignoring the group velocity dispersion in the radial dimensions, the wave equation \((\nabla^2 + k^2) E_t = 0\), with \( k = \omega n/c \) can be solved using an ansatz of the form \( E_t = \psi(r)e^{i\phi}e^{-i(\omega t - \beta z)} \). This yields

\[ \left[ \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} + p^2(r) \right] \psi(r) = 0 \] (31)

where

\[ p^2(r) = k_0^2 n^2(r) - \beta^2 - \frac{l^2}{r^2} \] (32)

with \( k_0 = \omega/c \). Number of modes supported by the condensate can be estimated by a simple WKB treatment. Quantization condition

\[ \int_{r_1}^{r_2} p(r) dr = \left( m + \frac{1}{2} \right) \pi m = 0, 1, 2, \ldots \] (33)
leads to the propagation constant

$$\beta_{lm} = n_0 k_0 \left[ 1 - \frac{2\sqrt{2\kappa}}{n_0 k_0 R_{TF}} (l + 2m + 1) \right]^{1/2},$$  \hspace{1cm} (34)$$

so that the number of modes are determined from the constraint

$$2(l + 2m + 1) \leq n_0 k_0 R_{TF} \sqrt{2\kappa}.$$  \hspace{1cm} (35)$$

Single mode condition for a BEC can be imposed in terms of radial size of the atomic cloud. Using experimental parameters we find that single mode condition at $T = 42 \text{nK}$ can be expressed in terms of the radial radius of the cloud being $R < 1.13 \text{\mu m}$. Around $T_c$ this radius can be about $\sim 3 \text{\mu m}$.

Multimode propagation could be useful for handling many transmission channels on one material. Short axial size of BEC and its unique density profile can be exploited to keep modal dispersion weak. In such short distance applications, this may be utilized to capture optimum amount of power from a light source and enhance the optical coupling capability of BEC. The modes closer to the thermal envelope, where the cloud is less dense, have faster group velocities than those in the central region. This results in pulse spread less than the case of a more uniform cloud. Because of Thomas-Fermi density profile, modal dispersion in the condensate becomes more significant at lower temperatures. Finally we propose that more index contrast and hence support of more modes in the BEC can be possible by some other quantum coherent effects such as index enhancement, alternative to off-resonant EIT, if one demands multiple mode excitations in the BEC.

IV. CONCLUSION

Summarying, we have studied short pulse propagation under EIT conditions within an atomic semi-ideal BEC. Third and higher order dispersive effects were found to be negligible in current BEC systems for propagation of pulses of widths up to nanoseconds. For pulses of widths greater than microseconds second order dispersion was also found to be insignificant. We have found that just below the critical temperature, broadening of the pulse becomes maximum. Besides, for a semi-ideal BEC, we show that broadening decreases with increasing scattering length. Taking into account spatial inhomegeneity of the cloud, we conclude that axial spatial variation of the cloud guards the pulse against spreading due to group velocity
dispersion. As the pulse broadens while it gets closer and closer to central region, it suffers less and less dispersive effects. Analytical studies indicated that there is a critical pump power which gives maximum achievable bit storage capacity. This value was shown to be independent of initial pulse width, but strongly depends on density profile of the atomic cloud. Multimode pulse propagation due to finite radial size was also examined. Single mode condition for light propagation was established for atomic BEC. Either off-resonant EIT or quantum coherent index enhancement can be used to realize and enhance multi-mode excitations. We conclude that radial density profile of BEC protects the pulse against spreading due to modal dispersion.

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