Light quark masses and CKM matrix elements from lattice QCD

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I give a brief overview of recent results from lattice QCD calculations which are relevant for the phenomenology of the Standard Model. I discuss, in particular, the lattice determination of light quark masses and the calculation of those hadronic quantities, such as semileptonic form factors, decay constants and $B$-parameters, which are of particular interest for the analysis of the CKM mixing matrix and the origin of CP violation.

1. LIGHT QUARK MASSES

Calculations of light quark masses are becoming, at present, one of the most important subject of investigation for lattice QCD. Indeed, in spite of their relevant phenomenological interest, light quark masses are still among the less known fundamental parameters of the Standard Model.

In the study of quark masses, the lattice method provides a unique approach: quark masses, defined as effective couplings renormalized at short distances, can be in fact determined on the lattice from non-perturbative calculations of hadronic quantities. An inspection of recent lattice results for quark masses suggests, however, that in such calculations a better understanding and quantification of the systematic errors is still an important requirement. To be specific, let us consider the lattice determinations of the strange quark mass, $m_s$. A compilation of results for this mass, in the $\overline{\text{MS}}$ scheme, at the renormalization scale $\mu = 2\text{ GeV}$, is shown in Table 1. All the results have been obtained in the quenched approximation. As can be seen from the table, although the lattice predictions all lie in the range $100 \lesssim \overline{m}_s \lesssim 160\text{ MeV}$, however, within the errors quoted by the authors, the results obtained from different numerical studies are often in disagreement.

The main two sources of systematic errors, which are responsible for the discrepancies discussed above, are easily identified. The quark mass that is directly computed in lattice simulations is the (short-distance) bare lattice quark mass $m(a)$, where $a$ is the lattice spacing. This quantity is typically determined by fixing, to its experimental value, the mass of a hadron containing a quark with the same flavour. Since the non-perturbative lattice calculation of hadronic quantities is affected by discretization errors (in general of $O(a)$), the same systematic errors will also propagate into the final determination of the quark masses.

The second step in these calculations consists in relating the bare quark mass $m(a)$ to the renormalized mass $m(\mu)$, in a given renormalization scheme. The connection is provided by a multiplicative renormalization constant, $m(\mu) = \overline{m}_s(\mu)$. 

| Yr       | $\overline{m}_s$(2 GeV)/MeV |
|----------|------------------------------|
| APE [1]  | 94 128 ± 18                  |
| LANL [2] | 96 100 ± 21 ± 10             |
| FNAL [3] | 96 95 ± 16                   |
| APE [4]  | 96 122 ± 20                  |
| SESAM [5]| 97 166 ± 15                  |
| CP-PACS [6]| 97 135 ± 7 (from $m_\phi$) |
|         | 97 111 ± 4 (from $m_K$)      |
| JLPACD [7]| 97 97 ± 9                   |
| QCDSF [8]| 97 112 ± 5 $O(a^2)$          |
| APETOV [9]| 97 111 ± 15 $O(a^2)$        |
| APE [10] | 98 130 ± 2 ± 18 NPR          |
| APE [11] | 98 121 ± 13 $O(a^2) +$ NPR  |
discretization errors are reduced to out at the non-perturbative level in [15]-[17]. The turbatively in [13,14] and, more recently, carried operators. This program has been realized per-
and consists in improving the lattice action and errors has been suggested by Symanzik [12]
cially reduced. The way to deal with discretiza-
masses, can be (and in fact have been) drasti-
tors, in the lattice determination of light quark
discretization and higher orders perturbative er-
the quark mass.

Higher orders perturbative errors in the evaluation of the quark mass renormalization constant are eliminated by adopting the non-perturbative renormalization prescription proposed in ref. [18]. The idea consists on imposing the renormal-
ization conditions directly on non-perturbatively calculated correlation functions between external off-shell quark states. This prescription defines the mass in a specific scheme, called RI-MOM scheme. Lattice calculations in this scheme are thus completely non-perturbative. Perturbation theory only enters if one wants to convert the re-
term to the mass in the theory only enters if one wants to convert the re-
results of ref. [11]:

$$m_l^{NLO} = 4.9(4) \text{ MeV}$$

$$m_s^{NLO} = 121(13) \text{ MeV}$$

(1)

and

$$m_l^{N^2LO} = 4.5(4) \text{ MeV}$$

$$m_s^{N^2LO} = 111(12) \text{ MeV}$$

(2)
at the NLO and N^2LO respectively. In these estimates discretization and higher orders perturbative errors are expected to be negligible, and the remaining uncertainty is mainly due to the quenched approximation. The values in eq. (1)}
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Figure 1. $Z_S$, obtained by using the non-perturbative method, as a function of the renormalization scale. The dashed and solid curves represent the solutions of the renormalization group equations at the LO and NLO respectively.

are also in good agreement with recent results from QCD sum rules [20].

2. CKM MATRIX ELEMENTS

In the analysis of the CKM mixing matrix, a particularly interesting issue is represented by the study of the unitarity triangle in the $\rho$-$\eta$ plane. Indeed, a non trivial shape of this triangle is the signature of a CKM source of CP violation.

At present, the phenomenological analysis of the unitarity triangle is constrained (mainly) by the existing measurements of $B_{d,s}^0 - \bar{B}_{d,s}^0$ mixings, $K^0 - \bar{K}^0$ mixing and semileptonic $b \to u$ transitions. Once these measurements are compared with the theoretical predictions, constraints on the $\rho$-$\eta$ parameters are derived. The status of the art is shown in Figure 2 from ref. [21]. The theoretical predictions are based on the non-perturbative evaluation of some relevant hadronic matrix elements, which, in turn, are parameterized in terms of form factors, decay constants and $B$-parameters. The lattice method provide a reliable approach to perform such calculations. In the following, I will present a short compilation of these lattice results.

2.1. $V_{ub}$ from $B \to \pi, \rho$ semileptonic decays

The determination of the $V_{ub}$ mixing angle is allowed by the analysis of $B \to \pi, \rho$ semileptonic decays. The reliability of lattice calculations in the study of heavy mesons semileptonic decays is provided by the analysis of $D$-meson decays, $D \to K, K^* l\nu$. In this processes the relevant mixing angle $V_{cs}$ is well constrained by the unitarity of the CKM matrix, so that the theoretical predictions can be compared with the experimental measurements. The comparison is shown in Figure 3 where the lattice results for the four relevant form factors, at zero momentum transfer, are presented together with their experimental values [22]-[28]. A summary of these results is also given in Table 2.

Figure 2. The unitarity triangle in the $\rho$-$\eta$ plane and the constraints derived from $\Delta m_d$, $\Delta m_s/\Delta m_d$, $|\epsilon_K|$ and $|V_{ub}|/|V_{cb}|$.

Figure 3. Lattice results for the $D \to K, K^* l\nu$ form factors, at zero momentum transfer, [22]-[28]. The horizontal band indicates the present experimental average.

For the $B \to \pi, \rho$ semileptonic decays, four different lattice groups have presented results so far.
Table 2
Summary of lattice and experimental results for $D \to K, K^*$ semileptonic decays form factors at zero momentum transfer. The results for $B \to \pi, \rho$ semileptonic decays, from ref. [30], are also presented.

|         | $D \to K, K^*$ | $B \to \pi, \rho$ |
|---------|----------------|------------------|
| Lattice | Exp.          | Lattice         |
| $f_+(0)$ | 0.72(7)       | 0.76(3)         | 0.27(11) |
| $V(0)$  | 1.14(18)      | 1.07(9)        | 0.35(6) |
| $A_1(0)$ | 0.64(8)       | 0.58(3)       | 0.27(5) |
| $A_2(0)$ | 0.53(13)      | 0.41(5)      | 0.26(3) |

Table 3
Summary of lattice results for $D$ and $B$-meson decay constant and $B$-parameters

| $f_D$/(MeV) | $f_{D_s}$/(MeV) | $f_B$/(MeV) | $f_{B_s}/f_{B_B}$ | $f_{B_B}/f_B$ |
|-------------|----------------|-------------|------------------|---------------|
| 200(30)     | 220(30)       | 170(35)     | 1.14(8)          | 1.00(3)       |

2.3. $\Delta m_d$ and $\Delta m_s$ from $B$-meson decay constants and $B$-parameters

The theoretical estimates of the $B$-meson mass differences, $\Delta m_d$ and $\Delta m_s$, can be expressed in terms of the pseudoscalar decay constants, $f_{D,s}$, and $B$-parameters. A recent compilation of lattice results for these quantities has been presented in ref. [34] and it is shown in Table 3. Notice that the lattice value for $f_{D_s}$, which has been predicted well before the first experimental measurement, is in good agreement with the present experimental average, $f_{D_s} = 243 \pm 36$ MeV [21]. The lattice results for this quantity have always been stable in the time. This is shown in Figure 4 where these results, obtained over a period of 10 years, are presented together with the current experimental value. The last point in

Figure 4. Lattice results for the pseudoscalar decay constant $f_{D_s}$. The horizontal band indicates the present experimental average. We refer to ref. [7] for a compilation of results and corresponding references.
sured yet in the experiments. From the (overconstrained) fit of the Standard Model, in ref. [21] the value
\[ f_B \sqrt{B_B} = 213(21)(20) \text{ MeV} \] (5)
is obtained, in good agreement with the lattice determination (see Table 3). An independent determination of \( f_B \) can be also derived by combining the lattice determination of the ratio \( f_B/f_{D_s} \), in which many of the systematic errors are expected to cancel, with the experimental measurement of \( f_{D_s} \). Two recent lattice calculations give:
\[ \frac{f_B}{f_{D_s}} = \left\{ \begin{array}{c}
0.77(8) \\
0.75(5)(7)
\end{array} \right. \text{ APE [35]} \\
\text{ MILC [36] (6)} \]
From these values I find:
\[ f_B = (185 \pm 27 \pm 19) \text{ MeV} \] (7)
where the first error comes from the experimental uncertainty on \( f_{D_s} \) and the second one from the theoretical uncertainty on the ratio \( f_B/f_{D_s} \). Eq. (7) is also in good agreement with the direct determination of \( f_B \) given in Table 3.

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