Testing extended theories of gravity with GRBs

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We present our studies on the neutrino pairs annihilation into electron-positron pairs ($\nu\bar{\nu} \rightarrow e^-e^+$) near the surface of a neutron star in the framework of extended theories of gravity. The latter modifies the maximum energy deposition rate near to the photonsphere and it might be several orders of magnitude greater than that computed in the framework of General Relativity. These results provide a rising in the Gamma-Ray Bursts energy emitted from a close binary neutron star system and might be a fingerprint of modified theories of gravity, changing our view of astrophysical phenomena.

Keywords: Extended theories of gravity, GRB, Neutrino energy deposition, Black Holes, Neutron Stars

1. Introduction

General Relativity (GR) is without any doubt the best theory of gravitational interaction. Its predictions have been tested with high accuracy on scales of the solar system (for example, the precession of the Mercury perihelion, the photons deviation and the gravitational leasing effect), on astrophysical scales (the gravitational waves), and cosmological scales (the cosmic microwave background radiation (CMBR) and the formation of primordial light elements (Big Bang Nucleosynthesis)). Despite these results, there are still open questions that make GR incomplete. The latter arise at short distances and small time scales (black hole and cosmological singularities, respectively), or at large distance scales, the rotational curve of the galaxies and the observed accelerated phase of the present Universe, for which any predictability is lost.

To solve these issues, deviations from the GR (hence from the Hilbert-Einstein action on which GR is based) are needed, and new ingredients, such as dark matter and dark energy, are required for fitting the present picture of our Universe. Indeed, in the last years, several alternative or modified theories of gravity have been proposed, which try to answer all the opened questions of GR and the Cosmological Standard Model. To give an example, higher-order curvature invariants than the simple Ricci scalar $R$, allow getting inflationary behaviour, removing the primordial singularity, as well as explaining the flatness and horizon problems. On the other hand, one can tend to preserve the GR and the Hilbert-Einstein action, adding only some two unidentified components: dark matter and dark energy. These two different approaches try to solve the same problems but at the moment there is not a final solution for that (modified gravity...
theories do not manage to solve all the problems while dark matters elements are still missing). One of the consequences of these approaches is that the metric tensor \( g_{\mu\nu} \) describing the gravitational field generated by a massive source gets modified with respect to GR metric, in particular, the Schwarzschild or Kerr metric. The latter are recovered in the limit in which the parameters characterizing some specific theory of gravity beyond GR are set to zero.

In this proceeding, we highlight the differences between GR and extended theories of gravity arising from the mechanism of generation of Gamma-Ray Bursts (GRBs). We focus, in particular, on GRBs powered by neutrino annihilation processes \( \nu \bar{\nu} \rightarrow e^+ e^- \).

The neutrinos annihilation process is relevant in many astrophysical frameworks: in the stellar envelope, as well as on the delay shock mechanism into the Type II Supernova (at late time, from the hot proto-neutron star, the energy is deposited into the supernova envelope via neutrino pair annihilation and neutrino-lepton scattering). These processes augment the neutrino heating of the envelope generating a successful supernova explosion\(^44\). Moreover, the neutrino annihilation process has been also proposed as the mechanism that power GRB in a binary neutron star system, which is the topic of this work. Simulations and analytical estimations performed within GR show that the mechanism is not sufficient to generate the required energy for explaining short GRB. Such a conclusion changes, as we will show, if the gravitational background is described by modified theories.

2. Neutrino energy deposition formulation

In this section we recall the main features to treat the energy deposition in curved spacetime\(^45,46\). Previous calculations of the \( \nu \bar{\nu} \rightarrow e^- e^+ \) reaction in the vicinity of a neutron star have been first based on Newtonian gravity\(^47,48\), then the effect of gravity has been incorporated for static stars\(^44,49\), and then extended to rotating stars\(^45,50\). We consider the spacetime around a black hole described by the following diagonal metric

\[
g_{\mu\nu} = \text{diag}\left(-f(r), h(r), r^2, r^2 \sin^2 \theta\right),
\]

The energy deposition per unit time and per volume is given by (considering \( c = \hbar = 1 \))\(^51\)

\[
\dot{q}(r) = \int \int f_\nu(p_\nu, r)f_\tau(p_\tau, r) \times \left[ \sigma |V_\nu - V_\tau| \right] \frac{\varepsilon_\nu^{+} \varepsilon_\tau^{-}}{\varepsilon_\nu^{+} \varepsilon_\tau^{-}} d^3p_\nu d^3p_\tau,
\]

\(^4\)Indeed the state of the art of NS-NS merger simulations is special relativistic, axisymmetric hydrodynamic simulations for the final black hole-torus system\(^38\) from which emerge that neutrino-pair annihilation in ordinary GR models seems to be not efficient enough to power GRBs and the Blandford-Znajek process is currently considered a more promising mechanism for launching. In extended theories of gravity, the situation can be different as we will show in this proceeding.
where $f_{\nu, \bar{\nu}}$ are the neutrino number densities in phase space, $v_\nu$ the neutrino velocity, and $\sigma$ is the rest frame cross section. Since the term $\sigma|v_\nu - v_{\bar{\nu}}|\varepsilon_\nu\varepsilon_{\bar{\nu}}$ is Lorentz invariant, it can be calculated in the center-of-mass frame, and turns out to be

$$\sigma|v_\nu - v_{\bar{\nu}}|\varepsilon_\nu\varepsilon_{\bar{\nu}} = \frac{DG_F^2}{3\pi}(\varepsilon_\nu\varepsilon_{\bar{\nu}} - p_\nu \cdot p_{\bar{\nu}} c^2)^2,$$

with $G_F = 5.29 \times 10^{-44} \text{ cm}^2 \text{ MeV}^{-2}$ the Fermi constant,

$$D = 1 \pm 4 \sin^2 \theta_W + 8 \sin^4 \theta_W,$$

$\sin^2 \theta_W = 0.23$ the Weinberg angle, and the plus sign for electron neutrinos and antineutrinos while the minus sign for muon and tau type. $T(r)$ is the temperature measured by the local observer and $\Theta(r)$ is the angular integration factor. At these energies, the mass of the electrons can be neglected and it is possible to obtain that the general expression of the rate per unit time and unit volume of the $\nu\bar{\nu} \rightarrow e^+e^-$ process

$$\dot{q} = \frac{7DG_F^2 \pi^3 \zeta(5)}{2}[kT(r)]^9\Theta(r).$$

The evaluation of $T(r)$ and $\Theta(r)$ account for the gravitational redshift and path bending. To write the latter in terms of observed luminosity $L_\infty$ one has to have to consider that temperature, like energy, varies linearly with red-shift and following the procedure of Ref.\textsuperscript{13} one finds that:

$$\Theta(r) = \frac{2\pi^3}{3}(1 - x)^4(x^2 + 4x + 5),$$

$$T(r) = \frac{\sqrt{f(R)}}{\sqrt{f(r)}}T(R),$$

$$L_\infty = f(R)L(R),$$

$$L(R) = L_\nu + L_\bar{\nu} = \frac{7}{4}a\pi R^2 T^4(R).$$

Here $R$ is the neutrinosphere radius (the spherical surface where the stellar material is transparent to neutrinos and from which neutrinos are emitted freely), $L(R)$ is the neutrino luminosity, $a$ the radiation constant, $x = \sin^2 \theta_r$, $\theta_r$ is the angle between the trajectory and the tangent velocity in terms of local radial and longitudinal velocities\textsuperscript{15} for which one can obtain that

$$\cos \theta_r = \frac{R}{r} \sqrt{\frac{f(r)}{f(R)}}.$$ 

This relation comes from the fact that the impact parameter $b$ is constant on all the trajectory and is related to $\cos \theta_r$ by the relation

$$b = \left( \frac{f(R)}{r \cos \theta_r} \right)^{-1}.$$
Moreover, photosphere radius $R_{ph}$ exists below which a massless particle can not be emitted tangent to the stellar surface. The present discussion is therefore restricted to $R > R_{ph}$. The neutrino emission properties hence mainly depend on the geometry of spacetime. From the equation of the velocities

$$\dot{r}^2 = \left( E \dot{t} - L \dot{\phi} \right) f(r), \quad \dot{\phi} = \frac{L}{r^2}, \quad i = -\frac{E}{f(r)},$$

where $E$ and $L$ are the energy and angular momentum at the infinity, one gets the effective potential $V_{eff}$, such that the photonsphere radius follows from the condition $\frac{\partial V_{eff}}{\partial r} = 0$. The circular orbit is derived by imposing $\dot{r}^2 = 0$. We note en passant that these results reduce to ones derived in the case of the Schwarzschild geometry, $R_{ph} = 3M$, as calculated in [44].

The integration of $\dot{q}$ from $R$ to infinity gives the total amount of local energy deposited by the neutrino annihilation process (for a single neutrino flavour) for time units

$$\dot{Q} = 4\pi \int_R^\infty \frac{r^2}{\sqrt{f(r)}} \dot{q}. \quad (12)$$

According to [44], the total energy deposition from the neutrinosphere radius $R_6$ to infinity (for a symmetric spherical star that emits neutrinos from a spherical neutrinosphere) is given by

$$\dot{Q}_{51} = 1.09 \times 10^{-5} F \left( \frac{M}{R} \right) D L_{51}^{9/4} R_6^{-3/2}. \quad (13)$$

Here $\dot{Q}_{51}$ is expressed in units of $10^{51}$ erg s$^{-1}$, $L_{51}$ is neutrino luminosity in units of $10^{51}$ erg s$^{-1}$, $D = 1 \pm 4\sin^2 \theta_W + 8\sin^2 \theta_W$, $\sin^2 \theta_W = 0.23$ and the plus sign is for electron neutrinos and antineutrinos while the minus sign is for muon and tau type, $R_6$ is the radius in units of 10 km and, for a generic diagonal metric of the form $g_{\mu\nu} = (g_{00}, -1/g_{00}, -r^2, -r^2 \sin^2 \theta)$, the function $F (\frac{M}{R})$ is given by

$$F \left( \frac{M}{R} \right) = 3g_{00}(R)^{9/4} \int_1^{R_{ch}} (x - 1)^4(x^2 + 4x + 5)y^2g_{11}(yR)^{1/2}dy \frac{g_{00}(yR)^{3/2}}{g_{00}(yR)^{9/2}}. \quad (14)$$

In the Newtonian limit one gets $F(0) = 1$ so that it is convenient, for our analysis, to consider the ratio $\dot{Q}_{GR}/\dot{Q}_{Newt} = F(M/R)$. Usually, almost all energy is carried out by electron neutrino thus we can approximate Eq. (13) considering $D = 1.23$.

Equations (13) and (14) allow obtaining the deposited energy of neutrinos and the energy that can be emitted to powering GRB. In the next Section, we will study the ratio (14) for astrophysical objects in various modified gravity theories.

3. Neutrino deposition in modified gravity

In what follows, we present three relevant cases that explain how relevant could be the modification to GR in this context. First of all, we take into consideration the
Einstein dilaton Gauss-Bonnet gravity. The action is:

\[ S = \frac{1}{8\pi} \int d^4x \sqrt{-g} \left( \frac{R}{2} - \frac{1}{2} \partial_\mu \psi \partial^\mu \psi + \alpha \psi L_{GB} \right), \quad (15) \]

where \( \psi \) is a scalar field and \( L_{GB} \) is the Gauss-Bonnet invariant:

\[ L_{GB} = R^2 - 4R^{\alpha\beta} R_{\alpha\beta} + R^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta}. \]

The solution considered is the Sotiriou-Zhau solution (solution in perturbation theory), which lead to results in Fig. 1:

\[ ds^2 = -f(r)dt^2 + h(r)dr^2 + r^2d\Omega^2, \quad (16) \]

with

\[ f(r) = \left(1 - \frac{2m}{r}\right) \left(1 + \sum_n A_n a^n\right); \]
\[ h(r) = \left(1 - \frac{2m}{r}\right)^{-1} \left(1 + \sum_n B_n a^n\right). \]

where, to the second order:

\[ A_1 = B_1 = 0; \]
\[ A_2 = -\frac{49}{40m^3r} - \frac{49}{20m^2r^2} - \frac{137}{30mr^3} - \frac{7}{15r^4} + \frac{52m}{15r^5} + \frac{40m^2}{3r^6}; \]
\[ B_2 = \frac{49}{40m^3r} + \frac{29}{20m^2r^2} + \frac{19}{10mr^3} - \frac{203}{15r^4} - \frac{436m}{15r^5} - \frac{184m^2}{3r^6}. \]

The maximum value taken for \( \alpha \), considering the perturbative regime of the solution, shown an increase of the 50% for the maximum amount of energy deposition respect to GR.

The second case that we want to analyze is the BransDicke theory. It represents a generalization of general relativity, where gravitational effects are in part due to geometry, in part due to a scalar field. The action is:

\[ S = \int d^4x \sqrt{-g} \left[ \psi R + \frac{16\pi c^4}{e^2} L - \omega(\psi) \right], \quad (17) \]

where \( L \) is the Lagrangian density of all the matter, including all non-gravitational field, \( \psi \) is a scalar field and \( \omega \) is its Lagrangian density.

With this Lagrangian, expressing the line element in the isotropic form, we obtain the solution:

\[ ds^2 = -e^{2\alpha} dt^2 + e^{2\beta} [dr^2 + r^2d\Omega^2], \quad (18) \]
Fig. 1. Ratio of energy deposition $\dot{Q}$ for the Sotiriou-Zhau metric to total Newtonian energy deposition $\dot{Q}_{\text{Newt}}$ for different values of $\alpha$. The green curve shows the GR energy deposition for comparison.

where

$$ \lambda = \sqrt{(C + 1)^2 - C(1 - \frac{C \omega}{2})} $$

$$ e^{2\alpha} = e^{2\alpha_0} \left[ \frac{1 - \frac{B}{r}}{1 + \frac{B}{r}} \right]^{\frac{\omega}{2}} $$

$$ e^{2\beta} = e^{2\beta_0} \left( 1 + \frac{B}{r} \right)^4 \left[ 1 - \frac{B}{r} \right]^{4(1 + \frac{B}{r}) \frac{2(\lambda - C + 1)}{\lambda}} $$

$$ \psi = \psi_0 \left[ 1 - \frac{B}{1 + \frac{B}{r}} \right]^{-C} $$

with $\omega$ positive constant and

$$ \alpha_0 = \beta_0 = 0 $$

$$ \psi_0 = \frac{4 + 2\omega}{3 + 2\omega} $$

$$ C \sim -\frac{1}{2 + \omega} $$

$$ B \sim \frac{M}{2\sqrt{\psi_0}} $$
Using this metric, we obtain the shape for energy deposition in Fig. 2. Even with this model we have an enhancement of about 50% respect to the maximum value of $\dot{Q}/Q_{\text{Newt}}$ 30 in GR.

Finally, we discuss the case of a BH surrounded by quintessence field. In this case

$$g_{00}(r) = 1 - \frac{2M}{r} - \frac{c}{r^{\omega_q+1}},$$  \hspace{1cm} (19)

where $c$ is a positive constant and $-1 < \omega_q < -1/3$. The quintessence parameter is constrained by the fact that increasing $c$, the model passes from describing a black hole with an event horizon to representing a naked singularity.

We chose to show only the case with $\omega = -0.4$, for which we obtain results in Fig. 3 and an enhancement of a factor 25 with respect to GR.

### 3.1. GRB enhancement

The above results show that modified gravity provides an enhancement of the neutrino annihilation process as compared to GR. Such an enhancement is relevant
Fig. 3. Ratio of total energy deposition $\dot{Q}$ for $\omega = -0.4$ to total Newtonian energy deposition $\dot{Q}_{\text{Newt}}$ for three values of the parameter $c$. The green curve shows the GR energy deposition for comparison.

for powering GRBs for the model given by a closed neutron stars binary merging system. Neutrino emission happens in the last phase of the merging and the final configuration is a black hole (or neutron star) with an accretion disk. With the developed formalism, we can not describe the disk emission (we are considering a spherical system), so we restrict ourselves to the central BH. Taking into account total energy emitted into neutrinos from the central BH of $O(10^{52})$ erg, a radius of $R = 20$ km, the maximum possible total energy released in GRB is

$$Q_{\text{GR}} \sim 2.5 \times 10^{49} \text{ erg},$$

which is too small to explain the short GRB from neutron star merging. Instead, considering the maximum enhancement that modified gravity induces shown in Fig. 1 and 2, we have that the total possible energy released in GRB can exceed the maximum energy of $O(10^{52})$ erg (we have to remind that we are considering only the neutrino emission from the central BH and therefore the true emitted energy is larger considering the whole BH+disk configuration). Moreover, considering that the deposited energy is converted very efficiently to the relativistic jet energy, we infer a constraint on the quintessence model. It is possible to obtain that the maximum allowed value of $F(R_{\text{ph}})$ is $O(10^4)$. The contour plots given in Figs. 4 and 5 show the value of $F(R_{\text{ph}})$ for allowed value of $\omega_q$ and $c$. Therefore, one can infer, for $F(R_{\text{ph}}) \sim O(10^4\text{-}10^5)$, the values of the parameter $c$ that are not allowed in the considered scenario.
Fig. 4. Contour plot for $\omega_q \in ]-1,-0.65[$. On the y axis are reported the excluded values of $c$ due to the creation of a naked singularity (white part) and, on the right, the values of $F(R_{ph})$. It can be also seen the values of the parameter $c$, for which $F(R_{ph}) \sim \mathcal{O}(10^{-4} - 10^{-6})$, that are excluded by the energy deposition bounds.

4. Conclusion

In conclusion, we have analyzed the neutrino pair annihilation process $\nu \bar{\nu} \rightarrow e^+ e^-$ near a BH in some modified gravitational theories. We have shown that, owing to a shift of the photosphere radius, there is an enhancement of the deposited energy rate ratio with respect to GR. Such an enhancement could be a relevant mechanism for the generation of GRBs in close neutron star binary merging, for which neutrino pairs annihilation has been proposed as a possible source. Moreover, the released energy may be larger than the Short GRB maximum energy observed of $\mathcal{O}(10^{52})$ erg as it happens in the quintessence model (cfr. Ref. [55]). In that case, one can constrain the value of $\omega$ and $c$ such that the released energy is inferior to $\mathcal{O}(10^{52})$ erg. The $\nu \bar{\nu} \rightarrow e^+ e^-$ processes are of great importance in astrophysics, as well as in modified gravity, since they lead to considerable differences with respect to GR and its phenomenology. Therefore, the results presented in this proceeding could provide a new astrophysical framework to search for a signature of gravitational theories beyond GR.

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