Coulomb blockade in a quantum wire with long-range interactions

Hélène Maurey and Thierry Giamarchi

Laboratoire de Physique des Solides, Université Paris–Sud, Bât. 510, 91405 Orsay, France

Abstract

We study the transport through two impurities or “barriers” in a one-dimensional quantum wire, taking into account the long-range $\frac{1}{r}$ Coulomb interactions. We compute the temperature-dependent conductance $G(T)$ of this system. Long-range forces lead to a dramatic increase of weak barrier potentials with decreasing temperature, even in the “resonant” case. The system thus always reaches a “strong barrier” regime in which only charge is pinned, contrary to the standard LL case. $G(T)$ vanishes faster than any power as $T$ goes to zero. In particular, resonant tunneling is suppressed at zero temperature.
Hope for experimental characterization of Luttinger Liquids (LL) has been renewed by the fabrication of nanostructures such as quantum wires. A LL is expected for short-range Coulomb interactions. However, in an isolated quantum wire, interactions can be long-range, leading to a Wigner Crystal (WC) with dominant $4k_F$ fluctuations and quasi-long-range order. For long wires containing many impurities, one expects universal $T^2$ dependence of the conductivity for the WC versus interaction-dependent power-law for the LL. Attention focused recently on short wires with a few impurities. For one impurity and repulsive interactions, conductance vanishes at zero $T$, as a power-law in the LL, faster than a power-law in the WC. With two impurities, only LL was studied. In the strong impurity regime, Coulomb Blockade and resonant tunneling phenomena occur. On resonance, perfect transmission is expected at $T = 0$ for moderately repulsive interactions.

We examine here the double-barrier in presence of long-range interactions, a situation relevant to realistic quantum wires. We show that in this case, contrarily to the LL, the physics is drastically different depending on whether one applies directly a very strong impurity potential $V \gg E_F$, or one starts from an initially weak potential ($V \ll E_F$) which is renormalized to large values as $T$ decreases. We show that the latter case leads, for the WC or LL with strongly repulsive interactions, to blockade of charge, spin being free to flow, whereas in the former both charge and spin are locked. We focus here on two initially “weak” impurities, which is the relevant case for most experimental situations (due to intrinsic disorder or artificial constrictions), the “strong” impurity case corresponding to tunnel junctions in the wire. We show that only charge degree of freedom plays a role in transport and that a WC exhibits Coulomb Blockade and charge resonances and compute microscopically the charging energy of the island formed between the two barriers. We draw the parallel with the phenomenological charging energy term usually added to explain Coulomb Blockade in standard mesoscopic systems. Unlike in a LL this charging energy is dominated by electrostatic effects. Both for weak and strong barriers, using respectively perturbation theory and an instanton method, we obtain the temperature-dependent conductance on and away...
from resonance. The conductance always vanishes faster than any power with decreasing $T$, even in the resonant case. At $T = 0$ resonant tunneling is suppressed. These effects could provide an experimental signature of the importance of long-range Coulomb interactions and the existence of the one-dimensional WC.

We consider a narrow wire of interacting electrons, of width $d \sim \lambda_F$ and of length $L \gg d$ so that the system is regarded as one-dimensional. The Hamiltonian of the pure system is

$$H = \sum_{\nu = \rho, \sigma} \frac{u_{\nu}}{2\pi} \int_{-\frac{L}{2}}^{\frac{L}{2}} dx \left[ K_{\nu}(\pi \Pi_{\nu})^2 + \frac{1}{K_{\nu}} (\partial_x \Phi_{\nu})^2 \right] + \frac{1}{\pi^2} \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{L}{2}}^{\frac{L}{2}} dx dx' V(x - x') (\partial_x \Phi_{\rho}(x)) (\partial_x \Phi_{\rho}(x'))$$

(1)

We have taken $\hbar = 1$, $\nu = \rho, \sigma$ indicate the charge and spin degrees of freedom. $\Pi_{\nu}$ is the conjugate momentum of $\Phi_{\nu}$, $K_{\rho}$ a number giving the strength of the short-range part of Coulomb forces, $u_{\rho}$ is the renormalized Fermi velocity due to the same interactions. We assume spin-isotropic case, $K_{\sigma} = 1$. $V(x) = \frac{e^2}{\kappa \sqrt{x^2 + d^2}}$ is the long range part of the Coulomb interaction ($\kappa$ is the dielectric constant). (1) describes a one dimensional WC, dominated by $4k_F$ charge fluctuations. Indeed $4k_F$ charge correlation functions are always the slowest decaying ones, with much slower decay than power laws [$\mathbb{I}$]. $2k_F$ ones are still power-laws due to the spin part. Screened long-range interactions correspond to $V = 0$ in (1), which in that case describes a LL: charge and spin correlations decay as power laws, with interaction-dependent exponents. $2k_F$ fluctuations dominate for $K_{\rho} > \frac{1}{3}$ and $4k_F$ ones for $K_{\rho} < \frac{1}{3}$.

We model two impurities by delta functions $V_1 \delta(x + \frac{a}{2})$ and $V_2 \delta(x - \frac{a}{2})$. As long as the renormalized barriers are weak ($\ll E_F$), we trivially keep only coupling to the slowest decaying part of the density

$$H_{imp} \simeq V_{1,4k_F} \cos(\sqrt{8} \Phi_{\rho}(\frac{a}{2}) + 2k_F a) + V_{2,4k_F} \cos(\sqrt{8} \Phi_{\rho}(\frac{a}{2}) - 2k_F a)$$

(2)

where $V_{i,4k_F} = V_{i\rho}(4k_F)$. Furthermore, even if such weak barriers flow to strong coupling, working with (2) only is still valid. In particular we needn’t take into account the terms $V_{2k_F} \cos(\sqrt{2} \Phi_{\rho}(\pm \frac{a}{2})) \cos(\sqrt{2} \Phi_{\sigma}(\pm \frac{a}{2}))$. For simplicity, let us show this for a single impurity
case, a full derivation being given in [14]. Since $V_{4k_F}$ is the most relevant term in the potential
[10], $V_{4k_F}/E_F$ becomes of order one while $V_{2k_F}/E_F$ is still very small and can be treated
perturbatively. The minimization of $V_{4k_F} \cos(\sqrt{8} \Phi_\rho)$ imposes that $V_{2k_F} \cos(\sqrt{2} \Phi_\rho) \cos(\sqrt{2} \Phi_\sigma)$
is zero. More precisely, one can show using the instanton method introduced in the following
that effective $V_{2k_F}$ in this regime decreases exponentially as $T \to 0$ [14]. Taking initially a
strong impurity would be very different: one then has to minimize the sum of all harmonics
in the potential. Minima are connected by $(\Phi_\rho, \Phi_\sigma) \to (\Phi_\rho \pm \pi \sqrt{2}, \Phi_\sigma \pm \pi \sqrt{2})$: thus here both
charge and spin are locked, giving different $G(T)$ [10]. For the WC a weak link is not the
asymptotic limit of a weak impurity problem. An analog distinction was made in the context
of the X-ray edge singularity [13].

Depending on temperature, the two barriers in (2) act in conjugation or as independent
scatterers, since finite $T$ introduces a thermal length above which correlations are destroyed.
Using the dispersion relation, to link energy and positional quantities, one obtains for the
WC, the thermal length $L_T \sim \frac{\sqrt{\alpha}}{a} \ln^{1/2} \frac{T}{T_d}$ where $\alpha_c = \frac{4Ke^2}{\pi \kappa}$ (from now on we drop the
subscript $\rho$ since only charge excitations will matter). $T_d = \frac{\sqrt{\alpha_c}}{d}$ is a cut-off temperature
($L_T \sim \frac{\sqrt{T}}{T}$ for a LL). When $L_T < a$ the two barriers are uncorrelated and transport is very
similar as for a single barrier, whereas if $L_T > a$ the barriers exhibit new behavior due to
mutual coupling. In this regime, and for very weak barriers the WC is almost not distorted
but chooses a relative position with respect to the barriers minimizing potential energy. $\Phi$
is therefore pinned, except for $\frac{4k_F a}{2\pi} = N + \frac{1}{2}$ ($N$ integer) and $V_{1,4k_F} = V_{2,4k_F} \equiv V_0$ for which
there is no preferred position: the WC slides freely at $T = 0$. Such “resonance” occurs
only for symmetric barriers, situation considered in the following (weak asymmetric barriers
can be assimilated to the off resonance case [14]). Using perturbation in powers of $V_0$ one
computes the corrections $\delta G$ to the conductance $G_0$ of the pure case [16].

At $T > T_a \sim \frac{\sqrt{\alpha}}{a} \ln^{1/2} \frac{a}{d}$ ($T_a \sim \frac{\sqrt{T}}{a}$ for the LL) only correlations involving one barrier
contribute, one is in the single-barrier regime:

$$
\delta G(T) \sim -\gamma g^2 T^{-2} \ln^{-1/2} \frac{T_d}{T} e^{-4\nu \ln^{1/2} \frac{T_d}{T}}
$$

(3)
where γ is a constant of order $e^2/h$ and $\nu = \sqrt{\frac{\pi u x K}{e}}$. The temperature dependence is identical to the one found for many weak impurities at high temperature [5,17]. A WC is more strongly pinned by a single barrier than a LL for which divergence of $\delta G$ at low T is slower ($T^{K_\rho-1}$ for electrons with spin [7,8]). When $\delta G$ becomes of the order of $G_0$, there is crossover to a strong-coupling regime. Two cases occur depending on $T_a$: either each barrier first flows to strong coupling at $T_{cr} > T_a$ while the system remains in the single barrier regime, or there is first a crossover to the double barrier regime at $T_a$. In such “weak-double-barrier” regime $T < T_a$ one has to consider correlations between the two barriers. Out of resonance, $\delta G$ grows as in the single-barrier case (3) times a factor $(1 + \cos 4k_Fa)$ up to crossover to strong coupling. Close enough to resonance the $8k_F$ component of $H_{\text{imp}}$ gives the main contribution

$$G(T)_{\text{on-res}} \sim G_0 - \gamma(1 + \cos 8k_Fa)g^2T^{-2}\ln^{-1/2}\frac{T_d}{T}e^{-16\nu \ln^{1/2}\frac{T}{d}}$$

$G$ decreases though slower than off-resonance. Contrary to a LL with moderate repulsive interactions $1/2 < K_\rho < 1$ for which $\delta G$ vanishes till perfect resonant transmission at $T = 0$ (see fig.1 and [18]), the WC always crosses over to strong coupling. In this regime the barriers coincide with minima of the $4k_F$ part of the charge density, by compression or dilatation of the WC. Smallest distortion then defines the ground state on the island. It is unique and corresponds to $N$ electrons, except on resonance ($\frac{4k_Fa}{2\pi} = N + \frac{1}{2}$, as for weak symmetric barriers), with two degenerate ground states for $N$ and $N + 1$ electrons. Again we treat symmetric barriers. Transport through the double-barrier is determined by the evolution of $\Phi$ at the locations of the barriers and one can integrate all other modes to obtain the action for $\Phi(\pm a/2)$. When $T_a > T_a$, $\Phi(-\frac{a}{2})$ and $\Phi(\frac{a}{2})$ decouple in the effective action and we recover the “single-barrier” case: $G(T) \sim t^2 T^{-2}\ln^{-1}\frac{T_d}{T}e^{-\frac{1}{3}ln^{3/2}\frac{T_d}{T}}$ where $t$ is the amplitude of tunneling through one barrier (a power $T^{-1}$ was found in [11], which is correct for initially strong ($V \gg E_F$) impurity as said above, due to a large $V_{2k_F} \gg E_F$ term). Such decrease is faster than any power (for a LL $G(T) \sim T^{K_\rho-1}$). For $T < T_a$ the two barriers are coupled and it is simpler to introduce $\tilde{\Phi} \equiv (\Phi(-\frac{a}{2}) - \Phi(\frac{a}{2}))$ describing the state on the island and $\Phi \equiv \frac{1}{2}(\Phi(-\frac{a}{2}) + \Phi(\frac{a}{2}))$ describing the island seen from the leads. The
effective action reads:
\[
S_{\text{eff}} = \frac{1}{2} \left( \frac{m_\Lambda}{2} \right) \int d\tau \dot{\Phi}(\tau)^2 + \frac{1}{2} (2m_\Lambda) \int d\tau \ddot{\Phi}(\tau)^2 + 2g \int d\tau \cos(\sqrt{8}\Phi(\tau)) \cos(\sqrt{2}\Phi(\tau) + 2kFa) + S_\Phi + S_{\bar{\Phi}} + S_c
\]

\[
S_c = \frac{1}{2C} \int d\tau e^{\sqrt{8}\Phi(\tau)}
\]

\[
S_\Phi = \frac{1}{4\pi^2\nu} \ln^2 \frac{a}{\tau_c} \int \int d\tau d\tau' \ln^{-3/2} \left( \frac{\tau - \tau'}{\tau_c} \right) \frac{(\Phi(\tau) - \Phi(\tau'))^2}{(\tau - \tau')^2}
\]

\[
S_{\bar{\Phi}} = \frac{1}{\pi^2\nu} \int \int d\tau d\tau' \ln^{1/2} \left( \frac{\tau - \tau'}{\tau_c} \right) \frac{(\bar{\Phi}(\tau) - \bar{\Phi}(\tau'))^2}{(\tau - \tau')^2}
\]

\(\Lambda\) is a momentum cut-off, \(\tau_c^{-1} = u\Lambda\), \(m_\Lambda = \frac{4\sqrt{\alpha_c}}{\pi^2\Lambda uK}\), \(C = \frac{a}{\tau_c} \ln^{-1} \frac{a}{\tau_c}\). For large \(g\), \((\sqrt{8}\Phi, \sqrt{2}\bar{\Phi})\) is constrained to minima of the potential \((\pi + n_0\pi, \pi + m_0\pi - 2kFa)\), with \(n_0 + m_0\) odd. Transport is possible only by quantum tunneling of electrons through the potential barriers, corresponding to instantons of the phases \((\sqrt{8}\Phi, \sqrt{2}\bar{\Phi}), (\pm\pi, \pm\pi)\) if an electron crosses one barrier, \((\pm2\pi, 0)\) if two electrons tunnel simultaneously, one through each barrier. This can be described in a WKB approximation \([8]\), or instanton method: the equation of motion for \(\Phi\) obtained from (5) is solved on each barrier independently. The general solution for \(\Phi(\pm\frac{\phi}{2}, \tau)\) is a linear combination of instantons and anti-instantons.

The first part (5) of \(S_{\text{eff}}\) describes a “gas of non-interacting” instantons. But \(S_{\bar{\Phi}}, S_\Phi\) and \(S_c\), due to electron-electron interactions, give correlations between tunneling events. \(S_{\bar{\Phi}}\) and \(S_\Phi\) provide instanton-instanton repulsion and instanton-anti-instanton attraction. \(S_c\) limits the number of electrons added to the island and is of the form \(\int Q^2/2C\). \(Q = \frac{e\sqrt{8}\Phi}{2\pi}\) is variation of charge in a segment of length \(a\) of the perfect WC. \(Q^2/2C\) is the electrostatic energy of the island and \(C\) its capacitance. Such charging energy is responsible for Coulomb blockade. It is due to Coulomb repulsion cost of creating an excess or lack of charge and to the elastic cost of the corresponding distortion. In the WC, Coulomb repulsion cost dominates and the blockade is genuinely of electrostatic nature. In the spinless LL, most of the “charging” energy comes from kinetic energy of the electrons, i.e. the quantization of levels on the island \([7]\), and short range interactions only provide small enhancement. Moreover for the WC both island and leads contribute (in equal part) to the charging energy, whereas in the LL only the inner island is involved. Indeed due to long-range interactions the island charge
creates an image one on the leads which contributes to the charging energy.

After substitution of the instanton form of \( \tilde{\Phi} \) and \( \Phi \) in (5)-(8) the partition function can be computed:

\[
Z = \sum_{\{n_1, n_2\}} \sum_{\{q_i\}, \{r_i\}} t^{2n_T} \int_0^\beta d\tau_2 \cdots \int_0^{2\beta} d\tau_1 e^{\frac{4}{\pi} \left( \sum_{i<j} q_i q_j \ln^{3/2} \frac{|r_i-r_j|}{d} - (3 \ln^2 \frac{a}{2}) r_i r_j \ln^{-1/2} \frac{|r_i-r_j|}{d} \right) - S_c} \tag{9}
\]

\( t = e^{-s_0} \) is the amplitude of tunneling of an electron through one barrier, \( s_0 = \sqrt{8m_A g} \) the action of a single instanton. \( Z \) is summed over all possible paths containing \( n_1 \) (resp. \( n_2 \)) events (instantons or anti-instantons) through the left (resp. right) barrier. \( n_T = n_1 + n_2 \).

At time \( \tau_i \), \( q_i \) electron(s) cross the double-barrier from left to right and \( 2r_i \) are added to the island. (9) is perturbatively expanded in terms of the number of tunneling events. Writing the current as \( I = \frac{e}{2\pi} \dot{\Phi} \), this allows to compute the conductance.

Two regimes occur depending on whether Coulomb blockade is determinant or not. If the cost of an additional electron is higher than thermal energy, the number of electrons on the island is locked to \( N \) and the main transport process is simultaneous tunneling in which one electron is transferred directly from one lead to the other \( (q_1 = \pm 1, r_1 = 0) \), giving

\[
G_{\text{off res.}}(T) \sim t^4 T^{-2} \ln^{-1} \frac{T_d}{T} e^{-\frac{2}{s_0} \ln^{3/2} \frac{T_d}{T}} \tag{10}
\]

Although temperature dependence is the same as in the one barrier regime, tunneling is strongly reduced since the amplitude goes as \( t^4 \) instead of \( t^2 \). Oppositely, when Coulomb Blockade term is negligible compared to thermal fluctuations, transport mechanism is two-step tunneling where an electron crosses one barrier after the other \( (q_1, r_1) = (+\frac{1}{2}, \frac{1}{2}) \), and \( (q_2, r_2) = (+\frac{1}{2}, -\frac{1}{2}) \), i.e. charge on the island hops between \( N \) and \( N + 1 \) giving

\[
G_{\text{on res.}}(T) \sim t^2 T^{-2} \ln^{-1} \frac{T_d}{T} e^{-\frac{2}{s_0} \ln^{3/2} \frac{T_d}{T}} \tag{11}
\]

\( G(T) \) vanishes faster than any power, though much slower than off resonance and with a factor \( t^2 \) instead of \( t^4 \), owing to two-step tunneling.

The physics of the WC is thus very different from that of a LL with spin in which both charge and spin are locked, and despite the spin degrees of freedom closer to a spinless
LL. In the LL with spin, lowest-order resonances are spin instead of charge ones, analog to Kondo resonances, allowing perfect transmission of both charge and spin at \( T = 0 \) if \( K_\rho > 1/2 \) (fig.1b) \([7]\). The conditions for resonance are also different: half-integer number of electrons between the barriers for the charge resonance in the WC, odd number for the spin resonance in the LL. Conductance oscillations are expected to vanish in a WC when lowering \( T \), whereas they should deepen in a LL with not too strong interactions. These experimentally testable effects could therefore allow to probe the strength and nature of interactions in a one-dimensional wire.
FIG. 1. Temperature dependence of the conductance through a symmetric double-barrier for a WC (a) and a LL with $K_\rho > \frac{1}{2}$ (b), starting from weak barriers at a given $T$ (right of the figure). We show here the situation $T_a < T_{cr}$ when the first crossover is to the strong-single-barrier regime. In the LL case there is a second crossover to weak coupling for $T < T_{cr2}$. Dashed (full) lines give off (on-)resonance cases. (Charge resonance for WC, spin (Kondo) resonance for LL). In “1 or 2/W or S”, 1 stands for one impurity or two impurity case off resonance, 2 for two impurities on resonance, W and S mean weak or strong coupling regimes. Expressions for $G(T)$ are (3), (4), (10), (11) for the WC and in Ref.[7] for the LL.
REFERENCES

[1] Laboratoire associé au CNRS. emails: maurey@lps.u-psud.fr and giam@lps.u-psud.fr.

[2] F. D. M. Haldane, J. Phys. C 14, 2585 (1981).

[3] T. J. Thornton et al., Phys. Rev. Lett. 56, 1198 (1986); J. H. F. Scott-Thomas et al., Phys. Rev. Lett. 62, 583 (1989); A. R. Goñi et al., Phys. Rev. Lett. 67, 3298 (1991); S. Tarucha, T. Saku, Y. Tokura, and Y. Hirayama, Phys. Rev. B 47, 4064 (1993).

[4] H. J. Schulz, Phys. Rev. Lett. 71, 1864 (1993).

[5] H. Maurey and T. Giamarchi, Phys. Rev. B 51, 10833 (1995).

[6] L. P. Gorkov and I. Dzyaloshinskii, JETP Lett. 18, 401 (1973). D. C. Mattis, J. Math. Phys. 15, 609 (1974); A. Luther and I. Peschel, Phys. Rev. B 9, 2911 (1974); W. Apel and T. M. Rice, Phys. Rev. B 26, 7063 (1982); T. Giamarchi and H. J. Schulz, Phys. Rev. B 37, 325 (1988).

[7] C. Kane and M. P. A. Fisher, Phys. Rev. B 46, 15233 (1992).

[8] A. Furusaki and N. Nagaosa, Phys. Rev. B 47, 4631 (1993).

[9] L. I. Glazman, I. M. Ruzin, and B. I. Shklovskii, Phys. Rev. B 45, 8454 (1992).

[10] M. Fabrizio, A. O. Gogolin, and S. Scheidl, Phys. Rev. Lett. 72, 2235 (1994); A. Furusaki and N. Nagaosa, J. Phys. Soc. Jpn. 63, 413 (1994).

[11] A. Furusaki and N. Nagaosa, Phys. Rev. B 47, 3827 (1993).

[12] H. Grabert and M. Devoret, Single Charge Tunneling (Plenum Press, New York and London, 1991); D. V. Averin and K. K. Likharev, in Mesoscopic Phenomena in Solids, edited by B. L. Altshuler, P. A. Lee, and R. A. Webb (North-Holland, Amsterdam, 1991).

[13] J. Sólyom, Adv. Phys. 28, 209 (1979); V. J. Emery, in Highly Conducting One-Dimensional Solids, edited by J. T. D. et al. (Plenum, New York, 1979), p. 327.
[14] H.Maurey and T.Giamarchi, in preparation.

[15] Y.Oreg and A.M.Finkel'stein, Phys. Rev. B 53, 10928 (1996).

[16] According to recent studies (A.Kawabata, J. Phys. Soc. Jap. 65, 30 (1996); Y.Oreg and A.M.Finkel'stein, preprint) for a quantum wire $G_0$ is $\frac{e^2}{h}$ per channel, not renormalized by the interactions. See also I.Safi and H.J.Schulz, Phys. Rev. B 52, R17040 (1995); D.Maslov and M.Stone, Phys. Rev. B 52, R5539 (1995)).

[17] T.Giamarchi and H.Maurey, in Correlated Fermions and Transport in Mesoscopic Systems, edited by G.Montambaux et al. (Editions Frontières, Gif-sur-Yvette, 1996).

[18] A.Schmid, Phys. Rev. Lett. 51, 1506 (1983).