Numerical calculation of elements of thin-walled structures under alternating loading taking into account damage

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Abstract. The statement of the problem and the algorithms for calculating thin-walled rods of arbitrary section under spatially variable loading based on the theory of small elastoplastic deformations and the refined theory of rods are presented. Using the variational Lagrange principle, mathematical models are developed for the deformation and damage of thin-walled rods in a cylindrical coordinate system. A system of differential equilibrium equations for a rod with spatially variable loading in vector form is obtained. To solve the boundary-value problem, the central difference scheme of the second order of accuracy and the matrix sweep method are used. An example of calculation is given.

1. Introduction
Increasing strength, reducing material consumption, intensifying the operating parameters of thin-walled structures are the most important conditions for increasing the efficiency of their use in the construction, engineering, and energy sectors. The work of most of the supporting elements of thin-walled structures occurs against the background of materials going beyond the elastic limits in the most stressed areas, which under the action of cyclic loads leads to a number of additional phenomena, such as the appearance of secondary plastic deformations, a change in deformation diagrams from cycle to cycle, and manifestations of properties cyclic hardening-softening, damage accumulation and crack propagation [1-3].

The issues of modeling the processes of deformation and accumulation of damage, the development of methods and algorithms, as well as the creation of a set of programs for calculating structural elements and structures beyond elasticity under alternating loads have become particularly relevant.

The concept of damage function was introduced in connection with studies of creep, long-term and cyclic strength by A A Ilyushin, L M Kachanov, Yu N Rabotnov, V V Moskvitin, V V Bolotin, C Truesdell, W Noll, Y Hult and etc.

The general formulation of the phenomenological approach to the description of damage accumulation was given by A A Ilyushin. In the works of V V Moskvitin, the damage function was introduced to assess the strength of elastoplastic and viscoelastic materials under variable loads [2].

Various versions of the criteria for damageability and low-cycle strength were proposed by V S Bondar, A V Berezin, Yu G Korotkikh, V N Kukudzhanov, A A Lebedev, A M Lokoshenko, A P Gusenkov and G V Moskvitin [4-10].
We present the statement of the problem and the numerical calculation of thin-walled structures of rods under repeated alternating elastoplastic loading based on the theory of small elastoplastic deformations by A A Ilyushin and the refined theory of rods proposed by V Z Vlasov, G Yu Dzhanelidze and V K Kabulov. As is known, with the joint longitudinal, transverse and torsional forces, the laws of the distribution of displacements, strains, and stresses in the cross sections of the rod are complex, therefore, the refined theory is based on a number of static hypotheses [11].

2. Materials and methods
Consider a thin-walled rod (pipes) of arbitrary section under the influence of external variable forces. The displacements of the center line of the bar under the nth loading will be denoted by $u^{(n)}, v^{(n)}, w^{(n)}$ and the components of the deformations and stresses by $e_{ij}^{(n)}, \sigma_{ij}^{(n)}$.

Following [2], we introduce the differences

$$
\overline{u}_i^{(n)} = (-1)^n (u_i^{(n+1)} - u_i^{(n)}), \quad \overline{e}_i^{(n)} = (-1)^n (e_i^{(n+1)} - e_i^{(n)}),
\overline{\sigma}_i^{(n)} = (-1)^n (\sigma_i^{(n+1)} - \sigma_i^{(n)}),
$$

(1)

Based on the assumptions and hypotheses [12], we represent the general displacements of the structure in the form (we omit the trait):

In Cartesian coordinates

$$
u_i^{(n)} = u_i^{(n)} - y\alpha_i^{(n)} - z\alpha_2^{(n)} + \phi_i^{(n)} + a_i\beta_1^{(n)} + a_i\beta_2^{(n)},
\nu_2^{(n)} = v_i^{(n)} - z\theta_i^{(n)},
\nu_3^{(n)} = w_i^{(n)} + y\theta_i^{(n)},
$$

(2)

In cylindrical coordinates ($x=x, y=r \cos \gamma, z=r \sin \gamma$):

$$
u_i^{(n)} = u_i^{(n)} - \alpha_1^{(n)} r \cos \gamma - \alpha_2^{(n)} r \sin \gamma + \phi_i^{(n)} + a_i(r, \gamma)\beta_1^{(n)} + a_i(r, \gamma)\beta_2^{(n)},
\nu_2^{(n)} = v_i^{(n)} - \theta_i^{(n)} r \sin \gamma,
\nu_3^{(n)} = w_i^{(n)} + \theta_i^{(n)} r \cos \gamma
$$

(3)

where $\alpha_1^{(n)}, \alpha_2^{(n)}$ are the angles of rotation of the cross section under pure bending under nth loading; $\beta_1^{(n)}, \beta_2^{(n)}$ are the angles of transverse shear, $\theta_i^{(n)}$ is the torsion angle, $V_1^{(n)}$ is the linear spin under loading, $\phi$ is the Sen-Venon torsion function.

To derive the equilibrium equations of thin-walled structures during repeated loading, we use the Lagrange variational principle [2]:

$$
\delta(A - \Pi) = 0,
$$

(4)

where $\delta \Pi$ is the variation of potential energy, in this formulation has the form:

$$
\delta \Pi = \int \sum_{i=1}^{3} \sigma_{ii}^{(n)} \delta e_{ii}^{(n)} dV = \int \left[ \sigma_{11}^{(n)} \delta e_{11}^{(n)} + \sigma_{12}^{(n)} \delta e_{12}^{(n)} + \sigma_{13}^{(n)} \delta e_{13}^{(n)} \right] dV,
$$

(5)

The variation of the work of external forces $\delta \overline{A}$ is taken in the form

$$
\delta A = \int \sum_{i=1}^{3} \overline{p}_i^{(n)} \delta \overline{u}_i^{(n)} dV + \int \sum_{x=1}^{3} \overline{q}_i^{(n)} \delta \overline{u}_i^{(n)} ds + \int \sum_{s_i=1}^{3} \overline{f}_i^{(n)} \delta \overline{u}_i^{(n)} ds,
$$

(6)

where $\overline{p}_i^{(n)}$ – volume forces, $\overline{q}_i^{(n)}$ – surface forces, $\overline{f}_i^{(n)}$ – end forces.
From the variational equation (4), the following system of differential equilibrium equations for the rod under variable loads with the corresponding boundary conditions in vector form is obtained:

1. When using diagrams of cyclic deformation of Masing-Moskvitin:

\[
\frac{d}{dx}\left[ (A^{e} - A^{p(l)}) \frac{d\bar{V}^{(l)}}{dx} + (B^{e} - B^{p(l)}) \bar{V}^{(l)} \right] + \left( C^{e} - C^{p(l)} \right) \frac{d\bar{U}^{(l)}}{dx} + \left( D^{e} - D^{p(l)} \right) \bar{U}^{(l)} = \bar{Q}^{(l)}
\]

(7)

\[-\left( (A^{e} - A^{p(l)}) \frac{d\bar{U}^{(l)}}{dx} + (B^{e} - B^{p(l)}) \bar{U}^{(l)} - \bar{Q}^{(l)} \right)\left( \mathbb{R} \right) \frac{d\bar{U}^{(l)}}{dx} = 0
\]

(8)

where \( \bar{Q}^{(l)} \), \( \bar{Q}^{(l)} \) are vectors; \( \bar{U}^{(l)} \) are the sought vectors of the ninth-order function of the fictitious coordinates: \( \bar{U}^{(l)} = \{ \bar{u}^{(l)}, \bar{u}^{(l)}_{1}, \bar{u}^{(l)}_{2}, \bar{w}^{(l)}, \bar{\theta}^{(l)}, \bar{\psi}^{(l)} \} \)

To determine the true values of the calculated values, we use the formula:

\[
U^{(l)} = U^{0} + \sum_{k=2}^{n} (-1)^{k-1} U^{(k)}, \quad \sigma^{(k)} = \sigma^{0} + \sum_{k=2}^{n} (-1)^{k-1} \tilde{\sigma}^{(k)}
\]

(9)

2. When using diagrams of cyclic deformation in the current coordinates of T. Buriyev [3]:

\[
\frac{d}{dx}\left[ (A^{el} - A^{pl(l)}) \frac{d\tilde{V}^{(l)}}{dx} + (B^{el} - B^{pl(l)}) \tilde{V}^{(l)} \right] + \left( C^{el} - C^{pl(l)} \right) \frac{d\tilde{U}^{(l)}}{dx} + \left( D^{el} - D^{pl(l)} \right) \tilde{U}^{(l)} = \tilde{Q}^{(l)}
\]

\[
+ \left( D^{el} - D^{pl(l)} \right) \tilde{V}^{(l)} = \tilde{F}^{(l)} + \frac{d}{dx}\left( A^{el} \frac{d\tilde{V}^{(l-1)}}{dx} + B^{el} \tilde{V}^{(l-1)} \right) \right) + C^{pl(l)} \frac{d\tilde{V}^{(l-1)}}{dx} + \left( D^{pl(l)} \tilde{V}^{(l-1)} \right) \left( \mathbb{R} \right) \frac{d\tilde{V}^{(l-1)}}{dx}
\]

\[
= \sum_{m=1}^{k-1} \frac{d}{dx}\left[ A^{el} \frac{d\tilde{V}^{(l-m)}}{dx} \right] + \left( B^{el} - B^{pl(l)} \right) \tilde{U}^{(l-m)} - \tilde{Q}^{(l-m)} + \left( B^{pl(l)} \tilde{U}^{(l-m)} - A^{el} \frac{d\tilde{V}^{(l-m)}}{dx} \right) - \sum_{m=1}^{k-1} \frac{d}{dx}\left[ A^{el} \frac{d\tilde{V}^{(l-m)}}{dx} \right] + \left( B^{pl(l)} \tilde{U}^{(l-m)} - \tilde{Q}^{(l-m)} \right) \left( \mathbb{R} \right) \frac{d\tilde{V}^{(l-m)}}{dx}
\]

\[
= \left[ \frac{d}{dx}\left[ (A^{el} - A^{pl(l)}) \frac{d\tilde{V}^{(l)}}{dx} + (B^{el} - B^{pl(l)}) \tilde{V}^{(l)} \right] + \left( C^{el} - C^{pl(l)} \right) \frac{d\tilde{U}^{(l)}}{dx} + \left( D^{el} - D^{pl(l)} \right) \tilde{U}^{(l)} = \tilde{Q}^{(l)} \right]
\]

\[
- \sum_{m=1}^{k-1} \frac{d}{dx}\left[ A^{el} \frac{d\tilde{V}^{(l-m)}}{dx} \right] + \left( B^{el} - B^{pl(l)} \right) \tilde{U}^{(l-m)} - \tilde{Q}^{(l-m)} + \left( B^{pl(l)} \tilde{U}^{(l-m)} - A^{el} \frac{d\tilde{V}^{(l-m)}}{dx} \right)
\]

\[
- \sum_{m=1}^{k-1} \frac{d}{dx}\left[ A^{el} \frac{d\tilde{V}^{(l-m)}}{dx} \right] + \left( B^{pl(l)} \tilde{U}^{(l-m)} - \tilde{Q}^{(l-m)} \right) \left( \mathbb{R} \right) \frac{d\tilde{V}^{(l-m)}}{dx}
\]

\[
= 0
\]

(10)

(11)

here \( \tilde{V}^{(l)} \) are the desired vectors of the ninth order function in the current coordinates.

Matrices A, B, C, D are quadratic matrices of the ninth order, \( \tilde{F}^{(k)}, \tilde{Q}^{(k)} \) - vectors of external forces.

In the case of the generalized Masing principle \( \lambda_{s} = \lambda, \ \tilde{e}^{(n)} = \alpha_{s} \tilde{e}_{s} \), when using the Gusenkov - Schneiderovich strain diagrams \( \tilde{e}^{(n)} = 2\tilde{e}_{s} \lambda_{s} = 1 - g_{n} \), where \( g_{n} \) is determined experimentally [10], and when damage accumulation is taken into account:

\[
e_{c}^{(n)}(\eta) = \begin{cases} 0, & \text{if } \tilde{e}^{(n)}(\eta) \leq \tilde{e}_{s}^{(n)}(\eta) \\ \lambda_{s} \left( 1 - \frac{\tilde{e}^{(n)}(\eta)}{\tilde{e}_{s}^{(n)}} \right), & \text{if } \tilde{e}^{(n)}(\eta) > \tilde{e}_{s}^{(n)}(\eta) \end{cases}
\]

(12)

\[
\tilde{e}_{s}^{(n)}(\eta) = \alpha_{s}^{(n)}(1 + \alpha_{n}) \tilde{e}_{s} + (3G)^{4} B^{\frac{1}{4}} \left[ 1 - 0.5(1 + \alpha_{n}) \tilde{e}_{s}^{2} \right]^{\frac{1}{2}} \left[ 1 - (1 - \eta)^{\alpha_{n}} \right]^{\frac{1}{2}} (n-1)^{1/2}
\]

(13)
The damage function $\eta$ is determined from the kinetic equation [2]:

$$\frac{d\eta}{dn} = f \left( \sigma_u^{(n)}, \eta_n \right) \text{ or } \eta = \int_0^n F(n-m)\nu(\sigma_u^{(n)}) \, dm$$

(14)

under conditions $\eta(0) = 0$, $\eta(N) = 1$, where $N$ is the number of half-cycles before the onset of the limiting state (destruction). In this problem, the finite difference method is used and in the process of their approximation, the central difference scheme of the second order of accuracy is used [13].

To solve the formulated algebraic equations with the corresponding boundary conditions, the matrix sweep method is used, using the following recurrence formula:

$$V_i^{(k)} = \alpha_i^{(k)} V_{i+1}^{(k)} + \beta_i^{(k)}; \quad i = N-1, ..., 1$$

(15)

$$\alpha_i^{(k)} = \left( \overline{B}_i^{(k)} - \overline{C}_i^{(k)} \alpha_i^{(k)} \right)^{-1} \overline{A}_i^{(k)}; \quad \beta_i^{(k)} = \left( \overline{B}_i^{(k)} - \overline{C}_i^{(k)} \alpha_{i-1}^{(k)} \right)^{-1} \left( \overline{C}_i^{(k)} \beta_{i-1}^{(k)} - \overline{F}_i^{(k)} \right)$$

(16)

at $i = 1, 2, ..., N-1$.

To implement the above algorithm, a modified integrated program was compiled in an object-oriented language.

3. Results and discussion

As an example, we present the results of calculating thin-walled rods of rectangular cross-section, pinched at the ends under repeated-alternating loading (Figure 1). The problem is solved with the following initial data: geometric and mechanical characteristics of the rod: $l=2.5m$; $h=0.1m$; $b_0=0.1m$; $E=2 \cdot 10^5$ MPa; $\varepsilon_s=0.0015$ uniformly distributed external loads:

$$f_0^0 = 25; \quad f_0^1 = 50; \quad f_0^* = 10; \quad f_0^c = 5 \text{ (kg } / \text{ sm}^2); \quad \overline{\tau} = \pi / 4; \quad \alpha = \pi / 3; \quad \gamma_s = \pi / 6; \quad \alpha_s = \pi / 2; \quad q^{(k)} = (-1)^{k+1}$

![Figure 1. The law of distribution of external load.](image)

Table 1 shows the maximum values of the rod displacement vector under cyclic loading ($k=5$) according to the generalized Masing principle for various materials (V-96, D-16T and St. TS).

To determine the true values of the calculated values, we used the variable loading theorem. The condition for the appearance of secondary, tertiary, etc. plastic areas is

$$\sigma_u^{(k)} \geq \alpha_4 \sigma_u$$

where $\alpha_4$ is the scale factor.

This problem is also solved taking into account the accumulation of damage with the following initial data: material constants of the kinetic equation of damage: $A=1.2 \cdot 10^4$; $\alpha = \beta = 5$; $\gamma = 0.8$; $\alpha_s = 0.97$; $B=1.4 \cdot 10^3$. The damage function $\eta(n)$ is determined from the kinetic equation:

$$\frac{d\eta}{dn} = A \left( \frac{\overline{\sigma}_u}{\sigma_u} \right)^\alpha \left( 1 - \gamma\eta^n \right)^\beta$$

(17)
Table 1. Calculation results according to the generalized Masing principle.

| Displacement vector $(V_i^{(k)})$ | Cyclically hardened | Cyclically softened |
|----------------------------------|---------------------|---------------------|
| $W^{(5)}(0.5)$                   | $Q = 2.08; \chi = 0.047$ | $Q = 2.02; \chi = 0.03$ |
| $\alpha_1^{(5)}(0.2)$           | 0.82524549          | 0.82526567          |
| $\beta_1^{(5)}(0.1)$            | 0.01984984          | 0.01985029          |
| $V^{(5)}(0.5)$                  | 0.25517460          | 0.25518087          |
| $\alpha_2^{(5)}(0.2)$           | 0.77374265          | 0.77376167          |
| $\beta_2^{(5)}(0.1)$            | 0.01861432          | 0.01861482          |
| $U^{(5)}(0.5)$                  | 0.00171965          | 0.00171973          |

If $\gamma = 1, r = 1, \alpha = \beta$ from (17) we obtain equations of the Kachanov-Rabotnov type, if $\gamma = 1, r \neq 1$ then we obtain the Shestrikov model. The calculation results are given ($\gamma = 0; \zeta = b_0$) at the points of the cross-section of the rod $x = 0.0; x = 0.2; x = 0.5$ under cyclic loading. Table 2 shows the kinetics of changes in the plasticity function $\phi^{(k)}$ and damage $\eta^{(k)}$, as well as the strain intensity $\varepsilon_u^{(k)}(\eta)$ and stresses $\sigma_u^{(k)}(\eta)$ depending on the loading cycle.

Table 2. Kinetics of changes in some physical and mechanical parameters depending on the loading cycle.

| $k$ | $x$ | $\phi^{(k)}(\eta)$ | $10^2 \eta^{(k)}$ | $10^2 \varepsilon_u^{(k)}(\eta)$ | $10^3 \sigma_u^{(k)}(\eta)$ |
|-----|-----|--------------------|-----------------|-------------------------------|-----------------------------|
|     |     |                    |                 |                               |                             |
| 1   | 0.0 | 0.8677             | 0.0000          | 1.7333                        | 4.7733                      |
|     | 0.2 | 0.7422             | 0.0000          | 0.6859                        | 3.7259                      |
|     | 0.5 | 0.7160             | 0.0000          | 0.6091                        | 3.6491                      |
| 10  | 0.2 | 0.8231             | 3.7556          | 1.7341                        | 6.4282                      |
|     | 0.5 | 0.6293             | 2.6155          | 0.6862                        | 5.3803                      |

Changes in the displacement components along the length of the rods are shown in Figure 2-a-f. The graphs of the calculated values $W^{(k)}$, $\alpha_1^{(k)}$, $\beta_1^{(k)}$, $V^{(k)}$, $\alpha_2^{(k)}$, $\beta_2^{(k)}$ for various values of the intensity of the load ($\delta = 1, \delta = 2, \delta = 3$) are given.

An analysis of a numerical experiment shows that with an increase in the number of loading cycles, the values of the plasticity and damage function change, and this, in turn, affects the strained and deformed state kinetics of thin-walled structures.
Figure 2. Changes to the components of movements along the length of the bar (a-f).

Figure 3. shows the ductility and damage zone for the cross section $x=0.0$, $x=0.5$ for $k=2$ (a, b) and $k=10$ (c, d), respectively.

Figure 3. The kinetics of changes in the ductility zone.
It can be seen from Figure 3 and Figure 4 that the end part of the rod completely passes into the plastic region. With distance from the left end of the rod, the plastic and damaged zones in the sections are extended to the upper right and lower left corners.

This calculation scheme was used to analyze the strained and deformed state of axisymmetric cylindrical shells and round plates [14-16].

4. Conclusion
The results of a numerical experiment show that with an increase in the number of loading cycles, the values of the plasticity and damage function change, and this, in turn, affects the kinetics of the strained and deformed state rod.

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