Commensuration Effects on Skyrmion Hall Angle and Drag for Manipulation of Skyrmions on Two-Dimensional Periodic Substrates

C. Reichhardt and C. J. O. Reichhardt
Theoretical Division and Center for Nonlinear Studies, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA
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We examine the dynamics of an individually driven skyrmion moving through a background lattice of skyrmions coupled to a 2D periodic substrate as we vary the ratio of the number of skyrmions to the number of pinning sites across commensurate and incommensurate conditions. As the skyrmion density increases, the skyrmion Hall angle is nonmonotonic, dropping to low or zero values in commensurate states and rising to an enhanced value in incommensurate states. Under commensuration, the driven skyrmion is channeled by a symmetry direction of the pinning array and exhibits an increased velocity. At fillings for which the skyrmion Hall angle is zero, the velocity has a narrow band noise signature, while for incommensurate fillings, the skyrmion motion is disordered and the velocity noise is broad band. Under commensurate conditions, multi-step depinning transitions appear and the skyrmion Hall angle is zero at low drives but becomes finite at higher drives, while at incommensurate fillings there is only a single depinning transition. As the gyrotropic component of the skyrmion dynamics, called the Magnus force, increases, peaks in the velocity that appear in commensurate regimes cross over to dips, and new types of directional locking effects can arise in which the skyrmion travels along other symmetry directions of the background lattice. At large Magnus forces, and particularly at commensurate fillings, the driven skyrmion can experience a velocity boost in which the skyrmion moves faster than the applied drive due to the alignment of the Magnus-induced velocity with the driving direction. In some cases, an increase of the Magnus force can produce regimes of enhanced pinning when the skyrmion is forced to move along a non-symmetry direction of the periodic pinning array. This is in contrast to systems with random pinning, where increasing the Magnus force generally reduces the pinning effect. We demonstrate these dynamics for both square and triangular substrates, and map out the different regimes as a function of filling fraction, pinning force, and the strength of the Magnus force in a series of dynamic phase diagrams.

I. INTRODUCTION

Magnetic skyrmions are particle-like spin textures found in numerous systems\textsuperscript{3,4}, including materials in which the skyrmions are stable at room temperature\textsuperscript{5,6,7,8}. Skyrmions can also be generated by an applied current\textsuperscript{9–12}. Due to their size scale, mobility, and stability, skyrmions are promising candidates for a variety of applications\textsuperscript{13–19}. Many of which require the ability to control how the skyrmions move, how they interact with defects or nanostructures, and how to manipulate them on the individual level. The skyrmions also interact with quenched disorder in the system, giving rise to a pinning effect and establishing a threshold driving force that must be applied for the skyrmion to be set in motion\textsuperscript{20–24}. The magnitude of this threshold depends on the properties of the disorder\textsuperscript{25,26,27,28}, collective interactions with other skyrmions\textsuperscript{29}, and thermal effects\textsuperscript{30,31}. There are also other types of skyrmions and skyrmion-like textures\textsuperscript{32}, including ferromagnetic\textsuperscript{33,34}, antiferromagnetic\textsuperscript{35–37}, skyrmions, antiskyrmions\textsuperscript{38,39}, and merons\textsuperscript{40}.

For ferromagnetic skyrmions subjected to an applied drive, the skyrmion motion exhibits a skyrmion Hall effect along the skyrmion Hall angle\textsuperscript{\[\theta_{sk}\]}\textsuperscript{3,4,45,46}, which arises from the topology of the skyrmions. The intrinsic skyrmion Hall angle\textsuperscript{\[\theta_{sk}^{int}\]} is proportional to the ratio of the Magnus or gyrotropic term to the dissipative term of the skyrmion dynamics. Many proposed skyrmion applications require the reduction or absence of the skyrmion Hall effect, so there have been numerous studies focused on understanding how to control the skyrmion Hall angle, such as by the use of nanostructures\textsuperscript{25,31}. The magnitude of the skyrmion Hall effect is modified both by the pinning landscape and by the velocity of the skyrmions. Quenched disorder induces a side jump effect that reduces the skyrmion Hall angle below its intrinsic value. The side jump is largest for small velocities just above depinning, giving a skyrmion Hall angle that is zero at depinning and increases with increasing drive until it saturates at a value near\textsuperscript{\[\theta_{sk}^{int}\]}\textsuperscript{39,30,38,47–55}. The skyrmion Hall angle can also be affected by shape distortions of the skyrmions and collisions with other skyrmions\textsuperscript{30,38,56,57}. In addition to the importance of understanding skyrmion dynamics in the presence of disorder for applications, the strong gyrotropic nature of the dynamics means that skyrmions represent a new class of systems that can show collective dynamics when driven over random or periodic substrates. Most previous studies of such behavior have involved overdamped systems\textsuperscript{25}.

One approach to generating well-controlled skyrmion motion is to couple the skyrmions to nanostructures, such as a periodic array of defects or other types of engineered landscapes\textsuperscript{21,22,25,58–61}. In this case, it is important to understand how individual skyrmions interact with both the defect array and with the other skyrmions. An example of such a system is skyrmions interacting with a two-
dimensional (2D) periodic substrate, where the system can be characterized by a filling factor $f$ corresponding to the ratio of the number of skyrmions $N_s$ to the number of pinning sites $N_p$. When $f = N_s/N_p$ is an integer, the skyrmions form a commensurate or ordered crystalline structure. Commensuration effects for particles on 2D substrates have been studied extensively in other condensed matter systems, such as the ordering of atoms or molecules on surfaces, sliding friction, colloidal particles on patterned substrates, dusty plasmas, Wigner crystal ordering in moiré systems, vortices in Bose-Einstein condensates, and cold atoms coupled to optical traps. The closest match to the skyrmion system, however, is vortices in type-II superconductors coupled to 2D pinning arrays. At commensuration, the superconducting vortices form an ordered lattice and exhibit a strong enhancement of the pinning effect observable as peaks in the depinning threshold as a function of changing superconducting vortex density.

Commensurate-incommensurate systems display a rich variety of dynamical phases since the collective motion differs at commensurate and incommensurate fillings. For example, there can be multiple step depinning, transitions between ordered and disordered flow, and soliton motion. Particle flow on 2D periodic substrates can be modified significantly depending on the direction of drive with respect to symmetry directions of the underlying substrate. For example, in directional or symmetry locking, the particles preferentially move along certain symmetry directions of the pinning lattice even when the drive is not aligned with those directions, and as a result, for changing drive orientation a series of steps appear in the velocity versus driving angle curves. Since the skyrmion Hall angle depends on the magnitude of the drive in systems with pinning, when individual skyrmions move over 2D periodic substrates, numerical studies have shown that the skyrmion motion locks to different substrate symmetry directions as the magnitude of the drive increases. For a square array, such locking directions include 0° and 45° from a primary lattice vector. In general, locking of skyrmions moving on a square array can occur at angles $\phi = \arctan(n/m)$ from the primary symmetry axis, with integer $n$ and $m$; however, the size of the pinning sites as well as interactions with other skyrmions can limit which symmetry directions are accessible. Other studies for skyrmions on 2D pinning arrays indicate that different types of crystalline ordering occur at the matching fields and that large scale collective flow states can arise under bulk driving. In micromagnetic simulations, skyrmions moving on a 2D pinning array exhibit a number of different dynamic phases that could be useful for applications. In addition, there are now various experiments on skyrmion states in periodic one-dimensional (1D) and 2D pinning arrays.

In this work, we examine the dynamics of skyrmions on a 2D square or triangular pinning lattice. We drive a single skyrmion that interacts both with the other skyrmions and directly with the substrate, and measure the velocity and direction of motion of the driven skyrmion as the system passes through a series of commensurate-incommensurate transitions at varied pinning strength and varied ratios of the Magnus term to the dissipative term. This work builds upon our previous studies examining the dynamics of individually driven superconducting vortices and skyrmions interacting with either a background lattice of particles or with pinning. In the case of superconducting vortices where the motion is overdamped, numerous methods to drive individual vortices, including nanotips and optical trapping, have been studied in experiments and simulations. Individual skyrmions can also be driven with different types of tips, local magnetic field gradients, and with optical trapping. The method of driving individual particles though a background of other particles while measuring the drag on the driven particle from fluctuations is known as active rheology and has been studied experimentally and theoretically for colloidal particles, granular matter, active matter, and superconducting vortex systems.

In most active rheology studies, under a constant driving force the velocity of the driven particle decreases as the density of the system increases due to an increase in the frequency of collisions with background particles, and there can be a sudden drop to zero motion or a pinning transition when the system passes through a critical density into a glass, jammed, crystalline, or amorphous solid state. In our previous work on active rheology in a skyrmion system, we considered a single skyrmion driven through a background of other skyrmions in the absence of pinning. For a constant driving force, we found that the skyrmion Hall angle decreases with increasing skyrmion density due to enhanced collisions; however, particularly for systems with a strong Magnus force, we also found a counter-intuitive increase in the velocity, or a boost effect, in which the skyrmion velocity increases with increasing system density. In some cases, the skyrmion velocity is larger than what it would be in the absence of collisions with other skyrmions. This boost effect arises from a combination of the skyrmion Hall effect and density fluctuations created in the surrounding skyrmions by the driven skyrmion. The density gradient forms perpendicular to the direction of the drive and exerts a repelling force on the driven particle along this direction, but the Magnus term generates a velocity perpendicular to this repelling force and parallel to the drive. This is example of what is known as an odd-viscosity effect of the type observed in chiral systems with gyroscopic forces. We have also considered single driven skyrmions interacting with other skyrmions in the presence of random quenched disorder, where in addition to velocity boost phenomena, we observe several pinned and jammed phases as well as stick-slip motion.

For overdamped systems we have also numerically examined the active rheology of superconducting vortices and colloidal particles interacting with 2D periodic pin-
ning arrays as the filling factor of the system is varied. Here we observe what we call an anti-commensuration effect in which the drag on the driven particle is reduced at commensurate matching conditions\textsuperscript{42}, opposite from the behavior found in bulk driven systems\textsuperscript{70,72-75}. The drag reduction at commensuration appears when the surrounding particles become strongly coupled to the substrate at matching conditions and cannot be dragged along by the driven particle, whereas at incommensurate fillings, the surrounding particles are much more weakly coupled to the substrate, permitting the driven particle to drag background particles and increasing the effective viscosity it experiences. The result is a strongly non-monotonic drag that shows a series of peaks at the matching conditions. In the case of an individual skyrmion driven over a 2D periodic array at commensurate and incommensurate conditions, the Magnus force produces much more complex dynamics than are found in the overdamped superconducting vortex system\textsuperscript{22}.

In this work we demonstrate that active rheology for a skyrmion driven through a background lattice in the presence of a 2D periodic pinning array produces very different behavior from that found for random pinning\textsuperscript{23} or in the absence of pinning\textsuperscript{22}. The skyrmion Hall angle is non-monotonic, falling to zero at commensurate conditions where the skyrmions form an ordered lattice and the skyrmion velocity peaks. For strong Magnus forces, the skyrmion Hall angle remains finite but is still reduced at the matching conditions. In general, the skyrmion motion is ordered at commensurate conditions and disordered at incommensurate fillings. For increasing Magnus force, there are more extended regions of finite drive for which the skyrmion remains pinned, a behavior that is the opposite of what is found for bulk driven systems with random pinning. At strong Magnus forces, under commensurate conditions we observe a pronounced velocity boost effect where the driven skyrmion moves faster than it would if there were no substrate and no collisions with other skyrmions. This boost occurs when the direction of motion of the driven skyrmion becomes locked along an interstitial channel and the driven skyrmion experiences a perpendicular repulsion from the skyrmions trapped in the pinning sites, which is converted by the Magnus term to a velocity in the direction of drive. This effect is similar to the velocity enhancement found for skyrmions moving along sample edges\textsuperscript{120,121}. For higher Magnus forces, the motion becomes increasingly chaotic and the effect of the substrate is strongly reduced.

\section{II. Simulation and System}

We consider a 2D system of size \(L \times L\) with periodic boundary conditions in the \(x\) and \(y\) directions containing a square pinning array with lattice constant \(a\). The total number of pinning sites is \(N_p\), giving a pinning density of \(n_p = N_p/L^2\). The sample contains \(N_s\) interacting skyrmions that are modeled as point particles according to a modified Thiele equation\textsuperscript{36,49}, in which the skyrmions have repulsive interactions with each other and attractive interactions with the pinning sites. We characterize the system by a filling factor \(f = N_s/N_p\). For integer values of \(f\), the system is commensurate and adopts a defect-free crystalline ordering, while for incommensurate fillings, the system either becomes amorphous or forms a crystalline state containing interstitials or vacancies\textsuperscript{22}. For certain fractional fillings, such as \(f = 1/2\) or \(f = 3/2\), the system can be partially ordered\textsuperscript{122,123}. Each pinning site has a finite spatial extent, so individual skyrmions can either sit inside a pinning site or in the interstitial regions between the pinning sites depending on the filling factor and the pinning strength. From previous studies of superconducting vortices interacting with 2D square periodic pinning arrays, it is known that the particles will form a square lattice with all of the pinning sites occupied for \(f = 1.0\), a checkerboard pattern with half of the particles in interstitials or vacancies for \(f = 0\), and a square lattice with all of the pinning sites occupied for \(f = 3.0\), and a hexagonal lattice at \(f = 4.0\)\textsuperscript{71,73,74,124}. The initial skyrmion positions are obtained by performing simulated annealing from a high temperature molten state down to \(T = 0\), as in previous work\textsuperscript{22}. After the system is initialized, we insert an additional interstitial skyrmion that is coupled to an applied driving force. This driven particle interacts both with the pinning sites and with the other skyrmions.

The equation of motion for skyrmion \(i\) is given by

\[ \alpha_d \ddot{v}_i + \alpha_m \dot{z} \times \dot{v}_i = F^\text{ss}_i + F^p_i + F^D_i \]  \hspace{1cm} (1) \]

where the skyrmion velocity is \(v_i = dr_i/dt\). The first term on the left with damping constant \(\alpha_d\) is the dissipation that aligns the skyrmion motion in the direction of the net applied force. The second term on the left is the Magnus force of magnitude \(\alpha_m\) that generates a velocity component perpendicular to the net force. The skyrmion-skyrmion interaction is described by \(F^p_i = \sum_{j=1}^{N_s} K_1(r_{ij})\dot{r}_{ij}\), where the distance between skyrmion \(i\) and skyrmion \(j\) is \(r_{ij} = |r_i - r_j|\), \(\dot{r}_{ij} = (r_i - r_j)/r_{ij}\), and \(K_1\) is the modified Bessel function which decays exponentially for large \(r\). The pinning sites are modeled as finite-range parabolic potential traps of radius \(r_p\) that exert a maximum pinning force of \(F_p\), giving \(F^D_i = \sum_{k=1}^{N_p} \Theta(|F_p/r_p - |r_i - r_k^p|)|\dot{r}_k^p|\), where \(\Theta\) is the Heaviside step function. We fix the pinning density to \(n_p = N_p/L^2 = 0.4882\) throughout this work. The driving force is \(F^D = F_D \dot{x}\) for the driven skyrmion and \(F^D = 0\) for all of the other background skyrmions, and the driving is always applied along the positive \(x\) direction. In the absence of other skyrmions or pinning, the driven skyrmion will move with an intrinsic skyrmion Hall angle of \(\theta_{sk}^\text{int} = \arctan(\alpha_m/\alpha_d)\). Pinning and skyrmion-skyrmion collisions can modify the observed skyrmion Hall angle, \(\theta_{sk} = \arctan(\langle V_x\rangle/\langle V_y\rangle)\), where \(\langle V_x\rangle\) is the average velocity along the driving direction and \(\langle V_y\rangle\) is the average velocity perpendicular to the
skyrmion moves in a disordered manner along where the background skyrmions are disordered. The driven always have \( \langle V \rangle \). For convenience, we use the normalization condition in the absence of pinning regardless of the value of \( \theta \). It also makes a velocity boost easy to detect, since for example if \( F_D = 1.0 \), there is a boost whenever \( \langle V \rangle > 1.0 \). We increment \( F_D \) from zero to a maximum value, spending \( 2 \times 10^6 \) to \( 5 \times 10^6 \) simulation time steps at each driving force increment in order to obtain a stationary state average velocity measurement.

### III. SKYRMION HALL ANGLE AT COMMENSURATE AND INCOMMENSURATE FILLINGS

In Fig. 1 we show a snapshot of a system containing a square pinning lattice with \( F_p = 0.25 \), \( \alpha_m/\alpha_d = 1.0 \), \( \theta_{sk}^{\text{int}} = -45^\circ \), and \( F_D = 1.0 \) at an incommensurate filling of \( f = 0.62 \) where the background skyrmions are disordered. The driven skyrmion follows a disordered trajectory with \( \theta_{sk} = -27^\circ \), indicating that the interactions with the pinning and the other skyrmions have depressed the magnitude of \( \theta_{sk} \) below that of its intrinsic value \( \theta_{sk}^{\text{int}} \).

In Fig. 2(a) we plot \( \langle V_x \rangle \) and \( \langle V_y \rangle \) versus filling fraction \( f \) for the system in Fig. 1. Figure 2(b) shows the net skyrmion velocity \( \langle V \rangle \) and Fig. 2(c) illustrates the corresponding \( \theta_{sk} = \arctan(\langle V_y \rangle/\langle V_x \rangle) \).

FIG. 1. Image of a sample containing a square pinning lattice showing background skyrmions (blue filled circles), pinning site locations (brown circles), the driven skyrmion (red filled circle), and the skyrmion trajectories (black lines) during a fixed time window. Here \( F_p = 0.25 \), \( \alpha_m/\alpha_d = 1.0 \), \( \theta_{sk}^{\text{int}} = -45^\circ \), and \( F_D = 1.0 \) at an incommensurate filling of \( f = 0.62 \) where the background skyrmions are disordered. The driven skyrmion moves in a disordered manner along \( \theta_{sk} = -27^\circ \).

In Fig. 2, we use the normalization condition \( (\alpha_x^2 + \alpha_y^2)^{1/2} = 1.0 \). This constraint ensures that we always have \( \langle V \rangle = (\langle V_x \rangle^2 + \langle V_y \rangle^2)^{1/2} = 1.0 \) for \( F_D = 1.0 \) in the absence of pinning regardless of the value of \( \theta_{sk}^{\text{int}} \). It also makes a velocity boost easy to detect, since for example if \( F_D = 1.0 \), there is a boost whenever \( \langle V \rangle > 1.0 \). We increment \( F_D \) from zero to a maximum value, spending \( 2 \times 10^6 \) to \( 5 \times 10^6 \) simulation time steps at each driving force increment in order to obtain a stationary state average velocity measurement.

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In Fig. 3 we show a blow up of the skyrmion motion for the system from Fig. 1 with \( F_p = 0.25 \), \( \alpha_m/\alpha_d = 1.0 \), \( \theta_{sk}^{\text{int}} = -45^\circ \), and \( F_D = 1.0 \). (a) A portion of the sample at \( f = 1.0 \), where \( \theta_{sk} = 0^\circ \). Here the driven skyrmion is outside of the visible frame. (b) The entire sample at \( f = 1.14 \), where \( \theta_{sk} = -10^\circ \).

FIG. 2. Behavior under varied filling fraction \( f = N_s/N_p \) for the system shown in Fig. 1 with a square pinning array, \( F_p = 0.25 \), \( \alpha_m/\alpha_d = 1.0 \), \( \theta_{sk}^{\text{int}} = -45^\circ \), and \( F_D = 1.0 \). (a) \( \langle V_x \rangle \) (red) and \( \langle V_y \rangle \) (blue) versus \( f \). (b) \( \langle V \rangle = (\langle V_x \rangle^2 + \langle V_y \rangle^2)^{1/2} \) versus \( f \). (c) \( \theta_{sk} = \arctan(\langle V_y \rangle/\langle V_x \rangle) \) versus \( f \), which goes to zero at commensurate fillings \( f = 1.0 \) and \( f = 2.0 \).

FIG. 3. Images of samples containing a square pinning lattice showing background skyrmions (blue filled circles), pinning site locations (brown circles), the driven skyrmion (red filled circle), and the skyrmion trajectories (black lines) during a fixed time window for the system from Fig. 1 with \( F_p = 0.25 \), \( \alpha_m/\alpha_d = 1.0 \), \( \theta_{sk}^{\text{int}} = -45^\circ \), and \( F_D = 1.0 \). (a) A portion of the sample at \( f = 1.0 \), where \( \theta_{sk} = 0^\circ \). Here the driven skyrmion is outside of the visible frame. (b) The entire sample at \( f = 1.14 \), where \( \theta_{sk} = -10^\circ \).
at \( f = 1.0 \) for the system in Fig. 2. The background skyrmions form an ordered commensurate square lattice, while the driven skyrmion moves along a 1D interstitial channel between two adjacent pinning rows. As \( f \) is further increased, \( \langle V_y \rangle \) and \( \theta_{sk} \) become finite again over the range \( 1.0 < f < 2.0 \) and the skyrmion Hall angle becomes finite, as illustrated in Fig. 3(b) at \( f = 1.14 \) where \( \theta_{sk} = -10^\circ \). The magnitude of the skyrmion Hall angle is smaller than that of \( \theta_{sk}^{int} \) due to the larger number of skyrmion-skyrmion collisions that occur at the higher densities. At \( f = 2.0, \theta_{sk} = 0.0 \) when the background skyrmions form an ordered checkerboard state and the driven skyrmion follows a 1D path along the \( y \) direction. For higher fillings, \( \langle V \rangle \) and the magnitude of \( \theta_{sk} \) gradually decrease.

In Fig. 4(a,b,c) we plot \( \langle V_x \rangle, \langle V_y \rangle, \langle V \rangle, \) and \( \theta_{sk} \) versus \( f \) for the system from Fig. 2 but with stronger dissipation of \( \alpha_m/\alpha_d = 0.204 \), where \( \theta_{sk}^{int} = -11.54^\circ \). We find that \( \theta_{sk} \) is equal to zero at \( f = 1.0 \) and \( f = 2.0 \), and is small in magnitude for all \( f > 0.5 \). There is still a strong peak in \( \langle V_x \rangle \) and \( \langle V \rangle \) at \( f = 1.0 \), with a smaller peak appearing at \( f = 2.0. \) In Fig. 4(d,e,f), similar behavior appears at \( \alpha_m/\alpha_d = 0.436 \) with \( \theta_{sk}^{int} = -21.9^\circ \), where there is a strong peak in \( \langle V \rangle \) at \( f = 1.0 \).

The behavior of \( \langle V_x \rangle, \langle V_y \rangle, \langle V \rangle, \) and \( \theta_{sk} \) versus \( f \) in a system with \( \alpha_m/\alpha_d = 0.57 \) and \( \theta_{sk}^{int} = -30^\circ \) appears in Fig. 5(a,b,c). There are peaks in \( \langle V \rangle \) at \( f = 1.0, 2.0, \) and \( 3.0 \), coinciding with fillings for which \( \theta_{sk} = 0^\circ \). Figure 5(d,e,f) shows the same quantities for a sample with \( \alpha_m/\alpha_d = 1.33 \) and \( \theta_{sk}^{int} = -53^\circ \). At the matching fillings, \( \langle V_y \rangle \) has a dip in magnitude but does not drop completely to zero. There is a cusp in \( \theta_{sk} \) to \( \theta_{sk} = -4^\circ \) at \( f = 1.0 \), and another more rounded cusp appears at \( f = 2.0. \) For these parameters, there is no peak in \( \langle V \rangle \) at \( f = 1.0, \) but there is a smaller peak at \( f = 0.66 \). In Fig. 5(e) we observe regions over which \( \langle V \rangle > 1.0 \), indicated by locations where the curve rises above the dashed line. This is a signature of a velocity boost produced by interactions with the background skyrmions and the pinning sites. In general, as the density increases, there are more collisions with the background skyrmions that increase the effective dissipative velocity; however, since the collisions can also contribute to the odd viscosity, there is a competition between the viscosity components. When the odd viscosity term dominates, the velocity boost effect can emerge.

For a sample with \( \alpha_m/\alpha_d = 2.065 \) and \( \theta_{sk}^{int} = -64^\circ \), we plot \( \langle V_x \rangle, \langle V_y \rangle, \langle V \rangle, \) and \( \theta_{sk} \) versus \( f \) in Fig. 5(a,b,c). The peaks that appeared in \( \langle V \rangle \) at \( f = 1.0 \) and \( f = 2.0 \) for lower \( \alpha_m/\alpha_d \) have not only disappeared, but there is now a dip in \( \langle V \rangle \) at \( f = 1.0 \) accompanied by a small increase in the magnitude of \( \theta_{sk} \). A velocity boost appears over the entire range \( 0.1 < f < 1.0 \), as indicated by the curve rising above the dashed line \( \langle V \rangle = 1.0 \) line in Fig. 5(b). There is an increase in \( \langle V_x \rangle \) over the same range of fillings since the boosted velocity is aligned with the driving or \( +x \) direction, and, when the Magnus force is large enough, the magnitude of the boost increases as the magnitude of the skyrmion Hall angle \( \theta_{sk} \) decreases. We plot the same quantities for a sample with \( \alpha_m/\alpha_d = 4.92 \) and \( \theta_{sk}^{int} = -78^\circ \) in Fig. 5(d,e,f). Here there is no large feature at \( f = 1.0; \) instead, strong peaks or dips appear in \( \langle V_x \rangle, \langle V_y \rangle \) and \( \langle V \rangle \) at \( f = 2.0. \) For this filling, \( \theta_{sk} = -45^\circ \), indicating that the commensurate structure is directionally locked with one of the major symmetry angles of the pinning array. As \( f \) increases further, the magnitude of \( \theta_{sk} \) resumes its decrease after passing through the locking step at \( \theta_{sk} = -45^\circ \). At \( f = 2.0, \) the...
system forms an ordered checkerboard state with a well defined symmetry direction; however, away from $f=2.0$, the structure becomes disordered again and the directional locking is lost. In Fig. 6(d), $\langle V_x \rangle$ increases with increasing $f$ while the magnitude of $\langle V_y \rangle$ undergoes a moderate decrease. Figure 6(e) shows that a velocity boost is present over the entire range of $f$ except at very small values of $f$, with the largest boost appearing at $f=2.0$. At this filling, $\langle V \rangle \approx 1.8$, nearly twice as large as the expected velocity in the limit of a single skyrmion and no substrate.

Taken together, the results in this section indicate that commensurate effects can both reduce the skyrmion Hall angle and also speed up the skyrmion motion, two features that are desirable for applications. For values of $\alpha_m/\alpha_d$ higher than what is shown here, we observe similar trends; however, as the relative strength of the damping term becomes small, the motion becomes increasingly disordered and fluid-like, the curves become smooth, and the directional locking effects disappear. In addition, for values of $f$ higher than those shown here, the velocity boost is eventually destroyed. For lower $F_D$ and high fillings, a jamming effect can occur in which the driven skyrmion becomes pinned or jammed due to its strong interactions with the background skyrmions, similar to what is found in other active rheology systems.\(^{105, 125}\)

A. Fluctuations and Noise

We can also characterize the behavior of the commensurate and non-commensurate states using the fluctuation properties of the time series of the driven skyrmion velocity. In general, we find ordered motion at commensurate fillings for which $\theta_{sk} = 0^\circ$ and disordered motion at incommensurate fillings where $\theta_{sk}$ is finite. In Fig. 7(a,b,c) we plot the time series of the instantaneous velocities $V_x$ and $V_y$ for the system in Fig. 2 with $\theta_{sk}^{int} = -45^\circ$. At $f = 0.62$ in Fig. 7(a), where $\theta_{sk} = -28^\circ$, the velocities fluctuate rapidly and, as shown in the corresponding velocity distribution plots $P(V_x)$ and $P(V_y)$ in Fig. 7(d), the distributions have a broadened Gaussian form that is characteristic of random fluctuations. At $f = 1.0$ in Fig. 7(b,c), the motion is periodic and $P(V_y)$ is centered at zero. Here the velocity distributions are non-Gaussian and have sharp features indicative of the repeated periodic motion. In Fig. 7(c,d) at $f = 1.14$, corresponding to the motion with $\theta_{sk} = -10^\circ$ illustrated in Fig. 7(b), the flow is once again random with Gaussian velocity distributions. We find similar behaviors for commensurate and incommensurate fillings at other values of $\alpha_m/\alpha_d$, including the $\langle V_y \rangle = 0$ states at $f = 1.0$ and $f = 2.0$ where the motion is periodic. For higher values of $\alpha_m/\alpha_d$ such as $\alpha_m/\alpha_d = 1.33$ in Fig. 5(f), where $\theta_{sk}$ does not reach zero but is strongly reduced at $f = 1.0$ and $f = 2.0$, the motion is mostly periodic with occasional jumps in the transverse direction.

The skyrmion motion can also be characterized by examining the velocity noise, as considered in both simulations\(^{52}\) and experiments.\(^{126}\) From the time series in Fig. 7 we can extract the power spectrum $S_a(\omega) = \left| \int V_a(t)e^{-i\omega t}dt \right|^2$, where $a = x, y$. In Fig. 8 we plot the

![Figure 6](image_url)  
**FIG. 6.** Behavior under varied $f$ for the system from Fig. 2 with a square pinning array, $F_p = 0.25$, and $F_D = 1.0$, but with (a,b,c) $\alpha_m/\alpha_d = 2.065$ and $\theta_{sk}^{int} = -64^\circ$ and (d,e,f) $\alpha_m/\alpha_d = 4.92$ and $\theta_{sk}^{int} = -78^\circ$. (a,d) $\langle V_x \rangle$ (red) and $\langle V_y \rangle$ (blue) versus $f$. (b,e) $\langle V \rangle$ versus $f$. (c,f) $\theta_{sk}$ versus $f$. The dashed lines in (b) and (e) indicate the value $\langle V \rangle = 1.0$; boost effects are present when $\langle V \rangle$ rises above this value.

![Figure 7](image_url)  
**FIG. 7.** Velocity fluctuation data for the system from Fig. 2 with a square pinning array, $F_p = 0.25$, $F_D = 1.0$, $\alpha_m/\alpha_d = 1.0$, and $\theta_{sk}^{int} = -45^\circ$ at (a,d) $f = 0.62$, (b,e) $f = 1.0$, and (c,f) $f = 1.14$. (a,b,c) Time series of the instantaneous velocities $V_x$ (red) and $V_y$ (blue). (d,e,f) The corresponding velocity distributions $P(V_x)$ (red) and $P(V_y)$ (blue). The distributions have a Gaussian character at $f = 0.62$ in (d) and $f = 1.14$ in (f) when the motion is disordered, but develop sharp peaks at $f = 1.0$ in (e) when the motion is periodic.
FIG. 8. The power spectra $S_x(\omega)$ (red) and $S_y(\omega)$ (blue) for the velocity time series $V_x$ and $V_y$, respectively, in Fig. 2 from samples with a square pinning array, $F_p = 0.25$, $F_D = 1.0$, $\alpha_m/\alpha_d = 1.0$, and $\theta_{\text{sk}}^m = -45^\circ$. (a) $f = 0.62$. (b) $f = 1.0$, where there is a strong narrow band noise signature. (c) $f = 1.14$.

power spectra from the time series of both $V_x$ and $V_y$ for the system in Fig. 2. At $f = 0.62$ in Fig. 8(a) there are no peaks in the power spectra, indicating that the motion is random, while at $f = 1.0$ in Fig. 8(b), there is a strong narrow band noise signal as indicated by the sequence of peaks. In this case there are two overlapping periodic signals. The first, at higher frequencies, arises from the periodic motions of the driven skyrmion as it interacts with the ordered commensurate pinned skyrmions. The second, at lower frequencies, is produced by a small number of defects that are present in the commensurate configuration, which generate a time-of-flight velocity signature. For the disordered motion at $f = 1.14$ in Fig. 8(c), the noise signature has a broad band character. There is a weak periodic signal at higher frequencies due to the low value of $\theta_{sk}$ at this filling, which causes the skyrmion to flow through an interstitial channel between two adjacent pinning rows with infrequent but roughly periodic hops occurring from one interstitial channel to the next.

In Fig. 8 we use the behavior of the transport curves and $\theta_{sk}$ to construct a dynamic phase diagram as a function of filling fraction $f$ versus $\alpha_m/\alpha_d$ for the system in Fig. 2. In the red region, the skyrmion moves strictly along the direction of drive and $\theta_{sk} = 0^\circ$. In the blue region, the skyrmion Hall angle is finite but there is no velocity boost, and in the green region, there is both a finite skyrmion Hall angle and a finite velocity boost. Near

FIG. 9. Dynamic phase diagram as a function of filling fraction $f$ versus $\alpha_m/\alpha_d$ for the system in Fig. 2 with a square pinning array, $F_p = 0.25$, and $F_D = 1.0$ constructed using the features in the transport curves and the behavior of $\theta_{sk}$. In the red region, $\theta_{sk} = 0.0^\circ$, a condition that extends out to higher $\alpha_m/\alpha_d$ at commensurate fillings. In the blue region, $\theta_{sk}$ is finite but there is no velocity boost, while in the green region, $\theta_{sk}$ remains finite and a velocity boost appears.

the commensurate conditions of $f = 1.0$ and $f = 2.0$, the window of zero skyrmion Hall angle extends out to larger values of $\alpha_m/\alpha_d$. For these same commensurate conditions, even when the value of $\theta_{sk}$ becomes finite, it is still suppressed relative to its value away from commensuration, a feature that is not illustrated in the plot.

IV. VELOCITY FORCE CURVES

We next consider the effect of varying the driving force $F_D$ on the driven particle in the system from Fig. 2. In Fig. 10(a) we plot $\langle V_x \rangle$ and $\langle V_y \rangle$ versus $F_D$ at $f = 0.62$. There is a single depinning threshold at $F_D = 0.6$, above which motion occurs simultaneously in both directions. Figure 10(b) shows the corresponding $\theta_{sk}$ versus $F_D$, where $\theta_{sk} = 0^\circ$ at the depinning threshold. As $F_D$ increases, the magnitude of $\theta_{sk}$ gradually increases until it approaches the intrinsic value $\theta_{\text{sk}}^m = -45^\circ$ at higher drives. In Fig. 10(c), $\langle V_x \rangle$ and $\langle V_y \rangle$ versus $F_D$ for a sample with $f = 1.0$ reveal the presence of a two step depinning threshold. The first depinning transition occurs at $F_D = 0.55$, and for $0.55 < F_D < 1.25$ the motion is only along the $x$-direction with $\theta_{sk} = 0^\circ$. The second depinning transition occurs at $F_D = 1.25$, above which finite motion in the $y$ direction appears. In the corresponding plot of $\theta_{sk}$ versus $F_D$ in Fig. 10(d), $\theta_{sk}$ is zero above the first depinning transition and begins to increase in magnitude above the second depinning transition. There are also several steps or plateaus in the velocity, including one with $\theta_{sk} \sim -27^\circ$ and another with $\theta_{sk} = -45^\circ$. These correspond to directional lock-
The skyrmion moves in an orderly periodic fashion strictly along $-45^\circ$.

In Fig. 12(a) we plot the time series of $V_y$ on the 1/1 step at $F_D = 6.0$ in the system from Fig. 10(c,d), while in Fig. 12(c) we show the corresponding Fourier transform $F(\omega) = \int V_y e^{-i\omega t} dt$, where there is a strong narrow band signal. The time series of $V_y$ for a non-step region at $F_D = 3.25$ with partially periodic motion appears in Fig. 12(b), and the broadened signature of the corresponding $F(\omega)$ is shown in Fig. 12(d). In general, as a function of increasing $F_D$ we find that in regimes of directional locking where there are steps in $\theta_{sk}$, the periodic motion produces narrow band velocity noise, while in regimes where no steps are present, the motion is more disordered and the velocity noise is either broad band or contains weakly periodic signals.

For individual skyrmions moving over a square substrate in the absence of background skyrmions, previous work showed that the velocity-force curve and skyrmion Hall angle exhibit a series of steps corresponding to directional locking where there are steps in $\theta_{sk}$, the periodic motion produces narrow band velocity noise, while in regimes where no steps are present, the motion is more disordered and the velocity noise is either broad band or contains weakly periodic signals.

In Fig. 12(a) we plot the time series of $V_y$ on the 1/1 step at $F_D = 6.0$ in the system from Fig. 10(c,d), while in Fig. 12(c) we show the corresponding $\theta_{sk}$ versus driving force $F_D$ for the system from Fig. 10(c,d) with a square pinning array, $F_p = 0.25$, $\alpha_m/\alpha_d = 1.0$, and $\theta_{sk}^{\text{int}} = -45^\circ$ at a filling fraction of $f = 0.62$. (b) The corresponding $\theta_{sk}$ versus $F_D$. (c) $V_y$ (red) and $V_y$ (blue) versus $F_D$ for the same system at $f = 1.0$, where there is a two step depinning process. (d) The corresponding $\theta_{sk}$ versus $F_D$ contains two directional locking steps along the angles $\theta_{sk} = \arctan(1/2)$ and $\theta_{sk} = \arctan(1/1)$.

FIG. 10. (a) $\langle V_x \rangle$ (red) and $\langle V_y \rangle$ (blue) versus driving force $F_D$ for the system from Fig. 2 with a square pinning array, $F_p = 0.25$, $\alpha_m/\alpha_d = 1.0$, and $\theta_{sk}^{\text{int}} = -45^\circ$ at a filling fraction of $f = 0.62$. (b) The corresponding $\theta_{sk}$ versus $F_D$. (c) $\langle V_x \rangle$ (red) and $\langle V_y \rangle$ (blue) versus $F_D$ for the same system at $f = 1.0$, where there is a two step depinning process. (d) The corresponding $\theta_{sk}$ versus $F_D$ contains two directional locking steps along the angles $\theta_{sk} = \arctan(1/2)$ and $\theta_{sk} = \arctan(1/1)$.

FIG. 11. Images of samples containing a square pinning lattice showing background skyrmions (blue filled circles), pinning site locations (brown circles), the driven skyrmion (red filled circle), and the skyrmion trajectories during a fixed time window for the system in Fig. 10(c,d) with $F_p = 0.25$, $\alpha_m/\alpha_d = 1.0$, and $\theta_{sk}^{\text{int}} = -45^\circ$ at a filling fraction of $f = 1.0$. (a) $F_D = 2.75$ near the 1/2 step. (b) $F_D = 6.0$ along the 1/1 step.
transition near \( F_D = 0.5 \). Figure 13(c,d) shows the same quantities for \( f = 2.0 \), where there is a two step depinning transition in which \( \langle V_x \rangle \) becomes finite at \( F_D = 0.5 \) and \( \langle V_y \rangle \) does not become finite until \( F_D = 1.15 \). There is a reduced 1/2 locking step, but the 1/1 locking step remains robust. In Fig. 13(a) we illustrate the skyrmion trajectories in a subsection of the sample at \( F_D = 6.0 \) on the 1/1 locking step, where the driven skyrmion moves at \(-45^\circ\) through an interstitial channel in the checkerboard lattice formed by the background skyrmions. The 1/1 step is associated with a cusp in \( \langle V_x \rangle \) and \( \langle V_y \rangle \) as indicated in Fig. 13(c). For driving individual skyrmions over a periodic substrate in the absence of background skyrmions, similar cusps in the velocity-force curves appear at several of the transitions into and out of the directional locking steps. Figure 14(b) shows the same system in a non-step region at \( F_D = 3.25 \), where the motion is more disordered and \( \theta_{sk} \) is smaller in magnitude.

V. VARIED PINNING STRENGTH

We next consider the effect of increasing the pinning strength for the system in Fig. 2 with fixed \( \alpha_m/\alpha_d = 1.0 \) and \( F_D = 1.0 \). We show the \( \langle V_x \rangle, \langle V_y \rangle, \langle V \rangle \), and \( \theta_{sk} \) versus \( f \) for a sample with \( F_p = 0.125 \), which is half the value of \( F_p \) used in Fig. 2. The general trends remain unchanged, with the magnitude of \( \theta_{sk} \) dropping to zero at \( f = 1.0 \) and reaching a value close to zero at \( f = 0.1 \). Figure 15(d,e,f) shows the same quantities in a sample with stronger pinning, \( F_p = 0.5 \). The shapes of the curves remain similar to those at lower \( F_p \), but there is now a finite window of \( f \) near \( f = 2.0 \) where \( \theta_{sk} \) is close to zero.

In Fig. 15(a,b,c) we plot \( \langle V_x \rangle, \langle V_y \rangle, \langle V \rangle \), and \( \theta_{sk} \) versus \( f \) for the same system in Fig. 15 but at stronger pinning of \( F_p = 0.625 \). There are now regions, such as near \( f = 1.0 \), where the system is pinned. There are also extended regions near \( f = 2.0 \) and \( f = 3.0 \) where \( \theta_{sk} \) is close to zero. The pinned phase arises from the interaction between the driven skyrmion and the pinning sites, which occurs both directly when the driven skyrmion encounters a pinning site, and indirectly when the driven skyrmion experiences a repulsion from a pinned skyrmion. At low \( f \), the driven particle has only direct interactions with the pinning sites, and as long as \( F_D/F_p > 1.0 \), it will not become pinned. At large \( f \), all of the pinning sites are occupied by background skyrmions and the driven skyrmion never encounters a pinning site directly; however, for certain incommensurate fillings at which the positions of the background skyrmions become disordered,
appearing in between those fillings, while the magnitude with \( \alpha \) Fig. 17 we construct a dynamic phase diagram as a function of the same system at \( f < f_0 \) for \( \langle V_x \rangle \) (red) and \( \langle V_y \rangle \) (blue) versus \( f \). (c,f) \( \theta_{sk} \) versus \( f \). The pinning strength is (a,b,c) \( F_p = 0.625 \) and (d,e,f) \( F_p = 0.75 \).

Fig. 16. Behavior under varied filling fraction \( f \) for the system from Fig. 4 with a square pinning array, \( \alpha_m/\alpha_d = 1.0 \), \( \phi^o_{sk} = -45^\circ \), and \( F_D = 1.0 \). (a,d) \( \langle V_x \rangle \) (red) and \( \langle V_y \rangle \) (blue) versus \( f \). (b,e) \( \langle V \rangle \) versus \( f \). (c,f) \( \theta_{sk} \) versus \( f \). The pinning strength is (a,b,c) \( F_p = 0.625 \) and (d,e,f) \( F_p = 0.75 \).

For low \( f \) where there are few interactions with background skyrmions, the brown region indicates that the motion is locked to ground skyrmions are disordered enough that the driven skyrmion begins to divert into the background skyrmions. For \( f > f_0 \), there are no velocity fluctuations, and the driven skyrmion can become pinned via interactions with skyrmions located at pinning sites. The plots of \( \langle V_x \rangle \), \( \langle V_y \rangle \), \( \langle V \rangle \), and \( \theta_{sk} \) versus \( f \) in Fig. 10(d,e,f) for the same system at \( F_p \) = 0.75 show that motion only occurs for \( f < 0.05 \) and \( f > 2.0 \) with a broad pinned window appearing in between those fillings, while the magnitude of \( \theta_{sk} \) remains below \( 10^\circ \) for the higher fillings.

Using the features in the transport curves and \( \theta_{sk} \), in Fig. 17 we construct a dynamic phase diagram as a function of \( F_p \) versus \( f \) for the system in Figs. 13 and 10 with \( \alpha_m/\alpha_d = 1.0 \) and \( F_D = 1.0 \). A large pinned region appears at larger \( F_p \). There are some smaller pinned regimes (not shown) above \( f = 2.1 \). The intervals of pinning also depend strongly on \( F_D \). When \( \theta_{sk} \approx 0^\circ \), the motion is locked to the \( x \) direction, shown as red regions. For low \( f \) where there are few interactions with background skyrmions, the brown region indicates that the motion is locked to \( \theta_{sk} = 45^\circ \). At these low values of \( f \), pinned states occur only when \( F_p > F_D = 1.0 \). In the blue region, \( \theta_{sk} \) is finite and there is no boost effect. Near the border between the flowing and pinned regimes, there are some windows of stick-slip motion which produce \( 1/f \) velocity fluctuation noise and a bimodal velocity distribution with a peak at \( \langle V \rangle = 0 \) and a second peak at higher velocities.

We next consider a sample with strong pinning of \( F_p \) = 0.75 and \( F_D = 1.0 \) where we vary \( \alpha_m/\alpha_d \). In Fig. 13(a) we plot \( \langle V_x \rangle \) and \( \langle V_y \rangle \) versus \( f \) for a system with \( \alpha_m/\alpha_d = 0.1 \) and \( \theta_{sk}^{int} = -5.74^\circ \). In this case, \( \langle V_y \rangle = 0 \) over the entire range of \( f \) measured. There are two pinned intervals with \( \langle V_x \rangle = 0 \) at \( 0.075 < f < 0.92 \) and \( 1.6 < f < 2.07 \). For low \( f \), the driven skyrmion moves along the pinning rows in the \( x \)-direction. As \( f \) increases, the driven skyrmion begins to collide with other skyrmions and becomes pinned by the combination of the pinning and the interactions with the background skyrmions. Near \( f = 1.0 \) where the system is more ordered, the driven skyrmion channels along the \( x \) direction between the pinned skyrmions. Near \( f = 0.8 \), the background skyrmions are disordered enough that the driven skyrmion trajectory begins to divert into the \( y \) direction, but the driven skyrmion quickly becomes trapped among the background skyrmions. For \( f > 2.0 \), there are re-
regions where the driven skyrmion depins and shepherds some of the background skyrmions along the x direction. Figure 18(b) shows $\langle V_x \rangle$ and $\langle V_y \rangle$ for a sample with $\alpha_m/\alpha_d = 0.58$ and $\theta_{sk}^{\text{int}} = -30^\circ$. For low $f$, the skyrmion Hall angle is finite. There is also a reduced window near $f = 1.0$ where the driven skyrmion channels along the $x$ direction. Figure 18 demonstrates that increasing the Magnus component of the dynamics enhances the effectiveness of the pinning, a behavior opposite from what is typically observed in systems with random pinning.

The high Magnus force causes the driven skyrmion to attempt to move partially in the $y$ direction rather than strictly along the $x$ direction, and since the direction of motion no longer coincides with a symmetry direction of the pinning lattice, encounters with pinning and pinned skyrmions happen more frequently and increase the effectiveness of the pinning.

In Fig. 19(a,b,c) we plot $\langle V_x \rangle$, $\langle V_y \rangle$, $\langle V \rangle$, and $\theta_{sk}$ versus $f$ for a sample with $\alpha_m/\alpha_d = 2.06$ and $\theta_{sk}^{\text{int}} = -64.15^\circ$. From $0.41 < f < 1.08$, there is an unpinned region with motion locked to $-45^\circ$, as indicated by the 1/1 label in Fig. 19(c). There is a pinned region for $1.08 < f < 2.16$ followed by another region in which motion occurs in both the $x$ and $y$ directions. Notice that when $f = 0$, $\langle V_x \rangle = 0.44$, but that in the range $0.41 < f < 1.0$, $\langle V_x \rangle \approx 0.9$, indicating a substantial boost of the velocity in the direction of driving. There is also a boost in the overall velocity for $1.08 < f < 2.16$, as shown in Fig. 19(b) where $\langle V \rangle$ rises above the dashed line marking $\langle V \rangle = 1.0$. Figure 19(d,e,f) shows $\langle V_x \rangle$, $\langle V_y \rangle$, $\langle V \rangle$, and $\theta_{sk}$ versus $f$ for a sample with $\alpha_m/\alpha_d = 3.04$ and $\theta_{sk}^{\text{int}} = -71.8^\circ$. The pinned regions are reduced in width and occur away from the commensurate fillings. There is still directional locking to $\theta_{sk} = -45^\circ$ near $f = 1.0$, as indicated in Fig. 19(f) by the 1/1 label, and the velocity boosted regime in Fig. 19(e) is more extended.

In Fig. 19(a,b,c) we plot $\langle V_x \rangle$, $\langle V_y \rangle$, $\langle V \rangle$, and $\theta_{sk}$ versus $f$ for a system with $\alpha_m/\alpha_d = 4.39$ and $\theta_{sk}^{\text{int}} = -77.16^\circ$. There is a single pinned region near $f = 0.5$ and a peak in the magnitude of $\langle V_x \rangle$, $\langle V_y \rangle$, and $\langle V \rangle$ at $f = 2.0$, but there is no peak near $f = 1.0$. The peak near $f = 2.0$ falls at the end of an extended window of locking to $-45^\circ$, shown as the 1/1 step in $\theta_{sk}$ in Fig. 20(c). Over most of the range of $f$ shown, there is a strong velocity boost in $\langle V \rangle$, as indicated by $\langle V \rangle$ running above the $\langle V \rangle = 1.0$ line in Fig. 20(b). Figure 20(d,e,f) shows $\langle V_x \rangle$, $\langle V_y \rangle$, $\langle V \rangle$, and $\theta_{sk}$ for a sample with $\alpha_m/\alpha_d = 13.28$ and $\theta_{sk}^{\text{int}} = -85.7^\circ$, where there is no longer a pinned phase and only a small 1/1 locking step appears near $f = 3.0$. Here the magnitude of $\theta_{sk}$ decreases nearly monotonically with increasing $f$, while $\langle V \rangle$ shows an increasing velocity boost as $f$ becomes larger. For large Magnus forces such as this, the dynamics become increasingly disordered, destroying the directional locking.

From the features in the transport curves and the behavior of $\theta_{sk}$ for the system in Figs. 18, 19 and 20 we construct a dynamic phase diagram as a function of $f$ versus $-\theta_{sk}^{\text{int}}$ in Fig. 21. We highlight the pinned phase, locking to the $x$ direction with $\theta_{sk} = 0^\circ$, locking to $\theta_{sk} = -45^\circ$ or the 1/1 direction, locking to $\theta_{sk} = \arctan(1/2)$ or the 1/2 direction, and motion at a finite skyrmion Hall angle. No distinction is made between regions with and without a velocity boost in this figure. For $-\theta_{sk}^{\text{int}} < 45^\circ$, there are extended regions of locking in the $x$-direction, while the additional directional locking effects appear only for higher Magnus forces with $-\theta_{sk}^{\text{int}} > 60^\circ$. Near $f = 1.0$ and 2.0, we find extended regions where $\theta_{sk} = 0^\circ$. There are extended regions of velocity boosting (not shown) which appear when $-\theta_{sk}^{\text{int}} > 45^\circ$. 

![Figure 19](image1.png)  
![Figure 20](image2.png)
FIG. 21. Dynamic phase diagram as a function of filling fraction \( f \) versus intrinsic skyrmion Hall angle \(-\theta_{sk}^{\text{int}}\) for the system from Figs. 18-20 with a square pinning array, \( F_p = 0.75 \), and \( F_D = 1.0 \). The pinned region is light blue, the red region is motion with \( \theta_{sk} = 0.0^\circ \), the brown region is motion with \( \theta_{sk} = -45^\circ \), the dark brown region is motion with finite \( \theta_{sk} \). Regions with and without velocity boost are not distinguished in this diagram.

FIG. 22. Behavior under varied filling fraction \( f \) for a system with a triangular pinning array, \( F_p = 0.5 \), and \( F_D = 1.0 \). \( \langle V_x \rangle \) (red) and \( \langle V_y \rangle \) (blue) versus \( f \). (a,d) \( \theta_{sk} \) versus \( f \). (b,e) \( \langle V \rangle \) versus \( f \). (c,f) \( \alpha_m/\alpha_d \) and \( \theta_{sk}^{\text{int}} \). (a,b,c) \( \alpha_m/\alpha_d = 1.73 \) and \( \theta_{sk}^{\text{int}} = -60^\circ \).

VI. TRIANGULAR PINNING ARRAYS

If the square pinning array is replaced with a triangular pinning array, similar behavior occurs. In Fig. 22(a,b,c) we plot \( \langle V_x \rangle \), \( \langle V_y \rangle \), \( \langle V \rangle \), and \( \theta_{sk} \) versus \( f \) for a sample with triangular pinning, \( F_p = 0.5 \), \( F_D = 1.0 \), \( \alpha_m/\alpha_d = 1.0 \), and \( \theta_{sk}^{\text{int}} = -45.0^\circ \). Here \( \theta_{sk} \) goes to zero at \( f = 1.0 \) but not at \( f = 2.0 \). For a triangular pinning array, the background skyrmions form a commensurate triangular lattice at \( f = 1.0 \); however, at \( f = 2.0 \) the skyrmions form a honeycomb structure rather than a triangular lattice. A similar structure has been observed for superconducting vortices on a 2D triangular pinning array. Since the honeycomb arrangement is less stable than the triangular arrangement, the background skyrmions are not as strongly pinned at \( f = 2.0 \) compared to \( f = 1.0 \), and there is less reduction of the drag on the driven skyrmion at \( f = 2.0 \). Figure 22(d,e,f) shows \( \langle V_x \rangle \), \( \langle V_y \rangle \), \( \langle V \rangle \), and \( \theta_{sk} \) versus \( f \) for the same sample at \( \alpha_m/\alpha_d = 1.73 \) and \( \theta_{sk}^{\text{int}} = -60^\circ \), where the skyrmion Hall angle does not show any feature at \( f = 1.0 \). There is a small velocity boost for \( f < 1.0 \), as indicated by the excursion of \( \langle V \rangle \) above the value \( \langle V \rangle = 1.0 \) in Fig. 21(c).

In Fig. 23(a,b,c) we plot \( \langle V_x \rangle \), \( \langle V_y \rangle \), \( \langle V \rangle \), and \( \theta_{sk} \) versus \( f \) for a triangular pinning system with \( \alpha_m/\alpha_d = 0.374 \) and \( \theta_{sk}^{\text{int}} = -20.5^\circ \). There is a pinned region near \( f = 0.4 \) along with a region of \( \theta_{sk} = 0.0^\circ \) centered at \( f = 1.0 \) that coincides with a peak in the magnitudes of \( \langle V_x \rangle \) and \( \langle V_y \rangle \). Figure 23(d,e,f) shows the same quantities in a sample with \( \alpha_m/\alpha_d = 0.658 \) and \( \theta_{sk}^{\text{int}} = -33.4^\circ \), where a peak appears in \( \langle V_x \rangle \) and \( \langle V \rangle \) for \( f = 1.0 \). We do not see the same commensurate effects at \( f = 2.0 \) due to the honeycomb ordering.

Figure 23(a,d) illustrates the skyrmion trajectories for the system in Fig. 22(a,b,c) at \( f = 0.6 \) where the skyrmion motion is along \( \theta_{sk} \approx -18^\circ \). The driven skyrmion interacts both directly with the pinning sites and with the background skyrmions, and its trajectory is disordered. In Fig. 23(b), the same system at \( f = 1.0 \) exhibits channeling motion along the \( \theta_{sk} = 0^\circ \) symmetry direction of the pinning array.
or even quasiperiodic \cite{130,131} exhibit a variety of phase locking phenomena. We also expect that this system could and experience shape distortions, or if internal skyrmion multiple skyrmions become trapped at a single pinning site, point particle approximation breaks down, such as if multiple effects could enhance some of the phenomena we observe. For example, at commensurate conditions the system forms an ordered structure that should be more resistant to thermal fluctuations, while at incommensurate fillings, the effectiveness of the pinning is reduced so that creep effects should be more prominent. In superconducting vortex systems with periodic pinning, commensuration effects are generally enhanced at higher temperatures. This effect could also be important when the point particle approximation breaks down, such as if multiple skyrmions become trapped at a single pinning site and experience shape distortions, or if internal skyrmion modes are excited. We also expect that this system could exhibit a variety of phase locking phenomena \cite{113,132} if ac driving were introduced. Such effects should be strongly enhanced near commensurate conditions. Our results should be general to other systems coupled to periodic substrates where gyroscopic effects come into play.

VIII. SUMMARY

We have numerically examined the dynamics of individual skyrmions driven through an assembly of other skyrmions in the presence of a two-dimensional periodic pinning array. The Magnus force causes the driven skyrmion to move with a finite skyrmion Hall angle. Under a constant driving force, we find a non-monotonic dependence of the skyrmion Hall angle on the density of the background skyrmions. In general, the skyrmion Hall angle drops to zero or is reduced in magnitude at commensurate conditions when the number of skyrmions is equal to an integer multiple of the number of pinning sites. There is also a peak in the net skyrmion velocity at the commensurate filling. At incommensurate fillings, the skyrmion Hall angle becomes finite again, but there is generally a decrease in the skyrmion Hall angle with increasing filling fraction. At commensurate fillings we find a two step depinning process and the motion of the driven skyrmion is well ordered, while at incommensurate fillings there is a single step depinning transition with disordered motion. For larger Magnus forces, additional locking effects can appear in which the motion of the driven skyrmion locks to different symmetry directions of the pinning lattice. In some cases, we find that increasing the Magnus force can enhance the effectiveness of the pinning, which is opposite to the behavior observed in a system with random pinning. This occurs because the Magnus force reduces the amount of channeling along the $x$ direction. We show that our results are robust for both square and triangular pinning arrays. We also map out dynamical phase diagrams as a function of varied pinning strength, Magnus force contribution, and filling fraction.

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