Hydraulic-biological substantiation fish distributions in the border layer

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Abstract. The aim of the research is the search and development of methods (tools) for regulating fish behavior in the conditions of changing the water stream’s kinematic structure. This is possible only through a thorough study of the kinematic, hydrodynamic features of the biological object’s behavior and their correlation with the kinematics and hydrodynamics of the habitat (river, boundary layer). The availability of such information undoubtedly opens up a broad prospect for creating the fish protection zones near water intakes or in the lower pools of hydroelectric facilities before entering the fish passage facilities, in which the movement, habitat of fish will occur in the most natural comfortable way with minimal energy consumption. Using the method of “comparing” the bio-hydrodynamic characteristics of fish with the water flow structural elements’ parameters, a number of general laws are established. On the basis of these laws the hydraulic calculation of the optimal vortex structures’ kinematic parameters for fish is performed.

Introduction

The solution of the acute environmental problem of preserving and reproducing the fish stocks under the conditions of the river flow artificial regulation with increasing water consumption from the surface water sources is associated with the need to study the kinematic structure of the water flow and the hydrodynamic features of a biological object’s behavior in their relationship and interdependence. However, the lack of such information is holding back the solution to this problem. The purpose of this study is to develop the tools to control fish behavior by changing the water stream kinematic structure.

Using the method of “comparing” the bio-hydrodynamic characteristics of fish with the water flow structural elements’ parameters, a number of general laws are established. On the basis of these laws the hydraulic calculation of the optimal vortex structures’ kinematic parameters for fish is performed.

The conclusions made on the Prandtl hypothesis basis confirm the thesis of the advisability of inducing fish in a stream of the scales vortices already existing in the river stream turbulent medium.
Once in a particular water-exchange (boundary) layer, the fish, in search of suitable (energy-saving) conditions, chooses the most vortex scale for itself.

Two tasks are of practical importance: in the first, the characteristic sizes of the vortices are determined by the given fish size; in the second, the characteristic fish size is determined by the given size of the vortex (in the boundary layer). The size and estimated location of the fish in the boundary layer are determined in accordance with its swimming ability in terms of the vortices’ scale formed by it.

In the course of the theoretical study, a method allowing to determine the probabilistic location in the cylinder wake and in the boundary layer, formed near the vertical wall for a given size of the fish having a subcarangiform or carangiform locomotion type was developed.

The presented relationship of the biological object with the water flow hydrodynamics opens up the real possibilities for the better methods’ development, promising the designs of devices and structures for fishery hydraulic engineering, the creation of fish protection zones near the water intakes or in the lower pools of hydroelectric facilities before entering the fish passage constructions in which the fish would move and inhabit most naturally comfortable with minimal energy.

Ensuring the ichthyofauna protection is based primarily on the scientific substantiation of the fish behavior in the river flow. The study of the fish various ecological groups adaptation to movement in the aquatic environment is of considerable interest in both theoretical and practical aspects. The absence of this kind of knowledge hinders the solution of acute environmental problems of the conservation and the fish stocks reproduction in the conditions of artificial regulation of river flow with an increasing volume of water consumption from the surface water sources.

The aim of the research is the search and development of methods (tools) for regulating fish behavior by changing the kinematic structure of the water stream. This is possible only through a thorough study of the kinematic, hydrodynamic features of a biological object’s behavior and their correlation with the kinematics and hydrodynamics of the habitat (river, boundary layer). The availability of such information undoubtedly opens up a broad prospect for creating the fish protection zones near water intakes or in the lower pools of hydroelectric facilities before entering the fish passage facilities, in which the movement, habitat of fish will occur in the most natural comfortable way with minimal energy consumption.

This work is a continuation and development of the conclusions made by the authors earlier in this direction [1-4].

Materials and Methods
To solve this problem, it is necessary: based on the fish swimming mechanism’s analysis, to study the phenomenon of the vortex structures’ formation by fish in an undisturbed aquatic environment; substantiate the kinematic parameters of the characteristic structural (vortex) elements of the river flow; compare the bio-hydrodynamic characteristics of the fish with the turbulent flow structural elements’ characteristics; to develop a method for determining the kinematic parameters of the vortex structures optimal for fish in a water stream.

Analysis of the fish swimming mechanism
Swimming is caused by the wave-like movement of the body and / or fins. With this body movement, a locomotor wave passing along the propulsor (a part of the body with a group of muscles) generates a driving force. The wave-like movement is created by the body and tail fin, acting as a whole; fins connected to the body. One or more half-waves travel from head to tail at a speed exceeding the fish speed relative to the water.

With the passage of such a wave, all segments of the body report acceleration of the adjacent water mass. The muscular force of the fish acts on the water, and an equal opposite force, the reaction force, acts on the propulsive element (Newton’s third law). On the other hand, $F = ma$, where $F$ – is the reaction force perpendicular to the propulsive element, $a$ – defines the water acceleration, $m$ – is the water mass receiving this acceleration.
During the locomotor wave passage, the back of the body, bending, moves a greater distance than the front. A greater caudal deviation of the para-caudal segments of the body is created in comparison with the peri-stem segments and the longitudinal force created by the tail elements of the body exceeds in magnitude the force created by the elements at the head.

Therefore, for the translational motion, the para-caudal elements are more important, since a higher velocity of the cauda equina transverse motion provides greater acceleration to the water.

The method of traction occurring as a result of the propulsive element movements during the passage of a locomotor wave through the body and the caudal fin largely determines the body shape that is most suitable for the needs of this fish species.

The fish silhouette is an important factor due to which the forward movement occurs. In the induction movement, maximum traction is created precisely by fish with a high silhouette. In the process of continuous swimming, the bending of the body significantly affects the caudal fin movement. The fish silhouette shape, which will provide the greatest efficiency of movement, depends on such segments’ interaction [5].

Kinematic and morphological models

In Figure 1, kinematic-morphological models of the biological objects’ movers’ types in the aquatic environment and the fish body are presented in graphical form [4].

Figure 1. Kinematic and morphological models: a) - types of the biological objects’ movers in the aquatic environment: 1 - eel, 2 - shark, 3 - body kit, 4 - molehill, 5 - stingray, 6 - moon fish, 7 - sea turtle, 8 - frog, 9 - shrimp, 10 - crayfish, 11 - jellyfish, 12 - squid, 13 - cuttlefish; b) - bodies of fish: 1 - eel, 2 - pike, 3 - swordfish, 4 - tuna, 5 - butterfly fish, 6 - salmon (pink salmon)

The pentagram in Figure 1, a represents the geometric model continuum, limited to five basic types of bio motors in the aquatic environment, namely: "locomotor wave"; “oscillation” (by the tail fin vibration); fins, paddle; jet stream. Each type of move is associated with a biological representative. So, for: locomotor wave the “specialist” is an eel or moray eel; tail fin oscillations - bodies; oscillations by other paired and unpaired fins - stingray (manta ray), moon fish, seahorse; rowing - shrimp, frog; jet stream - jellyfish, squid.

At each end of the pentagonal star between the two adjacent “specialists” there is a representative of the mover combined type. Between the eel and the body is a representative of the classic (scombroid) type of swimming - a shark; between the ramp and the car body - junk; between stingray and shrimp - sea turtle, penguin (amphibians); between shrimp and squid - crayfish; between squid and eel - cuttlefish with fin wings. This model clearly illustrates the kinematic interdependence of all the basic types of movers used in the aquatic environment by the biological objects.

In the central part of the kinematic-morphological model of the fish body (see Figure 1, b) salmon fish, characterized by a sub-carangiform locomotion type, are the most typical (classical) for most
river fish, and by the mover type (see Figure 1), a) corresponds to the upper end of a five-pointed gbh star. For this fish, the analysis of the vortex structures used by the fish in swimming will be performed further.

The hydrodynamic approach allows us to conclude that most of the traction force arises from the transfer of energy in space using a low-speed wave. In this case, unidirectional energy transfer is created not with the help of a moving mass (jet), but with the help of a wave. “The caudal fin is not regarded as a special specialized mover, all parts of the fish body make approximately the same contribution to the overall movement, and the caudal fin is one of the parts of the general distributed motor system of the fish in terms of force” [6, 7]. With an increase in the size of an individual, the maximum oscillation frequency of its tail gradually decreases, so that its ultimate cruising speed per unit length of the body decreases, although the translational speed itself increases.

Two of the above-mentioned approaches are the basis for substantiating the dynamic principle of the optimal organization of wave propulsors. So in [8], a model-theoretical substantiation of a bionic solution to the problem of creating an optimal wave propulsion system for water transport was presented: the propulsion device should have amplitude and phase characteristics of bending locomotor fish waves. The hydrodynamic problem of synthesizing the optimal wave propulsion, which provides the formation of a motive force by means of a bending wave running along a multilink chain, has been solved.

The mechanism of a locomotor vortex formation

The locomotor apparatus of a greater part of fish consists of specialized forms elements’ characteristic combinations. In a stationary and mobile aquatic environment (river), the fish locomotion has some significant differences. So, to create vortex structures in an unperturbed medium (ocean), the fish spends energy, and while in a turbulent medium (river flow), it is forced to react to active local vortex structures already existing in the boundary layer and use them optimally for their movement. Therefore, lateral receptors (lines) in river fish are more developed than in the marine ones, which indicates their greater sensitivity to external stimuli — pressure pulsations [9, 10], and can be characterized as “hydrodynamic vision”.

In the river flow, the change in speed is in a wide range. The same fish in its movement on different parts of the river can use one or another combination of body movement - from undulation to oscillation.

Animals artificially create such a disturbance around the body in which the wave propagates at low speed. The fish and the snake brought this principle to perfection, forming with their body a wave that moves along their body, but the rest skills are relative to the environment (water, sand, grass ...).

In water, the fish pushes the whole body back at different time intervals, and it moves forward, not allowing fluctuations to spread along the lateral directions. It turns out a zigzag (wedge-shaped) form of the object introduction into the thickness of undisturbed water. During this process, the side vortices with vertical axes of rotation are formed on both sides of the fish. Knowledge of their formation features is of particular practical interest.

This wave principle unites all types of locomotion indicated on the kinematic-morphological model (Figure 1, a). According to this model, the swimming ability of fish is characterized by five forms of locomotion: “turn”, “throw”, “sprint”, “marathon”, and “maneuver” [4]. Their swimming functions (motility) are correlated with the body shape [11]. The most striking representatives of these extreme forms (“specialists”) are located at the corners of the pentagon, these are: eel (moray eel), pike, swordfish, tuna, butterfly fish.

The mechanism of formation of lateral vortices

As a result of the bending and extension of the trunk (in the horizontal plane), a locomotor wave arises, which in the mooring mode forms and transports lateral (locomotor) vortices to the tail fin. This contributes to the traction force creation by unidirectional energy transfer in space [6, 7].

In total, in this harmonious process, traction is created by pushing the fish away from both the wave and the vortices with vertical, horizontal axes of rotation formed in the wake of the fish. The mechanism such vortex structures’ formation is described in [4, 12-15].
Figure 2 shows a two-dimensional diagram explaining the formation of vortex structures by a locomotor wave.

![Figure 2. Scheme of the formation of vortex structures by a locomotor wave](image)

The assumption that the wave propagates along the ox axis with a constant phase velocity, amplitude and wavelength is made, i.e.: \( V = \text{const}; \quad a_0 = \text{const}; \quad \lambda = \text{const} \). Solid bold line \((a_1, b_1, c_1, d_1, e_1)\) the initial position of the wave is indicated, and the dotted \((a_2, b_2, c_2, d_2, e_2)\) – her position after a time interval \(\Delta t\). During this period of time, the phase points the wave move over a distance \(\Delta l = V \cdot \Delta t\), equal to the segments’ length:

\[
\Delta l = a_1a_2 = b_1b_2 = c_1c_2 = d_1d_2 = e_1e_2 = \text{const},
\]

and the movement of the physical points of the propulsor (fish body) occurs in the transverse direction parallel to the axis oy \((b_1b_3 \text{ or } d_1d_3)\). Thus, each physical point of the wave oscillates (transverse), the phase points translate longitudinally, and in general the propulsive element acts as a flapping wing.

In the space before the wave \((a_1b_1c_1b_2a_2)\) an increased pressure zone, indicated by a “+” sign, is created, and behind the wave \((e_1d_1c_2b_2a_2)\) – the rarefaction zone indicated by the “-” sign. These two “antagonistic” zones are in contact at the top of the wave, which causes the displacement of the moved fluid from the “+” zone to the “-” zone along the path of least resistance, as shown by the arrow. Such a fluid movement can be compared with the phenomenon of the collapse of the sea wave, emerging in shallow water. An exception will be the case when the drag force on the plate surface (occurring in the boundary layer in the wave depression) at low amplitude will exceed the hydrodynamic pressure strength due to the pressure difference in the indicated zones.

Such a fluid movement in latent form initiates a phenomenon similar to the collapse of a sea wave emerging in shallow water. In the recess of the wave, a circulation flow is initiated, which is conventionally indicated in the figure (within the wave height) by a small developed vortex with a circle radius \(r_1\). At the same time, the tangent (in the arrow direction) flow represents the region of a possible shift (at the stress field level) along a closed circle with a radius \(r_2 > r_1\), that is, contoured in a continuous medium along the path of the least resistance (by analogy with the shear curve in cohesive soils). Large circle with radius \(r_2\) denotes the contour of the undeveloped future (downward) “vortex-avatar” at the voltage field level.

The motion of the propulsor at point b1 occurs at an acute angle, which contributes to the occurrence of a cumulative effect, which would manifest itself on the water surface as the jet splash. We call this vortex-avatar induced at the voltage field level (not always manifested in dynamics) “locomotor”.

**The fish locomotor eddies characteristic**

In the general case, as a result of fish locomotion, the locomotor vortices of different scale and
intensity are induced on both sides of it (Figure 3).

![Figure 3](image)

**Figure 3.** Induction by the fish locomotor vortices characteristic

It is logical to assume that the presence of such a (ready-made) combination of characteristic vortices in the turbulent flow of the river will facilitate the fish movement with less energy.

Despite the fact that the theory of forced movements does not reveal the biological significance of the whole spectrum of the fish behavioral reactions, however, it confirms the principle of correspondence of fish movement to the kinematics of turbulent flow, which is especially important when considering the issues related to fish protection and fish passage.

This important conclusion is based on the analysis of the fish movement characteristic features in the stream. So, in conditions of darkness or high turbidity, fish are able to stay in the thickness of the river stream without tactile contact with the bottom, wall [9, 10]. In this case, the reference points-stimuli for the fish are the vortex structural elements (pulsations) of the turbulent flow generated at the surface of the river bottom or in the wake of the streamlined object, side line receptors and touch remain in the work. With their help, the fish catches hydrodynamic and tactile stimulus-landmarks. This complex dowsing element can be characterized as “hydrodynamic vision” [2].

In undisturbed water, a certain amount of energy is spent on the vortices’ “unwinding” by fish, and it’s a completely different matter when such vortices are already created by the river in the natural manner. The presence of an ordered system of vortices forces the fish to interact with it. Being in the dark in the turbulent flow thickness, the fish, experiencing fluctuations in the water environment (pulsations) is forced to respond, optimally bending around or starting from them, that is, the ready-made vortices are used as supports for their movement [9, 16].

By adjusting to the kinematics of the vortex flow of the river (performing appropriate movements), the fish uses the opportunity to save energy. Otherwise, for example, in case of danger, the instinct of self-preservation is activated, the fish, saving its life, is forced to quickly, but briefly move at maximum (throwing) speeds without choosing the optimal route, sometimes jumping out of the water.

Thus, the mechanism of fish movement reflects the general tendency toward the release of an optimum energy, which for the river fish can be manifested in the tendency to move in the turbulent space of the vortex matrix in accordance with the kinematic structure of the flow along the path of least resistance. The fish swimming mechanism developed during evolution is a reflection of the internal mechanism of fluid movement in the boundary layer. Therefore, for its comfortable advancement from the entire spectrum of frequencies and scales of vortices in the river flow, the fish prefers those that are most related to the geometry of the wave bending of the body, being characteristic of it. Further, under the term “characteristic vortex” we mean the locomotor vortices that provide the maximum energy saving (in contrast to uncomfortable ones, which require increased energy consumption from fish).

**Justification of the optimal (characteristic) parameters of vortex structures in a turbulent flow**

From the point of view of controlling the fish behavior, all styles and modes of swimming of fish (see Figure 1) are of practical interest. For example, to create fish stores for fish protection or fish access, the attractive modes providing the comfortable (relaxation) and optimal fish swimming (at cruising speeds) during spawning migrations are preferable. The regime of “throw” and “sprint” (as forced high-speed) must be borne in mind when developing the fish passageways, in the tract of which there
are the sections with increased speeds, in particular, to calculate their length. In this case, along the length of the fish passage tract, it is necessary to arrange the recreation areas.

In the natural conditions of the river channel, such zones are formed behind the bottom, ridge deposits of different scales, which provide an opportunity for fish of different sizes to relax. So, for the conditions with increased flow rates, a design was developed in [17] and the geometric parameters of the bottom flow-forming element for controlling the fish behavior of the “dune” type were justified, and an optimal layout scheme for the elements’ placement in the river bed was established.

The fish “marathon” movement mode is optimal for swimming fish for a long time at cruising speeds. For this mode, the optimal amplitude of the fish pulsing $a_0$ compose 0,2-0,23 by the fish length $l_p$, i.e $k_p = a_0/l_p = 0,2 \div 0,23 \approx \text{const}$, where $k_p$ – is the relative amplitude of the optimal vibrations of the caudal fin of the fish [18, 19]. The jet expansion angle formed in the fish wake in the mooring mode, determined by this constant, is:

$$\theta_1 = \arctg\left(\frac{a_0}{l_p}\right) = 11,3 \div 13^\circ.$$

The value of this angle coincides with the expansion of a conventional flooded turbulent jet [20]. It seems that the amplitude of the fish caudal fin deviation due to the need to form the optimal (to create reactive thrust) jets.

On the other hand, it is known that the fry from a very early age most of the time lives in the coastal zone. The most attractive (from the standpoint of energy saving) for it is the area of the reduced speeds behind the vertical fairings. Being here, the fish can be held for a long time, making forced vibrations after the movement of streams (vortices) in the space between the reed stalks.

Figure 4 from [21] shows an illustration of the dropping vortices’ process from the caudal fin when the fish move and the formation of the jet flow induced by these vortices.

**Figure 4.** Illustration for the dropping vortices’ process from the tail fin by fish moving

The mechanism of fish movement reflects the general tendency to release an optimum energy, which for river fish manifests itself in a tendency to move in turbulent space in accordance with the kinematic structure of the flow along the path of the least resistance.

It is no accident that Karman and Burgers in their work [22] emphasize that the Karman path can produce a positive effect for fish. “If the direction of all the vortices changes, the Karman path will contribute to the emergence of the traction force, and will not lead to an increase in resistance to movement. When the rotation vortices’ direction changes, the jet stream created by them transfers an impulse that determines the traction fish force. And a much closer relationship arises than it may seem at first glance, between the wave-like mode of movement and jet propulsion”.

To establish this relationship, the theoretical studies results are presented in [2], a method for determining the kinematic and hydrodynamic parameters optimal for fish in the wake of a streamlined cylinder is proposed. Such a wake flow is a satellite stream, reminiscent of a turbulent structure, however, the vortex wake in it remains at a much greater distance from the body (Figure 5).
Figure 5. Scheme of the flow behind the cylinder: 1 - cylinder; 2 - vortices (Karman path); 3 - the boundary of the turbulent region; 4, 5 - fish

The following proportions were found in the structure of the Karman vortex track, which allow to determine the geometric parameters in relation to the wavelength \( \lambda \):

\[
Y_0 = \frac{\lambda}{4} \tan \beta_1; \quad r_0 = \frac{a - 2Y_0}{2}; \quad k_i = \frac{r_0}{b} = 0.22.
\]  

(1)

where \( r_0, \beta_1, a \) – accordingly, are the radius of a large vortex in the mixing layer, an acute angle and a hip of an isosceles triangle.

By comparing the amplitude of the caudal fin deviation with the upper (outer) vortex boundary \( a_0 = k_p l_p = Y_0 + r_0 = \text{const} \), the following calculated dependences which allow for a given fish size \( (l_p) \) determine the characteristic cylinder diameter \( (D) \) and the location coordinate of the fish behind it \( (x_p) \) are obtained:

\[
D = \frac{1.8}{C_p} \pi_{1m} b_p; \quad x_p = \frac{b_p}{18 \beta_2^2 \pi_{1m}}.
\]  

(2)

The fish bio hydrodynamic characteristics’ comparison with the structural elements of a water stream

For convenience, the size of the fish, determined by the radius of the locomotor vortex \( r_{p1} \) formed behind the caudal fin (see Figure 3), will be called the “first characteristic”, i.e. \( l_{p1} = f(r_{p1}) \). This fish length is extremely maximum for the width of the water exchange layer at the coordinate \( (x_p) \).

If the size of the largest in the water exchange layer behind the cylinder of fish \( l_{p1} \) due to the coincidence of the vortices’ size behind the fish and the vortices in the wake of the cylinder \( r_1 \), then the next characteristic fish size \( l_{p2} \) due to the ability of the fish to bend when passing between two diagonal vortices in the Karman path. At this point, the fish follows the path of least resistance (see Figure 5, position 4), and the symmetric rotation of the vortices only contributes to the accelerated fish movement. The second characteristic bending the fish radius is equal to half the diagonal of the triangle, i.e.

\[
r_{p2} = r_2 = \frac{a}{2} = \frac{Y_0}{\sin \beta_1} = \frac{k_{ru}}{k_i} \cdot \frac{r_i}{\sin \beta_i},
\]  

(3)

or
where \( r_{n} = Y_{i}/b = 0.3 \) – is the relative coordinate of the stability loss point in the velocity profile (vortex center) [19]. Figure 6 shows in geometric form a comparison of the first two characteristic vortices.

\[
\frac{r_{p2}}{r_{p1}} = \frac{r_{2}}{r_{1}} = \frac{k_{n}}{k_{1}} \cdot \frac{1}{\sin \beta_{1}}. \tag{4}
\]

Denoting the expansion half-angle tangent \( \left( \theta_{p} \right) \) with a symbol “\( \kappa_{p} \)”, let us define it from the geometric considerations

\[
\kappa_{p} = \tan \theta_{p} = \frac{r_{p2} - r_{p1}}{r_{p2} + r_{p1}} = \frac{k_{n} - k_{1} \cdot \sin \beta_{1}}{k_{n} + k_{1} \cdot \sin \beta_{1}}. \tag{5}
\]

This parameter depends on the angle \( \beta_{1} \), the value of which, according to [23], varies in the range \( 31.5^\circ \leq \beta_{1} \leq 39^\circ \). Figure 7 shows this function’s graph \( \kappa_{p} = f(\beta_{1}) \).

The characteristic vortices induced by the fish are interconnected by the common kinematics - the tangential wave motion of the fish curved body (see Figure 3). With such a close relationship, the vortex intensity (rotation frequency) is determined by the size of the vortices. The most intense will be a vortex with a minimum radius \( r_{p1} \), which forms behind the caudal fish fin. From it, as from the elastic body, the fish is repelled by the tail’s wave. From the rest of the larger vortices, the fish can be repelled by the body’s wave-like bending.

On the other hand, a similar mechanism of the multiscale vortex structures’ formation underlies the
kinematics of the boundary layer near the streamlined wall [24, 25]. Figure 8 shows a diagram of the torsion hydrodynamic structures in the boundary layer according to the theory described in [26].

Figure 8. Scheme of the torsion hydrodynamic structures in the boundary layer formed near the vertical wall: 1 - wall; 2 - plot of averaged speeds; 3 - orbit of a hydro quantum (analog of the Prandtlevsky way of mixing); 4 is a cross section of a dynamic fiber hydro quantum; 5 - the lowest high-frequency hydro quantum; 6 - quantum beam; 7 - fish

Hydraulic calculation of the kinematic parameters of vortex structures optimal for fish in a water stream

In accordance with the Prandtl hypothesis, the ratio of adjacent vortex layers (hydro quanta) in the boundary layer is determined by the following dependence:

\[
\frac{r_{q2}}{r_{q1}} = \frac{1 + \kappa}{1 - \kappa}.
\] (6)

In [26], the Karman constant is derived theoretically and is equal to \(\kappa = \tan \theta_{qa} = 0,391 \approx 0,4\), where \(\theta_{qa} = 21,37^\circ\) – the torsion bar half angle. For comparison, we note the value of the Karman constant defined in [27] for the case of a smooth wall \(\kappa = 0,405\), and for the roughness zone \(\kappa = 0,396\). For \(\kappa = 0,391\) the ratio of the hydro-quantum radii in the boundary layer according to the formula (6) is:

\[
\frac{r_{q2}}{r_{q1}} = \frac{1 + \kappa}{1 - \kappa} = \frac{1 + 0,391}{1 - 0,391} = 2,28.
\] (7)

On the graph (see Figure 7) at an angle \(\beta_1 = 35 \div 37^\circ \approx 36^\circ\) the values \(\kappa_p\) and the Karman constant coincide, i.e. \(\kappa_p \approx \kappa\). Then the ratio of the characteristic (for fish) radii of the vortices according to the formula (4) is:

\[
\frac{r_{p2}}{r_{p1}} = \frac{k_{mu}}{k_l} \cdot \frac{1}{\sin \beta_1} = \frac{0,3}{0,22} \cdot \frac{1}{\sin 36^\circ} = 2,32.
\] (8)

The ratios of the radii found by the formulas (7) and (8) actually coincide in magnitude. Equating the right sides of these equations, we obtain the following expression:
This formula connects the kinematic structures of the following water exchange layers: in the fish wake and behind the cylinder (Karmana path); in a turbulent jet \[24\]; in the boundary layer (near the streamlined wall). Thus, a general pattern has been established in the development of water-exchange layers of a different nature, which (in accordance with the principle of analogy) will also be valid for the third radius characteristic vortex \( r_{p3} \), i.e.:

\[
\frac{r_{p3}}{r_{p2}} = \frac{r_{p2}}{r_{p1}} = \frac{k_{ru}}{k_l} \frac{1}{\sin \left( \pi/5 \right)} = \frac{1+\kappa}{1-\kappa} \approx 2,3. 
\]

(9)

The conclusions made once again confirm the inducing fish advisability thesis in a flow of vortices of the scale that are already present in the turbulent medium of the river flow.

Once in one or another water-exchange (boundary) layer, the fish, in search of the suitable (energy-saving) conditions, selects one (of those already created by the river flow) the most optimal vortex scale (close in size to one of the three characteristic ones). A vortex of the same size can be used by the fish of at least three relative calibers: large, medium and small. Large fish use vortices of the first characteristic size \( (r_{p1}) \), middle – the second \( (r_{p2}) \), and small - the third \( (r_{p3}) \).

Two tasks are of practical importance.

**In the first task** according to the given fish size \( (l_p = \text{const}; r_{pj} = \text{var}) \) the characteristic radii are determined by the following formula:

\[
r_{pj} = l_p \cdot \frac{k_p \cdot k_j}{k_p + k_{ru}} \left( \frac{1+\kappa}{1-\kappa} \right)^{j-1},
\]

where \( j = 1, 2, 3 \) – is the serial number of the characteristic vortex, fish.

**Second task** the reverse. For a given radius of the vortex \( (r_p = \text{const}; l_{pj} = \text{var}) \) the characteristic size (length) of the fish is determined:

\[
\frac{l_{pj}}{r_p} = \frac{k_p + k_{ru}}{k_p \cdot k_j} \left( \frac{1-\kappa}{1+\kappa} \right)^{j-1}.
\]

(12)

The fish size \( (l_{pi}) \) and its estimated location in the boundary layer near the vertical wall is determined in accordance with the swimming ability of the fish, according to the scale of the vortices (see Figure 8). To do this, for a given width of the boundary layer \( (\delta) \), the maximum flow rate \( u_m = u_x \) and a known number of hydro quantum layers \( (N_q) \) \[26\], the following parameters are determined: radii of hydro-quantas \( (r_{qi}) \); coordinates of their centers \( (y_i) \) and the borders \( (Y_i) \); flow rates \( (u_i) \). To conduct the calculation, it is convenient to number the layers in the reverse order from the largest (external) hydro quantum to the wall \( (i = 1, 2 \ldots N_q) \). From the geometric considerations, taking into account (12) and (13), the following dependences are obtained:
\[ r_{qi} = \frac{\kappa}{1 + \kappa \left(1 + \kappa\right)^{i-1}}, \quad y_i = \frac{1}{\kappa}, \quad Y_i = y_i + r_{qi}; \tag{13} \]

\[ r_{pi_i} \equiv r_{qi}, \quad l_{pji} \equiv r_{qi}, \quad \frac{l_{pji}}{r_{pji}} = \frac{k_p + k_m \left(1 - \kappa\right)^{i-1}}{k_p, k_j \left(1 + \kappa\right)}; \tag{14} \]

where \( l_{pji} \) – is the characteristic length of fish (for \( i = \text{const}, \ j = 1, 2, 3 \)); \( r_{pi_i} \) – is the radius of the first characteristic (for fish) vortex.

The local flow velocity in the boundary layer is determined by the velocity profile. For this we use the universal wave velocity profile from [26], which is represented in parametric form by the following equations:

\[ \frac{u_i}{u_m} = \left(1 + \cos \left(\pi - \theta_i\right)\right)^{\frac{1}{2N_q}}; \quad \frac{Y_i}{\delta} = \frac{\theta_i - t8 \varphi_m \cdot \sin \theta_i}{\pi}, \tag{15} \]

where \( \theta_i \) – is the angular parameter (\( \theta_i = 0 \div \pi \)); \( \varphi_m = \pi/6 \) – is the torsion angle of the voltage torsion bar. Further along the known coordinate \( Y_i \) the angular parameter is calculated \( \theta_i \), and further on - the corresponding speed \( (u_i) \).

**Results**

As a result of the research, an analysis of the mechanism of fish swimming was performed on the basis of which the phenomenon of the vortex structures’ formation by fish in an undisturbed aquatic environment was studied. The kinematic parameters of the characteristic structural (vortex) elements of the river flow are substantiated. The correspondence of the bio hydrodynamic characteristics of fish with the characteristics of the turbulent flow structural elements is established. A method for determining the kinematic parameters of vortex structures optimal for fish in a water stream has been developed.

Figure 8 schematically shows the estimated distribution over the width of the boundary layer of medium-sized fish (with a characteristic radius \( r_{p2} \)), and in Figure 9 as an example for \( N_q = 6 \) in relative coordinates graphs of the distribution of speeds and three characteristic sizes of fish in the boundary layer are presented.
Figure 9. The plots of fish distribution (for three characteristic sizes) and velocities in the boundary layer

Discussion

The central idea of the presented hypothesis is the possibility of using fish of the various sizes vortex structures already created in the river flow for energy-intensive swimming. The research of many reputable scientists testifies to this possibility. However, the lack of adequate information on the discrete, quantum structure of the vortex kinematic structures of the boundary layer impeded the possibility of establishing this important connection. Now that the laws of fluid movement in the boundary layers are more known, the connection with the peculiarities of fish locomotion has become more obvious and, thanks to the technique presented in this work, can be expressed in quantitative characteristics.

A fish located in the boundary layer of a river stream has the ability to use its vortex structures of at least three characteristic sizes, scales (see Fig. 8) in any of the possible combinations - one, two, or three vortices at once. To a certain extent, this determines (from a hydrodynamic point of view) the configuration of the fish trajectory.

If fish swims in a boundary layer parallel to the wall (shore), then the vortex structures (hydro-quanta) of the same scale will prevail on its path. Making the movement at an angle, it can use (rely) on adjacent vortices at the same time of two or three different scales.

Since the fish is given the opportunity to instantly select one or another scale of the vortices, its movement trajectory shape can be multivariate, but generally limited by the range of hierarchical arrangement of three adjacent vortex structures, mixing layers in the transverse plane.

From this it follows that to use the entire arsenal of active vortices (three characteristic scales), the fish must move, deviating from side to side, along a sinusoidal, tack-like, spiral (three-dimensional)
trajectory with an amplitude of deviation (in the transverse plane), that is, in some the corridor of the water area as part of the river flow boundary layer favorable for movement, that is, with the least energy expenditure.

Knowledge of these fish behavior features and the methodology for determining the characteristic boundaries presented in the work allows predicting the places of the most preferred fish habitat in the general river flow, for example, during spawning migrations or for scientifically based organization of attracting fish to the fish passage structures’ entrance, as well as for scaring away juvenile fish from the zone of the water intakes influence.

Summary
In the course of the theoretical study, a method that allows to determine the probabilistic location of the fish in the wake of the cylinder and in the boundary layer, formed near the vertical wall for a given size of a fish having a sub-carangiform or carangiform type of locomotion, was developed.

The presented relationship of the biological object with the hydrodynamics of the water flow opens up the real possibilities for the development of more advanced methods, promising the devices and structures’ designs for fishery hydraulic.

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