Inverted neutrino mass hierarchies from $U(1)$ symmetries.

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Abstract

Motivated by effective low energy models of string origin, we discuss the neutrino masses and mixing within the context of the Minimal Supersymmetric Standard Model supplemented by a $U(1)$ anomalous family symmetry and additional Higgs singlet fields charged under this extra $U(1)$. In particular, we interpret the solar and atmospheric neutrino data assuming that there are only three left-handed neutrinos which acquire Majorana masses via a lepton number violating dimension-five operator. We derive the general form of the charged lepton and neutrino mass matrices when two different pairs of singlet Higgs fields develop non–zero vacuum expectation values and show how the resulting neutrino textures are related to approximate lepton flavor symmetries. We perform a numerical analysis for one particular case and obtain solutions for masses and mixing angles, consistent with experimental data.
1. Introduction

Analysis of the atmospheric and solar neutrino oscillation data \[1, 2, 3\] imply tiny neutrino squared mass differences and large mixing. Moreover, Yukawa couplings related to neutrino masses are highly suppressed compared to those for quarks and charged leptons.

Various authors \[4\]-\[18\], have claimed that the neutrino mass matrix structure and the (almost) maximal $\nu_\mu - \nu_\tau$ mixing -as the atmospheric data suggest- could be interpreted in terms of a symmetry beyond the standard model. Motivated by the fact that the majority of string models constructed so far include several (possibly anomalous) additional $U(1)$’s, we consider that this picture may indicate an underlying structure of the mass matrix determined by such a $U(1)$ broken at some high scale $M$. An apparently different approach has shown that the neutrino data can be interpreted using a restricted class of mass matrices \[16\] where at most two of its entries vanish. It is likely that these zeros can be naturally generated in the context of a symmetry principle obeyed by the neutrino Yukawa couplings.

Light Majorana neutrino masses in the range below the $eV$ scale -as experiments suggest- can be obtained either through a see-saw mechanism (in the presence of right handed-neutrinos), or from the dimension-five non-renormalizable operator $(LH)^2/M$, where $L, H$ are the lepton and Higgs doublets respectively, while $M$ is an appropriate large scale. In previous work\[6, 11\] we have suggested that the Minimal Supersymmetric Standard Model (MSSM) extended by a single $U(1)$ anomalous family symmetry spontaneously broken by non–zero vacuum expectation values (vevs) of a pair of singlet fields $\Phi, \bar{\Phi}$ with $U(1)$ charges $Q = \pm 1$ can provide acceptable masses and large mixing. It was found that such a symmetry retains the above mentioned dimension-five operator which provides Majorana masses to the left-handed neutrino states. Assuming symmetric lepton mass matrices, it was shown that the neutrino sector respects $L_e - L_\mu - L_\tau$ symmetry, implying inverse hierarchical neutrino mass spectrum, large solar ($\theta_{12}$) and atmospheric ($\theta_{23}$) mixing angles whilst $\theta_{13} = 0$ and degeneracy between the first two neutrino mass eigenstates. In view of these shortcomings, it was suggested that second order effects -possibly originating from additional singlet field vevs- might lift the mass degeneracy and reconcile the neutrino data accurately. In the present work, we explore in detail the structure of the lepton mass matrices in the presence of non-renormalizable contributions originating from more than one singlet Higgs pairs. This extension is motivated by the fact that string models usually predict a large number of singlet fields charged differently under the extra $U(1)$ symmetry. We start with a systematic study of the mass matrix structures obtained from non-renormalizable contributions coming from $\Phi, \bar{\Phi}$ singlet vevs only and establish the consistency of the resulting neutrino texture forms with flavor symmetries of the type $L_e \pm (L_\mu \mp L_\tau)$. Next, we consider non-renormalizable, hierarchically lesser contributions from additional singlets, and work out the particular case $L_f = L_e - L_\mu - L_\tau$ in detail. We show that contributions from a proper second Higgs singlet pair, generate additional mass entries, breaking ‘softly’ the symmetry $L_f$. We find that -under natural assumptions for the undetermined Yukawa coefficients- it is possible to correlate the resulting neutrino mass matrices with particular texture zeros discussed in \[16\] and give a consistent set of models which are in agreement with recent data.

The paper is organized as follows. In section 2, we develop a useful texture form of the neutrino mass matrix in terms of the eigenmasses and mixing angles and give a brief description of the constraints implied by recent oscillation data. In section 3, we present the extension of the model \[11\] with the inclusion of the second singlet Higgs pair and derive the relevant charged lepton and Majorana neutrino mass matrices. In section 4, we perform a test calculation and give solutions consistent with current neutrino data. Our conclusions are presented in section 5.

2. Neutrino mass matrix constraints and oscillation data

In this work, we assume that the neutrino data can be interpreted in terms of a Majorana mass matrix of the left-handed neutrino components. There are at most nine independent parameters in this matrix, while experiments can only measure two squared mass differences, three angles, one CP-phase and the double beta decay parameter. Thus, the neutrino mass matrix cannot be fully determined by the present
experiments. A general neutrino mass matrix will look as follows:

\[ M^\nu = U_n M_d U_n^\dagger \]  

(1)

where \( M_d \) is the diagonalized Majorana neutrino mass matrix

\[ M_d = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \]  

(2)

and \( U_n \) is the diagonalizing matrix parametrized in terms of three angles \( (c_{ij} = \cos \theta_{ij}, s_{ij} = \sin \theta_{ij}) \)

\[ U_n = \begin{bmatrix} c_{12} & c_{13} & c_{13} s_{12} & s_{13} e^{-i\delta} \\ -c_{23} s_{12} & -c_{12} s_{13} & s_{23} e^{i\delta} & c_{12} c_{23} - s_{12} s_{13} s_{23} e^{i\delta} \\ -c_{12} & c_{23} & c_{13} e^{i\delta} + s_{12} s_{23} & -c_{12} s_{12} s_{13} - c_{12} s_{23} e^{i\delta} \end{bmatrix} \cdot \]  

(3)

We observe that the matrix elements of \( \mathbf{1} \) can be written as inner products

\[ M^\nu = \begin{bmatrix} \vec{m} \cdot \vec{\nu}_1 \\ \vec{m} \cdot \vec{\nu}_2 \\ \vec{m} \cdot \vec{\nu}_3 \\ \vec{m} \cdot \vec{\nu}_4 \\ \vec{m} \cdot \vec{\nu}_5 \\ \vec{m} \cdot \vec{\nu}_6 \end{bmatrix} . \]  

(4)

where \( \vec{m} = (m_1, m_2, m_3) \) is the vector of the mass eigenstates (complex in general) and the vectors \( \vec{\nu}_i \) are functions of the mixing angles \( \theta_{12}, \theta_{13}, \theta_{23} \) and the phase \( \delta \).

\[ \vec{\nu}_1 = \begin{bmatrix} e^{2i\delta} \end{bmatrix}, \vec{\nu}_2 = \begin{bmatrix} c_{12}^2 c_{13}^2, c_{12}^2 s_{12}^2, s_{13}^2 e^{-2i\delta} \end{bmatrix}, \vec{\nu}_3 = \begin{bmatrix} -c_{12} c_{13} \left( c_{23} s_{12} + c_{12} s_{13} s_{23} e^{i\delta} \right), c_{13} s_{12} \left( c_{12} c_{23} - s_{12} s_{13} s_{23} e^{i\delta} \right), c_{13} s_{13} s_{23} e^{-i\delta} \end{bmatrix}, \vec{\nu}_4 = \begin{bmatrix} \left( c_{23} s_{12} + c_{12} s_{13} s_{23} e^{i\delta} \right)^2, \left( c_{23} s_{12} s_{13} s_{23} e^{i\delta} \right)^2, c_{13}^2 s_{23}^2 \end{bmatrix}, \vec{\nu}_5 = \begin{bmatrix} \left( c_{23} s_{13} e^{i\delta} + s_{13} s_{23} \right) \left( c_{23} s_{12} + c_{12} s_{13} s_{23} e^{i\delta} \right), \left( c_{23} s_{13} e^{i\delta} + c_{13} s_{23} \right) \left( s_{12} s_{13} s_{23} e^{i\delta} - c_{12} c_{23} \right), c_{13}^2 c_{23} s_{23} \end{bmatrix}, \vec{\nu}_6 = \begin{bmatrix} \left( c_{23} s_{13} e^{i\delta} - s_{13} s_{23} \right)^2, \left( c_{23} s_{12} s_{13} e^{i\delta} + c_{13} s_{23} \right)^2, c_{13}^2 c_{23}^2 \end{bmatrix}. \]

Pairs of \( \vec{\nu}_i \) vectors can be checked to be in general linearly independent thus, in a three dimensional space, \( \vec{m} \) can be made orthogonal to at most one pair. In this case, \( \mathbf{1} \) assumes a texture form i.e., \( \vec{m} \cdot \vec{\nu}_i = 0 \), \( \vec{m} \cdot \vec{\nu}_j = 0 \) and we get a constraint on the actual values of the neutrino masses

\[ \vec{m} = m_0 \vec{\nu}_i \times \vec{\nu}_j \]  

(5)

where \( m_0 \) is a (complex) mass parameter which characterizes the neutrino mass scale. The parameter \( |m_0| \) takes values in a closed interval which can be determined by requiring the moduli of the mass eigenstates to satisfy the experimental bounds

\[ 5.4 \times 10^{-5} \leq |m_2|^2 - |m_1|^2 \leq 9.5 \times 10^{-5} \]  

(6)

\[ 1.4 \times 10^{-3} \leq |m_3|^2 - |m_2|^2 \leq 3.7 \times 10^{-3} . \]

Experimentally allowed values for the elements of \( M^\nu \) can be determined as follows: We first form the dimensionless ratio

\[ 14.73 \leq \frac{|m_3|^2 - |m_2|^2}{|m_3|^2 - |m_1|^2} \leq 68.51 \]  

(7)
which depends only on the values of the mixing angles \( \theta_{12}, \theta_{13}, \theta_{23} \). Then, we look for values of the angles within the experimental bounds

\[
\begin{align*}
0.23 & \leq \sin^2 \theta_{12} \leq 0.39 \\
0.31 & \leq \sin^2 \theta_{23} \leq 0.72 \\
0.00 & \leq \sin^2 \theta_{13} \leq 0.054
\end{align*}
\]  

satisfying the ratio constraint (8), which are depicted in figures 1-4. These angle values can now be used to construct the matrix \( M^\nu \). Note that, the experimentally allowed values for \( \theta_{13} \) are close to zero, also those for \( \theta_{23} \) are close to \( \pi/4 \). In the limit where \( \theta_{13} = 0 \) and \( \delta = 0 \) we get the additional constraints \( \overrightarrow{\vartheta}_2 + \overrightarrow{\vartheta}_3 = 0 \) and \( \overrightarrow{\vartheta}_2 \cdot (\overrightarrow{\vartheta}_4 \times \overrightarrow{\vartheta}_5) = 0 \). If we further require that \( \theta_{23} = \pi/4 \), we get in addition \( \overrightarrow{\vartheta}_4 = \overrightarrow{\vartheta}_6 \). As an example, we assume that \( \overrightarrow{m} \cdot \overrightarrow{\vartheta}_4 = 0 \) and \( \overrightarrow{m} \cdot \overrightarrow{\vartheta}_6 = 0 \), so that the Majorana neutrino mass matrix takes the form

\[
M^\nu = \begin{pmatrix}
\overrightarrow{m} \cdot \overrightarrow{\vartheta}_4 & \overrightarrow{m} \cdot \overrightarrow{\vartheta}_2 & \overrightarrow{m} \cdot \overrightarrow{\vartheta}_3 \\
\overrightarrow{m} \cdot \overrightarrow{\vartheta}_2 & 0 & \overrightarrow{m} \cdot \overrightarrow{\vartheta}_5 \\
\overrightarrow{m} \cdot \overrightarrow{\vartheta}_3 & \overrightarrow{m} \cdot \overrightarrow{\vartheta}_5 & 0
\end{pmatrix}.
\]  

If this texture is to be compatible with experimental data, we must expect that \( M^\nu_{12} = M^\nu_{21} \approx -M^\nu_{13} \) and \( M^\nu_{23} \ll 1 \).

The above formulation reveals another interesting property of the matrix \( M^\nu \). If the mixing \( \nu_\mu - \nu_\tau \) is exactly maximal, i.e., if \( \theta_{23} = \frac{\pi}{4} \), then the texture \( M^\nu \) automatically implies degeneracy of the two eigenvalues. One finds

\[
\begin{align*}
m_1 \equiv m_2 &= -m_0 \cos^2 \theta_{13} \\
m_3 &= m_0 \left( 1 + e^{2\iota \delta} \sin^2 \theta_{13} \right)
\end{align*}
\]  

Thus, exactly maximal mixing in this case is not allowed since it would imply \( \Delta m^2_{\text{sol.}} = 0 \), in clear disagreement with the experimental data.

3. Description of the Model

In what follows we shall assume that there is no CP violation in the neutrino sector, and all fermionic mass eigenstates are real, but not necessarily positive. Our intention is to interpret the oscillation neutrino data using only the Standard Model fermion spectrum (without right handed neutrinos) and two pairs of singlet Higgs fields with appropriate \( U(1) \) charges. The particles, together with their \( U(1)_X \) charge notation are presented in Table 1. The \( U(1)_X \) charges of \( \Phi, \Phi^c \) are taken to be \( \pm 1 \), while the charges of the other representations will be fixed from low energy physics considerations.

In the absence of the right handed neutrinos, the neutrino masses arise from the lepton number violating \( (\Delta L = 2) \) operator

\[
\frac{y^\mu\nu_{\alpha\beta}}{M} (L^\alpha_i H^j \epsilon_{3i})(H^j L^\beta_i \epsilon_{1k}) \equiv \frac{y^\mu\nu_{\alpha\beta}}{M} \bar{\nu}_{L,\alpha} \nu_{L,\beta}
\]  

Here, \( y^\mu\nu_{\alpha\beta} \) is an effective Yukawa coupling depending on the details of the theory, \( v = \langle H \rangle \) is the Higgs doublet vev which is of the order of the electroweak scale and \( M \) stands for a large scale \( M \sim 10^{13-14} \) GeV, which could be identified with the effective gravity scale in theories with large extra dimensions obtained in the context of Type I string models. \(^1\)

To start with, we first review the construction with the introduction of the first singlet Higgs field pair only, i.e., \( \Phi, \Phi^c \) with charges \( \pm 1 \) and define for convenience the ratios \( \lambda = \frac{\Phi}{M} \) and \( \bar{\lambda} = \frac{\Phi^c}{M} \). We choose

\(^1\)We also notice that models with intermediate scales of the above order can be obtained in the context of D-brane scenarios, see for example [21].
The fermion \(U(1)_X\) charges of the Standard Model particles presented in Table 1, so that only the third generation tree-level couplings are present at the tree level potential

\[
W_{\text{tree}} = y_u^3 Q_3 U_3^e H_2 + y_d^3 Q_3 D_3^u H_1 + y_3^3 L_3 E_3^c H_1
\]

with \(y_{u,d,e}\) being the order-one Yukawa couplings. The two lighter generations get masses through non renormalizable Yukawa-type interactions. Thus, for the up, down quarks and charged leptons these are

\[
W^{(1)}_{\text{n.r.}} \propto Q_i U_i^c H_2 \varepsilon^{C_{ij}} + Q_i D_i^c H_1 \varepsilon^{C_{ij}^{C}} + L_i E_i^c H_1 \varepsilon^{C_{ij}}
\]

where, \(C^a_{ij}, (a = u, d, l)\) are appropriate integer powers depending on the \(U(1)_X\)-charge of the corresponding Yukawa term and \(\varepsilon\) is defined as follows

\[
\varepsilon^k = \begin{cases} 
\lambda^k & \text{if } k = [k] < 0 \\
\lambda^k & \text{if } k = [k] > 0 \\
0 & \text{if } k \neq [k]
\end{cases}
\]

Neutrino masses are generated by the operator \(12\), suppressed by the appropriate powers of the parameter \(\varepsilon\).

We assume symmetric mass matrices and take into account the fact that the third generation appears at tree-level. This way, the charged lepton \(U(1)_X\)-charge matrix takes the form

\[
C_e = \begin{pmatrix}
\frac{n'}{2} & \frac{m'+n'}{2} & \frac{n'}{2} \\
\frac{m'+n'}{2} & m' & \frac{m'}{2} \\
\frac{m'}{2} & \frac{m'}{2} & 0
\end{pmatrix}
\]

(15)

Here, \(m', n'\) are taken to be integers and we have defined \(\frac{m'}{2} = \ell_1 - \ell_3\) and \(\frac{n'}{2} = \ell_2 - \ell_3\), where \(\ell_i\) are the \(U(1)_X\)-charges of the leptons. Given the form of the operator \(12\) we find that the \(U(1)_X\)-charge entries for the light Majorana neutrino mass matrix takes the form

\[
C_\nu = \begin{pmatrix}
\frac{m'+n'}{2} + A & \frac{m'+n'}{2} + A & \frac{n'}{2} + A \\
\frac{m'}{2} + A & m' + A & \frac{m'}{2} + A \\
\frac{m'}{2} + A & \frac{m'}{2} + A & A
\end{pmatrix}
\]

(16)

where \(A\) is another constant, expressed in terms of \(U(1)\) fermion charges, \(A = 2(\ell_3 + h_2)\). Thus, the neutrino \(U(1)_X\)-charge entries differ from the corresponding charged leptonic entries by the constant \(A\).

### 3.1 Lepton Mass Matrices

We start now our investigation of the \(M^e\) and \(M^\nu\) mass matrices considering first the implications of singlets \(\Phi, \bar{\Phi}\). A mass entry \(m_{ij}^e\) of \(M^e\) is non-zero whenever a power \(C_{ij}^e\) of the expansion parameter \(\varepsilon\) in \(14\) matches the charge of the corresponding entry of \(15\). Since the charges with respect to \(U(1)_X\) of the latter are \(Q_X = \pm 1\), an entry \(m_{ij}^e\) is non-zero whenever the corresponding entry in \(15\) is also integer.
Since \( m', n' \) are integers, in order to obtain non-zero entries in the Majorana matrix too, the parameter \( A \) has to be either integer or half-integer. The case where \( A \) is integer implies that the charged lepton and neutrino mass matrices are proportional \( M^c \sim e^A M^\nu \) (up to Yukawa coefficients) and it is hard to obtain large mixing. Thus, we will assume that \( A \) is half integer and we will analyze this case in the sequel.

As far as \( m', n' \) parameters are concerned, we distinguish the following cases with respect to neutrino mass matrix: (i): if we take \( m' \) even and \( n' \) odd, i.e. \( m' = 2p \) and \( n' = 2q + 1 \), (where \( p, q \) are integers), then only the elements \( m_{12}^\nu = m_{21}^\nu, m_{13}^\nu = m_{31}^\nu \) are non-zero in the neutrino mass matrix. In this case, the neutrino mass matrix respects an \( L_f = L_e - L_\mu - L_\tau \) flavor symmetry, (where \( L_{e,\mu,\tau} \) are the lepton flavor numbers of the three generations), implying bimaximal (\( \theta_{12}, \theta_{23} \)) mixing, inverted neutrino mass hierarchy and, in particular, degeneracy of the two first neutrino eigenstates \( m_1^\nu = m_2^\nu \) while \( m_3^\nu = 0 \). The corresponding charged lepton mass matrix, however, does not conserve \( L_f \). (ii): if both \( m' \) and \( n' \) are odd, then \( m_{13}^\nu = m_{31}^\nu \neq 0 \) as well as \( m_{23}^\nu = m_{32}^\nu \neq 0 \), while all other neutrino mass entries are zero. We note that this neutrino texture exhibits an \( L_e + L_\mu - L_\tau \) symmetry. (iii): in the case \( m' = 2p + 1, n' = 2q \), the non-zero elements are \( m_{12}^\nu = m_{21}^\nu \neq 0, m_{23}^\nu = m_{32}^\nu \neq 0 \), while the symmetry is \( L_e - L_\mu + L_\tau \), and finally (iv): if both parameters are even integers, \( m' = 2p, n' = 2q \), all the entries of \( C_e \) become half-integers and the neutrino matrix is zero in this case, respecting thus an \( L_e + L_\mu + L_\tau \) symmetry. On the contrary, all \( M^e \) mass matrix elements are non-zero, filled-in by non-renormalizable terms \( \epsilon^{C_i} \). All four cases are summarized in Table 2.

![Table 2](image_url)

The above analysis shows that, if only one singlet acquires non-zero vev, the neutrino data cannot be accommodated. Assuming however, that the low energy spectrum of the higher theory contains various neutral singlet scalars - as it is the case in string constructions-, we expect that contributions from an appropriate second singlet vev will suffice to reconcile the predictions of the modified mass matrices with the experimental data.

From inspection of the mixing angles in the above four cases, we infer that (ii) and (iii) are unlikely to interpret the neutrino data. On the contrary, for case (i) small contributions may modify the neutrino masses and mixing and lead to acceptable results. Finally, as already noted, neutrino masses are all zero in case (iv) at this level, however, a second singlet with proper \( U(1)_X \) charge can in principle generate also a viable \( M^\nu \) matrix.

We will analyze in some detail case (i), i.e., \( n' = \text{odd}, m' = \text{even} \) and show that a viable set of lepton mass matrices and mixing naturally arise. If we write \( m' = 2p, n' = 2q + 1, A = r + 1/2 \) with \( p, q, r \) integers, the \( U(1)_X \)-charge entries of the charged-lepton mass matrix are

\[
C_e = \begin{pmatrix}
2q + 1 & p + q + \frac{1}{2} & q + \frac{1}{2} \\
p + q + \frac{1}{2} & 2p & p \\
q + \frac{1}{2} & p & 0
\end{pmatrix}.
\] (17)

The corresponding charge entries for the neutrino mass matrix are

\[
C_\nu = \begin{pmatrix}
2q + r + \frac{3}{2} & p + q + r + 1 & q + r + 1 \\
p + q + r + 1 & 2p + r + \frac{3}{2} & p + r + \frac{3}{2} \\
q + r + 1 & p + r + \frac{3}{2} & r + \frac{3}{2}
\end{pmatrix}.
\] (18)

The corresponding quark matrices have been analyzed in ref [13].
The parameters $m', n', \mathcal{A}$ (or $p, q, r$ respectively), determine also the $U(1)_X$-charges of the SM particles of Table 1. A systematic search shows that a natural set of $U(1)_X$ charges is obtained [11] for $m' = 2, n' = 7, \mathcal{A} = -\frac{7}{2}$. (For the lepton doublets in particular, choosing for example $b_2 = \frac{1}{7}$ we find $\ell_1 = 2, \ell_2 = -\frac{1}{2}, \ell_3 = -\frac{3}{2}$.) In this case, the leptonic charge entries become

$$C_e = \begin{pmatrix} 7 & 9 & 7 \\ \frac{7}{2} & 2 & 1 \\ 2 & 0 & 0 \end{pmatrix}, \quad C_\nu = \begin{pmatrix} 9 & 2 & 1 \\ \frac{7}{2} & -\frac{1}{2} & -\frac{3}{2} \\ 2 & -\frac{1}{2} & -\frac{3}{2} \end{pmatrix}.$$  \hspace{1cm} (19)

When only the singlet Higgs pair $\Phi, \bar{\Phi}$ with $Q_X = \pm 1$ obtain vevs, as already pointed out, only the integer charge mass entries are filled in. In this case, there are more than two zeros in the neutrino mass matrix thus, the experimental data can not be accurately interpreted [16]. Indeed, one finds $\Delta m_{12}^2 = \Delta m_{13}^2 = 0$, i.e., the first two neutrino mass eigenstates are degenerate. Furthermore, the leptonic mixing matrix $U^0_l = V^l_l V^\nu$ implies that the solar neutrino mixing angle is maximal, a situation disfavored by recent data.

As noted above, at this stage, the neutrino mass matrix respects a symmetry of the form $L_f = L_e - L_\mu - L_\tau$, where $L_{e,\mu,\tau}$ are the lepton flavor numbers of the three generations. We could consider this structure of the neutrino matrix as a starting point and consider additional secondary effects which break the $L_f$ symmetry softly and lead to a texture form consistent with experimental data. Indeed, the above drawbacks could be cured if additional non-zero entries are generated by additional effects. As noted in the introduction, in a string model, the low energy spectrum contains more than one Higgs singlets. A new singlet pair $\chi, \bar{\chi}$ with appropriate $U(1)_X$-charges might develop a vev, (provided that the flatness conditions are satisfied) so that additional mass entries could be filled-in by additional non-renormalizable terms.

Checking the neutrino charge entries in the case under consideration [19], we conclude that in order to have a viable neutrino mass matrix by symmetry principles, we need to generate non-zero values at least for the $\{23\}, \{32\}$ and $\{11\}$ elements, so that a matrix of the form $[11]$ of section 2 could be obtained. For example, in order to get a non-zero $\{32\}$ entry, the charge of the second singlet should satisfy $s Q_\chi - \frac{1}{2} = 0$, where $s$ is integer. This implies $Q_\chi = \frac{5}{2} \rightarrow \{ \frac{3}{2}, \frac{5}{2}, \frac{1}{2}, \frac{7}{2}, \ldots \}$. A reasonable choice of $U(1)_X$ charges of this extra singlet Higgs pair is $Q_\chi = \frac{5}{2}$ and $Q_{\bar{\chi}} = -\frac{3}{2}$.

Let us now assume that the new singlets $\chi, \bar{\chi}$, obtain vevs and denote $\eta = \frac{\chi}{\bar{\chi}}, \tilde{\eta} = \frac{\bar{\chi}}{\chi}$. In analogy to [13] we define $\vartheta^k = \eta^k$ if $k = [k] < 0$, $\vartheta^k = \tilde{\eta}^k$ if $k = [k] > 0$ and $\vartheta^k = 0$ otherwise. Then, for $Q_\chi = \frac{5}{2}$, a mass matrix element $m_{ij}$ with charge entry $N_{ij} + \frac{1}{2}$, (where $N_{ij}$ the integer part) is

$$m_{ij} = \sum_{\ell=-\ldots,0,1 \ldots} y_{ij}^{\ell} \epsilon^{N_{ij}-3\ell-1} \vartheta^{2\ell+1}$$  \hspace{1cm} (20)

where $y_{ij}^{\ell}$ are order-one Yukawa coefficients. Let $p = 1, q = 3$ so that $m' = 2, n' = 7$ as in case [19]. Then,

$$m_{12}^{'} = y_{12}^1 \tilde{\lambda}^3 \bar{\eta} + y_{12}^2 \bar{\eta}^3 + \cdots, \quad m_{13}^{'} = y_{13}^1 \bar{\lambda}^2 \eta + \cdots$$

and similarly for the rest of the mass matrix elements. We should point out however, that in a realistic string construction, due to additional (discrete) symmetries -remnants of the original string symmetry and various selection rules-, several non-renormalizable terms in the sum [20] will vanish.

To construct the mass matrices, we assume that $\lambda, \tilde{\lambda}$ and $\eta, \bar{\eta}$ are in the perturbative region, so we retain non-renormalizable terms only up to fourth order. Ignoring for simplicity the coefficients $y_{ij}$, the mass matrices are written in terms of $\lambda, \tilde{\lambda}$ and $\eta, \bar{\eta}$ as follows. For the charged leptons we find

$$M_e \approx m_0^2 \begin{pmatrix} \tilde{\lambda}^7 & \tilde{\lambda}^3 \bar{\eta} + \eta^3 & \tilde{\lambda}^3 \bar{\eta} + \tilde{\lambda} \eta^3 \\ \tilde{\lambda}^3 \bar{\eta} + \eta^3 & \tilde{\lambda}^2 \eta + \lambda \bar{\eta}^3 \\ \tilde{\lambda}^2 \eta + \bar{\lambda} \eta^3 \end{pmatrix},$$  \hspace{1cm} (21)
and for the neutrinos

\[ M_\nu \approx m_0 \begin{pmatrix} \bar{\lambda}^3 \bar{\eta} + \bar{\eta}^3 & -\bar{\lambda}^2 + \bar{\eta}^2 \lambda & \bar{\lambda} + \lambda^2 \bar{\eta}^2 \\ -\bar{\lambda}^2 + \bar{\eta}^2 \lambda & \lambda \eta + \lambda^2 \bar{\eta} & \eta \\ \bar{\lambda} + \lambda^2 \bar{\eta}^2 & \eta & \lambda \eta \end{pmatrix} \cdot (22) \]

Thus, in the presence of two singlet Higgs pairs, in principle, all the entries of the charged lepton and neutrino mass matrices become non-zero. We may assume however, hierarchies between the vevs \( \lambda, \bar{\lambda}, \eta, \bar{\eta} \), and derive approximate texture zero mass matrices [16] in the limiting cases where some of the vacuum expectation values are taken to be zero or negligible compared to others. Note that the corresponding charged lepton mass matrices are not diagonal for these neutrino textures, thus a straightforward comparison with the results of [16] is not possible. We will see however, that off-diagonal entries and mixing effects of the charged lepton sector in our models are not substantial, thus the neutrino mixing angles receive only small contributions from the charged lepton mass matrix. Thus, if we assume that \( \lambda \rightarrow 0 \), \( \eta \ll \bar{\eta} \) and \( \bar{\eta} < \lambda \), we obtain the analogue of texture (9)

\[ M_\nu = m_0 \begin{pmatrix} \bar{\lambda}^3 \bar{\eta} + \bar{\eta}^3 & -\bar{\lambda}^2 + \bar{\eta}^2 \lambda & \bar{\lambda} + \lambda^2 \bar{\eta}^2 \\ -\bar{\lambda}^2 + \bar{\eta}^2 \lambda & \lambda \eta + \lambda^2 \bar{\eta} & \eta \\ \bar{\lambda} + \lambda^2 \bar{\eta}^2 & \eta & \lambda \eta \end{pmatrix} \cdot (23) \]

If we now take \( \bar{\lambda} \ll 1 \), and ignoring fourth order terms, if \( \lambda < \bar{\eta} \) we can approximate

\[ M_\nu = m_0 \begin{pmatrix} \bar{\eta}^3 & \bar{\eta}^2 \lambda & \sim 0 \\ \bar{\eta}^2 \lambda & \sim 0 & \bar{\eta} \\ \sim 0 & \bar{\eta} & \lambda \eta \end{pmatrix} \cdot (24) \]

If on the other hand we take \( \lambda > \bar{\eta} \), we get the approximate form

\[ M_\nu = m_0 \begin{pmatrix} \sim 0 & \bar{\eta}^2 \lambda & \sim 0 \\ \bar{\eta}^2 \lambda & \lambda^2 \bar{\eta} & \eta \\ \sim 0 & \eta & \lambda \eta \end{pmatrix} \cdot (25) \]

For \( \bar{\eta} \ll 1 \), while assuming \( \bar{\lambda}^2 \ll 1 \) we obtain

\[ M_\nu = m_0 \begin{pmatrix} \sim 0 & \sim 0 & \bar{\lambda} \\ \sim 0 & \bar{\eta} \bar{\lambda} & \eta \\ \bar{\lambda} & \bar{\eta} & \lambda \eta \end{pmatrix} \cdot (26) \]

All textures [23, 24] are of the form discussed in [16]. In the next section, we are going to further analyze one specific case out of the many possible that may occur. Similar results can be obtained for the other cases too.

We give now a brief account of case (iv). A viable case arises if we choose \( n' = 8, m' = 2 \) and \( A = -\frac{\eta}{4} \). The choice \( h_2 = -\frac{1}{4} \) for the Higgs \( U(1)_X \)-charge, implies also a natural set of lepton charges in this case, \( \ell_1 = 2, \ell_2 = 1, \ell_3 = -2 \). Furthermore, for \( Q_{\chi, \bar{\chi}} = \pm \frac{1}{2} \), we obtain the matrices

\[ M^e = m_0 \begin{pmatrix} \bar{\lambda}^8 & \bar{\lambda}^5 \bar{\lambda}^4 & \bar{\lambda}^4 \\ \bar{\lambda}^5 & \bar{\lambda}^2 \bar{\lambda} & \bar{\lambda} \\ \bar{\lambda}^4 & \bar{\lambda} & 1 \end{pmatrix}, \quad M_\nu = m_0 \begin{pmatrix} \bar{\eta}^8 & \bar{\eta}^5 \bar{\eta}^4 & \bar{\eta}^4 \\ \bar{\eta}^5 & \bar{\eta}^2 \bar{\eta} & \bar{\eta} \\ \bar{\eta}^4 & \bar{\eta} & \bar{\eta} \end{pmatrix}, \quad (27) \]

which also lead to inverted neutrino mass hierarchy and large mixing for \( \eta \approx \bar{\eta} \).

4. Numerical Analysis

In this section we will calculate the mass spectrum and the leptonic mixing angles for the texture [23] which is of the type \( \bar{m} = m_0 \bar{\eta} \times \bar{\eta} \) of section 2. A texture of this type leads to inverted hierarchy values

\(^3\)If only contributions of \( \eta, \bar{\eta} \) are taken into account, the neutrino mass matrix exhibits an \( L_\mu - L_e \) symmetry.
for $m_1 m_2 m_3$, i.e., $m_1 \sim m_2 > m_3$. Note also that the sign of $m_2$ always comes out negative. In the approximation $\eta \ll \bar{\eta}$ and $\lambda \ll 1$ the model matrix \cite{22} becomes

$$M^\nu = m_0 \begin{bmatrix} \eta^2 & -\bar{\lambda}^2 & \xi \bar{\lambda} \\ -\bar{\lambda}^2 & 0 & \bar{\eta} \\ \xi \bar{\lambda} & \bar{\eta} & 0 \end{bmatrix}. \quad (28)$$

where $\xi$ is a Yukawa parameter of order one. In the limit $\eta, \bar{\eta} \to 0$ we have the simpler one singlet case discussed previously. This is the case where the neutrino sector respects the symmetry $L_f = L_e - L_\mu - L_\tau$ discussed extensively in the literature \cite{22, 23} which implies bimaximal ($\theta_{21}, \theta_{23}$) mixing. Mixing effects of the charged lepton matrix and the second singlet field result to a “soft breaking” of the quantum number $L_f$. Using the analysis of section 2, it is straightforward to generate numerical values for $\chi, \xi, \bar{\lambda}$ that satisfy the bounds \cite{47, 48}. It is to be noted however, that only a subset of the so generated values can satisfy the constraints implied by the charged-leptons mass matrix.

We now turn on to the charged lepton mass matrix. Since the leptonic mass matrix has fewer degrees of freedom than the corresponding neutrino matrix, additional Yukawa parameters have to be introduced. The minimal number required in order to reach a solution is three ($m_0^e, b, d$). This way (using in the same approximation $\eta \ll \bar{\eta}, \lambda \ll 1$), the leptonic mass matrix becomes

$$M^e = m_0^e \begin{bmatrix} \bar{\lambda}^2 & b \bar{\eta}^2 & 0 \\ b \bar{\eta}^2 & \bar{\lambda}^2 & d \bar{\lambda} \\ 0 & d \bar{\lambda} & 1 \end{bmatrix}. \quad (29)$$

If we are to be working in the perturbative region, we must require the values of $\bar{\eta}$ and $\bar{\lambda}$ to be smaller than one, while the values of the constants $b$ and $d$ to be at the most of order one. The trace of the matrix $M^e$ equals the sum of the lepton masses, thus, we expect that the value of the constant $m_0^e$ to be around the value of the tau mass. Upon further numerical investigation, solutions that satisfy our criteria were found. In table \ref{tab3} we present a selected subset. The columns (2 – 5) of this table present the numerical values of $m_0^e, b, d, \bar{\eta}$ and $\bar{\lambda}$. All these numbers correspond to lepton masses $m_e = -0.511$ MeV, $m_\mu = 105.66$ MeV and $m_\tau = 1777.05$ MeV. The matrix $M^e$ can be diagonalized by means of an orthogonal matrix $U_l$ so that

$$\text{diagonal } [m_e, m_\mu, m_\tau] = U_l M^e U_l^\dagger$$

The strong hierarchy of the leptonic masses $|m_e| < m_\mu < m_\tau$ and the structure of the charged lepton mass matrix obtained, put strong constraints on the elements of $U_l$. Since $M^e_{11}$ and $M^e_{22}$ are negligible, the eigenvectors corresponding to the eigenvalues $m_\tau$ and $m_\mu$ must be of the form $e_3 = [\sim 0, \sim 0, \sim 1]$ and $e_2 = [\sim 0, \sim 1, \sim 0]$ by orthogonality. Orthogonality then implies that the third eigenvector $e_1 = [\sim 1, \sim 0, \sim 0]$. Numerics confirm this statement. Indeed, choosing from Table \ref{tab3} the first solution we get

$$U_l = \begin{bmatrix} 0.993337 & -0.112408 & 0.0254002 \\ 0.115231 & 0.965657 & -0.23287 \\ 0.00164859 & 0.234245 & 0.972176 \end{bmatrix}$$

while for the eleventh solution we get

$$U_l = \begin{bmatrix} 0.997154 & -0.0753675 & 0.00201336 \\ 0.0753943 & 0.996752 & -0.0283198 \\ 0.000127579 & 0.028391 & 0.999597 \end{bmatrix}.$$ 

Thus, $U_l$ being close to the unit matrix in all cases, transfers only a small mixing to the neutrino sector. The neutrino mixing matrix is given by

$$M^\nu = U_l^\dagger M_n U_l$$

and can be diagonalized by means of an orthogonal matrix which is to be identified with $U_\nu$ \cite{49}. Columns (6, 7) of Table \ref{tab4} show the values of the remaining parameters $\eta, \xi$ fixed by the neutrino mass matrix. From this identification the values of the mixing angles $\theta_{12}, \theta_{13}, \theta_{23}$ can be calculated.
Table 3: The values of the parameters for 11 selected cases satisfying experimental data for lepton masses and mixing angles. \( m_0^e, b, d, \bar{\eta} \) and \( \bar{\lambda} \) are determined from the charged lepton mass matrix. The remaining \( \xi \) and \( \eta \) are fixed by \( m_{\nu} \).

| \( n_0 \) | \( m_0^c \) | \( b \) | \( d \) | \( \bar{\eta} \) | \( \bar{\lambda} \) | \( \xi \) | \( \eta \) |
|---|---|---|---|---|---|---|---|
| 1 | 1685.27 | 0.106904 | 0.662742 | 0.410916 | 0.341056 | 0.241615 | 0.072256 |
| 2 | 1685.33 | 0.114839 | 0.662618 | 0.401166 | 0.340994 | 0.24267 | 0.064982 |
| 3 | 1694.54 | 0.109728 | 0.643608 | 0.418435 | 0.326475 | 0.234507 | 0.074418 |
| 4 | 1700.30 | 0.09143 | 0.630442 | 0.418435 | 0.326475 | 0.234507 | 0.074418 |
| 5 | 1717.35 | 0.088453 | 0.583331 | 0.397698 | 0.309176 | 0.229562 | 0.068114 |
| 6 | 1735.09 | 0.090929 | 0.518201 | 0.390316 | 0.290874 | 0.223921 | 0.058959 |
| 7 | 1755.26 | 0.116885 | 0.401999 | 0.346965 | 0.268733 | 0.217379 | 0.040692 |
| 8 | 1759.96 | 0.124525 | 0.362853 | 0.337367 | 0.263384 | 0.215189 | 0.037264 |
| 9 | 1772.21 | 0.103441 | 0.203602 | 0.353084 | 0.24901 | 0.205992 | 0.044141 |
| 10 | 1775.19 | 0.110105 | 0.128032 | 0.344582 | 0.245416 | 0.204041 | 0.039409 |
| 11 | 1775.70 | 0.127301 | 0.109161 | 0.328114 | 0.24479 | 0.204962 | 0.033917 |

Table 4: The range of the neutrino mass scale \( m_0 \) defined in (29) is shown in the first two columns. The values of \( \sin^2 \theta_{ij} \) for the eleven cases discussed in the text are also presented in the next three columns.

| \( n_0 \) | \( m_{0\text{min}} \) | \( m_{0\text{max}} \) | \( \sin^2 \theta_{12} \) | \( \sin^2 \theta_{13} \) | \( \sin^2 \theta_{23} \) |
|---|---|---|---|---|---|
| 1 | 0.377324 | 0.421945 | 0.37482 | 0.053167 | 0.549865 |
| 2 | 0.235305 | 0.312101 | 0.389747 | 0.044787 | 0.555663 |
| 3 | 0.227853 | 0.302218 | 0.379206 | 0.042058 | 0.552185 |
| 4 | 0.259091 | 0.343651 | 0.342147 | 0.055435 | 0.539129 |
| 5 | 0.280139 | 0.371568 | 0.329176 | 0.046816 | 0.531798 |
| 6 | 0.301829 | 0.400338 | 0.325867 | 0.036055 | 0.521448 |
| 7 | 0.367913 | 0.487989 | 0.358503 | 0.019655 | 0.503853 |
| 8 | 0.385085 | 0.510765 | 0.365593 | 0.017262 | 0.496102 |
| 9 | 0.670619 | 0.753771 | 0.315931 | 0.022377 | 0.453056 |
| 10 | 0.377407 | 0.500582 | 0.34582 | 0.245416 | 0.204041 | 0.039409 |
| 11 | 0.404905 | 0.537054 | 0.353621 | 0.24479 | 0.204962 | 0.033917 |

We finally discuss in brief the prediction for the neutrinoless double-beta \( (\beta\beta_0) \) decay. Operator (12) violates lepton number by two units, allowing thus the \( \beta\beta_0 \) process \((A, Z) \rightarrow (A, Z + 2) + 2e^-\). The parameter relevant to experiments for this process is

\[
m_{ee} = \sum_i U_{ei}^2 m_{\nu_i} \tag{30}
\]

while the experimental constraint is \( m_{ee\text{exp}} \leq [0.55 - 1.10] \text{eV} \). The exact predictions for all eleven cases for \( m_0 = m_{0\text{max}} \) are compatible with the experimental bounds and are presented in Table 5.

We proceed with some remarks on the solutions obtained. We first observe that all vevs are in the perturbative region, while \( \eta \ll \bar{\eta} \sim \bar{\lambda} \) as already assumed. It can be checked that for all solutions \( m_{11}^e \approx m_{23}^e \), since \( \bar{\eta}^3 \sim \eta \). Furthermore, we observe that around solution 8 we get the approximate relation \( \theta_{12} + \theta_{13} \approx \theta_{23} \). This could be related to a recent conjecture about quark-lepton complementarity (QLC) [26] which states that \( \theta_{c12} \approx \theta_{c13} \) and \( \theta_{c12} + \theta_{c13} \approx \pi/4 \) where \( \theta_{c12} \) is the Cabbibo angle.
Table 5: Neutrino masses and the double beta decay parameter \(m_{ee} = \sum U_{ei}^2 m_i\) for the maximum value of the neutrino mass scale parameter \(m_0\) of \(29\). All masses are expressed in \(eV\) units.

| \(m_{0_{\text{max}}}\) | \(m_{\nu_1}\) | \(m_{\nu_2}\) | \(m_{\nu_3}\) | \(m_{ee}\) |
|-------------------------|---------------|---------------|---------------|-----------|
| 0.421945               | 0.0677911     | -0.0672912    | 0.0287763     | 0.0178    |
| 0.312101               | 0.0496165     | -0.0486496    | 0.0191827     | 0.0117    |
| 0.302218               | 0.0460616     | -0.0450185    | 0.0181325     | 0.0118    |
| 0.343651               | 0.0525161     | -0.0516036    | 0.0242644     | 0.0173    |
| 0.371568               | 0.0516527     | -0.0507248    | 0.024252      | 0.0182    |
| 0.400338               | 0.049542      | -0.0485738    | 0.0228372     | 0.0178    |
| 0.487989               | 0.0501541     | -0.0491979    | 0.0194268     | 0.0146    |
| 0.510765               | 0.0502146     | -0.0492598    | 0.0186574     | 0.0139    |
| 0.753771               | 0.0695465     | -0.0690545    | 0.0326878     | 0.0259    |
| 0.500582               | 0.0446707     | -0.0435943    | 0.0194047     | 0.0158    |
| 0.537054               | 0.0464765     | -0.0454431    | 0.0179377     | 0.0140    |

Figure 1: The allowed region for the three neutrino mixing angles
Figure 2: The dimensionless neutrino mass ratio as a function of the corresponding angles.

Figure 3: The dimensionless neutrino mass ratio as a function of the corresponding angles.
5. Conclusions

In this work we have explored the possibility of deriving viable neutrino mass matrix textures capable of interpreting the recent neutrino data for masses and mixing. This was realized in the context of the Minimal Supersymmetric Standard Model (MSSM) extended by an Abelian symmetry with a minimal fermion content and two additional pairs of singlet Higgs fields. We have developed a useful texture formalism where the matrix elements are represented as inner products of the neutrino mass eigenstate vector $\vec{m} = (m_1, m_2, m_3)$ with six vectors $\vec{\theta}_i$ which are functions of the mixing angles. Texture matrices having two vanishing elements [16], $\vec{m} \cdot \vec{\theta}_i = 0, \vec{m} \cdot \vec{\theta}_j = 0$ imply a neutrino eigenmass vector of the form $\vec{m} = m_0 \vec{\theta}_i \times \vec{\theta}_j$. We have attempted to relate these structures with textures obtained from symmetry principles in the context of the above proposed extended MSSM model. It was found that Yukawa mass terms, generated when the two singlet Higgs pairs with appropriate $U(1)$ charges obtain vevs, can generate to a good approximation, several texture zero forms. We have analyzed the particular case where the corresponding neutrino mass texture form respects the $L_f = L_e - L_\mu - L_\tau$ lepton flavor symmetry implying bimaximal neutrino mixing when only one singlet Higgs pair receives a non-zero vev. Contributions from the second singlet Higgs pair vev break the $L_f$ symmetry and generate new mass entries leading to acceptable neutrino masses and mixing. A numerical analysis, confirms that for natural values of the –yet undetermined by the theory– Yukawa couplings, there exist solutions satisfying all experimental constraints.

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