THE HAMILTONIAN APPROACH TO GENERAL RELATIVITY
AND CMB PRIMORDIAL SPECTRUM

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Approaches to solutions of problems of the energy, time, Hamiltonian operator quantization of the General Relativity, the creation of the Universe from vacuum are considered in the frame of reference associated with the CMB radiation in order to describe parameters of this radiation in terms of the parameters of the Standard Model of elementary particles.

Keywords: Cosmic Microwave Background; Quantization; General Relativity.

1. Introduction

The measurements of the Cosmic Microwave Background (CMB) radiation temperature [1] revealed its dipole component testifying that an Earth observer (i.e. his “Hubble” Telescope) moves to Leo with a velocity about 400 km/c. This velocity can be treated as a parameter of the Lorentz transformation between a rest frame of an Earth observer and the comoving frame of the Universe distinguished by the unit time-like vector \( l_\mu = (1, 0, 0, 0) \).

The revelation of the comoving frame of our Universe allows us to seek explanations of cosmological problems, or part of them, with the help of the ordinary Laplace-type questions: What are primordial values of the cosmological scale factor and field variables in the comoving frame? What are the units of measurement of the initial data which can give us the simplest fit of all observational data?

In order to describe the Universe creation and its evolution in the comoving
reference frame, we apply the Hamiltonian approach to General Relativity (GR) and Standard Model (SM) of elementary particles developed in the case of QED by Dirac [2] who distinguished the comoving frame by the unit time-like vector $l_\mu$ and separated vector field components $A_\mu$ into time-like $A_0 = l_\mu A_\mu$ and the space-like ones $A_j$. The latter are split on two transverse degrees of freedom $A_\mu(T) = (\delta_{ij} - \partial_i \partial_j) A_j$ (called the “photon”) and the longitudinal part $A_\mu(C) = \partial_\mu 1 - \partial_j A_j$ which together with the time-like component $A_0$ form the gauge-invariant Coulomb potential $A_0(T) = A_0 - \partial_\mu 1 \partial_j A_j$. Dirac [2] called these gauge-invariant functionals $A_0(T), A_\mu(T), \Psi(T) = \exp\{i e \partial_\mu \partial_j A_j\} \Psi$ as the “dressed” fields (where $e$ is a coupling constant). We can quantize only two gauge-invariant transversal components, while the instantaneous Coulomb potential $A_0(T)$ forms instantaneous atoms and molecules.

In the context of a consistent description of bound states and collective evolution of the type of cosmological expansion, one can ask what the instantaneous Newton potential and the operator quantization of GR are, if the cosmological evolution is considered as one of dynamic variables that can be quantized?

2. Hamiltonian Approach to GR in Finite Space-Time

Einstein’s GR [6] is associated with Hilbert’s action [7]

$$S_{GR}[\phi_0] = \int d^4x \sqrt{-g} \left[ -\frac{\phi_0^2}{6} R(g) + \mathcal{L}(\mathcal{M}) \right],$$

where $\phi_0 = M_{\text{Planck}} \sqrt{3/8\pi}$ is the Planck mass parameter. Recall that Hilbert [7] formulated GR so that Einstein’s generalization of the Lorentz frame group $x^\mu \rightarrow \bar{x}^\mu = \bar{\mathcal{F}}^\mu(x^\mu)$ becomes a gauge group. There is the principal difference between the frame transformations and the gauge ones. Parameters of frame transformations (of type of initial data) are treated as measurable quantities, whereas parameters of the gauge transformations are not measured. Gauge symmetries lead to constraints decreasing number of degrees of freedom.

In the context of the problem of the initial data one needs to separate the frame transformations (here the Lorentz ones) from the gauge transformations (here the general coordinate ones). Just this separation is the main difference of the Hamiltonian approach to GR considered here in finite space-time [8,9] from the Dirac – ADM one [10]. This separation can be fulfilled by using the gauge-invariant components of Fock’s symplex $\omega(\alpha)$ defined as $ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \ldots$
\begin{equation}
\omega^{(0)\omega(0)} - \omega^{(b)\omega(b)}; \quad \omega^{(\alpha)} = e^{(\alpha)\nu}dx^\nu \quad \text{where} \quad e^{(\alpha)\nu} \quad \text{are the Fock tetrad the components of which are marked by the general coordinate index without a bracket and the Lorentz index in brackets (} \alpha \text{)[11].}
\end{equation}

The choice of the time axis \(\imath_{(\alpha)} = (1, 0, 0, 0)\) as the CMB comoving frame allows us to construct an irreducible representation of the Poincare group by decomposition of Fock’s vector simplex field \(\omega^{(\alpha)}\) in accordance with the definition of the Dirac–ADM Hamiltonian approach to GR [10]

\begin{equation}
\omega^{(0)} = \psi^6 N_0 dx^0 \equiv \psi^2 \omega^{(L)}_{(0)}; \quad \omega^{(b)} = \psi^2 e_{(b)j}(dx^j + N^i dx^0) \equiv \psi^2 \omega^{(L)}_{(b)},
\end{equation}

where \(N^i\) is shift vector, \(N_0\) is Dirac’s lapse function, \(\psi\) is the spatial metric determinant, \(e_{(b)j}\) is a triad with the unit determinant \(|e| = 1\), and \(\omega^{(L)}_{(0)}, \omega^{(L)}_{(b)}\) are the scale-invariant Lichnerowicz simplex [12] forming the scale-invariant volume \(dV_0 = \omega^{(L)}_{(1)} \wedge \omega^{(L)}_{(2)} \wedge \omega^{(L)}_{(3)} = d^3x^{(L)}\) that coincides with the spatial coordinate volume.

It is well known [13] that the Hamiltonian approach to GR is invariant with respect to the general spatial coordinate transformations \(x^0 \rightarrow \tilde{x}^0 = \tilde{x}^0(x^0)\) and \(x^j \rightarrow \tilde{x}^j = \tilde{x}^j(x^0, x^j)\).

The gauge invariance \(x^j \rightarrow \tilde{x}^j = \tilde{x}^j(x^0, x^j)\) allows us to remove longitudinal components of tensor triads \(e_{(b)j}\) and keep two transversal gravitons distinguished by the constraint \(\partial_\imath e_{(b)j} \approx 0\) in complete analogy with the Dirac construction of QED with the one-to-one correspondence \([A_0^{(T)}, A_k^{(T)}] \rightarrow [N_{(b)}, e_{(b)k}]\). This means that the spatial coordinates and the Lichnerowicz finite volume \(V_0 = \int d^3x^{(L)} = \int d^3x\) can be identified with gauge-invariant observables.

The invariance of GR in the finite volume \(V_0\) (considered in the modern cosmology for description of the Universe evolution) with respect to the reparametrizations of the coordinate evolution parameter: \(x^0 \rightarrow \tilde{x}^0 = \tilde{x}^0(x^0)\) [13] allows us to convert one of variables into the time-like evolution parameter in the comoving frame. This means that the coordinate evolution parameter \(x^0\) is not observable. Wheeler and DeWitt [14] proposed considering the reparametrization invariance in GR by analogy with Special Relativity (SR), where the role of a timelike variable is played by one of the dynamic variables \(X_0\) in the World space of events \([X_0][X_k]\).

Wheeler and DeWitt [14] proposed considering the reparametrization invariance in GR in a similar manner, i.e. they proposed to generalize the construction of representations of the Poincare group to the field space of events in GR, where the role of reparametrization-invariant evolution parameter (i.e. a time-like variable in the field space of events) is played by a cosmological scale factor \(a(x^0)\). This factor is separated by the scale transformation of all fields with a conformal weight \(n\) including the metric components \(F = a^n(x^0)\tilde{F}^{(n)}, \quad g_{\mu\nu} = a^2(x^0)\tilde{g}_{\mu\nu}\). This transformation keeps the momentum constraint \(T_0^k = \tilde{T}_0^k = 0\), so that the cosmological scale factor \(a(x^0)\) can be considered as the zero mode solution of the momentum constraint. In order to conserve the number of variables in GR, the logarithm of the cosmological scale factor is identified with Lichnerowicz spatial averaging the spatial determi-
nant logarithm \( \log a = \langle \log \psi^2 \rangle \equiv \frac{1}{V_0} \int d^3x \log \psi^2 \). This finite volume generalization of the Dirac–ADM approach [10] was called the “Hamiltonian cosmological perturbation theory” [9]. This approach demonstrates that the naive perturbation theory \( g_{\mu\nu} = g^{\text{Minkowski}}_{\mu\nu} + h_{\mu\nu} \), where the coordinate evolution parameter \( x^0 \) is identified with measurable quantities, is incorrect because this theory contradicts to the gauge symmetry of GR. The definition of a measurable gauge-invariant coordinate evolution parameter in GR in finite volume will be considered below.

A scale transformation \( \sqrt{-g}R(g) = a^2 \sqrt{-g}R(\bar{g}) - 6a\partial_0 \left[ \partial_0 a \sqrt{-g} \bar{g}^{00} \right] \) converts action (1) into

\[
S = \bar{S} - \int d^3x \left( \frac{d\varphi}{d\varphi_0} \right)^2 \int d^3x \tilde{N}_d^{-1} = \int d^3x L, \tag{3}
\]

where \( \bar{S} \) is the action (1) in terms of metrics \( \bar{g} \) and the running scale of all masses \( \varphi(x^0) = \varphi_0 a(x^0) \). The variation of this action with respect to the new lapse function \( \tilde{N}_d \) leads to a new energy constraint

\[
T^0_0 = \bar{T}^0_0 - \left( \frac{d\varphi}{d\varphi_0} \right)^2 \frac{1}{\tilde{N}_d^2} = 0 \quad \left( \frac{\delta \bar{S}}{\delta \tilde{N}_d} \right). \tag{4}
\]

Spatial averaging the square root \( \sqrt{T^0_0} = \pm |d\varphi/(\tilde{N}_d dx^0)| \) over the Lichnerowicz volume \( V_0 = \int d^3x \) gives the Hubble-like relation

\[
\zeta(\pm) = \int d^3x \langle \tilde{N}_d^{-1} \rangle^{-1} = \pm \int d^3x \langle (\bar{T}^0_0)^{1/2} \rangle^{1/2}, \tag{5}
\]

where \( \langle F \rangle = \int d^3xF F \) and \( (d\zeta)^{-1} = \langle (dx^0 \tilde{N}_d)^{-1} \rangle \) is an inverse time-interval invariant with respect to time-coordinate transformations \( x^0 \rightarrow \tilde{x}^0 = \bar{x}^0(\tilde{x}^0) \). We see that the Hubble law in the exact GR appears as spatial averaging the energy constraint (4). Thus, in the contrast with the generally accepted Lifshits theory [15] its Hamiltonian version [9] distinguishes the time-coordinate \( x^0 \) as an object of reparametrizations from the reparametrization-invariant time interval (5).

Just this distinction help us to separate the local part of the energy constraint (4) and determine unambiguously the gauge-invariant Dirac lapse function

\[
N_{\text{inv}} = \langle (\tilde{N}_d)^{-1} \rangle \tilde{N}_d = \langle (\bar{T}^0_0)^{1/2} \rangle (\bar{T}^0_0)^{-1/2}. \tag{6}
\]

\(^{\text{c}}\)The separation of the cosmological scale factor is well-known as the “cosmological perturbation theory” (where \( \psi = 1 - \Psi/2, \bar{\psi}^0 \bar{N}_d = 1 + \Phi \)) proposed by Lifshits [15] in 1946 and applied now for analysis of observational data in modern astrophysics and cosmology (see [16]). However, in this theory, the scale factor is an additional variable without any constraint for the deviation \( \Psi \), so that \( \int d^3x \psi \neq 0 \), and there are two zero Fourier harmonics of the determinant logarithm instead of one. This doubling does not allow to express the velocities of both variables \( \log a \) and \( \int d^3x \psi \) through their momenta and to construct the Hamiltonian approach to GR [8,9].
The explicit dependence of $\tilde{T}_0^0$ on $\tilde{\psi}$ can be given in terms of the scale-invariant Lichnerowicz variables [12] $\omega^{(L)} = \tilde{\psi}^{-2}\omega^{(\mu)}$, $F^{(Ln)} = \tilde{\psi}^{-n}F^{(n)}$

$$\tilde{T}_0^0 = \tilde{\psi}^7 \tilde{\Delta} \tilde{\psi} + \sum_l \tilde{\psi}^l a^{1/2 - 2} \tau_I,$$  \hspace{1cm} (7)

where $\tilde{\Delta} \tilde{\psi} = (4\phi^2/3)\partial_{(b)} \partial_{(b)} \tilde{\psi}$ is the Laplace operator and $\tau_I$ is partial energy density marked by the index $I$ running a set of values $I = 0$ (stiff), 4 (radiation), 6 (mass), 8 (curvature), 12 (Λ-term) in accordance with a type of matter field contributions, and a is the scale factor [8].

The expression $(\tilde{T}_0^0)^{1/2}$ is Hermitian if a negative contribution of the local determinant momentum

$$p_{\tilde{\psi}} = \frac{\partial \mathcal{L}}{\partial (\partial \log \tilde{\psi})} \equiv \frac{-4\phi^2}{3} \cdot \frac{\partial (\tilde{\psi}^6 N^I)}{\tilde{\psi}^6N_d},$$  \hspace{1cm} (8)

is removed from the energy density (4) by the minimal surface constraint [10]

$$p_{\tilde{\psi}} \simeq 0 \Rightarrow \partial_j (\tilde{\psi}^6 N^j) = (\tilde{\psi}^6)' \quad (N^j = N^j(\tilde{N}_d^{-1})).$$  \hspace{1cm} (9)

One can see that the scalar sector $N_{\text{int}} \tilde{\psi}, \partial_j [\tilde{\psi}^6 N^j]$ is completely determined in terms of gauge-invariant quantities by the equations (6), (9) and the equation of the local part of the spatial determinant logarithm $\log \tilde{\psi}^2 \equiv \log \psi^2 - \langle \log \psi^2 \rangle$

$$\frac{\delta S}{\delta \log \tilde{\psi}} = -T_\psi + \langle T_\psi \rangle = 0, \quad \int d^3x \log \tilde{\psi}^2 \equiv 0,$$  \hspace{1cm} (10)

where $T_\psi$ is given as

$$T_\psi|_{(\tilde{\psi} = 0)} = 7N_{\text{inv}} \tilde{\psi}^7 \tilde{\Delta} \tilde{\psi} + \tilde{\psi} \tilde{\Delta} [N_{\text{inv}} \tilde{\psi}^7] + \sum_l I \tilde{\psi}^l a^{1/2 - 2} \tau_I.$$  \hspace{1cm} (11)

These equations are in agreement with the Schwarzschild-type solution for the potentials $\triangle \tilde{\psi} = 0$, $\triangle [N_{\text{inv}} \tilde{\psi}^7] = 0$ in the empty space $\tau_I = 0$, but they strongly differ from the “gauge-invariant” version [16] of the Lifshits perturbation theory [15].

The Lifshits theory [16] does not take into account both the Dirac constraint (9) removing the negative energy of the spatial determinant and the potential scalar perturbations formed by the determinant $\tilde{\psi} = 1 + (\mu - \langle \mu \rangle)$ in (7), where $\sum_l I l a^{1/2 - 2} \tau_I = \sum_n c_n (\mu - \langle \mu \rangle)^n \tau_n$, $\tau_n \equiv \sum_I I^n a^{1/2 - 2} \tau_n \equiv \langle \tau_0 \rangle + \tau_n$, and $\tau_n = \langle \tau(n) \rangle$. The Hamiltonian cosmological perturbation theory [8,9] leads to the scalar potentials

$$\tilde{\psi} = 1 + \frac{1}{2} \int d^3y \left[ D_{(+)}(x,y) \mathcal{T}^{(\psi)}_{(+)}(y) + D_{(-)}(x,y) \mathcal{T}^{(\psi)}_{(-)}(y) \right],$$  \hspace{1cm} (12)

$$N_{\text{inv}} \tilde{\psi}^7 = 1 - \frac{1}{2} \int d^3y \left[ D_{(+)}(x,y) \mathcal{T}^{(N)}_{(+)}(y) + D_{(-)}(x,y) \mathcal{T}^{(N)}_{(-)}(y) \right],$$  \hspace{1cm} (13)

where

$$\mathcal{T}^{(\psi)}_{(+)} = \tau_0 \pm 7\beta[\tau(0) - \tau(1)], \quad \mathcal{T}^{(N)}_{(+)} = [7\tau(0) - \tau(1)] \pm (14\beta)^{-1}\tau(0)$$  \hspace{1cm} (14)
are the local currents, \( D_{i(±)}(x, y) \) are the Green functions satisfying the equations\(^d\)

\[
[±\hat{m}_i^2(±) - \hat{A}]D_{i(±)}(x, y) = \delta^i(x - y) - 1/V_0,
\]

where \( \hat{m}_i^2(±) = 14(β ± 1)\langle τ(0) \rangle ± \langle τ(1) \rangle \). \( \hat{A} = \sqrt{1 + [(\langle τ(2) \rangle - 14\langle τ(1) \rangle)/(98\langle τ(0) \rangle)]} \).

These Hamiltonian solutions (12) and (13) do not contain the Lifshits-type kinetic scalar perturbations explaining the CMB spectrum in the Inflationary Model [16]; they disappear due to the positive energy constraint (9). Therefore, the problem arises to reproduce the CMB spectrum by the fundamental operator quantization based on the Hamiltonian approach to GR.

One can construct the Hamiltonian form of the action (3)

\[
S = \int dx^0 \left[ \int d^3x \left( \sum_F P_F \partial_0 F + C - \tilde{N}_a \tilde{T}_0^a \right) - P_x \partial_0 \varphi + \frac{P_x^2}{4 \int dx^3(N_a)^{-1}} \right],
\]

in terms of momenta \( P_F = [p_{\varphi}, p_i] \) given by (8) and \( P_{\varphi} = \partial L/\partial (\partial_0 \varphi) \), where \( C = N^i \tilde{T}_0^i + C_0 p_{\varphi} + C(b) \partial_0 e_b \) is the sum of constraints with the Lagrangian multipliers \( N^i, C_0, C(b) \) and the energy–momentum tensor components \( \tilde{T}_0^i \); these constraints include Dirac’s constraints (9) and the transversality \( \partial_0 e_b \approx 0 \) [10].

One can find evolution of all field variables \( F(\varphi, x^i) \) with respect to \( \varphi \) by the variation of the “reduced” action obtained as values of the Hamiltonian form of the initial action (16) onto the energy constraint (4):

\[
S|_{P_{\varphi}=±E_{\varphi}} = \int_{\varphi_1}^{\varphi_0} d\tilde{\varphi} \left\{ \int d^3x \left[ \sum_F P_F \partial_\varphi F + \tilde{C} \mp 2\sqrt{\tilde{T}_0^a(\tilde{\varphi})} \right] \right\},
\]

where \( \tilde{C} = C/\partial_0 \tilde{\varphi} \) and \( \varphi_0 \) is the present-day datum that has no relation to the initial data at the beginning \( \varphi = \varphi_1 \).

3. Observational Data in Terms of Scale-Invariant Variables

Let us assume that the density \( T^0_0 = \rho(0)(\varphi) + T_t \) contains a tremendous cosmological background \( \rho(0)(\varphi) \). The low-energy decomposition of “reduced” action (17)

\[
2d\varphi \sqrt{T^0_0} = 2d\varphi \sqrt{\rho(0)} + T_t = d\varphi \left[ 2\sqrt{\rho(0)} + T_t/\sqrt{\rho(0)} \right] + ... \text{ over field density} \ T_t \text{ gives the sum} S|_{P_\varphi=+E_\varphi} = S^{(\text{cosmic})}_{\text{field}} + \ldots, \text{where the first term of this sum} \ S^{(\text{cosmic})}_{\text{field}} = +2V_0 \int d\varphi \sqrt{\rho(0)}(\varphi) \text{ is the reduced cosmological action, whereas the second one is the standard field action of GR and SM} \ S^{(\text{field})}_{\text{field}} = \int_{\zeta_i}^{\zeta_0} \zeta d\zeta \int d^3x \left[ \sum_F P_F \partial_\zeta F + \tilde{C} - T_t \right]
\]

in the space with the interval \( ds^2 = d\xi^2 - \sum_a [e_{(a)i}(dx^i + N^i d\xi)]^2; \partial_i e_{(a)i} = 0, \partial_a N^a = 0 \)

\(^d\)In contrast to the Lifshits theory, the solutions (12) and (13) contain the nonzero shift-vector \( N^0 \) of the coordinate origin with the spatial metric oscillations that lead to a new mechanism of formation of the large-scale structure of the Universe [8,9].
and conformal time \( d\eta = d\zeta = d\varphi/\rho_0^{1/2} \) as the gauge-invariant quantity, coordinate distance \( r = |x| \), and running masses \( m(\zeta) = a(\zeta)m_0 \). We see that the correspondence principle leads to the theory, where the scale-invariant conformal variables and coordinates are identified with the observable ones and the cosmic evolution with the evolution of masses:

\[
\frac{E_{\text{emission}}}{E_0} = \frac{m_\text{atom}(\eta_0 - r)}{m_\text{atom}(\eta_0)} = \frac{\varphi(\eta_0 - r)}{\varphi_0} = a(\eta_0 - r) = \frac{1}{1 + z}.
\]

The conformal observable distance \( r \) loses the factor \( a \), in comparison with the nonconformal one \( R = ar \). Therefore, in this case, the redshift–coordinate-distance relation \( d\eta = d\varphi/\sqrt{\rho_0(\varphi)} \) corresponds to a different equation of state in comparison with the standard one [17]. The best fit to the data including cosmological SN observations [18] requires a cosmological constant \( \Omega_A = 0.7 \), \( \Omega_{\text{CDM}} = 0.3 \) in the case of the Friedmann “scale-variant quantities” of standard cosmology, whereas for the “scale-invariant conformal quantities” these data are consistent with the dominance of the stiff state of a free scalar field \( \Omega_{\text{Stiff}} = 0.85 \pm 0.15 \), \( \Omega_{\text{CDM}} = 0.15 \pm 0.10 \) [17].

If \( \Omega_{\text{Stiff}} = 1 \), we have the square root dependence of the scale factor on conformal time \( a(\eta) = \sqrt{1 + 2H_0(\eta - \eta_0)} \). Just this time dependence of the scale factor on the measurable time (here – conformal one) is used for a description of the primordial nucleosynthesis [17]. Thus, the relative units can describe all epochs including the creation of a quantum universe at \( \varphi(\eta = 0) = \varphi_I \), \( H(\eta = 0) = H_I \) by the stiff state [17]. This homogeneous stiff state can be formed by a free scalar field.

4. GR “Energy” and Creation of Universe with the Time Arrow

The “reduced” action (17) and the correspondence principle considered in the previous Section show us that the energy is the value of the scale factor canonical momentum

\[
P_\varphi = \frac{\partial L}{\partial (\partial_\varphi \varphi)} = -2V_0\partial_\varphi \left\langle \left( \bar{N}_I \right)^{-1} \right\rangle = -2V_0 \frac{d\varphi}{d\zeta} \equiv -2V_0\varphi'
\]

obtained by the spatial averaging the energy constraint (4) that takes the form \( P_\varphi^2 - E_\varphi^2 = 0 \), where \( E_\varphi = 2 \int d^3x (\bar{T}_0^0)^{1/2} \). Finally, we get the field space of events \([\varphi, \bar{F}]\), where \( \varphi \) is the evolution parameter, and its canonical momentum \( P_\varphi \) plays the role of the Einstein-type energy.

The primary quantization of the energy constraint \( \hat{P}_\varphi^2 - \hat{E}_\varphi^2 = 0 \) leads to the unique wave function \( \Psi_L \) of the collective cosmic motion. The secondary quantization \( \Psi_L = -\frac{1}{\sqrt{2E_\varphi}}[A^+ + A^-] \) describes creation of a “number” of universes \( <0|A^+A^-|0> = N \) from the stable Bogoliubov vacuum \( B^-|0> = 0 \), where \( B^- \) is Bogoliubov’s operator of annihilation of the universe obtained by the transformation

\[
A^+ = \alpha B^+ + \beta^* B^- \text{ in order to diagonalize equations of motion.}
\]

This causal quantization with the minimal energy restricts the motion of the universe in the field space of events \( E_\varphi > 0, \varphi_0 > \varphi_I \) and \( E_\varphi < 0, \varphi_0 < \varphi_I \), and it leads to the arrow of the time interval \( \zeta \geq 0 \) as the quantum anomaly [8].
5. Creation of Matter and Initial Data of the Universe

The initial data $\varphi_I, H_I$ of the universe can be determined from the parameters of matter cosmologically created from the stable quantum vacuum at the beginning of the universe.

The Standard Model in the framework of the perturbation theory and the operator quantization of SM [5] shows us that W-,Z-vector bosons have maximal probability of the cosmological creation due to their mass singularity. The uncertainty principle $\Delta E \cdot \Delta \eta \geq 1$ (where $\Delta E = 2M_I, \Delta \eta = 1/(2H_I)$) testifies that these bosons can be created from vacuum at the moment when their Compton length defined by the inverse mass $M_I^{-1} = (q_I/M_W)^{-1}$ is close to the universe horizon defined in the stiff state as $H_I^{-1} = a_I^2(H_0)^{-1}$. Equating these quantities $M_I = H_I$ one can estimate the initial data of the scale factor $a_I^2 = (H_0/M_W)^{2/3} = 10^{-29}$ and the Hubble parameter $H_I = 10^{29}H_0 \sim 1 \text{ mm}^{-1} \sim 3K$ [19].

The collisions and scattering processes with the cross-section $\sigma \sim 1/M_I^2$ lead to conformal temperature $T_c$. This temperature can be estimated from the condition that the relaxation time is close to the life-time of the universe, i.e., from the equation in the kinetic theory $\frac{\sigma}{\eta} \sim n(T_c) \times \sigma \sim H$. As the distribution functions of the longitudinal vector bosons demonstrate a large contribution of relativistic momenta [19] $n(T_c) \sim T_c^3$, this kinetic equation gives the temperature of relativistic bosons $T_c \sim (M_I^2H_I)^{1/3} = (M_I^2H_0)^{1/3} \sim 3K$ as a conserved number of cosmic evolution compatible with the SN data [17]. We can see that this value is surprisingly close to the observed temperature of the CMB radiation $T_c = T_{CMB} = 2.73 \text{ K}$. The equations describing the longitudinal vector bosons in SM, in this case, are close to the equations that are used in the Inflationary Model [16] for description of the “power primordial spectrum” of the CMB radiation.

The primordial mesons before their decays polarize the Dirac fermion vacuum and give the baryon asymmetry frozen by the CP – violation so that $n_b/n_\gamma \sim X_{CP} \sim 10^{-9}, \Omega_b / \Omega_{\text{CMB}} \sim \alpha_{\text{QED}} / \sin^2 \theta_{\text{Weinberg}} \sim 0.03, \text{ and } \Omega_R / \Omega_{\text{CMB}} \sim 10^{-4}$ [19].

All these results testify to that all visible matter can be a product of decays of primordial bosons, and the observational data on CMB reflect rather parameters of the primordial bosons, than the matter at the time of recombination. In particular, the length of the semi-circle on the surface of the last emission of photons at the life-time of W-bosons in terms of the length of an emitter (i.e. $M_W^{-1}(\eta_L) = (\alpha_W/2)^{1/3}(T_c)^{-1}$) is $\pi \cdot 2/\alpha_W$. It is close to the value of orbital momentum with the maximal $\Delta T$: $l_{(\Delta T_{\text{max}})} \sim \pi \cdot 2/\alpha_W \sim 210$, whereas $(\Delta T/T)$ is proportional to the inverse number of emitters $(\alpha_W)^3 \sim 10^{-5}$.

In relative units the temperature history of the expanding universe looks like the history of evolution of masses of elementary particles in the cold universe with the constant conformal temperature $T_c = a(\eta)T = 2.73 \text{ K}$ of the cosmic microwave background.

In relative units the nonzero shift vector and the scalar potentials given by Eqs. (12) and (13) determine [17] the parameter of spatial oscillations $\eta_{(\rho)}^2 =
The redshifts in the recombination epoch $z_r \sim 1100$ and the clustering parameter $r_{\text{clust.}} = \pi/m(-) \sim \pi/[H_0 \Omega_R^{1/2} (1 + z_r)] \sim 130$ Mpc recently found in the searches of large scale periodicity in redshift distribution [20] lead to a reasonable value of the radiation-type density (including the relativistic baryon matter one) $10^{-4} < \Omega_R \sim 3 \cdot 10^{-3}$ at the time of this epoch.

Conclusions

The observational astrophysical data on CMB radiation revealed that our Universe can be an ordinary physical object moving with respect to the Earth observer with occasional initial data. This revelation returns us back to representations of the Poincare group as the basis of operator quantization that includes occasional gauge-invariant and frame-covariant initial data and their units of measurements.

In order to explain the World, a modern Laplace should ask for the initial data of the gauge-invariant variables measured in the relative units in the comoving reference frame of this World. The statement of the problem proposes a complete separation of frame transformations from the gauge ones. This separation is the main difference of our Hamiltonian approach to GR from all other ones. The result is the exact resolution of the energy constraint in terms of gauge-invariant variables that mean here the application of the standard theory of the unitary irreducible representations of the Poincare group based on the time-like unit vector that distinguishes the comoving frame in the invariant space-time where components of the Fock simplex are given. Another frame means a choice of another time-like unit vector connected with the first one by the Lorentz transformation that leads to the dipole component of the CMB temperature.

Here we listed a set of numerous arguments in favor of that the fundamental operator quantization can be a real theoretical basis for a further detailed investigation of astrophysical observational data, including CMB fluctuations.

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