ASTEROSEISMOLOGY OF SUN-LIKE STARS – A PROPOSAL

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ABSTRACT

In the past decade, helioseismology has revolutionized our understanding of the interior structure of the Sun. In the next decade, asteroseismology will place this knowledge into context, by providing structural information for dozens of pulsating stars across the H-R diagram. Solar-like oscillations have already been detected from the ground in a few stars, and several current and planned satellite missions will soon unleash a flood of stellar pulsation data. Deriving reliable seismological constraints from these observations will require a significant improvement to our current analysis methods. We are adapting a computational method, based on a parallel genetic algorithm, to help interpret forthcoming observations of Sun-like stars. This approach was originally developed for white dwarfs and ultimately led to several interesting tests of fundamental physics, including a key astrophysical nuclear reaction rate and the theory of stellar crystallization. The impact of this method on the analysis of pulsating white dwarfs suggests that seismological modeling of Sun-like stars will also benefit from this approach.

1. OBSERVATIONAL CONTEXT

Most of what we can learn about stars comes from observations of their outermost surface layers. We are left to infer the properties of the interior based on our best current understanding of the constitutive physics. The exception to this general rule arises from observations of pulsating stars, where seismic waves probe deep through the interior and bring information to the surface in the form of light and radial velocity variations. The most dramatic example is the Sun, where such observations have led to the identification of millions of unique pulsation modes, each sampling the solar interior in a slightly different and complementary way. The radial profile of the sound speed inferred from these data have led to such precise constraints on the standard solar model that the observations and theory now agree to better than a few parts per thousand over 90 percent of the solar radius (Christensen-Dalsgaard, 2002).

If we were to move the Sun to the distance of even the nearest star, most of the pulsation modes that we now know to be present would be rendered undetectable. We would lose most of our spatial resolution across the disk of the star, and only those modes of the lowest spherical degree ($l \leq 3$) would produce significant variations in the total integrated light or the spectral line profiles. This would reduce the number of detectable modes from millions to dozens, leading to a corresponding reduction in the ability of the observations to constrain the internal structure (e.g., see Kjeldsen et al., 1999). Even so, such data would still allow us to determine the global properties of the star and to probe the gross internal composition and structure, providing valuable independent tests of stellar evolution theory.

Recent improvements in our ability to make high-precision radial velocity measurements from the ground have been driven largely by efforts to detect extra-solar planets. These advances in technology have simultaneously led to the first unambiguous detections of solar-like oscillations in other stars (see Bedding & Kjeldsen, 2003). Scintillation in the Earth’s atmosphere severely limits our ability to detect the corresponding light variations due to these pulsations. But the observational requirements are similar to programs for detecting extra-solar planet transits, so we can expect continued rapid progress in this area—primarily from space.

2. COMPUTATIONAL ASTEROSEISMOLOGY

Several present (WIRE, MOST) and future (COROT, Kepler) satellite missions will soon yield nearly uninterrupted long-term coverage of many types of pulsating stars. We will then face the challenge of determining the fundamental properties of these stars from the data, by attempting to match them with the output of our computer models. The traditional approach to this task is to make informed guesses for each of the model parameters, and then to adjust them iteratively until an adequate match is found. This subjective method is particularly troublesome when combined with a local approach to iterative improvement of the defining parameters.

An optimization scheme based on a genetic algorithm (Charbonneau, 1995; Metcalfe & Charbonneau, 2003) can avoid the problems inherent in many traditional approaches. Using only observations and the constitutive physics of the model to restrict the range of possible values for each parameter, genetic algorithms provide a rel-
It is efficiently efficient means of searching globally for the optimal model. They were inspired by Charles Darwin’s notion of biological evolution through natural selection. The basic idea is to solve an optimization problem by evolving the global solution, starting with an initial set of purely random guesses. The evolution takes place within the framework of the model, and the individual parameters serve as the genetic building blocks. Selection pressure is imposed by some goodness-of-fit measure between the model and the observations.

Metcalfe (2001) developed a fully parallel and distributed hardware/software implementation of the popular PIKAIA genetic algorithm written by Paul Charbonneau and Barry Knapp. He used this modeling tool in the context of white dwarf asteroseismology, leading to a number of interesting physical results including a precise estimate of the astrophysically important $^{12}$C$(\alpha,\gamma)^{16}$O nuclear reaction rate (Metcalfe, 2003), and an empirical test of the theory of stellar crystallization (Metcalfe et al., 2004). The impact of this method on the analysis of pulsating white dwarfs suggests that seismological modeling of other types of stars could also benefit from this approach.

We propose to extend this powerful new analysis method to treat some of the most important problems in asteroseismology, exploiting the full potential of the observations. After developing an interface between the existing parallel code and models of main-sequence stars, the initial applications will be to the well-characterized $\delta$-Scuti stars and to the analysis of solar-like oscillations, using existing observations and synthetic data. We describe the plans for these investigations in greater detail below.

### 2.1. Interfacing with Main-sequence Models

The basic idea behind a genetic algorithm is fairly simple: it is just an iterative Monte Carlo method that samples the model space randomly, but keeps a sort of memory of what worked well in the past. It accomplishes this through a computational analogy with the idea of biological evolution through natural selection. It starts just like a simple Monte Carlo, where we generate $N$ random sets of parameters, evaluate the model for each set, and then compare them to the observations. The genetic algorithm treats each set of parameters as an individual in a population, and assigns each a fitness based on how well it matches the observations. Next, it selects from this population at random, with the fittest individuals more likely to survive. It then encodes the parameters into simple strings of numbers, sort of like chromosomes; it pairs them up and performs operations that are analogous to breeding and mutation, and then decodes the strings back into numerical values for the parameters. This produces a new population, so we evaluate the model for each case again, and continue the whole process until some termination criterion is met. Although it may seem a rather contorted way of exploring the model space, there is a firm theoretical basis known as the schema theorem to explain why it actually works in practice (Goldberg, 1989).

**Automating the code.** Although genetic algorithms are often more efficient than other comparably global optimization methods, they are still quite demanding computationally. Fortunately, the procedure is inherently parallelizable: we need to calculate many models, and each one of them is independent of the others. So the number of available processors determines the number of models that can be calculated in parallel. Also, there is very little communication overhead; parameter values are sent to each processor, and they return either a list of pulsation periods or just a goodness-of-fit measure if the computed periods have already been compared to the observations. The parallel version of the PIKAIA genetic algorithm is perfectly general, and will not require any structural modifications to accommodate the application to main-sequence models.

Some customization will be required, however, for the main-sequence models that we adopt for this project. J. Christensen-Dalsgaard’s models were developed for the analysis of helioseismic data, and were used to produce ‘Model S’ of Christensen-Dalsgaard et al. (1996), which has been used extensively as a reference model for inversions. Using these models for the analysis of pulsations in Sun-like stars will provide a certain degree of internal consistency in our understanding of solar-like oscillations.

Many of the required modifications will be similar to what was necessary for the white dwarf code. The main challenge will be to develop a mode of operation that will allow the input model to evolve to a specified temperature (or luminosity) automatically. A small grid of starter models with different masses covering the range of interest can be constructed independently of the optimization. Other interesting parameters, like the helium mass fraction ($Y$) and the metallicity ($Z$), can be specified at the beginning of the evolution. When the model has evolved to the parameter values requested by the genetic algorithm, the adiabatic pulsation frequencies can be calculated and compared to the observed periods. This will lead to a goodness-of-fit measure that the genetic algorithm will attempt to maximize.

**Optimizing the efficiency.** The efficiency of genetic-algorithm-based optimization is defined as the number of model evaluations required to yield the global solution, relative to the number of models that would be required for enumerative search of the grid at the same sampling density. In practice, a genetic algorithm is usually hundreds or even thousands of times more efficient than a complete grid, and its performance is fairly insensitive to the few internal parameters that control its operation. We will initially set these internal parameters based on our experience with white dwarf models, but we will run synthetic data through the optimization procedure (a so-called ‘hare & hound’ exercise) to ensure that the input parameters are recovered faithfully. We will repeat these exercises with variations in the control parameters until the efficiency of the algorithm is optimal.
2.2. Application to δ-Scuti Stars

Extending the Cepheid and RR Lyrae instability strip down to the main sequence, we find a group of A and early-F type stars with pulsation periods between roughly half an hour and half a day. This class of pulsating stars, with several dozen known members, has been given the name of the prototype, δ-Scuti. The amplitude of the observed variation ranges from a few milli-magnitudes to a few tenths of a magnitude, and they often exhibit a mixture of both radial and non-radial modes.

Parameter sampling density. The interpretation of pulsation data for δ-Scuti stars is currently facing challenges quite similar to those faced by white dwarf modelers five years ago. The observational requirements for long-term photometric monitoring have been satisfied by successful multi-site campaigns on several stars (e.g., see (Breger et al., 1998, 1999; Handler et al., 2000). But exploration of the most important physical parameters in theoretical models has been limited to very coarse grids, making it difficult to establish a unique best-fit model for a particular set of observations. The most extensive attempt at model-fitting to date, in terms of the number of computed models, was published by (Pamyatnykh et al., 1998), who explored three parameters with a grid of 120 models. Using this basic grid, they attempted to interpolate the pulsation frequencies and model parameters to produce a much finer grid with 40,000 points. In the end they found that the initial grid density was inadequate, leading to significant differences between the interpolated periods and those resulting from a complete evolutionary calculation with the same parameters.

We will use the genetic algorithm to explore δ-Scuti models over a broad range of masses (M) and chemical compositions (Y, Z), covering the full extent of the instability strip and allowing roughly 100 possible values for each parameter. With various assumptions about convective core overshooting (α_{ov}) and rotation (v_{rot})—or including them as adjustable parameters if warranted—the genetic algorithm method will effectively sample the parameters with 10-100 times greater density than any previous model-fitting attempt. It will do so without the necessity of calculating the complete grid, because it samples primarily those areas of the model space that it objectively finds to produce better fits to the observations.

Mode identification. The problem with grid density has been exacerbated by uncertainties in the identification of the spherical degree and azimuthal order (l, m) of the pulsation modes. The pattern of hot and cool regions on the surface of a non-radially pulsating star can be decomposed into spherical harmonic functions described by these two indices, leading to distinguishable patterns of radial velocity and light variation. The frequency of the variation also depends on the radial overtone (n), which is not directly observable. The excitation mechanism for the pulsations observed in δ-Scuti stars is not well understood; only a small fraction of the pulsation modes that are theoretically possible appear to be excited to detectable amplitudes. This creates some difficulty for a unique interpretation of the observed frequencies, a problem which has only recently started to be resolved using multi-color photometric techniques (Dupret et al., 2003). As space-based data start to become available, allowing the reliable detection of more low amplitude pulsation modes, this issue is likely to be less of a problem. For example, recent WIRE observations of δ² Tauri (Poretti et al., 2002) nearly tripled the number of pulsation modes detected in this star, relative to earlier multi-site observations from the ground.

We will investigate the feasibility of incorporating mode identification directly into the model-fitting procedure, whenever it has not been determined independently from observations. The idea is to make no a priori assumptions about the (n, l, m) values in the absence of observational constraints, and just try to match the frequencies. Within the range of frequencies where pulsations are observed, the number of modes that are theoretically possible increases quickly with the value of l; rotation and magnetic fields split non-radial modes into 2l+1 components. So the goodness-of-fit measure must be weighted to correct for the mode density, greatly enhancing the fitness of models that match one of the observed frequencies with a radial mode (because there are few) and only slightly enhancing the fitness when an l=3 mode matches the observations (because there are many).

Convective core overshooting. Despite these difficulties, δ-Scuti stars are currently the most promising candidates for asteroseismology near the main-sequence, and objective global model-fitting to their pulsation frequencies promises to yield important insights into their interior structure and evolutionary history. For example, because some of these stars are slightly evolved, an exciting possibility is to use them as a direct test for the presence and degree of convective core overshooting, which is expected to leave a dramatic imprint on the pulsation spectrum. (Templeton et al., 2001) showed that changes to the convective core overshooting parameter in their models had a fundamentally different effect on the theoretical pulsation frequencies than any of the other parameters (M, Y, Z).

We will initially perform the model-fitting by considering only two possible values for the core overshooting parameter: α_{ov} = 0.0 (no overshooting), and α_{ov} = 0.2, the value inferred by (Ribas et al., 2000) from an analysis of eclipsing binary data. This will reveal the degree to which the frequencies can be fit better by a model that includes convective overshooting, and it may motivate additional fits that allow overshooting to be a fully adjustable parameter. This is analogous to the way we approached the sensitivity of white dwarf models to core composition: first allowing several discrete values to demonstrate the potential of the parameter to improve the fits (Metcalfe et al., 2000), and then following up with a full scale exploration leading to an estimate that was much more precise than was earlier thought possible (Metcalfe et al., 2001).
2.3. Solar-like Oscillations

Solar-like oscillations have now been detected in two main-sequence stars (α Cen A and B), several sub-giants (η Boo, Procyon, ζ Hyi), and several giants (ζ Hya, Arcturus, α UMa). The oscillation amplitudes and the frequency of maximum power in these stars agree reasonably well with our theoretical expectations. The field is progressing very rapidly, and there is good reason to believe that many new observations will become available in the next few years, particularly after the launch of COROT and Kepler. With this in mind, it will be a distinct advantage to have in place the analysis methods that can make sense of these data efficiently, leading us quickly to a deeper understanding of the solar oscillations in the context of similar pulsations in other stars.

Extending the method. After adapting the genetic algorithm fitting method to main-sequence models for the analysis of δ-Scuti stars, it will be straightforward to extend the method to solar-like pulsators. This primarily requires a redefinition of the range for each of the relevant parameters. We have experience making this kind of transition with white dwarf models—extending the genetic algorithm method from the helium-atmosphere (DB) to the hydrogen-atmosphere (DA) white dwarfs by simply changing the allowed temperature range and adding one adjustable parameter for the hydrogen layer mass. Though not trivial, the time-scale for this development was short compared to the time that it took to adapt the model to interface with the parallel genetic algorithm.

A simplifying circumstance for the analysis of solar-like oscillations, compared to δ-Scuti pulsations, is the relative ease of mode identification. The excitation mechanism for solar-like oscillations is convection near the surface, creating a broad envelope of power with a peak that scales with the acoustic cutoff frequency (Brown et al. [1991]). Within this envelope a large fraction of the theoretically possible pulsation modes are excited to detectable amplitudes, leading to readily identifiable patterns. Without any detailed modeling, these overall patterns (the so-called large and small separations, Δν and δν) immediately lead to an estimate of the mean density of the star and can indicate the presence of interior chemical gradients. But a full analysis must include a detailed comparison of the individual frequencies with theoretical models. With a sufficiently long time baseline, it should also be possible to resolve the rotationally split m-components of the non-radial (l ≥ 1) modes. The frequency separation between these components can yield information about the internal rotation rate, since each mode samples the interior in a slightly different manner.

Hare & Hound exercises. Before any new observations are available, we will perform theoretical investigations using synthetic data to document the results expected to emerge from the various space missions, and to benchmark and fine tune the optimization method with ‘hare & hound’ exercises to maximize its efficiency. When new observations become available that may benefit from the global exploration of models made possible by the genetic algorithm, we will use this powerful tool to extract the complete physical insight that the data can provide.

3. CALL FOR COLLABORATORS

The proposal outlined above will initially be supported for three years by an NSF Fellowship at the High Altitude Observatory, to begin in the fall of 2004. Comments, suggestions, and collaborators are welcome. Please contact Travis Metcalfe <travis@hao.ucar.edu> for more information about the current status of the project.

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