Structure of Thermal Quasifermion in the QCD/QED Medium

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In this paper we carried out a nonperturbative analysis of a thermal quasifermion in the chiral symmetric thermal QCD/QED medium by studying its self-energy function through the Dyson-Schwinger equation (DSE) with the HTL resummed improved ladder kernel.

Our analysis reveals several interesting results, some of which may force us to change the image of thermal quasifermions; (1) The thermal mass of a quasifermion begins to decrease as the strength of the coupling gets stronger and finally disappears in the strong coupling region, thus showing a property of a massless particle, and (2) its imaginary part (i.e., the decay width) persists to have $O(g^2 T \log(1/g))$ behavior. These results suggest that in the recently produced sQGP, the thermal mass of a quasiquark should vanish. Taking into account the largeness of the imaginary part, it seems very hard for a quark to exist as a quasiparticle in the sQGP phase.

Other important findings are as follows; (3) The collective plasmino mode disappears also in the strongly coupled system, and (4) there exists an ultrasoft third peak in the quasifermion spectral density at least in the weakly coupled QED/QCD plasma, indicating the existence of the ultrasoft fermionic mode.

I. INTRODUCTION

It is believed that the Relativistic Heavy Ion Collider (RHIC) at BNL and the Large Hadron Collider (LHC) at CERN have produced the primordial state of matter, namely the quark-gluon plasma (QGP), and liberated the quark and gluon degrees of freedom. Subsequent analyses have shown that the produced QGP medium shows the property close to that of a perfect fluid. This fact leads us to the understanding that the QGP produced in the energy region of the RHIC and LHC is a strongly interacting system of quarks and gluons, namely, the strongly coupled QGP (sQGP) [1].

Since the discovery of sQGP phase, the behavior and the properties of quasiquark in the new sQGP phase have attracted much attention; does the quark still work as the basic degree of freedom in the new phase or not? It is also pointed out theoretically that the hadronic excitation affects the spectral density of quasiquark even in the chiral symmetric phase, thus showing some characteristic structures near the phase boundary [2].

Up to now, most of the theoretical findings on thermal quasiquarks in the QGP are obtained through analyses with the assumption of weakly coupled QGP at high temperature, i.e., analyses through the hard-thermal-loop (HTL) resummed effective perturbation calculation [3], or those through the one-loop calculation with the massive bosonic mode, or by replacing the thermal gluon with the massive vector boson [4]. Kitazawa et al. [5] have pointed out the three-peak structure of the quasifermion spectral density and the existence of the massless third mode. Such analyses, however, cannot be justified in studying the thermal quasiparticle in the sQGP created in the energy region of RHIC. What we need is the nonperturbative analysis to explore the properties of strongly coupled system.

Nonperturbative calculations of correlators within lattice QCD are performed in Euclidean space and give interesting results [6]. However, strictly speaking it is not possible to carry out an analytic continuation that is necessary to determine the spectral function. In addition, it is difficult on the lattice to respect the chiral symmetry that should be restored in the sQGP phase, though we are interested in the property of thermal quasiparticles in the chiral symmetric sQGP phase.

In this paper we perform a nonperturbative analysis of a thermal quasifermion in thermal QCD/QED by studying its self-energy function through the Dyson-Schwinger equation (DSE) with the HTL resummed improved ladder kernel. Our analysis may overcome the problems in the previous analyses listed above for the following reasons: (1) it is a nonperturbative QCD/QED analysis, (2) we study the DSE in the real-time formalism of thermal field theory, which is suitable for the direct calculation of the propagator, or the spectral function, (3) we use the HTL resummed thermal gauge boson (gluon/photon) propagator as an interaction kernel of the DSE, and take into account the quasiparticle decay processes by accurately studying the imaginary part of the self-energy function, and finally, (4) we present an analysis based on the DSE that respects the chiral symmetry and describes its dynamical breaking and restoration. Our analysis is nothing but an application of our formalism employing the DSE to the study of thermal quasifermions on the strongly coupled QCD/QED medium with chiral symmetry [7].

With the solution of the DSE with the HTL resummed improved ladder kernel, we study the properties of the thermal quasifermion spectral density and its peak structure, the dispersion law of the physical modes corresponding to the poles of thermal quasifermion propagator, through which we elucidate the properties of thermal mass and the decay width of fermion and plasmino modes and also pay attention to properties of the possible third mode, both especially in the sQGP phase.

Analogous studies employing the DSE are carried out
by several groups [8]. All these analyses solve the DSE in the imaginary-time formalism, and try to perform an analytic continuation. Harada et al. study the DSE with a ladder kernel in which the tree-level gauge boson propagator is used, while Qin et al. and Mueller et al. use the maximum entropy method to compute the quark spectral density. Qin et al. also paid a special attention to the massless third mode.

Our analysis reveals several interesting results, some of which may force us to change the image of thermal quasifermions: (1) The thermal mass of a quasifermion begins to decrease as the strength of the coupling gets stronger and finally disappears in the strong coupling region, thus showing a property of a massless particle, and (2) its imaginary part (i.e., the decay width) persists to have $O(g^2T \log(1/g))$ behavior. These results suggest that in the recently produced sQGP, the thermal mass of a quasiquark should vanish. Taking into account the largeness of the imaginary part (i.e., the decay width), it seems very hard for a quark to exist as a quasiparticle in the sQGP phase.

Focusing on the fact (1) above, we have already reported briefly in Ref. [9], and in the present paper we give results of detailed analysis. The fact (4) has also been pointed out briefly in Ref. [9], and will be studied further in detail in a separate paper [10].

Other important findings are as follows: (3) The collective plasmino mode disappears also in the strongly coupled system, and (4) there exists an ultrasoft third peak in the quasifermion spectral density at least in the weakly coupled QED/QCD plasma, indicating the existence of the ultrasoft fermionic mode.

This paper is organized as follows: In Sec. II we present the HTL resummed improved ladder Dyson-Schwinger equation for the quasifermion self-energy function, with which we investigate the property of thermal quasifermion in the chirally symmetric QGP phase, and give the results in Sec. III. In Sec. IIIA properties of the quasifermion spectral density is studied, and in Sec. IIIB we discuss the problem in the relation between the peak-position of spectral density and the zero-point of the inverse quasifermion propagator. The dispersion law of the quasifermion is studied in Sec. IIIC and the vanishment of thermal mass and the disappearance of the plasmino mode in the strongly coupled system are pointed out. Properties of the thermal mass and the existence of the third peak or the ultrasoft mode are discussed in Sec. IIID and Sec. IIIE respectively. Finally in Sec. IIIF properties of the decay width of quasifermion is studied. Summary of the paper and discussion is given in Sec. IV. Several appendices are also given. In Appendix A we explain the approximations to get the HTL resummed improved ladder DSE to be solved. The cutoff dependence of our analysis is discussed in Appendix B, and the phase boundary between the chirally symmetric and broken phases in the Landau gauge is briefly explained in Appendix C. Finally Appendix D is devoted to explain why we do not use the peak position of the spectral density as the condition to determine the particle onshellness.

II. THE HTL RESUMMED IMPROVED LADDER DYSON-SCHWINGER EQUATION

In this paper we study the thermal QCD/QED in the real time closed time-path formalism [11], and solve the Dyson-Schwinger equation (DSE) for the retarded fermion self-energy function $\Sigma_R$, to investigate the property of thermal quasifermion in the chiral symmetric QGP phase. Throughout this paper we study the massless QCD/QED in the Landau gauge.

As is well-known, at zero-temperature the Landau gauge plays an essential role to ensure the gauge invariance of the solution of the ladder DSE because it is proved that in the Landau gauge the wave-function receives no renormalization, i.e., $A(P) = 1$ [12, 13]. At finite temperature, however, $A(P) \neq 1$ even in the Landau gauge, and there is no special reason to choose Landau gauge any more. In this analysis we choose the Landau gauge firstly for the sake of simplicity, and secondly for the sake of comparison with other works.

In this section we present the HTL resummed DSE for $\Sigma_R$, and also give an explication about the improved ladder approximation we make use of to the HTL resummed gauge boson propagator. We also calculate the effective potential for the retarded fermion propagator $S_R$ in order to find the “true solution” when we get several “solutions” of the DSE.

A. HTL resummed improved ladder DSE for fermion self-energy function $\Sigma_R$

The retarded quasifermion propagator $S_R(P), P = (p_0, \mathbf{p})$, is expressed by

$$S_R(P) = \frac{1}{P + i\epsilon\gamma^0 - \Sigma_R(P)}. \quad (1)$$

The retarded fermion self-energy function $\Sigma_R$ can be tensor-decomposed in a chiral symmetric phase at finite temperature as follows:

$$\Sigma_R(P) = (1 - A(P))p_i\gamma^i - B(P)\gamma^0. \quad (2)$$

$A(P)$ is the inverse of the fermion wave function renormalization function, and $B(P)$ is the chiral invariant mass function. The c-number mass function does not appear in the chiral symmetric phase.

In the real time closed time-path formalism, by adopting the tree vertex and the HTL resummed gauge boson propagator for the interaction kernel of the DSE, we obtain, in the massless thermal QED/QCD, the HTL resummed improved ladder DSE for retarded fermion self-energy function $\Sigma_R$ [6, 14] (coupling $\alpha \equiv g^2/4\pi$ :
\[ g^2 = g_s^2 C_f \] for QCD, \( g = e \) for QED

\[ -i \Sigma_R(P) = -\frac{g^2}{2} \int \frac{d^4K}{(2\pi)^4} \times \left[ \star \Gamma_R^{\mu\nu}(-P, K, P - K) S_{RA}(-K, K) \right. \]
\[ \times \Gamma_R^{\mu\nu}(-K, P, K - P) \star \Gamma_R^{\mu\nu}(-P, P - K) + \Gamma_R^{\mu\nu}(-P, K, P - K) S_{RR}(-K, K) \]
\[ \times \Gamma_{AAR}(-K, P, K - P) \star \Gamma_{AAR}(-P, P - K) \right] . \]

Here \( \star \Gamma_{\mu\nu} \) is the HTL resummed gauge boson propagator \[15, 16\] where \( R \equiv RA \) and \( C \equiv RR \) denote the retarded and the correlation components, respectively, and \( \star \Gamma_{\mu} = \gamma_{\mu} \) in the present approximation.

There are many attempt to carry out the higher order calculation within the HTL resummed effective perturbation theory and to get information beyond the applicability region of the HTL approximation \[3, 17, 18\]. The DSE with the HTL resummed gauge boson propagator as an interaction kernel can take the dominant effects of thermal fluctuation of \( O(gT) \) into account nonperturbatively. Thus we expect the HTL resummed improved ladder DSE to enable us to study wider regions of the couplings and temperatures, e.g., the strongly coupled QGP medium, than those restricted by the HTL approximation, i.e., the regions of weak couplings and high temperatures.

The explicit expression of the HTL resummed improved ladder DSE in the Landau gauge to determine the scalar invariants \( A(P) \) and \( B(P) \) in the chiral symmetric QGP phase becomes coupled integral equations as follows;

\[ p^2[1 - A(P)] = g^2 \int \frac{d^4K}{(2\pi)^4} \left[ 1 + 2n_B(p_0 - k_0) \right] \text{Im} [\star \Gamma_R^{\rho\sigma}(P - K)] \times \]
\[ \left\{ \{ K_\sigma P_\rho + K_\rho P_\sigma - p_0 (K_\sigma g_{\rho\sigma} + K_\rho g_{\sigma\rho}) - k_0 (P_\sigma g_{\rho\sigma} + P_\rho g_{\sigma\rho}) + pk_z g_{\rho\sigma} \right. \]
\[ + 2p_0 k_0 g_{\rho\sigma} g_{\rho\sigma} \} \frac{A(K)}{[k_0 + B(K)] + i\epsilon^2 - A(K)^2 k^2} + \{ P_\sigma g_{\rho\sigma} + P_\rho g_{\sigma\rho} \}
\[ - 2p_0 k_0 g_{\rho\sigma} g_{\rho\sigma} \} \frac{k_0 + B(K)}{[k_0 + B(K)] + i\epsilon^2 - A(K)^2 k^2} \right] , \]

\[ B(P) = g^2 \int \frac{d^4K}{(2\pi)^4} \left[ 1 + 2n_B(p_0 - k_0) \right] \text{Im} [\star \Gamma_R^{\rho\sigma}(P - K)] \times \]
\[ \left\{ \{ K_\sigma g_{\rho\sigma} + K_\rho g_{\sigma\rho} - 2k_0 g_{\rho\sigma} g_{\rho\sigma} \} \frac{A(K)}{[k_0 + B(K)] + i\epsilon^2 - A(K)^2 k^2} \right. \]
\[ + 2g_{\rho\sigma} g_{\rho\sigma} - g_{\sigma\rho} \} \frac{k_0 + B(K)}{[k_0 + B(K)] + i\epsilon^2 - A(K)^2 k^2} \right] + \{ 1 - 2n_F(k_0) \} \times \]
\[ \star \Gamma_R^{\rho\sigma}(P - K) \text{Im} \left\{ \{ K_\sigma g_{\rho\sigma} + K_\rho g_{\sigma\rho} \} \frac{A(K)}{[k_0 + B(K)] + i\epsilon^2 - A(K)^2 k^2} \left( k_0 + B(K) \right) \right. \]
\[ - 2k_0 g_{\rho\sigma} g_{\rho\sigma} \} + \{ 2g_{\rho\sigma} g_{\rho\sigma} - g_{\sigma\rho} \} \frac{k_0 + B(K)}{[k_0 + B(K)] + i\epsilon^2 - A(K)^2 k^2} \right] , \]

where \( n_B(x) \) and \( n_F(x) \) are the Bose-Einstein and the Fermi-Dirac equilibrium distribution functions, respectively,

\[ n_B(x) = \frac{1}{\exp(x/T) - 1}, \quad n_F(x) = \frac{1}{\exp(x/T) + 1}. \]

The above DSEs, Eq. (3), are still very tough to be attacked, and need further approximations to be solved. We thus adopt the instantaneous exchange approximation to the longitudinal gauge boson propagator, i.e., we
set the zero-th component of the longitudinal gauge boson momentum \( q_0 \) to zero. Details of the approximation we use are explained in Appendix A.

In solving the DSEs, Eq. (4), we are forced to introduce a momentum cutoff in the integration over the four-momentum \( \int d^4K, K = (k_0, \mathbf{k}) \). We use the following cutoff method (\( \Lambda \) denotes an arbitrary cutoff parameter and plays a role to scale any dimensionful quantity, e.g., \( T = 0.3 \) means \( T = 0.3 \Lambda \));

| three momentum \( k \) : \( k = |\mathbf{k}| \leq \Lambda \) | energy \( k_0 \) : \( |k_0| \leq \Lambda_0 \) (\( \Lambda_0 = \Lambda \sim 5 \Lambda \)) |

We determine the parameter \( \Lambda_0 \) so as to get a stable solution for the fermion spectral density. Over the range of the temperature and the coupling we study in the present analysis, we set \( \Lambda_0 = 2 \Lambda \). In fact solution is totally stable for \( \Lambda_0 \geq 2 \Lambda \). In Appendix B we give details of the cutoff dependence of the solution.

### III. Solution of the HTL Resummed Improved Ladder Dyson-Schwinger Equation

In this section we give the solution of the HTL resummed improved ladder DSE for the retarded self-energy function \( \Sigma_R \), and study its consequences in the chiral symmetric phase. Special attention is paid to the consequences in the strongly coupled QCD/QED medium, in order to get an information on the thermal quasifermion in the strongly coupled QGP (sQGP) phase, recently discovered through the experiments at RHIC and LHC [1]. Part of the results are already briefly reported [2].

#### A. Quasifermion Spectral Density

1. Spectral density of quasifermion \( \rho_\pm \)

In the chiral symmetric QCD/QED phase, the quasifermion propagator can be expressed as

\[
S_R(p) = \frac{1}{2} \left[ \frac{1}{D_+} \left( \gamma^0 + \frac{p_0 \gamma^i}{p} \right) + \frac{1}{D_-} \left( \gamma^0 - \frac{p_0 \gamma^i}{p} \right) \right]
\]

where

\[
D_\pm(p) = p_0 + B(p_0, p) \mp pA(p_0, p)
\]

#### B. The Effective Potential \( V[S_R] \) for the Retarded Full Fermion Propagator \( S_R \)

Above DSEs, Eq. (4), may have several solutions, and we choose the “true” solution by evaluating the effective potential \( V[S_R] \) for the fermion propagator function \( S_R \), then finding the lowest energy solution. The effective potential is expressed as [19]

\[
V[S_R] = \text{Tr}[\mathcal{P}S_R] + \text{Tr} \ln[S_R^{-1}] + \frac{g^2}{2} \int \frac{d^4K}{(2\pi)^4} \int \frac{d^4P}{(2\pi)^4} \frac{1}{2} \text{tr} \left[ \gamma_\mu S_R(K) \gamma_\nu S_R(P) G^{\mu\nu}_C(P - K) + \gamma_\mu S_C(K) \gamma_\nu S_R(P) G^{\mu\nu}_R(P - K) + \gamma_\mu S_R(K) \gamma_\nu S_C(P) G^{\mu\nu}_A(P - K) \right],
\]

where the 1st and the 2nd terms correspond to the one-loop effective potential, while the 3rd term corresponds to the two-loop effective contribution.

The spectral density of quasifermion \( \rho_\pm \) is defined by

\[
\rho_\pm(p_0, p) = -\frac{1}{\pi} \text{Im} \frac{1}{D_\pm(p)} = -\frac{1}{\pi} \text{Im} \frac{1}{p_0 + B(p_0, p) \mp pA(p_0, p)},
\]

which satisfies the sum rules [20]

\[
\begin{align*}
\int_{-\infty}^{\infty} dp_0 \rho_+(p_0, p) &= 1, \\
\int_{-\infty}^{\infty} dp_0 \rho_-(p_0, p) &= \pm p, \\
\int_{-\infty}^{\infty} dp_0 p_0^2 \rho_\pm(p_0, p) &= p^2 + m_f^2.
\end{align*}
\]

It also satisfies the symmetry property

\[
\rho_\pm(p_0, p) = \rho_\mp(-p_0, p).
\]

We therefore study only \( \rho_+(p_0, p) \) throughout this paper.

To study the spectral properties of quasifermion in the chiral symmetric QGP phase, we must at first make sure that we are in fact studying inside the chiral symmetric phase. We have already studied the phase structure of thermal QCD/QED through the same DSE approach [2], and determined the phase boundary between the chiral symmetric and the chiral symmetry broken phases. Thus we are sure that the region of temperatures and couplings...
we study in the present paper are well within the chiral symmetric phase. For details, see Appendix C where we give the results of our analyses to determine the phase boundary.

It is to be noted that the temperature $T$ in the present analysis represents the temperature scaled by the momentum cutoff $\Lambda$, thus $0 \leq T \leq 1$, and it does not denote the real temperature itself. With the temperature thus defined, we can determine the phase boundary curve in the two-dimensional $(\alpha, T)$-plane, separating the chiral symmetry broken/restored phase \[6\]. If we measure the temperature $T$ relative to the critical temperature $T_c$, high or low temperature has a definite meaning.

2. Structure of quasifermion spectral density

Now let us present the properties of quasifermion spectral density $\rho_+$ computed by the solution of DSEs, Eq. (4). At first we should note the fact that the quasifermion spectral density $\rho_+$ thus determined well satisfies the sum rules Eqs. (10a) and (10b) withing in a few present error. This fact proves a posteriori the adequacy of our choice of cutoff parameter $\Lambda_0 = 2\Lambda$ in the region of the couplings and the temperatures we study. The third sum rule is heavily depends on the HTL calculation, and the agreement depends on the couplings and the temperatures.

Then let us show the structure of $\rho_+(p_0, p)$ as a function of $p_0$ and $p$ in the two-dimensional $(p_0, p)$-plane. For convenience we study the dimensionless quantity $\rho_+(p_0, p)m_f^*$, where $m_f^*$ denotes the thermal mass determined through the next-to-leading order calculation of the HTL resummed effective perturbation theory \[21\],

$$\left(\frac{m_f^*}{m_f}\right)^2 = 1 - \frac{4g}{\pi} \left[\frac{g^2}{2\rho} + \frac{1}{3}\right], \quad (12)$$

$$m_f^2 = \frac{g^2T^2}{8}. $$

In measuring at moderately high temperature $T = 0.4$, we can see in Fig. 1 the three typical peak-structures depending on the strength of the coupling $\alpha = g^2/4\pi$;

i) At weak coupling $\alpha = 0.005$ we can see three peaks as a function of $p_0$ at $p = 0$. Two sharp peaks of them at positive and negative $p_0$ represent the fermion and the collective plasmino modes, respectively \[15\], and the slight third “peak” barely recognizable around $p_0 = 0$ corresponds to the massless, or the ultrasoft mode \[4\]. The plasma mode and the massless mode rapidly decrease and disappear as the size of momentum $p$ becomes large.

ii) At the intermediate strength $\alpha = 0.2$, we can see only two peaks at $p_0 \neq 0$ as a function of $p_0$ even at $p = 0$, and unable to recognize the existence of the third peak corresponding to the massless pole in this region of the coupling. The peak at the negative side of $p_0$ axis that may corresponds to the collective plasmino pole rapidly disappears as $p$ gets large.

iii) At the strong coupling $\alpha = 1.0$ we can only recognize, at any size of the momentum $p$, the existence of a broad “peak” that may represent the massless pole. No massive pole exists in the strong coupling region.

It should be noted the vast differences of the height of the peak and the spread of the spectral density in the three cases of the coupling strength i), ii) and iii), clearly showing the broadness of the “peak” in the strong coupling environment.

Figure 1 shows the coupling $\alpha$-dependence of the quasifermion spectral density $\rho_+m_f^*$ at fixed temperature $T = 0.3$. We can also see the temperature $T$-dependence, which is given in Fig. 2.

In Fig. 1 as noted above, we see the transition of the peak structure of spectral density as the strength of the coupling varies with the temperature kept fixed: triple peaks at small couplings, double peaks at intermediate couplings and finally single peak at strong couplings.

Figure 2 shows that the analogous behavior is also observed when the temperature of the environment varies with the strength of the coupling kept fixed. At small couplings ($\alpha = 0.01$), the three-peak structure at low temperature ($T = 0.2$) changes to the double-peak structure at high temperature ($T = 0.8$), and at intermediate couplings ($\alpha = 0.45$), the double-peak structure at low temperature ($T = 0.2$) tends to the single-peak structure at high temperature ($T = 0.8$).

Here let us see more carefully the structure of spectral density at $p = 0$, $\rho_+(p_0, p = 0)$, as a function of $p_0$. The $p_0$-coordinate of the peak position of $\rho_+(p_0, p = 0)$ will give the mass of the corresponding mode. In Fig. 3 shown are the spectral densities, $\rho_+(p_0, p = 0)$, at moderately high temperature $T = 0.3$, in the weak coupling region $\alpha \leq 0.01$, in the region of intermediate coupling strength $\alpha \approx 0.1 \sim 0.2$, and in the strong coupling region $\alpha \sim 1$.

In order to see what actually happens during the transition from the triple peak structure in the weak coupling region to the double peak one in the region of intermediate coupling strength, and finally to the single peak one at strong couplings, we present in Figs. 4 and 5 the Re$[D_+(p_0, p = 0)]$ and Im$[B(p_0, p = 0)]$ at $T = 0.3$, respectively, both of which show three curves corresponding to the three regions of the coupling as in Fig. 3.

At weak coupling $\alpha = 0.005$, we can clearly see in Fig. 3 two sharp peaks at positive and negative $p_0$, representing the quasifermion and the plasmino poles. The center-positions of both of peaks are in fact at $p_0/m_f^* \sim \pm 1$, which in fact almost coincide with the solution of the on-shell condition Re$[D_+(p_0, p = 0)] = 0$, as can be easily seen in Fig. 4. Thus at the weak coupling and high temperature both the quasifermion and the plasmino modes have a common thermal mass $m_f^*$, Eq. 12 which is, as
already noted, determined through the next-to-leading order calculation of HTL resummed effective perturbation theory \[2, 21\].

We can also barely recognize the existence of a slight "peak" around \( p_0 = 0 \), corresponding to a massless pole. The existence of this massless mode is also indicated by the onshell condition \( \text{Re}[D_+(p_0, p = 0)] = 0 \), in which \( p_0 = 0 \) is always a solution. The problem on this third "peak" will be discussed in the Sec. III E.

It is also worth noticing that at weak coupling \( \alpha = 0.005 \) \( \text{Im}[B(p_0, p = 0)] \) shows a three peak structure; a sharp steep peak centered at \( p_0 = 0 \), and two slight peaks centered at \( |p_0| = m_f^* \) \[23\]. All three peaks in \( \text{Im}[B] \) appear corresponding to the onshell point \( \text{Re}[D_+(p_0, p = 0)] = 0 \), as were the case in the peaks of the spectral density, the sharpness of the peaks being completely turned over.

At intermediate coupling \( \alpha = 0.2 \), the fermion spectral density exhibits a typical double peak structure. The existence of these peaks, however, is not easy to understand. As we can easily make sure by comparing Fig. 3 and Fig. 4, they do not have exact correspondence to the poles of the fermion propagator, i.e., the zero-point of the inverse propagator \( \text{Re}[D_+(p_0, p = 0)] = 0 \).

At strong coupling \( \alpha = 1.0 \), the situation becomes very simple. There exists only a broad single peak, whose existence is indicated by the onshell condition \( \text{Re}[D_+(p_0, p = 0)] = 0 \), in which \( p_0 = 0 \) is the only
solution in the strong coupling region, see Fig. It is not clear whether the massless peak at strong couplings is exactly the same one at weak couplings noted above, or not. We will discuss on this massless mode in Sec. III B.

To understand the typical structure of the fermion spectral density explained above, it is always important to correctly take notice of the height of the peak and the width of the corresponding pole, namely the height of the peak and the width of $\text{Im}[B(p_0)]$, given in Fig. 5 in connection with the solution of the onshell condition $\text{Re}[D_+(p_0, p = 0)] = 0$.

In this sense the appearance of the double-peak structure at intermediate couplings is somewhat confusing. It is because, while there is a clear correspondence between the triple peaks at small couplings and the physical poles or modes (i.e., the quasifermion, the plasmino and the massless or ultrasoft modes), the double peaks at intermediate couplings do not have an obvious correspondence to the physical poles or modes. They may correspond to the quasi-fermion and the plasmino modes, but the center-positions of the peaks are apparently bigger than expected from the value of thermal mass, see, Fig. 5.

In addition, as explained above, the third peak corresponding to the massless or the ultrasoft mode can not be recognized at all. This problem might arise from the broad-peak structure of the imaginary part of the mass function, $\text{Im}[B(p_0, p)]$, centered at $p_0 = 0$, see, Fig. 3 and will be discussed later in Sec III B, Sec. III C and Appendix D.

With the appearance of this problem, it seems better to determine the position of the quasi-fermion pole by the solution of the onshell condition $\text{Re}[D_+(p_0, p)] = 0$ than to determine by the peak-position of the spectral density.

**B. What determines the peak-position of spectral density? Or, the relation between the peak-position of spectral density $\rho_+(P)$ and the zero-point of the inverse propagator $\text{Re}[D_+(P)]$**

Here we study the problem: What determines the peak-position of spectral density? At the end of the last Sec III A 2 we have briefly commented on the problem by focusing on the relation between the peak-position of spectral density $\rho_+(P)$ and the zero-point of the inverse propagator $\text{Re}[D_+(P)]$. There we also noticed that we should correctly take into account the information on the $\text{Im}[B(P)]$.

Let us summarize what we have disclosed. a) At weak couplings there are two sharp peaks located at $p_0/m_f^* \sim \pm 1$, which in fact almost coincide with the solution of the onshell condition $\text{Re}[D_+(p_0, p = 0)] = 0$. The third slight “peak” around $p_0 = 0$ is also indicated by the onshell condition $\text{Re}[D_+(p_0, p = 0)] = 0$, in which $p_0 = 0$ is always a solution. b) Typical double peaks at intermediate strength of coupling do not have an obvious correspondence to the zero-point of the inverse propaga-
tor $\text{Re}[D_+(p_0, p = 0)] = 0$. c) At strong couplings, there exists only one broad “peak”, whose existence is indicated by the onshell condition $\text{Re}[D_+(p_0, p = 0)] = 0$, in which $p_0 = 0$ is the only solution in the strong coupling region.

There appear two questions. 1) What happens at intermediate strength of coupling? 2) What causes the huge difference of the peak height between sharp peaks of fermion and plasmino modes and the slight peak of massless mode at weak couplings?

We can add one more question: Does the massless peak (or the pole) at strong couplings represent the same massless mode at weak couplings? This third question, however, will be discussed in a separate paper.

Now let us study questions 1) and 2) in order.

On the question 1): First let us see the solution of the onshell condition $\text{Re}[D_+(p_0, p = 0)] = 0$. In the range of intermediate couplings around $\alpha \sim 0.2$ (temperature is fixed at $T = 0.3$), the real part of the chiral symmetric mass function at $p = 0$, $\text{Re}[B(p_0, p = 0)]$, as a function of $p_0$ exhibits a subtle structure around the origin. In studying the small $p_0$ region, it has a steep valley/peak structure at weak couplings, but as the coupling becomes stronger this valley/peak structure eventually diminishes in size and begins to behave almost as a straight line.

Intermediate coupling region is the transition region; As the coupling gets stronger the two solutions of the onshell condition $\vert p_0 \vert \neq 0$ eventually approach to $p_0 = 0$ and coincide with the solution at $p_0 = 0$ that always exists irrespective of the strength of the coupling. Thus the number of solutions of the onshell condition $\text{Re}[D_+(p_0, p = 0)] = 0$ changes suddenly from three to one.

We should check here, in the considered region at $T = 0.3$ with couplings around $\alpha = 0.2$ and stronger, where the real part of the inverse propagator vanishes, i.e., where the solutions of $\text{Re}[D_+(p_0, p = 0)] = 0$ exist. There are three solutions, two of them sit at $p_0 \neq 0$, i.e., $\vert p_0 \vert \approx 0.6$ and the third one at $p_0 = 0$, see Fig. 4 thus indicating the existence of three poles, or the appearance of three peaks in the spectral density.

We should then see the shape and the position of the peak of $\text{Im}[B(p_0, p = 0)]$. $\text{Im}[B(p_0, p = 0)]$ always has a single broad peak around $p_0 = 0$ in the corresponding region of temperatures and couplings, i.e., $T = 0.3$ and couplings around $\alpha = 0.2$ and stronger, see Fig. 4.

With these fact we understand that, at intermediate coupling $\alpha = 0.2$, the peak structure of $\text{Im}[B(p_0, p = 0)]$ plays an important role, scratching out (washing away) the peak of the spectral density at $p_0 = 0$, and the not-so-steep but still gaussian decreasing structure of $\text{Im}[B(p_0, p = 0)]$ makes the positions of the peaks of the spectral density at $\vert p_0 \vert \approx 0.6$ shift to larger $\vert p_0 \vert$ values.

On the question 2): To understand this question, let us see Figs. 5, 6 and 7 in the weak coupling region. The spectral density exhibits two sharp peaks at $p_0 \sim \pm m_f^*$, and one barely slight peak at $p_0 = 0$. The positions of these three peaks exactly agree with the three solutions of the onshell condition $\text{Re}[D_+(p_0, p = 0)] = 0$ at $p_0 \sim \pm m_f^*$ and at $p_0 = 0$, thus these three mode rigidly correspond to the fermion, plasmino and massless modes, respectively 4 12 16 24.

In contrast with the spectral density, the structure of $\text{Im}[B(p_0, p = 0)]$ (= $\text{Im}[D_+(p_0, p = 0)]$) is simple. $\text{Im}[B(p_0, p = 0)]$ at weak coupling $\alpha = 0.005$, as can be seen in Fig. 5, exhibits a sharp peak at $p_0 = 0$ and two slight peaks at $\vert p_0 \vert / m_f^* \approx 1$. These peaks have clear correspondence to the three solutions of the onshell condition $\text{Re}[D_+(p_0, p = 0)] = 0$. At the positions of two sharp peaks in the spectral density it is essential that $\text{Re}[D_+]$ is zero, and the imaginary part of it, or $\text{Im}[B]$, is so small that it does not play any essential role in the structure of the spectral density. At the position of massless pole $p_0 = 0$, however, $\text{Im}[B]$ is so large that it plays an important role to almost scratch out the fact that $\text{Re}[D_+]$ is zero, thus the peak structure almost disappears at around the origin.

The peak height at the pole is determined by its pole-residue. This fact means that, by measuring the ratio of peak-heights between the sharp peak representing the quasifermion mode and the slight peak at the origin representing the massless or ultrasoft mode, we can determine the ratio between the corresponding pole-residues. This analysis will be carried out in a separate paper 10.

C. The quasifermion pole and quasifermion dispersion law

1. How to define the quasifermion pole

Generally speaking, the pole of the propagator or the point where the propagator inverse vanishes defines the corresponding particle and its dispersion law. In the case of the thermal quasi-particle, however, its mass term usually has finite, not small but in most occasion quite large imaginary part, namely, the pole position of thermal quasiparticle sits deeply inside the complex $p_0$ plane.

Because it is not very simple to study the structure of such a pole sitting deeply inside the complex $p_0$ plane, we usually study such a pole by defining the condition so that the real part of propagator inverse vanishes as the onshell condition. We adopt this definition of onshell throughout this analysis, then the quasifermion pole is defined by the zero-point of the real part of chiral invariant fermion propagator inverse $\text{Re}[D_\pm(p_0, p)] = D_\pm(p_0, p)]$, 

$$\text{Re}[D_\pm(p_0, p)] = 0 \quad \text{at} \quad (p_0 = \omega_\pm, p), \quad (13)$$

which determines the dispersion law of this pole, $\omega_\pm(p)$.

There is of course another definition of onshell and its corresponding pole. One of such definition is to use the peak position of spectral density as the pole position of the corresponding particle. With this definition we can also determine the dispersion law of this pole, $\omega'_\pm(p)$. Though in most cases these two definitions give the same results, i.e., the dispersion law determined through the
onshell condition $\text{Re}[D_{\pm}(p_0 = \omega_{\pm}, p)] = 0$ agrees with the one determined by the peak position of the spectral density, in some cases two definitions give different results. We have also discussed in the Sec. III A above, the possibility that the peak position of the spectral density in the region of intermediate coupling strength may not correctly represent the physical modes. On this problem we will discuss in Appendix D. Therefore, as mentioned above, we adopt Eq. (13) as the definition of onshell.

2. Quasifermion dispersion law in the weakly coupled QCD/QED medium and the fermion thermal mass

Now let us study the quasifermion dispersion law determined through the onshell condition, Eq. (13), i.e., $\text{Re}[D_{\pm}(p_0 = \omega_{\pm}, p)] = 0$. In Fig. 6 we give the quasifermion dispersion law $\omega = \omega_{\pm}(p)$ at small coupling and at moderately high temperature. It should be noted that, as can be seen in Fig. 6 in the region of weak coupling strength $\alpha \lesssim 0.01$ the dispersion law lies on a universal curve determined by the HTL calculations 15, 16. Thus the result shows a good agreement with the HTL resummed effective perturbation calculation.

The important point is that both the quasifermion energy $\omega_{+}(p)$ and the plasmino energy $\omega_{-}(p)$ approach to the same fixed value $m_f^*$, Eq. (12), as $p \to 0$, namely, in Fig. 6 the normalized energy $\omega_{+}(p) \equiv \omega_{\pm}(p)/m_f^*$ approach to 1 as $p \to 0$. This fact clearly shows that the quasifermion as well as the plasmino have a definite thermal mass $m_f^*$ of $O(gT)$ determined through the next-to-leading order calculation of HTL resummed effective perturbation theory 3, 21. We should also note that the collective plasmino mode exhibits a minimum at $p \neq 0$ and vanishes rapidly on to the light cone as $p$ gets large.

It is also worth noticing that at weak coupling and moderately high temperature, dispersion law in the small-$p$ region determined by the zero-point of $D_+$ agrees well and almost coincides with that determined by the peak of the spectral density $\rho_+$. This fact can be understood by the one already noted in Sec. III A 2 that in the weak coupling region the thermal mass of quasifermion determined by the peak position of spectral density almost coincides with the one determined by the solution of the onshell condition $\text{Re}[D_{\pm}(p_0 = 0)] = 0$, and that the thermal mass thus determined is $m_f^*$. The sharp peak structure of $\text{Im}[B(p_0, p = 0)]$, or the narrow width structure of the quasifermion pole in the corresponding region may guarantee this fact. As the momentum $p$ becomes large, however, there appears a discrepancy between them, especially in the dispersion law of the plasmino branch, see Appendix D.

FIG. 6: The normalized quasifermion dispersion law $\omega^*(p) \equiv \omega(p)/m_f^*$ at $T = 0.3$ in the small coupling region.

3. Vanishing of the thermal mass in the strongly coupled QCD/QED medium

Next let us study how the result shown in Fig. 6 changes as the coupling gets stronger, namely, in the region of intermediate to strong couplings. For this purpose, let us see carefully the fermion branch of the quasifermion dispersion law in the small momentum region. (N.B. Temperatures and couplings we are studying belong to the chiral symmetric phase.)

Figure 7 shows the $\alpha$-dependence of the normalized dispersion law at $T = 0.3$ as the coupling $\alpha$ becomes stronger, where the normalization scale is the next-to-leading order thermal mass $m_f^*$. In this Fig. 7 we can clearly see, though in the weak coupling region we get the solution in good agreement with the HTL resummed perturbation analyses, as the coupling becomes stronger from intermediate to strong coupling region the normalized thermal mass $\omega^*(p = 0) \equiv \omega_+(p = 0)/m_f^*$ begins to decrease from 1 and finally tends to zero ($\alpha \gtrsim 0.27$ in Fig. 7). Namely, in the thermal QCD/QED medium, thermal mass of quasifermion begins to decrease as the strength of coupling gets stronger and finally disappears in the strong coupling region. This fact strongly suggests that in the recently produced strongly coupled QGP, thermal mass of quasifermion should vanish or at least become significantly lighter compared to the value in the ideal weakly coupled QGP.

To see the above behavior of thermal mass more clearly, in Fig. 8 we show the normalized mass $\omega^*_f (p = 0)$ as a function of $\alpha$. In the small coupling region ($\alpha \lesssim 0.1$) and around the temperature $T = 0.1 \sim 0.2$, results of thermal mass agree well with those of the HTL resummed perturbation calculation. As the coupling gets stronger from intermediate to strong coupling regions, however,
the normalized thermal mass \( \omega^*_+(p = 0) \) begins to decrease from 1 and finally goes down to zero, i.e., the thermal mass vanishes.

FIG. 7: The \( \alpha \)-dependence of the normalized quasifermion dispersion law \( \omega^*(p) \equiv \omega(p)/m_f \) at \( T = 0.3 \), as the coupling becomes stronger.

FIG. 8: The \( \alpha \)-dependence of the normalized thermal mass \( \omega^*_+(p = 0) \), see Text.

Analogous behavior of thermal mass \( \omega^+(p = 0) \) appears in the temperature-dependence. As can be seen in Fig. 9 with any coupling \( \alpha \), thermal mass decreases from \( m_f^* \) as the temperature becomes higher, and finally at extreme high temperature \( \omega^+(p = 0) \) becomes zero, thus the thermal mass vanishes. Figure 9 shows another characteristic behavior as \( T \to \) small. Almost at any coupling the thermal mass decreases and finally tends to vanish as temperature becomes lower. This behavior is consistent with the fact that at zero-temperature the thermal mass must vanish. The unexpected behavior is that, as the coupling becomes stronger, the thermal mass \( \omega^+(p = 0) \) vanishes at low but non-zero finite temperature.

Here it is to be noted that the ratio \( \omega^+(p = 0)/T \) is not necessarily a constant and the \( T \)-dependence at not-so-high \( T \) observed in our analysis is a consequence of the nonperturbative DSE analysis. It is because the additional dimensionful parameter, such as the regularization (or the cutoff) scale or the renormalization scale comes into the theory through the regularization and/or the renormalization of massless thermal QCD/QED. In our case the cutoff scale \( \Lambda \) is introduced into the theory. The thermal mass, in fact, has a logarithmic \( T \)-dependence in the effective perturbation calculation, see, e.g., Rebhan’s lecture in Ref. [3].

The behavior of thermal mass is determined by the behavior of chiral invariant mass function \( \text{Re}[B(p_0, p)] \). In Fig. 10 we show, for the sake of convenience, the \( \alpha \)-dependence of \( \text{Re}[D_+(p_0, p = 0)] = \text{Re}[p_0 + B(p_0, p = 0)] \) at \( T = 0.3 \). At small coupling \( \text{Re}[D_+(p_0, p = 0)] \) has a steep valley/peak structure in the small \( p_0 \) region, but as the coupling becomes stronger this structure eventually disappears and \( \text{Re}[D_+(p_0, p = 0)] \) belongs to behavior almost as a straight line with a slope +1.

FIG. 9: The \( \alpha \)-dependence of the normalized thermal mass \( \omega^*_+(p = 0) \), see Text.

FIG. 10: The \( \alpha \)-dependence of the real part of the inverse fermion propagator at \( p = 0 \), \( \text{Re}[D_+(p_0, p = 0)] = \text{Re}[p_0 + B(p_0, p = 0)] \), at \( T = 0.3 \).]

Thermal mass is given by the solution of \( \text{Re}[D_+(p_0, p = 0)] = 0 \), i.e., \( p_0 \)-coordinate of intersection point of the drawn curve of \( \text{Re}[D_+(p_0, p = 0)] \) and the \( p_0 \) axis. At first
we can see with this figure that at small couplings there are three intersection points, the one with positive \( p_0 \), the one with negative \( p_0 \), and the one at the origin \( p_0 = 0 \), which correspond to the quasifermion, the plasmino and the massless (or, ultrasoft) modes \( p \), respectively.

As the coupling becomes stronger (\( \alpha \gtrsim 0.27 \) at \( T = 0.3 \)), however, the number of the intersection points suddenly reduces and there appears only one intersection point at \( p_0 = 0 \), which may correspond to the massless pole in the fermion propagator. Thus we can understand the behavior in Fig. \( \text{S} \) namely, in the weak coupling region \( \omega_+^\alpha(p = 0) \equiv \omega_-(p = 0)/m_f^\alpha \) is almost unity, and reduces to zero in the strong coupling region (\( \alpha \gtrsim 0.27 \) at \( T = 0.3 \)), showing that the fermion thermal mass vanishes completely in the corresponding strong coupling region.

4. Disappearance of the plasmino mode in strongly coupled QCD/QED medium

Finally we study what happens in the plasmino mode in the strongly coupled QCD/QED medium, by explicitly examining the plasmino branch of the dispersion law. In the last Sec. \( \text{III} \text{C} \text{3} \) where we see the thermal mass vanishes in the strong coupling region, we only studied the structure of the fermion branch of the dispersion law, and of the propagator inverse at \( p = 0 \), \( \text{Re}[D_+(p_0, p = 0)] = 0 \). We can not exactly see what happens in the plasmino mode without explicitly studying the plasmino branch of the dispersion law.

Figure \( \text{II} \) shows the \( \alpha \)-dependence of the normalized dispersion law of the plasmino branch at \( T = 0.3 \) as the coupling \( \alpha \) becomes stronger. (At weak couplings we already saw its structure in Fig. \( \text{S} \).)

Paying attention to the plasmino branch, we recognize that, as the coupling gets stronger, the valley structure of the plasmino dispersion law, or, the existence of the minimum in the plasmino dispersion law, observed in the weak coupling region, eventually disappears, and that the plasmino dispersion law gets to sharply dropping onto the light cone as the momentum \( p \) becomes large.

At \( T = 0.3 \) in the small coupling region \( \alpha \lesssim 0.01 \) the plasmino branch lies on the universal curve determined by the HTL calculation. Around \( \alpha \approx 0.02 \) the dispersion law of the plasmino branch begins to change its structure: firstly the behavior as \( p \rightarrow \) large begins to show sudden decrease onto the light cone, then secondly around \( \alpha \approx 0.05 \) the valley structure of the plasmino dispersion law eventually disappears and the plasmino dispersion law monotonically drops sharply onto the light-cone, and finally in the region \( \alpha \gtrsim 0.27 \) (at \( T = 0.3 \)) the plasmino branch totally disappears.

If the coupling gets further stronger, the thermal mass begins to decrease and eventually disappears at \( \alpha \gtrsim 0.27 \), as noted in Sec. \( \text{III} \text{C} \text{3} \) before. The plasmino branch disappears at \( \alpha \gtrsim 0.27 \) also, which we can see in Fig. \( \text{II} \) and the three modes, i.e., the fermion, the plasmino and the ultrasoft modes, finally merge and become a single massless mode that can be hardly detected as a real physical mode in the strongly coupled QGP, as noted before because of its large decay width.

D. Thermal mass of the quasifermion

In the last Sec. \( \text{III} \text{C} \) we have disclosed unexpected behavior of the thermal mass of quasifermion in the strong coupling QCD/QED, namely the fact that the thermal mass vanishes in the strongly coupled QCD/QED medium (or, the recently produced strongly coupled QGP). Also we have pointed out that at weak coupling and high temperature both the quasifermion and the plasmino have a common thermal mass \( m_f^\alpha \), Eq. (12), determined through the next-to-leading order calculation of HTL resummed effective perturbation theory [3, 21].

In this section we examine how accurately the thermal mass \( m_f^\alpha \), Eq. (12), can describe the thermal mass calculated in our analysis. Fig. \( \text{S} \) showing the coupling dependence of the thermal mass presented in the last section, covers a wide range of couplings and gives us only a rough image, thus is not suited to the present purpose.

Here we present Fig. \( \text{II} \) the rescaled version of Fig. \( \text{S} \) showing the thermal mass calculated in our analysis in the weak coupling region \( \alpha \lesssim 0.1 \). Now we can see clearly that in the whole region of the temperature \( 0.100 \leq T \lesssim 0.200 \) at weak couplings \( \alpha \lesssim 0.1 \), the normalized thermal mass \( \omega_+(p = 0) \equiv \omega_-(p = 0)/m_f^\alpha \) is almost unity, namely, the thermal mass calculated in our analysis, \( \omega_-(p = 0) \), is well described by \( m_f^\alpha \). As the temperature becomes higher, discrepancy becomes evident and larger; the normalized thermal mass \( \omega_+(p = 0) \)
becomes to deviate from unity and gets smaller, namely, \( \omega_+(p = 0) \) begins to decrease from \( m_f^* \) and becomes smaller.

It should be noted, however, while at very small couplings \( \alpha \lesssim 0.01 \) a common tendency can be recognized in Fig. 5 that the normalized thermal mass \( \omega_+(p = 0) \) approaches unity, except at extreme high temperatures.

In studying the temperature dependence of the thermal mass, which is shown in Fig. 6, another fact can be recognized. The first thing that attracts our attention is that, except in the small coupling region, the normalized thermal mass \( \omega_+(p = 0) \) shows a peak structure, namely that \( \omega_+(p = 0) \) decreases as both the temperature becomes higher and becomes lower. This fact, on the former case we already noticed in Sec. III C 3, is unexpected and not easy to be understood with the knowledge we have learned through the effective perturbation analyses. The behavior in the lower temperature region may indicate that the thermal mass shows a behavior proportional to \( T/\log(1/T) \), while in the high temperature region the thermal mass shows a behavior proportional to \( T \log(1/T) \). (N.B.: The temperature \( T \) varies in the range \( 0 \leq T \leq 1 \).)

As for the temperature dependence in the higher temperature region, only we can say definitely at present that the ratio \( \omega_+(p = 0)/T \) is not necessarily a constant and the \( T \)-dependence of the normalized thermal mass \( \omega_+(p = 0) \) observed in our analysis is a consequence of the nonperturbative DSE analysis.

At small couplings the normalized thermal mass \( \omega_+(p = 0) \) seems to approach unity as the temperature becomes lower, showing the well-known behavior of the thermal mass being proportional to the temperature \( T \), and is easy to be understood.

E. Existence of the third peak, or the ultrasoft mode

The quasifermion and the plasmino modes are well understood in the HTL resummed analyses, the latter being the collective mode to appear in the thermal environment. What is the third peak? Is it nothing but convincing evidence of the existence of a massless or an ultrasoft mode? Is there any signature in our analysis?

Existence of the massless or the ultrasoft fermionic mode has been suggested first in the one-loop calculation [4] when a fermion is coupled with a massive boson with mass \( m \). The spectral function of the fermion gets to have a massless peak in addition to the normal fermion and the plasmino peaks. Recently a possible existence of collective fermionic excitation in the ultrasonic energy-momentum region \( p \lesssim g^2 T \) is investigated analytically through perturbative calculation [4, 22]. Both of these analyses are confined to the weak coupling regime, and nothing is known what happens in the sQGP we are interested in. In this sense first we will study the structure of the third mode, i.e., of the massless or the ultrasoft mode, in the weakly coupled QCD/QED medium, then will proceed to the intermediate and strong coupling region to investigate how the ultrasoft mode behaves in such an environment [4].

Now let us study the structure of the third mode, i.e., of the massless or the ultrasoft mode, in the weakly coupled QCD/QED medium. First we give in Fig. 13(a) the structure of spectral density \( \rho_+(p_0, p = 0) \) at weak coupling region \( (\alpha = 0.001, T = 0.3) \). Two sharp peaks, representing the quasifermion and the plasmino poles are clearly seen, and the existence of a slight “peak” can also be recognized around \( p_0 = 0 \). To see more clearly, in Fig. 13(b) shown is rescaled version of Fig. 13(a), where we can clearly see the “peak” structure around \( p_0 = 0 \). This third peak is nothing but a convincing evidence of the existence of a massless or an ultrasoft mode [4, 9]. This peak is indistinguishably slight compared to the sharp quasifermion and plasmino peaks.

Here we should take notice of the fact that the peak-height of the ultrasoft mode centered at \( p_0 = 0 \) is, roughly speaking, \( O(q) \) lower than the peak-height of the normal fermion or the plasmino peak centered at \( p_0 = \pm m_f^* \). This result does not exactly agree with what Hidaka, Satow and Kunihiko have shown in their works [22] concerning the residue of the ultrasoft fermion mode, and we will perform more detailed analysis on this problem in a separate paper.

F. Decay width of the quasifermion, or the imaginary part of the chiral invariant mass function \( B \)

Finally let us study the decay width of the quasifermion or the imaginary part of the chiral invariant mass function \( \text{Im}[B(p_0, p)] \) at \( p = 0 \). The decay width of the quasifermion is extensively studied through the HTL resummed effective perturbation calculation [23], giving a gauge-invariant result of \( O(g^2 T \log(1/g)) \). However, as is shown above, the quasiparticle exhibits an unexpected behavior, such as the vanishing of the thermal mass in the
strongly coupled QCD/QED medium, completely different from that expected from the HTL resummed effective perturbation analyses. How does the decay width of the quasifermion exhibit its property in the corresponding strongly coupled QCD/QED medium?

In both figures, the fitting straight line represents\[ \gamma(p = 0) = \frac{1}{3} \alpha T \left( \log \frac{1}{g^2} + c \right), \]
where\[ c \approx 3.38. \]

In the weak coupling and high temperature QGP, the decay width $\gamma(p = 0)$, Eq. (15), agrees with the HTL resummed effective perturbation calculation up to a numerical factor, see Fig. 14.

Quite unexpectedly even in the strongly coupled QGP, the resulting decay width $\gamma(p = 0)$, Eq. (15), shows the totally same behavior of $O(g^2 T \log(1/g^2))$ as in the weakly coupled QGP up to the numerical factor and the $O(g^2 T)$ correction term, Fig. 15.

What happens in the intermediate coupling region? The results of the decay width at $T = 0.150$ is given in Fig. 16. From this figure we can understand how the decay width in the weak coupled QGP and the one in the strongly coupled QGP coincide. In the intermediate coupling region, the decay width of the quasifermion shows a “rich” structure. The decay width $\gamma(p = 0)$ diverges at the vanishing point of the thermal mass $\omega_+(p = 0)$, namely, the point where $\omega_+(p = 0)$ first hits zero as the coupling changes, see Eq. (14).

This behavior is again not expected, because the quasifermion in the small coupling and high temperature QGP and the one in the strong and high temperature QGP are totally different; in the former case the quasifermion has a thermal mass of $O(gT)$ and the plasmino branch exists in a fermion dispersion law, while in

FIG. 13: (a) Quasifermion spectral density $\rho_+(p_0, p = 0)$ at small coupling region ($\alpha = 0.001, T = 0.3$). (b) Quasifermion spectral density $\rho_+(p_0, p = 0)$ enlarged around the origin.

FIG. 14: The decay width of the quasifermion at rest $\gamma(p = 0)$ in the weakly coupled QGP. The dotted straight line Eq. (15) represents the result from the HTL resummed calculation, see Text.

FIG. 15: The decay width of the quasifermion at rest $\gamma(p = 0)$ in the strongly coupled QGP. The horizontal straight line Eq. (15) represents the result from the HTL resummed calculation, see Text.

In Fig. 14 and Fig. 15 we show the decay width of the quasifermion $\gamma(p)$ at $p = 0$, in the weakly coupled and in the strongly coupled QGP, respectively, where

\[ \gamma(p) = \frac{1}{2} \text{Im}[D_+(p_0 = \omega_+(p), p)] \times \left[ \frac{\partial}{\partial p_0} \text{Re}[D_+(p_0, p)] \right]_{p_0 = \omega_+(p)}^{-1}. \]

In both figures, the fitting straight line represents
the latter case thermal mass of the quasifermion vanishes and the plasmino branch disappears.

The temperature-dependence of the decay width is again described by Eq. (15), namely, the decay width of the quasifermion is linearly proportional to the temperature, both in the weak and strong coupling QGP.

This behavior can be clearly seen in Figs. 17 and 18. The former fact indicates that in the strongly coupled QGP, recently produced at RHIC and LHC, the predicted massless or the ultrasoft pole is very hard to be detected as a real physical mode.

IV. SUMMARY AND DISCUSSION

In this paper we carried out a nonperturbative analysis of a thermal quasifermion in thermal QCD/QED by studying its self-energy function through the Dyson-Schwinger equation (DSE) with the HTL resummed improved ladder kernel. With the solution of the DSE we studied the properties of the thermal quasifermion spectral density and its peak structure, the dispersion law of the physical modes corresponding to the poles of thermal quasifermion propagator. Through the study of quasifermion we elucidated the properties of thermal mass and the decay width of fermion and plasmino modes, and also paid attention to properties of the possible third mode, both especially in the strongly coupled QCD/QED medium.

What we have revealed in this paper is the drastic change of properties of the “quasifermion” depending on the strength of the interaction among constituents of the QCD/QED medium;

i) In the weak coupling region, or in the weakly coupled QCD/QED medium: \( \alpha < \sim 0.02 \) or \( g < \sim 0.5 \) at \( T = 0 \). The onshell conditions through the peak structure of spectral density and from the zero point of the quasifermion inverse propagator give the same structure and properties of the quasifermion. A rigid quasiparticle picture holds with the thermal mass \( m_f^* \), Eq. (12), and a small imaginary part or the decay rate \( \gamma \sim g^2 T \log(1/g) \) and the fermion acts as a basic degree of freedom of the medium. The thermal mass \( m_f^* \) is nothing but the next-to-leading order result of the HTL resummed effective perturbation calculations [3]. A fermion and the plasmino mode appear. Thus the results in the weak coupling region well reproduce those of the HTL resummed effective perturbation calculations.

The triple peak structure of the quasifermion spectral density is clearly observed, indicating the existence of the fermionic ultrasoft third mode, which is absent from the HTL resummed effective perturbation analyses.

ii) In the strong coupling region, or in the strongly coupled QCD/QED medium: \( \alpha \gtrsim 0.1 \) or \( g \gtrsim 1 \) at \( T = 0.3 \). Both the spectral density and the inverse
fermion propagator tell the single massless peak structure with large imaginary part, or, the decay rate $\gamma \simeq g^2 T \log(1/g)$. The quasiparticle picture a la Landau has been broken down in the strongly coupled QCD/QED medium. The thermal mass vanishes and there appears only the fermion mode, the plasmino mode disappears in the strongly coupled medium.

iii) In the intermediate coupling region: $0.02 \leq \alpha \leq 0.1$ or $0.5 \leq g \leq 1$ at $T = 0.3$. In this region the spectral density and the inverse fermion propagator tell completely different structure. The spectral density tells that there should be two particle modes with large decay rates, while the inverse fermion propagator tells that there should be three poles in the propagator, thus may exist three modes in this coupling region, just as in the case in the weakly coupled medium. We conclude that the indication of the inverse fermion propagator tells the truth, see the text. Anyway the intermediate coupling region is the transitional region for the fermion in the medium to behave as a rigid quasiparticle, acting as a basic degree of freedom in the medium.

Here we give several comments and discussion on the results of the present analysis.

1) It is not a priori very clear which one really defines the onshellness of the physical particle and its dispersion law, the peak position of the quasifermion spectral density $\rho_{\pm}(p_0, p)$ or the zero-point of the real part of the inverse fermion propagator $\text{Re}[D_{\pm}(p_0, p)] = 0$, especially when the imaginary part is not very small. If we adopt the peak position of the quasifermion spectral density $\rho_{\pm}(p_0, p)$ as the onshell point of the particle, then the corresponding dispersion law exhibits a branch developing into the space-like domain of space-time. There is also the problem of double peak structure of the quasifermion spectral density in the transitional intermediate coupling region, as noted above in iii). With these facts we adopt $\text{Re}[D_{\pm}(p_0, p)] = 0$ as the onshell condition of the physical particle to study its dispersion law and various properties, such as the thermal mass and the decay width, etc.

2) With the onshell condition $\text{Re}[D_{\pm}(p_0, p)] = 0$ we select the particle mode and study its dispersion law and the particle properties. The onshell condition at $p = 0$, $\text{Re}[D_{\pm}(p_0, p = 0)] = 0$, always has solution at $p_0 = 0$, which may correspond to the ultrasoft mode. This correspondence is, however, not so simple. The structure of the imaginary part around the onshell point of the propagator plays an important role to make this correspondence exact. This relationship was pointed out by Kitazawa et al. [4]; if in the medium the bosonic mode with nonzero mass (with small decay rate) couples with the fermion, then the quasifermion spectral density shows a triple peak structure corresponding to the ultrasoft third mode together with the fermion and the plasmino modes. The appearance of the peak at $p_0 = 0$ corresponding to the ultrasoft third mode is guaranteed with the vanishing of the imaginary part, $\text{Im}[D_{\pm}(p_0, p = 0)] = 0$, at $p_0 = 0$, which happens because of the coupling of the fermion with the massive bosonic mode. Such a mechanism may not work in the QED medium since no massive bosonic excitations in the QED medium are expected.

In the present DSE analysis, the imaginary part of the fermion propagator does not vanish at $p_0 = 0$, $\text{Im}[D_{\pm}(p_0, p = 0)] \neq 0$, but rather shows a peak structure at $p_0 = 0$. This peak structure actually suppresses the peak-height of the spectral density, as noted in the text, Sec. IV C. In this sense the appearance of the ultrasoft third peak with very low peak-height in the present analysis may have different origin from that of Kitazawa et al. [4] and from Satow et al. [22]. This problem will be discussed further in a separate paper [10].

3) We have noted that the thermal mass of quasifermion decreases as the coupling gets stronger, and finally vanishes in the strong coupling region. This fact indicates that the thermal mass of quasiquark vanishes and behaves as massless fermion with large decay rate in the recently discovered strongly coupled QGP. It is not so simple, however, whether such a particle can be experimentally observed as a massless quasifermion or not.

4) As noted above, the decay rate $\gamma(p)$ of the quasifermion in the QCD/QED medium shows a typical $g^2 T \log(1/g)$ behavior both in the weakly and the strongly coupled medium. In the transitional intermediate coupling environment, however, the decay rate shows a rich structure and $\gamma(p = 0)$ even diverges at the vanishing point of the thermal mass. It is quite exciting if we can find some methods to be able to verify experimentally the unexpected behavior of the thermal mass and the decay rate.

Appendix A: Approximations to get the HTL resummed improved ladder DSE

In the present analysis, we solve the DS equation for the retarded fermion self-energy function $\Sigma_r$, with the HTL resummed gauge boson propagator Eq. (3), by adopting further the following two approximations to get Eq. (4), i) the point-vertex approximation, and ii) the modified instantaneous exchange approximation to get the final DSEs we solve, on which we give brief explanations below.

i) Point-vertex approximation to get Eq. (4)

As for the vertex function $\Gamma^\mu$ we adopt the point-vertex approximation, namely we simply set $\Gamma^\mu = \gamma^\mu$ disregarding the HTL corrections to $\Gamma^\mu$. Thus we investigate the ladder (point-vertex) DS equation with the HTL resummed gauge boson propagator.

There are two reasons: Firstly, without the point-vertex approximation the numerical calculation we should carry out becomes so complicated that we can not manage with the power of the computer we use, because the HTL resummed contribution to the vertex function is the non-local interaction term, and also because it be-
we are forced to introduce a momentum cutoff in the in-
the present analysis.

for the massless gauge term in proportion to 
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al HTL resummed improved ladder DS equations for

transverse propagation disappear.

important thermal effect, i.e., the dynamical screening of

mode to the pure massless propagation, thus makes the

mode is that the IE approximation reduces the transverse

reason why we do not take the IE limit to the transverse

boson self-energy, respectively \[15\]. The parameter

transverse and logitudinal modes of the retarded gauge

DS equation with the HTL resummed vertex function, it

problem of double counting of di-
agrams \[26\], especially at the level of numerical analyses.

Being free from this problem in the numerical analysis

is the main reason why we make use of the point-vertex

approximation.

ii) Modified Instantaneous Exchange (MIE) approxi-

mation to get the final DSEs to solve

The next approximation we make use of is the modified

instantaneous exchange (IE) approximation (i.e., set the

energy component of the gauge boson to be zero) to the

gauge boson propagator \( \ast G^{\mu\nu} \). The retarded \((R \equiv RA)\)

and correlation \((C \equiv RR)\) components of the HTL re-
summed gauge boson propagator \( \ast G^{\mu\nu} \) are given by \[16\]

\[
\begin{align*}
\ast G^{\mu\nu}_R(K) & \equiv \ast G^{\mu\nu}_{RA}(-K,K) = \frac{1}{\ast \Pi^R_T(K) - K^2 - i\epsilon k_0} A^{\mu\nu} + \frac{1}{\ast \Pi^R_L(K) - K^2 - i\epsilon k_0} B^{\mu\nu} - \frac{\xi}{K^2 + i\epsilon k_0} D^{\mu\nu}, \quad (A1a) \\
\ast G^{\mu\nu}_C(K) & \equiv \ast G^{\mu\nu}_{RR}(-K,K) = (1 + 2n_B(k_0)) \left[ \ast G^{\mu\nu}_R(K) - \ast G^{\mu\nu}_A(K) \right], \quad (A1b)
\end{align*}
\]

with \( \ast \Pi^R_T \) and \( \ast \Pi^R_L \) being the HTL contributions to the

transverse and logitudinal modes of the retarded gauge

boson self-energy, respectively \[15\]. The parameter \( \xi \) is

the gauge-fixing parameter (\( \xi = 0 \) in the Landau gauge).

In the above, \( A^{\mu\nu}, B^{\mu\nu} \) and \( D^{\mu\nu} \) are the projection
tenors given by \[16\]

\[
\begin{align*}
A^{\mu\nu} & = g^{\mu\nu} - B^{\mu\nu} - D^{\mu\nu}, \quad (A2a) \\
B^{\mu\nu} & = -\frac{\tilde{K}^\mu \tilde{K}^\nu}{K^2}, \quad (A2b) \\
D^{\mu\nu} & = \frac{K^\mu K^\nu}{K^2}, \quad (A2c)
\end{align*}
\]

where \( \tilde{K} = (k,k_0\hat{k}) \), \( k = \sqrt{k^2} \) and \( \hat{k} = k/k \) denotes the

unit three vector along \( k \).

The modified IE approximation we make use of con-
sists of taking the IE limit in the HTL resummed logi-
tudinal (electric) gauge boson propagator, \( \ast G^{\mu\nu}_L \), that is propor-
tional to \( B^{\mu\nu} \), while keeping the exact HTL re-
summed form for the transverse (magnetic) gauge boson

propagator, \( \ast G^{\mu\nu}_T \), that is proportional to \( A^{\mu\nu} \), and also for

the massless gauge term in proportion to \( D^{\mu\nu} \). The

reason why we do not take the IE limit to the transverse

mode is that the IE approximation reduces the transverse

mode to the pure massless propagation, thus makes the

important thermal effect, i.e., the dynamical screening of

transverse propagation disappear.

With the above two approximations, we obtain the fi-
nal HTL resummed improved ladder DS equations for

ther invarinat scalar functions \( A, B \) and \( C \), to solve.

Appendix B: Cutoff dependence

In this appendix we explain the cutoff dependence of

the present analysis.

As explained in Sec. II A in solving the DSEs, Eq. [1],

we are forced to introduce a momentum cutoff in the in-

tegration over the four-momentum \( \int d^4 K, K = (k_0,k) \)

is the fermion four-momentum. The cutoff method we

make use of is as follows (\( \Lambda \) denotes an arbitrary cut-

off parameter and plays a role to scale any dimensionful

quantity, e.g., \( T = 0.3 \) means \( T = 0.3\Lambda \));

\[
\text{three momentum } k : k = |k| \leq \Lambda \\
\text{energy } k_0 : |k_0| \leq \Lambda_0
\]

In the present analysis we make the ratio \( r \equiv \Lambda_0/\Lambda \) vary

\( r = 1 \sim 5 \), and fix it so as to get a stable solution to the

fermion spectral density.

In Fig. 19 we show how the spectral density changes as a function of \( k_0 \) as we vary the ratio in the range

\( r = 1 \sim 5 \). We can easily recognize that at \( T = 0.4 \) and

\( \alpha = 0.1 \) we can get a stable solution if we choose \( r \geq 2 \).

Situation is almost the same but slightly differs at dif-

ferent \( T \) and \( \alpha \), see Fig. 20 at \( T = 0.4, \alpha = 1.0 \) and

compare with Fig. 19 at \( T = 0.4, \alpha = 0.1 \). As the cou-

pling becomes stronger it is safe to choose larger values of

\( r \).

The stability of the solution can be checked by the

saturation of the sum rules, Eqs. [10a] and [10b]. As already

noted, the sum rule Eq. [10c] heavily relied on the

HTL calculation, thus we do not use this sum rule.

Result is given in Table II again showing the stability of

the solution when we choose \( r \geq 2 \) (or more safely \( r \geq 3 \)).

With the above results we choose, in most cases ex-
cept at very strong couplings, an appropriate value of the

ratio in the range \( r \geq 2 \), depending on the region of

temperatures and/or couplings we study. The extreme high
temperature may cause another problem, namely, the

problem of simulation artifact, therefore we restrict the
temperature to the region \( T \lesssim 0.6 \) and do not per-
form our analysis in an extreme high temperature region

\( T \gtrsim 0.8 \).
Appendix C: Phase boundary in the Landau gauge

In order to study the phase transition and to determine the phase boundary of thermal QCD/QED, we should solve the DSE for the retarded fermion self-energy function $\Sigma_R$, Eq. (2). For the present purpose, however, we must study the $\Sigma_R$ that has a $c$-number scalar mass function $C(P)$,

$$\Sigma_R(p) = (1 - A(P))p_i\gamma^i - B(P)\gamma^0 - C(P). \quad (C1)$$

The DSE in the Landau gauge to determine the three scalar invariants $A(P)$, $B(P)$ and $C(P)$ becomes coupled integral equations as follows;

TABLE I: The cutoff dependence of the saturation of the sum rules, Eqs. (10a) and (10b): $r = \Lambda_0/\Lambda$.

| $T = 0.2$ | $\alpha = 0.01$ | $T = 0.4$ | $\alpha = 0.1$ | $T = 0.4$ | $\alpha = 1.0$ |
|------------|----------------|------------|----------------|------------|----------------|
| Eq. (10a)  | $p = 0$ 0.949 0.994 0.996 1.003 | Eq. (10a)  | $p = 0$ 0.974 1.014 1.021 1.021 |
|            | $p = 0.02$ 0.986 0.985 0.983 0.982 0.990 |            | $p = 0.02$ 0.979 1.006 1.006 1.006 |
|            | $p = 0.04$ 0.0392 0.0404 0.0404 0.0407 0.0404 |            | $p = 0.04$ 0.0354 0.0406 0.0408 0.0408 |
|            | $p = 0.1$ 0.0990 0.1009 0.1009 0.1004 0.1013 |            | $p = 0.1$ 0.0948 0.1011 0.1016 0.1016 |
| Eq. (10b)  | $p = 0$ 0.927 1.004 1.009 1.009 | Eq. (10b)  | $p = 0$ 0.759 0.974 1.018 1.021 1.021 |
|            | $p = 0.02$ 0.929 1.006 1.006 1.006 |            | $p = 0.02$ 0.761 0.975 1.017 1.021 1.021 |
|            | $p = 0.04$ 0.0354 0.0406 0.0408 0.0408 |            | $p = 0.04$ 0.0218 0.0350 0.0407 0.0413 0.0413 |
|            | $p = 0.1$ 0.0948 0.1011 0.1016 0.1016 |            | $p = 0.1$ 0.0546 0.0879 0.1024 0.1037 0.1038 |

FIG. 19: The cutoff dependence of the quasifermion spectral density at $p = 0$, $\rho_+(p_0, p = 0)$, at $T = 0.4$ and $\alpha = 0.1$. $r = \Lambda_0/\Lambda$.

FIG. 20: The cutoff dependence of the quasifermion spectral density at $p = 0$, $\rho_+(p_0, p = 0)$, at $T = 0.4$ and $\alpha = 1.0$. $r = \Lambda_0/\Lambda$. 

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\[ p^2[1 - A(P)] = g^2 \int \frac{d^4K}{(2\pi)^4} \left[ \{1 + 2n_B(p_0 - k_0)\} \text{Im}[*G_{R}^{p\sigma}(P - K)] \times \right. \\
\left. \left\{ K_\sigma P_\rho + K_{\rho}P_\sigma - p_0(K_\sigma g_{\rho\sigma} + K_{\rho}g_{\sigma\rho}) - k_0(P_\sigma g_{\rho\sigma} + P_\rho g_{\sigma\rho}) + pkzg_{\rho\sigma} \\
+ 2p_0k_0g_{\sigma\rho} \right\} \frac{A(K)}{[k_0 + B(K) + i\epsilon]^2 - A(K)^2k^2 - C(K)^2} + \{P_\sigma g_{\rho\sigma} + P_\rho g_{\sigma\rho} \\
- 2p_0g_{\sigma\rho} + p_0k_0g_{\sigma\rho} \} \frac{A(K)}{[k_0 + B(K) + i\epsilon]^2 - A(K)^2k^2 - C(K)^2} \right] \times \]
\[ *G_{R}^{p\sigma}(P - K) \text{Im} \left( \left\{ K_\sigma P_\rho + K_{\rho}P_\sigma - p_0(K_\sigma g_{\rho\sigma} + K_{\rho}g_{\sigma\rho}) - k_0(P_\sigma g_{\rho\sigma} + P_\rho g_{\sigma\rho}) \\
+ pkzg_{\rho\sigma} + 2p_0k_0g_{\sigma\rho} \right\} \frac{A(K)}{[k_0 + B(K) + i\epsilon]^2 - A(K)^2k^2 - C(K)^2} \right] \right\} \times \]
\[ B(P) = g^2 \int \frac{d^4K}{(2\pi)^4} \left[ \{1 + 2n_B(p_0 - k_0)\} \text{Im}[*G_{R}^{p\sigma}(P - K)] \times \right. \\
\left. \left\{ K_\sigma g_{\rho\sigma} + K_{\rho}g_{\sigma\rho} - 2k_0g_{\sigma\rho} \right\} \frac{A(K)}{[k_0 + B(K) + i\epsilon]^2 - A(K)^2k^2 - C(K)^2} + \{1 - 2n_F(k_0)\} \times \right. \\
\left. *G_{R}^{p\sigma}(P - K) \text{Im} \left( \left\{ K_\sigma g_{\rho\sigma} + K_{\rho}g_{\sigma\rho} \\
- 2k_0g_{\sigma\rho} + k_0B(K) \right\} \frac{A(K)}{[k_0 + B(K) + i\epsilon]^2 - A(K)^2k^2 - C(K)^2} \right] \right\} \times \]
\[ C(P) = g^2 \int \frac{d^4K}{(2\pi)^4} \left[ \{1 + 2n_B(p_0 - k_0)\} \text{Im}[*G_{R}^{p\sigma}(P - K)] \times \right. \\
\left. \left\{ K_\sigma g_{\rho\sigma} + K_{\rho}g_{\sigma\rho} \right\} \frac{A(K)}{[k_0 + B(K) + i\epsilon]^2 - A(K)^2k^2 - C(K)^2} + \{1 - 2n_F(k_0)\} \times \right. \\
\left. *G_{R}^{p\sigma}(P - K) \text{Im} \left( \left\{ K_\sigma g_{\rho\sigma} + K_{\rho}g_{\sigma\rho} \right\} \frac{A(K)}{[k_0 + B(K) + i\epsilon]^2 - A(K)^2k^2 - C(K)^2} \right] \right\} \times \]

Above DSEs, Eq. (C2), may have several solutions, and we choose the “true” solution by evaluating the effective potential \( V[S_R] \) for the fermion propagator function \( S_R \), then finding the lowest energy solution. The effective potential \( V[S_R] \) we evaluate, is given in Sec. [11B, Eq. (3)].

Now we present Fig. 21, showing the phase boundary curve in \((T, 1/\alpha_c)\)-plane in Landau gauge, that separates the chiral symmetric phase from the broken one. This critical curve shows that the critical coupling inverse \( 1/\alpha_c \) is a monotonically decreasing function of the temperature \( T \) slightly concave upwards, and displays two characteristic behaviors: (1) as \( T \) becomes lower, the critical coupling inverse \( 1/\alpha_c \) becomes larger and seems to increase from below to the zero temperature value \( \alpha_c^{MS}(T = 0) = 1.52\pi \approx 1.152\) [13], and our result predicts slightly larger critical coupling \( \alpha_c(T = 0) = 1.32 \pm 0.14 \). The errors are given in the 3\( \sigma \) accuracy level of the least \( \chi^2 \) fit. Phase transition occurs only in the region \( \alpha \geq \alpha_c(T = 0) \approx 1.32 \) and \( T \leq T_0 \approx 0.21 \), i.e., chiral symmetry broken phase is restricted to the region of the \((T, 1/\alpha_c)\)-plane lower than the critical curve in Fig. 21. Therefore it is obvious that the region of the coupling and temperature where we study the property of the quasifermion, is well inside the chiral symmetric phase.

How does the property of the quasifermion change inside the chiral symmetry broken phase? This is an interesting question. Does the quasifermion mode still exist in the broken phase? These questions will be discussed in a separate paper.
through the analysis of the spectral density itself is not neither. These facts indicate that information obtained does not exactly represent the true position of the pole thus cannot be observed. The position of the two peaks completely hidden under the big tails of the broad two peaks, third peak representing the ultrasoft third pole is completely hidden from the broken one. The error-bar assigned to the best fit curve at \( T = 0 \) denotes the error in the 3\( \sigma \) accuracy level.

**Appendix D: Dispersion law \( \omega^0(p) \) determined through the peak position of the spectral density \( \rho^+ \)**

Throughout this paper we determined the dispersion law of the thermal quasifermion with the onshell condition \( \text{Re}[D_+(p_0, p = 0)] = 0 \). As was explained in Sec. III C generally speaking, the pole of the propagator or the point where the propagator inverse vanishes defines the corresponding particle and its dispersion law, and we can use another definition of onshellness. One of such definition is to use the peak position of the spectral density as the pole position of the corresponding particle, with which we can also determine the dispersion law of this particle.

These two definitions of onshellness almost agree with each other when the imaginary part of the mass term is small. In fact, in the weak coupling region at high temperature, the fermion branch of the dispersion law determined through the peak position of the spectral density \( \rho^+ \) almost completely coincides with the dispersion law, Fig. 6 determined through the onshell condition \( \text{Re}[D_+(p_0, p = 0)] = 0 \).

There are mainly two reasons why we adopt the onshell condition \( \text{Re}[D_+(p_0, p = 0)] = 0 \) rather than that given by the peak position of the spectral density in the present analysis. The first reason is already explained in Sec. III A 2. It is pointed out there that at the intermediate coupling strength the spectral density exhibits a typical double peak structure, indicating the existence of two poles in the quasifermion propagator. This is, however, not the case. There are actually three poles in the propagator in the corresponding coupling region. The third peak representing the ultrasoft third pole is completely hidden under the big tails of the broad two peaks, thus cannot be observed. The position of the two peaks does not exactly represent the true position of the pole neither. These facts indicate that information obtained through the analysis of the spectral density itself is not complete but even misunderstanding.

The second reason why did not adopt simply the peak position of the spectral density as the pole position of the corresponding particle, is the appearance of the plasmino branch continuing to exist in the space-like domain, namely the existence of the space-like plasmino solution. In Fig. 22 we show the dispersion law \( \omega^0(p) \) determined through the peak position of the spectral density \( \rho^+ \), which exactly corresponding to Fig. 6 showing the dispersion law determined through the onshell condition \( \text{Re}[D_+(p_0, p = 0)] = 0 \). Though the fermion branch almost completely agrees with each other, the plasmino branch exhibits a typical difference. In Fig. 6 the plasmino branch exhibits a minimum at \( p \neq 0 \) and vanishes rapidly on to the light cone as \( p \) gets large. In Fig. 22 the plasmino branch also exhibits a minimum at \( p \neq 0 \), and approaches rapidly to the light cone, \( \text{then crosses the light cone and continues to exists in the space-like domain} \), of the world sheet.

With these two reasons, in the present analysis we do not adopt to define the onshell condition through the peak position of the spectral density.

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