Three-parameter Model for Predicting the Earth pole Trajectory

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Abstract. A three-parameter model for forecasting the Earth pole motion is considered - a model with the minimum possible number of independent parameters and a satisfactory forecasting accuracy. The results of numerical simulation of the Earth pole oscillatory motion are compared with the observational data from the International Earth Rotation and Reference Systems Service. An analysis of the model accuracy is given.

Along with the adaptive models for calculating the Earth orientation parameters, which make it possible to give the most accurate short-time forecast (up to a year) [1], the development of autonomous forecast models is of practical interest for navigation problems [2]. Due to the non-stationary nature of the main components of the Earth pole oscillations (Chandler and annual components), that can lead to a sharp change in the average modulation frequency, the accuracy of a completely autonomous model turns out to be low. But the requirement for autonomy can be weakened if we consider the model for a given time interval (autonomy interval) without parameter correction. In this case, the correction of quasi-constant coefficients (amplitudes of the fundamental harmonics), that are subject to changes due to the non-stationary nature of the disturbing factors, is carried out at the beginning and at the end of this interval. These models are simple and, although they are not completely autonomous and can be considered as such only at specified time intervals, they have a higher forecasting accuracy. In practice, the choice of a particular model is the result of a compromise choice between the forecast accuracy, the duration of the forecasting interval (autonomy) and the computational complexity of the model.

For most applications [2], it is sufficient to use a few-parameter model of the Earth pole motion, which gives a satisfactory forecasting result compared by the observational data of the International Earth Rotation and Reference Systems Service (IERS) [1]. For calculating the Earth pole coordinates \( x_p \) and \( y_p \) it is rational to use a two-frequency model of its motion with annual and Chandler components [3, 4]:

\[
\begin{align*}
x_p &= c_x - a_x^c \cos 2\pi N \tau + a_x^p \sin 2\pi N \tau - N d_x^c \cos 2\pi \tau - d_x^p \sin 2\pi \tau; \\
y_p &= c_y + a_y^c \cos 2\pi N \tau + a_y^p \sin 2\pi N \tau - N d_y^c \cos 2\pi \tau + d_y^p \sin 2\pi \tau.
\end{align*}
\]
The value of the Chandler frequency $N$ is assumed to be a constant and equal to 0.843 cycles per year. Unknown coefficients $c_{x,y}, a_{x,y}^{y,c}, d_{x,y}^{x,c}$ are the quantities to be calculated from the IERS observations using the least squares method.

According to statistics, an increase in the forecast time interval leads to an increase in the mean square error, which is natural. With a fixed time interval, the forecast is usually stable and its accuracy depends on the accumulated error while determining the pole position. The resulting deviation of the forecast from the actual observational data, in most cases, is a result of unpredictable fluctuations - both external gravitational-tidal disturbances and perturbations from large-scale phenomena in the geomedia (atmosphere and hydrosphere). When integrating the Earth pole differential equations of motion, such disturbances lead to irregular changes in the fundamental harmonics amplitudes of the oscillatory process. It should be noted that the presence of short-period fluctuations in perturbations does not have a noticeable effect on the estimates of the parameters of the Chandler and annual oscillations, since in the resulting process their amplitudes are small.

However, in some cases, one can observe an increase in the forecast error over time and at prediction intervals with a fixed duration. At the same time, adjusting the model parameters to improve its accuracy characteristics does not lead to the required result, and even to a noticeable change in the nature of the error growth.

![Fig. 1. Absolute deviations of the Earth pole calculated trajectory from the observed one with (red line) and without (blue line) changes in the average frequency of its motion.](image)

The Earth pole motion has a number of interesting features [5, 6], which cannot be attributed to either regular effects or effects of a stochastic nature. So, studying the main oscillatory process of the Earth pole (the addition of the Chandler and annual components), it can be established that the average frequency of its motion changes abruptly when the amplitude ratio changes - the ratio of the amplitudes of the Chandler and annual components. On the one hand, this effect is described within the framework of the deterministic approach, and on the other hand, it occurs irregularly, since the amplitudes of the fundamental harmonics are subject to irregular or quasiregular changes. When such effects are not taken into account, it can lead to significant discrepancies in the Earth pole trajectory calculations. In fig. 1 a graph shows the absolute deviations of the calculated Earth pole position according to model (1) without parameters correction from the observed one according to the IERS data. Also, the graph shows the deviations of model (1) without parameter correction, but taking into
account the change in the pole oscillation average frequency over the modulation period. Note that the six-year cycles in the absolute deviation are caused when the long-period trend component is not taken into account or, in other words, by the error in predicting the constants $c_x, c_y$ in (1).

Let us consider a few-parameter model for predicting the Earth pole position, that makes it possible to take into account the changes in the amplitudes of the Chandler and annual components with the minimum possible number of adjustable parameters. Sufficient stability of the phases’ values of the fundamental oscillations harmonics makes it possible to choose a three-parameter model for each of the pole coordinates:

$$
x_p = c_x + k_1(-a_x^c \cos 2\pi N\tau + a_x^s \sin 2\pi N\tau) +
+k_2(Nd_x^c \cos 2\pi \tau + d_x^s \sin 2\pi \tau);
$$

$$
y_p = c_y + k_3(a_y^c \cos 2\pi N\tau + a_y^s \sin 2\pi N\tau) +
+k_4(-Nd_y^c \cos 2\pi \tau + d_y^s \sin 2\pi \tau).
$$

The coefficients of model (1) obtained by using the least squares method [4] approximate the model to the IERS observational data in the best way in the mean square sense on the given interval. However, without regular adjusting the model (1) parameters, its forecast will be calculated with a significant error. To improve the forecasts accuracy, it is possible to correct only the constant components and amplitudes of the annual and Chandler components. For this, the factors $k_1, k_2, k_3, k_4$ are introduced in (2). The number of these factors can be reduced to the two parameters $k_1, k_2$ with an accuracy sufficient for an approximate forecast, if it is assumed that the amplitudes of the main components are the same in the $x$ and $y$ coordinates, and their phases differ by 90°.

The values of the $k_{1,2,3,4}$ and $c_{x,y}$ coefficients are refined for each forecast by the least squares method on an interval of 6 years preceding the forecast date, and the coefficients $a_{x,y}^{c,s}$ were determined on a long test interval preceding the verification interval. Time $\tau$ is measured in MJD (MJD - modified Julian date), pole coordinates $x_p$ and $y_p$ are in arc seconds.

To estimate the accuracy of the model (2) a series of sequential forecasts were for time intervals: a year, 3 months, a month, a week, and the results were compared with the IERS observational.

In fig. 2, the graphs of the $a_{x,y}^{c,s}$ are shown - the standard deviation of the coordinates $x_{fp}, y_{fp}$ and the trajectory of the pole forecasts, respectively, for 365 days ($i = 1$); 90 days ($i = 2$); 30 days ($i = 3$) and 7 days ($i = 4$). The value of the forecast standard deviation is referred to the first day of the forecast and is measured in arc seconds.

From a comparative analysis of the simulation results it can be concluded that in order to maintain a satisfactory forecasting accuracy of model (1), it is necessary to correct at least a set of three parameters $c_x, k_1, k_2$ and $c_y, k_3, k_4$ for each of the pole coordinates according to (2) or four parameters $c_x, c_y, k_1, k_2$ for the trajectory forecast.

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Fig. 2. Comparison of standard deviations $\sigma$ at different time intervals of the consecutive forecast of $x_p$ coordinate (top); $y_p$ coordinates (middle); pole trajectories (bottom): ── – consecutive forecast by 365 days; ── – by 90 days; ── – by 30 days; ── – by 7 days.
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