General Relativity and the Cuprates

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GH and J. Santos, 1302.6586
Gauge/gravity duality can reproduce many properties of condensed matter systems, even in the limit where the bulk is described by classical general relativity:

1) Fermi surfaces
2) Non-Fermi liquids
3) Superconducting phase transitions
4) é

It is not clear why it is working so well.
Can one do more than reproduce qualitative features of condensed matter systems?

Can gauge/gravity duality provide a quantitative explanation of some mysterious property of real materials?

We will argue that the answer is yes!
Plan: Calculate the optical conductivity of a simple holographic conductor and superconductor with lattice included.

Earlier work on the effects of a lattice by many groups, e.g., Kachru et al; Maeda et al; Hartnoll and Hofman; Zaanen et al.; Siopisis et al., Flauger et al

Main result: We will find surprising similarities to the optical conductivity of some cuprates.
\[ \text{Re}(\sigma) = \frac{K\tau}{1 + (\omega\tau)^2}, \quad \text{Im}(\sigma) = \frac{K\omega\tau^2}{1 + (\omega\tau)^2} \]

Note:

(1) For \( \omega\tau \gg 1, \quad |\sigma| \approx K/\omega \)

(2) In the limit \( \tau \to \infty \):

\[ \text{Re}(\sigma) \propto \delta(\omega), \quad \text{Im}(\sigma) = K/\omega \]

This can be derived more generally from Kramers-Kronig relation.
Our gravity model

We start with just Einstein-Maxwell theory:

\[ S = \int d^4x \sqrt{-g} \left[ R + \frac{6}{L^2} - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} \right] \]

This is the simplest context to describe a conductor. We require the metric to be asymptotically anti-de Sitter (AdS)

\[ ds^2 = \frac{-dt^2 + dx^2 + dy^2 + dz^2}{z^2} \]
Want finite temperature: Add black hole

Want finite density: Add charge to the black hole. The asymptotic form of $A_t$ is

$$A_t = \mu - \rho z + O(z^2)$$

$\epsilon$ is the chemical potential and $\rho$ is the charge density in the dual theory.
Introduce the lattice by making the chemical potential be a periodic function:

$$\mu(x) = \bar{\mu} \left[ 1 + A_0 \cos(k_0 x) \right]$$

We numerically find solutions with smooth horizons that are static and translationally invariant in one direction.
Charge density for $A_0 = \frac{1}{2}, \ k_0 = 2, \ T/\epsilon = 0.055$

Solutions are rippled charged black holes.
To compute the optical conductivity using linear response, we perturb the solution

\[ g_{\mu \nu} = \hat{g}_{\mu \nu} + \delta g_{\mu \nu}, \quad A_{\mu} = \hat{A}_{\mu} + \delta A_{\mu} \]

Boundary conditions:

ingoing waves at the horizon

\[ \tilde{u} g_{\varepsilon 3} \] normalizable at infinity

\[ \tilde{u} A_{t} \sim O(z), \quad \tilde{u} A_{\chi} = e^{-i\chi t} [E/i\chi + J z + \epsilon] \]

induced current
Review: optical conductivity with no lattice

\( \frac{T}{\varepsilon} = .115 \)
With the lattice, the delta function is smeared out.
The low frequency conductivity takes the simple Drude form:

\[ \sigma(\omega) = \frac{K\tau}{1 - i\omega\tau} \]
Intermediate frequency shows scaling regime:

$$|\sigma| = \frac{B}{\omega^{2/3}} + C$$

Lines show 4 different temperatures:

$$0.033 < T/\varepsilon < 0.055$$
Comparison with the cuprates
(van der Marel, et al 2003)

\[ |\sigma(\omega)| = C \omega^{-0.65} \]

\[ Bi_2Sr_2Ca_{0.92}Y_{0.08}Cu_2O_{8+\delta} \]
What happens in the superconducting regime?

We now add a charged scalar field to our action:

\[
S = \int d^4x \sqrt{-g} \left[ R + \frac{6}{L^2} - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} - 2(\partial - ieA)\Phi |^2 + \frac{4|\Phi|^2}{L^2}\right]
\]

Gubser (2008) argued that at low temperatures, charged black holes would have nonzero $\bar{\Phi}$.

Hartnoll, Herzog, GH (2008) showed this was dual to a superconductor (in homogeneous case).
The scalar field has mass $m^2 = -2/L^2$, since for this choice, its asymptotic behavior is simple:

$$
\Phi = z\phi_1 + z^2\phi_2 + O(z^3)
$$

This is dual to a dimension 2 charged scalar operator $O$ with source $\phi_1$ and $\langle O \rangle = \phi_2$. We set $\phi_1 = 0$.

For electrically charged solutions with only $A_t$ nonzero, the phase of $\bar{u}$ must be constant.
We keep the same boundary conditions on $A_t$ as before:

$$\mu(x) = \bar{\mu} [1 + A_0 \cos(k_0 x)]$$

Start with previous rippled charged black holes with $\bar{\mu} = 0$ and lower $T$. When do they become unstable?

Onset of instability corresponds to a static normalizable mode of the scalar field. This can be used to find $T_c$. 


Having found $T_c$, we now find solutions for $T < T_c$ numerically.

These are hairy, rippled, charged black holes.

From the asymptotic behavior of $\bar{\Phi}$ we read off the condensate as a function of temperature.
Condensate as a function of temperature

Lattice amplitude grows from 0 (inner line) to 2.4 (outer line).
We again perturb these black holes as before and compute the conductivity as a function of frequency.

Find that curves at small $\omega$ are well fit by adding a pole to the Drude formula

$$\sigma(\omega) = i \frac{\rho_s}{\omega} + \frac{\rho_n \tau}{1 - i\omega \tau}$$

The lattice does not destroy superconductivity (Siopsis et al, 2012; Iizuka and Maeda, 2012)
Fit to: \[ \sigma(\omega) = i \frac{\rho_s}{\omega} + \frac{\rho_n \tau}{1 - i\omega\tau} \]
The dashed red line through $j_n$ is a fit to:

$$\rho_n = a + be^{-\Delta/T}$$

with $\Phi = 4 T_c$.

This is like BCS with thermally excited quasiparticles but:

(1) The gap $\Phi$ is much larger, and comparable to what is seen in the cuprates.
(2) Some of the normal component remains even at $T = 0$ (this is also true of the cuprates).
Intermediate frequency conductivity again shows the same power law:

\[ |\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C \]

Coefficient B and exponent 2/3 are independent of T and identical to normal phase.
8 samples of BSCCO with different doping.

Each plot includes $T < T_c$ as well as $T > T_c$.

No change in the power law.

(Data from Timusk et al, 2007.)
Preliminary results on a full 2D lattice \((T > T_c)\) show very similar results to 1D lattice.

The optical conductivity in each lattice direction is nearly identical to the 1D results.
Our simple gravity model reproduces many properties of cuprates:

- Drude peak at low frequency
- Power law fall-off $\propto \omega^{-2/3}$ at intermediate frequency
- Gap $2\phi = 8 T_c$
- Normal component doesn't vanish at $T = 0$
But key differences remain

- Our superconductor is s-wave, not d-wave
- Our power law has a constant off-set C