Stress-induced modification of gyration dynamics in stacked double-vortex structures studied by micromagnetic simulations

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Abstract
In this paper, using micromagnetic simulations, we investigate the stress-induced frequency tunability of double-vortex nano-oscillators comprising magnetostrictive and non-magnetostrictive ferromagnetic layers separated vertically by a non-magnetic spacer. We show that the relative orientations of the vortex core polarities $p_1$ and $p_2$ have a strong impact on the eigen-frequencies of the dynamic modes. When the two vortices with antiparallel polarities have different eigen-frequencies and the magnetostatic coupling between them is sufficiently strong, the stress-induced magnetoelastic anisotropy can lead to the single-frequency resonant gyration mode of the two vortex cores. Additionally, for the case of parallel polarities, we demonstrate that for sufficiently strong magnetostatic coupling, the magnetoelastic anisotropy leads to the coupled vortex gyration in the chaotic regime and to the lateral separation of the vortex core trajectories. These findings offer a path for achieving a fine control over gyration frequencies and trajectories in vortex-based oscillators via adjustable elastic stress, which can be easily generated and tuned electrically, mechanically or optically.

Supplementary material for this article is available online

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(Some figures may appear in colour only in the online journal)

1. Introduction

The ground state of nanoscale circular magnetic disks of certain geometric aspect ratios is a spontaneously forming stable vortex configuration with circulating in-plane magnetization and a vortex core pointing out-of-plane. Resonantly exciting the vortex core via either an rf magnetic field or an rf spin-polarized current yields a gyrotropic motion around its equilibrium position, characterized by a specific eigen-frequency. The gyrotropic frequency depends on the material parameters and the disk geometry [1]. Such oscillations, which can be read out via periodic magnetoresistance changes, generate rf signals with output powers in the nW range and...
linewidth below 1 MHz, centered at the frequencies of few hundred MHz, thus yielding high quality factors (~1000) [2]. The quality factor can be further improved by one order of magnitude when the coupled mode between the vortices in stacked double-layer structures is excited [3, 4]. The complex dynamics of coupled vortex modes was studied in detail in [5], revealing a fine splitting of the corresponding resonant frequencies. The recent demonstration of phase noise reduction in vortex-based spin-torque oscillators locked to an external rf current [6, 7] provides the potential for system-level integration of vortex-based devices for such applications as spectrum analysis [8, 9], microwave receivers/transmitters [10] and spintronics-based neuromorphic computing [11, 12]. While all these features make vortex-based nano-oscillators interesting as nanoscale rf sources, the major drawback remains their low frequency tuneability when operated in the linear regime.

Recently, the magnetoelastic effect has been proposed as a path to enhance the tuneability of the eigen-frequencies of individual vortices. It was shown both theoretically [13] and experimentally [14] that introducing an additional magnetoelastic anisotropy term to the vortex core dynamics leads to the softening of the restoring force constants and thus to a decrease of the gyrotropic frequency. This enables the stress-induced control of the vortex eigen-frequency e.g. via the bending of the flexible membrane-like substrate [14], an application of an electric field to the piezoelectric substrate [15, 16] or by using optically generated deformation of photostrictive materials [17, 18].

Here, we present a micromagnetic study of the interplay between the strain-induced magnetic anisotropy and the magnetoelastic interaction in vortex pairs separated vertically by a non-magnetic (NM) spacer. Specifically, we consider a double-disk structure comprising magnetostrictive (CoFe) and non-magnetostrictive (Py) ferromagnetic layers separated vertically by a NM spacer. We show that, when the two vortices with anti-parallel polarities have different eigen-frequencies and the magnetoelastic coupling between them is sufficiently strong, the stress-induced magnetoelastic anisotropy can lead to the single-frequency gyration mode of the two vortex cores. For the case of parallel vortex core polarities, the stress-induced magnetoelastic anisotropy affects considerably not only the resonant frequencies, but also the vortex core gyration trajectories leading to the chaotic dynamics of the magnetoelastically coupled vortex pairs. These findings offer a means of manipulating the frequency tuneability of double-vortex-based oscillators via elastic stress, which can be generated and controlled electrically, mechanically, or optically.

2. Simulation details

In this paper, we report on simulations of the vortex dynamics in double-vortex structure in the presence of magnetoelastic coupling and stress-induced magnetoelastic anisotropy. We use the GPU-accelerated MuMax3 software package [19] for the simulations of the magnetization dynamics in nanosized magnetic disks under the presence of the stress-induced magnetoelastic anisotropy. We consider a tri-layer system (see figure 1) comprising disk-shaped magnetostrictive (MS) and non-magnetostrictive (nMS) ferromagnetic layers separated by a NM spacer with the thickness \(d_{NM}\). We study the evolution of the frequency and the trajectory of the gyrotropic mode of the vortex core as a function of the in-plane uniform uniaxial stress \(\sigma\). Due to the inverse MS effect, the stress \(\sigma\) introduces an additional anisotropy energy \(K_\sigma = \frac{1}{2} \lambda_\sigma \sigma\), where \(\lambda_\sigma\) is the magnetostriction constant of the magnetic material [20]. When \(\lambda_\sigma\) is positive (negative), the tensile stress imposes a magnetoelastic anisotropy along (perpendicular to) the direction of the applied stress.

In our simulations, we choose CoFe as MS material providing significant magnetostriction (\(\lambda_{CoFe} = 65 \times 10^{-6}\)) [21, 22], and Permalloy (Ni80Fe20 or Py) as nMS material with negligible magnetostriction (\(\lambda_{Py} \approx 0\)) [23]. We use the following material parameters for the magnetization dynamics simulations: saturation magnetizations \(M_{sCoFe} = 1700\) kA m\(^{-1}\) for CoFe and \(M_{sPy} = 800\) kA m\(^{-1}\) for Py, exchange constants \(A_{CoFe} = 21\) pJ m\(^{-3}\) for CoFe and \(A_{Py} = 13\) pJ m\(^{-3}\) for Py, and damping constant \(\alpha = 0.008\) for both layers. For all simulated data presented here, we consider the MS/NM/nMS tri-layer of 300 nm diameter discretized into 128 \(\times\) 128 \(\times\) \(\frac{t_{tot}}{10}\) cells (where \(t_{tot}\) is the total thickness of the tri-layer). The magnetic vortex cores are excited by an in-plane sinusoidal rf magnetic field \(h_{rf} = h \sin(2 \pi f t)\), where \(f\) is the excitation frequency, \(h = (h_x;0;0)\) and the amplitude \(h_x = 1\) mT. The magnetization dynamics in both magnetic disks is simulated over a time \(T\) of 60 periods \((T = \frac{2 \pi}{f})\), and in the final magnetization state of each disk is captured for each value of the excitation frequency \(f\) in the chosen range. In order to ensure that the steady-state dynamics is reached, the excitation frequency is swept in small steps and the initial magnetization state for every frequency point (starting from the second point of the spectrum) is the last state simulated at the previous frequency point. Therefore, the excitation at every subsequent frequency will induce only a small perturbation to the previously simulated state [24]. Thus, we obtain an FMR-like absorption spectrum.
with the resonance peak corresponding to the eigen-frequency of the vortex core gyration. We show that in such double-vortex structures, the resonance frequencies of the individual and coupled modes, which in general are defined by the vortex polarities and the spacer thickness (i.e. magnetostatic coupling strength), can be modified by introducing the magnetoelastic energy $K_\sigma$.

3. Results and discussion

3.1. Vortex core dynamics in CoFe/NM/Py double-vortex structures: spacer thickness dependence

An individual vortex is described by the in-plane magnetization and the core polarity $p$, defining four distinct vortex configurations: $(c; p) = (1; 1), (c; p) = (1; -1), (c; p) = (-1; 1)$ and $(c; p) = (-1; -1)$. The eigen-frequencies of the vortex core gyration are mainly defined by the magnetic parameters of the material (i.e. saturation magnetization, exchange constant, anisotropies, etc) and the geometrical thickness-to-diameter ratio of the circular disk [1] but do not depend on the $(c; p)$ configuration. Similarly, for the case of double-vortex structure, four non-degenerate configurations of $c$ and $p$ can be distinguished: $(c_1 c_2; p_1 p_2) = (1; 1), (c_1 c_2; p_1 p_2) = (1; -1), (c_1 c_2; p_1 p_2) = (-1; 1)$ and $(c_1 c_2; p_1 p_2) = (-1; -1)$. However, the dynamics of the double-vortex structure is more complex and depends not only on the material parameters and geometry, but also on the mutual orientation of the vortex polarities and circulations as well as on the efficiency of the coupling between the vortices. In this section, we discuss the resonant dynamics of the CoFe/NM/Py double-vortex structure, excited by the in-plane rf magnetic field $h_{rf} = h_0 \sin(2 \pi f t)$ as a function of the thickness of the NM spacer, for the different $(c_1 c_2; p_1 p_2)$ configurations.

3.1.1. Antiparallel polarities. Figure 2(a) shows the simulated FMR spectra of the CoFe(10)/NM(d$_{NM}$)/Py(20) cylindrical tri-layer (the thicknesses are in nm) with a diameter $d = 300$ nm, as a function of the excitation frequency $f$, for different thicknesses $d_{NM}$ of the NM layer, ranging from 10 to 160 nm. Here, we consider the case of antiparallel polarities and circulations, i.e. $(c_1 c_2; p_1 p_2) = (-1; -1)$. All spectra feature double-resonance behavior with one low- (high-) frequency peak $f_{lo}$ ($f_{hi}$), corresponding to the Py (CoFe) layer resonance (figure 2(c)), respectively. This is confirmed by simulations of the vortex core configurations of the corresponding layer (figures 2(d) and (e)). At $f_{lo}$ (f$_{hi}$) the Py (CoFe) resonant trajectory is quasi-circular and the vortex core exhibits steady-state gyration. On the contrary, the second layer, i.e. the CoFe (Py), experiences periodic low-amplitude off-resonance deflection of the vortex core from the equilibrium position. Interestingly, a slight deformation of both trajectories is present for the two resonance frequencies; this is attributed to the non-zero magnetostatic coupling between the two layers.

For $d_{NM} \geq 200$ nm, the resonances occur at $f_{lo} = f_{Py} = 0.56$ GHz and $f_{hi} = f_{CoFe} = 0.62$ GHz, corresponding to the resonances of the uncoupled CoFe(10) and Py(20) disks, respectively. Decreasing the spacer thickness leads to a red-shift of the eigen-frequencies of both layers, as well as to a decreased frequency separation $\Delta f = f_{hi} - f_{lo}$, due to the increased magnetostatic coupling between the magnetic layers, which pulls the frequencies closer to each other. One has to comment here, that for extremely thin spacer ($t_{NM} \lesssim 5$ nm), the interlayer exchange interaction becomes dominant, and the dynamics of such double-vortex structure exhibits a single resonance corresponding to the dynamical mode of the exchange-coupled magnetic layers.

The resonant dynamics of the double-vortex structure is mostly defined by the mutual orientation of the core polarities. As seen from figure 2(b), changing the circulation direction in one of the magnetic layers introduces only minor changes to the resonant frequencies of the magnetostatically coupled disks. The direction of the gyrotropic motion, according to the Thiele’s equation [1, 25], is defined by the vortex core polarity and, for our geometry, is counter-clockwise for the CoFe layer resonance ($p_1 = 1$) and clockwise for the Py layer resonance ($p_2 = -1$).

3.1.2. Parallel polarities. For the case of parallel polarities ($p_1 p_2 = 1$), the evolution of the resonant dynamics of the double-vortex structure as a function of $d_{NM}$ is qualitatively dissimilar from the $p_1 p_2 = 1$ case. Figure 3(a) shows the simulated dynamical spectra of the CoFe(10)/NM(d$_{NM}$/)Py(20) tri-layer ($d = 300$ nm) for different $d_{NM}$ for the $(c_1 c_2; p_1 p_2) = (1; 1)$ case. Similar to the $p_1 p_2 = -1$ case, the dynamics of the double-vortex structure corresponds to the resonant gyration of the uncoupled isolated CoFe(10) and Py(20) vortices when the magnetostatic coupling is negligible, i.e. the NM spacer is thick ($t_{NM} \geq 200$ nm). Contrary to the $p_1 p_2 = -1$ case, when the spacer thickness is decreased, i.e for $t_{NM} \lesssim 160$ nm, the high-frequency resonance $f_{hi}$ is blue-shifted towards higher frequencies, whereas low-frequency resonance $f_{lo}$ is red-shifted towards lower frequencies thus increasing the frequency separation $\Delta f$. The frequencies of the excited dynamic modes are defined by the vortex core polarities with only marginal contribution from the in-plane circulations $c_1$ and $c_2$ of the vortices (see figure 3(b)).

In the case of parallel polarities, the increased magnetostatic interaction leads to the ‘repulsion’ of the frequencies of the excited modes and to the decreased amplitude of the low-frequency mode. For example, at $d_{NM} = 60$ nm, the low-frequency mode amplitude is more than one order of magnitude smaller than the amplitude of the high-frequency mode (see figure 3(c)). By examining the resonant trajectories of the corresponding modes, we attribute the low-frequency mode to the so-called small amplitude ‘out-of-phase’ gyration of the CoFe and Py vortex cores, which both gyrate counter-clockwise with the constant lateral separation (see figure 3(d)), i.e. their azimuthal components are $\pi$-shifted. The high-frequency mode corresponds to the large amplitude ‘in-phase’ resonant gyration where both vortex cores gyrate with the same azimuthal components (see figure 3(d)). The observed dynamics is in full agreement with the analytical
Figure 2. (a) Simulated FMR spectra of a CoFe(10)/NM(dNM)/Py(20) double vortex structure as a function of spacer thickness d_{NM} for the \((c_1, c_2; p_1p_2) = (-1; -1)\) case. The plots are vertically offset for better visibility. (b) Resonant frequencies as a function of d_{NM} for the two combinations of in-plane circulations \(c_1\) and \(c_2\). Open stars (solid squares) are used to denote parallel (anti-parallel) circulation directions, respectively. Horizontal bars correspond to the eigen-frequencies of the isolated CoFe(10) and Py(20) disks. (c) FMR spectra of the CoFe(10)/NM(40)/Py(20) tri-layer featuring Py-dominated and CoFe-dominated resonances. The arrow indicates the frequency sweep direction. Lines in (a)–(c) are guides to the eye. (d), (e) Simulated trajectories of the CoFe (black) and Py (red) vortex cores recorded at the resonant excitation frequencies of the plot (c). The trajectories were recorded after 100 gyration periods to ensure that the steady-state dynamics is set.

and micromagnetic description provided in [26] for the case of magnetostatically coupled symmetric double-vortex structure with parallel core polarities.

Hence, the dynamics of two stacked vortices coupled magnetostatically features a complex double resonance behavior with the possibility to manipulate the excited mode type and frequency by adjusting the geometry of the structure (i.e. disk aspect thickness-to-diameter aspect ratio and the spacer thickness) and by properly initiating the mutual orientation of the vortex core polarities.

3.2. Stress-tunable gyration dynamics in CoFe/NM/Py double-vortex structures

In this section, we present the stress-dependent tuning of the resonant frequencies and trajectories of the magnetostatically coupled MS/NM/nMS double-vortex structure. We introduce the magnetoelastic energy \(K_\sigma = \frac{1}{2} \lambda_\sigma \sigma\) to the dynamic simulations of the gyration of the MS vortex core (here CoFe). We show that, for the \(p_1p_2 = -1\) configuration of the vortex polarities, the stress-induced magnetoelastic anisotropy can switch the system from the double-resonance to the single-resonance regime. Moreover, for the case of parallel polarities \((p_1p_2 = 1)\) and strong magnetostatic coupling, we demonstrate the stress-induced lateral shift of the vortex core gyration trajectories.

3.2.1. Antiparallel polarities. Figure 4(a) shows the simulated spectra of the CoFe(10)/NM(60)/Py(20) double vortex structure for the \((c_1, c_2; p_1p_2) = (-1; -1)\) case and for different values of the uniaxial tensile stress \(\sigma_x\) applied to the tri-layer along the \(x\)-axis. The stress introduces an additional magnetoelastic energy \(K_\sigma\) along the \(x\)-axis of the CoFe layer via inverse MS effect. In our study, the in-plane uniaxial stress \(\sigma_x\) is varied in the range from 0 to 600 MPa, yielding the maximum value of the uniaxial anisotropy energy in the CoFe layer \(K_\sigma = 58.5\ \text{kJ m}^{-3}\). Since Py has a negligible magnetostriction constant, no stress-induced anisotropy is exerted on the Py layer. As the \(\sigma_x\) magnitude increases from 0 to 400 MPa, we observe a clear decrease of the resonance frequencies of both CoFe and Py vortex cores. The decrease of the CoFe resonance frequency at moderate stress values is attributed to the stress-induced additional magnetoelastic anisotropy \(K_\sigma\). The Py resonance frequency also follows the decreasing trend, as it is magnetostatically coupled to the CoFe layer via the spacer. Notably, the amplitude of the CoFe (Py) vortex resonance increases (decreases) with increased stress indicating a transition from double-resonance towards CoFe-dominated dynamics. This transition is also confirmed by visualizing the trajectories of the CoFe and Py vortex cores for different values of the applied stress and for different values of the excitation frequency.
Figure 3. (a) Simulated FMR spectra of a CoFe(10)/NM(d_{NM})/Py(20) double-vortex structure as a function of spacer thickness d_{NM} for the (c_1, c_2; p_1, p_2) = (1; 1) case. The plots are vertically offset for better visibility. The amplitudes of the low-frequency branch for d_{NM} = 20; 40 and 60 nm are multiplied by a factor of 10. (b) Resonant frequencies as a function of d_{NM} for different combinations of in-plane circulations c_1 and c_2 denoted by open stars (solid squares) for parallel (anti-parallel) circulation directions, respectively. Horizontal bars correspond to the eigen-frequencies of the isolated CoFe(10) and Py(20) disks. (c) FMR spectra of the CoFe(10)/NM(60)/Py(20) tri-layer featuring low-frequency out-of-phase gyration and high-frequency in-phase gyration resonances. The arrow indicates the frequency sweep direction. Lines in (a)–(c) are guides to the eye. (d), (e) Simulated trajectories of the CoFe (black) and Py (red) vortex cores recorded at the resonant excitation frequencies of the plot (c).

Figure 4. (a) Simulated spectra of a CoFe(10)/NM(60)/Py(20) double-vortex structure for the (c_1, c_2; p_1, p_2) = (−1; −1) case, taken at different uniaxial tensile stress σ_x. The plots are vertically offset for better visibility. (b) Trajectories of the CoFe (black) and Py (red) vortex cores recorded for excitation frequencies f = 0.51; 0.53; 0.55 and 0.57 GHz, without applied stress (σ_x = 0), and for σ_x = 200; 400 and 600 MPa. The table cell size corresponds to the square area of 80 × 80 nm^2 centered in the middle of the disk.

magnetic field frequency (figure 4(b)). For σ_x = 0 and 200 MPa (corresponding to K_σ = 19.5 kJ m^{-3}), the excitations at 0.51 and 0.57 GHz correspond to the Py and CoFe vortex core resonances respectively separating the frequency ranges of the Py-dominated (lower frequency) and the CoFe-dominated (higher frequency) gyration. For σ_x = 400 MPa (corresponding to K_σ = 39 kJ m^{-3}), an onset of the transition towards the CoFe-dominated dynamics is observed, where CoFe is at resonance around f = 0.56 GHz (see green graph in figure 4(a)), while the gyration of the Py vortex core is suppressed as
evidenced by the gyration trajectories of figure 4(b). Notably, when the stress is further increased up to 600 MPa (corresponding to $K_{\text{ex}} = 58.5$ kJ m$^{-3}$), the dynamics of the double-vortex features single-resonance behavior with increased amplitude of the resonant peak (see the bottommost graph in figure 4(a)). This resonance is attributed to the synchronized gyration of both CoFe and Py vortex cores, which is confirmed by examining the gyration trajectories (figure 4(b)).

The observed transition from the double-resonance to the single-resonance dynamics can be qualitatively understood as follows. For the magnetostatically coupled MS/NM/nMS double-vortex structure, the frequency splitting is defined by the resonant eigen-frequencies of the isolated vortices and the strength of the interlayer magnetostatic coupling (as shown in section 3.1.1). Stress-induced magnetoelastic anisotropy leads to the redshifting of the CoFe vortex eigen-frequency. When the magnetostatic coupling between the layers is negligible (i.e. the spacer is thick), both vortices gyrate independently and the redshifting of the CoFe eigen-frequency do not imprint any modification of the Py vortex dynamics (see the simulated dynamics for $d_{\text{NM}} = 160$ nm in the suppl. data (available online at stacks.iop.org/JPD/54/475002/ mmedia)). For decreased spacer thickness, the increased magnetostatic interaction becomes sufficient to overcome the separation between the CoFe and Py eigen-frequencies and to bring the system to large-amplitude gyration of both vortex cores in the synchronized regime.

One also has to comment on the non-linear behavior featured by the peak foldover effect revealed when sweeping the excitation frequency upwards and downwards through the resonance. The hysteretic behavior indicates that the excitation is in the non-linear regime due to a significant mode coupling for a given $d_{\text{NM}}$ and rf field magnitude. The observed hysteresis of the CoFe (Py) related mode increases (decreases) with increasing the stress-induced magnetoelastic anisotropy indicating a transition towards the synchronized dynamics with increased coupling between the Py and CoFe gyration modes. Nevertheless, this non-linearity does not impede the stress-induced effects since in vortex-based nano-oscillators, it is rarely the case that the excitation is in the linear regime. Otherwise only a small gyration of the vortex core around its equilibrium position would be excited and therefore result in a small change of the magnetoelastic dissipation of the nano-oscillator (which is directly related to the output rf power).

3.2.2. Parallel polarities. For the $p_1p_2 = 1$ case (i.e. for parallel polarities), the dynamics of magnetostatically coupled vortices under stress is qualitatively different from the $p_1p_2 = -1$. The dynamics features a high-frequency, large-amplitude in-phase mode and a low-frequency, small-amplitude out-of-phase mode, as described in detail in section 3.1.2. Figure S2(a) in the suppl. data shows the simulated spectra of the CoFe(10)/NM(60)/Py(20) double vortex structure for the $(c_1, c_2; p_1p_2) = (1; 1)$ case and for different values of the uniaxial tensile stress $\sigma_x$ applied along the x-axis. The spacer thickness of $d_{\text{NM}} = 60$ nm corresponds to the moderate magnetostatic coupling (figure 3(b)). When the stress increases, a clear redshift of the in-phase gyration mode is observed. This redshift of the coupled CoFe/NM/Py system is driven by the CoFe vortex eigen-frequency decrease, due to the softening of the restoring force spring constants in the presence of stress-induced magnetoelastic anisotropy. The low-frequency, out-of-phase mode also exhibits a redshifting behavior, caused by the frequency ‘repulsion’ behavior for the magnetostatically coupled vortices with parallel polarities. The resonant gyration trajectories of the high-frequency are concentric, have equal azimuthal components over one period (e.g. both vortices gyrate ‘in-phase’) and their circumference and shape depend only marginally on the applied stress (see figure S2(b) in the suppl. data).

However, when the spacer thickness is reduced to $d_{\text{NM}} = 20$ nm, an unexpected stress-induced behavior of the resonant trajectories is observed. With increasing stress, the observed peak foldover effect with increased hysteresis features the stress-induced transition to the strongly non-linear oscillatory regime due to enhanced static and dynamical coupling between the layers. Moreover, while the resonant frequencies follow the redshifting trend (see figure 5(a)) (similar to the $d_{\text{NM}} = 60$ nm case), the resonant trajectories are significantly modified as the stress increases from 0 to 600 MPa (figure 5(b)). Notably, starting from $\sigma_x = 200$ MPa, the trajectories of CoFe and Py vortex cores are no longer concentric but exhibit a lateral separation while conserving the ‘in-phase’ gyration character with equal azimuthal components. In the presence of the magnetoelastic anisotropy, both CoFe and Py vortex core trajectories of the in-phase mode deviate from the steady state circular orbits (see figure 5(b) for $\sigma_x = 0$) and exhibit low-frequency beatings around the equilibrium point (see figure 5(b) for $\sigma_x = 200, 400$ and 600 MPa). Figure 6(a) shows the time-varying y-coordinate of the CoFe and Py vortex cores recorded for 100 ns (corresponding to 84 gyration periods at $f = 0.84$ GHz) at $\sigma_x = 200$ MPa. The time traces reveal the periodic quasi-sinusoidal modulation of the circular vortex core trajectory, with the beating frequency of 40 MHz, extracted by fast Fourier transform of the time-varying y-coordinate (see figure 6(c)). The beatings of CoFe and Py vortex cores are in ‘anti-phase’, i.e. when, at certain moments in time, the y-coordinate of the CoFe vortex core reaches the maximum (minimum), the corresponding y-coordinate of the Py core is at its minimum (maximum), respectively. Thus, the maximum separation between the trajectories increases from ~32 nm for $\sigma_x = 200$ MPa to ~43.5 nm for $\sigma_x = 600$ MPa. Notably, when the stress is increased up to 600 MPa, the beatings become chaotic without any visible periodic pattern (see figure 6(b)). It is obvious that the frequency of these beatings is almost one order of magnitude lower comparing to that of the low-frequency out-of-phase gyration mode $f_{\omega}$, therefore one can conclude that these ‘beatings’ essentially differ from the low-frequency variation of the gyration trajectories described in [25] as a result of strong dynamic interlayer magnetostatic interaction.

Notably, the observed effects of the trajectories separation and beatings are present only for thin NM spacers ($d_{\text{NM}} \leq 20$ nm) and for the $c_1; c_2 = 1$ case, i.e. when the stacked vortices have the same in-plane circulations. This
Figure 5. (a) Simulated spectra of the 300 nm diameter CoFe(10)/NM(20)/Py(20) double-vortex structure for the \((c_1, c_2; p_1, p_2) = (1; 1)\) case taken at different uniaxial tensile stress \(\sigma_x\). The plots are vertically offset for better visibility. The amplitudes of the low-frequency branch are multiplied by a factor of 10. (b) Trajectories of the CoFe (black) and Py (red) vortex cores recorded for excitation frequencies \(f = 0.78, 0.82, 0.84\) and \(0.86\) GHz without applied stress \((\sigma_x = 0)\) and for \(\sigma_x = 200, 400\) and \(600\) MPa. The table cell size corresponds to the square area of \(80 \times 80\) nm\(^2\) centered in the middle of the disk.

Figure 6. (a), (b) Time traces of the \(y\)-coordinate of the resonant vortex core trajectories of figure 5(b) taken for \(\sigma_x = 200\) MPa (a) and \(\sigma_x = 600\) MPa (b). (c) FFT spectra of the time traces of the \(y\)-coordinate as a function of stress.

‘chaotic repulsion’ of the trajectories appears due to the increased interaction between the stress-induced magnetoelastic anisotropy and the strong magnetostatic coupling between two closely stacked magnetic vortices in the repulsion potential regime \((c_1, c_2; p_1, p_2) = (1; 1)\). Such complex dynamics of the stacked vortex pair, modulated by the stress-induced magnetoelastic anisotropy, allows controlling not only the gyration frequencies but also the lateral separation of the vortex core trajectories. This leads to the non-zero net magnetization change over the single gyration period, thus allowing the magnetoresistive readout of the in-phase gyration mode, which is often elusive when no analyzer layer is used. The observation of the chaotic dynamics with the ‘degree of chaoticity’ tunable by stress may be potentially utilized in non-Boolean computing schemes using arrays of spin-torque oscillators with additional tuneability parameter provided by stress.
4. Analytical model of the dynamics of MS/NM/nMS double-vortex under stress

To obtain a more general insight into the dynamics of a magnetoelastically coupled double-vortex structure in the presence of magnetostatic anisotropy, we have extended the analytical model described in [26] by including stress-dependent energy terms. More specifically, we consider a system of two magnetoelastically coupled disks with the following parameters: saturation magnetization $\mathit{M}_s$, disk thickness $\mathit{L}_1$, vortex core polarity $\mathit{p}_n$, magnetization circulation $\mathit{c}_i$, exchange length $\mathit{l}_{ex,i}$, disk radius $R = R_1 = R_2$ ($i = 1, 2$). The indices 1 and 2 stand for the CoFe and the Py layer, respectively. Thus, a total energy of the CoFe/NM/Py tri-layer including the demagnetizing energy, the magnetostatic interlayer interaction energy, the magnetostatic coupling to the interlayer interaction, can be expressed in the following form (see the derivation in the supplementary material):

$$W_{tot} = \frac{1}{2}(\kappa_1 - \kappa_2)\mathit{L}_1^2 + \frac{1}{2}(\kappa_1 - \kappa_2)\mathit{L}_2^2 + \frac{1}{2}\kappa_2(\mathit{x}_1^2 + \mathit{y}_1^2) + (\mu + \mu_2^\sigma)c_1\mathit{c}_2\mathit{x}_1\mathit{x}_2 + (\mu + \mu_2^\sigma)c_1\mathit{c}_2\mathit{y}_1\mathit{y}_2,$$

(1)

with:

$$\kappa_i = \mu_0 \mathit{L}_i^2 \left[4\pi F_x(\beta_i) - \frac{1}{2} \left(\frac{\mathit{l}_{ex,i}}{\mathit{R}}\right)^2\right],$$

$$\mu = \pi \mu_0 \mathit{M}_1 \mathit{M}_2 R \left[\frac{\mathit{L}_1}{\mathit{L}_2} F(\beta_2, d_i) + \frac{\mathit{L}_2}{\mathit{L}_1} F(\beta_1, d_i)\right],$$

(2)

$$\mu_2^\sigma = \frac{4}{3}\pi \mathit{L}_1 \lambda_1 \mathit{c}_1 \mathit{c}_2 \mathit{x}_1 \mathit{x}_2,$$

$$\mu_2^\sigma = \frac{6}{\pi} \mathit{L}_2 \mathit{M}_2 \left(\frac{\mathit{L}_2}{\mathit{L}_1} F(\beta_1, d_i)\right) \lambda_1 \mathit{c}_1 \mathit{c}_2 \mathit{x}_1 \mathit{x}_2,$$

(3)

and,

$$F_x(\beta) = \int_0^{\infty} \frac{1}{t} \left[1 - \frac{e^{-\beta t}}{\beta t}\right] \mathit{I}^2(t) dt,$$

$$F(\beta, d_i) = \int_0^{\infty} \frac{e^{-\beta t}}{t^2} \left[1 - e^{-\beta t}\right] \mathit{F}(t) dt,$$

$$\mathit{I}(t) = \int_0^{1} \mathit{x} \mathit{J}_1(\mathit{tx}) dx,$$

(4)

where $(\mathit{x}_i; \mathit{y}_i)$ are the coordinates of a displaced vortex core of the $i$th layer, $\beta_i = \mathit{L}_i/R$ is the disk aspect ratio, $d_i = \mathit{d}_{iNM}/R$ is the normalized separation between the disks, $\sigma_{x,y}$ are the in-plane stress components and $\mathit{J}_1$ is the first-order Bessel function of the first kind.

The system of Thiele equations for a pair of coupled vortices subjected to a uniaxial in-plane stress $\mathit{\tau}$, which follows from $W_{tot}$, can be solved analytically (see supplementary material) and yields the resonance frequencies:

$$f_\pm = \frac{1}{2\pi} \sqrt{\tau \pm \sqrt{\tau^2 - \rho^2}},$$

(5)

$$\tau = \frac{1}{2} \left[\kappa_1 \kappa_2 + \rho_i^2 \kappa_1 \kappa_2 + \rho^2 \kappa_1 \kappa_2\right] + \hat{\kappa}_1 \mu^\sigma, \quad \rho^2 = \kappa_1 \kappa_2 \rho_i^2 \kappa_1 \kappa_2 + \kappa_2 \mu^\sigma \rho^2 \kappa_2,$$

(6)

and,

$$\hat{\kappa}_1 = \frac{\kappa_1 - \kappa_2}{p_1 G_1}, \quad \hat{\kappa}_2 = \frac{\kappa_2}{p_2 G_2},$$

$$\hat{\mu}_1 = \frac{(\mu + \mu_2^\sigma) c_1 c_2}{p_1 G_1},$$

(7)

where $G_i = \frac{2\pi}{\pi} \mathit{L}_i^2$ is the gyromagnetic ratio ($\gamma/2\pi = 28$ GHz/T is the reduced gyromagnetic ratio).

Figures 7(a) and (b) show the resonance frequencies of the CoFe(10)/NM(d$_{iNM}$)/Py(20) double-vortex structure as a function of $d_{iNM}$ calculated using equation (5) for $\mathit{\sigma}_x = 0$. (c) Resonant frequencies as a function of the uniaxial stress $\mathit{\sigma}_x$ calculated using equation (5) for (c) $p_1p_2 = -1$ and $d_{iNM} = 60$ nm and (d) $p_1p_2 = 1$ and $d_{iNM} = 20$ nm.

![Figure 7](image-url)
structure as a function of the stress applied to the tri-layer in the range from 0 to 300 MPa. For the $p_1p_2 = -1$ case (figure 7(c)), when the stress increases from 0 to 100 MPa, the frequency of the CoFe-dominated mode decreases rapidly whereas the frequency of the Py-dominated mode remains almost constant, in qualitative agreement with the simulations data. As the stress value reaches $\sim$150 MPa, a hybridization of the CoFe and Py modes is observed. This region (marked by a blue circle in figure 7(c)) corresponds to the trajectories synchronization revealed by the micromagnetic simulations. The observed behavior is attributed to the competition between the magnetoelastic energy and interlayer magnetoelastic interaction energy. More specifically, with increasing stress, the CoFe restoring force spring constant $\tilde{\kappa}_1$ softens (see equation (7)) which leads to the decrease of the CoFe-related resonant frequency. On the contrary, the magnetoelastic interaction $\mu$ between CoFe and Py layers increases (see equation (8)) with increased stress, enhancing the pulling of the Py-related frequency by the CoFe resonance. In the simulations, the stress values needed to pull the CoFe frequency to match the Py frequency are few times higher as compared to the values derived analytically due to the fact that in the simulations, the vortex cores were excited in strongly non-linear regime in order to mimic the real experimental conditions.

For the $p_1p_2 = 1$ case (figure 7(d)), the interplay between the stress-induced magnetoelastic anisotropy and interlayer magnetoelastic interaction acts in a similar way leading to the decrease of the frequencies of both high-frequency and low-frequency coupled modes due to the softening of the CoFe restoring force spring constant $\tilde{\kappa}_1$. These results reproduce the data obtained micromagnetically in section 3.2 confirming that the observed effects in stacked double-vortex structures originate from the interplay between the stress-induced magnetoelastic anisotropy and interlayer magnetoelastic interaction.

5. Conclusions and outlook

In this paper, we focused on the dynamics of the MS/NM/nMS double-vortex structure with different $(c,p)$ combinations as a function of the uniaxial stress, and showed that complex dynamics of the coupled vortices occurs when the interlayer magnetoelastic coupling is present in addition to the magnetoelastic anisotropy. The micromagnetic simulation results reveal that, depending on the vortex core polarities, the effect of the magnetoelastic anisotropy is qualitatively different and leads to the synchronization of the gyration frequencies for the case of parallel polarities, or to the lateral separation of the gyration trajectories for the case of anti-parallel polarities. In this paper, we considered the double-vortex structure with one MS layer. However, when the double-vortex structure consists of two MS layers with magnetostriction constants of opposite sign (e.g. CoFe ($\lambda_0 > 0$) and Ni ($\lambda_0 < 0$)), more complex stress-induced behavior is expected, which is beyond of the scope of this paper.

The experimental detection of the effects predicted here can be performed, for example, by measuring the microwave absorption spectra of the double-vortex structures excited by a microwave Oersted field from the adjacent stripe line antenna. Another detection method consists of passing the rf current laterally through the structure using contact pads and measuring the microwave rectification voltage resulting from mixing of the oscillating magnetoresistance and injected rf current [27, 28]. The latter method is considered more efficient due to the selective addressing and presence of an additional spin transfer torque acting on the vortex core. However, it may also introduce the unwanted Joule heating and non-linear effects due to large driving forces hindering the efficient excitation of the vortex cores in such a tri-layer structure. The initial states with different mutual orientations of the core polarities can be initiated on-demand by proper sequence of out-of-plane magnetic fields applied to the tri-layer and/or dc electrical currents passed vertically through the structure [5].

The stress-induced magnetoelastic anisotropy can be introduced via converse piezoelectric effect when the MS/NM/nMS tri-layer is grown and patterned on a ferroelectric relaxor substrate with large piezoelectric coefficients. For example, for commercially available lead magnesium niobate-lead titanate (Pb$\text{Mg}_{1/3}\text{Nb}_{2/3}$)$_2\text{Ti}_3\text{O}_{12}$, PMN-PT) single crystals, the piezoelectric coefficient can be as high as 1500 ppm [29]. Considering that the Young’s modulus $Y$ for CoFe is 250 GPa [30], the corresponding electric fields needed to obtain the stress values used in this work (up to 600 MPa) are close to 1 MV m$^{-1}$ which can be easily achieved by proper electrodes design.

Data availability statement

The datasets simulated and analyzed in this work as well as the Python-based code for analytical computing of the resonant frequencies are available from the corresponding author upon reasonable request.

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