MODEL-INDEPENDENT LIMITS ON THE LINE-OF-SIGHT DEPTH OF CLUSTERS OF GALAXIES USING X-RAY AND SUNYAEV–ZELODovich DATA

ANDISHEH MAHDAVI1,2 AND WEIHAN CHANG1
1 Department of Physics and Astronomy, San Francisco State University, San Francisco, CA 94131, USA
2 Kavli Institute for Theoretical Physics, Kohn Hall, University of California, Santa Barbara, CA 93106–4030, USA

Received 2011 March 25; accepted 2011 May 11; published 2011 June 3

ABSTRACT
We derive a model-independent expression for the minimum line-of-sight extent of the hot plasma in a cluster of galaxies. The only inputs are the 1–5 keV X-ray surface brightness and the Comptonization from Sunyaev–Zel’dovich (SZ) data. No a priori assumptions regarding equilibrium or geometry are required. The method applies when the X-ray emitting material has temperatures anywhere between 0.3 keV and 20 keV and metallicities between 0 and twice solar-conditions fulfilled by nearly all intracluster plasma. Using this method, joint APEX-SZ and Chandra X-ray Observatory data on the Bullet Cluster yield a lower limit of 400 ± 56 kpc on the half-pressure depth of the main component, limiting it to being at least spherical, if not cigar-shaped primarily along the line of sight.

Key words: galaxies: clusters: general – galaxies: clusters: individual (Bullet Cluster) – galaxies: clusters: intracluster medium – X-rays: galaxies: clusters

1. INTRODUCTION

The hot intracluster medium (ICM) in clusters of galaxies emits X-rays via thermal bremsstrahlung and line emission; the very same atmosphere blueshifts cosmic microwave background photons via the Sunyaev–Zel’dovich (SZ) effect (for a review see Birkinshaw 1999; Carlstrom et al. 2002, and references therein). The X-ray emissivity scales as electron density squared, but the SZ spectrum modification depends on the integral of pressure along the line of sight (and hence only linearly on the electron density). As a result, if the cosmology is known, it is possible to recover structural information about the cluster atmosphere (Myers et al. 1997; Hughes & Birkinshaw 1998; Grego et al. 2000). Such “deprojection,” however, has always required drastic assumptions regarding the three-dimensional structure of the ICM; in general, imposing hydrostatic equilibrium is often required (Zaroubi et al. 2001). Most importantly, spherical or triaxial symmetry have always been necessary assumptions as well (Pointecouteau et al. 2002; Lee & Suto 2004; De Filippis et al. 2005; Ameglio et al. 2007, 2009; Nord et al. 2009; Basu et al. 2010; Allison et al. 2011). So far, forward convolution of simple, smooth, parameterized ICM thermodynamic variables has been the only way to tie the X-ray and SZ observables together to determine cluster geometry.

However, a large fraction of clusters are not relaxed and are poorly described by hydrostatic or spherically symmetric models (e.g., Eckert et al. 2011). The most spectacular and interesting of these systems are violent mergers such as the Bullet Cluster (Clowe et al. 2006), A520 (Mahdavi et al. 2007), or MACSJ0025 (Brdarč et al. 2008). These systems clearly resist any attempt at description using symmetric equilibrium models. At the same time, it would be highly valuable to recover some information regarding the structure of such objects along the line of sight. Such information could be used to constrain the parameter space of supercomputer N-body models that seek to replicate and thus better understand the collisions (e.g., Mastropietro & Burkert 2008; Randall et al. 2008; Forero-Romero et al. 2010).

In this Letter, we show that the Cauchy–Schwartz integral inequality can yield useful information on the depth of a cluster of galaxies without assumptions regarding the geometry or thermodynamic state of the ICM. In Section 2 we derive the inequality, and in Section 3 we interpret it. In Section 4 we apply it to the Bullet Cluster as a test case, and in Section 5 we conclude our discussion.

2. THEORETICAL FRAMEWORK

In a cluster of galaxies, the outgoing surface brightness of a column of hot, single-phase X-ray emitting plasma is

\[ \Sigma = \int_{E_1}^{E_2} dE \int_{-\infty}^{\infty} n_e n_H \Lambda_E(T, Z) dz, \]  

(1)

where the emission is in a rest-frame bandpass between photon energies \(E_1\) and \(E_2\). The electron and hydrogen densities are \(n_e\) and \(n_H\), \(T\) is the plasma temperature, \(Z\) is the plasma metallicity in solar units, and \(\Lambda_E\) is the cooling function incorporating the bremsstrahlung continuum as well as line emission. For convenience we can recast this formula as

\[ \Sigma = \int_{-\infty}^{\infty} n_e^2 \Lambda_{BP}(T, Z) dz, \]  

(2)

where \(\Lambda_{BP}(T, Z)\) is the cooling function integrated over the X-ray bandpass and multiplied by the electron-to-proton ratio, \(n_e/n_H\), of the plasma, which varies less than 1% for typical ICM metallicities.

In the non-relativistic limit, the fractional SZ decrement is proportional to the Compton y parameter:

\[ y = \frac{1}{m_e c^2} \int_{-\infty}^{\infty} n_e \sigma_T k T dz, \]  

(3)

where \(\sigma_T\) is the Thomson cross section, \(m_e\) is the electron mass, \(c\) is the speed of light, and \(k\) is Boltzmann’s constant. We now also define the half-pressure depth \(\delta_H\) such that

\[ y = \frac{2}{m_e c^2} \int_0^{\delta_H} n_e \sigma_T k T dz, \]  

(4)
where the origin of the \( z \)-axis is arbitrary as long as the above equation is fulfilled. In other words, half the integrated pressure of the cluster along the line of sight is confined to a region of size \( z_H \).

The X-ray surface brightness and Compton \( y \) parameter involve integrals of the thermodynamic variables along the line of sight. They therefore cannot be inverted without assumptions regarding the geometry of the cluster. However, consider the Cauchy–Schwartz inequality:

\[
\left( \int f(x)g(x)dx \right)^2 \leq \int f(x)^2dx \int g(x)^2dx. \tag{5}
\]

Here, we show that this integral is useful in recasting the depth measurement problem in model-independent terms. Specifically, consider the choice \( f = n_eT, \ g = 1 \); then, integrating only along the half-pressure depth, we have

\[
\int_0^{z_H} dz \geq \left[ \int_0^{z_H} n_eT dz \right]^2 \left/ \int_0^{z_H} n_e^2T^2dz \right. \tag{6}
\]

Furthermore, since the square of any function is more compact than the function itself, Equations (3) and (4) together imply that

\[
\int_{-\infty}^{\infty} n_e^2T^2dz \geq 2 \int_0^{z_H} n_e^2T^2dz. \tag{7}
\]

The above inequality follows from the fact that the variance of any square-integrable function \( f(x) \) is equal to or larger than the variance of \( f(x)^2 \). Therefore, it is always possible to choose a \( z \)-axis origin such that \([0, z_H] \) contains half the integrated pressure but more than half the integral of the pressure squared. We therefore obtain

\[
\int_0^{z_H} dz \geq 2 \left[ \int_0^{z_H} n_eTdz \right]^2 \left/ \int_{-\infty}^{\infty} n_e^2T^2dz \right. \tag{8}
\]

Combining Equation (8) with the Comptonization parameter (Equation (3)), and multiplying by \( \Sigma/\Sigma = 1 \), we can write

\[
z_H \geq \frac{m_e^2e^4y^2}{2\sigma_T^2 \int_{-\infty}^{\infty} n_e^2T^2dz \times \Sigma} \tag{9}
\]

This constraint on cluster depth can be rewritten as

\[
z_H \geq \ell \times \xi, \tag{10}
\]

where we define \( \ell \) as the fiducial depth of the cluster

\[
\ell \equiv \frac{m_e^2e^4y^2}{2\sigma_T \Sigma} \tag{11}
\]

\[
\approx 0.56 \left( \frac{y}{10^{-4}} \right)^2 \left( \frac{\Sigma}{10^{16} \text{erg s}^{-1} \text{Mpc}^{-2}} \right)^{-1} \text{Mpc} \tag{12}
\]

and where we define the dimensionless, bandpass-dependent thermodynamic uncertainty factor

\[
\xi \equiv m_e \int n_e^2 \Lambda_{BP}(T, Z) dz \left/ \sigma_T \int n_e^2k^2T^2dz \right. \tag{13}
\]

If we were dealing with a medium of constant temperature and metallicity, we could now pull out the functions of \( T \) and \( Z \), cancel the emission measure integrals, and be done; but we do not make this assumption and continue with the general case.

So far we have shown that given a measurement of the Compton \( y \) parameter and the X-ray surface brightness, the half-pressure depth of the cluster along the line of sight is at least \( \ell \) (Equation (12)) times a correction factor \( \xi \) (Equation (13)).

We now attempt to derive useful bounds on \( \xi \). Suppose that, for any given X-ray bandpass, \( \xi > \xi_{low} \) for all plausible values of \( T \) and \( Z \). In that case without any loss of generality we can write

\[
z_H \geq \ell \times \xi_{low}. \tag{14}
\]

To find \( \xi_{low} \), imagine three arbitrary one-dimensional positive functions \( n_e(z), p(z), q(z) \), and \( g(z) \). If \( p/q > \xi_{low} \) everywhere, then necessarily \( p > \xi_{low}q \), and so \( n_e^2p > \xi_{low}n_e^2q \) everywhere. It follows that \( \int_{z_H}^{\infty} n_e(z)^2p(z) > \xi_{low} \int_{z_H}^{\infty} n_e(z)^2q(z) \). In other words, the ratio of two one-dimensional integrals is always greater than the smallest value taken on by the ratio of the integrands, as long as the integrands are positive and the limits of integration are the same.

Choosing \( p = m_e \Lambda_{BP}(T, Z) \) and \( q = \sigma_T k^2T^2 \), it follows that \( \xi > \xi_{low} \) if

\[
\xi_{low} = \text{minimum of} \frac{m_e \Lambda_{BP}(T, Z)}{\sigma_T k^2T^2}. \tag{15}
\]

for all \( Z \) and \( T \) of interest. Our task then becomes one of minimizing the quantity

\[
\gamma = \frac{m_e \Lambda_{BP}(T, Z)}{\sigma_T k^2T^2}. \tag{16}
\]

3. Interpretation

The lower limit on the half-pressure depth of a cluster of galaxies becomes \( \ell \times \xi_{low} \), where \( \xi_{low} \) is the minimum of the bandpass-dependent quantity \( \gamma \) over all temperatures and metallicities contributing to the SZ and X-ray fluxes. Since the cooling function for thermal bremsstrahlung \( \Lambda(T, Z) \sim \sqrt{T} \) for \( T \gg 10 \text{ keV} \), it follows that \( \gamma \sim T^{-3/2} \) for large \( T \), and the \( z_H \) constraint requires the assumption of a high-temperature cutoff in order to be relevant (i.e., to avoid the trivial result \( z_H > 0 \)).

We call this high-temperature cutoff \( T_{max} \). We leave \( T_{max} \) flexible on a cluster-by-cluster basis. Its value can be constrained via modeling of sufficiently high signal-to-noise X-ray spectra, or else set to a sufficiently high value (e.g., no cluster is believed to possess thermal gas beyond 20 keV).

A low temperature cutoff, \( T_{min} \), is also needed, because \( \xi_{low} \rightarrow 0 \) whenever \( \Lambda_{BP}(T, Z) \rightarrow 0 \), i.e., when there is a column of material emitting outside the X-ray bandpass. In a bandpass centered on photon energy \( E \), all emission with \( T \gtrsim E/3 \) contributes to the X-ray flux, but emission with \( T \lesssim E/3 \) contributes negligibly. For most common X-ray bandpasses, this translates to \( T < 0.3 \text{ keV} \). Searches for very cool, \( T < 0.3 \text{ keV} \) material—typically referred to as the warm-hot intergalactic medium or WHIM—have thus far been inconclusive. While the WHIM must exist, it likely has densities \( \lesssim 5 \times 10^{-5} \text{ cm}^{-3} \) (Nicasio et al. 2005), and thus would contribute negligibly to the overall SZ and X-ray flux in the direction of a massive cluster, where the typical gas densities are \( 10^{-4} \text{ cm}^{-3} \) or higher. We therefore assume that in any source with both a significant SZ decrement and a measured
temperature is above 0.3 keV. We note that beyond the relativistic correction to Equation (3), which we neglect here, Chandra
The Astrophysical Journal Letters
T\text{at} < T\text{at} \geq T_{\text{min}}. We extract a region of radius 182 kpc
set (ObsID 5356). We reanalyze an archived 100 ks Chandra X-ray Observatory data set (ObsID 5356). We extract a region of radius 182 kpc
(0′691 at $H_0 = 71 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_M = 0.23$) centered on the main portion (i.e., the true X-ray center of the diffuse component of the cluster). The aperture was selected to match the beam size of the APEX-SZ experiment. We refer to Mahdavi et al. (2007) for all X-ray data reduction procedures, including particle background subtraction, point-source masking, and other details.
Using a redshift $z = 0.296$, we obtain a best-fit temperature of 11.5 ± 0.6 keV, best-fit metallicity of 0.23 ± 0.05 solar, and best-fit absorbing column of $(7.1 \pm 0.5) \times 10^{20} \text{ cm}^{-2}$. The X-ray flux in the 0.77–3.9 keV band (corresponding to the 1–5 keV rest-frame band) is $(1.89 \pm 0.01) \times 10^{-12} \text{ erg cm}^{-2} \text{ s}^{-1}$. Multiplying by $4\pi$ times the square of the luminosity distance at the above cosmology (1517 Mpc) and dividing by the projected area of 0.104 Mpc², we obtain a rest-frame surface brightness of $(5.03 \pm 0.03) \times 10^{45} \text{ erg s}^{-1} \text{ Mpc}^{-2}$.
Via Equation (12) the observations therefore imply an upper limit
\[ z_H \geq [0.56 \times (3.2)^2 \times (5.0)^{-1}] \zeta_{\text{low}}. \] (18)
To constrain $\zeta_{\text{low}}$, we need an estimate of the highest temperature gas along the line of sight. To be conservative, we assume that temperatures up to 20 keV contribute significantly to the emission measure (this is consistent with the measured X-ray spectrum). Plugging 20 keV into Equations (17) and (18) yields a lower limit
\[ z_H \geq 400 \pm 56 \text{ kpc}, \] (19)
where the dominant source of error is the statistical uncertainty on the Compton y parameter. The (unknown) systematic bias in y is not included.
Now we attempt to calculate the axial ratio of the main (eastern) component of the cluster, comparing the line-of-sight half-pressure depth $z_H$ to the plane-of-sky half-pressure width, which we call $w_H$. Halverson et al. (2009) find that
\[ \Delta T/T \propto (1 + \theta^2/\beta^2)^{1-3\beta/2}, \] (20)
where $\beta = 1.2 \pm 0.13$ and $\theta_e = 142'' \pm 18''$. Numerically integrating this profile, we find that half the flux is contained within a circular aperture of radius 0.67$k_e$, which translates into a half-pressure width of $w_H \approx 2 \times 0.67 \times 325 \text{ kpc} \approx 435 \text{ kpc}$. Note that this is likely more an upper limit than an accurate measurement, given the mixing of the bullet and main component signals due to poor resolution of the APEX–SZ instrument. However, the direction of this inequality works in our favor, because a lower limit on $z_H$ divided by an upper limit on $w_H$ still yields a valid lower limit on $z_H/w_H$:
\[ \frac{z_H}{w_H} \geq 0.92. \] (21)
Thus, the ratio of the half-pressure diameter of the Bullet Cluster to its half-pressure width in the sky is constrained to be greater than approximately one.

4. APPLICATION TO THE BULLET CLUSTER
To demonstrate this technique, we jointly analyze X-ray and SZ observations of the Bullet Cluster. This cluster is a particularly apt candidate for this type of analysis, due to the clear incorrectness of typical spherically symmetric models in describing its morphology. The Bullet Cluster consists of a “main” diffuse component plus a “bullet” to the west. Using 150 GHz APEX-SZ, Halverson et al. (2009) infer a Comptonization for the main component of the cluster of $\gamma_0 = (3.4 \pm 0.3) \times 10^{-4}$. This Compton y measurement is not model-independent—it relies to some extent on the precise two-dimensional shape of the cluster in the plane of the sky, reflecting the limited angular resolution of APEX-SZ experiment. Further uncertainties arise from the fact that some leakage from the Bullet 1.5 to the west into the central APEX-SZ beam is likely. It is difficult to estimate the bias in y introduced here, first because the Bullet is both denser and cooler than the main component of the cluster, and second because the Compton y model adopts the X-ray profile as a strong prior. A further issue is that the azimuthally averaged profile has better statistics at radii which include the Bullet (and therefore the fit may be particularly driven by those regions). These shortcomings highlight the need for higher angular resolution SZ experiments if bias-free three-dimensional measurements of cluster structure are to be achieved. Such observations would allow for more complicated two-dimensional Compton y models and thus overcome much of the systematic problems of the smooth symmetric models.
To measure the rest-frame 1–5 keV surface brightness, we reanalyze a set of 100 ks Chandra X-ray Observatory data set (ObsID 5356). We extract a region of radius 182 kpc

\[ \zeta_{\text{low}} = 1.883 + \frac{54.12}{T_{\text{max}}/\text{keV}} - \frac{19.05}{(T_{\text{max}}/\text{keV})^2}, \] (17)

\[ \text{i.e.,} \ \zeta_{\text{low}} \text{ has no dependence on the coolest material as long as its temperature is above 0.3 keV. We note that beyond } T = 20 \text{ keV the relativistic correction to Equation (3), which we neglect here, becomes important. The above formula is therefore only valid between } T_{\text{max}} = 3.5 \text{ and 20 keV.} \]
Figure 1. Dimensionless thermodynamic uncertainty factor $\zeta_{\text{low}}$ as a function of maximum plasma temperature $T_{\text{max}}$, calculated for the rest-frame 1–5 keV bandpass. The curve is given by Equation (17).

where $\zeta_{\text{low}}$ is a dimensionless number which depends on the specific bandpass used and the maximum ICM temperature that contributes significantly to the SZ effect. Equation (17) gives an empirical fitting formula for $\zeta_{\text{low}}$ in the 1–5 keV rest-frame bandpass (also shown in Figure 1).

This lower limit is a function of largely model-independent observables: $y$ (the Compton $y$ parameter) and $\Sigma$ (the X-ray surface brightness). No deprojection, and no modeling beyond standard reduction of the SZ decrement and X-ray spectrum, is required. The method neglects relativistic SZ corrections and assumes that the bulk of the pressure and cooling is done by gas with temperatures between 0.3 keV and 20 keV, and metallicities between 0 and 2 times solar. Otherwise, the gas can have an arbitrary distribution along the line of sight. The constraint can be made more stringent by relaxing the upper limit of gas temperatures allowed; one way to do this in the future is via measurements of the differential emission measure $dn_e^2/dT$.

This requires some more sophisticated spectral modeling, but still no assumptions regarding geometry or equilibrium.

These results are useful in a number of scenarios. For merging clusters of galaxies such as the Bullet Cluster, they can help constrain the geometry of the merger. Using this method, our analysis of joint APEX–SZ and Chandra data for the Bullet Cluster imply a minimum line-of-sight half-pressure depth of $>400$ kpc, implying a minimum line-of-sight to plane of the sky axial ratio of $\approx 1$.

Thus, the Bullet Cluster is constrained to be at least spherical, if not cigar-shaped primarily along the line of sight. For larger samples of X-ray emitting clusters, this technique can help quantify possible selection biases toward line-of-sight elongations, as well as possibly aid in determining the geometry of shock and cold fronts.

We thank James Allison and the anonymous referee for useful comments. This research was made possible by NASA through Chandra award no. AR0-11016A, issued by the Chandra X-ray Observatory Center, which is operated by the Smithsonian Astrophysical Observatory for and on behalf of NASA under contract NAS8-03060. This research was supported in part by the National Science Foundation under Grant No. NSF PHY05-51164.

REFERENCES

Allison, J. R., Taylor, A. C., Jones, M. E., Rawlings, S., & Kay, S. T. 2011, MNRAS, 410, 341
Ameglio, S., Borgani, S., Pierpaoli, E., & Dolag, K. 2007, MNRAS, 382, 397
Ameglio, S., Borgani, S., Pierpaoli, E., Dolag, K., Ettori, S., & Morandi, A. 2009, MNRAS, 394, 479
Basu, K., et al. 2010, A&A, 519, A29
Birkinshaw, M. 1999, Phys. Rep., 310, 97
Bradač, M., Allen, S. W., Treu, T., Ebeling, H., Massey, R., Morris, R. G., von der Linden, A., & Applegate, D. 2008, ApJ, 687, 959
Carlstrom, J. E., Holder, G. P., & Reese, E. D. 2002, ARA&A, 40, 643
Clowe, D., Bradač, M., Gonzalez, A. H., Markevitch, M., Randall, S. W., Jones, C., & Zaritsky, D. 2006, ApJ, 648, L109
De Filippis, E., Serena, M., Bautz, M. W., & Longo, G. 2005, ApJ, 625, 108
Eckert, D., Molendi, S., & Paltani, S. 2011, A&A, 526, A79
Forero-Romero, J. E., Gottlöber, S., &Yepes, G. 2010, ApJ, 725, 598
Grego, L., Carlstrom, J. E., Joy, M. K., Reese, E. D., Holder, G. P., Patel, S., Cooray, A. R., & Holzapfel, W. L. 2000, ApJ, 539, 39
Halverson, N. W., et al. 2009, ApJ, 701, 42
Hughes, J. P., & Birkinshaw, M. 1998, ApJ, 501, 1
Lee, I., & Suto, Y. 2004, ApJ, 601, 599
Mahdavi, A., Hoekstra, H., Babul, A., Sievers, J., Myers, S. T., & Henry, J. P. 2007, ApJ, 664, 162
Mastropietro, C., & Burkert, A. 2008, MNRAS, 389, 967
Mewe, R., Gronenschild, E. H. B. M., & van den Oord, G. H. J. 1985, A&AS, 62, 197
Myers, S. T., Baker, J. E., Readhead, A. C. S., Leitch, E. M., & Herbig, T. 1997, ApJ, 485, 1
Nicastro, F., Elvis, M., Fiore, F., & Mathur, S. 2005, Adv. Space Res., 36, 721
Nord, M., et al. 2009, A&A, 506, 623
Pointecouteau, E., Hattori, M., Neumann, D., Komatsu, E., Matsuo, H., Kuno, N., & Böhringer, H. 2002, A&A, 387, 56
Randall, S. W., Markevitch, M., Clowe, D., Gonzalez, A. H., & Bradač, M. 2008, ApJ, 679, 1173
Zaroubi, S., Squires, G., de Gasperis, G., Evrard, A. E., Hoffman, Y., & Silk, J. 2001, ApJ, 561, 600