The Influence of LC Filter on the Current Control of PWM-Fed Induction Motor Considering the Effect of Back-EMF

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Abstract. The high switch speed pulse width modulation (PWM) significantly improves the performance of induction motor. However, this mode of control generates PWM ripples and a wide spectrum of over-voltages at the motor terminal which stress the motor insulation and cause bearing current and electromagnetic interference (EMI). To solve this problem, LC passive filter is widely used because of its effectiveness on limit overvoltage and PWM ripples. This paper discuss the influence of LC filter on PWM-fed induction motor in a different view. The impedance of the motor is decided by its inductance and resistance in traditional research. In this paper, a more accuracy model is built up involving the effect of back electromotive force (EMF). Based on which, the situation of current control is investigated when the LC filter is employed. It is revealed that the inductance of the filter is the key factor of control bandwidth and a method to enhance current control bandwidth while avoid the loss of suppression ability of differential-mode PWM frequency components is given.

1. Introduction

The growing use of induction motors for high power adjustable speed applications is essentially due to the quick technological evolution of fast switching electronic devices, such as the insulated gate bipolar transistors (IGBT), which are nowadays widely adopted in medium voltage, medium power converters, for their performances in terms of driving, switching behaviour, etc\([1-4]\).

However, PWM inverter generates a large frequency spectrum of over-voltages and voltage ripples because of the high rate of voltage rise (\(dv/dt\)). This contributes to insulation deterioration and subsequent failure of the motor. If a long cable is employed between the inverter and the motor, damped high frequency ringing at the motor terminals occurs resulting in excessive over voltage, which further stresses the motor insulation. The \(dv/dt\) also contributes to damaging bearing currents and EMI problem\([5-7]\).

Methods to mitigate these effects have been proposed by adding passive filtering between the inverters and induction motor. The differential-mode LC filter is a common used filter topology\([8-10]\). As a kind of differential-mode sine filter, the cut-off frequency of the LC filter is below the PWM frequency, so it eliminates the differential-mode PWM frequency components and main switching ripple current, and makes motor terminal voltages sinusoidal. However, the control bandwidth of the inverter is limited by the filter. Thus, there is a trade-off between the differential mode suppression and the control bandwidth of the motor\([11-12]\).
When LC filter is considered, a lot of research pay attention to the motor terminal voltage control and few of them consider the performance of motor current. However, as the torque is directly controlled by the motor current, not the voltage, the performance issue of current is equal important. Moreover, in most equivalent circuits of the research, the effect of back-EMF is not considered.

This paper investigates the performance issue of LC filter in a different view. Firstly, a more detailed model is built up where motor is not only equal to pure impedance simply, but also involves the effect of back-EMF. Based on the new model, the condition when the effect of back-EMF can be ignored is discuss and the influence of LC filter on the current control bandwidth is detected. The research reveals that the inductance value of the filter is a key factor influencing the current control bandwidth and point out the method to enhance suppression ability without control bandwidth loss.

2. The situation without LC filter
Traditionally, when the transfer function of the PWM-fed induction motor system is investigated, the motor is equivalent to the combination of resistance and inductance. The back-EMF effect is usually not considered. In this paper, a more accuracy model of the induction motor is build up, which consider the back-EMF effect. To the purpose of explaining the influence of the LC filter, the motor without the filter is firstly built up in this section and the situation of system with a LC filter will be discussed in next section.

Figure 1 shows the induction motor system where $u_i$ and $i_e$ represents the inverter output voltage and the motor current respectively. $R_m$ and $L_m$ represent the equivalent resistance and inductance of the motor. $\omega$ is the motor’s speed. $G_m$ represents mechanical character, and $K_t$, $K_b$ and $M_c$ represent moment coefficient, back-EMF constant and load torque respectively. $E = K_b \omega$ represents the voltage generated by the back-EMF effect.

$G_m(s) = (L_m s + R_m)^{-1}$ represents the electrical characteristic of the motor, it is also the transfer function from $u_i$ to $i_e$ if the back-EMF is not considered.

According to Figure 1 we have:

\[ i_e = (u_i - E)G_e(s) \]  
\[ \omega = (i_e K_t - M_c)G_m(s) \]

Practically, $M_c$ has little influence on the motor current, therefore it is assumed that $M_c = 0$. Then from equation (1) and (2) we have the transfer function from $u_i$ to $i_e$:

\[ \frac{i_e(s)}{u_i(s)} = \frac{1}{G_e(s)^{-1} + K_b K_t G_m(s)} \]  

Equation (3) implies that when the back-EMF effect is considered, the transfer function of the motor is not $G_e$ anymore, because the influence of $G_m$ is also involved. Thus, the equivalent circuit of motor is not a simply combination of inductance and resistance.
3. Situation with a LC filter
In this section, the model of motor system with LC filter is built up considering the effect of back-EMF, and the transfer function from \( u_e \) to \( i_e \) is researched, by which the influence of LC filter on current control can be investigated.

3.1 Model of PWM-fed induction motor with LC filter considering back-EMF
Figure 2 shows the structure of PWM-fed motor drive system with a LC filter, where \( L_f \) and \( C_f \) represent the inductance and the capacitance of the filter respectively. When the back-EMF is considered, the differential-mode equivalent circuit is shown in Figure 3.

From Figure 3 we have:

\[
\frac{i_2}{C_f s} = \frac{i_e (L_m s + R_m) + E(s)}{i_1} = i_e + i_2
\]

According to equation (4), (5) and (6), we have:

\[
i_e(s) = \frac{u_e(s) - (1 + C_f L_f s^2)E(s)}{L_f C_f L_m s^3 + L_f C_f R_m s^2 + (L_m + L_f) s + R_m}
\]

According to equation (7) and (2), we have:

\[
G_e'(s) = \frac{i_e(s)}{u_e(s)} = \frac{1}{G_e'(s)^{-1} + K_b' K_m(s)}
\]

Where

\[
G_e'(s) = \frac{1}{L_f C_f L_m s^3 + L_f C_f R_m s^2 + (L_m + L_f) s + R_m}
\]

\[
K_b' = (1 + C_f L_f s^2) K_b
\]

Comparing equation (8) with equation (3), it is shown that when the LC filter is added, model of the induction motor in Figure 1 is replaced by model in Figure 4. The influence of the LC filter include two aspects: 1) the electrical characteristic of the motor is changed from \( G_e \) to \( G_e' \). 2) The back-EMF coefficient is also changed. It is no longer a constant, but a function of \( s \).
Figure 4. The model of induction motor with a LC filter considering back-EMF

To have a better understanding of $G_e'$, the differential-mode equivalent circuit of Figure 2 without back-EMF is shown in Figure 5. It is easy to get that the transfer function from $u_i$ to $i_e$ in Figure 5 is the same with $G_e'$. Then we can rewrite equation (8) as:

$$G_e''(s) = \frac{G_e'(s)}{1 + K_b' K_i G_m'(s) G_e'(s)}$$  

$$G_e'' \approx G_e'(s)$$ \text{for} \quad K_b' K_i G_m'' \approx 0$$  

Equation (12) implies that if the equation of $K_b' K_i G_m'(s) G_e'(s) \approx 0$ is satisfied, the back-EMF on the induction motor can be ignored when we consider the influence of LC filter on the current control, and the relationship between PWM output voltage and the motor current is given by $G_e'$.

Figure 5. Equivalent differential-mode circuit without back-EMF

3.2 Analysis on the influence of current control of LC filter

As we know, in practical system, the value of the value of $L_f$, $C_f$ and $L_m$ is very small. The unit of inductance is $mh$ and the unit of capacitance is $uf$. Thus there is a frequency range $\omega \in [0, \alpha_1]$ where $L_f C_f L_m \approx 0$ and $L_f C_f R_m s^2 \approx 0$. Also, the mechanical control bandwidth is usually narrower than that of current, thus there is a frequency range $\omega \in [\alpha_2, \infty]$ that $G_m \approx 0$. As the result, according to equation (8), then we have:

$$G_e''(j\omega) \approx \frac{1}{(L_m + L_f) j\omega + R_m \mid_{\omega \in [\alpha_2, \alpha_1]}}$$  

Equation (13) implies that the control bandwidth of the motor current is mainly decide by the value of the inductance of the filter and the motor which can be calculated as:
This conclusion is very useful because traditionally it is considered that there is a trade-off between the control bandwidth and the suppression ability of LC filter. However, equation (14) indicates that it is useful to get a higher suppression ability control bandwidth without degrading control bandwidth by declining the value of \( L_f \).

4. Simulation results:
In this section, simulation is carried out to verify the theory in section 3. The parameters of the system are listed as follow: the frequency of PWM is \( 10 \text{kHz} \); \( L_m = 8 \text{mH} \) and \( R_m = 5 \Omega \); \( K_p = K_r = 1 \) and \( G_m = (0.05s + 1)^{-1} \). There are three combinations of \( L_f \) and \( C_f \) considered, as listed in Table 1. It is obviously that the resonant frequency of the filters is the same.

| Table 1. Combinations of \( L_f \) and \( C_f \) |
|-----------------|------------------|
| Filter 1        | \( 1 \text{mH} \) | \( 10 \text{μF} \) |
| Filter 2        | \( 5 \text{mH} \) | \( 2 \text{μF} \)  |
| Filter 3        | \( 10 \text{mH} \)| \( 1 \text{μF} \)  |

Figure 6 shows the transfer function from \( u_i \) to \( u_e \), where \( u_e \) is the voltage on the motor terminal. It is clear that the suppression ability of differential-mode PWM frequency components of the three filter at \( 10 \text{kHz} \) is same.

![Bode Diagram](image)

Figure 6. The transfer function from \( u_i \) to \( u_e \)

Figure 7 shows the transfer function from \( u_i \) to \( i_e \) in different situation. Obviously the current control bandwidth of the motor is influence by employing the LC filter, especially the Filter 3. Table 2 compares the current control bandwidth calculated by equation (14) with by simulation and the results are found in a good agreement. Therefore, it is verified that the control bandwidth of the motor is mainly determined by the inductance of motor and the filter. If the resonant frequency of the filter keep unchanged, by declining the \( L_f \), the control bandwidth can be enhanced without suppression ability loss.
Figure 7. The transfer function from $u_i$ to $i_e$

Table 2. Current control bandwidth for different LC filters

| Situation       | $L_f$ (mh) | $C_f$ (uf) | Results by equation (14) | Simulation results |
|-----------------|------------|------------|--------------------------|--------------------|
| Without Filter  | --         | --         | 99.5                     | 99.5               |
| Filter1         | 1          | 10         | 88.4                     | 88.9               |
| Filter2         | 5          | 2          | 61.2                     | 61.4               |
| Filter3         | 10         | 1          | 44.2                     | 44.3               |

5. Conclusion
In this paper, the influence of LC filter on the PWM-fed the motor is investigated in the view of current control. Based on a more accurate model of the induction motor involving the effect of back-EMF, the transfer function from inverter output voltage to motor-drive current is acquired. Based on the model, it is shown that the LC filter influent the current control by changing both the electrical characteristic and the back-EMF coefficient of the motor. It is also revealed that the filter inductance is the factor determining the influence of motor-drive current control bandwidth, thus it is reasonable to enhance the control bandwidth of motor without suppression ability loss by declining the filter inductance. The simulation results are in good agreement of the theory.

Acknowledgments
Thanks for the guidance of my teachers Mss Yongmei Huang and Mr Yao Mao. Thanks for my colleague Mr Xinglong Chen and other colleagues for their selfless help.

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