THEORETICAL ANALYSIS OF COLLECTIVE MODES IN METAL-MOLten-SALT MIXTURES AND MOLTEN-SALT MIXTURES

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ABSTRACT

We present the results of a theoretical analysis of long wavelength collective modes in mixtures of metal-molten salts and mixtures of salts. In the former case the calculations are made based on the assumption of a rigid background for the conduction electrons, and extended to the metal-rich concentration range by taking into account the polarisability of the electron gas. The most interesting prediction of our analysis is that sound waves should be overdamped below a critical wavenumber for a certain range of concentrations in the salt-rich region of metal-salt solutions.

I. INTRODUCTION

In the following sections we give an analysis of collective modes both in metal-salt mixtures and molten-salt mixtures. We use linearized hydrodynamics, i.e. we focus on the long-wavelength, low-frequency limit, and extend our analysis for higher frequencies, using the generalized hydrodynamics, in the K→0 limit. The long wavelength domain is the region where collective modes occur. Moreover, in a strongly coupled ionic fluid, the collision frequency is large compared to all other characteristic frequencies and maintains the system locally in thermodynamic equilibrium, so that the hydrodynamic description of this high frequency mode is, at least qualitatively, correct. For the sake of simplicity, temperature fluctuations will be neglected throughout, so that the heat diffusion mode will never appear in our analysis. This is also a reasonable approximation in strongly coupled ionic systems where the potential energy dominates the kinetic energy. In other words, we assume the specific heat ratio $\gamma = c_p/c_v$ to be of order 1 (which is well verified for pure liquid metal).

For the metal-salt mixtures, the analysis is first carried out for the case of a rigid background, which applies in the salt-rich phase, and then extended to the situation where the polarisability of the gas of conduction electrons can no longer be neglected (metal-rich phase).

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For all these cases we have solved the equations of hydrodynamics for a charged fluid (1, 2), which express the conservation of mass density, charge density, momentum and energy density. This latter has been ignored, the coupling between temperature fluctuations and fluctuations of the other variables being neglected, as already mentioned. The resolution of these equations yields the hydrodynamic matrix of which the zeros of the determinant determine the dispersion relation of the different collective modes. Our purpose here is not to reproduce these calculations but just to summarize our results.

II. METAL-SALT SOLUTIONS - RIGID BACKGROUND MODEL

We consider mixtures of the type M_x(MX)_1-x where x is the mole fraction of the alkali metal. The metallic ions have a mass M_1 and carry a positive charge Z_1e, while the halogen have a mass m_2 and carry a negative charge Z_2e. The number densities of both species are respectively \( \rho_1 \) and \( \rho_2 \). It is convenient to choose as independent variables the temperature \( T \) and the mass and charge densities:

\[
\rho_m = \frac{\xi}{i=1} \rho_i^m; \quad \rho_z = \frac{\xi}{i=1} \rho_i^z Z_i^e
\]  

[1]

The thermodynamic potential associated with these variables is the Helmholtz free energy density \( f \), from which all thermodynamic quantities are derived. The two characteristic frequencies of our two-component system are the hydrodynamic plasma frequency

\[
\omega_p = \left( \frac{4\pi \rho_z^z}{\rho_m} \right)^{1/2}
\]  

[2]

which occurs naturally in the long-time, collision-dominated regime, and a frequency \( \omega_0 \), associated to the mutual diffusion between the two species or, equivalently, the ionic conductivity \( \sigma \):

\[
\omega_0 = \frac{4\pi \sigma}{\rho_m}
\]  

[3]

The determinant of the hydrodynamics matrix results, in that case, in a cubic equation (3):

\[
Z^3 + Z^2(\omega_k + \omega_p^2) + Z(\omega_k^2 + \omega_p^2 + c^2k^2) + \frac{k^2c^2}{\omega_0^2} \frac{1}{k_i} = 0
\]  

[4]

with

\[
\omega_k = \omega_0 + k \sigma \left( \frac{\partial \nu_z}{\partial \rho_z} \right)_{\tau_i \rho_m} = \sigma \left[ 4\pi + k \left( \frac{\partial \nu_z}{\partial \rho_z} \right)_{\tau_i \rho_m} \right]
\]  

[5]
\( v \) is the electrochemical potential conjugate to the charge density, \( b \) is the longitudinal kinematic viscosity, \( k \) is the ionic screening length, \( C = \gamma \left( \frac{\rho_m}{\rho_m - \rho_i} \right) \) is the actual velocity of the mixture (in the assumption \( \gamma = 1 \)), and \( c^* = \rho_m \left( \frac{\rho_m}{\rho_m - \rho_i} \right) \) reduces to the sound velocity \( c \) in the pure salt.

In the long-wavelength (k\( \rightarrow 0 \)) limit, the three roots of (4) are:

\[
Z_1 = 0; \quad Z_{2,3} = -\frac{1}{2} \left[ \omega_0 \pm \left( \omega_0^2 - 4 \omega_p^2 \right)^{1/2} \right]
\]  

[6]

Near the pure metal (\( \omega < 2 \omega_p \)), the roots \( z_2 \) and \( z_3 \) are complex conjugate and correspond to two plasmon modes of frequency \( \omega_p \), while the root \( z_1 \) corresponds to the k\( \rightarrow 0 \) limit of collective diffusion mode.

In the opposite limit of the pure molten salt (\( \omega > 2 \omega_p \)), the doubly degenerate root \( z_1 = z_3 = 0 \) is the zero wavenumber remnant of propagating sound modes, while \( z_2 = -\omega_0 \) corresponds to a fully damped, non-hydrodynamic, charge relaxation mode, with a relaxation time \( \tau = 1/\omega_0 \).

II.1. METAL RICH MIXTURES (\( \omega_p > \omega_0 \))

Perturbing around the non zero-root, we solve equation (4), and find a purely diffusive mode (\( z_1 \) in [6]) \( z = -D_M k^2 \), with a collective diffusion constant given by:

\[
D_M = c^* \left( \frac{\omega_0}{\omega_p^2} \right)
\]  

[7]

and two plasmon modes with the dispersion

\[
\omega = \pm \omega_p \left[ (1 - \frac{\omega_k^2}{8 \omega_p^2}) + \frac{k^2}{2 \omega_p^2} \left( \frac{c^*}{4 \omega_p^2} + \frac{(b - D_M) \omega_0}{4 \omega_p^2} \right) \right]
\]  

[8a]

and the damping

\[
\Gamma = \frac{1}{2} \left[ \omega_k + \left( \frac{D_M}{2} - \frac{b}{\omega_p^2} \right) k^2 \right]
\]  

[8b]

In the limit of the pure metal (\( \omega = 0 \)), we recover the result for the OCP (9). The two main effects of the inclusion of salt (\( \omega > 0 \)) is to lower the plasma frequency \( \omega_p \) and to make the damping of the plasmons finite even at zero wave number, due to inter-diffusion of the two ionic species which dominates the purely viscous damping in the pure metal. Moreover, as \( \omega_0 \) decreases due to the addition of salt, the \( k \)-dependent term in [8a] becomes more and more important, and the
plasmon-like dispersion will eventually turn into an acoustic one.

II.2 SALT RICH MIXTURES \((\omega_0 \gg \omega_p)\)

In this case, we solve the eqn. [4] by looking for small perturbations around the non-hydrodynamic fully damped charge relaxation mode \(z = -\omega\). Following the analysis of Giaguinta et al. [4], we distinguish two wavenumber regimes according to the relative values of the two characteristic frequencies \(\omega_o\) and \(c_k\) associated with charge relaxation and sound propagation respectively.

(i) At sufficiently long wavelengths, such that \(k < k_o = \omega_o/c\), the frequency \(\omega_o\) associated with charge relaxation dominates all other characteristic frequencies of the mixture. The fully damped mode has a relaxation time

\[
G_s^{-1} = (\omega_k - \omega_k^2) - k^2 \left( \frac{c_o^2 - c_k^2}{\omega_o} \right) \tag{9}
\]

where the two last terms represent corrections relative to the pure molten salt, due to the presence of a finite concentration of metal. The two remaining roots correspond to acoustic modes \(z = \pm ic_k k - \Gamma/2\) with a sound velocity

\[
c_s^2 = \frac{c^2 - b (\omega_p^2/2\omega_0^2)}{1 - (\omega_p^2/\omega_0^2)} \tag{10}
\]

and an attenuation

\[
\Gamma = \frac{\omega_p^2}{\omega_k} + k^2 \left( \frac{c^2 - c_k^2}{\omega_o} \right) + k^2 b \tag{11}
\]

Only the third term on the RHS of [11] contributes in the pure molten salt, where charge and density fluctuations are decoupled. The addition of some metal introduces a small coupling between these fluctuations, and the attenuation remains finite as \(k \to 0\). The coupling between charge and mass fluctuations is also responsible for the deviation of the sound velocity [10] from its value \(c'\) in the pure salt. The most important result in this long wavelength regime is that sound propagation is not possible at all concentrations. It is possible only if the general dispersion relation [4] has two complex
conjugate roots. The condition for sound propagation is hence:
\[ r^2 - 4 c'^2 \omega_0 c s k_i^2 < 0 \]  

[12]

Retaining only terms up to order \( k^2 \), we obtain the following condition, from \([9], [11], [12]\):
\[ \left( \frac{\omega_p}{\omega_o} \right)^4 < 4 \frac{\sqrt{c'^2}}{\omega_o^2} \Rightarrow \frac{\kappa^2}{\kappa_0^2} \]

[13]

This means that for a given wavenumber \( k \) (smaller than \( k_o \)), there exists a range of concentrations, delimited by the inequalities:
\[ \left( \frac{k}{k_o} \right)^{1/2} < \frac{\omega_p}{\omega_o} < \frac{1}{\kappa} \]

[14]

where all collective modes are overdamped or equivalently, for a given metallic concentration, sound propagation is possible only if the wavenumber \( k \) is larger than a critical value \( k_o = k_c \left( \omega_p/\omega_o \right) \). At higher metallic concentrations, plasmon like modes propagate (see \([6]\)), while at lower metallic concentrations the mixture sustains sound waves, but due to concentration gap \([14]\), there is no crossover between the two types of propagating modes.

(ii) At wavelengths sufficiently short so that \( k > k_0 \), the sound wave frequency \( c_k \) dominates the charge relaxation frequency \( \omega_o \). In that case the relaxation time of the fully damped charge mode is:
\[ c_s^{-1} = \omega_o \frac{c'^2}{c^2} \left( 1 + \frac{\kappa^2}{\kappa_i^2} \right) \]

[15]

The two remaining modes are sound waves of velocity
\[ c_s = c \]

[16]

and attenuation
\[ \Gamma = b k^2 + \omega_k - \omega_o \left( \frac{c'^2}{c^2} \left( 1 + \frac{\kappa^2}{\kappa_i^2} \right) \right) \]

[17]

The main difference between the two regimes i) and ii) lies in the sound attenuation \( \Gamma \) which is dominated by mutual diffusion for \( k > k_o \).
and by viscous forces for \( k < k_* \). Moreover sound waves propagate at any metal concentration as soon as \( k > k_0 \) and crossover directly into plasmon modes in the metal-rich phase. The difference between the two regimes is shown in figure 1.

### III. METAL–SALT SOLUTIONS, EFFECT OF ELECTRON SCREENING

In the metal rich phase the electron density is high and the associated screening length \( \lambda_e \) is of the order of the inter-ionic spacing. Hence, whenever the wavenumber \( k \) is smaller than this electronic screening wavenumber \( k_e \sim 1/\lambda_e \), the coupling of the conduction electrons to the collective modes of the solution cannot be ignored. But, because of the large ion to electron mass ratio, the adiabatic approximation for the electron gas is justified, so that frequencies characteristic of the fast electronic motions will never appear in the equations. Moreover, in the metal rich phase, we expect the ion-electron interaction to be weak so that we consider the response of the electron gas in the linear screening approximation.

The electronic contributions to the total free energy density \( f \) must be taken into account; this leads to a renormalization of the chemical potentials \( \mu_m \) and \( \mu_z \). The characteristic frequency [4] is changed to:

\[
\omega_k = \sigma \left[ \frac{4\pi}{\varepsilon_o(k)} + k^2 \left( \frac{\partial \mu_z}{\partial \rho_z} \right) \right] \quad [18]
\]

where \( \mu_\omega \) is now the renormalized electrochemical potential conjugate to the charge density and \( \varepsilon_o(k) \) is an effective electronic dielectric function which accounts for the short-range non-coulombic part of the ion-electron coupling (5). Its long wavelength form determines the electron screening wavenumber by the relation \( \varepsilon_o(k) = 1 + k_e^2/k^2 \). For \( k \gg k_* \), we recover the case of a rigid background described previously. In the polarisable background \( k >> k_* \), the fundamental difference arises from the fact that \( \omega_k \) now vanishes with \( k \), as:

\[
\omega_k = \sigma \left[ \frac{4\pi}{k_e^2} + \left( \frac{\partial \mu_z}{\partial \rho_z} \right) \right] k^2 = \frac{d}{k^2} \quad [19]
\]

so that the dispersion relation [4] takes the form:

\[
z^3 + z^2 \left[ (d + b)k^2 \right] + z \left[ (c^2 + \frac{\omega_R}{k_e^2}) k^2 + bd k^4 \right] + \omega_o c^2 (k_i^2 + k_e^2) k^4 = 0 \quad [20]
\]
which is reminiscent of that for a neutral one component fluid, the thermal diffusivity in the latter being replaced by the ionic conductivity in the present case. Over the whole range of concentrations, eqn [20] has two complex conjugate roots of the form [10], corresponding to propagating sound waves, and a purely diffusive, real root, \( z = -Dk \). The sound velocity is given by:

\[
C_s^2 = C^2 + \frac{\omega_P^2}{k_e^2} = \frac{1}{\rho_m \chi_T}
\]

where \( \chi_T \) is the total (i.e. ionic plus electronic contributions) compressibility of the mixture. The sound attenuation is:

\[
\Gamma = \left( b + d - D \right) k^2
\]

where the collective diffusion constant is given by:

\[
D = 4\pi \sigma_c \left( \frac{C_s'}{C_s} \right)^2 \left( \kappa_i^{-2} + \kappa_e^{-2} \right)
\]

As expected, the main effect of electronic polarisability, which occurs in the metal-rich side of the diagram is to transform the plasmon-like modes of the rigid background into acoustic modes, as in the pure metal [4].

IV. METAL-SALT SOLUTIONS. HIGH FREQUENCY BEHAVIOUR

In order to extend our analysis at higher frequencies, we outline a generalized hydrodynamics description of longitudinal collective modes in metal-salt solutions in the \( k \to 0 \) limit. In this limit, the frequency \( \omega_0 \) must be replaced by its frequency-dependent generalization, i.e. after Laplace transform:

\[
\omega_o(z) = 4\pi \sigma(z) = \delta^2 / \left[ z + M(z) \right]
\]

where \( M(z) \) is the memory function associated with the frequency dependent ionic-conductivity \( \sigma(z) \), and \( \delta = \Omega_p - \omega_c \) is the difference between the kinetic plasma frequency which occurs in the short-time, Vlaslov type, description of ionic dynamics (\( \Omega_p = 4\pi \sigma_c (z e)^2 / m \)), and the hydrodynamic plasma frequency.

Single relaxation time approximation for \( M(t) \) yields:
\[ M(t) = M(0) e^{-t/\tau} = \Omega_J^2 e^{-t/\tau} ; \quad M(z) = \Omega_J^2/(z + \mathcal{C}^{-1}) \] \[ \text{[25]} \]

where \( \Omega_J^2(k) \) is easily calculated, using standard procedures (6,3), and the relaxation time \( \tau \) can be determined from the static conductivity via:

\[ \sigma = \sigma(z = 0) = \frac{\delta^2}{4\pi M(0)} = \frac{\delta^2}{4\pi \Omega_J^2 \mathcal{C}} \] \[ \text{[26]} \]

The dispersion relation [4] becomes:

\[ Z^3 [z + M(z)] + \delta^2 z^2 + \omega_p^2 z [z + M(z)] = 0 \] \[ \text{[27]} \]

hence

\[ Z^3 + \frac{\delta^2 z^2}{z + \Omega_J^2/(z + \mathcal{C}^{-1})} + \omega_p^2 z = 0 \] \[ \text{[28]} \]

In the pure metal we recover the undamped plasmon modes \( z = \pm i \omega_p \). A perturbation calculation based on [28] shows that if some salt is added, the plasmon frequency is slightly shifted to:

\[ \omega = \Omega_p + \frac{1}{2} \frac{\delta^2 \Omega_J^2}{\Omega_p^3} \] \[ \text{[29]} \]

while the damping is given by

\[ \mathcal{C}_M^{-1} = \frac{\delta^2 \Omega_J^2}{2 \mathcal{C} \Omega_p^4} = \frac{\omega_p}{2} \left( \frac{\Omega_J}{\Omega_p} \right)^4 \] \[ \text{[30]} \]

We see from [29] and [30] that the generalised hydrodynamics calculation leads to a shift above the kinetic plasma frequency \( \Omega_p \), whereas the linearized hydrodynamics predicts a frequency below the hydrodynamic plasma frequency. This is a wellknown failure of the linearized hydrodynamics (7).
On the salt-rich side, the main consequence of introducing a frequency-dependent conductivity is to predict the existence of plasmon modes, reminiscence of the 'optic' modes of the crystal. The frequency of these modes is given by:

$$\omega = \pm \Omega_p \left[ 1 + \frac{\Omega^2_0}{2 \Omega^4_p} \right]$$  \[31\]

Comparing with eqns [29], [30], we conclude that the plasmon modes of the metal-rich region do not vanish at a salt concentration, such as $\omega = 2\omega_0$, as predicted by linearized hydrodynamics, but survive all the way into the salt-rich phase where they coincide with the familiar 'optic' modes of ionic systems.

V. MIXTURE OF MOLTEN-SALTS, $MX_2$–$MX_3$

We concentrate on mixtures of molten salts with the same cation $M$. The independent thermodynamic variables are chosen to be the temperature $T$, the mass density $\rho$, the charge density $\rho_m$ and the concentration $x$ of particles $X_3$, defined respectively as:

$$\rho = \sum_i \rho_i Z_i e; \quad \rho_m = \sum_i \rho_i m_i; \quad x = \rho_3 Z_3 / \rho$$  \[32\]

where $\rho = \rho_2 e + \rho_3 e$ is the density of negative ions. The global electroneutrality condition yields $\rho_2 = 0$.

We solve the equations of linearized hydrodynamics in a similar way to that in previous sections. We only have to add a current $\mathbf{j}_2(\mathbf{r}, t) = \mathbf{j}_3(\mathbf{r}, t) + \mathbf{j}_4(\mathbf{r}, t)$. The convective part of $\mathbf{j}_4$ is due to the total current of negative charges, $\mathbf{j}_3$ being the mutual diffusion current between species 2 and 3. Ignoring again the energy conservation law, the determinant of the hydrodynamic matrix, is given now by a quartic equation:

$$Z^4 + Z^3 \left[ \omega_0 + \frac{K^2}{2} (b + \gamma_{zz} + \gamma_{xx}) \right]$$

$$+ Z^2 \left[ K^2 (c^2 + b \omega_o + c_{xx} - c_{zz}) + o(K^4) \right]$$

$$+ Z \left[ K^2 \omega_0 c^2 + o(K^4) \right] + o(K^4) = 0$$  \[33\]

The $\gamma_{ij}$ are the elements of the generalized diffusion matrix $\nu_{ab} (a = z, x; b = z, x, m)$. This latter is the product of the mobility
matrix $\Lambda_{\alpha\beta}$, of which elements are expressed in the Kubo linear response theory as a correlation of the currents:

$$\Lambda_{\alpha\beta} = \frac{1}{3\Omega T} \int_0^\infty <J_\alpha(t)J_\beta(t)> dt \quad \alpha,\beta = (z, x)$$

and the inverse susceptibility matrix, related to the chemical potentials via:

$$\chi_{\alpha\beta}^{-1} = \frac{\partial \mu_{\alpha}}{\partial \mu_{\beta}} \quad (\alpha = z, x; \beta = z, x, m; \mu_x = \mu_x - dx)$$

Then

$$\chi_{\alpha\beta}^{-1} = \Lambda_{\alpha\beta} \chi_{\alpha\beta}^{-1}$$

$\omega_0$ and $\omega_1$ are the two characteristic relaxation frequencies given by:

$$\omega_0 = 4\pi \sigma$$

$$\omega_1 = 4\pi \Lambda_{xx}$$

In the infinite wavelength limit, the four roots are easily determined:

$$z_1 = z_3 = z_4 = 0 \quad z_2 = -\omega_0$$

The dispersion and damping of these modes are determined like previously by standard perturbation up to order $k^2$, starting from these zero wavenumber solutions. The fully damped mode ($z_2 = -\omega_0$) has a relaxation time $\tau$ such that:

$$\frac{1}{\omega_0} = 4\pi c \tau$$

Factoring out this mode yields a cubic equation which gives the three other modes dispersion relation. The discriminant of this equation, $\Delta = 4k^2 - o(k^4)$, is always positive, indicating the existence of two complex conjugate roots corresponding to two propagating sound waves $z_{2,3} = \pm ick - r/2$ and to a real root corresponding to a fully damped diffusive mode $z_1 = -DK$. The velocity of the sound waves at zero wavenumber is
and the sound attenuation is given by:

\[ \Gamma = \kappa^2 \left[ b + \frac{1}{\rho} \frac{\partial P}{\partial x} \frac{\partial \nu_x}{\partial (\Lambda - \frac{\Lambda^2 z z}{\sigma})} \right] + o(\kappa^4) \]  

where \( \Lambda = \Lambda_{XX} \) is the mutual mobility of the two salts, dominated by viscosity, charge diffusion and interspecies diffusion.

The most interesting result is that this sound attenuation is of the order \( \kappa \) and does not include a constant \( (\kappa \text{ independent term}) \), contrary to the case of molten-salt embedded in a rigid background, described in section I. Actually this constant term, proportional to the free electrons density \( \rho_e \), was responsible for the overdamping of sound propagation [see eqn. (1)]. In this case, although the overall electroneutrality is insured, the local electroneutrality condition is violated by any low frequency motion of the ions. The rigid background then overdamps the oscillation in order to prevent this local electroneutrality. Obviously, this phenomenon disappears in the case of a polarizable background because of the spontaneous rearrangement of the electrons, and in a mixture of molten salts because of the absence of free electrons.

The other typical result in our mixture of molten-salts is the appearance of a new diffusive mode \( z_2 = -D_2 \), a consequence of the extra degree of freedom introduced by allowing mutual diffusion between the species \( MX_2 \) and \( MX_3 \).

A very crude analysis within the framework of generalized hydrodynamics in the limit of zero wavenumber, like in III, confirms the existence of plasmon modes, remnant of the crystal optical modes. Let us note as an interesting result that, if the ions \( X^3 \) are replaced by "massive" electrons, this model can be viewed as a special case of metal-molten salts on the very salt-rich side of the phase diagram where the electrons are known to be trapped in localized defects as F centers (8).

VI. CONCLUSION

The main predictions of our different analysis are the following ones. For metal-salt solutions, in the rigid background model (valid in the salt rich side), linearized hydrodynamics predict propagating plasmon modes in the metal-rich region, up to a concentration such that \( \omega \approx 2 \omega \), while the generalized hydrodynamics show that these 'optic' modes persist up to the pure salt. Their frequency lies above...
the kinetic plasma frequency $\Omega$, and their damping is dominated by ionic conductivity (interdiffusion) and vanishes in the $k \rightarrow 0$ limit only for the pure metal. Interdiffusion of the two species leads to a fully damped charge relaxation mode which has a finite lifetime at zero wavenumber for any non-vanishing salt concentration. Sound waves propagate in the pure molten salt at long wavelengths but, as metal is added, the acoustic modes cease to propagate below a critical wavenumber $k_c$ because of the coupling between charge and mass density. This overdamping of propagating waves is the consequence of the presence of a rigid electronic background of finite density. As soon as either the electron gas is more dense, i.e. in the region of higher metallic concentrations, or the electrons are trapped in "F-centers' defects and can be considered as massive negative particles, the sound waves keep propagating. However there could be a small region in the phase diagram (for metallic concentrations between 5 and 20%) where the rigid electron background might be valid and where an overdamping of the sound waves might be observed.

Screening by conduction electrons modifies the predictions of the rigid background model in the metal-rich region, where the plasmon modes are changed into hydrodynamic sound waves.

For a mixture of molten salts, most of the results of the rigid background are recovered, i.e. the charge relaxation mode and the two plasmon modes remnant of the crystal optic modes. The two main differences lie in the persistence of sound wave propagation at long wavelength and in the apparition of a central ($\omega=0$) non-thermal diffusive mode, due to the mutual diffusion of the two species.

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Fig. 1 Qualitative Variation of the Frequency of the Propagating Mode as a Function of $\frac{\omega_P}{\omega_0}$ (i.e. Concentration) for Two Wavenumbers $K$. 