An Improved Synchronization Scheme for Non-Integer Multiple Spread Spectrum

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Abstract. Non-integer multiple spread spectrum systems are gradually being widely used in civil and military communications due to their superior communication security and signal privacy. For the difficulties in the capture of pseudo-codes in the non-integer multiple DSSS-DQPSK system under the large Doppler shift, a fourth power PMF-FFT (4th-Power-PMF-FFT) scheme is proposed. The system is optimized by zero-padding and batch FFT algorithms to eliminate scallop loss, improve carrier frequency estimation accuracy, and reduce hardware resource consumption. The simulation results proved the effectiveness of the method.

1. Introduction
The direct sequence spread spectrum (DSSS) system has been widely used in military communication, satellite navigation and other systems due to its low interception rate and anti-interference [1]. DSSS system synchronization is necessary to capture the Doppler shift as well as to ensure code phase synchronization, so joint two-dimensional search method concerns. M. Sust et al. solved the problem of large frequency offset by using FFT compensation. This structure is the rudiment of matching filtering (MF) combined with fast Fourier transform (FFT) [2]. G.J.R. Povey et al. first proposed the PMF-FFT algorithm [3] on the basis of [2], which segmented the received signals and solved the signal frequency by using the fast Fourier transform method. Based on the PMF-FFT algorithm, many scholars have proposed some improvement schemes [4-7]. However, these methods are mostly used to study integer multiple spread spectrum system, and they are basically ineffective to non-integer multiple spread systems. A spreading period of the non-integer multiple DSSS signal includes a non-integer number of information symbols, that is, a pseudo code corresponding to different information symbols is different, which is different from the conventional DSSS signal. Furthermore, different information codes are independent of each other, which destroy the correlation of the pseudo-code [8]. When synchronizing the non-integer multiple DSSS signals, the integral time is greater than one data bit, and the captured integral peak value will be affected by the information symbol flipping, which may lead to inaccessibility. This paper will study on this issue.

Aiming at the problem of non-integer multiple spread spectrum synchronization, this paper proposes a four-power PMF-FFT (4th-Power-PMF-FFT) synchronization algorithm based on the characteristics of DQPSK signal. And the system is optimized to eliminate the scallop loss, improve the carrier frequency estimation precision as well as reduce the hardware resource consumption by zero padding and batch FFT algorithm. The simulation results also proved the effectiveness of the method. The paper has the following outline. The principle and implementation method of 4th-Power-PMF-FFT are introduced in the next Section. Section 3 describes the optimization method of 4th-Power-PMF-FFT. The simulation results have been shown in Section 4 to verify the theoretical analysis and some conclusions are provided in Section 5.
2. **4th-Power-Pmf-Fft Algorithm**

The PMF-FFT algorithm is a time-frequency two-dimensional search algorithm that can quickly complete the code phase search and accurately estimate the frequency offset [5]. The traditional PMF-FFT algorithm performs the matched filtering correlation processing on the received signal, and uses the fast Fourier transform method to solve the signal frequency and the pseudo code phase. However, for non-integer multiple spread spectrum synchronization, the peak of the capture integral will be affected by the information symbol flipping if the traditional PMF-FFT acquisition scheme is adopted, which would result in the capture failure. For the non-integer multiple DSSS-DQPSK system, at least fourth power processing is required to eliminate the phenomenon of information symbol flipping due to the four-phase characteristics of DQPSK signal. Therefore, this paper proposes a fourth power PMF-FFT (4th-Power-PMF-FFT) scheme.

2.1 **Algorithm Principle**

The structure of 4th-Power-PMF-FFT scheme is shown in Fig. 1. This algorithm performs the M-point FFT transformation on the output of the PMF by the fourth power. Next, it is time to find the maximum molded value of the M-point FFT and compare it with the preset threshold to determine whether the capture is successful. The specific implementation process of 4th-Power-PMF-FFT is as follows:

![Figure 1: 4th-power PMF-FFT algorithm schematic diagram](image)

2.1.1 **Partial Matching Filtering.** After balanced modulation of DQPSK signal, the two input data (I and Q) of the matched filter are:

\[
I(n) = \frac{1}{2} A_I(n)c(n) \cos(2\pi f_d nT_c + \theta_0) - \frac{1}{2} A_Q(n)c(n) \sin(2\pi f_d nT_c + \theta_0)
\]

\[
Q(n) = \frac{1}{2} A_I(n)c(n) \sin(2\pi f_d nT_c + \theta_0) - \frac{1}{2} A_Q(n)c(n) \cos(2\pi f_d nT_c + \theta_0)
\]

(1)

Where \( A_I(n) \) and \( A_Q(n) \) are the information code elements, \( c(n) \) is the signal spread spectrum GOLD sequence, \( f_c \) is the Doppler frequency offset existing in the channel, \( f_d \) is the PN code rate, \( T_c = 1/ f_c \) is the pseudo-code width, and \( \theta_0 \) is the initial phase.

As shown in Fig 1, I and Q signals turn into M number of X stage partially matched filter bank, and then, after the output data is added, we can have:

\[
g_{PMF}(m,x) = I_{PMF}(m,x) + jQ_{PMF}(m,x), \quad m = 0,1,\cdots,M-1, x = 0,1,\cdots,X-1
\]

(2)

2.1.2 **Fourth Power Processing.** After the matched filtered data \( g_{PMF}(m,x) \) is processed by the fourth power, the result is:
\[ g^4_{PMF}(m, x) = [I_{PMF}(m, x) + jQ_{PMF}(m, x)]^4, \quad m = 0, 1, \ldots, M-1, x = 0, 1, \ldots, X-1 \]  

(3)

Combined with the autocorrelation property of GOLD code, when the received signal is completely matched with the local pseudo-code phase, \( c_m(n-x)=1 \) is established, where \( c_m(n-x) \) is the local pseudo code sequence. Therefore, (3) can be simplified as:

\[ g^4_{PMF}(m) = \sum_{n=0}^{M-1} [I_x(m) + jQ_x(m)]^4 = \sum_{n=0}^{M-1} [I_x^2(m) - Q_x^2(m) + j2I_x(m)Q_x(m)]^2, m = 0, 1, \ldots, M-1 \]  

(4)

Among them:

\[ I_x(n) = \frac{1}{2} A_x(n) \cos(2\pi f_d n T_C + \theta_0) - \frac{1}{2} A_Q(n) \sin(2\pi f_d n T_C + \theta_0) \]

\[ Q_x(n) = \frac{1}{2} A_x(n) \sin(2\pi f_d n T_C + \theta_0) + \frac{1}{2} A_Q(n) \cos(2\pi f_d n T_C + \theta_0) \]

Further simplification can be obtained:

\[ g^4_{PMF}(m) = \sum_{n=0}^{M-1} \left[-\frac{1}{4} e^{j2\pi f_d n T_C + \phi_0}\right], m = 0, 1, \ldots, M-1 \]  

(5)

For the convenience of discussion, the initial phase \( \theta_0 \) can be set to 0. The above formula can be considered as the DFT transform with sequence number 0 and length X, and the result after normalization is:

\[ g^4_{PMF}(m) = \left[\frac{1}{X} \sin(\pi 4 f_d T_c X) e^{-j\pi 4 f_d T_c X^{(X-1)}}\right] e^{j2\pi f_d m X T_c}, m = 0, 1, \ldots, M-1 \]  

(6)

2.1.3 FFT. When there is Doppler shift, the output of some part of the matched filter will produce peak value due to frequency compensation. Therefore, FFT can be used to estimate the frequency deviation. The output of M part matched filter is transformed by N-point FFT:

\[ G^4_{PMF-FFT}(k) = \sum_{m=0}^{M-1} g^4_{PMF}(m) W^{mn}_M, k = 0, 1, \ldots, M-1 \]  

(7)

Where \( W^{mn}_M = e^{-j2\pi mn/M} \) is shown. The first constant term in (6) is denoted as \( g_0 \). Equation (6) can be written as: \( g^4_{PMF}(m) = g_0 \cdot e^{j2\pi f_d m X T_c}, m = 0, 1, \ldots, M-1 \)

Derivation of (7) is:

\[ G^4_{PMF-FFT}(k) = \sum_{m=0}^{M-1} g_0 e^{j2\pi f_d m X T_c} e^{-j2\pi mn/M}, k = 0, 1, \ldots, M-1 \]

\[ = \sum_{m=0}^{M-1} g_0 e^{j2\pi m(4 f_d M X T_c - k) / M} \]  

(8)

\[ = g_0 \left[\frac{\sin(\pi 4 f_d T_c X M - \pi k)}{\sin(\pi 4 f_d T_c X - \pi k / M)} e^{-j4 f_d T_c X M} e^{j k (M-1)}}\right] \]

After substituting \( g_0 \) into (8) and normalizing the result, we can get:

\[ G^4_{PMF-FFT}(k) = \frac{1}{XM} \sin(\pi 4 f_d T_c X) \sin(\pi 4 f_d T_c X M - \pi k)} \sin(\pi 4 f_d T_c X \sin(\pi 4 f_d T_c X - \pi k / M) e^{j\phi(k)}, k = 0, 1, \ldots, M-1 \]  

(9)
Where \( \psi(k) = -j\pi \left[ 4f_d T_c (X - 1) + (4f_d T_c X - k / M)(M - 1) \right] \) is the phase characteristic of PMF-FFT.

### 2.1.4 Threshold Judgment

The maximum molded value, which is selected from the output of FFT, is compared with the preset threshold to determine whether the capture is successful.

### 2.2 Range and Accuracy of the Algorithm

In the 4th-Power-PMF-FFT structure, the correlation peak values of \( k \) and \( x \) were compared to determine the frequency bias of Doppler and pseudo-code.

#### 2.2.1 Range

PMF-FFT output has dual characteristics. The capture system can still work normally and effectively, as
\[
\left\{ \frac{c}{2}, \frac{-c}{2} \right\}
\]
and the maximum value of correlation output will be generated at the point as \( M - k \). The Doppler interval that the system can successfully capture is:
\[
\left\{ \frac{-c (4M)}{2}, \frac{c (8X)}{2}, \frac{c (4X)}{2} + \frac{c (8X)}{2} \right\} , k \in \left[ -M / 2 - 1, M / 2 \right] \tag{10}
\]
The range of phase offset captured by the system is:
\[
\left\{ X(l - 1), Xl \right\} , l \in [1, M] \tag{11}
\]

#### 2.2.2 Accuracy

The pseudo-code phase accuracy determined by the system according to the value of \( x \) is one chip, and the estimation error is \( 1/2 \) chip. The Doppler frequency offset determined according to \( k \) value is:
\[
\hat{f}_d = \frac{k f_c}{(4X M)} \tag{12}
\]
The estimated maximum Doppler error is:
\[
\Delta f_{d\text{max}} = \frac{f_c}{(8X M)} \tag{13}
\]
The resolution of M-point FFT is:
\[
\Delta f_{\text{FFT}} = \frac{f_c}{(4X M)} \tag{14}
\]
Therefore, in the case where the number of PMFs and the length of PMF are constant, the frequency offset estimation range of 4th-Power PMF-FFT algorithm is only \( 1/4 \) of PMF-FFT algorithm, while the estimation accuracy is improved to 4 times the traditional one.

### 3. Improved 4th-Power-PMF-FFT Scheme

In order to ensure the stable operation of COSTAS loop, the carrier frequency difference needs to be further reduced. At the same time, input data of FFT needs to be processed due to the existence of scallop loss [5]. One of the schemes is zero padding [6]. This method can not only reduce the scallop loss but also improve the recognition accuracy of frequency shift. However, with the increase of FFT points after zero padding, the consumption of hardware resources will also increase, and more space is needed to store input and output data of large FFT, leading to a significant decrease in effective throughput. Therefore, this paper proposes an improved scheme based on time domain batch FFT (Batch-In-Time) algorithm.

#### 3.1 Zero Padding

In the previous discussion, the number of points in the FFT was defaulted to the number of PMFs (\( N = M \)), in which case there was a scallop loss phenomenon. The scallop loss is defined as the difference between the intersection of two adjacent FFT output gains and the FFT maximum point, as shown in Fig. 2. The scallop loss can be reduced by zero padding. The number of points of FFT after zero-padding is increased from \( M \) to \( M \cdot NP \) and the accuracy of frequency offset estimation is correspondingly increased from \( \Delta f_{\text{FFT}} = \frac{f_c}{(4X M)} \) to \( \Delta f_{\text{FFT}} = \frac{f_c}{(4X M \cdot NP)} \), while the range of frequency offset estimation remains unchanged. Where \( NP \) is the multiple factor of zero-padding.
Figure 2: Scallop loss

3.2 Batch-In-Time (BIT) Algorithm

For BIT algorithm, the large point FFT is processed in batches. First, the N point FFT is divided into N/P points FFT and block-wise P point FFT. In this case, N can be divided into P. The corresponding process is shown in Fig. 3.

Figure 3: Schematic diagram of BIT

4. Simulation Results and Performance Analysis

4.1 Effect of Different Zero-Padding Points

MATLAB was used to simulate the effect of zero-padding length on synchronization performance in BIT algorithm in this section. The simulation parameters are shown in Table 1.

Table 1: Simulation Parameters of BIT Algorithm

| Parameters                        | Value  |
|-----------------------------------|--------|
| Symbol rate /bps                  | 64K    |
| Spread spectrum code rate /cps    | 10.23M |
| Spread code type                  | GOLD  |
| Period length of GOLD code        | 1023   |
| Doppler frequency offset /Hz      | -110K~110K |
| Pseudo-code offset /chip          | 2      |
| SNR /dB                           | 5      |
| Number of PMF(M)                  | 128    |
| Length of PMF(X)                  | 8      |

The effect of different zero-padding points on the FFT normalized output amplitude and frequency offset estimation error in the BIT algorithm are shown in Fig. 4 and Fig. 5, where NP represents the zero-add factor of FFT. As can be seen from Fig. 4, the scallop loss phenomenon will be very serious if there is no zero-padding (NP=1), while the phenomenon is negligible when NP is 16. As can be seen from Fig. 5, the frequency offset error decreases accordingly as the multiple factor NP increases. It is known that the BIT algorithm can not only reduce the scallop loss but also improve the recognition...
accuracy of the frequency offset. In addition, it will save a lot of hardware resources and facilitate the improvement of effective throughput because of the batch processing.

Figure 4: Relation between FFT normalized output amplitude and zero-padding points

Figure 5: Relation between the estimation error of frequency offset and zero-padding points

4.2 Capture Probability

The simulation parameters of capture probability are shown in Table 2. Time-frequency joint estimation 3D graphs of 4th-Power-PMF-FFT algorithm combined with BIT for different signal-to-noise ratio are shown in Fig. 6(a)(b). As can be seen from Fig.6, this algorithm can successfully estimate the Doppler shift and phase offset when the SNR is higher than -5dB. The relationship between the acquisition probability and the input signal-to-noise ratio at different symbol rates are shown in Fig. 7(a)(b), where Method 1 is the ordinary zero-padding method, method 2 is the BIT batch-filling zero method, and method 3 is quadratic estimation algorithm [9]. Obviously, the capture effects of methods 1 and 2 are the same, but the batch processing in BIT reduces the number of points of the FFT, which saves a lot of hardware resources and is beneficial to the improvement of effective throughput. Therefore, method 2 is superior to method 1 in hardware implementation. Method 3 has a poor performance in capturing because of the secondary estimation. Therefore, the simulation results prove the effectiveness of the proposed scheme.

Figure 6: Time-frequency joint estimation 3D graph

Table 2: Simulation Parameters of Captures Probability

| Parameters                  | Value                |
|-----------------------------|----------------------|
| Symbol rate /bps            | 128K /256K           |
| Doppler frequency offset /Hz| 150K                 |
| Pseudo-code offset /chip    | 2                    |
| SNR /dB                     | -30~0                |
| Number of PMF(M)            | 128                  |
| Length of PMF(X)            | 8                    |
| Factor of zero-padding(NP)  | 16                   |
5. Conclusion
This paper proposes a 4th-Power-PMF-FFT synchronization scheme for the non-integer multiple DSSS-DQPSK system, which is established on the phase characteristics of DQPSK. The new algorithm is superior to the existing algorithm in the accuracy of the estimation since the fourth power nonlinearity is used to eliminate symbol inversion in the integration period. In addition, the system is optimized by zero-padding and batch FFT algorithm, which reduces the scallop loss, improves the accuracy of carrier frequency estimation, and reduces the hardware resource consumption. Simulations were also performed to verify theoretical conclusions.

6. References
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