Best Relay Selection Schemes for NOMA Based Cognitive Relay Networks in Underlay Spectrum Sharing

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ABSTRACT Cognitive Radio and the emerging Non-Orthogonal Multiple Access (NOMA) techniques hold the potential to fulfill the increasing demands of radio spectrum by 5G-and-B5G wireless systems. It is a usual practice in underlay cognitive radio networks to take assistance from intermediate relays to reach the remote destination. Putting together, NOMA based Cognitive Relay Networks (NCRNs) have recently gained tremendous research attention to improve the spectrum utilization efficiency. This article studies the application of best relay selection (BRS) in downlink scenario of NCRNs in which a base station (BS) being unable to communicate directly with the far user $U_2$ takes assistance from the near user $U_1$ and from the best Decode-and-Forward (DF) relay selected from the potential NCRN operating in underlay environment. Three BRS schemes are proposed selecting a relay which (i) maximizes signal-to-noise ratio (SNR) on relay-$U_2$ link, (ii) minimizes interference on relay-primary user (PU) link, (iii) maximizes the quotient of relay-$U_2$ link’s SNR and relay-PU link’s interference power. Scheme 3 considers the existence of $U_1-U_2$ link whereas the other two schemes do not. To characterize the performance of the proposed schemes assuming Rayleigh channel model, closed-form expressions of probability distribution function (PDF) of received SNR at $U_2$, average number of reliable relays, outage probability and bit error rate are obtained. For insight analysis, analytical results are validated through simulations which reveal that relay selection in NCRNs is different from non-cognitive networks and is feasible in low-to-medium SNR regions. The signal from near user further improved the outage performance.

INDEX TERMS Cognitive relay networks, non-orthogonal multiple access, performance analysis, relay selection, underlay spectrum sharing.

I. INTRODUCTION

It is predicted that by 2030, more than 125 billion wireless devices will be inter-connected exchanging about 5 zettabytes (ZB) of data per month at an individual data rate of up to 100 Gbps [1]. Due to such unprecedented growth rate, 5G and B5G mobile communication systems are facing the challenges of increased connection density, spectrum utilization efficiency, mobility, seamless connectivity, energy efficiency and user fairness [2]–[4]. Many wireless transmission techniques have been proposed in recent years to meet the requirements. Towards this, cognitive radio stands as a well-established and acknowledged technology that can effectively alleviate the spectrum shortage and spectrum under-utilization problems through dynamic re-use of frequency bands assigned to the PUs. The emergence of cognitive radio technology seems to provide a perfect answer to address the requirements and challenges of future wireless networks [5].

The dynamic spectrum access modes in cognitive radio networks are broadly classified as interweave, overlay and underlay [6]. The working principle of these modes is summarized as follows. Interweave mode uses interference avoidance mechanism to provide opportunistic access of the radio spectrum to the SUs in the absence of PU. In overlay and underlay mode, PUs and SUs transmit simultaneously in the same frequency band, however in overlay mode SU acquires apriori knowledge of the PU’s signal for relaying purpose while simultaneously transmitting its own message. In underlay mode on the other hand, SU transmits simultaneously...
with the PU at a restricted transmit power level to satisfy some predefined interference threshold. Underlay mode is the simplest and straightforward dynamic spectrum access mode of cognitive radio networks however it suffers from reduced coverage area of SUs.

To further exploit cognitive radio capabilities, the joint application of cognitive radio and cooperative relay has already gained enormous research interest. Cooperative communication is an acknowledged technique to combat multipath fading and to enhance the reliability and capacity of wireless networks. The spectrum utilization efficiency, system’s outage performance and coverage area can be significantly increased by incorporating multiuser diversity and cooperative relaying into cognitive radio networks [7]. AF and DF protocols are regarded as the most commonly employed cooperative relaying protocols. Dual-hop CRNs extend the coverage area of SUs in underlay operating mode and enable the transmitters to reach the distant destination [8].

Besides cognitive radio technique, NOMA being the latest technology trend is promising to realize the aggressive 5G goal through multiplexing multiple users in power domain over the same resource block (frequency/time/code). Conventional mobile networks have been relying on OMA techniques that follow the principle of user orthogonality in frequency/time/code domain and enable single user in each resource block [9], [10]. The disjoint resource allocation in OMA minimizes inter-user interference however does not hold the spectral efficiency required to support the demands of future generation networks. NOMA systems provide high spectral efficiency and support super-high data rate over OMA.

The basic working principle of NOMA is as follows. At transmitter side, NOMA employs SC to superimpose (multiplex) the signals from multiple users. For this purpose, NOMA usually classifies the users as strong (near) users and weak (far) users based on their channel qualities and allocate them different power levels accordingly [11]. Unlike traditional water-filling algorithm, NOMA achieves user fairness by allocating more power to the far user. At receiver side, SIC is used for multi-user signal separation by first decoding the stronger signal while treating the other signals as interference. NOMA is recognized as a breakthrough technology to cope up with the challenges of future mobile networks and many NOMA techniques have come up in recent years. The most commonly employed techniques are PD-NOMA [12] and CD-NOMA [13].

In NOMA systems as well, seeking assistance from intermediate relays increases the spatial degree of freedom and significantly enhances the communication reliability of far users. A collaborative NOMA assisted relaying system was introduced in [14], in which users with good channel conditions towards BS served as relays to improve the communication performance of the users with poor channel conditions. Another cooperative NOMA transmission scheme with a dedicated relay was introduced to improve the communication reliability of far users [15]. Combining the advantages of NOMA and CRNs, NOMA based Cognitive Relay Networks (NCRNs) are promising to improve the spectrum utilization efficiency and communication reliability of far users.

A. RELATED WORK

The downlink scenario of two-user cooperative NOMA systems comprising a BS, two destination users and a relay has been actively studied and deeply analyzed. A fixed-gain AF relay was employed along with the direct link transmissions in [11]. Two scenarios were discussed based on the presence or absence of intermediate relay between the BS and the users, however the near user did not assist the far user in its communication in both cases. Outage analysis revealed better fairness of NOMA as compared to OMA. In order to reduce the system complexity while maintaining full diversity gain of multi-relay networks, relay selection has been regarded as an efficient resource utilization technique which needs very careful designing. Relay selection in cooperative NOMA has been recently studied to improve the transmission reliability of users with poor channel condition or long distance [9], [16]–[21].

Ding et al. studied the impact of relay selection in cooperative NOMA networks [16]. The authors proposed a two-stage relay selection strategy in the absence of direct path between BS and the destination users. First stage guaranteed to satisfy the targeted data rate of one user, whereas the second stage satisfied the rate of other user opportunistically. The proposed scheme achieved maximal diversity gain and minimal outage probability. Wang et al. proposed a joint relay selection and antenna selection strategy for AF-based NCRNs [17]. The analytical results demonstrated that the number of relays and antennas as well as proper selection of fading parameters improved the outage probability and achieved significant performance gain over OMA systems. An optimal joint user and relay selection method has been proposed [18], where AF relays assisted multiple users in transmitting information to two destinations. Performance analysis illustrated that keeping the target data rate small enough, the diversity order of the outage probability was equal to the number of intermediate relays. At the other end, outage probability approached one when the target data rate tend to infinity. Yue et al. proposed a random relay selection scheme in which multiple DF relays were deployed in a given area according to uniform distribution [19]. The proposed technique was able to achieve low implementation complexity, however the performance of randomly selected node could not be guaranteed.

Cooperative NOMA with PRS has also been studied. In reference [20], the authors proposed a PRS scheme for AF-based cooperative NOMA networks. Through outage analysis, the authors showed that deploying more relays enhanced the performance gain and proper selection of power allocation factor improved the ergodic capacity of weak user. Throughput analysis of two-phase cooperative NOMA system with relay selection was performed in [9]. A decodable relay was selected to forward the far user’s
signal and it was verified that cooperative NOMA with relay selection achieved higher system throughput than non-cooperative NOMA and conventional OMA networks. The authors in [21] proposed an opportunistic multi-relay selection scheme using max-min criteria to satisfy the QoS of destination users. Sum rate and outage probability were derived in closed-form. The application of relay selection in downlink scenario of NCRNs operating in underlay spectrum sharing environment also exists [22]–[26]. Bariah et. al. proposed a power allocation scheme incorporated with PRS [22]. A single AF relay having the strongest link towards the BS was selected to forward the signal to the secondary receivers. The pairwise error probability of the SUs with imperfect SIC was derived. Another PRS scheme was proposed for DF-based NCRNs in underlay environment [23]. The mutual interference of both primary and secondary networks was considered and closed-form expressions for outage probability were obtained. Performance analysis demonstrated that outage probability can be significantly reduced by increasing the number of relays and through careful selection of power allocation factor. For downlink NCRNs in the absence of direct path between BS and the destination users, Hoa and Bao proposed a PRS scheme with fixed relaying [24]. The outage probability was derived in closed-form over Rayleigh fading channels and the effect of imperfect CSI on relay selection was investigated. Another cooperative scheme for underlay NCRNs was proposed in [25], where an SU having strong channel gain was selected by a multi-antenna BS to relay the far user’s signal in the presence of direct path. The near user received its signal from the BS directly whereas relay selection was employed to improve the fairness of cell-edge user only. The authors obtained approximated expressions for outage probability. In the presence of PU, Chamisa et. al. performed PRS to select the best DF relay in order to enable the end-to-end communication [26]. The outage analysis illustrated that system performance can be significantly enhanced by increasing the number of relay nodes.

All the highlighted research contributions laid solid foundation for understanding relay selection in cognitive and non-cognitive cooperative NOMA networks. However, relay selection in underlay NCRNs is still in its infancy and the aforementioned schemes [23]–[26] select the best DF relay based on SNR of first-hop only which greatly simplifies the analysis. Although a DF relay network with multiple relays supports PRS only, however PRS can be applied on either the first or the second hop [27]. Keeping this in mind, relay selection needs significant research attention in NCRNs operating in underlay mode by considering both the SNR of second-hop as well as interference offered by the relays towards the PU. Furthermore, none of the highlighted schemes investigated the system performance taking into consideration the existence of near user-far user link in addition to the best relay link. This article aims to partly fill these research gaps through various BRS schemes.

B. CONTRIBUTIONS
As discussed above, the motivation behind this work is to partly fill the gap that exists in literature regarding relay selection in dual-hop NCRNs operating in underlay environment. Thus, the main contributions of this article are summarized as follows:

We propose and investigate various BRS schemes for NCRNs comprising a BS, two users and multiple half-duplex cognitive relays operating in underlay mode in the vicinity of a PU. The near user receives its signal from BS on direct link only, however far user takes assistance of near user and a single best relay to receive BS’s signal while direct link is non-existent due to high obstructions or deep fading. Considering DF relaying over Rayleigh fading channels with I.I.D. channel coefficients, two NOMA transmission scenarios are discussed under QoS and interference constraints.

- Scenario 1: Far user receives signal from the best relay only. Two BRS schemes are proposed for this scenario. Scheme 1 selects the best relay based on maximum SNR on relay-to-far user link and Scheme 2 picks up the relay based on minimum interference towards the PU.
- Scenario 2: Far user receives signal from the best relay as well as from the near user. Scheme 3 proposed for this scenario picks up the relay which maximizes the quotient of relay-link SNR and relay-PU interference power. The QoS constraint plays a significant role in this scheme to address the hidden node problem as explained in Section III below. The PDF of total SNR received at the far user is derived using MGF approach [28].

The performance of all the proposed BRS schemes (max SNR, min interference, max quotient of SNR-to-interference) has been investigated in terms of average number of retransmit relays, outage probability and BER. Simulation results supported the derived closed-form expressions of performance metrics and provided important insights. Scheme 3 outsmarts the other two schemes due to the signal received from near user as well.

C. ORGANIZATION AND NOTATIONS
The rest of the paper is structured as follows. System model along with related assumptions and the mathematical formulation of BRS problem are discussed in Section 2. Section 3 presents the three proposed BRS schemes. Next, performance analysis of all the proposed schemes is carried out in Section 4. Section 5 presents the simulation results while Section 6 concludes this work. For convenience, the list of notations and abbreviations used in this article are provided in Table 1.

II. SYSTEM MODEL AND PROBLEM FORMULATION
Consider a dual-hop NCRN operating in underlay environment as shown in Fig. 1 in which a base station BS communicates with two destination users, a near-user \( U_1 \) and a far-user \( U_2 \). BS is located at the cell-center and \( U_1 \) is located
TABLE 1. Notations.

| Notation | Definition |
|----------|------------|
| AF | Amplify-and-Forward |
| AWGN | Additive White Gaussian Noise |
| BER | Bit Error Rate |
| BPSK | Binary Phase Shift Keying |
| BS | Base Station |
| CDF-NOMA | Code Domain-NOMA |
| CRN | Cognitive Relay Network |
| CSI | Channel State Information |
| DF | Decode-and-Forward |
| I.I.D. | Independent and Identically Distributed |
| MGF | Moment Generating Function |
| NOMA | Non-Orthogonal Multiple Access |
| NCRN | NOMA based Cognitive Relay Network |
| OMA | Orthogonal Multiple Access |
| PDF | Probability Distribution Function |
| PRS | Partial Relay Selection |
| PU | Primary User |
| QoS | Quality-of-Service |
| SIC | Successive Interference Cancellation |
| SC | Superposition Coding |
| SNR | Signal-to-Noise Ratio |
| SU | Secondary User |
| J | Total number of relays |
| h_{u,j} | Channel coefficient between BS and U_j |
| h_{j} | Channel coefficient between BS and j_th relay |
| g_{u,j} | Channel coefficient between U_i and U_j |
| g_{j} | Channel coefficient between j_th relay and U_2 |
| f_j | Channel coefficient between j_th relay and Q |
| P_S | Transmit power of BS |
| P_R | Transmit power of U_1 and j_th relay |
| \mu | Modulation Index |
| \gamma | SNR threshold of U_2 |
| \lambda_1 | Message intended for U_1 |
| \lambda_2 | Message intended for U_2 |
| \lambda_1 | Power allocation factor of U_1 |
| \lambda_2 | Power allocation factor of U_2 |
| \Gamma | Average strength of any communication link |
| \bar{I} | Average strength of any interference link |
| \Gamma_0 | Average strength of U_1 - U_2 link |
| N_0 | Noise Variance |

From this situation, it is assumed that the direct link between BS close to the BS, while U_2 is located close to the cell-edge. In this situation, it is assumed that the direct link between BS and U_2 suffers from deep fading, however BS takes assistance from U_1 and from a single best relay selected from a potential DF-based CRN consisting of J potential relays to reach U_2. In this context, two scenarios have been considered and multiple BRS schemes are proposed which will be discussed in subsequent sections.

Without loss of generality, the important assumptions in the system model are as follows:

- Each terminal is equipped with a single antenna therefore end-to-end communication is completed in two-time-slots owing to the traditional half-duplex mode of cooperative relaying systems.
- The channel coefficients are I.I.D. random variables normalized as Rayleigh fading channel thus the channel gains follow exponential distribution.
- BS does not create interference towards PU due to large physical separation between them.
- Each j_th relay is aware of the CSI of its corresponding communication and interference links. The relay network collects the information about the primary network when the PU is either in transmit mode or when it acknowledges the received signal.
- The relays do not carry out detection for U_1 and do not harm U_1 either.
- All communication and interference links are subjected to AWGN with zero mean and variance N_0.
- The relays are located physically close to each other thus they experience the same average strengths on the communication and interference links.

Proceeding further, since end-to-end communication between BS and U_2 is completed in two-time-slots therefore in first phase, BS employs SC and broadcasts its composite signal x_S which is received by U_1 and the potential relay network, where,

\[ x_S = \sqrt{\lambda_1} P_S x_1 + \sqrt{\lambda_2} P_S x_2 \]  

In the above Eq., \( \lambda_1 + \lambda_2 = 1 \), \( E[|x_1|^2] = E[|x_2|^2] = 1 \). The signal y_{SU_1} received at U_1 from BS is given by:

\[ y_{SU_1} = h_u x_S + n_{SU_1} \]

= \( h_u (\sqrt{\lambda_1} P_S x_1 + \sqrt{\lambda_2} P_S x_2) + n_{SU_1} \)  

For the proposed BRS schemes, it is assumed that the users are not arranged by their channel conditions. Therefore, U_1 can first detect x_2 followed by SIC to detect its own signal x_1 [29]. The instantaneous SINR at U_1 to detect x_2 is expressed as:

\[ \Gamma_{SU_1,x_2} = \frac{\lambda_2 P_S |h_u|^2}{\lambda_1 P_S |h_u|^2 + N_0} \]  

After removing x_2 through SIC, the instantaneous SNR at U_1 to detect its own signal x_1 is given as:

\[ \Gamma_{SU_1,x_1} = \frac{\lambda_1 P_S |h_u|^2}{N_0} \]  

The signal received at j_th relay from BS is given as:

\[ y_{SR_j} = h_j x_S + n_j \]

= \( h_j (\sqrt{\lambda_1} P_S x_1 + \sqrt{\lambda_2} P_S x_2) + n_j \)  

where, \( j = 1, 2, \ldots, J \). The instantaneous SINR at j_th relay to decode x_2 is given as:

\[ \Gamma_{SR_j,x_2} = \frac{\lambda_2 P_S |h_j|^2}{\lambda_1 P_S |h_j|^2 + N_0} \]  

The selection of proper value of \( \lambda_2 \) plays a critical role in correct decoding. It is assumed that \( P_{R_1} = P_{R_2} = \ldots = P_{R_J} = P_{U_1} = P_{R} \). In second phase, a best relay is selected to forward the decoded signal x_2 to U_2. The signal received at the cell-edge user U_2 from any j_th relay link is given as:

\[ y_{R_j U_2} = g_j \sqrt{P_{R} x_2} + n_{R_j U_2} \]  

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Therefore, the instantaneous SNR at $U_2$ to detect $x_2$ during second-hop transmission is expressed as:

$$0 R_{j U_2} x_2 = \frac{P_R |g_j|^2}{N_0}$$  \hspace{1cm} (8)$$

From (6) and (8), owing to the dual-hop communication, the end-to-end SNR $\Gamma_j$ received at $U_2$ due to any $j^{th}$ DF relay link is upper bounded by \[ (9), \]

$$\Gamma_j \leq \min(\Gamma_{SR_2 U_2}, \Gamma_{R_j U_2}) \leq \Gamma_{R_j U_2}$$

where, the utility function $\min(\Gamma_{SR_2 U_2}, \Gamma_{R_j U_2})$ is a tight approximation of $\Gamma_j$ and represents the bottleneck effect along the $BS \to R_j \to U_2$ path. The upper bound SNR is more suitable for medium-SNR and high-SNR analysis.

In this work, since the best relay enables the end-to-end communication for $U_2$ only, therefore to proceed further, $U_2$ and $x_2$ can be dropped in the above Eq., for the simplicity of notation, i.e.

$$\Gamma_j \leq \Gamma_{R_j}$$  \hspace{1cm} (10)$$

where, $\Gamma_{R_j}$ is the SNR of the $j^{th}$ $R_j - U_2$ link. Proceeding towards the mathematical formulation of the BRS problem, the conventional max-min relay selection in the traditional non-cognitive relay networks can be conducted based on the single rule of $j^\ast = \max_{j=1,2,...,J} (\Gamma_j)$, which also gives $j^\ast = (J)$, hence the diversity order of the non-cognitive relay network is $J$. However, this may not be the only selection criterion for CRNs operating in underlay mode since a relay maximizing the end-to-end SNR on one hand might generate high interference to the PU on the other hand, therefore SUs strictly monitor the interference towards PU and adjust their transmit powers accordingly. Hence, transmission process at the relay network takes place only if interference threshold $\beta$ towards the PU $Q$ is satisfied. However, for some relays it would not be possible to satisfy this constraint even they provide maximum SNR on their respective relay-$U_2$ link, therefore the best relay will be picked up to maximize the performance at its corresponding relay-$U_2$ link while adhering to the interference limit. The interference offered by any $j^{th}$ relay towards the PU can be mathematically expressed as:

$$I_j = P_r |f_j|^2$$  \hspace{1cm} (11)$$

Summing up the whole discussion, we define two constraints for any $j^{th}$ relay. First, interference constraint towards $Q$ i.e. $I_j \leq \beta$ and second, QoS constraint of $U_2$ which requires $\Gamma_j \geq \zeta$, where $\zeta$ is the SNR threshold. Thus, our aim is to select the best relay with index $j^\ast$ that maximizes $\Gamma_j$ while satisfying the above two constraints. Mathematically,

$$j^\ast = \max_j (\Gamma_j)$$

subject to,

$$C_1 : I_j \leq \beta$$

$$C_2 : \Gamma_j \geq \zeta$$  \hspace{1cm} (12)$$
III. PROPOSED BEST RELAY SELECTION SCHEMES

As discussed above, both $\Gamma_j$ and $I_j$ are I.I.D. random variables, therefore, their PDFs are expressed by (13) and (14) respectively as:

\[ p_{\Gamma_j}(\Gamma) = \frac{1}{\Gamma} e^{-\frac{\Gamma}{\Gamma}} \]  \hfill (13)

\[ p_{I_j}(x) = \frac{1}{\hat{l}} e^{-\frac{x}{\hat{l}}} \]  \hfill (14)

and their corresponding CDFs are given in (15) and (16) respectively as:

\[ P_{\Gamma_j}(\Gamma) = 1 - e^{-\frac{\Gamma}{\Gamma}} \]  \hfill (15)

\[ P_{I_j}(x) = 1 - e^{-\frac{x}{\hat{l}}} \]  \hfill (16)

where $\hat{\Gamma}$ and $\hat{l}$ are the average strengths of communication links and interference links respectively. We consider two scenarios to propose the BRS schemes. First, far user receives signal from best relay only and second, far user receives signal from best relay and $U_1$. For all the proposed schemes, first we define a universal set $\Psi$ consisting of $J$ potential relays and a subset $\Omega \subseteq \Psi$ containing those relays which satisfy the interference threshold. Let us define another subset $\Phi \subseteq \Psi$ containing all relays which meet the SNR threshold of $U_2$. Thus, the best relay with the index $l^*$ lies in the subset $\Upsilon = \Omega \cap \Phi$ consisting of $L \leq J$ shortlisted relays.

A. FAR USER RECEIVES SIGNAL FROM SELECTED BEST RELAY ONLY

In this scenario, two schemes are proposed to select the best relay in order to enable the end-to-end communication between BS and $U_2$.

SCHEME 1

Scheme 1 picks up the best relay from the set $\Upsilon$ which maximizes SNR on the corresponding $R_1 \rightarrow U_2$ link conditioned on the two constraints given in (12). Mathematically,

\[ l^* = \max_{l \in \Upsilon} (\Gamma_{R_l}) \]
\[ \text{subject to,} \]
\[ C_1 : I_l \leq \beta \]
\[ C_2 : \Gamma_l \geq \xi \]  \hfill (17)

where, $l^*$ is the index of best relay and $l = 1, 2, \ldots, L$. Let us denote the SNR of the best $R_1 \rightarrow U_2$ link be $\Gamma^{1}_{l^*}$, where, 1 denotes Scheme 1. The conditional PDF of $\Gamma^{1}_{l^*}$ given $L$ available relays is expressed as:

\[ p_{\Gamma^{1}_{l^*}}(\Gamma/L) \]
\[ = p_{\Gamma^{1}_{l^*}}(\Gamma)Pr[\Gamma_{R_l} < \Gamma_{R_{l^*}}] \ldots Pr[\Gamma_{R_{l-1}} < \Gamma_{R_l}] \]
\[ + p_{\Gamma^{1}_{l^*}}(\Gamma)Pr[\Gamma_{R_1} < \Gamma_{R_{l^*}}] \ldots Pr[\Gamma_{R_{l-1}} < \Gamma_{R_l}] \]
\[ + \ldots + p_{\Gamma^{1}_{l^*}}(\Gamma)Pr[\Gamma_{R_1} < \Gamma_{R_{l^*}}] \ldots Pr[\Gamma_{R_{L-1}} < \Gamma_{R_l}] \]  \hfill (18)

Based on the assumptions made in Section II, during a hop transmission, the relays experience the same average strengths on all communication and interference links, hence, $\Gamma^{1}_{l^*} = \Gamma^{2}_{l^*} = \ldots = \Gamma^{k}_{l^*} = \hat{\Gamma}$, where $k = 1, 2$ and $\hat{I}_1 = \hat{I}_2 = \ldots = \hat{I}_k$ = $\hat{I}$, thus the above Eq. can be simplified as:

\[ p_{\Gamma^{1}_{l^*}}(\Gamma/L) = \frac{L}{\Gamma} \left( e^{-\frac{\Gamma}{\hat{\Gamma}}} \right) \left( 1 - e^{-\frac{\Gamma_{R_{l^*}}}{\hat{\Gamma}}} \right)^{L-1}, \quad l \neq l^* \]  \hfill (19)

In the above Eq., $p_{\Gamma^{1}_{l^*}}(\Gamma)$ is the PDF of best relay-link SNR and $Pr[\Gamma_{R_l} < \Gamma_{R_{l^*}}]$ is the CDF of $\Gamma_{R_k}$ evaluated at $\Gamma_{R_{l^*}}$. Using (13) and (15), the above Eq. becomes:

\[ p_{\Gamma^{1}_{l^*}}(\Gamma/L) = \frac{L}{\hat{\Gamma}} \left( e^{-\frac{\Gamma}{\hat{\Gamma}}} \right) (1 - e^{-\frac{\Gamma_{R_{l^*}}}{\hat{\Gamma}}})^{L-1} \]  \hfill (20)

The communication cannot take place at $L = 0$ because $U_2$ receives signal from best relay only. If $L = 1$, there will be no relay selection. Thus, the best relay will be selected when $L \geq 2$. Let us define $P_\beta$ and $P_\zeta$ as the probability of satisfying $C_1$ and $C_2$ respectively. Then the probability of finding $L$ reliable relays from $J$ potential relays can be expressed as binomial distribution given by:

\[ p_L(L; J; P_q) = \frac{J!}{L!(J-L)!} (P_q)^L (1-P_q)^{J-L} \]  \hfill (21)

where, \(\frac{J!}{L!(J-L)!}\) and $P_q = P_\beta P_\zeta$.

Finally, the unconditional PDF of $\Gamma^{1}_{b}$ can be found by averaging (20) over (21) as given below:

\[ p_{\Gamma^{1}_{b}}(\Gamma) = \frac{\frac{e^{-\frac{\Gamma}{\hat{\Gamma}}}}{\hat{\Gamma}} \left[ \sum_{L=1}^{J} \frac{J!}{L!(J-L)!} (P_q)^L \right]}{(1-P_q)^{J-L}} \left[ 1 - e^{-\frac{\Gamma_{R_{l^*}}}{\hat{\Gamma}}} \right]^{L-1} \]  \hfill (22)

SCHEME 2

There exists a non-zero probability that a particular relay which maximizes the end-to-end SNR at $U_2$ according to Scheme 1 does not offer the minimum interference towards $Q$ at the same time. Keeping this in mind, the primary objective of Scheme 2 is to investigate the system performance in the case when a relay offering the minimum interference is given preference for selection. Therefore, Scheme 2 picks up the best relay which minimizes interference on the corresponding $R_1 \rightarrow Q$ link. Mathematically,

\[ l^* = \min_{l \in \Upsilon} (I_l) \]
\[ \text{subject to,} \]
\[ C_1 : I_l \leq \beta \]
\[ C_2 : \Gamma_l \geq \xi \]  \hfill (23)

Let us denote the SNR of the best $R_1 \rightarrow U_2$ link be $\Gamma^{2}_{l^*}$ for Scheme 2. The conditional PDF of $\Gamma^{2}_{b}$ given $L$ available relays is expressed as:

\[ p_{\Gamma^{2}_{l^*}}(\Gamma/L) \]
\[ = p_{\Gamma^{2}_{l^*}}(\Gamma)Pr[ I_1 < I_2] Pr[ I_1 < I_3] \ldots Pr[ I_1 < I_L] \]
Following the same assumptions and same procedure as \( \eta \) only.

This scenario is considered to include the case when \( L = 0 \) i.e. none of the relays satisfies both constraints and end-to-end communication between BS and \( U_2 \) is enabled via \( U_1 \) only.

\section*{B. FAR USER RECEIVES SIGNAL FROM NEAR USER AND SELECTED BEST RELAY}

\begin{equation}
\begin{aligned}
p_{\Gamma_0}^{\gamma}(\Gamma/L) &= Lp_{\Gamma_0}^{\gamma}(\Gamma)\left[Pr[I_{i^*} < I_1]\right]^{L-1}, \quad l \neq l^*
\end{aligned}
\end{equation}

which then becomes,

\begin{equation}
\begin{aligned}
p_{\Gamma_0}^{\gamma}(\Gamma/L) &= \frac{L}{\Gamma}\left(e^{-\Gamma/L}\right)\left(1 - e^{-(\Gamma/L)}\right)^{L-1}
\end{aligned}
\end{equation}

Following the same assumptions and same procedure as Scheme 1, the unconditional PDF of \( \Gamma_0^{\gamma} \) takes the following form,

\begin{equation}
\begin{aligned}
p_{\Gamma_0}^{\gamma}(\Gamma) &= \frac{e^{-\Gamma/\Gamma}}{\Gamma} \left( \sum_{L=1}^{J} \binom{J}{L} L(P_d)^{L} \times (1 - P_d)^{J-L} \left(1 - e^{-(\Gamma/L)}\right)^{L-1} \right)
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
\text{The above Eq. can be simplified as:}
p_{\Gamma_0}^{\gamma}(\Gamma/L) &= Lp_{\Gamma_0}^{\gamma}(\Gamma)\left[Pr[\eta_1 > \eta_l]\right]^{L-1}
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
= Lp_{\Gamma_0}^{\gamma}(\Gamma)\left[Pr\left[\frac{\Gamma_0^{\gamma}}{I_{i^*}} > \frac{\Gamma}{I_1}\right]\right]^{L-1}, \quad l \neq l^*
\end{aligned}
\end{equation}

or we can write,

\begin{equation}
\begin{aligned}
p_{\Gamma_0}^{\gamma}(\Gamma/L) &= Lp_{\Gamma_0}^{\gamma}(\Gamma)\left[Pr\left[\frac{\Gamma_0^{\gamma}}{I_{i^*}} < \frac{\Gamma}{I_1}\right]\right]^{L-1}, \quad l \neq l^*
\end{aligned}
\end{equation}

In the above Eq., \( Pr[\frac{\Gamma_0^{\gamma}}{I_{i^*}} < \frac{\Gamma}{I_1}] \) is the CDF of \( \frac{\Gamma_0^{\gamma}}{I_{i^*}} \) evaluated at \( \Gamma_0^{\gamma} \). We will derive this CDF in two steps. First, we consider \( \frac{\Gamma_0^{\gamma}}{I_{i^*}} = \eta_l \) for all the selected relays. Next, we will consider \( W_l = \eta_l I_{i^*} \), in which \( I_{i^*} \) denotes the interference offered by the selected best relay towards the PU. The conditional CDF of \( \eta_l \) with the constraints \( C_1 \) and \( C_2 \) will be evaluated as:

\begin{equation}
\begin{aligned}
P_{\eta_l}(\eta; \Gamma_1 \geq \zeta, I_1 \leq \beta) &= \int_0^{\beta} \left[ \int_{\zeta}^{\beta} p_{\Gamma_0}^{\gamma}(\eta) d\Gamma_1 \right] p_{\eta_l}(\eta) dx = P_{\beta} P_{\eta_l} = \frac{1 - a \epsilon^{-\eta}}{1 + c \eta}
\end{aligned}
\end{equation}

where, \( P_{\zeta} = Pr(\Gamma_1 \geq \zeta) = e^{-\frac{\zeta}{\beta}} \), \( P_{\eta_l} = Pr(I_1 \leq \beta) = 1 - e^{-\frac{\beta}{a}} \), \( r = \frac{\beta}{a} \), \( a = e^{-\frac{\beta}{c}} \), \( c = \frac{1}{\Gamma} \). The PDF of \( \eta_l \) after differentiating the above Eq. will be:

\begin{equation}
\begin{aligned}
P_{\eta_l}(\eta; \Gamma_1 \geq \zeta, I_1 \leq \beta) &= \frac{c - (mn + n) \epsilon^{-\eta}}{1 + c \eta
\end{aligned}
\end{equation}

The distribution of \( \eta_l \) is given in (33) and \( I_{i^*} \) is exponentially distributed with the parameter \( I \). Substituting the required expressions in the above eq. and doing mathematical manipulations using [31, Eqs. 3.352.1, 3.462.16-17, 8.359.1] we get,

\begin{equation}
\begin{aligned}
P_{W_l}(w; \Gamma_1 \geq \zeta, I_1 \leq \beta) &= \int_0^{\beta} \left[ \int_{\frac{w}{\beta}}^{\infty} p_{\eta_l}(\eta) d\eta \right] p_{\eta_l}(x^*) dx = 
\end{aligned}
\end{equation}

The distribution of \( \eta_l \) is given in (33) and \( I_{i^*} \) is exponentially distributed with the parameter \( I \). Substituting the required expressions in the above eq. and doing mathematical manipulations using [31, Eqs. 3.352.1, 3.462.16-17, 8.359.1] we get,

\begin{equation}
\begin{aligned}
P_{W_l}(w; \Gamma_1 \geq \zeta, I_1 \leq \beta) &= \int_0^{\beta} \left[ \int_{\frac{w}{\beta}}^{\infty} p_{\eta_l}(\eta) d\eta \right] p_{\eta_l}(x^*) dx
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
= \frac{w}{\beta} e^{\frac{\zeta}{\beta}} \times \left[ E\left(- \frac{\beta}{\beta} - \frac{w}{\beta}\right) - E\left(- \frac{w}{\beta}\right) \right]
\end{aligned}
\end{equation}

where, \( Z = -\frac{c e^{-\eta_l}}{I + \beta} \) and \( E(.) \) is the exponential integral. Substituting (35) in (31), we finally get:

\begin{equation}
\begin{aligned}
P_{\Gamma_0}^{\gamma}(\Gamma/L; \Gamma_1 \geq \zeta, I_1 \leq \beta) &\approx \frac{L L^{L-1}}{\frac{L}{\Gamma}} e^{-\Gamma/L}
\end{aligned}
\end{equation}

The above approximation is obtained by replacing \( w = \Gamma \) in (35) and ignoring the higher powers of
Finally, the unconditional PDF of $\Gamma$ can be obtained by averaging (36) over (21) which gives:

$$p_{\Gamma}(\Gamma; \Gamma \geq \zeta, I_1 \leq \beta) = \frac{e^{-(\Gamma/\hat{P})}}{\hat{P}} \sum_{L=1}^{J} \left( \frac{J}{L} \right) L Z^{L-1} \times (P_q)^L (1 - P_q)^{J-L}$$  \hspace{1cm} (37)

Since the selected relay must guarantee the end-to-end SNR $\Gamma$ above the threshold $\zeta$ according to constraint $C_2$, this condition can be expressed as truncated MGF of $\Gamma^3_b$ as:

$$M_{\Gamma_b^3}(s; \Gamma \geq \zeta, I_1 \leq \beta) = \int_0^{\infty} e^{-st} p_{\Gamma} (\Gamma; \Gamma \geq \zeta, I_1 \leq \beta) d\Gamma$$

where,

$$M_{\Gamma_b^3}(s; \Gamma \geq \zeta, I_1 \leq \beta) = \frac{e^{-s \hat{P} \zeta}}{\hat{P}} \sum_{L=1}^{J} \left( \frac{J}{L} \right) L Z^{L-1} \times (P_q)^L (1 - P_q)^{J-L}$$  \hspace{1cm} (38)

Now, the signal received at $U_2$ via $U_1 - U_2$ link is expressed as:

$$y_{U_1 U_2} = g_n \sqrt{P_{R}} X_2 + n_{U_2}$$  \hspace{1cm} (39)

The SNR on $U_1 - U_2$ link is also exponentially distributed with average value $\Gamma_0$, therefore, its MGF can be easily obtained as $\frac{1}{1 + \lambda \Gamma_0}$. Finally, the selected relay link and $U_1 - U_2$ link are independent, therefore the CDF of total SNR $\Gamma_T$ at the cell-edge user $U_2$ can be expressed as:

$$P_{\Gamma_T} (\Gamma; \Gamma \geq \zeta, I_1 \leq \beta) = L^{-1} \left[ \frac{M_{\Gamma_b^1 U_2} (s) M_{\Gamma_b^3} (s)}{s} \right]_{s = \Gamma}$$  \hspace{1cm} (40)

where, $L^{-1}$ denotes the inverse Laplace transform operator. Substituting (38) in (40) and solving using [31, Table 17.3.25], we get,

$$P_{\Gamma_T} (\Gamma) = \sum_{L=1}^{J} \left( \frac{J}{L} \right) L Z^{L-1} (1 - P_q)^{J-L} P_q \times \left[ \Gamma_0 e^{-(\Gamma/\hat{P} \zeta)} - \hat{\Gamma} e^{-(\Gamma/\hat{P} \zeta)} + (\hat{\Gamma} - \Gamma_0) \right]$$  \hspace{1cm} (41)

In the above eq., the constraints are dropped for the sake of simplicity of notation.

### IV. PERFORMANCE ANALYSIS

The network performance highly depends on the resource allocation appropriateness. Among the most important metrics to evaluate system performance and QoS is the outage probability. Outage occurs when the achievable rate falls below a predefined threshold. For better system performance, a system requires higher throughput and lower outage probability.

#### A. AVERAGE NUMBER OF RELIABLE RELAYS

For performance analysis, first we find an expression for the average number of reliable relays available for retransmission. As discussed above, several constraints affect the performance of the secondary network in NCRNs operating in underlay environment such as interference level from relay to PU and SNR threshold from relay to $U_2$. All these elements affect the average number of reliable relays. Therefore, we consider two conditions. First, deriving an expression for average number of active relays in the presence of transmission threshold only. Second, NCRNs taking into consideration both SNR and interference constraints. Let us denote $L_1$ and $L_2$ be the average number of reliable relays for the first and second condition respectively. Therefore,

$$L_1 = \sum_{j=1}^{J} \left( \frac{J}{j} \right) j (P_{r_1}) (1 - P_{r_1})^{J-j}$$  \hspace{1cm} (42)

where, $P_{r_1}$ is the reliability probability for the first condition given by:

$$P_{r_1} = Pr[\Gamma \geq \zeta] = \frac{e^{-\zeta}}{\hat{P}}$$  \hspace{1cm} (43)

Similarly, $L_2$ is expressed as:

$$L_2 = \sum_{j=1}^{J} \left( \frac{J}{j} \right) j (P_{r_2}) (1 - P_{r_2})^{J-j}$$  \hspace{1cm} (44)

where the reliability probability $P_{r_2}$ for the second case is given by:

$$P_{r_2} = Pr[\Gamma \geq \zeta] AND Pr[I_1 \leq \beta] = e^{-\frac{\zeta}{\hat{P}}} \left( 1 - e^{-\frac{\beta}{\hat{P}}} \right)$$  \hspace{1cm} (45)

Plugging the above Eq. in (44), we get,

$$L_2 = \sum_{j=1}^{J} \left( \frac{J}{j} \right) j (e^{-\frac{\zeta}{\hat{P}}} (1 - e^{-\frac{\beta}{\hat{P}}})^j) \times \left( 1 - e^{-\frac{\zeta}{\hat{P}}} (1 - e^{-\frac{\beta}{\hat{P}}}) \right)^{J-j}$$  \hspace{1cm} (46)

#### B. OUTAGE PROBABILITY

Outage analysis of all the proposed BRS schemes is performed below.

**SCHEME 1**

The outage probability $P_0^1$ can be expressed by the following expression:

$$P_0^1 = 1 - \sum_{j=1}^{J} \left( \frac{J}{j} \right) (P_{r}) (1 - P_{r})^{J-j} \times Pr[\Gamma_1 > \Gamma_2] Pr[\Gamma_1 > \Gamma_3] \ldots Pr[\Gamma_1 > \Gamma_j] + Pr[\Gamma_2 > \Gamma_1] Pr[\Gamma_2 > \Gamma_3] \ldots Pr[\Gamma_2 > \Gamma_j] + \ldots + Pr[\Gamma_j > \Gamma_1] Pr[\Gamma_j > \Gamma_2] \ldots Pr[\Gamma_j > \Gamma_{j-1}]$$  \hspace{1cm} (47)
In compact form, 
\[ P_0^1 = 1 - \left[ \sum_{j=1}^{J} \left( \frac{J}{j} \right) (P_x y (1 - P_x)^{J-j} \right] \times \sum_{j=1}^{J} \left\{ Pr[\Gamma_j > \Gamma_j^*] \right\} \] (48)
where, \( J \) and \( J' \) refer to the relay under consideration and the remaining relays respectively.

SCHEME 2
Following the same approach, outage probability \( P_0^2 \) is given as:
\[ P_0^2 = 1 - \left[ \sum_{j=1}^{J} \left( \frac{J}{j} \right) (P_x y (1 - P_x)^{J-j} \right] \times \sum_{j=1}^{J} \left\{ Pr[I_j < I_j^*] \right\} \] (49)

SCHEME 3
The outage probability \( P_3^3 \) can be directly obtained by substituting \( \Gamma = \Gamma_{ih} \) in the expression of CDF of total SNR derived in (41) where \( \Gamma_{ih} \) is the outage threshold. It is quite possible that none of the relays is available for selection due to the interference constraint, i.e. \( L = 0 \) and \( U_2 \) receives signal from \( U_1 \) only. The probability of this event is given by:
\[ Pr[L = 0] = (1 - P_q)^L \] (50)
Hence, the outage probability of Scheme 3 is expressed as:
\[ P_3^3 = Pr[\Gamma] + (1 - P_q)^L \] (51)
where, \( (1 - e^{\Gamma_{ih}/\Gamma_0}) \) is probability of the event when the received SNR is less than \( \Gamma_{ih} \) for \( L = 0 \).

C. AVERAGE BIT ERROR PROBABILITY
In order to derive average BER, it is an acknowledged method to use standard \( Q \) function to express the probability of error conditioned over given SNR in AWGN and then taking its average over the PDF of desired total SNR [32]. Therefore, BER of all the proposed schemes is derived below.

SCHEME 1
The probability of bit error \( P_e^1 \) is given by:
\[ P_e^1 = \int_{0}^{\infty} P_e(\Gamma) d\Gamma \] (52)
where, \( P_e(\Gamma) \) denotes the PDF of \( \Gamma_b^1 \) derived in (22) and \( P_e(\Gamma_{b1}^1) \) is the conditional probability of error. Based on the above discussion, \( P_e(\Gamma_{b1}^1) = Q(\sqrt{\mu}) \). Plugging in the required expressions in (52) and after doing some mathematical manipulations, the probability of bit error \( P_e^1 \) is given by:
\[ P_e^1 = \frac{1}{4} \sqrt{\frac{2\mu \Gamma}{\pi (2 + \mu \Gamma)}} \sum_{L=1}^{J} \left[ \left( \frac{J}{L} \right) L(P_q)^L \times (1 - P_q)^{J-L} \right] \times \left( e^{-(\Gamma - \Gamma_0)} - 1 \right)^{L-1} \] (53)

SCHEME 2
Similarly, probability of bit error \( P_e^2 \) is given by:
\[ P_e^2 = \frac{1}{4} \sqrt{\frac{2\mu \Gamma}{\pi (2 + \mu \Gamma)}} \sum_{L=1}^{J} \left[ \left( \frac{J}{L} \right) L(P_q)^L \times (1 - P_q)^{J-L} \right] \times \left( e^{-(\Gamma - \Gamma_0)} - 1 \right)^{L-1} \] (54)

SCHEME 3
The probability of bit error \( P_e^3 \) can be expressed as:
\[ P_e^3 = Pr[L = 0] \int_{0}^{\infty} P_e(\Gamma) \times \left[ \Pi U_1 U_2 \right] \times \Gamma_0 \times \Gamma_0 \] (55)
where, \( P_e(\Gamma) = Q(\sqrt{\mu}) \). Again using the technique in [32], \( P_e^3 \) can be obtained using the derived CDF of SNR as:
\[ \int_{0}^{\infty} P_e(\Gamma) \times \Gamma_0 \times \Gamma_0 \times \Gamma_0 \times \Gamma_0 \times \Gamma_0 \times \Gamma_0 \] (56)
Using (41) and solving using [31,3,321,2], we get,
\[ P_e^3 = \frac{(1 - P_q)^L}{2} \left[ 1 - \frac{\mu \Gamma U_1 U_2}{2(1 + \Gamma U_1 U_2)} \right] + \sqrt{\frac{2P_e}{P_e}} \] (57)
where, \( erfc(.) \) is the complementary error function, \( \phi_1 = \sqrt{\frac{\Gamma_{ih} + \mu}{2\Gamma_{ih}}} \) and \( \phi_2 = \sqrt{\frac{\Gamma_{ih} + \mu}{2\Gamma_{ih}}} \).

V. SIMULATION RESULTS AND DISCUSSION
This section examines the performance of BRS schemes proposed for two different scenarios of NCRNs in underlay environment. The performance is evaluated based on the average number of retransmit relays, outage probability and BER by varying the average SNR per hop, power allocation factor and transmit power of the relay. Simulations are performed to verify the analytical results. First, the simulation setup is defined followed by discussion about the results.
A. SIMULATION SETUP

For Scenario 1, \( U_1 - U_2 \) link does not exist as discussed above. Hence, the average SNR of first and second hop is set as \( \Gamma = 10\Gamma_0 \). For Scenario 2, \( U_1 - U_2 \) link is simulated with \( \Gamma_0 = 0.8\Gamma \). For both scenarios, the interference channels between the relays and the PU are generated with parameter \( I = 0.7\Gamma \). Other common parameter values for both scenarios are as follows, unless mentioned otherwise. The transmit power of BS, \( U_1 \) and relay is set as, \( P_S = P_R = P = 10\)dbW. The noise in each hop is AWGN with zero mean and unit variance. BPSK modulation scheme is used with \( \mu = 2 \). The maximum number of potential relays \( J \) is set to 10. Regarding power allocation factors, \( \lambda_1 = 0.3 \) and \( \lambda_2 = 0.7 \).

B. DISCUSSION

In Fig. 2, the average number of retransmit relays is plotted for two cases to study the significance of interference constraint in underlay environment. First case considers SNR constraint only and it is observed that the average number of reliable relays increases with increase in average SNR and at high SNR all relays are satisfying the SNR threshold so they are able to retransmit. The second case considers both constraints and the figure contains plots for \( \beta = 0.5, 10, 15\)dB. For low interference level, it is much harder for the relays to satisfy the constraints and the average number of retransmit relays is the lowest. In fact, when the interference threshold is relaxed, the average number of active relays increases. However, the number of retransmit relays gradually decreases with increase in average SNR due to strong interference offered to the PU and vice versa.

Fig. 3 and 4 illustrate the outage probability of Scenario 1 and Scenario 2 respectively for \( \beta = 10 \) and \( \zeta = 1 \). Fig. 3 shows that both Scheme 1 and Scheme 2 have the same outage probability. In fact, the relays offer less interference in the low SNR region which increases the number of active relays, hence the outage probability decreases.

However, as the average SNR increases, the interference from the relays becomes stronger resulting in decrease in the number of active relays. Thus, a relay with maximum SNR may not be selected in the high SNR region which increases the outage probability. In Fig. 4, outage probability of Scheme 3 is plotted. Outage probability of the system when it is operating on \( U_1 - U_2 \) link only is also plotted for comparison purpose, i.e. when none of the relays satisfied both constraints. It can be observed that interference constraint is not an issue in low-to-medium SNR region and outage probability follows a normal trend. As the average SNR per hop increases, the relays start violating the interference constraint and get excluded from the selection pool. It results in performance degradation and eventually at high SNR the system operates on \( U_1 - U_2 \) link only.

Fig. 5 illustrates BER of two BRS schemes proposed for the first scenario. Two values of interference threshold are analyzed, i.e. \( \beta = 0\)dB and \( \beta = 10\)dB. As the figure shows,
Scheme 1 outperforms Scheme 2 in low SNR region as selecting a relay with maximum SNR provides the best performance. However, both schemes get closer in the high SNR region due to strict interference constraint and form their unique curves. Relaxing the interference threshold further improves the system’s performance for both schemes. BER of Scheme 3 is depicted in Fig. 6 under different sets of constraints. BER shows similar trends as observed in Fig. 4. The variation in the threshold levels suggest different optimal operating points of the system to achieve the minimum BER.

Fig. 7 presents BER of all schemes versus the number of potential relays as a function of average SNR per hop. All the remaining parameters are kept the same. As discussed above, interference constraint is not an issue at low-to-medium SNR levels. Hence, BER follows the normal trend and decreases significantly for all schemes with increase in the number of potential relays since more choices are available to perform relay selection. In addition, Scheme 3 outsmarts the other two schemes in all SNR regions due to the existence of $U_1 - U_2$ link as well. The BER of traditional non-cognitive cooperative NOMA network, i.e. without interference constraint is also plotted for comparative analysis only. In NCRN under consideration, the best relay is chosen amongst $L \leq J$ shortlisted relays which satisfy the interference constraint. In contrast, the traditional non-cognitive networks perform relay selection amongst $J$ potential relays and hence the diversity order of the system is $J$ at all SNR values. Therefore on the average, diversity order of cognitive network is less than that of the non-cognitive network because relay selection is controlled by the imposed constraints. This phenomenon results in better BER in non-cognitive cooperative NOMA networks at any value of SNR and $J$.

Fig. 8 plots the outage probability of $U_2$ by varying the power allocation factor $\lambda_2$ keeping $J = 10$, $\zeta = 1$ and SNR = 10dB. The simulation results again strictly match
with the analytical lines which confirms the accuracy of our
derivation. It can be observed that highest outage event occurs when \( \lambda_2 \) is too high or too low since interference constraint
effects the SNR of the system. Lower outage probability is
obtained keeping \( \lambda_2 \) in the range of 0.55 \( \rightarrow 0.75 \) with the
minimum value achieved at \( \lambda_2 = 0.7 \).

Finally, Fig. 9 provides an interesting insight to BER analysis.
The figure illustrates BER of the proposed schemes versus maximum
allowable transmit power \( P \) for two interference threshold levels, i.e. \( \beta = 10\text{dB} \) and \( \beta = 25\text{dB} \). The
figure shows that when interference threshold is limited to \( \beta < P \) for all values of \( \beta \), BER keeps on decreasing over the
full range of \( P \). However when \( \beta \approx P \), BER approaches a
constant value as observed for the case of \( \beta = 10\text{dB} \) which
clearly shows that even increasing the maximum allowable
transmit power beyond 10dB, the transmit power will be
limited to \( \beta \).

VI. CONCLUSION
This paper presents various BRS schemes for NCRNs
operating in underlay environment with DF relays. Three
selection criterion were considered; maximum SNR, minimum
interference and maximum quotient of SNR to interference.
Through closed-form expressions derived for outage proba-
bility and BER it was shown that relay selection in NCRNs
is feasible in low-to-medium SNR region due to interference
constraint. Scheme 1 was proposed to enhance the system
performance. Scheme 2 presented low interference to the
PU while maintaining an acceptable performance level and
Scheme 3 illustrated the significance of \( U_1 - U_2 \) link in
addition to the selected best relay link. Analysis and simula-
tions presented a good match. It was observed that either by
increasing the SNR threshold or decreasing the interference
threshold, it becomes very difficult for the relays to qualify
for selection thus increasing BER of the system. In contrast,
when the constraints are relaxed, more freedom is given to
relay selection reducing BER.

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