Robust PI-Based Non-Singular Terminal Synergetic Control for Nonlinear Systems via Hybrid Nonlinear Disturbance Observer

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ABSTRACT This paper proposes a novel robust proportional integral non-singular terminal synergetic control (PI-NTSC) for a class of affine nonlinear systems. To counteract the effects of the disturbances, a hybrid nonlinear disturbance observer (HNDOB) scheme is proposed to estimate the matched and mismatched disturbances. A novel proportional integral non-singular terminal synergetic control (PI-NTSC) is proposed for a class of affine nth-order systems. The stability of the PI-NTSC approach is proven using the Lyapunov stability theory. Its effectiveness and applicability is evaluated through simulations of a four-bar linkage mechanism and a servo-hydraulic system. The proposed HNDOB-based PI-NTSC is characterized by a finite time convergence rate, simple implementation, better disturbance rejection ability to both matched and mismatched uncertainties, and chattering free dynamics compared to conventional sliding mode control.

INDEX TERMS Robust control, nonlinear disturbance observer, servo-hydraulic system, four-bar linkage mechanism, Lyapunov stability criteria.

I. INTRODUCTION

The need to design new control methods for modern technologies and complex systems capable of handling unknown dynamics, uncertainties, and external disturbances has recently led to the introduction of the synergetic control (SC) theory [1]. SC is a state-space method originating from modern mathematics and synergetics. This control method is similar to the sliding mode control (SMC) and shares the advantages of the SMC techniques, such as order reduction, robustness to external disturbances, and easy implementation. However, there are some vulnerabilities in SMC, and different methods have been proposed over the past years to overcome them. For instance, [2]–[7] proposed different sliding mode-based controllers with nonlinear surfaces to overcome the infinite settling time issue of the conventional SMC which are applied in various applications, such as robotic manipulator control [3], [5], active fault-tolerant control [2], [7], fusion reactor vacuum vessel assembly robot control [6], and in combination with intelligent control methods, such as neural networks [4]. These controllers improve the performance of the system but are not fully capable of addressing the chattering phenomenon of the sliding mode-based controllers. To retain the states of the system in the boundary layer region of the sliding surface, a discontinuous switching control law is adopted. However, owing to its high-frequency nature, the discontinuous part causes a phenomenon called chattering [8], imposing high stresses on the mechanical actuators and the system.

Having the advantages of the SMC while simultaneously addressing the main drawback precluding its practical implementation, namely, the chattering phenomenon, the SC theory has gained a considerable amount of interest over the last few years [9], [10]. A fractional nonlinear synergetic controller was proposed and implemented to a wind turbine system in [11]. A decentralised nonlinear SC with strong robustness to external disturbances and uncertainties was proposed in [12] for power system stabilisers. The applications of this theory ranges from permanent-magnet synchronous motors [13], discrete-time linear time varying systems [14], flexible joint mechanical systems [15] to vaccination in epidemic systems [16]. However, there are still some drawbacks. The infinite time convergence to zero of the states of the system and the suppression of external disturbances
and uncertainties are two of the main issues of the synergetic theory that need to be addressed. Therefore, to guarantee the finite time convergence of the states to zero, a finite time synergetic theory was proposed in [17]. This could also be achieved with the novel non-singular terminal SC that was introduced in [18], [19] with applications in robotic manipulators and nth-order systems, such as servo-hydraulic systems.

Another major issue in the control of nonlinear systems is the effective compensation of external disturbances. These latter can be classified as matched (enter the system through the same channels as the control input), or mismatched (enter the system from different channels than the control input). SMC is, to some extent, robust to matched disturbances but loses its nominal control performance in the presence of mismatched disturbances [20], [21]. Classical robust control design methods that consider an $H_2$ norm-bounded condition with approaches, such as the Lyapunov-based control [22], linear matrix inequality (LMI)-based approaches [23], intelligent control [24], [25], and adaptive methods [26] are quite effective in dealing with mismatched disturbances, however, they are restrictive to this type of disturbance. However, it is obvious that not all disturbances satisfy this condition. More recently, approaches that combine a control law with a mechanism to estimate the disturbances have gained more attention. By doing so, a wide range of disturbances and uncertainties can be handled. Studies, such as [20], [27]–[30], have attempted to propose methods to counteract the effects of the disturbances. For instance, [20] proposed a new sliding surface that can compensate for an estimation of the mismatched disturbances. Although the proposed method offers better performance than conventional and integral SMC, the chattering problem of the SMC still remain. Additionally, only mismatched disturbances were considered in that design. [28] proposed a novel sliding surface for fractional-order mismatched uncertain systems. Similarly, this approach was only limited to mismatched disturbances and did not fully alleviate the chattering problem. Recent studies [31], [32] proposed a novel disturbance compensator with finite-time convergence to effectively compensate for matched disturbances.

This paper introduces a novel robust proportional integral non-singular terminal synergetic control (PI-NTSC) for a class of affine nonlinear systems. This method has the advantages of the SMC, guarantees that the states of the system converge to the origin in finite time, and offers a control signal with no chattering. Additionally, owing to the incorporation of the estimated matched and mismatched disturbances in its manifold, it can also handle external disturbances and uncertainties better than conventional synergetic-based controllers.

The contributions of this paper are threefold:

- Implementation of a novel robust proportional integral non-singular terminal SC, which has features, such as a finite-time convergence, chattering-free responses, simple implementation, robustness against disturbances, and model and parameter uncertainties robustness.
- The proposed robust PI-NTSC is expanded for a general system of order n, subject to both matched and mismatched disturbances.
- Proposition of a hybrid nonlinear disturbance observer to estimate the matched and mismatched disturbances.
- Incorporation of the estimation of the matched and mismatched disturbances in the proposed PI-NTSC to counteract the effects of the disturbances.

The rest of the paper is organised as follows. The proposed PI-NTSC approach is detailed in Section II. The principles of the Sr-UKF and the PI-NTSC are discussed, and the development of the hybrid nonlinear disturbance observer is described in this section. Section III highlights the performance of the proposed PI-NTSC approach when implemented to a second-order system, i.e. a four-bar linkage mechanism and an illustrative third-order system. Section IV concludes the paper.

II. ROBUST CONTROLLER DESIGN

In this section, first, a nonlinear Kalman filter is implemented to estimate the full states from partial state. Then, the principles of the control law is proposed based on PI non-singular terminal synergetic control and disturbance compensation. Finally, the scheme of the proposed disturbance observer that can estimate both matched and mismatched disturbances is discussed.

A. SYSTEM STATE OBSERVER

In this section the principles of a nonlinear Kalman filter is given for a noised system states combined with a disturbance observer. Consider the following nonlinear system:

$$\dot{x}(t) = f(x(t), u(t)) + w(t) + d(x, t)$$
$$y(t) = g(x(t)) + v(t) \quad (1)$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^m$ and $u \in \mathbb{R}^l$ represent the state vector, the outputs and inputs, respectively. The nonlinear dynamics and measurements of the system are represented by the functions $f$ and $g$, $w \sim N(0, Q)$ and $v \sim N(0, R)$ are Gaussian white process and measurement noise distributions, respectively. $d(x, t)$ indicates the system mismatched and matched uncertainties and disturbances.

The principles of the proposed square-root unscented Kalman filter is given in the Appendix A.

**Remark 1 ([33]):** Disturbances in the system are decomposed into two parts. A Gaussian white noise $w(t)$ and an unknown time-varying disturbance $d(x, t)$. The proposed Kalman filter is responsible to eliminate the effects of noises and also to estimate systems’ full states with partial states information.

**Remark 2:** The implementation of the proposed Sr-UKF ensures the stability of state estimation errors [34].
B. PROPORTIONAL INTEGRAL NTSC

As mentioned before, one of the main drawbacks of the SMC(s) is the chattering phenomenon, and SC is a newly introduced alternative to solve this problem. In this section, a new synergetic-based control is introduced. It has the advantages of conventional SC and guarantees the finite-time convergence of the system states to zero. In what follows, we first detail the synergetic control design for second order systems, then, generalize it to nth-order systems.

1) PI-NTSC FOR SECOND-ORDER SYSTEMS

Let us consider the nonlinear system given in Eq. (1) in the state-space representation as follows:

\[
\begin{align*}
\dot{x}_1 &= \dot{x}_2 + d_1(x, t) \\
\dot{x}_2 &= f(\hat{x}) + b(\hat{x})u + d_2(x, t) \\
y &= \hat{x}_1,
\end{align*}
\]

in which the estimated \( \hat{x} = [\hat{x}_1, \hat{x}_2]^T \in \mathbb{R}^2 \), the nonlinear functions are defined as \( f(\hat{x}) \) and \( b(\hat{x}) \neq 0 \), and \( y \) is the controlled output. The mismatched and matched uncertainties and disturbances are defined as \( d_1(x, t) \) and \( d_2(x, t) \), respectively.

Assumption 1: The disturbances and their derivatives are bounded and the derivative of the mismatched disturbance \( d_1(x, t) \) vanishes as \( t \to \infty \).

The objective is to design the control signal \( u \) in such a way that the output of the system can track the desired trajectory in the presence of both a mismatched uncertainty \( d_1(x, t) \) and an external matched disturbance \( d_2(x, t) \).

Define the trajectory tracking error and its derivative as \( e = \hat{x}_1 - \hat{x}_{1d} \) and \( \dot{e} = \dot{\hat{x}}_1 - \dot{\hat{x}}_{1d} = \dot{\hat{x}}_2 + \dot{\hat{d}}_1 - \dot{\hat{x}}_{1d} \), respectively. Using the PI surface defined as

\[
s_1 = K_p e + K_I \int_0^t e,
\]

where \( K_p \) and \( K_I \) are positive integers that should be tuned properly. The second non-singular terminal surface for a second-order system is defined as

\[
s_{2,SO} = s_1 + \frac{1}{\beta} (\hat{s}_1)^\alpha + \hat{d}_1
\]

Then, based on the tracking errors defined previously, and considering the fact that \( \dot{\hat{e}} = \dot{\hat{x}}_1 - \dot{\hat{x}}_{1d} \), which is equivalent to \( \dot{\hat{e}} = f(\hat{x}) + b(\hat{x})u + \dot{\hat{d}}_2 + \dot{\hat{d}}_1 - \dot{\hat{x}}_{1d} \), it can be concluded that

\[
\mu \left( K_p \dot{\hat{e}} + K_I e + \dot{\hat{d}}_1 + \frac{1}{\beta} (K_p \dot{\hat{e}} + K_I e + \dot{\hat{d}}_1)^{-\alpha} \right) ^{p/q} + \frac{\alpha}{\beta} (K_p \dot{\hat{e}} + K_I e + \dot{\hat{d}}_1) + \frac{1}{\beta} (K_p \dot{\hat{e}} + K_I e)^{\alpha} = 0
\]

Now, with basic mathematical calculations the control law can be derived as:

\[
u = -b^{-1}(\hat{x}) \left[ f(\hat{x}) - \hat{x}_{1d} + \dot{\hat{d}}_1 + \dot{\hat{d}}_2 \\
+ \frac{1}{K_p \alpha} (K_p \dot{\hat{e}} + K_I e)^{1-\alpha} \left( K_p \dot{\hat{e}} + K_I e + \dot{\hat{d}}_1 \right) \\
- \left( \frac{1}{\mu} \left[ (K_p e + K_I \int_0^t e + \dot{\hat{d}}_1) + \frac{1}{\beta} (K_p \dot{\hat{e}} + K_I e)^\alpha \right] \right)^{q/p} \\
+ \frac{\alpha}{\beta} (K_p e + K_I e)^{\alpha-1} \right) \right]
\]

Theorem 1: The application of the proposed PI-NTSC law in Eq.(9) causes the errors of the system in Eq.(2) to converge to zero in finite time.

Proof: The stability of the proposed controller is proven by considering the Lyapunov candidate function \( V_1 = 0.5 \psi^2(e) \)). Its derivative is \( \dot{V}_1 = \psi(e) \psi'(e) \). Therefore,

\[
\dot{V}_1 = \psi(e) \left[ \frac{\alpha}{\beta} (K_p \dot{\hat{e}} + K_I e)^{\alpha-1} \left( K_I e \right) \\
+ \frac{\alpha}{\beta} (K_p \dot{\hat{e}} + K_I e)^{\alpha-1} \left( f(\hat{x}) + b(\hat{x})u + \dot{\hat{d}}_2 \\
+ \dot{\hat{d}}_1 - \hat{x}_{1d} + K_p \dot{\hat{e}} + K_I e + \dot{\hat{d}}_1 \right) \right]
\]
substituting the designed control law from Eq. (9) to Eq. (10), it can be concluded that
\[ \dot{V}_1 = \psi(e) \left(-\mu^{-1} \psi(e)\right)^{q/p} \]
\[ = \left(-\mu^{-1}\right)^{q/p} \psi(e) \psi^{q/p}(e) \]
\[ = -\left(-\mu^{-1}\right)^{q/p} \psi(e(q+p)/p) \]
\[ \frac{1 < p/q < 2}{\mu > 0} \dot{V}_1 \leq 0 \] (11)

furthermore,
\[ \dot{V}_1 = \left(-\mu^{-1}\right)^{q/p} (\psi(e))^{(q+p)/p} \]
\[ \leq -\mu^{-q/p} \psi(e)^{2(q+p)/2p} \]
\[ \leq -\Pi V_1^{(q+p)/2p} \] (12)
in which \( \Pi = \mu^{-q/p}(q+p)/2p \).

**Lemma 1** ([35], [36]): If a continuous, positive-definite function satisfy:
\[ \dot{V}(t) \leq -\xi V^{n}(t), \quad \forall t \geq t_0, \quad V(t_0) \geq 0 \] (13)
in which \( \xi > 0 \) and \( 0 < \eta < 1 \). Then, for any given \( t_0 \), \( V(t) \) satisfies the following inequality:
\[ V^{1-\eta}(t) \leq V^{1-\eta}(t_0) - \xi(1-\eta)(t-t_0), \quad t_0 < t < t_1; \]
\[ V(t) \equiv 0, \quad \forall t \geq t_1 \] (14)
where \( t_1 = t_0 + V^{1-\eta}(t_0)/\xi(1-\eta) \).

From **Lemma 1**, it can be concluded that having \( \xi = \Pi \) and \( \eta = (q+p)/2p \), the synergetic manifold can converge to zero at the finite time:
\[ t_1 = \frac{V_1^{1-(q+p)/2p}(0)}{\Pi(1-(q+p)/2p)} \] (15)

Based on **Lemma 1**, the proposed macro-variable \( \psi(e) \) will converge to zero in finite-time. Thus, it can be concluded that \( \psi(e) = 0 \) for \( t > t_1 \) and the following equation can be achieved from Eq. (4):
\[ 0 = s_1 + \frac{1}{\beta} (\hat{s}_1)^{\alpha} + \hat{d}_1 \]
\[ \Rightarrow \hat{s}_1 = (1/\alpha) \left( s_1 + \hat{d}_1 \right)^{1/\alpha} \] (16)

Now, considering \( V_2 = 0.5(s_1 + \hat{d}_1)^2 \), it can be shown that \( s_1 = 0 \) is attractor as follows:
\[ \dot{V}_2 = (s_1 + \hat{d}_1)(\hat{s}_1 + \hat{d}_1) \]
\[ = (s_1 + \hat{d}_1)(-(\beta/\alpha)^{1/\alpha}(s_1 + \hat{d}_1)^{1/\alpha} + \hat{d}_1) \]
\[ = -(\beta/\alpha)^{1/\alpha}(s_1 + \hat{d}_1)^{1+1/\alpha} + (s_1 + \hat{d}_1) \hat{d}_1 \]
\[ \overset{\text{Assumption 1}}{\Rightarrow} \dot{V}_2 \leq -\left(\beta/\alpha\right)^{1/\alpha} 2 \frac{a+1}{2a} (V_2)^{a+1/2a} \]
\[ \overset{1 < a < 2}{\Rightarrow} \dot{V}_2 \leq 0 \] (17)

Having \( s_1 = 0 \) as an attractor, it can be concluded from Eq. (3) that \( K_p e + K_t \int_0^t e = 0 \). By taking the derivative of this equation and defining \( V_3 = 0.5e^2 \), it can be shown that \( e = 0 \) is an attractor.

Hence, the system states will converge to zero in finite time with the rate of convergence depending on the tuning parameters of \( \mu, \beta, \alpha, p \) and \( q \). This completes the proof. ■

2) PI-NTSC for Nth-order Systems

Let us consider an nth-order state space representation of the nonlinear system given in Eq. (2) as
\[ \ddot{x}_1 = \dot{x}_2 + d_1(x, t) \]
\[ \ddot{x}_2 = \dot{x}_3 + d_2(x, t) \]
\[ \vdots \]
\[ \ddot{x}_n = f(\hat{x}) + b(\hat{x})u + d_0(x, t) \]
\[ y = \hat{x}_1 \] (18)

where the state vector is defined as \( \hat{x} = [\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n]^T \in \mathbb{R}^n \), and the nonlinear functions \( f(\hat{x}) \) and \( b(\hat{x}) \neq 0 \) are the same as in Eq. (2). The mismatched disturbances are defined as \( d_i(x, t), i = 1, 2, \ldots, n-1 \), whilst \( d_0(x, t) \) represent the matched disturbance.

It is worth noting that a wide range of systems ([8], [37], [38]) can be modeled using equation (18).

**Assumption 2:** The disturbances and their derivatives are bounded and the derivatives of the mismatched disturbances \( d_i(x, t) \) (for \( i = 1, 2, \ldots, n-1 \)) vanish as \( t \to \infty \).

The tracking errors and their derivatives are as follows:
\[ e = \hat{x}_1 - x_{1d} \]
\[ \dot{e} = \dot{x}_1 - \dot{x}_{1d} \]
\[ \vdots \]
\[ e^{[n]} = f(\hat{x}) + b(\hat{x})u + \hat{d}_0(x, t) + \sum_{i=1}^{n-1} d_i^{[n-i]}(x, t) - x^{[n]}_{1d} \] (19)

The PI surface is considered as in Eq. (3), with the tuning parameters of \( K_p > 0 \) and \( K_t > 0 \). The novel non-singular terminal surface for the nth-order system is proposed as
\[ s_{2,NO} = \sum_{i=1}^{n} \frac{1}{\beta_i} \left(s_1^{[i-1]}\right)^{\alpha_i} + \sum_{i=2}^{n-1} \hat{d}_i^{[n-i-1]} \] (20)

Considering the synergetic macro variable as \( \psi = s_{2,NO} \) and substituting Eq. (20) in the synergetic manifold of Eq. (5), the control law can be concluded as
\[ \mu s_{2,NO}^{p/q} + s_{2,NO} = 0 \]
\[ \mu \left[ \sum_{i=1}^{n} \frac{\alpha_i}{\beta_i} \left(s_1^{[i-1]}\right)^{\alpha_i} + \sum_{i=1}^{n-1} \hat{d}_i^{[n-i]} \right]^{p/q} \]
\[ + \sum_{i=1}^{n} \frac{1}{\beta_i} \left(s_1^{[i-1]}\right)^{\alpha_i} + \sum_{i=1}^{n-1} \hat{d}_i^{[n-i-1]} = 0 \] (21)
To extract the $e^{[n]}$ term from Eq.(21), this equation can be rewritten as

$$
\mu \left[ \frac{\alpha_n}{\beta_n} \left( s_1^{[n]}(\alpha_{n-1}) s_1 \right) + \sum_{i=1}^{n-1} \frac{\alpha_i}{\beta_i} \left( s_1^{[i]}(\alpha_{i-1}) s_1 \right) + \sum_{i=1}^{n-1} \hat{d}_i^{[n-i]} \right]^{1/q} + \sum_{i=1}^{n} \frac{1}{\beta_i} \left( s_1^{[i-1]} \right) a_i + \sum_{i=1}^{n-1} \hat{d}_i^{[n-i-1]} = 0 \tag{22}
$$

Substituting $s_1^{[n]} = K_P e^{[n]} + K_I e^{[n-1]}$ and $e^{[n]}$ from Eq. (19) in Eq.(22), the control law can be calculated as

$$
u = -b^{-1}(\dot{x}) \left( f(x) + \dot{\hat{d}}_n(x, t) + \sum_{i=1}^{n-1} \hat{d}_i^{[n-i]}(x, t) - x_d^{[n]} \right) + \frac{K_I}{K_P} e^{[n-1]} + \frac{\beta_n}{K_P e^{[n]}} \left( s_1^{[n-1]} \right)^{1-\alpha_n} \left\{ \sum_{i=1}^{n-1} \hat{d}_i^{[n-i]} \right\}
$$

in which $\beta_i > 0$ so that $\pi(r) = r^n + (1/\beta_n)r^{n-1} + \ldots + (1/\beta_2)r + (1/\beta_1), r \in \mathbb{R}$ becomes a Hurwitz polynomial. The tuning parameters of $\alpha_i$ for nth-order systems can be selected as

$$
\begin{align*}
\alpha_{n+1} &= 1 \\
\alpha_n &= \alpha_0 \in (1 - \epsilon, 1), \quad \epsilon \in (0, 0.3) \\
\alpha_{i-1} &= \frac{\alpha_i \alpha_{i+1}}{2\alpha_{i+1} - \alpha_i}, \quad i = n, n-1, \ldots, 2 
\end{align*}
$$

Theorem 2: The control law represented by Eq.(23) causes the errors of the nonlinear system (18), which is subjected to matched and mismatched disturbances, to converge to zero in finite time.

Proof: The proof of the theorem is straightforward and similar to Theorem 1 and is thus omitted in this paper.

1) MISMATCHED DISTURBANCE OBSERVER

A mismatched disturbance observer [30] is considered to estimate the mismatched disturbances of the nth order system of Eq.(18). The proposed observer is illustrated using Algorithm 1.

Algorithm 1 Mismatched Disturbance Observer

1: for $i=1, 2, \ldots, (n-1)$ do
2: for $j=1, 2, \ldots, n$ do
3: $\hat{d}_i^{[j]} = P_{ij} + L_{ij} \hat{x}_i$
4: $\hat{P}_{ij} = -L_{ij} (\hat{x}_{i+1} + \hat{d}_i) + \hat{d}_i^{[j]}$
5: $\hat{P}_{in} = -L_{in} (\hat{x}_{i+1} + \hat{d}_i)$
6: end for
7: end for

Lemma 2 ([30]): Suppose that Assumption 2 is satisfied for the nth-order nonlinear system given in Eq. (18) and $e_d = d_i - \hat{d}_i (i = 1, \ldots, n - 1)$ is the disturbance estimation error. The estimation of disturbances using Algorithm 1 can track the mismatched disturbances $d_i(x, t)$ (lim$_{t \rightarrow \infty} e = 0$) if the observer gains are selected properly such that $L_{ij} > 0$ and $L_{ir} > 0$ holds.

2) MATCHED DISTURBANCE OBSERVER

Let us consider the nonlinear system subjected to the matched disturbance $d_i(x, t)$ in Eq. (18). The matched disturbance in this equation can be estimated using a nonlinear disturbance observer ([20]) as follows:

$$
\begin{align*}
\dot{\hat{d}}_n &= z + p(\hat{x}, \hat{\dot{x}}) \\
\dot{z} &= -L_d(\hat{x}, \hat{\dot{x}})z - L_d(\hat{x}, \hat{\dot{x}}) \left( f(\hat{x}) + b(\hat{x})u + p(\hat{x}, \hat{\dot{x}}) \right),
\end{align*}
$$

(25)

where $z \in \mathbb{R}^l, \hat{d}_n \in \mathbb{R}^l$, and $p(\hat{x}, \hat{\dot{x}})$ represent the internal state vector, estimated disturbance, and auxiliary vector, respectively. In this equation, $L_d(\hat{x}, \hat{\dot{x}})$ is the gain matrix of the nonlinear disturbance observer, which should be designed properly. To do so, the tracking error of the disturbance is expressed as $e_d = \hat{d}_n - d_n$. The estimation of $e_d$ can be calculated from Eq.(25). The derivative of the tracking error is

$$
\begin{align*}
\dot{e}_d &= \dot{\hat{d}}_n - \dot{d}_n = \dot{z} + p(\hat{x}, \hat{\dot{x}}) - \hat{d}_n \\
&= -L_d(\hat{x}, \hat{\dot{x}})z + p(\hat{x}, \hat{\dot{x}}) - \hat{d}_n \\
&= -L_d(\hat{x}, \hat{\dot{x}}) \left( f(\hat{x}) + b(\hat{x})u + p(\hat{x}, \hat{\dot{x}}) \right) - L_d(\hat{x}, \hat{\dot{x}}) \left( \hat{x}_n - d_n + p(\hat{x}, \hat{\dot{x}}) \right) \\
&= -L_d(\hat{x}, \hat{\dot{x}}) \left( \hat{x}_n - d_n + p(\hat{x}, \hat{\dot{x}}) \right)
\end{align*}
$$

(26)

Now, considering that $p(\hat{x}, \hat{\dot{x}}) = L_d(\hat{x}, \hat{\dot{x}})\hat{x}_n$ eliminates the dependence of the disturbance tracking error on the $\hat{x}$. On the
other hand, it is obvious that
\[
\dot{\hat{x}} = \frac{\partial p}{\partial \hat{x}} + \frac{\partial p}{\partial \hat{x}} = L_d(\hat{x})\hat{x}_n
\]
\[
\Rightarrow \frac{d}{dt} = \frac{\partial p}{\partial \hat{x}} = L_d(\hat{x}).
\]

Lemma 3 ([29], [39]): Consider the nonlinear system represented in Eq. (18) subjected to the matched disturbance of \(d_n(x, t)\). Defining the disturbance estimation error as \(e_d = d_n - d_n\), the disturbance estimation proposed in Eq. (25) can track the disturbance \(d_n(x, t)\) asymptotically \((\lim_{t \to \infty} e_d(t) = 0)\) if the observer gain is selected such that \(L_d(\hat{x}) > 0\) holds.

Remark 3: A general systematic approach to design the auxiliary vector \(p(\hat{x})\) and the NDOB gain matrix was introduced in [29]. In this paper, the LMI formulation is introduced to design the auxiliary vector and the NDOB gain matrix. The same method is used in this study to design the \(p(\hat{x})\) vector and the \(L_d(\hat{x})\) gain matrix.

D. OVERALL SCHEME OF THE PROPOSED HYBRID NDOB PI-NTSC

The overall scheme of the proposed robust PI-NTSC is illustrated in Fig.1.

The proposed approach can be described as follows:
1) Definition of the dynamic model of the system using Eq.(2) or Eq.(18).
2) Implementation of a nonlinear state observer based on the proposed square-root unscented Kalman filter given in Algorithm 2.
3) Implementation of the hybrid nonlinear disturbance observer for mismatched and matched disturbances based on Algorithm 1 and Eq.(25), respectively.
4) Define the PI surface, non-singular terminal surface, and synergetic manifold based on Eqs.(3),(4), and (5) for second-order systems, and Eqs.(3), (20), and (5) for nth-order systems.
5) Acquisition of the control signals based on Eq.(9) and Eq.(23) for the second-order and nth-order systems, respectively.

III. SIMULATION RESULTS

This section presents the performances of the proposed PI-NTSC and the HNDOB for a four-bar linkage mechanism and a servo-hydraulic system.

A. EXAMPLE 1: FOUR-BAR LINKAGE MECHANISM

For the second-order system, a four-bar linkage mechanism is considered. The schematic of the planar four-bar linkage mechanism is illustrated in Fig.2.

Four-bar linkage mechanism is a well-known system with one degree of freedom; however, expressing the dynamics of the closed system using a single equation is challenging. [41] proposed a procedure to reduce the number of coordinates to only one coordinate and expressed the dynamics of the system as
\[
M(\theta)\ddot{\theta} + V(\theta, \dot{\theta}) = \tau
\]

The detailed information regarding the values of the matrices \(M(\theta)\) and \(V(\theta, \dot{\theta})\) can be found in [41]. The dynamic equation of motion in Eq.(28) can be represented as a second-order nonlinear system formulated in Eq.(2), where \(x = [x_1, x_2]^T = [\theta, \dot{\theta}]^T\), (\(f(x) = -M^{-1}(\theta)V(\theta, \dot{\theta})\) and \(b(x) = M^{-1}(\theta)\). The system output is \(\theta\), hence the proposed Sr-UKF provides an estimation of \(\theta\) and \(\dot{\theta}\) to be used in the controller and the hybrid disturbance observer. For this purpose, it is supposed that the process and measurement noises are zero mean white noise with the covariance of \(Q = \text{diag}(10^{-7}, 10^{-7})\) and \(R = 10^{-6}\), respectively. It should be noted that in order to study the performance of the proposed HNDOB and PI-NTSC, a very small process and measurement noises are considered in this example. Therefore, it is possible to study the effect of matched and mismatched disturbances much better.

The parameter values of the links are presented in Appendix B. The mismatched disturbance is assumed to be \(d(t) = 10\) for \(t < 10\)sec. The matched disturbance, modelled

| TABLE 1. Parameter values for the simulation of the four-bar linkage. |
|---------------------------------------------------------------|
| PI-NTSC | \(\beta = 0.04\), \(\alpha = p/q = 1.02\), \(\mu = 0.10\), \(K_p = 1\), \(K_i = 5\) |
| HNDOB   | \(L_{11} = 10\), \(L_{12} = 20\), \(p(\hat{x}) = 121.13\hat{x}_2\) |
as the friction torques acting on the joints, is based on the model proposed in [42] as
\[
d_2(x, t) = F_c \text{sgn}(x_2) \left[ 1 - \exp \left( \frac{-x_2^2}{v_s^2} \right) \right] + F_s \text{sgn}(x_2) \exp \left( \frac{-x_2^2}{v_s^2} \right) + F_v x_2, \tag{29}
\]
where \(F_c = 0.49(N \cdot m)\), \(F_s = 3.5(N \cdot m)\), \(F_v = 0.15(kg \cdot m/s)\) are the Coulomb friction, static friction, and viscous friction coefficients. The Stribeck parameter is defined as \(v_s = 0.19(rad/s)\). It is assumed that the matched disturbance occurs for \(t \leq 10\) sec. The parameters of the proposed PI-NTSC and HNDOB are presented in Tab.1.

![Figure 3](image1.png)

**FIGURE 3.** Performance comparison of robust PI-NTSC and PI-NTSC (Desired trajectory: ---, PI-NTSC: --, Robust PI-NTSC: ---).

The performance of the robust PI-NTSC is depicted in Fig.3. In this figure, a robust PI-NTSC, which is a combination of a PI-NTSC and a HNDOB, is compared with a PI-NTSC with no disturbance observer. As can be seen, the PI-NTSC is not capable of converging to the trajectory in the first 10 seconds of the simulation due to the impact of the disturbances on the four-bar linkage mechanism. However, the performance of the proposed robust PI-NTSC is considerably better than that of the normal PI-NTSC right from the beginning of the simulation. The percent normalised mean square error (PNMSE) of the desired trajectory tracking is calculated, to attain a better understanding of these two methods. For the robust PI-NTSC, the value of PNMSE is 9.71%, whereas it is 28.02% for the PI-NTSC, which indicates the significance of using a disturbance observer in the PI-NTSC.

![Figure 4](image2.png)

**FIGURE 4.** Estimated disturbances of the four-bar mechanism based on the proposed HNDOB (Real disturbances: ---, Estimated disturbances using the HNDOB: ---).

To analyse the performance of the HNDOB, the estimated disturbances are depicted in Fig.4. It is obvious that the proposed method can estimate the mismatched and matched disturbances. However, the preference of using a separate observer to estimate matched disturbances can still be a question. The method introduced for the mismatched disturbance observer in [30] is also used to estimate the matched disturbances. In this section, the matched disturbance is assumed as Eq.(29) from the beginning of the simulation and no mismatched disturbance is considered. Fig.5 depicts the performance of the robust PI-NTSC based on the disturbances observer in [30] and the proposed HNDOB. From Fig.5, it can be concluded that the proposed HNDOB is better at estimating the matched disturbance than the method proposed in [30]. Therefore, the performance of the HNDOB-based PI-NTSC is better than that of the [30]-based PI-NTSC.
B. EXAMPLE 2: SERVO-HYDRAULIC SYSTEM

In this subsection, the proposed robust PI-NTSC is applied to a servo-hydraulic actuator. Its schematic is illustrated in Fig. 6. Considering the defined parameters in Appendix C, the equation of motion of the actuator can be written as

$$M \ddot{x}_p = -b \dot{x}_p + A_1 p_1 - A_2 p_2 - F_e,$$  \hspace{1cm} (30)

For a state space representation of the system, the derivative of Eq. (30) is calculated as follows:

$$M \dddot{x}_p = -b \ddot{x}_p + A_1 \dot{p}_L - \dot{F}_e,$$  \hspace{1cm} (31)

where $P_L = p_1 - \frac{A_1}{A_2} p_2$ is the load pressure. Hence, defining the state vector as $X = [x_1, x_2, x_3]^T = [x_p, \dot{x}_p, \ddot{x}_p]^T$, the state space representation of the system can be expressed as

$$\begin{align*}
\dot{x}_1 &= x_2 + d_1(x, t) \\
\dot{x}_2 &= x_3 + d_2(x, t) \\
\dot{x}_3 &= \frac{1}{M} (-b x_2 + A_1 \dot{p}_L - \dot{F}_e) + d_3(x, t),
\end{align*}$$

(32)

where the pressures can be calculated as [34]

$$\dot{p}_1 = \frac{\beta_e}{V_1} (Q_1 - A_1 \dot{x}_p + Q_1 - Q_{E1}),$$

$$\dot{p}_2 = \frac{\beta_e}{V_2} (Q_2 - A_2 \dot{x}_p - Q_2 - Q_{E2}),$$

(33)

where the volumes on each side of the cylinder are $V_1 = A_1 x_p + v_{01}$ and $V_2 = A_2 (L - x_p) + v_{02}$. The flows in each of the cylinders can be calculated as

$$Q_1 = \begin{cases} C_i u \sqrt{p_1 - p_0} & u \geq 0 \\ C_i u \sqrt{p_1 - p_0} & u < 0 \end{cases},$$

$$Q_2 = \begin{cases} C_i u \sqrt{p_2 - p_0} & u \geq 0 \\ C_i u \sqrt{p_2 - p_0} & u < 0 \end{cases},$$

$$Q_l = K_i (p_2 - p_1),$$

$$Q_{E1} = K_{E1} (p_1 - p_0),$$

$$Q_{E2} = K_{E2} (p_2 - p_0).$$

(34)  

(35)  

(36)

The actuator is affected by mismatched disturbances $d_1(t) = 0.5$ and $d_2(t) = 2$, and matched disturbance $d_3(x, t) = 10^6 (-3.85g(x_p) \dot{x}_p^3 + 2.85g(x_p) \ddot{x}_p^2)$ starting at $t = 10$ sec.
For practical reasons, it is assumed that there is only one sensor in $x_1$ channel which is the piston position. Hence, the proposed Sr-UKF provides state estimation of the other states based on the values of the piston position. Therefore, an additive Gaussian process and measurement noises with zero mean and a covariance of $Q = \text{diag}(10^{-5}, 10^{-5}, 10^{-5})$ and $R = 10^{-3}$ are assumed to affect the system during the entire simulation. The parameters of the control law are set as $K_I = 0.001$, $K_p = 1$, $\alpha_3 = 0.9$, $\mu = 0.001$, $\beta_1 = 0.0013$, $\beta_2 = 0.0025$, and $\beta_3 = 0.4545$. The tuning parameters of the HNDOB are $L_{11} = L_{21} = 10$, $L_{12} = L_{22} = 100$, $L_{13} = L_{23} = 1$, and $p(\hat{x}) = 231.33\hat{x}_3$. Fig.7 depicts the performance of the robust PI-NTSC with HNDOB in comparison with that of the PI-NTSC without HNDOB. As can be seen, the proposed robust PI-NTSC is capable of maintaining the stable behaviour of the system in
FIGURE 9. Estimated disturbances of the servo-hydraulic actuator based on the proposed HNDOB (Real disturbance: \(\cdot\), Estimated disturbances based on HNDOB: \(\circ\)).

The presence of the matched and mismatched disturbances, whereas the conventional control method diverges from the desired trajectory shortly after the occurrence of the disturbances. Due to the process and measurement noises in the system, the changes in the control signal to counteract the imposed disturbances are not distinguishable in Fig. 7c. That is why, a simulation with low noise covariances are also provided in this figure to indicate the changes in the control signal at \(t = 10\text{sec}\).

As indicated earlier, there is only one sensor which measures the position of the piston. The application of the proposed state observer makes it possible to estimate the other states \((\dot{x}_p \text{ and } \ddot{x}_p)\) from the values of this signal.

The performance of the Sr-UKF state observer is depicted in Fig. 8. It should be noted that the performance of the proposed robust PI-NTSC is related to the accuracy of the estimated states. As it can be seen from Fig. 8, the proposed Kalman filtering method can estimate the system states with high accuracy.

The estimated disturbances using the proposed HNDOB are illustrated in Fig. 9. The proposed method can estimate the matched and mismatched disturbances under the noisy conditions of the measurements.

To further study the performance of the proposed controller, the PI-NTSC is compared with the non-singular terminal sliding mode control (NTSMC) proposed in [44]. The main purpose is to compare their ability to deal with matched and mismatched uncertainties. To this end, it is assumed that

\[ 
\begin{align*}
\dot{d}_1(t) &= 0.7, \\
\dot{d}_2(t) &= 0.8, \\
d_3(t) &= 10^5(20\sin(12.83\dot{x}_p) - 2\text{sgn}(x_p)).
\end{align*}
\]

All the disturbances occur from \(t = 10\text{sec}\). The designing parameters of the PI-NTSC are set to be \(\beta_1 = 0.1, \beta_2 = 0.001, \beta_3 = 8.4, \alpha_3 = 0.9, K_p = 5, K_I = 10, \mu = 0.1\). Additive process and measurement noises with zero means and a covariance of \(Q = (10^{-5}I_{3x3})\) and
$R = 10^{-4}$ are also considered. All the mutual tuning parameters between PI-NTSC and NTSMC are set to be the same.

Fig.10 depicts the performance comparison of the proposed robust PI-NTSC versus the NTSMC proposed in [44]. As can be seen from Fig.10e, the proposed robust PI-NTSC converges to the desired trajectory faster than the NTSMC, due to the integral term used in the PI section. Additionally, the superiority of this method is obvious after $t = 10$ sec when the disturbances occur. The proposed PI-NTSC deals with the disturbances considerably better, and after a small deviation from the trajectory, it converges back to the trajectory faster than NTSMC. Furthermore, it can be seen from Fig.10d that this result is achieved with significantly small values of the control signal. The control signal of the PI-NTSC has higher values at the initial phase of the simulation; however, the chattering phenomenon of the NTSMC results in a higher control signal than that of the PI-NTSC in the rest of the simulation.

**IV. CONCLUSION**

This paper addresses the issue of control design for nonlinear systems subject to both matched and mismatched disturbances.
Algorithm 2 Square-Root Unscented Kalman Filter (Sr-UKF) [34] With Disturbance Observer Compensation

1: Initialization:
\[ \hat{x}_0 = E[x_0], \]
\[ S_0 = CH \left( E \left[ (x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T \right] \right) \]

2: for all samples do
3: Time Updating:
\[ x_k^{(i)}_{k-1} = \begin{bmatrix} \hat{x}_{k-1|k-1}, \\ \hat{x}_{k-1|k-1} + L + \lambda S_{k-1|k-1}, \\ \hat{x}_{k-1|k-1} - L + \lambda S_{k-1|k-1} \end{bmatrix} \]
\[ y_k^{(i)}_{k|k-1} = f \left( x_k^{(i)}_{k-1|k-1}, u_{k-1} \right) + \hat{d}_{k-1} \]
\[ \hat{x}_{k-1|k-1} = \sum_{i=0}^{2n} w_i^{(m)} y_k^{(i)}_{k-1|k-1} \]
\[ S_k^{(1)}_{k-1|k-1} = QR \left\{ \left[ \sqrt{w_i^{(c)}} \left( y_k^{(i)}_{k|k-1} - \hat{x}_{k|k-1} \right) \right] \right\} \]
\[ S_k^{(2)}_{k-1|k-1} = CU \left( S_k^{(1)}_{k-1|k-1}, y_k^{(i)}_{k|k-1} - \hat{x}_{k|k-1}, w_0^{(c)} \right) \]
\[ x_k^{(i)}_{k|k-1} = \begin{bmatrix} \hat{x}_{k|k-1}, \\ \hat{x}_{k|k-1} + L + \lambda S_{k|k-1}, \\ \hat{x}_{k|k-1} - L + \lambda S_{k|k-1} \end{bmatrix} \]
\[ y_k^{(i)}_{k|k-1} = g \left( x_k^{(i)}_{k|k-1} \right) \]
\[ \hat{y}_{k|k-1} = \sum_{i=0}^{2n} w_i^{(m)} y_k^{(i)}_{k|k-1} \]

4: Measurement Update:
\[ S_k^{yy} = QR \left\{ \left[ \sqrt{w_i^{(c)}} \left( y_k^{(i)}_{k|k-1} - \hat{y}_{k|k-1} \right) \right] \right\} \]
\[ S_k^{by} = CU \left( S_k^{yy}, y_k^{(i)}_{k|k-1} - \hat{y}_{k|k-1}, w_0^{(c)} \right) \]
\[ P_k^{yy} = \sum_{i=0}^{2n} \left[ w_i^{(c)} \left( y_k^{(i)}_{k|k-1} - \hat{y}_{k|k-1} \right) \times \left( y_k^{(i)}_{k|k-1} - \hat{y}_{k|k-1} \right)^T \right] \]

5: States Enhancement:
\[ K = P_k^{by} / S_k^{yy}, \quad K \]
\[ \hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (y_k - \hat{y}_{k|k-1}) \]
\[ U = K_k S_k^{yy}, \quad S_k^{yy} = \text{cholupdate} \left( S_k^{yy}, U, -1 \right) \]

6: end for

disturbances. To this end, it proposes a hybrid nonlinear disturbance observer-based robust PI-NTSC approach with Sr-UKF state observer. The proposed PI-NTSC has the advantages of the conventional SMC but evades the chattering phenomenon of SMC-based controllers. Its principles are proposed for second-order systems and further generalized to the nth-order nonlinear systems. Additionally, its stability is proven through the Lyapunov stability criteria.

To counteract the effects of the disturbances, an estimation of the matched and mismatched disturbances is incorporated in the proposed controller with the use of a proposed HNDOB. Results stemming from the implementation of the proposed approach to two nonlinear systems confirmed its efficiency in handling both matched and mismatched disturbances. Our future work will focus on further implementing the proposed design to other dynamical systems and carrying out additional performance analysis.

APPENDIX A
Algorithm 2 presents the implementation of the proposed square-root unscented Kalman filter.

Remark 4: It should be noted that considering $T_s$ as the sampling period, the variable values can be considered at each sampling time $kT_s$; however, for simplification reasons, $k$ notation will be used instead of $kT_s$ to describe the Square-root Unscented Kalman Filter.

Remark 5: In Algorithm 2, $CH$ represents the Cholesky factor $QR(A)$ returns the matrix $S$ that satisfies the equation $A = \begin{bmatrix} I & S \end{bmatrix}$ in which $S$ is an upper triangular matrix and $I$ is a unitary matrix and $CU$ represents the rank-one update of the Cholesky factorization. The number of states is indicated by $L$ and $\lambda$ is defined as $\lambda = L (\alpha^2 - 1)$ in which, $10^{-4} < \alpha < 1$ is a weighting factor. $w_0^{(m)}$ and $w_0^{(c)}$ used in Algorithm 2 are the predefined weights which can be calculated as:
\[ w_0^{(m)} = \frac{\lambda}{L + \lambda}, \quad w_0^{(c)} = \frac{\lambda}{L + \lambda} + \left( 1 - \alpha^2 + \beta \right), \quad w_i^{(m)} = w_i^{(c)} = \frac{1}{2(L + \lambda)}; \quad i = 1, \ldots, 2L \] (37)
in which $\beta$ is equal to 2 for a Gaussian distributions.

Remark 6: It should be noted that $d(x, t)$ is estimated online through the proposed hybrid nonlinear disturbance observer in Section II-C.

APPENDIX B
See Table 2.

APPENDIX C
See Table 3.

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