Strong First Order Phase Transition and $B$ Violation in the Compact 341 Model

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Baryogenesis in the context of the compact $SU(3)_C \otimes SU(4)_L \otimes U(1)_X$ model is investigated. Using the finite temperature effective potential approach together with unitarity, stability and no ghost masses constraints, the existence of a strong first order electroweak phase transition (EWPT) was shown and checked numerically during all steps of the spontaneous breakdown of the gauge symmetry of the model. Higgs masses regions fulfilling the EWPT criteria are also discussed. Moreover, and as a byproduct of our study, the $B$-violation via sphaleron was also emphasized.

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I. INTRODUCTION

Electroweak baryogenesis (EWBG) remains a theoretically attractive and experimentally testable scenario for explaining the cosmic baryon asymmetry. Particular attention is paid to Standard Model extensions [1], [2], [3], [4] that may provide the necessary ingredients for EWBG, and searches for the corresponding signatures at high energy limits. Within the Standard Model (SM) [5], [6], [7], [8], [9], the EWBG cannot explain the observed baryonic asymmetry of the universe. Indeed, the SM electroweak phase transition is of first order (in order to have large deviations from thermal equilibrium) only if the mass of the Higgs boson is less than 70 GeV [10], [11]. This is in contradiction with the current experimental value which is around 125 GeV [12]. Moreover, the CP violation induced by the CKM phase does not appear to be sufficient to generate the observed baryonic asymmetry [13], [14], [15]. Thus, an extended SM theory is needed. One of such possibilities is the widening of the gauge group symmetry leading to new interactions and particle spectrum and which can be achieved at the TeV scale, containing natural dark matter candidates [16], [17] and explaining the generation problem in the so-called 341 model [18], [19], [20], [21].

Electroweak phase transition (EWPT) is a type of symmetry breaking that plays an important role at the early stage of the expanding universe where the scalar potential is responsible for this. It is the transition between symmetrical and asymmetrical phases, generating masses to elementary particles.

In order to describe the EWPT, it is better to use the technique of the effective potential. It is a function containing the contributions coming from fermions, bosons, and depends on temperature and vacuum expectation values (VeVs) [22], [23], [24]. It is worth mentioning
that the first order EWPT has to be strong, that is, the true vacuum expectation value (VeV) \( \nu_c \) has to be larger than the critical temperature, \( \frac{\nu_c}{T_c} \geq 1 \) (in the unit where Boltzmann’s constant \( k_B = 1 \)) \[25\].

Among the extended models of our interest is the compact 341 model, which is based on the gauge symmetry group tensor product \( SU(3)_C \otimes SU(4)_L \otimes U(1)_X \). In addition to the SM particles spectra, the model contains twelve new gauge bosons, six exotic quarks, two charged Higgses, a doubly charged Higgs and two neutral Higgses. Some of the intriguing features of the 341 model are the automatic existence of the standard model Higgs, and the ability to contain a candidate for cold dark matter \[16\], \[17\].

This paper is organized as follows, in Sec. II, we briefly present the compact 341 model. In Sec. III, we introduce the effective potential technique and show the structure of phase transition in the compact 341 model. In Sec. IV, we present our numerical results taking into account the theoretical constraints imposed to the scalar potential. In Sec. V, we discuss the B-violation via the sphaleron approach. Finally, in Sec. VI we draw our conclusions.

II. THE COMPACT 341 MODEL

A. Particle content

The compact 341 model is described by the gauge group \( SU(3)_C \otimes SU(4)_L \otimes U(1)_X \), and contains all the particles of the SM with new gauge bosons, exotic quarks, and three Higgs scalar quartets \[18\], \[19\]. Like the SM, we have three generations of fermions represented by the quartets:

\[
L_{aL} = \begin{pmatrix} u_a \\ l_a \\ \nu_a \\ l_a^c \\ \nu_a^c \end{pmatrix} \sim (1, 4, 0), \quad a = e, \mu, \tau
\]

\[
Q_{1L} = \begin{pmatrix} u_1 \\ d_1 \\ U_1 \\ J_1 \end{pmatrix} \sim (3, 4, 2/3), \quad \begin{cases} u_{1R} \sim (3, 3, 1/3) \\ d_{1R} \sim (3, 1, -1/3) \\ U_{1R} \sim (3, 1, 2/3) \\ J_{1R} \sim (3, 1, 5/3) \end{cases}
\]

\[
Q_{iL} = \begin{pmatrix} d_i \\ u_i \\ D_i \\ J_i \end{pmatrix} \sim (3, 4, -1/3), \quad \begin{cases} u_{iR} \sim (3, 1, 2/3) \\ d_{iR} \sim (3, 1, -1/3) \\ U_{iR} \sim (3, 1, -1/3) \\ J_{iR} \sim (3, 1, -4/3) \end{cases}, \quad i = 2, 3
\]

with \( u_1, d_1 \) are the up and down quarks, \( U_1, J_1, J_i, D_i \) are the new exotic quarks with electric charges \( 2/3, 5/3, -4/3, -1/3 \) respectively. We remind that the \( U(1) \) charge \( X \) is related to the fermions electric charge by the relation:

\[
Q_e = (X, X - 1, X, X + 1)
\]
The scalar sector contains three Higgs scalar quartets

\[
\eta = \begin{pmatrix}
\eta_1^0 \\
\eta_2^0 \\
\eta_2^-
\end{pmatrix} = \begin{pmatrix}
1 \sqrt{2} (R_{\eta_1} + i I_{\eta_1}) \\
1 \sqrt{2} (v_\eta + R_{\eta_2} + i I_{\eta_2}) \\
\eta_2^-
\end{pmatrix} \sim (1, 4, 0) \tag{3}
\]

\[
\rho = \begin{pmatrix}
\rho_1^+ \\
\rho_1^0 \\
\rho_2^+ \\
\rho_2^+
\end{pmatrix} = \begin{pmatrix}
1 \sqrt{2} (\rho_1^+ + R_{\rho} + i I_{\rho}) \\
1 \sqrt{2} (v_\rho + R_{\eta_2} + i I_{\eta_2}) \\
\rho_2^+ \\
\rho_2^+
\end{pmatrix} \sim (1, 4, 1)
\]

\[
\chi = \begin{pmatrix}
\chi_1^- \\
\chi_2^- \\
\chi_2^- \\
\chi_0^0
\end{pmatrix} = \begin{pmatrix}
1 \sqrt{2} (\chi_1^- + R_{\chi} + i I_{\chi}) \\
1 \sqrt{2} (v_\chi + R_{\chi} + i I_{\chi}) \\
\chi_2^- \\
\chi_0^0
\end{pmatrix} \sim (1, 4, -1)
\]

the following neutral components develop three nontrivial vacuum expectation values (VeVs)

\[
\eta = \frac{1}{\sqrt{2}} \begin{pmatrix}
0 \\
0 \\
v_\eta
\end{pmatrix}, \quad \rho = \frac{1}{\sqrt{2}} \begin{pmatrix}
v_\rho \\
0 \\
0
\end{pmatrix}, \quad \chi = \frac{1}{\sqrt{2}} \begin{pmatrix}
0 \\
0 \\
v_\chi
\end{pmatrix} \tag{4}
\]

with \(R_{\eta_1}, R_{\eta_2}, R_{\rho}, R_{\chi}\) are the CP-even scalar (real), \(I_{\eta_1}, I_{\eta_2}, I_{\rho}, I_{\chi}\) are the CP-odd scalar (imaginary), the reason one chooses the eta quadruplet to develop VeV only in the 3rd component is related to the fact that we do not want to mix among ordinary and exotic quarks in the Yukawa lagrangian, which guarantees the usual CKM mixing in the quark sector. Equivalently, this is also possible if one also adds a new \(Z_3\) discrete symmetry to the model. The later will also allow for an appropriate scenario for generating masses through effective operators. Spontaneous symmetry breaking takes place in three different steps:

* The first step:
  \[
  SU(4)_L \otimes U(1)_X \rightarrow SU(3)_L \otimes U(1)_X \tag{5}
  \]

* The second step:
  \[
  SU(3)_L \otimes U(1)_X \rightarrow SU(2)_L \otimes U(1)_Y \tag{6}
  \]

* The third step:
  \[
  SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{QED} \tag{7}
  \]

The VeVs \(v_\chi, v_\eta, v_\rho\) satisfy the constraints:

\[
v_\chi > v_\eta > v_\rho \tag{8}
\]

So, in this model, there are two quite different scales of vacuum expectation values: \(v_\eta \sim O(\text{TeV}), v_\chi \sim O(\text{TeV}), \) and \(v_\rho \approx 246 \text{ GeV}.\) Following ref. [19], the relationship between \(SU(4)_L\) and \(U(1)_X\) coupling constants \(g_L\) and \(g_X\) respectively is:

\[
\frac{g_X^2}{g_L^2} = \frac{s_w^2}{1 + 4s_w^2} \tag{9}
\]
the relation \((9)\) exhibits a landau pole when \(s_w^2 = \frac{1}{4}\) where \(g_X \to \infty\) (comes infinite and \(g_L\) finite) \([26], [27], [28]\). The existence of a Landau pole for the compact 341 model at a scale of around 5 TeV implies a natural cut-off for the model where one can circumvent the long standing hierarchy problem.

### B. The Higgs sector

The scalar potential of the compact 341 model \([19]\) is given by

\[
V(\eta, \rho, \chi) = \mu_\eta^2 \eta^+ \eta + \mu_\rho^2 \rho^+ \rho + \mu_\chi^2 \chi^+ \chi \tag{10}
\]

\[
+ \lambda_1 (\eta^+ \eta)^2 + \lambda_2 (\rho^+ \rho)^2 + \lambda_3 (\chi^+ \chi)^2
\]

\[
+ \lambda_4 (\eta^+ \eta) (\rho^+ \rho) + \lambda_5 (\eta^+ \eta) (\chi^+ \chi) + \lambda_6 (\rho^+ \rho) (\chi^+ \chi)
\]

\[
+ \lambda_7 (\rho^+ \eta) (\eta^+ \rho) + \lambda_8 (\chi^+ \eta) (\eta^+ \chi) + \lambda_9 (\rho^+ \chi) (\chi^+ \rho) + h.c
\]

where \(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8, \lambda_9\) are dimensionless coupling constants, \(\mu_{\eta, \rho, \chi}^2\) are the mass dimension parameters satisfying the following relations when the potential is minimized. These relations are given by the tadpole conditions:

\[
\begin{align*}
\mu_\eta^2 + \lambda_1 v_\eta^2 + (\lambda_4 v_\eta^2 + \lambda_5 v_\chi^2)/2 &= 0 \tag{11} \\
\mu_\rho^2 + \lambda_2 v_\rho^2 + (\lambda_4 v_\rho^2 + \lambda_6 v_\chi^2)/2 &= 0 \\
\mu_\chi^2 + \lambda_3 v_\chi^2 + (\lambda_5 v_\eta^2 + \lambda_6 v_\rho^2)/2 &= 0
\end{align*}
\]

The scalar potential depending on VeVs \((10)\) can be written as follows:

\[
V(\eta, \rho, \chi) = \mu_\eta^2 v_\eta^2/2 + \mu_\rho^2 v_\rho^2/2 + \mu_\chi^2 v_\chi^2/2
\]

\[
+ \lambda_1 v_\eta^2/4 + \lambda_2 v_\rho^2/4 + \lambda_3 v_\chi^2/4
\]

\[
+ \lambda_4 v_\eta^2 v_\rho^2/4 + \lambda_5 v_\eta^2 v_\chi^2/4 + \lambda_6 v_\rho^2 v_\chi^2/4
\]

Moreover, the CP-even elements of the mass matrix \((4 \times 4)\) in the basis \((R_{\eta_1}, R_{\eta_2}, R_{\rho}, R_{\chi})\) are written as

\[
\begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 2\lambda_1 v_\eta & \lambda_4 v_\rho v_\eta & \lambda_5 v_\eta v_\chi \\
0 & \lambda_4 v_\rho v_\eta & 2\lambda_2 v_\rho^2 & \lambda_6 v_\rho v_\chi \\
0 & \lambda_5 v_\eta v_\chi & \lambda_6 v_\rho v_\chi & 2\lambda_3 v_\chi^2
\end{pmatrix}
\tag{13}
\]

The eigenvalues of \(13\) are the masses of the neutral Higgses \(H_1^0, H_2^0, H_3^0\)

\[
M_{H_1^0}^2 = \left( \lambda_2 + \frac{[\lambda_3 \lambda_5^2 + \lambda_6 (\lambda_1 \lambda_6 - \lambda_4 \lambda_5)]}{\lambda_5^2 - 4\lambda_1 \lambda_3} \right) v_\rho^2 \tag{14}
\]

\[
M_{H_2^0}^2 = \frac{1}{2} \left( \lambda_1 + \lambda_3 - \sqrt{(\lambda_1 - \lambda_3)^2 + \lambda_5^2} \right) v_\chi^2
\]

\[
M_{H_3^0}^2 = \frac{1}{2} \left( \lambda_1 + \lambda_3 + \sqrt{(\lambda_1 - \lambda_3)^2 + \lambda_5^2} \right) v_\chi^2
\]

\[
M_{G_1}^2 = 0
\]
The CP-odd elements of the mass matrix \((4 \times 4)\) in the basis \((I_{\eta_1}, I_{\eta_2}, I_{\rho}, I_{\chi})\) vanish and therefore, the neutral CP-odd Higgses are all massless

\[
M_{I_{\eta_1}}^2 = M_{I_{\eta_2}}^2 = 0, \quad M_{I_{\rho}}^2 = M_{I_{\chi}}^2 = 0 \tag{15}
\]

The mass matrices of the simply charged Higgs can be expressed according to three bases:

1) In the basis \((\eta^\pm_1, \rho^\pm_2)\) we have

\[
\frac{1}{2} \lambda_7 \begin{pmatrix}
v_{\eta}^2 & v_{\rho}v_{\eta} \\
v_{\rho}v_{\eta} & v_{\rho}^2
\end{pmatrix}
\]

with the eigenvalues

\[
M_{G^+_1}^2 = 0, \quad M_{h^+_1}^2 = \frac{1}{2} \lambda_7 (v_{\eta}^2 + v_{\rho}^2) \tag{16}
\]

2) In the basis \((\eta^\pm_2, \chi^\pm_2)\) we have

\[
\frac{1}{2} \lambda_8 \begin{pmatrix}
v_{\eta}^2 & v_{\chi}v_{\eta} \\
v_{\chi}v_{\eta} & v_{\chi}^2
\end{pmatrix}
\]

and the eigenvalues

\[
M_{G^+_2}^2 = 0, \quad M_{h^+_2}^2 = \frac{1}{2} \lambda_8 (v_{\eta}^2 + v_{\chi}^2) \tag{17}
\]

3) In the basis \((\rho^\pm_1, \chi^\pm_1)\) we have

\[
M_{\rho^+_1}^2 = 0, \quad M_{\chi^+_1}^2 = 0 \tag{18}
\]

For the doubly charged Higgses, we have the mass matrix in the basis \((\chi^{\pm\pm}, \rho^{\pm\pm})\)

\[
\frac{1}{2} \lambda_9 \begin{pmatrix}
v_{\chi}^2 & v_{\chi}v_{\rho} \\
v_{\chi}v_{\rho} & v_{\rho}^2
\end{pmatrix}
\]

with the eigenvalues

\[
M_{G^{\pm\pm}}^2 = 0, \quad M_{h^{\pm\pm}}^2 = \frac{1}{2} \lambda_9 (v_{\rho}^2 + v_{\chi}^2) \tag{19}
\]

So, in this model, we have three neutral Higgses \((H_1^0\text{ the Higgs of the SM})\) and three charged Higgses \((h_1^\pm, h_2^\pm, h^{\pm\pm})\), the Goldstone bosons \(G^{\pm\pm}\) eaten by the doubly charged gauge bosons \(V^{\pm\pm}\), the Goldstone bosons \(G^+_1, G^+_2, \rho_1^\pm, \chi_1^\pm\) eaten by the charged gauge bosons \(W^\pm, K^\pm, Y^\pm, X^\pm\) and \(I_{\eta_1}, I_{\eta_2}, I_{\rho}, I_{\chi}, G_1\) eaten by the neutral gauge bosons \(Z, Z', Z'', K, K'\).

### C. The Fermions sector

To obtain the fermion masses, we need the Yukawa interactions given by the Lagrangian density [29]:

\[
\mathcal{L}_Y = \lambda_{ij}^L \bar{Q}_{iL} X_j R + \lambda_{ij}^L \bar{Q}_{iL} \chi_j R + \lambda_{ij}^U \bar{Q}_{iL} \eta U_{1R} + \lambda_{ij}^D \bar{Q}_{iL} \eta D_{1R} + \lambda_{ij}^d \bar{Q}_{1L} \rho_d R + \lambda_{ij}^u \bar{Q}_{iL} \rho^* u_{aR} \tag{23}
\]
with $\lambda_{11}^f, \lambda_{1j}^f, \lambda_{11}^D, \lambda_{1j}^D$ and $\lambda_{1a}^u, \lambda_{1j}^u$ are respectively the Yukawa couplings of the exotic and the ordinary quarks. Like in the $SM$, the masses of the usual quarks are proportional to the VeV $v_\rho$, they are involved in the third step of the spontaneous symmetry breaking (SSB) $SU(2)_L \to U(1)_Q$. In what follows, we consider only the mass of the top quark:

$$ m_t = \frac{\sqrt{2}}{2} \lambda_{33}^u v_\rho $$

The masses of the exotic quarks $J_1, J_2, J_3$ are proportional to the VeV $v_\chi$, and they are involved in the first step of SSB $SU(4)_L \to SU(3)_L$, the mass of $J_1$ is:

$$ m_{J_1} = \frac{\sqrt{2}}{2} \lambda_{11}^{J_1} v_\chi $$

the mass matrix of $J_2, J_3$ in the basis $(J_2, J_3)$ is written as:

$$ \frac{\sqrt{2}}{2} v_\chi \begin{pmatrix} \lambda_{22}^{J_2} & \lambda_{23}^{J_2} \\ \lambda_{32}^{J_2} & \lambda_{33}^{J_2} \end{pmatrix} $$

with the eigenvalues

$$ m_{J_2} = \frac{\sqrt{2}}{2} \lambda_{22}^{J_2} v_\chi, \quad m_{J_3} = \frac{\sqrt{2}}{2} \lambda_{33}^{J_2} v_\chi $$

The masses of the exotic quarks $U_1, D_2, D_3$ are proportional to the VeV $v_\eta$, and they are involved in the second step of SSB $SU(3)_L \to SU(2)_L$, The mass of $U_1$ is:

$$ m_{U_1} = \frac{\sqrt{2}}{2} \lambda_{11}^{U_1} v_\eta $$

the mass matrix of $D_2, D_3$ in the basis $(D_2, D_3)$ is written as

$$ \frac{\sqrt{2}}{2} v_\eta \begin{pmatrix} \lambda_{22}^{D_2} & \lambda_{23}^{D_2} \\ \lambda_{32}^{D_2} & \lambda_{33}^{D_2} \end{pmatrix} $$

with the eigenvalues

$$ m_{D_2} = \frac{\sqrt{2}}{2} \lambda_{22}^{D_2} v_\eta, \quad m_{D_3} = \frac{\sqrt{2}}{2} \lambda_{33}^{D_2} v_\eta $$

A summary of the quarks masses formulation $m^2(v_\eta, v_\rho, v_\chi) = m^2(v_\eta) + m^2(v_\rho) + m^2(v_\chi)$ is shown in table 1:

| Quarks | $m^2(v_\eta)$ | $m^2(v_\rho)$ | $m^2(v_\chi)$ |
|--------|---------------|---------------|---------------|
| $m_{D_2}^2$ | $\frac{1}{2} \lambda_{22}^{D_2} v_\eta^2$ | $\frac{1}{2} \lambda_{23}^{D_2} v_\eta^2$ | 0 |
| $m_{D_3}^2$ | $\frac{1}{2} \lambda_{32}^{D_2} v_\eta^2$ | $\frac{1}{2} \lambda_{33}^{D_2} v_\eta^2$ | 0 |
| $m_{U_1}^2$ | $\frac{1}{2} \lambda_{11}^{U_1} v_\eta^2$ | $\frac{1}{2} \lambda_{12}^{U_1} v_\eta^2$ | 0 |
| $m_{J_2}^2$ | $\frac{1}{2} \lambda_{22}^{J_2} v_\chi^2$ | 0 | $\frac{1}{2} \lambda_{22}^{J_2} v_\chi^2$ |
| $m_{J_3}^2$ | $\frac{1}{2} \lambda_{32}^{J_2} v_\chi^2$ | 0 | $\frac{1}{2} \lambda_{33}^{J_2} v_\chi^2$ |
| $m_{J_1}^2$ | $\frac{1}{2} \lambda_{11}^{J_1} v_\chi^2$ | 0 | $\frac{1}{2} \lambda_{12}^{J_1} v_\chi^2$ |
| $m_t^2$ | $\frac{1}{2} \lambda_{33}^u v_\rho^2$ | 0 | $\frac{1}{2} \lambda_{33}^u v_\rho^2$ |

Table 1: the quarks masses formulation
D. The gauge Bosons

Considering the Lagrangian density $\mathcal{L}^B$ of the gauge bosons

$$\mathcal{L}^B = (D_\mu \chi)^+ \chi + (D_\mu \eta)^+ \eta + (D_\mu \rho)^+ \rho$$

(30)

where $D_\mu$ is the covariant derivative given by

$$D^\mu = \partial^\mu - \frac{ig_L}{2} \lambda_\alpha A_\alpha^\mu - ig_\rho X B^\mu = \partial^\mu - i P^\mu$$

and

$$P^\mu = \frac{g_L}{2} \begin{pmatrix} W_3^\mu + W_8^\mu/\sqrt{3} & (W_1^\mu + i W_2^\mu) & (W_6^\mu + i W_7^\mu) & (W_{13}^\mu + i W_{14}^\mu) \\ +W_{15}^\mu/\sqrt{6} & (W_4^\mu + i W_5^\mu) & (W_9^\mu + i W_{10}^\mu) \\ +2g_\rho X B^\mu & -W_3^\mu + W_8^\mu/\sqrt{3} & (W_4^\mu + i W_5^\mu) & (W_{11}^\mu + i W_{12}^\mu) \\ (W_1^\mu - i W_2^\mu) & +W_{15}^\mu/\sqrt{6} & (W_9^\mu + i W_{10}^\mu) & -2W_8^\mu/\sqrt{3} + 2g_\rho X B^\mu \\ (W_{13}^\mu - i W_{14}^\mu) & (W_9^\mu - i W_{10}^\mu) & (W_{11}^\mu - i W_{12}^\mu) & -3W_{15}^\mu/\sqrt{6} + 2g_\rho X B^\mu \end{pmatrix}$$

(32)

$W_i^\mu (i = 1...15)$ and $B_\mu$ are the gauge fields.

For the charged gauge bosons, in the basis $(W_1^\mu, W_2^\mu)$, $(W_4^\mu, W_5^\mu)$, $(W_6^\mu, W_7^\mu)$, $(W_9^\mu, W_{10}^\mu)$, $(W_{11}^\mu, W_{12}^\mu)$, $(W_{13}^\mu, W_{14}^\mu)$ one has the following mass matrices:

$$\frac{g_L^2}{4} \begin{pmatrix} v_\rho^2 & 0 \\ 0 & v_\rho^2 \end{pmatrix}, \quad \frac{g_L^2}{4} \begin{pmatrix} v_\rho^2 + v_\eta^2 & 0 \\ 0 & v_\rho^2 + v_\eta^2 \end{pmatrix}$$

(33)

$$\frac{g_L^2}{4} \begin{pmatrix} v_\eta^2 & 0 \\ 0 & v_\eta^2 \end{pmatrix}, \quad \frac{g_L^2}{4} \begin{pmatrix} v_\eta^2 + v_\chi^2 & 0 \\ 0 & v_\eta^2 + v_\chi^2 \end{pmatrix}$$

(34)

$$\frac{g_L^2}{4} \begin{pmatrix} v_\eta^2 + v_\chi^2 & 0 \\ 0 & v_\eta^2 + v_\chi^2 \end{pmatrix}, \quad \frac{g_L^2}{4} \begin{pmatrix} v_\chi^2 & 0 \\ 0 & v_\chi^2 \end{pmatrix}$$

To obtain the masses:

$$M_{W^\pm}^2 = \frac{g_L^2 v_\rho^2}{4}, \quad M_{K^\pm}^2 = \frac{g_L^2 (v_\rho^2 + v_\eta^2)}{4}$$

(35)

$$M_{K^0, K^\pm}^2 = \frac{g_L^2 v_\eta^2}{4}, \quad M_{Y^\pm}^2 = \frac{g_L^2 (v_\rho^2 + v_\chi^2)}{4}$$

$$M_{Y^\pm}^2 = \frac{g_L^2 (v_\eta^2 + v_\chi^2)}{4}, \quad M_{X^\pm}^2 = \frac{g_L^2 (v_\chi^2)}{4}$$
respectively. For the neutral bosons in the basis \((W^\mu_3, W^\mu_8, W^\mu_{15}, B^\mu)\) we have the following mass matrix:

\[
\frac{g^2_L}{4} \begin{pmatrix}
\frac{-v^2_\rho}{\sqrt{3}} & \frac{-v^2_\rho}{\sqrt{3}} & \frac{-v^2_\rho}{\sqrt{3}} & -2X_{\alpha}v^2_\rho \\
\frac{-v^2_\rho}{\sqrt{3}} & \frac{3}{3\sqrt{2}} & \frac{3}{3\sqrt{2}} & \frac{2X_{\alpha}}{g_L}\frac{v^2_\rho}{\sqrt{3}} \\
\frac{-v^2_\rho}{\sqrt{3}} & \frac{3}{3\sqrt{2}} & \frac{3}{3\sqrt{2}} & \frac{2X_{\alpha}}{g_L}\frac{v^2_\rho}{\sqrt{3}} \\
-2X_{\alpha} v^2_\rho & 2X_{\alpha} \frac{v^2_\rho(2v^2_\rho-3v^2_{\gamma})}{\sqrt{3}} & 2X_{\alpha} \frac{v^2_\rho(v^2_\rho-2v^2_{\gamma})}{\sqrt{3}} & \frac{4X_\alpha}{g_L^2}(v^2_\rho+v^2_{\gamma}+3v^2_{\chi})
\end{pmatrix}
\]

and therefore, the masses of \(\gamma, Z, Z', Z''\) are:

\[
M^2_\gamma = 0, \quad M^2_Z = g^2_L v^2_\rho/(3 - 4\sin^2\theta_W) \quad \text{(37)}
\]

\[
M^2_{Z'} = g^2_L v^2_{\eta}(1 - 4\sin^2\theta_W) + (3 - 4\sin^2\theta_W)/(3 - 4\sin^2\theta_W)(1 - 4\sin^2\theta_W)
\]

Here \(\theta_W\) is the Weinberg angle. Table 2 summarizes the gauge bosons and Higgs masses formulation \(m^2(v_\eta, v_\rho, v_\chi) = m^2(v_\eta) + m^2(v_\rho) + m^2(v_\chi)\):

**Table 2 : The gauge bosons and Higgs masses formulation**

| Bosons | \(m^2(v_\eta, v_\rho, v_\chi)\) | \(m^2(v_\eta)\) | \(m^2(v_\rho)\) | \(m^2(v_\chi)\) |
|--------|-------------------------------|----------------|----------------|----------------|
| \(m^2_{W^\pm}\) | \(\frac{g^2_L v_\rho^2}{4}\) | \(0\) | \(80.40 \text{ GeV}^2\) | 0 |
| \(m^2_{K^\pm}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) | 0 |
| \(m^2_K\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) | 0 |
| \(m^2_{Z^\pm}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) |
| \(m^2_{Y^\pm}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) |
| \(m^2_{X^\pm}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) |
| \(m^2_{Z'}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) |
| \(m^2_{Z''}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) |
| \(m^2_{h_1}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) |
| \(m^2_{h_2}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) |
| \(m^2_{H_1}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) |
| \(m^2_{H_2}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) |
| \(m^2_{H_3}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) | \(g^2_L \frac{v_\rho^2}{4}\) |

### III. ELECTROWEAK PHASE TRANSITION

#### A. Effective Potential

In a perturbative analysis of the electroweak phase transition, the important tool is the effective potential at finite temperature \([30, 31, 32, 33]\). It is the contribution coming from
fermions, bosons and Higgses. This function also depends on the VeV’s and temperature and is given at one loop order by \[34\]

\[
V_{\text{eff}}(\phi, T) = V_0(\phi) + V_1(\phi) + \Delta V_1^{(T)}(\phi, T)
\] (38)

where \(V_0\) and \(V_1\) are respectively the tree-level and the one-loop effective potential at \(T = 0\). The third term \(\Delta V_1^{(T)}(\phi, T)\) contains the one loop thermal corrections. The expression of \(V_1(\phi)\) reads \[30\], \[31\]

\[
V_1(\phi) = \sum_i \frac{\tilde{n}_i}{64\pi^2} m_i^4(\phi) \left[ \ln \frac{m_i^2(\phi)}{\mu^2} - C_i \right]
\] (39)

where \(\mu\) is the renormalization scale and \(\tilde{n}_i = n_i(-1)^{S_i}\), with \(S_i\) and \(n_i\) stand for the spin and the degrees of freedom of the particle \(i\). The sum is over all particles of the model having a mass \(m_i(\phi)\). It is worthwhile mentioning, that if we work in the Landau gauge and use the \(\overline{DR}\) renormalization scheme \[35\], the coefficients \(C_i\) take the same value $\frac{3}{2}$ for all kind of particles. Following ref. \[30\] \(\Delta V_1^{(T)}(\phi, T)\) has the form:

\[
\Delta V_1^{(T)}(\phi, T) = \sum_{i=\text{boson}} n_i \frac{T^4}{2\pi^2} J_B \left( \frac{m_i^2}{T^2} \right) - \sum_{j=\text{fermion}} n_j \frac{T^4}{2\pi^2} J_F \left( \frac{m_j^2}{T^2} \right)
\] (40)

where \(J_B(x), J_F(x)\) are respectively the bosonic and fermionic thermal functions, with the following expansions (for \(x \ll 1\)) \[36\]

\[
J_B(x^2) = -\frac{\pi^4}{45} + \frac{\pi^2}{12} x^2 - \frac{\pi}{6} x^3 - \frac{1}{32} x^4 \ln\left(\frac{x^2}{a_b}\right) + O(x^5)
\] (41)

and

\[
J_F(x^2) = -\frac{7\pi^2}{360} + \frac{\pi^2}{24} x^2 - \frac{1}{32} x^4 \ln\left(\frac{x^2}{a_f}\right) + O(x^5)
\] (42)

with \(\ln(a_b) \simeq 5.4076\) and \(\ln(a_f) \simeq 2.6351\). For \(x \gg 1\) one has \[36\]

\[
J_B(x^2) \simeq J_F(x^2) = \left( \frac{x}{2\pi} \right)^{\frac{3}{2}} e^{-x} \left( 1 + \frac{15}{8x} + O\left(\frac{1}{x}\right) \right)
\] (43)

Regarding the particles content of the compact 341 model, the effective potential \(V_{\text{eff}}(\phi, T)\) can be shown as having the following expression

\[
V_{\text{eff}}(\phi, T) = \frac{\mu_b^2 v_y^2 + \mu_{\rho}^2 v_\rho^2 + \mu_x^2 v_x^2}{2} + \frac{\lambda_1 v_y^2 + \lambda_2 v_\rho^2 + \lambda_3 v_x^2 + \lambda_4 v_y^2 v_\rho^2 + \lambda_5 v_y^2 v_x^2 + \lambda_6 v_\rho^2 v_x^2}{4} + \sum_{i=\text{bosons, fermions}} \tilde{n}_i m_i^4 \left[ \ln \frac{m_i^2}{\mu^2} - \frac{3}{2} \right] + \frac{T^4}{2\pi^2} \sum_{i=\text{bosons, fermions}} \tilde{n}_i J_{B,F} \left( \frac{m_i}{T} \right),
\] (44)

with

\[
\tilde{n}_h = 1, \quad \tilde{n}_{e, \text{arg} ed} = 2, \quad \tilde{n}_{\text{quark}} = -12, \quad \tilde{n}_{z, z', z'', \kappa, \kappa'} = 3, \quad \tilde{n}_{W, K_1, \gamma, X, \nu} = 6.
\] (45)
B. Electroweak Phase Transition in the compact 341 Model

The electroweak baryogenesis (EWBG) is one of the most attractive and important ways of accounting for the observed baryon asymmetry of the universe. In order to explain the problem of baryogenesis, Sakharov posited three conditions for generating this asymmetry (matter-antimatter) [37], [38]. The process causing the violation of the baryon number must come from a rapid transition of the sphalons into the symmetrical phase. It must violate C and CP since its conservation would lead to the creation of the same amount of matter-antimatter particles. The involved interactions should take place out-of-equilibrium so that the last Sakharov condition is reached by a strong first order phase transition.

Using the high temperature expansions in eqs. (41), (42) one can rewrite eq. (38) in a simplified form illustrating the thermal corrections in the effective potential, as in ref. [36]

\[
V_{\text{eff}}(\phi, T) = D(T^2 - T_0^2)\phi^2 - ET\phi^3 + \frac{\lambda}{4}\phi^4
\]

where \(D, E, T_0\) are temperature independent coefficients and \(\lambda\) is a slowly-varying function of \(T\). If \(E = 0\) the phase transition is of a second order type [39] with a transition temperature \(T_0\) and the Higgs expectation value \(\langle \phi \rangle_0\) such that

\[
\langle \phi \rangle_0 = T_0 \sqrt{\frac{2D}{\lambda} \left(1 - \frac{T^2}{T_0^2}\right)}
\]

For \(E \neq 0\) the phase transition becomes first order, at very high temperatures \(T \gg T_1\). The only minimum of the effective potential is \(\langle \phi \rangle_0 = 0\). When \(T = T_1\) the effective potential acquires an extra minimum at a value

\[
\langle \phi \rangle_1 = \frac{3ET}{2\lambda}
\]

which appears as an inflection point at a temperature

\[
T_1^2 = \frac{8\lambda D}{8\lambda D - 9E^2}T_0^2
\]

As the temperature drops, the second minimum in eq. (48) becomes deeper, and it degenerates with the other one \(\langle \phi \rangle_1 = 0\). The two minima are separated by a potential barrier at a critical temperature \(T_c\) such that

\[
T_c^2 = \frac{\lambda DT_0^2}{\lambda D - E^2}
\]

The value of the minimum of the critical temperature is given by

\[
\langle \phi \rangle_c = \frac{2ET_c}{\lambda}
\]

The first order phase transition is characterized by the ratio \(\frac{\langle \phi \rangle_c}{T_c}\). The transition from the local minimum at \(\langle \phi \rangle = 0\) to a deeper minimum at \(\langle \phi \rangle \neq 0\) proceeds via the thermal tunnelling [40]. It can be understood in terms of bubbles formation of the broken phase in the sea of the symmetrical phase. When enough large bubble forms and expands until it collides with other bubbles, then the universe becomes filled with the broken phase. The
The electroweak phase transition occurs at a temperature \( T_w \sim 100 \text{ GeV} \) and must be strongly first order to achieve a successful EWBG and its quantitative condition is \( \frac{\sigma_{\text{ET}}}{T^2} \geq 1 \). The height of the barrier between the two minima measures of the strength of the transition. Thus, the strong first order phase transition presents a high barrier.

The EWBG would not be possible at the critical temperature if a second order phase transition occurred when there was no barrier and the Higgs field would continuously drop from zero to a non-zero expectation value and that there was no bubble nucleation which is the source of non-thermal equilibrium. We would expect a very large baryon violation rate because there is no barrier between the two vacua, so the sphaleron transition is quick in this case.

In our work, we only consider the contributions of the gauge bosons, top and exotic quarks, three neutral and three charged Higgses to the effective potential during the three steps of the spontaneous symmetry breaking.

### 1. The first step phase transition

The effective potential at finite temperature of the first step of phase transition \((SU(4)_L \otimes U(1)_X) \rightarrow SU(3)_L \otimes U(1)_X\) has the compact form

\[
V_{\text{eff}}(v_\chi) = D(T^2 - T_0^2)v_\chi^2 - ET v_\chi^3 + \frac{\lambda(T)}{4} v_\chi^4
\]

where this phase transition occurs at the TeV scale. The parameters of the above equation are shown to have the following expressions:

\[
D = \frac{1}{24v_\chi^2} \left( 6m_{V^\pm}^2 + 6m_{Y^\pm}^2 + 6m_{h_2^\pm}^2 + 2m_{h_+}^2 + 2m_{h^-}^2 \right)
\]

\[
E = \frac{1}{12\pi v_\chi^2} \left( 6m_{V^\pm}^2 + 6m_{Y^\pm}^2 + 6m_{h_2^\pm}^2 + 2m_{h_+}^2 + 2m_{h^-}^2 + m_{H_2^0}^2 + m_{H_3^0}^2 \right)
\]

\[
T_0^2 = \frac{m_{H_2^0}^2 + m_{H_3^0}^2}{4D} - \frac{1}{32D\pi^2 v_\chi^2} \left( 6m_{V^\pm}^4 + 6m_{h_2^\pm}^4 + 6m_{h_2^\pm}^4 \right)
\]

\[
\lambda(T) = \frac{(m_{H_2^0}^2 + m_{H_3^0}^2)}{2v_\chi^2} \left\{ 1 - \left( \frac{1}{8\pi^2 v_\chi^2 (m_{H_2^0}^2 + m_{H_3^0}^2)} \right) \right\} \left[ 6m_{V^\pm}^4 \log \frac{m_{V^\pm}^2}{ABT^2} + 6m_{h_2^\pm}^4 \log \frac{m_{h_2^\pm}^2}{ABT^2} - 12m_{J_1}^4 \log \frac{m_{J_1}^2}{AT^2} - 12m_{J_2}^4 \log \frac{m_{J_2}^2}{AT^2} + 12m_{J_3}^4 \log \frac{m_{J_3}^2}{AT^2} - 12m_{J_4}^4 \log \frac{m_{J_4}^2}{AT^2} + m_{H_2^0}^4 \log \frac{m_{H_2^0}^2}{ABT^2} + 2m_{h_2^\pm}^4 \log \frac{m_{h_2^\pm}^2}{ABT^2} + 2m_{h_1^\pm}^4 \log \frac{m_{h_1^\pm}^2}{ABT^2} \right\}
\]
where the critical temperature $T_{c_1}$ is given by

$$T_{c_1} = \frac{T_0}{\sqrt{1 - E^2/\lambda D}}$$  \hspace{1cm} (54)$$

and the condition of first EWPT is

$$E > \frac{\lambda(T_{c_1})}{2}$$  \hspace{1cm} (55)$$

2. The second step phase transition

The effective potential at finite temperature of the second step of phase transition $(SU (3)_L \otimes U (1)_X \rightarrow SU (2)_L \otimes U (1)_Y)$ reads

$$V_{\text{eff}}(v_\eta) = D'(T^2 - T_0^2)v_\eta^2 - E'Tv_\eta^3 + \frac{\lambda'(T)}{4}v_\eta^4$$  \hspace{1cm} (56)$$

where this phase transition also occurs at the TeV scale. The parameters $D'$, $E'$, $T_0'$ and $\lambda'(T)$ are:

$$D' = \frac{1}{24v_\eta^2} \left( 3m_{K}^2 + 3m_{K'}^2 + 6m_{K}^2 + 6m_{Z'}^2 + 3m_{Z''}^2 \right)$$  \hspace{1cm} (57)$$

$$+ 6m_{U_1}^2 + 6m_{D_2}^2 + 6m_{D_3}^2 + 2m_{h_1^+}^2 + 2m_{h_2^+}^2$$

$$E' = \frac{1}{12\pi v_\eta^3} \left( 6m_{Y}^3 + 6m_{K}^3 + 3m_{Z'}^3 + 3m_{Z''}^3 \right)$$  \hspace{1cm} (58)$$

$$+ 3m_{K}^3 + 3m_{K'}^3 + 2m_{h_1^+}^3 + 2m_{h_2^+}^3$$

$$T_0'^2 = -\frac{1}{32\pi^2 D'v_\eta^2} \left( 6m_{Y}^4 + 6m_{K}^4 + 3m_{Z'}^4 + 3m_{Z''}^4 + 3m_{K}^4 + 3m_{K'}^4 - 12m_{U_1}^4 - 12m_{D_2}^4 - 12m_{D_3}^4 + 2m_{h_1^+}^4 + 2m_{h_2^+}^4 \right)$$

$$\lambda'(T) = \frac{1}{2v_\eta^2} \left[ v_\eta^2 - \frac{1}{8\pi^2 v_\eta^2} \left( 6m_{Y}^4 \log \frac{m_{Y}^2}{A_B T^2} + 6m_{K}^4 \log \frac{m_{K}^2}{A_B T^2} + 3m_{Z'}^4 \log \frac{m_{Z'}^2}{A_B T^2} + 3m_{Z''}^4 \log \frac{m_{Z''}^2}{A_B T^2} + 3m_{K'}^4 \log \frac{m_{K'}^2}{A_B T^2} - 12m_{U_1}^4 \log \frac{m_{U_1}^2}{A_F T^2} - 12m_{D_2}^4 \log \frac{m_{D_2}^2}{A_F T^2} - 12m_{D_3}^4 \log \frac{m_{D_3}^2}{A_F T^2} + 2m_{h_1^+}^4 \log \frac{m_{h_1^+}^2}{A_B T^2} + 2m_{h_2^+}^4 \log \frac{m_{h_2^+}^2}{A_B T^2} \right) \right]$$

where the critical temperature $T_{c_2}$ is given by

$$T_{c_2} = \frac{T_0'}{\sqrt{1 - E'^2/\lambda' D'}}$$  \hspace{1cm} (58)$$
and the condition of first EWPT is

$$E' > \frac{\lambda(T_{c1})}{2}$$  \hspace{1cm} (59)

3. The third step phase transition

The effective potential at finite temperature of the third step of phase transition $(SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{QED})$ has the form

$$V_{eff}(\upsilon_\rho) = D''(T^2 - T''^2)v_\rho^2 - E''T\upsilon_\rho^3 + \frac{\lambda''(T)}{4}v_\rho^4$$  \hspace{1cm} (60)

where this phase transition happens at the GeV scale and the coefficients $D''$, $E''$, $T''_0$ and $\lambda''(T)$ are:

$$D'' = \frac{1}{24v^2_\rho} \left( 6m_{W^\pm}^2 + 6m_{K^0}^2 + 6m_{V^{\pm\pm}}^2 + 3m_Z^2 \\
+ m_{H^0_1}^2 + 2m_{h^+_1}^2 + 2m_{h^{\pm\pm}_1}^2 + 6m_t^2 \right)$$  \hspace{1cm} (61)

$$E'' = \frac{1}{12\pi v^3_\rho} \left( 6m_{W^\pm}^3 + 6m_{K^0_1}^2 + 6m_{V^{\pm\pm}}^3 \\
+ 3m_Z^3 + m_{H^0_1}^4 + 2m_{h^+_1}^4 + 2m_{h^{\pm\pm}_1}^4 \right)$$

$$T''_0^2 = \frac{m_{H^0_1}^2}{4D''} - \frac{1}{32D''\pi^2 v^2_\rho} \left( 6m_{W^\pm}^4 + 6m_{K^0_1}^4 + 6m_{V^{\pm\pm}}^4 \\
+ 3m_Z^4 + m_{H^0_1}^6 + 2m_{h^+_1}^6 + 2m_{h^{\pm\pm}_1}^6 - 12m_t^4 \right)$$

$$\lambda''(T) = \frac{m_{H^0_1}^2}{2v^2_\rho} - \frac{1}{16\pi^2 v^4_\rho} \left( 6m_{W^\pm}^4 \log \frac{m_{W^\pm}^2}{A_B T^2} + 6m_{K^0_1}^4 \log \frac{m_{K^0_1}^2}{A_B T^2} \\
+ 6m_{V^{\pm\pm}}^4 \log \frac{m_{V^{\pm\pm}}^2}{A_B T^2} + 3m_Z^4 \log \frac{m_Z^2}{A_B T^2} + m_{H^0_1}^6 \log \frac{m_{H^0_1}^2}{A_B T^2} \\
+ 2m_{h^+_1}^6 \log \frac{m_{h^+_1}^2}{A_B T^2} + 2m_{h^{\pm\pm}_1}^6 \log \frac{m_{h^{\pm\pm}_1}^2}{A_B T^2} - 12m_t^4 \log \frac{m_t^2}{A_F T^2} \right)$$

with the critical temperature $T_{c3}$ is given by

$$T_{c3} = \frac{T''_0}{\sqrt{1 - E''^2/\lambda'' D''}}$$  \hspace{1cm} (62)

and the condition of the third step of phase transition is

$$E'' > \frac{\lambda(T_{c3})}{2}$$  \hspace{1cm} (63)
IV. NUMERICAL RESULTS

Before proceeding, we must first diagonalize first the scalar potential $V(\eta, \rho, \chi)$. To do so, we must find the eigenvalues of the block matrix $M_{6 \times 6}$ written in the basis $\Phi_1 = \eta^+, \Phi_2 = \rho^+, \Phi_3 = \chi^+, \Phi_4 = \rho^+, \Phi_5 = \chi^+, \Phi_6 = \chi^+$:

$$M_{6 \times 6} = \begin{pmatrix}
\lambda_1 & \lambda_4/2 & \lambda_5/2 \\
\lambda_4/2 & \lambda_2 & \lambda_6/2 \\
\lambda_5/2 & \lambda_6/2 & \lambda_3 \\
0 & 0 & 0
\end{pmatrix} (64)$$

Straightforward but tedious calculations give the following eigenvalues

$$V_1 = \pm \lambda_0/2, \quad V_2 = \pm \lambda_8/2, \quad V_3 = \pm \lambda_7/2$$

$$V_4 = \left\{ \begin{array}{c}
-\left( \frac{2}{27} \beta^3 - \frac{\gamma}{3} + \delta \right) / 2 \\
+ \left[ \left( \frac{4(\gamma - \beta^3/3)^3}{27} \right)^{1/3} + \\
\frac{27(\frac{2}{27} \beta^3 - \frac{\gamma}{3} + \delta)^2}{27} \right]^{1/2} \end{array} \right\}^{1/3}$$

$$V_5 = (-V_4 + \sqrt{\Delta})/2, \quad V_6 = (-V_4 - \sqrt{\Delta})/2$$

where

$$\beta = -(\lambda_1 + \lambda_2 + \lambda_3)$$

$$\gamma = \left( (\lambda_2^2 + \lambda_5^2 + \lambda_3^2)/4 + \lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_2 \lambda_3 \right)$$

$$\delta = \left( (\lambda_2^2 \lambda_1 + \lambda_5^2 \lambda_2 + \lambda_3^2 \lambda_3)/4 - \lambda_1 \lambda_2 \lambda_3 - (\lambda_4 \lambda_5 \lambda_6)/3 \right)$$

Now, in order to make our study clear, viable and self-consistent, some theoretical constraints have to be imposed on the scalar potential dimensionless couplings $V_i (i = 1, 6)$.

1)- $V_i$ 's have to be real such that the masses of all particles are real and positive (no ghost masses in the model).

2)- The potential has to be bounded from below and this will impose the stability conditions $V_i > 0 \ (i = 1, 6)$.

3)- In order to preserve the perturbative unitarity, $V_i$ 's have to verify the constraints $0 < V_i < 8\pi \ (i = 1, 6)$.

In what follows, we identify the lightest scalar as neutral Higgs like boson with mass $m_{H^0} = 125.616 \text{ GeV}$, and take the top quark mass $m_t = 172.4466 \text{ GeV}$, the $W^\pm, Z$ gauge bosons masses $m_{W^\pm} = 80.40 \text{ GeV}, m_Z = 91.6838 \text{ GeV}$, the VeV $v_\rho = 246 \text{ GeV}$ and the gauge coupling of the standard model $g_L \simeq g_s \simeq 0.65$. Taking the Yukawa coupling randomly and using a Monte-Carlo simulation after imposing the constraint of a strong first order phase transition, we obtain

$$\lambda_{33}^U = 0.9913, \quad 0.385 \leq \lambda_{11}^U \leq 0.54, \quad 0.382 \leq \lambda_{22}^U \leq 0.536, \quad 0.386 \leq \lambda_{33}^D \leq 0.542$$

$$0.42 \leq \lambda_{11}^D \leq 0.61, \quad 0.4 \leq \lambda_{22}^D \leq 0.54, \quad 0.41 \leq \lambda_{33}^D \leq 0.56$$
Moreover, using the fact that the VeVs take the values \( \nu_\chi \sim \nu_\eta \sim 2 \text{ TeV} \), (in order to avoid the Landau pole \([26], [27], [28]\)), and the masses of the exotic quarks \( U, J_1, J_2, J_3, D_2, D_3 \) are in the range of 600 – 700 GeV which are compatible with the LHC results giving the lower band reproducing the experimental results (bands from Monte-Carlo simulation is displayed in table 3), and imposing the theoretical constraints mentioned previously, we get the following stangent conditions on the scalar potential dimensionless coupling constants:

\[
\left( \lambda_2 + \frac{[\lambda_3 \lambda_4^2 + \lambda_6 (\lambda_1 \lambda_6 - \lambda_4 \lambda_5)]}{\lambda_5^2 - 4 \lambda_1 \lambda_3} \right) = 0.258
\]

\[
0.15 \leq \left( \lambda_1 + \lambda_3 - \sqrt{(\lambda_1 - \lambda_3)^2 + \lambda_5^2} \right) \leq 0.38
\]

\[
0.34 \leq \left( \lambda_1 + \lambda_3 + \sqrt{(\lambda_1 - \lambda_3)^2 + \lambda_5^2} \right) \leq 0.86
\]

\[
0.24 \leq (\lambda_1 + \lambda_3) \leq 0.62, \quad 0.009 \leq ((\lambda_1 - \lambda_3)^2 + \lambda_5^2) \leq 0.057
\]

\[
0.156 \leq \lambda_7 \leq 0.31, \quad 0.16 \leq \lambda_8 \leq 0.27, \quad 0.167 \leq \lambda_9 \leq 0.32
\]

The later constraints lead to the following intervals on the new heavy Higgses and gauge bosons masses shown in the table 3.

Figure 1 represents the allowed region of the first step of the EWPT for the ratio \( R_1 = v_{c_1}/T_{c_1} \) as a function of the heavy neutral Higgs boson mass \( m_{h_3} \), where the critical temperature \( T_{c_1} \) takes values in the interval \( T_{c_1} \sim 1200 - 2000 \text{ GeV} \). It is worth mentioning that the confidence band (the density plot) comes from the fact that at a given value of \( m_{h_3} \) we still have many choices of the scalar couplings \( \lambda_i \)'s combined to give the same value of the ratio \( R_1 \). Figure 2 displays the variation of the ratio \( R_1 \) within the allowed region of the parameters space in terms of the critical temperature \( T_{c_1} \) fulfilling the strong first order EWPT. Figure 3 is similar to figure 2 but for the second step of EWPT, where \( T_{c_2} \sim 989 - 1549 \text{ GeV} \). Notice that in this case the allowed region is narrower than the one of the first step, this is due to the fact that the parameters space is more constrained. Figure 4 is similar to figure 2 but for the third step of the EWPT, where \( T_{c_3} \sim 120 - 235 \text{ GeV} \).

For the sake of illustration and to get an idea if we take \( \lambda_1 \approx 0.2538, \lambda_2 \approx 0.2809, \lambda_3 \approx 0.1882, \lambda_4 \approx 0.1063, \lambda_5 \approx 0.155, \lambda_6 \approx 0.155, \lambda_7 \approx 0.2, \lambda_8 \approx 0.2128, \lambda_9 \approx 0.2339, \nu_\chi \approx 1987.72 \text{ GeV}, \nu_\eta \approx 1988.915 \text{ GeV}, \) we obtain \( m_{h_3} \approx 0.1988.8153 \text{ GeV}, m_{h_2} \approx 735.9784 \text{ GeV}, T_{c_1} \approx 1820 \text{ GeV}, T_{c_2} \approx 1000 \text{ GeV}, T_{c_3} \approx 122 \text{ GeV} \) and the ratio \( v_{c_1}/T_{c_1} \approx 1.09283, v_{c_2}/T_{c_2} \approx 1.98776, v_{c_3}/T_{c_3} \approx 2.00568 \). Figures 5, 6 and 7 represent the variation of the heavy charged and double charged Higgs masses \( (m_{h_1^\pm}, m_{h_2^\pm}) \), and \( m_{h_3^{+++}} \) respectively of the model as a function of the heavy neutral Higgs mass \( m_{h_3} \) fulfilling the theoretical constraints and the strong first order EWPT.

V. SPHALERON RATE IN THE COMPACT 341 MODEL

As it is well established in the literature \([29], [41], [42], [43] \) and in order to describe correctly the EWB one has to know:

(i) the type of the phase transition (first, second order...);

(ii) the critical and bubbles nucleation temperatures (giving informations about the occurrence of phase transitions);
(iii) the sphaleron rate necessary to generate the baryon number asymmetry.

Table 3: Various bands of the different particle masses compatible with strong EWPT and theoretical constraints.

| particle | $m_{\min}\,(\text{GeV})$ | $m_{\max}\,(\text{GeV})$ |
|----------|--------------------------|--------------------------|
| $m_{h_1^\pm}$ | 561.29 | 700 |
| $m_{h_3^\pm}$ | 834.4 | 939.76 |
| $m_{h_3^{\pm\pm}}$ | 634.064 | 749.46 |
| $m_{H_2^0}$ | 600 | 805 |
| $m_{H_3^0}$ | 896.36 | 1211.52 |
| $m_U$ | 590.85 | 760.11 |
| $m_J_1$ | 592.96 | 702.67 |
| $m_J_2$ | 588.685 | 697.59 |
| $m_J_3$ | 594.58 | 704.58 |
| $m_D_2$ | 570.5 | 683.55 |
| $m_D_3$ | 585.5 | 705 |
| $m_{k_1^\pm}$ | 576.79 | 651.67 |
| $m_{k_1,k_2'}$ | 571.22 | 646.75 |
| $m_{Y^{\pm\pm}}$ | 602.61 | 712.28 |
| $m_{Y^\pm}$ | 826.46 | 958.77 |
| $m_{X^\pm}$ | 597.28 | 707.78 |
| $m_{Z'}$ | 695.11 | 787.01 |
| $m_{Z''}$ | 1455.27 | 1647.67 |

Figure 1. (color online only) The ratio $\frac{\nu_{c_1}}{T_{c_1}}$ in terms of $m_{h_3^0}$ for the allowed strong first order EWPT region (density plot).
In the previous sections, we have discussed extensively the phase transition in the context of the compact 341 model. Furthermore, and as it was mentioned before, sphalerons are one of the most important ingredients in the study of EWB because the spaleron rate controls the rate of the baryon number density in the early universe. It should be noted that, the sphaleron phenomenon occurs if one has the transition from the zero VeV crossing the barrier to a non-zero VeV without tunnelling (classically).

Following ref. [29], the sphaleron rate $\Gamma$ by unit time is related to the sphaleron energy $\varepsilon$.
Figure 4. (color online only) The ratio $\frac{\nu_{c3}}{T_{c3}}$ in terms of $T_{c3}$

Figure 5. (color online only) Variation of $m_{h_1^+}$ as a function of $m_{h_3}$ verifying EWPT and theoretical constraints

via the relation

$$\frac{\Gamma}{V} = \alpha^4 T^4 \exp(-\varepsilon/T)$$

(69)

with $T$ is the temperature, $\varepsilon$ is the sphaleron energy, $\alpha = 1/30$ is a constant and $V$ is the volume of the EWPT region, $V = \frac{4\pi r^3}{3} \sim \frac{1}{T^2}$. To calculate the energy $\varepsilon$, we start from the
gauge-Higgs Lagrangian density of the compact 341 model

\[ \mathcal{L}_{gauge-Higgs} = -\frac{1}{4} F_{\mu \nu}^a F^{a \mu \nu} + (D_\mu \chi)(D^\mu \chi) + (D_\mu \eta)(D^\mu \eta) + (D_\mu \rho)(D^\mu \rho) - V(\chi, \eta, \rho) \]  \hspace{1cm} (70)

then we derive the Hamiltonian density and deduce the energy \( \varepsilon \) to get at the end the
following form \[29\]

\[
\varepsilon = \int d^3x \left[ (D_\mu \chi)^+ (D^\mu \chi) + (D_\mu \eta)^+ (D^\mu \eta) + (D_\mu \rho)^+ (D^\mu \rho) + V(\chi, \eta, \rho) \right]
\]

(71)

using the effective potential analysis at finite temperature with VeVs \( v_\chi, v_\eta \) and \( v_\rho \) discussed in section III, eq. (71) can be rewritten as

\[
\varepsilon = 4\pi \int d^3x \left[ \frac{1}{2} (\nabla_v \chi)^2 + \frac{1}{2} (\nabla_v \eta)^2 + \frac{1}{2} (\nabla_v \rho)^2 + V_{\text{eff}}(\chi, \eta, \rho) \right]
\]

(72)

using the static field approximation \[29\]

\[
\frac{\partial v_\chi}{\partial t} = \frac{\partial v_\eta}{\partial t} = \frac{\partial v_\rho}{\partial t} = 0 \tag{73}
\]

together with the equations of motion of the VeVs \( v_\chi, v_\eta \) and \( v_\rho \), leads to the following expressions of the sphaleron energy \( \varepsilon \) for each step of the phase transition:

\[
\varepsilon_{\text{sph}}(SU(4),SU(3),SU(2)) = 4\pi \int \left[ \frac{1}{2} d^3v_{\chi,\eta,\rho} + V_{\text{eff}}(v_{\chi,\eta,\rho}, T) \right] r^2 dr
\]

(74)

To estimate the sphaleron rate \( \Gamma \), and as it was done in ref. \[29\] concerning the 331 model, we proceed with the following approximation:

1. Static approximation: We assume that the VeVs of Higgs fields do not change from point to point of the universe, that is \( \vec{\nabla} v_{\chi,\eta,\rho} = 0 \) corresponding to the extremal of \( V_{\text{eff}} \) when using the field equations. The later are shown to be reduced to

\[
\frac{\partial V_{\text{eff}}(v_{\chi,\eta,\rho})}{\partial v_{\chi,\eta,\rho}} = 0.
\]

(75)

Thus, the sphaleron energies of eq. (74) are simplified and obtain

\[
\varepsilon_{\text{sph}}(SU(4),SU(3),SU(2)) = \frac{4\pi r^3}{3} V_{\text{eff}}(v_{\chi,\eta,\rho}, T).
\]

(76)

Using eqs. (52), (56) and (60) together with \( \vec{\nabla} v_{\chi,\eta,\rho} = 0 \), we get the following expressions for the sphaleron rates

\[
\Gamma = \begin{cases} 
\alpha^4 T \exp\left(-\frac{E^4}{4\lambda^4}\right) & \text{for } SU(4) \to SU(3) \\
\alpha^4 T \exp\left(-\frac{E'^4}{4\lambda'^4}\right) & \text{for } SU(3) \to SU(2) \\
\alpha^4 T \exp\left(-\frac{E''^4}{4\lambda''^4}\right) & \text{for } SU(2) \to U(1)
\end{cases}
\]

(77)

It is worth mentioning that, for the heavy particles of the 341 model within the allowed regions where the strong first order phase transitions occur, the quantities \( E \) (resp. \( E', E'' \)) and \( \lambda \) (resp. \( \lambda', \lambda'' \)) are almost constant. Thus, in this approximation \( \Gamma \) becomes linear function of \( T \) as illustrated in fig. 8. Moreover, numerical results show that for temperatures below that of the phase transition \( T_c \), where the universe switches to the symmetry breaking phase, the sphaleron rate is still much larger than the Hubble parameter and this leads to the washout of the \( B \)-violation. Fig. 8 displays the sphaleron rate as a function of the temperature for the three steps of
the SSB. Notice that for the first step, if we take $T = 1700$ GeV $< T_{c1} \sim 1800$ GeV, $\Gamma \sim 2.0985 \times 10^{-3} >> H$, and for the second step, if we take $T = 900$ GeV $< T_{c2} \sim 1000$ GeV, $\Gamma \sim 1.1401 \times 10^{-3} >> H$. Likewise for the third step, if we take $T = 70$ GeV $< T_{c3} \sim 122$ GeV, $\Gamma \sim 8.654 \times 10^{-4} >> H$. Consequently, in this approximation the sphaleron decoupling condition cannot be satisfied (same result was obtained by the authors of ref. [29] in the case of the 331 model).

![Figure 8](color online only) The Sphaleron rate $\Gamma$ as a function of the temperature $T$ for the three steps of the phase transition.

2. Thin wall approximation: Following ref. [29], we assume that,

$$\frac{\partial V_{\text{eff}}(v_{\chi,\eta,\rho})}{\partial v_{\chi,\eta,\rho}} = C_{\chi,\eta,\rho} = \text{const.}$$

(78)

where $v_{\chi,\eta,\rho}$ are the second minimum of the effective potential $V_{\text{eff}}$ in the bubble phase transition $SU(4) \rightarrow SU(3)$, $SU(3) \rightarrow SU(2)$ and $SU(2) \rightarrow U(1)$ respectively. In this case the field equations of the VeVs $v_{\chi,\eta,\rho}$ read

$$\frac{d^2 v_{\chi,\eta,\rho}}{dr^2} + \frac{2}{r} \frac{dv_{\chi,\eta,\rho}}{dr} = C_{\chi,\eta,\rho}$$

(79)

with the boundary conditions

$$\lim_{r \rightarrow \infty} v_{\chi,\eta,\rho}(r) = 0, \quad \left. \frac{dv_{\chi,\eta,\rho}(r)}{dr} \right|_{r=0} = 0$$

(80)

The solutions of eqs. (79) and (80) are given by [29]

$$v_{\chi,\eta,\rho} = \frac{C_{\chi,\eta,\rho}}{6} r^2 - \frac{A_{\chi,\eta,\rho}}{r} + B_{\chi,\eta,\rho}$$

(81)

where $A_{\chi,\eta,\rho}, B_{\chi,\eta,\rho}$ are integration constants. To be more specific if the sphaleron has a radius $R_{\chi,\eta,\rho}$ and a thickness $\Delta l_{\chi,\eta,\rho}$ the solution $v_{\chi,\eta,\rho}$ can be expressed as

$$v_{\chi,\eta,\rho}(r) = \begin{cases} v_{\chi,\eta,\rho,c} \frac{C_{\chi,\eta,\rho}}{6} r^2 - \frac{A_{\chi,\eta,\rho}}{r} + B_{\chi,\eta,\rho} & \text{when } R_{\chi,\eta,\rho} < r \leq R_{\chi,\eta,\rho} + \Delta l_{\chi,\eta,\rho} \\ 0 & \text{when } R_{\chi,\eta,\rho} + \Delta l_{\chi,\eta,\rho} < r \end{cases}$$

(82)
Here \( \nu_{\chi, \eta, \rho_c} \) stands for the second minimum for the 3 steps of the phase transition. In order to proceed further the constant \( C_{X, \eta, \rho} \) can be approximated as

\[
C_{X, \eta, \rho} \sim \frac{\Delta V_{	ext{eff}}(\nu_{X, \eta, \rho_c})}{\Delta \nu_{X, \eta, \rho}} \tag{83}
\]

where \( \Delta V_{	ext{eff}} = V_{	ext{eff}}(\nu_{X, \eta, \rho_c}) \) and \( \Delta \nu_{X, \eta, \rho} = \nu_{X, \eta, \rho_c} \). Now, for the numerical results and in order to avoid the washout of the baryonic asymmetry after the phase transition one has to assume that the sphaleron rate \( \Gamma \) has to be equal to the Hubble parameter \( H \) at the critical temperature \( T_c \). Of course \( \Gamma \) has to be larger than \( H \) at \( T > T_c \) and smaller at \( T < T_c \) see ref. [29], [13], [15].

Analyzing figures [9] to [11] a general behavior was observed for all the EWPT \( SU(4) \to SU(3), SU(3) \to SU(2) \) and \( SU(2) \to U(1) \): as \( T \) decreases from the bubble nucleation temperature \( T_1 \) where the strong first-order phase transition starts the radius \( R \) and the energy \( \varepsilon \) of the bubble increase while the sphaleron rate \( \Gamma \) decreases such that the ratio \( \frac{\Gamma}{H} \) is bigger (resp. smaller) than 1 for \( T_c < T < T_1 \) (resp. \( T_0 < T < T_c \)). To be more precise, we notice that the gauge symmetries respectively \( SU(4) \to SU(3), SU(3) \to SU(2) \) and \( SU(2) \to U(1) \) start to be broken spontaneously at the bubble nucleation temperature \( T_1 \sim 1823, 1309.35, 194.94 \) GeV (\( h = c = k = 1 \)). Then, a small bubble with radius \( R \sim 13.53 \times 10^{-4}, 4.5 \times 10^{-5}, 11.97 \times 10^{-4} \) GeV\(^{-1} \) and thickness \( \Delta l \sim 10^{-4}, 10^{-5}, 10^{-4} \) (GeV\(^{-1} \)) appears and stores the nonvanishing VeVs \( \nu_{X, \eta, \rho} \) inside. It is very important to mention that as it was pointed out in section [11], if the two minima are separated by a potential barrier the phase transition will occur with bubbles nucleations governed by thermal channeling from a local minimum at \( \phi = 0 \) (false vacuum), to a deeper minimum at \( \phi \neq 0 \) (true vacuum). The non-thermal equilibrium is induced by the rapidly expanding bubble walls through the cosmological plasma, and \( B \)-violation arises from the rapid sphaleron transition in the symmetrical phase. At this temperature, the sphaleron rate \( \Gamma \) gets the values \( 4.8456 \times 10^{11}, 2.8493 \times 10^{11}, 3.11674 \times 10^{10} \) GeV, which are larger than the values of \( H \sim 4.6769 \times 10^{-12}, 2.4126 \times 10^{-12}, 5.34795 \times 10^{-12} \) GeV. When the temperature decreases from \( T_1 \sim 1823, 1309.35, 194.94 \) GeV to \( T_c \sim 1820, 1306, 167.5 \) GeV, the bubble volume and energy increases and decreases respectively. Moreover, the rate \( \Gamma \) decreases but the ratio \( \frac{\Gamma}{H} \) remain greater than 1 allowing the bubbles to collide and fill all the space. This phenomenon is very violent leading to a huge deviation from thermal equilibrium. The baryon production takes place in the neighborhood of the expanding bubbles walls generating CP and C violation (which is not the scoop of our paper). In fact, for an illustration if \( T = 1822, 1308, 180 \) GeV, \( R = 13.91 \times 10^{-4}, 9.5 \times 10^{-5}, 13.55 \times 10^{-4}, \varepsilon = 17398.496, 24853.266, 3711.643 \) GeV, \( \Gamma = 3.7084 \times 10^{7}, 2091.05642, 56.98872 \) GeV and \( \frac{\Gamma}{H} = 7.9379 \times 10^{18}, 8.6848 \times 10^{14}, 1.2498 \times 10^{15} \). Of course, as it was assumed before at \( T = T_c \) the sphaleron rate \( \Gamma = H = 4.66178 \times 10^{-12}, 2.3913 \times 10^{-12}, 3.94764 \times 10^{-14} \) GeV. When \( T \) becomes smaller than \( T_c \) (\( T = 1811.5, 1300, 158 \) GeV) \( \Gamma \) decreases rapidly and the ratio \( \frac{\Gamma}{H} \) becomes less than 1 (\( \frac{\Gamma}{H} = 3.19962 \times 10^{-210}, 3.2444 \times 10^{-19}, 1.8254 \times 10^{-109} \)). As the temperature reaches the transition ending temperature \( T_0 = 1796.625, 1280.02, 151.89 \) GeV, only the broken phase remains and the sphaleron transitions \( SU(4) \to SU(3), SU(3) \to SU(2) \) and \( SU(2) \to U(1) \) are totally shut off.
Figure 9. (color online only) The sphaleron energy $\varepsilon$ as a function of the temperature $T$ for the three steps of the phase transition.

Figure 10. (color online only) The radii of the bubbles $R$ as a function of the temperature $T$ for the three steps of the phase transition.

VI. CONCLUSION

Throughout this paper, and in order to be self-consistent, theoretical constraints on the potential parameters such as the unitarity, stability, and boundness have been imposed. Moreover, using a Monte-Carlo simulation, we have bound the various allowed regions of the parameter space verifying the first-order phase transition criteria at $v_{\chi} \sim v_{\eta} \sim 2$ TeV and $v_{\rho} = 246$ GeV for the three steps $SU(4) \rightarrow SU(3)$, $SU(3) \rightarrow SU(2)$, and $SU(2) \rightarrow U(1)$ respectively, leading to an effective potential and confidence bands where masses of the heavy Higgs bosons are in the range of $700 - 1300$ GeV. Moreover, we have derived the expressions of the effective potential, nucleation, critical, and ending temperatures in terms of particle masses and temperature for each step of the EWPT. Furthermore, the baryogenesis study using the sphaleron approach was also investigated, where it is shown that the static approximation cannot give consistent results for the ratio $\frac{T}{\Gamma}$ for $T < T_c$ why the thin wall approximation does. Finally, we can conclude that we have obtained the same conclusions as those of ref. [29] but within the framework of the 331 model. It is very important to stress out that, the authors of ref. [29] did not impose the theoretical constraints on the potential parameters as we did. Further investigations on CP violation in this model are under consideration.
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