On a Generalization of Jones Polynomial and its Categorification for Legendrian Knots

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Based on a joint work with Dr. Dheeraj Kulkarni
Legendrian Knots and Their Front Projections

- A **Legendrian** knot in the contact manifold \((\mathbb{R}^3, \xi_{st} = \ker(dz - ydx))\) is a smooth embedding of \(S^1\), which is always tangent to \(\xi_{st}\).

- A **front projection** of a Legendrian knot is its projection onto the \(xz\)-plane.

\[
K(x, dz/dx, z)
\]
Legendrian Jones Polynomial

**Legendrian Bracket polynomial** of a front projection of a Legendrian link is defined by the following rules:

1. \[ \langle \quad \rangle = -A^2 r^{-1} - A^{-2} r \]
2. \[ \langle K_F \sqcup \quad \rangle = (-A^2 r^{-1} - A^{-2} r) \langle K_F \rangle \]
3. \[ \langle \quad \quad \rangle = A \langle \quad \rangle + A^{-1} r \langle \quad \quad \rangle \]

The polynomial defined as

\[ P_{K_F}(A, r) = (-A)^{-3\omega(K_F)} r^{\frac{c}{2}} - l(K_F) \langle K_F \rangle \]

is an invariant of Legendrian knots up to Legendrian isotopy\(^1\).

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\(^1\)Dheeraj Kulkarni and Monika Yadav, On a generalization of Jones polynomial and its categorification for Legendrian knots, Bull. Sci. Math. 182 (2023), https://doi.org/10.1016/j.bulsci.2022.103212