Chiral thermodynamics of dense hadronic matter

Chihiro Sasaki\textsuperscript{1,*}

\textsuperscript{1}Frankfurt Institute for Advanced Studies, D-60438 Frankfurt am Main, Germany

We discuss phases of hot and dense hadronic matter using chiral Lagrangians. A two-flavored parity doublet model constrained by the nuclear matter ground state predicts chiral symmetry restoration. The model thermodynamics is shown within the mean field approximation. A field-theoretical constraint on possible phases from the anomaly matching is also discussed.

I. PARITY DOUBLED NUCLEONS

Model studies of hot and dense matter have suggested a rich phase structure of QCD at temperatures and quark chemical potentials of order $\Lambda_{\text{QCD}}$. Our knowledge on the phase structure however remains limited and the description of strongly interacting matter does not reach a consensus yet \cite{1}. In particular, properties of baryons near the chiral symmetry restoration are poorly understood. The realistic modeling of dense baryonic matter must take into account the existence of the nuclear matter saturation point, i.e. the bound state at baryon density $\rho_0 = 0.16 \text{ fm}^{-3}$, like in Walecka type models \cite{2}. Several chiral models with pure hadronic degrees of freedom \cite{3, 4} have been constructed in such a way that the nuclear matter has the true ground state. An alternative approach is to describe a nucleon as a dynamical bound-state of a diquark and a quark \cite{5}.

In the mirror assignment of chirality to nucleons \cite{6, 7}, dynamical chiral symmetry breaking generates a mass difference between parity partners and the chiral symmetry restoration does not necessarily dictate the chiral partners to be massless. Mirror baryons embedded in linear and non-linear chiral Lagrangians have been applied to study their phenomenology in vacuum \cite{6-8}, nuclear matter \cite{9, 10} and neutron stars \cite{11}. Identifying the true parity partner of a nucleon is also an issue. In the mirror models $N(1535)$ is usually taken to be the

\textsuperscript{*}Electronic address: sasaki@fias.uni-frankfurt.de
negative parity state. This choice however fails to reproduce the decay width to a nucleon and $\eta$. This might indicate another negative parity state lighter than the $N(1535)$, which has not been observed so far.

The parity doublet model has been applied to a hot and dense hadronic matter and the phase structure of a chiral symmetry restoration as well as a liquid-gas transition of nuclear matter was explored. In Fig. we show the phase diagram for two different masses of the negative parity state, $m_{N^-} = 1.5$ GeV and 1.2 GeV. The latter is considered to be an phenomenological option. At zero temperature the system experiences a first-order liquid-gas transition at $\mu_B = 923$ MeV and the baryon density shows a jump from zero to a finite value $\rho \neq 0$. Roughly speaking chiral symmetry restoration occurs when the baryon chemical potential reaches the mass of the negative parity state, $\mu_B \sim m_{N^-}$. The order of chiral phase transition and its location depend on the set of parameters, especially on mass of the negative parity state. If we take the most frequently used value $m_{N^-} = 1500$ MeV, then in addition to the nuclear liquid-gas phase transition we obtain a weak first-order chiral transition at $\rho \sim 10 \rho_0$. With a lower mass $m_{N^-} = 1200$ MeV we get no true chiral phase transition but only a crossover at much lower density $\rho \sim 3 \rho_0$. The liquid-gas transition survives up to $T = 27$ MeV. Above this temperature there is no sharp phase transition but the order parameter is still attracted by the critical point: the order parameter typically shows a double-step structure and this makes an additional crossover line terminating at the liquid-gas critical point. Another crossover line corresponding to the chiral symmetry restoration follows the steepest descent of the second reduction in $\langle \sigma \rangle$. With increasing temperature the two crossover lines become closer and finally merge.

In contrast, the trajectory of a meson-to-baryon “transition” defined from the ratio of particle number densities is basically driven by the density effect with the hadron masses being not far from their vacuum values. The line is almost independent of the parameter set and goes rather close to the liquid-gas transition line. The chiral crossover and the meson-baryon transition lines intersect at $(T, \mu_B) \sim (150, 450)$ MeV. The parity doublet model thus describes 3 domains: a chirally broken phase with mesons thermodynamically dominating, another chirally broken phase where baryons are more dominant and the chirally restored phase, which can be identified with quarkyonic matter. It is worthy to note that this intersection point is fairly close to the estimated triple point at which hadronic matter, quarkyonic matter and quark-gluon plasma may coexist.
II. ANOMALY MATCHING IN MATTER

How does deconfinement of colors enter to the chiral thermodynamics at finite temperature and density? Although there exist a variety of studies using chiral Lagrangian approaches and holographic QCD models and a conjecture given in the large $N_c$ limit, no conclusive picture on the actual QCD phase diagram is reached so far [14].

The anomaly matching is often used to constrain possible massless excitations in quantum field theories [15]. External gauge fields, e.g. photons, interacting with quarks lead to anomalies in the axial current (see Fig. 2). When the chiral symmetry is spontaneously broken in confined phase, the anomalies are saturated by the Nambu-Goldstone bosons. On the other hand, in chiral restored phase, the anomalous contribution must be generated from the triangle diagram in which the baryons are circulating. In two flavors the anomalies are matched with massless baryons, however, in three flavors, the baryons forming an octet do not contribute to the pole in the axial current because of the cancellations [16]. Therefore, a system with restored chiral symmetry should be in deconfined phase when the physics does not depend on the number of quark flavors is imposed. For a system composed only from up and down quarks, the anomaly matching does not exclude the chiral restored phase with confinement. Nucleons with mirror assignment do not generate the anomalies since the axial couplings to the positive and negative parity states have the same strength and their signs are relatively opposite.

III. SUMMARY AND DISCUSSIONS

The parity doublet model within the mean field approximation describes the nuclear matter ground state at zero temperature and a chiral crossover at zero chemical potential at a reasonable temperature, which are the minimal requirements to describe the QCD thermodynamics. The first-order phase transitions appear only at low temperatures, below $T \sim 30$ MeV. Nevertheless, at higher temperature they still affect the order parameter which exhibits a substantial decrease near the liquid-gas and chiral transitions. If the chiral symmetry restoration is of first order, criticality around the end points of the two first-order phase transitions will be the same due to the identical universality class [17].

We have also discussed the anomaly matching and possible model-selection. In the pres-
ence of hot and dense matter the situation is more involved due to the lack of Lorentz covariance and the existence of thermal masses [18]. Besides, at high density gapless excitations on the Fermi surface might appear, which can be either bosons or fermions. Since relevant low-energy excitations in matter are not perfectly known, those degrees of freedom must be introduced in such a way that the model certainly saturates the correct anomalies. The anomaly matching thus has a role of the working hypothesis in modeling QCD matter. It is indispensable to any rigorous argument for this taking account of the physics with medium effects, which could lead to a possibility of the chirally restored phase with confinement.

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FIG. 1: The phase diagram in the parity doublet model [12]. The mass of the negative parity nucleon was taken to be 1.5 GeV (left) and 1.2 GeV (right).

FIG. 2: Saturations of the anomalies in terms of elementary quarks (left) and of hadrons (right).
FIGURE CAPTIONS

1. The phase diagram in the parity doublet model [12]. The mass of the negative parity nucleon was taken to be 1.5 GeV (left) and 1.2 GeV (right).

2. Saturations of the anomalies in terms of elementary quarks (left) and of hadrons (right).