A Progress Report on the SO(5) Theory of High $T_c$

Superconductivity

Shou-Cheng Zhang

Department of Physics, Stanford University, Stanford CA 94305

In this talk I give a brief update on the recent progress in the SO(5) theory of high $T_c$ superconductivity \cite{1}. Reviewed topics include SO(5) ladders, the unification of BCS and SDW quasi-particles in the SO(5) theory and the microscopic origin of the condensation energy.

First of all I would like to thank the Taniguchi foundation and the organizers of the Grand Finale Taniguchi Symposium for inviting me to this wonderful conference. This Symposium is appropriately entitled “The Physics and Chemistry of Transition Metal Oxides”, and it covers a vast and broad range of topics whose intimate relations remain to be discovered. In order to achieve a coherent grand synthesis in this subject, we must first overcome the language barrier which separates workers in different sub-fields. At the welcome party of the Symposium, Professor George Swatzsky and I were casually chatting about the SO(5) theory of high $T_c$ superconductivity. A distinguished chemistry professor looked more and more puzzled as he overheard our conversation. Finally, he couldn’t hold his curiosity and asked “SO\textsubscript{5}? I didn’t know that sulfur-pentoxide is a superconductor!”

In a series of conference proceedings, I tried to give a on-going update about the status of the SO(5) theory of HTSC \cite{2,3}. This is the third one in this series. In the mean time, Auerbach wrote a pedagogical review \cite{4} explaining the SO(5) theory in terms of more familiar concepts in SO(3) quantum magnetism, and Hanke \textit{et al} wrote a extensive review \cite{5} on the numerical calculations within the SO(5) theory. Currently, the SO(5) approach to HTSC is actively being investigated by many groups, focusing both on the microscopic origins and phenomenological consequences. Much progress has been made in understanding the logical structure and examining the internal consistency of the theory.
In this report, I would first like to summarize recent developments in understanding the microscopic realization of the $SO(5)$ symmetry by using ladder systems as a theoretical laboratory \[6–15\] and address the unification of BCS and SDW quasi-particles in the $SO(5)$ theory \[16\]. These developments reveal a fascinatingly rich internal structure of the $SO(5)$ theory and could ultimately lead to a microscopic foundation of the theory. Although the $SO(5)$ theory appears to be a natural framework to understand many experiments of HTSC in a unified fashion, no experiment has directly tested the fundamental validity of the theory. Some more tests have been recently proposed in \[17,18\]. However, I would like to focus on some recent works \[19,20\] concerning the microscopic origin of the condensation energy in HTSC, which could lead to a direct and quantitative understanding of the microscopic mechanism of HTSC.

Shortly after the $SO(5)$ proposal, various groups have constructed microscopic models with exact $SO(5)$ symmetry \[21–23\]. However, all this models involve long range interactions which are not familiar and natural. It appears that as long as there is only one orbital per unit cell, this problem is unavoidable. For this reason the $SO(5)$ symmetry was investigated for the two-legged ladder system, which has two orbitals per unit cell, so that long ranged interaction can be avoided \[6\]. Another reason for investigating $SO(5)$ symmetry in the ladder system is because it is an example of a Mott insulator without any long range order at half-filling. $SO(5)$ symmetry was originally proposed as a theory to unify antiferromagnetism (AF) with superconductivity (SC). It would be an interesting question to see how it applies to Mott insulators without any (quasi-) AF long range order. The third reason for investigating the $SO(5)$ symmetry in the ladder system is to address the question of how this symmetry could emerge at long wave length without being present at the microscopic level \[7–9,15\]. Because of quasi-one-dimensionality, well controlled weak coupling RG calculations can be performed to address this issue.

The fundamental quantity in the microscopic $SO(5)$ models is the concept of a $SO(5)$ spinor \[21\], which has four components. On a two-legged ladder, one could naturally combine the two sites on a rung to form such a spinor, and $SO(5)$ invariant models can be easily
constructed by writing down the most general invariant interactions [6]. The parameter space for $SO(5)$ models is surprisingly large. Among the usual five local parameters $t, t_\perp, U, V$ and $J$, only one condition is required to satisfy the $SO(5)$ symmetry ($U + V = J/4$). The Mott insulating state at half-filling is not only a total spin singlet, but also a $SO(5)$ singlet. The lowest energy excitation on top of this singlet ground state is a five fold degenerate manifold of triplet magnons and Cooper pairs. A uniform magnetic field or chemical potential can lower the energy for one of these bosons, leading to a condensate with AF or SC quasi-long-range order. Within this framework, Mott insulator, AF and SC states are intimately related and can be understood in a unified way. Many theoretical ideas about $SO(5)$ symmetry can be tested in the ladders system. The eigenstates of the $SO(5)$ ladder models can all be classified into general irreducible representations of the $SO(5)$ group. These states form a beautiful and revealing pattern and have been identified in numerical calculations by Eder, Dorneich, Zacher, Hanke and the author [11]. The photoemission spectrum of the $SO(5)$ ladder has been studied in details [11,13]. The single electron Green’s functions can be related by exact Ward identities and are shown by direct diagonalization to evolve continuously from the Mott insulator to the superconducting state [11]. The structure of the exact $\pi$ resonance can also be understood in detail both analytically and numerically [14,11]. Due to the quasi-long-range order of the superconducting state, the $\pi$ resonance is not a delta function peak, but a threshold singularity at energy $-2\mu$ [6,14]. Other physical quantities can also be calculated [25].

While the $SO(5)$ models offer a nice testing ground for many interesting theoretical ideas, the parameters of these models are not “realistic”. It is therefore desirable to understand whether they share some common features with more realistic ladder models. For example, in the strong coupling limit, the ground state of a generic ladder model is a product state of singlet rungs. Rather surprisingly, this state is not only a total spin singlet, but also a $SO(5)$ singlet, since this product state is annihilated by the $\pi$ operators. In any $SO(5)$ singlet state, the behavior of the static AF and SC correlation functions are identical. Therefore, one would expect these correlations to scale towards a common behavior in the strong
coupling limit of a generic ladder model. Unfortunately, this argument does not apply to the dynamic correlation functions. One can use the exact $SO(5)$ models as a point of departure to systematically vary the symmetry violating perturbations and compare the results with the partial multiplet structures found in the $t-J$ model \cite{24}. Investigation towards this direction has been taken in ref. \cite{10,11}.

Fortunately, in the weak coupling regime, one can perform controlled RG calculations to see how exact symmetries emerge from nonsymmetric interactions \cite{7,8,9}. Lin, Balents and Fisher \cite{9} showed that there are in general 9 marginal operators for a two-legged ladder at half-filling, 5 of them preserve $SO(5)$ symmetry, while 4 of them violate it. Surprisingly, the symmetry violating interactions scale to zero. Within the remaining $SO(5)$ manifold, there are four stable fix points, with even higher symmetry, namely $SO(8)$. Arrigoni and Hanke \cite{8} also studied the symmetry violating band structure effects, for example the next-nearest-neighbor hopping $t'$, and demonstrated that after a suitable redefinition, the symmetry violating effects can be completely absorbed. More recently, Schulz \cite{15} extended the RG calculations from half-filling to general filling, and showed that $SO(6)$ and $SO(5)$ symmetries can emerge dynamically. These developments are very exciting, and offer hope that similar symmetry restoration effects can occur in higher dimensional systems near a quantum critical point \cite{1,26}.

The original $SO(5)$ theory was formulated purely in terms of bosonic collective degrees of freedom \cite{1}. It is clear that a complete theory has to include the fermionic sector. The bosonic theory explains how various collective modes connect to each other at the transition between the AF and SC states, it is natural to ask how the fermionic BCS and SDW quasi-particles connect to each other. The answer to this question turns out to be surprisingly rich and beautiful. In elementary quantum mechanics, we learned about a remarkable demonstration of the spinor nature of the electron. If two strong magnetic fields polarize the electron spins in two orthogonal directions, then the electron wave functions corresponding to these two orthogonal fields are not orthogonal to each other. This non-orthogonality of the electron wave functions allows transmission of a electron beam through the orthogonal
field regions. It is also the origin of the Berry’s phase. Quite similarly, even though a anti-ferromagnet and a superconductor have “orthogonal” order parameters, the quasi-particles associated with these two states are not orthogonal to each other. This non-orthogonality leads to a novel generalization of Berry’s phase to a SU(2) holonomy \[10\]. The generalization from the U(1) Berry’s phase of a SO(3) spinor to the SU(2) holonomy of a SO(5) spinor is a unique generalization in a precise mathematical sense, and involves some of the most beautiful, and yet seeming disconnected mathematical concepts such as Hopf maps, quaternions and the Yang monopole. So far this work is still at a mathematical stage, and the precise physical implications can only be anticipated at this moment. Potential applications include a generalized Bogoliubov-deGennes type of formalism to discuss novel fermionic excitations near topological defects involving twists of the SO(5) superspin vector \[27–30\], a novel type of Andreev reflection at the AF/SC boundary, a non-abelian Bohm Aharonov effect associated with regions with non-trivial superspin twists and novel understanding of the single particle properties in the pseudogap regime. The remarkably rich fermionic structure in the SO(5) model shows that the SO(5) theory is much more than a expanded version of the Landau-Ginzburg theory, it can fully address single particle excitations and their coupling to the collective modes.

The ideas on the SU(2) holonomy can possibly be extended to other physical systems as well, especially transition metal oxide systems with orbital degeneracy. Soon after the discovery of quantum mechanics, Wigner and von Neumann studied the problem of generic level crossings in quantum mechanics. They and later Dyson classified generic level crossings into three categories, now called the orthogonal, unitary and symplectic ensembles. The familiar U(1) Berry’s phase occur in the unitary ensemble. In the symplectic ensemble, one deals with time reversal invariant systems with Kramers degeneracy. The level crossing between two Kramers doublets can be described by the four dimensional (since two doublets give four states in total) symplectic group Sp(4), which happens to be isomorphic to SO(5). This type of level crossing phenomenon can not only occur at AF/SC transition, but also in generic problems involving spin-orbit couplings. The ideas on SU(2) holonomy can not
only lead to deeper understandings of these systems, but may also help to understand their (formal) relationship to the high $T_c$ problem.

Although the $SO(5)$ theory was originally proposed as an effective theory to understand the interplay between AF and SC, it implicitly points to a microscopic mechanism for HTSC. Within this theory, the microscopic mechanism for SC is basically the “same” as the microscopic mechanism for AF, namely the lowering of the exchange energy $J \sum_{i,j} \vec{S}_i \cdot \vec{S}_j$. Recently, Scalapino and White [19] argued that the lowering of the exchange energy can be quantitatively correlated with the SC condensation energy. This insightful observation allows for quantitative test of various mechanisms of HTSC which should make detailed prediction on how the exchange energy is saved in the SC state. The $SO(5)$ theory predicts a $\pi$ resonance mode [1,31,32] which is identified with the neutron resonance mode observed in the SC state. The theory of the neutron resonance mode is based on a particle particle collective mode near momentum $(\pi, \pi)$, which exists both in the normal and SC state. Since a particle particle mode can only make a contribution to the spin correlation function in the SC state, the neutron resonance mode is observed only below $T_c$. Recently, Demler and I noticed [20] that this argument also provides a concrete microscopic mechanism for HTSC. It is straightforward to see that the coupling to a particle particle collective mode around $(\pi, \pi)$ gives a negative difference between the exchange energy $J \sum_{i,j} \vec{S}_i \cdot \vec{S}_j$ in the SC and the normal state. Therefore, the SC saves more exchange energy compared to the normal state, and the amount of saving is precisely given by $J$ times the (dimensionless) integrated spectral weight of the $\pi$ resonance. From the neutron scattering experiments by Fong et al [33], one can see that the change in the dimensionless quantity $\vec{S}_i \cdot \vec{S}_j$ due to the $\pi$ resonance is on the order of few per cent, which gives a saving of exchange energy of $35K$ per unit cell. On the other hand, the condensation energy of optimally doped $YBCO$ superconductor is about $5K$ [34]. Therefore, we see that the emergence of the $\pi$ resonance could be the dominant mechanism responsible for the superconducting condensation energy. Upon going to the SC state, the kinetic energy usually increases, so that the saving in exchange energy could be balanced by the cost in kinetic energy to give the right condensation energy.
therefore highly desirable to find direct ways to measure the change in kinetic energy when the system enters the SC state.

This line of reasoning leading to a microscopic mechanism of SC is somewhat unfamiliar, since most people equate the SC mechanism with a form of attractive interaction between electrons. However, I would like to argue that in a strongly correlated system, this new line of thinking is much more fruitful and experimentally accessible. *The central idea here is to identify a energy saving process which is forbidden in the normal state but possible in the SC state.* In our mechanism, the particle particle resonance is just such a process. The only other example I can think of is the interlayer tunneling mechanism [35]. In this case, the energy saving process is the c axis tunneling, which is forbidden in the normal state if the normal state is not a fermi liquid, but is allowed in the SC state. In both examples we see that once such a process is identified, experimentally falsifiable prediction about the condensation energy follow immediately. Following this line of thinking, we can hopefully move the debate about the microscopic mechanism of HTSC to a new level, where direct comparison with experiments becomes possible.

Since this argument seem to strongly rely on the onset of the $\pi$ resonance at $T_c$ for the optimally doped superconductors, a alert reader may wonder how this argument applies to the underdoped superconductors, where a broadened resonance peak is observed above $T_c$ but below the pseudogap temperature $T_{MF}$ [36,37]. Let us first see how the $SO(5)$ theory could explain the broadened peak in the pseudogap regime. The basic process in the SC state is given by the following Feymann diagram.
FIG. 1. Feynmann diagram for the $\pi$ resonance in the SC (left panel) and normal (right panel) state. In the SC state, a particle hole pair created by the neutron is converted into a particle particle pair by the SC condensate (marked by two crosses). In the normal state, such a anomalous process is forbidden, but one can open up the crosses and reconnect them to obtain a Cooper pair propagator. In the pseudogap regime, there is no sharp pole due to a single $\pi$ pair, but a broadened convolution of a $\pi$ and a Cooper pair.

We see that the spin vertex creates a particle hole pair, but the hole can be converted into a particle by the Gorkov $F$ function, and the multiple scattering in the particle particle channel gives rise to a sharp collective mode in the dynamic spin correlation function. In the normal state, the Gorkov $F$ function vanishes identically so that such a process is not possible. However, one could cut two Gorkov $F$ functions and glue them together in the normal state as shown in Fig 1. Such a process does not vanish in the normal state, and represents the preformed Cooper pair fluctuations in the pseudogap regime. In this case, rather than a single resonance peak, we obtain a convolution between the triplet momentum ($\pi, \pi$) particle particle resonance and the singlet zero momentum Cooper pair resonance. If these resonances are weakly dispersive, the convolution spectrum will be reasonably sharp. Since the basic processes below and above $T_c$ are different, we would generally expect some discontinuous behavior at $T_c$. The experimental plots of the intensity as a function of temperature does appear to consist of two separate curves joining with a discontinuous derivative at $T_c$. Having understood the SC fluctuation in the normal state as the origin of the neutron intensity above $T_c$, we would use the general arguments outlined above to correlate the neutron scattering intensity with the condensation energy in the pseudogap regime. Remarkably, the condensation energy obtained from Loram’s specific data (Fig. 8 of reference [34]) and $\pi$ resonance intensity measured in neutron scattering [38] in the underdoped regime have the same qualitative behavior, namely consisting of two separate curves joining with a discontinuous derivative at $T_c$. The remarkable similarity between these two seemingly different experiments lend strong support to our interpretation, and
could not only lead to a quantitative understanding of the microscopic mechanism of HTSC, but also the origin of the pseudogap physics.

I would to thank E. Demler, R. Eder, A. Furusaki, W. Hanke, S. Rabello and D. Scalapino for close collaborations on projects reported above. This work is supported by the NSF under grant numbers DMR-9400372 and DMR-9522915.

[1] Shou-Cheng Zhang. Science, 275:1089, 1997.
[2] Shou-Cheng Zhang. Physica C, 282-287:265, 1997.
[3] Shou-Cheng Zhang. cond-mat/9709289, 1997.
[4] A. Auerbach. cond-mat/9801294, 1998.
[5] W. Hanke et al. cond-mat/9807015, 1998.
[6] D. Scalapino, S.C. Zhang, and W. Hanke. Phys. Rev. B, 58:443, 1998.
[7] D.G. Shelton and D. Sénéchal. cond-mat/9710251, 1997.
[8] E. Arrigoni and W. Hanke. cond-mat/9712143, 1997.
[9] H. Lin, L. Balents, and M. Fisher. Phys. Rev. B, 58:1794, 1998.
[10] Daniel Duffy, Stephan Haas, and Eugene Kim. cond-mat/9804221, 1998.
[11] R. Eder et al. cond-mat/9805121, 1998.
[12] P. Bouwknegt and K. Schoutens. cond-mat/9805232, 1998.
[13] S-P Hong and S-H. Salk. cond-mat/9807154, 1998.
[14] A. Furusaki and Shou-Cheng Zhang. cond-mat/9807373, 1998.
[15] H. J. Schulz. cond-mat/9808167, 1998.
[16] Eugene Demler and Shou-Cheng Zhang. cond-mat/9805404, 1998.

[17] H. Bruus et al. cond-mat/9807167, 1998.

[18] C. Burgess, J. Cline, and A. Lutken. Phys. Rev. B, 57:8549, 1998.

[19] D.J. Scalapino and S.R. White. cond-mat/9805073, 1998.

[20] Eugene Demler and Shou-Cheng Zhang. cond-mat/9806339, 1998.

[21] S. Rabello, H. Kohno, E. Demler, and S.C. Zhang. Phys. Rev. Lett., 80:3586, 1998.

[22] C. Henley. Phys. Rev. Lett., 80:3590, 1998.

[23] C. Burgess, J. Cline, R. MacKenzie, and R. Ray. Phys. Rev. B, 57:8549, 1998.

[24] R. Eder, W. Hanke, and S.C. Zhang. Phys. Rev. B, 57:13781, 1998.

[25] Eugene Pivovarov. cond-mat/9807297, 1998.

[26] R. B. Laughlin. cond-mat/9709195, 1997.

[27] D. Arovas, A.J. Berlinsky, C. Kallin, and S.C. Zhang. Phys. Rev. Lett., 79:2871, 1997.

[28] Y. Bazaliy, E. Demler, and S.-C. Zhang. Phys. Rev. Lett., 79:1921, 1997.

[29] E. Demler, A.J. Berlinsky, C. Kallin, G. Arnold, and M. Beasley. Phys. Rev. Lett., 80:2917, 1998.

[30] P.M. Goldbart and D. Sheehy. Phys. Rev. B, 58:5731, 1998.

[31] E. Demler and Shou-Cheng Zhang. Phys. Rev. Lett., 76:4126, 1995.

[32] E. Demler, H. Kohno, and S.-C. Zhang. cond-mat/9710139, 1997.

[33] H.F. Fong et al. Phys. Rev. B, 54:6708, 1996.

[34] J. Loram et al. Journal of Superconductivity, 7:243, 1994.

[35] S. Chakravarty et al. Science, 261:337, 1993.
[36] P. Dai et al. *Phys. Rev. Lett.*, 77:5425, 1996.

[37] H.F. Fong et al. *Phys. Rev. Lett.*, 78:713, 1997.

[38] Mook et al. *to be published*, 1998.