Numerical simulation of the oil reservoir with regulated disturbances. Oil recovery stability simulation

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Abstract. The article discusses a direction for improvement of the oil recovery stability simulation method. It is shown that the problem of stable productive layer development is solved by setting appropriate goals. To solve the first task, it is necessary to substantiate RFC plugging in the hard mode. The main calculated ratio is the modified Dupuis-Forchheimer and Bernoulli equations based on linear hydraulic resistances of the RFC. To solve the second task, it is necessary to substantiate the choice of resonant frequencies of the oil recovery process in the elastic water-pressure mode. The dependences allow us to distinguish the upper and lower limiting frequencies of the stable oil displacement mode. The solutions make it possible to isolate main conclusions in a complex form.

1. Introduction

The article deals with the numerical simulation of oil recovery process to increase reliability of the current prediction of reservoir properties [1]. Improvement of the secondary methods of influence on the near-wellbore zones of the layers [2–4] requires control of the stability of the oil displacement process. This problem is exacerbated by the use of controlled disturbances in remote areas of the RFC. This problem is of particular relevance due to the use of metatechnology for extraction of hard-to-recover hydrocarbons [5–8] based on discontinuous control of the RFC.

The problem of numerical modeling of oil recovery sustainability is considered in solving two main tasks. The first task involves conditions of delivery of the grouting material in the most critical hard mode of the RFC. The second task is calculation of boundary resonant frequencies of the oil displacement process in the elastic water-pressure mode.

The aim is to increase sustainability of the oil extraction under coordinate-discontinuous control.

The models of oil recovery with steady elastic RFC modes are used [9]. Improvement of mathematical models of RFC sustainability [10, 11] contributes to advanced methodological [12] and technical [13] controls and complete development of productive layers.

Fulfillment of the RFC plugging conditions in the hard mode of operation. To isolate the developed areas of the RFC, the most preferable method of numerical simulation is the use of values of specific hydraulic
resistance \( R_0 \) [1]. For this parameter, linear fictitious radius \( R \) of the current tube for the interwell RFC and plane-parallel flow \( v \) are introduced. The value of variable fictitious radius \( R \) due to the heterogeneity of the reservoir in the hard mode of operation is determined by the value of in-situ pressure of the plugging process according to ratio \( R_{\text{in}} < R_m < R_h \), where \( R_0 \) and \( R_{\text{in}} \) are in-situ pressures of the fracture and reservoir operation. In this regard, the RFC operation performed in the elastic water-pressure mode is characterized by the linear reduced radius of the current tube in the ratio \( r < R \).

The value of the hydrostatic resistance of the RFC is determined by the ratios of the linear term of the Dupuis-Forchheimer and Bernoulli equations

\[
R_h = \frac{\mu}{kR} = \frac{\Delta P}{\Delta L} \frac{R}{v_0 m (R^2 - r^2)}.
\] (1)

One can calculate the ratio for radii \( R \) and \( r \) of the RFC in hard and elastic water-pressure modes depending on the spatially distributed pressure field and its FEP in the following form

\[
R^2 = \frac{(R^2 - r^2) \Delta L}{\Delta P} \frac{\mu \cdot m}{k} v_0.
\] (2)

In accordance with equation (2), the minimum value of the fictitious current tube radius \( (R_{\text{min}}) \) characteristic of the space-time delivery of plugging material should be taken into account [6]. There is no fluid in the filtration process. Multiplying both sides of equation (2) by dimensionless number \( \pi \), for its left side one can find the cross-sectional area of the RFC (for \( R_0 \rightarrow \min \)) at the end point of the cement delivery \( S_m = \pi R^2 \bigg| _{R_0 \rightarrow \min} \). In this case, it is also possible to determine the volumes of injected plugging portions \( V_m = n R_m S_m \), where \( n = 4 \ldots 20 \) is the number of fictitious radii \( R_m \) along the cylinder of the RFC current tube depending on its heterogeneity.

After plugging of the developed area of the RFC, the insulated area experiences high hydraulic resistance (at \( R_0 \rightarrow \max \)). Filtered fluid is injected in the area. Thus, during the transition from the plug to the RFC operation, the current tube narrows to form an elastic fluid-containing area. Due to the fact that the right-hand side of equation (2) is also multiplied by \( \pi \), the additional fluid-containing cross-sectional area \( S_f = \pi (R^2 - r^2) \bigg| _{R_0 \rightarrow \max} \) can be defined as the coefficient of fluid permeability.

Calculation of resonant frequencies in the elastic mode of operation. The solution of the second task is connected with the need to clarify the previously obtained results of the solutions [9]. General solution of the Cauchy problem using hyperbolic equations for a chain current tube

\[
\tilde{v}(y, \tau) = (A \cos \lambda y + B \sin \lambda y) \times \exp\left\{ (-\mu/2 \pm j \lambda) \tau \right\}
\] (3)

serves as a basis for determining the boundary values of resonant frequencies when deriving the bipolar RFC in the elastic optimization mode for oil displacement according to the results of its analysis of the exponential part (3).

In this case, the following notation of function \( \tilde{v}(y, \tau) \) was used for dependencies on dimensionless time \( \tau = t/l \sqrt{L_h C_h} \); RFC boundary states \( y = \pm l \), when at \( x = 0 \), input load \( y = 0 \) is obtained, and at \( x = l \), output \( y = 1 \). Moreover, \( v = p/p_o \) is dimensionless relative pressure in the ratio of reservoir pressures in the elastic mode turning into the hard mode; \( \mu = R/l \sqrt{L_h C_h} \) is relative parameter of resistance of the two-pole RFC considered in the ratio of active hydraulic resistance to its wave hydraulic resistance.

Parameter \( \lambda \) is a key on the right-hand side of equation (3). First, the wave equation taking into account the boundary conditions is constructed:

\[
\frac{\partial^2 v}{\partial y^2} - \frac{\partial^2 v}{\partial \tau^2} = \mu \frac{\partial v}{\partial \tau}, \quad (4, \text{a})
\]
\begin{equation}
\frac{\partial^2 v}{\partial y^2} - \frac{\partial^2 v}{\partial \tau^2} = \mu \frac{\partial v}{\partial \tau},
\end{equation}
\begin{equation}
v = 0, \text{ if } y = 1, \tag{4, b}
\end{equation}
where \( \overline{S} = \overline{S_0} \tau / \overline{L_0} \). This linear problem is solved by the method of separation of variables; the desired solution is represented as
\begin{equation}
v(y, \tau) = Y(y)T(\tau) \tag{5}
\end{equation}
after substituting expression (5) into equation (4, a), the following expressions can be found for \( Y(y) \) and \( T(\tau) \):
\begin{equation}
\frac{d^2 Y}{\partial y^2} + \overline{\lambda}^2 Y = 0, \quad \frac{d^2 T}{\partial \tau^2} = \mu \frac{\partial T}{\partial \tau} + \overline{\lambda}^2 T = 0, \tag{6}
\end{equation}
Where \( \overline{\lambda}^2 \) is the own value of the boundary problem.

Thus, the general solution of equation (4, a) is written as (3). Substituted in the first boundary condition (4, b) it determines the ratio of amplitudes \( A \) and \( B \) as
\begin{equation}
B / A = \pm j \mu \overline{S} - \overline{\lambda} \overline{\gamma}, \tag{7}
\end{equation}
and from the second boundary condition (4, b) a different ratio of these amplitudes can be derived
\begin{equation}
B / A = - \text{ctg} \overline{\lambda} \overline{\gamma}. \tag{8}
\end{equation}
Equating these relations (7) and (8), one can obtain
\begin{equation}
\text{ctg} \overline{\lambda} = \overline{\lambda} \overline{\gamma} \pm j \mu \overline{S} \tag{9}
\end{equation}
By dividing \( \overline{\lambda} \) in a series of powers \( \mu \) the following expression is obtained:
\begin{equation}
\overline{\lambda} = \omega + \mu \overline{\lambda}_1 + \ldots, \tag{10}
\end{equation}
where \( \omega \) is one of the natural frequencies of the corresponding conservative system. Using member of order \( \mu \), we can substitute it into (9)
\begin{equation}
\text{ctg} \omega - \mu \overline{\lambda}_1 (1 + \text{ctg}^2 \omega) = = \overline{\gamma} \omega + \mu \overline{\lambda}_1 \overline{\gamma} \pm j \mu \overline{S}, \tag{11}
\end{equation}
Then,
\begin{equation}
\overline{\lambda}_1 = \pm \frac{j \overline{S}}{1 + \overline{\gamma} + \overline{\gamma}^2 \omega^2}. \tag{12}
\end{equation}
Due to the fact that \( \overline{\lambda}_1 \) it is a purely imaginary value, there is no correction to the frequency in the first approximation.

2. Discussion
From general solution (3) and its conclusions (4) - (11), it follows that the solution increasing with time will be at \( |\overline{\lambda}_1| > 1/2 \). In the exponent of expression (3), \( e \) term \( \mu/2 \) characterizes the energy loss (decrement), and term \( \mu |\overline{\lambda}_1| \) is the increment of the system. The requirement for the increment exceeding the decrement (at small amplitudes) implies the condition of bringing the optimization oil-displacement (system self-excitation) to the soft mode.
\begin{equation}
S > (1 + \overline{\gamma} + \overline{\gamma}^2 \omega^2) / 2. \tag{13}
\end{equation}
For this case, it should be taken into account that at cross-hole interactions of the FEP, the RFCs have unpredictable manifestations. Therefore, for different reservoir rocks, the production-pressure characteristic (by analogy with the volt-ampere one) can be used to correct the resulting non-linearities for the soft oil displacement mode by adjusting them with a cubic polynomial:
Let us consider the study of inequality (14) using the graphic. Let us construct the right side of its expression as a function of frequency \( f(\omega) \), as shown in the figure.

\[
p_{dc}(q) = S_q q(1 - q^2 / 3q_0^2). \tag{14}
\]

The main characteristic of the given graph is direct self-excitation line \( S \). The indicated family of curves is due to the difference in the active and reactive parameters of the chained RFC current tube. On the abscissa axis, discrete frequencies \( \omega_i (i = 1, 2, \ldots) \) correspond to the eigenfrequencies of the system that satisfy equation (13). As follows from the figure, with an increase in natural frequency increases, the excess of the increment over the decrement decreases. Starting with frequency \( (\omega_8) \), the system is not self-excited. At the same time, frequency \( \omega_1 \) determines the upper boundary values of indicators of remote areas of the RFC, and the lower boundary values of indicators of sections of the RFC - in the near-wellbore space of the formation (near the injection well).

Thus, the characteristics of the cube of an oil reservoir with its discontinuous controls determined on the basis of numerical methods can reflect the values of current permeability coefficients of the RFC

\[
\begin{array}{cccccc}
\kappa_{11}^1 & \kappa_{12}^1 & 0 & 0 & 0 \\
\kappa_{21}^1 & \kappa_{22}^1 & \kappa_{23}^1 & 0 & 0 \\
0 & \kappa_{32}^1 & \kappa_{33}^1 & \kappa_{34}^1 & 0 \\
0 & 0 & \kappa_{43}^1 & \kappa_{44}^1 & \kappa_{45}^1 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\end{array}
\tag{15}
\]

In the matrix array (14), its first sheet shows evolutionary development of RFC\(_1\) with four conventionally selected plugging points. The \( n \)-sheet structure of the matrix array in the same cube of an operational object is determined in a similar way. These representations are confirmed by technical feasibility [12, 13] and make it possible to successfully use data arrays in the MatLab software system.
3. Conclusions
1. The solution of the first problem of oil recovery sustainability on the basis of the modified Dupuis-Forchheimer and Bernoulli equations for the derived hydrostatic resistance allows for determining the boundaries of the point of plugging parts delivery to different areas of the RFC taking into account its rigid and elastic modes of operation.
2. The general solution of the Cauchy problem in terms of hyperbolic equations for a chain RFC current tube is based on its running parameters and determines the conditions for stability of oil displacement depending on linearizability of the processes.
3. Stability of the solutions meet the requirements of the stability of systems according to A.M. Lyapunov. The solution of the problem of conditions of RFC self-excitation helps determine effectiveness of the hydropulse technology influenced on the reservoir. It has been established that the use of discontinuous controls is characteristic for the low-frequency spectrum of action, and acoustic hydro-pulse actions are typical of the high-frequency spectrum.
4. The method for the numerical RFC simulation with dimensionless linear parameters can be used in various areas of thermal and gas dynamics, petrochemistry, and experimental physics.

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