3DoF Parallel robot analysis

D V Artemov, V M Masyuk, S Y Orekhov and I V Pchelkina
Kaluga Branch of Bauman Moscow State Technical University, Bazhenova st. 4,
Kaluga, 248004, Russia
colt50rus@mail.ru

Abstract. This paper addresses a 3DoF parallel robot. A kinematic scheme and joint types are proposed. Kinematic analysis of the mechanism is done. Aspects of robot control are considered: a power supply, a controller.

1. Introduction
Parallel robots are known by stiffness of their mechanical structure, high position accuracy and high load carrying capacity. At the same time they have some drawbacks such as limited working space and difficult forward position kinematics because of the closed structure. In contrast, the inverse position kinematics is quite simple. Parallel mechanisms have been studying since the 60s of the last century but in this field there is a large area that needs to be enhanced, namely, dynamic analysis. When designing a parallel mechanism, a designer has many ways for assembly organization. Most common structures have been described in detail in [1], [2]. The main requirements that the equipment must meet are the possibility of easy installation and controlling and using of universal components for assembly. For practical reasons, in this work a parallel robot was developed with the ability to work in three and six degrees of freedom (DoF) depending on the modification of the structure. The 3D CAD models in SolidWorks are shown in figure 1. Below we consider a mechanism with 3DoF.

Figure 1. Robot modelling in SolidWorks: tripod (a) and hexapod (b).
Although 3DoF robots are considered well known [3], the proposed mechanism is comprehensible and suitable for educational and research purposes because it allows to illustrate the robot modelling and control tasks.

2. Assumptions

In this research, the objective is a 3DoF robot based on three legs. It consists of a fixed base and a moving platform connected by three serial chains. We consider two cases:

1) each leg has a revolute-revolute-translational-revolute-revolute (RRTRR) joint structure (figure 2, a);
2) each leg has a revolute-translational-spherical (RTS) joint structure (figure 2, b).

The revolute joints are passive, only the three translational joints are actuated. According to the kinematic schemes, in the first case the mechanism has 15 kinematic pairs of the fifth class (12 rotational and 3 translational). In the second case the robot has 6 kinematic pairs of the fifth class (3 rotational and 3 translational) and 3 kinematic pairs of the third class (3 rotational). The DoF for a closed-loop mechanism is examined by using the Somov-Malyshev formula:

\[ W_{np} = n p_5 - 5 p_3 = 12 - 3 \cdot 3 = 9, \]

where \( n \) is a number of all links, \( p_5 \) and \( p_3 \) denote the class of a kinematic pairs, \( W_1 \) is DoF for the RRTRR case, \( W_2 \) is DoF for RTS case. Interestingly, the first proposed kinematic scheme is very similar to the universal-prismatic-universal (UPU) structure [4].

If we fix the all three translational kinematic joints in RRTRR and RTS joint structures, the derived mechanisms will have the following DoF:

\[ W_{np} = n p_5 - 5 p_3 - 3 \cdot 3 = 12 - 3 \cdot 3 = 3, \]

Two prototypes are built (see figure 4), and it turned out that the RRTRR prototype collapses while RTS one behaves properly. It is clear that a RRTRR mechanism has unaccounted DoF. Therefore, further a RTS mechanism is considered.

![Figure 2. Kinematic schemes of the robot with RRTRR (a) and RTS (b) joint structure: specification of kinematic pairs (Greek numerals), links (Roman numerals) and axes. OXYZ – fixed coordinate system (CS), O_1X,Y,Z_1 – moving CS.](image)
For the RTS robot the position of the $i$-th support chain is given by the values of $L_i$ and angles of the upper and lower hinges $\alpha_i^{up}=\alpha_i^{low}=\alpha_i$, $\beta_i^{up}=\beta_i^{low}=0$ (figure 3). The angles $\alpha_i, \beta_i$ determine the orientation of the $i$-th support chain relative to the plane of the base, $L_i$ – the length of the support chain.

3. Kinematics analysis

Let’s introduce a fixed coordinate system (CS) $O_{xyz}$ and a moving CS $O_{1x_{1}y_{1}z_{1}}$. The latter is rigidly connected to the moving platform (figure 3). Three rods of variable length $A_iB_i$ are modelling the translational kinematic pairs. They are attached to the base platform at points $A_i$ and to the mobile platform at points $B_i$. Points $A_i, B_i$ lie on circles of radii $R_A$ and $R_B$, respectively. Wherein points $A_i, B_i$ lie in planes parallel to the platforms planes, but separated from them by some distance $d_s$, so the size of the hinge mountings has been taking into account. Position of $A_i, B_i$ are fixed in the corresponding CS:

$$A^0 = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} \frac{R_A}{2} \sqrt{3} - \frac{R_A}{2} \\ 0 \\ \frac{R_A}{2} \end{bmatrix} d_s \quad ; \quad B^1 = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} = \begin{bmatrix} \frac{R_B}{2} \sqrt{3} - \frac{R_B}{2} \\ 0 \\ \frac{R_B}{2} \end{bmatrix} d_s ,$$

(3)

where the upper index shows the coordinate system relative to which the vectors are considered: 0 for the base CS $O_{xyz}$ and 1 for CS of the moving platform $O_{1x_{1}y_{1}z_{1}}$. In the nominal position, the robot legs are symmetrical with a symmetry angle of $\frac{2\pi}{3}$, and the planes in which the points $A_i, B_i$ are located are parallel, the distance between them is $z_{nom}$.

![Figure 3. Kinematic scheme of the robot with RTS joint structure.](image-url)
\[ \xi = \begin{bmatrix} x, y, z, \phi, \theta, \psi \end{bmatrix}^T, \]

where \( \{x, y, z\} \) is the position of the point \( O_1 \) (the origin of CS \( O_1X_1Y_1Z_1 \)); \( \{\phi, \theta, \psi\} \) are the yaw, roll and pitch angles defining the orientation of CS \( O_1X_1Y_1Z_1 \) relative to CS \( OXYZ \). The inputs are the rod lengths \( A_iB_i \):

\[ L = [L_1; L_2; L_3], \quad L_i = A_iB_i, \quad i = 1, 2, 3. \]

Usually the inverse kinematics of parallel mechanism can be accomplished by the transformation matrix \[5\]. Introduce the transformation matrix \( T_0^1 \):

\[ T_0^1 = \begin{bmatrix} R_{\phi\theta\psi} & p \\ 0 & 0 & 0 & 1 \end{bmatrix}, \]

where \( R_{\phi\theta\psi} \) is the \( 3 \times 3 \) submatrix describing rotation by angles \( \{\phi, \theta, \psi\} \), and \( p = [x_{O_1}; y_{O_1}; z_{O_1}] \) is the \( 3 \times 1 \) submatrix describing translation.

As far as we know the position of the points \( B_i \) in CS \( O_1X_1Y_1Z_1 \), and \( A_i \) in CS \( OXYZ \), we can find the position of the \( i \)-th support leg:

\[ L_i = \overline{A_iB_i}, \quad \alpha_i = \cos^{-1} \left( \frac{A_iB_i^{2}_{XY} + A_iB_i^{2}_{i} - B_iB_i^{2}_{i,XY}}{2 \cdot A_iB_i^{2}_{i,XY} \cdot A_iB_i} \right), \quad \beta_i = \cos^{-1} \left( \frac{A_iB_i^{2}_{XY} + A_iO^{2}_1 - O'B_i^{2}_{i,XY}}{2 \cdot A_iB_i^{2}_{i,XY} \cdot A_iO_1'} \right), \]

where \( B_i^{XY} \) is the projection of \( B_i \) on the plane \( O'A_iA_iA_i \) that is parallel to the base plane with distance \( d_i \) between them; position of the point \( O' \) in CS \( O_1X_1Y_1Z_1 \) is \( O' = [0; 0; d_i] \); \( A_iB_i, B_iB_i^{XY}, A_iO_1', O'B_i^{XY} \) and \( A_iB_i^{XY} \) are the lengths of corresponding vectors.

3.2. Forward Kinematics

The direct kinematic problem is solved by establishing the kinematics constraint equations for the mechanism. Thus, the output vector \( \xi = [x, y, z, \phi, \theta, \psi]^T \) is obtained from equations

\[ \overline{A_iB_i}^{2} = L_i^{2}, \quad \overline{B_iO_1'}^{2} = R_{\beta}^{2}, \quad i = 1, 2, 3. \]

where \( L = [L_1; L_2; L_3] \) is the input vector; the point \( O_1' \) lies in the plane \( B_iB_iB_i \) with its position \( O_1' = [0; 0; -d_i] \) in CS \( O_1X_1Y_1Z_1 \).

3.3. Modelling

Mathematical modelling of (3)-(8) was developed by using the MATLAB software and Optimization Toolbox package. The geometric parameters of the parallel robot are given as \( R_A = 0.7, \quad R_B = 0.990, \quad d_A = 0.15, \quad z_{nom} = 0.610 \). Results are shown in figure 4.
4. Prototype development

4.1. Robot

The prototype of 3DoF parallel manipulator has been developed and assembled at the department “Mechatronic and Robotics” at Kaluga Branch of BMSTU (see figure 5). As a linear drive joint electric drives SuperJack HARL3612+ were used. These linear actuators have high load characteristics. Factory mountings of the actuators are presented by two spherical hinges that do not correspond to the proposed kinematic scheme. Therefore two sets of metal bushings for each actuator were designed and manufactured. The bushings are installed on spherical hinges by special fasteners and block two unwanted degrees of rotation. The fasteners are installed in specially designed brackets which act as universal joints. Thus, the final mechanism meets the proposed kinematic scheme.

As the framework of the robot universal 25 mm metal pipes and Joker/Uno fastening systems were used which include various connectors, clamps and flanges; this ensures uncomplicated and universal installation of the robot.

![Figure 4. Simulation results of robot kinematics in Matlab.](image)

![Figure 5. Prototypes of the robot with RRTRR (a) and RTS (b) joint structure.](image)
4.2. Control System
In this study, the task was to develop and create an H-bridge debug board based on MOSFET transistors with an allowable power of 110 W for the implementation of a DC motor control stand. As part of the project, we considered existing methods for constructing mechatronic control systems for the H-bridge [6], and analysed the advantages and disadvantages of various approaches. Also a patent search has been conducted, and relevant literature has been studied.

![Figure 6. Model diagram of the control system (a) and its implementation (b).](image)

At the current stage of the work, the H-bridge control circuit was designed using the Bootstrap circuit as power supply for the upper parts of the bridge. The system is designed on two IR2110 controllers and high-power field-effect transistors IRF 3205, a model diagram of the system is shown in figure 6.

5. Conclusions
In this paper the design of 3DoF parallel robot based on three kinematic chains is proposed. It is supposed to have the ability of changing the robot to 6DoF kinematic scheme by mounting three additional kinematic chains.

For a mechanism with 3DoF: kinematic analysis of two schemes (RRTRR and RTS) was done; an experimental setup was developed; control system for the linear actuator was designed. It was found that a RRTRR robot collapses while a RTS robot behaves properly. The next step of the study is to implement dynamic analysis of 3DoF model with RTS joint structure and refine the control system. In the future, we plan to develop a 6-UPS parallel robot.

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