Quantum-gravity-motivated Lorentz-symmetry tests with laser interferometers

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Abstract

We consider the implications for laser interferometry of the quantum-gravity-motivated modifications in the laws of particle propagation, which are presently being considered in attempts to explain puzzling observations of ultra-high-energy cosmic rays. We show that there are interferometric setups in which the Planck-scale effect on propagation leads to a characteristic signature. A naive estimate is encouraging with respect to the possibility of achieving Planck-scale sensitivity, but we also point out some severe technological challenges which would have to be overcome in order to achieve this sensitivity.

1 Introduction

Lorentz symmetry plays a key role in our present description of the fundamental laws of physics, and, as a result, there is a tradition [1, 2] of interest in testing this symmetry to the highest possible precision. During the last few years the sensitivity of laboratory tests of the principles underlying Lorentz symmetry has improved significantly [3, 4, 5, 6] and this has energized efforts aimed at improving these tests even further [7, 8]. In addition, there has also been strong interest [9, 10, 11, 12, 13] in precision tests of Lorentz symmetry that are based on certain types of observations in astrophysics. However, just at a time when, using these refined techniques, Lorentz symmetry is being verified experimentally at a much improved level of accuracy, there is growing interest in theoretical models with only an approximate Lorentz symmetry. Part of this research is based on the realization that certain types of phenomenologically viable modifications of present particle-physics models can be based on renormalizable field theories in which indeed there is only an approximate Lorentz symmetry [14, 15]. Perhaps even more significant is the fact that in the quantum-gravity literature models based on an approximate Lorentz symmetry have been recently
considered in most research lines, including models based on “spacetime foam” pictures [10, 16], “loop quantum gravity” models [17, 18], certain “string theory” scenarios [19], and “noncommutative geometry” [20, 21].

While the experimental accuracies at which Lorentz symmetry has been so far verified are remarkable in absolute terms, and constrain very significantly general parametrizations of possible descriptions of Lorentz symmetry as an approximate symmetry [14, 15], the studies motivated by quantum gravity predict departures from Lorentz symmetry that are naturally governed by the minute quantum-gravity length scale $L_{\text{QG}}$, which is usually identified, up to a few orders of magnitude, with the Planck length $L_P \sim 10^{-33}$ cm (the inverse of the huge Planck energy scale $E_P \sim 1/L_P \sim 10^{28}$ eV) and this leads to effects that are too small for testing with present Lorentz-symmetry laboratory tests.

Interest in tests of Planck-scale modifications of Lorentz symmetry has also increased recently with the realization [22, 23] that these modifications of Lorentz symmetry provide one of the possible solutions of the so-called “cosmic-ray paradox”. The spectrum of observed cosmic rays was expected to be affected by a cutoff at the scale $E_{\text{GZK}} \sim 5 \cdot 10^{19}$ eV. Cosmic rays emitted with energy higher than $E_{\text{GZK}}$ should interact with photons in the cosmic microwave background and loose energy by pion emission, so that their energy should have been reduced to the $E_{\text{GZK}}$ level by the time they reach our Earth observatories. However, the AGASA observatory has reported several observations of cosmic rays with energies exceeding the $E_{\text{GZK}}$ limit by nearly an order of magnitude [34]. This experimental puzzle will only be established when confirmed by other observatories, and solutions which do not rely on Planck-scale physics have been discussed in the literature, but it is noteworthy that the type of Planck-scale modification of Lorentz symmetry described in Ref. [10] can produce [22, 23] an increase in the threshold energy for pion production in collisions between cosmic rays and microwave photons, and the increase is sufficient to explain away the puzzle raised by the mentioned ultra-high-energy cosmic-ray observations.

In discussions of this possible relevance of Planck-scale (quantum-gravity) modifications of Lorentz symmetry for the cosmic-ray paradox it is commonly assumed that those same modifications of Lorentz symmetry could not be tested in controlled experiments\footnote{There is however interest in testing them through their implications for other classes of astrophysics observations [9, 10, 12, 11, 13].}. Indeed this is the case for all presently explored techniques for laboratory tests of Lorentz symmetry, in which the relevant Planck-scale effects would fall below sensitivity. In this paper we consider the possibility of Planck-scale Lorentz-symmetry tests in laser interferometry. We give a schematic description of possible setups for these interferometers which could be used to search for the relevant effects.

We take as our starting reference point LIGO/VIRGO-type [24, 25] and LISA-type [26] interferometers, but we also point out that several technological improvements should be achieved in order to reach the required sensitivity levels in the setups we consider. Since (as we shall show) the type of signal that represents a signature of the Planck-scale effects here considered can be “on” for a time of choice of the experimenter (it is not a short-duration signal), we will, at least in first instance, focus on the level of sensitivity that advanced interferometers can achieve in the study of a stably-periodic signal observed over a reference time of one year. The ultimate sensitivity goal of the LIGO and VIRGO interferometers is such that short-duration (bursting) effects inducing strain\footnote{The strain is here defined, as conventional, according to $h \equiv \Delta L/L$, where $L$ is the reference length of the interferometer arms and $\Delta L$ is the change of the length of the arms due to the effect under study.}, $h$, at the level $h \sim 10^{-22}$ could be detected. If the effect under study effectively induces steady pe-
periodic variations of the optical length of the arms of the interferometer with period \((100 \text{ Hz})^{-1}\) (the strain sensitivity of LIGO/VIRGO interferometers is at its maximum around 100 Hz), and if these periodic variations can be observed for a full year, one then obtains a year-integrated sensitivity of order \(h \sim 10^{-27}\). [For smaller interferometers, such as GEO600, this year-integrated sensitivity would be [27] at the level \(h \sim 10^{-26}\).]

In the next Section we present a naive “back-of-the-envelope” analysis comparing the sensitivity of advanced interferometers to the magnitude of the relevant Planck-scale departures from Lorentz symmetry. Although this estimate is admittedly simplistic, it is noteworthy that, in spite of the smallness of the Planck length, the comparison is encouraging for the goal of Planck-scale sensitivity. In Section 3 we discuss some interferometric setups in which the Planck-scale effects lead to a characteristic signature. Since the relevant quantum-gravity-motivated models predict an energy-dependent modification of the laws of light propagation, we find appropriate to analyze our proposal also from the perspective of a search for a photon mass [28, 29], which would lead to the same qualitative effect\(^3\) (but, as emphasized in the following, significant quantitative differences). While in Sections 3 our discussion relies on an idealization in which certain experimental limitations are ignored, these experimental limitations are considered in Section 4, and represent an (ambitious) agenda for those who might be interested in pursuing the proposal we are here putting forward. Closing remarks are offered in Section 5.

2 Planck-scale modifications of the laws of light propagation and advanced interferometers

The mentioned debate on the possibility that the puzzling observations of UHE cosmic rays might be due to a Planck-scale deformed dispersion relation is focusing on the first phenomenological proposal of a quantum-gravity-motivated dispersion relation put forward in Ref. [10]. There it was argued that such deformed dispersion relations might characterize quite a few approaches to the quantum-gravity problem. In proposing a phenomenological program looking for these new effects it appeared natural [10, 30] to start these investigations with the initial simple ansatz\(^4\)

\[
m^2 = E^2 - \vec{p}^2 + f(E, \vec{p}^2; E_P) \simeq E^2 - \vec{p}^2 + \eta \left( \frac{E^3}{E_P} \right),
\]

where \(\eta\) is a dimensionless coefficient, which is usually assumed [10] to be \(10^{-3} \leq \eta \leq 1\), reflecting the intuition that the quantum-gravity scale should be somewhere between the grand unification scale (\(\sim 10^{25}\text{eV}\)) and the Planck scale \(E_P\). The approximation \(f(E, \vec{p}^2; E_P) \simeq \eta E^3/E_P\) is to be valid for \(m \ll E \ll E_P\) and assumes that the leading-order difference between the exact dispersion relation and the ordinary classical-spacetime dispersion relation comes in with \(E/E_P\) overall suppression factor.

Dispersion relations that are closely related to (1) as well as some alternative Planck-scale-deformed dispersion relations are being discussed in the context of the “loop quantum gravity” theory, in studies [17, 18] which are motivated by the proposals put forward in Refs. [10, 22, 23].

\(^3\)We shall not comment instead on models based on other types of modifications of the Maxwell equations [14], in which the departures from Lorentz symmetry are not energy dependent.

\(^4\)We adopt conventions with \(\hbar = c = 1\).
More recently quantum-gravity-deformed dispersion relations were also discussed in the string-theory literature (see, e.g., Ref. [19]). We will however focus on the ansatz (1), since it is the one being considered for solutions of the cosmic-ray puzzle, and anyway experimental strategies which are found to be well suited for testing (1) should then be easily adaptable for testing other possible quantum-gravity-deformed dispersion relations.

In our analysis of light beams in interferometers we will study the relation (1) through its implications for the relation between frequency, $\omega$, and wave vector, $k$. We adopt the notation $\omega_{\text{QG}}$ for the frequency scale that corresponds to the energy scale $E_P/\eta$, and therefore $10^{41}$ Hz $\leq |\omega_{\text{QG}}| \leq 10^{44}$ Hz. Since one could in principle consider both positive and negative $\eta$, we shall remain open to both possibilities: our (dimensionful) parameter $\omega_{\text{QG}}$ can be positive or negative. With this notation, from (1) one obtains

$$k \simeq \omega + \frac{1}{2} \frac{\omega^2}{\omega_{\text{QG}}}$$

where $k \equiv \sqrt{k^2}$. Accordingly the group velocity for a wave packet of photons takes the form

$$v_g = d\omega/dk \simeq 1 + \omega/\omega_{\text{QG}}$$

and the phase velocity $v_p = \omega/k \simeq 1 - \frac{1}{2} \omega/\omega_{\text{QG}}$.

The physical mechanism that is basically exploited in the interferometric tests here proposed originates from the fact that, with the Planck-scale deformation of the dispersion relation, in an interferometric setup photons of different energies would be affected differently by the deformation: the Planck-scale deformation introduces a (minute but nonnegligible) dependence of the phase velocity on the energy/frequency of the photon. It has already been suggested [30, 32] that the remarkable sensitivities of these advanced interferometers might render them useful for studies of Planck-scale effects, but the implications of the Planck scale for Lorentz symmetry were not previously considered in this respect. Postponing to the next Sections a more detailed analysis, in this Section we just want to make a crude estimate of the contribution to the phase of a light wave that would come from this Planck-scale effect, and we want to compare it with the phase-sensitivity levels that are expected to be reached with LIGO/VIRGO-type and LISA-type interferometers.

Since the velocity is modified at the level $\omega/\omega_{\text{QG}}$ one can easily estimate (and this is confirmed by our analysis in the next Sections) that the corresponding Planck-scale contribution to the phase would be at the same level $\phi_{\text{QG}} = \phi_{\text{noQG}}(1 + \omega/\omega_{\text{QG}})$, and this would effectively correspond to a change in our estimate of the optical length $L$ of the interferometer which is again at the same $\omega/\omega_{\text{QG}}$ level: $L_{\text{QG}} = L(1 + \omega/\omega_{\text{QG}})$. Since for visible light $10^{-28} \leq |\omega/\omega_{\text{QG}}| \leq 10^{-25}$ one estimates $10^{-28} \leq |L_{\text{QG}} - L|/L \leq 10^{-25}$.

We want to compare this crude characterization of the magnitude of the Planck-scale effects with the expected sensitivity of LIGO/VIRGO- and LISA-type interferometers. The most publicized sensitivity characterization for these interferometers is the one for bursting/short-duration gravity-wave signals. For an incoming gravity wave of 100 Hz frequency (ideal for the LIGO/VIRGO setup, with optical length $\sim 1000$ km) the amplitude/strain of a short-duration gravity wave must be at least at the level $h \sim 10^{-22}$ in order for it to be revealed by the advanced phase [31] of LIGO/VIRGO interferometers. As mentioned in the Introduction, if instead of a bursting signal the interferometer is affected by a steady gravity wave of period 100 Hz, which remains steady for a full year, one should consider the year-integrated strain sensitivity, which would be at the level $h \sim 10^{-27}$. Since $h$ can be seen as $|L_{\text{GW}} - L|/L$, where $L$ is the optical length of the arms of

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5We are here loosely referring to the speed of the wave, since an effect of order $\omega/\omega_{\text{QG}}$ affects both the phase velocity and the group velocity in the framework we are here considering. In the more detailed analysis reported in the Section 3 we express everything directly in terms of $k$ and $\omega$, rather than velocities.
the interferometer when not affected by a gravity wave and \( L_{GW} \) is the maximum (or minimum) optical length of the arms of the interferometer in presence of the gravity wave, this sensitivity at the \( h \sim 10^{-27} \) level can be encouraging for the search of quantum-gravity effects at the level \( 10^{-28} \leq |L_{QG} - L|/L \leq 10^{-25} \).

For LISA similar or better sensitivity to the effects we are here considering can be expected. In searches of high-frequency (e.g. 100 Hz) gravity waves, LISA’s remarkable phase sensitivity does not translate into an equally remarkable strain sensitivity because the key advantage of a LISA-type interferometer, its huge arms length \( (L \sim 10^{10} \text{ m}) \), is not fully exploited in presence of high-frequency gravity waves. In fact, for high-frequency gravity waves the linearity between arms length and observed phase difference is lost. Instead in our searches of the effects induced by a Planck-scale deformation of the dispersion relation the relevant phase differences are always proportional to the arms length, and LISA’s remarkable phase sensitivity can be fully exploited.

Of course, the estimates we are providing in this section are crude and idealized. It is somewhat encouraging that at least at this level the suppression induced by the smallness of the Planck length does not appear to be unsurmountable, since, even at the same crude level of analysis, most other experimental setups would immediately prove to be inadequate for Planck-scale studies). In the next Section we discuss some interferometric setups that could be used to find evidence of the Planck-scale effects we are considering. This will provide the basis for our more realistic discussion, in Section 4, of the challenges that must be faced in order to render meaningful the encouraging naive estimate we obtained here.

As a corollary, in our analysis we also consider possible applications of our strategy in searches of a photon mass. Since a photon mass would affect the dispersion relation for light waves, the effect is qualitatively analogous to the one we are considering from the Planck-scale perspective. However, at the quantitative level the two effects are clearly distinguishable. In presence of a nonvanishing photon mass the standard \( k = \omega \) dispersion relation takes the form

\[
k \approx \sqrt{\omega^2 - m^2} \approx \omega - \frac{m^2}{2\omega}.
\]

When the Planck-scale effect is present (and there is no photon mass) the deformation of the dispersion relation becomes more and more significant as the frequency of the wave increases. The opposite is true for the case of a photon mass (and no Planck-scale effects): the photon-mass deformation of the dispersion relation becomes more and more significant as the frequency of the wave decreases.

3 Proposal of interferometric setups for a Planck-scale-induced phase difference

In the preceding Section we just compared the “phase sensitivity” of advanced interferometers to the magnitude of the contribution to the phase of a light wave due to the Planck-scale effects we are investigating. In light of the tempting result of this comparison we are encouraged to look for ways to test Planck-scale-deformed dispersion relations using laser-light interferometers.

\[\text{And correspondingly group and phase velocity take the form } v_g \approx 1 - m^2/(2\omega^2) \text{ and } v_p = 1 + m^2/(2\omega^2) \text{ respectively.}\]
There are probably a large number of interferometric setups that could be considered for our quantum-gravity tests. To render our proposal more specific we will discuss in some detail two such interferometric setups. A common feature of our interferometric setups is that they involve two beams at different energies/frequencies. As we shall show, the energy-dependence characteristic of Planck-scale deformed dispersion relations is such that the presence of beams at different energies in the interferometer naturally gives rise to Planck-scale-dependent phase differences. We specifically consider two frequencies, a reference frequency $\omega$ and the doubled frequency $2\omega$, but interferometric setups that exploit other types of pairs of frequencies should be possible. We choose to consider frequency doubling because of the relatively wide availability of second harmonic generation (SHG) or “frequency doublers” (see e.g. [33]).

The discussion in this Section is “idealized”, largely ignoring various potential challenges that would be encountered in actual realizations of our interferometric setups. As announced, we will comment on some of these challenges in Section 4.

### 3.1 Phase difference through splitting in energy and configuration space

Let us consider an interferometer with LIGO/VIRGO or LISA-type setup (and dimensions) with two orthogonal arms, respectively of length $L$ and $L'$ (see Fig. 1). We keep $L$ and $L'$ distinct because our signal will turn out to be proportional to $|L - L'|$. This will come at some cost in the sensitivity of the interferometer, but probably not more\(^7\) than a factor 10.

The scheme of this interferometric setup is shown in Fig. 1. Before entering the interferometer, a monochromatic wave with frequency $\omega$ goes through a frequency doubler. Both emerging beams, of frequencies $\omega$ and $2\omega$, are then split into a part that goes through the arm of length $L$ and a part that goes through the arm of length $L'$. When the beams are finally back (after reflection by a mirror) at the point where the interference patterns are formed, one then has access to two interference patterns: one interference pattern combines two waves of frequency $\omega$ and the other interference pattern is formed combining analogously two waves of frequencies $2\omega$. In an idealized setup (ignoring for example a possible wavelength dependence in beam-mirror interactions) the observed intensities\(^8\) would be governed by

\[
I_{\omega} \propto \frac{1}{2} (1 + \cos \phi_{\omega}), \quad \phi_{\omega} = k(L' - L), \\
I_{2\omega} \propto \frac{1}{2} (1 + \cos \phi_{2\omega}), \quad \phi_{2\omega} = k'(L' - L),
\]

where $k'$ is the wavelength associated with the doubled frequency $2\omega$. In Minkowski spacetime, with its standard dispersion relation, one has that $k' = 2k$, but in the case of the Planck-scale-deformed dispersion relation

\[
k' \simeq 2k + \frac{k^2}{\omega_{\text{QG}}},
\]

\(^7\)See, e.g., Ref. [35] for a comparison between an unequal-arm interferometer and a comparable equal-arm interferometer. Indeed, the analysis reported in Ref. [35] leads to the conclusion that the unequal-arm interferometer has sensitivity reduced by about a factor 10 with respect to the equal-arm interferometer.

\(^8\)In practice, it may be convenient to use frequency filters in such a way that at any given instant only one of the two intensities is observed. It might also be possible to insert another beam splitter into the beam going to the apparatus measuring the intensity. Both resulting beams may be frequency analyzed, one for the frequency $\omega$, the other for $2\omega$. By measuring the intensity of both of these beams both intensities may be measured simultaneously.
Figure 1: The unequal–arm interferometer which is capable to search for dispersion effects. The paths for the original ($\omega$) and the frequency doubled ($\omega'$) beams photons are identical in configuration space, but we draw separate lines for conceptual clarity.

We can then rewrite $\phi_{2\omega} - \phi_\omega$, from (4) and (5), using again the Planck-scale-deformed dispersion relation

$$\phi_{2\omega} - \phi_\omega = k'(L' - L) - k(L' - L) = \omega \left(1 + \frac{3}{2} \frac{\omega}{\omega_{\text{QG}}} \right) (L' - L).$$

(7)

A proper description of the meaning of this phase-difference relation requires a few considerations. This phase-difference relation characterizes a key difference between the standard dispersion relation and the Planck-scale-deformed dispersion relation in the interferometric setup we are considering. In the case of the standard classical-spacetime dispersion relation one expects, as illustrated in Fig. 2a, a specific type of correlations, which follow straightforwardly from $k' = 2k$, between the values of $I_\omega$ and $I_{2\omega}$ for given values of $L' - L$. For example, clearly one expects that
the intensity $I_{2\omega}$ of the wave at frequency $2\omega$ has a maximum whenever $L' - L = 2n\pi/\omega$ (with $n$ any integer number), and that correspondingly the intensity $I_{\omega}$ of the wave at frequency $\omega$ has either a maximum or a minimum. One therefore predicts, without any need to establish the value of $n$, that the configurations in which there is a maximum or a minimum of $I_{2\omega}$ must also be configurations in which there is a maximum or a minimum of $I_{\omega}$, see Fig. 2a. The Planck-scale-deformed dispersion relation modifies this prediction: for example, as codified by Eq. (7), the dispersion relation (2) predicts that configurations in which there is a maximum of $I_{\omega}$ is in the neighborhood but not exactly at one of its maximum/minimum values (see Fig. 2b). More precisely, when $L' - L$ is such that $I_{2\omega}(L' - L)$ is at a maximum value the quantum-gravity effect predicts that $I_{\omega}(L' - L)$ should differ from a maximum/minimum value of $I_{\omega}$ as if for being out-of-phase by an amount

$$\phi_{2\omega} - \phi_{\omega} = \frac{3}{2} \frac{\omega^2}{\omega_{QG}} (L' - L).$$

(8)

The comparison of maxima/minima of the functions $I_{2\omega}(L' - L)$ and $I_{\omega}(L' - L)$ is not necessarily the best feature of the general comparison between $I_{2\omega}(L' - L)$ and $I_{\omega}(L' - L)$ from the point of view of experimental searches, because close to a stationary point of course functions vary very slowly (only quadratic term is effective). But the standard classical-spacetime picture established a connection between the $I_{2\omega}(L' - L)$ and $I_{\omega}(L' - L)$ graphs even away from the stationary points. Moreover one could easily revise this setup in such a way to have $\omega$ and $3\omega$ beams (rather than $\omega$ and $2\omega$ beams) in which case standard classical-spacetime picture established that some points of maximum positive slope of $I_{3\omega}(L' - L)$ should correspond to points of maximum (minimum) slope of $I_{\omega}(L' - L)$.

This type of characteristic feature would be easily looked for by, for example, taking data at values of $L' - L$ that differ from one another by small (smaller than $1/\omega$) amounts in the neighborhood of a value of $L' - L$ that corresponds, say, to a maximum of $I_{2\omega}$. Perhaps, techniques for the active control of mirrors which are already being used in modern interferometers might be adapted for this task, and the development of dedicated techniques does not appear beyond our reach.

In closing this Subsection let us return to Eqs. (4) and (5) and remove one of the key idealizations in those formulas. As mentioned, it appears likely that in addition to the contributions $k(L' - L)$ and $k'(L' - L)$, respectively to $\phi_{\omega}$ and $\phi_{2\omega}$, there would also be some different $(L' - L)$-independent contributions to the phases due for example to the different response of the mirrors (and the beam splitter) to the $\omega$ wave and the $2\omega$ wave. Such a phase difference would itself induce a misalignment between the maxima of $I_{2\omega}(L' - L)$ and the maxima and minima of $I_{\omega}(L' - L)$, which could of course have negative implications for the observability of the analogous quantum-gravity effect. One might try to determine these unwanted phase differences to high precision in the laboratory before carrying out the final experiment described above. In alternative one could consider the possibility of exploiting the fact that the misalignment due to the quantum-gravity effect is proportional to $(L' - L)$ while the other potential source of misalignment is $(L' - L)$-independent. One could for example repeat the entire procedure necessary to establish the amount of misalignment between the maxima of $I_{2\omega}(L' - L)$ and the maxima and minima of $I_{\omega}(L' - L)$ for two macroscopically different values of $(L' - L)$. If for the two macroscopically different values of $(L' - L)$ one found the same amount of misalignment the quantum-gravity effect would be excluded, whereas in the opposite case one would have evidence in favour of the quantum-gravity effect.

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9This point was emphasized to us by A. Rüdiger.
Figure 2: a) Qualitative description of the dependence on $L' - L$ of the intensities $I_\omega$ and $I_{2\omega}$, according to ordinary Lorentz symmetry. We show different maximum values for $I_\omega$ and $I_{2\omega}$ to reflect the fact that the intensities of the two beams that emerge from SHG in general are not identical. However, this is not of concern for us: independently of the values of the maximum intensities exact Lorentz symmetries introduces a correlation between the values of $I_\omega$ and $I_{2\omega}$ at given values of $L' - L$. For example, whenever $L' - L$ is such that there is a maximum of $I_{2\omega}$ there must also necessarily be a maximum or a minimum of $I_\omega$. b) Qualitative description of the dependence on $L' - L$ of the intensities $I_{2\omega}$ and $I_\omega$, according to the quantum-gravity-induced departure from Lorentz symmetry here considered. By comparing with a) the implications of the Planck-scale effects are visible. For example, one notices that the quantum-gravity effect induces a misalignment between the maxima of $I_{2\omega}$ and the maxima and minima of $I_\omega$. As shown in our analysis this misalignment is proportional to $(\omega^2/\omega_{QG})(L' - L)$.

### 3.2 Interferometry in energy space

As another possible realization of an interferometric setup that is sensitive to deformations of the dispersion relation we propose a setup in which the frequency (or energy) is the parameter characterizing the splitting of the photon state. The main experimental procedure for doing that is SHG. (Again one may consider a more general setup where the frequency may have other than doubled values.) If an incoming wave has a frequency $\omega$ then, after passing through the frequency doubler, the outgoing wave in general consists of two components, one possessing a frequency $\omega$ and the other a frequency $2\omega$. 
frequency

\[ u_0(x,t) \sim e^{-i\omega t} \]

\[ u_0(x,t) \sim e^{-i\omega t} \]

\[ u_2(x,t) \sim e^{-2i\omega t} \]

\[ u_2(x,t) \sim e^{-2i\omega t} \]

\[ u_2(x,t) \sim e^{-3i\omega t} \text{ (forbidden)} \]

**Figure 3:** The waves leaving two frequency doublers. The two waves leaving the second doubler with frequency \(2\omega\) form an interferometer since they move between the two doublers at different frequencies. Waves not contributing to our interferometer are displayed by dashed lines. Dotted waves are forbidden due to the matching condition. Note that the different paths in the figure only distinguish different frequencies; all beams follow the same path in configuration space.

We intend to consider an interferometric setup, described in Fig. 3, where two SHG are positioned in a row at a distance \(L\). After the second SHG the photonic state consists of two components with frequency \(\omega\) and two components with frequency \(2\omega\). The first contribution to the emerging \(2\omega\) beam is the transmission of the component which left the first SHG as a \(2\omega\) wave, while the other contribution to the emerging \(2\omega\) beam is the doubled component of that part which went through the first SHG without change in the frequency. Therefore, the final \(2\omega\) component represents an interferometer in energy space. Similarly there is contribution to the emerging \(\omega\) beam which is the transmission of the component which left the first SHG still as a \(\omega\) wave (part of the beam which is not affected by any of the two SHG), while the other contribution to the emerging \(\omega\) beam is the halved component of a part of the beam which left the first SHG as a \(2\omega\) wave. Therefore, also the final \(\omega\) component represents an interferometer in energy space. (All beams travel always along the same path in configuration space. There is no need for a beam splitting in configuration space. Beam splitting takes place with respect to energy.)

We use a general model in order to describe the process of frequency doubling, or more generally, addition and subtraction of frequencies. This is called second harmonic generation, SHG. Thereby, light enters a nonlinear material, that is, a material whose dielectric function depends on the frequency. Thus, inside the material, the polarization depends on the square of the incoming electric field. Consequently, if the incoming wave consists of two monochromatic parts

\[ u_0(x,t) = A_{01}e^{i\omega_1 t}e^{i(k(\omega_1)(x-x_0) - \omega_1 t)} + A_{02}e^{i\omega_2 t}e^{i(k(\omega_2)(x-x_0) - \omega_2 t)} \]

then in the nonlinear medium the polarization field consists of components possessing the frequencies \(\omega_{\text{shg}} = \{2\omega_1, 2\omega_2, \omega_- = \omega_1 - \omega_2, \omega_+ = \omega_1 + \omega_2\}\), and the corresponding wave vectors \(k_{\text{shg}} = \{2k(\omega_1), 2k(\omega_2), k(\omega_1) + k(\omega_2), k(\omega_1) - k(\omega_2)\}\), see e.g. [33]. However, due to possible destructive interference not all of these waves survive. (The reason for that is that the frequencies and the corresponding wave vectors of the polarization field do not fulfill a wave equation and thus possess a periodicity different from the propagating waves.) The destructive interference of these waves is prohibited only if a phase matching condition \(k(\omega_{\text{shg}}) - k_{\text{shg}} = 0\) for the corresponding
pair of frequencies and wave vectors, is fulfilled. This is a condition on the refraction index of the medium. If this condition is not met exactly, then this will only influence the intensities (rather than the frequencies) of the beams leaving the SHG only.

The combinations we are interested in are doubling, $\omega + \omega \rightarrow 2\omega$, and subtraction, $2\omega - \omega \rightarrow \omega$. It turns out, that for these two processes the phase matching conditions are identical. The condition for a further process $2\omega + \omega$ in general cannot be fulfilled at the same time. The frequency doubler only converts a part of the incoming beam to a beam with twice the original frequency. From this it is clear that a SHG acts as a beam splitter in frequency space and two SHGs in a row act as an interferometer in frequency space (Fig. 3).

Based on these considerations, we propose the following procedure: A monochromatic plane wave

$$u_0(x,t) = A_0 e^{i\alpha_0 e^{i(k(x-x_0)-\omega t)}} \quad (10)$$

with a (real) amplitude $A_0$ and an extra phase $\alpha_0$ enters the first SHG placed at position $x_1$. The outcome is a frequency-doubled wave and a part which just goes through the crystal:

$$u_1(x,t) = A_1 e^{i\alpha_1 e^{i(k(x-x_0)-\omega t)}} e^{i(k(x-x_1)-\omega t)} + B_1 e^{i\beta_1 e^{i(2k(x-x_0)+2k(k(x-x_1)-\omega t)}} \quad (11)$$

where $k'$ again is related to $2\omega$ through the modified dispersion relation and $A_1$, $B_1$, $\alpha_1$, and $\beta_1$ are real functions of the amplitude and the frequency of the incoming wave.

These two waves enter the second identical SHG placed at position $x_2 = x_1 + L$. With the phase matching condition, only waves with frequency $\omega$ and $2\omega$ come out. Each of these two frequencies consists of two parts, one from the wave which went through the material, and the other which is either frequency added (doubled) or frequency subtracted:

$$u_2(x,t) = A e^{i(k(x-x_2)-\omega t)} + B e^{i(k'(x-x_2)-2\omega t)} \quad (12)$$

with

$$A = A_{21} e^{i\alpha_{21} e^{i(\alpha_1+\alpha_0+k(x-x_0)+k(x-x_1))}} e^{i(k(x-x_1)}}$$

$$B = B_{21} e^{i\beta_{21} e^{i(\beta_1+2\alpha_0+k(x-x_0)-(k_1+k_2))}} e^{i(k'(x-x_1)-k(x-x_1))} \quad (13)$$

$$+ A_{22} e^{i\alpha_{22} e^{i(\beta_1+2\alpha_0+k(x-x_0))}} e^{i(k'(x-x_1)-k(x-x_1))}$$

$$+ B_{22} e^{i\beta_{22} e^{i(\beta_1+2\alpha_0+k(x-x_0))}} e^{i(k'(x-x_1)-2\omega t)} \quad (14)$$

Here all amplitudes $A_{2i}$ and $B_{2i}$ as well as all phases $\alpha_{2i}$ and $\beta_{2i}$ depend on the amplitudes and phases of the waves entering the second SHG only and not on the distance between the two SHGs.

By means of a frequency filter, we can select the doubled frequency component which is

$$u_2^{(2\omega)}(x,t) = \left( B_{21} e^{i\beta_{21} e^{i(\beta_1+2\alpha_0+2k(x-x_0))}} e^{i(k'(x-x_1)}}$$

$$B_{22} e^{i\beta_{22} e^{i(\beta_1+2\alpha_0+k(x-x_0))}} e^{i(k'(x-x_1)-2\omega t)} \quad (15)$$

The interference pattern shows up in the intensity $I^{(2\omega)} = |u_2^{(2\omega)}(x,t)|^2$ which is

$$I^{(2\omega)} = B_{21}^2 + B_{22}^2 + 2B_{21}B_{22} \cos \phi^{(2\omega)} \quad (16)$$
with
\[
\phi^{(2\omega)} = \beta_{21} + \beta_1 + 2\alpha_0 + 2k(x_1 - x_0) + k'(x_2 - x_1) - \beta_{22} - 2(\alpha_1 + \alpha_0 + k(x_1 - x_0)) - 2k(x_2 - x_1) \\
= \beta_{21} - \beta_{22} + \beta_1 - 2\alpha_1 + (k' - 2k)(x_2 - x_1),
\] (17)

where we set \( x = x_2 \). Since the generation of doubled frequencies is not very effective (may be up to 30%), \( B_{22} \) is of the order of \( B_{21} \), so that the visibility of the \( 2\omega \)-interference pattern may reach 1.

If we select the \( \omega \)-component, then we get
\[
I^{(\omega)} = A_{21}^2 + A_{22}^2 + 2A_{21}A_{22} \cos \phi^{(\omega)},
\] (18)

with
\[
\phi^{(\omega)} = \alpha_{21} + \alpha_1 - \alpha_{22} - \beta_1 + \beta_{11} - (k' - 2k)(x_2 - x_1).
\] (19)

Selection of the frequency \( \omega \) part is possible, and maybe useful to improve the quality of the analysis (e.g. as an opportunity to double-check the results obtained with the \( 2\omega \)-wave interference studies), but of course the contrast of the \( \omega \)-wave interference pattern is by far not as good as for the doubled part. (The \( \omega \)-wave which just goes through the two SHG is dominating with respect to the \( \omega \)-wave which is affected twice by the SHG: \( A_{21} \gg A_{22} \).

On the basis primarily of Eq. (17), for the interference of the doubled-frequency waves, we do have in this setup sensitivity to the Planck-scale deformation of the dispersion relation, which is encoded in the term \((k' - 2k)L\),
\[
k' - 2k = \frac{\omega^2}{\omega_{QG}}.
\] (20)

However, the term \((k' - 2k)L\) might be obscured by the additional phases in Eq. (17). This can be avoided exploiting the fact that the other phases in Eq. (17) do not depend on the distance \( L \) between the SHGs. It is therefore again useful to introduce some form of controlled variation of \( L \) in the interferometric setup.

### 3.3 Associated photon-mass analysis

The examples of interferometric setups which we discussed in the preceding two Subsections are in general sensitive to any type of modification of the \( \omega(k) \) dispersion relation. In particular, in addition to the study of possible Planck-scale-induced effects on which we focused, our interferometric setups are also sensitive to the modification of the \( \omega(k) \) dispersion relation that would be induced by a hypothetical photon mass \( m_\gamma \).

The analysis of the photon-mass scenario can be done in complete analogy with the one reported in the preceding two Subsections, with the only difference that in the case of a photon mass one finds
\[
k' - 2k = \frac{3}{4}m_\gamma^2/\omega.
\] (21)
rather than the Planck-scale effect \( k' - 2k = \omega^2/\omega_{QG} \) which we have been considering so far.

Clearly the two candidate effects, the ones due to Planck-scale physics and the ones due to a photon mass, can be easily distinguished (they would not represent significant backgrounds for one another). If anomalous results are found in interferometric studies of the type we are proposing it will be easy to establish whether the anomaly is due to the Planck-scale effects or to a photon...
mass. In fact, that Planck-scale modifications of the dispersion relation lead to an effect which is proportional to the frequency of the laser source, while the photon-mass-induced effect decreases as the frequency is increased.

If interferometers working with light frequencies of order, say, $10^{15}$ Hz, achieved, following the strategy here outlined, sensitivity to the quantum-gravity deformed dispersion relation, this would correspond to sensitivity to a photon mass of order $m_\gamma \sim 10^{-47}$ g, 3 orders of magnitude better than recent astrophysical analyses [12] and one order of magnitude worse\footnote{This is actually not surprising: experiments such as the ones discussed in Ref. [36] are not propagation experiments, but rather they infer limits on the photon mass on the basis of measurements of processes associated with the electromagnetic interactions. Gravitational interactions are much weaker than electromagnetic ones, and therefore a similar strategy would not be fruitful in searches of quantum-gravity effects (which might well be seen first in propagation studies and only at a much weaker level in contexts involving interactions).} than the one obtained in laboratory experiments based on a dynamic torsion balance [36].

4 Key challenges for achieving the needed experimental accuracy

Interferometric techniques have improved continuously and substantially from decade to decade from the last quarter of the 19th century to the present times. Sensitivities that appeared inimaginable at one point in this development were eventually reached and surpassed. An example that is presently at the center of the attention of the scientific community is the one of the interferometric searches of gravity waves: we expect to detect gravity waves within a few years, but this will be achieved through remarkable sensitivity improvements with respect to the sensitivities actually achievable when the first ideas on such studies appeared in the literature of the 1960s and 1970s. The proposal we are putting forward in this paper is to be intended in the same spirit as those early discussions of interferometric detection of gravity waves: interferometric studies of Planck-scale deformations of the dispersion relation may require a significant improvement of the interferometric techniques with respect to the ones that are presently available, but this improvement does not appear to be beyond reach, if indeed our mastery in interferometry keeps progressing at the pace of these past decades.

From this perspective we should stress here at least a few of the key challenges that must be faced in attempting to realize interferometric setups of the type we considered, with the level of sensitivity that is needed for the study of effects that are truly at the Planck-scale. A first key concern comes from the fact that the type of interferometric setups we considered in Section 3 requires that the interferometer functions with beams at two different frequencies, and the sensitivity estimate here reported in Section 2 shows that in order to achieve Planck-scale sensitivities the interferometer should work with these two frequencies at a level of accuracy that is comparable to the accuracy presently achieved in dealing with a single frequency. The modern interferometers that achieve these remarkable accuracies rely on optimization of all experimental devices to the frequency emitted by the laser. It should be studied how significant a loss of accuracy would result from working with both the laser frequency and a doubled frequency, and how to compensate for this loss of accuracy by introducing new techniques and new devices.

In particular, properties of some elements of our interferometric setups may be different for the two frequencies; for example, the reflection time for a mirror may depend on the frequency. One may attempt to study these properties in advance in laboratory at the needed level of accuracy, and then
develop some appropriate compensation techniques in the interferometric setups. One can also exploit the differences between the two possibilities of a terrestrial interferometer (LIGO/VIRGO-type) which relies on a large ratio optical-length/arm-length (large number of reflections) and of a gigantic space interferometer (LISA-type) in which one does not use any reflections (since the arm’s length is already huge to begin with). This difference might render LISA-type setups less sensitive to problems due to lack of performance of mirrors used at two different frequencies.

Another key challenge is posed by the fact that, at least in the two setups considered in the previous section, it appears to be necessary to implement controlled variations of a macroscopic distance $L$ (e.g. length of one of the arms of the interferometer). For the setup of Subsection 3.1, with splitting of beams both in configuration and in energy space, a key element of the analysis relies on variations of a macroscopic distance which are of the order of the wavelength of the laser beam, which might be easily implementable with appropriate use of active mirror control techniques already in use in interferometry. In the closing remarks of Subsection 3.1 we also pointed out that, unless (as just mentioned) one manages to test in advance to high accuracy some key elements of the interferometric setup, such as a possible dependence on wavelength of a mirror reflection time, the quantum-gravity analysis based on the setup of Subsection 3.1 may require repeating the experiment with a macroscopically different choice of the length of one of the arms, keeping all other aspects of the setup unchanged. A realization of the strategy described in Subsection 3.1 may therefore require both microscopic and macroscopic changes of the length of one of the arms of the interferometer, in different stages of the measurement procedure. The setup of Subsection 3.2, with splitting of beams only in energy space, relies from the onset on repeating the experiment at macroscopically different values of the length $L$ of the single arm of that interferometric setup.

In the setup of Subsection 3.2 it appears however natural to contemplate possible ways to replace the actual macroscopic variations of $L$ with some other strategy giving the sought effect. It is in fact conceivable that in the setup of Subsection 3.2 one might obtain a time-varying phase difference without actually varying $L$, but rather using one of these possibilities: (i) by introducing a time dependence in the SHG-like mechanism that generates the two different frequencies, (ii) by using three identical SHGs at three positions where alternatively one of the second and third ones is deactivated, (iii) by introducing a medium along the path followed by the light rays and implementing some form of electrical manipulation of the optical properties of the medium. The advantage of these possibilities (i),(ii),(iii) is that one might conceivably implement them in such a way that there is no actual macroscopic variation of the length $L$ (it would be changed only “effectively”) and that the change of configuration (e.g., from the configuration with the third SHG “on” to one in which the third SHG is “off”) might be implemented without actually intervening mechanically/manually on the interferometric setup, by using for example remote electrical controls affecting the properties of the third SHG of another medium.

5 Closing remarks

We have proposed that a strategy for the use of laser-light interferometers in tests of models of Planck-scale departures from conventional Lorentz symmetry, which are attracting interest in the literature also in relation with the emerging puzzle of observations of ultra-high-energy cosmic rays. A first-level simple-minded comparison between the magnitude of the Planck-scale effect and the sensitivity of modern interferometers was found (Sec. 2) to provide encouragement. The key element for the relevant interferometric studies is the capability to work simultaneously with two
beams of different wavelength. This is certainly doable but with present technologies would imply a (possibly severe) loss in overall sensitivity. Since technical developments will be required (and their specific nature might affect the structure of the interferometric setup which is eventually adopted), our discussion focused on illustrating two examples (Sec. 3) of interferometric setups that would allow to convert the relevant Planck-scale effects into signals for which interferometers are well suited. The fact that the two setups considered here differ in several significant aspects encourages us to think that, should technical obstructions be encountered in the development of one such setup, it should be possible to eventually find alternative interferometric setups in which the technical challenges can be handled.

As stressed in Sec. 4, an actual realization of our proposal may require a relatively long time. It appears however that present outlook of our proposal should be viewed at least from the perspective that was adopted when the first ideas on gravity-wave interferometric studies appeared in the literature of the 1960s and 1970s. The first preliminary analysis reported here appears to suggest that the objective is achievable, and we hope that this may provide encouragement for future studies.

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