Pressure Corrections to the Equation of State in the Nuclear Mean Field.

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We show the connection between stiffness of Equation of State (EoS) in a Relativistic Mean Field (RMF) of Nuclear Matter (NM) and the existence of a strong violation of longitudinal Momentum Sum Rule (MSR) in RMF for a finite pressure. The increasing pressure between nucleons starts to increase the ratio of a nucleon Fermi to average single particle energy and according to the Hugenholtz-van Hove (HvH) theorem valid for NM, the MSR is broken in the RMF approach. We propose changes which modify the nucleon Partonic Distribution Function (PDF) and make (EoS) softer to fulfill MSR sum rule above a saturation density. The course of EoS in our modified RMF model is very close to a semi-empirical estimation and to results obtained from extensive DBHF calculations with a Bonn A potential which produce EoS enough stiff to describe neutron star properties (mass-radius constraint), especially the mass of "PSR J16142230" the most massive known neutron star, which rules out many soft equations of state including exotic matter. Other features of the model without free parameters includes good values of saturation properties including spin-orbit term. An admixture of additional hyperons are discussed in our approach.

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I. INTRODUCTION

Experimentally, in the Deeply Inelastic electron Scattering (DIS) on nuclear targets, photons with large negative momentum square \(-q^2 = Q^2 > 1 GeV^2\) and large energy transfer \(\nu\), interacting with partons, probe bound hadrons - a kind of moving sub-targets. Start with the picture of a nucleus with mass \(M_A\) (\(A -\) a mass number). Björken scaling allows to describe nuclear dynamics by the Structure Function (SF) \(F^h_2(x_A)\) which depends on the Lorentz invariant Björken variable \(x_A \equiv Q^2/(2M_A\nu)\). Generally the PDF and SF depends also on the resolution \(Q^2\) which is particularly important for \(x_A < 0.01/A\) where a nuclear shadowing takes place. Shadowing should be included in any treatment of the EMC effect. However the shadowing is described as a multi-scattering process with diffraction between different nucleons. If the Momentum Sum Rule (MSR) has to be analyzed, the simple convolution of nucleon PDF with nuclear distribution preserves the Longitudinal Momentum (LM) of this partonic system. For \(x_A > 0.1/A\) we know that nuclear shadowing is unimportant.

In the Light Cone (LC) formulation, \(x_A\) corresponds to the nuclear fraction of a quark LM \(k^+ = k^0 + k^3\) and is equal (in the nuclear rest frame) to the ratio \(x_A = k^+ / P_A^+ \equiv \sqrt{k^+} / M_A\) - Lorentz invariant. But the composite nucleus is made of hadrons which are distributed with longitudinal momenta \(p_h^+\), where \(h = N, \pi, \ldots\) stands for nucleons, virtual pions, ... . In the convolution model a fraction of parton LM \(x_A\) in the nucleus is given as the product \(x_A = x_h \cdot y_h / A\) of fractions: a parton LM in hadrons \(x_h \equiv Q^2/(2M_h\nu) = k^+ / p_h^+\) and a hadron LM in the nucleus \(y_h = p_h^+ / P_A^+\). The nuclear dynamics of given hadrons in the nucleus is described by the distribution function \(f_h(y_h)\) and PDF \(F^h_2(x = x_h)\) describes its partonic structure. Remember that there are two different scales of interactions: long range nuclear scale which forms hadron distribution functions in nuclear matter and a much shorter partonic scale which is responsible for their PDF’s.

A. Kinematics in the Björken limit

Consider DIS on hadrons, \(eH \rightarrow e'X\) as an introduction. In the final state we measure the electron energy and scattering angle \(\Theta\) of outgoing electron. The virtual photon momentum transfer is:

\[q = (\nu, 0, 0, -\sqrt{\nu^2 + Q^2}).\]  

The differential cross-section

\[d\sigma \sim L^{\mu\nu}W_{\mu\nu},\]  

for electron scattering, in which hadrons in X states are not observed, is proportional to the contraction of a lepton tensor \(L^{\mu\nu}\) with the hadron tensor \(W_{\mu\nu}\) given by:

\[W_{\mu\nu} \equiv \sum_X (2\pi)^4 \delta^4(p + q - px) < p|J_\mu(0)\rangle\langle X|J_\nu(0)|p>\]

where \(J_\mu\) is a hadronic electromagnetic current operator. For low \(Q^2\) one could expect the corrections from strong interaction but for our nuclear purpose this approximation is sufficient. Shifting the current \(J_\mu(0)\) to the spacetime point \(z\) (see Fig.) and assuming completeness of intermediate states \(X\) we get:

\[W_{\mu\nu} = \int d^4\xi e^{iq\xi} < p|\langle J_\mu(\xi)J_\nu(0)\rangle|0>.\]  

In the Björken limit, \(Q^2 = -q^2 \rightarrow \infty\) and \(q^2 /\nu^2 \rightarrow 0\), the scaling variable \(x = Q^2 / 2M_h\nu \approx k^+ / p^+\) is fixed. For

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They are given by LC fields \( q \cdot p = \sqrt{q^2} \rightarrow \infty \) but \( q^+ = -M_h q/\sqrt{2} \) remains finite. These imply for a conjugate variable \( \xi \) in Eq.3 \( \xi^+ \rightarrow 0 \) and \( \xi \leq \sqrt{2}/M_h x \) from which one gets the following restrictions for components:

\[
\xi_0 \leq 1/M_h x \quad \xi_\parallel \equiv z \leq 1/M_h x.
\]

The spatial variable \( \xi \) is connected directly to the correlation length in elementary the subprocess where the electron interacts with a quark and changes its four momentum by \( q \). We have therefore two resolutions scales in a deep inelastic scattering:\( 1/\sqrt{Q^2} \) which is connected with the virtuality of a foton probe and \( z = 1/M_h x \) which is the distance how far the intermediate quark can propagate in the medium, see Fig.1 and Eq.4. Small \( x \) means a relatively large correlation length \( z \). The hadron tensor \( W_{\mu\nu} \) can be expressed in terms of two structure functions \( W_1 \) and \( W_2 \) depending of two Lorentz invariants: \( q^2 \) and \( q \cdot p = M_h \nu \). In the Björken scaling, the DIS is described by the PDF - \( F_2(x) \) defined in a target rest frame, in terms of quark \( q_n (x) \) and antiquark \( \bar{q}_{n_f} (x) \) distributions:

\[
F_2(x) = \nu \lim_{b \rightarrow 0} W_2 (q^2, \nu) = x \sum_n (q_n (x) + \bar{q}_{n_f} (x)) \quad (5)
\]

They are given by LC fields \( \psi_+ \) with the sum over flavors \( n_f \):

\[
\sum_\mu \left< \psi_+ | \right. \delta(p^+ - xp^+ - p_X^+) \left| \psi_+ \right> = \sum_X \left< \psi_+ | q_n (x) + \bar{q}_{n_f} (x) \right|^2 \quad (5)
\]

Functions \( q_n (\bar{q}_{n_f}) \) are probabilities to remove the quark (antiquark) with flavor \( n_f \) from target living behind some remnant with the momentum \( (1 - x)p^+ \). Resulting sum rule for a total number of quarks \( N_q \) in a hadron and MSR for its total LM fraction \( M_q \) are:

\[
\int_0^1 F_2(x) \frac{dx}{x} = N_q^0 \quad \text{and} \quad \int_0^1 F_2(x) dx = M_q.
\]

Using an additional phenomenological observation that in hadrons and in good approximation in nuclei (EMC effect) the LM is equally distributed between quarks and gluons we can normalize \( M_q = 1 \) in order to get the total LM. Such a MSR should be satisfied because only partons, constituents of the strongly interacting system, carry the whole LM of a nucleon or a nucleus.

B. A Convolution Model in the Nuclear Mean Field

Each parton carries its \( x_A \) fraction of LM and the nuclear SF \( F_2^A (x_A) \) which described the distribution of this fractions is normalized. The Björken scaling in \( x_A \) corresponds in the LC dynamics to the scaling in \( x \) for a nucleon PDF \( F_2^N(x) \). This scaling is caused by the relativistic contraction of nucleons. In the convolution model restricted to nucleons and pions (lightest virtual mesons) the nuclear SF \( F_2^A \) is described by:

\[
F_2^A(x_A) = \sum_{h=N,\pi^\pm} \int y dy \int dx_h \delta(x_A - x_h y) f_h(y) F_2^B(x) \quad (7)
\]

where \( y = A y_N \) and \( F_2^N \) will be later replaced by \( F_2^B \) the PDF of bound nucleon. Both quark and nucleon distributions are manifestly covariant and can be expressed by Green’s functions, \( S(p, P_A) \). The trace is over the Dirac and isospin indices and the single nucleon Green’s function in the nuclear medium is given i.e. in Eq.8:

\[
S(p, P_A) = -i (\gamma \cdot (p - U_V) + M_N^*) \times (8)
\]

The effective mass \( M_N^* \) is substantially lower from the bare nucleon mass \( M_N \) (vacuum value). The values of vector \( U_V \) and scalar \( g_S U_S \) potentials are discussed for example in four specific mean-field models. The connected part (second term) of Eq.8 inserted into Eq.7 for \( f_N(y) \) gives, after taking the trace and using the delta function to integrate over \( p^0 \), the result which can be simplified in the RMF to the form:

\[
f_N(y) = \frac{1}{(p - U_V)^2 - M_N^2 + i\epsilon} \delta (p^0 - E_N^* (p) - g_S U_S^0) \quad (9)
\]

Here the nucleon spectral function was taken in the impulse approximation: \( S_N = n(p) \delta (p^0 - (E_N^* (p) + U_V^0)) \), \( \varepsilon_A = M_A/|A| \). \( E_F^A \) is the nucleon Fermi energy and \( y \) takes the values determined by the inequality \( (E_F^A - p_F)/\varepsilon_A < y < (E_F^A + p_F)/\varepsilon_A \). Finally the nucleon distribution function \( f(y) \) depends on its Fermi momentum \( p_F \), Fermi energy \( E_F^A \) and a single particle energy \( \varepsilon_A \) but only two of them or independent in RMF. Using Eqs.7,9 with \( F_2^B = F_2^N \) and neglecting pion contributions we obtain:

\[
\int d x_A f_2^A (x_A) = \int dy y f_N(y) = \frac{E_F^A}{\varepsilon_A} \geq 1 \quad (10)
\]
where the last inequality obtained for \( p_H \geq 0 \) comes from the following HvH relation between \( E_2^B, \varepsilon_A \) and NM pressure \( p_H \) (see for example \( 14 \)) which was proven in the self consistent RMF approach \( 12 \). According to HvH theorem a Fermi energy given as baryon density derivative of a energy \( E^A = A\varepsilon_A \) in a constant volume \( \Omega \) is:

\[
E_F^A = \frac{\partial}{\partial A} (E^A)_\Omega = \frac{d}{d\rho} \left( \frac{E^A}{\rho} \right) = \varepsilon_A + \frac{p_H}{\rho} = \varepsilon_A + E_{\text{press}}
\]

(11)

where \( \rho = A/\Omega \) and the hadron pressure \( p_H \) is given by the thermodynamic relation:

\[
p_H = -\left( \frac{\partial E_A}{\partial \Omega} \right)_A = \rho^2 \frac{d}{d\rho} (\varepsilon_A).
\]

(12)

The integral \( 10 \) is equal to 1 at the saturation point. Taking only a nucleon contribution in \( 7 \) it would mean that nucleons carry at equilibrium the whole \( F_N^A \) of the nucleus although mesonic fields \( (U_S, U_V) \) are very strong (few hundred MeV) \( 6 \). But we know \( 13 \) that pure Fermi motion can not describe the EMC effect therefore \( F^B(x) \neq F^N(x) \). When the resolution \( z = 1/(M_N x) \) \( (\text{Eq.} 3) \) is smaller then one half of the NN distance \( d \approx 1/\sqrt{s} \), the single particle area is "visible" in DIS \( 15 \). Let us determine the limiting value \( x_L \) of \( x \) from:

\[
x_L = 2/(M_N d) \simeq 2\rho^5/M_N \sim 0.25.
\]

(13)

The EMC ratio \( (\sigma_A/\sigma_D \simeq F^A_2(x)/F^N_2(x)) \) shows \( 16 \) for \( 0.3 \lesssim x \lesssim 0.6 \), that \( F^A_2 \) and consequently \( F^B_2 \), is gradually smaller from a free PDF \( F^N_2 \). The average \( (p_\pi = 0) \) LM fraction \( x_{max} \) taken by partons localized in nuclear pions should not exceed the mass ratio \( x_{max} \approx M_\pi/M_N \approx 0.15 \). Clearly for small \( x \leq 0.25 \) where we expect contributions from pionic partons we can observe a small excess in EMC ratio \( 16 \). If MSR is satisfied, the excess for small \( x \) can be interpreted as contributions of nuclear pions carrying LM while being exchanged between separate nucleons seeing in the large \( x \geq x_L \) region mentioned before. The resulting average pion excess \( <p_\pi^+> \) included formally in \( 7 \) is given by the difference:

\[
\frac{<p_\pi^+>}{<p_A^+>} = \int_{x_L}^1 (F^B_2(x) - F^N_2(x)) \, dx
\]

(14)

Phenomenologically this ratio is sufficiently small \( 9 \) \( 16 \) \( 17 \) \((\sim 1\%) \) to describe also the nuclear Drell-Yan reactions. So finally the nuclear MSR:

\[
\int dx_A F^A_2(x_A) = 1
\]

(15)

is satisfied \( 18 \) for \( p_H = 0 \) with a modified PDF \( F^B_2 \neq F^N_2 \) and a small admixture of virtual pions. For \( x \approx 0.5 \), where eventually the excess of heavier meson would be present there is a clear reduction of a nuclear SF. (The Fermi motion starts to increase EMC ratio for \( x \geq 0.7 \).)

**FIG. 2**: The nucleon energy \( \varepsilon_A - M_N \) as a function of NM density for two RMF models; \( \sigma - \omega \) Walecka (dot lines) and our Modified Mass approach (solid). Both RMF models are calculated for two parameterizations: \( S_1 \) version \( 10 \) \((\rho_0 = \cdot16 \text{fm}^{-3}) \) and version \( S_2 \) \((\rho_0 = \cdot19 \text{fm}^{-3}) \). Results for full DBHF \( 20 \) (dotted marked line) calculation using Bonn A NN interaction are displayed for comparison, also nucleon energy in ZM model \( 11 \) is in the plot (dotted marked line).

**II. NON-EQUILIBRIUM CORRECTIONS TO NUCLEAR DISTRIBUTION**

A nucleon repulsion and increasing pressure distorts the parton distribution in nucleons. For example, describing nucleons as bags, the finite pressure will influence their surfaces \( 19 \ 22 \). In the paper we show how such a modification of PDF will influence the EoS.

Consider the nuclear pion contributions above the saturation point. In Dirac-Brueckner calculations the pion effective cross section in a reaction \( N + N = N + N + \pi \) is strongly reduced at higher nuclear densities above the threshold \( 24 \) (also with RPA insertions to self energy of \( N \) and \( \Delta \) \( 25 \) included). Moreover the average distances between nucleons are smaller and nucleon approach eventually close packing limit. In fact the limiting parameter \( x_L \) in \( 11 \) will increase with a density and the room for nuclear pions given by \( 13 \) will be reduced. Summarizing, the separated nuclear pions carry possibly less then 1% of the nuclear LM for positive pressure and dealing with a non-equilibrium correction to the nuclear distribution we will restrict considerations to the nucleon part \( \hbar = N \) in Eq. \( 7 \) without additional virtual pions. The eventual admixture of additional pions makes the violation of longitudinal momentum even stronger.

The Equation of State (EOS) for NM has to match the saturation point with compressibility \( K^{-1} = 9\rho^3 \frac{d^2 E}{d\rho^2} \) for \( \rho = \rho_0 \) but then the behavior for higher densities is different for different RMF models. Generally, the
choice of initial Lagrangian in nuclear RMF models and the dependence of nucleon masses from a density is not unique. Let us compare two density dependent effective masses from extremely different examples of RMF models. It is well known that in the linear W model the compressibility defined at the saturation density is too large $K^{-1} \approx 560$ MeV. The non-linear Zimanyi-Moszkowski (ZM) model produces the soft EoS with a good value of $K^{-1} = 225$ MeV. In both models, two coupling constants of the theory are fixed at the semi-empirical saturation density of NM. In the stiff W model $M_N^* \approx 0.6M_N$ at equilibrium. Here effective mass $M_N^*$ is obtained by a subtraction of strong scalar field from the nucleon mass at a saturation point (for a respective EOS see marked lines in fig.2). In a model $[11, 27, 28]$ the nucleon mass at a saturation point (for a respective is obtained by a subtraction of strong scalar field from the nucleon mass). The nucleon mass unchanged. 

In medium $M_{med}$, defined as the total LM of partons in a nucleon rest frame. If at the equilibrium weekly bond nucleons carry almost whole average $<P^+_A>$ then the nucleon mass in medium should be close $M_{med} \approx M_N$ to its vacuum value. Above the saturation point the increasing pressure between nucleons starts to increase the $E^2_B/\varepsilon_A$ $[11, 10]$ thus MSR $[15]$ is broken by $E_{press}/\varepsilon_A [10]$. $E_{press}$ calculated for those two models is shown on Fig.3. It is relatively small (therefore the violation of MSR is weaker) in ZM model (for $\rho = 0.3$ fm$^{-3}$ only 25 MeV in comparison to 150 MeV in W model) but not negligible. This "unexpected" strong departure of the MSR [11] from 1 for a positive pressure $p_H$ in the NM originates from a relativistic flux factor$^1$

$\frac{\sum_{medium} k^+_i}{N_q} \leq \frac{\sum_{medium} k^0_i}{N_q} < M_N = \sqrt{2p^+}$ or $\int_0^1 dx F^B_2(x) < 1.$

(16)

Let us compare two inequalities [10] induced by a pressure, with the total MSR [15]. We propose to use [10] the inequality for the parton LM distribution in a nucleon in medium in order to meet the total LM momentum sum rule [15] violated linearly [10] along with a positive pressure. To this end the inequality [10] with the PDF $F^B_2(x)$ has to fulfill the condition:

$\int_0^1 dx F^B_2(x) = \frac{\varepsilon_A}{E_F} \leq 1$ for $p_H \geq 0$ (17)

A. The nucleon PDF for finite pressure

In the RMF the nucleons are approximated by point like objects, which interact exchanging mesons. But in fact nucleons have a finite volume therefore a positive pressure should influence internal parton distributions. This process can not be described clearly by a perturbative QCD[30] but a next subsection contains a simple bag model estimate. Partons - gluons and quarks inside compressed nucleon will start to adjust their momenta to nucleon properties like a surface, volume and a mass. The particularly energetic weakly bound partons give a large contribution to the nucleon rest energy. Simultaneously, partons take part in the increasing Fermi motion of nucleons. These squeezed extended objects exist in NM under a positive pressure and the amount of energy is required to make a room for nucleons by displacing its environment. It will reduce the sum of $N_q$ parton LM momenta in medium:

$$\sqrt{2} \sum_{medium} k^+_i = \sum_{medium} k^0_i < M_N = \sqrt{2p^+} \quad \text{or} \quad \int_0^1 dx F^B_2(x) < 1.$$  

FIG. 3: Corrections $E_{press} = \frac{\mu}{\rho}$ - in the evolution of the PDF inside NM for stiff W($S_1$) and soft ZM models (comp. Fig.1).

$\rho = 0.3$ fm$^{-3}$ only 25 MeV in comparison to 150 MeV in W model) but not negligible. This "unexpected" strong departure of the MSR [11] from 1 for a positive pressure $p_H$ in the NM originates from a relativistic flux factor.

$^1$ In a criticized non-relativistic approach always $\int dy y f_N(y) = 1$.

FIG. 3: Corrections $E_{press} = \frac{\mu}{\rho}$ - in the evolution of the PDF inside NM for stiff W($S_1$) and soft ZM models (comp. Fig.1).
in order to satisfy, with the help of (10), the MSR:

$$\int_0^1 F_2^A(x_A) dx_A = \frac{E_F^A}{\varepsilon_A} \int_0^1 F_2^B(x) dx = 1 \quad (18)$$

where \( x = k^+ / p^+ \). To estimate main effects in medium, we assume the following parametric form of a PDF in NM: \( F_2^B(x) = a F_2^N(x) \). The required condition (17) determines the relation \( b = a E_F^A / \varepsilon_A \) therefore we have:

$$F_2^B(x) = a F_2^N \left( \frac{E_F^A}{\varepsilon_A} x \right) \quad (19)$$

with a free parameter \( a(p_H) \). The number of valence quarks should not be changed. However a total parton number \( N_q = a(p_H) N_q^0 \) will eventually increase in a more energetic compressed medium, thus \( a \geq 1 \); e.g.

a simple choice \( F_2^B(x) = (\varepsilon_A / E_F^A) F_2^N(x) \) is not suitable; although satisfies Eq. (17), it decreases \( N_q \). The number of quark constituents \( N_q \) (which includes sea quarks) is preserved for \( a = 1 \). The scaling of Björken x by the factor \( a E_F^A / \varepsilon_A \) squeezes a PDF in NM towards smaller \( x \), consequently the sum of the quark longitudinal momenta given by the PDF integral (17) is smaller. The actual upper limit in (17) is diminished (see (19)) to \( x_{up} = \varepsilon_A / (a E_F^A) \). Thus \( F_2^B(x) \) is assumed to be negligible for large \( 1 > x > x_{up} \).

In a RMF approach the detailed form of the nucleon PDF (19) with the specified parameter "\( a \)" or \( N_q(p_H) \) is not important for EOS; important is the condition (17) which defines in the nucleon rest frame a nucleon mass in medium \( M_{med} \) with the following decrease along with the increasing pressure \( p_H \):

$$M_{med} \equiv \sqrt{2} \sum_{medium} k_r^+ = M_N \int dx F_2^B(x) = M_N \varepsilon_A / E_F^A \quad (20)$$

$$= M_N \left( 1 + \frac{p_H}{\varepsilon_A} \right) \approx M_N \left( 1 - \frac{p_H}{\varepsilon_A} \right) \quad \text{for} \quad p_H > 0$$

Concluding, changes of the nucleon PDF (17) affect the nucleon mass in a medium (20) setting \( M_{med} \approx M_N \).

Please note that for \( \varrho < \varrho_0 \) nucleons are well separated, therefore we assume that the nucleon PDF and mass remain unchanged. However for \( p_H < 0 \) the MSR integral (10) \((1 + E_{press} / \varepsilon_A) < 1 \) (see Fig. 3). The missing negative part: \( (E_{press} / \varepsilon_A) \) of LM is taken in our approach by nuclear pions (7). Its biggest contribution (\( \sim 5\% \)) is obtained (depending from the RMF model) for \( \varrho \approx (0.05 - 0.1) \) and disappears along with the \( p_H \) for \( \varrho \rightarrow 0 \).

### B. The bag model estimate

Let us discuss these mass modifications in the simple bag model[31] where the nucleon in the lowest state of three quarks is a sphere of volume \( \Omega_N \) and its energy \( E_{Bag} \) is given in a vacuum as a function of a radius \( R \) with phenomenological constants - \( \omega_0 \), \( Z_0 \) and \( B \):

$$E_{Bag}^0(R) = \frac{3 \omega_0 - Z_0}{R} + \frac{4 \pi B R^3}{3} \approx 1 / R \quad (21)$$

The following condition for the pressure \( p_B = 0 \) inside a bag in equilibrium gives the relation between \( R \) and \( B \) which was used in the last relation of (21):

$$p_B = (\partial E_{Bag} / \partial \Omega_N)_n = 0 \quad (22)$$

\( E_{Bag}^0 \) differs from the nucleon mass by the c.m. correction (21) in the partonic model of a nucleon.

However in a compressed medium the pressure generated by free quarks inside the bag is balanced at the bag surface[31] not only by an intrinsic confining force represented by the bag "constant" \( B(\varrho) \) (which depends on \( \varrho \)) but additionally by a NM pressure \( p_H \) generated by elastic collisions with other hadrons[12][22] bags or a NN pressure derived in QMC/QHD model in medium[23]. Using a spherical bag solution (the first relation in (21)) with the formula (22) for finite \( p_B \) we can obtain the expression for the radius \( R \) of a compressed nucleon. Now the pressure \( p_B \) inside a bag is equal on the bag surface to an external pressure \( p_H \) and finally:

$$p_H = p_B = \frac{3 \omega_0 - Z_0}{4 \pi R^4} \cdot B(\varrho) \quad \rightarrow \quad (B(\varrho) + p_H) R^4 = const \quad (23)$$

The pressure \( p_H \) between hadrons acts on the bag surface similarly to the bag constant \( B \). At the saturation \( p_H = 0 \) and the bag "constant" \( B(\varrho_0) \) is determined by the value of the nucleon radius \( R \sim 1 \text{fm} \). Above the saturation point when the NM pressure \( p_H \) would be not taken into account \( (p_H = 0 \text{ in } (23)) \) the nucleon radius \( R \) increases in a NM. It is shown[23] that an decreasing of the B constant from a saturation density \( \varrho \) up to \( 3 \varrho_0 \) by \( 60 \text{MeV fm}^{-3} \) is accompanied be similar increase of the pressure \( p_H \). The changes in medium depend on the EoS. The QMC model in medium (21) takes into account the \( p_H \) contributions to the bag radius. In particular for the ZM model which has the realistic value of \( K^{-1} = 225 \text{MeV} \) the nucleon radius remains almost constant[23] up to density \( \varrho = 10 \varrho_0 \). Such a solution of a slowly varied \( R \) with \( \varrho \) is probably the property of the relatively soft EoS[20]; e.g. a ZM model shown in Fig. 3. Also in our estimate[23] when the bag radius weakly depends from the increasing density the sum \( (B(\varrho) + p_H) \) remains approximately constant.

The nucleon rest energy \( E_{Bag} \) under the compression \( p_H \) can be finally obtain from (21,22,23):

$$E_{Bag} = 4 \pi R^3 \left( \frac{4}{3} (B + p_H) \right) \left( \frac{p_H}{R} \right) = E_{Bag}^0 \frac{R_0}{R} - p_H \Omega_N \quad (24)$$

where \( R_0 \) and \( E_{Bag}^0 \) denote a radius and a bag energy fit to the nucleon mass for \( p_H = 0 \). The scaling factor \( R_0 / R \) comes from the well known model dependence (21).
(E_{bag}^0 \sim 1/R) in a spherical bag \cite{31}. This simple radial dependence is now lost in \cite{24}.

Responsible for that is the pressure dependent correction to a mass of the nucleon given by the product of \( p_H \) and the nucleon volume \( \Omega_N \). Now we can compare it with a similar correction to the nucleon mass from \cite{20}. They have a common linear behavior with the pressure and they are equal for \( \varrho \simeq (M_N/\varepsilon_A)\varrho_{\text{max}} \) - where \( \varrho_{\text{max}} = 1/\Omega_N \) denotes the greatest density of not overlapping (approximately) nucleon bags. These corrections are express above all by a product of pressure and a single particle volume \( \Omega_N \) \cite{24} or \( \Omega/A \) \cite{20} which physically means the necessary work \( W = p_H \Omega_N \) to be done in order to create a space for this extended system - the nucleon in a compressed NM. These RMF results shows that the nucleon mass \( M_N \) can be consider generally as the enthalpy \( H = U + p_H \Omega_N \) - equal to the total energy which includes the \( M_{\text{med}} \) (as an internal energy \( U \)) and a work \( W \). Thus \( M_N \simeq M_{\text{med}} + p_H \Omega_N \). In the nuclear medium in equilibrium \( p_H = 0 \) therefore \( M_{\text{med}} = M_N \).

### III. RESULTS

Our calculations show how changes in nucleon mass, will soften the stiff EoS of linear W model \cite{8} shown on Fig.2. In our calculations we replace the nucleon mass \( M_N \) by the mass in medium \( M_{\text{med}} \), see Fig.4. To accomplish it, our explicit mass dependence \cite{20} from density, energy \( \varepsilon_A \) and pressure \( p_H \) is combined with the standard linear RMF equations \cite{8} for the energy per nucleon \( \varepsilon_A \) in terms of effective mass \( M^*_{\text{med}} \) analogous to \( M_N^* \) in Eq.8:

\[
\varepsilon_A = C_F^2 \partial + \frac{C_0^2}{\varrho} (M_{\text{med}} - M^*_{\text{med}})^2 + \frac{\gamma}{\varrho}\int_{0}^{p_{\text{PPR}}} d^3p \frac{\varrho_{\text{PPR}}}{(2\pi)^3} \sqrt{p^2 + M^*_{\text{med}}}.
\]

\[
M^*_{\text{med}} = M_{\text{med}} - \frac{\gamma}{2C_2} \int_{0}^{p_{\text{PPR}}} d^3p \frac{M^*_{\text{med}}}{(2\pi)^3} \sqrt{p^2 + M^*_{\text{med}}}.
\]

where \( \gamma \) denotes a level degeneracy (\( \gamma = 2 \) for a neutron matter) and two (coupling) constants: vector \( C_F^2 \) and scalar \( C_0^2 \), were fitted \cite{3,10}, at the saturation point of nuclear matter (in the formula \( 2C_F^2 = C_0^2/M_N^2, \quad 2C_0^2 = M_N^2/C_F^2 \) with \( g_1U_0^0 = 2C_0^2 \)). In the direct coupled W model the nucleon mass \( M_N \) is constant. In our modified version the finite pressure corrections to \( M_{\text{med}} \) \cite{20} convert the recursive equation \cite{25} to a differential-recursive set of equations above the saturation density \( \varrho_0 \) in a general form:

\[
f(\varepsilon_A, \frac{d}{dQ}(\varepsilon_A)) = 0 \quad \text{for} \quad \varrho \geq \varrho_0
\]

Note that equation \cite{26} is obtained from the energy-momentum tensor for the model Hamiltonian with a constant nucleon mass \cite{8}. Here we assume that the same equation with a medium mass \( M_{\text{med}} \) will be satisfied. It should be a good approximation, at least not very far from the saturation density. The pressure \( p_H \) is obtained from the thermodynamic relation \cite{16}. The final results were obtained by solving numerically differential recursive equations \cite{26}, starting from standard solutions of Eq.\cite{25} at the saturation density for two version of the Walecka model: a first version \( S_1 \) \cite{10} \( (\varrho_0 = 16\, fm^{-3}, \quad C_V^2 = 273.8, \quad C_s^2 = 357.4) \) and a second version \( S_2 \) \cite{8} have a minimum at \( \varrho_0 = 19\, fm^{-3} \) (parameters \( C_V^2 = 195.9, \quad C_s^2 = 267.1 \)). They are displayed in Figs.\( (2,4,5) \). In Fig.2 our values of the energy per nucleon \( \varepsilon_A \) calculated for two version are denoted by solid lines (Mass Mod.) and solutions of the ordinary Walecka model with constant mass, denoted by dashed lines, are presented for comparison. Our EoS’s are generally much softer - from the unrealistic value of \( K^{-1} = 560\, MeV \) for the Walecka model \( (S_2) \) to the reasonable \( K^{-1} = 290\, MeV \) obtained in our model. Below saturation density these solutions are of course identical for a given version \( (\text{solid lines}) \). Our energy and pressure results for \( S_2 \) parametrization are similar to the DBHF results Figs.\( (2,3) \). The EoS for ZM model seems to be too soft for high densities. The nucleon masses: \( M_N \) and \( M_{\text{med}} \) in medium with their effective masses \( M_N^* \) and \( M_{\text{med}}^* \) (used in Walecka and our model respectively) are compare in Figs.\( 4 \).

Our pressure results (lower and upper panel of Fig.\( 5 \)) are compared with a semi-experimental estimate \cite{33} from

\footnote{It is important to mention that in solutions of Eqs.\( 25,26 \) the Fermi energy from definition Eq.\( 11 \) has a different value then the one calculated from the usual form \( E_F^2 = \sqrt{M_N^2 + p_F^2 + U_0} \) used in Eq.\( 11 \). The discrepancy vanish near the saturation density, increases with the density and reach the 15% of the total vector repulsion in Eq.\( 24 \). Similar problems \cite{32} are connected with the proper choice of single particle potential, which in our case should be adjusted to the changes of nucleon mass. This discrepancy can be removed, here e.q. by the less repulsive momentum dependent vector potential for nucleons however such a correction has no influence on presented results.}
heavy ion collisions and indeed they correct (solid lines) Walecka results (dashed) quite well, making the EoS significantly softer. We have good course of EOS in NM (lower panel) for a set $S_2$ up to density $\rho = 5 \text{ fm}^{-3}$. Our results are close (slightly below for lower density) DBHF results (dotted line) which produce the EOS enable to describe the mass of PSR J16142230 star\cite{34}. In fact, for this density, the (partial) de-confinement is expected which will change EOS above the phase transition\cite{36}. Therefore it is interesting how strong, in the realistic NN calculations with off-shell effects, is violation of the longitudinal MSR. It is worth to mention that in DBHF method, there are additional corrections\cite{26} from self energy which diminish the nucleon mass with density.

Our neutron matter results Fig.5 (upper panel) for $S_2$ parametrization fit well the allowed course of EOS and can be compare with another RMF models\cite{37,38}. Anyway, in case of an additional large softening of EOS the $S_1$ parametrization, which is much stiffer but near the allowed range, can be consider.

Strangeness corrections will be present in the strange nuclear matter\cite{22,39} which supposedly exist also in the neutron stars\cite{38}. Because the coupling of the hyperon to the omega mesons is weaker then that of the nucleon a shift in baryon content from nucleons to hyperons occurs only when the shift softens the equation of state. The generalized Hugenholz van Hove theorem concerns\cite{40} different barions in a nuclear matter; for example additional $S$ strange barions. Analogously to (11) a sum of all Fermi energies, including Fermi energies of strange barions $E_F^S$, is equal:

$$AE_F^A + SE_F^S = (A+S)(\varepsilon_{A+S} + p_H/\rho) \quad (27)$$

$$\text{with } p_H = \rho^2 \frac{d}{d\rho} (\varepsilon_{A+S}) \quad (28)$$

Therefore, the medium corrections to the mass of strange barions like $\Lambda$ and $\Sigma$ based on Eq.27 will be similar to (20) depending mainly from a pressure and a total energy of the system. The strong repulsion\cite{41} of the $\Sigma$ particle in medium will delay the appearance (in increasing density) of the first hyperon $\Sigma$. The corresponding EOS with the strangeness and nucleon resonances will be calculated and published elsewhere. The basic conclusions however will remain the same.

Our mass corrections reduce the violation of longitudinal MSR from 50% (in the linear Walecka model) to 10% in our model\cite{2} with $K^{-1} = 290 \text{ MeV}$. Other features of the Walecka model, including a good value of the spin-orbit force remain in our model unchanged. The presented EoS is relatively stiff above $\rho = 5 \text{ fm}^{-3}$ which is desirable in the investigation of neutron and compact stars\cite{12}. The strangeness\cite{39} will probably not spoil the allowed course or the phase order transition to the quark matter might happen earlier\cite{36}. Our softening correction to the nucleon mass will disappear naturally with deconfinement. However in the interacting system a part of nucleons occupy states above the Fermi level. Therefore our formula (9) and MSR should be treated as the RMF approximation. Alternatively the mean field scenario should be supplemented by neutron-proton short range correlations which have the remarkably similar $A$ dependence as the EMC effect\cite{43}. On the other hand the simultaneous description of the nuclear Drell-Yan reaction\cite{17} and the EMC effect\cite{18} provides that the RMF model is working correctly.

IV. CONCLUSIONS

The conservation of a parton MSR for a positive pressure modifies the nuclear SF and enables\cite{3} to obtain the

$$AE_F^A + SE_F^S = (A+S)(\varepsilon_{A+S} + p_H/\rho) \quad (27)$$

$$\text{with } p_H = \rho^2 \frac{d}{d\rho} (\varepsilon_{A+S}) \quad (28)$$
good compressibility of NM without free parameters using the simple linear scalar-vector W model in the RMF approach. Particularly, it was shown that a violation of a longitudinal MSR for partons in compressed NM can be removed by finite volumes corrections to the nucleon mass in medium, which reduce the nuclear stiffness to the acceptable value giving the good course of EOS for higher densities. Also we have argued, that the rather weak dependence of the nucleon radius (or a size of confining region) from density gives the proper EOS fit to heavy ion collisions and neutron star properties (a mass-radius constraint), especially the most massive known neutron star[37] recently discussed in the application to the nuclear EoS in compact and neutron stars.

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[1] S.V. Akulinichev, S. Shlomo, S.A. Kulagin,G.M. Vagradov, Phys. Rev. Lett. 55, 2239 (1985); G.V. Dunne and A.W. Thomas, Phys. Rev. D 33, 2061 (1986); Nucl.Phys. A 455, 701 (1986), R.P. Bickerstaff, M.C. Birse and G.A. Miller, Phys. Rev. D33, 3228 (1986); M. Birse, Phys. Lett. B 299, 188 (1993); K. Saito, A. W. Thomas, Nucl. Phys. A 475, 659 (1994); H. Mineo at al., Nucl. Phys. A735, 482 (2004) J. R. Smith and G. A. Miller, Phys. Rev. Lett. 91 (2003) 213201.

[2] N. N. Nikolaev and V. I. Zakharov, Phys. Lett. B 55, 397 (1975), L. L. Frankfurt and M. I. Strikman, Nucl. Phys. B, 316 (1989).

[3] F. Gross and S. Liuti, Phys. Rev. C45, 1374 (1992), S. A. Kulagin, G. Piller, W. Weise, Phys. Rev. C50, 1154 (1994).

[4] S. A. Kulagin and R. Petti, Nucl.Phys. A 765, 126-187 (2006).

[5] R. L. Jaffe, Los Alamos School on Nuclear Physics, CTP 1261, Los Alamos, July 1985.

[6] L. L. Frankfurt and M. I. Strikman, Phys. Rep. 160, 235 (1988).

[7] G. Itzykson, J.-B. Zuber, "Quantum Field Theory", McGraw-Hill Inc. 1986.

[8] B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. Vol. 16 (Plenum, N. Y. 1986).

[9] J. R. Smith and G. A. Miller, Phys. Rev. C65, 015211, 055206 (2002).

[10] R. J. Furnstahl and B. D. Serot, Phys. Rev. C 41, 262 (1990).

[11] J. Zimanyi and S.A. Moszkowski, Phys. Rev. C 42, 1416 (1990).

[12] J. Boguta, Phys. Lett. B 106B, 188 (1993).

[13] M. Birse, Phys. Lett. B 299, 188 (1993).

[14] N.M. Hugenholzt and L.M. van Hove, Physica 24 (1958), K. Kumar, "Perturbation Theory and the Many Body Problem", North Holland, Amsterdam 1962. L.L. Frankfurt and M.I. Strikman, Phys. Lett. 183B, 254 (1987).

[15] J. Rożynek, Nucl. Phys. A 755, 357c (2004).

[16] J. Arrington Jlab coll., nucl-ex/0701017, J.Phys.Conf.69, 012024(2007).

[17] D.M. Alde et al., Phys. Rev. Lett. 64, 2479 (1990).

[18] J. Rożynek, G.Wilk, Phys. Rev. C 71, 068202 (2005).

[19] L. Ferroni and V. Koch, Phys. Rev. C 79, 034905 (2009).

[20] Guo Hua, J. Phys. G25, 1701 (1999). P.A. Guichon, Phys. Rev. Lett. B290, 235, (1988).

[21] X. Jin and B. K. Jennings, Phys. Rev. C 54, 1427 (1996), H. Müller, and B. K. Jennings, Nucl. Phys. A 626, 966 (1997).

[22] J.I. Kapusta and Ch. Gale, "Finite Temperatures Field Theory", Cambridge University Press, New York 2006.

[23] Y. Liu, D. Gao, H. Guo, Nucl. Phys. A695, 353 (2001).

[24] B. ter Haar and R. Malfilet, Phys. Rev. C 36, 1611 (1987), Phys. Rep. 149, 287 (1987).

[25] E. Oset, L.L. Saucedo, Nucl. Phys. 468, 631 (1987), "The Nuclear Methods and the Nuclear Equation of State", ed. M. Baldo, World Scientific 1999.

[26] T. Gross-Boelting, C. Fuchs, A. Faessler, Nuclear Physics A 648, 105 (1999); E. N. E. van Dalen, C. Fuchs, A. Faessler, Phys. Rev. Lett. 95, 022302 (2005); Fuchs J. Phys. G 35, 014049 (2008). (1995), D.P. Menezes, C. Providencia, M.Ciapparini, M.E. Bracco, A. Delfino, M. Malheiro, Phys. Rev. C 76, 064902, (2007).

[27] A. Delfino, C.T. Coelho and M. Malheiro, Phys. Rev. C 51, 2188.

[28] N.K. Glendenning, F. Weber, S.A. Moszkowski, Phys. Rev. C 45, 844 (1992).

[29] J. Boguta, H. Stocker, Phys. Lett. B120, 289 (1983).

[30] G. E. Brown, M. Rho, Phys. Rev. Lett. 66, 2729 (1991).

[31] K. Johnson, Acta Phys. Pol. B6, 865 (1975) and references therein, A. Chodos et al., Phys. Rev. D 9, 3471 (1974).

[32] K.A. Brueckner and J.L. Gammel, Phys. Rev. 109, 1023 (1958); M. Baldin, I. Bombaci, G. Gianfrancesco, U. Lombardo, C. Mahaux, R. Sartor, Phys. Rev. C 41, 1748 (1990); P. Czerski, A. De Pace and A. Molinari Phys. Rev. C 65, 044317 (2002); P. Danielewicz, R. Lacey, W. G. Lynch, Science 298,1592 (2002).

[33] T. Klähn et al., Phys. Rev. C 74, 035802 (2006).

[34] P. B. Demorest et al., Nature 467, 7319 (2010).

[35] T. Klähn et al., Phys.Lett. B654, 170, (2007).

[36] P. Haensel, A.Y. Potekhin, D.G. Yakovlev, "Neutron Stars 1", 2007 Springer.

[37] N.K. Glendenning, S.A. Moszkowski, Phys. Rev. Lett. 67, 2414 (1991), N.K. Glendenning, "Compact Stars", Springer-Verlag, New York, 2000.

[38] Bombaci, M. Prakash, P. J. Ellis, J.M. Lattimer and R. Knorren, Phys. Rep. 280 (1997), S. Balberg A. Gal, Nucl.Phys. A625, 435 (1997).

[39] L. Satpathy, R. Nayak, Phys. Rev. Lett. 51, 1243, (1983).

[40] J. Dabrowski, J. Rożynek Phys. Rev. C 78, 037601 (2008).

[41] T. Klähn, D. Blassche, R. Lastowiecki, Acta Phys. Pol. B Proc. Suppl. 5, 757 - 772 (2012).

[42] L.B. Weinstein et al., Phys. Rev. Lett. 106, 052301 (2011).