Stability of the discrete time-crystalline order in spin-optomechanical and open cavity QED systems

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Discrete time crystals (DTC) have been demonstrated experimentally in several different quantum systems in the past few years. Spin couplings and cavity losses have been shown to play crucial roles for realizing DTC order in open many-body systems out of equilibrium. Recently, it has been proposed that eternal and transient DTC can be present within an open Floquet setup in the thermodynamic limit and in the deep quantum regime with few qubits, respectively. In this work, we consider the effects of spin damping and spin dephasing on the DTC order in spin-optomechanical and open cavity systems in which the spins can be all-to-all coupled. In the thermodynamic limit, it is shown that the existence of dephasing can destroy the coherence of the system and finally lead the system to its trivial steady state. Without dephasing, eternal DTC is displayed in the weak damping regime, which may be destroyed by increasing the all-to-all spin coupling or the spin damping. By contrast, the all-to-all coupling is constructive to the DTC in the moderate damping regime. We also focus on a model which can be experimentally realized by a suspended hexagonal boron nitride (hBN) membrane with a few spin color centers under microwave drive and Floquet magnetic field. Signatures of transient DTC behavior are demonstrated in both weak and moderate dissipation regimes without spin dephasing. Relevant experimental parameters are also discussed for realizing transient DTC order in such an hBN optomechanical system.

I. INTRODUCTION

In recent years, periodically driven (Floquet) quantum many-body systems have attracted considerable attention since they are crucial for understanding new nonequilibrium Floquet many-body localization (MBL)1 phase and may have potential applications in quantum metrology.2 One example of a non-equilibrium Floquet-MBL phase is the discrete time-crystalline (DTC) order, which is different from a continuous time crystal and is characterized by the breaking of discrete time-translation symmetry (TTS).3. The DTC order has been realized experimentally in several quantum systems in the past few years.4-14. Under driving with a period $T$, the system can exhibit a strobo-scope response with a period $nT$ and it is expected to be robust against imperfection of the driving.15-16. Recently, the DTCs in open Floquet systems have been reported.17-22. Since any realistic systems will be unavoidably coupled to its surroundings and the influences of baths can be either negative or positive, the mechanisms of stabilizing DTC in dissipative systems will be important to explore.

Meanwhile, recent development of optomechanical systems24-28 has facilitated breakthroughs of quantum technologies such as ground state cooling29-30, optical sensing31-33, and quantum information processing34-35. With nanoscale cavity optomechanical devices, the coupling between light and motion of mechanical resonators can be flexibly modulated with controllable loss36, which may even reach ultrastrong coupling regime37. A natural choice of mechanical modes is to use membranes of two-dimensional materials due to their excellent mechanical properties.38-40. Recently, hexagonal boron nitride (hBN) has drawn great interest and served as a promising platform for exploring both quantum and nanophotonic effects41-45. hBN has a wide bandgap and outstanding chemical and thermal stability beyond that of graphene. As a type of van der Waals materials, hBN can be integrated with plasmonic, nanophotonic, and potentially more complex structures46-50. The hBN membranes have low mass, small out-of-plane stiffness, high elasticity modulus and strong tensile strength, which make them a promising candidate for high-Q mechanical resonators and high-sensitivity sensors51-52. A spin-mechanical system based on color centers in a suspended hBN mechanical resonator has been proposed,53-54, which can even simulate the Rabi model in the ultrastrong coupling regime. Very recently, optically addressable spin defects were observed in hBN55-57. As the DTC order has been found in $N$ atoms in a lossy cavity19-21, it is interesting to explore the DTC in such spin-optomechanical systems with incoherent noise (spin damping or dephasing).
The Floquet driving protocol is that the spin-cavity coupling will be switched on (off) in the first (second) half of a Floquet period. For convenience, a more compact version can be derived as

$$\hat{H}(h, \lambda) = \omega_0 \hat{J}_z + \omega \hat{a}^\dagger \hat{a} + \frac{h}{N} \hat{J}_z^2 + \frac{2\lambda}{\sqrt{N}} (\hat{a} + \hat{a}^\dagger) \hat{J}_x,$$

(2)

by introducing the collective angular moment operator $\hat{J}_\mu = \sum_i \hat{s}_i^\mu$ and neglecting a constant term. We consider a general decoherent model by including both the spin and cavity losses. Then, the dynamics of the system can be described by the master equation (setting $\hbar = 1$)

$$\frac{d\hat{\rho}}{dt} = -i[\hat{H}, \hat{\rho}] + \gamma D[\hat{a}\hat{\rho}] + \frac{\Gamma}{N} D[\hat{J}_-\hat{\rho}] + \frac{\tilde{\Gamma}}{N} D[2\hat{J}_z\hat{\rho}],$$

(3)

where $\hat{J}_- = \hat{J}_x - i\hat{J}_y$ is the collective lowering operator and $D[\hat{a}\hat{\rho}] = \hat{a}^\dagger\hat{\rho} - (\hat{a}^\dagger\hat{a}\hat{\rho} + \hat{\rho}\hat{a}^\dagger\hat{a})/2$. Here, $\gamma = \omega/Q$ is the cavity damping rate with $Q$ the quality factor. In addition, $\Gamma$ and $\tilde{\Gamma}$ are the spin relaxation and dephasing rate, respectively. Previous works mainly focused on the DTC in cavity QED systems with merely the cavity loss or the nearest-neighbor (short-range) spin coupling \cite{13, 21}. They have neither discussed stabilizing DTC in dissipative systems with all-to-all coupling nor considered the effects of spin damping and spin dephasing.

First, we would like to consider the robustness of DTC behavior in the thermodynamic limit $N \rightarrow \infty$. By performing the mean-field approximation and factorizing the means of operator product, we obtain a closed set of semiclassical equations as

$$\dot{x} = p - \frac{\gamma}{2} x,$n

$$\dot{p} = -\omega^2 x - \frac{\gamma}{2} p - 2\lambda \sqrt{2\omega} j_x,$n

(4)

where $j_x = \langle \hat{J}_x \rangle / j$ with $j = N/2$ and $\sum_x j_x^2 = 1$, $x = \langle \hat{a} + \hat{a}^\dagger \rangle / \sqrt{2N\omega}$, and $p = \langle \hat{a}^\dagger - \hat{a} \rangle / \sqrt{2N\omega}$. The set of Equation (4) is a generalization of that in Reference \cite{12}, which is a special case as $h = 0$ here. The introduction of spin-spin coupling $h$ breaks the original stable attractors $(j_x, j_y, j_z)_{st} = (\pm \sqrt{1 - \mu^2}, 0, -\mu)/2$ and $(x, p)_{st} = \mp [\lambda \sqrt{2\omega}(1 - \mu^2)/(\omega^2 + \gamma^2/4)](1, \gamma/2)$, with $\mu = (\lambda_e/\lambda)^2$ and the critical spin-cavity coupling strength $\lambda_e = \sqrt{(\omega_0/\omega)(\omega^2 + \gamma^2/4)/2}$. We would also like to focus on the steady-state solutions as Reference \cite{12}, which is instead numerically found out due to the more complexity considered. It is clear that there exist trivial
steady-state solutions as $x = p = j_x = j_y = 0$ and $j_z = \pm 1$. Besides, as long as the dephasing exists ($\Gamma \neq 0$), the steady-state solutions will fall into be trivial. This can be understood as that the existence of dephasing will finally destroy the coherence (non-diagonal terms of density matrix) and leads to the final state as either $|+N/2\rangle$ or $|−N/2\rangle$ when the $\mathbb{Z}_2$ symmetry is broken at $\lambda > \lambda_c$. Here, $|±N/2\rangle$ are the eigenstates of $\hat{J}_z$ with $\hat{J}_z |±N/2\rangle = ±N/2 |±N/2\rangle$. Therefore, we set $\Gamma = 0$ in the following discuss, unless specifically mentioned. Besides, we assume the spins are initially in the eigenstate $|→→⋯→→\rangle$ with $j_x|t=0\rangle = 1$, $j_y|t=0\rangle = 0$, and $j_z|t=0\rangle = 0$ and the cavity mode is initially in a coherent state $|α\rangle$ with $x|t=0\rangle = p|t=0\rangle = 0$. If we consider the symmetry-broken regime $\lambda > \lambda_c$, it is clear that the final state will fall into either one of the two nontrivial stable states. To observe a DTC order, we perform the Floquet driving protocol similar to Reference [19]: the spin-cavity coupling $\lambda$ is artificially switched off in the second-half period, i.e., $\lambda = 0$ for $(n + 1/2)T \leq t < (n + 1)T$ with $n = 0, 1, 2, \ldots$. From an alternative viewpoint, the Floquet driving is that we let the spins periodically driven by a leaky cavity in every first-half period $nT \leq t < (n + 1/2)T$. We introduce the imperfection parameter $\varepsilon$ via a detuning between $\omega$ and $\omega_0$ as $\omega = (1 - \varepsilon)\omega_T$ and $\omega_0 = (1 + \varepsilon)\omega_T$ with $\omega_T = 2\pi/T$. In the perfect case ($\varepsilon = 0$), it is not difficult to check that the unitary dynamics during the second-half period contributes a parity operator $P = e^{-i\pi(a^\dagger a + J_z)}$ which flips the stable state to the other one. If certain observables of the spins (say $j_\mu$) or the cavity mode (say $x, p$) exhibit period doubling oscillations which are robust against imperfection driving $\varepsilon$, then a DTC order may be identified. We also consider nonunitary imperfections due to decoherence of the system. To observe the long-time behavior, we numerically solve a Floquet–Lindblad master equation (setting $\lambda$ in Equation (4) be periodically time-dependent as characterized above) up to 500 periods $T$ by means of the Runge-Kutta method. We shall remark that we have also tried more periods such as 5000 periods as in Reference [19] but there is no qualitative difference. For convenience, we set $\omega_T = 1$ and $\lambda = 1$ to illustrate the perfect DTC in the $\lambda > \lambda_c$ regime.

In Figures 2 and 3 we plot the stroboscopic dynamics of the scaled angular momentum vector $\tilde{\mathbf{j}} = (j_x, j_y, j_z)$ (red, green, blue) in the thermodynamic limit for the perfect driving case $\varepsilon = 0$. The top shows typical stroboscopic dynamics of $j_x$ (solid red curve), $j_y$ (dashed green curve) and $j_z$ (dotted blue curve) for the last 30 periods of the entire 500-period evolution. The bottom displays the stroboscopic trajectories on the Bloch sphere for the entire 500 periods (green) and for the last 100 periods (red) sphere. We consider no spin-cavity coupling $h = 0$ in (a-d) and increasing spin-cavity coupling strength in (e-h) with $h = 0.05, 0.1, 0.3, 1$, respectively. The parameters are set as: (a,e) $\gamma = \Gamma = 0.05$, (b,f) $\gamma = 0.05, \Gamma = 0.3$, (c,g) $\gamma = \Gamma = 0.3$ and (d,h) $\gamma = 1.5, \Gamma = 0.3$. 

Figure 2. Stroboscopic dynamics (top) and stroboscopic trajectories (bottom) of the scaled angular momentum vector $\tilde{\mathbf{j}} = (j_x, j_y, j_z)$ (red, green, blue) in the thermodynamic limit for the perfect driving case $\varepsilon = 0$. The top shows typical stroboscopic dynamics of $j_x$ (solid red curve), $j_y$ (dashed green curve) and $j_z$ (dotted blue curve) for the last 30 periods of the entire 500-period evolution. The bottom displays the stroboscopic trajectories on the Bloch sphere for the entire 500 periods (green) and for the last 100 periods (red) sphere. We consider no spin-cavity coupling $h = 0$ in (a-d) and increasing spin-cavity coupling strength in (e-h) with $h = 0.05, 0.1, 0.3, 1$, respectively. The parameters are set as: (a,e) $\gamma = \Gamma = 0.05$, (b,f) $\gamma = 0.05, \Gamma = 0.3$, (c,g) $\gamma = \Gamma = 0.3$ and (d,h) $\gamma = 1.5, \Gamma = 0.3$. 

In Figures 2 and 3 we plot the stroboscopic dynamics of the scaled angular momentum vector $\tilde{\mathbf{j}} = (j_x, j_y, j_z)$ as well as their stroboscopic trajectories on the Bloch sphere for the perfect driving ($\varepsilon = 0$) and imperfect driving ($\varepsilon \neq 0$) cases, respectively. By comparing the first row a-d where there is no spin-spin coupling with $h = 0$, we clearly observe different stroboscopic dynamics in different dissipation regimes. First, the DTC order is well
preserved by the existence of weak spin damping $\Gamma$ as shown in Figure 2a, and robust again imperfection $\varepsilon$ as shown in Figure 3a. As the spin damping rate $\Gamma$ increases, the DTC dynamics becomes irregular with the trajectory of $\vec{j}$ scattered on the Bloch sphere (Figures 2b and 3b). However, the dynamics will become more regularly with the area of stroboscopic trajectories reduced if the cavity loss rate $\gamma$ increases, the DTC dynamics becomes irregular with the trajectory of $\vec{j}$ scattered on the Bloch sphere (Figures 2b and 3b). The eternal stroboscopic oscillations with doubling period $2T$ do survive in few atom cases, the so-called deep quantum regime [19]. In this regime, we do not perform semiclassical approximation so that all the quantumness of the system is well maintained. The interplay among spin-spin coupling, spin-cavity coupling and dissipations may give rise to more subtle behaviors for transiently long DTC in this deep quantum regime. By transiently long we mean that the DTC lasts much longer than the decay time $\gamma^{-1}$. The initial state is chosen to be $|\gamma\rangle$ or $|\alpha\rangle$, where $|\gamma\rangle \equiv \otimes_{j=1}^{N}|j\rangle$ is the eigenstate of $J_x$ with the eigenvalue $N/2$ and $|\alpha\rangle$ is a coherent state with $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$. The Floquet–Lindblad dynamics extended from Equations (11) and (3) is directly solved under a truncation of 16 photons for $N = 2$.

Figure 3a shows the stroboscopic dynamics of the scaled angular momentum vector components $j_x$, $j_y$, $j_z$ (solid red curve), $j_x$ (dashed green curve), $j_z$ (dotted blue curve) in the thermodynamic limit for the imperfect driving case $\varepsilon = 0.05$. Other parameters are the same as Figure 2. The robustness of DTC against imperfection is clearly shown in (a,d,e). Besides, it is interesting to find that the DTC can even benefit from the imperfection as comparing Figure 3g, with Figure 2a.

III. TRANSIENT DTC BEHAVIOR IN THE DEEP QUANTUM REGIME

We proceed to focus on the few-atom cases [$N \sim O(1)$], which corresponds to the hBN optomechanical system as displayed in Figure 1a. It is expected that a DTC behavior may still survive in few atom cases, the so-called deep quantum regime [19]. In this regime, we do not perform semiclassical approximation so that all the quantumness of the system is well maintained. The interplay among spin-spin coupling, spin-cavity coupling and dissipations may give rise to more subtle behaviors for transiently long DTC in this deep quantum regime. By transiently long we mean that the DTC lasts much longer than the decay time $\gamma^{-1}$. The initial state is chosen to be $|\gamma\rangle$ or $|\alpha\rangle$, where $|\gamma\rangle \equiv \otimes_{j=1}^{N}|j\rangle$ is the eigenstate of $J_x$ with the eigenvalue $N/2$ and $|\alpha\rangle$ is a coherent state with $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$. The Floquet–Lindblad dynamics extended from Equations (11) and (3) is directly solved under a truncation of 16 photons for $N = 2$.

Figure 3a shows the stroboscopic dynamics of the scaled angular momenta $j_x$, $j_y$, $j_z$ (solid red curve), $j_x$ (dashed green curve), $j_z$ (dotted blue curve) in the thermodynamic limit for the imperfect driving case $\varepsilon = 0.05$. Other parameters are the same as Figure 2. The robustness of DTC against imperfection is clearly shown in (a,d,e). Besides, it is interesting to find that the DTC can even benefit from the imperfection as comparing Figure 3g, with Figure 2a.

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Figure 4. Stroboscopic dissipative dynamics of the scaled angular momenta of \(j_x\) (red solid curve), \(j_y\) (green dashed curve), and \(j_z\) (blue dotted curve) in the two-qubit \(N = 2\) case. The inset shows quadratures \(x\) (purple solid) and \(p\) (black dashed) behaviors. We consider weak dissipation in (a) \(h = \gamma = \Gamma = 0.05, \Gamma = 0\) and moderate dissipation in (b) \(h = \gamma = \Gamma = 0.3, \Gamma = 0\) but without spin dephasing as the thermodynamic limit case. Contrast to (a) and (b), (c) and (d) includes spin dephasing \(\Gamma \approx 2\Gamma\) as suggested in Reference [53].

Figure 5. Stroboscopic dissipative dynamics of the scaled angular momenta of \(j_x\) (red solid curve), \(j_y\) (green dashed curve), and \(j_z\) (blue dotted curve) for the three-qubit \(N = 3\) case. The parameter setups are the same as those in Figure 4.

In some sense, a transient DTC order is established in the deep quantum regime before reaching the stationary state. In Figure 4(d), we plot the stroboscopic dynamics in the moderate dissipation regime \((\gamma = \Gamma = 0.3)\). In this case, the decay time can be estimated as \(\gamma^{-1} = \Gamma^{-1} \sim 0.5T\) so that the stroboscopic dynamics occur immediately and lasts over \(10T\), which still maintains a transient DTC order. Moreover, if the spin dephasing \(\Gamma \approx 2\Gamma\) as predicted in Reference [53] is additionally considered, as shown in Figures 4(d), we find that the oscillation time is merely comparable to the decay time and thus no transient DTC order exists. Besides, we observe similar phenomena if more spins are involved such as the case of \(N = 3\) shown in Figure 5. One effect of increasing the spin number \(N\) is that the transient oscillations evolve into an eternal one as predicted at the thermodynamic limit \(N \to \infty\).
Another effect of increasing $N$ may be that the stroboscopic oscillations is more robust against the spin dephasing as comparing the oscillation dynamics of quadrature $x$ (purple solid) in Figures 4 and 5.

Before ending, we would like to discuss the setup of experimental parameters for realizing TDC order in the optomechanical system of hBN monolayer membrane. According to Reference 53, a maximum magnetic field gradient 270 G/nm may be reached such that the spin-cavity coupling $\lambda$ may become comparable or even larger than the oscillator frequency $\omega$. In this work, we consider $\lambda = \omega_T$, $\omega = (1 - \varepsilon)\omega_T$ and $\omega_c = (1 + \varepsilon)\omega_T$ with $\varepsilon \leq 10\%$, and the cavity loss rate $\gamma \leq 1.5\omega_T$, which indicates $\lambda_c \leq 0.65\omega_T$. To insure the occurrence of transient DTC dynamics, we need operate in the regime of $\lambda > \lambda_c$. Then, the minimal spin-cavity coupling is to achieve $\lambda > 0.65\omega_T$, which is realizes in a suspended circular hBN membrane with radius $R \sim 1$ $\mu m$. Another important aspect is to control the dephasing rate $\Gamma$ which is detrimental to the DTC order. According to Reference 53, the spin dephasing mainly stems from optical polarization $\Gamma_{\text{pol}}$ and membrane vibrations $\Gamma_\nu$, which is proportional to the vibration frequency $\omega$. Therefore, to suppress the dephasing rate, it is suggested to reduce the cavity frequency $\omega$, which also corresponds to enhance the membrane radius $R$. Last, but not least, the cavity loss promotes spin cooling and localization, which is crucial to the emergence of DTC. However, as can be indicated by comparing Figure 1 with Figure 4 (or Figure 5 with Figure 5), a too strong cavity loss $\gamma$ (corresponding to extremely low $Q$) may overdamp the system dynamics and destroy the DTC order. Besides, a stronger $\gamma$ leads to a higher critical spin-cavity coupling $\lambda_c$ such that stronger spin-cavity coupling $\lambda$ is needed, which imposes a challenge to its experimental realization. For cavity loss rate $\gamma = 0.05\omega_T$ as considered in Figures 4 and 5, the quality factor $Q$ is about 20, which provides a balance between spin cooling and loss to make experimental realization more feasible 52. Overall, negligible spin dephasing, weak spin damping and appropriate cavity loss are suggested in realizing transient DTC order in such an optomechanical system.

**IV. CONCLUSIONS**

In summary, we have investigated DTC order in a Floquet open system composed of $N$ qubits trapped in a (mechanical) cavity. The influences of all-to-all spin interactions, spin damping, spin dephasing as well as cavity loss are explored both in the thermodynamic limit and in the deep quantum regime. It is shown that the existence of dephasing will destroy the coherence of the system and finally leads the system to its trivial steady state. Without dephasing and all-to-all spin coupling, different stroboscopic dynamics in different dissipation regimes is demonstrated. First, with weak spin damping and weak all-to-all coupling, eternal DTC oscillations are observed and robust against imperfection. As the spin damping rate increases, the stroboscopic dynamics evolves irregularly accompanied by the trajectory of the scaled angular momentum vector scattered on the Bloch sphere. However, with enhancement of cavity loss, the dynamics will become more regularly and the eternal eternal DTC order will reemerge at strong cavity loss. Besides, by growing the all-to-all coupling, we demonstrate that stroboscopic oscillations are gradually destroyed in the weak damping regime. It is interesting to show that the DTC order may be rebuilt by appropriate all-to-all coupling in the moderate damping regime.

We also focus on the few-atom cases, the so-called deep quantum regime, the model of which describes a suspended hBN monolayer membrane with a few spin defects under a microwave drive and a Floquet magnetic field. A transient DTC lasting much longer than the decay time can be found in both weak and moderate dissipation regimes when there is no spin dephasing. Nonetheless, the existence of dephasing will destroy transient oscillations and leads the system fast to a trivial steady state, which is consistent with the results obtained by semiclassical approximation in the thermodynamic limit. We also find that stroboscopic oscillations may be more robust against the spin dephasing by increasing the spin number. Finally, the parameters in the experimental aspect are briefly discussed and how to realize transient DTC order in such an hBN optomechanical system is suggested.

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