Self-Gravitating Accretion Disks

G. Bertin* and G. Lodato†
Scuola Normale Superiore, Piazza dei Cavalieri, 7, 56126, Pisa, Italy

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Abstract

We consider a class of fully self-gravitating accretion disks, for which efficient cooling mechanisms are assumed to maintain the disk close to the margin of Jeans stability. For such self-regulated disks the equations become very simple in the outer regions, where the angular momentum convective transport approximately balances the viscous transport. These latter equations have been shown [G. Bertin, Astrophys. J. 478, L71 (1997)] to lead naturally to a self-similar solution with flat rotation curve, with circular velocity proportional to (Mr)^1/3 and fixed opening angle. Here we report on a recent extensive investigation [G. Bertin and G. Lodato, Astron. Astrophys. 350, 694 (1999)] of the general properties of self-regulated accretion disks.

1. Introduction

Traditionally, accretion disk models neglect the self-gravity associated with the disk and consider disks in Keplerian rotation around a central object [3]. On the other hand, recent observations of Active Galactic Nuclei [4] and of protostellar disks [5] point out that in some cases the disk self-gravity can play a major role.

A key feature of standard accretion disk models is the recognition that anomalous transport processes, needed to account for the high accretion rates observed, are present. The description of this anomalous transport is usually asymmetric, in the sense that the angular momentum transport is described with an empirical prescription for the viscosity [6], while the energy equations are kept in their ideal form (e.g. see [7]).

Here we present a model for self-gravitating accretion disks where both momentum and energy transport are handled heuristically: the standard prescription for the viscosity is retained and the energy equations are replaced by a physically based prescription, best suited for the regime where the disk self-gravity dominates.

The self-regulation prescription considered in this paper (Eq. (8) below) can be argued to result from the competitive balance of a number of highly non-linear collective processes. In spite of the physical plausibility of the arguments that can be put forward for it [2], much like for the mechanisms that might justify the use of the viscosity prescription (Eq. (5) below), the challenging issue of providing a detailed derivation from basic equations is yet unresolved.

2. Self-regulated accretion disks

2.1. Equations and parameter space

We consider a steady, axisymmetric, geometrically thin accretion disk, with constant mass and angular momentum accretion rates, which we call M and J, respectively. The conservation laws for mass and angular momentum, in cylindrical coordinates, are:

\[ \dot{M} = -2\pi r u, \]

\[ J = M r^2 \Omega + 2\pi \nu \sigma v^3 \frac{d\Omega}{dr}, \]

where \( \sigma \) is the surface density of the disk, \( u \) is the radial velocity, \( \Omega \) is the angular velocity, and \( \nu \) is the viscosity coefficient. Note that we take \( M > 0 \) in the case of inflow. Usually on the right hand side of Eq. (2) the two terms have opposite signs; the first term describes convection of angular momentum, while the second (negative in the common situation where \( d\Omega/dr < 0 \)) accounts for the angular momentum transport due to viscous torques. Thus \( J > 0 \) corresponds to a net angular momentum influx.

For a cool, slowly accreting disk, the radial balance of forces requires:

\[ \Omega^2 \sim \frac{1}{r} \frac{d\Phi_e}{dr} + \frac{G M_*}{r^2}, \]

where \( M_* \) is the mass of the central object and \( \Phi_e \) is the disk contribution to the gravitational potential. The radial gravitational field generated by the disk can be written as:

\[ \frac{\partial \Phi_e}{\partial r}(r, z) = \frac{G}{r} \int_0^\infty \left( K(k) - \frac{1}{4} \left( \frac{k^2}{1-k^2} \right) \right) \left( \frac{r}{r' - \frac{z^2}{r'}} \right) \sqrt{\frac{r'}{r}} e(k) \left( \frac{r'}{r} + \frac{z^2}{r'} \right) E(k) \left( \frac{r}{r'} - \frac{z^2}{r'} \right) \, dk, \]

where \( E(k) \) and \( K(k) \) are complete elliptic integrals of the first kind, and \( k^2 = 4r'/[(r'+r)^2+z^2] \) (see [8]). The field \( d\Phi_e/dr \) in the equatorial plane is obtained by taking the limit \( z \to 0 \).

For the viscosity we follow the standard prescription [6]:

\[ v = z c h, \]

where \( c \) is the effective thermal velocity and \( h \) the half-thickness of the disk. Here \( x \) is a dimensionless parameter, with \( 0 < x \leq 1 \); in the following we take it to be a constant.

In the case of dominant disk self-gravity, one may adopt the requirement of hydrostatic equilibrium in the \( z \) direction.
for a self-gravitating slab model, which gives

\[ h = \frac{\dot{\Omega}}{\pi G \sigma}. \] (6)

(A more refined analysis of the vertical structure is given in [2].) Substituting Eq. (6) into Eq. (5) we obtain \( v_\sigma = (\alpha/\pi G) \sigma^3 \), which, inserted in Eq. (2), yields:

\[ G \dot{J} = GM^2 r^2 \Omega + 2 \pi \sigma^3 r^3 \frac{d\sigma}{dr}. \] (7)

Standard studies usually close the set of equations by means of the energy transport equations (see [3]). Here we consider an alternative scenario [1], where we assume that efficient cooling processes are able to maintain the disk in a condition of marginal Jeans stability:

\[ c_K \frac{c_K}{\pi G^2} = \tilde{Q} \approx 1, \] (8)

where \( \kappa \) is the epicyclic frequency. (For a detailed description of this self-regulation mechanism, see [2].) For a given value of \( \alpha \) and \( \tilde{Q} \), we now have a complete set of equations, dependent on the three parameters \( M_s, M_s, \) and \( J \).

In the special case of \( J = 0 \) and \( M_s = 0 \), the problem is solved by the self-similar solution with flat rotation curve [1], characterized by:

\[ 2 \pi G \sigma r = r^2 \Omega^2 = V^2 = \text{const.} \] (9)

The same solution is also valid asymptotically at large radii even when \( M_s \) and \( J \) do not vanish. In fact, the effect of a non-zero \( M_s \) should be unimportant for \( r \gg r_s = 2GM_s(\tilde{Q}/4)^2(GM/2\pi)^{-3/2} \) while the effects of a non-zero \( J \) should be unimportant for radii much greater than \( r_J = \sqrt{2(\pi J/M)(\tilde{Q}/4)(GM/2\pi)^{-1/2}} \).

2.2. Properties of the models

In the presence of a central point mass, the problem has a well defined lengthscale, \( r_s \), and a dimensionless parameter, \( \xi = \sgn(J) r_J / r_s \), related to the angular momentum flux.

In Fig. 1 we show the rotation curves of models for several values of \( \xi \), along with the Keplerian curve \( V_K \) associated with a non-gravitating disk. As can be seen, discrepancies from Keplerian rotation are significant even at radii \( \mathcal{O}(r_s) \). For example, for the \( \xi = 0 \) case we find \( (V - V_K)/V_K \approx 100\% \) at \( r = 2r_s \).

Disks where the angular momentum is transported outwards (\( \xi < 0 \)) tend to develop a warmer core, while the opposite trend occurs for disks where the angular momentum is carried inwards (\( \xi > 0 \)). This is shown in Fig. 2.

In Fig. 3 we show the cumulative mass of the disk relative to that of the central object. Note that in all cases \( M_{\text{disk}} / M_s \to 0 \) for \( r \to 0 \). Although in the inner regions the disk mass becomes negligible with respect to the central point mass, the disk self-gravity is important all the way down to the center. In fact, if the inner disk were fully Keplerian, the vertical scaleheight \( h_v = cr / V_K \) should become much smaller than the thickness \( h = c^2 / (2 \pi G) \) of our models. Instead the two scales only become comparable to each other, as long as the self-regulation prescription is retained. This is illustrated in Fig. 4.

3. Extensions

For specific astrophysical systems, the self-regulation constraint may fail outside a certain radial range, either near the central object, where the cooling processes may not be efficient, or near the outer edge of the disk, where strong surface density gradients may lead to higher values of \( \tilde{Q} \). In order to check what effects on the disk structure may arise...
from relaxing the self-regulation prescription in the inner disk, we have considered disks where \( Q \) is not constant and follows the profile:

\[
\frac{C K}{\pi G \rho} = \tilde{Q}(1 + (r/r_0)^{-9/8} \exp[-(r/r_0)])
\]  

(10)

The mathematical form of this profile is suggested by recent studies of the transition zone from non-gravitating to self-gravitating disks [9]. In Fig. 5 we show the rotation curves of some partially self-regulated disks for the \( \xi = 0 \) case, compared to those of the original model and to the Keplerian curve. We have then studied another extension of the model, in view of possible applications to some astrophysical systems, such as AGN disks, or to the general galactic context, by considering disks in the presence of an “external” spherical field, modeled as that associated with an isothermal sphere with a finite core radius. In Fig. 6 we show the rotation curves for partially self-regulated disks for different values of the transition radius \( r_0 \), between the outer and the inner non self-regulated disk, compared to those of the original model and to the Keplerian curve, for the \( \xi = 0 \) case.

Fig. 3. Cumulative mass of the disk relative to that of the central object. Note that in all cases \( M_{\text{disk}}/M_* \to 0 \) for \( r \to 0 \).

Fig. 4. Ratio of the vertical scaleheight \( h_z = c/\alpha V_K \) to the thickness \( h = c^2/\pi G \rho \) of the disk for various models.

Fig. 5. Rotation curves for partially self-regulated disks for different values of the transition radius \( r_0 \), between the outer and the inner non self-regulated disk, compared to those of the original model and to the Keplerian curve, for the \( \xi = 0 \) case.

Fig. 6. Rotation curves for accretion disks in an external spherical field, for different values of the parameter \( f^2 \), measuring the relative strength of the "external" field.
curves for these models for different values of the relative strength of the external field.

4. Possible astrophysical applications

As mentioned earlier, recent observations have shown that in some cases the disk self-gravity should play a major role in some astrophysical systems, such as AGN disks and protostellar disks. For example, in the context of AGN disks, in the Seyfert galaxy NGC 1068, the rotation curve appears to display significant deviations from the Keplerian law, as the rotational velocity declines as $r^{-0.35}$ at distances $\approx 1$ pc from the “center” [4]. It is worth noting that if we adopt the numbers suggested by the data for NGC 1068, in our model we would have, for $\xi = -5$, a gradient of the rotation curve at $r \approx 1$ pc compatible with $V \sim r^{-0.37}$.

If we refer to protostellar disks, typically quoted parameters are $M \approx 10^{-8} M_\odot$/yr and $M_* \approx 0.5 M_\odot$ [10]. Under these circumstances we find $r_c \approx 1000$ AU. Interestingly, protostellar disks have been observed to extend out to a radius from $\approx 100$ AU to $\approx 1000$ AU. As for the case of AGN’s, a check on the values of the temperatures anticipated on the basis of the effective thermal speeds predicted by the self-regulated models shows that the numbers fall reasonably well within the range suggested by the observations.

In conclusion, in this paper we have presented a model for self-gravitating disks independently of the conditions characterizing a specific astrophysical system. Further work is desired in order to test the viability of this model for specific cases. In principle, this model could also be applied to the context of protogalactic disks or to the extended HI disks sometimes observed in elliptical galaxies [11].

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