SOME QUESTIONS AND ANSWERS ABOUT CPT VIOLATION

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Minuscule violations of CPT and Lorentz invariance might arise in an extension of the standard model as suppressed effects from a more fundamental theory. In this contribution to the CarruthersFest, I present and answer some questions about CPT and the possibility of its violation.

1 Introduction

It is a pleasure for me to join in celebrating Pete’s 61st birthday. In the spirit of Pete’s approach to physics and to life, here are answers to some of the questions you always had about CPT and its violation (but were afraid to ask).

2 Basics

What is the CPT theorem?
The CPT theorem states that the product CPT of three discrete transformations, namely, charge conjugation C, parity reflection P, and time reversal T, is an exact symmetry of local relativistic field theories of point particles. The theorem has withstood numerous high-precision experimental tests. Indeed, CPT remains to date the only combination of C, P, T that is observed as an exact symmetry of nature.

Why consider CPT violation if the theorem says it isn’t broken?
The CPT theorem is a general result holding for relativistic particle theories, and it can be experimentally tested to great accuracy. These facts make CPT violation an excellent candidate signature for unconventional physics, such as might arise in a fundamental theory based on extended objects like, say, strings.

How could CPT be violated?
Theories disobeying any of the assumptions that enter the CPT theorem could violate CPT. In fact, one of the earliest explicit examples of CPT violation was presented by Pete. It is a class of particle theories that naively appear
normal but that entail nonlocal interactions, excluded by assumption in the CPT theorem.

It is also possible to produce theories violating CPT for relatively subtle technical reasons. For example, the CPT theorem assumes that fields appear in finite-dimensional representations of the Lorentz group. Certain theories involving infinite-dimensional representations break CPT. Also, since the CPT theorem holds within quantum field theory, it may be theoretically feasible to violate CPT if conventional quantum mechanics fails. This possibility has been suggested in the context of quantum gravity.

Are there cases where CPT violation might be physically interesting? A somewhat unexpected example where the CPT theorem does not appear to apply directly is ordinary quantum chromodynamics (QCD). The CPT theorem makes assumptions about the correspondence between the asymptotic Hilbert space and the fields in the theory that are open to question in QCD because of confinement. Rob Potting and I thought about this issue back in 1990 during our investigations of CPT symmetry in strings, but we didn’t find an explicit proof or refutation of CPT invariance in QCD. To my knowledge, this aspect of CPT in QCD (and other confining theories) remains an open issue at present.

A particularly interesting and conceivably physical situation is spontaneous CPT violation. In this case the dynamics of the action remains CPT invariant, which means many desirable properties of the theory are preserved. The violation occurs spontaneously in the solutions of the equations of motion, like the spontaneous breaking of the electroweak gauge group in the standard model. This type of CPT violation is a possibility in string theory, where the usual axioms of the CPT theorem may be modified because strings are extended objects.

3 Spontaneous CPT Violation

How could spontaneous CPT violation occur? Suppose a higher-dimensional action that is Lorentz- and CPT-invariant underlies nature. The higher-dimensional Lorentz group would presumably be spontaneously broken by the solution to this theory, since it must represent our apparently four-dimensional world. This may induce spontaneous CPT breaking.

As an example, strings naturally exist in higher dimensions. Spontaneous Lorentz violation is possible in string theory because string interactions exist that can trigger nonzero expectation values for Lorentz-tensor fields. Comparable interactions don’t appear in conventional four-dimensional renormalizable
gauge theories. If one or more of these tensors has an odd number of spacetime indices, CPT is also spontaneously broken.

Can these ideas be verified explicitly in string theory?
For the field theory of the open bosonic string, the explicit action and equations of motion can be derived analytically for particle fields below some fixed level number $N$. Solutions have been found and compared for different $N$, in some circumstances to a depth of over 20,000 terms in the static potential. These solutions include ones spontaneously breaking Lorentz and CPT invariance that persist as $N$ is increased.

Does spontaneous CPT violation have to come from a string theory?
If spontaneous CPT violation emerges within a higher-dimensional theory then a string origin would presently seem to be the only possibility, since to my knowledge no other consistent candidate theories exist.

Does spontaneous CPT violation imply Lorentz violation or vice versa?
If the spontaneous CPT breaking arises from nonzero expectations of Lorentz tensors, then Lorentz invariance is necessarily spontaneously violated too. However, the converse is false, because expectation values of Lorentz tensors with an even number of indices preserve CPT.

Does spontaneous Lorentz violation imply causality is destroyed?
To my knowledge, there are no theoretical (or experimental) reasons to exclude (small) spontaneous Lorentz violation. Unlike other kinds of Lorentz breaking that do violence to accepted notions, spontaneous breaking is merely a feature of the solutions to the theory. The underlying dynamics remains Lorentz invariant. Indeed, it is possible to verify explicitly that microcausality is preserved in certain simple models arising from spontaneous Lorentz breaking.

Other considerations make it seem very unlikely that a fundamental problem exists with the notion of spontaneous Lorentz violation. For example, the physics of a particle moving inside a biaxial crystal need not be (rotation or boost) Lorentz covariant, but this is merely a reflection of the presence of the background crystal fields and does not affect causality. Nonzero Lorentz-tensor expectation values throughout spacetime are similar in some respects and therefore also might be expected to have benign effects.

If spontaneous CPT/Lorentz breaking occurs, where are the Goldstone bosons?
Goldstone’s theorem does not apply to discrete symmetries like CPT. If Lorentz invariance is treated as a global symmetry, its spontaneous breaking would indeed produce massless excitations, carrying quantum numbers related to the graviton. However, Lorentz invariance is believed to be local. In vector gauge
theories, Goldstone bosons would be absorbed by the gauge fields, which become massive through the Higgs mechanism. In the present case, the graviton propagator is affected but no graviton mass is generated.

4 Standard-Model Extension

If spontaneous CPT breaking occurs in higher dimensions, is it observable?

If the mechanism of spontaneous Lorentz and CPT violation occurs in a higher-dimensional theory, it would seem likely to involve the four physical dimensions too. However, as neither Lorentz nor CPT breaking have been experimentally observed, any effects at the level of the standard model must be highly suppressed.

What would be the scale of the suppression?

Taking the scale governing the fundamental theory as the Planck mass $m_{Pl}$ and denoting the electroweak scale by $m_{ew}$, the natural suppression factor for Planck-scale effects in the standard model is $m_{ew}/m_{Pl} \approx 10^{-17}$. A factor this small means that only a few Lorentz and CPT-violating effects are likely to be observable.

How would effects in the fundamental theory appear in a low-energy theory?

The fermionic sector of the four-dimensional low-energy effective theory might, for example, contain terms of the form

$$\mathcal{L} \sim \frac{\lambda}{M^k} \langle T \rangle \cdot \overline{\psi} \Gamma(i\partial)^k \chi + h.c.$$  \hspace{1cm} (1)

Here, a fermion bilinear involving a gamma-matrix structure $\Gamma$ and derivatives $i\partial$ is coupled to the expectation value of a Lorentz tensor $T$, which breaks Lorentz and CPT symmetry. The coupling coefficient involves a dimensionless coupling constant $\lambda$ and an appropriate power of some large scale $M$, such as the Planck or compatification scale.

Is there an extension of the standard model that includes these effects?

A general extension of the standard model, including Lorentz-breaking terms both with and without CPT violation, has been obtained. The extra terms maintain the usual SU(3) $\times$ SU(2) $\times$ U(1) gauge invariance and are power-counting renormalizable. A framework has also been given for treating theoretically the effects of spontaneous CPT and Lorentz breaking.

5 Experimental Tests

How can CPT be tested experimentally to high precision?

Oscillations of neutral mesons $P$, where $P$ is one of $K$, $D$, $B_d$, or $B_s$, are sen-
sitive probes of CPT violation by virtue of their interferometric nature.\footnote{14} The time evolution of the oscillations is governed by a $2 \times 2$ effective hamiltonian $\Lambda$. Conventional quantum mechanics allows in principle two complex CP-violating parameters to appear in $\Lambda$: the usual CP- and T-violating parameter $\epsilon_P$ that preserves CPT, and a CP- and CPT-violating parameter $\delta_P$ that preserves T. Experiments bounding the value of $\delta_P$ can test CPT to high precision.

**How do the theoretical modifications affect experimental observables?**

Within the CPT-violating extension of the standard model, nonzero values of $\delta_P$ emerge from small corrections to conventional perturbative calculations. For a given $P$ system, it turns out that $\delta_P$ is given by

$$\delta_P = i \frac{h_{q_1} - h_{q_2}}{\sqrt{\Delta m^2 + \Delta \gamma^2/4}} e^{i \phi}. \quad (2)$$

Here, the experimental observables $\Delta m$ and $\Delta \gamma$ are mass and rate differences, with $\phi = \tan^{-1}(2\Delta m/\Delta \gamma)$. The parameters $h_{q_j} = r_{q_j} \lambda_{q_j} \langle T \rangle$ are determined by coefficients of terms in the standard-model extension and by factors $r_{q_j}$ from the quark-gluon sea.

**Are there definite signals from spontaneous CPT violation?**

Assuming hermiticity of the standard-model extension, the $h_{q_j}$ are real. This implies the condition

$$\text{Im} \delta_P = \pm \frac{\Delta \gamma}{2 \Delta m} \text{Re} \delta_P. \quad (3)$$

Moreover, the severity of the suppression factor for Planck-scale effects suggests direct CPT violation in $P$-meson decay amplitudes is unobservable. The relation $(3)$ for indirect CPT violation and the absence of direct CPT violation are signatures for spontaneous CPT violation in any $P$ system.

In addition, the CPT-violating couplings in the standard-model extension seem likely to differ substantially for distinct quarks, as do the Yukawa couplings. The CPT-violating quantities $\delta_P$ could therefore vary significantly for different $P$. This means CPT should be tested in more than one neutral-meson system. Given the sparsity of present bounds on CP violation in the $B_d$ system, it is even possible that $|\delta_{B_d}| > |\epsilon_{B_d}|$, in which case CPT effects would dominate conventional CP ones in the proposed $B$ factories.

**What are the current limits and prospects for future tests?**

The kaon system offers the best CPT bound from neutral mesons. The published limits on $|\delta_K|$ are of order $10^{-3}$. Completed experiments (e.g.,...
CPLEAR at CERN), ongoing ones (e.g., KTeV at Fermilab), and ones currently being designed are likely to improve the bounds in the near future.

Mixing has not yet been seen in the $D$ system, and dispersive effects make theoretical predictions uncertain. In favorable conditions some tests of CPT symmetry could be feasible, perhaps even with current data and probably with statistics available within the next decade.

The $B_d$ system might involve the largest CPT violation because it includes the heavy $b$ quark. Enough data to bound $\delta_{B_d}$ at the level of order 10% have already been obtained in the CERN LEP experiments and in CLEO experiments at Cornell. Indeed, the OPAL collaboration at CERN has very recently placed a bound on Im $\delta_{B_d}$ of about $2 \times 10^{-2}$. The many $B$-dedicated experiments now being developed are likely to improve this bound considerably.

Are there any tests in systems other than neutral mesons?
Several possibilities exist, including signals that might emerge from the CPT-violating extension of quantum electrodynamics implied by the standard-model extension. For example, CPT violation can potentially be tightly constrained by experiments establishing the difference between the electron and positron anomalous magnetic moments. Bounds could be placed on leptonic parameters for CPT violation that are comparable to those in neutral mesons. Further bounds may emerge from photon properties. It is important to consider a variety of tests because the standard-model extension allows distinct parameters to control effects in the different sectors.

What does CPT violation imply about the observed baryon asymmetry?
Conventional baryogenesis requires nonequilibrium processes and C- and CP-breaking interactions. However, an acceptable mechanism for baryogenesis in thermal equilibrium might emerge from terms of the form under suitable conditions. A large asymmetry could be produced at grand-unification scales, subsequently being diluted to the observed value through sphaleron or other effects.

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