Ways to access transversity through asymmetry measurements are reviewed. The recent first extraction and possible near future extractions are discussed.

Keywords: Transverse spin physics, cross section asymmetries

1. Transversity from 1978 to 2008

The year 1978 marks the birth of transversity as a quark distribution with the submission of the seminal paper \(^1\) by Ralston and Soper on November 14, 1978. Transversity was a pre-existing term, but is meant here as the distribution of transversely polarized quarks inside a transversely polarized hadron. It depends on the lightcone momentum fraction \(x\) carried by the quark and is often denoted by \(h_1(x)\). Theoretically it is defined as a hadronic matrix element of a nonlocal operator:

\[
\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P, S_T, \bar{\psi}(0) \mathcal{L}[0, \lambda] i\sigma^i \gamma_5 \psi(\lambda n^-) | P, S_T \rangle = S_T^i h_1(x),
\]

(1)

where \(P\) and \(S_T\) denote the momentum and the transverse spin vector of the hadron, \(n^-\) is a lightlike direction, and \(\mathcal{L}\) is a path-ordered exponential that renders the nonlocal operator color gauge invariant. Transversity is a chiral-odd or helicity-flip quantity, hence, in observables it always appears accompanied by another chiral-odd quantity, several of which will be discussed below. Ralston and Soper considered the double transverse spin asymmetry in the Drell-Yan process (reconsidered in detail at a later stage in Refs.\(^2-4\)), i.e. the asymmetry in the azimuthal angular distribution of a produced lepton pair in the collision of two hadrons, in this case protons,
with transverse spins parallel minus antiparallel:

\[ A_{TT} = \frac{\sigma(p^1 p^1 \to \ell \bar{\ell} X) - \sigma(p^1 p^1 \to \ell \bar{\ell} X)}{\sigma(p^1 p^1 \to \ell \ell X) + \sigma(p^1 p^1 \to \ell \ell X)} \propto \sum_q e_q^2 h_T^q(x_1) h_T^\bar{q}(x_2). \]  

(2)

However, polarized Drell-Yan is very challenging experimentally, as witnessed by the fact that even 30 years later it has not yet been performed. RHIC at BNL is at present the only place that can do double polarized proton-proton scattering, but \( A_{TT} \) is expected to be small at RHIC. It involves two unrelated transversity functions: the one for quarks and the one for antiquarks for which likely holds that \( h_T^\bar{q} \ll h_T^q \). An upper bound on \( A_{TT} \) can be obtained by using Soffer’s inequality,

\[ |h_1(x)| \leq \frac{1}{2} [f_1(x) + g_1(x)]. \]  

(3)

The upper bound on \( A_{TT} \) was shown\(^5\) to be small at RHIC, of the percent level, requiring an accuracy that will not be reached soon.

Already before the advent of RHIC, people started to search for alternative ways of probing transversity. The first suggestion was made by Collins.\(^6\) The idea was to exploit what is now referred to as the Collins effect, which is parameterized by the transverse momentum dependent fragmentation function \( H_1^\perp(z, k_T^2) \). It is a spin-orbit coupling effect in the fragmentation of a transversely polarized quark, resulting in an asymmetric azimuthal angular distribution of produced hadrons around the quark polarization direction, a \( \sin \phi \) distribution. \( H_1^\perp \) is also a chiral-odd quantity.

Collins pointed out that there would be a \( \sin(\phi_h + \phi_S) \) asymmetry in semi-inclusive DIS (SIDIS) proportional to \( h_1 \otimes H_1^\perp \). Here \( \phi_h \) and \( \phi_S \) are the azimuthal angles of a final state hadron and the transverse spin of the initial state polarized hadron, respectively. The angles are measured w.r.t. the lepton scattering plane, which fixes the polarization state of the virtual photon such that the helicity flip state of the probed quark is selected.

The HERMES experiment at DESY was the first to measure a clearly nonzero, percent level, \( \sin(\phi_h + \phi_S) \) Collins asymmetry in SIDIS.\(^7,8\) This asymmetry has afterwards also been measured by the COMPASS experiment at CERN using a deuteron target\(^9,10\) and, as first reported at this Transversity 2008 workshop, also on a proton target.\(^11\) These measurements allow for an extraction of transversity once the Collins function \( H_1^\perp \) is known. This exemplifies the crucial role played by electron-positron annihilation experiments. In Ref.\(^12\) it was pointed out how \( H_1^\perp \) can be extracted from a \( \cos 2\phi \) asymmetry in \( e^+ e^- \to \pi^+ \pi^- X \) (see also the recent, more extended Ref.\(^13\)). This turned out to be the method that has actually been
employed. The measurement of this $\cos 2\phi$ asymmetry has been performed at KEK using BELLE data.\textsuperscript{14,15} This allowed for the first extraction of transversity by Anselmino \textit{et al}.\textsuperscript{16} in 2007, cf. Fig. 1.

Next we turn to a discussion of the magnitude of the extracted transversity functions for $u$ and $d$ quarks. Often $h_1$ is compared with its Soffer bound in Eq. (3) or with $g_1$, which is interesting for theoretical reasons, but for phenomenology it is more relevant to compare it to $f_1$, since that is what determines the magnitude of asymmetries. The first extraction, the best fit, indicates that $h_1(x) \approx f_1(x)/3$, which means that transversity is not particularly small. Whether it is of the expected magnitude is a different matter though. One way of quantifying this is to compare it to expectations from lattice QCD and from models for the tensor charge,

\[
\delta q = \int_0^1 dx \left[ h_1^q(x) - h_1^{\bar{q}}(x) \right],
\]

which is a fundamental charge, like the electric and the axial charge. Transversity is the only known way of obtaining the tensor charge experimentally. Using the central fit and assuming antiquark transversity to be small, the first extraction yields\textsuperscript{17} (at $Q^2 = 2.4 \text{ GeV}^2$)

$\delta u \simeq +0.39$, $\delta d \simeq -0.16$, s.t. $\delta u - \delta d \simeq 0.55$
A lattice determination with two dynamical quark flavors yields\(^{18}\) (at \(\mu^2 = 4\) GeV\(^2\))

\[
\delta u = +0.857 \pm 0.013, \quad \delta d = -0.212 \pm 0.005, \quad \text{s.t.} \quad \delta u - \delta d = 1.068 \pm 0.016
\]

The combination \(\delta u - \delta d\) is given, because it has the advantage of cancellation of disconnected contributions which, although expected to be small, are not calculated.

Most models find tensor charges roughly in the range:

\[
\delta u = +1.0 \pm 0.2, \quad \delta d = -0.2 \pm 0.2
\]

All of this is consistent with the bounds derived by Soffer\(^{19}\)

\[
|\delta u| \leq 3/2, \quad |\delta d| \leq 1/3
\]

The recent extraction via the Collins effect asymmetries seems to indicate a \(u\)-quark tensor charge that is smaller than expected from lattice QCD and most models. However, at this workshop we learned that a new fit using newer and more accurate data yields a larger \(\delta u\), which seems more in line with expectations\(^{20}\).

There is nevertheless another issue concerning the magnitude of the extracted transversity functions. The BELLE and SIDIS data are obtained at different scales: \(Q^2 = 110\) GeV\(^2\) and \(\langle Q^2 \rangle = 2.4\) GeV\(^2\), respectively. The extraction uses two Collins effect asymmetries, which are not like ordinary leading twist asymmetries. Both azimuthal asymmetries involve transverse momentum dependent functions (TMDs) and beyond tree level this becomes quite involved. The formalism that deals with TMDs beyond tree level is that of Collins-Soper (CS) factorization, initially considered for (almost) back-to-back hadron production in \(e^+e^-\) annihilation\(^{21}\), and later for SIDIS and Drell-Yan\(^{22,23}\). In principle, CS factorization dictates how azimuthal asymmetries depend on \(Q^2\), but in practice this has not been implemented in the \(h_1\) extraction analysis\(^{16}\). Evolution is taken into account only partially in the following way. The Collins function is parameterized in terms of the unpolarized fragmentation function \(D_1\),

\[
H_1^T(z, k_T^2) \equiv D_1(z) F(z, k_T^2), \tag{5}
\]

and the evolution is taken to be the one of the collinear function \(D_1(z)\). This does not take into account that beyond tree level also the transverse momentum dependence requires modification with changing energy scale.

Collins effect asymmetries involve convolution integrals, for example the SIDIS asymmetry as a function of the observed transverse momentum \(q_T\)
(with absolute value $Q_T)$,

$$\frac{d\sigma(e^p \rightarrow e'hX)}{d^2q_T} \propto \left| \frac{S_T}{Q_T} \right| \sin(\phi_h + \phi_S) \mathcal{F} \left[ \frac{q_T \cdot k_T}{M} h_1 H_1^1 \right].$$  \hspace{1cm} (6)

involves a convolution that at tree level is of the form:

$$\mathcal{F}[w f D] = \int d^2 p_T \ d^2 k_T \delta^{(2)}(p_T + q_T - k_T) w(p_T, q_T, k_T)$$

$$\times f(x, p_T^2) D(z, z^2 k_T^2).$$  \hspace{1cm} (7)

In general however it involves another factor $U$ (called $S$ in Refs. 22,23):

$$\mathcal{F}[w f D] = \int d^2 p_T \ d^2 k_T \ d^2 l_T \delta^{(2)}(p_T + l_T + q_T - k_T) w(p_T, q_T, k_T)$$

$$\times f(x, p_T^2) D(z, z^2 k_T^2) U(l_T^2).$$  \hspace{1cm} (8)

In terms of diagrams the difference is expressed in Fig. 2. As a side remark,

we note that it is possible to get rid of the convolutions by weighted integration over the observed transverse momentum of the asymmetry. This requires that the asymmetry is well-described for all values of the transverse momentum, which means that one has to connect the CS factorization expressions to the collinear factorization ones that are valid at large transverse momenta. For the $Q_T$-weighted Collins asymmetry in SIDIS this works, but for the $Q_T^2$-weighted $\cos 2\phi$ asymmetry in $e^+e^-$ annihilation a direct extraction of the Collins function is not possible in this way.\textsuperscript{13,24}

Beyond tree level the soft factor $U$ dilutes the asymmetry, and increasingly so as $Q^2$ increases. Differently stated, for the same functions $f$ and $D$ and weight $w$, the quantity $\mathcal{F}[w f D]$ is smaller beyond tree level. This effect

Fig. 2. The left figure shows pictorially the tree level expression $(H = 1$ and $U(l_T^2) \propto \delta(l_T^2))$, the right figure (by F. Yuan) shows the all-order expression (with $S = U$).
becomes stronger as $Q$ increases and is referred to as Sudakov suppression. Conversely, if $\mathcal{F}[w f D]$ is obtained from experiment and if for instance $f$ is extracted from it for given $w$ and $D$, then $f$ will be larger when using the expression beyond tree level. In Ref.\textsuperscript{25} this was studied numerically and a rule of thumb for the $Q^2$ dependence of azimuthal asymmetries was put forward: asymmetries involving one $k_T$-odd TMD, such as the Collins effect asymmetry in SIDIS, approximately fall off as $1/\sqrt{Q}$; asymmetries involving two $k_T$-odd functions, such as the Collins effect asymmetry in $e^+e^-$ annihilation, approximately fall off as $1/Q$. This behavior was obtained in the investigated range of $Q = 10 - 100$ GeV and is to a very large extent independent of model assumptions, even though the magnitude of the asymmetries does depend heavily on them.

This Sudakov suppression implies that tree level extractions of the Collins function from the $\cos 2\phi$ asymmetry at BELLE, leads to an underestimation of $H_1^\perp$ (since beyond tree level it will be larger). Hence, using that underestimated function to extract transversity from SIDIS data at a lower $Q^2$ (less Sudakov suppression), leads to an overestimation of $h_1$. Based on the results of Refs.\textsuperscript{13,25}, this overestimation may be as large as a factor of 2, although there are many uncertainties in this estimate and it does not take into account that in Ref.\textsuperscript{16} some $Q^2$ dependence of $H_1^\perp$ is included through the scale dependence of $D_1$, as explained above.

To get clarity about the magnitude and about the reliability of the Collins effect extraction method, of course the best would be to do another independent measurement of transversity. Ideally one wants this to be a non-TMD, self-sufficient transversity measurement. These are the cleanest transversity asymmetries that consist of a single observable that only involves collinear distributions and do not require experimental input from other experiments done at different scales and/or using different processes.

Before addressing this topic in detail, it may be worth recalling that the scale dependence of $h_1(x)$ itself is quite well-known, i.e. to next-to-leading order.\textsuperscript{26–28} The evolution of $h_1(x, Q^2)$ is very different from that of $g_1(x, Q^2)$, in part, because there is no gluon transversity distribution. $h_1$ grows with increasing $Q^2$ towards smaller $x$, to eventually become proportional to $\delta(x)$, but with a proportionality constant that decreases to zero as $Q^2 \to \infty$, hence $h_1(x, Q^2) \to 0$. Therefore, also the tensor charge decreases with $Q^2$, but it should be emphasized that it is only very mildly energy scale dependent. At the Planck scale the tensor charge is still only reduced by a factor 2 w.r.t. $Q^2 = 1$ GeV under next-to-leading order (NLO) evolution.
2. Transversity asymmetries

There is an obvious classification of transversity asymmetries into double and single transverse spin asymmetries, but from a theoretical point of view there is a more important distinction based on whether TMDs are involved or not. Cases where collinear factorization can be applied are much safer than cases for which CS factorization is expected to apply. The latter usually require some as yet unknown nonperturbative input and one has to resort to model assumptions, for instance about the transverse momentum shape of the TMDs, as was done for the first transversity extraction in Ref.16. For the analysis it also matters whether one has to combine information from several observables, either obtained under the same experimental conditions or different ones. Below these aspects will be discussed for the explicit routes to transversity.

2.1. Double transverse spin asymmetries

Almost no experiment aiming to extract \( h_1 \) will be self-sufficient. Most cleanly this requires experiments probing a single \( "(h_1)^2" \) observable. There are only two such processes:

- \( \bar{p}^\uparrow p^\uparrow \rightarrow \ell \bar{\ell} X \)
- \( p^\uparrow p^\uparrow \rightarrow \text{high}-p_T \text{ jet} + X \)

Both processes were discussed by Artru and Mekhfi.2 But the first process was only recently considered in detail, because of plans to use the future FAIR facility at GSI for its measurement. The second process was extensively discussed by Jaffe and Saito29, who concluded that it is likely too challenging to be done at RHIC, because it leads to a permille level asymmetry (a result confirmed by Vogelsang30).

A somewhat less clean observable is \( \bar{p}^\uparrow p^\uparrow \rightarrow \pi X \), which is \( \propto (h_1)^2D_1 \), considered by Mukherjee, Stratmann and Vogelsang.31 Also \( \bar{p}^\uparrow p \) or \( \bar{p}p^\uparrow \) Drell-Yan experiments are self-sufficient, but these involve TMDs and will be discussed in the next subsection on TMD single spin asymmetries.

First we look at double transverse spin asymmetries in \( \bar{p}^\uparrow p^\uparrow \) collisions, in particular in Drell-Yan. It is ideally suited for \( h_1 \) extraction, because \( h_1^{\bar{p}/p} = h_1^{q/p} \), leading to:

\[
A_{TT} = \frac{\sigma(\bar{p}^\uparrow p^\uparrow \rightarrow \ell \bar{\ell} X) - \sigma(\bar{p}^\uparrow p^\uparrow \rightarrow \ell \ell X)}{\sigma(\bar{p}^\uparrow p^\uparrow \rightarrow \ell \ell X) + \sigma(\bar{p}^\uparrow p^\uparrow \rightarrow \ell \bar{\ell} X)} \propto \sum_q e_q^2 h_1^q(x_1) h_1^q(x_2) \quad (9)
\]

As said, this can perhaps be done at GSI-FAIR. Some of the considered options32,33 are a collider mode at \( \sqrt{s} = 14.5 \text{ GeV} \) (the currently preferred...
asymmetric collider option of 15 GeV antiprotons on 3.5 GeV protons) and a fixed target mode at $\sqrt{s} = 6.7$ GeV (usually quoted as $s = 45$ GeV$^2$).

A chiral quark soliton model calculation$^{34}$ of $h_1$ indicates that large asymmetries of 40-50% can be expected in the fixed target mode at $s = 45$ GeV$^2$. The asymmetry grows with increasing $Q^2$, but is generally smaller for the higher-$\sqrt{s}$ collider mode. Upper bounds on the asymmetry of approximately 17% at $Q = 2$ GeV to 38% at $Q = 12$ GeV for the collider mode of $\sqrt{s} = 14.5$ GeV have been obtained by Shimizu et al.$^{35}$ The first extraction of $h_1$ indicates that transversity is not much smaller than its upper bound, so asymmetries of order 10% at GSI kinematics should be expected. A Monte Carlo study regarding the feasibility of measuring $A_{TT}$ at GSI-FAIR is promising.$^{36,37}$ But in the end the success of double polarized Drell-Yan at GSI-FAIR depends predominantly on whether significant polarization of the antiproton beam can be achieved.

From the study by Shimizu et al.$^{35}$ of the upper bound on $A_{TT}$ it has also become clear that perturbative corrections hardly affect the asymmetry. The transition from leading to next-to-leading order pQCD is small and also resummation of large logs hardly has an effect. A similar robustness can be observed for a closely related asymmetry investigated in Ref.$^{38}$, the unintegrated asymmetry $A_{TT}(Q_T)$, which depends on the transverse momentum $Q_T$ of the lepton pair w.r.t. the beam axis. Although resummation is essential for this observable, which is described within the CSS formalism$^{39}$ that derives from the CS formalism discussed before, resummation beyond the leading-logarithmic approximation (LL) has little effect on the asymmetry. As explained in Ref.$^{38}$ this is particular to $\bar{p}p$ scattering in the valence region.

In Ref.$^{38}$ the upper bound of the asymmetry $A_{TT}(Q_T)$ for GSI kinematics was shown to be of similar magnitude as the integrated asymmetry $A_{TT}$ (which is obtained from $A_{TT}(Q_T)$ by integrating its numerator and denominator separately). Remarkably, $A_{TT}(Q_T)$ is very flat as a function of $Q_T$ and remains flat under $Q^2$ evolution.

The asymmetry $A_{TT}(Q_T)$ for $pp$ scattering,$^{40}$ which is considerably smaller for RHIC ($\sqrt{s} = 200$ GeV) than J-PARC ($\sqrt{s} = 10$ GeV) kinematics, shows a very different behavior compared to $\bar{p}p$ scattering for potential GSI kinematics. The asymmetry is flat at LL level, but not at next-to-leading log. Resummation beyond LL clearly matters in $pp$ collisions.

In conclusion, the double transverse spin asymmetries in $\bar{p}^1 p^1$ Drell-Yan offer clean, direct and unique probes of transversity and in the valence region they are very robust under perturbative corrections.
As mentioned, one could also consider $A_T^{\pi^0}$ in $\bar{p} \uparrow p \rightarrow \pi^0 X$, which is a slightly less clean observable as it requires input on the pion fragmentation function, which however is quite well-known. Upper bounds for assumed beam polarizations of 30% for $\bar{p}$ and 50% for $p$ yield asymmetries of a few percent. The difference between LO and NLO is a bit larger in this case.

2.2. TMD single spin asymmetries

What if one only has one polarized beam? This question is relevant for GSI if the antiproton beam cannot be polarized significantly. For one polarized beam there is a self-sufficient measurement of transversity which involves TMDs, namely the single spin asymmetry in $\bar{p} \uparrow p \rightarrow \ell \bar{\ell} X$ or $p \uparrow p \rightarrow \ell \bar{\ell} X$. Both options are equally useful, there is no difference theoretically.

In the case of one transversely polarized hadron beam, there is a possible spin angle $\phi_S$ dependence of the differential cross section:

$$\frac{d\sigma}{d\Omega} d\phi_S \propto 1 + \lambda \cos^2 \theta + \sin^2 \theta \left[ \frac{\nu}{2} \cos 2\phi - \rho |S_T| \sin(\phi + \phi_S) \right] + \ldots$$

In a measurement of $\rho$ (from $p \uparrow - p \downarrow$) also $\nu$ can be extracted from the same data (from $p \uparrow + p \downarrow$), i.e. under exactly the same experimental conditions. This is in contrast to the previously discussed Collins effect asymmetries.

At tree level one has

$$\nu \propto h_1^T h_1^T$$

analogue of $\cos 2\phi$ asymmetry in $e^+e^-$

$$\rho \propto h_1 T h_1^T$$

analogue of Collins asymmetry in SIDIS

These two expressions involve the TMD distribution function $h_1^T$, which in some respects is very similar to the Collins effect fragmentation function, but can be quite different from it. It is depicted in Fig. 3.

Fig. 3. Nonzero $h_1^T$ means that the transverse polarization $S_T$ of quarks (with momentum $q \approx xP + k_T$) inside an unpolarized hadron (with momentum $P$) is nonzero. It is a $k_T$-odd and chiral-odd TMD.

The asymmetry $\nu$ has been measured in $\pi^- N \rightarrow \mu^+\mu^- X$ by the NA10 Collaboration at CERN and the E615 Collaboration at Fermilab.
roughly 20 years ago. The data show an anomalously large asymmetry, which differs much from the perturbative QCD $O(\alpha_s)$ Lam-Tung relation $\nu = (1 - \lambda)/2$ and the $O(\alpha_s^2)$ corrections to it.\textsuperscript{45,46} Nonzero $h_1^\perp$ offers an explanation for this discrepancy.\textsuperscript{47} Assuming $u$-quark dominance, Gaussian $k_T$-dependence for $h_1^\perp$ and $x$-dependence $\propto f_1(x)$, $\rho$ can be related to $\nu$:\textsuperscript{48}

$$\rho = \frac{1}{2} \sqrt{\frac{\nu}{\nu_{\text{max}}} \frac{h_1^\perp}{f_1^0}}$$

(10)

The result is displayed in Fig. 4.

Fig. 4. Analyzing power $\nu$ of $\cos 2\phi$ asymmetry as fitted to NA10 data using a model Ansatz\textsuperscript{47,48} for $h_1^\perp$ and the resulting prediction of the single spin asymmetry $\rho$ using Eq. (10) for the case $h_1 = f_1/3$.

The asymmetry $\nu$ for $pp$ (e.g. at RHIC, where also $\rho$ can be measured) is expected to be smaller than for $\pi p$, due to absence of valence antiquarks. Preliminary $pp$ data from Fermilab were shown at this workshop\textsuperscript{49} and confirm this expectation. Earlier $pd$ data\textsuperscript{50} also show a small asymmetry, probably for the same reason.

The asymmetry $\nu$ for $\bar{p}p$ on the other hand is expected to be very similar to $\pi p$, due to the presence of valence antiquarks. Therefore, unpolarized $\bar{p}p$ Drell-Yan at GSI-FAIR will likely show a large anomalous $\cos 2\phi$ asymmetry, providing crucial information about its origin. As explained above, the measurement of $\nu$ and $\rho$ at GSI-FAIR with only one polarized beam (either $\bar{p}^1$ or $p^1$) offers a probe of transversity. In this case predominantly $h_{1u/p}$ and $h_{1n/p}$ are accessed, due to the charge-squared factor in Eq. 9.

The COMPASS experiment plans to do $\pi^\pm p^1$ Drell-Yan\textsuperscript{51}, which although not self-sufficient would provide valuable information on the flavor dependence of $h_1$ and $h_1^\perp$. Especially $\pi^+ p^1$ is of interest, as there is no
data available on it yet and it provides information on the $d$-quark ratio $h_1^{d/p}/h_1^{u/p}$, without suppression by a charge-squared factor. The ratio $\nu/\rho$ for $\pi^\pm p$ Drell-Yan in valence approximation namely provides the ratios $h_1^{u/p}/h_1^{u/p}$ and $h_1^{d/p}/h_1^{d/p}$ for $\pi^-$ and $\pi^+$, respectively. Using the input on $h_1^\perp$ from for example unpolarized $p\bar{p}$ Drell-Yan (either from the Tevatron or from GSI-FAIR) would allow for an extraction of $h_1$ from $\pi^\pm p$ Drell-Yan at COMPASS.

The function $h_1^\perp$ may be extracted from $\nu$ at the Tevatron, but the high $\sqrt{s}$ leads to high $Q^2$ on average. This can result in considerable Sudakov suppression, which would be disadvantageous but interesting to verify. One may also probe $h_1^\perp$ via a $\cos 2\phi$ asymmetry in photon-jet production $p\bar{p} \rightarrow \gamma \text{jet} X$ at the Tevatron,

$$d\sigma^{h_1 h_2 \rightarrow \gamma \text{jet} X}/dp_\gamma dq_\perp \propto (1 + \nu_{DY} R \cos 2(\phi_\perp - \phi_\gamma))$$

(11)

where $\phi_\perp$ is the angle of the transverse momentum $q_\perp$ of the photon-jet system and $\phi_\gamma$ is the angle of the transverse momentum $K_\gamma \perp$ of the photon. The analyzing power consists of a proportionality factor $R$ times $\nu_{DY}$, the $\cos 2\phi$ asymmetry of Drell-Yan probed at the scale $|K_\gamma \perp|$ which in general is different from $Q$, which might make a difference from the perspective of Sudakov suppression. The proportionality factor $R$ is only a function of $f_1$.

For typical Tevatron kinematics in the central region, recently investigated for the angular integrated case by the DØ Collaboration, $\nu_{DY} R$ was estimated to be $\sim 5 - 15\%$. That could be large enough to allow transversity related TMD studies at the Tevatron too.

Another “helper” process is the $\cos 2\phi$ asymmetry $\nu$ in unpolarized SIDIS $e p \rightarrow e' \pi X$, which would be proportional to $h_1^\perp H_1^\perp$. Given the Collins function it could in principle be used to extract $h_1^\perp$ too. The asymmetry in SIDIS turns out to be of quite different size compared to Drell-Yan. It has been investigated using model calculations in e.g. Refs.\textsuperscript{52–54}. The asymmetries as a function of observed transverse momentum of the pion are typically on the percent level and are very similar in size for HERMES kinematics and JLab kinematics (the 12 GeV upgrade). Interestingly, the $\pi^-$ asymmetries are positive and according to Ref.\textsuperscript{52} roughly four times as large as the $\pi^+$ asymmetries which is of opposite sign. This factor of four is not related to the charge-squared factor ratio of $u$ and $d$ quarks.

The available data on the $\cos 2\phi$ asymmetry in unpolarized SIDIS are from EMC and COMPASS; the latter were presented at this workshop for the first time\textsuperscript{55} (soon also HERMES data should become available). The data show that $\nu_{SIDIS} \ll \nu_{DY}$. The SIDIS data are obtained for not too
large values of $Q^2$, where also higher twist contributions, such as the Cahn effect, can be relevant. The recent model calculation of Ref.\textsuperscript{54} for instance shows this very clearly. This limits the usefulness of this observable for transversity related investigations. Nevertheless, it is interesting to study the importance of higher twist effects at HERMES and COMPASS energies through this observable.

It should be added that there is also high $Q^2$ data on the unpolarized azimuthal asymmetries in SIDIS from ZEUS ($\langle Q^2 \rangle = 750$ GeV$^2$). Within the sizeable errors the ZEUS data are consistent with pQCD expectations, but they have been presented with a lower cut-off on the transverse momentum of the final state hadron, cutting out contributions of interest here. Apart from that, high $Q^2$ is not favorable to probe the $h_{1T}^\perp H_1^+$ contribution due to the Sudakov suppression discussed earlier.

### 2.3. Non-TMD single spin asymmetries

If one only has one transversely polarized proton beam, then there are two further possibilities to probe $h_1$ which do not involve TMDs, i.e. to use:

- transverse $\Lambda$ polarization
- two hadron systems within a jet

Both options are not self-sufficient, at least not in a straightforward way; they involve unknown fragmentation functions, which most cleanly can be obtained from $e^+e^-$ data. The big advantage is though that collinear factorization applies, therefore, one only deals with non-TMD functions.

Transverse $\Lambda$ polarization enters with the transversity fragmentation function $H_1^\perp(z)$. It is still unknown, but can be measured in $e^+e^- \rightarrow \Lambda^+ \bar{X}$: $\propto (H_1^\perp)^2$.\textsuperscript{58} Subsequently, $h_1$ can be accessed via the spin transfer asymmetry $D_{NN} \propto h_1 H_1^\perp$ in either $e^- p \rightarrow e' \Lambda^ \perp X$ or $p p \rightarrow \Lambda^ \perp X$. The latter has been measured by the E704 Collaboration\textsuperscript{59}, yielding a $D_{NN}$ of order 20-30% at a transverse momentum $p_T$ of the $\Lambda$ of around 1 GeV/c ($\sqrt{s} \approx 20$ GeV). However, because of the low $p_T$, this result can probably not be used to extract $h_1$ in a trustworthy manner. This should be different at RHIC. Upper bounds for $D_{NN}$ calculated\textsuperscript{60} for RHIC at $\sqrt{s} = 500$ GeV show promisingly large asymmetries at much larger $p_T$.

The other option is to use the Interference Fragmentation Function $H_1^\perp$, or more generally, chiral-odd two-hadron fragmentation functions. Consider for definiteness the final state $|\pi^+ \pi^- X\rangle$, i.e. a $\pi^+ \pi^-$ pair inside a jet. The corresponding fragmentation correlation function $\Delta(z)$ of this final state
can be parameterized as
\[ \Delta(z) \propto \left[ D_1 P + iH_1^\perp \frac{R_T P}{2M_\pi} \right], \tag{12} \]

where the two-hadron fragmentation functions \( D_1 \) and \( H_1^\perp \) depend on the sum \( z \) of the momentum fractions \( z^\pm \) of the \( \pi^\pm \) and on the invariant mass of the two-pion system (not necessarily in a factorized way as assumed in Ref.\textsuperscript{61}). The momenta appearing are \( P = P_{\pi^+} + P_{\pi^-} \) and \( R_T = (z^+ P_{\pi^-} - z^- P_{\pi^+})/z \). The \( k_T \) of the pion pair w.r.t. the fragmenting quark is integrated over. See Ref.\textsuperscript{62} for details.

Nonzero \( H_1^\perp \) can arise due to interference between different partial waves of the \((\pi^+ \pi^-)\) system and leads to single spin asymmetries \( \sin(\phi_{S_T}^R + \phi_{R_T}^R) \) in\textsuperscript{61,63,64}

\[
\begin{align*}
ep &\rightarrow e'(\pi^+ \pi^-) X \quad \propto h_1 \otimes H_1^\perp \\
p p &\rightarrow (\pi^+ \pi^-) X \quad \propto f_1 \otimes h_1 \otimes H_1^\perp
\end{align*}
\]

HERMES SIDIS data\textsuperscript{65} below and above the \( \rho \) mass show a nonzero single spin asymmetry (with the same sign), which is another indication that transversity is nonzero. From the comparison\textsuperscript{66} of the data to various model predictions for HERMES kinematics using different \( h_1 \) functions, we conclude that the two-hadron asymmetry data are compatible with \( h_1 \approx f_1/3 \), albeit with considerable room for other values too. COMPASS data could narrow this range down.

As said, both options discussed here have the advantage that one is dealing with collinear factorization. This means no Sudakov suppression and no process dependence. The latter topic will not be addressed here, but is intimately connected with the gauge invariant definition of TMDs, cf. e.g. Ref.\textsuperscript{67}. Another advantage is that the evolution equations for \( H_1(z) \) and \( H_1^\perp(z) \) are known to next-to-leading order\textsuperscript{68,69}, they are in fact the same. Therefore, from a theoretical point of view exploiting the transversely polarized \( \Lambda \) or two-hadron fragmentation functions currently offers the safest and most straightforward way to extract transversity.

Like \( H_1 \), \( H_1^\perp \) can most cleanly be extracted from electron-positron annihilation, in this case from \( e^+ e^- \rightarrow (\pi^+ \pi^-)_{\text{jet}1} (\pi^+ \pi^-)_{\text{jet}2} X \) via a \( \cos(\phi_{R_T}^R + \phi_{R_T}^R) \) asymmetry\textsuperscript{70} \( \propto (H_1^\perp)^2 \). Since pions are easier to measure than polarized \( \Lambda \)'s, this is probably the easiest route to transversity at this moment. BELLE can once again play a crucial role here (as BABAR could). Its data would allow for a non-TMD extraction of transversity in the not too far future. Therefore, all eyes are on BELLE again in this respect.
3. Routes to transversity

In the previous section several different routes to transversity were discussed. They can be classified into four types, which are summarized in Table 1 for the processes discussed before. On the one hand, there are the options that use collinear (non-TMD) functions, which are safer from a theoretical point of view. Some of these options are self-sufficient, but others require additional input, which most cleanly comes from $e^+e^-$ collisions. On the other hand, there are the TMD options, which are theoretically challenging and it is somewhat ironic to note that what is theoretically the most challenging option, i.e. exploiting the Collins effect, is the one that has been done first experimentally.

|                  | non-TMD                      | TMD                        |
|------------------|------------------------------|-----------------------------|
| self-sufficient  | $p\uparrow p\uparrow \rightarrow \ell \bar{\ell} X$ | $p\bar{p}\uparrow \rightarrow \ell \bar{\ell} X$ |
|                  | $p\uparrow p\uparrow \rightarrow (\text{high-}p_T\text{ jet}) X$ | $\bar{p} p\uparrow \rightarrow \ell \bar{\ell} X$ |
| using external input | $e p\uparrow \rightarrow e' \Lambda\uparrow X$ | $e p\uparrow \rightarrow e' \pi X$ |
|                  | $p p\uparrow \rightarrow \Lambda\uparrow X$ | $\pi p\uparrow \rightarrow \ell \bar{\ell} X$ |
|                  | $e p\uparrow \rightarrow e'(\pi^+ \pi^-) X$ | $e p\uparrow \rightarrow (\pi^+ \pi^-) X$ |
|                  | $p p\uparrow \rightarrow (\pi^+ \pi^-) X$ |                                |

Several remarks have to be added in relation to this table. As pointed out by Bacchetta and Radici\textsuperscript{71} $H_1^S$ can also be extracted from $p p \rightarrow (\pi^+ \pi^-) (\pi^+ \pi^-) X$. Similarly, $H_1$ could be extracted from $p p \rightarrow \Lambda\uparrow \bar{\Lambda}\uparrow X$. This makes $p p\uparrow$ experiments in principle self-sufficient too. But clearly this would be less clean and more involved than using $e^+e^-$ extractions of $H_1^S$ and $H_1$, due to the appearance of additional distribution and fragmentation functions, and contributions from multiple partonic subprocesses. Here, different subprocesses can enter in numerator and denominator of the asymmetries, because of which the observables in $p p$ are likely to be considerably smaller than in $e^+e^-$ annihilation. Nevertheless, it is important to keep in mind that one can make the experiments that require a separate extraction of an unknown fragmentation function, self-sufficient by considering more complicated $p p$ or $e p$ processes. This may not apply to the Collins function however. It is currently not clear whether $p p \rightarrow \pi\pi X$, where the two pions are in separate jets, can be used to safely extract the Collins function. Concerns regarding factorization have been raised in e.g. Refs.\textsuperscript{72,73}. 

Note that the process $pp \rightarrow \pi X$ is absent from the table. This is because the single spin asymmetry $A_N$ is described by a twist-three expression that consists of several contributions, not all proportional to transversity. Therefore, it is not clear how to safely extract it from this observable. Instead, $pp \rightarrow \gamma \pi X$ or $pp \rightarrow \pi \text{jet} X$ (cf. also Ref.\textsuperscript{74,75}) could be used, although also here factorization is yet to be established (which is the reason for not including them in the table).

Finally, it is worth adding that there is a special role for Drell-Yan at RHIC: $p^\uparrow p^\uparrow \rightarrow \ell \bar{\ell} X$. It offers a clean way to learn about transversity of antiquarks $h_\bar{q}^T$. Its contribution to the tensor charge is important to know. It would not be satisfactory to always have to assume that antiquark transversity is small, without knowing how small. Therefore, $A_{TT}$ at RHIC is still worth measuring.

### 3.1. Transversity GPD

Now we turn to a completely independent way of accessing transversity that falls outside the framework of collinear functions and TMDs discussed thus far.

There are four chiral-odd Generalized Parton Distributions (GPDs),\textsuperscript{76} which includes the transversity GPD\textsuperscript{77} $H_T(x, \xi, t)$. In the forward limit: $H_T(x, 0, 0) = h_1(x)$, which offers an alternative way to access transversity and the tensor charge. Of course, the latter requires not only the extrapolation to the forward limit, but also integration over all $x$ values. This will be quite challenging, but it may be worth pursuing this route too because it can be measured, for instance at JLab or a future electron-ion collider, without the need to polarize the proton.

Suggestions to probe $H_T$ in exclusive electroproduction have been put forward, for instance, via the production of two vector mesons in particular polarization states\textsuperscript{78,79}, $\gamma^* p \rightarrow \rho^0_L \rho^+_T n$. Very recently it was suggested\textsuperscript{80,81} that transversity could be measured via $\gamma^* p \rightarrow \pi^0 p'$. In both cases the idea is that the spin states of the photon and the meson(s) enforce a helicity flip of the quarks inside the proton. In this way there is no need to polarize the proton. Helicity conservation requires the helicity flip on the proton side. This is similar to how the axial charge can be measured in unpolarized elastic $ep$ or $\nu p$ scattering.

It should be mentioned that in case of single vector meson production, e.g. $\gamma^* p \rightarrow \rho_T p'$, problems regarding factorization arise. Unfortunately this process cannot be used to extract transversity.\textsuperscript{82,83}

Some information on chiral-odd GPDs has already been obtained from
Besides yielding results for the tensor charge, they also show there to be nonzero transverse polarization of quarks inside unpolarized hadrons, hinting at nonzero $h^\perp_1$.

4. Conclusions

Although transversity is a very difficult quantity to measure, several transversity asymmetries have come within reach of present day and near-future experiments. Thanks to SIDIS data by HERMES and COMPASS, and $e^+e^-$ annihilation data by BELLE the first extraction of transversity, exploiting the Collins effect, has been possible. This is an important step forward. The Collins effect asymmetries involve $k_T$-dependent functions, TMDs, and are consequently more difficult to analyze theoretically. Therefore, an independent, preferably non-TMD extraction of transversity is desired. For this, $\bar{p}^\uparrow p^\uparrow$ Drell-Yan would be the ideal process, but two hadron fragmentation functions currently offer the most straightforward way. Many more observables could contribute to our knowledge of transversity, the tensor charge, and other chiral-odd quantities, such as $h^\perp_1$. Unpolarized Drell-Yan data and lattice QCD results strongly suggest that the transverse polarization of quarks inside unpolarized hadrons, which is encoded by $h^\perp_1$, is nonzero and large. If so, especially $\bar{p}^\uparrow p$ or $\bar{p}^\uparrow p$ Drell-Yan offers another promising opportunity to probe transversity. Perhaps this will be possible at GSI-FAIR. It is a self-sufficient way of measuring transversity, in the sense that no information from other experiments or even other processes needs to be included in the analysis. Other, more demanding self-sufficient or nearly self-sufficient options that do not involve TMDs exist too. Amazingly most of these possibilities are in principle possible with existing accelerators. Further transversity measurements are therefore expected in the coming years, contributing valuably to our understanding of hadron spin.

Acknowledgments

I wish to thank the organizers for their kind invitation to give the opening talk at this exciting workshop, where so many new results were presented. I thank Markus Diehl for a useful discussion on vector meson production and Marco Contalbrigo for some feedback on the text.

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