Gödel’s Incompleteness Theorem and Problems Outside Quantum Mechanics

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Abstract. Gödel’s Incompleteness Theorem is considered a significant finding in logic and mathematics and has many indications in physics. This paper analyzes and concludes that Gödel’s Incompleteness theorem potentially parallels problems of reference frame in classical physics and other issues considering self-inclusion with regard to consciousness, which not necessarily relevant to quantum mechanics. It also demonstrates further investigations on Gödel’s Incompleteness theorem, which can add to our understanding of problems in classical physics and philosophy of mind that still require in-depth research.

1. Introduction
Gödel’s incompleteness theorem is widely considered an essential accomplishment in logic, mathematics, and it possibly contains many implications for science. A common formulation of Gödel’s Incompleteness Theorem suggests the impossibility of forming a consistent and finite system of theories if it is sufficient to include arithmetic [1]. For a consistent mathematical system to work, there must be axioms that needs to be treated as absolutely correct, and then all other theorems can be deduced from them. To prove the axioms within the system, however, is not possible. Gödel’s Incompleteness Theorem potentially insinuates the impossibility to develop a theoretical system that encompasses all phenomena in the universe. In reality, while scientists have formulated the standard model to describe electromagnetic, strong, and weak forces, it still cannot take account of gravity [2]. Is it even possible to include gravity in the standard model? Is Gödel’s Incompleteness Theorem preventing the scientists from doing so? Inspired by questions like such, many researchers are continuously searching for further connections between Gödel’s Incompleteness Theorem and physics, from the Heisenberg Uncertainty Principle to a possible route of solving the Yang-Mills mass gap problem [3]. However, in most of the research the connection between Gödel’s Incompleteness theorem and science and philosophy has been centered around quantum mechanics, rather than classical physics and other fundamental philosophical issues, possibly because Gödel’s Incompleteness Theorem was advanced near the time when quantum theory emerged, and Newtonian physics seemingly implies the fundamental determinism of the world – the motion of all particles in the world can be calculated and predicted using Newton’s laws. In this paper, I explained several connections already made between Gödel’s Theorem and physics, drew philosophical connections from the theorem to other physics and epistemological problems without considering quantum ideas and call people to do more research with regard to the non-QM ways Gödel’s theory may come into place.
2. Analysis

2.1. Previous Research with connections between physics and Gödel’s theorem

The overall idea behind Gödel’s incompleteness theorem is straightforward [1]. Euclidean geometry is an excellent example for demonstration. Euclidean Geometry is defined by the five axioms. Namely, the five axioms are considered absolutely correct when using Euclidean Geometry. All other theorems are derived based on these axioms. Within the system, it is impossible to prove these 5 axioms, because they are where all logical deductions start. To prove the five axioms, new axioms (which also makes the system not Euclidean Geometry anymore) has to be introduced, but then the new axioms become unprovable again unless we introduce other new axioms, and the process goes on repeatedly.

![Figure 1. An illustration of Euclidean Geometry.](image)

Euclidean Geometry is defined by the 5 axioms, and all theorems are derived using them. Within Euclidean Geometry, it is impossible to prove the 5 axioms.

Initially, the great mathematician David Hilbert thought it is possible to form a complete axiomatic system that includes everything in mathematics, and he thought as long as more axioms are added to Euclidean Geometry, the system would become complete ultimately [1]. However, the reasoning above shows reaching such a goal is not possible. Gödel rigorously proved the Incompleteness Theorem mathematically and first showed the impossibility for complete axiomatics.

Many researchers have attempted to unveil the connection between science and Gödel’s Incompleteness Theorem. The reason is probably science, especially physics, has a strong base in mathematics; and many physicists wish to form a complete theory for the world as well [1]. Researchers have come across different ways Gödel’s Incompleteness theorem can relate to physics, but almost all of them are considering modern physics, especially branches involving quantum mechanics, as opposed to classical physics and other fundamental issues, resulting in the demonstrated understanding of the connection between Gödel’s Incompleteness Theorem and reality to be somewhat narrow. It appears that one of the earliest connections made between the Incompleteness Theorem and physics is about the Heisenberg Uncertainty Principle, which suggests the impossibility to know the exact position and velocity of a particle at the same time. This connection makes sense in the way they both somehow addresses the partially agnostic nature of reality, suggesting the limited capacity of humans to perceive the world. Interestingly, however, Gödel opposes this type of connection, possibly because he was greatly drawn to Einstein’s relativity theory and choose to follow his “God does not play dice” argument on quantum mechanics as well, according to Wheeler, a scientist who had the chance to meet Gödel in the office.[1].

Gödel’s Incompleteness theorem also has its place in solving more modern issues [3]. According to research done by Cubitt’s team, it is unfeasible to know if there is a spectral gap between the two
lowest energy levels of an electron. When they carry out the calculations, they found out it is impossible to figure out if the calculations reach an end, which matches perfectly with Turing’s formulation of the Incompleteness theorem [3], which points out the impossibility to determine if some computer algorithms to be finished in a finite amount of time.

Another more recent research by Myers and Madjid extends the Incompleteness theorem [4], considers Turing’s formulation and relates it with more advanced quantum mechanics. They proved that, for any given evidence, such as an assigned probability, could have an uncountably infinite number of non-equivalent explanations. While physicists cannot come up with an infinite number of explanations because of their limitation of knowledge, it is possible to prove by pure logic that uncountably infinite numbers of solutions exist in a metaphysical sense. They also deduced that it is impossible to rule out any of these explanations by logic. If we need to, we need to “guess”, or arbitrarily introduce previously not established possibilities. Such a process is almost identical to the canon Gödel’s incompleteness theorem, where we cannot form a finite system to account for everything.

2.2. The relationship between Gödel’s Incompleteness Theorem and Non-quantum problems

Among the past research mentioned, it is clear that most physicists relate Gödel’s Incompleteness Theorem more with quantum mechanics. This, at least partially, can be attributed to the “undecidable” nature of quantum mechanics – ultimately, we cannot know the exact momentum and position of a particle simultaneously. Instead, we can only solve for wave functions and get probability distributions of the state a particle may have.

\[
\Delta x \Delta p \geq \frac{\hbar}{2} \tag{1}
\]

\[
\imath \hbar \frac{\partial \psi}{\partial t} = \frac{\hbar^2}{2m} \nabla^2 \psi + \nu \psi \tag{2}
\]

The Heisenberg Uncertainty Principle (1) and the Schrodinger Equation (2). We cannot find the momentum and position of a particle simultaneously to arbitrary accuracy. We can only solve the Schrodinger equation for probability distributions of the state a particle may have.

This potentially aligns with Turing’s formulation about the Incompleteness theorem, claiming the existence of some undecidable algorithms [3]. The way quantum mechanics behave presents a limitation of scientific deduction, probably because of the inherent physiological limitations of human brain. Gödel did a similar thing by showing the limitation of human logic, only the process is done by mere logical deductions rather than shown by phenomena in reality.

On the other hand, classical physics seems to have no such issue, because it is “deterministic” at first glance. If we take a Laplacian view of classical physics, uncertainty occurs just because of our inability to get precise measurements [5]. Therefore, it is metaphysically possible to obtain the true state of any object of our measurements. If we know the true states of every particle in the universe and have arbitrarily large calculation capacity, we would be able to compute how every particle has evolved and will evolve, and therefore we can know the past and predict the future, becoming “Laplace’s Demon”. In this logic, everything is deterministic and explanatory by Newton’s laws.

However, there are scientific and philosophical issues with the deterministic view of the world, and Gödel’s theorem may have implications for them. Even without considering quantum mechanics, the validity of Laplace’s Demon can be questioned. First, Newton’s laws are considered absolute truth in classical physics, and there are no ways to mathematically prove them, but there are further parts to this problem that can be examined. In fact, the legitimacy of such an omniscient being to emerge was already rebutted by the Chinese Philosopher Huishi [6] from the School of Names. Huishi advances the concept of “everything” (the Chinese word “wan-you”) and “unity” (the Chinese word “yi”). To get to know “everything”, we may postulate this intellect has to somehow experience it at first. To experience it, this intellect has to be outside of it. However, “everything” by definition encompasses everything, including this intellect. If such an intellect exists, we came to a contradiction to “everything”. Therefore, by this logic, this kind of omniscient being cannot exist.
Here, the problem emerges because of self-inclusion within a system. To make the claim of knowing “everything”, one must first understand everything about itself. The feasibility of a mind to do so is a highly debatable question. This mind-related issue has been studied by physicists, psychologists, and biologists for thousands of years. On one hand, from a human perspective, understanding the exact mechanism of the relationship between our consciousness and body movements still seems so difficult by any scientific means. Although neuroscience has already progressed much, and people are able to relate different body parts with respective brain parts and see the electroencephalogram when people make different decisions, we still have no idea how our consciousness appears and governs our body parts. This is something neither foreseen by philosophers nor understood by scientists. An argument by Thomas Nagel suggests the problem may be even more difficult for humans to contemplate because of the subjectivity of the mind [7]. We couldn’t state what mind is because we are constantly experiencing it using our mind. It is impossible to imagine what the mind is in separation from people’s subject experiences. In this sense, we may not be able to understand consciousness in the same way as we understand basic motions of objects. This elicits a set of questions that are not answerable if we take the notion of determinism. If our consciousness is just a hallucination occurring because of the deterministic motions of our particles, why do we experience free will? If our consciousness is a real thing beyond the states of particles in our body, how do they interact with our body?

Discussing these questions in-depth is beyond the scope of this paper. This problem of self-inclusion is intriguing because it is potentially analogous to Gödel’s Incompleteness theorem. Gödel’s problem occurs because the system is defined by several axioms. Then we build every corollary from these axioms using logic. Just like using physics requires reference frames, to experience something requires us to have a reference, usually ourselves. If we want to experience ourselves, the problem gets larger because we are unlikely to get any useful “motion”. The axioms in a system behave in a similar way. A similar analogy could be drawn to the concept of internal force in classical mechanics. Just like one cannot lift himself up, we cannot contribute anything to the axioms using them. Unless we pick another reference, the axioms are not provable. Nonetheless, if we do so, the problem just transfers to that reference we pick recently.

This phenomenon may have different manifestations in different realms. In pure logic and math, it is Gödel’s Incompleteness theorem; in the philosophy of mind, it is the possibility of experiencing one’s own consciousness; in classical physics, it resembles the problem of requiring a reference frame for any statements to have meanings; and in advanced physics, it can relate to the undecidability of low energy level gaps and the problem with integrating the M theory [2]. Instead of staying in merely esoteric realms, such as pure math and quantum mechanics, Gödel’s incompleteness theorem has implications with our perceptions and ways to describe the world, even though not in an exactly homogeneous manner.

3. Discussion and suggestion

This research is primarily based on academic journals, alongside other sources, including a talk of Professor Stephen Hawking. Gödel’s Incompleteness theorem and the latest research on its implications on physics are researched. In the process, the fact that the majority of research done is related to quantum mechanics and its successors is noticed. Cubitt’s research team relates it to the low mass gap problem and sees it as a door to the Yang-Mills mass gap problem; Stephen Hawking relates it to its connections with M theory, suggesting we may never have an ultimate theory that works well in all situations.

In the course researching these, the lack of classical physics or anything not related to quantum mechanics in these arguments is noticed. From a phenomenological point of view, the behaviors of tiny particles do imply the agnostic nature also insinuated by Gödel’s Incompleteness Theorem, which could explain the concentration of research on quantum physics. However, I do find there are issues long debated by philosophers that can have implications with Gödel’s Incompleteness Theorem. So, this research connected philosophical problems with regard to Laplace’s Demon and effectiveness of
consciousness to Gödel’s Incompleteness theorem, hoping to expand the scope to which this theory applies beyond quantum mechanics. Further research can be done by adding Gödel’s Perspective into some long-held philosophical questions. The issue of self-referencing, for example, is long debated throughout history and can have significant implications with the philosophy of mind. Our understanding of these questions may be enlarged if we add Gödel’s perspective into problems of the existence of free will and the connection between our consciousness and body. At least, we may gain insights about whether these problems are decidable or not.

Gödel’s perspective may also play a role in further research examining unsolved problems in classical physics. Currently, scientists have yet to understand turbulence in fluid dynamics, and the understanding of the Navier-Stokes equation is limited [8]. If computer simulation is done, there are numerical problems giving the flow in the inner corner of a right angle turn infinite velocity, which is impossible in reality, and scientists and mathematicians still cannot solve this puzzle. Why does this kind of problem happen? This is a problem that may need to be considered from a meta-scientific perspective, and Gödel’s Incompleteness theorem could play a role in this kind of meta-scientific formulations, telling us if this kind of problem is even decidable.

4. Conclusion
Gödel’s Incompleteness Theorem is valued so much by logicians, mathematicians, and scientists because of its ground-breaking impact on axiomatics and profound implications in our understanding of the world. However, in late scientific research and contemplations regarding Gödel’s incompleteness theorem, Gödel’s findings are usually related to only quantum mechanics related topics. In this paper, I analyzed and showed Gödel’s Theorem actually has a connection to the self-referencing issue connecting to the basis of Newtonian mechanics, and it has implications with problems emergent long before the development of quantum mechanics, such as problems with respect to the philosophy of mind. Therefore, I hope further research to be done on the relationship between the theorem and classical physics and philosophy questions. Such an endeavor may contribute to our understanding of what we may don’t understand enough, such as the behavior of turbulence.

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