Scaling and Spontaneous Symmetry Restoring in Reconnecting Nematic Disclinations

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Liquid crystal has been a platform for studying and visualizing topological defects, yet it has been a challenge to resolve three-dimensional structures of dynamically evolving singular topological defects. Here we report a confocal observation of dynamics of disclination lines – the most basic kind of defects in nematic liquid crystal – relaxing from an electrically driven turbulent state. We focus in particular on reconnections, characteristic of such line defects. We find a scaling law for in-plane reconnection events, by which the distance between reconnecting disclinations decreases by the square root of time to the reconnection. Moreover, we show that apparently asymmetric dynamics of reconnecting disclinations is actually symmetric in a co-moving frame, in marked contrast to the two-dimensional counterpart whose asymmetry is established. We argue, with experimental supports, that this is because of symmetric twist configurations that disclinations take spontaneously, thanks to the topology that allows rotation of winding axis. Our work illustrates a general mechanism for such spontaneous symmetry restoring, which can take place if topologically distinct asymmetric objects in lower dimensions become homeomorphic in higher dimensions.

Introduction. Topological defects constitute a universal concept in physics that arises in a vast variety of areas, ranging from cosmology and field theories [1, 2] to condensed matter [3, 4], liquid crystal [3, 5], biophysics [6, 7], and so forth. While there exist various kinds of defects characterized by different symmetries and properties, defects may also enjoy common properties across different disciplines. In this context, liquid crystal has the advantage that it is amenable to direct optical observations, various compounds and experimental techniques exist, and, as a soft matter system, it shows large response to external fields, being suitable for studying nonequilibrium and nonlinear effects [3, 5]. This advantage has been recognized and indeed used for decades, with a notable example of observing liquid crystal defects to test predictions for cosmic strings [8].

Despite this history, resolving fully three-dimensional (3D) structures of liquid crystal defects has not been straightforward, even for the simplest kind of defects, namely nematic disclination lines. Well-known techniques for 3D observation of defects and other orientational structures are the fluorescence confocal polarizing microscopy [9, 10] and two- or three-photon excitation fluorescence polarizing microscopy [11–13]. Both techniques allow one to reconstruct the 3D structure of the director field, by which one can determine the position and structure of defects in principle. To do so, however, one needs to reduce the effect of defocusing and polarization changes due to the birefringence of liquid crystal. For singular defects such as nematic disclinations, scattering at the core gives another difficulty. The effect of birefringence can be significantly reduced by partial polymerization of the medium [14], but this cannot be used to study dynamics of defects.

Here we propose a method to capture dynamically evolving 3D structures of nematic disclination lines, by using a recently reported accumulation of fluorescent dyes around the singular core of defects [15]. This method allows us to visualize the disclinations directly, without reconstructing and analyzing the director field. We focus in particular on reconnections of disclinations [Fig. 1(a,b)], which are characteristic of such line defects. Observing pairs of disclinations reconnecting in nearly horizontal planes [Fig. 1(a)], we identify a scaling law for the time evolution of the distance between reconnecting disclinations.

![FIG. 1. Reconnections and loop shrinkage. (a-c) Sketches of an in-plane reconnection (a), an intersecting reconnection (b), and a loop shrinkage (c). (d-f) Confocal observations of an in-plane reconnection (d), an intersecting reconnection (e), and a loop shrinkage (f). The inset of (e) displays a side view of the event shown in the main panel. The scale bars represent 50 µm for the main panels and 20 µm for the inset of (e). See also Videos S1-S4.](image-url)
disclinations. We also observe that reconnecting disclinations approach each other in an apparently asymmetric manner, reminiscent of the well-known asymmetry reported for pair annihilations of point disclinations in two dimensions (2D) [16, 17]. We however reveal that, for disclination lines in 3D observed here, this is deceptively so, when observed in the laboratory frame. Using a comoving frame to remove the effect of drift induced by other defects and circumstances, we find that the dynamics of reconnecting disclinations is almost perfectly symmetric. We show that this is because disclination lines spontaneously take symmetric twist configurations when reconnecting, which are favored from the energetic viewpoint. These observations suggest a general mechanism of such spontaneous symmetry restoring, which can take place if topologically distinct asymmetric objects in lower dimensions become homeomorphic in higher dimensions.

**Method.** Here we describe the experimental methods employed for the main results on the in-plane reconnections, while changes for other experiments are described in Supplemental Text (Sec. II A). The liquid crystal sample was nematic compound MLC-2037 (Merck, discontinued product), doped with 0.5 wt% of tetra-n-butylammonium bromide and 0.005 wt% of fluorescent dye Coumarin 545T. The mesogen MLC-2037 was chosen for its low birefringence $\Delta n = 0.0649$ and negative dielectric anisotropy $\Delta \epsilon < 0$ (Table S1), the latter of which was used to induce electroconvection to generate disclinations [5]. The sample was introduced to a homemade cell, which consists of a coverslip and a glass plate both coated with indium tin oxide, and 130 $\mu$m thick Kapton tapes used as spacers. The inner surfaces were coated with polyvinyl alcohol and rubbed to realize a homogeneous planar alignment.

To study disclination dynamics, we generated a large density of disclinations by applying an alternating electric field (root-mean-square amplitude 150 V, frequency 50 Hz) to the sample, inducing an electrohydrodynamic turbulence called the dynamic scattering mode 2 [5, 18]. Then we removed the electric field and observed relaxation of disclinations by a confocal laser scanning microscope (Leica SP8, objective 20x, NA 0.75, oil immersion), equipped with a resonant scanner working at 8 kHz and a piezo objective scanner. The fluorescent dyes were excited at 488 nm by laser light polarized in the $x$-direction, respectively. The fluorescence signal in the range between 500 and 600 nm was confocally detected by a photomultiplier tube detector. The voxel size in the $xy$ plane was 0.91 $\mu$m and the spacing between $z$ slices was 1 $\mu$m. The number of voxels was 512, 128, 21 in the $x, y, z$ directions, respectively. The time interval between consecutive confocal images was 0.255 s. Compared with this, the time needed for fluorescent dyes to follow the evolution of disclination lines is expected to be much shorter, which we evaluate to be roughly 1 ms, using a length scale 0.33 $\mu$m reported as the apparent size of dye accumulation in [15] and the typical value of the diffusion coefficient of dye molecules, $10^{-10}$ m²/s [10, 19].

**Observations of reconstructions and loop shrinkage.** Upon removal of the applied voltage, we observed the formation of a large density of singular disclinations (winding number $\pm 1/2$). It was followed by coarsening dynamics including reconnections and loop shrinkage (Fig. 1 and Videos S1-S4), similar to those observed previously by bright-field microscopy after temperature or pressure quench [8, 20, 21]. We also observed non-singular disclinations (winding number $\pm 1$) terminating at singular ones (Fig. S1), as well as other kinds of defect structure, as reported in the past bright-field studies [20, 21]. Most disclination lines were found near the mid plane between the top and bottom surfaces and extended mostly horizontally, because of the homogeneous boundary condition we imposed. As a result, we were able to classify reconstructions into two kinds: in-plane reconnections [Fig. 1(a,d) and Video. S2] and intersecting reconstructions [Fig. 1(b,e) Video. S3]. An in-plane reconstruction consists of a pair of curved disclinations in a nearly single horizontal plane, which approach in that plane and reconnect [Fig. 1(d) and Video. S2]. An intersecting reconstruction consists of a pair of disclinations crossing at different $z$ positions, which approach vertically and reconnect [Fig. 1(e) and Video. S3]. In this case, the upper disclination appeared dark above the intersection [Fig. 1(e) inset] and apparently bent when the pair is close enough, presumably because of the lensing effect due to the lower disclination. Since this prevented quantitative analysis, in the following we focus on the in-plane reconstructions and study their reconnection dynamics. We analyzed a total of 38 in-plane reconstructions without any noticeable non-singular disclinations in the field of view.

**Scaling law for in-plane reconstructions.** Using the confocal images of the in-plane reconstructions, we extracted the 3D positions of the two disclinations, until the moment of the reconnection (see Supplemental Text, Sec. II B). Measuring how the minimum distance between the two disclinations, $\delta(t)$, decreases with time $t$ (Fig. 2), we found the following scaling law for all in-plane reconstructions:

$$\delta(t) \simeq C|t - t_0|^{1/2},$$

with a coefficient $C$. This power law is identical to that for annihilation of point disclinations in two-dimensional nematics [17, 22], as well as for reconstructions of quantum vortices in quantum fluids [23, 24]. It is interesting to note that, recently, Long et al. [25] theoretically evaluated force between straight disclinations in 3D nematics, showing that the power law (1) arises only when the two disclinations are strictly parallel. In our experiments, the disclinations are not straight but curved inward [Fig. 1(d)]. This suggests that only local directions at the closest points of the two disclinations, which are
Parallel for in-plane reconnections, influence on this scaling law.

**Apparent asymmetry in the laboratory frame.** In the case of point disclinations in 2D nematics, it is well known that pairs of +1/2 and −1/2 disclinations approach in an asymmetric manner, due to the different backflow generated by the two defects [16, 17]. It would be then natural to expect analogous asymmetry to arise for line disclinations in 3D nematics. However, this is not so trivial from the viewpoint of topology, because +1/2 and −1/2 disclinations are topologically equivalent (homeomorphic) in 3D nematics [1, 3, 5]. Besides, unlike point disclinations, line disclinations have shapes and are deformable, giving additional potential sources of asymmetry.

Here we inspected this asymmetry experimentally. Instead of the distance $\delta(t)$ between reconnecting disclinations, we measured the distance between each disclination line and the reconnection point, $D_1(t)$ and $D_2(t)$ [Fig. 3(a)]. Plotting $D_i(t)^2$ against $t - t_0$, we found a power law $D_i \approx C_i |t - t_0|^{1/2}$ analogous to Eq. (1), with coefficients $C_i$ that are typically asymmetric between the two disclinations [see Fig. 3(c) for an example]. Using this, we define the asymmetry parameter $A$ by

$$A = \frac{\max\{C_1, C_2\}}{\min\{C_1, C_2\}}$$

and determined it for each reconnection event. By definition, $A = 1$ for symmetric reconnections, and $A > 1$ for asymmetric ones. The histogram of $A$ [blue bars in Fig. 3(d)] shows that most in-plane reconnections appear to be significantly asymmetric. We suspected that different curvature of the two disclination lines may contribute to this asymmetry, but this effect turned out to be minuscule (Supplementary Text, Sec. III A).

**Disappearance of asymmetry in the co-moving frame.** Let us now recall the fact that disclinations have extended line structures and also that the studied pairs were not the only defects present in the system. It is therefore reasonable to consider that the reconnection dynamics may be affected by such extrinsic factors, which may induce flow and director changes superimposed to the intrinsic reconnection dynamics. These effects are expected to add a drift to the intrinsic motion of reconnecting disclinations. To evaluate this drift, we used the midpoint $\vec{M}(t)$ of the points on the disclinations closest to the reconnection point $\vec{X}_0$. (c) Distance $D_i$ measured in the laboratory frame, for an example pair of reconnecting disclinations. (d) Histograms of the asymmetry parameter $A$ (the square root of the ratio of the two slopes in (c), see text) measured in the laboratory frame (blue) and the co-moving frame (red). Note that two outliers are not displayed in the blue histogram, taking $A \approx 5.5$ and $A \approx 40$ in the laboratory frame, but in the co-moving frame all data including those outliers fell in the first bin ($\max A = 1.03 \pm 0.01$). (e) Distance $D_i$ measured in the co-moving frame, for the pair shown in (c). The inset shows the distance between the midpoint $\vec{M}(t)$ and the reconnection point $\vec{X}_0$ seen in the laboratory frame.
FIG. 4. Director configurations and asymmetry. (a) Sketch of the director field around a disclination, for different $\Omega$ (or $\beta$). (b) Sketch of the director field around a wedge (top) and twist (bottom) disclination pair.

dependence on time, suggesting that the intrinsic dynamics of reconnection is actually symmetric. Moreover, using the drift velocity $\tilde{V}$ evaluated from $\frac{d\vec{t}}{dt}$, we define a co-moving frame and measure the distance $\tilde{D}$, between the closest point and the reconnection point in this co-moving frame. The result shows that, remarkably, the reconnection dynamics in this co-moving frame is nearly perfectly symmetric [Fig. 3(e)]. We carried out this analysis for all reconnection events and for all cases the asymmetry parameter became very close to 1 [red bar in Fig. 3(d)], the largest deviation being $A = 1.03 \pm 0.01$.

Spontaneous symmetry restoring. We have found that the asymmetry present in the 2D pair annihilation of $\pm 1/2$ point disclinations disappears for the in-plane reconnections of 3D disclination lines. Obviously, if two disclination lines were straight and had $\pm 1/2$ director configurations around, this pair would exhibit the same asymmetry as its 2D counterpart, unless their director configurations change with time. However, since $+1/2$ and $-1/2$ disclinations are homeomorphic in 3D [1, 3, 5], the director configuration can change continuously between these two limiting structures. More precisely, the winding of the director around a disclination line is not characterized by the sign of the winding number, but a unit vector that specifies the rotation axis of the director, denoted by $\vec{\Omega}$ (see, e.g., [25]). With the unit tangent vector $\vec{t}$ whose head and tail are set arbitrarily, the director rotates by 180° in the plane perpendicular to $\vec{\Omega}$ counterclockwise, along a closed path that turns about the disclination counterclockwise with respect to $\vec{t}$ [Fig. 4(a)].

If $\vec{\Omega} = \vec{t}$ (or the angle $\beta = \cos^{-1}(\vec{\Omega} \cdot \vec{t}) = 0$), the director is essentially in the plane perpendicular to the disclination line and the defect is equivalent to a $+1/2$ point disclination in that plane. Similarly, if $\vec{\Omega} = -\vec{t}$ ($\beta = \pi$), it is equivalent to a $-1/2$ point disclination. These two limiting structures, called the wedge disclinations, are interpolated continuously by intermediate $\beta$. In particular, if $\vec{\Omega} \perp \vec{\Omega}$ ($\beta = \pi/2$), the director twists around the defect; hence, it is called a twist disclination.

Now, for a pair of reconnecting disclinations in plane, we have two tangent vectors $\vec{t}_1$ and $\vec{t}_2$ which are parallel near the reconnection point, so that we choose $\vec{t}_1 = \vec{t}_2$. Then it is reasonable to assume $\vec{\Omega}_1 = -\vec{\Omega}_2$ ($\beta_2 = \pi - \beta_1$) so that the disclinations may attract each other. This leaves one free parameter, $\beta_1$ (or $\Omega_1$). If $\beta_1 = 0$ or $\pi$, we have a pair of $\pm 1/2$ wedge disclinations [Fig. 4(b) top], which is equivalent to a pair of annihilating point disclinations in two-dimensional nematics and therefore asymmetric [16, 17]. By contrast, if $\beta = \pi/2$, we have a pair of twist disclinations [Fig. 4(b) bottom], for which the director field is symmetric and so is the dynamics.

Our experimental results of the vanishing asymmetry suggest that all disclination pairs we observed spontaneously took the symmetric twist configurations. This can be attributed to the anisotropic elasticity of liquid crystal: bulk deformation of the director can be decomposed into splay, twist, and bend deformations, characterized by different elastic constants denoted by $K_1$, $K_2$, and $K_3$, respectively [5]. For the mesogen used here, MLC-2037, these are $K_1 = 11.6$ pN, $K_2 = 6.1 \pm 0.5$ pN, $K_3 = 13.2$ pN (Table S1 and Supplementary Text, Sec. II C). Similarly to other typical mesogens, the elastic constant for twist deformations is lower than that for splay and bend deformations. Then it follows that the twist configuration of the disclination pair [Fig. 4(b) bottom] is energetically favored over the wedge configuration [Fig. 4(b) top]. This explains why the twist configuration seemed to be exclusively observed in our experiments, accounting for the disappearance of the asymmetry. We indeed confirm the realization of the twist configuration via the coefficient $C$ of the power law (1). Balancing the drag force $J\gamma_1(\delta/2)$ according to Geurts et al. [26], with a dimensionless coefficient $J \approx 1.9$, and the attractive force $\pi K/2\delta$ exerted to the pair with $\vec{\Omega}_1 = -\vec{\Omega}_2$ under the one-constant approximation $K_1 = K_2 = K_3 \equiv K$ [25], we obtain

$$C^2 = \frac{2\pi K}{J\gamma_1}.$$  

However, since the actual elastic constant is anisotropic, Eq. (3) is expected to hold with $K \approx K_1, K_3$ for the wedge configuration and $K \approx K_2$ for the twist one. From our data (Fig. 2), we obtain $C^2 = 152 \pm 4 \mu m^2/s$, which is close to the value for the twist configuration, $C^2 \approx 3.1 \times 10^2 \mu m^2/s$, instead of that for the wedge one, $C^2 \approx 1.5 \times 10^2 \mu m^2/s$.

It is important to note that such a lowest energy configuration is to describe the equilibrium state, while our
observations deal with relaxation to it. Upon quenching from the turbulent state, we expect that there exist various types of disclinations, from wedge to twist and in between. However, since all these configurations are homeomorphic, disclinations are allowed to change the configurations continuously, toward the lowest energy state, i.e., the twist configuration. This is not possible for 2D nematics, for which +1/2 and −1/2 disclinations are topologically distinct. One can generalize this argument as follows. Topological defects are mathematically characterized in terms of homotopy group [1], which generally depends on the dimensionality. In the case where topological defects of interest have two asymmetric structures that are topologically distinct in a lower dimension but become homeomorphic in a higher dimension, such as the case of ±1/2 nematic disclinations in the higher dimension is allowed to take an intermediate structure that continuously interpolates the two asymmetric analogues of those in the lower dimension. Then it is likely that a symmetric intermediate structure exists and, if it is energetically favored, the asymmetry present in the lower dimension will tend to disappear in the higher dimension spontaneously. Our results on reconnecting nematic disclinations constitute a clear example of such spontaneous restoring of symmetry.

**Concluding remarks.** We carried out a direct confocal observation of disclination dynamics in 3D nematics, using the accumulation of fluorescent dyes to disclinations. Our method successfully resolved characteristic dynamics of disclination lines, such as reconnections and loop shrinkage. Studying in-plane reconnection events in depth, we demonstrated the distance-time scaling law (1) predicted for parallel disclination pairs, despite the curved shape of the observed disclinations. Moreover, we revealed that the dynamics of reconnecting disclinations is only deceptively asymmetric in the 3D case, being actually symmetric in the co-moving frame. This is explained by the spontaneous realization of symmetric twist configurations, which is energetically favored because of the lower twist elasticity. These observations led us to propose a mechanism of such spontaneous symmetry restoring from a general viewpoint of topology.

Since the concept of topological defects is universal, it is important to think of similarities and dissimilarities in defect properties across different disciplines of physics. For example, quantum vortices in superfluid are known to have similar interaction energy and the same scaling law (1) follows, as observed experimentally [23, 24], while the corresponding homotopy group is different and the rotation axis $\vec{\Omega}$, if defined analogously, is fixed to $\vec{\Omega} = \hat{t}$ or $-\hat{t}$. Generally, the homotopy group for different symmetries of the order parameter field has been tabulated [1]. We hope that general mechanisms such as the spontaneous symmetry restoring illustrated in this work will accelerate multidisciplinary understanding of topological defects and that the visualization of nematic disclination dynamics reported here will be a useful tool in this line.

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Supplementary Information for
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I. SUPPLEMENTAL TABLES AND FIGURES

TABLE S1. Material parameters of MLC-2037 at temperature 20 °C, provided by Merck. Note that MLC-2037 is a discontinued product.

| Property                      | Value          |
|-------------------------------|----------------|
| refractive index (extraordinary) | $n_e = 1.5371$ |
| refractive index (ordinary)   | $n_o = 1.4722$ |
| refractive index (anisotropy) | $\Delta n = n_e - n_o = 0.0649$ |
| dielectric constant (parallel) | $\epsilon_\parallel = 3.6$ |
| dielectric constant (perpendicular) | $\epsilon_\perp = 6.7$ |
| dielectric constant (anisotropy) | $\Delta \epsilon = \epsilon_\parallel - \epsilon_\perp = -3.1$ |
| elastic constants (splay)     | $K_1 = 11.6$ pN |
| elastic constants (bend)      | $K_3 = 13.2$ pN |
| rotational viscosity          | $\gamma_1 = 132$ mPa·s |

FIG. S1. Non-singular disclination bridging a pair of singular disclinations. Confocal (a) and bright-field (b) images taken at the same moment are shown. The number of voxels was 512, 128, 31 in the x, y, z directions, respectively. The applied voltage before removal was of 150 V and 50 Hz. Other experimental conditions were the same as those used for the intersecting reconnection in Fig. 1(e), except that the time interval of the confocal images was 0.374 s here.

FIG. S2. Analysis of the case of point disclinations in 2D nematics. The data in Fig.2(a) of Ref. [1] were extracted and analyzed here, with permission of the authors. (a) Time evolution of the positions $x_i$ of a $+1/2$ (red) and a $-1/2$ (blue) point disclination before the pair annihilation. (b) The squared distance $D_i^2$ of each defect from the reconnection point. The asymmetry parameter is estimated at $A = 2.0 \pm 0.4$. (c) The distance $D_M$ between the midpoint $(x_1 + x_2)/2$ and the reconnection point. The inset shows $D_M^2$. 

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II. METHODS

A. Experimental conditions

All results on the in-plane reconnections presented in this work were obtained under the experimental conditions described in the main text. Below we describe experimental conditions used for other observations. Conditions and parameters that are not specified below were kept unchanged from those for the in-plane reconnections.

The intersecting reconnection displayed in Fig. 1(e) was observed in a sample that contained 0.25 wt% of tetra-n-butylammonium bromide and 0.2 wt% of Coumarin 545T. After an alternating voltage of root-mean-square amplitude 150 V and frequency 100 Hz was removed, we observed the intersecting reconnection in a manner similar to the case of the in-plane reconnections, except that the number of voxels was 512 × 64 × 36 and the time interval was 0.300 s.

The loop shrinkage displayed in Fig. 1(f) was observed in a sample that contained 0.05 wt% of tetra-n-butylammonium bromide and 0.005 wt% of Coumarin 545T.

B. Image analysis

The data acquired by the confocal microscope were the fluorescence intensity detected at 3D position (x, y, z) and time t. Using the 3D image at each time, we obtained cross sections and extracted the coordinates of the disclinations as follows. First we chose the cross sections to use, either in the xz plane or in the yz plane, chosen so that the cross sections become closer to perpendicular to the disclination lines. In each cross section, the two disclinations appear as bright spots. These bright spots were fitted by a Gaussian function to obtain the coordinates of the spot centers. Repeating this over all cross sections, we obtained a set of 3D coordinates along each disclination line.

The time and the position of each reconnection event, as well as the distance $D_i(i=1,2)$ of a disclination from the reconnection point, were determined as follows. For the reconnection time $t_0$, we determined it from the 2D image constructed from the transmitted excitation laser, to benefit from the finer time resolution than that of the confocal images. For the position $X_0 = (X_0, Y_0, Z_0)$ of the reconnection point, we first approximately located it from the series of transmitted and confocal images ($X_0$ and $Y_0$ from the transmitted images, $Z_0$ from the confocal images). Using this and the coordinates of the disclinations, we could evaluate the distance $D_i$ in the laboratory frame, but for the analysis presented in the paper, we evaluated $D_i$ more precisely in the following manner. First, we fitted the 3D coordinates of disclinations by smoothing splines, to reduce the noise and to interpolate the lines appropriately. In general, smoothing splines $s(x)$ for a data set $(x_i, y_i)$ are such a function that minimizes

$$p \sum_i w_i (y_i - s(x_i))^2 + (1-p) \int \left( \frac{d^2 s}{dx^2} \right)^2 dx, \quad (S1)$$

with a smoothing parameter $p$ and a weight $w_i$, which is set to be 1 here. By adjusting $p$, we obtained smoothing splines that reproduced the defect shape without high wave number components, for the two coordinates that spanned the cross sections (i.e., for $xz$ cross sections, the obtained smoothing splines were $x(y)$ and $z(y)$). Then, we also refined the estimate of the reconnection point $\hat{X}_0$, by using the coordinates of the disclinations before the moment of the reconnection. Specifically, we determined $\hat{X}_0$ in such a way that the scaling $D_i \simeq C_i |t - t_0|^{1/2}$ is satisfied most precisely in a time period before the reconnection, under the constraint that $\hat{X}_0$ is not changed by more than 3 µm from the first rough estimate. This was done by evaluating $D_i$ for each candidate position $\hat{X}_0$ in the range within 3 µm, fitting it to $D_i^2 = A_i |t - t_0| + B_i$, and choosing the candidate $\hat{X}_0$ that minimizes $B_i^2 + B_i^2$. The distance $\hat{D}_i$ in the co-moving frame was also determined analogously, by using the position $\hat{X}_0$ that drifts with the velocity of the co-moving frame.

C. Estimation of $K_2$

The twist elastic constant $K_2$ of MLC-2037 was evaluated by using the Fréedericksz transition under an external magnetic field [2]. The Fréedericksz transition point $H_i^c$ corresponding to the elastic constant $K_i$ is given by

$$H_i^c = \frac{\pi}{d} \sqrt{\frac{K_i}{\Delta \chi}}, \quad (S2)$$

where $d$ is the cell thickness and $\Delta \chi$ is the magnetic anisotropy. For MLC-2037, $\Delta \chi$ was unknown but $K_1$ is known (see Table S1). Therefore, we measured the Fréedericksz transition point for both the splay and twist configurations, obtaining $H_1^c$ and $H_2^c$, and used the ratio

$$\frac{H_2^c}{H_1^c} = \sqrt{\frac{K_2}{K_1}}, \quad (S3)$$

to determine $K_2$ from $K_1$.

We used a ready-made cell with homogeneous planar alignment (EHC, KSRO-25/B107M6NTS, $d = 25 \mu m$) filled with MLC-2037. Using a superconducting magnet, we applied a magnetic field perpendicular to the cell surface for the splay configuration, and parallel to the cell surface but perpendicular to the easy axis for the twist configuration. The Fréedericksz transition point was determined by measuring the retardation change, through the transmitted light intensity that changed in a swept magnetic flux density $B$ under crossed Nicols (Fig. S3).
FIG. S3. Observation of the Fréedericksz transition of MLC-2037, for the splay (a) and twist (b) configurations. The intensity of the transmitted light $I_{TM}$ is shown against the applied magnetic flux density $B$. The data shown here were obtained for increasing $B$ and by using a halogen lamp as a light source. The dashed lines indicate $B_c$ from these data.

The measurement for the twist configuration was performed at oblique incidence ($5^\circ$) to reduce the effect of polarization rotation [3, 4].

We measured the Fréedericksz transition eight times for the splay configuration and three times for the twist configuration. The light source was either a halogen lamp or a light-emitting diode. For each measurement, we determined the transition point twice, when the magnetic field was increased and decreased. As a result, we obtained a total of 16 estimates of the transition point $B_1^c$ and 6 estimates of $B_2^c$. By using all of them, we determined our final estimates at $B_1^c = 4.4 \pm 0.1$ T and $B_2^c = 3.2 \pm 0.1$ T. Then it follows, by using Eq. (S3) and $K_1 = 11.6$ pN (Table S1), that $K_2 = 6.1 \pm 0.5$ pN.

### III. SUPPLEMENTAL ANALYSIS

#### A. Evaluation of the curvature effect

Here we evaluate the effect of different curvature of the two disclinations, as a potential source of the apparently asymmetric dynamics observed in the laboratory frame. It is well known that disclinations have a line tension $T$ apart from a logarithmic correction, which is roughly the elastic constant $K$ under the one-constant approximation, $T \approx K$ [2, 5]. This implies curvature-driven force per length, $T/R$, exerted to a curved disclination, where $R$ is the local radius of curvature. Geurst et al. [5] described this phenomenologically and proposed that the contribution of this force to the disclination velocity is approximately $v_{\text{curv}} \approx K/\gamma_1 R$, where $\gamma_1$ is the rotational viscosity. We used this to compensate for the effect of curvature. The radius of curvature at each point on the disclination line was determined from the smoothing splines we obtained in the image analysis (Sec. II B).

First, we used the power law $D_i \approx C_i|t-t_0|^{1/2}$ in the laboratory frame and evaluated the velocity $\frac{dD_i}{dt}$. The solid lines in Fig. S4(b) display $|\frac{dD_i}{dt}|$ for the pair shown in Fig. S4(a) [same as Fig. 3(c) in the main paper]. We then evaluated the curvature contribution $v_{\text{curv}} = K/\gamma_1 R_i$, with $K = K_3$ and the radius of curvature $R_i$ measured at the point closest to the reconnection point, and plotted $|\frac{dD_i}{dt}| + v_{\text{curv}}$ in Fig. S4(b) (open symbols). This figure also shows, by the dashed lines, the results of fitting by

$$|\frac{dD_i}{dt}| + v_{\text{curv}} \approx C_i^2|t-t_0|^{-1/2}.$$  

(S4)

Comparing the speed with and without compensation of the curvature effect [dashed and solid lines in Fig. S4(b), respectively], we find that the contribution of the curvature effect is much smaller than the observed asymmetry. We confirmed this for the ensemble of the reconnection events, by re-evaluating the asymmetry parameter $A$ [Eq. (2) in the main paper], i.e.,

$$A \equiv \frac{\max \{C_1, C_2\}}{\min \{C_1, C_2\}},$$  

(S5)

and making a histogram [red bars in Fig. S4(c)]. Therefore, we conclude that the asymmetry observed in the laboratory frame is far stronger than the asymmetry induced by curvature.
FIG. S4. Evaluation of curvature effect. (a) Distance $D_i$ measured in the laboratory frame, for an example pair of reconnecting disclinations. This panel is identical to Fig. 3(c). (b) Speed $|\frac{dD_i}{dt}|$ for the disclination pair shown in (a). The dashed lines indicate the speed evaluated from the fits shown in (a), without compensation of the curvature effect. The results after compensation of the curvature effect (see text) are shown by open symbols (data) and dashed lines [fit by Eq. (S4)]. (c) Histograms of the asymmetry parameter $A$, with (red) and without (blue) the compensation of the curvature effect. Note that the two outliers described in the caption of Fig. 3(e) are also outside the range displayed here, for both of the histograms.

IV. VIDEO CAPTIONS

**Video S1:** A relaxation process from the electrohydrodynamic turbulence upon removal of the applied voltage. Real playback speed. The spacing between $z$ slices was 1.5 µm. The number of voxels was 512, 512, 20 in the $x, y, z$ directions, respectively. The time interval between consecutive confocal images was 0.760 s. The other conditions were the same as those for the in-plane reconnections.

**Video S2:** The in-plane reconnection shown in Fig. 1(d). Real playback speed.

**Video S3:** The intersecting reconnection shown in Fig. 1(e). Real playback speed.

**Video S4:** The loop shrinkage shown in Fig. 1(f). Real playback speed.

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