Near-Thermal Radiation in Detectors, Mirrors and Black Holes: A Stochastic Approach

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Abstract

In analyzing the nature of thermal radiance experienced by an accelerated observer (Unruh effect), an eternal black hole (Hawking effect) and in certain types of cosmological expansion, one of us proposed a unifying viewpoint that these can be understood as arising from the vacuum fluctuations of the quantum field being subjected to an exponential scale transformation in these systems. This viewpoint, together with our recently developed stochastic theory of particle-field interaction understood as quantum open systems described by the influence functional formalism, can be used effectively to address situations where the spacetime possesses an event horizon only asymptotically, or none at all. Examples studied here include detectors moving at uniform acceleration only asymptotically or for a finite time, a moving mirror, and a two-dimensional collapsing mass. We show that in such systems radiance indeed is observed, albeit not in a precise Planckian spectrum. The deviation therefrom is determined by a parameter which measures the departure from uniform acceleration or from exact exponential expansion. These results are expected to be useful for the investigation of non-equilibrium black hole thermodynamics and the linear response regime of backreaction problems in semiclassical gravity.

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1 Introduction

Particle production \[1\] with a thermal spectrum from black holes \[2, 3, 4\], moving mirrors \[5\], accelerated detectors \[6\], observers in de Sitter Universe \[7\] and certain cosmological spacetimes \[8\] has been a subject of continual discussion since the mid-seventies because of its extraordinary nature and its basic theoretical value. The mainstream approach to these problems relied on thermodynamic arguments \[9, 10\], finite temperature field theory techniques \[11, 12, 13\], or geometric constructions (event horizon as a global property of spacetime) \[14\], or pairwise combinations thereof. The status of work on quantum field theory in curved spacetimes up to 1980 can be found in \[15\]. The eighties saw attempts and preparations for the backreaction problem \[16\] (for cosmological backreaction problems, see \[17\]), i.e., the calculation of the energy momentum tensor (see \[18\] and earlier references), the effect of particle creation on a black hole (in a box, to ensure quasi-equilibrium with its radiation) \[14\], and the dynamical origin of black hole entropy \[19\]. These inquiries are mainly confined to equilibrium thermodynamics or finite-temperature field theory conditions.\[1\] To treat problems of a dynamical nature such as the backreaction of Hawking radiation on black hole collapse, one needs a new conceptual framework and a more powerful formalism for tackling non-equilibrium conditions and high energy (trans-Planckian) processes. A new viewpoint which stresses the local, kinematic nature of these processes rather than the traditional global geometric properties has been proposed \[30, 31, 32, 33\] which regards the Hawking-Unruh thermal radiance observed in one vacuum as resulting from exponential redshifting of quantum noise of another. This view puts the nature of thermal radiance in the two classes of spacetimes on the same footing \[34\], and empowers one to tackle situations which do not possess an event horizon at all, as the examples in this and a companion paper will show.

Such a formalism of statistical field theory has been developed by one of us and co-workers in recent years \[35, 36, 37, 38, 39\]. This approach aims to provide the quantum statistical underpinnings of field theory in curved spacetime, and strives at a microscopic and elemental description of the structure and dynamics of matter and spacetime. The starting point is the quantum and thermal fluctuations for fields, the focus is on the evolution of the reduced density matrix of an open system (or the equivalent distribution or Wigner functions); the quantities of interest are the noise and dissipation kernels contained in the influence functional \[10\], and the equation of motion takes the form of a master, Langevin, Fokker-Planck, or stochastic Schrödinger equation describing the evolution of the quantum statistical state of the system, including, in addition to the quantum field effects like radiative corrections and renormalization, also statistical dynamical effects like decoherence, correlation and dissipation. Since it contains the causal (Schwinger-Keldysh) effective action \[11\] it is a generalization of the traditional scheme of thermal field theory \[13\] and the ‘in-out’ (Schwinger-DeWitt) effective action \[12\], and is particularly suited for treating fluctuations and dissipation in backreaction problems in semiclassical gravity \[19\].

The foundation of this approach has been constructed recently based on the open system

\[1\]Among other notable alternatives, we’d like to mention Sciama’s dissipative system approach \[20\], Unruh’s work on sonic black holes \[21\] (see also Jacobson \[22\]), Zurek and Thorne’s degree of freedom counts \[23\], Sorkin’s geometric or ‘entanglement’ entropy \[24, 25\] (see also \[26\]), and Bekenstein-Page’s information theory approach \[27, 28\]. See also the views expressed earlier by Stephens, t’Hooft and Whiting \[29\].
concepts and the quantum Brownian model [43, 44]. The method has since been applied to particle creation and backreaction processes in cosmological spacetimes [45, 46, 47]. For particle creation in spacetimes with event horizons, such as for an accelerated observer and black holes, this method derives the Hawking and Unruh effect [48, 44] from the viewpoint of exponential amplification of quantum noise [33]. It can also describe the linear response regime of backreaction viewed as a fluctuation-dissipation relation. [47, 49].

This paper is a continuation of our earlier work [44, 48, 49] to present two main points: 1) A unified approach to treat thermal particle creation from both spacetimes with and without event horizons [34] based on the interpretation that the thermal radiance can be viewed as resulting from quantum noise of the field being amplified by an exponential scale transformation in these systems (in specific vacuum states) [33]. In contradistinction to viewing these as global, geometric effects, this viewpoint emphasizes the kinematic effect of scaling on the vacuum in altering the relative weight of quantum versus thermal fluctuations.

2) An approximation scheme to show that near-thermal radiation is emitted from systems undergoing near-uniform acceleration or in slightly perturbed spacetimes. We wish to demonstrate the relative ease in constructing perturbation theory using the statistical field theory methods. Let us elaborate on these two points somewhat.

It may appear that this approximation can be equally implemented by taking the conventional viewpoints (notably the geometric viewpoint), and the perturbative calculation can be performed by other existing methods (notably the thermal field theory method). But as we will show here, it is not as easy as it appears. Conceptually, the geometric viewpoint assumes that a sufficient condition for the appearance of Hawking radiation is the existence of an event horizon, which is considered as a global property of the spacetime or the system. (Note for the case of an extreme Reissner-Nordstrom spacetime, this is not the case, as there exists an event horizon but no radiation. 2) When the spacetime deviates from the eternal black hole, or that the trajectory deviates from the uniformly accelerated one, physical reasoning tells us that the Hawking or Unruh radiation should still exist, albeit with a non-thermal spectrum. But the event horizon, if exists, of the deformed spacetime may not be so easily described in geometric terms. And for time-dependent perturbations of lesser symmetry, or for situations where uniform acceleration occurs only for a finite interval of time, it is not easy to deduce the form of Hawking radiation in terms of purely global geometric quantities (see, however, Wald [50] and Teitelboim et al [51]). The concept of an approximate event horizon, which exists for a finite period of time or only asymptotically, is difficult to define and even if it is possible (by apparent horizons, e.g., [52]), rather unwieldy to implement in the calculation of particle creation and backreaction effects.

Technically one may think calculations via the thermal field theory are equally possible. Indeed this has been tried before by one of us and others. One way is to assume a quasi-periodic condition on the Green function, making it near-thermal [13]. But this is not a good solution, as the deviation from eternal black hole or uniform acceleration disables the Euclidean section in the spacetimes (Kruskal or Rindler), and the imaginary time finite temperature theory is not well defined any more. Besides, to deal with the statistical

This is the one case which arose in the discussion between Hu and Unruh (private communication, 1988) who shared the somewhat unconventional view that the exponential redshifting is a more basic mechanism than event horizons responsible for thermal radiance.
dynamics of the system, one should use an in-in boundary condition and work with causal Green functions. The lesson we learned in treating the backreaction problems of particle creation in cosmological spacetimes \[35, 45\] is that one can no longer rely on methods which are restricted to equilibrium conditions (like the imaginary-time or thermo-field dynamics methods), but should use nonequilibrium methods such as the Schwinger-Keldysh (closed time path) effective action \[11\] for the treatment of backreactions. Its close equivalent, the influence functional method \[40\] is most appropriate for investigating the statistical mechanical aspects of matter and geometry, like the entropy of quantum fields and spacetimes, information flow, coherence loss, etc. \[53\].

In this and a companion paper \[44\] we shall use these methods to analyze particle creation in perturbed situations whose background spacetime possess an event horizon, such as an asymptotically uniformly-accelerated detector, or one accelerated in a finite time interval (Sec. 3), the moving mirror (Sec. 4) and the asymptotically Schwarzschild spacetime (Sec. 5). In the follow-up paper we shall study near-thermal particle creation in an exponentially expanding universe, a slow-roll inflationary universe, and a universe in asymptotically-exponential expansion. What ties the problem of thermal radiation in cosmological as well as black hole spacetimes together is the exponential scale-transformation viewpoint expressed earlier \[30, 31, 32, 33, 34\]. The stochastic theory approach is capable of implementing this view. One can describe all these systems with a single parameter measuring the deviation from uniformity or stationarity, and show that the same parameter also appears in the near-thermal behavior of particle creation in all these systems. This result will be used later in our exploration of the linear-response regime of the backreaction problem in semiclassical gravity.

1.1 General Formalism

Consider a particle detector linearly coupled to a quantum field. The dynamics of the internal coordinate \(Q\) of the detector in a wide class of spacetimes is derived in \[44\], and can be described by Langevin equations of the form:

\[
\frac{\partial L}{\partial Q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{Q}} - 2 \int_{t_i}^{t} \mu(t,s) Q(s) ds = \xi(t)
\]  

(1.1)

where \(\xi(t)\) is a stochastic force with correlator \(\langle \xi(t)\xi(t') \rangle = \hbar \nu(t,t')\). The trajectory \(x^\mu(s)\) of the detector, parametrized by a suitable parameter \(s\), will be denoted simply by \(x(s)\) for convenience.

For the special case of linear coupling between a field \(\phi\) and the detector of the form \(L_{\text{int}} = eQ\phi(x(s))\), the kernels \(\mu\) and \(\nu\), called the dissipation and noise kernels respectively, are given by

\[
\mu(s,s') = \frac{e^2}{2} G(x(s),x(s')) \equiv -i \frac{e^2}{2} \langle \hat{\phi}(x(s)),\hat{\phi}(x(s')) \rangle >
\]  

(1.2)

\[
\nu(s,s') = \frac{e^2}{2} G^{(1)}(x(s),x(s')) \equiv \frac{e^2}{2} \{\hat{\phi}(x(s)),\hat{\phi}(x(s'))\} >
\]  

(1.3)

where \(G\) and \(G^{(1)}\) are the Schwinger and the Hadamard functions of the free field operator \(\hat{\phi}\) evaluated for two points on the detector trajectory, \(<\>\) denotes expectation value with
respect to a vacuum state at some arbitrarily chosen initial time $t_i$, and $[,]$ and $\{,\}$ denote the commutator and anticommutator respectively. This result may be obtained either by integrating out the field degrees of freedom as in the Feynman-Vernon influence functional approach [40] or via manipulations of the coupled detector-field Heisenberg equations of motion in the canonical operator approach.

It will often be convenient to express the kernels $\mu$ and $\nu$ as the real and imaginary parts of a complex kernel $\zeta = \nu + i\mu$, called the influence kernel. For linear couplings, it follows from the above expressions that $\zeta$ is given by the Wightman function $G^+$:

$$\zeta(s, s') = e^2G^+(x(s), x(s')) \equiv e^2 < \hat{\phi}(x(s))\hat{\phi}(x(s'))>.$$  \hfill (1.4)

The influence kernel thus admits the mode function representation

$$\zeta(s, s') = e^2 \sum_k u_k(x(s))u_k^*(x(s')),$$ \hfill (1.5)

the $u_k$’s being the fundamental modes satisfying the field equations and defining the particular Fock space whose vacuum state is the one chosen above. This method of evaluating the kernels $\mu$ and $\nu$ is only applicable for linear coupling cases.

An alternative approach [44], consists of decomposing the field Lagrangian into parametric oscillator Lagrangians at the very outset, thus converting a quantum field-theoretic problem to a quantum mechanical one. Denoting the parametric oscillator degrees of freedom by $q_k$ (and their masses and frequencies by $m_k$ and $\omega_k$ respectively), the detector-field interaction mentioned earlier is generally given by $L_{\text{int}} = \sum_k c_k(s)Qq_k$, where the coupling “constants” $c_k$ now become time-dependent, and contain information about the detector trajectory. In this approach, the influence kernel is given in terms of the oscillator mode functions $X_k$, as

$$\zeta(s, s') = \int_0^\infty dk I(k, s, s')X_k(s)X^*_k(s')$$ \hfill (1.6)

where the $X_k$’s satisfy the parametric oscillator equations

$$\ddot{X}_k + \omega_k^2(t)X_k = 0$$ \hfill (1.7)

satisfying the initial conditions $X_k(t_i) = 1$ and $\dot{X}_k(t_i) = -i\omega(t_i)$. The spectral density function $I(k, s, s')$ is defined as

$$I(w, s, s') = \sum_k \delta(w - \omega_k)\frac{c_k(s)c_k(s')}{2m_k(t_i)\omega_k(t_i)}.$$ \hfill (1.8)

One may decompose the influence kernel into its real and imaginary parts, thus obtaining the dissipation and noise kernels:

$$\mu(s, s') = \frac{i}{2} \int_0^\infty dk I(k, s, s')\left[X_k(s)X_k(s') - X_k^*(s')X^*_k(s)\right]$$ \hfill (1.9)

$$\nu(s, s') = \frac{1}{2} \int_0^\infty dk I(k, s, s')\left[X_k(s)X_k(s') + X_k^*(s')X^*_k(s)\right].$$ \hfill (1.10)

By expressing the field as a collection of parametric oscillators, it can be explicitly verified that the two approaches mentioned above lead to the same result for the influence kernel $\zeta$. 5
For the purpose of calculating it in a specific case, we will find it more convenient to use the second approach.

To study the thermal properties of the radiation measured by a detector, the influence kernel is compared to that of a thermal bath of static oscillators each in a coherent state [44]:

\[
\zeta = \int_0^\infty dk \ I_{\text{eff}}(k, \Sigma) \ [C_k(\Sigma) \cos k \Delta - i \sin k \Delta] \tag{1.11}
\]

where

\[
\Sigma = (t + t')/2, \quad \Delta = t - t'
\]

and the function \( C_k = \coth \frac{\hbar k}{2k_B T} \). We will show in the specific cases discussed below that the unknown function \( C_k \) indeed has a coth form, and can then deduce the temperature of the radiation seen by the detector. Here \( I_{\text{eff}}(k, \Sigma) \) is the effective spectral density, also to be determined by formal manipulations of (1.6). We can always write \( \zeta \) in this way since \( \nu \) is even in \( \Delta \) while \( \mu \) is odd. By equating the real and imaginary parts of the two forms of \( \zeta \) and fourier inverting, we obtain

\[
I_{\text{eff}} C_k = \frac{1}{\pi} \int_{-\infty}^{\infty} d\Delta \ \cos k \Delta \ \nu(\Sigma, \Delta) \tag{1.12}
\]

\[
I_{\text{eff}} = -\frac{1}{\pi} \int_{-\infty}^{\infty} d\Delta \ \sin k \Delta \ \mu(\Sigma, \Delta). \tag{1.13}
\]

We will now consider various examples where \( \zeta \) is evaluated and shown to have, to zeroth order, a thermal form. Higher-order corrections to \( \zeta \) give a near-thermal spectrum. In principle, the real and imaginary parts of the influence kernel may be substituted in the Langevin equation (1.1) to yield stochastic near-thermal fluctuations of the detector coordinate \( Q \). This procedure will be demonstrated in the example of a finite time uniformly accelerated detector (Sec. 3 below). In this way, the methodology presented above describes a stochastic approach to the problem of detector response, as opposed to the usual perturbation theory approach (where the perturbation parameter is \( e^2 \)) involving the calculation of detector transition probabilities. It should be emphasized that equation (1.1) is exact for linear coupling and does not involve a perturbation expansion in \( e^2 \) (for linear systems, such an expansion is, in fact, unnecessary because they are exactly solvable).
2 Asymptotically Uniformly-Accelerated Observer

We first consider the case of a non-uniformly accelerated monopole detector in $1 + 1$ dimensions. For a general detector trajectory $(x(\tau), t(\tau))$ parametrized by the proper time $\tau$, it has been shown [49] that the function $\zeta(\tau, \tau')$ is

$$\zeta(\tau, \tau') \equiv \nu + i\mu = \frac{e^2}{2\pi} \int_0^\infty \frac{dk}{k} e^{-ik(t(\tau)-t(\tau'))} \cos k(x(\tau) - x(\tau')).$$

(2.1)

Here $e$ is the coupling constant of the detector to a massless scalar field (initially in its ground state). The initial state of the detector is unspecified at the moment and would appear as a boundary condition on the equation of motion of the detector. Here, however, we are primarily interested in the noise and dissipation kernels themselves, as properties of the field, and not in the state of the detector.

First, we note that the function $\zeta$ can be separated into advanced and retarded parts, in terms of the advanced and retarded null coordinates $v(\tau) = t(\tau) + x(\tau)$ and $u(\tau) = t(\tau) - x(\tau)$ respectively:

$$\zeta^a(\tau, \tau') = \frac{e^2}{4\pi} \int_0^\infty \frac{dk}{k} e^{-ik(v(\tau)-v(\tau'))},$$

(2.2)

$$\zeta^r(\tau, \tau') = \frac{e^2}{4\pi} \int_0^\infty \frac{dk}{k} e^{-ik(u(\tau)-u(\tau'))},$$

$$\zeta(\tau, \tau') = \zeta^a(\tau, \tau') + \zeta^r(\tau, \tau').$$

In the case when the detector is uniformly accelerated with acceleration $a$, its trajectory is given by:

$$v(\tau) = \frac{1}{a} e^{a\tau}; \quad u(\tau) = \frac{1}{a} e^{-a\tau}.$$

(2.3)

Substitution of the above trajectory into equations (2.2) yields a thermal, isotropic detector response at the Unruh temperature $a/(2\pi)$ [44, 49].

2.1 Perturbation increasing with time

The above analysis is now applied to the case of near-uniform acceleration by introducing a dimensionless $h$ parameter which measures the departure from exact uniform acceleration:

$$h = \frac{\dot{a}}{a^2}$$

(2.4)

where the overdot indicates derivative with respect to the proper time. The trajectory of the detector is now chosen to be:

$$v(\tau) = \frac{1}{a(\tau)} e^{\int a(\tau) d\tau}; \quad u(\tau) = \frac{1}{a(\tau)} e^{-\int a(\tau) d\tau}.$$

(2.5)

One can expand $a(\tau)$ in a Taylor series about the acceleration at $\tau = 0$:

$$a(\tau) = a_0 + \sum_{n=1}^\infty \frac{\tau^n}{n!} a_0^{(n)}$$

(2.6)
where \(a_0^{(n)}\) denotes the \(n\)-th derivative of \(a\) at \(\tau = 0\). We make the assumption of ignoring second and higher derivatives of \(a\). This implies

\[
a(\tau) = a_0 + h_0 \tau a_0^2
\]

(2.7)

where \(h_0 = a_0/a_0^2\).

Hereafter, we shall also make the further assumption of evaluating quantities to first order in \(h_0\). In this approximation, \(h = h_0\) to first order in \(h_0\). Then there is no distinction between \(h\) and \(h_0\) (\(h\) is essentially constant), and we can safely drop the subscript and work with \(h\) alone. It should be noted that expanding quantities to first order in \(h_0\) actually involves expansion of quantities to first order in \(h\tau a_0\), and hence, for arbitrary trajectories, the final results are to be considered valid over time scales \(\tau\) such that \(\tau \ll (h a_0)^{-1}\). Alternatively, equation (2.7) can be taken to define a family of trajectories for which this analysis applies.

Using the linearized form of \(a(\tau)\), one can now obtain the trajectory explicitly, to first order in \(h_0\). The result is:

\[
v(\tau) = a_0^{-1}e^{a_0 \tau} \left( 1 + h\tau a_0 \left( \frac{a_0 \tau}{2} - 1 \right) \right) \\
u(\tau) = -a_0^{-1}e^{-a_0 \tau} \left( 1 - h\tau a_0 \left( \frac{a_0 \tau}{2} + 1 \right) \right)
\]

(2.8)

One also finds, to first order in \(h_0\),

\[
e^{-ik(v(\tau) - v(\tau'))} = e^{-\frac{2ik}{a_0}e^{a_0 \Sigma} \sinh(\frac{a_0 \Delta}{2})} \left[ 1 - ikhe^{a_0 \Sigma} \left( \frac{a_0 \Delta^2}{4} + a_0 \Sigma^2 - 2\Sigma \right) \sinh(\frac{a_0 \Delta}{2}) + \Delta(a_0 \Sigma - 1) \cosh(\frac{a_0 \Delta}{2}) \right]
\]

(2.9)

\[
e^{-ik(u(\tau) - u(\tau'))} = e^{-\frac{2ik}{a_0}e^{-a_0 \Sigma} \sinh(\frac{a_0 \Delta}{2})} \left[ 1 + ikhe^{-a_0 \Sigma} \left( \frac{a_0 \Delta^2}{4} + a_0 \Sigma^2 + 2\Sigma \right) \sinh(\frac{a_0 \Delta}{2}) - \Delta(a_0 \Sigma + 1) \cosh(\frac{a_0 \Delta}{2}) \right]
\]

(2.10)

where \(\Delta = \tau - \tau', \Sigma = \frac{1}{2}(\tau + \tau')\).

Using the identities

\[
e^{-2i\frac{k}{a_0}e^{-a_0 \Sigma} \sinh(\frac{a_0 \Delta}{2})} = \frac{4}{\pi} \int_0^\infty d\nu K_{2inu}(2k/a_0 e^{-a_0 \Sigma})[\cosh(\pi \nu) \cos(\nu a_0 \Delta) - i \sinh(\pi \nu) \sin(\nu a_0 \Delta)]
\]

(2.11)

and

\[
|\Gamma(i\nu)|^2 = \frac{\pi}{\nu \sinh \pi \nu}; \quad |\Gamma(\frac{1}{2} + i\nu)|^2 = \frac{\pi}{\cosh \pi \nu}
\]

(2.12)

one finally obtains, after some simplification,

\[
\zeta^a(\tau, \tau') = \frac{e^2}{4\pi} \int_0^\infty \frac{dk}{k} \left[ \coth(\frac{\pi k}{a_0}) \cos(k\Delta)(1 + h_1) - i \sin(k\Delta) \right]
\]

(2.13)

\[
\zeta^r(\tau, \tau') = \frac{e^2}{4\pi} \int_0^\infty \frac{dk}{k} \left[ \coth(\frac{\pi k}{a_0}) \cos(k\Delta)(1 + h_2) - i \sin(k\Delta) \right]
\]

(2.14)
with

\[
\Gamma_1 = -k \tan(k\Delta) \tanh^2\left(\frac{\pi k}{a_0}\right) \left[ \left(\frac{a_0\Delta^2}{4} + a_0\Sigma^2 - 2\Sigma\right) \sinh\left(\frac{a_0\Delta}{2}\right) + \Delta(a_0\Sigma - 1) \cosh\left(\frac{a_0\Delta}{2}\right) \right]
\]

\[
\Gamma_2 = k \tan(k\Delta) \tanh^2\left(\frac{\pi k}{a_0}\right) \left[ \left(\frac{a_0\Delta^2}{4} + a_0\Sigma^2 + 2\Sigma\right) \sinh\left(\frac{a_0\Delta}{2}\right) - \Delta(a_0\Sigma + 1) \cosh\left(\frac{a_0\Delta}{2}\right) \right]
\]

(2.15)

The advanced and retarded parts of \( Re(\zeta) \) being unequal, the noise is anisotropic. Adding expressions (2.12) and (2.13), we have

\[
\zeta(\tau, \tau') = \frac{e^2}{2\pi} \int_0^\infty \frac{dk}{k} \left[ \coth\left(\frac{\pi k}{a_0}\right) \cos(k\Delta)(1 + \alpha \Gamma_1) - i \sin(k\Delta) \right] (2.16)
\]

where

\[
\Gamma = \frac{\Gamma_1 + \Gamma_2}{2} = k\Sigma \tan(k\Delta) \tanh^2\left(\frac{\pi k}{a_0}\right)(2 \sinh\left(\frac{a_0\Delta}{2}\right) - a_0\Delta \cosh\left(\frac{a_0\Delta}{2}\right)).
\]

(2.17)

The noise experienced by the detector is thus identical to the noise experienced in a heat bath, with a small correction, \( \Gamma \). The accelerated detector therefore has a near-thermal response at the Unruh temperature \( a_0/(2\pi) \) with an order \( h \) correction which increases with time.

### 2.2 Perturbation exponentially decreasing with time

We will now consider a trajectory for the accelerated detector which exponentially approaches the uniformly accelerated trajectory at late times. This trajectory, in null coordinates, has the form

\[
v(\tau) = a_0^{-1}e^{a_0 \tau}(1 + \alpha e^{-\gamma \tau}); \quad u(\tau) = -a_0^{-1}e^{-a_0 \tau}(1 + \alpha e^{-\gamma \tau}).
\]

(2.18)

In this case, the magnitude of the proper acceleration is, to first order in \( \alpha \),

\[
a(\tau) = a_0\{1 + \alpha e^{-\gamma \tau}(1 + \frac{\gamma^2}{a_0^2})\} + O(\alpha^2).
\]

(2.19)

The influence kernel is obtained in a manner similar to the treatment of the previous subsection. Here, we get

\[
\zeta^a(\tau, \tau') = \frac{e^2}{4\pi} \int_0^\infty \frac{dk}{k} \left[ \coth\left(\frac{\pi k}{a_0}\right) \cos(k\Delta)(1 + \alpha \Gamma_1) - i \sin(k\Delta) \right] (2.20)
\]

\[
\zeta^r(\tau, \tau') = \frac{e^2}{4\pi} \int_0^\infty \frac{dk}{k} \left[ \coth\left(\frac{\pi k}{a_0}\right) \cos(k\Delta)(1 + \alpha \Gamma_2) - i \sin(k\Delta) \right] (2.21)
\]

with

\[
\Gamma_1 = -2ka_0^{-1}e^{-\gamma \Sigma} \sinh\left(\frac{a_0 - \gamma}{2}\right) \tan(k\Delta) \tanh^2\left(\frac{\pi k}{a_0}\right)
\]

\[
\Gamma_2 = -2ka_0^{-1}e^{-\gamma \Sigma} \sinh\left(\frac{a_0 + \gamma}{2}\right) \tan(k\Delta) \tanh^2\left(\frac{\pi k}{a_0}\right)
\]

(2.22)
The noise is again seen to be anisotropic. Adding $\zeta^a$ and $\zeta^r$, we have

$$
\zeta(\tau, \tau') = \frac{e^2}{2\pi} \int_0^{\infty} \frac{dk}{k} \left[ \coth(\frac{\pi k}{a_0}) \cos(k\Delta)(1 + \alpha\Gamma) - i \sin(k\Delta) \right]
$$

(2.23)

where

$$
\Gamma = \frac{\Gamma_1 + \Gamma_2}{2} = -2ka_0^{-1} e^{-\gamma \Sigma} \sinh \frac{a_0 \Delta}{2} \cosh \frac{\gamma \Delta}{2} \tan(k\Delta) \tanh^2 \left( \frac{\pi k}{a_0} \right).
$$

(2.24)

In this case, the correction to the thermal spectrum is exponentially suppressed at late times. This feature will distinguish the behavior of quantum fields in the vicinity of a moving mirror and a collapsing mass, as shown in later sections.
3 Finite-Time Uniformly-Accelerated Detector

In this section, we consider a detector trajectory which is a uniformly accelerated one for a finite interval of time \((-t_0, t_0)\). Before and after this interval, the trajectory is taken to be inertial, at uniform velocity. To ensure continuity of the proper time along this trajectory, the velocity of the detector is assumed to vary continuously at the junctions \(\pm t_0\).

With these constraints, the trajectory is chosen to be

\[
x(t) = \begin{cases} 
  x_0^{-1}(a^{-2} - t_0 t), & t < -t_0 \\
  (a^{-2} + t^2)^{1/2}, & t > -t_0, \ t < t_0 \\
  x_0^{-1}(a^{-2} + t_0 t), & t > t_0.
\end{cases}
\] (3.1)

The trajectory is symmetric under the interchange \(t \to -t\). \(a\) is the magnitude of the proper acceleration during the uniformly accelerated interval \((-t_0, t_0)\) of Minkowski time and \(x_0\) is the position of the detector at time \(t_0\). \(x_0\) and \(t_0\) are related by \(x_0^2 - t_0^2 = a^{-2}\).

Before the uniformly accelerated interval, the detector has a uniform velocity \(-t_0/x_0\) (we have chosen units such that \(c = 1\); if one keeps factors of \(c\), the velocity is \(-c^2t_0/x_0\)) and after this interval, its velocity is \(t_0/x_0\). This trajectory thus describes an observer traveling at constant velocity, then turning around and traveling with the same speed in the opposite direction. The “turn-around” interval corresponds to the interval of uniform acceleration.

We may also define null coordinates \(u = t - x\) and \(v = t + x\). In terms of these, the time at which the trajectory crosses the future horizon \(u = 0\) of the uniformly accelerated interval is \(t_H = -(a^2u_0)^{-1}\).

If we choose to parametrize the trajectory by the proper time \(\tau\), it can be expressed as (with the zero of proper time chosen at \(t = 0\))

\[
u(\tau) = \begin{align*}
  &\theta(-\tau_0 - \tau)v_0\{a(\tau + \tau_0) - 1\} - a^{-1}\theta(\tau_0 + \tau)\theta(\tau_0 - \tau)e^{-a\tau} \\
  &+ \theta(\tau - \tau_0)u_0\{1 + a(\tau_0 - \tau)\}
\end{align*}
\] (3.2)

\[
v(\tau) = \begin{align*}
  &\theta(-\tau_0 - \tau)v_0\{a(\tau + \tau_0) + 1\} + a^{-1}\theta(\tau_0 + \tau)\theta(\tau_0 - \tau)e^{a\tau} \\
  &+ \theta(\tau - \tau_0)v_0\{1 - a(\tau_0 - \tau)\}
\end{align*}
\] (3.3)

where \(\pm \tau_0\) is the proper time of the trajectory when it exits (enters) the uniformly accelerated phase. It satisfies the relations

\[
\begin{align*}
u_0 &\equiv t_0 + x_0 = a^{-1}e^{a\tau_0} \\
u_0 &\equiv t_0 - x_0 = -a^{-1}e^{-a\tau_0}.
\end{align*}
\] (3.4)

Another convenient definition is the horizon - crossing proper time \(\pm \tau_H = \pm (a^{-1} + \tau_0)\).

The function \(\zeta(\tau, \tau')\) can be found in a standard way. If both points lie on the inertial sector of the trajectory, it has the usual zero-temperature form in two-dimensional Minkowski space. If both points lie on the uniformly accelerated sector, it has a finite temperature form exhibiting the Unruh temperature. It is therefore straightforward to obtain the following:

If \(\tau, \tau' > \tau_0\) or \(\tau, \tau' < -\tau_0\),

\[
\zeta(\tau, \tau') = \frac{\epsilon^2}{2\pi} \int_0^\infty \frac{dk}{k} e^{ik(\tau' - \tau)}. \] (3.5)
If $-\tau_0 < \tau, \tau' < \tau_0$,

$$
\zeta(\tau, \tau') = \frac{e^2}{2\pi} \int_0^\infty \frac{dk}{k} \left\{ \coth\left(\frac{\pi k}{a}\right) \cos k(\tau' - \tau) + i \sin k(\tau' - \tau) \right\}. \tag{3.6}
$$

Also, if $\tau < -\tau_0, \tau' > \tau_0$,

$$
\zeta(\tau, \tau') = \frac{e^2}{2\pi} \int_0^\infty \frac{dk}{k} \cos(\tau' + \tau) \tanh(\tau_0) e^{ik(\tau' - \tau + 2(a^{-1} \tanh(\tau_0) - \tau_0))}. \tag{3.7}
$$

Of interest is this function evaluated for one point on the inertial sector and the other on the uniformly accelerated sector. We will show that this function has a thermal form if the point on the inertial sector is sufficiently close to $(t_0, x_0)$ and departs smoothly from the thermal form away from it. It is also found that the horizons of the uniformly accelerated sector (which are not horizons for the entire trajectory) are the points where the near-thermal expansion breaks down.

Consider, for example, the case when $-\tau_0 < \tau' < \tau_0$ and $\tau < -\tau_0$. Then the function $\zeta$ is expressed as

$$
\zeta(\tau, \tau') = \frac{e^2}{4\pi} \int_0^\infty \frac{dk}{k} \left\{ e^{-ik(\tau + \tau_0)} + e^{ik(\tau + \tau_0) + \ln(1 - a(\tau + \tau_0))} \right\}. \tag{3.8}
$$

Introducing the Fourier transforms

$$
e^{\frac{ik}{a} e^{-\tau}} = \frac{1}{2\pi a} \int_{-\infty}^{\infty} d\omega e^{i\omega \sigma} \Gamma\left(\frac{k}{a}\right) e^{\frac{i\omega}{a} e^{\frac{\pi}{a}}}, \quad k > 0
$$

$$
e^{-\frac{ik}{a} e^{-\tau}} = \frac{1}{2\pi a} \int_{-\infty}^{\infty} d\omega e^{i\omega \sigma} \Gamma\left(\frac{k}{a}\right) e^{-\frac{i\omega}{a} e^{\frac{\pi}{a}}}, \quad k > 0
$$

we obtain, after some simplification,

$$
\zeta(\tau, \tau') = \frac{e^2}{4\pi} \int_0^\infty \frac{dk}{k} \left\{ \cos k \left( \tau' + \tau_0 + \frac{1}{a} \ln(1 - a(\tau + \tau_0)) \right) \coth\left(\frac{\pi k}{a}\right) \right. \\
+ \left. i \sin k \left( \tau' + \tau_0 + \frac{1}{a} \ln(1 - a(\tau + \tau_0)) \right) \right\} + \cos k(\tau' + \tau_0 - \frac{1}{a} \ln | a(\tau + \tau_H) |) \left( \coth\left(\frac{\pi k}{a}\right) \theta(\tau_H + \tau) + \theta(-\tau_H - \tau) \right) \\
+ \left. i \sin k(\tau' + \tau_0 - \frac{1}{a} \ln | a(\tau + \tau_H) |) \theta(\tau_H + \tau) \right\}. \tag{3.10}
$$

If we further restrict our attention to the case $\tau > -\tau_H$, i.e. both points lie inside the Rindler wedge, the above expression simplifies to the following:

$$
\zeta(\tau, \tau') = \frac{e^2}{2\pi} \int_0^\infty \frac{dk}{k} \cos \left( \frac{k}{2a} \ln(1 - a^2(\tau + \tau_0)^2) \right) \times \\
\left\{ \coth\left(\frac{\pi k}{a}\right) \cos k \left( \tau' + \tau_0 + \frac{1}{2a} \ln(1 - 2\frac{\tau + \tau_0}{\tau + \tau_H}) \right) \\
+ \left. i \sin k \left( \tau' + \tau_0 + \frac{1}{2a} \ln(1 - 2\frac{\tau + \tau_0}{\tau + \tau_H}) \right) \right\}. \tag{3.11}
$$
It is clear from this expression that an exact thermal spectrum is recovered in the limit of \( \tau \rightarrow -\tau_0 \), as expected. Suppose we now define \( \tau + \tau_0 \equiv \epsilon \) as the time difference between the proper time \( \tau \) and the proper time of entry into the accelerated phase, \(-\tau_0\). Then \( \alpha \epsilon \) will be the appropriate dimensionless parameter characterizing a near-thermal expansion. Note that \( \epsilon < 0 \).

From the above expression for \( \zeta \), we find that there is no correction to the thermal form of \( \zeta(\tau, \tau') \) to first order in \( \epsilon \). This can be understood from the fact that the coordinate difference between the point \( \tau = -\tau_0 - \epsilon \) and a corresponding point on a globally uniformly accelerated trajectory with the same proper time is of order \( \epsilon^2 \). Indeed, we may define Rindler coordinates \((\psi, \eta)\) on the right Rindler wedge by \( v = \psi^{-1} e^{\psi \eta} \) and \( u = -\psi^{-1} e^{-\psi \eta} \). Then the Rindler coordinates for the point \( \tau = -\tau_0 - \epsilon \) on the trajectory we consider are found to be \( \psi = a + O(\epsilon^2) \) and \( \eta = -\tau_0 - \epsilon + O(\epsilon^2) \), which are exactly the coordinates, to order \( \epsilon \), of a corresponding point with the same proper time on a globally uniformly accelerated trajectory with acceleration \( a \). It is thus no surprise that the spectrum is exactly thermal up to order \( \epsilon \).

Furthermore, it can be shown in a straightforward way from the above expression that the spectrum is also thermal up to \( O(\epsilon^2) \), although the above-mentioned coordinate difference does have terms of order \( \epsilon^3 \). Then the first correction to the thermal spectrum is of order \( \epsilon^3 \) and has the form

\[
\zeta(\tau, \tau') = \frac{\epsilon^2}{2\pi} \int_0^\infty \frac{dk}{k} \left\{ \coth\left(\frac{\pi k}{a}\right) \cos k(\tau' - \tau + \frac{a^2 \epsilon^3}{3}) - i \sin k(\tau' - \tau + \frac{a^2 \epsilon^3}{3}) \right\} + O(\epsilon^4). \tag{3.12}
\]

The validity of such a near-thermal expansion is characterized by the requirement that \(|\alpha \epsilon|\) is small. This translates to \(-1 < a(\tau + \tau_0)\) or equivalently, \(\tau > -\tau_H\). The expansion thus breaks down for \(\tau < -\tau_H\), for which case the two-point function may be called strictly non-thermal. This is the case when one of the points lies outside the right Rindler wedge while the other point is still inside it. The two-point function in such a situation will contain non-trivial correlations across the Rindler horizon, as was pointed out before \([49]\).

The response of the detector is governed by the Langevin equation (1.1). This equation may be formally integrated to yield

\[
\langle Q(\tau)Q(\tau') \rangle = \frac{\hbar}{\Omega^2} \int_{-\infty}^\tau ds \int_{-\infty}^{\tau'} ds' \nu(s, s') e^{-\gamma(\tau - s)} e^{-\gamma(\tau' - s')} \sin \Omega(\tau - s) \sin \Omega(\tau' - s') \tag{3.13}
\]

where \( \Omega = (\Omega_0^2 - \gamma^2)^{1/2} \), \( \Omega_0 \) is the natural frequency of the internal detector coordinate and \( \gamma = \epsilon^2 / 4 \) is the dissipation constant arising out of the detector’s coupling to the field. The double integral in the above equation may be computed by splitting each integral into a part which lies completely in the uniformly accelerated sector and parts which lie in the inertial sectors. For example, suppose we wish to compute the above correlation function for the case \(-\tau_0 < \tau, \tau' < \tau_0\), i.e. both points lie in the uniformly accelerated sector. Then each integral can be split into two parts \( (\int_{-\infty}^{\tau_0} - \int_{-\tau_0}^{\tau} + \int_{-\tau_0}^{\tau_0} + \int_{-\infty}^{-\tau_0}) \) and the resulting double integral therefore has four terms:

\[
\langle Q(\tau)Q(\tau') \rangle = F_1 + F_2 + F_3 + F_4. \tag{3.14}
\]
Writing $\nu \equiv Re(\zeta)$, we obtain, after straightforward manipulations,

$$
F_1 \equiv \frac{2h\gamma}{\pi\Omega^2} Re \int_{-\infty}^{\tau_0} ds \int_{-\infty}^{\tau_0} ds' \int_{0}^{\infty} \frac{dk}{k} e^{ik(s'-s)} e^{-\gamma(\tau-s)} e^{-\gamma(\tau'-s')} \times \\
\sin \Omega(\tau-s) \sin \Omega(\tau'-s') \\
= \frac{h\gamma}{\pi\Omega^2} e^{-\gamma(\tau+\tau'+2\tau_0)} \int_{0}^{\infty} \frac{dk}{k} [(\gamma^2 - k^2 + \Omega^2)^2 + 4\gamma^2 k^2]^{-1} \times \\
\{(\Omega^2 + \gamma^2 + k^2) \cos \Omega(\tau - \tau') + (\Omega^2 - \gamma^2 - k^2) \cos \Omega(\tau + \tau' + 2\tau_0) \\
+ 2\gamma \Omega \sin \Omega(\tau + \tau' + 2\tau_0) \}, \\
(3.15)
$$

and

$$
F_4 \equiv \frac{2h\gamma}{\pi\Omega^2} Re \int_{\tau_0}^{\tau} ds \int_{\tau_0}^{\tau'} ds' \int_{0}^{\infty} \frac{dk}{k} e^{ik(s'-s)} \coth \left(\frac{\pi k}{a}\right) e^{-\gamma(\tau-s)} e^{-\gamma(\tau'-s')} \times \\
\sin \Omega(\tau-s) \sin \Omega(\tau'-s') \\
= \frac{h\gamma}{\pi\Omega^2} e^{-\gamma(\tau+\tau'+2\tau_0)} \int_{0}^{\infty} \frac{dk}{k} \coth \left(\frac{\pi k}{a}\right) [(\Omega^2 + \gamma^2 - k^2)^2 + 4\gamma^2 k^2]^{-1} \times \\
\{(\gamma^2 + k^2 + \Omega^2) \cos \Omega(\tau - \tau') + (\Omega^2 - \gamma^2 - k^2) \cos \Omega(\tau + \tau' + 2\tau_0) \\
+ 2\gamma \Omega (\sin \Omega(\tau + \tau' + 2\tau_0) - \sin \Omega(\tau + \tau_0) \sin \Omega(\tau' + \tau_0)) \\
- \Omega^2 (\cos \Omega(\tau + \tau_0) + \cos \Omega(\tau' + \tau_0) - 1) \}, \\
(3.16)
$$

where $Re$ stands for the real part.

The functions $F_2$ and $F_3$, in which one of the integration variables runs over the inertial sector and the other over the uniformly accelerated sector, are difficult to evaluate. We shall simply express them here in the following form:

$$
F_2 = \frac{h\gamma}{\pi\Omega^2} e^{-\gamma(\tau+\tau')} Re \int_{-\infty}^{\tau_0} ds e^{\gamma s} \sin \Omega(\tau-s) \times \\
\int_{0}^{\infty} \frac{dk}{k} \left[ e^{ik(au+s+au(1+a\tau_0))} A_1(k; \tau') + e^{-ik(au+s-au(1-a\tau_0))} A_2(k; \tau') \right] \\
(3.17)
$$

$$
F_3 = \frac{h\gamma}{\pi\Omega^2} e^{-\gamma(\tau+\tau')} Re \int_{-\infty}^{\tau_0} ds e^{\gamma s} \sin \Omega(\tau'-s) \times \\
\int_{0}^{\infty} \frac{dk}{k} \left[ e^{-ik(au+s+au(1+a\tau_0))} A_1(k; \tau) + e^{ik(au+s-au(1-a\tau_0))} A_2(k; \tau) \right] \\
(3.18)
$$

where the functions $A_1$ and $A_2$ are

$$
A_1(k; s) = \int_{-\tau_0}^{s} ds' e^{ika-1} e^{as} e^{\gamma s'} \sin \Omega(s-s') \\
A_2(k; s) = \int_{-\tau_0}^{s} ds' e^{-ika-1} e^{-as} e^{\gamma s'} \sin \Omega(s-s'). \\
(3.19)
$$

Similarly, if one wishes to compute the detector correlation function for two points in the late inertial sector $(\tau, \tau' > \tau_0)$, then one has nine terms similar in form to the ones displayed above.

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4 Moving mirror in Minkowski space

In this section, we treat the motion of a mirror following a trajectory $z(t)$ in Minkowski space. A massless scalar field $\phi$ is coupled to the mirror via a reflection boundary condition. It obeys the Klein-Gordon equation

$$\frac{\partial^2 \phi}{\partial t^2} - \frac{\partial^2 \phi}{\partial x^2} = 0 \quad (4.1)$$

subject to the boundary condition

$$\phi(t, z(t)) = 0. \quad (4.2)$$

For a general mirror path this equation is difficult to solve; however we can exploit the invariance of the wave equation under a conformal transformation to change to simpler coordinates. We follow the treatment of [5]. To this end, we introduce a transformation between the null coordinates $u, v$ and $\bar{\mu}, \bar{\nu}$ defined as

$$u = t - x, \quad v = t + x, \quad u = f(\bar{\mu}), \quad v = \bar{\nu} \quad (4.3)$$

The function $f$ is chosen such that the mirror trajectory is mapped to $\bar{z} = 0$. To do this, we relate the two sets of coordinates as follows:

$$t = \frac{1}{2}[\bar{\nu} + f(\bar{\mu})]$$

$$x = \frac{1}{2}[\bar{\nu} - f(\bar{\mu})] \quad (4.4)$$

On the mirror path, setting $\bar{z} = 0$ means that the trajectory can be expressed as

$$\frac{1}{2} [\bar{t} - f(\bar{t})] = z \left(\frac{1}{2} [\bar{t} + f(\bar{t})]\right) \quad (4.5)$$

which allows $f$ to be implicitly determined. In the new coordinates the wave equation is unchanged, however it now has a time independent boundary condition, meaning the mirror is static, while the detector moves along some more complicated path. Thus the wave equation with boundary condition can easily be solved to give

$$\phi(\bar{t}, \bar{\nu}) = \int_0^\infty (2\pi k)^{-1/2} \sin k\bar{\nu} e^{-ik\bar{t}} \, dk \quad (4.6)$$

where the mode functions are orthonormal in the Klein-Gordon inner product. In these barred coordinates, $\bar{\zeta}$ is proportional to the two point function in the presence of a static reflecting boundary at $\bar{z} = 0$.

Also, in these coordinates, the time dependent modes of the field are just exponentials. That is, they can be described by oscillators with unit mass and frequency $k$. So $X_k(\bar{t})$ is a solution to the oscillator equation (1.4), and by satisfying the initial conditions $X_k(0) = 1, X'_k(0) = -ik$ we obtain

$$X_k(\bar{t}) = e^{-ik\bar{t}} \quad (4.7)$$
We now consider a detector placed in the vicinity of the mirror. The spectral density function $I$ is determined by the path of the detector and its coupling to the field. Denoting the detector position by $r(t)$ and the field modes by $q_k(t)$ and assuming the monopole interaction

$$L_{\text{int}} = -\int eQ\phi(\tau, \bar{\tau}) \delta(\tau - \bar{\tau}) \, d\tau$$

$$= -eQ\phi(\tau, \bar{\tau})$$

$$= -\int eQq_k(\tau) \, \sin k\tau \, dk,$$

we have

$$I(k, \tau, \bar{\tau}) = \int \frac{dk_n}{2k_n} \delta(k - k_n) e^2 \sin k_n\tau(\tau) \sin k_n\tau(\bar{\tau})$$

$$= \frac{e^2}{2k} \sin k\tau(\tau) \sin k\tau(\bar{\tau}).$$

(4.8)

Defining $\eta = \tau - \bar{\tau}(t)$ and $\nu = \tau + \bar{\tau}$, we can now express the function $\zeta$ as

$$\zeta = -\frac{e^2}{8\pi} \int_0^\infty \frac{dk}{k} \left[ e^{ik(\eta - \nu)} - e^{ik(\nu - \nu)} - e^{ik(\eta - \nu)} + e^{ik(\nu - \nu)} \right]$$

(4.10)

Since only the outgoing modes have reflected off the mirror, only the outgoing part of the correlations $\zeta$ will give appropriate thermal behavior. Thus, from now on, we focus on the correlation

$$\zeta_{uu} = -\frac{e^2}{8\pi} \int_0^\infty \frac{dk}{k} e^{ik(\eta - \nu)}.$$

(4.11)

It remains to evaluate the above function. To do this, we specify the function $f$ by considering a specific mirror trajectory. A convenient choice of the mirror path is the following:

$$z(t) = -t - A e^{-2\kappa t} + B$$

(4.12)

for $A$, $B$, $\kappa$ positive. This path possesses a future horizon in the sense that there is a last ingoing ray which the mirror will reflect; all later rays never catch up with the mirror and so are not reflected. It is this aspect which enables the moving mirror to emulate a black hole. Eq. (4.5) can now be solved to give

$$f(\tau) = -\tau - \frac{1}{\kappa} \ln \frac{B - \tau}{A}.$$ 

(4.13)

In the late time limit ($\tau \simeq B$), we consider the following ansatz for $f^{-1}$

$$f^{-1}(x) \simeq B - A e^{-\kappa(B + x)} + \alpha$$

(4.14)

where $\alpha$ is taken to be small in the sense that terms of order $\alpha^2$ are ignored. In this approximation, one finds

$$\alpha = -\kappa A^2 e^{-2\kappa(B + x)}$$

(4.15)

and the transformation from barred to unbarred coordinates becomes

$$\nu = B - A e^{-\kappa(B + u)} - \kappa A^2 e^{-2\kappa(B + u)}$$

(4.16)
plus terms of higher powers in $e^{-\kappa(B+u)}$.

We now need an explicit form for the detector trajectory $u(t)$ since this is what appears in the function $\zeta$. Choosing it to be inertial, we have $r(t) = r_\ast + ut$, which gives $u(t) = t(1-w) - r_\ast$. In terms of the proper time of the detector, this becomes $u(\tau) = \tau\sqrt{\frac{1-w}{1+w}} - r_\ast$.

Defining the sum and difference $\Sigma = \frac{1}{2}(\tau + \tau')$ and $\Delta = \tau - \tau'$, and $z = \sqrt{\frac{1-w}{1+w}}$, we obtain

$$\bar{u} - \bar{u} = -2\kappa e^{-\kappa(B+r_\ast+\Sigma\nu)} \sinh\left(\frac{K\nu\Delta}{2}\right) - 2\kappa A^2 e^{-2\kappa(B+r_\ast+\Sigma\nu)} \sinh(K\nu\Delta).$$

This is substituted in $\zeta_{uu}$, and, after some simplification we obtain the near-thermal form

$$\zeta_{uu}(\tau, \tau') = \frac{e^{2}}{8\pi} \int_{0}^{\infty} \frac{dk}{k} \left[ \coth\left(\frac{k\pi}{K\nu}\right) \cos(k\Delta)(1 + h\Gamma) - i\sin(k\Delta) \right]$$

with

$$\Gamma = -2\kappa e^{-\kappa(B+r_\ast+\Sigma\nu)} \tanh^2\left(\frac{\pi k}{K\nu}\right) \tan k\Delta.$$

Thus a thermal detector response, at the temperature $\frac{\kappa}{2\pi}$, Doppler-shifted by a factor $\nu$ depending on the speed of the detector, is observed, with a correction that exponentially decays to zero at late times.
5 Collapsing Mass in Two Dimensions

In this section we study radiance from a collapsing mass, analogous to the moving mirror model. We essentially follow the method of [15], but using stochastic analysis, and generalizing it to include higher order terms in the various Taylor expansions involved, thus exhibiting the near-thermal properties of detector response.

We will exploit the conformal flatness of two dimensional spacetime in the subsequent analysis. Outside the body the metric is expressed as

$$ds^2_o = C(r) du \, dv$$  \hspace{1cm} (5.1)

where $u, v$ are the null coordinates

$$u = t - r^* + R_0^*$$ \hspace{1cm} (5.2)

$$v = t + r^* + R_0^*$$

and $r^*$ is the Regge-Wheeler coordinate:

$$r^* = \int^r \frac{dr'}{C(r')}$$  \hspace{1cm} (5.3)

with $R_0^*$ being a constant. The metric outside the body is thus assumed to be static in order to mimic the four dimensional spherically symmetric case (for which Birkhoff’s theorem holds). The point at which the conformal factor $C = 0$ represents the horizon, and the asymptotic flatness condition is imposed by $C \to 1$ as $r \to \infty$.

On the other hand, the metric inside the ball is for now assumed to be a completely general conformally flat metric:

$$ds^2_i = A(U,V) dU dV$$  \hspace{1cm} (5.4)

with

$$U = \tau - r + R_0$$

$$V = \tau - r - R_0$$  \hspace{1cm} (5.5)

and $R_0$ and $R_0^*$ are related in the same way as $r$ and $r^*$. The surface of the collapsing ball will be taken to follow the worldline $r = R(\tau)$, such that, for $\tau < 0$, $R(\tau) = R_0$. Thus, at the onset of collapse, $\tau = t = 0$, $U = V = u = v = 0$ on the surface of the ball.

We will let the two sets of coordinates be related by the transformation equations

$$U = \alpha(u)$$

$$v = \beta(V).$$  \hspace{1cm} (5.6)

The functions $\alpha$ and $\beta$ are not independent of each other because one coordinate transformation has already been specified by equation (5.3).

Without as yet determining the precise form of $\alpha$ and $\beta$, we will consider a massless scalar field $\phi$ propagating in this spacetime subject to a reflection condition $\phi(r = 0, \tau) = 0$. 

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Such a field propagates in a similar fashion to the field in the vicinity of a moving mirror. To make this explicit, we introduce a new set of barred coordinates

\[
\begin{align*}
\mathbf{u} &= \beta [\alpha(u) - 2R_0] \\
\mathbf{v} &= v.
\end{align*}
\] (5.7)

In terms of these, we also define the coordinates \( \mathbf{r} = \frac{1}{2}(\mathbf{v} - \mathbf{u}) \), and \( \mathbf{t} = \frac{1}{2}(\mathbf{v} + \mathbf{u}) \).

These new coordinates have the properties: a) \( r = 0 \Rightarrow \mathbf{r} = 0 \), and b) the field equations have incoming mode solutions of the form \( e^{ikv} \). Thus the left-moving parts of the correlation functions of the ‘in’ vacuum defined in terms of barred coordinates are identical to those of the vacuum defined with respect to unbarred coordinates.

Keeping these properties in mind, we may expand the field in terms of standard modes obeying the reflection boundary condition (by conformal invariance of the massless scalar field equation) as

\[
\phi(\mathbf{r}, \mathbf{t}) = \sqrt{\frac{2}{L}} \sum_{k>0} q_k(t) \sin k\mathbf{r},
\] (5.8)

just as in the moving mirror case.

We now consider a detector placed outside the collapsing ball at fixed \( \mathbf{r} \) (or \( \mathbf{r}^* \)), namely \( \mathbf{r} = \mathbf{r}_0 \) (or \( \mathbf{r}^* = \mathbf{r}^*_0 \)). The interaction between detector and field is described by the interaction Lagrangian

\[
L_{\text{int}} = -\epsilon Q \phi(\mathbf{s}, \mathbf{r}),
\] (5.9)

where

\[
\begin{align*}
\mathbf{r} &= \frac{1}{2}(v - \beta(\alpha(u) - 2R_0)) \\
&= \frac{1}{2}(t + r^*_0 - R^*_0 - \beta(\alpha(t - r^*_0 + R^*_0) - 2R_0)) \\
\mathbf{s} &= \frac{1}{2}(t + r^*_0 - R^*_0 + \beta(\alpha(t - r^*_0 + R^*_0) - 2R_0))
\end{align*}
\] (5.10)

and \( Q \) is the internal detector coordinate.

The influence kernel \( \zeta \), due to a reflection condition at \( \mathbf{r} = 0 \), has the same form as the moving mirror case, in barred coordinates. Its outgoing part is therefore given by

\[
\zeta_{uu} = \frac{e^2}{8\pi} \int_0^\infty \frac{dk}{k} e^{ik(\mathbf{r} - \mathbf{r})}.
\] (5.11)

where

\[
\mathbf{r} = \mathbf{s} - \mathbf{r} = \beta(\alpha(t - r^*_0 - R^*_0) - 2R_0)
\] (5.12)

and \( \mathbf{r} \) is the same function of \( t' \).

We will now determine the functions \( \alpha \) and \( \beta \) and show that, to zeroth order in an appropriate parameter, \( \mathbf{r} \) is an exponential function of \( t \), and thus \( \zeta_{uu} \) has a thermal form. The correction to the exponential form, obtained by including higher order terms, will lead to a near-thermal spectrum.
To determine $\alpha$ and $\beta$ we match the interior and exterior metrics at the collapsing surface $r = R(\tau)$. Then we have
\[
\alpha'(u) \equiv \frac{dU}{du} = -C \left(1 - \frac{\dot{R}}{R}\right) \left[1 + \left(1 + \frac{AC}{R^2} \left(1 - \dot{R}^2\right)\right)^{\frac{1}{2}}\right]^{-1} \quad (5.13)
\]
\[
\beta'(V) \equiv \frac{dv}{dV} = C^{-1} \frac{\dot{R}}{1 + \dot{R}} \left[1 - \left(1 + \frac{AC}{R^2} \left(1 - \dot{R}^2\right)\right)^{\frac{1}{2}}\right] \quad (5.14)
\]
where $\dot{R} = \frac{dR}{d\tau}$.

Now we expand these quantities about the horizon. We recall the definition of the horizon radius $R_h$ as $C(R_h) = 0$. We may further define $\tau_h$ as $R(\tau_h) = R_h$. Then we obtain the following Taylor expansions:
\[
R(\tau) = R_h + \nu(\tau_h - \tau) + \beta(\tau_h - \tau)^2 + \cdots \quad (5.15)
\]
where $\nu = -\dot{R}(\tau_h)$, $\beta = \frac{1}{2} \ddot{R}(\tau_h)$, and
\[
C = \left. \frac{\partial C}{\partial r} \right|_{R_h} (R - R_h) + \frac{1}{2} \frac{\partial^2 C}{\partial r^2} \left|_{R_h} (R - R_h)^2 + \cdots \right. \quad (5.16)
\]
where $\kappa = \left. \frac{1}{2} \frac{\partial C}{\partial r} \right|_{R_h}$, the surface gravity, and $\gamma = \frac{1}{2} \frac{\partial^2 C}{\partial r^2} \left|_{R_h}$. Since the ball is collapsing, $\nu > 0$.

Substituting the above expansions in the expression for $\alpha'(u)$, we obtain, to order $(\tau_h - \tau)^2$,
\[
\frac{dU}{du} = a(R_0 - R_h + \tau_h - U) + b(R_0 - R_h + \tau_h - U)^2 \quad (5.17)
\]
where
\[
a = (\nu + 1)\kappa \quad (5.18)
\]
\[
b = \frac{\kappa}{\nu} \left\{ (3 + \nu)\beta + (1 + \nu) \frac{\gamma \nu^2}{2\kappa} - \frac{1}{2} A\kappa(1 - \nu^2)(1 + \nu) \right\}. \quad (5.19)
\]
Note that, for a slowly collapsing ball, $\nu \ll 1$, and hence $a$ reduces to the surface gravity $\kappa$.

Also, to order $(\tau_h - \tau)$,
\[
\frac{dv}{dV} = c + d(\tau_h + R_h - R_0 - V) \quad (5.20)
\]
where
\[
c = \frac{A(1 + \nu)}{2\nu} \quad (5.21)
\]
\[
d = \frac{A}{\nu^2} (\beta - \frac{A\kappa}{4}(1 - \nu^2)(1 + \nu)). \quad (5.22)
\]
We consider a regime in which $(\tau_h - \tau)d \ll c$ so that we may ignore the second term in (5.20). Then we can integrate this equation to give
\[
v(V) \equiv \beta(V) = c_1 + cV \quad (5.23)
\]
where $c_1$ is an integration constant.

Similarly to lowest order in $\frac{b}{a^2}$ (which turns out to be the appropriate dimensionless parameter describing deviations from exact exponential scaling or exact thermal behavior), we integrate equation (5.17) to give

$$U(u) \equiv \alpha(u) = R_0 - R_h + \tau_h + a^{-1}e^{-(u-c_2)}(1 + \frac{b}{a^2}e^{-(u-c_2)}), \quad (5.24)$$

$c_2$ being another integration constant.

We are now in a position to obtain explicitly the transformation between barred and unbarred coordinates, to lowest order in $\frac{b}{a^2}$. Thus we have

$$\pi = \beta [\alpha(u) - 2R_0]$$

$$= M_1 + M_2e^{-(u-c_2)}(1 + \frac{b}{a^2}e^{-(u-c_2)}), \quad (5.25)$$

where

$$M_1 = c_1 - c(R_0 + R_h - \tau_h) \quad (5.26)$$

$$M_2 = \frac{c}{a}. \quad (5.27)$$

At the position $r_0^*$ of the detector, $u = t - r_0^*$. Therefore, defining $\Delta = u' - u$ and $\Sigma = \frac{1}{2}(u' + u) + r_0^*$, we may perform the above transformation to obtain

$$\pi' - \pi = -2M_2e^{ac_2}\{e^{-(\Sigma-r_0^*)}\sinh \frac{a\Delta}{2} + \frac{b}{a^2}e^{-2a(\Sigma-r_0^* - \frac{a}{2})}\sinh a\Delta\}. \quad (5.28)$$

Invoking the identities (2.11) and (2.12), the function $\zeta_{uu}$ can now be simplified to yield the near-thermal form

$$\zeta_{uu} = \frac{e^2}{8\pi} \int_0^\infty \frac{dk}{k} \{\coth(\frac{\pi k}{a}) \cos k\Delta(1 + \Gamma) - i \sin k\Delta\} \quad (5.29)$$

where

$$\Gamma = -\frac{2bk}{a^3}e^{a(c_2 - r_0^*)}\tanh^2(\frac{\pi k}{a}) \tan k\Delta \sinh a\Delta. \quad (5.30)$$

The function $\Gamma$ vanishes at late times ($\Sigma \to \infty$). Thus the exact thermal spectrum is recovered at the Hawking temperature redshifted by the velocity of the surface of the ball, on a time scale defined by the surface gravity $a$. 

21
6 Discussion

We now summarize our findings and discuss their implications. There are four main points made or illustrated here:

1) This paper gives a stochastic theoretical derivation of particle creation, in the class of spacetimes which possess an event horizon in some limit. This approach generalizes the established methods of quantum field theory and thermal field theory (in curved spacetimes) to statistical and stochastic field theory. The exact thermal radiance cases arising from an exact exponential scale transformation such as is found in a uniformly-accelerated detector, the Schwarzschild black hole and the de Sitter universe, have been treated in the stochastic theoretical method before [44]. Here we give the treatment of the moving mirror and the collapsing mass as further examples. (Thermal radiation in certain classes of cosmological spacetimes inflationary universe will be studied in a following paper.)

2) We have shown that in all the examples considered in this class of spacetimes, i.e., accelerated observers, moving mirrors and collapsing masses (black holes), those which yield a thermal spectrum of created particles all involve an exponential scale transformation. Thermal radiance observed in one vacuum arises from the exponential scaling of the quantum fluctuations (noise) in another vacuum. This view espoused by one of us [30, 31, 32, 33] is illustrated in the examples treated here.

3) The main point of this paper is to show how one can calculate particle creation in the near-exponential cases, yielding near-thermal spectra. These cases are not so easy to formulate conceptually using the traditional methods: the geometric picture in terms of the properties of the event horizons as global geometric entities works well for equilibrium thermodynamics (actually thermostatics) conditions, so does thermal field theory which assumes a priori a finite temperature condition (e.g., periodic boundary condition on the imaginary time). They cannot be generalized to non-equilibrium dynamical conditions so easily. In the stochastic theory approach we used, the starting point is the vacuum fluctuations of quantum fields subjected to kinematical or dynamical excitations. There is no explicit use of the global geometric properties of spacetimes: the event horizons are generated kinematically by exponential scaling. (Thus, for example, this method can describe the situations where a detector is accelerated only for a short duration, whereas one cannot easily describe in geometric terms the scenario of an event horizon appearing and disappearing.) There is also no a priori assumption of equilibrium conditions: the concept of temperature is neither viable nor necessary, as is expected in all non-equilibrium conditions. Thermal or near-thermal radiance is a result of some specific conditions acting on the vacuum fluctuations in the system.

4) We restrict our attention in this paper to near-thermal conditions because of technical rather than conceptual limitations. In the near-thermal cases treated here, we want to add that the stochastic theoretical method is not the only way to derive these results. One can alternatively approach with the global geometric or thermal field methods, say, by working with generalized definitions of event horizons or quasi-periodic Green’s functions. However, we find it logically more convincing and technically more rigorous to use the
stochastic method to define and analyze statistical concepts like fluctuations and dissipation, correlation and coherence. Certainly in the fully dynamical and non-equilibrium conditions, such as will be encountered in the full backreaction problem (not just confined to the linear response regime) this method is, in our opinion, more advantageous than the existing ones. Even though the technical problems will likely be grave, (because of the built-in balance between dissipation and fluctuations, as demanded by a self-consistent treatment), there are no intrinsic conceptual pitfalls or shortcomings. These issues and problems are currently under investigation.

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