Optomechanical transduction of an integrated silicon cantilever probe using a microdisk resonator

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Sensitive transduction of the motion of a microscale cantilever is central to many applications in mass, force, magnetic resonance, and displacement sensing. Reducing cantilever size to nanoscale dimensions can improve the bandwidth and sensitivity of techniques like atomic force microscopy, but current optical transduction methods suffer when the cantilever is small compared to the achievable spot size. Here, we demonstrate sensitive optical transduction in a monolithic cavity-optomechanical system in which a sub-picogram silicon cantilever with a sharp probe tip is separated from a microdisk optical resonator by a nanoscale gap. High quality factor ($Q\approx10^5$) microdisk optical modes transduce the cantilever’s MHz frequency thermally-driven vibrations with a displacement sensitivity of $\approx4.4\times10^{-16}$ m/$\sqrt{\text{Hz}}$ and bandwidth $>1$ GHz, and a dynamic range $>10^5$ is estimated for a 1 s measurement. Optically-induced stiffening due to the strong optomechanical interaction is observed, and engineering of probe dynamics through cantilever design and electrostatic actuation is illustrated.

Micro- and nanoscale cantilevers are at the heart of many applications in mass, force, magnetic resonance, and displacement sensing. In atomic force microscopy (AFM), the push towards smaller cantilevers is motivated by the ability to increase mechanical frequencies while maintaining a desired level of stiffness. This influences the force sensitivity and measurement bandwidth, in turn determining the image acquisition rate and ability to resolve time-dependent forces and acquire additional information about the tip-sample interaction potential. Standard optical methods for transducing cantilever motion include beam deflection and laser interferometry, and in macroscopic devices that are 1 mm $\times$ 1 mm $\times$ 60 $\mu$m (length, width, and height), quantum-limited displacement sensitivity of $4\times10^{-19}$ m/$\sqrt{\text{Hz}}$ has been achieved. Interferometric approaches using a high numerical aperture objective have also been used in micro-scale devices, resulting in displacement sensitivities of $3\times10^{-14}$ m/$\sqrt{\text{Hz}}$ for cantilevers that are 20 $\mu$m $\times$ 4 $\mu$m $\times$ 0.2 $\mu$m and $1\times10^{-15}$ m/$\sqrt{\text{Hz}}$ for larger conventional cantilevers (223 $\mu$m $\times$ 31 $\mu$m $\times$ 6.7 $\mu$m). However, as the cantilever dimensions are pushed below the detection wavelength, diffraction effects limit the sensitivity of these approaches, and near-field optics and/or integrated on-chip detection methods can be of significant benefit.

To that end, researchers have recently used evanescently coupled on-chip waveguides acting as doubly-clamped cantilevers to demonstrate displacement sensitivities of $3.5\times10^{-14}$ m/$\sqrt{\text{Hz}}$, while end-to-end waveguides acting as singly-clamped devices have achieved similar performance. Although these waveguide-based approaches are optically broadband, the strong, multi-pass interaction provided by optical cavities can be of considerable advantage. Cavity optomechanics has seen substantial recent progress, where in many cases the optical resonator also acts as a mechanical oscillator, and its internal vibrations have been transduced with measurement imprecision at or below the standard quantum limit and with absolute displacement sensitivities in the $10^{-17}$ m/$\sqrt{\text{Hz}}$ to $10^{-18}$ m/$\sqrt{\text{Hz}}$ range. In contrast, here we focus on transducing the motion of a cantilever probe, requiring a design in which the cantilever can be brought near a surface and its fluctuations sensed by a nearby optical cavity without inducing excessive optical loss.

A similar approach was presented in Ref. 26 where doubly-clamped SiN$_x$ nanobeams were brought into the near-field of SiO$_2$ microtoroid cavities fabricated on a separate chip. In comparison, here we fabricate a cantilever-optical cavity system on a single silicon device layer, while tailoring the cantilever geometry for both strong optomechanical interactions and applicability to AFM. Beyond demon-
stratting sub-fm/√Hz sensitivity to cantilever motion, this approach has many potential benefits for AFM. Silicon’s high refractive index allows for significantly smaller optical cavities to be used, yielding stronger cantilever-cavity coupling rates and permitting higher bandwidth operation. Moving to silicon opens up potentially advanced device functionality, including electrostatic actuation and integrated optical waveguide readout. By largely separating the mechanical and optical designs, engineering of the cantilever geometry to achieve desired parameters can be accomplished without adversely affecting the optical readout mechanism. In addition, the strong optomechanical interaction can allow for optical control of cantilever mechanics, through effects such as optically-induced stiffening and optically-driven mechanical vibrations. Finally, this platform provides simplifications with respect to free-space detection systems that may improve measurement stability and be of importance in parallelized multi-probe measurements or environments with limited optical access. This work lays the foundations for a class of practical nanoscale mechanical sensors enabled by cavity optomechanics.

**Device geometry and simulation**

A simple device geometry is shown in Fig. 1(a), with fabrication details given in the Methods. A semicircular cantilever of width $W$ is suspended at its ends and separated by a gap $G$ from a $10 \mu m$ diameter silicon microdisk. The silicon is $260$ nm thick, and the cantilever has been designed to support a sharp tip at its midpoint. Devices are fabricated with $W=65$ nm, $100$ nm, and $200$ nm, and nominal values $G=50$ nm, $75$ nm, and $100$ nm. Scanning electron microscope (SEM) images indicate that $W$ is typically within $\pm 5$ nm of its nominal value, while $G$ is often smaller than the nominal value by a couple tens of nanometers, though charging effects due to the electron beam limit this estimate. The cantilever geometry is chosen to maximize its interaction with microdisk optical modes while minimizing the scattering loss induced by its presence. Optical modes are labeled $TE_{p,n}$ and $TM_{p,n}$, according to polarization (transverse electric or transverse magnetic) and radial ($p$) and azimuthal ($n$) order. Three-dimensional finite element method (FEM) eigenfrequency simulations indicate that, for $W=65$ nm or $W=100$ nm, cavity quality factors ($Q_s$) in excess of $10^6$ can be achieved for $TE_{1,n}$ and $TE_{2,n}$ modes, and $Q_s$ in excess of $10^5$ can be achieved for $TM_{1,n}$ modes, even as $G$ decreases to $\approx 30$ nm. In comparison and as a baseline, fabricated microdisks without cantilevers exhibit $Q_s$ in the mid-$10^5$ to low-$10^6$ range.

Mechanical modes of a $W=65$ nm cantilever are determined from FEM simulations (see Methods), with representative modes shown in Fig. 1(b). We have focused on the $h_{xx}$ modes, which are even symmetry in-plane modes whose primary displacement direction is normal to the gap ($x$ direction), as they are the dominant modes that are optically transduced and are of particular relevance to AFM work. The predicted stiffness ($k$), resonant frequency ($\Omega_m$), and effective mass ($m$) of these modes are compiled in Table I. Focusing on the $h_{xx}$ mode at $\Omega_m/2\pi = 2.23$ MHz, its optomechanical coupling to $p=1$ and $p=2$ optical modes in the $1550$ nm band, defined as $g_{OM} = d\omega_c/dG$ ($\omega_c$ is the cavity mode frequency), is calculated by FEM simulation and displayed in Fig. 1(c). For the range of gaps studied in this work, $g_{OM}/2\pi \approx 0.5$ GHz/nm to $g_{OM}/2\pi \approx 3.0$ GHz/nm. This is about two orders of magnitude larger than $g_{OM}$ for Si$_3$O$_2$ cantilevers coupled to SiO$_2$ microtoroids, and is due to the more tightly confined optical modes supported by the silicon microdisks.

**Transduction of cantilever motion**

We measure the fabricated devices using a fiber taper coupling method shown schematically in Fig. 2(a) (see Methods). A $1550$ nm band tunable diode laser is attenuated and coupled into the devices using an optical fiber taper waveguide, a single mode optical fiber whose minimum diameter has been adiabatically and symmetrically reduced to around $1 \mu m$. At this diameter, the waveguide mode’s spatial profile extends well beyond the glass core into the surrounding air cladding, and this evanescent tail is used to excite and collect light from the microdisk modes. The signal exiting the cavity is split by a $90:10$ fiber coupler, with $10$ % of the light used for monitoring the transmission level and recording swept-wavelength transmission spectra, and $90$ % sent into a radio frequency (RF) photodetector, after which an electronic spectrum analyzer measures RF oscillations in the detected signal.

Normalized transmission spectra over the full wavelength band for TE and TM polarized modes of a $W=65$ nm, $G=50$ nm device are shown in Fig 2(b), along with zoomed-in scans of individual modes. The polarization of the modes is determined by comparing the free spectral ranges for modes of a given radial order with those predicted from simulation. Loaded cavity $Q_s$ of $8.0 \times 10^4$ and $1.8 \times 10^5$ are observed for this device (corresponding intrinsic $Q_s$ of $1.1 \times 10^5$ and

### Table I: Calculated and measured properties of the $h_{xx}$ cantilever modes.

| Mode | $k$ (calc.) | $m$ (calc.) | $\Omega_m/2\pi$ (calc.) | $\Omega_m/2\pi$ (expt.) | $\Gamma_m/2\pi$ (expt.) | $Q_m$ (expt.) |
|------|-------------|-------------|--------------------------|--------------------------|--------------------------|-------------|
| $h_{xx}$ | 0.14 N/m | 0.73 pg | 2.23 MHz | 2.35 MHz | 479 ± 8 kHz | 4.9 |
| $h_{yx}$ | 1.41 N/m | 0.58 pg | 7.82 MHz | 7.89 MHz | 598 ± 11 kHz | 13.1 |
| $h_{yy}$ | 5.72 N/m | 0.35 pg | 20.37 MHz | 20.51 MHz | 533 ± 2 kHz | 38.5 |
| $h_{zx}$ | 12.43 N/m | 0.32 pg | 31.17 MHz | 31.36 MHz | 706 ± 4 kHz | 44.4 |
| $h_{zy}$ | 41.95 N/m | 0.44 pg | 49.36 MHz | 49.86 MHz | 815 ± 4 kHz | 61.2 |
| $h_{xy}$ | 74.05 N/m | 0.40 pg | 68.13 MHz | 68.71 MHz | 752 ± 43 kHz | 91.0 |
2.1 × 10^5, respectively), which supports doublet modes due to surface-roughness-induced backscattering that couples the clockwise and counterclockwise modes of the cavity. Over all devices, Qs of 5 × 10^4 to 2 × 10^5 are typically observed for TE_{1,n}, TE_{2,n} and TM_{1,n} modes, though occasional devices have Qs as high as ≈ 6 × 10^5 (see supplemental data). Optical transduction of the cantilever’s motion due to thermal noise is performed by fixing the laser on the blue-detuned shoulder of a TE-polarized cavity mode. A 1 MHz to 600 MHz spectrum for a W = 65 nm, G = 100 nm device is shown in Fig. 2(c), and contains several peaks. Those below 100 MHz originate from motion of the cantilever, while those at higher frequencies (364.63 ± 0.35 MHz and 577.20 ± 0.25 MHz) are from motion of the disk. This is confirmed by measuring the RF spectrum of a disk without a cantilever (Fig. 2(d)) through a high-Q cavity mode (loaded Q = 5.7 × 10^5 ± 0.5 × 10^5, intrinsic Q ≈ 1.0 × 10^6), which yields RF peaks at near-identical frequencies (364.74 ± 0.03 MHz and 576.20 ± 0.03 MHz). FEM simulations indicate that the higher frequency mode is the disk’s radial breathing mode (RBM); its measured linewidth is Γ_M/2π = 21.68 ± 0.06 MHz, corresponding to Q_M ≈ 27.

Focusing on the frequency range between 100 kHz and 100 MHz, a higher resolution RF spectrum at 223 μW of input power (P_{in}) into the cavity is shown in Fig. 2(e). The frequencies of the transduced modes (Table I) correspond well with the previously described simulation results. The mechanical quality factors of these modes are between Q_M ≈ 5 for the h_{1x} mode and Q_M ≈ 61 for the h_{5x} mode (Table I); these values are likely limited by air damping. The detection background, shown in Fig. 2(c) in black, is found by placing the laser off-resonance while maintaining a fixed detected power. Focusing on the h_{1x} mode, its calculated effective mass and measured frequency correspond to a peak displacement amplitude of x_{rms} = √{k_B T/ɛ} ≈ 160 pm when driven by thermal noise at 300 K. We use x_{rms} and Γ_M to convert the RF amplitude in Fig. 2(e) to displacement sensitivity. Corresponding photodetector-limited sensitivity is 4.4 × 10^{-16} ± 0.3 × 10^{-16} m/√Hz. This value is consistent with that determined by a phase modulator calibration (Methods) to within our uncertainty in the disk-cantilever gap. It represents an improvement by about a factor of 100 with respect to other on-chip silicon cantilever experiments, is at the same absolutely sensitivity level demonstrated for SiN cantilevers transduced by silica microtoroids, and is about a factor of 5 times larger than the standard quantum limit for our system. Along with the sensitivity, two other important quantities that characterize this system for its use as a displacement sensor are its dynamic range and bandwidth. The maximum detectable dis-

FIG. 2: (a) Setup for device characterization. (b) Broad wavelength scan (left) for TE (top) and TM (bottom) modes of a typical disk-cantilever device (W=65 nm, G=50 nm). Zoomed-in scans (right) show data (green) along with a doublet model fit (black). (c) Broad RF spectrum of a disk-cantilever device (W=65 nm, G=100 nm), transduced by fixing the probe laser on the short wavelength side of the TE-polarized mode shown in the inset (black=low power, P_{in} = 14.1 μW, red=high power, P_{in} = 223 μW). Mechanical modes below 100 MHz (blue) are due to the cantilever, while modes at 364.63 MHz and 577.20 MHz (green) are due to the disk. (d) RF spectrum of a disk without the cantilever, displaying modes at 364.74 MHz and 576.20 MHz. The inset shows a high-Q TE optical mode of the disk (blue) with fit (black). (e) Zoomed-in RF spectrum of the disk-cantilever, showing the h_{mc} modes (blue), calibration peak (purple), and detection background (black).
placement is approximately the ratio of the cavity linewidth ($\Gamma/2\pi = 2.44$ GHz) to $g_{OM}$, and is $\approx 4$ nm, giving a dynamic range $> 10^6$ (60 dB) for a 1s measurement. The bandwidth (BW) is limited by the cavity’s response time, which determines how quickly it can transduce mechanical motion. We therefore expect a BW $> 1$ GHz, and this is substantiated by transduction of the 575 MHz oscillations of the disk as previously described in Fig. 2(c)-(d). Adjusting the BW (e.g., through the waveguide coupling) allows for gain/BW trade-off within the fixed gain-BW product. The large BW of these devices is one advantage of relying on large $g_{OM}$ rather than ultra-high-$Q$ for displacement detection.

Optically-induced stiffening

Increasing the optical power coupled into the cavity causes several notable changes in the RF spectrum, as seen in Fig. A(a)-(b) for a $W=65$ nm, $G = 75$ nm device, where the coupled power is changed by fixing $P_{in} = 446 \, \mu W$ and varying the detuning $\Delta \lambda$ between the laser and cavity mode. First, the spectral position of the $h_{1x}$ mode changes from $\Omega_M/2\pi \approx 2.24$ MHz at large $\Delta \lambda$ to $\Omega_M/2\pi \approx 3.26$ MHz at $\Delta \lambda = -21$ pm before returning to close to its original value at near-zero $\Delta \lambda$ (Fig. A(c)). One explanation for this is the optical spring effect, an optically-generated rigidity of the mechanical oscillator, as seen in other works. In particular, if we take the measured values for $\Omega_M$, $\Omega_M'$, $\Delta \lambda$, $\Gamma$, and internal cavity energy $U$ (determined by $P_{in}$, transmission contrast, and $\Gamma$), the value of $g_{OM}$ that best matches the maximum frequency shift is $g_{OM}/2\pi = 1.4$ GHz/nm, corresponding to a gap $G \approx 60$ nm for the TE$_{2,45}$ mode. Similarly, Fig. A(d) shows a shift from $\Omega_M/2\pi \approx 2.26$ MHz at large $\Delta \lambda$ to $\Omega_M/2\pi \approx 4.25$ MHz at $\Delta \lambda = -41$ pm, in this case for the $h_{1x}$ mode of a $W=65$ nm, $G = 50$ nm device. This shift is consistent with $g_{OM}/2\pi = 3.0$ GHz/nm, corresponding to a gap $G \approx 32$ nm for the TE$_{2,45}$ mode. Both of these gaps are smaller than the nominal values, but are reasonable given the variation observed in SEM images of fabricated devices.

Along with the change in frequency, the linewidth of the $h_{1x}$ mode changes from $\Gamma_M/2\pi \approx 410$ kHz at $\Delta \lambda = -61$ pm to $\Gamma_M/2\pi \approx 860$ kHz at $\Delta \lambda = -21$ pm, indicating damping. In addition, the increase in RF amplitude of the $h_{mx}$ modes is accompanied by a broad background which, in certain detuning ranges, produces peaks in the RF spectrum not seen at lower powers and at frequencies that are not predicted by mechanical simulations of the cantilever. The precise nature of these effects is not understood, though a likely cause is the interplay between free-carrier and thermal effects that takes place in silicon microdisks as the intracavity energy is increased. Measurements of devices with and without cantilevers (supplementary information) show behavior consistent with previous observation of such effects. It should also be noted that thermal effects have been observed to generate damping for blue-detuned excitation in other optomechanical systems.

**Cantilever engineering and outlook**

While optically-induced stiffening provides real-time control of the cantilever properties over a certain range, a number of modifications to its geometry can improve its applicability to different AFM applications. The sub-N/m spring constant of the $h_{1x}$ mode is suitable for weak force measurements in which the cantilever is undriven, but in dynamic techniques for which the best imaging conditions have been achieved, such as frequency modulation AFM, spring constants in the tens of N/m to hundreds of N/m range (or more) are desirable for small amplitude operation. In our geometry, the cantilever stiffness may be increased by increasing its width; figs. A(a)-(b) show the mechanical mode spectra for $W=100$ nm and $W=200$ nm devices. The $h_{1x}$ modes at $\Delta \lambda_M/2\pi = 3.33$ MHz and $\Omega_M/2\pi = 6.96$ MHz agree well with the simulated values of 3.42 MHz and 7.22 MHz. Based on the calculated effective masses, these values correspond to a cantilever stiffness of 0.52 N/m and 4.11 N/m, respectively, with the latter being a $30\times$ increase in stiffness relative to the $h_{1x}$ mode of the $W = 65$ nm device. Stiffer cantilevers can be produced by a further increase in $W$, though degradation in the optical $Q$ is expected unless $G$ is increased, which can then limit the displacement sensitivity due to a reduced $g_{OM}$. Another option is to use smaller diameter microdisks, to reduce the cantilever length between its suspension points. Bare microdisks have radiation-limited $Q_s > 10^6$ until their diameters are just a couple of micrometers, and simulations predict that the $h_{1x}$ mode of a $W=100$ nm cantilever coupled to a 4.5 $\mu m$ diameter disk will occur at 7.96 MHz ($k = 1.8$ N/m). Another important consideration is the modal structure of the cantilever. Though we have focused on the $h_{1x}$ mode due to its displacement profile and transduction under thermal noise, in an AFM setting, the cantilever motion will be defined by both its actuation mechanism and the surface it is interrogating, and its motion will be a superposition of its modes. This includes out-of-plane ($\perp$ direction) and orthogonal in-plane ($\parallel$ direc-
Reducing the cantilever spring constant by as much as two or-tion mechanism for driving the cantilever’s motion. As an il-
surrounding areas. By combining this approach with
further modifications may be made to increase the stiffness of
frequencies. Figure 4(c) shows an optically-transduced RF
of-plane mode
4(c) has
double cantilever structure shown in the SEM images of Fig.
37
v
h
µ
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1
1
significantly stiffened and shifted to higher
x
y
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100 nm, G=200 nm. (c) RF spectrum from a disk-double-cantilever with
W=65 nm, G=50 nm. Inset shows SEMs of the device geometry.
FIG. 4: (a) RF spectrum from a disk-cantilever with
W=100 nm, G=200 nm. Inset shows displacement profiles for cantilever modes
not shown in Fig. (b). (b) RF spectrum from a device with W=200 nm, G=200 nm. (c) RF spectrum from a disk-double-cantilever with
W=65 nm, G=50 nm. Inset shows SEMs of the device geometry.
Future dynamic AFM measurements will require an actua-
tion level should be achievable for an applied voltage near
5 V. We can then estimate the performance of this system in
a frequency modulation AFM scheme using the results from
Ref. 35 along with the h1x mode frequency ΩM/2π=2.48
MHz and linewidth ΓM/2π=603±4 kHz for the device of Fig.
4(c). The minimum detectable force Fmin and force gradient
δF′min are Fmin = (4kkBτB)/(ΩM QM)=5.1×10^{-14} N and
δF′min = (4kkBτB)/(ΩM QM A^2)=1.2×10^{-5} N/m, where A is
the cantilever oscillation amplitude (4 nm), and B is the
measurement bandwidth, taken to be 50 Hz for comparison
to other experiments. Despite operating in an ambient envi-
ronment with QM ≈ 4, the estimated Fmin and δF′min values are
competitive with a range of systems operated in an ultra-high
vacuum environment. In particular, silicon cantilevers5 with
k=2 N/m, ΩM/2π=75 kHz, and QM = 1.0×10^4 have achieved
Fmin = 5.9×10^{-15} N and δF′min = 3.0×10^{-7} N/m, while
quartz tuning forks in the qPlus configuration6,7 with k=1800
N/m, ΩM/2π=20 kHz, and QM = 2.5×10^3 have achieved
Fmin = 2.1×10^{-12} N and δF′min = 1.1×10^{-2} N/m. More
recently,9,12,18 ultra-stiff piezoelectric quartz length-extension
resonators with k=5.4×10^5 N/m, ΩM/2π=1 MHz, and QM =
2.5×10^5 have achieved Fmin = 1.6×10^{-12} N and δF′min =
8.3×10^{-3} N/m. The disk-cantilever system demonstrated
here operates in an attractive region of parameter space that
differs from the above sensors, in combining a MHz oscilla-
tion frequency with a 0.1 N/m to 10 N/m stiffness (with stiffer
geometries potentially feasible).

In summary, we have demonstrated sensitive transduction of the motion of a nanoscale cantilever using a high quality factor microdisk cavity fabricated on the same device layer. Future work will be aimed at understanding the capabilities of this system in AFM measurements. This will include measurements under vacuum to determine ultimate mechanical Q\textsubscript{ls} of the devices, and to ascertain whether effects such as optical cooling and regenerative oscillations\textsuperscript{19} are accessible. Functional devices for AFM will be fabricated to expose the probe tips to allow close proximity to other surfaces, and will be fully integrated systems combining electrostatic or optical actuation with on-chip resonators and waveguides\textsuperscript{41}.

Methods

Device Fabrication Devices were created in a silicon-on-insulator wafer with a 260 nm thick device layer, 1 \( \mu \text{m} \) thick buried oxide layer, and specified device layer resistivity of 13.5-22.5 ohm-cm (p-type). Fabrication steps included electron-beam lithography of a 400 nm-thick positive-tone resist, an SF\textsubscript{6}/C\textsubscript{4}F\textsubscript{8} inductively-coupled plasma reactive ion etch through the silicon device layer, a stabilized H\textsubscript{2}SO\textsubscript{4}/H\textsubscript{2}O\textsubscript{2} etch to remove the remnant resist and other organic materials, an HF wet etch to undercut the devices and release the cantilevers, and a critical point dry to finish the processing. The etch time required to go through the silicon device layer is a function of cantilever-disk gap, with an \( \approx 30\% \) increase in etch time required for \( G = 50 \text{ nm} \) devices relative to \( G = 200 \text{ nm} \) devices.

Device Simulation Mechanical eigenfrequencies and eigenmodes of the cantilever and disk were studied using a commercial finite element software package. Silicon was modeled as an elastic cubic material using three independent elastic constants\textsuperscript{12} with (100) orientation, and clamped boundary conditions were assumed at the cantilever ends. Mesh refinement studies indicate that numerical errors are below the uncertainty resulting from imperfect knowledge of the cantilever geometry, which is generally a few percent of the reported values. For the reported mode frequencies \( \Omega \) and effective masses \( m \), zero residual stress was assumed. In a separate numerical study, all cantilever mode frequencies were shown to be approximately independent (within a few percent) of the residual stress for stress values under \( \pm 100 \text{ MPa} \). The mode stiffness was calculated as \( k = m \Omega^2 \).

Electrostatic actuation was modeled by iteratively solving a coupled three-dimensional static-mechanical problem and a three-dimensional electrostatic problem. The former fixes the elastic properties and clamped boundary conditions at the four double-cantilever ends to be the same. The mechanically fixed electrodes are 260 nm thick, 500 nm wide, and 3 \( \mu \text{m} \) long, and the gap between them and the cantilever is 350 nm. The same fixed voltage is applied to both electrodes (doped silicon is assumed to be a perfect conductor), while the cantilever is assumed to be at the ground potential. Given the applied voltages and shape of the deformable cantilever and fixed electrodes, the electrostatic force densities on all cantilever surfaces are calculated using a boundary element method. The calculated force densities were then applied as boundary conditions and the mechanical problem was solved to find the new deformed beam shape. The electrostatic and mechanical solvers were iterated until the solution converged to a stable value for each applied voltage. Mesh refinement studies were conducted on the electrostatic surface mesh to ensure numerical accuracy. The microdisk and substrate were assumed to be at ground potential and not included in this model for simplicity. This is justified because the cantilever-electrode gap is much smaller than the distances between the electrodes and either the microdisk or substrate.

Optical eigenfrequencies and eigenmodes of the disk-cantilever system were found numerically using a second commercial finite element software package. The silicon layer was modeled as having an index of refraction \( n = 3.4 \) surrounded by air (\( n = 1 \)), and both materials were assumed lossless and non-magnetic. The model size was chosen to be large enough to fully contain the modes studied, with scattering boundary conditions on the outside surfaces. A mesh refinement study was conducted to ensure numerical accuracy. \( g_{OM} \) for the \( h_1 \) mechanical mode as a function of the gap \( G \) was obtained by linearly translating the cantilever with respect to the disk along the \( x \)-axis from the initial cantilever-disk gap \( G = 100 \text{ nm} \). For each value of \( G \) between 30 nm and 300 nm the cantilever was further deformed using the calculated \( h_1 \) mode shape. The modal deformations were 0 and \( \pm d \), where \( d \) varied from 2 nm for \( G = 30 \text{ nm} \) to 10 nm for \( G > 100 \text{ nm} \). For each \( G \) and deformation the frequencies and \( Q \)s for multiple optical modes were numerically calculated. For each optical mode \( G \) and the derivative of the frequency with respect to modal deformation was obtained using the slope of a linear fit. In all cases, the gap changes and cantilever deformations were implemented by numerically deforming the same original mesh to obtain the desired cantilever shape and position before solving the optical eigenvalue problem.

Device Characterization Devices were characterized using a swept-wavelength external cavity tunable diode laser with a time-averaged linewidth < 90 MHz and absolute stepped wavelength accuracy of \( \pm 1 \text{ pm} \). The wavelength tuning range and linearity are calibrated using an acetylene reference cell, so that the uncertainty in optical cavity \( Q \)s is dominated by fits to the data. Light is coupled into and out of the cavities using an optical fiber taper waveguide in a \( \text{N}_2 \)-purged environment at atmospheric pressure and room temperature. Cavity transmission spectra were recorded using a variable gain InGaAs photoreceiver with a typical bandwidth of 775 kHz, noise equivalent power (NEP) of 1.25 pW/\( \sqrt{\text{Hz}} \), and gain of \( 4.5 \times 10^4 \text{ V/W} \). RF spectra were recorded using either a 0 MHz (DC) to 125 MHz InGaAs photoreceiver (NEP=2.5 pW/\( \sqrt{\text{Hz}} \), gain=4\times10^4 \text{ V/W}) or DC to 1.1 GHz InGaAs avalanche photodiode (NEP=1.6 pW/\( \sqrt{\text{Hz}} \), gain=1.4\times10^4 \text{ V/W}) whose output was sent into a 9kHz to 3.0 GHz electronic spectrum analyzer with resolution bandwidth typically set at 30 kHz. RF frequencies and linewidths are determined by Lorentzian fits to the data, with uncertainties given by the 95 % confidence intervals of the fit (uncertainties are not written if they are smaller than the number of digits to which the value is quoted). Optical frequencies and linewidths are determined by a least squares fit to the data using a doublet model that takes into account both clockwise and counterclockwise whispering.
gallery modes and their coupling due to backscattering.\textsuperscript{30} Phase modulator calibration As a consistency check on the calibration of displacement sensitivity,\textsuperscript{25} we use an electro-optic phase modulator (Fig. 2(a)) of known modulation depth $\delta \phi$ and frequency $\Omega_{\text{mod}}$ to generate a tone in the RF spectrum, at 44 MHz in Fig. 2(e). This modulation peak is equivalent to an effective mechanical oscillation amplitude $s_{\text{mod}} = \delta \phi (\Omega_{\text{mod}} / \Omega_{\text{OM}})$, and can provide a check on $x_{\text{rms}}$, but is limited by the accuracy to which $\Omega_{\text{OM}}$ is known. For Fig. 2 assuming $G = 100$ nm and that the optical mode used for transduction is the TE$_{2,45}$ mode, $\Omega_{\text{OM}} / 2\pi = 0.61$ GHz/nm produces a value $x \approx 192$ pm that is $\approx 20 \%$ greater than $x_{\text{rms}} = 160$ pm. A likely source for the discrepancy is imperfect knowledge of the gap; for example, a 10 nm decrease in it would completely account for the difference between the two values.
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Supplementary Information

1. Optical cavity modes and optomechanical coupling

Finite-element method (FEM) simulations indicate that the \( p=1 \) and \( p=2 \) modes of TE polarization and \( p=1 \) modes of TM polarization have high \( Q_s \) values (\( >10^5 \)) for sufficiently thin cantilevers. Simulation results for \( W=65 \) nm cantilevers are shown in Fig. S1(a). Similar simulations for \( W=100 \) nm cantilevers indicate a reduction in \( Q_s \) by as much as a factor of 3, though it nevertheless remains above \( 10^3 \). As discussed in the text, while most fabricated devices have cavity \( Q_s \)s in the range of \( 5 \times 10^4 \) to \( 2 \times 10^5 \); a few exhibit \( Q_s \)s as high as \( \approx 6 \times 10^5 \) (Fig. S1(b)). The optically transduced RF spectra in such devices often show a strong amplitude for not only the \( h_{1x} \) modes, but also \( h_{my} \) and \( v_{n} \) modes. This suggests some amount of asymmetry in the cantilever structure not found in the majority of the devices (such as those studied in the main text).

Generally, the measured optical \( Q_s \)s decrease with decreasing gap and increasing cantilever width. Smaller gaps can also be problematic because the time required to etch through the silicon layer goes up as the gap size is reduced, potentially leading to mask erosion and a roughening of the disk sidewalls. The optomechanical coupling \( g_{OM} \), on the other hand, increases with decreasing gap and increasing cantilever width. The calculated \( g_{OM} \) for \( p=1 \) modes with a \( W=100 \) nm cantilever is shown in Fig. S1(c), and can be \( \approx 25 \% \) larger than the values calculated for \( W=65 \) nm in Fig. S1(d).

2. Hansch-Couillaud polarization spectroscopy

For future experiments (including AFM applications) it will likely be necessary to lock the probe laser to the cavity. This can be done by beating the signal exiting the cavity with a strong local oscillator (LO), thereby measuring phase fluctuations due to cantilever motion and giving access to a dispersive signal needed for locking. A particularly convenient approach, Hansch-Couillaud polarization spectroscopy as described in Refs. S2 and S3 and shown schematically in Fig. S2(a), sets the polarization so that only part of the input field couples to the cavity, with the orthogonal polarization serving as the LO. The interference signal is analyzed using a \( \lambda/4 \) waveplate and polarizing beam splitter, whose outputs are measured on a 100 MHz balanced photodetector. The interference signal produced by scanning the laser over a cavity resonance is shown in the inset to Fig. S2(b). Positioning the laser on resonance and measuring the RF fluctuations in this signal produces the thermal noise spectrum shown in Fig. S2(b).

3. Self-induced optical modulation and free carrier effects

Two-photon absorption is well-known to play an important role in silicon nanophotonics\(^{54} \), with the subsequent generation of phonons and free carriers giving rise to both optical dispersion and loss, and with the associated lifetimes affecting the speed of devices intended to exploit these effects. In Ref. S5, Johnson et al. observed that under sufficiently strong continuous wave input, silicon microdisks of similar dimensions to those studied in this work exhibited steady-state oscillations in their transmitted power. The authors attributed this to competing thermal and free-carrier effects, as the dispersion in the refractive index caused by the two effects are opposite in sign (red-shift for thermal, blue-shift for free carriers), and as the cavity mode position shifts due to this change in refractive index, the circulating power in the cavity changes, thereby changing the rate at which heat and free carriers are created. Looking in the frequency domain, the RF spectrum of the transmitted signal displayed a number of sharp peaks with a spacing of a few hundred kHz. We have observed similar phenomena in our bare (no cantilever) microdisks. Fig. S3(a) shows both a broad (up to 200 MHz) and zoomed-in (up to 15 MHz) spectrum of the transmitted signal from a microdisk with a \( Q \approx 3 \times 10^5 \) mode coupled to by a fiber taper.
FIG. S2: (a) Schematic of the setup used for Hansch-Couillaud homodyne spectroscopy. The input polarization into the cavity is set so that a small fraction of the signal is coupled into the mode of interest, while the remainder acts as a local oscillator. (b) RF spectrum measured using Hansch-Couillaud homodyne spectroscopy. Inset shows the difference signal produced when the laser is scanned over the cavity resonance.

waveguide with $P_{in} \approx 440 \mu W$ at 1533.6 nm. A comb of sharp peaks is observed in the RF spectrum, with a nearest-neighbor spacing that is typically $\approx 3.23$ MHz.

The devices shown in the main body of the text have somewhat lower optical $Q$s than the above device, which likely explains why similarly sharp RF peaks are not observed at similar input powers. Instead, the RF spectra (Fig. S2a)) look to be a superposition of the spectrum due to mechanical oscillations of the cantilever and/or disk and the spectrum due to competing free carrier and thermal effects within the disk, albeit below the threshold for oscillation. To consider this point further, we look at the disk-cantilever device of Fig. S1b), coupling to a $Q \approx 1.3 \times 10^5$ cavity mode. If we initially restrict $P_{in} \approx 60 \mu W$ and vary the laser-cavity detuning $\Delta \lambda$, we generate fig. S3b). We see that for initial large detunings ($\Delta \lambda = -35$ pm), the RF spectrum is dominated by the mechanical modes of the cantilever, but as the detuning decreases, a background with broad resonances is superimposed ($\Delta \lambda = -31$ pm) and dominates ($\Delta \lambda = -23$ pm), before eventually the mechanical modes re-appear for small enough detunings ($\Delta \lambda = -15$ pm). It is believed that the broad background is due to the same thermal/free-carrier effects seen in Ref. S5 and in the bare disk of Fig. S2a). Indeed, if $P_{in}$ is increased to a few hundred $\mu W$, a qualitatively similar RF spectrum (Fig. S3c)) is observed - here the mechanical modes are completely dominated by the thermal/free-carrier effects.

Quantitative modeling of this behavior can be accomplished in a manner similar to that of Ref. S6, where the equations of bare optomechanics (evolution of the intracavity optical field amplitude and mechanical position) were augmented by an equation for the cavity temperature increase. We now have to add a fourth differential equation, to account for the change in free carrier population. Following the treatment of thermal
and free-carrier terms presented in Ref. S5, we have:

\[
\frac{da}{dt} = -\frac{1}{2} \left( \Gamma + \alpha_{\text{TPA}} |a(t)|^2 + \beta_{\text{FCA}} N(t) \right) a(t) + i \left( \delta \omega_c + g_{\text{GOM}} x + g_{\text{th}} \Delta T(t) + g_{\text{fc}} N(t) \right) a(t) + \kappa_s
\]

\[
\frac{dx}{dt} = -\Gamma_M \frac{dx}{dt} - \Omega_M^2 x - \frac{|a(t)|^2 g_{\text{GOM}}}{\omega_c m}
\]

\[
\frac{d\Delta T}{dt} = -\gamma_{\text{th}} \Delta T(t) + c_{\text{th}} \left( \Gamma_{\text{abs}} + \alpha_{\text{TPA}} |a(t)|^2 + \beta_{\text{FCA}} N(t) \right) |a(t)|^2
\]

\[
\frac{dN}{dt} = -\gamma_{\text{fc}} N(t) + \chi_{\text{FCA}} |a(t)|^4
\]

where for simplicity we have assumed a single-mode cavity - a more detailed treatment would include both modes of the microdisk and their coupling via backscattering. The first equation describes the intracavity field amplitude \(a(t)\), where the first term on the right is its decay due to intrinsic and waveguide loss \(\Gamma\), two-photon absorption \(\alpha_{\text{TPA}}\) and free-carrier absorption \(\beta_{\text{FCA}}\), while the second term includes the laser-cavity detuning \(\delta \omega_c\) and dispersion due to optomechanical coupling \(g_{\text{GOM}}\), thermo-optic tuning \(g_{\text{th}}\), and free-carrier dispersion \(g_{\text{fc}}\). The second equation describes the mechanical motion \(x(t)\) with frequency \(\Omega_M\) and damping \(\Gamma_M\) and driven by the coupling to the optical field. The third equation describes the cavity temperature change \(\Delta T(t)\), where the cavity has a heat capacity \(c_{\text{th}}\), the temperature decays with a rate \(\gamma_{\text{th}}\), and is generated by linear absorption \(\Gamma_{\text{abs}}\) is the portion of total optical loss that contributes, two-photon absorption \(\alpha_{\text{TPA}}\), and free-carrier absorption \(\beta_{\text{FCA}}\). Finally, the fourth equation describes the modal free carrier population \(N(t)\), which decays at a rate \(\gamma_{\text{fc}}\) and is generated in proportion to the square of intracavity energy, with proportionality \(\chi_{\text{FCA}}\). The various coefficients in the above equations are described in detail in Ref. S5, and are a combination of physical properties such as the two-photon absorption coefficient of silicon and the absorption cross-section for free carriers, as well as cavity mode properties such as its group index and different confinement factors and modal volumes weighted according to the electric-field dependence of the given process (e.g., two-photon absorption or free-carrier absorption).

An analysis of the above equations will produce correction terms to the mechanical frequency \(\Omega_M\) and linewidth \(\Gamma_M\), and may help provide a better understanding of the power-dependent RF spectra shown in the main text (Fig. 3). For example, Eichenfield et al. determined that in their SiN crystal nanobeam devices, heating significantly affected the linewidth as a function of detuning, but not the mechanical frequency. In comparison, in addition to linewidth modification (observation of damping for blue-detuned excitation), the frequency dependence on detuning for our devices (Fig. 3(c)-(d)) does appear to show some effect, in that the shape of the curves near zero-detuning is not nearly as sharply-sloped as the equations of bare optomechanics predict.

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