Unification of gauge coupling constants in the minimal supersymmetric model with 
\(\alpha_s \approx 0.11\)

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We have studied the gauge unification with the recent electroweak data as a function of the higgsino mass. It was shown that if the strong coupling constant is small \(\approx 0.11\), consistent picture of gauge unification is not possible in the minimal supersymmetric standard model.

PACS number(s): 12.60.Jv, 12.10.Kt

I. INTRODUCTION

One of the testing ground of grand unified theories is the strict unification of the three coupling constants \(\alpha_1, \alpha_2\) and \(\alpha_s\) of the standard model \(SU(3)_c \times SU(2)_L \times U(1)_Y\) at some high scale \(M_{GUT}\), using as inputs their experimental values at the Z-pole mass. The coupling constants \(\alpha_1\) and \(\alpha_2\) can be determined from the accurate experimental measurement of \(\alpha_{em}\) and \(\sin^2 \theta_W\) at \(M_z\) pole mass. The world average values for \(M_z, \alpha_{em}\) and \(\sin^2 \theta_W\) are

\[
\begin{align*}
M_z &= 91.184 \pm 0.0022 \\
\alpha_{em}^{-1} &= 127.9 \pm 0.09 \\
\sin^2 \theta_W &= 0.2315 \pm 0.0002 \pm 0.0003
\end{align*}
\]

The coupling constants \(\alpha_1\) and \(\alpha_2\) of the \(U(1)_Y\) and \(SU(2)_L\) gauge are related to \(\alpha_{em}\) and \(\sin^2 \theta_W\) as,

\[
\begin{align*}
\alpha_1 &= \frac{5}{3} \frac{\alpha_{em}}{\cos^2 \theta_W} \\
\alpha_2 &= \frac{\alpha_{em}}{\sin^2 \theta_W}
\end{align*}
\]

The strong coupling constant \(\alpha_s\) has also been measured, its world average value is; \(\alpha_s(M_z) = 0.123 \pm 0.005\). Recently, it has been pointed out that QCD can not tolerate such a large \(\alpha_s\). Several low energy experiments also indicate that \(\alpha_s\) must be close to 0.11, \(3\sigma\) below the Z-peak value measured at collider experiments. Theoretically clean deep inelastic scattering experiments give \(\alpha_s(M_z) = 0.112 \pm 0.005\). A new analysis of the \(Y\) sum rule yield \(\alpha_s(M_z) = 0.109 \pm 0.001\). Similar values are obtained in lattice QCD; \(\alpha_s(m_Z) = 0.110 \pm 0.006\) (\(c\bar{c}\) spectrum) and \(0.115 \pm 0.002\) (\(b\bar{b}\) spectrum). The apparent conflict between the low energy and the collider energy Z-peak determination of \(\alpha_s\) may be due to the higher order corrections to the LEP values. Preliminary analysis suggests that these higher order correction may bring down the collider values to be consistent with the low energy measurements. Indeed it has also been argued that the systematic error usually quoted in LEP number is grossly underestimated, and the LEP can only claim to determine the strong coupling constant within the limit, \(0.10 \leq \alpha_s \leq 0.15\).

It is now established that the standard model is inconsistent with gauge unification, with the experimentally measured coupling constants. At early 90, the measurements were consistent with the gauge unification in the minimal supersymmetric standard model (MSSM). Recently Langacker and Polonsky using the recent electroweak data found that in MSSM, \(\alpha_s(M_z) \approx 0.129\) is required for strict gauge unification. This value is considerably higher than the world average value of \(\alpha_s(M_z) = 0.123\). Also it is much above the low energy measurements and QCD motivated value of \(\alpha_s(M_z) \approx 0.11\). In the context of MSSM the lower \(\alpha_s\) require raising the SUSY particle masses considerably higher than the 1 TeV scale. However, SUSY mass spectra considerably higher than 1 TeV scale will have problem of diverging radiative correction. Also, heavy SUSY masses will be inconsistent with our expectation that the lightest supersymmetric particle (LSP) will be neutral and the candidate for the dark matter. The other possibility, to reconcile \(\alpha_s \approx 0.11\) with gauge unification, is to assume very large negative heavy threshold correction at GUT scale and NRO’s which could decrease \(\alpha_s(M_z)\) by 10% \(\alpha_s(M_z) \approx 0.109\). Recently, Roszkowski and Shifman argued that in the simple supersymmetric extension of the standard model, one need not have the constraint of universal gaugino masses at the GUT scale, which is, in general assumed in MSSM. To fully specify the MSSM, one needs to invoke the supersymmetry breaking pattern. The soft breaking term, which in essence couple the MSSM with the N=1 supergravity give rise to the constraint of universal gaugino masses at GUT scale. Thus in
a pure phenomenological approach, one can relax the condition of universal gaugino masses. They showed that by relaxing the constraint small $\alpha_s$ compatible with low energy measurements can be obtained \[18\]. In that case, gluino becomes lighter than the wino and can be well below 200 GeV \[18\]. However, the approach is unsatisfactory, as there is no theoretical scheme for the symmetry breaking term.

In the present paper, we show that consitent gauge unification with small $\alpha_s$ is not possible within the framework of MSSM. Our approach is phenomenological. We keep the constraint of universal gaugino masses. We treat the higgsino masses as a parameter of unification, and tune it to obtain unification for a given value of $\alpha_s$. Gauge unification with $\alpha_s \approx 0.11$, require higgsino masses in the range of $10^6$ GeV, three order of magnitude larger than other sparticle masses. Higgsino masses in the range of $10^6$ GeV will require that the supersymmetry break at that scale only and the model will face the unsatisfactory aspect of gauge hierarchy problem.

The paper is organised as follows. In section 2, we describe the model briefly, in section 3, the results obtained will be discussed. Summary and conclusions will be given in section 4.

II. MINIMAL SUPERSYMMETRIC STANDARD MODEL (MSSM)

The renormalisation group equations for the gauge couplings are given by (neglecting the small Yukawa couplings),

$$\frac{d\alpha_i^{-1}}{dt} = -\frac{b_i}{2\pi} - \sum_j \frac{b_{ij} \alpha_j}{8\pi^2}, i = 1, 3$$  \hspace{1cm} (3)

where $t = \ln(Q/M_{GUT})$, with Q the running scale and $M_{GUT}$ the unification scale mass.

The one loop coefficients $b_i$ of the $\beta$ functions for the gauge couplings change across each new running mass threshold. In the MSSM they can be parameterized as \[19\] \[21\],

$$b_1 = \frac{41}{10} + \frac{2}{5}\theta_H + \frac{1}{10}\theta_{H_2} + \frac{1}{5}\sum_i \left[ \frac{1}{12}(\theta_{u_{L_i}} + \theta_{d_{L_i}}) + \frac{4}{3}\theta_{\tilde{u}_{R_i}} + \frac{1}{3}\theta_{\tilde{d}_{R_i}} + \frac{1}{4}(\theta_{\tilde{e}_{L_i}} + \theta_{\tilde{e}_{R_i}} + \theta_{\tilde{E}_{R_i}}) \right]$$  \hspace{1cm} (4a)

$$b_2 = -\frac{19}{6} + \frac{4}{3}\theta_W + \frac{2}{3}\theta_H + \frac{1}{6}\theta_{H_2} + \frac{1}{2}\sum_i \left[ \theta_{u_{L_i}} \theta_{d_{L_i}} + \frac{1}{3}\theta_{\tilde{e}_{L_i}} \theta_{\tilde{e}_{R_i}} \right]$$  \hspace{1cm} (4b)

$$b_3 = -7 + 2\theta_g + \frac{1}{6}\sum_i \left[ \theta_{u_{L_i}} \theta_{d_{L_i}} + \theta_{u_{R_i}} \theta_{\tilde{u}_{R_i}} \right]$$  \hspace{1cm} (4c)

where $\theta_x = \theta(Q^2 - m_x^2)$. In the above equations $\tilde{H}$ stands for the mass degenerate Higgsino fields, $\tilde{W}$ for the winos, the partner of the SU(2) gauge bosons ($M_{\tilde{W}} = M_2$), $\tilde{g}$ is the gluino, the partner of the gluon, all are assumed to be mass eigen states in this approximation. The $H_2$ stands for the heavy Higgs boson doublet \[21\].

The effect of low mass threshold on two loop beta function is expected to be small. In the weak scale, we thus use the SM values:

$$b_{ij} = \begin{pmatrix} 199&27&44 \\ 9&35&12 \\ 10&9&26 \end{pmatrix}$$  \hspace{1cm} (5)

and from the weak to the GUT scale, we use,

$$b_{ij} = \begin{pmatrix} 199&27&88 \\ 9&25&24 \\ 1&9&-14 \end{pmatrix}$$  \hspace{1cm} (6)

The RGE equations can be integrated in a step function approximation to obtain $\alpha_i(\mu)$ at a scale $\mu$ for a given $\alpha_i(\mu)$,

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(\mu)} + \beta_0 \ln \frac{\mu}{\mu} + \frac{\beta_1}{\beta_0} \ln \frac{1/\alpha_i(\mu)}{1/\alpha_i(\mu)}$$  \hspace{1cm} (7)

with
\[ \beta_0 = -\frac{1}{2\pi} (b_i + \frac{b_{ij}}{4\pi} \alpha_j (\mu) + \frac{b_{ik}}{4\pi} \alpha_k (\mu)) \]  
(8a)
\[ \beta_1 = -\frac{2b_{ii}}{(4\pi)^2} \]  
(8b)

The above eq. can be solved iteratively to obtain the coupling constants at any arbitrary energy, knowing their value at a given energy.

In addition to the above equations, we consider the evolution of gaugino masses. There evolution equations are simple,

\[ \frac{dM_i}{dt} = -\frac{b_i}{4\pi} \alpha_i M_i \]  
(9)

with the boundary condition at \( M_{GUT} \): \( M_i(t = 0) = M_{1/2} \). The solution for the gluino and the winos can be written as,

\[ M_{\tilde{W}} = \frac{\alpha_2 (M_{\tilde{W}})}{\alpha_5} M_{1/2} \]  
(10a)
\[ M_{\tilde{g}} = \frac{\alpha_5 (M_{\tilde{g}})}{\alpha_3} M_{1/2} \]  
(10b)

where \( \alpha_5 \) is the unified coupling constant at the GUT scale. Combining the two equation we obtain,

\[ M_{\tilde{g}} = \frac{\alpha_5 (M_{\tilde{g}})}{\alpha_2 (M_{\tilde{W}})} M_{\tilde{W}} \]  
(11)

For a given \( M_{\tilde{W}} \), \( M_{\tilde{g}} \) can be obtain iteratively from eq. [1].

The simple step-function approximation used to integrate the RG equations is justified only in \( \overline{DR} \) scheme. However, the experimental \( \alpha_{em} \) and \( \sin^2 \theta_W \) were obtained in the \( \overline{MS} \) scheme. We therefore convert the coupling constants into the \( \overline{DR} \) scheme by,

\[ \frac{1}{\alpha_i} = \frac{1}{\alpha_i^{\overline{MS}}} \frac{C_i}{12\pi} \]  
(12)

where \( C_1 = 0, C_2 = 2 \) and \( C_3 = 3 \).

III. RESULTS

A. Higgsinos and gauge unification

We note that apart from the winos and the gluino, the beta function coefficients are most affected by the higgsinos. Not only they appear in two beta function \( b_1 \) and \( b_2 \), compared to other sparticles, their coefficients are also larger. We therefore choose the higgsino mass as the parameter which will be tuned to obtain strict gauge unification with recent electroweak data. The wino and the gluino will be treated separately as they are connected by the universal gaugino mass at GUT scale. All the other SUSY masses will be assumed to be degenerate at a common mass (\( M_c \)). This is certainly an assumption, however, it helps to identify the role of higgsinos in the unification process.

We define the GUT scale (\( M_{GUT} \)) as the scale where all the three coupling constants unify at some value \( \alpha_5 \). The RG equations for \( \alpha_1, \alpha_2 \) and \( \alpha_3 \) are run simultaneously from \( M_c \) to \( M_{GUT} \). The input \( \alpha_1 \), \( \alpha_2 \), and \( \alpha_3 \) were calculated from the experimental \( \alpha_{em} \) and \( \sin^2 \theta_W \) (eq. [4]). We have used the central value for \( \alpha_{em} \). Unification of three forces is sensitive to the input value of \( \sin^2 \theta_W \). This sensitivity is due to the fact that \( \alpha_2 \) does not change much between \( M_c \) and \( M_{GUT} \), as do the other two couplings. Thus a small change in \( \sin^2 \theta_W \) has an enhanced effect on the unification.

We therefore consider two input values of \( \sin^2 \theta_W \): \( \sin^2 \theta_W = 0.231 \) and \( \sin^2 \theta_W = 0.232 \), covering the 1\( \sigma \) variance. For the input strong coupling \( \alpha_s \), we choose a value between 0.10-0.13. For a given wino mass (\( M_{\tilde{W}} \)) and the common SUSY mass \( M_c \) we vary the higgsino mass (\( M_{\tilde{H}} \)) 100 GeV onwards to find the minimum \( M_{\tilde{H}} \) required for the strict unification of the three couplings. The gluino mass was obtained iteratively from eq. [1].

In fig. 1a, we have shown the higgsino mass \( M_{\tilde{H}} \) required to obtain strict gauge unification, as a function of the input \( \alpha_s \). The common SUSY particle mass (\( M_c \)) and the wino mass were fixed at 1000 GeV. The black dots and the open triangles corresponds to \( \sin^2 \theta_W = 0.231 \) and \( \sin^2 \theta_W = 0.232 \) respectively. The higgsino mass required for unification
shows a sensitive dependence on the input \( \alpha_s \). It also depends on the input \( \sin^2 \theta_W \). For \( \sin^2 \theta_W = 0.231 \), the higgsino mass varies between \( 6.4 \times 10^9 \) GeV to \( 4.5 \times 10^3 \) GeV, as \( \alpha_s \) changes from 0.10 to 0.13. The higgsino mass is lowered approximately by a factor of 10, if \( \sin^2 \theta_W = 0.232 \). It then varies between \( 8.2 \times 10^8 \) GeV to \( 6 \times 10^2 \) GeV. In fig.1b and 1c, we have shown the unification scale \( M_{\text{GUT}} \) and the inverse of the unified coupling \( (\alpha_5^{-1}) \). We note that they are anti-correlated. While \( M_{\text{GUT}} \) increases, \( \alpha_5^{-1} \) decreases with the input \( \alpha_s \). In comparison to the higgsino mass, \( M_{\text{GUT}} \) is quite insensitive to the input \( \alpha_s \). As \( \alpha_s \) changes from 0.10 to 0.13, the GUT scale is increased by a factor of 2 only. Similarly, \( \alpha_5^{-1} \) also shows weak dependence on the input \( \alpha_s \). It is changed by less than 10%. Dependence of the GUT scale on the input \( \sin^2 \theta_W \) is also manifest. Within 1\( \sigma \) variation of \( \sin^2 \theta_W \), it is changed approximately by 10%. Interestingly, the unified coupling do not show appreciable dependence on the input \( \sin^2 \theta_W \). In fig.1d, the ratio \( M_5/M_{\tilde{W}} \) is shown. Within 1\( \sigma \) variation of \( \sin^2 \theta_W \) the gluino mass also do not show appreciable dependence on the input value of the Weinberg angle. However, as input \( \alpha_s \) varies from 0.10 to 0.13, it increases from 2300 GeV to 2800 GeV, showing weak dependence on the input \( \alpha_s \). In fig.1e, we have depicted the variation of the universal gaugino mass \( M_{1/2} \). \( M_{1/2} \) also do not show appreciable dependence on the input \( \sin^2 \theta_W \). It shows weak dependence on the input \( \alpha_s \), increasing from 1.15 \( \times 10^5 \) GeV to 1.23 \( \times 10^3 \) GeV, as \( \alpha_s \) changes from 0.10 to 0.13.

The results discussed so far indicate that while the higgsino mass shows a sensitive dependence to the input \( \alpha_s \) for strict unification, \( (\text{it changes by 6 order of magnitude as } \alpha_s \text{ changes from } 0.10 \text{ to } 0.13) \) other parameters of the model, e.g. \( M_{\text{GUT}}, \alpha_s, M_\tilde{g} \) and \( M_{1/2} \) shows a weak dependence on the input \( \alpha_s \). Also, unlike the higgsino mass, the input dependence of \( \sin^2 \theta_W \) is also less in those parameters. With the world average value of \( \alpha_s = 0.123 \), strict unification is possible with Higgsino mass as well as all the other sparticle masses in the range of TeV scale. However, if we consider the QCD motivated small \( \alpha_s \approx 0.11 \), strict unification is obtained with Higgsinos masses in the range of \( 10^6 - 10^7 \) GeV. All the other SUSY masses can be in the TeV scale. To be specific, for \( \alpha_s = 0.11 \), strict gauge unification is obtained with \( M_\tilde{H} = 1.46 \times 10^6(2.15 \times 10^6) \) GeV for \( \sin^2 \theta_W = 0.231(0.232) \).

In the above calculations, the mass of the winos \( (M_{\tilde{W}}) \) were fixed at 1000 GeV. To observe the effect of wino mass on the unification, with small \( \alpha_s \), we now vary \( M_\tilde{W} \) from 100 GeV to 2000 GeV. We fix the strong coupling at \( \alpha_s = 0.11 \). The common SUSY mass \( M_c \) was fixed at 1000 GeV. As before, while the central value was used for \( \alpha_s \), \( \text{for } \sin^2 \theta_W \), we consider variation within 1\( \sigma \). In fig.2, the results are shown. In fig.2a, the higgsino mass required for strict unification is shown as a function of the wino mass. As observed earlier, the higgsino mass depends on the input \( \sin^2 \theta_W \). Within its 1\( \sigma \) variation, it is changed approximately by a factor of 10. However, the higgsino mass required for unification shows a weak dependence on the wino mass. As the wino mass is varied from 100 to 2000 GeV, the higgsino mass is decreased by a factor of 5 only. Interestingly, we find that after 1000 GeV, the the higgsino mass remains nearly same. Variation of the GUT scale and inverse of the unified coupling with the wino mass is shown in fig.2b and 2c. As before, they are anti-correlated. While \( M_{\text{GUT}} \) shows weak dependence on the input \( \sin^2 \theta_W \), \( \alpha_5^{-1} \) do not exhibit any such dependence. There dependence on the wino mass is also weak. Here also, after \( M_\tilde{W} = 1000 \) GeV, rate of variation of \( M_{\text{GUT}} \) and \( \alpha_5^{-1} \) slows down. The variation of the ratio \( M_5/M_{\tilde{W}} \) with the wino mass is shown in fig.2d. The ratio do not depend on the input \( \sin^2 \theta_W \). It decreases with the wino mass. Again after 1000 GeV, the rate of decrease is slowed down and the relation \( M_5 \approx 2.5 M_{\tilde{W}} \) become valid. In fig.2e, the universal gaugino mass \( M_{1/2} \) and its variation with the wino mass is shown. \( M_{1/2} \) do not show any dependence on the input \( \sin^2 \theta_W \). As expected, it increases linearly with the wino mass. The results indicate that the unification of three forces with \( \alpha_s = 0.11 \) depend weakly upon the wino mass. Increasing the wino mass beyond 1000 GeV, does not affect the unification appreciably.

We have also studied the dependence of the common SUSY mass \( M_c \) on the unification process with \( \alpha_s = 0.11 \). The wino mass was fixed at 1000 GeV. In fig.3a, the variation of the higgsino mass with \( M_c \) is shown. The input dependence of \( \sin^2 \theta_W \) on the higgsino mass is again manifest. The higgsino mass shows a very weak dependence on the common SUSY mass \( M_c \). Similarly, the GUT scale \( (\text{fig.3b}) \), the unified coupling \( (\text{fig.3c}) \), the ratio \( M_5/M_{\tilde{W}} \) \( (\text{fig.3d}) \) and \( M_{1/2} \) \( (\text{fig.3e}) \) shows very weak dependence on the input \( M_c \). Thus the unification of three forces with \( \alpha_s = 0.11 \) is insensitive to the common SUSY mass \( M_c \) within the range 500-2500 GeV. Thus a more accurate SUSY mass spectra will not alter the present results significantly.

The present analysis indicate that low energy QCD motivated strong coupling constant \( \alpha_s \approx 0.11 \) is consistent with gauge unification in MSSM, if the higgsino masses are in the range of \( 10^6 - 10^7 \) GeV and other SUSY masses are in the TeV range. In the following we will examine its effect on some other features of the model.

### B. Color triplet Higgs mass

Any GUT theory must consider the nucleon decay rate in the model. As the GUT scale obtained presently exceeds \( 10^{15} \) GeV, we do not expect problem from the direct proton decay \( p \rightarrow e^+ \pi^0 \). However, nucleons can decay via exchange of color-triplet Higgs multiplet \([22,23]\), through the dimension five operator. The color triplet Higgs mass
for the dark matter. Thus if only the higgsinos are in the mass range 10

wino do not mix with each other nor they mix with the higgsinos. Thus bin os can still be the LSP and the candidate

1/2 particle) to give the neutralino’s. In the limiting case when

M

−

accuracy of 10

small and large mass scale in nature. In SU(5), as well known, the tr ee level parameters needed to be fine tuned to an

problem of gauge hierarchy in supersymmetric model is not as sever e as in SU(5). Also, in supersymmetric theories, the expectation

renormalisation but they also do not receive finite renormalisation in h igher orders. Thus the parameters, once fine

is valid for exact or softly broken supersymmetry, the parameter s of the super potential do not only receive infinite

logarithmic dependence of radiative corrections makes the emerge nce of high mass scale from a low mass scale quite

D. Gauge hierarchy problem

One of the motivation of introducing the supersymmetry is the gauge hierarchy problem, i.e. existence of very small and large mass scale in nature. In SU(5), as well known, the tree level parameters needed to be fine tuned to an accuracy of 10

26 or so, in order to obtain the mass ratio \( \frac{M_Z}{m_W} \approx 10^{12} \). However, tree level fine tunings are upset at higher order due to quadratic radiative corrections and SU(5) is in problem. This need not happen in supersymmetric theories due to the nonrenormalisation theorem of Grisaru, Rocek and Siegel [26]. According to this theory, which is valid for exact or softly broken supersymmetry, the parameters of the super potential do not only receive infinite renormalisation but they also do not receive finite renormalisation in higher orders. Thus the parameters, once fine tuned at tree level to obtain hierarchy, the radiative correction donot disturb the hierarchy at higher order. Thus problem of gauge hierarchy in supersymmetric model is not as severe as in SU(5). Also, in supersymmetric theories, logarithmic dependence of radiative corrections makes the emergence of high mass scale from a low mass scale quite natural.

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However, with the Higgsino mass in the scale of 10

GeV, effective breaking of supersymmetry will occur at that scale only. The effective theory below that scale will not be protected by the non-renormalisation theory. Quadratic divergences will be generated in Higgs mass due to absence of Higgs-Higgsino-gaugino coupling at low energy. With
Higgsino mass in the range of $10^6$ GeV, the corrections to the Higgs mass will be of that scale and correct electroweak scale can not be achieved without fine tuning. Thus we find that if the strong coupling constant is small as obtained in QCD experiments, gauge unification with recent electroweak data can be achieved, alongwith universal gaugino masses, only if we abandon one of the primary motivation of supersymmetry, namely the resolution to the gauge hierarchy problem. Also generating three order of hierarchy among the sparticles will be difficult to obtain in any theoretical framework. The discussion suggests that consistent gauge unification with $\alpha_s \approx 0.11$ is not possible within the frame work of MSSM.

IV. SUMMARY AND CONCLUSIONS

We have studied the gauge unification in the MSSM with respect to the higgsino masses. The purpose of our study was to find whether with $\alpha_s \approx 0.11$ gauge unification is possible in the MSSM with universal gaugino mass at GUT scale. Our approach is phenomenological. We choose the higgsino mass as a parameter. For a given wino mass, it was tuned to obtain gauge unification. The gluino mass was obtained iteratively from the condition that at GUT scale the winos and the gluino have a common mass $M_{1/2}$. All the other sparticle masses were assumed to be degenerate at a common mass $M_c$. The RG equations were run from $M_z$ to the GUT scale, with the beta function coefficients changing at appropriate masses. We find that if $\alpha_s$ ranges between 0.10 to 0.13, with the recent electroweak data, it is possible to unify all the three forces by varying the higgsino mass between $10^9 - 10^3$ GeV. All the other sparticles can be in the TeV scale. The result is insensitive to the wino mass or the common mass $M_c$. Thus a more accurate SUSY spectra will not alter the results significantly. It was found that if $\alpha_s \approx 0.11$, as measured in QCD experiments, gauge unification require higgsino masses in the range of $10^6$ GeV, three order of magnitude higher than other sparticle masses. Colour triplet Higgs were also calculated and found to be heavy enough to forbid rapid proton decay. However, with Higgsino masses in the scale of $10^6$ GeV, supersymmetry breaks at that scale only and problem of gauge hierarchy resurface. One of the fundamental motivation of introducing supersymmetry is then lost.

To conclude, if $\alpha_s$ is indeed small, as measured in QCD experiments, then consistent gauge unification within MSSM is not possible. However, MSSM is merely the simplest extension of standard model with supersymmetry. It is possible that with a more complicated model a consistent picture will emerge.

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FIG. 1. Variation of (a) the higgsino mass ($M_{\tilde{H}}$), (b) the unification scale ($M_{GUT}$), (c) the inverse of the unified coupling ($\alpha_{s}^{-1}$), (d) the ratio $M_3/M_{\tilde{H}}$ and (e) the universal gaugino mass ($M_{1/2}$), with the strong coupling $\alpha_s(M_Z)$ is shown. The wino mass $M_W$ and the common SUSY mass $M_c$ was fixed at 1000 GeV. The black dots are for $\sin^2 \theta_W = 0.231$ and the open triangles are for $\sin^2 \theta_W = 0.023$. 

FIG. 2. Variation of (a) the higgsino mass ($M_{\tilde{H}}$), (b) the unification scale ($M_{GUT}$), (c) the inverse of the unified coupling ($\alpha_{s}^{-1}$), (d) the ratio $M_3/M_{\tilde{H}}$ and (e) the universal gaugino mass ($M_{1/2}$), with the wino mass ($M_{\tilde{W}}$) is shown. The strong coupling was fixed at $\alpha_s(M_Z) = 0.11$. The common SUSY mass $M_c$ was fixed at 1000 GeV. The black dots are for $\sin^2 \theta_W = 0.231$ and the open triangles are for $\sin^2 \theta_W = 0.232$. 

FIG. 3. Variation of (a) the higgsino mass ($M_{\tilde{H}}$), (b) the unification scale ($M_{GUT}$), (c) the inverse of the unified coupling ($\alpha_{s}^{-1}$), (d) the ratio $M_3/M_{\tilde{H}}$ and (e) the universal gaugino mass ($M_{1/2}$), with the common SUSY mass ($M_c$) is shown. The strong coupling was fixed at $\alpha_s(M_Z) = 0.11$. The wino mass $M_{\tilde{W}}$ was fixed at 1000 GeV. The black dots are for $\sin^2 \theta_W = 0.231$ and the open triangles are for $\sin^2 \theta_W = 0.232$. 

FIG. 4. Variation of the color triplet Higgs mass ($M_{H_c}$), with the wino mass ($M_{\tilde{W}}$) is shown. The strong coupling was fixed at $\alpha_s(M_Z) = 0.11$. The common SUSY mass $M_c$ was fixed at 1000 GeV. The black dots are for $\sin^2 \theta_W = 0.231$ and the open triangles are for $\sin^2 \theta_W = 0.232$. 

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$M_c = M_{\tilde{W}} = 1000$ GeV

(a) $M_{\tilde{H}}$ (GeV)

(b) $M_{\text{GUT}}/10^{16}$ (GeV)

(c) $\alpha_5^{-1}$

(d) $M_g/M_{\tilde{W}}$

(e) $M_{1/2}/10^2$ (GeV)

$\alpha_s(M_Z)$
\( \alpha_s = 0.11, M_c = 1000 \text{GeV} \)

- (a) \( M_{\tilde{H}} \) (GeV)
- (b) \( M_{\text{GUT}}/10^{18} \) (GeV)
- (c) \( \alpha_s^{-1} \)
- (d) \( M_{g}/M_{\tilde{W}} \)
- (e) \( M_{1/2}/10^2 \) (GeV)

Graphs showing various quantities as functions of \( M_{\tilde{W}} \) (GeV).
\( \alpha_s = 0.11, \ M_{\tilde{W}} = 1000 \ \text{GeV} \)

\[ \begin{align*}
\text{(a)} \quad & M_{H^\pm} \ (\text{GeV}) \\
\text{(b)} \quad & M_{\text{GUT}}/10^{16} \ (\text{GeV}) \\
\text{(c)} \quad & \alpha_5^{-1} \\
\text{(d)} \quad & M_{M_g}/M_{\tilde{W}} \\
\text{(e)} \quad & M_{1/2}^3/10^3 \ (\text{GeV})
\end{align*} \]

\[ M_c \ (\text{GeV}) \]
$\alpha_s = 0.11, \ M_c = 1000 \text{GeV}$