A new $Z$-eigenvalue localization set for tensors

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Abstract
A new $Z$-eigenvalue localization set for tensors is given and proved to be tighter than those in the work of Wang et al. (Discrete Contin. Dyn. Syst., Ser. B 22(1):187-198, 2017). Based on this set, a sharper upper bound for the $Z$-spectral radius of weakly symmetric nonnegative tensors is obtained. Finally, numerical examples are given to verify the theoretical results.

MSC: 15A18; 15A69

Keywords: $Z$-eigenvalue; localization set; nonnegative tensors; spectral radius; weakly symmetric

1 Introduction
For a positive integer $n$, $n \geq 2$, $N$ denotes the set $\{1, 2, \ldots, n\}$. $\mathbb{C}$ ($\mathbb{R}$) denotes the set of all complex (real) numbers. We call $A=(a_{i_1i_2\cdots i_m})$ a real tensor of order $m$ dimension $n$, denoted by $\mathbb{R}^{[m,n]}$, if

$$a_{i_1i_2\cdots i_m} \in \mathbb{R},$$

where $i_j \in N$ for $j=1, 2, \ldots, m$. $A$ is called nonnegative if $a_{i_1i_2\cdots i_m} \geq 0$. $A=(a_{i_1\cdots i_m}) \in \mathbb{R}^{[m,n]}$ is called symmetric [2] if

$$a_{i_1\cdots i_m} = a_{\pi(i_1\cdots i_m)}, \quad \forall \pi \in \Pi_m,$$

where $\Pi_m$ is the permutation group of $m$ indices. $A=(a_{i_1i_2\cdots i_m}) \in \mathbb{R}^{[m,n]}$ is called weakly symmetric [3] if the associated homogeneous polynomial

$$Ax^m = \sum_{i_1i_2\cdots i_m \in N} a_{i_1i_2\cdots i_m}x_{i_1}x_{i_2}\cdots x_{i_m}$$

satisfies $\nabla Ax^m = mAx^{m-1}$. It is shown in [3] that a symmetric tensor is necessarily weakly symmetric, but the converse is not true in general.

Given a tensor $A=(a_{i_1\cdots i_m}) \in \mathbb{R}^{[m,n]}$, if there are $\lambda \in \mathbb{C}$ and $x=(x_1, x_2, \ldots, x_n)^T \in \mathbb{C}^n \setminus \{0\}$ such that

$$Ax^{m-1} = \lambda x \quad \text{and} \quad x^Tx = 1,$$

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then $\lambda$ is called an $E$-eigenvalue of $A$ and $x$ an $E$-eigenvector of $A$ associated with $\lambda$, where $Ax^{m-1}$ is an $n$ dimension vector whose $i$th component is

$$(Ax^{m-1})_i = \sum_{i_2,\ldots,i_m \in N} a_{i_2\ldots i_m}x_{i_2}\cdots x_{i_m}.$$ 

If $\lambda$ and $x$ are all real, then $\lambda$ is called a $Z$-eigenvalue of $A$ and $x$ a $Z$-eigenvector of $A$ associated with $\lambda$; for details, see [2, 4].

Let $A = (a_{i_1\ldots i_m}) \in \mathbb{R}^{[m,n]}$. We define the $Z$-spectrum of $A$, denoted $\sigma(A)$ to be the set of all $Z$-eigenvalues of $A$. Assume $\sigma(A) \neq 0$, then the $Z$-spectral radius [3] of $A$, denoted $\varrho(A)$, is defined as

$$\varrho(A) := \sup\{|\lambda| : \lambda \in \sigma(A)\}.$$ 

Recently, much literature has focused on locating all $Z$-eigenvalues of tensors and bounding the $Z$-spectral radius of nonnegative tensors in [1, 5–10]. It is well known that one can use eigenvalue inclusion sets to obtain the lower and upper bounds of the spectral radius of nonnegative tensors; for details, see [1, 11–14]. Therefore, the main aim of this paper is to give a tighter $Z$-eigenvalue inclusion set for tensors, and use it to obtain a sharper upper bound for the $Z$-spectral radius of weakly symmetric nonnegative tensors.

In 2017, Wang et al. [1] established the following Geršgorin-type $Z$-eigenvalue inclusion theorem for tensors.

**Theorem 1** ([1], Theorem 3.1) Let $A = (a_{i_1\ldots i_m}) \in \mathbb{R}^{[m,n]}$. Then

$$\sigma(A) \subseteq K(A) = \bigcup_{i \in N} K_i(A),$$

where

$$K_i(A) = \left\{z \in \mathbb{C} : |z| \leq R_i(A) \right\}, \quad R_i(A) = \sum_{i_2,\ldots,i_m \in N} |a_{i_2\ldots i_m}|.$$ 

To get a tighter $Z$-eigenvalue inclusion set than $K(A)$, Wang et al. [1] gave the following Brauer-type $Z$-eigenvalue localization set for tensors.

**Theorem 2** ([1], Theorem 3.2) Let $A = (a_{i_1\ldots i_m}) \in \mathbb{R}^{[m,n]}$. Then

$$\sigma(A) \subseteq L(A) = \bigcup_{i \in N} \bigcap_{j \in N, j \neq i} L_{ij}(A),$$

where

$$L_{ij}(A) = \left\{z \in \mathbb{C} : \left(|z| - (R_i(A) - |a_{ij-j}|)\right)|z| \leq |a_{ij-j}||R_j(A)| \right\}.$$ 

In this paper, we continue this research on the $Z$-eigenvalue localization problem for tensors and its applications. We give a new $Z$-eigenvalue inclusion set for tensors and prove that the new set is tighter than those in Theorem 1 and Theorem 2. As an application of this set, we obtain a new upper bound for the $Z$-spectral radius of weakly symmetric nonnegative tensors, which is sharper than some existing upper bounds.
2 Main results

In this section, we give a new Z-eigenvalue localization set for tensors, and establish the comparison between this set with those in Theorem 1 and Theorem 2. For simplification, we denote

\[ \Delta_j = \{(i_2, i_3, \ldots, i_m) : i_k = j \text{ for some } k \in \{2, \ldots, m\}, \text{ where } j, i_2, \ldots, i_m \in N \}, \]

\[ \overline{\Delta}_j = \{(i_2, i_3, \ldots, i_m) : i_k \neq j \text{ for any } k \in \{2, \ldots, m\}, \text{ where } j, i_2, \ldots, i_m \in N \}. \]

Then \( R_j(A) = r_j^{\Delta_j}(A) + r_j^{\overline{\Delta}_j}(A) \).

Theorem 3 Let \( A = (a_{i_1 \cdots i_m}) \in \mathbb{R}^{[m,n]} \). Then

\[ \sigma(A) \subseteq \Psi(A) = \bigcup_{i \in N} \bigcap_{j \in N, j \neq i} \Psi_{ij}(A), \]

where

\[ \Psi_{ij}(A) = \{ z \in \mathbb{C} : |z| - r_j^{\overline{\Delta}_j}(A) |z| \leq r_j^{\Delta_j}(A) R_j(A) \}. \]

Proof. Let \( \lambda \) be a Z-eigenvalue of \( A \) with corresponding Z-eigenvector \( x = (x_1, \ldots, x_n)^T \in \mathbb{C}^n \setminus \{0\} \), i.e.,

\[ Ax^{m-1} = \lambda x, \quad \text{and} \quad \|x\|_2 = 1. \tag{1} \]

Assume \( |x_t| = \max_{i \in N} |x_i| \), then \( 0 < |x_{t}|^{m-1} \leq |x_t| \leq 1 \). For \( \forall j \in N, j \neq t \), from (1), we have

\[ \lambda x_t = \sum_{(i_2, \ldots, i_m) \in \Delta_j} a_{i_2 \cdots i_m} x_{i_2} \cdots x_{i_m} + \sum_{(i_2, \ldots, i_m) \in \overline{\Delta}_j} a_{i_2 \cdots i_m} x_{i_2} \cdots x_{i_m}. \]

Taking the modulus in the above equation and using the triangle inequality give

\[ |\lambda| |x_t| \leq \sum_{(i_2, \ldots, i_m) \in \Delta_j} |a_{i_2 \cdots i_m}| |x_{i_2}| \cdots |x_{i_m}| + \sum_{(i_2, \ldots, i_m) \in \overline{\Delta}_j} |a_{i_2 \cdots i_m}| |x_{i_2}| \cdots |x_{i_m}| \]

\[ \leq \sum_{(i_2, \ldots, i_m) \in \Delta_j} |a_{i_2 \cdots i_m}| |x_j| + \sum_{(i_2, \ldots, i_m) \in \overline{\Delta}_j} |a_{i_2 \cdots i_m}| |x_j| \]

\[ = r_j^{\Delta_j}(A)|x_j| + r_j^{\overline{\Delta}_j}(A)|x_j|, \]

i.e.,

\[ (|\lambda| - r_j^{\overline{\Delta}_j}(A)) |x_t| \leq r_j^{\Delta_j}(A) |x_j|. \tag{2} \]
If \(|x_j| = 0\), by \(|x_i| > 0\), we have \(|\lambda| - r_i^\lambda (A) \leq 0\). Then
\[
(\lambda - r_i^\lambda (A))|\lambda| \leq r_i^\lambda (A)R_j(A).
\]

Obviously, \(\lambda \in \Psi_1(A)\). Otherwise, \(|x_j| > 0\). From (1), we have
\[
|\lambda||x_i| \leq \sum_{i_1, \ldots, i_m \in N} |a_{j_1 \ldots i_m}||x_{i_1}| \cdots |x_{i_m}| \leq \sum_{i_1, \ldots, i_m \in N} |a_{j_1 \ldots i_m}||x_i|^{m-1} \leq R_j(A)|x_i|.
\]

Multiplying (2) with (3) and noting that \(|x_i||x_j| > 0\), we have
\[
(\lambda - r_i^\lambda (A))|\lambda| \leq r_i^\lambda (A)R_j(A),
\]
which implies that \(\lambda \in \Psi_1(A)\). From the arbitrariness of \(j\), we have \(\lambda \in \bigcap_{j \in N, j \neq i} \Psi_j(A)\). Furthermore, we have \(\lambda \in \bigcup_{i \in N} \bigcap_{j \in N, j \neq i} \Psi_i(A)\).

Next, a comparison theorem is given for Theorem 1, Theorem 2 and Theorem 3.

**Theorem 4** Let \(A = (a_{i_1 \cdots i_m}) \in \mathbb{R}^{[m,n]}\). Then
\[
\Psi_j(A) \subseteq \mathcal{L}(A) \subseteq \mathcal{K}(A).
\]

*Proof* By Corollary 3.1 in [1], \(\mathcal{L}(A) \subseteq \mathcal{K}(A)\) holds. Here, we only prove \(\Psi_j(A) \subseteq \mathcal{L}(A)\). Let \(z \in \Psi_j(A)\). Then there exists \(i \in N\), such that \(z \in \Psi_i(A), \forall j \in N, j \neq i\), that is,
\[
(\lambda - r_i^\lambda (A))z \leq r_i^\lambda (A)R_j(A), \quad \forall j \in N, j \neq i.
\]

Next, we divide our subject in two cases to prove \(\Psi_j(A) \subseteq \mathcal{L}(A)\).

Case I: If \(r_i^\lambda (A)R_j(A) = 0\), then we have
\[
(\lambda - (R_i(A) - |a_{j\ldots j}|))z \leq (\lambda - r_i^\lambda (A))z \leq r_i^\lambda (A)R_j(A) = 0 \leq |a_{j\ldots j}|R_j(A),
\]
which implies that \(z \in \bigcap_{j \in N, j \neq i} \mathcal{L}_i(A) \subseteq \mathcal{L}(A)\) from the arbitrariness of \(j\), consequently, \(\Psi_j(A) \subseteq \mathcal{L}(A)\).

Case II: If \(r_i^\lambda (A)R_j(A) > 0\), then dividing both sides by \(r_i^\lambda (A)R_j(A)\) in (4), we have
\[
\frac{|z| - r_i^\lambda (A)}{r_i^\lambda (A)} \leq \frac{|z|}{R_j(A)} \leq 1,
\]
which implies
\[
\frac{|z| - r_i^\lambda (A)}{r_i^\lambda (A)} \leq 1,
\]
or
\[
\frac{|z|}{R_j(A)} \leq 1.
\]
Let \( a = |z|, b = r_i^{\Delta_j}(A), c = r_i^{\Delta_j}(A) - |a_{ij}| \) and \( d = |a_{ij}| > 0 \), when (6) holds and \( d = |a_{ij}| > 0 \), from Lemma 2.2 in [11], we have

\[
\frac{|z| - (R_i(A) - |a_{ij}|)}{|a_{ij}|} = \frac{a - (b + c)}{d} \leq \frac{a - b}{c + d} = \frac{|z| - r_i^{\Delta_j}(A)}{r_i^{\Delta_j}(A)}. \tag{8}
\]

Furthermore, from (5) and (8), we have

\[
\frac{|z| - (R_i(A) - |a_{ij}|)}{|a_{ij}|} \frac{|z|}{R_i(A)} \leq \frac{|z| - r_i^{\Delta_j}(A)}{r_i^{\Delta_j}(A)} \frac{|z|}{R_i(A)} \leq 1,
\]
equivalently,

\[
(|z| - (R_i(A) - |a_{ij}|)|z| \leq |a_{ij}| R_i(A),
\]

which implies that \( z \in \bigcap_{j \in N \setminus \{i\}} \mathcal{L}_j(A) \subseteq \mathcal{L}(A) \) from the arbitrariness of \( j \), consequently, \( \Psi(A) \subseteq \mathcal{L}(A) \). When (6) holds and \( d = |a_{ij}| = 0 \), we have

\[
|z| - r_i^{\Delta_j}(A) - r_i^{\Delta_j}(A) \leq 0, \quad \text{i.e.,} \quad |z| - (R_i(A) - |a_{ij}|) \leq 0,
\]

and furthermore

\[
(|z| - (R_i(A) - |a_{ij}|)|z| \leq 0 = |a_{ij}| R_i(A).
\]

This also implies \( \Psi(A) \subseteq \mathcal{L}(A) \).

On the other hand, when (7) holds, we only prove \( \Psi(A) \subseteq \mathcal{L}(A) \) under the case that

\[
\frac{|z| - r_i^{\Delta_j}(A)}{r_i^{\Delta_j}(A)} > 1. \tag{9}
\]

From (9), we have \( \frac{a}{b + c + d} = \frac{|z|}{R_i(A)} > 1 \). When (7) holds and \( |a_{ji}| > 0 \), by Lemma 2.3 in [11], we have

\[
\frac{|z|}{R_i(A)} = \frac{a}{b + c + d} \leq \frac{a - b}{c + d} = \frac{|z| - r_i^{\Delta_j}(A)}{r_i^{\Delta_j}(A)}. \tag{10}
\]

By (7), Lemma 2.2 in [11] and similar to the proof of (8), we have

\[
\frac{|z| - (R_i(A) - |a_{ji}|)}{|a_{ji}|} \leq \frac{|z|}{R_i(A)}. \tag{11}
\]

Multiplying (10) and (11), we have

\[
\frac{|z| - (R_i(A) - |a_{ji}|)}{|a_{ji}|} \frac{|z|}{R_i(A)} \leq \frac{|z| - r_i^{\Delta_j}(A)}{r_i^{\Delta_j}(A)} \frac{|z|}{R_i(A)} \leq 1;
\]
equivalently,
\[(|z| - (R_j(A) - |a_{j\cdots i}|)|z| \leq |a_{j\cdots i}|R_i(A)).\]

This implies \( z \in \bigcap_{i \in \mathbb{N}, i \neq j} L_j(A) \subseteq L(A) \) and \( \Psi(A) \subseteq L(A) \) from the arbitrariness of \( i \).

When (7) holds and \( |a_{j\cdots i}| = 0 \), we can obtain
\[|z| - R_j(A) \leq 0, \quad \text{i.e.,} \quad |z| - (R_j(A) - |a_{j\cdots i}|) \leq 0\]
and
\[(|z| - (R_j(A) - |a_{j\cdots i}|)|z| \leq |a_{j\cdots i}|R_i(A)).\]

This also implies \( \Psi(A) \subseteq L(A) \). The conclusion follows from Case I and Case II. \( \square \)

**Remark 1** Theorem 4 shows that the set \( \Psi(A) \) in Theorem 3 is tighter than \( \mathcal{K}(A) \) in Theorem 1 and \( L(A) \) in Theorem 2, that is, \( \Psi(A) \) can capture all \( Z \)-eigenvalues of \( A \) more precisely than \( \mathcal{K}(A) \) and \( L(A) \).

Now, we give an example to show that \( \Psi(A) \) is tighter than \( \mathcal{K}(A) \) and \( L(A) \).

**Example 1** Let \( A = (a_{ijkl}) \in \mathbb{R}^{[4,2]} \) be a symmetric tensor defined by
\[a_{1222} = 1, \quad a_{2222} = 2, \quad \text{and} \quad a_{ijkl} = 0 \quad \text{elsewhere}.\]

By computation, we see that all the \( Z \)-eigenvalues of \( A \) are \(-0.5000, 0 \) and \( 2.7000 \). By Theorem 1, we have
\[\mathcal{K}(A) = \mathcal{K}_1(A) \cup \mathcal{K}_2(A) = \{z \in \mathbb{C} : |z| \leq 1\} \cup \{z \in \mathbb{C} : |z| \leq 5\} = \{z \in \mathbb{C} : |z| \leq 5\}.\]

By Theorem 2, we have
\[L(A) = L_{12}(A) \cup L_{23}(A) = \{z \in \mathbb{C} : |z| \leq 2.2361\} \cup \{z \in \mathbb{C} : |z| \leq 5\} = \{z \in \mathbb{C} : |z| \leq 5\}.\]

By Theorem 3, we have
\[\Psi(A) = \Psi_{12}(A) \cup \Psi_{21}(A) = \{z \in \mathbb{C} : |z| \leq 2.2361\} \cup \{z \in \mathbb{C} : |z| \leq 3\} = \{z \in \mathbb{C} : |z| \leq 3\}.\]

The \( Z \)-eigenvalue inclusion sets \( \mathcal{K}(A) \), \( L(A) \), \( \Psi(A) \) and the exact \( Z \)-eigenvalues are drawn in Figure 1, where \( \mathcal{K}(A) \) and \( L(A) \) are represented by blue dashed boundary, \( \Psi(A) \) is represented by red solid boundary and the exact eigenvalues are plotted by ‘+’, respectively. It is easy to see \( \sigma(A) \subseteq \Psi(A) \subseteq L(A) \subseteq \mathcal{K}(A) \), that is, \( \Psi(A) \) can capture all \( Z \)-eigenvalues of \( A \) more precisely than \( L(A) \) and \( \mathcal{K}(A) \).
3 A new upper bound for the $Z$-spectral radius of weakly symmetric nonnegative tensors

As an application of the results in Section 2, we in this section give a new upper bound for the $Z$-spectral radius of weakly symmetric nonnegative tensors.

**Theorem 5** Let $A = (a_{i_1 \cdots i_m}) \in \mathbb{R}^{[m,n]}$ be a weakly symmetric nonnegative tensor. Then

$$\varrho(A) \leq \max_{i \in \mathbb{N}} \min_{j \in \mathbb{N}, j \neq i} \Phi_{ij}(A),$$

where

$$\Phi_{ij}(A) = \frac{1}{2} \left\{ r_{ij}^{\Delta}(A) + \sqrt{(r_{ij}^{\Delta}(A))^2 + 4r_{ij}^{\Delta}(A)R_j(A)} \right\}.$$

**Proof** From Lemma 4.4 in [1], we know that $\varrho(A)$ is the largest $Z$-eigenvalue of $A$. It follows from Theorem 3 that there exists $i \in \mathbb{N}$ such that

$$(\varrho(A) - r_{ij}^{\Delta}(A))\varrho(A) \leq r_{ij}^{\Delta}(A)R_j(A), \quad \forall j \in \mathbb{N}, j \neq i.$$  \hspace{1cm} (12)

Solving $\varrho(A)$ in (12) gives

$$\varrho(A) \leq \frac{1}{2} \left\{ r_{ij}^{\Delta}(A) + \sqrt{(r_{ij}^{\Delta}(A))^2 + 4r_{ij}^{\Delta}(A)R_j(A)} \right\} = \Phi_{ij}(A).$$

From the arbitrariness of $j$, we have $\varrho(A) \leq \min_{j \in \mathbb{N}, j \neq i} \Phi_{ij}(A)$. Furthermore, $\varrho(A) \leq \max_{i \in \mathbb{N}} \min_{j \in \mathbb{N}, j \neq i} \Phi_{ij}(A).$  \hspace{1cm} $\square$
By Theorem 4, Theorem 4.5 and Corollary 4.1 in [1], the following comparison theorem can be derived easily.

**Theorem 6** Let \( A = (a_{i_1 \ldots i_m}) \in \mathbb{R}^{[m,n]} \) be a weakly symmetric nonnegative tensor. Then the upper bound in Theorem 5 is sharper than those in Theorem 4.5 of [1] and Corollary 4.5 of [5], that is,

\[
\varrho(A) \leq \max_{i \in N} \min_{j \in N, j \neq i} \Phi_{ij}(A)
\]

\[
\leq \max_{i \in N} \min_{j \in N, j \neq i} \frac{1}{2} \left( R_i(A) - a_{ij} - j + \sqrt{(R_i(A) - a_{ij} - j)^2 + 4a_{ij}R_j(A)} \right)
\]

\[
\leq \max_{i \in N} R_i(A).
\]

Finally, we show that the upper bound in Theorem 5 is sharper than those in [1, 5–8, 10] by the following example.

**Example 2** Let \( A = (a_{ijk}) \in \mathbb{R}^{[3,3]} \) with the entries defined as follows:

\[
A(:,:,1) = \begin{pmatrix}
3 & 3 & 0 \\
3 & 2 & 2.5 \\
0.5 & 2.5 & 0
\end{pmatrix}, \quad A(:,:,2) = \begin{pmatrix}
3 & 2 & 2 \\
2 & 0 & 3 \\
2.5 & 3 & 1
\end{pmatrix},
\]

\[
A(:,:,3) = \begin{pmatrix}
1 & 3 & 0 \\
2.5 & 3 & 1 \\
0 & 1 & 0
\end{pmatrix}.
\]

It is not difficult to verify that \( A \) is a weakly symmetric nonnegative tensor. By both Corollary 4.5 of [5] and Theorem 3.3 of [6], we have

\[
\varrho(A) \leq 19.
\]

By Theorem 3.5 of [7], we have

\[
\varrho(A) \leq 18.6788.
\]

By Theorem 4.6 of [1], we have

\[
\varrho(A) \leq 18.6603.
\]

By both Theorem 4.5 of [1] and Theorem 6 of [8], we have

\[
\varrho(A) \leq 18.5656.
\]

By Theorem 4.7 of [1], we have

\[
\varrho(A) \leq 18.3417.
\]
By Theorem 2.9 of [10], we have

$$\varrho(A) \leq 17.2063.$$ 

By Theorem 5, we obtain

$$\varrho(A) \leq 15.2580,$$

which shows that the upper bound in Theorem 5 is sharper.

### 4 Conclusions

In this paper, we present a new $Z$-eigenvalue localization set $\Psi(A)$ and prove that this set is tighter than those in [1]. As an application, we obtain a new upper bound $\max_{i \in \mathbb{N}} \min_{j \in \mathbb{N}, j \neq i} \Phi_{ij}(A)$ for the $Z$-spectral radius of weakly symmetric nonnegative tensors, and we show that this bound is sharper than those in [1, 5–8, 10] in some cases by a numerical example.

### Competing interests

The author declares that they have no competing interests.

### Author’s contributions

The author read and approved the final manuscript.

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