COMPRESSIBLE STREAMING INSTABILITIES IN ROTATING THERMAL VISCOUS OBJECTS

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ABSTRACT

We study electromagnetic streaming instabilities in thermal viscous regions of rotating astrophysical objects, such as protostellar and protoplanetary magnetized accretion disks, molecular clouds, their cores, and elephant trunks. The obtained results can also be applied to any regions of interstellar medium, where different equilibrium velocities between charged species can arise. We consider a weakly and highly ionized three-component plasma consisting of neutrals and magnetized electrons and ions. The vertical perturbations along the background magnetic field are investigated. The effect of perturbation of collisional frequencies due to density perturbations of species is taken into account. The growth rates of perturbations are found in a wide region of wave number spectrum for media, where the thermal pressure is larger than the magnetic pressure. It is shown that in cases of strong collisional coupling of neutrals with ions the contribution of the viscosity is negligible.

Key words: accretion, accretion disks – instabilities – magnetic fields – plasmas – waves

1. INTRODUCTION

In a series of papers, Nekrasov (2007, 2008a, 2008b, 2009a, 2009b), a general theory for electromagnetic compressible streaming instabilities in multicomponent rotating magnetized objects, such as accretion disks and molecular clouds has been developed. In equilibrium of accretion disks, different background velocities of different species (electrons, ions, dust grains, and neutrals) have been found from the momentum equations with taking into account anisotropic thermal pressure and collisions of charged species with neutrals. Due to velocity differences, compressible streaming instabilities have been shown to arise having growth rates much larger than the rotation frequencies. New fast instabilities found in these papers have been suggested to be a source of turbulence in accretion disks and molecular clouds.

In papers cited above, the viscosity has not been considered. However, numerical simulations of the magnetorotational instability show that this effect can have influence on the magnitude of the saturated amplitudes of perturbations and, correspondingly, on the turbulent transport of the angular momentum of the saturated amplitudes of perturbations and, correspondingly, on the turbulent transport of the angular momentum. We consider a weakly and highly ionized three-component plasma consisting of electrons, ions, and neutrals. The charged species are supposed to be magnetized, i.e., their cyclotron frequencies are considered to be much larger than their orbiting frequencies and collisional frequencies with neutrals. We will investigate the vertical perturbations along the background magnetic field. The presence of static (in perturbations) dust grains can be invoked through the quasineutrality condition. We take into account the effect of perturbation of collisional frequencies due to density perturbations of species, which takes place at different background velocities of species. We find expressions for the perturbed velocity of any species that also contain the perturbed velocity of other species due to collisions. For magnetized charged species, we derive the dispersion relation, which is solved in the thermal regime when the pressure force dominates the inertia. The conditions of strong or weak collisional coupling of neutrals with charged species and the role of the viscosity will be considered. The growth rates due to different azimuthal velocities of electrons and ions will be found.

This paper is organized as follows. In Section 2, the basic equations are given. In Section 3, we shortly discuss the equilibrium state. Solutions for the perturbed velocities of species for the vertical perturbations are obtained in Section 4. The dispersion relation in the general form is derived in Section 5. In Section 6, this dispersion relation is solved in the thermal regime in the specific cases and unstable solutions are found. In Section 7, we give an expression needed for determining the polarization of perturbations. Some problems concerning the contribution of the resistivity in the standard magnetohydrodynamics (MHD) are discussed in Section 8. Discussion of the obtained results and their applicability to protostellar and protoplanetary disks are given in Section 9. The main points of the paper are summarized in Section 10.

2. BASIC EQUATIONS

We will consider weakly ionized rotating objects consisting of electrons, ions, and neutrals. Here, we do not treat the presence of dust grains. However, the latter may be involved as static (in perturbations) species through the condition of quasineutrality. The electrons and ions are supposed to be magnetized, i.e., their cyclotron frequencies are larger than their collisional...
frequencies with neutrals. Self-gravity is not included. We will study one-dimensional perturbations along the background magnetic field $B_0$. Then the momentum equations for species in the inertial (nonrotating) reference frame accounting for the viscosity (Braginskii 1965) take the form,

$$\frac{\partial v_j}{\partial t} + v_j \cdot \nabla v_j = - \nabla U - \frac{\nabla P_j}{m_j \rho_j} + \frac{q_j}{m_j} E + \frac{q_j}{m_j} v_j \times B - \frac{\mu_j}{\omega_{cj}} \frac{\partial^2 v_j}{\partial \xi^2} + 6 \frac{\mu_j}{\omega_{cj}^2} \frac{\partial^2 v_{j+}}{\partial \xi^2} + 4 \frac{\mu_j}{3 \omega_{cj}^2} \frac{\partial^2 v_{j-}}{\partial \xi^2},$$

where the index $j = e, i$ denotes the electrons and ions, respectively, and the index $n$ denotes the neutrals. In Equations (1) and (2), $q_j$ and $m_j$ are the charge and mass of species $j$ and neutrals, $v_j, n$ is the hydrodynamic velocity, $n_{j, n}$ is the number density, $P_{j, n} = n_{j, n} T_{j, n}$ is the thermal pressure, $T_{j, n}$ is the temperature, $v_{j, n} = \gamma_{jn} m_{jn} n_{jn} (v_{nj} = \gamma_{jn} m_{jn} n_{jn})$ is the collisional frequency of species $j$ ( neutrals) with neutrals (species $j$). The indices $\xi$ and $\eta$ denote directions across and along the magnetic field, respectively. The velocity $v_{j, n} = \langle r v_j \rangle j/m_j + n_j$, where $\langle r v_j \rangle$ is the rate coefficient for momentum transfer, and $\mu_j = v_{j, n}^2 / v_{j, n}^{\xi} n_{j, n}$ is the coefficient of the kinematic viscosity ($\nu_{jn}$ is the neutral–neutral collisional frequency), where $v_{j, n} = (T_{j, n} / m_{j, n})^{1/2}$ is the thermal velocity. Further, $\omega_{cj} = q_j B_0 / m_j c$ is the cyclotron frequency and $\tau_{jn} = v_{jn}^{-1}$. Other notations are the following: $U = -G M/R$ is the gravitational potential of the central object having mass $M$ (when it presents), $G$ is the gravitational constant, $R = (r^2 + z^2)^{1/2}$, $E$ and $B$ are the electric and magnetic fields, $b = B/B_0$, and $c$ is the speed of light in vacuum. The magnetic field $B$ includes the external magnetic field $B_{ext}$ of the central object and/or interstellar medium, the magnetic field $B_{cstr}$ of the stationary current in a steady state, and the perturbed magnetic field. We use the cylindrical coordinate system $(r, \theta, z)$, where $r$ is the distance from the symmetry axis $z$ and $\theta$ is the azimuthal direction. We assume that the background magnetic field is directed along the $z$ axis, $B_0 = B_{cstr} + B_{cstr}$. In Equation (1), the condition $\omega_{cj} \gg v_{jn}$ is satisfied for the viscous regime. For unmagnetized charged particles of species $j$, $\omega_{cj} \ll v_{jn}$, the viscosity coefficient has the same form as that for neutrals. We adopt the adiabatic model for the temperature evolution when $P_{j, n} \sim n_{j, n}^{2/3}$. Then the momentum Equations (1) and (2) in the linear approximation take the form,

$$\frac{\partial v_{j, n}}{\partial t} = - c_s^{\xi} \nabla n_{j, n} + \frac{q_j}{m_j} E + \frac{q_j}{m_j} v_{j, n} \times B + \frac{q_j}{m_j} v_{j, n} \times B - \frac{\mu_j}{\omega_{cj}} \frac{\partial^2 v_{j, n}}{\partial \xi^2} + 6 \frac{\mu_j}{\omega_{cj}^2} \frac{\partial^2 v_{j, n+}}{\partial \xi^2} + 4 \frac{\mu_j}{3 \omega_{cj}^2} \frac{\partial^2 v_{j, n-}}{\partial \xi^2},$$

(6)

where $j = \sum j q_j n_{j, n}$. We consider the wave processes with typical timescales much larger than the time the light spends to cover the wavelength of perturbations. In this case, one can neglect the displacement current in Equation (5) that results in quasineutrality both for the electromagnetic and purely electrostatic perturbations.

3. EQUILIBRIUM

We suppose that electrons, ions, and neutrals rotate in the azimuthal direction of the astrophysical object (accretion disk, molecular cloud, its cores, elephant trunk, and so on) with different, in general, velocities $v_{j, n, o}$. The stationary dynamics of light charged species, electrons and ions, is undergone by the effect of the background magnetic field and collisions with neutrals. In their turn, the neutrals also experience the collisional coupling with charged species influencing on their equilibrium velocity. Some specific cases of equilibrium have been investigated in papers by Nekrasov (2007, 2008a, 2008b, 2009b), where the expressions for stationary velocities of species in the gravitational field of the central mass have been found at the absence of collisions as well as with taking into account collisions for cases of weak and strong collisional coupling of neutrals with ions.

Due to different stationary velocities of charged species, the electric currents are generated in the equilibrium state.

4. LINEAR REGIME

In the present paper, we do not treat perturbations connected with the background pressure gradients. Thus, we exclude the drift and internal gravity waves from our consideration. We take into account the induced reaction of neutrals on the perturbed motion of charged species. The neutrals can be involved in electromagnetic perturbations, if the ionization degree of medium is sufficiently high. We also include the effect of perturbation of the collisional frequencies due to density perturbations of charged species and neutrals. This effect emerges when there are different background velocities of species. Then the momentum Equations (1) and (2) in the linear approximation take the form,

$$\frac{\partial v_{j, n}}{\partial t} = - c_s^{\xi} \nabla n_{j, n} + \frac{q_j}{m_j} E + \frac{q_j}{m_j} v_{j, n} \times B - \frac{\mu_j}{\omega_{cj}} \frac{\partial^2 v_{j, n}}{\partial \xi^2} + 6 \frac{\mu_j}{\omega_{cj}^2} \frac{\partial^2 v_{j, n+}}{\partial \xi^2} + 4 \frac{\mu_j}{3 \omega_{cj}^2} \frac{\partial^2 v_{j, n-}}{\partial \xi^2},$$

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of perturbation of the collisional frequencies due to density perturbations. The index 1 denotes the quantities of the first order of magnitude. The neutrals participate in the electromagnetic dynamics due to collisional coupling with the charged species, mostly with ions (in the absence of dust grains). However, we keep, for generality, collisions of neutrals with electrons mostly with ions (in the absence of dust grains). However, we keep, for generality, collisions of neutrals with electrons (in the absence of dust grains). However, we keep, for generality, collisions of neutrals with electrons.

The continuity Equation (3) in the linear regime is the following:

$$\frac{\partial n_{j,1}}{\partial t} + n_{j,0} \frac{\partial v_{j,1}}{\partial z} = 0. \quad (8)$$

We further apply the Fourier transform to Equations (6)–(8), supposing the perturbations of the form $\exp(ik_z z - i\omega t)$. Using Equation (8) and solving Equation (7), we find the expressions for components of the Fourier amplitude $v_{n1k}$, where (and below) the index $k = \{k_z, \omega\}$.

$$-i\omega_{n\perp} v_{n1k} = \sum_j v_{nj} v_{j1rk},$$
$$-i\omega_{n\perp} v_{n10k} = \sum_j v_{nj} v_{j10k} - \sum_j \frac{k_z (v_{j0} - v_{j0})}{\omega} v_{j1zk},$$
$$-i\omega_{n\perp} v_{n1zk} = \sum_j v_{nj} v_{j1zk}. \quad (9)$$

Here and below, the index 0 by $v_{nj}^0$ and $v_{j0}^0$ is, for simplicity, omitted. In Equations (9), the notations are introduced,

$$\omega_{n\perp} = \omega + i\nu_n + \frac{3}{4} \chi_{njz},$$
$$\omega_n = \omega - \frac{k_z^2 c_n^2}{\omega} + i\nu_n + i\chi_{njz},$$

where $\nu_n = \sum_j v_{nj}$, $\chi_{njz} = \frac{3}{4} \chi_{njz} k_z^2$.

Now we substitute the expressions for $v_{n1k}$ and $n_{j,n1k} = n_{j,0} k_z v_{j1nk}/\omega$ in Equation (6). Then we obtain the following expressions for components of $v_{j1k}$:

$$-i\omega_{j\perp} v_{j1rk} = \frac{q_j}{m_j} E_{1rk} + \omega_j v_{j10k} + i\frac{\nu_{jn}}{\omega_{n\perp}} \sum_l v_{nl} v_{l1rk},$$
$$-i\omega_{j\perp} v_{j10k} = \frac{q_j}{m_j} E_{10k} - \omega_j v_{j1zk} + i\frac{\nu_{jn}}{\omega_{n\perp}} \sum_l v_{nl} v_{l10k} - i \sum_l \beta_{jlz} v_{l1zk},$$
$$-i\omega_{j\perp} v_{j1zk} = \frac{q_j}{m_j} (E_{1zk} + n_z \frac{v_{j0}}{c} E_{10k}) + i\frac{\nu_{jn}}{\omega_{n\perp}} \sum_l v_{nl} v_{l1zk}. \quad (10)$$

Here the notations are introduced

$$\omega_{j\perp} = \omega + i\nu_n + i\chi_{j\perp},$$
$$\omega_{jz} = \omega - \frac{k_z^2 c_{jz}^2}{\omega} + i\nu_n + i\chi_{jz},$$
$$\omega_j = \omega_{cj} - \chi_{j\perp},$$
$$\beta_{jlz} = v_{j0} v_{nj} k_z \left( \frac{v_{j0} - v_{j0}}{\omega_{n\perp}} + \frac{v_{j0} - v_{n0}}{\omega_{njz}} \right),$$

where $\chi_{j\perp} = \mu_j k_z^2 / \omega_{cj} \tau_{jn}$, $\chi_{j\perp2} = 1.2 \mu_j k_z^2 / \omega_{cj}^2 \tau_{jn}^2$.

Faraday’s Equation (4), $B_{1rk} = -n_z E_{10k}$ ($B_{10k} = n_z E_{1rk}$), $B_{1zk} = 0$, where $n_z = k_z c / \omega$.

From Equation (10) for $v_{j1zk}$, we can find the longitudinal (along $B_0$) velocities of electrons and ions,

$$D_z v_{j1zk} = i n_j b_{jz} E_{10k} + i a_{jz} E_{1zk}. \quad (11)$$

Here,

$$D_z = \omega c \omega_{cj} \omega_n + \omega c v_{en} v_{ne} + \omega c v_{en} v_{ni} / \omega_n,$$
$$a_{c} = \alpha_j \frac{q_j}{m_j} - v_{en} v_{ni} / \omega_n,$$
$$b_{c} = \alpha_j \frac{q_j}{m_j} v_{j0} / \omega_n,$$
$$a_{c} = \alpha_j \frac{q_j}{m_j} / \omega_n,$$
$$b_{c} = \alpha_j \frac{q_j}{m_j} v_{j0} / \omega_n,$$
$$\alpha_{c} = \frac{q_j}{m_j} / \omega_n.$$

Let us find now the transverse (across $B_0$) velocities of charged species. When solving Equations (10), we will consider the case in which the charged species are magnetized, i.e., we will suppose that the following conditions are satisfied:

$$\omega c_{j\perp} \gg \omega_j, v_{jn} v_n / \omega_{n\perp}, \chi_{j\perp}. \quad (12)$$

Conditions (12) signify that the Lorentz force is dominant. Then, using Equation (11), we find solutions,

$$v_{j1rk} = -i \frac{q_j}{m_j j\omega_{c_j}} b_{jz} E_{1rk} + \frac{1}{\omega_{c_j}} \left( \frac{q_j}{m_j} a_j + \frac{n_z}{D_z} \chi_{jz} \right) E_{10k}$$
$$v_{j1zk} = -i \frac{q_j}{m_j j\omega_{c_j}} a_j E_{1zk} - i \frac{q_j}{m_j j\omega_{c_j}} b_{jz} E_{10k}. \quad (13)$$

Here,

$$a_j = 1 + \chi_{j\perp1} / \omega_{c_j}, b_j = \omega_{j\perp} + v_{jn} v_n / \omega_{n\perp},$$
$$\delta_{c} = \frac{q_j}{m_j} v_{j0} / \omega_n,$$
$$\lambda_{c} = \frac{q_j}{m_j} v_{j0} / \omega_n,$$
$$\gamma_{c} = \beta_{jz} a_{c} - \beta_{jz} v_{en} v_{ne},$$
$$\gamma_{jz} = \beta_{jz} a_{c} - \beta_{jz} v_{en} v_{ne}.$$
equations for determining the components of the perturbed electric field $\mathbf{E}_{1k} = (E_{1xk}, E_{1yk}, E_{1zk})$:

$$\mathbf{\hat{A}}_k \mathbf{E}_{1k} = 0,$$

(15)

where the matrix $\mathbf{\hat{A}}_k$ is equal to

$$\mathbf{\hat{A}}_k = \begin{bmatrix} n_z^2 - \varepsilon_1 + \varepsilon_5, & i(\varepsilon_2 + \varepsilon_\perp), & \varepsilon_6, \\ -i(\varepsilon_2 + \varepsilon_\perp + \varepsilon_\parallel), & n_x^2 - \varepsilon_1, & -i\varepsilon_5, \\ \varepsilon_4, & 0 & -\varepsilon_3 \end{bmatrix}.$$ 

The components of the matrix $\mathbf{\hat{A}}_k$ are the following:

$$\varepsilon_1 = \sum_j \frac{\omega_{pj}}{\omega_{cj}}^2 b_j, \quad \varepsilon_2 = -\frac{\omega_{pd}}{\omega_{cd}}^2, \quad \varepsilon_3 = -1 \frac{\omega_D}{\omega}^2 \sum_j \frac{m_j}{q_j} b_j,$$

$$\varepsilon_4 = n_x \frac{\omega_D}{\omega}^2 \sum_j \frac{m_j}{q_j} v_0^2 \frac{b_j}{c^2}, \quad \varepsilon_5 = n_z^2 \frac{\omega_D}{\omega}^2 \sum_j \frac{m_j}{q_j} v_0 \frac{a_j}{c^2}, \quad \varepsilon_6 = n_n \frac{\omega_D}{\omega}^2 \sum_j \frac{m_j}{q_j} v_0 \frac{d_j}{c^2},$$

$$\varepsilon_\perp = \sum_j \frac{\omega_{pj}}{\omega_{cj}}^2 \delta_j, \quad \varepsilon_\parallel = \frac{1}{n_z} \frac{\omega_D}{\omega}^2 \sum_j \frac{m_j}{q_j} v_0 \frac{a_j}{c^2},$$

where $\omega_{pj} = (4 \pi n_j q_j^2 / m_j)^{1/2}$ is the plasma frequency and the index $d$ denotes dust grains (if they are present and are static in perturbations).

The dispersion relation is obtained by equating the determinant of the matrix $\mathbf{\hat{A}}_k$ to zero. As a result, we obtain

$$\left( \begin{array}{c} n_z^2 - \varepsilon_1 \\ n_x^2 - \varepsilon_1 + \varepsilon_5 \\ n_y^2 - \varepsilon_1 - (\varepsilon_2 + \varepsilon_\perp) \\ n_z^2 - \varepsilon_1 - (\varepsilon_2 + \varepsilon_\parallel) \end{array} \right) \times \left( \begin{array}{c} n_z^2 - \varepsilon_1 \\ n_x^2 - \varepsilon_1 + \varepsilon_5 \\ n_y^2 - \varepsilon_1 - (\varepsilon_2 + \varepsilon_\perp) \end{array} \right) = 0.$$

(16)

In this equation, it is easy to see that the value $\varepsilon_2 + \varepsilon_\perp$ can be, in general, small in comparison to the value $n_z^2 - \varepsilon_1$. Therefore, we consider below the case in which the following conditions are satisfied:

$$n_z^2 - \varepsilon_1 \approx (\varepsilon_2 + \varepsilon_\perp),$$

$$n_x^2 - \varepsilon_1 + \varepsilon_5 \approx (\varepsilon_2 + \varepsilon_\parallel),$$

(17)

Then Equation (16) will take the form

$$n_z^2 - \varepsilon_1 \approx (\varepsilon_3 + \varepsilon_5 + \varepsilon_6).$$

(18)

We do not consider the damping Alfvén perturbations $n_y^2 - \varepsilon_1 = 0$. When Equation (18) is satisfied, the component of the electric field $E_{10,z} \neq 0$ and $E_{1xk} \ll E_{10,k}$. The second and third equations of a set (Equation (15)) determine the components $E_{1xk}$ and $E_{1zk}$.

$$n_z^2 - \varepsilon_1 \varepsilon_3 E_{1xk} = i(\varepsilon_2 + \varepsilon_\perp) \varepsilon_3 + \varepsilon_3 \varepsilon_\parallel + \varepsilon_4 \varepsilon_\parallel |E_{10,k}|,$$

$$\varepsilon_3 E_{1zk} = \varepsilon_4 E_{10,k}.$$

(19)

Below, we will investigate solutions of Equation (18).

6. DISPERSION RELATION (18) AND ITS SOLUTIONS IN THE SPECIFIC CASE

We further will not take into account the presence of dust grains. Then the quasineutrality condition has the form $q_n n_0 = q_i n_i = 0$. Using this condition, we will calculate the quantities $\varepsilon_1, \varepsilon_3, \varepsilon_4, \varepsilon_5$, and $\varepsilon_6$. We will be interested in the case in which $\omega \ll \nu_n$. When $\chi_{nz} \ll \nu_n$ there is strong collisional coupling of neutrals with charged species in the transverse direction to the $z$ axis. This coupling along the magnetic field depends on the relation between $k_z^2 c_s^2$ and $\omega_D$ (see Equations (9)). If $\chi_{nz} \ll \nu_n$, then the neutrals have a weak coupling with charged species. We suppose also that $\omega \ll \nu_v, \nu_n$ (this inequality is willingly satisfied in weakly ionized plasma if $\chi_{nz} \ll \nu_n$ and $\omega \ll \nu_v$). In this case, the contribution of the viscosity $\chi_{nz}$ in the expressions for $\omega_{j,nz}$ is much smaller in comparison to the thermal term $k_z^2 c_s^2 / \omega_D$ and can be ignored. We further will investigate the case in which $k_z^2 c_s^2 \gg \omega^2$ when the thermal pressure dominates the inertia. Below, we will obtain the dispersion relations for two regions of small and large wave number $k_z$.

6.1. Dispersion Relation (18) for Small $k_z$

At first, suppose that the following condition is satisfied:

$$\omega d_j \gg k_z^2 c_s^2 / \nu_n.$$

(20)

where

$$d_j = c_{sj} v_n + c_{sn} v_{in}.$$

Under conditions at hand we will find,

$$\varepsilon_1 = \sum_j \frac{\omega_{pj}}{\omega_{cj}}^2 v_n = \frac{c^2}{c_A},$$

$$\varepsilon_3 = i \frac{\omega_{pj}^2}{\omega_D^2} \left( d_i - \frac{q_m \nu_v}{v_n} \right),$$

$$\varepsilon_4 = -i \frac{\omega_{pn}^2 n_z^2}{\omega_D^2} d_j,$$

$$\varepsilon_5 = \frac{\omega_{pe}^2 n_z^2}{\omega_D^2} d_j \left( \frac{v_0}{u_{in}} - \frac{v_{in}}{u_{in}} \right),$$

$$\varepsilon_6 = \frac{\omega_{pe}^2 n_z^2}{\omega_D^2} u_{in} \left( \frac{v_{in}}{u_{in}} - \frac{v_{in}}{u_{in}} \right),$$

(21)

where

$$d_1 = \frac{v_0}{c} d_i - \frac{q_m v_0}{q_r m_i} d_e, \quad d_2 = \frac{v_0}{c^2} d_i - \frac{q_m v_0}{q_r m_i} d_e,$$

$$u_{in} v_n + \frac{q_m v_0}{q_r m_i} v_{in},$$

$$u_{in} v_n + \frac{q_m v_0}{q_r m_i} v_{in} - \frac{v_0}{c}.$$

In the expression for $\varepsilon_1$, we have supposed that $\nu_n \gg \chi_{nz}$ that is compatible with condition (20).
The value $D_z$ has the form,

$$D_z = g \frac{k_z^2}{\omega}, \quad (22)$$

where

$$g = c_{z\nu}^2 v_{\nu
 e} + c_{z\nu}^2 v_{\nu
 i} + c_{z\nu}^2 v_{\nu
 e}.$$  

Substitute now the expressions (21) and (22) in Equation (18). Then we obtain the following dispersion relation under conditions given above:

$$(n_z^2 - \epsilon_1) \left( \frac{q_e}{m_e} d_i - \frac{q_i}{m_i} d_e \right) g + \omega^2 \left( \frac{q_i}{m_i} \right) \times \left[ \left( k_z^2 d_e d_i + \omega (v_{\nu
 e} v_{\nu
 i} d_i + v_{\nu
 e} v_{\nu
 e} d_e) \right) (v_{i0} - v_{e0})^2 \right] = 0. \quad (23)$$

6.1.1. Solution of Dispersion Relation (23) for Sufficiently Large $k_z$

Let us find a solution of Equation (23) in the case

$$k_z^2 d_e d_i \gg \omega (v_{\nu
 e} v_{\nu
 i} d_i + v_{\nu
 e} v_{\nu
 e} d_e). \quad (24)$$

Then Equation (23) takes the form

$$\omega^3 - \omega k_z^2 c_A^2 + i h^2 k_z^2 c_A^2 \frac{(v_{i0} - v_{e0})^2}{c^2} = 0, \quad (25)$$

where

$$h^2 = \omega^2 \frac{q_i}{m_i} d_i \left( \frac{q_i}{m_i} d_i + \frac{q_e}{m_e} d_e \right).$$

The sign $\parallel$ denotes an absolute value. We see that due to the last term on the left-hand side of Equation (25) there is an unstable solution.

The solution of Equation (25) in the region $\omega^2 \ll k_z^2 c_A^2$ is equal to

$$\gamma = \frac{h^2}{\sqrt{2}} \frac{(v_{i0} - v_{e0})^2}{c^2}, \quad (26)$$

where $\gamma = \text{Im} \omega$ is the growth rate. Thus, for $k_z > k_z^* = h^2 (v_{i0} - v_{e0})^2 / c^2$, the growth rate is maximal and independent from the wave number. In the region $\omega^2 \gg k_z^2 c_A^2$, we obtain

$$\gamma = \left[ h k_z c_A (v_{i0} - v_{e0}) / c \right]^{2/3}. \quad (27)$$

This growth rate increases with increasing of $k_z$ in the region $k_z < k_z^*.$

6.1.2. Solution of Dispersion Relation (23) for Sufficiently Small $k_z$

In the case

$$k_z^2 d_e d_i \ll \omega (v_{\nu
 e} v_{\nu
 i} d_i + v_{\nu
 e} v_{\nu
 e} d_e), \quad (28)$$

Equation (23) has a solution,

$$\omega = \left[ k_z^2 - s^2 \frac{(v_{i0} - v_{e0})^2}{c^2} \right]^{1/2} c_A, \quad (29)$$

where

$$s^2 = \omega^2 \frac{q_i}{m_i} \left( v_{\nu
 e} v_{\nu
 i} d_i + v_{\nu
 e} v_{\nu
 e} d_e \right) g.$$

We see that perturbations with the wave number $k_z < s |v_{i0} - v_{e0}| / c$ will be unstable.

### 6.2. Solutions of Dispersion Relation (18) for Large $k_z$

Now consider the dispersion relation (18) in the case of large $k_z$ when

$$k_z^2 c_A^2 \gg \omega d_i. \quad (30)$$

Then we obtain the following expressions:

$$\epsilon_j = -\frac{\omega^2}{\omega D_z} k_z^4 c_A^2 \left( \frac{q_i m_i}{q_e m_e} c_A^2 \right), \quad \epsilon_k = \frac{\omega^2}{\omega D_z} k_z^4 c_A^2 \left( \frac{q_i m_i}{q_e m_e} c_A^2 \right),$$

$$\epsilon_s = \frac{\omega^2}{\omega D_z} k_z^4 c_A^2 \left( \frac{q_i m_i}{q_e m_e} c_A^2 \right)^2 + \frac{\omega^2}{\omega D_z} \frac{v_{i0} - v_{e0}}{c},$$

$$\epsilon_e = \epsilon_j + \frac{\omega^2}{\omega D_z} \frac{v_{i0} - v_{e0}}{c}, \quad (31)$$

The expression for $\epsilon_1$ in the case $v_n \gg \chi_{n\nu}$ has the form of Equation (21). In the opposite case, $v_n \ll \chi_{n\nu},$ we obtain

$$\epsilon_1 = i \sum_j \frac{\omega^2}{\omega \omega_{ej}} \left( v_{jn} + \chi_{j\pm2} \right). \quad (32)$$

Substituting the expressions in Equation (31) and $\epsilon_1$ from Equation (21) in Equation (18), we obtain in the case $v_n \gg \chi_{n\nu}$ a dispersion relation which has a solution,

$$\omega = \left[ k_z^2 - \frac{\omega^2}{c_s^2} \frac{(v_{i0} - v_{e0})^2}{c^2} \right]^{1/2} c_A, \quad (33)$$

where $c_s = [3 (|q_e| T_e + |q_i| T_i) / |q_e| m_e]^{1/2}$ is the ion sound velocity. Using expressions (31) and (32), we find in the case $\chi_{n\nu} \gg v_n$ the following solution of the dispersion relation (18):

$$\omega = i \sum_j \frac{\omega^2}{\omega_{ej}} \left( v_{jn} + \chi_{j\pm2} \right) \left[ \frac{\omega^2}{c_s^2} \left( v_{i0} - v_{e0} \right)^2 - k_z^2 c_A^2 \right]. \quad (34)$$

The perturbations (33) and (34) with wave numbers $k_z$ such that $k_z < k_{zh},$ where $k_{zh} = \omega_d \mu |v_{e0} - v_{i0}| / c \kappa,$ are unstable due to different equilibrium velocities of electrons and ions.

### 7. On Calculation of $E_{\nu
 e\nu
 i}$

To find the radial component of the electric field $E_{\nu
 e\nu
 i}$ defined by Equation (19), it is necessary to calculate the value $\epsilon_{3\nu
 e\nu
 i} + \epsilon_{4\nu
 e\nu
 i}.$ The terms $\epsilon_{3\nu
 e}$ and $\epsilon_{4\nu
 e}$ have been arisen due to perturbation of collisional frequencies proportional to the number density perturbations. Using the expressions for these terms containing in Equation (15) and the expressions for $\epsilon_3$ and $\epsilon_4$ from Equations (20), we obtain

$$\epsilon_{3\nu
 e\nu
 i} + \epsilon_{4\nu
 e\nu
 i} = i \frac{\omega^2}{\omega D_z} n_z \left( \frac{q_i}{m_i} \right) \frac{v_{i0} - v_{e0}}{c} \sum_j \omega_{ej} \frac{m_j}{q_j} \times \left[ (\alpha_e, d_e - v_{\nu
 e} v_{\nu
 i} d_i) \beta_{j\nu
 e} + (\alpha_e, d_i - v_{\nu
 e} v_{\nu
 e} d_e) \beta_{j\nu
 e} \right].$$
and using Equations (4) and (5), we obtain the well-known solutions to the standard two MHD equations, is substituted by the ion (or neutral) velocity. As a result, one obtains the dispersion relations, where \( \beta_{ij} \), \( l = e, i \), is defined in Section 4. We see that the value \( \varepsilon_3 \varepsilon_2 + \varepsilon_4 \varepsilon_6 \sim (v_{0e} - v_{0i})(v_{0e} - v_{0n}) + (v_{0i} - v_{0n})(v_{0n} - v_{00}) \). Thus, polarization of perturbations also depends on the difference of equilibrium velocities of species.

8. ON THE RESISTIVITY IN THE STANDARD MHD

In the papers by Nekrasov (2007, 2008a, 2008b, 2009a, 2009b) and in the present paper, we study streaming instabilities of multicomponent rotating magnetized objects, using the equations of motion and continuity for each species. From Faraday’s and Ampere’s laws we obtain equations for the electric field components (see Equation (15)). Such an approach allows us to follow the movement of each species separately and obtain rigorous conditions of consideration and physical consequences in specific cases (see, e.g., Section 9). This approach permits us to include various species of ions and dust grains having different charges and masses. In some cases, the standard methods used in the MHD leads to conclusions that are different from those obtained by the method using the electric field of perturbations. Indeed, let us consider the magnetic induction equation. For simplicity, we take a two-component electron–ion plasma embedded in the background magnetic field directed along the \( z \) axis. The magnetic induction equation with the resistivity is obtained from the momentum equation for the electrons at neglecting the electron inertia,

\[
0 = -\frac{e}{m_e} \mathbf{E} - \frac{e}{m_e} v_e \times \mathbf{B} - v_e (v_e - v_i),
\]

where \( v_e \) (\( v_{i,e} \)) is the electron–ion (ion–electron) collisional frequency, and \( -e \) is the electron charge. Replacing \( v_i - v_e \) by the current \( j/en \) (\( n_e = n_i = n \)), applying \( \nabla \times \) to Equation (35), and using Equations (4) and (5), we obtain the well-known magnetic induction equation,

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times v_e \times \mathbf{B} + \eta_m \nabla^2 \mathbf{B},
\]

where \( \eta_m = v_{ei} c^2/\omega_{pe}^2 \) is the coefficient of the magnetic diffusion or the resistivity. The momentum equation for the ion has the form,

\[
\frac{d v_{i}}{d t} = -\frac{e}{m_i} \mathbf{E} + \frac{e}{m_i} v_i \times \mathbf{B} - v_{ie} (v_i - v_e),
\]

where \( d/dt = \partial/\partial t + v_i \cdot \nabla v_i \). It is seen from Equations (35) and (37) that if the electrons and ions are magnetized, then \( v_{e,\perp} \approx v_{i,\perp} \) (the sign \( \perp \) denotes the transverse direction relatively to the magnetic field). Usually, the electron velocity in Equation (36) is substituted by the ion (or neutral) velocity. As a result, one solves the standard two MHD equations,

\[
\rho \frac{d \mathbf{v}}{d t} = \frac{1}{4\pi} \nabla \times \mathbf{B} \times \mathbf{B},
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{v} \times \mathbf{B} + \eta_m \nabla^2 \mathbf{B},
\]

where \( \rho = m_i n, \mathbf{v} = v_i \).

From Equations (38) in the linear approximation, one obtains the dispersion relations,

\[
\omega^2 + i \omega \eta_m k^2 - k^2 c_A^2 = 0,
\]

for magnetosonic waves, and

\[
\omega^2 + i \omega \eta_m k^2 - k^2 c_A^2 = 0,
\]

for Alfvén waves. Here, \( k^2 = k^2_\perp + k^2_z \), \( c_A = (B_0^2/4\pi \rho)^{1/2} \) is the Alfvén velocity.

Two consequences are followed from Equations (39) and (40).

1. Magnetosonic and Alfvén waves are damped due to the resistivity.
2. The resistivity is isotropic since it is proportional to the total wave number.

We derive now the dispersion relation, calculating the perturbed velocities of electrons and ions through the electric field. For magnetized electrons and ions, we find from Equations (35) and (37),

\[
j_{1rk} = \frac{\text{enc} \omega}{B_0 \omega_{ci}} \left( -i E_{1rk} + \frac{\omega}{\omega_{ci}} E_{10k} \right),
\]

\[
j_{10k} = \frac{\text{enc} \omega}{B_0 \omega_{ci}} \left( -i E_{10k} - \frac{\omega}{\omega_{ci}} E_{1rk} \right),
\]

\[
v_{1ez} = -\frac{e}{m_e v_{ei}} E_{1z}, \quad v_{11z} = 0.
\]

We see that the collisional frequency \( v_{ei} \) is absent in the transverse current and determines only the electron velocity along the background magnetic field. Using Equations (4) and (5) and neglecting the small terms proportional to \( \omega/\omega_{ci} \) in the round brackets of Equations (41), we obtain the following dispersion relation:

\[
(\omega^2 - k^2 c_A^2) \left( \omega^2 + i \omega \eta_m k^2 - k^2 c_A^2 \right) = 0.
\]

We can conclude from Equation (42) the following.

1. Magnetosonic waves are not damped because they have no field \( E_z \).
2. Alfvén waves are damped due to resistivity. The resistivity is proportional to \( k^2_\perp \) and is anisotropic. When \( k^2_\perp = 0 \) Alfvén waves also are not damped.

These results differ, in principle, from the one that follows from Equations (39) and (40). From our viewpoint, Equation (42) takes into account the physical mechanism of the collisional damping correctly.

9. DISCUSSION

The neutrals participate in the electromagnetic perturbations only due to collisions with the charged particles, electrons, ions, and dust grains. One can say that the neutrals are as passive agent, ballast making difficult perturbations of charged species. At the same time, the latter are only active agents generating electromagnetic perturbations. Therefore, an adequate description of multicomponent plasmas including the neutrals is to express the neutral dynamics through the dynamics of charged species and substitute induced velocities of neutrals into collisional terms of the momentum equations for charged species. Then using Faraday’s and Ampere’s laws, we can derive the dispersion relation in the linear approximation and/or investigate nonlinear structures.

From expressions for induced velocities of neutrals, one can easily find rigorous conditions when the neutral dynamics is important or not, i.e., when there is strong (or sufficiently strong) or weak collisional coupling of neutrals with charged species. In the last case, the neutrals are immobile in the electromagnetic perturbations.
Let us write out conditions, which have been used for obtaining the dispersion relation (18), and consider parameters of astrophysical objects, for which these conditions can be satisfied. We will apply our results to protostellar and protoplanetary disks. We have assumed that the electrons as well as ions are magnetized, \( \omega_{ci} \gg \nu_{in} \). The condition \( \omega_{ci} (> 0) \gg \nu_{en} \) is, in general, satisfied in astrophysical objects (Wardle & Ng 1999). The rate coefficient for momentum transfer by elastic scattering of electrons with neutrals is \( \langle \sigma v \rangle \nu_{en} = 4.5 \times 10^{-6} (T_e/30 \, \text{K})^{1/2} \text{cm}^3 \, \text{s}^{-1} \) (Draine et al. 1983). As for ions, we should consider parameters to satisfy conditions \( \omega_{ci} \gg \nu_{in} \) and \( c_i^2 > > c_A^2 \), or \( c_i^2 \gg \omega_{ci}^2 \) (for the last two conditions see below). Thus, the magnitude of the magnetic field must be in some limits. We take the standard values \( m_i = 30 m_p \) and \( m_n = 2.33 m_p \) (the proton mass). The rate coefficient for momentum transfer \( \langle \sigma v \rangle \nu_{in} \) is equal to \( \langle \sigma v \rangle \nu_{in} = 1.9 \times 10^{-9} \text{cm}^3 \, \text{s}^{-1} \) (Draine et al. 1983). Then, for example, from conditions \( \omega_{ci} \gg \nu_{in} \) and \( c_i^2 > > c_A^2 \) we obtain \( 3 \nu_{in} T_i^{1/2} \gg \nu_{in} \gg 0.43 \times 10^{-12} n_n \) \( (T_i \) is in the energetic units, \( B_0 \) is in G, and \( n_n \) is in \( \text{cm}^{-3} \)). We have used \( q_i = -q_e \). Note that it is followed from the last inequalities that the number density of neutrals is limited for a given ion temperature. If we take \( T_i(K) = 700 \, \text{K} \), then we obtain \( n_n \gg 1.5 \times 10^{-2} \text{cm}^{-3} \). In this case, the neutral mass density \( \rho_n = m_n n_n \gg 6.1 \times 10^{-12} \text{g} \, \text{cm}^{-3} \). This condition is applicable for early stage of protoplanetary disks or in the surface layers, where the density is lower and temperature is higher. In the dense inner parts of a disk, where \( \rho_n \sim 10^{-10} \sim 10^{-9} \text{g} \, \text{cm}^{-3} \) (Hayashi et al. 1985; Wardle & Ng 1999), the ions can be unmagnetized and their viscosity will be of the same form as that for neutrals. Our model is not applicable to such dense regions.

For rotating protostellar cores of molecular clouds (protostellar disks), we will adopt the following parameters: \( n_n = 10^7 \sim 10^{10} \text{cm}^{-3} \), \( n_i/n_n = 10^{2} \sim 10^{5} \) (e.g., Caselli et al. 1998; Ruffle et al. 1998; Pudritz 2002), and \( B_0 = 10 \mu \text{G} \) (e.g., Goodman et al. 1993; Crutcher et al. 1999; Caselli et al. 2002). For this magnetic field we obtain \( \omega_{ci} = 1.76 \times 10^2 \text{s}^{-1} \) and \( \omega_{ce} = 3.19 \times 10^{-3} \text{s}^{-1} \).

Now, we give some relationships that are useful at analysis of conditions of consideration. For protoplanetary disks, we take \( T_e = 700 \, \text{K} \). Then we obtain \( \nu_{en}/\nu_{in} = 158.73 \), \( \nu_{en}/\nu_{vn} = 2.34 \times 10^{-4} n_n/n_i \), \( \nu_{in}/\nu_{vn} = 12.88 n_n/n_i \), \( \nu_{vn}/\nu_{ne} = 3.47 \times 10^2 n_i/n_e \), \( \nu_{en}/\nu_{vn} = 12.32 n_n/n_i \), \( m_i / m_e \nu_{in}/\nu_{vn} = 3.47 \times 10^2 \), and \( \nu_{vn}/\nu_{ne} = 6.94 (\langle \sigma v \rangle n_i / \langle \sigma v \rangle n_n) \). For protostellar disks, we take \( T_e = 70 \, \text{K} \). Then \( \nu_{en}/\nu_{in} = 50.19 \), \( \nu_{vn}/\nu_{ne} = 1.1 \times 10^2 n_i/n_e \), \( \nu_{en}/\nu_{vn} = 1.9 n_n/n_i \), and \( m_i / m_e \nu_{in}/\nu_{vn} = 1.1 \times 10^3 \).

Let us consider solution (44) for specific parameters in protostellar and protoplanetary disks. For protostellar disks we take \( n_n = 10^8 \text{cm}^{-3} \) and \( n_i/n_n = 10^{-2} \). When we have obtained \( d_e > c_i^2 \nu_{vn} \), then we obtain \( \nu_{en}/\nu_{vn} = 1.37 \times 10^{-5} \text{s}^{-1} \) and \( \nu_{vn}/\omega_{ci} = 4.29 \times 10^{-3} \). In this case, the condition \( |\nu_{in} - \omega_{ci}| \ll c_i \nu_{in}/\omega_{ci} \) takes the form

\[
\gamma = \frac{\omega_{pi}^2}{\nu_{in}} \frac{(v_{in} - v_{i0})^2}{c_i^2}.
\]

Collecting all conditions given above, we find that solution (43) is satisfied for wave numbers in the region

\[
\frac{\nu_{vn}}{c_i^2} > > \frac{k_z^2}{\gamma} \max \left\{ \frac{\nu_{vn}}{c_i^2} \frac{c_i \nu_{in}}{\omega_{ci}}, \frac{\nu_{vn}}{c_i^2} \nu_{in} \right\}.
\]

Note that we consider the case in which \( c_i^2 \gg c_A^2 \). Using condition \( \gamma/\nu_{vn} \ll 1 \), we obtain from Equation (43) that inequality 16.29 \( |\nu_{in} - \omega_{ci}| \ll \sqrt{\rho_n} (\nu_{in} \text{ in cm}^{-3}) \) must be satisfied. We see that for protostellar disks this condition is not realized. For protoplanetary disks we take \( n_n = 5 \times 10^{11} \text{cm}^{-3} \). Then we obtain \( |\nu_{in} - \omega_{ci}| \ll 4.34 \times 10^6 \text{cm}^{-1} \). We further take \( \nu_{en} / \nu_{in} = 3 \times 10^5 \text{ms}^{-1} \), \( n_i / n_n = 10^{-9} \), and \( B_0 = 0.25 \text{G} \). In this case, \( \nu_{in} = 5.37 \times 10^3 \text{s}^{-1} \), \( \omega_{ci} = 79.75 \text{s}^{-1} \), \( n_n = 68.47 \text{s}^{-1} \), \( \nu_{en} = 8.82 \times 10^{-7} \text{s}^{-1} \), \( \nu_{in} = 1.09 \times 10^4 \text{s}^{-1} \). The Alfven velocity \( c_A = 5.05 \times 10^2 \text{m s}^{-1} \), \( c_{sn} = 2.73 \text{ km s}^{-1} \), \( c_{i0} = 7.6 \times 10^2 \text{m s}^{-1} \), and \( c_{s0} = 1.78 \times 10^4 \text{ km s}^{-1} \). The growth rate \( \gamma \) is equal to \( \gamma \approx 4.2 \times 10^{-7} \text{s}^{-1} \). The wave number \( k_z \) is in the limits, \( 3.8 \times 10^{-7} \text{m}^{-1} \) \( > \approx 7 \times 10^{-3} \text{m}^{-1} \). The wavelength of unstable perturbations \( \lambda_z = 2 \pi/k_z \) has a range

\[
7.57 \times 10^6 \text{ km} \gg \lambda_z \gg 1.65 \times 10^6 \text{ km}.
\]
|v_n - v_0| \ll 1.03 \times 10^2 \text{ cm s}^{-1}, \text{ where } \nu_{ci} = 2.4 \times 10^{10} \text{ m s}^{-1}. In real situation, the value |v_n - v_0| is most probably larger and does not satisfy the last condition. Thus, solution (44) is not realized in protostellar disks under conditions at hand.

Let us determine parameters of protoplanetary disks for which solution (44) is satisfied. This solution exists if |v_{e0} - v_0|/\nu_{ci} < v_{ni}/\omega_{ci}. In the case when collisions of neutrals with ions do not influence the equilibrium velocity of neutrals \nu_{00}, the difference |v_{e0} - v_0| can be estimated as |v_{e0} - v_0| \approx \nu_{00}^2 m^2/\nu_{ci}^2 (Nekrasov 2009b). Thus, the last inequality takes the form, v_{ni}/\omega_{ci} < \omega_{ci}/v_n, where \nu_{ci} = 7.59 \times 10^2 \text{ m s}^{-1} at T_i = 700 K. We consider the region of rotating (pre)protoplanetary disk where v_{ni} \sim 10^3 \text{ km s}^{-1}. In this case, we obtain condition v_{ni}/\omega_{ci} < 7.59 \times 10^{-2}. We take v_{ni}/\omega_{ci} = 3 \times 10^{-2} and B_0 = 0.5 \times 10^{-2} G. It is followed from this that \omega_{ci} = 1.6 \text{ s}^{-1}, v_{ni} = 4.8 \times 10^{-2} \text{ s}^{-1}, and n_n = 3.5 \times 10^6 \text{ cm}^{-3}. We take the ionization degree to be n_i/n_n = 10^{-4}. Then v_n = 6.17 \times 10^{-9} \text{ s}^{-1}. Other parameters are equal: \omega_{pi} = 4.49 \times 10^2 \text{ s}^{-1}, |v_{e0} - v_0| = 9 \text{ m s}^{-1}, c_s = 3.82 \times 10^1 \text{ m s}^{-1}. Then we obtain k_z^2 = 2.05 \times 10^{-11} \text{ m}^{-2}. The wave number k_z satisfies conditions

\left( \frac{v_{ni}}{k^2 c_s} \right)^3 \gg k_z \gg \left( \frac{v_{ni}}{k^2 \nu_{ci}} \right)^3.

Under parameters at hand, we have 0.48 \gg k_z/k_s^* \gg 0.06. Thus, the wavelength of perturbations is in the band

5.13 \times 10^9 \text{ km} \gg \lambda_z \gg 6.38 \times 10^8 \text{ km}.

For k_z/k_s^* = 0.4 the growth rate (Equation (43)) is equal to

\gamma = 0.54 v_{ni} = 3.33 \times 10^{-3} \text{ s}^{-1}.

Perturbations with more small k_z, k_z^2 c_s^2 \ll \omega v_{ni} (see inequality (28), have the growth rate (29). When additionally k_z \ll \nu_{ni}/c_s, where s = (\omega_{pi}/c_s)(v_{ni}/v_{ni})^{1/2}, this growth rate is

\gamma = v_{ni} \omega_{ci} |v_{e0} - v_0|/v_{ni} c_s.

The wave number k_z is in the region

\min \left\{ \frac{(\nu_{ni} v_{ni}^{1/2})}{c_s}, \frac{s |v_{e0} - v_0|}{c_s} \right\} \gg k_z \gg \gamma /c_s.

It is followed from here that conditions v_{ni} \gg 12.88 \gamma and c_s^2 \gg c_A^2 must be satisfied. In these perturbations, the neutrals are strongly coupled with ions both along and across the magnetic field. The viscosity of species is also negligible in the case of Equation (45). For parameters above condition v_{ni} \gg 12.88 \gamma results in a large magnetization, v_{ni}/\omega_{ci} \ll 10^{-2}, for which the value |v_{e0} - v_0| is too small. Thus, solution (45) is not available for protostellar and protoplanetary disks.

Let us now consider the large k_z when k_z^2 c_s^2 \gg \omega v_{ni} (see inequality (30)). Under this condition the inequality k_z^2 c_s^2/c_{ni} \gg \omega_{ci} is satisfied at n_i \lesssim n_n. In this case, the neutrals are weakly coupled with ions in the direction along the magnetic field (see Equations (9)). Solutions considered below exist in media where c_s^2 \gg c_A^2 or 12m_n T_i \gg B_0^2. This condition is not satisfied for typical parameters in protostellar disks given above. However, near the central star, an innermost part of the protoplanetary disk can be highly ionized due to high temperature and have a high density. Note that a highly ionized dense plasma disks exist around neutron stars, stellar black holes, active galactic nuclei, and white dwarfs (see, e.g., Jin 1996 and references therein). The last condition together with \omega_{ci} \gg v_{ni} results in (12\pi n_i T_i)^{1/2} \gg B_0 \gg 0.43 \times 10^{-12} n_n. We see from here that at T \sim 10^9 K and n_i \sim n_n the number density must be n_n \sim 2.62 \times 10^{14} \text{ cm}^{-3}. If we take n_n \sim 10^{13} \text{ cm}^{-3}, we obtain the range of the magnetic field, 22.81 G \gg B_0 \gg 4.3 G. Let, at first, \chi_{nz} \ll v_{ni} or k_z^2 c_s^2 \ll v_{ni}v_{ni} when the neutral viscosity does not play a role. Then the neutrals are strongly coupled with ions across the magnetic field. Under conditions at hand we see that v_{ni} \gg 1.86\omega should be satisfied. In this case, the solution for \omega has the form (33). For k_z such that k_z \ll k_{ih} = \omega_{pi}/v_{ni} - v_0/c_s, the growth rate is equal to

\gamma = \omega_{pi} c_A |v_{e0} - v_0|/c_s.

The wave number k_z is in the region

\min \left\{ k_{ih}, (v_{ni} v_{ni})^{1/2} \right\} \gg k_z \gg (k_{ih} c_A v_{ni})^{1/2}.

Using parameters given above and taking B_0 = 7 G, we find: \omega_{pi} = 7.59 \times 10^8 \text{ s}^{-1}, \omega_{ci} = 2.23 \times 10^3 \text{ s}^{-1}, v_{ni} = 1.37 \times 10^3 \text{ s}^{-1}, v_{ni} = 1.76 \times 10^4 \text{ s}^{-1}, c_A = 3.16 \text{ km s}^{-1}, c_{ni} = 0.88 \text{ km s}^{-1}, c_s = 4.06 \text{ km s}^{-1}, c_{ni} = 2.87 \text{ km s}^{-1}, and c_{ni} = 10.3 \text{ km s}^{-1}. From condition v_{ni} \gg 1.86\omega we can find the limit on the value |v_{e0} - v_0| \ll 4.81 \text{ km s}^{-1}. In the case |v_{e0} - v_0| = 1 \text{ km s}^{-1}, we obtain k_{ih} = 0.62 m^{-1}. The growth rate (46) is equal to \gamma = 1.96 \times 10^{3} \text{ s}^{-1}. The wavelength of unstable perturbations has a range 1.1 \times 10^3 m \gg \lambda_z \gg 10.06 m. The viscosity of electrons and ions is negligible since k_z^2 c_s^2 \ll 1.

The strong coupling of neutrals with ions in the transverse direction is maintained at \chi_{nz} \ll v_{ni}. When \chi_{nz} \gg v_{ni} or k_z^2 c_s^2 \gg v_{ni}v_{ni} the neutral–ion coupling is weak. In this case, the solution (34) at k_z \ll k_{ih} can be written in the form

\gamma = \frac{\omega_{ci}^2}{v_{ni} (1 + 1.2k_z^2 c_s^2)} |v_{e0} - v_0|^2/ c_s.

The condition \chi_{nz} \gg \gamma is satisfied. The wave number is in the region

k_{ih} \gg k_z \gg \frac{\min \left\{ (\nu_{ni} v_{ni}^{1/2})/c_s, (v_{ni} v_{ni})^{1/2} \right\}}{c_{ni}}.

From these inequalities we see that c_s^2 \gg c_A^2 (we assume k_z^2 c_s^2 \leq 1). The growth rate (46) can be larger than v_{ni}. At the same time, condition of consideration is \gamma \ll v_{ni} (see Section 6). Thus, the condition |v_{e0} - v_0|/c_s \ll v_{ni}/\omega_{ci} must be satisfied. Note that solution (47) is the only case when the viscosity of neutrals and ions is important. However, the neutrals are weakly coupled with ions in this case and are immobile in perturbations.

10. CONCLUSION

In the present paper, we have studied electromagnetic streaming instabilities in thermal viscous regions of rotating astrophysical objects, such as protostellar and protoplanetary magnetized accretion disks, molecular clouds, their cores, and elephant trunks. However, the obtained results can be applied to any regions of interstellar medium, where different background velocities between electrons and ions can arise.

We have considered a weakly and highly ionized three-component plasma consisting of electrons, ions, and neutrals. The cyclotron frequencies of charged species are supposed to be much larger than their collisional frequencies with neutrals. The
vertical perturbations along the background magnetic field have been investigated. We have included the effect of perturbation of collisional frequencies due to density perturbations of species. We have shown that due to collisions of charged species with neutrals and neutrals with charged species the latter experience the back reaction on their perturbations. So far as the neutrals participate in the electromagnetic perturbations only because of collisions with the charged species, an adequate description of multicomponent plasmas including neutrals is to express the neutral dynamics through the dynamics of charged species and substitute induced velocities of neutrals into collisional terms of the momentum equations for charged species. Then using Faraday’s and Ampere’s laws, we can derive the dispersion relation and/or investigate nonlinear structures.

The viscosity of magnetized species is important when \( k^2 \rho_j^2 \gtrsim 1 \), where \( \rho_j = v_Tj / \omega j \) is the Larmor radius. The viscosity of neutrals is negligible in comparison to the thermal pressure for the low frequency perturbations, \( \omega \ll v_{in} \). For the one-dimensional perturbations along the magnetic field the thermal pressure is present in the longitudinal perturbed velocities of species. In the transverse velocity of neutrals, the viscosity of neutrals is important when \( \chi_{nz} \gg \nu ni \) or \( k^2 c_{sn}^2 \gg \nu ni v_{in} \). However, the neutrals are weakly coupled with ions in this case and are immobile in perturbations.

The growth rates of perturbations in the wide region of the wave number have been found. We have derived that the long wavelength part of spectrum can be excited in medium, where \( c_{AI} \gg c_{si} \gg c_A \), and the short wavelength perturbations are excited at \( c_{si} \gg c_{AI} \), where \( c_{si} \), \( c_A \), and \( c_{AI} \) are the ion thermal velocity, Alfvén velocity including the mass density of neutrals, and ion Alfvén velocity including the ion mass density, respectively. We have shown that the viscosity plays the role only for the short wavelength edge of spectrum when the neutrals have a weak collisional coupling with ions. In the cases of strong neutral–ion coupling, the viscosity is negligible. The resistivity is absent in the one-dimensional perturbations along the magnetic field.

On the simple example, we have demonstrated that there are discrepancies between the standard MHD results and multicomponent approach at consideration of the resistivity. From the latter approach it is followed that the resistivity has an anisotropic nature. If the resistivity can play an important role in damping of perturbations, this effect can result in anisotropic turbulence. Electromagnetic streaming instabilities considered in the present paper can be a source of turbulence in thermal regions of astrophysical objects.

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