Non-Abelian Stokes Theorem for Wilson Loops Associated with General Gauge Groups

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Abstract

A formula constituting the non-Abelian Stokes theorem for general semi-simple compact gauge groups is presented. The formula involves a path integral over a group space and is applicable to Wilson loop variables irrespective of the topology of loops. Some simple expressions analogous to the ’t Hooft tensor of a magnetic monopole are given for the 2-form of interest. A special property in the case of the fundamental representation of the group $SU(N)$ is pointed out.
§1. Introduction

The non-Abelian Stokes theorem (NAST) equates the Wilson loop variable

\[ W[\gamma] = \text{tr} \left( P e^{i\lambda \int_0^1 A_\mu(x(t)) \frac{dx^\mu(t)}{dt}} \right) \]

(1.1)

with a quantity described by the surface integral of the field strength,

\[ F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) - i\lambda[A_\mu(x), A_\nu(x)]. \]

(1.2)

In (1.1), \( \gamma \) is a closed loop parametrized by \( t \) \((\gamma = \{x(t) \mid 0 \leq t \leq 1, x(0) = x(1)\})\), \( \lambda \) is a gauge coupling constant, \( P \) denotes a path ordering, and \( A_\mu(x) \) is a non-Abelian gauge field. Roughly speaking, there have been proposed two kinds of NAST. The first kind of NAST\(^{1)}\)\(^{13)}\) has a rather long history and involves a duplicate path ordering. When the loop \( \gamma \) is trivial, i.e. unknotted and unlinked, it is described as

\[ W[\gamma] = \text{tr} \left( P_t e^{i\lambda \int_0^1 dt \int_0^1 ds F_{\mu\nu}(x(s,t)) \partial_x^\mu(x(s,t)) \partial_s^\nu(x(s,t))} \right), \]

(1.3)

\[ F_{\mu\nu}(x) = w(x)F_{\mu\nu}(x)w^{-1}(x), \]

(1.4)

where \( P_t \) denotes the \( t \)-ordering, \( w(x) \) is a unitary matrix depending on the path from \( x(0,0) \) to \( x(s,t) \), and the boundary \( \partial S \) of the simply connected surface \( S = \{x(s,t) \mid 0 \leq s, t \leq 1\} \) is assumed to coincide with \( \gamma \). The point \( x(0,0) \) in (1.3) should be identical with the point \( x(0) = x(1) \) in (1.1). Since the matrix \( w(x) \) contains the ordering for a path connecting \( x(0,0) \) and \( x(s,t) \), the r.h.s. of (1.3) involves a duplicate path ordering. The consistency of the formula (1.3) is guaranteed\(^{10)}\) by the Bianchi identity,

\[ [D_\rho, F_{\mu\nu}] + [D_\mu, F_{\nu\rho}] + [D_\nu, F_{\rho\mu}] = 0. \]

(1.5)

Generalization of (1.3) to the cases that the loop \( \gamma \) is topologically nontrivial, i.e. knotted and/or linked, was realized recently.\(^{11)}\)

For the second kind of NAST, the path ordering is replaced with a path integral over a group manifold.\(^{14)}\)\(^{20)}\) In the formulation of Diakonov and Petrov, it takes the form\(^{14)}\)

\[ W[\gamma] = \int [dg]_\gamma e^{i\int_\gamma \zeta(x)}, \]

(1.6)

where \( \zeta(x) \) is the 1-form defined by

\[ \zeta(x) = \lambda\langle A|A _\mu^{-1}(x)|A\rangle dx^\mu, \]

(1.7)

\[ A _\mu^{-1}(x) = g^{-1}(x) A_\mu(x) g(x) + \frac{i}{\lambda} g^{-1}(x) \partial_\mu g(x). \]

(1.8)
In (1.7) and (1.8), \( g(x) \) is a unitary representation of the gauge group, \(|A\rangle\) is the highest weight state in the representation and satisfies the normalization condition \( \langle A|A \rangle = 1 \). It can be shown that there exists a group-invariant measure \( dg \) satisfying the condition

\[
\int dg \ g|A\rangle \langle A|g^{-1} = 1. \tag{1.9}
\]

The integral measure \([dg]_\gamma\) in (1.6) is given by

\[
[dg]_\gamma = \prod_{x \in \gamma} dg(x). \tag{1.10}
\]

It is known that the integral over the group \( G \) can be replaced by an integral over the quotient group \( G/H \), where \( H \) is the stability group of \(|A\rangle\) defined by

\[
h|A\rangle = e^{i\varphi(h)}|A\rangle, \quad \varphi(h) \in \mathbb{R}, \quad h \in H. \tag{1.11}
\]

With the aid of the conventional Stokes theorem,

\[
\int_\gamma \zeta(x) = \int_S d\zeta(x), \quad \partial S = \gamma, \tag{1.12}
\]

the integrand of the exponent on the r.h.s. of (1.6) can be expressed as a surface integral of a function containing the field strength. For the fundamental representation of the gauge group \( SU(2) \), Diakonov and Petrov\(^{17}\) observed that the 2-form \( d\zeta(x) \) in (1.12) can be described by a gauge invariant tensor field which has a structure similar to the ’t Hooft tensor.\(^{22}\) The latter tensor field is given by

\[
T_{\mu\nu}(x) = \sum_{a=1}^{3} F_{\mu\nu}^a(x)\hat{\phi}^a(x) - \frac{1}{\lambda} \sum_{a,b,c=1}^{3} \epsilon^{abc} \hat{\phi}^a(x)(D_\mu\hat{\phi}(x))^b(D_\nu\hat{\phi}(x))^c, \tag{1.13}
\]

where \( \hat{\phi}^a(x) \) \((a = 1, 2, 3)\) is a unit isovector field constructed with the \( SO(3) \) Higgs field \( \phi^a(x) \) as \( \hat{\phi}^a(x) = \phi^a(x)/|\phi(x)| \). It is well known that the electromagnetic field produced by the \( SO(3) \) magnetic monopole is given by \( T_{\mu\nu}(x) \). If we replace \( \hat{\phi}^a(x) \) in (1.13) by

\[
n^a(x) = \frac{1}{2} \text{tr}(g^{-1}(x)\tau^a g(x)\tau^3), \tag{1.14}
\]

with \( \tau^a \) \((a = 1, 2, 3)\) being Pauli matrices, we obtain the tensor field appearing in \( d\zeta(x) \).

The purpose of this paper is to pursue the line of thought of the second kind of NAST\(^{14,17}\) further. We first discuss the case of the general semi-simple compact gauge groups. We find some interesting expressions for the integrand \( d\zeta(x) \) of the above-mentioned surface integral. We also find that there exists a simpler expression for \( d\zeta(x) \) if we restrict ourselves to a fundamental representation of \( SU(N) \). All the above expressions for \( d\zeta(x) \) involve natural generalizations of the ’t Hooft tensor.
This paper is organized as follows. In §2 we obtain some expressions for the 2-form $d\zeta(x)$ which can be used for general semi-simple compact gauge groups. In §3 we discuss the case of the fundamental representation of $SU(N)$ and clarify its special property. Section 4 is devoted to summary and discussion.

§2. Some expressions for the integrand of the surface integral

2.1. Preliminaries

We consider a fixed $D$-dimensional representation of a semi-simple compact Lie group $G$. The representations \{${T^a | a = 1, 2, \cdots, k}$\} = \{${H_i, E_\alpha | i = 1, 2, \cdots, l, \alpha: \text{root of } G}$\} of the generators of $G$ are assumed to satisfy

\[
[H_i, H_j] = 0, \quad i, j = 1, 2, \cdots, l,
\]

\[
[H_i, E_\alpha] = \alpha_i E_\alpha, \quad i = 1, 2, \cdots, l,
\]

\[
\text{tr}(T^a T^b) = \kappa \delta^{ab}, \quad a, b = 1, 2, \cdots, k,
\]

where $\alpha = (\alpha_1, \alpha_2, \cdots, \alpha_l) \in \mathbb{R}^l$ is a root vector and $\kappa$ is a positive constant. Thus, we are working in a $D$-dimensional representation of a group of dimension $k$ and rank $l$. The basis vectors $|\mu(n)\rangle (n = 1, 2, \cdots, D)$ of the representation space satisfy

\[
H_i |\mu(n)\rangle = \mu_i(n) |\mu(n)\rangle, \quad i = 1, 2, \cdots, l,
\]

\[
\langle \mu(n)|\mu(m)\rangle = \delta_{mn}, \quad \sum_{n=1}^{D} |\mu(n)\rangle \langle \mu(n)| = 1,
\]

where $\mu(n) = (\mu_1(n), \mu_2(n), \cdots, \mu_l(n)) \in \mathbb{R}^l$ is a weight vector. We denote the highest weight of the representation and the highest weight state by $\Lambda$ and $|\Lambda\rangle$, respectively:

\[
\Lambda = \mu(1) = (\Lambda_1, \Lambda_2, \cdots, \Lambda_l),
\]

\[
|\Lambda\rangle = |\mu(1)\rangle.
\]

We define a group-theoretic coherent state $|g\rangle$ by

\[
|g\rangle = g |\Lambda\rangle,
\]

where $g$ is a $D$-dimensional representation of an element of $G$. Then, as discussed by many authors, the Wilson loop variable defined by (1-1) can be written as (1-6). If we make use of (1-6) and (1-12), we are led to the formula

\[
W[\gamma] = \int [dg]_{\gamma} e^{i \int_{\gamma} \omega},
\]

where $\omega$ is given by

\[
\omega = d\zeta(x) = \lambda d \left( \langle \Lambda | A_{\mu}^{\phi^{-1}}(x) | \Lambda \rangle dx^\mu \right).
\]
2.2. A formula for $\langle A|K|A \rangle$

Diakonov and Petrov\textsuperscript{14} argued that the 1-form $\zeta(x)$ in (1.7) is given by the trace of the product of the quantity $A \cdot H = \sum_{i=1}^{l} A_{i}H_{i}$ and $A_{\mu}^{-1}(x)$ in (1.8). We begin our discussion by explicitly deriving their conclusion. If we define diagonal matrices $J_{n}$ ($n = 1, 2, \cdots, D$) by

$$|\mu(n)\rangle\langle \mu(n)| = \frac{1}{\kappa}\mu(n) \cdot H + J_{n}, \quad (2.9)$$

it can be shown that, they satisfy

$$\text{tr}(H_{i}J_{n}) = \text{tr}(E_{\alpha}J_{n}) = 0. \quad (2.10)$$

Then, we have

$$\langle A|K|A \rangle = \text{tr}(|A\rangle\langle A|K) = \text{tr}\left(\left\{ \frac{1}{\kappa}A \cdot H + J_{i}\right\}K\right) = \frac{1}{\kappa}\text{tr}(A \cdot HK), \quad K \in G_{D}, \quad (2.11)$$

where $G_{D}$ is a $D$-dimensional representation of the Lie algebra of $G$, i.e. a linear span of the $H_{i}$ and the $E_{\alpha}$. The formula (2.11) reproduces the conclusion of Diakonov and Petrov. We note that the formula

$$H_{i} = \sum_{n=1}^{D} \mu_{i}(n)|\mu(n)\rangle\langle \mu(n)| \quad (2.12)$$

yields the relations

$$\sum_{n=1}^{D} \mu_{i}(n)\mu_{j}(n) = \kappa\delta_{ij},$$

$$\sum_{n=1}^{D} \mu_{i}(n)J_{n} = 0 \quad (2.13)$$

and the expression

$$J_{n} = \sum_{m=1}^{D}\left\{ \delta_{nm} - \frac{1}{\kappa}\mu(n) \cdot \mu(m)\right\}|\mu(m)\rangle\langle \mu(m)|. \quad (2.14)$$

2.3. An Abelian-like expression for $\omega$

From (2.8) and (2.11), we obtain the formula

$$\omega = \frac{\lambda}{\kappa}\text{tr}\left((A \cdot H)d(A_{\mu}^{\gamma^{-1}}(x)dx^{\mu})\right). \quad (2.15)$$
If we define a vector field $B_\mu(x)$ by
\[
B_\mu(x) = \frac{1}{\kappa} \text{tr}\left( (A \cdot H)(A_\mu^{-1}(x)) \right),
\] (2.16)
$\omega$ becomes
\[
\omega = \lambda G_{\mu\nu}(x) d\sigma^{\mu\nu},
\] (2.17)
\[
G_{\mu\nu}(x) = \partial_\mu B_\nu(x) - \partial_\nu B_\mu(x),
\] (2.18)
\[
d\sigma^{\mu\nu} = \frac{1}{2} dx^\mu \wedge dx^\nu.
\] (2.19)

We can interpret the factor $\int_S \omega$ in (2.7) as the flux of the field strength produced by the potential $B_\mu(x)$. It should be noted that the transformation property of $B_\mu(x)$ under $A_\mu^{-1}(x) \rightarrow (A_\mu')^{-1}(x) = A_\mu^{-1}g'(x)$ is not simple and that, under the transformation
\[
(A_\mu^{-1})(x) \rightarrow (A_\mu')^{-1}(x) = A_\mu^{-1}g^{-1}(x)
\] (2.20)
with
\[
g'(x) = \exp[i \theta(x) \cdot H], \quad \theta_i(x) \in \mathbb{R}, \quad (i = 1, 2, \cdots, l)
\] (2.21)
$B_\mu(x)$ transforms in the following way:
\[
B_\mu(x) \rightarrow B_\mu(x) + \frac{1}{\lambda} \partial_\mu (A \cdot \theta(x)).
\] (2.22)

2.4. A ’t Hooft tensor-like expression for $\omega$

To obtain another expression for $\omega$, we separate $A_\mu^{-1}(x)$ into two terms as in (1.7). Then we have
\[
\omega = \xi - \eta,
\] (2.23)
\[
\xi = \frac{1}{\kappa} \sum_{i=1}^{l} A_i \xi_i, \quad \eta = \frac{1}{\kappa} \sum_{i=1}^{l} A_i \eta_i,
\] (2.24)
\[
\xi_i = \lambda \text{tr} \left( H_i d(g^{-1}(x)A_\mu(x)g(x))dx^\mu \right),
\] (2.25)
\[
\eta_i = i \text{tr} \left( H_i g^{-1}(dg)g^{-1}dg \right).
\] (2.26)

The 2-form $\eta_i$ is called the Kirillov-Kostant 2-form. It is known\textsuperscript{24} that, making use of $Q(g)$ defined by
\[
Q(g) = g(x)|A\rangle \langle A| g^{-1}(x) = \frac{1}{\kappa} g(x) A \cdot H g^{-1}(x) + g(x) J_1 g^{-1}(x),
\] (2.27)
the 2-form $\eta$ can be written as
\[
\eta = -i \text{tr} \left( Q(g) [\partial_\mu Q(g), \partial_\nu Q(g)] \right) d\sigma^{\mu\nu}.
\] (2.28)
The first term $\xi$ on the r. h. s. of (2·23) can be rewritten as

$$\xi = \lambda \text{tr} \left( Q(g)[F_{\mu\nu}(x) + i\lambda[A_{\mu}(x), A_{\nu}(x)] - 2\{A_{\mu}(x), \partial_{\nu}Q(g)\}] \right)d\sigma^{\mu\nu}. $$

After some manipulations, it turns out that $G_{\mu\nu}(x)$ in (2·17) can be expressed as

$$G_{\mu\nu}(x) = \text{tr} \left( Q(g) \left\{ F_{\mu\nu}(x) + \frac{i}{\lambda}[D_{\mu}Q(g), D_{\nu}Q(g)] \right\} \right),$$

$$D_{\mu}Q(g) = \partial_{\mu}Q(g) - i\lambda[A_{\mu}(x), Q(g)].$$

(2·29)

The similarity of $G_{\mu\nu}(x)$ to $T_{\mu\nu}(x)$ of (1·11) is obvious. We note that the formulas

$$\{Q(g)\}^2 = Q(g)$$

(2·30)

and

$$Q(g)\{\partial_{\mu}Q(g)\}Q(g) = 0$$

(2·31)

were repeatedly utilized in the derivation of (2·29). We also note that the operator $Q(g)$ appearing in $G_{\mu\nu}(x)$ does not belong to $G_D$ of (2·11) since it contains $g(x)J_1g^{-1}(x)$, while $M_i(g)$ defined by

$$M_i(g) = g(x)H_i g^{-1}(x), \quad i = 1, 2, \cdots, l,$$

(2·32)

belongs to $G_D$.

2.5. The third expression for $\omega$

In terms of $M_i(g)$ introduced in the last subsection, the tensor field $G_{\mu\nu}(x)$ can be written as

$$G_{\mu\nu}(x) = \frac{1}{\kappa} \text{tr} \left( (A \cdot M)\{F_{\mu\nu}(x) + H_{\mu\nu}(x)\} \right),$$

(2·33)

where $H_{\mu\nu}(x) \in G_D$ is defined by

$$H_{\mu\nu}(x) = \frac{i}{\lambda}[K_{\mu}(x), K_{\nu}(x)],$$

(2·34)

$$K_{\mu}(x) = \lambda A_{\mu}(x) + i(\partial_{\mu}g(x))g^{-1}(x).$$

(2·35)

It can be readily seen that $K_{\mu}(x)$ and $H_{\mu\nu}(x)$ transform covariantly under gauge transformations. In the next section, we consider the special case of the fundamental representation of $SU(N)$. There, we find an expression for $G_{\mu\nu}(x)$ which is also similar to (2·29) and (1·13).
§3. The case of the fundamental representation of $SU(N)$

In the previous section, we discussed the path integral formulation of the NAST by making use of group-theoretic coherent states. It turned out that the integrand $\omega$ of the surface integral, or $G_{\mu\nu}(x)$ in (2.17), can be expressed by the operator $Q(g)$ of (2.27) in a form analogous to the 't Hooft tensor $T_{\mu\nu}(x)$, (1.13), of the $SO(3)$ magnetic monopole. We stress that $Q(g)$ is not Lie algebra valued because it has the piece $g(x)J_1g^{-1}(x)$. In this section, we show that, in the case of the fundamental representation of $SU(N)$, we can rewrite $G_{\mu\nu}(x)$ in terms of the Lie algebra valued quantity $M(g)$ defined by

$$M(g) = \frac{1}{\kappa} g(x) A \cdot H g^{-1}(x) = \frac{1}{\kappa} \sum_{i=1}^{N-1} A_i M_i(g). \quad (3.1)$$

By definition, $M(g)$ can be written as

$$M(g) = \frac{1}{\kappa} \sum_{a=1}^{N^2-1} M^a(g) T^a, \quad (3.2)$$

$$M^a(g) = \tr(T^a M(g)) \quad \quad \quad = \langle A|g^{-1}(x)T^a g(x)|A \rangle, \quad (3.3)$$

where $N^2 - 1$ is the dimension of the group $SU(N)$. The rank of $SU(N)$ is $N - 1$ and hence there are $N - 1$ fundamental representations. The following discussion is applicable to all of them. Since the dimension of the fundamental representation of $SU(N)$ is $N$, we are led to the conclusion that all of the diagonal matrices $J_n$ in (2.9) are given by

$$J_n = \frac{1}{N} \mathbf{1}, \quad n = 1, 2, \ldots, N, \quad (3.4)$$

where $\mathbf{1}$ is the $N \times N$ unit matrix. The above conclusion is deduced from the requirements $\tr(H_iJ_n) = 0$, $\tr(H_i) = 0$ ($i = 1, 2, \ldots, N - 1$) and $\tr(|\mu(n)\rangle\langle\mu(n)|) = 1$. From (2.27), (3.1) and (3.4), we obtain

$$Q(g) = M(g) + \frac{1}{N} \mathbf{1}, \quad (3.5)$$

and hence

$$\partial_\mu Q(g) = \partial_\mu M(g). \quad (3.6)$$

With the help of (2.10), (2.29), (3.5) and (3.6), we easily find the formula

$$G_{\mu\nu}(x) = \tr \left( M(g) \left\{ F_{\mu\nu}(x) + \frac{i}{\lambda} [D_\mu M(g), D_\nu M(g)] \right\} \right), \quad (3.7)$$

$$D_\mu M(g) = \partial_\mu M(g) - i\lambda [A_\mu(x), M(g)]. \quad (3.8)$$
In terms of the components $M^a(g)$ in (3.2), the tensor field $G_{\mu\nu}(x)$ is expressed as

$$
G_{\mu\nu}(x) = \sum_{a=1}^{N^2-1} F^a_{\mu
u}(x)M^a(x) - \frac{1}{\lambda} \sum_{a,b,c=1}^{N^2-1} f^{abc}M^a(g)(D_\mu M(g))^b(D_\nu M(g))^c,
$$

(3.9)

where $f^{abc}$ is the structure constant of $SU(N)$. Comparing (3.9) with (1.13), we observe that there exists a complete parallelism between the 't Hooft tensor $T_{\mu\nu}(x)$ and the $G_{\mu\nu}(x)$ in the fundamental representation of $SU(N)$.

To end this section, we compare our result with the expression for $\eta_i$ in (2.24) given by Faddeev and Niemi:

$$
\eta_i = -i \text{tr} \left( M_i(g) \sum_{k=1}^{N-1} [\partial_\mu M_k(g), \partial_\nu M_k(g)] \right) d\sigma^{\mu\nu}.
$$

(3.10)

On the other hand, through analysis similar to the above, we obtain

$$
\eta = -i \text{tr} (M(g)[\partial_\mu M(g), \partial_\nu M(g)]) d\sigma^{\mu\nu}.
$$

(3.11)

The fact that the expressions (3.10) and (3.11) are consistent with the relation $\eta = (\sum_{i=1}^{N-1} A_i \eta_i) / \kappa$, (2.24), can be checked in the following way. With the help of the formulas

$$
M_i(g)M_j(g) = \frac{1}{2} \sum_{k=1}^{N-1} d_{ijk} M_k(g) + \frac{1}{2N} \delta_{ij}
$$

(3.12)

$$
\sum_{i=1}^{N-1} d_{iik} = 0,
$$

(3.13)

with $d_{ijk} \equiv 4 \text{tr}(H_i H_j H_k)$, which are used in Ref. 26), we can rewrite (3.10) as

$$
\eta_i = i \text{tr} (M_i(g)[J_\mu, J_\nu]) d\sigma^{\mu\nu},
$$

(3.14)

$$
J_\mu = (\partial_\mu g(x))g^{-1}(x).
$$

(3.15)

As for $\eta$ in (3.11), we make use of the formula

$$
\{M(g)\}^2 = \frac{N-2}{N} M(g) + \frac{N-1}{N^2} 1,
$$

(3.16)

derived from (2.30) and (3.5). Then we have

$$
\eta = i \text{tr} (M(g)[J_\mu, J_\nu]) d\sigma^{\mu\nu}.
$$

(3.17)

Recalling the definitions (2.24) and (3.1), it is clear that (3.14) and (3.17), and hence (3.10) and (3.11), are consistent. It is, of course, possible to rewrite $G_{\mu\nu}(x)$ of (3.7) and (3.9) in terms of $M_i(g)$ instead of $M(g)$. In our opinion, however, the expressions (3.7) and (3.9) are simplest.
§4. Summary and discussion

We have presented some expressions for the 2-form $\omega$ appearing in the NAST (2·7). It turns out that the simple formulas (2·9) and (2·10), which lead us to (2·11), are useful. We hope that the formulas (2·18), (2·29) and (2·33) for the general case and (3·7) and (3·9) for the case of the fundamental representation of $SU(N)$ are helpful for future investigations of the Wilson loop. The NAST (2·7) can be applied to any closed loop $\gamma$, since only the conventional Stokes theorem, (1·11), has been used in the derivation. In contrast to Abelian cases, the expressions (2·29) and (3·9) for $G_{\mu\nu}(x)$ still contain the gauge potential $A_{\mu}(x)$ through the covariant derivative $D_{\mu}$. The formula (2·7) is, in a sense, of a curious structure: the integral measure $[dg]_\gamma$ concerns the loop $\gamma = \partial S$, while the exponent $\int \omega$ concerns the interior $S^0$ of $S$ as well as $\gamma$. If necessary, we can insert the path integral $\int [\bar{dg}]_{S^0} = 1$ over the interior $S^0$ with $[\bar{dg}]_{S^0} = \prod_{x \in S^0} \bar{d}g(x)$, $\bar{d}g(x) = |\langle A|g(x)|\bar{A}\rangle|^2 dg(x)$ into (1·6), since we obtain $\int \bar{d}g(x) = 1$, $\forall x$, from (1·8). If we make use of (1·12) after the insertion, we are led to the formula $W[\gamma] = \int [dg]_\gamma [\bar{dg}]_{S^0} e^{i\int_S \omega}$.

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