CAPITAL-CONSTRAINED SUPPLY CHAIN WITH MULTIPLE DECISION ATTRIBUTES: DECISION OPTIMIZATION AND COORDINATION ANALYSIS

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ABSTRACT. A Supply Chain Finance (SCF) system involving a commercial bank and a capital-constrained retailer is designed in the imperfect capital market with non-zero bankruptcy costs. A decentralized borrower-lender game is analyzed, and the optimal centralized strategy is developed for SCF from the perspective of multi-attribute utility (MAU) maximization, including maximizing the expected profit and the service level, as well as minimizing the bankruptcy cost. Furthermore, we analytically and numerically explore the coordination condition for SCF and conclude that the bank financing scheme with a suitable combination of decision preferences can realize coordination, even super coordination. Through sensitivity analyses and numerical experiments, we discuss the impacts of the borrower’s initial capitals on the upstream firm’s pricing decision and dig out why he has incentives to support the retailer’s choice of adopting SCF. The findings of this study reveal that the capital-constrained retailer would require more initial capital when maximizing MAU than maximizing the expected profit, and thus the equilibrium order quantity and the bankruptcy risk would also be higher. Moreover, based on a suitable combination of decision preferences, our proposed bank financing scheme can realize coordination, even super coordination.

1. Introduction. Recent years have witnessed a phenomenal growth in the successful deployment of innovative supply chain financing strategies in almost every industry as well as active academic studies in the supply chain management (SCM) field. Supply chain finance (SCF) is viewed as one of innovative credit facilities for capital-constrained supply chains, especially for the small and medium-sized enterprise (SMEs) ones. SCF is all about optimizing the cash flow in and out of the SME supply chain as well as the flow of physical inventories (Zhao and Huchzermeier, 2010

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However, SCF is a complex decision problem involving operational considerations (such as production, procurement, inventory, service, and so on) and financial considerations (such as revenue, profit, interest rate, transaction cost, etc.). While it is relatively easy to deliver a consistent product across the supply chain, maintaining high quality services with low risk and enhancing the working capital resources are much more difficult to achieve. Moreover, it’s reported that managers have difficult decisions and consider an increasingly wide range of criteria in making daily decisions. In the past, such decisions were often judged only based on a single attribute, such as profit or cost. However, cost or profit alone often does not fully capture the desirability of a decision alternative (Warren et al., 2012). Thus, trade-offs exist among profitability, sustainability, services, and the like. The significant impacts of the service level of supply chain operations necessitate the attention of conflicting objectives during decision-making processes in order to ensure a high satisfaction level. Further, much consideration should be given to bankruptcy risk mitigation, and any continual development must reconcile conflicting financial and operational goals and criteria, since bankruptcy is generally a costly process in itself and not only a transfer of ownership implies that these costs negatively affect the total value of the firm.

Today’s decision makers, especially in SMEs, always do not encounter certainty conditions in which there is only one attribute for decision making. Contrarily, they confront difficult decisions daily and must consider an increasingly wide range of criteria in making those decisions, because the single attribute, such as profit or cost alone, often does not fully capture the characteristics of decision objectives. In other words, decisions should be made under complex conditions of several criteria/attributes which contrast to each other and under uncertainty conditions. (Rao, 2013). Therefore, the approach of multiple criteria decision-making has a lot to offer for reinforcing this need.

Motivated by the emerging practices and the extant literature on SCF, this paper focuses on decision-making with multiple criteria for a capital-constrained retailer in an imperfect capital market. Several interesting research questions may arise. (i) How do we characterize the influence of the degree of capital constraints on the imperfections of the capital market, such as transaction cost, tax, asymmetric information, limited commitment, etc.? (ii) How do decision preferences affect optimal decisions and coordination mechanisms? (iii) What is the difference between the multi-attribute utility (MAU) maximization and the traditional expected profit maximization in terms of operational strategies? These three questions represent the major difficulties identified in empirical evidence on SCF practices when SMEs simultaneously face financial decisions and operational decisions; further, these questions denote the theoretical challenges in the SCF field when dealing with multiple decision attributes in formulating a model.

To answer these questions, we formulate a SCF system involving a commercial bank and a capital-constrained retailer. The systematic objective is to maximize the multi-attribute utility, including the expected profit, bankruptcy cost, and the service level. To optimize SCF decisions, the interactions between the borrower and the lender should be properly characterized. Well-developed financing schemes have proven inestimable for improving inventory turnovers and receivables and then free up more working capital. Moreover, since the banks could gain more effective information when engaging in SCF practices, they could be closely connected to the
borrower’s operations, thus reduce the bankruptcy risk. Hence, it is more instructive to study SCF operations in terms of decision optimization.

Lately recently, a growing body of research puts emphasizes on studying the operations-finance interface. As one of the landmark studies to formulate the capital-constrained newsvendor model in a perfect capital market without consideration of transaction costs, taxes and bankruptcy costs, Buzacott and Zhang (2004) discuss the interactions between the bank and the retailer with different capital constraints. It concludes that retailers can increase their returns with asset-based financing compared to when they only use their own capitals. Similarly, Dada and Hu (2008) construct a bank loan model to design a partial coordination mechanism under the bank credit financing. Yang and Birge (2017) analyze how to utilize the inventory-financing scheme including trade credit, debt and cash to figure out a financial diversification solution for the supply chain with capital constraints. Srinivas and Mishra (2011) formulate a two-echelon supply chain with a manufacturer and a retailer, in which both companies encounter capital constraints. They explore how the lender’s earnings are dependent on the borrower’s cash positions. Jing et al. (2012) explore the game equilibrium for bank credit financing (BCF) or trade credit financing (TCF). They suggest that TCF generally charges a higher wholesale price, thus, having less attractions than BCF. Yan et al. (2013) formulate the interaction among the SCF participants as a bi-level Stackelberg game in which both the manufacturer and the retailer have capital constraints and borrow from the commercial bank independently. Additionally, they consider the capital market to be perfect. While part of our analysis has some similarity to these prior studies, the significant distinction is that we extend the present work to a more complex and more practical SCF setting in which the capital market is imperfect with bankruptcy costs, and the systematic decision objective is to maximize multi-attribute utility (MAU) rather than only consider the expected profit maximization.

To address the research gap, several SCF studies began to concentrate on the imperfect capital market. Xu and Birge (2004) explore how the retailer’s inventory decision is affected by his capital structure and capital constraints; these decisions are proportional to the bankruptcy cost. Alan and Gaur (2018) analyze the importance of asset-based lending for the bankruptcy risk, operational investment, and the borrower’s capital structure. They formulate the bankruptcy cost as a function of the difference between the bankruptcy threshold demand and the realized demand. Our model differs from Alan and Gaur’s (2018) model in the formulation of bankruptcy cost; our formulation is similar to that of Kouvelis and Zhao (2011, 2016). Kouvelis and Zhao (2011, 2016) formulate the supply chain of a supplier selling to a retailer with capital constraints. They consider the bankruptcy cost in an imperfect capital market as the sum of a fixed part and a variable part. In this study, we characterize bankruptcy costs in a similar manner.

Moreover, according to SCF practices, we consider a more realistic scenario where the decision maker has multiple decision attributes, such as expected profit maximization, bankruptcy cost minimization, and service level maximization. The methodology we use in this study is the multi-attribute utility (MAU) theory, which is one of our key contributions to SCF literature. In this work, we pay greater attention to MAU maximization than to the traditional expected profit maximization, and we aim to study the implications of decision preferences on SCF equilibrium.
The most closely related study is Brito and Almeida (2012). They present a multi-attribute utility (MAU) for the newsvendor model, considering the profit, the impacts of the service level on customers’ goodwill and enterprise image, as well as the environment impact caused by the disposal of unsold goods. We extend their traditional newsvendor model to an innovative SCF setting in which the retailer with capital constraints may adopt bank credit financing; subsequently, we use the MAU theory to analyze this model. This approach is conspicuously scarce in present research about SCF modeling, which is our primary motivation us to formulate such a model.

We contribute to the literature on SCF optimization by incorporating multiple decision attributes in the decision objective. The goals of our study are to integrate operation decisions and financing decisions and to provide insights about the impact of decision preferences and coordination mechanisms for SCF. Our research contributes to the extant research in three aspects. First, we formulate a borrower-lender game involving a commercial bank and a retailer with capital constraints in the SCF setting. The key distinction from the present research is that we consider the non-zero bankruptcy cost of the imperfect capital market. Second, to explore the impacts of the preferences for each decision attribute on game equilibriums, we mathematically analyze the optimal interest rate and optimal order quantity. Third, through numerical examples, we comparatively analyze MAU optimization and the traditional expected profit maximization, and explore the impacts of capital constraints, bankruptcy cost parameters, and attribute preferences on the corresponding decisions.

The rest of our paper is outlined as follows. In section 2 we present the modeling framework, including variable notations, modeling assumptions, and threshold analysis. Section 3 develops the SCF game and investigates optimal decisions for the borrower and the lender respectively. From the view of the point of systematic optimization, Section 4 discuss the MAU analysis, coordination analysis and sensitivity analysis. In section 5, from the manufacturer’s point of view, we further explore the impact of capital constraints and coordination mechanisms on the optimal pricing decisions. Section 6 concludes our research.

2. Modeling framework. We formulate a retailer in a similar classical newsvendor setting, in which she/he procures products from the upstream firm and sells them to customers. A practical problem that distinguishes supply chain practices is that the retailer has capital constraints in procurement. To avoid discontinuity in supply chain, the retailer seeks to adopt bank credit financing. The bank can help facilitate supply chain trade by easing financial needs and helping settle payments on time. Moreover, SCF also offers banks a great opportunity to tap burgeoning corporate demand, whilst reaping the rewards of this funding source. From a financier’s perspective, more attractive margins can be achieved than from traditional bank financing (Demica Report, 2007).

In this study, we assume that the retailer might encounter bankruptcy risk, which is depending on whether or not the borrower’s liquid assets could cover the corresponding loan obligations. When bankruptcy occurs, the borrower would have no profit after clearing the bank dues.

2.1. Notations and assumptions. To describe our model formulations clearly, we outline the variable notations and modeling assumptions first.

Parameters
Retail price per unit
Wholesale price per unit
Production cost per unit
The retailer’s initial capital
Risk-free interest rate
The fixed administrative cost of bankruptcy
Portion of variable bankruptcy cost to the retailer’s revenue
Decision attribute preference factor
Demand threshold for bankruptcy
Administrative bankruptcy threshold
Quantity threshold with no bankruptcy
Bank’s expected repayment from the retailer
Expected profit
Expected total bankruptcy cost
Expected service level
Multi-attribute utility function

Decision variables
Order quantity
The lender’s endogenous interest rate; \( R_r \geq R_f \)

The subscript \( b, r, m \) and \( s \) denotes the bank, the retailer, the manufacturer and the SCF system, respectively.

The stochastic demand is denoted by \( x \) which is a non-negative random variable with mean \( \mu \). \( f(\cdot) \) and \( F(\cdot) \) are the probability density function (PDF) and the cumulative distribution function (CDF), respectively. The complementary CDF is \( \hat{F}(\cdot) = 1 - F(\cdot) \). We assume that \( F(x) \) is strictly increasing, differentiable, and continuous. We respectively use \( z(x) = f(x)/F(x) \) and \( Z(x) = xz(x) \) to denote the hazard rate and the generalized failure rate, which are increasing in \( x \), namely, the demand distribution has a strictly increasing failure rate (IFR). (see Lariviere, 2006).

In our research, the capital market is assumed to be competitive, which is a common-used in SCF research (see Jing et al., 2012; Jing and Seidmann, 2014; Kouvelis and Zhao, 2011). Moreover, we assume \( p \geq w(1 + R_r) \geq c(1 + R_r) \) without loss of generality.

2.2. Bankruptcy threshold. Using the initial capital \( K_r \), the retailer order \( q \) at a unit price \( w \) from the upstream firm, such as the manufacturer, with unknown demand. If the retailer has capital constraints (i.e., \( K_r \leq wq \)), she/he may seek to borrow from the bank at \( R_r \). Assume that the retailer has a full credit line, and the loan size is determined by the retailer’s capital-gap; i.e., the loan amount is \( \max \{ wq - K_r, 0 \} \), and the repayment is \( (1 + R_r) \cdot \max \{ wq - K_r, 0 \} \) when there is no bankruptcy.

In the end of the selling season, if the borrower’s liquid assets from sales can cover her/his loan obligations, i.e., \( p \min(q, x) \geq (wq - K_r)(1 + R_r) \), she/he would have no bankruptcy risk. Thus, in light of this inequality, the maximal order quantity \( \hat{q}_{NR} \) without capital constraint and the minimal realized demand \( \hat{x}(q) \) without bankruptcy can be obtained as follows:

\[
\hat{q}_{NR} = K_r/w.
\]
\[ \hat{x}(q, w, R_r) = \begin{cases} 0, & 0 \leq q < \hat{q}_{NR} \\ (wq - K_r)(1 + R_r)/p, & q \geq \hat{q}_{NR} \end{cases} \] (2)

Note, if the retailer plays too much order or the realized demand is relatively low, he would will retain a lot of inventory backlog, resulting in lower earnings. In such cases, the retailer could not fulfill the repayment and would be at bankruptcy risk. Furthermore, the risk of bankruptcy might transfer to the lender to a certain extent, and the retailer would default with probability \( F(\hat{x}(q)) \).

3. Borrower-lender game.

3.1. Lender’s decision. In our modeling, we assume that the capital market is imperfect with non-zero bankruptcy cost, which is similar to the assumption in Kouvelis and Zhao (2011). However, our model and the resulting solutions differ because of MAU with a different expression of bankruptcy cost, which will be discussed in Section 4. If the realized demand is relatively low, i.e., \( x < \hat{x}(q, w, R_r) \), the retailer goes bankrupt. During the bankruptcy process, the retailer’s residual assets are used to first pay the fixed administrative cost, followed by the variable cost, which is a portion of revenue. Therefore, the bank will receive the repayment as follows:

\[
M(x) = \begin{cases} 0, & x < k \\ (1 - \alpha)px - B, & x \geq k \end{cases}
\] (3)

where \( k = B/(1 - \alpha)p > 0 \) is the administrative bankruptcy threshold, which refers to the minimal realized demand required for covering the fixed administrative bankruptcy cost alone. The bank’s expected repayment can be given as follows.

\[
\Delta_b(\hat{x}(q, w, R_r)) = \int_0^{\hat{x}(q, w, R_r)} M(x) dF(x) + \int_{\hat{x}(q, w, R_r)}^{+\infty} p\hat{x}(q, w, R_r)dF(x) - \int_0^{\hat{x}(q, w, R_r)} M(x) dF(x) + p\hat{x}(q, w, R_r)\bar{F}(\hat{x}(q, w, R_r))
\] (4)

Given the retailer’s order, if \( q > \hat{q}_{NR} \), the bank’s expected repayment can be rewritten into Eq.(5).

\[
\Delta_b(\hat{x}(q, w, R_r)) = \begin{cases} p\hat{x}(q, w, R_r)\bar{F}(\hat{x}(q, w, R_r)), & 0 \leq \hat{x}(q, w, R_r) < k \\
(1 - \alpha)p \int_k^{\hat{x}(q, w, R_r)} \bar{F}(x) dx \hat{x}(q, w, R_r) \geq k \\
+ (\alpha p\hat{x}(q, w, R_r) + B) \bar{F}(\hat{x}(q, w, R_r)), & \hat{x}(q, w, R_r) \geq k \end{cases}
\] (5)

When \( 0 \leq \hat{x}(q, w, R_r) \leq k \), the bank bears a lower bankruptcy risk, whereas \( \hat{x}(q, w, R_r) \geq k \) translates to a higher bankruptcy risk.

From Eq.(5), the partial derivative of the bank’s expected repayment with respect to the demand threshold can be calculated:

\[
\partial \Delta_b(\hat{x}(q)) / \partial \hat{x}(q) = p \cdot \bar{F}(\hat{x}(q)) \cdot G(\hat{x}(q))
\] (6)

where

\[
G(\hat{x}(q)) = \begin{cases} 1 - \hat{x}(q) \cdot z(\hat{x}(q)), & 0 \leq \hat{x}(q) < k \\
1 - (\alpha \hat{x}(q) + B/p)z(\hat{x}(q)), & \hat{x}(q) \geq k
\end{cases}
\]

Since the capital market is assumed to be perfectly competitive, the lender’s expected profit would be zero. Thus, this fairly-priced loan could be formulated as Eq.(7).

\[
\pi_b(\hat{x}(R_r^*, q, w)) = \Delta_b(\hat{x}(R_r^*, q, w)) - \pi^f_b = 0
\] (7)
where \( \pi^f_t = (wq - K_r)(1 + R_f) \) is the bank’s average return on investments. Given \( q \) and \( w \), the equilibrium interest rate \( R^*_r \) can be deduced by combining Eq.(7) and Eq.(2).

**Lemma 3.1.** Given \( R^*_r \), \( G(\hat{x}(R^*_r, q)) \) decreases with \( q \), i.e., \( dG(\hat{x}(R^*_r, q))/dq \leq 0 \).

**Proof of Lemma 3.1.** Applying the Implicit Function Theorem of Eq.(7), given \( R^*_r \), we have
\[
\frac{d\sigma_k(\hat{x}(R^*_r, q))}{dq} = \frac{d\sigma_k(\hat{x}(R^*_r, q))}{d\hat{x}(R^*_r, q)} \cdot \frac{d\hat{x}(R^*_r, q)}{dq} - w(1 + R_f) = 0.
\]
Therefore, for \( q > \hat{q}_{NR} \),
\[
\frac{d\hat{x}(R^*_r, q)}{dq} = \frac{\sigma_k(\hat{x}(R^*_r, q))}{\frac{d\sigma_k(\hat{x}(R^*_r, q))}{d\hat{x}(R^*_r, q)}w(1 + R_f)} > 0.
\]
From Eq.(6), we have \( G(\hat{x}(q)) = 1 - \hat{x}(q) \cdot z(\hat{x}(q)) \) for \( 0 \leq \hat{x}(q) \leq k \). Differentiating it with respect to \( q \) on both sides, we can deduce that \( dG(\hat{x}(q))/dq = dG(\hat{x}(q))/d\hat{x}(q) \cdot d\hat{x}(q)/dq = -[z(\hat{x}(q)) + \hat{x}(q) \cdot dz(\hat{x}(q))/d\hat{x}(q)] \cdot d\hat{x}(q)/dq \). Since the demand distribution function is IFR, we have \( dz(\hat{x}(q))/d\hat{x}(q) \geq 0 \). Therefore, the inequality of \( dG(\hat{x}(R^*_r, q))/dq \leq 0 \) holds. Otherwise, if \( \hat{x}(q) \geq k \), \( G(\hat{x}(q)) = 1 - (\alpha \hat{x}(q) + B/p) z(\hat{x}(q)) \). Similarly, differentiating it with respect to \( q \), we have \( dG(\hat{x}(q))/dq = -[\alpha z(\hat{x}(q)) + (\alpha \hat{x}(q) + B/p) \cdot dz(\hat{x}(q))/d\hat{x}(q)] \cdot d\hat{x}(q)/dq \). It is obvious that \( dG(\hat{x}(R^*_r, q))/dq \leq 0 \) is also true. \( \square \)

### 3.2. Borrower’s decision.
In the end of the selling season, the retailer can yield \( p \cdot E(\min \{q, x\}) \) and has to repay principal and interest to the bank as \((wq - K_r)(1 + R_f)\). The cash flow of a capital-constrained retailer in different scenarios is summarized as follows:

| Table 1. Retailer’s Cash Flow |
|-----------------------------|
| Cash Flow | \( t = 0 \) | \( t = T \) |
|-----------|--------------|--------------|
| \( 0 \leq q < \hat{q}_{NR} \) | INFLOWS (+) | 0 | \( p \cdot \min \{q, x\} \) |
| | OUTFLOWS (-) | \( wq \) | 0 |
| \( q \geq \hat{q}_{NR} \) | INFLOWS (+) | \( wq - K_r \) | \( p \cdot \min \{q, x\} \) |
| | OUTFLOWS (-) | \( wq \) | \( p \cdot \min \{\hat{x}(q), x\} \) |

If \( 0 \leq q \leq \hat{q}_{NR} \), the retailer has no capital constraint; therefore, her/his expected cost is equal to the procurement cost, considering the time value of capital, i.e., \( wq(1 + R_f) \). Thus, the retailer’s expected profit could be formulated as following.

\[
\pi_r(q) = p \cdot E(\min \{q, x\}) - wq(1 + R_f) = p \cdot \int_0^q \hat{F}(x)dx - wq(1 + R_f) \cdot (8)
\]

Note, in this scenario, the borrower’s decision comes down to the traditional newsvendor model and the optimal solution can be expressed as the classic form of \( q^* = \bar{F}^{-1}[w(1 + R_f)/p] \). Let \( q^{N*} \) be the optimal decision for \( 0 \leq q \leq \hat{q}_{NR} \), i.e., \( q^{N*} = \bar{F}^{-1}(\Omega) \), where \( \Omega = w(1 + R_f)/p \) and the superscript \( N \) denotes the case of no financing.

Otherwise, if \( q > \hat{q}_{NR} \), the retailer has a bankruptcy risk, and the expected repayment is \( p \cdot E(\min \{x, \hat{x}(q)\}) \). Thus, considering the time value of the initial capital, the borrower’s expected profit can be expressed as Eq.(9).

\[
\pi_r(q) = -K_r(1 + R_f) + p \cdot E(\min \{q, x\}) - p \cdot E(\min \{x, \hat{x}(q)\}) + p \cdot \int_{\hat{x}(q)}^q \hat{F}(x)dx - K_r(1 + R_f) \cdot (9)
\]

**Proposition 1.** Given the optimal interest rate \( R^*_r \), the retailer’s optimal order quantity is
\[
q^* = \bar{F}^{-1}[\Omega/G(\hat{x}(q^*))], \quad \text{where} \quad G(\hat{x}(q^*)) = \begin{cases} 1 - \hat{x}(q^*) \cdot z(\hat{x}(q^*)) & , 0 \leq \hat{x}(q^*) < k \\ 1 - (\alpha \hat{x}(q^*) + B/p) z(\hat{x}(q^*)) , \hat{x}(q^*) \geq k \end{cases}
\]
Proof of Proposition 1. According to Eq.\((9)\), if \(q > \hat{q}_{NR}\), taking the first-order derivative of \(\pi_r\) with respect to \(q\), it follows that
\[
d\pi_r (q)/dq = p\tilde{F} (q) - p\tilde{F} (\hat{x} (q)) \cdot d\hat{x} (q)/dq.  \quad \text{(A.1)}
\]

From Lemma 3.1, substituting \(d\hat{x} (q)/dq\) in (A.1), it can be obtained that
\[
d\pi_r (q)/dq = p\bar{F} (q) - w (1 + R_f)/G (\hat{x} (q)).
\]
Furthermore, taking the second-order derivative of \(\pi_r\) with respect to \(q\), we have
\[
d^2 \pi_r (q)/dq^2 = -pf (q) + [w (1 + R_f)/G^2 (\hat{x} (q))] \cdot dG (\hat{x} (q))/dq \leq 0
\]

since \(dG (\hat{x} (R^*_r, q))/dq \leq 0\) proved in Lemma 3.1. Therefore, the retailer’s expected profit function is concave in \(q\).

From the first-order condition of \(d\pi_r (q)/dq^* = 0\), we have \(\bar{F} (q^*) = w (1 + R_f)/pG (\hat{x} (q^*))\). \(\square\)

According to Proposition 1 and Eq.\((9)\) we have \(d\pi_r (q^*)/dK_r = (1 - G (\hat{x} (q^*)) (1 + R_f)/G (\hat{x} (q^*)) \geq 0\) since \(0 \leq G (\hat{x} (q^*)) \leq 1\). Furthermore, taking the second-order derivative of \(\pi_r (q^*)\) in terms of \(K_r\), we have
\[
d^2 \pi_r (q^*)/dK_r^2 = -\frac{(1 + R_f) \cdot \partial G (\hat{x} (q^*))}{G^2 (\hat{x} (q^*))} \cdot \frac{d\hat{x} (q^*)}{dK_r} \leq 0,
\]

since \(\partial G (\hat{x} (q^*)) < 0\) and \(d\hat{x} (q^*) > 0\) (as proved above). This means that the borrower’s optimal profit at equilibrium is concave in \(K_r\). Therefore, according to the first-order optimality condition, i.e., \(d\pi_r (q^*)/dK_r = 0\), the maximum profit can be obtained at the point of \(G (\hat{x} (q^*)) = 1\), i.e., \(K_r = K_1\). In other words, the constrained retailer’s optimal expected profit with SCF cannot exceed the optimal one without capital constraint, i.e., \(\pi_r (q^*) \leq \pi_r (q^*_N)\).

4. Systematic optimization for SCF.

4.1. MAU analysis. Multi-attribute utility (MAU) theory is used to formulate the utilities of multiple attribute consequences or outcomes as a function for the utilities of each decision attribute taken separately (Keeney and Raiffa 1976; Farquhar et al., 2009). In this section, we first formulate each decision attribute for the SCF system, and then optimize the MAU function.

In this section, we mainly focus on the integrated decision for both the borrower and the lender, which compose the SCF system. Since they have conflicting objectives, we aim to explore how to maximize the systematic utility function with respect to multiple decision attributes, including the expected profit, the bankruptcy cost and the service level. Here, we use \(q_s\) to be the systematic order and \(U_s\) to be the MAU function of the SCF system.

Recall that the capital market is perfectly competitive and therefore the bank’s profit is zero. Thus, the expected systematic profit is equivalent to the retailer’s expected sales profit. According to the analysis in Section 3, the expected systematic profit function is formulated as follows.
\[
\pi_s (q_s) = \left\{ \begin{array}{ll} p \int_{q_s}^{q_{NR}} \bar{F} (x) \, dx - wq_s (1 + R_f), & 0 \leq q_s \leq \hat{q}_{NR} \\ p \int_{q_s}^{\hat{x} (q_s)} \bar{F} (x) \, dx - K_r (1 + R_f), & q_s > \hat{q}_{NR} \end{array} \right. \quad \text{(10)}
\]

The expected bankruptcy cost can be formulated according to the following two cases. If \(0 \leq q_s \leq \hat{q}_{NR}\), the retailer has no bankruptcy risk. Hence, the bankruptcy cost is zero; i.e., \(BC (q_s) = 0\). Otherwise, if \(q_s > \hat{q}_{NR}\), the retailer may encounter bankruptcy; thus, the bankruptcy cost is \(BC (q_s) = pE (\min \{x, \hat{x} (q_s)\}) - \)
\[ \Delta_b (\hat{x} (q_s)) \]

where \( \Delta_b (\hat{x} (q_s)) \) is the bank’s expected repayment given by Eq.(4).

Thus, the expected bankruptcy cost can be expressed as follows:

\[
BC (q_s) = \begin{cases} 
0, & 0 \leq q_s \leq \hat{q}_{NR} \\
p \int_0^{\hat{x} (q_s)} \bar{F} (x) \, dx - (wq_s - K_r) (1 + R_f), & q_s > \hat{q}_{NR}
\end{cases}
\]  

(11)

Furthermore, the service level is widely-used in SCM to measure the performance of inventory-ordering policies. There are several definitions of service level used in both academia and practice. They might differ with respect to not only the scope and the number of considered goods but also the time interval to which are related (Bowersox et al., 2012). In this study, we use the expected fill rate to measure the inventory effectiveness at meeting demands. Fill rate is a well-known logistics measure of ordering performance, which is defined by the number of units filled as a percentage of total ordered units. Fill rate can be formulated as Eq.(12).

\[
SL (q_s) = E (\min \{x, q_s\}) / E (x) = \int_0^{q_s} \bar{F} (x) \, dx / \mu.
\]

(12)

Therefore, we use an additive utility function to indicate the independence of preferences among the decision attributes in the SCF system, formulated as Eq.(13).

\[
\max_{q_s} U_s (q_s) = \lambda_1 \pi_s (q_s) - \lambda_2 BC (q_s) + \lambda_3 SL (q_s).
\]

(13)

where \( \lambda_i \) is the independent decision preference for each attribute and determined by decision-makers, which satisfies \( 0 \leq \lambda_i \leq 1, \) \( i = 1, 2, 3 \). In addition, \( \pi_s (q_s) \geq 0 \) is the expected systematic sales profit given by Eq.(10), \( BC (q_s) \geq 0 \) is the expected bankruptcy cost given by Eq.(11), and \( 0 < SL (q_s) \leq 1 \) is the expected service level given by Eq.(12), respectively. Note that \( BC (q_s) \) is an aversion attribute with negative effect on the objective function and then a minus sign used before \( \lambda_2 \).

Note, the general process for MAU analysis of the above problem, in its simplest form, consists of the following two stages. Firstly, the decision makers frame the objective goals formulated in Eq.(13), and then identify the decision alternatives and any related attributes, formulated in Eq.(10-12), which address the decision making objectives. Then the decision makers weight the preferences between those attributes, establish the independent preferences \( \lambda_i (i = 1, 2, 3) \) for each attribute, and form the basis for making rational decisions (Hahn, 2015). It is worth noting that this step, unique to decision analysis, involves the creation of a model of values to evaluate the alternatives. This is done in a structured discussion between a decision analyst and the decision makers to quantify value judgments about possible consequences in the problem (Keeney, 1982). Moreover, there exist scale differences in decision attributes because the expected sales profit and bankruptcy cost are monetary factor and service level is non-monetary factor. Hence, we define that \( 0 \leq \lambda_i \leq 1, (i = 1, 2, 3) \), and it depends on the specific value of each attribute in the specific decision problem. In this paper, we have a presumption that the preference for each attribute is ex ante information, namely, an exogenous parameter, and we mainly focus on how to use MAU maximization as a criterion for decision optimization and coordination analysis in SCF setting.

We now present numerical examples to demonstrate the monotonicity of \( U_s (q_s) \) and the impact of initial capitals on optimal order quantities, depicted in Figure 1.

According to the parameter settings used in Buzacott and Zhang (2004), we assume that \( R_f = 0.05, \ K_r \sim [0, 5], \ p = 1.3, \ c = 0.4 \) and the demand follows
exponential distribution with a mean of 10. Furthermore, we assume $\lambda_1 = \lambda_2 = \lambda_3 = 1/3$ for simplicity.

Figure 1 demonstrates the system’s utility with multi-decision attributes under different initial capitals; the dash curve refers to the case with no capital constraint. Regardless of how $K_r$ changes, $U_s(q_s)$ remains continuous and unimodal in $q_s$. The optimal order depends on two threshold values $K_1$ and $K_2$. The two subfigures in Figure 1 suggest that the optimal order is reached at the thresholds, i.e., $K_r = K_1 \mid q_s^* = q_{NR}$ and $K_r = K_2 \mid q_s^* = q_k$, $\hat{x}(q_k) = k$. More generally, $q_s^*$ is reached in the zone

Figure 1. SCF’s MAU with different $K_r$
of $q_s \leq \hat{q}_{NR}$ when $K_r \geq K_1$ for the retailer without capital constraints. Further, for the retailer with capital constraints, $q_s^\ast$ is reached in the zone of $\hat{q}_{NR} \leq q_s \leq q_k$ when $K_2 \leq K_r \leq K_1$ with lower bankruptcy risk ($0 \leq \hat{x}(q_s^\ast) \leq k$), whereas it is reached in $q_s \geq q_k$ with higher risk when $K_r \leq K_2$. Thus, we obtain the expression of the optimal order quantity with financing, which is presented in Proposition 2.

**Proposition 2.** If the SCF system’s objective is to maximize MAU by incorporating the expected profit, bankruptcy cost, and service level, the unique optimal order quantity $q_s^\ast$ would be dependent on the capital constraint and the decision preference for each attribute. Let $K_2 = \max \{0, \hat{K}_2\}$, where

$$
\hat{K}_2 = w\tilde{K}^{-1} \left[ \Omega \left( \theta_1 + \theta_2 (1 - G(k)) / G(k) \right) - pk\tilde{K} / (1 + R_f) \right] \quad \text{and} \quad \theta_2 = \lambda_2 \mu p / (\lambda_1 \mu p + \lambda_3).
$$

The optimal order can be obtained as $q_s^\ast = F^{-1} \left[ \Omega \left( \theta_1 + \theta_2 (1 - G(\hat{x}(q_s^\ast))) / G(\hat{x}(q_s^\ast)) \right) \right]$, where $G(\hat{x}(q_s^\ast)) = \left\{ \begin{array}{ll}
1, & K_r \geq K_1 \\
1 - \hat{x}(q_s^\ast) \cdot z(\hat{x}(q_s^\ast)), & K_2 \leq K_r < K_1 \text{ and } 0 \leq \hat{x}(q_s^\ast) < k \\
1 - \left( \alpha \hat{x}(q_s^\ast) + B / p \right) \cdot z(\hat{x}(q_s^\ast)), & 0 \leq K_r < K_2 \text{ and } \hat{x}(q_s^\ast) \geq k
\end{array} \right.$

**Proof of Proposition 2.** Similar to proof of Proposition 1, we have $d\pi_r(q_s)/dq_s = p\tilde{F}(q_s) / (1 + R_f) / G(\hat{x}(q_s))$.

According to Eq. (11), taking the first-order derivative of $BC(q_s)$ with respect to $q_s$, it follows that

$$
dBC(q_s)/dq_s = \left\{ \begin{array}{ll}
0, & 0 \leq q_s \leq \hat{q}_{NR} \\
\left[ 1 / G(\hat{x}(q_s)) - 1 \right] w (1 + R_f), & q_s > \hat{q}_{NR}
\end{array} \right. \quad (A.2)
$$

Meanwhile, differentiating $SL(q_s)$ with respect to $q_s$ from Eq. (12), we can deduce that $dSL(q_s)/dq_s = \tilde{F}(q_s) / \mu$. According to Eq. (13), taking the first-order derivative of $U_s(q_s)$ with respect to $q_s$,

$$
dU_s(q_s)/dq_s = \lambda_1 \cdot d\pi_r(q_s)/dq_s - \lambda_2 \cdot dBC(q_s)/dq_s + \lambda_3 \cdot dSL(q_s)/dq_s
$$

$$
= \left\{ \begin{array}{ll}
[F(q_s) \cdot (\lambda_1 \mu p + \lambda_3) / \mu - \lambda_1 w (1 + R_f)], & 0 \leq q_s \leq \hat{q}_{NR} \\
\tilde{F}(q_s) \cdot (\lambda_1 \mu p + \lambda_3) / \mu - w (1 + R_f) \left[ \lambda_1 + \lambda_2 (1 - G(\tilde{x}(q_s))) / G(\tilde{x}(q_s)) \right], & \hat{q}_{NR} < q_s
\end{array} \right. \quad (A.3)
$$

It’s obvious that $dU_s(q_s)/dq_s$ decreases in $q_s$ when $0 \leq q_s \leq \hat{q}_{NR}$. On the other hand, if $q_s > \hat{q}_{NR}$, it can be obtained that $d\tilde{x}(q_s)/dq_s < 0$ from Eq. (2) and $dG(\hat{x}(q_s))/d\hat{x}(q_s) < 0$ for IFR distribution. Thus, we also have $dU_s(q_s)/dq_s$ decreasing in $q_s$ when $q_s > \hat{q}_{NR}$. Therefore, if $q_s = \hat{q}_{NR}$, then $\hat{x}(\hat{q}_{NR}) = 0$ and $G(\hat{x}(\hat{q}_{NR})) = G(0) = 1$. Moreover, evaluating $dU_s(q_s)/dq_s$ at the left side and right side of $\hat{q}_{NR}$, respectively, we have $dU_s(q_s)/dq_s|_{q_s=\hat{q}_{NR}} = \tilde{F}(\hat{q}_{NR}) \cdot (\lambda_1 \mu p + \lambda_3) / \mu - \lambda_1 w (1 + R_f)$ and $dU_s(q_s)/dq_s|_{q_s=\hat{q}_{NR}} = \tilde{F}(\hat{q}_{NR}) \cdot (\lambda_1 \mu p + \lambda_3) / \mu - \lambda_1 w (1 + R_f)$. Thus, $dU_s(q_s)/dq_s|_{q_s=\hat{q}_{NR}} = dU_s(q_s)/dq_s|_{q_s=\hat{q}_{NR}} = dU_s(q_s)/dq_s|_{q_s=\hat{q}_{NR}}$, and $dU_s(q_s)/dq_s$ is continuous. Thus, $U_s(q_s)$ is quasi-concave in $q_s$ and the unique optimal order quantity $q_s^\ast$ can be obtained from the first-order optimality condition of $dU_s(q_s^\ast)/dq_s = 0$.

Let $\theta_1 = \lambda_1 \mu p / (\lambda_1 \mu p + \lambda_3)$ and $\theta_2 = \lambda_2 \mu p / (\lambda_1 \mu p + \lambda_3)$. If $K_r \geq K_1$, $\hat{q}_{NR} = K_r / w \geq q_s^\ast = F^{-1} [\theta_1 \Omega]$. Because $F^{-1}(\cdot)$ is monotonously decreasing, $dU_s(q_s)/dq_s|_{q_s=\hat{q}_{NR}} \leq 0$ and thus $q_s^\ast \geq \hat{q}_{NR}$. Therefore, the equation of $\tilde{F}(q_s^\ast) \cdot (\lambda_1 \mu p + \lambda_3) / \mu - \lambda_1 w (1 + R_f) = 0$ holds, and it can be written as $q_s^\ast = F^{-1} [\theta_1 \Omega]$. Otherwise, if $K_2 \leq K_r < K_1$, we have $dU_s(q_s)/dq_s|_{q_s=\hat{q}_{NR}} > 0$. Hence, we have $q_s^\ast \geq \hat{q}_{NR}$ and $\tilde{F}(q_s^\ast) \cdot (\lambda_1 \mu p + \lambda_3) / \mu - w (1 + R_f) \cdot (\lambda_1 + \lambda_2 (1 - G(\tilde{x}(q_s^\ast))) / G(\tilde{x}(q_s^\ast)) = 0$, i.e., $F(q_s^\ast) = \Omega \cdot [\theta_1 + \theta_2 (1 - G(\tilde{x}(q_s^\ast))) / G(\tilde{x}(q_s^\ast))]$. 

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Since $G(\hat{x}(q))$ is a piecewise function, there exists a piecewise point $q = \hat{q}_B$ satisfying $\hat{x}(\hat{q}_B) = k$. Next, we discuss how to determine the threshold value of $K_2$. Note, $\hat{q}_B$ satisfies the identical equation of deducing the interest rate. From Eq. (11), we have $(w\hat{q}_B - K_2)(1 + R_f) = \Delta_b(\hat{x}(\hat{q}_B))$, i.e., $(w\hat{q}_B - K_2)(1 + R_f) = px(\hat{q}_B)\hat{F}(\hat{q}_B)$. Thus, $K_2 = w\hat{q}_B - pk\hat{F}(k)/(1 + R_f)$.

Thus, if $K_r = K_2$, $q^*_r = \hat{q}_B$ and $\hat{q}_B$ satisfies $dU_s(q_s)/dq_s|_{\hat{q}_B} = 0$, i.e.,

$$dU_s(q_s)/dq_s|_{\hat{q}_B} = \hat{F}(\hat{q}_B) \cdot (\lambda_1 \mu p + \lambda_3)/\mu - w(1 + R_f)\left[\lambda_1 + \lambda_2(1 - G(k))\right]/G(k) = 0. \quad (A.4)$$

Hence, if $pG(k) > [\theta_1 + \theta_2(1 - G(k))] \cdot w(1 + R_f)$, there exists an optimal solution for Eq. (A.4), i.e., $\hat{q}_B = \hat{F}^{-1}\left[\Omega(\theta_1 + \theta_2(1 - G(k)))/G(k)\right]$. Substituting it into $K_2$, we have $\hat{K}_2 = w\hat{q}_B - pk\hat{F}(k)/(1 + R_f)$. On the other hand, if $pG(k) \leq [\theta_1 + \theta_2(1 - G(k))] \cdot w(1 + R_f)$, it’s obvious that $K_2 = 0$.

Furthermore, from Eq. (2), we know that $\hat{x}(q)$ decreases with $K_r$. Since $\hat{x}(q^*_r) = k$ for $K_r = K_2$, we have the inequality of $\hat{x}(q^*_r) \leq k$ for $K_r \geq K_2$, and $\hat{x}(q^*_r) \geq k$ for $K_r \leq K_2$, which corresponds to different cases for $G(\hat{x}(q^*_r))$. Therefore, Proposition 2 can be proved.

Proposition 2 suggest the differences between the MAU decision and the traditional decision objective of only maximizing the expected profit. When we aim to realize the MAU maximization, the preference for each decision attribute has an inevitable impact on the optimal systematic order quantity.

4.2. Coordination analysis. Like traditional supply chain systems with no capital constraints, the SCF system also has the problem of double marginalization related to the objective conflict and channel inefficiency. It is well-known that the wholesale price contract cannot realize coordination in most traditional SCM models, unless the wholesale price is equivalent to the production cost. Obviously, it could not be optimal from a systematic perspective since the manufacturer could not have profits. So, we seek to design a coordination mechanism to ensure that the optimal decision in the decentralized system is accordant with the optimal systematic decision, i.e., $q^* = q^*_s$.

**Proposition 3.** Given $R^*_r$, if the demand follows IFR distribution, then (1) there exist an optimal combination of decision preferences by $(\hat{\lambda}_1, \hat{\lambda}_2, \hat{\lambda}_3)$, which could coordinate the SCF system with MAU, i.e., $q^* \left(\hat{\lambda}_1, \hat{\lambda}_2, \hat{\lambda}_3\right) = q^*_s \left(\hat{\lambda}_1, \hat{\lambda}_2, \hat{\lambda}_3\right)$ when $\hat{\lambda}_2 \mu p (1 - G(\hat{x}(q^*_r))) = \hat{\lambda}_3$ and $\hat{\lambda}_1 = 1 - \hat{\lambda}_2 - \hat{\lambda}_3$; (2) when coordination is realized, the optimal order is independent of $\lambda_1$, increasing with $\lambda_2$ and decreasing with $\lambda_3$, i.e., $dq^*/d\lambda_1 = 0$, $dq^*/d\lambda_2 > 0$ and $dq^*/d\lambda_3 < 0$.

**Proof of Proposition 3.** (1) According to Proposition 1 and 2, we have

$$\hat{F}(q^*_r)/\hat{F}(q^*_s) = G(\hat{x}(q^*_r))/G(\hat{x}(q^*_s))(\theta_1 + \theta_2(1 - G(\hat{x}(q^*_r)))) \quad (A.5)$$

Therefore, if $\hat{\lambda}_2 \mu p (1 - G(\hat{x}(q^*_r))) = \hat{\lambda}_3$, we have $\hat{F}(q^*_r) = \hat{F}(q^*_s)$. Since $F(x)$ is assumed to be strictly increasing, $q^* = q^*_s$ holds for $\hat{\lambda}_2 \mu p (1 - G(\hat{x}(q^*_r))) = \hat{\lambda}_3$ and $\hat{\lambda}_1 = 1 - \hat{\lambda}_2 - \hat{\lambda}_3$.

(2) From Proposition 1, we have $\hat{F}(q^*_r) = \Omega/G(\hat{x}(q^*_r))$. Substituting it in A.5, it can be rewritten as $\hat{F}(q^*_r) = w(1 + R_f)\lambda_2 \mu p/\left(\lambda_2 \mu p - \lambda_3\right)$ under the condition
of coordination. Hence, it is easily to find that $dq^*/d\lambda_1 = 0$, $dq^*/d\lambda_2 > 0$ and $dq^*/d\lambda_3 < 0$.

It is worth nothing from Proposition 3 that in the SCF setting with MAU, the optimal performance could be achieved when the participants in SCF system coordinate with suitable preference to different decision attributes. A more inspiring insight is that there might exists a super-coordination effect, which implies the decentralized system could outperform the centralized one, i.e., $q^* > q^*_s$ may be achieved if $\theta_1 + \theta_2 (1 - G(\hat{x}_s^*)) \geq G(\hat{x}_s^*)/G(\hat{x}^*)$ (see Yan et.al. 2016). It suggests that this outstanding feature is worth studying in academia and practice.

Next, under the same parameter settings as earlier and assuming $\alpha = 0.1$, $B = 0.2$, we present numerical analysis for coordination conditions under different decision preferences and capital constraints, as shown in Figure 2.

Figure 2. Coordination effect under different $K_r$ and decision

It’s straightforward from Figure 2 that the optimal order quantity of the decentralized and centralized system, i.e., $q^*$ and $q^*_s$, are increasing in $K_r$ in the financing zones (Zone I and Zone II) and maintain constant in the zone of no capital constraints (Zone III). Moreover, Figure 2 compares $q^*$ and $q^*_s$ under different $K_r$. Obviously, in the financing zone, the retailer’s optimal quantity cannot exceed the systematic quantity. But given some decision preferences, such as $\lambda_1 = 1/4, \lambda_2 = 3/4, \lambda_3 = 0$, $q^*$ may equal to $q^*_s$. It suggests that under the MAU decision the coordination effect could be realized for the non-financing scenarios, which commonly failed in the traditional supply chain system with sole decision objective. More importantly, in the financing zones, $q^*$ may even be higher than $q^*_s$ under a specific combination of decision preferences, namely, the super-coordination effect may occur when $(\lambda_1, \lambda_2, \lambda_3)$ are suitably set.

4.3. Sensitivity analysis. To investigate the magnitude of the impacts of some operational and financial parameters on optimal decisions, we perform some sensitivity analysis as follows.
4.3.1. Impact of capital constraints.

**Proposition 4.** Given $R^*_r$, if the retailer adopts bank credit financing,

1. the optimal order quantity $q^*_s$ for maximizing MAU in SCF is continuous in $K_r$. It is non-decreasing with $K_r$ when the retailer borrows from the bank, i.e., $dq^*_s/dK_r \geq 0$ for $K_r < K_1$;

2. the bankruptcy cost $BC(q^*_s)$ is non-increasing with $K_r$; further, both the expected profit $\pi_s(q^*_s)$ and the service level $SL(q^*_s)$ are non-decreasing with $K_r$, i.e., $d\pi_s(q^*_s)/dK_r \geq 0, dBC(q^*_s)/dK_r \leq 0, dSL(q^*_s)/dK_r \geq 0$ and $dU_s(q^*_s)/dK_r \geq 0$.

**Proof of Proposition 4.** (1) According to Proposition 2, if $K_r < K_1$, for notation simplification, we denote $V_q(q_s) = \mathcal{F}(q_s) G(\hat{x}(q_s)) - \Omega[\theta_1 + \theta_2 (1 - G(\hat{x}(q_s)))]$. Hence, from Proposition 2, we can rewrite it as $V_q(q_s) = 0$.

Applying the Implicit Function Theorem on $V_q$ and $\pi_b(\hat{x}(R_r))$ in Eq.(7) yields

$$
\partial V_q/\partial K_r + [\mathcal{F}(q_s) + \theta_2 \Omega] \cdot \partial G(\hat{x}(q^*_s))/\partial \hat{x}(q^*_s) 
- \partial \hat{x}(q^*_s)/dK_r \cdot d\pi_b(\hat{x}(q^*_s))/dK_r - f(q^*_s) G(\hat{x}(q^*_s)) \cdot dq^*_s/dK_r = 0.
$$

Combining Eq.(A.6) and Eq.(A.7), we have

$$
dq^*_s/dK_r = \frac{\mathcal{F}(\hat{x}(q^*_s)) \cdot \partial V_q/\partial K_r + f(q^*_s) G(\hat{x}(q^*_s)) \cdot \partial \pi_b/\partial K_r}{\mathcal{F}(\hat{x}(q^*_s)) G^2(\hat{x}(q^*_s)) f(q^*_s) - w(1 + R_f) (\mathcal{F}(q^*_s) + \theta_2 \Omega) \cdot \partial G(\hat{x}(q^*_s))/\partial \hat{x}(q^*_s)}.
$$

For IFR distribution, $G(\hat{x}(q^*_s))$ is a non-decreasing function of $\hat{x}(q^*_s)$, i.e., $\partial G(\hat{x}(q^*_s))/\partial \hat{x}(q^*_s) \leq 0$. Furthermore, due to $0 \leq G(\hat{x}^*) \leq 1$, the denominators of Eqs. (A.8) and (A.9) are positive. Moreover, since $\partial V_q/\partial K_r = 0$ and $\partial \pi_b/\partial K_r = - (1 + R_f) \leq 0$, the numerator of Eq.(A.8) is non-negative and the numerator of Eq.(A.9) is non-positive. Therefore, we have $dq^*_s/dK_r \geq 0$ and $d\hat{x}^*/dK_r \leq 0$.

(2) According to Eq.(11), taking the first-order derivates of $BC(q^*_s)$ with respect to $K_r$, we have

$$
dBC(q^*_s)/dK_r = \mathcal{F}(\hat{x}(q^*_s)) \cdot d\hat{x}(q^*_s)/dK_r - w(1 + R_f) \cdot dq^*_s/dK_r + \partial BC(q^*_s)/\partial K_r.
$$

Substituting $\partial BC(q^*_s)/\partial K_r = 1 + R_f$ in Eq.(A.10),

$$
\frac{dBC(q^*_s)}{dK_r} = \left(\frac{(1 + R_f) f(q^*_s) \mathcal{F}(\hat{x}(q^*_s)) G(\hat{x}(q^*_s)) [G(\hat{x}(q^*_s)) - 1]}{f(q^*_s) \mathcal{F}(\hat{x}(q^*_s)) G^2(\hat{x}(q^*_s)) - \Omega [\mathcal{F}(q^*_s) + \theta_2 \Omega] \cdot \partial G(\hat{x}(q^*_s))/\partial \hat{x}(q^*_s)}\right).
$$

Since $0 \leq G(\hat{x}^*) \leq 1$ and $\partial G(\hat{x}(q^*_s))/\partial \hat{x}(q^*_s) \leq 0$ proved in Proposition 2, the denominator of Eq.(A.11) is positive and the numerator Eq.(A.11) is non-positive. Therefore, we have $dBC(q^*_s)/dK_r \leq 0$. 

According to Eq.(12), we have $SL(q^*_s) = \int_0^{q^*_s} \bar{F}(x)dx/\mu$. Differentiating it with respect to $K_r$ on both sides yields $dSL(q^*_s)/dK_r = \bar{F}(q^*_s)/\mu \cdot dq^*_s/dK_r + \partial SL(q^*_s)/\partial K_r = \bar{F}(q^*_s)/\mu \cdot dq^*_s/dK_r$. Recall that $dq^*_s/dK_r \geq 0$ when $K_r < K_1$ was proved in Proposition 4(1), the inequality of $K_r < K_1$ holds.

In addition, According to Eq.(10), differentiating it with respect to $K_r$ on both sides of $\pi_s(q^*_s)$, we have $d\pi_s(q^*_s)/dK_r = p\bar{F}(q^*_s)\cdot dq^*_s/dK_r - p\bar{F}(\hat{x}(q^*_s))\cdot d\hat{x}(q^*_s)/dK_r + \partial \pi_s(q^*_s)/\partial K_r$. Substituting $\partial \pi_s(q^*_s)/\partial K_r = -(1 + R_f)$ into it,

$$d\pi_s(q^*_s)/dK_r = p\bar{F}(q^*_s)\cdot dq^*_s/dK_r - p\bar{F}(\hat{x}(q^*_s))\cdot d\hat{x}(q^*_s)/dK_r + \partial \pi_s(q^*_s)/\partial K_r$$

$$= (1 + R_f)f(q^*_s)\bar{F}(\hat{x}(q^*_s)) G(\hat{x}(q^*_s)) (1 - G(\hat{x}(q^*_s)))$$

$$- \frac{f(q^*_s)\bar{F}(\hat{x}(q^*_s)) G^2(\hat{x}(q^*_s)) - \Omega (\bar{F}(q^*_s) + \theta_2\Omega) \cdot \partial G(\hat{x}(q^*_s))/\partial \hat{x}(q^*_s)}{(1 + R_f) [\bar{F}(q^*_s) - \Omega] (\bar{F}(q^*_s) + \theta_2\Omega) \cdot \partial G(\hat{x}(q^*_s))/\partial \hat{x}(q^*_s)} \cdot G(\hat{x}(q^*_s))$$

(A.12)

Since $0 \leq G(\hat{x}^*) \leq 1$ and $\partial G(\hat{x}(q^*_s))/\partial \hat{x}(q^*_s) \leq 0$ proved above, the denominator of Eq.(A.12) is positive. Moreover, since $dq^*_s/dK_r \geq 0$ proved in Proposition 4(1), the inequalities of $\bar{F}(q^*_s) \geq \bar{F}(q^*_X) = \Omega$ can be true and thus the numerator of Eq.(A.12) is non-negative. Therefore, $d\pi_s(q^*_s)/dK_r$ is non-negative.

Furthermore, according to Eq.(13), it is obvious that $dU_s(q^*_s)/dK_r \geq 0$.

The main inference from Proposition 4 (1) is that the optimal order quantity in SCF would be maximum when $K_r \geq K_1$, i.e., $q^*_s = q^*_s|_{K_r=K_1}$. This property will be useful for the retailer with capital constraints to determine the appropriate capital structure.

Moreover, the intuitive insight from Proposition 4 (2) is that when the retailer is constrained, the monotonic property of the decision attributes is dependent on the characteristics of the attributes. As shown before, the expected profit and the service level belong to the incentive attribute, which is positive correlated with the MAU function. The bankruptcy cost belongs to the punitive attribute, which is negatively correlated with the MAU function. Proposition 3 implies that as the capital constraint decreases (i.e., as $K_r$ increases), the incentive attribute at equilibrium increases, whereas the punitive attribute decreases. Overall, the equilibrium MAU is non-decreasing with $K_r$.

Using parameter settings similar to those used in Section 3, we carry out some computational examples to numerically illustrate our theoretical analysis discussed earlier. Figure 3 illustrates the thresholds of capital constraints that differentiate the zone of financing with higher bankruptcy ([0, $K^\text{mau}_2$]), that of financing with lower bankruptcy ([K^mau_2, $K^\text{mau}_1$]), and the zone of no capital constraints ([K^mau_1, $\infty$]) in the case of maximizing MAU. We use $K^2_f$ and $K^2_r$ to express the counterparts of the capital thresholds when the expected profit maximization is the only objective.

Figure 3 demonstrates the impact of different amounts of initial capital on the optimal MAUs $U_s(q^*_s(R_2))$ and the constituent parts, namely, expected profit $\pi_s(q^*_s)$, bankruptcy cost $BC(q^*_s)$, and service level $SL(q^*_s)$. It suggests that when $0 \leq K_r \leq K^\text{mau}_1$ (i.e., in financing zones Zone I and Zone II), the optimal bankruptcy cost decreases with $K_r$ and both the service level as well as the optimal MAU increases with $K_r$. The optimal MAU is concave, and the maximum value is
achieved in the financing zone with lower bankruptcy risk (Zone II). Additionally, the optimal expected profit when there is borrowing would be no higher than that without financing, regardless of the retailer’s decision objective.

4.3.2. Impact of bankruptcy cost parameters.

Proposition 5. Given $K_r$ and $R^*_r$, the optimal systematic order $q^*_s$ is non-increasing with fixed bankruptcy cost $B$ (i.e., $dq^*_s/dB \leq 0$ if $\hat{x}(q^*_s) \geq k$); otherwise, it remains constant (if $0 \leq \hat{x}(q^*_s) < k$).

Proof of Proposition 5. Similar to the derivation of Eq.(A.8), taking the first-order derivative of $q^*_s$ with respect to $B$,

$$\frac{dq^*_s}{dB} = \frac{p\mathbb{F} (\hat{x}(q^*_s)) G (\hat{x}(q^*_s)) \cdot \partial V_q / \partial B + (\mathbb{F}(q^*_s) + \theta_2 \Omega) \cdot \partial G (\hat{x}(q^*_s)) / \partial \hat{x}(q^*_s) \cdot \partial \pi_b / \partial B}{pf(q^*_s) \mathbb{F} (\hat{x}(q^*_s)) G^2 (\hat{x}(q^*_s)) - w(1 + R_f)(\mathbb{F}(q^*_s) + \theta_2 \Omega) \cdot \partial G (\hat{x}(q^*_s)) / \partial \hat{x}(q^*_s)}.$$  

(A.13)

Note, the denominator of Eq.(A.13) is as same as that of Eq.(A.8), which has been proved to be positive. Hence, the sign of Eq.(A.13) depends on the sign of its numerator. For $0 \leq \hat{x}(q^*_s) < k$, since $\partial V_q / \partial B = 0$ and $\partial \pi_b / \partial B = 0$, it can be seen that $dq^*_s/dB = 0$ by substituting them into Eq.(A.13). Alternatively, if $\hat{x}(q^*_s) \geq k$, taking the first-order derivate of $V_q$, and $\pi_b$ with respect to $B$, respectively, we have $\partial V_q / \partial B = -z (\hat{x}(q^*_s) \mathbb{F}(q^*_s) + \theta_2 \Omega) / p \leq 0$, and $\partial \pi_b / \partial B = \mathbb{F}(k) - \mathbb{F}(\hat{x}(q^*_s)) \geq 0$.

Therefore, the numerator of Eq.(A.13) is non-positive, and thus $dq^*_s/dB \leq 0$. □

Proposition 5 demonstrates the non-increasing monotonic property of the optimal order quantity in the context of fixed bankruptcy cost. This is because the bank’s expected repayment is independent of the fixed bankruptcy cost. As $B$ increases, the bank’s expected repayment drops; therefore, the bank would compensate for the higher bankruptcy cost by charging a higher interest rate, which would result in a lower order.
Proposition 6. Given $K_r$ and $R^*_r$, the optimal systematic order $q^*_s$ is non-increasing with a portion of the variable bankruptcy cost $\alpha$ when $\hat{x}(q^*_s) \geq k$ (i.e., $dq^*_s/d\alpha \leq 0$); otherwise, it remains constant when $0 \leq \hat{x}(q^*_s) < k$ (i.e., $dq^*_s/d\alpha = 0$).

Proof of Proposition 6. The monotone properties of $q^*_s$ with respect to $\alpha$ can be proved in the same fashion.

\[
\frac{dq^*_s}{d\alpha} = \frac{pF(\hat{x}(q^*_s))G(\hat{x}(q^*_s))\cdot \partial V_{q_s}/\partial \alpha + (\hat{F}(q^*_s) + \theta_2 \Omega) \cdot \partial G(\hat{x}(q^*_s))}{pf(q^*_s)F(\hat{x}(q^*_s))G^2(\hat{x}(q^*_s)) - w(1 + R_1)(\hat{F}(q^*_s) + \theta_2 \Omega) \cdot \partial G(\hat{x}(q^*_s))/\partial \alpha}.
\]

(A.14)

Note, the denominator of Eq.(A.14) is also as same as that of Eq.(A.8) which has been proved to be positive. Hence, the sign of Eq.(A.14) depends on the sign of its numerator. For $0 \leq \hat{x}(q^*_s) < k$, since $\partial V_{q_s}/\partial \alpha = 0$ and $\partial \pi_s/\partial \alpha = 0$, it can be seen that the numerator of Eq.(A.14) is zero and thus $dq^*_s/d\alpha = 0$. Alternatively, if $\hat{x}(q^*_s) \geq k$, taking the first-order derivative of $V_{q_s}$ and $V_{q_s}$ with respect to $\alpha$, respectively, we have $\partial V_{q_s}/\partial \alpha = -\hat{x}(q^*_s)z(\hat{x}(q^*_s)) [\hat{F}(q^*_s) + \theta_2 \Omega] \leq 0$, and $\partial \pi_s/\partial \alpha = p_k \hat{x}(q^*_s) x f(x) dx \geq 0$. Substituting the above two inequalities in the numerator of Eq.(A.14), we have $dq^*_s/d\alpha \leq 0$.}

Similar to Proposition 5, Proposition 6 demonstrates that the non-increasing monotonicity of $q^*_s$ with $\alpha$ is guaranteed. The explanation for this is quite similar to the earlier analysis, differing only by involving a portion of the variable bankruptcy cost rather than the fixed bankruptcy cost.

Observing both Propositions 5 and 6, it is interesting to find that the threshold of the retailer’s revenue that can cover the bankruptcy cost in terms of both $B$ and $\alpha$ is determined by $\hat{x}(q^*_s) = k$, i.e., $B \leq \hat{B} = (1 - \alpha)p\hat{x}(q^*_s)$ and $\alpha \leq \hat{\alpha} = 1 - B/p\hat{x}(q^*_s)$. In other words, the optimal order would be the minimum value if $\hat{x}(q^*_s) \leq k$, i.e., $q^*_s = q^*_s \bigg|_{\hat{x}(q^*_s) = k}$.

Given these parameter settings, we aim to illustrate the impacts of the bankruptcy parameter $B$ and the portion of the variable bankruptcy cost $\alpha$ on optimal decisions (Figures 4-5). In this section, we focus on the bankruptcy costs; thus, we consider only the capital-constrained scenarios and ignore the trivial cases without constraints. Specifically, we conduct numerical analyses under three different initial capital levels ($K_r = 0, 1, 2$).

Figure 4 illustrates the impact of the dependence of the fixed bankruptcy cost $B$ on the optimal order quantity $q^*_s$. It indicates that regardless of the degree of capital constraints, the optimal order linearly decreases with the fixed bankruptcy cost when the borrower faces higher bankruptcy risk, and it remains constant when the bankruptcy risk is lower. Recall that the repayment the bank receives decreases with $B$. Hence, the higher the fixed bankruptcy cost, the higher the interest rate would be, and the less order the retailer would play. In addition, when $B$ exceeds a certain threshold when the borrower’s revenue cannot repay the fixed bankruptcy cost and the bank cannot obtain repayment, the optimal order will approach the minimum, and it will never decrease with $B$.

Moreover, given the fixed bankruptcy cost, a comparison of the different capital constraints suggests that the optimal order with SCF increases with $K_r$, which substantiates Proposition 4.

Similar to the analysis demonstrated through Figure 4, Figure 5 implies a correlation between the variable bankruptcy cost and the optimal order quantity. Given
$K_r$, $q^*_s$ decreases with $\alpha$ if the retailer faces a higher bankruptcy risk, and remains constant when the bankruptcy risk is lower. The reason is quite similar to that described in the previous analysis. However, the optimal order quantity $q^*_s$ has different decreasing properties with $B$ and $\alpha$. A comparison of Figure 4 and Figure 5 indicates that $q^*_s$ is more sensitive to variable bankruptcy cost than it is to the fixed bankruptcy cost. This is because $\alpha$ is the portion of the variable bankruptcy cost to the retailer’s revenue that depends on demand uncertainties, whereas the fixed bankruptcy cost is independent of it. Additionally, Figure 5 demonstrates the
increasing property of the optimal order with initial capitals. This is also consistent with Proposition 4.

5. Discussion: Manufacturer’s pricing decision. Furthermore, from the perspective of the capital-constrained supply chain, we aim to analyze how capital constraints and coordination effect in SCF affect the upstream firm’s pricing decision.

In this study, we consider the upstream firm (for example, the manufacturer) has ample capital to fulfill the retailer’s order and would charge an optimal wholesale price to maximize his profits. For consistency, we also consider the time value of capital; i.e., the manufacturer’s production cost is \( C = cq (1 + R_f) \). In this section, we would consider the scenario of SCF coordination, i.e., \( q^* = q^*_w \). Thus, the manufacturer’s decision objective is Eq.(14)

\[
\max_w \pi_m (w) = (w - c) q^*_w (w). \tag{14}
\]

where \( q^*_w (w) \) is the optimal order quantity in SCF, which is given by Proposition 2.

Note, the retailer’s capital thresholds \( K_1 \) and \( K_2 \) are the functions of \( w \); i.e., \( K_1 (w) \) and \( K_2 (w) \). To pursue the optimal wholesale price for the manufacturer, we need to investigate the interactions between them, as shown in the following lemmas.

Lemma 5.1. Let \( K_1^* = \max \{ K_1 (w) \}; \ K_2^* = \max \{ K_2 (w) \}; \varphi_1 = [\theta_1 + \theta_2 k \cdot z (k)]/(1 - k \cdot z (k)); \varphi_2 = pkF (k)/(1 + R_f) \) with \( q \) satisfying \( \varphi_1 \) satisfying \( \varphi_2 \):

1. \( K_1^* = p\hat{q}^2F (\hat{q})/\theta_1 (1 + R_f) \) and \( K_2^* = p\hat{q}^2F (\hat{q})/\varphi_1 (1 + R_f) - \varphi_2 \).

2. For \( K_r \geq K_1^* \), \( q^*_w (w) \) satisfies \( q^*_w (w) = q^*_w (w) = F^{-1} (\theta_1 \Omega) \).

3. For \( \varphi_2 \leq \varphi_1 \), there exist at most two values of \( w \) (i.e., \( w_1 \) and \( w_2 \)) that satisfy \( 0 \leq w_1 \leq w_2 \leq p/(1 + R_f) \), such that \( K_r = K_1 (w) \). If \( w \leq w_1 \) and \( w \geq w_2 \), when \( K_r \geq K_1^* \), then \( q^*_w (w) \) satisfies \( q^*_w (w) = F^{-1} (\theta_1 \Omega) \). If \( w_1 \leq w \leq w_2 \) for \( K_2^* \leq K_r \leq K_1^* \), \( q^*_w (w) \) satisfies \( G (\hat{q}^*_w (w)) = 1 - \hat{x} (\hat{q}^*_w (w)) \cdot z (\hat{q}^*_w (w)) \).

4. For \( K_r \leq K_2^* \), there exist at most two values of \( w \) (i.e., \( w_3 \) and \( w_4 \)) that satisfy \( w_3 \leq w \leq w_4 \), such that \( K_r = K_2 (w) \). If \( w_3 \leq w \leq w_4 \), \( q^*_w (w) \) satisfies \( G (\hat{q}^*_w (w)) = 1 - \hat{x} (\hat{q}^*_w (w)) \cdot z (\hat{q}^*_w (w)) \).

Proof of Lemma 5.1. Since \( K_1 (w) = wF^{-1} (\theta_1 \Omega) \) is a unimodal function in \( w \), we have \( dK_1/dw = -F (K_1 /w f (K_1/w) + K_1/w through applying the Implicit Function Theorem. From the first-order optimality condition, let \( dK_1/d\hat{w} = 0 \), the maximum value of \( K_1 \) is \( K_1^* = \max \{ K_1 \} = \tilde{w} \hat{q} \), where \( \tilde{q} \cdot z (\hat{q}) = 1 \) and \( K_1/\tilde{w} \cdot z (K_1/\hat{w}) = 1 \). Moreover, from \( F (\hat{q}) = \theta_1 \tilde{w} (1 + R_f)/p \), we have \( \tilde{w} = pF (\hat{q})/\theta_1 (1 + R_f) \). Then, for \( K_r \leq K_1^* \), there exist at most two values of which solve \( K_r = K_1 (w) \), i.e., \( w_1 \) and \( w_2 \), and \( 0 \leq w_1 \leq w_2 \leq p/(1 + R_f) \).

Similarly, since \( K_2 (w) = wF^{-1} (\varphi_2 \Omega) - \varphi_2 \) is also concave in \( w \), we have \( dK_2/dw = -F ((K_2 + \varphi_2)/w) f ((K_2 + \varphi_2)/w) + (K_2 + \varphi_2)/w \). From the first-order optimality condition, let \( dK_2/d\hat{w}' = 0 \) we have \( K_2 = \tilde{w}' \hat{q}' - \varphi_2 \), where \( \tilde{w}' = pF (\hat{q})/\varphi_1 (1 + R_f) \). Hence, for \( K_r \leq K_2^* \), there exist at most two roots of \( K_r = K_2 (w) \), i.e., \( w_3 \), \( w_4 \) and \( w_3 \leq w_4 \). Then \( w_1 \leq w_3 \leq w_4 \leq w_2 \) holds since \( K_1 (w) \leq K_2 (w) \). \( \square \)
Lemma 5.2. The best response function \( q^*_r(w) \) is continuous and decreasing with the wholesale price \( w \).

Proof of Lemma 5.2. For \( K_r \geq K_1 \), \( q^*_r \) satisfies \( \tilde{F}(q^*_r) = \theta_1 w (1 + R_f) / p \). Applying the Implicit Function Theorem, we have \( dq^*_r / dw = -\theta_1 (1 + R_f) / pf(q^*_r) \), which is obviously less than or equal to zero.

Alternatively, similar to the proof of Proposition 2, if \( K_r < K_1 \), we have \( dq^*_r / dw = 1 / w \cdot dq^*_r [F(q^*_r) + \theta_2 \Omega] / \theta_2 (\tilde{F}(q^*_r) + \tilde{F}(\bar{x})) \). Applying the Implicit Function Theorem of \( \tilde{F}(q^*_r) = \Omega \{ \theta_1 + \theta_2 (1 - G(\hat{x})) \} / G(\hat{x}) \). The denominator is straightforward positive and the numerator is negative since \( \partial G(\hat{x}) / \partial \hat{x} < 0 \) and \( G(\hat{x}) \leq 1 \). Therefore, \( dq^*_r / dw \leq 0 \).

The monotonic decreasing property is intuitive since a rational retailer would place fewer orders when the manufacturer charges a higher price.

Lemma 5.3. (1) In the case of no financing (Zone III), there exists a unique equilibrium \((w_m, q_m)\) that maximizes the utility function with multi-decision attributes and the manufacturer’s expected profit, which satisfies \( w_m = p\tilde{F}(q_m)/\theta_1 (1 + R_f) \) and \( \tilde{F}(q_m) - \theta_1 c (1 + R_f)/p - q_m f(q_m) = 0 \); \((w_m, q_m)\) is independent of \( K_r \).

(2) In the case of bank financing (Zone I and Zone II), there exists a unique equilibrium \((w_b, q_b)\) that maximizes the utility with multi-decision attributes and the manufacturer’s expected profit, which satisfies \( c q_b [\tilde{F}(q_b) + \theta_2 \Omega] \cdot \partial G(\hat{x}_b) / \partial \hat{x}_b + \tilde{F}(\hat{x}_b) G(\hat{x}_b) \{ \theta_1 + \theta_2 (1 - G(\hat{x}_b)) \} \cdot [w_b (1 - q_b z(q_b)) - c] = 0 \)
and \( \tilde{F}(q_b) G(\hat{x}_b) - \Omega \{ \theta_1 + \theta_2 (1 - G(\hat{x}_b)) \} = 0 \), where \( G(\hat{x}_b) = \{ 1 - q_b z(\hat{x}_b), 0 \leq \hat{x}_b \leq k \} \).

(3) \( q_b \leq q_m < \tilde{q} < q_1 \) where \( \tilde{q} \) satisfies \( \tilde{q} \cdot z(\tilde{q}) = 1 \), and \( q_1 \) satisfies \( \tilde{F}(q_1) = \theta_1 w_1 (1 + R_f) / p \). \( w_1 \) is the smaller root of the equation \( K_1 (w) = K_r \).

Proof of Lemma 5.3. From Eq.(14), taking the first-order derivative of \( \pi_m \) with \( w \), we have \( dw_m / dw = q^* + (w - c) \cdot dq^*_r / dw \).

(1) According to Proposition 1 and Lemma 5.2, \( \tilde{F}(q^*_r) = \theta_1 \Omega \) and \( dq^*_r / dw = -\theta_1 (1 + R_f) / pf(q^*_r) \) for \( K_r \geq K_1 \). Then, let \( d\pi_m / dw = 0 \), the optimal wholesale price can be obtained as \( w^* = c + pq^*_r f(q^*_r) / \theta_1 (1 + R_f) \). Combining these equations together, let \((w_m, q_m)\) be the equilibrium solution, they satisfy \( w_m = p\tilde{F}(q_m)/\theta_1 (1 + R_f) \) and \( \tilde{F}(q_m) - \theta_1 c (1 + R_f)/p - q_m f(q_m) = 0 \). In addition, it can be rewritten as \( \tilde{F}(q_m) [1 - q_m z(q_m)] - \theta_1 c (1 + R_f)/p = 0 \).

Hence, because \( c > 0 \), the inequality of \( q_m \cdot z(q_m) < 1 \) holds, i.e., \( q_m \cdot z(q_m) < \tilde{q} \cdot z(\tilde{q}) = 1 \). Thus, \( q_m < \tilde{q} \) holds based on the assumption of IFR. Therefore, we have \( q_m < \tilde{q} < q_1 \) for \( w_1 \leq w \leq w_m \).

(2) Otherwise, if \( K_r < K_1 \), from Lemma 5.2, we have \( q^*_r + (w - c) \cdot dq^*_r / dw = -1/w \cdot c q_b [\tilde{F}(q_b) + \theta_2 \Omega] \cdot \partial G(\hat{x}_b) / \partial \hat{x}_b + \tilde{F}(\hat{x}_b) G(\hat{x}_b) \{ \theta_1 + \theta_2 (1 - G(\hat{x}_b)) \} \cdot [w_b (1 - q_b z(q_b)) - c] \). Since the denominator is positive, the first-order optimality condition can be satisfied if the numerator is zero.

Let \((w_b, q_b)\) be the equilibrium point, they satisfies \( \tilde{F}(q_b) G(\hat{x}_b) - \Omega \{ \theta_1 + \theta_2 (1 - G(\hat{x}_b)) \} = 0 \) and \( c q_b [\tilde{F}(q_b) + \theta_2 \Omega] \cdot \partial G(\hat{x}_b) / \partial \hat{x}_b + \tilde{F}(\hat{x}_b) G(\hat{x}_b) \{ \theta_1 + \theta_2 (1 - G(\hat{x}_b)) \} \cdot [w_b (1 - q_b z(q_b)) - c] = 0 \). Moreover, let \( w_b (1 - q_b z(q_b)) - c = 0 \), we have \( \tilde{q} \cdot z(\tilde{q}) = 1 - c/w_b < 1 = \tilde{q} \cdot z(\tilde{q}) \).
which implies that $\tilde{q} < \bar{q}$. In addition, $(w_b, q_b)$ also satisfies

$$w_b(1 - q_b z(q_b)) - c = -\frac{c p F(q_b) + \theta_2 \Omega}{G(\tilde{x}_b) [1 - q_b z(q_b)]},$$

which is obvious non-negative since $G(\tilde{x}_b)/\partial \tilde{x}_b$ lies in $[0, 1]$. Hence, $q_b z(q_b) \leq 1 - c/w_b = \tilde{q} z(\bar{q})$, i.e., $q_b z(q_b) \leq 1 - c/w_b = \tilde{q} z(\bar{q})$ and thus $w_b > \bar{w}$.

(3) Lemma 5.1 proves that $K_1^* = w q$ and $0 \leq w_1 \leq w_2 \leq p/(1 + R_f)$ for $K_r \leq K_1^*$. Hence, it is straightforward that $q_2 < \bar{q} < q_1$ for $0 \leq w_1 \leq \bar{w} \leq w_2$, since $q_2^*$ decreases in $w$. Furthermore, combining the above two equations which solve $(w_b, q_b)$, we have $p F(q_b) [1 - q_b z(q_b)] = \left\{ \frac{c (1 + R_f)}{G(\tilde{x}_b)} \cdot [\theta_1 + \theta_2 (1 - G(\tilde{x}_b))] \cdot \left( 1 - \frac{q_b z(q_b)}{G(\tilde{x}_b) [1 - q_b z(q_b)]} \right) \right\} \geq \frac{c (1 + R_f)}{G(\tilde{x}_b)}$.

$\left[ \theta_1 + \theta_2 (1 - G(\tilde{x}_b)) \right]$ since $0 < G(\tilde{x}_b) \leq 1$. Therefore, we have $p F(q_b) [1 - q_b z(q_b)] - c (1 + R_f) [\theta_1 + \theta_2 (1 - G(\tilde{x}_b))]^2 \geq 0$.

$\left( 1 - \frac{q_b z(q_b)}{G(\tilde{x}_b) [1 - q_b z(q_b)]} \right)$ since 0 $\leq G(\tilde{x}_b) \leq 1$. Thus, $q_b \leq q_m^*$ holds based on the assumption of IFR. Thus, $q_b \leq q_m < \bar{q} < q_1$.

Lemmas 5.1-5.3 suggest that if the retailer has ample capital (i.e., $K_r \geq K_1^*$), the utility with multi-decision attributes can be maximized without financing, and the optimal decisions for the retailer and the manufacturer can be reached at $w = w_m$ and $q_2^* = q_m^*$, which are independent of $K_r$. If $K_r \leq K_1^*$, the equilibrium point in the zone of no constraints is $(w_m, q_m)$, whereas the equilibrium in the financing zone would be $(w_1, q_1)$. Moreover, based on Lemma 5.3, we find that $w_1 \leq \bar{w} \leq w_m$ for $q_m < \bar{q} < q_1$. If $w_2 \leq w_m$ (i.e., $q_m \leq q_2$), $q_1^* (w)$ falls into the zone of no capital constraints (Zone III), and the equilibrium would be $(w_m, q_m)$, i.e., $w^* = w_m$ and $q_2^* = q_m$. Let $K_m^* = w_m q_m$. The retailer has ample capital to order $q_m$ to realize the utility maximization only if $K_r \geq K_m^*$. Alternatively, if $w_2 > w_m$ (i.e., $q_m > q_2$), $q_1^* (w)$ falls into the financing zone (Zone I and Zone II). In this case, the equilibrium would be $(w_b, q_b)$; i.e., $w^* = w_b$, and $q_2^* = q_b$. Thus, the optimal decisions for both the retailer and the manufacturer at equilibrium can be summarized in Proposition 7.

Proposition 7. If $K_r \geq K_1^*$, the equilibrium solution $(w^*, q_1^*)$ equals $(w_m, q_m)$. If $K_r < K_1^*$, $(w^*, q_1^*)$ is $(w_m, q_m)$ for $q_m \leq q_2$, and it is $(w_b, q_b)$ for $q_m > q_2$, where $q_2 = F^{-1} [\theta_1 w_2 (1 + R_f)/p]$, and $w_2$ is the bigger root of the equation $K_1 (w) = K_r$.

Further, using the above parameter settings, we also present numerical examples to illustrate the best order response with respect to $w$ under different $K_r$, depicted in Figure 6. We also assume that $\lambda_1 = 1/4, \lambda_2 = 3/4, \lambda_3 = 0$ for simplicity.

Figure 6 demonstrates the convexity of $q_2^* (w)$ in different cases, which are in accordance with Lemmas 5.1-5.3. The monotonic decreasing property of $q_2^*$ with is clearly depicted in Figure 6. Moreover, we find that although the capital-constrained retailer can mitigate the capital gap through bank financing, the optimal order quantity in this case cannot be higher than that without constraints; i.e., $q_2^* (w) \leq q_2^* (w)$. Further, the higher the bankruptcy risk that the retailer bears, the more orders she/he would place; i.e., $q_2^* (w)_{\bar{i} \leq k} \leq q_2^* (w)_{\bar{i} \geq k}$. The subfigure in the bottom right corner of Figure 6 demonstrates the relationship between the manufacturer’s pricing decision and the retailer’s ordering decision. Interestingly, the retailer’s choice about whether or not to opt for financing is dependent on her/his


Retailer’s best response to the wholesale price under different $K_r$ initial capital as well as on the manufacturer’s price. That is, if the manufacturer charges more, the retailer would order less with her/his own cash.

Next, we perform some computational examples to numerically demonstrate the sensitivity analysis of the impact of the capital constraints on the manufacturer’s optimal price and expected profit, respectively depicted in Figure 7 and Figure 8.

Figure 7 shows the impact of the initial capital on the manufacturer’s pricing decision, which is in accordance with Proposition 7. If $K_r \geq K_r^*$, the equilibrium price remains constant in the zone of no capital constraints. If $K_r < K_r^*$, whether or not the retailer adopting SCF depends on the relationship between $q_m$ and $q_2$. 
According to the preceding analysis, the equilibrium would be \((w_m, q_m)\) only if 
\[ K_r \geq K_r^* = w_m q_m. \] Note that the optimal wholesale price is piecewise continuous
since \(\partial G(\hat{x})/\partial \hat{x}\) is not continuous at \(\hat{x} = k\).

Figure 8 clearly suggests that the manufacturer’s optimal profit in the case of SCF is not higher than that in the case of no capital constraints, since the existence of capital constraints and bankruptcy costs greatly affects the retailer’s profit as well as the manufacturer’s profit. However, if the capital-constrained retailer has no choice of financing, she/he can use only the initial capital to procure goods; i.e., 
\[ q^* (w) = \min \{ q^{N+} (w), K_r / w \}, \] where \(q^{N+} (w) = \bar{F}^{-1} [\theta_1 \Omega] \). A threshold value of \(w\) exists that satisfies \(q^{N+} (w) = K_r / w\), which is equivalent to 
\[ K_r = K_1 (w). \] In this case, for
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\[ w_1 \leq w \leq w_2, \text{ the manufacturer’s profit would be } \pi_m(w) = K_r (1 + R_f) (1 - c/w), \]

which increases with \( w \). This result implies that the manufacturer would charge more if the retailer has less capital, since \( w_2 \) decreases with \( K_r \). Further, Figure 8 shows that the manufacturer’s optimal profit in the case of SCF is higher than that in the case without financing. It shows that although there is non-zero bankruptcy cost, the manufacturer has incentives to support the capital-constrained retailer’s choice of adopting SCF in order to reduce her/his loss of profit to some extent.

6. Conclusions. Not considering the practical imperfections of the capital market (such as bankruptcy cost, transaction cost, tax, and so on) will result in the non-optimality of supply chain financing decisions and operational decisions, thereby restricting its wide implementation. Optimal strategies for supply chain financing to SMEs under an imperfect capital market have increasingly been the focus of international and domestic industries as well as SCM academic research. However, the research so far is not yet mature. In this study, we include the bankruptcy cost in a SCF game and analyze the financing strategies at equilibrium from a multi-attribute utility perspective. We present several analytical results and highlight the challenges and opportunities. Specifically, we construct a multi-attribute decision model for maximizing the expected systematic profits and the service level, and simultaneously minimizing the bankruptcy costs. Finally, we carry out sensitivity analyses and numerical analyses to validate the theoretical outcomes.

Our analyses provide some major results as follows. First, in the capital-constrained supply chain under consideration of MAU, with a suitable combination of decision preferences, the proposed bank financing scheme can realize coordination, even super coordination. Second, in the case of maximizing MAU, the capital-constrained retailer would require more initial capital than when she/he seeks to maximize the expected profit; therefore, the equilibrium order quantity and the bankruptcy risk would also be higher. If the retailer is less constrained, the optimal order quantity would be independent of her/his decision preference for the expected bankruptcy cost. Further, if the decision maker pays more attention to the service level for customers, she/he should place more orders to increase the fill rate. Finally, although both the optimal order and the optimal price decrease with the bankruptcy cost, the manufacturer has incentives to support the capital-constrained retailer’s decision to adopt SCF, which would be more profitable than the case without financing, even when multiple decision attributes are considered.

Our analytical and numerical results have several managerial insights with respect to SCF decision optimization and coordination. First, the multi-attribute decision is a relatively more complicated problem in the SCF context, which requires an appropriate tradeoff among all kinds of decision attributes, including the incentive attribute and the punitive one. A more important insight is that the tradeoff is dependent not only on the operational parameters (such as wholesale price, retail price, and order quantity) but also on the financial parameters (such as interest rate, fixed bankruptcy cost, and various bankruptcy costs). Second, SMEs with capital constraints should seek to customize the optimal SCF decision for themselves according to their preferences for each decision attribute. In other words, the optimal decision is unique and cannot be replicated by others. For example, if the SCF system prefers to mitigate the bankruptcy cost above all else, the retailer should order less to avoid bankruptcy risk. On the other hand, if the SCF system would rather enhance the service level to decrease the stockout rate, more
orders should be placed. The managerial insight is that the decision process should be comprehensive, and various decision factors should be given balanced consideration, without weighing on a single indicator too heavily and ignoring others. Finally, as important SCF partners, the commercial bank and the manufacturer should pay more attention to the borrowers’ decision process, which would have a significant impact on their own wealth. Specifically, if the decision objective is to maximize MAU rather than maximize the expected profit level only, the bank should charge a higher interest rate to prevent from bankruptcy risk. Moreover, a rational manufacturer would set a reasonable price to help the retailer obtain bank loans in order to mitigate the capital gap to improve profitability for both of them.

Thus, our research findings can enrich and improve supply chain finance theories and methods and can provide some theoretical guidance and reference for the healthy improvement of SME supply chain management. However, several research directions remain to be investigated. First, this study considers only the bankruptcy cost as one of the important imperfections of the capital market. How to characterize other imperfections requires further research. How to use other utility functions to formulate the objective function with multiple decision attributes is another topic for future research.

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