Energy Efficient Control of Electric Motors

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Abstract

This paper presents development of an optimal feedback linearization control (OFLC) for interior permanent magnet (PM) synchronous machines operating in a non steady-state operating point, i.e., varying torque and speed, to achieve precision tracking performance and energy saving by minimizing the copper loss. An isomorphism mapping between the dq-axes phase voltages and two auxiliary control inputs over full ranges of torque and speed is established by the linearization controller using the notion of orthogonal projection. The auxiliary control inputs are defined to be exclusively responsible for torque generation and power consumption. Subsequently, an analytical solution for the optimal-linearization control is derived in a closed-form by applying the Hamiltonian of optimal control theory in conjunction with the Pontryagin’s minimum principle. The optimal controller takes the maximum voltage limit and torque tracking constraint into account while maximizing machine efficiency for non-constant operational load torque and speed. Unlike the convectional quadratic regulator-based control of electric motors, the proposed control approach does not rely on steady-state operation conditions and hence it is suitable for such applications as electric vehicles and robotics.

1 Introduction and motivation

The market for motor drives encompasses a wide range of applications areas from consumer appliances and electric cars to industrial robotics and machine tools. Precise and fast torque tracking performances and energy-efficiency over the entire speed/torque range of motors is highly required in many of these applications [1,5]. PM synchronous motors (PMSMs) are often suitable choices for these applications as they exhibit high torque-to-mass ratio, fast dynamic responses, and high power efficiency compared to other types of motors. In fact, the recent surge in demand for high efficient PM motor drives is mainly caused by rapid proliferation of motor drives into the automobile industry. The performance and efficiency of PMSMs are constantly being improved through not only optimal motor design and construction but also by implementation of advanced control methods [2,4,6,7,9] for electric vehicles as well as industrial automation and robotics within the manufacturing sector. Such applications often require motor drives to work in wide speed and torque ranges while maintaining high efficiency [8].

There is a variety of control methods in literature for computer control of electric machines. Minimization of power dissipation has been considered in some of these control approaches for electric machines working in steady-state operation conditions, i.e., constant torque or speed.

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A modification to the conventional vector-controlled drive system for special Synchronous machines is proposed in [10] to minimize copper losses based on a voltage-constraint-tracking in the field-weakening control. However, the control is designed based on steady-state voltage equations of a machine in the rotor reference frame, i.e., time-derivatives of currents are ignored. There are also other direct control possibilities such as feedback linearization control (FLC) [6,11,12] or direct torque control (DTC) [3,13–16]. The DTC schemes have been further developed to minimize copper loss or to defer voltage saturation using flux-weakening control in order to extend the range of operational speed of PMSMs [17–20]. A torque regulator and a flux regulator for PMSMs operating in a wide speed range requiring high torque/power accuracy and a fast dynamic response is proposed in [8]. Linear quadratic regulator (LQR)-based optimal vector control of PMSM in dq-axes reference frame using linear state feedback is reported in [21–24]. Again, these optimal control approaches are designed for torque/speed regulation of PMSMs working in steady-state operating point. It is worth noting that pulse width-modulation inverter nonlinearities affect high-frequency carrier-signal voltage and that motivated researchers to developed a number of techniques to compensate for inverter’s nonlinearities [25–27] or limited bandwidth [28]. It is also known that the motor parameters are also prone to change in time mainly due to temperature changes and that can cause performance-degradation of the control system. On-line identification schemes have been adopted for control of PMSMs to cope with the parametric uncertainty [29–31]. Alternatively, sliding mode observer or sliding model control techniques can be adopted for control of PMSMs to achieve fast response and robustness w.r.t. the effect of the variations of motor parameters [32–34].

Optimal-current determination for multiphase PMSMs in real time are reported in [28,35–39]. These indirect optimal torque controller typically requires current source inverter equipped with a large bandwidth current controller in order to be able to inject currents into the inductive windings without introducing significant phase lag and for smooth torque production. The armature current vector can be controlled using Maximum Torque Per Ampere (MTPA) control technique for maximizing torque for a given magnitude of current vector [40–42]. High-performance current regulator to improve the current responses in high-speed flux-weakening region by a feedforward compensator is developed in [43]. This control strategy has been widely adopted in constant torque operating range to achieve fast transient and high-efficiency operation of PMSM drive systems [10,44]. A control scheme for a PMSM drive that can automatically switch to the MTRA or the field-weakening control modes is proposed in [10] to maintain minimum copper-loss operation. The switching controller relies on an empirical method to properly select the values of a couple of tuning parameters for a stable operation [10]. MTPA control scheme for PM motors considering the temperature dependence of the magnet field and saturation effect is proposed in [45]. These MPTA controllers require current source power converters which amplify the instantaneous phase current commands by means of phase current feedback to provide high-bandwidth closed-loop current regulation [40].

This work applies the Hamiltonian of optimal control theory to achieve optimal feedback linearization control (OFLC) of PM synchronous machines operating with varying speed/torque [9]. A feedback linearization scheme with two distinct control inputs is developed based on notion of orthogonal projection. The stationary dq-axes phase voltages are related to two auxiliary control inputs through an isomorphism mapping in such a way that the control inputs are exclusively responsible for torque generation and energy minimization. Subsequently, an optimal controller is developed based on the maximum principle formulation for minimize power
consumption subject to accurate torque production and voltage limits. The important features of the optimal feedback linearization scheme is that it works for time-varying torque or variable-speed drive applications.

2 Generalized linearization model of synchronous Motors

The voltage equations of synchronous motors with salient-pole can be written in the $d$, $q$ reference frame by

$$L_d \frac{di_d}{dt} = -Ri_d + L_q i_q \omega + v_d \tag{1a}$$

$$L_q \frac{di_q}{dt} = -Ri_q + L_d i_d \omega - \psi \omega + v_q \tag{1b}$$

where $L_q$ and $L_d$ are the $q$- and $d$-axis inductances, $i_q$, $i_d$, $v_q$, and $v_d$ are the $q$- and $d$-axis currents and voltages, respectively, $\psi$ is the motor back EMF constant, and $\omega$ is motor speed \[46–48\]. The equation of motor torque, $\tau$, can be described by

$$\tau = \frac{3}{2} p (\psi i_q + (L_d - L_q)i_d i_q), \tag{2}$$

where $p$ is the number of pole pairs \[47\]. Suppose

$$\eta = \frac{L_q}{L_d} - 1 \quad \text{and} \quad \mu = \frac{L_q}{R} \tag{3}$$

are defined as the inductance ratio and the machine time-constant, respectively, and vector $i = [i_d \ i_q]^T$ contains the stationary dq-axes phase currents. Then, the time-derivative of (2) takes the form

$$\dot{\tau} = \frac{3}{2} p (\psi \dot{i}_q + (L_d - L_q)(i_d \dot{i}_q + i_d \dot{i}_q)), \tag{4}$$

After substituting the time-derivative of the dq-axes phase currents from (1) into (4) and rearranging the resulting equation, we arrive at

$$\tau + \mu \dot{\tau} = b^T(i) \psi + \phi(i, \omega) \tag{5}$$

where

$$b(i) = [b_d \ b_q]^T,$$

$$b_d(i) = -\frac{3p}{2R} \eta L_d i_q,$$

$$b_q(i) = \frac{3p}{2R} (\psi - \eta L_d i_d),$$

$$\phi(i, \omega) = \frac{3p}{2} \frac{\omega}{R} (L_q i_d \psi - \eta L_q^2 i_q^2 - \eta L_d^2 i_d^2 - \psi^2) + \frac{3p}{2} \eta L_q i_d i_q.$$
Notice that the motor phase currents $i_a, i_b,$ and $i_c$ are related to stationary dq-axes phase currents by

$$
\begin{bmatrix}
i_d \\
i_q
\end{bmatrix} = K(\theta) \begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix}
$$

(6)

where $\theta$ is the mechanical angle and

$$
K(\theta) = \frac{2}{3} \begin{bmatrix}
\cos(p\theta) & \cos(p\theta - \frac{2\pi}{3}) & \cos(p\theta + \frac{2\pi}{3}) \\
\sin(p\theta) & \sin(p\theta - \frac{2\pi}{3}) & \sin(p\theta + \frac{2\pi}{3})
\end{bmatrix}
$$

is the Park-Clarke transformation.

Now, assume that the power converter is a voltage source inverter in order to set the stator voltage. Suppose vector $v = [v_d v_q]^T$ represents the stationary dq-axes phase voltages. Then, by virtue of the dynamics model of motor torque (5), we propose the following linearization control law in terms of control inputs $u$ and $z$

$$
v = \frac{b}{\|b\|^2} (u - \phi) + z,
$$

(7)

where scaler $u$ can take any value but $z$ is specifically defined to satisfy the following constraint

$$
b^T z = 0.
$$

(8)

One can readily verify that substitution of the voltage control law (7) into the time-derivative of motor torque in (5) yields the following first-order linear system

$$
\tau + \mu \dot{\tau} = b^T \frac{b}{\|b\|^2} (u - \phi) + b^T z + \phi = u
$$

(9)

It is apparent from (9) that input $z$ does affect the motor torque generation and hence control input $u$ is exclusively responsible for the torque production. Consequently, we treat $u$ and $z$ as the torque control input and energy minimizer control input, respectively. Equation (7) can be interpreted as a transformation from the auxiliary inputs $u$ and $z$ to the dq-axes phase voltages. Control input $u$ constitutes the reference torque, i.e., $\tau + \mu \dot{\tau} = u$. However, as will be shown in the following section, the other control input $z$ can be utilized to minimize power dissipation due to the copper loss.

### 3 Optimal control to minimize energy consumption

#### 3.1 Optimal control

By substituting the linearization control law (7) into the machine voltage equations (1), we arrive at the following time-varying linear system describing the dynamics of the currents in response to the control inputs $u$ and $z$

$$
\frac{db}{dt} = L^{-1} \left( \frac{b}{\|b\|^2} (u(t) - \phi) + h + z \right),
$$

(10)

$$
= f(i, t, z)
$$

(11)
where $L = \text{diag}\{L_d, L_q\}$ and vector $h$ is defined as

$$h(i, \omega) = \begin{bmatrix} -Ri_d + L_qi_q \omega \\ -Ri_q + L_d i_d \omega - \psi \omega \end{bmatrix}$$

The cost function to minimize is power dissipation $J = \|i\|^2$ over interval $h$, i.e.,

$$\int_t^T \|i(\zeta)\|^2 d\zeta$$

(12)

where $T = t + h$ is the terminal time of the system. Then, the Hamiltonian function can be constructed from (10) and (12) as

$$H = J + \lambda^T f = \|i\|^2 + \lambda^T \frac{di}{dt} = \|i\|^2 + \lambda^T L^{-1} \left( \frac{b}{\|b\|^2}(u - \phi) + h + z \right),$$

(13)

where $\lambda \in \mathbb{R}^2$ is the costate vector. The optimality condition stipulates that the time-derivative of costate satisfies

$$\dot{\lambda} = -\frac{\partial H}{\partial i},$$

(14)

and the transversally condition dictates

$$\lambda(T) = 0$$

(15)

Therefore, using using the Hamiltonian expression from (13) in (14), one can show that evolution of the system costate is governed by the following time-varying differential equation

$$\frac{d}{dt} \lambda = A^T \lambda - 2i$$

(16)

where

$$A = L^{-1} \left( - (u - \phi) \frac{\partial}{\partial i} \frac{b}{\|b\|^2} + \frac{b}{\|b\|^2} \frac{\partial \phi^T}{\partial i} - \frac{\partial h}{\partial i} \right) = (u - \phi) \Lambda + \Gamma$$

and

$$\Lambda = \frac{3p \eta L_d}{2R \|b\|^4} L^{-1} \left[ \begin{array}{ccc} 2b_q b_d - b_q^2 & b_q^2 & b_d^2 \\ b_q^2 & 2b_q b_d - b_d^2 & b_q^2 \\ b_d^2 & b_q^2 & 2b_q b_d \end{array} \right]$$

$$\Gamma = L^{-1} \left( \frac{b}{\|b\|^2} \frac{\partial \phi^T}{\partial i} - \frac{\partial h}{\partial i} \right)$$

$$\frac{\partial \phi}{\partial i} = \frac{3p}{2} \left[ \mu \omega \psi + \eta L_q i_q - 2R \eta L_d^2 i_d \right]$$

$$\frac{\partial h}{\partial i} = \left[ \begin{array}{ccc} -R & L_q \omega \\ L_d \omega & -R \end{array} \right]$$
Dynamics equation (16) can be used as an observer to estimate the costate $\lambda$. We can write the equivalent discrete-time model of the continuous system (16) as

$$\frac{1}{h}(\lambda_{k+1} - \lambda_k) = A_k^T \lambda_k - 2i_k$$

(17)

The boundary condition can be inferred from (15) as $\lambda_{k+1} = 0$. Using the latter identity in the above equation, we get

$$\lambda_k = 2(\frac{1}{h}I + A_k^T)^{-1}i_k$$

(18)

Notice that computation of the costate from (18) does not involve its time-history. Therefore, for the sake of notational simplicity, we will drop the $k$ subscript of the variables in the following analysis without causing ambiguity.

Moreover, according to the Pontryagin’s minimum principle, the optimal control input minimizes the Hamiltonian over the set of all permissible controls and over optimal trajectories of the state $i^*$ and costate $\lambda^*$, i.e.,

$$z = \arg\min_{z \in Z} H(i^*, \lambda^*, z)$$

(19)

where

$$Z = \{z : z \perp b\}$$

(20)

is the set of admissible control inputs. Recall that the constraint in (20) ensures that the energy minimizer control input $z$ does not contribute to the motor torque generation. It can be inferred from the expression of Hamiltonian (13) and identity (8) that (19) is tantamount to minimizing $\lambda^T L^{-1}z$ subject to the equality and inequality constraints of admissible $b^T z = 0$. Therefore, the problem of finding optimal permissible $z$ can be transcribed to the following constrained linear programming

$$\begin{align*}
\text{minimum} & \quad \lambda^T L^{-1}z \\
\text{subject to} & \quad b^T z = 0
\end{align*}$$

(21)

Let us define projection matrix

$$B = I - \frac{bb^T}{\|b\|^2},$$

(22)

which projects vector from $\mathbb{R}^2$ to a vector space perpendicular to $b$, i.e., $Bb = 0$. Then, it can be inferred from (21) and (22) that optimal $z$ should be aligned with vector $BL^{-1}\lambda$ in opposite direction, i.e.,

$$z = -\gamma BL^{-1}\lambda$$

(23)

and $\gamma > 0$ is a positive scaler. Finally, the energy minimizer control command is obtained from (23) and (18).

### 3.2 Voltage saturation limit

A power converter has a maximum voltage limit, which is the bus voltage. The variable $\gamma$ in (23) should take a large value in order to minimize energy consumption. However, a very large
\[ \gamma \text{ tends to increase the inverter voltage towards the saturation limit. Therefore, the value of } \gamma \text{ should be selected large as much as possible as long as the voltage vector does not reach its saturation limit, i.e.,} \]
\[ \|v\| \leq v_{\text{max}} \quad (24) \]

where \( v_{\text{max}} \) is the maximum phase voltage magnitude [10]. From (7), we can say
\[ \|v\|^2 = \frac{(u - \phi)^2}{\|b\|^2} + \|z\|^2 \quad (25) \]

In view of (25) and (24), the maximum allowable magnitude of control input \( z \) is
\[ \|z\| \leq z_{\text{max}} \quad (26) \]

where the limits of the auxiliary control input \( z \) is
\[ z_{\text{max}} = \sqrt{v_{\text{max}}^2 - \frac{(u - \phi)^2}{\|b\|^2}} \quad (27) \]

In other words, the set of admissible control input (21) in the presence of voltage saturation limit becomes
\[ Z = \{ z : z \perp b \land \|z\| \leq z_{\text{max}} \} \quad (28) \]

By virtue of (23), (27), and (26), we can conclude the optimal value of variable \( \gamma \) minimizing power losses given the voltage limit \( v_{\text{max}} \) to be
\[ \gamma = \frac{\sqrt{v_{\text{max}}^2 - (u - \phi)^2/\|b\|^2}}{\|BL^{-1}\lambda\|} \quad (29) \]

### 3.3 Torque limit

It should be pointed out that the expression under the square-root in (27) must be positive to ensure real-valued solution for the control input \( z \). That requires
\[ v_{\text{max}}\|b\| \geq |u - \phi|. \]

Therefore, the value of the torque command should be within the following bands
\[ u_{\text{min}} \leq u \leq u_{\text{max}}, \quad (30) \]

where \( u_{\text{min}} = \phi - \|b\|v_{\text{max}} \) and \( u_{\text{max}} = \phi + \|b\|v_{\text{max}} \) are the lower- and upper-bounds. Therefore, the torque control input \( u \) can be modified according to the following to ensure feasible solution
\[
\begin{cases} 
  u_{\text{max}} & \text{if } u > u_{\text{max}} \\
  u & \text{if } u_{\text{min}} \leq u \leq u_{\text{max}} \\
  u_{\text{min}} & \text{if } u < u_{\text{min}} 
\end{cases} \quad (31)
\]

Now with \( u \) and \( z \) in hand, one may use (7) to calculate stationary dq-axes phase voltages. Finally, the inverter phase voltages can be obtained from the inverse Park-Clarke transform, i.e.,
\[
\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = K^{-1}(\theta) \begin{bmatrix} v_d \\ v_q \end{bmatrix} \quad (32)
\]
where

\[
K^{-1}(\theta) = \begin{bmatrix}
\cos(p\theta) & \sin(p\theta) \\
\cos(p\theta - \frac{2\pi}{3}) & \sin(p\theta - \frac{2\pi}{3}) \\
\cos(p\theta + \frac{2\pi}{3}) & \sin(p\theta + \frac{2\pi}{3})
\end{bmatrix}.
\] (33)

### 3.4 Torque bandwidth

The linearization control voltage vector \( v \) is computed based on auxiliary input \( u(t) \) and the optimizing control \( z \). Upon applying the optimal feedback linearization control to the motor, the input/output of the closed-loop system in the Laplace domain is simply given by

\[
\frac{\tau(s)}{u(s)} = \frac{1}{\mu s + 1}
\] (34)

where \( s \) is the Laplace variable and \( \mu \) is the machine time-constant as previously defined. Since the linearized system (34) is strictly stable, the proposed feedback linearization control scheme is inherently robust without recurring to external torque feedback loop. Nevertheless in order to increase the bandwidth of the linearized system, one may consider a PI feedback loop closed around the linearized system, where \( \tau^* \) is the desired input torque and \( \tau(i) \) is the motor torque estimated from d-q currents according to (2).

In summary, the control algorithm may proceed with the following steps:

i. Acquire data pertaining to shaft position and speed, and the phase currents from sensors. Then, compute d-q currents from Park-Clarke transform (6).

ii. Estimate motor torque according to (2) and then obtain \( u \) from the PI feedback loop.

iii. Given torque control input \( u \) and maximum voltage limit \( v_{\text{max}} \), limit the magnitude of the command according to (31).

iv. Use (18) to estimate the costate vector \( \lambda \).

v. Compute the energy minimizer control input \( z \) from (23) and (29).

vi. With \( u \) and \( z \) in hand, use the linearization transformation (7) to obtain the stationary d-q-axes phase voltages. Then, compute the inverter phase voltages from the inverse Park-Clarke transform (33).

### 4 Conclusions

The Hamiltonian of optimal control theory in conjunction with the Pontryagin’s minimum principle have been rigorously applied to derive an optimal feedback-linearization control scheme for energy-efficient and accurate torque control of PMSMs subject to time-varying operational speed/torque and voltage limit. Analytical solution for the optimal control problem has been found based on the maximum principle formulation for convenience of real-time implementation. The important feature of the optimal controller was that it could admit non-constant reference torques (or velocity) as oppose to the conventional regulation control approaches requiring the motor to operate at constant torque or velocity (regulation). The optimal controller
could achieve accurate torque tracking with minimize power consumption given inverter voltage limit.

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