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M. Defurne,1,* M. Mazouz,2 Z. Ahmed,3 H. Albataineh,4 K. Allada,5 K. A. Aniol,6 V. Bellini,7 M. Benali,8 W. Boeglin,9 P. Bertin,8,10 M. Brossard,8 A. Camsonne,10 M. Canan,11 S. Chandavar,12 C. Chen,13 J.-P. Chen,19 C.W. de Jager,10,1 R. de Leoo,14 C. Desnault,15 A. Deur,10 L. El Fassi,16 R. Ent,10 D. Flay,17 M. Friend,18 E. Fuchey,8 S. Frullani,19 F. Garibaldi,19 D. Gaskell,10 A. Giusa,7 O. Giamadzlin,20 S. Golge,21 J. Gomeez,10 O. Hansen,10 D. Higinbotham,10 T. Holmstrom,22 T. Horn,23 J. Huang,5 M. Huang,24 G.M. Huber,25 C.E. Hyde,11,8 S. Iqbal,6 F. Itard,8 Ho. Kang,26 Hy. Kang,26 A. Kelleher,27 C. Keppel,10 S. Koizala,11 I. Korover,28 J.J. LeRose,10 R. Lindgren,29 E. Long,30 M. Magne,8 J. Mammei,31 D.J. Margaziotis,6 P. Markowitz,9 A. Marti Jiménez-Argüello,32,15 F. Meddi,19 D. Meekins,10 R. Michaels,10 M. Mihovilovic,33 N. Muangma,5 C. Muñoz Camacho,8,15 P. Nadel-Turonski,10 N. Nuruzzaman,13 R. Paremuzyan,15 A. Puckett,34 V. Punjabi,35 Y. Qiang,10 A. Rahman,3 M.N.H. Rashad,11 S. Riordan,29 J. Roche,12 G. Russo,7 F. Sabatié,1 K. Saenboonruang,29,37 A. Saha,10,1 B. Sawatzky,10,17 L. Selvy,30 A. Shahinyan,33 S. Sirca,33 P. Solvignon,10,1 M.L. Sperduto,7 R. Subedi,39 V. Sulkosky,5 C. Sutera,7 W.A. Tobias,29 G.M. Urciuoli,40 D. Wang,29 B. Wojtsekhowski,10 H. Yao,17 Z. Ye,29 L. Zana,3 X. Zhan,41 J. Zhang,10 B. Zhao,27 Z. Zhao,29 X. Zheng,29 and P. Zhu29

(The Jefferson Lab Hall A Collaboration)

1Irifu, CEA, Université Paris-Saclay, 91191 Gif-sur-Yvette, France
2Faculté des sciences de Monastir, 5000 Tunisia
3Syracuse University, Syracuse, New York 13244, USA
4Texas A&M University-Kingsville, Kingsville, Texas 78363, USA
5Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
6California State University, Los Angeles, Los Angeles, California 90032, USA
7INFN/Sezione di Catania, 95125 Catania, Italy
8Clermont université, Université Blaise Pascal, CNRS/IN2P3, Laboratoire de physique corpusculaire, FR-63000 Clermont-Ferrand, France
9Florida International University, Miami, Florida 33199, USA
10Thomas Jefferson National Accelerator Facility, Newport News, Virginia 23606, USA
11Old Dominion University, Norfolk, Virginia 23529, USA
12Ohio University, Athens, Ohio 45701, USA
13Hampton University, Hampton, Virginia 23668, USA
14Università di Bari, 70121 Bari, Italy
15Institut de Physique Nucléaire CNRS-IN2P3, Orsay, France
16Rutgers, The State University of New Jersey, Piscataway, New Jersey 08854, USA
17Temple University, Philadelphia, Pennsylvania 19122, USA
18Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA
19INFN/Sezione Sanità, 00161 Roma, Italy
20Kharkov Institute of Physics and Technology, Kharkiv 61108, Ukraine
21North Carolina Central University, Durham, North Carolina 27701, USA
22Longwood University, Farmville, Virginia 23909, USA
23The Catholic University of America, Washington, DC 20064, USA
24Duke University, Durham, North Carolina 27708, USA
25University of Regina, Regina, Saskatchewan S4S 0A2, Canada
26Seoul National University, Seoul, South Korea
27College of William and Mary, Williamsburg, Virginia 23187, USA
28Tel Aviv University, Tel Aviv 69978, Israel
29University of Virginia, Charlottesville, Virginia 22904, USA
30Kent State University, Kent, Ohio 44242, USA
31University of Massachusetts, Amherst, Massachusetts 01003, USA
32Facultad de Física, Universidad de Valencia, Valencia, Spain
33University of Ljubljana, 1000 Ljubljana, Slovenia
34Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA
35Norfolk State University, Norfolk, Virginia 23529, USA
36Stony Brook University, Stony Brook, New York 11794, USA
37Kasetsart University, Chatuchak, Bangkok, 10900, Thailand
38Yerevan Physics Institute, Yerevan 375036, Armenia
39George Washington University, Washington, DC 20052, USA
40INFN/Sezione di Roma, 00185 Roma, Italy
41Argonne National Laboratory, Lemont, Illinois 60439, USA
We present deeply virtual $\pi^0$ electroproduction cross-section measurements at $x_B=0.36$ and three different $Q^2$-values ranging from 1.5 to 2 GeV$^2$, obtained from Jefferson Lab Hall A experiment E07-007. The Rosenbluth technique was used to separate the longitudinal and transverse responses. Results demonstrate that the cross section is dominated by its transverse component, and thus is far from the asymptotic limit predicted by perturbative Quantum Chromodynamics (QCD). Nonetheless, an indication of a non-zero longitudinal contribution is provided by the measured interference term $\sigma_{LT}$. Results are compared with several models based on the leading twist approach of Generalized Parton Distributions (GPDs). In particular, a fair agreement is obtained with models in which the scattering amplitude includes convolution terms of chiral-odd (transversity) GPDs of the nucleon with the twist-3 pion distribution amplitude. This experiment, together with previous extensive unseparated measurements provide strong support to the exciting idea that transversity GPDs can be accessed via neutral pion electroproduction in the high $Q^2$ regime.

Deep exclusive reactions have been the subject of intense experimental and theoretical work in the last decades, as they provide clean probes of the internal three-dimensional structure of hadrons. We present here measurements of the differential cross section for the forward exclusive electroproduction reaction $ep \to ep\pi^0$. These results are the first separation of the differential cross section for longitudinally and transversely polarized virtual photons of exclusive $\pi^0$-electroproduction in the electron scattering kinematics of Deep Inelastic Scattering (DIS). A diagram of this process, including definitions of the kinematic variables, is presented in Fig. 1.

The Quantum Chromodynamics (QCD) factorization theorems predict that deep virtual meson production should be dominated by the longitudinal virtual photon-production cross section [1]. In the Bjorken limit $Q^2 \to \infty$ and $t/Q^2 \ll 1$ at fixed $x_B$, the longitudinal scattering amplitude factorizes into a hard perturbative contribution, the leading twist Generalized Parton Distributions (GPDs) of the nucleon and the pion distribution amplitude (DA) [1–3]. GPDs describe the three-dimensional structure of hadrons by correlating the internal transverse position of partons to their longitudinal momentum [4–6]. In the case of a nucleon and at leading twist, four chiral-even GPDs conserve the helicity of the parton, whereas four chiral-odd GPDs, also referred to as transversity GPDs, flip its helicity. While a rigorous factorization proof has not been established for the transverse virtual photo-production amplitude, it is proven to be suppressed by a factor of $1/Q$ with respect to its longitudinal counterpart [1].

The leading-twist approximation is in good agreement with high $Q^2$ electroproduction data for photon [7–9] and vector meson production [10, 11]. On the other hand, the co-linear approximation underestimates by about one order of magnitude the total $\pi^0$ electroproduction cross sections measured at $Q^2 \simeq 2$ GeV$^2$ by the Hall A [12] and CLAS [13? ] collaborations at Jefferson Lab (JLab). It was suggested in [14, 21] that for neutral meson production the twist-3 quark-helicity flip DAs coupled with the transversity GPDs of the proton would create a large cross section for transversely polarized virtual photons, without violating the QCD factorization theorem. These calculations are in good agreement (within model uncertainties) with the unseparated cross sections of [13? ]. The present study is undertaken to verify whether or not the separated cross sections for longitudinally and transversely polarized virtual photons can be accurately described by a formalism based on leading twist GPDs.

The Deeply Virtual Meson Production (DVMP) cross section can be written in the following form [15]:

$$
\frac{d^4\sigma}{dQ^2dx_Bdtd\phi} = \frac{1}{2\pi} \frac{d\Gamma}{dx_BQ^2} (Q^2, x_B, E) \left[ \frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt} \right] + \sqrt{2\epsilon(1+\epsilon)} \frac{d\sigma_T}{dt} \cos \phi + \epsilon \frac{d\sigma_{TT}}{dt} \cos 2\phi ,
$$

where $E$ is the incident lepton energy in the target rest frame and $\phi$ the angle between the leptonic and hadronic plane defined according to the Trento convention [16].

The factor $\frac{d\Gamma}{dx_BQ^2} (Q^2, x_B, E)$ is the virtual photon flux and $\epsilon$ is the degree of longitudinal polarization defined as $\epsilon = [\sqrt{q \cdot P}/|k \cdot P|]$:

$$
\frac{d\Gamma}{dx_BQ^2} (Q^2, x_B, E) = \frac{\alpha}{8\pi} Q^2 \frac{1 - x_B}{x_B} \frac{1}{1 - \epsilon} ,
$$

$$
\epsilon = \frac{1 - y - Q^2/4E^2}{1 - y + y^2/2 + Q^2/(4E^2)} ,
$$

$M$ being the proton mass.

![FIG. 1. Diagram of the exclusive $\pi^0$ electroproduction reaction, identified by the $\pi^0 \to \gamma\gamma$ decay mode. The value of $t$ with minimal $|t|$ can be evaluated as $t_{\text{min}} = (Q^2 + m_{\pi^0}^2)/(4W^2) = (|q_{CM}^e| - |q_{CM}^\gamma|)^2$, with $|q_{CM}^e|$ and $|q_{CM}^\gamma|$ the norms of $\vec{q}, \vec{q}'$ in the $p^\pi^0$ final state center-of-mass frame.](image)
Experiment E07-007 ran in JLab Hall A from October to December 2010. One of its goals was to separate the exclusive transverse and longitudinal \( \pi^0 \) electroproduction cross sections using the Rosenbluth technique, consisting of measurements at two different values of the incident electron energy at each setting. Table I lists the three \( Q^2 \) settings measured, each of them at two different values of \( \epsilon \).

The electron beam was incident on a 15-cm-long liquid \( \text{H}_2 \) target, for a typical luminosity of \( 2 \times 10^{37} \text{cm}^{-2}\text{s}^{-1} \). Scattered electrons were detected in a high resolution spectrometer (HRS), with \( 10^{-4} \) momentum resolution and better than 2 mr horizontal angular resolution [17]. The two photons of the \( \pi^0 \) decays were detected in a PbF\(_2\) electromagnetic calorimeter consisting of a \( 13 \times 16 \) array of \( 3 \times 3 \times 18.6 \text{cm}^3 \) crystals, coupled to mesh-dynode photomultipliers. Each calorimeter channel was continuously sampled by a 1 GHz flash ADC system that recorded the signal over 128 ns. The high resolution in electron kinematics accurately determined the event-by-event \((Q^2, x_B)\) values. The fast Cherenkov signal from the calorimeter allowed a coincident time resolution between the electron and \( \pi^0 \) detections of 0.6 ns. The vertex resolution of the HRS and position resolution of the calorimeter accurately determined the \( \pi^0 \) direction and thus the kinematical variables \( t \) and \( \phi \). The measured energy in the calorimeter is used to identify \( \pi^0 \) events through the 2-photon invariant mass \( m_{\gamma\gamma} = \sqrt{(q_1 + q_2)^2} \) and to ensure the exclusivity of the reaction using the \( ep \rightarrow e\gamma\gamma X \) missing mass squared \( M_{ep\rightarrow e\gamma\gamma X}^2 \).

The calibration of the calorimeter was done in two steps. Firstly, we used elastic scattering \( H(e, e\gamma) \) events. This calibration required dedicated runs, since the polarity of the HRS had to be reversed to allow proton detection. We performed elastic calibrations at the beginning, middle and end of the experiment. A resolution of 3.1% at 3.16 GeV was measured, with a position resolution of 3 mm at 110 cm from the target. Between elastic calibrations, channel gains were observed to drift up to 10%. We attributed these changes to radiation damage of the PbF\(_2\) crystals. In order to correct for the calibration coefficient drifts between the elastic run periods we used exclusive \( \pi^0 \) data from our \( H(e, e\gamma\gamma) \) sample. By assuming \( M_{ep\rightarrow e\gamma\gamma X}^2 = M^2 \) and \( m_{\gamma\gamma} = m_{\pi^0} \), the sum of the energies of the two decay photons was determined and used to compute the calibration coefficients. The combination of both elastic and exclusive \( \pi^0 \) calibrations provided a continuous invariant mass resolution of 9.5 MeV through the full run period.

The data acquisition was triggered by an electron detection signal in the HRS, formed by the coincidence of the gas Cherenkov detector and the plastic scintillator plane S2m of the HRS [17]. In order to select neutral pions, we studied 2-cluster events in the calorimeter with an energy deposit larger than 500 MeV in each cluster and within 3 ns of the electron detection. To account for the natural correlation between the measured \( M_{ep\rightarrow e\gamma\gamma X}^2 \) and \( m_{\gamma\gamma} \) values, we define a corrected missing mass squared:

\[
M_X^2 = M_{ep\rightarrow e\gamma\gamma X}^2 + C \times (m_{\gamma\gamma} - m_{\pi^0}) \tag{4}
\]

with the empirical value \( C = 12 \text{ GeV} \). Fig. 2 shows the \( M_X^2 \) distribution of the \( H(e, e\gamma\gamma) \) events. Exclusive events are selected by requiring \( 100 < m_{\gamma\gamma} < 170 \text{ MeV} \) and \( M_X^2 < 0.95 \text{ GeV}^2 \). Contamination from the diffractive \( ep \rightarrow ep\omega \) reaction with a 8.5% branching-ratio \( \omega \rightarrow \pi^0\gamma \) decay was estimated to be smaller than 0.3% within our acceptance and cuts based on the measurements performed in [?]. The inclusive yield obtained by subtracting the simulation from the data is below 2% for \( M_X^2 < 0.95 \text{ GeV}^2 \) and treated as a point-to-point systematic uncertainty. The number of accidental \( H(e, e\gamma\gamma) \) triple coincidences is estimated by measuring the number of 2-photon events detected in the calorimeter for each of the three possible timings with respect to the scattered electron: one photon in-time and one out-of-time, both out-of-time but in-time between themselves, and both out-of-time with the electron and with each other. Finally, an analysis of 3-cluster events was performed in order to correct for the fraction of exclusive \( \pi^0 \) events where one of the 3 clusters was an accidental photon coincidence. This correction was applied bin-by-bin and found to be 5% on average.

The different terms of the unpolarized \( \pi^0 \) cross section are extracted by minimizing the following \( \chi^2 \) defined between the number of experimental and simulated events:

\[
\chi^2 = \sum_{i=0}^{N} \left( \frac{N_i^{\text{exp}} - N_i^{\text{sim}}}{\sigma_i^{\text{exp}}} \right)^2 \tag{5}
\]

where the sum runs over all experimental bins of one \( Q^2 \) setting, including data at two different values of \( \epsilon \). The variable \( N_i^{\text{exp}} \) is the number of events in the experimental bin \( i \), with \( \sigma_i^{\text{exp}} \) being its corresponding uncertainty. The number of simulated events \( N_i^{\text{sim}} \) is given by:

\[
N_i^{\text{sim}} = \mathcal{L} \int \frac{d\sigma}{dtdQ^2dx_Bd\phi} \int dQ^2dx_Bd\phi \tag{6}
\]

with \( \mathcal{L} \) the experimental integrated luminosity, corrected by the data acquisition dead-time. The integration is.
FIG. 2. (Color online) Distribution of the $H(e, e'\gamma)X$ events within cuts for Kin3 at $E_{beam}^e = 5.55$ GeV and $t_{min} - t < 0.15$ GeV$^2$. The Monte-Carlo (MC) simulation is represented by the open crosses, whereas the triangles show the estimated inclusive yield obtained by subtracting the simulation from the data. The vertical dotted line illustrates the $M_X < 0.95$ GeV$^2$ cut applied in the analysis.

![Image](image-url)

FIG. 3. $2\pi \frac{d^2\sigma}{dt d\phi}$ for $Q^2=1.5$ (triangles), 1.75 (squares) and 2 GeV$^2$ (circles) at $x_B=0.36$ and $t_{min} - t=0.025$ GeV$^2$. The cross sections extracted at low/high $\epsilon$ are shown in open/filled symbols (and dashed/solid lines).

TABLE II. $Q$–dependence obtained from the fit of the $t$–integrated responses, with statistical and systematics uncertainties added in quadrature, by the function $A/Q^{n_{theo}}$. The QCD asymptotic limit of each term is $\sim Q^{-n_{theo}}$.

| Term          | $n_{exp}$ | $n_{theo}$ |
|---------------|-----------|------------|
| $d\sigma_T/dt$ | 9 ± 2     | 8          |
| $d\sigma_{TT}/dt$ | 4 ± 2     | 8          |
| $d\sigma_{TL}/dt$ | 26 ± 5    | 7          |

performed with a MC simulation, convoluting the known kinematical dependences of the cross section with the experimental acceptance. We limit the analysis to the overlapping $(Q^2,x_B)$–phase space between the two beam–energy settings. After minimization of Eq. (5), the unknown $Q^2$–dependences of $d\sigma_T/dt, d\sigma_L/dt, d\sigma_{TT}/dt$ and $d\sigma_{TL}/dt$ are fitted to the results and included into the MC simulation in order to account for the leading variations of the cross section within bins. A second $\chi^2$ minimization is performed, which provides stable results over further iterations and yield the final results we present herein, with $\chi^2/dof=76/60$, 83/80 and 61/60 respectively for $Q^2=1.5$, 1.75 and 2.0 GeV$^2$. No $Q^2$–dependence is included for $d\sigma_T/dt$ as results are found compatible with zero in all experimental bins. Tab. II shows the $Q$–dependences obtained. The small HRS acceptance does not allow for an $x_B$–dependence study.

The Monte-Carlo (MC) simulation is based on the GEANT4 toolkit. It includes radiative corrections following the procedure described in [8] based on calculations by Vanderhaeghen et al. [18]. A comparison with the radiative calculations of [?] at our central kinematics showed agreement within 2%. The HRS acceptance is modeled by an R-function that defines the distance of the particle from the HRS acceptance bound [19]. Our cut on $M_X^\gamma$ to ensure exclusivity removes a significant fraction of exclusive $\pi^0$ events. This is compensated by applying an identical cut on the simulated data. For this to be accurate, the experimental and MC simulated $M_X^\gamma$ (and $n_{\gamma\gamma}$) distributions should have exactly the same widths and positions. These parameters are dominated by the calibration and resolution of the electromagnetic calorimeter crystals. Thus, great care was taken to locally reproduce the calorimeter energy and position resolutions in the MC simulation. While the number of $\pi^0$ events removed by the $M_X^\gamma$ cut depends on $\phi$ and $t$, its systematic uncertainty was found almost independent of the kinematics, with a value of 2% estimated by varying the applied cut. In order to propagate this point-to-point uncertainty to the extraction of the four structure functions, we added it in quadrature to the statistical uncertainty when computing the $\sigma_i^{exp}$ of each bin in Eq. (5).

Tab. III lists the different sources of correlated systematic uncertainties. A check of our global normalization was made by extracting the DIS cross section in each of our kinematic settings. Results agree within the uncertainty listed in Tab. III with the parametrization of the DIS cross section in [20].

Fig. 3 presents the electroproduction cross section $2\pi \frac{d^2\sigma}{dt d\phi}$ for the three different $Q^2$–values and the lowest $t' = t_{min} - t$ bin, as a function of $\phi$. The cross section is almost independent of $\epsilon$, indicating that most of the signal is coming from its transversely polarized component.

The uncertainties of the Rosenbluth separated $\frac{d\sigma}{dt}$ and
preventing collinear factorization in the case of transversely polarized virtual photons are regularized by the transverse momentum $k_t$ of the quarks and antiquarks making up the meson. These models are in good agreement with our results for both $d\sigma_T/dt$ and $d\sigma_L/dt$ within the experimental uncertainties. However, they predict the opposite sign for $d\sigma_T/dt$ and do not reproduce the $Q^2$-dependence of the interference terms listed in Tab. II, especially for $\sigma_T/dt$.

In conclusion, we have performed the L/T separation of $\pi^0$ electroproduction cross section for $Q^2 = 1.5, 1.75$ and $2.0$ GeV$^2$ at $x_B = 0.36$. $d\sigma_T/dt$, though compatible with zero, is also consistent with the leading-twist predictions of a model of the chiral even GPDs [2]. We observe fair agreement (particularly at our largest $Q^2$ kinematic) between these results and two models incorporating transversity-GPD. This supports the prediction of a chirally enhanced helicity-flip pion distribution amplitude [14, 21] and the exciting possibility of accessing transversity GPDs of the nucleon through exclusive $\pi^0$ electroproduction for $Q^2 \geq 1.5$ GeV$^2$.

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* Corresponding author: maxime.defurne@cea.fr
† Deceased

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![FIG. 4. (Color online) $d\sigma_T$ (full circles), $d\sigma_L$ (open circles), $d\sigma_{TL}$ (triangles) and $d\sigma_{TT}$ (squares) as a function of $t_{min} - t$ for $Q^2 = 1.5$ (left), 1.75 (center) and 2 GeV$^2$ (right) at $x_B = 0.36$. The full lines are predictions from [21] and the long-dashed lines from [22]. The short-dashed line show the VGG model [2] for $d\sigma_L$. Filled boxes around the points show normalization systematic uncertainties; for $d\sigma_L$ and $d\sigma_T$ these uncertainties are strongly anti-correlated. Previous unseparated measurements ($\sigma_{UL} = \sigma_T + \sigma_{UL}$) at similar, but not equal, kinematics are also shown and described in the text. ](image)

| Systematic uncertainty | Value |
|------------------------|-------|
| HRS acceptance cut     | 1%    |
| Gas Cherenkov detector efficiency | 0.5% |
| HRS tracking efficiency | 0.5% |
| $\pi^0$ detection efficiency | 0.5% |
| Radiative corrections  | 2%    |
| Deadtime and luminosity | 2%   |

Total 3.12%
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