A simple method for theoretical determination of the radius- and time-dependent electron temperatures in nanosecond pulsed longitudinal discharges in helium and neon assuming a bi-Maxwellian electron energy distribution function

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Abstract. Assuming a bi-Maxwellian electron energy distribution function, the temporal and radial distribution of the electron temperature is determined in nanosecond pulsed longitudinal discharges used for excitation of two prospective high-power gas-discharge lasers, namely, a deep ultraviolet Cu+ Ne-H2-CuBr laser and a He-Sr+ recombination laser. For this purpose, the parameters of the set of nonstationary heat-conduction equations for the two groups of electrons, namely, the electrical power density and the specific heat capacity, are determined for each group. A 2D(r,t) numerical model is also developed in order to solve the set of the equations.

1. Introduction

In the course of several decades, huge efforts have been directed to studying metal and metal halide vapor lasers excited by a nanosecond pulsed longitudinal discharge (NPLD) in various gas mixtures. The high output laser characteristics obtained on a number of atomic and ionic metal transitions in the visible spectral range [1, 2], such as transitions of the copper atom (Cu) [1] and strontium ion (Sr+) [2], and the greater market competitiveness have justified these investigations. In view of the enormous potential for application of Cu atom lasers oscillating on the 510.6-nm and 578.2-nm Cu lines, and of Sr+ recombination lasers oscillating on the 430.5-nm and 416.2-nm Sr+ lines with multi-watt average output power and high beam-quality (beam divergence close to the diffraction limit), experimental [3-6] and theoretical [6-9] determination of the plasma parameters have been an inseparable part of this research. Our statement on the inapplicability of the well-known experimental techniques for electron temperature determination, namely probe measurement, laser Thomson scattering, and optical emission spectroscopy (line-ratio method), in NPLDs [10], has not been so far adequately disproved. Moreover, the published experimental results for electron temperature determination are very scarce and, hence, considered untrustworthy. All experimental methods for electron temperature determination, excluding Druyvesteyn’s procedure of Langmuir probe measurement, assume a Maxwellian electron energy distribution function (EEDF). Unlike the gas temperature calculation,
determining theoretically the electron temperature requires a year-consuming development of complex kinetic models, which include tens of energy levels per particle and the corresponding coupled tens or hundreds of partial or ordinary differential equations to be solved numerically. In spite of all this, there exist several well-known self-consistent kinetic models of varying degrees of complexity, which simulate the discharge kinetics and laser characteristics of Cu vapor [9] and CuBr vapor lasers [6] oscillating on self-terminating Cu atom transitions, and Sr⁺ recombination laser oscillating on Sr⁺ transitions [7, 8]. These kinetic models predict, among other parameters, the electron temperature values with considerable discrepancies. Moreover, all of those complex models assume a Maxwellian EEDF, with the exception of the one presented in [9], where a bi-Maxwellian EEDF is used.

The radial and temporal dependences of the electron temperature \( T_e(r,t) \) is obtained in [10] for Maxwellian and Druyvesteyn EEDFs through a new simple method based on solving numerically a single partial differential equation, namely, the nonstationary heat-conduction equation for the electron gas, in two NPLDs under considerably different gas-discharge conditions.

In this paper, the radius- and time-dependent electron temperature \( T_e(r,t) \) is calculated for a bi-Maxwellian EEDF by the abovementioned method numerically solving a set of two partial differential equations, i.e. the nonstationary heat conduction equations for the two groups of electrons in two NPLDs under considerably different gas-discharge conditions. With this aim, parameters of the nonstationary heat conduction equations, such as the electrical power density and the specific heat capacity, are determined for each group of electrons. A 2D\((r,t)\) numerical model is also developed by means of a conservative homogeneous difference scheme [11] and MATLAB implementation, in order to solve the set of equations.

2. Theoretical results and discussion

Following [9], the electrons are divided in two energy groups with cutoff energies \( \varepsilon_1 \) of 16.7 eV and 19.8 eV for neon and helium, respectively, corresponding to the first excitation energy of these atoms.

The vast majority of the electrons are in the so-called “bulk” group; each variable concerning this group is denoted by subscript \( b \). The EEDF for electron energies \( \varepsilon < \varepsilon_1 \), the first excitation energy of the buffer-gas, is represented by part of a Maxwellian curve with a temperature \( T_b \):

\[
 f_b(\varepsilon) = \frac{2}{\sqrt{\pi}} \left( k_b T_b \right)^\frac{3}{2} e^{-\frac{\varepsilon}{k_b T_b}}
\]  

Similarly, electrons in the high-energy “tail” segment (\( \varepsilon > \varepsilon_1 \)) are characterized by variables marked by index \( t \) and have a Maxwellian profile with a temperature \( T_t \):

\[
 f_t(\varepsilon) = \frac{2S}{\sqrt{\pi}} \left( k_b T_t \right)^\frac{3}{2} e^{-\frac{\varepsilon}{k_b T_t}}
\]

where \( S = \left( \frac{T_l}{T_b} \right)^\frac{3}{2} e^{-\frac{\varepsilon_1}{k_b T_l}} \) is a normalization factor at the cutoff energy \( \varepsilon_1 \) such that \( f_b(\varepsilon_1) = f_t(\varepsilon_1) \).

The two energy-conservation equations for the bulk and tail electrons are:

\[
 C_b(t,T_b) \frac{\partial T_b}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( r k_e(T_b) \frac{\partial T_b}{\partial r} \right) + q_v^b(t,T_b), \quad 0 \leq r \leq R \quad \text{and} \quad 0 \leq t \leq T,
\]

\[
 C_t(t,T_b,T_t) \frac{\partial T_t}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( r k_e(T_t) \frac{\partial T_t}{\partial r} \right) + q_v^t(t,T_b,T_t),
\]

Where \( k_e(T) = \frac{\varepsilon}{2} \left( 1 + \frac{\varepsilon}{\varepsilon_1} \right) \) is the electron energy, \( \varepsilon \) is the energy of the absorbed light, \( \varepsilon_1 \) is the first excitation energy, and \( T_b \) and \( T_t \) are the temperatures of the bulk and tail groups, respectively.
where $k_e$ is the heat conductivity of the electron gas depending on the respective electron temperature, $C_b$ and $C_t$ are the heat capacities of each group of electrons, $q_v^b(t)$ and $q_v^t(t)$ are the electric power densities deposited into the NPLD for heating each group of electrons, $R$ is the discharge zone radius, $T$ is the period between the excitation pulses at a given pulse repetition rate. The parameters of the two equations are determined as follows:

$$q_v^b(t) = \frac{N_b}{N_e} q_v(t) \text{ and } q_v^t(t) = \frac{N_t}{N_e} q_v(t),$$

where $N_b = N_e \int_{0}^{\xi_b} f_b(\epsilon) d\epsilon$ and $N_t = N_e \int_{0}^{\xi_t} f_t(\epsilon) d\epsilon$. (4)

$$C_b = \frac{dU_b}{dT_b} = N_b \int_{0}^{\xi_b} \frac{\epsilon}{\epsilon_t} \frac{df_b(\epsilon)}{dT_b} d\epsilon \text{ and } C_t = \frac{dU_t}{dT_t} = N_t \int_{0}^{\xi_t} \frac{\epsilon}{\epsilon_t} \frac{df_t(\epsilon)}{dT_t} d\epsilon.$$ (5)

The time-dependent electric power and electron concentration are experimentally determined in [12] with time-resolved measurement of the electrical discharge parameters, namely, the tube voltage and discharge current. The $q_v(t)$ pulse is derived from the electric power pulse and is fitted with a Gaussian-type function, while the time-dependent electron concentration $N_e(t)$ is fitted with an exponential function:

$$q_v(t) = q_v^0 + A e^{-\left(\frac{t-t_0}{w}\right)^2} \quad \text{and} \quad N_e(t) = N_e^0 + A e^{-\frac{t-t_0}{w}}.$$ (6)

The fitting parameters for both functions are shown in Table 1.

| Function | $q_v^0$ or $N_e^0$ | $A$ | $w$ | $t_0$ |
|----------|--------------------|-----|-----|-------|
| $q_v(t)$ | $1.61588 \times 10^8$ | $1210.21118$ | $3.60545 \times 10^8$ | $9.48298 \times 10^{-8}$ |
| $N_e(t)$ | $-3.52795 \times 10^{18}$ | $1.06363 \times 10^{19}$ | $2.97375 \times 10^8$ | $2.07038 \times 10^{-8}$ |

Figure 1. Temporal dependence of electric powers (a) and electron densities (b) under gas-discharge conditions optimal for Cu$^+$ and Sr$^+$ lasers.

Figure 1 presents the results for the electric power density (a) and the electron concentration (b) under gas-discharge conditions optimal for laser oscillation at Cu$^+$ and Sr$^+$ lines, respectively.

The power index scanning ($k_e = T_e$) undertaken in [10] to determine the electronic heat conductivity yields values for the power index $a$ of 0.02 and 0.25 for neon and helium, respectively.

The set of equations (3) is numerically solved under initial conditions:

$$T_b(r,0) = T_b^0(r) = \left[ T_b(R,0) \right]^{1+a} + \frac{1+a}{4B} q_v^{av} \left( R^2 - r^2 \right)^{\frac{1}{1+a}},$$
\[ T_i(r,0) = T_i^0(r) = \left[ T_i(R,0)^{1+a} + \frac{1+a}{4B} q_v^{\infty} (R^2 - r^2) \right]^{1/(1+a)}, \]

and boundary conditions:
\[ \lim_{r \to 0} r k_b(T_b) \frac{\partial T_i}{\partial r} = 0 \quad \text{and} \quad T_i(R, t) = T_i(R, 0), \]
\[ \lim_{r \to 0} r k_e(T_e) \frac{\partial T_i}{\partial r} = 0 \quad \text{and} \quad T_i(R, t) = T_i(R, 0). \]

The gas-discharge parameters optimal for laser oscillation on the corresponding atom and ion transitions are given in table 2.

| Laser                  | Buffer | \( d_a = 2R_a \) (cm) | \( l_a \) (cm) | \( p \) (Torr) | \( q_v^{\infty} \) (W cm\(^{-3}\)) | \( B \) | \( a \) | \( T_i(R,0) \) (K) |
|------------------------|--------|------------------------|---------------|--------------|---------------------|-------|-------|------------------|
| Ne-Cu\(^{+}\) laser   | Ne     | 0.71                   | 100           | 16.7         | 20                  | 9.7\times10^{-4} | 0.685 | 880              |
| He-Sr\(^{+}\) laser    | He     | 1.6                    | 50            | 230          | 8                   | 34.9\times10^{-4} | 0.670 | 920              |

Figure 2 shows the time-resolved electron temperature in an NPLD used to excite a deep ultraviolet Cu\(^{+}\)-Ne-H\(_2\)-CuBr laser for different radial distances for the first (a) and the second (b) electron groups. Figure 3 illustrates the time-resolved electron temperature in an NPLD used to excite a He-Sr\(^{+}\) recombination laser for different radial distances for the first (a) and the second (b) electron groups.

**Figure 2.** Temporal dependence of the electron temperature in an NPLD used to excite a DUV Cu\(^{+}\)-Ne-H\(_2\)-CuBr laser for different radial distances for the first (a) and the second (b) electron groups.

**Figure 3.** Temporal dependence of electron temperature in an NPLD used to excite a He-Sr\(^{+}\) recombination laser for different radial distances for the first (a) and the second (b) electron groups.
3. Conclusions
The temporal and radial distributions of the electron temperature are determined in Ne-H$_2$-CuBr and He-Sr NPLDs in the case of a bi-Maxwellian EEDF. The results obtained are in fair agreement with the time-dependent electron temperature experimentally obtained by time-resolved measurements of the electrical discharge parameters [12] and by the existing self-consistent models developed for Ne-Cu, Ne-CuBr and He-Sr$^+$ lasers excited in NPLDs [6-9].

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