DETERMINATION OF STANDARD MODEL PARAMETERS
FROM LATTICE QCD

Vittorio Lubicz

Dip. di Fisica, Università di Roma Tre and INFN, Sezione di Roma
Via della Vasca Navale 84, I-00146 Roma, Italy

Abstract

In this talk I review some recent results from lattice QCD calculations which aim to a more accurate determination of the Standard Model parameters in the quark sector. After a review of our current knowledge of these parameters, I present lattice results for the strange, charm and bottom quark masses, the CKM matrix element $V_{ub}$ and the complex CKM phase $\delta$, which induces CP violation effects in the Standard Model with three generations.
1 Introduction

The Standard Model (SM) of the electroweak and strong interactions contains a quite large number (19) of free parameters. These parameters can be only determined by comparing the result of some theoretical prediction with the corresponding experimental measurement. The majority of these parameters enter in the so-called quark sector of the Model, and represent the least known quantities appearing in the theory. On the contrary, in the pure electroweak sector, all SM parameters have been quite accurately determined, with the only exception of the Higgs boson mass: this particle has not yet been observed and its mass is only marginally constrained by the electroweak radiative corrections and precision measurements [1].

A good representation of our present knowledge of the SM free parameters in the quark sector (although possibly not the most up-to-dated) can be obtained by looking at the values quoted by the PDG [2]. For instance, the strong coupling constant $\alpha_s$ is currently relatively well known. One finds that its value, in the $\overline{\text{MS}}$ scheme, at the $m_Z$ mass scale, is given by:

$$\alpha_s(m_Z) = 0.117 \pm 0.005$$

where the error represents the $1\sigma$-level of uncertainty.

Among the most poorly known fundamental parameters in the SM are the masses of the three light quarks. The PDG quotes their values in the $\overline{\text{MS}}$ scheme at a reference scale $\mu \approx 1 \text{ GeV}$:

$$\overline{m}_u(1 \text{ GeV}) = 5 \pm 3 \text{ MeV}$$
$$\overline{m}_d(1 \text{ GeV}) = 10 \pm 5 \text{ MeV}$$
$$\overline{m}_s(1 \text{ GeV}) = 200 \pm 100 \text{ MeV}$$

(2)

Thus, according to the PDG, uncertainties on these quantities are still of the order of 50%. Estimates of the quark masses are significantly more accurate for the three heavy quarks (charm, bottom, top). These masses are usually given in terms of the $\overline{\text{MS}}$ running masses, $\overline{m}_q$, renormalized at the scale $\mu = \overline{m}_q$, which lies well inside the perturbative region in QCD. For the charm and bottom quark masses, the PDG quotes:

$$\overline{m}_c(\overline{m}_c) = 1.3 \pm 0.3 \text{ GeV}$$
$$\overline{m}_b(\overline{m}_b) = 4.3 \pm 0.2 \text{ GeV}$$

(3)

The discovery of the top quark was not yet well established at the time the latest version of the PDG has been published. Moreover, the determination of the top quark mass is at
present rapidly improving. Thus, I present here the more recent estimate $m_t = 175 \pm 6$ GeV \cite{3}. This is the average value obtained by the CDF and D0 experiments at Fermilab, and corresponds to the so-called pole definition of the quark mass\cite{3}. By converting this value to the $\overline{\text{MS}}$ definition of the quark mass, one obtains:

$$\overline{m}_t(m_t) = 167 \pm 6 \text{ GeV}$$  \hspace{1cm} (4)

The top quark mass is then one of the best measured free parameters in the quark sector of the SM.

For three generations of elementary particles, four free parameters of the SM enter in the CKM mixing matrix. They represent three angles and one phase. The sines of these angles, $s_{12}$, $s_{23}$ and $s_{13}$, correspond, with an excellent approximation, to three elements of the matrix. The PDG quote their values at the 90\% of CL. In terms of estimates at the 1σ-level one has:

$$s_{12} \simeq |V_{us}| = \lambda = 0.221 \pm 0.002$$
$$s_{23} \simeq |V_{cb}| = A\lambda^2 = 0.040 \pm 0.005$$
$$s_{13} \simeq |V_{ub}| = A\lambda^3\sigma = 0.0035 \pm 0.0010$$  \hspace{1cm} (5)

For later convenience, we have also given in eq. (5) the expressions of the matrix elements in the Wolfenstein parameterization, with $\sigma = \sqrt{\rho^2 + \eta^2}$.

The above discussion shows that, in the quark sector, the SM free parameters are typically affected by large errors. These are of the order of 50\% for the three light quark masses, and of $20 - 30\%$ for the charm quark mass and the CKM matrix element $V_{ub}$. Notice that a value of the CKM phase $\delta$, which is expected to be the source of CP violation, is not even quoted by the PDG.

The uncertainties on the values of SM free parameters in the quark sector mainly reflect our poor theoretical understanding of the strong interactions in the low-energy non-perturbative region. As we have seen, there are three noteworthy exceptions in this context. These are the strong coupling constant $\alpha_s$, which can be determined from purely perturbative theoretical calculations; the sine of Cabibbo angle, $V_{us} \simeq s_{12} = \sin \theta_c$, which is well determined from the study of kaon semileptonic decays and the corresponding QCD chiral Lagrangian predictions; and the matrix element $V_{cb}$, which is constrained by the analysis of $B \to D^*\ell\nu$ semileptonic decays within the framework of the Heavy Quark Effective Theory (HQET).

\footnote{A more detailed discussion of the different definitions of quark masses will be given in sect. \cite{3}.}
At present, there is increasing evidence that a reliable non-perturbative approach to the study of quark and gluon strong interactions is provided by lattice QCD calculations. Quantitative determinations of all the fundamental SM free parameters in the quark sector are in fact either already available from lattice calculations or they are expected to be provided by the lattice in the near future. The quark masses of the can be extracted on the lattice from the calculation of some physical hadron mass, the CKM matrix elements can be obtained by studying the semileptonic and radiative decays of mesons, and the phase $\delta$ can be extracted by using the lattice results combined with the experimental measurements of $K^0 - \bar{K}^0$ and $B_d^0 - \bar{B}_d^0$ mixing.

In this talk I will review some of these lattice results in more details. A personal choice of the topics has been necessarily introduced, and I apologize for the many interesting and recent results which will not be covered in the discussion. Among them, I should recall the lattice calculations of the strong coupling constant $\alpha_s$, which have been considered by the PDG to obtain the corresponding world average, and the interesting studies of exclusive $B \rightarrow D$ and $B \rightarrow D^*$ semileptonic decays (see ref. for a recent compilation of lattice results), which play a crucial role in the determination of the CKM mixing angle $V_{cb}$. The interested reader can find a more comprehensive discussion of recent lattice results in the proceedings of the latest Lattice conferences.

Moreover, I will not discuss in any details the lattice technique itself and the sources of statistical and systematic errors which affect these calculations. These errors will be simply quoted as final uncertainties in the various results. The aim of this talk is to illustrate the present accuracy of the lattice calculations and the impact that they already have on the phenomenology of the electroweak and strong interactions and in the determination of the SM free parameters.

2 The quark masses

In the SM quark masses and lepton masses are fundamental parameters. However, quark masses cannot be measured directly in the experiments, since quarks do not appear as physical states and are confined into colour-singlet hadrons. Thus, the kinematical concept of on-shell mass is meaningless for quarks. The values of the quark masses then depends on precisely their definitions.

\footnote{A possible exception is represented by the top quark mass, which is too heavy for the top quark to constitute bounded hadronic states. For the same reason, however, this mass can be determined from the experimental measurements of top decays with a quite high level of accuracy.}
Among the several theoretical definitions of quark masses, two of them are mostly used. These are the \( \overline{\text{MS}} \) running quark mass, \( \overline{m}_q(\mu) \), evaluated at a given reference scale \( \mu \), and the pole mass \( m_q^{\text{pole}} \). It should be noted that the constituent quark masses, which are obtained from phenomenological quark models, have no deep meaning, since they cannot be related in any sensible way to the parameters of the QCD Lagrangian.

The \( \overline{\text{MS}} \) quark mass is defined as the value of the renormalized quark mass in the \( \overline{\text{MS}} \) scheme. It is obtained from the perturbative expansion of the bare quark propagator after the divergences have been removed according to the \( \overline{\text{MS}} \) prescription, and the bare coupling constant has been replaced by the renormalized coupling in the same scheme. The \( \overline{\text{MS}} \) mass \( \overline{m}_q \) is a running coupling. For heavy quarks, \( \overline{m}_q \) is usually considered at the scale \( \mu = m_q \) itself. This choice is not possible, however, in the case of light quarks, since such a scale would lie outside the perturbative region in QCD. For this reason, the \( \overline{\text{MS}} \) masses for light quarks are usually defined at a conventional scale (typically \( \mu = 1 \) or \( 2 \) GeV). Notice that \( \overline{m}_q(\mu) \) is a short distance quantity.

For heavy quarks only, the pole mass \( m_q^{\text{pole}} \) can also be considered. This is defined as the value of the pole of the quark propagator, computed in perturbation theory. The pole mass would correspond to the kinematical on-shell mass in the leptonic case. For confined quarks, however, there should not be any pole in the full propagator and the definition of pole mass makes sense only in perturbation theory. On the other hand, the pole mass is affected by a renormalon ambiguity of \( \mathcal{O}(\alpha_s^{\text{QCD}}) \) [7, 8], which prevent the possibility of precisely define its perturbative expansion. For this reason, and also because the pole mass can be only defined in the heavy quark case, a short distance definition, as the \( \overline{\text{MS}} \) quark mass, should be preferred.

The relation between the \( \overline{\text{MS}} \) mass \( \overline{m}_q \) and the pole mass \( m_q^{\text{pole}} \) can be calculated in perturbation theory:

\[
\overline{m}_q(\overline{m}_q) = m_q^{\text{pole}} \left[ 1 - \frac{4}{3} \frac{\alpha_s(m_q)}{\pi} + \mathcal{O}(\alpha_s^2) \right]
\]

The numerical differences between the two definitions are always important. For instance, as we have already noted, the experimental value \( m_t^{\text{pole}} = 175 \pm 6 \text{ GeV} \) corresponds to the \( \overline{\text{MS}} \) value \( \overline{m}_t(\overline{m}_t) = 167 \pm 6 \text{ GeV} \). Thus, a particular care should be taken to specify which definition of quark mass one is currently using.

Even though quark masses cannot be directly measured in the experiments, their values are particularly important for the phenomenology of weak interactions. For instance, in the SM the theoretical prediction for the parameter \( \varepsilon'/\varepsilon \), which measures the effects of direct CP violation in kaon decays, is given, at a quite good level of accuracy, by the
simple expression [4]:

\[
\left( \frac{\varepsilon'}{\varepsilon} \right)^{\text{th}} \approx \frac{C_6 B_6 + C_8 B_8}{m_s^2}
\]

(7)

Here \(B_{6,8}\) are the \(B\)-parameters of the local four-fermion operators \(O_{6,8}\) in the \(\Delta S = 1\) effective weak Hamiltonian and \(C_{6,8}\) are the corresponding Wilson coefficients. In eq. (7), \(m_s\) is the strange quark mass. Thus, it should be clear that an accurate determination of this mass is a crucial ingredient for a precise theoretical prediction of \(\varepsilon'/\varepsilon\).

An important, non-perturbative theoretical tool to investigate the low-energy structure of QCD is chiral perturbation theory (ChPT). However, ChPT can only determine the ratios of light quark masses and does not provide predictions for their absolute values. The ratios of quark masses are scale and, to a good approximation, renormalization scheme independent quantities. A recent analysis by Leutwyler [10], performed at the next-to-leading order in the chiral expansion, indicates for these ratios the values:

\[
\frac{m_u}{m_d} = 0.553 \pm 0.043, \quad \frac{m_s}{m_d} = 18.9 \pm 0.8, \quad \frac{m_s}{m_u} = 34.4 \pm 3.7
\]

\[
\frac{m_s - \hat{m}}{m_d - m_u} = 40.8 \pm 3.2, \quad \frac{m_s}{\hat{m}} = 24.4 \pm 1.5
\]

(8)

where \(\hat{m}\) is the average value of the two lightest quark masses, \(\hat{m} = (m_u + m_d)/2\).

These predictions are in remarkable agreement with the results of a recent lattice study [11] of the electromagnetic properties of hadrons. The contribution from electromagnetic effects to the hadronic mass splittings within isomultiplets (e.g. to the difference \(m_{\pi^+} - m_{\pi^0}\)) is comparable to the size of the \(u-d\) quark mass splitting. Therefore, to evaluate the latter, the electromagnetic effects must be taken into account in the context of the non-perturbative QCD dynamics. In a preliminary numerical study, the authors of ref. [11] find:

\[
\frac{m_u}{m_d} = 0.512(6), \quad \frac{m_d - m_u}{m_s} = 0.0249(3)
\]

(9)

From eq. (8) I obtain \((m_d - m_u)/m_s = 0.0236(38)\). Thus, the lattice results are in excellent agreement with the predictions [8] of ChPT.

To investigate the absolute value of light quark masses, ChPT is not helpful. At present, the only first principle, non-perturbative techniques available for such a study are QCD sum rules (SR) and lattice calculations. The advantage of lattice calculations is that no additional model parameters have to be introduced, besides those present in the SM. Moreover, the statistical and systematic errors in lattice calculations (e.g. quenched approximations or finite cutoff effects) can be estimated and systematically corrected in time, with increasing computer resources.
Lattice QCD is in principle able to predict the mass of any quark by fixing, to its experimental value, the mass of a hadron containing a quark with the same flavour. The quark mass that is directly determined in lattice simulations is the (short distance) bare lattice quark mass $m(a)$, where $a$ is the lattice spacing, the inverse of the UV cutoff. The connection between $m(a)$ and the renormalized quark mass $m(\mu)$, in a given renormalization scheme, is provided by a multiplicative renormalization constant:

$$m(\mu) = Z(a\mu)m(a) \quad (10)$$

The perturbative expression of $Z(a\mu)$, renormalization group-improved at the next-to-leading order, has been given in ref. [12]. A non-perturbative renormalization prescription for the quark mass has been also proposed in ref. [12]. The basic idea consists on imposing the renormalization conditions directly on non perturbatively calculated correlation functions between external quark states, at momentum $p^2 = \mu^2$ and in a fixed gauge [13]. Once the lattice bare quark mass $m(a)$ has been fixed and the renormalization constant $Z(a\mu)$ has been computed, the renormalized quark mass can be derived from eq. (10). A continuum perturbative calculation can be finally performed to convert the result to the renormalized quark mass in the $\overline{\text{MS}}$ scheme.

Among the light quark masses, the strange quark mass is of particular interest. The values of the lightest quark masses, $m_u$ and $m_d$, can be derived from it by considering the values of the quark mass ratios given by ChPT, cf. eq. (8), or by the lattice calculations, eq. (9). In addition, the value of the strange quark mass is more easily accessible to lattice calculations, since the up and down quark masses are too small to be directly computed in numerical simulations, because of finite volume effects. An extrapolation is in fact required for them to be measured.

The values of the strange quark mass, obtained from the most recent lattice calculations, are shown in table 1. These are defined as the $\overline{\text{MS}}$ mass, $\overline{m}_s$, at the reference scale $\mu = 2$ GeV. For a comparison, two recent QCD SR determinations, evolved at the same scale, are also shown in the table.

The lattice determinations of the strange quark mass give all results in the range of approximately 100-150 MeV. Notice that, within the standard $2\sigma$ level of uncertainty, the central values are not always compatible one to each other. The discrepancies are mainly due to finite cutoff effects present in the lattice calculations, and to the way they are taken into account by the several groups (e.g. the authors of refs. [14, 15] attempted an extrapolation to $a \to 0$). These differences might give an estimate of the systematical uncertainties present in the lattice calculation of the strange quark mass. A systematic uncertainty of the same size is also found in the QCD SR determinations, as shown by
Table 1: Values of the strange quark mass $m_s(\mu = 2 \text{ GeV})$ as given by the most recent lattice calculations. All the results have been obtained in the quenched approximation, with the exception of the unquenched result of ref. [17], denoted by a star in the table. Two recent QCD SR determinations are also shown for comparison.

| $m_s$ (MeV) | Ref.   |
|-------------|--------|
| 128 ± 18    | [12] Lattice |
| 100 ± 23    | [14] Lattice |
| 95 ± 16     | [15] Lattice |
| 122 ± 20    | [16] Lattice |
| 166 ± 15    | [17] Lattice |
| 140 ± 20    | [17] Lattice* |
| 146 ± 14    | [18] QCD SR |
| 102 ± 13    | [19] QCD SR |

In this case, the discrepancy comes from non-resonant contributions to the relevant hadronic spectral functions, which have been included in the calculation in ref. [19]. From the average of the lattice results, I quote:

$$m_s(\mu = 2 \text{ GeV}) = 125 \pm 25 \text{ MeV} \quad (11)$$

which is also compatible with the recent QCD SR determinations. The value of the strange quark mass quoted by the PDG, see eq. (2), once evolved to the reference scale of 2 GeV, corresponds to $m_s(\mu = 2 \text{ GeV}) = 140 \pm 70 \text{ MeV}$. Given the better consistency of the recent theoretical calculations, I think that the estimated uncertainty of 70 MeV might be at present too conservative.

The mass of the charm quark has been calculated on the lattice in ref. [12], by using the results of several lattice simulations. The lattice bare quark mass has been connected to the renormalized mass in the $\overline{\text{MS}}$ scheme by using the perturbative formula, with included next-to-leading logarithmic corrections. The final result is:

$$m_c(\mu = 2 \text{ GeV}) = 1.48 \pm 0.28 \text{ GeV} \quad (12)$$

This can be compared with the QCD SR determination $m_c^\text{pole} = 1.46 \pm 0.07 \text{ GeV}$ of ref. [20], which corresponds to $m_c(\mu = 2 \text{ GeV}) = 1.27 \pm 0.06 \text{ GeV}$.

From a phenomenological point of view, the mass of the $b$-quark is also a very important quantity. On the lattice, the $b$-quark cannot be directly simulated, since its mass is larger than the lattice cutoff, $a^{-1} \sim 2 - 4 \text{ GeV}$ in current simulations. Therefore, one has to rely on additional theoretical tools. The HQET has proven to be a very useful tool for studying heavy flavour physics, also within the context of the lattice regularization. In this approach, the mass of the $B$-meson can be expanded in inverse powers of the $b$-quark
mass, $m_b^{\text{pole}}$, and the result has the form:

$$M_B = m_b^{\text{pole}} + \Lambda + \mathcal{O}\left(\frac{\bar{\Lambda}}{q}\right)$$

(13)

where $\Lambda$ is a parameter of the order of $\Lambda_{QCD}$ whose value cannot be predicted on the basis of the HQET only. The lattice formulation of the HQET offers the possibility of a numerical, non-perturbative calculation of $\bar{\Lambda}$. This calculation has been recently performed in ref. [21]. The result, once inserted in eq. (13), can be eventually translated into an estimate for the $b$-quark mass in the $\overline{\text{MS}}$ scheme. Ref. [21] finds:

$$m_b(m_b) = 4.15 \pm 0.05 \pm 0.20 \text{GeV}$$

(14)

where the last systematic error is an estimate of higher order perturbative corrections in the matching between the full and the heavy quark effective theory.

A different approach to the calculation of the $b$-quark mass, still within the lattice QCD regularization, has been followed in ref. [22]. This is based on the use of a nonrelativistic QCD Lagrangian for $b$-quarks. The bare $b$-quark mass, entering in this Lagrangian, is fixed to reproduce on the lattice the experimental value of the $\Upsilon$-meson. Perturbation theory is then used to convert the result into a value for the $b$-quark mass in the $\overline{\text{MS}}$ scheme. It is found:

$$\bar{m}_b(m_b) = 4.0 \pm 0.1 \text{GeV}$$

(15)

in good agreement with the result of eq. (14). Notice that the error coming from higher order corrections in the matching between the full and the effective theory is not included in eq. (13).

A recent QCD SR calculation of the $b$-quark mass gives $\bar{m}_b(m_b) = 4.13 \pm 0.06 \text{GeV}$ [23], in remarkable agreement with the two lattice determinations. However, since the value of the $b$-quark mass is crucial for the phenomenology of weak interactions (e.g. the decay rates of bottom hadrons are proportional to the fifth power of this mass), a still more accurate determination of the $b$-quark mass would be very important.

3 The semileptonic decays of $D$ and $B$-mesons and the determination of $V_{ub}$

Semileptonic decays $D$- and $B$-mesons are relatively simple to investigate, both theoretically and experimentally. For this reason, they play a crucial role in our understanding of
the CKM mixing matrix and of the interplay between strong and weak interactions. Recently, the exclusive semileptonic decays of $B$-mesons into charmless final states, $B \to \pi l\nu$ and $B \to \rho l\nu$, have been observed by the CLEO Collaboration [24]. A comparison between the experimental branching ratios and the corresponding theoretical predictions then allows a clean extraction of the relevant mixing angle $V_{ub}$. This is one of the least known entries in the CKM matrix, see eq. (5).

At present, there is increasing evidence that quantitative calculations of semileptonic decays can be obtained from lattice QCD simulations. Among these, $D$-meson semileptonic decays provide a good test of the lattice method, since the relevant CKM matrix element is well constrained by unitarity in the SM, $V_{cs} \simeq 0.975$. Thus, theoretical predictions for these decays can be compared directly with the corresponding experimental measurements.

Over the last years, the invariant form factors which govern the $D \to K$ and $D \to K^*\ell\nu$ semileptonic decays, have been computed in various lattice calculations, [25]-[31]. The results, at momentum transfer $q^2 = 0$ are shown in fig. I, together with the corresponding experimental average [2]. Notice that the lattice calculations, as well as the QCD SR calculations, can determine the dependence of the form factors on the momentum transfer $q^2$. This is in contrast to quark models, which can compute the form factors only at a given value of $q^2$, typically $q^2 = q_{\text{max}}^2$. A summary of the lattice results for the form factors, which are shown in fig. I, is also given in table 2, together with the corresponding experimental average. The central values, quoted in the table, are the weighted average of the lattice results; the errors are my personal estimates which take into account both...
Table 2: Lattice and experimental results for the $D \to Kl\nu$ and $D \to K^*l\nu$ form factors, at momentum transfer $q^2 = 0$.

|            | Lattice | Exp.  |
|------------|---------|-------|
| $f_+(0)$   | 0.70(7) | 0.74(3) |
| $V(0)$     | 1.13(17)| 1.0(2) |
| $A_1(0)$   | 0.63(7) | 0.55(3) |
| $A_2(0)$   | 0.52(12)| 0.40(8) |

the statistical and the systematic uncertainties. The agreement between lattice calculations and experimental measurements, indicated by table 2, is remarkable, and strongly supports the reliability of lattice calculations in the study of semileptonic decays of heavy-light mesons. This conclusion is crucial for the studies of $B$-meson semileptonic decays on the lattice, which are important for the determination of the mixing angle $V_{ub}$. On the other hand, the lattice results are still affected by uncertainties which are typically of the order of 15%. Reducing such uncertainties is a primary goal of future lattice calculations.

For $D$-meson semileptonic decays, similar results have been also obtained by QCD SR calculations [32]. In contrast, some popular quark models, like the ISGW [33] and WSB [34] models, fail to correctly describe the $D \to K^*$ semileptonic decays: the predicted values for the form factors $A_1(0)$ and $A_2(0)$ in these models are in the range between 0.8 and 1.0, well above the present experimental values.

With respect to $D$-meson semileptonic decays, lattice calculations of $B$-meson decays are affected by larger uncertainties. The reason is that, as already noted, the $b$-quark is too heavy to be directly simulated on the lattice. Thus, an extrapolation of the lattice form factors is required, from the charm quark mass region to the bottom mass. The form of this extrapolation is dictated by the HQET, which predicts the dependence of the form factors on the heavy meson mass [35]. However, the extrapolated form factors are always obtained at large momentum transfer, $q^2 \sim q_{max}^2$, and an assumption is then necessary in order to reconstruct the form factor in the whole $q^2$ range. In order to improve the situation, it is necessary to work with larger lattice cutoff and heavier quark masses.

So far, the exclusive charmless semileptonic decays of $B$-mesons, $B \to \pi l\nu$ and $B \to \rho l\nu$, have been studied on the lattice by three groups: the ELC [27], APE [29] and UKQCD [36] collaborations. Preliminary results have been also presented in ref. [31]. The values of the form factors for these decays, extrapolated at $q^2 = 0$, are shown in table 3. We find that the lattice results for the two form factors $f_+$ and $A_1$, which are those affected by the smallest statistical errors, are in quite good agreement one to each other. The preliminary results of ref. [31] may suggest some discrepancy, but the quoted uncertainties are larger in this case. The form factors $f_+$ and $A_1$ are also those which dominate the corresponding semileptonic decay rates. Thus, a reliable determination of $V_{ub}$ can be
Table 3: Lattice results for the $B \to \pi l \nu$ and $B \to \rho l \nu$ form factors, at momentum transfer $q^2 = 0$.

|        | $f_+(0)$  | $V(0)$  | $A_1(0)$ | $A_2(0)$ |
|--------|-----------|---------|-----------|-----------|
| ELC 27 | 0.30(14)(5) | 0.37(11) | 0.22(5) | 0.49(21)(5) |
| APE 29 | 0.35(8) | 0.53(31) | 0.24(12) | 0.27(80) |
| UKQCD 36 | 0.23(2) | —— | 0.27(+$\frac{7}{3}$)(3) | —— |
| GSS 31 | 0.50(14)(+$\frac{7}{6}$) | 0.61(23)(+$\frac{9}{6}$) | 0.16(4)(+$\frac{22}{16}$) | 0.72(35)(+$\frac{10}{16}$) |

Figure 2: Values for $V_{ub}$ obtained by the CLEO Collaboration [24] using different theoretical models.

obtained by comparing the lattice predictions with the corresponding experimental values of branching ratios.

A compilation of results for $V_{ub}$, obtained by the CLEO Collaboration using different theoretical models, is shown in fig. 2 [24]. In this analysis, the theoretical predictions, which are needed to extract $V_{ub}$ from the measured values of decay rates, are also used in input to estimate the experimental detection efficiencies. The CLEO Collaboration has evaluated their efficiencies by using several theoretical models. The resulting values of $V_{ub}$, from these models, are presented above the horizontal line in fig. 2. It is interesting to note that one can test the validity of a given model by comparing the measured ratio of semileptonic widths $\Gamma(B \to \rho)/\Gamma(B \to \pi)$, which is independent on the unknown $V_{ub}$,
to the prediction of that model. For instance, the Korner and Schuler phenomenological quark model has been found to be consistent only at the 0.5% level, so it has been excluded by CLEO from any model averages \cite{24}.

The final CLEO estimate of $V_{ub}$, from exclusive semileptonic $B$ decays, is $V_{ub} = (3.3 \pm 0.2^{+0.3}_{-0.4} \pm 0.7) \cdot 10^{-3}$. The last error, coming from the estimated theoretical uncertainties, is at present the dominant one. However, I believe that an improvement of the theoretical predictions in this analysis is likely to occur in the near future, especially from lattice calculations.

4 The phase $\delta$ and CP violation

After more than thirty years since its discovery, CP violation has been only observed in the neutral kaon system. It comes from the mixing of opposite CP eigenstate in the $K_{L,S}$ mass eigenstates. This mixing, which is controlled by the $\varepsilon$ parameter, is responsible for the so-called “indirect” CP violation in kaon decays. On the other hand, the experimental observation of “direct” CP violation, which is realized via a direct transition from two opposite CP eigenstates, are far to be conclusive. The two well known experimental results for the relevant parameter, $\varepsilon'/\varepsilon$, are \cite{37, 38}:

$$\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right) = \begin{cases} (7.4 \pm 6.0) \cdot 10^{-4} & [\text{NA31}] \\ (23.0 \pm 6.5) \cdot 10^{-4} & [\text{E731}] \end{cases}$$  \hfill (16)

Since experimental evidence for direct CP violating transitions is still lacking, the only observed source of CP violation in the $K^0 - \bar{K}^0$ mixing could still be explained within the old Wolfenstein theory of a superweak interaction \cite{39}.

In the SM, with three generations of quarks, all CP violating effects are due to the existence of a single, complex phase $\delta$, entering in the CKM mixing matrix. Thus, needless to say, to determine the value of this phase is crucial for our understanding of CP violation in the SM. The natural place to look at for a determination of $\delta$ is the value of the $\varepsilon$ parameter itself. Within the SM, the theoretical expression of $\varepsilon$ is given by:

$$|\varepsilon| = C_\varepsilon \hat{B}_K A^2 \lambda^4 \sigma \sin \delta \left\{ F(x_c, x_t) + F(x_t) \left[ A^2 \lambda^4 (1 - \sigma \cos \delta) \right] - F(x_c) \right\}$$  \hfill (17)

where $C_\varepsilon$ is a known dimensionless coefficient, $x_q = m_q^2/m_W^2$ and $F(x_i)$ and $F(x_i, x_j)$ are the so-called Inami-Lim functions, including QCD corrections. In eq. (17), $\lambda$, $A$ and $\sigma$ are the parameters of the CKM matrix in the Wolfenstein parameterization, cf. eq. (5). $\hat{B}_K$ is the renormalization group-invariant $B$-parameter to be discussed in the following.
Table 4: Lattice results for pseudoscalar decay constants of heavy-light mesons and for $B$-mesons $B$-parameters. The quoted values are from ref. [45].

In principle, a comparison between the theoretical expression of $\varepsilon$ with the corresponding experimental value, allows one to obtain an estimate of the phase $\delta$. However, since $\varepsilon$ is basically proportional to $\sin \delta$, such a comparison implies two possible solutions of opposite sign for $\cos \delta$, see for example [40]. Thus, an additional experimental input is needed for $\delta$ to be fixed. A convenient quantity to be considered in the analysis is the mass difference of neutral $B_d$-mesons. In the SM, this difference is given by:

$$\Delta M_{B_d} = C_B \frac{f_B^2 \hat{B}_B}{M_B} A^2 \lambda^6 \left(1 + \sigma^2 - 2 \sigma \cos \delta\right) F(x_t)$$  \hspace{1cm} (18)

Here $C_B$ is a known dimensionless coefficient, $f_B$ is the $B$-meson pseudoscalar decay constant and $\hat{B}_B$ the renormalization-invariant parameter relevant for the $B - \bar{B}$ system, the analogous of $\hat{B}_K$. A combined analysis of $\varepsilon$ and $\Delta M_{B_d}$ is thus the procedure to constrain the phase $\delta$ [9, 41].

The theoretical expression of $\varepsilon$ and $\Delta M_{B_d}$, eqs. (14) and (18), are obtained combining the relevant matrix elements of local operators, entering in the weak effective Hamiltonian, with the corresponding Wilson coefficients. In this approach, the matrix elements of local operators contain all the non-perturbative, long distance effects of the strong interactions. In eqs. (14) and (18), this contribution is parameterized through the pseudoscalar decay constant $f_B$ and the two $B$-parameters, $\hat{B}_K$ and $\hat{B}_B$. A fully non-perturbative determination of these quantities is then required for $\varepsilon$ and $\Delta M_{B_d}$ to be determined.

The latest average lattice determination for the $\hat{B}_K$ parameter is [42]:

$$\hat{B}_K = 0.87 \pm 0.03 \pm 0.14$$  \hspace{1cm} (19)

where the last error is systematic, and takes into account the effect of quenched approximation. Notice that significatively lower estimates of $\hat{B}_K$, such as those obtained by using the QCD Hadronic Duality approach ($\hat{B}_K = 0.39 \pm 0.10$) [43] or using the $SU(3)$ symmetry and PCAC ($\hat{B}_K = 1/3$) [44] are presently excluded by the experiments: they require $m_t > 200$ GeV in order to explain the experimental value of $\varepsilon$ [9].

A compilation of lattice results for the pseudoscalar decay constants of heavy-light mesons and the $B$-mesons $B$-parameters has been recently given by Martinelli [45] and it is presented in table 4. Concerning the values of the pseudoscalar decay constants, it is reassuring to find that the lattice value for $f_{D_s}$, which has been predicted well before the first experimental measurement, is in excellent agreement with the present experimental average from $D_s \rightarrow \mu \nu \mu$ decays, $f_{D_s} = 241 \pm 21 \pm 30$ MeV [46].

| $f_D$ (MeV) | $f_{D_s}$ (MeV) | $f_B$ (MeV) | $f_B \sqrt{\hat{B}_{B_d}}$ (MeV) | $f_B/f_B$ | $\hat{B}_{B_s}/\hat{B}_{B_d}$ |
|-------------|----------------|-------------|--------------------------------|-----------|-----------------------------|
| 205 ± 15    | 235 ± 15       | 175 ± 25    | 207 ± 30                       | 1.15 ± 0.05 | 1.01 ± 0.01               |
Thus, the values of the non-perturbative parameters which are necessary to estimate the CKM phase $\delta$ are predicted by lattice calculations. The most recent phenomenological analysis are mainly based on these predictions. Here, I present two recent estimates for $\cos \delta$ and the related value of $\sin 2\beta$, where $\beta$ is the angle of the unitary triangle which controls the CP asymmetry in $B^0_d \rightarrow J/\Psi K_s$ decays. These estimates are:

$$\begin{align*}
\cos \delta &= 0.43 \pm 0.19 \\
\sin 2\beta &= 0.66 \pm 0.13 & \text{Ref.[9]} \\
\cos \delta &= 0.38 \pm 0.23 \\
\sin 2\beta &= 0.68 \pm 0.10 & \text{Ref.[41]}
\end{align*}$$

in very satisfactory agreement one to each other.

I have shown that the results from lattice calculations are extremely important for a determination of the CKM complex phase $\delta$ and our understanding of CP violation in the SM. I have also shown that the same conclusion applies to the determination of other fundamental parameters in the SM, like the quark masses and mixing angles. In the last years, the lattice technique is rapidly improving. It is increasing the accuracy of its predictions (e.g. by systematically correcting finite cutoff errors [17]) and it is extending the range of its applicability (e.g. the possibility of studying non-leptonic hadronic decays on the lattice [18] is currently under investigation). For these reasons, I believe that in the near future new results from lattice calculations will have considerable impact on our understanding of the phenomenology of the electroweak and strong interactions.

**Acknowledgements**

I wish to thank the organizers of such a stimulating and interesting conference. I am also grateful to Guido Martinelli for an enjoyable and long lasting collaboration, from which I learnt many of the topics covered in this talk.

**References**

[1] For a recent review see G.Altarelli, Lectures delivered at the 3rd International Symposium on Radiative Corrections, Cracow, Poland, August 1996. [hep-ph/9611239](http://arxiv.org/abs/hep-ph/9611239).

[2] The Particle Data Group, R.M. Barnett et al., Phys.Rev. **D54** (1996) 1.
[3] P.Tipton, Proceedings of the 28th International Conference on High-energy Physics (ICHEP 96), Warsaw, Poland, 25-31 Jul 1996.

[4] J.Shigemitsu, Nucl.Phys. B (Proc.Suppl.) 53 (1997) 16. hep-lat/9608058.

[5] H.Wittig, Invited talk at the 3rd German-Russian Workshop on Progress in Heavy Quark Physics, Dubna, Russia, 20-22 May 1996. hep-ph/9606371.

[6] Lattice’95, Proceedings of the 13th International Symposium on Lattice Field Theory, Melbourne, Australia, 11 - 15 July 1995, Nucl.Phys. B (Proc.Suppl.) 47 (1996). Lattice’96, Proceedings of the 14th International Symposium on Lattice Field Theory, St. Louis, USA, 4 - 8 June 1996, Nucl.Phys. B (Proc.Suppl.) 53 (1997).

[7] I.I.Bigi, M.A.Shifman, N.G.Uraltsev and A.I.Vainshtein, Phys.Rev. D50 (1994) 2234. hep-ph/9602360.

[8] M.Beneke and V.M.Braun, Nucl.Phys. B246 (1994) 301. hep-ph/9602364.

[9] A.Buras, Proceedings of the Workshop on K Physics, Orsay, France, 30 May - 4 Jun 1996. hep-ph/9609324.

[10] H.Leutwyler, Phys.Lett. B378 (1996) 313. hep-ph/9602366.

[11] A.Duncan, E.Eichten and H.Thacker, Phys.Rev.Lett. 76 (1996) 3894. hep-lat/9602005.

[12] C.R.Allton, M.Ciuchini, M.Crisafulli, E.Franco, V.Lubicz and G.Martinelli, Nucl.Phys. B431 (1994) 667. hep-ph/9406263.

[13] G.Martinelli, C.Pittori, C.T.Sachrajda, M.Testa and A.Vladikas, Nucl.Phys. B445 (1995) 81. hep-lat/9411010.

[14] R.Gupta and T.Bhattacharya, Phys.Rev. D55 (1997) 7203. hep-lat/9605039.

[15] B.J.Gough, G.M.Hockney, A.X.El-Khadra, A.S.Kronfeld, P.B.Mackenzie, B.P. Mertens, T.Onogi and J.N.Simone, FERMILAB-PUB-96-283-T. hep-ph/9610223.

[16] C.R.Allton, V.Gimenez, L.Giusti and F.Rapuano, Nucl.Phys. B489 (1997) 427. hep-lat/9611021.

[17] SESAM Collaboration, N.Eicker et al., WUB-97-14. hep-lat/9704013.
[18] K.G.Chetyrkin, D.Pirjol and K.Schilcher, MZ-TH-96-27. hep-ph/9612394.

[19] P.Colangelo, F.De Fazio, G. Nardulli and N.Paver, BARI-TH-97-262. hep-ph/9704249.

[20] C.A.Dominguez, G.R.Gluckman and N.Paver, Phys.Lett. B333 (1994) 184. hep-ph/9406329.

[21] M.Crisafuli, V.Gimenez, G.Martinelli and C.T. Sachrajda, Nucl.Phys. B457 (1995) 594. hep-ph/9506210.
V.Gimenez, G.Martinelli and C.T. Sachrajda, Phys.Lett. B393 (1997) 124. hep-lat/9607018.

[22] C.T.H.Davies, K.Hornbostel, A.Langnau, G.P.Lepage, A.Lidsey, C.J.Morningstar, J.Shigemitsu and J.Sloan, Phys.Rev.Lett. 73 (1994) 2654. hep-lat/9404012.

[23] M.Jamin and A.Pich, IFIC-97-06. hep-ph/9702276.

[24] L.K.Gibbons, UR-1494. Invited talk at the 28th International Conference on High-energy Physics (ICHEP 96), Warsaw, Poland, 25-31 Jul 1996. hep-ex/9704017.

[25] V.Lubicz, G.Martinelli and C.T.Sachrajda, Nucl.Phys. B356 (1991) 301.
V.Lubicz, G.Martinelli, M.S.McCarthy and C.T.Sachrajda, Phys.Lett. B274 (1992) 415.

[26] C.W.Bernard, A.X.El-Khadra and A.Soni, Phys.Rev. D43 (1991) 2140; Phys.Rev. D45 (1992) 869.

[27] ELC Collaboration, A.Abada et al., Nucl.Phys. B416 (1994) 675. hep-lat/9308007.

[28] UKQCD Collaboration, K.C.Bowler et al., Phys.Rev. D51 (1995) 4905. hep-lat/9410012.

[29] APE Collaboration, C.R.Allton et al., Phys.Lett. B345 (1995) 513. hep-lat/9411011.

[30] T.Bhattacharya and R.Gupta, Nucl.Phys. B (Proc.Suppl.) 47 (1996) 481. hep-lat/9512007.

[31] S.Gusken, K.Schilling and G.Sieger, Nucl.Phys. B (Proc.Suppl.) 47 (1996) 485. hep-lat/9510007.
[32] P. Ball, V.M. Braun and H.G. Dosch, Phys. Rev. D44 (1991) 3567.

[33] N. Isgur, D. Scora, B. Grinstein and M.B. Wise, Phys. Rev. D39 (1989) 799.
D. Scora and N. Isgur, Phys. Rev. D40 (1989) 1491.

[34] M. Wirbel, B. Stech and M. Bauer, Z. Phys. C29 (1985) 637.

[35] N. Isgur and M.B. Wise, Phys. Rev. D42 (1990) 2388.

[36] UKQCD Collaboration, D.R. Burford et al., Nucl. Phys. B447 (1995) 425.
hep-lat/9503002 J. M. Flynn et al., Nucl. Phys. B461 (1996) 327. hep-ph/9506398.

[37] NA31 Collaboration, G.D. Barr et al., Phys. Lett. B317 (1993) 233.

[38] E731 Collaboration, L.K. Gibbons et al., Phys. Rev. Lett. 70 (1993) 1203.

[39] L. Wolfenstein, Phys. Rev. Lett. 13 (1964) 562.

[40] M. Lusignoli, L. Maiani, G. Martinelli and L. Reina, Nucl. Phys. B369 (1992) 139.

[41] M. Ciuchini, E. Franco, G. Martinelli, L. Reina and L. Silvestrini, Z. Phys. C68 (1995) 239. hep-ph/9501265.
M. Ciuchini, Invited talk at the 4th KEK Topical Conference on Flavor Physics, Tsukuba, Japan, 29-31 Oct 1996. hep-ph/9701278.

[42] J. M. Flynn, Presented at the 28th International Conference on High-energy Physics (ICHEP 96), Warsaw, Poland, 25-31 Jul 1996. hep-lat/9611016.

[43] A. Pich and E. de Rafael, Phys. Lett. B158 (1985) 477.
J. Prades et al., Z. Phys. C51 (1991) 287.

[44] J. F. Donoghue, E. Golowich and B. R. Holstein, Phys. Lett. B119 (1982) 412.

[45] G. Martinelli, Presented at Beauty 96, Rome, Italy, 17-21 Jun 1996.
Nucl. Instrum. Meth. A384 (1996) 241. hep-ph/9610455.

[46] J. D. Richman, Invited talk at the 28th International Conference on High-energy Physics (ICHEP 96), Warsaw, Poland, 25-31 Jul 1996. hep-ex/9701014.

[47] M. Luscher, S. Sint, R. Sommer and P. Weisz, Nucl. Phys. B478 (1996) 365.
hep-lat/9605038.

[48] M. Ciuchini, E. Franco, G. Martinelli and L. Silvestrini, Phys. Lett. B380 (1996) 362.