Escaping the Large Fine-Tuning and Little Hierarchy Problems in the Next to Minimal Supersymmetric Model and $h \to aa$ Decays

Radovan Dermišek and John F. Gunion

Department of Physics, University of California at Davis, Davis, CA 95616

We demonstrate that the NMSSM can have small fine-tuning and modest light stop mass while still evading all experimental constraints. For small $\tan \beta$ (large $\tan \beta$), the relevant scenarios are such that there is always (often) a SM-like Higgs boson that decays to two lighter — possibly much lighter — pseudoscalar Higgses.

In the CP-conserving Minimal Supersymmetric Model (MSSM), large soft-supersymmetry-breaking mass parameters are required in order that the one-loop corrections to the tree-level prediction for the lightest Higgs boson ($m_h \leq m_Z$) increase $m_h$ sufficiently to avoid conflict with lower bounds from LEP data. The large size of these soft-SUSY breaking masses compared to the weak scale, the natural scale where supersymmetry is expected, is termed the little-hierarchy problem. This hierarchy implies that a substantial amount of fine-tuning of the MSSM soft-SUSY breaking parameters is needed. The severity of these problems has led to a variety of alternative approaches. For instance, little Higgs models \cite{1} can be less fine tuned. Or, one can argue that large fine-tuning is not so bad, as in "split-supersymmetry" \cite{2}. In this letter, we show that the Next to Minimal Supersymmetric Model (NMSSM \cite{3}) can avoid or at least ameliorate the fine-tuning and little hierarchy problems. In addition, we find that parameter choices that are consistent with all LEP constraints and that yield small fine-tuning at small $\tan \beta$ (large $\tan \beta$) are nearly always (often) such that there is a relatively light SM-like CP-even Higgs boson that decays into two light, perhaps very light, pseudoscalars. Such decays dramatically complicate the Tevatron and LHC searches for Higgs bosons.

The NMSSM is very attractive in its own right. It provides a very elegant solution to the $\mu$ problem of the MSSM via the introduction of a singlet superfield $\hat{S}$. For the simplest possible scale invariant form of the superpotential, the scalar component of $\hat{S}$ naturally acquires a vacuum expectation value of the order of the SUSY breaking scale, giving rise to a value of $\mu$ of order the electroweak scale. The NMSSM is the simplest supersymmetric extension of the standard model in which the electroweak scale originates from the SUSY breaking scale only. A possible cosmological domain wall problem \cite{4} can be avoided by introducing suitable non-renormalizable operators \cite{5} that do not generate dangerously large singlet tadpole diagrams \cite{6}. Hence, the phenomenology of the NMSSM deserves to be studied at least as fully and precisely as that of the MSSM.

Radiative corrections to the Higgs masses have been computed \cite{7,8,9,10} and basic phenomenology of the model has been studied \cite{11}. The NMHDECAY program \cite{12} allows easy exploration of Higgs phenomenology in the NMSSM. In particular, it allows for the possibility of Higgs to Higgs pair decay modes (first emphasized in \cite{13} and studied later in \cite{14}) and includes the associated modifications of LEP limits. Of greatest relevance are $h \to aa$ decays, where $h$ is a SM-like CP-even Higgs boson and $a$ is a (mostly singlet) CP-odd Higgs boson. The relevant limits come from the analysis \cite{15} of the $Zh \to Zaa \to Zb\bar{b}S\bar{S}$ channel and the analysis \cite{16} of the $Zh \to Zaa \to Z\tau^+\tau^-\tau^+\tau^-$ channel. The weaker nature of the limits from LEP on such scenarios will play an important role in what follows.

The extent to which there is a no-lose theorem for NMSSM Higgs discovery at the LHC has arisen as an important topic \cite{13,17,18,19,20}. In particular, it has been found that the Higgs to Higgs pair decay modes can render inadequate the usual MSSM Higgs search modes that give rise to a no-lose theorem for MSSM Higgs discovery at the LHC. And, it is by no means proven that the Higgs to Higgs pair modes are directly observable at the LHC, although there is some hope \cite{18,19}.

Earlier discussions of fine-tuning in the NMSSM have been given in \cite{21,22}.

We very briefly review the NMSSM. Its particle content differs from the MSSM by the addition of one CP-even and one CP-odd state in the neutral Higgs sector (assuming CP conservation), and one additional neutralino. We will follow the conventions of \cite{12}. Apart from the usual quark and lepton Yukawa couplings, the scale invariant superpotential is

$$\lambda \hat{S}\hat{H}_u\hat{H}_d + \frac{\kappa}{3} \hat{S}^3$$

(1)

depending on two dimensionless couplings $\lambda, \kappa$ beyond the MSSM. [Hatted (unhatted) capital letters denote superfields (scalar superfields components).] The associated trilinear soft terms are

$$\lambda A_a SH_u H_d + \frac{\kappa}{3} A_s S^3.$$  (2)

The final two input parameters are

$$\tan \beta = h_u/h_d, \quad \mu_{\text{eff}} = \lambda s,$$  (3)

where $h_u \equiv \langle H_u \rangle$, $h_d \equiv \langle H_d \rangle$ and $s \equiv \langle S \rangle$. These, along with $m_Z$, can be viewed as determining the three SUSY breaking masses squared for $H_u$, $H_d$ and $S$ (denoted $m_{H_u}^2$, $m_{H_d}^2$ and $m_S^2$) through the three minimization equations of the scalar potential.

Thus, as compared to the three independent parameters needed in the MSSM context (often chosen as $\mu$, $\tan \beta$, and $m_{\tilde{g}}$), the NMSSM requires only two independent parameters $\lambda$ and $\kappa$. However, the additional singlet superfield $\hat{S}$ allows a vast array of properties for the neutral Higgs sector, subject to only the constraint \cite{13,17,18,19,20}.

The implications for $h \to aa$ decays are considered in \cite{13,17,18,19,20}. The relevance of these processes to new physics searches at the LHC is determined by the assumption $m_a < m_H < m_S < m_{\tilde{g}}$, which is the case for the NMSSM. In this limit, the $a$ can only decay to $H_u$ and the $H_u$ to two lighter Higgses $h$ and $a$.

The $h \to aa$ decays contribute to several observables which are sensitive to the lightest Higgs mass $m_a$. This will lead to a no-lose theorem when the LHC reaches the appropriate energy (estimated to be around $10^{16}$ eV).

The NMSSM is also of great importance for the interpretation of the Tevatron Higgs search. The lightest Higgs boson $h$ can decay to $\tau\tau$ pairs, which are very difficult to detect due to $W$ and $Z$ background. In this case, the constraint from the Tevatron Higgs search can be relaxed as $m_h > 115$ GeV.

The NMSSM thus provides a very attractive model for the search for new physics at the LHC, and we look forward to the results of the LHC experiments.
tan β and \(M_A\), the Higgs sector of the NMSSM is described by the six parameters
\[
\lambda, \kappa, A_0, A_{\kappa}, \tan \beta, \mu_{\text{eff}}.
\] (4)
We will choose sign conventions for the fields such that \(\lambda\) and \(\tan \beta\) are positive, while \(\kappa, A_0, A_{\kappa}\) and \(\mu_{\text{eff}}\) should be allowed to have either sign. In addition, values must be input for the gaugino masses and for the soft terms related to the (third generation) squarks and sleptons that contribute to the radiative corrections in the Higgs sector and to the Higgs decay widths.

Sample discussions of the fine-tuning issues for the MSSM appear in [24]. We will define
\[
F = \text{Max}_a F_a = \text{Max}_a \left| \frac{d \log m_Z}{d \log a} \right|,
\] (5)
where the parameters \(a\) comprise \(\mu, B_\mu, B_a\) and other GUT-scale soft-SUSY-breaking parameters. (In some papers, \(\frac{d \log m_Z}{d \log a}\) is employed.) In our approach, we choose \(m_Z\)-scale values for all the squark soft masses squared, the gaugino masses, \(M_{1,2,3}(m_Z)\), \(A_0(m_Z)\) and \(A_{\kappa}(m_Z)\) (with no requirement of universality at the GUT scale).

We also choose \(m_Z\)-scale values for \(\tan \beta, \mu\) and \(M_A\); these uniquely determine \(B_{\beta}(m_Z)\). The vevs \(h_u, h_d\) at scale \(m_Z\) are fixed by \(\tan \beta\) and \(m_Z\) via \(m_Z^2 = \mathcal{G}(h_u^2 + h_d^2)\) (where \(\mathcal{G} = g^2 + g'^2\)). Finally, \(m_{\tilde{H}_u}(m_Z)\) and \(m_{\tilde{H}_d}(m_Z)\) are determined by the two potential minimization conditions. We then evolve all parameters to the MSSM GUT scale (including \(\mu\) and \(B_{\mu}\)). Next, we shift each of the GUT-scale parameters in turn, evolve back down to scale \(m_Z\), and re-minimize the Higgs potential using the shifted values of \(\mu, B_{\mu}, m_{\tilde{H}_u}\) and \(m_{\tilde{H}_d}\). This gives new values for \(h_u, h_d\) yielding new values for \(m_Z\) and \(\tan \beta\).

Results will be presented for \(\tan \beta(m_Z) = 10, M_{1,2,3}(m_Z) = 100, 200, 300\) GeV. We scan randomly over \(|A_0(m_Z)| \leq 500\) GeV and 3rd generation squark and slepton soft mass-squared above (200 GeV)\(^2\), as well as over \(|\mu(m_Z)| \geq 100\) GeV, \(\text{sign}(\mu) = \pm\) and over \(m_A > 120\) GeV (for which LEP, MSSM constraints require \(m_h \gtrsim 114\) GeV [24]). On the left side of Fig. 1 we plot \(F\) as a function of the mean stop mass \(\sqrt{m_{\tilde{t}_1}m_{\tilde{t}_2}}\), which enters into the computation (we use HDECAY [27] with \(m_{\text{pole}} = 175\) GeV of the radiative correction to the SM-like light Higgs mass \(m_h\). Points plotted as +’s (×’s) have \(m_h < 114\) GeV (\(m_h \geq 114\) GeV) and are excluded (allowed) by LEP data. Very modest values of \(F\) (of order \(F \sim 5\)) are possible for \(m_h < 114\) GeV but the smallest \(F\) value found for \(m_h \geq 114\) GeV is of order \(F \sim 185\) [27]. The very rapid increase of the smallest achievable \(F\) with \(m_h\) is illustrated in the right plot of Fig. 1. This is the essence of the current fine-tuning problem for the CP-conserving MSSM. Also, to achieve \(m_h > 114\) GeV, \(\sqrt{m_{\tilde{t}_1}m_{\tilde{t}_2}} \geq 1\) TeV is required, an indicator of the little hierarchy problem. We now contrast this to the NMSSM situation. One combination of the three potential minimization equations yields the usual MSSM-like expression for \(m_Z^2\) in terms of \(\mu^2, \tan \beta, m_{\tilde{H}_u}^2\) and \(m_{\tilde{H}_d}^2\), with \(\mu\) replaced by \(\mu_{\text{eff}}\). However, a second combination gives an expression for \(\mu_{\text{eff}}\) in terms of \(m_Z^2\) and other Higgs potential parameters. Eliminating \(\mu_{\text{eff}}\), we arrive at an equation of the form
\[
m_1^2 + 2Bm_2^2 + C = 0,
\]
with solution \(m_Z^2 = -B \mp \sqrt{B^2 - C}\), where \(B\) and \(C\) are given in terms of the soft susy breaking parameters, \(\lambda, \kappa\) and \(\tan \beta\). Only one of the solutions to the quadratic equation applies for any given set of parameter choices. Small fine-tuning is typically achieved when \(C < B^2\) and derivatives of \(m_Z^2\) with respect to a GUT scale parameter tend to cancel between the \(-B\) and \(\mp \sqrt{B^2 - C}\) for \(B > 0\) (for \(B < 0\)).

To explore fine-tuning, we proceed analogously to the manner described for the MSSM. At scale \(m_Z\), we fix \(\tan \beta\) and scan over values of \(\lambda \leq 0.5\) (\(\lambda \leq 0.7\) is required for perturbativity up to the GUT scale), \(|\kappa| \leq 0.3\), \(\text{sign}(\kappa) = \pm\) and 100 GeV \(\leq |\mu_{\text{eff}}| \leq 1.5\) TeV, \(\text{sign}(\mu_{\text{eff}}) = \pm\). We also choose \(m_Z\)-scale values for the soft-SUSY-breaking parameters \(A_\lambda, A_\kappa, A_t = A_h, M_1, M_2, M_1^2, m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2, m_{\tilde{h}_d}^2, \) and \(m_{\tilde{H}_d}^2\), all of which enter into the evolution equations. We process each such choice through NMHDECAY (using \(m_{\text{pole}} = 175\) GeV) to check that the scenario satisfies all theoretical and available experimental constraints (including \(m_{\tilde{t}_1}^2 \geq 100\) GeV). For accepted cases, we then evolve to determine the GUT-scale values of all the above parameters. The fine-tuning derivative for each parameter is determined by shifting the GUT-scale value for that parameter by a small amount, evolving all parameters back down to \(m_Z\), redetermining the potential minimum (which gives new values \(h_u^*\) and \(h_d^*\)) and finally computing a new value for \(m_Z^2\) using \(m_Z^2 = \mathcal{G}(h_u^* h_u^2 + h_d^2)\).

Our results for \(\tan \beta = 10\) and \(M_{1,2,3}(m_Z) = 100, 200, 300\) GeV and randomly chosen values for the soft-SUSY-breaking parameters listed earlier are dis-
We note that all points with $F < \text{a few}$. For both classes of points, the $h$ points with $F$ have somewhat suppressed; $h_1 \to a_1 a_1$ decays are dominant; this makes it possible for the naturally less fine-tuned values of $m_{h_1} \geq 114$ GeV to be LEP-allowed. Second, small $F$ is frequently (nearly always) achieved for $m_{h_1} < 114$ GeV ($m_{h_1} \geq 114$ GeV) via the cancellation mechanism noted earlier, where $C \ll B^2$, and this mechanism generally works mainly for small $A_\kappa$. Indeed, there are many phenomenologically acceptable parameter choices with $m_{h_1} > 114$ GeV that have large $A_\kappa$, but these all also have very large $F$.

For lower $\tan \beta$ values such as $\tan \beta = 3$, extremely large $\sqrt{m_{h_1} m_{h_2}}$ is required for $m_h > 114$ GeV in the MSSM, leading to extremely large $F$. Results in the NMSSM for $\tan \beta = 3$ are plotted in Fig. 4 for $M_{1,2,3}(m_Z) = 100, 200, 300$ GeV and scanning as in the $\tan \beta = 10$ case. We see that $F \sim 15$ is achievable for $\sqrt{m_{h_1} m_{h_2}} \sim 300$ GeV. No points with $m_{h_1} > 114$ GeV were found. All the plotted points escape LEP limits because of the dominance of the $h_1 \to a_1 a_1$ decay. For very large $\tan \beta$ (e.g. $\tan \beta \sim 50$), it is possible to obtain $m_h > 114$ GeV with relatively small $\sqrt{m_{h_1} m_{h_2}}$ in the MSSM as well as in the NMSSM. We have not yet studied fine-tuning at very large $\tan \beta$ in either model.
contribution to $m_h^2$, proportional to $\lambda^2$, thereby allowing $m_h > 114$ GeV for somewhat smaller $\sqrt{m_t m_b}$ than in the MSSM. This, in turn, reduces the fine-tuning and little hierarchy problems, but not nearly to the extent achieved by our parameter choices. In our plots, the SM-like $h$ is always the $h_1$. The points with very small $F$ have low $\sqrt{m_t m_b}$, modest $\lambda$ and $\kappa$, and escape LEP constraints not because $m_h$ is large but because $h \to aa$ decays are dominant.

In conclusion, we reemphasize that the NMSSM provides a rather simple escape from the large fine-tuning and (little) hierarchy problems characteristic of the CP-conserving MSSM. However, the relevant NMSSM models imply a high probability for $h_1 \to a_1 a_1$ decays to be dominant. We speculate that similar results will emerge in many supersymmetric models where the Higgs sector is more complicated than that of the MSSM. Higgs detection in such a decay mode should be pursued with greatly increased vigor. Existing work [18, 19] which suggests a very marginal LHC signal for $\gamma\gamma \to h$ when $m_{a_1} > 2m_h$ should be either refuted or improved upon. In addition, the $a_1 a_1 \to \tau^+ \tau^- \tau^+ \tau^-$ channel that dominates for $2m_r < m_{a_1} < 2m_h$ (an entirely acceptable and rather frequently occurring mass range in our parameter scans and not excluded by $\Upsilon$ decays since the $a_1$ has a large singlet component) should receive immediate attention. Hopefully, we will not have to wait for Higgs discovery at an $e^+e^-$ linear collider via the inclusive $Zh \to \ell^+\ell^-X$ reconstructed $M_X$ approach (which allows Higgs discovery independent of the Higgs decay mode) or at a CLIC-based $\gamma\gamma$ collider [22] in the $\gamma\gamma \to h \to b\bar{b} \nu^+\nu^-$ or $\tau^+\tau^-\tau^+\tau^-$ modes.

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