The Depression Produced by Impellers and the Similarity Problem

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Abstract. The authors intend to apply original analytical formulas to estimate the depression produced by different types of impellers. The problem has been approached in different studies, especially by numerical simulations and not from the analytical point of view. The study is useful to estimate not only the magnitude of the depression, but also the dimensions of the depression zone. Unpleasant effects can be produced if cavitations appear, leading to propelling unit damage and consequently increasing the operation costs of different applications, such as power industry, chemical industry equipment or naval transport. The authors test impellers of different sizes in two experimental setups in order to characterize the phenomenon.

1. Introduction
Analytical formulas are always preferred by engineers because they are easier to use. The previous original contributions of the authors [1; 2] are used in this paper for other kind of impellers and practical conclusions are obtained. A short numerical example is also presented and a similarity study is added. Experimental results are used to complete the theoretical original formulas to estimate the behavior of impellers in different practical cases.

2. The theoretical characteristics of the depression zone
For a depression zone as in figure 1, one can see the equilibrium of the forces involved in defining the depression zone, filled usually by a two phase fluid. The parameters which can characterize the depression zone were described in detail in [1, 2]. These parameters are the radius R and the height H of the parabolic depression zone and also the pressure inside the depression zone.

As previously computed [1; 2], the radius of the depression zone is:

\[
R = \sqrt{\frac{2}{\rho e^2 t^2}}(p_e - p_v)
\]

(1)

In the previous formula, at the radius R the pressure becomes equal with the vaporisation pressure \( p_v \), the exterior liquid (water) has the pressure \( p_e \), \( \varepsilon \) is the tangential acceleration, \( \rho \) the liquid density and \( t \) the time variable.
The pressure inside the depression zone can be expressed depending on the pressure in the exterior with:

$$p = p_e - \rho \varepsilon^2 t^2 r^2$$

(2)

The height of the parabolic depression zone is given by:

$$H = \frac{\varepsilon^2 t^2}{2g} R^2$$

(3)

3. Case study

The following amounts for a hydropower plant are considered: $p_e = 10^5$ Pa, $p_v = 2 \cdot 10^3$ Pa, $\rho = 1000$ kg/m$^3$, the total necessary time to bring the turbine to synchronism $t_{total} = 5$ min = 300 s and the final rotational speed as the rated speed of the runner $n = 750$ rot/min.

One can determine the rotational speed corresponding to synchronism, respectively the acceleration of the circular movement. So, the final radius R can be obtained by applying Eq. (1).

Retaining two decimal fractions, R and D, respectively (the diameter of the depression zone) become: $R = 0.18$ m, $D = 0.36$ m.

$$\omega_{final} = \frac{\pi n}{30} = 25\pi \text{ rad/s}, \quad \varepsilon = \frac{\omega_{final}}{t_{total}} = \frac{\pi}{12} \text{ rad/s}^2.$$  

For the numerical values previously specified one obtains:

$$p = 10^5 \left(1 - \frac{25\pi^2}{8} r^2\right),$$

so the pressure is a two degree function of the radius r.

Using the previous numerical values and substituting in (3), from figure 1 it results the depression zone’s height, $H = 10$ m.

4. Experimental results

The first experimental measurement was made on an experimental setup (figure 2) operating at a rated speed of the impeller $n = 2520$ rot/min. The angle of the blades is $45^\circ$, so the rotating impeller has two effects.

The first is a centrifugal force which produces the depression zone and the second is an increased height of the parabola caused by an increased vertical flow.
After reaching the rated rotational speed, the movement of the impeller is permanent, so

$$\varepsilon t = \omega, \quad \omega = \frac{\pi n}{30} \approx 264 \text{rad/s.}$$

Substituting in (3), for example, a correspondence between the radius and the depression zone height is obtained:

$$H = 3552 \cdot R^2$$

(4)

By comparing the results of the carried out experiments, namely the values $R = 5 \text{ mm}$ and $H = 9 \text{ cm}$, with the theoretical value $H = 0.089 \text{ m}$ obtained by using the relation (4), a good agreement of the results can be noticed.

The second experimental setup (figure 3) consists of a transparent cylindrical vessel equipped with a variable speed impeller, electronic speed control and speed measurement system.

The aim of the second set of experiments was to assess the influence of the width of impeller blades used to circulate the fluid (figure 4), on the depression magnitude.

The measured depression values and friction moments at different speeds for the four types of impellers are presented in Table 1.
Table 1.

| n [rot/min] | Impeller with 2 thin blades (10 mm width) | Impeller with 2 medium blades (20 mm width) | Impeller with 2 wide blades (40 mm width) | Impeller with 2 shorter blades (20 mm width) |
|------------|------------------------------------------|--------------------------------------------|--------------------------------------------|---------------------------------------------|
|            | $\Delta H$ [mmH$_2$O] | $M$ [N·cm] | $\Delta H$ [mmH$_2$O] | $M$ [N·cm] | $\Delta H$ [mmH$_2$O] | $M$ [N·cm] | $\Delta H$ [mmH$_2$O] | $M$ [N·cm] |
| 60         | 0 | 2.3 | 0 | 2.5 | 1 | 2.4 | 0 | 2.1 |
| 100        | 1 | 2.4 | 1 | 3.0 | 5 | 2.9 | 0 | 2.3 |
| 150        | 2 | 2.8 | 5 | 3.5 | 11 | 3.7 | 0 | 2.5 |
| 200        | 4 | 3.2 | 11 | 4.3 | 21 | 4.7 | 0 | 2.8 |
| 250        | 8 | 3.8 | 18 | 5.3 | 35 | 6.0 | 2 | 3.0 |
| 300        | 11 | 4.3 | 27 | 6.3 | 49 | 7.4 | 7 | 3.2 |
| 350        | 16 | 5.0 | 38 | 7.6 | 67 | 9.3 | 10 | 3.5 |
| 400        | 20 | 5.7 | 52 | 9.2 | 88 | 11.3 | 15 | 3.8 |
| 450        | 28 | 6.6 | 64 | 10.8 | 18 | 4.1 |
| 500        | 36 | 7.6 | 80 | 12.6 | 23 | 4.4 |
| 550        | 48 | 8.5 | 96 | 14.4 | 28 | 4.7 |
| 600        | 58 | 9.7 | 110 | 16.8 | 33 | 5.0 |
| 650        | 69 | 11.0 | 126 | 19.2 | 39 | 5.4 |
| 700        | 81 | 12.3 | 144 | 21.6 | 46 | 5.9 |
| 750        | 93 | 13.8 | 164 | 24.0 | 55 | 6.3 |

The impellers have a blade length of 100 mm at a hub diameter of 20 mm. The impeller with shorter blades has the length of 60 mm at a width of 20 mm. All the blades are vertical, the same as the hub. The value of the depression $\Delta H$, expressed in mmH$_2$O, is shown in figure 5.

Figure 5. The depression produced by the four types of impellers

By analyzing figure 5, it results that the depressions are parabola functions and correspond with the theoretical difference of pressure as a function of second degree of the variable radius $r$, as in (2).

The centrifugal force is increasing with the width of the blade and, as a consequence, the depression follows the same trend. Also, it can be noticed that the impeller with shorter blades realizes the smallest depression, as in figure 5.
The momentum has a similar variation, as in figure 6.

![Figure 6. The momentum produced by the four types of impellers](image)

These types of graphs can be used at the computation of the power needed by impellers to perform certain types of tasks. As previously seen, the momentum grows with the increase of the blades width and decreases in the case of a shorter blade impeller.

5. Similitude criteria

The centrifugal forces are preponderant in the studied phenomenon and they depend on the tangential acceleration. They act as inertia forces. For this reason, it is appropriate to chose as a similitude criterion the Newton criterion:

\[ Ne = \frac{F}{\rho V^2 l^2} \]  
\[ \text{(5)} \]

Then, one can obtain:

\[ Ne = Ne', \text{ respectively } \frac{F}{\rho V^2 l^2} = \frac{F'}{\rho' V'^2 l'^2}. \]  
\[ \text{(6)} \]

Because water is used in both experimental setups, we consider that \( \rho = \rho' \); substituting \( V = \omega r \) and replacing the characteristic length with the variable radius \( r \), it results:

\[ \frac{F}{\omega^2 r^4} = \frac{F'}{\omega'^2 r'^4}. \]  
\[ \text{(7)} \]

By considering \( \omega \) and \( r \) for the turbine, the centrifugal force can be obtained as:

\[ F = m\omega^2 r \]  
\[ \text{(8)} \]

Taking into account formula (7), for a certain \( \omega' \) used in the model installation, it results \( r' \); thus, by using the similitude criterion, one can obtain the force \( F' \):

\[ F' = F \left( \frac{\omega'}{\omega} \right) \left( \frac{r'}{r} \right)^4 \]  
\[ \text{(9)} \]

Now, all the amounts for the model installation are known. Obviously, if the amounts used in the model installation are known, the amounts corresponding to the prototype installation can be calculated.
6. Conclusions
Using different types of analyses one can conclude that the original theoretical simplified formulas of
the authors, (1), (2) and (3) correspond with the phenomenon and, as a consequence, they can be used
in practical applications.

The magnitude of the depression under the impellers and the magnitude of the resistant moment
can be increased by varying the length and width of the blades, as shown in figure 5 and figure 6.

The results of the research are useful in practical applications which require a preliminary estimate
of the phenomena that characterize the real case operation. Moreover, the means for performing the
necessary assessment of the real applications are provided by the current research, through the original
similitude calculus synthesized by formula (9).

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