Large Multiplicity Gluon Production in QCD

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Abstract

We compute the approximate cross section $\sigma_n^{(0)}$ for producing $n-2$ resolved gluons in a gluon-gluon collision, using the Parke-Taylor formula regularized in a Lorentz invariant manner. We find, in double leading logarithm approximation, that

$$\sigma_n^{(0)} \approx \frac{1}{s} \left( \frac{N_c \alpha_s}{2\pi \sqrt{12}} \ln^2 \frac{s}{s_{\text{cut}}} \right)^{n-2},$$

where $\sqrt{s_{\text{cut}}}$ is the minimum invariant mass for a resolved gluon pair. There is no factor of $1/(n-2)!$ multiplying the expression. We present additional numerical results, and comment on their implications for perturbative calculations of $n$-jet cross sections at colliders.
Introduction

There is little doubt that QCD, the $SU(3)$ gauge theory describing the interactions of quarks and gluons, is the underlying theory of strong interactions. Various experimental results from both electron-positron machines [1] and hadron colliders [2] are in good agreement with QCD predictions. Use of sophisticated Monte Carlo generators based on coherent branching processes which can take into account both initial and final state radiation [3] are able to reproduce the CDF data [4] up to six jets.

Multiple QCD jets will, of course, constitute important background for new physics discovery at the future SSC/LHC colliders. Thus, there is practical importance in being able to calculate differential cross sections for $n$-jet processes in QCD at collider energies, with cuts appropriate to experiments and detectors of interest. At a more theoretical level, recent work on the behavior of multiparticle amplitudes in field theories of massive bosons [5, 6, 7, 8, 9] has revealed non-perturbative behavior for tree level cross sections near threshold. It is an interesting question whether the most established bosonic field theory (QCD) maintains a perturbative behavior at tree level for multiple jet production.

In this Letter, we will focus on the behavior of the exclusive cross section for the process $gg \rightarrow (n-2)g$ [10]. All final state partons will be separated in a Lorentz-invariant way, by a given fixed minimum amount $(p_i + p_j)^2 \geq s_{\text{cut}}$. We will dispense with regions of phase space where our cuts are not obeyed, so that we do not merge unresolved gluons into jets; it is in this sense that our result will be an exclusive cross section. We will concentrate on the limiting case of the production of a large number of final state gluons. Our results will depend on the ratio $\Delta = s_{\text{cut}}/s$, where $s$ is the parton-parton center-of-mass (c.m.) energy squared (usually denoted by $\hat{s}$). Different values of $\Delta$ discriminate whether or not the produced jets constitute important backgrounds for new physics; it is also possible that multi-minijet jet cross sections (small $\Delta$) may be the most important QCD contribution to the total hadronic cross section. A typical value of $\Delta$ for interesting jets at the SSC ($p_T > 50$ GeV, $\theta_{ij} > 30^\circ$, $\sqrt{s} \approx 4$ TeV) is $\Delta \approx 10^{-5}$.

We will also be concerned with the validity of perturbation theory in the approximate tree-level calculation we will perform. Naively, each time there is an extra parton in the final state one should pay a price of a coupling constant. However, it is possible that in calculating certain quantities the expansion parameter becomes effectively the coupling constant multiplied by a number that may be large. In this case, care must be taken either by summing up these large contributions to all orders or, more modestly, performing perturbative calculations only when the effective expansion parameter is small.

Exact multi-parton QCD amplitudes may be generated through use of the Berends-
Giele recursion relations [11]; amplitudes for \( n = 8 \) (six final state gluons) have been given explicitly [12]. However, the complexity of these amplitudes makes their usefulness rather limited by the computer time required for their evaluation. Since it is the aim of this work to examine cross sections for large numbers of final state gluons, we will search for a reliable approximation for these exact multiparticle cross sections.

**The Parke-Taylor Formula**

As a first step to such an approximation, we will use as the squared amplitude for the process \( gg \rightarrow (n-2)g \) the so-called Parke-Taylor (PT) formula [13], which was conjectured in Ref. [13], and later proved in Ref. [11]. Quite surprisingly, the PT result can also be derived from soft-gluon factorization techniques [14]. This formula is the result in leading order in the number of colors \((N_c = 3)\) for the square of the amplitude describing the process \( gg \rightarrow (n-2)g \), summed over colors and over a particular set of gluon helicity configurations:

\[
|A_{PT}^n|^2 = 2g_s^{2n-4}N_c^{n-2}(N_c^2 - 1) \sum_{i>j} s_{ij}^4 \sum_{P'} \frac{1}{s_{12}s_{23}s_{34}...s_{n1}}
\]

where \( g_s \) is the QCD coupling constant, \( s_{ij} = (p_i + p_j)^2 = 2p_i \cdot p_j \) and the primed sum is over the \((n-1)!/2\) non-cyclic permutations of \((1, 2, ..., n)\). The relevant set of configurations is the one where all gluons but two have the same helicities, the so-called maximally helicity-violating (MHV) amplitudes. There are \( n(n-1) \) MHV amplitudes for \( n > 4 \).

This leading \( N_c \) result contains all the leading collinear and soft divergencies in it and furthermore it preserves all the coherence effects and includes initial and final state radiation interference.

In comparing the PT formula with exact results one should devise methods to take into account the contributions from amplitudes other than MHV ones. Here we’ll adopt the simplest approach, due to Kunszt and Stirling [15] which is to assume that all the amplitudes are of comparable size. We’ll call this procedure the KS approximation. Hence, since there are \( 2^n - 2(n + 1) \) non-zero helicity amplitudes in total we simply multiply the PT formula by a combinatorial factor which counts the number of non-vanishing amplitudes:

\[
|A_{KS}^n|^2 = \frac{2^n - 2(n + 1)}{n(n-1)} |A_{PT}^n|^2.
\]
transverse momentum jets which may turn out to be backgrounds for interesting new physics. Typical cuts are: $p_T \geq 50 \text{ GeV} ; \; |\eta| \leq 3 ; \; \theta_{ij} \geq 30^\circ$ for the LHC. Kunszt and Stirling [13] have found good agreement between the exact $gg \to gggg$ cross section and the PT formula in the KS approximation and they have also used the PT result to compute up to $gg \to gggggg$ and found a broad agreement with the multiplicity of large $p_T$ jets up to six jets seen at the UA1 detector. The exact $gg \to gggg$ result has also been tested against various approximations schemes [12, 16, 17] for different sets of cuts. The overall conclusion is that the PT formula in the KS approximation overestimates the exact results by as much as 50% for tight cuts but becomes a better approximation for looser cuts. This can be understood since for loose cuts the event rate is dominated by the infrared and collinear singularities which are taken into account by the PT formula. Since all the gluons produced are supposed to be well separated in these calculations, the merging of two or more gluons into jets was not taken into account.

**Phase Space Parametrization**

The form in which the PT formula is written suggests a parametrization of the $(n-2)$-body massless phase space in terms of the variables $s_{i, i+1} = (p_i + p_{i+1})^2$ [18] :

$$
\int d\, (PS) = \frac{(4\pi)^{-2n+7}}{2} \frac{1}{s} \int_0^s \frac{dM^2_{n-3}}{s - M^2_{n-3}} \int_{s_{n-2, n-1}}^{s_{n-2, n-1}} ds_{n-2, n-1} \times \\
\int_0^{M^2_{n-3}} \frac{dM^2_{n-4}}{M^2_{n-3} - M^2_{n-4}} \int_{s_{n-3, n-2}}^{s_{n-3, n-2}} ds_{n-3, n-2} \times \cdots \times \\
\int_0^{M^2_3} \frac{dM^2_2}{M^2_3 - M^2_2} \int_{s_{3, 4}}^{s_{3, 4}} ds_{3, 4} \times \int_{s_{2, 3}}^{s_{2, 3}} ds_{2, 3}
$$

(3)

where $M^2_i = (p_1 + p_2 + \cdots + p_i)^2$, $M^2_{n-2} = s$ , $M^2_{n-1} = M^2_i = 0$ and the limits of integration on the variables $s_{i, i+1}$ are given by :

$$
s_{i, i+1}^+ = \frac{1}{M^2_i} |M^2_{i+1} - M^2_i| (M^2_i - M^2_{i-1}).
$$

(4)

There are $(n-3)$ independent $s_{i, i+1}$ variables, $(n-4)$ independent $M^2_i$ variables and the integration over the remaining $(n-3)$ azimuthal angles was already performed.
We can write down schematically the cross section for the process \( gg \rightarrow (n-2)g \) :

\[
\sigma_n = \frac{1}{2s} \int d(P_S)_{n-2} |A_n^{KS}|^2
\]  
(5)

In order to avoid collinear and infrared divergencies we introduce a Lorentz invariant cut-off by requiring \( s_{ij} = (p_i + p_j)^2 \geq s_{\text{cut}} \) for all particles involved. In particular, we will not consider merging processes, i.e., contributions to the \( n \)-gluon cross section from processes with a larger number of gluons where one or more gluons fail to pass our cut.

We are interested in the large \( n \) behavior of this cross section. Due to the factorially growing number of permutations entering in the calculation of the PT formula and the large dimensionality of phase space, we make two approximations that we feel are plausible in the large-\( n \) limit:

- **Ordering Independence**: For large \( n \), the symmetry of the PT formula and of our cuts suggests that each term makes a similar contribution to the cross section after integrating over phase space \([19]\). It is the presence of the two fixed initial momenta which violates this hypothesis, and it is plausible that the effect of these momenta is less important at large \( n \). In fact, we have verified this approximation numerically for several arbitrary permutations of the momenta, in the cases \( n = 10, n = 12 \), using the phase space generators GENBOD and RAMBO. This simplifies our calculation tremendously since we now have to compute the contribution from the basic string only.

- **Neglect of end point effects**: there are \((n-3)\) independent variables \( s_{i, i+1} \) parametrizing the \((n-2)\)-body phase space whereas there are \( n \) such variables appearing in a given string in the PT formula. It is very difficult to write out the 3 dependent quantities in term of the independent variables. Since the number of dependent variables is \( n \) independent, we expect that in the large-\( n \) limit their contribution is not relevant. Therefore, here we assume that we can conservatively substitute \( s^3 \) for the dependent variables.

In addition to these, we also approximate the numerator of the PT formula by \((\sum_{i>j} s_{ij}^4) \approx s^4 \). This we call \( s^4 \) dominance.

In the context of these approximations and with the above cuts we find:

\[
\sigma_{gg \rightarrow (n-2)g} = \frac{1}{2s} \cdot \frac{1}{4(N_c^2 - 1)^2} \cdot \frac{1}{(n-2)!} \cdot \frac{2^n - 2(n+1)}{n(n-1)} \cdot \frac{(4\pi)^{-2n+7}}{2s} \cdot (2g_s^{2n-4} N_c^{n-2}(N_c^2 - 1)) \cdot s^4 \cdot (n-1)! \cdot \frac{1}{s^3} \cdot I_{n-2}(s_{\text{cut}}/s) 
\]  
(6)
where

\[ I_{n-2}(s_{\text{cut}}/s) = \int_{(M^2)_{\text{min}}}^{(M^2)_{\text{max}}} \frac{dM^2_{n-3}}{s - M^2_{n-3}} \int_{(M^2)_{\text{min}}}^{(M^2)_{\text{max}}} \frac{dM^2_{n-4}}{M^2_{n-3} - M^2_{n-4}} \cdots \int_{(M^2)_{\text{min}}}^{(M^2)_{\text{max}}} \frac{dM^2_n}{M^2_3 - M^2_2} \times \]

\[ \int_{s_{\text{cut}}}^{s_{n-2}} ds_{n-2} \int_{s_{n-2}}^{s_{n-1}} ds_{n-3} \cdots \int_{s_{\text{cut}}}^{s_{n-2}} ds_{n-2} \int_{s_{\text{cut}}}^{s_{n-2}} ds_{n-2} \]

\[ (7) \]

with \((M^2)_{\text{min}} = \frac{i}{2(i-2)} s_{\text{cut}}\) and \((M^2)_{\text{max}} = M^2_{i+1} - i s_{\text{cut}}\). We wrote out explicitly the different factors entering in Eq. (7) in order to point out their origin. Respectively these factors account for: the usual flux factor, the average over helicities and color of the initial gluons, the Bose symmetry factor for the \((n - 2)\) identical particles in the final state, the KS approximation, the phase-space overall factor, the overall factor in the PT formula, the \(s^4\) dominance, the ordering independence approximation and the neglecting of end point effects.

Let us now focus on the estimation of the multiple integral \(I_{n-2}(\Delta = s_{\text{cut}}/s)\) given by Eq. (7), which after a trivial integration over the \(s_i, i+1\) variables can be written as:

\[ I_{n-2}(\Delta) = \int_{(x_1)_{\text{min}}}^{(x_1)_{\text{max}}} \frac{dx_1}{1 - x_1} \int_{(x_2)_{\text{min}}}^{(x_2)_{\text{max}}} \frac{dx_2}{1 - x_2} \cdots \int_{(x_{n-4})_{\text{min}}}^{(x_{n-4})_{\text{max}}} \frac{dx_{n-4}}{1 - x_{n-4}} \times \ln\left(\frac{1 - x_1}{\Delta}\right) \]

\[ \cdot \ln\left(\frac{1 - x_1(1 - x_2)}{\Delta}\right) \cdot \ln\left(\frac{x_1}{\Delta}(1 - x_2)(1 - x_3)\right) \cdot \ln\left(\frac{x_1}{\Delta} x_2 (1 - x_3)(1 - x_4)\right) \cdots \]

\[ \ln\left(\frac{x_1}{\Delta} x_2 x_3 \cdots x_{n-6}(1 - x_{n-5})(1 - x_{n-4})\right) \cdot \ln\left(\frac{x_1}{\Delta} x_2 x_3 \cdots x_{n-5}(1 - x_{n-4})\right) \]

\[ (8) \]

where the new variables \(x_i\) are defined by:

\[ x_1 = \frac{M^2_{n-3}}{s}, \quad x_2 = \frac{M^2_{n-4}}{sx_1}, \quad x_3 = \frac{M^2_{n-5}}{sx_1x_2}, \quad \ldots, \quad x_{n-4} = \frac{M^2_2}{sx_1x_2 \cdots x_{n-6}x_{n-5}} \]

\[ (9) \]

and the limits of integration are:

\[ (x_i)_{\text{max/min}} = \frac{(M^2)_{\text{max/min}}}{sx_1x_2 \cdots x_{i-1}} \]

\[ (10) \]

At this point we could proceed by simply performing a numerical integration of Eq. (7) for different values of \(\Delta = s_{\text{cut}}/s\); this we did for a number of gluons up to \(n = 14\), and the results will be discussed shortly. However, in this section we would like to gain some analytic understanding of the cross section for small enough \(\Delta\) and large \(n\). For these reasons, we explore ways of finding the leading terms of \(I_{n-2}(\Delta)\) in this limit.
Double Leading Log Behavior

In order to have an approximate analytical form for $I_{n-2}(\Delta)$, we find that the following simplifications in the integration limits are useful:

$$(x_i)_{\text{min}} = 0, \quad (x_i)_{\text{max}} = 1 - \frac{\Delta}{x_1 x_2 \cdots x_{i-1}}$$  \hspace{1cm} (11)

\[
\int_{0}^{1-\Delta} \frac{dx}{1-x} \ln^m (1-x) \ln^n (x) \approx \int_{0}^{1} \frac{dx}{1-x} \ln^m (1-x) \ln^n (x) \equiv c(m, n) \hspace{1cm} (12)
\]

\[
\int_{0}^{1-\Delta} \frac{dx}{1-x} \ln^m (1-x) \approx \int_{0}^{1-\Delta} \frac{dx}{1-x} \ln^m (1-x) = \frac{1}{m+1} \ln^{m+1}(1/\Delta) \hspace{1cm} (13)
\]

where the numbers $c(m, n)$ can be easily computed numerically. Within these approximations and using MATHEMATICA we are able to find an analytical form for $I_{n-2}(\Delta)$ for $n$ up to 11. For $n > 11$ the computer time required becomes too large. Nevertheless, the analytical approximation provides us with the double leading-logarithm term (DLL) in $1/\Delta$, $I_{n-2}^{\text{DLL}}(\Delta)$ for arbitrary $n$:

$$I_{n-2}^{\text{DLL}}(\Delta) = \begin{cases} 
\left(\frac{1}{12}\right)^{\frac{n-4}{2}} \ln^{2n-7}(1/\Delta) & \text{n even} \\
\left(\frac{1}{3}\right)^{\frac{n-5}{2}} \ln^{2n-7}(1/\Delta) & \text{n odd}
\end{cases} \hspace{1cm} (14)
$$

which becomes dominant at small $\Delta$. The power of $\ln^2$ in (14) reflects the kinematic constraint that not every final state gluon momentum can be within $s_{\text{cut}}$ of another one. This consideration becomes unimportant at large $n$.

We have compared the results from the numerical integration with the approximate analytical and DLL results for different values of $\Delta$ and $n$. We find that the analytical and numerical integration results agree over a wide range of $n$ and $\Delta$. For fixed $n$, our DLL approximation is valid only at small enough $\Delta$, the range of validity being related to the number of partons $n$. For our results, a rough measure of the range of validity of the DLL is given by:

$$\Delta \ll \exp[-n] \hspace{1cm} (15)$$

Therefore, from the above considerations we finally arrive at the asymptotic behavior of the small $\Delta$, large $n$-gluon exclusive cross section by combining Eqs. (11) and
In the large $n$ limit:

$$\sigma_n^{(0)} \approx \frac{1}{s} \left( \frac{N_c \alpha_s}{2\pi \sqrt{12}} \ln^2(1/\Delta) \right)^{n-2}$$

(16)

In Eq. (16) the superscript refers to the number of unresolved gluons. We believe this result is valid in the range $n_0 < n \lesssim n > \ln(1/\Delta)$, where $n_0$ is the minimum number of gluons such that our approximations become reliable. We don’t have a definite estimate for $n_0$ but it is not unreasonable to assume $n_0 \approx 10$.

**Results for Hard Jets**

Even at moderate values of $n$ and $\Delta$, where the DLL approximation is not valid, our numerical results may be of relevance to SSC physics. This can be seen by writing Eq. (6) as:

$$\sigma_n = \frac{1}{s} (z(n, \Delta))^{n-2},$$

(17)

where

$$z(n, \Delta) = \frac{N_c \alpha_s}{2\pi} (I_{n-2}(\Delta))^{\frac{1}{n-2}}.$$  

(18)

When $z(n, \Delta) \approx 1$ one would be suspicious of the use of perturbation theory. In Table 1 we show $z(n, \Delta)$ for some interesting values of $n$ and $\Delta$, where we conservatively used $\alpha_s(Q^2 = s_{cut}) = 0.12$. Our results suggest that at the SSC, even reasonably “hard” jet cuts place $\Delta$ at a value ($\approx 10^{-5}$) which may be approaching a threshold of uncertainty for perturbation theory, in the sense that $\sigma_{n+1}^{(0)} > \sigma_n^{(0)}$. In the DLL approximation, $z(n, \Delta)$ becomes the effective $n$-independent expansion parameter of Eq. (16):

$$z(n, \Delta) \overset{\text{DLL}}{\rightarrow} z(\Delta) = \frac{N_c \alpha_s}{2\pi \sqrt{12}} \ln^2(1/\Delta) .$$

(19)

**Discussion and Concluding Remarks**

(1) It is not difficult to understand how our basic result Eq. (16) comes about: The factor $\ln^{2n}(1/\Delta)$ can be accounted for by simply counting the integrals in Eq. (6); it
is a consequence of the infrared and collinear divergences contained in the PT formula that are regulated by $s_{\text{cut}}$. However, the crucial feature of Eq. (10) is the absence of a $1/n!$ factor. This can be traced to the approximate ordering independence of the phase space integrated PT formula; the different $n!$ permutations compensate for the Bose symmetry factor. It should be noted that this cancellation of the $n!$ does not occur for QED: in that case, the $n!$ terms in the amplitude add to a single term which incorporates the permutation symmetry [20].

(2) The Parke-Taylor formula drops terms in the cross section which are non-leading in $1/N_c$. Non-leading terms in $1/N_c$ do not contribute in DLL [21], so that we do not consider them further.

(3) As a result of the absence of the $1/n!$ or other similar factor, the cross sections $\sigma_n^{(0)}$ for the production of $n-2$ resolved gluons in the DLL form a geometric series in $z$, and the exclusive total cross section

$$\sigma_{\text{tot}}^{(0)} = \sum_n \sigma_n^{(0)}$$

is not summable (except for a phase space cutoff) for $z(\Delta) \geq 1$. In particular, there is no sign of the exponential summation $\sigma \sim \exp(-\frac{4}{\pi} \ln s)$ proposed by Lipatov [22]. We will comment shortly on possible implications for inclusive $n$-jet cross sections.

(4) The factor $\sqrt{12}$ in our small-$\Delta$ result (Eq. [16]) contains the the KS approximation (Eq. [2]). If this approximation overestimates the cross section by a factor $(1 + \delta)^n$, then the substitution $\sqrt{12} \to \sqrt{12}(1 + \delta)$ should be made in Eq. [16].

(5) Our results suggest that caution should be exercised before using tree-level exact multi-gluon amplitudes to compute mini-jet cross sections. When convoluting the hard cross section with the gluon distribution functions, it is possible that (for $n$ not too large) the important contributions to the cross section come from low values of the initial gluons’ invariant mass $\sqrt{s}$ (i.e., larger values of $\Delta$), in which case the use of perturbation theory will be valid. However, there will always be potentially large contributions from non-perturbative regions of $s/s_{\text{cut}}$, and one must be certain that they are under control.

(6) Can higher order corrections restore the validity of perturbation theory? Schematically, we would have to compute contributions to the inclusive cross section $\sigma_n$ from processes with a larger number of gluons where one or more gluons would fail to pass our cuts and would be merged to another gluon:

$$\sigma_n = \sigma_n^{(0)} + \sigma_n^{(1)} + \sigma_n^{(2)} + \ldots$$  \hspace{1cm} (20)

where the superscripts indicate the number of “unresolved” merged gluons and
$\sigma_n^{(0)} = A_0 \alpha_s^n \ln^{2n}(1/\Delta) $

$\sigma_n^{(1)} = A_1 \alpha_s^n \ln^{2n}(1/\Delta) \alpha_s (\ln^2(1/\epsilon) - \ln^2(1/\Delta))$

$\sigma_n^{(2)} = A_2 \alpha_s^n \ln^{2n}(1/\Delta) \alpha_s^2 (a \ln^4(1/\epsilon) + b \ln^2(1/\epsilon) \ln^2(1/\Delta) + c \ln^4(1/\Delta))$

$\vdots \vdots$

(21)

$\epsilon$ is an infrared cut-off whose contributions will be canceled by loop corrections, and the various constants depend on the details of the merging procedure. An explicit discussion in the case of $e^+ e^- \rightarrow q\bar{q} +$ gluons has been given by Brown and Stirling [23], and will be commented on below. In our case,

$\sigma_n = A_0 \alpha_s^n \ln^{2n}(1/\Delta) \left(1 - \frac{A_1}{A_0} \alpha_s \ln^2(1/\Delta) + \frac{A_2 c}{A_0} \alpha_s^2 \ln^4(1/\Delta) + \ldots\right) = f_n(z) \sigma_n^{(0)}.$

(22)

In order that perturbation theory be restored with respect to $\sigma_n$, it would be necessary that $f_n z^n$ be bounded for large enough $x$ and $n$. As an example, the usual $n$-independent Sudakov factor, $f_n(z) = \exp[-K z]$, is not enough to solve our problem. In their study of $e^+ e^- \rightarrow$ jets, Brown and Stirling [23] have examined inclusive jet production using a jet merging algorithm similar to ours (the JADE algorithm), and have not found evidence for the presence of Sudakov summation. At this time, we have nothing to say about the function $f_n(z)$, but its evaluation will be important in proceeding beyond the results of this work.

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Table 1: Numerical values of the effective expansion parameter $z(n, \Delta)$ for different values of $\Delta$ and $n$.

| $\Delta$ | $n = 10$ | $n = 11$ | $n = 12$ | $n = 13$ |
|----------|----------|----------|----------|----------|
| $10^{-4}$ | 0.69     | 0.70     | 0.70     | 0.70     |
| $10^{-5}$ | 1.1      | 1.1      | 1.1      | 1.1      |
| $10^{-6}$ | 1.5      | 1.5      | 1.6      | 1.7      |
References

[1] T. Hebbeker, Phys. Rep. 217, 69 (1992).

[2] J. E. Huth and M. L. Mangano, FERMILAB report PUB-93/19-E, to be published in Annu. Rev. Nucl. Part. Sci. 43.

[3] See, e.g., B. Webber in QCD at 200 TeV, edited by L. Cifarelli and Yu. Dokshitzer, Plenum Press, 1992.

[4] F. Abe et al., Phys. Rev. D45, 2249 (1992).

[5] J. M. Cornwall, Phys. Lett. 243B, 271 (1990).

[6] H. Goldberg, Phys. Lett. 246B, 445 (1990).

[7] M. B. Voloshin, Nucl. Phys. B383, 233 (1992).

[8] E. N. Argyres, R. Kleiss, and C. G. Papadopoulos, Phys. Lett. 296B, 139 (1992).

[9] L. S. Brown, Phys. Rev. D46, 4215 (1992).

[10] A realistic cross section could then be obtained by convoluting the parton level result with the gluon distribution function inside the initial hadrons, and with the final state hadronization functions. This will not be done in the present work.

[11] F. A. Berends and W. T. Giele, Nucl. Phys. B306, 759 (1988).

[12] F. A. Berends, W. T. Giele and H. Kuijf, Nucl. Phys. B333, 120 (1990).

[13] S. J. Parke and T. R. Taylor, Phys. Rev. Lett. 56, 2459 (1986).

[14] F. Fiorani, G. Marchesini and L. Reina, Nucl. Phys. B309, 439 (1988).

[15] Z. Kunszt and W. J. Stirling, Phys. Rev. D37, 2439 (1988).

[16] R. Kleiss and H. Kuijf, Nucl. Phys. B312, 616 (1989).

[17] C. J. Maxwell, Nucl. Phys. B316, 321 (1989).

[18] See, e.g., E. Byckling and K. Kajantie, Particle Kinematics (John Wiley, N.Y.,1973).

[19] There are in fact $O(n!)$ terms in which the permutations are strictly of the final state gluons. These do give identical contributions on integration over phase space. (We would like to thank C. Maxwell for comments on this point.)
[20] See, e.g., M. Mangano, *Nucl. Phys.* **B309**, 461 (1988).

[21] M. Mangano, S.J. Parke, and Z. Xu, in *Proc. of “Rencontres de Physique de la Vallée d’Aoste”*, La Thuile, Italy, (1987), ed. M. Greco, Editions Frontières, p. 513.

[22] L. N. Lipatov, *Sov. J. Nucl. Phys.* **23**, 338 (1976).

[23] N. Brown and W. J. Stirling, *Phys. Lett.* **B252**, 657 (1990).