Spin-dependent transport in lateral periodic magnetic modulations: a scheme for spin filters

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A scheme for spin filters is proposed by studying the coherent transport of electrons through quantum wires with lateral magnetic modulations. Unlike other schemes in the literature, the modulation in our scheme is much weaker than the Fermi energy. Large spin polarization through the filter is predicted. Further study suggests the robustness of this spin filter.

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The realization of spintronic devices relies on the ability to inject a spin-polarized current into a semiconductor. Progress has been made in injecting polarized electrons from ferromagnets\(^\text{1,2,3}\) or semimagnetic semiconductors\(^\text{4,5,6,7}\) into semiconductors. Besides these efforts, generating spin polarization (SP) through a spin filter has aroused more and more attention.\(^\text{4}\) In these works, spin-selective barriers or stubs\(^\text{5,6,7}\) are essential to realize the SP. Other method such as generating SP inside semiconductors by reflection at the interface with a ferromagnet has also been proposed.\(^\text{1}\) In this letter, we propose a scheme for a spin filter where the SP is generated during the transport without tunneling through any barrier or being mode-selected by any stub.

We consider the electron ballistic transport through a semiconductor nanowire under the periodic spin dependent modulation shown in Fig. 1. Here the spin dependent potential has the Zeeman-like form: \(V_0(x) = \sigma V_0 g(x)\) with \(g(x) = 1\) if \(x\) is located at the \(A\)-layer, and 0 otherwise. \(\sigma = \pm 1\) for spin-up and -down electrons respectively. \(V_0\) denotes a spin-independent parameter for the strength of the potential. Therefore, spin-up and -down electrons experience different periodic potentials: spin-up electrons coherently transport under the modulation of periodic barriers while spin-down ones under the modulation of periodic wells. The transmission and reflection coefficients can be easily obtained and are spin dependent. Some earlier schemes for spin filters are also reported based on similar spin dependent modulation.\(^\text{8}\) However, they only work in the assumption of the large potential \(V_0 > E_F\) with \(E_F\) representing the Fermi energy. This is because a large barrier for the spin-up electrons strongly suppresses the transmission of spin-up electrons, therefore the spin-up current decays exponentially. Nevertheless the spin-down electrons can easily transmit through the large wells. Generally speaking, the spin dependent modulation is weak except in the extreme conditions such as applying a strong magnetic field or using a heavily-doped ferromagnetic semiconductor which still remains a challenge and will cause new problems that limit the application. In our scheme, we focus on the weak modulation case, i.e., \(V_0/E_F \ll 1\). In this case, even spin-up electrons do not see any true barriers but rather “transparent” barriers (TB’s). Therefore a single TB (well) affects the transmission coefficient of a spin-up (-down) electron only a little bit. Hence, the SP is negligible when the spin-up/down electrons coherently transport through a single TB/well. However, when electrons transmit through a set of weakly periodic TB’s or wells, under right conditions, a new feature appears: electrons with different spin may pick up these small SP’s and accumulate to a large one after they transmit through a large number \(N\) of TB’s (wells). Moreover, we find the spin polarization shows oscillations with \(N\). Furthermore, 100 % SP is also predicted in our scheme. This is because with the periodic modulations, there exists an energy gap. The positions of the gap for the spin-up and -down electrons are separated because the modulations are different. Hence, when the Fermi energy of the leads is within the gap regime of the spin-up (-down) electrons, the transmission coefficient for spin-up (-down) will decay exponentially with \(N\) while that for the spin-down (-up) still oscillates with \(N\).

![FIG. 1: The modulation for spin-up (solid curve) and spin-down (dashed curves) electrons.](image)

We describe a quantum wire along the \(x\)-direction with the tight-binding approach.\(^\text{11}\) By taking a two dimensional grid with \(N_y\) grid sites along the transverse direction and \(N_x\) sites along the wire, we have

\[
H = \sum_{l,m,\sigma} \epsilon_{l,\sigma} c_{l,m,\sigma}^\dagger c_{l,m,\sigma} + t \sum_{l,m,\sigma} \left( c_{l+1,m,\sigma}^\dagger c_{l,m,\sigma} + h.c. \right),
\]

where \(l\) and \(m\) denote the coordinates along the \(x\) and \(-\)\(y\) axis respectively. \(\epsilon_{l,\sigma} = \epsilon_0 + \sigma V_0 (= \epsilon_0)\) when \(l\) locates at the \(A\) (B) layer, denotes the on-site energy with \(\epsilon_0 = -4t\). \(t = -\hbar^2 / 2m^* a^2\) is the hopping energy with \(m^*\) and \(a\) standing for the effective mass and the “lattice” constance respectively.

The spin dependent conductance is calculated using the Landauer-Büttiker\(^\text{12}\) formula with the help of the Green function method.\(^\text{13}\) The two-terminal spin-resolved conductance is given by \(G^{+,-\sigma} = \langle e^2 / h \rangle \text{Tr}[\Gamma_L^{+\sigma} G^{+\sigma}_{N_x} \Gamma_R^{-\sigma} G^{-\sigma}_{N_x+1}]\) with \(\Gamma_{L,R}^{+\sigma}\) representing the self-energy function for the isolated ideal leads.\(^\text{13}\) We choose the perfect ideal ohmic contact between the leads and the semiconductor, \(G_{N_x}^{+\sigma}\) and \(G_{N_x+1}^{+\sigma}\) are the retarded and advanced Green functions for the conductor, but with the effect of the leads included. The trace is performed over the spatial degrees of freedom along the \(y\)-
axis. Without the spin-flip process, one can define the SP as
\[ P = \frac{G_{↑↑} - G_{↓↓}}{G_{↑↑} + G_{↓↓}}. \]

We perform a numerical calculation for a quantum wire with fixed width \( N_δ = 40a \). The hard wall potential is applied in this transverse direction which makes the lowest energy of the \( n \)-th subband (channel) to be \( \varepsilon_n = 2(t + |t| \cos[n\pi/(N_δ + 1)] \). \( a = 20 \) Å throughout the computation. The total length of a single unit (an A-layer plus a B-layer) is fixed at 30a. We take the Fermi energy \( E_f = 0.01697|t| \) and the Zeeman splitting energy \( V_0 = 0.001|t| \). Such a choice of the Fermi energy guarantees not only the lowest subband (single mode) contribute to the conductance but also gives the SP. It is noted that \( V_0/E_f \sim 0.06 \) is very small.

In Fig. 2(a) the SP \( P \) is plotted as a function of the length of the semiconductor wire \( L_x \) for two different modulations: Case I, the length of the A-layer \( L_A \) is the same as that of the B-layer \( L_B \), (curve 1); Case II, \( L_A = 17a \) (curve 2). \( L_A + L_B \) is always fixed to be 30a. It is seen from the figure that unpolarized injection from the left lead acquires SP when it reaches to the right one if the length of the modulation is long enough. When \( L_x \) is around 1 \( \mu \)m, the SP’s for the two cases all reach to 10 %. Oscillations appear when the filter length further increases. The maximum SP differs for different modulations. For case I, nearly 70 % SP can be achieved when \( L_x \) is around 4 \( \mu \)m. In order to understand this SP oscillation, we plot the spin dependent conductances \( G_{↑↑} \) and \( G_{↓↓} \) versus the filter length \( L_x \) in Fig. 2(b) for Case I. It is clearly observed that both conductances oscillate with \( L_x \), nevertheless with different periods. The period for spin-up conductance is around 8.6 \( \mu \)m and that for spin-down one is about 2.4 \( \mu \)m. Therefore, through several periods, large mismatch accumulates and the peak SP is reached when the position of the peak transmission of one spin is around the position of the valley transmission of the opposite spin.

To further elucidate the effect of the SP oscillation, we consider an exact one dimensional scattering problem of an electron with large kinetic energy \( E_F \) passing through the same spin dependent modulations as shown in Fig. 1. After

\[ N \]-units, the transmission coefficient is
\[ T_s(N) = \left\{ 1 + \left| \frac{|\gamma_{0}|^2 - \sin^2(\delta \theta_{\sigma})|\sin^2(N \delta \theta_{\sigma})/\sin^2(\delta \theta_{\sigma})|^{-1} \right|^2 \right\} \]

with \( \delta \theta_{\sigma} = \arccos[-\cos(\kappa_{\sigma}L_A + k_EL_B) + 1/2\sin(\kappa_{\sigma}L_A)\sin(k_EL_B)(\kappa_{\sigma}/k_E + k_F/\kappa_{\sigma} - 2)].\) \( k_F = \sqrt{2m^*E_F/\hbar^2} \) and \( \kappa_{\sigma} = \sqrt{2m^*(E_F + \sigma V_0)/\hbar^2} \) are the free electron momentum at B-layers and the electron momentum under the potential of the TB’s or wells at A layers respectively. \( \gamma_{0} = i[\sin(k_EL_B)\cos(\kappa_{\sigma}L_A) + \sin(\kappa_{\sigma}L_A)\cos(k_EL_B)(\kappa_{\sigma}/k_E + k_F/\kappa_{\sigma})/2] \) is a pure imaginary number. Equation (2) clearly shows that \( T_s(N) \) is a periodic function of \( N \). When \( N\delta \theta_{\sigma} = m_1\pi \) or \( N\delta \theta_{\sigma} = (m_2 + 1/2)\pi \) is satisfied \( (m_1 \) and \( m_2 \) here represent integers), the peak or valley appears respectively with the value of the peak and the valley being 1 and \( \sin^2(\delta \theta_{\sigma})/|\gamma_{0}|^2 \). The length corresponding to the first large SP is determined approximately by matching the peak of one spin with the valley of the opposite one: \( (|m_1 - m_2 + 1/2\pi|/(\delta \theta_{\sigma} - \delta \theta_{\sigma}))|(L_A + L_B) \) by choosing the smallest \( m_1 \) and \( m_2 \) to satisfy \( (m_1/\delta \theta_{\sigma}) \sim (m_2 + 1/2)/\delta \theta_{\sigma} \). In order to have large SP, the oscillation of each spin transmission should be large enough. The oscillation amplitude of each spin transmission can be determined by subtracting the valley transmission from the peak: 
\[ 1 - \sin^2(\delta \theta_{\sigma})/|\gamma_{0}|^2 = \frac{(\kappa_{\sigma}/k_E - k_F/\kappa_{\sigma})^2\sin^2(\kappa_{\sigma}L_A)/4|\sin(\kappa_{\sigma}L_A + k_EL_B) + \sin(\kappa_{\sigma}L_A)\cos(k_EL_B)(\kappa_{\sigma}/k_E + k_F/\kappa_{\sigma} - 2)/2|^2}{\sin(\kappa_{\sigma}L_A)\cos(k_EL_B)(\kappa_{\sigma}/k_E + k_F/\kappa_{\sigma} - 2)/2} \]
true for case II.

FIG. 4: (a) Spin dependent conductance $G^{\uparrow\downarrow}$ (curve 1) and $G^{\downarrow\uparrow}$ (curve 2) as well as SP vs. the length of the filter for case I in the presence of a Anderson disorder ($W = 0.01|t|$). It is noted that the scale of the SP is at the right side of the figure. (b) SP vs. the filter length for the same modulation as in (a) but without disorder. Chain curve: without Rashba effect; Solid curve: with Rashba effect.

It is interesting to see that 100 % SP can be obtained if one chooses $E_F = 0.01701|t|$ as shown in Fig. 3(a). When $L_s$ is longer than 10 μm, $P$ stays 100 %. This can be understood from Eq. (2). The phase shift $\delta \theta_{\sigma}$ for spin-up electron is 0 while the one for spin-down electron is 0.085. Therefore $T_1(N) = 1/[|\gamma|^2 N^2]$ which decays to 0 when $N$ increases. The length period of the oscillations of transmission coefficient for spin-down electron is 2.2 μm, exactly as shown in the figure. If one further increases $E_F$ slightly, $\delta \theta_{\sigma}$ becomes a pure imaginary number which implies that $E_F$ is within the regime of the gap of the spin-down electrons. As $\sin(n|\delta \theta_{\sigma}|N) = i\sin(n|\delta \theta_{\sigma}|N)$, from Eq.(2) one can see that that $T(N)$ decays exponentially with $N$. The gap can be determined by keeping $\delta \theta_{\sigma}$ imaginary: $\{-\cos(k_{\sigma}L_A + k_F L_B) + 1/2\sin(k_{\sigma}L_A)\sin(k_F L_B)(k_F/k_{\sigma} + k_F/k_{\sigma} - 2)\} \geq 0$. To the second order of $(V_0/E_F)$, the gap for the spin $\sigma$ is given by $h^2 |K_{\sigma} - \frac{1}{2\pi} \frac{k^2_F}{k^2_F \sqrt{\sin(k^2_F L_A) \sin(k^2_F L_B)}} / (2m^*)|$. This is because the contribution to the SP only comes from the third mode.

In summary, we have proposed a scheme for spin filters by studying the coherent transport of electrons through quantum wires with the lateral magnetic modulations. Unlike other schemes in the literature, the modulation in our scheme is much weaker than the Fermi energy. A large SP is predicted if the condition $k_F(L_A + L_B) \sim \pi$ is satisfied. Further study also shows the robustness of this scheme. The magnetic modulation can be realized by sticking the magnetic strips on top of the sample or using magnetic semiconductor as A-layer.

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15 Here the SP is defined as $P = (G_{↑↑} + G_{↓↑} - G_{↓↓} - G_{↑↓})/(G_{↑↑} + G_{↓↑} + G_{↓↓} + G_{↑↓})$. 