Inspired by the works of Rickard on splendid equivalences ([Ri2]) and of Chuang and Rouquier on perverse equivalences ([ChRou2]), we are here interested in the combination of both, i.e. a splendid perverse equivalence. This is naturally the right framework to understand the relations between global and local perverse equivalences between blocks of finite groups, as a splendid equivalence induces local derived equivalences via the Brauer functor. We prove that under certain conditions, we have an equivalence between a perverse equivalence between the homotopy category of $p$-permutation modules and local derived perverse equivalences, in the case of abelian defect group.

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obtain local derived perverse equivalences. In our attempt to go back up from local to global, we will introduce the refined notion of perverse equivalence relative to a partial order. Finally, we will illustrate this by a careful study of the cyclic case, we shall see that Rouquier’s splendid complex ([Rou1]) does not necessarily realize a global perverse equivalence although it always induces locally perverse equivalences.

The aim of Section 1 is to develop this theme of global versus local along Boltje and Xu’s notion of $p$-permutation equivalence. In Section 2 and 3, we will make precise a result of Rickard on splendid complexes connecting a splendid tilting complex $X$ with its image by the Brauer functor $\text{Br}_{\Delta Q}(X)$. In Section 4, we make the connection between perverse equivalences at the level of the centralizer of a $p$-group and at the level of the corresponding normalizer. Then in Section 5, we show that a global perverse homotopy equivalence yields to local derived perverse equivalence. We then introduce the notion of perverse equivalence relative to a partial order in order to go back up, from the data of local perverse derived equivalence, to a global perverse homotopy equivalence. Then, we gather all of the above results and prove our main result.

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0.1. Notations. For $G$ a finite group, we choose to define the diagonal of $G$ as $\Delta G := \{(g, g^{-1})|g \in G\}$. For $p$ a prime number, the $p$-core of $G$ is defined to be the largest normal $p$-subgroup of $G$ and is denoted by $O_p(G)$. We denote by $(\mathcal{O}, K, k)$ a modular system, i.e. $\mathcal{O}$ is a discrete valuation ring, with field of fractions $K$ of characteristic 0, large enough for all groups considered here, and residue field $k$ of characteristic $p > 0$.

For $A$ a symmetric $R$-algebra ($R$ either $k$ or $\mathcal{O}$), $A$-mod denotes the category of finitely generated $A$-modules. Let $\mathcal{C}$ be an additive category and $\mathcal{A}$ an abelian category. Then $\text{Comp}^b(\mathcal{C})$ denotes the category of bounded complexes of objects of $\mathcal{C}$, $\text{Ho}^b(\mathcal{C})$ the homotopy category of $\text{Comp}^b(\mathcal{C})$ and $\mathcal{D}^b(\mathcal{A})$ denotes the bounded derived category of $\mathcal{A}$.

By $K_0(RG)$, we denote the Grothendieck group of finitely generated $RG$-modules. The isomorphism classes $[[S]]$ of simple $RG$-module form a $\mathbb{Z}$-basis of $K_0(RG)$. The space of class functions of $G$ taking values in $K$ will be denoted by $\text{CF}(G)$. Let $e$ and $f$ denote the principal block idempotents of $kG$ and $kH$ respectively. For $Q \leq P$, we denote by $e_Q$ (resp. $f_Q$) the principal block idempotent of $kC_G(Q)$ (resp. $kC_H(Q)$).

0.2. Reminder. We will now recall some classical definitions.

Definition 0.1. Two finite groups $G$ and $H$ with a common Sylow $p$-subgroup $P$ share the same $p$-local structure if for every $Q_1$ and $Q_2$ subgroups of $P$ with $\theta: Q_1 \rightarrow Q_2$ an isomorphism, then there is an element $g \in G$, such that $\theta(q) = q^g$ for all $q \in Q_1$ if and only if there is an element $h \in H$ such that $\theta(q) = q^h$ for all $q \in Q_1$.

For instance, if $H$ is a subgroup of $G$, $P$ is abelian and $H$ contains $N_G(P)$ then $H$ and $G$ have the same $p$-local structure. From now on, we will always assume that we are in this particular situation.
Definition 0.2. Let $G$ be a finite group and $Q$ a $p$-subgroup of $G$. We denote by $\text{Br}_Q$ the Brauer functor $\text{Br}_Q : kG\text{-mod} \to kN_G(Q)\text{-mod}$ defined for $M$ a $kG$-module by

$$\text{Br}_Q(M) = M^Q/(\sum_{P < Q} \text{Tr}_P^Q M^P),$$

the quotient of $Q$-fixed points of $M$ by the relative traces from all proper subgroups $P$ of $Q$ of the $P$-fixed points. We will also write $M(Q)$ for $\text{Br}_Q(M)$.

We will be mostly interested in a particular type of modules, the $p$-permutation modules (see, for example, [Br1]). These are direct summands of permutation modules. In view of Green's theory of vertices and sources, they are also known as trivial source modules. We denote by $kG$-perm the full subcategory of $kG$-mod of $p$-permutation modules. In what follows, we will consider the restriction of the Brauer functor $\text{Br}_Q : kG\text{-perm} \to kN_G(Q)\text{-perm}$. Note that if $M$ is a permutation module with a $G$-stable basis $X$, we have that $M(Q) \simeq k[X^Q]$. Considering the group algebra $kG$ as a $k[G \times G]$-module, $kG(\Delta Q)$ is naturally isomorphic to the group algebra $kC_G(Q)$. An important key feature of the Brauer construction is the following isomorphism, for $M$ and $N$ $p$-permutation $kG$-modules, not only do we have

$$M(Q) \otimes_k N(Q) \sim (M \otimes_k N)(Q),$$

but we also have the isomorphism

$$M(Q) \otimes_{kG(Q)} N(Q) \sim (M \otimes_{kG} N)(Q).$$

Another important feature is that the vertex of an indecomposable $p$-permutation $kG$-module $M$ is precisely the maximal $p$-subgroup of $G$ such that $\text{Br}_Q(M) \neq 0$. We now recall the very useful Broué-Puig’s parametrization of $p$-permutation modules. We put $\bar{N}_G(Q) = N_G(Q)/Q$.

Proposition 0.3. ([Br1, Theorem 3.2]) The correspondence $M \mapsto (Q, \text{Br}_Q(M))$ for $Q$ a vertex of $M$ defines a bijection from the set of isomorphism classes of indecomposables $p$-permutation $kG$-modules to the set of conjugacy classes of pairs $(Q, N)$, where $Q$ is a $p$-subgroup of $G$ and $N$ is an isomorphism class of indecomposable projective $k\bar{N}_G(Q)$-modules.

Note that this parametrization is well compatible with the decomposition of the group algebras into blocks. In [Br2], Broué introduces the notion of perfect character between $OGe$ and $OHf$ as follows.

Definition 0.4. A perfect character is an element $\mu \in K_0(KGe, KHf)$ satisfying the following:

- $\forall g \in G$, $\forall h \in H$, $\frac{\mu(g,h)}{|C_G(g)|} \in O$ and $\frac{\mu(g,h)}{|C_H(h)|} \in O$.
- if $\mu(g,h) \neq 0$, then $g$ has order prime to $p$ if and only if $h$ has order prime to $p$.

To $\mu \in K_0(KGe, KHf)$, we associate isomorphisms $I_{\mu} : K_0(KHf) \to K_0(KGe)$ and $R_{\mu} : K_0(KFe) \to K_0(KHf)$ and we say that $I_{\mu}$ is a perfect isometry if $\mu$ is a perfect character. If the above is considered as the standard definition of a perfect isometry, we would like to notice that one might prefer (as we do) the equivalent definition of [Br2, Proposition 4.1]. By laziness, we refer the reader to [Br2] for the precise definition of an isotypy. Kindly, we will set the mind of the anxious reader at rest by pretending that one can think of an isotypy as a “nicely” compatible family of perfect isometries.

By $T(RG)$, we denote the representation ring of $p$-permutations $RG$-module. Also, we will
write $T(RG, RH)$ for $T(RG \otimes_R RH^{opp})$. The isomorphism classes $[M]$ of indecomposable $p$-permutation $RG$-modules form a $\mathbb{Z}$-basis of $T(RG)$. For $Q$ a $p$-subgroup of $G$ we denote by $T^Q(RG)$ the subgroup of $T(RG)$ generated by relatively $Q$-projective $p$-permutation modules. For convenience, if $\gamma \in T(RG)$, we will put $\gamma(Q) = Br_Q(\gamma)$. To $\gamma \in T(OG)$, we associate its character $\mu(\gamma) \in K_0(KG)$.

The tensor product $- \otimes_{RH} -$ induces a $\mathbb{Z}$-bilinear map

$$T(RG, RH) \times T(RH, RL) \to T(RG, RL), \ (\gamma, \delta) \mapsto \gamma \cdot H_\delta,$$

for any third group $L$. Also, taking the $R$-dual induces an isomorphism

$$T(RG, RH) \to T(RH, RG), \ \gamma \mapsto \gamma^*.$$

We define two type of equivalences between $A$ and $B$, two symmetric $R$-algebras: the so-called Rickard equivalences and the stable equivalence. We say that an $(A, B)$-bimodule $M$ is exact if it is projective as a left $A$-module and as a right $B$-module.

**Definition 0.5.** A bounded complex $X$ of exact $(A, B)$-bimodules induces a Rickard equivalence if

- $X \otimes_B X^* \simeq A \oplus Z_1$ as complexes of $(A, A)$-bimodules
- $X^* \otimes_A X \simeq B \oplus Z_2$ as complexes of $(B, B)$-bimodules,

where $A$ and $B$ are concentrated in degree 0, and $Z_1$ and $Z_2$ are homotopy equivalent to 0.

A Rickard complex $X$ then induces a derived equivalence $X \otimes_B - : D^b(B\text{-mod}) \to D^b(A\text{-mod})$.

**Definition 0.6.** A bounded complex $X$ of exact $(A, B)$-bimodules induces a stable equivalence if

- $X \otimes_B X^* \simeq A \oplus Z'_1$ as complexes of $(A, A)$-bimodules
- $X^* \otimes_A X \simeq B \oplus Z'_2$ as complexes of $(B, B)$-bimodules,

where $A$ and $B$ are concentrated in degree 0, and $Z_1$ and $Z_2$ are homotopy equivalent to complexes of projectives bimodules.

Let $RA$ and $RB$ be block algebras of $RG$ and $RH$ respectively. There is a specific type of Rickard complexes between block algebras, called splendid complex introduced in Rickard [Ri2].

**Definition 0.7.** A complex $X \in \text{Comp}^b(RA\text{-mod}\cdot RB)$ is splendid if its terms (view as $R[G \times H^{opp}]$-modules) are direct summands of finite direct sums of modules of the form $\text{Ind}_{\Delta Q}^{G \times H^{opp}}(R)$ for $Q \leq P$ and $X$ realizes a Rickard equivalence between $D^b(RA)$ and $D^b(RB)$.

It is shown in [Ri2] that a splendid equivalence induces an isotypy at the level of the Grothendieck group, and so with Broué’s abelian defect group conjecture in mind (cf. [Br2]). Hence, this might lead us to believe that the derived equivalence predicted by Broué, between the derived category of a block with abelian defect and its Brauer correspondent, should be splendid.

There is another specific type of derived equivalences, introduced by Chuang and Rouquier (cf. [ChRou2]), called perverse equivalences. They can be seen as filtered derived equivalences, i.e. as a patching of Morita equivalences on an each stratum of the filtration.

Let $S$ (resp. $S'$), the set of isomorphism classes of simple objects of $RA$ (resp. $RB$). Consider

- a filtration $S_\bullet = (\emptyset = S_{-1} \subset S_0 \subset \ldots \subset S_r = S)$
• a filtration $S'_r = (\emptyset = S'_{-1} \subset S'_{0} \subset \ldots \subset S'_r = S')$
• and a function $p : \{0, \ldots, r\} \rightarrow \mathbb{Z}$.

**Definition 0.8.** An equivalence $F : \mathcal{D}^b(RA) \xrightarrow{\sim} \mathcal{D}^b(RB)$ is perverse relative to $(S_\bullet, S'_\bullet, p)$ if the following holds:

- given $V \in S_i - S_{i-1}$, then the composition factors of $H^r(F(V))$ are in $S'_{i-1}$ for $r \neq -p(i)$ and there is a filtration $L_1 \subset L_2 \subset H^{-p(i)}(F(V))$ such that the composition factors of $L_1$ and of $H^{-p(i)}(F(V))/L_2$ are in $S'_{i-1}$ and $L_2/L_1 \in S'_i - S'_{i-1}$.
- The map $V \mapsto L_2/L_1$ induces a bijection $S_i - S_{i-1} \xrightarrow{\sim} S'_i - S'_{i-1}$.

Note that if $F$ is a perverse equivalence with $p = 0$, then $F$ restricts to a Morita equivalence $RA\text{-mod} \xrightarrow{\sim} RB\text{-mod}$.

In the context of $\mathfrak{sl}_2$-categorification defined by the previous two authors in [ChRou1], the Rickard complex $\Theta$ that gives a self-derived equivalence $\Theta : \mathcal{D}^b(A) \xrightarrow{\sim} \mathcal{D}^b(A)$ for $A$ a $\mathfrak{sl}_2$-categorification is perverse. Another famous example of a perverse equivalence is the complex of Rickard and Cabanes ([CaRi]) which gives the Alvis-Curtis duality at the level of Grothendieck groups. Note also that the splendid complex $X$ of the Section 3 of [Ri2] between the principal block of $kA_3$ and $A_4$ gives a perverse derived equivalence. This is a prototype of the so-called elementary perverse equivalences.

In Section 5, we will define another type of perverse equivalence, at the level of the homotopy category of additive categories.

### 1. The Classical Dynamic of Global and Local

Within this paradigm of connecting global and local properties, a first natural question is what can be said homologically with no binds of perversity. More precisely, if $X$ is a complex of relatively $\Delta P$-projective $p$-permutations $(kG, kH)$-bimodules which locally induces derived equivalences, i.e. at the level of centraliser of $p$-element, does $X$ realize a derived equivalences between $kG$ and $kH$? The answer is not exactly. Indeed, using a result of Bouc-Rouquier [Rou3, Theorem 5.6] showed that one can only hope for a global stable equivalence:

**Proposition 1.1.** ([Rou3, Theorem 5.6]) Let $X \in \text{Comp}^b(kGe\text{-mod}-kHf)$ is a complex of relatively $\Delta P$-projective $p$-permutations $(kG, kH)$-bimodules. The following assertions are equivalent:

1. $X$ induces a stable equivalence between $kGe$ and $kHf$.
2. For every non-trivial subgroup $Q \leq P$, the complex $\text{Br}_{\Delta Q}(X)$ induces a Rickard equivalence between $kC_G(Q)e_Q$ and $kC_H(Q)f_Q$.
3. For every subgroup $Q$ of order $p$ in $P$, the complex $\text{Br}_{\Delta Q}(X)$ induces a Rickard equivalence between $kC_G(Q)e_Q$ and $kC_H(Q)f_Q$.

Boltje and Xu introduced an intermediate notion, that lies between a splendid equivalence and an isotypy, the so-called $p$-permutation equivalence. Indeed, they showed in [BoXu] that, not only, a splendid equivalence induces a $p$-permutation equivalence but also that a $p$-permutation equivalence induces an isotypy.

**Definition 1.2.** A $p$-permutation equivalence between $OGe$ and $OHf$ is an element $\gamma \in T^{\Delta P}(OGe, OHf)$ satisfying

$$\gamma \cdot \gamma^* = [OGe] \in T(OGe, OGe)$$
and

\[ \gamma^* \cdot \gamma = [OHf] \in T(OHf, OHf). \]

**Lemma 1.3.** Let \( \gamma \in T^P(OGe, OHf) \) with \( \mu(\gamma) \) an isometry. Then \( \gamma \) is a \( p \)-permutation equivalence between \( OGe \) and \( OHf \) if and only if for every \( p \)-subgroup \( Q \neq 1 \), \( \gamma(\Delta Q) \) is a \( p \)-permutation equivalence between \( OC_{G}(Q)_{eQ} \) and \( OC_{H}(Q)fQ \).

**Proof.** One direction is straightforward, as we have \( \text{Br}_{\Delta Q}(\gamma \cdot \gamma^*) \cong \text{Br}_{\Delta Q}(\gamma) \cdot \text{Br}_{\Delta Q}(\gamma^*) \).

We proceed to prove the other direction, from local to global. We write \( \hat{\gamma} := \gamma \cdot \gamma^* - [OGe] = [M] - [N] \) where \( M \) and \( N \) are \( p \)-permutation \( OGe \)-bimodules and we prove that if for every \( p \)-subgroup \( Q \neq 1 \), \( \hat{\gamma}(\Delta Q) = 0 \) then \( \hat{\gamma} = 0 \). We have \( [M(\Delta Q)] = [N(\Delta Q)] \) for every \( Q \). By Broué’s parametrization of permutation modules (cf. [Br1, Theorem 3.2]) and the Krull-Remak-Schmidt theorem, we have \( [M] + [L] = [N] + [L] \), where \( L \) and \( L' \) are projective \( OGe \)-bimodules. So that \( \gamma \cdot \gamma^* - [OGe] = [L] - [L'] \). But if we take the associated character over \( K \), and as \( \mu \cdot \mu^* = [kG]\), we have \( [L] = [L'] \). However by injectivity of the Cartan homomorphism (cf. [Se, Chapter 16]) \( c : K_0(kG-proj) \twoheadrightarrow K_0(kG) \), we conclude that \( [L] = [L'] \) and hence \( \gamma \cdot \gamma^* = [OGe] \in T(OGe, OGe) \). □

The next stronger result relies on the same technique and gives us a converse to Boltje and Xu’s theorem [BoXu, Theorem 1.11].

**Proposition 1.4.** Let \( \gamma \in T^P(OGe, OHf) \). If \( (\mu_{\Delta Q})_{Q \leq P} \) is an isotypy, then \( \gamma \) is a \( p \)-permutation equivalence.

**Proof.** In fact, we do not actually need to have an isotypy but only a family of perfect isometries. We prove that if for every \( p \)-subgroup \( Q \), \( \mu(\Delta Q) = 0 \) then \( \gamma(\Delta Q) = 0 \), the rest will follow according to the proof of the previous lemma. We proceed by decreasing induction as follow. First for the Sylow \( P \), \( \hat{\gamma}(\Delta P) \) is constituted of \( k[N_{G \times H}^{opp}(\Delta P)/\Delta P]-\text{projective module} \) and so as \( \mu_{\Delta Q} = 0 \) we use again the injectivity of the Cartan homomorphism to conclude that \( \hat{\gamma}(\Delta P) = 0 \). Now suppose we proved the above property for any \( Q < R \) and let’s prove it for \( R \). We mimic the proof of the previous lemma to \( \gamma(R) \) as \( \gamma(\Delta R)(\Delta Q) = \gamma(\Delta Q) = 0 \) for every \( Q < R \). Hence, we can finally conclude that \( \gamma \) is in deed a \( p \)-permutation equivalence. □

**Remark 1.5.** According to our proof, it is enough to require the local property only for every \( p \)-subgroup \( Q \leq P \) of order \( p \). This way, our result has a similar flavor as Proposition 1.1.

2. A commutative diagram

Here our goal is to understand splendid equivalences locally, i.e. at the level of centralizers of \( p \)-elements, following the fundamental article of Rickard [Ri2]. If a splendid complex yields to local derived equivalences, our desire as algebraists to realize it in a commutative diagram cannot then be assuaged. Indeed, the Brauer functor cannot be defined on derived categories, as it is neither left or right exact. This suggests that the right framework of study might be the homotopy category of \( p \)-permutations modules.

**Lemma 2.1.** For \( X \) a \( \Delta P \)-projective \( k[G \times H] \)-module and \( Y \) a \( kH \)-module, we have that \( \text{Res}_{G \times H}^{\Delta P \times H}(X \otimes_k Y) \) is \( \Delta P \)-projective, where \( \Delta P \) is canonically embedded into \( G \times \Delta H \).
Proof. By assumption, $X|\text{Ind}_{\Delta P}^{G \times H^{\text{opp}}}(X')$ for $X'$ a $k\Delta P$-module and hence we can write

$$\text{Res}_{G \times \Delta H^{\text{opp}}}(\text{Ind}_{\Delta P}^{G \times H^{\text{opp}}}(X') \otimes_k Y) = \text{Res}_{G \times \Delta H^{\text{opp}}}(\text{Ind}_{\Delta P}^{G \times H^{\text{opp}}}(X' \otimes_k Y)).$$

By Mackey’s theorem, the latter is equal to:

$$\bigoplus_{x \in \Delta P \times H \backslash (G \times H^{\text{opp}} \times H)} \text{Ind}_{(G \times \Delta H^{\text{opp}}) \cap (\Delta P \times H^{\text{opp}})}^{(G \times \Delta H^{\text{opp}})} x^*(X' \otimes_k Y).$$

However, $\Delta P \times H \backslash (G \times H^{\text{opp}} \times H)/G \times \Delta H^{\text{opp}} = \{1\}$ and so there is only one term in the previous direct sum.

Hence as $(G \times H)$-module, $\text{Res}_{G \times \Delta H^{\text{opp}}}(X \otimes_k Y)$ is $\Delta P$-projective.

We can now state the following result which is taken from [Ri2].

**Proposition 2.2** (à la Rickard). *Let $X$ be a complex whose terms are relatively $\Delta P$-projective $p$-permutation $kG \times kH$-bimodules, then for every subgroup $Q \leq P$, we have a commutative diagram:

$$
\begin{array}{ccc}
\text{Ho}^b(kH-\text{perm}) & \xrightarrow{X \otimes_{kH} -} & \text{Ho}^b(kG-\text{perm}) \\
\text{Br}_Q & \downarrow & \text{Br}_Q \\
\text{Ho}^b(kC_H(Q)-\text{perm}) & \xrightarrow{X^Q \otimes_{kC_H(Q)} -} & \text{Ho}^b(kC_G(Q)-\text{perm})
\end{array}
$$

Here $X_Q := \text{Br}_Q(X)$.

**Proof.** Everything can be carried on termwise: we first apply the previous lemma and we notice that if $M$ and $N$ are $p$-permutations $kG$-modules, then so is $M \otimes_k N$.

Hence for $Y$ a $p$-permutation $kH$-module, we have that the terms of $X \otimes_k Y$ is a $\Delta P$-projective $p$-permutation $k[G \times H]$-module. We can then apply an analogous version of the Lemma 4.2 and Lemma 4.3 of [Ri2] on tensor products rather than on Hom-spaces, in order to find that $\text{Br}_Q(X \otimes_{kH} Y) \simeq \text{Br}_Q(X) \otimes_{kC_H(Q)} \text{Br}_Q(Y)$.

However, we shall have been more careful here as the Brauer functor is actually defined from $\text{Ho}^b(kH-\text{perm})$ to $\text{Ho}^b(kN_H(Q)-\text{perm})$ (or even more precisely $\text{Ho}^b(k(N_H(Q)/Q)-\text{perm})$), and so we need to clarify things here. More concretely, we have to be careful when going from the normaliser down to the centraliser. This is the purpose of the following section.

3. A WHITE LIE AND ANOTHER COMMUTATIVE DIAGRAM

We shall start here by setting some notations. Firstly, recall that $e$ and $f$ are principal blocks idempotents and $P$ is an abelian $p$-Sylow. Since as $P$ is abelian, we have $p \nmid [N_G(Q) : C_G(Q)]$ for any $p$-subgroup $Q$.

If $X$ is a splendid tilting complex of $kG\times kH$-$f$-bimodules, we denote by $X_Q := \text{Br}_Q(X)$, the corresponding complex of $kN_{G \times H^{\text{opp}}} \Delta Q]$-module. By restriction to $C_G(Q) \times C_H^{\text{opp}}(Q)$, $X_Q$ still gives a Rickard equivalence.

More interestingly, according to a lemma of Marcus ([Ma]), we can lift this Rickard complex so that $X'_Q := \text{Ind}_{N_{G \times H^{\text{opp}}} \Delta Q}(X_Q)$ is also a Rickard equivalence between $kN_G(Q)e_Q$ and $kN_H(Q)f_Q$. From now on, we will write $N := N_{G \times H^{\text{opp}}} \Delta Q]$. 
In fact, we claim that we can link together the Rickard equivalences for the centralizers with the Rickard equivalences for the normalizers.

**Proposition 3.1.** Let $X$ be a splendid tilting complex of $k Ge \cdot k H f$-bimodules, then for every subgroup $Q \leq P$, we have the commutative diagram:

$$
\begin{array}{ccc}
\text{Ho}^b(kHf_{\text{perm}}) & \xrightarrow{\times kH_{\text{perm}}} & \text{Ho}^b(kGe_{\text{perm}}) \\
\downarrow_{\text{Br}_Q} & & \downarrow_{\text{Br}_Q} \\
\text{Ho}^b(kN_H(Q)_{\text{perm}}) & \xrightarrow{\times kN_H(Q)_{\text{perm}}} & \text{Ho}^b(kN_G(Q)_{\text{perm}}) \\
\downarrow_{\text{Res}_{C_H(Q)}} & & \downarrow_{\text{Res}_{C_G(Q)}} \\
\text{Ho}^b(kC_H(Q)_{\text{perm}}) & \xrightarrow{\times kC_H(Q)_{\text{perm}}} & \text{Ho}^b(kC_G(Q)_{\text{perm}})
\end{array}
$$

where for the sake of simplicity, we set $\tilde{X}_Q := \text{Res}_{C_G(Q) \times C_H(Q)^{opp}}(X_Q)$.

**Proof.** The bigger square is in fact the right formulation of the previous proposition à la Rickard, we have a canonical isomorphism of functors:

$$
\begin{align*}
\tilde{X}_Q \otimes_{kC_H(Q)} & \text{Res}_{C_H(Q)}(\text{Br}_Q(-)) \xrightarrow{\sim} \text{Res}_{C_G(Q)}(\text{Br}_Q(\Delta_Q(X \otimes kH -))) \\
\text{Res}_{C_G(Q)}(\text{Ind}_N^{N_G(Q) \times N_H(Q)^{opp}} X_Q \otimes_{kN_H(Q)} -) & \xrightarrow{\sim} \tilde{X}_Q \otimes_{kC_H(Q)} \text{Res}_{C_H(Q)}(-)
\end{align*}
$$

The commutativity of the bottom square can be expressed as

$$
\text{Res}_{C_G(Q)}(\text{Ind}_N^{N_G(Q) \times N_H(Q)^{opp}} X_Q) \xrightarrow{\sim} \text{Ind}_N^{C_G(Q) \times C_H(Q)^{opp}} X_Q
$$

Applying Mackey’s formula and as $1 \times N_H(Q)^{opp} \backslash N_G(Q) \times N_H(Q)^{opp} / N = 1$, we get:

$$
\text{Res}_{C_G(Q)}(\text{Ind}_N^{N_G(Q) \times N_H(Q)^{opp}} X_Q) \xrightarrow{\sim} \text{Ind}_N^{C_G(Q) \times C_H(Q)^{opp}} X_Q
$$

So that the previous expression now becomes:

$$
\text{Ind}_N^{C_G(Q) \times C_H(Q)^{opp}} X_Q \otimes_{kN_H(Q)} - \xrightarrow{\sim} \tilde{X}_Q \otimes_{kC_H(Q)} \text{Res}_{C_H(Q)}(-)
$$

Now, this is just expressing the adjunction between induction and restriction as an isomorphism between $k$-vector spaces. This is actually also an isomorphism of $kC_G(Q)$-modules.

It remains to prove that the upper square is commutative. In order to do so, it is enough to prove that both (1) and (3) are $N_G(Q)$-isomorphisms.

For the rest of this proof, we will work with Hom’s spaces rather than tensor product, as we rather deal with fixed than cofixed point. Of course, thanks to the adjunction between Hom-functor and tensor product functor, this does not change anything.

First of all, we shall point out that it is in no way easy to see that for $Y \in \text{Ho}^b(kHf_{\text{perm}})$, $N_G(Q)$ acts on $\text{Hom}_{C_H(Q)}(X_Q, Y_Q)$. What follows is a particularly nice trick to unveil the action. We denote by $Z := \text{Hom}_k(X_Q, Y_Q)$ the $(N^{opp} \times N_H(Q))$-bimodule and we try to provide $Z^{\Delta C_H(Q)} = \text{Hom}_{C_H(Q)}(X_Q, Y_Q)$ with an action of $N_G(Q)$. If one can find $N' \subseteq N^{opp} \times N_H(Q)$, such that the following short exact sequence holds:

$$
1 \longrightarrow \Delta C_H(Q) \longrightarrow N' \longrightarrow N_G(Q) \longrightarrow 1
$$


then we could endowed $\big(\text{Res}_{N}Z\big)^{\Delta_{CH}(Q)}$ with a natural action of $N_{G}(Q)$. We will proceed in two steps, the first one defines $\tilde{N}$ as follow:

$$N^{\text{opp}} \times N_{H}(Q) \to \text{Aut}(Q)^{\text{opp}} \times \text{Aut}(Q)$$

We invite the attentive reader to check that $N' = \tilde{N} \cap (N_{G}(Q) \times \Delta_{N_{H}(Q)})$ suits us. Concretely, the action of $g \in N_{G}(Q)$ on $\text{Hom}_{kC_{H}(Q)}(X_{Q}, Y_{Q})$ is defined as the action of $(g, h) \in N'$: for $\beta \in \text{Hom}_{kC_{H}(Q)}(X_{Q}, Y_{Q})$ and $g \in N_{G}(Q)$, $g.\beta := \beta(g \cdot h)^{-1}$.

We now turn to see that the adjunction (3) is also a $kN_{G}(Q)$-morphism. The Mackey’s isomorphism (2) provides $\text{Ind}_{CH(Q)^{\text{opp}}}^{N_{H}(Q)^{\text{opp}}}(X_{Q})$ with an action of $N'$:

$$k(N_{G}(Q) \times N_{H}(Q)^{\text{opp}}) \otimes_{kN} X_{Q} \xrightarrow{\sim} X_{Q} \otimes_{kC_{H}(Q)} N_{H}(Q)$$

where $k \in N_{H}(Q)$ and $g \in N_{G}(Q)$ induce the same automorphism of $Q$, so that $g \otimes h = \big(\sum_{e \in N} (g \otimes k)(1 \otimes k^{-1}h)\big)$.

That way we define an action of $N_{G}(Q) \times N_{H}(Q)$ on $\text{Ind}_{CH(Q)^{\text{opp}}}^{N_{H}(Q)^{\text{opp}}}(X_{Q})$. For $(g, h) \in N_{G}(Q) \times N_{H}(Q)$ and $x \otimes n \in \text{Ind}_{CH(Q)^{\text{opp}}}^{N_{H}(Q)^{\text{opp}}}(X_{Q})$, we have $(g, h). (x \otimes n) := \phi((g, h). \phi^{-1}(x \otimes n)) = ((g \otimes k)x) \otimes k^{-1}nh$.

Hence for $g \in N_{G}(Q)$ such that $(g, h, h) \in N'$, $g. (x \otimes n) = (\phi(g \otimes h)x) \otimes h^{-1}nh$.

Now recall that we had the adjunction (3):

$$\text{Hom}_{kC_{H}(Q)}(X_{Q}, \text{Res}_{CH(Q)^{\text{opp}}}(Y_{Q})) \xrightarrow{\sim} \text{Hom}_{kN_{H}(Q)}(\text{Ind}_{CH(Q)^{\text{opp}}}^{N_{H}(Q)^{\text{opp}}}(X_{Q}), Y_{Q}))$$

For $g \in N_{G}(Q)$, $h \in N_{H}(Q)$ such that $(g, h, h) \in N'$, $F(g.\beta) = (x \otimes n \mapsto \beta(gxh)^{-1}h^{-1}n^{-1}) = (x \otimes n \mapsto \beta(gxh)(h^{-1}n^{-1}h)^{-1}) = g. F(\beta)$.

We conclude, as promised, that (3) is a $N_{G}(Q)$-isomorphism.

Finally, a little diagram chasing lead us to our desired conclusion: the upper square is also commutative.

\[ \square \]

4. Perverse Equivalences and Clifford Theory

Let $G$ a finite group and $H$ a normal subgroup of $G$ of index prime to $p$ with $G = H \times L$. Let $S_{G}$ (resp. $S_{H}$) the set of isomorphism classes of simple $kG$-module (resp. $kH$-module). We define an equivalence relation on $S_{G}$ by $M \sim N$ if $\text{Hom}_{kH}(\text{Res}_{H}(M), \text{Res}_{H}(N)) \neq 0$.

**Lemma 4.1.** Induction and restriction yields to a bijection $S_{H}/L \sim S_{G}/\sim$.
Consider $S_\bullet := (\emptyset = S_0 \subset S_1 \subset \ldots \subset S_r = S_G)$ a filtration of $S_G$ and a perversity function $p : \{1, \ldots, r\} \to \mathbb{Z}$. This perversity datum $(p, S_\bullet)$ is said to be $H$-compatible if it is compatible with $\sim$.

We adapt to our situation a result of [CraRou]:

**Proposition 4.2.** Let $X$ be a complex of $\mathcal{K}N_G \times H\text{-}\text{proj}(\Delta Q)$-module and let $(p, S_{C_G(Q), \bullet}, S_{C_H(Q), \bullet})$ a $H$-invariant perversity datum with corresponding $H$-compatible datum $(p', S_{N_G(Q), \bullet}, S_{N_H(Q), \bullet})$. Then $\text{Res}_{C_G(Q) \times C_H(Q)\text{-}\text{proj}}(X)$ induces a perverse equivalence relative to the datum $(p, S_{C_G(Q), \bullet}, S_{C_H(Q), \bullet})$ if and only if $\text{Ind}_{C_G(Q) \times C_H(Q)\text{-}\text{proj}}(X)$ induces a perverse equivalence relative to $(p', S_{N_G(Q), \bullet}, S_{N_H(Q), \bullet})$.

**Proof.** The equivalence part is given by Marcus’ lemma. For the perversity, we have to use the previous lemma. We refer to [CraRou] for a more detailed proof.

\[\square\]

5. ABOUT SPLendid PERVERSE EQUIVALENces

Splendid equivalence gives rise to derived equivalences at the level of centralizers of $p$-elements. It then seems natural to wonder if a splendid perverse equivalence would also gives perverses equivalences locally. Under some mild assumptions, the answer is positive.

Firstly, we recall the definition of a perverse equivalence for the homotopy category [ChRou2]. Let $C, C'$ additive categories satisfying the Krull-Schmidt property. Given $C \in \text{Comp}^b(C)$, we denote by $C_{\text{min}} \subset \text{Comp}^b(C)$ the complex, unique up to isomorphism such that $C \simeq C_{\text{min}}$ in $\text{Ho}^b(C)$ and has no non-zero direct summand that is homotopy equivalent to 0.

Let $I$ be the set of isomorphism classes of indecomposable objects of $C$. We have a bijective correspondence $I \leftrightarrow [I]$ from Serre subcategories of $C$ to subsets of $I$. Finally, we denote by $I_\bullet$ (resp. $I'_\bullet$) a filtration of $C$ (resp. $C'$) of length $r$ by Serre subcategories and consider $p : \{1, \ldots, r\} \to \mathbb{Z}$.

**Definition 5.1.** An equivalence $F : \text{Ho}^b(C) \sim \text{Ho}^b(C')$ is perverse relative to $(C_\bullet, C'_\bullet, p)$ if and only if

- for $M \in [I_i] - [I_{i-1}]$, we have $(F(M)_{\text{min}})^r \in I'_{i-1}$ for $r \neq -p(i)$ and $(F(M)_{\text{min}})^{-p(i)} = M' \oplus L$ for some $M' \in [I'_i] - [I'_{i-1}]$ and $L \in I'_{i-1}$.
- The map $M \mapsto M'$ gives a bijection $[I_i] - [I_{i-1}] \sim [I'_i] - [I'_{i-1}]$.

The following lemma [ChRou] establishes the connection between perverse equivalence for the homotopy category of projectives $kG$-modules and perverse equivalence for the derived category of $kG$-mod. Consider $S$ and $S'$ filtrations of $kG$-mod and $kH$-mod of length $r$. We denote by $P_i$ the additive full subcategory of $kG$-proj generated by the projective covers of $V$, $V \in S - S_{r-1}$. We define $\tilde{p}$ by $\tilde{p}(i) = p(r - i + 1)$.

**Lemma 5.2.** Consider an equivalence $F : \mathcal{D}^b(kG) \sim \mathcal{D}^b(kH)$ that restricts to an equivalence $\tilde{F} : \text{Ho}^b(kG\text{-proj}) \sim \text{Ho}^b(kH\text{-proj})$. The equivalence $F$ is perverse relative to $(S_\bullet, S'_\bullet, p)$ if and only if $\tilde{F}$ is perverse relative to $(P_\bullet, P'_\bullet, \tilde{p})$.

5.1. From global to local perversities. We now have all the tools we need to answer our original question. We consider $X$ a splendid Rickard complex of $kG e kH f$-bimodules that restricts to a perverse equivalence between the homotopy category of $p$-permutation modules.

This is an important assumption as this is stronger than asking for a perverse equivalence between derived categories.
We denote by $I_\bullet$ a filtration of $I$, the set of isomorphism classes of indecomposables $p$-permutation $kHf$-modules. For $Q$ a $p$-subgroup of $H$, we denote by $I_{Q\bullet}$ the corresponding filtration consisting only of permutation modules of vertex $Q$. Then $Br_Q(I_{Q\bullet})$ gives us a filtration on $kN_H(Q)f_Q\text{-proj}$. Let us see if the induced equivalence $Ho^b(kN_H(Q)f_Q\text{-proj}) \sim Ho^b(kN_G(Q)e_Q\text{-proj})$ is perverse. For that, we consider the following commutative diagram.

$$
\begin{array}{ccc}
Ho^b(kHf_{\text{perm}}) & \xrightarrow{X \otimes_{kH}} & Ho^b(kGe_{\text{perm}}) \\
\downarrow{Br_Q} & & \downarrow{Br_Q} \\
Ho^b(kN_H(Q)f_Q_{\text{perm}}) & \rightarrow & Ho^b(kN_G(Q)e_Q_{\text{perm}}) \\
\downarrow{Br_Q} & & \downarrow{Br_Q} \\
Ho^b(kN_H(Q)f_Q_{\text{proj}}) & \rightarrow & Ho^b(kN_G(Q)e_Q_{\text{proj}})
\end{array}
$$

Collecting all of the above, we have the following statements.

**Lemma 5.3.** For $M$ an indecomposable $p$-permutation $kHf$-module of vertex $Q$, the terms of $X \otimes_{kH} M$ have, up to conjugacy, vertices smaller than $Q$.

**Proof.** Let $M$ an indecomposable $p$-permutation $kHf$-module of vertex $Q$ with $M \in [I_i] - [I_{i-1}]$. Then $Br_Q(M)$ is a $kN_H(Q)f_Q$-projective indecomposable. Reciprocally, we know that any $kN_H(Q)f_Q$-projective indecomposable comes us this way. Now a quick diagram chasing gives us the desired result. \hfill $\Box$

**Proposition 5.4.** Let $X$ a splendid $kGe-kHf$-complex that restricts to a perverse equivalence $Ho^b(kHf_{\text{perm}}) \sim Ho^b(kGe_{\text{perm}})$. Then for any $p$-subgroup $Q$ of $P$, $Br_{AQ}(X)$ induces a perverse equivalence $D^b(kN_H(Q)f_Q) \sim D^b(kN_G(Q)e_Q)$.

**Proof.** We have to show that we have perverse homotopy equivalence between the corresponding homotopy category of projective modules thanks to Lemma 5.2. The only thing we need to check is that the bijections $M \mapsto M'$ (as in Definition 5.1) between each stratum, given by the global perverse equivalence, respects the vertex. A projective module is sent by a splendid complex on a perfect complex and hence if $M$ has vertex 0, so does $M'$. We can now proceed by induction. With the previous lemma, we know that if $M$ has vertex $Q$, then the terms of $X \otimes_{kH} M$ has vertex smaller than $Q$. However for every vertex $Q' < Q$ we have, up to isomorphism, as many $kG$-permutation modules with vertex $Q'$ than $kH$-permutation modules with vertex $Q'$. As the assignment $M \mapsto M'$ is already a bijection, we conclude by induction that $M'$ is also of vertex $Q$.

According to Broué-Puig’s parametrization, we have $I \sim \{(Q, N_H(Q)\text{-simples})\}_Q$. The induced perversity datum for elements of vertex $Q$ is denoted by $(I_{Q\bullet}, p_Q)$, where $p_Q$ is the restriction of $p$ to $I_{Q\bullet}$. We say it is $C_H(Q)$-compatible if for any $S, S'$ simple $kN_H(Q)$-modules such that $Res_{C_H(Q)}(S) \simeq Res_{C_H(Q)}(S')$, then $p(S) = p(S')$.

**Definition 5.5.** If for all $p$-subgroups $Q$, the induced perversity data $(I_{Q\bullet}, p_Q)$ are $C_H(Q)$-compatibles, we say that $(I_\bullet, p)$ is locally compatible.

Now, using Proposition 4.2 and the previous proposition, we can state the following:
Proposition 5.6. Let $X$ a splendid $kGe$-$kHf$-complex that restricts to a perverse equivalence $\Ho^b(kHf_{-\text{perm}}) \sim \Ho^b(kGe_{-\text{perm}})$ with cool perversity datum. Then for all $Q$, $X$ induces perverses equivalences $\mathcal{D}^b(kC_H(Q)\mathcal{f}_Q) \sim \mathcal{D}^b(kC_G(Q)\mathcal{e}_Q)$.

5.2. From local to global? We would now want a converse to Proposition 5.4, we need to see how can from the data of local perverse equivalence, we could obtain a global perverse equivalence. First, we need to slightly extend the notion of perverse equivalence for a partial order. Let $\mathcal{C}, \mathcal{C}'$ additive categories and $I$ (resp. $I'$) the set of indecomposable objects of $\mathcal{C}$ (resp. $\mathcal{C}'$). We consider $\leq$ (resp. $\leq'$) partial order on $I$ (resp. $I'$) such that $I$ and $I'$ are isomorphic as posets through a map $\phi$. A perversity function is then $p : I \to \mathbb{Z}$.

Definition 5.7. An equivalence $F : \Ho^b(\mathcal{C}) \sim \Ho^b(\mathcal{C}')$ is perverse relative to $(\leq, \leq', p)$ if and only if for $M \in I$, we have $(F(M)_{\min})^r \in I_{\leq'\phi(M)}$ for $r \neq -p(M)$ and $(F(M)_{\min})^{-p(M)} = \phi(M) \oplus L$ for $L \in \mathcal{E}$.

Of course, if the order is total, we find the original definition of perverse equivalence on homotopy categories.

Let’s go back to our classical settings and consider $\leq$ a partial order on $\{ (Q, N_H(Q)\text{-simples}) \}_Q$, or equivalently on $I$, the set of isomorphisms classes of indecomposable $p$-permutation $kH$-module. The next lemma tells us that we can refine any partial order in a particularly interesting way, so that it respects the inclusion of vertices.

Lemma 5.8. Consider an equivalence $X \otimes_{kH} - : \Ho^b(kHf_{-\text{perm}}) \sim \Ho^b(kGe_{-\text{perm}})$ given by a splendid complex $X$. Suppose that $X \otimes_{kH} -$ is perverse relative to $(\leq, p)$.

Then it is perverse relative to $(\leq^+, p)$ where $\forall Q, P$ $p$-subgroups of $G$, and $M \in N_H(Q)\mathcal{f}_Q$-simple, $N \in N_H(P)\mathcal{f}_Q$-simple, $(Q, M) \leq^+ (P, N)$ if $(Q, M) \leq (P, N)$ and $Q \leq_G P$.

Proof. This should be straightforward as we have already noticed in Lemma 5.2 that if $M$ has vertex $Q$, then the terms of $X \otimes_{kH} M$ have, up to conjugacy, smaller vertices.

Proposition 5.9. Suppose given $\leq$ on $I$ as before so that we can suppose $\leq$ respects the inclusions of vertices and let $\leq^Q$ denotes the local partial order induced by $Br_Q$ on $\Ho^b(kN_H(Q)\mathcal{f}_Q\text{-proj})$.

Let $X$ a splendid $kGe$-$kHf$-complex such that for all $Q$ it induces perverses equivalences $\mathcal{D}^b(kN_H(Q)\mathcal{f}_Q) \sim \mathcal{D}^b(kN_G(Q)\mathcal{e}_Q)$ relative to $\leq^Q$. Then $\Ho^b(kHf_{-\text{perm}}) \sim \Ho^b(kGe_{-\text{perm}})$ is a perverse equivalence relative to $\leq$.

Proof. We refer the reader to our previous commutative diagram and use that if $P < Q$ then $(P, N) < (Q, M)$ so that the terms of $X \otimes_{kH} M$ with vertex strictly smaller than $Q$ belongs to $\mathcal{I}_{\leq^Q(I,M)}$.

We have now proved enough to state the following equivalence:

Theorem 5.10. Suppose given a partial order $\leq$ on $I$ that respects the inclusions of vertices and a locally compatible perversity datum $(\leq, p)$. Then the splendid equivalence $X \otimes_{kH} - : \Ho^b(kHf_{-\text{perm}}) \sim \Ho^b(kGe_{-\text{perm}})$ is perverse relative to $\leq$ if and only if for all $Q \leq P$, it induces perverses equivalences $\mathcal{D}^b(kC_H(Q)\mathcal{f}_Q) \sim \mathcal{D}^b(kC_G(Q)\mathcal{e}_Q)$ relative to $\leq^Q$.

Remark 5.11. The attentive reader might have spotted that we took in our previous result all subgroups of $P$... hence also the trivial subgroup! This amounts to include in our ‘local’ data the perverse equivalence $\mathcal{D}^b(kHf) \sim \mathcal{D}^b(kGe)$. In the following section, we shall try to understand how much of an obstruction this condition on the trivial subgroup really is.
At this stage, one is not entirely satisfied with the previous result as our hope was to obtain a result in the flavour of Bouc-Rouquier’s theorem (cf. Proposition 1.1). However, we do not know yet how to make sense of a ‘perverse stable equivalence’. We will conclude by giving an example of a splendid equivalence which induces local perverse equivalences but is not a global perverse equivalence, as this shall give a measure of our previous obstruction.

Examples 5.12. Let $G$ be a finite group, $OG_e$ a block of $G$ with cyclic defect $p$-group $D$ and $ON_G(D) f$ the block of $N_G(D)$ Brauer correspondent of $OG_e$. Rouquier showed that the blocks $OG_e$ and $ON_G(D) f$ are splendidly Rickard equivalent (cf. [Rou1]). However, the splendid equivalence of Rouquier is not perverse in general. Indeed let us set $p = 2$ and consider the principal blocks of $G = SL_2(8)$ and its Borel subgroup $H = \mathbb{Z}/8\mathbb{Z} \rtimes \mathbb{Z}/7\mathbb{Z}$. We now have a splendid equivalence between the principal blocks $OG_e$ and $OH f$. However as the decomposition matrix of $G$ is not unitriangular, this can not be a perverse equivalence (cf. [ChRou2]). Now we claim that locally, the induced derived equivalences are just shifted Morita equivalences and hence they are perverse (cf [Dre]).

References

[BoXu] R. Boltje and B. Xu, On $p$-permutation equivalences: between Rickard equivalences and isotypies, Trans. Amer. Math. Soc. 360, 2008, 5067-5087.

[Br1] M. Broué, On Scott modules and $p$-permutation modules: an approach through the Brauer morphism, Proc. Amer. Math. Soc 93, 1985, 401-408.

[Br2] M. Broué, Isométries parfaites, types de blocs, catégories dérivées, Astérisque 181-182, 1990, 61-92.

[Br3] M. Broué, Equivalences of Blocks for Group Algebras, V.Dlab and L.L. Scott (Eds.), Finite dimensional algebras and related topics, Kluwer, 1994, 1-26.

[CaRi] M. Cabanes and J. Rickard, Alvis-Curtis duality as an equivalence of derived categories, Modular Representation Theory of Finite Groups, de Gruyter, 2001, 157-174.

[ChRou1] J. Chuang and R. Rouquier, Derived equivalences for symmetric groups and $\mathfrak{sl}_2$-categorification, Annals of Math. 167, 2008, 245-298.

[ChRou2] J. Chuang and R. Rouquier, Calabi-Yau algebras and perverse equivalences, in preparation.

[CraRou] D. Craven and R. Rouquier, Perverse equivalences and Broué’s conjecture, Adv. Math. 248, 2013, 1-58.

[Da] E.C. Dade, Blocks with cyclic defect groups, Ann. of Math. 84, 1966, 936-958.

[Dre] Léo Dreyfus-Schmidt, PhD thesis, Université Paris-Diderot, 2014.

[Li] M. Linckelmann, On splendid derived and stable equivalences of blocks of finite groups, J. Algebra 242, 2001, 819-843.

[Ma] A. Marcus, On equivalences between blocks of group algebras: reduction to the simple components, J. Algebra 184, 1996, 372-396.

[Ri1] J. Rickard, Derived categories and stable equivalences, J. Pure Appl. Algebra 64, 1989, 303-317.

[Ri2] J. Rickard, Splendid equivalences: derived categories and permutation modules, Proc. London Math. Soc. 72, 1996, 331-358.

[Rou1] R. Rouquier, The derived category of blocks with cyclic defect groups, Lectures given at the Workshop on Derived Equivalences, Pappenheim, 1994.

[Rou2] R. Rouquier, Local constructions in block theory, in preparation.

[Rou3] R. Rouquier, Block theory via stable and Rickard equivalences, Modular representation of finite groups, de Gruyter, 2001, 101-146.

[Se] J.-P. Serre, Représentations linéaires des groupes finis, 3rd edition, Herman, 1978.