Three Neutrino $\Delta m^2$ Scales and Singular Seesaw Mechanism

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It is shown that the singular seesaw mechanism can simultaneously explain all the existing data supporting nonzero neutrino masses and mixing. The three mass-squared differences that are needed to accommodate the atmospheric neutrino data (through $\nu_\alpha - \nu_\alpha$ oscillation), the solar neutrino data via MSW mechanism (through $\nu_e - \nu_x$ oscillation), and the positive result of $\nu_\mu - \nu_\tau$ oscillation from LSND can be generated by this mechanism, whereas the vacuum oscillation solution to the solar neutrino problem is disfavored. We find that the electron and tau neutrino masses are of order $10^{-3}$ eV, and the muon neutrino and a sterile neutrino are almost maximally mixed to give a mass of order 1 keV. Two heavy sterile neutrinos have a mass of order 1 keV which can be obtained by the double seesaw mechanism with an intermediate mass scale $\sim 10^5$ GeV. A possible origin of such a scale is discussed.

I. INTRODUCTION

There are several neutrino oscillation experiments which have indicated the nonzero neutrino masses and mixing. The solar neutrino problem is the first to be noted. The deficit of the solar neutrinos predicted by the Standard Solar Model (SSM) can be explained by neutrino oscillation between $\nu_e$ and $\nu_x$. The $\nu_x$ can be $\nu_\mu$, $\nu_\tau$, or a sterile neutrino. In the case of resonant MSW transitions, it was found that the oscillation parameters $\Delta m^2$ and $\sin^2 2\theta$ ($\theta$ is the mixing angle) given by

\[
3 \times 10^{-6} \lesssim \Delta m_{\text{solar}}^2 \lesssim 1.2 \times 10^{-5}
\]
\[
4 \times 10^{-3} \lesssim \sin^2 2\theta_{\text{atm}} \lesssim 1.2 \times 10^{-2}
\]

(1)
can explain the solar neutrino problem. The solar neutrino problem can also be solved by invoking vacuum neutrino oscillations, in which case the neutrino mass-squared difference is about $\Delta m^2 \sim 10^{-11} eV^2$.

Another hint of the neutrino masses and mixing comes from the experiments on the atmospheric neutrinos. The indications in favor of $\nu_{\mu} \leftrightarrow \nu_{\tau}$ oscillations ($x \neq \mu$) have been found in the Kamiokande, IMB, and recent Super-Kamiokande, Soudan-II atmospheric neutrino experiments. From the analysis of the Super-Kamiokande and the other data the allowed ranges for the oscillation parameters were obtained, assuming $\nu_\mu \leftrightarrow \nu_\tau$,

\[
3 \times 10^{-4} \lesssim \Delta m_{\text{atm}}^2 \lesssim 7 \times 10^{-3}
\]
\[
0.8 \lesssim \sin^2 2\theta_{\mu\tau} \lesssim 1.
\]

(2)

The recent results from the Chooz reactor experiment appear to exclude the $\nu_\mu \leftrightarrow \nu_e$ oscillation as a solution to the atmospheric neutrino problem. The neutrino oscillations $\nu_\mu \leftrightarrow \nu_\tau$ and $\nu_\mu \leftrightarrow \nu_\alpha$ give rise to a similar result for the atmospheric neutrino anomaly since $\nu_\alpha$ and $\nu_\tau$ are not distinguishable in the current experiments. New experiments with high statistics in Super-Kamiokande and ICARUS can provide the discrimination by measuring the neutral current events appearing, for instance, in neutral pion productions.

Finally, indications in favor of $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations have been found recently in the LSND experiment, in which antineutrinos originating from the decays of $\mu^+$ at rest were detected. The KARMEN experiment will be able to crosscheck the positive result of LSND in the near future. From the analysis of the data of the LSND experiment and the negative results of other short-baseline experiments (in particular, the Bugey and BNL E776 experiments), one obtains the oscillation parameters:

\[
0.3 \lesssim \Delta m_{\text{LSND}}^2 \lesssim 2.2 eV^2
\]
\[
10^{-3} \lesssim \sin^2 2\theta_{e\mu} \lesssim 4 \times 10^{-2}.
\]

(3)

All the existing neutrino experiments which are in favor of neutrino oscillations cannot be accommodated by a scheme with mixing of ordinary three neutrinos. That is, the allowed range of the mass-squared differences which explain the solar neutrino, atmospheric neutrino, and LSND neutrino experiments do not overlap at all. In order to obtain three independent mass-squared differences which can explain the known three experiments, we need at least four massive neutrinos.

Furthermore, only two schemes of neutrino mass-squared differences are compatible with the results of all the experiments. Four neutrino masses are divided into two pairs of almost degenerate masses separated by a gap of $\sim 1$ eV which is indicated by the result of the LSND experiments:
In (A), $\Delta m_{21}^2$ is relevant for the explanation of the atmospheric neutrino anomaly and $\Delta m_{43}^2$ is relevant for the suppression of solar $\nu_e$’s. In (B), the roles of $\Delta m_{21}^2$ and $\Delta m_{43}^2$ are interchanged.

If there exists a fourth neutrino, it has to be a sterile neutrino as indicated by the invisible decay width of $\nu_s$. In section III, we will find the region of the two mass parameters by which all the neutrino data mentioned above can be accommodated. It is then required that the Dirac mass scale is $\sim 1$ eV and the heavy Majorana mass scale is $\sim 1$ keV. Such low mass scales are shown to imply the presence of an intermediate scale $\sim 10^5$ GeV and the grand unification scale $\sim 10^{16}$ GeV in the double seesaw mechanism introduced in section IV. Finally, we conclude in section V.

II. SINGULAR SEEWSAWE MECHANISM

In the singular seesaw mechanism [26], the neutrino mass matrix is written by

$$\mathcal{M} = \begin{bmatrix} 0 & \epsilon M_D^T \\ \epsilon M_D & M_M \end{bmatrix}$$

(5)

where $M_D$ is the usual Dirac neutrino mass matrix and $M_M$ is the right-handed Majorana neutrino mass matrix taken to be a singular (rank-two) $3 \times 3$ matrix. Here, we assume that there is no hierarchical structure in the mass matrices $M_D$ and $M_M$ whose elements are of the same order of magnitude, denoted by $M$. The small number $\epsilon$ encodes the hierarchical structure of the Dirac and sterile (right-handed) neutrino masses. The value $M$ is related to physics of lepton number violation and new physics. Two parameters $\epsilon$ and $M$ are to be determined later. We write the neutrino states in the interaction basis as

$$\Psi = \begin{bmatrix} \psi_{(e,\mu,\tau),l} \\ \psi_{(e,\mu,\tau),S} \end{bmatrix}$$

(6)

where $\psi_{(e,\mu,\tau),l}$ and $\psi_{(e,\mu,\tau),S}$ represent, respectively, the three standard active and sterile neutrinos. In the context of the singular seesaw mechanism, a combination of $\psi_{(e,\mu,\tau),S}$ becomes light. In order to obtain physical neutrino states and mass eigenvalues, we perform diagonalization by several steps. First step is to diagonalize the Majorana part by a rotation matrix $R$ such as

$$\mathcal{M} = \begin{bmatrix} 1 & 0 \\ 0 & R^T \end{bmatrix} \begin{bmatrix} 0 & \epsilon M_D^T \\ \epsilon R M_D & M_M \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & R \end{bmatrix}$$

(7)

where

$$\tilde{M}_M = R M_M R^T = \text{diagonal matrix}$$

(8)

Since $M_M$ is a rank-two singular matrix, we can take zero for the (11)-element of the diagonal matrix $M_M$. Then, we rewrite the mass matrix as

$$\begin{bmatrix} 0 & \epsilon M_D^T \\ \epsilon R M_D & M_M \end{bmatrix} = \begin{bmatrix} \epsilon M_\alpha & \epsilon M_\beta \\ \epsilon M_\beta & \tilde{M}_m \end{bmatrix}$$

(9)

where $M_\alpha$ is a $4 \times 4$ matrix, $M_\beta$ is a $4 \times 2$ matrix, and $\tilde{M}_m$ is a $2 \times 2$ diagonal matrix. In particular, the matrix elements $M_{\alpha i j}$ for $i, j = 1, 2, 3$ are zero. The values of the other matrix elements can be taken to be arbitrary. The next step is to block-diagonalize this mass matrix as follows;
neutrino mass eigenstates are
\[
\begin{bmatrix}
\epsilon M_\alpha & \epsilon M_\beta \\
\epsilon M_\beta & \tilde{M}_m
\end{bmatrix}
\]
(10)
\[
\begin{bmatrix}
1 & \epsilon P \\
-\epsilon P^T & 1
\end{bmatrix}
\begin{bmatrix}
Q & 0 \\
0 & \tilde{M}_m
\end{bmatrix}
\begin{bmatrix}
1 & -\epsilon P \\
\epsilon P^T & 1
\end{bmatrix}
\]

Here the \(4 \times 2\) matrix \(P\) and the light neutrino mass matrix \(Q\) are given by
\[
P = M_\beta \tilde{M}_m^{-1} \quad Q = \epsilon M_\alpha - \epsilon^2 P \tilde{M}_m P^T = \epsilon M_\alpha - \epsilon^2 M_\beta \tilde{M}_m^{-1} M_\beta^T.
\]
Finally, the light neutrino mass matrix \(Q\) is diagonalized by a \(4 \times 4\) unitary rotation matrix \(U\). The mass eigenstates (physical states) are given by
\[
\Psi_P = \begin{bmatrix} U & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\epsilon P \\ \epsilon P^T & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} \psi_{(\nu_e,\mu,\tau),L} \\ \psi_{(\nu_e,\mu,\tau),S} \end{bmatrix}
\]
(12)

Let us now determine the masses of six physical neutrinos. The mass matrix relevant for heavy neutrinos is \(M_m\). The nonzero values of the matrix elements are of order \(M\). And the physical neutrino fields are given by
\[
\nu_5 = \sum_{\alpha=e,\mu,\tau} \epsilon \left(P_{\alpha,1} \nu_{\alpha,1} + P_{\alpha,4} R_{\alpha,1} \nu_{\alpha,S}\right) + R_{2,\alpha} \nu_{\alpha,S}
\]
\[
\nu_6 = \sum_{\alpha=e,\mu,\tau} \epsilon \left(P_{\alpha,2} \nu_{\alpha,1} + P_{\alpha,4} R_{\alpha,1} \nu_{\alpha,S}\right) + R_{3,\alpha} \nu_{\alpha,S}
\]
(13)
\[
\nu_5 \simeq R_{2,\alpha} \nu_{\alpha,S}, \quad \nu_6 \simeq R_{3,\alpha} \nu_{\alpha,S}.
\]
The masses of the light neutrinos come from diagonalizing mass matrix \(Q\) given by
\[
Q = \epsilon M_\alpha - \epsilon^2 M_\beta \tilde{M}_m^{-1} M_\beta^T.
\]
Neglecting \(\epsilon^2\) term, the most general matrix is
\[
Q = \epsilon M_\alpha = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}
\]
(15)
The eigenvalues of \(Q\) are two zeros and \(\pm \sqrt{a^2 + b^2 + c^2}\). These two-fold degeneracies are lifted by the \(\epsilon^2\) term \(-\epsilon^2 M_\beta \tilde{M}_m^{-1} M_\beta^T\). Therefore, the light neutrinos are, in general, two with mass \(\epsilon^2 M\) and two with mass \(\epsilon M\), their mass difference being of order \(\epsilon^2 M\).

When we neglect the \(\epsilon^2\) term in the mass matrix, the neutrino mass eigenstates are
\[
\nu_{3,4} = \frac{1}{\sqrt{2}} \left( \begin{array}{c}
a \nu_e + b \nu_\mu + c \nu_\tau \\
\sqrt{a^2 + b^2 + c^2} \\
a \\ b \\ c
\end{array} \right) \pm \nu_S
\]
(16)
where \(\nu_S = R_{1,\alpha} \nu_{\alpha,S}\) is the massless component of the sterile neutrino mass matrix. Other two neutrino states (whose masses are of order \(\epsilon^2 M\)) are orthogonal combinations of \(\nu_{3,4}\). Including the \(\epsilon^2\) term in the light neutrino mass matrix \(Q\), it will give an order of \(\epsilon\) to the neutrino mixing. Therefore, we know that the lightest pair of the neutrinos with mass \(\epsilon^2 M\) are most likely active neutrinos. However, the compositions of the \(\nu_3\) and \(\nu_4\) are dependent on the form of the mass matrix. For example, let \(a = b = c\), then the electron, muon and tau neutrino component in \(\nu_{3,4}\) is \(1/6\), respectively. The other half of these neutrinos is the sterile neutrino. As another example, consider \(c = 0\) and \(a = b\), then the electron and muon neutrino component in \(\nu_{3,4}\) is \(1/4\), respectively, and the other half of \(\nu_{3,4}\) is sterile.

We summarize the neutrino mass eigenvalues and their mass-squared differences which are relevant to the known neutrino experiments for our discussions. The neutrino masses are determined as follows:
\[
\begin{align*}
\nu_{e} & \approx m_{\nu_e} \approx \epsilon^2 M \\
\nu_{\mu} & \approx m_{\nu_\mu} \approx \epsilon M \\
\nu_{\tau} & \approx m_{\nu_\tau} \approx M.
\end{align*}
\]
(17)

Two lightest neutrinos, \(\nu_1\) and \(\nu_2\), are mostly active neutrinos. Two medium-weighted neutrinos, \(\nu_3\) and \(\nu_4\), are almost equal combinations of active and sterile neutrinos. Two heaviest, \(\nu_5\) and \(\nu_6\), are almost sterile neutrinos. Therefore, the mass-squared differences are given by
\[
\Delta m_{21}^2 = (m_{\nu_2} - m_{\nu_1})(m_{\nu_2} - m_{\nu_1}) \approx \epsilon^4 M^2
\]
\[
\Delta m_{43}^2 = (m_{\nu_4} - m_{\nu_3})(m_{\nu_4} - m_{\nu_3}) \approx \epsilon^4 M^2
\]
\[
\Delta m_{42}^2 = (m_{\nu_4} + m_{\nu_2})(m_{\nu_4} - m_{\nu_2}) \approx \epsilon^2 M^2
\]
and \(\Delta m_{12}^2 \approx \Delta m_{31}^2 \approx \Delta m_{32}^2 \approx \Delta m_{31}^2\).

III. DETERMINATION OF \(\epsilon\), M AND NEUTRINO MASSES

As shown above, the singular seesaw mechanism has three \(\Delta m^2\) scales which provide a possibility of explaining the three known experiments. It is, however, a priori uncertain whether three experimental data (1), (2) and (3) can be accommodated simultaneously since the three \(\Delta m^2\) are parametrized by only two numbers \(\epsilon\) and \(M\). We wish to see if the scheme (B) in Eq. (1) can indeed be realized. Note first that the large mixing required for the atmospheric neutrino oscillation is built in the singular seesaw mechanism. That is, a combination of active neutrinos and a sterile neutrino have the maximal mixing to yield degenerate neutrinos \(\nu_3, \nu_4\). Therefore, the atmospheric neutrino problem can be explained by the large mixing \(\nu_\mu \leftrightarrow \nu_\tau\) oscillation with the mass-squared difference \(\Delta m_{43}^2 \approx \epsilon^4 M^2\). The solar neutrino problem is then to be solved by the \(\nu_e \leftrightarrow \nu_\tau\) oscillation with the smallest mass-squared difference \(\Delta m_{21}^2 \approx \epsilon^2 M^2\). As a consequence, the \(\nu_\mu \leftrightarrow \nu_\tau\) oscillation occurs automatically with the largest mass-squared difference \(\Delta m_{12}^2 \approx \epsilon^2 M^2\).
which may yield observable signals in the $\nu_\mu \leftrightarrow \nu_e$ oscillation experiments.

Let us now look for the parameter region of $(\epsilon, M)$ determined by the neutrino experiments, that is, $\epsilon^4 M^2 = \Delta m^2_{\text{solar}}, \epsilon^2 M^2 = \Delta m^2_{\text{atm}}$ and/or $\epsilon^2 M^2 = \Delta m^2_{\text{LSND}}$. From the first two identities based on the MSW solution to the solar neutrino problem and the atmospheric neutrino oscillation, one finds the region of a crescent shape inside $\epsilon = (0.43 \sim 9.6) \times 10^{-3}$ and $M = (0.02 \sim 9.3)$ keV. Remarkably, this region give rise to $\Delta m^2_{12} = \epsilon^2 M^2$ in the sensitivity range of the LSND and KARMEN experiments. Imposing the LSND positive result, the allowed region of $(\epsilon, M)$ is further reduced, and in fact determined solely by the solar neutrino and LSND data. The common region of $(\epsilon, M)$ which can explain all three known experiments lies inside

$$\epsilon = (1.1 \sim 6.4) \times 10^{-3}, \quad M = (0.086 \sim 1.3) \text{ keV}, \quad (19)$$

which is shown in Fig. 1. From Eqs. (17) and (14), we find the following typical values for the neutrino masses;

$$m_{\nu_{\mu,2}} \sim 10^{-3} \text{ eV}$$

$$m_{\nu_{\mu,4}} \sim 1 \text{ eV}$$

$$m_{\nu_{\mu,6}} \sim 1 \text{ keV}. \quad (20)$$

If the solar neutrino deficit is due to the vacuum oscillation, the solar neutrino data and the atmospheric neutrino data yields $\epsilon = \Delta m^2_{21}/\Delta m^2_{32} \lesssim 3 \times 10^{-7}$. Then the heavy right-handed neutrinos ($\nu_{\mu,6}$) get mass 100 MeV $\lesssim M \lesssim M_0 = 30$ eV, and 30 eV $\lesssim m_{\nu_{\mu,6}}$ $\lesssim 700$ eV. The right-handed neutrinos with such masses may overclose the universe as they cannot decay into the standard model particles. The muon and light sterile neutrinos can be candidates for hot dark matter satisfying the overclosure bound $\sum m_{\nu_i} \lesssim 94h^2\Omega_\nu$ eV. But they are too heavy to provide a good fit for the structure formation as $\Omega_\nu \lesssim 0.3$ is required in mixed dark matter models [27,28]. Furthermore, one finds no region of $(\epsilon, M)$ which accommodates all the three neutrino experiments. Therefore, our model disfavors the possibility of solving the solar neutrino problem in terms of the vacuum oscillation.

It follows from the result (21) that almost degenerate muon and sterile neutrinos can be good candidates for hot dark matter [25], and the heavy sterile neutrino can be a candidate for the warm dark matter. Thus, the existence of hot dark matter desirable for structure formation in the mixed dark matter scenario is a natural consequence of the singular seesaw mechanism under consideration.

IV. DISCUSSIONS ON THE MASS SCALE OF THE STERILE NEUTRINOS

It seems unnatural to have sterile (right-handed) neutrinos with mass scale $M \approx$ keV. In the usual seesaw mechanism its scale is about $10^{12}$ GeV or GUT scale. Such a low mass scale of lepton number violation is not acceptable. In order to raise the lepton number violation scale, we introduce the “double seesaw mechanism” in which extra sterile neutrinos are needed in addition to the ordinary right-handed neutrinos. A simple realization of the double seesaw mechanism can be found in grand unified theory (GUT) with intermediate step breakings. The minimal (non-supersymmetric) grand unification model is ruled out from the study of the gauge coupling evolution [29], but non-minimal GUT models and supersymmetric GUT models can successfully accommodate the gauge coupling unification and the absence of proton decay. As an example, let us consider a GUT model with $E_6$ unification group. It has five neutral particles in each generations of the fermion [27] representation: Two (16 under SO(10)) of them are the neutrinos in the discussion, other three neutrinos (1 + 10 under SO(10)) are heavy neutrinos with GUT scale masses. Suppose that the mass matrix of the active neutrinos, the right-handed neutrinos and the extra sterile neutrinos takes the form;

$$\mathcal{M} = \begin{bmatrix} 0 & 0 & M_L \\ M_L^T & M_R^T & M_S \\ 0 & 0 & M_R \end{bmatrix}. \quad (21)$$

Here $M_L$ is the Dirac mass matrix originating from the electroweak symmetry breaking, and $M_R, M_S$ are generated from a higher symmetry breaking. Note that $M_L, M_R$ are $3 \times 9$ matrices, and $M_S$ is $9 \times 9$ matrix in the three generation model. It should be mentioned that the mass matrix (23) requires fine-tunings which may not be a serious problem in supersymmetric theories. Given the hierarchy $M_L << M_R << M_S$, the seesaw mechanism with the ultra heavy neutrino masses $M_S$ gives rise to the $6 \times 6$ matrix,

$$\mathcal{M}_{\text{sub}} = - \begin{bmatrix} M_L \\ M_R \end{bmatrix} \cdot S^{-1} \cdot \begin{bmatrix} M_L^T \\ M_R^T \end{bmatrix}$$

$$= - \begin{bmatrix} M_L M_S^{-1} M_L^T & M_L M_S^{-1} M_R^T \\ M_R M_S^{-1} M_L^T & M_R M_S^{-1} M_R^T \end{bmatrix} \quad (22)$$

which can be identified with the matrix (8) apart from the upper-left corner. Note that the singular seesaw mechanism requires the lower-right submatrix of Eq. (22) to be singular. The nonzero contribution in the upper-left part of the matrix (22) is order of the solar neutrino mass scale $\epsilon^2 M \sim 10^{-3} \text{ eV}$ which does not alter our conclusion.

Following the discussion in the previous section, we can find the typical scales of $M_R$ and $M_S$. Namely, requiring $\epsilon = M_L/M_R, M = M_R^2/M_S$, and $M_L \sim 100$ GeV, one obtains

$$M_R \approx 10^5 \text{ GeV}, \quad M_S \approx 10^{16} \text{ GeV}. \quad (23)$$

It is worth emphasizing that the heaviest mass scale $M_S$ coincides with the conventional GUT scale. On the other
hand, the intermediate scale $M_R$ turns out to be considerably lower than the conventional scale $10^{12}$ GeV desirable for the usual seesaw mechanism. In GUT models, various intermediate scales of gauge symmetry breaking can be made consistent with the unification of gauge coupling constants. In particular, such a low scale $M_R$ may be obtained by introducing certain exotic particles to the GUT model \[24\].

The scale $M_R$ may be related to physics of supersymmetry breaking. In the supersymmetric standard model, supersymmetry breaking can be mediated either by gravitation or by gauge interactions \[31\]. The latter scheme provides a natural suppression of flavor violation in the supersymmetric sector and yields distinctive phenomenological and cosmological consequences \[22,33\]. The minimal type of such theories requires the existence of vector-like quarks and leptons (messengers) at the mass scale ($10^4 \sim 10^7$) GeV. This is a just right scale for $M_R$ under discussion. Therefore, in the supersymmetric standard model with gauge-mediated supersymmetry breaking, it is conceivable that some messengers are vector-like sterile neutrinos with the mass matrix given in Eq. \[21\].

V. CONCLUSIONS

Motivated by recent experimental evidences for nonzero neutrino masses and mixing, we have examined the consequences of the singular seesaw mechanism. The three mass-squared scales required for simultaneously explaining the solar and atmospheric neutrino anomalies, and the LSND data can be realized if the Majorana mass matrix of right-handed neutrinos is rank-two. Without assuming any hierarchies in the Dirac and Majorana mass matrices, three mass-squared values are found to be determined by two mass parameters: the Dirac mass for the active neutrinos and the Majorana mass for heavy right-handed neutrinos. The singular seesaw mechanism cannot accommodate simultaneously the vacuum oscillation explanation of the solar neutrino deficit, and the atmospheric neutrino oscillation. However, the MSW solution to the solar neutrino problem is consistent with the model and the existence of the LSND mass scale is also explained. The almost maximal mixing of a sterile neutrino with the muon neutrino (having the Dirac mass $m_{\nu_{\mu s}} \sim 1$ eV) explains the atmospheric neutrino anomaly, and the mixing of the electron and tau neutrino explains the solar neutrino anomaly (having the lightest mass $m_{\nu_{e s}} \sim 10^{-3}$ eV). Two massive right-handed neutrinos turn out to be rather light (having the Majorana mass $m_{\nu_{\mu s}} \sim 1$ keV). We stress that the existence of hot dark matter (consists of $\nu_{\mu s}$) desirable for the structure formation of the universe is a natural consequence of our scheme. In addition, warm dark matter can be provided by the heavy right-handed neutrinos. We have introduced the double seesaw mechanism in which the two low mass scales, $m_{\nu_{\mu s}}$ and $m_{\nu_{\mu s}}$, are generated by the weak scale $M_L \sim 100$ GeV and an intermediate scale $M_R \sim 10^5$ GeV together with the usual grand unification scale $M_S \sim 10^{16}$ GeV. A candidate for the intermediate scale $M_R$ can be found in the GUT models with an intermediate step breaking, or in gauge-mediated supersymmetry breaking models.

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Fig. 1. The regions between the dashed lines, the dotted lines, and the solid lines are allowed by the solar neutrino data, the atmospheric neutrino data, and the LSND data, respectively. The shaded region accommodates all the three neutrino experiments.