Comparison of topological optimization methods on the example of column press traverse

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Abstract. Based on the example of a real three-dimensional object (the traverse of a column press) the results of applying the most common topological optimization methods (ESO and SIMP) in terms of accuracy and speed are presented. Two formulations of the topological optimization problem are considered: minimization of compliance with a limit on volume and minimization of volume with a limit on equivalent stresses. Engineering interpretations of optimized structures are proposed and the results of their comparison with the existing version of the traverse are presented.

1. Introduction
In recent decades, the topological optimization (TO) methods are widely applied in industry due to the rapid development of computers and software. At the stage of search for the structure (topology) of an object, the methods of traditional parametric optimization show low efficiency due to the complexity of formalizing the optimization problem and a large number of variables, while the maintenance methods make it possible to get closer to the best distribution of material in a given area in a reasonable time.

The choice of optimization method often represents a non-trivial task. Currently there are more than 10 varieties of maintenance methods. The article [1] reflects the history of the development of this scientific direction and provides a brief description of the main methods, highlights the advantages and disadvantages of each of them, but there is no data for a quantitative comparison of the methods.

SIMP (Solid Isotropic Material with Penalization) and types of evolutionary methods such as ESO (Evolutionary Structural Optimization), BESO (Bi-directional Evolutionary Structural Optimization) are most widely used in engineering practice. SIMP method involves finding the pseudo-density field of a material in a computational area using gradient methods (as a rule, the moving asymptotes method – MMA). The idea of evolutionary methods is to gradually remove excess material according to specified criteria for the effectiveness of its implementation. In contrast to ESO-method, BESO allows the addition of material in high-loaded areas.

In this paper, using the example of a real technical object (the traverse of a column press, see figure 1), SIMP method is compared with ESO method, as the simplest of all existing ones. This problem is considered in a three-dimensional formulation, in contrast to [2], where the indicated methods, as well as BESO, are compared by the example of the calculation of a cantilever beam in a two-dimensional formulation.
The analysis of publications on the implementation of ESO method leads to conflicting conclusions. In article [3], ESO method is subjected to serious criticism based on theoretical reasoning and the solution of some test problems. In particular, the author refers to the study of Zhou and Rozvany [4], where ESO method leads to an unsatisfactory result. However, in a later study [5], it was found that this result was due to the application of a too rough computational finite element mesh. Moreover, it was demonstrated that on a more accurate mesh ESO method provides the best solution (compared to SIMP method and the authors’ solution [4]). The article [3] also notes the irregular nature of the convergence of the objective function, the disadvantages of the computational algorithm of ESO method. The main argument of the critique of this method is the use of heuristics: it is not proven that the removal of inefficient finite elements (FE) leads to the optimal solution.

There is a lack of analysis of the manufacturability of the resulted structures for the majority of works in the field of TO. As a successful example of applying maintenance methods to a complex technical object, we note the paper [6], which is dedicated to solving the problem of optimizing the frame press using SIMP method. The authors of [6] come to the conclusion that the engineering interpretation of the results of the direct solution of the problem is difficult; therefore, they use additional constraints in the formulation of the problem, obtaining plane-parallel elements (stiffening ribs) instead of a non-technological three-dimensional structure. At the second stage, stiffening ribs are subjected to optimization without limitations on technological effectiveness.

The article [7] also considers the problem of optimizing the lower and upper traverse of the column press, however, the designs of the existing traverse are used as the initial area of optimization.

2. Problem formulation

The compliance of the structure or its volume can be used as the objective function at TO of the stress-strain state. In this regard, there are two main problem formulations:

1) minimizing the compliance of the structure with a restriction on its volume;
2) minimizing the volume with a limit on the maximum equivalent stress.

We consider each of the statements in more detail. The compliance of the static system characterizes quantitatively the work of external forces, which is completely transformed into the potential energy of deformations. The reduction of the compliance of the structure leads to an increase in its rigidity. In the general case, the compliance is determined according to the expression [8] under the action of volume (gravitational and inertial forces), surface and point loads:

\[
C = \int \mathbf{g} \cdot \mathbf{u} dV + \int \mathbf{q} \cdot \mathbf{u} dS + \sum_{i=1}^{n} \mathbf{F}_i \cdot \mathbf{u}_i, \tag{1}
\]

where \(\mathbf{g}\) – volume load vector, N/m³;
\(\mathbf{u}\) – displacement vector, m;
\(\mathbf{q}\) – surface load vector, Pa;
\(\mathbf{F}_i\) – force vector, acting in \(i\) point.

As noted above, the implementation of SIMP method requires the determination of the field of an auxiliary quantity, the relative pseudo-density, which is connected by a power law with the calculated modulus of elasticity:

\[
E_j = x_j^p E_0, \tag{2}
\]

where \(E\) – calculated modulus of elasticity of \(j\) of finite element (FE), Pa;
\( x_\text{j} \) – relative pseudo-density of \( \text{j} \) FE, \( x \in (0,1] \);
\( p \) – penalty parameter;
\( E_0 \) – elastic modulus of the used material, Pa.

The penalty parameter \( p \) is applied to minimize areas with intermediate values of relative pseudo-density (in this paper, the value 3 is assumed).

Thus, according to the first option of the problem formulation of TO, it is necessary to find a field of relative pseudo-density, in which the compliance of the system reaches a minimum value when the constraint on its volume is satisfied:

\[
C(x) \rightarrow \min;
\]

\[
\frac{1}{V_0} \int x \, dV \leq V_{\text{max}},
\]

where \( V_0 \) – initial volume of the optimized area, m\(^3\);
\( V_{\text{max}} \) – maximum allowable relative volume.

The second option of the problem formulation, which assumes minimization of the volume, is written in the form

\[
\int_{V} x \, dV \rightarrow \min;
\]

\[
\max \sigma_{\text{equiv}} \leq [\sigma],
\]

where \( \sigma_{\text{equiv}} \) – equivalent stress field, Pa;
\( [\sigma] \) – allowable stress, Pa.

In evolutionary TO methods, the set of active FE \( \Omega_{\text{live}} \) is used instead of the concept of pseudo-density as a variable quantity. Exclusion of FE from the global hardness matrix is a time-consuming procedure, which in practice is usually simulated by multiplying its hardness by a small amount. For example, in ANSYS system of finite element analysis, this approach is implemented using the so-called «FE-birth and death technology». Thus, the variable \( x \) in equations (3)-(5) with the use of evolutionary methods can be considered as a discrete rate of FE activity, taking values 0 or 1. In ESO method, the procedure for deactivating FE is irreversible.

Traverse of the press, shown in figure 1, performs the function of support, transferring during pressing the load from the upper plates on the columns. In addition to a significant tensile force, the columns perceive a bending moment due to deformation of the traverse. In order to prevent the critical loads and ensure the verticality of the columns during the pressing process, the traverse is made as rigid as possible, which leads to an increase in the metal intensity of the press as a whole. For example, the mass of the traverse of the press shown in figure 1, is 559 kg, which reaches 15% of the mass of the entire press. In this regard, the optimization of the traverse design in order to reduce its metal consumption becomes an urgent task.

The design domain is shown in figure 2. Since the column press has two planes of symmetry, its quarter is considered with the corresponding conditions:

\[
u_{\text{x}}(0,y,z) = 0;\]
\[
u_{\text{y}}(x,0,z) = 0.\]

The overall dimensions of the optimized area 1 (960×600×345 mm) correspond to the dimensions of the existing traverse design. In order to reduce the amount of computation, the computational area is limited to the traverse, thermal insulation and heating plates (2 and 3, respectively), the upper nut 4, the column 5 and the lower nut 6. The columns are connected to the massive base of the press with the help of the nut 6, therefore it is assumed that that they are rigidly fixed:
where $\Omega_{\text{fix}}$ – lower surfaces of the nut 6 and the column 5.

The lower nut of the traverse (see figure 1) is excluded from the calculation, since it serves only to fix the traverse at the required height and does not take the loads in the process of pressing. As it was shown in [9], a further reduction of the computational area leads to significant errors, since it is not possible with the help of boundary conditions (BC) to describe the effect of column bending on the stress-strain state of the traverse. On the lower surface of the heating plate 3, the dimensions of $600 \times 600 \times 50$ mm are set to be non-deformable to simulate the effect of molds that evenly fill the working surfaces of the plates:

$$ u_z(x, y, 0) = \text{const} \big|_{\Omega_3} $$

where $\Omega_3$ – heating plate region 3.

The compressive stress also acts on the lower surface of the heating plate 3:

$$ \sigma_z(x, y, 0) = -\frac{F}{S} \big|_{\Omega_3} $$

where $F = 2.5 \text{ MN}$ – press force;
$S = 0.36 \text{ m}^2$ – surface working area of the heating plate.

Taking into account the formulations (7), (9) and (10), the equation (1) for the calculation of compliance is simplified to the following form:

$$ (1') $$

$$ C = Fu_z(0, 0, 0). $$

Between the traverse 1 and column 5, the BC of ideal slide is set (without friction):

$$ u \cdot n|_{\Omega_1^{\text{slide}}} = u \cdot n|_{\Omega_5^{\text{slide}}}, $$

where $\Omega_1^{\text{slide}}$ – cylindrical surface of traverse 1;
$\Omega_5^{\text{slide}}$ – region of cylindrical surface of column 5 in contact with the traverse 1;
$n$ – unit vector of normal to the contact surface.

Similarly, the ideal sliding state BC is recorded between the traverse 1 and the nut 4. The condition of rigid connection of the plates with the traverse is ensured by the cohesiveness of the meshes (see figure 3, lower detail), therefore additional BC in the contact areas of components 1, 2 and 3 are not required. In the contact areas of unconnected meshes of nuts and columns (see figure 3, the upper detail), the rigid coupling BC is given:

$$ u|_{\Omega_4^{\text{bond}}} = u|_{\Omega_5^{\text{bond}}}, $$

where $\Omega_4^{\text{bond}}$ – region of the column 5, contacting with the nuts;
$\Omega_5^{\text{bond}}$ – internal cylindrical surfaces of nuts 4 and 6.

The estimated height of the column 5 comprises 1550 mm, the main diameter is 110 mm, the diameter of the threaded parts is 100 mm. The height of the upper nut is 4 - 120 mm, the height of the lower nut is 6 - 70 mm. The external diameters of the nuts are 160 mm. Thickness of a heat-insulating plate is 2 - 40 mm. The material of all components of the design scheme except for the heat-insulating plate is steel 30 with a modulus of
elasticity $E = 2 \cdot 10^{11}$ Pa and Poisson’s ratio $\nu = 0.3$. Properties of the heat insulating plate: $E_{\text{ins}} = 3 \cdot 10^{10}$ Pa, $\nu_{\text{ins}} = 0.18$.

3. Solution, Results and Discussion

The calculation of stress-strain state was carried out in ANSYS system of finite element analysis using linear equations of the theory of elasticity using FE of first order SOLID185. Linear BC of contact interaction (11) and (12) were modeled using contact FE CONTA174 and TARGE170 according to MPC method (multipoint constraint, see [10]).

When pressing, a significant part of the strain energy accumulates in the columns due to their elongation. Absolutely rigid traverse transfers only tensile force to the columns. According to a preliminary calculation, the elongation of the column under the action of a load $F/4$, evenly distributed on the lower surface of the upper nut, is 0.49 mm. Consequently, the compliance of the four columns $C_0$ with an absolutely rigid traverse is 1225 J. Further, the value of the difference in ductility will be used, which characterizes the accumulation of the deformation energy of the system due to a decrease in the hardness of the traverse:

$$C_u = C - C_0.$$

Two series of calculations were performed on the same FE model in accordance with two problem formulations of TO for a comparative analysis of SIMP and ESO methods. When implementing the I problem formulation (3)-(4), the calculated relative value of the volume of the existing traverse structure (see figure 1), which was 0.38, was used as a limitation of $V_{\text{max}}$.

Deactivation of FE in ESO method was performed by the criterion of equivalent stress, i.e. at each design stage the hardness of the least loaded FE was multiplied by a small amount ($10^{-6}$), the total volume of which was calculated based on the equation

$$V_{k+1} = V_k (1 - \alpha),$$

where $V_k$ – volume of «live» FE at $k$ iteration, m$^3$;

$\alpha = 0.01$ – deactivation coefficient.

The allowable stress of steel 30, used in the II problem formulation (5)-(6), according to [11] is $[\sigma] = 98$ MPa with a safety factor of 2.5. The algorithm for FE deactivating in ESO method does not depend on the option of the problem formulation, which affects only the criterion for the completion of the optimization process: the achievement of a set volume (formulation I) or the excess of allowable stress (formulation II).

The calculation results are summarized in table 1. Data analysis allows us to conclude that SIMP method is significantly superior to ESO method in speed when solving the task I TO – the calculation time is 35 times less. At the same time, SIMP method is slightly inferior by the value of the objective function by 5.2%.

Using the II problem formulation, the advantage of SIMP method in terms of speed is significantly reduced – the duration of the calculation is only 2.6 times less. According to the value of the objective function (relative to volume), ESO method is inferior by 5.4%, however, it exceeds significantly the solution of SIMP method in compliance – by 26%.

Thus, the convergence of SIMP method strongly depends on the option of the problem formulation: minimizing the volume with a limit on the maximum equivalent the stress required 7.3 times more iterations than on minimizing compliance with a limit on the volume. ESO method provides more rigid structures, which is especially evident when using the II problem formulation (5)-(6).

Figure 4 shows the graphs of the change of the objective function (1') and the restrictions on the volume when solving problem I of TO. The first iteration of the methods is no different, since it corresponds to the calculation of the initial construction, the compliance of which equals to $C = 1435$ J. Next in SIMP method, the specified volume value is set (0.38) and does not change further,
from the second iteration. Compliance at the second iteration increases sharply due to a decrease in volume and gradually decreases during subsequent iterations, approaching some optimum. SIMP optimization process continues until the convergence condition is satisfied:
\[ |C_{k+1} - C_k|/C_k < \varepsilon, \]

where \( \varepsilon = 0.001 \).

### Table 1. Comparison of calculation results

| Task | Rate | ESO | SIMP |
|------|------|-----|------|
| I    |      |     |      |
|      | Calculation time, min | 110.1 | 3.1 |
|      | Number of iterations | 98 | 12 |
|      | \( C_{tr}, J \) | 294.3 | 309.6 |
|      | \( \text{max } \sigma_{eqv}, \text{ MPa} \) | 82.7 | 88.0 |
| II   |      |     |      |
|      | Calculation time, min | 179.3 | 69.0 |
|      | Number of iterations | 163 | 88 |
|      | \( C_{tr}, J \) | 427.0 | 538.3 |
|      | Relative volume | 0.196 | 0.185 |

In ESO method, the volume gradually decreases in accordance with the set law (14), which leads to an increase in the objective function. Optimization continues until the condition on volume constraints (4) is satisfied. Thus, the concept of convergence of the objective function to ESO method does not apply.

The topologies of the traverse, obtained by solving the problem of TO (3)-(4), on average, exceed by 28% the rigidity of the existing traverse with relative compliance \( C_{tr} = 9.418 C_0 \) with equal masses.

The traverse structures obtained by solving the second TO task using SIMP and ESO methods have significant differences in the shape of the upper transverse beam (see table 1). As a result of engineering refinement, the manufacturability of these structures was significantly improved. A comparison of the final engineering interpretations of the results of task II solution of the maintenance task is given in table 2.
Despite the significant external differences, the presented optimized structures differ by less than 6% in terms of mass, stiffness and maximum equivalent stresses. The mass of the existing structure of the traverse on average exceeds the mass optimized by 1.57 times with comparable rigidity. An equivalent stress of 278 MPa is achieved in the area of the stress concentrator. It should be noted that the presented optimized traverses are efficient only under the condition that the heating plate is not deformable (9). The general case of loading, including the transfer of force through a single small-sized mold, is planned to be considered in further work.

Table 2. Comparison of engineering interpretations

|                  | ESO       | SIMP      | Original traverse |
|------------------|-----------|-----------|-------------------|
| Weight, kg       | 350       | 363       | 559               |
| $C_n$, J         | 436.7     | 457.1     | 418.9             |
| max $\sigma_{eqv}$, MPa | 121       | 115       | 278               |

4. Conclusion

- When minimizing compliance with a limit on the volume (I task TO), SIMP method is faster than ESO method by tens of times (in the considered example it is 35 times).
- The structures, obtained by ESO method, have greater rigidity. The most significant difference is observed when solving task II of TO (minimizing the volume with a limit on the maximum equivalent stress).
- The convergence of SIMP method significantly deteriorates when solving task II (in the considered example, 7.3 times more iterations were required).
- The number of iterations of ESO method depends only on the final volume and the specified deactivation coefficient of FE. When solving problem I of TO, the number of iterations is known in advance.
- Depending on the option of the problem formulation, SIMP method, in contrast to the ESO method, provides solutions that differ greatly in topology.

Based on the presented conclusions, SIMP method can be recommended for solving I task of TO. Additional studies of convergence are required to conclude on the effectiveness of this method when solving task II TO. Testing of ESO method using the traverse as an example did not reveal any problems other than time consuming.

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