Dynamic localization in quantum dots: analytical theory

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We analyze the response of a complex quantum-mechanical system (e.g., a quantum dot) to a time-dependent perturbation $\phi(t)$. Assuming the dot energy spectrum and the perturbation to be described by the Gaussian Orthogonal Ensemble of random matrices we find the quantum corrections to the energy absorption rate as a function of dephasing time $t_\phi$. If $\phi(t)$ is a sum of $d$ harmonics with incommensurate frequencies, the quantum corrections behave similarly to those of conductivity $\delta\sigma$ for the $d$-dimensional Anderson model of the orthogonal symmetry class. For periodic perturbations, the leading quantum corrections are generically absent as in the systems of the unitary symmetry class. Exceptions are the harmonic perturbation and all other periodic perturbations $\phi(t)$ that obey the generalized time-reversal condition $\phi(-t+\tau) = \phi(t+\tau)$, where $\tau$ is a certain shift. Such cases fall into the quasi-1d orthogonal universality class.

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The process of energy absorption by a quantum system with a time-dependent Hamiltonian underlies a large part of modern physics, both fundamental and applied. A generic Hamiltonian can be written in the form:

$$\hat{H}(t) = \hat{H}_0 + \hat{V}\phi(t),$$

(1)

where we explicitly separated the time-independent part $\hat{H}_0$ and the external perturbation $\hat{V}$ with the time dependence specified by a given function $\phi(t)$. Most often the relevant case is that of the classical ohmic Joule absorption. The simplest nonlinear effects that restrict the absorption rate are the saturation effects that originate from an upper bound on the spectrum of $\hat{H}_0$ (as in the textbook example of a two-level system).

In the past two decades attention of the scientific community was drawn to a different and much less trivial example of saturation when the spectrum of $\hat{H}_0$ is essentially unlimited in the energy space, yet, after a certain time the absorption stops. This so-called dynamic localization (DL) in the energy space was observed in numerical simulations on the kicked quantum rotor (KQR) – particle on a circle with $\hat{H}_0 = -\partial^2/\partial\theta^2$ and $\phi(t)$ being a periodic sequence of $\delta$-pulses [1], as well as in an actual experimental realization of the KQR – trapped ultracold atoms in the field of a modulated laser standing wave [2]. The mapping of the KQR to the quasi-random 1d Anderson model has been done in Ref. [3]. A similar analogy was exploited by Gefen and Thouless [4] to demonstrate the DL in a mesoscopic disordered ring threaded by a magnetic flux growing linearly in time. In Ref. [5] an analogy between the KQR and band random matrices was pointed out, the latter have been reduced to a 1d nonlinear $\sigma$ model [6]. In Ref. [7] the direct correspondence between the KQR and a 1d nonlinear $\sigma$ model was demonstrated.

On the other hand, numerical simulations for a $\delta$-kicked particle in an infinite potential well that differs from the KQR only by the boundary conditions, revealed no DL [9]. This example shows that DL is not a consequence of the one-dimensional character of the energy space but it depends on the details of both the unperturbed system and the perturbation. It is clear that the results on DL [5] obtained in the framework of the orthodox kicked rotor (standard map) model [8] cannot be automatically applied to a generic quantum mechanical system under arbitrary time-dependent perturbation. Even less clear is the status of a peculiar KQR model with the time-dependent perturbation characterized by three incommensurate frequencies where the phenomenon similar to the 3d Anderson localization-delocalization transition has been found numerically [9].

The most general assumption about a complex quantum-mechanical system would be the randomness of the Hamiltonian. It is well-known [10] that at energies smaller than the Thouless energy $E_c$ description of complex quantum systems falls into one of the three universality classes, each corresponding to a Gaussian ensemble of random matrices. Considering a non-magnetic electron system with the spin-rotation symmetry, we arrive at Eq. (1) with $\hat{H}_0$ and $\hat{V}$ from the Gaussian orthogonal ensemble (GOE). An analogous Hamiltonian was considered in connection with laser pumping of complex organic molecules [11], but DL was not a target at that stage. The problem of DL in systems described by the Hamiltonian [11] has been addressed in Ref. [12]. However, most of results of this study are qualitative in character and DL has been demonstrated numerically only for the case where the typical amplitude of harmonic perturbation $V$ and its frequency $\omega$ are of the order of the mean level spacing $\delta$ for the unperturbed system. This numerics has also shown that DL is unstable with respect to adding even a small amount of noise in the time dependence.

In this Letter we develop an analytical theory of DL for a closed system described by the Hamiltonian [11].
with $\hat{H}_0$ and $\hat{V}$ from the GOE of random matrix theory (RMT) and a general time dependence $\phi(t)$. We show that in this model the existence of DL and even the classical ohmic Joule regime with constant absorption rate depends strongly on the character of $\phi(t)$. For periodic $\phi(t) = \sum_n A_ne^{i\omega_n t}$ with $|A_n|^2$ decreasing as $1/n^1$ or slower, the absorption rate diverges and the ohmic regime does not exist in the limit of infinite matrix size $N$. In particular, this happens if $\phi(t)$ is a periodic $\delta$-function as in the orthodox KQR model. This result alone demonstrates the difference between the KQR and the time-dependent random matrix theory.

For a periodic $\phi(t)$ with rapidly decreasing $A_n$, we obtain that the ohmic Joule absorption rate $W_0$ is modified by the quantum interference effects responsible for the DL. We calculate the first (one-loop) quantum correction, which determines the weak DL, and show that it is only nonzero if the condition

$$\phi(-t + \tau) = \phi(t + \tau) \tag{2}$$

is fulfilled for a certain choice of $\tau$. In this case the correction $\delta W(t) \propto \sqrt{t}$ grows with the time $t$ of the action of the time-dependent perturbation, like in the quasi-1d Anderson localization in the orthogonal symmetry class. At a certain time scale $t_x$ the first correction $\delta W(t_x)$ becomes comparable with the Joule absorption rate $W_0$ indicating a crossover from the weak to strong localization in the energy space. On the other hand, if dephasing time $t_x$ is shorter than $t_x$, then dissipation remains ohmic, with the rate being smaller than $W_0$ by the value of $\delta W(t_x)$.

The condition (2) is a generalization of the time-reversal symmetry condition $\phi(t) = \phi(-t)$ for the case when a shift of the time origin does not matter [14]. For periodic functions with several harmonics Eq. (2) can be fulfilled only for special choice of relative phases. However any pure harmonic function obeys this condition and thus the monochromatic perturbation is rather an exception than a paradigm of a periodic perturbation. For generic periodic functions which do not obey Eq. (2) the quantum correction emerges in the two-loop approximation, as in systems of the unitary symmetry class.

Finally, we find that for $\phi(t)$ being a sum of $d$ harmonic functions with incommensurate frequencies the first quantum correction to the absorption rate is similar to the weak localization correction to conductivity of a $d$-dimensional disordered system of the orthogonal symmetry class. This gives an analytical support of the numerical observation of Ref. [15] for the KQR model.

Qualitative picture and mapping.—The similarity and difference between DL and AL, as well as between KQR and RMT can be qualitatively understood using the exact correspondence between a quantum system under a (multi-)periodic time-dependent perturbation and a tight-binding lattice model with the time-independent Hamiltonian. Consider a system with energy levels ("orbitals") $E_l$ under a harmonic perturbation $V_l\phi e^{i\omega lt}$.

$$V_l\phi e^{i\omega lt}. \quad \text{As follows directly from the Schrödinger equation, its time evolution can be expressed in terms of eigenfunctions and eigenvalues of another system (see Fig. 1), obtained from the original one by replicating it into a one-dimensional lattice and shifting the levels of each consequent site by $\hbar \omega$ so that the energy of the "orbital" $l$ on the site $s$ is given by $E_{l,s} = E_l - s\hbar\omega$, and introducing the coupling between the $l$th orbital of the site $s$ and the $l'$th orbital of the site $s+1$ by the stationary perturbation matrix element $V_{l'}$. In the same way one can show that higher harmonics in the perturbation $(2\omega, 3\omega, \ldots)$ correspond to the coupling to next neighboring sites (second, third, etc.), while the presence of several incommensurate frequencies $\omega_1, \ldots, \omega_d$ requires a $d$-dimensional lattice with sites $s = (s_1, \ldots, s_d)$, on-site energies $E_1 - s_1\omega_1 - \ldots - s_d\omega_d$, and the matrix element $V_{l'}$ at the $i$th frequency corresponding to the coupling along the $i$th dimension.

The $d$-dimensional space with "orbitals" on different "sites" coupled by the matrix elements of perturbation, arises quite naturally from the notion of quasi-energy [16]. However, this model is far from the conventional Anderson model, where there is only one orbital per site with a random energy and from the Wegner’s $N$-orbital model [17] where there is a fixed number of orbitals with random energies and random hopping integrals. The point is that being a copy of the original spectrum, the set of orbitals’ energies are not independent at different sites. Another difference is that the number of orbitals is infinite in the relevant limit of the infinite matrix size.

From the above mapping it is clear why slowly decreasing amplitudes of harmonics kill localization in RMT. It happens because of the possibility of long-range hops over the sites. It is also clear why the long-range hops are not so dangerous for localization in the KQR case. The point is that in the orthodox KQR model, the time-dependent perturbation is coupled to $\cos \theta$ which in the basis of eigenfunctions $e^{il\theta}$ of the unperturbed system corresponds to matrix elements $V_{l,l+1}$ that may connect only neighboring orbitals. For long enough distance between sites the coupled orbitals run out of resonance and the hopping is suppressed. Should the basis change to $\sin(l\theta)$ as in the case of infinite potential well in Ref. [15],
the matrix elements $V_{ll'} \propto 1/|l - l'|$ become long-range in the orbitals’ space, so that the resonance hopping between remote sites may still occur, though only between remote orbitals. For the RMT case $V_{ll'}$ does not decrease with the distance $|l - l'|$ and the condition for the long-range resonance hopping is further improved.

This consideration shows that the KQR with $\delta$-kicks and the time-dependent RMT with harmonic $\phi(t)$ are in fact two complementary models. In the former remote sites are connected but only for neighboring orbitals, while in the latter remote orbitals are connected only on neighboring sites. Sometimes – as in the case of several harmonics with incommensurate frequencies – this complementarity makes two models equivalent but this equivalence is fragile and needs to be checked in every particular case.

Description of the formalism.—Consider a closed system of non-interacting fermions with the single-particle Hamiltonian $H_0$, $H_0$ and $V$ being real symmetric random $N \times N$ matrices distributed with probabilities

\[ P_{H_0} \propto \exp\left[-\frac{\pi^2}{4N\delta^2} T H_0^2\right], \quad P_V \propto \exp\left[-\frac{\pi}{4T} \delta^2 V^2\right], \]

respectively, $\delta$ being the mean level spacing at the center of the band, and $\Gamma$ measuring the sensitivity of the energy levels $E_t$ to the variation of $\phi$: $\langle (\partial E_t/\partial \phi)^2 \rangle = (2/\pi)\Gamma\delta^2$. Such a form of time-dependent random matrix theory corresponds to a quantum dot under a perturbation with characteristic frequencies $\omega \ll E_c$.\[10\]

To treat the dynamical problem specified by the Hamiltonian $H$, we employ the nonlinear-\(\sigma\)-model approach recently developed in Ref. $11$ on the basis of the Keldysh nonequilibrium formalism $13$ $14$. In the limit $N \to \infty$, the effective action of the $\sigma$-model reads:

\[ S[Q] = \int \left[ \frac{\pi i}{2\delta} \text{tr}\{i\tau_3\sigma_0\delta t_\nu\partial t Q_{\nu\nu}\} \right. \\
\left. + \frac{\Gamma}{\delta^2} \langle \phi(t) - \phi(t')\rangle^2 \text{tr}\{Q_{\nu\nu}Q_{\nu'\nu'}\} \right] dt dt'. \tag{3} \]

Here $Q_{\nu\nu'}$ is a $4 \times 4$ matrix in the direct product of the $2 \times 2$ particle-hole and Keldysh spaces. Pauli matrices in these spaces are denoted by $\tau_i$ and $\sigma_i$, respectively. The first term in $S$ is the standard random-matrix action $20$ responsible for the whole spectral statistics, while the second – kinetic – term accounts for the effects of the time-dependent perturbation. The matrix $Q$ is subject to the constraints $(Q^2)_{\nu\nu'} = \tau_0\sigma_0\delta_{\nu\nu'}$ and $\tau_2\sigma_1 Q^\dagger \tau_2 = Q^\dagger$, where the product and the transpose involve the time arguments too. The saddle point of the action $3$ is

\[ A_{\nu\nu'} = \left( \begin{array}{cc} \delta_{\nu\nu'} & 2F_{\nu\nu'}^{(0)} \\ 0 & -\delta_{\nu\nu'} \end{array} \right) \otimes \tau_3, \tag{4} \]

where the function $F_{\nu\nu'}^{(0)}$ satisfies the kinetic equation

\[ \left[ \partial_t + \partial_\nu + \Gamma (\phi(t) - \phi(t'))^2 \right] F_{\nu\nu'}^{(0)} = 0. \tag{5} \]

$F_{\nu\nu'}$ is the electron distribution function in the time representation; a more familiar quantity is its Wigner transform $F_E(t) = \int d\eta e^{iE\eta} F_{\nu\nu'}/\Delta_{\nu\nu'}/\Delta_{\nu\nu'}$. In equilibrium with temperature $T$ it is $F_E = \tanh(E/2T)$. Out of equilibrium it satisfies the Wigner-transformed Eq. $5$, which after averaging over fast oscillations in $t$ reduces to the diffusion equation in the energy space:

\[ \left[ \partial_t - D\delta^2_{\nu\nu'} \right] F_E^{(0)}(t) = 0, \quad D = \Gamma \langle (\delta \phi/\partial t)^2 \rangle, \tag{6} \]

the overline meaning the average over time. Equation $5$ gives ohmic Joule absorption rate

\[ W_0 = D/\delta. \tag{7} \]

The saddle-point expression $4$ is valid provided that (i) the perturbation is sufficiently fast, $\partial_\nu \phi \gg \delta^3/\Gamma^2\Gamma^2$, which is the anti-adiabaticity condition ensuring that the spectrum is quasi-continuous and (ii) interference effects responsible for DL are neglected.

The perturbative correction to Eq. $6$ for the case of linear bias $\phi(t) = vt$ was calculated in Ref. $17$. There, the interference is ineffective and quantum corrections describe the high-$\nu$ tail of the crossover from the Kubo to the Landau-Zener regimes of dissipation $21$. Contrary, in this Letter we concentrate on the case of large $\partial_\nu \phi \gg \delta^3/\Gamma^2\Gamma^2$ but bounded $\phi(t)$, when the saddle-point approximation is invalidated by interference effects.

Quantum corrections to the mean-field absorption rate $6$ can be obtained in the regular way by expanding over Gaussian fluctuations near the saddle point. The deviation of the $Q$ matrix from $\Lambda$ is parametrized $17$ by the diffusion and cooperon modes $b_{\nu\nu'}$ and $a_{\nu\nu'}$ with the bare propagators: $\langle b_{\nu\nu'} b_{\nu'\nu'}^\dagger \rangle = (2\delta/\pi) \delta(\eta - \eta') D_\eta(t, t')$ and $\langle a_{\nu\nu'} a_{\nu'\nu'}^\dagger \rangle = (\delta/\pi) \delta(t - t') C_\eta(\eta, \eta')$, where

\[ D_\eta(t, t') = \theta(t - t') \exp\left[-\int_t^{t'} \gamma_\eta(\tau) d\tau\right], \tag{8} \]

\[ C_\eta(\eta, \eta') = \theta(\eta - \eta') \exp\left[-\frac{1}{2} \int_{\eta}^{\eta'} \gamma_\eta(t) d\xi\right], \tag{9} \]

and we have denoted $t_{\pm} = t \pm \eta/2$, $t'_{\pm} = t' \pm \eta'/2$, and $\gamma_\eta(t) \equiv \Gamma \langle \phi(t_{\pm}) - \phi(t_-) \rangle^2$.

In the presence of fluctuations, the average matrix $Q$ still has the form $3$ but with the saddle-point $F_E^{(0)}$ substituted by the renormalized electron distribution $F$ which determines the energy absorption rate:

\[ W(t) \equiv \partial_t \langle E(t) \rangle = -\frac{ie}{\delta} \lim_{\eta \to 0} \partial_\eta \partial_\eta F_{\nu\nu'}/\Delta_{\nu\nu'}/\Delta_{\nu\nu'} \tag{10} \cdot \]

The one-loop quantum correction to $Q$ contains a diffusion and one cooperon loop coupled by a Hikami box. We evaluate this diagram for a generic perturbation $\phi(t)$ switched on at $t = 0$. Using the asymptotics $F_{\nu\nu'}/\Delta_{\nu\nu'} \sim (i\pi\eta)^{-1}$ following from the property $\lim_{E \to \pm \infty} F_E(t) = \tanh(E/2T)$
\[ W(t) = W_0 + \frac{\Gamma}{\pi} \int_0^t \phi'(t) \phi'(t - \xi) C_{t-\xi/2}(\xi, -\xi) d\xi. \]  

(11)

The second term in Eq. (11) is the one-loop quantum interference correction to the ohmic Joule heating. Expression (11) is the main result of this part of the paper and will be the base for the subsequent considerations. It can be also obtained from the conventional diagrammatic technique (16, 22).

**Results.** First, we consider a periodic perturbation: 
\[ \phi(t) = \sum_n A_n \cos(n\omega t - \varphi_n), \]
with the diffusion coefficient 
\[ D = (\omega^2 \Gamma/2) \sum_n n^2 A_n^2. \]
If \( A_n \) decrease as \( n^{-3/2} \) or slower (e. g. for \( \delta \)-kicks), the diffusion coefficient and dissipation rate diverge. This divergence is cut either by a finite size \( N \) of the matrix or by the interaction effects that is beyond our analysis.

To study the long-time, period-averaged dynamics at \( t, \xi \gg 1/\omega \) we can approximate
\[ C_{t-\xi/2}(\xi, -\xi) \approx e^{-2\xi \sum n A_n^2 \sin[2n(\omega t - \xi/2) - \varphi_n].} \]

(12)

For a particular choice of phases, \( \varphi_n = n\varphi \), there exists a set of points \( \xi_k = 2t - 2(\varphi + \pi k)/\omega \) with integer \( k \) where the cooperon (2) is equal to unity (Fig. 2a). Existence of such no-dephasing points (3) is equivalent to the generalized time-reversal symmetry (2) of the perturbation. At large \( \xi \) only \( \xi \approx \xi_k \) contribute to the integral (11) (otherwise the cooperon is exponentially small) which can then be calculated with the steepest descent method. Performing summation over the no-dephasing points \( \xi_k \) we obtain a negative and growing in time quantum interference correction to the ohmic absorption rate (7):

\[ \frac{W(t)}{W_0} = 1 - \sqrt{\frac{t}{t_s}}, \quad t_s = \frac{\pi^2 \Gamma n^2}{2 \delta^2}; \]

(13)

where \( n^2 = \sum_n n^2 A_n^2 \) and the limit \( t \gg 1/\omega, 1/\Gamma \) is implied. The \( \sqrt{t} \) dependence is remarkably similar to the \( \sqrt{t} \) dependence of the first quantum correction to the conductivity of a particle in a quasi-one-dimensional disordered sample with the phase relaxation time \( t_s \).

In our case the relative quantum correction becomes comparable to unity at time \( t_s \). This is an indication of DL at the characteristic energy scale \( E_* \sim \sqrt{\Gamma n^2} \). Taking \( \delta = 3 \) \( \mu eV, \hbar \omega = 40 \) \( \mu eV (\omega/2\pi \approx 10 \) GHz), \( \Gamma n^2 = 10 \mu eV \) (corresponding to the microwave electric field of a few V/m over the dot size of 1 \( \mu m \)) we obtain \( t_s \sim 10 \) ns, \( E_* \sim 400 \mu eV \sim 5 \) K.

When the generalized time-reversal symmetry (2) is absent for \( \phi(t) \), the dephasing rate is positive and separated from zero by a finite gap (Fig. 2b). In this case the integral over \( \xi \) converges exponentially, so the first quantum correction stays small even at \( t \to \infty \), as in the systems of the unitary symmetry class.

Next, we consider the case of \( d \) incommensurate frequencies \( \omega_n, \phi(t) = \sum_{n=1}^d A_n \cos(\omega_n t - \varphi_n) \), for which

\[ D = (\Gamma/2) \sum_{n=1}^d \omega_n^2 A_n^2. \]

For incommensurate frequencies the relationship between the phases \( \varphi_n \) does not matter. The reason is that in this case there can be only one no-dephasing point at most which cannot make a decisive contribution to the integral, in contrast to the periodic case with an infinite number of such points. On the other hand, in the incommensurate case the dephasing rate can become arbitrarily small (pseudo-gap) in an infinite number of points regardless of phases (Fig. 2c), and the integral is dominated by these points.

To calculate the first quantum correction in Eq. (11) we use the cooperon (2) with \( n\omega \) substituted by \( \omega_n \), expand it into a \( d \)-dimensional sum involving the modified Bessel functions \( I_\nu(z) \), and average over \( t \) which is significantly simplified due to incommensurability. The analytical expression for the case of arbitrary relation between the amplitudes \( A_n \) is bulky. We write the resulting expression valid at \( t \gg 1/\omega, 1/\Gamma \) for the case when all \( A_n = 1 \):

\[ \frac{W(t)}{W_0} = 1 - \frac{\delta}{\pi \Gamma} \int_0^{\Gamma t} dz e^{-z^2}[I_0(z)]^{d-1} \frac{dI_0(z)}{dz}. \]

(14)

Using the asymptotic form \( I_0(z) \approx dI_0(z)/dz \approx e^z/\sqrt{2\pi z} \) at \( z \gg 1 \), we recover Eq. (13) for a monochromatic perturbation. For \( d = 2 \), the relative correction is \( (\delta/2\pi^2 \Gamma) \ln \Gamma t \), whereas for \( d > 2 \) it saturates \( \propto t^{1-d/2} \) in the limit of large \( t \), in complete analogy with the behavior of the quantum correction of Ref. (23) in \( d \) dimensions.

In conclusion, we have developed an analytical approach based on the zero-dimensional, time-dependent nonlinear sigma-model and obtained the weak dynamical localization in complex quantum systems under time-dependent perturbation described by the random matrix theory. The character of energy absorption in such systems is determined entirely by the frequency spectrum of a time-dependent perturbation. In particular we obtained no DL for the time-periodic \( \delta \)-function perturba-
tion, and the dynamical localization corrections similar to the $d$-dimensional weak localization corrections to conductivity if the perturbation is a sum of $d$ incommensurate harmonic functions.

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