Kelvin–Helmholtz versus Tearing Instability: What Drives Turbulence in Stochastic Reconnection?

Grzegorz Kowal1, Diego A. Falceta-Gonçalves1, Alex Lazarian2, and Ethan T. Vishniac3

1 Escola de Artes, Ciências e Humanidades, Universidade de São Paulo, Av. Arlindo Bétnio, 1000—Vila Guará, CEP: 03828-000, São Paulo—SP, Brazil; grzegorz.kowal@usp.br
2 Department of Astronomy, University of Wisconsin, 475 North Charter Street, Madison, WI 53706, USA
3 Department of Physics & Astronomy, Johns Hopkins University, 3400 N. Charles Street, Baltimore, MD 21218, USA

Abstract

Over the last few years it became clear that turbulent magnetic reconnection and magnetized turbulence are inseparable. It was not only shown that reconnection is responsible for violating the frozen-in condition in turbulence, but also that stochastic reconnection in 3D generates turbulence by itself. The actual mechanism responsible for this driving is still unknown. Processes such as the tearing mode or Kelvin–Helmholtz, among other plasma instabilities, could generate turbulence from irregular current sheets. We address the nature of the driving mechanism for this process and consider the relative role of tearing and Kelvin–Helmholtz instabilities for the process of turbulence generation. In particular, we analyze the conditions for development of these two instabilities within 3D reconnection regions. We show that both instabilities can excite turbulence fluctuations in reconnection regions. However, the tearing mode has a relatively slow growth rate, and at later times it becomes partially suppressed by a component of the magnetic field that runs transversely to the current sheet, which is generated during the growth of turbulent fluctuations. In contrast, the Kelvin–Helmholtz instability quickly establishes itself in the outflow region, and at later times, it dominates the turbulence generation compared to the contribution from the tearing mode. Our results demonstrate that the tearing instability is subdominant to the the Kelvin–Helmholtz instability in terms of generation of turbulence in the 3D reconnection layers, and therefore the self-driven reconnection is turbulent reconnection, and the tearing instability is only important at the initial stage of the reconnection.

Unified Astronomy Thesaurus concepts: Plasma astrophysics (1261); Solar magnetic reconnection (1504); Interplanetary turbulence (830)

1. Introduction

Magnetic reconnection is a fundamental problem that is essential for understanding the magnetohydrodynamic (MHD) flows, especially turbulent ones. Within such flows, magnetic flux tubes cross each other, and therefore the properties of the flow depend on whether the tubes can or cannot cross each other.

The answer that follows from the Sweet–Parker theory of magnetic reconnection (Parker 1957; Sweet 1958) is that in typical astrophysical situations the magnetic flux tubes cannot reconnect and change the magnetic field topology. Indeed, the Sweet–Parker reconnection rate is $V_{\text{rec,SP}} \approx V_A S_L^{-1/2} \ll V_A$, with $S_L = L V_A / \eta$ being the Lundquist number, $L$—a longitudinal scale of the reconnecting flux tube, $V_A$ the Alfvén speed, and $\eta$ the magnetic resistivity. Given the large scales of magnetic fields involved in astrophysical flows and the highly conductive nature of astrophysical plasmas, it is obvious that $S_L$ is so large that the rates predicted by the Sweet–Parker mechanism are absolutely negligible. This, however, is in gross contradiction with observational data, e.g., the data on solar flares. The Sweet–Parker reconnection is an example of a slow laminar reconnection, while much faster reconnection is required to explain observations. Formally, the fast reconnection is one that does not depend on $S_L$, or if depends on it, depends on it logarithmically.

For many years, the fast reconnection research was focused on the X-point reconnection, i.e., the reconnection at which the magnetic field is brought at a sharp angle in the reconnection zone. This is opposed to the Sweet–Parker reconnection, which is an example of the Y-point reconnection. The X-point reconnection was proposed by Petschek (1964) and required that the inflow and outflow of the matter into the reconnection zone are comparable. Indeed, the slow rate of reconnection with the Y-point can be viewed as a direct consequence of the disparity of the scale of astrophysical inflow of the fluid and the scale of outflow determined by microphysics, i.e., the resistivity or plasma effects. The latter, however, was challenged by Lazarian & Vishniac (1999, henceforth LV99).

The most significant point of the LV99 theory was that in the presence of 3D turbulence, the reconnection outflow is determined by the magnetic field wandering, and the width of the outflow is the function of turbulence intensity rather than the resistivity of plasma effects. The dependence of the reconnection rate on the level of turbulence as predicted by the LV99 theory was successfully tested in the numerical studies of Kowal et al. (2009, 2012). More recently, these predictions received additional support from relativistic MHD simulations by Takamoto et al. (2015). The most important
consequence of the LV99 theory, in contrast to all the previous theories of fast reconnection, was the prediction that the reconnection does not require special settings, but occurs everywhere in turbulent media. As a result, this violates flux freezing in astrophysical fluids, which are generically turbulent (see Eyink 2011; Eyink et al. 2011). This remarkable breakdown of the classical magnetic flux-freezing theorem (Alfvén 1942) was numerically demonstrated in Eyink et al. (2013).\(^5\)

Turbulence can be both externally driven, as is testified from observations of the interstellar medium (ISM) and molecular clouds (see Armstrong et al. 1995; Padoan et al. 2009; Chepurnov & Lazarian 2010; Chepurnov et al. 2015), but it can also be driven by the reconnection process, as first discussed in LV99 and further elaborated in Lazarian & Vishniac (2009). The first numerical study of magnetic reconnection induced by turbulence that is generated by reconnection was performed in 2013 (published in Beresnyak 2017) with an incompressible code, and later in Oishi et al. (2015) and Huang & Bhattacharjee (2016) taking into account compressibility. A detailed numerical study of reconnection with self-generated turbulence was performed in Kowal et al. (2017).

One of the most important questions of the current research on 3D reconnection is the nature of turbulence in the reconnection events. Our earlier study in Kowal et al. (2017) demonstrated that the turbulence generated in the reconnection events follows the Goldreich–Sridhar statistics (Goldreich & Sridhar 1995). However, an open question is related to the driving mechanism of the observed turbulent motions. The literature has suggested that tearing modes, plasmoid instabilities, and shear-induced instabilities could mediate the energy transfer from coherent to turbulent flows. The problem of the relative importance of different driving processes has not been explored quantitatively.

In our numerical experiments we do not identify tearing modes, although filamentary plasmoid-like structures are present. Visual inspection shows, however, that their filling factor is small. Sheared flows, on the other hand, are present around and within the whole current sheet. As the field lines reconnect, the Lorentz force increases, accelerating plasma and creating the current sheet. This process is patchy and bursty in 3D. Therefore, the accelerated flows are strongly sheared. The statistical importance of these burst flows is large, as has been shown in our previous work (Kowal et al. 2017), where we compared the velocity anisotropy of reconnecting events to that of decaying turbulence without the reversed field. The Kelvin–Helmholtz instability due to the sheared velocities in reconnecting layers has previously been conjectured as a possible origin of turbulence by Beresnyak (2017). In Kowal et al. (2017), we provided solid evidence for the self-generated turbulence driven by the velocity shear. Here we perform a proper analysis of the growth rates of such instabilities.

Velocity shear is a global process that occurs in regular magnetized and unmagnetized fluids. The nonlinear evolution of the related instabilities, such as the Kelvin–Helmholtz instability, is known to be one of the main contributors to the energy transfer between wave modes, i.e., the energy cascade. If the energy cascade in reconnection layers is led by similar mechanisms, it is straightforward to understand why the statistics observed resemble those of Kolmogorov-like turbulence and Goldreich–Sridhar anisotropy scaling. In other words, our claim is that the turbulent onset and cascade in reconnection is not different to those found in regular MHD and hydrodynamic systems.

In this work we consider two possible instabilities that have both been extensively studied analytically and numerically since 1960s, namely the tearing-mode instability in a slab geometry (see Furth et al. 1963; Somov & Vernetta 1989), which naturally develops in a thin elongated current sheets, and the Kelvin–Helmholtz instability (see, e.g., Chandrasekhar 1961; Fejer 1964; Michalke 1964; Sen 1964; Gerwin 1968; Ong & Roderick 1972; Miura & Pritchett 1982; Frank et al. 1996; Faganello & Califano 2017; Berlok & Pfrommer 2019), which could result from the local velocity shear that is produced by the outflows from reconnection sites. Both instabilities are able to generate turbulence near current sheets. In this work we investigate which of the two is responsible for or dominates the generation of turbulence in stochastic reconnection, i.e., the reconnection without an externally imposed turbulence resulting from weak initial plasma irregularities.

In what follows we provide a brief review of the theoretical analysis of Kelvin–Helmholtz and tearing instabilities in Section 2, together with dispersion relations derived for different magnetic field and velocity profiles and conditions for their suppression. In Section 3 we describe the numerical simulation and our approach in analyzing both instabilities. In Section 4 we present the results of our analysis and compare the maximum growth rates of the two instabilities at different times, which we discuss in Section 5. Finally, in Section 6 we state our conclusions.

2. Analyzed Instabilities

2.1. Tearing-mode Instability

With tokamaks in mind, many works on the tearing-mode instability considered a cylindrical plasma column in so-called “pinch configuration” or simply toroidal geometry. In this configuration, the current density pinch, which is due to the change in the azimuthal component of the magnetic field, is located at some distance from the column center and the axial component is usually stronger than the azimuthal one (see, e.g., Coppi et al. 1976; Ara et al. 1978). In this subsection we briefly review the dispersion relations for the tearing mode derived in a slab geometry, where one component of the magnetic field changes sign over a short distance 2δ along its perpendicular direction. It is know that independently of the configuration, the plasma is stable in the ideal MHD framework. The stability occurs when a finite resistivity is taken into account.

Furth et al. (1963) developed the theory of resistive MHD instabilities for the case of a neutral current layer with an arbitrary magnetic field profile. However, in deriving the dispersion relations, they considered piecewise linear or hyperbolic tangent profiles. From their theory, we know that the tearing mode develops under condition \( \alpha = k \delta < 1 \), where \( k \) is the perturbation wavenumber (along the sheet plane) and \( \delta \) is the current sheet half-width (see Table 1 in Furth et al. 1963). Under the assumption of the so-called “constant \( \Psi \)” in the region of discontinuity (see Section 5 in Furth et al. 1963), where \( \Psi \) is the flux function of the in-plane magnetic field,
\[ \mathbf{B} = (-\partial_t \Psi, \partial_x \Psi) \], the growth rate \( \gamma \), expressed by the renormalized growth rate \( p = \gamma \tau_{\alpha} \) of the instability for the piecewise linear profile is given by

\[ p \approx \alpha^{-2/5} \xi^{2/5}, \tag{1} \]

with \( \tau = \tau_{\alpha}/\tau_{\Sigma} \), where \( \tau_{\alpha} = 4\pi B_0^2/\eta \) is the resistive diffusion time, \( \tau_{\Sigma} = \delta/v_A \) is the Alfvén time, and \( v_A = B_0/\sqrt{4\pi p_0} \) is the upstream Alfvén speed. This growth rate corresponds to short-wavelength modes, \( \alpha S^{1/4} \ll 1 \). For the long-wavelength modes, \( \alpha S^{1/4} \gg 1 \), the function \( \Psi \) is no longer constant in the region of discontinuity, and the dispersion relation takes form

\[ p \approx \alpha^{2/3} S^{2/3}. \tag{2} \]

The existence of these two regimes with different scalings indicates that the growth rate of tearing mode has its maximum \( p_{\text{max}} \) with respect to \( \alpha \), which according to Furth et al. (1963) scales as \( p_{\text{max}} \sim S^{1/2} \). Coppi et al. (1976), analyzing the internal kink instability of a plasma column, and Loureiro et al. (2007), considering a finite-length current sheet, derived complete dispersion relations covering both regimes.

The tearing mode is usually studied in the 2D incompressible MHD framework, and since the theory by Furth et al. (1963), an additional mechanism was searched for that could stabilize or destabilize the instability. Furth et al. (1963) already considered effects of fluid compressibility, thermal conductivity, or finite viscosity and concluded that they do not affect the instability. Nevertheless, the tearing instability is a subject to suppression under some circumstances. For instance, the presence of traverse, i.e., normal to the current sheet plane, the magnetic field component stabilizes the tearing mode. The stabilization effect of this component was demonstrated by Schindler (1974) and Galeev & Zelenyi (1975, 1976).

Somov & Verneta (1989) have shown that the stabilization effect of the transverse field \( B_n \) becomes essential when \( \xi \equiv B_n/B_0 \gg S^{-3/4} \), decreasing the growth rate \( p \) with increasing value of \( \xi \). When \( \xi \gg S^{-3/4} \), the instability is completely stabilized. Somov & Verneta (1989) derived the dispersion relation for the tearing instability in the presence of \( B_n \), which is given by

\[ \Delta S^{1/4} \left( \alpha^2 S^2 \right)^{1/4} \left( 1 - \frac{p_{\text{max}}^{3/2}}{\alpha S} \right) - p \alpha \sqrt{\frac{\pi}{2}} = 0, \tag{3} \]

where \( \Delta = (1 + \xi^2 S^2/p) \). The above dispersion relation is valid for \( \xi < \alpha \). When \( \xi \rightarrow 0 \), Equation (3) reduces to

\[ \left( \alpha^2 S^2 \right)^{1/4} \left( 1 - \frac{p_{\text{max}}^{3/2}}{\alpha S} \right) - p \alpha \sqrt{\frac{\pi}{2}} = 0, \tag{4} \]

properly recovering the scalings in the short- and long-wavelength regimes discussed before. When we apply an implicit derivative to Equation (3) with respect to \( \alpha \) and setting \( dp_{\text{max}}/d\alpha = 0 \), we obtain the following relationships between the maximum growth rate \( p_{\text{max}} \) and the corresponding \( \alpha_{\text{max}} \)

\[ \alpha_{\text{max}} = \frac{2}{\sqrt{\pi}} \left( \frac{p_{\text{max}}}{S^2} + \xi^2 \right)^{1/6} \quad \text{and} \quad p_{\text{max}} = \left( \frac{\pi^2}{64} \alpha_{\text{max}}^6 - \xi^2 \right) S^2. \tag{5} \]

Inserting the equation for \( \alpha_{\text{max}} \) into the dispersion relation Equation (3) gives us an expression for the maximum growth rate \( p_{\text{max}} \), which depends only on \( \xi \) and \( S \),

\[ p_{\text{max}}^4 + \xi^2 S_{\text{max}}^3 - \frac{1}{\pi} \left( \frac{2}{3} \right)^3 S^2 = 0. \tag{6} \]

The resulting expression is a quartic function that has a negative determinant for any possible value of \( \xi \), thus it has two real roots, one negative and one positive, with the positive one being the physical maximum growth rate. Provided the estimates of \( \xi \) and \( S \) from the simulations, we can therefore determine the maximum growth rate \( p_{\text{max}} \). For \( \xi = 0 \) this equation directly gives the maximum growth rate \( p_{\text{max}} = (2/3)^{3/4} \pi^{-1/4} S^{1/2} \) that is compatible with the estimate given by Furth et al. (1963). After \( p_{\text{max}} \) is determined, we can also retrieve the corresponding parameter \( \alpha_{\text{max}} \) from the left expression in Equation (5), from which we can calculate the wavenumber \( k_{\text{max}} = \alpha_{\text{max}}/\delta \) at which the maximum growth rate \( p_{\text{max}} \) takes place.

\[ \gamma = k \sqrt{U^2 - v_A^2}, \quad U > v_A, \tag{7} \]

where \( U \) is the velocity shear amplitude, assuming it changes from \(-U \) to \( U \) between two layers, \( v_A \) is the local Alfvén speed, and \( k \) is the wavenumber of perturbation. This expression, however, is valid only for a simplified case of a “vortex sheet,” where the velocity has a discontinuity in the sheet location, and gives an unrealistic growth rate that increases with \( k \) to arbitrarily high values. A more realistic continuous shear layer was already considered a long time before. In the hydrodynamic framework, Rayleigh (1879) derived a dispersion relation for a linear piecewise profile of velocity. Chandrasekhar (1961) also considered continuous variations of density and velocity profiles (see Section 102–104 and Figure 120 there), while Michalke (1964) numerically analyzed a hyperbolic tangent profile. All these works concluded that the growth rate reaches its maximum value \( \gamma_{\text{max}} \) for \( k_{\text{max}} \delta < 1 \), where \( \delta \) is the half-width of the shear region. Following the Rayleigh (1879) derivation, Drazin & Reid (1981, see Section 23) provided the growth rate for the Kelvin–Helmholtz instability with a piecewise linear velocity profile in the hydrodynamic framework,

\[ \gamma^2 = \frac{U^2}{4 \delta^2} \left( e^{-4\alpha} - (1 - 2\alpha)^2 \right), \tag{8} \]

where \( \alpha \equiv k \delta \). The expression indicates that the instability is completely stabilized for \( \alpha \gtrsim 0.639 \), and the maximum growth rate occurs at \( \alpha \approx 0.398 \). In the incompressible MHD framework, Ong & Roderick (1972), also considering a piecewise
linear profile, derived the following dispersion relation:

\[
\Omega^4 + \Omega^2(\alpha - 2\alpha^2(1 + 1/M^2_U)) - \frac{1}{4}(1 - e^{-4\alpha} + \alpha^2(1 - 1/M^2_u)^2 - \alpha^2(1 - 1/M^2_u)^2 = 0, \tag{9}
\]

where \(\Omega = \omega/U\), \(M_u^2 = U^2/v_A^2\) is the Alfvénic Mach number related to the shear amplitude \(U\), and \(\omega = \phi + i\gamma\), with \(\phi\) and \(\gamma\) being the phase change and growth rate, respectively. The dispersion relation expressed by Equation (9) is valid, however, in the limit \(M_u^2 \gg 1\) for \(\alpha < 1.0\). In the hydrodynamic limit, i.e., when \(M_u\) tends to infinity, the dispersion relation becomes compatible with the relation provided by Drazin & Reid (1981). Similar analytical studies of the magnetized Kelvin–Helmholtz instability with a linear velocity profile were made by Gudkov & Troshichev (1996) and resulted in compatible conclusions.

Ong & Roderick (1972) also studied the Kelvin–Helmholtz instability in the compressible MHD framework. In this regime, they derived the following dispersion relation:

\[
8\alpha \mu^2 \Omega^4 + \Omega^2 \left[-4 + \mu^2 \left(-1 - 4\alpha + 8\alpha^2 - \frac{10}{8} \alpha^3\right)\right] + (1 - 2\alpha)^2 - e^{-4\alpha} - \mu^2 (\alpha^2 + \frac{4}{3} \alpha^3 - \frac{8}{3} \alpha^4 + \frac{8}{5} \alpha^5) = 0, \tag{10}
\]

which is valid when \(\mu^2 \equiv M^2/\alpha^2 \ll 1\), with \(M = U/c_m\), \(c_m = \sqrt{\alpha^2 + v_A^2}\), and \(\alpha\) being the magnetosonic Mach number, the magnetosonic speed, and the sound speed, respectively. When \(\mu^2 \rightarrow 0\), Equation (10) reduces to the relation compatible with Equation (8). The two dispersion relations (Equations (9) and (10)) indicate the existence of a maximum growth rate \(\gamma_{\text{max}}\) at a corresponding wavenumber \(k_{\text{max}}\) also in the magnetized version of the Kelvin–Helmholtz instability, with \(k_{\text{max}} \delta \ll 1\). In order to find \(\gamma_{\text{max}}\) and \(k_{\text{max}}\), these relations have to be solved numerically for a given \(\delta\), \(U\), \(v_A\), and \(\alpha\), which is somewhat simplified because both equations are biquartic functions.

Miura & Pritchett (1982) studied the Kelvin–Helmholtz instability numerically in a general configuration, i.e., taking into account the compressibility and arbitrary angles between the shear direction, magnetic field, and perturbation propagation. They derived a sufficient condition for stability, \(U \lesssim v_A(k \cdot \hat{B}_0)/(k \cdot \hat{U})\), which, assuming that the perturbation wavevector \(k\) and magnetic field \(B_0\) both lie in the plane of the maximum shear and form angles \(\phi\) and \(\theta\) with the shear direction, respectively, can be written as

\[
U \lesssim v_A (\cos \theta + \sin \theta \tan \phi). \tag{11}
\]

The condition states that if the magnetic field is aligned with the shear \((B_0 \perp U\), i.e., \(\theta = 0\)), the angle of the perturbation propagation \(\phi\) does not matter and the mode is stable if \(U \lesssim v_A\). When the magnetic field is perpendicular to the shear \((B_0 \perp U\), i.e., \(\theta = 90^\circ\)), the perturbations are stabilized only for modes nearly perpendicular to the shear \((\phi \approx 90^\circ)\). Moreover, considering the perturbation propagating strictly along the shear \((k \parallel U\), for the perpendicular field \((B_0 \perp U\), the instability develops only if \(M < 1\), which is equivalent to \(U < c_m\). For the parallel magnetic field \((B_0 \parallel U\), the instability is completely suppressed for \(M_g = U/a > 1\) when \(M_u > 1\) has to be fulfilled for instability.\(^6\) However, the restrictions for \(M\) and \(M_u\) are relaxed for perturbations propagating obliquely with respect to the sheared velocity component and magnetic field (see Equation (37) in Miura & Pritchett (1982)). Moreover, it is important to note that even though the system may be strongly magnetized overall, in the regions where reconnection occurs, the local degree of magnetization decreases considerably, allowing the growth of Kelvin-Helmholtz-unstable modes (see, e.g., Loureiro et al. (2013)).

3. Methodology and Modeling

3.1. Numerical Simulations

In this work we analyze a numerical simulation of the reconnection-driven turbulence in a 3D rectangular domain with physical dimensions \(L \times 4L \times L\) (with assumed \(L = 1\)) by solving nonideal isothermal compressible MHD equations using the high-order shock-capturing adaptive refinement Godunov-type code AMUN.\(^7\) The exact numerical setup of the model analyzed here is the same as for the models presented in Kowal et al. (2017), with the sound speed \(a = 1.0\) \((\beta = 2.0)\), and explicit viscosity \(\nu\) and resistivity \(\eta\), both equal to \(10^{-5}\), in order to control the effects of numerical diffusion. The model was run with the base resolution of \(32 \times 128 \times 32\), and the local mesh refinement up to seven levels, with the refinement criterion based on the normalized value of the vorticity and current density, resulting in an effective resolution \(2048 \times 8192 \times 2048\) or an effective grid size \(h = 1/2048\) (the same for all directions). The initial density \(\rho_0\) and the strength of the reconnecting component of the magnetic field \(B_0\) were set to unity, resulting in the velocity and the simulation time units being \(v_A = 1\) and \(t_x = L/v_A = 1\), respectively. It is important to note that the code works with the normalized magnetic field multiplied by a factor of \(\sqrt{4\pi}\). Therefore, the Alfvén speed using the code units is given by \(V_A = B_0/\sqrt{\rho_0}\). In addition to the reconnecting magnetic field component along the X-direction, we also set the uniform guide field along the Z-direction of the strength of 0.1. The velocity field was initialized with fluctuations represented by 100 Fourier modes of random phases and directions, an amplitude equal to \(10^{-2}\), and a fixed wavenumber of \(k = 64\pi\). The simulation was terminated after \(t = 7.0\). For a complete description of the numerical setup of the model, the boundary conditions, and the numerical methods used, we refer to Kowal et al. (2017).

3.2. Shear Detection in Vector Fields

In order to detect locations of the current sheet we analyze a quantity that is correlated with the local change of the polarization of magnetic field, or simply the magnetic shear. Several techniques have been proposed in the literature to

\(^6\) Miura & Pritchett (1982) considered the velocity change from \(-v_0/2\) to \(v_0/2\) and calculated Mach numbers using \(v_0\), which resulted in respective conditions for the Mach numbers \(M_u < 2\) and \(M_u > 2\), respectively.

\(^7\) The code is freely available from http://amuncode.org.
determine locations where the reconnection takes place. The most straightforward is the amplitude of the current density \( |\mathbf{J}| \). We can also use the magnetic shear angle, i.e., the change in the magnetic field direction along a line, or the method called partial variance of increments (PVI), which measures the variation in the magnetic field across the current sheet (see Greco et al. 2008; Servidio et al. 2011). Similarly, for the velocity shear, we can consider, for example, the vorticity \( \omega = \nabla \times \mathbf{v} \) as the shear detector. In this work we analyze the local maxima of the Euclidean norm of the shear rate tensor \( S_{ij} = \partial u_i / \partial x_j + \partial u_j / \partial x_i \) for \( i \neq j \) and \( S_{ij} = 0 \) for \( i = j \), \( S = ||S|| \equiv \sqrt{\sum_i S_{ii}^2} \), where \( x_i, x_j \in \{x, y, z\} \) and \( \mathbf{u} \) means either \( \mathbf{B} \) or \( \mathbf{v} \), depending on the analyzed instability. Therefore, \( S = S(x, y, z) \) is a function of position.

The algorithm we used to determine the local geometry of the shear consists of the following steps:

1. First we detect the local maxima of \( S(x, y, z) \). We scan over the domain cells and verify for each cell the position \( \mathbf{p} = (x, y, z) \) along each direction \( X, Y, \) and \( Z \), if the first-order partial derivative of \( S(x, y, z) \) vanishes and the second-order partial derivative is negative. We do this by calculating the backward and forward partial derivatives along each direction using finite differences. If along all directions the sign of the backward derivative is positive and the forward one is negative, which also guarantees the negativity of the second-order derivative, the cell is selected as the local maximum of \( S \) and is further analyzed. Otherwise, the remaining steps of our analysis are skipped and the next cell is considered.

2. For each cell of the local shear maximum determined in the previous step, i.e., the cell where all directional first-order partial derivatives vanish and the second-order derivatives are negative, we calculate the complete Hessian of the analyzed shear detector \( S \),

\[
H_{ij} = \frac{\partial^2 S}{\partial x_i \partial x_j},
\]

where again \( x_i, x_j \in \{x, y, z\} \). By solving the eigenproblem for the Hessian, we can obtain the direction of the steepest decay of \( S \) at the given location, which is determined by the eigenvector \( \hat{e}_s = \hat{e}(\lambda_{\text{min}}) \), which corresponds to the minimum eigenvalue \( \lambda_{\text{min}} = \min \{\lambda_i\} \).

3. The eigenvector \( \hat{e}_s \) is perpendicular to the component that suffers the strongest shear. However, we would like to determine a complete vector base corresponding to the directions along the strongest shear, \( \hat{e}_s \), across that shear, \( \hat{e}_n \), and perpendicular to both, \( \hat{e}_g = \hat{e}_s \times \hat{e}_n \) (see the left panel of Figure 1). Because a pure shear is a combination of the normal stress and rotation, is it not trivial to determine its precise direction. Moreover, in a generic case, the stress may not be planar, acting along all three principal axes. In order to deal with this situation, we can assume that we are only interested in a plane defined by two principal axes, corresponding to the strongest positive and strongest negative normal stresses. In order to determine these, we need to solve the eigenproblem for the shear rate tensor \( S_{ij} \). The eigenvector, \( \hat{e}_n \), corresponding to the middle eigenvalue, determines the plane of the maximum shear. Therefore, the direction of the sheared component \( \hat{e}_s \) is

\[
\hat{e}_s = \hat{e}_n \times \hat{e}_g. \tag{13}
\]

There is no reason for \( \hat{e}_n \) and \( \hat{e}_g \) to be perfectly orthogonal. Therefore, after normalizing \( \hat{e}_s, \hat{e}_n \) is corrected by setting it to \( \hat{e}_n \times \hat{e}_g \). This completes the determination of the orthonormal base of the shear.

4. After determining our orthonormal base in the local shear maximum, we perform the interpolation of all three components of the analyzed vector field, \( u_x, u_y, \) and \( u_z \) along the normal direction determined by the vector \( \hat{e}_n \) within a distance of several cell sizes (e.g., \( s = -20h, ..., 20h \), where \( s \) is the distance in units of the cell size \( h \)). Projecting the resulting vectors on \( \hat{e}_s \), we obtain the sheared component of the analyzed vector field,

\[
u_i(s) = \hat{e}_s \cdot \mathbf{u}(\mathbf{p} + s\hat{e}_n). \tag{14}
\]

For the interpolation, we use a piecewise quintic Hermite
interpolation that preserves the continuity of the first and second derivatives (see, e.g., Dougherty et al. 1989).

5. At this point, we fit the sheared component \( u_s(s) \) to the hyperbolic tangent function 
\[
f(s) = \hat{u}_0 \tanh[(s - s_0)/\delta] + u_0 \text{ (or a piecewise linear function)}
\]
to the obtained profile \( u_s(s) \). The fitting is made within the region \( s_l \leq s \leq s_u \), where \( s_l < 0 \) and \( s_u > 0 \), and \( s_l \) and \( s_u \) indicate distances along \( \hat{e}_x \) at which the derivative \( du_s/ds \) changes sign. The estimated values \( \hat{u}_0 \) and \( u_0 \) correspond to the amplitudes of the sheared and uniform components, respectively, \( s_0 \) is the shift with respect to the current position, and \( \delta \) is the half-width of the profile. The right panel in Figure 1 shows an example of the determined sheared component of the magnetic field \( B_s(s) \) together with its fitting (blue and dashed red lines, respectively). The fitting parameters are shown at the top of the plot.

6. Along the normal direction, \( \hat{e}_n \), we can also project other quantities. For example, in the case of the tearing mode, we estimate the transverse component of the magnetic field \( B_t(s) = \hat{e}_x \cdot \mathbf{B}(p + \delta \hat{e}_n) \). By averaging \( B_t(s) \) within the local current sheet, i.e., within the interval \( s_l \leq s \leq s_u \) of the fitted function, we can derive the mean value of the transverse component of the magnetic field \( \langle B_t \rangle \). Similarly, the upstream Alfvén speed can be determined from, e.g., 
\[
\nu_A = [\nu_A(p - \delta \hat{e}_n) + \nu_A(p + \delta \hat{e}_n)]/2,
\]
which is necessary to estimate the Lundquist number \( S \).

7. Finally, in order to estimate the length of the shear region, i.e., the longitudinal dimension of the shear, we project the detector \( \mathbf{S} \) along the vector parallel to the sheared component of the field, \( \mathbf{S}(s) = \mathbf{S}(p + \delta \hat{e}_n) \), and analyze the decay of \( \mathbf{S} \) along \( s \). We measure the distance \( l \) between the points where \( \mathbf{S} \) drops to half its maximum value in position \( p \), treating \( l \) as the longitudinal length of the shear region.

The procedure described above allows us to estimate the thickness \( \delta \) and the longitudinal dimension \( l \) of all shear regions found in the domain. For each region we can determine the local direction of the shear \( \hat{e}_n \) and estimate other parameters required for determining the local growth rate \( \gamma \), which are described in more detail in the next subsection.

For illustration, in the left panel of Figure 1 we show a sketch of a shear region (with arbitrary orientation) with vector field lines of the opposite polarization (red and blue) with the local reference frame used to project the field components on three axes \( \hat{e}_x \), \( \hat{e}_n \), and \( \hat{e}_\xi \), corresponding to the shear, transverse, and guide components. For the case of magnetic shear, an extracted profiles of sheared (reconnecting) \( B_t(s) \), transverse \( \hat{e}_n \), and guide \( \hat{e}_\xi \) components (blue, orange, and green, respectively) along the direction normal to the current sheet are shown in the right panel of Figure 1. The plot also shows the fitted parameters \( \delta \mathbf{B} \), \( B_0 \) and \( \delta \) of the sheared component (red dashed line) together with the estimated stabilizing parameter \( \xi \).

3.3. Estimation of Conditions and Growth Rates for the Tearing Instability

Estimating the growth rate of the tearing mode in fluid simulations is not trivial. First of all, it is necessary to detect the locations of the current sheets, and when this is done, to estimate the local Lundquist number \( S \) from the upstream Alfvén speed \( \nu_A \) and the thickness \( \delta \) of each sheet region. In order to take into account the effect of the transverse field, we need to estimate \( \xi \) from the transverse and total magnetic fields, \( B_\perp \) and \( B \), respectively. Nevertheless, all these parameters are not sufficient because we still need information about the local perturbations, which might be especially difficult to characterize.

In the turbulent case, considered here, we usually have a packet of waves of different amplitudes, wavenumbers, and directions traveling through the considered region. First of all, we can estimate the bounds for the perturbation wavenumber \( k \) from the geometry of the current sheet region. Estimating the longitudinal length of the sheet, \( l \), (see step 7 in the previous section), we can determine the lower limit for the wavenumber \( k_1 \approx 2\pi/l \). The upper wavenumber limit is determined by the resolution of the simulation, i.e., \( k_\max \approx 2\pi/h \). Therefore, the permitted range of wavenumbers for local perturbations is \( k_1 \leq k \leq k_\max \). For our analysis, it is enough, however, to consider \( k_\max \) corresponding to the maximum growth rate \( \gamma_\max \), assuming that because we have a turbulent region, there is a high probability of finding fluctuations with a wavenumber equal to \( k_\max \) present in the region. Obviously, if \( k_\max \) lies beyond the permitted range, the limited value of \( k_\max = \min(\max(k_\max, k_1, k_\min)) \) should be considered in the estimation of the tearing growth rate \( \gamma_\max \). We also take into account that \( k_\max \delta \) should be smaller than unity.

For the case of the tearing instability, the algorithm described in Section 3.2 allows us to determine the thickness of the local current sheet \( \delta \), the local upstream Alfvén speed \( \nu_A \), the transverse and total components of the magnetic field, \( B_\perp \) and \( B \), respectively, and the longitudinal scale of the sheet region \( l \). From \( \delta \) and \( \nu_A \) we calculate \( S \) (using the explicit value of resistivity \( \eta \) for which the simulation was made), and from \( B_\perp \) and \( B \) we calculate \( \xi \). The estimated values of \( S \) and \( \xi \) are inserted into Equation (6), which is solved numerically in order to obtain \( p_\max \). Then, using the left expression in Equation (5), we estimate \( \alpha_\max \) from \( p_\max \). If \( \alpha_\max = k_\max \delta < k_\delta \), however, its value is limited by setting \( \alpha_\max = k_\delta \) and estimating \( p \) numerically directly from Equation (3) for a given \( \alpha = \alpha_\max \), using, e.g., the Newton iterative root finder with the initial guess set \( p_\max \) obtained from Equation (6). In this way, we estimate the local growth rate of the tearing instability, \( \gamma_\max = p_\max / \tau_R \), and the corresponding wavenumber \( k_\max \). The results of this analysis are presented in Sections 4.1 and 4.3.

3.4. Estimation of Conditions and Growth Rates for the Kelvin–Helmholtz Instability

The Kelvin–Helmholtz instability is analyzed in a similar manner as the tearing mode. Here, we determine the positions of the velocity shear using the algorithm described in Section 3.2. When a shear region is detected, its thickness \( \delta \) and longitudinal dimension \( l \) are estimated. These two parameters allow us to estimate the permitted range of the wavenumbers of the perturbation, \( k_1 \lesssim k \lesssim k_\max \), where \( k_1 \) and \( k_\max \) are calculate in the same way as in the tearing instability, i.e., \( k_1 = 2\pi/l \), and \( k_\max = 2\pi/h \), however, \( l \) refers here to the longitudinal dimension of the velocity shear region. From Equation (11) we see that the stability condition depends on the relative angle between the direction of the perturbation propagation \( \hat{k} = k/k \), the direction of the local magnetic field \( \hat{B}_0 = B_0/B_0 \) both with respect to the direction of the shear \( \hat{U} = U/U \). Discussing the distribution of \( U \) in Section 4.2, we consider two cases, \( k \parallel B \parallel U \) and \( k \parallel B \perp U \). However, in
deriving the maximum growth rates described below, we always assume the case of three vectors parallel to each other.

In order to determine the growth rate of the Kelvin–Helmholtz instability in each detected region, we estimate the shear velocity $U = \nu_A$ from the interpolated transverse component of velocity (see step 5 in Section 3.2). We also estimate the upstream Alfvén speed $v_A$ in a similar way as in the tearing mode. In this way, we build a vector of samples for the shear width $\delta$, the velocity shear amplitude $U$, and the Alfvén speed $v_A$, which is necessary to verify the stability conditions and estimate the growth rate from Equations (7), (9), and (10). In estimating the maximum growth rate $\gamma_{\text{max}}$ in the case of discontinuous shear and incompressible MHD (Equation (7)), we estimate the wavenumber $k_{\text{max}}$ from the condition $k_{\text{max}} = \max(0.4/\delta, k_l)$. The value 0.4 has been chosen based on the $k_{\text{max}}\delta$ for the piecewise linear profile for the hydrodynamic case (see Equation (8) and the description below). In the case of MHD Kelvin–Helmholtz instability dispersion relations with the piecewise linear profile (Equations (9) and (10)), both $\alpha_{\text{max}}$ and $\Omega_{\text{max}}$ have to be found numerically, using, for example, the golden section method (Kiefer 1953). When they are estimated, we calculate the maximum growth rate from $\gamma_{\text{max}} = \Im(\Omega_{\text{max}}) U/\delta$ and $k_{\text{max}} = \alpha_{\text{max}}/\delta$. The results of the above described analysis of the Kelvin–Helmholtz instability are presented in Sections 4.2 and 4.3.

4. Analysis and Results

Before we describe our results, we stress that we present the statistical results corresponding to the populations of cells that belong to magnetic (current sheet) or velocity shear maxima and not individual shear regions. This means that we do not group cells as belonging to the same shear region or not, or analyze any connectivity between these cells. This does not mean either that we demonstrate the statistics of individual shear regions. The number of samples shown in, e.g., histograms, has only a comparative and not an absolute meaning, and because we analyze cells at shear maxima, it should not be interpreted as a filling factor, after it is divided by the total effective number of cells in the system. The main objective of this work is to verify the conditions that are provided for the development of analyzed instabilities in the reconnection-driven turbulence, and if they can be responsible for the energy input driving local turbulence.

4.1. Tearing Instability Analysis

Before we estimate the growth rate of the tearing instability, we analyze the properties of current sheets in the system. From Equations (1) and (2) we see that the growth rate $\gamma$ increases with the decrease in the current sheet thickness $\delta$. It also increases with the perturbation wavenumber $k$ for $k < k_{\text{max}}$ (i.e., for long-wavelength modes), and decreases with $k$ for $k > k_{\text{max}}$ (i.e., for short-wavelength modes). These two quantities, $\delta$ and $k$, also determine the instability condition $k\delta < 1$. Therefore we analyze them first.

In the left plot of Figure 2 we show the distribution of the current sheet thicknesses $\delta$ for all detected current sheets at times $t = 0.1, 1.0, 3.0, 5.0$, and $7.0$. The two vertical lines correspond to the effective cell size $h = 1/2048$ (left) and the initial current sheet thickness $\delta_{\text{ini}} = 3.16 \times 10^{-3}$. We see that at initial times, the current sheet thickness broadens from the initial thickness $\delta_{\text{ini}}$ up to values that are several times higher (see the blue, orange, and green lines in the left plot of Figure 2, which correspond to $t = 0.1, 1.0,$ and $3.0$, respectively). At later times $t > 3.0$, however, the current sheets tend to become thinner, with a significant fraction of the detected sheets with thicknesses $\delta$ comparable to (or slightly below) the effective cell size $h$, indicating a sharp change in magnetic field orientation (only two cells to change the polarization of magnetic field lines), probably related to turbulent dynamos near the sheet plane. On the other hand, the number of current sheet samples with $\delta > 10^{-2}$ quickly decays, indicating that thick current sheets become rare events (see the red and purple distributions in the left plot of Figure 2, which correspond to $t = 5.0,$ and $7.0$, respectively).

In the right plot of Figure 2 we show the evolution of the distribution of the lengths of current sheet regions at the same times. As expected, initially we have one current sheet plane that extends over the whole box. This is indicated by a significant number of samples of $l \approx 1.0$ at $t = 0.1$ (blue line). However, we can also see that at this very early time, a less significant population of samples with lengths that are a fraction of $L$ forms. We would interpret them as the points belonging to parts of the current sheet that are already significantly deformed because using our analysis, we cannot determine whether these points belong to the same or to a separated current sheets. It is important, however, that this population increases with time, as seen at times $t = 1.0$ and $2.0$ (orange and green lines, respectively). At later times, $t > 3.0$, nearly all points belong to current sheet regions whose longitudinal dimension $l$ is significantly shortened compared to the box size $L$, with values spread between $l = 10^{-2}$ and $10^{-1}$ at $t = 7.0$ (purple line).

Analyzing Figure 2, we can deduce that at initial times, the tearing mode should be a preferential instability for fluctuation generation because the current sheets are characterized by relatively thin and extended current sheets with $\delta \approx 0.003–0.01$ and $l \lesssim 1.0$. Because $k\delta < 1.0$ for an instability, it indicates that the limit for allowed wavenumbers is $k \lesssim 300$. We recall from Section 3.1 that the imposed perturbation wavenumber $k = 64\pi \approx 201$ at $t = 0.0$ indicates that the system should be initially unstable. At the final times, $\delta$ decreases to values $\delta \approx 0.0004–0.01$, which should significantly enhance the development of the tearing mode, but the fragmentation or deformations of the current sheet significantly decrease the length $l$ of the current sheet regions, which impedes the development of long wave perturbations. Still, according to the instability condition $k\delta < 1$, perturbations with wavenumbers up to $k \approx 2500$ could be unstable.

The analysis above did not give a clear response to the question whether turbulence might initially be generated by tearing mode. At later times, the turbulence develops in regions where the current sheet is thinner, potentially increasing the growth rate of the instability. At the same time, however, it is possible that the same turbulence generates a component of the magnetic field normal to the current sheet, which according to Equation (3), may suppress the instability. In order to analyze the stabilizing effect of this component, we show the correlations between the normalized transverse component of the magnetic field $\xi = B_y/B$ and the Lundquist number $S = v_A\delta/\eta$ in Figure 3 for two moments, $t = 3.0$ (left panel) and $t = 7.0$ (right panel). The red line, corresponding to the relation $\xi = S^{-3/4}$, divides the plot into two regions: one below...
the line, where $\xi$ has negligible effect, and another above the line, where the stabilization by $\xi$ starts to be significant and increases with distance from the red line. In the left panel ($t = 3.0$) we see that $\xi$ is not important for most of the detected cells; their $S$ values are concentrated slightly below $10^3$ and the parameter $\xi$ spreads up to value $10^{-2}$. A partial stabilization in the upper tail, i.e., for $\xi \approx 10^{-2}–10^{-1}$, already takes place.

At a later time, $t = 7.0$, shown in the right panel of Figure 3, the situation is very different. The points of the distribution spread toward lower values of $S$, roughly between $10^0$ and $10^3$, and across many orders of magnitude along the stabilization parameter $\xi$, nearly up to $10^{1}$. We see a significant concentration of detected samples somewhat along the red line that divides two regions of $\xi$ importance. The spread to the left along the horizontal direction should be attributed to the decrease in current sheet thicknesses caused by the action of turbulence, which is also responsible for the generation of the transverse component $B_n$. From the distributions for two different times shown in Figure 3 we see that the generation of $\xi$ by turbulence cannot be ignored in estimating the growth rate of the tearing instability.

An interesting question to ask is what the principal direction of magnetic shear in the detected current sheet cells is at different moments, considering the presence of a guide field and weak initial perturbations. In Figure 4 we show the angular distribution of the shear direction only for the unstable cells (for which $\gamma_{\text{max}} > 10^{-3}$) at two moments, $t = 3.0$ (left panel) and 7.0 (right panel). We see that at $t = 3.0$, the shear direction is still strongly concentrated along the X-direction, the direction of the reconnecting component, spreading roughly from $-10^0$ to $10^0$ in the azimuthal and from $-5^0$ to $5^0$ in the vertical directions, with a small tendency toward negative azimuthal angles, resulting probably from the presence of a weak guide field. At the final time, $t = 7.0$, however, the distribution of the directions, although still strongly concentrated along the X-axis, is characterized by a much larger spread in both directions. This indicates that the turbulence acting on the

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**Figure 2.** Statistics of the current sheet thickness $\delta$ (left) and length $l$ (right) for different evolution moments, $t = 0.1, 1.0, 3.0, 5.0, 7.0$ (blue, orange, green, red, and purple, respectively). The right vertical dashed line (teal) shows the initial thickness of the current sheet $\delta_{\text{ini}} = 3.16 \times 10^{-3}$, and the left line (gray) shows the effective grid size $h$.

**Figure 3.** Correlations between the ratio of the transverse component of the magnetic field to the magnetic field amplitude within the current density $\xi = B_n/B$ against the Lundquist number $S = \delta v_A/\eta$ at two evolution moments $t = 3.0$ (left) and $t = 7.0$ (right). Above the red dashed line the transverse component starts to decrease the growth rate, eventually stabilizing the tearing mode.
current sheet can significantly bend it, which modifies its local topology.

4.2. Kelvin–Helmholtz Instability Analysis

Similarly to the tearing mode analysis, we start by showing the distributions of the thickness $\delta$ and length $l$ of the velocity shear regions in Figure 5. The first interesting observation is that there are no detected velocity shear regions for times $t = 0.1$ and 1.0, or the shear strength is too weak, below the threshold value $U_{\text{min}} = 10^{-4}$ set in the shear detection algorithm. These distributions are of all cells at detected velocity shear maxima. At $t = 3.0$ we already see a number of cells belonging to shear regions of thicknesses $\delta$ between $3 \times 10^{-3}$ and about 10% of the length unit $L$, with distributions peaking at values below $10^{-2}$ for $t = 3.0$ and around $10^{-2}$ at $t = 7.0$. There are some velocity shear structures detected at $t = 7.0$ that have thicknesses comparable to the cell size $h$ (see the purple distribution in the left plot of Figure 5). The right panel of Figure 5 shows that the longitudinal dimensions of these shear regions spread from several cells to nearly $L$, indicating a generation of nearly global shear in the computational domain. Transforming these lengths into wavenumbers indicates that perturbations of any $k$ from $k = 2\pi$ up to nearly $k \sim 3000$, may grow due to the Kelvin–Helmholtz instability if appropriate conditions are fulfilled in the local shear region. The peak value for the longitudinal dimension $l$ of the shear regions is between 0.1 and 0.2, decreasing to a range between 0.02 and 0.1 for later times, corresponding to the wavenumbers of $k \sim 30–300$.

The most important parameter in the development of the Kelvin–Helmholtz instability is the shear amplitude $U$. In the left panel of Figure 6 we show the evolution of the distribution of $U$ for all cells where the maximum shear was detected (solid lines) and only for cells that are unstable, with the assumption that the angle between the wavevector $k$ and the shear direction is $0^\circ$ (dashed) and $75^\circ$ (dotted), according to the sufficient condition for the Kelvin–Helmholtz instability (Equation (11)). We note that although the shear is relatively common after $t = 3.0$, only cells with the strongest $U$ are in fact unstable (i.e.,
the corresponding growth rates determined from Equation (10) are higher than zero. We see that for these unstable cells, the shear strength spreads between $10^{-2}$ to slightly above 0.1, measured in Alfvén speed $v_A$. At later times, the distribution peaks at a fraction of $v_A$. This plot clearly indicates that strong shear can be generated in such systems in a relatively short time.

In the right panel of Figure 6 we verify the prediction for the compressible system by Miura & Pritchett (1982), stating that if the magnetic field is parallel to the shear ($B_\parallel U$), if the sonic Mach number $M_s > 1.0$ or $M_A < 1.0$, the instability is stabilized. We show distributions of both Mach numbers for the last snapshot of our simulation, at $t = 7.0$. Clearly, all unstable cells have a sonic Mach number $M_s < 1.0$ and an Alfvénic Mach number $M_A > 1.0$, which is in perfect agreement with the theoretical prediction. Moreover, when we assume a perpendicular magnetic field ($B_\perp U$), $M < 1.0$ for all unstable cells. As we can see in the right panel of Figure 6, this case also agrees with the predictions.

Similarly to the tearing mode analysis, we show the distribution of the velocity shear directions in Figure 7 for two moments: at $t = 3.0$ (left panel), when the Kelvin–Helmholtz unstable cells start to appear, and at the final moment $t = 7.0$ (right panel), when the instability is already developed. We see that at $t = 3.0$, the Kelvin–Helmholtz instability can operate only in the plane that is significantly extended in the horizontal direction from $-80^\circ$ to $60^\circ$ and by a few degrees in the vertical direction with respect to the X-axis. At the final time ($t = 7.0$), the velocity shear directions are scattered over all angles in both directions, azimuthal and vertical, although the most statistically significant part is within $20^\circ$ from the X-axis, extended somewhat more in the azimuthal direction across all angles.
4.3. Evolution of the Growth Rates: Tearing versus Kelvin–Helmholtz

Supported by the results from the previous subsections, which show the analysis of the factors that are important for the development of the tearing and Kelvin–Helmholtz instabilities, we can now compare the estimated maximum growth rates for both instabilities. This will help us to determine which instability dominates. The way we estimate $\gamma_{\text{max}}$ (and corresponding $k_{\text{max}}$) was already described in detail in Sections 3.3 and 3.4 for the tearing and Kelvin–Helmholtz instabilities, respectively. As we have shown above, the range of possible wavenumbers of perturbations, $k$, for both instabilities can be estimated from the longitudinal dimensions of individual shear regions, $l$, and the effective cell size, $h$. Of course, if the estimated $k_{\text{max}}$ lies beyond the allowed range, $k_l < k < k_u$, it is limited to $k_{\text{max}} = \min(\max(k_{\text{max}}, k_l), k_u)$ and a new $\gamma_{\text{max}}$ is calculated accordingly. It is also important to mention that because the dispersion relations provided in Section 2 were derived under certain assumptions, i.e., Equation (6) is valid for $\xi < \alpha$ for the tearing mode and Equations (9) and (10) (incompressible and compressible MHD, respectively) are valid for $\lambda^2 \gg 1$ and $\mu^2 \ll 1$, respectively, for the Kelvin–Helmholtz instability, only cells that fulfilled them were processed to obtain the results presented here.

In Figure 8 we show the distributions of the estimated maximum growth rates for both instabilities. The statistics for the tearing mode and the Kelvin–Helmholtz instability are shown in the left and right column, respectively. Three different moments in time were chosen, $t = 3.0, 5.0$, and 7.0, shown in the upper, middle, and lower rows, respectively. We see that at earlier times (upper left panel), the tearing mode, even though it is expected to be the dominant mode, has relatively low maximum growth rates, $\gamma_{\text{max}} < 0.7$, with the distribution peaking at $\gamma_{\text{max}} \approx 0.2$. We should mention that considering the initial setup, the maximum growth rate of the tearing mode was estimated to be a value $\sim 0.7$ in the initial current sheet. Therefore we see a small decrease in $\gamma_{\text{max}}$ at early stages. Nevertheless, the number of the maximum magnetic shear cells increases at later time, with the distribution of $\gamma_{\text{max}}$ reaching values in the range between 0.1 and 0.2 (see the middle and lower left panels in Figure 8). We should note that the distributions of the maximum growth rates for the tearing instability are not too sensitive for the traverse component, which is represented by the parameter $\xi$. Even at the final time of the simulation, $t = 7.0$ (the lower left panel in Figure 8), when the turbulence is already developed in the initial current sheet region, the growth rate $\gamma_{\text{max}}$ for the estimated value of $\xi$ is reduced only in a small fraction of cells.

The Kelvin–Helmholtz instability is initially relatively negligible (see the $U$ distribution in the left plot in Figure 6 for $t < 30.0$). At $t = 30$ (the upper right plot in Figure 8), the velocity shear regions have already developed and can initiate the turbulence production through the instability. As seen in the right column of Figure 8, the maximum values of $\gamma_{\text{max}}$ reach 0.3–10 depending on the dispersion relation that is used, with the number of cells comparable (blue and red) for incompressible MHD or even larger (gray) for compressible MHD. When the compressibility is taken into account, $\gamma_{\text{max}}$ decreases, as expected. However, even though the tearing mode is preferential from the beginning, its maximum growth rates are lower than those related to the Kelvin–Helmholtz instability at later times. See the middle and lower panel in Figure 8, were the $\gamma_{\text{max}}$ distributions for the Kelvin–Helmholtz mode somewhat dominate those for the tearing mode, both in terms of the growth rates that are reached and the number of detected cells.

At the final moment ($t = 7.0$), the distributions of the growth rates for the Kelvin–Helmholtz instability when we apply the compressible MHD derivation of the dispersion relation with a smooth profile and the incompressible MHD with the discontinuous profile (Equations (7) and (10), respectively) are very similar. When the compressibility is ignored and a smooth shear profile is considered, the instability can reach values slightly over 100. When we recall the distribution of the Mach numbers shown in Figure 6, all unstable cells are characterized by low sonic Mach numbers, indicating that indeed $\gamma_{\text{max}}$ could be enhanced by a factor of a few.

5. Discussion

5.1. Limitations of Our Approach

Our approach in analyzing the tearing and Kelvin–Helmholtz instabilities is robust, but it has its drawbacks, which should be pointed out. First of all, in the presence of growing turbulent fluctuations, it is nearly impossible to determine the characteristics of the perturbations that are present in the analyzed local shear region. In order to determine the growth rate precisely, we would have to possess information about the wavenumber and direction of each local perturbation. In order to compensate for the lack of these data, we determine the plausible range of wavenumbers for which the instabilities would have been unstable. Therefore, they are limited to the range $k \in (k_l, k_u)$ as described in Section 3.3 for the tearing and Section 3.4 for the Kelvin–Helmholtz instabilities. In deriving the maximum growth rates $\gamma_{\text{max}}$, we determine the corresponding perturbation wavenumber $k_{\text{max}}$. If this wavenumber lies beyond the range $(k_l, k_u)$, it is limited and the growth rate is calculated directly from the dispersion relations. Moreover, we apply all the conditions related to stability and the validity of the dispersion relations. However, the condition for the stability of the Kelvin–Helmholtz depends on the direction of the perturbation and the upstream magnetic field. In the derivation of $\gamma_{\text{max}}$, we assume $k \parallel U$, using the fact that the initial perturbation was isotropic, and after the turbulence is developed, it is possible to find modes parallel to the shear.

As we described in the beginning of Section 4, our results are based on the statistics that we extracted from the cell-by-cell analysis, and the data were collected only for cells belonging to shear maxima. This means that the measurement does not represent the individual shear regions; the analysis is done separately for these. For example, in the case of the tearing instability analysis, we have one current sheet that initially crossing the whole computational box, therefore the points represent the cells along the plane of the maximum of current density. Due to the developing turbulence, this current sheet is being deformed and eventually broken into a number of current sheet regions that are not necessarily separated, but are interlinked in a complex manner. Therefore our analysis should be understood as a comparative analysis between two instabilities. This should be kept in mind, especially when the statistics of the longitudinal dimensions of the shear regions are interpreted.

The derivation of the analytical dispersion relations for the tearing and Kelvin–Helmholtz instabilities requires significant
simplifications. Two uniform regions of density, pressure, magnetic field, and velocity are typically considered to be connected by a transition layer. This layer can be discontinuous or smooth. The smooth case is usually approximated by a piecewise linear or hyperbolic tangent profile. The dispersion relations are different for these three different profiles, giving a different dependence of the growth rate $\gamma$ on the perturbation wavenumber $k$ (see, e.g., Chandrasekhar 1961; Furth et al. 1963; Ong & Roderick 1972; Walker 1981; Miura & Pritchett 1982; Somov & Verneta 1989; Chen et al. 1997;
Berlok & Pfrommer 2019). For a discontinuous profile \((\delta \approx 0)\) there is no limit on the wavenumber. The condition \(k_0 \delta < 1\) is always fulfilled in the case of the tearing mode, and the growth rate of the Kelvin–Helmholtz instability depends linearly on \(k\), resulting in arbitrarily high values of \(\gamma\). When we take a smooth transition within the shear region into account, we obtain a wavenumber \(k_{\text{max}}\) for which the growth rate has its maximum \(\gamma_{\text{max}}\), and which is typically related to the thickness of the region \(k_0 \delta \approx 1\). This maximum growth rate is usually a fraction of the growth rate that corresponds to the discontinuous shear. It is relatively difficult to determine the exact profile in the shear regions that is detected in our simulation. Especially in the case of broad shear and in the presence of turbulence, the profile can be very different from piecewise linear or hyperbolic tangent ones. Nevertheless, after determining the thickness \(\delta\) and upstream fields, \(B_0\) and \(U\), we assumed that they represent the finite-width profiles used in deriving the relation corresponding to each instability dispersion. Moreover, in calculating the tearing growth rates, we did not take the compressibility into account, although its effect should be relatively negligible because the velocity fluctuations do not reach velocities higher than the sound speed \(a\).

5.2. Turbulent Reconnection as a Dominant Process

Suggested 20 yr ago, the turbulent reconnection model has received significant support from subsequent numerical (see Kowal et al. 2009, 2012, 2017; Eyink et al. 2013; Oishi et al. 2015; Takamoto et al. 2015; Beresnyak 2017; Takamoto 2018; theoretical (Eyink 2011, 2015; Eyink et al. 2011; Lazarian et al. 2015, 2019), as well as observational studies (see Claravalla & Raymond 2008; Sych et al. 2009, 2015; Kharrova & Obridko 2012; Lazarian et al. 2012; Leão et al. 2013; Santos-Lima et al. 2013; González-Casanova et al. 2018). At the moment of its introduction, the model was an alternative to the Hall-MHD models that predicted a Petschek X-point geometry of the reconnection point, i.e., a very regular type of reconnection. The latter model required plasma to be collisionless, which is in contrast to the turbulent one, which did not depend on any plasma microphysics and was applicable to both collisional and collisionless media. It was later understood that the X-point geometry is not tenable in realistic settings. Instead, the tearing reconnection (see Syrovatskii 1981; Loureiro et al. 2007; Bhattacharjee et al. 2009) became the main alternative scenario for the turbulent model. So far, 2D simulations demonstrated fast reconnection for both MHD and kinetic regimes. Compared to earlier Hall-MHD reconnection that necessarily required a collisionless plasma condition, this was definitely an important improvement. The tearing reconnection shares many features with the turbulent one. For instance, Hall-MHD Petschek-type reconnection, due to the presence of a slow shock crossing the boundary, required a particular set of boundary conditions that was difficult to preserve in the realistic setting with random external perturbations (see, e.g., Forbes 2001). In the presence of random or turbulent motions, which are inevitable in astrophysical settings, the Petschek solution would not be satisfied. The tearing plasmoid reconnection therefore is a more robust scheme.

With two reconnection processes providing fast reconnection, it is important to understand the applicability of each. It has been numerically demonstrated in Kowal et al. (2009) that including additional microscopic effects that simulate enhanced plasma resistivity does not change the turbulent reconnection rate. This agrees well with the theoretical expectations in turbulent reconnection (see LV99 and Eyink 2011), in particular with the generalized Ohm law derived in Eyink (2015). As a result, if a medium is already turbulent, one does not expect to see effects of tearing reconnection. With the existing observational evidence about the turbulence of astrophysical fluids, this means that the turbulent reconnection is dominant for most of the cases. For instance, we expect the turbulent reconnection to govern violation of the flux freezing in turbulent fluids. This results in reconnection diffusion that governs star formation (Lazarian et al. 2012), which induces the violations of the structure of the heliospheric current sheet, and the Parker spiral (Eyink 2015). The numerical results on flux-freezing violation that follows from the LV99 theory cannot possibly be explained with the tearing reconnection. This clearly demonstrates that there are situations when the turbulent reconnection is at work, while tearing reconnection is not expected.

The pure problem of self-driven turbulent reconnection was the focus of our study in Kowal et al. (2017). There we showed that in the absence of the external turbulence driving, the turbulence develops in the reconnection region, and this turbulence has properties that correspond to the expectations of the MHD turbulence. This was in contrast to Huang & Bhattacharjee (2016), who claimed that turbulence produced in reconnection regions is radically different from the Goldreich & Sridhar (1995) turbulence. The properties of turbulence are important, as the LV99 magnetic reconnection, and closely connected to it, Richardson dispersion (Eyink et al. 2011), are proven to work in conditions where no tearing instability is expected. Therefore, if this type of turbulence is present in the reconnection regions, it is expected to induce fast reconnection. The correspondence of the reconnection rates in self-driven reconnection with the expectations of the LV99 theory was demonstrated in Lazarian et al. (2015), where the results of earlier simulations, e.g., Beresnyak (2017), initially available on arXiv in 2013, were analyzed.

The present paper is a step forward in understanding the process of self-induced fast reconnection. Here we explore the nature of turbulence driving in the reconnection region. If the tearing mode is absolutely essential for driving turbulence, one may still argue that the actual reconnection occurs through tearing, while the turbulence plays only an auxiliary role for the process. Our results indeed testify that the actual picture is very different. The process of the tearing mode plays in 3D a role at the earliest stage of reconnection. As the system evolves in time, the outflows induced by the reconnection region become turbulent, and the Kelvin–Helmholtz instability plays the dominant role. As the reconnection grows, the region becomes increasingly turbulent, and the tearing instability is overtaken or even suppressed, and does not play a role on the reconnection process overall.

Our simulations are performed in the high-beta plasma regime, and in these conditions, the reconnection outflow does not induce sufficient turbulence to trigger the self-accelerating process of “reconnection instability” (see Lazarian & Vishniac 2009), however.

While the MHD simulations show a very different picture for 2D and 3D self-driven reconnection, the particles-in-cell (PIC) simulations tend to show similar tearing patterns both in 3D and 2D. One possible explanation is related to the limitations of
present-day PIC simulation, given that these do not present enough particles in the reconnection regions to result in developed turbulence. Therefore, in such a “viscous” regime, the Kelvin–Helmholtz instability is suppressed and cannot operate, and the only signatures that can be seen arise from the tearing instability. In other words, the viscous outflow does not feel the additional degrees of freedom that would allow high Reynolds turbulent behavior to take place. Nevertheless, the high-resolution PIC simulations presented by Hui Li in a reconnection review by Lazarian et al. (2019) show the signatures of developing turbulence, e.g., a Richardson dispersion of the magnetic field lines was reported. Therefore we expect that the results we now obtained with MHD modeling can be also obtained or confirmed with very high particle number PIC simulations.

There is, however, another puzzle that is presented by the comparison of the 3D kinetic and MHD simulations. The kinetic simulations show higher reconnection rates, and it is important to understand whether these differences persist for reconnection at all scales or if they are just a transient feature of reconnection processes taking place for small-scale reconnection. This issue was recently addressed in Beresnyak (2018) using the Hall-MHD code. The results there testify that the reconnection rates for self-driven 3D turbulent reconnection obtained with Hall-MHD gradually converge to the results obtained for the 3D MHD self-driven reconnection. This is expected from theory (see LV99; Eyink et al. 2011; Eyink 2015). Nevertheless, in terms of our present study, the convergence of the results obtained with MHD and the Hall-MHD code testify that the results in the present paper do not change in the presence of additional plasma effects.

Our confirmation of the predictions of turbulent reconnection theory formulated in the LV99, and subsequent theoretical studies, also has a bearing on the ongoing discussion of the so-called “reconnection-mediated turbulence” idea presented in a number of theoretical papers (see Boldyrev & Loureiro 2017, 2018; Loureiro & Boldyrev 2017a, 2017b; Mallet et al. 2017a, 2017b; Comisso et al. 2018; Vech et al. 2018; Walker et al. 2018). For sufficiently large Reynolds numbers, due to both the process of “dynamical alignment” and the effect of magnetic fluctuations becoming more anisotropic with decreasing scale, current sheets that are prone to the tearing instability can develop. These changes of the turbulence at a scale λc in the vicinity of the dissipation scale do not change the nature of the turbulent cascade, which lies on scales of the inertial range ∝λc. Our study therefore suggests that if reconnection takes place at small scales comparable to λc, it will also be turbulent, as demonstrated by our simulations. However, because reconnection does not occur at the larger eddy scales, it is preferred to refer to this hypothetical regime as “tearing-mediated turbulence” instead. The objective reality is, however, that in the reconnection community, historically only bursts of reconnection were considered.

6. Conclusions

In this work we analyzed two MHD instabilities: the tearing mode and the Kelvin–Helmholtz instability, which are candidates for processes that are responsible for turbulence generation in spontaneous reconnection, e.g., reconnection without externally imposed turbulent driving. The generated turbulence is due to the initially imposed weak noise that is present in the vicinity of the Harris current sheet. We analyzed factors important for the growth of both instabilities, but also those that suppress them. The analysis presented in this work has shown important results, which can be synthesized as follows:

1. The region of the current sheet with the presence of initial noise develops into a region with conditions that are favorable for development of MHD instabilities, such as the tearing mode or the Kelvin–Helmholtz instability. Although the tearing instability is natural for thin elongated current sheets, the conditions for Kelvin–Helmholtz instability have never been verified before in systems with stochastic reconnection.

2. Evolution of stochastic reconnection leads to the formation of shear regions, both magnetic and velocity, with a broad range of thicknesses and longitudinal dimensions. Our studies indicate that the regions of magnetic shear for later times are somewhat thinner than those of velocity shear, while the longitudinal scales of these regions are shorter for velocity shear than for the magnetic one.

3. The tearing instability is expected to develop at earlier stages, while after a sufficient amplitude of turbulence is generated near the current sheet, it can be suppressed due to the presence of the transverse component of magnetic field Bτ. As shown in Somov & Verneta (1993), for ξ = Bτ/B > S−3/4 this instability is suppressed. We demonstrate that in our models ξ can be sufficiently large but does not completely suppress the instability. When we take the contribution of the transverse component Bτ into account, the instability can develop with a dynamical time that is shorter than the Alfvénic time tA under favorable circumstances. The estimated maximum growth rate γmax, initially below unity, reaches values of a few tens at later times with most of the detected points lying in the range 0.1–10.

4. Due to misalignment of the outflows from neighboring reconnection events, they can generate enough sheared flows to induce a Kelvin–Helmholtz instability. The Mach numbers we calculated with respect to the shear velocity U satisfy necessary conditions for the instability to develop. Our analysis indicates the presence of sheared flows with a broad range of amplitudes, 10−2 ≤ U ≤ 1, and thicknesses, 10−3 ≤ δ ≤ 0.5. The distributions of the estimated maximum growth rates γmax peak at values between 1 and 10 and reach maximum values close to 100, which suggests that the growth of the Kelvin–Helmholtz instability (within the dynamical time) is much shorter than the time tλc.

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ORCID iDs
Grzegorz Kowal @ https://orcid.org/0000-0002-0176-9909
Diego A. Falceta-Gonçalves @ https://orcid.org/0000-0002-1914-6654
Alex Lazarian @ https://orcid.org/0000-0002-7336-6674
Ethan T. Vishniac @ https://orcid.org/0000-0002-2307-3857

References

Alfvén, H. 1942, Natur, 150, 405
Ara, G., Basu, B., Coppi, B., et al. 1978, AnPhy, 112, 443
Armstrong, J. W., Rickett, B. J., & Spangler, S. R. 1995, ApJ, 443, 209
Beresnyak, A. 2017, ApJ, 834, 147
Beresnyak, A. 2018, JPhCS, 1031, 012001
Bhattacharjee, A., Huang, Y.-M., Yang, H., & Rogers, B. 2009, PhFl, 16, 112102
Boldyrev, S., & Loureiro, N. F. 2017, ApJ, 844, 125
Boldyrev, S., & Loureiro, N. F. 2018, JPhCS, 1100, 012003
Chandra Sekhar, S. 1961, Hydrodynamic and Hydromagnetic Stability (Oxford: Clarendon)
Chepurnov, A., Galvao, R., Pellat, R., Rosenbluth, M., & Rutherford, P. 1976, SvJPP, 2, 533
Dougherty, R. L., Edelman, A. S., & Hyman, J. M. 1997, JGR, 102, 151
Drizin, P. G., & Reid, W. H. 1981, STIA, 82, 17950
Eyink, G., Vishniac, E., & Lai, D. 2002, JGR, 107, 1372
Eyink, G. L. 2015, ApJ, 807, 137
Eyink, G. L. 2011, PhRvE, 83, 056405
Eyink, G. L. 2015, ApJ, 807, 137
Eyink, G. L. 2014, JPlPh, 83, 535500601
Fejer, J. A. 1964, PhFl, 7, 499
Forbes, T. G. 2001, EPAS, 53, 423
Frank, A., Jones, T. W., Ryu, D., & Gaalas, J. B. 1996, ApJ, 460, 777
Furth, H. P., Killeen, J., & Rosenbluth, M. N. 1963, PhFl, 6, 459
Galeev, A. A., & Zeleny, M. 1975, JETPL, 22, 170
Galeev, A. A., & Zeleny, M. 1976, JETP, 43, 1113
Gerwin, R. A. 1968, PlMP, 6, 45
Goldreich, P., & Sridhar, S. 1995, JPhFl, 83, 763
González-Casanova, D. F., Lazarian, A., & Cho, J. 2018, MNRAS, 475, 3324
Greco, A., Chuychpai, P., Matthaeus, W. H., Servidio, S., & Dmitruk, P. 2008, GeoRL, 35, L19111
Gudikov, M. G., & Troshichev, O. A. 1996, JATP, 58, 613
Huang, Y.-M., & Bhattacharjee, A. 2016, ApJ, 818, 20
Jacobson, A. R., & Moses, R. W. 1984, PhRvA, 29, 3335
Khabarova, O., & Obridko, V. 2012, ApJ, 761, 82
Kiefer, J. 1953, Proc. Am. Math. Soc., 4, 502
Kowal, G., Falceta-Gonçalves, D. A., Lazarian, A., & Vishniac, E. T. 2017, ApJ, 838, 91
Kowal, G., Lazarian, A., Vishniac, E. T., & Otmianowska-Mazur, K. 2009, ApJ, 700, 63
Kowal, G., Lazarian, A., Vishniac, E. T., & Otmianowska-Mazur, K. 2012, NPGeo, 19, 297
Lazarian, A. 2005, in AIP Conf. Ser. 784, Magnetic Fields in the Universe: From Laboratory and Stars to Primordial Structures, ed. E. M. de Gouveia Dal Pino, G. Lugones, & A. Lazarian (Melville, NY: AIP), 42
Lazarian, A., Esquivel, A., & Crutcher, R. 2012, ApJ, 757, 154
Lazarian, A., Eyink, G., Jafari, A., et al. 2019, PhFl, 27, 012305
Lazarian, A., Eyink, G., Vishniac, E., & Kowal, G. 2015, RSPTA, 373, 20140144
Lazarian, A., & Vishniac, E. T. 1999, ApJ, 517, 700
Lazarian, A., & Vishniac, E. T. 2009, RMxAC, 36, 81
Leão, M. R. M., de Gouveia Dal Pino, E. M., Santos-Lima, R., & Lazarian, A. 2013, ApJ, 777, 46
Loureiro, N. F., & Boldyrev, S. 2017a, PhRvL, 118, 245101
Loureiro, N. F., & Boldyrev, S. 2017b, ApJ, 850, 182
Loureiro, N. F., Schekochihin, A. A., & Cowley, S. C. 2007, PhFl, 14, 100703
Loureiro, N. F., Schekochihin, A. A., & Uzdensky, D. A. 2013, PhRvE, 87, 013102
Mallet, A., Schekochihin, A. A., & Chandran, B. D. G. 2017a, JPhFl, 83, 905830609
Mallet, A., Schekochihin, A. A., & Chandran, B. D. G. 2017b, MNRAS, 468, 4862
Matthaeus, W. H., & Lamkin, S. L. 1985, PhFl, 28, 303
Matthaeus, W. H., & Lamkin, S. L. 1986, PhFl, 29, 2513
Michalke, A. 1964, JFM, 19, 543
Miura, A., & Pritchett, P. L. 1982, JGR, 87, 7431
Oishi, J. S., Mac Low, M.-M., Collins, D. C., & Tamura, M. 2015, ApJL, 806, L12
Ong, R. S. B., & Roderick, N. 1972, P&SS, 20, 1
Padoan, P., Juvela, M., Kritsuk, A., & Norman, M. L. 2009, ApJL, 707, L153
Parker, E. N. 1957, JGR, 62, 509
Petschek, H. E. 1964, NASSP, 50, 425
Rayleigh, L. 1879, Proc. London Math. Soc., s1-11, 57
Santos-Lima, R., de Gouveia Dal Pino, E. M., & Lazarian, A. 2013, MNRAS, 429, 3371
Santos-Lima, R., Lazarian, A., de Gouveia Dal Pino, E. M., & Cho, J. 2010, ApJ, 714, 442
Schindler, K. 1974, JGR, 79, 2803
Sen, A. K. 1964, PhFl, 7, 1293
Serrín, S., Dmitruk, P., Greco, A., & Demin, V. 2011, NPGeo, 18, 675
Somov, B. V., & Vernet, A. 1989, SoPh, 120, 93
Somov, B. V., & Vernet, A. 1993, SSRv., 65, 253
Sweet, P. A. 1958, Obs, 78, 30
Syrovatskii, S. I. 1981, ARA&A, 19, 163
Takamoto, M. 2018, MNRAS, 476, 4263
Takamoto, M., Inoue, T., & Lazarian, A. 2015, ApJ, 815, 16
Vech, D., Mallet, A., Klein, K. G., & Kasper, J. C. 2018, ApJL, 855, L27
Walker, A. D. M. 1981, P&SS, 29, 1119
Walker, J., Boldyrev, S., & Loureiro, N. F. 2018, PhRvE, 98, 033209

Kowal et al.