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Streamfunction-Vorticity Flow Reconstruction for Color-Doppler Ultrasound Velocimetry

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Abstract—A novel method for reconstructing the two-component velocity fields from one-component planar color-Doppler ultrasound images, termed Doppler Velocity Reconstruction (DoVeR), was developed. We validated DoVeR using artificial Doppler images generated from computational fluid dynamics (CFD) models of left ventricle (LV) and carotid artery flows. DoVeR results were compared to those from the traditional intraventricular Vector Flow Mapping (iVFM) algorithm. Results from the LV model analysis indicate that DoVeR is robust to both noise and image resolution changes, yielding constant reconstruction error across all conditions (MAE_{DoVeR} = 5.5%, RMSE_{DoVeR} = 7%). Furthermore, DoVeR maintained a two- to three-fold improvement in reconstructed velocity accuracy over iVFM (RMSE_{iVFM} = 11.4% - 23.8%). Overall, 95% of DoVeR reconstructed velocities were within 14% error, whereas iVFM velocities fell within 33% error. In the carotid flow model analysis, 95% of DoVeR reconstructed velocities were within 18% error, whereas iVFM velocities were within 32% error. Subsequently, DoVeR and iVFM were compared using in vivo images from mouse LV and carotid ultrasound scans. In vivo measurements agreed with results from the artificial datasets. The DoVeR method yielded physiologically accurate reconstructions, which will enable more robust quantitative cardiovascular metrics and improve diagnostic capabilities.

Index Terms—Ultrasound, Heart, Vessels, Visualization

I. INTRODUCTION

Doppler ultrasound is a standard imaging modality that cardiologists and sonographers use to assess cardiac function and blood flow in the heart and vasculature. Color-Doppler imaging measures blood velocity along multiple scan lines, producing a 2-dimensional (2D) map of blood flow which captures flow patterns throughout the cardiac cycle. However, Doppler imaging measures velocity of blood flow toward or away from the transducer, but not flow transverse to the transducer. Therefore, interpreting the Doppler flow patterns is challenging as the scan velocities provide an incomplete description of the underlying velocity vector field [1]. Reconstruction methods to obtain a 2D velocity vector field from color-Doppler images are needed, as they can provide a more complete visualization and analysis of blood flow.

Currently, the most widely employed method for reconstructing transverse velocities inside the left ventricle (LV) is intraventricular vector flow mapping (iVFM), which is based on the mass conservation equation [2], providing a reconstruction algorithm that requires little computing cost. Today iVFM is a standard modality on Hitachi ultrasound systems [3]. Despite widespread use, iVFM only yields acceptable results for left ventricle (LV) flow reconstruction. Moreover, iVFM oversimplifies the influence of wall and bulk fluid motion on local reconstructed velocities [4], which causes large, non-physical velocity gradients [5]. Correcting the velocity gradients requires excessive smoothing [2, 6] which introduces errors. Assi et al. recently published a generalized iVFM formulation [7], but results still showed elevated relative errors (15-20%) during diastole. Furthermore, free-slip and fixed-wall boundary conditions were used to perform reconstruction, but the former are not realistic and the later are violated when heart wall motion is large, adding significant error during isovolumic flow [7]. Although iVFM is not mathematically limited to LV flows, no studies adopting this method for vascular flow reconstruction have been reported.

There are additional methods that have been used for color-Doppler velocity vector reconstruction. Echodynamography reconstruction, which separates the flow in order to solve for two components: ‘base’ flow and divergence-free flow [6]. However, the ‘base’ flow is not physically or mathematically justified and only performs well in rotating flows, limiting use to cardiac chamber flow analysis [7]. Pedrizzetti and Tonti proposed a reconstruction method based on the irrotational flow assumption [8]; but this formulation underestimates the strength of rotating flows. Arigovindan et al. published a 2D reconstruction method based on registering several Doppler views [9], which Gomez extended to 3-dimensional (3D) scans [10]. However, acquiring unique views to optimize registration is still a challenge during routine imaging.

In this work we developed a Doppler velocity vector reconstruction algorithm based on the streamfunction–vorticity ($\psi-\omega$) formulation. Our approach ensures smooth velocity fields and accurate velocity gradients, eliminates the need for excessive smoothing, and is adaptable for any region of the cardiovascular network where boundary conditions are known. This paper presents the new method—termed Doppler velocity reconstruction (DoVeR)—for 2D color-Doppler flow visualization. We analyzed the method using artificial datasets and demonstrated its capabilities from animal data. DoVeR was also compared with the traditional iVFM algorithm [2].
II. METHODS

A. Doppler Vector Reconstruction

As blood flows through the heart and blood vessels it interacts with the myocardial walls, valves, and vessels, which are in constant motion. This results in the flow constantly changing directions, producing unsteady, three-dimensional (3D) flow patterns [11]. Scans using 2D color-Doppler imaging captures the flow through these regions. Because this study utilizes 2D color-Doppler, the planar flow assumption (no out-of-plane motion) must be made. This simplification introduces error (~15%) [2, 11] but still enables determination of clinically useful measurements [2, 12].

A 2D color-Doppler image provides velocities \( v_{\text{doppler}}(x, y) \) for blood motion, as shown in Figure 1a and 1b. Each Doppler velocity is a projection of an underlying velocity vector \( \mathbf{u} = [u(x, y), v(x, y), 0]^T \) at a pixel location \((x, y)\). It can be assumed \( v_{\text{doppler}}(x, y) = v(x, y) \), such that values of \( u(x, y) \) must be reconstructed.

The velocity component \( u(x, y) \) can be recovered by computing the volume flux throughout a region of blood flow. Volume flux can be described by a scalar quantity known as the streamfunction, \( \psi \). Isolines of the streamfunction, shown in Figure 1c, describe a path, or streamline, along which volume flows with a constant rate. When the \( \psi \) values corresponding to two streamlines are subtracted the volume flow rate between these two lines is obtained. Therefore, a relationship between \( \mathbf{u} \) and \( \psi \) can be identified through the mass conservation principle

\[
\psi = \int u \, dy - \int v \, dx = \int \mathbf{u} \cdot d\mathbf{s}. \tag{1}
\]

Equation 1 can be written in vector potential (differential) form

\[
\mathbf{u} = \nabla \times \mathbf{\psi}, \tag{2}
\]

where \( \mathbf{\psi} = [0, 0, \psi] \). The curl of the velocity vector \( \mathbf{u} \) quantifies local rotation, known as vorticity, \( \omega \), and describes how \( \mathbf{u} \) changes orientation. The equation for \( \omega \) is written as

\[
\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \nabla \times \mathbf{u}. \tag{3}
\]

Through vector \( \mathbf{u} \) Equation 2 can be substituted into Equation 3, providing a relationship between \( \psi \) and \( \omega \) written as

\[
\omega = -\nabla^2 \psi = -\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2}. \tag{4}
\]

Equation 4 reveals that rotation is present when there is volume flux between streamlines. This is a Poisson equation and is at the heart of the \( \psi - \omega \) formulation used for Doppler vector field reconstruction.

The DoVeR algorithm implements Equation 2, Equation 3, and Equation 4 in an iterative framework to improve velocity reconstruction accuracy. DoVeR performs three recursive steps: (1) solving \( \omega \) from \( \mathbf{u} \) via Equation 2, (2) solving \( \psi \) via Equation 4 with proper boundary conditions, and (3) updating \( \mathbf{u} \) via Equation 2, shown in Figure 1d. We imposed that any non-zero Doppler velocities \( v(x, y) \) remain constant during reconstruction. To solve for \( \psi \) (Equation 4), DoVeR uses an LU-decomposition on a collocated Cartesian grid with Dirichlet boundary conditions at the inflow, outflow, and along the walls. To ensure volume flux is conserved, these boundary conditions are no slip and free penetration flux for \( \psi \) along all boundaries. In order to initialize the solver, \( \mathbf{u} \) is assumed to be the Doppler velocities. The algorithm is considered to have converged once the normalized square difference for \( \psi \) between passes is less than \( 1 \times 10^{-8} \).

![Figure 1: Steps of Doppler vector reconstruction. (a) A color Doppler image is extracted from an ultrasound scan. (b) Doppler velocity values are interpolated from the color Doppler images. (c) Doppler vector reconstruction resolves the streamfunction, \( \psi \), from the Doppler velocity values and proper boundary conditions. (d) The reconstructed velocity field is computed from the derivatives of the \( \psi \) values.](image-url)
ranging from low to high image resolution. Noise was modulated from 0% to 20% of each image’s maximum velocity [9, 20, 21]. Coherent structure identification was done using the λc criterion with a 5% threshold of the swirl [22]. We quantified mean absolute error (MAE) and root mean square (RMS) error for each test case to analyze reconstruction accuracy under variable noise and resolution. Error quantities were normalized using the CFD model peak inflow velocity.

Carotid artery model artificial color-Doppler images were rendered using a simulated linear array configuration (Figure 2d). Duplex scanning was performed to generate images with both color flow and tissue speckle RF data. For this work, only the color flow settings were necessary. Refer to Swillens et al. [14] for a complete description of the duplex configuration. A 192-element linear array probe with a 5.0 MHz center frequency and four pulse periods was simulated. The linear array focal point was set to a depth of 20mm and PRF was set at 4 kHz. Table 1 provides a complete list of transducer settings. Doppler images (Figure 2e and 2f) were formed from the RF images using standard autocorrelation with 10 packets for each line [19]. Pulse wave velocity measurements for the common carotid, internal, and external vessels were provided to initialize reconstructions. Flow streamtraces were calculated using a Runge-Kutta 45 ODE solver.

DoVeR algorithm reconstructions were performed on the raw artificial Doppler image datasets. For LV reconstructions, temporal bootstrapping of the velocities at the inflow and outflow boundaries was performed to remove noise in inlet and outlet boundary conditions. iVFM reconstructions were performed on filtered artificial Doppler images according to Garcia et al. [2]. A median filter with a 3x3 neighborhood window was applied to reduce the number of outlier velocity values followed by a 2D Gaussian filter with a standard deviation of 1.5mm.

Bland-Altman difference analysis and cumulative density functions (CDFs) of absolute error on reconstructed velocities for DoVeR and iVFM methods compared to the CFD model were computed for each flow model. Mean difference and 95% limits of agreement (LoA) for each reconstruction were calculated during the Bland-Altman analysis. CDF values at 10% absolute error for each method and 95% CDF limits were computed to compare convergence between reconstruction methods.

D. Animal Ultrasound Data

The DoVeR and iVFM algorithms were further demonstrated on in vivo imaging data using small animal color-Doppler ultrasound scans. All animal work was approved by the Purdue Animal Care and Use Committee. Imaging was performed with a Vevo2100 small animal ultrasound system (FUJIFILM VisualSonics Inc., Toronto, Ontario, Canada) with a 22-50MHz linear array probe (40 MHz center frequency; MS550D). Two healthy male C57BL/6J wild-type mice were purchased from Jackson Laboratories (Bar Harbor, ME, USA). One animal was 21 weeks old, while the other was 70 weeks old. Before imaging, the mice were anesthetized with 2–3% isoflurane-to-room air mixture at 1.5 L/min using a low-flow integrated digital anesthetic vaporizer (Somnosume, Kent Scientific, Torrington, CT, USA). Hair was removed from the ventral thorax using depilatory cream and ointment was applied to the eyes to prevent drying. Respiratory and ECG monitoring was collected by securing the animal’s paws to leads embedded in the stage in the supine position (Vevo Imaging Station). The
animal’s body temperature was monitored using a rectal temperature probe, which ensured a constant body temperature between 35-37°C. A heating lamp was also used to aid in maintaining body temperature during imaging.

Two separate imaging sessions were conducted, one for each mouse. One mouse was imaged to collect apical 2-chamber (A2C) LV flow, while the other was imaged along the right carotid artery bifurcation. Recordings were gated using the ECG signal which provided a 50-frame image series for during one cardiac cycle time for 50 different cycles. Each recording was averaged to generate a representative color flow image. In total, 60 distinct gated recordings were collected for the LV dataset and 13 distinct gated recordings were collected for the carotid bifurcation dataset. Additionally, pulse-wave Doppler was recorded in the main carotid, internal, and external arteries, which we used to initialize inlet and outlet boundary conditions for each reconstruction algorithm in carotid flow reconstruction.

III. RESULTS

A. LV Model Simulated Doppler Reconstruction

Figure 3 presents DoVeR and iVFM algorithm reconstruction vector fields alongside CFD model vector fields for the 128-line density, 5% added noise test case. This figure shows four distinct cardiac cycle time points: isovolumic contraction (IVC, Figure 3a-1 thru 3a-3), peak systole (Figure 3b-1 thru 3b-3), isovolumic relaxation (IVR, Figure 3c-1 thru 3c-3), and end diastole (Figure 3d-1 thru 3d-3). The instantaneous vector field is overlaid onto the vorticity field and includes coherent structures as closed contours. Overall, both reconstruction methods capture the bulk flow seen in equivalent CFD model flow fields for all time points. However, low velocity frames and high velocity frames show distinct differences in reconstruction quality. During IVC (Figure 3a) and IVR (Figure 3c) phases, low velocity cardiac cycle intervals, the reconstruction vorticity fields appear smooth. In IVC, a rotating structure form near the ventricle center, which is captured by both the DoVeR (Figure 3a-1) and iVFM (Figure 3a-3) reconstructions. However, the primary structure in the iVFM reconstruction appears elongated, is broken into multiple structures, and is coupled with other non-physical structures resolved in the surrounding flow. Both reconstructions produce similar results to the CFD model in the IVC phase.

In peak systolic ejection (Figure 3b), high velocity flow and high shear are present along the outflow track. Both DoVeR (Figure 3b-1) and iVFM (Figure 3b-3) reconstructions resolve similar structures and vorticity to the corresponding CFD field (Figure 3b-2). However, the bands of vorticity in the iVFM reconstruction are due to noise and boundary conditions. In peak diastolic filling, high velocity flow and high shear were present along the inflow track, with a large rotating structure formed near the ventricle center (Figure 3d-2). DoVeR reconstruction (Figure 3d-1) resolved similar features to the CFD field (Figure 3d-2), whereas iVFM (Figure 3d-3) showed both non-physical separation of the large rotating structure, similar to the IVC, and banded vorticity similar to peak systolic ejection.

Reconstructed velocity difference plots for DoVeR and iVFM as compared to CFD are presented in Figure 4a and 4b.

The DoVeR difference scatter point spread was within ±50% difference, whereas iVFM had a difference spread of ±100%. The DoVeR 95% LoA were ±14%, whereas iVFM 95% LoA were ±34%. This means the DoVeR estimate precision compared to iVFM improved by a factor of 2.4.

The CDFs in Figure 4c show the distribution of absolute error for DoVeR and iVFM reconstructed velocities. For the probability that measurements are within 10% absolute error, 87% of all DoVeR reconstructed velocities fall within this limit as compared to 62% of all iVFM reconstructed velocities. Furthermore, 95% of DoVeR reconstructed velocities fall within 14% absolute error, whereas iVFM error was 33%.
These values mirror the limits of agreement (LoA) determined in the difference analysis.

Figure 5 shows relative MAE and RMS error. DoVeR relative MAE and RMS error are about 5.5 and 7.0% for all cases, respectively. iVFM relative MAE varies between 9.7 and 16.5%, while relative RMS varies between 14.4 and 23.8%, with both errors increasing at lower resolution and elevated noise. Because the DoVeR MAE and RMS error measurements are consistent between all test conditions, these results indicate that the DoVeR method is robust to noise and is unaffected by image resolution.

B. Carotid Model Simulated Doppler Reconstruction

Figure 6 presents DoVeR and iVFM reconstructions alongside the CFD field for systolic flow and diastolic flow within the carotid artery model. During peak systole and peak diastole, unidirectional flow in the main branch of the carotid artery, followed by flow redirection near the bifurcation to the outlets along the external artery (top branch) and internal artery (bottom branch) was expected. This behavior was observed in the CFD simulation streamtraces, shown in Figure 6a-2 and 6b-2. The DoVeR reconstruction (Figure 6a-1 and Figure 6b-1) streamtraces paralleled the CFD result, exhibiting unidirectional flow along the main branch followed by redirection near the bifurcation. Additionally, DoVeR reconstruction vorticity fields showed similar patterns and intensity to the CFD result. Conversely, the iVFM reconstruction (Figure 6a-3 and Figure 6b-3) exhibited flow stagnation along the main branch and external carotid wall, and vorticity measurements had large fluctuations that did not mirror the CFD result.

The carotid flow reconstruction difference plots, Figure 7a and 7b, show similar difference ranges compared to the LV model. All DoVeR reconstructed velocities lie within ±50% difference of the CFD, whereas iVFM velocities spread past ±100% difference. Thus, DoVeR estimate precision compared to iVFM again shows improvement by a factor of 2. The iVFM differences also exhibit a bias of approximately 6.7% whereas DoVeR differences remains at 0% bias.

Additionally, the iVFM scatter has a gap in measurement around 50% mean velocity, primarily due to failed reconstruction in the external branch. This, in turn, increases the number of scatter points near 0% mean velocity which have 100% error and can be observed in the difference plot presented in Figure 7b.

The absolute error CDFs for the DoVeR and iVFM reconstructions of the carotid artery flow model are presented in Figure 7c. The probability that reconstructed velocities are within 10% absolute error for DoVeR are 81% and iVFM are 58%. Furthermore, 95% of DoVeR reconstructed velocities fall within 18% absolute error where as 95% of iVFM velocities fall within 32% absolute error. These values deviate slightly from the difference analysis LoA values.

C. Animal Color-Doppler Scan Reconstruction

Figure 8 presents DoVeR and iVFM algorithm reconstruction of vector fields from the in vivo mouse LV scan at four distinct cardiac cycle points: IVC (Figure 8a-1 and 8a-2), systolic ejection (Figure 8b-1 and 8b-2), IVR (Figure 8c-1 and 8c-2), and peak diastolic filling (Figure 8d-1 and 8d-2). In systole (Figure 8b) a vortex near the middle of the ventricle with
Figure 6: Carotid artery flow patterns for the (1) DoVeR algorithm, (2) CFD model, and (3) iVFM algorithm during (a) deceleration after peak systole and (b) peak diastole. The DoVeR algorithm agrees with the CFD model, whereas iVFM fails to resolve flow redirection along the external artery.

Figure 7: Difference plots for (a) DoVeR and (b) iVFM reconstruction velocities against CFD model velocities, and (c) relative absolute error cumulative density functions for DoVeR and iVFM reconstructions for the carotid artery model. The DoVeR method shows a two-fold improvement in measurement precision based on the LoA values. DoVeR measurements are within 18% error where as iVFM measurements are within 32% error at the 95% CDF threshold.

IV. DISCUSSION

We have introduced a new algorithm, DoVeR, for color-Doppler scan vector field reconstruction that recovers transverse velocities and provides smooth velocity gradients thereby improving additional measurement estimates obtained in data post-processing. Through this work we have identified

high vorticity along the outflow track was observed, similar to the LV model (Figure 3b). Although there was no clear outflow track in the mouse data due to the probe placement, the shear layer and vortex shape agreed well with the LV model. During the diastolic phase (Figure 8d) large vorticity values were observed and a vortex ring formed on the exterior of the inflow. This structure is expected to form based on a priori knowledge of LV filling mechanics and is therefore physically consistent. The low velocity IVC (Figure 8a) and IVR phases (Figure 8c) showed small vortex structures that are remnants of the larger, stronger structures which formed during the systolic and diastolic phases.

In the systolic ejection phase, the DoVeR and iVFM results (Figure 8b-1 and 8b-2, respectively) capture a vortex near the LV center. However, the identified vortex structure varies between reconstruction method. In the DoVeR result, the vortex contour is smooth, whereas the iVFM result vortex contour shows large spatial variations.

Although some spatial variation in the contour may be present, the observed large variations would indicate large changes in the velocity gradients between pixel locations.

Later, in diastolic filling, the DoVeR and iVFM results (Figure 8d-1 and 8d-2, respectively) capture a vortex pair along the inflow jet that is part of the expected vortex ring. DoVeR results capture primarily a dominant vortex pair, whereas the iVFM result captures several vortex pairs with similar size and intensity in close proximity to each other. In this case the iVFM result is not physically consistent; where we expect to see a single vortex pair, several pair are present. These pairs are artificially produced because of large changes in the velocity gradients in this field, similar to what we observed in the systolic ejection phase.

In the carotid artery dataset, uniform flow through the main branch into the internal and external arteries was observed, similar to the results observed in the carotid artery model (Figure 6). DoVeR results capture uniform flow with smooth redirection at the bifurcation in both systolic and diastolic phases (Figure 9a-1 and 9b-1, respectively). In contrast, the iVFM results showed flow passing through the main branch and redirecting primarily into the external artery for both systolic and diastolic phases (Figure 9a-2 and 9b-2, respectively). Furthermore, redirection into the internal artery occurred abruptly at the center of the vessel and stagnated along the vessel walls.
limitations of the traditional iVFM algorithm reported by Garcia [2] and Itatani [5] and shown how they affect reconstruction results. DoVeR was compared with the traditional iVFM formulation which is the current methodology used in Hitachi ultrasound machines [3].

A. Limitations on iVFM addressed by DoVeR

The traditional iVFM algorithm uses a 1D line integral to reconstruct transverse velocities. This formulation ensures smooth velocity reconstruction along the integral path but does not ensure smooth velocity gradients between lines. Gradients between lines and vorticity therefore become corrupted by noise, manifesting as “bands” in these fields—a limitation reported by Itatani [5, 6]. This effect appears in the artificial LV vorticity fields (Figure 3) and artificial carotid artery vorticity fields (Figure 6). Garcia’s original formulation [2] attempts to address gradient smoothness by using a median filter and Gaussian average filter combination; however, Itatani et al. [5] have shown that this original formulation may not be sufficient.

After assessing artificial reconstruction results, it is clear that vorticity, specifically transverse velocity gradients between lines, affected the quality of secondary measurements. In the artificial LV case, velocity gradients were used to compute the coherent flow structures. Because the traditional iVFM gradients are not smooth, large singular structures were identified as multiple, ellipsoid-shaped regions, as seen in Figure 3a-3 and Figure 3d-3.

Itatani chose to address this issue by implementing a more aggressive Gaussian smoothing kernel to improve vorticity field smoothness; however, more aggressive smoothing introduces larger bias errors on both the axial and transverse velocity components [5]. This method, therefore, sacrifices accuracy in order to obtain reliable velocity gradient estimates. DoVeR’s change in formulation to the 2D area integral effectively reduced error and ensured smooth velocity gradients simultaneously.

In Garcia’s paper, a weighted averaging operation was added to the iVFM formulation [2] in order to reduce noise, under the assumption that only axisymmetric flows are being reconstructed. This weighting is incorporated on the Hitachi software [3]. However, this averaging step introduces an error that violates the main assumption of the iVFM formulation: mass is conserved within the plane. Mass conservation is invalidated due to path dependence for each line integral; to have conservation the left-hand and right-hand integrals must be equal. Noise, boundary condition error, and out-of-plane motion will cause the left-hand and right-hand integrals to have different values. Thus, when the results from these integrals of different values are summed, mass conservation is no longer preserved.

The carotid bifurcation reconstructions (Figure 6 and Figure 9) emphasizes this issue. The averaging step alters flow direction along the external artery on the artificial dataset, causing the streamtraces that approach the outlet to turn and terminate along the walls; this same condition was observed along the internal artery in the experiment data. Through the 2D area integral, DoVeR addresses this issue by enforcing continuity based on the boundary conditions and velocity field provided while being robust to noise.

B. Boundary Condition Considerations

Reconstruction accuracy using numerical methods depends on boundary conditions. In biological flows, no-slip conditions are preferred, because they enforce tissue and fluid to have the same velocity along their interface. Intracardiac reconstruction methods impose this condition by using B-mode speckle tracking to estimate tissue velocities [4, 10], including the iVFM algorithm currently on Hitachi systems [3]. This wall tracking-based boundary condition is viable in 3D reconstruction configurations [4, 10] because wall motions are fully defined, and two unknown velocity components allow a divergence-free condition to be achieved. In the 2D
configuration this is not possible, as acknowledged by Assi [7]. Instead, Garcia and Assi adopt a tangential velocity condition estimated from velocities along the wall, assuming that the boundary layer is not resolvable [2, 7]. However, this condition is only practical for inertia-dominated flows such as LV diastolic filling and systolic ejection and even then, the results shows that these frames have elevated error compared to the entire cardiac cycle.

DoVeR uses a Dirichlet condition which mimics the speckle tracking concept without the need for estimating the motion. The Poisson equation solved in DoVeR enforces the divergence-free condition, thereby overcoming the 2D configuration limitation in previous formulations. We assume that tissue motion is uniform between frames for high velocity cases, which is not always physically consistent. However, despite this boundary condition inaccuracy, DoVeR improved reconstruction accuracy, as shown in the artificial dataset results.

C. Carotid Artery Flow Reconstruction

To the best of our knowledge, this is the first publication to demonstrate color-Doppler-based numerical vector reconstruction of flow in the heart and blood vessels. Our method is thus an improved approach for reconstructing blood flow from ultrasound images.

D. Implementation considerations

The DoVeR method reconstructs velocity vector fields from color Doppler images, requiring Doppler velocities and boundary conditions as inputs. As such, input quality plays a critical role in reconstructions. Although this work shows that image resolution and noise do not affect accuracy, considerations for PRF and Doppler aliasing must be made.

The PRF governs the Nyquist frequency or the scan velocity dynamic range. A low PRF setting will resolve higher velocities and prevent velocity aliasing; however, low velocities will not be resolved or will be unreliable. A high PRF setting will capture lower velocities but cause the scan to be aliased several times over. Although not a scope of this work, reliable de-aliasing algorithms can enable high PRF settings to be used in order to resolve more Doppler velocity information.

Doppler velocity fields and boundary conditions can be gathered on board scanner systems. Thus, the DoVeR algorithm can be practically implemented on board scanners as well. No additional equipment would need to be developed, and the algorithm implementation is primarily software programming.

E. Limitations

The work presented here compares DoVeR and iVFM methods using artificial data derived CFD models and small animal ultrasound images. Although results appear physiologically consistent, further comparison against experiments and in vivo measurements should be completed. EchoPIV, which measures the motion of ultrasound-opaque contrast bubbles, is ideally suited for this work. Furthermore, collecting several datasets sequentially, whether in silico or in vivo, can allow for more robust validation as well as improved reconstructions. Currently, the DoVeR algorithm is limited to 2D flows. However, the planar flow assumption has previously been validated [2, 11, 12, 23] with minimal error (about 15%) due to out-of-plane motion. Advancements in ultrasound technology are motivating more frequent use of 3D imaging, therefore this method should be generalized for 3D reconstructions in future work. DoVeR uses a piece-wise linear model for balancing mass flux as the boundary condition for reconstruction, uniformly distributing wall motion is across the entire boundary. However, wall motion is not uniform in healthy or diseased patients, suggesting further study of boundary condition selection should be performed to validate the current boundary conditions and to understand how the selection of these change reconstruction in the presence of disease.

V. Conclusion

We present a new algorithm for color-Doppler ultrasound velocity flow reconstruction, DoVeR, which is applicable in a wide variety of regions in the cardiovascular network. The DoVeR formulation is a 2D numerical method that enforces in-plane mass conservation under appropriate boundary conditions. We validated DoVeR using artificial color-Doppler data derived from computation models and compared results to iVFM, the most widely-used color-Doppler reconstruction technique. Datasets included LV flow and carotid artery flow. Artificial data error analysis indicates that DoVeR reconstruction shows at least a two-fold improvement in both reconstruction estimate accuracy and precision compared to the conventional iVFM method. Furthermore, DoVeR
reconstruction is agnostic to noise and scan resolution. We further demonstrate utility by reconstructing in vivo data from mouse scans of LV and carotid flows. In vivo results matched well with observations made from the validation data, highlighting the wide range of applications that have the potential to improve current flow reconstruction methods.

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