Determination of Optical Rotation Based on Liquid Crystal Polymer Vortex Retarder and Digital Image Processing

SIJIA HUANG1,2, SITE LUO1,2, YANG YANG1,2, TING LI1,2, YAN WU1,2, QIONGFANG ZENG3, AND HUIHUI HUANG1,2

1Key Laboratory for Micro/Nano Optoelectronic Devices of Ministry of Education, School of Physics and Electronics, Hunan University, Changsha 410082, China
2Hunan Provincial Key Laboratory of Low-Dimensional Structural Physics and Devices, School of Physics and Electronics, Hunan University, Changsha 410082, China
3School of Public Administration and Human Geography, Hunan University of Technology and Business, Changsha 410205, China

Corresponding authors: Site Luo (lost147258@163.com), Qiongfang Zeng (2681@hutb.edu.cn), and Huihui Huang (huangh@hnu.edu.cn)

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ABSTRACT A polarimeter for optical rotation (OR) detection and optical active media measurement based on m=1 Zero-Order Liquid Crystal Polymer Vortex Half-Wave Retarder (LCPVR) and Digital Image Processing (DIP) was proposed in this letter. In this research, we demonstrate the principle of measuring optical rotation with LCPVR polarimeter system which is based on Jones vectors and matrices. Two opposing wedge-shaped dark areas will appear in the output image when the uniform distribution polarized light passes the LCPVR. DIP is used to find the darkest line in the opposing wedge-shaped dark areas, then the optical rotation angle is calculated, which is from -90 degree to +90 degree. The polarimeter is used to measure the concentration of sucrose solutions at the range of 1000-10000mg/dl L. We also present the fabricated method of LCPVR by Digital Micro-mirror Device (DMD) system, and the LCPVR will be low price in the fabricated processing. Compared with existed polarimeter, our polarimeter with LCPVR has simpler optical structure and less size. The key innovation of this system is that the optical rotation angle can be displayed with visualized image except digital. And the rotation angle is calculated by DIP without mechanical adjust a 1/4 wave plate, which lead to higher detection speed and stronger stability.

INDEX TERMS Liquid crystal polymer vortex half-wave retarder (LCPVR), optical rotation, digital image processing.

I. INTRODUCTION

In recent years, Liquid crystal metasurfaces with special microstructures have achieved widely discussed. They show good application prospects in many fields, such as the Liquid crystal polarization gratings can use in multistage optical memory [1], [2] and the Liquid crystal flat lens (Pancharatnam-Berry lens) employs in optical edge detection [3], [4] and so on. The vortex retarder is also a kind of liquid crystal metasurface device with unique microstructure. The vortex retarder can convert linearly polarized light with different polarization states into a vector polarized beam or a vortex beam with orbital angular momentum [5]. The applications of these beams in previous literature include but are not limited to: an ellipsometer for measuring SiO₂ thickness and index of refraction using vectorial polarized beams was designed and validated [6]. Radially polarized beam near-field imaging characteristics applied in optical data storage technology [7]. The refractive index distribution can capture by using a lighting system using a zero-order Bessel beam [8].

Typically, the following methods are commonly used to measure optical rotation. The symmetric angle method polarimeter based on the accuracy of human vision which is the most widely used [9]. By using Faraday’s magneto-optical effect principle, the plane polarized light rotates under the axial magnetic field by applying the current to control the
magneto-optic components. The optical rotation is calculated by the amount of current applied. Such polarimeters presents high precision, however, it has relatively limited measuring range and complex photoelectric structure [10], [11]. Polarization gratings are also used to construct polarimeters. This polarimeter system measures the ratio of the diffraction intensity of the gratings to the rotation angle by measuring the ratio of the positive and negative first order. It can measure glucose concentrations ranging from 50 to 1000 mg/dL. However, the bulky optical setup used remains a problem (the tube used was 100 cm long) [12], [13]. The sinusoidal voltage drives liquid crystal variable retarder to control the tilt angle of LC to determine the optical rotation [14]. Azimuthally polarization axis finder sensors can measure polarization angle. The concentration of the optical active media was determined by the measured polarization angle. This azimuthally polarization axis finder system enables the measurement of the concentration of optical active media in a range of 5%-35%, but with a resolution of 5%. The image obtained is fuzzy and low quality [15]. Pixelated-polarization-camera uses pixelated micropolarizer array to measure the polarization angle and form polarimeters, but it just shows the polarized light image under polarizer array to measure the polarization angle and form polarimeters. This azimuthally polarization axis finder system enables the measurement of the concentration of optical active media in a range of 5%-35%, but with a resolution of 5%. The image obtained is fuzzy and low quality [15].

We report a polarimeter system which is constructed by sodium lamp, glass diffuser, linear polarizer, sample tube, LCPVR, and CMOS. The light, from sodium lamp, passed the glass diffuser will be a natural light. The linear polarizer is used to turn the natural light to linearly polarized light. The output image on CMOS will present the two opposing wedge-shaped dark areas when the linearly polarized light passes the LCPVR and another linear polarizer. Furthermore, the opposing wedge-shaped dark areas suggest the optical rotated angle, which can be calculated by finding out the darkest line in the image. Furthermore, the DIP process could be an automatic recognition method by computer program. This polarimeter system with LCPVR has less optical elements and less size, compared with existed polarimeters. Moreover, the optical rotation angle in this system can be displayed with visualized image, and the rotation angle is calculated by automatic DIP without mechanical adjust a 1/4 wave plate, which lead to higher detection speed and stronger stability.

II. METHODS

When This letter proposes a method of using the DMD to fabricate a \( m = 1 \) Zero-Order Liquid Crystal Polymer Vortex Half-Wave Retarder (LCPVR). \( m = 1 \) represents a fast axis distribution cycle period of the vortex retarder along the center of a pi. The difference between the different orders \( m \) is the fast axis distribution over the clear aperture of the retarder. Zero-order indicates that the design delay of the plate is 1/2 of the design wavelength. Then, we use LCPVR, Digital Image Processing (DIP) and cheap, commonly used optical devices to build a polarimeter. Our polarimeter have straight and simple optical paths, the length of the test tube is 100 mm. There is no active control device in the optical path, such as magneto-optical module, LC voltage controller, etc. There is also no spectroscopic device in the optical path, such as Wollaston prism or polarizing beam splitter. There is a great prospect of miniaturization, and the results can be visualized and saved. It has relatively high accuracy and is able to identify the optical rotation from a single camera image—allowing the potential for low-cost and real-time application. In the experiment, we set the initial group 0mg/dL and multiple groups of Concentrations of sucrose solution, 1000mg/dL, 2000mg/dL, 4000mg/dL, 6000mg/dL, 8000mg/dL, and 10000mg/dL, respectively.

The principle that the LCPVR and DIP can be used to detect the optical rotation angle is depicted in Fig.1(a) and explained as follows. Vortex retarder is a passive liquid crystal optical element. The Jones matrix for a vortex retarder is [17]

\[
J[\phi] = \begin{bmatrix}
\cos(m\phi) & \sin(m\phi) \\
\sin(m\phi) & -\cos(m\phi)
\end{bmatrix}.
\]

(1)

where \( \phi \) is the azimuthal angle of the vortex retarder and \( m \) is the output mode of the field. Its fast axis rotates continuously in the optical region. The fast axis orientation of the vortex retarder can be express as a linear relationship [17]:

\[
\phi = \frac{m}{2} \psi + \delta.
\]

(2)

Here \( \phi \) is the orientation of the fast-axis as a given azimuthal angle \( \psi \) on the vortex retarder, \( \delta \) is the orientation of the fast axis at \( \psi = 0 \). Thus different vortex retarder will have a different distribution of the fast axis about the center of the device. In this letter, the parameters are \( m = 1, \delta = 0 \). Fig.1(b) depict the fast axis pattern on this letter used LCPVR. \( G_{LCPVR} \) can be represented the LCPVR used in this experiment by the Jones matrix as

\[
G_{LCPVR} = \begin{bmatrix}
\cos\psi & \sin\psi \\
\sin\psi & -\cos\psi
\end{bmatrix}
\]

(3)

First, we discuss when there are no optically active media in our polarimeter system, only a LCPVR is placed between two crossed polarizers. The Polarizer P1 transmission axis direction is horizontal. As is shown in Fig.1, natural light can form linearly polarized light in the horizontal polarization direction after passing through the polarizer P1, which is horizontal linear polarization. The Jones vector and light intensity of the incident light are defined as \( E_0 \) and \( I_0 \). The Jones vector after P1 (\( E_{int} \)) is given by:

\[
E_{int} = E_0 \begin{bmatrix} 1 \\ 0 \end{bmatrix}
\]

(4)

The Jones matrix of the polarizers (\( P_1 \) and \( P_2 \)) with the transmission axis can be written as:

\[
P_1 = \begin{bmatrix}
1 & 0 \\
0 & 0
\end{bmatrix}
\]

\[
P_2 = \begin{bmatrix}
0 & 0 \\
0 & 1
\end{bmatrix}
\]

(5)

As is shown in Fig.1, the Jones vector and light intensity of the output light without placing sample are defined as...
**FIGURE 1.** Principle of LCPVR and DIP for measuring optical rotation. (a) Optimal rotation measurement system using LCPVR. (b) Distributions of Liquid Crystal directors in LCPVR. (c) Ideal initial state image (no Sample); “hourglass” intensity ideal image. (d) Output light intensity image after Sample (e.g., $\theta = 35^\circ$). (e) Optical rotation $\theta$: the angle of polarization plane rotated caused by optically active media (the angle difference between the red line and the blue line).

It is convenient to express the Jones vector ($E_{\text{out1}}$) of the output light using normalized Jones vectors and matrices as:

$$E_{\text{out1}} = P_2 \times G_{\text{LCPVR}} \times E_{\text{int}} = E_0 \begin{bmatrix} 0 \\ \sin \phi \end{bmatrix}$$  \hspace{1cm} (6)

The output light intensity ($I_{\text{out1}}$) can be obtained from Eq(7) as

$$I_{\text{out1}} = E_{\text{out1}} \cdot E_{\text{out1}}^* = I_0 \sin^2(\phi)$$  \hspace{1cm} (7)

We can know from $I_{\text{out1}}$ that when the $\phi$ azimuthal angle is equal to 0 or 180 degrees, the intensity presents two opposing wedge-shaped dark areas which obtain a pattern with an “hourglass” intensity profile shown in Fig.1(c) [18]. We can read the initial state angle $\phi_0 = 0$ from the Fig.1(c).

Then we discuss the polarization state of the output light after placing sample (containing optically active media) between LCPVR and P1. Due to the existence of optically active media and optical rotation effects: When a beam of linearly polarized light passes through the optically active media, the polarization plane is rotated because of the specific configuration. Here we express the angle of polarization plane rotated caused by optically active media matter as $\theta$. Optically active media can be represented by the Jones matrix $G_{\text{Sample}}$ as [19]:

$$G_{\text{Sample}} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$  \hspace{1cm} (8)

The Jones vector of the output light (after placing sample) using normalized Jones vectors and matrices ($E_{\text{out2}}$) is expressed as:

$$E_{\text{out2}}=P_2 \times G_{\text{LCPVR}} \times G_{\text{Sample}} \times E_{\text{int}} = E_0 \begin{bmatrix} 0 \\ \sin(\phi - \theta) \end{bmatrix}$$  \hspace{1cm} (9)

and the corresponding output light intensity is expressed as:

$$I_{\text{out2}} = E_{\text{out2}} \cdot E_{\text{out2}}^* = I_0 \sin^2(\phi - \theta)$$  \hspace{1cm} (10)

From Eq (11), it is easy to find that when the LCPVR azimuth angle ($\phi$) is equal to the deflection angle caused by the optically active media ($\theta$), the light intensity is the darkest shown in Fig.1(d). We can read the azimuth angle $\varphi_1 = \theta$ from the Fig.1(d). So, we only require the relative difference between these two azimuth angles, that is, the difference between the dark line angles of the corresponding pictures. Fig.1(e) is superposed by Fig.1(c) and Fig.1(d). The blue line represents the darkest line (initial state) in Fig.1(c), and the red line represents the darkest line (measured result) in Fig.1(d). This difference is the optical rotation.

$$\varphi_0 = \varphi_1 - \varphi_0$$  \hspace{1cm} (11)

A $m=1$ Zero-Order Liquid Crystal Polymer Vortex Half-Wave Retarders with a center wavelength of 589.3 nm was fabricated in this letter. The LCPVR manufacturing process is shown in Fig.2.(a). LCPVR same as the ordinary liquid crystal structure [20], [21]. Glass substrates are UV cleaned and spin-coated with SD1 (0.5 wt.% polarization photosensitive alignment sulphonyl azo-dye) films. SD1 molecules tend to align their absorption oscillators perpendicular to the polarization of the activating light. Then, the substrate coated with the SD1 dried on a heating platform at a temperature of 100 degrees Celsius for about five minutes.

After drying, it was removed and returned to room temperature, and then placed in a DMD mask exposure system for exposure and the $m=1$ zero-order vortex half-wave retarders mask designed by MATLAB has been loaded into the DMD before exposure. The mask generated by MATLAB is an
image of only black and white, and the angle of the DMD micro mirror can be changed through the distribution of black and white images. Fig. 2 (c) shows a mask map image used As shown in Fig. 2.(b), The DMD mask exposure system consists of approximately four subsystems, the luminescence part, the image focusing part, the dynamic pattern generation part, and the monitoring part, respectively [22]. All these subsystems work together to provide the correct exposure on the substrate. After exposure, we prepared monomer LC mixtures with 10 wt.% RM257 and spin-coat the prepared RM257 solution onto the sample exposed under DMD. The sample was cured by UV illumination after the spin coating is completed and let it stand for 2.5 minutes to cure. After curing, apply a layer of RM257 to the sample and cure it again. Completing the above steps, we obtained a LCPVR sample, as shown in Fig. 2.(b).

In LCPVR polarimeter, reading the digital image is the method to get the optical rotation angle and then to get the concentration of the solution. We have designed a special algorithm for this: first, find the center point of the picture: the point with the highest contrast. Then, select the 300 × 300 image area according to the center point. Lastly, find the darkest point, fit the darkest point, and find the angle. Obviously, the quality of the digital image is the most important factor to obtain better accuracy and data stability.

As a matter of fact, the principal element affecting the quality of images is the noise introduced by the environment and devices to the digital images. The image noise that needs to be filtered mainly includes impulse noise, Gaussian white noise, and spatially correlated noise. In this paper, three kinds of digital image processing methods are selected to deal with these three types of noise. They are Fuzzy operation(FO), bilateral filtering(BF), Gaussian filtering(GF). Fig 3 shows the images taken in the initial state of the polarimeter and the DIP images. Fig. 3(a) shows the original image, and the darkest pixel line is drawn out in Fig. 3(b). As shown in Fig. 3(b), the darkest pixel line in dotted box is not concentrated, especially at both ends of the picture, which is not conducive to subsequent fitting processing. While the darkest pixel line after image processing in Fig. 3(c) have obvious angular and straight-line trends. After image processing, the pixel values are excessively evenly smooth and have good image quality. And it can be seen that the impurity black spots introduced by the optical path have disappeared.
after DIP. Therefore, we only need to fit the darkest row of pixels in Fig. 3(c) to get the rotation angle that we need.

Through three different DIP filtering algorithms, 10 groups have been analyzed for comparisons, results shown in Fig.4. All the experimental data below are measured without any optically active media (only air in the polarimeter system). The FO is a kind of intelligence algorithm, and it is suitable when we have not clearly information to this mode. It is used in image enhance. Bilateral filtering is a non-linear filter that can achieve edge-preserving, noise-reducing, and smoothing effects, which is complemented based on the euclidean distance of pixels and similarity between error and center transition in convolution kernel, color intensity, depth distance [23], [24]. Gaussian filtering(GF), suitable for eliminating Gaussian noise, is a process of weighted averaging the entire image [25]. The Max-Min: the difference between the maximum and minimum angles and the Average: average value among the 10 angles is calculated, and the comparison results are listed in Table 1. By comparing the data in Table 1, it can be found that after DIP, the stability of the data becomes significantly better.

Ideally, two polarizers (P1,P2) are orthogonal, as shown in Fig.1.(a). The 0-degree azimuth of LCPVR shall completely coincide with the 0 degree azimuth of polarizer P1 and P2. The angle obtained after taking and analyzing the picture should be 0 degrees. However, due to the existence of certain errors in optical path construction and device installation, the standard 0 degree cannot be achieved. The initial state of our whole system should be -0.7099 degree after filtering.

III. EXPERIMENTAL SETUP AND RESULTS
The polarimeter experimental setup pertinent to this study is shown in Fig.5. A stabilized sodium lamp was used to supply = 589.3 nm as the light source. Firstly, A ground glass diffuser (GD)(LBTEK, DW110-600) with 600 grit polishes and aspheric condenser lens (AC lens)(LBTEK, MAC1909-A) is in utilized to establish a more uniform and stable light illuminating field. Since the measurement results are related to the uniformity of the light source, GD can help us to improve the uniformity of the light field. AC Lens collimates the light source. Then, Set linear polarizer Pol(1) (LBTEK, FLP20-VIS) behind the lens closely to obtain a linearly polarized beam with a light vector along the x-axis is formed linearly polarized light field. The test tube is placed between the aperture and LCPVR, which generates a rotation angle for measurement. The length of the test tube made by three-dimensional(3D) printing is 100mm. Behind the test tube is an LCPVR and a linear polarizer Pol(2) (LBTEK, FLP20-VIS) with the transmission axis along the y-axis and an imaging lens. To collect and analyze the intensity profile, an excellent capability CMOS camera is employed and converted into a digital image. CMOS camera is MV-SUA501GM-T black

![FIGURE 4. Measured results(the initial state). FO:Fuzzy operation; BF: bilateral filtering; GF: Gaussian filtering.](image)

![FIGURE 5. Schematic diagram of the experimental setup for LCPVR polarimeter. GD: ground glass diffuser; AC lens: aspheric condenser lens; Pol(1): polarizer with the transmission axis along the x-axis; Test Tube: rotation generating tube with sucrose solution; LCPVR: m=1 Zero-Order Liquid Crystal Polymer Vortex Half-Wave Retarder; Pol(2): polarizer with the transmission axis along the y-axis.](image)

| DIP Algorithm                        | Average(°) | Max-Min(°) |
|--------------------------------------|------------|------------|
| Fuzzy operation                      | -0.6248    | 0.1015     |
| bilateral filtering+Gaussian filtering | -0.4140    | 0.0338     |
| Fuzzy operation + bilateral filtering+Gaussian filtering | -0.7099    | 0.0008     |

![TABLE 1. Comparisons of three different DIP filtering algorithms.](image)
FIGURE 6. Image of experimental results. (a) - (f) 1000mg/dL, 2000mg/dL, 4000mg/dL, 6000mg/dL, 8000mg/dL and 10000mg/dL Processed images.

TABLE 2. The Experimental Data of Different Sucrose Concentration.

| C (mg/dL) | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | Ave (°) | SD (°) | Max-min (°) |
|----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|---------|--------|------------|
| 0        | -0.8388 | -0.8477 | -0.8433 | -0.8398 | -0.8393 | -0.8437 | -0.8363 | -0.8448 | -0.8321 | -0.8453 | -0.8411 | 0.00427 | 0.0156     |
| 1000     | -0.0929 | -0.0965 | -0.0983 | -0.0951 | -0.0982 | -0.0796 | -0.0845 | -0.0773 | -0.1056 | -0.106 | -0.0934 | 0.00903 | 0.0287     |
| 2000     | 0.3433  | 0.3474 | 0.3388 | 0.3439 | 0.3386 | 0.3482 | 0.3518 | 0.3442 | 0.3508 | 0.3622 | 0.3469  | 0.00632 | 0.0236     |
| 4000     | 1.8436  | 1.8624 | 1.8621 | 1.8659 | 1.8692 | 1.8661 | 1.8627 | 1.8669 | 1.8686 | 1.8684 | 1.8636  | 0.00678 | 0.0256     |
| 6000     | 2.9885  | 2.9822 | 2.9868 | 2.9912 | 2.986 | 2.9912 | 2.9876 | 2.9897 | 2.9976 | 2.9937 | 2.9876  | 0.00406 | 0.0151     |
| 8000     | 4.264  | 4.2578 | 4.2545 | 4.2574 | 4.2587 | 4.2629 | 4.2604 | 4.2573 | 4.257 | 4.2566 | 4.2587  | 0.00266 | 0.0095     |
| 10000    | 5.7381 | 5.7454 | 5.7426 | 5.7288 | 5.7459 | 5.7347 | 5.7377 | 5.7374 | 5.7397 | 5.7396 | 5.7390  | 0.00457 | 0.0171     |

and white industrial camera of MindVision Company. The camera has a resolution of 2592 × 1944, 5 million effective pixels and a pixel size of 3.45 x 3.45 µm. Finally, the image is transmitted and saved in a computer.

In the experiment, the measured sucrose solution made up using chemically pure sucrose (ALADDIN, S112226) and distilled water. We mainly research the measurement performance of this polarimeter at a low concentration, so we equipped with 1000mg/dL, 2000mg/dL, 4000mg/dL, 6000mg/dL, 8000mg/dL and 10000mg/dL sucrose solution for measurement. A total of ten sets of data are recorded for each group of concentration, a set of data contains ten consecutive digital images and collects every thirty seconds, at 20 ± 1°C. We also specify the measurement angle range from -90 degree to +90 degree. The test results are presented at Fig.6.

As shown in Fig.6, (a)∼(f) present the images of sucrose solutions at 1000mg/dL, 2000mg/dL, 4000mg/dL, 6000mg/dL, 8000mg/dL and 10000mg/dL. The darkest pixel line is marked as a white line in these images to present the rotation angle. And the rotation angle of distilled water is used as a standard, whose rotation angle is 0 degree. The white lines in Fig.6 (a)∼(f) show the relative angles to distilled water. It is presented clearly in Fig.6 that the rotation angles are increasing with the concentration. The number of the rotation angles can be calculated by DIP with computer program, which is built as automatic recognition system.

The data of rotation angle at different concentrations is recorded in Table 2. It can be seen from Table 2. Table 2 shows that the data stability of the polarimeter is good and the polarimeter system is reliable. C is the standard concentration.
According to formula (13) and Table 2, we can calculate the concentration data of each group. The data of the angle and concentration is recorded at Fig.7 and Table 3. As shown in Fig.7, the concentrations of sucrose solution and optical rotation angles have a high linearity. Table 3 suggests that there is a large error at 1000mg/dL and 2000mg/dL, while a good performance is presented at higher concentrations more than 2000mg/dL. The reason is that the low concentration solution of 1000mg/dL is near the resolution of the LCPVR. The experimental results show that LCPVR and DIP polarimeters can be used to determine the optical active media measurement system.

IV. CONCLUSION
In this letter, a polarimeter system based on LCPVR and Digit Image Processing (DIP) is proposed to measure optical rotation. Jones matrices and optical rotation effect explain that the intensity image passing a polarimeter based on LCPVR will produce a rotation, and the DIP is used to calculate the rotation angle. This polarimeter is used to detect the sucrose concentration because there is a linear relation between optical rotation angle and sucrose concentration. Optical rotation angles can be obtained by marking the darkest pixel line since the image will generate two opposing wedge-shaped dark areas. DMD, SD1, and RM257 are employed in the fabrication of an LCPVR. A variety of filtering algorithms are used to process the obtained pictures to ensure the stability of the data. It overcomes the complexity of optical path and huge equipment caused by the need of additional magneto-optical components, rotating electromechanical components and electro-optical components in traditional optical rotation measurement. The polarimeter optical route is simple and straight, the optical devices are cheap, commonly used and off-the-shelf. Moreover, the optical rotation angle can be displayed with visualized image except digital, and the rotation angle is calculated by automatic DIP without mechanical adjust a 1/4 wave plate, which lead to higher detection speed and stronger stability.

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