A new glitch-rejection algorithm forged in the spherical harmonic basis

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Abstract. The noise in the output of a gravitational-wave interferometer is known not to be due to stationary Gaussian processes. In fact, even after whitening, it will contain spikes of very large power (called glitches), due to environmental disturbances, instabilities in detector systems, and other factors. These glitches can sometimes mimic, and, due to their loudness, obfuscate real signals close in time. In this article we outline a new method of discriminating these glitches from gravitational-wave signals which works on the principle of the spherical harmonic decomposition of the correlation of the data streams in the detector network. It is intrinsically a coherent all-sky method (working, as it does, in the spherical harmonic domain) which can produce a time-series showing the glitchiness of the underlying data. We demonstrate this spherical-harmonic technique using real interferometer data and compare it to another glitch-rejection method in current use.

1. Introduction
Detecting or reducing the effect of glitches in the output of gravitational-wave interferometers is an important part of any search for transient gravitational-wave signals [1, 2]. A loud glitch is nothing but a nuisance; it can mimic a signal when coincident with glitches in the other data streams, it can wash out the effect of a real signal present, and it can affect background estimation too. Thus, there is a need to identify the presence of glitches accurately and quickly without giving false readings - mischaracterising a signal as a glitch or vice versa. This article will outline a flexible, and simple, discriminator that can be used to categorise transients as signals or glitches. Specifically, we perform a spherical harmonic decomposition of the sky map constructed from the pairwise correlation of the whitened data streams from the detector network. We find that the ratio of the power in the \( \ell > 0 \) modes to that in the \( \ell = 0 \) mode provides a powerful discriminator for rejecting background glitches. We demonstrate this technique using real interferometer data from the fourth LIGO Science Run [3]. We also compare it to another glitch-rejection technique used in coherent searches for gravitational-wave bursts.

2. Gravitational-wave burst searches
We focus on searches for gravitational-wave bursts; transient signals whose waveforms are not known \textit{a priori}. (Our method does not require knowledge of the waveform, but it should be equally useful in a matched-filter search where the waveform is known.) Burst searches are usually carried out using a coherent analysis, in which the data from a network of detectors
is combined in amplitude and phase to form one or more statistical measures of the likelihood of a signal being present. These statistics are commonly computed as time-frequency maps, where candidate signals are identified as clusters of pixels of large value; see Figure 1 for an example. One commonly used search pipeline is coherent WaveBurst (cWB) [4], which computes a constrained likelihood measure of the gravitational wave signal-to-noise ratio, and combines this with a measure of the correlation across the network to reject or down-weight glitches. X-Pipeline [5] computes time-frequency maps of the energy in the reconstructed plus, cross and null [7] polarisations of a gravitational wave, and rejects glitches based on the relative contribution of the detector cross-correlations to these energies [6]. Similarly, the STAMP algorithm [8, 9] uses the auto-power in each detector to discriminate between Gaussian noise (possibly including a gravitational wave) and glitches.

![Time-frequency map](image)

**Figure 1.** A time-frequency map of whitened data; to the upper left we see a short-duration white noise burst (WNB) signal, to the lower middle a glitch. The color scale is logarithmic in coherent network power, as defined in equation 4.

3. **The Spherical Radiometer pipeline**

A common problem with the coherent analysis is its speed. It takes time to form a skymap of coherent energy - each direction is usually calculated independently, so the higher the resolution required, the longer the generating time. To improve the speed of all-sky, all-time burst searches, we have been exploring a fundamentally new approach: spherical radiometry. This approach takes advantage of the fact that sky maps in gravitational-wave searches show strong correlations over large angular scales in a pattern closely tied to the network geometry. Computing sky maps indirectly through their spherical harmonics avoids much of the redundant calculations.

The core of our spherical radiometer pipeline, SphRad, is a set of fast cross-correlator codes written by Cannon [10]. This engine transforms the problem of computing correlations between data streams into the spherical harmonic domain, allowing correlation between detectors in a network to be performed extremely quickly. These codes are wrapped by the Ω pipeline [11]; Ω finds, loads, and whitens the data, optionally generating simulated glitch and injection signals (for testing). We also use the suite of post-processing codes in Ω to process the output event triggers.
3.1 Output

To generate the cross-correlation for each detector pair in our network, we need to compute the integral

\[ \xi_{1,2}(\hat{s}) = \frac{1}{N} g_1^T(\hat{s}) \cdot \hat{g}_2(\hat{s}) = \frac{1}{N} \hat{g}_1 \cdot \mathbb{T}^T(\hat{r}_1 - \hat{s}) \cdot \mathbb{T}(\hat{r}_2 - \hat{s}) \cdot \hat{g}_2. \]  

(3)

Note that the dependence on sky position, \( \hat{s} \), has been decoupled from the data, \( \hat{g}_i \). By using an Earth-fixed coordinate system for \( \hat{s} \), Fourier transforming \( \hat{g}_i \) and expanding \( \mathbb{T} \) to finite order in spherical harmonics we can efficiently calculate this integral over the sky (for more details see [10]). This calculation must be performed for each pair of detectors that we have in our network, so the total coherent network power is therefore given by

\[ N_{total} = \xi_{1,2} + \xi_{1,3} + \xi_{2,3} + \xi_{1,1} + \xi_{2,2} + \xi_{3,3} \]  

(4)

which we use as our statistic when we are forming time-frequency and time-harmonic maps.

For each ‘time bin’ (our FFT length), the detector data is whitened, Fourier transformed and fed into the correlator. The output is a 2-dimensional ‘grid’ (see Figure 2) - the spherical harmonic expansion for each frequency of interest. Along one axis we have the frequency span of our data, along the other we have the spherical harmonic coefficients, \( c_{lm} \), of that expansion (the coefficients are ordered according to SpharmKit [12]).

Similarly, if we sum along the coefficients, we get the power at each frequency for that time bin, averaged over the whole sky (as we are dealing with spherical harmonics). If we then generate this for each of the time bins in our segment (usually 64 or 128 seconds, divided into 1 second bins), we get a coherent time-frequency map. Here though, the value in the time-frequency pixel is the power in the correlations for our detector network, \( N_{total} \), defined by equation (4) above. See Figure 1 for an example time-frequency map.

Similarly, if sum along the frequencies for each spherical harmonic coefficient, we get a single set of coefficients for all frequencies, relating to the power over the sky. Generating this for each time bin in our segment we get a time-harmonic map. Each set of coefficients tell us something about the distribution of power over the sky for that time (see Figure 3). This ‘dual view’ of the data is very important, and is one of the major advantages of decomposing the integral into spherical harmonics.
Figure 2. Frequency-spherical harmonic map in log-scale colouring for the time bin containing an injected sine-Gaussian. The injection is clearly visible as a horizontal line at $f = 235$ Hz, the central frequency of the injection.

Figure 3. Time-harmonic map for a segment containing a glitch and an injection. Only the injection is visible (the vertical line at 10s) - compare to Figure 1.

Figure 4. Skymap of the coefficients corresponding to the time bin containing the injection, see Figure 3 at 10s.

3.2. Skymaps

For easier interpretation, the spherical harmonic coefficients can be transformed into the image domain using a Fast Spherical Harmonic Transform (FSHT), which is the spherical harmonic equivalent of the Fast Fourier Transform. The FSHT is defined in [13] and found in [12]. Transforming a set of coefficients into the image domain using the FSHT generates a skymap showing the localisation of power in the sky; see Figure 4 for an example. This allows for the formation of any statistic that can be constructed from the correlation between the detectors in the network. For the Livingston, Hanford, Virgo (HLV) network, the cross-correlations $HL$, $HV$, $LV$ are available, as well as the auto-correlations of $H$, $L$ and $V$. Thus, it is possible to easily construct (for instance) the coherent energy in the plus polarisation (equation 5; see [5]), by multiplying the correlation of each detector pair, $\tilde{d}_{ij}$, by the appropriate antenna factors, $e_{ij}^\pm$, in the image domain and summing:

$$E_+ = \sum |e^+ \cdot d|^2$$

$$= e_{11}^+ \cdot \tilde{d}_{11} \cdot e_{11}^+ \cdot \tilde{d}_{11} + e_{11}^+ \cdot \tilde{d}_{12} \cdot e_{12}^+ \cdot \tilde{d}_{12} + e_{11}^+ \cdot \tilde{d}_{13} \cdot e_{13}^+ \cdot \tilde{d}_{13} + e_{21}^+ \cdot \tilde{d}_{21} \cdot e_{21}^+ \cdot \tilde{d}_{21} + e_{21}^+ \cdot \tilde{d}_{22} \cdot e_{22}^+ \cdot \tilde{d}_{22} + e_{21}^+ \cdot \tilde{d}_{23} \cdot e_{23}^+ \cdot \tilde{d}_{23} + e_{31}^+ \cdot \tilde{d}_{31} \cdot e_{31}^+ \cdot \tilde{d}_{31} + e_{31}^+ \cdot \tilde{d}_{32} \cdot e_{32}^+ \cdot \tilde{d}_{32} + e_{31}^+ \cdot \tilde{d}_{33} \cdot e_{33}^+ \cdot \tilde{d}_{33} \quad (5)$$

$$= e_{11}^+ \cdot \tilde{d}_{11} \cdot e_{11}^+ \cdot \tilde{d}_{11} + e_{11}^+ \cdot \tilde{d}_{12} \cdot e_{12}^+ \cdot \tilde{d}_{12} + e_{11}^+ \cdot \tilde{d}_{13} \cdot e_{13}^+ \cdot \tilde{d}_{13} + e_{21}^+ \cdot \tilde{d}_{21} \cdot e_{21}^+ \cdot \tilde{d}_{21} + e_{21}^+ \cdot \tilde{d}_{22} \cdot e_{22}^+ \cdot \tilde{d}_{22} + e_{21}^+ \cdot \tilde{d}_{23} \cdot e_{23}^+ \cdot \tilde{d}_{23} + e_{31}^+ \cdot \tilde{d}_{31} \cdot e_{31}^+ \cdot \tilde{d}_{31} + e_{31}^+ \cdot \tilde{d}_{32} \cdot e_{32}^+ \cdot \tilde{d}_{32} + e_{31}^+ \cdot \tilde{d}_{33} \cdot e_{33}^+ \cdot \tilde{d}_{33} \quad (6)$$. 

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Similar calculations can be carried out for the cross polarisation and null stream [5, 7, 14], or any other statistic that can be written as the sum of pairwise cross- and auto-correlations between detectors.

3.3. Projections
The time-frequency and time-harmonic maps derive from a higher order map, so they contain the same total power. The distribution is different however, which implies something about the underlying structure. For simulated signals injected with a prescribed central frequency and peak time, the time-frequency map lights up for the duration and bandwidth expected. Similarly, for glitches, the time-frequency map also lights up the pixels around the expected time and frequency (see Figure 1). In contrast, the time-harmonic map shows structure along the coefficients at the time of an injection, but nothing at the time of a glitch (see Figure 3).

This leads to the conjecture that the spherical harmonic coefficients could be used as some form of glitch discriminator, and a quick aside to think about how details on the sky map relate to the spherical harmonic coefficients should convince why this is so.

3.4. Sky localisation
Using a network of gravitational wave detectors to localise an event is a process of triangulation - the time that the event is seen in each detector gives you a measure of its location [15, 16].

- Behaviour with signal: In this case, there is a preferred time delay for each detector that will match the signal up, corresponding to a preferred sky direction. In the time-frequency representation, we have essentially summed over the whole sky, so direction/time delays are not important. However, in the time-harmonic representation a signal is resolved to a patch of the sky meaning that power will be present in most of the coefficients - generally none will stand out more than any other.

- Behaviour with glitch: There is no preferred time-delay for a glitch, as there is nothing coherent for it to couple to in the other detector streams. And, as the time-frequency representation is independent of the time delay, we see similar behaviour to that with a signal. In spherical harmonic terms, the zeroth component, $c_{00} Y_{00}$, corresponds to the amount of energy present over the whole sphere, while the higher order terms relate to fine details. Thus, the zeroth component is a measure of amount of incoherent energy there is in the sky map, which directs relates to the glitch energy, as this is mostly present in the $c_{00} Y_{00}$ coefficient.

4. Spherical harmonic statistic
The new coherent statistic works on the principle that we can decouple the ‘incoherent’ and ‘coherent’ energy using the decomposition into spherical harmonics. Using the nomenclature $c_{lm}$ to represent the spherical harmonic coefficients, the $|c_{00}|$ coefficient corresponds to the incoherent energy, while the sum of the rest of the coefficients corresponds to the coherent energy. For a signal (which has a preferred sky location), all of the coefficients have some measure of power, however, for a glitch, most of the energy is located in the $c_{00} Y_{00}$ coefficient.

The algorithm is as follows:

(i) Decompose a coherent skymap into the spherical harmonic domain using the FSHT (if required).

(ii) Sum all of the $\ell > 0$ coefficients, $\sum |c_{lm}|$, as a measure of coherent energy.

(iii) Divide the sum by $|c_{00}|$, the measure of incoherent energy.

(iv) Repeat for all time bins in a data segment. Any bin that rises above the background is a signal, any falling below a glitch. See Figure 5 for an example.
Essentially, for each time bin we calculate:

$$Z_{\text{ratio}} = \frac{\sum_{l=1}^{l_{\text{max}}} \sum_{m=-l}^{l} |c_{lm}|}{|c_{00}|}$$

(7)

where $l_{\text{max}}$ is defined using a combination of baseline length and the data sample frequency (see [10] for more details). Figure 5 shows the statistic for data from the 4th LIGO science run (S4).

***Figure 5.*** The $Z_{\text{ratio}}$ statistic of equation (7) for the 64 seconds of data displayed in Figures 1 and 3. The injected gravitational-wave signal rises well above the background noise at $t = 10$ s, while the glitch drops below at $t = 31$s.

5. Results

To test the robustness of the new statistic, it was applied to both simulated and real data, sampled at 2048 Hz, and using the Hanford-Livingston-Virgo (HLV) network. White Noise Bursts (WNB) containing 2 independent polarisations of approximately equal power were injected into the data with a central frequency of 300 Hz, a bandwidth of 50 Hz and a duration of 0.1 seconds. A range of root-sum-square injection amplitudes were used, from $3.7 \times 10^{-23} \text{s}^{-1/2}$ to $1.5 \times 10^{-19} \text{s}^{-1/2}$, defined as:

$$h_{\text{RSS}} = \sqrt{\int_{-\infty}^{\infty} (|h_+(t)|^2 + |h_\times(t)|^2) dt}$$

(8)

where $h_+(t)$ and $h_\times(t)$ are the plus and cross-polarisation strain functions, and $h_{\text{RSS}}$ has units of $\text{Hz}^{-1/2}$.

5.1. Simulated noise data

We analysed approximately one month of simulated noise data using the SPHRAx pipeline. Obviously, this data is actually Gaussian and does not contain any glitches, so, we used a very
simple glitch model to generate a population of roughly one glitch per hour for the analysis. This consisted of a family of sine-Gaussians with random parameters (central frequency, duration, peak time, amplitude) inserted into the simulated data. In the course of the analysis, all of the standard coherent energies are generated (plus, cross, null and standard likelihood), as well as a timeline of the amount of power in the spherical harmonic coefficients. This allows us to generate scatter plots of the coherent/incoherent energy for each trigger.

In the following scatter plots, $I_{sh}$ corresponds to the power in the zeroth spherical coefficient (i.e. $|c_{00}|$, the incoherent energy), while $E_{sh}$ corresponds to the sum of the $\ell > 0$ coefficients. The energy in the null stream (i.e. the projection of the data in which a gravitational wave signal should be cancelled, $E_{null}$ and $I_{null}$), is computed using the cross-product of the orthonormal antenna factors for the cross and plus-polarisation, see [5].

![Figure 6](image-url)  

**Figure 6.** Scatter plot of the coherent spherical harmonic energy, $E_{sh}$, against the incoherent, $I_{sh}$, for simulated data.

Figure 6 shows such a scatter plot. The bifurcation pattern is the defining feature of these type of plots, and shows good separation between the black crosses of the background (generated using the standard time slides method), and the simulated signals.

5.2. S4 data

In order to test the statistic on real data containing a real glitch population, the same analysis was run over a month of S4 data [3] when three detectors were taking science data. In reality, the network used to take this data was the Hanford-Livingston network (H1L1H2), whereas the network we are interested in is Hanford-Livingston-Virgo (H1L1V1). To simulate this, we ‘relabeled’ the H2 data stream as V1 (i.e. once the H2 data was loaded, treat it as if it had been generated by the Virgo detector). As we were only interested in using the data as a source of noise and glitches, this was sufficient. We note, however, that in reality the H1 and H2 detectors were co-located; meaning noise between the data streams could be correlated, while we are treating them as independent.

Once again, there is a clear separation between the background and the injected signals (Figure 7), but notice that the angle between them has reduced. This is a common feature of
these plots when using real data and appears to be related to the amount of correlation in the noise between different detectors. This is more of an issue here, because there will be coupling between H1 and ‘V1’ (actually the relabelled H2).

In Figure 8 we see a scatter plot of the null energy calculated by SphRad. It is clear that the spherical harmonic power compares favourably to this - the clear separations, the well defined background etc. But one thing to note here is the null energy is calculated for a single sky direction, whereas the spherical harmonic power is all sky, and we get it as a by-product of the way the pipeline works. It can be generated exceptionally quickly and be used as a first pass discriminator of signal/glitch.

Figure 7. Scatter plot of the coherent part of the statistic versus the incoherent part for S4 data. Figure 8. Scatter plot of the coherent and incoherent null energy for S4 data.

Figure 9. The SH statistic for 4000 seconds of data starting at GPS time 793502657.
To demonstrate, Figure 9 shows the plot of spherical harmonic power, $Z_{\text{ratio}}$, for a large stretch of data. There is a horizontal band at $Z_{\text{ratio}} \approx 1.4$ which is used to discriminate glitches and signals. Here, the injected signals have a hrss of $2.4 \times 10^{-21}$ s$^{-1/2}$ and can be seen rising above the band, while the three large, clear, glitches can be seen dropping below the band (at approximately 550s, 1600s and 2900s).

6. Conclusions

The spherical harmonic statistic has a ‘clean’ background, pushing up against the line at approximately $I_{sh} = 0.8E_{sh}$. This means that the parameters of a cut can easily be calculated and applied, it is merely a straight line. As can be seen from comparing Figures 6 and 7 to 8, there is good separation between the injections and the background. What these plots do not demonstrate is the computational cost associated with calculating the cuts - the method of [5] has to loop over different parameters (generating the curve in the log-log space) and polarisations, while the SPHRASt statistic simply has to generate a straight line. Another major advantage of using the spherical harmonic information as a discriminator is that if the sky pointing statistic fails to select the correct sky location, it has no effect, whereas the standard coherent energy can be calculated incorrectly as it is highly dependent on getting the timing information correct.

In summary, we believe this statistic has all the advantages of the standard coherent statistics, but very few of their disadvantages. Timing errors do not have a significant effect on its value, and it strongly discriminates between signals and glitches as shown by Figures 5 and 9.

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