Visualization of logistic algorithm in Wilson model

A S Glushchenko¹, V A Rodin², S V Sinegubov²

¹ Voronezh State University, 1, Lenina Sq., Voronezh, 394000, Russia
² Voronezh Institute of Ministry of Internal Affairs of Russia, 53, Prospect Patriotov, Voronezh, 394065, Russia

E-mail: sinusdvm@mail.ru

Abstract. Economic order quantity (EOQ), defined by the Wilson's model, is widely used at different stages of production and distribution of different products. It is useful for making decisions in the management of inventories, providing a more efficient business operation and thus bringing more economic benefits. There is a large amount of reference material and extensive computer shells that help solving various logistics problems. However, the use of large computer environments is not always justified and requires special user training. A tense supply schedule in a logistics model is optimal, if, and only if, the planning horizon coincides with the beginning of the next possible delivery. For all other possible planning horizons, this plan is not optimal. It is significant that when the planning horizon changes, the plan changes immediately throughout the entire supply chain. In this paper, an algorithm and a program for visualizing models of the optimal value of supplies and their number, depending on the magnitude of the planned horizon, have been obtained. The program allows one to trace (visually and quickly) all main parameters of the optimal plan on the charts. The results of the paper represent a part of the authors’ research work in the field of optimization of protection and support services of ports in the Russian North.

1. Introduction

The mathematical theory of inventory management is one of the major fields of economic and mathematical research, applied since the 1950s. The economic order quantity (EOQ) is a model that is used to calculate the optimal quantity that can be purchased or produced to minimize the cost of both the carrying inventory and the processing of purchase orders or production set-ups. The model was first developed by Ford W. Harris in 1913, but R.H. Wilson, a consultant, is given credit for his in-depth analysis. The economic order quantity (EOQ), obtained in the Wilson’s model, is widely used at various stages of products production and distribution. It is practically useful for making decisions in stocks management, bringing noticeable economic benefits. Currently a large amount of reference materials [1, 2] and computer programs [3] that help to solve various logistics problems are available.

However, the use of universal computer environments is not always justified and requires special user training. It is known that the intensive supply schedule in any logistics model is optimal, if and only if, the planning horizon coincides with the beginning of the next delivery possible.

For all other planning horizons, this plan is not optimal. It is significant that when the planning horizon changes, the plan changes immediately throughout the entire supply chain, thus making it difficult to predict the type of this change.

In this research work, the algorithm of visualization of the optimal plan in a form of a graph, which
changes its appearance depending on the horizon planned, is obtained. This work continues the studies of optimization of protection and support services of Russian North ports (see [4, 5]). Due to the specific climatic and geographical conditions, the timing and delivery plan in the areas of Northern Sea Route can change abruptly. Thus it could be necessary to create (in shortest possible time) a new optimized delivery plan. In this paper, the algorithm of visualization of the graph of optimal plan for such planning horizon is suggested.

The problem of planning horizon does not arise only in logistics. It is common for any long-term planning; therefore it is very important for the strategic management development.

2. Classical model of inventory management

Let \( y(t) \) be the size of supplies stock at a warehouse at time \( t, t > 0 \). While the shortage is not permitted, \( y(t) \geq 0 \) for all \( t > 0 \); goods are in even demand with intensity \( \mu \) (i.e. during time interval \( \Delta t \), a part of reserved supplies \( \mu \Delta t \) is taken from the warehouse and delivered to consumers). At times \( t_0 = 0, t_1, \ldots, t_n \), the stock at a warehouse is replenished with supplies amounts \( Q_0, Q_1, \ldots, Q_n \), respectively. Let \( S \) be the storage fee for the unit of goods per unit of time and \( g \) be the delivery charge for one consignment of goods. Let us assume (for simplification) that it does not depend on the delivery size. Let \( n(T) \) be the number of deliveries that came within time interval \( [0, T) \). It is known (see [1]) that the optimal plan is a tight schedule with the equal intervals between deliveries, where all consignments have the same volume:

\[
Q_0 = \sqrt{\frac{2\mu g}{S}}.
\]

This is the Square Root formula or Wilson’s formula.

The average costs (per time unit) can be expressed as a function of the size of the batch of goods \( Q \):

\[
f(T, y) = g \frac{n(T)}{T} + \frac{\mu ST}{2n(T)} = f_1(Q) = \frac{\mu g}{Q} + \frac{SQ}{2}.
\]

Minimization of the average costs by \( Q \) is a discrete optimization task. The fact is that almost always \( Q_0 \not\in \left\{ \frac{n(T)}{n}; n = 1, 2, \ldots \right\} \); however, non-negative integer \( n_0 \) can be always specified:

\[
Q_1 = \frac{\mu g}{n_0 + 1} < Q_0 \leq \frac{\mu g}{n_0} = Q_2.
\]

3. Algorithm for the optimal plan creation

1. Find \( Q_0 \) using the Square Root formula (1).

2. Find \( n_0 \) from condition (3).

3. Calculate \( f_1(Q) \) using formula (2) for \( Q = Q_1 \) and \( Q = Q_2 \), where \( Q_1 \) and \( Q_2 \) are defined in inequalities (3).

4. The smallest of two numbers \( f_1(Q_1) \) and \( f_1(Q_2) \) is the required minimum. Let us denote this solution as \( Q_{opt} \). The optimal delivery plan is a tight plan, in which the volumes of all supplies are \( Q_{opt} \).

In this case, this optimal plan is usually not realized in practice due to arbitrariness and not
discreteness of the planning horizon. The fact is that almost always \( Q_0 \notin \left\{ \frac{\mu T}{n}; n = 1, 2, \ldots \right\} \). Under the conditions where it is impossible to change the time of the last delivery (i.e. – changing of the weather conditions), it is important to take into account the short-term changes throughout the entire time interval. The changes are shown in the next paragraph in the form of graphs, created with the help of the computer program.

4. The program testing

![Figure 1. The best is one plan](image1)

![Figure 2. Two plans are optimal](image2)
5. Explanation of the program operation
The program for visualization of the models of optimal supplies value and their number (with the concept of the optimal plan in mind) demonstrates a smooth graph change. When the small planning interval is set for one delivery, then the graph of a supplies stock level at a warehouse consists of only one prong (Figure 1). As the planning horizon increases, the size of a prong gradually increases and, at a certain point (Figure 2), transits from one prong to two. At this point, two delivery plans are optimal: one with a single prong and the second with two prongs. During the transition to plans with two prongs, the size of each prong decreases abruptly. With the further increase of planning horizon, the optimal plan is turning into a graph with the two identical prongs, which size increases gradually. Then, at some point (Figure 3), an optimal plan with three prongs becomes effective, in which the size of each prong decreases abruptly (Figure 3) and so on.

Figure 3 shows the principal difference between the operation of the proposed program and the standard accounting software. The upper part of the prong is obtained with the use of a standard program. The lower triangle is obtained with the help of the program obtained. Thus considerable savings have been achieved, both in the process of goods delivery and in storage payment. A significant cost savings has been achieved using small planning horizons and small delivery volumes.

6. Means of computer realization
This combination of means of implementing the integrated development environment of Microsoft Visual Studio 2015 (i.e. the C# language and the “Dot Net Framework” virtual machine) chosen allows one to write graphical interface software without any difficulties.

The program itself consists of the function that takes as an argument a planning horizon and a timer with a period of 0.1 second and a step of 0.01 days (this is about 14 minutes 24 seconds). This allows one to build a smooth visual animation that illustrates the situation on the specified planning horizon.
Figure 4. Development environment

Figure 4 shows the development environment with a part of the program source code. Keeping in mind that the user can use a different computer language over time, the authors do not give a detailed listing of their existing program in C# and the virtual machine “Dot Net Framework”.

7. The program scheme

Figure 5 describes the main parts of the program. The use of timer allows building a smooth and visual graph animation. In case when the planning horizon ends before the entire cycle (Figure 3), a special subprogram is applied.
8. Conclusions
In practice, the input data consist of $s, \mu, g$ parameters (same as in the tested example), the definition of which is given in the second paragraph of this paper. The control parameter is $Q = Q_{\text{min}}$. Here $Q_{\text{min}}$ is the parameter value, which minimizes the costs by formula (2). The program gives out a drawing in the form of Figures 1-3. In terms of the implementation of the program, it can be considered that the decision-maker can select either one delivery (Figure 1) of a large volume or that of two smaller volumes, but for a longer storage term (Figure 2). If the horizon needs to be increased, then incomplete deliveries within the last cycle might be the optimal solution. The diagram allows tracing and comparing fast and visually different types of supplies deliveries. The purpose of the algorithm presented is to provide the operational planning tool, without attracting complex volumetric literature on this subject (Figure 3).

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