The Influence of the Initial Spot Size of a Double Half-Gaussian Hollow Beam on Its Propagation Characteristics in a the Turbulent Atmosphere

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In this paper, by using the Rayleigh-Sommer field theory and the cross-spectral density function, the analytical expression for the intensity distribution of a double half-Gaussian hollow beam in a turbulent atmosphere is obtained. The influence of the initial spot size of this beam on its propagation properties in a turbulent atmosphere is simulated, and the intensity distributions for such beams with different spot sizes are obtained. The results show that the initial spot size has an important influence on the propagation properties in the near field, while this influence in the far field is very weak.

Keywords: Double half-Gaussian hollow beam, Rayleigh-Sommerfield theory, Spot size
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I. INTRODUCTION

A hollow laser beam, for which the center's intensity equals zero, is a special kind of higher-order laser beam. Not only can it be applied in biology, laser processing, atomic cooling, etc. [1-3], but it also has potential applications in the fields of remote sensing, tracking, and long-distance optical communication. Thus the properties of its propagation have aroused extensive attention. For example, the propagation properties of a controllable dark hollow beam and a vortex hollow beam in free space, for which the refractive-index structure constant equals zero, were researched using the ABCD transfer matrix [4, 5]; the propagation properties of an inverse-Gaussian beam were researched based on diffraction integral theory [6]; and the elliptical-symmetry hollow beam and partially coherent hollow beam also was studied [7, 8].

Up to now, hollow laser beams with different intensity distributions were generated by different methods [9-15], and several mathematic models have been used to describe them, such as the TEM01 beam[16], the high-order Bessel beam [17], the Laguerre-Gaussian modes, etc [18]. The common characteristic is that the light intensity of the beam from the edge of the annulus to the center gradually decreases, and according to theory is zero only at the center, which would limit the applications for this kind of beam in invisible detection, invisible control, and so on.

Recently, a new kind of hollow beam, the double half-Gaussian hollow beam, was put forward by the author’s group. The intensity distribution of the vertical cross section of its annulus in the near field is such that the intensity of the entire hollow area is zero, and rises at the inner edge of the annular beam, which turns out to be a step function. The intensity diminishes as a Gaussian function from the inner margin to the outer margin, forming the double half-Gaussian distribution. Due to the steep intensity distribution at the inner edge, it can be used to great effect in the fields of atomic cooling and remote sensing. The propagation properties of a double half-Gaussian hollow beam in free space were researched in [19]. Where a model of the propagation properties in a turbulent atmosphere was constructed, and the influence of the initial spot size of the beam on
its propagation properties was numerical simulated.

II. THEORETICAL ANALYSIS

The optical field of a double half-Gaussian hollow beam at \( z = 0 \) is written as follow: [19]

\[
E_\nu(r,0) = G_\nu \left[ \frac{r^2}{\alpha_0^2} \right]^n \exp \left[ -\frac{r^2}{\alpha_0^2} \right] \times \left[ 1 - \text{circ} \left( \frac{r}{\alpha_0 \sqrt{n}} \right) \right]
\] (1)

where \( G_\nu \) is a constant related to the intensity, \( \omega_0 \) is the initial spot radius, and \( n \) is the order number. The normalized intensity distributions for double half-Gaussian hollow beams when \( n=1, 3, 5, \) and \( 8 \) are shown in Fig. 1, which indicates that the hollow part of the beam grows as the order number increases.

The series expansion for the circ function is written as [20]

\[
\text{circ} \left( \frac{r}{\alpha} \right) = \sum_{j=1}^{\infty} A_j \exp \left( -\frac{B_j}{\alpha^2} r^2 \right)
\] (2)

where \( A_j \) is the expansion coefficient and \( B_j \) is the Gaussian coefficient. Substituting Eq. (1) into Eq. (2),

\[
E_\nu(r,0) = G_\nu \left[ \frac{r^2}{\alpha_0^2} \right]^n \exp \left[ -\frac{r^2}{\alpha_0^2} \right] \times \left[ 1 - \sum_{j=1}^{\infty} A_j \exp \left( -\frac{B_j}{\alpha_0 \sqrt{n}} r^2 \right) \right]
\] (3)

The cross-spectral density function of a double half-Gaussian hollow beam at the source plane is

\[
W_\nu(r_1,r_2,0,z) = \langle E(r_1,0,\omega) E^* (r_2,0,\omega) \rangle
\] (4)

where \( E(r_1,0,\omega) \) is the electric-field component of the beam, \( \langle > \) indicates an ensemble average, \( * \) indicates a conjugate, \( r_1, r_2 \) are position vectors at the source plane, and \( \omega \) is the angular frequency of the beam. Substituting Eq. (3) into Eq. (4), then

\[
W_\nu(r_1,r_2,0) = G_\nu \left[ \frac{(r_1^1)^2 (r_2^1)^2}{\alpha_0^2 \alpha_0^2} \right] \exp \left[ -\frac{(r_1^1)^2}{\alpha_0^2} \right] \exp \left[ -\frac{(r_2^1)^2}{\alpha_0^2} \right] \exp \left[ -\frac{(r_{12})^2}{2\sigma_0^2} \right]
\]

\[
\left[ 1 - \sum_{j=1}^{\infty} A_j \exp \left( -\frac{B_j}{\alpha_0 \sqrt{n}} (r_{12})^2 \right) \right] \left[ 1 - \sum_{j=1}^{\infty} A_j \exp \left( -\frac{B_j}{\alpha_0 \sqrt{n}} (r_{12})^2 \right) \right]
\] (5)

where \( r_{10} = (x_{10}, y_{10}) \) and \( r_{20} = (x_{20}, y_{20}) \) are the coordinates of two correlation points in the source plane. The propagation of a double half-Gaussian hollow beam in turbulent atmosphere follows Rayleigh-Sommerfeld theory:

\[
W(t_1,t_2,z) = \left( \frac{z}{\lambda} \right)^3 \int \int \left[ W_\nu(r_{10},r_{20},0) \exp \left( \frac{ik(R_{10}^2 - R_{20}^2)}{R_{10}^2 R_{20}^2} \right) d^2 r_{10} d^2 r_{20} \right]
\] (6)

where \( t_1 = (x_1, y_1) \) and \( t_2 = (x_2, y_2) \) are the coordinates of two correlation points in the far-field receiving surface, \( k = \frac{2\pi}{\lambda} \) is the wave number, \( \lambda \) is the wavelength. The cross-spectral density function of a double half-Gaussian hollow beam when \( z > 0 \) is

\[
W_\nu(x_{10}, y_{10}, x_{20}, y_{20}, z) = \frac{k^2}{4\pi z^2} \int \int \int \int \left[ W_\nu(x_1, y_1, x_2, y_2, 0) \times \exp \left( \frac{i k}{2z} (x_{10}^2 + y_{10}^2 - x_{10}^2 - y_{10}^2) \right) \times \exp \left( \frac{i k}{2z} (x_{20}^2 + y_{20}^2 - x_{20}^2 - y_{20}^2) \right) \right] d^2 x_1 d^2 y_1 d^2 x_2 d^2 y_2
\] (7)

where

\[
\langle \exp (i k (x_1 x_2 + y_1 y_2)) \rangle = \exp \left( \frac{-1}{\rho_0^2} \left( x_1 - x_2 \right)^2 \right)
\]

\[
= \exp \left( \frac{-1}{\rho_0^2} \left( y_1 - y_2 \right)^2 \right)
\]

\( \rho_0 = (0.545 C_\nu^2 k^2 z)^{\frac{3}{5}} \)

is the coherence length of a spherical wave propagating in a turbulent medium, and \( C_\nu^2 \) is refractive-index structure constant, which represents the intensity of the turbulence. Then we have:

\[
W_\nu(x_{10}, y_{10}, x_{20}, y_{20}, 0) = G_\nu \left[ \frac{(x_{10}+y_{10})^2 (x_{20}+y_{20})^2}{\alpha_0^2 \alpha_0^2} \right] \exp \left[ -\frac{(x_{10}+y_{10})^2}{2\sigma_0^2} \right] \exp \left[ -\frac{(x_{20}+y_{20})^2}{2\sigma_0^2} \right] \exp \left[ -\frac{(x_{20}+y_{20})^2}{2\sigma_0^2} \right] \times \left[ 1 - \sum_{j=1}^{\infty} A_j \exp \left( -\frac{B_j}{\alpha_0 \sqrt{n}} (x_{12}+y_{12})^2 \right) \right] \left[ 1 - \sum_{j=1}^{\infty} A_j \exp \left( -\frac{B_j}{\alpha_0 \sqrt{n}} (x_{12}+y_{12})^2 \right) \right]
\] (8)

FIG. 1. The intensity distribution of double half-Gaussian hollow beam.
where $\sigma_0$ is the spatial coherence length in the source plane ($z = 0$). When $x = x_1 = x_2$ and $y = y_1 = y_2$, the intensity distribution of a double half-Gaussian hollow beam propagating in turbulent atmosphere is

$$W(x_0, y_0, x_2, y_2, z) = W(x_0, y_0, z)$$

$$= G_0^2 \frac{k^2}{4\pi^2 z^2} \frac{1}{\sigma_0^n} \left\{ A_2 + A_3 + A_4 \right\}$$

As hollow beam has circular symmetry, the propagation properties in the $xoz$ plane can represent its complete properties, and the intensity distribution of a double half-Gaussian hollow beam propagating in turbulent atmosphere can be written:

$$W(x_0, 0, x_2, 0, z) = W(x_0, 0, z)$$

$$= G_0^2 \frac{k^2}{4\pi^2 z^2} \frac{1}{\sigma_0^n} \left\{ A_2 + A_3 + A_4 \right\}$$

where

$$A_2 = \sum_{j=1}^{n} A_j$$

$$2n \exp \left[ \frac{i k}{z} \right] \frac{\pi}{4 S_z} \frac{1}{\sqrt{S_z \int (2n-2a) d\lambda}}$$

$$\sum_{b=0}^{2n} \frac{(2n-2a + b) \left( \pi \right)^{2n-2a}}{S_z} \left\{ \frac{1}{\sigma_0^n} \left( \frac{2}{\rho_0^n} \right) \right\}$$

$$\left(2n+b \right) \exp \left[ \frac{S_z^2}{4 S_z} \right] \frac{\pi}{4 S_z} \sum_{b=0}^{2n} \frac{1}{\int (2n+b-2c) d\lambda}$$

$$\left(2n+b \right) \exp \left[ \frac{S_z^2}{4 S_z} \right] \frac{\pi}{4 S_z} \sum_{b=0}^{2n} \frac{1}{\int (2n+b-2c) d\lambda}$$

where

$$a, b, c, and n are positive integers, and a \leq n, b \leq 2(n-a)$$
FIG. 2. The normalized intensity distribution of a double half-Gaussian beam at different distances when \( n = 1 \). (a) \( z = 0.6 \) km, (b) \( z = 1.5 \) km, (c) \( z = 2 \) km, (d) \( z = 4 \) km.

III. THE PROPAGATION PROPERTIES FOR DIFFERENT SPOT SIZES

Setting \( n = 1 \), \( \sigma_0 = 6 \) mm, \( C_n^2 = 10^{-14} \text{m}^{-2/3} \), and \( \lambda = 0.6328 \) µm, and substituting these parameters into the Eq. (10), then the propagation properties are obtained by numerical calculation. The two-dimensional normalized intensity distribution of the double half-Gaussian beam at different distances is shown as Figs. 2 and 3, with spot size \( \omega_0 = 0.02, 0.05, \) and 0.08 mm under the conditions of \( n = 1 \) and \( n = 3 \), respectively.

From Figs. 2 and 3 we can see that the hollow area of the beam increases with increasing spot size (shown in Fig. 2 (a) and Fig. 3 (a)), and with increasing order number. In Fig. 3, the hollow area of the double half-Gaussian beam with smaller spot size (\( \omega_0 = 0.02 \) mm) first begins to turn out the light intensity at \( z = 1.5 \) km (shown in (b)), and then the hollow area of the beams with smaller spot sizes \( \omega_0 = 0.05 \) mm and \( \omega_0 = 0.08 \) mm also turn out light intensity successively (shown in (c) and (d)). The distance need to devolve into a solid beam is different in each case. Comparing these figures, we find that (i) in the near field, the sharp inner edges of the original beam turn Gaussian, and the wider the original hollow region, the longer it takes for the Gaussian beam to form; (ii) the longer the distance, the more the beam starts to resemble a Gaussian one; and (iii) the bigger spot size, the longer the distance and smaller the intensity distribution. After the hollow beam devolves into a Gaussian beam, the spot size extends outward, but still keeps a Gaussian intensity distribution.

Comparing the Figs. 2 and 3, we also find that: the propagation properties can be influenced not only by the spot size, but also by the order number, and the distance need...
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FIG. 3. The normalized intensity distribution of a double half-Gaussian beam at different distance when \( n=3 \). (a) \( z=0.6 \) km, (b) \( z=1.5 \) km, (c) \( z=2 \) km, (d) \( z=4 \) km.

FIG. 4. The center axial intensity distribution of the double half-Gaussian beam.

to devolve into a solid beam will increase as the order number increases.

The center-axial intensity distribution of a double half-Gaussian beam with different spot sizes is shown in Fig. 4. From this figure we can see that the smaller the spot size, the faster the center intensity reaches its maximum value, and that the maximum value decreases as the spot size increases. The differences among the curves of for the center-axial intensity distribution are relatively large in the near field, while the curves gradually tend to overlap as the propagation distance increases, which indicating that the role of the spot size reduces gradually.

IV. FORMATION OF A DOUBLE HALF-GAUSSIAN BEAM

The device used to form a double half-Gaussian beam is shown in the Fig. 5 [21].

The collimated Gaussian beam passes through the center of a spherical reflector, and impinges on a splitting cone, then reflected by the inner surface of the spherical reflector, and then focuses on one point, located on the axis. When that point coincides with the focus of an output coupling lens, a hollow double half-Gaussian beam is formed at the
exit surface of the output coupling lens. The hollow double half-Gaussian beam and its relative intensity distribution are shown in Fig. 6 [19].

V. CONCLUSION

Based on the Rayleigh-Sommerfield theory, the cross-spectral density function is used to deduce the analytical expression for intensity distribution of a double half-Gaussian hollow beam in a turbulent atmosphere, and the propagation properties are obtained by numerically calculating this expression. The results show that the spot size of a double half-Gaussian beam has a significant influence on the distance needed for it to devolve into a solid beam; the bigger the spot size, the longer the distance and smaller the intensity distribution. After the hollow beam has devolved into a Gaussian beam, the spot size extends outward, but still keeps a Gaussian intensity distribution. The difference among the curves of center axial intensity distribution is relatively large in the near field, but the influence role of the spot size gradually weakens as the propagation distance increases, because the axial intensity distribution curves tend to gradually overlap.

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