Research on Real-time Monitoring of Launch Vehicle Deformation Using Fiber Bragg Grating Sensor

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Abstract. To solve the problem that the curvature information of the large flexible rocket body affects the attitude control of the rocket body, a shape reconstruction method based on optical fiber strain sensing is proposed in this paper, which can measure the bow deformation and vibration state of the rocket in real time as the input data for adjusting the attitude control of the rocket. In this scheme, the dynamics of flexible rocket can be solved stably and quickly. In order to verify the reconstruction method, the beam model was used as the experimental analysis object, and the vibration information such as deformation of the beam model was identified through the strain value measured by fiber Bragg grating (FBG) sensors. The results show that the deformation reconstruction method can well restore the deformation and rotation angle of the beam system and has good stability and real-time performance. This method can provide a theoretical basis for the real-time deformation calculation and high-precision attitude control of future flexible launch vehicle.

1. Introduction

To reduce the structural mass and improve the delivery efficiency, the development trend of launch vehicle is to increase the slenderness ratio of the vehicle for launching. However, as the slenderness ratio of the rocket increases, structural vibration and flexibility characteristics become more obvious, leading to inaccurate pose information measured by the attitude sensing device [1]. In addition, low-order elastic vibration modes of the vehicle for launching are easily excited by external interference, resulting in large vibration, which adversely affects the stability of the attitude motion of the rigid vehicle for launching [2].

At present, to reduce the effect exerted by structural vibration on attitude controlling system, notch filters are mostly used in the design of controlling devices to filter low-order modal signals. However, in the design of notch filters, a large range of frequency variation must be considered, and design parameters are usually conservative, leading to low control accuracy. In addition, the notch filter can decrease the phase margin, and it faces difficulty in ensuring that the rigid vehicle for launching and the elastic vehicle for launching have sufficient stability margin at the same time [3]. It is more and more difficult to control the rocket attitude only with attitude information to meet the development needs of launch vehicle, so it is necessary to measure the strain and deformation of the body of the vehicle for launching and introduce the deformation information of the body of the vehicle for launching into the controlling mechanism, to realize the high-precision attitude controlling and stable flight of the rocket.
The fiber Bragg grating sensing device has the characteristics of small volume, light weight, corrosion resistance, electromagnetic interference resistance, miniaturization, embedding composite materials and so on [4], which has become the most promising sensing technology in the field of space sensing. Since the rise of optical fiber sensing technology at the end of last century, NASA, ESA [5] and other space agencies have paid great attention and made great efforts in research. By installing fiber Bragg grating sensing device array on the rocket body to measure the strain of the rocket, the stress and deformation of the rocket structure is yielded in real time, and the attitude and elastic deformation information can be decoupled, to obtain the accurate attitude information of the rocket. Combined with the advanced control algorithm, the accuracy and stability of the rocket attitude controlling can achieve an effective improvement.

The core of rocket deformation reconstruction technology is reconstruction algorithm. In recent years, more and more research has been invested in the development of structural deformation reconstruction technology, which makes it possible to reconstruct the displacement field by discrete strain measurement. At present, such mature structural deformation algorithms mainly fall into three categories: inverse finite element method (IFEM) [6-8], modal method [9-11] and Ko displacement theory [12,13].

The inverse finite element method uses different types of error functions and problem related finite elements to solve the inverse problem of full field shape reconstruction. Shkarayev et al. [6] used the least square algorithm in 2001 to approximately express the load through a polynomial. The coefficients of the polynomial are determined according to the minimum least square error between the predicted strain and the measured strain, and the displacement is predicted by reconstructing the load. Tessler and Spangler of NASA Langley Research Center developed the inverse finite element method based on the shear deformable plate and shell structure in 2003. This method expresses the strain field based on the variational principle of the least square function and reconstructs the deformation of the structure by using the relationship between strain and displacement [7]. Zhao et al. [8] used the inverse finite element method and particle swarm optimization (PSO) algorithm to solve the model to monitor the wing state in real time; The modal method takes the modal superposition method as the main idea, combined with the component strain and displacement information, obtains the corresponding strain mode matrix and displacement mode matrix, obtains the displacement strain conversion matrix based on the mechanical relationship between them, and reconstructs the displacement field of the structure according to the measured strain information. Bernasconi et al. [9] established the normalized modal strain field based on the modal measurement test of early components in 1988. Borgert et al. [10] verified the effectiveness of the modal method by comparing it with the finite element analysis method based on the modal experiment in 2003. Based on the modal method, Kim et al. [11] studied the problem of large deformation of wind turbine blade structure in work, and finally realized the real-time monitoring of blade structure deformation; Ko displacement theory is to reconstruct the deformation of the structure based on the classical beam theory, the component surface strain information, and the discrete idea. A. Derkevorkian et al. [12] conducted deformation reconstruction experiments on wing like plates using Ko displacement theory, and the experimental results were compared with the modal method and finite element analysis values; W. L. Richards et al. [13] monitored the deformation of the beam structure of the UAV, and successfully reconstructed the longitudinal displacement information of the beam by using the strain information data collected in the flight test and Ko displacement algorithm. In addition to the above three main methods, Su et al. [14] based on Euler Bernoulli beam theory, combined with the characteristics of Ko algorithm and modal method, in which the discrete beam modal shape is extracted and represented by using continuously shifted Legendre polynomials, which allows the spatial derivatives of mode shape to represent rotation and curvature modes.

The reconstruction algorithm suitable for rocket deformation reconstruction technology should have the characteristics of high computational efficiency, strong robustness, and small reconstruction error. The inverse finite element method can realize the transformation from the measured strain information of the component to the unknown displacement information when the material attribute parameters and load elements of the component are unknown. However, the inverse finite element method is difficult to construct the inverse element, the element boundary conditions are difficult to obtain accurately, and
the programming of the algorithm is complex and the workload is large; The advantage of modal method is that it can reconstruct the deformation of measured components in a wide area based on a relatively small amount of strain measurement information. However, this reconstruction method is sensitive to the direction of sensor layout, component shape and load loading. Any condition will obviously affect the accuracy of the algorithm; Ko displacement theory can realize the transformation from the measured strain information to the unknown displacement information when the load on the measured component is not clear, but this method requires a large number of sensors under high reconstruction accuracy and can only be applied to simple structures. In comparison, the continuous shifted Legendre polynomial is used to extract and represent the discrete beam modal shape, which can not only restore the vibration information such as deformation, rotation angle and velocity of the tested member under the condition of relatively small amount of strain measurement information, but also has high calculation efficiency and less calculation resources, it is suitable for reverse reconstruction of a large number of strain sensing data on rocket.

For solving the issue of the bend information of large flexible rocket affects the attitude control of rocket, a shape reconstruction method based on optical fiber strain sensor is proposed in this paper, which can measure the bow deformation and vibration state of the rocket in real time. Firstly, this paper reviews the Euler Bernoulli beam theory and the deformation reconstruction method of Legendre polynomial fitting mode based on continuous displacement, and then introduces the FBG sensing principle and the research on measurement error compensation algorithm. Finally, to verify the reconstruction method, the beam system was used as the simulation and experimental analysis object, and the experimental platform is built to identify the deformation value of the beam model through the strain value measured by the FBG sensor. The results show that the deformation reconstruction method can well fit the deformation and velocity of the beam system, has good stability and real-time performance.

2. Principle of Strain Sensing and Error Correction

Fiber Bragg grating is the core component of fiber grating sensor, and its sensing principle is shown in Figure 1.

Using fiber grating sensing theory and its corresponding transmission mode, the FBG reflection wavelength \( \lambda_B \) that meets the Bragg phase matching condition can be expressed as:

\[
\lambda_B = 2n_{\text{eff}}\Lambda
\]  

where \( n_{\text{eff}} \) is the effective refractive index of the optical fiber, and \( \Lambda \) is the grid period of the FBG. From the above formula, the change in Bragg reflection wavelength can be further expressed as

\[
\Delta\lambda_B = 2\Delta n_{\text{eff}}\Lambda + 2n_{\text{eff}}\Delta\Lambda
\]
Obviously, the reflection wavelength $\lambda_B$ of FBG is related to the effective refractive index and grid period of the optical fiber. When the external quantities (such as temperature, strain, etc.) changes, the reflection wavelength of FBG used for measurement will drift. Therefore, by detecting the change of the reflection wavelength of FBG, the external quantities can be measured.

Assuming the temperature keeps constant, the grate period is proportional to the strain induced by axial uniform stress:

$$\varepsilon = \frac{\Delta \Lambda}{\Lambda}$$  \hspace{1cm} (3)

At the same time, strain also changes the density of FBG, which further changes the effective refractive index $n_{\text{eff}}$. The relationship between the effective refractive index change and the strain is:

$$\Delta n_{\text{eff}} = \frac{n_{\text{eff}}^2}{2} \left( \nu (p_{11} + p_{12}) - p_{12} \right) \varepsilon$$  \hspace{1cm} (4)

where $\nu$ is the Poisson's ratio of the fiber, and $p_{11}$ and $p_{12}$ are the photoelastic coefficients of the fiber. To simplify the formula, a variable called FBG photoelastic coefficient is introduced:

$$p_e = \frac{n_{\text{eff}}^2}{2} \left[ (p_{12} - \nu (p_{11} + p_{12})) \right]$$  \hspace{1cm} (5)

$$\varepsilon = \frac{1}{(1 - p_e)} \frac{\Delta \lambda_B}{\lambda_B}$$  \hspace{1cm} (6)

The Bragg wavelength change is proportional to the external stress (for single-mode SiO$_2$ fiber, $p_e = 0.22$). When the fiber grating sensor measures the target test piece, the sensor is often pasted on the measuring point position on the surface of the measured component. Cut out a small section of the fiber grating sensor pasted on the component under test and set the distance of this small section as shown in Figure 2. According to the assumptions in material mechanics, the front and back sections of this interception section are separated from each other. These two sections do not undergo deformation before and after the component is deformed. They only rotate around the neutral plane respectively. The final relative angle formed by the two sections is: So that the short length of $w$ from the neutral layer becomes:

$$\overline{mm'} = (\rho + w)d\theta$$  \hspace{1cm} (7)

Figure 2. Sectional view of components.

Among them, $\rho$ is the radius of curvature of the neutral plane. The original length of $mm'$ is $dx$, and $mm'=oo'$. Since the length of the neutral layer $oo'$ does not change before and after the component is deformed, so:
According to the definition of strain, the strain of short length $mm'$ from the neutral layer is calculated as:

$$\varepsilon = \frac{\overline{mm'} - mm}{mm} = \frac{(\rho + w)d\theta - \rho d\theta}{\rho d\theta} = \frac{w}{\rho}$$

(9)

For the fiber Bragg grating strain sensor with tubular package, its support shall be pasted on the surface of the tested component during operation, as shown in Figure 3. Since the thickness of the tested component is $h$, the bending curvature radius of the tested component is $R$, the height from the circular hole center of the sensor tubular package to the bonding surface is $h_1$ (including the radius of bare optical fiber, the thickness of single-layer protective layer, the thickness of single-layer adhesive layer, the thickness of single-layer outer protective sleeve and the height of outer support), the measured value of the sensor strain is $\varepsilon_{FBG}$, the true value of the measured point strain of the tested component is $\varepsilon_S$, and the compensation coefficient is $\alpha$. When the sensor is pasted on the extended surface of the bending member, the compensation coefficient is:

$$\begin{cases} 
\varepsilon_S = \frac{h}{2R} \\
\varepsilon_{FBG} = \frac{h/2 + h_1}{R} \\
\alpha = \frac{\varepsilon_{FBG}}{\varepsilon_S} = 1 + \frac{2h_1}{h}
\end{cases}$$

(10)

Figure 3. Tube-type packaged fiber Bragg grating strain sensor.

For the fiber Bragg grating strain sensor packaged with substrate, when working, the sensor substrate shall be pasted on the surface of the tested component, and the fiber Bragg grating sensor is embedded in the groove on the substrate surface, as shown in Figure 4. Since the thickness of the tested member is $h$, the bending curvature radius of the tested member is $R$, the optical fiber radius is $r$, the substrate thickness is $h_1$, the grooving depth is $h_2$, the adhesive layer thickness between the substrate and the tested member after pasting is $h_3$, the measured value of the sensor strain is $\varepsilon_{FBG}$, the true value of the measured point strain of the tested member is $\varepsilon_S$, and the compensation coefficient is $\alpha$, when the sensor is pasted on the extension surface of the bending member, the compensation coefficient is:

$$\begin{cases} 
\varepsilon_S = \frac{h}{2R} \\
\varepsilon_{FBG} = \frac{h/2 + h_1 - h_2 + h_3 + r}{R} \\
\alpha = \frac{\varepsilon_{FBG}}{\varepsilon_S} = 1 + 2 \frac{h_1 - h_2 + h_3 + r}{h}
\end{cases}$$

(11)
According to different measurement conditions, select the corresponding parameters and bring them into (10) (11) formula to calculate the corresponding strain compensation coefficient of fiber Bragg grating sensor and compensate the measured strain of the sensor.

3. Deformation Reconstruction Method of Flexible Launch Vehicle

3.1. Euler-Bernoulli Beam Model

According to Figure 5, a rocket model with large aspect ratio can be regarded as a non-homogeneous Euler–Bernoulli beam [9] with free ends, which is affected by gravity, aerodynamics, and the rocket’s own thrust during flight. In structural vibration analysis, its elastic vibration can be described by the following differential equation:

\[
EJ(x) \frac{\partial^2 y(x,t)}{\partial x^2} + m(x) \frac{\partial^2 y(x,t)}{\partial t^2} = P(x,t)
\]  

where \( y(x,t) \) denotes the lateral displacement of the beam model regarding the \( x \)-axis, \( m(x) \) expresses the rocket’s mass distribution function, \( EJ(x) \) is the bending stiffness, and \( P(x,t) \) represents the transverse force under a unit beam length. The below should be met for the free-beam boundary condition:

\[
EJ(0)y''(0) = 0 \quad EJ(L)y''(L) = 0
\]

\[
(EJ(0)y''(0))' = 0 \quad (EJ(L)y''(L))' = 0
\]  

When studying the elastic vibration of flexible rocket, in order for generating an n-degree of freedom approximate differential equation model in terms of the continuous mechanism, the displacement at a certain point of the body of the vehicle for launching is extended to a linear combination of N shape functions, and the shape function of the body of the vehicle for launching approximated by the deformation \( y(x,t) \) is expressed as:

\[
y(x,t) = \sum_{j=1}^{\infty} \varphi_j(x) \eta_j(t)
\]  

where \( \varphi_j(x) \) is the shape of the \( j \)-th mode shape, and \( \eta_j(t) \) is the time-dependent amplitude of the \( j \)-th mode. \( \varphi_j(x) \) can be determined by the finite element approach. Similarly, the rotation angle of the vehicle for launching \( \theta(x,t) \) at a point can be expressed as:

\[
\theta(x,t) = \sum_{i=1}^{\infty} \varphi_i(x) \eta_i(t)
\]  

In terms of a beam vibration mechanism, the finite element model of the Euler–Bernoulli beams is:

\[
M\ddot{X}(t) + C\dot{X}(t) + KX(t) = F(t)
\]  

where \( M, C, \) and \( K \) are the inertia matrix, damping matrix, and stiffness matrix, separately; \( X(t), \dot{X}(t), \) and \( \ddot{X}(t) \) are the displacement, speed, and acceleration of the system, separately; and \( F(t) \) is the excitation force vector.
3.2. Deformation Reconstruction of Flexible Rocket by complying with Legendre Polynomials

Since the mass of the rocket changes and the mass distribution is not uniform during the flight, the mode of the equivalent beam model changes with time, and there is no fixed analytic function. It is necessary to use continuous function to fit discrete mode points. Legendre polynomials are a set of orthogonal sequence functions on \([-1,1]\). Legendre continuous functions are used to approximate the discrete points extracted from theoretical calculations or finite element software. Legendre polynomials are expressed as follows:

$$P_i(x) = \frac{2i+1}{2i+1} \frac{(2i+1)(2x-1)P_i(x) - iP_{i-1}(x)}{i+1} \quad (17)$$

The mode shape \(\varphi_j(x)\) can be fitted by the linear combination of \((m+1)\) Legendre polynomials as follows:

$$\varphi_j = \sum_{i=0}^{\infty} a_{ij} P_i(x) \approx a_{0j} P_0(x) + a_{1j} P_1(x) + \cdots + a_{mj} P_m(x) \quad (18)$$

where \(a_{ij}\) is the Legendre polynomial coefficient. For the first \(n\)-order modes, the approximate values can be fitted by the linear combinations of the first \(m+1\) shifted Legendre polynomials:

$$\Phi(x) = [\varphi_1 \varphi_2 \ldots \varphi_n]$$

$$= [P_0(x) P_1(x) P_2(x) \cdots P_n(x)]$$

$$= A \Phi(x) \quad (19)$$

The matrix \(A\) of feature coefficient is yielded:

$$A = P(x)^{-1} \Phi(x) \quad (20)$$

Once the matrix \(A\) of feature coefficient pertaining to the model is achieved, the nodal rotation mode shape is able to be achieved based on \(P(x)\) derivative:

$$\Phi_\theta(x) = [\varphi_1 \varphi_2 \cdots \varphi_n] = P'(x)A \quad (21)$$

Given kinematics, the tensile strain as impacted by the beam bending \((\varepsilon)\) shows a relation with the nodal displacement:

$$\varepsilon(x,t) = -z_0 y''(x,t) \quad (22)$$
where $z_0$ denotes the distance from beam reference line (here, the beam center axis is adopted to be the baseline) to the of the FBG sensing device position, with the general arrangement on the beam surface for measuring the strain. When $\varepsilon(x,t)$ receives the measurement with FBG sensing devices, the instantaneous modal coordinates $\eta(t)$ are acquired:

$$\varepsilon(x,t) = -z_0 P^*(x) A \eta(t)$$  \hspace{1cm} (23)

If the current instantaneous mode coordinate $\eta(t)$ is obtained, the acceleration, velocity and displacement of the respective beam node are able to be achieved by polynomial $P(x)$ and matrix $A$ of feature coefficient, as shown below:

$$y(x,t) = P(x) A \eta(t)$$

$$v(x,t) = P(x) A \dot{\eta}(t) = \frac{1}{\Delta t} P(x) A (\eta_i - \eta_{i-\Delta t})$$  \hspace{1cm} (24)

$$a(x,t) = P(x) A \ddot{\eta}(t) = \frac{1}{\Delta t^2} P(x) A (\eta_i - 2\eta_{i-\Delta t} + \eta_{i-2\Delta t})$$

Through the insertion of the Legendre polynomials $P(x)$ to (24), angular velocity $\dot{\theta}$, the beam deflection angle $\theta$, and the angular acceleration $\ddot{\theta}$ attributed to vibration is yielded as follows:

$$\theta(x,t) = P'(x) A \eta(t)$$

$$\dot{\theta}(x,t) = P'(x) A \dot{\eta}(t)$$

$$\ddot{\theta}(x,t) = P'(x) A \ddot{\eta}(t)$$  \hspace{1cm} (25)

4. Simulation and experimental verification

In order to verify the effect of the deformation reconstruction algorithm used in this article, this chapter conducts finite element simulation and physical model experiments. In Section 4.1, the finite element model of the cantilever beam was constructed by the finite element analysis software (ANSYS workbench 18.2), which was used to evaluate the appropriate Legendre fitting terms and modal order. In Section 4.2, a rocket deformation reconstruction experimental platform was built, and the beam model was tested under different static and dynamic loads to evaluate its reconstruction accuracy.

4.1. Simulation verification of beam model

The Euler-Bernoulli cantilever beam was used as the simulation object. The model parameters are shown in Table 1. The beam was discretized into 13 units, and one FBG sensor was pasted at the midpoint of each unit, and all sensors were along the center line of the beam surface Paste at equal intervals to convert the displacement and rotation angle variables of the element nodes into strain values. The strain values measured by 13 FBG sensors attached to the surface of the beam were used as observation values to identify the displacement and inclination angle at each point of the beam. The node division and measurement point positions of the cantilever beam structure are shown in Figure 6. The vertical distance from the sensor to the beam reference line was $b/2$.

| Property               | Value   | Unit   |
|------------------------|---------|--------|
| Span, $L$              | 1.000   | m      |
| Cross-section thickness, $b$ | 0.005  | m      |
| Cross-section width, $h$   | 0.020  | m      |
| Material density, $\rho$  | 2667   | kg/m$^3$ |
| Young’s Modules, $E$     | $6.350 \times 10^{10}$ | Pa    |
After establishing the finite element model, it needs to select the appropriate number of Legendre polynomial fitting modes accordingly. Figure 7 uses the mode based on the first 5th and 15th order Legendre polynomials to fit the first-order beam. The blue box represents the discrete modal data obtained through the finite element analysis software. The red solid line represents the Legendre polynomial fitting curve. The fitting effect is in the state of "underfitting" and "overfitting", even if part of the data points are appropriate, the root and tip areas will also see a large difference between the fitting model and the FEM results. The first four-order mode curve fitted by the 11th-order Legendre polynomial is shown in Figure 8, which shows that the fitting effect was good.

Figure 7. Legendre polynomial "underfitting" and "overfitting" first-order modal diagrams
Figure 8. Analytical and finite element solutions for the first four modes of beam and the fitted continuous mode shapes.

By fitting FEM, the characteristic coefficient matrix A of the cantilever beam model is calculated. When the characteristic coefficient matrix A is known, based on the strain data "measured" by FBG, the vibration displacement of the beam model under the action of an external load can be restored.

In order to verify the accuracy of the reconstruction algorithm in the simulation environment, a load of 10g, 20g and 50g was applied at the midpoint of the end of the cantilever beam model, and the beam model displacement obtained by the finite element simulation and the reconstruction algorithm was compared. In the real-time simulation process, the optical fiber sensor measures the bending strain of the beam at a specific position to verify the accuracy of using real-time data to simulate beam deformation. Theoretically, the input of the system should be the measurement data of the optical fiber sensor. In the current study, the transient response of the beam model is used as the "measurement" data. In order to further describe the fitting effect of the rocket body and quantify the reconstruction error of the rocket body, the mean absolute error (MAE) and root mean square error (RMSE) are used to describe the fitting error.

\[
\begin{align*}
\text{MAE} & = \frac{1}{N} \sum_{i=1}^{N} |Y_i - \hat{Y}_i| \\
\text{RMSE} & = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (Y_i - \hat{Y}_i)^2}
\end{align*}
\]

where \(Y_i\) is the simulated real lateral vibration information (displacement and rotation angle) of the beam structure, \(\hat{Y}_i\) is the vibration information identified by the algorithm, and \(N\) is the total number of samples.

Analyzing the data in Table 2 and Figure 9, for the beam finite element model, under the simulation of different loads, the Legendre fitting modal reconstruction method can better restore the displacement and inclination angle, and the average displacement error was at about 5e-05m, the root mean square error was about 8e-05m, and the larger average error of the inclination reduction was about 3e-04°, and the root mean square error is about 4e-04°.

The reconstruction algorithm of the above simulation is based on the first 11-order Legendre polynomial fitting the modal of the first fourth-order beam model. In general, all the reconstruction results are very close to the displacement and angle results of the finite element simulation. But in addition to calculation accuracy, calculation time is also a key factor in the development of rocket real-
time control. In the same computing environment, compare the reconstruction displacement, inclination error and calculation time of the different modal fitting schemes of the beam model which are shown in Table 3. It can be seen from the results that although the higher the number of fitted modalities, the higher the accuracy of the reconstruction, But the corresponding time cost is greatly increased and the reconstruction accuracy is not significantly increased, so it is more appropriate to use the first four-order modal fitting.

Table 2. Simulation calculation accuracy of deformation reconstruction

| Load (g) | MAE_{strain} | RMSE_{strain} | MAE_{disp} | RMSE_{disp} | MAE_{theta} | RMSE_{theta} |
|----------|--------------|---------------|------------|-------------|--------------|---------------|
| 10       | 6.446e-07    | 1.357e-06     | 5.579e-05  | 7.614e-05   | 2.953e-04   | 3.832e-04     |
| 20       | 8.081e-07    | 1.587e-06     | 5.523e-05  | 7.818e-05   | 2.877e-04   | 3.748e-04     |
| 50       | 1.326e-06    | 2.318e-06     | 5.906e-05  | 8.131e-05   | 2.768e-04   | 3.708e-04     |

Figure 9. Fitted strain, displacement, and model angle with the load of 10g, 20g and 50g.
Table 3. Comparison of reconstruction error and calculation time under different modes

| Modal number | $MAE_{\text{strain}}$ ($\varepsilon$) | $MAE_{\text{disp}}$ ($m$) | Cpu-time (s) |
|--------------|--------------------------------------|---------------------------|--------------|
| 3            | 7.225e-07                            | 8.487e-05                 | 0.124        |
| 4            | 6.446e-07                            | 5.579e-05                 | 0.357        |
| 5            | 6.324e-07                            | 5.347e-05                 | 0.579        |
| 6            | 6.298e-07                            | 5.273e-05                 | 0.827        |

4.2. Experimental verification of beam model

The static load and dynamic load experimental tests are carried out on the rocket equivalent beam model to verify the feasibility and effectiveness of Legendre's reconstruction algorithm in the experimental environment. The beam model is made of aluminum alloy. For the convenience of comparison with the simulation data, the specific parameters of the beam model and the position of the measuring point of the FBG sensor are set in section 4.1 (see Table 1 and Figure 6 for details).

An experimental verification platform for the beam model is established, which includes a strain sensing system, a laser displacement sensing system, an IMU sensing system, and an excitation system. The strain sensing system includes FBG sensors, demodulators, and analysis software. 13 FBG strain sensors (initial wavelength range [1531nm, 1554nm]) were placed at different positions along the main beam to obtain strain information on the beam surface. Among them, the parameters of FBG demodulator are as follows:

Table 4. FBG measurement system parameters

| FBG measuring system model | The main technical parameters |
|---------------------------|------------------------------|
| FT1611                    | Working wavelength: 1529–1569 nm |
|                           | Number of optical channels: 16 |
|                           | Measurement wavelength range: 40 nm |
|                           | Resolution: 2 pm               |
|                           | Scanning frequency: 1000Hz     |

The displacement sensing system includes 4 laser displacement sensors, data collector and power supply. The sensors were arranged at $L/4$, $L/2$, $3L/4$, and $L$ along the beam (see Figure 10), and the laser is sent vertically to the surface of the beam. The displacement at the measuring point was calculated to reflect the structural deformation of the beam model, and the displacement was compared with the beam model displacement obtained by the deformation reconstruction. The displacement sensor (Panasonic HG-C1200-P) has a measuring center distance of 200mm, a measuring range of ±80mm, and an accuracy of 20 $\mu$m in its measuring range.

The inertial sensor system includes an IMU sensor and PC-side data receiving software, which measures the inclination angle and angular velocity of the beam model under dynamic load and compares it with the angle and rotation angle obtained by deformation reconstruction, so as to obtain the performance recognition accuracy of FBG. The IMU sensor was located at $3L/4$. In addition, the excitation system is mainly composed of a signal generator, a power amplifier, and a vibration exciter to apply the dynamic load of the specified signal.

For the load test, two different loading conditions are considered (see Figure 11(a) (b)): 1) One end of the beam is fixed, and 10g, 20g and 50g weights are loaded at the end node of the beam; 2) A specific signal is applied to the fixed end of the beam. The exciting force is to simulate the vibration source of the rocket engine.
From the Table 5 and Figure 12, it can be obtained that when the load of 10g, 20g and 50g was loaded under the experimental environment, the average absolute error and the root mean square error were both within 0.001m, and the maximum percentage error was about 3%. The maximum error occurs at the root of the beam. Under static load conditions, the deformation reconstruction...
algorithm used has higher reconstruction accuracy and stability, but the accuracy of the model experiment is worse than that of the simulation, which also shows that the optical fiber actual noise of the sensor and measurement error will affect the accuracy of the reconstruction model.

Figure 12. Comparison of FBG reconstruction and displacement sensor under different static loads.

Table 5. MAE and RMSE under different loads

| Load (g) | $MAE_{disp}$ (m) | $RMSE_{disp}$ (m) | Maximum Percentage Error |
|----------|-----------------|-------------------|---------------------------|
| 10       | 7.874e-04       | 7.233e-04         | 3.1%                      |
| 20       | 7.985e-04       | 7.562e-04         | 3.4%                      |
| 50       | 8.501e-04       | 8.165e-04         | 3.2%                      |

Figure 13. Comparison between FBG reconstruction and displacement sensor under sinusoidal load.
Table 6. MAE and RMSE under different position between 0 and 4 seconds.

| Position (m) | $MAE_{\text{disp}}$ (m) | $RMSE_{\text{disp}}$ (m) | Maximum Percentage Error |
|--------------|--------------------------|--------------------------|--------------------------|
| 0.25         | 1.332e-03                | 1.567e-03                | 5.3%                     |
| 0.50         | 1.736e-03                | 2.325e-03                | 4.2%                     |
| 0.75         | 2.345e-03                | 2.798e-03                | 3.4%                     |
| 1.00         | 2.474e-03                | 2.562e-03                | 4.5%                     |

From Figure 13 and Table 6, it can be obtained that when a sinusoidal excitation signal is loaded at the root of the beam, the average absolute error and the root mean square error of the deformation reconstruction were maintained at about 0.002m, and the maximum percentage error was about 3%-5%. It can be concluded from Figure 14 that under the condition of loading the sinusoidal excitation signal at 750mm of the beam model, the inclination angle obtained by the reconstruction algorithm is basically consistent with the IMU, and the average absolute error and root mean square error were 0.112° and 0.134°, respectively.

Figure 14. Comparison between FBG reconstruction and IMU between 0 and 4 seconds.

5. Conclusions
In order to deal with the problem that the bending information of the large flexible rocket affects the attitude control of the rocket body and improve the control accuracy of the controller, this paper proposes a launch vehicle deformation reconstruction method based on the measurement of the FBG strain sensor. In order to verify the accuracy and real-time performance of the reconstruction algorithm, the beam is used as the experimental object for simulation and experiments. The results show that the first 11-order Legendre polynomial is used to fit the first four-order model of the beam under the simulation conditions. The calculation efficiency is improved under the premise of accuracy; under the experimental conditions, the displacement and deflection angle of the beam can be estimated more accurately under both static and dynamic loads. The proposed scheme provides an effective method for the deformation and reconstruction of flexible rockets. In the subsequent work, multiple degrees of freedom and torsion will be considered to provide a more accurate simulation model of the launch vehicle, and the angular velocity and angular acceleration caused by the reduction vibration will be considered.

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