SPINNING PARTICLE AS A
SUPER BLACK HOLE

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Abstract

A natural combined model of the Kerr spinning particle and superparticle is obtained leading to a non-trivial super black hole solution.

By analogue with complex structure of the Kerr solution we perform a supershift on the Kerr geometry, and then select a ”body”-submanifold of superspace that yields a non-trivial supergeneralization of the Kerr metric with a nonlinear realization of (2,0)-supersymmetry.

For the known parameters of spinning particles this ”black hole” is to be in a specific state without horizons and very far from extreme.

The naked Kerr singular ring has to be hidden inside a rotating superconducting disk, built of a supermultiplet of matter fields.

Stringy wave excitations of the singular ring (traveling waves) yield an extra axial singular line modulated by de Broglie periodicity.

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1 Introduction

The Kerr-Newman BH-solution has paid an attention as a model of spinning particle since 1968 (Carter, Israel, Burinskii, López). Recently, an important role of the Kerr black hole in string theory has been also obtained and there has appeared a point of view that black holes should be treated as fundamental string states [1], [2], or as elementary particles [3]–[6].

On the other hand, the models of spinning particles based on anticommuting parameters are well known.

The aim of this paper is to discuss one very natural way to combine the Kerr spinning particle and superparticle models. The resulting background has a metric of a non-trivial super black hole with a nonlinear realization of supersymmetry [6]. For the known parameters of spinning particles the horizons disappear, and naked singularity has to be covered by a disk-like source built of a multiplet of superfields.

2 Complex shift method

Starting point is a "complex shift method" allowing to generate spinning solutions from spherical symmetrical ones. In complex representation (initiated by Newman, 1974) Kerr solution can be obtained from Schwarzschild one by a TRIVIAL complex SHIFT \((x, y, z) \rightarrow (x, y, z + ia)\).

The Kerr metric \(g_{ik} = \eta_{ik} + 2hk_i k_k\) is defined by the principal null (P.N.) congruence \(k^i(x)\), and can be represented as a retarded-time field generated by a "complex point source" which propagates in complex Minkowski space \(CM^4\) along a complex "world line" \(x'_0(\tau)\), \((i = 0, 1, 2, 3)\), parametrized by a complex time parameter \(\tau = t + i\sigma = x'_0(\tau)\).

The non-trivial twisting structure of P.N. congruence appears on a REAL SLICE as a consequence of nonlinear (light cone) constraints \((x - x_0(\tau))^2 = 0\), where \(x_0(\tau)\) is position of a mysterious "complex source", and \(x\) are points of the real submanifold.

In the gauge \(x^0_0 = \tau\) this equation may be split as a complex retarded-time equation

\[
t - \tau = \tilde{r} = -(x_i - x_{0i})\dot{x}^i_0, \tag{1}
\]

where \(\tilde{r} = r + ia \cos \theta\) is a complex radial distance.
The complex light cone is split into two families of null planes: ”right” ($\psi_R = \text{const}; \bar{\psi}_L - \text{var.}$) and ”left” ($\bar{\psi}_L = \text{const}; \psi_R - \text{var.}$).

The rays of the principal null congruence $K$ of the Kerr geometry are the tracks of these complex null planes (right or left) on the real slice of Minkowski space.

3 Body-slice, and geometry generated by superworldline

Now we would like to generalize this complex retarded-time construction to the case of complex ”supersource” propagating along a super-world-line. By analogy with above construction we consider a TRIVIAL SUPERSHIFT leading to a complex ”supersource”

$$X^i_0(\tau) = x^i_0(\tau) - i\theta\sigma^i\bar{\zeta} + i\zeta\sigma^i\bar{\theta}. \quad (2)$$

Instead of the nonlinear real slice conditions we introduce now the ”BODY-SLICE” constraints

$$(x_i - X_{0i})(x^i - X^i_0) = 0, \quad (3)$$

where $X_{0i}$ are coordinates of ”supersource”, and coordinates $x_i$ belong to a ”body” of superspace, that means a submanifold of superspace where the nilpotent part of $x_i$ is equal to zero. Selecting the nilpotent parts of this equation we obtain the above real slice condition and the extra Body-slice conditions

$$[x^i - x^i_0(\tau)](\theta\sigma_i\zeta + \zeta\sigma_i\bar{\theta}); \quad (4)$$

$$(\theta\sigma_i\zeta + \zeta\sigma_i\bar{\theta})^2 = 0. \quad (5)$$

The eq. (4) may be rewritten by using the spinor null plane parameters $\psi, \bar{\psi}$ in the form

$$(\theta^\alpha\sigma_{i\alpha\dot{\alpha}}\bar{\zeta}^{\dot{\alpha}} \rightleftharpoons \zeta^\alpha\sigma_{i\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}})\psi^\beta\sigma^i_{\beta\dot{\beta}}\bar{\psi}^{\dot{\beta}} = 0 \quad (6)$$

which yields

$$\bar{\psi}\bar{\theta} = 0, \quad \bar{\psi}\bar{\zeta} = 0 \quad (7)$$

which in turn is a condition of proportionality of the commuting spinors $\psi(x)$ and anticommuting spinors $\bar{\theta}$ and $\bar{\zeta}$ providing the left null superplanes to reach B-slice.
Taking into account that $\tilde{\psi}^2 = Y(x), \quad \tilde{\psi}^1 = 1$ we obtain

$$
\tilde{\theta}^2 = Y(x)\tilde{\theta}^1, \quad \tilde{\theta}^\alpha = \tilde{\theta}^1 \tilde{\psi}^\alpha, \quad \tilde{\zeta}^2 = Y(x)\tilde{\zeta}^1, \quad \tilde{\zeta}^\alpha = \tilde{\zeta}^1 \tilde{\psi}^\alpha.
$$

(8)

It also gives that $\tilde{\theta} \tilde{\theta} = \tilde{\zeta} \tilde{\zeta} = 0$, and equation (5) is satisfied automatically.

Therefore, the Body-slice condition fixes a correspondence between the coordinates $\tilde{\theta}, \tilde{\zeta}$ and twistor null planes forming the Kerr congruence.

The retarded time eq. (1) takes the form $t - T = \tilde{R} = \tilde{r} + \eta$, where $\tilde{R} = -(x_i - X_{0i})\tilde{X}_i^j$ is a superdistance. $T = \tau + \eta$ is a supertime containing the nilpotent term

$$
\eta = i\theta \sigma^0 \tilde{\zeta}(\tau) - i\zeta \sigma_0 \tilde{\theta}.
$$

(9)

## 4 Supershift of the Kerr solution

By performing supershift to the Kerr solution one can note that the Kerr solution is a particular solution of supergravity field equations with vanishing spin-3/2 field. The solution with supersource (2) can be obtained from the Kerr solution by supershift

$$
x'^i = x^i - i\theta \sigma^i \tilde{\zeta} + i \zeta \sigma^i \tilde{\theta}; \quad \theta' = \theta + \zeta, \quad \tilde{\theta}' = \tilde{\theta} + \tilde{\zeta},
$$

(10)

which is a ”trivial” supergauge transformation. However, the subsequent imposition of B-slice constraints is a nonlinear operation breaking the original supersymmetry, so the arising spin-3/2 field can not be gauged away. However, there survives a nonlinear realization of (2,0)- supersymmetry.

Starting from tetrad form of the Kerr solution $ds^2 = e^1e^2 + e^3e^4$, where

$$
e^1 = d\xi - Y dv; \quad e^2 = d\tilde{\xi} - \tilde{Y} dv; \quad e^3 = du + \tilde{Y} d\xi + Y d\tilde{\xi} - Y\tilde{Y} dv; \quad e^4 = dv - he^3,
$$

(11)

(12)

(13)

and using the coordinate transformations (10) under constraints (8), and also substitution $\tilde{R} \to \tilde{r}$, one obtains the following tetrad

$$
e'^1 = e^1 + (A - C^1\tilde{\theta}^1)dY, \quad e'^2 = e^2 + Ad\tilde{Y} - B^2 d\tilde{\theta}^1, 
$$

$$
e'^3 = e^3 - C^3\tilde{\theta}^1 dY, \quad e'^4 = dv + \tilde{h}e^3 + dA - B^4 d\tilde{\theta}^1,
$$

(14)

(15)
where \(dY = \tilde{R}^{-1}(Pe^1 - P\gamma e^3)\) and

\[
A = i\sqrt{2}(\theta^1\tilde{\zeta}), \quad B^a = ie^a_\sigma(\zeta\sigma^i\tilde{\psi}), \quad C^a = ie^a_i(\zeta\sigma^i\partial_y\tilde{\psi}),
\]

\[
\tilde{h} = m(Re\tilde{R}^{-1})/P^2.
\]

This is a metric of a NON-TRIVIAL super black hole, that means it cannot be turned into a solution of the Einstein gravity by a supergauge transformation. The non-triviality is provided by the B-slice constraints.

Similar treatment can be carried out for the Kerr-Newman solution and for the Kerr solution generalized by Sen to low energy string theory.

For the parameters of spinning particles \(|a| \gg m\), and this ”super black hole” is to be in a specific state without horizons and very far from extreme. The naked Kerr singular ring has to be covered by a rotating superconducting (and probably also superfluid) disk, built of a supermultiplet of matter fields.

Stringy excitations of the Kerr singular ring (traveling waves) yield an extra axial singular line modulated by de Broglie periodicity.

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