Construction Process Duration Predicted by Statistical Method

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Abstract. Many construction projects today are planned and managed using computer technology. An integral part of the management of these projects is sophisticated software, which includes statistical probabilistic methods. The main task in this area is direct verification of the validity of planned labour productivity values during the construction process according to the recorded average performance values. Using selected statistical methods and analyses, a case study can document this type of undertaking, for example, in a selected masonry process in which the upper and lower limits of performance, i.e. the optimistic and pessimistic bounds, may be calculated with 95% probability. Evaluation of these performance parameters in the construction software used for this study showed a difference in time of 11 days at the end of the process. The figures indicated a 9.6% and 14.3% decrease in labour productivity, respectively, for the optimistic and pessimistic values compared to the construction software’s planned values. Repeated evaluation of performance can aid in improving labour productivity and attaining project milestones and subsequent construction deadlines during the construction process. This paper aims to confirm or refute this theoretical balance using probabilistic statistical methods and to emphasize the importance of statistical analysis in the real construction process with the use of the software.

1. Introduction
Many international authors have historically explored the productivity of individual construction works using construction software or combination stochastic methods with the applications of construction software.

1.1. Construction software
One of the first tasks dealing with this problem was fulfilled by the creation of a model, which looks into a new testing method and assessments [1]. The reduction of the difference between the planned and actual work productivities is one of the following aims how to decrease construction costs [2]. Differentiation among labour productivity deviances has been also frequently discussed [3]. The elaboration of standard performance dates, was being used to predict the duration of the construction process [4]. The similar problem dealing with standard performance is solved by the presentation of the analysis for direct work rate improvement in construction [5]. The EAC index method frequently used abroad is applied for the prediction of the cost over time to reduce it [6][7]. The critical construction delay factors have been determined with the help of statistical methods relative importance index and mean score [8]. The construction delay factors are reviewed in the various analytical and computerized schedule analysis methods [9][10].
1.2. Combination stochastic methods with the applications of construction software

Statistical analysis were used for identification of delays in construction projects. With the help of various observations have been suggested measures that had reduced the delays [11]. The method proposed to construct the distribution function of the SMS/SSMS in a natural way can quantify the reliability of the forecast [12]. Stochastic Project Scheduling Simulation (SPSS), developed to measure the probability to complete a project in a certain time is one of the following methods how to predict the construction time period [13]. Further can be mentioned the analysis exploring the use of the Weibull method in stochastic assessments of a construction project development using the CPI (Cost Performance Index) and SPI (Schedule Performance Index) data files [14]. The linear regression models to predict the construction cost of buildings, based on 286 sets of data collected in the United Kingdom is interesting way of construction process management [15]. Two models for predicting cost overrun percentage in construction projects based on the principle of the regression analysis extends number of the works dealing with this problem [16]. The results revealed a strong linear relationship between cost overrun percentage and the previous 11 causes that significantly affect cost overrun of projects. Parsimonious multiple linear regression (MLR) model for predicting the percentage of cost overruns based on parameters that are known before the contract award phase [17] together with the one-factor ANOVA test present method determine whether construction cost and schedule overruns significantly. The following investigation showed the large, long duration projects to have significantly higher cost and schedule overruns than smaller ones [18]. According to the publications [19], the application of the EAC index method can cause an omission of some important information, related to the volume of the conducted work, by a given construction site management. The result of this research work is an improvement of the traditional EVM (Earned Value Management) method, and the methodology, connected to CPI and SPI, which shows certain limitations in the area of defining the given critical path, work quality and risk impact. Yet other studies that improve performance of the EVM method by implementing the SPC control statistical process were published [20]. Many studies emphasizes the importance of the SPC and EVM integration because of the EVM sensitivity upon discovering abnormal signals, i.e. big differences between the planned and real values in the case of projects that are controlled by software systems [21]. The many systems have been developed to improve the EVM prediction ability of abnormal deviance applying mathematical statistics [22]. Due to poor accuracy of the EVM in predicting project durations and, a new forecasting method is developed based on Kalman filter and the earned schedule method [23]. Because of forecasting total project duration was extensively reviewed and evaluated, earned value (EV) is one another developed methods improving its ability compared to EVM [24]. The control analyzes has been accomplished comparing the efficiency of the Earned Value Management technique and its Earned Schedule extension, as means of forecasting costs and deadlines [25].

The manner of using the stochastic S curve (SS – curve) as an alternative method in relation to the determining S curve is often used to control construction process [26].

| Symbol | Description |
|--------|-------------|
| P      | interval with a 95% probability |
| s      | performance standard deviation |
| n      | number of workers |
| l - a  | confidence of interval prediction |
| α      | significance level |
| X̄     | sample average |
| s²     | sample dispersion |
| z₁−α/2 | selected quantile of the standardized distribution |
| F(x)   | continuous distribution function |
| p      | sample relative frequency |
| π      | relative frequency |
| P₁     | left-sided interval with a 95% probability |
| x₁−α,n−1 | corresponding distribution quantile |
| n - l  | degrees of freedom |
| P(X)   | probability function |

k number of occurrences of unfavourable climatic days

EX mean value

λ mean number of events
t time event

Tn,1 random quantity

µ mean value

H₀ zero hypothesis

H₁ alternative hypothesis

X_OBS monitored value of the test statistics

X̄₁, X̄₂ the averages of sample sets

S₁, S₂ sample standard deviations

n₁, n₂ the ranges of the sample sets

t₁,n₁+n₂−2 degree of freedom

t₁/2,n−1 quantile of the Student’s distribution
SS – curve is based on the cost deviance of individual activities within the frame of suspending the WBS (Work Breakdown Structure) works. This system allows for determining the cost over in the required time. The S – curve envelope [27] consists of two curve and can be used as an early warning system if actual construction process doesn’t correspond to the time schedule. A new method was created by a combination of the EAC index method and statistical analyses [28]. This method eliminates the problem with delaying the signal that indicates the cost being exceeded. An interesting probability model [29] is based on a comparison of the degree of probability of the predicted and, subsequently, implemented performances in the case of projects implemented in the past and similar projects that are currently in the planning stage. The construction time of these comparable future products is then predicted based on a probability calculation applied to the already implemented projects. A correlation analyzing the impact of rainy days on labour productivity is applied on construction site of the Highway Pavement Operations [30]. Another method that models labour productivity based on dynamic approach was used for assessment productivity in construction projects [31]. A way of the construction process management exploring potential factors and having an impact on the process achieved good results [32].

1.3. Simulation technique

Simulation techniques can be considered as alternative solutions, the so-called intermediate construction management development stage between the application of the classic method using construction software and a combination of the statistical methods and construction software [33]. According to the works [34], The index calculation using EAC does not consider the consequences of the given future order risk. That is why a risk management implementation is proposed within the frame of the classic EVM calculation [35][36], modifying the classic solution using a Monte Carlo simulation and the logic fuzzy theory in the form of introducing linguistic variables, implementation of which minimizes the occurrence of possible errors. Internally generated risks in projects is investigated as factor influencing the construction process [37]. Fuzzy approach is used for modelling schedule overrun and cost escalation percentages of highway projects [38]. A fuzzy Mamdani inference method was proposed for the selection of the elements to be evaluated. The model can be helpful mainly for the contractor who wants to know not only the probability of the total cost overrun but also the possibility and amount of increase in the costs of individual packages of works or detailed construction works necessary to complete a construction project. [39], Simulation applications are the subject of the studies [40], which are based on the data recorded at the given construction site that were used for the construction process modelling. This system is based on an analogous principle as the model [41], which was used in the Czech Republic.

1.4. Motivation and aims

The proposed construction process control method is intended primarily for the management of construction processes. Its main aim is to verify the planned mean labour productivity values for bricklaying work on which the time schedule (HMG) is based by performing statistical analyses of recorded worker performance data, directly at the construction site. By collecting real performance data, the course of this work in time and thus the entire construction process can be predicted. The values obtained for labour productivity are compared to the data achieved on similar construction projects (Chapter 3.1.8.), and the possible reasons for drop in performance are then examined. By creating and updating the performance data, the probabilities of the course of construction processes in similar future construction works may be estimated.

The study consists of the following parts: Chapter 2 includes a list of the used statistical methods. Chapter 3 addresses data collection and subsequent tests conducted based on Chapter 2. Discussion 4 discusses the significance of statistical methods when managing construction processes. Chapter 5 Conclusions assesses the conclusions arising from the conducted tests.
2. Method and materials

2.1 Used statistical method

The applied statistical methods will explore the mean value prediction $\mu$ and the corresponding standard deviation $s$ based on the random selection principle. Examination of the sample relative frequency with a 95% confidence interval will provide a percentage prediction of the fulfilment of the mean value range of the volume of the works conducted by individual workers during a one-month period. Then, based on the principle of the Poisson process, a percentage estimate of the probability of occurrence of adverse climatic conditions in the form of days with temperature $< -5 ^\circ C$, which can cause interruptions of the work, will be carried out. Finally, a test of the hypothesis of the estimated mean value and relative frequency will be performed, based on the probability calculation of the occurrence of days of extreme climatic conditions that cause the work interruptions.

2.1.1 Estimation of mean value $\mu$.

When calculating the mean value estimate, it is assumed standard distribution, not knowing standard deviation $\sigma$, i.e. performance deviation of individual workers per work shift, in the case of construction processes where performance of $\geq 30$ workers can be recorded per one day, making sure the procedural condition pursuant to the Lindemberg – Lévy and Moivre – Laplace Theorem is observed. The following formula is used for determining the corresponding interval prediction. (where $\sigma = s$)

\[
P\left(\bar{x} - \frac{s}{\sqrt{n}} \cdot z_{1-\frac{\alpha}{2};n-1} < \mu < \bar{x} + \frac{s}{\sqrt{n}} \cdot z_{1-\frac{\alpha}{2};n-1}\right) = 1-\alpha \tag{1}
\]

\[
\bar{x} = \frac{\sum x_i}{n} \tag{2}
\]

\[
s = \sqrt{s^2} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} \tag{3}
\]

$P$ - the interval with a 95% probability, $s$ - the performance standard deviation of a worker per a given unit, $n$ - the number of workers, performance of whom was measured,

- Kolmogorov – Smirnov test.

Based on the information related to the monitored data, it was decided about a theoretical division of the given population (standard distribution) - see the previous Chapter 2.1.1. Using this test, it can be verified if this prediction is correct. The Kolmogorov – Smirnov test will be applied to test the concurrence of the sample and theoretical distributions. The test is used for verifying the hypothesis that the obtained selection comes from a distribution with a continuous distribution function $F(x)$.

2.1.2 Relative frequency $\pi$.

Percentage information, such as the probability of the fulfilment of the mean value range $\mu$ pursuant to Chapter 2.1.1., is important for the construction work contractors. The selected time interval, during which performance of the workers was measured, was 7 days. The percentage fulfilment of the mean value range $\mu$ was then assessed. When executing the calculation, it is assumed that the condition of the Moivre – Laplace Theorem is fulfilled. To find the 95% confidence interval for the relative frequency, is used the following formula (4) [42]:

\[
P(p - \sqrt{\frac{p(1-p)}{n}} \cdot z_{1-\frac{\alpha}{2}} < \pi < p + \sqrt{\frac{p(1-p)}{n}} \cdot z_{1-\frac{\alpha}{2}}) = 1-\alpha \tag{4}
\]

$p$ - the sample relative frequency of workers fulfilling mean value, $\pi$ - the relative frequency, $n$ - the total number of workers with a performance corresponding to 7 days, $1-\alpha$ - the confidence interval estimation $= 0.95$, $\alpha$ - the significance level, $z_{\alpha}$ - the selected quantile of the standardized normal distribution [43].
2.1.3. Standard deviation confidence interval

The subject of the examination in the given chapter will be a determination of the 95% estimate of the left-sided confidence interval for dispersion and standard deviation of the achieved performance standard average, which can, in a real situation at a construction site, represent the lowest value of the given performance. (5)[42]

\[ P_1 \left( \frac{(n-1)}{x_{1-\alpha, n-1}} . s^2 < \sigma^2 \right) \]  

\( P_1 \) - the left-sided interval with a 95% probability, \( s_2 \) - sample dispersion, \( \sigma \) - dispersion. Reliability of the interval prediction: \( 1 - \alpha = 0.95 \), \( x_{1-\alpha, n-1} \) - the corresponding distribution quantile \( s \) with \( n - 1 \) degrees of freedom [43].

2.1.4. Poisson process. Poisson process was used to assess the probability (calculation of the probability function \( P(X) \)) of the occurrence of adverse climatic conditions during the execution of this work. The process describes the occurrence of random events in a certain fixed time interval. The probability function of occurrence of failures, in our case the number of days with unfavourable climatic conditions, can be then defined by the following formula. (6)

\[ P(X = k) = \left( \frac{\lambda t}{k!} \right) \cdot e^{-\lambda t}, \] \( 0 \leq k < \infty, \ EX = \lambda t \)

\( P(X) \) - the probability function, \( k \) - the number of occurrences of unfavourable climatic days, and \( EX \) - the mean value. As it is clear from formula (6), the mean value of \( EX \) equals to \( \lambda t \). It can be stated that the Poisson distribution parameter is equal to the mean number of events \( \lambda \) during time event \( t \) [42].

2.1.5. Mean value hypothesis test. Determining the mean value hypothesis consists of the following steps:

- **Formulation of null and alternative hypothesis**
  
  When formulating alternative hypothesis \( H_a \), it will be assumed, based on the calculation of the percentage occurrence of days with extreme temperatures pursuant to Chapter 2.1.4, a drop of mean value \( \mu \), resp. worker performance, corresponding to the work suspension caused by unfavourable climatic conditions. Balanced state \( H_0 \) corresponds to the standard hour value pursuant to the CONTEC database. We will thus assume a state where mean value \( \mu < \mu_0 \).

- **Selection of the test statistics**
  
  When determining the tests statistics, it will be assumed of not knowing the standard deviation similarly to the calculation of the standard deviation calculation - see Chapter 2.1.3. (7)[42]

\[ T(X) = T_{n-1} = \frac{\bar{x} - \mu}{s} \cdot \sqrt{n} \rightarrow t_{n-1} \]  

Where \( T_{n-1} \) is a random quantity with \( n \) degrees of freedom, \( s \) is the standard deviation, \( \bar{x} \) is the sample average, \( \mu \) is the mean value, and \( n \) is the number of degrees of freedom (number of workers).

- **Calculation of the monitored value of the test statistics \( x_{OBS} \)**
  
  The calculation will use formula (8), based on which the \( p-value \) will be determined and decision adopted if \( H_0 \) should be accepted or rejected.
2.1.6. Relative frequency hypothesis test. In the case of the construction of the apartment complex in Prague 13, Klementova Street, has been presented a hypothesis that there had been a drop in the relative frequency of the fulfilment of the performance standard, assessed by a subsequent pure significance test, where zero hypothesis $H_0$ represents the value not reduced due to work suspension; alternative hypothesis $H_A$ represents the value that corresponds to the performance reduction caused by unfavourable climatic conditions [42]. The hypothesis specification process consists of the same steps as described in Chapter 2.1.5. The difference is in the selection of the given test statistics.

- Selection of the test statistics.
  We will proceed in accordance with the following formula (8)

$$T(X) = P_2 = \frac{P - \pi}{\sqrt{\pi (1-\pi)}} \cdot \sqrt{n} \rightarrow N(0,1)$$

$P$ - the sample relative frequency, $\pi$ - the relative frequency, $n$ - the number of workers (number of random attempts). The test statistics is then calculated on the basis of the net significance test – $x_{\text{OBS}}$[42].

2.1.7. Interval prediction of the difference of the mean values of two populations. The construction of the apartment complex in Prague 13, Western City, consists of 2 buildings, J12 and J34. Due to the mutual interconnection of the buildings, it is necessary to conduct a statistical comparison of the conducted performances by executing the basic characteristics, such as estimates of the mean values. Explored quantities $\bar{x}_1$ and $\bar{x}_2$ with a range of $n_1$ and $n_2$, and with standard deviations $s_1$ and $s_2$. Random quantity $T$ with Student’s t-distribution with $(n_1 + n_2 - 2)$ and degrees of freedom $t_{n_1 + n_2 - 2}$ looks like this (9)[42]

$$P( \bar{x}_1 - \bar{x}_2 ) - s_p \cdot \left(\frac{1}{\sqrt{n_1}} + \frac{1}{\sqrt{n_2}}\right) \cdot t_{1-\frac{\alpha}{2},n_1+n_2-2} < (\mu_1 - \mu_2) < (\bar{x}_1 - \bar{x}_2) + s_p \cdot \left(\frac{1}{\sqrt{n_1}} + \frac{1}{\sqrt{n_2}}\right) \cdot t_{1-\frac{\alpha}{2},n_1+n_2-2} = 1 - \alpha$$

$\bar{x}_1$ and $\bar{x}_2$ are the averages of sample sets, $s_1, s_2$ sample standard deviations, $n_1, n_2$ are the ranges of the sample sets, $\mu_1, \mu_2$ are the mean values, degree of freedom $t_{n_1 + n_2 - 2}$, $t_{1-\frac{\alpha}{2},n_1}$, $t_{1-\frac{\alpha}{2},n_1}$ are the corresponding quantile of the Student’s distribution with $n - 1$ degrees of freedom [43].

3. Results and discussion
The main subject of the examination is the performance mean value and its deviations when it comes to bricklaying works. Data measurement of individual workers' performance over a given period of time was conducted to determine the range of bricklayer mean values $\mu$ (performances), assuming a 95% confidence interval. By evaluating these random experiments in the form of the daily
3.1. Survey results

3.1.1. Productivity of the brick structures. The performance of workers, a probabilistic estimate of the completion date of the given process was achieved with the option of consequent strengthening of the work-force capacities needed to complete the process in accordance with the originally planned deadlines. Subsequent tests will explore relative frequency, the difference of the mean values and the occurrence probability of days with unfavourable climatic conditions. Furthermore, the hypothesis that assesses performance drops caused by work suspension will be also tested.

Figure 1 illustrates a typical above-ground story of building structure J1 – J2 in Prague, Western City, Klementova Street 3.

Most of the brick structures at SO J34 and J12 are formed by brick lining of the perimeter sheathing, partitions among individual apartments, partitions among apartments and hallways, and internal partitions of individual apartments on above-ground storeys (1st through 7th floor). The locations of individual workers on individual floors is specified in Table 1.

| Storey | Number of bricklayers | Total planned performance m³/shift | Total planned performance m³/week |
|--------|------------------------|------------------------------------|----------------------------------|
| 1st floor | 16                     | 29.8                               | 148.8                            |
| 2nd floor | 16                     | 29.8                               | 148.8                            |
| 3rd floor | -                      | -                                  | -                                |
| 4th floor | -                      | -                                  | -                                |
| 5th floor | -                      | -                                  | -                                |
| 6th floor | -                      | -                                  | -                                |
| 7th floor | -                      | -                                  | -                                |
| Σ        | 16                     | 175.2                              | 3,504                            |

3.1.2. Estimated range of mean value μ incl. standard deviation s.

A. Building structure J34

The performance standard according to the CONTEC database for partition walls is 0.66 m²/h. In the case of external peripheral masonry it is 4.18 m³/h. Based on the assumption of an 8.5 - hour working
shift, the performance per shift in the case of the perimeter masonry is 1.91 m$^3$ of porotherm masonry with the thickness of 300 mm. In the case of erecting partition walls using the performance standard of 0.66 m$^2$/h according to the CONTEC database, the performance per shift should amount to 12.12 m$^2$, which, when converted to m$^3$, amounts to 1.81 m$^3$ per shift. Due to the similarity of the performance standard it will be used the performance per m$^3$ with a mean performance value of 1.86 m$^3$ per shift to facilitate the statistical assessment. Table 2 shows the performance of the workers per shift.

Table 2. Bricklaying works SO J34 – estimated mean value

| Worker | Performance |
|--------|-------------|
| 1      | 1.6         |
| 2      | 1.5         |
| 3      | 1.7         |
| 4      | 1.5         |
| 5      | 1.6         |
| 6      | 1.7         |
| 7      | 1.6         |
| 8      | 1.5         |
| 9      | 1.7         |
| 10     | 1.5         |
| 11     | 1.6         |

| Worker | Performance |
|--------|-------------|
| 12     | 1.7         |
| 13     | 1.6         |
| 14     | 1.7         |
| 15     | 1.6         |
| 16     | 1.5         |
| 17     | 1.7         |
| 18     | 1.6         |
| 19     | 1.7         |
| 20     | 1.6         |
| 21     | 1.7         |
| 22     | 1.6         |

| Worker | Performance |
|--------|-------------|
| 23     | 1.5         |
| 24     | 1.7         |
| 25     | 1.6         |
| 26     | 1.7         |
| 27     | 1.5         |
| 28     | 1.7         |
| 29     | 1.6         |
| 30     | 1.5         |
| 31     | 1.7         |
| 32     | 1.6         |

• **Kolmogorov–Smirnov test**

The objective of this test is to determine if the results of the random tests stated in Table 1 are subject to a standard distribution and if it will thus be possible to proceed in accordance with the given mathematical formulas - see Chapters 2.1.1 - 2.1.7.

• **Selection of the of null and alternative hypotheses**

$H_0: F(x) = F_0(x)$, where $F_0(x)$ is the distribution function of a standard distribution with parameters $\mu = 1.61; s = 0.078$, assuming that the given data come from N (1.61; 0.078$^2$)

$H_A: F(x) \neq F_0(x)$, where $F_0(x)$ is the distribution function of a standard distribution with parameters $\mu = 1.61; s = 0.078$, assuming that the given data do not come from N (1.61; 0.078$^2$)

• **Selection of the test statistics $T(X)$ incl. zero distribution**

Test statistics $D_n$ is defined as a maximal deviation of the theoretical and empirical distribution function. The calculation of the distribution function values and their deviations are stated in Table 3.

Table 3. Calculation of the monitored value $X_{obs}$

| Values | Ord. (i) | (i-1)/n | i/n | F(x) | D for i/n | D for (i-1)/n | D |
|--------|----------|---------|-----|------|-----------|---------------|---|
| 1.5    | 1        | 0       | 0.031| 0.081| 0.05 | 0.081         | 0.081|
| 1.5    | 2        | 0.031   | 0.063| 0.081| 0.018 | 0.05 | 0.05 |
| 1.6    | 21       | 0.625   | 0.656| 0.448| 0.208 | 0.177 | 0.208 |
| 1.7    | 22       | 0.656   | 0.688| 0.875| 0.187 | 0.219 | 0.219 |
| 1.7    | 23       | 0.688   | 0.719| 0.875| 0.156 | 0.187 | 0.187 |
| 1.7    | 30       | 0.906   | 0.938| 0.875| 0.063 | 0.031 | 0.063 |
| 1.7    | 31       | 0.938   | 0.969| 0.875| 0.094 | 0.063 | 0.094 |
| 1.7    | 32       | 0.969   | 1    | 0.875| 0.125 | 0.094 | 0.094 |

$X_{obs} = 0.219$
Calculation of p-value:

\[ H_A: F(x) \neq F_0(x) \]

\[ p-value = 2 \cdot \min \{ F_0(x_{\text{OBS}}) - F_0(x) \} \]

\[ F_0(x_{\text{OBS}}) = F_0(0.219) \]

\[ F_0(0.219) < 0.75 \]

\[ 0.25 < 1 - F_0(0.219) \]

\[ p-value > 0.5 \text{ see Table 4 [43]} \]

Conclusion - p-value > 0.5, which means we do not reject the zero hypothesis, i.e. the data in the form of worker performances are subject to a standard distribution. Calculation of the sample characteristics can be conducted in compliance with Chapter 2.1.1.

Sample characteristics:

Sample average: \( \bar{x} = \frac{\sum_{i=1}^{12} x_i}{n} = \frac{(1.6+1.5+1.7+1.5+1.6+1.7+\ldots)}{32} = 1.605 \)

Sample standard deviation: \( s = \sqrt{s^2} = \sqrt{0.0061} = 0.078 \)

Sample distribution: \( s^2 = \frac{\sum_{j=1}^{n} (x_i - \bar{x})^2}{n-1} = \frac{(1.6-1.61)^2 + (1.5-1.61)^2 + \ldots + (1.7-1.61)^2}{32} = 0.0061 \)

\[ P \left( \bar{x} - \frac{s}{\sqrt{n}} \cdot Z_{1-\frac{\alpha}{2}} < \mu < \bar{x} + \frac{s}{\sqrt{n}} \cdot Z_{1-\frac{\alpha}{2}} \right) \]

(10)

\[ P \left( 1.61 - \frac{0.078}{\sqrt{32}} \cdot 1.96 < \mu < 1.61 + \frac{0.078}{\sqrt{32}} \cdot 1.96 \right) = 1 - \alpha \]

\[ 1.582 < \mu < 1.637 \]

Evaluation

Based on the calculation of a two-directional interval with a 95% confidence, can be stated that the estimate of the mean value is within the range of 1.582 m³ and 1.637 m³, with the stipulation that the median of the range applied to the CONTEC database is lower by 13.5%. To eliminate this performance drop, it is proposed to strengthen the capacity by 2 bricklayers, who will be assigned to each floor of a typical story of the minimal work queue.

A. Building structure J12

To implement the brick lining of the perimeter sheathing and internal partitions, the same number of workers as in the previous case will be used, but has been deployed different groups, that’s why is to expect the various range of the mean value can be achieved in the Building structure. Following calculations proved this assumption, thus was achieved different interval range of the mean value of the performance. Table 4 shows the performance of the workers per shift.

| Table 4. Bricklaying works SO J12 – estimated mean value |
|----------------------------------------------------------|
| Worker | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| Perform. | 1.6 | 1.5 | 1.4 | 1.5 | 1.6 | 1.4 | 1.6 | 1.5 | 1.3 | 1.5 | 1.6 |
Worker 12 13 14 15 16 17 18 19 20 21 22
Perform. 1.4 1.6 1.7 1.6 1.5 1.7 1.6 1.6 1.4 1.6

Worker 23 24 25 26 27 28 29 30 31 32
Perform. 1.3 1.7 1.6 1.6 1.7 1.5 1.4 1.6 1.5 1.7

Sample characteristics:

Sample average: \( \bar{x} = \frac{\sum_{i=1}^{12} x_i}{n} = \frac{(1.6+1.5+1.4+1.5+1.6+1.4+1.3+1.7+1.6+1.6+1.4+1.5)}{32} \)

\[ \bar{x} = 1.53 \]

Sample standard deviation: \( s = \sqrt{s^2} = \sqrt{0.0061} = 0.091 \)

Sample distribution: \( s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1} = \frac{(1.6-1.61)^2+(1.6-1.61)^2+...+(1.7-1.61)^2+(1.7-1.61)^2}{32} = 0.0083 \)

\[ P \left( \frac{\bar{x} - s}{\sqrt{n}} \cdot z_{1-\alpha} < \mu < \bar{x} + \frac{s}{\sqrt{n}} \cdot z_{1-\alpha} \right) \]

\[ P \left( 1.53 - \frac{0.091}{\sqrt{32}} \cdot 1.96 < \mu < 1.53 + \frac{0.091}{\sqrt{32}} \cdot 1.96 \right) = 1 - \alpha \]

\[ 1.498 < \mu < 1.562 \]

Evaluation

Based on the calculation J12 in compliance with Chapter 2.1.1, the estimation of the mean value between 1.498 m³ and 1.562 m³, with the stipulation that the median of the range applied to the CONTEC database is lower by 18%. To comply with the partial deadlines related to this building, it is not only necessary to increase the number of workers from 16 to 18, but also to increase the overtime work standards, which will compensate for small deviations of the given work performance.

3.1.3. Estimate of the standard deviation confidence interval. This information, implementing in computerized system CONTEC can be used by the construction contractor as the pessimistic variant of the worker performance per shift with a 95% probability, i.e., in our case, the lowest performance conducted by 1 worker per shift. The right-sided deviation would represent the highest performance with a 95% confidence. Left-sided deviations will be calculated for SO J34 and SO J12.

B. Building structure J34

We base our assumption on the results of the calculations of the performance mean value range of bricked structures - see Chapter 3.1.2. Table 2. The calculation will be based on mathematical formula (5) (see Chapter 2.1.3), with the stipulation that \( s^2 = 0.0061, n = 32 \). We base our assumption on the results of the calculations of the performance mean value range of bricked structures.

\[ P_f \left( \frac{(n-1)}{x_{1-\alpha,n-1}} \cdot s^2 < \sigma^2 \right) = 1 - \alpha \]

\[ P_f \left( \frac{32-1}{19,3} \cdot 0.0061 < \sigma^2 \right) = 1 - \alpha \]

\[ P_f (0.0098 < \sigma^2) = 1 - \alpha \]

\[ (0.099 < \sigma) = 1 - \alpha \]
• **Evaluation**

With a 95% confidence can be stated that the maximal left-sided standard deviation from the sample average is 0.099 m³/sm. This information can be used by the construction contractor for planning the latest acceptable deadlines for commencing the following processes using its implementation in the CONTEC construction software [44].

3.1.4. **Relative frequency.** Relative frequency is an expression of the probability of the given phenomenon based on the Bernoulli’s principle [42]. In the given chapter, we will examine relative frequency of the fulfilment of the performance mean value range of the bricklaying works for a period of 7 days (3 shifts), which represents about 50% of the expected time needed for executing the minimal work queue of 2 typical storeys of this construction process, assuming a 95% validity of the confidence interval.

A. Building structure J34

Using mathematical formulas (4) from Chapter 2.1.2. it will be investigated, with a 95% confidence the percentage fulfilment of the estimate of the performance mean value range by a work squad over 7 days (3 shifts). Successful experiments are marked (U), unsuccessful (N) see Table 5.

| Table 5. Building Structure J34 – Relative frequency |
|-----------------------------------------------|
| Work. Nr. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| Exper. N/U | U | U | U | N | U | U | U | U | U | U | U |

| Work. Nr. | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
| Exper. N/U | U | U | U | U | U | U | U | U | U | U | U |

| Worker No. | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 |
| Experim. N/U | U | U | U | U | U | U | U | U | U | U |

• **Selection file**

\[ \begin{align*}
    P &= \frac{29}{32} = 0.91 \\
    n &= 32 \\
    P(0.91 - \sqrt{\frac{0.91(1-0.91)}{32}} \cdot 1.96 < & \text{U} < \sqrt{\frac{0.91(1-0.91)}{32}} \cdot 0.91 + \sqrt{0.91(1-0.91)} \cdot 1.96) = 0.95 \\
    P(0.91 - \sqrt{0.0025} \cdot 1.96 < & \text{U} < 0.91 + \sqrt{0.0025} \cdot 1.96) = 0.95 \\
    0.817 < & \text{U} < 1.008 \\
\end{align*} \]

• **Evaluation**

Percentual probability of the fulfilment of the estimate of the performance mean value range by a work squad is between 81.7% and 100.8%, with the stipulation that the median of this range is 91.2%.

B. **Building structure J12**

The calculation process is identical as in previous case of the Building structure J34. In the Table 6 are recorded successful (U) and unsuccessful (N) experiments.
Table 6. Building Structure J12 – Relative frequency

| Work. Nr. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|-----------|---|---|---|---|---|---|---|---|---|----|----|
| Exper. N/U| U | U | U | U | U | U | U | U | U | U | U |

| Work. Nr. | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
|-----------|----|----|----|----|----|----|----|----|----|-----|----|
| Exper. N/U| U  | U  | U  | U  | U  | U  | U  | U  | U  | U   | U  |

| Worker No. | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 |
|------------|----|----|----|----|----|----|----|----|----|----|
| Experim. N/U| U | U | U | U | U | U | U | U | U | U |

- **Selection file**
  
  \[ p = \frac{30}{32} = 0.94 \]
  
  \[ n = 32 \]
  
  \[ P(0.94 - \sqrt{0.94(1-0.94)} \cdot 1.96 < \mu_1 - \mu_2 < 0.94 + \sqrt{0.94(1-0.94)} \cdot 1.96) = 0.95 \]
  
  \[ P(0.94 - \sqrt{0.0018} \cdot 1.96 < \mu_1 - \mu_2 < (0.94 + \sqrt{0.0018} \cdot 1.96) = 0.95 \]
  
  \[ 0.856 < \mu_1 - \mu_2 < 1.023 \]

- **Evaluation**

  Percentual probability of the fulfilment of the estimate of the performance mean value range by a work squad is between 85.6 % and 102.3 %, with the stipulation that the median of this range is 93.9 %.

3.1.5. **Interval prediction of the difference of the mean values** of two populations. When implementing the bricklaying structures of the perimeter sheathing and internal brick linings in SO J12, completely different bricklayer work squads were used compared to SO J34. It can be thus assumed that different mean value ranges of the performance of individual workers in these buildings will be recorded. This fact was demonstrated by executing statistical estimates - see Chapter 3.1.2. It is thus important to compare these data in order to ensure compliance with the commencement of the PSV works pursuant to the given time schedule. To calculate the difference of the mean values of two populations will be used mathematical formulas (10) from Chapter 2.1.7.

- **Selection file**

  \[ n_1 = 32, s_1^2 = 0.0061, n_2 = 32, s_2^2 = 0.0083, \bar{x}_1 = 1.61, \bar{x}_2 = 1.53 \]
  
  \[ s_p = \sqrt{\frac{(32-1)^2 \cdot 0.0061 + (32-1)^2 \cdot 0.0083}{32+32-2}} = \sqrt{\frac{5.862+7.976}{62}} = 0.472 \]
  
  \[ P(1.61 - 1.53) - 0.472 \cdot (\sqrt{\frac{1}{32} + \frac{1}{32}} \cdot 1.998 < (\mu_1 - \mu_2) < (1.61 - 1.53) + 0.472 \cdot (\sqrt{\frac{1}{32} + \frac{1}{32}} \cdot 1.998) = 0.95 \]
  
  \[ P(0.08 - 0.472 \cdot 0.25 \cdot 1.998) < (\mu_1 - \mu_2) < (0.98 + 0.472 \cdot 0.25 \cdot 1.998) - 0.156 < (\mu_1 - \mu_2) < 0.316 \]
• Evaluation

When the difference of the mean value ranges \((\mu_1 - \mu_2) < 0\), we can state that the mean value of the worker performance in SO 34 is < SO J12. When the difference of the values \((\mu_1 - \mu_2) > 0\), the mean value in SO 34 is > SO J12. The calculations conducted in compliance with Chapter 2.1.7 demonstrated that the interval reaches negative as well as positive values. That is why it cannot not be unambiguously determine, which of the performance mean values in SO J34 and in SO J12 represents a statistically higher value. Negative values represent 33% of the range of the interval estimate of the difference of the mean values, therefore it is to expect that the bricklaying works in SO J34 will reach higher mean values than in SO J12.

3.1.6. Climatic influences on labour productivity. In this chapter it will be estimated the climatic impacts on labour productivity, or more precisely, the impact on the magnitude of the range of mean value \(\mu\) during the winter months. In certain cases we can take measures with help of which is possible to continue construction works. A calculation of the probability of the occurrence of days with unfavourable climatic conditions, i.e. days with temperatures < - 5 °C, which can result in work suspensions, will be executed.

A. Probability of the occurrence of days with unfavourable climatic conditions

The Poisson process was applied for calculating the probability of the occurrence of unfavourable climatic conditions. The process describes occurrence of random events over a fixed time interval \([42]\). The subject of the calculation of the occurrence of days with temperatures below 5 °C will be a time period between December 2020 and March 2021. Work suspensions ≥ 2 days will be assessed, which is the expected number of days with unfavourable climatic conditions in 12/2020 based on the meteorological office reports for the last 5 years. We will proceed according to mathematical formula (6) stated in Chapter 2.1.4.

• Mean value

\[ EX = \lambda t = \frac{13}{4} = 3,25 \]

• Probabilistic function

\[
P(X \geq 2) = 1 - P(X < 2) = 1 - [P(X = 0) + P(X = 1)] = 1 - \sum_{k=0}^{1} \frac{(\lambda t)^k e^{-\lambda t}}{k!} = 1 - \frac{3,25^2 \cdot 2,718^{-3,25}}{2!} = 80,8\% \]

• Evaluation

The given calculation demonstrated that the occurrence probability ≥ 2 days in one of the months between 12/2017 and 3/2018 is 80.8%. It is thus necessary to eliminate the drop of the performance mean value caused by work suspensions that last for up to 10% of the number of the work shifts/month by increasing the work capacity by 2 brick-layers.

3.1.7. Relative frequency hypothesis test. Based on the calculation the probability of occurrence of unfavourable climatic conditions, see Chapter 3.1.6., Paragraph A), it was stated that there will be more that 2 days in the months of December 2020 to March 2021, with a probability of 80.8%, with a temperature of < - 5 °C, which means that it will not be possible to work for about 10% of the total working time per month (20 shifts/month). Relative frequency of the fulfilment of the range of mean value \(\mu\) for SO J34 is between 81.7% and 100.8% with the range median at 91.2%. Theoretically, the median of the relative frequency range can drop by 10% from 91.25% to approximately 82.1% due to work suspensions ≥ 2 days, caused by unfavourable climatic conditions. This drop will be verified by a
pure performance test, determining if it is statistically significant information that requires a capacity increase.

- **Evaluation**
  
  Based on the calculation of the \( p - \text{value} = 0.035 \) using the pure significance test [42], the zero hypothesis was not rejected. As a result of this, the extent of the examined population had to be expanded by additional monitoring of yet another cycle of 32 random tests (3 shifts) with an identical result of the relative frequency range. Based on the measured extent \( n = 64 \), the value of \( x_{\text{OBS}} = -2.57 \) with the \( p - \text{value} = 0.005 < 0.01 \), [43] which means that the drop of mean value \( \mu \) of the relative frequency range from 91.2% to approximately 82.1% is statistically significant. That is why it is necessary to strengthen the bricklaying capacities from 18 to 20 workers in December 2020, thus making sure the original construction completion deadline is complied with.

3.1.8. **Determination of the optimistic and pessimistic values for construction.** The highest and lowest labour productivity values for individual construction processes in accordance with the test performed to estimate the left-hand and right-hand intervals with 95% reliability (Chapter 3.1.3.) were gradually processed by the CONTEC construction software, thereby determining the longest and shortest execution times for the bricklaying work process.

- **Evaluation**
  
  By processing the worker performance data in the CONTEC construction software according to the left-hand and right-hand intervals (pessimistic vs optimistic variant), it was shown that the difference in duration between the two process variants was 11 days. It should be emphasized that the optimistic labour productivity value is 9.6% lower than the planned value according to the CONTEC time standards database. In the case of the pessimistic variant, this value is 14.3% lower, which again confirms the need to strengthen the workforce capacity.

3.1.9. **Comparison of sample averages of construction processes.** Bricklaying and plastering work are among the most demanding construction activities in terms of labour intensity, which is why labour productivity in these processes has been monitored on construction sites for a long time.

- **Bricklaying work**
  
  In this chapter, recent works on masonry structures for various types of buildings, mostly residential, are compared. The average performance of a bricklaying worker according to the amount of bricks laid in \( \text{m}^3 \) per shift was monitored on an ongoing basis. This data is one of the basic data which forms the selection characteristics for estimating the range of the mean value \( \mu \). According to the terminology (Chapter 2.1.1.), this is referred to as the sample average \( \bar{x} \). The compiled graph (Figure 2) charts a comparison of sample averages, i.e. median worker performance achieved in individual projects. The graph shows the highest sample average of 1.75 \( \text{m}^3/\text{sm} \) was achieved at the Jégho alej construction site in Bratislava, which is 7% lower than the performance value in the CONTEC database. A high sample average such as this is due to the size of the building and straightforward layout of individual flats. Similar labour productivity was achieved at a housing project in Prague Prosek, where the achieved average was 13% lower than the CONTEC database standard time value. The difference between the mean values of these projects was due to the adverse climatic conditions of 2006, which experienced a long-lasting winter which affected the value of the sample average \( \bar{x} \). In the Západní Město housing project in Prague, the lack of a qualified labour force as a factor is already beginning to show. It has unfortunately been an emerging problem in the Czech construction industry in recent years. Regrettably, this type of work is executed by foreign workers whose professional qualifications do not correspond with the required standards. Although the financial volume and dispositional repeatability of the housing project has the prerequisites to achieve similar sample averages as those achieved in the two above-mentioned construction projects, the project has experienced a 15% reduction compared to the CONTEC...
database standard time values. In the case of a smaller housing project located in Vsetín, the sample average showed a reduction of 20% in labour productivity compared to the CONTEC database time standards. This drop is attributable to the relatively small financial volume and rather complex internal layout. For comparison, the results of the sample average values achieved during the construction of the Pavilon péče o Matku a dítě in FNSP Ostrava, which represents a different type of construction but with a high demand on the quality of work performed, are presented. The internal layout of this construction project is not as complex as the Západní Město housing project in Prague, however the size of the project is significantly smaller. Comparable performance was achieved here with a sample average only 1% higher than in the case of the Západní Město housing project in Prague.

![Comparison of sample averages](image)

**Figure 2.** Comparison of sample averages–masonry structures

4. Discussion
Capital construction management and modelling of capital construction processes are key areas in any economy. An integral component in this process is the planning model for the course of capital construction, which in the construction industry is now often created with the use of sophisticated software which incorporates standard time values in its database. The main theoretical basis of this type of software is probabilistic methods which allow us to monitor the effectiveness of the construction process through selected statistical methods. Software modelling and the use of set models allows us to verify work productivity and construction deadlines during the given construction process in real time. By calculating the relative frequency of individual construction processes, the probability of completion of the entire project by a certain time can then be obtained as a percentage value. The purpose of this paper is to examine the topic of how effectively statistical methods can contribute not only to improving but also shortening the construction process and how we can incorporate these methods into practice. Systematic monitoring of labour productivity recorded during each construction process creates and continuously updates a set of real performance data which lets us predict, both technologically and typologically, the probability of completion of construction of similar projects.

5. Conclusion
Using selected statistical methods in combination with the sophisticated construction software CONTEC, the paper examined a case study of the management of a selected construction process. The study applied the principle of direct collection and evaluation of data during a given activity on-site, i.e.
by taking into account real performance at the construction site. This method would have been a major advantage in the given project’s entire construction process compared to management of the project’s construction using software only based on a database of standard time values which, in many cases, were not met. Another advantage of the proposed method in managing capital construction is the usefulness of static analyses conducted at the beginning of the construction process in order to allocate sufficient labour to satisfy the expected deadlines. The pessimistic and optimistic values of the monitored work obtained by calculating the estimated left-hand or right-hand intervals with 95% reliability were then retroactively incorporated into the construction software to obtain the shortest or longest execution times for the given construction processes. Incorporation of these performance parameters into the construction software indicated a difference of 11 days to complete the process. Compared to the construction software’s planned value, the optimistic and pessimistic labour productivity values were 9.6% and 14.3% lower, respectively. A relative frequency test showed 91.2% fulfilment of the range of the mean value in SO J34 and 93.9% in SO J12. Calculation of the occurrence of days with unfavourable climatic conditions between 12/2020 and 3/2021 indicated an 80.8% probability of interruption to work which would exceed two days. To ensure completion of the masonry structures by the planned date, four additional workers would need to have been hired to supplement the original 20. This hypothesis was confirmed with a relative frequency test. Continuous monitoring and evaluation of the effectiveness of construction activities can significantly contribute to attaining or even shortening construction times and thereby save associated costs. Current construction and project management methods used in capital construction can fully apply modern concepts based on theoretical probabilistic modelling of the construction process in real time.

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