D-Brane Probe and Closed String Tachyons

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ABSTRACT

We consider a D-brane probe in unstable string background associated with flux branes. The twist in spacetime metric responsible for the supersymmetry breaking is shown to manifest itself in mixing of open Wilson lines with the phases of some adjoint matter fields, resulting in a non-local and nonsupersymmetric form of Yang-Mills theory as the probe dynamics. This provides a setup where one can study fate of a large class of unstable closed string theories that include as a limit type 0 theories and various orbifolds of type II and type 0 theories. We discuss the limit of \( C/Z_n \) orbifold in some detail and speculate on couplings with closed string tachyons.

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1 Introduction and Summary

Recently, study of unstable string theories \cite{1, 2, 3, 4, 5, 6} has added important new understanding of string theory in general. General philosophy is nothing new. One looks for tachyonic degrees of freedom in perturbative string sector and interprets its condensation as a sign that the unstable theory is moving toward a more stable theory. In some cases, such as open string sector associated with unstable D-branes, there are fairly convincing arguments that the end point of the condensation is a supersymmetric vacuum. These studies were possible in part because open strings per se do not contain gravitational degrees of freedom. The background geometry of spacetime, for instance, may be regarded as given, and one studies a field theory of some sort in that given background.

When it comes to closed string theories with tachyonic degrees of freedom, things are more involved. Since closed strings contain gravity at tree level, condensation of tachyon induces immediate reaction by the spacetime geometry, study of which seems considerably more difficult. A recent work by Adams et. al.\cite{7} tried to alleviate this problem by considering unstable closed string theories, but with tachyons living in lower dimensional subspace, namely at orbifold fixed points. Effect of tachyon condensation on spacetime geometry is studied via D-brane probe analysis and also from beta function analysis, which seems to point toward a supersymmetric string theory in a flat background as the end point. The models they considered are nonsupersymmetric and noncompact orbifolds of 2 and 4 dimensions.

In this note, we would like to offer D-brane probe analysis of a continuous family of unstable closed string theories, which include the orbifolds of Adams et. al. as special limits \cite{3, 9}. Consider type II string theory on an orbifold of type

$$\mathbb{R}^{1+6} \times (\mathbb{R}^2 \times \mathbb{R}^1)/\mathbb{Z}, \quad (1)$$

where the integer group acts as a rotation on $\mathbb{R}^2$ and a shift on $\mathbb{R}^1$. Denoting the coordinates on each factor as $re^{i\phi} = x^7 + ix^8$ and $y = x^9$, we define the action as

$$\phi \leftrightarrow \phi + 2\pi \epsilon, \quad y \leftrightarrow y + 2\pi R. \quad (2)$$

The string spectrum in such background has been analyzed by Russo and Tseytlin \cite{10}, who found tachyonic degrees of freedom for sufficiently small $R$. Type IIA string in this background is related to type IIA string on the so-called Melvin universe via
9-11 flip, and is also related to type 0 theory \cite{11, 12, 13}. For instance, \( R \to 0 \) limit with \( \epsilon = 1 \) is supposed to yield a type 0 string theory in flat 10D background.

In order to keep an angular coordinate of period \( 2\pi \), we may introduce a new coordinate by \( \phi = \theta + b y \) with \( b \equiv \epsilon / R \). With respect to the new coordinate system, the metric of the spacetime is

\[
-(dt)^2 + (dx^1)^2 + \cdots + (dx^6)^2 + dr^2 + r^2(d\theta + b dy)^2 + (dy)^2,
\]

which is locally flat.

According to the perturbative string spectrum of these models \cite{10}, tachyons reside in winding sector(s) along \( y \). The fact that tachyons are in winding sectors also implies that tachyons are confined to the region near \( r = 0 \); A string that winds around \( y \) must also wander in \( \mathbb{R}^2 \) and traverse a net angle of \( 2\pi \epsilon \). Because of this, a string is longer and thus heavier when its endpoints are located away from the origin. In effect, the tachyon lives in a lower dimensional subspace, and one may hope that D-brane probe analysis may yield some answers on the fate of this unstable theory upon tachyon condensation. In this note, we construct and analyze D-brane probe theory in the above background for general twist, and try to make contact with various limits such as type 0 theories and nonsupersymmetric orbifolds of type II theories. For CFT construction of D-branes on this background, see \cite{14, 15}.

For convenience, let us summarize the probe theory action. We probe the background by \( N \) D0 branes. The resulting worldvolume theory is a 2-dimensional \( U(N) \) Yang-Mills theory on the dual circle of circumference \( 1/R \). There are eight adjoint Higgs fields denoted by six hermitian \( \mathcal{X}^I \)'s corresponding to \( \mathbb{R}^6 \) and a complex field and its conjugate, \( Z \) and \( \bar{Z} \) associated with \( \mathbb{R}^2 \). Fermions are in an 16 component spinor \( \Psi \) also in the adjoint. The Lagrangian is formally identical to that of standard \( N = 8 \), maximally supersymmetric Yang-Mills theory

\[
\frac{1}{2} \text{tr} \left( -\mathcal{F}_{01}^2 - \sum_I ([\mathcal{D}, \mathcal{X}^I])^2 - |[\mathcal{D}, Z]|^2 \\
+ \sum_{I> J} [\mathcal{X}^I, \mathcal{X}^J]^2 + \sum_I [\mathcal{X}^I, Z][\mathcal{X}^I, \bar{Z}] + [Z, \bar{Z}][Z, Z]/4 \\
+ i\bar{\Psi} \Gamma^I [D_i, \Psi] - \sum_I \bar{\Psi} \Gamma_I [\mathcal{X}^I, \Psi] - \bar{\Psi} \Gamma_Z [\bar{Z}, \Psi] - \bar{\Psi} \Gamma_Z [Z, \Psi] \right),
\]

with \( SO(9,1) \) Dirac matrices \( \Gamma_{0,1,2,\ldots,9} \) except for one important detail: \( Z \) and \( \Psi \) are always accompanied by a Wilson line. For instance, everywhere in the above
expressions, one should replace \( Z \) by
\[
Z(\sigma) \rightarrow [Pe^{-i\int_\sigma^\sigma - b A}] Z(\sigma - b),
\]
(5)
where \( A \) is the gauge field along the dual circle. Also, breaking up the Fermions in terms of chirality with respect to \( R^2 \), and calling them \( \Psi_\pm \), one must make the replacement
\[
\Psi_\pm(\sigma) \rightarrow [Pe^{-i\int_\sigma^\sigma \mp b/2 A}] \Psi_\pm(\sigma \mp b/2).
\]
(6)
Subsequently, the argument of the field that multiplies to the right (to the left of \( \bar{Z} \) and of \( \bar{\Psi} \)) must be accordingly shifted as is necessary to maintain covariance.

2 D0 Probe

2.1 Generality

Let us probe this background by D0 branes. Following Taylor [16], we may derive the effective dynamics of D0’s by introducing infinite number of images upon the action of \( Z \). Introducing Chan-Paton factor \( |n\rangle \), the lowest lying mode of open strings ending on the D0’s can be written as infinite matrices. Denote the 9 transverse scalars by upper case symbols,
\[
X^I_{mk}, \quad I = 1, 2, 3, 4, 5, 6, \\
Z_{mk}, \\
\bar{Z}_{mk}, \\
Y_{mk}.
\]
(7)
The matrices satisfy a reality condition, which requires in particular,
\[
\bar{Z}_{mk} = Z^*_{km},
\]
(8)
while \( Y \) and \( X^I \) are Hermitian.

Action of the orbifold group is such that
\[
Y_{m+1,k+1} \leftrightarrow Y_{mk} + 2\pi R \delta_{mk}, \\
Z_{m+1,k+1} \leftrightarrow e^{2\pi i} Z_{mk}, \\
\bar{Z}_{m+1,k+1} \leftrightarrow e^{-2\pi i} \bar{Z}_{mk}, \\
X^I_{m+1,k+1} \leftrightarrow X^I_{mk},
\]
(9)
\footnote{This action can also be found in Ref. [7].}
where one should take each component as representing a square block. The block would be $N \times N$ if there are $N$ D-brane probes. Configurations invariant under such mapping can be solved for and give

$$
Y_{mk} = A_{m-k} + 2\pi m R \delta_{mk},
$$
$$
Z_{mk} = f_{m-k}^{(\beta)} e^{\pi i (m-k) \epsilon} e^{\pi i (m+k) \epsilon},
$$
$$
X^I_{mk} = X^I_{m-k},
$$
(10)

where $\beta$ is an arbitrary real number. This number does not enter the physics and we will later take it to be $-1$ as a matter of convenience.

Reality condition implies that

$$
A_m = (A_{-m})^*,
$$
$$
X^I_m = (X^I_{-m})^*,
$$
(11)

and also that

$$
Z_{mk} = (Z_{km})^\dagger = (f_{k-m}^{(\beta)})^\dagger e^{\pi i (m-k) \epsilon} e^{-\pi i (m+k) \epsilon}.
$$
(12)

Finally, gauge transformation is also reduced similarly to satisfy,

$$
U_{m+1,k+1} = U_{mk}.
$$
(13)

In principle, one could derive low energy effective action of D-branes by restricting open string fields to the above constraints.

A little more illuminating procedure can be found by trading off the discrete indices in favor of a continuous variable on a circle of circumference $1/R$. In more concrete terms, we define

$$
C(\sigma, \sigma') \equiv R \sum_{mk} C_{mk} e^{-2\pi i R m \sigma} e^{2\pi i R k \sigma'},
$$
(14)

for each object in adjoint representation, and find

$$
Y(\sigma, \sigma') = [A(\sigma) + i \partial_\sigma] \delta(\sigma - \sigma'),
$$
$$
Z(\sigma, \sigma') = f(\sigma) \delta(\sigma - \sigma' - \epsilon/R),
$$
$$
\bar{Z}(\sigma, \sigma') = f(\sigma')^\dagger \delta(\sigma' - \sigma - \epsilon/R),
$$
$$
X^I(\sigma, \sigma') = X^I(\sigma) \delta(\sigma - \sigma'),
$$
(15)
with $\beta = -1$ suppressed in the notation. The delta function is normalized by
\[
\int_0^{1/R} \delta(\sigma - \sigma_0) d\sigma = 1. \quad (16)
\]
Similarly we find gauge transformation is encoded in a local quantity,
\[
U(\sigma, \sigma') = U(\sigma) \delta(\sigma - \sigma'). \quad (17)
\]
With these, it is not difficult to convince oneself that all of the above fields transform under the gauge transformation bilocally. That is,
\[
C(\sigma, \sigma') \Rightarrow U(\sigma) C(\sigma, \sigma') U(\sigma')^\dagger, \quad (18)
\]
which is in reality local except for $f$. That is,
\[
[A(\sigma) + i \partial_\sigma] \Rightarrow U(\sigma)[A(\sigma) + i \partial_\sigma] U(\sigma)^\dagger, \\
\mathcal{X}^I(\sigma) \Rightarrow U(\sigma)\mathcal{X}^I(\sigma) U(\sigma)^\dagger, \quad (19)
\]
where $A(\sigma)$ is the gauge field on the dual circle and transforms as such. On the other hand, we have
\[
f(\sigma) \Rightarrow U(\sigma)f(\sigma)U(\sigma - \epsilon/R)^\dagger. \quad (20)
\]
We can trade off this $f$ in favor of $Z$ satisfying
\[
Z(\sigma) \Rightarrow U(\sigma)Z(\sigma) U(\sigma)^\dagger, \quad (21)
\]
with a field redefinition involving open Wilson line
\[
W(\sigma, \sigma') \equiv Pe^{-i \int_{\sigma'}^{\sigma} A}, \quad (22)
\]
such that
\[
Z(\sigma, \sigma') = W(\sigma, \sigma - \epsilon/R) Z(\sigma - \epsilon/R) \delta(\sigma - \sigma' - \epsilon/R), \quad (23)
\]
and similarly
\[
\bar{Z}(\sigma, \sigma') = Z(\sigma - \epsilon/R, W(\sigma, \sigma + \epsilon/R) \delta(\sigma - \sigma' + \epsilon/R). \quad (24)
\]
2.2 Commutators and the Probe Theory

Matrix multiplications involving $Y, Z, X^I$ can be mapped to integrals along the dual circle as follows,

$$
\sum_l C_{mk} D_{kl} \rightarrow \int d\sigma'' C(\sigma, \sigma'') D(\sigma'', \sigma').
$$

(25)

An extra ingredient that we so far neglected is the gauge field along time direction. Using same procedure as above, we may define $D_0 = \partial_t - iA_0$ the time-like covariant derivative on the dual side, which shows up in commutators with $D_0$,

$$
[D_0, Y] \rightarrow (\mathcal{F}_{01}(\sigma)) \delta(\sigma - \sigma'),
$$

$$
[D_0, X^I] \rightarrow ([D_0, X^I(\sigma)]) \delta(\sigma - \sigma'),
$$

$$
[D_0, Z] \rightarrow (D_0(\sigma)W(\sigma, \sigma - b) Z(\sigma - b) - W(\sigma, \sigma - b) Z(\sigma - b)D_0(\sigma - b)) \delta(\sigma - \sigma' - b).
$$

(26)

Recall $b \equiv \epsilon/R$. $\mathcal{F}$ is the field strength of $(A_0, A_1 \equiv A)$. As evident in the above expression, the effect of the twisting is in the extra Wilson line factor that comes with $Z$. Commutators with $Y$ becomes covariant derivative along the dual circle,

$$
[Y, X^I] \rightarrow \left(i\partial_\sigma X^I(\sigma) + [A(\sigma), X^I(\sigma)]\right) \delta(\sigma - \sigma'),
$$

$$
[Y, Z] \rightarrow \left([i\partial_\sigma + A(\sigma)]W(\sigma, \sigma - b) Z(\sigma - b) - W(\sigma, \sigma - b) Z(\sigma - b)A(\sigma - b)\right) \delta(\sigma - \sigma' - b).
$$

(27)

Finally commutators among “scalar fields” are

$$
[X^I, X^K] \rightarrow ([X^I(\sigma), X^K(\sigma)]) \delta(\sigma - \sigma'),
$$

$$
[X^I, Z] \rightarrow (X^I(\sigma)W(\sigma, \sigma - b) Z(\sigma - b) - W(\sigma, \sigma - b) Z(\sigma - b)X^I(\sigma - b)) \delta(\sigma - \sigma' - b),
$$

$$
[X^I, Z] \rightarrow (X^I(\sigma)Z(\sigma)^\dagger W(\sigma, \sigma + b) - Z(\sigma)^\dagger W(\sigma, \sigma + b)X^I(\sigma + b)) \delta(\sigma - \sigma' + b),
$$

$$
[Z, Z] \rightarrow (Z(\sigma)^\dagger Z(\sigma) - W(\sigma, \sigma - b) Z(\sigma - b)Z(\sigma - b)W(\sigma - b, \sigma)) \delta(\sigma - \sigma'),
$$

(28)

respectively.
The action of the D0 probe descends from that of the D0 Yang-Mills quantum mechanics whose Lagrangian is of the form,

\[
\frac{1}{2} \text{Tr} \left( - \sum_A [D_0, X^A][D_0, X^A] + \sum_{A>B} [X^A, X^B][X^A, X^B] 
+ i\bar{\Phi}\Gamma^0[D_0, \Phi] - \sum_A \bar{\Phi}\Gamma_A[X^A, \Phi] \right),
\]

where \(X^A\) includes \(Y, Z, \bar{Z},\) as well as \(X^I.\)

Summations over the indices transform into integral over the dual circle. In the action, all commutators must be squared and traced, but the latter operator produces a factor of \(\delta(0).\) The infinity associated with this indicates to us that we are over-counting by treating as if all images of the D0 under the orbifold action are real. One must divide by the total number of D0 images, which is simply taken care of by dropping \(\delta(0).\) The Lagrangian is then integral of square of the densities appearing the commutators above, weighted according to the D0 action. The resulting bosonic part of the action is that of a standard 1 + 1 dimensional Yang-Mills theories with adjoint scalars, except that one holomorphic adjoint scalar comes with a Wilson line of length \(b\) to the left.

While we did not explicitly discuss fermionic quantities, they can be treated similarly. The only difference is that fermions rotate under the shift \(2\pi \epsilon\) differently. Fermions will rotate under the twist as

\[
e^{i\pi \Gamma_{78}}
\]

which translates to a Wilson line

\[
[Pe^{-i\int a^\sigma \mp b/2 A}]
\]

attached to the left of \(\Psi(\sigma \mp b/2)_{\pm},\) where \(\pm\) denotes chirality under the \(\Gamma_{78}.\) Shifting the argument of quantities that multiply \(\Psi_{\pm}\) from the right in the same manner as above, one finds the probe action summarized in the introduction.

### 2.3 Probe Geometry

Since Hamiltonian of the system is proportional to the sum of absolute squares of the above commutators, the vacuum configuration is found by setting them to zero. For simplicity, let us take the case of a single D0 probe and take the temporal gauge \(A_0 = 0.\) Commutators with \(D_0\) reduces to partial derivatives with respect to time.
First of all, vanishing of \([Y, X^I]\) forces \(X^I\) uniform. On the other hand, the only gauge invariant information in \(A\) at any given time is the Wilson line, so we may render \(A_0 = A\) to be uniform and take value in the range of \([0, 2\pi R]\). Then, vanishing of \([X^I, Z]\) with uniform \(X^I\), in turn forces \(Z\) to be uniform also. Thus, we seem to find vacuum moduli space of the form,

\[
\mathbb{R}^6 \times \mathbb{R}^2 \times S^1.
\]  

(32)

However, this is only a topological statement and the actual moduli space metric is determined by slow motion along the moduli space. Recalling that we adopted the temporal gauge \(A_0 = 0\), we may write these moduli as

\[
\begin{align*}
X^I &= x^I, \\
Z &= r e^{i\theta}, \\
A &= y,
\end{align*}
\]  

(33)

from which it also follows that \(F_{01} = \partial_0 y\). The only nontrivial aspect of the kinetic terms for these fields is that the Wilson line mixes with the phase of \(Z\), and in fact the two always appears in the combination of \(\theta + b y\). In effect, the moduli space metric, which is extracted from time derivative part of the kinetic terms, is

\[
(dx^1)^2 + \cdots + (dx^6)^2 + dr^2 + r^2(d\theta + b dy)^2 + (dy)^2,
\]  

(34)

reproducing the spacetime that is being probed by the D0 brane.

### 2.4 A Consistency Check: T-duality

On the other hand, T-duality takes the flat Melvin background to one with a non-trivial spacetime metric, dilaton, and antisymmetric tensor \([10]\),

\[
g + B = -(dt)^2 + (dx^1)^2 + \cdots + (dx^6)^2 + dr^2 + \frac{r^2}{\Lambda}(d\theta - b d\tilde{y})(d\theta + b d\tilde{y}) + (d\tilde{y})^2,
\]  

(35)

where \(\tilde{y}\) parameterizes the dual circle. Curved nature of this background is summarized in the function,

\[
\Lambda \equiv 1 + b^2 r^2,
\]  

(36)

which also enters the dilaton:

\[
e^{2(\Phi_0 - \Phi)} = \Lambda.
\]  

(37)
Since we could consider the D0 probe above as a D-string probing the dual background, one might wonder why the nontrivial curvature of the dual geometry is not seen by the probe. Here, we will clarify this issue by considering D-string effective action in the dual background.

A single D-string probing this geometry is governed by a Born-Infeld action,

$$e^{-\Phi}\sqrt{-\text{Det}(g + B + \mathcal{F})}. \quad (38)$$

T-dual of the D0 would be a D-string wrapped around $\tilde{y}$. Set the worldvolume coordinate such that

$$t(\tau, \sigma) = \tau, \quad \tilde{y}(\tau, \sigma) = \sigma, \quad (39)$$

and estimate the determinant up to two derivative terms.

$$-\frac{1}{\Lambda} + \frac{1}{\Lambda} \left( (\partial_0 x)^2 + (\partial_0 r)^2 + (r^2/\Lambda)(\partial_0 \theta)^2 \right) - \left( (\partial_1 x)^2 + (\partial_1 r)^2 + (r^2/\Lambda)(\partial_1 \theta)^2 \right) + (\mathcal{F}_{01} + br^2 \partial_0 \theta/\Lambda)^2 + \cdots. \quad (40)$$

Taking into account the dilaton factor, we find the following leading kinetic terms from derivative expansion of Born-Infeld action,

$$-\frac{1}{2}(\mathcal{F}_{01})^2 - \frac{1}{2} \left( (\partial_0 x)^2 + (\partial_0 r)^2 + r^2(\partial_0 \theta + b\mathcal{F}_{01})^2 \right) + \frac{1}{2} (\partial_1 x)^2 + (\partial_1 r)^2 + r^2(\partial_1 \theta)^2 + \frac{1}{2} b^2 r^2 \left( (\partial_1 x)^2 + (\partial_1 r)^2 \right), \quad (41)$$

up to an overall factor of $e^{-\Phi_0}$.

Now it is a matter of straightforward exercise to check the same action results from the $U(1)$ probe theory. For instance, the first line reproduces the probe metric. The mixing between $\partial_0 \theta$ and $\mathcal{F}_{01}$ is precisely what happened from the gauge theory side. The last two terms may seem unfamiliar. But, recalling that we performed a derivative expansion on Born-Infeld side, we should also take the limit where $\partial_1$ derivatives of fields $\mathcal{X}$ and $\mathcal{Z}$ are small. In this limit, two sets of nontrivial commutator terms are

$$\frac{1}{2} \text{Tr}[X^I, Z][X^I, Z] \rightarrow \frac{1}{2} \int d\sigma \, \mathcal{Z}(\sigma) \bar{\mathcal{Z}}(\sigma)(\mathcal{X}^I(\sigma + b) - \mathcal{X}^I(\sigma))^2 \simeq \frac{1}{2} \int d\sigma \, \mathcal{Z}(\sigma) \bar{\mathcal{Z}}(\sigma) \left( b^2(\partial_1 \mathcal{X}^I(\sigma))^2 \right)$$
\[ \rightarrow \frac{1}{2} \int d\sigma b^2 r^2 (\partial_1 \vec{x})^2, \]

\[ \frac{1}{2} \text{Tr}([Z, \bar{Z}]/2)/([\bar{Z}, Z]/2) \rightarrow \frac{1}{2} \int d\sigma \left( [\bar{Z}^\dagger(\sigma)Z(\sigma) - Z(\sigma-b)\bar{Z}^\dagger(\sigma-b)]/2 \right)^2 \]

\[ \simeq \frac{1}{2} \int d\sigma \left( b \partial_1 (Z\bar{Z})/2 \right)^2 \]

\[ \rightarrow \frac{1}{2} \int d\sigma b^2 r^2 (\partial_1 r)^2, \quad (42) \]

which results from the nonlocal nature of each term. The upshot is that while the full Born-Infeld action knows about the nontrivial geometry, one must truncate to two-derivative terms in order to make comparison with the gauge theory result. At this level, the two approaches produce identical results.

3 **C/Z\(_n\) Orbifold Limit and Closed String Tachyon**

It has been noted that, when \( \epsilon = 1 + 1/n \) with odd integer \( n \), the above orbifold in the limit \( R \rightarrow 0 \), reduces to a singular orbifold \( C/Z_n \times \mathbb{R}^1 \) \[8, 9\]. In this section, this reduction to the orbifold limit is also confirmed from the D-brane probe side by making contact with the probe theory found by Adams et.al. This comparison should, in part, serve as a useful tool for understanding how tachyon couples to the probe dynamics.

### 3.1 Orbifold Limit of Probe Theory

The probe theory of the \( C/Z_n \) orbifold is a \( U(N)^n \) quiver theory \[7\]. One may wonder how the present \( U(N) \) theory can “reduce” to a theory with apparently larger gauge group. The keypoint is that we may think of the case \( \epsilon = 1 + 1/n \) as

\[ (\mathbb{R}^2 \times \mathbb{S}^1)/Z_n \quad (43) \]

where \( \mathbb{S}^1 \) is a circle of \( nR \). The dual circle is then of circumference \( 1/nR \), which may be thought of as a \( 1/n \) part of the dual circle we used above. In the following, each factor of \( U(N)^n \) will be simply \( U(N) \) restricted to each of these intervals of length \( 1/nR \). Cutting the dual circle of length \( 1/R \) into \( n \) intervals of length \( b = 1/nR \) each,\(^2\) This is not obvious from the background geometry itself, but was rather deduced from the form of perturbative string spectrum. We suspect that some nontrivial field redefinition is at work, in much the same way as in Ref. \[11\].
say, \([-1/2nR, 1/2nR), [1/2nR, 3/2nR), [3/2nR, 5/2nR), \] etc, each of these intervals grows to be an infinite line of its own.

Instead of working with \(n\) copies of a real line, a more sensible thing to do is treat the dynamical fields in each segment line as a field of its own. Namely split each of local field, say \(A\), into \(n\) independent quantities \(A_a\) with a cyclic label \(a = 1, \ldots, n, n+1 \sim 1\), such that

\[
A_a(\sigma) = \lim_{R \to 0} A(\sigma + a/nR). \tag{44}
\]

Here we invoked the periodicity of the dual circle, \(1/R\), i.e., \(\sigma \sim \sigma + 1/R\) on the right hand side prior to taking the limit. The range of \(\sigma\) in this definition is restricted to be between \(-1/2nR\) and \(1/2nR\), which eventually becomes the entire real line upon the limit. This gives us \(n\) gauge fields that propagate along \(\mathbb{R}^1\) of \(C/Z_n \times \mathbb{R}^1\). The net effect is to have \(U(N)_1 \times U(N)_2 \times \cdots \times U(N)_n\) gauge group instead of \(U(N)\).

Similar decomposition works for \(X_I\),

\[
(\mathcal{X}_I)'_a(\sigma) = \lim_{R \to 0} \mathcal{X}_I'(\sigma + a/nR). \tag{45}
\]

giving us six adjoint fields for each \(U(N)_a\) factor.

For \(Z_a\)'s, we want to include the Wilson line intact, so that

\[
Z'_a(\sigma) = \lim_{R \to 0} W(\sigma + a/nR, \sigma + (a-1)/nR)Z(\sigma + (a-1)/nR), \tag{46}
\]

transforms as a bifundamental under \(U(N)_a \times U(N)_{a-1}\) and is effectively a local field in this limit. Finally fermions come with a Wilson line of length \(\mp \epsilon/2R = \mp(n+1)/2nR\). For example,

\[
(\Psi_{\pm})'_a(\sigma) = \lim_{R \to 0} W(\sigma + a/nR, \sigma + (a \mp (n+1)/2)/nR) \times \Psi_{\pm}(\sigma + (a \mp (n+1)/2)/nR), \tag{47}
\]

are each in the bifundamental representation under \(U(N)_a \times U(N)_{a \mp (n+1)/2}\). Note that \((n+1)/2\) is always an integer since \(n\) is an odd integer. The combined field content is exactly that of the probe theory on \(C/Z_n\) as found by Adams et.al. [7].

### 3.2 Reconstructing Twisted Sector Tachyons

Coming back to the case of \(N = 1\), an interesting observation Adams et.al. made is that closed string tachyons couple to these worldvolume fields in a manner reminiscent
of Fayet-Iliopoulos terms in probe theories for supersymmetric orbifolds. For $U(1)^n$ case, the leading coupling can be rewritten as

$$\sum_a \zeta_a (|Z'_a|^2 - |Z'_{a+1}|^2),$$

(48)

where parameters $\zeta_a$ obey $\sum_a \zeta_a = 0$ and are turned on by the condensation of tachyons [7, 20]. The role of such $\zeta$'s was seen to resolve the singularity at the conical tip of $\mathbf{C}/\mathbb{Z}_n$, which leads them to conjecture that the condensation of tachyons leads the theory back to some supersymmetric critical string theory.

The closed string tachyons of the $\mathbf{C}/\mathbb{Z}_n$ orbifold reside in $n - 1$ twisted sectors of the orbifold. The vacuum energy of the $k$-th twisted sector (in Green-Schwarz formalism) is

$$-\frac{k}{2n} \quad \text{for } k \text{ even},$$

$$-\frac{n-k}{2n} \quad \text{for } k \text{ odd},$$

(49)

which, given an odd $n$, produces a pair of tachyons for each value of mass squared

$$-\frac{l}{n} \times \frac{4}{\alpha'}, \quad l = 1, 2, \ldots, \lfloor n/2 \rfloor.$$

(50)

where we restored the explicit factor of $\alpha'$. These tachyons live in $\mathbf{R}^{1+7}$ and, in particular, propagate along the $\sigma$ direction arising from the dual circle.

From the Melvin viewpoint these tachyons can be reconstructed as follows. In the limit of $R \to 0$, a sector with the winding number $w$ has the vacuum energy that depends on non-integer part of $\epsilon w$ (in NSR formalism),

$$-\frac{1}{2} (\epsilon w - [\epsilon w]),$$

(51)

or

$$-\frac{1}{2} + \frac{1}{2}(\epsilon w - [\epsilon w]),$$

(52)

depending on whether the quantity inside the parenthesis is smaller or larger than $1/2$. One must further take into account the GSO projection which changes depending

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3 This vacuum energy is with respect to the Fock vacuum defined by $\psi_r |0\rangle = \psi^\dagger_{-r} |0\rangle = \tilde{\psi}_r |0\rangle = \tilde{\psi}^\dagger_{-r} |0\rangle = 0 \quad (r > 0)$, where $\psi_r$ and $\tilde{\psi}_r$ are modes of the free worldsheet fermions with the twisted boundary condition[10]: $\psi(z) = \sum_r \psi_r z^{-r-1/2}$, $\tilde{\psi}(\bar{z}) = \sum_r \tilde{\psi}_r \bar{z}^{-r-1/2}$
on whether the integer part of $\epsilon w$ is odd or even. The lowest lying state after GSO projection gets contribution from left(right) moving oscillators as follows. With $\epsilon = 1 + 1/n$,
\begin{equation}
-\frac{2l}{2n}
\end{equation}
for $w = 2l$ and
\begin{equation}
-1 + \frac{(n-2l)}{2n} = -\frac{2l}{2n}
\end{equation}
for $w = n - 2l$. The pattern repeats itself for $w = kn \pm 2l$ for all integer $k$. Thus there are two tachyonic towers for each integer $l$ less than $[n/2]$, whose mass squared is
\begin{equation}
-\frac{l}{n} \times \frac{4}{\alpha'}, \quad l = 1, 2, \ldots, [n/2].
\end{equation}
On the T-dual side, the winding numbers translate to quantized momenta along the dual circle. Successive momentum modes in any given tower are separated by $n$ basic quanta, so each tower is appropriate for building up a complete field along a $1/n$ segment of the dual circle. Thus, we find $n-1$ tachyonic fields that may propagate along $\mathbf{R}^1$ of $\mathbf{C}/Z_n \times \mathbf{R}^1$. This agrees with findings by Adams et.al. Also, it is fairly obvious that the winding number $w$ modulo $n$ labels the $n-1$ twisted sectors in this orbifold limit.

### 4 Conclusion

We have considered D0 brane probe in the background associated with Melvin type twisting. The probe dynamics is written down explicitly, and is almost identical to a maximally supersymmetric $U(N)$ theory except that a complex adjoint Higgs and all fermions are accompanied by a open Wilson line. This renders the Lagrangian density to be a nonlocal expression. The flat directions are studied and the probe geometry has been shown to agree with the background, where the open Wilson line plays a crucial role. We have further considered a $Z_n$ orbifold limit that has been contained by that of $w = 1$. If we take $\epsilon = 1 + \frac{1}{n}$, $w = 1$ mode becomes massless when $R = \sqrt{2\alpha'(1-1/n)}$. [10, 8]

\begin{equation}
\left\{
\begin{array}{ll}
\frac{(wR)^2}{\alpha'} - \frac{2}{\alpha'}(\epsilon w - [\epsilon w]) & \text{for } [\epsilon w] = \text{even,} \\
\frac{\alpha}{\alpha'}R^2 (1 - (\epsilon w - [\epsilon w])) & \text{for } [\epsilon w] = \text{odd,}
\end{array}
\right.
\end{equation}

where the first term is the contribution of winding number, and the second term is the sum of zero point energy and oscillator number which gives the result of (56). Tachyonic region for $w \neq 1$ modes is contained by that of $w = 1$. If we take $\epsilon = 1 + \frac{1}{n}$, $w = 1$ mode becomes massless when $R = \sqrt{2\alpha'(1-1/n)}$. 

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4 The mass squared of the lowest mode with winding number $w$ with finite $R$ is [10, 8]
argued to reduce to ones in Ref. [7], and reproduced the probe dynamics of the latter by taking an appropriate limit.

An important question that was not addressed in this note is how the closed string tachyon vev couples to the probe in general Melvin background. Among the constraints the coupling must satisfy is that it should reproduce that of Adams et.al. in the orbifold limit. For instance, if a term of type

\[ \int d\sigma \, \zeta(\sigma) \times \left( |Z(\sigma)|^2 - |Z(\sigma - b)|^2 \right), \]

with constraint

\[ \int d\sigma \zeta(\sigma) = 0. \]

is present for a single D0 probe, it will reproduce (58) in the C/Zn limit. In fact, if \( \zeta \) is linear in the tachyon, this constraint is automatically satisfied since tachyon vev must have nontrivial momentum along the dual circle. For any finite \( R \), at most a finite number of winding modes will be tachyonic and the profile of condensed tachyon is never uniform along the dual circle. What must happen in the C/Zn orbifold limit is that an infinite number of winding modes become tachyonic, so a piece-wise uniform tachyon vev becomes possible along the dual circle and produces \( \{\zeta_a\} \). Computation of the precise coupling of tachyon is beyond the scope of this note, however, and we would like to come back to the issue of tachyon condensation elsewhere [18].

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