No Free Lunch Theorem for Security and Utility in Federated Learning

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In a federated learning scenario where multiple parties jointly learn a model from their respective data, there exist two conflicting goals for the choice of appropriate algorithms. On one hand, private and sensitive training data must be kept secure as much as possible in the presence of semi-honest partners; on the other hand, a certain amount of information has to be exchanged among different parties for the sake of learning utility. Such a challenge calls for the privacy-preserving federated learning solution, which maximizes the utility of the learned model and maintains a provable privacy guarantee of participating parties’ private data.

This article illustrates a general framework that (1) formulates the trade-off between privacy loss and utility loss from a unified information-theoretic point of view, and (2) delineates quantitative bounds of the privacy-utility trade-off when different protection mechanisms including randomization, sparsity, and homomorphic encryption are used. It was shown that in general there is no free lunch for the privacy-utility trade-off, and one has to trade the preserving of privacy with a certain degree of degraded utility. The quantitative analysis illustrated in this article may serve as the guidance for the design of practical federated learning algorithms.

CCS Concepts: • Computer systems organization → Embedded systems; Redundancy; Robotics; • Networks → Network reliability;

Additional Key Words and Phrases: Federated learning, privacy-preserving computing, security, utility, trade-off, divergence, optimization

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1 INTRODUCTION

Modern machine learning techniques are data-hungry, and it is not uncommon to use a trillion bytes of data in developing large pre-trained machine learning models, such as for natural language processing [16, 18] or image processing [13, 47]. For a large variety of machine learning applications, for instance, in social media or finance use cases, data are often distributed across multiple devices or institutions. Collecting such data onto a central server for training will incur additional communication overhead, management and business compliance costs, privacy issues, and even regulatory and judicial issues such as GDPR (General Data Protection Regulation).\footnote{GDPR is applicable as of May 25, 2018 in all European member states to harmonize data privacy laws across Europe (https://gdpr.eu/).}

**Federated learning (FL)** has been introduced as an effective technology to allow multiple parties to jointly train a model *without gathering or exchanging private training data among different parties*. Moreover, it is also required that information exchanged during the learning or inference stages *do not disclose private data owned by respective parties to semi-honest adversaries* (for a formal definition, see Section 2.2.1), who aim to espy or infer other parties’ private data from exchanged information. The requirements as such constitute the primary mandate for a novel statistical framework of privacy-preserving FL as illustrated in this article.

From an information theory point of view, the amount of information about private data that a semi-honest party can infer from exchanged information is inherently determined by the statistical dependency between private data and publicly exchanged information. Recent studies have shown that semi-honest adversaries can exploit this dependency to recover the private training images with pixel-level accuracy from exchanged gradients of learned models, such as by using Bayesian inference attacks (see Section 3.3 and other works [41, 89, 90] for details). Taking such threat models into account, we therefore reiterate that a secure federated learning (SFL) scheme must (1) support multiple parties to jointly train models and to make joint inferences without exchanging private data, and (2) have clearly defined threat models and security guarantees under these models.

In this article, we consider a *horizontal FL* [81], or a *cross-device FL* [46], setting, in which multiple clients upload respective local models or model updates to an aggregator,\footnote{We assume the existence of a centralized aggregator for the brevity of analysis. Yet the analysis is also applicable to the decentralized FL setting in which model aggregation is performed without an aggregator. Detailed discussion on this setting is outside the scope of this article and will be reported elsewhere.} who is responsible for aggregating multiple local models into a global model. There are a variety of application scenarios that use this scheme for FL [56, 57, 82]. The fundamental privacy-preserving requirement is to maintain potential privacy loss below an acceptable level. This is achieved by reducing the dependency between the exchanged model information and private data. The reduced dependency, however, makes the aggregated global model less accurate and leads to utility loss, such as a loss in model accuracy, as compared to a global model trained without using any protection. Severely degraded model utility indeed defeats the purpose of FL in the first place. We therefore propose in this article a unified statistical framework to analyze the privacy-utility trade-off on a rigorous theoretical foundation. The main results of our research are summarized as follows (Figure 1 presents a pictorial summary):

- First, we formulate the privacy leakage attack [41, 89, 90] through a *Bayesian inference attack* perspective (Definition 3.1) [29, 79]. A privacy metric called *Bayesian privacy* is then proposed to quantify the additional amount of information about private data that a semi-honest adversary might gain by observing publicly exchanged information. Specifically, the information gain for adversaries—that is, *Bayesian privacy leakage* (Definition 3.2)—is...
measured by distances between distributions of adversaries’ prior and posterior beliefs about private data. In addition, an upper bound of Bayesian privacy leakage called $\epsilon$-Bayesian privacy provides the privacy-preserving guarantee regardless of any Bayesian inference attack that may be launched by semi-honest adversaries.

• Second, we put forth a statistical framework to cast the optimal privacy-utility trade-off as a constrained optimization problem in which the utility loss (Definition 3.3) is minimized subject to a preset upper bound of Bayesian privacy leakage (i.e. $\epsilon$-Bayesian privacy) constraint (see Section 4.2). A theoretical analysis of the trade-off is then manifested as Theorem 4.1, which dictates that (1) a weighted sum of the privacy loss and utility loss is greater than a problem-dependent non-zero constant, and (2) relative weights between the privacy loss and utility loss depend on multiple factors, including Bayesian inference attacks that adversaries may launch, and protection mechanisms adopted as well as distributions of the training and testing data. In other words, in principle, _one has to trade a decrease of the privacy loss with a certain degree of increase of the utility loss, and vice versa._

• Third, the general principle of Theorem 4.1 is applied to give quantitative analysis of the trade-off under specific privacy-protection mechanisms, including randomization [1, 34, 75], sparsity [38, 69, 70] and homomorphic encryption (HE) [31, 84] (see Theorems 4.2, 4.4, and 4.6). These theoretically justified results are of interest in their own rights. However, they are of pragmatic value in serving as a practical guidance to the design of FL algorithms under various scenarios (see Section 4.3 for applications).

To our best knowledge, the proposed Bayesian privacy formulation is the first unified framework that explicitly considers both threat models (attacking mechanisms) and security models (protection mechanisms). It is also the first that is applicable to different privacy-protection mechanisms including randomization [1, 34, 75], sparsity [38, 69, 70], and HE [31, 84]. The rest of the article is organized as follows. Section 2 reviews existing works related to privacy measurements and FL. Section 3 illustrates the statistical framework based on Bayesian inference attacks and protection mechanisms. Section 4 illustrates the quantification of the privacy-utility trade-off (Theorem 4.1), and the applications of the trade-off on different protection mechanisms are further presented in Section 4.3. The comparison of Bayesian privacy with other privacy measurements is presented in Section 5. Finally, Section 6 concludes the article with discussions on future research plans.

2 RELATED LITERATURE

This section gives a brief review of related work from distinct aspects, including privacy measurements, privacy attacking and protection methods in FL, and the privacy-utility trade-off.

2.1 Privacy Measurement

The need of privacy-preserving computing arises in various applications ranging from database mining, communication of secret information over public channels, and, more recently, machine learning. Numerous definitions of privacy or privacy measurements proposed in the literature can be broadly categorized as follows as prior-independent and prior-dependent ones:

• **Differential privacy (DP)** is a celebrated prior-independent definition proposed by Dwork et al. to protect individual privacy in response to queries about databases [21, 23, 25]. A series of DP variants were subsequently proposed to tighten the privacy budget under various conditions, including Renyi DP with the natural relaxation of DP [59] and Gaussian DP

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3This principle bears an intriguing similarity to the no free lunch theorem for optimization and machine learning [80], which inspire us to name Theorem 4.1 “No free lunch theorem (NFL) for security and utility” in FL.
with a closed-form privacy bound and stronger guarantee [11]. Moreover, a large number of algorithms have been designed to achieve DP in FL [67, 75], and Abadi et al. [1] proposed the moments accountants of DP suitable for deep learning. We refer readers to other works [22, 39] for a thorough coverage of privacy preserving based on DP. It is worth mentioning that DP-based measurements do not take into account the prior distributions of private data—that is, DP and its variants are prior-independent except for Bayesian differential privacy (BDP) [73].

• Among different prior-dependent privacy measurements [3, 19, 45], Information Privacy (IP) [19] is proposed to model the privacy leakage in terms of the “amount of knowledge” learned by an adversary about the private data after observing the user’s output. The optimal privacy-utility trade-off is then cast, following the rate-distortion theory [5, 71], as an optimization problem to minimize information leakage subject to utility loss constraints. The divergences adopted to quantify information leakages include Kullback-Leibler (KL) divergence (mutual information) [19], chi-squared divergence [43], total variation divergence [62], and Renyi divergence [53].

### 2.2 Federated Learning

The notion of FL was initially proposed by McMahan et al. with the aim to build a machine learning model based on datasets that are distributed across multiple devices [49, 50, 56, 57]. The main idea is to aggregate local models learned on multiple devices but without sending private data to a semi-honest third-party server or other devices. Bearing in mind the privacy concern of secret data distributed across multiple institutions, Yang et al. [81] extended applications of FL to a wide spectrum of use cases and classified FL scenarios into three categories:

1. **Horizontal FL**: Datasets share the same feature space but different space in samples.
2. **Vertical FL**: Two datasets share the same sample ID space but differ in feature space.
3. **Federated transfer learning**: Two datasets differ not only in samples but also in feature space.

Yang et al. [81] also proposed the notion of SFL to highlight the importance of protecting the privacy of training data from various kinds of adversary attacks. Horizontal and vertical FL are also referred to as cross-device and cross-silo FL, respectively, in the work of Kairouz et al. [46]. A wealth of literature as follows has been proposed to improve the privacy-preserving capability and model utility.
2.2.1 Threat Model in FL. For the sake of privacy security, it is often necessary to consider the existence of semi-honest (honest-but-curious) adversaries in FL. The adversary is honest in the sense that he or she faithfully follows the collaborative learning protocol and does not submit any malformed message, but he or she may launch privacy attacks to espy the training data of other participants, by analyzing periodic updates to the joint model (e.g., gradients) during training. Such kind of attacks is referred to as Bayesian inference attack (please refer to Definition 3.1), which can be broadly classified according to the information source exploited by a semi-honest adversary:

- **Gradient inversion attack**: Zhu and Han [89] and Zhu et al. [90] demonstrated that deep leakage attacks allow adversaries to restore the private data up to pixel-level accuracy from exchanged deep neural network model gradients. Following this seminal work, it was shown that adversaries can launch even more effective attacks by further exploiting various kinds of prior such as image smoothness prior by total variation loss [28], image label prior [87], and group consistency of estimated images [83].

- **Model inversion attack from outputs**: Split learning [38] allows a machine learning model to be separated and trained on multiple clients with low computing resources where each client only trains a small portion of the split models (e.g., a few layers of neural networks). However, adversaries could still infer private data from the model output in this case [27, 41]. Gu et al. [37] later applied model inversion attack in a federated split learning (called SplitFed) setting [70] in which clients only upload partial information to the server.

- **Generative adversarial network based attack**: Generative adversarial networks were used to infer clients’ private data. Hitaj et al. [42] viewed the aggregate model as a discriminator of the generative adversarial network to generate a distribution of specific classes. Wang et al. [78] proposed to learn a generator, which could recover user-specified private data in addition to the data distribution of a specific class.

2.2.2 Protection Mechanisms in FL. To protect private data from being disclosed by adversarial attacks, the following protection mechanisms have been adopted for SFL:

- **Randomization**: DP has been widely adopted to protect exchanged information in FL [1, 34, 75] by adding to model information either Laplace noise or Gaussian noise [23]. Local DP [21] was also proposed to randomize the response in FL [67, 75, 88]. Despite its simplicity and popularity, the randomization approach inevitably leads to compromised performances in terms of slow convergence, low model utility, and loose privacy guarantee as documented elsewhere [44, 48, 73], and so forth.

- **Encryption**: Encryption is a widely adopted technique for protecting sensitive information. In particular, HE and its variants [31, 32, 64] allow the aggregation of FL models to be performed directly on encrypted local models without decryption needed [2, 74, 85, 86]. However, extremely heavy computation and communication overhead incurred by HE prevent it from being readily applicable to large models such as deep neural networks with billions of model parameters. It remains an active research topic to come up with efficient HE algorithms to improve FL efficiency as demonstrated elsewhere [2, 84].

- **Sparsity-based methods**: These methods protect clients’ private data by hiding part of the information from being exchanged with other parties [69]. In particular, split learning in a federated setting proposed to separate and hide part of neural network models from other parties [38], and SplitFed [70] combined split learning and FL to improve the efficiency and utility at the same time.

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4 Note that the privacy analysis illustrated in Section 4.3 applies to randomization, sparsity, and HE protection mechanisms and secret sharing.
Secret sharing: Secret sharing [4, 6, 68] was developed to distribute a secret among a group of participants. However, it requires extensive exchange of messages and entails a communication overhead not viable in many FL settings.

Some protection mechanisms (e.g., [51]) do not belong to the field of FL and are beyond the scope of our article.

2.3 Privacy-Utility Trade-Off
In the past decade, there has been wide interest in the literature in understanding the privacy-utility trade-off:

- Some work focuses on privacy-utility trade-off, where utility is quantified (inversely) via distortion (accuracy), and privacy via IP. Sankar et al. [65] provided a precise quantification of the trade-off in databases between the privacy needs of the individuals and the utility of the published data. Makhdoumi and Fawaz [55] modeled the privacy-utility trade-off according to the framework proposed by Calmon and Fawaz [19]. They regard the trade-off as a convex optimization problem, which aims at minimizing the mutual information between the private data and released data under the constraint of utility (distortion) of released data. Moreover, Rassouli and Gündüz [62] further defined the privacy using total variation distance and illustrated that the optimal privacy-utility trade-off could be solved using a standard linear program. In addition, Wang and Calmon [77] provided a trade-off when utility and privacy were both evaluated using $\chi^2$-based information measures.

- The trade-off of utility with relaxed privacy-preserving capability was considered necessary by Dwork and Naor [24] for queries to be useful, and she also gave a quantitative analysis that relates $(\epsilon, 0)$-DP with the maximal error (in utility) bounded by $\Omega(\frac{1}{\epsilon})$ [25]. The DP budget $\epsilon$ of privacy-preserving algorithms was later related to the convergence (utility) of privacy-preserving algorithms [35, 76].

To sum up, in existing work we witnessed a compelling need to lay down a solid foundation for the design of novel FL algorithms. Especially, the recent Bayesian inference type of threats [28, 83, 87, 89, 90] were not explicitly modeled by existing privacy definitions, such as DP, which is considered the gold standard definition of privacy. This shortage of rigorous formulations makes it unclear for practitioners how to determine the optimal choice of algorithm design concerning the privacy-utility trade-off. There are no existing frameworks that simultaneously take into account different protection mechanisms, including randomization and HE. The rest of the present article therefore will illustrate such a novel statistical framework based on Bayesian inference attack, which aims to address aforementioned issues not properly resolved in existing work.

3 GENERAL SETUP AND FRAMEWORK

3.1 Notations
We follow the convention of representing the random variables using uppercase letters such as $S$, representing the particular values they take on using lowercase letters, and representing the support of random variables using copperplate such as $s$ and $D$. The distributions are represented using uppercase letters such as $F$ and $P$, and the probability density functions are represented using lowercase letters such as $f$ and $p$. We use $f_{D_{s}}(d)$ to represent the value of the probability density function $f$ at $s$, and the subindex represents the random variable. We use the notation $f_{D_{s}|W_{s}}(d|w)$ to represent the conditional density function. For continuous distributions $P$ and $Q$ over $\mathbb{R}^n$, the KL divergence is defined as $\text{KL}(P||Q) = \int p(x) \log(p(x)/q(x))dx$, where $p$ and $q$ denote the probability densities of $P$ and $Q$. The Jensen-Shannon (JS) divergence is a smoothed
Table 1. Table of Notation

| Notation | Meaning |
|----------|---------|
| $\epsilon_p$ | Privacy leakage (Definition 3.2) |
| $\epsilon_u$ | Utility loss (Definition 3.3) |
| $W^O_k$ | Unprotected model information of client $k$ |
| $W^S_k$ | Protected model information of client $k$ |
| $P^O_k$ | Distribution of unprotected information of client $k$ |
| $P^S_k$ | Distribution of protected information of client $k$ |
| $F^B_k$ | Adversary’s prior belief distribution about the private information of client $k$ |
| $F^A_k$ | Adversary’s belief distribution about client $k$ after observing the protected information |
| $F^O_k$ | Adversary’s belief distribution about client $k$ after observing the unprotected information |
| $\text{JS}(\cdot||\cdot)$ | Jensen-Shannon divergence between two distributions |
| $\text{TV}(\cdot||\cdot)$ | Total variation distance between two distributions |

version of the KL divergence, which is defined as $\text{JS}(P||Q) = \frac{1}{2} \left[ \text{KL}(P,M) + \text{KL}(Q,M) \right]$, where $M = (P + Q)/2$. The total variation distance between $P$ and $Q$ is $\text{TV}(P||Q) = \sup_{A \subseteq \mathbb{R}^n} |P(A) - Q(A)|$. Table 1 presents detailed descriptions of notations.

3.2 General Setup

In this work, we consider a horizontal FL scenario, or cross-device FL [46], where multiple ($K$) participants\(^5\) collaboratively learn a global model without exposing their private training data [56, 81]. The requirement of adopting certain protection of private data is due to the threat model. Some semi-honest adversaries may launch privacy attacks on exchanged information to infer private data of other participants (for a formal definition of semi-honest adversaries, see Section 2.2.1). Initially, the server distributes the global model information to all of the clients. The overall SFL procedures are illustrated in the left panel of Figure 2 and summarized as follows:

1. With the global model information from the server, each client $k$ trains the local model using his or her own dataset $D_k$, and obtains the local model information $W^O_k$, which follows a distribution $P^O_k$. The objective of learning $W^O_k$ is to maximize the model utility $U_k$.
2. To prevent semi-honest adversaries from inferring other clients’ private information $D_k$ according to $W^O_k$, the client adopts a protection mechanism $M$ to convert model information $W^O_k$ to protected model information $W^S_k$.
3. Each client uploads the protected information $W^S_k$ to the server and aggregates as a new global model $W^S_a$.
4. The clients download the global model $W^S_a$ and continue to update the local model information.

Processes 1 through 4 iterate until the utility of the aggregated model does not improve.

Remark: In SFL, the local model information includes the model parameters, model gradients, and model outputs, all of which may optionally be exchanged to the aggregator and get exposed to semi-honest adversaries (see details in Appendix B). The goal of the protection mechanism $M$ is to protect private data such that the dependency between $W^S_k$ and $D_k$ is reduced, as compared to the dependency between the unprotected information $W^O_k$ and $D_k$.

\(^5\)In this article we use participant, party, and client interchangeably to refer to a device or institute that participates in FL.
Fig. 2. An illustration of the setup for SFL. The left panel demonstrates the four procedures. (1) The $k_{th}$ client learns the local model information $W_k^O$ using his or her own dataset $D_k$. (2) The client adopts a protection mechanism $M$ to convert model information $W_k^O$ to protected model information $W_k^S$. (3) Each client uploads the protected information $W_k^S$ to the aggregator, who aggregates all $W_k^S, k = 1, 2, \ldots, K$ into the global model $W_a^S$. (4) The clients download the global model $W_a^S$ and continue to update the local model information. The right panel illustrates the privacy leakage that adversaries may infer the private data $D_k$ according to the protected model information $W_k^S$, and the utility loss that protection mechanism induces the aggregated global model to be less accurate with the decreased utility.

In this SFL setting, the privacy loss and utility loss as illustrated in the right panel of Figure 2 are considered:

(1) For the sake of privacy preserving, a Bayesian privacy leakage measure (see Definition 3.2) is used to quantify the amount of information about private data that semi-honest adversaries may still infer despite the protection mechanisms applied to publicly exposed information. The Bayesian privacy leakage allows one to evaluate the security of a secure federated learning scheme and justifies that the adopted protection mechanism is secure in thwarting Bayesian inference attacks if the Bayesian privacy leakage is less than an acceptable threshold (see Section 4.2).

(2) The protection mechanism modifies the original model information $W_k^O$ to its protected counterpart $W_k^S$ and induces the local model to behave less accurately. Consequently, the aggregated global model is less accurate, and the incurred utility loss is defined as the difference of utilities with and without protections (Definition 3.3).

### 3.3 Bayesian Inference Attack in SFL

It was shown that semi-honest adversaries could recover private training images up to pixel-level accuracy from unprotected gradients of learned models, such as by using efficient Bayesian inference attacks ([41, 89]). We assume that an adversary aims to recover the $k_{th}$ client’s private variable $D_k$ from exposed variable $W_k^S$, which is the output of applying certain protection mechanisms on model information (i.e., $W_k^S = M(W_k^O)$). Such a Bayesian inference attack is formally defined as follows.
**Definition 3.1 (Bayesian Inference Attack).** A Bayesian inference attack\(^6\) is an optimization process that aims to infer the private variable \(D_k\) to best fit the exposed information \(W_k^S\) as

\[
d^* = \arg\max_d \log \left( f_{D_k|W_k^S}(d|w) \right)
= \arg\max_d \log \left( \frac{f_{W_k^S|D_k}(w|d)f_{D_k}(d)}{f_{W_k^S}(w)} \right)
= \arg\max_d \left[ \log f_{W_k^S|D_k}(w|d) + \log f_{D_k}(d) \right],
\]

where \(f_{D_k|W_k^S}(d|w)\) is the posterior of \(D_k\) given the protected variable \(W_k^S\). According to Bayes’ theorem, maximizing the log-posterior \(f_{D_k|W_k^S}(d|w)\) on \(D_k\) involves maximizing summation of \(\log(f_{W_k^S|D_k}(w|d))\) and \(\log(f_{D_k}(d))\). The former one aims to find \(D_k\) to best match \(W_k^S\) (maximize the likelihood of \(W_k^S\)), and the latter one aims to make the prior of \(D_k\) more significant. In short, Bayesian inference attack establishes the posterior belief of \(D_k\) conditioned on \(W_k^S\), denoted as \(f_{D_k|W_k^S}\).

The learned conditional distribution \(f_{D_k|W_k^S}\) from the Bayesian inference attack reflects the dependency between \(W_k^S\) and \(D_k\), which determines the amount of information that adversaries may infer about \(D_k\) after observing \(W_k^S\). In case the exposed information \(W_k^S\) is independent of the private data \(D_k\), then the posterior belief \(f_{D_k|W_k^S}\) is guaranteed to be indistinguishable from the prior \(f_{D_k}\) and the Bayesian privacy leakage is zero. This extreme case corresponds to the semantic security in cryptosystems [36].

**Remark:**

1. Bayesian inference attack defined here is a concrete realization of the semi-honest participant assumption. The attacker learns information from other parties and attempts to infer the private information held by other parties. The attacker does not actively inject information on exchanged messages to gain more private information or “poison” the joint model, however.

2. It is required that the time cost of the Bayesian inference attack is polynomial. Given this requirement, adversaries cannot gain any additional information about the private data of clients if the attacking cost increases exponentially, such as against HE [31]. From a practical point of view, in Appendix B.4 we give a detailed analysis of Bayesian inference attack including gradient inversion attack, model inversion attack, and brute-force attack against different protection mechanisms.

3. We formulate three attacks including gradient-inverse attack [28, 83, 87, 89], model inversion attack [27, 41], and brute-force attack (especially for encryption) from the Bayesian inference perspective (see details in Appendix B). These attacking approaches differ in the types of exposed information, including model, model gradients, and model outputs in SFL. Moreover, the prior (denoted as \(F_k^B\)) is an important factor for inferring the private data. For example, if the attacker knows the label of an image is “cat,” he or she may use an averaged “cat” image to initialize the restored image and significantly improve the accuracy of restored data (the influence of prior for the attack is shown in Appendix B.1.1).

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\(^6\)The Bayesian inference framework has been applied to the image restoration problem since the 1960s [7, 15, 30].
3.4 Bayesian Privacy Leakage

With the Bayesian inference attack (introduced in Definition 3.1), we now define Bayesian privacy leakage as a privacy leakage measurement, which quantifies the discrepancy between the posterior belief (with exposure $W^S_k$) and prior belief (without exposure) under the Bayesian inference attack. Let $F^A_k$, $F^O_k$, and $F^B_k$ represent the attacker’s belief distribution about $D_k$ upon observing the protected information, the original information, and without observing any information, respectively. Let $f^{A}_{D_k}$, $f^{O}_{D_k}$ and $f^{B}_{D_k}$ represent the probability density function of $F^A_k$, $F^O_k$, and $F^B_k$. Specifically, $f_{D_k}(d) = \int_{W_k} f_{D_k|W_k}(d|w) dP^S_k(w)$, $f_{D_k}(d) = \int_{W_k} f_{D_k|W_k}(d|w) dP^O_k(w)$, and $f_{D_k}(d) = f_{D_k}(d)$.

Definition 3.2 (Bayesian Privacy Leakage). Let $\epsilon_{p,k}$ represent the privacy leakage of client $k$, which is defined as

$$\epsilon_{p,k} = \sqrt{JS(F^A_k||F^B_k)} = \left[ \frac{1}{2} \int_{D_k} f^{A}_{D_k}(d) \log \frac{f^{A}_{D_k}(d)}{f^{M}_{D_k}(d)} d\mu(d) + \frac{1}{2} \int_{D_k} f^{B}_{D_k}(d) \log \frac{f^{B}_{D_k}(d)}{f^{M}_{D_k}(d)} d\mu(d) \right]^\frac{1}{2},$$

where $f^{M}_{D_k}(d) = \frac{1}{2}(f^{A}_{D_k}(d) + f^{B}_{D_k}(d))$. Furthermore, the Bayesian privacy leakage in SFL resulted from releasing the protected model information is defined as

$$\epsilon_p = \frac{1}{K} \sum_{k=1}^{K} \epsilon_{p,k}. \quad (3)$$

The Bayesian privacy leakage measures the discrepancy between the adversaries’ belief with and without leaked information. Moreover, the Bayesian privacy leakage is averaged with respect to the protected model information variable that is exposed to adversaries.

Remark: Unlike KL divergence, JS divergence is symmetrical and its square root satisfies the triangle inequality\(^7\) [26]. This property allows the derivation of main results illustrated in Theorem 4.1. The definition of Bayesian privacy leakage could be generalized as the maximum privacy leakage over clients.

3.5 Utility Loss in SFL

In addition to privacy leakage, another important concern investigated in our work is the utility loss, which is defined as follows.

Definition 3.3 (Utility Loss). The utility loss is defined as the discrepancy between the utility with the unprotected model information drawn from distribution $P^O_a$ and that drawn from the protected distribution $P^S_a$,

$$\epsilon_u = \frac{1}{K} \sum_{k=1}^{K} \epsilon_{u,k} = \frac{1}{K} \sum_{k=1}^{K} \left[ U_k(P^O_a) - U_k(P^S_a) \right],$$

where $P^O_a$ and $P^S_a$ represent, respectively, the distributions of convergent models with or without any protection mechanism used, and $U_k(P) = E_{D_k} E_{W_k} \frac{1}{|D_k|} \sum_{d \in D_k} U(W_k, d)$ is the expected utility taken with respect to $D_k \sim P_k$ and $W_k \sim P$.

Remark: The model utility evaluates model performance for a variety of learning tasks. For example, model utility is classification accuracy in a classification problem and prediction accuracy in a regression problem.

\(^7\)We employ the property of triangular inequality to establish main results illustrated in Section 4.1. Also see Figure 3 for a pictorial illustration.

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4 A GENERAL FRAMEWORK FOR THE PRIVACY-UTILITY TRADE-OFF

In Section 3, we introduced the definition of privacy leakage and utility loss in SFL. In this section, we first lay down the rigorous analysis of the trade-off between Bayesian privacy leakage and utility loss (no free lunch theorem). Then we formulate the privacy-utility trade-off as an optimization problem, which minimizes the utility loss under the $\epsilon$-Bayesian privacy constraint. Finally, we apply the proposed no free lunch theorem between Bayesian privacy leakage and utility loss into four protection mechanisms in SFL.

4.1 Theoretical Analysis for Privacy-Utility Trade-Off

The idea of privacy preserving is to modify the unprotected information and guarantee that the private data cannot be disclosed to semi-honest adversaries. To protect privacy, the protected information denoted as $W^S_k$ is shared. The utility loss quantifies the increase of expected average loss incurred by the released information $W^S_k$ compared with the utility caused by $W^O_k$. Note that $W^S_k$ and $W^O_k$ respectively follow distributions $P^S_k$ and $P^O_k$. In this setting, we want to characterize the limitation of privacy-preserving mechanisms. It requires to measure the amount of utility that is inevitable to lose to protect privacy. Intuitively, the larger the change exerted on $W^O_k$ is, the more secure the privacy protection is, and meanwhile, the less accurate the output is. We first measure the extent of modification using the total variation distance between the unprotected distribution and the protected distribution. Then we use this distance as a key element for connecting the privacy leakage and the utility loss. Note that the union of $W^S_k$ (the support of $P^S_k$) and $W^O_k$ (the support of $P^O_k$) is denoted as $W_k$.

Definition 4.1 (Optimal Parameters). Let $W^*_a$ represent the set of parameters achieving the maximum utility. Specifically,

$$W^*_a = \operatorname{argmax}_{w \in W^*_a} \frac{1}{K} \sum_{k=1}^{K} U_k(w),$$

where $U_k(w) = \mathbb{E}_{D_k} \left[ \frac{1}{|D_k|} \sum_{d \in D_k} U(w, d) \right]$ is the expected utility taken over $D_k$ sampled from distribution $P_k$.

Definition 4.2 (Near-optimal Parameters). Let $W^S_a$ represent the support of the protected distribution of the aggregated model information. Given a non-negative constant $c$, the near-optimal parameters is defined as

$$W^*_c = \left\{ w \in W^S_a : \left| \frac{1}{K} \sum_{k=1}^{K} U_k(w^*) - \frac{1}{K} \sum_{k=1}^{K} U_k(w) \right| \leq c, \forall w^* \in W^*_a \right\}.$$

Assumption 4.1. Let $\Delta$ be the maximum constant that satisfies

$$\int_{W^S_a} p^S_{W^*_a}(w) 1 \{ w \in W^*_a \} dw \leq \frac{TV(P^O_a||P^S_a)}{2}, \quad (4)$$

where $p^S_{W^*_a}$ represents the probability density function of the protected model information $W^S_a$. We assume that $\Delta$ is positive (i.e., $\Delta > 0$).

Remark:

(1) This assumption implies that the cumulative density of the near-optimal parameters as defined in Definition 4.2 is bounded. This assumption excludes the cases where the utility function is constant or indistinguishable between the optimal parameters and a certain fraction of parameters.
(2) Note that $\Delta_k$ is independent of the threat model of the adversary and $\Delta_k$ is a constant when the protection mechanism, the utility function, and the datasets are fixed.

The following theorem represents the central result of this work. As a form of the no free lunch theorem, it states that privacy protection and utility enhancement of a joint model in our semi-honest SFL setting is bounded by a constant. If one increases privacy protection, one risks losing some utility of the joint model, and vice versa.

**Theorem 4.1 (No Free Lunch Theorem (NFL) for Security and Utility).** Let $\epsilon_p$ be defined in Definition 3.2. We have that

$$C_1 \leq \epsilon_p + \frac{1}{K} \sum_{k=1}^{K} \left( e^{\xi_k} - 1 \right) \cdot TV\left( p^{O}_k || p^{S}_k \right).$$

Furthermore, let $\epsilon_u$ be defined in Definition 3.3 at the convergence step, and with Assumption 4.1 we have that

$$C_1 \leq \epsilon_p + C_2 \cdot \epsilon_u,$$

in which

- $\xi = \max_{k \in [K]} \xi_k$, where $\xi_k = \max_{w \in W_k, d \in D_k} | \log \left( \frac{f_{S_k}(d|w)}{f_{O_k}(d)} \right) |$ represents the maximum privacy leakage over all possible information $w$ released by client $k$, and $[K] = \{1, 2, \ldots, K\}$. $\xi$ is a constant independent of the protection mechanism.
- $C_1 = \frac{1}{K} \sum_{k=1}^{K} \sqrt{JS\left( F^{O}_k || F^{B}_k \right)}$ is a constant representing the averaged square root of JS divergence between the adversary’s belief distribution about the private information of client $k$ before and after observing the unprotected parameter. This constant is independent of the protection mechanisms.
- $C_2 = \frac{\gamma}{4\alpha} \left( e^{\xi_k} - 1 \right)$ is a constant once the protection mechanisms, the utility function, and the datasets are fixed, where $\gamma = \sum_{k=1}^{K} TV\left( p^{O}_k || p^{S}_k \right) \cdot 8$.

Theorem 4.1 illustrates that the summation of the average privacy leakage of the clients and the utility loss is lower bounded by a problem-dependent constant. As shown in Figure 3, Equation (6) is based on the triangle inequality of the JS measure adopted in Bayesian privacy leakage (Theorem 4.1). It essentially dedicates that *one has to trade a decrease of the privacy leakage ($\epsilon_p$) with a certain degree of the increase of the utility loss ($\epsilon_u$), and vice versa*. This principle bears an intriguing similarity to the no free lunch theorem for optimization and machine learning [80], which dedicated that “if an algorithm performs well on a certain class of problems then it necessarily pays for that with degraded performance on the set of all remaining problems.” We therefore name Equation (6) “No free lunch theorem (NFL) for security and utility” in FL.

**Remark:** $TV\left( p^{O}_k || p^{S}_k \right)$ in Equation (5) provides an upper bound of the modification brought by the protection mechanism, in terms of the distance between distributions of model information with and without protection methods being applied.

### 4.2 Privacy-Preserving Optimization

In the privacy-preserving machine learning scenario, clients aim to maintain maximal possible model utility without disclosing private information beyond an acceptable level. We can view this trade-off as an optimization problem. The goal is to minimize the utility loss subject to the

See details of analyzing of the value of $\gamma$ in Appendix C.1.

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Fig. 3. $\epsilon_{p,k} = \sqrt{JS(F^A_k || F^B_k)}$ as is introduced in Definition 3.2. Notice that the square root of the JS divergence satisfies the triangle inequality. It implies that the summation of $\epsilon_{p,k}$ and $\sqrt{JS(F^A_k || F^O_k)}$ is at least $\sqrt{JS(F^O_k || F^B_k)}$, which is a problem-dependent constant. Moreover, $\sqrt{JS(F^A_k || F^O_k)}$ is at most $\frac{1}{2}(e^{2\xi} - 1) \cdot TV(P^O_k || P^S_k)$. Consequently, the summation of $\epsilon_{p,k}$ and $\frac{1}{2}(e^{2\xi} - 1) \cdot TV(P^O_k || P^S_k)$ is at least $\sqrt{JS(F^O_k || F^B_k)}$. The total variation distance between the unprotected distribution and protected distribution is then used as a key element for connecting the privacy leakage and the utility loss.

privacy-preserving constraint. Let the Bayesian privacy leakage $\epsilon_p$ be defined as Definition 3.2. We say an SFL system guarantees $\epsilon$-Bayesian privacy if it holds that

$$\epsilon_p \leq \epsilon. \quad (7)$$

$\epsilon$-Bayesian privacy guarantees that the Bayesian privacy leakage of the SFL system never exceeds a prescribed threshold $\epsilon$ as the security requirement. For a federated machine learning system, the goal is to find a protection mechanism $M$ that achieves the minimum aggregated utility loss. These two requirements can be cast as a constrained optimization problem over the protection mechanism:

$$\min_M \epsilon_u := \frac{1}{K} \sum_{k=1}^{K} \left[ U_k \left( P^O_k \right) - U_k \left( P^S_k \right) \right],$$

subject to $\epsilon_p = \frac{1}{K} \sum_{k=1}^{K} \sqrt{JS(F^A_k || F^B_k)} \leq \epsilon. \quad (8)$

Remark:

(1) The constant $\epsilon$ imposed in Equation (8) provides a privacy security guarantee that one can achieve, regardless of a variety of Bayesian inference attacks that may be launched by adversaries.

(2) This constrained optimization problem is closely related to the optimization formulated [19], which instead aims to minimize the information leakage while the utility loss is guaranteed to be less than a given threshold.

4.3 Applications of Privacy-Utility Trade-Off

In this section, we apply the no free lunch theorem between the Bayesian privacy leakage and utility loss on four privacy-preserving mechanisms: randomization [1, 34, 75], sparsity [38, 69, 70],
HE [31, 84], and secret sharing [4, 6, 68]. We analyze the influence of specific protection mechanisms’ parameters on the fluctuation of privacy leakage and utility loss.

4.3.1 Randomization Mechanism. Randomization mechanism is a natural choice since its noise parameter (e.g., $\sigma$) allows a flexible control of the trade-off. In SFL, one of the randomization methods is to add random noise such as Gaussian noise to model gradients [1, 34, 75]. Let $W_k^O \in \mathcal{W}_k^O$ be the parameter sampled from distribution $P_k^O = \mathcal{N}(\mu_0, \Sigma_0)$, $k = 1, \ldots, K$, where $\mu_0 \in \mathbb{R}^n$, $\Sigma_0 = \text{diag}(\sigma_1^2, \ldots, \sigma_n^2)$ is a diagonal matrix. Adding noise in the parameter could be expressed as $W_k^S = W_k^O + \epsilon_k$, where $\epsilon_k \sim \mathcal{N}(0, \Sigma_\epsilon)$ and $\Sigma_\epsilon = \text{diag}(\sigma_\epsilon^2, \ldots, \sigma_\epsilon^2)$. Therefore, $W_k^S$ follows the distribution $P_k^S = \mathcal{N}(\mu_0, \Sigma_0 + \Sigma_\epsilon)$. The server aggregates all protected parameters as $W_a^S = \frac{1}{K} \sum_{k=1}^K (W_k^O + \epsilon_k)$ following the distribution $P_a^S = \mathcal{N}(\mu_0, \Sigma_0/K + \Sigma_\epsilon/K)$. The following theorem establishes bounds for the privacy leakage and utility loss using the variance of the noise $\sigma_\epsilon^2$ (see Appendix C.1 for the full proof).

**Theorem 4.2.** For the randomization mechanism by adding Gaussian noise, the privacy leakage and utility loss are bounded using variance of the Gaussian noise $\sigma_\epsilon^2$,

$$C_1 \leq \epsilon_p + \frac{C_3}{K} \min \left\{ 1, \sigma_\epsilon^2 \sum_{i=1}^n \frac{1}{\sigma_i^2} \right\}, \quad (9)$$

and

$$\epsilon_u \leq C_4 \min \left\{ 1, \sigma_\epsilon^2 \sum_{i=1}^n \frac{1}{\sigma_i^2} \right\}, \quad (10)$$

where $C_1 = \frac{1}{K} \sum_{k=1}^K \sqrt{JS(P_k^O || P_k^{O^2})}$ and $C_3 = (e^{\frac{\sigma_\epsilon^2}{2}} - 1)/2$ are two constants independent of the protection mechanisms adopted, and $C_4$ is a constant satisfying that $U(w, d) \leq C_4$ for any $w \in \mathcal{W}_k$ and $d \in \mathcal{D}_k$.

Remark: In this application, we assume $W_k^O$ follows Gaussian distribution $\mathcal{N}(\mu_0, \Sigma_0)$. In particular, when $\Sigma_0$ tends to zero, the Gaussian distribution degenerates into the one-point distribution when the algorithm converges. Moreover, the estimation of the variable $\gamma$ in Theorem 4.1 is shown in Appendix C.1.

**Proposition 4.3.** If $\sigma_\epsilon^2 \sum_{i=1}^n \frac{1}{\sigma_i^2} \leq 1$, then we have

- The Bayesian privacy leakage $\epsilon_p$ is at least $C_1 - \frac{C_1}{K} \sigma_\epsilon^2 \sum_{i=1}^n \frac{1}{\sigma_i^2}$, which is a decreasing function of $\sigma_\epsilon$.
- The utility loss $\epsilon_u$ is at most $C_4 \sigma_\epsilon^2 \sum_{i=1}^n \frac{1}{\sigma_i^2}$, which is an increasing function of $\sigma_\epsilon$.

Proposition 4.3 demonstrates that when the variance of adding noise $\sigma_\epsilon$ decreases, the lower bound of Bayesian privacy leakage $\epsilon_p$ increases. In particular, when $\sigma_\epsilon = 0$, $\sigma_\epsilon^2 \sum_{i=1}^n \frac{1}{\sigma_i^2} = 0$. Therefore, $\epsilon_u = 0$ and $\epsilon_p \geq C_1$. As a result, at least $C_1$ privacy leakage might be incurred due to the exposure of unprotected model information to adversaries who may launch Bayesian inference attacks. This theoretical analysis is in accordance with the empirical evidence demonstrated elsewhere [41, 89].

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\(^9\)Secure hardware [54] was used in FL, but we do not include it in our analysis since it requires to send private data to a trustworthy third party that violates the semi-honest adversaries setting considered in this article.
4.3.2 Sparsity Mechanism. The trade-offs between privacy and utility change with distinct extents of sparsity. In SFL, clients launch the sparsity mechanism by uploading the partial parameters to the server. Specifically, each client uploads $d$ of total $n$ dimensions to the server. Let $W^O_k \in \mathcal{W}^O_k$ represent the parameter sampled from distribution $F^O_k = \mathcal{N}(\mu_0, \Sigma_0)$, where $\mu_0 = (\mu_u, \mu_o), \mu_u \in \mathbb{R}^d, \mu_o \in \mathbb{R}^{n-d}$ and $\Sigma_0 = \text{diag}(\Sigma_u^{d \times d}, \Sigma_o^{(n-d) \times (n-d)})$ is a diagonal matrix. Without loss of generality, we assume each client uploads the first $d$ dimensions to the server. We assume the vector composed by the last $(n - d)$ dimensions follows a Gaussian distribution denoted as $\mathcal{N}(\mu_g, \Sigma_g)$, where $\Sigma_g$ is a diagonal matrix. In this setting, the protected model information follows $P^S_k \sim \mathcal{N}(\mu, \Sigma)$, where $\mu = (\mu_u, \mu_g), \Sigma = \text{diag}(\Sigma_u^{d \times d}, \Sigma_g^{(n-d) \times (n-d)})$. Then the server aggregates all uploaded parameters of clients as $W^S_a = \frac{1}{K} \sum_{k=1}^K W^S_k$ following the distribution $\mathcal{N}(\mu, \Sigma/K)$, and $W^O_k = \frac{1}{K} \sum_{k=1}^K W^O_k$ follows the distribution $\mathcal{N}(\mu_0, \Sigma_0/K)$. The following theorem illustrates bounds for privacy leakage and utility loss using the dimension of the uploaded model information $d$. The full proof is deferred to Appendix C.2.

**Theorem 4.4.** For the sparsity mechanism by uploading partial information to the server, denote $h(\mu_1, \Sigma_1, \mu_2, \Sigma_2) = (1 - \frac{\det(\Sigma_1)^{1/4} \det(\Sigma_2)^{1/4}}{\det(\Sigma_{12}^{(d \times d)})^{1/2}}) \exp(-\frac{1}{2} (\mu_1 - \mu_2)^T (\Sigma_{12}^{(d \times d)})^{-1} (\mu_1 - \mu_2))^{1/2}$, we have

$$C_1 \leq c_p + C_3 \cdot h(\mu_o, \mu_g, \Sigma_o, \Sigma_g),$$

and

$$\epsilon_u \leq C \cdot h(\mu_o, \mu_g, \Sigma_o, \Sigma_g),$$

where $C_1 = \frac{1}{K} \sum_{k=1}^K \sqrt{JS(F^O_k || F^S_k)}$, $C_3 = \frac{\sqrt{2}(e^{d/2} - 1)}{2}$ are two constants independent of the protection mechanisms adopted, and $C$ is a constant satisfying that $\sqrt{2}U(w, d) \leq C$ for any $w \in \mathcal{W}_k$ and $d \in \mathcal{D}_k$.

**Proposition 4.5.** For the sparsity mechanism by uploading partial information to the server, we have the following:

- The Bayesian privacy leakage $\epsilon_p$ is at least $C_1 - C_3 \cdot h(\mu_o, \mu_g, \Sigma_o, \Sigma_g)$, which is an increasing function of the dimension of uploaded model information $d$.

- The utility loss $\epsilon_u$ is at most $C \cdot h(\mu_o, \mu_g, \Sigma_o, \Sigma_g)$, which is a decreasing function of the dimension of uploaded model information $d$.

Proposition 4.5 demonstrates that when the dimension of uploaded model information $d$ increases, the lower bound of Bayesian privacy leakage $\epsilon_p$ increases, and the upper bound of utility loss $\epsilon_u$ decreases. In particular, $d = n$ implies that the server uploads all parameters to the server, then $h(\mu_o, \mu_g, \Sigma_o, \Sigma_g) = 0$. Therefore, $\epsilon_p \geq C_1$ and $\epsilon_u = 0$. It shows that at least $C_1$ Bayesian privacy leakage occurs, but the utility is not sacrificed.

4.3.3 Homomorphic Encryption. HE [31, 32, 64] allows certain computation (e.g., addition) to be performed directly on ciphertexts, without decrypting them first. Such characteristic allows applying HE into FL to protect privacy because the server only accesses the uploaded parameters of users on ciphertexts instead of clients’ plaintext directly [2, 74, 85, 86]. Specifically, in FL, the uploaded model weights or gradients have been encrypted by the clients themselves. Therefore, it is hard for adversaries to espy the exposed information and infer clients’ private data. The following theorem evaluates the privacy leakage and utility loss of the approximate eigenvector method [33], which is a widely used method of HE.

**Theorem 4.6.** For the encryption mechanism that encrypts the model information ($W^O_k = \text{Enc}(W^O_k)$ using the approximate eigenvector method),
If the private key is unknown for the server (adversary), then $\epsilon_p = 0$ and $\epsilon_u \geq C_1 C_2$.

If the private key is known for the server (adversary), then $\epsilon_u = 0$ and $\epsilon_p \geq C_1$.

Remark:

1. On one hand, if the private key is unknown, the Bayesian privacy leakage $\epsilon_p$ is zero because encryption satisfies semantic security, with which $D_k$ is independent of $W^S_k$. Moreover, since the private key of HE is unknown and the global model $\text{Enc}(W^O_a)$ remains encrypted, the utility evaluated by server is far away from the optimal utility (utility loss is large).

2. On the other hand, if the private key is mistakenly disclosed to server (adversary), then the server could decrypt the global model and utility loss $\epsilon_u = 0$ in the ideal case. Moreover, the privacy leakage $\epsilon_p$ is large.

4.4 Secret Sharing

Secret sharing [4, 6, 68] was developed to distribute a secret among a group of participants. However, it requires extensive exchange of messages and entails a communication overhead not viable in many FL settings. Let $W^O_k$ represent the original model information that follows a uniform distribution over $[c^1_k - \delta, c^1_k + \delta] \times [c^2_k - \delta, c^2_k + \delta] \times \cdots \times [c^n_k - \delta, c^n_k + \delta]$, $W^S_k$ represent the distorted model information that follows a uniform distribution over $[c^1_k - a^1_k + b^1_k, c^1_k + b^1_k] \times [c^2_k - a^2_k, c^2_k + b^2_k] \times \cdots \times [c^n_k - a^n_k + b^n_k, c^n_k + b^n_k]$, and $0 < \delta < a^i_k, b^i_k, \forall i = 1, 2, \ldots, n$. The following theorem measures utility loss and provides lower bounds for privacy leakage.

**Theorem 4.7.** For the secret sharing mechanism, the privacy leakage

$$\epsilon_{p,k} \geq \sqrt{\text{JS}(F^O_k || F^S_k)} - \frac{1}{2}(e^{2\xi} - 1) \cdot \left(1 - \prod_{j=1}^{m} \left(\frac{2\delta}{b^j_k + a^j_k}\right)\right).$$

Furthermore, we have that

$$\epsilon_u = 0.$$  \hspace{1cm} (14)

Remark: For the secret sharing mechanism, there does not exist a continuous trade-off between privacy and utility, but Equation (5) in Theorem 4.1 still holds.

5 DISCUSSION

The section illustrates the comparison of the proposed Bayesian privacy with respect to DP [25], BDP [73], Local DP [20], and IP [19]. A brief summary of the comparison is summarized in Table 2.

5.1 The Comparison with DP and Its Variants

We illustrate the comparison of Bayesian privacy with respect to DP and its variants as follows:

- First, the privacy-utility trade-off analyzed in DP is essentially characterized by a reciprocal relation as shown by Theorem 8.7 of Dwork and Roth [25], which dedicates that the outputs protected by an $\epsilon$-differentially private mechanism have the maximum error $\Omega(\frac{1}{\epsilon})$. This trade-off analysis is in contrast to the no free lunch theorem (Theorem 4.1) in the present article, which dictates that the sum of privacy loss and utility loss is greater than a problem-dependent constant.

- Second, it is shown that a $\xi$-maximum Bayesian privacy-preserving mapping $f_{W|S}(\cdot)$ is $(2\xi)$-differentially private (see Appendix D for details).
Table 2. Comparison of Privacy-Utility Trade-Offs with Bayesian Privacy and Related Privacy Measurements, Namely DP [25], Local DP (LDP) [20], BDP [73], and IP [19]

| Protection         | Privacy-Utility Trade-Off | Applied to FL |
|-------------------|---------------------------|---------------|
| DP [25]           | Error \( n \in \Omega(\frac{1}{\epsilon^2}) \) (Theorem 8.7 [25]) | [67, 73] |
| Local DP [20]     | \( c_1 \min\{1, \frac{1}{\sqrt{n}a^2}, \frac{d}{n\alpha^2}\} \leq \Omega_n[48, 73] \) \[67, 88\] |
| BDP [73]          | N.A.                      | [72] |
| IP [19]           | Min IP s.t. distortion    | No            |
| Bayesian Privacy  | C_1 \leq \epsilon_{P} + C_{2}\epsilon_{u} No free lunch theorem | This article |

- Third, the privacy-utility trade-off established by the local DP dedicates that, for an \( \alpha \)-private estimator, its minimax error bound (in utility) is greater than a constant that is proportional to \( \frac{1}{\alpha^2} \) (see Theorem 1 in the work of Duchi et al. [20]).
- Fourth, BDP was also proposed to measure the privacy loss following Bayesian rule [20]. Nevertheless, BDP estimated the posterior of query outputs (denoted by \( S \)) given private data (denoted by \( x \)). This is in sharp contrast to the proposed Bayesian privacy, which models adversaries’ posterior belief of private data (denoted by \( S \)) given publicly exchanged information (denoted by \( W^S \)). Moreover, no privacy-utility trade-off analysis was provided for BDP in the work of Duchi et al. [20].
- Last but not least, DP and its variants are not amenable to include encryption as a protection mechanism. To our best knowledge, the SFL framework based on Bayesian inference attack is the first that is applicable to different privacy-protection mechanisms, including randomization in DP [1, 34, 75], sparsity [38, 69, 70], HE [31, 84], and secret sharing [4, 6, 68].

5.2 The Relationship Between Bayesian Privacy and IP
There are three fundamental differences between Bayesian privacy and IP [19], as shown in the following:
- One fundamental difference between Bayesian privacy and IP lies in the fact that Bayesian privacy is formulated within an SFL setting as illustrated in Section 3, although it is unclear how to apply IP for delineating the privacy-utility trade-off in distributed learning scenarios.
- As per the privacy definitions, Bayesian privacy involves the averaged gain of information about private data that an adversary may obtain by observing publicly exposed model information (see Definition 3.2 in this article). However, IP is defined based on the bound of such information gain (see Definition 6 in the work of Calmon and Fawaz [19]). Note that we also give the bounds of Bayesian privacy in Equation (7), which is nevertheless based on the average gain.
- In terms of the privacy-utility trade-off, the optimal trade-off in Bayesian privacy is cast as the solution of a constrained optimization where the goal is to minimize utility loss subject to the hard constrain of maintaining Bayesian privacy leakage less than an acceptable threshold (the problem in Equation (8)). In contrast, the optimal trade-off in IP is to minimize the
privacy leakage subject to a given utility loss constrain (Definition 3 in the work of Calmon and Fawaz [19]).

Remark: It is worth noting that the Bayesian privacy is measured using JS divergence instead of KL divergence. Unlike KL divergence, JS divergence is symmetrical and its square root satisfies the triangle inequality. The property of triangular inequality of JS divergence facilitates the derivation of our main results illustrated in Section 4.1.

6 CONCLUSION
In this article, we proposed a statistical framework in SFL based on Bayesian inference attack to take into account two compelling requirements: the protection of private data as well as the maximization of model utility in SFL. The proposed framework is based on the cornerstone definition of Bayesian privacy leakage that quantitatively measures the amount of information about private data that semi-honest adversaries may infer from the espied information during the SFL learning process. Then the trade-off privacy and utility is formulated as an optimization problem in which the goal is to find a modified distribution of exchanged model information that achieves the smallest utility loss without incurring privacy loss above an acceptable level. The theoretical analysis of the trade-off then leads us to the no free lunch theorem for FL, which dictates that one has to trade a high privacy persevering guarantee with a certain degree of utility loss, and vice versa. This trade-off analysis is applicable to protection mechanisms including randomization [1, 34, 75], sparsity [38, 69, 70], HE [31, 84], and secret sharing [4, 6, 68], which are special cases of the general theorem proved in Theorem 4.1.

The analysis of privacy-utility trade-off in SFL, to our best knowledge, is the first work that takes into account different specific protection mechanisms, including DP and HE, in a unified framework. Although the main result disclosed in this article is inspiring, we plan to apply the technique we developed during this research endeavor to explore in the future the trade-off between efficiency and privacy, which provides a more comprehensive view of SFL.

APPENDICES
A THEORETICAL ANALYSIS USING JS DIVERGENCE
Before introducing the analysis in detail, we first illustrate the key characteristics of the metric for measuring such a relationship.

Property 1. The square root of the JS divergence satisfies the triangle inequality. Specifically,

\[ \sqrt{\text{JS}(F^O_k || F^B_k)} - \sqrt{\text{JS}(F^A_k || F^B_k)} \leq \sqrt{\text{JS}(F^A_k || F^O_k)}. \]

Property 2. From the AM-GM inequality, we have

\[ \frac{f^A_{D_k}(d)}{f^M_{D_k}(d)} \geq \frac{f^B_{D_k}(d)}{f^B_{D_k}(d)}. \]

A.1 The Quantitative Relationship Between TV\((P^O_k || P^S_k)\) and \(\epsilon_{p,k}\)

Lemma A.1. Let \(P^O_k\) and \(P^S_k\) represent the distribution of the parameter of client k before and after being protected. Let \(F^A_k\) and \(F^O_k\) represent the belief of client k about D after observing the protected and original parameter. Then we have

\[ \text{JS}(F^A_k || F^O_k) \leq \frac{1}{4}(e^{2\xi} - 1)^2 \text{TV}(P^O_k || P^S_k)^2. \]
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Proof. Let $F_k^M = \frac{1}{2}(F_k^A + F_k^O)$. We have

$$JS\left(F_k^A || F_k^O\right) = \frac{1}{2} \left[ KL\left(F_k^A, F_k^O\right) + KL\left(F_k^O, F_k^M\right)\right]$$

$$= \frac{1}{2} \left[ \int_{D_k} f_{D_k}^A(d) \log \frac{f_{D_k}^A(d)}{f_{D_k}^M(d)} \, d\mu(d) + \int_{D_k} f_{D_k}^O(d) \log \frac{f_{D_k}^O(d)}{f_{D_k}^M(d)} \, d\mu(d)\right]$$

$$\leq \frac{1}{2} \int_{D_k} \left| f_{D_k}^A(d) - f_{D_k}^O(d) \right| \log \frac{f_{D_k}^M(d)}{f_{D_k}^A(d)} \, d\mu(d),$$

where the inequality is due to $\frac{f_{D_k}^A(d)}{f_{D_k}^M(d)} \leq \frac{f_{D_k}^M(d)}{f_{D_k}^A(d)}$.

Bounding $|f_{D_k}^A(d) - f_{D_k}^O(d)|$.

Let $\mathcal{U}_k = \{w \in \mathcal{W}_k : dP_k^S(w) - dP_k^O(w) \geq 0\}$, and $\mathcal{V}_k = \{w \in \mathcal{W}_k : dP_k^S(w) - dP_k^O(w) < 0\}$. Then we have

$$|f_{D_k}^A(d) - f_{D_k}^O(d)| = \int_{\mathcal{U}_k} f_{D_k|\mathcal{W}_k}(d|w)[dP_k^S(w) - dP_k^O(w)]$$

$$= \int_{\mathcal{U}_k} f_{D_k|\mathcal{W}_k}(d|w)[dP_k^S(w) - dP_k^O(w)] + \int_{\mathcal{V}_k} f_{D_k|\mathcal{W}_k}(d|w)[dP_k^S(w) - dP_k^O(w)]$$

$$\leq \left(\inf_{w \in \mathcal{W}_k} f_{D_k|\mathcal{W}_k}(d|w) - \sup_{w \in \mathcal{W}_k} f_{D_k|\mathcal{W}_k}(d|w)\right) \int_{\mathcal{U}_k} [dP_k^S(w) - dP_k^O(w)]. \tag{15}$$

Notice that

$$\sup_{w \in \mathcal{W}_k} f_{D_k|\mathcal{W}_k}(d|w) - \inf_{w \in \mathcal{W}_k} f_{D_k|\mathcal{W}_k}(d|w) = \inf_{w \in \mathcal{W}_k} f_{D_k|\mathcal{W}_k}(d|w) \left(\frac{\sup_{w \in \mathcal{W}_k} f_{D_k|\mathcal{W}_k}(d|w)}{\inf_{w \in \mathcal{W}_k} f_{D_k|\mathcal{W}_k}(d|w)} - 1\right).$$

From the definition of $\xi$, we know that for any $w \in \mathcal{W}_k$,

$$e^{-\xi} \leq \frac{f_{D_k|\mathcal{W}_k}(d|w)}{f_{D_k}(d)} \leq e^{\xi}.$$

Therefore, for any pair of parameters $w, w' \in \mathcal{W}_k$, we have

$$\frac{f_{D_k|\mathcal{W}_k}(d|w)}{f_{D_k|\mathcal{W}_k}(s|w')} = \frac{f_{D_k|\mathcal{W}_k}(d|w)}{f_{D_k}(d)} \frac{f_{D_k|\mathcal{W}_k}(s|w')}{f_{D_k}(d)} \leq e^{2\xi}.$$

Therefore, the first term of Equation (15) is bounded by

$$\sup_{w \in \mathcal{W}_k} f_{D_k|\mathcal{W}_k}(d|w) - \inf_{w \in \mathcal{W}_k} f_{D_k|\mathcal{W}_k}(d|w) \leq \inf_{w \in \mathcal{W}_k} f_{D_k|\mathcal{W}_k}(d|w)(e^{2\xi} - 1). \tag{16}$$

From the definition of total variation distance, we have

$$\int_{\mathcal{U}_k} [dP_k^S(w) - dP_k^O(w)] = \text{TV}\left(P_k^O || P_k^S\right). \tag{17}$$
Combining Equations (16) and (17), we have

\[
|f_{D_k}^{\mathcal{A}}(d) - f_{D_k}^{\mathcal{O}}(d)| = \left( \sup_{w \in \mathcal{W}_k} f_{D_k \mid \mathcal{W}_k}(d \mid w) - \inf_{w \in \mathcal{W}_k} f_{D_k \mid \mathcal{W}_k}(d \mid w) \right) \int_{\mathcal{U}_k} \left[ dP_k^S(w) - dP_k^O(w) \right] \\
\leq \inf_{w \in \mathcal{W}_k} f_{D_k \mid \mathcal{W}_k}(d \mid w)(e^{2\xi} - 1)TV(P_k^O \mid P_k^S).
\]  

(18)

Bounding \(|\log \frac{f_{D_k}^{\mathcal{M}}(d)}{f_{D_k}^{\mathcal{O}}(d)}|\).

We have that

\[
\left| \log \frac{f_{D_k}^{\mathcal{M}}(d)}{f_{D_k}^{\mathcal{O}}(d)} \right| \leq \frac{|f_{D_k}^{\mathcal{M}}(d) - f_{D_k}^{\mathcal{O}}(d)|}{\min\{f_{D_k}^{\mathcal{M}}(d), f_{D_k}^{\mathcal{O}}(d)\}} \\
= \frac{|f_{D_k}^{\mathcal{A}}(d) - f_{D_k}^{\mathcal{O}}(d)|}{2 \min\{f_{D_k}^{\mathcal{M}}(d), f_{D_k}^{\mathcal{O}}(d)\}} \\
\leq \frac{1}{2}(e^{2\xi} - 1)TV(P_k^O \mid P_k^S),
\]

(19)

where the first inequality is due to Lemma A.3, and the third inequality is due to \(\min\{f_{D_k}^{\mathcal{M}}(d), f_{D_k}^{\mathcal{O}}(d)\} \geq \min\{f_{D_k}^{\mathcal{A}}(d), f_{D_k}^{\mathcal{O}}(d)\} \geq \inf_{w \in \mathcal{W}_k} f_{D_k \mid \mathcal{W}_k}(d \mid w)\).

Combining Equations (18) and (19), we have

\[
JS\left(F_k^{\mathcal{A}} \mid \mid P_k^S\right) \leq \frac{1}{2} \int_{\mathcal{D}_k} \left| (f_{D_k}^{\mathcal{A}}(d) - f_{D_k}^{\mathcal{O}}(d)) \right| \left| \log \frac{f_{D_k}^{\mathcal{M}}(d)}{f_{D_k}^{\mathcal{O}}(d)} \right| d\mu(d) \\
\leq \frac{1}{4}(e^{2\xi} - 1)^2TV(P_k^O \mid P_k^S)^2 \int_{\mathcal{D}_k} \inf_{w \in \mathcal{W}_k} f_{D_k \mid \mathcal{W}_k}(d \mid w) d\mu(d) \\
\leq \frac{1}{4}(e^{2\xi} - 1)^2TV(P_k^O \mid P_k^S)^2.
\]

\[\square\]

**Lemma A.2.** Let \(\epsilon_{p,k}\) be defined in Definition 3.2. Let \(P_k^O\) and \(P_k^S\) represent the distribution of the parameter of client \(k\) before and after being protected. Let \(F_k^{\mathcal{B}}\) and \(F_k^{\mathcal{A}}\) represent the belief of client \(k\) about \(D\) before and after observing the released parameter. Then we have

\[
\sqrt{JS(F_k^O \mid \mid F_k^B)} \leq \epsilon_{p,k} + \frac{1}{2}(e^{2\xi} - 1)TV(P_k^O \mid P_k^S).
\]

(20)

Consequently, we have

\[
\frac{1}{K} \sum_{k=1}^{K} \sqrt{JS(F_k^O \mid \mid F_k^B)} \leq \epsilon_{p} + \frac{1}{K} \sum_{k=1}^{K} \frac{1}{2}(e^{2\xi} - 1)TV(P_k^O \mid P_k^S).
\]

**Proof.** From Lemma A.1, we know that

\[
JS(F_k^{\mathcal{A}} \mid \mid F_k^O) \leq \frac{1}{4}(e^{2\xi} - 1)^2TV(P_k^O \mid P_k^S)^2.
\]

This further implies that

\[
\sqrt{JS(F_k^{\mathcal{A}} \mid \mid F_k^O)} \leq \frac{1}{2}(e^{2\xi} - 1)TV(P_k^O \mid P_k^S).
\]
Notice that the square root of the JS divergence satisfies the triangle inequality. Then we have that
\[ \sqrt{JS(F^O_k || F^B_k)} - \sqrt{JS(F^A_k || F^B_k)} \leq \sqrt{JS(F^A_k || F^O_k)} \leq \frac{1}{2}(e^{2\xi} - 1)TV(P^O_k || P^S_k). \]
Therefore, we have
\[ \sqrt{JS(F^O_k || F^B_k)} \leq \sqrt{JS(F^A_k || F^B_k) + \frac{1}{2}(e^{2\xi} - 1)TV(P^O_k || P^S_k)} = \epsilon_{p,k} + \frac{1}{2}(e^{2\xi} - 1)TV(P^O_k || P^S_k). \]

**Lemma A.3 ([20]).** For two positive numbers \(a\) and \(b\), we have that
\[ \left| \log \left( \frac{a}{b} \right) \right| \leq \frac{|a - b|}{\min\{a, b\}}. \]

### A.2 The Quantitative Relationship Between \(TV(P^O_a || P^S_a)\) and \(\epsilon_u\)

**Lemma A.4.** Let Assumption 4.1 hold and \(\epsilon_u\) be defined in Definition 3.3. Let \(P^O_a\) and \(P^S_a\) represent the distribution of the aggregated parameter before and after being protected. Then we have
\[ \epsilon_u \geq \frac{\Delta}{2} \cdot TV(P^O_a || P^S_a). \]

**Proof.** Let \(\mathcal{U}_a = \{w \in \mathcal{W}_a : dP^S_a(w) - dP^O_a(w) \geq 0\}\), and \(\mathcal{V}_a = \{w \in \mathcal{W}_a : dP^S_a(w) - dP^O_a(w) < 0\}\), where \(\mathcal{W}_a\) represents the union of the supports of \(P^S_a\) and \(P^O_a\).

For any \(w \in \mathcal{V}_a\), the definition of \(\mathcal{V}_a\) implies that \(dP^O_a(w) > dP^S_a(w) \geq 0\). Therefore, \(w\) belongs to the support of \(P^O_a\), which is denoted as \(\mathcal{W}^O_a\). Therefore, we have that
\[ \mathcal{V}_a \subset \mathcal{W}^O_a. \] (21)

Similarly, we have that
\[ \mathcal{U}_a \subset \mathcal{W}^S_a. \] (22)

It is assumed that the utility of the aggregated model information achieves the maximal value at the convergence step [40, 52]. Therefore, we have that
\[ \mathcal{W}^O_a \subset \mathcal{W}^*_a. \] (23)

Notice that from the definition of \(\mathcal{W}^*_a\), for any \(w \in \mathcal{W}_a\) and \(w^* \in \mathcal{W}^*_a\) we have that
\[ \frac{1}{K} \sum_{k=1}^{K} U_k(w^*) \geq \frac{1}{K} \sum_{k=1}^{K} U_k(w). \] (24)

Let \(\Delta\) be a positive constant defined in Assumption 4.1. From Definition 4.2, we have
\[ \mathcal{W}_\Delta = \left\{ w \in \mathcal{W}^S_a : \left| \frac{1}{K} \sum_{k=1}^{K} U_k(w^*) - \frac{1}{K} \sum_{k=1}^{K} U_k(w) \right| \leq \Delta, \forall \mathcal{W}^* \in \mathcal{W}^*_a \right\}, \]
which implies that for any \(w \in \mathcal{W}^S_a \setminus \mathcal{W}_\Delta\) and \(w^* \in \mathcal{W}^*_a\) it holds that
\[ \left| \frac{1}{K} \sum_{k=1}^{K} U_k(w^*) - \frac{1}{K} \sum_{k=1}^{K} U_k(w) \right| > \Delta. \] (25)
Combining Equations (24) and (25), for any \(w \in \mathcal{W}_a^S \setminus \mathcal{W}_\Delta\) and \(w^* \in \mathcal{W}_a^*\) we have

\[
\frac{1}{K} \sum_{k=1}^K U_k(w^*) - \frac{1}{K} \sum_{k=1}^K U_k(w) > \Delta. \tag{26}
\]

Recall that \(W_a\) represents the aggregated parameter, \(P_a^S\) represents the distribution of the aggregated parameter after being protected, and \(p_{W_a}^S(w)\) represents the corresponding probability density function. We denote \(U_a(w) = \frac{1}{K} \sum_{k=1}^K U_k(w)\), then we have

\[
\epsilon_u = \frac{1}{K} \sum_{k=1}^K \epsilon_{u,k}
\]

\[
= \frac{1}{K} \sum_{k=1}^K \left[ U_k(P_a^O) - U_k(P_a^S) \right]
\]

\[
= \frac{1}{K} \sum_{k=1}^K \left[ \mathbb{E}_{w \sim P_a^O}[U_k(w)] - \mathbb{E}_{w \sim P_a^S}[U_k(w)] \right]
\]

\[
= \frac{1}{K} \sum_{k=1}^K \left[ \int_{\mathcal{W}_a} U_k(w)dP_a^O(w) - \int_{\mathcal{W}_a} U_k(w)dP_a^S(w) \right]
\]

\[
= \frac{1}{K} \sum_{k=1}^K \left[ \int_{\mathcal{U}_a} U_k(w)[dP_a^O(w) - dP_a^S(w)] - \int_{\mathcal{U}_a} U_k(w)[dP_a^S(w) - dP_a^O(w)] \right]
\]

\[
= \frac{1}{K} \sum_{k=1}^K \left[ \int_{\mathcal{V}_a} U_k(w)\mathbb{1}\{w \in \mathcal{W}_a^*\}[dP_a^O(w) - dP_a^S(w)] - \int_{\mathcal{U}_a} U_a(w)\mathbb{1}\{w \in \mathcal{W}_a^S\}[dP_a^S(w) - dP_a^O(w)] \right]
\]

where \(\bullet\) is due to \(\mathcal{V}_a \in \mathcal{W}_a^O \subseteq \mathcal{W}_a^*\) from Equations (21) and (23), and \(\star\) is due to \(\mathcal{U}_a \subseteq \mathcal{W}_a^S\) from Equation (22).

We decompose \(\int_{\mathcal{U}_a} U_a(w)\mathbb{1}\{w \in \mathcal{W}_a^S\}[dP_a^S(w) - dP_a^O(w)]\) as the summation of

\[
\int_{\mathcal{U}_a} U_a(w)\mathbb{1}\{w \in \mathcal{W}_a^S\}[dP_a^S(w) - dP_a^O(w)] = \int_{\mathcal{U}_a} U_a(w)\mathbb{1}\{w \in \mathcal{W}_a^S\}[dP_a^S(w) - dP_a^O(w)] + \int_{\mathcal{U}_a} U_a(w)\mathbb{1}\{w \notin \mathcal{W}_a^S\}[dP_a^S(w) - dP_a^O(w)]
\]

Then we have

\[
\epsilon_u = \frac{1}{K} \sum_{k=1}^K \epsilon_{u,k}
\]

\[
= \int_{\mathcal{V}_a} U_a(w)\mathbb{1}\{w \in \mathcal{W}_a^*\}[dP_a^O(w) - dP_a^S(w)] - \int_{\mathcal{U}_a} U_a(w)\mathbb{1}\{w \in \mathcal{W}_a^S\}[dP_a^S(w) - dP_a^O(w)]
\]

\[
\geq \Delta \cdot \left[ \text{TV}(P_a^O || P_a^S) - \int_{\mathcal{U}_a} \mathbb{1}\{w \in \mathcal{W}_a^S\}[dP_a^S(w) - dP_a^O(w)] \right]
\]

\[
\geq \Delta \cdot \left[ \text{TV}(P_a^O || P_a^S) - \int_{\mathcal{W}_a^S} \mathbb{1}\{w \in \mathcal{W}_\Lambda\} p_{W_a}^S(w)dw \right]
\]

\[
\geq \frac{\Delta}{2} \cdot \text{TV}(P_a^O || P_a^S),
\]
in which we have the following:

- The first inequality is due to $U_a(w) \leq U_a(w^*)$ for any $w \in \mathcal{W}_a$ and $w^* \in \mathcal{W}_a^*$ according to Equation (24), and $U_a(w^*) - U_a(w) > \Delta$ for any $w \in \mathcal{W}_a^* \setminus \mathcal{W}_a$ and $w^* \in \mathcal{W}_a^*$ from Equation (26).

- The second inequality is due to $\int_{\mathcal{W}_a} \inf \{w \in \mathcal{W}_a^* \} \{w \in \mathcal{W}_a\} [dP_a^S(w) - dP_a^O(w)] \leq \int_{\mathcal{W}_a} \inf \{w \in \mathcal{W}_a^* \} \{w \in \mathcal{W}_a\} dP_a^S(w) \leq \int_{\mathcal{W}_a^*} \inf \{w \in \mathcal{W}_a\} dP_a^S(w) = \int_{\mathcal{W}_a^*} \inf \{w \in \mathcal{W}_a\} p_{w_a}^S(w) dw$.

- The third inequality is due to $\int_{\mathcal{W}_a^*} \inf \{w \in \mathcal{W}_a\} p_{w_a}^S(w) dw \leq \frac{TV(p_a^O||p_a^S)}{2}$. □

### A.3 Analysis of Theorem 4.1

With Lemmas A.2 and A.4, it is now natural to provide a quantitative relationship between the utility loss and the privacy leakage (Theorem 4.1).

**Theorem A.5 (No Free Lunch Theorem (NFL) for Security and Utility).** Let $\epsilon_p$ be defined in Definition 3.2. We have that

$$C_1 \leq \epsilon_p + \frac{1}{K} \sum_{k=1}^{K} \frac{1}{2} (e^{2\xi} - 1) \cdot TV(p_k^O||p_k^S).$$

(27)

Furthermore, let $\epsilon_u$ be defined in Definition 3.3 at the convergence step. With Assumption 4.1, we have that

$$C_1 \leq \epsilon_p + C_2 \cdot \epsilon_u,$$

(28)

in which

- $\xi = \max_{k \in [K]} \xi_k$, where $\xi_k = \max_{w \in \mathcal{W}_k, d \in D_k} \log(\frac{\int_{\mathcal{D}_k} dP_k \{d \mid w\}}{\int_{\mathcal{D}_k} dP_k})$ represents the maximum privacy leakage over all possible information $w$ released by client $k$, and $[K] = \{1, 2, \ldots, K\}$. $\xi$ is a constant independent of the protection and attack mechanisms.

- $C_1 = \frac{1}{K} \sum_{k=1}^{K} \sqrt{JS(F_k^O||F_k^B)}$ is a constant representing the averaged square root of JS divergence between the adversary’s belief distribution about the private information of client $k$ before and after observing the unprotected parameter. This constant is independent of the protection mechanisms.

- $C_2 = \frac{\gamma}{4\Delta} (e^{2\xi} - 1)$ is a constant once the protection mechanisms, the utility function, and the datasets are fixed, where $\gamma = \frac{\sum_{k=1}^{K} TV(p_k^O||p_k^S)}{TV(p_a^O||p_a^S)}$.\footnote{\textsuperscript{10}See details of analyzing the value of $\gamma$ in Section 4.3.}

**Proof.** From Lemma A.2, we have

$$\frac{1}{K} \sum_{k=1}^{K} \sqrt{JS(F_k^O||F_k^B)} \leq \frac{1}{K} \sum_{k=1}^{K} \epsilon_{p,k} + \frac{1}{K} \sum_{k=1}^{K} \frac{1}{2} (e^{2\xi} - 1) TV(p_k^O||p_k^S).$$

(29)

From Lemma A.4, we have

$$\epsilon_u \geq \frac{\Delta}{2} \cdot TV(p_a^O||p_a^S).$$

(30)
Combining Equations (29) and (30), we have that
\[
\frac{1}{K} \sum_{k=1}^{K} \sqrt{\text{JS}(F^O_k \mid \mid F^B_k)} \leq \epsilon_p + \frac{Y}{4\Delta} (e^{2\xi} - 1) \epsilon_u,
\]
where \( \gamma = \frac{1}{K} \sum_{k=1}^{K} \text{TV}(P_{O_k}^k \mid \mid P_{S_k}^k) \), and \( \epsilon_p = \frac{1}{K} \sum_{k=1}^{K} \epsilon_{p,k} \).

The preceding equation could be further simplified as
\[
C_1 \leq \epsilon_p + C_2 \epsilon_u,
\]
where \( C_1 = \frac{1}{K} \sum_{k=1}^{K} \sqrt{\text{JS}(F^O_k \mid \mid F^B_k)} \) and \( C_2 = \frac{Y}{4\Delta} (e^{2\xi} - 1) \).

\[\text{□}\]

\section{B \quad BAYESIAN INFERENCE ATTACK IN SFL}

It was shown that semi-honest adversaries can recover the private training images with pixel-level accuracy from unprotected gradients of learned models. We assume that an adversary aims to recover the \( k_{th} \) client’s private variable \( D_k \) from exposed variable \( W^S_k \), which is the output of applying certain protection mechanisms on model information—that is, \( W^S_k = M(W^O_k) \), where \( M \) is a protection mechanism.

In this section, we formulate three privacy leakage attacks of SFL including gradient-inverse attack [28, 83, 87, 89], model inversion attack [27, 41], and brute-force attack (especially for encryption) from the Bayesian inference perspective. Such a Bayesian inference attack mechanism is defined as follows.

\text{Definition B.1 (Bayesian Inference Attack).} A Bayesian inference attack\footnote{The Bayesian inference framework has long been applied to the image restoration problem (as early as the 1960s) [7, 15, 30].} is an optimization process that aims to infer the private variable \( D_k \) to best fit the exposed information \( W^S_k \)—that is,

\[
d^* = \arg\max_{d} \log f_{W^S_k \mid D_k}(d|w) = \log f_{W^S_k}(w) \cdot \frac{\log f_{D_k}(d)}{f_{W^S_k}(w)} = \arg\max_{d} \log f_{W^S_k \mid D_k}(w|d) + \log f_{D_k}(d),
\]

where \( L_1(d, W^S_k) = \log f_{W^S_k \mid D_k}(w|d) \) and \( L_2(d) = \log f_{D_k}(d) \).

There are three types of Bayesian inference attack, as follows.

\text{B.1 Gradient Inversion Attack}

In SFL, the private model updates (gradients) and the model needs to be uploaded to the server, which may be used to reconstruct the input data of clients by semi-honest adversaries. In this setting, gradient-inverse attack is proposed in the work of Zhu and Han [89] to infer the private gradients.
data $D_k$ from the model gradients (exposed information $W_{k}^{S}$ here is model gradients $G$). The form of $L_1(d, W_{k}^{S})$ is

$$ L_1 \left( d, W_{k}^{S} \right) = L_1(d, G) = C - \left\| \nabla W(d) - G \right\|^2, \quad (32) $$

in which $\nabla W(d)$ is the gradient of training loss w.r.t. model parameters for the estimated data $d$, and $G$ is the observed gradients of model parameters.

### B.1.1 Influence of Prior in Gradient Inversion Attack

Moreover, the prior $L_2(d) = \log f_{D_k}(d)$ is an important factor for Bayesian inference, and a series of work was developed to use different priors in gradient inversion attack:

- **No prior**: Zhu and Han [89] proposed the gradient inversion attack in which reconstructed private data is randomly initialized.
- **Smoothness prior**: Geiping et al. [28] improved the attack with the smoothness of data (e.g., image). Specifically, they viewed the TV loss of the data as the smoothness: $L_2(d) = \text{TV}(d)$.
- **Label prior**: Zhao et al. [87] developed the prior with the label information of the data—that is, $L_2(d) = \text{Label}(d)$.
- **Group consistency prior**: Besides smoothness and label of data, Yin et al. [83] applied group consistency of estimated data to the training process of gradient inversion attack, where $L_2(d) = \text{Group}(d)$.

Moreover, the results of other works [28, 83, 87, 89] demonstrate that a different prior would cause a different type of privacy leakage. A stronger prior of private data lets the adversaries attack the private data of clients more effectively. For example, the attack method proposed by Yin et al. [83] combined three priors—smoothness, label, and group consistency—into gradient inversion attack. It is shown that their attack performs best in images with a large batch size compared to other gradient inversion methods.

### B.2 Model Inversion Attack

In some cases of SFL, such as split learning [38] or vertical FL [81], the model outputs $O$ may be leaked. A new reconstruction attack is proposed according to the exposed model outputs $O$ in other works [27, 41], called *model inversion attack*, with the following $L_1(d, W_{k}^{S} = O)$ loss function:

$$ L_1 \left( d, W_{k}^{S} \right) = L_1(d, O) = C - \left\| \hat{O}(d) - O \right\|^2, \quad (33) $$

in which $\hat{O}(d)$ is the output of the training model with respect to the data $d$ and $O$ is the observed output of models for the private data.

### B.3 Brute-Force Attack for Encryption

In the cryptosystem, the attack problem is formulated to restore a private Key given the chosen plaintext and ciphertext (CPA). The brute-force method in a cryptosystem is to try all possible private keys until the chosen plaintext matches plaintext. Similarly, this formulation could also be written as a Bayesian inference attack with $L_1(d, W_{k}^{S})$ as

$$ L_1 \left( d, W_{k}^{S} \right) = L_1 \left( d, \left( W_{k}^{O}, \text{Enc} \left( W_{k}^{O} \right) \right) \right) = \begin{cases} 1, & \text{decrypt} \left( d, \text{Enc} \left( W_{k}^{O} \right) \right) = W_{k}^{O} \\ 0, & \text{decrypt} \left( d, \text{Enc} \left( W_{k}^{O} \right) \right) \neq W_{k}^{O} \end{cases}, $$

where $d$ is the estimation of private key, and $W_{k}^{S} = (W_{k}^{O}, \text{Enc}(W_{k}^{O}))$ is pair of plaintext model information $W_{k}^{O}$ and ciphertext $\text{Enc}(W_{k}^{O})$. 

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B.4 Time Complexity of Bayesian Inference Attack

For gradient inversion and model inversion attack, it could be solved by the optimization loss as

\[
\max_d L(d) = \max_d \left[ L_1(d, W_k^S) + L_2(d) \right] = \max_d \left[ \| J(d) - w_k^S \|^2 + L_2(d) \right],
\]

where \( J \) represents the mapping from data to model gradients in gradient inversion attack, and the mapping from data to model outputs in model inversion attack.

**Proposition B.1.** The time complexity to solve Equation (34) is polynomial, if any of the following conditions are satisfied:

- \( L \) is convex and smooth (moreover, the convergence rate is \( O(1/t) \)).
- \( L \) is convex but non-smooth, and \( L \) is Lipschitz continuous (moreover, the convergence rate is \( O(1/\sqrt{t}) \)).
- \( L \) is non-convex and \( L \)-smooth (moreover, the convergence rate is \( O(1/t^2) \)).

Remark: Proposition B.1 demonstrates three conditions of \( L \) that the optimization problem Equation (34) is polynomial time. For the analysis of this proposition, please refer to the work of Boyd and Vandenberghe [8].

For Bayesian inference methods of encryption, Gentry et al. [33] proposed a simple fully homomorphic encryption (FHE) scheme (approximate eigenvector method) based on the LWE (learning with errors) problem [63]. They prove the security of the proposed scheme on GapSVP\( _\gamma \) via a classical reduction from LWE [9, 10, 61]. The best algorithms [58, 66] for GapSVP\( _\gamma \) require at least \( 2^{\Omega(n_d/\log y)} \) time. The approximate eigenvector method reduces from \( y = n_d^{O(\log(n_d))} \), for which the best known algorithms run in time \( 2^{\Omega(n_d)} \). Paillier is a probabilistic asymmetric algorithm for public key cryptography based on the decisional composite residuosity assumption [60]. Since it has an additive homomorphic property, Paillier could be applied into FedAverage [2, 14, 74, 86] in FL, which only needs addition operations. As far as we know, the most efficient algorithm against Paillier encryption is based on factoring of a large number [60]. That is also the best known attack against RSA encryption [64]. One of the widely known efficient algorithms for factoring the large number is general number field sieve (GNFS) [12], the time complexity of which is sub-exponential \( (\exp((\sqrt{64/9} + o(1))(\ln n)^{1/3}(\ln \ln n)^{1/3}) \).

C PROOF FOR APPLICATIONS OF PRIVACY-UTILITY TRADE-OFF

C.1 Proof for the Randomization Mechanism

**Lemma C.1 (Total Variation Distance between Gaussians with the Same Mean [17]).** Let \( \mu \in R^n, \Sigma_1, \Sigma_2 \) be diagonal matrix, and let \( \lambda_1, \ldots, \lambda_n \) denote the eigenvalues of \( \Sigma_1^{-1}\Sigma_2 - I_n \). Then,

\[
\frac{1}{100} \leq \frac{TV(N(\mu, \Sigma_1), N(\mu, \Sigma_2))}{\min \left\{ 1, \sqrt{\sum_{i=1}^n \lambda_i^2} \right\}} \leq \frac{3}{2}.
\]

(35)

According to the Lemma C.1, we could estimate the range of \( \gamma \).

**Lemma C.2. (Estimation of \( \gamma \)) Let \( P_{a_0}^S \sim N(\mu_0, \Sigma_0/K + \Sigma_\epsilon/K) \), \( P_{a_0}^O \sim N(\mu_0, \Sigma_0/K) \), \( P_k^S \sim N(\mu_0, \Sigma_0 + \Sigma_\epsilon) \) and \( P_k^O \sim N(\mu_0, \Sigma_0) \), where \( \Sigma_0 = \text{diag}(\sigma_0^2, \ldots, \sigma_0^2) \) and \( \Sigma = \text{diag}(\sigma^{2}_\epsilon, \ldots, \sigma^{2}_\epsilon) \). Then we have

\[
\frac{1}{150} \leq \gamma = \frac{TV(P_k^O||P_k^S)}{TV(P_{a_0}^O||P_{a_0}^S)} \leq 150.
\]

(36)
Proof. According to Lemma C.1, we have

\[
\frac{1}{100} \min \left\{ 1, \sigma^2 \sum_{i=1}^{n} \frac{1}{\sigma_i^4} \right\} \leq \text{TV} \left( P^O_k \parallel P^S_k \right) \leq \frac{3}{2} \min \left\{ 1, \sigma^2 \sum_{i=1}^{n} \frac{1}{\sigma_i^4} \right\}
\]  \tag{37}

\[
\frac{1}{100} \min \left\{ 1, \sigma^2 \sum_{i=1}^{n} \frac{1}{\sigma_i^4} \right\} \leq \text{TV} \left( P^O_a \parallel P^S_a \right) \leq \frac{3}{2} \min \left\{ 1, \sigma^2 \sum_{i=1}^{n} \frac{1}{\sigma_i^4} \right\}. \tag{38}
\]

Combining Equations (37) and (38), we have

\[
\frac{1}{150} \leq \gamma = \frac{\text{TV} (P^O_k \parallel P^S_k)}{\text{TV} (P^O_a \parallel P^S_a)} \leq 150. \tag*{□}
\]

Lemma C.3. If \( U(w, d) \in [0, C_4] \) for any \( w \in W_k \) and \( d \in D_k \), where \( k = 1, \ldots, K \), then we have

\[
\epsilon_u \leq C_4 \cdot \text{TV} \left( P^O_a \parallel P^S_a \right).
\]

Proof. Let \( \mathcal{U}_k = \{ w \in W_k : dP^S_a(w) - dP^O_a(w) \geq 0 \} \), and \( \mathcal{V}_k = \{ w \in W_k : dP^S_a(w) - dP^O_a(w) < 0 \} \). Then we have

\[
\epsilon_u = \frac{1}{K} \sum_{k=1}^{K} \epsilon_{u,k} = \frac{1}{K} \sum_{k=1}^{K} \left[ U_k (P^O_a) - U_k (P^S_a) \right]
\]

\[
= \frac{1}{K} \sum_{k=1}^{K} \left[ \mathbb{E}_{w \sim P^O_a} [U_k(w)] - \mathbb{E}_{w \sim P^S_a} [U_k(w)] \right]
\]

\[
= \frac{1}{K} \sum_{k=1}^{K} \left[ \int_{W_k} U_k(w) dP^O_a(w) - \int_{W_k} U_k(w) dP^S_a(w) \right]
\]

\[
= \frac{1}{K} \sum_{k=1}^{K} \left[ \int_{\mathcal{V}_k} U_k(w) [dP^O_a(w) - dP^S_a(w)] - \int_{\mathcal{U}_k} U_k(w) [dP^S_a(w) - dP^O_a(w)] \right]
\]

\[
\leq \frac{C_4}{K} \sum_{k=1}^{K} \left[ dP^O_a(w) - dP^S_a(w) \right] = C_4 \cdot \text{TV} \left( P^O_a \parallel P^S_a \right). \tag*{□}
\]

Theorem C.4. For the randomization mechanism adding Gaussian noise, we have the following relation between privacy leakage, utility loss, and variance of Gaussian noise \( \sigma^2_e \):

\[
C_1 \leq \epsilon_p + C_3 \cdot \frac{3}{2} \min \left\{ 1, \sigma^2 \sum_{i=1}^{n} \frac{1}{\sigma_i^4} \right\}, \tag{39}
\]

\[
\epsilon_u \leq C_4 \min \left\{ 1, \sigma^2 \sum_{i=1}^{n} \frac{1}{\sigma_i^4} \right\}, \tag{40}
\]

where \( C_1 = \frac{1}{K} \sum_{k=1}^{K} \sqrt{JS(F^O_k \parallel F^S_k)} \) and \( C_3 = (e^{2\xi} - 1)/2 \) are two constants, which are independent of the protection mechanism.
Proof. According to the Theorem 4.1, we have

\[ C_1 \leq \epsilon_p + \frac{1}{K} \sum_{k=1}^{K} \frac{1}{2} (e^{2\xi} - 1) \cdot \text{TV} \left( P_k^O || P_k^S \right). \]

From Lemma C.1 and definitions of \( P_k^O, P_k^S \), we further obtain

\[ C_1 \leq \epsilon_p + C_3 \cdot \min \left\{ 1, \sigma^2 \sqrt{\frac{1}{\sum_{i=1}^{n} \frac{1}{\sigma_i^4}}} \right\}, \]

where \( C_3 = (e^{2\xi} - 1)/2 \). Therefore, Equation (39) is proved. Moreover, combining Lemmas C.1 and C.3, we have

\[ \epsilon_u \leq C_4 \cdot \text{TV} \left( P_a^O || P_a^S \right) \leq C_4 \min \left\{ 1, \sigma^2 \sqrt{\frac{1}{\sum_{i=1}^{n} \frac{1}{\sigma_i^4}}} \right\}. \]

Therefore, Equation (40) is proved. □

C.2 Proof for the Sparsity Mechanism

Lemma C.5 (Total Variation Distance between Gaussians with Different Means [17]). Assume that \( \Sigma_1, \Sigma_2 \) are positive definite, and let

\[ h = h(\mu_1, \Sigma_1, \mu_2, \Sigma_2) = \left( 1 - \frac{\det(\Sigma_1)^{1/4} \det(\Sigma_2)^{1/4}}{\det(\frac{\Sigma_1 + \Sigma_2}{2})^{1/2}} \exp\left\{ -\frac{1}{8} (\mu_1 - \mu_2)^T \left( \frac{\Sigma_1 + \Sigma_2}{2} \right)^{-1} (\mu_1 - \mu_2) \right\} \right)^{1/2}. \] (41)

Then we have

\[ h^2 \leq \text{TV} (N(\mu_1, \Sigma_1), N(\mu_2, \Sigma_2)) \leq \sqrt{2} h. \] (42)

Theorem C.6. For the sparsity mechanism by uploading partial information to the server, suppose \( P_k^O = N(\mu_0, \Sigma_0) = N((\mu_u, \mu_0), \text{diag}(\Sigma_u, \Sigma_o)), P_k^S = N(\mu, \Sigma) = N((\mu_u, \mu_g), \text{diag}(\Sigma_u, \Sigma_g)), P_a^O = N(\mu_0, \Sigma_0/K) \) and \( P_a^S = N(\mu, \Sigma/K) \). We have

\[ C_1 \leq \epsilon_p + C_3 h(\mu_o, \mu_g, \Sigma_o, \Sigma_g) \] (43)

and

\[ \epsilon_u \leq C h(\mu_o, \mu_g, \Sigma_o, \Sigma_g), \] (44)

where \( C_1 = \frac{1}{K} \sum_{k=1}^{K} \sqrt{f(S(F_k^O || F_k^B))}, C_3 = \frac{\sqrt{2} (e^{2\xi} - 1)}{2} \) are two constants independent of the protection mechanisms adopted, and \( C \) is the upper bound of \( \sqrt{2} U(\omega, d) \) for any \( \omega \in \mathcal{W}_k \) and \( d \in \mathcal{D}_k \).
Proof. According to the Lemma C.5 and Equation (5) of Theorem 4.1, we have
\[
\frac{1}{K} \sum_{k=1}^{K} \sqrt{JS(F_k^O||F_k^B)} \leq \epsilon_p + \frac{1}{K} \sum_{k=1}^{K} \frac{1}{2} (e^{2\xi} - 1) \cdot TV\left( P_k^O || P_k^S \right)
\]
\[
= \epsilon_p + \frac{1}{2} (e^{2\xi} - 1) \cdot TV\left( P_k^O || P_k^S \right)
\]
\[
\leq \epsilon_p + \frac{\sqrt{2}(e^{2\xi} - 1)}{2} h(\mu_0, \mu, \Sigma_0, \Sigma)
\]
\[
= \epsilon_p + \frac{\sqrt{2}(e^{2\xi} - 1)}{2} h(\mu_0, \mu, \Sigma_0, \Sigma_g),
\]
which could be simplified as Equation (43), where \( C_3 = \frac{\sqrt{2}(e^{2\xi} - 1)}{2} \), \( C_1 = \frac{1}{K} \sum_{k=1}^{K} \sqrt{JS(F_k^O||F_k^B)} \) are two constants, which is independent of the protection mechanism. Moreover, according to Lemma C.3, we have
\[
\epsilon_u \leq C_4 TV(P^O_\theta || P^S_\theta)
\]
\[
= \sqrt{2} C_4 h(\mu_0, \mu, \Sigma_0/K, \Sigma/K) = Ch(\mu_0, \mu, \Sigma_0/K, \Sigma_g/K)
\]
\[
\leq Ch(\mu_0, \mu, \Sigma_0, \Sigma_g),
\]
where \( C \) is the upper bound of \( \sqrt{2} U(w, d) \) for any \( w \in W_k \) and \( d \in D_k \). The last inequality is because \( h(\mu_0, \mu, \Sigma_0/K, \Sigma_g/K) \leq h(\mu_0, \mu, \Sigma_0, \Sigma_g) \).

Proposition C.7. For the sparsity mechanism by uploading partial information to the server, we have

- The Bayesian privacy leakage of federated system \( \epsilon_p \) has the lower bound: \( C_1 - C_2 \cdot h(\mu_0, \mu, \Sigma_0, \Sigma_g) \), which is an increasing function of \( d \).
- The utility loss \( \epsilon_u \) has the upper bound: \( C \cdot h(\mu_0, \mu, \Sigma_0, \Sigma_g) \), which is a decreasing function of \( d \).

Proof. It just needs to prove \( h(\mu_0, \mu, \Sigma_0, \Sigma_g) \) is a decreasing function of \( d \)—that is, when \( \mu_0 \in \mathbb{R}^{n-d} \) changes to \( \mu'_0 \in \mathbb{R}^{n-(d-1)} \) (\( d \rightarrow d - 1 \)), \( h(\mu_0, \mu, \Sigma_0, \Sigma_g) < h(\mu'_0, \mu, \Sigma_0, \Sigma_g) \). Let \( \mu'_0 = (\mu_0, t_0), \Sigma'_0 = \text{diag}(\Sigma_0, s_0^2), \mu'_g = (\mu_g, t_g), \Sigma'_g = \text{diag}(\Sigma_g, s_g^2) \). Then we have
\[
h\left( \mu'_0, \mu'_g, \Sigma'_0, \Sigma'_g \right) = \left( 1 - \frac{\det(\Sigma_0)^{1/4} \det(\Sigma_g)^{1/4}}{\det(\Sigma_0^2 + \Sigma_g^2)^{1/2}} \right) \exp\left( -\frac{1}{8} \left( \mu'_0 - \mu'_g \right)^T \left( \frac{\Sigma_0 + \Sigma_g}{2} \right)^{-1} \left( \mu'_0 - \mu'_g \right) \right)^{1/2}
\]
\[
= \left( 1 - \frac{\det(\Sigma_0)^{1/4} \det(\Sigma_g)^{1/4}}{\det(\Sigma_0^2 + \Sigma_g^2)^{1/2}} \right) \cdot \exp\left( -\frac{1}{8} \left( \mu'_0 - \mu'_g \right)^T \left( \frac{\Sigma_0 + \Sigma_g}{2} \right)^{-1} \left( \mu'_0 - \mu'_g \right) \right)^{1/2}
\]
\[
\geq \left( 1 - \frac{\det(\Sigma_0)^{1/4} \det(\Sigma_g)^{1/4}}{\det(\Sigma_0^2 + \Sigma_g^2)^{1/2}} \right) \exp\left( -\frac{1}{8} \left( \mu_0 - \mu_g \right)^T \left( \frac{\Sigma_0 + \Sigma_g}{2} \right)^{-1} \left( \mu_0 - \mu_g \right) \right)^{1/2}
\]
\[
= h(\mu_0, \mu, \Sigma_0, \Sigma_g).
\]

The last equation is due to \( \frac{(s_0 s_g)^{1/2}}{(s_0^2 + s_g^2)^{1/2}} \leq 1 \) and \( \exp\left( -\frac{1}{8} \left( t_0 - t_g \right)^T \left( \frac{s_0 + s_g}{2} \right)^{-1} \left( t_0 - t_g \right) \right) = 1 \) and \( \exp\left( -\frac{1}{8} \left( t_0 - t_g \right)^T \left( \frac{s_0 + s_g}{2} \right)^{-1} \left( t_0 - t_g \right) \right) \leq 1 \).
C.3 Proof for Encryption Mechanism

**Theorem C.8.** For the encryption mechanism that encrypts the model information \( W_k^S = Enc(W_k^O) \) using the approximate eigenvector method,

- If the private key is unknown for the server (adversaries), then \( \epsilon_p = 0 \) and \( \epsilon_u \geq \frac{C_1}{C_2} \).
- If the private key is known for the server (adversaries), then \( \epsilon_p = 0 \) and \( \epsilon_u \geq C_1 \).

**Proof.** If the private key is unknown for the server (adversaries), since the approximate eigenvector method \([33]\) based on LWE is proved to be semantic security (Lemma 5.4 in the work of Regev \([63]\)), any probabilistic, polynomial-time algorithm that is given the ciphertext \( W_k^S = (Enc(W_k^O)) \) of a certain message and the message’s length cannot determine any partial information on the message with probability non-negligibly higher than all other probabilistic, polynomial-time algorithms that only have access to the message length. Thus, we have \( f_{D_k | W_k} (d | w) = f_{D_k} (d) \) for \( w \in W_k^S \). Consequently,

\[
f_{D_k}^A (d) = \int_{W_k^S} f_{D_k | W_k} (d | w) dP_k^S (w)
= \int_{W_k^S} f_{D_k} (d) dP_k^S (w)
= f_{D_k} (d)
= f_{D_k}^B (d).
\]

As a result,

\[
f_{D_k}^M (d) = \frac{1}{2} \left( f_{D_k}^A (d) + f_{D_k}^B (d) \right) = f_{D_k}^B (d).
\]

Therefore,

\[
\epsilon_p = \frac{1}{K} \sum_{k=1}^{K} \epsilon_{p,k} \sqrt{JS(F_k^A || F_k^B)}
= \frac{1}{K} \sum_{k=1}^{K} \left[ \frac{1}{2} \int_{D_k} f_{D_k}^A (d) \log \frac{f_{D_k}^A (d)}{f_{D_k}^M (d)} d\mu (d) + \frac{1}{2} \int_{D_k} f_{D_k}^B (d) \log \frac{f_{D_k}^B (d)}{f_{D_k}^M (d)} d\mu (d) \right]^{\frac{1}{2}}.
= 0,
\]

Moreover, according to Equation (6) of Theorem 4.1, we have

\[
\epsilon_u \geq \frac{C_1}{C_2}.
\]

If the private key is known for the server (adversaries), then the server could decrypt the global model accurately so that utility loss \( \epsilon_u = 0 \). Moreover, according to Equation (6) of Theorem 4.1, we have

\[
\epsilon_p \geq C_1.
\]

\[\square\]
C.4 Proof for the Secret Sharing Mechanism

**Theorem C.9.** For the secret sharing mechanism, we have that
\[ \epsilon_{p,k} \geq \sqrt{\text{JS}(F^O_k || F^S_k)} - \frac{1}{2}(e^{2\bar{v}} - 1) \cdot \left( 1 - \prod_{j=1}^{m} \left( \frac{2\bar{v}}{b_j' + a_j'} \right) \right). \] (45)

Furthermore, we have that
\[ \epsilon_u = 0. \] (46)

**Proof.** Let \( W^O_k \) represent the original model information that follows a uniform distribution over \([c^1_k - \delta, c^1_k + \delta] \times [c^2_k - \delta, c^2_k + \delta] \times \cdots \times [c^n_k - \delta, c^n_k + \delta]\). \( W^S_k \) represent the distorted model information that follows a uniform distribution over \([c^1_k - a^1_k, c^1_k + b^1_k] \times [c^2_k - a^2_k, c^2_k + b^2_k] \times \cdots \times [c^n_k - a^n_k, c^n_k + b^n_k]\), and \( 0 < \delta < a'_k, b'_k \), \( \forall i = 1, 2, \ldots, n \). Then we have that
\[
\text{TV}(P^O_k || P^S_k) = \int_{[c^1_k - \delta, c^1_k + \delta]} \int_{[c^2_k - \delta, c^2_k + \delta]} \cdots \int_{[c^n_k - \delta, c^n_k + \delta]} \left( \frac{1}{2\delta} \right)^m \prod_{j=1}^{m} \left( \frac{1}{b_j' + a_j'} \right) \, dw_1 \, dw_2 \cdots \, dw_m
= \left( \frac{1}{2\delta} \right)^m \prod_{j=1}^{m} \left( \frac{1}{b_j' + a_j'} \right) \cdot (2\delta)^m.
\]

Therefore, we have that
\[
\epsilon_{p,k} \geq \sqrt{\text{JS}(F^O_k || F^S_k)} - \frac{1}{2}(e^{2\bar{v}} - 1)\text{TV}(P^O_k || P^S_k)
= \sqrt{\text{JS}(F^O_k || F^S_k)} - \frac{1}{2}(e^{2\bar{v}} - 1) \cdot \left( 1 - \prod_{j=1}^{m} \left( \frac{2\bar{v}}{b_j' + a_j'} \right) \right),
\] (47)
where the first inequality is due to Equation (20) in Lemma A.2.

For the secret sharing mechanism, the aggregated parameter does not change after being protected, which implies that \( P^O_k = P^S_k \). Therefore, we have that
\[
\epsilon_u = \frac{1}{K} \sum_{k=1}^{K} \epsilon_{u,k}
= \frac{1}{K} \sum_{k=1}^{K} \left[ U_k \left( P^O_a \right) - U_k \left( P^S_a \right) \right]
= \frac{1}{K} \sum_{k=1}^{K} \left[ \mathbb{E}_{w \sim P^O_a} [U_k(w)] - \mathbb{E}_{w \sim P^S_a} [U_k(w)] \right]
= 0. \]

D PROOF OF THE CONNECTION BETWEEN BAYESIAN PRIVACY AND DP

We may establish the connection between DP and Bayesian privacy as follows.

**Lemma D.1.** Let \( f_{W|D}(\cdot) \) be a privacy-preserving mapping that guarantees \( \bar{v} \)-maximum Bayesian privacy. In other words, \( \bar{v} = \max_{w \in W, d \in D} | \log \left( \frac{f_{W}(d|w)}{f_D(d)} \right) | \). Then, the mapping \( f_{W|D}(\cdot) \) is \((2\bar{v})\)-
differentially private:

\[
\frac{f_{D|W}(d|w)}{f_{D|W}(d|w')} \in [e^{-2\xi}, e^{2\xi}].
\]

**Proof.** From the definition of maximum privacy leakage, we know that

\[
\frac{f_{D|W}(d|w)}{f_{D}(d)} \in [e^{-\xi}, e^{\xi}],
\]

for any \( w \in \mathcal{W} \) and \( d \in \mathcal{D} \).

Therefore, for any \( w, w' \in \mathcal{W} \) we have

\[
\frac{f_{D|W}(d|w)}{f_{D|W}(d|w')} = \frac{f_{D|W}(d|w)}{f_{D}(d)} \cdot \frac{f_{D}(d)}{f_{D}(d)} \in [e^{-2\xi}, e^{2\xi}].
\]

From the definition of differential privacy, we know that it is \((2\xi)\)-differentially private. \(\square\)

**REFERENCES**

[1] Martin Abadi, Andy Chu, Ian Goodfellow, H. Brendan McMahan, Ilya Mironov, Kunal Talwar, and Li Zhang. 2016. Deep learning with differential privacy. In Proceedings of the 2016 ACM SIGSAC Conference on Computer and Communications Security. ACM, New York, NY, 308–318.

[2] Le Trieu Phong, Yoshinori Aono, Takuya Hayashi, Lihua Wang, and Shibo Moriai. 2017. Privacy-preserving deep learning via additively homomorphic encryption. *IEEE Transactions on Information Forensics and Security* 13, 5 (2017), 1333–1345.

[3] Shahab Assoodeh, Mario Diaz, Fady Alajaji, and Tamás Linder. 2018. Estimation efficiency under privacy constraints. *IEEE Transactions on Information Theory* 65, 3 (2018), 1512–1534.

[4] G. R. Blakley. 1979. Safeguarding cryptographic keys. In *Proceedings of the 1979 AFIPS National Computer Conference*. 313–317.

[5] Yochai Blau and Tomer Michaeli. 2019. Rethinking lossy compression: The rate-distortion-perception tradeoff. In *Proceedings of the International Conference on Machine Learning*. 675–685.

[6] Keith Bonawitz, Vladimir Ivanov, Ben Kreuter, Antonio Marcedone, H. Brendan McMahan, Sarvar Patel, Daniel Ramage, Aaron Segal, and Karn Seth. 2017. Practical secure aggregation for privacy-preserving machine learning. In *Proceedings of the 2017 ACM SIGSAC Conference on Computer and Communications Security*. 1175–1191.

[7] George E. P. Box and George C. Tiao. 2011. *Bayesian Inference in Statistical Analysis*. Vol. 40. John Wiley & Sons.

[8] Stephen Boyd and Lieven Vandenberghe. 2004. *Convex Optimization*. Cambridge University Press.

[9] Zvika Brakerski. 2012. Fully homomorphic encryption without modulus switching from classical GapSVP. In *Proceedings of the Annual Cryptology Conference*. 868–886.

[10] Zvika Brakerski, Adeline Langlois, Chris Peikert, Oded Regev, and Damien Stehlé. 2013. Classical hardness of learning with errors. In *Proceedings of the 45th Annual ACM Symposium on Theory of Computing*. 575–584.

[11] Zhiqi Bu, Jinshuo Dong, Qi Long, and Weijie J Su. 2020. Deep learning with Gaussian differential privacy. *Harvard Data Science Review* 2020, 23 (2020).

[12] Joe P. Buhler, Hendrik W. Lenstra, and Carl Pomerance. 1993. Factoring integers with the number field sieve. In *Proceedings of the 1979 AFIPS National Computer Conference*. 50–94.

[13] Hanting Chen, Yunhe Wang, Tianyu Guo, Chang Xu, Yiping Deng, Zhenhua Liu, Siwei Ma, Chunjing Xu, Chao Xu, and Wen Gao. 2021. Pre-trained image processing transformer. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*. 12299–12310.

[14] Kewei Cheng, Tao Fan, Yilun Jin, Yang Liu, Tianjian Chen, Dimitrios Papadopoulos, and Qiang Yang. 2021. SecureBoost: A lossless federated learning framework. *arXiv:1901.08755* (2021).

[15] Arthur P. Dempster. 1968. A generalization of Bayesian inference. *Journal of the Royal Statistical Society: Series B (Methodological)* 30, 2 (1968), 205–232.

[16] Jacob Devlin, Ming-Wei Chang, Kenton Lee, and Kristina Toutanova. 2018. Bert: Pre-training of deep bidirectional transformers for language understanding. *arXiv preprint arXiv:1810.04805* (2018).

[17] Luc Devroye, Abbas Mehrabian, and Tommy Reddad. 2018. The total variation distance between high-dimensional Gaussians. *arXiv preprint arXiv:1810.08693* (2018).

[18] Li Dong, Nan Yang, Wenhui Wang, Furu Wei, Xiaodong Liu, Yu Wang, Jianfeng Gao, Ming Zhou, and Hsiao-Wuen Hon. 2019. Unified language model pre-training for natural language understanding and generation. In *Advances in Neural Information Processing Systems* 32 (2019).
No Free Lunch Theorem for Security and Utility in Federated Learning

[19] Flávio du Pin Calmon and Nadia Fawaz. 2012. Privacy against statistical inference. In Proceedings of the 2012 50th Annual Allerton Conference on Communication, Control, and Computing (Allerton ’12). IEEE, Los Alamitos, CA, 1401–1408.

[20] John C. Duchi, Michael I. Jordan, and Martin J. Wainwright. 2013. Local privacy and minimax bounds: Sharp rates for probability estimation. arXiv preprint arXiv:1305.6000 (2013).

[21] Cynthia Dwork. 2006. Differential privacy. In Proceedings of the International Colloquium on Automata, Languages, and Programming. 1–12.

[22] Cynthia Dwork. 2008. Differential privacy: A survey of results. In Proceedings of the International Conference on Theory and Applications of Models of Computation. 1–19.

[23] Cynthia Dwork, Frank McSherry, Kobbi Nissim, and Adam Smith. 2006. Calibrating noise to sensitivity in private data analysis. In Proceedings of the Theory of Cryptography Conference. 265–284.

[24] Cynthia Dwork and Moni Naor. 2010. On the difficulties of disclosure prevention in statistical databases or the case for differential privacy. Journal of Privacy and Confidentiality 2, 1 (Sept. 2010), 93–107.

[25] Cynthia Dwork and Aaron Roth. 2014. The algorithmic foundations of differential privacy. Foundations and Trends in Theoretical Computer Science 9, 3–4 (2014), 211–407.

[26] Dominik Maria Endres and Johannes E. Schindelin. 2003. A new metric for probability distributions. IEEE Transactions on Information Theory 49, 7 (2003), 1858–1860.

[27] Matt Fredrikson, Somesh Jha, and Thomas Ristenpart. 2015. Model inversion attacks that exploit confidence information and basic countermeasures. In Proceedings of the 22nd ACM SIGSAC Conference on Computer and Communications Security. 1322–1333.

[28] Jonas Geiping, Hartmut Bauermeister, Hannah Dröge, and Michael Moeller. 2020. Inverting gradients—How easy is it to break privacy in federated learning? arXiv preprint arXiv:2003.14053 (2020).

[29] Stuart Geman and Donald Geman. 1984. Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images. IEEE Transactions on Pattern Analysis and Machine Intelligence 6 (1984), 721–741.

[30] Stuart Geman and Donald Geman. 1984. Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images. IEEE Transactions on Pattern Analysis and Machine Intelligence PAMI-6, 6 (1984), 721–741. https://doi.org/10.1109/T Pattern.1984.476796

[31] Craig Gentry. 2009. A Fully Homomorphic Encryption Scheme. Stanford University, Stanford, CA.

[32] Craig Gentry. 2010. Toward basing fully homomorphic encryption on worst-case hardness. In Proceedings of the Annual Cryptology Conference. 116–137.

[33] Craig Gentry, Amit Sahai, and Brent Waters. 2013. Homomorphic encryption from learning with errors: Conceptually-simpler, asymptotically-faster, attribute-based. In Proceedings of the Annual Cryptology Conference. 75–92.

[34] Robin C. Geyer, Tassilo Klein, and Moin Nabi. 2017. Differentially private federated learning: A client level perspective. arXiv preprint arXiv:1712.07557 (2017).

[35] Antonious Girgis, Deepesh Data, Suhais Diggavi, Peter Kairouz, and Ananda Theertha Suresh. 2021. Shuffled model of differential privacy in federated learning. In Proceedings of the International Conference on Artificial Intelligence and Statistics. 2521–2529.

[36] Shafi Goldwasser and Silvio Micali. 1984. Probabilistic encryption. Journal of Computer and System Sciences 28, 2 (1984), 270–299.

[37] Hanlin Gu, Lixin Fan, Bowen Li, Yan Kang, Yuan Yao, and Qiang Yang. 2021. Federated deep learning with Bayesian privacy. arXiv preprint arXiv:2109.13012 (2021).

[38] Oktkrist Gupta and Ramesh Raskar. 2018. Distributed learning of deep neural network over multiple agents. Journal of Network and Computer Applications 116 (2018), 1–8.

[39] Trung Ha, Tran Khanh Dang, Tran Tri Dang, Tuan Anh Truong, and Manh Tuan Nguyen. 2019. Differential privacy in deep learning: An overview. In Proceedings of the 2019 International Conference on Advanced Computing and Applications (ACOMP’19). IEEE, Los Alamitos, CA, 97–102.

[40] Farzin Haddapour and Mehrdad Mahdavi. 2019. On the convergence of local descent methods in federated learning. arXiv preprint arXiv:1910.14425 (2019).

[41] Zecheng He, Tianwei Zhang, and Ruby B. Lee. 2019. Model inversion attacks against collaborative inference. In Proceedings of the 35th Annual Computer Security Applications Conference. 148–162.

[42] Briland Hitaj, Giuseppe Ateniese, and Fernando Perez-Cruz. 2017. Deep models under the GAN: Information leakage from collaborative deep learning. In Proceedings of the 2017 ACM SIGSAC Conference on Computer and Communications Security. 603–618.

[43] Hsiang Hsu, Shahab Asoodeh, Salman Salamatian, and Flavio P. Calmon. 2018. Generalizing bottleneck problems. In Proceedings of the 2018 IEEE International Symposium on Information Theory (ISIT’18). IEEE, Los Alamitos, CA, 531–535.

[44] Xueyang Hu, Mingxuan Yuan, Jianguo Yao, Yu Deng, Lei Chen, Qiang Yang, Haibing Guan, and Jia Zeng. 2015. Differential privacy in telco big data platform. Proceedings of the VLDB Endowment 8, 12 (2015), 1692–1703. https://doi.org/10.14778/2824032.2824067

ACM Transactions on Intelligent Systems and Technology, Vol. 14, No. 1, Article 1. Publication date: November 2022.
[45] Ibrahim Issa, Aaron B. Wagner, and Sudeep Kamath. 2019. An operational approach to information leakage. *IEEE Transactions on Information Theory* 66, 3 (2019), 1625–1657.

[46] Peter Kairouz, H. Brendan McMahan, Brendan Avent, Aurélien Bellet, Mehdi Bennis, Arjun Nitin Bhagoji, Kallista A. Bonawitz, et al. 2021. Advances and open problems in federated learning. *Foundations and Trends in Machine Learning* 14, 1–2 (2021), 1–210. https://doi.org/10.1561/2200000083

[47] Tero Karras, Samuli Laine, and Timo Aila. 2019. A style-based generator architecture for generative adversarial networks. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*. 4401–4410.

[48] Muah Kim, Onur Günlü, and Rafael P. Schaeffer. 2021. Federated learning with local differential privacy: Trade-offs between privacy, utility, and communication. In *Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP’21)*. IEEE, Los Alamitos, CA, 2650–2654.

[49] Jakub Konečný, H. Brendan McMahan, Daniel Ramage, and Peter Richtárik. 2016. Federated optimization: Distributed machine learning for on-device intelligence. *arXiv preprint arXiv:1610.02527* (2016).

[50] Jakub Konečný, H. Brendan McMahan, Felix X. Yu, Peter Richtárik, Ananda Theertha Suresh, and Dave Bacon. 2016. Federated learning: Strategies for improving communication efficiency. *arXiv preprint arXiv:1610.05492* (2016).

[51] Ang Li, Yixiao Duan, Huarnri Yang, Yiran Chen, and Jianlei Yang. 2020. TIPRDC: Task-independent privacy-respecting data crowdsourcing framework for deep learning with anonymized intermediate representations. In *Proceedings of the 26th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*. 824–832.

[52] Xiang Li, Kaixuan Huang, Wenhao Yang, Shusen Wang, and Zhihua Zhang. 2019. On the convergence of FedAvg on non-IID data. *arXiv preprint arXiv:1907.02189* (2019).

[53] Jiachun Liao, Oliver Kosut, Lalitha Sankar, and Flavio di Pui Calmon. 2019. Tunable measures for information leakage and applications to privacy-utility tradeoffs. *IEEE Transactions on Information Theory* 65, 12 (2019), 8043–8066.

[54] Ji Liu, Jizhou Huang, Yang Zhou, Xuhong Li, Shilei Ji, Haoyi Xiong, and Dejing Dou. 2022. From distributed machine learning to federated learning: A survey. *Knowledge and Information Systems* 64, 4 (2022), 885–917.

[55] Ali Makhdoumi and Nadia Fawaz. 2013. Privacy-utility tradeoff under statistical uncertainty. In *Proceedings of the 2013 51st Annual Allerton Conference on Communication, Control, and Computing (Allerton’13)*. IEEE, Los Alamitos, CA, 1627–1634.

[56] Brendan McMahan, Eider Moore, Daniel Ramage, Seth Hampson, and Blaise Agüera y Arcas. 2017. Communication-efficient learning of deep networks from decentralized data. In *Proceedings of the 20th International Conference on Artificial Intelligence and Statistics*. 1273–1282.

[57] H. Brendan McMahan, Eider Moore, Daniel Ramage, and Blaise Agüera y Arcas. 2016. Federated learning of deep networks using model averaging. *arXiv preprint arXiv:1602.05629* (2016).

[58] Daniele Micciancio and Panagiotis Voulgaris. 2013. A deterministic single exponential time algorithm for most lattice problems based on Voronoi cell computations. *SIAM Journal on Computing* 42, 3 (2013), 1364–1391.

[59] Ilya Mironov. 2017. Rényi differential privacy. In *Proceedings of the 2017 IEEE 30th Computer Security Foundations Symposium (CSF’17)*. IEEE, Los Alamitos, CA, 263–275.

[60] Pascal Paillier. 1999. Public-key cryptosystems based on composite degree residuosity classes. In *Proceedings of the International Conference on the Theory and Applications of Cryptographic Techniques*. 223–238.

[61] Chris Peikert. 2009. Public-key cryptosystems from the worst-case shortest vector problem. In *Proceedings of the 41st Annual ACM Symposium on Theory of Computing*. 333–342.

[62] Borzoo Rassouli and Deniz Gündüz. 2019. Optimal utility-privacy trade-off with total variation distance as a privacy measure. *IEEE Transactions on Information Forensics and Security* 15 (2019), 594–603.

[63] Oded Regev. 2009. On lattices, learning with errors, random linear codes, and cryptography. *Journal of the ACM* 56, 6 (2009), 1–40.

[64] Ronald L. Rivest, Len Adleman, and Michael L. Dertouzos. 1978. On data banks and privacy homomorphisms. *Foundations of Secure Computation* 4, 11 (1978), 169–180.

[65] Lalitha Sankar, S. Raj Rajagopalan, and H. Vincent Poor. 2013. Utility-privacy tradeoffs in databases: An information-theoretic approach. *IEEE Transactions on Information Forensics and Security* 8, 6 (2013), 838–852.

[66] Claus-Peter Schnorr. 1987. A hierarchy of polynomial time lattice basis reduction algorithms. *Theoretical Computer Science* 53, 2-3 (1987), 201–224.

[67] Mohamed Seif, Ravi Tandon, and Ming Li. 2020. Wireless federated learning with local differential privacy. In *Proceedings of the 2020 IEEE International Symposium on Information Theory (ISIT’20)*. IEEE, Los Alamitos, CA, 2604–2609.

[68] Adi Shamir. 1979. How to share a secret. *Communications of the ACM* 22, 11 (Nov. 1979), 612–613. https://doi.org/10.1145/359168.359176

[69] Chandra Thapa, Mahawaga Arachchige Pathum Chamikara, and Seyit Cantepe. 2020. SplitFed: When federated learning meets split learning. *arXiv preprint arXiv:2004.12088* (2020).

ACM Transactions on Intelligent Systems and Technology, Vol. 14, No. 1, Article 1. Publication date: November 2022.
[71] Oleh J. Tretiak. 1974. Rate distortion theory: A mathematical basis for data compression, Toby Berger. Prentice-Hall, Urbana, IL (1971).

[72] A. Triastcyn and B. Faltings. 2019. Federated learning with Bayesian differential privacy. In Proceedings of the 2019 IEEE International Conference on Big Data (Big Data’19). IEEE, Los Alamitos, CA, 2587–2596. https://doi.org/10.1109/BigData47090.2019.9065465

[73] Aleksei Triastcyn and Boi Faltings. 2020. Bayesian differential privacy for machine learning. In Proceedings of the 37th International Conference on Machine Learning, Hal Daumé III and Aarti Singh (Eds.). Proceedings of Machine Learning Research, Vol. 119. PMLR, 9583–9592. https://proceedings.mlr.press/v119/triastcyn20a.html.

[74] Stacey Truex, Nathalie Baracaldo, Ali Anwar, Thomas Steinke, Heiko Ludwig, Rui Zhang, and Yi Zhou. 2019. A hybrid approach to privacy-preserving federated learning. In Proceedings of the 12th ACM Workshop on Artificial Intelligence and Security. 1–11.

[75] Stacey Truex, Ling Liu, Ka-Ho Chow, Mehmet Emre Gursoy, and Wenqi Wei. 2020. LDP-Fed: Federated learning with local differential privacy. In Proceedings of the 3rd ACM International Workshop on Edge Systems, Analytics, and Networking. 61–66.

[76] Di Wang, Minwei Ye, and Jinhui Xu. 2017. Differentially private empirical risk minimization revisited: Faster and more general. In Advances in Neural Information Processing Systems 30 (2017).

[77] Hao Wang and Flavio P. Calmon. 2017. An estimation-theoretic view of privacy. In Proceedings of the 2017 55th Annual Allerton Conference on Communication, Control, and Computing (Allerton’17). IEEE, Los Alamitos, CA, 886–893.

[78] Zhibo Wang, Mengkai Song, Zhifei Zhang, Yang Song, Qian Wang, and Hairong Qi. 2019. Beyond inferring class representatives: User-level privacy leakage from federated learning. In Proceedings of the IEEE Conference on Computer Communications (IEEE INFOCOM’19). IEEE, Los Alamitos, CA, 2512–2520.

[79] Larry Wasserman. 2004. Bayesian inference. In All of Statistics. Springer, 175–192.

[80] David H. Wolpert and William G. Macready. 1997. No free lunch theorems for optimization. IEEE Transactions on Evolutionary Computation 1, 1 (1997), 67–82.

[81] Qiang Yang, Yang Liu, Tianjian Chen, and Yongxin Tong. 2019. Federated machine learning: Concept and applications. ACM Transactions on Intelligent Systems and Technology 10, 2 (2019), 1–19.

[82] Qiang Yang, Yang Liu, Yong Cheng, Yan Kang, Tianjian Chen, and Han Yu. 2019. Federated learning. Synthesis Lectures on Artificial Intelligence and Machine Learning 13, 3 (2019), 1–207.

[83] Hongxu Yin, Arun Mallya, Arash Vahdat, Jose M. Alvarez, Jan Kautz, and Pavlo Molchanov. 2021. See through gradients: Image batch recovery via GradInversion. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition. 16337–16346.

[84] Chengliang Zhang, Suyi Li, Junzhe Xia, Wei Wang, Feng Yan, and Yang Liu. 2020. BatchCrypt: Efficient homomorphic encryption for cross-silo federated learning. In Proceedings of the 2020 USENIX Annual Technical Conference (USENIX ATC’20). 493–506. https://www.usenix.org/conference/atc20/presentation/zhang-chengliang.

[85] Chengliang Zhang, Suyi Li, Junzhe Xia, Wei Wang, Feng Yan, and Yang Liu. 2020. BatchCrypt: Efficient homomorphic encryption for cross-silo federated learning. In Proceedings of the 2020 USENIX Annual Technical Conference (USENIX ATC’20). 493–506.

[86] Jiale Zhang, Bing Chen, Shui Yu, and Hai Deng. 2019. PEFL: A privacy-enhanced federated learning scheme for big data analytics. In Proceedings of the 2019 IEEE Global Communications Conference (GLOBECOM’19). IEEE, Los Alamitos, CA, 1–6.

[87] Bo Zhao, Konda Reddy Mopuri, and Hakan Bilen. 2020. iDLG: Improved deep leakage from gradients. arXiv preprint arXiv:2001.02610 (2020).

[88] Yang Zhao, Jun Zhao, Mengmeng Yang, Teng Wang, Ning Wang, Lingjuan Lyu, Dusit Niyato, and Kwok-Yan Lam. 2020. Local differential privacy-based federated learning for Internet of Things. IEEE Internet of Things Journal 8, 11 (2020), 8836–8853.

[89] Ligeng Zhu and Song Han. 2020. Deep leakage from gradients. In Federated Learning. Springer, 17–31.

[90] Ligeng Zhu, Zhihian Liu, and Song Han. 2019. Deep leakage from gradients. In Proceedings of the Annual Conference on Neural Information Processing Systems (NeurIPS’19).