Research on the profile modification of power skiving tool for internal gears

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Abstract
Power skiving provides an effective solution and considerable machining efficiency for the machining of internal gears. The tool profile design and the reusability after resharpening is crucial in gear manufacturing. In this paper, a novel method of tool profile correction based on the inverse error-complement of involute profile is proposed. Firstly, the mathematical model of involute cutter with rake angle and relief angle is established. The profile error relative to the target gear is calculated by the mathematical model. Then, the distribution of the gear profile errors is fitted by a fifth-order multinomial. The fitted multinomial function is attached to the cutter profile. The maximum theoretical error of the target gear profile is in 10e-7 mm order of magnitude through the calculation of fewer iterations. Finally, the distribution of the multinomial coefficients along the resharpening direction is obtained by linear programming. The result shows that the cutter designed by the proposed method possess almost negligible error to the gear profile and good repeatability of resharpening.

Keywords Power skiving · Profile modification · Gear manufacturing · Profile error

1 Introduction

Power skiving is an efficient gear machining method, which is specifically suitable for machining internal gears. Power skiving was proposed in the early twentieth century by Pittler [1], but it was not developed until modern times due to the limitations of machine tools and cutting tools at that time. The traditional machining of internal gears usually adopts gear shaping and gear broaching, which are inefficient gear machining methods. The application of power skiving, which operated in continuous cutting mode, as well as the development of tool coating technology has greatly improved the machining efficiency of internal gears.

In terms of machining theory, many scholars have carried out a series of studies on the principle, mathematical model, cutting angle, slice shape, and cutting force of power skiving method [2–7]. Over the past decade, power skiving set off a boom in the world; machine tool manufacturers have also shown people all kinds of power skiving machine tools [8]. In fact, the motorial structure of gear machine tools only needs to meet the required degree of freedom of the product. Some researches on the motion of the machine tool with respect to power skiving have been carried out. For example, Guo et al. [9] analyzed the error of machining internal gears by shaper cutter and corrected the machine setting parameter using the principle of linear superposition and the linear regression method. Tsai and Lin [10] took power skiving method applied to the traditional six-axis CNC center and the automatic generation of machining code was realized.

However, the ability to modify tooth profile by adjusting machine parameters is limited. The design of cutters will much directly affect the cutting performance and the gear profile. Thus, more attentions should be paid to the design of cutters because the cutter determines the basic profile of gears. Power skiving cutters can be divided into two categories according to the shape of rake face. For some cutters with smaller helical angle, the rake face is often designed as a conical surface which apex is on the cutter axis.

References

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kind of cutter is efficient and convenient to resharpening. As to other cutters with larger spiral angle, the rake surface is designed to be a plane which is approximately perpendicular to the helix line. That will balance the relief angle of the two cutting edges. The resharpening of all power skiving cutters is to move the rake face along the axis of the cutter to erase the original cutting edge.

The traditional power skiving cutters are usually made into a shaper-cutter-shaped structure, whose cutting performance will be affected by the working angle of the cutter tool. For example, a larger working rake angle will enhance the ability to remove chips, while a smaller working relief angle will increase the strength of the cutter. In view of this, Tsai studied the influence of tool parameters on the working angle, such as rake angle and relief angle, in the process of gear machining and optimized the design scheme of tool parameters with considering resharpening [11, 12]. However, the actual tooth surface machined by the traditional tool is not the standard involute profile. Guo et al. [13] found that the inherent errors of the skiving cutters, the longitudinal deviations of cutting edge and the extra errors caused by skiving cutter resharpening will result in tooth profile errors. Furthermore, resharpening amount of skiving cutter has obvious influence on the tooth profile error of workpiece. But the solution of eliminating these errors is not mentioned. Shih and Li [14] proposed a method of fitting B-spline curve to modify the generating gears which is conjugated with the work gear, so as to obtain a more accurate tool model compared with the traditional tool. However, when the cutter is designed based on the generating tool surfaces conjugated with the work gear, it is difficult to obtain the standard involute gear profile due to the fact that the cutting edge can hardly be guaranteed to coincide with the conjugate line, except for the case of the cutters with zero helical angle. Huang et al. [15] introduced a method for correcting the transverse profile of the gear shaper cutter enveloping gear up to a third-order correction. The transverse profile errors after third-order modification have been reduced to 0.1 μm. But this method is not used in power skiving cutters. These studies have obtained considerable results in the design of power skiving cutters and analyzed the effect of resharpening on cutting result. However, there are few of researches on resharpened cutters of power skiving for gear profile accuracy.

With the rapid development of CNC machine tools and cutting tool materials, power skiving is more and more widely used in the machining of internal gears. The resulting problems began to present a high demand for machining accuracy and tool reuse. However, resharpening will make the gear profile processed by power skiving lose constant accuracy. In this paper, a novel method of tool design is proposed based on the profile design of the cutter tooth, which can meet the profile accuracy requirements of the internal gears after resharpening the cutter. The research contents of this paper are as follows:

1. The machining principle of power skiving aiming at internal gears is expounded.
2. The mathematical model of the standard involute cutter tool is established, and the profile error state of the manufactured gear is obtained.
3. The profile of the cutter tool is compensated in reverse through the distribution of the gear profile errors, so as to obtain more accurate gear profile curve.
4. According to the resharpening law of the cutter, the compensation coefficients of the cutter profile are regarded as distributed linearly along the longitudinal direction of the cutter. The cutter tooth surface with theoretical errors less than 5e-4 mm to the target profile is obtained.

See Appendix 1 and Appendix 2 for coordinate transformation matrix [16] and some nomenclatures, respectively. Bold symbols represent vectors.

\section{2 Principle of power skiving}

The machining of internal gears by power skiving is carried out according to the gear meshing theory and the principle of line enveloping surface. In the process of machining internal gears, there is a shaft angle $\Sigma$ in the direction between the cutter axis and the workpiece axis (see Fig. 1). The value of $\Sigma$ is determined by the superposition of the helix angle of the cutter tool and the helix angle of the workpiece. $a_c$ is the axis of the cutter. $a_1$ is the axis of the workpiece. $\omega_c$ is the angular velocity of the cutter. $\omega_1$ is the angular velocity of the workpiece. The pitch circle of the cutter and the workpiece is tangent at point $O$. The positional relationship

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{power_skiving.png}
\caption{Power skiving}
\end{figure}
between the cutter and the workpiece should follow the same tooth direction. For spur gear, the helix angle is 0, and the cutter can be regarded as a helical gear. The cutting edge is the transverse tooth profile of the helical gear.

The action of power skiving is composed of two parts. The linkage-rotation of the cutter and the workpiece provides the relative linear velocity $V_c$ from $V_c$ and $V_1$, which come from the cutter and the workpiece at point $O$, respectively. $V_c$ provides cutting action part. At the same time, the movement of the cutter along the axis of the workpiece provides the feed motion part.

Power skiving needs to meet the following motion relation as

$$\omega_1 z_c = \omega_2 z_1$$ (1)

### 3 Cutter tool profile equation

The power skiving cutter itself is a cylindrical gear. However, in order to make the cutter possess cutting ability, we should provide a rake angle and relief angle to avoid producing interference and chip accumulation between the cutter tooth and the gear tooth. The rake angle is usually provided by the rake face perpendicular to the helix curve of the cutter. The relief angle is provided from the continuous reducing tooth profile shift coefficient $x_c$ along the axis of the cutter. Thus, the outline of the cutter is a conical shape with a taper angle $\alpha_f$. The design of the relief angle and the rake angle of the cutter prevents the friction between the cutter tooth surface and the machined tooth surface, which can affect the finish of the tooth surface. Power skiving cutter will become gradually blunt with the increase of processing. The cutter tooth requires resharpening for repeated using.

The resharpening of the cutter is to translate the rake face along the axial direction of the cutter so as to grind the old cutting edge and provide new cutting edge for subsequent processing. We will adopt the same resharpening principle for the cutter design (see Fig. 2).

The side face of the cutter is shown in Fig. 3. $O_0$ is the reference point at the intersection of the reference circle and plane $A_0$, where the tooth profile shift coefficient of the cutter is $x_0$. Normally, the value of $x_0$ is the same as the value of the cylindrical gear being machined. The reference point moves to the point $O$ when the cutter tooth was resharpened by $d_g$. The radius of the reference circle of the cutter at this point can be expressed by Eq. (2).

$$r_0 = \frac{m z_c}{2 \cos \beta_0} + x_0 m$$ (2)

Plane $T_c$ is the section plane perpendicular to the cutter axis at any position of the gear cutter. $\mu$ represents the position of plane $T_c$. When $T_c$ is located near the tail of the cutter, $\mu$ is positive value. When plane $T_c$ is located away from the tail of the tool, $\mu$ is negative value. The shift coefficient $x_c$ of the transverse tooth profile of the cutter can be expressed in Eq. (3).

$$x_c = -\frac{(\mu + d_g) \tan \alpha_f}{m \cos \beta_0} + \frac{x_0 \cos \beta_0}{\cos \beta_0}$$ (3)

Figure 4 shows the profile of the cutter in plane $T_c$. The tooth profile of the helical cylindrical gear is an involute with $r_b$ as the base circle. Point $P$ is arbitrary point on the involute. The coordinates of $P$ can be expressed in Eq. (4) and the unit normal vector can be expressed in Eq. (5).
where $\phi = \tan \alpha_p$ presents the profile parameter when the transverse pressure angle of the cutter is $\alpha_p$.

The cutting edge of the cutter is provided by the intersection line between the rake face and the tooth surface of the cutter. In the process of machining, the influence factors on the theoretical gear profile shape are all from the cutting edge. Therefore, it is necessary to model the cutting edge accurately to obtain the accurate profile curve of work gears. The cutting edge needs to satisfy the equation of the cutter tooth surface as well as the rake surface.

Figure 5 is the coordinate system of the cutting edge. $O_3$ is the intersection point of the cutter axis and the transverse plane which contains the point $O$. Coordinate $Z_3$ coincides with the axis of the cutter. $X_3$ crosses the point $O$. Plane $T_a$ coincides with the rake face. $Z_a$ is perpendicular to the plane $T_a$. $Y_a$ and $X_a$ are on the plane $T_a$. $Y_a$ is tangent to the reference circle of the cutter. $n_a$ is the unit normal vector of the rake face at point $O$. Plane $T_b$ is the auxiliary plane perpendicular to the direction of the helical curve of the cutter. The angle between $T_b$ and plane $T_a$ is $\alpha_k$. Coordinate system $S_b$ is the auxiliary coordinate system. $X_b$ coincides with $X_3$. Plane $T_c$ is on the plane $X_2Y_2$.

We transform the expression of the cutting edge of the cutter into coordinate system $S_3$, which can be expressed in Eq. (6).

$$r_3(\varphi, \mu) = M_{32}(S_c, \theta, \mu)M_{21}(S_a, \theta) r_1(\varphi)$$

(6)

where

$$\begin{aligned}
\theta &= \frac{x}{2z} + \tan \alpha_r - \alpha_r + \frac{2s_r \tan \alpha_l}{z} \\
\theta &= \frac{x}{2z} + \tan \alpha_l - \alpha_l + \frac{2s_r \tan \alpha_l}{z}
\end{aligned}$$

(7)

The deduced $r_3(\varphi, \mu)$ is the coordinate of any point on the tooth surface of the cutter. However, we have already discussed that the cutting edge of the cutter is derived from the intersection of the rake face and the tooth surface of the cutter. The above Eq. (6) represents a surface rather than a curve. A boundary conditions should be provided for Eq. (6) so that the above expression can represent the cutting edge.

We can represent $n_a a = [0;0;-1]$ in $S_3$ coordinates by transformation of coordinates in Eq. (8). Superscript $a$ indicates that $n_a a$ belongs to the plane $T_a$.

$$n'_a = T_{3b}(\beta, S_c) T_{ba}(\alpha_k)n_a = \begin{bmatrix} -\sin \alpha_k \\ -S_c \cos \alpha_k \sin \beta_c \\ -\cos \alpha_k \cos \beta_c \end{bmatrix}$$

(8)

The constraint equations to Eq. (6) can be expressed in Eq. (9).
\[ f_i(\varphi, \mu) = - (r_{3x} - r_0) \sin \alpha_k - S_x r_{3y} \cos \alpha_k \sin \beta_c - r_{3z} \cos \alpha_k \cos \beta_c = 0 \]

**4 Mathematical model of power skiving**

Figure 6 is the coordinate system frame of power skiving for internal gears. \( S_x \), which describes the rotation of the tool, is the initial position of \( S_x, S_y, \) which describes the rotation of the workpiece, is the initial position of \( S_r, S_x \) and \( S_y \) are auxiliary coordinate systems parallel to each other in space. \( Z_5 \) coincides with the axis of the workpiece. \( X_5 \) intersects the axis of the cutter at point \( O_5 \). \( \Sigma \) is the shaft angle of the cutter axis and \( \phi \) is the offset of the cutter to the direction of \( Y_6 \) relative to the standard position. The cutter rotates around its axis by angle \( \varphi_c \) while the workpiece rotates around the axis of the workpiece by \( \varphi_1 \). The following conditions should be met as in Eq. (10).

\[
\begin{align*}
\varphi_1 &= \frac{z}{z_1} \varphi_c \\
n &= \frac{m_1}{2 \cos \beta} + x_m - r_0 + d_s \tan \alpha_p \\
\Sigma &= \beta_c + \beta
\end{align*}
\]

The tooth surface equation of the internal gear machined by power skiving can be represented as Eq. (11).

\[ r_1(d_x, \varphi_c, \varphi, \mu) = M_{76}(\varphi_1) M_{65}(d_x, a, \gamma) M_{54} \left( \sum_i S_i M_{43}(\varphi_c) r_3(\varphi, \mu) \right) \]

Power skiving also meet the theory of gear meshing. The meshing equation can be represented as in Eq. (12) [10].

\[ f_2(\varphi, \varphi_c, \varphi, \mu) = \left( \frac{\partial r_1(1 : 3)}{\partial \varphi} \right) \cdot \left( \frac{\partial r_1(1 : 3)}{\partial \varphi_c} \right) = 0 \]

Equation (11) represents the coordinate expression of the internal gear surface by power skiving. Given radial position \( R \) and the axial position \( Z \) of the workpiece can we obtain 2 conditional equations as Eqs. (13) and (14).

\[ f_3(d_x, \varphi_c, \varphi, \mu) = \sqrt{r_1^2(d_x, \varphi_c, \varphi, \mu) + r_2^2(d_x, \varphi_c, \varphi, \mu)} - R = 0 \]
\[ f_4(d_x, \varphi_c, \varphi, \mu) = r_3(d_x, \varphi_c, \varphi, \mu) - Z = 0 \]

The coordinates of the tooth surface expressed in Eq. (11) contains 4 unknown parameters: \( d_x, \varphi_c, \varphi, \) and \( \mu \). Meanwhile, we also obtain 4 boundary constraint equations \( f_1 \sim f_4 \) which can form a system of four-element nonlinear equations. A unique set of exact solutions \( (d_x, \varphi_c, \varphi, \mu) \) can be obtained by providing a set of initial values for the four nonlinear equations. We can obtain the three-dimensional coordinates of the gear surface by substitute the exact solutions into Eq. (11).

**5 Optimization algorithm of cutter profile**

According to the principle of power skiving, the transverse profile of the cylindrical gear machined is constant along the axial direction. By this, we only need to analyze and calculate the profile of a certain transverse face. We will set \( Z = 0 \) in the following section.

To facilitate the optimization and outputting the final results, the error of the gear profile is defined as the distance between the gear profile machined and the standard involute curve of the cylindrical gear at radial position \( R \). The sign of error that is negative or positive depends on whether the calculated point of the tooth profile is inside or outside the tooth. We can rotate the gear profile curve around the axis to move the point \( P_m \) on the gear profile curve machined to the point \( P_m' \) on the target profile curve by angle \( \theta_m \). The error at the reference radius \( R_m \) will be 0. As shown in Fig. 7, \( P_i' \) and \( P_i \) are the points on the profile curve machined and the target profile curve, respectively. They are all located on the radius \( R_i \) of the cylindrical gear. Point \( P_i \) is the initial position of the point \( P_i' \).
where $E_i$ is the error at point $P_i$ of the work gear. Then, the new profile will be compared with the target profile curve so as to obtain the new profile of the work gear. Then, the new profile will be closer to the target profile after compensating errors. However, this method may require several iterations to make the gear profile closer to the target profile because the change rule of errors of the cutter profile and the gear profile is not strictly linear. In the practical engineering production, the common gear tooth surface accuracy is within a certain limit as well as the manufacture of the cutter tool. Therefore, the modification of the cutter profile only needs to meet the requirements of engineering applications.

We define point $P_i$ in Fig. 7 is processed by the point $P$ in Fig. 4. To make the point $P_i$ on the gear profile of the cylindrical gear close to the target profile as much as possible, the error value $E_i$ can be compensated along the normal direction at the corresponding point $P$ of the cutter. The error at point $P$ will be reversed to the profile of the work gear during machining.

Since the error value calculated is tiny relative to the gear size, so we can take the currently calculated $E_i$ as the step size in current iteration process for the calculation of the cutter profile (Eq. 19).

\[
E_c = E_i
\]  

The profile curve of the gear can be averagely divided into $n$ (in this paper $n = 17$) points from the addendum to the dedendum of the gear tooth. By solving the error value $E_i$ of every point at radial position $R_c$, we can make the error amount smoothly distributed on the profile of the cutter by fitting a high-order multinomial. $R_i$ can be represented as in Eq. (20).

\[
R_i = R_{\text{min}} + \frac{(i - 1)(R_{\text{max}} - R_{\text{min}})}{(n - 1)}
\]  

In fact, $R_i$ and the tool parameter $\varphi$ are one-to-one correspondence. For the convenience of calculation, $(\varphi, E_i)$ can be used as the observation point. The fifth-order multinomial of the tool parameter $\varphi$ can be established by the least square method for the defined error of the cutter in Eq. (21).

\[
E_i(\varphi) = C_{1k}\varphi^3 + C_{2k}\varphi^4 + C_{3k}\varphi^5 + C_{4k}\varphi + C_{5k}
\]

The modified profile of the cutter can be expressed as in Eq. (22).

\[
r_{i_c}(\varphi) = r_i(\varphi) + E_i(\varphi)u_1(\varphi)
\]

We plug $r_{i_c}(\varphi)$ into the Eqs. (6) and (11) to recalculate the new profile of the work gear. Then, the new profile will be compared with the target profile curve so as to obtain the next set of error $E_i$. The new set of error $E_i$ can be used for obtaining the coefficients of another set of fifth-order multinomial again through iterative calculation. The more accurate gear profile will be obtained. When the profile
error obtained meets the designed requirements by \( k \) times iteration, the final fifth-order multinomial coefficients can be expressed as in Eq. (23).

\[
C = \left[ \sum_{j=1}^{l} C_{0j} \sum_{j=1}^{k} C_{1j} \sum_{j=1}^{k} C_{2j} \sum_{j=1}^{k} C_{3j} \sum_{j=1}^{k} C_{4j} \sum_{j=1}^{k} C_{5j} \right]
\] (23)

The process of this optimization algorithm is shown in Fig. 8. Firstly, the power skiving cutter with involute in transverse plane is used as the initial cutter tool. Through the mathematical model in the previous sections, the expression of the tooth profile of the internal gear \( (r) \) can be obtained. Then, the calculated profile error \( E_i \) generated by the initial cutter will be obtained by the calculation method in this section. The distribution of \( E_i \) respect to the profile parameter \( \phi \) can be fitting as a fifth-order multinomial. Attaching the fifth-order multinomial to the initial profile of the cutter tooth, the new tooth profile error \( E_i \) will be obtained again. When the maximum of \( E_i \) is less than the designed tolerance error \( E_t \), the current expression of the cutter tooth should be the final optimized profile of the cutter tooth.

6 Example of the algorithm

The tooth surface of the cylindrical gear machined by power skiving is generated by two cutting edges. However, the standard tooth profile of cylindrical gears is involute. Because the gear cutter and the cylindrical gear are placed by a shaft angle position, the meshing relationship between the cutter and the cylindrical gear is no longer the meshing of standard involute gear pair. It is necessary.

to analyze the tooth shape of the cylindrical gear machined by power skiving. Taking the involute as the target tooth profile, the deviations calculated using the above method between the gear profile machined by the cutter with involute profile and the target profile is obtained in Fig. 9. The data of calculation examples used are shown in Table 1.

As shown in Fig. 9, the abscissa represents the profile parameter \( \phi \) of the cutter. The ordinate represents the error of the internal gear profile. The nonzero error of the internal gear profile explains the tooth profile of the internal gear generated by the cutter with standard involute profile is not involute. Moreover, the tooth profile of the right tooth surface \( (S_a = -1) \) shows the maximum negative error \((-0.0053 \text{ mm})\) is at the addendum, the maximum positive error \((+0.0063 \text{ mm})\) is at the dedendum. The left tooth surface \( (S_a = 1) \) shows that the maximum negative error

![Fig. 9 Error by initial cutter profile](image)

**Table 1 Illustrative example**

| Items             | Value         |
|-------------------|---------------|
| \( m \)           | 3             |
| \( z_c \)         | 41            |
| \( \beta_c \)     | 20°RH         |
| \( \beta \)       | 0             |
| \( a_0 \)         | 7°            |
| \( x_0 \)         | -0.3          |
| \( x \)           | -0.3          |
| \( \alpha_r \)    | 5°            |
| \( r_b \)         | 61.0289 mm    |
| \( d_g \)         | 20°           |
| \( a \)           | 86            |
| \( z_1 \)         | 64.547 mm     |
| \( R_m \)         | 128.1 mm      |
| \( \gamma \)      | 0             |
| \( R_{\text{min}} \) | 125.1 mm     |
| \( R_{\text{max}} \) | 131.1 mm    |
(−0.0285 mm) is at the dedendum and the maximum positive error (0.0097 mm) is at the addendum. The profile calculated has obviously deviated from the target profile, which will lead to the instability of the gear in the transmission process and increasing the transmission error and noise.

Figure 10 shows the distribution of the tooth profile errors of the internal gear calculated after one iteration \((k = 1)\). As can be seen that the profile errors of the right tooth surface decreases from the uncorrected 0.0116 mm to 2.4e-4 mm, and the profile errors of the left tooth surface decreases from the uncorrected 0.0382 mm to 4.2e-4 mm. The modification algorithm produces a significant effect on error reduction. However, since the iteration step adopted is the deviation of the profile along the circumferential direction, the compensation cannot reduce the errors of the tooth profile by the same extent. The ability of error reduction of iteration is limited in a way.

Figures 11 ~ 13 show the distribution of the profile errors after multiple iterations. The profile errors of the tooth surface are reduced to 0.9e-5 mm and 2.3e-5 mm in the second iteration (see Fig. 11), respectively. In Fig. 12, the tooth profile errors are reduced to 2.2e-5 mm and 0.4e-5 mm by the third iteration. In terms of the order of magnitude, when the third iteration is carried out, the reduction of the errors is not obvious enough and even tends to increase in reverse. However, when we manage the fourth iteration (see Fig. 13), the order of magnitude of the tooth profile errors was reduced to 10e-7 mm. Such error levels can be negligible in gear manufacturing. Therefore, we can stop the calculation after the fourth iteration.

The relevant multinomial coefficients are shown in Table 2.

In the first three iterations, the fitting polynomial curve of the error was very close to the discrete points. However, in the fourth iteration, the fitting polynomial curve had seriously deviated from the distribution of the discrete points. It is related to the order of polynomial adopted in this paper. The convergence will be greatly reduced if the iterative calculation is continued after the fourth iteration that will increase unnecessary calculation costs. In general, higher
order of the fitting polynomial will make the fitting curve more similar to the discrete points. However, the profile error of the gear with the order of magnitudes by 0.001 mm is sufficient for most occasions of gear manufacturing. Therefore, the modification results obtained by using the fifth-order polynomial coefficients are sufficient to meet engineering requirements. Moreover, we can also stop the calculation with a signal that the fitting polynomial curve deviated from the distribution of the discrete points.

The correctness of the algorithm can be verified by simulation with modeling the cutter tool in VERICUT software. The modeling of the cutter tool is shown in Fig. 14. The tool modeling can be imported into the simulation software of virtual machining (see Fig. 15). The overcut amount and the residual amount of the tooth surface can be obtained by comparing the machinated tooth surface and the target tooth surface in VERICUT software (see Figs. 16 and 17). As shown in Figs. 16 and 17, it can be seen that the error distribution of the simulated tooth surface is consistent with the calculation results in Figs. 9 and 13. Thus, we can use the algorithm proposed for subsequent study.

### 7 Topological modification of cutter tooth surface

As we know, cutters should be designed with reuse features after resharpening. In above section, we obtained the profile shape of the designed cutter without resharpening, that is, $d_g = 0$. However, in the actual production process, the tool needs to be polished when the cutter is wearing by a certain degree. The position of the reference point of the cutter tooth will be changed along the axial direction, that is, $d_g > 0$.

To make the cutter possess excellent repeatability, the whole tooth flank surface of the cutter should be taken into account. We can modify the profile at the maximum resharpening amount of the cutter in the same way as the initial

### Table 2 Multinomial coefficients

| $k$ | $S_a$ | $C_{5k}$ | $C_{4k}$ | $C_{3k}$ | $C_{2k}$ | $C_{1k}$ | $C_0$ |
|-----|-------|---------|---------|---------|---------|---------|-------|
| 1   | 1     | -0.03679| -0.208659| 0.48143| -0.31038| 0.10904| 0.01772|
| 2   | 1/2   | -0.0236 | -0.1004 | -0.2675 | -0.1820 | 0.1033 | -0.0014 |
| 3   | 1/3   | 0.0110  | -0.0159 | 0.1482 | -0.0592 | 0.0125 | -0.0011 |
| 4   | 1/4   | -0.00416| 0.0439  | -0.0128 | -0.0005 | 0.0006 | -0.0000 |

**Fig. 14** Cutter tool modeling
transverse profile mentioned above. Consequently, another set of fifth-order multinomial coefficients can be obtained in Eq. (24).

\[
A = \left[ \sum_{j=1}^{k} A_{ij} \sum_{j=1}^{k} A_{ij} \sum_{j=1}^{k} A_{ij} \sum_{j=1}^{k} A_{ij} \right]^{(24)}
\]

Since the change of the profile error is very slight relative to the resharpening amount, we can regard the tool errors influenced by resharpening as approximately linear change. The error compensation at any position can be indicated as in Eq. (25).

\[
E_c = \frac{\sum C_i d_{g}}{\sum A_i - \sum C_i} \quad \text{for} \quad i = 0 \sim 5
\]

where \(i = 0 \sim 5\).

Based on the example in Table 1, we suppose that the final resharpening amount is 10 mm, meaning \(d_{g_{\max}} = 10\) mm. Figure 18 shows the error curves of the gear tooth profile after resharpening the cutter by 10 mm. It can be seen that the errors of the right tooth profile increase to 0.034 mm, while the profile
errors of the left tooth surface increase to 0.040 mm. It has seriously exceeded the accuracy requirements of the gear. The same method can be used for modifying the tooth profile on the end transverse plane of the cutter tooth. The corrected error curves after resharpening with 5 iterations are shown in Fig. 19. The final multinomial coefficients are output in Table 3.

By substituting multinomial coefficients $A_i$ into Eqs. (26) and (21) can we obtain the distribution of the tooth profile errors as the changing of the resharpening position.

Figure 20 shows the error distribution of gear tooth profile with change of resharpening after adopting linear distribution for multinomial coefficients. As can be seen from the result calculated, with the increase of the resharpening amount, the tooth profile errors are gradually increased and then gradually decreased to minification. The pressure angle is slightly larger than the target gear profile. However, the maximum of the tooth profile errors of the internal gear is 2.3e-4 mm after modification.

Figure 19  Error by $d_k = 10$ after modification

8 Conclusions

In this paper, a method of error compensation is proposed to modify the profile of the power skiving cutter. This method has the advantages of good convergence performance, simple calculation, and high accuracy. The algorithm can also be applied to the design of other non-involute profile gear cutters. The following conclusions are summarized through the researches on the correction of power skiving cutters.

1. Processing internal gears by power skiving using the involute profile cutter will cause profile errors. The distribution of these errors of the two surfaces is not equal.
2. The proposed method to modify the cutter profile by compensating the transverse profile errors of the internal gear can obtain the accuracy of 10e-7 mm order of magnitude.
3. Since resharpening cutter impact on the profile error is relatively obvious, distributing the modification coefficient by linear program can effectively improve the profile errors which are generated by resharpening. The errors obtained in the example can be controlled within 2.3e-4 mm. The position of the maximum error is in the middle of the length of the cutter tooth.

| $S_0$ | $A_0$ | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ |
|-------|-------|-------|-------|-------|-------|-------|
| -1    | -0.03571 | -0.06276 | 0.32075 | -0.24489 | -0.01364 | 0.01731 |
| 1     | 0.01122 | -0.01567 | -0.34965 | -0.21014 | -0.00797 | 0.02899 |

Table 3 Multinomial coefficients after 5 iterations
Appendix 1. Nomenclature

\( m \)

normal module of gear

\( z_c \)

teeth number of cutter

\( z_i \)

teeth number of internal gear

\( \alpha \)

normal pressure angle of cutter

\( \alpha_r \)

Transverse pressure angle of cutter

\( \beta_c \)

helical angle of cutter

\( \beta \)

helical angle of gear

\( \alpha_f \)

tooth profile shift coefficient of gear

\( x_0 \)

Initial tooth profile shift coefficient of cutter

\( \alpha_k \)

Rake angle of cutter

\( S_a \)

Cutting edge mark (+1 L, −1 R)

\( S_e \)

hand of spiral mark of cutter (+1 R, −1 L)

\( r_b \)

base radius of cutter

\( r_\mu \)

radius of reference circle of cutter

\( R_m \)

radius of reference circle of gear

\( R_{\max} \)

radius of internal gear for dedendum

\( R_{\min} \)

radius of internal gear for addendum

\( d_c \)

resharpening length

\( E_r \)
	error tolerance

\( \gamma \)

offset of cutter

Appendix 2

\[
M_{21} = \begin{bmatrix}
\cos \theta & S_a \sin \theta & 0 & 0
\end{bmatrix},
M_{22} = \begin{bmatrix}
\cos \theta & -S_e \sin \theta & 0 & 0
\end{bmatrix},
M_{43} = \begin{bmatrix}
\cos \varphi_e & \sin \varphi_e & 0 & 0
\end{bmatrix},
M_{54} = \begin{bmatrix}
1 & 0 & 0 & 0
\end{bmatrix},
M_{65} = \begin{bmatrix}
1 & 0 & 0 & 0
\end{bmatrix},
M_{76} = \begin{bmatrix}
0 & 0 & 0 & 0
\end{bmatrix},
\]

\[
T_{ba} = \begin{bmatrix}
\cos \alpha_k & 0 & \sin \alpha_k & 0
\end{bmatrix},
T_{30} = \begin{bmatrix}
1 & 0 & 0 & 0
\end{bmatrix},
\]

\[
\begin{align*}
\text{Appendix 2} \\
\text{References} \\
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\]

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