Mathematical model of a ferromagnetic lubricant in the presence of a porous coating in the bearing structure

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Abstract. Based on the flow equation of a ferromagnetic liquid lubricant for a "thin layer", the continuity equation and Darcy's equation describing the flow of a lubricant in a porous body, an exact self-similar solution of wedge-shaped sliding support with a porous coating of the surface of the support ring is found, taking into account the dependence of the viscosity of the ferromagnetic lubricant and the permeability of the porous coating with the incomplete filling of the working gap. Analytical dependencies for the velocity and pressure fields in the lubricating and porous layer are obtained. Also, the main operating characteristics are determined load-bearing capacity and friction force. The numerical analysis of the theoretical results showed that the bearing capacity of the bearings can be increased by 8-12% in the range of the studied load-speed modes. At the same time, the coefficient of friction is reduced by 14-16%. To verify and confirm the effectiveness of the obtained theoretical models, an experimental study of a modified wedge-shaped sliding support on TP-22C, MS-20 oil and their mixture with various additives was carried out. As a result of theoretical and experimental studies, tribotechnical characteristics were determined that allow us to judge the presence of a long-term friction mode.

1. Introduction

A sufficient number of works have been devoted to the development of a computational model of thrust plain bearings with a porous coating on the support ring surface [1-8]. However, they did not take into account the rheological properties of ferromagnetic lubricants and the incomplete (pre-emergency state) filling of the working gap [8-16].

The proposed paper presents a mathematical calculation model of the hydrodynamic flow regime of the lubricant in the working gap, taking into account the dependence of the viscosity of the lubricant, which has wedge-shaped sliding support and in the porous body of the material, which prevails in the laminar flow regime with ferromagnetic rheological properties with incomplete (pre-emergency state) filling of the working gap and the permeability of the porous coating on pressure.

2. Materials and methods

The laminar flow of an incompressible ferromagnetic fluid in the working gap is considered. The wedge-shaped sliding support is stationary, and the support ring with a porous coating moves at a speed $u'$ (Figure 1).
The contours presented in the calculation scheme in the coordinate system are:

- \( C_0 \) - an inclined contour;
- \( C_1 \) - a support ring;
- \( C_2 \) - a support ring with a porous coating.

These contours are designated as follows:

\[
\begin{align*}
C_0: & \ y' = h_0 + x' t g \alpha = h(x') \\
C_1: & \ y' = 0 \\
C_2: & \ y' = -\bar{\bar{H}}
\end{align*}
\]

where \( h_0 \) is the thickness of the lubricant; \( \alpha \) is the angle of the inclined slider with the \( ox \) axis; \( \bar{\bar{H}} \) - the thickness of the porous coating.

The dependence of the viscosity of the rheological properties of a ferromagnetic lubricant on the pressure is taken into account in the following form:

\[
\mu = \mu_0 e^{\alpha \rho}; \quad \sigma' = \sigma_0 e^{\alpha \rho}; \quad k' = k_0 e^{\alpha \rho}
\]

Where \( \mu ' \) is the dynamic viscosity coefficient;
- \( \mu_0 \) - dynamic viscosity of the lubricant;
- \( \sigma \) - electrical conductivity of the lubricant;
- \( \sigma_0 \) - characteristics of the electrical conductivity of the lubricant;
- \( p ' \) - hydrodynamic pressure in the lubricating layer;
- \( \alpha ' \) - a parameter that characterizes the dependence of the lubricant viscosity on the pressure.
- \( k' \) - permeability of the porous coating.

To solve this problem, the well-known equations for the "thin layer" of the movement of a ferromagnetic lubricant, the continuity equation, and Darcy’s equation describing the flow of a lubricant in a porous body are used, taking into account the influence of additional factors, namely, such as the lubricant viscosity, the permeability of the porous coating, the Hartmann number, the electromagnetic field strength.

\[
\frac{\partial p}{\partial y} = 0, \quad \frac{\partial^2 v}{\partial y^2} = e^{-\alpha p} \frac{\partial p}{\partial x} - NB^2 - ABE, \quad \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = 0, \quad \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial x^2} = 0.
\]

By generally accepted simplifications, the boundary conditions for this problem will be written as:

\[
\begin{align*}
\text{at} \ y = 0, \quad & u = 0, \quad t_g = 1 + \eta x = h(x), \quad v = -1 at y = 0, \\
\text{at} \ y = \bar{y}, \quad & u = 0, \quad \bar{\bar{M}} = -\kappa \frac{\partial p}{\partial y^*}, \quad A = \frac{\sigma_0 B^* e^{h_0}}{\mu_0}\mu u^*.
\end{align*}
\]

where \( N \) is the value due to the presence of an electromagnetic field;
- \( \eta \) - the Hartmann number;
- \( E \) - the vector of the electromagnetic field strength;
- \( B \) - magnetic induction vector.
\(\mu\) – dynamic viscosity coefficient; 
\(\sigma\) - electrical conductivity; 
\(x_1\) and \(x_2\)-respectively, the angular coordinates of the beginning and end of the free surface of the lubricant; 
\(\bar{M}\) – a parameter that characterizes a porous body; 
\(\eta\) - relative eccentricity; 
The transition to dimensionless variables in the lubricant layer was carried out according to the standard method:
\[
\begin{align*}
\psi' &= \epsilon u^* u, \quad \psi'' = u^* v, \quad \epsilon = \frac{h_0}{L}, \\
y' &= h_0 y, \quad x' = Lx, \quad p' = p^* p, \quad p^* = \frac{\mu u^* L}{h_0^2}, \\
E' &= E^* E, \quad B' = B^* B, \quad p^* = \frac{\mu u^* L}{h_0^2}, \\
\mu' &= \mu_0 e^{\alpha' p'}, \quad \sigma' = \sigma_0 e^{\alpha' p'}, \quad \alpha' = \frac{\alpha}{p^*}
\end{align*}
\]
In a porous layer:
\[
n' = Lx', \quad y' = Ly', \quad \psi' = P^* P, \quad k' = k_0 e^{\alpha' p'}.
\]
When setting the problem, several simplifying assumptions are made:
\(\text{a) Values } B = \{0; B_p; 0\}, \quad E = \{0; 0; E_x\},\)
\(\text{b) Maxwell's equations are fulfilled when } E_z = E = \text{const}, \quad B_p = B = \text{const}\)
\(\text{c) The influence of the flow on the electric and magnetic fields is neglected, taking into account the magnitude of the parameters } E', \quad B' \text{ and the flow velocity of the ferromagnetic liquid. Let's introduce the notation } z = e^{-\alpha p'}.
\]
We differentiate both parts of the equality and substitute equation (3), as a result, equation (3) and the boundary conditions (6) will take the form:
\[
\begin{align*}
\frac{\partial p}{\partial y} &= 0, \\
\frac{\partial^2 \nu}{\partial y^2} - \frac{1}{a} \frac{dx}{dz} - NB^2 - ABE, \\
\frac{\partial \psi}{\partial x} + \frac{\partial u}{\partial y} &= 0, \\
\frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial x^2} &= 0.
\end{align*}
\]
\[
p = p|_{y' = 0}, \quad u|_{y' = 0} = \bar{M} \frac{\partial \psi}{\partial y}|_{y' = 0}, \quad \frac{\partial \psi}{\partial y}|_{y' = -\frac{h}{L}} = 0, \quad z(x_1) = z(x_2) = 1.
\]
The direct solution is found in the form of a self-similar solution taking into account Darcy’s equation.
We are looking for an exact self-similar solution of system (2) in the form:
\[
\psi' = \bar{C}_2', \quad \psi''(\xi) = \bar{C}_1 \bar{u}'(\xi) + \xi \bar{v}'(\xi) = 0, -\frac{1}{a} \frac{dx}{dz} - NB^2 - ABE = \frac{\bar{C}_1}{h^2(x)} + \frac{\bar{C}_2}{h^3(x)}.
\]
Substituting (9) into (7), (8), we get:
\[
\frac{\partial \psi}{\partial y}(\xi) = \bar{C}_2', \quad \bar{v}'(\xi) = \bar{C}_1 \bar{u}'(\xi) + \xi \bar{v}'(\xi) = 0, -1 \frac{dx}{dz} - NB^2 - ABE = \frac{\bar{C}_1}{h^2(x)} + \frac{\bar{C}_2}{h^3(x)}.
\]
\[
\frac{\partial \bar{u}}{\partial x}(\xi) = 0, \quad \frac{\partial \bar{v}}{\partial y}(\xi) = 0 \text{ at } \xi = 1; \quad \frac{\partial \bar{v}}{\partial y}(\xi) = 0 \text{ at } \xi = 0,
\]
\[
\xi = 1 \frac{\partial \bar{u}}{\partial x}(\xi) = -1 \text{ at } \xi = 0; \quad \bar{u}|_{y' = 0} = \bar{M} \frac{\partial \psi}{\partial y}|_{y' = 0} = 0.
\]
The result of task solving (10)–(11) is found by direct integration:
\[
\bar{v}_0(\xi) = \frac{\bar{C}_1}{L^2} - \frac{\bar{C}_1}{L} \xi + 1, \quad \bar{v}_0(\xi) = \frac{\bar{C}_1}{L^2} \xi - \frac{1}{2} (\xi^2 - \xi).
\]
From (4) the equation of the system (10) for the hydrodynamic pressure, we have:
\[
z = -\alpha (x - x_1) [\bar{C}_1 (1 - \eta (x + x_1)) + \bar{C}_2 (1 - \frac{3}{2} \eta (x + x_1)) + (NB^2 + ABE)] + 1
\]
The condition \(z(x_1) = z(x_2) = 1\) follows:
\[
\bar{C}_2 = -\bar{C}_1 \left(1 + \frac{3}{2} (x_2 + x_1)\right) - (NB^2 + ABE) \left(1 + \frac{3}{2} \eta (x + x_1)\right)
\]
With (14) taken into account for $z$, we get:

$$z = -\alpha \frac{\eta}{2} (x - x_1)(x - x_2) \left( \tilde{C}_1 + 3(NB^2 + ABE) \right) + 1. \quad (15)$$

Using the asymptotic expansion $\varepsilon$ for the pressure from equation (12), we have as a result:

$$p = \frac{\eta}{2} (x - x_1)(x - x_2) \left( \tilde{C}_1 + 3(NB^2 + ABE) \right) \quad (16)$$

With (16) taken into account, the solution of Darcy's equation is presented as:

$$P(x, y') = R(y') + \frac{\eta}{2} (x - x_1)(x - x_2) \left( \tilde{C}_1 + 3(NB^2 + ABE) \right). \quad (17)$$

Introducing the expression (17) into the fourth equation of the system (7), we come to the form of the function $R(y')$:

$$R'(y') + \eta \left( \tilde{C}_1 + 3(NB^2 + ABE) \right), \quad (18)$$

$$R(0) = 0, \quad \frac{\partial R}{\partial y'} y' = -\frac{\mu}{\eta} = 0 \quad (19)$$

As a solution result (18)-(19), we obtain the expression:

$$R(y') = -3(NB^2 + ABE) \left( \frac{y'^2}{2} + \frac{\mu}{\eta} y' \right) - \tilde{C}_1 \eta \left( \frac{y'^2}{2} + \frac{\mu}{\eta} y' \right). \quad (20)$$

Integrating the continuity equation $\tilde{M} \frac{\partial \tilde{P}}{\partial y'} y' = \int_{0}^{1} \tilde{v}(\xi) d\xi$, we find the expression for $\tilde{C}_1$

$$\tilde{C}_1 = \frac{6(1 + 6(NB^2 + ABE)^{\frac{\mu}{\eta}})}{-12\beta \eta^{\frac{\mu}{\eta} + 1}} \quad (21)$$

Then, for the hydrodynamic pressure, we finally get:

$$P = \frac{3\eta}{2} (x - x_1)(x - x_2) \left( \frac{2(NB^2 + ABE)}{-12\beta \eta^{\frac{\mu}{\eta} + 1}} \right) \quad (22)$$

3. Results
The expressions for the velocity and pressure fields in the lubricating and porous layers are obtained as a study result, taking into account the dependence of the viscosity of the ferromagnetic lubricant and the permeability of the porous layer on the pressure. New multiparametric expressions for load capacity and friction force have been developed.

For the bearing capacity and the friction force, we get:

$$W = p^* L \int_{x_1}^{x_2} p dx = \frac{3 \mu_0 u^* L N}{2k_0} (x_1 - x_2)^3 \left( (NB^2 + ABE) + \frac{3}{12\beta \eta^{\frac{\mu}{\eta} + 1}} \right) \quad (23)$$

$$L_{TP} = \mu \int_{x_1}^{x_2} \frac{\tilde{v}(0)}{h^2(x)} + \frac{\tilde{v}(0)}{h(x)} dx$$

$$= \mu_0 (x_1 - x_2) \left( (NB^2 + ABE) + \left(1 + \eta \left( x_1 + x_2 \right) \right) + 1 - \eta \left( x_1 + x_2 \right) \right) \times$$

$$\times \left( 1 + \alpha p - \frac{\alpha p^2}{2} \right) \quad (24)$$

4. Experimental studies
In an experimental study, wedge-shaped sliding support with a porous guide coating on a ferromagnetic lubricant is considered.

When conducting experimental studies for the base oils TA-22 C and MS-20 and their mixtures (at a ratio of 1:1) substances were added:
- graphite and fine powders of metals having electrically conductive properties, ferromagnetic properties of liquid lubricants are given by iron oxides Fe$_3$O$_4$ and its compounds having magnetic properties C$_{32}$H$_{16}$N$_8$Fe and citric acid.
Table 1. Composition of additives for the preparation of the studied liquid lubricants

| Identification of additives | Additive components                                      | Volume quantity, % |
|-----------------------------|----------------------------------------------------------|--------------------|
| I                           | Graphite (GOST 8295-73), Fe$_3$O$_4$, Citric acid        | 8-12               |
|                             |                                                          | 4-6                |
|                             |                                                          | 9-12               |
| II                          | CaF$_2$, Fe$_3$O$_4$, Citric acid                        | 7-12               |
|                             |                                                          | 5-7                |
|                             |                                                          | 10-12              |
| III                         | Phthalocyanine (GOST 32335-2013), Fe$_3$O$_4$, Citric acid| 6-8                |
|                             |                                                          | 5-8                |
|                             |                                                          | 8-11               |

Figure 2. The effect of MS-20 aviation oil with additive III on the friction coefficient

Figure 3. The effect of MS-20 aviation oil with additive III on the temperature in the friction zone

Graph 2.3 shows the addition effects III on the coefficient of friction and the temperature in the friction zone.

Calculations for MS-29 aviation oil with additive III reduce the maximum friction coefficient ≈ by 11%, and the temperature in the friction zone by 15%.

An experimental study to determine the load-bearing capacity was performed under the conditions of lubrication with the considered lubricants with additives (Figures 4 and 5)
Figure 4. Bearing capacity of radial plain bearings: standard, lubricated with TA-22C oil without additives and bearings with a porous shaft coating, lubricated with TA-22C oil with additive I

Figure 5. Bearing capacity of radial plain bearings: standard, lubricated with MS-20 oil with additive II, and bearings with a two-layer porous coating of the support surface, lubricated with MS-20 oil with additive III

5. Conclusions and discussion
The analysis of experimental studies has shown that the use of a porous coating in the design of bearings significantly increases their bearing capacity. For turbine TA-22C with an additive, this is 14%, for MS-20 aviation oil with an additive-12.3%, and for their 1:1 mixture with an additive of 13.3%, therefore, the less viscous the liquid lubricant used, the more effective the use of a porous coating in the bearing design.

Figures 4 and 5 show graphs of experimental studies of changes in the coefficient of friction and temperature in the friction zone of MS-20 oil with the additive phthalocyanine (GOST 32335-2013) - (6-8%), Fe₃O₄ - (5-8%) and citric acid - (8-11%).

Thus, experimental research data confirm the results of the theoretical development of calculation models for bearings lubricated with ferromagnetic lubricants.

Theoretical studies have shown that the bearing capacity of the porous coating of the guide surface, taking into account the rheological properties of the ferromagnetic lubricant, increases by ≈ 8-12% with an increase in the parameters A, due to the presence of an electromagnetic field, the
Hartmann number $N$, the magnetic induction vector $B$ and the length of the loaded region, and the coefficient of friction decreases by $\approx 14\text{-}16\%$.

As a result of theoretical and experimental studies, tribotechnical characteristics were determined that allow us to judge the presence of the duration of the hydrodynamic friction regime. As a result of the experimental study, the reliability of the developed theoretical calculation models and the data of their numerical analysis is determined.

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