Remarks on the high-energy behaviour of cross-sections in weak-scale string theories

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(March 25, 2022)

We consider the high-energy behaviour of processes involving Kaluza-Klein (KK) gravitons of weak-scale string theories. We discuss how form-factors derived within string theory modify the couplings of KK gravitons and thereby lead to an exponential fall-off of cross sections in the high-energy limit. Further, we point out that the assumption of Regge behaviour for a scattering amplitude in the high energy limit, \( T \propto s^{\alpha(t)} \), combined with a linear growth of the total cross-section, \( \sigma_{\text{tot}}(s) \propto s \), violates elastic unitarity. Regge behaviour leads to a stringent bound on the growth of the total cross-section, \( \sigma_{\text{tot}}(s) \leq 32\pi\alpha' \ln(s/s_0) \).

PACS numbers: 98.70.Sa, 14.60.Lm, 11.25.Mj

I. INTRODUCTION

Neutrinos are the only known stable particles that can traverse extragalactic space without attenuation even at energies \( E \sim 10^{20} \text{ eV} \), thus avoiding the Greisen–Zatsepin–Kuzmin cutoff [1]. Since neutrinos are not deflected by (extra-) galactic magnetic fields, this primary candidate could also explain possible correlations between the arrival directions of observed ultrahigh energy cosmic rays and astrophysical objects at cosmological distances [2]. Therefore, it has been speculated that the ultrahigh energy primaries initiating the observed air showers are not protons, nuclei or photons but neutrinos [3,4]. However, in the Standard Model (SM) neutrinos are deeply penetrating particles producing only horizontal not vertical extensive air showers. Consequently, either the neutrino has to be converted locally into strongly interacting particles [5] or one has to postulate new interactions that enhance the ultrahigh energy neutrino-nucleon cross-section.

A particular realization of the latter possibility are string theories with \( \delta \) large extra dimensions [7]. If the SM particles are confined to the usual \( 3 + 1 \)-dimensional space and only gravity propagates in the higher-dimensional space, the compactification radius \( R \) of the extra dimensions can be large, corresponding to a small scale \( 1/R \) of new physics. From a four-dimensional point of view the higher dimensional graviton in these theories appears as an infinite tower of Kaluza-Klein (KK) excitations with masses squared \( m_n^2 = \vec{n}^2/R^2 \). Since the weakness of the gravitational interaction is partially compensated by the large number of KK states and cross-sections of reactions mediated by spin 2 particles are increasing rapidly with energy, it has been argued in Refs. [8,9] that neutrinos could initiate the observed vertical showers at the highest energies.

In Refs. [8,9], the neutrino-nucleon cross-section via the exchange of KK gravitons was calculated within the effective field-theoretic model valid below the string scale \( M_{\text{st}} \). Since amplitudes involving virtual exchange of KK gravitons diverge for more than one large extra dimension, a form-factor suppressing the KK modes above \( M_{\text{st}} \) was employed in [9]. Moreover, the bound \( \sigma_{\text{tot}}(s) \propto \ln^2(s/s_0) \) was derived with the eikonalization method assuming the validity of the Regge picture as an effective theory above \( M_{\text{st}} \) [9]. Recently, both the use of a form-factor in the field-theoretic framework as well as the Froissart-like bound in the Regge picture was criticized in Ref. [10]. It is the purpose of this short article to discuss these criticisms. In Sec. II, we review how form-factors suppressing the couplings of KK gravitons with large squared four-momentum appear in string theory, and in Sec. III we discuss bounds on the neutrino-nucleon cross-section if the validity of the Regge picture above \( M_{\text{st}} \) is assumed.$^1$

$^1$We do not consider the production of black holes in neutrino-nucleon scattering in this paper [11].
II. FORM-FACTORS FOR KK GRAVITON COUPLINGS FROM STRING THEORY

The amplitude for the exchange of KK gravitons in the $t$ channel between two particles with four momentum $p$ and $k$ is

$$T(s,t) = \frac{1}{M_{Pl}^2} \sum_{\vec{n}=0}^{\infty} \bar{T}_{\vec{n}}(k) \frac{f_{\vec{n}}^2}{t-m_{\vec{n}}} T^{\mu\nu}(p),$$  \hspace{1cm} (1)

where $\bar{T}_{\vec{n}} = T_{\vec{n}} - \frac{1}{2} \eta_{\vec{n}} T_{\alpha\alpha}$, $T_{\mu\nu}$ is their energy-momentum tensor, $\eta_{\vec{n}}$ is the metric tensor, $M_{Pl}^2 = G_N^{-1}$ is the reduced Planck mass and $s,t$ are the usual Mandelstam variables.

In the calculation of Ref. [2], an exponential suppression of the effective coupling of the level $\vec{n}$ KK mode to four-dimensional fields was used as form-factor $f_{\vec{n}}$,

$$f_{\vec{n}} = \exp \left( -\frac{c m_{\vec{n}}^2}{2 M_{Pl}^2} \right),$$  \hspace{1cm} (2)

where $c$ is a constant of order 1. Then the summation over $n$ which for $f_{\vec{n}} = 1$ only converges in the case of one extra dimension becomes well-defined for all $\delta$. However, a legitimate question to ask is whether this modification is well-motivated. To address this point, we first note that the suppression of the couplings sets in only for $t \gtrsim M_{Pl}^2$. Thus, the form-factor (2) does not modify the effective theory in its range of validity, $-t \lesssim M_{Pl}^2$.

On the other hand, string theories generically predict an exponential suppression of the coupling to higher KK modes in the regime $-t \gg M_{Pl}^2$. This suppression is also consistent with the idea that recoil effects due to the finite tension of D-branes become important in the emission of KK gravitons with large momentum transfer, $-t \gg M_{Pl}^2$ [3].

Let us now discuss in detail how form-factors to the coupling of KK gravitons appear as string corrections to the well-known field theoretical result [14]. The effective theory in its range of validity, $-t \lesssim M_{Pl}^2$.

The Veneziano amplitude has the usual Regge-pole structure. In the hard-scattering limit, $s \to \infty$ with fixed scattering angle $t/s = -\sin^2 \frac{\phi}{2}$, the amplitude falls off exponentially,

$$S(s,t) \propto \exp \left[ \frac{s}{M_{Pl}} \left( \sin^2 \frac{\phi}{2} \ln \sin^2 \frac{\phi}{2} + \cos^2 \frac{\phi}{2} \ln \cos^2 \frac{\phi}{2} \right) \right].$$  \hspace{1cm} (5)

It was pointed out in Ref. [5] that this exponential fall-off can be interpreted as an effective thickness of D-branes of the order of the string scale, which would give rise to a form-factor of the kind (3).

However, this tree-level result is due to string Regge excitations. Contributions from KK gravitons first appear at the one-loop level [14,18]. The Type I string diagram which contains $s$-channel gravitational exchange in the low-energy limit is the non-planar cylinder diagram, which gives rise to contributions to the amplitude of the form

$$A_{1-loop} = -\frac{1}{2} s t A(1,2,3,4) \text{tr}(t^i t^j) \text{tr}(t^k t^l) f_T(s,t),$$  \hspace{1cm} (6)

where the dots indicate permutations of the indices $1,\ldots,4$ and the Mandelstam variables $s$, $t$ and $u$.

The non-planar cylinder amplitude $f_T(s,t)$ has recently been analysed in detail [19,20]. It is given by

$$f_T(s,t) = \frac{g_2^2}{M_{Pl}^6} \int_0^\infty d\nu_1 \int_0^\nu_1 d\nu_2 \int_0^\nu_2 d\nu_3 \int_0^{\nu_3} d\nu_4 \frac{1}{M_{Pl}^{2s+2l}} \left( \frac{\nu_1^{\frac{s}{M_{Pl}}}}{\psi_{12}^{\frac{s}{M_{Pl}}} \psi_{13}^{\frac{s}{M_{Pl}}} \psi_{14}^{\frac{s}{M_{Pl}}}} \right) \left( \frac{\nu_2^{\frac{s}{M_{Pl}}}}{\psi_{12}^{\frac{s}{M_{Pl}}} \psi_{23}^{\frac{s}{M_{Pl}}} \psi_{14}^{\frac{s}{M_{Pl}}}} \right) F_6(l,R),$$  \hspace{1cm} (7)

where

$$F_6(l,R) = \frac{M_{Pl}^6}{(RM_{Pl})^6} g_3^6 \left( 0, \frac{1}{2} l(RM_{Pl})^2 \right)$$  \hspace{1cm} (8)

are the winding mode contributions from the toroidal compactification. For simplicity we have assumed that all six extra dimensions have the same compactification radius $R$. Further,

$$\psi_{ij} = \frac{1}{l} \frac{\partial_1 (v_j - v_i, i)}{\eta^3 (i)},$$  \hspace{1cm} (9)

$$\psi_{ij}^T = \frac{1}{l} \frac{\partial_1 (v_j - v_i, i)}{\eta^3 (i)},$$  \hspace{1cm} (10)

where $\partial_i$ are the usual Jacobi $\theta$ functions and $\eta$ is the Dedekind function.

In Eq. (9), $l$ is the modulus of the cylinder and in the low-energy limit the contribution from exchange of KK
gravitons in the $s$-channel can be extracted by considering the $l \to \infty$ limit of the integrand in Eq. (7). In this limit one finds [19]:

$$f_{T}^{(1)}(s,t) \approx \left[ \frac{2^{-s/M_{st}^{2}} \Gamma \left( \frac{1}{2} - \frac{s}{2M_{st}^{2}} \right)}{\sqrt{\pi M_{Pl}^{2}} \Gamma \left( 1 - \frac{s}{2M_{st}^{2}} \right)} \right]^{2}$$

(11)

$$\times \frac{1}{M^{2}_{st}} \int_{0}^{\infty} dl \sum_{\vec{n}} \exp \left[ -\pi l \left( -\frac{s}{2M_{st}^{2}} + \frac{l^{2}}{2} (RM_{st})^{2} \right) \right].$$

The integral in the second line is just the proper-time parametrization of the sum over the winding mode propagators. The $l \to 0$ limit of the integration is ill-defined, however, this divergence corresponds only to a harmless IR divergence of a box diagram [19]. By the modular transformation $l \rightarrow \frac{1}{l}$ the amplitude (7) can be represented as a sum of box diagrams giving a well-defined result in the limit $l \to 0$.

Here we are mainly interested in the KK graviton contributions. The prefactor in Eq. (11) can be interpreted as the squared form factor for the emission of a KK graviton with momentum-squared $s > 0$,

$$f_{\vec{n}} = \frac{2^{-s/M_{st}^{2}} \Gamma \left( \frac{1}{2} - \frac{s}{2M_{st}^{2}} \right)}{\sqrt{\pi M_{Pl}^{2}} \Gamma \left( 1 - \frac{s}{2M_{st}^{2}} \right)}.$$ 

(12)

In the hard scattering limit this gives an exponential suppression of the KK graviton coupling

$$f_{\vec{n}} \propto \exp \left( -\frac{s}{M_{st}^{2}} \ln 2 \right) \quad \text{for} \quad s \gg M_{st}^{2}.$$ 

(13)

This form-factor is valid for KK gravitons emitted either on-shell or for virtual graviton exchange in the $s$-channel. In these cases we recover Eq. (2) with $s = m_{n}^{2}$.

Next, we discuss the $t$-channel exchange of KK gravitons which is important for neutrino-nucleon scattering. Contributions from the $t$-channel exchange are contained in the planar cylinder diagram, which is a one-loop correction to the tree-level amplitude [3] of the form

$$A_{1-loop} = -\frac{1}{2} s t A(1,2,3,4) \text{tr}(t^{1} t^{2} t^{3} t^{4}) f_{T}^{(1)}(s,t) + \ldots$$ 

(14)

The planar cylinder amplitude $f_{T}^{(1)}(s,t)$ is given by [18,21]

$$f_{T}^{(1)}(s,t) = \frac{g_{s}}{M_{st}} \int_{0}^{\infty} dl \int_{0}^{\infty} d\nu_{2} \int_{0}^{\infty} d\nu_{1} \int_{0}^{\infty} d\nu_{3} \psi_{13} \psi_{24} \psi_{12} \psi_{34}^{*} \frac{1}{s/M_{st}^{2}} \left( \frac{\psi_{13} \psi_{24}}{\psi_{12} \psi_{34}} \right)^{l/M_{st}} F_{6}(l,R).$$

(15)

Again considering the $l \to \infty$ limit of the integrand one obtains a contribution to the amplitude which can be written as a derivative of the tree-level Veneziano amplitude [18],

$$f_{T}^{(1)}(s,t) \approx \frac{M_{st}^{5}}{4 \pi^{2} s t M_{Pl}^{4}} \left( \frac{\partial S(s,t)}{\partial M_{st}} \right) \times \frac{1}{M^{2}_{st}} \int_{0}^{\infty} dl \sum_{\vec{n}} \exp \left[ -\pi l \frac{l^{2}}{2} (RM_{st})^{2} \right].$$

(16)

It is easy to see that the prefactor in Eq. (16) has the same Regge-pole structure as the Veneziano amplitude. In the hard-scattering limit, $s \to \infty$, $t/s$ fixed, the amplitude again falls off exponentially, as in Eq. (4). Thus, form-factors derived within string theory indeed modify the couplings of virtual KK gravitons and, hence, also the cross-sections in the high-energy limit.

### III. Unitarity Limits in the Regge Picture

The authors of Ref. [8] pointed out that the Regge picture is a reasonable approximation to string theory valid above $M_{st}$. As motivation for this assumption we note that the Regge picture takes into account not only the KK modes of the graviton but also those from lower lying trajectories and misses only genuine string modes like winding modes. In the following, we will therefore also use this assumption and derive bounds on the total cross-sections valid within this framework.

A general Regge amplitude $T_{R}$ can be represented by

$$T_{R}(s,t) = \beta(t) \left( \frac{s}{s_{0}} \right)^{\alpha(t)},$$

(17)

where the exponent $\alpha(t)$ is given by the Chew-Frautschi plot of the spin against the mass of the particles lying on the leading Regge trajectory contributing to the reaction. In our case, the intercept $\alpha(0)$ of this trajectory is equal to the spin $j$ of the massless graviton, $\alpha(0) = 2$.

We first note that a Regge amplitude with intercept $\alpha(0) = 2$ gives via the optical theorem a total cross-section growing linearly with $s$,

$$\sigma_{tot}(s) = \frac{1}{s} \text{Im} \{ T_{R}(s,0) \} \propto s^{\alpha(0)-1}.$$ 

(18)

Thus the assumed Regge-behaviour alone, without any unitarization, reduces the growth of the total cross-section by one power of $s$ compared to the naive expectation $\sigma \propto s^{j} = s^{2}$. On the other hand, the elastic cross-section
\[ \sigma_{el}(s) = \frac{1}{16\pi s^2} \int_{-s}^{0} dt \left| T_R(s,t) \right|^2 \propto \frac{s^2}{\ln(s/s_0)} \]  

(19)

increases even faster than the total cross-section. Therefore, elastic unitarity, \( \sigma_{tot} \geq \sigma_{el} \), is violated above a certain energy for any Regge amplitude with \( \alpha(0) > 1 \) — as is well-known from the case of the pomeron. These findings are in clear contradiction to the ones of Ref. [8,10]; there, it was claimed that a linear growth of \( \sigma_{tot}(s) \) for \( T_R(s,0) \propto s^2 \) respects unitarity.

Next, we derive the maximal total cross-section allowed for an arbitrary Regge amplitude by elastic unitarity. Following Leader [22], we rewrite \( T_R(s,t) \) as

\[ T_R(s,t) = \beta \left( \frac{s}{s_0} \right)^{\alpha(t)} = T_R(s,0) \left( \frac{s}{s_0} \right)^{\alpha(t) - \alpha(0)} \]  

(20)

and expand the amplitude around \( t = 0 \),

\[ \left( \frac{s}{s_0} \right)^{\alpha(t) - \alpha(0)} = \exp\{\alpha't\ln(s/s_0) + O(\alpha'' t^2)\} \]  

(21)

Here, \( \alpha' \) denotes the derivative of \( \alpha(t) \) evaluated at \( t = 0 \) and we have neglected for clarity possible non-linear terms in \( t \) and the subdominant \( t \) dependence of \( \beta \). Then we evaluate \( \sigma_{el} \),

\[ \sigma_{el} = \frac{1}{16\pi s^2} \int_{-s}^{0} dt \left| T_R(s,t) \right|^2 = \frac{\left| T_R(s,0) \right|^2}{16\pi s^2} \frac{1}{2\alpha'\ln(s/s_0)}. \]  

(22)

Requiring now elastic unitarity

\[ \sigma_{el} \leq \sigma_{tot} = \frac{1}{s} \ln \{ T_R(s,0) \} < \frac{1}{s} \left| T_R(s,0) \right|, \]  

(23)

it follows

\[ \frac{1}{32\pi\alpha'\ln(s/s_0)} \frac{\left| T_R(s,0) \right|^2}{s^2} \leq \sigma_{tot} \]  

(24)

or

\[ \frac{\sigma_{tot}^2}{32\pi\alpha'\ln(s/s_0)} \leq \sigma_{tot} \]  

(25)

and finally

\[ \sigma_{tot}(s) \leq 32\pi\alpha'\ln(s/s_0). \]  

(26)

Thus the assumption of a Regge amplitude results in a stronger bound for the total cross-section than the Froissart bound. A more general derivation of such a bound can be found in Ref. [23].

Some remarks are now in order: First, we have always used formulae valid for \( d = 4 \) dimensions. This is appropriate because the main contribution to the cross-sections comes from the small \( t \) region and therefore does not probe the extra dimensions. Second, we note that this bound applies on the parton not the hadron level. Third, the bound (26) contains two parameters, the slope of the Regge trajectory \( \alpha' \) and the unknown scale \( s_0 \), and is therefore still not useful for a numerical evaluation.

To proceed, we use that

\[ \sigma_{tot}^{N\nu}(s) = [N(s) + \delta N] \sigma_{tot}(s) \leq N(s)\sigma_{tot}(s), \]  

(27)

where \( N(s) \propto s^{0.4} \) takes into account the increasing number of target partons in the nucleon. The term \( \delta N < 0 \) corrects that each parton carries only a fraction \( x < 1 \) of the nucleon momentum, i.e. that \( \ln(xs/s_0) < \ln(s/s_0) \). A numerical value for the bound (27) can now be determined by joining the field-theoretic result and the Regge result on the hadron level at that scale \( s' \sim M_{st}^2 \), where the field-theoretic result starts to violate s-wave unitarity on the parton level. We find that the KK contribution to the total cross-section at UHE is at most of the same order of magnitude as the SM cross-section.

Finally, we want to comment briefly on the suggestion of Ref. [4] that the exponential increase of (lepto-quark like) KK resonances in the s channel could enhance the neutrino-nucleon cross-section. Since the Horn-Schmid duality [25] connects s and t channel Regge/String amplitudes, our discussion above can be applied immediately to this case. The \( n = 0 \) lepto-quarks can have either spin \( j = 0 \) or 1. Even in the later case, the intercept will be smaller than 1 and the partonic cross-section will be asymptotically decreasing with \( s \).

### IV. CONCLUSIONS

Couplings of KK states derived within a field-theoretic model valid below \( M_{st} \) are modified by form-factors calculable in string theory [13,14]. The use of these form-factors makes the sums over KK states well-defined without simply cutting-off KK modes with \( m_n \lesssim M_{st} \). For the case of neutrino-nucleon scattering, the tree-level string cross-section was calculated in Ref. [24]: even for a string scale as low as 1 TeV, the cross-section found there is only of the same order as the SM cross-section at energies \( s \gtrsim M_{st}^2 \). Taking KK graviton exchange into account as one-loop correction yields a cross-section that is very different from the nucleon-nucleon cross-section, i.e. neutrino-nucleon scattering cannot explain the observed ultrahigh energy cosmic rays.

We have addressed the question how the assumption of Regge behaviour, \( T \propto s^{\alpha(t)} \), for the neutrino-nucleon scattering amplitude above \( M_{st} \) bounds the growth of \( \sigma_{tot} \). We have shown that a linear growth of \( \sigma_{tot} \), as advocated in [8,10], is incompatible with elastic unitarity. Regge behaviour allows instead only logarithmic growth of the partonic cross-sections.
ACKNOWLEDGMENTS

We are grateful to E. Dudas, A. Faraggi, J. Mourad and D. Ross for useful comments. M.P. was supported by the EU network “Supersymmetry and the early universe” under contract no. HPRN-CT-2000-00152.

[1] For a general discussion of ultrahigh energy cosmic rays see e.g. M. Nagano and A.A. Watson, Rev. Mod. Phys. 72, 689 (2000); M. Kachelrieß, in “Cosmology and Particle Physics”, Eds. R. Durrer, J. Garcia-Bellido, M. Shaposhnikov, AIP Conference Proceedings 555, p. 156–169, astro-ph/0011231, and references therein.

[2] A. Virmani et al., astro-ph/0010235; P.G. Tinyakov and I.I. Tkachev, astro-ph/0102476; but see also G. Sigl, D.F. Torres, L.A. Anchordoqui and G.E. Romero, Phys. Rev. D 63, 081302 (2001).

[3] V.S. Berezinsky and G.T. Zatsepin, Phys. Lett. B 28, 423 (1969); G. Domokos and S. Nussinov, Phys. Lett. B 187, 372 (1987); J. Bordes, H. Chan, J. Faridani, J. Pfaunder and S.T. Tsou, Astropart. Phys. 8, 135 (1998).

[4] G. Domokos and S. Kovesi-Domokos, Phys. Rev. Lett. 82, 1366 (1998).

[5] P. Jain, D.W. McKay, S. Panda and J.P. Ralston, Phys. Lett. B 484, 267 (2000).

[6] D. Fargion, B. Mele and A. Salis, Astrophys. J. 517, 725 (1999); T.J. Weiler, Astropart. Phys. 11, 303 (1999).

[7] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B 429, 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B 436, 257 (1998), Phys. Rev. D 59, 086004 (1999).

[8] S. Nussinov and R. Shrock, Phys. Rev. D 59, 105002 (1999).

[9] M. Kachelrieß and M. Plümacher, Phys. Rev. D 62, 103006 (2000).

[10] S. Nussinov and R. Shrock, Phys. Rev. D 64, 047702 (2001).

[11] S. B. Giddings and S. Thomas, hep-ph/0106219; J. L. Feng and A. D. Shapere, hep-ph/0109106.

[12] L. Dixon, D. Friedan, E. Martinec and S. Shenker, Nucl. Phys. B 282, 13 (1987); S. Hamidi and C. Vafa, Nucl. Phys. B 279, 465 (1987); I. Antoniadis and K. Benakli, Phys. Lett. B 326, 69 (1994).

[13] M. Bando, T. Kugo, T. Noguchi and K. Yoshioka, Phys. Rev. Lett. 83, 3601 (1999); J. Hisano and N. Okada, Phys. Rev. D 61, 106003 (2000); H. Murayama and J. D. Wells, hep-ph/0109004.

[14] G. F. Giudice, R. Rattazzi and J. D. Wells, Nucl. Phys. B 544, 3 (1999); T. Han, J. D. Lykken and R. Zhang, Phys. Rev. D 59, 105006 (1999).

[15] A. Hashimoto and I. R. Klebanov, Phys. Lett. B 381 (1996) 437; A. Hashimoto and I. R. Klebanov, Nucl. Phys. Proc. Suppl. 55B (1997) 118.

[16] C. Lovelace, Phys. Lett. B 34 (1971) 500.

[17] K. Bardakci, M. B. Halpern and J. A. Shapiro, Phys. Rev. 185 (1969) 1910; G. Frye and L. Susskind, Phys. Lett. B 31 (1970) 589; D. J. Gross, A. Neveu, J. Scherk and J. H. Schwarz, Phys. Rev. D 2 (1970) 697.

[18] M. B. Green and J. H. Schwarz, Nucl. Phys. B 198 (1982) 441; M. B. Green, J. H. Schwarz and L. Brink, Nucl. Phys. B 198 (1982) 474.

[19] E. Dudas and J. Mourad, Nucl. Phys. B 575 (2000) 3.

[20] S. Cullen, M. Perelstein and M. E. Peskin, Phys. Rev. D 62 (2000) 055012.

[21] K. Kikkawa, B. Sakita and M. A. Virasoro, Phys. Rev. 184 (1969) 1701; G. Frye and L. Susskind, Phys. Lett. B 31 (1970) 537.

[22] E. Leader, Phys. Lett. 5, 75 (1963).

[23] R.J. Eden, High energy collisions of elementary particles, Cambridge University Press 1967.

[24] R. Gandhi, C. Quigg, M.H. Reno, I. Sarcevic, Astropart. Phys. 5, 81 (1996).

[25] R. Dolen, D. Horn and C. Schmid, Phys. Rev. Lett. 19, 402 (1967).

[26] F. Cornet, J. I. Illana and M. Masip, Phys. Rev. Lett. 86, 4235 (2001).