Constraints on a scalar-tensor model with Gauss–Bonnet coupling from SN Ia and BAO observations

S. Bellucci1 · A. Banijamali2 · B. Fazlpour3 · M. Solbi2

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Abstract In the present work, the observational consequences of a subclass of the Horndeski theory have been investigated. In this theory a scalar field (tachyon field) is non-minimally coupled to the Gauss–Bonnet invariant through an arbitrary function of the scalar field. By considering a spatially flat FRW universe, the free parameters of the model have been constrained using a joint analysis from observational data of the Type Ia supernovae and Baryonic Acoustic Oscillations measurements. The best fit values obtained from these datasets are then used to reconstruct the equation of state parameter of the scalar field. The results show the phantom, quintessence and phantom divide line crossing behavior of the equation of state and also cosmological viability of the model.

Keywords Dark energy · Gauss–Bonnet coupling · Observational cosmology

1 Introduction

The current accelerated expansion of the universe is one of the great problems of modern cosmology. This acceleration was first suggested by Type Ia supernovae (SN Ia) surveys (Riess et al. 1998, 1999; Perlmutter et al. 1999) and then by measurements of the cosmic microwave background (CMB) (Komatsu et al. 2011; Ade et al. 2014), the Hubble constant (Riess et al. 2009), Baryonic Acoustic Oscillations (BAO) (Lampeitl et al. 2009) and more measurements of Type Ia supernovae (Kowalski et al. 2008). Although observational cosmology confirm the acceleration of the universe, explaining this issue from theoretical point of view is a big challenge. The simplest way to obtain an accelerated universe is adding a cosmological constant to the standard cosmological model. However, a cosmological constant suffers from fine-tuning problem that is due to its extremely small observed value compared to predictions from theoretical considerations (Martin 2012). As a result one can follow two ways to explain the late-time behavior of the universe: modification of general relativity at large scale (Capozziello and De Laurentis 2011) or introducing a new content in the universe such as canonical scalar field, phantom scalar, both scalars, vector fields etc. that is introducing the concept of dark energy (Bassett et al. 2006; Copeland et al. 2006; Cai et al. 2010).

Furthermore, dynamical dark energy models can be extended in a huge class of models. Among them, non-minimally coupled dark energy models in which scalar fields coupled to the curvature terms dubbed scalar-tensor theories have been extensively studied in the literature. The most famous example of such theories is known as the Brans and Dicke (1961) theory in which the gravitational constant is replaced by a scalar field \( \phi \) entering into the action as \( \phi^2 R \), \( R \) is the Ricci scalar. Another well-known example of non-minimally coupled system is provided by \( (1 - \xi \phi^2) R \) coupling in which \( \xi \) is a constant measuring the strength of non-minimal coupling (Bezrukov and Shaposhnikov 2008).
Moreover, due to the novel features of non-minimally coupled scalar field system, such as allowing the phantom divide crossing and having the cosmological scaling solutions, these models are of great interest to the community (Sahni and Starobinsky 2000, 2006; Padmanabhan 2006; Peebles and Ratra 2003; Perivolaropoulos 2006; Straumann 2003; Frieman 2008; Sami 2007; Sami 2009; Bamba et al. 2012; Tsujikawa 2011; Linder 2008; Caldwell and Kamionkowski 2009; Silvestri and Trodden 2009; Frieman et al. 2008). On the other hand, in 1974 Horndeski (1974) found the most general class of scalar-tensor theories which lead to the second order differential equations similar to the Einstein general relativity. The Horndeski gravity has been considered in many papers in the context of the inflationary cosmology (Deffayet et al. 2011; De Felice et al. 2011). An interesting subclass of the Horndeski theory is given by the non-minimal coupling of the scalar field to the Gauss–Bonnet invariant in four dimensions (Antoniadis et al. 1994; Gasperini et al. 1997; Brustein and Madden 1998; Kawai et al. 1998; Cartier et al. 2000; Nojiri et al. 2005; Calcagni et al. 2006). Such a non-minimal coupling originates from the string theory and the trace anomaly and may play an important role in cosmological context. For example, this coupling has been proposed to address the dark energy problem in Nojiri et al. (2005) and various aspects of accelerating cosmologies with Gauss–Bonnet correction have been discussed in Nojiri et al. (2006), Tsujikawa and Sami (2007), Capozziello et al. (2013). Indeed, these studies yield the result that the scalar-curvature coupling predicted by fundamental theories may become important at current, low-curvature universe. It deserves mention that the modifications of gravity from the Gauss–Bonnet invariant have been often considered as the result of quantum gravity effects (Chiba 2001; Faraoni 2002; Elizalde et al. 2004).

In the present work, we will consider a model in which the scalar field playing the role of dark energy is coupled to the Gauss–Bonnet invariant. Here we derive constraints on the model parameters from a combination of available SN Ia data as well as available BAO data and $\chi^2$ minimization technique.

The outline of the paper is as follows: In the next section we present the basic formalism of our model in a flat FRW background along with the definition of different cosmological parameters. We then discuss the observational dataset and methodology in Sect. 3. Our main results in data analysis are summarized in Sect. 4. Finally, Sect. 5 is devoted to our conclusions.

## 2 The model and cosmological background

The model we examine in this paper is described by the following action:

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{2\kappa^2} - V(\phi)\sqrt{1 - \partial_\mu\phi\partial^\mu\phi} - \eta(\phi)\mathcal{G} + \mathcal{L}_m \right], \quad (1)$$

where $g$ is the determinant of the metric tensor, $\kappa^2 = 8\pi G$, $G$ is the gravitational constant and $\mathcal{L}_m$ is the matter Lagrangian density. The second term in the brackets is the Lagrangian of tachyon field with the potential $V(\phi)$, while the third term represents a non-minimal coupling between the scalar field and curvature through a general function $\eta(\phi)$. $\mathcal{G}$ is the Gauss–Bonnet invariant which is given by:

$$\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho}, \quad (2)$$

where $R$, $R_{\mu\nu}$, $R_{\mu\nu\lambda\rho}$ are the Ricci scalar, the Ricci tensor and the Riemann tensor, respectively.

Notice that not only tachyon field originates from the string theory but also the term proportional to the Gauss–Bonnet invariant $\mathcal{G}$ is considered as a stringy correction in the action. These are our main motivations to study the model. Furthermore, in the theory considered in the present manuscript, the propagation speed of gravitational waves is different from the speed of light. This can be seen clearly by looking at some formulas given e.g. in Kobayashi et al. (2011). Since the model is supposed to describe the present Universe, it must satisfy the stringent constraint on the propagation speed of gravitational waves imposed by GW170817. In spite of this, we think the model in the manuscript cannot be ruled out and, more in general, we do not believe that the model cannot be ruled out and, more in general, we do not believe that the $f(R, G)$ models are deceased in the light of GW170817; it is enough to compel them more and fine-tune them to save certain classes, inside the whole paradigm. In fact, we recall that there is a debate regarding the cutoff scale and LIGO’s observation frequency (de Rham and Melville 2018).

To analyse the model it is more convenient to use the following redefinition, as proposed in Quiros et al. (2010) for studying the tachyon dynamics,

$$\phi \rightarrow \phi = \int d\phi \sqrt{V(\phi)} \quad \iff \quad \partial_\phi = \frac{\partial \phi}{\sqrt{V(\phi)}}. \quad (3)$$

Applying (3) in (1) yields to our starting action as follows:

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{2\kappa^2} - V(\phi)\sqrt{1 - \partial_\mu\phi\partial^\mu\phi} - \eta(\phi)\mathcal{G} + \mathcal{L}_m \right], \quad (4)$$

The variation of the action (4) with respect to the metric leads to the following gravitational equations:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa^2(T_{\mu\nu} + T_{\mu\nu}^{GB} + T_{\mu\nu}^m), \quad (5)$$
where \( T^m_{\mu \nu} \) is the usual energy-momentum tensor for the matter, \( T^\phi_{\mu \nu} \) corresponds to the energy-momentum tensor of minimally coupled tachyon scalar field and \( T^GB_{\mu \nu} \) is the contribution of the non-minimal Gauss–Bonnet coupling. These last two components are given by

\[
T^\phi_{\mu \nu} = -u \nabla_\mu \phi \nabla_\nu \phi - g_{\mu \nu} u^{-1} V(\phi)
\]

and

\[
T^GB_{\mu \nu} = 4\left( \nabla_\mu \nabla_\nu (\phi) - g_{\mu \nu} \nabla_\rho \nabla_\sigma (\phi) \right) R - 2\left[ \nabla_\rho \nabla_\nu (\phi) \right] R_{\rho \nu} - 2\left[ \nabla_\rho \nabla_\sigma (\phi) \right] R_{\rho \sigma} + 2\left[ \nabla_\rho \nabla_\nu (\phi) \right] R_{\mu \rho \nu \sigma},
\]

(7)

where \( u = \sqrt{1 - \frac{\dot{\phi} \phi}{V(\phi)}} \).

In derivation of \( T^GB_{\mu \nu} \), the properties of the 4-dimensional Gauss–Bonnet invariant have been used (see Nojiri et al. 2005; Farhoudi 2009 for details). The energy density and pressure derived from these energy-momentum tensors will be considered as effective ones and we represent them by \( \rho_{DE} \) and \( p_{DE} \), respectively.

Now, we assume the spatially flat Friedmann–Robertson–Walker (FRW) metric,

\[
ds^2 = -dt^2 + a^2(t) \left( dr^2 + r^2 d\Omega^2 \right),
\]

(8)

where \( a(t) \) is the scale factor. Considering this metric in (5)–(7) we obtain the following Friedmann equations:

\[
H^2 = \frac{k^2}{3} (\rho_{DE} + \rho_m),
\]

(9)

\[
\dot{H} = -\frac{k^2}{2} (\rho_{DE} + p_{DE} + \rho_m + p_m),
\]

(10)

where \( \rho_m \) and \( p_m \) are the energy density and pressure of the matter, \( \rho_{DE} \) and \( p_{DE} \) are given by

\[
\rho_{DE} = u V(\phi) + 24H^2 f(\phi) \dot{\phi},
\]

(11)

and

\[
p_{DE} = -u^{-1} V(\phi) - 8H^2 \left[ f, \phi \right]^2 + f(\phi) \right]

\[ - 16H f(\phi) \dot{\phi} (H + H^2),
\]

(12)

where \( H = \frac{\dot{a}}{a} \) is the Hubble parameter, and we have also defined \( f(\phi) = \frac{dV}{d\phi} \).

Further, by varying the action (4) over \( \phi \) and assuming that \( \phi \) only depends on time, we obtain the equation of motion for \( \phi \), which in FRW background takes the following form

\[
\ddot{\phi} + 3u^{-2} H \dot{\phi} + \left( 1 - \frac{3\dot{\phi}^2}{2V} \right) V, \phi + 24H^2 (\dot{H} + H^2) f(\phi) = 0.
\]

(13)

Note that in deriving equation (13), we have used the following expression for the Gauss–Bonnet invariant in FRW background

\[
G = 24H^2 (\dot{H} + H^2).
\]

In addition, the energy conservation equations for dark energy and the matter are expressed in the following forms, respectively

\[
\dot{\rho}_{DE} + 3H (1 + \omega_{DE}) \rho_{DE} = 0,
\]

(15)

and

\[
\dot{\rho}_m + 3H (1 + \omega_m) \rho_m = 0,
\]

(16)

where \( \omega_{DE} = \frac{p_{DE}}{\rho_{DE}} \) and \( \omega_m = \frac{p_m}{\rho_m} \) are the equation of state parameters of dark energy and matter respectively. Here, we just focus on the late-time eras, so that we can neglect the radiation contribution and assume a pressureless fluid for the matter content \( \omega_m = \frac{p_m}{\rho_m} = 0 \). Then, the continuity equation (16) can be easily integrated to yield

\[
\rho_m = \rho_{m0} \left( \frac{a_0}{a} \right)^{-3} = \rho_{m0} (1 + z)^3,
\]

(17)

where \( \rho_{m0} \) denotes the present value of the matter energy density and \( z \) is the redshift parameter \( z + 1 = \frac{a_0}{a} \).

In addition, we define the density parameters of dark energy and the matter by \( \Omega_{DE} = \rho_{DE}/(3H^2) \) and \( \Omega_m = (k^2 \rho_m)/(3H^2) \) and here after a subscript “0” for a parameter stands for the present value of that parameter.

Before closing this section it is worthwhile to mention that since we are going to constrain the model using the observational data, the Friedmann equations can be straightforwardly expressed in terms of the redshift instead of the cosmic time, by the following replacements in (11) and (12),

\[
\dot{H} = -H H' (1 + z), \quad \dot{\phi} = -H (1 + z) \phi',
\]

\[
\ddot{\phi} = H^2 (1 + z) \phi'' + HH' (1 + z)^2 \phi' + H^2 (1 + z)^2 \phi'''.
\]

(18)

3 Methods

Here, we explain the methodology that we use to constrain the model by using the recent observational datasets from Type Ia Supernova (SNe Ia) and Baryon Acoustic Oscillations (BAO).

We use the Markov-chain Monte Carlo (MCMC) method for the minimization of \( \chi^2 \) to perform the statistical analysis.
We have tested the model using the publicly available codes by S. Nesseris et al. (see for example Nesseris 2013; Spyros et al. 2013) and making the necessary changes in case of our model. Now, we briefly explain the method for elaboration of the observational data.

Our study follows the likelihood \( \mathcal{L} \propto \exp(-\chi^2/2) \), where the total \( \chi^2 \) for combined datasets reads:

\[
\chi^2_{\text{total}} = \chi^2_{\text{SN}} + \chi^2_{\text{BAO}}.
\]

In the following subsections, the way by which, one can calculate each of \( \chi^2 \) is described.

### 3.1 Type Ia Supernova (SN Ia)

The \( \chi^2 \) function for the SNe Ia is given by Perivolaropoulos (2005),

\[
\chi^2_{\text{SN}} = A - 2 \mu_0 B + \mu_0^2 C,
\]

where \( A, B \) and \( C \) are defined by

\[
\begin{align*}
A &= \sum_i \left[ \frac{\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i; \mu_0 = 0)}{\sigma_i^2} \right]^2, \\
B &= \sum_i \frac{\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i; \mu_0 = 0)}{\sigma_i^2}, \\
C &= \sum_i \frac{1}{\sigma_i^2}.
\end{align*}
\]

The definition of the distance modulus is

\[
\mu_{\text{th}}(z) \equiv 5 \log_{10} D_L(z) + \mu_0,
\]

where \( \mu_0 \equiv 42.38 - 5 \log_{10} h \), with \( h \equiv H_0/100/\text{[km sec}^{-1}\text{Mpc}^{-1}] \) (Komatsu et al. 2011) and the subscript “th” and “obs” stand for theoretical and the observed distance modulus. Also, the quantity \( \sigma_i \) represents the statistical uncertainty in the distance modulus.

The dimensionless luminosity distance \( D_L \) for the flat universe is given by:

\[
D_L(z) = (1+z) \int_0^z \frac{dz'}{E(z')},
\]

where

\[
E(z) = \frac{H(z)}{H_0} = \sqrt{\Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_{DE}(1+z)^3(1+w_{DE})}.
\]

Here, \( \Omega_r \) is the radiation density parameter and \( \Omega_r^{(0)} = \Omega_r^0 (1 + 0.2271 N_{\text{eff}}) \), where \( \Omega_r^0 \) is the present fractional photon energy density and \( N_{\text{eff}} = 3.04 \) is the effective number of neutrino species (Komatsu et al. 2011).

Now, the minimizing of \( \chi^2_{\text{SN}} \) with respect to \( \mu_0 \) yields to

\[
\chi^2_{\text{SN}} = A - \frac{B^2}{C}.
\]

In our statistical analysis we use (25) for SNe Ia dataset and the Union 2.1 compilation data (Suzuki et al. 2012) of 580 data points have been used to constraint the model parameters.

### 3.2 Baryon Acoustic Oscillations (BAO)

Next, we have used BAO measurement dataset to put the BAO constraints on the model parameters. The BAO observable is the distance ratio \( d_z \equiv r_s(z_d)/D_V(z) \), where \( r_s \) is the comoving sound horizon, \( z_d \) is the redshift at the drag epoch (Percival et al. 2010) and \( D_V \) is the volume-averaged distance which is defined as follows (Eisenstein et al. 2005),

\[
D_V(z) \equiv \left[ (1+z)^2 D_A(z) \frac{z}{H(z)} \right]^{1/3}.
\]

In (26) \( D_A(z) \) is the proper angular diameter distance for the flat universe.

Here we have considered six BAO data points (see Table 1). The WiggleZ collaboration (Blake et al. 2011) has measured the baryon acoustic scale at three different redshifts, while SDSS and 6DFGS surveys provide data at lower redshift (Percival et al. 2010).

The \( \chi^2 \) function of the BAO data is defined as,

\[
\chi^2_{\text{BAO}} = \left( \chi_{\text{th}, \text{BAO}} - \chi_{\text{obs}, \text{BAO}} \right) (C^{-1}_{\text{BAO}})^{-1} \left( \chi_{\text{th}, \text{BAO}} - \chi_{\text{obs}, \text{BAO}} \right)^T.
\]

where the indices \( i, j \) are in growing order in \( z \), as in Table 1 and \( C^{-1}_{\text{BAO}} \) can be obtained by the covariance data (Blake et al. 2011) in terms of \( d_z \) as follows:

\[
C^{-1}_{\text{BAO}} = \begin{pmatrix}
4444 & 0 & 0 & 0 & 0 & 0 \\
0 & 30318 & -17312 & 0 & 0 & 0 \\
-17312 & 87046 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 23857 & -22747 & 10586 \\
0 & 0 & -22747 & 128729 & -59907 & 125536 \\
0 & 0 & 10586 & -59907 & 122536 & 125536
\end{pmatrix}.
\]

One can now obtain the best fit values of the model parameters by minimizing \( \chi^2_{\text{total}} \) in (19).

| Table 1 | The BAO data used in our analysis |
|---------|----------------------------------|
| \( d_z \) | 0.336 | 0.1905 | 0.1097 | 0.0916 | 0.0726 | 0.0592 |
| \( \Delta d_z \) | 0.015 | 0.0061 | 0.0036 | 0.0071 | 0.0034 | 0.0032 |
4 Observational constraints on the model parameters

Following the $\chi^2$ analysis (as presented in the previous section), in this section, we obtain the constraints on the free parameters of the model.

Two important functions in our analysis are the function $f(\phi)$ and the scalar field potential $V(\phi)$. We will consider power-law and exponential forms for $f(\phi)$ and $V(\phi)$ and thus our study is categorized into the following four different cases:

Case I: Exponential $f(\phi)$ and Power-law $V(\phi)$
\[ f(\phi) \propto e^{\alpha \phi}, \quad V(\phi) \propto \phi^\beta \]

Case II: Power-law $f(\phi)$ and Exponential $V(\phi)$
\[ f(\phi) \propto \phi^\alpha, \quad V(\phi) \propto e^{\beta \phi} \]
In the present work, we identify the parameters of the model as the parameters $\alpha$ and $\beta$, the present matter density parameter $\Omega_m$, and the dark energy equation of state parameter $\omega_{DE}$. Thus, the model parameters are $(\alpha, \beta, \Omega_m, \omega_{DE})$.

Now, performing the combined ($SN\text{Ia} + BAO$) analysis using MCMC method for the cases I–IV, yields to the constraint results as what summarized in Table 2.

In this table, the reader may see a compact presentation of the best fit values of the model parameters as well as $\chi^2_{min}$ for each case, separately.
Fig. 3 1σ (68.3%), 2σ (95.4%) and 3σ (99.7%) confidence level contour plots for different combinations of the model parameters with also 1-dimensional posterior distributions in the case III for combined observational dataset from SN Ia + BAO. The black dot in each contour plot represents the best fit values of the corresponding pair.

Furthermore, in Figs. 1, 2, 3, 4 we present the 1σ, 2σ and 3σ confidence level contour plots for several combinations of the model parameters as well as their likelihood analysis for the cases I–IV, respectively. Additionally, in Fig. 5, using the same combined analysis SN Ia + BAO, we have shown the qualitative evolution of the dark energy equation of state parameter. Figure 6 shows the Hubble diagram for 580 SN Ia from (Union 2.1) sample. The curves represent the distance modulus predicted by the four cases I–IV in our model.

Further, the case with the lowest value of $\chi^2_{\text{min}}$ is the case I and as it is clear from Table 2 the cases with the exponential coupling function $f(\phi)$ (cases I and IV) have a lower $\chi^2_{\text{min}}$ than the cases with power-law $f(\phi)$ (cases II and III).

The joint analysis on cases I–IV shows that the best fit values of the dark energy equation of state parameter, exhibit phantom behavior although very close to the cosmological constant boundary. As one see from Table 2, in case I, we have the nearest value of the equation of state parameter to the cosmological constant ($\omega_A = -1$) and
in case IV, the phantom character of the current dark energy equation of state is the clearest one. Notice that the quintessence behavior of $\omega_{\text{DE}}$ is excluded in all cases of Table 2. Further, from Fig. 5, it is clear that the transition from quintessence phase ($\omega_{\text{DE}} > -1$) to the phantom phase ($\omega_{\text{DE}} < -1$) or the so-called phantom divide line crossing, occurs in all four cases, which is in agreement with observational results (Zhao et al. 2005; Caldwell and Doran 2005; Feng et al. 2005).

From the constraints on $\alpha$ and $\beta$ as shown in Table 2, it is clear that the combination of SN Ia + BAO data favors negative values for $\alpha$ and positive values for $\beta$ in cases I–IV.

Between the best-fit values of $\alpha$, the case II i.e. when the coupling function is in power-law form, has the minimum value, while the case I in which $f(\phi)$ is exponential has the maximum value of $\alpha$. On the other hand, the best-fit value of the free parameter $\beta$, has its minimum and maximum values

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**Fig. 4** $1\sigma$ (68.3%), $2\sigma$ (95.4%) ans $3\sigma$ (99.7%) confidence level contour plots for different combinations of the model parameters with also 1-dimensional posterior distributions in the case IV for combined observational dataset from SN Ia + BAO. The black dot in each contour plot represents the best fit values of the corresponding pair.
The evolution of the dark energy equation of state parameter, for the best fit values of \((\alpha, \beta)\) that arises from the analysis of SN Ia + BAO datasets, for the cases I (blue), II (green), III (orange) and IV (pink).

The Hubble diagram for 580 data of SN Ia from Union 2.1 sample (Suzuki et al. 2012). The curves correspond to the distance modulus predicted by the four cases I–IV with the best fit values coming from the joint analysis of SN Ia + BAO as presented in Table 2.

### Table 2

| Model | \(\chi^2_{\text{min}}\) | \(\Omega_{\text{m0}}\) | \(\omega_{\text{DE}}\) | \(\alpha\) | \(\beta\) |
|-------|-----------------|-----------------|-----------------|---------|---------|
| Case I | 594.216         | 0.26 ± 0.006    | -1.099 ± 0.002  | -0.995 ± 0.012 | 0.023 ± 0.011 |
| Case II | 663.505         | 0.26 ± 0.003    | -1.199 ± 0.003  | -2.998 ± 0.025 | 0.077 ± 0.013 |
| Case III | 601.833        | 0.26 ± 0.005    | -1.199 ± 0.004  | -1.998 ± 0.025 | 0.23 ± 0.016  |
| Case IV | 595.747         | 0.26 ± 0.028    | -1.25 ± 0.06    | -1.96 ± 0.25   | 0.43 ± 0.081  |

for the cases I and IV, that is for power-law and exponential potentials, respectively.

It deserves mention here that the values of the dark matter density parameter at present \(\Omega_{\text{m0}}\) for all four cases are very close to the desired value in cosmology.

### 5 Conclusion

The Gauss–Bonnet gravity in which the Gauss–Bonnet invariant \(\mathcal{G}\) or a general function of \(\mathcal{G}\) in the form of \(f(\mathcal{G})\) play a prominent role in cosmology has been largely studied in the literature. Inflationary cosmology in the framework of \(f(R, \mathcal{G})\) gravity has been discussed in De Laurentis et al. (2015) where a double inflationary scenario naturally emerges. Observational viability of power-law solutions for a class of \(f(R, \mathcal{G})\) gravity using CMB, SN Ia and local \(H_0\) measurement has been examined in Benetti et al. (2018). The weak field limit of \(f(R, \mathcal{G})\) gravity as specifically the Newtonian and post Newtonian limits of it have been investigated in De Laurentis and Lopez-Revelles (2014) and its phase-space analysis has been studied in Santos da Costa et al. (2018). In addition, Kleidis and Oikonomou (2018) have investigated which Loop Quantum Cosmology cor-
rected $f(\mathcal{G})$ gravity may realize the intermediate inflation and the singular bounce cosmologies.

In this paper, we have focused on the analysis of a non-minimally coupled scalar field theory in which the scalar field is considered as a candidate of dark energy. In this model tachyon field non-minimally coupled to the Gauss–Bonnet invariant via a general coupling function $f(\phi)$ as in action (4). The cosmological evolution of the model is studied by assuming a flat FRW universe. Then, we placed constraints on the free parameters of the model by performing a joint statistical analysis using the recent cosmological data from SN Ia and BAO measurements. We have considered the exponential and power-law forms for the scalar field potential, as well as the non-minimal coupling function. Then, we have obtained the best fit values of the free parameters $\alpha$ and $\beta$ in the potential and coupling function, respectively. The equation of state parameter of dark energy, $\omega_{DE}$ and the present value of the matter density $\Omega_{m0}$ have also been fitted.

According to the contents of Table 2, in which our results are summarized, the joint analysis of SN Ia + BAO, favors the negative values for $\alpha$ and positive values for $\beta$. In addition, by constraining with the data sets of SN Ia and BAO, we found that $\omega_{DE} < -1$, for all cases, which means our universe slightly biases towards phantom behavior while the values of $\Omega_{m0}$, the present-day dark matter density is very close to the desired value $\Omega_{m0} \approx 0.27$. Finally, using the best fits of the model parameters in Table 2, we have evolved the equation of state parameter, $\omega_{DE}$ for all cases I–IV in Fig. 5. The so-called phantom divide line crossing phenomenon has been clearly depicted in this figure. All in all, according to our analysis a theory with non-minimal coupling between tachyon scalar field and the Gauss–Bonnet invariant is in agreement with cosmological observations and can be considered as a good candidate for dark energy.

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