Consensus Control for Wheeled Mobile Robots Under Input Saturation Constraint

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ABSTRACT
In this article, the consensus control problem for the nonholonomic wheeled mobile robots under the input saturation constraint is addressed. The specified-time observer is designed to estimate the root node’s linear speed and angular speed by only using the configuration of the robot, and the convergence time when the observer obtains the real values can be specified in advance according to the specific task requirements. Based on the observed velocity states, the consensus controller subjects to input saturation is constructed to steer a wheeled mobile robot to keep track of the root node. Last, to verify the feasibility and effectiveness of the proposed control strategy, both the numerical simulation and physical experiment results are conducted.

INDEX TERMS
The specified-time observer, input saturation, nonholonomic wheeled mobile robots.

I. INTRODUCTION
Consensus control of multiple underactuated mobile robots has been a hot topic over the past few decades in the field of advanced nonlinear control [1]–[7], and there is a rich collection of the results for the underactuated robot control, such as Bullo et al. [1], Kim and Ahn [2], Aguiar and Hespanha [3], Pettersen and Egeland [4], and Bechlioulis et al. [5]. Consider its real applications in practical engineering, it’s always required that the robot could work steadily and effectively even in a non-ideal environment, i.e., there exists some practical constraints need to be considered for a robot to execute the given tasks.

Generally speaking, the first constraint need to be considered in consensus control for multi robots is the availability of all system states. For example, for the attitude dynamics of a robot which is described by a rigid body on special orthogonal group SO(3), only the orientation and the torque information are available for the rigid body so that an angular speed observer is then designed to estimate the real angular speed within specified time [8]. This non-ideal condition gives a velocity-free consensus control algorithm for the rigid body, which works well when the gyro is damaged or the measurements of the angular speed are not accessible. To solve the problem that only part of the states information are accessible, observer-based control strategy are well introduced in [9]–[11]. With only the configuration (i.e., position and orientation) of the robot, an elegant velocity observer is designed in [11] to estimate the necessary velocity information (linear speed and angular speed) for the needs of consensus controller design. However, compared to a state observer with exponential or finite time convergence, it is more desirable that the observer could converge to the real values within a specified time $T$, which is determined by the specific task requirements. The second constraint need to be considered is the control saturation. For the consensus control problem, there exists input saturation constrains normally, which implies that the magnitude of the control signals is confined to a given upper bound, which also implies that only the velocity with limited value can be used to drive the system in the practical engineering application. To solve this problem, many related works have been reported in literature. For example, considering second-order multi-agent systems, Zhai and Xia [12] gives a discussion on this issue and designed the distributed controller under the input saturation constraint. And an adaptive finite-time control scheme is developed for noncooperative spacecraft fly-around subject to input saturation in [13]. However, to the best of our knowledge, there are few results considering the input saturation constraint for a wheeled mobile robot, whose dynamics is described by a rigid body on Lie group SE(2).

Motivated by the discussions above and existing relevant references, such as [14] and [15], we address the consensus control problem for wheeled mobile robots under the input saturation constraint, together with the constraint that only part of robots (followers) have access to the configuration...
and velocity of root node (leader). The main contributions of this article are as follows. Firstly, inspired by the observer frame proposed in [11], we give a velocity observer with specified time convergence to estimate the velocity of the root node (leader) for all of its child nodes, whose dynamics can be described by the rigid body on SE(2). Based on the designed specified time velocity observer, all the child nodes (followers) can obtain the velocity information of root node within a specified time \( T \), where \( T \) can be determined according to the task requirements. Hence, before executing the given tasks, we can obtain the unknown velocity information of the root node (leader) within time \( T \) in advance, and hence the control action can be set to zero before time \( T \). Secondly, based on the designed observer, we construct a distributed controller for each follower on model SE(2) with linear speed saturation constraint. The controller proposed can steer every follower to keep track of the leader asymptotically and the follower’s linear speed has an upper bound which can be specified in advance.

The paper is organized as follows. Section II introduces system model of nonholonomic mobile vehicles on kinematics level and some necessary preliminaries. Then the control problem we considered in this article is addressed. Section III gives the specified-time observer and the distributed controller for each follower under the input saturation. The simulation results and the practical experiments that verify the feasibility and effectiveness of the proposed approach are provided in Section IV. Finally, conclusion is drawn in Section V.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. SYSTEM MODEL ON LIE GROUP SE(2)

In this section, we introduce the system model on Lie group SE(2) firstly. In general, the wheeled mobile robots can be viewed as planar rigid bodies, whose time behavior can be described by a trajectory on Lie group SE(2). The element \( g \) in SE(2) represents a robot’s configuration consisting of position and orientation, and it has the following form

\[
g = \begin{bmatrix}
\cos(\theta) & -\sin(\theta) & x \\
\sin(\theta) & \cos(\theta) & y \\
0 & 0 & 1
\end{bmatrix},
\]

where \((x, y)^T\) denotes the position of a robot, and \(\theta \in (-\pi, \pi]\) is its orientation in the initial frame.

Then the kinematics of the wheeled mobile robot is given by

\[
\dot{g} = g\dot{\xi} = \begin{bmatrix}
\cos(\theta) & -\sin(\theta) & x \\
\sin(\theta) & \cos(\theta) & y \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
0 & -\omega & v_x \\
\omega & 0 & v_y \\
0 & 0 & 0
\end{bmatrix},
\]

where \(\dot{\xi}\) is the control input, \(\omega\) is the angular speed, and \(v_x\) and \(v_y\) are the translational speeds along the x-axis and y-axis in the robot’s body frame, separately. We set the mapping \(\sigma(\cdot) : SE(2) \to \mathbb{R}^3\) and \(P = \sigma(g) = [x, y, \theta]^T\), and the inverse mapping is \(\sigma^{-1}\). Owing to the nonholonomic constraint existing for the wheeled mobile robots, the translational velocity along the y-axis \(v_y\) satisfies \(v_y = 0\), i.e., and the robot cannot move sideways. Thus we use \(v\) to denote the linear speed along the x-axis, and \((v, \omega)^T\) to denote the control input.

B. PROBLEM STATEMENT

Consider \(n\) wheeled mobile robots with kinematics \(g_i = g_i(e_t)\), \(g_i = \sigma^{-1}((x_i, y_i, \theta_i)^T)\) denotes its configuration and \((v_i, \omega_i)\) is the linear and angular speeds in \(\xi_i\). A directed graph can be represented by \(G = (V, E, A)\), where \(V = \{1, 2, \ldots, n\}\) is the node set and \(E\) is a set of edges for pairs of nodes. \(A = (a_{ij})_{n \times n}\) is the adjacency matrix where we have \(a_{ii} = 0, a_{ij} = 1\) if robot \(j\), the parent node, has an information flow to robot \(i\), the child node, otherwise \(a_{ij} = 0\). \(L\) denotes its Laplacian matrix, where \(l_{ij}\) represents the number of \(i\)th robot’s parent nodes, and \(l_{ij} = -1\) denotes that there is an information flow from node \(j\) to node \(i\), otherwise \(l_{ij} = 0\).

The consensus problem is considered in this article for multiple robots under the communication topology given by a directed acyclic graph (DAG) with only one root. Let the robot \(1\) be the root and controlled by \((v_1, \omega_1)\), where \(v_1\) is the linear speed and \(\omega_1\) is the angular speed, and the robot \(i\) be the follower, \(i = 2, \ldots, n\). If node \(i\) can get information from node \(j\), then we call node \(j\) parent node and node \(i\) child node, and the DAG indicates that the root only has children and no robot can simultaneously be child and parent node for the same one node.

In order to make our study to be more concrete, and confined to the specific non-ideal situations, we make the following assumption.

**Assumption 1:** The root node’s velocities are unknown, and the linear speed satisfies \(0 < v_1 \leq k_1\), where \(k_1\) is a positive constant. Simultaneously, these velocities are \(C^2\) functions of time \(t\), and both the velocities and their time derivatives are bounded for all time.

**Remark 1:** This assumption may seems restrictive at first glance, however it has its real case in work. As mentioned in Reference [16], UAV (unmanned aerial vehicle) with fixed wings can only move forward with its linear speed, but it cannot move backward. Hence, the leader in our manuscript could be a UAV with fixed wings under positive linear speed, the mobile robot on the ground can receive the distribution information of the leader and then can conduct the air-ground cooperation task.

Moreover, only the configuration, i.e., the position and orientation, of the root node is available to its child nodes. However, the rest of robots whose parent nodes are not the root node, can get both the configurations and the velocities from their parents if they are connected with a directed edge. The objective of this article is to design a specified-time observer to estimate the root node’s velocities only using the position and orientation, i.e., for the robot \(i, i \in \{a_{i1} = 1\}\), for a specified finite time \(T > 0\), the observed value of the root node’s velocities should be equal to the exact values \((v_1, \omega_1)\) when \(t = T\). And based on the observed values, the consensus problem is addressed with a proposed controller under the saturation constraint of the
linear speed, which implies that \( g_1 \) should asymptotically converge to the configuration \( g_1 \), and the control input \( v_i \) should satisfy \( |v_i| \leq k_i \) with a positive constant \( k_i \) pre-specified by the task requirements.

### III. MAIN RESULTS

In this section, the consensus problem for multiple wheeled mobile robots with linear speed saturation constraint is addressed.

#### A. THE SPECIFIED-TIME OBSERVER

Note that the children of the root node are unable to obtain the information of the root node's velocities, so that a velocity observer using only the position and orientation is necessary. In order to obtain the exact values of the root node's velocities within specified time \( T \), a time scaling function \( \tau \) in [8] is introduced, which is a continuously differentiable function \( \tau : [0, T) \rightarrow [0, \infty) \), and satisfies \( \tau(0) = 0 \) and \( \lim_{\tau \rightarrow T} \tau = \infty \). Motivated by [11], all the children which can get the root node's configurations directly, can estimate the velocities with the following specified-time observer

\[
\begin{align*}
\dot{\tilde{x}}_i &= S_i + K_0 \tilde{x}_i, \\
\dot{\tilde{y}}_i &= K_1 \tilde{\tau}^2 \text{sgn}(\tilde{f}_i) + K_2 \tilde{\tau}^2 \tilde{f}_i,
\end{align*}
\]  

(3)

where \( \tilde{x}_i \) represents the \( i \)th robot's estimations of root node's configuration \( P_1 = \sigma(g_1) \). \( S_i \in \mathbb{R}^3 \) is the auxiliary variable, and \( K_0, K_1, K_2 \in \mathbb{R}^{3 \times 3} \) are positive, diagonal, constant matrices. \( \tilde{x}_i = P_1 - f_i \) is the configuration error between \( i \)th robot and the root node. Then the key step is to choose a continuously differentiable function \( \tau \) satisfying \( \tau(0) = 0 \) and \( \lim_{\tau \rightarrow T} \tau = \infty \). This kind of functions is called the time scaled function, and we choose the following function in this article with a positive constant \( a \)

\[
\tau = a \ln(\frac{T - \tilde{\tau}}{T - t}),
\]  

(4)

and its time derivative is \( \dot{\tau} = \frac{a}{T - \tilde{\tau}} \), and \( \tau^{-1} = \frac{T - t}{T - \tilde{\tau}} \).

Then we give one of the main results in the following theorem.

**Theorem 1:** Consider the wheeled mobile robots under the Assumption 1, the children of the root node can estimate the exact velocities within the given time \( T \) with the specified-time observer (3) if the coefficient \( K_1 \) is chosen to satisfy the following condition

\[
K_{1ii} > \sup_{\tau} (|N_o| + |\dot{N}_o|),
\]  

(5)

where

\[
N_o = \begin{bmatrix}
\omega_1 v_1 \cos(\theta_1) \\
-\omega_1 v_1 \sin(\theta_1) \\
0
\end{bmatrix} + \begin{bmatrix}
\sin(\theta_1) & 0 & 0 \\
\cos(\theta_1) & 0 & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
\dot{v}_1 \\
0 \\
\dot{\theta}_1
\end{bmatrix}.
\]

(6)

**Proof:** The observer (3) can be rewritten as

\[
\begin{align*}
\dot{\tilde{x}}_i &= \tilde{\tau}^{-1} S_i + K_0 \tilde{x}_i, \\
\dot{\tilde{y}}_i &= \tilde{\tau} K_1 \text{sgn}(\tilde{f}_i) + \tilde{\tau} K_2 \tilde{f}_i,
\end{align*}
\]

(7)

where the subscript \( \tau \) represents the derivative with respect to \( \tau \). With \( \tilde{P}_1 = N_o \), we have \( \dot{\tilde{P}}_1 = \tilde{\tau}^{-2} N_o \), and it is easy to obtain that

\[
\dot{f}_i = K_1 \text{sgn}(f_i) + K_2 \tilde{f}_i + K_0 \tilde{\tau} \tilde{f}_i,
\]

(8)

and the second derivative of \( \tilde{f}_i \) with respect to \( \tau \) is \( \ddot{f}_i = \ddot{\tilde{P}}_1 - K_1 \text{sgn}(f_i) - K_2 \tilde{f}_i - K_0 \tilde{\tau} \tilde{f}_i \).

Then we choose the variable \( r \) as \( \dot{r} = \tilde{f}_i + \dot{\tilde{f}}_i \), and its derivative with respect to \( \tau \) is

\[
\dot{r} = \dot{\tilde{P}}_1 - K_1 \text{sgn}(f_i) - K_2 \tilde{f}_i - (K_0 - I_3) \tilde{f}_i,
\]

(9)

and the positive diagonal matrix \( K_2 \) can be chosen as \( K_2 = K_0 - I_3 \), where \( I_3 \) denotes the \( 3 \times 3 \) identity matrix. Then we have

\[
\dot{r} = N_o - K_2 r - K_1 \text{sgn}(f_i).
\]

(10)

With the proof of the Theorem 2 in [11], we can obtain that \( \tilde{f}_i = (\tilde{P}_1 - f_i) \rightarrow 0 \) as \( \tau \rightarrow \infty \), which is equivalent to estimate the exact values of the velocities when \( \tau \rightarrow \infty \). As the definition of the function \( \tau \), we can conclude that the observer proposed in (3) will obtain the exact values of the velocities when \( \dot{\tau} \rightarrow T \). \( \square \)

**Remark 2:** The observer (3) requires that the leader’s velocities should be differentiable and bounded, and the observed values \( \tilde{f}_i \), however, are no longer differentiable. For the reason of the specified time, the observed values might be so large that they can not be used during the process of consensus control. Therefore, the controller for the follower is set to zero before the time \( T \), which will be discussed further in the next section.

**Remark 3:** Comparing to the observed proposed in [16], there are two main characters distinguish the observer in [16] with our observer. Firstly, owing to the frame of observer design, the observer we proposed is a distributed observer, and the convergence analysis has also been done in our manuscript, which is missing in reference [16]. Secondly and importantly, the distributed observer we proposed is capable of specified time estimation, so that followers can obtain the estimated leader’s information within a short time to conduct the cooperation task, which benefits the control performance. In addition, since the specified time estimation would unavoidably cause large value of estimation, we further took the input saturation into consideration, so that the controller can be used in real application.

#### B. DISTRIBUTED CONTROLLER DESIGN UNDER THE CONSTRAINT OF INPUT SATURATION

Based on the observer above, we can use the observed values when \( \tau \geq T \). Then we consider the consensus control problem for the nonholonomic wheeled mobile robots under a DAG when their linear velocities are limited due to the task requirements and the limitations of their own structures. Note that there may be several parent nodes for one robot under the DAG, and the parent nodes of \( i \)th robot can be combined
into a virtual parent, whose configuration can be denoted by \( g_i(x_{vi}, y_{vi}, \theta_{vi}) \). As discussed in [15], the convex of \( l_{ii} \) parent nodes can be generated iteratively as

\[
g_{1,2} = g_1 \exp(\lambda_1 \log(g^{-1}_1 g_2))
\]

\[
g_{1,2,\ldots,l_{ii}} = g_{1,\ldots,l_{ii-1}} \exp(\lambda_{l_{ii}} \log(g^{-1}_{l_{ii}, l_{ii-1}} g_{l_{ii}})),
\]

where \( \lambda_m, m = 1, 2, \ldots, l_{ii} - 1 \) is a positive constant satisfying \( 0 \leq \lambda_j \leq 1 \) and \( g_{1,\ldots,l_{ii}} \) is the virtual parent node \( g_{vi} \). Then we define a series of positive constants \( \gamma_q = \frac{\lambda_l}{\lambda_{l_{ii} - 1}} \) for the virtual parent node of \( i \)th robot with \( q = 1, 2, \ldots, l_{ii} \), and \( \lambda_m \) and \( \gamma_q \) satisfy the following relationships

\[
\lambda_1 = \frac{\gamma_2}{\gamma_1 + \gamma_2}, \lambda_2 = \frac{\gamma_3}{\gamma_1 + \gamma_2 + \gamma_3}, \ldots, \lambda_{l_{ii} - 1} = \gamma_{l_{ii}}.
\]

Moreover, the kinematics of the virtual parent node can be formally written as

\[
\dot{g}_{vi} = g_{vi} \xi_{vi},
\]

where \( \xi_{vi} = \sum^{l_{ii}}_{q=1} \gamma_q \xi_q \) is the virtual control input, and \( \xi_q \) is the control input of the parent node.

Lemma 1: The network of \( n \) robots on SE(2) in a DAG achieves consensus with the root node, i.e., \( \lim_{t \to \infty} g_i = g_1 \) \( \forall i = 2, \ldots, n \), if \( i \)th robot achieves consensus with the convex combination of the configurations of its all parent nodes [15].

As discussed above, we need to design the distributed consensus control \((v_i, \omega_i)^T\) for each robot \( i \) and make sure that the \( i \)th robot can track the convex combination of the configurations of its all parent nodes. Firstly, we consider the control design for robot \( i \) and its virtual leader, and its relative positions are defined as \( x_{vi} = \cos(\theta_i)(x_{vi} - x_i) + \sin(\theta_i)(y_{vi} - y_i), y_{vi} = -\sin(\theta_i)(x_{vi} - x_i) + \cos(\theta_i)(y_{vi} - y_i) \). Then we define the relative distance \( l_i \) and angle \( \alpha_i \) of the \( i \)th robot with respect to its virtual leader as

\[
l_i = \sqrt{(x_{vi}^2 + y_{vi}^2)}, \alpha_i = \text{arctan2}(y_{vi}, x_{vi}).
\]

The relative angle \( \beta_i \) in the initial frame, which is the auxiliary variable, can be defined as \( \beta_i = \text{arctan2}(y_{vi} - y_i, x_{vi} - x_i) \), and these three angles satisfy \( \alpha_i = \beta_i - \theta_{vi} \).

Now we assume that \( i \)th robot has \( l_{ii} \) parent nodes, and the convex combination of the configurations of its parent nodes is denoted by \( g_{vi} \). We set the upper bound of \( i \)th robot linear speed control input as \( k_i \), where \( k_i \) is a positive constant satisfying some constraint conditions discussed in the sequel.

We propose the following control input \((v_i, \omega_i)\) in \( \xi \) for \( i \)th robot to solve the consensus control problem under the linear speed saturation constraint:

\[
\begin{bmatrix}
  v_i \\
  \omega_i
\end{bmatrix} = \begin{bmatrix}
  k_i \text{sat}_{k_b}(\alpha_i \cos(\alpha_i)) \\
  k_i \text{sat}_{k_b}(\alpha_i \sin(\alpha_i)) - v_i \sin(\alpha_i) \\
  + k_i \text{sat}_{k_b}(\alpha_i \cos(\alpha_i)) \sin(\alpha_i) \end{bmatrix} - l_i v_i \cos(\alpha_i),
\]

where \( k_{sat} \) is a positive constant, \( \alpha_i \) is the relative angle in the local frame of the virtual leader and satisfies \( \alpha_{vi} = \beta_i - \theta_{vi} \).

The saturation function \( \text{sat}_{k_b}(\cdot) \) is defined as

\[
\text{sat}_{k_b}(x) = \begin{cases}
  k_b, & x \geq k_b \\
  -k_b, & x \leq -k_b \\
  x, & |x| < k_b
\end{cases}
\]

where the constant \( k_{b} \) is chosen to be 1 for the purpose of limiting the size of the linear speed, and the coefficient \( k_i \) represents the upper bound of \( v_i \). As the definition of the convex combination, the virtual leader’s linear speed is

\[
v_{vi} = \sum_{j=\{j|y_{aj}=1\}}^{\infty} \gamma_{y_{ij}} v_{y_{ij}}.
\]

With the upper bound \( k_i \) of the linear speed of \( i \)th robot’s parent nodes, \( k_i \) is chosen to satisfy the following inequality

\[
k_i \geq \sum_{j=\{j|y_{aj}=1\}}^{\infty} \gamma_{y_{ij}} k_j.
\]

We are now ready to present our main results in the following theorem.

Theorem 2: The control input described by (15) ensures that the followers can achieve consensus asymptotically with the root node under the Assumption 1, i.e., \( \lim_{t \to \infty} g_i = g_1 \), \( j = 2, \ldots, n \), provided the coefficient \( k_i \) satisfies the inequality (18).

Proof: The velocities of the root node can not be obtained by its out-neighbors, i.e., the child nodes, while the controller (15) requires the velocities of the parent nodes. With the specified time observer (3), the exact velocities are accessible after the specified time \( T \).

After several simple calculations and rearrangements, the derivatives of the relative parameters with respect to time \( t \) are \( \dot{l}_i = v_i \cos(\alpha_i) - v_i \cos(\alpha_i), \dot{\alpha}_i = (v_i \sin(\alpha_i) + v_i \sin(\alpha_i)) l_i^{-1} - \omega_i \). Then the Lyapunov function is defined as follows:

\[
V = \sum_{i=2}^{n} \frac{1}{2} (l_i^2 + \alpha_i^2).
\]

After taking the time derivative of (19) and substituting (15) into it, one can obtain

\[
\dot{V} = \sum_{j=2}^{n} (-k_i \text{sat}_{k_b}(\alpha_i \cos(\alpha_i)) l_i \cos(\alpha_i) + l_i v_i \cos(\alpha_i)
\]

\[
- k_{sat} \alpha_i^2 - k_i \sin(\alpha_i) \alpha_i l_i).
\]

By choosing \( k_b = 1 \), \( \dot{V} \) is divided into the following two cases

- case 1: if \(-1 \leq (\alpha_i \cos(\alpha_i)) \leq 1 \), the time derivative of \( V \) satisfies \( \dot{V} = \sum_{j=2}^{n} (-k_i \frac{\alpha_i}{\sin(\alpha_i)} - k_{sat} \alpha_i^2 + l_i v_i \cos(\alpha_i)) \leq \sum_{j=2}^{n} (-k_i \frac{\alpha_i}{\sin(\alpha_i)} - \delta_i l_i - k_{sat} \alpha_i^2) \), where \( \delta_i = \sum_{j=\{j|y_{aj}=1\}}^{\infty} \gamma_{y_{ij}} k_j \) is the upper bound of the linear speed of the \( i \)th robot’s virtual leader in (17). With the coefficient \( k_i \) satisfying (18) and the fact \( \frac{\alpha_i}{\sin(\alpha_i)} \geq 1 \), we can conclude that \( \dot{V} \leq 0 \) and the errors \( l_i \), \( \alpha_i \) converge to zero when \( t \to \infty \).
• case 2: if the term \((\alpha_i\cot(\alpha_i)) \leq -1\), we can obtain that
\[
sat_k(\alpha_i\cot(\alpha_i)) = -1.
\]
Then the time derivative \(\dot{V} = \sum_{j=2}^{n}(-k_i l_i\cos(\alpha_i) - k_i s\sin(\alpha_i) - \frac{\omega_i}{k_i}) - k_\omega \omega_i^2 \leq 0\). Note that the term \(-\cos(\alpha_i) + \alpha_i \sin(\alpha_i)\) is always greater than 1 when \(\alpha_i \in \{x | x\cot(x) < -1 \text{ and } |x| \leq \pi\} \) and \(\frac{\omega_i}{k_i} \leq 1\).

Therefore, \(\dot{V} \leq 0\) still holds in this case.

In conclusion, the linear speed of each robot is designed to be smaller than the positive constant \(k_i\), and multiple wheeled mobile robots in a DAG can achieve consensus by the proposed controller (15).

Remark 4: The controller (15) can be put into effect after the observer (3) converges to the exact values, i.e., after the time \(T\). Therefore, the controller for each robot can be set to zero when \(t < T\), and it will be switched to (15) when \(t \geq T\).

There is a term \(l_i^{-1}\) in the angular speed, and as discussed in [15], it is not a serious limitation for practical applications. For position tracking, a practical stability is achieved if \(l_i\) converges to an arbitrarily small nonzero value.

IV. SIMULATION AND EXPERIMENT RESULTS

In this section, we give some simulation and experiment results to verify the efficiency of the proposed control strategy. Denote robot 1 as the leader, and other five as the followers, called robot \(i\), \(i = 2, 3, 4, 5, 6\). The communication graph for the six robots is a DAG in Fig. 1. The initial configurations of these robots are \((x_1, y_1, \theta_1) = (0, 0, 0), (-2, 5, \frac{\pi}{4}), (-2, 0, \pi), (-2, -5, -\frac{\pi}{4}), (-4, 2, -\frac{\pi}{4})\), and \((-4, -2, \frac{\pi}{4})\), and the controller for leader is \((v_1, \omega_1) = (1.5, \cos(2t))\).

A. SIMULATION RESULTS OF THE SPECIFIED-TIME OBSERVER

The time \(T\) sets 4, and the matrices are \(K_0 = diag(2, 2, 2), K_1 = diag(10, 10, 10), K_2 = K_0 - I_3 = (1, 1, 1)\). Then the observers of robot \(i\), \(i = 2, 3, 4\), is designed to obtain the exact values within 4 seconds.

From the simulation results in Fig. 2, it can be concluded that the child nodes of the robot 1, i.e., robot 2, 3, 4 can obtain the exact values of the first robot’s velocities within a specified time \(T = 4\), and the errors between the observed values and the real ones converge to zero eventually. Meanwhile, the time derivatives of robot’s configurations called generalized velocities are bounded, and we can use the observed values to substitute the real ones when \(t > 0\), rather than \(t > T\).

B. SIMULATION RESULTS OF THE DISTRIBUTED CONTROLLER UNDER INPUT SATURATION CONSTRAINT

Here we consider that the linear speed of each follower is constrained by an upper bound, and these coefficients are chosen as \(k_2 = k_3 = k_4 = 2, l_5 = k_6 = 2.5, k_\omega = 3\), \(i = 2, 3, 4, 5, 6\). Simulation results are depicted in Fig. 3.

It can be concluded that the followers can keep track of the robot 1, and due to the saturation constraint, it takes the followers more time to make the tracking errors converge to zero.

C. EXPERIMENT RESULTS OF THE DISTRIBUTED CONTROLLER UNDER INPUT SATURATION CONSTRAINT

In order to verify the controller’s efficiency in the practical engineering application, we use three wheeled mobile robots as the followers with a virtual leader. The leader’s velocities are \(v_1 = 0.07, \omega_1 = 0.2\cos(0.6t)\), and its initial configuration is \((x_1, y_1, \theta_1) = (0, 0, 0)\). The three followers’ initial configurations are \((x_2, y_2, \theta_2) = (-0.6, 0.8, 0), (-4.5, 5, \frac{\pi}{4})\), and \((-7, -4, -\frac{\pi}{4})\) respectively.

FIGURE 2. Errors of the specified-time observer for three robots.

FIGURE 3. Consensus control under the saturation constraint of the linear velocities.
After getting the real values, we propose a consensus con-

\[ \begin{align*}
(x_3, y_3, \theta_3) &= (-0.6, -0.6, 0), \quad \text{and} \quad (x_4, y_4, \theta_4) = (-1, -0.4, 0). \end{align*} \]

The communication between robots can be depicted with adjacent matrix \( A \), and it has that \( a_{21} = a_{31} = a_{42} = a_{43} = 1 \), i.e., robot 2 and robot 3 are the child nodes of robot 1, and robot 4 is the child node of robot 2 and 3. The coefficients can be set as \( k_2 = 0.08 \), \( k_3 = 0.08 \), \( k_4 = 0.09 \), and \( k_{\text{col}} = 0.5 \). With the controller (15), the experiment results are shown in Fig. 4. It can be concluded that robot 2 and 3 can perform a consensus with the virtual leader, and robot 4 can keep track of its parent nodes, i.e., robot 2 and 3.

From the above simulation and experiment results, it can be observed that the proposed specified time observer and control law with saturation are proved to be effective under the E-puck platform. The estimation error converge to zero within specified time, so that the followers can keep track of the leader even if the velocity of the leader is not accessible, and due to saturation constraint, it takes the followers more time to make the tracking errors converge to zero.

V. CONCLUSION

In this article, consensus control for multi wheeled mobile robots with linear speed saturation has been studied in this article. Since the followers can not get the velocity information of the leader, a specified-time observer is given to obtain the exact values of the leader’s velocities within the time \( T \). After getting the real values, we propose a consensus controller in a DAG to drive all the robots to keep track of the root node (leader) under the linear velocity saturation. Currently the proposed controller is designed on the kinematics level, but it is undeniable that dynamic effect plays an important role in consensus control, hence in our future work, we will consider the consensus control problem for multi wheeled mobile robots based on the dynamic models.

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