Broken Symmetry Ground States in $\nu = 2$ Bilayer Quantum Hall Systems

A.H. MacDonald$^1$, R. Rajaraman$^{1,2}$ and T. Jungwirth$^{1,3}$

$^1$Department of Physics, Indiana University, Bloomington, IN 47405
$^2$School of Physical Sciences, Jawaharlal Nehru University, New Delhi 110067, India
$^3$Institute of Physics ASCR, Cukrovarnická 10, 162 00 Praha 6, Czech Republic

Abstract

We report on a study of a bilayer two-dimensional electron gas at Landau level filling factor $\nu = 2$. The system exhibits both magnetic and spontaneous interlayer phase coherence broken symmetries. We propose a 3-parameter Slater determinant variational wavefunction which describes the ground state over the full range of bias potential ($\Delta V$) Zeeman coupling ($\Delta z$) and interlayer tunneling amplitude ($\Delta t$) strengths. Broken symmetry states occur inside a volume in this three-dimensional phase diagram near the $\Delta_z^2 = \Delta_V^2 + \Delta_t^2$ surface. We have obtained analytic results for the intersections of the phase diagram with the $\Delta_t = 0$, $\Delta_z = 0$, and $\Delta_V = 0$, planes and show that the differential capacitance of the bilayer system is singular at the phase boundary.

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I. INTRODUCTION

The interaction physics of two-dimensional electrons in the quantum Hall regime is enriched by the macroscopic degeneracy of zero-width Landau bands. Among the consequences of electron-electron interactions in the quantum Hall regime are broken symmetry ground states that are especially robust at integer Landau level filling factors. In a single-layer two-dimensional electron gas, the ground state at Landau level filling factor $\nu = 1$ is a strong ferromagnet with total spin quantum number $S = N/2$ where $N$ is the number of electrons. When mixing between different orbital Landau levels is neglected, the exact ground state is known and consists of a fully occupied set of lowest-Landau-level (LLL) orbitals sharing a common spinor characterized by an arbitrary spin orientation. This broken symmetry ground state is conveniently described using the lexicon of the Hartree-Fock mean-field approximation, which happens to be exact in this instance. In this language, the broken symmetry is a consequence of exchange interactions which spontaneously spin-split the orbital Landau levels. The Stoner criterion for ferromagnetism in itinerant-electron mean-field theory, $Ig(\epsilon_F) > 1$ where $I$ is the exchange integral and $g(\epsilon_F)$ is the density-of-states at the Fermi energy, is always satisfied because the density-of-states is infinite for a zero-width band.

For bilayer quantum Hall systems, described schematically in Fig. 1, the Hartree-Fock approximation ground state at $\nu = 1$ breaks both spin-rotational invariance and the individual-layer charge conservation symmetry by fully occupying an orbital with an arbitrary spin orientation and spontaneously developed phase coherence between top and bottom layers even in the absence of interlayer tunneling and Zeeman coupling. Bilayer systems are richer still for Landau level filling factor $\nu = 2$. In the Hartree-Fock approximation, two orbital Landau levels are fully occupied, the spin and bilayer state of each described by a four-component spinor. In realistic circumstances electron-electron interactions can be more important than the one-particle interaction terms corresponding to Zeeman coupling, bias potential, or inter-layer tunneling energy in determining the form of these spinors, opening
the possibility for broken symmetries in the ground state. In previous work it has been established that the Hartree-Fock ground state at zero bias potential and weak Zeeman couplings is a canted antiferromagnet, with spin ordered moments having opposing tilts away from the Zeeman field direction in opposite quantum wells. Theoretical predictions for the canted antiferromagnet state are in good agreement with the findings of inelastic light scattering experiments. Although, unlike the single layer case, the Hartree-Fock approximation does not yield the exact ground state in bilayers, it can be improved upon systematically and should reliably indicate the types of broken symmetry phases which occur. In this paper, we use the Hartree-Fock approximation to describe the dependence of the ground state of $\nu = 2$ bilayer quantum Hall system on a bias voltage which moves the two layers away from density balance. A bias potential is readily applied in situ and, as we show, can be used to tune the system either in to or out of the broken symmetry region of the phase diagram. We point out that the differential capacitance of the bilayer system, which is relatively easy to measure experimentally, will show singular behavior at the phase boundaries. In Section II we introduce a four-component spinor representation of bilayer quantum Hall ferromagnets at $\nu = 2$. We propose a general 3-parameter Slater determinant variational wavefunction which accurately describes the ground state over the entire phase space. We present an explicit expression for the ground state energy functional and the four-dimensional Hartree-Fock Hamiltonian whose eigenvalues can be used to estimate the gap for charged excitations. Sections III, IV and V report analytical results for the cases of zero Zeeman coupling, zero bias potential, and zero interlayer tunneling, respectively. Section III and V also include analyses of the singularity which occurs in the bias-potential-dependent internal capacitance at the order-disorder phase transition. In Section VI we report numerical results for the dependence of several quantities which are open to experimental study on model parameters. Finally in Section VII, we conclude with a discussion of the dimension of the order parameter space.
II. HARTREE-FOCK VARIATIONAL WAVEFUNCTIONS

Bilayer models for quantum well electron systems apply whenever the growth-direction degree of freedom of the electron is adequately described by specifying the well in which an electron resides, or equivalently whenever the two lowest electric subbands are well separated from higher subbands. We adopt the bilayer model throughout this article. The single-particle Hilbert space is then a direct product of the two-dimensional orbital continuum space, the two-levels associated with the electronic spin degree of freedom, and a second two-level system which constitutes the growth-direction layer degree of freedom. It is convenient and has become a standard notation to describe orbital two-level systems using a pseudospin language. In this paper we will treat the layer degree of freedom as a pseudospin and use a convention in which an electron in the top layer is said to have pseudospin up while an electron in the bottom layer is said to have pseudospin down. The magnetic field is assumed to be sufficiently strong that only 2D electron orbitals in the LLL can be occupied. We further assume that translational invariance is not broken in the ground state. We seek the single-Slater-determinant many-particle wavefunction which minimizes the expectation value of the Hamiltonian. It follows from the above that it has the form

\[
|\Psi[z]\rangle = \prod_{i,X} \left( \sum_{k=1,4} z_i^k c_{k,X}^{\dagger} \right) |0\rangle.
\]  

Here \(c_{k,X}^{\dagger}\) is a creation operator for a Landau gauge LLL orbital. The index \(k\) specifies the spin and pseudospin state; in our convention \(k = 1\) for an up-spin electron in the top layer, \(k = 2\) for a down-spin electron in the top layer, \(k = 3\) for an up-spin electron in the bottom layer and \(k = 4\) for a down-spin electron in the bottom layer. Translational invariance requires that the coefficients \(z_i^k\) are independent of the label \(X\) which specifies the guiding center state within LLL. At \(\nu = 2\), the index \(i = 1, 2\) and the ground state consists of two filled Landau levels specified by four-component spinors \(z^1\) and \(z^2\), respectively. This is an enormous simplification but, at first sight, still leaves us with a sixteen-dimensional space specified by eight complex numbers in which to search for a minimum. The considerations
we use to reduce this dimension are dependent in part on the form of the Hamiltonian which we now specify.

The model we examine allows for external bias potential, tunneling between the quantum wells, and Zeeman coupling, in addition to electron-electron interactions. We write the single-particle Hamiltonian in the form

\[ h^0 = -\left(\Delta_V/2\right)\tau^z - \left(\Delta_t/2\right)\tau^x - \left(\Delta_z/2\right)\sigma^x. \]  

(2)

We have introduced the notation \( \tau^\alpha \) for the Pauli spin matrices acting on the pseudospin degree of freedom and \( \sigma^\alpha \) for the those acting on the spin degree of freedom. In the current context these are four-dimensional matrices whose explicit form is easily constructed from our convention for the \( k \) indices specified above. The parameter \( \Delta_V \) represents the electric potential drop between the two quantum wells, due to electric field created by charges external to the bilayer system, \( \Delta_t/2 \) is the amplitude for an electron tunneling between top and bottom quantum wells which can be measured by studying weak-field Schubnikov-deHaas oscillations, and \( \Delta_z \) is the single-particle spin-splitting due to Zeeman coupling. The single-particle terms yield four macroscopically degenerate Landau levels whose energies are

\[ E^{0}_{1,2,3,4} = \pm \frac{1}{2} \left[ \sqrt{\Delta_t^2 + \Delta_V^2} \pm \Delta_z \right] \]  

(3)

Broken symmetry states for \( \nu = 2 \) occur only when the relative and absolute strengths of the three single-particle terms is such that the gap between the two lower energy eigenvalues and the two higher energy eigenvalues is smaller than or comparable to electron-electron interaction strengths.

The many-particle Hamiltonian whose expectation value we minimize is

\[ \hat{H} = \sum_{k',k,X} c_{k',X}^\dagger h^0_{k',k} c_{k,X} + \frac{1}{2} \sum_{k_1,k_2,X_1,X_2,X'_1,X'_2} c_{k_1,X_1}^\dagger c_{k_2,X_2}^\dagger c_{k_1,X_1} c_{k_2,X_2} \langle X'_1,X'_2 | V_+ | X_1,X_2 \rangle \]
\[ + \tau^z_{k_1,k_1} \tau^z_{k_2,k_2} \langle X'_1,X'_2 | V_- | X_1,X_2 \rangle. \]  

(4)

Here \( V_\pm = (V_S \pm V_D)/2 \) and \( V_S \) and \( V_D \) are the 2D interactions between electrons in the same layer and in different layers. The qualitative physics of bilayer quantum Hall systems at
\( \nu = 2 \) is controlled by the property that \( V_D \) is weaker than \( V_S \). Note that this model explicitly drops the, normally small, terms in which interactions scatter electrons between layers. The model also assumes that the two quantum wells in the bilayer system are identical, usually true of in experimental systems. This assumption can easily be relaxed and does not change any qualitative physics.

The properties of orbital states within a Landau level can be used to derive a general expression for the variational energy of the Slater determinant trial wavefunctions:

\[
\epsilon = -\frac{1}{2} \sum_{i=1}^{2} \left[ \Delta_V \langle z^i|z^i \rangle + \Delta_s \langle z^i|z^i \rangle + \Delta_z \langle z^i|z^i \rangle \right] + F_{\pm} - \sum_{j=1}^{2} \left( H \langle z^i|z^j \rangle - F_{\pm} \langle z^i|z^j \rangle \right) \]  

(5)

Here \( \epsilon \) is energy per Landau level orbital. (The number of orbitals in a Landau level is \( N_\phi = \frac{AB}{\Phi_0} \) where \( A \) is the cross-sectional area and \( \Phi_0 \) is the electron magnetic flux quantum.) In Eq.\([5]\) we have adopted Dirac notation for the two four-component spinors and introduced the three interaction energy parameters which appear in the Hartree-Fock theory of bilayer quantum Hall systems:

\[
H = \frac{1}{2} V_{\pm} \langle \vec{q} = 0 \rangle,  
\]  

(6)

and

\[
F_{\pm} = \int \frac{d^2\vec{q}}{(2\pi)^2} \exp\left(-q^2\ell^2/2\right) V_{\pm} \langle \vec{q} \rangle.  
\]  

(7)

The parameter \( H \) characterizes the electrostatic energy associated with charge transfer between the wells, while \( F_{\pm} \) and \( F_- \) are exchange energies associated with the sum and difference of intralayer and interlayer interactions. Note that \( F_+ \) contributes only a constant to the energy so that the ground state depends only on the three single-particle parameters and on \( H \) and \( F_- \). In the limit of infinitely narrow quantum wells and for the LLL, \( H = \frac{e^2}{\varepsilon \ell} d/2\ell \) where \( \varepsilon \) is the dielectric constant and \( \ell = \frac{\hbar c}{eB} \) is the magnetic length. The interaction parameters depend in general on details of the bilayer system geometry and on the index of the orbital Landau level at the Fermi energy. The values of these parameters are easily calculated in the LLL.
evaluated once the geometry has been specified. We assume in what follows\[10\] that \( H > F \), an inequality which is always satisfied when the interacting 2D electron layers are identical.

Extrema of the Hartree-Fock variational energy functional satisfy the Hartree-Fock single particle equations. For the present problem, the spinors we seek are the two lowest energy eigenstates of the four-dimensional matrix

\[
h^{HF} = h^0 - F_+ \rho + H \tau^z [\text{Tr}(\rho \tau^z)] - F_- \tau^z \rho \tau^z
\]

where \( \rho \) is the four-dimensional density matrix which must be constructed self-consistently:

\[
\rho_{k',k} = \sum_{i=1}^{2} \langle b_{k'} | z^i \rangle \langle z^i | b_k \rangle
\]

Here \( b_k \) denote the four four-component basis spinors \((b_1=(1,0,0,0), b_2=(0,1,0,0), b_3=(0,0,1,0), b_4=(0,0,0,1))\) corresponding to label-\(k\) spin and pseudospin states as introduced in Eq. (1). The Hartree-Fock variational state can be determined by finding solutions of the Hartree-Fock equations or directly by minimizing the energy functional \((5)\) with parameterized spinors. The former approach is usually more convenient when numerical solutions are required and has the advantage of yielding an estimate of the quasiparticle excitation gap, which is important in interpreting transport experiments\[11\]. The latter approach is often more convenient when the calculation can be done analytically. In this paper we will try to proceed as far as possible analytically before resorting to numerical calculations to illustrate and extend the analytical results.

We start from a variational \textit{ansatz} for the two four-component spinors \( z^1 \) and \( z^2 \) appearing in the \( \nu = 2 \) translationally invariant trial wavefunction (Eq.[1]). (A more complete discussion of the most general translationally invariant \( \nu = 2 \) wavefunction and of the number of parameters it can carry will be given in the concluding Section VII.) We have checked numerically to ensure that our ansatz, characterized by three variational parameters, is sufficiently general to capture the minimum energy state for any values of the parameters which appears in the one-body Hamiltonian \((2)\), namely, the bias potential, the Zeeman coupling and the tunneling amplitude. Our wavefunctions are specified by the following two four-component spinors:
\[ z^1 = (Cc\uparrow, Ss\downarrow, Cs\uparrow, -Sc\downarrow) \]
\[ z^2 = (-Ss\uparrow, Cc\downarrow, Sc\uparrow, Cs\downarrow) \] (10)

Here \( C = \cos(\theta/2) \), \( S = \sin(\theta/2) \), \( c_\sigma = \cos(\chi_\sigma/2) \), and \( s_\sigma = \sin(\chi_\sigma/2) \) with \( \sigma = \uparrow, \downarrow \). The variational energy of these wavefunctions is

\[
\begin{align*}
\epsilon &= -\Delta_V \cos(\theta) \cos(\chi_+) \cos(\chi_-) - \Delta_t \cos(\theta) \sin(\chi_+) \cos(\chi_-) - \Delta_z \sin(\theta) \sin(\chi_-) \\
& \quad - F_+ - F_- [\sin^2(\theta) \sin^2(\chi_-) + \cos^2(\theta) (\cos^2(\chi_+) \cos^2(\chi_-) + \sin^2(\chi_+) \sin^2(\chi_-)) ] \\
& \quad + 2H \cos^2(\theta) \cos^2(\chi_+) \cos^2(\chi_-)
\end{align*}
\] (11)

Here \( \chi_\pm \equiv (\chi_\downarrow \pm \chi_\uparrow)/2 \). We have not been able to minimize this energy expression analytically for the case when all three parameters \( \Delta_V \), \( \Delta_t \) and \( \Delta_z \) are non-zero. In the following sections we present analytic results for the cases in which one of the three parameters is set to zero.

The density matrix corresponding to our variational wavefunction is

\[
2\rho = \mathbf{1} + \cos(\theta) \cos(\chi_+) \cos(\chi_-) \tau^z + \sin(\theta) \sin(\chi_-) \sigma^z \\
+ \cos(\theta) \sin(\chi_+) \cos(\chi_-) \tau^x + \sin(\theta) (\cos(\chi_-) - \cos(\chi_+)) \sigma^x \tau^x \\
- \cos(\theta) \cos(\chi_+) \sin(\chi_-) \sigma^z \tau^x + \cos(\theta) \sin(\chi_+) \sin(\chi_-) \sigma^z \tau^x
\] (12)

Expectation values of spin and pseudospin operators and their products can be read off from Eq. (12) by using the familiar properties of Pauli matrices. For example, the difference between the \( \hat{z} \) direction projection of the spin in the top and bottom layers, \( \langle \tau^z \sigma^z \rangle = Tr(\rho \tau^z \sigma^z) \) which was identified as the order parameter in the work of Zheng et al. is given by

\[ O_{zz} = 2 \cos(\theta) \sin(\chi_+) \sin(\chi_-). \] (13)

This order parameter is like that of a canted antiferromagnet; indeed this is the rubric used by Zheng et al. to describe the broken symmetry state. We will show that

\[ O_{xz} \equiv -\langle \tau^x \sigma^z \rangle = 2 \cos(\theta) \cos(\chi_+) \sin(\chi_-) \] (14)
can also be non-zero. We discuss the meaning of this new order parameter, which is non-local in the growth-direction spatial coordinate, at greater length later. Finite expectation values for either operator require broken symmetry since the Hamiltonian is invariant under rotations about the \( \hat{x} \) axis in spin space.

Given the density matrix (12), the Hartree-Fock single particle Hamiltonian is readily evaluated:

\[
2h^{HF} = -F_S + (4H - F_S - \Delta_V) \cos(\theta) \cos(\chi_+) \cos(\chi_-) \tau^x - (F_S + \Delta_z) \sin(\theta) \sin(\chi_-) \sigma^x
- (F_D + \Delta_t) \cos(\theta) \cos(\chi_-) \tau^x + F_D \cos(\theta) \cos(\chi_+) \sin(\chi_-) \sigma^z \tau^z
- F_D \sin(\theta)(\cos(\chi_+) - \cos(\chi_-)) \sigma^x \tau^x - F_S \cos(\theta) \sin(\chi_+) \sin(\chi_-) \sigma^z \tau^z,
\]

(15)

where \( F_S = F_+ + F_- \) and \( F_D = F_+ - F_- \). When the variational parameters are at an extremum of the Hartree-Fock energy functional (11), \( z^1 \) and \( z^2 \) are eigenvectors of matrix (15) and if the extremum is the minimum these eigenvectors will correspond to the two lowest energy eigenvalues, \( E_1^{HF} \) and \( E_2^{HF} \). The Hartree-Fock approximation for the charged excitation energy gap, measured by quantum Hall transport experiments, is \( E_{\text{gap}} = E_3^{HF} - E_2^{HF} \).

**III. THE VANISHING ZEEMAN COUPLING LIMIT**

At \( \Delta_z = 0 \), we expect the ground state spin-magnetization directions in the two quantum wells to be in opposition. The collinear spin configuration corresponds to taking the parameter \( \theta \) to be zero in the ansatz (10). This reduces the number of free parameters in the variational wavefunction to two, namely, the polar angles for the up-spin and down-spin pseudospin orientations. With \( \theta = 0 \), the spinors (10) reduce to \( z^1 = (\cos(\chi^+/2), 0, \sin(\chi^+/2), 0) \) and \( z^2 = (0, \cos(\chi^+/2), 0, \sin(\chi^+/2)) \), and the energy per orbital state (11) reads

\[
\epsilon = -\Delta_V \cos(\chi_+) \cos(\chi_-) + 2H \cos^2(\chi_+) \cos^2(\chi_-)
- \Delta_t \sin(\chi_+) \cos(\chi_-) - F_+ - F_-(\cos^2(\chi_+) \cos^2(\chi_-) + \sin^2(\chi_+) \sin^2(\chi_-)).
\]

(16)

The first two terms on the right hand side of this equation represent the electrostatic energy which is minimized when the ‘flat band’ condition, \( \cos(\chi^+) \cos(\chi^-) = \Delta_V/4H \), is
satisfied. If these were the only terms in the energy expression, the charge transfer between layers would perfectly screen the external interlayer electric field parameterized here by $\Delta_V$. Since we have not included Zeeman coupling, any difference between the pseudospin angles of up-spins and down-spins, i.e. $\chi_- \neq 0$, represents a broken symmetry. The exchange energy is maximized in magnitude by localizing both up-spins and down-spins in one of the layers; at $\Delta_V = 0$ this objective can be made consistent with minimizing the electrostatic energy by choosing opposite layers for the two spins, i.e., $\chi_+ = \pi/2$. Minimizing with respect to $\chi_-$ then gives $\cos(\chi_-) = \Delta_t/2F_-$ for $\Delta_t < 2F_-$. Otherwise $\cos(\chi_-) = 1$ and there is no broken symmetry. The spin-up and spin-down pseudospins for $\Delta_z = 0$, $\Delta_V = 0$ are schematically illustrated in Fig. 2. Here and in all figures below, we plot results for layer separation $d = 1$ in units of the magnetic length $\ell$ yielding the following interaction strengths: $H = 0.5$, $F_+ = 0.9545$, and $F_- = 0.2988$, where energy is measured in units of $e^2/\varepsilon \ell$. The mean pseudospin orientation is in the $\hat{x}$ direction which corresponds to equal distribution of charge between the layers. For $\Delta_t > 2F_-$ the two pseudospins are parallel ($\chi_- = 0$) and the spin polarization in each well is zero, as also indicated in Fig. 4. For $\Delta_t < 2F_-$, $z^1$ and $z^2$ have pseudospin orientations tilted from the $\hat{x}$ direction upwards and downwards, respectively, $\chi_- \neq 0$ and, hence, the order parameter $O_{zz}$ is non-zero. As the tilt angle increases with decreasing $\Delta_t$, opposite spin polarizations builds up in the two layers reaching a maximum as $\Delta_t \to 0$. Then the two pseudospins are in opposition and aligned in the $\hat{z}$ direction. The non-zero value of $O_{zz}$ in the ordered region reflects the broken SU(2) symmetry (full rotational symmetry) of spin space. Note that the order parameter $O_{xz}$ is zero along the $\Delta_z = 0$, $\Delta_V = 0$ line since $\chi_+ = \pi/2$.

We now turn our attention to $\Delta_z = 0$, $\Delta_t = 0$ line in the phase diagram. In this case the energy is minimized when one of the spinors, e.g. $z^1$, is localized in the top well, i.e., $\chi_+ = 0$ so that $\chi_- = \chi_+ = \chi_\downarrow$. Minimizing the energy functional (16) with respect to $\chi_-$ gives

$$\cos^2(\chi_-) = \frac{\Delta_V - 2F_-}{4(H - F_-)}$$ (17)

for $2F_- < \Delta_V < 4H - 2F_-$, $\cos^2(\chi_-) = 0$ for $\Delta_V < 2F_-$ and $\cos^2(\chi_-) = 1$ for $\Delta_V > 4H -$.
For bias potential $\Delta_V < 2F_-$ the up and down spin orbitals are localized in opposite wells and no charge transfer is produced, as illustrated in Fig. 2. For $\Delta_V > 4H - 2F_-$, the up and down spin orbitals are localized in the top well and the charge transfer is complete. At intermediate values of $\Delta_V$, Eq. (17) applies. In the interval $2F_- < \Delta_V < 4H - 2F_-$, both $O_{zz} \neq 0$ and $O_{xz} \neq 0$. The non-zero value of $O_{zz}$ reflects broken $SU(2)$ spin-symmetry in the ground state while the non-zero value of $O_{xz}$ at $\Delta_t = 0$ reflects a $U(1)$ broken pseudospin symmetry. At $\Delta_t = 0$ the Hamiltonian is invariant under rotations about the $\hat{z}$ axis in pseudospin space; the number of particles in each layer is a good quantum number. This symmetry is broken since the orbital states with variational spinor $z^2$ possess interlayer phase coherence (see Fig. 2). As in the $\nu = 1$ case, spontaneous interlayer phase coherence will lead to a variety of interesting effects which we do not pursue further here. We return to this limit again in Section V.

We have been unable to obtain general analytic expression for the minimum energy state for both $\Delta_V$ and $\Delta_t$ non-zero. Instead we define $C = \cos(\chi_+ \cos(\chi_-)$, $S = \sin(\chi_+) \sin(\chi_-)$ and expand around the disordered state for which $S = 0$. We consider $S$ as an order parameter and perform a Ginzburg-Landau analysis.

$$\epsilon = \epsilon_0(C) + \epsilon_2(C)S^2 + \frac{1}{2} \epsilon_4(C)S^4 + \ldots$$

Comparing with Eq. (16), we obtain the following results for the coefficients of this Ginzburg Landau expansion:

$$\epsilon_0(C) = -\Delta_V C + 2HC^2 - \Delta_t \sqrt{(1 - C^2) - F_+ - F_- C^2}$$

$$\epsilon_2(C) = \frac{\Delta_t}{2(1 - C^2)^{3/2}} - F_-$$

$$\epsilon_4(C) = \frac{\Delta_t}{4} \frac{1 + 4C^2}{(1 - C^2)^{7/2}}.$$  

In the normal state, $S$ is zero and $C$ is determined my minimizing $\epsilon_0(C)$, implying the following relationship between $\Delta_V$ and $C$:

$$\Delta_V = (4H - 2F_-)C + \Delta_t \frac{C}{(1 - C^2)^{1/2}}.$$
The normal state becomes unstable when $\epsilon_2(C)$ becomes negative; on the phase boundary, $C$ equals

$$C^* = (1 - (\Delta t/2F_\text{t})^{2/3})^{1/2}.$$  \hspace{1cm} (21)

As the bias potential $\Delta V$ increases, $\epsilon_2(C)$ increases, crossing zero at the critical bias potential

$$\Delta V^* = [4H - 2F_\text{t} + \Delta t^{2/3}(2F_\text{t})^{1/3}]^{1/2}(1 - (\Delta t/2F_\text{t})^{2/3})^{1/2}.$$  \hspace{1cm} (22)

Eq. (22) describes the projection of the mean-field phase boundary onto the $\Delta z = 0$ plane, illustrated in Fig. 4.

Among the observables which are singular at this $T = 0$ phase transition, the most experimentally accessible\textsuperscript{12} is the internal differential capacitance of the bilayer system:

$$C_{\text{int}} \equiv \frac{d\sum_{i=1}^{2}\langle z_i^2|z_i^2\rangle}{d\Delta V} = 2 \frac{dC}{d\Delta V}.$$  \hspace{1cm} (23)

In the normal state, an equation for $C_{\text{int}}$ can be obtained by differentiating both sides of Eq. (20) with respect to $\Delta V$:

$$C_{\text{int}}^\text{norm} = \frac{2}{4H - 2F_\text{t} + \Delta t(1 - C^2)^{-3/2}}.$$  \hspace{1cm} (24)

Only the first term in the denominator appears in the electrostatic approximation for this capacitance. A similar expression, valid near the phase boundary, may be derived for the ordered state by minimizing the Ginzburg-Landau energy expression with respect to $S$. We find that

$$\epsilon \approx \epsilon_0(C) - \Lambda (C^* - C)^2.$$  \hspace{1cm} (25)

where

$$\Lambda = \frac{18F_\text{t}(1 - (\Delta t/2F_\text{t})^{2/3})}{5 - 4(\Delta t/2F_\text{t})^{2/3}}.$$  \hspace{1cm} (26)

The additional condensation contribution to the energy alters the inverse capacitance. Differentiating Eq. (23) first with respect to $C$ and then with respect to $\Delta V$ we find that as the phase boundary is approached from the ordered state side
\[ C_{\text{order}}^{\text{int}} = \frac{2}{4H - 2F_\gamma + \Delta_t (1 - C^2)^{-3/2} - \Lambda}. \]  

In mean-field-theory the capacitance has a jump discontinuity at the phase boundary. This observable is the analog, for this bias tuned quantum phase transition, of the specific heat at a temperature tuned phase transition. The differential capacitance is singular at all points in the two-dimensional boundary of the ordered state region in the three-dimensional phase space. This singular behavior should be accessible to experiment and may be interesting to study, especially since the nature of the broken symmetry in the ordered state crosses over from SU(2) to U(1) on going away from the \( \Delta_z = 0 \) plane. The mean-file critical exponent will be replaced by the specific heat exponent of a \textit{three-dimensional} classical system, presumably that of the 3D Heisenberg model at \( \Delta_z = 0 \) and that of the 3D XY model at \( \Delta_z \neq 0 \). In Section V we give an analytical expression for the capacitance jump for finite Zeeman coupling in the limit of vanishing interlayer tunneling.

### IV. THE VANISHING BIAS POTENTIAL LIMIT

The Hartree-Fock equations for the zero bias potential limit have been solved previously by Das Sarma \textit{et al.} in their pioneering work on \( \nu = 2 \) broken symmetry ground states. In this section we report analytic solutions of the Hartree-Fock equations which provide some additional insight. When \( \Delta V = 0 \), we expect the energy minimum to occur for equal charge density in the two layers, i.e., at \( \langle \tau^z \rangle = 2 \cos(\theta) \cos(\chi_+) \cos(\chi_-) = 0 \). It is clear that the energy is minimized by \( \chi_+ = \pi/2 \), since this choice does not frustrate any of the other energy terms. The Hartree energy then vanishes and the total energy expression \((11)\) simplifies to

\[ \epsilon = -\Delta_t \cos(\theta) \cos(\chi_-) - \Delta_z \sin(\theta) \sin(\chi_-) - F_+ - F_- \sin^2(\chi_-). \]  

Minimizing with respect to \( \theta \) we find that at the extremum

\[ \Delta_t \cos(\chi_-) \sin(\theta) = \Delta_z \cos(\theta) \sin(\chi_-). \]  

Minimizing with respect to \( \chi_- \) and using Eq. \((29)\) we find that
\( (\Delta_t - \Delta_z^2/\Delta_t) \cos(\theta) = 2F_- \cos(\chi_-). \)  \( (30) \)

These two equations can be combined to find explicit equations for \( \cos(\theta) \) and \( \sin(\chi_-) \):

\[
\cos^2(\theta) = \frac{\Delta_t^2[(\Delta_z^2 - \Delta_t^2)^2 - (2F_- \Delta_z)^2]}{(\Delta_t^2 - \Delta_z^2)^3}
\]

\[
\sin^2(\chi_-) = \frac{4\Delta_t^2 F_-^2 - (\Delta_z^2 - \Delta_t^2)^2}{4F_-^2(\Delta_t^2 - \Delta_z^2)}. \]  \( (31) \)

Broken symmetry states occur when both of these equations have solutions. For \( \Delta_t \) larger than \( 2F_- \) and \( \Delta_z \) smaller than

\[
\Delta_z^{\text{min}} = \sqrt{\Delta_t(\Delta_t - 2F_-)} \]  \( (32) \)

the expression for \( \sin^2(\chi_-) \) is negative. The minimum energy then occurs for \( \sin(\chi_-) = 0 \) and \( \theta = 0 \) and the variational spinors reduce to the up-spin and down-spin symmetric states, as illustrated in Fig. 3. These states are expected to have the lowest two energies for non-interacting electrons for \( \Delta_z < \Delta_t \). For \( \Delta_z \) larger than

\[
\Delta_z^{\text{max}} = \sqrt{F_-^2 + \Delta_t^2 - F_-} \]  \( (33) \)

the expression for \( \cos^2(\theta) \) is negative. The minimum energy then occurs for \( \theta = \pi/2, \chi^\uparrow = 0 \), and \( \chi^\downarrow = \pi \). In this case the variational spinors are both positive eigenvalue eigenstates of \( \sigma^x \). The wavefunction in this region is the non-interacting electron state for \( \Delta_z > \Delta_t \). In the ordered region, \( O_{zz} \) is non-zero and the spin U(1) symmetry is broken. The magnetic configuration (see Fig. 3) is that of a canted antiferromagnet as observed by Zheng et al. Each of the two spinors \( z^1 \) and \( z^2 \) has non-zero spin up and spin down components in the ordered region for \( \Delta_z > 0 \). However, the weights of spin down component of \( z^1 \) and spin up component of \( z^2 \) decrease with decreasing Zeeman coupling and the state continuously develops into the one illustrated in Fig. 2 in the limit of \( \Delta_z = 0 \). Note that \( \Delta_z^{\text{max}} \propto \Delta_t^2 \) at small \( \Delta_t \) so the broken symmetry state is not robust against Zeeman coupling when \( \Delta_t \) is small. Note also that when \( \Delta_t \) is large \( \Delta_z^{\text{min}} = \Delta_t - F_- - F_-^2/2\Delta_t + \ldots \) while \( \Delta_z^{\text{max}} = \Delta_t - F_- + F_-^2/2\Delta_t + \ldots \); the broken symmetry state interpolates between the fully
spin-polarized and fully pseudospin polarized states and occurs only over a narrow interval of Zeeman fields surrounding $\Delta_z = \Delta_t - F_-$. The field at which the crossover occurs is shifted from $\Delta_t$ because of the exchange energy is more favorable for the full spin-polarized state than for the fully pseudospin-polarized state.

V. THE VANISHING INTERLAYER TUNNELING LIMIT

For $\Delta_t = 0$, we expect $\chi_+ = 0$. Numerical calculations presented in the next section verify that this choice of $\chi_+$ gives the optimal variational energy for vanishing interlayer tunneling. Given $\chi_+ = 0$, it can be seen that the energy is minimized by $\theta = \chi_-$, leaving a single parameter to be determined by minimizing $-\Delta_V x - \Delta_z (1 - x) - F_+ - F_- ((1 - x)^2 + x^2) + 2H x^2$ with respect to $x = \cos^2(\theta) = \cos^2(\chi_-)$. We find that $\cos(\theta) = \cos(\chi_-) = 0$ for $\Delta_V < 2F_- + \Delta_z$, $\cos(\theta) = \cos(\chi_-) = 1$ for $\Delta_V > 4H - 2F_- + \Delta_z$, and that

$$\cos^2(\theta) = \frac{\Delta_V - \Delta_z - 2F_-}{4(H - F_-)}$$

in the intervening interval. The low bias state is fully spin polarized along the Zeeman field direction and has the charge equally distributed between the two layers, as indicated in Fig. 4. The high bias state is spin unpolarized and has all electrons in the top layer. For non-interacting electrons, the transition between these two states is first order and occurs at $\Delta_V = \Delta_z$. In the Hartree approximation, charge transfer occurs linearly with bias voltage, since the double layer system acts like a capacitor. In this approximation, the minority spin Landau level in the top layer and the majority spin Landau level in the bottom layer are degenerate throughout the interval of bias voltages where charge transfer takes place. We show below that an energy gap exists throughout this interval, and that the gap is supported by the development of a broken symmetry state. However, the broken symmetry is not the canted antiferromagnet order of the $\Delta_V = 0$ plane but rather something closely akin to the spontaneous interlayer phase coherence which occurs at $\nu = 1$ in double-layer systems. The projection of the phase diagram onto the $\Delta_t = 0$ plane is illustrated in Fig. 4.
In the ordered portion of the $\Delta_t = 0$ plane, $O_{zz} = 0$ and
\[
O_{xz} = \frac{1}{2} \left[ 1 - \left( \frac{\Delta_V - \Delta_z - 2H}{2(H - F_-)} \right)^2 \right]^{1/2}.
\] (35)

In the ordered state, the Landau level orbitals with spinors $z^1$ and $z^2$ spontaneously develop interlayer phase coherence and spin-polarization in the $\hat{z}$ direction. (The Zeeman field is oriented in the $\hat{x}$ direction.) One Landau level orbital has up and down spin projections whose pseudospin orientations are proportional to $[\sin(\chi_-), 0, \cos(\chi_-)]$ and $[\sin(\chi_-), 0, -\cos(\chi_-)]$ respectively; the spin projections of the other Landau level orbital have opposing pseudospin orientation and altered weights (see Fig. 4). Spontaneous interlayer phase coherence corresponds to non-zero pseudospin components in the $\hat{x}$ direction. The occurrence of this type of order is driven by interlayer interactions; the spatial correlations between electrons in different layers are improved when it is established.

We note that the solutions we have found for the $\Delta_t \to 0$ limit of the $\Delta_z = 0$ plane and the $\Delta_z \to 0$ limit of the $\Delta_t = 0$ plane differ. Evidently, the order of limits matters in this instance. It turns out that when both these terms are absent, energy minimization fixes a value only for the product $\cos(\theta) \cos(\chi^+)$ and not for these factors individually. In the $\Delta_t = 0$ plane, the state is invariant under a simultaneous rotation of all pseudospins around the $\hat{z}$ axis and under a rotation of spins around the $\hat{x}$ axis. These invariances are explicitly accounted for in limiting the number of parameters in our variational wavefunction. For $\Delta_z = 0$, the state is invariant under rotation about any spin-axis. When both external fields are set to zero, all symmetries are present, leading to additional soft modes and a set of variational wavefunctions which have the same energy. When these field parameters approach zero, $O_{zz} \gg O_{xz}$ if $\Delta_t \gg \Delta_z$, and $O_{xz} \gg O_{zz}$ if $\Delta_z \gg \Delta_t$. From the above solution for the variation parameters, it is possible to show that for $\Delta_t = 0$, $E_{\text{gap}} = F_D$, a constant throughout the ordered region of the phase diagram. The gap arises entirely because of the interlayer interactions which give rise to spontaneous interlayer phase coherence.

In the $\Delta_t = 0$ plane, $\langle \tau^z \rangle = 2 \cos^2(\theta)$. Using Eq. (34) we can derive an analytical expression for the internal capacitance in the ordered phase:
\begin{align}
C_{\text{int}} = \frac{d\langle \tau^z \rangle}{d\Delta_V} &= \frac{1}{2(H - F_-)}.
\end{align}

In the normal state, no charge transfer is produced by changing $\Delta_V$ in either low or high bias regions of the phase diagram. The internal capacitance is hence zero throughout the normal state region and has a jump, given by (36), along the $\Delta_V = \Delta_z + 2F_-$ and $\Delta_V = \Delta_z + 4H - 2F_-$ lines.

**VI. NUMERICAL RESULTS**

We supplement the analytical results discussed in the previous sections with results obtained from numerical solutions of the four-dimensional Hartree-Fock equation outlined in Section II. An overview on the order-disorder physics of bilayer quantum Hall systems at $\nu = 2$ is provided in Fig. 3, a three-dimensional phase diagram whose axes are the three one-body external fields, $\Delta_V$, $\Delta_t$, and $\Delta_z$. In this diagram, the region with spontaneous order is enclosed by two surfaces. In the limit of no interactions these collapse onto the single surface, $\Delta_V^2 + \Delta_t^2 = \Delta_z^2$, where the 2nd and 3rd Landau levels are degenerate. Broken symmetry states can occur at integer filling factors when interactions dominate the energetic splitting of nearby Landau levels. The width of the ordered region tends to expand when the single-particle field parameters are small and all four single-particle Landau levels are close to degeneracy. The intersection of this phase diagram with the $\Delta_V = 0$ plane recovers the results obtained originally by Das Sarma et al. and derived analytically in Section IV. In the ordered region within the $\Delta_V = 0$ plane, $\sin(\chi_+) = 1$, and $O_{zz} \neq 0$ while $O_{xz} = 0$. When $\Delta_t \neq 0$, interlayer phase coherence is not spontaneous so $O_{zz} \neq 0$ represents only the $\hat{x}$-axis spin-rotation ($U(1)$) broken symmetry. In the ordered portion of the $\Delta_t = 0$ plane and for $\Delta_z > 0$, $O_{xz} \neq 0$, but $O_{zz} = 0$. The order here involves spontaneous interlayer phase coherence and spin order; as at $\nu = 1$ we can expect interesting physics to occur for small $\Delta_t$ in tilted fields in this region of the phase diagram. The ordered state has $\hat{x}$-axis spin-rotation and $\hat{z}$-axis pseudospin-rotation broken symmetries ($U(1) \otimes U(1)$ broken symmetry) so we expect two distinct Goldstone modes to appear. At $\Delta_z = 0$ the broken symmetry changes
to SU(2)⊗U(1) in the ordered region of $\Delta_t = 0$ plane. The phase diagram intersections with the $\Delta_z = 0$ and $\Delta_t = 0$ planes are in agreement with the analytic results presented in Sections III and V respectively. In the ordered region of the $\Delta_z = 0$ plane, both $O_{zz}$ and $O_{xz}$ are non-zero and are indices of the same broken symmetry. In this case, the Hamiltonian is spin-rotationally invariant so we have a SU(2) broken symmetry.

In experimental samples, $\Delta_V$ and $\Delta_t$ can easily be made large but $\Delta_z$ is usually small. Since it is easy to tune the bias voltage $\Delta_V$ over large ranges experimentally, we examine below the dependence of physical observables on this parameter with the other two parameters held fixed.

In Figs. 6(a)-(c) we plot the bias dependence of the following quantities, all of which are experimentally accessible: the two order parameters $O_{zz}$ and $O_{xz}$, the Hartree-Fock energy gap $E_{\text{gap}}$, the difference between top layer and bottom layer charge densities $\langle \tau^z \rangle$, and the differential capacitance of the bilayer system, $d\langle \tau^z \rangle/d\Delta_V$. The bias dependence of the energy gap has been extensively studied experimentally; we expect the present Hartree-Fock theory to capture the main trends in these observations although we know that spin-texture excitations can be important and that disorder, not accounted for here, plays an important role in experimental samples. The most interesting quantity in our view is the internal differential capacitance, which shows singular behavior at the phase boundary. Results are plotted here for $(\Delta_t, \Delta_z) = (0, 0.02)$, $(0.02, 0.02)$, and $(0.1, 0.02)$. These choices are motivated by the fact that it is difficult to fabricate samples with values of the Zeeman coupling in interaction units $e^2/\ell$ which differ from 0.02 by more than a factor of two at $\nu = 2$, while it is relatively easy to vary $\Delta_t$ from a large value to one which is immeasurably small, by changing the Aluminum content in the barrier separating the quantum wells.

Numerical calculations for $\Delta_t = 0$ and $\Delta_z = 0.02$, presented in Fig. 6(a), confirm analytical results of Section V: the order parameter $O_{zz} = 0$ while $O_{xz}$ is non-zero for $\Delta_V > \Delta_z + 2F_- = 0.6176$ and $\Delta_V < \Delta_z + 4H - 2F_- = 1.4024$; the quasiparticle excitation gap $E_{\text{gap}}$ is constant in the ordered region; the charge is equally distributed between layers in the low $\Delta_V$ disordered region, all electrons are in the top layer in the high $\Delta_V$ disordered
region and the charge is transferred between wells linearly with $\Delta_V$ in the ordered region. In Fig. 6(b) we show numerical results for $\Delta_t = \Delta_z = 0.02$. Both low and high $\Delta_V$ disordered regions are still present for this choice of external fields. The ordered region with both $O_{xz}$ and $O_{zz}$ non-zero is slightly shifted to lower bias potentials and the step in $C_{int}$ is reduced at the right phase boundary. Fig. 6(c) corresponds to large enough tunneling energy ($\Delta_t = 0.1$) so that the $\Delta_V = 0$ point falls into the ordered region of the zero-bias plane of the phase diagram (see Fig. 3). Hence, only the high $\Delta_V$ phase boundary is present and is shifted further to lower $\Delta_V$, qualitatively consistent with our previous analysis of the $\Delta_z = 0$ phase plane.

VII. DISCUSSION

The variational ansatz (10) on which we have based the analytic portion our analysis carries only 3 parameters. One might wonder if more general wavefunctions with more parameters could unearth lower energy states. Let us consider how many parameters at most can be involved in the most general translationally invariant LLL state of bilayer electrons with spin, at a filling $\nu = 2$. Such a state will correspond, at each spatial point or LLL orbital, to some 2-fermion state where each fermion is described by a 4-spinor. More generally, consider $N$ fermions, each a d-spinor. There are $d!/(N!(d - N)!)$ combinations for placing $N$ fermions in $d$ states. Each such combination corresponds to an independent $N$-fermion state. These form a basis and a general state will be a linear combination of these with complex coefficients. Since each complex coefficient carries two real parameters the total number of parameters in the most general state will be $\hat{n} \equiv 2d!/(N!(d - N)!)$, where 2 parameters have been subtracted out which respectively account for normalisation and an overall phase. For our problem where $d=4$ and $N=2$, $\hat{n} = 10$; much larger than the number (three) of parameters in our trial spinors.

However not all these $\hat{n}$ independent states can be written as a single outer product of two 4-spinors, as is required for the Slater determinant form (1) of the Hartree-Fock trial
states. For example, the linear combination of 2-electron states

\[ |T \uparrow, T \downarrow\rangle + |B \uparrow, B \downarrow\rangle \]  \tag{37}

(where T,B refer to the two layers) cannot be written as an outer product of two 4-spinors of the form

\[
\begin{pmatrix}
  a_1 \\
  a_2 \\
  a_3 \\
  a_4 \\
\end{pmatrix} \otimes \begin{pmatrix}
  b_1 \\
  b_2 \\
  b_3 \\
  b_4 \\
\end{pmatrix}
\]  \tag{38}

for any values of the constants \(a_i\) and \(b_i\) which would need to satisfy the conflicting conditions \(a_1 b_2 = a_3 b_4 = 1\), while \(a_1 b_4 = a_3 b_2 = 0\). Whether such variational states which, for each spatial point, are sums of products of spinors (as for instance in (37)) can be candidates for the ground state of different phases is an interesting question. \textit{Apriori} they are not ruled out by any basic superselection rules. However, broken symmetry states with which we are familiar, for example superconducting and magnetic states, can be described using an order parameter field functional integral language in which a saddle point approximation to the order parameter functional integral corresponds to a Hartree-Fock approximation for the broken symmetry ground state. We assume that to be the case for double-layer quantum Hall states as well.\[\text{[7]}\]. In particular, we believe that the set of possible broken symmetries can be identified by looking only at single Slater determinant states. The positions of the phase boundaries will be altered by quantum and, at finite temperature, also thermal fluctuations in the order parameter field. We therefore are motivated to identify the number of parameters needed to characterize all possible Hartree-Fock states.

A product of \(N\) orthonormal \(d\)-plets will carry \(Nd\) complex parameters to start with. This will be reduced by \(N\) normalization constraints, \(N(N - 1)/2\) complex orthogonality constraints, and \(N^2\) symmetry constraints since unitary transformations among the \(N\) the single-particle states will not yield a distinct \(N\)-fermion state. This leaves behind \(n \equiv 2dN - N - 2N(N - 1)/2 - N^2 = 2N(d - N)\) real parameters. Notice that this is just the
dimensionality of the coset space $U(d)/\left(U(N) \times U(d-N)\right)$. (See Arovas et al. who also do this counting). For $d = 4$ and $N = 2$ the number of free parameters in Hartree-Fock states is $n = 8$; for $N = 1$, the case relevant to double-layer systems at $\nu = 1$, $n = 6$. For our problem, the number of free parameters is further reduced by symmetries of the interaction Hamiltonian which is invariant under arbitrary (3-parameter) rotations in spin-space and under (1 more parameter) rotations about the $\hat{z}$ axis in pseudospin space. Of course these symmetries are partially broken by one-body terms in the Hamiltonian. However, such breaking simply selects a known preferred direction for the total spin-polarization (along the Zeeman coupling direction) and for the projection of the total pseudospin polarization onto the $\hat{x} - \hat{y}$ plane (the $\hat{x}$ direction.) The end result is that the number of free parameters is 4 for $\nu = 2$. We have used 3 parameters based on the assumption that in the grounds state pseudospin projections in each layer will be in the $\hat{x} - \hat{z}$ plane.

VIII. CONCLUDING REMARK

The richness of order-disorder physics in double-layer quantum Hall systems at $\nu = 2$ suggests that this should be an inviting system for experiment. To date, only transport and optical studies, which probe the state rather indirectly have been completed. We believe that capacitance studies of double-layer systems as a function of an interlayer bias potential will prove more interesting. In particular we expect the differential capacitance, an experimentally accessible quantity which is the analog for these zero-temperature phase transitions of the specific heat in thermal phase transitions, to be a very useful probe in studying changes in singular dependence on bias potential with temperature, Zeeman coupling strength, and interlayer tunneling amplitude.

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FIGURES

FIG. 1. Schematic of the conduction band edge profile and one-particle energy levels in a biased double quantum well structure. The four one-particle states are products of orbitals split due to both interlayer tunneling and a bias potential, and spinors split by the Zeeman coupling. Interactions become important and broken symmetry states occur at $\nu = 2$ when the second and third levels, and sometimes also the first and fourth levels, are close to degeneracy.

FIG. 2. $\Delta_z = 0$ plane of the phase diagram. The ground state has SU(2) broken symmetry throughout the shaded region and U(1)$\otimes$SU(2) broken symmetry along the darkened portion of the $\Delta_z = 0 - \Delta_t = 0$ line. The dashed-line arrows labeled $(i, \sigma)$ illustrate schematically the pseudospin of spin-$\sigma$ projections of the spinor $z^i$ at a given point in the phase diagram. An upward arrow represents an orbital localized in the top layer, downward arrow an orbital in the bottom layer, while a finite tilt from the vertical direction indicates interlayer phase coherence. The solid-line arrows show schematically the orientation of the spin polarization in top (upper box) and bottom (lower box) quantum wells. The length of the arrow represents the magnitude of the spin polarization in that layer while its thickness represents the fraction of the charge density.

FIG. 3. $\Delta_{V} = 0$ plane of the phase diagram. The ground state has a U(1) broken spin symmetry in the shaded region. The dashed-lines and solid-lines summarize respectively the spin and pseudospin structures of the ground state as in Fig. 2.

FIG. 4. $\Delta_t = 0$ plane of the phase diagram. The dashed-lines and solid-lines summarize respectively the spin and pseudospin structures of the ground state as in Fig. 2. Spontaneous interlayer phase coherence is developed in the shaded region.

FIG. 5. 3D phase diagram for bilayer system at $\nu = 2$. Ordered states occur in the volume enclosed by the shaded and transparent surfaces.
FIG. 6. Order parameters, quasiparticle excitation gap, charge distribution, and internal capacitance as a function of bias potential for $\Delta_t = 0$ and $\Delta_z = 0.02$ (a), $\Delta_t = \Delta_z = 0.02$ (b), and $\Delta_t = 0.1$ and $\Delta_z = 0.02$ (c).
\((\Delta_t^2 + \Delta_V^2)^{1/2}\)
$\Delta z = 0$

Diagram showing the relationship between $\Delta t$ and $\Delta v$ with various points marked and arrows indicating movement between them.
\[ \Delta_t = 0 \]

A diagram showing a shaded region on a coordinate plane with axes labeled \( \Delta_y \) and \( \Delta_v \). The region is bounded by lines and arrows indicating transitions labeled \((1, \uparrow)\), \((1, \downarrow)\), \((2, \uparrow)\), and \((2, \downarrow)\). Points and arrows suggest pathways or states within this region.
\( \Delta_t = 0, \Delta_z = 0.02 \)
$\Delta_t = 0.02, \Delta_z = 0.02$

![Graph showing various parameters](image_url)
$\Delta_t = 0.1, \Delta_z = 0.02$

- $O_{zz}$
- $O_{xz}$
- $E_{\text{gap}}$
- $\langle \tau^z \rangle$
- $C_{\text{int}}$