Detailed description and analysis of the cross-coupling magnetic saturation on permanent magnet synchronous motor

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Abstract: Permanent-magnet synchronous motor (PMSM) has obtained more attention, acquired more applications, and received great reputation due to its high torque density and simple structure. While exploiting its high torque capability, PMSM’s control strategy faces great challenge at high overload scenario due to core-saturation effect. This study presents a detailed physical description and analysis on the effect of cross-coupling magnetic saturation in PMSM. Applying the conventional \( i_d = 0 \) control strategy on a 1.5 MW surface mounted permanent-magnet synchronous generator, this study discusses the cause and influence of cross-coupling effect, presents the unbalanced magnetic permeance between direct-\((d)\)- and quadrature-\((q)\)-axis magnetic circuit. Either \( d \)- or \( q \)-axis electromagnetic force (mmf) would generate both \( d \)- and \( q \)-axis flux, especially those induced by permanent magnets. Then a permeance-fixed finite-element analysis is applied to separate the influence between permanent magnets and armature current. Finally, this study proposes a revised maximum-torque-per-ampere trajectory for surface-PMSM at high overload situation.

1 Introduction

Permanent-magnet synchronous motors (PMSMs) deployed in the industrial field are originally controlled using the conventional linear direct-quadrature-axis \((dq)\)-axis) model, which is based on the assumption that the parameters of the motor are constant [1, 2]. When the effect of the magnetisation BH curve of the iron core is taken into consideration, the solution, most commonly, is the two-reaction method that divides the air-gap magnetomotive force (mmf) \( F_a \) into two components, one along the \( d \)-axis and the other along the \( q \)-axis, and calculates the core-saturation effect separately [3, 4].

In the recent several decades, the issue of cross-coupling effect in the synchronous motor, which described as the coupling between the windings in rectangular coordinate systems [5, 6], like the \( dp \)-axis windings came into being. A mutual incremental inductance between \( d \)- and \( q \)-axis would appear in the PMSM as a consequence of the asymmetry of the magnetic circuit between \( d \)- and \( q \)-axis saturation resulted by core-saturation [7, 8].

However, most papers that discuss the cross-coupling effect, consider the field flux induced by the permanent magnets would locate exactly at the \( d \)-axis [9–11], few papers have noticed the phenomenon that the field-winding flux produced by the permanent magnets would have \( q \)-axis components. Even if the phenomenon is noticed in [12], there is still no detail physical description and analysis on that problem.

This paper first presents a detailed physical description and analysis of the cross-coupling effect in the PMSM based on a surface mounted PSM (SPM). Then, using finite-element analysis (FEA) to distinguish the armature reaction flux linkages (induced by the armature current) and the excitation flux linkages (induced by the permanent magnets) under load condition so as to perform the time phasor diagram of electromotive force \((emf)\) and to discuss the cross-coupling effect. Finally, the paper gives an optimal current control method of the PMSM.

2 Conventional motor analysis method

2.1 PMSM model for theoretical analysis

In this section, a two-pole surface mounted permanent-magnet motor schematic view is set as a preparation for the physical description on cross-coupling magnetic saturation. The conventional \( i_d = 0 \) control strategy is applied to the motor since the cross-coupling effect will be more significant at high overload situation.

As shown in Fig. 1, the excitation mmf \( F_f \) which orients along the magnetic axis of the rotor, direct-axis (commonly referred to as the \( d \)-axis) and the armature reaction mmf \( F_a \) which lies along the quadrature-axis (\( q \)-axis), the air-gap mmf \( F_{ag} \), sum of \( F_f \) and \( F_a \), locates between the two axes.

2.2 Two-reaction method

When the conventional two-reaction method is applied to take the effect of saturation into account, the \( d \)-axis mmf \( F_d \) and the \( q \)-axis mmf \( F_q \) are analysed individually. With \( i_d = 0 \) control, \( F_d \) represents excitation mmf \( F_f \), and \( F_q \) is the armature reaction mmf \( F_a \). As the 2 mmfs are analysed individually, there will not be any phase difference between flux density and the mmf which produces it. Thus the excitation flux density \( B_f \) will orient along the \( d \)-axis and the armature reaction flux density \( B_a \) lies along the \( q \)-axis. As long as the position of the rotor is confirmed, the space vector diagram of \( emf \) and the time phasor diagram of \( emf \) are confirmed. Additionally, the excitation \( emf \) \( E_0 \) remains constant.

Take phase A as an example, neglecting the harmonics, and with rotor rotating in the counterclockwise direction, at the moment shown in Fig. 1, the magnetic axis of the rotor is perpendicular to the axis of phase A. So, the armature reaction mmf \( F_a \) locates on the axis of phase A at this moment, the current of phase A will reach its maximum value and coincide with the axis \( +j \). The excitation \( emf \) \( E_0 \) and current \( I \) are in same phase and the armature reaction \( emf \) \( E_0 \) is perpendicular to them, as shown in Fig. 2.

where \( E_0 \) is the \( emf \) induced by the air-gap flux, \( jX_a \) and \( IR \) is the voltage drop on the leakage reactance and the armature resistance, \( U \) is the terminal voltage of the phase.
According to Fig. 2, the power (which is the sum of the phases’ dot product of terminal voltage $U$ and phase current $I$) can also be expressed in (1).

$$P = m(E_0I - f'R_1).$$  

(1)

where, $P$ is the power; $R_1$ is the phase resistance; and $m$ is the number of the phase. This expression will be used in the following sections for current calculation.

### 3 Theoretical analysis on cross-coupling effect

When PMSM is working overload and the cross-coupling magnetic saturation effect is taken into consideration, the analysis of saturation should be based on the air-gap $mmf$ $F_0$. The magnetic permeability of the tooth in the pole width between the excitation $mmf$ $F_1$ and the armature reaction $mmf$ $F_a$, where the air-gap $mmf$ $F_0$ is located, would be smaller than that of the adjacent area. That is to say, the magnetic permeability of the area lies on both sides of the $d$- or $q$-axis will not be constant anymore. The excitation flux density $B_1$, therefore, will not orient along the corresponding $mmf$ $F_1$ which lies along the $d$-axis in space. Similar scenario happens between armature reaction flux density $B_a$ and the armature reaction $mmf$ $F_a$.

The excitation flux density $B_1$ will be ahead of the $mmf$ $F_1$ by a small angle as the area ahead of the $mmf$ $F_1$ in space has a greater magnetic permeability (see Fig. 3c). Moreover, the armature reaction flux density $B_a$ would be lag behind the $mmf$ $F_a$ by a small angle as the area behind $mmf$ $F_a$ has a greater magnetic permeability (see Fig. 3d). This phenomenon indicates that the flux linkage produced by the $mmf$ that lies along $d$- or $q$-axis would results in both $d$- or $q$-axis flux density components.

The excitation $emf$ $E_0$ would be ahead of the axis $+j$ by the same angle as that between the field-winding flux $B_t$ and $mmf$ $F_t$ and the armature reaction $emf$ $E_a$ would lag the axis $+j$ by an electric angle of $90 + \theta$, where $\theta$ equals the space angle between armature reaction flux density $B_a$ and the armature reaction $mmf$ $F_a$, as shown in Fig. 4.

It should be noted that, the $emf$ $E_a$ calculated by the 2D FEA model includes the effect of the slot leakage reactance and the differential leakage reactance of the total leakage reactance $X_{la}$ except end leakage reactance $X_e$.

According to Fig. 4, the load current should be calculated by the following two equations:

$$P = m(E_0j\cos(\gamma) - f'R_1).$$  

(2)

$$P = m(E_0j\cos(\delta) + E_aj\cos(\theta) - f'R_1).$$  

(3)

Obviously, the calculation equation of the load current has a big difference with or without consideration of the cross-coupling magnetic saturation.

### 4 FEA calculation and experiment

The motor that the paper analyses and experiments on is a six-phase 1.5 MW exterior-rotor surface mounted permanent-magnet synchronous generator designed for the wind-power application that adopts the conventional $i_d=0$ control strategy with a full power inverter.

#### 4.1 FEA validation

FEA calculation model of the motor is applied here to analyse the effect of the cross-coupling magnetic saturation in the motor by evaluating the armature current under certain torque demands.

The no-load $emf$ $E_0$ that would be used in the conventional two-reaction method mentioned above is calculated first, additionally test and verify the validity of the FEA model. The FEA calculation method of $E_0$ mainly includes two steps. First, rotates the rotor in the counter-clockwise direction for an electric angle of 360° (two-pole width), during which, calculate the phase flux linkage of the motor at certain equidistant points. Then, derive the fundamental component of the no-load $emf$ $E_0$ from the phase flux linkage calculated by differentiation and Fourier decomposition.

The calculation and experiment result of the no-load $emf$ $E_0$ is plotted in Fig. 5 (at the temperature of 20°C), the solid line represents the calculation result and the dotted line represents the experiment result. The flat-topped wave is the calculated result and the sinusoidal wave is its fundamental component respectively.
the calculation of the three emf under load is to make a distinction between the armature reaction flux linkages and the excitation flux linkages under saturation condition since the saturation is caused by both the armature current and the permanent magnets.

The FEA calculation method used to solve this problem is to fix the magnetic permeance at the operation point. The key point is to keep the saturation condition (see the comparison in Fig. 6) unchanged during each calculation step. The detailed calculation steps of the method are listed below.

i. Apply all the excitations including both the permanent magnets and the rated armature current to the FEA model, calculate the magnet permeability of all the elements of the FEA model. Additionally, calculate the air-gap flux linkage \( \Psi_b \).

ii. Fix the magnet permeability of all the elements calculated at step (i), and apply only the permanent magnets as excitation to the FEA model, calculate the flux linkage \( \Psi_i^n \) that is induced by the field flux.

iii. Fix the magnet permeability of all the elements calculated in step (i), and apply only the rated current to the FEA model, calculate the flux linkage \( \Psi_i^n \) that is induced by the magnetising reactance of the armature winding.

iv. With the result from step (ii) and (iii), put the calculated \( E_0^n, \theta_i^n, E_a^n \), and \( \theta_i^n \) into (3) and calculate a new torque current \( I_{n+1} \).

v. Repeat (i) till torque current \( |I_{n+1} - I_n| \) less than a certain error.

The no-load emf of the motor

\[ E_0 = 357 \, \text{V} \]

The theoretical calculation of the rated current based on Fig. 4 could be achieved by (2) and (3) now according to Fig. 7.

A clear resemblance arises between Figs. 4 and 7. Compared to the calculation based on the two-reaction method, the amplitude of \( emf \) \( E_0 \) decreases from 357 to 348.9 V due to the cross-coupling effect in the motor and a degree of 3.31° exists between \( emf \) \( E_0 \) and current \( I_n \).

The theoretical calculation of the rated current based on Fig. 4 could be achieved by (2) or (3) now according to Fig. 7.

The rated current calculated using the three methods and measured from the experiment are listed in Table 1.

From Table 1, the rated current calculated with conventional two-reaction method has a comparative big error nearly 6.84% compared to the experiment result. However, the result calculated by the method accounting the cross-coupling effect has a better coincidence with the experiment result. A smaller difference of 1.45% is assumed mainly due to the result from neglecting of the iron loss and the mechanical loss.

Based on the physical description and FEA calculation of the cross-coupling effect, the \( i_d = 0 \) control strategy adopted by the generator may not be the optional optimum method to minimise the armature current for SPMSM, which is to say, the rated current may be smaller under the same rated conditions (rated power, rotation speed etc) if it involves in demagnetisation current. In order to prove the assumption, the phase angle of the load current that differs from the original angle which applied in the \( i_d = 0 \) control strategy is calculated. The results are, with the current delay angle increases the rated current will first decrease and then increase after a certain angle.

It could be concluded from Fig. 8 that a demagnetisation component of the armature current would help reduce the saturation level of the motor, and then reduce the amplitude of the rated current under the rated conditions. This means while generating same power, if the armature current consists of a certain demagnetisation component, the armature current would be smaller. The minimum armature current compared to the value used by \( i_d = 0 \) control strategy has a 0.65% advantage. Although the reduction of the armature current is small, the guidance of a certain compensation angle applied for the case without the position transducer may increase the system’s stability at high overload.
The cross-coupling effect would be more obvious if motor's core saturation goes deeper. The load current under different calculation methods and the phasor diagram of emf with the power load up to 2625 kW are calculated and listed in Table 2 and Fig. 9.

The difference of the rated armature current between the results calculated by the conventional two-reaction method and with consideration of the cross-coupling effect is 5.43%, as listed in Table 1, and the value will be up to 20.55% when the power load is up to 2625 kW as a result of more obvious saturation level.

Comparing Figs. 4 and 9, the angle between the armature reaction emf $E_a$ and the current $I$ increases further from 90°, which indicates that the $d$-axis component of the armature reaction flux produced by the corresponding mmf $F_a$ that lies along the $q$-axis becomes bigger. The angle between the excitation emf $E_0$ and the current $I$ increases, which indicates that the $q$-axis component of the field flux produced by the excitation mmf $F_a$ along the $d$-axis increases when the saturation problem becomes more severe.

Same as Fig. 8, the armature current under 2625 kW is calculated and presented with different delay angles in Fig. 10.

The reduction of the current rise to 2.62% compared to the value of 0.65% in Fig. 7, which indicates that an optimum current control guide becomes more obvious when the degree of saturation in the motor goes deeper to maintain stability at overload situation and to achieve higher overload output ability.

Generally, the generator will not be working at such overload condition, but the principle discussed here would be helpful to the drive motor used in some cases that usually working at overload situation for a short time. When the drive motor is overloading and operating in deep saturation, to generate the same torque, a certain demagnetisation component of the armature current would help to reduce the amplitude of the current effectively. The best way to find the maximum-torque-per-ampere trajectory on SPMSM and the best current delay angle is to generate the map of torque contour line and current contour line and seek the points of tangency (see Fig. 11).

The difference between maximum torque per ampere (MTPA) trajectory in Fig. 11 and the traditional MTPA trajectory (for SPMSM MTPA trajectory overlaps with its $i_d=0$ trajectory) is that the trajectory brought forward by this paper has considered the nonlinear cross-saturation effect. Another significant meaning for this MTPA trajectory is this trajectory is defined without the inductance parameters, which will be more accurate than those abducted with either constant or nonlinear motor parameters as inductances ($L_{d}$, $L_{q}$) and no-load flux ($\psi_0$).

5 Conclusion

This paper presents a detailed physical explanation of the effect of cross-coupling magnetic saturation in the PMSM. The armature reaction flux linkages and the excitation flux linkages at saturation situation are distinguished with help of fixing magnetic permeance FEA at the certain working point. The analysis shows that either

| Calculation Method | Result, A |
|--------------------|-----------|
| the two-reaction method (based on Fig. 2 and calculated by (1)) | 768.6 |
| considering cross-coupling effect (based on Fig. 4 and calculated by (2)) | 812.9 |
| considering cross-coupling effect (based on the FEA model) | 813.0 |
| experiment result | 825.0 |

| Calculation Method | Result, A |
|--------------------|-----------|
| the two-reaction method (based on Fig. 2 and calculated by (1)) | 1326.1 |
| considering cross-coupling effect (based on Fig. 4 and calculated by (2)) | 1669.2 |
| considering cross-coupling effect (based on the FEA model) | 1671.8 |

Fig. 7 Phasor diagram calculated from the FEA model of the motor

Table 1 Rated current comparison between calculation and experiment

| Method | Current, A |
|--------|------------|
| the two-reaction method (based on Fig. 2 and calculated by (1)) | 768.6 |
| considering cross-coupling effect (based on Fig. 4 and calculated by (2)) | 812.9 |
| considering cross-coupling effect (based on the FEA model) | 813.0 |
| experiment result | 825.0 |

Fig. 8 Rated current under different delayed angles

Fig. 9 Phasor diagram calculated from the FEA model with a power load of 2625 kW

Fig. 10 Current under different delayed angles

Fig. 11 PMSM's maximum torque per ampere trajectory

Table 2 Calculation of load current under 2625 kW

| Calculation Method | Result, A |
|--------------------|-----------|
| the two-reaction method (based on Fig. 2 and calculated by (1)) | 1326.1 |
| considering cross-coupling effect (based on Fig. 4 and calculated by (2)) | 1669.2 |
| considering cross-coupling effect (based on the FEA model) | 1671.8 |
the $d$- or $q$-axis mmf would produce both the $d$- and $q$-axis flux, especially those generated by permanent magnet. This ‘leaked’ mmf is one of the causes that the current acquired to achieve a certain torque might be insufficient. The effect will be more obvious if motor’s core saturation goes deeper. Furthermore, the paper also presents a certain demagnetisation current appending method to reduce the amplitude of the current in the generator compared to the conventional $i_d = 0$ control strategy.

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