Rotational alignment near $N=Z$ and proton-neutron correlations

S. Frauendorf$^{(1)}$ and J.A. Sheikh$^{(1,2,3)}$

$^{(1)}$IKHP, Research Center Rossendorf, PF 510119, D-01314 Dresden, Germany
$^{(2)}$Department of Physics, University of Surrey, Guildford, Surrey GU2 5XH, UK
$^{(3)}$Tata Institute of Fundamental Research, Bombay, 400 005, India

Abstract

The effects of the residual proton-neutron interactions on bandcrossing features are studied by means of shell model calculations for nucleons in a high-$j$ intruder orbital. The presence of an odd-nucleon shifts the frequency of the alignment of two nucleons of the other kind along the axis of rotation. It is shown that the anomalous delayed crossing observed in nuclei with aligning neutrons and protons occupying the same intruder subshell can be partly attributed to these residual interactions.
I. INTRODUCTION

The cranked shell model (CSM) approach \[1,2\] accounts well for the overall systematics of bandcrossing phenomena in rapidly rotating medium and heavy mass nuclei. In this model the protons and neutrons move independently in a fixed rotating potential. As a consequence of this assumption, the alignment of a pair of neutrons with the rotational axis occurs at the same rotational frequency for different proton configurations. However, recently substantial deviations from this simple picture have been reported using the CSM approach in the high-spin study of nuclei near \(N = Z\) \[3–10\] and for nuclei with aligning neutrons and protons occupying the same intruder subshell \[11–16\]. These deviations, concerning the crossing between the \(g\)-band and the \(s\)-band in the even-neutron systems referred to as the (AB)-crossing, include:

- the crossing in the odd-proton nuclei is considerably delayed as compared to the neighboring even-even nuclei,

- in the case of \(N = Z\) even-even nuclei, the (AB)-crossing is substantially delayed as compared to the neighboring \(N \neq Z\).

Fig. 1 summarizes the experimental evidence.

The observed delay of the neutron alignment has been attributed to an increase of the deformation induced by the odd high-\(j\) proton. This mechanism has first been pointed out in ref. \[2\] as the possible origin of the delayed \(i_{13/2}\) neutron backbend if an \(h_{9/2}\) proton is present. It has been substantiated by systematic calculations of the equilibrium deformations for bands with an odd-proton in the \(h_{11/2}, h_{9/2}\) and \(i_{13/2}\) orbitals (cf. e. g. \[15,12,14,16\]). Though being in the right order of magnitude, the calculations tend to underestimate the experimental shifts. The p-n residual interaction has been invoked to be responsible for the remaining discrepancy \[15,12\]. In a study of a QQ-type p-n interaction acting in the particle-hole channel ref. \[16\] finds only very small shifts. In the present paper we further investigate the role of the residual interaction. At variance with ref. \[16\], we use a short
range force and calculate the exact solutions for the model system of interacting protons and neutrons in a \( j \) shell, exposed to a deformed potential.

The strong short-range proton-neutron (p-n) attraction probes the relative orientation of the high-\( j \) orbitals, because of their strongly anisotropic, torus like density distributions. It favors the anti parallel (\( J = 0,1 \)) and the parallel (\( J = 2j \)) coupling of a p-n pair, the energies being comparable. Since there are many low-\( J \) pairs available, they tend to form a correlated state (p-n pairing). Processes in which the high-\( j \) particles change their relative orientation should be good means to study these aspects of the p-n interaction. In this manuscript, we investigate a special kind of such reorientation, the alignment of a pair of neutrons with the axis of rotation. By means of a shell model calculation it will be demonstrated that the rotational frequency at which the neutron alignment occurs changes when additional protons are present and that the frequency shift are sensitive to the p-n interaction and the correlations it generates. It will also be shown that the p-n interaction causes a delay of the first double-bandcrossing in \( N = Z \) model systems studied as compared to cases with \( N \neq Z \) which may be related to the delayed bandcrossing observed in \( N = Z \) nuclei [6].

II. THE SHELL MODEL

In our model the protons and neutrons in a \( j \)-shell interact via a delta force and move in the deformed rotating potential generated by the nucleons outside the \( j \)-shell. The model hamiltonian is the same as used in refs. [7,8], where one may look for the details, and is given by

\[
H' = -\sum_i \left[ 4\left(\frac{4\pi}{3} \right)^{1/2} \kappa Y_{20}(\hat{r}_i) + \omega j_i^{(x)} \right] - G \sum_{i<j} \delta(|\hat{r}_i - \hat{r}_j|). \tag{1}
\]

The first term is the deformed quadrupole field where \( \kappa \) is related to the deformation parameter \( \beta \) by \( \kappa = 51.5A^{1/3}\beta \) in units of the coupling strength \( G \) of the delta force. The second, cranking term describes the uniform rotation about the x-axis with the frequency \( \omega \) and the
third term is the interaction. The eigenfunctions of the hamiltonian are found by numerical diagonalization. They can be classified by the isospin and the signature (symmetry with respect to $R_x(\pi)$) which are conserved by the model hamiltonian.

We would like to point that the present model predictions should only be considered as qualitative indications of the physics expected from more realistic studies. The major restriction of our model is that we consider only few particles occupying the intruder subshell with rest of the nucleons giving rise to a deformed potential. The assumption that the nucleons outside the intruder shell do not take part in the correlations generated by the short range residual interaction seems to be the most serious restriction of the model. Since we choose a realistic value of the deformation for a given nucleus and estimate the number of particles occupying the high-$j$ orbital by looking at the realistic Nilsson diagrams, the estimates of the frequency where the rotational alignment occurs should be qualitatively right. As a consequence of the restriction to a single $j$-shell, the total angular momentum is underestimated. This is of little importance, because we study the frequency of the alignment processes, which will not be changed by the missing angular momentum from the nucleons outside the $j$-shell.

III. EXAMPLES FOR DELAYED CROSSINGS

Fig. 2 shows the expectation value of the $x$ component of the angular momentum calculated by assuming a deformation $\beta = 0.25$ of a well deformed nucleus. The situation with few protons and neutrons occupying the $h_{11/2}$ shell is studied. The alignment of a pair of neutrons with the axis of rotation ($x$), caused by the Coriolis force, shows up as the step like rise of the angular momentum from small values to about 10. It corresponds to the crossing of g-band with the neutron s-band, where the inflection point defines the crossing frequency. It is seen that the presence of the odd proton delays the alignment of the neutron pair. As demonstrated by fig. 3 both in the $h_{11/2}$ and in the $i_{13/2}$ shells a similar delay of the crossing frequency is seen for the combination ($Z = 1, N = 2, 4, 6$). The estimated number
of protons and neutrons in the high-$j$ shell, denoted by $Z_j$ and $N_j$, must be compared with $Z$ and $N$ of the calculations, respectively.

Fig. 3 demonstrates that for near symmetric filling of the $g_{9/2}$ shell ($Z = 2, N = 2$) and ($N = 3, Z = 2$) the crossing frequency in the odd-A nuclei lies below the first crossing in the even-even nuclei ($Z = N = 2$), which corresponds to the simultaneous alignment of protons and neutrons [17]. This inversion of the order of the bandcrossings seems to be seen in the Br- and Kr-isotopes, as illustrated by fig. 1. The figure also shows that the inverse ordering for near symmetric filling as compared to asymmetric filling can be attributed to a delay of the crossing in the even even $N = Z$ nuclei as compared to the ones with $N \neq Z$ [6], which is stronger than the delay in the odd-$Z$ system.

IV. SYSTEMATICS OF THE CROSSING FREQUENCY

The features discussed in the previous section reflect a systematic tendency that is illustrated in fig. 4 and 5, showing $J_x(\omega)$ and $\mathcal{J}^{(2)}(\omega) = dJ_x/d\omega$ for the $f_{7/2}$ shell. (We have chosen $f_{7/2}$ shell since the dimensions of the matrices to be diagonalized are lower and a systematic study is possible.) The alignments show up as peaks in $\mathcal{J}^{(2)}$. We are interested in the alignment of a pair of $f_{7/2}$ neutrons, which is displayed in lower panels of the figs. for the pure neutron $N = 2$ and $N = 4$ cases. If there were no p-n correlations, the frequency at which these alignments appear would not change with the number of protons. However, this is not seen. Instead, the critical frequency for the neutron alignment first grows with the number of added protons. The shift reaches its maximum when $Z = N$ and then decreases when $Z$ becomes larger than $N$. Of course, the role of $N$ and $Z$ could be exchanged.

Qualitatively, one may ascribe the progressive delay of the crossing to p-n correlations that disfavor the simultaneous alignment of protons and neutrons (e.g. the presence of p-n pairs with a low $J$), generated by the attractive p-n interaction. The more protons are added the stronger the correlations become, until $Z = N$. As discussed in [18], the character of the p-n interaction between the aligned protons and neutrons changes from attractive to
repulsive when \( Z > N \), because the character of the aligned configuration changes from particle-like to hole-like. This shifts the p-n correlations to the highly excited states, whereas the yrast states are not much modified by the p-n interaction.

The shift of the crossing frequency is a consequence of the \( T = 1 \) part of the \( \delta \)-interaction. The dashed dotted line in fig. 4 shows a calculation where the \( T = 0 \) part of the \( \delta \)-interaction has been removed. The function \( J_x(\omega) \) almost coincides with the one calculated with the full force. The question if the p-n correlations that cause the shifts belong to the particle-hole channel or the particle-particle channel (or to both) cannot be decided on the basis of the shell model calculations. For this, comparisons with mean field calculations are necessary, which will be addressed in a forthcoming paper. We state here only the existence of strong p-n correlations of the \( T = 1 \) type between nucleons in the same \( j \)-shell, which cause a delay of the first bandcrossing.

A delay of the onset of the bandcrossing for \( N = Z = 36 \) has recently been observed in the Kr isotopes \(^{252}\text{Kr}\) (cf. fig. 1). It is suggested in the paper that the delay may be a consequence of \( T = 0 \) pair correlations. As already mentioned, the dashed dotted line in fig. 4 shows a calculation where we have switched off the \( T = 0 \) part of the interaction. It is seen that the results are almost the same as for the full force, i.e. the \( T = 0 \) correlations do not influence the alignment significantly in the present model analysis and it is interesting to see whether this conclusion will hold in a realistic study.

Comparing in figs. 4 and 5 the alignment of the odd-proton in the \( Z = 3 \) system with the one in the \( Z = 3, N = 2, 4 \) one notices also the the alignment of an odd-proton is hindered by the presence of neutrons in the same \( j \)-shell.

In the previous work \(^{17}\) it was mentioned that strong p-n correlations can alter the bandcrossing substantially and the double-bandcrossing can occur below the individual proton and neutron crossings. However, most of the analysis was performed in the spherical approximation and the dominant component of the p-n interaction which changes the band-crossing was not fully explored. It is shown in the present work using the proper deformed basis that it is \( T = 1 \) component of the n-p interaction which is responsible for the particle
number dependence of the bandcrossing. This is in accordance with a recent mean field study \[19\] of the $j$-shell model system, which attributes the delay of the crossing frequency in the even-even $N = Z$ system to the $T = 1$ monopole pair field.

V. CONCLUSIONS

It is clear from the present simple model study that the alignment of high-$j$ nucleons with the rotational axis is sensitive to the p-n interaction. The alignment of one kind of particles is delayed when the other kind of particles is present in the same $j$-shell. The effect reaches its maximum for $N = Z$ and is a consequence of $T = 1$ correlations between protons and neutrons. For $Z = N$ the crossing frequency in the even system is higher than for asymmetric filling. The experimentally observed delay of the first backbend in $Z = N$ even-even $^{72}$Kr can be attributed to these correlations. The experimentally observed delay of the first backbend in even-neutron nuclei by the presence of an odd-proton in the same $j$-shell could be a combination of these correlations and the change of deformation induced by the odd-nucleon. By measuring the deformation and/or calculating it one may try to disentangle the two effects and probe the p-n correlations. Finally, we would like to mention that while the results of the present model analysis seem to point into the right direction they should not be compared with the experimental data in a quantitative form. It would be quite interesting to study the effects in more realistic shell model configuration spaces.
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FIG. 1. Experimental crossing frequencies, $\hbar \omega_c$, for the first alignment of a pair of neutrons in the $g_{9/2}$, $h_{11/2}$ and $i_{13/2}$ shells. The odd-proton occupies the same $j$ shell as the neutrons. The number $N_j$ is an estimate of the number of neutrons in the $j$ shell, obtained by counting Nilsson levels at the deformation $\beta = 0.25$. For the $h_{11/2}$ and $i_{13/2}$ shells the proton occupancy is zero or one, for the $g_{9/2}$ shell it is close to the neutron occupancy. The arrows indicate that only lower limits are known. For $N = 98$ the very gradual rise of the function $I(\omega)$ does not permit a reliable determination of $h\omega_c$, however the shift of the two curves gives a reasonable estimate of the difference between the crossing frequencies. In the case of $Z = 50$ and 51 the deformed excited excited bands are shown. The experimental data are from ref.s 3-14. The even Sr - isotopes and the $g_{9/2}$ bands of the Rb - isotopes do not show a $g_{9/2}$ alignment what might be a consequence of the $Z = N = 38$ gap in the single particle spectrum.
FIG. 2. Angular momentum $J_x$ as a function of the rotational frequency for particles in a $h_{11/2}$ shell with deformation $\beta=0.25$. The arrows indicate the crossing frequencies. For odd-$Z$ a constant of $11/2$ is subtracted from $J_x$. 
FIG. 3. Angular momentum $J_x$ as a function of the rotational frequency for particles in a $h_{9/2}$ shell with deformation $\beta=0.25$. The case $Z = 2, N = 3$ is identical. The arrows indicate the crossing frequencies. For odd-$Z$ a constant of $11/2$ is subtracted from $J_x$. 
FIG. 4. Angular momentum $J_x$ as a function of the rotational frequency for particles in a $f_{7/2}$ shell with deformation $\beta=0.25$.

Upper panel: The particle numbers are quoted in the form $Z+N$. Full lines: even $A$, dotted: odd-$A$, dashed dotted: only $T=1$ part of the $\delta$ force.

Lower panel: Only one kind of particles, the number of which is quoted.
FIG. 5. Dynamic moment of inertia $\mathcal{J}^{(2)}$ as a function of the rotational frequency for particles in a $f_{7/2}$ shell with deformation $\beta=0.25$.

Upper panel: The particle numbers are quoted in the form $Z + N$. Full lines: even $A$, dotted: odd-$A$.

Lower panel: Only one kind of particles, the number of which is quoted.