Prediction of Strength and Modulus of Jointed Rocks Using P-wave Velocity

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Abstract Strength and deformation characteristics of jointed rocks are important parameters for the design of many civil and mining engineering structures. These parameters are difficult to obtain from direct testing of jointed rocks; hence it is usual practice to derive them through indirect approaches. In the present study, an attempt is made to obtain rough estimates of strength and modulus of jointed rocks through P wave velocity. Jointed specimens of a model rock were prepared for laboratory testing. Joints at different orientations and comprising of different joint roughness coefficients were used (JRC = 2–4, 12–14 and 14–16). Ultrasonic P-wave velocity and UCS tests were conducted on the specimens. The effect of joint orientation and joint wall roughness on P-wave velocity was studied. An index, Joint Factor was used to quantify the effect of joint attributes on strength and modulus of jointed rock. It was observed that P-wave velocity is closely linked with Joint Factor. This study suggests a correlation to obtain Joint Factor \( J_f \) from P-wave velocity. P-wave velocity may be measured in the field and \( J_f \) may be computed through the suggested correlation. The computed \( J_f \) may then be used to get the strength and modulus values of jointed rocks. Charts are also suggested to roughly assess the shear strength parameters, \( c_{mass} \) and \( \phi_{mass} \) of the jointed rock.

Keywords Strength · Modulus · P-wave velocity · Jointed rocks · Non-linear strength · Mohr–Coulomb shear strength parameters

1 Introduction

Geologists and engineers often encounter rocks while working on various projects like underground excavations, mines, tunnels, and caverns. These rocks invariably consist of discontinuities and the overall engineering behaviour of rocks is governed by the discontinuities (Ghosh and Daemen 1993; Bhasin and Kaynia 2004). Joint is the most common discontinuity encountered in the field and its orientation, roughness and frequency play major role in governing the engineering response of the jointed rock. To obtain the strength and deformational response of jointed rock, ideally, it is desirable that tests be performed on undisturbed rock specimens extracted from the field. However, extracting undisturbed specimens of jointed rocks is expensive, time-consuming and sometimes not feasible. Hence indirect approaches are preferred to derive the engineering characteristics of jointed rocks rather than testing undisturbed specimens (Singh and Rao 2005).
Joint Factor concept (Ramamurthy 1993; Ramamurthy and Arora 1994; Singh et al. 2002) is one such approach that can be used to roughly assess the strength and modulus of jointed rocks based on the characteristics of joints and intact rock. The concept was developed based on extensive studies conducted on jointed rock specimens in the laboratory (Ramamurthy 1993; Ramamurthy and Arora 1994; Singh 1997; Singh et al. 2002). A weakness index termed as Joint Factor \( J_f \) was defined which depends on joint attributes. The Joint Factor quantifies weakness brought by jointing into an intact rock. The \( J_f \) was found to correlate the strength and modulus of jointed rock with those of intact rock.

P-wave velocity is a non-destructive technique that is used to characterise the jointed rock mass encountered in the field. Literature suggests several correlations for assessing the strength and modulus of intact or jointed rocks from observed P-wave velocity (Entwisle et al. 2005; Diamantis et al. 2011; Sarkar et al. 2012; Karakus and Akatay 2013; Kurtulus et al. 2016; Shen et al. 2017; Jamshidi et al. 2018). Studies are also available which have focused on the influence of rock joints and their attributes on the seismic wave propagation (Pyrak-Nolte et al. 1990; Boadu and Long 1996; Wu et al. 1998; Zhao and Cai 2001; Zhao et al. 2006a, b; Li et al. 2010; Zhu et al. 2011a, b; Zhu and Zhao 2013; Li 2013; Fan and Sun 2015). Some statistical and novel neuro-fuzzy models are also presented in the past to predict the strength and deformation of rocks from P-wave velocity (Khandelwal 2013; Sharma et al. 2017 and Singh et al. 2022). These studies do corroborate the fact that \( V_p \) gets affected by the attributes of joints in rocks. Some investigators have analyzed the effect of different joint characteristics on \( V_p \) through laboratory studies. Some of these studies are by Kahraman 2001, 2002; Altundağ and Güney 2005; Kurtulus et al. 2012; Li et al. 2010; Wu et al. 1998, and Wu and Zhao 2015; Huang et al. 2014; Mohd-Nordin et al. 2014; Pappalardo 2015; Zuo et al. 2020.

On appraising the work performed in past, it is observed that not many studies are available, which have considered the joint attributes altogether to study their effect on P-wave velocity. In the present study, the combined influence of joint roughness and joint orientation on the variation of P-wave velocity is studied. Following it, a correlation is suggested to assess the Joint Factor corresponding to a given P-wave velocity. The P-wave velocity of the rock mass in the field can be obtained as per Vilhelm et al. 2016; Jiang et al. 2014; Kianpour et al. 2020; Guoliang and Jun 2022. Using this P-wave velocity, the Joint Factor can be obtained by using the suggested correlation. The computed \( J_f \) may then be used to assess the strength and modulus values of the jointed rock mass. The suggested correlations were validated by using an independent data source. Easy to use charts are also suggested to assess the shears strength parameters of the jointed rocks in the field.

## 2 Experimental Programme

A laboratory study was performed by conducting ultrasonic pulse wave velocity tests and uniaxial compressive strength tests on jointed specimens of a model rock. The prepared specimens comprised of joint with variable roughness and orientation. The joint orientation \( \theta \) is taken as the angle of normal to the joint plane with respect to loading direction. The joint orientation was varied from 0° to 75° for each orientation; three roughness values, equal to JRC 2–4, 12–14 and 14–16 were used as per Barton and Choubey chart (1977). The details of the experimental program are shown in Fig. 1.

### 2.1 Model Material

Extracting specimens to reproduce results from natural rocks is very difficult. Due to this reason, model materials are extensively used to simulate intact rock. In the present study, a model material called Ultrarock, which is a by-product of gypsum, is used as a model rock. The material has Uniaxial Compressive Strength (UCS) of 46.55 MPa and tangent modulus (\( E_{50} \)) of 9.09 GPa respectively. The model rock can be classified as “DL” (low strength and low modulus ratio) group, according to Deere-Miller (1966) Classification Chart. On testing the specimens for a range of confining pressure of 0–10 MPa, Mohr–Coulomb shear strength parameters of intact rock, \( c_i \) and \( \phi_i \) are observed to be 12.15 MPa and 39.91° respectively. The properties of the material are shown in Table 1.

[Table 1](#)
2.2 Configurations of Model Jointed Rocks

The configurations of the jointed specimens are shown in Fig. 2. The specimens were prepared with joint orientation angle $\theta$ equal to 0, 15, 30, 45, 60 and 75° respectively. For each orientation, three distinct surface profiles (with estimated JRC = 2 to 4, 12 to 14 and 18 to 20) were used. Zhao (1997a, b) has stated that the joint walls may not perfectly match with each other and hence effective JRC may not be equal to that obtained from the surface profile. Concept of Joint Matching Coefficient (JMC) was introduced by Zhao (1997a, b) to account for the mismatch of joints. For present profiles, direct shear tests were performed on joint surfaces and the JMC values for the three profiles were obtained. The JMCs were observed to be about 1, 1 and 0.8 respectively. The effective JRC values for the surface profiles were therefore considered as 2–4, 12–14 and 14–16 respectively (Fig. 3).

2.3 Preparation of Specimens and Testing Programme

To achieve the desired joint orientation and joint wall roughness, the specimen was prepared in two steps. In the first step, the first half of the specimen was prepared. A cylindrical segment of perspex was prepared as shown in Fig. 4. This segment was used to prepare one half of the jointed specimen. The segment had its inclined surface at a specific orientation, say

Table 1 Physical and engineering properties of model rock

| Property                   | Symbol | Value | Unit    |
|----------------------------|--------|-------|---------|
| Unit weight                | $\gamma$ | 21.3  | kN/m$^3$|
| Cohesion (for $\sigma_3 = 0$ to 10 MPa) | $c_i$ | 13.34 | MPa     |
| Angle of internal friction (for $\sigma_3 = 0$ to 10 MPa) | $\phi_i$ | 35.1 | °       |
| UCS of intact rock         | $\sigma_{ci}$ | 46.55 | MPa     |
| Tangent modulus            | $E_{t50}$ | 9.09  | GPa     |
| Brazilian tensile strength | $\sigma_t$ | 3.14  | MPa     |
| Deere-miller classification (1966) | DL (low strength and low modulus ratio) | | |
30°. In order to get imprint of natural joint surface, natural rock chunks having three different selected profiles were procured (Fig. 5). A special polymer named Moldsil-15 was used to take the imprint of natural rock’s roughness. The Moldsil-15 polymer is a high-performance silicon rubber, a flowable grade with high mechanical strength. It was prepared by mixing with CAT-16 medium speed catalyst at a ratio of 100:5. The mixture prepared was then poured over the surface of rock chunks, kept for 24 h to be air-dried and then removed. The desired roughness imprint achieved is shown in Fig. 6a. This membrane was pasted over one half of the perspex sheet specimen (Fig. 6b). Now, this prepared half perspex segment was put inside an aluminium cylindrical mould of 55 mm diameter and 112 mm height as shown in Fig. 6b. The water and Ultrarock powder were mixed in a ratio of 1:5. This paste was then poured in the cylindrical mould. The poured paste was kept for half an hour to be hardened. This way one part of the jointed specimen was prepared. This prepared one-half specimen of ultrarock was again kept into the steel mould and the mould was filled with Ultrarock paste. The two halves were prepared to form one jointed specimen having specific joint angle and JRC.

![Image](attachment:image.jpg)

**Fig. 2** Test samples with joint orientation (0°, 15°, 30°, 45°, 60°, 75°) and (JRC = 2–4, 12–14, 14–16) a. JRC = 2–4, b. JRC = 12–14, c. JRC = 14–16

**Fig. 3** Joint surface profiles and effective JRC used in present study
The specimen was removed from the mould when it gets hardened after 30 min. In the similar manner, all the specimens of required joint orientation and roughness were prepared. The prepared jointed specimens were cured for 21 days in a water tank. After 21 days, specimens cast were taken out and were air-dried. The cylindrical-shaped specimens were cut and polished according to ISRM standards (1981) in the Geotechnical Engineering Laboratory, IIT Roorkee.

The prepared specimens were tested for Ultrasonic Pulse wave velocity and UCS tests. The P-wave velocity (Vp) was measured by using ULTRASONIC CONCRETE TESTER UX 4600 M having a pulse generator, a pair of transducers (transmitter and receiver) with a frequency of 100 kHz (Fig. 7a). For conducting Ultrasonic Pulse wave velocity tests, in order to provide good coupling, the end surfaces of specimens were assured to be smooth enough. The transducers were kept along the axis of the sample and P-wave velocity was measured as shown in Fig. 7b.

After conducting P-wave velocity measurement, the specimens were tested for Uniaxial Compressive Strength (UCS) tests. The tests were performed in a displacement-controlled mode at a displacement rate of 0.002 mm/s using a servo-controlled Cyclic cum Static Rock Triaxial Testing machine (Fig. 8). The displacement rate was set so that failure might occur within
15 min of the commencement of the experiment. The machine consists of a loading unit with a hydraulic actuator, a linear motion device intended to apply the load up to a capacity of 2000kN, and the stroke length of 75 mm to apply strain at a rate of 0.01–5 mm/s. The jointed rock specimen is loaded on the basis of a command signal from the data acquisition system attached to the machine. The electronic unit consists of a data acquisition system responsible for the actuator movement to apply the load in an axial direction. The unit connected to the computer receives signals from LVDTs, load cell, and pressure transducers and displays the output in the form of the Load–Displacement curve. The axial load was applied monotonically until it reached its peak followed by failure and was continued until residual strength.

3 Results and Discussion

3.1 Results of Ultrasonic Pulse Wave Velocity Tests

Figure 9 shows plots of variation of $V_p$ with orientation $\theta$ for varying roughness values. For all JRC
values, starting from $\theta$ equal to 0°, there is some increase in $V_p$ up to $\theta$ equal to 15°, beyond which $V_p$ decreases and becomes minimum at $\theta$ equal to 60° and again increases at $\theta$ equal to 75°. For JRC equal to 2–4, at $\theta=60^\circ$, $V_p$ was found to be 3939 m/s, and at 75°, it was 4390 m/s. The value of $V_p$ is highest at 75°, followed by 15° and 0°. It is inferred that $V_p$ has an undulated concave upward trend with joint orientation angle $\theta$, being lowest at 60° and highest at 75° (Fig. 9). The reason for $V_p$ to be lowest at 60° might be due to low stiffness at this joint orientation and higher $V_p$ at 75° might be due to higher stiffness and no obstruction in the propagation of wave as joint is nearly parallel to the wave propagation. A similar inference was drawn by Eitzenberger (2012). High stiffness values tend to nullify the effect of discontinuity, thereby resulting in higher wave velocity propagation through the jointed rock. On the other hand, low stiffness values of jointed rock attenuate the wave propagation (Eitzenberger 2012). $V_p$ has been low at $\theta=45^\circ$ and $\theta=60^\circ$ due to low stiffness and low elastic modulus value. The same trend has been observed by Kim et al. (2012) and Min and Jing (2003). The higher roughness (JRC = 14–16) offers the maximum obstruction in the wave propagation due to the presence of secondary asperities, followed by medium roughness (JRC = 12–14) and then the smoothest one. $V_p$ decreases with an increase in JRC. The same observation was made by Kahraman (2002). Kahraman (2002) stated that P-wave velocity decreases with an increase in Fracture Roughness Coefficient (FRC). This trend is seen more prominently when JRC is high, i.e., JRC is 14–16. The diminution of $V_p$ is in an irregular way in the specimens with higher joint wall roughness, whereas for a smoother surface (JRC = 2–4) the curve is steady. Kahraman (2002), Li and Zhu (2012), Huang et al. (2014) and Mohd-Nordin et al. (2014) have also observed that with increase in roughness of joints, $V_p$ decreases. Boadu and Long (1996) attributed the reduction in P-wave velocity at higher roughness to low fractional contact area between the joint surfaces due to secondary asperities.

For all joint orientations, the effect of JRC on $V_p$ is systematic. For higher JRC, the $V_p$ is low, and for low JRC, the $V_p$ is high. It is seen that both JRC and joint orientation have substantial influence on $V_p$. However, if the effect of JRC is compared with the effect of joint orientation, the effect of orientation is observed to be significantly higher.
3.2 Results of Uniaxial Compressive Strength (UCS) Tests

After performing Ultrasonic Pulse Wave velocity tests, the specimens were tested for UCS. The stress–strain plots for all the joint orientation angles (i.e. θ=0, 15, 30, 45, 60 and 75° respectively) and three different JRCs (2–4, 12–14 and 14–16) are shown in Fig. 10. From the stress–strain plots, the UCS (σ cj) is obtained as the peak stress. The tangent modulus (E tj) was obtained by drawing tangent at 50 percent of failure stress. The variation of σ cj and E tj with joint orientation θ° is shown in Figs. 11 and 12 respectively.

From Fig. 11, it is observed that UCS has a falling trend with orientation θ, upto at θ ≈ 60° and is again increasing to a maximum at θ = 75°. The minimum value is observed near θ = 60°, and the specimen at this angle, fails in sliding along the joint plane. For smooth joints, the shape of the plot is u-shaped. With increasing roughness, the plot tends to develop a shoulder near θ = 15 to 30° and shape of the plot shifts towards ‘ν’ shape. Substantially higher strength is exhibited by rough joints. The strength behaviour is highly anisotropic which is similar to that observed by Ramamurthy (1993) and Singh et al. (2002). Singh et al. (2002) also observed shoulder for orientation θ=15 to 30° due to increase in interlocking.

Variation of the tangent modulus E tj with θ and JRC is shown in Fig. 12. Similar to strength behaviour, the modulus values are also anisotropic exhibiting u-shaped plot for smooth joints and ν-shaped plot for rough joints. Lowest values of E tj have been observed near θ≈60°.

Three types of dominant failure modes were observed in the present study: a) vertical splitting and spalling of intact material, b) shearing of intact material, and c) sliding along the joint (Figs. 13, 14 and 15). Many a times the failure was due to combination of more than one mode.

In the case of orientations θ=0°and 75°, the jointed rock more or less behaves like an intact rock, and the failure is governed by the intact rock matrix. There is vertical splitting and little spalling too, as happened in case of intact rock. For θ=15° and 30° cases, the specimen fails in a combination of shearing and vertical splitting. For θ=30°, the specimen with JRC equal to 2–4, failure mode is a combination of shearing and vertical splitting. As JRC increases (JRC=14–16), the interlocking of joint wall increases, and specimens with joints oriented at θ=30° fail in longitudinal splitting.

4 Analysis of the Test Data

4.1 Effect of Joint Orientation and Joint Roughness on Vp

Figure 16 shows combined plots of Vp and UCS vs. θ. It is noticed that both Vp and UCS have a concave upward relationship with joint orientation. A similar type of behaviour was observed for Vp and E tj vs. θ as shown in Fig. 17. It is concluded that UCS, E tj, θ, Vp, and roughness parameters are inter-related to each other.

The main objective of the present study has been to quantify the effect of joint orientation and joint wall roughness on the variation of Vp and then assess the strength and deformation behavior of jointed rock. For this purpose, a weakness coefficient, Joint Factor Jf has been employed.

4.2 Joint Factor

Joint Factor concept was introduced by Ramamurthy and co-workers (Ramamurthy 1993; Ramamurthy and Arora 1994) based on extensive laboratory tests conducted on natural and modeled rocks. Joint Factor is an index that reflects the weakness or incompetence of jointed rocks due to presence of joints. Jf is governed by the three parameters namely, Jn (number of joints/meter depth in the loading direction), n (joint inclination parameter), which depends on the orientation of the joint with respect to the direction of axial loading (Table 2), and r (joint shear strength parameter), which is taken as equal to tan φj, where φj is joint friction angle at very low normal stress (as experienced in UCS loading condition).

These parameters were clubbed together (Ramamurthy 1993; Ramamurthy and Arora 1994) to define Jf as follows:

\[
J_f = \frac{J_n}{n \times r}
\]

As per Jf concept, the strength and modulus of jointed rock are assessed in their simplest form, i.e.,
Fig. 10 Stress–strain plots for jointed specimens
unconfined condition. It was shown that the strength and modulus values of jointed rocks under unconfined state are strongly correlated with those of intact rocks through $J_f$ (Table 3). Typical variations of $\sigma_{cj}$ and $E_j$ with $J_f$ as per Ramamurthy (1993) are shown in Fig. 18.

### 4.3 Application of Joint Factor Concept to the Present Study

To apply Joint Factor concept, three parameters namely joint frequency $J_n$, joint inclination parameter $n$ and joint friction angle $\phi_j$ are required. Considering one joint in 110 mm height of specimen, $J_n$ works out to be equal to 9 joints/m. The inclination parameter $n$ was assigned from Table 2. The friction angle of joints ($\phi_j$) is a normal stress-dependent parameter. The concept recommends that $\phi_j$ should be obtained under those normal stress conditions on the joint plane, which prevail during the uniaxial loading of the specimen at the time of failure. The joint is also assumed to have zero cohesion and hence secant value of $\phi_j$ is used. For the present study, it was considered prudent to compute the normal stress at joint plane at the time of failure and use Barton’s model (Barton and Choubey 1977) to get $\phi_j$ at that normal stress. Consider a jointed specimen (Fig. 19) with joint oriented at $\theta^\circ$, failing at uniaxial compressive stress equal to $\sigma_{cj}$.

Normal stress acting on the joint plane at the time of failure is computed as:

$$\sigma_n = \frac{\sigma_{cj}}{2}(1 + \cos2\theta)$$

Joint friction angle $\phi_j$ corresponding to normal stress $\sigma_n$ acting on the joint plane is obtained as Barton and Choubey (1977):

$$\phi_j = \phi_r + JRC \log_{10}\left(\frac{JCS}{\sigma_n}\right)$$

where, $\phi_r$ is the residual friction angle of joint (29° in the present case) and JRC is joint roughness coefficient and JCS is joint wall compressive strength (MPa). In the present case, JCS has been taken equal to UCS of the intact rock as joint plane surface is fresh and unweathered. The computed values of $\phi_j$ are shown in Table 4. These values were used to obtain $J_f$ for each specimen.

### 4.4 Correlation of $J_f$ with $V_p$

It is attempted to see how $V_p$ varies with $J_f$. It has already been seen that $V_p$ gets influenced by joint orientation and joint roughness. In an attempt to get a correlation of $V_p$ with $J_f$, the observed $V_p$ values for jointed specimens were normalized by dividing them by $V_p$ of intact rock, and several trials were made to get correlation between Joint Factor and normalised $V_p$. Figure 20 shows variation of $J_f$ with normalised $V_p$. The following correlation is obtained between $J_f$ and normalised $V_p$:

$$J_f = 7.037 \left(\frac{V_{pj}}{V_{pi}} \times n\right)^{-0.974}$$
To see how the strength is influenced by $J_f$, normalized UCS for each jointed specimen was also plotted against $J_f$ (Fig. 21). The plot shows a strong correlation between $\sigma_{cj}$ and $J_f$ as:

$$\frac{\sigma_{cj}}{\sigma_{ci}} = 17.522 J_f^{-1.418}$$ \hspace{1cm} (5)

Similarly, the modulus values of the jointed specimens were normalized by dividing them by intact rock modulus. The normalized values of modulus were plotted against $J_f$ as shown in Fig. 22 and following correlation was obtained:

$$\frac{E_j}{E_i} = 5.007 J_f^{-0.839}$$ \hspace{1cm} (6)

It is seen that reasonably strong correlation exists between $J_f$ and normalised $V_p$. Also good correlations are observed for strength and modulus values of jointed rock with those of intact rock through Joint Factor $J_f$. In the field, P-wave velocity may be obtained as per procedure prescribed in ISRM: 1998 and IS:13372 (1992). The inclinations of the major joints may also be obtained in the field. The upper bound values of P-wave velocity may be obtained by performing tests on intact rock specimens. The normalised P-wave velocity may then be used to obtain Joint Factor $J_f$. The value of Joint Factor gives an assessment of how much weakness has occurred in the rock due to jointing. The intact rock strength and intact rock modulus values may now be scaled down by using correlations (Eqs. 4, 5 and 6) in the form of Joint Factor to get the strength and modulus of jointed rock.

![Fig. 13 Failure modes of jointed specimens (JRC = 2–4)](image-url)
4.5 Validation of Suggested Correlations

The correlations proposed from the present study have been validated using results of an independent experimental program. Widening of the roads is being done on National Highway NH-58 near the town Rishikesh (India). The side slopes comprise of Shale. The rock chunks were procured from the site, and six specimens of NX size were prepared and tested for $V_p$ and UCS. Figure 23 shows the photograph of the failed specimens. The specimens were having natural joints oriented at an angle $\theta = 13$, 35, 43, 67 and 90° respectively and joint walls had roughnesses in the range of JRC = 4 to 10. P-wave velocity, $V_p$ was observed to be the lowest at 67° and highest at 90°. Using observed $V_p$ values, $J_f$ was obtained through Eq. 4, and then UCS and modulus values of jointed rock were estimated through Eqs. 5 and 6. The computations are shown in Table 5. Laboratory UCS and modulus values are also mentioned in Table 5.

The observed and predicted values of UCS and modulus are plotted in Figs. 24 and 25 respectively. It is seen that the predicted values of the strength and modulus of the jointed rocks are close to those obtained from laboratory tests. It is concluded that the approach may be used in the field with confidence to roughly assess the strength and modulus of jointed rocks.

4.6 Estimates on Mohr–Coulomb Shear Strength Parameters of Jointed Rocks

This section demonstrates how an estimate on Mohr–Coulomb shear strength parameters $c_{\text{mass}}$ and $\phi_{\text{mass}}$ for the jointed rock may be made. It is well established fact that the triaxial strength of intact and jointed rock varies non-linearly with increase
in confining pressure. The Mohr–Coulomb shear strength parameters are not constant for a given rock and vary with the confining pressure range used to derive these parameters. Alternatively, non-linear strength criteria are preferred (Hoek and Brown 1980, 1997; Ramamurthy 1993; Singh et al. 2011; Singh and Singh 2012). Based on extensive analysis of triaxial test data on jointed rocks, Singh and Singh (2012) suggested modification to the conventional linear Mohr–Coulomb criterion and proposed Modified Mohr Coulomb (MMC) criterion as follows:

\[
\begin{align*}
\sigma_1 - \sigma_3 &= \sigma_{cj} + \frac{2 \sin \phi_{j0}}{1 - \sin \phi_{j0}} \sigma_3 \\
&- \frac{1}{\sigma_{ci}} \frac{\sin \phi_{j0}}{1 - \sin \phi_{j0}} \sigma_3^2 \quad \text{for} \quad 0 \leq \sigma_3 \leq \sigma_{ci}
\end{align*}
\]  

(7)

where \(\sigma_1\) and \(\sigma_3\) are the major and minor principal stresses acting on jointed rock at the time of failure; \(\sigma_{cj}\) and \(\sigma_{ci}\) are the UCS of jointed and intact rock respectively; \(\phi_{j0}\) is the limiting value of angle of internal friction of the jointed rock for the condition \(\sigma_3 \to 0\). The value of the friction angle \(\phi_{j0}\) may be obtained as (Singh and Singh 2012):

\[
\sin \phi_{j0} = \frac{(1 - SRF) + \frac{\sin \phi_{i0}}{1 - \sin \phi_{i0}}}{(2 - SRF) + \frac{\sin \phi_{i0}}{1 - \sin \phi_{i0}}}
\]

(8)

where SRF is strength reduction factor = \(\sigma_{cj}/\sigma_{ci}\); \(\phi_{i0}\) is the limiting value of angle of internal friction of the intact rock for the condition \(\sigma_3 \to 0\). The value of the limiting value \(\phi_{i0}\) was suggested to be obtained by following the computation as follows (Singh et al. 2011):

Fig. 15 Failure modes of jointed specimens (JRC = 14–16)
Despite the fact that non-linear behaviour is accepted by geotechnical engineers and geologists, many softwares still use conventional linear Mohr–Coulomb failure criterion to model the strength behaviour of the jointed rocks. The rough equivalent linear Mohr–Coulomb parameters for practical applications may be obtained by generating triaxial strength values of the jointed rock for a range of minor principal stress $\sigma_3$ and fitting linear failure envelope. The equivalent shear strength parameters obtained from this analysis are very sensitive to the range of values of the minor principal stress $\sigma_3$ used to generate the simulated triaxial tests results (Hoek and Brown 1997). On the basis of several trials, Hoek and Brown (1997) have suggested that most

\[
A_j = \frac{\sum (\sigma_1 - \sigma_2 - \sigma_3)}{\sum (\sigma_2^2 - 2\sigma_2\sigma_3)}; 0 < \sigma_3 \leq \sigma_{ci} \tag{9}
\]

\[
B_j = -2A_j\sigma_{ci} \tag{10}
\]

\[
\sin \varphi_0 = \frac{B_j}{2 + B_j} \tag{11}
\]

![Diagram of JRC 2-4](image1)

![Diagram of JRC 12-14](image2)

![Diagram of JRC 14-16](image3)

**Fig. 16** Variation of observed $V_p$ and UCS of jointed specimens with joint orientation $\theta$ for three JRCs
**Fig. 17** Plots showing variation of experimental $V_p$ and $E_t$ with joint orientation $\theta$ for specimens having different surface profiles as JRC = 2–4, 12–14 and 14–16 respectively

**Table 2** Joint inclination parameter, $n$ (Ramamurthy 1993)

| Joint orientation, $\theta^\circ$ | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
|----------------------------------|---|----|----|----|----|----|----|----|----|----|
| Inclination Parameter, $n$       | 1 | 0.814 | 0.634 | 0.465 | 0.306 | 0.071 | 0.046 | 0.105 | 0.46 | 0.81 |

**Table 3** Some correlations for $\sigma_{cj}$ and $E_j$ with $J_f$ given previously

| Authors                  | Correlations                                           |
|--------------------------|--------------------------------------------------------|
| Ramamurthy (1993)        | $\sigma_{cj} = \sigma_i(\exp(-0.008J_f))$             |
|                          | $E_j = E_i(\exp(-1.15 \times 10^{-2}J_f))$             |
| Sitharam et al. (2001)   | $\sigma_{cj} = \sigma_i(0.04 + 0.89\exp\left(-\frac{J_f}{161.0}\right))$ |
|                          | $E_j = E_i(0.035 + 0.879\exp\left(-\frac{J_f}{92.69}\right))$ |
| Singh et al. (2002)      | $\sigma_{cj} = \sigma_i(\exp(-0.0123J_f))$             |
|                          | $E_j = E_i(\exp(-0.020J_f))$                           |
| Splitting                | $\sigma_{cj} = \sigma_i(\exp(-0.011J_f))$             |
|                          | $E_j = E_i(\exp(-0.020J_f))$                           |
| Shearing                 | $\sigma_{cj} = \sigma_i(\exp(-0.025J_f))$             |
|                          | $E_j = E_i(\exp(-0.04J_f))$                            |
| Rotation                 | $\sigma_{cj} = \sigma_i(\exp(-0.018J_f))$             |
|                          | $E_j = E_i(\exp(-0.035J_f))$                           |
consistent results for practical use are obtained when eight equally spaced values of $\sigma_3$ are used in the range $0 < \sigma_3 < 0.25\sigma_{ci}$. For present study, eight equally spaced values of $\sigma_3$ were used in MMC criterion (Singh and Singh 2012) to generate simulated triaxial test results. The equivalent shear strength parameters for the jointed rock were then obtained by fitting linear failure envelopes. The values are presented in the form of charts (Figs. 26 and 27). The equivalent MC parameters of the jointed rock are designated as

**Table 4** Computation of joint friction angle $\phi_j$

| Average JRC | $\theta^\circ$ for which sliding occurred at failure | Resolved normal stress component on sliding plane (MPa) | Computed joint friction angle |
|-------------|---------------------------------|---------------------------------|-----------------------------|
| 3           | 30                              | 4.13                            | 35.24°                      |
|             | 45                              | 0.70                            |                             |
|             | 60                              | 0.02                            |                             |
| 13          | 45                              | 1.02                            | 55.9°                       |
|             | 60                              | 0.16                            |                             |
| 15          | 45                              | 2.14                            | 57.13°                      |
|             | 60                              | 0.18                            |                             |
cmass and ϕ mass to distinguish them from joint shear strength parameters. The friction angle of the jointed rock ϕ mass is plotted as a function of Strength Reduction Factor (SRF = σ cj/σ ci) and ϕ i0 (Fig. 26). Few triaxial test results on intact rock will be required to obtain ϕ i0. The cohesion of the jointed rock (c mass) is normalised and is shown as function of SRF (Fig. 27). It should be noted that ϕ mass will be higher than ϕ i0 for low confining pressure due to high dilation which reflects through high non-linearity of the failure criterion of jointed rock at low σ 3 values. The charts have been prepared for ϕ i0 ranging between 20 to 60° at an interval of 10° and interpolation may be done for intermediate values.

5 Conclusion

Rocks in the field are invariably jointed and exhibit anisotropic and incompetent engineering response as compared to intact rocks. P-wave velocity provides an economic non-destructive tool to qualitatively assess the competence of jointed rock in the field. In the present study, an attempt is made to get a qualitative assessment of the weakness of jointed rock in terms of strength and modulus of deformation. Joint Factor concept has been utilized for this purpose. It is found that P-wave velocity in jointed rock is closely linked with the Joint Factor Jf. Based on the laboratory test results, a correlation is suggested to obtain Joint Factor depending on observed P-wave velocity and orientation of discontinuities in the field. The Joint Factor (Jf) concept is well established in literature and can be used to get the strength and modulus of jointed rock under unconfined state. Correlations have also been suggested in this study to get strength and modulus values of jointed rock by using Jf. Validation of the suggested correlations has been performed through testing of specimens from a project site. The strength of jointed rock under confined condition can be obtained by using a non-linear failure criterion. In the present study, Modified Mohr–Coulomb failure criterion (Singh and Singh 2012) has been used and charts have been produced to get equivalent linear Mohr–Coulomb parameters c mass and ϕ mass. The charts can be used for quick assessment of shear strength parameters in the field by engineers and geologists.
Fig. 23 Photographs of failed specimens of shale in UCS tests

Table 5 Observed and predicted strength and modulus values for shale

| Sample name | Intact | 5300 | V_p (m/s) | V_p/V_p0 × n | Computed Jf (Observed) | θ° | V_p (m/s) | V_p/V_p0 × n | Computed Jf (Predicted) | Observed | Predicted | Jf (Observed) | Predicted | E_t (GPa) | Observed | Predicted |
|-------------|--------|------|-----------|---------------|------------------------|----|-----------|---------------|------------------------|----------|-----------|--------------|-----------|----------|-----------|-----------|
| TB3-S1      |        |      |           |               |                        | 13 | 4156      | 0.60          | 11.65                  | 49.58    | 61.69     | 49.58        | 61.69     | 10.44   | 8.12       |
| TB5-S1      |        |      |           |               |                        | 35 | 4933      | 0.36          | 18.88                  | 37.35    | 31.10     | 37.35        | 31.10     | 6.34     | 5.42       |
| TB1-S2      |        |      |           |               |                        | 43 | 4605      | 0.20          | 33.00                  | 31.16    | 14.09     | 31.16        | 14.09     | 7.12     | 3.39       |
| TB8-S2      |        |      |           |               |                        | 67 | 4156      | 0.07          | 96.19                  | 10.87    | 3.09      | 10.87        | 3.09      | 1.34     | 1.38       |
| TB5-S3      |        |      |           |               |                        | 90 | 5375      | 0.82          | 8.52                   | 100.76   | 96.09     | 100.76       | 96.09     | 11.76   | 10.56      |
Fig. 24  Comparison of observed and predicted value of UCS of shale

Fig. 25  Comparison of observed and predicted value of modulus of shale

Fig. 26  Chart suggested for computing angle of shearing resistance for jointed rock

Fig. 27  Chart suggested for computing cohesion of jointed rock
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Data Availability Raw data were generated at Geotechnical Engineering Laboratories, Department of Civil Engineering, IIT Roorkee. Derived data supporting the findings of this study are available from the corresponding author [M. Singh] on request.

Code Availability No special algorithm/ computer code (Except MS Office) has been used in this study.

Declarations

Conflicts of interest No conflict of interest.

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