Model-Independent Determination of Photon Helicity in Radiative $B \to K_1 \gamma$ Decays

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Photons emitted in exclusive $B \to K_1(\to K\pi\pi)\gamma$ decay are predominantly left-handed and therefore measuring photon polarizations in this mode can test the standard model and probe new physics effects. However such an approach demands the knowledge on the $K_1 \to K\pi\pi$ decay. In this work, we propose to use semileptonic $D \to K_1(\to K\pi\pi)e^+\nu_e$ decays and determine the photon helicity in a model-independent way. In particular a ratio of up-down asymmetries is formulated to extract the input for $K_1 \to K\pi\pi$. Experimental facilities like BESIII and Belle-II can measure this ratio, and accordingly access the photon helicity in $B \to K_1\gamma$.

Introduction.— It is generally believed that the search for new physics (NP) beyond the standard model (SM) can proceed in two distinct directions. On the one side, new particles can be directly produced in high energy collisions for instance at LHC. On the other side, it is of great interest to study various low-energy observables with high precision that can give an indirect search for NP. The photon helicity in inclusive and exclusive $s\gamma$ decays is predominantly left-handed in the SM and thereby its measurement plays a unique role in probing right-handed couplings. A representative example is the left-right symmetric model (LR), in which the photon helicity is sensitive to the interference between left-handed and right-handed couplings. A ratio of up-down asymmetries in semileptonic $D \to K_1(\to K\pi\pi)e^+\nu_e$ can provide all the essential inputs without any hadronic contamination. In particular we point out that a ratio of up-down asymmetries in semileptonic $D$ decays exactly mimics the hadronic input in $B \to K_1(\to K\pi\pi)\gamma$. The measurement on this ratio by BESIII and Belle-II will help to access the photon polarization, and its right-handed coupling.

Photon polarization in $B \to K_1(\to K\pi\pi)\gamma$.—Let us start with a brief review of the determination of the photon helicity in $B \to K_1(\to K\pi\pi)\gamma$. The effective Hamiltonian for $b \to s\gamma$ has the general form:

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* (C_{1R}O_{7R} + C_{7L}O_{7L}),$$

where $C_{7L,R}$ are the Wilson coefficients for $O_{7L,R}$. Due to the chiral structure of $W^\pm$ couplings to quarks in the SM, the emitted photon from $b \to s\gamma$ is mostly left-handed and the right-handed configuration is suppressed by $C_{7R}^\text{SM}/C_{7L}^\text{SM} \approx m_s/m_b$. For $b \to s\gamma$, it is vice versa.

The differential decay rate for $B \to K_1(\to K\pi\pi)\gamma$ can be expressed as a sum of contributions from left- and right-polarized photons, and turns out to be

$$\frac{d\Gamma_{K_1\gamma}}{d\cos\theta_K} = \frac{|A|^2 |\vec{J}|^2}{4} \times \left[ 1 + \cos^2\theta_K + 2\lambda_7 \cos\theta_K \frac{\text{Im}[\vec{n} \cdot (\vec{J} \times \vec{J}^*)]}{|\vec{J}|^2} \right].$$

Hereafter the $\theta_K$ is chosen as the relative angle between the normal direction $\vec{n}$ of the $K_1$ decay plane and the opposite flight direction of photon in $K_1$ rest frame.
The coefficient $A$ is a nonperturbative amplitude. The $\vec{J}$ characterizes the $K_1 \to K\pi\pi$ decay amplitude with $\mathcal{A}(K_1 \to K\pi\pi) = \vec{c}_{K_1} \cdot \vec{J}$. The polarization parameter $\lambda_{\gamma}$ is defined as

$$\lambda_{\gamma} = \frac{|\mathcal{A}(B \to K_{1R}\gamma R)|^2 - |\mathcal{A}(B \to K_{1L}\gamma L)|^2}{|\mathcal{A}(B \to K_{1R}\gamma R)|^2 + |\mathcal{A}(B \to K_{1L}\gamma L)|^2}.$$  (3)

which is $\lambda_{\gamma} \simeq -1$ for $b \to s\gamma$ and $\lambda_{\gamma} \simeq +1$ for $b \to s\gamma$ and in the SM. Any significant deviation from the above values would imply a signal of new physics.

To measure $\lambda_{\gamma}$, one way is to analyze the angular distribution based on Eq. (2), which however requires large number of data. With the limitation of the current experiments, an integrated up-down asymmetry is more convenient to scrutinize the photon helicity:

$$\mathcal{A}_{\text{UD}} \equiv \left[ \int_0^1 d \theta_B \frac{d\Gamma(B \to K \gamma \gamma)}{d \cos \theta_{BK}} \right] \left[ \int_0^1 d \theta_B \frac{d\Gamma(B \to K \gamma \gamma)}{d \cos \theta_{BK}} \right] = \lambda_{\gamma} \frac{3\text{ Im}[\vec{n} \cdot (\vec{J} \times \vec{J}^*)]}{|\vec{J}|^2}.$$  (4)

The LHCb collaboration has measured the up-down asymmetry in $B^+ \to K^+ \pi^- \pi^+\pi^-$ [10], and for the first time established the polarization of photons, with $\mathcal{A}_{\text{UD}} = (6.9 \pm 1.7) \times 10^{-2}$ at the range of $m_K \pi \pi = [1.1, 1.3]\text{GeV}$. Nevertheless it can be seen that, in order to extract $\lambda_{\gamma}$, it is also essential to fathom the hadronic factor $\text{Im}[\vec{n} \cdot (\vec{J} \times \vec{J}^*)]/|\vec{J}|^2$. Many estimations on this input factor have been made in either model-dependent or phenomenological approaches [13-15, 17-19]. Unfortunately, the current understanding of $K_1 \to K\pi\pi$ is very limited, due to the complicated intermediate states of $K^*\pi$, $K\rho$, $(K\pi)_{S\text{-wave}}\pi$ and $K(\pi\pi)_{S\text{-wave}}$, and their phases for interferences. Considerable hadronic uncertainties are thus inevitably introduced and beyond control. Therefore, the accurate result of $\lambda_{\gamma}$ has never been achieved so far, even though the up-down asymmetry has been well measured. The key issue in the following is to reliably determine the hadronic input of $\text{Im}[\vec{n} \cdot (\vec{J} \times \vec{J}^*)]/|\vec{J}|^2$.

**Determination of Hadronic Inputs in $D \to K_{\text{res}} e^+\nu_e \to K\pi\pi e^+\nu_e$ Decays.**—We now proceed with the angular distribution for $D \to K_{\text{res}} (\to K\pi\pi) e^+\nu_e$, and demonstrate that the hadronic factor, $\text{Im}[\vec{n} \cdot (\vec{J} \times \vec{J}^*)]/|\vec{J}|^2$, can be determined in a model-independent way. With the above conventions, it is undemanding to obtain the angular distribution for $D \to K_1 (\to K\pi\pi) e^+\nu_e$ as:

$$\frac{d\Gamma_{K_1 e^+\nu_e}}{d \cos \theta_K d \cos \theta_{\pi\nu}} = d_1 [1 + \cos^2 \theta_K \cos^2 \theta_{\pi\nu}]$$

$$+ d_2 [1 + \cos^2 \theta_K \cos \theta_{\pi\nu} + d_3 \cos \theta_K [1 + \cos^2 \theta_{\pi\nu}]]$$

$$+ d_4 \cos \theta_K \cos \theta_{\pi\nu} + d_5 [\cos^2 \theta_K + \cos^2 \theta_{\pi\nu}].$$  (5)

An additional angle $\theta_{\pi\nu}$ is introduced as the relative angle between the flight directions of $e^+$ in the $e^+\nu$ rest frame and the $e^+\nu$ in the $D$ rest frame, as shown in Fig. 1. The angular coefficients are given as:

$$d_1 = \frac{1}{2} |\vec{J}|^2 [4|c_0|^2 + |c_-|^2 + |c_+|^2],$$

$$d_2 = -|\vec{J}|^2(|c_-|^2 - |c_+|^2),$$

$$d_3 = -\text{Im}[\vec{n} \cdot (\vec{J} \times \vec{J}^*)](|c_-|^2 - |c_+|^2),$$

$$d_4 = 2\text{Im}[\vec{n} \cdot (\vec{J} \times \vec{J}^*)]|(c_-|^2 + |c_+|^2),$$

$$d_5 = -\frac{1}{2} |\vec{J}|^2[4|c_0|^2 - |c_-|^2 - |c_+|^2].$$  (6)

In the above, $c_{0,+,−}$ corresponds to the nonperturbative amplitudes for different polarizations of the $K_1$. The above formula is corresponding to the semi-leptonic decays with electrons whose mass is negligible. For the muon channels, there are some additional terms regarding to the muon mass, but our method is still valid. It is evident that once the angular distributions are experimentally determined, the hadronic input $\text{Im}[\vec{n} \cdot (\vec{J} \times \vec{J}^*)]/|\vec{J}|^2$ can be obtained by taking a ratio of $d_3$ and $d_2$, or $d_4$ and $d_1 + d_5$.

Measuring the full angular distribution in Eq. (5) requests a large number of data sample. We also propose to explore a ratio of up-down asymmetries (or forward-backward asymmetries):

$$\mathcal{A}_{\text{UD}}^2 \equiv \left[ \int_0^1 d \theta_K \frac{d\Gamma_{K_1 e^+\nu_e}}{d \cos \theta_K} \right] \left[ \int_0^1 d \theta_K \frac{d\Gamma_{K_1 e^+\nu_e}}{d \cos \theta_K} \right] = \frac{3\text{ Im}[\vec{n} \cdot (\vec{J} \times \vec{J}^*)]}{|\vec{J}|^2}.$$  (7)

It is straightforward to find

$$\mathcal{A}_{\text{UD}}^2 = \frac{\text{Im}[\vec{n} \cdot (\vec{J} \times \vec{J}^*)]}{|\vec{J}|^2}.$$  (8)
which is the exact hadronic factor to be explored. Apparently quantifying the $A_{2D}^{2e}$ in experiment will help to extract the photon helicity, as shown in Eq. (1). In $D \rightarrow K_1(\rightarrow K\pi\pi)e^+\nu$, the $c_+$ and $c_-$ corresponds to the positive and negative polarization of $K_1$, respectively. On the one side, through the $\epsilon \cdot J$ coupling, the transversely polarized $K_1$ provides the $\cos \theta_K$ dependence in a similar way with the $B \rightarrow K_1(\rightarrow K\pi\gamma)$. Such a term takes the form \( \text{Im} [\bar{\nu} \cdot (\vec{J} \times \vec{K})] (|c_-|^2 - |c_+|^2) \). On the other side, the lepton pair is produced by a virtual $W$ boson which couples to the lepton pair through the $V - A$ current. The two transverse polarizations of $W$ boson gives the $\cos \theta_1$ dependence that is proportional to $|\vec{J}|^2 (|c_-|^2 - |c_+|^2)$. The up-down asymmetries, the numerator and denominator in Eq. (10), are constructed to project them out, whose ratio naturally gives \( \text{Im} [\bar{\nu} \cdot (\vec{J} \times \vec{K})]/|\vec{J}|^2 \).

Including more $K_J$ resonances.— Though the LHCb measurement of $B \rightarrow K\pi\pi\gamma$ has shown that the $K_1(1270)$ contributions dominates over other resonances [16], it is also meaningful to include contributions from more $K_J$ resonances, like $K_1(1400)$, $K_2(1430)$, $K^*(1410)$ with $J^P = 1^-$. In particular the $K_1(1400)$, $K_2(1430)$ contribution will interfere with that from the $K_1(1270)$.

With the above resonating contributions, the $B \rightarrow K\pi\pi\gamma$ angular distribution now becomes:

\[
\frac{d\Gamma(B \rightarrow K\pi\pi\gamma)}{d cos \theta_K} = \frac{d\Gamma(B \rightarrow K\pi\pi\gamma)}{d cos \theta_K} + C\left(\frac{1}{2} |\vec{K}|^2 \cos \theta_K + \cos^2 \theta_K \right) + \frac{1}{2} \lambda_2 \text{Im}[\bar{\nu} \cdot (\vec{K} \times \vec{K}^*)] \cos \theta_K \cos \theta_K \right) + \frac{1}{2} \lambda_2 \text{Im}[AB^* \bar{\nu} \cdot (\vec{J} \times \vec{K}^*)] + \text{Re}[AB^* \bar{\nu} \cdot (\vec{J} \times \vec{K}^*)] \cos \theta_K \right) .
\]

(9)

The $(B, \vec{K})$ and $C$ are nonperturbative coefficients relating to $K_2(1430)$ and $K^*(1410)$ respectively, and their explicit forms can be found in Ref. [14]. Apparently, the photon polarization $\lambda_2$ can also be extracted through the $K_2$ contribution or the $K_1 - K_2$ interference terms, but again such determinations request the knowledge of nonperturbative matrix elements, $\text{Im}[\bar{\nu} \cdot (\vec{K} \times \vec{K}^*)]/|\vec{K}|^2$, and $\text{Re}[AB^* \bar{\nu} \cdot (\vec{J} \times \vec{K}^*)]/|\vec{K}|^2$.

Including the different resonances, we find that the angular distribution for $D \rightarrow K_{res}(\rightarrow K\pi\pi)e^+\nu$ reads:

\[
\frac{d\Gamma(D \rightarrow K_{res}(\rightarrow K\pi\pi)e^+\nu)}{d cos \theta_K d cos \theta_1} = \sum_{K_j = K_1, K_2, K_2'} \frac{d\Gamma(B \rightarrow K\pi\pi\gamma)}{d cos \theta_K d cos \theta_1} .
\]

(10)

Here the $K_1^*$ denotes the vector $K^*(1410)$ meson and its contribution is:

\[
\frac{d\Gamma(K_1^*\bar{\nu})}{d cos \theta_K d cos \theta_1} = \frac{1}{2} |c_+|^2 \cos \theta_K \right) + \frac{1}{2} \lambda_2 \text{Re}[AB^* \bar{\nu} \cdot (\vec{J} \times \vec{K}^*)] + \frac{1}{2} \lambda_2 \text{Im}[AB^* \bar{\nu} \cdot (\vec{J} \times \vec{K}^*)] \cos \theta_K \right) .
\]

(11)

while the $D \rightarrow K_2(\rightarrow K\pi\pi)e^+\nu$ contribution is:

\[
\frac{d\Gamma(K_2\bar{\nu})}{d cos \theta_K d cos \theta_1} = \frac{1}{2} |c_+|^2 \cos \theta_K \right) + \frac{1}{2} \lambda_2 \text{Re}[AB^* \bar{\nu} \cdot (\vec{J} \times \vec{K}^*)] + \frac{1}{2} \lambda_2 \text{Im}[AB^* \bar{\nu} \cdot (\vec{J} \times \vec{K}^*)] \cos \theta_K \right) .
\]

(12)

The $\lambda_2$ correspond to the nonperturbative amplitudes involving the $K_1^*$ and $K_2$ resonances. It is necessary to note that neglecting the higher order corrections, $c_+(c_+)^* \propto AB^*$, and $c_-(c_-)^* \propto AB^*$. As a consequence, the quantity $\text{Re}[AB^* (\vec{J} \times \vec{K})]/|\vec{K}|^2$ can be extracted through the angular analysis.

Discussions.— A measurement of the up-down asymmetry by the LHCb collaboration gives $A_{UD} = (6.9 \pm 1.7) \times 10^{-2}$ in the kinematic region $1.1 \text{GeV} < m_{K\pi\pi} < 1.3 \text{GeV}$ with the $\theta_K$ chosen as the relative angle between the normal direction of $\vec{p}_{\pi, \text{slow}} \times \vec{p}_{\pi, \text{fast}}$ and the photon flight direction. In the SM $\lambda_2 \sim 1$ and therefore the above result corresponds to $A_{UD}^2 = (9.2 \pm 2.3) \times 10^{-2}$. A significant deviation from the above value would imply the $\lambda_2$ is not unity and thereby is a clear signal for new physics beyond SM.

On the experimental side, an earlier evidence for $D^0 \rightarrow K_1^*(1270)e^+\nu$ has been found by CLEO in Ref. [20]. Quite recently, using the 2.93 fb$^{-1}$ data sample of $e^+e^-$ collision at the center of mass energy of 3.773 GeV, BESIII collaboration has observed $D^+ \rightarrow K_1^*(1270)e^+\nu$ for the first time with a statistical significance greater than
$10\sigma$ and the measured branching fraction is \cite{21}:

$$B(D^+ \to K^0_1 e^+\nu_e) = (2.3 \pm 0.26 \pm 0.18 \pm 0.25) \times 10^{-3}. \tag{14}$$

From the above result for branching fraction, one may infer that a direct measurement of the $A_{UL}^2$ is feasible in the near future. If so, the photon polarization in $B \to K_1^0 \gamma$ would be determined reliably. We also expect that a high precision could be achieved taking into account the fact that much more data will be accumulated at BESIII, Belle-II, and LHCb, leaving aside the Super tau-charm factory in the future.

Although the lepton mass has been neglected, we have checked that our method is still valid when the lepton is massive. Thus this analysis also applies to $D \to K_{res}(\to K \pi \pi) \mu^+ \nu\mu$.

In the above, we have elucidated the method using the angular distribution of $D \to K_{res}(\to K \pi \pi)e^+\nu$ decay, but one can also use the $B_s \to K_{res}(\to K \pi \pi)\ell\bar{\nu}$ and $D_s \to K_{res}(\to K \pi \pi)e^+\nu$ decays. An estimate of the branching fraction of $D_s \to K_1^0 \ell\bar{\nu}$ is about \( (3.65^{+2.27}_{-1.87}) \times 10^{-4} \) \cite{22} and might be measured in the future. The $D_s \to K_{res}e^+\nu$ is a $c\to d$ transition suppressed by the CKM matrix element, and needs more data.

Photon helicity and the right-handed couplings are likely different in $b \to s\gamma$ and $b \to d\gamma$. Thus the $B \to a_1(1260)\gamma \to \pi \pi \pi \gamma$ is also of great interest. Similarly, together with $D \to a_1 e^+\nu$, the photon helicity in $b \to d\gamma$ can be determined in a model-independent way.

Conclusions.— After the discovery of the Higgs boson, the primary objective in particle physics now switches to search for new physics. In complementary with the direct search at high energy frontier, the indirect search in low-energy processes can provide valuable information/constraint to the underlying new physics scenario.

Since photons emitted in exclusive $\bar{B} \to K_1^0(\to \bar{K} \pi \pi)\gamma$ decay are predominantly left-handed, measuring photon polarizations in this mode can test the standard model and probe the new physics effects with large right-handed couplings. However such a procedure requests the knowledge on the $K_1^0 \to K \pi \pi$ decay, and introduces the uncontrollable model-dependence. In this work, we have proposed to use semileptonic $D \to K_1^0(\to K \pi \pi)e^+\nu$ decays and determine the photon helicity in a model-independent way. Particularly we have shown that a ratio of up-down asymmetries can mimic the $K_1^0 \to K \pi \pi$ input. Experimental facilities like BESIII and Belle-II can measure this ratio, and accordingly the photon helicity in $B \to K_1^0 \gamma$ that provides a direct test of standard model. This analysis can also be generalized to measure the photon helicity in $b \to d\gamma$ in a model-independent way.

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\begin{thebibliography}{99}
\bibitem{1} D. Atwood, M. Gronau and A. Soni, Phys. Rev. Lett. \textbf{79}, 185 (1997) doi:10.1103/PhysRevLett.79.185 [hep-ph/9704272].
\bibitem{2} D. Becirevic, E. Kou, A. Le Yaouanc and A. Tayduganov, JHEP \textbf{1208}, 090 (2012) doi:10.1007/JHEP08(2012)090 [arXiv:1206.1502 [hep-ph]].
\bibitem{3} A. Paul and D. M. Straub, JHEP \textbf{1704}, 027 (2017) doi:10.1007/JHEP04(2017)027 [arXiv:1608.02556 [hep-ph]].
\bibitem{4} E. Kou, C. D. L and F. S. Yu, JHEP \textbf{1312}, 102 (2013) doi:10.1007/JHEP12(2013)102 [arXiv:1305.3173 [hep-ph]].
\bibitem{5} N. Haba, H. Ishida, T. Nakaya, Y. Shimizu and R. Takahashi, JHEP \textbf{1503}, 160 (2015) doi:10.1007/JHEP03(2015)160 [arXiv:1501.00668 [hep-ph]].
\bibitem{6} D. Atwood, T. Gershon, M. Hazumi and A. Soni, Phys. Rev. D \textbf{71}, 076003 (2005) doi:10.1103/PhysRevD.71.076003 [hep-ph/0410036].
\bibitem{7} S. Akar, E. Ben-Haim, J. Hebling, E. Kou and F. S. Yu, JHEP \textbf{1909}, 034 (2019) doi:10.1007/JHEP09(2019)034 [arXiv:1802.09433 [hep-ph]].
\bibitem{8} Y. Grossman and D. Pirjol, JHEP \textbf{0006}, 029 (2000) doi:10.1088/1126-6708/2000/06/029 [hep-ph/0005069].
\bibitem{9} F. Kruger and J. Matias, Phys. Rev. D \textbf{71}, 094009 (2005) doi:10.1103/PhysRevD.71.094009 [hep-ph/0502060].
\bibitem{10} D. Becirevic and E. Schneider, Nucl. Phys. B \textbf{854}, 321 (2012) doi:10.1016/j.nuclphysb.2011.09.004 [arXiv:1106.3283 [hep-ph]].
\bibitem{11} Y. Amhis et al. [HFLAV Collaboration], Eur. Phys. J. C \textbf{77}, no. 12, 895 (2017) doi:10.1140/epjc/s10052-017-5058-4 [arXiv:1612.07233 [hep-ex]].
\bibitem{12} P. Ball, G. W. Jones and R. Zwicky, Phys. Rev. D \textbf{75}, 054004 (2007) doi:10.1103/PhysRevD.75.054004 [hep-ph/0612081].
\bibitem{13} M. Gronau, Y. Grossman, D. Pirjol and A. Ryd, Phys. Rev. Lett. \textbf{88}, 051802 (2002) doi:10.1103/PhysRevLett.88.051802 [hep-ph/0107254].
\bibitem{14} M. Gronau and D. Pirjol, Phys. Rev. D \textbf{66}, 054008 (2002) doi:10.1103/PhysRevD.66.054008 [hep-ph/0205065].
\bibitem{15} E. Kou, A. Le Yaouanc and A. Tayduganov, Phys. Rev. D \textbf{83}, 094007 (2011) doi:10.1103/PhysRevD.83.094007 [arXiv:1011.6593 [hep-ph]].
\bibitem{16} R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. \textbf{112}, no. 16, 161801 (2014) doi:10.1103/PhysRevLett.112.161801 [arXiv:1402.6852 [hep-ex]].
\bibitem{17} A. Tayduganov, E. Kou and A. Le Yaouanc, Phys. Rev. D \textbf{85}, 074011 (2012) doi:10.1103/PhysRevD.85.074011 [arXiv:1111.6307 [hep-ph]].
\bibitem{18} M. Gronau and D. Pirjol, Phys. Rev. D \textbf{96}, 054004 (2017) doi:10.1103/PhysRevD.96.054004 [arXiv:1704.03239 [hep-ph]].
\end{thebibliography}
[19] E. Kou, A. Le Yaouanc and A. Tayduganov, Phys. Lett. B 763, 66 (2016) doi:10.1016/j.physletb.2016.10.013 [arXiv:1604.07708 [hep-ph]].

[20] M. Artuso et al. [CLEO Collaboration], Phys. Rev. Lett. 99, 191801 (2007) doi:10.1103/PhysRevLett.99.191801 [arXiv:0705.4276 [hep-ex]].

[21] M. Ablikim et al. [BESIII Collaboration], arXiv:1907.11370 [hep-ex].

[22] R. H. Li, C. D. Lu and W. Wang, Phys. Rev. D 79, 034014 (2009) doi:10.1103/PhysRevD.79.034014 [arXiv:0901.0307 [hep-ph]].