Miura, Tatsuya

Elastic curves and phase transitions. (English) [Zbl 1436.49060]
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Summary: This paper is devoted to a classical variational problem for planar elastic curves of clamped endpoints, so-called Euler’s elastica problem. We investigate a straightening limit that means enlarging the distance of the endpoints, and obtain several new results concerning properties of least energy solutions. In particular we reach a first uniqueness result that assumes no symmetry. As a key ingredient we develop a foundational singular perturbation theory for the modified total squared curvature energy. It turns out that our energy has almost the same variational structure as a phase transition energy of Modica-Mortola type at the level of a first order singular limit.

MSC:
49S05 Variational principles of physics
74B05 Classical linear elasticity
49J40 Variational inequalities

Keywords:
planar elastic curves; Euler’s elastica; least energy solution

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