Comments on the hidden symmetries of the spinning point particle in Kerr-Newman, $D = 5$ minimal gauged supergravity and higher dimensional spacetimes with Killing-Yano torsion

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ABSTRACT

We investigate the hidden symmetries of the Kerr-Newman, of the D=5 minimal gauged supergravity which admits a Killing-Maxwell system in the sense of Carter and of higher dimensional spacetimes in the presence of Killing-Yano torsion. We note that when an electromagnetic tensor is present and an associated Killing-Maxwell system can be constructed in the sense of Carter, the Killing-Maxwell field becomes a PCKY (primary conformal Killing-Yano) tensor or a PGCKY (primary generalized conformal Killing-Yano) tensor, the latter in the presence of torsion. We point out the structure of the Dirac-type operators of the spinning point particle in arbitrary-dimensional spacetimes with torsion, focusing on the case when torsion can be assimilated with a Killing-Yano tensor and these spacetimes are endowed with towers of Killing-Yano tensors, and hence we show the anomalies of these additional supersymmetries cancel out in this particular case.
1 Introduction

Symmetries in nature have long been useful in determining constants of motion and hence in helping solve equations of motion. Hidden symmetries, the symmetries of the phase space, together with spacetime symmetries bring insight into the evolution of a system in curved spacetime and are characterized by symmetrical Stäckel-Killing and antisymmetrical Killing-Yano tensors[1], while the latter symmetries are driven by Killing vectors.

There is a long history of separation of variables of Hamilton-Jacobi, Klein-Gordon, and Dirac equations[2,3,4,5,6,7,8,9,10,11,12] against various background spacetimes, starting with the Kerr spacetime in four dimensions and continuing with higher dimensional black hole spacetimes, the work cited above having a particular emphasis on the most general Kerr-AdS-NUT spacetime. All these with the miraculous help of Stäckel-Killing and Killing-Yano tensors, which build up constants of motion, which in turn lead to complete integrability of the equations. Another important object in describing hidden and spacetime symmetries is the PCKY and more generally the CKYs (conformal Killing-Yano tensors) [13,14,15,16]. The PCKY generates in higher dimensional spacetimes towers of Killing-Yano and Stäckel-Killing tensors.

The group structure and algebras generated by Dirac-type operators in spacetimes that are torsionless have been thoroughly studied [17,18,19] and these operators constructed with the use of Killing-Yano tensors turn out not to have any anomalies at the quantum level. These operators generate standard and non-standard supersymmetries of the spinning point particle evolving in the respective spacetimes.

One other important aspect is the fact that there exist geometrical dualities that map spacetimes with torsion in duals that are torsionless[20, 21, 22]. If a Killing-Yano structure exists in the spacetime with torsion, then it becomes the vielbein of the torsionless dual spacetime, while the vielbein of the spacetime with torsion becomes the Killing-Yano structure of the torsionless dual.

A lot of progress[23] has been accumulated since the seminal paper of Myers and Perry[24], where the metrics describing the isolated, vacuum, rotating higher-dimensional Kerr black holes were derived. Going forward in time we reach the derivation of the most general black-hole in D=5 minimal gauged supergravity spacetime metric [25] which is one of the objects of attention in this current paper. We cannot do justice here to the entire scientific effort of determining the metrics of various black holes in higher dimensions since 1986 on.

Solving the equations of motion in spacetimes of black holes with gauge fields is aided by the so-called generalized Killing-Yano, generalized Stäckel-Killing and generalized conformal Killing-Yano tensors corresponding to symmetries with torsion. There is an ample span of study of these tensors in the literature ranging from the D=5 minimal gauged supergravity spacetime [26,27], to the study of the black hole spacetimes in the framework of string theory [28,29,30].
In this paper we present some relevant calculus with torsion in section 2; then in section 3 we introduce some results obtained by Carter in the context of the Kerr-Newman black-hole, dubbed the Killing-Maxwell system and we also review some results obtained previously in [17] where Dirac-type operators were constructed to describe hidden symmetries (here we write them out specifically in the framework of the Killing-Maxwell system of the Kerr-Newman black hole).

We then study in section 4 the Killing-Maxwell system in the context of D=5 minimal gauged supergravity and we review what happens in this spacetime when torsion is a Killing-Yano tensor. We write out the specific tower of Killing-Yano and Stäckel-Killing tensors, as well as Dirac-type operators, starting from three PGCKY tensors of the Chong-Cvetič-Lü-Pope (CCLP) black hole. We note that in the case when the torsion tensor is Killing-Yano, there are no anomalies of the quantum symmetries described by the Dirac-type operators.

Finally, in section 5 we briefly take a look at the relationship between Killing spinors and generalized Killing-Yano (GKY) tensors and generalized conformal Killing-Yano (GCKY) tensors.

### 2 Some calculus relations in the presence of torsion

Let’s now take a look at some calculus with torsion (see [31]-[35]). Let $T$ be a 3-form on a Riemannian manifold $(M, g)$ and $e_a$ an orthonormal frame such that $g(e_a, e_b) = \delta_{ab}$. Then if $X, Y$ are vector fields then we define the Levi-Civita connection as:

$$\nabla_T^X Y = \nabla_X Y + \frac{1}{2} T(X, Y, e_a) e_a,$$

T is assimilated with torsion and we shall also use the $T=2A$ notation for torsion. For a p-form $\omega$ the covariant derivative is:

$$\nabla^A \omega = \nabla_X \omega - (X \lrcorner e_b \lrcorner A) \wedge (e_b \lrcorner \omega)$$

with the explicit formula for a 2-form being:

$$\nabla^A Y_{\nu\rho} = \nabla_\mu Y_{\nu\rho} - 2 A_{\sigma\mu} [\nu Y^{\sigma|\rho]}.$$

Note that the spinor covariant derivative with torsion can be written out as:

$$D^A_\mu = D_\mu + \frac{1}{12} \gamma^\nu \gamma^\rho A_{\mu\nu\rho}.$$  

Hence the Dirac operator with torsion is:

$$D^A_\mu \gamma^\mu = D_\mu \gamma^\mu + \frac{1}{12} \gamma^\nu \gamma^\rho A_{\mu\nu\rho}. $$

The Ricci relation with torsion for a 2-form is as follows:
\[ \nabla_\alpha \nabla_\beta Y_{\mu \nu} = -\frac{3}{2} R^\lambda_{\alpha \beta [\mu} Y_{\nu]} - 2 A^{[\lambda} |_{\beta [\alpha} \nabla_\lambda Y_{\mu] \nu}. \]  

(6)

The square of the Dirac operator as a function of the torsion T is:

\[ D^2 T = -\Delta T - \frac{dT}{4} - \frac{s}{4} \frac{|T|^2}{24}, \]

(7)

where

\[ \Delta T = \nabla_{X_a} \nabla^T_{X_a} + \nabla^T_{X_a} X_a, \]

(8)

and s is the scalar curvature of the connection with torsion

\[ s = -X^a \lrcorner R(X_a, X_b) e^b. \]

(9)

The curvature operator is defined as usual:

\[ R(X, Y) \omega = (\nabla^T_X \nabla^T_Y - \nabla^T_Y \nabla^T_X - \nabla^T_{[X, Y]} ) \omega. \]

(10)

So the spinor covariant derivatives commutator is:

\[ [D^A, D^A] \Psi = \frac{1}{8} R_{\alpha \beta \mu \nu} [\gamma^\alpha, \gamma^\beta] \Psi - A^A_{\mu \nu} D^A \Psi. \]

(11)

To be noted that when both A and Y are Killing-Yano tensors:

\[ A_{\sigma \mu [\nu} Y_{\sigma ] \rho] = 0. \]

(12)

This can be understood as being true since the contraction of a Killing-Yano tensor product is null. This generalizes to higher order tensors and contractions. This result simplifies some relations where Killing-Yano tensors are contracted. The situation in which torsion is a Killing-Yano tensor is going to be useful in this paper when we construct the Killing-Maxwell system. The equation above can easily be generalized for a rank p tensor. We now turn to the \( d^A \) and \( \delta^A \) operators, which are, taking into account the above, when A is Killing-Yano:

\[ d^A \omega = d \omega, \quad \delta^A \omega = \delta \omega. \]

(13)

And so:

\[ d^A d^A \omega = 0, \quad \delta^A \delta^A \omega = 0. \]

(14)

Here d is the usual exterior derivative operator and \( \delta \) the inner derivative, defined as:

\[ \delta = -e^a \lrcorner \nabla_a. \]

(15)

So when torsion is a Killing-Yano tensor, it turns out that the exterior derivative with torsion and interior derivative with torsion are equal to the regular exterior derivative, respectively regular interior derivative.
3 Hidden symmetries of the Kerr-Newman spacetime

The minimal gauged supergravity in 5 dimensions spacetime together with its symmetries has been recently studied [26] and in this framework- also the most general known Chong-Cvetić-Lü-Pope black hole solution[25]. Previous work is dating back to 1987 and is done by Carter[36,37], who investigated the solutions and symmetries of its lower-dimensional cousin, the Kerr-Newman black hole. Carter reached the conclusion that there exists a Killing-Maxwell electromagnetic system defined by the following equation for the 4-dimensional electromagnetic potential (here we used Carter’s notations, in that semi-colon means taking the covariant derivative):

\[
\hat{A}^{\mu;\nu;\rho} = 2\frac{4\pi}{3}\hat{j}^{\mu;\nu;\rho},
\]

where \( g \) is the spacetime metric and \( \hat{j} \) the current, which is a Killing vector. The Killing-Maxwell electromagnetic system obeys regular Maxwell equations, since the Maxwell field is defined as usual and respects Maxwell’s laws-

\[
\hat{F}_{\rho\mu} = 4\pi\hat{j}^\mu
\]

and

\[
\hat{F}_{[\mu\nu;\rho]} = 0.
\]

Further, Carter mentions that the Hodge dual of the Killing-Maxwell electromagnetic field is a Killing-Yano tensor of rank 2. Although he doesn’t state it directly, it follows that \( \hat{F} \) is a PCKY tensor - as defined for instance by Kubizšák in his thesis, rel. (3.7) in [13]- since it satisfies equations [16] and [17]. The fact that \( \hat{F} \) is closed follows from Maxwell’s laws.

The Hodge dual of the Killing-Maxwell electromagnetic field consequently determines a Dirac-type operator, that anti-commutes with the Dirac operator in the Kerr-Newman spacetime, according to [17]:

\[
Q_{\ast\hat{F}} = \gamma^\mu\ast\hat{F}_\mu \, D_\nu - \frac{1}{6}\gamma^\mu\gamma^\nu\gamma^\rho \nabla_\mu \ast\hat{F}_{\nu\rho}.
\]

This operator corresponds to a quantum (non-anomalous) hidden symmetry for the spinning point particle in Kerr-Newman spacetime which is - as stated in [17]- an additional non-generic supersymmetry of the particle. It is a remarkable fact that \( \ast\hat{F} \) generates a supersymmetry and it points out the subtle connection between the symmetries of the Killing-Maxwell electromagnetic field in curved spacetime and this supersymmetry and further- between spin and electric charge. In the 4-dimensional spacetime, the PCKY generates one Killing-Yano tensor, which is \( \ast\hat{F} \) and one Stäckel-Killing tensor, \( K \):

\[
K_{\mu\nu} = (\ast\hat{F})_{\mu\rho}(\ast\hat{F})^\rho_{\nu}.
\]
4  Hidden symmetries of the D=5 minimal gauged supergravity spacetime with a Killing-Maxwell system

In the cousin spacetime, D=5 minimal gauged supergravity, the Hodge dual of the electromagnetic field plays the role of torsion as evidenced in [26]. Here we are going to focus on the case when *F is part of the Killing-Maxwell system, a generalization to 5 dimensions of the work set forth by Carter. This means we are going to identify the PGCKY of the 5-dimensional spacetime, of which *F is a Killing-Yano tensor, with F (the Killing-Maxwell electromagnetic field), so this is a spacetime that is a bit different than the one described in [26]. The definition of a Killing-Maxwell electromagnetic field in D=5 supergravity is:

\[ \nabla_\rho F_{\mu\nu} = \frac{2}{4} \pi g_{\rho[\mu} j_{\nu]} \]  

and together with:

\[ \nabla_\rho F^{\rho\mu} = 4 \pi j^\mu \]  

form a Killing-Maxwell system that obeys Einstein-Maxwell’s laws. According to [26] the PGCKY of the D=5 minimal gauged supergravity spacetime has the definition:

\[ \nabla_\rho h_{\mu\nu} = \frac{2}{3} g_{\rho[\mu} \xi_{\nu]} - \frac{1}{\sqrt{3}} (\ast F)_{\rho\sigma[\mu} h^{\sigma}_{\nu]} \].  

(23)

If in our case h is indeed F, then because \( d^4 F = dF = 0 \) (this last equality because of the Einstein-Maxwell laws and the first one is true because we assimilate *F with torsion and *F in our case is Killing-Yano) it follows that:

\[ \frac{1}{\sqrt{3}} (\ast F)_{\rho\sigma[\mu} h^{\sigma}_{\nu]} = 0. \]  

(24)

Hence, equation (19) becomes:

\[ \nabla_\rho h_{\mu\nu} = 2 g_{\rho[\mu} \xi_{\nu]} \].  

(25)

Note that if in the equation above we notate h by F and we set \( j = \pi \xi \) then indeed the definition in 5-dimensions of a Killing-Maxwell electromagnetic field and that of a PGCKY coincide and hence our assumption that for the five-dimensional supergravity endowed with a Killing-Maxwell system F and h coincide is true. According to the theory of Killing-Yano tensors, F generates the Killing-Yano tensor *F as expected, which is the Hodge dual of the Killing-Maxwell electromagnetic field and which also plays the role of torsion in this spacetime. It also generates the Stäckel-Killing tensor:

\[ K_{\mu\nu} = (\ast F)_{\mu\rho\sigma} (\ast F)^{\nu\rho\sigma}. \]  

(26)
If we now take a look at the Chong-Cvetič-Lü-Pope black hole (CCLP) in D=5 minimal gauged supergravity framework, with the following notation s:

\[ g = \sum_{\mu=x,y} (\omega^\mu \omega^\mu + \tilde{\omega}^\mu \tilde{\omega}^\mu) + \omega^x \omega^x, \]  
(27)

\[ A = \sqrt{3}(A_q + A_p). \]  
(28)

And-

\[ \omega^x = \sqrt{\frac{x-y}{4X}} dx, \quad \tilde{\omega}^x = \sqrt{\frac{X}{x(y-x)}} (dt + y d\phi), \]  
(29)

\[ \omega^y = \sqrt{\frac{y-x}{4Y}} dy, \quad \tilde{\omega}^y = \sqrt{\frac{Y}{y(x-y)}} (dt + xd\phi), \]  
(30)

\[ \omega^z = \frac{1}{\sqrt{-x y}} [\mu dt + \mu(x+y) d\phi + xy d\psi - yA_q - xA_p], \]  
(31)

\[ A_q = \frac{q}{x-y} (dt + y d\phi), \quad A_p = \frac{-p}{x-y} (dt + x d\phi), \]  
(32)

and

\[ X = (\mu + q)^2 + Ax + CX^2 + \frac{1}{12} \Lambda x^3, \]  
(33)

\[ Y = (\mu + p)^2 + By + Cy^2 + \frac{1}{12} \Lambda y^3. \]  
(34)

Please note that it was proven in [38] that CCLP is the unique minimal gauged supergravity spacetime with torsion such that the torsion tensor is both closed \((dT = 0)\) and co-closed \((\delta T = 0)\). We then find following [26] that, \(F\), the newly discovered PGCKY in our case is:

\[ F = \sqrt{-x \tilde{\omega}^x \wedge \omega^x + \sqrt{-y \tilde{\omega}^y \wedge \omega^y}} \]  
(35)

and the corresponding Killing tensor-

\[ K = y(\omega^x \omega^x + \tilde{\omega}^x \tilde{\omega}^x) + x(\omega^y \omega^y + \tilde{\omega}^y \tilde{\omega}^y) + (x+y)\omega^x \omega^x. \]  
(36)

Note that \(F\), the Killing-Maxwell field above, is different from the electromagnetic field in the CCLP black hole spacetime, as cited in [26]. Note that \(K\) is directly involved in separating the Hamilton-Jacobi and Klein-Gordon equations in this spacetime.

We can now proceed to write down a couple of new GCCKY tensors for the above metric:

\[ h_{ab} = 4\omega^{\{a}(\partial_{b})_{\}c) \]  
(37)

which obeys the equations \((\xi = (\partial_{\psi}))\)
\[ \nabla_c h_{ab} = 2 g_{c[a} \xi_{b]} \]  
and
\[ \xi_b = \frac{1}{D - 1} \nabla_d h^d_b. \]

In a similar way we can retrieve:
\[ h_{ab} = 4 \omega_{[a} (\partial_{\psi})_{b]}. \]

Consequently the tensors above generate 2 new Stäckel-Killing tensors via the following relation:
\[ K_{ab} = h_{ac} h_{b}^c - \frac{1}{2} g_{ab} h^2 \]

which are -
\[ K^{\psi}_{ab} = 16 \omega_{[a} (\partial_{\psi})_{c]} \omega_{[b} (\partial_{\psi})^{c]} - 4 g_{ab} (\omega_d \omega^d (\partial_{\psi})_c (\partial_{\psi})^c - \omega_d \omega^c (\partial_{\psi})_c (\partial_{\psi})^d), \]

respectively
\[ K^{\phi}_{ab} = 16 \omega_{[a} (\partial_{\phi})_{c]} \omega_{[b} (\partial_{\phi})^{c]} - 4 g_{ab} (\omega_d \omega^d (\partial_{\phi})_c (\partial_{\phi})^c - \omega_d \omega^c (\partial_{\phi})_c (\partial_{\phi})^d). \]

Also for spacetimes with \( D \geq 6 \) where an electromagnetic field is present and a Killing-Maxwell system can be constructed, the PGCKY is, naturally, in these spacetimes still the Killing-Maxwell electromagnetic tensor, \( F \), which generates a tower of Killing-Yano and Stäckel-Killing tensors as follows:
\[ F^{(j)} = F \wedge \cdots \wedge F. \]

Above the wedge is taken \( j \) times, \( F^{(j)} \) is a \((2j)\)-form and \( F^{(1)} = F \). \( F^{(j)} \) is a set of \((n-1)\) non-vanishing closed CKY (conformal Killing-Yano) tensors, where \( D \), the dimension of the spacetime is:
\[ D = 2n + \epsilon. \]

Here \( \epsilon = 0, 1 \) depending on whether \( D \) is even or odd. And this generates the towers of \((n-1)\) rank \((D-2j)\) Killing-Yano tensors:
\[ Y^{(j)} = * F^{(j)} \]
and \((n-1)\) rank-2 Stäckel-Killing tensors:
\[ K^{(j)}_{\mu \nu} = Y^{(j)}_{\mu \rho_1 \cdots \rho_{D-2j-1}} Y^{(j)\rho_1 \cdots \rho_{D-2j-1}}_{\nu}. \]
Now let’s turn our attention to the spinning point particle in the presence of torsion. The rank-2 quantum phase space Dirac-type operator in the presence of torsion when torsion is a Killing-Yano tensor (Y is the Killing-Yano tensor that generates the corresponding Dirac-type operator) writes:

\[
\hat{Q}_Y^A = \gamma^\mu Y_\mu^\nu D_A^\nu - \frac{1}{6} \gamma^\mu \gamma^\nu \gamma^\rho \nabla_\mu Y_{\nu\rho}.
\]  

(48)

We found this operator having as a starting point the ansatz (A12) of [26]. It generates an additional supersymmetry of the spinning point particle in an arbitrary spacetime endowed with a Killing-Yano torsion that admits a rank-2 generalized Killing-Yano tensor. We note that this operator’s form is very close to equation (15), hence the Dirac-type operators in spacetimes with Killing-Yano torsion differ from the Dirac-type operators in torsionless spacetimes only in their spin derivatives. We can see that we have good agreement with equation (A20) of [26], taking into account equation (12). By taking the anticommutator with the Dirac operator with Killing-Yano torsion - equation (5) (see also [11]) - we can see by direct verification that there are no anomalies, which is the anticommutator vanishes.

The higher rank Dirac-type operators with Killing-Yano torsion anticommute/commute (for p even/odd) with the Dirac operator as well (by direct verification) and they are:

\[
\hat{Q}^{A,p}_Y = \gamma^{\mu_1} \ldots \gamma^{\mu_{p-1}} Y_{\mu_1 \ldots \mu_{p-1}}^\nu D^{A}_\nu - \frac{(-1)^p}{2(p+1)} \gamma^\nu \gamma^{\mu_1} \ldots \gamma^{\mu_p} \nabla_\nu Y_{\mu_1 \ldots \mu_p}.
\]  

(49)

By comparison with results in [27], we find agreement if we notice that in fact (relations (2.3) therein) are null, in the case of Killing-Yano tensors, according to relation (12) in this paper. Also note that \( T \wedge \delta^\tau \omega = 0 \) in this case and hence relations (4.8) and (4.9) in [27] cancel out. So there are no anomalies for the supersymmetries described by equation(30).

5 Killing spinors in D=5 minimal gauged supergravity

It is well-known that there is a close intertwining between the existence of Killing spinors and Killing-Yano tensors and other structures and that Killing spinors have been widely used to classify solutions of supergravity in various dimensions. The supersymmetric solutions of minimal gauged supergravity were classified in [39]. In particular for D=5 the Killing spinors obey the equation:

\[
[D_a + \frac{1}{4\sqrt{3}}(\gamma^\beta_\alpha - 4\delta^\beta_\alpha \gamma^\gamma)F_{\beta\gamma}]e^a - \chi e^{ab}(\frac{1}{4\sqrt{3}}\gamma_\alpha - \frac{1}{2}A_\alpha)e^b = 0,
\]  

(50)

where \( \chi \) is a real constant and \( e^{ab} \) the Levi-Civita tensor. We have shown in the previous section that this spacetime admits at least one Killing-Yano tensor.
Lemma. If a spacetime admits a (generalized) rank 2 Killing-Yano tensor, then it admits a Killing spinor as well.

Proof. The existence of a (generalized) rank 2 Killing-Yano tensor, implies the separability of the Dirac equation and hence the existence of a Majorana spinor. Since the manifold is symplectic, one can construct a symplectic spinor bundle, whose section is a symplectic spinor. Hence the Majorana spinor (which exists for this type of manifold) is symplectic and hence it is a Killing spinor for the spacetime.

The same holds true for a (generalized) conformal Killing-Yano tensor. If one exists, then a Killing spinor exists as well. To be noted that when a Killing spinor $\psi$ exists, then a tower of (generalized) Killing-Yano tensors can be constructed as (p odd):

$$ Y_{\mu_1...\mu_p} = \bar{\psi} \gamma_{\mu_1...\mu_p} \psi, $$

(51)

where

$$ \gamma_{\mu_1...\mu_p} = \gamma_{[\mu_1...\mu_p]}. $$

(52)

Similarly, if the Killing spinor exists, for p even a similar tower of (generalized) CKY tensors can be constructed for the respective spacetime, be it endowed with torsion or not.

6 Conclusions

We overviewed briefly some of the symmetries of the Kerr-Newman, D=5 minimal gauged supergravity endowed with a Killing-Maxwell system and higher dimensional spacetimes with Killing-Yano torsion, to find that if a Killing-Maxwell electromagnetic field is present, it becomes the PCKY or the PGCKY in the cases with torsion and that it generates towers of Killing-Yano and Stäckel-Killing tensors of the spacetime. For the studied 5-dimensional spacetime the Hodge dual of the Killing-Maxwell electromagnetic field plays the role of torsion and is at the same time a generalized Killing-Yano tensor of the spacetime, being derived naturally from the PGCKY. Some results regarding Killing spinors for the 5-dimensional minimal gauged supergravity are derived.

The generalized Killing-Yano tensors together with covariant derivatives form Dirac-type operators which are not anomalous and characterize additional supersymmetries of the spinning point particle in curved spacetimes with Killing-Yano torsion. It is very interesting that these supersymmetries are generated by the Killing-Maxwell electromagnetic field when this is present and this sparks further investigation of the correlation between the electromagnetic gauge symmetry and supersymmetry in curved spacetimes. Also another interesting track would be to determine the dual (torsionless) spacetime of the D=5 minimal gauged supergravity spacetime. Moreover, it would be interesting to see whether
other Killing-Yano tensors exist in the studied supergravity spacetime (of dimensionality 5) except for the Hodge dual of the Killing-Maxwell electromagnetic field.

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