Research Article

Research on Output Signal Controlling of an Asymmetric Hydraulic Cylinder Based on a Flexible Connection

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In industrial production, the structures of hydraulic servo system-connecting device-load systems are often simplified as rigid connections for the ease of calculation. However, this simplification is problematic when applied to flexible connections in hydraulic systems, which generally have multivariable and strong couplings; these characteristics affect the control accuracy of the hydraulic servo system and lead to serious distortion of the output waveform, which cannot be ignored. These problems cause greater lag and attenuation of the actual signal than those of the expected signal, leading to lower credibility. Therefore, it is important to study the waveform distortion caused by flexible connections. In this paper, according to the characteristics of a flexible connection, a corresponding mathematical model is established, and an adaptive controller, whose structure is simple and calculation cost is low, is used to adjust the amplitude and phase of the response signal and improve the accuracy of the system response. Treating the change in the response signal as the error value, the algorithm weights are adjusted until the error value is stable. Then, a more accurate output signal is obtained. Finally, the validity and practicability of the adaptive controller are verified by simulation experiments.

1. Introduction

Due to the application of servo valves, hydraulic servo systems are gradually being widely applied in industry. Generally, the displacement and control quantity are the controlled quantities of hydraulic servo systems. The control system is required to have highly dynamic characteristics and to reproduce the input signal accurately, which makes it a crucial component of industrial equipment and other applications [1, 2]. To facilitate the design and application of systems in related fields, the structure of a hydraulic servo system-connecting device-load system is often adopted, and it is often regarded as an entirely rigid system. That is, under this assumption, the middle connecting structure of the mechanical system does not deform, and its corresponding elastic coefficient is considered to be infinite. Although this simplified form offers convenience, the elasticity cannot be ignored in specific situations. Elasticity not only causes deterioration of the control performance and unexpected oscillation in the output but also increases the adjustment time and even causes loss of system stability. This leads to failure in obtaining accurate system position and speed data and affects the control condition [3, 4].

Nonlinear factors such as friction, clearance, and dead zones often reduce the accuracy and even cause instability. For electrohydraulic servo systems, various mechanical, hydraulic, electronic, and other nonlinear phenomena are integrated into one phenomenon, the flow-pressure characteristic. In hydraulic systems, the parameters related to the flow rate, pressure, and viscosity of oil vary with time [5, 6]. Therefore, when the electrohydraulic servo system is excited by a sine signal, the higher harmonics appear in the acceleration output, which seriously distorts the signal [7]. In addition, due to servo valve manufacturing, adjustment, and assembly error, zero offset is often inevitable. Although usually adjusted to a minimum, zero offset typically increases due to looseness or distortion of some centering components caused by vibration. For high-precision and
high-response systems, zero offset reduces the accuracy and response speed of the system. The influence of friction nonlinearity on the hydraulic system is also very complex, and Coulomb friction causes dead zone characteristics. At low speed, viscous friction produces additional nonlinearity such as hysteresis, while at high speed, the influence of viscous friction can be ignored.

In practical applications, the changes in load and flexible connections limit the improvement potential of hydraulic servo system performance. In the traditional design, the control parameters of the hydraulic servo system do not change with the load inertia. When the load inertia changes, the performance deteriorates. Therefore, realizing a fast response or high dynamic stiffness of the system is a common problem. Many theoretical studies have been performed on the optimization of the flexible connection load control. The research in this field involves a variety of control methods. Commonly used traditional methods include fuzzy control and PID robust control. In addition, there are many intelligent algorithms such as the network control, adaptive control, repetitive control, and genetic algorithm that provide a theoretical basis for solving problems [8–11]. Lee et al. proposed a controller algorithm using state observer to monitor the change in disturbance torque to realize signal tracking at the load side [12]. Umemo et al. generated a robust speed control system of a servo motor based on a parametric two-degrees-of-freedom controller, which improved the input response performance of the system [13]. Nayyerloo et al. studied the nonlinear structure under vibration and proposed an improved LMS-based SHM algorithm to estimate the output velocity signal more accurately and its influence on vibration suppression [14]. For a class of high-order nonlinear systems with mismatched uncertain parameters and external loads, Guan and Zhu proposed a multisliding mode-based robust adaptive control strategy to reduce the tracking error caused by system uncertainty [15]. Rozaimi et al. used sliding mode control (SMC) to evaluate the position tracking performance of an electrohydraulic servo (EHS) system, an approach that was established in the process of considering the nonlinear friction model [16].

Although the aforementioned methods can improve the performance of the output signal, they seldom consider the control situation of flexible connections based on asymmetric actuators. In this paper, the asymmetric hydraulic cylinder servo mechanism is used as an experimental platform to construct a two-mass flexible connection to improve the accuracy of the servo mechanism response signal and servo control strategy based on adaptive control is studied. The paper is primarily focused on the two-mass flexible connection state of an asymmetric hydraulic cylinder, for which the corresponding mathematical model is established. The mathematical factors of the coupling phenomenon can be understood from the model. The adaptive algorithm formula is used to construct an adaptive output control to ensure the accuracy of the output signal. From the derived adaptive formula, it is seen that the value of the step size parameter is important to determine for accurate control. Through simulations and experiments before and after the input of the sinusoidal signal, the fuzzy PID algorithm is compared with the adaptive algorithm. It is proven that the adaptive output control has good reproducibility for the signal and effectively reproduces the amplitude and phase characteristics of the input signal without identifying the system model. The adaptive output control is not specific to a certain kind of nonlinearity. In the application, the adaptive algorithm needs only to adjust the step size. Therefore, it has advantages of low computation costs, a simple structure, and wide application range.

2. Mathematical Modeling of Asymmetric Cylinder with Two-Mass Flexible Connection

Asymmetric hydraulic cylinders have advantages such as small space, a simple structure, convenient operation, and low cost. Therefore, asymmetric actuators have been effectively applied to some specific hydraulic control systems such as multipoint linkage compound hydraulic loading control systems, fatigue strength simulation tests of mechanical vibration parts, simulation operations of driving control systems, and simulation operation tests of hydraulic vibration systems based on servo mechanical shaking tables.

A sketch of the asymmetric hydraulic cylinder system including various parameters is shown in Figure 1. Ignoring the influence of friction on the load, the system is regarded as a two-mass system model for analysis under the effect of a flexible connection. It is controlled by an ideal zero-opening four-sided slide valve, and the hydraulic cylinder leakage and compressibility of the hydraulic oil are not taken into account. By default, the hydraulic cylinder load remains constant, the supply pressure remains unchanged, and the return oil pressure is zero. The mathematical modeling takes the research object of the hydraulic system as the main condition, while the secondary condition is not considered.

Considering the convenience and main factors of actual hydraulic modeling, it is necessary to make the following assumptions: there is no pressure loss in the pipeline; the pressure loss at the local and inflection points is not considered; the hydraulic oil temperature is in an ideal state; the hydraulic oil density is constant or the changes are small enough to be ignored; and the internal or external leakage is in a laminar flow state.

According to the actual model of an asymmetric hydraulic cylinder, the selected ratio is \( a = (\frac{A_2}{A_1}) \); therefore, the flow relationship between the left and right chambers is

\[
q_2 = aq_1. \tag{1}
\]

Based on hydraulic industry regulations, the right movement of the valve core is in a positive direction, and the movement direction of the valve core is divided into two kinds of motion situations. When the spool moves in the right direction, that is, when the spool displacement \( x_r > 0 \), the linearized flow continuity equation is

\[
q_L = K_f x_r - K_p P_L. \tag{2}
\]

When the slide valve moves in the left direction, that is, when the displacement \( x_r < 0 \), the piston is in the situation of
a left retraction. The linear flow continuity equation of the retraction motion is

\[ q'_L = K'_q x_v - K'_p p'_L. \]  

(3)

The flow equation in the two cavities of the hydraulic cylinder under ideal conditions is as follows. When the slide valve moves in the right direction, the friction and pressure loss of the hydraulic servo system are not considered. The quality factor of liquid flow is ignored.

\[ \begin{align*}
q_1 &= C_{ip} (p_1 - p_2) + C_{cp} p_1 + \frac{V_1}{\beta_c} \frac{dp_1}{dt} + \frac{A_1}{2} \frac{dy}{dt}, \\
q_2 &= C_{ip} (p_1 - p_2) - C_{cp} p_2 - \frac{V_2}{\beta_c} \frac{dp_2}{dt} + \frac{A_2}{2} \frac{dy}{dt}.
\end{align*} \]  

(4)

Based on the possible leakage of hydraulic parts and the compressibility of hydraulic oil, the equations of flow and load are established as follows:

\[ q_L = C_{ip} (p_1 - p_2) + C_{cp} p_1 + \frac{1 + \alpha^2}{1 + \alpha^2} \frac{V_e}{\beta_c} \frac{dp_1}{dt} + \frac{\alpha (A_1 + A_2)}{2} \frac{dy}{dt}, \]  

(5)

\[ \begin{align*}
q'_L &= C_{ip} (p'_1 - p'_2) + \frac{V_1}{\beta_c} \frac{dp'_1}{dt} + \frac{A_1}{2} \frac{dy}{dt}, \\
q'_2 &= C_{ip} (p'_1 - p'_2) - C_{cp} p'_2 + \frac{V_2}{\beta_c} \frac{dp'_2}{dt} + A_2 \frac{dy}{dt}.
\end{align*} \]  

(7)

Because the equivalent volume \( V_e \) is a function of displacement, it is necessary to simplify the constant in order to obtain a linear expression of the flow rate.

Similarly, considering the possible leakage of hydraulic parts and the compressibility of hydraulic oil, the equation of the load flow and load is established as follows:

\[ q'_L = C_{ip} (p'_1 - p'_2) + C_{cp} p'_2 + \frac{1 + \alpha^2}{1 + \alpha^2} \frac{V'_e}{\beta_c} \frac{dp'_1}{dt} + \frac{(A_1 + A_2)}{2} \frac{dy}{dt}. \]  

(8)

The equation relating load flow, load pressure, and hydraulic oil source is obtained as follows:

\[ q'_L = C_{ip} p'_2 + \frac{1 + \alpha^2}{1 + \alpha^2} \frac{V'_e}{\beta_c} \frac{dp'_1}{dt} + \frac{(A_1 + A_2)}{2} \frac{dy}{dt}. \]  

(9)

Considering the effect of the viscous damping force, the force balance equation based on two-degrees-of-freedom is as follows:

\[ \begin{align*}
A_1 p_1 - A_2 p_2 &= m_p \frac{d^2 y_p}{dt^2} + B_L \frac{dy_p}{dt} + K_L y_p - B_L \frac{dy_l}{dt} - K_L y_L \text{ right,} \\
A_2 p_2 - A_1 p_1 &= m_p \frac{d^2 y_p}{dt^2} + B_L \frac{dy_p}{dt} + K_L y_p - B_L \frac{dy_l}{dt} - K_L y_L \text{ left,}
\end{align*} \]  

(10)

According to the previous formula, the Laplace transform is carried out and sorted out. Consider when the spool valve core moves to the left and the piston of the hydraulic cylinder retracts, by comparing equations (3), (9), (11), and (12), the following expressions are obtained:

\[ \begin{align*}
Y_p &= \frac{K_q m_s^2 + B_L s + K_L}{A_i D(s)}, \\
D(s) &= \frac{1 + \alpha^2}{1 + \alpha^2} \frac{m_p m_p}{K_h s^5} + \left[ \frac{1 + \alpha^2}{1 + \alpha^2} \frac{B_L (m_l + m_p)}{K_h} + \frac{K_m m_p}{A_i} \right] s^4 \\
&+ \left[ \frac{a + 1}{2} B_L + \frac{K_q B_L (m_l + m_p)}{A_i} \right] s^2 + \frac{a + 1}{2} K_L s.
\end{align*} \]  

(13)
\[
\frac{Y_p}{X_v} = \frac{K'_q a(m_l s^2 + B_l s + K_l)}{A_l D(s)},
\]
\[
D(s) = \frac{1 + a^2 m_m s^5}{1 + a} K_h^{-1} + \left[ \frac{1 + a^2}{1 + a} \frac{B_l (m_l + m_p)}{K_h} + \frac{K'_c m_l m_p}{A_l} \right] s^3
\]
\[
+ \left[ \frac{1 + a}{2} \frac{K'_c B_l (m_l + m_p)}{A_l} \right] s^2 + \frac{1 + a}{2} K_l s.
\]

From this analysis, it is seen that the coupling effect of the hydraulic system constitutes internal resonance. Compared with the mathematical formula of an inertial system, the mathematical formula of a hydraulic system with a flexible connection adds two links, namely, an oscillation link and a second-order differential link. Due to the uncertainty and nonlinearity of the coupling characteristics, it causes oscillations in the system bandwidth.

### 3. Research on Control Strategy

#### 3.1. Background of the Problem

A sinusoidal signal is commonly used as a given signal in hydraulic control. However, due to the influence of nonlinear factors of a hydraulic system and the limitation of hydraulic valve characteristics, the closed-loop frequency characteristics of the hydraulic system are inferior, which make it is impossible to obtain an accurate output signal. The influence of the servo valve dead zone and flow nonlinearity on the system leads to large waveform distortion and phase lag. Both the positive overlap of the slide valve and the Coulomb friction cause a dead zone characteristic, which has a great influence on the static error of the system but generally does not damage its stability. The dead zone characteristic delays the response of the system, which is the main reason for the distortion of the system response waveform. Therefore, the peak value of the output sinusoidal response signal deviates. Even when a feedforward control is used, the sinusoidal signal cannot be accurately obtained.

In a servo hydraulic system, the sinusoidal response signal often leads to phase lag and amplitude attenuation, especially phase lag; neither high tracking accuracy of the sinusoidal response signal at each frequency point nor reaching the specified signal amplitude or phase is guaranteed [17]. Therefore, to realize the method of embedded controller, the amplitude and phase of the fundamental frequency response should be adjusted necessarily.

#### 3.2. Fuzzy PID Control

Generally, the fuzzy PID controller is used as a control method. It takes the variable error \( e \) and error change rate \( \Delta e \) as the inputs. The error signal \( e \) and its change rate \( \Delta e / \Delta t \) are obtained by using fuzzy rules in the feedback system to calculate the control quantity [18, 19].

The structure of a typical adaptive fuzzy PID controller is shown in Figure 2.

The setting process of the algorithm is summarized by three standard rules as follows:

1. When the error value \( e \) is too large, it is necessary to accelerate the response speed of the system. To avoid overshooting the system, a smaller \( K_p \) value is chosen and the value of \( K_d \) is increased as much as possible.

2. When the error value \( e \) is moderate, to prevent excessive overshoot, a smaller \( K_p \) value is usually selected. In addition, it is necessary to select a value in line with \( K_c \) and \( K_d \), otherwise, the running speed of the system is affected.

3. When the error value \( e \) is small, the selected \( K_p \) and \( K_c \) values should be increased to a certain extent to meet stability requirements. It is also necessary to select an appropriate value of \( K_d \) according to the error change rate; when the error change rate is small, the value of \( K_d \) is generally moderate to meet the requirements of the system. When the error change rate is large, \( K_d \) needs to be smaller.

In the actual operation process, the basic process of fuzzy PID control parameter tuning is shown in Figure 3. The fuzzy PID algorithm is based on the error and error rate of change, which are used to calculate the values of the adjustment control quantities \( \Delta K_p \), \( \Delta K_q \), and \( \Delta K_d \). The type of the controller is the input of two parameters and the output structure of three parameters. The structure of the constructed fuzzy controller is shown in Figure 4. In the process of fuzzification and defuzzification, a membership function is used as a tool to establish the relationship between the control parameters and control rules. Through the fuzzy universe of parameter variables in the algorithm, the definitions of error \( e \), error rate \( \Delta e \), \( \Delta K_p \), \( \Delta K_q \), and \( \Delta K_d \) are determined and expressed as a fuzzy set \( \{ NB, NM, NS, ZO, PS, PM, PB \} \), as shown in Table 1. Finally, the fuzzy controller is set up after the rule table is established [20, 21].

Although the fuzzy PID algorithm has many advantages, it has high requirements for control rules, a lack of systematic control design, and limited application scope. The simple fuzzy processing of information often leads to a reduction in control accuracy and the deterioration of dynamic quality, and the control purpose cannot be defined. Solving the problem of multivariable coupling and higher-order functions is not ideal. Therefore, it is necessary to introduce better methods to solve complex problems in hydraulic servo systems [22].

#### 3.3. Adaptive Filter Theory

To achieve higher output accuracy and ensure an online embedded controller to implement the control strategy, a control strategy based on linear adaptive filter theory is a more feasible scheme.

A detailed block diagram of a transverse filter that realizes the whole filtering process under the control of the adaptive control algorithm is shown in Figure 5. The tap input is the sequence of input vectors \( u(n) \) representing \( M \times 1 \), and the tap weight is the sequence of matrix \( M \times 1 \) tap...
weight vectors \( \omega_0(n) \). To obtain an estimated expected value that is close to the value of the Wiener solution, the number of iterations tends to infinity. Under the given input, the output is the estimated value of the expected response \( \hat{d}(n) \), that is, the estimated value of the expected response \( d(n) \). In fact, the difference between the output of the filter and the expected response is defined as the estimation error, and the estimation error \( e(n) \) and the tap input vector \( u(n) \) are inputs of the adaptive algorithm, thus forming closed-loop feedback [23, 24].

Assuming that the step size parameter is selected to take the value of \( \mu \) appropriately and the gradient vector \( \nabla J(n) \) of each iteration \( n \) is accurately measured, then the tap vector will converge to the Wiener solution [25, 26]. The estimation formula of the gradient vector \( \nabla J(n) \) is as follows:

\[
\nabla J(n) = -2u(n)d^*(n) + 2u(n)u^H(n)\hat{\omega}(n). 
\]

(15)

As the transient estimation of the expected response and tap input vector, the expression is as follows:

\[
\hat{R}(n) = u(n)u^H(n), \quad \hat{P}(n) = u(n)d^*(n). 
\]

(16)

Superscript \( H \) means Hermitian transpose and \( * \) means complex conjugate. \( \hat{\omega} \) represents the tap weight vector. The transient estimation of gradient vector is generally biased, and the expression is as follows:

\[
\hat{\nabla}J(n) = -2u(n)d^*(n) + 2u(n)u^H(n)\hat{\omega}(n). 
\]

(17)

Based on this, a new formula to update the tap vector is obtained. The recursive formula is as follows:

\[
\hat{\omega}(n + 1) = \hat{\omega}(n) + \mu u(n)[d^*(n) - u^H(n)\hat{\omega}(n)]. 
\]

(18)
Table 1: Representation meaning of parameters in fuzzy sets.

| Symbol | NB  | NM  | NS  | ZO  | PS  | PM  | PB  |
|--------|-----|-----|-----|-----|-----|-----|-----|
| Meaning | Negative big | Negative middle | Negative small | Zero | Positive small | Positive middle | Positive big |

Set the tap input vector of transverse filter as

\[ \mathbf{u}(n) = [u(n), u(n - \tau)]^T = [A \sin(\omega n), A \cos(\omega n)]^T, \]

where \( \tau \) is the sampling time of \( \cos(\omega n) \) lagging \( \sin(\omega n) \). The output of transverse filter is as follows:

\[ \mathbf{u}_c(n) = \tilde{\mathbf{w}}^H(n)\mathbf{u}(n), \]

where \( \tilde{\mathbf{w}} = [w_1 \ w_2]^T \). Equation (19) is introduced into equation (20) to obtain the following:

\[ u_c(n) = w_1 A \sin(\omega n) + w_2 A \cos(\omega n) \]

\[ = A \sqrt{w_1^2 + w_2^2} \left( \frac{w_1}{\sqrt{w_1^2 + w_2^2}} \sin(\omega n) + \frac{w_2}{\sqrt{w_1^2 + w_2^2}} \cos(\omega n) \right). \]

Make \( \cos(\phi) = (w_1/\sqrt{w_1^2 + w_2^2}), \sin(\phi) = (w_2/\sqrt{w_1^2 + w_2^2}), \) and \( A_c = A \sqrt{w_1^2 + w_2^2} \), so it is obtained that

\[ u_c(n) = A_c \sin(\omega n + \phi). \]

For the stability of the adaptive algorithm, the eigenvalues of the correlation matrix of the correlation matrix are closely related to the adaptive algorithm. For the input vector of equation (19), the corresponding characteristic equation is written as

\[ R = \begin{bmatrix} E[u_c^2(n)] & E[u_c(n)u_c(n - \tau)] \\ E[u_c(n - \tau)u_c(n)] & E[u_c^2(n - \tau)] \end{bmatrix}. \]

For the value of \( E[u_c^2(t)]\) and \( E[u_c(t - \tau)u_c(t)] \), \( u(t) \) is a periodic signal and \( E[u(t)] = \lim_{T \to \infty} \int_{-T/2}^{T/2} u(t) \, dt \), so there is

\[ E[u(t)u(t - \tau)] = E\left[A_c^2 \sin(\omega t)\cos(\omega t)\right] = A_c^2 \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \sin(\omega t)\cos(\omega t) \, dt = 0. \]

From this, a conclusion is obtained:

\[ E[u_c(n - \tau)u_c(n)] = E[u_c(n)u_c(n - \tau)]. \]
For the same principle, it is obtained that

\[ E[u^2(t)] = E[A^2\sin^2(\omega t)] = A^2 \lim_{T \to \infty} \frac{1}{T} \int_0^T \sin^2(\omega t) dt = \frac{A^2}{2}. \]  

(27)

It is seen from equation (27) that

\[ E[u^2(n)] = E[u^2(n-T)] = \frac{A^2}{2}. \]  

(28)

The correlation matrix can be obtained by substituting equations (26) and (28) into equation (24)

\[ R = \begin{bmatrix} \frac{A^2}{2} & 0 \\ 0 & \frac{A^2}{2} \end{bmatrix}. \]  

(29)

The eigenvalue of the matrix is obtained as follows:

\[ \lambda_{1,2} = \frac{A^2}{2}. \]  

(30)

Combining the output of \( y(n) = \mathbf{w}^H(n)u(n) \) with estimation error \( e(n) = d(n) - y(n) \) and the definition of the weight error vector, the estimation error is as follows:

\[ e(n) = d(n) - \mathbf{w}^H(n)u(n) = d(n) - \mathbf{w}_{0}^H(n) + \mathbf{e}^H(n)u(n) = e_0(n) + \mathbf{e}^H(n)u(n) = e_0(n) + \mathbf{e}_0^H(n)u(n). \]  

(31)

The sign \( e_0(n) \) in the formula represents the weight error vector, so its mean square error is as follows:

\[ J(n) = E[|e(n)|^2] = E\left[ e_0^2(n) + e_0^H(n)e_0(n) \right] = J_{\min} + 2\text{Re}\left[ E\left[ e_0(n)\mathbf{e}_0^H(n)u(n) \right] \right] + E\left[ \mathbf{e}_0^H(n)u(n)\mathbf{e}_0^H(n)u(n) \right]. \]  

(32)

According to the small step hypothesis, it is known that

\[ E[e_0^2(n)e_0^H(n)u(n)] = 0. \]  

(33)

It is deduced from the same principle that

\[ E[e_0^H(n)u(n)e_0(n)] = \sum_{k=1}^{M} \lambda_k E[|v_k(n)|^2]. \]  

(34)

\( v_k(n) \) is expressed as the \( k \)th natural mode of the filter [27]. By substituting equations (33) and (34) into equation (32), the following is obtained:

\[ J(n) = J_{\min} + \mu J_{\min} \sum_{k=1}^{M} \lambda_k \left( |v_k(0)|^2 - \frac{\mu J_{\min}}{2 - \mu \lambda_k} \right) (1 - \mu \lambda_k)^{2n}. \]  

(35)

When the value \( \mu \) is small, equation (35) is simplified as

\[ J(n) \approx J_{\min} + \mu J_{\min} \sum_{k=1}^{M} \lambda_k \left( |v_k(0)|^2 - \frac{\mu J_{\min}}{2} \right) (1 - \mu \lambda_k)^{2n}. \]  

(36)

The mean square value of estimation error consists of two parts: the independency depends on the steady-state component of iteration number \( n \) and the transient component. The cost function \( J(n) \) at \( n \) time is controlled by an exponential factor \( (1 - \mu \lambda_k)^{2n} \). When the step size is small, the exponential factor \( (1 - \mu \lambda_k)^{2n} \) will decay to zero as the iteration increases [28, 29]. In this case, the learning curve decays to a constant value, and there is no oscillation. The specific expression is as follows:

\[ J(\infty) \approx J_{\min} + \frac{\mu J_{\min}}{2} \sum_{k=1}^{M} \lambda_k. \]  

(37)

Therefore, it can be seen from equation (37) that \( \mu \) must satisfy the following inequality to make the cost function converge:

\[ 0 < \mu < \frac{2}{\lambda_{\max}}. \]  

(38)

where \( \lambda_{\max} \) is the largest eigenvalue of the tap vector correlation matrix \( R \). According to formula (38), the necessary conditions for stability are as follows:

\[ \mu < \frac{4}{A^2}. \]  

(39)

which is a necessary condition for the adaptive algorithm to achieve convergence, so in order to ensure the safety, the
step convergence condition is relatively strict, and range of weights is usually limited based on the prior knowledge.

4. Case Analysis

CQYZ-MDH high-pressure and large-flow electrohydraulic servo performance experiment table is a new type of electrohydraulic valve experiment table designed according to the characteristics of ten-way hydraulic valve. In order to simulate the actual industrial pressure and flow, the maximum pressure can reach 21 MPa and the maximum flow can reach 60 L/min. It is used to design the dynamic performance experiment of the servo system with numerical pressure less than 21 MPa. Based on this, the static and dynamic performance experiments of servo valve can also be completed. The experimental device is shown in Figure 7, and its specific parameters are shown in Table 2. The flexible components used for the connection of the hydraulic servo system are two slender hydraulic damping rods, which can provide the load stiffness slightly less than hydraulic spring stiffness. The two ends of the flexible rod are, respectively, connected with the mass blocks at both ends.

The principle of the test system is shown in Figure 8, which is composed of hydraulic valve, computer, PCI8335A-A/D, D/A acquisition card, proportional amplifier, flowmeter, sensor, and regulated power supply (+24 V). It can implement the function of data acquisition and processing and draw the processed image. To automate the testing process, the computer outputs control signals and controls the hydraulic system. As shown in Figure 9, the multifunction data acquisition card of PCI8335A has the functions such as digital input, analog input and output, and counter.

**Simulation Analysis.** When the step $\mu$ is 0.002, the expected response of the input adaptive controller is $2\sin 12t$ (m/s$^2$). Figure 10 is the comparison diagram of input signal, fuzzy PID control, and adaptive control output signal, and Figure 11 is the error convergence comparison curve. It is seen that the output curve of adaptive controller is basically consistent with the output after reaching stability, while the fuzzy PID control has a certain amplitude attenuation and phase lag, which needs a longer stability time and exists a certain amount of error.

When the step $\mu$ is increased to 0.005, the input signal response is still $2\sin 12t$ (m/s$^2$). Figure 12 is the diagram of

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**Table 2: Hydraulic servo system parameters.**

| Parameter name              | Parameter symbol | Unit            | Parameter value                      |
|-----------------------------|------------------|-----------------|-------------------------------------|
| Area of rodless cavity      | $A_r$            | m$^2$           | $1.256 \times 10^{-3}$              |
| Area of rod cavity          | $A_o$            | m$^2$           | $3.14 \times 10^{-4}$               |
| Flow gain of hydraulic valve| $K_q$            | m$^3$/s         | 1.2                                 |
| Effective bulk modulus of elasticity | $\beta_e$    | MPa             | 690                                 |
| Stiffness of hydraulic spring | $K_h$         | N/m             | $7.91 \times 10^6$                  |
| Total flow pressure         | $K_{ce}$         | m$^3$/(N·s)     | $10.9 \times 10^{-13}$              |
| Stiffness of flexible spring | $K_L$           | N/m             | $8.11 \times 10^{-13}$              |
| Viscous damping coefficient | $B_L$           | N/(m/s)         | 100                                 |
| Mass                        | $m_L$            | kg              | 8                                   |

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**Figure 7: Two-mass connection hydraulic servo system.**

**Figure 8: The principle of the test system.**
Figure 9: PCI8335A acquisition card.

Figure 10: Sine signal before and after control.

Figure 11: Error curve comparison.
curve after control, and the error convergence comparison curve is shown in Figure 13. The output curve of fuzzy PID control has little change, but the adaptive control output has a certain distortion, and the difference becomes larger, which indicates that the mean square error has increased significantly at this time. It is concluded that a larger step size should be chosen carefully; otherwise, a larger gradient noise will be produced.

**Experimental Analysis.** When the input signal is $2\sin t$ (V) test signal, the step parameter is 0.0003, and Figure 14 is the comparison diagram before and after control. It is seen from the figure that the adaptive controller has good control performance, quickly reaches the steady state, and better reflects the input signal than the fuzzy PID algorithm. The fuzzy PID controller has a certain error value at the peak value. From Figure 15, with the continuous adjustment of
adaptive control, the instantaneous value of error decreases rapidly which reaches a stable state and does not fluctuate in a small range. However, the error of fuzzy PID control is relatively large.

5. Conclusions

The model of hydraulic servo actuators and flexible connection systems is the most commonly used model in this field. It is usually regarded as a rigid connection for the convenience of modeling and calculation. However, there are many complex modeling states in the actual system that cannot be ignored. The system is usually affected by nonlinear factors and coupling, which lead to distortion and a large error in the input signal.

In this paper, an adaptive controller is used to control and adjust the output signal response. The formula shows that the parameter selection of the step size has a great influence on the control accuracy. The accuracy of the output signal is improved by choosing the step size appropriately. Based on simulation and experiment, the adaptive algorithm exhibits better control precision than the fuzzy PID algorithm.

However, there are still some shortcomings in this paper; the hydraulic servo system is only a single-degree-of-freedom platform. For complex hydraulic servo mechanisms, such as electrohydraulic servo shaking tables with multiple degrees of freedom, due to the existence of multiple hydraulic cylinders and redundancy, the output signal is not only distorted but may also exhibit multiple harmonics. Therefore, the algorithm needs to be verified by a greater number of experimental platforms.

Abbreviations

| Symbol | Description |
|--------|-------------|
| A1, A2 | Effective area of rodless cavity, (m²) |
| B1 | Viscous damping coefficient (N/(m/s)) |
| Cep, Cip, Ctc | Leakage coefficient (m³/s/Pa) |
| J(∞) | Final solution |
| Kc, Kce | Flow-pressure coefficient, (m³/s/Pa) |
| Kh | Hydraulic spring stiffness, (N/m) |
| Kq | Flow gain, (m²/s) |
| mL, mp | Mass (kg) |
| n | The number of iteration |
| p1, p2 | Pressure of rodless cavity (Pa) |
| q1, q2 | Flow rate into rodless cavity (L/min) |
| qL | Load flow (m³/s) |
| R | Correlation matrix of tap vector |
| V1, V2 | Liquid volume of rodless chamber (m³) |
| Vc | Liquid volume of rod cavity (m³) |
| xc, yc | Spool displacement (m) |
| yL, yp | Load displacement (m) |
| βe | Equivalent bulk modulus of elasticity (Pa) |
| μ | Step parameter, a normal number |
| ε0(n) | The weight error vector. |

Data Availability

The figures and equipment parameter data used to support the findings of this study are included within the article.
Conflicts of Interest

The authors declare that they have no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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