Direct CP violation in $\bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$

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Abstract

We study the direct CP violation in $\bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$ (with unpolarized $\rho^0(\omega)$) via the $\rho - \omega$ mixing mechanism which causes a large strong phase difference and consequently a large CP violating asymmetry when the masses of the $\pi^+\pi^-$ pairs are in the vicinity of the $\omega$ resonance. Since there are two $\rho(\omega)$ mesons in the intermediate state $\rho - \omega$ mixing contributes twice to the first order of isospin violation, leading to an even larger CP violating asymmetry (could be 30\% – 50\% larger) than in the case where only one $\rho(\omega)$ meson is involved. The CP violating asymmetry depends on the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and the hadronic matrix elements. The factorization approach is applied in the calculation of the hadronic matrix elements with the nonfactorizable effects being included effectively in an effective parameter, $N_c$. We give the constraint on the range of $N_c$ from the latest experimental data for the branching ratios for $\bar{B}^0 \rightarrow \rho^0\rho^0$ and $\bar{B}^0 \rightarrow \rho^+\rho^-$. We find that the CP violating asymmetry could be very large (even more than 90\% for some values of $N_c$). It is shown that the sensitivity of the CP violating asymmetry to $N_c$ is large compared with its smaller sensitivity to the CKM matrix elements. We also discuss the possibility to remove the mod ($\pi$) ambiguity in the determination of the CP violating phase angle $\alpha$ through the measurement of the CP violating asymmetry in the decay $\bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$.

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I. INTRODUCTION

Although $CP$ violation has been a central concern in particle physics since it was first observed in the neutral kaon system more than four decades ago [1], the dynamical origin of $CP$ violation still remains an open problem. $CP$ violation in the framework of the Standard Model (SM) is supposed to arise from a weak complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix which is based on quark flavor mixing [2, 3]. Therefore, the study of $CP$ violation is essential to the test of the CKM mechanism in the SM.

Besides the kaon system much more studies have been carried out on $CP$ violation in the $B$ meson system both theoretically and experimentally in the past few years. It was suggested theoretically that large $CP$ violating asymmetries should be observed in the experiments for $B$ mesons [4]. This important prediction has already been confirmed by the experiments of BaBar and Belle etc. through the measurements on $CP$ violation in several decay channels of $B$ mesons such as $B^0 \rightarrow J/\psi K^0_S$ and $B^0 \rightarrow K^+\pi^-$. From the summer of 2007, the Large Hadron Collider (LHC) at CERN will start to contribute to the exploration of $CP$ violation in the $B$ meson system in a more accurate way due to its much higher statistics. This will also provide an opportunity to discover new physics beyond the SM.

In the decay process we have the so-called direct $CP$ violation which occurs through the interference of two amplitudes with different weak phases and strong phases. The weak phase difference is directly determined by the CKM matrix. On the contrary, the strong phase is usually due to complicated strong interaction and hence difficult to control. Since a large strong phase difference is required for a large $CP$ asymmetry, one needs to appeal to some phenomenological mechanism to get such a large strong phase difference. The charge asymmetry violating mixing between $\rho^0$ and $\omega$ ($\rho - \omega$ mixing) has been applied for this purpose in the past few years. From a series of studies for $CP$ violation in some decay channels of heavy hadrons including $B$, $\Lambda_b$ and $D$, it has been found that $\rho - \omega$ mixing can provide a very large strong phase difference (usually 90 degrees) when the mass of the decay product of $\rho(\omega)$, $\pi^+\pi^-$, is in the vicinity of the $\omega$ resonance [6, 7, 8, 9, 10]. Furthermore, it has been shown that the measurement of the $CP$ violating asymmetry for these decays can be used to remove the mod ($\pi$) ambiguity in the determination of the $CP$ violating phase angle $\alpha$.

In this paper, we will investigate the $CP$ violating asymmetry for the decay $\bar{B}^0 \rightarrow$
\( \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-. \) This process is unique since it has two \( \rho(\omega) \) mesons in the intermediate state, each of them contributing \( \rho - \omega \) mixing. One can expect that there should be a bigger \( CP \) violating asymmetry than in the case where \( \rho - \omega \) mixing only contributes once. It will be shown from our explicit calculations that this is true indeed. The \( CP \) violating asymmetry in the case of double \( \rho - \omega \) mixing could be 30 – 50\% bigger than that in the case of single \( \rho - \omega \) mixing, depending on the value of \( N_c \) and \( q^2/m_b^2 \) (see the meaning of \( N_c \) and \( q^2/m_b^2 \) below).

In our calculations of the \( CP \) violating asymmetry, hadronic matrix elements for both tree and penguin operators in the effective Hamiltonian are involved. These matrix elements are controlled by the effects of nonperturbative QCD which are difficult to handle. In order to extract the strong phase difference we will use the factorization approximation, in which one of the currents in the Hamiltonian is factorized out and generates a meson, assuming the vacuum intermediate state saturation. In this way, the decay amplitude becomes the product of two matrix elements. Such factorization scheme was first argued to be plausible in energetic decays like bottom-hadron decays \cite{11,12}, then was proved to be the leading order result in the framework of QCD factorization when the radiative QCD corrections of order \( O(\alpha_s(m_b)) \) (\( m_b \) is the b-quark mass) and the \( O(1/m_b) \) corrections in the heavy quark effective theory are neglected \cite{13}. Since the nonfactorizable contributions are ignored in the factorization scheme we introduce an effective parameter, \( N_c \), in order to take into account nonfactorizable contributions effectively. In this way, the value of \( N_c \) is not the color number (3) any more, but should be determined by experimental data. In the present work, this will be done by comparing the theoretical results with the experimental data for the decay branching ratios for the processes \( \bar{B}^0 \rightarrow \rho^0\rho^0 \) and \( \bar{B}^0 \rightarrow \rho^+\rho^- \).

The remainder of this paper is organized as follows. In Sec. II we briefly present the effective Hamiltonian, the Wilson coefficients and the CKM matrix elements. In Sec. III we give the formalism for the \( CP \) violating asymmetry in \( \bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^- \) via \( \rho - \omega \) mixing. Then we give the calculation details of the strong phase difference and the numerical results for the \( CP \) violating asymmetry. In Sec. IV we calculate the branching ratios for \( \bar{B}^0 \rightarrow \rho^0\rho^0 \) and \( \bar{B}^0 \rightarrow \rho^+\rho^- \) and present the range of \( N_c \) allowed by the latest experimental data for these decays. In the last section, we give a summary and discussion.
II. THE EFFECTIVE HAMILTONIAN AND THE CKM MATRIX

In order to calculate the direct CP violating asymmetry one needs to use the following effective weak Hamiltonian based on the operator product expansion [14]:

\[ H_{\Delta B=1} = \frac{G_F}{\sqrt{2}} \sum_{q=d,s} V_{ub} V_{uq}^* (c_1 O_1^u + c_2 O_2^u) - V_{tb} V_{tq}^* \sum_{i=3}^{10} c_i O_i \] + H.c., \quad (1)

where \( c_i \) (i=1,...,10) are the Wilson coefficients, \( V_{ub}, V_{uq}, V_{tb} \) and \( V_{tq} \) are the CKM matrix elements. The operators \( O_i \) have the following form:

\[
\begin{align*}
O_1^u &= \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) u_\beta \bar{u}_\beta \gamma^\mu (1 - \gamma_5) b_\alpha, \\
O_2^u &= \bar{q} \gamma_\mu (1 - \gamma_5) u \bar{u} \gamma^\mu (1 - \gamma_5) b, \\
O_3 &= \bar{q} \gamma_\mu (1 - \gamma_5) b \sum_{q'} \bar{q}' \gamma^\mu (1 - \gamma_5) q', \\
O_4 &= \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} \bar{q}'_\beta \gamma^\mu (1 - \gamma_5) q'_\alpha, \\
O_5 &= \bar{q} \gamma_\mu (1 - \gamma_5) b \sum_{q'} \bar{q}' \gamma^\mu (1 + \gamma_5) q', \\
O_6 &= \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} \bar{q}'_\beta \gamma^\mu (1 + \gamma_5) q'_\alpha, \\
O_7 &= \frac{3}{2} \bar{q} \gamma_\mu (1 - \gamma_5) b \sum_{q'} e_q \bar{q}' \gamma^\mu (1 + \gamma_5) q', \\
O_8 &= \frac{3}{2} \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} e_q \bar{q}'_\beta \gamma^\mu (1 + \gamma_5) q'_\alpha, \\
O_9 &= \frac{3}{2} \bar{q} \gamma_\mu (1 - \gamma_5) b \sum_{q'} e_q \bar{q}' \gamma^\mu (1 - \gamma_5) q', \\
O_{10} &= \frac{3}{2} \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} e_q \bar{q}'_\beta \gamma^\mu (1 - \gamma_5) q'_\alpha, \quad (2)
\end{align*}
\]

where \( \alpha \) and \( \beta \) are color indices, and \( q' = u, d \) or \( s \) quarks. In Eq. (2) \( O_1^u \) and \( O_2^u \) are tree operators, \( O_3 - O_6 \) are QCD penguin operators, and \( O_7 - O_{10} \) arise from electroweak penguin diagrams.

The Wilson coefficients are known to the next-to-leading logarithmic order [14, 15]. They are renormalization scheme dependent since the renormalization prescription involves an arbitrariness in the finite parts in the renormalization procedure. The physical quantities should be renormalization scheme independent. Since the radiative QCD corrections are not included in the factorization approach we work, the hadronic matrix elements do not
carry any information about the renormalization scheme dependence\(^1\). Therefore, we choose to use the renormalization scheme independent Wilson coefficients which are defined in Refs. [15, 16, 17] so that the CP violating asymmetry we obtain is renormalization scheme independent. The renormalization scale \(\mu\) is chosen as the energy scale in the decays of the \(B\) meson, \(O(m_b)\). When \(\mu = 5\) GeV, these renormalization scheme independent Wilson coefficients take the following values [16, 17]:

\[
\begin{align*}
    c_1 &= -0.3125, \quad c_2 = 1.1502, \\
    c_3 &= 0.0174, \quad c_4 = -0.0373, \\
    c_5 &= 0.0104, \quad c_6 = -0.0459, \\
    c_7 &= -1.050 \times 10^{-5}, \quad c_8 = 3.839 \times 10^{-4}, \\
    c_9 &= -0.0101, \quad c_{10} = 1.959 \times 10^{-3}.
\end{align*}
\]

(3)

The matrix elements of the operators \(O_i\) should be renormalized to the one-loop order. This results in the effective Wilson coefficients, \(c'_i\), which satisfy the constraint

\[
c_i(m_b)\langle O_i(m_b) \rangle = c'_i\langle O_i \rangle_{\text{tree}},
\]

(4)

where \(\langle O_i \rangle_{\text{tree}}\) are the matrix elements at the tree level, which will be evaluated in the factorization approach. From Eq. (4), the relations between \(c'_i\) and \(c_i\) are [16, 17]

\[
\begin{align*}
    c'_1 &= c_1, \quad c'_2 = c_2, \\
    c'_3 &= c_3 - P_s/3, \quad c'_4 = c_4 + P_s, \\
    c'_5 &= c_5 - P_s/3, \quad c'_6 = c_6 + P_s, \\
    c'_7 &= c_7 + P_e, \quad c'_8 = c_8, \\
    c'_9 &= c_9 + P_e, \quad c'_{10} = c_{10},
\end{align*}
\]

(5)

where

\[
\begin{align*}
    P_s &= (\alpha_s/8\pi)c_2[10/9 + G(m_c, \mu, q^2)], \\
    P_e &= (\alpha_{em}/9\pi)(3c_1 + c_2)[10/9 + G(m_c, \mu, q^2)],
\end{align*}
\]

\(^1\) It has been shown that in the QCD factorization approach the renormalization scheme dependence of the Wilson coefficients and that of the hadronic matrix elements cancel [13].
with

\[ G(m_c, \mu, q^2) = 4 \int_0^1 dx(x - 1) \ln \frac{m_c^2 - x(1 - x)q^2}{\mu^2}, \]

where \( m_c \) is the \( c \)-quark mass and \( q^2 \) is the typical momentum transfer of the gluon or photon in the penguin diagrams. \( G(m_c, \mu, q^2) \) has the following explicit expression \[18\]:

\[
\text{Re}G = \frac{2}{3} \left( \ln \frac{m_c^2}{\mu^2} - \frac{5}{3} - \frac{4}{3} \frac{m_c^2}{q^2} + \left( 1 + 2 \frac{m_c^2}{q^2} \right) \sqrt{1 - \frac{4}{3} \frac{m_c^2}{q^2} \ln \frac{1 + \sqrt{1 - 4 \frac{m_c^2}{q^2}}}{1 - \sqrt{1 - 4 \frac{m_c^2}{q^2}}} \right),
\]

\[
\text{Im}G = -\frac{2}{3} \pi \left( 1 + 2 \frac{m_c^2}{q^2} \right) \sqrt{1 - \frac{4}{3} \frac{m_c^2}{q^2}}. \tag{6}
\]

The value of \( q^2 \) is chosen to be in the range \( 0.3 < q^2/m_b^2 < 0.5 \) \[6, 7\]. From Eqs. (3) (5) (6) we can obtain numerical values of \( c_i' \) which are listed in Table I, where we have taken \( \alpha_s(m_Z) = 0.118, \alpha_{em}(m_b) = 1/132.2, m_b = 5 \text{ GeV}, \) and \( m_c = 1.35 \text{ GeV} \).

**TABLE I: Effective Wilson coefficients for the tree operators, electroweak and QCD penguin operators** \[17, 18\]

| \( c_i' \) | \( q^2/m_b^2 = 0.3 \) | \( q^2/m_b^2 = 0.5 \) |
|---|---|---|
| \( c_1' \) | -0.3125 | -0.3125 |
| \( c_2' \) | 1.1502 | 1.1502 |
| \( c_3' \) | \( 2.433 \times 10^{-2} + 1.543 \times 10^{-3}i \) | \( 2.120 \times 10^{-2} + 5.174 \times 10^{-3}i \) |
| \( c_4' \) | \( -5.808 \times 10^{-2} - 4.628 \times 10^{-3}i \) | \( -4.869 \times 10^{-2} - 1.552 \times 10^{-2}i \) |
| \( c_5' \) | \( 1.733 \times 10^{-2} + 1.543 \times 10^{-3}i \) | \( 1.420 \times 10^{-2} + 5.174 \times 10^{-3}i \) |
| \( c_6' \) | \( -6.668 \times 10^{-2} - 4.628 \times 10^{-3}i \) | \( -5.729 \times 10^{-2} - 1.552 \times 10^{-2}i \) |
| \( c_7' \) | \( -1.435 \times 10^{-4} - 2.963 \times 10^{-5}i \) | \( -8.340 \times 10^{-5} - 9.938 \times 10^{-5}i \) |
| \( c_8' \) | \( 3.839 \times 10^{-4} \) | \( 3.839 \times 10^{-4} \) |
| \( c_9' \) | \( -1.023 \times 10^{-2} - 2.963 \times 10^{-5}i \) | \( -1.017 \times 10^{-2} - 9.938 \times 10^{-5}i \) |
| \( c_{10}' \) | \( 1.959 \times 10^{-3} \) | \( 1.959 \times 10^{-3} \) |

The CKM matrix, which should be determined from experiments, can be expressed in terms of the Wolfenstein parameters, \( A, \lambda, \rho \) and \( \eta \) \[19\]:

\[ \begin{bmatrix}
A & \lambda & \rho \\
\rho & \eta & \rho \\
\rho & \rho & \eta
\end{bmatrix} \]
\[
\begin{pmatrix}
1 - \frac{1}{2} \lambda^2 & \lambda & A \lambda^3 (\rho - i \eta) \\
-\lambda & 1 - \frac{1}{2} \lambda^2 & A \lambda^2 \\
A \lambda^3 (1 - \rho - i \eta) & -A \lambda^2 & 1
\end{pmatrix}, 
\]

(7)

where \( O(\lambda^4) \) corrections are neglected. The latest values for the parameters in the CKM matrix are \[20\]:

\[
\begin{align*}
\lambda &= 0.2272 \pm 0.0010, & A &= 0.818^{+0.007}_{-0.017}, \\
\bar{\rho} &= 0.221^{+0.064}_{-0.028}, & \bar{\eta} &= 0.340^{+0.017}_{-0.045}, 
\end{align*}
\]

(8)

where

\[
\begin{align*}
\bar{\rho} &= \rho (1 - \frac{\lambda^2}{2}), & \bar{\eta} &= \eta (1 - \frac{\lambda^2}{2}).
\end{align*}
\]

(9)

From Eqs. (8) (9) we have

\[
0.198 < \rho < 0.293, \quad 0.302 < \eta < 0.366.
\]

(10)

**III. CP VIOLATION IN \( \bar{B}^0 \to \rho^0(\omega)\rho^0(\omega) \to \pi^+\pi^-\pi^+\pi^- \)**

**A. Formalism**

Letting \( A (\bar{A}) \) be the amplitude for the decay \( \bar{B}^0 \to \pi^+\pi^-\pi^+\pi^- \) \( (B^0 \to \pi^+\pi^-\pi^+\pi^-) \) one has:

\[
\begin{align*}
A &= \langle \pi^+\pi^-\pi^+\pi^- | H^T | \bar{B}^0 \rangle + \langle \pi^+\pi^-\pi^+\pi^- | H^P | B^0 \rangle, \\
\bar{A} &= \langle \pi^+\pi^-\pi^+\pi^- | H^T | B^0 \rangle + \langle \pi^+\pi^-\pi^+\pi^- | H^P | B^0 \rangle,
\end{align*}
\]

(11)

with \( H^T \) and \( H^P \) being the Hamiltonian for the tree and penguin operators, respectively.

We can define the relative magnitude and phases between the tree and penguin operator contributions as follows:

\[
\begin{align*}
A &= \langle \pi^+\pi^-\pi^+\pi^- | H^T | \bar{B}^0 \rangle [1 + re^{i(\delta + \phi)}], \\
\bar{A} &= \langle \pi^+\pi^-\pi^+\pi^- | H^T | B^0 \rangle [1 + re^{i(\delta - \phi)}],
\end{align*}
\]

(13)

(14)

where \( \delta \) and \( \phi \) are strong and weak relative phases, respectively. The phase \( \phi \) can be expressed as a combination of the CKM matrix elements: \( \phi = \arg[(V_{tb}V_{td}^*)/(V_{ub}V_{ud}^*)] \). As a
result, $\sin \phi$ is equal to $\sin \alpha$ with $\alpha$ being defined in the standard way \cite{20}. The parameter $r$ is the absolute value of the ratio of penguin and tree amplitudes:

$$r \equiv \left| \frac{\langle \pi^+\pi^-\pi^+\pi^-|H^T|\bar{B}^0\rangle}{\langle \pi^+\pi^-\pi^+\pi^-|H^T|\bar{B}^0\rangle} \right|. \tag{15}$$

The $CP$ violating asymmetry, $a$, can be written as

$$a \equiv \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} = \frac{-2r \sin \delta \sin \phi}{1 + 2r \cos \delta \cos \phi + r^2}. \tag{16}$$

In order to obtain a large signal for direct $CP$ violation, we need some mechanism to make $\sin \delta$ large. It has been found that $\rho - \omega$ mixing has the dual advantages that it leads to a large strong phase difference and is well known \cite{7, 8, 9, 10}. With this mechanism, to the first order of isospin violation, we have the following results when the invariant masses of $\pi^+\pi^-$ pairs are near the $\omega$ resonance mass:

$$\langle \pi^+\pi^-\pi^+\pi^-|H^T|\bar{B}^0\rangle = \frac{2g^2_\rho}{s^2_\rho s_\omega} \tilde{\Pi}_{\rho\omega} t_{\rho\omega} + \frac{g^2_\rho}{s^2_\rho} t_{\rho\rho}, \tag{17}$$

$$\langle \pi^+\pi^-\pi^+\pi^-|H^T|\bar{B}^0\rangle = \frac{2g^2_\rho}{s^2_\rho s_\omega} \tilde{\Pi}_{\rho\omega} p_{\rho\omega} + \frac{g^2_\rho}{s^2_\rho} p_{\rho\rho}. \tag{18}$$

Here $t_{\rho\rho}(p_{\rho\rho})$ and $t_{\rho\omega}(p_{\rho\omega})$ are the tree (penguin) amplitudes for $\bar{B} \to \rho^0\rho^0$ and $\bar{B}^0 \to \rho^0\omega$, respectively, $g_\rho$ is the coupling for $\rho^0 \to \pi^+\pi^-$, $\tilde{\Pi}_{\rho\omega}$ is the effective $\rho - \omega$ mixing amplitude which also effectively includes the direct coupling $\omega \to \pi^+\pi^-$, and $s_V (V=\rho$ or $\omega$) is the inverse propagator of the vector meson $V$,

$$s_V = s - m^2_V + im_V \Gamma_V, \tag{19}$$

with $\sqrt{s}$ being the invariant masses of the $\pi^+\pi^-$ pairs (we let the invariant masses of the two $\pi^+\pi^-$ pairs be the same). Eqs. (17) (18) have different forms from the case where only single $\rho - \omega$ mixing is involved \cite{8, 9, 10}: there is a factor of 2 in front of the effective $\rho - \omega$ mixing amplitude, $\tilde{\Pi}_{\rho\omega}$, since $\rho - \omega$ mixing contributes twice to the first order of isospin violation. Furthermore, we have $g^2_\rho$ and $s^2_\rho$ instead of $g_\rho$ and $s_\rho$ as before due to two $\rho \to \pi\pi$ couplings and two $\rho$ propagators (note that $s^2_\omega$ term is of the second order of isospin violation and hence is ignored).

As mentioned before, the direct coupling $\omega \to \pi^+\pi^-$ has been effectively absorbed into $\tilde{\Pi}_{\rho\omega}$ \cite{21}. This leads to the explicit $s$ dependence of $\tilde{\Pi}_{\rho\omega}$. In practice, however, the $s$
dependence of $\tilde{\Pi}_{\rho\omega}$ is negligible. Making the expansion $\tilde{\Pi}_{\rho\omega}(s) = \tilde{\Pi}_{\rho\omega}(m_\omega^2) + (s - m_\omega)\tilde{\Pi}'_{\rho\omega}(m_\omega^2)$, the $\rho - \omega$ mixing parameters were determined in the fit of Gardner and O'Connell \[22\]:

\[
\Re \tilde{\Pi}_{\rho\omega}(m_\omega^2) = -3500 \pm 300 \text{MeV}^2,
\]

\[
\Im \tilde{\Pi}_{\rho\omega}(m_\omega^2) = -300 \pm 300 \text{MeV}^2,
\]

\[
\tilde{\Pi}'_{\rho\omega}(m_\omega^2) = 0.03 \pm 0.04.
\]

From Eqs. (11)(13)(17)(18) one has

\[
re^{i\delta}e^{i\phi} = \frac{2\tilde{\Pi}_{\rho\omega}p_{\rho\omega} + s_\omega p_{\rho\rho}}{2\tilde{\Pi}_{\rho\omega}t_{\rho\omega} + s_\omega t_{\rho\rho}},
\]

where the factor of 2 in front of $\tilde{\Pi}_{\rho\omega}$ arises from the involvement of double $\rho - \omega$ mixing. Defining

\[
p_{\rho\omega} \equiv r' e^{i(\delta_\omega + \phi)}, \quad t_{\rho\omega} \equiv \alpha e^{i\delta_\alpha}, \quad \frac{p_{\rho\rho}}{p_{\rho\omega}} \equiv \beta e^{i\delta_\beta},
\]

where $\delta_\alpha$, $\delta_\beta$ and $\delta_\omega$ are strong phases, one finds the following expression from Eqs. (21)(22):

\[
re^{i\delta} = r' e^{i\delta_\omega} \frac{2\tilde{\Pi}_{\rho\omega} + \beta e^{i\delta_\beta} s_\omega}{2\tilde{\Pi}_{\rho\omega} \alpha e^{i\delta_\alpha} + s_\omega}.
\]

In order to obtain the $CP$ violating asymmetry in Eq. (16), $\sin \phi$ and $\cos \phi$ are needed, where $\phi$ is determined by the CKM matrix elements. In the Wolfenstein parametrization \[19\], one has

\[
\sin \phi = \frac{\eta}{\sqrt{[\rho(1 - \rho) - \eta^2]^2 + \eta^2}},
\]

\[
\cos \phi = \frac{\rho(1 - \rho) - \eta^2}{\sqrt{[\rho(1 - \rho) - \eta^2]^2 + \eta^2}}.
\]

**B. Calculational details**

With the Hamiltonian given in Eq. (1) we can evaluate the matrix elements for $B^0 \rightarrow \rho^0(\omega)\rho^0(\omega)$. In the factorization approximation, $\rho^0(\omega)$ is generated by one current which has the appropriate quantum numbers in the Hamiltonian. For this decay process, the amplitude can be written as the product of two matrix elements after factorization, i.e. (omitting Dirac matrices and color labels): $\langle \rho^0(\omega)|(\bar{q}q)|0\rangle\langle \rho^0(\omega)|(\bar{d}b)|B^0\rangle$ ($q = u, d$), where $(\bar{q}q)$ and $(\bar{d}b)$ denote the $V - A$ currents, $\bar{q}\gamma_\mu(1 - \gamma_5)q$ and $\bar{d}\gamma_\mu(1 - \gamma_5)b$, respectively. Since $\rho^0$ and
ω are vector mesons the amplitude for $B^0 \rightarrow \rho^0(\omega)\rho^0(\omega)$ may be polarized or unpolarized. Here we investigate the later case. Defining

$$\langle \rho^0 |(\bar{u}u)|0\rangle \langle \rho^0 |(\bar{d}b)|\bar{B}^0 \rangle \equiv T,$$

one has

$$T = -\langle \rho^0 |(\bar{d}d)|0\rangle \langle \rho^0 |(\bar{d}b)|\bar{B}^0 \rangle$$
$$= -\langle \rho^0 |(\bar{u}u)|0\rangle \langle \omega |(\bar{d}b)|\bar{B}^0 \rangle$$
$$= \langle \rho^0 |(\bar{d}d)|0\rangle \langle \omega |(\bar{d}b)|\bar{B}^0 \rangle$$
$$= \langle \omega |(\bar{u}u)|0\rangle \langle \rho^0 |(\bar{d}b)|\bar{B}^0 \rangle.$$  

After factorization, the contribution to $t_{\rho\rho}$ from the tree level operator $O^u_T$ is

$$\langle \rho^0 \rho^0 |O^u_T|\bar{B}^0 \rangle = 2\langle \rho^0 |(\bar{u}u)|0\rangle \langle \rho^0 |(\bar{d}b)|\bar{B}^0 \rangle = 2T.$$  

Using the Fierz transformation the contribution of $O^u_T$ is $(1/N_c)T$. Hence we have

$$t_{\rho\rho} = 2\left(c_1' + \frac{1}{N_c}c_2'\right)T.$$  

It should be noted that in Eq. (29) we have neglected the color-octet contribution which is nonfactorizable and difficult to calculate. Therefore, $N_c$ should be treated as an effective parameter and may deviate from the naive value 3 [8, 9, 10]. In the same way we find that $t_{\rho\omega} = 0$. This lead to

$$\alpha e^{i\delta_\alpha} = 0,$$

from Eq. (22).

In a similar way, we can evaluate the penguin operator contributions $p_{\rho\rho}$ and $p_{\rho\omega}$ with the aid of the Fierz identities. From Eq. (22) we have

$$\beta e^{i\delta_\beta} = \frac{-2\left(c_4' + \frac{1}{N_c}c_3'\right) + 3\left(c_7' + \frac{1}{N_c}c_8'\right) + 3\left(c_9' + \frac{1}{N_c}c_{10}'\right) + \left(c_{10}' + \frac{1}{N_c}c_9'\right)}{2\left(c_4' + \frac{1}{N_c}c_3'\right) + 2\left(c_7' + \frac{1}{N_c}c_8'\right) + 2\left(c_9' + \frac{1}{N_c}c_{10}'\right) - \left(c_7' + \frac{1}{N_c}c_8'\right) - \left(c_9' + \frac{1}{N_c}c_{10}'\right) - \left(c_{10}' + \frac{1}{N_c}c_9'\right)},$$

$$r' e^{i\delta_q} = \frac{-2\left(c_3' + \frac{1}{N_c}c_4'\right) - 2\left(c_4' + \frac{1}{N_c}c_3'\right) - 2\left(c_5' + \frac{1}{N_c}c_6'\right) - \left(c_7' + \frac{1}{N_c}c_8'\right) - \left(c_9' + \frac{1}{N_c}c_{10}'\right) + \left(c_{10}' + \frac{1}{N_c}c_9'\right)}{2\left(c_1' + \frac{1}{N_c}c_2'\right)} \times \left| \frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*} \right|,$$
where
\[
\left| \frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*} \right| = \frac{\sqrt{(1 - \rho)^2 + \eta^2}}{(1 - \lambda^2/2)\sqrt{\rho^2 + \eta^2}}.
\] (33)

C. Numerical results

We have several parameters in the numerical calculations: \( q^2, N_c \), and the CKM matrix elements. Since \( N_c \) includes nonfactorizable effects, which cannot be evaluated accurately at present, we choose to treat it as a parameter and determine its range from the experimental data. Then, we can extract an allowed range for \( N_c \) from a comparison of the theoretical results and the experimental data. By doing this, we get the range of \( N_c \) as \( 2.74(2.81) < N_c < 4.77(4.92) \) for \( q^2/m_b^2 = 0.3(0.5) \). This will be discussed in detail in Sec. IV. The most uncertainties due to the CKM matrix elements come from \( \rho \) and \( \eta \) since \( \lambda \) is well determined (see Eq. (8)) and since the \( CP \) violating asymmetry is independent of the Wolfenstein parameter \( A \). Therefore, in our numerical calculations, we take the central value for \( \lambda \) and only let \( (\rho, \eta) \) vary between the limiting values \( (\rho_{\text{min}}, \eta_{\text{min}}) \) and \( (\rho_{\text{max}}, \eta_{\text{max}}) \). In fact, explicit numerical results show that the \( CP \) violating asymmetry is very insensitive to \( \lambda \).

In the numerical calculations, it is found that for a fixed \( N_c \) there is a maximum value, \( a_{\text{max}} \), for the \( CP \) violating parameter, \( a \), when the invariant masses of the \( \pi^+\pi^- \) pairs are in the vicinity of the \( \omega \) resonance. This is shown explicitly in Fig. 1. For \( q^2/m_b^2 = 0.3(0.5) \) and \( N_c = 2.74(2.81) \), the maximum \( CP \) violating asymmetry varies from around -91.1\% (-70.1\%) to around -96.1\% (-77.8\%) as \( (\rho, \eta) \) change from \( (\rho_{\text{max}}, \eta_{\text{max}}) \) to \( (\rho_{\text{min}}, \eta_{\text{min}}) \); For \( q^2/m_b^2 = 0.3(0.5) \) and \( N_c = 4.77(4.92) \), the maximum \( CP \) violating asymmetry varies from around 55.8\% (28.9\%) to around 53.2\% (22.9\%) when \( (\rho, \eta) \) change from \( (\rho_{\text{max}}, \eta_{\text{max}}) \) to \( (\rho_{\text{min}}, \eta_{\text{min}}) \).

Our results show that the \( \rho - \omega \) mixing mechanism produces a large \( \sin \delta \) in the allowed range of \( N_c \), which is necessary for a large \( CP \) violating asymmetry. The involvement of double \( \rho - \omega \) mixing in \( \bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^- \) gives rise to a factor of 2 in Eq. (21) in front of \( \tilde{\Pi}_{\rho\omega} \). This makes the \( CP \) asymmetry even larger than in the case where the \( \rho - \omega \) mixing contributes only once. Fig. 2 shows explicitly the comparison between these two cases for \( N_c = 2.74(2.81) \) and \( q^2/m_b^2 = 0.3(0.5) \). The maximum asymmetry with the involvement of single \( \rho - \omega \) mixing, for \( q^2/m_b^2 = 0.3(0.5) \) and \( N_c = 2.74(2.81) \), is around -55.2\% (-20.0\%) for the set \( (\rho_{\text{max}}, \eta_{\text{max}}) \) and -63.2\% (-23.8\%) for the set \( (\rho_{\text{min}}, \eta_{\text{min}}) \).
$q^2/m_b^2 = 0.3(0.5)$ and $N_c = 4.77(4.92)$, we find that $a_{\text{max}}$ is around 26.5% (-1.49%) for the set $(\rho_{\text{max}}, \eta_{\text{max}})$ and 25.1% (-1.50%) for the set $(\rho_{\text{min}}, \eta_{\text{min}})$ in the case of single $\rho - \omega$ mixing.

The reason that double $\rho - \omega$ mixing leads to a larger $CP$ violating asymmetry than in the case of single $\rho - \omega$ mixing is that $\sin \delta$ becomes bigger in the case of double $\rho - \omega$ mixing than in the case of single $\rho - \omega$ mixing. This can be seen explicitly from Fig. 3. The involvement of double $\rho - \omega$ mixing may also change the value of $r$ as can be seen from Fig. 4. However, as found from our detailed analysis for the influence of $r$ on the $CP$ violating asymmetry, the effect of the change of $r$ on $a$ is small compared with the change of $\sin \delta$ due to the involvement of double $\rho - \omega$ mixing.

It is noted that when $N_c$ is around 2.81 and 4.92 in the case $q^2/m_b^2 = 0.5$, we could also have large $CP$ asymmetries when $\sqrt{s}$ is far away from the $\omega$ resonance for all the allowed values of the CKM matrix elements (see Figs. 1(b) and 5(b)). In these cases, the effective $\rho - \omega$ mixing contributes little and the large $CP$ asymmetry is caused by the effective Wilson coefficients, which can also give a large strong phase, $\delta$, since they are complex numbers.

In most direct $CP$ violating decays such as $B^0 \to \rho^0(\omega)\pi^0 \to \pi^+\pi^-\pi^0$ [9, 10] and some other processes, the involvement of $\rho - \omega$ mixing leads to the result that the strong phase, $\delta$, passes through $90^\circ$ ($\sin \delta = 1$) at the $\omega$ resonance. However, in the decay we are discussing this does not happen in the allowed range of $N_c$. Instead, the absolute value of $\sin \delta$ just gets close to 1, but does not equal 1 (see Fig. 3), even though it is enough to give large $CP$ asymmetry, especially when double $\rho - \omega$ mixing is involved.

Figs. 5 and 6 show the dependence of the $CP$ violating asymmetry and $\sin \delta$, respectively, on both $N_c$ and $\sqrt{s}$. One can see that the $CP$ asymmetry strongly depends on $N_c$. Take Fig. 5(a) as an example (for $q^2/m_b^2 = 0.3$ and maximum $\rho$ and $\eta$): when $N_c < 3.68$, one gets minus asymmetry around the $\omega$ resonance, whereas when $N_c > 3.68$ the $CP$ violating asymmetry becomes positive.

It can be seen from Fig. 5 that when $N_c$ takes the critical value, $-\epsilon_2/\epsilon_1 \simeq 3.68$, the $CP$ violating asymmetry becomes zero. This is because $t_{\rho \rho} = 0$ at this point (as can be seen from Eq. (29) easily) and hence the penguin operator contributions dominate. Furthermore, the sign of $\sin \delta$ and hence the sign of the $CP$ violating asymmetry change at this point. It would be interesting to see whether or not $N_c$ can take this value in the future when more accurate experimental data are available. From most previous studies, it seems that $N_c$ is usually less than this critical value [7, 8, 9, 10, 23]. If this is true for $\bar{B}^0 \to \rho^0(\omega)\rho^0(\omega)$, then
the sign of \(\sin\delta\) would remain unchanged. Then, one could remove the mod \((\pi)\) ambiguity in the determination of the \(CP\) violating phase angle \(\alpha\) (through \(\sin 2\alpha\)) by measuring the \(CP\) violating asymmetry in \(\bar{B}^0 \to \rho^0(\omega)\rho^0(\omega) \to \pi^+\pi^-\pi^+\pi^-\).

IV. BRANCHING RATIOS FOR \(\bar{B}^0 \to \rho^0\rho^0\) AND \(\bar{B}^0 \to \rho^+\rho^-\)

A. Formalism

If the decay amplitude for \(B \to V_1V_2\) (\(V_1, V_2\) denote vector mesons) has the form
\[
A(B \to V_1V_2) = \alpha X^{(BV_1,V_2)},
\]
where \(X^{(BV_1,V_2)}\) denotes the factorizable amplitude with the form \(\langle V_2|\bar{q}_2q_3|0\rangle \langle V_1|\bar{q}_1b|B\rangle\), then the decay rate is given by \([23]\)
\[
\Gamma(B \to V_1V_2) = \frac{p_c}{8\pi m_B^2} |\alpha (m_B + m_1)m_2 f_{V_2} A^{BV_1}(m_2)|^2 H, \tag{34}
\]
where \(\alpha\) is related to the CKM matrix elements and Wilson coefficients, \(f_{V_2}\) is the decay constant of \(V_2\), \(p_c\) is the c.m. momentum of the decay particles, \(m_B\) and \(m_1(m_2)\) are the masses of the \(B\) meson and the vector meson \(V_1(V_2)\), respectively, and
\[
H = (a - bx)^2 + 2(1 - c^2y^2), \tag{35}
\]
where
\[
a = \frac{m_B^2 - m_1^2 - m_2^2}{2m_1m_2}, \quad b = \frac{2m_B^2p_c^2}{m_1m_2(m_B + m_1)^2}, \quad c = \frac{2m_Bp_c}{(m_B + m_1)^2},
\]
\[
x = \frac{A^{BV_1}(m_2)}{A^{BV_1}(m_2)}; \quad y = \frac{V^{BV_1}(m_2)}{A^{BV_1}(m_2)};
\]
\[
p_c = \frac{\sqrt{[m_B^2 - (m_1 + m_2)^2][m_B^2 - (m_1 - m_2)^2]}}{2m_B}. \tag{36}
\]

\(A^{BV_1}, A^{BV_2}\) and \(V^{BV_1}\) in Eqs. (35) and (36) are the form factors associated with \(B \to V_1\) transition.

The decay amplitudes for \(\bar{B}^0 \to \rho^0\rho^0\), \(\bar{B}^0 \to \rho^0\omega\) and \(\bar{B}^0 \to \rho^+\rho^-\) are
\[
A(\bar{B}^0 \to \rho^0\rho^0) = \alpha_1 X^{(B\rho^0,\rho^0)}, \tag{37}
\]
\[
A(\bar{B}^0 \to \rho^0\omega) = \alpha_2 X^{(B\rho^0,\omega)}, \tag{38}
\]
and
\[
A(\bar{B}^0 \to \rho^+\rho^-) = \alpha_3 X^{(B\rho^+,\rho^-)}, \tag{39}
\]
where

\[ \alpha_1 = \frac{G_F}{\sqrt{2}} [2a_1 V_{ub} V_{ud}^* - (-2a_4 + 3a_7 + 3a_9 + a_{10}) V_{tb} V_{td}^*], \]  

\[ \alpha_2 = -\frac{G_F}{\sqrt{2}} (2a_3 + 2a_4 + 2a_5 - a_7 - a_9 - a_{10}) V_{tb} V_{td}^*, \]  

\[ \alpha_3 = -\frac{G_F}{\sqrt{2}} [a_2 V_{ub} V_{ud}^* - (a_4 + a_{10}) V_{tb} V_{td}^*], \]

with \( a_i \ (i = 1, 2, \cdots, 10) \) being defined as:

\[ a_{2j} = c'_{2j} + \frac{c'_{2j-1}}{N_c}, \]

\[ a_{2j-1} = c'_{2j-1} + \frac{c'_{2j}}{N_c}, \quad \text{for } j = 1, 2, \cdots, 5. \]

When we calculate the branching ratios we should take into account the \( \rho - \omega \) mixing contribution for consistency since we are working to the first order of isospin violation. Then, we obtain the branching ratio for \( \bar{B}_0 \to \rho^0 \rho^0 \):

\[ BR(\bar{B}_0 \to \rho^0 \rho^0) = \frac{p_c}{8\pi m_B^2 \Gamma_B} \left| \left( \alpha_1 + \alpha_2 \frac{2\tilde{\Pi}_{\rho\omega}}{(s_\rho - m_\omega^2) + im_\omega \Gamma_\omega} \right) (m_B + m_{\rho^0}) m_{\rho^0} f_{\rho^0} A_1(m_{\rho^0}^2) \right|^2 H. \]

(44)

For \( \bar{B}_0 \to \rho^+ \rho^- \), we have

\[ BR(\bar{B}_0 \to \rho^+ \rho^-) = \frac{p_c}{8\pi m_B^2 \Gamma_B} \left| (\alpha_3 (m_B + m_{\rho^+}) m_{\rho^-} f_{\rho^-} A_1(m_{\rho^+}^2) \right|^2 H. \]

(45)

B. Form factor models

The form factors \( A_1(k^2) \), \( A_2(k^2) \) and \( V(k^2) \) depend on the inner structure of hadrons and consequently depend on the phenomenological models for hadronic wave functions. We adopt the following form factor models:

Model 1(2) \[24, 25\]:

\[ V(k^2) = \frac{V(0)}{1 - k^2/(m_{1_-}^2)}, A_1(k^2) = \frac{A_1(0)}{1 - k^2/(m_{1_+}^2)}, A_2(k^2) = \frac{A_2(0)}{1 - k^2/(m_{1_+}^2)}, \]

where \( V(0) = 0.33(0.395), A_1(0) = A_2(0) = 0.28(0.345), m_{1_-} = 5.32\text{GeV}, \) and \( m_{1_+} = 5.71\text{GeV} \).

Model 3(4) \[23, 24, 25\]:

\[ V(k^2) = \frac{V(0)}{[1 - k^2/(m_{1_-}^2)]^2}, A_1(k^2) = \frac{A_1(0)}{1 - k^2/(m_{1_+}^2)}, A_2(k^2) = \frac{A_2(0)}{[1 - k^2/(m_{1_+}^2)]^2}. \]

(47)
where the form factors have double pole dependence and the parameters take the same values as in Models 1 and 2.

Model 5 \[26\]:

for $V(k^2)$:

$$V(k^2) = \frac{V(0)}{(1 - k^2/m_V)[1 - \sigma_1 k^2/m_V^2 + \sigma_2 k^4/m_V^4]}, \quad (48)$$

for $A_i(k^2)$ ($i=1, 2$):

$$A_i(k^2) = \frac{A_i(0)}{1 - \sigma_1 k^2/m_V^2 + \sigma_2 k^4/m_V^4}, \quad (49)$$

where $m_V = m_{B^*} = 5.32$GeV; $V(0) = 0.31$, $\sigma_1 = 0.59$ and $\sigma_2 = 0$ for $V(k^2)$; $A_1(0) = 0.26$, $\sigma_1 = 0.73$ and $\sigma_2 = 0.10$ for $A_1(k^2)$; and $A_2(0) = 0.24$, $\sigma_1 = 1.40$ and $\sigma_2 = 0.50$ for $A_2(k^2)$.

Model 6 \[27, 28\]:

the form factors $A_1(k^2)$, $A_2(k^2)$ and $V(k^2)$ have the same form:

$$f(k^2) = \frac{f(0)}{1 - a_F k^2/m_B^2 + b_F k^4/m_B^4}, \quad (50)$$

where $f$ could be $A_1$, $A_2$, or $V$. The parameters $f(0)$, $a_F$ and $b_F$ for various form factors are: for $A_1$, $A_1(0) = 0.261$, $a_F = 0.29$, $b_F = -0.415$; for $A_2$, $A_2(0) = 0.223$, $a_F = 0.93$, $b_F = -0.092$; and for $V$, $V(0) = 0.338$, $a_F = 1.37$, $b_F = 0.315$.

### C. Numerical results

As mentioned before, $N_c$ includes the nonfactorizable effects effectively, which cannot be handled well at present. Therefore, we treat $N_c$ as a parameter to be determined by experimental data. Usually $N_c$ is assumed to be universal for all decay channels in the factorization approach. However, it certainly could be different for different channels. Therefore, we choose to determine the range of $N_c$ for $\bar{B}^0 \to \rho^0(\omega)\rho^0(\omega)$ from the experimental data for the branching ratios for the decays $\bar{B}^0 \to \rho^0\rho^0$ and $\bar{B}^0 \to \rho^+\rho^-$ (we expect the nonfactorizable contributions and hence the values of $N_c$ in these two channels are the same if isospin violation is ignored). In order to find the range allowed for $N_c$ we use the latest experimental data for the branching ratios for the two decay channels, $\bar{B}^0 \to \rho^0\rho^0$ and $\bar{B}^0 \to \rho^+\rho^-$ \[20\]:

$$BR(\bar{B}^0 \to \rho^0\rho^0) < 1.1 \times 10^{-6},$$

$$BR(\bar{B}^0 \to \rho^+\rho^-) = (2.5 \pm 0.4) \times 10^{-5}. \quad (51)$$
We calculate the branching ratios for $\bar{B}^0 \to \rho^0\rho^0$ and $\bar{B}^0 \to \rho^+\rho^-$ with the formulae given in Eqs. (34) – (45) in all the models for the weak form factors associated with $\bar{B}^0 \to \rho^0$ and $\bar{B}^0 \to \rho^+(\rho^-)$, which are mentioned in the previous subsection. In addition to the dependence on $N_c$, these two branching ratios also depend on the CKM matrix elements which are parameterized by $\lambda, A, \rho$ and $\eta$, with the experimental values of them being given in Eqs. (8) – (10). Since each of these parameters has some uncertainty, we let each of them vary in its allowed range when we calculate the branching ratios. Then, for each set of the values for the parameters $\lambda, A, \rho$ and $\eta$, we obtain a range of $N_c$ which is allowed by the experimental data for the branching ratios for both $\bar{B}^0 \to \rho^0\rho^0$ and $\bar{B}^0 \to \rho^+\rho^-$. This is shown in Figs. 7 and 8 for a special set of CKM matrix parameters when $q^2/m_b^2 = 0.3$. Repeating this process for various sets of the values for $\lambda, A, \rho$ and $\eta$ and taking the union of the ranges of $N_c$ for all these sets, we find a range for $N_c$ which covers the whole range for these CKM matrix parameters. We repeat this process for all the form factor models mentioned in Eqs. (46) – (50) and obtain the range of $N_c$ for each model as shown in Table II. Taking the union of all the ranges for these models we finally find the maximum possible range for $N_c$: $2.74 < N_c < 4.77$ and $2.81 < N_c < 4.92$ for $q^2/m_b^2 = 0.3$ and $q^2/m_b^2 = 0.5$, respectively.

| Model   | $q^2/m_b^2=0.3$ | $q^2/m_b^2=0.5$ |
|---------|-----------------|-----------------|
| Model 1 | (2.74, 4.74)    | (2.81, 4.90)    |
| Model 2 | (2.78, 4.47)    | (2.84, 4.64)    |
| Model 3 | (2.75, 4.77)    | (2.81, 4.92)    |
| Model 4 | (2.76, 4.54)    | (2.82, 4.72)    |
| Model 5 | (2.74, 4.69)    | (2.81, 4.84)    |
| Model 6 | (2.77, 4.50)    | (2.84, 4.68)    |
| maximum range | (2.74, 4.77) | (2.81, 4.92) |

V. SUMMARY AND DISCUSSION

We have calculated the $CP$ violating asymmetry in the process $\bar{B}^0 \to \rho^0(\omega)\rho^0(\omega) \to \pi^+\pi^-\pi^+\pi^-$ including the effect of $\rho - \omega$ mixing. The advantage of $\rho - \omega$ mixing is that it
makes the strong phase difference, $\delta$, between the hadronic matrix elements of the tree and penguin operators very large at the $\omega$ resonance for a fixed $N_c$. We have found that $\sin\delta$ becomes large and reaches the maximum point at the $\omega$ resonance. Consequently, the $CP$ violating asymmetry reaches the maximum value when the invariant masses of the $\pi^+\pi^-$ pairs in the decay product are in the vicinity of the $\omega$ resonance. Furthermore, since there are two $\rho(\omega)$ mesons in the intermediate state, $\rho-\omega$ mixing contributes twice when we work to the first order of isospin violation. This leads to an even larger $CP$ violating asymmetry than in the case where only single $\rho-\omega$ mixing is involved. This is unique for the process $\bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$. As a result, the largest $CP$ violating asymmetry could be more than 90% for some values of $N_c$. This could be observed in the future experiments at LHC. Now, we roughly estimate the possibility to observe this $CP$ violating asymmetry. If the branching ratio for $\bar{B}^0 \rightarrow \rho^0\rho^0$ is of order $10^{-6}$, then the number of $B^0\bar{B}^0$ pairs needed for observing the $CP$ violating asymmetry (90%) is roughly \[ \frac{1}{BR(B^0\rightarrow\rho^0\rho^0)} \frac{1}{a^2} \sim 10^6 \] for $1\sigma$ signature and $10^7$ for $3\sigma$ signature [29]. It has been pointed out that at LHC, the number of $B^0\bar{B}^0$ pairs could be around $4 \times 10^7$ (for ATLAS and CMS) and $4 \times 10^5$ (for LHCb) per year [30]. Therefore, it is possible to observe the $CP$ violation for $\bar{B}^0 \rightarrow \rho^0(\omega)\rho^0(\omega) \rightarrow \pi^+\pi^-\pi^+\pi^-$ via the double $\rho-\omega$ mixing mechanism at LHC.

In the calculation, we need the Wilson coefficients for the tree and penguin operators at the scale $m_b$. We work with the renormalization scheme independent Wilson coefficients. We have found that apart from the $\rho-\omega$ mixing mechanism, the Wilson coefficients themselves could also give observable $CP$ violating asymmetry in some cases. The errors in the CKM matrix elements lead to some uncertainties in the $CP$ violating asymmetry. Even bigger uncertainties come from the hadronic matrix elements of the tree and penguin operators due to the nonperturbative QCD effects. We have worked in the factorization approach, with the effective parameter $N_c$ being introduced to account for the nonfactorizable effects. We have shown that the $CP$ violating asymmetry in this decay process strongly depends on the parameter $N_c$.

In order to determine the range of $N_c$ we have compared the theoretical values and the experimental data for the branching ratios for $\bar{B}^0 \rightarrow \rho^0\rho^0$ and $\bar{B}^0 \rightarrow \rho^+\rho^-$. We have found that the latest experimental data constrain $N_c$ to be in the range $(2.74, 4.77)$ for $q^2/m_b^2=0.3$ and $(2.81, 4.92)$ for $q^2/m_b^2=0.5$, respectively, when we let the CKM matrix elements vary in the ranges determined by the current experiments. We have studied the sign of $\sin\delta$ in the
range of $N_c$ and found that $\sin \delta$ changes its sign at the point $N_c = 3.68$. This also leads to the change of the sign of the $CP$ violating asymmetry. Due to the large errors in the current experimental data for the branching ratios for $\bar{B}^0 \to \rho^0 \rho^0$ and $\bar{B}^0 \to \rho^+ \rho^-$ we cannot constrain $N_c$ more accurately at present. If the future experimental data could constrain $N_c$ to be less than 3.68 ($N_c$ is usually less than 3.68 in other studies), the sign of the $CP$ violating asymmetry would remain unchanged in the whole range of $N_c$. Then one could remove the mod ($\pi$) ambiguity in the determination of the $CP$ violating phase angle $\alpha$ by measuring the $CP$ violating asymmetry in $\bar{B}^0 \to \rho^0(\omega)\rho^0(\omega) \to \pi^+\pi^-\pi^+\pi^-$. 

For the decay process $\bar{B}^0 \to \rho^0(\omega)\rho^0(\omega)$, the factorization approach we have used is expected to be a good approximation since $B$ meson decays are energetic and since $\alpha_s(m_b)$ and $1/m_b$ corrections should be small in the QCD factorization scheme. One may also work in the QCD factorization scheme, taking the value of $N_c$ to be 3 and including corrections of order $\alpha_s(m_b)$ as done in Ref. [31]. However, the QCD factorization scheme suffers from endpoint singularities which are not well controlled. The $CP$ violating asymmetry depends on the unknown parameters which are associated with such endpoint singularities. This lead to very uncertain $CP$ violating asymmetries in the QCD factorization scheme [31]. As mentioned before, the uncertainty for the $CP$ violating asymmetry is also very large in the factorization approach we have used, i.e. from about -96% to about 56% depending on the value of $N_c$ and the CKM matrix elements. Furthermore, the $CP$ violating asymmetry may strongly depend on the factorization approach adopted [31]. All these issues need further and more careful investigations.

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Figure Captions

Fig. 1 The $CP$ violating asymmetry, $a$, for $\bar{B}^0 \to \rho^0(\omega)\rho^0(\omega) \to \pi^+\pi^-\pi^+\pi^-$. (a): for $q^2/m_b^2 = 0.3$, $N_c = 2.74$ and 4.77 and limiting values of the CKM matrix elements, $\rho$ and $\eta$; the solid line (dotted line) corresponds to $N_c = 2.74$ and maximum (minimum) $\rho$ and $\eta$; the dashed line (dot-dashed line) corresponds to $N_c = 4.77$ and maximum (minimum) $\rho$ and $\eta$. (b): for $q^2/m_b^2 = 0.5$, $N_c = 2.81$ and 4.92 and limiting values of $\rho$ and $\eta$: the solid line (dotted line) corresponds to $N_c = 2.81$ and maximum (minimum) $\rho$ and $\eta$; the dashed line (dot-dashed line) corresponds to $N_c = 4.92$ and maximum (minimum) $\rho$ and $\eta$.

Fig. 2 Comparison between the $CP$ violating asymmetries in the cases where single and double $\rho - \omega$ mixing is involved, respectively. (a): for $q^2/m_b^2 = 0.3$ and $N_c = 2.74$, the solid line (dotted line) corresponds to the case with double $\rho - \omega$ mixing and maximum (minimum) $\rho$ and $\eta$; the dashed line (dot-dashed line) corresponds to the case with single $\rho - \omega$ mixing and maximum (minimum) $\rho$ and $\eta$. (b): same as (a) but for $q^2/m_b^2 = 0.5$ and $N_c = 2.81$.

Fig. 3 $\sin \delta$ as a function of $\sqrt{s}$ for $\bar{B}^0 \to \rho^0(\omega)\rho^0(\omega) \to \pi^+\pi^-\pi^+\pi^-$. (a): for $q^2/m_b^2 = 0.3$ and $N_c = 2.74$ (4.77): the solid line (dashed line) corresponds to the case with double $\rho - \omega$ mixing; the dotted line (dot-dashed line) corresponds to the case with single $\rho - \omega$ mixing. (b): same as (a) but for $q^2/m_b^2 = 0.5$ and $N_c = 2.81$ (4.92).

Fig. 4 The ratio of penguin to tree amplitudes, $r$, as a function of $\sqrt{s}$, for limiting values of $\rho$ and $\eta$: the solid line (dotted line) corresponds to the case of double (single) $\rho - \omega$ mixing with maximum $\rho$ and $\eta$; the dashed line (dot-dashed line) corresponds to the case of double (single) $\rho - \omega$ mixing with minimum $\rho$ and $\eta$. In (a) $q^2/m_b^2 = 0.3$, $N_c = 2.74$ (left) and 4.77 (right) while in (b) $q^2/m_b^2 = 0.5$, $N_c = 2.81$ (left) and 4.92 (right).

Fig. 5 The $CP$ violating asymmetry, $a$, as a function of $N_c$ and $\sqrt{s}$, for $\bar{B}^0 \to \rho^0(\omega)\rho^0(\omega) \to \pi^+\pi^-\pi^+\pi$ for $\rho = \rho_{max}$ and $\eta = \eta_{max}$. (a) and (b) correspond to $q^2/m_b^2 = 0.3$ and $q^2/m_b^2 = 0.5$, respectively.

Fig. 6 $\sin \delta$ as a function of $\sqrt{s}$ and $N_c$. (a) and (b) correspond to $q^2/m_b^2 = 0.3$ and $q^2/m_b^2 = 0.5$, respectively.

Fig. 7 Branching ratio for $\bar{B}^0 \to \rho^0\rho^0$ for all the models when $q^2/m_b^2 = 0.3$, $\lambda = 0.2272$, and $\eta = \eta_{max}$. (a): for $N_c = 2.74$ and 4.77: the solid line (dotted line) corresponds to $N_c = 2.74$ and maximum (minimum) $\rho$ and $\eta$; the dashed line (dot-dashed line) corresponds to $N_c = 4.77$ and maximum (minimum) $\rho$ and $\eta$. (b): same as (a) but for $N_c = 2.81$. 
$A = 0.818, \rho = 0.246, \text{ and } \eta = 0.334$: the lower (upper) solid line corresponds to Model 1 (2), the lower (upper) dotted line corresponds to Model 3 (4) and the lower (upper) dashed line corresponds to Model 5 (6).

Fig. 8 Branching ratio for $\bar{B}^0 \rightarrow \rho^+\rho^-$ for all the models when $q^2/m_b^2 = 0.3, \lambda = 0.2272, A = 0.818, \rho = 0.246, \text{ and } \eta = 0.334$: the lower (upper) solid line corresponds to Model 1 (2), the lower (upper) dotted line corresponds to Model 3 (4) and the lower (upper) dashed line corresponds to Model 5 (6).
Fig. 5(b)

Fig. 6(a)

Fig. 6(b)
Fig. 7

Fig. 8