Ultrarelativistic boost of spinning and charged black rings

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Abstract. We study Aichelburg-Sexl ultrarelativistic limits in higher dimensions. After reviewing the boost of \( D \geq 4 \) Reissner-Nordström black holes as a simple illustrative example, we consider the case of \( D = 5 \) black rings, presenting new results for static charged rings.

1. Introduction and review of the boost of \( D \geq 4 \) Reissner-Nordström black holes

In 1959 Pirani argued that the geometry associated with a fast moving mass resembles a “plane” gravitational wave [1]. Later on, Aichelburg and Sexl (AS) [2] considered a limiting (“ultrarelativistic”) boost of the Schwarzschild line element to determine the exact impulsive pp-wave [3] generated by a lightlike particle. In a higher dimensional context, the AS limit of static black holes [4] has been known for some time [5] (also with a dilaton [6]). In the case of zero charge, the inclusion of an external magnetic field is straightforward [7]. The AS boost of rotating black holes [8] has been studied in [9]. Here we focus on \( D = 5 \) black rings [10]. In this section we pedagogically review the ultrarelativistic limit of \( D \geq 4 \) static black holes, as a simple example encompassing all the technical steps. Section 2 follows [11,12] in studying the case of vacuum black rings [10]. In section 3 we present new results for static charged rings [13–15].

A static charged black hole is described in any \( D \geq 4 \) by the line element [4,8]

\[
ds^2 = -f^2 dt^2 + f^{-2} dr^2 + r^2 d\Omega_{D-2}^2,
\]

(1)

\( d\Omega_{D-2}^2 \) being the standard the unit \((D-2)\)-sphere, while \( f \) and the electric potential are

\[
f^2 = 1 - \frac{\mu}{r^{D-3}} + \frac{e^2}{r^{2(D-3)}}, \quad A = -\sqrt{\frac{D-2}{2(D-3)}} \frac{e}{r^{D-3}} dt.
\]

(2)

The mass and the charge of the black hole are given by [8]

\[
M = \frac{\mu(D-2)\Omega_{D-2}}{16\pi}, \quad Q = e\sqrt{\frac{(D-2)(D-3)}{2}},
\]

(3)

and the condition \( 4e^2 \leq \mu^2 \) must be satisfied in order for the spacetime to be really “black”.

For our purposes, it is convenient to decompose the line element (1) as

\[
ds^2 = ds^2_0 + \Delta,
\]

(4)
in which $ds_0^2 = -dt^2 + dr^2 + r^2d\Omega_{D-2}^2$ (i.e., eq. (1) with $\mu = 0 = e$) is Minkowski spacetime, and
\[
\Delta = \left(\frac{\mu}{r^{D-3}} - \frac{e^2}{r^{2(D-3)}}\right) \left(dt^2 + \frac{1}{r^2}dr^2\right).
\] (5)

Note that for $r \to \infty$ one has $\Delta \to 0$ and $ds^2 \to ds_0^2$. This enables us to define a Lorentz boost using the symmetries of the flat $ds_0^2$. We first introduce cartesian coordinates $(z_1, \ldots, z_{D-1})$ via
\[
r = \sqrt{z_1^2 + \rho^2}, \quad \rho = \sqrt{z_i z^i} \quad (i = 2, \ldots, D - 1),
\] (6)
and then suitable double null coordinates $(u', v')$ by
\[
t = \frac{-u' + v'}{\sqrt{2}}, \quad z_1 = \frac{u' + v'}{\sqrt{2}}.
\] (7)

Now, a boost along the (generic) direction $z_1$ takes the simple form
\[
u' = \epsilon^{-1}u, \quad v' = ev.
\] (8)

An “ultrarelativistic” boost to the speed of light amounts in taking $\epsilon \to 0$ in eq. (8). We will also rescale the mass and the electric charge as $M = \gamma^{-1}p_M, Q^2 = \gamma^{-1}p_Q^2$ [2, 5], i.e. (for $\epsilon \to 0$)
\[
M \approx 2\epsilon p_M, \quad Q^2 \approx 2\epsilon p_Q^2 \quad (p_M, p_Q > 0).
\] (9)

We can now make the substitutions (6)–(9) in the black hole metric (1) (i.e., eqs. (4) and (5)). The flat background becomes $ds_0^2 = 2dudv + dz_idz^i$, and $\Delta$ depends parametrically on $\epsilon$
\[
\Delta_\epsilon = \left(\frac{32\pi}{(D - 2)\Omega_{D-2}} \frac{p_M}{\tilde{r}^{D-3}} - \frac{4}{(D - 2)(D - 3)} \frac{p_Q^2}{\tilde{r}^{2(D-3)}}\right)
\times \epsilon \left[\frac{1}{2}(-\epsilon^{-1}du + \epsilon dv)^2 + \frac{1}{\tilde{r}^2} \left(\frac{z_\epsilon}{\tilde{r}} \frac{1}{\sqrt{2}}(-\epsilon^{-1}du + \epsilon dv) + \frac{z_i}{\tilde{r}}dz^i\right)^2\right],
\] (10)
where $\tilde{r} = \tilde{r}(z_\epsilon, z_i) = \sqrt{z_\epsilon^2 + \rho^2}$, $z_\epsilon = \frac{1}{\sqrt{2}}(\epsilon^{-1}u + \epsilon v)$.

We have introduced the combination $z_\epsilon$ because it fully determines how $\Delta_\epsilon$ depends on $u$. In the limit $ds^2 = ds_0^2 + \lim_{\epsilon \to 0} \Delta_\epsilon$, the expansion of $\Delta_\epsilon$ in $\epsilon$ displays a peculiar behaviour at $u = 0$
\[
\Delta_\epsilon = \frac{1}{\epsilon} h(z_\epsilon)du^2 + \ldots,
\] (12)
\[
h(z_\epsilon) = \frac{32\pi}{(D - 2)\Omega_{D-2}} \frac{p_M}{2} \left(\frac{1}{\tilde{r}^{D-3}} + \frac{z_\epsilon^2}{\tilde{r}^{D-1}}\right) - \frac{4}{(D - 2)(D - 3)} \frac{p_Q^2}{2} \left(\frac{1}{\tilde{r}^{2(D-3)}} + \frac{z_\epsilon^2}{\tilde{r}^{2(D-2)}}\right),
\] (13)
and the dots denote terms which become negligible in the limit. We have emphasized here the essential dependence of the function $h$ on $z_\epsilon$ (and thus on $\epsilon$), but of course $h$ depends also on $\rho$.

In taking the limit $\epsilon \to 0$ of eq. (12), we apply the distributional identity
\[
\lim_{\epsilon \to 0} \frac{1}{\epsilon} g(z_\epsilon) = \sqrt{2} \delta(u) \int_{-\infty}^{+\infty} g(z)dz.
\] (14)

The final metric is thus
\[
ds^2 = 2dudv + dz_idz^i + H\delta(u)du^2,
\] (15)
and eq. (13). For any $D > 4$ one can readily employ the integral $2 \int_{0}^{\infty} x^{2a-1}/(1 + x^{2})^{a+b} dx = \Gamma(a)\Gamma(b)/\Gamma(a+b)$ (with appropriate values $a, b > 0$) to evaluate (16), and one obtains explicitly

$$H = C_{M} \frac{p_{M}}{p_{D-4}} - C_{Q} \frac{p_{Q}^{2}}{p_{D-3}^{2}}$$

\begin{equation}
(D > 4),
\end{equation}

where

$$C_{M} = \frac{16\pi\sqrt{2}}{(D - 4)\Omega_{D-3}}$$

$$C_{Q} = \frac{(2D - 9)!!}{(D - 3)!!} \frac{2D - 5}{(D - 3)(2D - 4)} \pi^{\frac{2}{2}}$$

\begin{equation}
(17)
\end{equation}

For $D = 4$ the integral of (13) is divergent, due (only) to the terms proportional to $p_{M}$. One can perform the (by now standard) infinite “gauge” subtraction of [2] to eventually obtain

$$H = -8\sqrt{2}p_{M} \ln \rho - \frac{3\pi\sqrt{2}p_{Q}^{2}}{2\rho}$$

\begin{equation}
(D = 4).
\end{equation}

A $D$-dimensional charged black hole boosted to the speed of light is thus described by the impulsive $pp$-wave (15) with eqs. (17) or (18). For $p_{Q} = 0$, this is just the (higher dimensional) AS vacuum solution. The spacetime (15) is flat everywhere except on the wave front $u = 0$. There is a singularity at $\rho = 0$, as a remnant of the curvature singularity $r = 0$ of the original metric (1). Because of eq. (9) for $Q$, the Maxwell field $F = dA$ (cf. eq. (2)) tends to zero when $\epsilon \to 0$, but its energy-momentum tensor is non-vanishing and proportional to $p_{Q}^{2}\delta(u)$ [5] (such a physically “peculiar configuration” is, however, mathematically sound [17]). On the other hand, if one replaced the rescaling (9) by $Q = \text{const.}$, one would end up with a non-zero $F_{\mu\nu} \sim Q\delta(u)$, but with an ill-defined $T_{\mu\nu} \sim Q^{2}\delta^{2}(u)$. Note also that the rescalings (9) imply a violation of the charge bound $4e^{2} \leq \mu^{2}$ for a small $\epsilon$. The $pp$-wave presented above thus corresponds to the boost of a charged naked singularity rather than a black hole. In order to preserve the event horizon(s) until the very final limit, one should rescale $Q$ (i.e., $\epsilon$) at least as fast as $Q^{2} \sim \epsilon^{2}$, but one would then simply recover the vacuum AS geometry [5, 6]. See, e.g., the reviews [19] and references therein for other AS limits and for general properties of impulsive waves.

2. Boost of the vacuum black ring

We now consider the AS boost of the black ring [10]. We start from the metric form of [20]

$$\text{d}s^{2} = \frac{F(y)}{F(x)} \left[ dt + C(\nu, \lambda) L \frac{1 + y}{F(y)} \text{d}\psi \right]^{2}$$

$$+ \frac{L^{2}}{(x - y)^{2}} F(x) \left[ - \frac{G(y)}{F(y)} \text{d}y^{2} - \frac{\text{d}x^{2}}{G(x)} + \frac{G(x)}{F(x)} \text{d}\phi^{2} \right],$$

\begin{equation}
(19)
\end{equation}

where

$$F(\zeta) = \frac{1 + \lambda\zeta}{1 - \lambda}, \quad G(\zeta) = (1 - \zeta^{2}) \frac{1 + \nu\zeta}{1 - \nu}, \quad C(\nu, \lambda) = \sqrt{\frac{\lambda(\lambda - \nu)(1 + \lambda)}{(1 - \nu)(1 - \lambda)^{3}}}.$$  

\begin{equation}
(20)
\end{equation}

The black ring (19) is asymptotically flat at spatial infinity is at $x, y \to -1$, and we refer [10,20] for more details. Mass, angular momentum and angular velocity (at the horizon) are

$$M = \frac{3\pi L^{2}}{4} \frac{\lambda}{1 - \lambda}, \quad J = \frac{\pi L^{3}}{2} \sqrt{\frac{\lambda(\lambda - \nu)(1 + \lambda)}{(1 - \nu)(1 - \lambda)^{3}}}, \quad \Omega = \frac{1}{L} \sqrt{\frac{(\lambda - \nu)(1 - \lambda)}{\lambda(1 + \lambda)(1 - \nu)}}.$$  

\begin{equation}
(21)
\end{equation}

\begin{footnote}
Ref. [16] pointed out that such a procedure contains some ambiguity, and presented a different approach based on boosting the energy-momentum tensor. In the case, e.g., of the boosted Schwarzschild metric, one can alternatively remove the ambiguity by using a simple symmetry argument, since the final $pp$-wave must be an axially symmetric vacuum solution (which uniquely determines the result of [2]). No ambiguity arises for $D > 4$.
\end{footnote}
With the choice \( \psi, \phi \in [0,2\pi] \) there are no conical singularities at \( y = -1 \) and \( x = -1 \). The black ring is “in equilibrium” if conical singularities are absent also at \( x = +1 \), which requires

\[
\lambda = \frac{2\nu}{1 + \nu^2}. \tag{22}
\]

To perform the AS limit of the black ring, we decompose eq. (19) as in eq. (4), where the flat \( ds_0^2 \) is now given by given by eq. (19) with \( \lambda = 0 = \nu \), and \( \Delta \) is a cumbersome expression given in [12]. In order to express the boosts of \( ds_0^2 \), we first define new coordinates \((\xi, \eta)\) via

\[
y = -\frac{\xi^2 + \eta^2 + L^2}{\sqrt{(\xi^2 + \eta^2 - L^2)^2 + 4L^2\eta^2}}, \quad x = -\frac{\xi^2 + \eta^2 - L^2}{\sqrt{(\eta^2 + \xi^2 - L^2)^2 + 4L^2\eta^2}}. \tag{23}
\]

Then, spatial cartesian coordinates \((x_1, x_2, y_1, y_2)\) are given by

\[
x_1 = \eta \cos \phi, \quad x_2 = \eta \sin \phi, \quad y_1 = \xi \cos \psi, \quad y_2 = \xi \sin \psi,
\]

so that \( ds_0^2 = -dt^2 + dx_1^2 + dx_2^2 + dy_1^2 + dy_2^2 \). This enables us to study a boost along a general direction \( z_1 \), which can be specified by a single parameter \( \alpha \) if we introduce rotated axes \((z_1, z_2)\)

\[
x_1 = z_1 \cos \alpha - z_2 \sin \alpha, \quad y_1 = z_1 \sin \alpha + z_2 \cos \alpha.
\]

Using null coordinates as in eq. (7), we can finally perform the boost (8). In the ultrarelativistic limit \( \epsilon \to 0 \), we again rescale the mass as \( M = \gamma^{-1}p_M \approx 2\epsilon p_M \). In addition, we wish to keep the angular velocity \( \Omega \) finite. From eq. (21), these requirements suggest the rescalings

\[
\lambda = \epsilon p_\lambda, \quad \nu = \epsilon p_\nu,
\]

where \( p_\lambda = 8p_M/(3\pi L^2) \), and \( p_\nu \) is another positive constant such that \( p_\lambda \geq p_\nu \). In terms of these parameters, for \( \epsilon \to 0 \) the equilibrium condition (22) becomes \( p_\lambda = 2p_\nu \).

All the ingredients necessary to evaluate how the metric (19) transforms under the boost (8) have been given above, and we skip intermediate lengthy calculations (see [11, 12] for more details). As in section 1, after the splitting (4) one obtains for \( \Delta \) an expression of the form (12) when \( \epsilon \to 0 \). With eq. (14), one finds that the final metric is a \( D = 5 \) impulsive \( pp \)-wave

\[
ds^2 = 2du dv + dx_1^2 + dy_1^2 + dx_2^2 + dy_2^2 + H_\perp(x_2, y_1, y_2)\delta(u)du^2, \tag{27}
\]

where

\[
H_\perp = \sqrt{2\frac{3p_\lambda L^2 + (2p_\nu - p_\lambda)\xi^2}{(\xi + L)^2 + x_2^2}} K(k) + \sqrt{2(2p_\nu - p_\lambda)}
\times \left[ -\frac{1}{(\xi + L)^2 + x_2^2}E(k) + \frac{\xi - L}{\xi + L} \frac{x_2^2}{\sqrt{(\xi + L)^2 + x_2^2}} \Pi(\rho, k) + \pi|x_2|\Theta(L - \xi) \right], \tag{28}
\]

in which

\[
k = \sqrt{\frac{4\xi L}{(\xi + L)^2 + x_2^2}}, \quad \rho = \frac{4\xi L}{(\xi + L)^2}, \quad \xi = \sqrt{y_1^2 + y_2^2}, \tag{29}
\]
and $\Theta(L - \xi)$ denotes the step function (cf. the appendix of [11] for definitions of the elliptic integrals $K$, $E$ and $\Pi$). The function (28) simplifies considerably for the more interesting case of black rings in equilibrium, i.e. $p_\lambda = 2p_\nu$. This corresponds to a spacetime which is vacuum everywhere except on the singular circle at $u = 0 = x_3$, $\xi = L$, a remnant of the curvature singularity ($y = -\infty$) of the original black ring (19). For $p_\lambda \neq 2p_\nu$, the discontinuous term proportional to $\Theta(L - \xi)$ is responsible for a disk membrane supporting the ring.

In the case of a parallel boost ($\alpha = \pi/2$) the final $pp$-wave, now propagating along $y_1$, is

$$ds^2 = 2du dv + dx_1^2 + dx_2^2 + dy_2^2 + H_{||}(x_1, x_2, y_2)\delta(u)du^2,$$

(30)

$$H_{||} = \left[2(2p_\lambda - p_\nu)L^2 + (2p_\nu - p_\lambda)a^2 \left(1 + \frac{L^2 + \eta^2}{a^2 - y_2^2}\right) + 2\sqrt{p_\lambda(p_\lambda - p_\nu)}L y_2 \left(1 - \frac{L^2 + \eta^2}{a^2 - y_2^2}\right)\right]^{\sqrt{2}/a}K(k') - 2\sqrt{2}(2p_\nu - p_\lambda)aE(k') + \frac{\sqrt{2}}{2}\left[(2p_\nu - p_\lambda)y_2 - 2\sqrt{p_\lambda(p_\lambda - p_\nu)}L\right]\left[-\frac{\eta^2 + L^2a^2 + y_2^2}{ay_2} - \frac{\eta^2 + L^2a^2 + y_2^2}{a^2 - y_2^2}\Pi(\rho', k') + \pi \text{sgn}(y_2)\right],$$

(31)

where

$$k' = \left[\frac{(a^2 - \eta^2 - y_2^2 + L^2)^{1/2}}{\sqrt{2a}}\right], \quad \rho' = -\left[\frac{(a^2 - \eta^2)^2}{4a^2y_2^2}\right],$$

$$a = \left[\left(\eta^2 + y_2^2 - L^2\right)^2 + 4\eta^2L^2\right]^{1/4}, \quad \eta = \sqrt{x_1^2 + x_2^2}.$$  

(32)

The profile (31) is simpler for $p_\lambda = 2p_\nu$, when it describes a vacuum spacetime singular on the rod at $u = 0 = \eta$ and $|y_2| \leq L$. In [11,12] we also pointed out distinct features with respect to boosted black holes [9], and we analyzed the AS limit of the supersymmetric black ring of [21].

3. Boost of the charged black ring

We present here new results describing the AS boost of static charged black rings. These were found in [13] (up to a misprint in $F_{uu}$) in the Einstein-Maxwell theory, generalized to dilaton gravity in [14], and rederived more systematically in [15]. The solution of [13] coincides with the static limit of certain supergravity black rings [22]. It can also be obtained by Harrison-transforming (along $\partial_t$) a static vacuum black ring (eq. (19) with $\nu = \lambda$), which gives

$$ds^2 = -\Lambda^{-2}\frac{F(y)}{F(x)}dt^2 + \frac{2\Lambda L^2}{(x - y)^2}F(x)\left[(y^2 - 1)dv^2 + \frac{1}{F(y)}\frac{dy^2}{y^2 - 1} + \frac{1}{F(x)}\frac{dx^2}{1 - x^2} + (1 - x^2)d\phi^2\right],$$

(33)

with $F(\zeta)$ as in eq. (20). The function $\Lambda$ and the electromagnetic potential are given by

$$\Lambda = \frac{1 - e^2F(y)}{F(x)}, \quad A = -\frac{\sqrt{3}eF(y)}{2\Lambda F(x)}dt,$$

(34)

and we assume $e^2 < 1$. Mass and electric charge of the black ring are

$$M = \frac{3\pi L^2}{4}\frac{\lambda}{1 - \lambda} \frac{1 + e^2}{1 - \lambda 1 - e^2}, \quad Q = 2L^2\frac{\lambda}{1 - \lambda 1 - e^2}.$$  

(35)

In order to perform the ultrarelativistic boost, one can follow the method outlined in sections 1 and 2. For $\epsilon \to 0$ we rescale the mass as $M \approx 2\epsilon p_M$, that is $\lambda = \epsilon p_\lambda$ as in eq. (26) (hence
\[ p_\lambda = p_M \frac{8}{3\sqrt{37}}(1-e^2)/(1+e^2), \]
and we keep \( e \) constant. This will automatically enforce \( Q \sim \epsilon \), which is different from the rescaling (9) for black holes (note that \( M/Q = \sqrt{2}\pi (1+e^2)/e \) here).

Referring to the key coordinate transformations (23)–(25) of section 2, we directly jump to the final result. For an orthogonal boost we obtain an impulsive \( pp \)-wave of the form (27), with

\[
H^c_\perp = \sqrt{2}p_\lambda \left[ \frac{3L^2 1 + e^2}{1-e^2} + 2\xi^2 + x_2^2 \xi + L \right] \frac{K(k)}{\sqrt{(\xi + L)^2 + x_2^2}}
- \sqrt{(\xi + L)^2 + x_2^2} E(k) - \frac{\xi + L}{\xi - L} \frac{(\xi - L)^2 + x_2^2}{\sqrt{(\xi + L)^2 + x_2^2}} \Pi(p_0, k) + \frac{\pi}{2} |x_2|, \tag{36}
\]

\( k \) and \( \xi \) as in eq. (29) and \( p_0 = -(\xi - L)^2/x_2^2 \). For a parallel boost, one finds a metric (30) with

\[
H^c_\parallel = \sqrt{2}p_\lambda \left[ \frac{2L^2 1 + 2e^2}{1-e^2} + a^2 + a^2 \frac{L^2 + \eta^2}{a^2 - y_2^2} \frac{1}{a} K(k') - 2aE(k') \right.
\left. - \frac{\eta^2 + L^2 a^2 + y_2^2}{2a} \frac{1}{a^2 - y_2^2} \Pi(p', k') + \frac{\pi}{2} |y_2| \right], \tag{37}
\]

where \( k', \rho', a \) and \( \eta \) are as in eq. (32). The above profiles reduce to those for boosted static vacuum black rings [11] when \( e = 0 \). The function \( H^c_\perp \) is singular on the circle \( u = 0 = x_2, \xi = L \), which is the boundary of a disk-shaped membrane (a remnant of the conical singularity of the spacetime (33)). The function \( H^c_\parallel \) is singular on the rod \( u = 0 = \eta, |y_2| \leq L \), where the membrane has also contracted now. The rescaling \( Q \sim \epsilon \) (discussed above) implies that \( H^c_\perp \) and \( H^c_\parallel \) represents vacuum solutions, i.e. both \( F_{\mu\nu} \) and \( T_{\mu\nu} \) tend to zero for \( \epsilon \to 0 \). Note, however, that in eqs. (36) and (37) there is still a signature of the original electric charge (via \( \epsilon \)). This enables us to perform the “BPS limit” [13, 14] \( e^2 \to 1^- \) (simultaneously, replace \( p_\lambda \) with \( p_M \)) of \( H^c_\perp \) and \( H^c_\parallel \), which leads to simple expressions with no membrane disk. Exactly the same expressions can be recovered from the boost of the supersymmetric black ring [12].

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