Phase Transition Dynamics and Its $\alpha'$ Corrections

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Abstract

We study the dynamics of the first order phase transition in the holographic hard wall model, namely, Polchinski-Strassler’s model and come to the conclusion that the phase transition is incomplete in large $\mathcal{N}$ limit with the natural boundary condition. We also consider the string length corrections to both hard wall model and Witten’s model, and find that the interesting transition configuration is preserved under the $\alpha'$ corrections.
1 Introduction

The duality proposed by Maldacena [1] between type IIB string theory in AdS$_5 \times$S$^5$ and $\mathcal{N} = 4$ supersymmetric Yang-Mills theory in four dimensions has been an arena for amazing theoretical advances in the past several years. This duality, often called AdS/CFT duality for simplicity, has been extended to more realistic situations and shown to provide a supergravity description of the field theory that displays confinement and chiral symmetry breaking [2] [3]. On the other hand, the fact that black holes have thermodynamic properties is one of the most striking features of classical and quantum gravity. Thus studying thermal aspects of AdS/CFT duality will give more insights into understanding of gauge theory, gravity and Holographic principle. At finite temperature, Witten [4] generalized this duality to relate the thermodynamics of gauge theories to that of gravity theories, and to give geometric descriptions to confining - deconfining phase transitions in certain gauge theories. The dictionary of the gauge-gravity duality is that the deconfined phase is described by a black hole, and the confined phase is described by a nonsingular spacetime without a horizon. On the gravity side, this transition is the so-called Hawking-Page phase transition [5] and is the first order one. In [6, 7], Polchinski and Strassler proposed another interesting phenomenological model by removing the small radius region of AdS which corresponds to introducing an IR cutoff in the dual conformal field theory.

Recently, Horowitz and Roberts [8] found an interesting scenario of the first order phase transition in the framework of gauge-gravity duality. Starting with supercooled deconfined phase (with temperature well below the critical temperature), there are three stages in the phase transition: (i) nucleation of bubbles of the confined phase, (ii) rapid growth of these bubbles, (iii) large plasma-ball phase. From then on the evolution depends on the boundary condition. With a natural description, microcanonical (fixed energy) description, the plasma-ball is stable and the phase transition is not complete. The authors of [8] investigated Witten’s model of gauge-gravity duality and found that in stage (iii) the plasma-ball occupies at least one quarter of the initial volume.

This note is organized as follows. In section two we study the first order
phase transition dynamics of Polchinski-Strassler’s model [6, 7], often called hard wall model, and we get a similar conclusion as that of Horowitz and Roberts [8]. In section three we pay attention to the string length corrections of Polchinski-Strassler’s model and we also discuss Witten’s model with the $\alpha'$ corrections. Finally in section four we present some discussions and conclusions.

2 Gravity analysis of the hard wall model

Recently, the hard wall model has been studied in many papers [11]-[20]. Let us first give a brief review of this model. Following [1], there are remarkable processes [6, 2, 3] in extending the duality to more realistic conditions in order to give us more insights into QCD and the nature of confinement. A general realization from these papers is that confinement is related to supersymmetric field theories whose gravitational descriptions cap off the geometry in a smooth way in the infrared at small radius. The gravitational descriptions of those approaches are cumbersome, but the geometric insight is clear: IR cutoff of geometry at small radius produces confinement in the dual gauge theory. Based on this insight, [7] proposed a far simpler model: the small radius region of AdS is removed. While such a removal is brutal, subsequent work has shown that one gets realistic, semi-quantitative descriptions of low energy QCD [9, 10]. Some aspects of thermodynamics of this model have been discussed in papers such as [11, 13]. In this note, dynamics of confining-deconfining phase transition of this model is concerned. In the following, we will consider black holes thermodynamics in AdS, so the five sphere will not play an important role and we will not write it explicitly in the discussion below. In gauge-gravity duality, the gravity solution describing the gauge theory at finite temperature can be obtained by taking the decoupling limit of the general black 3-brane and keeping the energy density above extremality finite. Then in the hard wall model the system in the deconfined phase is described by black 3-brane metric

$$ds^2 = \frac{r^2}{L^2}(-f dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + \frac{L^2}{r^2} f^{-1} dr^2$$  \hspace{1cm} (2.1)
where \( f = 1 - \frac{r_0^4}{r^4} \), \( L^4 = 2g_{YM}^2 N \alpha'^2 \), \( r_0 \) is the mass parameter of the solution, and 't Hooft coupling constant for the SU(\( N \)) gauge theory is \( \lambda = 2g_{YM}^2 N = 4\pi g_s N \). The black hole horizon is located at \( r = r_0 \). The temperature can be determined by Wick rotating to Euclidean time \( \tau = it \) and choosing a periodicity for Euclidean time \( \beta = \pi L^2 / r_0 \equiv 1/T \) to regularize the singularity at \( r = r_0 \). The space only exists for \( r > r_{IR} \) due to the hard wall model with IR cutoff \( r_{IR} \). The ground state of confined phase is described by thermal AdS space

\[
 ds^2 = \frac{r^2}{L^2} (-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + \frac{L^2}{r^2} dr^2 \tag{2.2}
\]

with the same IR cutoff \( r > r_{IR} \).

According to [5], one can identify the free energy of the theory with Euclidean gravitational action times temperature, i.e.

\[
 I = \beta F \tag{2.3}
\]

The 5-dimensional gravitational action obtained from D=10 type IIB supergravity action by compactifying on S\(^5\) is

\[
 I_5 = -\frac{1}{16\pi G_5} \int dx^5 \sqrt{g_5}(\mathcal{R}_5 + \frac{12}{L^2}) \tag{2.4}
\]

Note that, since \( \text{Vol}(S^5) = \pi^3 L^5 \), we have

\[
 \frac{1}{16\pi G_5} = \frac{\pi^3 L^5}{16\pi G_{10}} = \frac{\pi^3 L^5}{2\kappa^2} = \frac{\pi^3 L^5}{(2\pi)^7 g_s^2 \alpha'^4} \tag{2.5}
\]

In the following discussions, we will set \( L = 1 \) for simplicity. Calculation of the contributions from the 2-derivative terms is divergent at large distance and requires a subtraction. In the black hole phase, the space exists from \( r_{\text{max}} = \max(r_0, r_{IR}) \) to infinity. Therefore, one has the Euclidean action for the black hole phase

\[
 I_{5b} = -\frac{N^2}{8\pi^2} V_3 \beta_b \int_{r_{\text{max}}}^{R} dr \sqrt{g_5}(\mathcal{R}_5 + 12) = \frac{N^2}{4\pi^2} V_3 \beta_b (R^4 - r_{\text{max}}^4) \tag{2.6}
\]

and Euclidean action for thermal AdS space phase with IR cutoff \( r > r_{IR} \)

\[
 I_{5a} = -\frac{N^2}{8\pi^2} V_3 \beta_a \int_{r_{IR}}^{R} dr \sqrt{g_5}(\mathcal{R}_5 + 12) = \frac{N^2}{4\pi^2} V_3 \beta_a (R^4 - r_{IR}^4) \tag{2.7}
\]
In order to analyze the dynamics of the first order phase transition, a natural reference background is the AdS space in the zero limit IR cutoff. Thus the background contribution is

\[ I_{5r} = -\frac{N^2}{8\pi^2} V_3 \beta_r \int_0^R dr \sqrt{g_5} (R_5 + 12) = \frac{N^2}{4\pi^2} V_3 \beta_r R^4. \]  

(2.8)

We must choose the appropriate periodicities of \( \beta_a \) and \( \beta_r \) to ensure that the solutions can be embedded into the reference background

\[ \frac{r}{L} f^{1/2} \beta_b |_{r=R} = \frac{r}{L} \beta_a |_{r=R} = \frac{r}{L} \beta_r |_{r=R}. \]  

(2.9)

In the large R limit, we have the constraint as

\[ (1 - \frac{r_0^4}{2R^4}) \beta_b = \beta_a = \beta_r. \]  

(2.10)

Subtracting the contribution of the reference background and taking \( R \to \infty \), one obtains the Euclidean action of the black hole

\[ I'_{5b} = \frac{N^2}{4\pi^2} V_3 \beta_b (\frac{1}{2} r_0^4 - r_{\text{max}}^4) \]  

(2.11)

In the light of the results in [11, 18], that in the classical gravity side, the Hawking-Page phase transition can occur only when \( r_{\text{IR}} < r_0 \), we take \( r_{\text{max}} = r_0 \) here. Then the formula (2.11) can be simplified to

\[ I'_{5b} = \frac{-N^2}{8\pi^2} V_3 \beta_b r_0^4 = \frac{-N^2 \pi^2}{8} V_3 T^3 \]  

(2.12)

Using the general thermodynamics relations

\[ I = \beta F \]

\[ E = \frac{\partial I}{\partial \beta} \]

\[ F = E - TS \]  

(2.13)

we find

\[ F_b = \frac{-\pi^2 N^2}{8} V_3 T^4 \]

\[ E_b = \frac{3\pi^2 N^2}{8} V_3 T^4 \]  

(2.14)

\[ S_b = \frac{\pi^2 N^2}{2} V_3 T^3. \]
Taking the same approach to AdS space (2.2), we get

\[
I'_{5a} = -\frac{N^2}{4\pi^2} V_3 \beta b r_{IR}^4 \\
F_a = -\frac{N^2}{4\pi^2} V_3 r_{IR}^4 \\
E_a = -\frac{N^2}{4\pi^2} V_3 r_{IR}^4 \\
S_a = 0 .
\] (2.15)

These energies are measured relative to the pure thermal AdS space. The AdS space has no horizon and thus no intrinsic entropy. We can calculate the free energy difference between the two phases and find:

\[
F_b - F_a = \frac{N^2}{8\pi^2} V_3 (2r_{IR}^4 - r_0^4) .
\] (2.16)

So for \( T < T_c = 2\frac{1}{4} r_{IR}/\pi \), the thermal gas dominates, and at \( T > T_c = 2\frac{1}{4} r_{IR}/\pi \), the black brane dominates. In the gravitational language, this is the so-called Hawking-Page phase transition. The above discussion is just the result of [11]. Now we will consider the dynamics of the phase transition.

Starting with the deconfined phase and quickly lowering the temperature below \( T_c = 2\frac{1}{4} r_{IR}/\pi \), how does the phase transition proceed? Based on the argument of [8], bubbles of the confined phase should be nucleated and grow. On the gravity side, this corresponds to bubbles of AdS being nucleated on the black 3-brane. At large N limit, the growth of these bubbles are described by supergravity. We will first discuss the growth of these bubbles in a microcanonical ensemble (fixed energy) which is the natural boundary condition for asymptotically AdS solutions. After the bubbles are nucleated, only a fraction \( \alpha \) of initial volume \( V_3 \) will be occupied by black 3-brane and the rest will be AdS space. In this process we will fix the energy due to the microcanonical ensemble

\[
E = \alpha E_b + (1 - \alpha) E_a
\] (2.17)

The total energy \( E \) is characterized by the equivalent temperature \( T_0 \), and the black brane covering all of \( V_3 \) will have this total energy. Substituting
the relation (2.14) and (2.15) into (2.17), we have
\[ \frac{3\pi^2 N^2}{8} V_3 T_0^4 = \alpha \frac{3\pi^2 N^2}{8} V_3 T^4 - (1 - \alpha) \frac{N^2}{4\pi^2} V_3 r_{IR}^4 . \tag{2.18} \]
This gives
\[ T = \left[ \frac{1}{\alpha} T_0^4 + \frac{1 - \alpha}{3\alpha} T_0^4 \right]^{1/4} . \tag{2.19} \]
Thus the entropy of this configuration is just \( \alpha \) times the entropy of the black brane:
\[ S = \frac{\pi^2}{2} \alpha N^2 V_3 T^3 \]
\[ = \frac{\pi^2}{2} N^2 V_3 \left[ \frac{1}{\alpha} T_0^4 + \frac{1 - \alpha}{3\alpha} T_0^4 \right]^{3/4} . \tag{2.20} \]
The entropy of this configuration is maximized at
\[ \alpha = \frac{1}{4} \left[ 1 + 3 \frac{T_0^4}{T_c^4} \right] \tag{2.21} \]
with \( \alpha \in [0, 1] \) and \( T_0 \in [0, T_c] \). One notable feature of this result is that even as \( T_0 \to 0 \), the maximal entropy configuration is the localized black hole taking up a quarter of initial volume. In terms of gauge theory, there will be a large region of deconfined plasma. Another interesting feature of this maximal entropy configuration is that the temperature of the black hole is at critical temperature independent of initial temperature \( T_0 \).

3 String length corrections to models

In this section we will focus on contributions from the string length correction \( \alpha^2 R^4 \) to the supergravity action. In the Einstein frame, using the convention of including \( F_5^2 \) in the action and imposing the self duality constraint on fiveform field strength \( F_5 = F_5^* \) by hand, the tree level type IIB string effective action has the following structure:
\[ I = -\frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{g} \left[ R - \frac{1}{2} (\partial \phi)^2 - \frac{1}{4 \cdot 5!} F_5^2 + \cdots + \gamma e^{-\frac{4}{3} \phi} W + \cdots \right] \] \( \tag{3.1} \)
where
\[ \gamma = \frac{1}{8} \zeta (3) \alpha' \]
\[ W = \mathcal{R}^{hmk} \mathcal{R}_{pqmn} \mathcal{R}^{\tau sp} \mathcal{R}_{\tau rsk} + \frac{1}{2} \mathcal{R}^{hkmn} \mathcal{R}_{pqmn} \mathcal{R}^{\tau sp} \mathcal{R}_{\tau rsk} + \text{terms depending on Ricci tensor} \] (3.2)

Dots stand for other terms depending on antisymmetric tensor field strengths and derivatives of dilaton. The field redefinition ambiguity allows one to change the coefficients of terms involving the Ricci tensor, so there exists a scheme where \( W \) entirely depends on the Weyl tensor
\[ W = \mathcal{C}^{hmk} \mathcal{C}_{pqmn} \mathcal{C}^{\tau sp} \mathcal{C}_{\tau rsk} + \frac{1}{2} \mathcal{C}^{hkmn} \mathcal{C}_{pqmn} \mathcal{C}^{\tau sp} \mathcal{C}_{\tau rsk} \] (3.3)

The form of \( W \) is special in the sense that the AdS\(_5\times\)S\(_5\) is still the solution of the action (3.1) with self-dual \( F_5 \) and constant dilaton. The \( \alpha'^3 \mathcal{R}^4 \) correction to AdS\(_5\) black brane metric has been obtained in [21]. Considering 5-dimensional metric
\[ ds^2 = H^2 (K^2 d\tau^2 + P^2 dr^2 + dx_1^2 + dx_2^2 + dx_3^2) \] (3.4)
where \( H, K, \) and \( P \) are functions of \( r \) only, then from (3.3) we have
\[ W = \frac{5}{36} \frac{1}{K^8 H^4 P^4} \left[ \frac{K'}{P} \right]^4 \] (3.5)
where primes denote derivatives with respect to \( r \). \( H, K, \) and \( P \) are given by
\[ H = r \quad , K = e^{a+4b} \quad , P = e^b \] (3.6)
and
\[ a = -2 \log r + \frac{5}{2} \log (r^4 - r_0^4) - \frac{15}{2} \gamma (25 \frac{r_0^4}{r^4} + 25 \frac{r_0^8}{r^8} - 79 \frac{r_0^{12}}{r^{12}}) + \mathcal{O}(\gamma^2) \] (3.7)
\[ b = -\frac{1}{2} \log (r^4 - r_0^4) + \frac{15}{2} \gamma (5 \frac{r_0^4}{r^4} + 5 \frac{r_0^8}{r^8} - 19 \frac{r_0^{12}}{r^{12}}) + \mathcal{O}(\gamma^2) \] (3.8)
The contribution from term \( W \) to the action has been calculated in [21] and the result is
\[ \delta I = -\frac{1}{16 \pi G_5} \int dx^5 \sqrt{\mathfrak{g}_5} \gamma W \] (3.9)
3.1 High order corrections to hard wall model

Let us first consider the hard wall model with higher order corrections. In the following discussions, we will consider the configurations having the confining-deconfining phase transitions, in which we just take \( r_{max} = r_0 \). The Euclidean action for the black hole phase with high order corrections is:

\[
I_{5b} = -\frac{N^2}{8\pi^2} V_3 \beta_b \int_{r_0}^{R} dr \sqrt{g_5} (R_5 + 12 + \gamma W)
\]

\[
= -\frac{N^2}{8\pi^2} V_3 \beta_b \int_{r_0}^{R} dr [-8r^3 + \gamma \left( \frac{360r_0^{16}}{r^{13}} + \frac{960r_0^{12}}{r^9} \right) + \mathcal{O}(\gamma^2)]
\]

\[
= \frac{N^2}{4\pi^2} V_3 \beta_b [R^4 - r_0^4 + 15\gamma r_0^{12} (r_0^4 - \frac{1}{R^{12}} - \frac{1}{r_0^{12}}) + 4(\frac{1}{R^8} - \frac{1}{r_0^8})]
\]

and for the AdS space with the IR cut-off is:

\[
I_{5a} = -\frac{N^2}{8\pi^2} V_3 \beta_a \int_{R_{IR}}^{R} dr \sqrt{g_5} (R_5 + 12 + \gamma W)
\]

\[
= \frac{N^2}{4\pi^2} V_3 \beta_a (R^4 - r_{IR}^4) .
\] (3.11)

The natural reference background action takes the form:

\[
I_{5r} = \frac{N^2}{4\pi^2} V_3 \beta_r R^4 .
\] (3.12)

Under the \( \alpha' \) correction, the periodicity condition becomes

\[
[1 - \frac{(1 + 75\gamma)r_0^4}{2R^4}] \beta_b = \beta_a = \beta_r
\]

and the expression of temperature becomes

\[
T \equiv \frac{1}{\beta_b} = \frac{(1 + 15\gamma)r_0}{\pi} .
\] (3.14)

Then we can calculate the thermodynamics quantities of the black hole phase:

\[
I'_{5b} = -\frac{N^2\pi^2}{8\beta_b^3} V_3 (1 + 15\gamma)
\]

\[
E_b = \frac{3\pi^2 N^2}{8\beta_b^3} V_3 (1 + 15\gamma)
\]

\[
S_b = \frac{\pi^2 N^2}{2\beta_b^3} V_3 (1 + 15\gamma)
\] (3.15)
and that of the AdS phase

\begin{align}
I'_{5a} &= -\frac{N^2}{4\pi^2} V_{3b} r_{1R}^4 \\
E_a &= -\frac{N^2}{4\pi^2} V_{3l} r_{1R}^4 \\
S_a &= 0
\end{align}

(3.16)

Then using the same trick of section two, we will easily get the maximal entropy configuration with

\[ \alpha = \frac{1}{4} \left[ 1 + \frac{3(1 + 15\gamma)\pi^4 T_0^4}{2r_{1R}^4} \right] = \frac{1}{4} \left[ 1 + \frac{3T_0^4}{T_c^4} \right] \]

(3.17)

and the temperature of this configuration is just the critical temperature after \(\alpha'\) correction, which is a little lower than the uncorrected one.

### 3.2 High order corrections to Witten’s model

In the frame of Witten’s model, we will focus on \(\mathcal{N} = 4\) super Yang-Mills compactified on a circle with antiperiodic fermions. This breaks the supersymmetry, and gives mass to the fermions and scalars. The low energy limit is a confining theory, but not purely 2+1 dimensional Yang-Mills theory with regard to the argument of [22]. Considering \(\mathcal{N} = 4\) super Yang-Mills on \(\mathbb{R}^3 \times S^1_{\theta}\), the circle has antiperiodic fermions, but is spacelike. It does not represent a Euclidean time direction. The ground state of the confining phase is described by the AdS soliton [23]

\[ ds^2 = \frac{r^2}{L^2} (-dt^2 + dx^2 + dy^2 + f d\theta^2) + \frac{L^2}{r^2} f^{-1} dr^2 \]

(3.18)

where

\[ f = 1 - \frac{r_s^4}{r^4} \]

(3.19)

The space only exists for \(r > r_s\) and regularity at \(r = r_s\) requires \(r_s = \pi L^2 / \beta_s \theta\) where \(\beta_s \theta\) is the length of \(\theta\) direction. At sufficiently high temperatures \(T > T_c\), the system is in a deconfined phase described by the black 3-brane metric

\[ ds^2 = \frac{r^2}{L^2} (-f dt^2 + dx^2 + dy^2 + d\theta^2) + \frac{L^2}{r^2} f^{-1} dr^2 \]

(3.20)
where
\[ f = 1 - \frac{r_0^4}{r^4} \] (3.21)

To avoid the canonical singularity at \( r = r_0 \), we require \( r_0 = \pi L^2/\beta \equiv \pi L^2 T \).

The first order phase transition of this model has been discussed in [8] by means of the dual classical gravity analysis. In this subsection, we will discuss the string length corrections to the dynamics of the phase transitions of Witten’s model. Using the same procedure of the above subsection, we have

\[
I_b = -\frac{N^2}{8\pi^2} V_2 \beta \beta \int_{r_0}^{R} dr \sqrt{g_5} (R^5 + 12 + \gamma W)
= \frac{N^2}{4\pi^2} V_2 \beta [R^4 - r_0^4 + 15 \gamma r_0^{12} (r_0^4 \left( \frac{1}{R}\frac{1}{12} \right) - \frac{1}{r_0^4}) + 4 \left( \frac{1}{R^8} - \frac{1}{r_0^8} \right)]
\] (3.22)

for the black hole phase and

\[
I_s = -\frac{N^2}{8\pi^2} V_2 \beta \beta \int_{r_s}^{R} dr \sqrt{g_5} (R^5 + 12 + \gamma W)
= \frac{N^2}{4\pi^2} V_2 \beta [R^4 - r_s^4 + 15 \gamma r_s^{12} (r_s^4 \left( \frac{1}{R}\frac{1}{12} \right) - \frac{1}{r_s^4}) + 4 \left( \frac{1}{R^8} - \frac{1}{r_s^8} \right)]
\] (3.23)

for the AdS soliton phase. The natural reference background is taking \( r_0, r_s \rightarrow 0 \) limit, and then we get a thermal AdS space

\[
I_r = -\frac{N^2}{8\pi^2} V_2 \beta' \beta' \int_{0}^{R} dr \sqrt{g_5} (R^5 + 12 + \gamma W)
= \frac{N^2}{4\pi^2} V_2 \beta [R^4 \left( 1 - \frac{1 + 75 \gamma}{2} \frac{r_0^4}{R^4} \right) + \mathcal{O}(\frac{r_0^8}{R^8})]
\] (3.24)

In the above derivation, we have used the periodicity conditions

\[
\beta_s = \beta' = \beta \left[ 1 - \frac{1 + 75 \gamma}{2} \frac{r_0^4}{R^4} + \mathcal{O}(\frac{r_0^8}{R^8}) \right]
\] (3.25)

\[
\beta_\theta = \beta'_\theta = \beta_\theta \left[ 1 - \frac{1 + 75 \gamma}{2} \frac{r_0^4}{R^4} + \mathcal{O}(\frac{r_0^8}{R^8}) \right]
\] (3.26)
After the background subtraction, the gravitational actions become

\[
I'_b = -\frac{N^2\pi^2}{8\beta^3}V_2\beta_\theta(1 + 15\gamma) = -\frac{N^2\pi^2}{8}V_2\beta_\theta T^3(1 + 15\gamma) \quad (3.27)
\]

\[
I'_s = -\frac{N^2\pi^2}{8\beta^3_s\theta}V_2\beta_s(1 + 15\gamma) = -\frac{N^2\pi^2}{8\beta^3_s\theta T}V_2(1 + 15\gamma) \quad (3.28)
\]

In order to get (3.28), we have used the \(\alpha'\) correction to \(r_s = \pi L^2/\beta_\theta\)

\[
(1 + 15\gamma)r_s = \pi L^2/\beta_\theta \quad . \quad (3.29)
\]

Employing the same procedure as before, we can calculate the thermodynamics quantities

\[
E_b = \frac{3\pi^2 N^2}{8\beta^3}V_2\beta_\theta(1 + 15\gamma) = \frac{3\pi^2 N^2}{8}V_2\beta_\theta T^4(1 + 15\gamma)
\]

\[
S_b = \frac{\pi^2 N^2}{2\beta^3}V_2\beta_\theta(1 + 15\gamma) = \frac{\pi^2 N^2}{2}V_2\beta_\theta T^3(1 + 15\gamma) \quad (3.30)
\]

and that of the AdS phase

\[
E_a = -\frac{\pi^2 N^2}{8\beta^3_s\theta}V_2(1 + 15\gamma)
\]

\[
S_a = 0 \quad (3.31)
\]

Following the derivation of [8], we can easily get the maximal entropy configuration with

\[
\alpha = \frac{1}{4}[1 + 3\beta_\theta\beta^3_s T_0^4] = \frac{1}{4}[1 + 3\frac{T_0^4}{T_c^4}] \quad (3.32)
\]

\[
T = [\beta_\theta\beta^3_s T_0^4]^{-\frac{1}{4}} = T_c \quad (3.33)
\]

where \(T_c\) is the phase transition temperature of this process, which can be obtained by subtracting (3.28) from (3.27). So the form of \(\alpha\) is retained, though the critical temperature becomes lower under the corrections. The maximal entropy configuration is the same as that of [8].
4 Discussion and Conclusion

In this note, we have discussed the dynamics of the first order confining-deconfining phase transition in the hard wall model. Following the procedure of [8], we show a similar maximal entropy configuration. In the large $\mathcal{N}$ limit, starting with the supercooled deconfined phase, the process of the phase transition of the hard wall model with the natural boundary condition ends up with at least 1/4 of initial volume of deconfined phase. This agrees with the conjecture of [24] that there exists a plasma-ball region that undergoes the first order phase transition. We also discuss the string length corrections to the hard wall model and Witten’s model. As we expect, this interesting picture is preserved under the $\alpha'$ corrections.

Besides the models discussed above, there are many other interesting holographic phenomenological models with confining-deconfining phase transition phenomenon. Based on the early works [4] [25], recently Sakai and Sugimoto considered the dynamics of flavor D8 branes in the background geometry of color D4 branes in the probe approximation [26] [27] which gives a model of holographic QCD with matter. The higher derivative corrections to this model has been studied by Basu [28]. It is interesting to study phase transition dynamics in this more realistic model. There are many models [29] [30] [31] which exhibit confining-deconfining phase transitions in frame of gauge gravity duality. The higher derivative corrections of them have been studied in [32] [33]. That study the phase transition dynamics of these models to see whether the interesting picture discussed above is preserved is still an open question.

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