Deuteron spin oscillations and rotation as a universal method of the N-N scattering amplitude study.

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Abstract

We consider the effects of the deuteron spin rotation and oscillations at the matter which is proportional to the real part of the deuteron spin-dependent forward scattering amplitude. That gives a possibility of a direct measurement of this quantity. Spin-dependent forward scattering amplitude of the deuteron on an unpolarized proton is discussed in terms of the Glauber multiscattering theory. This amplitude is determined by the nucleon rescattering, nonsphericity of the deuteron and spin-dependent nucleon-nucleon scattering amplitude. Thus deuteron spin oscillation phenomenon represents a method for N-N scattering amplitude investigation, including its real part, over a broad energy range.

I. INTRODUCTION

The most fundamental principles of our understanding of particle physics are analyticity and unitarity. Through analyticity we can get dispersion relation between the real and imaginary parts of the forward scattering amplitude. Experimental checking of dispersion relations is of great importance. While the imaginary part of zero angle amplitude is expressed through the total cross section using unitarity relation the measurement of the real part presents difficulties. The traditional method of measurement of the real part of nuclear scattering amplitude utilizes interference with the presumed known Coulomb amplitude dominating at small scattering angles \([1,2]\). For measuring the elastic scattering cross section over such a small range of the momentum transferred the special methods based on the spectrometry of the recoil nucleons are applied. It is need to avoid rescattering by target for the accurate measurement of the momentum and the angle of the recoil nucleon. The low density targets \([1]\) (supersonic hydrogen jet) or special detectors \([2]\) are used for this aim.

It has been shown in \([3,4]\) that there is an unambiguous method which makes direct measurement of the real part of the spin dependent forward scattering amplitude possible. This technique based on the phenomena of particle beam spin rotation and oscillation in a matter \([3,4]\) uses measurement of spin rotational angle under the condition of a transmission experiment — the so-called spin rotation experiment.

For the particles with spin \(S \geq 1\) the spin oscillation and rotation exist even in unpolarized targets and its value does not decrease with particle energy grows \([3,4]\) at high
energies.

Spin rotation and oscillation can be described by the particle spin-dependent refractive index of a medium which is proportional to the forward scattering amplitude by a target particle. For particles with spin $S \geq 1$ ($\Omega$-hyperon, deuteron) forward scattering amplitude by an unpolarized nucleon has the form:

$$\hat{F}(0) = T_0 + T_2(sn)^2,$$

(1.1)

where $S$ is the particle spin operator and $n$ is the direction of the particle momentum. Analysis by the frame of the Glauber multiscattering theory shows that $T_2$ is determined by rescattering of colliding particle constituents only i.e. the first term of Glauber series (single scattering) does not contribute to $T_2$. So, one has the unique possibility to observe nucleon rescattering in deuteron-proton collision and constituent quarks rescattering in $\Omega$-hyperon proton collision.

There exist two different contributions to the $T_2$ term. The first one is due to the nonsphericity of the $S \geq 1$ particle and the second one is due to the spin-dependent elastic scattering amplitude of the constituents. At present work $T_2$ is calculated for deuteron-proton collision and it is found that the contribution due to spin dependence of N-N scattering amplitude decreases with energy increasing and at the deuteron energy $\gtrsim 5$ Gev only the contribution due to deuteron nonsphericity survives.

Deuteron spin oscillation phenomenon is proportional to the product of a real to imaginary parts of the N-N scattering amplitude and increases with the energy growth at high energies. So a new possibility for checking dispersion relations arises. In particular, at high and super high energies we may check derivative dispersion relations for a scattering amplitude.

II. THE ROTATION AND OSCILLATION PHENOMENON OF A DEUTERON SPIN

The motion of a particle with spin inside the matter can be described by the refractive index

$$\hat{n} = 1 + \frac{2\pi \rho}{k^2} \hat{F}(0).$$

(2.1)

Here $\rho$ is the density of scatterers in the matter (the number of scatterers in 1 cm$^3$), $k$ is the wave number of an incident particle. $\hat{F}(0)$ is the zero-angle elastic scattering amplitude which is an operator in spin space of the incident particle. The dependence of the amplitude $\hat{F}(0)$ on the orientation of colliding particle spins gives rise to quasioptic effects (spin rotation, spin oscillation and dichroism) under the passage of a particle through the medium.

For particles with spins $S \geq 1$ the spin rotation arises even in passing through unpolarized targets.

Consider the propagation of deuteron through the unpolarized medium in details. Let the spin state of a deuteron being incident on a target be characterized by an initial spin wave function $\psi_0$. Then the spin wave function in the target can be written as:

$$\psi(z) = \exp\{ik\hat{n}z\}\psi_0.$$  

(2.2)
If the target is unpolarized, the zero angle scattering amplitude is determined by the deuteron spin properties only \((1.1)\). In this case \(T_0 = F_0(0)\), \(T_2 = F_1(0) - F_0(0)\), where \(F_1(0)\) and \(F_0(0)\) correspond to zero-angle scattering amplitudes for the deuteron with spin projection \(|S_z| = \pm 1\) and \(S_z = 0\), \(n\) is the direction of the incident deuteron momentum.

Equation \((1.1)\) contains the terms only with square-law spin dependence (we consider T-invariant interactions, therefore the odd powers of spin in the amplitude should be absent). Taking the quantization z-axis parallel to \(n\) and denoting the magnetic quantum number through \(m\) we obtain from equation \((2.1)\) and \((1.1)\):

\[
n_m = 1 + \frac{2\pi \rho}{k^2} F_m(0), \quad n_m = n'_m + i n''_m, \tag{2.3}
\]

where \(n'_m, n''_m\) are the real and imaginary parts of the refractive indices for the particle in an eigenstate with spin operator projection \(S_z = m\).

It follows from \((1.1)\), that the states with quantum numbers \(m\) and \(-m\) are described by the same refraction indexes, however \(n_{\pm 1} \neq n_0\). This difference determines such effects as spin rotation and oscillation.

In general case the deuteron spin wave function at a medium entry can be written as:

\[
\psi_0 = \{a e^{i \delta_{-1}}, b e^{i \delta_0}, c e^{i \delta_1}\}. \tag{2.4}
\]

The polarization properties of the particle with spin \(S = 1\) are expressed through spin vector \(\hat{S}\) and rank 2 tensor \(Q_{ij} = 3/2(\hat{S}_i \hat{S}_j + \hat{S}_j \hat{S}_i - 4/3 \delta_{ij})\). Using equations \((2.1),(2.2),(2.4)\) it is possible to find their evolution as a function of a particle way length inside the target. In particular, for the polarization vector \(<\hat{S}|\psi> = \frac{|\langle \psi | \hat{S} | \psi \rangle|}{|\psi|^2}\) we have the following expressions:

\[
\begin{align*}
<S_x> &= \sqrt{2} e^{-(n''_0 + n''_1)z} b (a \cos[\delta_{-1} - \delta_0 + (n'_1 - n'_0) z] + c \cos[\delta_0 - \delta_1 + (n'_0 - n'_1) z]) / |\psi|^2 \\
<S_y> &= \sqrt{2} e^{-(n''_0 + n''_1)z} b (a \sin[\delta_{-1} - \delta_0 + (n'_1 - n'_0) z] \\
&\quad + c \sin[\delta_0 - \delta_1 + (n'_0 - n'_1) z]) / |\psi|^2 \\
<S_z> &= e^{-2z n''_0} (-a^2 + c^2) / |\psi|^2
\end{align*}
\tag{2.5}
\]

And for the tensor rank 2:

\[
\begin{align*}
<Q_{xx}> &= \left\{ -[a^2 + c^2] e^{-2n''_0 z} / 2 + b^2 e^{-2n''_0 z} + 3 e^{-2n''_0 z} ac \cos[\delta_{-1} - \delta_1] \right\} / |\psi|^2 \\
<Q_{yy}> &= \left\{ -[a^2 + c^2] e^{-2n''_0 z} / 2 + b^2 e^{-2n''_0 z} - 3 e^{-2n''_0 z} ac \cos[\delta_{-1} - \delta_1] \right\} / |\psi|^2 \\
<Q_{zz}> &= \left\{ [a^2 + c^2] e^{-2n''_0 z} - 2 b^2 e^{-2n''_0 z} \right\} / |\psi|^2 \\
<Q_{xy}> &= 3 e^{-2n''_0 z} a c \sin[\delta_{-1} - \delta_1] / |\psi|^2
\end{align*}
\tag{2.6}
\]
\[ < Q_{zz} > = \left\{ \frac{3}{\sqrt{2}} e^{-(n''_0 + n''_1)z} b (-a \cos[\delta_{-1} - \delta_0 + (n'_1 - n'_0)z] + c \cos[\delta_{0} - \delta_{1} + (n'_0 - n'_1)z]) \right\} / |\psi|^2 \]

\[ < Q_{yz} > = \left\{ \frac{3}{\sqrt{2}} e^{-(n''_0 + n''_1)z} b (-a \sin[\delta_{-1} - \delta_0 + (n'_1 - n'_0)z] + c \sin[\delta_{0} - \delta_{1} + (n'_0 - n'_1)z]) \right\} / |\psi|^2 , \]

where \(|\psi|^2 = [a^2 + c^2]e^{-2n''_1z} + b^2 e^{-2n''_0z}\) and \(z\) is the particle way length inside the medium. From these expressions it follows that in general case the spin dynamics of deuteron is characterized by a superposition of two rotations in clockwise and counter-clockwise directions. Let’s consider some particular cases. If the initial polarization vector is normal to the particle momentum so, the initial populations and phases of the states with the quantum numbers \(m\) and \(-m\) are equal the components \(< S_y >\) and \(< S_z >\) remain zero during the whole time of particle penetration through the medium and \(< S_x >\) oscillates. The components of quadrupole tensor of deuteron oscillate too. If the initial polarization vector is directed at an acute angle to the momentum direction the polarization vector motion looks like rotation in fig.[4]. If the initial polarization vector is directed at an obtuse angle to the momentum direction the spin rotates in the opposite direction.

### III. EIKONAL APPROXIMATION FOR THE SPIN PARTICLES.

The phenomena of spin oscillation and rotation are defined by the value of \(T_2\). The two factors give contribution to \(T_2\) : 1) nonsphericity of a deuteron; 2) spin dependence of nucleon - nucleon scattering amplitude. In (4, 5) the eikonal Glauber approximation (6, 7) was used for the study of oscillation and rotation phenomena. The spin dependence of scattering amplitude isn’t taken into account in the traditional Glauber theory. Therefore, only the contribution of 1) was studied. Now, for the account of 1) and 2) we should use the spin eikonal Glauber approximation. Consider proton-deuteron elastic scattering. According to (10) the scattering operator for two colliding particles is written as

\[ \mathcal{T} = V_{int} + V_{int} \Omega V_{int} \]

where

\[ \Omega = \frac{1}{E - H + i0}, \quad H = K + V_{int} \] is Hamiltonian of the system, \(V_{int}\) is the operator of deuteron - proton interaction, \(K\) is the noninteracting Hamiltonian part for the \(d-p\) system. Making the transformations

\[ \frac{1}{E - H + i0} = \frac{1}{E - K + i0} \left( 1 + V_{int} \frac{1}{E - K + i0} \right) \]

we derive

\[ \mathcal{T} = V_{int} + V_{int} \Omega_0 \mathcal{T}, \text{ where } \Omega_0 = \frac{1}{E - K + i0} \] is the free propagator function of ”deuteron + proton” system. The interaction part of Hamiltonian is written as \(V_{int} = V_1 + V_2\) where \(V_1, V_2\) are ”proton - proton” and ”neutron - proton” interaction operators. By analogy we can write:

\[ \mathcal{T}_1 = V_1 + V_1 \Omega_0 \mathcal{T}_1, \quad \mathcal{T}_2 = V_2 + V_2 \Omega_0 \mathcal{T}_2, \]

(3.2)
Where \( \Omega_{01}, \Omega_{02} \) are the free propagators of the "proton + proton" and "neutron + proton" systems.

Expressing \( V_1 \) and \( V_2 \) through \( \mathcal{T}_1 \) and \( \mathcal{T}_2 \) from (3.2): \( V_1 = \mathcal{T}_1(1 + \Omega_{01}\mathcal{T}_1)^{-1}, \ V_2 = \mathcal{T}_2(1 + \Omega_{02}\mathcal{T}_2)^{-1} \) and substituting results in (3.1) we obtain \( \mathcal{T} = (1 - V_{int}\Omega_0)^{-1}V_{int} = \{ 1 - (\mathcal{T}_1(1 + \Omega_{01}\mathcal{T}_1)^{-1} + \mathcal{T}_2(1 + \Omega_{02}\mathcal{T}_2)^{-1})\Omega_0 \}^{-1} \times (\mathcal{T}_1(1 + \Omega_{01}\mathcal{T}_1)^{-1} + \mathcal{T}_2(1 + \Omega_{02}\mathcal{T}_2)^{-1}) \approx \{ 1 + \mathcal{T}_1\Omega_0 + \mathcal{T}_2\Omega_0 \} \{ \mathcal{T}_1 - \mathcal{T}_1\Omega_0\mathcal{T}_1 + \mathcal{T}_2 - \mathcal{T}_2\Omega_0\mathcal{T}_2 + \mathcal{T}_3\Omega_0\mathcal{T}_1 \} \) (3.3)

Considering proton scattering by resting deuteron in the impulse approximation for the proton kinetic energy much greater than the bound energy of the deuteron we can neglect the difference between \( \Omega_0 \) and \( \Omega_{0i} \):

\[
\mathcal{T} \approx \mathcal{T}_1 + \mathcal{T}_2 + \mathcal{T}_1\Omega_0\mathcal{T}_2 + \mathcal{T}_2\Omega_0\mathcal{T}_1 .
\] (3.4)

It can be written for small angle scattering:

\[
<\text{out}|\mathcal{T}|\text{in}> = -\delta^{(3)}(\mathbf{p}_{\text{out}} - \mathbf{p}_{\text{in}}) \frac{p}{(2\pi)^2\varepsilon(p)} F(q, n) ,
\] (3.5)

where \( \mathbf{p}_{\text{in}} \) is the sum of incident particles momentums, \( \mathbf{p}_{\text{out}} \) is that after the collision, \( \varepsilon(p) \), \( p \) are the energy and momentum of projectile, \( n = \frac{p}{|p|} \), \( q \) is momentum transferred \( q = \mathbf{p}' - \mathbf{p} \), \( F(q, n) \) is the scattering amplitude normalized by the condition \( \sigma_{tot} = 4\pi ImF(0) \). It can be obtained from the ordinary amplitude through division by the wave number of the incident particle. Amplitude normalized in such a way is invariant relative to the Lorentz transforms along the incident particle momentum direction, so, the \( p - d \) amplitude is equal to the \( d - p \) amplitude when the deuteron moves with the same velocity as proton at the first case. It is more convenient to speak about \( p - d \) amplitude under calculation but we need \( d - p \) amplitude for the description of the spin oscillations effect. Calculating scattering matrix element we use the deuteron wave function in the following form

\[
|d \mathbf{p} > = (2\pi)^{-3/2}e^{i\mathbf{r}_1 \mathbf{r}_2} \phi(\mathbf{r}_1 - \mathbf{r}_2) = (2\pi)^{-3/2} \int \tilde{\phi}(\mathbf{k})|1\mathbf{k} + \frac{p}{2} - \mathbf{k} + \frac{p}{2} > d^3\mathbf{k} ,
\] (3.6)

where \( \tilde{\phi}(\mathbf{k}) = \int \phi(\mathbf{r})e^{-ik\mathbf{r}} d^3\mathbf{r} \), \( |1\mathbf{k}; \mathbf{p} > \equiv |\mathbf{k} > |2\mathbf{p} > \equiv (2\pi)^{-3/2}e^{ik\mathbf{r}_1}(2\pi)^{-3/2}e^{ip\mathbf{r}_2} \). Taking matrix element from (3.4) we obtain for the single scattering (moving nucleon and rest deuteron are considered):

\[
<0|\mathcal{T}|\mathbf{p}> = \frac{1}{(2\pi)^3} \int \tilde{\phi}^+(\mathbf{k}') <\mathbf{p'}|\mathcal{T}_1|\mathbf{k}|\mathbf{p} > \times \tilde{\phi}(\mathbf{k}) d^3\mathbf{k} = \int \tilde{\phi}^+(\mathbf{k}')\delta^{(3)}(\mathbf{k}' - \mathbf{k}) <\mathbf{p'}|\mathcal{T}_1|\mathbf{k}|\mathbf{p} > \tilde{\phi}(\mathbf{k}) d^3\mathbf{k}' \]
\[
= -\delta^{(3)}(\mathbf{p}' - \mathbf{p}) \frac{p}{(2\pi)^2\varepsilon(p)} S\rho_{\sigma_1\sigma_2}\{ f_1(0, n)G(0) \}
\] (3.7)
and for the double scattering:

\[
\langle d 0 | p' | T_1 \Omega_0 T_2 | p \rangle | d 0 \rangle = \frac{1}{(2\pi)^3} \int \tilde{\phi}(k') \delta^3(k)d^3k' \]

\[
\times \phi(k)d^3kd^3k' = \frac{1}{(2\pi)^3} \int \tilde{\phi}(k') < p',2'-k',1 | T_1 | k; q > \frac{1}{E - \varepsilon(q) + i0}
\]

\[
\times \times < q,2-k | T_2 | - k; p > \tilde{\phi}(k)d^3q d^3k d^3k'
\]

\[
\approx \frac{\delta^{(3)}(p - p')}{(2\pi)^3} \int \tilde{\phi}(k') \frac{p}{\varepsilon(p)} f_1(p - q, n) \frac{1}{E - \varepsilon(p + k' - k) + i0}
\]

\[
\times \frac{p}{\varepsilon(p)} f_2(q - p, n) \tilde{\phi}(k) \delta(q - k' + k_z - p) \delta^{(2)}(q, k' + k) d^3q d^3k d^3k'
\]

\[
\approx -i \delta^{(3)}(p - p') \frac{p}{(2\pi)^3} S p_{\sigma_1 \sigma_2} \left\{ \int d^2q f_1(-q, n) f_2(q, n) | G^{(+)}(2q) \right\}, \quad (3.8)
\]

where the form factor \( G^{(\pm)}(q) \) is defined as

\[
G^{(\pm)}(q) = \frac{1}{(2\pi)^4} \int \tilde{\phi}(k') \tilde{\phi}(k) \delta^{(2)}(k + q, k' + \frac{q}{2}) \frac{i}{k_z - k'_z} d^3k d^3k', \quad (3.9)
\]

\[
G(q) = G^{(+)}(q) + G^{(-)}(q).
\]

To derive these equations we have used some assumptions: 1) it is suggested that \( r_D > r_0 \) where \( r_D \) is the deuteron size and \( r_0 \) is the interaction size; 2) we remain only the energy of the incident nucleon at the denominator of \( \Omega_0 \) and bring \( \Omega_0 \) to the eikonal form: \( (E - \varepsilon(p + k - k) + i0)^{-1} = (-\frac{\partial \varepsilon(p)}{\partial p}(k' - k) + i0)^{-1} = \frac{\varepsilon(p)}{p(k_z - k'_z + i0)} \). Taking the matrix element from the remaining terms of (8.4) and comparing with (3.3) we find the forward scattering amplitude of the nucleon on the deuteron:

\[
F(0) = S p_{\sigma_1 \sigma_2} \left\{ (f_1(0) + f_2(0)) G(0) \right\} + \frac{i}{2\pi} S p_{\sigma_1 \sigma_2} \left\{ \int (f_1(-q) f_2(q)) G^{(+)}(2q) \right\}
\]

\[
+ f_2(q) f_1(-q)) G^{(-)}(2q) d^2q \}
\]

(3.10)

It is implied in (3.10) that \( q \) is two-dimensional vector.

**IV. FORWARD ELASTIC SCATTERING AMPLITUDE OF THE DEUTERON ON A NUCLEON.**

Rewriting (3.10) in terms of a profile function \( \Gamma(b) \) and density \( \rho(r) \) we have for the nucleon deuteron scattering:

\[
F(0) = \frac{i}{2\pi} \int S p_{\sigma_1 \sigma_2} \left\{ \Gamma_1(b - b_1) + \Gamma_2(b - b_2) - \Gamma_1(b - b_1) \Gamma_2(b - b_2) \theta(z_1 - z_2) - \right.
\]

\[
\left. \Gamma_2(b - b_2) \Gamma_1(b - b_1) \theta(z_2 - z_1) \right\} \delta(r_1 - r_2) \rho(r_1) d^3r_1 d^3r_2 =
\]

\[
\frac{i}{2\pi} \int S p_{\sigma_1 \sigma_2} \left\{ (\Gamma_1(b - b') + \Gamma_2(b + b')) \rho_{\perp}(b') - \Gamma_1(b - b') \Gamma_2(b + b') \rho_{\perp}^{(+)}(b') - \right.
\]

\[
\left. \Gamma_2(b + b') \Gamma_1(b - b') \rho_{\perp}^{(-)}(b') \right\}\}
\]

(4.1)
where \( \rho_{\perp}^{(+)}(b) = \int_{0}^{+\infty} \rho(r)dr, \quad \rho_{\perp}^{(-)}(b) = \int_{-\infty}^{0} \rho(r)dr, \quad \rho_{\perp}(b) = \rho_{\perp}^{(+)}(b) + \rho_{\perp}^{(-)}(b), \)
\( r \equiv \{b, z\}. \) \( \rho(r) \) is the nucleon density in deuteron and spin density matrix of deuteron nucleons simultaneously. The expression (3.10) implies that, if the incident particle firstly scatters on the nucleon 1 it spin wave function is acted by \( \Gamma_1, \) and then by the \( \Gamma_2. \) If the first collision happens with nucleon 2 the \( \Gamma_2 \) acts firstly. Profile-function \( \Gamma(b) \) connected with the \( N-N \) nucleon scattering amplitude by the relation: 
\[
f(q) = \frac{i}{2\pi} \int \Gamma(b)e^{-iqb}d^2b
\]
and form factors connected with the density as \( C^{(\pm)}(q) = \int \rho_{\perp}^{(\pm)}(b)e^{iqb}d^2b. \) If we look at the expression (3.10) we see that in the resulting expression the integration area over transferred momentum \( q \) is restricted by the deuteron form factor. In supposition of a sharper dependence of deuteron form factor on \( q \) we may carry out N-N scattering amplitude from the integration or using the model profile function:

\[
\Gamma_{\alpha}(b) = \frac{2\pi i}{m} \left\{ a_{\alpha} + v_{\alpha}(\sigma_{\alpha}) + e_{\alpha}(\sigma_{\alpha}n)(\sigma n) - \frac{c_{\alpha}}{m}(\sigma + \sigma_{\alpha}) \times n \right. \\
+ \left. e_{\alpha}(\sigma_{\alpha}n)(\sigma n) - \frac{c_{\alpha}}{m}(\sigma + \sigma_{\alpha}) \times n - \frac{d_{\alpha}}{m^2}(\sigma_{\alpha} \sigma \partial / \partial b)(\sigma \partial / \partial b) \right\} \delta^{(2)}(b).
\]

where \( \delta^{(2)}(b) \) is the two dimensional Dirac delta-function, \( m \) is the nucleon mass. This profile function corresponds to the scattering amplitude

\[
f_{\alpha}(q) = a_{\alpha} + v_{\alpha}(\sigma_{\alpha}) + e_{\alpha}(\sigma_{\alpha}n)(\sigma n) + \\
\frac{ic_{\alpha}}{m}(\sigma + \sigma_{\alpha})q \times n + \frac{d_{\alpha}}{m^2}(\sigma_{\alpha}q)(\sigma q),
\]

where \( a_{\alpha}, \ v_{\alpha} \ldots \) do not depend on \( q. \) Considering T and P invariance we can represent \( \rho(r) \) as:

\[
\rho(r) = \frac{1}{4} \left\{ A_0 + A_1 S(\sigma_1 + \sigma_2) + A_2 (S r)(\sigma_1 + \sigma_2) \cdot r + A_3 (S r)^2 + A_4 (\sigma_1 \sigma_2) \\
+ A_5 (\sigma_1 r)(\sigma_2 r) + A_6((\sigma_1 S)(\sigma_2 S) + (\sigma_2 S)(\sigma_1 S)) + A_7 (\sigma_1 r)(\sigma_2 r) (S r)^2 \\
+ A_8 (\sigma_1 \sigma_2)(S r)^2 + A_9((\sigma_1 \times S \cdot r)(\sigma_2 S + \sigma_2 \cdot S \times r) + (\sigma_2 \times S \cdot r)(\sigma_1 \times S \cdot r)) \\
+ A_{10}((\sigma_1 r)(\sigma_2 S)(S r) + (\sigma_1 S)(\sigma_2 r)(S r) + (\sigma_1 r)(\sigma_2 S) + (S r)(\sigma_2 r)(\sigma_1 S)) \right\}
\]

\( A_n \) are real functions of \( r \) everywhere in (4.4). \( A_0 - A_{10} \) are found from the deuteron wave function:

\[
\phi_m = \frac{1}{\sqrt{4\pi}} \left( \frac{U(r)}{r} + \frac{1}{\sqrt{8}} \frac{W(r)}{r} S_{12} \right) \chi_m,
\]

where \( U(r) \) is the radial deuteron S-wave function and \( W(r) \) is the radial D-function, \( \chi_m \) is the spin wave function of two nucleons with spin projection \( m, \) \( S_{12} = \frac{3(\sigma_1 r)(\sigma_2 r)}{r^2} - (\sigma_1 \sigma_2). \)

Taking into account \( \rho(r) = 8\phi(2r)\phi^+(2r) \) we find nucleon density matrix for the deuteron at the state corresponding to spin projection 1:
G scattering amplitude is written down as:

\[ G_{\text{ordinary}}(r) = \frac{1}{\sqrt{8}} \left( U(2r) + \frac{1}{\sqrt{8}} W(2r) S_{12} \right) \chi_1 \chi_1^{\dagger} \left( U(2r) + \frac{1}{\sqrt{8}} W(2r) S_{12} \right) \]

where \( e \) is the unit vector in the deuteron spin direction. From the other side \( \langle 1 | \rho | 1 \rangle \) can be obtained by taking the matrix element from (1,4) for the deuteron state corresponding to spin projection 1. It can be obtained by comparing the expressions:

\[ A_0(r) = v^2 - 8uw + 16w^2, \quad A_1 = v^2 - 2uw - 8w^2, \quad A_2 = 6uw + 12w^2, \quad A_3 = 12uw - 12w^2, \quad A_4 = 8w^2 + 8uw - u^2, \quad A_5 = -24w^2, \quad A_6 = u^2 - 2uw + 4w^2, \quad A_7 = 72w^2, \]
\[ A_8 = -12w^2, \quad A_9 = -6uw, \quad A_{10} = -12w^2. \]

Functions \( u \) and \( w \) are expressed through \( U \) and \( W \) by: \( W(2r) = 4\sqrt{\pi ru}(r), U(2r) = \sqrt{2\pi ru}(r) \). By substituting the expression for \( \rho(r) \) and for the profile-function to the (1.1) it is possible to calculate the forward \( p - d \) scattering amplitude (Appendix) that is also \( d - p \) scattering amplitude for the two times larger energy due to normalization used. The values \( a, b, c, \ldots \) have been taken from the SAID [1] phase shift analysis (solution SP98) and Nijmegen deuteron S- D- wave function [13] results have been obtained which are pictured on the figure [3]. We see that the contribution of the spin dependent N-N interactions is small for the deuteron energy greater than 5 GeV. Note, that the approximation of the constant \( a, v, e, c, \ldots \) used in (1.2) is very rough, because calculations show that spin-dependent deuteron form factors have less sharper \( q \) dependence than spin-less form factor. Consider, for instance, form factor with neglecting the dependence on spin of deuteron nucleons.

\[ G(q) = G_0(q) + (Sn)^2 G_s(q) \]
\[ = \int (A_0(r) + A_3(r)b^2)e^{iqb}d^2bdz + (Sn)^2 \int A_3(r)(z^2 - b^2) e^{iqb}d^2bdz. \]

Form factors \( G_0(q) \) and \( G_s(q) \) are shown in fig. 3 and we see that the \( G_s(q) \) is wider than ordinary \( G_0(q) \) considered. We still use above the approximation to simplify calculations.

V. DEUTERON PROTON SCATTERING AMPLITUDE AT HIGH ENERGIES.

It has been found above that we may neglect the spin dependence of the N-N scattering amplitude at high energies. In this case the \( T_2 \) term of the proton deuteron forward scattering amplitude is written down as:

\[ T_2 = \frac{i}{2\pi} \int f(t)f(t)G_s(2q)d^2q, \]

where \( t = -q^2 \). We take N-N scattering amplitude from the [13] (dipole pomeron is considered there):

\[ f(s,t) = P(s,t) + \Phi(s,t) \pm \omega(s,t), \]

where " + " sign corresponds to the \( p\bar{p} \) scattering and " - " to the \( pp \) one, \( s \) is the nucleons energy squared in their center-of-mass frame. The amplitude contains dipole pomeron contribution.
\[ P(s, t) = ig^2 \ln(s e^{-i\pi}/s_1)(s e^{-i\pi}/s_2)^{\alpha_p(t)-1} \exp(-b_p t), \] (5.3)

\[ \Phi \text{ reggeon} \]
\[ \Phi(s, t) = i\sigma(s e^{-i\pi}/s_0)^{\alpha_s(t)-1} \exp(-b_s t) \] (5.4)

and \( \omega \) reggeon
\[ \omega(s, t) = r_\omega(1 - t/t_\omega)(s e^{-i\pi}/s_0)^{\alpha_\omega(t)-1} \exp(-b_\omega t) \] (5.5)

differences. Parameters used are listed in [13,14]. The N-N scattering amplitude (5.2) satisfies the DDR at high energies.

Let us remember how the DDR arises. The requirement of analyticity is that the
\( s \) obeys the derivative dispersion relation (DDR) [15–17].

In the latest equality \( f^pp(u + i0, t) = f^pp(4m^2 - u - i0, t) \),
where the relation \( s + u + t = 4m^2 \) is used. For the large \( u \) we have:
\[ f^pp(u + i0, t) = f^pp(-u - i0, t) = f^pp^*(u + i0, t). \] (5.7)

In the latest equality \( f^pp(u) \) is assumed to be a real function of \( u \). Changing \( u \) by \( s \) we have
\[ f^pp(s + i0, t) = f^pp^*(-s + i0, t) \] (5.8)

Constructing combinations \( f_+ = \frac{f^pp + f^pp}{2} \) and \( f_- = \frac{f^pp - f^pp}{2} \) we find
\[ f_+(se^{i\pi}) = f_+^*(s), \quad f_-^*(se^{i\pi}) = -f_-^*(s), \] (5.9)

and for our amplitude normalization \( f \sim \frac{f}{s} \):
\[ f_+(se^{i\pi}) = -f_+^*(s), \quad f_-^*(se^{i\pi}) = f_-^*(s). \] (5.10)

Solution of this relations are \( f_+(s) = iK(se^{-\pi/2}) \) and \( f_-^*(s) = K(se^{-\pi/2}) \), where \( K(s) \) is a real function \( K^*(s) = K(s^*) \). It is easy to check that \( f_+(s) \) and \( f_-^*(s) \) are satisfied to the dispersion relations:

\[ \Re f_+(s, t) = \tan(\frac{\pi}{2} \frac{d}{d\ln(s)}) \Im f_+(s, t) \]
\[ \Im f_-^*(s, t) = -\tan(\frac{\pi}{2} \frac{d}{d\ln(s)}) \Re f_-^*(s, t). \] (5.11)

The difference between the particle and antiparticle cross sections goes to zero at high energies [14] and the single DDR for \( f \) remains. The \( T_2 \) term of the forward deuteron
-proton scattering amplitude satisfies the DDR at high energies
\[ \Re T_2(s) = \tan(\frac{\pi}{2} \frac{d}{d\ln(s)}) \Im T_2(s) \] (5.12)

because if \( f \) satisfies the first equation of (5.10), \( T_2 \) in (5.1) has the same form as \( f \) \((G(q) \)

is a real quantity). Certainly we can deduce this by straight way by considering \( dp \) and \( d\bar{p} \)
channels. So measuring real part through the effect of the deuteron spin oscillation we have
a new test for DDR checking. Oscillation phase (rotational angle) and dichroism calculated
with the amplitude (5.2) is shown in the Fig. [4]. We see that rotation angle goes through
zero in vicinity of \( E \sim 1 \text{ Tev} \) and then its absolute value increases with energy.
VI. CANCELATION OF THE COULOMB INTERACTION INFLUENCE ON SPIN OSCILLATION AND ROTATION PHENOMENA.

It is necessary to discuss the influence of electromagnetic interaction. Charged particle beam moving in a matter is under the action of Coulomb multiple scattering. Since the Coulomb effects at high energies are significant only at small angles, it is enough to consider the terms proportional to $q^{-2}$ and to $q^{-1}$. Divergent at small $q$ terms of the deuteron electromagnetic scattering amplitude on an unpolarized nuclei of charge $Z$ can be written as (another terms which is convergent at small $q$ is much smaller than the nuclear part of the scattering amplitude):

$$F_{\text{col}} = a_{\text{col}}(q) - \frac{ic_{\text{col}}(q)}{m_d}(S \cdot n \times q), \quad (6.1)$$

where $a_{\text{col}}(q) = -\frac{2Z\alpha}{q^2\sqrt{\gamma_d^2 - 1}}$, $c_{\text{col}} = \frac{Z\alpha}{q^2} \left( \mu_d - \frac{\gamma_d}{\gamma_d + m_d/m + \sqrt{1 + (m_d/m)^2 + 2m_d/m\gamma_d}} \right)$,

$\alpha = 1/137$ is the fine structure constant, $\gamma_d$ is the deuteron Lorentz factor, $m_d$ and $m$ are the deuteron and target nuclei masses correspondingly. The nuclear part of scattering amplitude has smooth $q$–behavior and much smaller than the Coulomb one at small $q$. Coulomb scattering results in the particle beam spreading over small angle but still it does not prevent spin rotation and oscillation if to speak about rotation of the polarization averaged over beam spreading angle. This problem is discussed earlier in terms of density matrix formalism [24,19] and it has been shown that the second term of the electromagnetic amplitude results in only insignificant depolarization of the beam lowering absolute value of the beam polarization $|\xi|$

$$\frac{d\xi^2}{dz} = -4\pi\rho z \frac{|c_{\text{col}}|^2}{2m_d^2} \xi^2, \quad (6.2)$$

where

$$|c_{\text{col}}|^2 = \int_{q_{\text{min}}}^{q_{\text{max}}} |c_{\text{col}}|^2 q^3 dq = (Z\alpha)^2 \times \left( \mu_d - \frac{\gamma_d}{\gamma_d + m_d/m + \sqrt{1 + (m_d/m)^2 + 2m_d/m\gamma_d}} \right) \ln \left( \frac{q_{\text{max}}}{q_{\text{min}}} \right).$$

Maximum momentum transferred $q_{\text{max}} = \frac{2\pi}{r_d}$ is restricted by the deuteron size $r_d$ or by the collimation angle $\theta_{\text{det}}$ of the detector $q_{\text{max}} = \theta_{\text{det}}k_d$, where $k_d$ is the deuteron momentum. Minimal momentum transferred $q_{\text{min}} = \frac{2\pi}{r_b}$ is determined by the screening radius approximately equaled to the radius of the first Born orbit $r_b$. First term of the amplitude (6.1) results in the correction due to Coulomb nuclear interference which can be taken into account [19] by changing $F(0) \to F(0) \exp(i\tilde{a}_{\text{col}})$ in (2.1), where
\[
\tilde{a}_{col} = \int_{q_{\text{min}}}^{q_{\text{max}}} a_{col}(q) dq = -2Z\alpha \frac{\gamma_d}{\sqrt{\gamma_d^2 - 1}}.
\]

However, nuclear amplitude also contains an electromagnetic correction due to Coulomb wave distortion which turns out to be \( F(0) = F_{\text{nuc}} \exp\left(-i\tilde{a}_{col}\right) \) (Bethe’s phase \[21, 30\]), where \( F_{\text{nuc}} \) is ”pure” nuclear scattering amplitude under the turned off electromagnetic interaction. We see that these corrections cancel each other and ”pure” nuclear amplitude can be measured with the help of spin oscillation and rotation and dihroism phenomenon.

### VII. DEUTERON SCATTERING BY NUCLEI.

Deuteron scattering by nuclei should be considered on the basis of the Glauber multiscattering theory with Gribov inelastic corretions \[31, 32\]. However, we give simplified analysis of the discussed effect. Notice, that two types of the diagrams of deuteron double scattering exist \[33\]. For the first type diagrams both deuteron nucleons strike the same nucleon of the nucleus. For the second type diagrams deuteron nucleons are scattered by different nucleons of a nucleus (figure 5).

Contribution of the first type diagrams with screening can be expressed through the deuteron-nucleon scattering amplitude. Let us write deuteron-nucleus scattering amplitude as

\[
F_{dA} = \frac{i}{2\pi} \int \left(1 - e^{-\Omega_{dA}(b)}\right) d^2 b, \quad (7.1)
\]

where eikonal function is expressed through the deuteron-nucleon forward scattering amplitude \( F \), the nucleus radius \( R \), the deuteron radius \( r_d \) and the atomic mass number \( A \):

\[
\Omega_{dA} = \frac{3iFA}{R^2 + r_d^2} \exp\left(-\frac{3b^2}{2(R^2 + r_d^2)}\right). \quad (7.2)
\]

Substituting \( \Omega_{dA} \) to the \( (7.1) \) we find

\[
F_{dA} = i \frac{R^2 + r_d^2}{3} \text{Ein} \left(-\frac{3iFA}{R^2 + r_d^2}\right), \quad (7.3)
\]

where exponential integral \[34\] function \( \text{Ein} \) is

\[
\text{Ein}(z) = \int_0^z (1 - e^{-t}) dt/t, \quad \text{Re } z > 0.
\]

Remembering that the values of deuteron-nucleon scattering amplitudes \( F \) for the deuteron spin projection on the momentum \( S_z = 0 \) and \( |S_z| = 1/2 \) are slightly different we express the \( T_2 \) term of the forward deuteron-nucleus scattering amplitude through the \( T_2 \)-term of the deuteron-nucleon amplitude

\[
T_{2dA} = \frac{\partial F_{dA}}{\partial F} T_2 = i \frac{R^2 + r_d^2}{3} \left(1 - \exp\left(\frac{3iFA}{R^2 + r_d^2}\right)\right) \frac{T_2}{F} \quad (7.4)
\]

Taking \( R = 1.12 \times A^{1/3} \) fm we conclude that \( T_{2dA} \) is proportional to \( A \) at small \( FA/(R^2 + r_d^2) \) and to the \( A^{2/3} \) at large one.
Second type diagrams can be described through the elastic scattering amplitude of the nucleon by nucleus. Nucleon-nucleus eikonal has the same form as (7.2) \( \Omega_{NA} = \frac{3i f A}{R^2} \exp \left( -\frac{3b^2}{2R^2} \right) \) and connected with the N-A amplitude \( f_{NA}(q) \) as in (7.1). \( T_2 \) term of d-A forward scattering amplitude is written down as

\[
T_2^{dA} = \frac{i}{\pi} \int f_{NA}(q)f_{NA}(-q)G_s(2q)d^2q.
\] (7.5)

We can approximate \( G_s(q) \approx a_s q^2 \) and substituting to (7.5) we find

\[
T_2^{dA} = -\frac{ia_s}{4} \left( \exp \left( \frac{6ifA}{R^2} \right) - E_{in} \left( -\frac{6ifA}{R^2} \right) \right)
\] (7.6)

Assymptotic

\[
E_{in}(z) \approx \begin{cases} 1 + \frac{z^2}{2}, & |z| \ll 1; \\ -0.5772 + \ln(z), & |z| \gg 1 \end{cases}
\]

shows that \( T_2^{dA} \sim A^{2/3} \) at small \( fA/R^2 \) and \( T_2^{dA} \sim ln(A) \) at large one. Relative contribution of the diagrams of the first and the second type is seen from in Fig. 6.

VIII. CONCLUSION

Let us estimate the effect value for hydrogen target of 0.0675 \( g/cm^3 \) density. The oscillation phase and dihroism \( A \) are described by expressions:

\[
\phi(z) = 2\pi \rho Re(F_1(0) - F_0(0))z,
\] (8.1)

and

\[
A(z) = \frac{I_0(z) - I_{\pm 1}(z)}{I_0(z) + I_{\pm 1}(z)} = 2\pi \rho Im(F_1(0) - F_0(0))z,
\] (8.2)

where \( I_m(z) \) is the intensity of deuterons with the spin projection \( m \) on the depth \( z \) if the incident beam is unpolarized. For an unpolarized beam \( I_{-1}(0) = I_0(0) = I_1(1) = I/3 \). It is obtained for hydrogen target \( \phi \sim 1.1 \times 10^{-2} \text{ rad/m}, \ A \sim -3.9 \times 10^{-2} \text{ m}^{-1} \) at the deuteron laboratory energy 1 \( Gev \) (Fig. 2) and \( \phi \sim -5.2 \times 10^{-4} \text{ rad/m}, \ A \sim -2.3 \times 10^{-3} \text{ m}^{-1} \) at \( E_{lab} = 10 \text{ Tev} \) (Fig. 3).

For carbon target of 2.2 \( g/cm^3 \) density we find with the help of (7.4) \( \phi \sim 0.22 \text{ rad/m}, \ A \sim -0.65 \text{ m}^{-1} \) at the deuteron laboratory energy 1 \( Gev \) and \( \phi \sim -5.8 \times 10^{-4} \text{ rad/m}, \ A \sim -3.1 \times 10^{-2} \text{ m}^{-1} \) at \( E_{lab} = 10 \text{ Tev} \).

Note, that above 1 \( Tev \) the absolute value of the spin rotational angle grows asymptotically with the energy (Fig. 3). Thus the effect of the deuteron spin oscillations and dichroism allow measurement of the \( T_2 \) term of the deuteron elastic scattering amplitude over a broad energy range. This gives information about rescattering of nucleons, deuteron wave function, spin-dependent scattering amplitude of nucleons, high energy dependence of the real part of N-N scattering amplitude.
The expression for the forward scattering amplitude of the deuteron on a proton has the form

\[
F(0) = a_1 + a_2 + \{(v_1 + v_2)(\sigma S) + (e_1 + e_2)(\sigma n)(S n)\}4\pi^2 \int_0^{+\infty} A_1(z)z^2dz
\]

\[
+ \pi i \int_0^{+\infty} \left\{ \{a_1a_2A_0 + v_1v_2(3A_4 + z^2A_5 + 4A_6 + 4z^2A_9) + (e_1e_2 + v_1e_2 \right. \\
+ v_2e_1)(A_4 + z^2A_5) - \frac{1}{m^2}(d_2v_1 + v_2d_1)\left(\frac{A'_4}{2z} + \frac{3}{2}A_5 + \frac{A'_6}{z} + zA'_9 \right) \\
+ A_8 + 6A_{10}) - \frac{c_1c_2}{2m^2}\left(\frac{A'_0}{z} + \frac{A'_1}{z} + A_5 + \frac{2A'_6}{z} + 2A_8 + 2zA'_9 - 4A_{10} \right) \\
+ 4A_3\right\} + (S n)^2\{a_1a_2z^2A_3 + v_1v_2(z^4A_7 + 3z^2A_8 - 2z^2A_9 \\
+ 4z^2A_{10}) + (e_1e_2 + v_1e_2 + v_2e_1)(2A_6 + z^4A_7 + z^2A_8 + 4z^2A_{10}) - \frac{1}{4m^2}(d_2v_1 \\
+ v_2d_1)(-\frac{2A'_6}{z} + 6z^2A_7 + 2zA'_8 - 2A_8 - 2zA'_9 - 4A_9 - 12A_{10}) \\
+ \frac{c_1c_2}{2m^2}(zA'_9 + 2A_3 + \frac{A'_6}{z} + z^2A_7 - zA'_8 + A_8 + zA'_9 - 6A_9 - 2A_{10})\} \\
+ (\sigma S)\{(a_1v_2 + v_1a_2)A_1 - \frac{1}{4m^2}(a_1d_2 + a_2d_1)(\frac{A'_1}{z} + 3A_2) \\
- \frac{1}{2m^2}c_1c_2(\frac{A'_1}{z} - A_2) - \frac{i}{2m}(c_1v_2 + v_1c_2 + c_1e_2 + e_1c_2)zA_2\} \\
+ (\sigma n)(S n)\{(a_1v_2 + v_1a_2)z^2A_2 + (a_1e_2 + e_1a_2)(A_1 + z^2A_2) \\
+ \frac{1}{4m^2}(a_1d_2 + a_2d_1)(\frac{A'_1}{z} + 3A_2) + \frac{1}{2m^2}c_1c_2(\frac{A'_1}{z} - A_2) \\
+ \frac{i}{2m}(3c_1v_2 + 3v_1c_2 + c_1e_2 + e_1c_2)zA_2\} \right\}dz ,
\]

where \(a, v, c, e, d\) are constant and \(A_n\) are function of \(z\), prime means differentiation on \(z\).
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FIG. 1. The example of spin rotation of the particle with spin $S=1$ moving in unpolarized medium.
FIG. 2. Spin rotation angle and dichroism in hydrogen target before 5 Gev. Solid curve corresponds to the calculation with the spinless $N - N$ amplitudes.
FIG. 3. Deuteron spinless and quadrupole form factors.
FIG. 4. Rotation angle, dichroism in hydrogen target at high energies.

FIG. 5. Diagrams corresponding to the two kinds of the deuteron nucleons double scattering by nucleus.
FIG. 6. Dependence of the $T_2$-term from atomic width for the first (solid line) and for the second (dashed line) kinds of diagrams.
FIG. 7. Absolute value of the rotational angle at high energies.