How do magnetic microwires interact magnetostatically?

A. Pereira, J. C. Denardin, J. Escrig
Departamento de Física, Universidad de Santiago de Chile, USACH, Av. Ecuador 3493, Santiago, Chile.

The magnetostatic interaction between two ferromagnetic microwires is calculated as a function of their geometric parameters and compared with those measured through magnetic hysteresis loops of glass-coated amorphous Fe$_{77.5}$Si$_{7.5}$B$_{15}$ microwires. The hysteresis loops are characterized by well-defined Barkhausen jumps corresponding each to the magnetization reversal of individual microwires, separated by horizontal plateaux. It is shown that the magnetostatic interaction between them is responsible for the appearance of these plateaux. Finally, using the expression for the magnetostatic interaction is trivial to obtain the interacting force between microwires. Our results are intended to provide guidelines for the use of these microwires with technological purpose such as the fabrication of magnetic sensors.

I. INTRODUCTION

In the last decades, soft magnetic materials have been deeply investigated. Besides the basic scientific interest in their magnetic properties, there are a great deal of technologic interest due to their use in sensing applications, particularly in the fields of automotive, mobile communication, medical and home appliance industries. Moreover, these materials are very promising for spintronic devices in magnetic recording media. Two types of soft magnetic microwire families are currently studied: in-water quenched amorphous wires with diameters of around 120 µm and quenched and drawn microwires with diameters ranging from around 2 to 20 µm covered by a protective insulating glassy coat. Bi-stable microwires are characterized by square-shaped hysteresis loops defined by the abrupt reversal of the magnetization between two stable remanent states.

Although an array composed of a few ferromagnetic wires could in principle seem a quite simple problem to study and model, it is striking to notice how complex this problem can turn out to be. Effects of interparticle interactions are in general complicated by the fact that the dipolar fields depend upon the magnetization state of each element, which in turn depends upon the fields due to adjacent elements. Therefore, the modelling of interacting arrays of wires is often subject to strong simplifications like, for example, modelling the wire using a one-dimensional modified classical Ising model. Zhan et al. used the dipole approximation including additionally a length correction. J. Velázquez and M. Vázquez considered each microwire as a dipole, in a way that the axial field generated by a microwire is proportional to its magnetization. Nevertheless, this model is merely phenomenological since the comparison of experimental results with a strict dipolar model shows that the interaction in the actual case is more intense. They have also calculated the magnetostatic field and expanded it in multipolar terms showing that the non-dipolar contributions of the field are non negligible for distances considered in experiments. Besides, the magnetostatic interaction energy between two magnetic elements of arbitrary shape was derived within the frame-work of a Fourier space approach by Beleggia et al. Recently, a detailed study of the magnetostatic interaction between two parallel wires placed side by side has been shown. In spite of the extended study of the dipolar interactions, the effect of magnetostatic interactions on the hysteresis loops, due to the vertical displacement of the wires, has not been studied yet.

![FIG. 1. (a) Geometric parameters used for the individual wire description. (b) Micrograph image that shows a glass coated amorphous microwire.](image-url)
expression for the interaction in which the lengths and radii of the wires are taken into account. We focus on
the interacting force in pairs of interacting wires, as a function of the distance between them, in order to gain
insight on the understanding of the role of interactions on the plateaux that appears in the hysteresis loops.

![Diagram of interacting wires](image)

**FIG. 2.** Relative position of interacting wires: \( d \) is the interaxial distance and \( s \) is the horizontal separation.

**II. EXPERIMENTAL METHODS**

The experimental measurements have been performed in glass coated amorphous bi-stable magnetic microwires
with nominal composition Fe\(_{77.5}\)Si\(_{7.5}\)B\(_{15}\), with a saturation magnetization \( M_0 = 1.2 \times 10^6 \) A/m, radii of \( R = 6.5 \) µm, and the thickness of the glass coating of \( t = 3.3 \) µm. They are fabricated by means of Taylor-Ulitovsky tech-
nique by which the molten metallic alloy and its glassy coating are rapidly quenched and drawn to a kind of com-
posite microwire. Two samples with length of \( L = 7 \) mm each were cut from a larger wire and fixed parallel by
means of vacuum grease on a glass holder, with a separa-
tion of \( d = 45 \) µm between the wires.

The morphology of microwires was investigated by a retro-optical microscope (Olimpus). The hysteresis
curves were measured on a specially designed vibrating
sample magnetometer (VSM), inserted within a pair of
Helmholtz coils. These coils have sufficient field to satu-
rate the wires and present the advantage of field homo-
geneity and the absence of remanent fields. We will focus
our attention on measurements performed at room tem-
perature because a low temperature there is a change in
the domain structure of the microwires, probably owing
to the increasing internal stresses induced by the differ-
ent thermal expansion coefficients of the ferromagnetic
alloy and the covering glass.\(^{10}\)

**III. THEORETICAL CALCULATIONS**

We adopt a simplified approach in which the discrete distribution of magnetic moments of microwire is re-
placed by a continuous one characterized by a slowly varying magnetization \( \mathbf{M}(r) \). We consider wires for
which \( L \gg R \) so it is reasonable to assume an axial magnetization due to shape anisotropy, defined by
\( \mathbf{M}(r) = M_0 \hat{z} \), where \( \hat{z} \) is the unit vector parallel to the wire axis. The magnetostatic interaction between wires
can be calculated from\(^{20}\)

\[
E_{int} = \mu_0 \int \mathbf{M}_2(r) \nabla U_1(r) \, dV ,
\]

where \( \mathbf{M}_2(r) \) is the magnetization of the wire 2 and \( U_1(r) \) is the magnetostatic potential of the microwire 1. The
expression for this potential is given by

\[
U = \frac{1}{4\pi} \left( \int V \frac{\nabla \cdot \mathbf{M}(r')}{|r-r'|} \, d^3r' + \int_S \frac{\hat{n} \cdot \mathbf{M}(r')}{|r-r'|} \, ds' \right).
\]

Note that the first term on the left-hand side of Eq. (1) vanishes, because the magnetization field is constant; fur-
thermore, in the surface integral of Eq. (1) the only contributions arise from the upper and lower bases of the wire:
the upper circle is located at \( z = L/2 \) and the lower on at \( z = -L/2 \). Due to the symmetry of the problem, we used
the adequate type of the cylindrical kernel for the integral\(^{21}\), and after few manipulations, one finds that the
integral expression for the scalar potential is given by

\[
U(r, z) = \frac{M_0 R}{2} \int_0^\infty \frac{dk}{k} \left( J_0(kr) - J_1(kr) \left( e^{-k|\frac{L}{2} - z|} - e^{-k|\frac{L}{2} + z|} \right) \right) ,
\]

where \( J_m \) is a Bessel function of first kind and \( m \) order.

Now it is possible to calculate the magnetostatic interaction energy between two identical microwires using the
magnetostatic field experienced by one of the wires due to
the other. The final results reads

\[
E_{int} = -\mu_0 M_0^2 \pi R^2 L \int_0^\infty \frac{dq}{q^3} e^{-q(1+\alpha)} J_0 \left( \frac{d}{q L} \right) \frac{1}{q} \left( \frac{R}{L} \right) \left\{ \begin{array}{ll}
(1-e^q)^2 & s \geq L \\
(1-2e^q+e^{2q}) & s \leq L
\end{array} \right. \ (3)
\]

Equation (3) has been previously obtained for nanotubes.\(^{22}\) The general expression for the interaction energy
between wires with axial magnetization, given by Eq. (3), can only be solved numerically. However, wires that
motivated this work satisfy \( R/L = \alpha \ll 1 \), in which case one can use that \( J_1(\alpha x) \approx \alpha x/2 \). With this
approximation, Eq. (3) can be written in a very simple form as

\[
E_{int} = -\frac{\mu_0 M_0^2 \pi R^4}{4} \left( \frac{1}{\sqrt{d^2+(L-s)^2}} - \frac{2}{\sqrt{d^2+s^2}} + \frac{1}{\sqrt{d^2+(L+s)^2}} \right) \ (4)
\]
Finally, the magnetostatic field can be written as a function of the magnetostatic interaction between the magnetic microwires and is given by:

\[ H_{\text{int}} = \frac{E_{\text{int}}}{\mu_0 M_0 V} = -\frac{M_0 R^2}{4L} \left( \frac{1}{\sqrt{d^2 + (L - s)^2}} - \frac{2}{\sqrt{d^2 + s^2}} + \frac{1}{\sqrt{d^2 + (L + s)^2}} \right) \]  

(5)

IV. RESULTS

Figure 3 shows the hysteresis loop for two microwires of radii of \( R = 6.5 \, \mu m \), length of \( L = 7 \, \text{mm} \), thickness of the glass coating of \( t = 3.3 \, \mu m \), and interaxial separation \( d = 45 \, \mu m \) at room temperature. As observed, the magnetization process takes place in two steps: a jump near 0.5 Oe, that must be ascribed to the reversion of a microwire; and an additional jump near 1.2 Oe, corresponding to the reversion of the other microwire. The flat region before the second step corresponds to the magnetic field felt by one wire due to the other.

A. Dependence of the interaxial separation of the wires on the magnetostatic field

Theoretical and experimental magnetostatic results are combined in Fig. 4. Experimental data for the magnetostatic field for different values of the interaxial separation \( d \) are depicted by gray dots and the theoretical prediction is represented by the solid line. For the analytical calculations we have used an effective radii of \( R^* = 4.7 \, \mu m \), which is smaller than the real radius, in order to compensate the approximation of consider a homogeneous magnetization in our model. It is clear that there is a strong dependence of the magnetostatic field on \( d \). Note the good agreement between experimental data and analytical results for distances smaller than 80 \( \mu m \). The coupling rapidly decreases by increasing the separation between the wires, and become negligible for interaxial distances bigger than 80 \( \mu m \), at least within our experimental sensitivity. It can be understood if we consider that the stray field felt by one wire due to the other changes considerably if the wires are not completely parallel to each other. Thus, when the separation between the wires is big enough, small deviations may reduce the stray field produced by the wires.

\[ \text{FIG. 3. Hysteresis loop for two parallel amorphous wires of Fe}_{77.5}\text{Si}_{7.5}\text{B}_{15}. \]

\[ \text{FIG. 4. Magnetostatic field as a function of the interaxial separation of the microwires. The gray dots correspond to experimental data and the solid line represents the values calculated analytically. Parameters: } R = 6.5 \, \mu m, L = 7 \, \text{mm}, t = 3.3 \, \mu m, s = 0 \, \mu m \text{ and } d \text{ ranging between 45 } \mu m \text{ and } 90 \, \mu m. \text{ For the analytical calculations we have used an effective radii of } R^* = 4.7 \, \mu m \text{ and } M_0 = 1.2 \times 10^6 \, \text{A/m.} \]

B. Dependence of the horizontal separation of the wires on the magnetostatic field

It is interesting to analyze the behavior of the magnetostatic plateaux as the interaxis distance, \( d \), is kept fixed and the horizontal separation, \( s \), is varied. Our results are combined in Fig. 5. Experimental data for the magnetostatic field of the system are depicted by gray dots and the solid line represents the analytical calculation. We observe a strong decrease of the magnetic field as the horizontal separation between wires is increased. Good agreement is obtained between the measured data and analytical calculations.
A couple of isolated magnetic microwires can attract or repel each other upon approach depending on their relative orientations. For the microwires investigated the interacting force, \( \mathbf{F} = -\nabla E_{\text{int}} \), has been estimated to be of the order of 0.017 (for \( d = 90 \mu m \) and \( s = 0 \mu m \)) to 0.0685 dynes (for \( d = 45 \mu m \) and \( s = 0 \mu m \)) for the case of interaxial separation in the range of parameters considered. For the horizontal separation, the interacting force is in the range between 1.63 \( \times 10^{-5} \) (for \( d = 45 \mu m \) and \( s = 3.5 \) mm) to 0.0685 (for \( d = 45 \mu m \) and \( s = 0 \) mm) dynes.

V. DISCUSSION AND CONCLUSION

In summary, we have investigated the magnetostatic interaction between magnetic microwires. Using a continuous model we have obtained a simple expression to model the magnetostatic interaction in these particles. From our calculations and measurements we can conclude that the magnetostatic plateaux strongly depends on the geometry of the system. A couple of isolated magnetic microwires can attract or repel each other upon approach depending on their relative orientations. Based on microwires investigated the interacting force, \( \mathbf{F} = -\nabla E_{\text{int}} \), has been estimated to be of the other of 1.63 \( \times 10^{-5} \) (for \( d = 45 \mu m \) and \( s = 3.5 \) mm) to 0.0685 dynes (for \( d = 45 \mu m \) and \( s = 0 \mu m \)) in the range of parameters considered. The performance of such experiment could provide the basis for testing the different theoretical models. Our results provide guidelines for the production of microstructures with tailored magnetic properties.

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