Electromagnetic mass differences of SU(3) baryons within a chiral soliton model

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We investigate the electromagnetic mass differences of SU(3) baryons, using an “model-independent approach” within a chiral soliton model. The electromagnetic self-energy corrections to the masses of the baryon are expressed as the baryonic two-point correlation function of the electromagnetic currents. Using the fact that the electromagnetic current can be treated as an octet operator, and considering possible irreducible representations of the correlation function, we are able to construct a general collective operator for the electromagnetic self-energies, which consists of three unknown parameters. These parameters are fixed, the empirical data for the electromagnetic mass differences of the baryon octet being employed. We predict those of the baryon decuplet and antidecuplet. In addition, we obtain various mass relations between baryon masses within the corresponding representation with isospin symmetry breaking considered. We also predict the physical mass differences of the baryon decuplet. The results are in good agreement with the existing data.

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1. Isospin symmetry breaking in mass splittings of hadrons has been one of the most fundamental issues historically even before the advent of QCD [1]. Effects of isospin symmetry breaking are well known that they are originated from two different sources: Electromagnetic (EM) self-energies (also known as photon cloud energies) and the mass difference of the up and down quarks. The electromagnetic (EM) self-energies contribute to masses of a hadron isospin multiplet, depending on the corresponding charge. The EM mass differences arising from the EM self-energies were extensively studied [2,3]. Gasser and Leutwyler analyzed in their seminal paper [4] the EM contribution to baryon masses. They estimated the isospin-breaking mass differences of the baryon octet with the EM self-energies and were able to separate the pure hadronic part of isospin mass splittings, which should arise from the up and down quark mass differences, subtracting the EM mass differences from the experimental ones.

Thus, the mass splittings of the SU(3) baryons within an isospin multiplet can fall into two different terms, i.e., the hadronic and electromagnetic parts

\[ \Delta M_B = M_{B_1} - M_{B_2} = (\Delta M_B)_{\text{H}} + (\Delta M_B)_{\text{EM}}, \]  

where the subscript \( B \) denotes the baryon isospin multiplet and \( M_{B_1} \) and \( M_{B_2} \) stand for masses of two different baryons belonging to the same isospin multiplet. The \( (\Delta M_B)_{\text{H}} \) and the \( (\Delta M_B)_{\text{EM}} \) represent the hadronic and the electromagnetic contributions to the mass splitting. There is a great amount of works on the isospin mass differences of baryons [5-17]. Praszalowicz et al. calculated isospin mass splittings of the baryon octet and decuplet [14] in the chiral quark-soliton model, considering the EM mass differences based on the Dashen Ansatz [18]. Though the analysis of Ref. [14] works well phenomenologically, the Dashen Ansatz was originally derived for mesons and is valid strictly in the chiral limit.

While the isospin mass splittings for the baryon octet are experimentally well known, those for the baryon decuplet are less known. In the Review of Particle Physics 2010 [19], only the mass difference \( M_{\Xi^0} - M_{\Xi^-} \) is given as \(-2.9 \pm 0.9\) MeV. The Gatchina group [20] reported many years ago the mass difference \( M_{\Delta^+} - M_{\Delta^-} = -5.9 \pm 3.1\) MeV. However, the experimental uncertainty is rather large.

In the present work, we want to investigate the EM mass differences \( (\Delta M_B)_{\text{EM}} \) of SU(3) baryons within the framework of a chiral soliton model (\( \chi \)SM) in a “model-independent approach”. The motivation lies in the fact that previous works on the mass splittings of the SU(3) baryons in chiral soliton models are hampered by some uncertainties [21, 22] in determining model parameters from the experimental data, since they are rather sensitive to the data. One way to discard these uncertainties is to turn on the isospin symmetry breaking so that one can utilize the whole data of the masses of the baryon octet. However, since the experimental data contain the effects of isospin symmetry breaking due to both hadronic and EM self-energy corrections, one has to extract the hadronic part from the data. In order to isolate that, we have to subtract the EM self-energy contributions from them so that we can fix unambiguously the model parameters that are purely hadronic.

The EM self-energy corrections to the masses of the baryon are obtained from the baryonic two-point correlation functions of the EM currents. Thus, in this work, we will first derive a general collective operator for the EM self-energies within a framework of the chiral soliton model, using the fact that the EM current can be treated as an octet operator, and considering possible irreducible representations of the correlation functions. We will see that the collective operator for the EM corrections consists of three unknown parameters. Instead of computing these parameters using a model, we will fix them by employing the EM mass differences of the baryon octet that were extracted in Ref. [10]. With these parameters fixed, we can determine not only the EM mass splittings of the SU(3) baryons but also the physical mass differences within the isospin multiplets.

2. The current quark mass term in the QCD Lagrangian can be expressed as

\[ -\mathcal{L}_m = \bar{\psi} \hat{m} \psi = m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s, \]

where the \( \psi \) represents the quark field \( \psi = (u, d, s) \). The \( \hat{m} \) denotes the quark mass matrix \( \hat{m} = \text{diag}(m_u, m_d, m_s) \). The Lagrangian can be expressed in terms of the SU(3) flavor matrices

\[ -\mathcal{L}_m = \bar{\psi} (m_0 I + m_3 \lambda_3 + m_8 \lambda_8) \psi \]

with \( m_0 = (m_u + m_d + m_s)/3, \) \( m_3 = (m_u - m_d)/2, \) and \( m_8 = (m_u + m_d - 2m_s)/2\sqrt{3} \). The \( I \) denotes the singlet flavor matrix \( \text{diag}(1,1,1) \) whereas \( \lambda_3 \) and \( \lambda_8 \) stand for the third and the eighth components of the flavor SU(3) Gell-Mann matrices, respectively. Since the first term in Eq. (3) is a flavor singlet, it does not contribute to the mass splittings and mixings. On the contrary, the second and the third ones in Eq. (3), which cause the isospin and SU(3) symmetry breakings, respectively, lead to the mass splittings inside SU(3) baryon multiplets and the mixings between the multiplets. They can be obtained by sandwiching the second and third mass terms between the baryon states:

\[ \langle B' | \psi (m_3 \lambda_3 + m_8 \lambda_8) \psi | B \rangle. \]
Taking into account the fact that \( \lambda_8 \) transforms as the eighth component of an octet operator, we can express the masses of the baryon octet in terms of two parameters, since there are two irreducible singlet representations from the direct product of \( 8 \otimes 8 \otimes 8 \). On the other hand, the decuplet and the antidecuplet are uniquely determined by a single parameter, respectively, because the products \( 8 \otimes 8 \otimes 10 \) and \( 8 \otimes 8 \otimes 10 \) contain only one irreducible singlet representation, respectively. Similarly, we can parametrize the hadronic contributions of isospin symmetry breaking as \( (M_8)_{iso}^H = a T_3 + b Y T_3 \), \( (M_{10})_{iso}^H = r T_3 \), and \( (M_{\overline{10}})_{iso}^H = s T_3 \), where the \( T_3 \) and the \( Y \) stand for the third component of the isospin and the hypercharge, respectively. Thus, the hadronic parts of the SU(3) baryon masses can be written as

\[
M_8 = \overline{M}_8 + \begin{bmatrix} (a + b) T_3 \\ a T_3 \\ (a - b) T_3 \end{bmatrix} + \begin{bmatrix} -3x + y \\ -x \\ 2x - y \end{bmatrix} \text{ for } \begin{bmatrix} N \\ \Lambda \\ \Sigma \end{bmatrix},
\]

\[
M_{10} = \overline{M}_{10} + \begin{bmatrix} r T_3 \\ r T_3 \\ r T_3 \end{bmatrix} + \begin{bmatrix} -z \\ 0 \\ z \end{bmatrix} \text{ for } \begin{bmatrix} \Sigma^* \\ \Xi^* \end{bmatrix},
\]

\[
M_{\overline{10}} = \overline{M}_{\overline{10}} + \begin{bmatrix} 0 \\ s T_3 \\ s T_3 \end{bmatrix} + \begin{bmatrix} -2v \\ -v \\ v \end{bmatrix} \text{ for } \begin{bmatrix} \Xi^* \\ \Xi_{3/2} \end{bmatrix},
\]

where \( M_8, M_{10}, \) and \( M_{\overline{10}} \) denote the mass matrices corresponding to the baryon octet, decuplet, and antidecuplet, respectively. The \( \overline{M}_8, \overline{M}_{10}, \) and \( \overline{M}_{\overline{10}} \) are the corresponding center values. The parameters \( x, y, z, \) and \( v \) are originated from the flavor SU(3) symmetry breaking \( (m_8) \). The contributions from SU(3) symmetry breaking are presented in the bases of \( (N, \Lambda, \Sigma, \Xi) \) for the octet, \( (\Delta, \Sigma^*, \Xi^*, \Omega^-) \) for the decuplet, and \( (\Theta^+, \Sigma^*, \Xi_{3/2}) \) for the antidecuplet. The isospin symmetric parts of Eq. (1) are identified as the well-known Gell-Mann-Okubo mass formula \( [22, 24] \).

Note that, however, the baryon masses given in Eqs. (4-6) do not contain the EM corrections. Thus, in order to get the physical baryon masses, we have to take into account the EM corrections as in Eq. (11). We will show in the following how to analyze these EM corrections to the baryon masses.

3. It is well known that the EM corrections to the baryon masses can be derived from the baryonic two-point correlation functions of the EM current \( J_\mu \) in the static limit \( [25] \):

\[
M_B^{EM} = \frac{1}{2} \int d^3x d^3y \langle B | T | J_\mu(x) J^\mu(y) | B \rangle D_\gamma(x, y) = \langle B | O^{EM} | B \rangle,
\]

where \( J^\mu \) is defined as

\[
J^\mu(x) = e \bar{\psi}(x) \gamma_\mu \hat{Q} \psi(x)
\]

with the electric charge \( e \) and the quark charge operator \( \hat{Q} \) defined as the Gell-Mann-Nishijima relation

\[
\hat{Q} = \frac{1}{2} \left( \lambda_3 + \frac{1}{\sqrt{3}} \lambda_8 \right).
\]

The static photon propagator \( D_\gamma \) is given as \( 1/4\pi|x - y| \), but it will be absorbed in parameters we will fit to experimental data.

The EM current is taken as an octet operator, so that we can write in the most general form the \( O_{EM} \) as a collective operator

\[
O_{EM} = \alpha_1 \sum_{i=1}^3 D_{Qi}^{(8)} D_{Qi}^{(8)} + \alpha_2 \sum_{p=4}^7 D_{Qp}^{(8)} D_{Qp}^{(8)} + \alpha_3 D_{Q8}^{(8)} D_{Q8}^{(8)},
\]

where \( D_{Qa}^{(8)} = (D_{Qa}^{(8)} + D_{Qa}^{(8)}/\sqrt{3})/2 \) in which \( D_{Qa}^{(8)} \) denote the SU(3) Wigner \( D \) functions in the octet representation. The parameters \( \alpha_i \) encode specific dynamics of a chiral soliton model. For example, one could use the chiral quark-soliton model (\( \chi QSM \)) to obtain Eq. (10) and determine \( \alpha_i \). The EM operator \( O_{EM} \) is expressed in the \( \chi QSM \) as

\[
O_{EM} = -\frac{e^2}{2} \int d^3x d^3y D_\gamma(x, y) \int \frac{d\omega}{2\pi} \text{tr} \left( x \left| \frac{1}{\omega + iH} \gamma_\mu \lambda^a \right| y \right) \left( y \left| \frac{1}{\omega + iH} \gamma_\mu \lambda^b \right| x \right) D_{Qa}^{(8)} D_{Qb}^{(8)},
\]
where \( D^{(8)} = (D^{(8)}_8 + D^{(8)}_6)/\sqrt{3} \). The parameters \( a_i \) depends on specific dynamics of a \( \chi SM \), which will be fitted to the empirical data of the EM mass differences. Since the EM current is regarded as an octet operator, the product of two octet operators can be expressed in terms of irreducible operators \( 1 \oplus 8_s \oplus 8_a \oplus 10 \oplus \mathbf{10} \oplus 27 \). However, because of Bose symmetry, we are left only with the singlet, the octet, and the ekosheptaplet, which are all symmetric. A similar structure for the EM corrections can be found in Ref. [30]. We rewrite \( O^{EM} \) in terms of a new set of parameters \( c^{(n)} \) as follows.

We can reduce the collective EM operator \( O^{EM} \) in terms of single \( D \) functions as follows

\[
O^{EM} = c^{(27)} \left( \sqrt{3} D^{(27)}_{\Sigma_2 A_2} + \sqrt{3} D^{(27)}_{\Sigma_1 A_2} + D^{(27)}_{A_2 A_2} \right) + c^{(8)} \left( \sqrt{3} D^{(8)}_{\Sigma_0 A} + D^{(8)}_{AA} \right) + c^{(1)} D^{(1)}_{AA},
\]

where

\[
c^{(27)} = \frac{1}{40} \left( \alpha_1 - 4\alpha_2 + 3\alpha_3 \right), \quad c^{(8)} = \frac{1}{10} \left( \alpha_1 - \frac{2}{3} \alpha_2 - \frac{1}{3} \alpha_3 \right), \quad c^{(1)} = \frac{1}{8} \left( \alpha_1 + \frac{4}{3} \alpha_2 + \frac{1}{3} \alpha_3 \right).
\]

The notations \( \Sigma_0, \Sigma_1, \Sigma_2, \Lambda, \) and \( A_27 \) in the subscripts of the \( D \) functions stand for the corresponding flavor quantum numbers, in given representations \( R \), \( Y, T, T_3 \) \((R) = (0, 1, 0)(8), (0, 1, 0)(27), (0, 2, 0)(27), (0, 0, 0)(8), \) and \((0, 0, 0)(27), \) respectively [29]. Note that \( O^{EM} \) in Eq. (12) consist only of the ekosheptaplet \((27)\), the octet \((8)\), and the singlet \((1)\) representations. Since the EM current is regarded as an octet operator, the product of two octet operators can be expressed in terms of irreducible operators \( 1 \oplus 8_s \oplus 8_a \oplus 10 \oplus \mathbf{10} \oplus 27 \). However, because of Bose symmetry, we are left only with the singlet, the octet, and the ekosheptaplet, which are all symmetric. Note that the first three terms of Eq. (12) (part of the ekosheptaplet) have the same parameter \( c^{(27)} \) of the contributions from the \( \Delta T = 0, 1, 2 \) transitions in ekosheptaplet. The last term in Eq. (12) does not contribute to the EM mass splittings, because it is the singlet and corresponding corrections will be canceled out for the EM mass differences. The parameters \( c^{(n)} \) will be fixed by the empirical data estimated in Ref. [10]. A similar structure for the EM corrections can be found in Ref. [30] in which the operator for the EM mass splitting is given as

\[
O^{EM}_{Swart} = O^{(27)}_{(0,2,0)} + O^{(27)}_{(0,1,0)} + O^{(8)}_{(0,1,0)} + O^{(1)}_{(0,0,0)}.
\]

The operator (14) is different from Eq. (12), because \( \Delta T = 1 \) and \( \Delta T = 2 \) contributions in the ekosheptaplet are treated separately in Eq. (14). On the other hand, they are identical in the EM operator of the present work. This is due to the fact that in the chiral soliton model the SU(2) soliton is embedded in SU(3) by Witten’s trivial embedding [31].

In order to calculate the matrix elements of Eq. (12), we need to know SU(3) baryon wave functions. In the \( \chi SM \), the baryon wave functions are found to be SU(3) Wigner functions in representation \( R \)

\[
|B\rangle = \sqrt{\text{dim}(R)} (-1)^{J_3 + Y'}/2 D^{(R)*}(Y,T,T_3)(-Y',J,J_3) (A),
\]

which diagonalize the collective Hamiltonian in the \( \chi SM \). The \( Y' \) denotes the eighth component of the SU(3) spin operator \( Y' = -2J_3/\sqrt{3} = N, B/3 = 1 \). The \( B \) is the baryon number. The constraint of the \( Y' \) in the Skyrme model arises from the Wess-Zumino term [32, 33] whereas it comes from the valence quarks filled in the discrete level in the \( \chi QSM \) [26, 24]. The collective baryon wave functions are not in a pure representation, when the SU(3) symmetry breaking effects are considered. However, since we are interested in the EM mass differences in the present work, we need not consider the wave-function corrections.

The EM mass can be obtained by sandwiching the collective operator \( O^{EM} \) in Eq. (10) between the baryon states. The corresponding results can be written for the baryon octet

\[
M^{EM}_N = \frac{1}{5} \left( c^{(8)} + \frac{4}{9} c^{(27)} \right) T_3 + \frac{3}{5} \left( c^{(8)} + \frac{2}{27} c^{(27)} \right) \left( T_3^2 + \frac{1}{4} \right) + c^{(1)},
\]
\[
M^{EM}_{\Lambda} = \frac{1}{10} \left( c^{(8)} - \frac{2}{3} c^{(27)} \right) + c^{(1)},
\]
\[
M^{EM}_{\Sigma} = \frac{1}{2} c^{(8)} T_3 + \frac{2}{9} c^{(27)} T_3^2 - \frac{1}{10} \left( c^{(8)} + \frac{14}{9} c^{(27)} \right) + c^{(1)},
\]
\[
M^{EM}_{\Xi} = \frac{4}{5} c^{(8)} - \frac{1}{9} c^{(27)} T_3 - \frac{2}{5} \left( c^{(8)} - \frac{1}{9} c^{(27)} \right) \left( T_3^2 + \frac{1}{4} \right) + c^{(1)},
\]

and for the baryon decuplet

\[
M^{EM}_\Delta = \frac{1}{4} \left( c^{(8)} + \frac{8}{63} c^{(27)} \right) T_3 + \frac{5}{63} c^{(27)} T_3^2 + \frac{1}{8} \left( c^{(8)} - \frac{2}{3} c^{(27)} \right) + c^{(1)},
\]
the Weinberg-Treiman mass formula
turn off the effects of isospin symmetry breaking, the mass formula $e$ of Eq.(22) are reduced to the following relation
unknown parameters, i.e.
in which they are canceled out. Therefore, the expressions of EM mass differences of SU(3) baryons have only two unknown parameters, i.e. $c^{(8)}$ and $c^{(27)}$.

As shown in Eqs.16 17, they are expressed in terms of the isospin third component $T_3$, its square $T_3^2$, and the constant terms arising from the hypercharge. Note that Eqs.16 17 in general can be rewritten in terms of the electric charge $Q$ and its square $Q^2$ with the Gell-Mann-Nishijima relation in Eq.9 used. We want to mention that the present results are distinguished from the Dashen ansatz for the EM mass splittings that shows $Q^2$ proportionality ($\sim Q_B^2 M_B$), which was employed in Ref.14. Moreover, it turns out that Eqs.16 17 have the same structures as the Weinberg-Treiman mass formula $M(T_3) = \alpha T_3^2 + \beta T_3 + \gamma$[27].

It is straightforward to obtain the EM mass differences for the baryon octet from Eq.(16)
\[
(M_p - M_n)_{EM} = \frac{1}{5} \left( c^{(8)} + \frac{4}{9} c^{(27)} \right), \quad (M_{\Sigma^+} - M_{\Sigma^-})_{EM} = c^{(8)}, \quad (M_{\Xi^0} - M_{\Xi^-})_{EM} = \frac{4}{5} \left( c^{(8)} - \frac{1}{9} c^{(27)} \right). 
\] 

Using Eq.18, we immediately obtain the following mass formula $c^{(8)} = (M_p - M_n)_{EM} + (M_{\Xi^0} - M_{\Xi^-})_{EM} = (M_{\Sigma^+} - M_{\Sigma^-})_{EM}$. This is just the well-known Coleman-Glashow mass formula[3] at the level of the EM corrections. Although these formulae indicate that these three mass differences are dependent on each other, one can adjust the values of the parameters $c^{(8)}$ and $c^{(27)}$ by the method of least squares. In order to determine the parameters $c^{(8)}$ and $c^{(27)}$, we will first use the empirical data estimated in Ref.10. Using these empirical and experimental data, we can determine the values of the parameters $c^{(8)}$ and $c^{(27)}$ as follows
\[
c^{(8)} = -0.15 \pm 0.23, \quad c^{(27)} = 8.62 \pm 2.39\]
in units of MeV. In Table I the reproduced EM mass differences for the baryon octet are listed.

|                | Inputs | Reproduced |
|----------------|--------|------------|
| $(M_p - M_n)_{EM}$ | $0.76 \pm 0.30$ | $0.74 \pm 0.22$ |
| $(M_{\Sigma^+} - M_{\Sigma^-})_{EM}$ | $-0.17 \pm 0.30$ | $-0.15 \pm 0.23$ |
| $(M_{\Xi^0} - M_{\Xi^-})_{EM}$ | $-0.86 \pm 0.30$ | $-0.88 \pm 0.28$ |

Employing the results of Eqs.16 and 19 and the masses of the baryon octet as input, we can determine the parameters for the hadronic isospin symmetry breaking as follows
\[
a = -3.63 \pm 0.09, \quad b = 2.86 \pm 0.12
\] 
in units of MeV. From Eq.16, we are able to reproduce various mass relations of the baryon octet, the EM corrections being taken into account
\[
M_p - M_n = (M_{\Sigma^+} - M_{\Sigma^-}) - (M_{\Xi^0} - M_{\Xi^-}).
\] 
Equation 21 is the Coleman-Glashow relation[3]. Note that even though we consider the EM corrections, the Coleman-Glashow relation is still preserved. Taking into account both mass splittings from the SU(3) and isospin symmetry breakings with the EM corrections, we derive the following relations
\[
2(M_p + M_{\Xi^0}) = 3M_A + M_{\Sigma^+} + (M_{\Sigma^+} - M_{\Sigma^-}) + \frac{2}{3} \Delta M_{\Sigma},
\]
\[
2(M_n + M_{\Xi^-}) = 3M_A + M_{\Sigma^-} - (M_{\Sigma^+} - M_{\Sigma^-}) + \frac{2}{3} \Delta M_{\Sigma},
\]
where $\Delta M_{\Sigma} = M_{\Sigma^+} + M_{\Sigma^-} - 2M_{\Xi^0}$. These two mass relations are well satisfied with the experiment data. If we turn off the effects of isospin symmetry breaking, the mass formulae of Eq.22 are reduced to the following relation
\[
2(M_N + M_\Xi) = 3M_A + M_\Sigma + \frac{2}{3} \Delta M_\Sigma,
\]
which generalizes Gell-Mann-Okubo mass formula for the baryon octet [23, 24] with the EM corrections presented in the last term. When the EM interaction is turned off, Eq. (23) leads to the Gell-Mann-Okubo mass formula for the baryon octet with the EM corrections presented.

The EM mass differences of the baryon decuplet can be read off from Eq. (17) as follows

\[
(M_{\Delta^+} - M_{\Delta^0})_{\text{EM}} = \frac{1}{4} \left( c^{(8)} + \frac{16}{21} c^{(27)} \right),
\]

\[
(M_{\Delta^0} - M_{\Delta^-})_{\text{EM}} = \frac{1}{4} \left( c^{(8)} + \frac{8}{63} c^{(27)} \right),
\]

\[
(M_{\Sigma^{*+}} - M_{\Sigma^{*0}})_{\text{EM}} = \frac{1}{4} \left( c^{(8)} + \frac{32}{63} c^{(27)} \right),
\]

for \( \Delta T_3 = 1 \),

\[
(M_{\Delta^{++}} - M_{\Delta^0})_{\text{EM}} = \frac{1}{2} \left( c^{(8)} + \frac{4}{9} c^{(27)} \right),
\]

\[
(M_{\Delta^+} - M_{\Delta^-})_{\text{EM}} = \frac{1}{2} \left( c^{(8)} - \frac{4}{21} c^{(27)} \right),
\]

for \( \Delta T_3 = 2 \), and

\[
(M_{\Delta^{++}} - M_{\Delta^-})_{\text{EM}} = \frac{3}{4} \left( c^{(8)} + \frac{8}{63} c^{(27)} \right),
\]

for \( \Delta T_3 = 3 \). The parameter \( r \) in Eq. (4) is found to be

\[
r = -2.19 \pm 0.08
\]

in units of MeV. Note that the mass relations shown in Eqs. (25)-[27] are also valid with the hadronic effects of the isospin symmetry breaking considered.

4. We now present the numerical results and discuss them. Since we have determined all relevant parameters for the EM mass differences of the SU(3) baryons, i.e. \( c^{(8)} \) and \( c^{(27)} \), we can proceed to calculate numerically the EM mass differences. Putting into Eq. (25) the numerical values of \( c^{(8)} \) and \( c^{(27)} \) in Eq. (19), we can predict the results of the EM mass differences of the baryon decuplet. In Table III we list the corresponding results.

Since we have also determined the mass parameter \( r \) for the hadronic isospin symmetry breaking, we can easily find the numerical results for the physical mass differences within the isospin multiplets of the baryon decuplet, which are listed in Table III. We compare the present results with the experimental data for the isospin mass splittings for the baryon decuplet. Note that the data from Ref. [35] are based on the \( \pi N \) phase-shift analysis. Moreover, one has to keep in mind that the experimental data suffer from a large amount of errors and are not completely free from model dependence. The present results are in good agreement with the experimental data within uncertainties. The results in Table III consist of the contributions from both the mass difference between the up and down quarks \( (m_u - m_d) \) and the EM corrections. Since we are able to extract the EM corrections from the physical mass differences, we can estimate how large isospin symmetry is hadronically broken. Moreover, we find that the physical mass relation

\[
(M_{\Sigma^{*0}} - M_{\Sigma^{*-}}) = (M_{\Xi^{0}} - M_{\Xi^{-}})
\]

is well satisfied with the experimental data, as shown in Table III.
Table III. Isospin mass splittings of the baryon decuplet in units of MeV.

| Mass Difference | This Work | Experimental Data |
|-----------------|-----------|--------------------|
| $M_{\Delta^+} - M_{\Delta^+}$ | $-0.59 \pm 0.47$ |                  |
| $M_{\Delta^-} - M_{\Delta^-}$ | $-1.95 \pm 0.13$ |                  |
| $M_{\Delta^0} - M_{\Delta^0}$ | $-3.32 \pm 0.32$ |                  |
| $M_{\Xi^-} - M_{\Xi^-}$ | $-1.95 \pm 0.13$ |                  |
| $M_{\Xi^0} - M_{\Xi^0}$ | $-3.32 \pm 0.32$ | $-3.1 \pm 0.6$ [36] |
| $M_{\Xi^0} - M_{\Xi^0}$ | $-2.54 \pm 0.57$ | $-2.86 \pm 0.30$ [35] |
| $M_{\Delta^+} - M_{\Delta^-}$ | $-5.28 \pm 0.30$ |                  |
| $M_{\Delta^+} - M_{\Delta^-}$ | $-5.86 \pm 0.38$ | $-5.9 \pm 3.1$ [20] |
| $M_{\Xi^+} - M_{\Xi^-}$ | $-5.28 \pm 0.30$ |                  |

For completeness, we also present the results of the EM mass differences for the baryon antidecuplet listed in Table IV. Though there are no experimental data for them, it is still of great importance to know them in order to determine the masses of the baryon antidecuplet unambiguously.

5. In the present work, we have investigated the electromagnetic mass differences of the SU(3) baryons, employing an “model-independent approach” within a chiral soliton model. The electromagnetic self-energy corrections to the masses of the baryon are expressed as the baryonic two-point correlation function of the electromagnetic currents. We first derived a general collective operator for the electromagnetic self-energies, using the fact that the electromagnetic current can be treated as an octet operator, and considering possible irreducible representations of the correlation function. The collective operator for the electromagnetic corrections were shown to have three unknown parameters. Instead of computing them using a model, we have fixed $c^{(8)}$ and $c^{(27)}$ by the empirical data [10] for the electromagnetic mass differences of the baryon octet. The parameters $c^{(8)}$ and $c^{(27)}$, which are responsible for the mass splittings of the isospin multiplet due to the electromagnetic self-energies, were found to be $-0.15 \pm 0.23$ MeV and $8.62 \pm 2.39$ MeV, respectively.

Having calculated the electromagnetic mass differences, we were able to extract the hadronic part of the isospin mass splittings from the physical mass differences. The results of the physical isospin mass differences of the baryon decuplet are in good agreement with the existing data. The mass relations are also well satisfied with the data. In addition, we also present the electromagnetic mass differences of the baryon antidecuplet for completeness.

Since we have determined the EM mass differences for the SU(3) baryons, we can continue to study the mass splittings of the SU(3) baryons unambiguously. The corresponding investigation will appear elsewhere.

Table IV. Electromagnetic mass differences of the baryon anti-decuplet in units of MeV.

| Mass Difference | This Work | Mass Difference | This Work |
|-----------------|-----------|-----------------|-----------|
| $(M_{I_2} - M_{I_2})_{EM}$ | $-1.31 \pm 0.31$ | $(M_{I_2} - M_{I_2})_{EM}$ | $-0.89 \pm 0.26$ |
| $(M_{I_2} - M_{I_2})_{EM}$ | $-1.31 \pm 0.31$ | $(M_{I_2} - M_{I_2})_{EM}$ | $-0.89 \pm 0.26$ |
| $(M_{I_2} - M_{I_2})_{EM}$ | $0.24 \pm 0.10$ | $(M_{I_2} - M_{I_2})_{EM}$ | $-1.84 \pm 0.54$ |
| $(M_{I_2} - M_{I_2})_{EM}$ | $-1.31 \pm 0.31$ | $(M_{I_2} - M_{I_2})_{EM}$ | $0.71 \pm 0.29$ |
| $(M_{I_2} - M_{I_2})_{EM}$ | $0.24 \pm 0.10$ | $(M_{I_2} - M_{I_2})_{EM}$ | $1.60 \pm 0.46$ |
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