Grinding Deviation Analysis of Offset Face Gear Based on Involute Disc Wheel

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Abstract. Offset face gear drive has shown many unique advantages in the area of crossed-axes transmission. In this paper, deviation analysis of an offset face gear using an involute disc wheel as a cutting tool has been discussed to achieve precise machining. Firstly, equations of involute disc wheel and offset face gear are derived based on meshing theory. Then, a grinding machine tool is designed and equations of offset face gear containing machining deviation are derived. At last, figures of deviation cloud of face gear surface are created with MATLAB and importance of different machining deviation is discussed. The results show that the deviation Δz along the axis of grinding wheel is the most influential deviation, while Δy along the axis of face gear is the least one.

1. Introduction
Face gear drive is a face gear driven with a spur pinion wherein their axes are intersected, which has unique advantages such as light weight, compact structure and strong bearing capacity. NASA, Boeing, and North Star have verified the unique advantages of face gear in engineering research. [1-3]. Scholars have done lots of research on precise manufacturing, profile modification and meshing analysis of orthogonal face gears[4-8]. However, little research has been done on offset face gears.

Offset face gear drive is a face gear drive where axes of face gear and spur pinion are not crossed. In this paper, grinding process of offset face gear using involute disc grinding wheel has been studied. Tooth surface equations containing machining deviation are derived and figures of deviation cloud are created with MATLAB, which provide a theoretical support for precise machining of offset face gear.

2. Tooth Surface Equation of Offset Face Gear
The tooth surface of offset face gear is formed by a complex generating motion (figure 1) of involute disc grinding wheel. Fixed coordinate systems S₀(x₀, y₀, z₀) and S₂(x₂, y₂, z₂) represent the initial position of grinding wheel and face gear respectively, where E is the offset distance and x₂=x₀+E. Moving coordinate system S₁(x₁, y₁, z₁), S₂(x₂, y₂, z₂) are fixed to grinding wheel and face gear respectively. When the grinding wheel rotates around the axis z₁ to the angle of φ₁ at the angular speed of ω₁, the face gear rotates around the axis z₂ to the angle of φ₂ at the angular speed of ω₂ accordingly. The tooth profile of offset face gear r₂ can be expressed as equation 1.

\[ r_2(u_e, \theta_e, \varphi_2) = M_{2S}(\varphi) \mathbf{r}_1(u_e, \theta_e) \]  

(1)

Coordinate transformation matrix M_{2S} can be expressed as equation 2.
The involute profile of grinding wheel can be expressed as equation 3.

\[
 r_s(u_s, \theta_{ks}) = \begin{bmatrix}
 \pm r_{m_s}[\sin(\theta_{vs} + \theta_{os}) - \theta_{vs} \cos(\theta_{vs} + \theta_{os})]
 \\
 -r_{m_s}[\cos(\theta_{vs} + \theta_{os}) + \theta_{vs} \sin(\theta_{vs} + \theta_{os})]
 \\
 u_s
 \\
 1
\end{bmatrix}
 k = (\gamma, \beta)
\]

where \( r_{m_s} \) is radius of base circle, \( \theta_{os} \) is initial angle, \( u_s \) and \( \theta_{ks} \) are variables of involute, and "\( \pm \)" stand for \( \gamma, \beta \) respectively, as shown in figure 2.

According to the meshing theory, the unit normal vector of involute tooth profile \( n_s \) and the velocity of meshing point \( v_s^{(s,2)} \) in ordinary system \( S_s \) are calculated. Substituting \( n_s \cdot v_s^{(s,2)} = 0 \) into Eq. 3 to eliminate \( u_s \), the tooth profile of offset face gear can be given as equation 4.

\[
 r_s(\phi_s, \theta_{ks}) = \begin{bmatrix}
 r_{m_s} \left[ \cos \phi_s (\sin \xi_{ks} m \theta_s \cos \xi_{ks}) - \frac{\sin \phi_s}{m_s \cos \xi_{ks}} \right] - E \cos \phi_s
 \\
 -r_{m_s} \left[ \sin \phi_s (\sin \xi_{ks} m \theta_s \cos \xi_{ks}) + \frac{\cos \phi_s}{m_s \cos \xi_{ks}} \right] + E \sin \phi_s
 \\
 -r_{m_s} (\cos \xi_{ks} \pm \theta_s \sin \xi_{ks})
 \\
 1
\end{bmatrix}
\]

where \( \xi_{ks} = \phi_s \pm (\theta_{ks} + \theta_{os}) \), and "\( \pm \)" stand for \( \gamma, \beta \) respectively.

3. Machining deviation analysis of offset face gear

A face gear grinding machine using involute disc wheel has been designed based on generating theory, as shown in figure 3.
Figure 3. Structure of a face gear grinding machine

The grinding wheel can move along Y and Z axes and rotate around A axis, meanwhile, the workpiece can move along X axis and rotate around B axis. Generation of face gear is achieved by 5-axes(X/Y/Z/A/B) linkage. The dresser is a diamond roller which moves along X axis, and dressing process is achieved by 3-axes(X1/Y/Z) linkage. When dressing, the trajectory of the roller is the offset curve of the involute.

The deviation of offset distance E and involute profile of grinding wheel can affect the accuracy of tooth profile of offset face gear. In this paper, equations of offset face gear containing machining deviation are derived. Figures of point cloud of offset face gear containing deviation are created by MATLAB and relationship between machining deviation and tooth surface deviation is analyzed, which provides theoretical basis for error compensation.

Parameters of the offset face gear for numerical example are as shown in Table 1.

### Table 1. Design parameters of face gear

| Parameter                           | Value       |
|-------------------------------------|-------------|
| Module                              | m=5mm       |
| Number of teeth of the shaper       | Ns=20       |
| Number of teeth of the face-gear    | N2=41       |
| Pressure angle of driving side and coast side | $\alpha=20^\circ$ |
| Shaft angle                         | $\gamma_m=90^\circ$ |
| Offset distance                     | E=25mm      |
| Inner radius of the face-gear       | r2=112mm    |
| Outer radius of the face-gear       | r1=130mm    |

(1) deviation of offset distance $\Delta E$

Equation of face gear tooth surface containing deviation of offset distance $\Delta E$ is given as equation 5.

$$r_s(\varphi, \theta_s) = \begin{bmatrix}
    r_s \left[ \cos \phi_s \left( \sin \xi_{s}, m \xi_{s}, m \cos \xi_{s} \right) - \frac{\sin \phi_s}{m_2, \cos \xi_{s}} \right] (E + \Delta E) \cos \phi_s \\
    -r_s \left[ \sin \phi_s \left( \sin \xi_{s}, m \xi_{s}, m \cos \xi_{s} \right) + \frac{\cos \phi_s}{m_2, \cos \xi_{s}} \right] + (E + \Delta E) \sin \phi_s \\
    -r_s \left( \cos \xi_{s} \pm \theta_{s}, \sin \xi_{s} \right) / 1
\end{bmatrix}$$

where $\xi_{s} = \varphi_s \pm (\theta_s + \theta_m)$

The deviation of face gear surface is uniformly distributed along the direction of tooth height (along z axis in figure 4, the same below), and smaller near outer circumference and larger near inner circumference along the direction of tooth width (along y axis in figure 4, the same below). The signal of deviation of right tooth( $\beta$ )is the same as $\Delta E$, but opposite to deviation of left tooth( $\gamma$ ). When $\Delta E=\pm 0.1mm$, maximum of deviation of face gear surface $\delta_E$ is about $\pm 0.085mm$ and minimum is about $\pm 0.07mm$, as shown in figure 4.
ΔE=0.1mm(surface γ)  ΔE=0.1mm(surface β)

**Figure 4.** Deviation of face gear surface δE containing deviation ΔE

(2) deviation of dressing along \( x_1 \) axis

When there is a deviation \( Δx \) along \( x_1 \) axis during dressing process, the involute profile containing deviation \( Δx \) can be given as equation 6.

\[
\mathbf{r}(u, \theta_a) = \begin{pmatrix}
\pm r_\gamma \left[ \sin (\theta_{\gamma} + \theta_a) - \theta_a \cos (\theta_{\gamma} + \theta_a) \right] \\
\mp \theta_a \cos (\theta_{\gamma} + \theta_a) + \theta_a \sin (\theta_{\gamma} + \theta_a) + \Delta x \cos \frac{\pi}{N_\gamma} \\

u,
\end{pmatrix}
\]

Equation of face gear tooth surface containing deviation \( Δx \) can be given as equation 7

\[
\mathbf{r}_2(\varphi_s, \theta_a) = \begin{pmatrix}
\text{near} \theta_a \\
-\Delta x \sin \left( \frac{\pi}{N_\gamma} \right) \cos \varphi_2 - E \cos \varphi_2 - \delta \sin \varphi_2 \\
\Delta x \sin \left( \frac{\pi}{N_\gamma} \right) \sin \varphi_2 + E \sin \varphi_2 - \delta \cos \varphi_2 \\
-\Delta x \cos \left( \frac{\pi}{N_\gamma} \right) \sin \varphi_2 + E \cos \varphi_2 + \delta \cos \varphi_2 \\
\end{pmatrix}
\]

where \( \xi_s = \varphi_s \pm (\theta_a + \theta_a) \), \( \delta_s = \frac{r_\gamma - \Delta x \cos \left( \frac{\pi}{N_\gamma} \right) (\theta_a + \theta_a)}{m_2 \cos \xi_s} \)

The deviation of face gear surface is uniformly distributed along the direction of tooth width, and smaller near tooth root and larger near tooth top along the direction of tooth height. The signal of deviation of tooth surface is the same as \( Δx \). When \( Δx=±0.1\text{mm} \), maximum of deviation of face gear surface \( δ_s \) is about ±0.075mm and minimum is about ±0.05mm, as shown in figure 5.

Δx=0.1mm(surface γ)  Δx=0.1mm(surface β)

**Figure 5.** Deviation of face gear surface δx containing deviation Δx
(2) deviation of dressing along y axis

When there is a deviation $\Delta y$ along y axis during dressing process, the deviation issue can be simplified to a plane geometry issue as shown in figure 6, where O represents the theoretical center of grinding wheel, $O_1$ represents the theoretical center of dresser, P represents the theoretical contact point between dresser and grinding wheel, $O_1'$ represents the center of dresser considering deviation $\Delta y$, and P' represents the contact point between dresser and grinding wheel considering deviation $\Delta y$. The deviation of involute profile of grinding wheel $\delta y$, which is caused by dressing deviation $\Delta y$, can be approximatively given as equation 8.

$$\delta y = |OP'| - |OP| = |OO_1'| - |OO_1| = \frac{(\Delta y)^2}{|OO_1'|+|OO_1'|}$$  \hspace{1cm} (8)

In general, $\Delta y \ll |OO_1'| + |OO_1'|$, so $\delta y$ can be considered as a higher order infinitesimal of $\Delta y$, which is an insensitive deviation.

![Figure 6. Analysis of deviation $\Delta y$](image)

(4) deviation of dressing along z axis

When there is a deviation $\Delta z$ along z axis during dressing process, the involute profile containing deviation $\Delta z$ can be given as equation 9.

$$r_0(\alpha, \theta_0) = \begin{bmatrix} \pm r_0[\sin(\theta_0 + \theta_0) - \cos(\theta_0 + \theta_0)] \pm \Delta z \cos \frac{\pi}{N_{i}} \sin \phi_2 \cos \varphi_2 \cos \varphi_2 \cos \zeta_2 \sin \varphi_2 \\ -r_0[\cos(\theta_0 + \theta_0) + \cos(\theta_0 + \theta_0)] + \Delta z \cos \frac{\pi}{N_{i}} \sin \phi_2 \sin \varphi_2 \\ + r_0[\cos(\theta_0 + \theta_0) + \cos(\theta_0 + \theta_0)] + \Delta z \cos \frac{\pi}{N_{i}} \sin \phi_2 \cos \varphi_2 \\ \end{bmatrix}$$  \hspace{1cm} (9)

The equation of face gear tooth profile containing deviation $\Delta x$ can be given as equation 10.

$$r_n(\phi_i, \theta_{0n}) = \begin{bmatrix} r_n[\cos \xi_n + \cos \xi_n] + \Delta z \cos \frac{\pi}{N_{y}} \sin \phi_2 \cos \phi_2 \cos \zeta_2 \sin \varphi_2 \\ -r_n[\cos \xi_n + \cos \xi_n] \sin \phi_2 \cos \phi_2 + E \sin \phi_2 \cos \phi_2 \\ -r_n[\cos \xi_n + \cos \xi_n] \sin \phi_2 \cos \phi_2 \cos \phi_2 \\ \end{bmatrix}$$  \hspace{1cm} (10)

where $\xi_n = \phi_n + (\theta_{0n} + \theta_0)$ and $\zeta_2 = \frac{m_{z_2} \cos \xi_n}{\Delta z}$

The deviation of face gear surface is uniformly distributed along the direction of tooth width, and smaller near tooth root and larger near tooth top along the direction of tooth height. The signal of deviation of tooth surface is the same as $\Delta z$. When $\Delta z = \pm 0.1 mm$, maximum of deviation of face gear surface $\delta z$ is about $\pm 0.10 mm$ and minimum is about $\pm 0.08 mm$, as shown in figure 7.
Δz=0.1mm (surface γ)
Δz=0.1mm (surface β)

Figure 7. Deviation of face gear surface δ, containing deviation Δz

By adjusting the value of deviation parameters ΔE, Δx, Δy from [-0.1, 0.1], a further simulation has been carried out (Figure 8.) which draws conclusions as follows:
(1) There is approximately a linear relationship between deviation of face gear surface δ and deviation of grinding process Δ.
(2) The importance of deviation of grinding process in descending order is Δz, ΔE, Δx and Δy, and Δy is an insensitive deviation.

Figure 8. Relationship between deviation of face gear surface and deviation of grinding process

4. Conclusions
In this paper, relationship between deviation of grinding process and deviation of offset face gear surface have been studied. According to the generating theory of offset face gear, a grinding machine has been designed, and influences caused by deviation of offset distance ΔE and deviation of dressing process Δx, Δy and Δz have been analysed. Equations of offset face gear tooth surface containing deviation have been derived and figures of deviation cloud have been created with MATLAB, which provide a theoretical support for precise machining of offset face gear. The results show that Δz is the most influential deviation while Δy is the least one.

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