The Large Numbers Hypothesis: Outline of a self-similar quantum-cosmological Model

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Abstract

In 1919 A. Einstein suspected first that gravitational fields could play an essential role in the structure of elementary particles. In 1937, P.A.M. Dirac found a miraculous link between the properties of the visible Universe and elementary particles. Both conjectures stayed alive through the following decades but still no final theory could be derived to this issues. The herein suggested fractal model of the Universe gives a consistent explanation to Dirac’s Large Numbers Hypothesis and combines the conjectures of Einstein and Dirac.

1 Introduction

1.1 The Large Numbers Hypothesis

Dirac[4, 5] first mentioned in 1937 a presumable connection between the properties of elementary particles and the Universe by multiplying elementary properties with certain large numbers. Since then this conjecture is called the Large Numbers Hypothesis. It states, that the cosmological quantities \( M, R, T \) can be related to the particle quantities \( m, r, t \) through the scale relation

\[
\frac{T}{t} = \frac{R}{r} = \sqrt{\frac{M}{m}} = \Lambda \sim 10^{38-41}
\]

which may easily be verified for example for the properties of the Universe and the proton. Mass, time and length is enough to construct a complete system of units. A simple dimensional analysis leads to a scaled quantum of action

\[
H = \Lambda^3 \cdot \hbar
\]

in order to handle the Universe like an elementary particle. Hence the Universe should have an angular momentum of about \( H/2\pi \) and it can be suspected to be a huge rotating black-hole close to its Kerr limit, or close to Gödel’s spin[1, 2, 35, 10], which is the value for the rotating cosmological solution to Einstein’s equations of General Relativity.

The recent discussion regarding the origin of and destiny of the Universe raises, for example, the question of where the Universe comes from. The main explanation for this is that time and space were created with the big-bang and therefore the question of where the Universe is embedded or what happened before the creation, is not admissible. This takes the assumption that there was no reason for the creation and that the Universe was created from ‘nothing’ by a type of quantum fluctuation and should have an overall-energy of zero. Although this explanation sounds plausible, it does not solve the real problem that if the Universe was created from ‘nothing’, why should there not be countless other Universes, and what relationship could there be between them? Numerous scientists have tried to give an explanation to this question (e.g. A. Linde[17, 18]). Whatever the correct answer is, the explanation must be a kind of infinite regress, as otherwise the embedding problem would recur.

An additional question that is also unsatisfactory answered is, what matter, such as elementary particles, really is. To answer this question, one tries to find the most elementary particles by experimentation and theory, all matter should be composed of these smallest ‘grains’ of matter. But this explana-
tion always raises the question that these 'grains' could be composed of something that could be described by an even more elementary description. This is where the so-called string-theory comes in; this gives a topological explanation, with the strings as the origin of matter. But what are these strings composed of, they should be pure topology, as space-time curvatures are \[ \Lambda \neq 0 \].

1.2 Can the Universe be described as a Black Hole?

The critical mass density of the Universe is the amount of mass which is needed to bring the universal expansion to a halt in the future and to a collapse in a final big crunch. This mass is, depending on the world model, about \( 10^{53} \) kg. The visible mass of the Universe is sufficient for only about some percent of the critical mass density. On the other hand the mass density of the baryons should be 10 to 12 percent of the critical mass density \[ \rho \] as can be derived from the theory of nucleosynthesis and the measured photon density. Therefore the so-called dark matter should be about 10 times larger than the visible bright mass. This matter shows itself in the extinction of light on its way through the Universe as well as by its gravitational force on galaxies and galaxy clusters. Including this, the mass density of baryonic matter seems to be not large enough to close the Universe. Besides the classical baryonic matter therefore also exotic matter is taken into consideration as far as the overall mass of the Universe is concerned.

The Schwarzschild-radius \( R_{\text{sch}} = \frac{2GM}{c^2} \) of a given mass \( M \) is derived from the Einstein field equations considering a point-like mass. The Schwarzschild solution is a static, homogeneous and isotropic solution for the region outside the Schwarzschild radius. However, the inside solution may look other. A very interesting solution is the solution of Oppenheimer and Snyder \[ [31, 26] \] which shows the astonishing result that the inside solution must be a Friedmann-Universe \[ 3 \]. This result is an outcome from the fitting of the outside to the inside solution of a collapsing star: after the burning out of the star every pressure vanishes when the star shrinks to 9/8-th of its Schwarzschild radius. When the collapsing star reaches the event-horizon one has to set \( p = 0 \) and one gets a Friedmann-Universe at its maximum expansion. So the Oppenheimer/Snyder-solution may be taken for a speculation of a Friedmann-Universe inside a black hole, shrinking down to a singularity and from there expanding back to its maximum expansion again.

If someone does not like such an interpretation, there is the problem to explain how the expanding Universe let behind its own event-horizon. As the standard Big-Bang Universe starts to expand at vanishing spatial dimensions, this leads to the following seemingly paradoxical context: Either the Schwarzschild-radius is greater-equal to the world radius \[ 4 \] of the Universe, as large estimations of the Universe mass assume, then the Universe may be called a black hole \[ 4 \]. Or, as small estimates of the mass of the Universe assume, the Schwarzschild-radius is about some 10 percent of the today world radius of the Universe. But then the Universe should have expanded beyond \[ 4 \] its Schwarzschild-radius in former times when it crossed a radius of at least 1.5 billion light-years, which is an event that is supposed to be not in casual harmony with general relativity. By this one could guess that our Universe should be indeed a black hole.

The theory of inflation \[ [18] \] tries to avoid such a paradox by a kind of extremely fast inflationary expansion. But finally the standard inflationary model also demands a critical mass density for the Universe. Recent models prefer cosmologies with a cosmological constant \( \Lambda \neq 0 \) and small densities, which are also in agreement with the interpretation of the luminosities of far away Type-Ia-Supernovae which seem to be about 0.3\( m \) darker than expected. These more complex \( \Lambda \neq 0 \)-models open a large

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\(^2\)Although the dimension of the world radius and the Schwarzschild-radius may be equal, there is a considerable difference: The world radius is the 'visible' dimension which is defined by an infinite red shift. On the other hand, the Schwarzschild-radius can never be seen. Even when an observer is placed close to this border (as seen from 'outside'): In this case the line of sight would be curved when seen from outside but an inside observer would observe it as free of any forces and straight. Because of this any number of world radius' can be placed into the area of the Schwarzschild-radius even if both have the same dimension.

\(^3\)A black hole, if watched from inside, may be called a white hole because it is guessed that from its inside singularity everything can come into existence \[ [3, 4] \]. A black hole actually needs an outer space which it is defined on, but the existence or non-existence of an outer space is first of all a question of belief.

\(^4\)A 'pushing in front' of the event-horizon is easily possible due to the increasing scale-factor \( R(t) \) of the Universe.
variety of possible large scale structures of the Universe. It shall be emphasized, that especial the non-critical, so-called open (hyperbolic) Universes, must not have an infinite volume in any case \[27\].

The correspondance between an open and an infinite Universe is only true for the very special case of a zero cosmological constant in a simple-connected topology of the Universe \[21\]. Also there exist competing theories to explain the luminosities of the Type-Ia-Supernovas, such as taking into account the nonhomogenity \[20\] of the Universe on medium scales and others. And the work of Gawiser and Silk \[7\], which numerically calculates the 10 most discussed cosmological models and relates them to the variation of the observed cosmic microwave background (CMB), shows the astonishing result that the critical standard model fits the observation of the CMB by far best.

Therefore, as a working-hypothesis, herein a black-hole-Universe is defined through a Universe of critical mass density. The possibility of a black-hole Universe was already taken into consideration in the literature. Just as examples, out of the uncounted works about this idea, are cited here the books of J-P.Luminet \[19\] and Smolin \[36\]. In the works of Carneiro \[1, 2\] the Universe is considered to be a huge rotating black hole \[1\] with remarkable links between particle physics and cosmology. A model of a Universe as a white hole is also provided by Gibbs \[8\].

2 Simple solutions for a black-hole-Universe

The main sections of the model\[8\] are repeated here for convenience. The coordinate frame used herein is, if not mentioned differently, for an observer at spatial infinity.

2.1 The Einstein-deSitter model

The Einstein-deSitter model (EdS model) follows from the Friedmann-models which are simplified with a curvature parameter \(\kappa = 0\) and a cosmological constant \(\Lambda = 0\). Therefore the Universe is approximated with an euclidic, isotropic and homogeneous world model. Hence the scale-factor \(R(t)\) is a solution of \(R\dot{R}^2 = \text{Const.}\), which gives

\[
R(t_e) = R_0 \cdot \left(\frac{t_e}{t_0}\right)^{2/3}.
\]

Therein \(t_e\) is the time of emission of a signal and \(t_e = 0\) is the time of the big bang and \(t_0\) is the present time. With the Hubble equation \(\frac{dR}{dt} = H(t)R(t)\) follow the relations for the age and size of the Universe:

\[
t_U = \frac{2}{3H_0} \quad \text{and} \quad R_U = \frac{2c}{H_0}.
\]

The actual distance \(r\) between two points with previous distance \(\rho\) is derived from the equation:

\[
r(t) = R(t) \cdot \rho.
\]

The standardization is given by reference to the present time \(R(t_0) = R_0 = 1\).

General Relativity demands a maximal velocity \(c\) only for the peculiar movement. The variation of the scale-factor

\[
\frac{dR}{dt_e} = \frac{2R_0}{3t_0^{2/3} t_e^{1/3}}
\]

runs to infinity for small times of emission. Therefore the overall-velocity of the Expansion

\[
\frac{dr}{dt} = \frac{dR}{dt} \cdot \rho + R \cdot \frac{d\rho}{dt}
\]

can be much greater than the speed of light. Hence, considering events for which the speed of light is a given limit, one has to look at variations of \(\rho\).

The fact that the EdS-model is simplified with a curvature parameter \(\kappa = 0\) seems to point out (as (4) never turns its sign to minus) that it is not suitable to treat a black-hole Universe which should have an \(\kappa = +1\) and should collapse in a finite time. The critical EdS-Universe is, in the scope of this simple model, a black-hole-Universe which collapses in an infinite time.

2.2 The Planck dimensions

If one equates the Planck energy \(E = \hbar \omega\) with the Einstein energy \(E = m_0 c^2\) of a mass charged particle one gets the de-Broglie wavelength of a resting particle \(m_0\), which is also known from the theory of photon scattering on electrons as the Compton
wavelength $\lambda_c = \frac{h}{mc}$. The wavelength of the particle is in inverse proportion to its mass than the Schwarzschild-radius of the corresponding black-hole $\rho_{ss} = \frac{2GM}{c^2}$. The identity of both lengths leads to the Planck dimensions \[m_{pl} = \frac{\sqrt{\hbar c}}{2G} = 1.54 \cdot 10^{-33} \text{kg}\]

\[l_{pl} = \frac{\hbar}{m_{pl}c} = \sqrt{\frac{2\hbar G}{c^3}} = 2.29 \cdot 10^{-35} \text{m}\]

\[t_{pl} = \frac{\hbar}{m_{pl}c^2} = \sqrt{\frac{2\hbar G}{c^5}} = 0.762 \cdot 10^{-43} \text{sec}\]

\[E_{pl} = m_{pl}c^2 = \sqrt{\frac{\hbar c}{2G}} = 1.38 \cdot 10^9 \text{J}\]  

(5)

The main property of the Planck dimension is the fact, that the energy of a wave with wavelength $\lambda$ equals to a mass which bends space to a black hole of Planck size.

### 2.3 The mass formula for the Einstein-de Sitter model

The Eds model delivers the coordinate distance of an event travelling with the speed of light from (9) through the integration of $d\rho = \frac{cdt}{H(t)}$:

\[\rho = \frac{3ct_0}{R_0} \cdot \left[1 - \left(\frac{t_e}{t_0}\right)^{1/3}\right] \]  

(6)

This distance should at maximum be equal to any given Schwarzschild-radius $\rho_{ss} = 2GM/c^2$ which results in:

\[M(t_e) = \frac{c^3}{H_0G} \cdot \left[1 - \left(\frac{3H_0t_e}{2}\right)^{\frac{1}{2}}\right] \]  

(7)

This time-dependent mass should be at least included by a gravitational spherewave starting at time $t_e$ and running with velocity $c$. Otherwise the wave would have to go beyond its own event-horizon. Inserting $t_e = 0$ for the origin of the Universe one gets:

\[M_{\text{min}} \geq M(0) = \frac{c^3}{H_0G} \]  

(8)

This is the mass which makes an EdS Universe critical. The value of $H_0$ is still controversial and differs depending on the source between approximately 50 and $100 \text{km/secMpc}$. So the mass of the Universe should be in the bounds $M_U \in [1.248,2.497] \cdot 10^{53} \text{kg}$. For any computational purpose herein the average value of $75 \text{km/secMpc}$ is used, which agrees also with recently elaborated values $(72 \pm 6) \text{km/(secMpc)}$ of the Hubble constant.

The mass formula (7) is now generalized to local gravitational waves running in a local flat spacetime. For that purpose (7) is expanded to a Taylor series at the time $t_0$ transforming the time coordinate to $t = t_0 - t_e$. By this one gets the mass formula for small masses implying small times $t << t_0$:

\[m(t) = \frac{1}{2G} \cdot \left[\frac{c^3}{6G} t + \frac{5}{54} \frac{c^3}{G} \left(\frac{t}{t_0}\right)^2 \cdot t + \ldots\right] \]  

(9)

The factor $\left(\frac{t}{t_0}\right)^n$ rapidly drops to zero for small times and therefore one can calculate further on with only the first part of the sum:

\[m(t) = \frac{1}{2G} \cdot c^3 \cdot t \]  

(10)

For the peculiar distance of events which travel with the speed of light one gets similar to the derivation of (3):

\[\rho(t) = ct + \frac{cH_0}{2} t^2 + \frac{5}{12} cH_0^2 t^3 + \ldots \]  

(11)

Hence the apparent force\(^7\) acting on a gravitational event running with $c$ is:

\[|\vec{F}_e| = \frac{d}{dt}(mv) = \frac{c^4}{2G} (1 + 3H_0 t + \ldots) \cong \frac{c^4}{2G} \]  

(12)

This almost constant force acts on the event over the area of the Schwarzschild-radius:

\[E \approx F_e \cdot \rho_{ss} = \frac{c^4}{2G} \cdot \frac{2GM}{c^2} = mc^2 \]  

(13)

It seems that a gravitational wave may run unhindered just if the mass included in its sphere is zero

\(^7\)\text{This force is seen only by an observer at spatial infinity.}\n
An observer travelling inside a black hole would never feel to hit against the event-horizon because any forced curvature of his line of sight would be sensed to be straight.\n
4
or infinite. Every distortion of space-time causing a mass creates an event-horizon proportional to this mass. Then the wave hits its self-made horizon. The rest energy of the so created (virtual) mass is borrowed from the energy of the gravitational wave and is transformed into a potential energy of the same quantity represented by a virtual-black-hole.

3 An approximate stationary solution for black hole particles

As a simple approximation one can consider the effect of a wave in a rectangular potential well. The gravitational wave runs unhindered in a small area until it is stopped, as mentioned from outside, by its self-made event-horizon which exerts a nearly infinite apparent force \( F \) which hinders the progress of the wave:

\[
V = 0 \quad \text{for} \quad r \in [0, \rho_{SS}]
\]

\[
V = \infty \quad \text{for} \quad r > \rho_{SS}
\]

The common known solution of the time-independent Schrödinger equation for a particle in such a rectangular potential well delivers the energy eigenvalues

\[
E_n = \frac{n^2 \pi^2 \hbar^2}{2ma^2}
\]

with \( n = 1, 2, 3 \ldots \) and \( a \) the diameter of the well. With the substitution of \( a = 2\rho_{SS} = \frac{4Gm}{c^2} \) and \( m = \frac{\hbar c}{\rho_{SS}} \) one gets the energy eigenvalues as the real roots of

\[
E_n = \pm \frac{\sqrt{n}}{2} \sqrt{n}E_{pl}
\]

The factor \( \frac{\sqrt{n}}{2} \sqrt{n} = 1.054 \) origins from the simplification of the potential \( \frac{\hbar c}{\rho_{SS}} \) and is set equal to 1.

Then the masses of the virtual Schwarzschild areas have the eigenvalues:

\[
m_n^\pm = \pm \sqrt{n} \cdot m_{pl}
\]

The difference of neighbouring positive eigenvalues is

\[
\Delta m_n = (\sqrt{n+1} - \sqrt{n})m_{pl} \approx \frac{1}{2\sqrt{n}} \cdot m_{pl}
\]

for large \( n \). From the equations \([17]\) and \([18]\) follows

\[
\Delta m_n = \frac{1}{2} \cdot \frac{m_{pl}^2}{m_n} \quad \text{with} \quad n = \frac{1}{4} \cdot \frac{m_{pl}^2}{\Delta m_n^2}
\]

and the relation between stimulated mass \( \Delta m_n \) and positive virtual mass \( m_n \) is:

\[
\frac{\Delta m_n}{m_n} = \frac{1}{2n}
\]

The radius of an elementary particle in this model is the event-radius of the virtual mass

\[
\rho_n := \rho_{SS}(m_n) = \frac{2Gm_n}{c^2}
\]

The formula \([19]\) therefore serves the equation

\[
\lambda_C = \frac{\hbar}{m}\quad \Leftrightarrow \quad m_0 \cdot \lambda_C = \frac{\hbar}{2}
\]

for the Compton wavelength of an elementary particle:

\[
\Delta m_n \cdot 2\rho_n = \frac{m_{pl}^2}{2m_n} \cdot \frac{4Gm_n}{c^2} = \frac{\hbar c}{2}
\]

with the substitutions \( m_0 = \Delta m_n \) and \( \lambda_C = 2\rho_n \). Heisenberg’s uncertainty relation is fulfilled, during the creation of the virtual black hole, for the stimulated mass:

\[
\Delta E \Delta t = \Delta E_n \cdot \frac{\Delta r}{c} = \Delta m_n c^2 \cdot \frac{2Gm_n}{c^2} = \frac{Gm_{pl}^2}{c} = \frac{\hbar}{2}
\]

The angular momentum \( \vec{L} = m \vec{\nu} \times \vec{r} \) of a wave with mass \( \Delta m_n \) which circulates at the speed of light \( c \) along the horizon of a black hole with mass \( m_n \) is:

\[
|\vec{L}| = \Delta m_n c \cdot \rho_n = \frac{m_{pl} c}{2\sqrt{n}} \cdot \frac{2G}{c} \cdot \sqrt{nm_{pl}} = \frac{Gm_{pl}^2}{c^2} \cdot \frac{c}{2G} = \frac{\hbar}{2}
\]

\[
\Rightarrow s_z = \pm \frac{\hbar}{2}
\]

\[\text{[11]The same quantization rule for black holes was found for example by Khriplovich [14] from a totally different point of view. For the handling of negative masses see also the article of Olavo [23].}\]
The spin of a stimulated black hole particle is, by this, half of Planck’s quantum, which corresponds to a fermion \[\uparrow.\] Particles relating to the energy of stimulation of a virtual-miniature-black-hole \[\downarrow\] will herein be referred to as SBH-particles.

### 3.1 SBH-Particles

The table \[\uparrow\] shows the values of some selected masses referring to the formulas of chapter \[\uparrow\] and gives an idea of an Universe which is built up in a fractal manner of black-hole-topologies. All masses origin from stimulated curvatures of spacetime and every black hole has its typical quantum. For macroscopic black holes these quantums have energies much smaller than the electron rest mass and spin.

The Hawking-times \[\tau\] are a rough hint for the lifetimes of SBH-particles and they show meaningful values for stable elementary particles. But effects of quantum gravitation should generate very different values for this lifetimes, namely those experimentally seen values which can be much less for instable and much more for stable particles.

The product of virtual mass and stimulated mass is always a constant for every SBH-particle:

\[
C_{m_n}^\pm := \Delta m_n \cdot m_n^\pm = \pm \frac{1}{2} m_{pl}^2 = \pm \frac{\hbar}{2\pi} \approx \pm 1.185 \cdot 10^{-16} \text{ kg}^2
\]

The basic eigenvalue \[n = 1\] of self-curvature is given by the Planck mass which has a stimulated mass of approximately the same quantity. The sizes of the self-curvatures of space-time increase with \[n\] and they reach their maximum at \[\sqrt{n} = 10^{61}\] with the Universe as the largest Schwarzschild area. The stimulated mass of this SBH-Universe is a rather massless particle, as are neutrino or photon. An extrapolation of the \[\Delta m_n/m_n\] dependence for an elementary SBH-particle to the (not allowed) eigenvalue \[n = 0\] gives cause to a speculation of a nearly massless particle of which the stimulated mass is an Universe with an incredibly small lifetime satisfying the uncertainty relation. As follows from chapter \[\uparrow\] these quantums may be fermions; the one with the lightest weight is a neutrino and the one with the heaviest weight is the Planck quantum. The speculative extrapolation to the eigenvalue \[n = 0\] gives as the heaviest fermion an Universe. Hence a possible fractal construction of the world is conceivable, based on black-areas of different sizes.

### 3.2 The entropy of black holes

A fundamental question in gravity physics is to explain the high entropy of black holes. The Bekenstein-Hawking-entropy \[\uparrow\] is

\[
S_{BH} = \frac{A}{4G}\text{.}
\]

The surface of the black hole \[A = 4\pi r_n^2\] can be deployed

\[
S_{BH} = \frac{2\pi}{c^3} \cdot n
\]

and relates the black hole entropy directly to its excitation level \[n\].

If one relates the rest-energy of a particle, using the herein derived mass-quantization \[\uparrow\] to the Hawking-temperature of a black-hole, which is

\[
T_h = \frac{\hbar c^3}{8\pi k_B GM},
\]

one gets:

\[
\frac{E_0}{T_h} = \frac{\Delta m_n c^2}{T_h(m_n)} = 2\pi k_B
\]

This shows that the SBH-particle is in thermodynamical equilibrium \[\uparrow\] with the Hawking-temperature of the stimulated-black-hole.

### 3.3 The mass of the proton

As pointed out in \[\uparrow\], the gravitational power of the evolving Universe is greater than the power needed to create virtual black holes for the first primordial instant of space

\[
t_B = \left( \frac{28}{3} \cdot \frac{21/3}{3} \right)^{3/10} \left( \frac{\hbar H_0}{c^3} \right)^{7} \cdot \frac{1}{t_H^6} \approx 2 \cdot 10^{-25} \text{ sec}
\]

\[\downarrow\]
The mass relation (19) of the Universe as the underlying virtual-fermion, which results from a fermionic excitation in this model the neutrino is the less weighted close to the mass of the proton.

This time may be interpreted as a kind of phase change as the Universe stops boiling. On the other hand the uncertainty equation for the Universe gives at this time a mass-equivalent of \( m_t = \frac{\hbar}{2c^5} \). Inserting (28) results in

\[
m_l = 0.239 \cdot \frac{\hbar^{13} H_0^5}{c^5 G^5} \approx 1.7 \cdot m_p \tag{29}
\]

which is a limiting upper value for SBH-masses close to the mass of the proton.

### 3.4 The mass of the neutrino and the rotation of the Universe

In this model the neutrino is the less weighted fermion, which results from a fermionic excitation of the Universe as the underlying virtual-mass-particle. The mass relation \( \Delta m_\nu = \frac{1}{2} \cdot \frac{m_\nu^2}{m_\nu} \) gives a minimum mass for the lightest neutrino of

\[
m_\nu \geq \frac{\hbar H_0}{4c^5} = 0.40 \cdot 10^{-33} \text{eV} \tag{30}
\]

which can also be interpreted as the minimum mass for a particle which is blurred over the whole Universe as shown by Heisenbergs uncertainty relation.

The relation (19) shows resemblance to the Dirac-mass- term \( \frac{\hbar}{2c^5} \), which follows from the so-called see-saw-mechanism in GUT-theory:

\[
m_\nu \approx \frac{m_\nu^2}{M_R} \tag{31}
\]

In this case, the Dirac-mass \( M_D \) could be identified with the Planckmass \( m_\nu \) and the mass of the right-handed neutrino \( M_R \) with the mass of the Universe.

If the Universe is assumed to be a kind of a fermion and the SBH-partner of the neutrino, it should have an intrinsic rotation of about

\[
\frac{\hbar}{2} = m_\nu \omega U R_U^2 \Rightarrow \omega_U = H_0/2 \tag{32}
\]

by substituting the herein derived mass of the neutrino and the EdS radius and mass of the Universe. As this is just a rough estimation, one should expect a rotational ratio of the order of some \( H_0 \):

\[
\omega_U \sim H_0 \tag{33}
\]

Past works about the claimed observation of a rotation of the Universe give estimates of \( \omega = (6.5 \pm 0.5)H_0 \). From equation (32) follows the scaled quantum of action \( \frac{H}{2} \) for the EdS-Universe as

\[
\frac{H}{2} = M_U \omega_U R_U^2 \Rightarrow H = \frac{4c^5}{GH_0} \approx 2.5 \cdot 10^{88} \text{kgm}^2/\text{sec} \tag{34}
\]
and with equation (2) the scale-factor of the Large Numbers Hypothesis can be expressed as:

\[ \Lambda = \sqrt{\frac{H}{\hbar}} = \left( \frac{4c^5}{\hbar G^2 H_0^2} \right)^{\frac{1}{4}} \cong (t_{pl} H_0)^{-\frac{2}{3}} = 3.3 \cdot 10^{40} \]  

(35)

### 3.5 The Large Numbers coincidences

We now can combine the formulas (3), (8), (35) to derive the masses of usual matter:

\[ m = \frac{M}{N^2} = \frac{c^3}{H_0 G N^2} \]  

(36)

which gives:

\[ m = \left( \frac{\pi^2 h^2 H_0}{4 G c} \right)^{1/3} \]  

(37)

This formula is equivalent to the empirical Weinberg formula (36) for the mass of the pion \( m_π \simeq \left( \frac{\pi^2 h^2}{G c} \right)^{1/3} \). The Large Numbers coincidence is the fact, that all elementary particles are close together considering the large number \( \Lambda \):

\[ m = \frac{c^3}{H_0 G N^2} \Rightarrow x = \frac{\ln (c^3/(n H_0 G))}{\ln \Lambda} \]  

(38)

Setting in the masses of the proton, pion and electron, \( x \) is clustered \(^\text{14}\) at values close to 2, which are 1.97411, 1.99458 and 2.05465 respectively. So all elementary particles are determined by the same \( \Lambda \).

The mass (36) can be compared with the herein derived mass relation \( \Delta m_n = \frac{\Delta m_n}{m_n} = \frac{1}{2^n} \). The Compton-radius of the mass is the Schwarzschild radius \( r = 2Gm_n/c^2 \) of the related virtual-mass, which gives \( \Delta m_n = m = rc^2/4n G \). Comparing the values for \( m \) gives \( r/4n = c/H_0 \Lambda^2 \), and as will be shown later \( \Lambda = 2n \), this results in:

\[ r = \frac{2c}{H_0 \Lambda} = \frac{R}{\Lambda} \]  

(39)

So the classical Dirac conjecture (4) is recovered.

### 3.6 The electric charge

Moreover a black hole has not to gravitate because as an effect of its event-horizon no gravitational waves or gravitons may leave it. An ordinary black hole gravitates because it leaves behind the gravitational field of a collapsing object. If for example a Daemon would cut out a stellar black hole exactly at the Schwarzschild-border and place it somewhere else in the Universe, such a black hole would not gravitate except by the small mass equivalent of its Hawking radiation. This mechanism may define elementary particles as the Hawking radiation of virtual-miniature-black-holes.

By this assumption the Compton-wavelength of a particle is associated with a virtual black hole, which means a quantum space-time topology with at least one curvature according to this size. Such a virtual-miniature-black-hole is charged with the constant \( C_{\pm} \). This charge is independent of the mass of the virtual black hole and could give rise to an electrical charge by quantum gravity effects: the gravitational force between a non-gravitating virtual-black-hole and a gravitating stimulated-mass (27) would be \( 2n \)-times stronger than the gravitational force between two stimulated masses.

The classical ratio between the gravitational force \( F_G = G \frac{m^2}{r^4} \) and the electromagnetic force \( F_Q = \frac{1}{4\pi \varepsilon_0} \frac{e^2}{r^2} \) for for example an electron in any given distance \( r \) is:

\[ \frac{F_Q}{F_G} = \frac{e^2}{4\pi \varepsilon_0 G m_e^2} = 4.167 \cdot 10^{42} \]  

(40)

This classical ratio is also a worth mentioning Large Number and it lies just between the eigenvalues for SBH-quarks and SBH-electrons. If the virtual mass of a SBH-particle would be present as a higher order effect in any unknown way, the gravitational force between a SBH-particle and the virtual-SBH-mass of its vis-a-vis would be:

\[ F_{G-virtual} = G \frac{\Delta m_n}{r^2} m_n = \frac{ch}{4\pi^2} \]  

(41)

As this force is independent of \( n \) and therefore independent of the mass of the particle it can be related to an electromagnetic charge:

\[ F_Q = \frac{1}{4\pi \varepsilon_0} \frac{Q^2}{r^2} = F_{G-virtual} = \frac{ch}{4\pi^2} \]  

(42)
which gives
\[ Q = \pm \sqrt{\pi \varepsilon_0 c \hbar} = \pm 5.853 \cdot e \] (43)

and is about 6 times the elementary charge. As this assumption is just a plain one (as it must be an effect of higher order), this charge is not so far away from unity as it seems at first sight. Hence the Large Number (44) represents the ratio of virtual mass to particle mass:
\[ \Lambda \simeq \frac{F_G \text{-virtual}}{F_G} = \frac{m_n}{\Delta m_n} = 2n \sim 10^{38-44} \] (44)

3.7 On the quantum-topology of the SBH

To consider elementary particle as a static sphere may be too much simplified. Especially one could expect a rotating black hole close to its Kerr-limit, which gives a more complicated model [23, 32, 34].

The equivalence principle of GR provides the assumption that matter should be equal to a related space-time curvature. Hence the quantum-topology of a SBH may be for example a torus, with one main-curvature related to the Planck-length and the other main-curvature related to the Compton-length. Then it would appear to be a closed string. Another consideration could be to imagine the hadrons as a bag model[16, 23] with bumps in it, so that the overall-curvature might be of Compton-size and the bumps providing curvatures which relates to the quark masses and fractional charges.

It was A. Einstein[6] himself who first suspected elementary particles to be build up by gravitational fields. An attempt in recent time was done by Recami et al.[28], describing hadrons as strong-blackholes by a concept of strong-gravity. The Recami-model describes hadrons and its constituents with the, according to the large numbers hypothesis, scaled Einstein-field equations of GR. This bi-scale theory of gravitational and strong interactions delivers a description of the confinement and asymptotic freedom of the quarks, the Yukawa-potential of strong interactions and derives a strong coupling constant and a mesonic mass spectra, which fit well the theoretical and experimental values.

A crucial requirement for SBH-particles is that this particles should behave as known particles do. So what happens if two SBH-particles collide? SBH-particles have at least two quantum-numbers in this plain model: The spin \( s_z = \pm \hbar/2 \) and the virtual-mass-charge \( C^+_n = \pm c \hbar / 4G \), which corresponds to the electrical charge \( \pm e \) and a virtual-mass of \( \pm m_n \). A SBH-positron has the opposite values of this quantum-numbers, so SBH-electron and SBH-positron annihilate to a radiation of 2-times the energy of the stimulated mass \( \Delta m_n \), just as electron and positron do. When two SBH-electrons meet, they will not merge to a double-massive SBH-electron, as the two \( \hbar/2 \)-spins would add to \( \hbar \) or 0, but the double-massive SBH-electron must have a \( \pm \hbar/2 \)-spin.

4 Conclusion

The Large Numbers Hypothesis was discovered by Dirac in 1937 in which he found that cosmological quantities can be related to particle quantities by simple scale relations of which he thought could not just be random in nature.

Gravitation is a sort of energy and in consequence itself is a source of gravitation. Such a self-relating characteristic is a typical element of any fractal structure. Here a blueprint of a fractal Universe composed of black holes was outlined by assuming that a gravitational wave should not overrun its own event-horizon. The model gives possible answers to still miraculous issues, such as the entropy of black holes, the origin and integer value of the electric charge, the spin, sizes and weights of elementary particles and the background of Dirac’s Large Numbers coincidences.

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