Effects of Chemical Potential on Hadron Masses at Finite Temperature

QCD-TARO Collaboration: Ph. de Forcrand, M. García Pérez, T. Hashimoto, S. Hioki, H. Matsufuru, O. Miyamura, A. Nakamura, I.-O. Stamatescu, T. Takaishi and T. Umeda

We study the effects of the chemical potential on the ρ meson mass at finite temperature. Our preliminary results show that some effects are seen in the vicinity of the phase transition point. Although the signal is still too noisy to obtain conclusive physical results within limited statistics, the mass susceptibility is consistent with zero.

1. Introduction and Motivation

QCD Sum Rule analysis has suggested that a mass-shift occurs in proportion to the baryon density of medium. In nuclear matter, a few to 10% reduction of vector meson masses is expected [2] and confirmation of this fact is of current experimental interest.

Although the importance of a quantitative investigation on this subject is obvious, it seems difficult to tackle it by present lattice technology. A new attempt to measure the derivative of mass M in terms of the chemical potential, ∂M/∂µ at µ = 0 has been made in this work. At T = 0, since the density depends cubically on µ, ∂M/∂µ vanishes. On the other hand, at finite temperature, a linear dependence on the chemical potential is expected and we will have a chance to see the density effect.

2. Formulation

We use 2 flavors of the Wilson fermion for the fermionic part of the action. The action is given by $S = S_G + S_F$ where $S_G$ and $S_F$ stand for the gauge and the fermionic parts respectively, and $S_F$ has the form,

$$S_F = \sum_{i=u,d} \bar{\psi}^i(n)D(n,m)\psi^i(m).$$

For $S_G$ we use the standard Wilson action. The Wilson matrix $D(n,m)$ with chemical potential is written as

$$D(n,m) = \delta_{n,m} - \kappa \sum_{i=1}^3 [(1 - \gamma_i)U_i(m)\delta_{m,n+i}]
+(1 + \gamma_i)U_i^\dagger(n-i)\delta_{m,n-i}]
-\kappa[(1 - \gamma_4)U_4(m)e^\mu\delta_{m,n+4}]
+(1 + \gamma_4)U_4^\dagger(n-4)e^{-\mu}\delta_{m,n-4}].$$

The hadron correlation function $C(t)$ is given by

$$C(t) = \int dU d\bar{\psi} H(t) H^\dagger(0)e^{-S}/Z$$

$$= \sum_i A_i \cosh(M_i(t - N_i/2)).$$

where $A_i$ may be written as $A_i = \tilde{A}_i/(2\sinh(M_iN_i/2))$.
To derive the expression of the mass susceptibility on the chemical potential, we take a derivative of $C(t)$ with respect to the chemical potential $\mu$. From eq. (3) one obtains
\[
\frac{\partial C(t)}{\partial \mu} = - <H(t)H^\dagger(0)\frac{\partial S_F}{\partial \mu} > + <H(t)H^\dagger(0)\frac{\partial S_F}{\partial \mu} >.
\]

On the other hand from eq. (4) one obtains
\[
\sum_i \left[ \frac{\partial A_i}{\partial \mu} \cosh(M_i(t - Nt/2) - \frac{\partial m_i}{\partial \mu}A_i(t - Nt/2) \sinh(M_i(t - Nt/2))] \right] = 0.\]

Eqs. (5) and (6) should be equal to each other. Eq. (5) is evaluated by Monte Carlo simulations. Eq. (6) contains 4 unknown fitting parameters for a given $i$. $A_i$ and $M_i$ are determined from a fit to the hadronic correlation function of eq. (4). Then these results are used for eq. (5) as fixed parameters and, thus the chemical potential dependent terms $\partial A_i/\partial \mu$ and $\partial M_i/\partial \mu$ are extracted from a two parameter fit to eq. (5).

3. Calculation of $<H(t)H^\dagger(0)\frac{\partial S_F}{\partial \mu} >$

In the present study we aim at obtaining the mass susceptibility at $\mu = 0$. At $\mu = 0$, $<\frac{\partial S_F}{\partial \mu} > = 0$. Thus the calculation of eq. (5) reduces to calculate the remaining term $<H(t)H^\dagger(0)\frac{\partial S_F}{\partial \mu} >$.

Now we specify the hadronic operator. For the present study we take the $\rho$ meson operator, which is given by,
\[
H(t) = \sum_x u(x,t)\gamma_\mu \bar{d}(x,t).
\]

We take $\nu = 2$ here. The derivative of $S_F$ with respect to $\mu$ is
\[
\frac{\partial S_F}{\partial \mu} = -\kappa \sum_{i = u,d} \sum_{n,m} \bar{\psi}(n)[(1 - \gamma_4)U_4(m)e^{\mu} \delta_{m,n+1}
\]
\[
- (1 + \gamma_4)U_4^\dagger(n - 4)e^{-\mu} \delta_{m,n-1}]\psi(m)
\]
\[
\equiv -\kappa \sum_{i = u,d} \sum_{n,m} \bar{\psi}(n)[B_1 - B_2]\psi(m),
\]

where
\[
B_1 = (1 - \gamma_4)U_4(m)e^{\mu} \delta_{m,n+1};
\]
\[
B_2 = (1 + \gamma_4)U_4^\dagger(n - 4)e^{-\mu} \delta_{m,n-1}.
\]

Let us consider
\[
<H(t)H^\dagger(0)\kappa \sum_{n,m} \bar{u}(n)B_1u(m) >
\]

where $u(m) = \psi^a(m)$. This term contains a connected and disconnected diagrams. We neglect the disconnected part, which is unimportant for the present study. Eq. (11) is written as
\[
- \sum_{x,z} \gamma_\mu D^{-1}(0,0 : z)(1 - \gamma_4)U_4(z)\times
\]
\[
D^{-1}(z + \hat{4} : x,t)\gamma_\mu D^{-1}(x,t : 0,0).
\]

where the chemical potential $\mu$ is set to zero.

Similar calculations are applied for the remaining terms ( for $B_2$ and for $i = d$ ).

![Figure 1. Polyakov loop and its susceptibility as a function of $\kappa$.](image.png)

4. Simulation

Simulations are done on a lattice size of $16 \times 8 \times 8 \times 4$ at $\beta = 5.30$. 

To find a finite temperature transition point, the Polyakov loop and its susceptibility are calculated, as shown in Fig. 1. The finite temperature transition happens around $\kappa = 0.158$. The chemical potential effect on hadron masses at $\mu = 0$ is expected to start in the vicinity of the transition. We take $\kappa = 0.153$ and $\kappa = 0.158$ for the present study. We use 30 configurations generated by the hybrid Monte Carlo algorithm for each $\kappa$. The configurations are separated by 40-50 trajectories.

5. Preliminary results

Figs. 2 and 3 show the derivative of the hadronic correlation function $C(x)$ with respect to $\mu$ as a function of the largest lattice direction: $x$. The hadronic correlation function is measured in the $x$-direction.

![Figure 2](image1.png)

Figure 2. The derivative of the hadronic correlator $C(x)$ with respect to $\mu$ at $\kappa = 0.153$ as a function of $x$.

Although the signals within the current statistics are still too noisy, we fit the data using eqs. (3) and (4). The mass susceptibility, $\partial M/\partial \mu$, is consistent with zero, while $\partial A/\partial \mu$ could be finite and positive.

![Figure 3](image2.png)

Figure 3. Same as in Fig. 2 but at $\kappa = 0.158$.

6. Discussions

The effect of chemical potential on the mass susceptibility is suppressed by $\exp(-M_B/T)$ where $M_B$ is the baryon mass. Our results may show that $M_B$ at our simulation parameter is too heavy to obtain a reasonable signal within limited statistics. Our hope is that signal may be enhanced if a small quark mass is used.

One of the authors (T.T.) would like to thank Electric Technology Research Foundation of Chugoku for financial support. Calculations reported here were done on AP1000 at Fujitsu Parallel Computing Research Facilities and SX-4 at RCNP, Osaka Univ.

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