Update on the lattice calculation of $B \rightarrow K^{*}\gamma$

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Abstract: We present updated results on the calculation of the matrix elements for $B \rightarrow K^{*}\gamma$ in the quenched approximation on a $24^3 \times 48$ lattice at $\beta=6.2$, using an $O(a)$-improved fermion action. The scaling behaviours of the form factors $T_1(q^2=0)$ and $T_2(q_{\max}^2)$ for the decay are examined and pole model ansätze tested.

1. Introduction

Theoretical interest in the rare decay $B \rightarrow K^{*}\gamma$ as a test of the Standard Model has been renewed by the experimental results of the CLEO collaboration [1]. The viability of calculating the relevant hadronic matrix elements on the lattice was first demonstrated by Bernard, Hsieh and Soni [2] in 1991.

The computational details and results of this work have been described in references [3] and [4].

2. Form Factor Definitions

The hadronic matrix elements can be parametrised by three form factors,

$$\langle K^{*}(k,\epsilon)\gamma\sigma_{\mu\nu}q^{\nu}b_R | B(p) \rangle = \sum_{i=1}^{3} C_{2i}^2 T_i(q^2),$$  \hspace{1cm} (1)

where,

$$C_{21}^i = 2\varepsilon_{\mu\nu\lambda\rho}e^\nu p^\lambda k^\rho,$$  \hspace{1cm} (2)

$$C_{22}^i = \epsilon_\mu (m_B^2 - m_{K^{*}}^2) - \epsilon \cdot q(p + k)_\mu,$$  \hspace{1cm} (3)

$$C_{23}^i = \epsilon \cdot q \left( q_\mu - \frac{q^2}{m_B^2 - m_{K^{*}}^2} (p + k)_\mu \right),$$  \hspace{1cm} (4)

and $q$ is the momentum of the emitted photon.

As the photon emitted is on-shell, the form factors need to be evaluated at $q^2=0$. In this limit,

$$T_2(q^2=0) = -iT_1(q^2=0),$$  \hspace{1cm} (5)

and the coefficient of $T_3(q^2=0)$ is zero in the on-shell matrix element. Hence, the branching ratio can be expressed in terms of a single form factor, for example $T_1(q^2=0)$.

3. Heavy Quark Scaling

We calculate with a selection of quark masses near the charm mass and extrapolate to the $b$-quark scale. In the heavy quark limit, heavy quark symmetry [5] tells us that,

$$T_1(q_{\max}^2) \sim m_P^{1/2},$$

$$T_2(q_{\max}^2) \sim m_P^{-1/2},$$  \hspace{1cm} (6)

where $m_P$ is the pseudoscalar mass. Combining this with the relation $T_2(q^2=0) = -iT_1(q^2=0)$ constrains the $q^2$ dependence of the form factors. However, it does not provide a scaling law for $T_1(q^2=0)$ without further assumptions about the actual $q^2$ behaviour of the form factors.

Pole dominance ideas suggest that,

$$T_i(q^2) = \frac{T_i(0)}{(1 - q^2/m_i^2)^{n_i}},$$  \hspace{1cm} (7)

for $i = 1, 2$, where $m_i$ is a mass that is equal to $m_P$ plus $1/m_P$ corrections and $n_i$ is a power. Since $1 - q^2_{\max}/m_i^2 \sim 1/m_P$ for large $m_P$, the combination of heavy quark symmetry and the form factor relation at $q^2=0$ implies that $n_1 = n_2 + 1$. For example, $T_2(q^2)$ could be a constant and $T_1(q^2)$ a single pole, or $T_2(q^2)$ could be a single pole and $T_1(q^2)$ a double pole. These two cases correspond to,

$$T_1(0) \sim \begin{cases} m_P^{1/2} & \text{single pole} \\ m_P^{-3/2} & \text{double pole} \end{cases}.$$  \hspace{1cm} (8)

The data appear visually to favour $T_2(q^2)$ constant in $q^2$ when $m_P$ is around the charm scale. However, we will consider both constant and single pole behaviours for $T_2(q^2)$ below.
4. Results

As demonstrated in a previous paper [3], the evaluation of \( T_1(q^2; m_P; M_K^* ) \) is relatively straightforward, and \( T_2 \) can be determined in a similar way. We fit \( T_1(q^2) \) to a pole or dipole model in order to obtain the on-shell form factor \( T_1(q^2 = 0) \),

\[
T_1(q^2) = \frac{T_1(q^2 = 0)}{1 - q^2/m^2}, \quad \frac{T_1(q^2 = 0)}{1 - q^2/m^2}.
\]

The difference between the two models was found to be negligible. The form factor \( T_2 \) was fitted to a pole model or constant.

The ratio \( T_1/T_2 \) at \( q^2 = 0 \) for dipole/pole and pole/constant fits is shown in Fig.(1). The magnitude is found to be consistent with 1 at low masses, in accordance with the identity \( T_1(0) = i T_2(0) \), Eq.(5). At higher masses, the dipole/pole fits for \( T_1/T_2 \) deviate less than the pole/constant fits.

5. Extrapolation of \( T_2(q^2_{\text{max}}) \) to \( m_B \)

In order to test heavy quark scaling, we also extracted the form factor \( T_2 \) at maximum recoil, where \( q^2 = q^2_{\text{max}} = (m_P - m_V)^2 \), in the same way as Bernard et al. [6]. In the heavy quark limit, \( T_2(q^2_{\text{max}}) \) is expected to scale as \( m_P^{-1/2} \), analogous to the scaling of \( f_B \). Higher order \( 1/m_P \) and radiative corrections will also be present. For convenience, we remove the leading scaling behaviour by forming the quantity,

\[
\hat{T}_2 = T_2(q^2_{\text{max}}) \sqrt{\frac{m_P}{m_B}} \left( \frac{\alpha_s(m_P)}{\alpha_s(m_B)} \right)^{2/\beta_0}.
\]

The normalisation ensures that \( \hat{T}_2 = T_2(q^2_{\text{max}}) \) at the physical mass \( m_B \). Linear and quadratic correlated fits for \( \hat{T}_2 \) were carried out with the functions,

\[
\hat{T}_2(m_P) = A \left( 1 + \frac{B}{m_P} \right), \quad \hat{T}_2(m_P) = A \left( 1 + \frac{B}{m_P} + \frac{C}{m_P^2} \right),
\]

and are shown in Fig.(2). Taking the quadratic fit of \( \hat{T}_2 \) at \( m_P = m_B \) as the best estimate, and the difference between the central values of the linear and quadratic fits as an estimate of the systematic error, \( T_2 \) was found to be,

\[
T_2(q^2_{\text{max}}; m_B; M_K^*) = 0.269_{-0.011}^{+0.017}.
\]

If the \( q^2 \) dependence of \( T_2 \) at \( m_P = m_B \) were known, this result could be related to \( T_1(q^2 = 0) \) via the identity \( T_1(0) = iT_2(0) \).

6. Extrapolation of \( T_1(q^2 = 0) \) to \( m_B \)

For \( T_1(q^2 = 0) \) we test the two possible scaling laws in the same way as for \( T_2 \), by forming the quantity,

\[
\tilde{T}_1 = T_1(q^2 = 0) \left( \frac{m_P}{m_B} \right)^n \left( \frac{\alpha_s(m_P)}{\alpha_s(m_B)} \right)^{2/\beta_0},
\]

where \( n = 1/2, 3/2 \). For \( n = 3/2 \), a similar scaling relationship has been found using light-cone sum rules by Ali, Braun and Simma [7]. The \( n = 1/2 \) case has been suggested by other sum rules calculations [8].
Linear and quadratic fits were carried out with the same functions as for $T_2$. The two cases $n = 1/2, 3/2$ are shown in Fig.(3). The $\chi^2$/d.o.f. are approximately 1 for the scaling laws, indicating that the models are statistically valid in the available mass range.

The final results for $T_1(q^2=0; m_B; m_{K^*})$ are taken from the quadratic fit for $T_1$, with the systematic error estimated as for $T_2$,

$$T_1(q^2=0) = \begin{cases} 0.159^{+34}_{-33} \pm 0.067 & n = 1/2 \\ 0.124^{+20}_{-18} \pm 0.022 & n = 3/2 \end{cases}$$ (15)

7. Conclusions

Further information on the $q^2$ dependence of $T_1$ and $T_2$ is required to remove the uncertainty in obtaining the form factors at the physical point $q^2=0$, $m_p=m_B$.

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REFERENCES

1. CLEO Collaboration, R. Ammar et al., Phys. Rev. Lett. 71, 674 (1993).
2. C. Bernard, P. Hsieh, and A. Soni, Nucl. Phys. (Proc Suppl.) B26, 347 (1992), note that there is a factor of 2 missing in eq. (4) of this paper.
3. UKQCD Collaboration, K. Bowler et al., Phys. Rev. Lett. 72, 1398 (1994), hep-lat 9311004.
4. UKQCD Collaboration, K. Bowler et al., hep-lat 9407014 (1994).
5. N. Isgur and M. Wise, Phys. Rev. D 42, 2388 (1990).
6. C. Bernard, P. Hsieh, and A. Soni, Phys. Rev. Lett. 72, 1402 (1994), hep-lat 9311011.
7. A. Ali, V. Braun, and H. Simma., hep-ph 9401277 (1993).
8. P. Ball, hep-ph 9308244 (1993). P. Colangelo, C. Dominguez, G. Nardulli, and N. Paver, Phys. Lett. B 317, 183 (1993), hep-ph 9308264.