Three Tank System Sensors and Actuators Faults Detection Employing Unscented Kalman Filter *

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Abstract — Fault detection is critical for industrial applications to maintain a stable operation and to reduce maintenance costs. Many fault detection techniques have been introduced recently to cope with the increasing demand for safer operations. One of the most promising fault detection algorithms is the Unscented Kalman Filter (UKF). UKF is a model-based algorithm that could be used to detect different fault types for a given system. On the other hand, the three-tank system is a well-known benchmark that simulates many industrial applications. The fault detection of the three-tank system is quite challenging as it is a Multi-Input Multi-Output (MIMO) nonlinear system. Therefore, UKF will be employed as a fault detection strategy for this system to detect sensor and actuator faults. The performance of the UKF will be investigated under different operating and fault conditions to show its merits for the given case study.

Keywords — Fault detection, Three-tank system, Unscented Kalman Filter (UKF)

I. INTRODUCTION

Fault detection and tolerance are gaining more interest recently especially for industrial dynamic systems. This is primarily because of the increasing demand for improved control system performance, in addition to higher safety and reliability standards [1]. Faults can originate from the main process elements such as sensors or actuators [2]. This can be noticed as an error in the accuracy of level, temperature, or flow measurements. Also, it could be represented by uncalibrated or defective actuators such as motors and valves [3], [4].

Fault detection is roughly categorized into model-based and model-free methods. As the name suggests, model-free methods do not require the mathematical model of the process. However, a state-space model is usually used for the model-based methods or the transfer function model in some processes. However, a state-space model is usually used for the model-based methods or the transfer function model in some processes. Although the parameters are of great importance as stated above, obtaining an exact value can be quite challenging. Therefore, one of the approaches is to formulate the parameter estimation as a state estimation problem by defining the fault parameter as an additional state. In the traditional Kalman Filter (KF) approach, the states are estimated through two distinct steps. First, the healthy state model is developed, then the faulty state one. The fault is estimated by analyzing the plant model mismatch [7]. However, the parameters estimation errors increase with the system uncertainty. So, the Unscented Kalman Filter (UKF) is introduced as a modified version of the KF such that the mathematical state-space model is used to get a precise estimation of the fault parameters [8]. Therefore using UKF can overcome some of the KF’s weaknesses, such as the requirement of differentiable state dynamics, and sensitivity to bias or divergence in the state estimates [9].

A three-tank system is used as a case study for this paper. It represents a typical system in the process industry, such as the fuel management system of airplanes and flight vehicles as well as applications in chemical and petrochemical industries. It is considered a valuable experimental setup for studying multivariable feedback control as well as fault diagnosis [10]–[12]. Some of the main parameters of this system such as the viscosity coefficients are uncertain due to the change in the liquid characteristics, aging effects, other environmental reasons as corrosion, scaling, and changing operating conditions. Some techniques have been previously proposed such as neural, fuzzy [13], [14], non-linear observers [15], the generalized likelihood ratio-based system [16], and the multiple model-based approaches [17]. Nevertheless, these approaches are unable to estimate the fault if the noise in the system is higher than the fault magnitude. The proposed UKF can estimate the faults with low magnitude compared to the presented noise in the system. Additionally, UKF utilizes the method of the direct nonlinear model as an alternative to linearizing it [9]. This eliminates the need to calculate Jacobian or Hessian. Also, it has a further advantage of a lower computational burden.

The paper structure is as follows: the three-tank system state-space model is given in Section II. Section III is devoted to the adaptation of the three-tank system model by the UKF approach. Results are presented in Section IV for sensor and actuator faults considering the noise effect and tuning the covariance and weighting matrices. The performance is
evaluated at different operating conditions to test the system's robustness. Finally, the conclusions are highlighted in Section V.

II. THREE TANK SYSTEM MATHEMATICAL MODEL

The three-tank system is illustrated in Fig. 1. It includes similar cylinder-shaped tanks with a cross-section area $A_t$. The tanks are linked through two cylindrical tubes of cross-section area $A_p$ with similar outflow coefficients $\mu_{13}$ and $\mu_{32}$. The outflow is placed at the second tank with a cross-section area $A_p$ with an outflow coefficient $\mu_{20}$. Two pumps supply the first and second tanks, while the third tank affects their levels. $q_1$ and $q_2$ are the flow rates of the pumps with a maximum value of $Q_{\text{max}}$. The system could be described by the following flow balance equations (1)-(3) [18], [19]:

$$A_t \frac{dL_t}{dt} = q_1(t) - q_{13}(t)$$  

$$A_t \frac{dL_t}{dt} = q_2(t) + q_{32}(t) - q_{20}(t)$$  

$$A_t \frac{dL_t}{dt} = q_{13}(t) - q_{32}(t)$$  

where $q_{mn}$ are the flow rate from tank $m$ to tank $n$ ($m, n = 1, 2, 3$ and $m \neq n$), $L_1$, $L_2$, and $L_3$ are the three-tank levels. Based on Torricelli law the flow rate equals (4):

$$q_{mn}(t) = \mu_{mn}A_p\sqrt{g(L_m(t) - L_n(t))}$$  

where $g$ is the gravity constant. Consequently, the flow rate for the three tanks will be (5)-(7):

$$q_{13} = \mu_{13}A_p\sqrt{2g(L_1 - L_3)}$$  

$$q_{32} = \mu_{32}A_p\sqrt{2g(L_3 - L_2)}$$  

$$q_{20} = \mu_{20}A_p\sqrt{2gL_2}$$

Thus, the flow balance equations for the first tank will be (8) and (9):

$$A_t \frac{dL_t}{dt} = q_1(t) - q_{13}(t)$$  

$$A_t \frac{dL_t}{dt} = q_1(t) - \mu_{13}A_p\sqrt{2g(L_1 - L_3)}$$

for the second tank the flow balance equation is written as (10) and (11):

$$A_t \frac{dL_t}{dt} = q_2(t) + q_{32}(t) - q_{20}(t)$$  

$$A_t \frac{dL_t}{dt} = q_2(t) + \mu_{32}A_p\sqrt{2g(L_3 - L_2)} - \mu_{20}A_p\sqrt{2gL_2}$$

and for the third tank rewritten as (12) and (13):

$$A_t \frac{dL_t}{dt} = q_{13}(t) - q_{32}(t)$$  

$$A_t \frac{dL_t}{dt} = \mu_{13}A_p\sqrt{2g(L_1 - L_3)} - \mu_{32}A_p\sqrt{2g(L_3 - L_2)}$$

As a result, the three-tank system model could be written as:

$$x(t) = r(x(t)) + s(x(t)) u(t)$$  

$$y(t) = x(t)$$
sensors. In particular, the states of the three-tank system will be extended by five parameters:

\[ \hat{x}(t) = [L_1, L_2, L_3, \Delta L_1, \Delta L_2, \Delta L_3, \Delta q_1, \Delta q_2] \]

The added estimated parameters are represented by \( \lambda_k \). The discrete extended model can be expressed as:

\[ \begin{align*}
    \hat{x}_{k+1} &= r_k + s_k \hat{u}_k + F_{sk} \lambda_k + w_k \\
    \hat{y}_k &= \hat{x}_k + F_{yk} \lambda_k + v_k
\end{align*} \]

where

\[ r_k = \left[ 1 + T_s \tau(\lambda_k) \right] \quad s_k = \left[ T_s \sigma(\lambda_k) \right] \quad 0_{5 \times 2} \]

and

\[ F_{sk} = [s_k \quad 0_{3 \times 3}] \quad F_{yk} = [I_{3 \times 3} \quad 0_{3 \times 3}] \]

b) Unscented Kalman Filter

For Kalman filter algorithms, UKF is a modified version of the Extended Kalman Filter (EKF). They are designed to deal with non-linear systems models in the presence of state and measurement noises. However, the UKF works with the concept of calculating the sigma point instead of using the Jacobian matrix for linearizing the non-linear model as in EKF [8]. The UKF procedure is stated in [8], the first step is the initialization of the parameters based on the initial values of the states \( \hat{x}_0 \) and the covariance matrix \( P_0 \) [20]:

\[ \begin{align*}
    \hat{x}_0 &= [\hat{x}_0^T \quad 0] \\
    P_0 &= E[(\hat{x}_0^T - \hat{x}_0)(\hat{x}_0^T - \hat{x}_0)^T]
\end{align*} \]

where

\[ \begin{align*}
    \hat{x}_{k-1} &= [\hat{x}_{k-1}^T \quad \hat{y}_{k-1}^T] \\
    X_{k-1} &= \left[ X_{k-1}^T \quad X_{k-1}^{yT} \right]
\end{align*} \]

and expressed as:

\[ \begin{align*}
    \hat{x}_k &= \sum_{i=0}^{2L} W_i^m X_{i,k-1} \\
    \hat{y}_k &= \sum_{i=0}^{2L} W_i^c (Y_{i,k-1} - \hat{x}_k)
\end{align*} \]

where \( W_i^m \) and \( W_i^c \) are weighting factors and they are equal to \( 1/2^L \). As in the EKF, the prediction phase is the next step for calculating the new covariance matrix \( P_k \). It can be expressed as:

\[ P_k = \sum_{i=0}^{2L} W_i^c (Y_{i,k-1} - \hat{x}_k)(Y_{i,k-1} - \hat{x}_k)^T \]

Finally, the measurement update equations where the Kalman gain \( K_k \) is calculated for the correction of the next state estimation \( \hat{x}_k \) and covariance matrix \( P_k \) is given by:

\[ \begin{align*}
    K_k &= (P_{yk} P_k)^{-1} P_{yk} \hat{y}_k \\
    \hat{x}_k &= \hat{x}_{k-1} + K_k (y_k - \hat{y}_k) \\
    P_k &= P_{k-1} - K_k P_{yk} K_k^T
\end{align*} \]

where \( P_{yk} \) and \( P_{yk} \) represent the covariance of the posterior sigma points of \( \hat{y}_k \) and \( \hat{x}_k \) and expressed as:

\[ \begin{align*}
    P_{yk} &= \sum_{i=0}^{2L} W_i^c (Y_{i,k-1} - \hat{y}_k)(Y_{i,k-1} - \hat{y}_k)^T \\
    P_{yk} &= \sum_{i=0}^{2L} W_i^c (X_{i,k-1} - \hat{x}_k)(X_{i,k-1} - \hat{x}_k)^T
\end{align*} \]

c) Covariance and weighting matrices tuning

The tuning of the weighting matrices, \( Q_k \) and \( R_k \), depends on the amount of the state and measurement noises in the system. Consequently, both matrices are diagonal in terms of the standard deviation variance of the state noise \( \sigma^2_x \) and measurements noises \( \sigma^2_y \). Based on [21], the \( Q_k \) and \( R_k \) are illustrated as:

\[ \begin{align*}
    Q_k &= \sigma^2_o \begin{bmatrix} 1 & 0 \\
                         0 & \sigma^2_o \end{bmatrix} \\
    R_k &= \sigma^2_y \begin{bmatrix} 1 & 0 \\
                                    0 & 1 \end{bmatrix}
\end{align*} \]

where \( m \) is the number of states \( x \) and \( n \) is the number of estimated parameters \( \lambda \). The ratio \( \sigma_y/\sigma_o \) is equal to:

\[ \begin{align*}
    \frac{\sigma_y}{\sigma_o} &= \frac{T_s}{\left( \sum_{i=1}^{n} \left( \frac{\sigma_y}{\sigma_o} \right) \right)^2}
\end{align*} \]

where \( \beta \) is the evaluation time constant of the estimated parameters and \( i \) represents the number of the estimated parameters. It is a factor that can be used to set the value of \( \sigma_y/\sigma_o \). Applying (32) to (31), the initial value of the error covariance matrix and weighting matrices are given in (33):

\[ \begin{align*}
    Q_0 &= \begin{bmatrix} 1 \times 10^{-2} & 0 \\
                         0 & 0 \end{bmatrix} \\
    R_0 &= \begin{bmatrix} 0.01 & 0 \\
                          0 & 0 \end{bmatrix} \\
    Q_k &= \begin{bmatrix} 1 \times 10^{-2} & 0 \\
                         0 & 0 \end{bmatrix} \\
    R_k &= \begin{bmatrix} 1 \times 10^{-2} & 0 \\
                         0 & 0 \end{bmatrix}
\end{align*} \]

IV. SIMULATION RESULTS AND DISCUSSION

The MATLAB/ Simulink program is used to simulate the process of the three-tank system. The system parameters and operating point are shown in Table I [11]. The sampling time \( T_s \) was chosen equal to 0.1 s. A closed loop PID control was implemented to the system with suitable gains that have been tuned based on the system model. The PID gains for both inputs are illustrated in Table II. The sensor and actuator faults are included and the UKF response is tested in both cases with and without considering the system noise.

a) UKF estimation response without the system noise

In this context, the UKF is tested without including the state or measurement noises, so it is considered as a noise-free operating condition for both types of system faults. The time constant for the parameter estimation \( \beta \) is chosen to be 0.1 s for all results in this case. \( \beta \) has a low value to increase the estimation response as much as possible, as the noise is not considered in these fault scenarios.

| Parameter | Value |
|-----------|-------|
| Tank cross-section area (\( \phi \)) | 0.0154 m² |
| Pipe cross-section area (\( \phi \)) | 5 \times 10^{-5} m² |
| Outflow coefficient (\( \mu_0 \)) | 0.5, \( \mu_0 = 0.675 \) |
| Maximum flow rate constraint (\( Q_{\text{max}} \)) | 1.2 \times 10^{-4} m³/sec |
| Maximum level (\( L_{\text{max}} \)) | 0.62 m |
| Operating point | \( L_{\text{op}} = 0.35 \times 10^{-4} m \) |
| \( L_{\text{op}} = 0.75 \times 10^{-4} m \) | 0.4 m |
| \( L_{\text{op}} = 0.2 m \) | 0.3 m |

TABLE I. THREE TANK SYSTEM PARAMETERS
TABLE II. PID PARAMETERS VALUES.

| Parameter   | Value  |
|-------------|--------|
| Proportional gain ($K_p$) | 28.78  |
| Integral gain ($K_i$)       | 2.4    |
| Differential gain ($K_d$)   | -13.89 |

1) Sensors’ fault

In such a scenario, the sensor faults are represented in adding an amount to the level measurement, which is undetermined by the utilized sensor. At $t = 250$ s, a decrease in the first tank level by 0.03 m is applied. The sensor still has the same reading without any change. The UKF estimates the actual level $L_1$ as one of the estimated states in the process as shown in Fig. 2. Consequently, the UKF calculates the difference between both readings $\Delta L_1$, which in this case is 0.03 m as shown in Fig. 3. The same scenario is applied for the second tank but as an increase in the actual level value of 0.04 m. This amount is not considered by the sensor reading $L_2$ as shown in Fig. 4. The UKF obtains the value of $\Delta L_2$ and it is equal to -0.04 m as shown in Fig. 5. The estimation response is very fast for both cases of sensor faults, it takes 4 s to reach the steady-state, and the values of the estimated parameters are error-free.

2) Actuators’ fault

In this case, the actual value of the level sensors $L_m$ is equal to the measured value $L_n$ in all three-tank level measurements and the fault will occur at the feeding pumps of the system. The impact of the fault will appear in the input flow rates values of the system. An increase at the flow rate $Q_1$ by 50% of its nominal value at $t = 250$ s will be imposed. The applied closed-loop control for the system will maintain the first tank level to its reference value as shown in Fig. 6. Consequently, The UKF estimates the added values to the flowrate difference $\Delta Q_1$ as shown in Fig. 7. Another test is executed by reducing the flowrate $Q_2$ by 20% at $t = 250$ s as shown in Fig. 8. The UKF evaluates the value of $\Delta Q_2$ which causes a reduction in the flow rate $Q_2$ as presented in Fig. 9. The estimation response for both cases of actuator faults takes 8 s to reach steady-state and values of the estimated parameters are without error as it is a noise-free case.
In order to simulate the real operating conditions, a white measurement noise with a standard deviation of 0.01 is added to the measured signal from the level sensors. In this case, the estimation response is affected by the added noise and the weighting matrices, $Q_k$ and $R_k$, is very important for the estimation response performance. The tuning of the weighting matrices depends on the expected values of the measurements and output noise in the system. Thus, for the proposed case, the evaluation time constant $\beta$ is chosen to be 100 s to deliver a suitable estimation response.

**1) Sensors’ fault**

Fig. 10 shows the UKF estimation response of $\Delta L_1 = 0.05$ m considering an imposed fault at $t = 250$ s. The noise effect appears clearly in the estimation response time. The estimation reaches the steady-state after 118 s and the value of the estimation tolerance is around 0.05 m. In this case, the mean value of the calculated error from the estimated signal for 1000 data points, which represents 100 s, is 1 %. The fault is repeated at various values for different tank levels and they have the same response as shown in Table III.

**2) Actuators’ fault**

Similarly, at the same noise condition, an increase in the flow rate $Q_1$ by 50% of the operating point is occurred, the UKF estimates the value of $\Delta q_1$ after 125 s, and the estimation tolerance is around the added value as shown in Fig. 11. The percentage of the mean value of the calculated error from the estimated signal for 1000 data points is 7 %. The fault is repeated for flow rate $Q_2$ with different values of $\Delta q_2$, and the estimation of the UKF shows the same response as shown in Table IV.
TABLE IV. UKF ESTIMATED $\Delta q$ VALUES

| Case | Reference $\Delta q_1$ value ($m^3/s$) | Estimated $\Delta q_1$ ($m^3/s$) | Reference $\Delta q_2$ value ($m^3/s$) | Estimated $\Delta q_2$ ($m^3/s$) |
|------|--------------------------------------|----------------------------------|--------------------------------------|----------------------------------|
| 1    | $10\% \times Q_1$ ($3.5 \times 10^{-6}$) | $4.7 \times 10^{-6}$ | $10\% \times Q_2$ ($3.75 \times 10^{-6}$) | $5.02 \times 10^{-6}$ |
| 2    | $20\% \times Q_1$ ($7 \times 10^{-6}$) | $8.212 \times 10^{-6}$ | $20\% \times Q_2$ ($7.5 \times 10^{-6}$) | $8.0 \times 10^{-6}$ |
| 3    | $30\% \times Q_1$ ($10.5 \times 10^{-6}$) | $11.71 \times 10^{-6}$ | $30\% \times Q_2$ ($11.2 \times 10^{-6}$) | $12.6 \times 10^{-6}$ |
| 4    | $40\% \times Q_1$ ($14 \times 10^{-6}$) | $15.21 \times 10^{-6}$ | $40\% \times Q_2$ ($15 \times 10^{-6}$) | $16.3 \times 10^{-6}$ |
| 5    | $50\% \times Q_1$ ($17.5 \times 10^{-6}$) | $18.71 \times 10^{-6}$ | $50\% \times Q_2$ ($18.7 \times 10^{-6}$) | $20 \times 10^{-6}$ |

3) The weighting matrices tuning

The change in the weighting matrices, $Q_k$ and $R_k$, affect both the estimation time response and the percentage of the estimation error. Fig. 12 shows the effect of changing the evaluation time constant $\beta$ in the estimation response. It is noticed that the increase in $\beta$ of the estimated parameters is followed by a decrease in the parameters estimation error percentage as shown in Fig. 13 and Fig. 14. However, the estimation time response increases with the increase of $\beta$ as illustrated in Fig. 15. The estimation error and time in the case of sensors’ fault is less than the actuators’ fault case due to the difference in the values of the estimated parameters.

C) UKF estimation response with multi-operating points

To assure the successful operation of the studied technique in various operating conditions, the first tank level reference is step changed every 500 s without noise in the presence of a sensor false reading of $\Delta L_1 = -0.03$ m applied at $t = 250$ s as shown in Fig. 16a. The UKF shows constant estimation response with the change of the operating point as shown in Fig. 16b. Likewise, the UKF has a constant estimation response for the level and flow rate difference in the case of the first pump fault of $\Delta q_1 = 17.5 \times 10^{-6}$ as illustrated in Fig. 17.
In this paper, the UKF technique has been presented as a fault diagnosis tool for industrial sensors and actuators faults. The three-tank system model has been used as a case study. The mathematical state-space model of the system has been introduced to be utilized by the UKF Technique. The sensor faults have been represented as a difference between the actual reading and the measured sensor reading \( \Delta r_m \) of the level sensors, while the actuator faults have been represented as a difference between the expected and the real flow rate \( \Delta q_m \) controlled by the system pumps. The technique has been employed for the fault diagnosis with and without system noise. In the noise-free condition, the UKF technique estimated the \( \Delta L_A \) and \( \Delta q_m \) accurately without noticeable error in all working conditions and fault values. Also, the estimation response was tuned to reach the steady-state after 4 s in the case of sensor fault and 8 s for actuator fault. However, the parameter estimation error increased with higher system noise. Thus, the weighting matrices, \( Q_k \) and \( R_k \), should be tuned to give a suitable estimation response. The evaluation time constant of the estimated parameters \( \beta \) has been used to tune the value of the \( Q_k \) matrix. Consequently, the estimation response accuracy and speed were adapted according to the expected measurement noise of the system. The estimation response in the presence of noise reached the steady-state after the tune the value of the noise. Thus, the weighting matrices, \( Q_k \) and \( R_k \), should be tuned to give a suitable estimation response. The evaluation time constant of the estimated parameters \( \beta \) has been used to tune the value of the \( Q_k \) matrix. Consequently, the estimation response accuracy and speed were adapted according to the expected measurement noise of the system.

V. CONCLUSION

In this paper, the UKF technique has been presented as a fault diagnosis tool for industrial sensors and actuators faults. The three-tank system model has been used as a case study. The mathematical state-space model of the system has been introduced to be utilized by the UKF Technique. The sensor faults have been represented as a difference between the actual reading and the measured sensor reading \( \Delta r_m \) of the level sensors, while the actuator faults have been represented as a difference between the expected and the real flow rate \( \Delta q_m \) controlled by the system pumps. The technique has been employed for the fault diagnosis with and without system noise. In the noise-free condition, the UKF technique estimated the \( \Delta L_A \) and \( \Delta q_m \) accurately without noticeable error in all working conditions and fault values. Also, the estimation response was tuned to reach the steady-state after 4 s in the case of sensor fault and 8 s for actuator fault. However, the parameter estimation error increased with higher system noise. Thus, the weighting matrices, \( Q_k \) and \( R_k \), should be tuned to give a suitable estimation response. The evaluation time constant of the estimated parameters \( \beta \) has been used to tune the value of the \( Q_k \) matrix. Consequently, the estimation response accuracy and speed were adapted according to the expected measurement noise of the system. The estimation response in the presence of noise reached the steady-state after the tune the value of the noise. Thus, the weighting matrices, \( Q_k \) and \( R_k \), should be tuned to give a suitable estimation response. The evaluation time constant of the estimated parameters \( \beta \) has been used to tune the value of the \( Q_k \) matrix. Consequently, the estimation response accuracy and speed were adapted according to the expected measurement noise of the system.