Optimization model of the hub airport schedule under uncertainty

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Abstract. On the basis of a combination of fuzzy-multiple and theoretical probabilistic approaches, a model has been developed for optimizing the timetable for the movement of aircraft at an airport operating in the framework of the air transport system “hub & spoke”. The model can be integrated into mathematical and software environment designed to support decision-making at the stages of the formation and reconstruction of the hub & spoke system. The source of the initial data of the model is the databases of information production systems of airports and expert assessments. The purpose of optimizing the schedule is to reduce the lost profits of the hub-forming airline, arising from the failure of its services to those passengers who are potentially ready to be transported with a change in a hub, who consider the duration of the transfer to be unacceptable. Optimization consists in determining, on a given time interval, the points of time of arrival and departure minimizing lost profits due to the preservation of the potential transfer passenger traffic by ensuring a comfortable time of transfer. Restrictions on time and resource parameters of ground service processes are taken into account. The presence of fuzzy values in the composition of initial data is explained by the use of expert assessments and the difficult formalization of the subjective opinions of passengers with respect to the time spent in the hub. The stochastic nature of ground handling processes in a hub makes it necessary to use random variables in the model. The task with a fuzzy objective function and probabilistic constraints is reduced to the problem of mathematical programming. A description of the model example results is described. The possibility of a significant increase in income from transfer traffic is shown only by optimizing the schedule.

1. Introduction
As part of the hub & spoke air transport system, the hub airport is used by the hub-forming airline as a point of mass transfer for passengers transported between the surrounding hub peripheral airports. To ensure a comfortable time for transfer passengers in the hub, the movement of the aircraft is given the character of a sequence of waves. Each wave begins with a massive arrival of aircraft into the hub, followed by transfer passengers on connecting flights with the final wave in the same mass departure from the hub. The article deals with the task of optimizing the schedule within a single wave, which consists in determining the arrival and departure points of time that deliver the minimum amount of loss to the hub-forming airline caused by the refusals to transport potential transfer passengers who find the stay in the hub unacceptable. Potential passengers are those who have the need and ability to travel with a transfer in the hub at the established fare. Restrictions on the capacity of the airfield and
production complexes of the hub, affecting the duration of ground service of aircraft, passengers, baggage, and not allowing to set the time of arrival and departure are taken into account.

The solution of the optimization problem is complicated by the presence of uncertainty in the source data, due to the stochasticity of both the air transport market and airport activities. The parameters of the ground service process can usually be described in terms of probability theory. To set their frequency distributions, statistical data can be used that are accumulated in computer databases of airport production information systems. The situation is different with the values of passenger traffic and tariffs affecting the value of the objective function, which at the scheduling stage, preceding the stage of transport operations, are not reliably known.

Statistical data necessary for determining distribution rules may be absent, for example, if a company opens a new airline or establishes a new air link through the hub, which will make expert assessment the only possible method for determining such values. The degree of attractiveness of the time spent in the hub from the point of view of the transfer passenger, which affects the level of passenger traffic, is also indefinite and difficult to formalize. In most cases, this value is also determined by experts. So, since in the initial data of the problem there are two types of uncertainty - fuzziness inherent in the determined by the expert price indicators and production results of the airline, and randomness inherent in the statistically determined technological parameters, the proposed method of solving the problem assumes parallel execution of both fuzzy calculations and numerical operations over random variables.

In contrast to some papers [1–4] devoted to the formation of an optimal schedule for one selected type of airport resources or operation, this work aims to optimize the hub aircraft’s flight schedules using a network schedule that links all the main ground service operations and taking into account the preferences of passengers about the transfer time. The approach to the solution is outlined in [5], where a similar problem was considered on the basis of an extremely simplified schedule of ground handling of aircraft, while ignoring the limitations on the technical capabilities of hub resources. In paper [5], as in the present work, the optimization problem with a fuzzy criterion and probabilistic constraints is reduced to a mathematical programming problem. The methods of the theory of fuzzy sets are not yet widely used in solving air transport problems. In this regard, such works as [6, 7] have to be mentioned, in the first of which the task of fuzzy optimization of aerodrome capacity was solved, in the second, a model of operational control of the airport production process using fuzzy information was described.

2. Optimization method
A time interval of duration $T$ («wave») in considered, during which $K$ number of aircraft arrive at the hub, undergo maintenance and take off. Let it be possible to set the time of arrival $t_{ij} \in T$ and departure $t_{ij} \in T$ k-th ($k = 1, ..., K$) of an aircraft. Let us denote a couple of aircraft as $ij$, the first of which $i$, arrives at the hub, and the second $j$ takes off from the hub $(i, j \in \{1, ..., K\})$. We will assume that for all couples $ij$ expert forecasts of the number of potential transfer passengers $\bar{v}_{ij}$ and tariffs $\bar{c}_{ij}$ are given in fuzzy form. To take into account possible refusals to transport those potential transfer passengers who are not satisfied with the length of stay in the hub, we introduce a fuzzy value $\tilde{w}_{ij}$ expressing the number of transfer passengers ready to change from $i$ to $j$, despite the need to wait in the hub. Conventionally, we will call these passengers "valid" and we will take their number depending on the time spent in the hub $\Delta t_{ij}$. Taking into account the obvious connection

$$\Delta t_{ij} = t_{ij} - t_{ij}, \quad i, j = 1, ..., K,$$

the value $\tilde{w}_{ij}$ should be considered as a function of two variables: $\tilde{w}_{ij} = \tilde{w}_{ij}(t_{ij}, t_{ij})$. The outflow of potential transfer passengers from flights of the hub-forming airline will lead to the formation of lost profits, expressed by a fuzzy value:
\[ \tilde{D} = \sum_{i=1}^{K} \sum_{j=1}^{K} C_{ij}^{T} \left[ \tilde{w}_{ij}^T - \tilde{w}_{ij}^T (t_i^*, t_j^*) \right]. \]

The criterion for the optimization problem is the minimum to which the optimal values of the elements of the vectors \( t' = (t'_1, ..., t'_k) \) and \( t'' = (t''_1, ..., t''_k) \) correspond. The following restrictions are considered:

- time intervals between take-off and landing operations (TLO) at the aerodrome, which should not be less than the specified minimum \( \Delta t^{\text{min}} \):

\[
\Delta t^{\text{min}} \leq \left| t_{n}^{\text{TLO}} - t_{m}^{\text{TLO}} \right|, \quad \forall t_{n}^{\text{TLO}}, t_{m}^{\text{TLO}} \in t', \cup t'', \quad t_{n}^{\text{TLO}} \neq t_{m}^{\text{TLO}},
\]

where \( t_{n}^{\text{TLO}}, t_{m}^{\text{TLO}} \) = any two points of TLO;

- the time of aircraft departure, which cannot be earlier than the scheduled (and therefore not accidental) time \( t_{k}^{d} \) of the end of its preparation for departure:

\[
t_{k}^{d} \geq t_{k}^{d}, \quad k = 1, ..., K;
\]

- the required number of hub resources, which at any time during the wave should not be greater than the one at the airport:

\[
N_{s}(t) \leq n_{i}^{e}, \quad h = 1, ..., H, \quad \forall t \in T,
\]

where \( H \) = number of hub technology resource types, \( N_{s}(t) \) and \( n_{i}^{e} \) = accordingly, the required at the moment \( t \) and the given number of resources of the \( h \)-th type that is available to the hub. The value \( N_{s}(t) \) is assumed to be random due to the stochasticity of the technological processes of the hub.

The minimization problem \( \tilde{D} \) with constraints (2) and (3) is not definite, because its objective function contains fuzzy variables, and constraint (3) contains random variables. To get rid of the uncertainty in the formulation of the problem, we use the value

\[
D = \text{defuz}[\tilde{D}],
\]

where \( \text{defuz}[\cdot] = \) fuzzy defuzzification operator. Restriction (3) will be presented in the form:

\[
P\{N_{s}(t) \leq n_{i}^{e}\} \geq P^{\rho}, \quad h = 1, ..., H, \quad \forall t \in T,
\]

where \( P\{N_{s}(t) \leq n_{i}^{e}\} \) = probability \( N_{s}(t) \leq n_{i}^{e} \) fulfilment, which should be not lower than a specified probability \( P^{\rho} \).

Let us define the parameters of the objective function \( \tilde{w}_{ij}^T \) and restrictions \( t_{k}^{d} \) and \( P\{N_{s}(t) \leq n_{i}^{e}\} \). In order to establish the dependence \( \tilde{w}_{ij}^T (t'_i, t'_j) \), we will introduce the fuzzy interval \( \Delta T^T \), which will be interpreted as “a comfortable time of stay of the transfer passenger in the hub”. Basing on the results of the analysis of passengers’ subjective preferences in relation to \( \Delta T^T \) [8], we will assume that function \( \mu_{\Delta T^T}(\Delta T) \) of the fuzzy interval \( \Delta T^T \) relates to trapezoidal type [9] and has four reference points \( 0 < t^{(1)} \leq t^{(2)} \leq t^{(3)} \leq t^{(4)} \). Pairs of aircraft with values \( \Delta T^T < t^{(1)} \) and \( \Delta T^T > t^{(4)} \) are not considered by passengers as possible connections due to too little or unacceptably long time spent on the transfer. The most comfortable transfer corresponds to the period \( [t^{(2)}, t^{(3)}] \) in which none of the potential passengers will refuse to fly. In the interval \( [t^{(1)}, t^{(2)}] \) the number of people willing to make the trip increases linearly, and in the interval \( [t^{(3)}, t^{(4)}] \) it decreases linearly. \( \mu_{\Delta T^T}(\Delta T) \) function character lets us to use it to determine \( \tilde{w}_{ij}^T \) using \( \tilde{v}_{ij}^T, t_{i}^* \) and \( t_{j}^* \) as:

\[
\tilde{w}_{ij}^T = \tilde{v}_{ij}^T \cdot \mu_{\Delta T^T}(\Delta T^T) = \tilde{v}_{ij}^T \cdot \mu_{\Delta T^T}(t_{i}^* - t_{j}^*).
\]
To determine \( t_{k}^{in} \) network schedule ground maintenance aircraft is used, establishing logical links between key technological operations, the duration of which combined into vectors \( \tau_{k} \), are assumed to be given for all \( k = 1, \ldots, K \). The time of plane expectation for transfer passengers leads to the establishment of interconnections between the network schedules of various planes. To calculate it two events in the network schedule of the plane are distinguished. The first event is the completion at the time \( t_{k}^{in} \) of the service of passengers transferring from the \( k \)-th plane at the moment of arrival. The second marks the beginning of the service of passengers transferring to the \( k \)-th aircraft, before its departure. The time of accomplishment of the second event \( t_{k}^{in} \), \( i = 1, \ldots, K \), of the network schedules of those planes from which passengers are transferred to the \( k \)-th plane:

\[
t_{k}^{in} = \max_{i} \left( t_{i}^{in} \right).
\]

Expression (6) is used in the composition of the developed calculation methodology, which can be represented as operators \( t^{in} (\cdot) \) and \( t^{r} (\cdot) \) establishing dependencies:

\[
t_{k}^{in} = t_{k}^{in} \left( t_{k}^{in}, \tau_{k} \right), \quad t_{k}^{r} = t^{r} \left( t_{k}^{in}, \tau_{k}, t_{k}^{in} \right), \quad k = 1, \ldots, K.
\]

The calculations \( P \{ N_{h} (t) \leq n_{h}^{*} \} \) take into account that the cross section \( N_{h} (t) \) of a random process at time \( t \) is the following sum of cross sections of random processes:

\[
N_{h} (t) = \sum_{k=1}^{K} n_{h} (t), \quad h = 1, \ldots, H,
\]

where \( n_{h} (t) \) is the number of technological resources of \( h \)-th type, required at the moment of time \( t \in T \) for servicing the \( k \)-th aircraft, defined as

\[
n_{h} (t) = \begin{cases} N_{h} (t), & \forall t \in T_{h}, \\ 0, & \forall t \not\in T_{h}, \end{cases} \quad h = 1, \ldots, H, \quad k = 1, \ldots, K,
\]

where \( N_{h} \) is the number of resources of \( h \) type, allocated by hub control services for \( k \)-th aircraft type; \( T_{h} \subseteq T \) is time (or many time periods), during which the resources of \( h \) type are used for servicing \( k \)-th aircraft. Probability estimates \( P \{ N_{h} = n_{h} \} \), with which \( n_{h} \) takes its possible values \( n_{h} = n_{h}^{*} \), are determined according to the hub statistics. To determine the boundaries of the intervals \( T_{h} \subseteq T \) the networks of ground handling of aircraft are used in conjunction with the model (6), (7). Taking into account (9), estimates of the probabilities \( P \{ N_{h} (t) = n_{h} \} \) with which \( n_{h} \) takes possible values \( n_{h} = 0, \ldots, n_{h}^{*} \) are easily determined for \( \forall t \in T \) by the known известным \( P \{ N_{h} = n_{h} \} \). The estimate of the probability \( P \{ N_{h} (t) = n_{h} \} \) of the sum (8) is calculated using the well-known formula for the numerical addition of random variables [10]:

\[
P \{ N_{h} (t) = n_{h} \} = \sum_{h_{1}} \sum_{n_{1}} P \left( \prod_{t=1}^{K} \{ n_{h} (t) = n_{h} \} \right), \quad n_{h} = 0, \ldots, n_{h}^{*}.
\]

As a result, the desired probability \( P \{ N_{h} (t) \leq n_{h}^{*} \} \) is determined as the sum:

\[
P \{ N_{h} (t) \leq n_{h}^{*} \} = \sum_{n_{h} = 0}^{n_{h}^{*}} P \{ N_{h} (t) = n_{h} \}.
\]
The use of the described calculation methods allowed reducing the task of optimizing the hub schedule to an uncertainty-free mathematical programming problem, which involves defining such $t^*$ and $t^f$ that ensure the minimum of the objective function (4) and satisfy constraints (1), (2) and (5). The solution discussed below was derived from the IBM ILOG OPL software.

3. Model example
The wave of arrivals and departures, formed by 15 aircraft of two types, is considered. A ground service schedule was used, which includes 16 operations for which resources of 13 types of various purposes are attracted. The duration of the critical path of the schedules of the aircraft was 76 minutes and 109 minutes by type. Expert estimates of values and are given by triangular fuzzy numbers. Due to the limited volume of the article table 1 shows only modal fuzzy values $\tilde{v}_{ij}^T$ for the pairs $ij$ for which interest from potential passengers is predicted. The values of the other initial parameters are taken as follows: $t^{(1)} = 45$ min., $t^{(2)} = 75$ min., $t^{(3)} = 90$ min., $t^{(4)} = 360$ min., $\Delta t^{\min} = 2$ min., $T = 180$ min., $P\{N_i(t)\leq n^*_i\} = 0.95$.

Table 1. Modal values $\tilde{v}_{ij}^T$.

|   | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 1 | -  | 4  | -  | -  | -  | -  | -  | 30 | 16 | 7  | 5  | -  | -  | -  | -  |
| 2 | 5  | -  | -  | 3  | -  | -  | -  | 4  | -  | -  | -  | 9  | -  | 7  | -  |
| 3 | -  | -  | -  | -  | -  | -  | -  | 14 | -  | -  | 18 | 14 | -  | -  | -  |
| 4 | -  | 3  | -  | -  | 10 | -  | -  | -  | -  | -  | -  | -  | -  | -  | -  |
| 5 | -  | -  | -  | 22 | -  | -  | -  | 13 | -  | -  | 18 | 7  | -  | -  | -  |
| 6 | -  | 6  | -  | -  | -  | -  | 13 | -  | -  | 12 | 15 | -  | -  | 8  | -  |
| 7 | -  | -  | -  | -  | -  | -  | 9  | -  | -  | 10 | -  | 12 | -  | -  | -  |
| 8 | 11 | 3  | -  | -  | -  | -  | -  | -  | 5  | -  | -  | -  | -  | -  | -  |
| 9 | 12 | -  | 14 | -  | 13 | -  | 8  | -  | 7  | 20 | -  | 12 | -  | -  | -  |
| 10| 8  | 2  | -  | -  | 8  | -  | -  | 5  | -  | 7  | 7  | -  | -  | -  | -  |
| 11| -  | -  | -  | 5  | -  | 22 | -  | -  | 7  | -  | 8  | 5  | -  | -  | -  |
| 12| -  | 30 | -  | 27 | -  | 11 | -  | -  | 23 | 22 | -  | 9  | -  | -  | -  |
| 13| -  | -  | -  | 6  | -  | 7  | -  | -  | 15 | 5  | -  | -  | -  | 34 | -  |
| 14| -  | -  | -  | -  | 14 | -  | 16 | -  | 12 | 19 | -  | -  | -  | -  | -  |
| 15| -  | 38 | -  | -  | -  | -  | -  | -  | -  | -  | -  | -  | -  | -  | -  |

The optimal schedule is shown in figure 1, where the length and offset along the horizontal axis of the rectangles reflect the temporal characteristics of the aircraft in the hub, and the signatures mean arrival time (from left to right), duration of stay in the hub and departure time of the aircraft. The schedule clearly reflects the group nature of the movement of aircraft. The later arrival of aircraft 4 and 5 is due to the insufficient number of resources that do not allow all 15 aircraft to be serviced simultaneously. Optimization of the schedule allowed, in general, to save potentially possible revenues from the transportation of transfer passengers. Thus, the de-optimized sum of revenues corresponding to the optimal schedule is only 6% less than the similar value obtained for the hypothetical transportation of all potential passengers. Evaluation of the effect of optimization allowed modeling for a number of source data sets that simulate the schedule of the above aircraft at a non-hub airport. In all cases, the loss of profit is 1.5-3 times higher than the optimal value.
Figure 1. The optimal schedule.

4. Conclusion
The model is of interest for small hubs, where airlines can schedule a favorable time for arrivals and departures of aircraft. In this case, the hub-forming airline has the opportunity to increase its own revenues only by optimizing the schedule of the hub without any financial costs. With the use of modern personal computing, the solution is achieved in a practically acceptable time, which makes the developed methodology suitable for inclusion into the software for decision-making support when designing the hub & spoke system.

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