Theory of Network Wave (On a Primary Path)

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Abstract: Aiming at the disorder problem (i.e. uncertainty problem) of the utilization of network resources commonly existing in multi-hop transmission networks, the paper proposes the idea and the corresponding supporting theory, i.e. theory of network wave, by constructing volatility information transmission mechanism between the sending nodes of a primary path, so as to improve the orderliness of the utilization of network resources. It is proved that the maximum asymptotic throughput of a primary path depends on its intrinsic period, which in itself is equal to the intrinsic interference intensity of a primary path. Based on the proposed theory of network wave, an algorithm for the transmission of information blocks based on the intrinsic period of a primary path is proposed, which can maximize the asymptotic throughput of the primary path. The research results of the paper lay an ideological and theoretical foundation for further exploring more general methods that can improve the orderly utilization of network resources.

I. Introduction

As we all know, in today’s networks with a certain scale (including wired networks and wireless networks), the multi-hop transmission mode based on the idea of “store and forward” is mainly used to transmit information from one end of the network to the other end (in fact, the idea of “store and forward” constitutes the core of packet switching technology [1]). When implementing multi-hop transmissions based on the idea of “store and forward”, the common practice is that an information forwarding node (commonly known as “a router”) schedules its access to network resources according to the distribution of its local resources and the dynamic requirements of
network traffic. The advantage of this method for accessing network resources is that it can help to reduce the design and implementation complexity of network protocols as much as possible. However, the problem it brings about is also prominent, that is, because each node only determines the usage of network resources according to its own needs, which can be compared to the way that each node uses shared network resources from the starting point of “myopia” and “selfishness” (Note: as a matter of fact, the multiple access protocol of nodes in a multi-hop network operates based on this basic idea). This will inevitably lead to the disorderly usage of network resources (Note: the “disorder” here actually refers to “uncertainty”, which can be measured by the concept of entropy proposed in information theory [2]). And the disorderly usage of network resources leads to two problems: first, the availability of network resources used to transmit network traffic presents great uncertainty, which makes it difficult to ensure the Quality of Services (QoS) guarantee; second, due to the possible disorderly competition for network resources among different nodes, collisions of resource occupations occur from time to time, which further reduces the utilization of network resources.

Based on the starting point of “myopia” and “selfishness”, with the continuous expansion of the network scale (i.e. the continuous increase of the number of nodes accessing into the network), the uncertainty of the availability of network resources will also increase. In other words, with the continuous expansion of the network scale, the disorderly usage of network resources will be further...
amplified. Consider the example shown in Fig. 1, in the subfigure (a), node 1 sends three packets, and to some extent the time intervals between the three packets present some uncertainty (i.e. disorder). It is particularly noteworthy that due to the uncertainty of the intervals of these transmissions, the remaining idle access resources also present some uncertainty. And then, another node, i.e. node 2, can only, by following the starting point of “myopia” and “selfishness”, access into the remaining resources which have already shown a certain degree of uncertainty (Note: in order to avoid mutual interference, the accesses and transmissions of node 1 and node 2 must be staggered in time), and sends its two packets. As can be seen from the subfigure (b), the access resources occupied by nodes 1 and 2 show greater uncertainty.

With the help of the concepts of entropy and conditional entropy defined in information theory [2], it is easy to explain why the disorderly usage of network resources is gradually amplified. Suppose that the three transmission time instances, which can be regarded as three random variables, corresponding to the three packets sent by node 1 in Fig. 1 are \( T_1, T_2, T_3 \), respectively. And the two transmission time instances corresponding to the two packets sent by node 2 are \( T_4, T_5 \) (which are also regarded as random variables), respectively. The following relationship can be obtained easily according to the information theory:

\[
H(T_1, T_2, T_3, T_4, T_5) = H(T_1, T_2, T_3) + H(T_4, T_5 | T_1, T_2, T_3) \\
\geq H(T_1, T_2, T_3)
\]

(1)

Among them, \( H(T_1, T_2, T_3) \) is the joint entropy of the transmission time instances corresponding to the three packets sent by node 1, i.e. the uncertainty presented by node 1 with its transmissions. And, \( H(T_4, T_5 | T_1, T_2, T_3) \) denotes the conditional entropy corresponding to the other two packets sent by node 2 on the premise that the access time instances of node 1 are known (that is, the uncertainty of the accesses of node 2 with the transmissions of node 1 being given). \( H(T_1, T_2, T_3, T_4, T_5) \) is the joint entropy of the random variables corresponding to the five transmission time instances of both node 1 and node 2, which measures the uncertainty of the usage of the network resources with the transmissions of the two nodes. It can be seen from equation (1), with the further accesses of node 2, the occupation of network resources shows greater uncertainty (i.e. \( H(T_1, T_2, T_3, T_4, T_5) \geq H(T_1, T_2, T_1) \)). What is worse, in addition to the gradual “amplification” of the above disorder, this kind of disorder can be rapidly spreaded throughout the network with the interactions between adjacent nodes (just like the spread of viruses!). In short, the access mode
based on the starting point of “myopia” and “selfishness” makes the disorderly usage of network resources to be continuously amplified and spreaded out in the whole network.

In the paper, the idea of introducing volatility into the network access mechanism is proposed, which reduces the disorder of the usage of network resources and accordingly improves the orderliness of the usage of network resources. Therefore, network throughput and performance for QoS guarantee can be improved. The contributions of the paper can be summarized as follows:

1) The idea of volatility is introduced into the access mechanism of multi-hop networks;
2) For a single multi-hop transmission path (i.e. a primary path which is defined in the paper), the corresponding basic theoretical framework of network wave is constructed. It is proved that the maximum asymptotic throughput of a primary path depends on its intrinsic period, and the intrinsic period of a primary path is equal to its intrinsic interference intensity;
3) An algorithm for the transmission of information blocks based on the intrinsic period of a primary path is proposed, which can maximize the asymptotic throughput of a primary path.

The paper is organized into eight parts: in Section I, the disorder of the usage of network resources in the current access mechanisms of the multi-hop networks is pointed out. In Section II, the basic idea of the paper is proposed, that is, volatility is introduced into the access mechanism of a multi-hop network. In Section III, a primary path and some related basic concepts are defined. In Section IV, the relationships of concurrency and interference between nodes of a primary path are defined. In Section V, the concepts of the intrinsic concurrency intensity and the intrinsic interference intensity of a primary path are defined, and the related basic properties are analyzed and proved. In Section VI, the concepts of the subset of equally spaced nodes and reachable period of a primary path are introduced, and the related basic properties are analyzed and proved. In Section VII, the average throughput and the asymptotic throughput of a primary path are defined, and the relevant basic properties are proved. Moreover, an algorithm for the transmission of information blocks based on the intrinsic period of a primary path is proposed. Finally, conclusions are given in Section VIII.

II. Basic idea

In order to reduce the disorderly usage of network resources, it is necessary to increase the certainty for behaviors of nodes accessing into network resources. In the example shown in Fig. 2,
the access behaviors of both node 1 and node 2 show apparent certainty (i.e. notable periodicity). If most of the nodes access into the network by following such kind of notable periodicity, network resources available to be used will inevitably show good certainty (i.e. orderliness).

![Figure 2: Orderliness of the usage of network resources](image)

So, how to achieve the above “attractive” orderliness in networks? There are many possible ways, but the authors’ suggestion is to follow nature! As we all know, wave phenomena occur almost everywhere in nature. We can enumerate many different types of waves, such as water wave, sound wave, light wave, seismic wave, gravitational wave, and even the “material wave” studied in quantum mechanics, and so on. Just as the famous physicist Feynman pointed out in his book “the Feynman structures on Physics (Vol. I)” [3], “…except that wave oscillations appear not only as time-oscillations at one place, but propagate in space as well”. In other words, if one looks along the propagation path of a wave, she will find that the wave is moving forward. Moreover, if a point at a specific position on the propagation path of the wave being observed is fixed, it will be found that the motion of that point shows a strong regularity, that is, the so-called “vibration phenomenon”.

As for information network, which is invented by human beings, its core mission is to transmit information from one end of the network to the other. Therefore, is it possible that we imitate the wave mechanism in nature and build a similar “wave” in the network (let us call it as “network wave”), and, based on that, realize the orderly and efficient transmission of information blocks? Based on the research of the paper, the answer we give is yes. Since the physical world itself operates
in this way, why can’t this mechanism give us some important enlightenments?!

In short, the core idea of building network wave can be summarized as follows: make each forwarding node on a multi-hop path regularly switch back and forth between “forwarding state” and “silence state”. And the state switching behaviors between adjacent nodes on the path are well coordinated in time (that is, in the same time beat, we make some nodes with a certain spatial distance on the path to be in the “forwarding state” and the other nodes to be in the “silence state”). In this way, the network wave can be established, and the information can be transmitted from one end to the other along the multi-hop path. Obviously, with the establishment of network wave, the access behaviors of nodes on the path do have good certainty (refer to the subsequent chapters of this paper for the detailed theory of network wave and the corresponding proposed algorithm).

III. The concept of a primary path and some related definitions

In this section, the concept of a primary path is given. The set and subsets of sending nodes contained in a primary path are defined. The identification method of any sending node in a primary path is given.

Definition 1: A primary path

Consider that in a multi-hop path consisting of \( N+1 \) nodes, information blocks are relayed from the node 1 (i.e. the source node) through the node 2, the node 3,..., the node \( i,... \), and the node \( N (\geq 1) \) to the final node \( N+1 \) (i.e. the destination node) (Note: 1, it is assumed that all the former \( N \) nodes have the function of sending/forwarding information blocks, and the last node \( N+1 \) only has the function of receiving information blocks. Therefore, for convenience and consistency of discussion, the node \( N+1 \) is generally not mentioned in the following discussions unless otherwise specified. 2, for the sake of convenience, it is assumed that the source node always has saturated traffic loads waiting to be transmitted). A multi-hop path, which transmits information blocks from the source node to the destination node, consisting of \( N \) sending nodes (whose sequence numbers are set in turn by using incrementing positive integers) arranged in sequence is called as a primary path, which is denoted as \( P \). In the following part of the paper, for the convenience of description, the node with a larger sequence number is said to be the “downstream node” of the node with a smaller sequence number, and, conversely, the node with a smaller
sequence number is the “upstream node” of the node with a larger sequence number. Without loss of generality, it is assumed that in a primary path the propagation of an information block from one node to its immediate downstream node takes only one time beat, which is denoted as \( \Delta t \).

**Definition 2: The set of sending nodes of a primary path**

The set of nodes, consisting of all the sending nodes in a primary path \( P \), is defined as the set of sending nodes of \( P \), which is denoted as \( S_p \). The number of elements in the set \( S_p \) is denoted as \( N_{S_p} \).

**Definition 3: A subset of sending nodes of a primary path**

A subset of sending nodes of a primary path \( P \) is defined as a subset of \( S_p \), which is denoted as \( S'_p \) (\( S'_p \subseteq S_p \)). The number of elements in the set \( S'_p \) is denoted as \( N_{S'_p} \).

**Definition 4: A specific sending node in a primary path**

Let \( n_i (1 \leq i \leq N_{S_p}) \) denote the node whose sequence number is \( i \) in a primary path \( P \).

**IV. Definitions of the relationships of concurrency and interference between nodes**

In this section, the possible relationships of concurrency and interference between nodes in a primary path are defined. The mutual exclusion rules between the relationships of concurrency and interference are defined. Moreover, the second-order concurrency subset consisting of some sending nodes in a primary path and the related second-order concurrency rules are defined.

**Definition 5: The concurrency relationship between two nodes**

If a node \( n_i (1 \leq i \leq N_{S_p}) \) and a node \( n_j (1 \leq j \leq N_{S_p}) \) (possibly they are the same node) in a primary path \( P \) can be activated within the same time beat (i.e., both of them are in the state of sending information blocks in the same time duration) without interfering with the correct receptions of information blocks of the corresponding receiving nodes, we say that the two nodes can be concurrent within the same time beat, i.e. there is a concurrency relationship between these two.
nodes, which is denoted as \( n_i \parallel n_j \). Specifically, it is defined that any single sending node has a concurrency relationship with itself; i.e. \( n_i \parallel n_i \).

**Definition 6: The interference relationship between two nodes**

If two different nodes \( n_i \ (1 \leq i \leq N_s) \) and \( n_j \ (1 \leq j \leq N_s) \) in a primary path \( P \) (Note: saying that two nodes are different means \( i \neq j \), which is also denoted as \( n_i \neq n_j \)) are activated within the same time beat, which brings about the interference with the correct reception of at least one of the corresponding receiving nodes, we say that the two nodes cannot be concurrent within the same time beat, i.e. there is an interference relationship existing between the two sending nodes, which is denoted as \( n_i \succ n_j \).

**Rule 1: The mutual exclusion rules between the relationships of concurrency and interference**

1) If two different nodes \( n_i \) and \( n_j \) in a primary path \( P \) have an interference relationship, i.e. \( n_i \succ n_j \) holds, then \( n_i \parallel n_j \) must not hold;

2) If two nodes \( n_i \) and \( n_j \) (possibly they are the same node) in a primary path \( P \) have a concurrency relationship, i.e. \( n_i \parallel n_j \) holds, then \( n_i \succ n_j \) must not hold.

**Definition 7: A second-order concurrency subset**

If there is a concurrency relationship between any two nodes in a nonempty subset \( S'_P \) of sending nodes in a primary path \( P \), i.e. \( n_i \parallel n_j \ (n_i, n_j \in S'_P) \), then \( S'_P \) is defined as a second-order concurrency subset of \( P \). In particular, a subset \( S'_P \) containing only one sending node is also regarded as a second-order concurrency subset.

**Rule 2: The second-order concurrency rule**

It is assumed that for any second-order concurrency subset of nodes concurrent transmissions with all the member nodes in the set participating in it at the same time beat can be achieved (i.e. no interference with each other), which is regarded as the second-order concurrency rule. (Note:
this rule is an approximation of higher-order concurrency relationships, and is essentially equivalent to the widely used “DISK” model for inter-link interferences).

V. Concepts of the intrinsic concurrency intensity and the intrinsic interference intensity and some related properties

In this section, the concepts of the intrinsic concurrency intensity for a subset of nodes in a primary path, the intrinsic interference intensity for a subset of nodes in a primary path, the intrinsic concurrency connection degree of a node and the intrinsic interference connection degree of a node are proposed. The maximum second-order concurrency subset, the second-order interference subset, the maximum second-order interference subset, the subset without interference, the subset with interference, and the subset of nodes with dominant intrinsic interference intensity are defined, respectively. The necessary and sufficient conditions for a nonempty subset of nodes to be a second-order concurrency subset are given (Theorem 1). The concurrency rules between nodes in a primary path are defined. Both the structural property of a second-order interference subset in a primary path (Theorem 2) and the continuity of a maximum second-order interference subset (Corollary 2-1) are proved. The splitting characteristics of a subset of nodes with dominant intrinsic interference intensity are proved (Theorem 3). Moreover, the relationship between the intrinsic interference intensity and the intrinsic concurrency intensity of a subset of nodes with dominant intrinsic interference intensity is given (Corollary 3-1).

Definition 8: The maximum second-order concurrency subset of a nonempty subset of nodes

In all of the second-order concurrency subsets contained in a nonempty subset $S'_p$ of nodes in a primary path $P$, the second-order concurrency subset with the largest number of elements is defined as the maximum second-order concurrency subset of $S'_p$. (Note: there may be more than one such second-order concurrency subset).

Definition 9: The intrinsic concurrency intensity of a nonempty subset of nodes

The number of elements contained in a maximum second-order concurrency subset of a nonempty subset $S'_p$ of nodes in a primary path $P$ is defined as the intrinsic concurrency
intensity of the set, which is denoted as $C'_{PS}$.

**Definition 10: The intrinsic concurrency intensity of a primary path**

The intrinsic concurrency intensity of the set $S_p$ composed of all the sending nodes of a primary path $P$ is defined as the intrinsic concurrency intensity of the primary path $P$, which is denoted as $C'_{S_p}$.

**Definition 11: The concurrency connection degree of a node**

As for a sending node $n_i \in S_p$, the number of sending nodes (excluding the node $n_i$ itself) in a nonempty subset $S'_p$ of nodes in a primary path $P$ having concurrency relationships with it is defined as the concurrency connection degree of the node $n_i$ in $S'_p$, which is denoted as $D_{S'_p:n_i}$.

**Definition 12: The intrinsic concurrency connection degree of nodes in a nonempty subset of nodes**

The maximum value among the concurrency connection degrees of all the sending nodes $n_i \in S'_p$ in a nonempty subset $S'_p$ of nodes in a primary path $P$ is defined as the intrinsic concurrency connection degree of nodes in $S'_p$, which is denoted as $D'_{S'_p}$.

**Definition 13: The intrinsic concurrency connection degree of nodes in a primary path**

The maximum value among the concurrency connection degrees of all the sending nodes $n_i \in S_p$ in the set $S_p$ of nodes in a primary path $P$ is defined as the intrinsic concurrency connection degree of nodes in $P$, which is denoted as $D'_{S_p}$.

**Definition 14: The second-order interference subset**

If any two different nodes in a nonempty subset $S'_p$ of nodes in a primary path $P$ have
interference relationship, i.e. \( n_i > n_j \) \( (n_i, n_j \in S'_p, n_i \neq n_j) \) holds, then the subset \( S'_p \) is defined as a second-order interference subset of \( P \) (Note: a second-order interference subset contains at least two elements).

**Definition 15: The subset without interference and the subset with interference**

If a nonempty subset \( S'_p \) of nodes in a primary path \( P \) does not contain any second-order interference subset, it is defined as a subset without interference, otherwise it is defined as a subset with interference.

**Lemma 1-1: The subset without interference forms a second-order concurrency subset**

If a nonempty subset \( S'_p \) of nodes is a subset without interference, all the sending nodes in it form a second-order concurrency subset.

**Proof:**

**Case 1:** Number of sending nodes in \( S'_p \) \( \geq \) 2

Here, reduction to absurdity is adopted. Assuming that all the sending nodes in \( S'_p \) cannot form a second-order concurrency subset, it is evident that at least two of these sending nodes must have interference relationship with each other. Therefore, these two nodes form a second-order interference subset, which is contrary to the proposition.

**Case 2:** Number of sending nodes in \( S'_p \) = 1

According to the definition of a second-order concurrency subset (Definition 7), a subset containing only one sending node forms a second-order concurrency subset.\( \blacksquare \)

**Lemma 1-2: A second-order concurrency subset is a subset without interference**

If a nonempty subset \( S'_p \) of nodes is a second-order concurrency subset, it is also a subset without interference.

**Proof:**

It can be directly proved based on the mutual exclusion rules between the relationships of concurrency and interference (Rule 1), the definition of a second-order concurrency subset (Definition 7) and the definition of a subset without interference (Definition 15).\( \blacksquare \)
Theorem 1: The necessary and sufficient condition for a nonempty subset to be a second-order concurrency subset

The necessary and sufficient condition for a nonempty node subset \( S'_P \) to be a second-order concurrency subset is that the subset is a subset without interference.

Proof:

It can be proved based on Lemma 1-1 and Lemma 1-2. ■

Definition 16: The maximum second-order interference subset of a subset with interference

Among all the second-order interference subsets contained in a subset \( S'_P \) with interference in a primary path \( P \), the subset with the maximum number of elements is defined as a maximum second-order interference subset of \( S'_P \). (Note: there may be more than one such second-order interference subset).

Definition 17: The intrinsic interference intensity of a nonempty subset of nodes

If a nonempty subset \( S'_P \) of nodes in a primary path \( P \) is a subset with interference, the number of elements contained in its maximum second-order interference subset is defined as its intrinsic interference intensity, which is denoted as \( I^*_s \). Moreover, if \( S'_P \) is a subset without interference, its intrinsic interference intensity is defined as \( I^*_s \equiv 1 \).

Definition 18: The intrinsic interference intensity of a primary path

The intrinsic interference intensity of the set \( S_p \) composed of all the sending nodes of a primary path \( P \) is defined as the intrinsic interference intensity of \( P \), which is denoted as \( I^*_s \).

Definition 19: The interference connection degree of a node

As for a sending node \( n_i (\in S_p) \), the number of sending nodes in a nonempty subset \( S'_P \) of nodes in a primary path \( P \) having interference relationships with it is defined as the interference connection degree of the node \( n_i \) in \( S'_P \), which is denoted as \( D_{i, >a} \).

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Definition 20: The intrinsic interference connection degree of nodes in a nonempty subset of nodes
The maximum value among the interference connection degrees of all the sending nodes $n_i \in S'_p$ in a nonempty subset $S'_p$ of nodes in a primary path $P$ is defined as the intrinsic interference connection degree of nodes in $S'_p$, which is denoted as $D'_{S'_p}$. 

Definition 21: The intrinsic interference connection degree of nodes in a primary path
The maximum value among the interference connection degrees of all the sending nodes $n_i \in S_p$ in the set $S_p$ of nodes in a primary path $P$ is defined as the intrinsic interference connection degree of nodes in $P$, which is denoted as $D'_{S_p}$. 

Definition 22: A nonempty subset of nodes with dominant intrinsic interference intensity
In a nonempty subset $S'_p$ of nodes, if its intrinsic interference connection degree $D'_{S'_p}$ is less than its intrinsic interference intensity $I'_{S'_p}$, i.e. $D'_{S'_p} < I'_{S'_p}$, the subset $S'_p$ is defined as a subset of nodes with dominant intrinsic interference intensity. 

Definition 23: A primary path with dominant intrinsic interference intensity
As for a primary path $P$, if its intrinsic interference connection degree $D'_{S_p}$ is less than its intrinsic interference intensity $I'_{S_p}$, i.e. $D'_{S_p} < I'_{S_p}$, the primary path $P$ is defined as a primary path with dominant intrinsic interference intensity. 

Rule 3: The concurrency rules between nodes in a primary path
If two different nodes $n_i$ and $n_j$ in a primary path $P$ have concurrency relationship (i.e. $n_i \parallel n_j$ ($n_i, n_j \in S_p, i \neq j$)), and node $n_{j+1}$ is another sending node of $P$ (i.e. $n_{j+1} \in S_p$), then it can be deduced that node $n_i$ and node $n_{j+1}$ also have concurrency relationship. (Note: this rule
can be summarized as: \( n_i \parallel n_j \Rightarrow n_i \parallel n_{j+1} \).

**Rule 4: The concurrency rules between nodes in a primary path**

If two different nodes \( n_i \) and \( n_j \) in a primary path \( P \) have concurrency relationship (i.e. \( n_i \parallel n_j (n_i, n_j \in S_p, i < j) \)), and node \( n_{i-1} \) is another sending node of \( P \) (i.e. \( n_{i-1} \in S_p \)), then it can be deduced that node \( n_{i-1} \) and node \( n_j \) also have concurrency relationship. (Note: this rule can be summarized as: \( n_i \parallel n_j \Rightarrow n_{i-1} \parallel n_j \)).

**Definition 24: The continuous second-order interference subset in a primary path**

Among all the sending nodes in a second-order interference subset \( S'_p \) of a primary path \( P \), let node \( n_i (\in S'_p) \) have the smallest sequence number (the sequence number of node \( n_i \) is \( i \)), and node \( n_j (\in S'_p, i \neq j) \) have the largest sequence number (the sequence number of node \( n_j \) is \( j \)). If \( S'_p \) includes all the nodes with their sequence numbers between \( i \) and \( j \) in \( P \), then \( S'_p \) is regarded as a continuous second-order interference subset in \( P \).

**Theorem 2: The structural property of a second-order interference subset in a primary path**

Assuming that a primary path \( P \) satisfies both Rule 3 and Rule 4, in which two different nodes \( n_i \) and \( n_j \) do not have concurrency relationship (i.e. \( n_i > n_j (n_i, n_j \in S_p, i < j) \)), then the node set \( S'_p \triangleq \{ n_k | n_k \in S_p; i \leq k \leq j \} \) composed of all the nodes (including \( i \) and \( j \)) with sequence numbers between \( i \) and \( j \) in \( P \) is a continuous second-order interference subset.

**Proof:**

Reduction to absurdity is adopted. Assuming that node subset \( S'_p = \{ n_k | n_k \in S_p; i \leq k \leq j \} \) is not a second-order interference subset, therefore, there are at least two nodes \( n_k \) and \( n_l \) \((i \leq k < l \leq j)\) that can be concurrent, i.e. \( n_k \parallel n_l (i \leq k < l \leq j) \). According to Rule 3, we know that \( n_k \parallel n_l \) holds through several iterations. Since \( n_i \parallel n_j \), according to Rule 4, \( n_i \parallel n_j \) can be
obtained through several iterations, which is contradictory to the premise of \( n_i > n_j \) in the proposition. Therefore, \( S'_p \) must be a second-order interference subset. Furthermore, combined with the definition of the continuous second-order interference subset in a primary path (Definition 24), it can be seen that \( S'_p \) is a continuous second-order interference subset in \( P \).

**Corollary 2-1: The continuity of a maximum second-order interference subset in a primary path**

On the premise of satisfying both Rule 3 and Rule 4, any maximum second-order interference subset in a primary path \( P \) containing at least one second-order interference subset is a continuous second-order interference subset.

**Proof:**

Reduction to absurdity is adopted. Consider a subset \( S'_p \) is a maximum second-order interference subset in a primary path \( P \), and the number of elements of \( S'_p \) is \( I'_{S'_p} \). Without losing generality, assuming that node \( n_i \) has the smallest sequence number (i.e. \( i \)) in \( S'_p \) and node \( n_j \) has the largest sequence number (i.e. \( j \)) in \( S'_p \). If \( S'_p \) is not continuous, there must be at least one node in \( P \) whose sequence number is between \( i \) and \( j \), and the node does not belong to \( S'_p \). Therefore, it can be deduced from Theorem 2 that all nodes with sequence numbers between \( i \) and \( j \) (including \( i \) and \( j \)) can form a continuous second-order interference subset, and the number of elements in this subset must be greater than \( I'_{S'_p} \), which contradicts the premise that \( S'_p \) is a maximum second-order interference subset.

**Theorem 3: The splitting characteristics of a subset of nodes with dominant intrinsic interference intensity**

If the intrinsic interference intensity of a subset \( S'_p \) of sending nodes with dominant intrinsic interference intensity is \( I'_{S'_p} \), all the nodes contained in it can be divided into \( I'_{S'_p} \) mutually disjoint second-order concurrency subsets of nodes.

**Proof:**

Because the intrinsic interference intensity of \( S'_p \) is \( I'_{S'_p} \), at least \( I'_{S'_p} \) sending nodes that
can jointly form a second-order interference subset can be found in $S'_p$. Starting from these $I^p_{S_p}$ sending nodes, we can construct $I^p_{S_p}$ subsets of sending nodes which contain only one sending node (from one of these $I^p_{S_p}$ sending nodes) each and are mutually disjoint to each other. On the premise of keeping these subsets of sending nodes as second-order concurrency subsets, by adding the remaining sending nodes in $S'_p$ into these subsets one by one, $I^p_{S_p}$ mutually disjoint second-order concurrency subsets of nodes can be finally formed. Assuming that at least one sending node $n$ in $S'_p$ cannot be added into any of these $I^p_{S_p}$ second-order concurrency subsets, which indicates that in each of these $I^p_{S_p}$ second-order concurrency subsets at least one sending node, which has interference relationship with the node $n$, can be found. Therefore, it can be seen that the interference connection degree of node $n$ in $S'_p$ must not be less than $I^p_{S_p}$, which is contrary to the assumption that the set $S'_p$ is dominated by intrinsic interference intensity. Therefore, such a node $n$ does not exist at all. That is to say, all nodes contained in $S'_p$ can be divided into $I^p_{S_p}$ mutually disjoint second-order concurrency subsets of nodes.

**Corollary 3-1: The relationship between the intrinsic interference intensity and the intrinsic concurrency intensity of a set of nodes with dominant intrinsic interference intensity**

Let the intrinsic interference intensity and intrinsic concurrency intensity of a set $S'_p$ of nodes with dominant intrinsic interference intensity be $I^p_{S_p}$ and $C^p_{S_p}$, respectively. If the number of elements in the set $S'_p$ is $N_{S'_p}$, there is $I^p_{S_p} \cdot C^p_{S_p} \geq N_{S'_p}$.

**Proof:**

According to Theorem 3, $S'_p$ can be divided into $I^p_{S_p}$ nonempty and mutually disjoint second-order concurrency subsets, and the number of elements in each subset will not exceed the intrinsic concurrency intensity $C^p_{S_p}$ (according to the definition of intrinsic concurrency intensity of nonempty subsets of nodes (Definition 9)), so $I^p_{S_p} \cdot C^p_{S_p} \geq N_{S'_p}$ is valid.

(Note: Theorem 3 and Corollary 3-1 do not depend on whether Rule 3 and Rule 4 are satisfied or...
Corollary 3-2: The maximum second-order interference subsets contained in a set of nodes with dominant intrinsic interference intensity do not intersect to each other.

If there are multiple different maximum second-order interference subsets in a set $S'_p$ of nodes with dominant intrinsic interference intensity, these maximum second-order interference subsets do not intersect to each other.

Proof:
Reduction to absurdity is adopted. Assuming that the intrinsic interference intensity of $S'_p$ is $I'_{S'_p}$. Both node subsets $S'_a$ ($\subseteq S'_p$) and $S'_b$ ($\subseteq S'_p$) are maximum second-order interference subsets of $S'_p$, and $S'_a \neq S'_b$, so there is at least one node $j (\notin S'_a, \in S'_b)$. Assuming that $S'_a \cap S'_b \neq \emptyset$, there is at least one node $k (\in S'_a \cap S'_b)$. According to the definition of the maximum second-order interference subset (Definition 16), there is an interference relationship between $k (\in S'_a)$ and $I'_{S'_p} - 1$ nodes in $S'_a$, and the node $k (\in S'_b)$ and the node $j (\notin S'_a, \in S'_b)$ also interfere with each other. Therefore, it can be deduced that the interference connection degree of node $k$ in $S'_p$ is not less than $I'_{S'_p}$, which contradicts with the premise that $S'_p$ is dominated by its intrinsic interference intensity, so $S'_a \cap S'_b = \emptyset$. ■

VI. The concepts and related basic properties of the subset of equally spaced nodes and reachable period

In this section, the subset of equally spaced nodes with a given initial phase in a primary path is defined. The concepts of reachable period and intrinsic period of a primary path are given. The reachability of the period of a primary path is proved, and the important conclusion that the intrinsic period of a primary path is equal to its intrinsic interference intensity is obtained (Theorem 4). It is further pointed out that subsets of equally spaced nodes corresponding to the intrinsic period of a primary path cannot form a second-order concurrency relationship between each other (Corollary 4-1).

Definition 25: The subset of equally spaced nodes with a given initial phase in a primary path
As for a primary path $P$, given the spacing between adjacent nodes (i.e. the difference of sequence numbers between adjacent nodes) as $T_{s_y} \ (T_{s_y} \in \mathbb{Z}^+, 1 \leq T_{s_y} \leq N_{s_y})$ and the initial phase as $\theta_{s_y} \ (\theta_{s_y} \in \mathbb{Z}^+, 1 \leq \theta_{s_y} \leq T_{s_y})$, respectively. The subset of nodes
\[ \Phi'_{s_y}(\theta_{s_y}, T_{s_y}) = \{ n_i | i = \theta_{s_y} + j \cdot T_{s_y}, 1 \leq i \leq N_{s_y}, j \in \mathbb{Z}, j \geq 0 \} \] in $P$ is defined as a subset of equally spaced nodes with its initial phase being $\theta_{s_y}$ and the spacing between adjacent nodes being $T_{s_y}$. If the set $\Phi'_{s_y}(\theta_{s_y}, T_{s_y})$ is also a second-order concurrency subset, it is defined as a second-order concurrency subset with equally spaced nodes in a primary path $P$. (Note: in order to make $\Phi'_{s_y}(\theta_{s_y}, T_{s_y})$ contain at least one valid element of node, only the case of $T_{s_y} \leq N_{s_y}$ is considered in the paper).

**Definition 26: The reachable period of a primary path**

For a certain value $T_{s_y} \ (T_{s_y} \in \mathbb{Z}^+, 1 \leq T_{s_y} \leq N_{s_y})$, if all the subsets $\Phi'_{s_y}(\theta_{s_y}, T_{s_y}) \ (1 \leq \theta_{s_y} \leq T_{s_y})$ of equally spaced nodes in a primary path $P$ with their initial phases between 1 and $T_{s_y}$ (including 1 and $T_{s_y}$) are second-order concurrency subsets, then $T_{s_y}$ is defined as a reachable period of $P$, otherwise $T_{s_y}$ is regarded as an unreachable period of $P$.

**Definition 27: The intrinsic period of a primary path**

The minimum value of all the reachable periods of a primary path $P$ is defined as the intrinsic period of $P$, and it is denoted as $T_{s_y}^\ast$.

**Lemma 4-1: The unreachability of the period of a primary path**

If the intrinsic interference intensity of a primary path $P$ is $I_{s_y}^\ast$, the period of $T_{s_y} < I_{s_y}^\ast$ cannot be reached.

**Proof:**

**Case 1:** $I_{s_y}^\ast > 1$
Let the primary path $P$ be composed of $N_{S_\theta}$ sending nodes, and the sequence numbers of these nodes are $1, 2, ..., N_{S_\theta}$, respectively. Consider the case of $T_{S_\theta} < I'_{S_\theta}$. First, a matrix $M$ is constructed:

$$M = \begin{pmatrix}
1 & 1 + T_{S_\theta} & 1 + 2T_{S_\theta} & \cdots & 1 + \left\lfloor \frac{N_{S_\theta}}{T_{S_\theta}} \right\rfloor T_{S_\theta} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
j & j + T_{S_\theta} & j + 2T_{S_\theta} & \cdots & N_{S_\theta} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
T_{S_\theta} & T_{S_\theta} + T_{S_\theta} & T_{S_\theta} + 2T_{S_\theta} & \cdots & \times \times \times
\end{pmatrix}$$

(2)

1) The matrix has $T_{S_\theta}$ ($1 \leq T_{S_\theta} \leq N_{S_\theta}$) rows and $\left\lfloor \frac{N_{S_\theta}}{T_{S_\theta}} \right\rfloor$ columns;

2) In ascending order, put the sequence number of the node $j$ ($1 \leq j \leq N_{S_\theta}$) in $P$ into the corresponding unit in row $(j - T_{S_\theta} + \left\lfloor \frac{j}{T_{S_\theta}} \right\rfloor)$ and column $\left\lfloor \frac{j}{T_{S_\theta}} \right\rfloor$ of the matrix one by one. That is, set the value of the unit in row $(j - T_{S_\theta} + \left\lfloor \frac{j}{T_{S_\theta}} \right\rfloor)$ and column $\left\lfloor \frac{j}{T_{S_\theta}} \right\rfloor$ of $M$ to be $j$;

3) Ignore the settings of matrix units that may not be filled by sequence numbers of nodes.

According to the definition of the subset of equally spaced nodes, it can be seen that row $j$ of matrix $M$ is composed of all the elements of the subset $\Phi'_{S_\theta}(j, T_{S_\theta})$ with initial phase being $j$ and spacing between adjacent nodes being $T_{S_\theta}$.

Now let us prove that if $T_{S_\theta} < I'_{S_\theta}$, it can be deduced that not all the $T_{S_\theta}$ subsets $\Phi'_{S_\theta}(\theta_{S_\theta}, T_{S_\theta})$ ($1 \leq \theta_{S_\theta} \leq T_{S_\theta}$) are second-order concurrency subsets, which suggests that $T_{S_\theta} < I'_{S_\theta}$ is not reachable. Assuming that $S'_{p}$ is a maximum second-order interference subset of $P$ with the number of its elements being $I'_{S_\theta}$, and the set is continuous according to Corollary 2-1. Since
the number of rows of matrix $M$ is $T_{S_p} < I_{S_p}^*$, the corresponding distributions of the elements of $S^*_{S_p}$ in $M$ will stretch across (i.e. occupy) at least two adjacent columns of $M$, so that there must be at least two adjacent elements on a row $j$ ($1 \leq j \leq T_{S_p}$) of matrix $M$ belonging to the set of $S^*_{S_p}$, which makes it is evident that the two nodes corresponding to these two adjacent elements in $M$ cannot be concurrent. That is to say, the subset $\Phi'_{S_p}(j,T_{S_p})$ of equally spaced nodes corresponding to the row $j$ of matrix $M$ is not a second-order concurrency subset, which indicates that among $T_{S_p} < I_{S_p}^*$ subsets $\Phi'_{S_p}(\theta_{S_p},T_{S_p}) (1 \leq \theta_{S_p} \leq T_{S_p})$ of equally spaced nodes, at least one subset is not a second-order concurrency subset. Therefore, according to the definition of the reachability of the period of a primary path, it can be concluded that the period $T_{S_p} < I_{S_p}^*$ is unreachable.

**Case 2:** $I_{S_p}^* = 1$

According to the definition of a reachable period, the reachable period should not be smaller than one. That is to say, as for the case of $I_{S_p}^* = 1$, the reachable period $T_{S_p}$ should not be smaller than $I_{S_p}^{eq}$. ■

**Lemma 4-2: The reachability of the period of a primary path**

If the intrinsic interference intensity of a primary path $P$ is $I_{S_p}^*$, it is reachable for the period of $I_{S_p}^* \leq T_{S_p} \leq N_{S_p}$.

**Proof:**

**Case 1:** $I_{S_p}^* > 1$

Reduction to absurdity is adopted. Suppose that a period $T_{S_p} (\geq I_{S_p}^*)$ is not reachable. First, let us construct the matrix $M$ (that is, the matrix has $T_{S_p}$ rows and $\left\lceil \frac{N_{S_p}}{T_{S_p}} \right\rceil$ columns) according to the same method adopted in Lemma 4-1. Because it is assumed that the period $T_{S_p} (\geq I_{S_p}^*)$ is unreachable, according to the definition of the unreachable period of a primary path (Definition 26),
it can be concluded that at least one subset \( \Phi'_{S_p}(j,T_{S_p}) \) corresponding to row \( j (1 \leq j \leq T_{S_p}) \) of the matrix \( M \) is not a second-order concurrency subset, which suggests that at least two nodes in the subset \( \Phi'_{S_p}(j,T_{S_p}) \) cannot be concurrent. According to Theorem 2, these two nodes and all the other nodes between them will form a continuous second-order interference subset, and the number of elements contained in the interference subset will not be less than \( T_{S_p} + 1 \). That is, the number of elements of the formed interference subset is larger than the intrinsic interference intensity \( I^*_{S_p} \) (Note: the case of \( T_{S_p} \geq I^*_{S_p} \) is considered here), which contradicts the premise that the intrinsic interference intensity of \( P \) is \( I^*_{S_p} \).

**Case 2:** \( I^*_{S_p} = 1 \)

In this case, all the \( N_{S_p} \) sending nodes of \( P \) constitute a second-order concurrency subset \( \Phi'_{S_p}(1,1) \) (i.e. \( T_{S_p} = 1 \) and \( \theta_{S_p} = 1 \)) with equally spaced nodes.

**Theorem 4:** The intrinsic period of a primary path is equal to its intrinsic interference intensity

If the intrinsic period of a primary path \( P \) is \( T^*_{S_p} \), and the intrinsic interference intensity is \( I^*_{S_p} \), then \( T^*_{S_p} = I^*_{S_p} \) holds.

**Proof:**

According to Lemma 4-1, the period of \( T_{S_p} < I^*_{S_p} \) is not reachable, and according to Lemma 4-2, the period of \( T_{S_p} \geq I^*_{S_p} \) is reachable. Therefore, the minimum reachable period, i.e. the intrinsic period of a primary path \( P \), is \( T^*_{S_p} = I^*_{S_p} \).

**Corollary 4-1:** The subsets of equally spaced nodes corresponding to the intrinsic period of a primary path cannot form a second-order concurrency relationship between each other

If the intrinsic period of a primary path \( P \) is \( T^*_{S_p} \) (> 1), among all of its \( T^*_{S_p} \) subsets of equally spaced nodes with initial phases being \( \theta_{S_p} (1 \leq \theta_{S_p} \leq T^*_{S_p}) \) and spacing between adjacent
nodes being \( T_s^* \), any two of them cannot jointly form a second-order concurrency subset.

**Proof:**

Firstly, a matrix \( M \) with the number of rows being \( T_s^* \) and the number of columns being \( N_s \) is constructed by following the method adopted in Lemma 4-1. According to Theorem 4, the intrinsic interference intensity of the primary path \( P \) is \( I^* = I^* \), and according to Corollary 2-1, all possible maximum second-order interference subsets of \( P \) are continuous. Therefore, if we take any two rows (corresponding to two subsets of equally spaced nodes with different initial phases) from the matrix \( M \), it can be found that these two rows must contain two different elements, each of which belongs to a common maximum second-order interference subset, and the corresponding two nodes must interfere with each other. Therefore, the two subsets of equally spaced nodes corresponding to these two rows selected arbitrarily from the matrix \( M \) contain nodes that interfere with each other. That is to say, the two arbitrarily selected subsets of equally spaced nodes cannot form a second-order concurrency subset. ■

**VII. Definition of throughput of a primary path and some related basic properties**

In this section, the average throughput and the asymptotic throughput of a primary path are defined. An algorithm (Algorithm 1) for the transmission of information blocks based on the intrinsic period of a primary path is proposed, and an important conclusion is drawn that the proposed transmission algorithm can maximize the asymptotic throughput of a primary path (Theorem 5).

**Definition 28: The specific time interval**

Starting from time \( t \), the time duration of lasting for \( d \ (d \in \mathbb{Z}, d \geq 0) \) time beats is defined as the specific time interval, which is denoted as \( T_d(t,d) \).

**Definition 29: The average throughput of a primary path in a specific time interval**

If the destination node of a primary path \( P \) receives \( m \) information blocks in the time interval \( T_d(t,d) \) (Note: for simplicity, assuming that all the information blocks carry the same
amount of information, so here only the number of information blocks is used to measure the amount
of information received), the value \( R_{AV}^{i} \frac{m_{i}}{d} \) (unit: number of information blocks/time beat) is
defined as the average throughput of a primary path \( P \) in this time interval.

**Definition 30: The asymptotic throughput of a primary path**

Assuming that the destination node of a primary path \( P_{i} \) \( (i = 1, 2) \) receives \( m_{i} \) information
blocks in the time interval \( T_{i}(t, d) \). The limitation of the average throughput of \( P_{i} \) in the case of
the duration \( d \) (measured by the number of time beats) of a time interval \( T_{i}(t, d) \) approaching
to infinity is defined as the asymptotic throughput of \( P_{i} \), which is denoted as \( R_{AV}^{i} \frac{m_{i}}{d} \).

**Lemma 5-1: The upper limit of the asymptotic throughput of a primary path**

Assuming that the intrinsic interference intensity of a primary path \( P \) is \( I_{S_{p}}^{*} \) (according to
Theorem 4, we have \( T_{S_{p}}^{*} = I_{S_{p}}^{*} \)), then the asymptotic throughput of \( P \) is \( R_{AV}^{e} \leq \frac{1}{I_{S_{p}}} = \frac{1}{T_{S_{p}}} \).

**Proof:**

**Case 1:** \( I_{S_{p}}^{*} > 1 \)

Assuming that the set \( S_{p}^{*} \) is a maximum second-order interference subset of \( P \). When
information blocks are transmitted among the nodes in \( P \), they must pass through all the nodes
contained in \( S_{p}^{*} \) successively. When information blocks are transmitted in \( S_{p}^{*} \) one hop after
another, in order to avoid interference between adjacent nodes, only one information block can be
transmitted simultaneously in \( S_{p}^{*} \), i.e. at most one node in \( S_{p}^{*} \) is at active state in one time beat.
Therefore, as for \( S_{p}^{*} \), at most one information block can be output per \( I_{S_{p}}^{*} \) time beats, so the
asymptotic throughput of a primary path \( R_{AV}^{e} \leq \frac{1}{I_{S_{p}}} = \frac{1}{T_{S_{p}}} \). Furthermore, from Theorem 4, we have
\( R_{AV}^{e} \leq \frac{1}{I_{S_{p}}} = \frac{1}{I_{S_{p}}} \).

**Case 2:** \( I_{S_{p}}^{*} = 1 \)

In this case, a primary path \( P \) can at most output one information block per time beat, so the
asymptotic throughput \( R_{sy}^c \leq \frac{1}{I_{sy}^c} \) is still valid. Furthermore, from Theorem 4, it can be obtained that \( R_{sy}^c \leq \frac{1}{I_{sy}^c} = \frac{1}{I_{sy}^*} \).

Algorithm 1: The algorithm for the transmission of information blocks based on the intrinsic period of a primary path

1: BEGIN
2: Set the intrinsic interference intensity of a primary path \( P \) as \( I_{sy}^* \) (\( T_{sy}^* = I_{sy}^* \)), the time beat for \( P \) starting the transmission of the first information block is regarded as the “starting beat”, and the corresponding sequence number of the “starting beat” is set to be 1;
3: WHILE (the sequence number of the beat \( \leq \) the upper limit of the sequence number of the beat) DO
4: IF (the sequence number of the beat \( = 1 + j \cdot T_{sy}^* \) (\( j = 0, 1, 2, ... \))) THEN
5: Activate all the nodes in the subset \( \Phi_{sy}^* (1, I_{sy}^*) \);
6: For node 1 in \( \Phi_{sy}^* (1, I_{sy}^*) \): gets a new information block from the information source (here, it is assumed that there is always an information block waiting to be transmitted at the information source) and sends it to its adjacent downstream node within the current beat; For all the other nodes in \( \Phi_{sy}^* (1, I_{sy}^*) \): send the information block (if any) received from their upstream nodes in the previous beat to their adjacent downstream nodes in the current beat;
7: ELSE IF (\( I_{sy}^* > 1 \))
\( \backslash \text{Note: in this case, the sequence number of the beat} = i + j \cdot T_{sy}^* \) (\( 1 < i \leq T_{sy}^*, j = 0, 1, 2, ... \))
8: Activate all the nodes in the subset \( \Phi_{sy}^* (i, I_{sy}^*) \);
9: For all the nodes in \( \Phi_{sy}^* (i, I_{sy}^*) \): send the information block (if any) received from their upstream nodes in the previous beat to their adjacent downstream nodes in the current beat;
10: END IF
Lemma 5-2: The reachable upper limit of the asymptotic throughput

Assuming that the intrinsic interference intensity of a primary path \( P \) is \( I_{sy}^P (T_{sy}^P = I_{sy}^P) \), then the asymptotic throughput \( R_{sy}^P = \frac{1}{I_{sy}^P} = \frac{1}{T_{sy}^P} \) of \( P \) can be reached.

Proof:

At least Algorithm 1 can be adopted. That is to say, by using Algorithm 1, in steady state, the primary path \( P \) will achieve the performance of outputting one information block per \( T_{sy}^P \) beats.

In this case, the asymptotic throughput of \( P \) is \( R_{sy}^P = \frac{1}{I_{sy}^P} = \frac{1}{T_{sy}^P} \). Therefore, the upper limit of the asymptotic throughput \( R_{sy}^P = \frac{1}{I_{sy}^P} = \frac{1}{T_{sy}^P} \) can be reached.

Theorem 5: The asymptotic throughput of a primary path can be maximized by using Algorithm 1

Assuming that the intrinsic interference intensity of a primary path \( P \) is \( I_{sy}^P (T_{sy}^P = I_{sy}^P) \), the maximum asymptotic throughput \( R_{sy}^P = \frac{1}{I_{sy}^P} = \frac{1}{T_{sy}^P} \) can be achieved by using Algorithm 1.

Proof:

It can be obtained by combining Lemma 5-1 and Lemma 5-2.

Definition 31: One time end-to-end delay of an information block on a primary path

On a primary path \( P \), the total number of time beats needed from the time an information block entering the source node to the time it finally reaching the destination node is defined as the one-time end-to-end delay of an information block on \( P \), which is denoted as \( T_{sy}^{ed} \).

Theorem 6: The one time end-to-end delay on a primary path can be minimized by using Algorithm 1
As for a primary path $P$, if Algorithm 1 is adopted, the one time end-to-end delay can be minimized, i.e. $T_{ed}^P = N_{sp}$.

Proof:

For a primary path $P$ with $N_{sp}$ sending nodes, starting from its node 1 (source node), at least $N_{sp}$ beats are needed for an information block reaching the final destination node (i.e. node $N_{sp} + 1$) through hop-by-hop transmissions. By checking the execution procedure of Algorithm 1, it is not difficult to see that for the transmission of each information block, Algorithm 1 can achieve the shortest one-time end-to-end delay. ■

VIII. Conclusions and discussions

Aiming at the disorder problem (i.e. uncertainty problem) of the utilization of network resources commonly existing in multi-hop transmission networks, this paper proposes the idea of network wave, that is, to improve the orderliness of the utilization of network resources by constructing volatility information transmission mechanism between sending nodes of a primary path, which lays the ideological and theoretical foundation for the follow-up researches of more general methods that can further improve the orderly utilization of access resources of the whole networks.

In the paper, some important and interesting conclusions are obtained. To give some examples:

1) In Theorem 4, it is found that the intrinsic period of a primary path is equal to its intrinsic interference intensity.

2) In Theorem 5, it is pointed out that by using Algorithm 1, the asymptotic throughput of a primary path can be maximized.

Based on the conclusions obtained in this paper, we can further summarize the advantages of volatility transmission method as: it can maximize the end-to-end asymptotic throughput of a primary path while maintaining the orderly utilization of network resources as much as possible. In the future work, based on the work proposed in this paper, we will further study how to extend the theory of network wave to more general cases, such as the cases including two primary paths (i.e. pair of paths) and any multiple primary paths (i.e. group of paths).
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