Hadronic Final States in Deeply Inelastic Scattering *

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Abstract

Results on hadronic final states in deeply inelastic scattering are reviewed. They comprise jet production and its interpretation in perturbative QCD, signatures to distinguish conventional QCD dynamics from possible new features of QCD at small \( x \), and measurements of inclusive charged particle production. Theoretical developments such as color dipole emission and instanton induced final states are reported on.

1. Introduction

The basic measurement in deeply inelastic scattering (DIS) is a measurement of the cross section \( ep \rightarrow eH \) in terms of the structure function \( F_2 \), where \( H \) stands for any hadronic system. A wealth of information upon the partonic structure of the proton and its dynamics have been obtained from structure function measurements. Measurements of the properties of the hadronic final state \( H \) provide complementary information which cannot be obtained from inclusive structure functions.

In the simple quark parton model (QPM) of DIS, a quark is scattered out of the proton by the virtual boson emitted from the scattering lepton. QCD modifies this picture. Partons may be radiated before and after the boson-quark vertex, and the boson may also fuse with a gluon inside the proton by producing a quark-antiquark pair (figure [fig:1]). In fact, the parton which is probed by the boson may be the end point in a whole cascade of parton branchings. This parton shower materializes in the hadronic final state, allowing experimental access to the dynamics governing the cascade.

HERA has opened a new kinematic domain to study QCD in DIS, and most contributions in this working group were concerned with HERA physics. In HERA electrons of \( E_e \approx 27 \) GeV collide with protons of \( E_p = 820 \) GeV, resulting in a centre of mass energy of \( \sqrt{s} \approx 300 \) GeV. The kinematic region covered with the present data is roughly \( 10^{-4} < x < 10^{-1}, 7 \) GeV$^2 <
Another area of recent interest is the kinematic regime leading to prominent jets observable in the final state. The large phase space available for hard QCD evolution of physics quantities over a large kinematic range. The processes contributing to DIS up to first order in $\alpha_s$ are shown in figure 1. The QPM process results in a so-called “1+1” jet topology, while the QCDC and BGF processes give “2+1” jet events, where the “+1” refers to the unobserved remnant jet. From a measurement of the 2+1 jet rate at large $x$ and $Q^2$, where the parton densities are well known, $\alpha_s$ can be measured. At small $x$ and $Q^2$, one can determine the largely unknown gluon density from the rate of 2+1 jet events, which is then dominated by the BGF graph (assuming $\alpha_s$ to be known). Complications arise from the fact that the initial state contains strongly interacting particles, leading to the evolution of parton showers. Such effects need to be taken into account with the help of MC simulations.

2. Jet physics

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2.1. The strong coupling constant $\alpha_s$

Both H1 and ZEUS use the modified JADE algorithm [1] with resolution parameter $y_{cut} = 0.02$ to define jets in the $\alpha_s$ analysis. A pseudoparticle is introduced to account for the unobserved remnant, and then all particles $i, j$ satisfying $m_{ij}^2 < y_{cut} \cdot W^2$ are merged into jets. The chosen $y_{cut}$ value is a compromise between statistical precision (small $y_{cut}$), and controllable higher order corrections (large $y_{cut}$). In the H1 analysis
an angular cut $\theta_{\text{jet}} > 10^\circ$ (w.r.t. the proton direction) protects against parton showers close to the remnant. The obtained jet rates are corrected for detector effects, remaining parton shower contributions and hadronization with the MEPS model. In order to extract $\alpha_s$ from the measured jet rates, it is important to take next to leading order (NLO) corrections into account to reduce dependencies upon $y_{\text{cut}}$ and the chosen renormalization and factorization scales $[12]$. Using PROJET $[14]$ as NLO calculation, the measured jet rate then yields measurements of $\alpha_s (Q^2)$ in the range $10 \text{ GeV}^2 < Q^2 < 3000 \text{ GeV}^2$, which can be seen to run according to the QCD expectation $[10]$. However, below $Q^2 = 100 \text{ GeV}^2$, the corrections are very model dependent (MEPS vs. CDM). Therefore only data at $Q^2 > 100 \text{ GeV}^2$ are used to extract $\alpha_s (m_Z^2) = 0.123 \pm 0.018$ $[14]$. For 2+1 jet events with $Q^2 > 160 \text{ GeV}^2$ and $x > 0.01$, ZEUS has measured the jet distribution in the Lorentz invariant $z_p$ variable $[13]$, which is in the centre of mass frame of the virtual photon and the incoming parton is an angular variable $z_p = \frac{1}{2} \cdot (1 - \cos \theta_{\text{jet}})$. Here $\theta_{\text{jet}}$ is the angle of the jet w.r.t. the direction of the incoming parton. Perturbation theory in next to leading order (NLO) $[14]$ is able to describe the jet angular distribution down to $z_p \approx 0.1$. For $z_p < 0.1$ an excess of jets is observed. Both, the MEPS (LO matrix element + parton showers) and ME (pure LO matrix element) simulations are similar to the NLO calculation $[15]$. The excess of jets at $z_p < 0.1$ is therefore unlikely to be cured by next to NLO calculations.

For the $\alpha_s$ extraction, a cut $z_p > 0.1$ restricts the data to a region well described by NLO perturbation theory and QCD models $[16]$. The preliminary $\alpha_s$ measurements $[16]$ for $100 \text{ GeV}^2 < Q^2 < 3600 \text{ GeV}^2$ demonstrate the potential of HERA to study the dependence of $\alpha_s$ upon the renormalization scale, and agree well with the QCD expectation (see figure 3). It is expected that already the analysis of the 1994 HERA data, once finalized, will yield a very competitive measurement of $\alpha_s (m_Z^2)$.

### 2.2. The gluon density in the proton

The 2+1 jet sample (defined with the cone algorithm in the CMS) in the range $10 \text{ GeV}^2 < Q^2 < 100 \text{ GeV}^2$ is used to extract the gluon density $g(x_g, Q^2)$, because there the BGF graph (figure 2) dominates (BGF:QCDC $\approx 4:1$ $[16, 17]$). The momentum fraction $x_g$ which the gluon carries is calculated from the invariant mass $s$ of the hard subsystem forming the 2 jets via $x_g = x(1 + s/Q^2) \approx s/W^2$. Special cuts remove events affected by parton showers $[16, 17]$. The MEPS model is used to unfold detector effects, the QCDC contribution, QPM background and remaining parton shower contributions.

The MEPS model employs a cut-off for invariant parton-parton masses $m_{ij}^2 > y_{\text{min}} \cdot W^2$ to regulate divergencies of its LO matrix element. In order to access $x_g$ as small as possible, $s$ is chosen as small as experimental resolution allows, and as problems with the diverging LO matrix element can be avoided. It has to be ensured that the BGF events to be analyzed are actually generated by the model and do not fall below that cut-off $[16, 17]$.

The H1 analysis $[16]$ uses a fixed cut-off $\hat{s} > 100 \text{ GeV}^2$ to define BGF events, and they parametrize the MEPS cut-off such as to follow the limit at which the order $\alpha_s$ contribution exceeds the total cross section within a margin of $\Delta \sqrt{\hat{s}} = 2 \text{ GeV}$ . ZEUS uses the standard $y_{\text{min}}$ cut-off scheme in the MEPS model and defines BGF events via $\hat{s} > y_{\text{min}} \cdot W^2$. The parameter $y_{\text{min}}$ is then varied between 0.0025 and 0.01 to study its influence on the result. The H1 and ZEUS results $[16]$ agree well with each other, but yield different size systematic errors (figure 3). The ZEUS errors receive large contributions from the $y_{\text{min}}$ variation. The rise of the measured gluon density towards small $x$ can be described by a LO gluon density $[18]$ following the DGLAP (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi) $[19]$ equations. The data are also consistent with the indirect determination of $g(x_g, Q^2)$ from the scaling violations of $F_2$ $[18]$, providing a non-trivial test of QCD.

### 2.3. Open Points

Lack of understanding of parton showers close to the remnant (model dependent corrections, failure of NLO calculations) currently prevents the $\alpha_s$ analysis to make full use of the large statistics data at $Q^2 < 100 \text{ GeV}^2$. 

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**Figure 2.** Preliminary $\alpha_s (Q)$ measurements from ZEUS, compared to the QCD predictions corresponding to $\Lambda_{\text{QCD}} = 100, 200$ and 300 GeV.
Though increasing HERA luminosity will allow the \( \alpha_s \) analysis to be restricted to higher \( Q^2 \) to reduce uncertainties, the understanding of the forward region remains a challenge.

So far the \( \alpha_s \) measurements rely solely upon the JADE algorithm, being the only algorithm for which NLO jet cross sections are calculated \(^\text{[22]}\). NLO calculations for other algorithms, such as the cone \(^\text{[23]}\) or the theoretically preferred \( k_T \) \(^\text{[24]}\) algorithm are desirable. Such a program, which would also be able to calculate event shape variables like energy-energy correlations, Thrust, etc., is being worked upon by D. Graudenz, but results cannot be expected in a short term. Theoretical uncertainties could also be reduced by resumming higher order corrections.

The validity of corrections from hadronic to partonic final states, defined either in LO or NLO, need to be checked with models based upon different parton shower and hadronization schemes. Unfortunately, a MC generator incorporating the QCD matrix elements beyond LO is missing.

The gluon density has so far been determined in LO. A method allowing a measurement in NLO is presently under study \(^\text{[12]}\).

How can \( \alpha_s \) be determined consistently, considering it is input for the evolution of parton densities which are used in the analysis \(^\text{[27]}\)?

3. Novel QCD dynamics

The observed strong rise of the structure function \( F_2 \) towards small \( x \) \(^\text{[20]}\) has caused much debate on whether the QCD evolution of the parton densities can still be described by the conventional DGLAP \(^\text{[21]}\) equations, or whether the HERA data extend into a new regime at small \( x \) where the dynamics is governed by the BFKL (Balitsky-Fadin-Kuraev-Lipatov) \(^\text{[26]}\) equation. It would be extremely interesting to test QCD in such a new regime. While the rise is consistent with the expectation from BFKL dynamics, it can however also be described by a DGLAP evolution \(^\text{[27]}\). At lowest order the BFKL and DGLAP equations resum the leading logarithmic \( (\alpha_s \ln 1/x)^n \) or \( (\alpha_s \ln(Q^2/Q_0^2))^n \) contributions respectively. In this approximation the leading diagrams are of the ladder type (figure 4). The leading log DGLAP ansatz corresponds to a strong ordering of the transverse momenta \( k_T \) (w.r.t. the proton beam) in the parton cascade \( (Q_0^2 \ll k_T 1 \ll ...k_T 1 \ll ...Q^2) \), while there is no such ordering in the BFKL ansatz \( (k_T 1 \approx k_T 2 \approx ...k_T n) \) \(^\text{[28]}\). Measurements on the hadronic final state emerging from the cascade therefore offer another handle to search for signatures of the BFKL behaviour. They are compared to analytical calculations as well as to the QCD models MEPS and CDM. The CDM description of gluon emission is similar to that of the BFKL evolution, because the gluons emitted by the dipoles do not obey strong ordering in \( k_T \) \(^\text{[29]}\). The MEPS model with its leading log parton shower is based upon DGLAP dynamics, and the emitted partons are thus ordered in \( k_T \).

3.1. Transverse Energy Production

As a consequence of the strong \( k_T \) ordering the DGLAP evolution is expected to produce less transverse energy \( E_T \) in a region between the current region and the proton remnant than the BFKL evolution \(^\text{[30]}\). H1 and ZEUS have measured the flow of transverse energy in the laboratory frame as a function of pseudorapidity \( \eta = -\ln \tan(\theta/2) \), where \( \theta \) is the angle of the energy deposition w.r.t. the proton beam axis \(^\text{[31]}\). The measurements are made for varying ranges in \( x \) \( (2 \times 10^{-4} < \langle x \rangle < 5 \times 10^{-3}) \) and \( Q^2 \) \( (7 \text{ GeV}^2 < \langle Q^2 \rangle < \)
30 GeV$^2$ and agree well between the experiments \[32\].

The $E_T$ flows for large $x$ and $Q^2$ are reasonably well described by MEPS and CDM. For smaller $x$ and $Q^2$ both models predict a more pronounced enhancement in the current fragmentation region than is seen in the data. Between the current system and the proton remnant (the lab. forward region), the data are reasonably well described by the CDM, while the MEPS model produces too little $E_T$ \[31, 32\]. This intermediate region is expanded in figure 5, because perturbative calculations, based either on DGLAP or on BFKL dynamics, are available \[30\]. The BFKL calculation comes out close to the data, while the DGLAP calculation predicts much less $E_T$. However, the non-perturbative hadronization phase is missing in these calculations.

H1 has determined the average $E_T$, measured centrally in the CMS as a function of $x$ and $Q^2$ (figure 5). They find an increase of $\langle E_T \rangle$ with decreasing $x$, which is a characteristic BFKL prediction \[30\]. The data are in agreement with the BFKL calculation \[34\], if one assumes an $E_T$ contribution from hadronization of about 0.4 GeV per unit rapidity (independent of $x$). That estimate is taken from the CDM, which agrees with the BFKL calculation at the parton level.

The apparent failure of the MEPS model has caused many questions about its ingredients: the way the parton shower is “matched” to the matrix element, the colour connection between the current and the remnant system and its effect upon hadronization, and the remnant fragmentation itself which is little tested. It seems that re-arranging colour configurations can produce enough $E_T$ through hadronization to compensate the $E_T$ deficit in the DGLAP cascade of the MEPS model \[35\]. A MEPS version thus modified should be available soon for detailed testing. The flexibility in the hadronization modelling presently precludes unambiguous tests of the DGLAP evolution through $E_T$ measurements. For the same reasons the intriguing success of the CDM without $k_T$ ordering may be fortuitous. A MC model invoking explicitly the BFKL evolution, currently being developed by K. Golec-Biernat et al., would help interpreting the data. In any case, the $E_T$ data provide important input for QCD phenomenology.

3.2. Forward Jets

At present strong conclusions upon the validity of the BFKL or DGLAP parton evolutions at small $x$ from the $E_T$ measurements are hampered by the uncertainties about hadronization. Jet production should be less affected by hadronization. A signature for BFKL dynamics proposed by \[36\] is the production of “forward jets” with $x_{\text{jet}} = E_{\text{jet}} / E_p$, the ratio of jet energy and proton beam energy, as large as possible, and with transverse momentum $k_T$ close to $Q$ in order to reduce the phase space for the $k_T$ ordered DGLAP evolution (see figure 5). An enhanced rate of events with such jets is thus expected in the BFKL scheme \[36, 37\]. The experimental difficulty is to detect these “forward” jets which are close to the beam hole in proton direction.

The rate of forward jets measured by H1 \[38, 33\] (figure 7) is larger at low $x$ than at high $x$. This
The rate of forward jets (selected with $x_{\text{jet}} > 0.025$, $0.5 < k_{T\text{jet}}^2/Q^2 < 4$ and $k_{T\text{jet}} > 5$ GeV $^2$) in the kinematic range $2 \cdot 10^{-4} < x < 2 \cdot 10^{-3}$ and $Q^2 \approx 20$ GeV $^2$. The measurement is compared to the CDM and MEPS models.

Figure 7. The rate of forward jets (selected with $x_{\text{jet}} > 0.025$, $0.5 < k_{T\text{jet}}^2/Q^2 < 4$ and $k_{T\text{jet}} > 5$ GeV $^2$) in the kinematic range $2 \cdot 10^{-4} < x < 2 \cdot 10^{-3}$ and $Q^2 \approx 20$ GeV $^2$. The measurement is compared to the CDM and MEPS models.

3.4. Dipole emission

An interesting ansatz to calculate final state observables was presented by R. Peschanski \cite{42}. The starting point is onium-onium scattering \cite{43} with onium wave functions which can be derived from QCD. Such a reaction is analogous to an interaction of the current system with the remnant system in DIS. Radiation is treated in the dipole picture, leading to a copious production of dipoles in the central rapidity region of the interaction. Once such an ansatz yields quantitative predictions, it could be tested in DIS, e.g. with $E_T$ flow measurements.

Bo Andersson \cite{40} discussed DIS final states in terms of a chain of radiating colour dipoles, and its connection with the Ciafaloni-Catani-Fiorini-Marchesini ansatz \cite{1}. In principle this model could provide a complete picture of the hadronic final state in DIS. The implementation in the Ariadne \cite{5} MC generator is in progress to allow detailed predictions.

3.5. QCD Instantons

The standard model contains processes which cannot be described by perturbation theory, and which violate classical conservation laws like baryon and lepton number in the case of the electroweak sector and chirality for the strong interaction \cite{44}. Such anomalous processes are induced by instantons \cite{45}. At HERA, QCD instantons may lead to observable effects in the hadronic final state in DIS \cite{46,47}, which were discussed by F. Schrempp. The instanton should decay isotropically into a high multiplicity state of gluons and all quark flavours simultaneously which are kinematically allowed. A MC program to simulate instanton events has become available \cite{48}. Due to the isotropic decay, one expects a densely populated region in rapidity, other than the current jet, which is isotropic.
in azimuth. The presence of strangeness and charm could provide an additional signature. However remote the a priori chances to see such signals may appear, here is a chance for a major discovery at HERA!

4. Charged Particle Spectra

The H1 and ZEUS measurements of inclusive charged particle spectra \[49\] are performed either in the Breit frame or in the CMS. In the Breit frame in- and outgoing quark have equal but opposite sign momenta \(Q/2\) (QPM picture), and in \(e^+e^-\) annihilation the outgoing quark and antiquark have equal but opposite momenta \(\sqrt{2}/2 = Q/2\). Due to this similarity it is interesting to compare particle spectra in the Breit current hemisphere in DIS with \(e^+e^-\) data. DIS experiments have the advantage over \(e^+e^-\) experiments that they cover a large span in \(Q\), presently from 3 GeV to 50 GeV, in a single experiment. The current mean charged multiplicity at HERA rises \(\sim \ln Q\) within errors, and agrees with \(e^+e^-\) data (divided by 2) where they overlap \[50\].

Colour coherence should lead to a suppression of soft gluon emission. The HERA data \[50, 51\] on the scaled charged particle momentum distribution \(\ln 1/p\) with \(x_p = 2 \cdot p_z/W\) exhibit the expected hump backed plateau \[51\], the evolution of which with \(Q\) is in agreement with the assumption of colour coherence. However, like in \(e^+e^-\) annihilation, this behaviour can also be mimicked through the Lund string fragmentation \[49\].

The scaled momentum spectrum of \(x_F\) in the CMS, where the particle longitudinal momenta \(p_z\) are divided by the maximal possible momentum, \(x_F = 2 \cdot p_z/W\), are shown in figure 3 for the current region (the target region is not observed). Comparing HERA data at \(W \approx 120\) GeV \[51, 50\] with fixed target data at \(W = 14\) and 18 GeV \[52, 53\], significant scaling violations are observed, in agreement with QCD expectations: the large value of \(W\) at HERA results in a large phase space for QCD radiation, softening the \(x_F\) spectrum w.r.t. data at lower \(W\). It can be expected that such data will be used to extract \(\alpha_s\) in the future.

The effect of QCD radiation is clearly seen in the “seagull plot” (figure 11), where the mean transverse momenta \(p_T^2\) squared of the particles is plotted as a function of \(x_F\). As a consequence of increased QCD radiation, much larger \(p_T^2\) are observed at HERA \[51, 50\] than at EMC \[53\] at smaller \(W\), again in agreement with QCD expectation. ZEUS has also compared DIS events with and without a large rapidity gap \[54\] in this respect \[54\]. Much smaller \(p_T^2\) than in normal DIS events are observed in events with a large rapidity gap, thought to stem from diffractive processes and accounting for approximately 10% of the total sample \[54\]. This indicates that the scale governing radiation is much smaller than \(W\) for rapidity gap events.

5. Conclusion

Two complementary approaches to the HERA data can be distinguished. In one approach, one tries to identify a region which is “well understood”, meaning that the observation agrees with the theory and the models. Under this condition, the data can be interpreted in the framework of the theory, and physical quantities which are defined within the theory can be extracted. The measurements of \(\alpha_s\) and \(g(x_F, Q^2)\) fall into this category. However, we have also seen data which are not yet understood theoretically, namely hadron and jet production in the forward region. Such data currently pose a challenge to the theory, and experimentalists should make every effort to provide theory with solid data to work with.

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Figure 10. The seagull plot. Shown are the mean transverse momenta squared \( \langle p_T^2 \rangle \) as a function of \( x_F \) in the CMS for HERA data with and without a rapidity gap (LRG/ZEUS) compared to the QPM prediction (dotted line) and the MEPS model (full line), and to EMC data at lower \( W \).

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