Sensorless control of SMPMSM drive system integrated the differential algebra with the algebraic parameter identification

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Abstract. The differential algebraic sensorless control has the technical features of clear concept and low calculation amount. However, this method is particularly sensitive to motor parameters. Therefore, a differential algebraic sensorless control strategy combined with algebraic parameter identification is proposed and applied to surface mounted permanent magnet synchronous motor (SMPMSM) drive system in this paper. Without the signal injection, the proposed method can promptly and accurately realize the online simultaneous identification of stator resistance and inductance based on the different torque conditions of system, and then the motor parameters used in the differential algebraic sensorless control are updated in time. In addition, the tracking differentiator is designed to accurately obtain the first-order differential of the stator current signal with noises for rotor position estimation. Finally, the effectiveness of the proposed sensorless control method is validated by the simulation results.

1. Introduction

Field Oriented Control (FOC) can achieve the current decoupling control of permanent magnet synchronous motor (PMSM), therefore, the field oriented controlled PMSM drive system enjoys better control performance. Real-time and accurate acquisition of the rotor position is essential for FOC. However, the traditional mechanical position sensor leads to an increase in the cost and volume of system, while also reducing its reliability. Therefore, a variety of sensorless control methods have been proposed [1].

There are two main categories of sensorless control schemes: model-based sensorless control and saliency-based high-frequency injection sensorless control [2]. By rotation signal injection [3] or pulsation signal injection [4], the saliency-based method realizes the position sensorless control of PMSM drive system. Nevertheless, at the same time the injection of high-frequency excitation signals will cause additional motor phase current harmonics and system losses.

Model-based method can be divided into closed-loop methods and open-loop methods. According to the designed structure, the closed-loop methods mainly include the extended Kalman filter (EKF) methods, the model reference adaptive system (MRAS) methods, and the sliding mode observer methods. The open loop method obtains the rotor position and speed by directly calculating the back electromotive force or flux linkage. It has the advantages of simple algorithm and fast dynamic response. As an open-loop method, the differential algebraic sensorless control method [5] directly obtains the rotor angle and speed through algebraic transformation of the mathematical model of the
motor. Furthermore, the observability of the rotor position is demonstrated by the differential algebra theory of the nonlinear system.

In PMSM drive system, the differential algebraic sensorless control faces two technical challenges. The first is to accurately obtain the differential signal of stator current. The tracking differentiator in active disturbance rejection control [6] is applied to achieve the accurate first-order differential of current signal containing noises. The second is this method is sensitive to variation of motor parameter, especially the stator resistance and inductance. Operating temperature and magnetic field saturation will cause significant changes in stator resistance and inductance [7]. In [8], MARS considering the nonlinearity of the inverter is designed to estimate the stator resistance and inductance. And in [9], the stator resistance and stator inductance are synchronously identified by injecting a short pulse of negative d-axis current, and then solving two sets of simplified PMSM state equations corresponding to \( id = 0 \) and \( id \neq 0 \) through an adaline neural network at the same time. However, the motor parameter identification methods in [8] and [9] both require the rotor position obtained by the position sensor. For the sensorless controlled PMSM system, the rank-deficient problem of the identification model is particularly acute, because the rotor position is also a parameter needs to be estimated. In [10], two time-scale affine projection algorithms (APAs) were proposed to estimate stator resistance and inductance respectively. But the fast and slow APAs need more time to execute and are not convenient to be implemented in ordinary digital controllers. An online parameter estimation method based on Lyapunov’s theory is proposed to calculate the stator resistance and inductance of the sensorless controlled SMPMSM system [11]. In the steady state, only the stator resistance is identified, and the stator inductance is regarded as a constant. On the contrary, in the dynamic state, the stator inductance becomes the identification object, and the stator resistance is assumed to be constant. By using adaptive full-stage state feedback current control, the stator resistance and inductance of the sensorless controlled SMPMSM system are simultaneously identified [12]. However, a sinusoidal d-axis current signal is injected, which brings additional disturbances. Based on the mathematical model of the motor, the algebraic identification method acquires the parameters to be identified through optimization theory with rapid convergence and high identification accuracy. It has been deduced in detailed theory [13] and has been applied to the parameter identification of brushless DC motors[14].

To improve the robustness to parameter uncertainties of differential algebraic sensorless control, a differential algebraic sensorless control strategy combined with algebraic parameter identification is proposed. Based on theoretical analysis, a simulation model of the sensorless controlled SMPMSM system is established, and the effectiveness of the proposed method is verified.

2. The sensorless control based on differential algebra

2.1. Rotor position estimation of SMPMSM based on differential algebraic method

The current state equation of SMPMSM in synchronous rotating d-q reference frame are given as

\[
\frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \frac{R_s}{L_s} & \omega \\ -\omega & \frac{R_s}{L_s} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} \frac{u_d}{L_s} \\ \frac{u_q}{L_s} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{\varphi_f}{L_s} \end{bmatrix} \omega
\]

(1)

where \( R_s, L_s, \varphi_f \) express stator resistance, stator inductance and the permanent-magnet flux linkage nominal parameters, respectively; \( u_d, u_q, i_d, i_q \) denote the stator voltages and currents in rotating frame, respectively; \( \omega \) is electrical angular velocity.

In order to prove the observability of the rotor angle \( \theta \), define the vector \( \mathbf{x}, \mathbf{y}, \mathbf{u} \) as
\[ x = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{bmatrix}^T = \begin{bmatrix} i_d & i_q & \cos \theta & \sin \theta & \tan \theta \end{bmatrix}^T \]

\[ y = \begin{bmatrix} y_1 & y_2 \end{bmatrix}^T = \begin{bmatrix} i_a & i_b \end{bmatrix}^T \]

\[ u = \begin{bmatrix} u_1 & u_2 \end{bmatrix}^T = \begin{bmatrix} u_\alpha & u_\beta \end{bmatrix}^T \]

where \( u_\alpha, u_\beta, i_\alpha, i_\beta \) express the stator voltages and currents in \( \alpha-\beta \) reference frame, respectively.

After coordinate transformation and equation derivation, the state equation of SMPMSM system can be obtained as

\[
\begin{align*}
\frac{dx_1}{dt} &= -\frac{R_s}{L_s} x_1 + \omega x_2 + \frac{1}{L_s} x_i u_1 + \frac{1}{L_s} x_i u_2 \\
\frac{dx_2}{dt} &= -\omega x_1 - \frac{R_s}{L_s} x_2 - \frac{1}{L_s} x_i u_1 + \frac{1}{L_s} x_i u_2 - \omega \phi_f \\
\frac{dx_3}{dt} &= -x_4 \\
\frac{dx_4}{dt} &= x_3 \\
\frac{dx_5}{dt} &= -x_3^2 \\
y_1 &= x_3 x_1 - x_4 x_2 \\
y_2 &= x_4 x_1 + x_3 x_2
\end{align*}
\]

(2)

For the algebraic theory [5], state variables \( x_i \) is observable if it can be written as follows

\[
x_i = f(u_1, u_\alpha, y_1, \ldots) \quad q(u_1, u_\alpha, y_1, \ldots) \quad (i=1, 2, 3, 4, 5)
\]

where \( f \) and \( q \) are the differential polynomials.

For SMPMSM, The current state equation in \( \alpha-\beta \) reference frame are given as

\[
\begin{bmatrix}
u_\alpha \\
u_\beta
\end{bmatrix} = R \begin{bmatrix}
i_\alpha \\
i_\beta
\end{bmatrix} + L_s \begin{bmatrix} 0 & 1 \\
1 & 0
\end{bmatrix} \begin{bmatrix}
i_\alpha \\
i_\beta
\end{bmatrix} + \omega \varphi_f \begin{bmatrix} -\sin \theta \\
\cos \theta
\end{bmatrix}
\]

(3)

According to equation (4), then

\[
\begin{align*}
\omega \varphi_f \sin \theta &= R_s i_\alpha + L_s \dot{i}_\alpha - u_\alpha \\
\omega \varphi_f \cos \theta &= -R_s i_\beta - L_s \dot{i}_\beta + u_\beta \\
\tan \theta &= \frac{R_s y_1 + L_s \dot{y}_1 - u_1}{-R_s y_2 + L_s \dot{y}_2 + u_2}
\end{align*}
\]

(4)

The rotor angle \( \theta \) of SMPMSM is observable and can be estimated according to equation (6).

2.2. Speed calculation

The speed is obtained by the estimated rotor angle to avoid the influence of motor parameter changes. According to the average filter theory, speed is calculated by accumulating \( \theta \) values over \( m \) samples and then multiplying the accumulated rotor angle difference by a constant. The formula is shown as
\[ \omega = \frac{1}{mT_s} \sum_{i=1}^{m} (\theta_i - \theta_{i-1}) \]  
\[ (7) \]

where \( \theta_i \) is current rotor angle, \( \theta_{i-1} \) is previous rotor angle, \( m \) denote the number of accumulation, \( T_s \) express sampling time.

2.3. Design of discrete tracking differentiator

In SMPMSM drive system, the \( i_\alpha \) and \( i_\beta \) obtained by sampling and coordinate changes contain noise signals. In order to get the first-order differential of \( i_\alpha \) and \( i_\beta \), a discrete tracking differentiator is designed as

\[
\begin{align*}
    y_1(k + 1) & = y_1(k) + T_s y_2(k) \\
    y_2(k + 1) & = y_2(k) + T_s f(y_1(k), y_2(k), u(k), b, h) 
\end{align*}
\]

where \( b \) is the speed factor, \( h \) is the filter factor, and \( T_s \) expresses sampling time. \( u(k) \) is the input signal, \( y_1(k) \) is the tracking signal of \( u(k) \), and \( y_2(k) \) is the first-order differential signal of \( u(k) \).

Define the \( f \) function as

\[
f = \begin{cases} 
    -ba / \delta, & |a| \leq \delta \\
    -b \text{sgn}(a), & |a| > \delta 
\end{cases}
\]

where \( \delta = bh \) and \( a \) is

\[
a = \begin{cases} 
    y_2 + (y_1 - u + hy_2) / h, & |y_1 - u + hy_2| \leq \delta h^2 \\
    y_2 + 0.5(\sqrt{\delta^2 + 8b|y_1 - u + hy_2|} - \delta) \text{sgn}(y_1 - u + hy_2), & |y_1 - u + hy_2| > \delta h^2
\end{cases}
\]

By adjusting the speed factor \( b \) and filter factor \( h \) of the tracking differentiator, the first-order differential signals of \( i_\alpha \) and \( i_\beta \) can be accurately obtained.

3. Algebraic parameter identification

Rewrite the q-axis voltage equation of SMPMSM as

\[ u_q = R_\text{ia} + L_\text{ia} \frac{di_a}{dt} + \omega L_\text{id} i_d + \omega \varphi_f \]  
\[ (9) \]

Multiply both sides of equation (9) by \( t \) at the same time, and integrate within the time \( [0, t] \), then equation (9) becomes

\[
\int_0^t \delta u_q(\delta) d\delta - \int_0^t (\varphi_f \delta \omega(\delta)) d\delta = L_\text{s} \left[ t_i_q(t) - \int_0^t i_q(\delta) d\delta + \int_0^t (\delta \omega(\delta) i_d(\delta)) d\delta \right] + R_\text{s} \int_0^t (\delta i_q(\delta)) d\delta
\]

Define the matrix \( \gamma, p, q \) as

\[
\gamma = \begin{bmatrix} L_\text{s} & R_\text{s} \end{bmatrix}, \quad p = \begin{bmatrix} t_i_q(t) - \int_0^t i_q(\delta) d\delta + \int_0^t (\delta \omega(\delta)i_d(\delta)) d\delta & \int_0^t (\delta i_q(\delta)) d\delta \end{bmatrix}, \quad q = \begin{bmatrix} \int_0^t \delta u_q(\delta) d\delta \end{bmatrix}
\]

Equation (10) can be rewritten as

\[ p \gamma = q \]

Because \( p \) is a singular matrix, an optimized method can be used to solve \( \gamma \). The cost function is constructed as

\[ J(\gamma, t) = \frac{1}{2} \int_0^t \epsilon^2(\gamma, \sigma) d\sigma \]

where \( \epsilon(\gamma, t) = p \gamma - q \) is error term.
The optimal solution of $\gamma$ can be defined as the value that minimizes the integral square error criterion, as

$$\hat{\gamma} = \arg\min_{\gamma} \frac{1}{2} \int_{0}^{t} (p(\sigma)\gamma - q(\sigma))^2 d\sigma$$

(13)

Here, the gradient method is used to find the minimum value of the cost function. After calculating the derivative of the unknown parameter $\gamma$ for $J(\gamma, t)$, then it becomes

$$\nabla_{\gamma} J(\gamma, t) = \frac{\partial}{\partial \gamma} \int_{0}^{t} e^2(\gamma, \sigma) d\sigma = \int_{0}^{t} \frac{\partial e^2(\gamma, \sigma)}{\partial \gamma} \varepsilon(\gamma, \sigma) d\sigma$$

Note the following equation

$$\frac{\partial e(\gamma, t)}{\partial \gamma} = \frac{\partial (p(t)\gamma - q(t))}{\partial \gamma} = p^T(t)$$

Equation (14) can be simplified to

$$\nabla_{\gamma} J(\gamma, t) = \int_{0}^{t} p^T(\sigma)[p(\sigma)\gamma - q(\sigma)]d\sigma$$

(15)

Let equation (15) be equal to 0, then

$$\int_{0}^{t} p^T(\sigma)p(\sigma)d\sigma \gamma = \int_{0}^{t} p^T(\sigma)q(\sigma)d\sigma$$

(16)

For formula (16), define the matrices $A$ and $B$ as

$$A = \int_{0}^{t} p^T(\sigma)p(\sigma)d\sigma, \quad B = \int_{0}^{t} p^T(\sigma)q(\sigma)d\sigma$$

Then $\gamma$ is obtained as

$$\gamma = A^{-1}B$$

(17)

When equation (6), (7), and (17) are combined, the differential algebraic sensorless control strategy integrated with algebraic identification method is designed, and the structure diagram of the SMPMSM drive system based on the proposed strategy is shown in figure 1.

**Figure 1.** Structure diagram of the proposed sensorless controlled SMPMSM drive system.

### 4. Modeling and simulation of SMPMSM sensorless control system

To demonstrate the effectiveness of the proposed method, the simulation model of SMPMSM drive system is established based on Matlab/Simulink. The performance at different speeds will be
investigated and compared with the conventional differential algebraic sensorless control. The nominal parameters of SMPMSM are given in table 1. The dc link voltage is controlled to 48V, and the sampling time of current loop is chosen as 100μs. In order to achieve the maximum torque current control of the SMPMSM drive system, the d-axis reference current is set to 0A. Then the q-axis reference current is calculated according to the reference torque. The tested SMPMSM operates in torque control mode, and an ideal dyno is coupled on the shaft with the SMPMSM and works under speed control mode.

Table 1. Nominal parameters of SMPMSM.

| Parameters           | Value         |
|----------------------|---------------|
| Pole pairs           | 12            |
| Stator resistance    | 0.0957 Ω      |
| Stator inductance    | 1 mH          |
| Intertia             | 0.01015 kg.m²|
| Magnetic flux        | 0.027Wb       |
| Rated torque         | 13 N.m        |
| Rated current        | 19A rms       |

The rotor speed of SMPMSM system is kept at 200r/min, furthermore, the q-axis current references are set to \(i_q^* = 0\)A (0N.m), a step change current reference \(i_q^* = 26.75\)A (13N.m) is given at 0.3s. When the stator resistance \(R_s\) increases to 1.25 times the rated value, and the stator inductance \(L_s\) decreases to 0.85 times the rated value, the dynamic identification results of motor parameters are shown in figure 2 and figure 3. When the q-axis step reference current is applied at 0.3s, \(R_s\) and \(L_s\) both converge to the actual value within 0.1s. The identification result of \(L_s\) is 0.821mH, and the steady-state error is 3.4%. The identification result of \(R_s\) is 0.1203Ω, and the steady-state error is 0.6%.

The stator resistance and stator inductance maintain the same deviation as before. The step command \(i_q^* = 26.75\)A is still given at 0.3s. First, the motor operates at 200rpm. Figure 4 clearly reveals that when the motor parameters change, the rotor position estimated by the traditional differential algebraic sensorless control method (without parameter update) has an electrical angle error of 0.14rad (8.02°). However, the proposed method (with parameter update) has a rotor electrical angle error close to 0, because it has the ability to automatically update the motor parameters. Figure 5 is the comparison results when the motor is running at 400rpm. The estimated position error of traditional method is increased to 0.31rad (17.76°). And the proposed method can still accurately estimate the rotor electrical angle.
Figure 4. Rotor angle and speed in 200rpm. Figure 5. Rotor angle and speed in 400rpm.

Figure 6 and figure 7 show the torque dynamic response of the two methods when the motor speed is 200rpm and 400rpm, respectively. When the traditional differential algebraic position sensorless control method is adopted, the output torque of SMPMSM will be reduced due to the changes of motor parameters. As the motor speed increases from 200rpm to 400rpm, the torque reduction becomes more serious. By contrast, the proposed control method can ensure that the output torque accurately tracks the torque command at different speeds.
5. Conclusion
This paper proposes a differential algebra based sensorless control method for SMPMSM drive system, which integrated with the algebraic parameter identification. The proposed method can accurately and rapidly identify the stator resistance and inductance simultaneously by utilizing the change of operating conditions of SMPMSM. Then, the motor parameters are adaptively updated, and the discrete tracking differentiator is also designed to achieve the first-order differential of the stator current for achieving the differential algebraic sensorless control. Compared with the traditional differential algebraic sensorless control method, the proposed method can estimate the accurate rotor position of the SMPMSM drive system with parameter uncertainties. Thus, the electromagnetic torque command of the motor is effectively tracked, and the proposed method has a technical advantage of strong robustness.

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