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Enhancement of Digital PID Controller Performance for Blood Glucose Level of Diabetic Patients using Disparate Tuning Techniques

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Abstract

Objectives: To design digital PID controller by using CHR-I and CHR-II tuning techniques, as it helps in finding out the tuning parameters of controllers for a specific system. Transformation of analog to digital PID controller using various transformation techniques like first order hold method, impulse-invariant mapping, Tustin approximation and zero-pole mapping equivalents and also the mathematical modeling of blood glucose level, such that a system injects the exact amount of insulin into the body of diabetic patient to maintain his/her glucose level to the normal range.

Method/Statistical Analysis: The differential equation of the blood glucose level is formulated and then it is converted to three-dimensional Laplace equation using forward Laplace transform. Using the Laplace transform the differential equation of the blood glucose is converted into a s-domain equation. Then, using the s-domain equation as the equation of the system and the Tuning techniques, CHR-I and CHR-II, the tuning parameters (Kp, Ki and Kd) are acquired. Then, it is converted into digital, i.e. in z-domain, by applying disparate transformation techniques. Findings: On analyzing the acquired equation, it is depicted that on tuning the controller with CHR-I tuning technique the system exhibits zero overshoot which is most reliable and efficient for diabetic patient. Also, a considerable settling time of 6.3362 seconds is also achieved. Application/Improvement: Therefore, a system that can inject the exact amount of insulin into the patient’s blood and bring the blood glucose level to the normal range, by automatically calculating the amount of insulin required, from the available status of blood glucose level, is being achieved.

Keywords: Blood Glucose, Diabetes, Diabetic Patients, Digital PID Controller Tuning Techniques

1. Introduction

From primordial time diabetes has been in existence in this world. This disease has always remained incurable since its emanation. However, there had been lots of research both in, biological and technological fields and it can be inferred from the indelible annals of history present. The basic way of normalizing the diabetes is to inject insulin. But the process is still done manually even after concoction of many instruments kindred to it.

In 2014, Centers for Disease Control and Prevention (CDC) National diabetes report says that 29.1 million US children and adults are affected with diabetes. Diabetes is a long-term disease which can happen when body does not respond or properly produce insulin. Insulin is a hormone which needs to absorb the body cell and use glucose as a fuel for body cells. Due to diabetes many problems occur, such as, coronary heart disease, weakness, kidney problem, non traumatic amputations, blindness, secondary infection and so on. There are 3 types of diabetes, Type-I diabetes, Type-II diabetes and Gestational diabetes.

Type-II diabetes occurs when our body cells have become resistant to the sway of insulin and hence this increases the blood glucose level and it mostly occurs in the persons with age over 45 years and also in corpulent persons. It can also emanate in person because of inherency and this type of diabetes is called as non-insulin dependent diabetes or adult onset.

Gestational diabetes ubiquitously occurs in pregnant
women during pregnancy. Aspects include pregnancy over the age of 25, overweight at the time of pregnancy and excessive intra uterine growth during pregnancy. During pregnancy, hyper glycemia increases which affects the offspring. Hyperglycemia occurs due to high blood glucose level above 120 mg/dl while hypoglycemia occurs when the blood glucose level is 60 mg/dl.

Type-I diabetes is caused by a death of beta cell in the pancreas. So, due to the absence of beta cell pancreas does not produce insulin as per the requirement of body (healthy blood glucose level is between 60mg/dl to 120mg/dl). This type of diabetes mostly occurs in childhood or adolescence, so it is also called childhood diabetes or insulin-dependent diabetes. Insulin-dependent diabetes can be easily diagnosed by injecting insulin so as to control the blood glucose level.

Here, using automatic control device (digital proportional integral derivative (PID) controller) controlling the appropriate amount of healthy blood glucose level of diabetic patient is done. If diabetic patient has blood glucose level above or below the set point then, the digital PID controller first senses the blood glucose level and if the level is not in the normal range then it automatically controls the blood glucose level by injecting the appropriate amount of external insulin required.

2. **Mathematical Modeling of Blood Glucose Level from Differential Equation**

The differential equation of blood glucose equation is

\[ r(t) = \frac{d^2 c}{dt^2} + \frac{6}{5} \frac{dc}{dt} + \frac{5}{dt} \]

(1)

Now, to convert the differential equation into Laplace domain, forward Laplace transform is used which can be mathematically expressed as

\[ C(s) \rightarrow L[c(t); t \rightarrow s] \]

(2)

\[ R(s) \rightarrow L[r(t); t \rightarrow s] \]

(3)

On applying equations (2) and (3), equation (4) comes out to be

\[ R(s) = s^2 C(s) + 6s^2 C(s) + 5s C(s) \]

(4)

Simplifying this above equation in transfer function form, the resultant is,

\[ G_c(s) = \frac{R(s)}{C(s)} = \frac{1}{s^2 + 6s^2 + 5s} \]

(5)

The equation (4) shows the transfer function of blood glucose level of diabetic patient. Simulating this equation, the step response of blood glucose level of diabetic patient shown in Figure 1 is obtained.

From Figure 1, it can be inferred that the blood glucose insulin system takes more time to settle or to attaining the steady state. Also, the steady state error is high. Therefore, using digital PID controller the steady state error is overcome and also an accurate step response with lesser rise time is obtained.

![Figure 1. Input Step Response of Blood Glucose Level of Diabetic Patient.](image1.jpg)

Figure 2 is the stability plot and Figure 3 is the bode plot of blood glucose level of diabetic patient. The stability plot shows the system is stable. So, the system performance can be improvised. Therefore, PID controller with various tuning methods like ziegler-nichols and cohen-coon method can be used.

![Figure 2. Input Step Stability Response of Blood Glucose Level of Diabetic Patient.](image2.jpg)


Figure 3. Bode Plot of Input Step Response of Blood Glucose Level of Diabetic Patient.

3. Designing of Controller

For designing the digital PID controller and for determining the error, where the error is the difference between glucose sensor’s measured value and desired value of glucose, the equation for PID controller is

\[ u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{d}{dt} e(t) \]  

(5)

Where, \( u(t) \) is output response, \( t \) is instantaneous time, \( \tau \) is the integration variable that varies from 0 to \( t \) and \( e \) is the error which is SP-MV, where SP is set point of glucose and MV is measured value of glucose. \( K_p \) is proportional gain and it depends on the present value of system. \( K_i \) is integral gain and it depends on past accumulated value of system. \( K_d \) is derivative gain and it depends on future or expected value.

For equation (5), the transfer function of PID controller is

\[ G_c(s) = K_p \left(1 + \frac{1}{sT_i} + sT_d \right) \]  

(6)

Where, \( T_i \) is integral time and \( T_d \) is derivative time.

The approximate modeling of blood glucose insulin system by using PID controller equation (5) is shown in the Figure 4 & 5.

For finding the gain parameters like \( K_p \), \( K_i \) and \( K_d \), the tuning methods like Ziegler –Nichols, Cohen-coon, Chien-Hrone-Reswick methods are used. Then the best response parameter performance between them is compared.

4. Chien-Hrones-Reswick Tuning Technique

The modified version of the Ziegler-Nichols method is Chien-Hrones-Reswick (CHR) method. In 1952, this method was developed by Chien-Hrones-Reswick which gives a better way to select a compensator for control applications. There are basically two forms of CHR and they are, Chien-Hrone-Reswick(set point regulation), also known as CHR-1 and the Chien-Hrone-Reswick(disturbance rejection), also known as CHR-2. According to Chien-Hrones-Reswick tuning technique, controller parameters are tuned in industry processes. The set point response method is summarized in Table 1 and disturbance regulation in Table 2, which show the controller parameters. These controller parameters have 0% and 20% overshoots which is again summarized in Table 1 and Table 2.
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4.1 Tuning of PID Controller by using Chien-Hrones-Reswik (Distribution Rejection) Method

By using the blood glucose equation (4) as a system and then tuning the PID controller using Chien-Hrones-Reswick (disturbance rejection) technique, the following tuning parameters for designing the appropriate controller are obtained:

\[ K_p = 3.25316, \quad K_i = 0.882044 \quad \text{and} \quad K_d = 2.51964 \]

Now, on solving the equation (4) and equation (6) the expression acquired is,

\[
Gc(s) = \frac{K_d s^2 + K_p s + K_i}{s^4 + 6s^3 + (5 + K_d)s^2 + K_p s + K_i}
\]  

(7)

On putting the values of \( K_p \), \( K_i \) and \( K_d \), the overall transfer function of blood glucose level is,

\[
H(s) = \frac{2.232s^2 + 2.575s + 2.00e^{-0.08}}{s^4 + 6s^3 + 7.232s^2 + 2.575s + 2.00e^{-0.08}}
\]  

(8)

5. Zero-Order Hold Conversion Method

For staircase inputs in time domain, we need to accurately discretized the system. The process of discretization of continuous-time system with applying the zero-order hold method as shown below,

By applying the constant value of \( U[k] \) over the zero order hold (ZOH) block and this ZOH block produce the continuous-time input \( U(t) \).

\[
U(t) = u[k], \quad kT_s \leq t \leq (k+1)T_s
\]  

(11)

\( U(t) \) is input signal to the continuous system \( H(s) \) and generates the output \( y(t) \) and the sampling of \( y(t) \) every \( T_s \) seconds generate the output \( y[k] \).

On the other hand, the discrete system \( H_d(z) \) is also converted into continuous system \( H(s) \). There is following limitation of ZOH discrete-to-continuous conversion.

5.1 Discrete to Continuous (d2c) cannot Converted into LTI Systems with Poles at \( z = 0 \)

In ZOH, discrete to continuous(d2c) conversion produce higher order continuous-time system while discrete-time LTI system has negative real poles.

This ZOH method can be used to discretize MIMO or SISO continuous time system with time delays. This method shows an accurate discretization for systems with output and input delays with no internal delays.

After this the conversion of the Chien-Hrones-Reswick (distribution rejection), equation (8), and Chien-Hrones-Reswick (set-point regulation), equation (10), into discrete domain by using zero-order hold with sampling time 0.1 sec, thus,
And for equation (10),
\[ G(z) = \frac{0.00954z^3 - 0.00926z^2 - 0.0068552z + 0.007237}{z^4 - 3.494z^3 + 4.538z^2 - 2.593z + 0.5488} \]  

6. **First-Order Hold (FOH) Method**

For piecewise linear inputs in time domain, accurately discretized system can be desired.

First-order hold method is slightly different from zero-order hold by the principle of hold mechanism. The first order hold method uses linear interpolation to convert the sample \( u[k] \) into continuous input \( u(t) \)
\[
 u(t) = u[k] + \frac{t-kTs}{Ts}(u[k+1]-u[k]), kTs \leq t \leq (k+1)Ts
\]

The First-order hold method is more suitable and more accurate system driven by smooth inputs than zero-order hold method. It differs from standard causal first-order hold method and is called as triangle approximations and it is also known as ramp-invariant approximation.

This FOH method for system time delay is same as the zero order hold method. This method can be also used to discretize MIMO or SISO continuous-time system with time delays.

After this the Chien-Hrones-Reswick (distribution rejection), equation (13), and Chien-Hrones-Reswick (set-point regulation), equation (10), are converted into discrete domain by using First-order hold with sampling time 0.1 sec, then \( G(z) \) for equation (8) is,
\[ G(z) = \frac{0.2006z^3 - 0.3762z^2 + 0.1763z - 9.786e^{-0.17}}{z^4 - 3.491z^3 + 4.534z^2 - 2.591z + 0.5488} \]  

And for equation (10) \( G(z) \) is,
\[ G(z) = \frac{0.1764z^3 - 0.3336z^2 + 0.1572z + 6.981e^{-0.17}}{z^4 - 3.494z^3 + 4.538z^2 - 2.593z + 0.5488} \]  

7. **Impulse-Invariant Mapping**

For impulse train inputs in time domain this impulse-invariant mapping conversion gives accurately discretized the systems.

Impulse-invariant mapping introduces a phase mismatch at higher frequencies and shift in DC gain of the discretized system. This phase mismatch occurs due to aliasing effects and this effect become more precise to increases the sampling time.

The aliasing effects become more prominent, when the shift in DC gain of the system decreases with decreasing the sampling time. Due to this aliasing, impulse-invariant mapping is not a good choice for matching the frequency response in continuous time system. For frequency response matching we choose the bilinear transform such as Tustin approximation.

This Impulse-invariant mapping with time delays can be used to discretize MIMO or SISO continuous time system. This method shows an accurate discretization for continuous-time systems.

After that we convert the Chien-hrones-reswick (distribution rejection) equation (8) and chien-hrones-reswick (set-point regulation) equation (10) into discrete domain by using Impulse-invariant mapping with sampling time 0.1 sec, we get \( G(z) \) for equation (8) is,
\[ G(z) = \frac{0.2006z^3 - 0.3762z^2 + 0.1763z - 9.786e^{-0.17}}{z^4 - 3.491z^3 + 4.534z^2 - 2.591z + 0.5488} \]  

And for equation (10) \( G(z) \) is,
\[ G(z) = \frac{0.1764z^3 - 0.3336z^2 + 0.1572z + 6.981e^{-0.17}}{z^4 - 3.494z^3 + 4.538z^2 - 2.593z + 0.5488} \]

8. **Tustin Approximation**

The bilinear approximation or Tustin approximation uses the following equation.
\[ Z = e^{st} = \frac{1 + \frac{sTs}{2}}{1 - \frac{sTs}{2}} \]  

The above equation is related to \( z \)-domain and \( s \)-domain functions. By using \( c2d \) conversion, the discretization \( H_s(z) \) of continuous transfer function \( H(s) \) is
\[ H_s(z) = H_s(s') \quad s' = \frac{2}{Ts} \frac{z - 1}{z + 1} \]

Similarly, by using \( d2c \) conversion, for the inverse transfer function, the continuous \( H(s) \) of discrete transfer function \( H_s(z) \) is
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\[ H(s) = H_s(z'), \quad z' = \frac{1 + \frac{sT_s}{2}}{1 - \frac{sT_s}{2}} \]  

(21)

Tustin approximation method for implicit matching of frequency domain between continuous-time system and the discretized system is used. Tustin approximation can be used to approximate discretize MIMO or SISO continuous-time systems with time delays (\(\tau\)) and \((\frac{K}{T_s})\), which is the time delay of integer portion, which maps to delay the \(k\)th sampling periods in the discretized system.

After this, conversion of the Chien-Hrones-Reswick (distribution rejection) equation (8) and Chien-Hrones-Reswick (set-point regulation) equation (10) into discrete domain by using it with Tustin approximation with sampling time 0.1 sec. For equation (13) is,

\[ G(z) = \frac{0.00508z^4 + 0.000639z^3 - 0.00952z^2 - 0.000600z + 0.00447}{z^4 - 3.486z^3 + 4.53z^2 - 2.579z + 0.54466} \]  

(22)

And for equation (15) \(G(z)\) is,

\[ G(z) = \frac{0.004477z^4 + 0.000408z^3 - 0.000468z^2 - 0.0003989}{z^4 - 3.489z^3 + 4.532z^2 - 2.579z + 0.5444} \]  

(23)

9. Zero-Pole Matching Equivalents

In zero-pole matching, equivalents are matched DC gain in discretized systems and continuous time system, they are only applied in SISO systems and their pole and zeros transformation are shown below:

\[ Z_i = e^{sT_s} \]  

(24)

Where, \(Z_i\) and \(S_i\) are the \(i\)th zero or pole in discrete-time system and continuous time system and the sampling time is \(T_s\).

This zero-pole matching with time delay can be used to discretize SISO continuous-time system. This zero-pole matching is similar to Tustin approximation in time delay.

After this, the Chien-Hrones-Reswick (distribution rejection) equation (8) and Chien-Hrones-Reswick (set-point regulation) equation (10) are converted into discrete domain by using Zero-pole matching equivalent with sampling time 0.1 sec., and hence, \(G(z)\) for equation (8) is,

\[ G(z) = \frac{0.01004z^3 - 0.008789z^2 - 0.01001z + 0.008822}{z^4 - 3.491z^3 + 4.534z^2 - 2.591z + 0.5488} \]  

(25)

And for equation (10), \(G(z)\) is,

\[ G(z) = \frac{0.008838z^3 - 0.007875z^2 - 0.008838z + 0.007875}{z^4 - 3.494z^3 + 4.538z^2 - 2.5932z + 0.5488} \]  

(26)

10. Result

10.1 By using Chien-Hrones-Reswick (Distribution Rejection) Method

Results

Figure 6 shows the step response of the complete blood glucose insulin system which is designed using the CHR (distribution rejection) tuning technique. The step response has been found out after placing each and every component in closed-loop system.

Figure 6. Step Response of Blood Glucose Insulin System.

Figure 7 demonstrates the bode plot of the system when kept in open loop after the whole system is designed using CHR (distribution rejection) method.
Figure 7. Open Loop Bode Plot of Blood Glucose System.

From Figure 8, it can be derived that it is the Bode plot of blood glucose insulin system for continuous-time approximation when CHR (disturbance rejection) tuning technique is used.

Figure 8. Continuous-Time Approximation Bode Plot of Blood Glucose Insulin.

From Figure 9, it can be inferred that it is the Bode plot of blood glucose insulin system for discrete-time approximation when CHR (disturbance rejection) tuning technique is used.

Figure 9. Discrete-Time Approximation Bode Plot of Blood Glucose Insulin System.

10.2 By using Chien-Hrones-Reswik (Set-point Regulation) Method Results

Figure 10 shows the step response of the complete blood glucose insulin system which is designed using the CHR (set point regulation) tuning technique. The step response has been found out after placing each and every component in closed-loop system.

Figure 10. Step Response of Blood Glucose Insulin System.
Figure 11 demonstrates the bode plot of the system when kept in open loop after the whole system is designed using CHR (set point regulation) method.

![Bode Diagram](image)

**Figure 11.** Open Loop Bode Plot of Blood Glucose System.

From Figure 12, it can be derived that it is the Bode plot of blood glucose insulin system for continuous-time approximation when CHR (set point regulation) tuning technique is used.

![Bode Diagram](image)

**Figure 12.** Continuous-Time Approximation Bode Plot of Blood Glucose Insulin.

From Figure 13, it can be inferred that it is the Bode plot of blood glucose insulin system for discrete-time approximation when CHR (set point regulation) tuning technique is used.

![Bode Diagram](image)

**Figure 13.** Discrete-Time Approximation Bode Plot of Blood Glucose Insulin System.

### 11. Discussion

From Table 3, the values of overshoot, settling time and rise time when the conventional techniques were used, can be deduced.

For Chien-Hrones-Reswick (Distribution Rejection) (CHR2), the overshoot, settling time and rise time are 29.41%, 17.0751 seconds and 2.2858 seconds respectively and for Chien-Hrones-Reswick (Set-point Regulation) (CHR1), they are, 0%, 6.3362 seconds and 4.8152 seconds respectively.

| Parameters          | Chien-Hrones-Reswick (Distribution Rejection) | Chien-Hrones-Reswick (Set-Point Regulation) |
|---------------------|---------------------------------------------|---------------------------------------------|
| Rise time (in seconds) | 2.2858                                      | 4.8152                                      |
| Overshoot (in percent) | 29.41                                       | 0                                           |
| Settling time (in seconds) | 17.0751                                     | 6.3362                                      |

### 12. Conclusion

Thus, we have successfully designed the Digital PID controllers for blood glucose level of diabetic patient, i.e.
PID controller using various efficient tuning algorithms. The integer order model of blood glucose level gave a very terrible response by using traditional tuning methods.

From the results, it is obvious that Chien-Hrones-Reswick(Distribution Rejection) method yields a very high Overshoot whereas Chien-Hrones-Reswick(Set-point regulation) method exhibits a zero overshoot and Chien-Hrones-Reswick(Set-point Regulation) method illustrates low settling time as compared to Chien-Hrones-Reswick(Distribution Rejection).

The overshoot in blood glucose level controller may create sudden high insulin level and endanger the life of patient. Similarly due to high settling time in Chien-Hrones-Reswick(Distribution Rejection) method the blood glucose level takes a very long time to maintain the steady state hence resulting in chances of life danger.

Finally these PIDs are converted into digital PIDs using various conversion methods. After tuning the PID it is essential to convert the analog PID to digital PID as we know hardware implementation of digital PID is very easy in minimized area. Further tuning of PID after implementation also become very easy and the system also becomes very accurate.

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