Quantized Consensus Under Data-Rate Constraints and DoS Attacks: A Zooming-In and Holding Approach

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Abstract—This article is concerned with the quantized consensus problem for uncertain nonlinear multiagent systems under data-rate constraints and denial-of-service (DoS) attacks. The agents are modeled in strict-feedback form with unknown nonlinear dynamics and external disturbance. Extended state observers (ESOs) are leveraged to estimate agents’ total uncertainties along with their states. To mitigate the effects of DoS attacks, a novel dynamic quantization with zooming-in and holding capabilities is proposed. The idea is to zoom-in and hold the variable to be quantized if the system is in the absence and presence of DoS attacks, respectively. The control protocol is given in terms of the outputs of the ESOs and the dynamic-quantization-based encoders and decoders. We show that, for a connected undirected network, the developed control protocol is capable of handling any DoS attacks inducing bounded consecutive packet losses with merely three-level quantization. The application of the zooming-in and holding approach to known linear multiagent systems is also discussed.

Index Terms—Data rate, denial-of-service (DoS) attacks, multiagent systems, quantized consensus, uncertain nonlinear systems.

I. INTRODUCTION

Consensus over digital networks is known as one of the most important issues in cooperative control of multiagent systems [1]. The success of consensus relies on efficient and reliable information exchange between the agents. According to the basic communication principles, the capacity of a real digital network is limited, i.e., at each time step or time interval, only limited data can be transmitted reliably between the agents. On the other hand, the digital network is vulnerable to malicious cyberattacks, such as denial-of-service (DoS) [2]. DoS attacks refer to destroying the information availability, which may deteriorate the consensus performance and even lead to instability. Thus, it is of theoretical significance and practical relevance to investigate the consensus problem for multiagent systems under data-rate constraints and DoS attacks.

A. Literature Review

Data-rate constraints for multiagent systems can be modeled by introducing a quantization-encoding-decoding process with a finite number of quantization levels [6]. At each time step, the sender encodes the quantized information and sends out the code via the digital network. When the neighbors receive the code, they use a decoding algorithm to reconstruct the information. Primary studies on the quantized consensus problem have been on first-order integrator multiagent systems [3], [4], [5], [6], [7]. In [3], the states of the agents were assumed to be integer-valued. In [4], a random dither was added to the sensor state before quantization to make the quantization error a “white” noise. In [6], with the use of a finite-level uniform quantizer and an exponentially decaying scaling function, it was shown that the average consensus can be achieved with one-bit data rate for a connected undirected network. The idea in [6] was further generalized to multiagent systems with higher order dynamics [8], [9], [10], [11], [12].

Recent years have also witnessed a growing interest toward control systems under DoS attacks. In [13], DoS attacks were characterized by their levels of average frequency and duration under which input-to-state stability (ISS) of the closed-loop system can be preserved. Such a characterization of DoS attacks has motivated many interesting results, e.g., [14], [15], [16], [17]. For multiagent systems under DoS attacks, Deng and Wen [18] developed a resilient fault-tolerant approach to handle the secure cooperative problem in the presence of actuator faults and DoS attacks. In [19] and [20], the event-triggered strategy was introduced to linear and nonlinear multiagent systems under DoS attacks, respectively. We mention that the communication data rate was implicitly assumed to be infinite in [18], [19], and [20], since the agents can communicate exact information with their neighbors.

Manuscript received 1 July 2022; accepted 29 October 2022. Date of publication 18 November 2022; date of current version 30 August 2023. This work was supported in part by the National Natural Science Foundation of China under Grant 61872925 and Grant 61833016 and in part by the Fundamental Research Funds for the Central Universities, China under Grant 30922010714. Recommended by Associate Editor S.-I. Azuma. (Corresponding author: Lihua Xie.)

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For the simultaneous consideration of data-rate constraints and DoS attacks, a few recent works have been done for centralized systems [21], [22], [23], [24], [25], and multiagent systems [26]. In [21], a dynamic encoding scheme was developed, and sufficient conditions on DoS frequency and duration for exponential state convergence were obtained. In [22] and [23], the tradeoff between system resilience against DoS attacks and data rate was analyzed, in which the data-rate conditions are dependent on the plant state matrix, and DoS frequency and duration. In [24], the ISS of linear systems with quantized state measurements under external disturbances and DoS attacks was studied. In [25], the scenario that both input (controller-to-plant) and output (plant-to-controller) channels are subject to data-rate constraints and DoS attacks was investigated. An interesting property of the approach in [25] is that when only the output channel is subject to DoS attacks, the widely adopted network acknowledgement signal is not required. For multiagent systems under data-rate constraints and DoS attacks, Feng and Ishii [26] proposed a dynamic quantization approach with zooming-in and zooming-out capabilities. Specifically, the scaling function used for dynamic quantization zooms-in and zooms-out the variable to be quantized in the absence and presence of DoS, respectively. It should be pointed out that the required data rate for each paired agents in [26] increases to infinity if the number of the agents $N \to \infty$. What is more, the tolerable level of DoS attacks specified by their average frequency and duration in [26] may be low, since the quantized control scheme needs to zoom-out under DoS for compensating the diverging consensus error.

By investigation of the existing literature, there are two important issues that are noteworthy. On one hand, it still lacks a solution to the quantized consensus problem of general uncertain nonlinear multiagent systems under data-rate constraints and DoS attacks. The approaches in [21], [22], [23], [24], [25], and [26] are for certain linear systems. However, most practical control systems are inherently uncertain and nonlinear [27], [28]. On the other hand, it is highly desirable to develop new techniques to achieve lower data rate and higher tolerance to DoS attacks.

### B. Main Contribution

The theme of this article is to investigate the quantized consensus problem for uncertain nonlinear multiagent systems under data-rate constraints and DoS attacks. The agents are modeled in strict-feedback form with unknown nonlinear dynamics and external disturbance. When DoS occurs, the information availability between the agents can be destroyed, and the consensus becomes difficult under DoS attacks. Therefore, an important technical difficulty we need to overcome is to efficiently govern the multiagent systems during the attack periods, with the uncertain nonlinear agent dynamics well-handled. Based on the intuitive consideration that it is expected to lead to a higher tolerance to DoS attacks if the multiagent system can hold its status instead of having a diverging trend when it is attacked, we develop a new dynamic quantization scheme which we refer to as zooming-in and holding. The new scheme is significantly different from that in the state-of-the-art result [26], as it zooms-in the variable to be quantized in the absence of DoS attacks, and holds the variable in the presence of DoS attacks. What is more, we employ an extended state observer (ESO) [29], [30] to estimate the system unknown nonlinear dynamics and external disturbance, and compensate them in real time in the control action. In summary, the main contribution of this article is the development of an ESO-based dynamic encoding–decoding scheme with zooming-in and holding capabilities for uncertain nonlinear multiagent systems under data-rate constraints and DoS attacks.

Compared with the existing literature, the advantages of the control scheme developed in this article include the following.

1) It is a more practical solution to the quantized consensus problem under data-rate constraints and DoS attacks. The developed scheme is capable of handling uncertain nonlinear dynamics and external disturbance. More importantly, it is tolerable to serious DoS attacks. We only require that the consecutive packet losses induced by DoS attacks be bounded. This is a minimum requirement since a multiagent system becomes $N$ single systems if an attack is always active.

2) It is capable of leading to better consensus performance. This is due to the nature of the zooming-in and holding mechanism, in which the consensus error keeps invariant during DoS attacks. On the contrary, the consensus error may diverge during DoS attacks for the zooming-in and zooming-out mechanism [26].

3) It achieves theoretically the lowest data rate. We show that merely three-level quantization suffices to guarantee quantized consensus, which is independent from the number of the agents and DoS attacks.

The rest of this article is organized as follows. Section II provides some preliminaries and problem formulation. The proposed control scheme for uncertain nonlinear multiagent systems is presented in Section III. In Section IV, for ease of comparison, we apply the zooming-in and holding approach to known linear multiagent systems. Two examples are provided in Section V. Finally, Section VI concludes the article.

### II. Preliminaries and Problem Formulation

#### A. Notation

In this article, we use $\mathbb{R}$, $\mathbb{R}^n$, and $\mathbb{R}^{n \times n}$ to denote the set of real numbers, $n$-dimensional real column vectors, and $n \times m$ real matrices, respectively. For a given vector or matrix $A$, its transpose, Euclidean norm, and $\infty$-norm are denoted by $A^T$, $\|A\|_2$, and $\|A\|_\infty$, respectively. For a given positive number $\nu$, the maximum integer less than or equal to $\nu$ is denoted by $\lfloor \nu \rfloor$; the minimum integer greater than or equal to $\nu$ is denoted by $\lceil \nu \rceil$. $I_n$ denotes the identity matrix of dimension $n$. $1_n$ and $0_n$ denote the $n$-dimensional column vector with all ones and zeros, respectively. Let $J_n = (1/n)1_n 1_n^T$. Big $O$-notation in terms of $\nu$ is denoted as $O(\nu)$. Let $\text{sat}(\cdot)$ be the unity saturation function defined by $\text{sat}(\nu) = \text{sign}(\nu) \cdot \min\{1, |\nu|\}$.

#### B. Graph Theory

For a group of $N$ agents, the corresponding graph $G$ can be expressed by a triple $\{V, E, A\}$, where $V = \{1, 2, \ldots, N\}$ is
the set of nodes, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ is the adjacency matrix with $a_{ij} = 1$ if $(i, j) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. It is assumed $a_{ii} = 0$, i.e., no self-loop in $\mathcal{G}$. The neighborhood of agent $i$ is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} | (i, j) \in \mathcal{E}\}$. A sequence of edges $(i_1, i_2), (i_2, i_3), \ldots, (i_{\ell-1}, i_{\ell})$ is called a path from agent $i_1$ to agent $i_{\ell}$. If for any two agents $i, j \in \mathcal{V}$, there exists a path from agent $i$ to agent $j$, $\mathcal{G}$ is called a connected graph. The in-degree of agent $i$ is represented by $\deg_i = \sum_{j \in \mathcal{N}_i} a_{ij}$, and $\delta^* = \max_{i \in \mathcal{N}} \deg_i$ is the degree of $\mathcal{G}$. Denote $\mathcal{D} = \text{diag}\{\deg_1, \ldots, \deg_N\}$ and the Laplacian matrix of $\mathcal{G}$ by $L = \mathcal{D} - \mathcal{A}$. If $\mathcal{A}$ is a symmetric matrix, $\mathcal{G}$ is called an undirected graph, and the eigenvalues of the Laplacian matrix, in an ascending order are denoted by $0 = \lambda_1 < \lambda_2 \leq \cdots \leq \lambda_N$ [32].

C. Problem Formulation

Consider a multiagent system consisting of $N$ agents described by the following single-input–single-output uncertain nonlinear dynamics:

$$
\begin{align*}
\dot{x}_{ij} &= f_{ij}(x_{i1}, \ldots, x_{ij}) + x_{i,j+1}, \ 1 \leq j \leq r - 1 \\
\dot{x}_{ir} &= f_{ir}(x_{i1}, z_{i}, \omega_i) + u_i \\
\dot{z}_i &= f_{i0}(x_i, z_i, \omega_i) \\
y_i &= x_{i1}, \ 1, \ldots, N
\end{align*}
$$

(1)

where $x_i = [x_{i1}, x_{i2}, \ldots, x_{ir}]^T \in \mathbb{R}^r$ is the state agent, $z_i \in \mathbb{R}^{n_i - r}$ is the state of the zero dynamics, $n_i$ is the dimension of agent $i$, $r$ is the relative degree, $y_i \in \mathbb{R}, u_i \in \mathbb{R},$ and $\omega_i \in \mathbb{R}^{\mu_i}$ are the measured output, control input, and external disturbance, respectively, and $f_{i0} \in C^1(\mathbb{R}^r \times \mathbb{R}^{n_i - r} \times \mathbb{R}^{\mu_i}, \mathbb{R}), f_{ij} \in C^{r+j-1}(\mathbb{R}^r, \mathbb{R}), \ 1 \leq j \leq r - 1,$ and $f_{ir} \in C^1(\mathbb{R}^r \times \mathbb{R}^{n_i - r} \times \mathbb{R}^{\mu_i}, \mathbb{R})$ are unknown locally Lipschitz functions. The strict-feedback system (1) represents a wide class of physical plants [27], such as the pendulum with motor functions. The strict-feedback system (1) represents a wide class of physical plants [27], such as the pendulum with motor functions. The strict-feedback system (1) represents a wide class of physical plants [27], such as the pendulum with motor functions. The strict-feedback system (1) represents a wide class of physical plants [27], such as the pendulum with motor functions. The strict-feedback system (1) represents a wide class of physical plants [27], such as the pendulum with motor functions. The strict-feedback system (1) represents a wide class of physical plants [27], such as the pendulum with motor functions. The strict-feedback system (1) represents a wide class of physical plants [27], such as the pendulum with motor functions. The strict-feedback system (1) represents a wide class of physical plants [27], such as the pendulum with motor functions.

Assumption A1: The communication graph $\mathcal{G}$ among the $N$ agents is undirected and connected.

Assumption A2: The external disturbances $\omega_i$ and their derivatives $\dot{\omega}_i$ are bounded.

Assumption A3: The internal dynamics $\dot{z}_i = f_{i0}(x_i, z_i, \omega_i)$ are bounded–input–bounded-state stable with respect to $(x_i, \omega_i)$.

For a real digital network connecting the multiagent system (1), information to be transmitted between the agents is to be first quantized and encoded at the transmitter, and then decoded at the receiver. In control protocols to be developed in the following, we use the following finite-level uniform quantizer:

$$
q(\nu) = \begin{cases} 
0, & -1/2 < \nu < 1/2 \\
1/2 + \frac{2i}{2}, & \nu \geq 2K + 1/2, \ i = 1, 2, \ldots, K - 1 \\
-1/2 - \frac{2i}{2}, & \nu \leq -1/2 \\
\text{ Otherwise } & 
\end{cases}
$$

where $K \in \{1, 2, \ldots, \}$. For a $(2K + 1)$-level quantizer specified by (2), the agent is required to be capable of transmitting $[\log_2(2K + 1)]$ bits at each time step. For subsequent use, let $q(\nu) = [q(\nu_1), \ldots, q(\nu_t)]^T \in \mathbb{R}^t$ with $\nu = [\nu_1, \ldots, \nu_t]^T \in \mathbb{R}^t$. On the other hand, it is assumed that the transmission is acknowledgement-based and free of delay. That is, the receivers send acknowledgments to the transmitters immediately if they receive encoded signals successfully [21], [22], [23], [24], [25], [26]. From a communication point of view, this can be realized by requiring that the acknowledgments are sent by a more powerful source.

In this article, similar to [26], we refer to DoS attacks as the event for which all the encoded signals cannot be received by the decoders and it affects all the agents. The attacks are assumed to be launched by an adversarial entity possibly having limited energy. Hence, here we characterize the attacker’s action by its capability of inducing consecutive transmission failures by launching DoS. In addition to DoS, we apply periodic transmission strategy, i.e., the transmission attempts of each agent takes place periodically at $t = kT$, in which $k$ is a nonnegative integer and $T > 0$ is the sampling interval. Due to DoS attacks, not all the transmission attempts at $kT$ succeed. Then, we let $\{v_iT\} = \{v_0T, v_1T, v_2T, \ldots\} \subseteq \{kT\}$ with $\ell = 0, 1, \ldots$ represent the sequence of time instants when the transmissions are successful.

Assumption A4: The duration of the interval $[v_{\ell+1}T - v_{\ell}T]$ calculated as $v_{\ell+1}T - v_{\ell}T$ with $\ell = 0, 1, \ldots$ is bounded.

Remark 1 (DoS Model): Under DoS attacks, the interval between two consecutive successful transmissions can be prolonged by DoS attacks. Assumption A4 constrains the maximal number of consecutive packet drops induced by DoS attacks. In [13], a DoS model was developed to describe an attacker’s action in terms of the DoS average frequency and duration, and the level of DoS was specified by $\frac{1}{T_D} + \frac{T_D}{2}$, where $T_D > 1$ and $T_D > T$ are DoS model parameters. More specifically, $\frac{1}{T_D} + \frac{T_D}{2} = 0$ indicates no DoS attacks, and $\frac{1}{T_D} + \frac{T_D}{2} = 1$ the attack can always active. Here, we mention that Assumption A4 has some connections with the DoS model in [13]. Based on the model in [13], the papers [15] and [16] presented a result concerning the upper bound of the intervals between two consecutive successful transmissions in the presence of DoS attacks. In particular, the value of the upper bound depends on the level of DoS attacks, that is, if $\frac{1}{T_D} + \frac{T_D}{2}$ is high, then the upper bound of the intervals between two consecutive successful transmissions can be long. In the context of our article, this means $v_{\ell+1}T - v_{\ell}T$ can be large. What is more, the results of the article [26] were developed based on the DoS model in [13], in which the upper bound of consecutive packet losses is involved in the design process. Here, since we will design a novel zooming-in and holding quantization strategy, we only require that the number of consecutive packet losses induced by DoS is upper bounded as in Assumption A4, and the exact upper bound is not necessary to be known for design. Our result is also applicable under other DoS models that can imply bounded consecutive packet losses.

In the following, we aim to solve the quantized consensus problem for uncertain nonlinear multiagent system (1) over a digital network under data-rate constraints and DoS attacks. Our main results will be stated in Section III. The special case that the multiagent system (1) is known and linear will be discussed in Section IV.

III. Quantized Consensus of Uncertain Nonlinear Multiagent Systems

In this section, we present the ESO-based dynamic encoding–decoding scheme with zooming-in and holding capabilities. The control protocol is based on the outputs of the observers,
encoders, and decoders. Conditions for quantized consensus under data-rate constraints and DoS attacks will be derived.

A. Protocol Design

To present the main results, we first conduct a state transformation to the agent dynamics. Let $\rho_i = [\rho_{i1}, \ldots, \rho_{ir}]^T \in \mathbb{R}^r$ with $\rho_{i1} = y_i$ and $\rho_{ij} = \rho_{i,j-1}, 2 \leq j \leq r$. Then, by an iterative procedure [35], the agent dynamics (1) can be transformed into the following normal form [27], [28]:

$$
\begin{align*}
\dot{\rho}_i &= A_p \rho_i + B_p [F_i(\rho_i, z_i, \omega_i) + u_i] \\
\dot{z}_i &= F_{i0}(\rho_i, z_i, \omega_i), \\
y_i &= \rho_{i1}, \quad i = 1, \ldots, N.
\end{align*}
$$

(3)

where $F_i \in C^1(\mathbb{R}^r \times \mathbb{R}^{n_i-r} \times \mathbb{R}^{n_i}, \mathbb{R})$, $F_{i0} \in C^1(\mathbb{R}^r \times \mathbb{R}^{n_i-r} \times \mathbb{R}^{n_i}, \mathbb{R}^{n_i-r})$, and matrices $A_p \in \mathbb{R}^{r \times r}$ and $B_p \in \mathbb{R}^{r \times 1}$ are given by

$$
A_p = \begin{bmatrix}
0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1 \\
0 & 0 & \cdots & 0
\end{bmatrix}, \quad B_p = \begin{bmatrix}
0 \\
\vdots \\
\vdots \\
1
\end{bmatrix}.
$$

Lemma 1: For $1 \leq i \leq N$, let

$$
h_i = k_1 \rho_{i1} + k_2 \rho_{i2} + \cdots + k_{r-1} \rho_{i,r-1} + \rho_{ir}
$$

(4)

where $k_1, \ldots, k_{r-1}$ are selected such that the polynomial $k_1 + k_2 \lambda + \cdots + k_{r-1} \lambda^{r-1} + \lambda^r$ is Hurwitz. If $h_i, 1 \leq i \leq N$, are bounded and $\lim_{i \to \infty} h_i = h^* + O(\nu)$ for some constant $h^*$ and small positive constant $\nu$, then $\lim_{i \to \infty} (y_i - y_j) = O(\nu)$, $1 \leq i \neq j \leq N$.

Proof: Let $\tilde{\rho}_i = [\rho_{i1}, \ldots, \rho_{i,r-1}]^T \in \mathbb{R}^{r-1}$. From (4), one has

$$
\dot{\tilde{\rho}}_i = \tilde{A} \tilde{\rho}_i + \tilde{B} h_i
$$

(5)

where the matrices $\tilde{A} \in \mathbb{R}^{(r-1) \times (r-1)}$ and $\tilde{B} \in \mathbb{R}^{(r-1) \times 1}$ are given by

$$
\begin{align*}
\tilde{A} &= \begin{bmatrix}
0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1 \\
-1 & -k_2 & \cdots & -k_{r-1}
\end{bmatrix}, \\
\tilde{B} &= \begin{bmatrix}
0 \\
\vdots \\
\vdots \\
1
\end{bmatrix}.
\end{align*}
$$

(6)

Note that system (5) is a stable linear time-invariant system in controllable canonical form with state $\tilde{\rho}_i$ and input $h_i$. Then, by [11, Lemma 1] and the superposition principle in linear control theory, one can readily conclude that $\lim_{i \to \infty} (y_i - y_j) = O(\nu), 1 \leq i \neq j \leq N$.

According to Lemma 1, the consensus of the variables $h_i, 1 \leq i \leq N$, implies the output consensus of the multiagent system (1). In the following, the control protocol $u_i$ will be designed to make $h_i$ achieve consensus. Hence, an attempt in transmitting the information of $h_i$ will be made between each paired agents. In [9] and [26], for higher order multiagent systems, the agent state in vector form was transmitted through the digital network. Recall that $h_i$ is a linear combination of the agent states and the agents are single output. We will show that transmitting the 1-D signal $h_i$ rather than the $r$-dimensional state enables us to achieve the lowest data rate.

Since for each agent, only the first state variable $y_i = \rho_{i1}$ is available, the unmeasurable agent states and unknown agent dynamics will be estimated online. Augmenting an extended integrator to the chain of integrators in (3), the ESO for the extended system is designed as

$$
\begin{align*}
\dot{\hat{\rho}}_i &= A_p \hat{\rho}_i + B_p (\hat{\rho}_{i,r+1} + u_i) + L(\varepsilon)(y_i - \hat{\rho}_{i1}) \\
\dot{\hat{\rho}}_{i,r+1} &= \frac{1}{\rho_{i1} + 1} (y_i - \hat{\rho}_{i1})
\end{align*}
$$

(7)

where $\hat{\rho}_i = [\hat{\rho}_{i1}, \ldots, \hat{\rho}_{ir}]^T \in \mathbb{R}^r$ is the estimate of the agent state $\rho_i$, and $\hat{\rho}_{i,r+1}$ is the estimate of the extended state $\rho_{i,r+1} \triangleq F_i(\rho_{i1}, z_i, \omega_i)$, which represents the total effect of the unknown nonlinear dynamics and external disturbance. For the observer (7), the initial condition is set as $0_{r+1}, \varepsilon < 1$ is a small positive constant, $L(\varepsilon) = \begin{bmatrix}
\frac{1}{\varepsilon}, \ldots, \frac{1}{\varepsilon}
\end{bmatrix}^T$, and $l_1, \ldots, l_{r+1}$ are selected such that the following matrix is Hurwitz:

$$
E = \begin{bmatrix}
-1 & l_1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
-1 & 0 & \cdots & 1 \\
-1 & l_{r+1} & \cdots & 0
\end{bmatrix}.
$$

(8)

In this article, we assume that the output $y_i$ is free of measurement noise. Otherwise, some other techniques, such as nonlinear-gain structure [31], should be adopted to improve the robustness to measurement noise. What is more, the peaking phenomenon is also known as an important feature of the ESO (7). More specifically, $|\rho_{ij} - \hat{\rho}_{ij}|, 2 \leq j \leq r + 1$, might peak to $O(1/\varepsilon^j - 1)$ value during the initial short transient period.

To overcome the peaking phenomenon, the saturation technique [30] is employed to saturate the estimates $\hat{\rho}_1$ to $\rho_{i,r+1}$ outside a compact set of interest. Let $\bar{\rho}_{ij} = M_j \text{sat}(\frac{\rho_{ij}}{M_j})$, $1 \leq j \leq r + 1$, where $M_j$ are bounds selected such that the saturations will not be invoked during the steady period of the observer [30]. Correspondingly, denote $\tilde{\rho}_i = [\bar{\rho}_{i1}, \ldots, \bar{\rho}_{ir}]^T$ and

$$
\tilde{h}_i = k_1 \tilde{\rho}_{i1} + k_2 \tilde{\rho}_{i2} + \cdots + k_{r-1} \tilde{\rho}_{i,r-1} + \tilde{p}_i.
$$

(9)

Based on the output of the ESO, the information of $\tilde{h}_i$ will be transmitted through the DoS affected digital network. Recall that $t = kT$ with $k = 0, 1, \ldots$ denotes the periodic sampling instants at which the multiagent system (1) attempts to exchange information. The data to be transmitted at time $t = kT$ are denoted by $\tilde{T}_i(kT)$, and some of them may not be successfully transmitted if DoS is present at $kT$. For the instants at which DoS is not present, as mentioned before Assumption A4, we let $v_0 T, v_1 T, v_2 T, \ldots \text{represent the sequence of time instants when the transmissions are successful.}$ Let $H_v \triangleq \{v_i; i \geq 0\}$.

According to the acknowledgement signal, the encoder for agent $i$ is designed as

$$
\begin{align*}
\chi_i(0) &= 0 \\
\chi_i((k+1)T) &= \chi_i(kT) + \beta((k+1)T) \Delta_i((k+1)T), \quad \text{if } k + 1 \in H_v \\
\chi_i((k+1)T) &= \chi_i(kT), \quad \text{if } k + 1 \notin H_v \\
\Delta_i((k+1)T) &= q \left( \frac{\overline{T}_{(k+1)T} - \chi_i(kT)}{\beta(kT)} \right)
\end{align*}
$$

(10)

where $\chi_i$ is the internal state of the encoder and $\Delta_i$ is binary data, which represents the output of the encoder. In (10), $\beta((k+1)T)$ is the scaling function that is initialized with $\beta(0) = \beta_0 > 0$ and updated by the following mechanism:

$$
\beta((k+1)T) = \max\{\gamma \beta((kT), \sqrt{\varepsilon}\}
$$

(11)

with

$$
\gamma = \begin{cases}
\gamma_1, & \text{if } k + 1 \in H_v \\
1, & \text{if } k + 1 \notin H_v
\end{cases}
$$

(12)
where $0 < \gamma_1 < 1$ will be specified latter. Note that here we let $eta(kT) \to \sqrt{\varepsilon}$ rather than 0 as in [26]. The explanation is that in the quantizer employed in (10), we use the ESO estimated information $\tilde{h}_i((k+1)T)$ rather than the accurate information $h_i((k+1)T)$. In the subsequent section, we will show that for any time interval $[\tau_1, \tau_2] \subseteq [0, \infty)$, $\frac{\partial}{\partial \gamma} (\tilde{h}_i(\gamma)-\tilde{h}_i(\gamma))_{\tau_1}^{\tau_2} \to 0$ as $\varepsilon \to 0$. Therefore, letting $\beta(kT) \to \sqrt{\varepsilon}$ prevents the saturation of the quantizer from the accumulated ESO estimation error. What is more, note that in (10), the scaling function $\beta(kT)$ appears in the denominator of the information to be quantized. This together with its update mechanism (11)–(12), for $\beta(kT) > \sqrt{\varepsilon}$, the sensitivity of the quantizer to the term $[\tilde{h}_i((k+1)T) - \chi_i(kT)]$ increases if the system is free of DoS attacks (since $\gamma = \gamma_1 < 1$), and keeps invariant if under DoS attacks (since $\gamma = 1$).

If agent $i$ is a neighbor of agent $j$, $\Delta_j$ (which is the output of the encoder for agent $j$) will be transmitted through the DoS affected digital network. Agent $i$ uses the following decoder to reconstruct $\hat{h}_j$:

$$
\begin{align*}
\hat{h}_j(0) &= 0 \\
\hat{h}_j((k+1)T) &= \hat{h}_j(kT) + \beta(kT)\Delta_i((k+1)T) \\
\hat{h}_j(\ell) &= \hat{h}_j(kT), \text{ if } k+1 \notin H_s \\
\end{align*}
$$

(13)

where $\hat{h}_j$ is the estimate of $h_j$ obtained by agent $i$. Since the initial conditions of the encoders and decoders are set as 0, we let $x_0 = 0$ henceforth.

With the aid of the ESO (7), encoder (10), and decoder (13), we propose the following protocol:

$$
\begin{align*}
u_i(t) &= -\sum_{\ell=1}^{r-1} k_{\ell}\rho_{\ell,t+1}(t) - \rho_{i,t+1}(t) \\
&\quad + c \sum_{j \in N_{i,DoS}(kT)} a_{ij} \left( \hat{h}_j(kT) - \chi_i(kT) \right) \\
&\quad + c \sum_{j \in N_{i,DoS}(kT)} a_{ij} \left( \hat{h}_j(kT) - \chi_i(kT) \right) \\
&\quad + \sum_{\ell=1}^{r} k_{\ell} \left( \rho_{i,t+1}(t) - \rho_{i,t+1}(t) \right), t \in [kT, (k+1)T]
\end{align*}
$$

(14)

where $c > 0$, and $N_{i,DoS}(kT)$ represents the set of agents that are able to exchange information with agent $i$ through the DoS affected digital network at $t = kT$. That is, $N_{i,DoS}(kT) = N_i$ if $k \in H_s$, and $N_{i,DoS}(kT) = \emptyset$ otherwise. In the control protocol (14), the term $-\sum_{\ell=1}^{r} k_{\ell}\rho_{\ell,t+1}(t) - \rho_{i,t+1}(t)$ is to compensate for the agent dynamics in real time, and the term $c \sum_{j \in N_{i,DoS}(kT)} a_{ij} \left( \hat{h}_j(kT) - \chi_i(kT) \right)$ satisfies

$$
\begin{align*}
&c \sum_{j \in N_{i,DoS}(kT)} a_{ij} \left( \hat{h}_j(kT) - \chi_i(kT) \right) \\
&= c \sum_{j \in N_{i,DoS}(kT)} a_{ij} \left( \hat{h}_j(kT) - \hat{h}_j(kT) \right) \\
&\quad - c \sum_{j \in N_{i,DoS}(kT)} a_{ij} \left( \hat{h}_j(kT) - \hat{h}_j(kT) \right) \\
&\quad + c \sum_{j \in N_{i,DoS}(kT)} a_{ij} \left( \hat{h}_j(kT) - \hat{h}_j(kT) \right)
\end{align*}
$$

(15)

where the second term represents the weighted sum of the estimation errors for agent $i$'s neighbors' information $\hat{h}_j(kT)$; and the last term represents the weighted sum of the estimation errors for $\hat{h}_i(kT)$ by agent $i$'s neighbors. The last term plays an error-compensation role in the protocol and is mainly based on the assumption that $a_{ij} = a_{ji}$. This mechanism was first considered in [37], and also employed in many previous quantized consensus results [5], [6], [8], [9], [10], [11]. The extension of the protocol (14) to direct communication graphs is not straightforward, and some extra techniques are needed [7].

The proposed control scheme for uncertain nonlinear multiagent systems under data-rate constraints and DoS attacks is depicted in Fig. 1.

B. Convergence Analysis

Before stating the convergence results, we first present the dynamics of the multiagent system in the absence and presence of DoS attacks. Denote $H(kT) = [h_1(kT), \ldots, h_N(kT)]^T$, $\overline{H}(kT) = [\overline{h}_1(kT), \ldots, \overline{h}_N(kT)]^T$, $\overline{H}(kT) = [\chi_1(kT), \ldots, \chi_N(kT)]^T$, $\mu(kT) = \overline{H}(kT) - \overline{H}(kT)$, and $\delta(kT) = H(kT) - J_N H(kT)$. Note that $\mu(kT)$ and $\delta(kT)$ represent the compact quantization error and consensus error, respectively.

If the multiagent system is in the absence of DoS attack, i.e., $k \in H_s$, then by (14) and the definition of $h_i$, one has

$$
\begin{align*}
\dot{h}_i(t) &= \sum_{\ell=1}^{r} k_{\ell}\rho_{\ell,t+1}(t) + u_i(t) \\
&= c \sum_{j \in N_i} a_{ij} \left( \hat{h}_j(kT) - \chi_i(kT) \right) \\
&\quad + \sum_{\ell=1}^{r} k_{\ell} \left( \rho_{i,t+1}(t) - \rho_{i,t+1}(t) \right), t \in [kT, (k+1)T]
\end{align*}
$$

(15)

where $k_r = 1$. It follows that

$$
\begin{align*}
h_i((k+1)T) &= h_i(kT) \\
&\quad + cT \sum_{j \in N_i} a_{ij} \left( \hat{h}_j(kT) - \chi_i(kT) \right) + \varsigma_i(kT)
\end{align*}
$$

(16)

where

$$
\varsigma_i(kT) = \int_{kT}^{(k+1)T} \left( \sum_{\ell=1}^{r} k_{\ell} \left( \rho_{i,t+1}(t) - \rho_{i,t+1}(t) \right) \right) dt.
$$

According to the structures of the paired encoder (10) and decoder (13), one has

$$
\hat{h}_j(kT) = \chi_j(kT), i \in N_j, j = 1, 2, \ldots, N.
$$

(17)

This together with (16) leads to

$$
\begin{align*}
H((k+1)T) &= H(kT) - cT \mathcal{L} \overline{H}(kT) + \varsigma_i(kT) \\
&= (I - cT \mathcal{L}) \overline{H}(kT) + cT \mathcal{L} \mu(kT) \\
&\quad + H(kT) - \overline{H}(kT) + \varsigma_i(kT)
\end{align*}
$$

(18)
Architecture of the proposed control scheme for uncertain nonlinear multiagent systems under data-rate constraints and DoS attacks.

where \( \varsigma_1(kT) = [\varsigma_{11}(kT), \ldots, \varsigma_N(kT)]^T \). Since \( \mathcal{L}J_N = J_N \mathcal{L} = 0 \), by (18), one gets
\[
\delta((k+1)T) = (I - J_N)H((k+1)T)
\]
\[
= (I - cT \mathcal{L})\delta(kT) + cT \mathcal{L}\mu(kT)
\]
\[
+ cT \mathcal{L}(H(kT) - \widehat{H}(kT))
\]
\[
+ (I - J_N)\varsigma_1(kT)
\]
\[
(19)
\]
and
\[
\widehat{H}((k+1)T) - \widehat{H}(kT)
\]
\[
= H((k+1)T) - \widehat{H}(kT) + \widehat{H}((k+1)T) - H((k+1)T)
\]
\[
= (I + cT \mathcal{L})\mu(kT) - cT \mathcal{L}\delta(kT)
\]
\[
+ (I + cT \mathcal{L})(H(kT) - \widehat{H}(kT))
\]
\[
+ \varsigma_1(kT) + \widehat{H}((k+1)T) - H((k+1)T)
\]
\[
(20)
\]
If the multiagent system is in the presence of DoS attack, i.e., \( k \not\in H_v \), then one has
\[
\hat{h}_i(t) = \sum_{\ell=1}^{r} k_{\ell}(\rho_{i,\ell+1}(t) - \bar{p}_{i,\ell+1}(t)), t \in [kT, (k+1)T). \]
\[
(21)
\]
It follows that
\[
H((k+1)T) = H(kT) + \varsigma_1(kT).
\]
\[
(22)
\]
By (22), one has
\[
\delta((k+1)T) = \delta(kT) + (I - J_N)\varsigma_1(kT)
\]
\[
(23)
\]
and
\[
\widehat{H}((k+1)T) - \widehat{H}(kT)
\]
\[
= \mu(kT) + \varsigma_1(kT) + H(kT) - \widehat{H}(kT)
\]
\[
+ \widehat{H}((k+1)T) - H((k+1)T).
\]
\[
(24)
\]
According to the update mechanisms of the encoders and decoders, the evolution equations of \( \mu(kT) \) are stated as follows:
\[
\mu((k+1)T) = \left[ \widehat{H}((k+1)T) - \widehat{H}(kT) \right] - \beta(kT)Q\left( \widehat{H}((k+1)T) - \widehat{H}(kT) \right)
\]
\[
if k + 1 \in H_v
\]
\[
(25)
\]
\[
\mu((k+1)T) = \widehat{H}((k+1)T) - \widehat{H}(kT)
\]
\[
if k + 1 \not\in H_v
\]
\[
(26)
\]
where \( \widehat{H}((k+1)T) - \widehat{H}(kT) \) is given by (20) if \( k \in H_v \), and given by (24) if \( k \not\in H_v \).

The dynamics of the multiagent system, in terms of the quantization error \( \mu(kT) \) and consensus error \( \delta(kT) \), are given by (19), (23), (25), and (26). The following remark explains the design philosophy of the proposed zooming-in and holding approach in the context of quantized consensus of multiagent systems under DoS attacks.

Remark 2 (Design Philosophy): When the transmissions are successful, the agents are driven to achieve consensus. In this case, similar to [6], [7], [8], [9], [10], [11], and [26], the scaling function \( \beta(kT) \) zooms-in the information to be quantized (i.e., the term \( \widehat{H}((k+1)T) - \chi_i(kT) \)) to decrease the consensus error. Therefore, \( \gamma_1 \) is selected less than 1. When the transmissions are unsuccessful, the consensus process is interrupted. The strategy in this article is to hold the previous status of the whole system. In the consensus error dynamics (23) under DoS attacks, anticipating that \( \varsigma_1(kT) \rightarrow 0 \) as \( \varepsilon ightarrow 0 \), one gets \( \delta((k+1)T) = \delta(kT) \) if \( k \not\in H_v \). What is more, the status of the encoders and decoders are also hold, as we let \( \chi_i(kT) = \chi_i((k-1)T) \), \( \tilde{h}_j((k-1)T) = \tilde{h}_j((k-1)T) \), and \( \beta(kT) = \beta((k-1)T) \) if \( k \not\in H_v \). Such a zooming-in and holding mechanism is expected to lead to higher tolerance to DoS attacks. We should point out that the zooming-in and holding approach is fundamentally different from the zooming-in and zooming-out approach in [26] (see Section IV for more details).

In the following, we will state the convergence results of the closed-loop system. Let \( \mathcal{X}^\dagger \) be a compact subset of \( \mathbb{R}^r \), and \( \mathcal{X} \) slightly smaller than \( \mathcal{X}^\dagger \) (i.e., \( \mathcal{X} \subseteq \mathcal{X}^\dagger \) and their boundaries
are disjoint). Denote $C_h = \max_{1 \leq i \leq N} |h_i(0)|$. Let $\omega_i \in \mathbb{W}_i$ for some compact set $\mathbb{W}_i \subseteq \mathbb{R}^{m_i}$, and let $z_i(0) \in Z_i$ for some compact set $Z_i \subseteq \mathbb{R}^{n_i}$. By Assumption A3, there exists a positive constant $N_{zi}$ such that $\sup_{t \in [0, \infty]} \|z_i(t)\| \leq N_{zi}$ for all $z_i(0) \in Z_i$, $\rho_i \in \mathbb{X}_i^1$, and $\omega_i \in \mathbb{W}_i$. Note that any compact subset of $\mathbb{R}^r \times \mathbb{R}^{m_i} \times \mathbb{R}^{n_i}$ can be put in the interior of $\mathbb{X}_i \times Z_i$. We first state the following lemma.

**Lemma 2 [6]:** If Assumptions A1 and A2 hold and $c \in (0, 2/(T\lambda_N))$, then $\rho_h < 1$, where

$$\rho_h = \max_{0 \leq i \leq N} \left\{ e^{-cT\lambda_i} \right\}.$$  

Furthermore, if $c \in (0, 2/(T(\lambda_2 + \lambda_N)))$, then $\rho_h = 1 - cT\lambda_2$.

Now, we are in a position to state our first main result.

**Theorem 1:** Consider the multiagent system (1) with a digital communication network subject to data-rate constraints and DoS attacks. Given the control protocol (14) with ESO (7), encoder (10), and decoder (13). Suppose Assumptions A1–A4 are satisfied, and the initial conditions of the agents $(\rho_i(0), z_i(0)) \in \mathbb{X}_i \times Z_i, 1 \leq i \leq N$. Let

$$c \in (0, 2/(T\lambda_N)),$$  

$$\gamma_1 \in (\rho_h, 1),$$  

$$K \geq \left[ \frac{K_1(c, \gamma_1)}{\gamma_1(1 + \gamma_1)} \right] + 1,$$  

$$\beta_0 \geq \max \left\{ \frac{2cT\lambda NC_h}{\gamma_1(1 + 2\rho_h)}, \frac{4C_h(1 - \rho_h)}{cT\lambda_N} \right\}$$

where

$$K_1(c, \gamma_1) = \frac{1 + 2cT\Delta^2}{2\gamma_1} + \frac{\sqrt{Nc}T^2\lambda_N^2}{2\gamma_1(1 - \rho_h)}.$$  

Then, for any $\sigma > 0$, there exists $\varepsilon^! > 0$, which is dependent on $\sigma$, such that for every $\varepsilon \in (0, \varepsilon^!)$:

1) The ESO (7) achieves practical convergence, i.e., there exists $\tau_0(\varepsilon) > 0$ satisfying $\lim_{t \to \infty} \tau_0(\varepsilon) = 0$, such that $\forall t \in [\tau_0(\varepsilon), \infty)$

$$|\rho_i(t) - \rho_i(t)| \leq \delta, 1 \leq i \leq N, 1 \leq j \leq r + 1;$$

2) The quantizer will never be saturated, and the multiagent system (1) achieves practical output consensus, i.e.,

$$\lim_{t \to \infty} |y_i(t) - y_j(t)| \leq \sigma, 1 \leq i \neq j \leq N.$$  

**Proof:** See the Appendix.

As stated in Theorem 1, the DoS attacks can be arbitrarily serious as long as their induced maximal number of consecutive packet losses is bounded. Also note that $K \to \infty$ as $N \to \infty$. However, the quantization level (or data rate) of a real digital network is limited. Our next theorem shows that the selection of $K$ can be made independent of the number of the agents. In particular, the lowest quantization level we achieve is $3$, i.e., $K = 1$. That is, the communication channel is required to be capable of transmitting merely 2 bits at each time step, no matter how large the number of the agents is.

**Theorem 2:** Consider the multiagent system (1) with a digital communication network subject to data-rate constraints and DoS attacks. Given the control protocol (14) with ESO (7), encoder (10), and decoder (13). Suppose Assumptions A1–A4 are satisfied, and the initial conditions of the agents $(\rho_i(0), z_i(0)) \in \mathbb{X}_i \times Z_i, 1 \leq i \leq N$. For any given $K \geq 1$, let $\beta_0$ be selected according to (31), and let

$$c \in (0, \min\{2/(T(\lambda_2 + \lambda_N)), c_m\})$$

$$\gamma_1 = 1 - (1 - \varepsilon^0)cT\lambda_2$$

where $\varepsilon^0 \in (0, 1)$ and $c_m = 2\rho_i\lambda_2 T^{-1}[\sqrt{N}\lambda_N^2 + 2\rho_i\lambda_2 d^\nu + (2K + 1)(1 - \varepsilon^0)cT\lambda_2]^{-1}$. Then, the statements in Theorem 1 hold.

**Proof:** By the definition of $c_m$, one can verify that $K_1(c, \gamma_1)$ specified by (32) satisfies $\frac{1}{2} < K_1(c, \gamma_1) < K + \frac{1}{2}$. This together with (35) and (36) yields that the conditions in (28)–(30) are all satisfied. Then, similar to Theorem 1, one can complete the proof of Theorem 2.

**Theorem 2** indicates that the proposed control protocol is capable of handling any DoS attacks inducing bounded consecutive packet losses for the uncertain nonlinear multiagent system (1) with merely 2 bits information exchange between each pair of adjacent agents for each transmission attempt. Also note that $\gamma_1 \to 1$ as $N \to \infty$, which means that, under limited data rate, the convergence rate becomes slow if the number of the agents increases.

**Remark 3 (Parameters Selection):** The saturation bounds $M_i$ to $M_{i+1}$ satisfying $M_j \geq \sup_{t \leq N} |\rho_j(t)|, 1 \leq j \leq r + 1$, are selected such that the saturations will not be invoked in the steady state of the observer [30]. The parameter $C_h$ is to prevent the saturation of the quantizer in the initial stage [6], [7], [8], [9], [10], [11], [26]. The selections of $M_j$ and $C_h$ might end up with some conservative values due to the uncertain nonlinear agent dynamics. For the parameter $\varepsilon$, theoretically, smaller $\varepsilon$ leads to higher estimation and consensus accuracy. Unfortunately, in the case of uncertain dynamics, concrete and constructive algorithms to calculate the upper bound of $\varepsilon$ (i.e., $\varepsilon^!$) are very challenging to obtain and still missing in the literature [30]. This is indeed a cost we pay for the uncertainties. In this article, the scope of the current work is to develop a control scheme with the existence of certain parameters to handle the quantized consensus problem under data-rate constraints and DoS attacks. In practice, such parameters can be decided by a simple trial and error procedure [30], or by a simple line search algorithm [36]. In fact, the ESO has been successfully adopted in many control engineering problems in recent years [38], [39]. At last, we mention that the sampling period $T$ is not a design parameter in Theorem 1. As $T$ is always multiplied by the constant $c$, the effects of the sampling period on the control design can be adjusted by the value of $c$.

**Remark 4 (Comparison with [12]):** In our previous work [12], a higher order ESO-based protocol was developed to handle the limited data-rate problem for a class of simpler nonlinear multiagent systems without DoS attacks. In this article, we consider more general uncertain nonlinear systems, and our focus is on the coexistence problem of data-rate constraints and DoS attacks. Due to the communication interruption caused by DoS, the consensus error may diverge during the DoS periods, while in [12], the communication network is always reliable.

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**References:**

[6], [7], [8], [9], [10], [11], [26], [30], [38], [39].
Therefore, in this article, new encoding–decoding scheme is needed, and the behavior of the system during the attack periods must be carefully governed. Accordingly, we developed a novel dynamic quantization with zooming-in and holding capabilities, and showed that it enables better transient consensus performance and higher level of tolerable DoS attacks. These render the contribution of this article with respect to [12] substantial.

IV. QUANTIZED CONSENSUS OF KNOWN LINEAR MULTIAGENT SYSTEMS

In this section, for the ease of comparison with state-of-the-art results, we revisit the quantized consensus problem for a class of known discrete-time linear multiagent systems under data-rate constraints and DoS attacks.

A. Protocol Design

Without loss of generality, it is assumed that the model of agent $i$, $1 \leq i \leq N$, is already in the following controllable canonical form:

$$x_i((k + 1)T) = Ax_i(kT) + Bu_i(kT)$$

(37)

where $x_i \in \mathbb{R}^r$ is the state, and matrices $A \in \mathbb{R}^{r \times r}$ and $B \in \mathbb{R}^{r \times 1}$ are given by

$$A = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -a_1 & -a_2 & \cdots & -a_r \end{bmatrix}, B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}.$$  

Here, we are interested in the case where $A$ has at least one eigenvalue on or outside the unit circle. Otherwise, the multiagent system (37) cannot achieve state consensus by setting $u_i(kT) \equiv 0$. This implies that any communication protocol among the agents and any pattern of DoS attacks do not influence the trivial consensus problem if $A$ is a stable matrix.

Denote

$$h_i(kT) = k_1x_{i1}(kT) + \cdots + k_{r-1}x_{i,r-1}(kT) + x_{ir}(kT).$$

(38)

For the discrete-time case, $k_1, \ldots, k_{r-1}$ are selected such that the eigenvalues of matrix $A$ specified by (6) are real and distinct, and located inside the unit circle. For subsequent use, let $\tilde{\lambda}_1, \ldots, \tilde{\lambda}_{r-1}$ be the eigenvalues of matrix $A$, and let $\Lambda = \text{diag}(\tilde{\lambda}_1, \ldots, \tilde{\lambda}_{r-1})$. It follows that there exists an invertible matrix $\Gamma \in \mathbb{R}^{(r-1) \times (r-1)}$ such that $\Gamma A\Gamma^{-1} = \Lambda$. Let $\tilde{A}_1 = \text{diag}(1/(1-\tilde{\lambda}_1), \ldots, 1/(1-\tilde{\lambda}_{r-1})$ and $\tilde{A}_{r-1} = [-k_1, \ldots, -k_{r-1}]$ be the $(r-1)$th row of matrix $A$.

Lemma 3: If $h_i(kT)$, $1 \leq i \leq N$, $k \geq 0$, are bounded and

$$\lim_{k \to \infty} \|x_i(kT) - x_j(kT)\| = 0, \quad 1 \leq i \neq j \leq N.$$  

(39)

Furthermore,

$$\lim_{k \to \infty} x_i(kT) = \begin{bmatrix} \Gamma \tilde{A}_1 \Gamma^{-1} \tilde{B} \\ 1 + \tilde{A}_{r-1} \Gamma \tilde{A}_1 \Gamma^{-1} \tilde{B} \end{bmatrix} h^*$$

(40)

where the matrices $\tilde{A}$ and $\tilde{B}$ are given by (6).

Proof: Let $\tilde{x}_i(kT) = [x_{i1}(kT), \ldots, x_{i,r-1}(kT)]^T \in \mathbb{R}^{r-1}$. By (38), one has

$$\tilde{x}_i((k + 1)T) = \tilde{A} \tilde{x}_i(kT) + \tilde{B} h_i(kT).$$

(41)

Note that the eigenvalues of matrix $\tilde{A}$ are inside the unit circle, and $h_i(kT)$ is bounded and converges to $h^*$. From the linear control theory [40], one can readily obtain (39). Furthermore, according to the response of the discrete-time linear system (41) and the relation $A^k = \Gamma \tilde{A}^k \Gamma^{-1}$, one has $\lim_{k \to \infty} \tilde{x}_i(kT) = \Gamma \tilde{A}^k \Gamma^{-1} B h^*$. This together with (38) yields (40).

By Lemma 3, the consensus of $h_i(kT)$ implies the consensus of $x_i(kT)$. The dynamics of $h_i(kT)$ is given by

$$h_i((k + 1)T) = \sum_{\ell=1}^{r-1} k_{\ell} x_{i,\ell+1}(kT) - \sum_{m=1}^{r} a_m x_{im}(kT) + u_i(kT).$$

(42)

Similar to Section III, the encoder for agent $i$ is designed as

$$h_i((k + 1)T) = \chi_i((k + 1)T)$$

$$\begin{bmatrix} \chi_i(0) \\ \chi_i((k + 1)T) = \chi_i(kT) + \beta(kT) \Delta_i((k + 1)T) & \text{if } k + 1 \in H_v \\ \chi_i((k + 1)T) = \chi_i(kT) + \beta(kT) \Delta_i((k + 1)T) & \text{if } k + 1 \not\in H_v \\ \Delta_i((k + 1)T) = q (h_i((k + 1)T) - \chi_i(kT)) \end{bmatrix}$$

(43)

where the scaling function is updated by

$$\beta((k + 1)T) = \gamma \beta(kT)$$

(44)

with $\gamma$ given by (12). Note that in (43), we use the accurate value of $h_i$, while the ESO estimated value $\hat{h}_i$ in Section III. That is, no additional ESO estimation error will be injected into the quantization process here. Therefore, in this section, we let the scaling function converge to 0 rather than $\sqrt{\gamma}$. The decoder for agent $i$ corresponding to the edge $(i,j)$ is as follows:

$$\begin{bmatrix} \hat{h}_{ji}(0) \\ \hat{h}_{ji}((k + 1)T) = \hat{h}_{ji}(kT) + \beta(kT) \Delta_j((k + 1)T) & \text{if } k + 1 \in H_v \\ \hat{h}_{ji}((k + 1)T) = \hat{h}_{ji}(kT) & \text{if } k + 1 \not\in H_v \end{bmatrix}$$

(45)

where $\hat{h}_{ji}$ is the estimate of $h_j$ obtained by agent $i$. Here, we remind that for the known linear multiagent systems in this section, $h_j$ is specified by (38). The protocol for the known linear multiagent system (37) is then given by

$$u_i(kT) = - \sum_{\ell=1}^{r-1} k_{\ell} x_{i,\ell+1}(kT) + \sum_{m=1}^{r} a_m x_{im}(kT) + h_i(kT)$$

$$+ cT \sum_{j \in \mathcal{N}_i^{\text{known}}(kT)} a_{ij} \left( \hat{h}_{ji}(kT) - \chi_i(kT) \right)$$

(46)

where $c > 0$.

B. Convergence Analysis

First of all, denote $H(kT)$ and $d(kT)$ as in Section III-B, and let $\tilde{H}(kT) = [\chi_1(kT), \ldots, \chi_N(kT)]^T$ and $\mu(kT) = H(kT) - \tilde{H}(kT)$ in this section.
If the multiagent system (37) is in the absence of DoS attack, i.e., $k \in H_v$, then by (42) and (46), one has
\[
h_i((k + 1)T) = h_i(kT) + cT \sum_{j \in N_i} a_{ij}(\hat{h}_{ji}(kT) - \chi_j(kT)).
\] (47)
This together with $\hat{h}_{ji}(kT) = \chi_j(kT)$, $i \in N_j$, gives
\[
H((k + 1)T) = H(kT) - cT\mathcal{L}\hat{H}(kT)
= (I - cT\mathcal{L})H(kT) + cT\mathcal{L}\mu(kT).
\] (48)
By $\mathcal{L}J_N = J_N\mathcal{L} = 0$, one has
\[
\dot{\delta}((k + 1)T) = (I - cT\mathcal{L})\delta(kT) + cT\mathcal{L}\mu(kT)
\] (49)
and
\[
H((k + 1)T) - \hat{H}(kT) = (I + cT\mathcal{L})\mu(kT) - cT\mathcal{L}\delta(kT).
\] (50)
If the multiagent system (37) is in the presence of DoS attack, i.e., $k \notin H_v$, then one has
\[
h_i((k + 1)T) = h_i(kT).
\] (51)
It follows that
\[
\dot{\delta}((k + 1)T) = \delta(kT)
\] (52)
and
\[
H((k + 1)T) - \hat{H}(kT) = \mu(kT).
\] (53)
The evolution equations of $\mu(kT)$ are given by
\[
\mu((k + 1)T) = \left[ H((k + 1)T) - \hat{H}(kT) \right]
- \beta(kT)Q \frac{H((k + 1)T) - \hat{H}(kT)}{\beta(kT)}
\] (54)
if $k + 1 \in H_v$
\[
\mu((k + 1)T) = \mu(kT), \text{ if } k + 1 \notin H_v
\] (55)
where $[H((k + 1)T) - \hat{H}(kT)]$ is given by (50) if $k \in H_v$, and given by (53) if $k \notin H_v$.

The dynamics of the multiagent system (37), in terms of the quantization error $\mu(kT)$ and consensus error $\delta(kT)$, are represented by (49), (52), (54), and (55). Now, we are ready to present the result concerning the state consensus of the known linear multiagent system (37) with the zooming-in and holding approach.

**Theorem 3:** Consider the known linear multiagent system (37) with a digital network subject to data-rate constraints and DoS attacks. Given the control protocol (46) with encoder (43) and decoder (45). Suppose Assumptions A1 and A4 are satisfied, and the initial conditions of the agents $x_i(0) \in X$, $1 \leq i \leq N$. For any given $K \geq 1$, let the parameter $c$ and $\gamma_1$ be selected as in Theorem 2, and the scaling function be updated by (44) and (12), and
\[
\beta_0 > \max \left\{ \frac{C_h}{K + \frac{1}{2}}, \frac{2C_h(\gamma_1 - \rho_h)(cT\kappa_N + 2\gamma_1)}{cT\kappa_N} \right\}.
\] (56)
Then, the quantizer will never be saturated, and the multiagent system (37) achieves state consensus specified by (39). Furthermore, the final consensus value is given by
\[
\lim_{k \to \infty} x_i(kT) = \left[ \frac{\Gamma A^{-1} \tilde{B}}{1 + \tilde{A}_v \Gamma A^{-1} \tilde{B}} \right] \frac{1}{N} \sum_{m=1}^{N} \sum_{j=1}^{N} k_j x_{mj}(0).
\] (57)

**Proof:** The proof is straightforward by following a similar technique line as in the Proposition 2 in the proof of Theorem 1. Here, we omit the details due to space limitation. The initial condition of the scaling function $\beta_0$ is given by (56) rather than (31) is because that in this section $\|\mu(0)\|_\infty = \|H(0) - \hat{H}(0)\|_\infty \leq C_h$ while in Section III $\|\mu(0)\|_\infty = \|\hat{H}(0) - \tilde{H}(0)\|_\infty = 0$.

**C. Discussion**

In this section, we first briefly introduce the approach in [26] for comparison purpose. In [26], a control scheme that adopts a zooming-in and zooming-out mechanism was proposed to tackle the quantized consensus problem of known linear multiagent systems under data-rate constraints and DoS attacks. The basic idea of this mechanism is to update the encoder as an open-loop system during DoS attacks. In this case, the scaling function needs to zoom-out the information to be quantized to prevent the quantizer being saturated. Specifically, for the multiagent system (37), the encoder in [26] is given by

\[
\begin{align*}
\dot{x}_i(0) &= 0, \\
\dot{x}_i((k + 1)T) &= A\dot{x}_i(kT) + \beta(kT)\Delta_i((k + 1)T) \\
\Delta_i((k + 1)T) &= Q \left( x_i((k + 1)T) - \dot{x}_i(kT) \right) \\
\end{align*}
\] (58)

where $\dot{x}_i$ is the estimate of the state of agent $i$ by agent $i$ itself and its neighbors. In (58), the scaling function $\beta(kT)$ is updated by (44) with
\[
\gamma = \begin{cases}
\gamma_1, & \text{if } k + 1 \in H_v \\
\gamma_2, & \text{if } k + 1 \notin H_v
\end{cases}
\] (59)
where $0 < \gamma_1 < 1$ and $\gamma_2 > 1$. Accordingly, the control protocol in [26] is given by
\[
u_i(kT) = K_c \sum_{j \in N_i} a_{ij} (\dot{x}_j(kT) - \dot{x}_i(kT))
\] (60)
where the feedback gain $K_c$ is selected such that the eigenvalues of the matrix $\text{diag} \{A - \lambda_2 BK_c, \ldots, A - \lambda_N BK_c\}$ locate inside the unit circle.

In the following, several remarks are presented that provide intuitive explanations on the properties and advantages of the approach developed in this article.

**Remark 5 (Design Complexity):** Compared with [26], the control design in this article is relatively simpler. When the system is subject to DoS attacks, $N^{\text{DoS}}(kT) = \emptyset$, and hence by (42) and (46), one gets (51), i.e., $h_i((k + 1)T) = h_i(kT)$. This implies that the variable $h_i(kT)$ keeps invariant under DoS attacks. Consequently, in (43), we are able to make the state of the encoder invariant during DoS attacks, and hence, the parameter of the scaling function $\gamma = 1$ if the transmission fails. By contrast, in (58), the control protocol is designed as (60) in
which the state estimation under DoS in the encoder is calculated according to the open-loop equation $\dot{x}(k+1)T = A\hat{x}(kT)$. In this case, the consensus error driven by the control protocol (60) may diverge during DoS attacks [26]. Therefore, in [26], the scaling function $\beta(kT)$ must be updated accordingly to make sure that the quantizer is unsaturated during DoS attacks. That is, zoom-out the term $x_i((k+1)T) - x_i(kT)$ by introducing an extra parameter $\gamma_2$. It should be mentioned that the calculation of $\gamma_2$ is not an easy work. In fact, the value of $\gamma_2$ is decided by the system matrices $A$ and $B$, the Laplacian matrix $L$, and $\Omega$, which represents the maximum number of consecutive packet losses induced by DoS attacks. Also note that the information of $\Omega$ is not required for controller design in this article. ■

**Remark 6 (Consensus Performance):** The consensus performance can be evaluated by the convergence speed, transient performance, and final consensus value. For the convergence speed and transient performance, it is mainly decided by the scaling function $\beta(kT)$. During DoS attacks, the consensus error in [26] may diverge while in this article it keeps invariant [see (52)]. In this sense, the approach developed in this article is able to lead to faster convergence speed and better transient performance under the comparable level of DoS attacks. For the final consensus value, in [26], it diverges to infinity when $A$ has at least one eigenvalue outside the unit circle. On the contrary, the final consensus value in Theorem 3 is given in terms of the initial condition of the multiagent system. This is important for some practical applications, such as distributed computation [41]. ■

**Remark 7 (Required Data Rate):** Compared with [26], the approach developed in this article is capable of achieving lower data rate. On one hand, we encode the information of a linear combination of the agent state (i.e., $h_i$) rather than the agent state $x_i$ itself as in (58). That is to say, the dimension of the encoding–decoding channel between each pair of adjacent agents in this article is 1 while in [26] is $r$ which is the dimension of $x_i$. On the other hand, we show that, no matter how many agents there are, our proposed protocol guarantees consensus with a fixed number of quantization level. More importantly, the lowest number of the quantization level we achieve is 3 (i.e., $K = 1$, a two-bit quantizer). In [26], the required quantization level is proportional to the value of $\sqrt{N}r$. This limits its capability for a large group of agents.

**Remark 8 (DoS Attack Tolerance):** For the zooming-in and zooming-out mechanism developed in [26], as the consensus error may diverge during DoS attacks, the level of tolerable DoS attacks is expected to be low. More specifically, the successful transmissions must occupy sufficient proportion of the total transmissions, to compensate for the increase of the consensus error caused by those unsuccessful transmissions. In [26], the tolerable DoS level is given by $\frac{1}{r^2} + \frac{T}{T} < \frac{-\ln 3}{m_\gamma^2 - \ln 3}$. Note that the term $\frac{-\ln 3}{m_\gamma^2 - \ln 3}$ can be much smaller than 1 in case the resulting $\gamma_2$ is large, which implies a lower tolerance to DoS. In this article, we only require that the number of consecutive packet losses induced by DoS attacks is upper bounded, that is, $\frac{1}{r^2} + \frac{T}{T} < 1$. As discussed in [15], the bound “$< 1$” is the best bound that one can achieve in the sense that if the bound is violated, a DoS attacker may corrupt all the transmission attempts and consensus is not possible by any controller design. From this perspective, the control scheme in this article largely increases the tolerance to DoS attacks with respect to [26]. ■

**V. EXAMPLES**

In this section, we present two simulation examples: a nonlinear academic one, demonstrating the full capabilities of the proposed control scheme, and a linear comparative example with the recent article [26].

### A. Nonlinear Example

Consider a group of five agents with the following dynamics:

\[
\begin{align*}
\dot{x}_{i1} &= x_{i1} + x_{i2} \\
\dot{x}_{i2} &= x_{i3} \\
\dot{x}_{i3} &= -p_{i3}x_{i3} - p_{i1}x_{i2} - p_{i2}x_{i2}^3 + u_i + z_i + \omega_i \\
\dot{z}_i &= -(x_{i3}^2 + \omega_i^2)z_i \\
y_i &= x_{i1}, \quad 1 \leq i \leq 5
\end{align*}
\]

(61)

where $p_{i1}$, $p_{i2}$, and $p_{i3}$ are system parameters, and $\omega_i$ is the external disturbance. System (61) can be regarded as a connection of a nonlinear Duffing equation [42] with stable zero dynamics and a first-order linear system. Let $\rho_{i1} = x_{i1}$, $\rho_{i2} = x_{i1} + x_{i2}$, and $\rho_{i3} = x_{i1} + x_{i2} + x_{i3}$. Then, system (61) can be written as

\[
\begin{align*}
\dot{\rho}_{i1} &= \rho_{i2} \\
\rho_{i2} &= \rho_{i3} \\
\dot{\rho}_{i3} &= \rho_{i3} - p_{i3}(\rho_{i3} - \rho_{i2}) - p_{i1}(\rho_{i2} - \rho_{i1}) \\
\dot{z}_i &= -(\rho_{i3} - \rho_{i2})^2 + u_i + z_i + \omega_i \\
y_i &= \rho_{i1}, \quad 1 \leq i \leq 5
\end{align*}
\]

(62)

The digital communication network among the five agents is depicted in Fig. 2. In this simulation study, we set $p_{i1} = -1.1 - 0.2i$, $p_{i2} = 1 + 0.2i$, $p_{i3} = 0.4 + 0.1i$, and $\omega_i = \frac{1}{27}\sin(it)$, $1 \leq i \leq 5$. The initial conditions of the agents $\rho_{i1}(0)$ and $z_i(0)$, $1 \leq i \leq 5$, $1 \leq j \leq 3$, are randomly located in $[0, 5]$. For the proposed control scheme, let $k_1 = k_2 = 4$, and $T = 0.05$; the bounds are selected as $M_1 = 10$, $M_2 = 10$, $M_3 = 10$, $M_4 = 100$, and $C_h = 45$; the ESOs are designed with $[I_{l2} l_3 I_{l4}]^T = [4 6 4 1]^T$.

1) **Simulation With High Quantization Level:** First, we simulate the high quantization level case. According to Theorem 1, let $c = 1$, $\gamma_1 = 0.96$, $\beta_0 = 10$, and $K = 10$. We investigate a randomly generated sustained DoS. Here, for simulation purpose, the maximal duration of the DoS is limited by 9 s, i.e., the number of consecutive packet losses is no larger than 180, and the DoS attacks are present for $\sim 90\%$ of total time. Fig. 3 shows the simulation results with $\varepsilon = 0.01$ and $\varepsilon = 0.001$. From this figure, one can observe that the multiagent system with unknown nonlinear dynamics, data-rate constraints, and

![Fig. 2. Digital communication network of the five agents.](image-url)
DoS attacks achieved practical convergence. Also note that smaller ε leads to smaller residual consensus error. Fig. 4 depicts the response of the ESO for agent 1 with ε = 0.01. One can see that both the agent states ρ1j, 1 ≤ j ≤ 3, and the extended state ρ14 = ρ13 − ρ13(ρ13 − ρ12) − ρ11(ρ12 − ρ11) + z1 + ω1 are well-estimated.

2) Simulation With Low Quantization Level: Next, we investigate the performance of the proposed control protocol with two-bit quantizers (i.e., K = 1). According to the conditions in Theorem 2, we select ε0 = 0.9, K = 1, c = 0.5, γ1 = 0.9975, and β0 = 35. Here, we simulate a more serious DoS scenario. The duration of each DoS attack is randomly selected between 0 and 95 s, while the duration without DoS attack is randomly selected between 0 and 5 s. That is, the DoS attacks are present for ~95% of total time. Fig. 5 shows the trajectories of yi, 1 ≤ i ≤ 5, with K = 1 and ~95% DoS attacks. One can see that the agents achieved output consensus with serious DoS attacks and merely two-bit information exchange between each pair of adjacent agents at each time step. The simulation results are consistent with the theoretical results in Theorem 2.

B. Comparative Example

We consider the diving consensus problem of five autonomous underwater vehicles (AUVs). The discrete-time model of the ith AUV with sampling period 1 s is as follows [43] (we have transformed the system into controllable canonical form):

\[
\begin{bmatrix}
    x_{11}(k+1) \\
    x_{12}(k+1) \\
    x_{13}(k+1)
\end{bmatrix} =
\begin{bmatrix}
    0 & 1 & 0 \\
    0 & 0 & 1 \\
    0.216 & -1.502 & 2.286
\end{bmatrix}
\begin{bmatrix}
    x_{11}(k) \\
    x_{12}(k) \\
    x_{13}(k)
\end{bmatrix}
\]

+ \begin{bmatrix}
    0 \\
    0 \\
    1
\end{bmatrix} u_i(k), 1 ≤ i ≤ 5. \quad (63)

The communication graph of the AUVs is also given by Fig. 2. The initial conditions are set as x1(0) = [57.55 181.56 113.06]T, x2(0) = [96.21 32.08 180.69]T, x3(0) = [26.76 75.41 197.66]T, x4(0) = [71.21 118.11 79.59]T, and x5(0) = [108.91 50.44 19.41]T, which are generated by the rand function in MATLAB.

1) Simulation With Low-Level DoS Attacks: First, we simulate the low-level DoS attacks case. For the control protocol in [26], let Kx = [0.0720 − 0.4426 0.4029], γ1 = 0.9, γ2 = 4.8, β0 = 100, and K = 1600 (see [26, Th. 1] for the procedure of the calculation of these parameters). For the control protocol in this article, according to Theorem 3, we select k1 = −0.25, k2 = 0, Cb = 150, ε0 = 0.5, e = 0.2, γ1 = 0.9, β0 = 100, and K = 1600. Note that the same quantization parameter K, initial value of the scaling function β0, and parameter γ1 are adopted to make a fair comparison. The low-level DoS attacks are present for ~5% of total time. Fig. 6 shows the responses of δ1(kT) = x11(kT) − 1 5 ∑j=15 xj1(kT), 1 ≤ i ≤ 5, under the two control protocols with transmissions failed at k = 10, 30, 45, 70, 85, 105, 125, 145, 165, 185. From this figure, one can observe that the control protocol in this article achieves much better consensus performance. The main reason leads to this improvement is that the consensus error diverges during the DoS attacks under the control protocol in [26], while is held unchanged in this article.

2) Simulation With High-Level DoS Attacks: We further simulate the high-level DoS attacks case, i.e., the DoS attacks are present for ~95% of total time. For the control protocol in [26],
the consensus cannot be achieved if the level of DoS attacks increases to ~0.07. In contrast to [26], the control protocol in this article can tolerate arbitrarily serious DoS attacks as long as their induced maximal number of consecutive packet losses is bounded (i.e., present less than 100% of total time). Fig. 7(a) illustrates the responses of $\delta_1(kT)$ under the proposed control protocol with same parameters as in the previous simulation.

Now we show the potentials of data-rate reduction by the control scheme developed in this article. For the multiagent system (63), the number of quantization levels in [26] must satisfy $2K + 1 \geq 2711$, while in this article $2K + 1 \geq 3$. According to Theorem 3, we now select $K = 1$, $\gamma_1 = 0.9925$, and $\beta_0 = 60$. The responses of $\delta_1(kT)$ with three-level quantizers and ~95% DoS attacks are depicted in Fig. 7(b), from which one can see that the consensus has been successfully achieved.

Finally, we mention that the final consensus value of the proposed control protocol in the simulations is $x_i = [133.40 133.40 133.40]^T$, which is consistent with the theoretical computation (57), and independent with the quantization parameter $K$ and DoS attacks. On the contrary, the AUVs’ states under the control protocol in [26] trend to infinity, as the state matrix in (63) is unstable.

VI. CONCLUSION

This article investigated the quantized consensus problem for uncertain nonlinear multiagent systems under data-rate constraints and DoS attacks. An ESO-based dynamic encoding–decoding scheme with zooming-in and holding capabilities was developed, which exhibits several intriguing properties such as better consensus performance, lower data-rate requirement, and higher tolerance to DoS attacks. In particular, it was shown that the quantized consensus can be achieved under two-bit data rate and any DoS attacks inducing bounded consecutive packet losses. Two numerical examples verified the capabilities and superiorities of the developed control scheme.

In this article, as a preliminary theoretical research, we made some necessary assumptions. Future research efforts will be devoted toward extending the developed control scheme to more challenging scenarios, such as directed networks [7], with network transmission delays [44], unreliable or no acknowledgement signal [24], [45], and DoS attacks that affect only a part of the communication graph.

APPENDIX: PROOF OF THEOREM 1

To facilitate the analysis, let $\alpha(kT) = \delta(kT)/\beta(kT)$ and $\zeta(kT) = \mu(kT)/\beta(kT)$. Before stating the proof of Theorem 1, we need two additional propositions. In the following, Proposition 1 is to analyze the convergence of the ESO, and to verify that the accumulared ESO estimation error will not lead to quantizer saturation if we let $\beta(kT) \to \sqrt{\varepsilon}$. Proposition 2 is to show the boundedness of $\alpha(vT)$.

Proposition 1: Consider the system (1) with ESO (7). Suppose Assumptions A2 and A3 are satisfied, and the agents’ states $(\rho_i, z_i)$ are bounded. Let $M_j \geq \sup_{1 \leq i \leq N} |\rho_{ij}|$, $1 \leq j \leq r + 1$. Then, there exists $\varepsilon^* > 0$ such that $\forall \varepsilon \in (0, \varepsilon^*)$:

1) there exists $\tau_0(\varepsilon) > 0$ satisfying $\lim_{\varepsilon \to 0} \tau_0(\varepsilon) = 0$, such that $\forall t \in [\tau_0(\varepsilon), \infty)$,

$$|\rho_{ij}(t) - \bar{\rho}_{ij}(t)| = O(\varepsilon^{r+2-j}), \quad 1 \leq j \leq r + 1; \quad (64)$$

2) for any time interval $[\tau_1, \tau_2] \subseteq [0, \infty)$,

$$\int_{\tau_1}^{\tau_2} (\rho_{ij}(\tau) - \bar{\rho}_{ij}(\tau)) \, d\tau \to 0 \quad \text{as} \; \varepsilon \to 0, \; 1 \leq j \leq r + 1. \quad (65)$$

Proof of Proposition 1: Consider the scaled ESO estimation error $\eta_i = [\eta_{i1}, \ldots, \eta_{i, r+1}]^T \in \mathbb{R}^{r+1}$ with $\eta_{ij} = \frac{\rho_{ij} - \bar{\rho}_{ij}}{\varepsilon^{r+2-j}}$, $1 \leq j \leq r + 1$. By (3) and (7), one has

$$\dot{\eta}_i = \varepsilon \left[ \sum_{j=1}^{r+1} \rho_{ij} \frac{\partial F_i}{\partial \rho_{ij}} + [F_i(\rho_i, z_i, \omega_i) + u] \frac{\partial F_i}{\partial \eta_i} + \bar{F}_i(\rho_i, z_i, \omega_i) \frac{\partial F_i}{\partial \omega_i} \right]. \quad (66)$$

By the boundedness of $\rho_i$, and Assumptions A2 and A3, there exists $\varepsilon$-independent positive constant $\delta_{i1}$ such that $|\bar{F}_i(\rho_i, z_i, \omega_i)| \leq \delta_{i1}$. Recall that the matrix $E$ is Hurwitz by design. One can select a Lyapunov function candidate $V_i(\eta_i) = \eta_i^T P \eta_i$, where $P \in \mathbb{R}^{(r+1) \times (r+1)}$ is the unique positive definite matrix solution to the equation $PE + E^T P = -I_{r+1}$. Let $\sigma_1$ and $\sigma_2$ be the minimal and maximal eigenvalues of the matrix $P$, respectively. It follows that $\sigma_1 \parallel \eta_i \parallel^2 \leq V_i(\eta_i) \leq \sigma_2 \parallel \eta_i \parallel^2$. Then, by (66), one has

$$\frac{dV_i(\eta_i)}{dt} = \frac{1}{\varepsilon} \left(E^T P + PE\right) \parallel \eta_i \parallel^2 + \left(F_i^T(\rho_i, z_i, \omega_i) + \eta_i^T P \bar{F}_i(\rho_i, z_i, \omega_i) \right) \leq -\frac{1}{\varepsilon} \parallel \eta_i \parallel^2 + 2\sigma_2 \delta_{i1} \parallel \eta_i \parallel$$

$$\leq -\frac{1}{\sigma_2 \varepsilon} V_i(\eta_i) + 2\sigma_2 \delta_{i1} \parallel \eta_i \parallel \leq -\frac{1}{\sigma_2 \varepsilon} V_i(\eta_i) + \frac{2\sigma_2 \delta_{i1}}{\sigma_1} \sqrt{V_i(\eta_i)}. \quad (67)$$

By $dV_i(\eta_i)/dt = 2\sqrt{V_i(\eta_i)} dV_i(\eta_i)/dt$, the theory of ordinary differential inequality, and some straightforward manipulations, one can obtain

Fig. 7. Responses of $\delta_1(kT)$ under the proposed control protocol with different quantization levels and ~95% DoS attacks. (a) $K = 1600$, (b) $K = 1$. 

Authorized licensed use limited to the terms of the applicable license agreement with IEEE. Restrictions apply.
\[
\|\eta_i\| \leq \frac{\sqrt{V_i(\eta_i)}}{\sigma_1} \\
\leq \left( \frac{\sqrt{V_i(\eta_i(0))} - 2\sigma_2^2 f_1\varepsilon}{\sigma_1} \right) e^{-\frac{\sigma_2^2 f_1\varepsilon}{\sigma_1} t} + \frac{2\sigma_2^2 f_1\varepsilon}{\sigma_1} \\
\leq \sqrt{\frac{\sigma_2}{\sigma_1}} (\|\eta_i(0)\|) e^{-\frac{\sigma_2^2 f_1\varepsilon}{\sigma_1} t} + \frac{2\sigma_2^2 f_1\varepsilon}{\sigma_1}.
\]

Note that the right-hand side of (68) is of the order of \(O(\varepsilon)\) for all \(t \geq \tau_0(\varepsilon) = -2(t + 1)\sigma_2^2 f_1\ln \varepsilon\). Since \(\tau_0(\varepsilon) \to 0\) uniformly as \(\varepsilon \to 0\), and \(|\rho_{ij} - \hat{\rho}_{ij}| = \varepsilon^{r+1-j}|\eta_{ij}|\), \(1 \leq j \leq r + 1\), the conclusion in the first bullet holds. What is more, the saturations will not be invoked after the transient period of the observer, i.e., \(\hat{\rho}_{ij} = \rho_{ij}, t \geq \tau_0(\varepsilon)\).

For the second bullet, consider three cases.

Case 1: \(\tau_2 \leq \tau_0(\varepsilon)\). In this case, one has
\[
\int_{\tau_2}^{\tau_1} (\rho_{ij}(\tau) - \hat{\rho}_{ij}(\tau)) \, d\tau \\
\leq 2M_j \tau_0(\varepsilon) \xrightarrow{\varepsilon \to 0} 0 \quad \text{as} \quad \varepsilon \to 0, \quad 1 \leq j \leq r + 1.
\]

Case 2: \(\tau_1 \geq \tau_0(\varepsilon)\). In this case, we start with \(j = 1\). By (66), one has
\[
\rho_1 - \hat{\rho}_1 = \sum_{i=1}^m (F_i(x_i, z_i, \omega_i) - \eta_{i,r+1}).
\]
It follows that
\[
\int_{\tau_1}^{\tau_2} (\rho_{i,1} (\tau) - \hat{\rho}_{i,1} (\tau)) \, d\tau \\
= \frac{\varepsilon^{r+1/2}}{r+1} \left[ (F_i(x_i, z_i, \omega_i) - \eta_{i,r+1}) \right]_{t=\tau_2} - (F_i(x_i, z_i, \omega_i) - \eta_{i,r+1}) \right|_{t=\tau_1}.
\]
Since \(F_i(x_i, z_i, \omega_i)\) is bounded, and \(\eta_{i,r+1} = O(\varepsilon) \forall t \geq \tau_0(\varepsilon)\), the right-hand side of (70) is of the order of \(O(\varepsilon^{r+1/2})\). For \(j = 2\), by (66), one has
\[
\rho_2 - \hat{\rho}_2 = \varepsilon^r \hat{\eta}_1 + \frac{\varepsilon^r}{\varepsilon} (\rho_1 - \hat{\rho}_1).
\]
It follows that
\[
\int_{\tau_1}^{\tau_2} (\rho_{i,2} (\tau) - \hat{\rho}_{i,2} (\tau)) \, d\tau \\
= \varepsilon^{r} (\eta_{i,2} - \eta_{i,1}) + \frac{\varepsilon^r}{\varepsilon} \int_{\tau_1}^{\tau_2} (\rho_{i,1} (\tau) - \hat{\rho}_{i,1} (\tau)) \, d\tau.
\]

By (70), the right-hand side of (71) is of the order of \(O(\varepsilon^{r+1/2})\). Then, by a simple iterative procedure, one can prove that
\[
\int_{\tau_1}^{\tau_2} (\rho_{ij}(\tau) - \hat{\rho}_{ij}(\tau)) \, d\tau = O(\varepsilon^{r+1/2-j}), \quad 1 \leq j \leq r + 1.
\]

Case 3: \(\tau_1 < \tau_0(\varepsilon) \leq \tau_2\). Note that
\[
\int_{\tau_2}^{\tau_0(\varepsilon)} (\cdot) \, d\tau = \int_{\tau_1}^{\tau_0(\varepsilon)} (\cdot) \, d\tau + \int_{\tau_0(\varepsilon)}^{\tau_2} (\cdot) \, d\tau.
\]
Then, by Cases 1) and 2), one can conclude that
\[
\int_{\tau_1}^{\tau_2} (\rho_{ij}(\tau) - \hat{\rho}_{ij}(\tau)) \, d\tau \to 0 \quad \text{as} \quad \varepsilon \to 0 \quad \text{in this case}.
\]
Finally, combining Cases 1)–3) leads to the conclusion in (65).

This completes the proof of Proposition 1.

**Proposition 2**: There exists \(\varepsilon^1 > 0\) such that \(\forall \varepsilon \in (0, \varepsilon^1),\n\|\alpha(v_i, T)\| < \varpi, \quad i = 0, 1, \ldots \quad (73)\]
where \( v_{m-1} + 1 \leq k \leq v_m - 2 \), and
\[
s_t(kT) = s_1(kT) + H(kT) - \mathcal{H}(kT) + \mathcal{H}((k+1)T) - H((k+1)T).
\]
Applying a simple iteration to (78)–(79) yields
\[
\alpha((v_m - 1)T) = \alpha((v_m - 1)T) + (I - J_N) \sum_{j=v_m-1+1}^{v_m-2} \frac{s_1(jT)}{\beta(v_m - 1T)}.
\]
(80)
\[
\zeta((v_m - 1)T) = \zeta((v_m - 1)T) + \frac{\sum_{j=v_m-1+1}^{v_m-2} s_4(jT)}{\beta(v_m - 1T)}.
\]
(81)

For the transmission at time instant \((v_m - 1)T\), by (23), (25), and \(\beta(v_mT) = \gamma_1 \beta((v_m - 1)T)\), one has
\[
\alpha(v_mT) = \frac{1}{\gamma_1} \alpha((v_m - 1)T) + \frac{(I - J_N) s_1((v_m - 1)T)}{\beta(v_mT)}.
\]
(82)
\[
\zeta(v_mT) = \frac{1}{\gamma_1} \zeta((v_m - 1)T) + \frac{s_4((v_m - 1)T)}{\beta(v_mT)} - \frac{1}{\gamma_1} Q \left( \frac{s_4((v_m - 1)T)}{\beta(v_mT)} \right).
\]
(83)

Therefore, by (76)–(77), (80)–(81), and (82)–(83), the equations that describe the evolution of \(\alpha(kT)\) and \(\zeta(kT)\) from \(k = v_{m-1}\) to \(k = v_m\) are expressed as
\[
\alpha(v_mT) = \frac{1}{\gamma_1} \alpha(v_{m-1}T) + \frac{(I - J_N) s_1(v_{m-1}T)}{\beta(v_mT)} + \frac{s_2(v_{m-1}T)}{\beta(v_mT)} + \frac{(I - J_N) \sum_{j=v_m-1+1}^{v_m-1} s_1(jT)}{\beta(v_mT)}.
\]
(84)
\[
\zeta(v_mT) = \frac{1}{\gamma_1} \left[ \Psi(v_{m-1}T) - Q(\Psi(v_{m-1}T)) \right] - \frac{1}{\gamma_1} Q \left( \frac{s_4((v_m - 1)T)}{\beta(v_mT)} \right).
\]
(85)

where
\[
\Psi(v_{m-1}T) = \left( I + cT\mathcal{L} \right) \zeta(v_{m-1}T) - cT\mathcal{L} \alpha(v_{m-1}T)
\]
\[
+ \frac{s_4(v_{m-1}T)}{\beta(v_mT)} + \frac{1}{\gamma_1} \sum_{j=v_m-1+1}^{v_m-1} s_4(jT).
\]
(86)

For \( m = 1 \), according to Proposition 1, (31), (74), and (75), one can readily verify that \(\|\alpha(v_1T)\| < \pi\) and \(\|\Psi(v_0T)\|_{\infty} < K + \frac{1}{2}\) for sufficiently small \(\epsilon\). In the following, we investigate the case that \(t = v_mT\), \(m \geq 2\), with the assumption that \(\|\alpha(v_1T)\| < \pi\) and the quantizer is unsaturated for all \(t = v_iT\), \(1 \leq i \leq m - 1\).

To facilitate the analysis, let \(U\) be a unitary matrix defined by \(U = [1_N / \sqrt{N}, \psi_2, \ldots, \psi_N]\), where \(\psi_i^T \mathcal{L} = \lambda_i \psi_i^T\), \(2 \leq i \leq N\). Denote \(\tilde{\alpha}(kT) = U^{-1} \alpha(kT) = iT \alpha(kT)\), and decompose \(\tilde{\alpha}(kT) = \tilde{\alpha}_1(kT) + \tilde{\alpha}_2(kT)\) with a scalar \(\tilde{\alpha}_1(kT)\).

One can verify that \(\tilde{\alpha}_1(kT) = 0\). Let \(\mathcal{M}_{\gamma_1,c} = \gamma_1^{-1} \text{diag} \{1 - cT \lambda_2, \ldots, 1 - cT \lambda_N\} \) and \(\psi = [\psi_2, \ldots, \psi_N]\). It follows from (84) that
\[
\tilde{\alpha}_2(v_mT) = \mathcal{M}_{\gamma_1,c} \tilde{\alpha}_2(v_{m-1}T) + \frac{cT \psi^T \mathcal{L}}{\gamma_1} \zeta(v_{m-1}T)
\]
\[
+ \frac{\psi^T \tilde{\alpha}_2(v_{m-1}T)}{\beta(v_mT)} + \frac{\psi^T (I - J_N) \sum_{j=v_m-1+1}^{v_m-1} s_1(jT)}{\beta(v_mT)}.
\]
(87)

By (87), one can establish the evolution equation of \(\tilde{\alpha}_2(kT)\) from \(k = v_0\) to \(k = v_m\). This together with \(\alpha(kT) = \psi \tilde{\alpha}_2(kT)\) and \(\tilde{\alpha}_2(kT) = \psi^T \alpha(kT)\) yields
\[
\alpha(v_mT) = \psi \left[ \mathcal{M}_{\gamma_1,c} \right] \psi^T \alpha(v_0T)
\]
\[
+ \frac{cT \psi^T \mathcal{L}}{\gamma_1} \zeta(v_0T)
\]
\[
+ \frac{\psi^T \tilde{\alpha}_2(v_{m-1}T)}{\beta(v_mT)} + \frac{\psi^T (I - J_N) \sum_{j=v_m-1+1}^{v_m-1} s_1(jT)}{\beta(v_mT)}.
\]
(88)

Next, we investigate the five terms on the right-hand side of (88) to seek for the upper-bound of \(\|\alpha(v_mT)\|\). Note that \(\|\mathcal{M}_{\gamma_1,c}\| \leq \frac{\rho_h}{\gamma_1}, \|\psi\| = 1\), and \(\|\mathcal{L}\| = \lambda_N\). By (74), the first term satisfies
\[
\left\| \psi \left[ \mathcal{M}_{\gamma_1,c} \right] \psi^T \alpha(v_0T) \right\| \leq \left( \frac{\rho_h}{\gamma_1} \right) \left\| \frac{2\sqrt{N}C_h}{\beta_0} \right\| \left( \frac{\rho_h}{\gamma_1} \right)^{m-1} \frac{2\sqrt{N}C_h}{\beta_0}.
\]
(89)

The second term in (88) vanishes due to (75). For the third term, recall that we have assumed that the quantizer is unsaturated for all \(t = v_iT, 1 \leq i \leq m - 1\). According to (85), one has \(\max_{1 \leq i \leq m-1} \|\psi_i\|_{\infty} \leq \frac{1}{\sqrt{\gamma_1}}\), and hence
\[
\left\| \psi \left[ \mathcal{M}_{\gamma_1,c} \right] \frac{cT \psi^T \mathcal{L}}{\gamma_1} \zeta(v_{m-1}T) \right\| \leq \left( \frac{\rho_h}{\gamma_1} \right) \left\| \frac{2\sqrt{N}C_h}{\beta_0} \right\| \left( \frac{\rho_h}{\gamma_1} \right)^{m-1} \frac{2\sqrt{N}C_h}{\beta_0}.
\]
(90)

For the fourth and fifth terms, by Proposition 1 and the fact that \(\frac{\rho_h}{\gamma_1} < 1\), they converge to 0 as \(\epsilon \to 0\). This together with (31), (89), and (90) yields \(\|\alpha(v_mT)\| < \pi\). What is more, by \(\|\alpha(v_{m-1}T)\| < \pi\), \(\|\zeta(v_{m-1}T)\|_{\infty} \leq \frac{1}{\gamma_1}\), (28), and (31), one has
\[
\left( I + cT \mathcal{L} \right) \left( ||v_{m-1}T - cT \mathcal{L} \alpha(v_{m-1}T)||_{\infty} \leq \left( I + cT \mathcal{L} \right) \left( ||v_{m-1}T||_{\infty} + cT \mathcal{L} \||\alpha(v_{m-1}T)||_{\infty}\right)\right.
\]
\[
\leq \left( I + cT \mathcal{L} \right) \left( \frac{2\sqrt{N}C_h}{\sqrt{\gamma_1}} + cT \mathcal{L} \||\alpha(v_{m-1}T)||_{\infty}\right)\right) \leq \left( I + cT \mathcal{L} \right) \left( \frac{2\sqrt{N}C_h}{\sqrt{\gamma_1}} + cT \mathcal{L} \frac{2\sqrt{N}C_h}{\sqrt{\gamma_1}} \right) \leq K_1(c, \gamma_1).
\]
(91)
Hence, in (86), by Proposition 1 and (91), one has 
\[ \|\Psi(v_{m-1}T)\| \leq K + \frac{1}{\beta} \] for sufficiently small \( \varepsilon \). That is, the quantizer is unsaturated when \( t = v_{m}T \). Therefore, by induction, we conclude that \( \|\alpha(v_{m}T)\| < \pi \) and the quantizer is unsaturated when \( t = v_{m}T, 1 \leq m < m^* \).

**Case 2:** \( v_{m} = v_{m^*} \). In this case, \( \beta(v_{m-1}T) \leq \sqrt{\varepsilon} \) and \( \beta(v_{m}T) = \sqrt{\varepsilon} \). By following the similar line as in (76)–(85), one can obtain
\[
\alpha(v_{m}T) = \frac{\beta(v_{m-1}T)}{\sqrt{\varepsilon}} (I - cTL)\alpha(v_{m-1}T) \\
+ \frac{\beta(v_{m-1}T)}{\sqrt{\varepsilon}} cTL\zeta(v_{m-1}T) \\
+ \frac{\varepsilon}{\sqrt{\varepsilon}} (I - JN) \sum_{j=v_{m-1}+1}^{v_{m}} s_{i}(jT) \\
+ \frac{\varepsilon}{\sqrt{\varepsilon}} (I - JN) \sum_{j=v_{m-1}+1}^{v_{m}} s_{i}(jT) \\
(92)
\]
\[
\zeta(v_{m}T) = \frac{\beta(v_{m-1}T)}{\sqrt{\varepsilon}} [\Psi(v_{m-1}T) - Q(\Psi(v_{m-1}T))].
\]

**Case 3:** \( v_{m} > v_{m^*} \). In this case, \( \beta(v_{m}T) = \beta(v_{m-1}T) = \sqrt{\varepsilon} \), and hence
\[
\alpha(v_{m}T) = (I - cTL)\alpha(v_{m-1}T) \\
+ \frac{\varepsilon}{\sqrt{\varepsilon}} (I - JN) \sum_{j=v_{m-1}+1}^{v_{m}} s_{i}(jT) \\
+ \frac{\varepsilon}{\sqrt{\varepsilon}} (I - JN) \sum_{j=v_{m-1}+1}^{v_{m}} s_{i}(jT) \\
(94)
\]
\[
\zeta(v_{m}T) = \Psi(v_{m-1}T) - Q(\Psi(v_{m-1}T)).
\]

Note that in Case 2), \( \frac{\beta(v_{m-1}T)}{\sqrt{\varepsilon}} \leq \frac{1}{\gamma} \), and in Case 3), \( 1 < \frac{1}{\gamma} \).

By Proposition 1 and conducting a similar analysis as in (87)–(91), one can verify that \( \|\alpha(v_{i}T)\| < \pi \) and the quantizer is unsaturated when \( t = v_{m}T, m \geq m^* \).

Finally, combining Cases 1)–3) leads to (73). What is more, by Cases 1)–3), (16) and (21), and the update mechanism of the encoder, one can conclude that the quantizer will never be saturated. This completes the proof of Proposition 2.

The proof of Proposition 2 is inspired by [6]. However, due to the uncertain nonlinear dynamics and the introduction of the DoS attacks, several new and important technique issues arise. On one hand, since the DoS destroys the information availability, the behavior of the system in the presence and absence of DoS attacks needs to be carefully analyzed. In fact, the evolution (80) and (82) clearly reveal the properties of the proposed zooming-in and holding approach. On the other hand, in this work, the agent uncertain nonlinear dynamics, the DoS affected quantization process, and the consensus process are highly coupled. In (88), we show that the evolution of the scaled consensus error is governed by the accumulated ESO estimation error, the accumulated error induced by quantization, and the behavior of the DoS attacker.

Finally, we are ready to state the proof of Theorem 1.

**Proof of Theorem 1:** By Propositions 1 and 2, one can readily conclude the statement in the first bullet of Theorem 1. For the second bullet, in Proposition 2, we have proved that the quantizer will never be saturated for all transmissions. According to the definition of \( \alpha(kT) \) and the update mechanism of the scaling function \( \beta(kT) \), for \( m \geq m^* \), one has \( \|\delta(v_{m}T)\| = \|\delta(v_{m}T)\| < \pi\sqrt{\varepsilon} \). Hence, \( h_{1}(v_{m}T) - h_{2}(v_{m}T) \rightarrow 0, 1 \leq i \neq j \leq N \), as \( m \rightarrow \infty \) and \( \varepsilon \rightarrow 0 \). This together with Proposition 1, (15), and (21) yields that \( h_{1}(t) - h_{2}(t) \rightarrow 0, 1 \leq i \neq j \leq N \), as \( t \rightarrow \infty \) and \( \varepsilon \rightarrow 0 \). Finally, according to Lemma 1, one can conclude that the multi-agent system (1) achieves the practical output consensus specified by (34). This completes the proof of Theorem 1. □

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