Electromagnetic Transitions and Structures of even–even $^{76−90}$Kr Isotopes within Interacting Boson Model

Saad Naji.Abood$^1$, Abdul.Kader.Saad$^1$ and Laith Ahmed.Najim$^2$*

$^1$Physics Department, College of Science, AL-Nahrain University, Bagdad, Iraq.
$^2$Physics Department, College of Science, Mosul University, Mosul, Iraq.

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ABSTRACT

The Interacting Boson Model is applied to the even $^{76−90}$Kr isotopes. Excitation energies, electromagnetic transition strengths, quadrupole and magnetic dipole moments, and $(E2/M1)$ multipole mixing ratios, monopole transitions and mixed symmetry states have been described systematically. It is seen that the properties of low-lying levels in these isotopes, for which the comparison between experiment and IBM-2 calculations is possible, can be satisfactorily characterized by the Interacting Boson Model-2.

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Introduction

The even–even $^{76−90}$Kr isotopes are the members of the chain existing around the mass region $A \approx 80$ and they are settled away from both the proton closed-shell number at 28 and neutron closed shell at 50. The interacting boson model-2 (IBM-2) has recently been applied to many light isotopes of Kr with emphasizing on the energy levels and on the electromagnetic transition probabilities rates.

Several some theoretical and experimental studies of even-even Kr isotopes have been carried out: Kaup and Gelberg [1], have performed systematic analysis of Kr isotopes in IBM-2, reproduced energy levels. Hellestein and Lieb [2] study the energy levels and Electric transition probability and compare with experimental data. Meyer et al., [3] investigated the nuclear structure of the $^{80}$Kr isotope, using in-beam spectroscopy studies and compare the experimental data with the results of IBM-2.

Giannatiempo et al., [4] studied the life-time of the $0^+_2$ level in the $^{80}$Kr isotope and compare with the calculated value of IBM-2.

Deibaksh et al., [5] have performed the IBM-2 calculation on Kr isotopes, using two different approaches. The first approach based on the energy of bosons that $\varepsilon_π = \varepsilon_π$, and the second approach based on the difference the energy of proton boson and energy of neutron boson $\varepsilon_π \neq \varepsilon_π$. The results of IBM-2 in a good agreement with experimental data accept for the state $2^+_3$. Giannatiempo et al., [6] have studied the symmetry property of the bands in $^{74−82}$Kr isotopes by calculating $F$-spin and the $n_d$ component of the wave function of the states of these bands.

Shi Zhu-Yii et al.,[7] have studied by using a microscopic sd IBM-2+2q.p. approach, the levels of the ground-band, $Y$-band and partial two-quasiparticle bands for $^{72−84}$Kr isotopes are calculated. The data obtained are in good agreement with the experimental results, and successfully reproduce the nuclear shape phase transition of $^{72−84}$Kr isotopes at zero temperature. The ground-state band is described successfully up to $J^\pi = 18^+$ and $E_\gamma = 10$ MeV. Based on this model, the aligned requisite minimum energy has been deduced. The theoretical calculations indicate that no distinct change of nuclear states is caused by the abruptly broken pair of a boson, and predict that the first backbending of Kr isotopes may be the result of aligning of two quasi-neutrons in orbit $g_{9/2}$, which gains the new experimental support of the measurements of $g$ factors in the $^{78−86}$Kr isotopes.

Al-Khudad and Gui-Lu [8] studied the level structure of $^{76,78,80}$Kr isotopes within framework of IBM-2, and performed that the $J^\pi = 2^+$ (one-phonon mixed symmetry state) and $J^\pi = 1^+,2^+,3^+$ (two-phonons mixed symmetry states), and have been identified by analyzing the wavefunction of $M1$ transition.

Turkan et al., in 2006 [9] have determined the most appropriate Hamiltonian that is needed for present calculations of nuclei about the $A \approx 80$ region by the view of Interacting Boson Model-2 (IBM-2). After obtaining the best Hamiltonian parameters, level energies and B(E2) probabilities of some transitions in $^{80}$Kr nuclei were estimated. Results are compared with previous experimental and theoretical data and it is observed that they are in good agreement.

Turkan et al., in 2009 [10] studied The quadrupole moments of $^{76,78,80}$Kr and $^{74,76,78,80}$Se isotopes are investigated in terms of the interacting boson model (IBM), and it was found that a good description of them can also be concluded in this model. Before
the quadrupole moments were calculated, the positive-parity states and electromagnetic-transition rates $B(E2)$ of even-mass $K$ nuclei have also been obtained within the framework of IBM. It was seen that there is a good agreement between the presented results and the previous experimental data. The quadrupole moments of the neighboring $Se$ isotopes were also obtained and it was seen that the results are satisfactorily agree well with the previous experimental data.

The aim of this work is to calculate the energy levels and electromagnetic transitions probabilities $B(E2)$ and $B(M1)$, multipole mixing ratios and monopole matrix elements in $Kr$ isotopes, using the IBM-2, mixed symmetry states for these isotopes have been studied in this research, and to compare the results with the experimental data.

The Model

In the IBM-2 the structure of the collective states in even-even nuclei is calculated by considering a system of interacting neutron ($\nu$) and proton ($\pi$) bosons($l = 0$) and d($l = 2$). The boson Hamiltonian can be written as [11]:

$$H = E_0(n_d + n_\nu) + \kappa Q_{\nu}^{(2)}Q_{\pi}^{(2)} + V_{\pi\pi} + V_{\nu\nu} + M_{\nu\pi} + \ldots$$

where

$$Q_{\rho} = (s_{\rho}^+d_{\rho} + d_{\rho}^+s_{\rho})^{(2)} + \chi_{\rho}(d_{\rho}^+d_{\rho})^{(2)}$$

$$\rho = \nu, \pi$$

$\kappa$ is the quadrupole-quadrupole strength and $V_{\rho\rho}$ is the boson-boson interaction, which is given by the equation:

$$V_{\rho\rho} = \frac{1}{2} \sum L_{\rho\rho} C_L^{(2)}\left[d_{\rho}^+d_{\rho}\right]^{(L)}

and

$$M_{\nu\pi} = L_{\pi}(s_{\nu}^+d_{\nu} + d_{\nu}^+s_{\nu})^{(2)}(d_{\pi}^+d_{\pi})^{(2)} - 2 \sum_{k=1,2} L_{k}(d_{\nu}^+d_{\pi}^+)^{(k)}(d_{\nu}d_{\pi})^{(2)} \ldots$$

The Majorana term $M_{\nu\pi}$ shifts the states with mixed proton-neutron symmetry with respect to the totally symmetric ones. Since little experimental information is known about such states with mixed symmetry, we did not attempt to fit the parameters appearing in eq. (3), but rather took constant values for all $Kr$ isotopes. :

$$T(E2) = e_\pi Q_{\pi} + e_\nu Q_{\nu} + \ldots$$

The quadrupole moment $Q_{\rho}$ is in the form of equation (2), for simplicity, the $\chi_{\rho}$ has the same value as in the Hamiltonian. This is also suggested by the single j-shell microscopy, $e_\pi$ and $e_\nu$ are proton and neutron boson effective charges respectively. In general, the E2 transition results are not sensitive to the choice of $e_\pi$ and $e_\nu$, whether $e_\pi = e_\nu$ or not.

The reduced electric quadrupole transition probability $B(E2)$ is given by:

$$B(E2; I_i \rightarrow I_j) = \frac{1}{2I_i + 1}\left|\langle I_i | T(E2) | I_j \rangle \right|^2$$

The M1 transition operator is given:

$$T(M1) = \sqrt{3/\pi}(g_\pi L_{\pi} + g_\nu L_{\nu})$$

where $L_{\nu}(L_{\pi})$ is the neutron (proton) angular momentum operator

$$L_{\rho}^{(1)} = \sqrt{10}(d_{\rho}^+d_{\rho})^{(1)}$$

where $g_\pi$ and $g_\nu$ are the effective boson (proton, neutron) gyromagnetic –factors.

The reduced magnetic dipole transition probability $B(M1)$ is given by:

$$B(M1; I_i \rightarrow I_j) = \frac{1}{2I_i + 1}\left|\langle I_i | T(M1) | I_j \rangle \right|^2$$

In the IBM-2, the monopole transition (E0) operator is given by [18]

$$T(E0) = \beta_{0\rho}(d_{\rho}^+\times d_{\rho})^{(0)} + \gamma_{0\rho}(s_{\rho}^+\times s_{\rho})^{(0)}$$

which is related to the transition matrix $\rho(E0)$ by the expression [12]:

$$\rho_{0\rho}(E0) = \frac{Z}{R_0^2} \sum \beta_{0\rho}^2 <\text{f}d_{\rho}^+\times d_{\rho}|^2$$

where $R_0$ is the nuclear radius constant ($R_0 = 1.25 \times 10^{-15}$ m).

The Monopole transition probability is defined by:

$$B(E0; I_i \rightarrow I_j) = e^2 R_0^2 \rho^2(E0)$$

The two parameters $\beta_{0\nu}$ and $\beta_{0\pi}$ in eq.9, may be estimated by fitting the isotopic shift, which is different in the mean square radius $\Delta <r^2>$ between neighboring isotopes in their ground state [2]:
\[ \Delta < r^2 > = 0 \chi_{\nu}, \chi_{\pi} > \Delta \]
\[ \Delta < r^2 > = \beta_{\nu} \left[ < 0 \left[ d^\nu d^\nu \right] 0_i >_{\nu} - < 0 \left[ d^\nu d^\nu \right] 0_i >_{\nu} \right] + \beta_{\pi} \left[ < 0 \left[ d^\pi d^\pi \right] 0_i >_{\nu} \right] \]
\[ = - < 0 \left[ d^\nu d^\nu \right] 0_i >_{\nu} - \gamma_{0}\nu \]

Results and Discussion

Hamiltonian Interaction Parameters

Since the Hamiltonian contain many parameters it is unpractical and not very meaningful to vary all parameters freely. Instead it is convenient to use the behavior of the parameters predicted by a microscopic point of view as a zeroth-order approximation. In a simple shell-model picture based upon degenerate single nucleon levels [13] the expected dependence of \( E, K, \chi_{\nu}, \chi_{\pi} \) on neutron \((N_\nu)\) and proton \((N_\pi)\) boson numbers can be expressed as:

\[ E = \text{constant}, \quad K = \chi_{\nu} \chi_{\nu}, \quad \kappa_\rho = \sqrt{\frac{\rho - N_\rho}{\rho - 1}} \chi_{\rho}, \quad \chi_{\rho} = \frac{\Omega_\rho - 2N_\rho}{\sqrt{\Omega_\rho - N_\rho}} \chi_{\rho}^{(0)} \rho = \pi, \nu \]  

Here \( \chi_{\rho}^{(0)} \) and \( \chi_{\rho}^{(0)} \) are constants, and \( \Omega_\rho \) is the pair degeneracy of the shell. We see that while \( \kappa_\rho \) has always the same sign, \( \chi_{\rho} \) changes sign in the middle of the shell.

In realistic cases the estimates of eq.1 are expected to be valid only approximately. In our approach we have imposed somewhat weaker constrains on the parameters: (i) it is assumed that within a series of isotones (isotones) \( \chi_{\nu}(\chi_{\pi}) \) does not vary at all; (ii) the parameters \( E, K \) and \( \chi_{\nu} \) are assumed to be smooth functions of \((N_\nu)\).

Concerning the sign of \( \chi_{\nu} \) and \( \chi_{\pi} \) a complication arises. From very simple microscopic consideration it follows that the \( \chi_{\nu} \)'s (which also determine to a large extent the sign of the quadrupole moment of the first excited state \( 2^+ \)) are negative in the region where the valence shell is less than half filled (particle-boson) and positive in the region where the valence shell is more than half filled (hole-boson). Quantitatively, such a behavior was confirmed in other phenomenological calculations with IBM-2. For example in a study of the Ba isotopes with \( 72 < N < 80 \), a good fit to the energy levels was obtained with \( \chi_{\nu} \approx 0.90 \) [14,15]. Since in the naïve shell-model picture in the Kr region both neutrons and protons are hole-like and therefore both \( \chi_{\nu} \) would be positive, there would be no way to obtain an \( SU(5) \) type spectrum, which requires opposite signs of \( \chi_{\nu} \) and \( \chi_{\pi} \). This indicates that the situation is not so simple and that more complicated effects play a role, such as a possible nonclosedness of the \( Z = 50 \) or the \( N = 82 \) core. Although the Hamiltonian invariant under simultaneous change in sign of both \( \chi_{\nu} \) and \( \chi_{\pi} \) and thus equally good fits to energy spectra can be obtained for both combinations \( \chi_{\nu} > 0 \) and \( \chi_{\pi} < 0 \). Namely, only with this choice the observed sign of the mass quadrupole moment of the \( 2^+ \) state in \( Kr \) can be reproduced.

The remaining parameters play a less important role and are used mainly to improve the fit with experiment. In this work only \( C_{0\nu} \) and \( C_{2\nu} \) representing part of the d-boson conserving interaction between neutron bosons, were used as free parameters independent of \((N_\pi)\). Finally, the values of \( \xi_2 \) and \( \xi_1 \) were vary from isotope to another, \( \xi_3 \) kept constant. The parameters used for the various isotopes are shown in table 1.

It seen that parameters are constant or vary smoothly: within a series of isotopes \( \chi_{\pi} \) does not vary, the variation in \( E \) is very small and there is a slight decrease of the value of \( K \) for the lighter Kr isotopes. The change in character of the spectra through a series of isotopes is essentially due to two effects: (i) the decrease of the value of \( \chi_{\nu} \), and (ii) the decrease of the number of neutron bosons \( N_\nu \). We note that the behaviors of \( E, K, \chi_{\nu}, \chi_{\pi} \) is I qualitative agreement with microscopic considerations. It was found that both \( C_{0\nu} \) and \( C_{2\nu} \) vary for the isotopes. Such a behavior agree with the trend found in other regions [16]. The positive value of \( \xi_2 \) guarantees that no low-lying anti-symmetric multiplets occur for which there is no experimental evidence.
Energy Spectra

The calculated excitation energies of positive parity levels to $^{76-90}$Kr isotopes are given in Table 1 and displayed in Figs.(1-7). The agreement between the calculated and experimental values is satisfactory.

Using the parameters in Table 1, the estimated energy levels are shown in the figures, along with experimental energy levels. As can be seen, the agreement between experiment and theory is quite good and the general features are reproduced well. We observe the discrepancy between theory and experiment for high spin states. However, the general features are reproduced well. We observe the discrepancy between theory and experiment for the $^2_1$ state relative to the $^2_0$ state in the $^{76-90}$Kr isotopes.

Table 1: IBM-2 Hamiltonian parameters for $^{90}$Kr isotopes, all parameters in MeV units except $\lambda_\pi$ and $\lambda_\nu$ are dimensionless

| $\xi_3$ | $\xi_2$ | $\xi_1$ | $C_{4\pi}$ | $C_{2\pi}$ | $C_{0\pi}$ | $C_{4\nu}$ | $C_{2\nu}$ | $C_{0\nu}$ | $\lambda_\pi$ | $\lambda_\nu$ | $^\pi J^\pi$ | $^\nu J^\nu$ | $E$ | $N$ | $N_\nu$ | $N_{\pi}$ | Isotopes |
|--------|--------|--------|------------|------------|------------|------------|------------|------------|--------------|--------------|---------------|---------------|-----|-----|-----|-----|---------|
| 0.11   | 0.032  | 0.26   | 0.30       | -1.2       | 0.11       | -1.20      | -0.60      | 0.41       | -0.080       | 0.701         | 9              | 5              | 4    | 43  | 53  | 46  | $^{88}$Kr$_{46}$ |
| 0.11   | 0.130  | 0.10   | 0.30       | -1.2       | 0.11       | 0.27       | -0.66      | 0.52       | -0.090       | 0.722         | 8              | 4              | 4    | 43  | 53  | 46  | $^{82}$Kr$_{46}$ |
| 0.11   | 0.050  | 0.24   | 0.05       | -0.21      | 0.11       | -0.22      | -0.60      | 0.60       | -0.081       | 0.890         | 7              | 3              | 4    | 43  | 53  | 46  | $^{80}$Kr$_{46}$ |
| 0.11   | 0.050  | 0.18   | 0.07       | -0.18      | 1.43       | -0.37      | -0.82      | 0.11       | -0.60         | 0.70          | 6              | 2              | 4    | 43  | 53  | 46  | $^{82}$Kr$_{46}$ |
| 0.11   | 0.440  | 0.60   | 0.38       | 0.25       | 0.14       | 0.00       | 0.00       | 0.60       | -0.60         | 0.81          | 0.949          | 5              | 1              | 4    | 43  | 53  | 46  | $^{84}$Kr$_{48}$ |
| 0.11   | 0.450  | 0.60   | 0.00       | -0.18      | -0.30      | 0.12       | 0.12       | 0.62       | -0.087       | 0.920         | 5              | 1              | 4    | 43  | 53  | 46  | $^{88}$Kr$_{52}$ |
| 0.11   | 0.510  | 0.63   | 0.17       | 0.17       | 0.11       | 0.11       | 0.11       | -0.60      | 0.59          | -0.992        | 0.862          | 6              | 2              | 4    | 43  | 53  | 46  | $^{90}$Kr$_{54}$ |

Table 2: Energy ratio $R_{4/2} = E(4_1^+) / E(2_1^+)$ for $^{76-90}$Kr isotopes

| $E(4_1^+) / E(2_1^+)$ | $^{76}$Kr$_{40}$ | $^{78}$Kr$_{42}$ | $^{80}$Kr$_{44}$ | $^{82}$Kr$_{46}$ | $^{84}$Kr$_{48}$ | $^{88}$Kr$_{52}$ | $^{90}$Kr$_{54}$ |
|------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Exp. [17]              | 2.44            | 2.459           | 2.327           | 2.345           | 2.375           | 2.121           | 3.043           |
| IBM-2                  | 2.49            | 2.467           | 2.312           | 2.354           | 2.329           | 2.116           | 3.043           |

In the Figures we show the results of our calculations for the energies of the ground state band ($2_1^+, 4_1^+, 6_1^+, 8_1^+$ and $10_1^+$) in the $^{76-90}$Kr isotopes. We observe the discrepancy between theory and experiment for $J^\pi = 6_1^+$, $8_1^+$ in $^{90}$Kr isotopes with neutron bosons ($N = 42,44,46,48$). However, one must careful in comparing theory with experiment, since all calculated low-lying states have a collective nature.

The order of the $0_2^+$ and $3_1^+$ is correctly predicted in $^{76-90}$Kr isotopes and we remark that the energy of the $3_1^+$ state is predicted systematically too high. This is a consequence of the presence of a Majorana term $M_{\pi\nu}$ in the Hamiltonian (eq. 3). We have chosen the parameters of the Majorana force in such a way that it pushes up states which are not completely symmetric with respect to proton and neutron bosons, since there is no experimental evidence for such states. However, experimental information becomes available about these states with mixed symmetry, this situation could possibly be improved. In the present case it would have been possible to further higher its energy by constant the value of $\xi_3$.

The position of the $2_3^+$ state relative to the $0_2^+$ state especially in $^{78}$Kr, $^{82}$Kr and $^{88}$Kr isotopes. The moment of inertia of the ground state band increase, the quasi $\gamma$-band is pushed up, and also $0_2^+$ state becomes a member of a $K = 0$ $\beta$-band. 

In the Figures we show the results of our calculations for the energies of the ground state band ($2_1^+, 4_1^+, 6_1^+, 8_1^+$ and $10_1^+$) in the $^{76-90}$Kr isotopes. We observe the discrepancy between theory and experiment for $J^\pi = 6_1^+$, $8_1^+$ in $^{90}$Kr isotopes with neutron bosons ($N = 42,44,46,48$). However, one must careful in comparing theory with experiment, since all calculated low-lying states have a collective nature.
Figures 1& 2 : A comparison between the experimental energy levels from IBM-2 calculations for $^{76}$Kr and $^{78}$Kr [18]

Figures 3& 4 : A comparison between the experimental energy levels from IBM-2 calculations for $^{80}$Kr and $^{82}$Kr [18]
Figures 5 & 6: A comparison between the experimental energy levels from IBM-2 calculations for $^{84}$Kr and $^{88}$Kr [18]

Figure 7: A comparison between the experimental energy levels from IBM-2 calculations for $^{90}$Kr [18]

Electric Transition Probability

The effective boson charges $e_\pi$ and $e_\nu$ were calculated by plotting $M$ against $N_\nu/N_\pi$,

where[19]:
for the $U(5)$ limit.........(13)

\[
M_1 = \left( \frac{1}{N_\pi} \right) \sqrt{NB(E2; 2^+_1 \rightarrow 0^+_1)} \frac{e b}{e_\pi + e_\nu N_\nu / N_\pi}
\]

\[
M_2 = \left( \frac{1}{N_\pi} \right) \sqrt{\frac{5N}{N+4} B(E2; 2^+_1 \rightarrow 0^+_1)} \frac{e b}{e_\pi + e_\nu N_\nu / N_\pi}
\]

for the $O(6)$ limit.........(14)

where $B(E2)$ is the reduced transition probability, $N_\pi$ and $N_\nu$ are the boson numbers of proton and neutron respectively, $N = N_\pi + N_\nu$ is the total boson number. The difference between the effective charge and the charge of the single nucleon is referred to as the polarization charge. The value of effective charge may depend somewhat on the orbit of the nucleon. In particular, the polarization effect decreases when the binding energy of the nucleon becomes small.

Figure: 8 represent the relation between $\sqrt{NB(E2; 2^+_1 \rightarrow 0^+_1) / N_\pi}$ and $e_\pi + e_\nu N_\nu / N_\pi$ for the SU(5) limit, $\sqrt{\frac{5N}{N+4} B(E2; 2^+_1 \rightarrow 0^+_1) / N_\pi}$ and $e_\pi + e_\nu N_\nu / N_\pi$ for O(6) limit. The linearity is indeed present giving $e_\pi = 0.0288\; eb$ and $e_\nu = 0.209\; eb$ in the SU(5) limit and $e_\pi = 0.0575\; eb$, $e_\nu = 0.1047\; eb$ in the O(6) limit. The best fit to $^{76}_{20}K_r$ isotopes was obtained $e_\pi = 0.04315\; eb$ and $e_\nu = 0.15685\; eb$. This result gives a clear indication that the rotational contribution in nuclear motion in this region is very high.

We used these results of effective charges to calculate the electric transition probabilities using the NPBEM code. The results are presented in table 3.

![Figure 8: The plot of the quantity $M_1$ and $M_2$ versus $N_\nu / N_\pi$ for several even-even nuclei.](image)

**Table 3: Electric Transition Probability for $K_r$ isotopes in $e^2b^2$ units**

| IBM-2 | Exp.[8,18] | $I_i' \rightarrow I_j$ | Isotopes |
|-------|------------|------------------------|----------|
| 0.165 | 0.164(57)  | $2_1 \rightarrow 0_1$  | $^{76}_{20}K_r$ |
| 0.00521| 0.0090    | $2_2 \rightarrow 0_2$  |
| 0.0087 | 0.0038    | $2_2 \rightarrow 2_2$  |
| 0.0071 | -         | $3_2 \rightarrow 2_2$  |
| 0.1322 | -         | $0_3 \rightarrow 2_2$  |
| 0.0065 | 0.0019    | $3_1 \rightarrow 2_2$  |
| 0.1871 | -         | $3_1 \rightarrow 2_2$  |
| 0.0004 | -         | $3_2 \rightarrow 2_2$  |
| 0.1413 | 0.198(190)| $3_2 \rightarrow 2_2$  |
| 0.0098 | 0.0011(4) | $4_1 \rightarrow 2_2$  |
| 0.1320 | 0.0858(28)| $4_2 \rightarrow 2_2$  |
| -     | -         | $4_3 \rightarrow 2_2$  |
| 0.0731 | 0.0209(76)| $4_3 \rightarrow 4_1$  |
| 0.0007 | -         | $1_1 \rightarrow 2_2$  |
| 0.0021 | -         | $1_1 \rightarrow 2_2$  |
| 0.1861 | -         | $1_1 \rightarrow 2_2$  |
| -0.501 | -         | $Q(2_1)$           |
| 0.1202 | 0.1206(79)| $2_1 \rightarrow 0_1$  |


| Energy (MeV) | Transition | Branching Ratio | 76Kr | 78Kr | 80Kr | 82Kr | 84Kr |
|-------------|------------|----------------|------|------|------|------|------|
| 0.0029      | 0.003(4)   | 2→0,0,2       |      |      |      |      |      |
| 0.0825      | 0.0118(39) | 2→2,2         |      |      |      |      |      |
| 0.00025     | -          | 2→2           |      |      |      |      |      |
| 0.1861      | -          | 0→2           |      |      |      |      |      |
| 0.1462      | -          | 3→2           |      |      |      |      |      |
| 0.0060      | -          | 3→2           |      |      |      |      |      |
| 0.00019     | -          | 3→2           |      |      |      |      |      |
| -           | -          | 3→2           |      |      |      |      |      |
| -           | -          | 3→2           |      |      |      |      |      |
| 0.1876      | 0.1740(138)| 4→2           |      |      |      |      |      |
| 0.1092      | 0.1147(158)| 4→2           |      |      |      |      |      |
| -           | -          | 4→2           |      |      |      |      |      |
| -           | -          | 4→2           |      |      |      |      |      |
| 0.0622      | 0.0474(118)| 4→4           |      |      |      |      |      |
| 0.0396      | -          | 1,1→2         |      |      |      |      |      |
| 0.1261      | -          | 1,1→2         |      |      |      |      |      |
| 0.0071      | -          | 1,1→2         |      |      |      |      |      |
| -0.361      | -          | Q(2;2)         |      |      |      |      |      |
| 0.0756      | 0.0727(43) | 2,1→0         |      |      |      |      |      |
| 0.0266      | 0.0038     | 2,2→0         |      |      |      |      |      |
| 0.0772      | 0.0511(102)| 2,2→2         |      |      |      |      |      |
| 0.0002      | -          | 2,2→2         |      |      |      |      |      |
| 0.0903      | -          | 0,1→2         |      |      |      |      |      |
| 0.0019      | 0.011(3)   | 3,1→2         |      |      |      |      |      |
| 0.0782      | 0.0695(102)| 3,1→2         |      |      |      |      |      |
| 0.0002      | -          | 3,2→2         |      |      |      |      |      |
| -           | -          | 3,2→2         |      |      |      |      |      |
| 0.0971      | 0.8999(122)| 4,1→2         |      |      |      |      |      |
| 0.0072      | 0.0005(3)  | 4,2→2         |      |      |      |      |      |
| 0.117       | 0.1021(613)| 4,2→2         |      |      |      |      |      |
| 0.041       | -          | 4,2→4         |      |      |      |      |      |
| 0.0287      | -          | 1,1→2         |      |      |      |      |      |
| 0.1291      | -          | 1,1→2         |      |      |      |      |      |
| 0.0431      | -          | 1,1→2         |      |      |      |      |      |
| -0.361      | -          | Q(2;2)         |      |      |      |      |      |
| 0.0469      | 0.0450(14) | 2,1→0         |      |      |      |      |      |
| 0.0006      | 0.0002     | 2,2→0         |      |      |      |      |      |
| 0.0072      | 0.0053     | 2,2→2         |      |      |      |      |      |
| -           | -          | 2,2→2         |      |      |      |      |      |
| 0.0428      | 0.0317(105)| 0,2→2         |      |      |      |      |      |
| 0.0008      | -          | 3,1→2         |      |      |      |      |      |
| 0.0631      | -          | 3,1→2         |      |      |      |      |      |
| 0.0002      | -          | 3,2→2         |      |      |      |      |      |
| -           | -          | 3,2→2         |      |      |      |      |      |
| 0.0620      | 0.0676(253)| 4,1→2         |      |      |      |      |      |
| 0.0081      | 0.0024(4)  | 4,2→2         |      |      |      |      |      |
| 0.0202      | 0.0195(4)  | 4,2→2         |      |      |      |      |      |
| 0.0320      | 0.0812(13) | 4,2→4         |      |      |      |      |      |
| 0.0005      | -          | 1,1→2         |      |      |      |      |      |
| 0.00024     | -          | 1,1→2         |      |      |      |      |      |
| 0.0627      | -          | 1,1→2         |      |      |      |      |      |
| -0.361      | -          | Q(2;2)         |      |      |      |      |      |
| 0.0239      | 0.0244(11) | 2,1→0         |      |      |      |      |      |
| 0.0002      | 0.0052(13) | 2,2→0         |      |      |      |      |      |
| 0.0301      | 0.0239(87) | 2,2→2         |      |      |      |      |      |
| 0.0005      | -          | 2,2→2         |      |      |      |      |      |
| 0.0297      | -          | 0,1→2         |      |      |      |      |      |
| 0.0002      | -          | 3,1→2         |      |      |      |      |      |
| 0.00019     | -          | 3,1→2         |      |      |      |      |      |
| 0.0422      | -          | 3,2→2         |      |      |      |      |      |
| -           | -          | 3,2→2         |      |      |      |      |      |
| 0.0402      | 0.0479(65) | 4,1→2         |      |      |      |      |      |
| 0.00019     | 0.0003     | 4,2→2         |      |      |      |      |      |
| 0.00420     | 0.0035(5)  | 4,2→2         |      |      |      |      |      |
| 0.0221      | -          | 4,2→4         |      |      |      |      |      |
| 0.0001      | -          | 1,1→2         |      |      |      |      |      |
| 0.0003      | -          | 1,1→2         |      |      |      |      |      |
| 0.0272      | -          | 1,1→2         |      |      |      |      |      |
| -0.23       | -          | Q(2;2)         |      |      |      |      |      |
Continued B(E2) for Kr isotopes

| IBM-2 | Exp. | $I_{i}^{+} \rightarrow I_{f}^{+}$ | Isotopes |
|-------|------|---------------------------------|----------|
| 0.0322 | - | 2$_{0}^{+} \rightarrow 0_{1}^{+}$ | $^{88}$Kr |
| 0.0003 | - | 2$_{0}^{+} \rightarrow 0_{1}^{+}$ |
| 0.0622 | - | 2$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0007 | - | 2$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0622 | - | 0$_{0}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0003 | - | 3$_{1}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0025 | - | 3$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0001 | - | 3$_{1}^{+} \rightarrow 2_{1}^{+}$ |
| - | - | 3$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0527 | - | 4$_{1}^{+} \rightarrow 2_{1}^{+}$ |
| 0.00781 | - | 4$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0321 | - | 4$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0737 | - | 4$_{1}^{+} \rightarrow 4_{1}^{+}$ |
| 0.0003 | - | 1$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.00009 | - | 1$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0632 | - | 1$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| -0.22 | - | $Q(2_j^0)$ |
| 0.0652 | - | 2$_{0}^{+} \rightarrow 0_{1}^{+}$ |
| 0.0137 | - | 2$_{0}^{+} \rightarrow 0_{1}^{+}$ |
| 0.0321 | - | 2$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0001 | - | 2$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0346 | - | 0$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0187 | - | 3$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0276 | - | 3$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.00015 | - | 3$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| - | - | 3$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0972 | - | 4$_{1}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0622 | - | 4$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0432 | - | 4$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0821 | - | 4$_{1}^{+} \rightarrow 4_{1}^{+}$ |
| 0.0002 | - | 1$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0001 | - | 1$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0197 | - | 1$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| -0.20 | - | $Q(2_j^0)$ |
| -0.361 | - | $Q(2_j^0)$ |

| IBM-2 | Exp. | $I_{i}^{+} \rightarrow I_{f}^{+}$ |
|-------|------|---------------------------------|
| 0.0527 | - | 4$_{1}^{+} \rightarrow 2_{1}^{+}$ |
| 0.00781 | - | 4$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0321 | - | 4$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0737 | - | 4$_{1}^{+} \rightarrow 4_{1}^{+}$ |
| 0.0003 | - | 1$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.00009 | - | 1$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0632 | - | 1$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| -0.22 | - | $Q(2_j^0)$ |
| 0.0652 | - | 2$_{0}^{+} \rightarrow 0_{1}^{+}$ |
| 0.0137 | - | 2$_{0}^{+} \rightarrow 0_{1}^{+}$ |
| 0.0321 | - | 2$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0001 | - | 2$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0346 | - | 0$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0187 | - | 3$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0276 | - | 3$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.00015 | - | 3$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| - | - | 3$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0972 | - | 4$_{1}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0622 | - | 4$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0432 | - | 4$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0821 | - | 4$_{1}^{+} \rightarrow 4_{1}^{+}$ |
| 0.0002 | - | 1$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0001 | - | 1$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| 0.0197 | - | 1$_{2}^{+} \rightarrow 2_{1}^{+}$ |
| -0.20 | - | $Q(2_j^0)$ |
| -0.361 | - | $Q(2_j^0)$ |

Experimental data are taken from [8,18]

It is well known that absolute gamma ray transition probabilities offer the possibility of a very sensitive test of nuclear models and the majority of the information on the nature of the ground state has come from studies of the energy level spacing. The transition probability values of the exited state in the ground state band constitute another source of nuclear information. Yrast levels of even-even nuclei ($I=2,4,6,.....$) usually decay by $E2$ transition to the lower lying yrast level with $I_f = I_i - 2$.

In table 3, we show the $B(E2;2^+_1 \rightarrow 0^+_1)$ and $B(E2;4^+_1 \rightarrow 2^+_1)$ values, which are of the same order of magnitude and display a typical decrease towards the middle of the shell.

As a consequence of possible $M1$ admixture the $B(E2;2^+_2 \rightarrow 2^+_1)$ quantity is rather difficult to measure. For $^{86}$Kr isotopes, we give the different, conflicting experimental results and we see that no general feature n be derived from them, from these values seems to increase for $^{76-78}$Kr and decrease for $^{82-84}$Kr.

In the table we show $B(E2;2^+_1 \rightarrow 0^+_1)$ values. Experimentally the results are radically different for the $^{82}$Kr isotopes, the value. In the some $^{90}$Kr isotopes the value seems to increased towards the middle of the shell, whereas in another $^{90}$Kr isotopes is decreased. Our calculations could not reproduce these contradictory features simultaneously.

The quantity $B(E2;0^+_2 \rightarrow 2^+_1)$, which is shown in table 3, provides a second clue for identifying intrude $0^+_2$ states. If the experimental $B(E2;0^+_2 \rightarrow 2^+_1)$ value largely deviates from the results of our calculation, it is very likely the observed $0^+_2$ states does not correspond to the collective state, but it is rather an intruder state. In $^{82}$Kr isotope, there is a good agreement between
experimental and calculated $B(E2;0^+_2 \rightarrow 2^+_1)$ value. This confirm our earlier statement about the nature of the lowest $0^+$ state in this isotope.

The electric transition probabilities from the mixed-symmetry state $I = 1^+$ to the symmetric states $(2^+_1, 2^+_2)$ is weak collective E2 transition. The E2 transition between the $1^+$ and the $2^+_1$ ground state is small, whereas E2 transitions are large between fully-symmetric states and between mixed-symmetry states.

To conclude this section on the E2 properties, we give the results for the quadrupole moments $Q(2^+_1)$ of the first excited $2^+$ state in table 3. We show complication of theoretical results. The general features of these results is clear, namely an increased in the negative quadrupole moment with increasing neutron number.

**Magnetic Transition Probability**

The $B(M1)$ reduced transition probabilities were calculated using eq.6, and the boson gyromagnetic factors $g_\pi$, $g_\nu$ were estimated using the fact that $g = Z/A$ and the relation [20]

$$g = g_\pi \frac{N_\pi}{N_\pi + N_\nu} + g_\nu \frac{N_\nu}{N_\pi + N_\nu} \ldots \ldots \ldots \ldots (15)$$

and one of the experimental $B(M1;2^+_2 \rightarrow 2^+_1) = 0.0429 \mu_N^2$ for $^{76}Kr$ isotope, were used to produce a suitable estimation for the boson gyromagnetic factors. These values are $g_\pi = 0.782 \mu_N$ and $g_\nu = 0.328 \mu_N$. They are different from those of the rare–earth nuclei, $(g_\pi - g_\nu = 0.65\mu_N)$, suggested by Van Isacker et al [21] also used $g_\pi = 1$ and $g_\nu = 1$ to reduce the number of the model parameters in their calculation of $M1$ properties in deformed nuclei. The results of our calculation are listed in table 4. A good agreement between the theory and the available experimental data is achieved. As can be seen from the table yields to a simple prediction that $M1$ matrix elements values for gamma to ground and transitions should be equal for the same initial and final spin. Also the size of gamma to ground matrix elements seems to decrease as the mass number increases.

The results shows that the transitions between low-lying collective states are relatively weak. This is because of the increase of the anti-symmetric component in the wave functions introduced by $F$-spin breaking in the Hamiltonian. The magnitude of $M1$ values increases with increasing spin for $\gamma \rightarrow g$ and $\gamma \rightarrow \gamma$ transitions and we see:

1-By fitting $B(M1)$ from $2_\gamma$ to $2_g$ we always get small value for $g_\pi - g_\nu$ compared with the value basis on the microscopic calculations $g_\pi = g_\nu = 1$.

2-There are evidences that $M1$ small mode exists in all spectra.

3-one cannot make decisive conclusions related to the agreement between theoretical and experimental data from the above table due to the lack of experimental data. However both experiments and theory predicts small $M1$ component which is due to symmetry and forbiddances of band crossing gamma transitions.

4-The $\gamma \rightarrow \gamma$ $M1$ matrix elements are larger than the $\gamma \rightarrow g$ $M1$ matrix elements by a factor of 2 to 3. Again, this agree qualitatively with the perturbation expressions derived in [22].

5-The size of the $\gamma \rightarrow g$ $M1$ matrix elements seems to decrease with increasing mass. Specially, a change in $\gamma \rightarrow g$ $M1$ strengths occurs when the gamma band crosses the beta band.

The $M1$ properties of collective nuclei are certainly very sensitive to various, even small, components in the wave functions either of collective or non-collective character. In the $^{88-90}Kr$ isotopes it was shown that the inclusion of excitations across the major shell and two quasi-particle states is important. One excepts that also for $^{88-90}Kr$ isotopes (which are near to closed shell for neutron) similar effects come into play. As above analysis suggests they can manifest in considerable renormalization of IBM-2 boson $g$-factors from their slandered values. The magnetic dipole moment for first excited state is given by

$$\mu(2^+_1) = g_\pi L_\pi + g_\nu L_\nu \ldots \ldots \ldots (16)$$

where $g_\pi (g_\nu)$ is the $g$-factor for the correlated proton (neutron) boson and $L_\pi (L_\nu)$ is the corresponding angular momentum operator. According to the microscopic foundation of the model, $g_\pi (g_\nu)$ is expected to depend, in first approximation on proton (neutron) number $N_\pi (N_\nu)$ only, $g_\pi = g_\pi(N_\pi)$ and $g_\nu = g_\nu(N_\nu)$. The IBM-2 calculations for $\mu(2^+_1)$ are listed in table 4, we see a good agreement with experimental data.

It is clear that the two effects contribute to the dependence of the magnetic moments on proton and neutron number: the dependence of $g_\pi$ and $g_\nu$ on proton and neutron number and the variation of the matrix elements of the operator $L_\pi (L_\nu)$ with $N_\pi$ and $N_\nu$. As will be better shown below, the former effect is the related to the shell structure of the orbits, while the latter is related to the average number of proton and neutron boson taking part in the collective motion.
Experimental data are taken from [8, 18].

Mixing Ratio \( \delta(E2/M1) \)

We evaluate the mixing ratio \( \delta(E2/M1) \) for Kr isotopes, depends on the equation [23].

\[
\delta(E2/M1) = \frac{<I|P(E2)|I'>}{<I|P(M1)|I'>} \quad \ldots \ldots (17)
\]

These are compared with experimental and theoretical results in Table 5, where one can see good agreement with estimated and experimental values. The variations in sign of the \( E2/M1 \) mixing ratios from one nucleus to another for the same class of transitions, and within a given nucleus for transitions from different spin states, suggest that a microscopic approach is needed to explain the data theoretically. For such reason, the sign of the mixing ratio is not taken into consideration. Sign convention of mixing ratios has been explained in detail by Lange et al. [23].

These results exhibit disagreement in some cases, with one case showing disagreement in sign. However, it is a ratio between very small quantities and any change in the denominator that will have a great influence on the ratio. The large calculated value for \( 2^+_2 \rightarrow 2^+_1 \) is not due to a dominate \( E2 \) transition, but may be under the effect of very small \( M1 \) component in the transition.

Moreover, the large predicted value for transition \( 2^+_2 \rightarrow 2^+_1 \) in \( ^{80}\text{Kr} \) compared with experimental value may be related to high predicted energy level value of the IBM-2: \( E(2^+_2) = 1.287 \text{ MeV} \), while the experimental value is 1.256 MeV. We are unable to bring the energy value of this state close to experimental value simply by changing the Majorana parameters.

Table 5: Mixing Ratios \( \delta(E2/M1) \) in Kr Isotopes in MeV \( \mu^2 \) units

| \( ^{90}\text{Kr} \) | \( ^{88}\text{Kr} \) | \( ^{84}\text{Kr} \) | \( ^{82}\text{Kr} \) | \( ^{80}\text{Kr} \) | \( ^{78}\text{Kr} \) | \( ^{76}\text{Kr} \) | \( I_1 \rightarrow I'_1 \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| IBM-2 Exp. | IBM-2 Exp. | IBM-2 Exp. | IBM-2 Exp. | IBM-2 Exp. | IBM-2 Exp. | IBM-2 Exp. | IBM-2 Exp. |
| -6.327 | -0.9107 | -0.914 | 10 | - | 0.717 | 0.08 ≤ \( \Delta E \) ≤ 0.64 | 5.480 | 16 | 0.1865 | - | 1.189 | - |
| 1.1176 | -0.835 | -0.1435 | - | -0.549 | - | 0.5317 | - | 0.165 | - | 0.087 | - |
| 4.805 | -0.912 | -0.647 | - | 1.144 | 4.2(2) | 0.4447 | - | 7.182 | - | 1.9935 | - |
| -10.255 | -0.3915 | -4.150 | - | 5.858 | 2.13(7) | 2.833 | - | 0.0989 | - | 0.4289 | - |
| 0.7333 | -3.792 | -7.372 | - | 0.0274 | - | 0.2710 | - | 0.1729 | - | 2.565 | - |
| 18.4123 | -10.121 | -2.979 | - | 0.3909 | - | 0.6478 | - | 2.376 | - | 1.630 | - |
| 4.942 | -22.701 | -0.5674 | - | 2.1323 | - | 1.605 | - | 1.3559 | - | 0.0887 | - |
| 3.617 | -21.432 | -0.0614 | - | 0.6455 | - | 2.104 | - | 1.7973 | - | 0.0492 | - |
| 0.527 | -0.0327 | -3.560 | - | 0.7104 | - | 0.855 | - | 0.0466 | - | 2.227 | - |

Experimental data are taken from [8, 23].
Electric Monopole Matrix Element $\rho(E0)$

The $E0$ transition occurs between two states of the same spin and parity by transferring the energy and zero units of angular momentum, and it has no competing gamma ray. The $E0$ transition is present when there is a change in the surface of the nucleus. For example, in nuclear models where the surface is assumed fixed, $E0$ transitions are strictly forbidden, such as in shell and IBM models. Electric monopole transitions are completely under the penetration effect of atomic electrons on the nucleus, and can occur not only in $0^+ \rightarrow 0^+$ transition but also, in competition with gamma multipole transition, and depending on transition selection rules that may compete in any $\Delta J = 0$ decay such as a $2^+ \rightarrow 2^+$ or any $I_1 = I_f$ states in the scheme. When the transition energy greater than $2\hbar\omega$, monopole pair production is also possible.

The $E0$ reduced transition probability is given in eq. 9. The parameters in equation 9, can be predicted from the isotope shift [20] (see table 7), since such data are not available for $\text{Kr}$ isotopes, we calculate these parameters by fitting procedure into two experimental values of isotopic shifts (equation (2-59)). The parameters which were subsequently used to evaluate the $\rho(E0)$ -values were: $\beta_0 = 0.062 \text{ \, eb}$, $\beta_0 = -0.021 \text{ \, eb}$ and $\gamma_0 = 0.032 \text{ \, fm}^2$. From the table 6, in general there is no experimental data to compare with the IBM-2 calculations.

The monopole matrix element is important for nuclear structure and the model predictions due to their sensitivity for the nuclear shape. We conclude that more experimental work is needed to clarify the band structure and investigate an acceptable degree of agreement between the predictions of the models and the experimental data.

We also find good agreement between the calculated and experimental values for isotope shifts for all mercury isotopes (table 7) but the isomer shift result for $\text{Kr}$ isotopes is in poor agreement with the experimental value.

Table 6: Monopole matrix element for $\text{Kr}$ isotopes

| $^{90}$Kr | $^{88}$Kr | 0.0082 | $^{82}$Kr | $^{80}$Kr | 0.0551 | 0.0431 | $^{76}$Kr | $^{78}$Kr | 0.0007 | 0.0092 | 0.0009 | 0.00083 | 0.00080 | 0.00073 | $^{0}$ | $^{1}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|--- |
| 0.075 | 0.00094 | 0.0082 | 0.0750 | 0.070 | 0.0551 | 0.0431 | $^{0}$ | $^{1}$ |
| 0.0009 | 0.00092 | 0.00089 | 0.00083 | 0.00080 | 0.00073 | 0.0067 | $^{0}$ | $^{1}$ |
| 0.0102 | 0.0099 | 0.00094 | 0.00089 | 0.00034 | 0.0045 | 3.18*10-3 | $^{0}$ | $^{1}$ |
| 0.013 | 0.00099 | 0.0096 | 0.0076 | 0.0026 | 0.0002 | | |
| 0.00070 | 0.00067 | 0.00065 | 0.00052 | 0.00046 | 0.00041 | 0.00034 | | |
| 0.00072 | 0.000531 | 0.000519 | 0.00395 | 0.00351 | 0.0027 | 0.0021 | | |
| 0.000742 | 0.00072 | 0.000611 | 0.00059 | 0.00057 | 0.00041 | 0.00035 | | |

Table 7: Isotopic Shifts for $\text{Kr}$ Isotopes

| Nucleus | $\Delta < r^2 > \, \text{fm}^2$ | IBM-2 |
|---|---|---|
| $^{76}_{36}$Kr $^{78}_{36}$Kr | - | -0.009 |
| $^{78}_{36}$Kr $^{80}_{36}$Kr | - | -0.0148 |
| $^{80}_{36}$Kr $^{82}_{36}$Kr | -0.028(5) | -0.0298 |
| $^{82}_{36}$Kr $^{84}_{36}$Kr | -0.040(4) | -0.055 |
| $^{84}_{36}$Kr $^{86}_{36}$Kr | 0.071(3) | 0.0810 |
| $^{86}_{36}$Kr $^{88}_{36}$Kr | 0.37997 | 0.431 |
| $^{88}_{36}$Kr $^{90}_{36}$Kr | 0.751 | 0.655 |

Mixed Symmetry States

One of the advantage of the IBM-2 is ability of reproducing the mixed symmetry states. These states are created by a mixture of the wave function of protons and neutrons that are observed in most even-even-even nuclei. This mixed symmetry states (MSS) has been observed in many nuclei. In more vibrational and $\gamma$-soft nuclei This mixed symmetry states (MSS) has been observed in many nuclei. In more vibrational and gamma soft nuclei. We expect the lowest MSS with $I^+ = 2^+$ state, while in rotational nuclei observed as the $I^+ = 1^+$ state. In $\text{Kr}$ isotopes we see that when the states...
\[ I^\pi = 2^+_3, 2^+_4 \] and \[ 3^+_1 \] are strongly dominated by the \( F=F_{\text{max}} \), the strongest contribution to the \( I^\pi = 2^+_1, 3^+_2 \) states is the one with \( F=F_{\text{max}}-1 \). We can see the \( I^\pi = 2^+_3, 3^+_2 \) states as a mixed symmetry states in Kr isotopes.

In this work, we proposed that the \( 2^+_3 \) state decays to the first excited state with an energy 1.598 MeV in \(^{76}\text{Kr}\) with a mixing ratio \( \delta(E2/M1) = 1.189 \) which means it is dominated by the M1 transition, with \( B(M1) \) equal to 0.0031 \( \mu_N^2 \). In \(^{78}\text{Kr}\) isotope, for the third \( I^\pi = 2^+ \) state at energy 1.685 MeV excitation is close to the experimental data for 1.755 MeV. The energy is well reproduced by the calculation, where the choice of the Majorana parameters plays a crucial role. This state is quite pure \( F_{\text{max}}=1 \) with \( R = 0.5 \) \( F2 \) \( I \) \( F_{\text{max}} = (F_{\text{max}} + 1) = 50\% \). The excitation energy of \( 3^+_2 \) state is 2.399 MeV with mixing ratio \( \delta(E2/M1; 3^+_2 \rightarrow 2^+_1) = 2.565 \cdot B(M1; 3^+_2 \rightarrow 2^+_1) = 0.00301 \mu_N^2 \). In the \(^{80}\text{Kr}\), the calculation predicted the \( 2^+_3 \) state at 2.251 MeV with \( R = 83\% \).

In other \(^{82-84,88-90}\text{Kr}\) isotopes the states \( 2^+_3 \) and \( 3^+_2 \) are mixed symmetry states their excitation energies are closed with available experimental data and the values of \( R = 73\%, 75\%, 72\% \) and 80\% respectively.

In all \( Kr \) isotopes that the second \( 3^+_1 \) states to be the lowest \( I^\pi = 3^+_1 \) mixed symmetry states with two phonon excitation. The low-lying levels with angular momentum greater than \( 3^+ \) with a large mixed symmetry states component are predicted in this work.

The energy fit to several levels is very sensitive to the parameters in the Majorana term which also strongly influence the magnitude and sign of the multipole mixing ratios of many transitions. In particular we find that the calculated energies of a number of states are affected in a very similar way and these might be considered to have a mixed-symmetry origin, or contain substantial mixed-symmetry components. Those with a mixed-symmetry origin have no counterpart in IBM-1. The energy dependence of the \( 2^+_3 \) and \( 2^+_4 \) levels is consistent with the mixed-symmetry character of the \( 2^+_3 \) level being shared with neighboring states[24,25].

The influence of the parameters on these states is shown in table 1. The \( \xi_2 \) term strongly affects the energies of all of the levels considered to have a mixed-symmetry character or to contain mixed-symmetry components. In obtaining this plot the \( \xi_1 \) and \( \xi_3 \) terms were maintained at their best-fit values. The mixing ratio data, discussed in the above section have a strong dependence on \( \xi_2 \) and show that \( \xi_2 \) cannot be zero in our fit.

The \( 1^+ \) level is strongly affected by changing \( \xi_1 \), while the \( 3^+_1 \) level energy depends on the \( \xi_3 \) value. The \( 2^+_3 \) mixed-symmetry state and the predominantly symmetric \( 2^+_2 \) and \( 3^+_4 \) levels are largely unaffected by changing \( \xi_1 \) or \( \xi_3 \) in contrast to their dependence on \( \xi_2 \).

Most experimentally observed low-spin levels, apart from \( 1^+ \) states below 5 MeV; have their counterpart in the IBM-2 level spectrum although the energy match is not good in every case. It also appears that we may identify the members of the family of mixed-symmetry states corresponding to the \([N-1,1]\) representation [12]. The small \( E2/M1 \) mixing ratios are consistent with this interpretation but level lifetimes are required for a firmer identification.

In \( K r \) isotopes, all hitherto discovered MSS have been reviewed in [9]. It has been shown that the lowest lying MSS is the one quadrupole phonon MSS labeled as \( 2^+_3 \) \( 3^+_2 \) \( l=1, m=l \) and characterized by a weakly-collective \( E2 \) transition probability to the ground state and a large \( M1 \) transition to the \( 2^+_1 \) state.

Table 6, contains the calculated \( \rho(E0) \) values. In general there is no experimental data to compare the IBM-2 results. It must also be remarked that the comparatively large \( \rho(E0) \) values for transitions from the \( 2^+_3 \) mixed-symmetry state and from the \( 2^+_2 \) and \( 2^+_4 \) states indicate that substantial E0 components occur in these decays from mixed-symmetry states. The E0 matrix element describing such decay is proportional to \( \beta_{0+} \) and \( \beta_{0+} \), although the \( \beta \) values are small, their sign difference results in the E0 matrix being greatest.

**Conclusion**

We have presented results of calculations of the properties of the \(^{76-90}\text{Kr}\) isotopes found in many cases good agreement between our calculations and experiment. However, we have also found that \(^{76-90}\text{Kr}\) isotopes, that there are several \( 0^+ \) states whose properties cannot be reproduced by our calculations. These intruder states are presumably associated with other with the other low-lying degrees of freedom. Since they appears to be present also in other mass regions, it is clear that they must be explicitly included in the calculations if one wants to describe all observed low-lying states.

The shape transition predicted by this study is consistent with the spectroscopic data for \(^{76-90}\text{Kr}\) are typical examples of isotopes that exhibit a smooth phase transition from vibrational nuclei to soft triaxial rotors U(5) → O(6). The comparison of some \( B(E2), B(M1) \), mixing ratios for these isotopes with the experimental data show that these isotopes exist along the U(5)-O(6) side of the IBM triangle.
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