NONEQUILIBRIUM CHIRAL DYNAMICS AND EFFECTIVE LAGRANGIANS

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We review our recent work on Chiral Lagrangians out of thermal equilibrium, which are introduced to analyse the pion gas formed after a Relativistic Heavy Ion Collision. Chiral Perturbation Theory is extended by letting $f_\pi$ be time dependent and allows to describe explosive production of pions in parametric resonance. This mechanism could be relevant if hadronization occurs at the chiral phase transition.

1 Introduction

In the last few years there has been a growing interest in nonequilibrium Quantum Field Theory, motivated by the experiments on Relativistic Heavy Ion Collisions seeking the Quark-Gluon Plasma (QGP) and its properties. After the promising results obtained by SPS at CERN the torch has been passed on to RHIC at the BNL. Roughly speaking, these experiments observe the final particle spectra, trying to extract information about the possible QGP formation and its subsequent expansion until freeze-out, during which the chiral phase transition and hadronization take place. Apart from equilibrium properties such as in-medium masses and decay widths or the phase diagram, nonequilibrium aspects of the expansion may also be relevant. An important example is the hadronization process, which is not yet fully understood. One of the scenarios suggested to explain the production of hadrons out of the expanding plasma is that of Disoriented Chiral Condensates, where pions develop strong fluctuations after the chiral phase transition. In this context, two mechanisms can give rise to pion production. The first is spinodal decomposition, which takes place in an early stage after the transition, when it is reasonable to assume an initial supercooled state. Thus, long wavelength modes grow exponentially yielding strong pion correlations. The second is parametric resonance, where the oscillations of the $\sigma$ mode in a later stage yield explosive pion production. The resulting reheating process yields a final temperature $T_f \simeq 135$ MeV compatible with the freeze-out of hadrons. Both approaches are complementary and have been studied numerically in great detail in the large-$N$ limit of the $O(N)$ model. Here we will discuss the extension of Chiral Perturbation Theory (ChPT) to include nonequilibrium effects and its application to pion production in parametric resonance.
2 Nonequilibrium effective chiral models

An effective model to describe QCD well below the chiral scale $\Lambda_\chi \simeq 1.2$ GeV must be based on the chiral Spontaneous Symmetry Breaking (SSB) pattern $SU_L(N_f) \times SU_R(N_f) \rightarrow SU_V(N_f)$ where $N_f$ is the number of light flavours. One example is the $O(N)$ model, which reproduces the chiral SSB only for $N = 4$ ($N_f = 2$) and, due to the strong interaction, is nonperturbative in the coupling constant, so that nonperturbative schemes such as large $N$ must be implemented. An alternative for low-energy QCD is ChPT\cite{9,10}, which provides a well-defined expansion in $p/\Lambda_\chi$ where $p$ is any meson energy. Such expansion is renormalizable order by order. For instance, the $O(p^4)$ lagrangian, the familiar Non-Linear Sigma Model (NLSM), generates $O(p^6)$ loops whose divergences are absorbed by adding to the NLSM a $O(p^4)$ lagrangian including all possible terms compatible with the symmetries, with undetermined coefficients, the so-called low-energy constants (LEC), which can be fixed with experimental information. Regarding nonequilibrium, the ChPT scheme is best suited for a stage where the system is into the broken phase. In that case, it provides us with the advantages commented above. For instance, it can be used both for $N_f = 2, 3$. Therefore, as far as pion production is concerned, it will be useful in the parametric resonance regime, whereas the $O(N)$ model is more adequate to describe the spinodal phase. The first step is then to extend the NLSM out of equilibrium. The approach we will follow is to perturb the system from an equilibrium state and analyze its subsequent evolution. Since the form of the lagrangian is dictated by the chiral symmetry, assuming spatial homogeneity and isotropy, the only way to introduce the perturbation to leading order is by letting the pion decay constant $f_\pi$ be time dependent\cite{7}. Thus, the $O(p^2)$ NLSM nonequilibrium action becomes

$$S_2[U] = \int_C d^4x \frac{f^2(t)}{4} \text{tr} \partial_\mu U^\dagger(\vec{x}, t) \partial^\mu U(\vec{x}, t)$$

where $f_C$ indicates that the time integration runs along a Schwinger-Keldysh contour in the complex plane, which is the standard technique to formulate the non-equilibrium path integral. We fix the initial time to $t = 0$, where the system is in thermal equilibrium at temperature $T_i$. Thus, $f(t \leq 0) = f$ where $f = f_\pi(1 + O(p^2))$ and $f_\pi \simeq 93$ MeV. We restrict to $N_f = 2$ so that the $SU(2)$ matrix $U = [(f^2(t) - \pi^2)^{1/2} + i\vec{\tau} \cdot \vec{\pi}] / f(t)$ where $\pi^a(\vec{x}, t)$ are the three pion fields. Note also that we work in the chiral limit, i.e, the $u, d$ masses vanish exactly, so that we have not included explicit symmetry breaking mass terms. One can then proceed with ChPT as usual, expanding the action\cite{4} in pion fields, which generates all possible interaction vertices. The chiral power counting remains
consistent if $\dot{f}/f \simeq O(p)$, $\ddot{f}/f \simeq O(p^2)$ and so on. In this sense, ChPT is more appropriate to describe a situation not far from equilibrium such as parametric resonance. As explained before, from the moment we consider loop diagrams the ChPT scheme requires the $O(p^4)$ lagrangian, whose nonequilibrium version is more involved to construct than the $O(p^2)$ NLSM in (3). For that purpose, it is useful to view the action (3) as an equilibrium NLSM in a spatially flat Robertson-Walker (RW) metric background whose scale factor is just $f(t)/f$. This is easily achieved by rescaling the pion fields to $\pi f(t)$ and allows to write the lagrangian to any order by rising and lowering indices with the RW metric and including all new tensor structures preserving the symmetry. The $O(p^4)$ lagrangian built in this way is given in (7). In addition to the equilibrium lagrangian, it involves two new terms and hence two new LEC, which have been determined experimentally in (11). These two terms are crucial since they allow to renormalize new time-dependent infinities that were not present in the equilibrium case. For instance, the ChPT one-loop diagrams include tadpoles proportional to the equal time correlator $\int \frac{d^{d-1}k}{(2\pi)^{d-1}} G_0(t, t, k)$, where $d$ is the space-time dimension and $G_0(t, t, k) = \langle T C \pi_k(t) \pi_k(t') \rangle$ is the pion two-point function. This quantity has a time-dependent divergent part which vanishes in equilibrium. Using dimensional regularization, these divergences can be absorbed in the new $O(p^4)$ parameters so that observables become finite and scale independent.

3 Parametric Resonance in ChPT

In the nonequilibrium $O(4)$ model, $\langle \sigma \rangle$ is time dependent and in the last stage of the nonequilibrium evolution it oscillates around the constant vacuum. Thus, $\sigma(x, t) = \sigma_0(t) + \delta \sigma(x, t)$ where $\sigma_0(t)$ is a time-dependent classical background which oscillates around $\sigma_0 = f$ and $\delta \sigma$ includes quantum corrections, which are subleading. The oscillations transfer energy to the pion fields, yielding exponential growth of the pion correlator via parametric resonance. The connection with ChPT is established through the observation that the NLSM corresponds, to leading order, to the $O(4)$ model with the constraint $\sigma^2 + \pi^2 = \sigma_0^2(t)$ and $f(t) \sim \sigma_0(t)$. Therefore, we will take

$$f(t) = f \left[ 1 - \frac{q}{2} \sin Mt \right] \quad (t > 0) \quad (2)$$

It is important to bear in mind that using (2) is valid for times $t < t_{BR}$ where we denote by $t_{BR}$ the back-reaction time such as the pion correlations become typically $\langle \pi^2 \rangle(t_{BR}) \sim f^2$. When that time scale is reached, (2) is no longer a solution of the equations of motion in the $O(4)$ model and loop
corrections in ChPT become of the same order as the tree level. It must be pointed out that for $t \sim t_{BR}$, dissipation makes particle production stop\footnote{1}. Therefore, the ChPT approach is able to account for nearly all the pion production before $t_{BR}$. Another point with is worth stressing is that to one loop in ChPT, the nonequilibrium chiral power counting demands $qM^2 = O(p^2)$. That is, we are studying small oscillations, which in inflationary Cosmology is called narrow resonance. The mass parameter $M$ does not need to be small and in fact numerically $M \sim m_\sigma$, although there is no need to invoke the $\sigma$ particle in the ChPT context. Once the assumptions and limitations of our approach are clear, let us sketch how the parametric resonance works in ChPT.

To $O(\pi^2)$, the action (1) yields the “free” lagrangian, which now contains a time-dependent mass term, so that, taking into account (2), $G_0$ satisfies
\begin{equation}
\{ \partial_t^2 + k^2 - (qM^2/2) \sin Mt \} G_0(t, t', k) = -\delta_C(t - t')
\end{equation}

The above differential equation has the form of the Mathieu equation, whose solutions develops instability bands in momentum space, centered at $k_n \simeq nM/2$ and of width $\Delta k_n = O(q^n)$. Therefore, to the order we are considering, i.e, $O(q)$, only the first band is resonant.

The first observable one can calculate in one-loop ChPT is $f_\pi(t)$, whose NLO corrections include tree level diagrams coming from the $O(p^4)$ lagrangian and tadpoles containing the exponentially growing function
\begin{equation}
\Delta_{unst}(t) = \frac{i}{2\pi^2} \int_{\Delta k_1} dk k^2 [G_0(t, t, k) - G_0(0, 0, k)]
\end{equation}

We have plotted $\Delta_{unst}(t)$ and $f_\pi(t)$ in Figure \ref{fig1} where $f_\pi^{s,t}(t)$ correspond to the spatial and time components of the axial current \footnote{2}. The parameters

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.pdf}
\caption{The left curve is $\Delta_{unst}(t)/f_\pi^2$ where the dashed line is the tree level contribution in \cite{4}. The right curve shows $f_\pi^s(t)/f_\pi(0)$ for the same parameters.}
\end{figure}
Figure 2: The left solid curve is the time average of \( n(k, t) \) from \( t = 0 \) to \( t_{BR} \) and the dashed line is that of \( (k^2/M^2)n(k, t) \). The right curve is \( \langle n(t) \rangle/V = (1/2\pi^2) \int dk k^2 n(k, t) \). The parameters \( M, q, T \) are the same as in Figure 1.

have been chosen so that \( t_{BR} \simeq 10 \text{ fm/c} \), the typical plasma lifetime. We have estimated \( t_{BR} \) as the time when \( \Delta_{\text{unst}} \) equals the tree level contribution. Observe that the \( f_\pi(t) \) oscillations are not damped, which is a reminder that we have not included the back-reaction properly and should not worry us as far as pion production is concerned. We also see that the central value decreases with time. This effect can be interpreted as a reheating of the system (since \( f_\pi \) decreases with the temperature) which here leads to a final temperature \( T_f \simeq 125-140 \text{ MeV} \), compatible with hadronic freeze-out.

The pion distribution function \( n(k, t) \) and hence the pion number may also be analysed in this context. First, one needs to provide an appropriate definition for \( n(k, t) \) at nonequilibrium since, similarly to QFT in curved space-time, the vacuum state is time-dependent. A consistent approach is to define it in terms of the energy-momentum tensor expectation value, using a suitable point-splitting technique. In this way, one can calculate the number of massless pions to one loop in ChPT\(^8\). After a careful evaluation of all the NLO contributions it turns out \( n^{NLO}(k, t) = n^{LO}(k, t) + \mathcal{O}(q^2) \). In Figure 2, we have plotted the time average of \( n(k, t) \) as well as the total pion density as a function of time. The pion spectral function is peaked around the resonant frequency \( k \simeq 300 \text{ MeV} \), which could indeed be of importance for RHIC\(^\text{13}\).

Note that the pion density grows without stop because the back-reaction is neglected. However, as commented before, for practical purposes we only need the number of pions produced by the end of the expansion, for which we get 0.2 per fm\(^3\). Our predictions agree with the \( \mathcal{O}(4) \) model\(^\text{14,15}\).
4 Conclusions and Outlook

Chiral Effective Models can be used to describe nonequilibrium phenomena such as particle production, which are relevant for the hadronization of a QGP. We have showed our results using ChPT, which provides a systematic perturbative expansion in energies and time derivatives. Explosive production of pions takes place when \( f_\pi(t) \) oscillates in the parametric resonance regime. The final pion spectra and pion density agree with recent determinations within the \( O(4) \) model. We believe that our results are promising and provide useful techniques for future theoretical analysis related to RHIC applications.

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References

1. A.Anselm, Phys. Lett. B 217, 169 (1989); A.Anselm and M.Ryskin, Phys. Lett. B 226, 482 (1991); J.P.Blaizot and A.Krzywicki, Phys. Rev. D 46, 246 (1992).
2. K.Rajagopal and F.Wilczek, Nucl. Phys. B 404, 577 (1993); S.Gavin, A.Gocksch and R.D.Pisarski, Phys. Rev. Lett. 72, 2143 (1994).
3. D.Boyanowsky, H.J. de Vega and R.Holman, Phys. Rev. D 51, 734 (1995).
4. F.Cooper, Y.Kluger, E.Mottola and J.P.Paz, Phys. Rev. D 51, 2377 (1995).
5. S.Gavin and B.Müller, Phys. Lett. B 329, 486 (1994); S.Mrowczynski and B.Müller, Phys. Lett. B 363, 1 (1995).
6. D.Boyanowsky, H.J. de Vega, R.Holman and J.F.J.Salgado, Phys. Rev. D 54, 7570 (1996).
7. A.Gómez Nicola and V.Galán-González, Phys. Lett. B 449, 288 (1999).
8. A.Gómez Nicola, Phys. Rev. D 64, 016011 (2001).
9. S.Weinberg, Physica A 96, 327 (1979).
10. J.Gasser and H.Leutwyler, Ann.Phys. (N.Y) 158, 142 (1984), Nucl. Phys. B 250, 465 (1985).
11. J.F.Donoghue and H.Leutwyler, Z. Phys. C 52, 343 (1991).
12. R.D.Pisarski and M.Tytgat, Phys. Rev. D 54, 2989 (1996).
13. J.Randrup, Phys. Rev. C 62, 064905 (2000).
14. H.Hiro-Oka and H.Minakata, Phys. Lett. B 425, 129 (1998); 434, 461 (1998) (Erratum); Phys. Rev. C 61, 044903 (2000).