Quark-hadron mixed phases in protoneutron stars

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Abstract. We consider the possible formation of the quark hadron mixed phase in protoneutron stars. We discuss two cases: the first one, corresponding to a vanishingly small value of the surface tension of quark matter, is the well known mixed phase in which the global electric charge neutrality condition is imposed. In turn, this produces a non-constant pressure mixed phase. In the second case, corresponding to very large values of the surface tension phase, the charge neutrality condition holds only locally. However, the existence in protoneutron star matter of an additional globally conserved charge, the lepton number, allows for a new type of non-constant pressure mixed phase. We discuss the properties of the new mixed phase and the possible effects of its formation during the evolution of protoneutron stars.
1. Introduction

The possibility of a first order phase transition from nuclear matter to quark matter at the high densities and moderate temperatures reached in Supernova events or, afterwards, in the still hot and lepton rich protoneutron stars, has attracted much attention in the last years [1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13]. In particular, possible interesting fingerprints of this phase transition within the neutrino signal emitted from the newly born neutron star, have been proposed in [6, 11, 8, 9] thus providing a new tool to investigate the properties of the QCD phase diagram at high densities.

The usual way to model the phase transition, after the seminal paper of Glendenning [14], is to impose the Gibbs conditions for a first order phase transition in multicomponent systems. For matter in neutron stars indeed the baryonic charge and the electric charge are conserved globally resulting in an extended mixed phase in which the pressure increases as a function of the baryon density. In turn, this implies the possible existence of a layer of mixed phase embedded between the pure nuclear and pure quark phases in hybrid stars. Within the mixed phase, due to the finite value of the surface tension between hadronic matter and quark matter, different structures appear depending on the density: first the quark pasta phases, immersed in nuclear matter, and then the nuclear pasta phases immersed in quark matter [15, 16]. However, as shown in Refs. [17, 18], sophisticated calculations which include finite size and charge screening effects together with more recent estimates for the value of the surface tension, have shown that the mixed phase obtained with the Gibbs construction is actually quite similar to the simpler Maxwell construction. Due to the large value of the surface tension, the charged structures have sizes larger than the electron Debye screening length and therefore their net charge is significantly reduced. One is back to the (almost) constant pressure Maxwell construction which implies that the mixed phase cannot be present in neutron stars where the pressure must be a monotonic function of the radius.

In protoneutron stars, however, an additional charge is conserved, the lepton number, during the stage of neutrino trapping. As we will show in the following, due to this additional conservation law, it is possible to form a new type of mixed phase in protoneutron stars. Since this mixed phase is related to the conservation of lepton number, this phase will gradually “disappear” as the neutrinos become untrapped.

2. The equation of state of mixed phases

We want to present here two extreme cases for the mixed phase in protoneutron star matter: the first, widely considered in the literature, corresponding to a vanishing surface tension (we call it Mixed Phase 1, MP1) and the second, corresponding to a very large value of the surface tension (we call it Mixed Phase 2, MP2). Let us discuss first the structure of the interface between hadronic matter and quark matter: a charge separated interface is formed with a size of the order of the Debye screening length, \( \sim 10 \text{ fm} \), with a layer of positively charged, electron depleted, hadronic matter on one
side and a layer of quark matter with an excess of the electrons on the other side (as discussed in [19] for the CFL phase). The interface is stabilized by the resulting electric field. Notice that neutrinos, being not affected by the electric field, can freely stream across the interface. Consequently, the lepton number is conserved globally even in the case of a large surface tension.

We consider the “standard” conditions of ProtoNeutron Star (PNS) matter [3]: fixed lepton fraction $Y_L = (n_e + n_\nu)/n_B = 0.4$ and fixed entropy per baryon $S/N_B = 1$ where $n_e$, $n_\nu$ and $n_B$ are the electron, neutrino and baryon number densities and $S$ is the entropy. The chemical equilibrium between the different species of particles within the two phases allows to write the following general relations:

$$\mu_n = \mu_B^H, \mu_p = \mu_B^H + \mu_C^H, \mu_e^H = \mu_L^H - \mu_C^H, \mu_\nu^H = \mu_L^H$$  \hspace{1cm} (1)

$$\mu_u^Q = \frac{\mu_B^Q + 2\mu_C^Q}{3}, \mu_d^Q = \frac{\mu_B^Q - \mu_C^Q}{3}, \mu_s^Q = \frac{\mu_B^Q - \mu_C^Q}{3}$$  \hspace{1cm} (2)

where $\mu_i^A$ ($i = n, p, e, \nu, u, d, s$ and $A = H, Q$) are the chemical potentials of neutrons, protons, electrons, neutrinos and up, down, strange quarks within the hadronic phase ($A=H$) and the quark phase ($A=Q$). The chemical potentials associated with globally conserved quantities are equal in the two phases [20]. In presence of a vanishingly small value of the surface tension, the electric charge is conserved globally as the baryonic number and the lepton number, therefore its chemical potential is continuous at the onsets of the phase transition and we can write the Gibbs conditions as follows:

$$P^H(\mu_B^H, \mu_C^H, \mu_L^H, T^H) = P^Q(\mu_B^Q, \mu_C^Q, \mu_L^Q, T^Q)$$  \hspace{1cm} (3)

$$(1 - \chi)n_C^H + \chi n_C^Q - n_e = 0$$  \hspace{1cm} (4)

$$n_e + n_\nu = Y_L n_B$$  \hspace{1cm} (5)

$$(1 - \chi)s^H + \chi s^Q = S/N_B$$  \hspace{1cm} (6)

$$T^H = T^Q$$  \hspace{1cm} (7)

$$\mu_B^H = \mu_B^Q, \mu_C^H = \mu_C^Q, \mu_L^H = \mu_L^Q$$  \hspace{1cm} (8)

where $P^{H,Q}$, $T^{H,Q}$, $s^{H,Q}$ and $n_C^{H,Q}$ are the pressure, the temperature, the entropy density and the electric charge density of the hadronic and quark phases respectively. $\chi$ is the volume fraction of the quark phase.

In presence of a large value of the surface tension, the electric charge is conserved only locally, therefore its chemical potential is different in the two phases, but the lepton number, as noticed before, is still conserved globally as the baryon number and its chemical potential must be continuous across the phase transition. In this case, we can write the Gibbs conditions as follows:
The systems of equations above presented have a general validity. To give some numerical examples, we consider, as customary, two models for strongly interacting matter: one at low density with nucleon degrees of freedom and one at large densities with quark degrees of freedom. For the former, we adopt the relativistic mean field model with the parameterization TM1 \[21\] and for the latter the MIT bag model. We set the masses of up and down quarks to zero and the mass of the strange quark to 100 MeV. The bag constant \(B\) is set to \(B^{1/4} = 165\) MeV. The phase transition is then computed by using the systems for the two mixed phases MP1 and MP2. In Fig. 1 we show the equations of state for protoneutron star matter and cold and beta stable matter for the two cases MP1 and MP2. Notice that for PNS matter, the equations of state are quite similar: an extended mixed phase with varying pressure is obtained also in the case of local charge neutrality due to the existence of an additional globally conserved quantities which is the lepton number. In cold and beta stable matter on the other hand, there are only two conserved charged, the baryonic and the electric charge, and

\[
P^H(\mu_B^H, \mu_C^H, \mu_L^H, T^H) = P^Q(\mu_B^Q, \mu_C^Q, \mu_L^Q, T^Q) \tag{9}
\]
\[
n_C^H - n_e^H = 0 \tag{10}
\]
\[
n_C^Q - n_e^Q = 0 \tag{11}
\]
\[
(1 - \chi)(n_e^H + n_\nu) + \chi(n_e^Q + n_\nu) = Y_L n_B \tag{12}
\]
\[
(1 - \chi)s^H + \chi s^Q = S/N n_B \tag{13}
\]
\[
T^H = T^Q \tag{14}
\]
\[
\mu_B^H = \mu_B^Q, \quad \mu_L^H = \mu_L^Q \tag{15}
\]
Figure 2. Density profiles for a $M_B = 1.7M_{\text{sun}}$ star in its protoneutron star stage and the cold and beta stable asymptotic state, the case of the mixed phase MP2 is considered. In the protoneutron star a 2km layer of mixed phase is present while in the cold configuration the mixed phase disappears and a sharp interface separating the pure quark and nuclear phase is obtained.

therefore in the case of MP1 one obtains an extended mixed phase but in MP2 the result is the simple constant pressure Maxwell construction. In general, the mixed phase MP2 extends over a smaller range of density and has a lower pressure gradient with respect to MP1, because the requirement of local charge neutrality is more restrictive.

3. Discussion and conclusions

The new mixed phase here proposed, MP2, is present as long as neutrinos are trapped. After the complete deleptonization and cooling of the star it becomes a constant pressure mixed phase which cannot appear in neutron stars. This implies that the deleptonization drives a gradual modification of the structure of the star: for sufficiently large masses of the protoneutron stars, one has a core of pure quark matter, a layer of mixed phase MP2, and then a layer/crust of nuclear matter. The layer of mixed phase disappears during deleptonization and, at the end of the deleptonization stage, a sharp interface will separate the pure quark and the pure nuclear phase, see Fig. 2.

Notice that the mixed phase MP2 has quite different properties with respect to MP1: in MP2 the two phases are locally charge neutral, therefore no Coulomb lattice with charged finite structures of the two phases can form. This implies also that only spherical pasta structures can be formed since the 1-D and 2-D structures can take place only in the presence of Coulomb interactions. Moreover the charge neutral structures have macroscopic sizes contrary to the MP1 case where the optimal size of the structures is limited by the Coulomb energy. We expect that within the MP2 mixed phase the process of diffusion of neutrinos can be significantly different with respect to the MP1 case: for instance, due the large sizes of the structures within MP2, no coherent
scattering of neutrinos with pasta structures can take place \([22]\), as the neutrino wavelength is of the order of tens of fermi. A detailed simulation of neutrino transport within this new mixed phase would be extremely interesting for the possible implications on the neutrino signal of the changes of the structure of the star during deleptonization. Also the motion and the interactions of the drops/bubbles within the mixed phase, in presence of turbulence, might represent an interesting source of gravitational waves. A more detailed study, which includes also superconducting quark matter \([23, 24, 25]\), is presently in progress.

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