Cosmological Constraints on the Sign-Changeable Interactions

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ABSTRACT

Recently, Cai and Su [Phys. Rev. D 81, 103514 (2010)] found that the sign of interaction $Q$ in the dark sector changed in the approximate redshift range of $0.45 \lesssim z \lesssim 0.9$, by using a model-independent method to deal with the observational data. In fact, this result raises a remarkable problem, since most of the familiar interactions cannot change their signs in the whole cosmic history. Motivated by the work of Cai and Su, we have proposed a new type of interaction in a previous work [H. Wei, Nucl. Phys. B 845, 381 (2011)]. The key ingredient is the deceleration parameter $q$ in the interaction $Q$, and hence the interaction $Q$ can change its sign when our universe changes from deceleration ($q > 0$) to acceleration ($q < 0$). In the present work, we consider the cosmological constraints on this new type of sign-changeable interactions, by using the latest observational data. We find that the cosmological constraints on the model parameters are fairly tight. In particular, the key parameter $\beta$ can be constrained to a narrow range.

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I. INTRODUCTION

In the dark energy cosmology [1], the well-known cosmological coincidence problem has an important position. This problem asks: why are we living in an epoch in which the densities of dark energy and matter are comparable? Since their densities scale differently with the expansion of our universe, there should be some fine-tunings. To alleviate this coincidence problem, it is natural to consider the possible interaction between dark energy and dark matter in the literature (see e.g. [2–12]). In fact, since the natures of both dark energy and dark matter are still unknown, there is no physical argument to exclude the possible interaction between them. On the contrary, some observational evidences of the dark sector interaction have been found recently [13, 14]. In the literature, it is usual to assume that dark energy and dark matter interact through a coupling term $Q$, according to

\begin{align}
\dot{\rho}_m + 3H\rho_m = Q, \\
\dot{\rho}_{de} + 3H(\rho_{de} + p_{de}) = -Q,
\end{align}

where $\rho_m$ and $\rho_{de}$ are the densities of dark matter and dark energy (we assume that the baryon component can be ignored); $p_{de}$ is the pressure of dark energy; $H \equiv \dot{a}/a$ is the Hubble parameter; $a = (1 + z)^{-1}$ is the scale factor (we have set $a_0 = 1$; the subscript “0” indicates the present value of corresponding quantity; $z$ is the redshift); a dot denotes the derivative with respect to cosmic time $t$. Note that Eqs. (1) and (2) preserve the total energy conservation equation $\dot{\rho}_{tot} + 3H(\rho_{tot} + p_{tot}) = 0$, where $\rho_{tot} = \rho_m + \rho_{de}$. Since there is no natural guidance from fundamental physics on the interaction $Q$, one can only discuss it to a phenomenological level. This is the realistic status of the interacting dark energy models so far.

The most familiar interactions extensively considered in the literature (see e.g. [2–12]) include $Q = 3\alpha H\rho_m$, $Q = 3\beta H\rho_{tot}$, and $Q = 3\eta H\rho_{de}$. It is easy to see that these interactions are always positive or negative and hence cannot give the possibility to change their signs in the whole cosmic history. However, recently Cai and Su [15] found that the sign of interaction $Q$ changed in the approximate redshift range of $0.45 \lesssim z \lesssim 0.9$, by using a model-independent method to deal with the observational data. Obviously, this result raises a remarkable problem. Motivated by the work of Cai and Su, we have proposed a new type of interaction in a previous work [16], which is given by

$$Q = q(\alpha \dot{\rho} + 3\beta H \rho),$$

where $\alpha$ and $\beta$ are both dimensionless constants; the energy density $\rho$ could be $\rho_m$, $\rho_{tot}$ and $\rho_{de}$ for examples; the deceleration parameter

$$q \equiv -\frac{\ddot{a}}{aH^2} = -1 - \frac{\dot{H}}{H^2}.\quad (4)$$

Obviously, this new type of interaction $Q$ can change its sign when our universe changes from deceleration ($q > 0$) to acceleration ($q < 0$). In fact, the deceleration parameter $q$ in $Q$ is the key ingredient of this new interaction, which makes our proposal different from the previous ones considered in the literature. Note that the term $\alpha \dot{\rho}$ in $Q$ is introduced from the dimensional point of view (we refer to [16] for details). One can remove this term by setting $\alpha = 0$, and then $Q$ becomes simply $Q = 3\beta q H \rho$ (in fact this is the very case which will be considered in the followings).

Since the appearance of the deceleration parameter $q$ in the interaction $Q$ looks speculative to some extent, we would like to say some words before going further. Firstly, as is well known, in the literature there is no natural guidance from fundamental physics on the interaction $Q$, one can only discuss it to a phenomenological level. In this sense, the other familiar interactions extensively considered in the literature have no better origin from the fundamental physics than the one proposed in Eq. (3). Secondly, we note that $q = -1 - \dot{H}/H^2$ from Eq. (4) and $H^2 \propto \rho_{tot}$ from the Friedmann equation. Thus, one can regard the deceleration parameter $q = f(\rho_{tot}, \rho_{tot})$ as a function of the total energy density $\rho_{tot} = \rho_m + \rho_{de}$ and its derivative. In this sense, the interaction $Q = q(\alpha \dot{\rho} + 3\beta H \rho) = f(\rho, \dot{\rho})$ is not so unusual, since it is reasonable to image that $Q$ depends on the energy densities of dark energy and matter. Finally, while the familiar interactions extensively considered in the literature (such as $Q = 3\alpha H\rho_m$, $Q = 3\beta H\rho_{tot}$, and $Q = 3\eta H\rho_{de}$) cannot give the possibility to change their signs in the whole cosmic history, our proposal in Eq. (3) provides a possible way out. So, we consider that it deserves further investigation.
In [16], we have studied the cosmological evolution of quintessence and phantom with this new type of sign-changeable interactions, and found some interesting results. In the present work, we would like to consider the cosmological constraints on this new type of sign-changeable interactions, by using the latest observational data. For simplicity, in this work, we restrict ourselves to the decaying Λ model (see e.g. [17] and references therein), namely, the role of dark energy is played by the decaying vacuum energy. In this case, Eq. (2) becomes

$$\dot{\rho}_\Lambda = -Q.$$  

(5)

The Friedmann and Raychaudhuri equations are given by

$$H^2 = \frac{\kappa^2}{3} \rho_{\text{tot}} = \frac{\kappa^2}{3} (\rho_\Lambda + \rho_m),$$  

(6)

$$\dot{H} = -\frac{\kappa^2}{2} (\rho_{\text{tot}} + p_{\text{tot}}) = -\frac{\kappa^2}{2} \rho_m,$$  

(7)

where $\kappa^2 \equiv 8\pi G$. Notice that we consider a flat Friedmann-Robertson-Walker (FRW) universe throughout this work. In Sec. II, we briefly introduce the latest observational data which will be used in this work. In Sec. III, we consider the cosmological constraints on three particular sign-changeable interactions, i.e.,

$$Q = q(\alpha \dot{\rho}_m + 3\beta H \rho_m),$$  

(8)

$$Q = q(\alpha \dot{\rho}_{\text{tot}} + 3\beta H \rho_{\text{tot}}),$$  

(9)

$$Q = q(\alpha \dot{\rho}_\Lambda + 3\beta H \rho_\Lambda).$$  

(10)

Finally, some brief concluding remarks are given in Sec. IV.

II. OBSERVATIONAL DATA

In the present work, we will consider the latest cosmological observations, namely, the 557 Union2 Type Ia Supernovae (SNIa) dataset [18], the shift parameter $R$ from the Wilkinson Microwave Anisotropy Probe 7-year (WMAP7) data [19], and the distance parameter $A$ of the measurement of the baryon acoustic oscillation (BAO) peak in the distribution of SDSS luminous red galaxies [20, 21].

The data points of the 557 Union2 SNIa compiled in [18] are given in terms of the distance modulus $\mu_{\text{obs}}(z_i)$. On the other hand, the theoretical distance modulus is defined as

$$\mu_{\text{th}}(z_i) \equiv 5 \log_{10} D_L(z_i) + \mu_0,$$  

(11)

where $\mu_0 \equiv 42.38 - 5 \log_{10} h$ and $h$ is the Hubble constant $H_0$ in units of 100 km/s/Mpc, whereas

$$D_L(z) = (1 + z) \int_0^z \frac{d\tilde{z}}{E(\tilde{z}; \mathbf{p})},$$  

(12)

in which $E \equiv H/H_0$, and $\mathbf{p}$ denotes the model parameters. Correspondingly, the $\chi^2$ from the 557 Union2 SNIa is given by

$$\chi^2_{\mu}(\mathbf{p}) = \sum_i \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i)]^2}{\sigma^2(z_i)},$$  

(13)

where $\sigma$ is the corresponding 1σ error. The parameter $\mu_0$ is a nuisance parameter but it is independent of the data points. One can perform an uniform marginalization over $\mu_0$. However, there is an alternative way. Following [22, 23], the minimization with respect to $\mu_0$ can be made by expanding the $\chi^2_{\mu}$ of Eq. (13) with respect to $\mu_0$ as

$$\chi^2_{\mu}(\mathbf{p}) = \hat{A} - 2 \mu_0 \hat{B} + \mu_0^2 \hat{C},$$  

(14)
where

$$\tilde{A}(p) = \sum_i \frac{[\mu_{\text{obs}}(z_i) - \mu_B(z_i; \mu_0 = 0, p)]^2}{\sigma_{\mu_{\text{obs}}}(z_i)},$$

$$\tilde{B}(p) = \sum_i \frac{[\mu_{\text{obs}}(z_i) - \mu_B(z_i; \mu_0 = 0, p)]}{\sigma_{\mu_{\text{obs}}}(z_i)}, \quad \tilde{C} = \sum_i \frac{1}{\sigma_{\mu_{\text{obs}}}(z_i)}.$$

Eq. (13) has a minimum for $\mu_0 = \tilde{B}/\tilde{C}$ at

$$\hat{\chi}_\mu^2(p) = \tilde{A}(p) - \frac{\tilde{B}(p)^2}{\tilde{C}}. \quad (15)$$

Since $\hat{\chi}_{\mu, \text{min}}^2 = \chi_{\mu, \text{min}}^2$ obviously, we can instead minimize $\hat{\chi}_\mu^2$ which is independent of $\mu_0$.

There are some other relevant observational data, such as the observations of cosmic microwave background (CMB) anisotropy [19] and large-scale structure (LSS) [20]. However, using the full data of CMB and LSS to perform a global fitting consumes a large amount of computation time and power. As an alternative, one can instead use the shift parameter $R$ from the CMB, and the distance parameter $A$ of the measurement of the BAO peak in the distribution of SDSS luminous red galaxies. In the literature, the shift parameter $R$ and the distance parameter $A$ have been used extensively. It is argued that they are model-independent [24], while $R$ and $A$ contain the main information of the observations of CMB and BAO, respectively.

As is well known, the shift parameter $R$ of the CMB is defined by [24, 25]

$$R \equiv \Omega_{m0}^{1/2} \int_{0}^{z_*} \frac{dz}{E(z)}, \quad (16)$$

where $\Omega_{m0}$ is the present fractional energy density of pressureless matter; the redshift of recombination $z_* = 1091.3$ which has been updated in the Wilkinson Microwave Anisotropy Probe 7-year (WMAP7) data [19]. The shift parameter $R$ relates the angular diameter distance to the last scattering surface, the comoving size of the sound horizon at $z_*$ and the angular scale of the first acoustic peak in CMB power spectrum of temperature fluctuations [24, 25]. The value of $R$ has been updated to $1.725 \pm 0.018$ from the WMAP7 data [19]. On the other hand, the distance parameter $A$ of the measurement of the BAO peak in the distribution of SDSS luminous red galaxies is given by [26]

$$A \equiv \Omega_{m0}^{1/2} E(z_b)^{-1/3} \left[ \frac{1}{z_b} \int_0^{z_b} \frac{dz}{E(z)} \right]^{2/3}, \quad (17)$$

where $z_b = 0.35$. In [21], the value of $A$ has been determined to be $0.469 (n_s/0.98)^{-0.35} \pm 0.017$. Here the scalar spectral index $n_s$ is taken to be 0.963, which has been updated from the WMAP7 data [19]. So, the total $\chi^2$ is given by

$$\chi^2 = \hat{\chi}_\mu^2 + \chi_{\text{CMB}}^2 + \chi_{\text{BAO}}^2, \quad (18)$$

where $\hat{\chi}_\mu^2$ is given in Eq. (13), $\chi_{\text{CMB}}^2 = (R - R_{\text{obs}})^2 / \sigma_R^2$ and $\chi_{\text{BAO}}^2 = (A - A_{\text{obs}})^2 / \sigma_A^2$. The best-fit model parameters are determined by minimizing the total $\chi^2$. As in [26, 27], the 68.3% confidence level is determined by $\Delta \chi^2 \equiv \chi^2 - \chi_{\text{min}}^2 \leq 1.0, 2.3$ and 3.53 for $n_p = 1, 2$ and 3, respectively, where $n_p$ is the number of free model parameters. Similarly, the 95.4% confidence level is determined by $\Delta \chi^2 \equiv \chi^2 - \chi_{\text{min}}^2 \leq 4.0, 6.17$ and 8.02 for $n_p = 1, 2$ and 3, respectively.

III. COSMOLOGICAL CONSTRAINTS ON THE SIGN-CHANGEABLE INTERACTIONS

In this section, we consider the cosmological constraints on the sign-changeable interactions given in Eqs. (8)–(10), by using the observational data mentioned in the previous section.
Before we consider the case of \( Q = q(\alpha \dot{\rho}_m + 3\beta H \rho_m) \) given in Eq. (8), we define the time \( t \) to scale factor \( a \) with the help of the universal relation \( \dot{f} = Ha f' \) for any function \( f \) (where a prime denotes the derivative with respect to scale factor \( a \)), and recast Eq. (22) as

\[
\ddot{H} = \frac{\beta q - 1}{1 - \alpha q} \cdot 3H \dot{H},
\]

which is a second-order differential equation for \( H(a) \). Note that the deceleration parameter

\[
q = -1 - \frac{\dot{H}}{H^2} = -1 - \frac{a}{H} \dot{H}',
\]

is also a function of \( H \) and \( H' \). Unfortunately, if \( \alpha \neq 0 \), there is no analytical solution for the second-order
differential equation (23), because one will encounter a transcendental equation. Therefore, we consider only the case of $\alpha = 0$ in this work. In this case, the sign-changeable interaction reads

$$Q = 3\beta q H \rho_m.$$  

(25)

By solving the second-order differential equation (23) with $\alpha = 0$, we find that

$$H(a) = C_{12} \left[ 3C_{11}(1 + \beta) - (2 + 3\beta) a^{-3(1+\beta)} \right]^{1/(2+3\beta)},$$  

(26)

where $C_{11}$ and $C_{12}$ are both integral constants, which can be determined in the following. From Eq. (21), we find that the fractional energy density of dark matter is given by

$$\Omega_m \equiv \frac{\kappa^2 \rho_m}{3H^2} = -\frac{2\dot{H}}{3H^2} = -\frac{2aH'}{3H}.$$  

(27)

Substituting Eq. (26) into Eq. (27), we have

$$\Omega_m = \frac{2(1 + \beta)}{2 + 3\beta - 3C_{11}(1 + \beta) a^{3(1+\beta)}}.$$  

(28)

Requiring $\Omega_m(a = 1) = \Omega_{m0}$, we obtain

$$C_{11} = \frac{\Omega_{m0}(2 + 3\beta) - 2(1 + \beta)}{3\Omega_{m0}(1 + \beta)}.$$  

(29)

On the other hand, requiring $H(a = 1) = H_0$, from Eq. (24) we can find that

$$C_{12} = H_0 \left[ 3C_{11}(1 + \beta) - (2 + 3\beta) \right]^{-1/(2+3\beta)}.$$  

(30)

Substituting Eqs. (29) and (30) into Eq. (26), we finally obtain

$$\frac{E}{H} = \left\{ 1 - \frac{2 + 3\beta}{2(1 + \beta)} \Omega_{m0} \left[ 1 - (1 + z)^{3(1+\beta)} \right] \right\}^{1/(2+3\beta)}.$$  

(31)
There are two free model parameters, namely $\Omega_{m0}$ and $\beta$. Note that when $\beta = 0$, Eq. (31) reduces to $E(z) = [\Omega_{m0}(1 + z)^3 + (1 - \Omega_{m0})]^{1/2}$, i.e., the one of $\Lambda$CDM model.

By minimizing the corresponding total $\chi^2$ in Eq. (18), we find the best-fit parameters $\Omega_{m0} = 0.2738$ and $\beta = -0.010$, whereas $\chi^2_{\text{min}} = 542.725$. In Fig. 1, we present the corresponding 68.3% and 95.4% confidence level contours in the $\Omega_{m0}-\beta$ plane. Obviously, the current observational data slightly prefer a negative $\beta$. We are also interested in the fractional energy densities $\Omega_m$ given in Eq. (28) and $\Omega_\Lambda = 1 - \Omega_m$, the deceleration parameter $q$ given in Eq. (24), and the effective equation-of-state parameter (EoS) $w_{\text{eff}} \equiv p_{\text{tot}}/\rho_{\text{tot}} = (2q - 1)/3$. We present them as functions of redshift $z$ with the best-fit model parameters in Fig. 2. It is easy to find the transition redshift $z_t = 0.7489$ where the universe changes from deceleration ($q > 0$) to acceleration ($q < 0$). Since the best-fit $\beta$ is negative, dark matter decays into dark energy ($Q < 0$) when $z > z_t$, and dark energy decays into dark matter ($Q > 0$) when $z < z_t$. The interaction $Q$ crosses the non-interacting line ($Q = 0$) at $z_t$.

Secondly, we consider the case of $Q = q(\alpha \dot{\rho}_{\text{tot}} + 3\beta H \rho_{\text{tot}})$ given in Eq. (9). From Eq. (6), it is easy to find $\rho_{\text{tot}} = 3H^2/\kappa^2$. Substituting into Eq. (9), we can finally obtain

$$Q = \frac{3qH^3}{\kappa^2} \left( 2\alpha \frac{\dot{H}}{H^2} + 3\beta \right). \quad (32)$$

Substituting Eqs. (21) and (32) into Eq. (1), we have

$$\ddot{H} + 3H \dot{H} (1 + \alpha q) + \frac{9}{2} \beta q H^3 = 0. \quad (33)$$

Similarly, we recast it as

$$aH'' + \frac{\alpha}{H} H'^2 + (4 + 3\alpha q) H' + \frac{9\beta qH}{2a} = 0, \quad (34)$$

FIG. 3: The same as in Fig. 1 but for the case of $Q = 3\beta q H \rho_{\text{tot}}$ with the condition $\beta \geq 0$. 

B. The case of $Q = q(\alpha \dot{\rho}_{\text{tot}} + 3\beta H \rho_{\text{tot}})$
which is a second-order differential equation for $H(a)$. Note that the deceleration parameter $q$ is also a function of $H$ and $H'$ [cf. Eq. (21)]. Similar to the case of $Q = q(\alpha \dot{\rho}_m + 3 \beta H \rho_m)$, we consider only the case of $\alpha = 0$ in this work. In this case, the sign-changeable interaction reads

$$Q = 3 \beta qH \rho_{tot}.$$ 

(35)

By solving the second-order differential equation (34) with $\alpha = 0$, we find that

$$H(a) = C_{22} \cdot a^{-3(2-3\beta+r_1)/8} \cdot \left[a^{3r_1/2} + C_{21}\right]^{1/2},$$

(36)

where $C_{21}, C_{22}$ are both integral constants, and

$$r_1 \equiv \sqrt{4 + \beta (4 + 9\beta)}.$$ 

(37)

Substituting Eq. (36) into Eq. (27), we have

$$\Omega_m = \frac{1}{4} \left[2 - 3\beta + \left(\frac{2C_{21}}{a^{3r_1/2} + C_{21}} - 1\right) r_1\right].$$

(38)

Requiring $\Omega_m(a = 1) = \Omega_{m0}$, we obtain

$$C_{21} = -1 + \frac{2r_1}{2 - 3\beta - 4\Omega_{m0} + r_1}.$$ 

(39)

On the other hand, requiring $H(a = 1) = H_0$, from Eq. (36) we can find that

$$C_{22} = H_0 (1 + C_{21})^{-1/2}.$$ 

(40)

From Eqs. (36) and (40), it is easy to obtain

$$E \equiv \frac{H}{H_0} = (1 + z)^{3(2-3\beta+r_1)/8} \cdot \left[\frac{(1 + z)^{-3r_1/2} + C_{21}}{1 + C_{21}}\right]^{1/2},$$

(41)
where $C_{21}$ and $r_1$ have been given in Eqs. (39) and (37), respectively. There are two free model parameters, namely $\Omega_{m0}$ and $\beta$. Note that when $\beta = 0$, Eq. (41) reduces to $E(z) = [\Omega_{m0}(1 + z)^3 + (1 - \Omega_{m0})]^{1/2}$, i.e., the one of $\Lambda$CDM model.

Imposing the condition $0 \leq \Omega_m \leq 1$ when $a \to 0$, we have $\beta \geq 0$ from Eq. (38). Under this condition, by minimizing the corresponding total $\chi^2$ in Eq. (18), we find the best-fit parameters $\Omega_{m0} = 0.2701$ and $\beta = 0$, whereas $\chi^2_{\text{min}} = 542.919$. In Fig. 3 we present the corresponding 68.3% and 95.4% confidence level contours in the $\Omega_{m0} - \beta$ plane. In Fig. 4 we also present the $\Omega_m$ given in Eq. (38), $\Omega_{\Lambda} = 1 - \Omega_m$, $q$ given in Eq. (24) and $w_{\text{eff}} \equiv \rho_{\text{tot}}/\rho_{\text{tot}} = (2q - 1)/3$ as functions of redshift $z$ with the best-fit model parameters. The universe changes from deceleration ($q > 0$) to acceleration ($q < 0$) at the transition redshift $z_t = 0.7549$.

However, the above best-fit model with $\beta = 0$ is in fact the $\Lambda$CDM model without interaction between dark energy and dark matter. So, we would like to give up the condition $\beta \geq 0$. This means that in the early universe we have $\Omega_m \geq 1$ and then $\Omega_{\Lambda} \leq 0$, namely, $\rho_{\Lambda}$ might be negative. Since the negative energy density can arise in quantum field theory (see e.g. [28] for a good review), it is reasonable to consider this possibility. Without the condition $\beta \geq 0$, by minimizing the corresponding total $\chi^2$ in Eq. (18), we find the best-fit parameters $\Omega_{m0} = 0.2764$ and $\beta = -0.0247$, whereas $\chi^2_{\text{min}} = 542.711$. In Fig. 5 we present the corresponding 68.3% and 95.4% confidence level contours in the $\Omega_{m0} - \beta$ plane. Obviously, the current observational data slightly prefer a negative $\beta$. In Fig. 6 we also present the $\Omega_m$ given in Eq. (38), $\Omega_{\Lambda} = 1 - \Omega_m$, $q$ given in Eq. (24) and $w_{\text{eff}} \equiv \rho_{\text{tot}}/\rho_{\text{tot}} = (2q - 1)/3$ as functions of redshift $z$ with the best-fit model parameters. The universe changes from deceleration ($q > 0$) to acceleration ($q < 0$) at the transition redshift $z_t = 0.7688$ where the universe changes from deceleration ($q > 0$) to acceleration ($q < 0$). Since the best-fit $\beta$ is negative, dark matter decays into dark energy ($Q < 0$) when $z > z_t$, and dark energy decays into dark matter ($Q > 0$) when $z < z_t$. The interaction $Q$ crosses the non-interacting line ($Q = 0$) at $z_t$.

**C. The case of $Q = q(\alpha\dot{\rho}_{\Lambda} + 3\beta H\rho_{\Lambda})$**

Finally, we consider the case of $Q = q(\alpha\dot{\rho}_{\Lambda} + 3\beta H\rho_{\Lambda})$ given in Eq. (18). Substituting it into Eq. (9), one can find that

$$\dot{\rho}_{\Lambda} = -\frac{3\beta q H \rho_{\Lambda}}{1 + \alpha q}. \quad (42)$$
Then, substituting into Eq. (10), we can finally obtain
\[ Q = \frac{3 \beta q \rho \Lambda a}{1 + \alpha q}. \]  
(43)

From Eqs. (6) and (7) [or equivalently Eq. (21)], we have
\[ \rho \Lambda = \frac{3}{\kappa^2} H^2 - \rho_m = \frac{1}{\kappa^2} \left(3H^2 + 2 \dot{H}\right). \]  
(44)

Substituting Eqs. (21), (43) and (44) into Eq. (1), we find that
\[ \ddot{H} + 3H \dot{H} \left(1 + \frac{\beta q}{1 + \alpha q}\right) + \frac{9 \beta q H^3}{2a(1 + \alpha q)} = 0. \]  
(45)

Similarly, we recast it as
\[ aH'' + \frac{a}{H} H'^2 + \left(4 + \frac{3 \beta q}{1 + \alpha q}\right) H' + \frac{9 \beta q H^3}{2a(1 + \alpha q)} = 0, \]  
(46)

which is a second-order differential equation for \( H(a) \). Note that the deceleration parameter \( q \) is also a function of \( H \) and \( H' \) [cf. Eq. (21)]. Unfortunately, if \( \alpha \neq 0 \), there is no analytical solution for the second-order differential equation (46), because one will encounter a transcendental equation. Therefore, we consider only the case of \( \alpha = 0 \) in this work. In this case, the sign-changeable interaction reads
\[ Q = 3 \beta q H \rho \Lambda. \]  
(47)

By solving the second-order differential equation (46) with \( \alpha = 0 \), we find that
\[ H(a) = C_{32} \cdot a^{-3(2-5\beta+\sqrt{2})/[4(2-3\beta)]} \cdot \left(a^{3\sqrt{2}/2} + C_{31}\right)^{1/(2-3\beta)}, \]  
(48)

where \( C_{31}, C_{32} \) are both integral constants, and
\[ r_2 \equiv \sqrt{(2 - \beta)^2} = |2 - \beta|. \]  
(49)
Substituting Eq. (48) into Eq. (27), we have

\[
\Omega_m = \frac{1}{2 (2 - 3\beta)} \left[ 2 - 5\beta + \left( \frac{2C_{31}}{a^{3r_2/2} + C_{31}} - 1 \right) r_2 \right].
\]  

(50)

Requiring \(\Omega_m(a = 1) = \Omega_{m0}\), we obtain

\[
C_{31} = -1 + \frac{2 r_2}{2 - 5\beta + r_2 + 2\Omega_{m0}(3\beta - 2)}.
\]  

(51)

On the other hand, requiring \(H(a = 1) = H_0\), from Eq. (48) we get

\[
C_{32} = H_0 (1 + C_{31})^{1/(3\beta - 2)}.
\]  

(52)

From Eqs. (48) and (52), it is easy to obtain

\[
E \equiv \frac{H}{H_0} = (1 + z)^{3(2 - 5\beta + r_2)/[4(2 - 3\beta)]} \left[ \frac{(1 + z)^{-3r_2/2} + C_{31}}{1 + C_{31}} \right]^{1/(2 - 3\beta)},
\]  

(53)

where \(C_{31}\) and \(r_2\) have been given in Eqs. (51) and (49), respectively. There are two free model parameters, namely \(\Omega_{m0}\) and \(\beta\). Note that when \(\beta = 0\), Eq. (53) reduces to \(E(z) = [\Omega_{m0}(1 + z)^3 + (1 - \Omega_{m0})]^{1/2}\), i.e., the one of \(\Lambda\)CDM model.

By minimizing the corresponding total \(\chi^2\) in Eq. (18), we find the best-fit parameters \(\Omega_{m0} = 0.2717\) and \(\beta = 0.0136\), whereas \(\chi^2_{\text{min}} = 542.778\). In Fig. 7, we present the corresponding 68.3% and 95.4% confidence level contours in the \(\Omega_{m0} - \beta\) plane. Obviously, the current observational data slightly prefer a positive \(\beta\). In Fig. 8 we also present the \(\Omega_m\) given in Eq. (50), \(\Omega_\Lambda = 1 - \Omega_m\), \(q\) given in Eq. (24) and \(w_{\text{eff}} \equiv p_{\text{tot}}/\rho_{\text{tot}} = (2q - 1)/3\) as functions of redshift \(z\) with the best-fit model parameters. It is easy to find the transition redshift \(z_t = 0.7398\) where the universe changes from deceleration (\(q > 0\)) to acceleration (\(q < 0\)). Since the best-fit \(\beta\) is positive, dark energy decays into dark matter (\(Q > 0\)) when \(z > z_t\), dark matter decays into dark energy (\(Q < 0\)) when \(z < z_t\). The interaction \(Q\) crosses the non-interacting line (\(Q = 0\)) at \(z_t\).
IV. CONCLUDING REMARKS

Recently, Cai and Su \[15\] found that the sign of interaction $Q$ in the dark sector changed in the approximate redshift range of $0.45 \lesssim z \lesssim 0.9$, by using a model-independent method to deal with the observational data. In fact, this result raises a remarkable problem, since most of the familiar interactions cannot change their signs in the whole cosmic history. Motivated by the work of Cai and Su, we have proposed a new type of interaction in a previous work \[16\]. The key ingredient is the deceleration parameter $q$ in the interaction $Q$, and hence the interaction $Q$ can change its sign when our universe changes from deceleration ($q > 0$) to acceleration ($q < 0$). In the present work, we consider the cosmological constraints on this new type of sign-changeable interactions, by using the latest observational data. We find that the cosmological constraints on the model parameters are fairly tight. In particular, the key parameter $\beta$ has been constrained to a narrow range.

Some remarks are in order. Firstly, we briefly consider the comparison of these models. For convenience, we also consider the well-known $\Lambda$CDM model in addition. In fact, it corresponds to the decaying $\Lambda$ model with $Q = 0$. Fitting $\Lambda$CDM model to the observational data considered in the present work, it is easy to find the corresponding best-fit parameter $\Omega_{m0} = 0.2701$, whereas $\chi^2_{\text{min}} = 542.919$. Of course, we would like to also consider the decaying $\Lambda$ model with a traditional interaction $Q = 3\beta H \rho_m$ which cannot change its sign in the whole cosmic history. The corresponding $E(z)$ can be found in e.g. \[12\], namely

$$E(z) = \left[ \frac{\Omega_{m0}}{1 - \beta} (1 + z)^{3(1 - \beta)} + \left( 1 - \frac{\Omega_{m0}}{1 - \beta} \right) \right]^{1/2}. \quad (54)$$

Fitting to the same observational data, we find the best-fit parameters $\Omega_{m0} = 0.2731$ and $\beta = -0.0021$, whereas $\chi^2_{\text{min}} = 542.735$. A conventional criterion for model comparison in the literature is $\chi^2_{\text{min}}/\text{dof}$, in which the degree of freedom $\text{dof} = N - k$, whereas $N$ and $k$ are the number of data points and the number of free model parameters, respectively. We present the $\chi^2_{\text{min}}/\text{dof}$ for all the 6 models in Table \[I\].

On the other hand, there are other criterions for model comparison in the literature, such as Bayesian Information Criterion (BIC) and Akaike Information Criterion (AIC). The BIC is defined by \[29, 31\]

$$\text{BIC} = -2 \ln \mathcal{L}_{\text{max}} + k \ln N, \quad (55)$$

where $\mathcal{L}_{\text{max}}$ is the maximum likelihood. In the Gaussian cases, $\chi^2_{\text{min}} = -2 \ln \mathcal{L}_{\text{max}}$. So, the difference in
Model $\Lambda$CDM $Q = 3\beta H \rho_m$ $Q = 3\beta q H \rho_m$ $Q = 3\beta q H \rho_{tot}$ with $\beta \geq 0$ $Q = 3\beta q H \rho_{tot}$ without $\beta \geq 0$ $Q = 3\beta q H \rho_A$

| Best fit | $\Omega_{m0} = 0.2701$ | $\Omega_{m0} = 0.2731$ | $\Omega_{m0} = 0.2738$ | $\Omega_{m0} = 0.2701$ | $\Omega_{m0} = 0.2764$ | $\Omega_{m0} = 0.2717$
|---|---|---|---|---|---|---|
| $\omega_{\text{min}}^2$ | 542.919 | 542.735 | 542.725 | 542.919 | 542.711 | 542.778 |
| $k$ | 1 | 2 | 2 | 2 | 2 | 2 |
| $\chi_{\text{min}}^2/dof$ | 0.9730 | 0.9744 | 0.9744 | 0.9747 | 0.9743 | 0.9745 |
| $\Delta \text{BIC}$ | 0.0021 | 0.0100 | 0.0100 | 0.0247 | 0.0136 | 0.0136 |
| $\Delta \text{AIC}$ | 0 | 2 | 2 | 2 | 2 | 2 |
| Rank | 1 | 4 | 3 | 6 | 2 | 5 |

$\chi^2_{\text{min}}$ is the minimum value of the likelihood function, $\Omega_{m0}$ is the initial matter density parameter, $\chi^2/dof$ is the reduced chi-squared. $\Delta \text{BIC}$ and $\Delta \text{AIC}$ are the differences in BIC and AIC between the models.

### TABLE I: Summarizing all the 6 models considered in this work.

| Model | $\Lambda$CDM | $Q = 3\beta H \rho_m$ | $Q = 3\beta q H \rho_m$ | $Q = 3\beta q H \rho_{tot}$ with $\beta \geq 0$ | $Q = 3\beta q H \rho_{tot}$ without $\beta \geq 0$ | $Q = 3\beta q H \rho_A$ |
|---|---|---|---|---|---|---|
| $\Omega_{m0} = 0.2701$ | $\Omega_{m0} = 0.2731$ | $\Omega_{m0} = 0.2738$ | $\Omega_{m0} = 0.2701$ | $\Omega_{m0} = 0.2764$ | $\Omega_{m0} = 0.2717$ |
| $\beta$ | $-0.0021$ | $-0.0100$ | $0.0100$ | $-0.0247$ | $0.0136$ |

The difference in AIC between two models is given by $\Delta \text{AIC} = \Delta \chi^2_{\text{min}} + 2\Delta k$. The AIC is defined by

$$\text{AIC} = -2 \ln L_{\text{max}} + 2k.$$  \hspace{1cm} \text{(56)}$$

The difference in BIC between two models is given by $\Delta \text{BIC} = \Delta \chi^2_{\text{min}} + \Delta k \ln N$. The BIC is defined by

$$\text{BIC} = -2 \ln L_{\text{max}} + k \ln N.$$  \hspace{1cm} \text{(57)}$$

The BIC and AIC are used to compare the models, and the model with the lowest BIC or AIC is considered the best. The $\beta$ parameter is used to determine the interaction between the dark energy and dark matter.

Secondly, we note that the case of $Q = 3\beta q H \rho_A$ is fairly different from the cases of $Q = 3\beta H \rho_m$ and $Q = 3\beta q H \rho_{tot}$. Comparing Fig. 7 with Figs. 1, 3 and 5, it is easy to see that the direction of contours in the $\Omega_{m0} - \beta$ plane is rightward for the case of $Q = 3\beta q H \rho_A$, whereas the ones are leftward for the cases of $Q = 3\beta H \rho_m$ and $Q = 3\beta q H \rho_{tot}$. From Table I we find that the best-fit $\beta$ is positive for the case of $Q = 3\beta q H \rho_A$, whereas the ones are negative (or zero) for the cases of $Q = 3\beta H \rho_m$ and $Q = 3\beta q H \rho_{tot}$. This means that in the case of $Q = 3\beta q H \rho_A$, the interaction $Q$ crosses the non-interacting line ($Q = 0$) from above to below, whereas in the cases of $Q = 3\beta H \rho_m$ and $Q = 3\beta q H \rho_{tot}$ the interaction $Q$ crosses the non-interacting line ($Q = 0$) from below to above. This is physically interesting, because $Q > 0$ means that the energy transfers from dark energy to dark matter, whereas $Q < 0$ means that the energy transfers from dark matter to dark energy.

Finally, in this work the role of dark energy is only played by the decaying $\Lambda$ (vacuum energy), whereas the parameter $\alpha$ in the sign-changeable interactions are chosen to be zero. So, the constraints obtained in this work cannot be directly used to the models different from the ones considered here. In fact, the interacting dark energy models with sign-changeable interactions can be generalized. For instance, one can choose dark energy to be the one with a constant or variable EoS (including parameterized EoS, or even the ones of quintessence, phantom, k-essence, Chaplygin gas, quintom, hessence, holographic or agegraphic dark energy, and so on). Of course, one can also let the parameter $\alpha$ be free and then constrain the models numerically.

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