A Review of SHACL: From Data Validation to Schema Reasoning for RDF Graphs

Paolo Pareti and George Konstantinidis

1 University of Winchester, Winchester, United Kingdom
paolo.pareti@winchester.ac.uk
2 University of Southampton, Southampton, United Kingdom
g.konstantinidis@soton.ac.uk

Abstract. We present an introduction and a review of Shapes Constraint Language (SHACL), the W3C recommendation language for validating RDF data. A SHACL document describes a set of constraints on RDF nodes, and a graph is valid with respect to the document if its nodes satisfy these constraints. We revisit the basic concepts of the language, its constructs and components and their interaction. We review the different formal frameworks used to study this language and the different semantics proposed. We examine a number of related problems, from containment and satisfiability to the interaction of SHACL with inference rules, and exhibit how different modelings of the language are useful for different problems. We also cover practical aspects of SHACL, discussing its implementations and state of adoption, to present a holistic review useful to practitioners and theoreticians alike.

1 Introduction

The Shapes Constraint Language (SHACL) [23] is a W3C recommendation language for the validation of RDF graphs. In SHACL, validation is based on shapes, which define particular constraints and specify which nodes in a graph should be validated against these constraints. The ability to validate data with respect to a set of constraints is of particular importance for RDF graphs, as they are schemaless by design. Validation can be used to detect problems in a dataset and it can provide data quality guarantees for the purpose of data exchange and interoperability. A set of constraints can also be interpreted as a "schema", functioning as one of the primary descriptors of a graph dataset, thus enhancing its understandability and usability. A set of SHACL shapes is called a shapes graph, but we refer to it as a SHACL document in order not to confuse it with the graphs that it is used to validate.

In this article we present a review of SHACL, which is composed of three main parts. In the first part, in Sections 2 and 3 we review the SHACL specification. This part focuses on how shapes are defined, and how they are used for the purpose of validation. We highlight the main peculiarities of this language, and discuss how SHACL validation can be expressed either in terms of SPARQL queries, to facilitate its implementation, or in terms of assignments [13], to make it...
amenable to theoretical study. The syntax of SHACL is outside the scope of this review, and for the precise details on how to encode particular constraints we refer the reader to the SHACL specification [23]. We also do not discuss the process that lead to the development of SHACL, but it should be noted that this specification was built on top of a number of previous constraint languages, the most influential of which is Shape Expressions (ShEx) [48].

In the second part of our review, in Sections 5 to 8 we present the formal properties of this language. This mainly revolves around a discussion of recursion. The semantics of recursion is not defined in the SHACL specification, and thus has been the subject of significant subsequent research [3]. The formal semantics of SHACL is given as a translation into SCL [37], a first order logic language that captures the entirety of the SHACL specification. Apart from validation, several standard decision problems are discussed, such as satisfiability and containment, along with an existing study on the interaction of SHACL with inference rules. We try to keep a consistent notation throughout this article and at times this notation might be different from the one in the original articles.

In Section 9 we review existing implementations of SHACL validators and their integration with mainstream graph databases. We also review prominent additional tools to manage SHACL documents, such as tools designed to automate or semi-automate the process of creating SHACL documents by exploiting graph data, ontologies, or other constraint languages. These approaches provide solutions to the cold start problem, and alleviate reliance on expert knowledge, which are typical problems of new technologies. We complement a discussion of these approaches with a review of prominent applications of SHACL in several domains; in summary, the abundance of SHACL related tools and applications highlights the remarkable level of maturity and adoption reached by this relatively new language.

2 Preliminaries

Before discussing SHACL, we briefly introduce our notation for RDF graphs [14]. With the term RDF graph (or just graph) we refer to a set of RDF triples (or just triples), where each triple <s, p, o> identifies an edge with label p, called predicate, from a node s, called subject, to a node o, called object. Subjects, predicates and objects of RDF triples are collectively called RDF terms. The RDF terms that appear as subject and objects in the triples of a graph are called the nodes of the graph. Graphs in this article are represented in Turtle syntax using common XML namespaces, such as sh, rdf and rdfs to refer to, respectively, the SHACL, RDF, and RDFS [7] vocabularies. Queries over RDF graph will be expressed as SPARQL [42] queries.

In the RDF data model, subjects, predicates and objects are defined over different but overlapping domains. For example, while RDF terms of the IRI type can occupy any position in a triple, RDF terms of the literal type (representing datatype values) can only appear in the object position. These differences are not central to the topics discussed in this review, and thus, for the sake of simplifying
A Review of shacl

:EmployeeShape a sh:PropertyShape ;
  sh:targetClass :Employee ;
  sh:path :hasOfficeNumber ;
  sh:minCount 1 .

:Anne a :Employee .
  :Bob a :Employee ;
    :hasOfficeNumber "18" ;
    :hasOfficeNumber "3" .
  :Carl a :Employee ;
    :hasOfficeNumber "171" .
  :David a :Customer .

Fig. 1. (Left) A sample shacl document (shape graph) stating the constraint that every employee must have at least one office number. (Right) A sample RDF graph (data graph).

notation, we will assume that all elements of a triple are drawn from a single and infinite domain of constants. This corresponds to the notion of generalized RDF [13].

3 Overview of ShACL

The main application of SHACL is data validation. Data validation in SHACL requires two inputs: (1) an RDF graph $G$ to be validated and (2) a SHACL document $M$ that defines the conditions against which $G$ must be evaluated. The SHACL specification defines the output of the data validation process as a validation report, detailing all the violations that were found in $G$ of the conditions set by $M$. If the violation report contains no violations, a graph $G$ is valid w.r.t. SHACL document $M$. The SHACL validation process can be abstracted into the following decision problem. Given a graph $G$ and a SHACL document $M$, we denote with $\text{VALIDATE}(G,M)$ the decision problem of deciding whether $G$ is valid w.r.t. SHACL document $M$, that we call validating $G$ against $M$.

For example, the graph on the left of Figure 1 represents a SHACL document $M_1$, that defines the condition that every employee must have an office number. Therefore, the validation report for a graph and $M_1$ would list all of the instances of :Employee in the graph that do not have an office number. The validation report for $M_1$ and the data graph $G_1$ on the right of Figure 1 contains a violation on node rdf:Anne, since she does not have an office number. Therefore $G_1$ is not valid w.r.t. $M_1$.

Formally, a SHACL document is a set of shapes. Validating a graph against a SHACL document involves validating it against each shape. Shapes restrict the structure of a valid graph by focusing on certain nodes and examining whether they satisfy their constraints. The main components of a shape are a constraint $d$ and a target definition $t$. Constraints can be evaluated on any RDF node to determine whether that node satisfies or not the given constraints. A node that satisfies the constraint of a shape it is said to conform to that shape, or not-conform otherwise. If a shape has an empty constraint, all nodes trivially conform to the shape. Not all nodes of a graph must conform to all the shapes in the
SHAQ document. The constraint definition of each shape defines which RDF nodes, called target nodes, must conform to that shape in order for the graph to be valid. A shape with an empty constraint definition does not have any target nodes. Through inter-shape referencing, as we will see below, additional nodes might be required to conform to certain shapes (or not, if negation is used) for the validation to succeed. Further irrelevant nodes within the graph do not play a role in validation of the shape, whether they conform to it or not. The SHAQ document $M_1$ of our previous example, contains shape :EmployeeShape, whose constraint captures the property of “having an office number”, and whose target definition targets only the RDF nodes of type Employee. Nodes of the Client type do not generate violations by not having an office number.

Formally, a shape is a tuple $\langle s, t, d \rangle$ defined by three components: (1) the shape name $s$, which uniquely identifies the shape; (2) the target definition $t$, and (3) the set of constraints which are used in conjunction, and hence hereafter referred to as the single constraint $d$. As demonstrated in Figure 1 a SHAQ document is itself an RDF graph. The graph representing a SHAQ document is called a shapes graph, while the graph being validated is called a data graph. This approach to serialisation is similar to how OWL ontologies are serialised, and it serves a similar purpose. Thanks to this approach, a SHAQ document does not require any dedicated infrastructure to be stored and shared. In fact, a SHAQ document can be embedded directly into the very graph it validates, thus combining the shape graph and data graph into a single graph. Interestingly, with this serialisation, a shapes graph, being an RDF graph, can be itself subject to validation. The SHAQ specification, in fact, defines a shapes graph that can be used to validate shapes graphs.

We will now look in more details at the two major components of shapes, namely target definitions and constraints.

3.1 SHAQ Target Definitions

A SHAQ target definition, within a constraint, is a set of target declarations. There are four types of target declarations defined in SHAQ, each one taking an RDF term $c$ as a parameter.

Node Targets A node target declaration on $c$ targets that specific node.

Class-based Targets If a shape has a class-based target on $c$, then all the nodes in the graph that are of type (rdf:type) $c$ are target nodes for that shape.

Subjects-of Targets If a shape has a subject-of target on $c$, then the target nodes for that shape are all the nodes in the graph that appear as subjects in triples with $c$ as the predicate.

Objects-of Targets If a shape has an object-of target on $c$, then the target nodes for that shape are all the nodes in the graph that appear as objects in triples with $c$ as the predicate.

The shape defined in Figure 1 demonstrates an example of a class-based target targeting class :Employee. Similarly, to target a subject of a property, e.g., :worksAt, the second line of the shape definition would be substituted with:
Typically, a target declaration is used to select, among all the nodes in a graph, the ones to target for constraint validation. The node target declaration, however, behaves differently, as it targets a particular node regardless of whether this node occurs in the graph or not. An important implication of this is that empty graphs are not trivially valid, since node targets can detect violations on nodes external to the graph. If a target definition of a shape is empty, then that shape will have no target nodes. However, this does not mean that the constraint of that shape will not be evaluated on any nodes since, as mentioned, other shapes can refer to it and “pass it” a node to check for conformance.

3.2 Focus nodes and property paths

When a target or another node is considered against a shape for conformity, we call it a focus node. Initially a shape focuses on its target nodes (these are the initial set of focus nodes). Additional focus nodes are obtained by following shacl property paths, which we also refer to as just paths. Shacl property paths are a subset of sparql property paths and, as the name suggests, define paths in the RDF graph. The simplest type of path, called predicate path, corresponds to a single property IRI c. This path identifies all the nodes that are reachable in the RDF graph from the current focus node by following a single edge c. In other words, this path identifies all the RDF nodes in the object position of triples that have c as the predicate and the current focus node as the subject. More complex paths can be constructed by inverting the direction of a path, by concatenating two different paths one after the other, or by allowing the repetition of a path for a minimum, maximum or arbitrary number of times.

Based on the use of property paths, SHACL specification distinguishes shapes into two types: node shapes and property shapes. Intuitively, the constraint of a node shape is evaluated directly on the focus nodes of the shape. Instead, when using a property path, shapes must be declared as a property shapes. These are characterised by a path, and their constraints are evaluated over all of the nodes that can be reached from the focus nodes following such path. For example, the constraint that every employee’s password must be at least 8 characters long can be represented by a property shape that targets employee nodes, and that has a relation such as :hasPassword as its path. In this way, the actual nodes that must satisfy the “at least 8 characters long” constraint are not the target nodes, but instead those that appear as objects in triples with an employee node as a subject, and :hasPassword as the predicate.

3.3 SHACL Constraints

The majority of the SHACL recommendation is dedicated to defining the different types of constraint components that can be used in SHACL constraints. The main type of constraint components are called core constraint components. These are the components that SHACL compliant systems typically support, and where
most of the existing literature focuses on. The other main type of components are the \textit{SPARQL-based constraint components}, that are used to embed SPARQL queries into SHACL constraints. This significantly increases the expressive power of such constraints. However, the inclusion of arbitrarily complex SPARQL queries can lead to performance issues, and can make such constraints harder to understand and use. It is also worth noting that, outside of the SHACL recommendation, a number of additional SHACL features\footnote{https://w3c.github.io/shacl/shacl-af/} are currently being designed, and some of them might be included in further versions of SHACL. In the rest of this paper we will focus on core constraint components.

In order to better understand SHACL core constraint components, we propose a broad categorisation of these components into three main categories, depending on how they are evaluated on the focus nodes. Notice that most constraint components can be used in both node shapes and property shapes.

### Graph Structure Components
These components define constraints that are evaluated at the level of triples of the graph, and focus on restrictions such as the minimum and maximum cardinality that the focus node must have for certain paths, or the RDF class that the focus node should be a type of. The shape defined in Figure\ref{fig:shacl_shape} demonstrates an example of a minimum cardinality constraint for predicate path \texttt{hasOfficeNumber}. Two other salient constraints in this category are the property pair equality and disjointedness, that specify whether the two sets of nodes reachable from two different paths must be equal or disjoint, respectively.

### Filter Components
These components define constraints that are evaluated at the level of nodes, and their evaluation is usually independent from the triples present in the graph. Filter constraints restrict the focus node (1) to be a particular RDF term, (2) to be of a particular type, such as IRI, blank node or literal, or (3) to be a literal that satisfies certain properties, such as being of the integer datatype, or a string produced by a certain regular expression.

### Logical Components
Logical components define the standard logical operators of conjunction, disjunction and negation over other constraints.

While most core constraint components fall into one of these categories, the pair of constraints \texttt{sh:lessThan} and \texttt{sh:lessThanOrEquals} is a notable exception, as it combines the properties of graph structure and filter components. These two constraints require all the nodes reachable by one path to be literals that are less than (resp. less than or equals) to the nodes reachable by a second path.

It is worth noting that all constraints but one, namely \texttt{sh:closed}, are not affected by triples with unknown predicates (i.e. predicates not occurring in the SHACL document). This means that if a graph is valid with respect to a set of those constraints, it would still remain valid if new triples with unknown predicates are added to the graph. Thus, given a non-empty graph $G$, valid w.r.t. a SHACL document $M$, graph $G \cup \langle s, p, o \rangle$ is also valid w.r.t. $M$ if (1) $p$ does not occur in $M$ and (2) $M$ does not contain the \texttt{sh:closed} constraint component.

\footnote{https://w3c.github.io/shacl/shacl-af/} accessed on 18/6/21
A Review of shACL

:\EmployeeShapeB a sh:PropertyShape ;
  sh:targetClass :Employee ;
  sh:path :hasOfficeNumber ;
  sh:qualifiedMinCount 1 ;
  sh:qualifiedValueShape :OfficeNumberShape .

:OfficeNumberShape a sh:NodeShape ;
  sh:minLength 3 .

Fig. 2. A sample shACL document stating the constraint that every employee must have at least one 3-characters or longer office number.

Intuitively, this means that those constraints restrict the usage of terms from a particular vocabulary, but they do not restrict in any way the graph from containing triples described using other vocabularies. The sh:closed component, on the other hand, restricts the predicates of the triples that have the focus node as a subject to belong to a predetermined finite set. Effectively, the sh:closed component can prohibit the use of unknown predicate relations for certain nodes in the graph, and thus prevent the inclusion of terms from other vocabularies. Interestingly, component sh:closed introduces an asymmetry in shACL, since it only affects triples where the focus node is the subject, and it is not possible to define a similar constraint for nodes in the object position.

A major feature of shACL is that constraints can use the name of a shape to require a particular set of nodes to conform to that shape. This is called a shape reference. An example of a shape reference is demonstrated by the shACL document in Figure 2. This document contains shape :EmployeeShapeB which references shape :OfficeNumberShape. The former shape restricts all of its target nodes to having an edge :hasOfficeNumber to a node that conforms to the latter shape, having a string length of at least three characters. Validating the data graph in Figure 1 with the shACL document in Figure 2 results in two violating nodes for shape :EmployeeShapeB. The first one is :Anne, who does not have an office number, and the second one is :Bob, whose office numbers all contain fewer than three digits.

Shape references can be recursive, that is, the constraint of a shape can reference the constraints of a second shape which, in turn, can reference the constraints of a third shape, and so on, creating a loop. Let \( S_i^d \) be the set of all the shape names occurring in a constraint \( d \) of a shape \( \langle s, t, d \rangle \); these are the directly referenced shapes of \( s \). Let \( S_i^{d+1} \) be the set of shapes in \( S_i^d \) union the directly referenced shapes of the constraints of the shapes in \( S_i^d \).

Definition 1. A shape \( \langle s, t, d \rangle \) is recursive if \( s \in S_i^\infty \); else it is non-recursive.

Definition 2. A shACL document \( M \) is recursive if it contains a recursive shape, and non-recursive otherwise.

The semantics of recursive shACL documents are not defined in the shACL specification. In Section 3 we review the official semantics of non-recursive shACL
documents, while in Section 5 we review the extended semantics for recursive SHAACL document that have been proposed in the literature.

4 SHAACL Validation

In this section we present the semantics of SHAACL data validation, that is, the $\text{VALIDATE}(G, M)$ decision problem, for any given graph $G$ and SHAACL document $M$. In Section 4.1 we review how validation is defined in the SHAACL specification, with the help of SPARQL queries. While this query-based description of SHAACL semantics can be easily translated into a concrete implementation, it does not lend itself well to theoretical investigation. In Section 4.2 we will discuss an alternative approach to defining SHAACL semantics that is instead amenable to a formal study.

4.1 SHAACL Validation by SPARQL queries

The validation of an RDF graph $G$ against a SHAACL document $M$ can be performed on a shape-by-shape basis. For each shape $\langle s, t, d \rangle$, this process involves verifying the fact that every node $n$, targeted by target definition $t$, satisfies constraint $d$. Intuitively, graph $G$ is valid w.r.t. $M$ if and only if this fact is true for every shape in $M$.

Given a graph $G$ and a target definition $t$, the set of target nodes for $t$ can be computed by evaluating a SPARQL query on $G$ for each target declaration in $t$, and taking the union of the values returned by these queries. Table 1 details the corresponding SPARQL query for each of the four types of target declarations defined in SHAACL. It should be noted that, by default, SHAACL does not enforce any particular entailment regime. If an entailment regime is being adopted, then this should be taken into account when developing a SHAACL validator. For example, if the RDFS entailment regime [7] is being considered, subclass inference should be accounted for when computing the set of entities of a given class. To accommodate for this entailment regime, the query for the node target in Table 1 could be updated to the following one.

```
SELECT ?x WHERE {
  ?x rdf:type/rdfs:subClassOf* c
}
```

Once an RDF term has been identified as being in the target of a shape, evaluating whether it conforms to the shape can be done using SPARQL queries. In the SHAACL specification, in fact, several core constraint components are defined with respect to SPARQL queries. Most notably, the semantics of SHAACL filter components is in direct dependence to the semantics of SPARQL filter functions. For example, the $\text{shminLength}$ constraint component restricts a focus node to having a string length equal or larger than a given number. Formally, a focus node $n$ has a $\text{shminLength}$ of $j$ if and only if the following SPARQL query evaluates to true.

```sparql
SELECT ?x WHERE {
  ?x shminLength 1
}
```
Table 1. Target declarations and their corresponding SPARQL queries to compute the set of target nodes on a given graph

| Target declaration         | SPARQL query                                      |
|----------------------------|---------------------------------------------------|
| Node target (node c)       | SELECT ?x WHERE { VALUES ?x { c } }                |
| Class target (class c)     | SELECT ?x WHERE { ?x rdf:type c . }               |
| Subjects-of target (predicate c) | SELECT ?x WHERE { ?x c ?y . }                   |
| Objects-of target (relation c) | SELECT ?x WHERE { ?y c ?x . }               |

ASK {
  FILTER (STRLEN(str(?x)) >= j) .
}

Not all SHACL constraints, however, can be easily verified by a single SPARQL query. Evaluating whether a constraint that contains shape references is satisfied by a focus node, in fact, might involve evaluating whether other constraints are satisfied by other nodes which, in turn, might require even further constraint evaluations. For example, in order to evaluate whether node rdf:Carl from the data graph in Figure 1 conforms to shape rdf:EmployeeShapeB from Figure 2, we would need to evaluate whether his office number, namely RDF term “171”, conforms to shape rdf:OfficeNumberShape. This is especially problematic in case of recursion, as it could generate an infinite series of constraint evaluations. For non-recursive SHACL documents, Corman et al. [12] showed that it is always possible to check the validity of a graph using a single SPARQL query. For example, a graph can be checked against the SHACL document of Figure 2 by evaluating the following SPARQL query.

SELECT ?x WHERE {
  ?x a :Employee .
  FILTER NOT EXISTS {
    ?x :hasOfficeNumber ?y .
    FILTER (STRLEN(str(?y)) >= 3) .
  }
}

This query selects all RDF nodes of type Employee that do not have an office number with at least three characters. Thus, any RDF term returned by this query is a node violating a shape of the SHACL document. If this query evaluates to an empty set, then the graph that it is evaluated on is valid with respect to the SHACL document.

4.2 Shape Assignments: A Tool for Defining SHACL Validation

The SPARQL-based approach to SHACL validation does not provide a concise and formal description of SHACL semantics. Moreover, it does not provide us with a terminating procedure to check graphs in the face of SHACL recursion. In this
section we review the concept of shape assignments (or just assignments)\[13\], which can be used to address the above mentioned problems.

As defined in Table\[1\] a target declaration $\textit{t}$ is a unary query over a graph $\textit{G}$. We denote with $\textit{G} \models t(\textit{n})$ that a node $\textit{n}$ is in the target of $\textit{t}$ with respect to a graph $\textit{G}$. If $\textit{t}$ is empty, no node in any graph is in the target of $\textit{t}$. The definition of whether a node conforms to a shape, as we previously discussed, does not only depend on the graph $\textit{G}$, but it might also depend, due to shape references, on whether other nodes conform to other shapes. Intuitively, the concept of assignments\[13\] is used to keep track, for every RDF node, of all the shapes that it conforms to, and all of those that it does not. Given a document $\textit{M}$ and a graph $\textit{G}$, we denote $\text{nodes}(\textit{G}, \textit{M})$ the set of nodes in $\textit{G}$ together with any extra ones referenced by the node target declarations in $\textit{M}$. With $\text{shapes}(\textit{M})$ we refer to all the shape names in a document $\textit{M}$.

**Definition 3.** Given a graph $\textit{G}$, and a SHACL document $\textit{M}$, an assignment $\sigma$ for $\textit{G}$ and $\textit{M}$ is a function mapping nodes in $\text{nodes}(\textit{G}, \textit{M})$, to subsets of $\text{shapes}(\textit{M})\cup \{\neg s | s \in \text{shapes}(\textit{M})\}$, such that for all nodes $\textit{n}$ and shape names $s$, $\sigma(\textit{n})$ does not contain both $s$ and $\neg s$.

Expression $[d]^{n,G,\sigma}$ denotes the evaluation of constraint $d$ on a node $\textit{n}$ w.r.t. a graph $\textit{G}$ under an assignment $\sigma$, as defined in\[13\]. If $[d]^{n,G,\sigma}$ is $\text{True}$ (resp. $\text{False}$) we say that node $\textit{n}$ satisfies (resp. does not satisfy) constraint $d$ w.r.t. $\textit{G}$ under $\sigma$. For any graph $\textit{G}$ and assignment $\sigma$, fact $s \in \sigma(\textit{n})$ (resp. $\neg s \in \sigma(\textit{n})$) denotes the fact that node $\textit{n}$ conforms (resp. does not conform) to $s$ w.r.t. $\textit{G}$ under $\sigma$. Expression $[d]^{n,G,\sigma}$ evaluates to $\text{True}$, $\text{False}$ or $\text{Undefined}$ values of Kleene’s 3-valued logic, and the truth value of any shape reference in $d$ is computed using the assignment (it should be noted that the $\text{Undefined}$ value never occurs in non-recursive shapes, but is used to define possible extended semantics in the face of recursion). In other words, whenever a truth value in the evaluation of $[d]^{n,G,\sigma}$ depends on whether another node $\textit{j}$ conforms to a shape $s'$, with constraints $d'$, this is not resolved by evaluating $[d']^{j,G,\sigma}$, but instead it is $\text{True}$ if $s' \in \sigma(\textit{j})$, $\text{False}$ if $\neg s' \in \sigma(\textit{j})$, or else $\text{Undefined}$. This, in turn, eliminates the problem of a potentially infinite series of constraint evaluations.

The semantics of SHACL validation can be defined with respect to a particular type of assignments, called faithful\[13\].

**Definition 4.** For all graphs $\textit{G}$, and SHACL documents $\textit{M}$ and assignments $\sigma$, assignment $\sigma$ is faithful w.r.t. $\textit{G}$ and $\textit{M}$, denoted with $(\textit{G}, \sigma) \models \textit{M}$, if the following two conditions hold for any shape $(s,t,d)$ in $\text{shapes}(\textit{M})$ and node $\textit{n}$ in $\text{nodes}(\textit{G}, \textit{M})$:

1. $s \in \sigma(\textit{n})$ iff $[d]^{n,G,\sigma}$ is $\text{True}$; and $\neg s \in \sigma(\textit{n})$ iff $[d]^{n,G,\sigma}$ is $\text{False}$;
2. if $\textit{G} \models t(\textit{n})$ then $s \in \sigma(\textit{n})$.

Condition (1) ensures that the facts denoted by the assignment are correct; while condition (2) ensures that the assignment is compatible with the target definitions. Condition (2) is trivially satisfied for SHACL documents where all target definitions are empty. Later we will want to discuss assignments where
the first property of Def. 4 holds, but not necessarily the second, in order to
reason about the existence of alternative assignments that are correct (as in,
they satisfy the first part of Def. 4) but that are not faithful. In fact, these
will be faithful assignments to a document that is “stripped empty” of target
definitions. Let $M^{\emptyset}$ denote the SHACL document obtained from substituting
all target definitions in SHACL document $M$ with the empty set. The following
lemma holds:

**Lemma 1.** For all graphs $G$, SHACL documents $M$ and assignments $\sigma$, condition (1) from Definition 4 holds for any shape $s$ in $\text{shapes}(M)$ and node $n$ in nodes($G, M$) iff $(G, \sigma) \models M^{\emptyset}$.

The existence of a faithful assignment is a necessary and sufficient condition
for validation for non-recursive SHACL documents [13]. As we will see later, this
is also necessary condition for all the other extended semantics.

**Definition 5.** A graph $G$ is valid w.r.t. a non-recursive SHACL document $M$ if there exists an assignment $\sigma$ such that $(G, \sigma) \models M$.

5 SHACL Recursion

The semantics of recursion in SHACL documents is left undefined in the SHACL
specification [23], and this gives rise to several possible interpretations. In this
section we consider extended semantics of SHACL that define how to validate
graphs against recursive SHACL documents. We focus on existing extended se-
manitics that follow monotone reasoning. These can be characterised by two
dimensions, namely the choice between partial and total assignments [13] and
between brave and cautious validation [3], which we will subsequently define.
Put together, these two dimensions define the four extended semantics of brave-
partial, brave-total, cautious-partial and cautious-total. We will not go into the
details of the less obvious dimension of stable-model semantics [3], which relates
SHACL to non-monotone reasoning in logic programs.

As mentioned in the previous section, assignments can specify a truth value
of True, False or Undefined to whether a node conforms to given shape. The truth
value of Undefined, which does not occur in non-recursive SHACL documents, can
instead play an important role in validating SHACL under recursion. Intuitively,
this happens during validation, when recursion makes it impossible for a node $n$
to either conform or not to conform to a shape $s$ but, at the same time, validity
does not depend on whether $n$ conforms to shape $s$ or not. Consider for example
the following SHACL document, containing a single shape $\langle s^*, \emptyset, d^* \rangle$ (with name
:InconsistentS in this example). This shape is defined as the negation of itself,
that is, given a node $n$, a graph $G$ and an assignment $\sigma$, fact $[d^*]_n^{G, \sigma}$ is true iff
$\neg s^* \in \sigma(n)$, and false iff $s^* \in \sigma(n)$.

:InconsistentS a sh:NodeShape ;
sh:not :InconsistentS .
It is easy to see that any assignment that maps a node to either $s^*$ or $\neg s^*$ is not faithful, as it would violate condition (1) of Definition 4. However, an assignment that maps every node of a graph to the empty set would be faithful for that graph and document $\{s^*\}$. Intuitively, this means that nodes in the graph cannot conform nor not conform to shape $s^*$, but since this shape does not have any target node to validate, then the graph can still be valid. The fact of whether nodes conform or not conform to shape $s^*$ can thus be left as “undefined”.

This type of validation, for recursive SHACL documents, is called validation with partial assignments. More specifically, validation under brave-partial semantics simply extends the criterion of Def. 5 to recursive SHACL documents. All other extended semantics are constructed by adding additional conditions to brave-partial semantics. The term “partial” should not be interpreted as the fact that it describes only “part” of nodes of a graph, or that it describes the relationship of a node to only “part” of the shapes. Within a partial assignment, the conformance of every node to every shape is precisely specified by one of three truth values, and the term “partial” only indicates that one of these three truth values is Undefined.

**Definition 6.** A graph $G$ is valid w.r.t. a SHACL document $M$ under brave-partial semantics if there exists an assignment $\sigma$ such that $(G, \sigma) \models M$.

In the SHACL specification, nodes either conform to, or not conform to a given shape, and the concept of an “undefined” level of conformance is arguably alien to the specification. It is natural, therefore, to consider restricting the evaluation of a constraint to the True and False values of boolean logic. This is achieved by restricting assignments to be total.

**Definition 7.** An assignment $\sigma$ is total w.r.t. a graph $G$ and a SHACL document $M$ if, for all nodes $n$ in $\text{nodes}(G, M)$ and shapes $(s, t, d)$ in $M$, either $s \in \sigma(n)$ or $\neg s \in \sigma(n)$.

For any graph $G$ and SHACL document $M$ we denote with $A^{G,M}$ and $A^{G,M}_T$, respectively, the set of assignments, and the set of total assignments for $G$ and $M$. Trivially, $A^{G,M}_T \subseteq A^{G,M}$ holds.

**Definition 8.** A graph $G$ is valid w.r.t. a SHACL document $M$ under brave-total semantics if there exists an assignment $\sigma$ in $A^{G,M}_T$ such that $(G, \sigma) \models M$.

Since total assignments are a more specific type of assignments, if a graph $G$ is valid w.r.t. a SHACL document $M$ under brave-total semantics, than it is also valid w.r.t. $M$ under brave-partial semantics. The reverse, instead, is only true for non-recursive SHACL documents. In fact, as shown in [13], if there exists a faithful assignment for a graph $G$ and a non-recursive document $M$, then there exists also a total faithful assignment for $G$ and $M$. Therefore, the definition of validity under brave-total semantics (Def. 8), for non-recursive SHACL documents, coincides with the standard definition of validation (Def. 5).
While total assignments can be seen as a more natural way of interpreting the SHACL specification, they are not without issues when recursive SHACL documents are considered. Going back to our previous example, we can notice that there cannot exist a total faithful assignment for the SHACL document containing shape :InconsistentS, for any non-empty graph. This is a trivial consequence of the fact that no node can conform to, nor not conform to, shape :InconsistentS. This, however, is in contradiction with the SHACL specification, which implies that a SHACL document without target declarations in any of its shapes (such as the one in our example) should trivially validate any graph. If there are no target declarations, in fact, there are no target nodes on which to verify the conformance of certain shapes, and thus no violations should be detected.

The second and last dimension that we consider is the difference between brave and cautious validation. When a SHACL document $M$ is recursive, there might exist multiple assignments $\sigma$ satisfying property (1) of definition 4. That is, such that $(G, \sigma) \models M^t$. Intuitively, these can be seen as equally “correct” assignments with respect to the constraints of the shapes, and brave validation only checks whether at least one of them is compatible with the target definitions of the shapes. Cautious validation, instead, represents a stronger form of validation, where all such assignments must be compatible with the target definitions.

**Definition 9.** A graph $G$ is valid w.r.t. a SHACL document $M$ under cautious-partial (resp. cautious-total) semantics if it is (1) valid under brave-partial (resp. brave-total) semantics and (2) for all assignments $\sigma$ in $A_{G,M}$ (resp. $A_{T,M}^G$), it is true that if $(G, \sigma) \models M^t$ holds then $(G, \sigma) \models M$ also holds.

To exemplify this distinction, consider the following SHACL document $M_1$. This document requires the daily special of a restaurant, node :DailySpecial, to be vegetarian, that is, to conform to shape :VegDishShape. This shape is recursively defined as follows. Something is a vegetarian dish if it contains an ingredient, and all of its ingredients are vegetarian, that is, entities conforming to the :VegIngredientShape. A vegetarian ingredient, in turn, is an ingredient of at least one vegetarian dish.

```shacl
:VegDishShape a sh:PropertyShape ;
   sh:targetNode :DailySpecial ;
   sh:path :hasIngredient ;
   sh:minCount 1 ;
   sh:qualifiedMaxCount 0 ;
   sh:qualifiedValueShape [ sh: not :VegIngredientShape ] .

:VegIngredientShape a sh:PropertyShape ;
   sh:path [ sh:inversePath :hasIngredient ] ;
   sh:node :VegDishShape .
```

Consider now a graph $G_1$ containing the following triple.

```sharc
:DailySpecial :hasIngredient :Chicken .
```
Due to the recursive definition of \texttt{VegDishShape}, there exist two different assignments $\sigma_1$ and $\sigma_2$, which are both faithful for $G_1$ and $M_1^{\top}$. In $\sigma_1$, no node in $G_1$ conforms to any shape, while $\sigma_2$ differs from $\sigma_1$ in that node \texttt{DailySpecial} conforms to \texttt{VegDishShape} and node \texttt{Chicken} conforms to \texttt{VegIngredientShape}. Essentially, either both the dish and the ingredient from graph $G_1$ are vegetarian, or neither is. Therefore, $\sigma_2$ is faithful for $G_1$ and $M_1$, while $\sigma_1$ is not. The question of whether the daily special is a vegetarian dish or not can be approached with different levels of “caution”. Under brave validation, graph $G_1$ is valid w.r.t. $M_1$, since it is possible that the daily special is vegetarian. Cautious validation, instead, takes the more conservative approach, and under its definition $G_1$ is not valid w.r.t. $M_1$, since it is also possible that the daily special is not vegetarian. When analysing such recursive definitions, one might want to exclude “unfounded” assignments, that is, assignments that assign certain shapes to a node for no other reason than to allow the validation of a graph. This is achieved by the recursive semantics for SHACL proposed in [3], which is based on the concept of \textit{stable models} from Answer Set Programming.

For each extended semantics, the definition of validity of a graph $G$ with respect to a SHACL document $M$, denoted by $G \models M$, is summarised in the following list.

- \textbf{brave-partial} there exists an assignment that is faithful w.r.t. $G$ and $M$;
- \textbf{brave-total} there exists an assignment that is total and faithful w.r.t. $G$ and $M$;
- \textbf{cautious-partial} there exists an assignment that is faithful w.r.t. $G$ and $M$, and every assignment that is faithful w.r.t. $G$ and $M^{\top}$ is also faithful w.r.t. $G$ and $M$.
- \textbf{cautious-total} there exists an assignment that is total and faithful w.r.t. $G$ and $M$, and every assignment that is total and faithful w.r.t. $G$ and $M^{\top}$ is also faithful w.r.t. $G$ and $M$.

6 Formal languages for SHACL

In this section we review the two main formal languages that have been proposed to model the semantics of SHACL. We first discuss a complete first-order formalisation of SHACL, which can be used to study a number of decision problems. We then present a simplified language that effectively models SHACL constraints for the purpose of validation.

6.1 SCL, A First-Order Language for SHACL

In order to formally study SHACL, it is convenient to abstract away from the syntax of its RDF and SPARQL representations. The SCL first order language [37][36] is currently the only complete formalisation of SHACL into a formal logical system. The expressiveness of this language covers all of the SHACL target declarations and all of the SHACL core constraint components, including the filter components, which are less commonly studied. This language captures the semantics
A Review of shacl

of whole shacl documents, and it can be used to study a number of related decision problems, including validation. The relation between shacl and SCL is given by translation \( \tau \) \([36]\), such that, given a shacl document \( M \), the first order sentence \( \tau(M) \) is the translation of \( M \) into SCL. We identify the inverse translation with \( \tau^- \).

Before defining SCL and its properties, we must define how RDF graphs and assignments are modelled in this logical framework. The domain of discourse is assumed to be the set of RDF terms. Triples are modelled as binary relations, with atom \( R(s, o) \) corresponding to triple \( <s, R, o> \). A minus sign identifies the inverse role, i.e. \( R^-(s, o) = R(o, s) \). Binary relation name \( \text{isA} \) represents class membership triples \( <s, \text{rdf:type}, o> \) as \( \text{isA}(s, o) \). Assignments are modelled with a set of monadic relations \( \Sigma \), called shape relations. Each shacl shape \( s \) is associated with a unique shape relation \( \Sigma_s \) in SCL. Facts \( \Sigma(x) \) (resp. \( \neg \Sigma(x) \)) describe an assignment \( \sigma \) such that \( s \in \sigma(x) \) (resp. \( \neg s \in \sigma(x) \)). Since this logical framework adopts boolean logic, \( \forall x. \Sigma(x) \lor \neg \Sigma(x) \) holds, by the law of excluded middle. Thus shape relations define total assignments.

Given a graph \( G \) and an assignment \( \sigma \), we now define their respective translations \( G^\tau \) and \( \sigma^\tau \) into first order structures.

**Definition 10.** Given a graph \( G \), fact \( p(s, o) \) is true in the first order structure \( G^\tau \) iff \( <s, p, o> \in G \).

**Definition 11.** Given a total assignment \( \sigma \), fact \( \Sigma_s(n) \) is true in the first order structure \( \sigma^\tau \) iff \( s \in \sigma(n) \).

**Definition 12.** Given a graph \( G \) and a total assignment \( \sigma \), the first order structure \( I \) induced by \( G \) and \( \sigma \) is the disjoint union of structures \( G^\tau \) and \( \sigma^\tau \). Given a first order structure \( I \): (1) the graph \( G \) induced by \( I \) is the graph that contains triple \( <s, p, o> \) iff \( I \models p(s, o) \) and (2) the assignment \( \sigma \) induced by \( I \) is the assignment such that, for all nodes \( n \) and shape relations \( \Sigma_s \), fact \( s \in \sigma(n) \) is true iff \( I \models \Sigma_s(n) \) and \( \neg s \in \sigma(n) \) iff \( I \not\models \Sigma_s(n) \).

The existence of faithful assignments using SCL and its standard model-theoretic semantics is presented in the following theorem \([37]\). Trivially, this also defines what condition, in SCL, corresponds to validation under the brave-total extended semantics (Def. \([8]\), which also defines validation for non-recursive shacl documents (Def. \([5]\)).

**Theorem 1.** For any graph \( G \), total assignment \( \sigma \) and shacl document \( M \), it is true that \( (G, \sigma) \models M \) iff \( I \models \tau(M) \), where \( I \) is the first order structure induced by \( G \) and \( \sigma \).

For any first order structure \( I \) and SCL formula \( \phi \), it is true \( I \models \phi \) iff \( (G, \sigma) \models \tau^- (\phi) \), where \( G \) and \( \sigma \) are, respectively, the graph and assignment induced by \( I \).

Sentences in the SCL language follow the \( \phi \) grammar in Definition \([13]\).

**Definition 13.** The shacl first order language (SCL, for short) is the set of first order sentences built according to the following context-free grammar, where
Table 2. Translation of a shape with name $s$ with a target definition $t$, into an SCL target axiom.

| Target declaration in $t$ | SCL target axiom |
|--------------------------|------------------|
| Node target (node $c$)   | $\Sigma_s(c)$    |
| Class target (class $c$) | $\forall x. isA(x, c) \rightarrow \Sigma_s(x)$ |
| Subjects-of target (relation $R$) | $\forall x, y. R(x, y) \rightarrow \Sigma_s(x)$ |
| Objects-of target (relation $R$) | $\forall x, y. R^-(x, y) \rightarrow \Sigma_s(x)$ |

$c$ is a constant from the domain of RDF terms, $\Sigma$ is a shape relation, $F$ is a filter relation, with shape relations disjoint from filter relations, $R$ is a binary-relation name, $\star$ indicates the transitive closure of the relation induced by $\pi(x, y)$, the superscript $\pm$ refers to a relation or its inverse, and $n \in \mathbb{N}$.

\[ \varphi := \top | \varphi \land \varphi \]
\[ | \Sigma(c) | \forall x. isA(x, c) \rightarrow \Sigma(x) | \forall x, y. R^{\pm}(x, y) \rightarrow \Sigma(x) \]
\[ | \forall x. \Sigma(x) \leftrightarrow \psi(x); \]
\[ \psi(x) := \top | \neg\psi(x) | \psi(x) \land \psi(x) | x = c | F(x) | \Sigma(x) | \exists y. \pi(x, y) \land \psi(y) \]
\[ | \neg\exists y. \pi(x, y) \land R(x, y) | \forall y. \pi(x, y) \leftrightarrow R(x, y) | \forall y. z. \pi(x, y) \land R(x, z) \rightarrow \varsigma(y, z) \]
\[ | \exists^\geq n y. \pi(x, y) \land \psi(y); \]
\[ \pi(x, y) := R^{\pm}(x, y) \]
\[ | \exists z. \pi(x, z) \land \pi(z, y) | x = y \lor \pi(x, y) | \pi(x, y) \lor \pi(x, y) | (\pi(x, y))^*; \]
\[ \varsigma(x, y) := x <^\pm y | x \leq^\pm y. \]

Symbol $\varphi$ corresponds to a SHACL document. An SCL sentence could be empty ($\top$), a conjunction of documents, a target axiom representing a target definition (a production of the 3rd, 4th and 5th production rule) or a constraint axiom representing a constraint (a production of the last production rule). Target axioms take one of three forms, based on the type of target declarations. The translation of SHACL target declarations into SCL target axioms is summarised in Table 2. Letters in square brackets are annotations for naming SCL components and thus are not part of the grammar. These letters are essentially first-letter abbreviations of prominent SHACL components (that together define fragments of SCL), and are also listed in Table 3.

The non terminal symbol $\psi(x)$ corresponds to the subgrammar of the SHACL constraints components. Within this subgrammar, $\top$ identifies an empty constraint, $x = c$ a constant equivalence constraint and $F$ a monadic filter relation.
### Table 3. Relation between prominent SHACL components and SCL expressions.

| Abbr. | Name                  | SHACL component               | Corresponding expression               |
|-------|-----------------------|-------------------------------|----------------------------------------|
| S     | Sequence Paths        | Sequence Paths                | $\exists z \cdot \pi(x, z) \land \pi(z, y)$ |
| Z     | Zero-or-one Paths     | sh:zeroOrOnePath              | $x = y \lor \pi(x, y)$                |
| A     | Alternative Paths     | sh:alternativePath            | $\pi(x, y) \lor \pi(y, x)$            |
| T     | Transitive Paths      | sh:zeroOrMorePath             | $\pi(x, y)$                           |
| D     | Property Pair Disjointness | sh:disjoint                | $\neg \exists y \cdot \pi(x, y) \land R(x, y)$ |
| E     | Property Pair Equality| sh:equals                     | $\forall y \cdot \pi(x, y) \leftrightarrow R(x, y)$ |
| O     | Property Pair Order   | sh:lessThan                   | $x \leq \pm y$ and $x < \pm y$         |
| C     | Cardinality Constraints| sh:qualifiedValueShape       | $\exists \geq n \cdot \pi(x, y) \land \psi(y)$ with $n \neq 1$ |

(e.g. $F^{\text{IRI}}(x)$, true iff $x$ is an IRI). Filters components are captured by $F(x)$ and the $O$ component. The $C$ component captures qualified value shape cardinality constraints. The $E$, $D$ and $O$ components capture the equality, disjointedness and order property pair components. The $\pi(x, y)$ subgrammar models SHACL property paths. Within this subgrammar $S$ denotes sequence paths, $A$ denotes alternate paths, $Z$ denotes a zero-or-one path and $T$ denotes a zero-or-more path.

Translation $\tau$ results in a subset of SCL formulas, called well-formed defined subsequently, and the inverse translation $\tau^{-1}$ only takes well formed sentences as an input. An SCL formula $\phi$ is well-formed iff for every shape relation $\Sigma$, formula $\phi$ contains exactly one constraint axiom with relation $\Sigma$ on the left-hand side of the implication. Intuitively, this condition ensures that every shape relation is “defined” by a corresponding constraint axiom. The translation of the document from Fig. 2 into a well-formed SCL sentence, via $\tau$, is the following. Arguably, this logic notation might seem easier to read and understand than the SHACL syntax of Fig. 2

$$\left( \forall x. \text{isA}(x, \text{:Employee}) \rightarrow \Sigma_{\text{:EmployeeShapeB}}(x) \right)$$

$$\land \left( \forall x. \Sigma_{\text{:EmployeeShapeB}}(x) \leftrightarrow \exists y. R_{\text{:hasOfficeNumber}}(x, y) \land \Sigma_{\text{:OfficeNumberShape}}(y) \right)$$

$$\land \left( \forall x. \Sigma_{\text{:OfficeNumberShape}}(x) \leftrightarrow F_{\text{length} \geq 3}(y) \right)$$

The language defined without any of these constructs is called the base language, denoted $\emptyset$. On top of the base language different syntactic fragments of SCL are defined by considering different combinations of features allowed. We name these fragments by concatenating the letters that represent the features allowed, into a single name. For example, SA identifies the fragment that only allows the base language, sequence paths and alternate paths. This means that in order to write an SCL document in SA, one can only use the production rules
of Def. 13 that are not annotated with any feature (base language) or those identified by abbreviations $S$ and $A$.

The SHACL specification presents an unusual asymmetry in the fact that equality, disjointedness and order components force one of their two path expressions to be an atomic relation. This can result in situations where the order constraints can be defined in just one direction, since only the less-than and less-than-or-equal property pair constraints are defined in SHACL. The $0$ fragment models a more natural order comparison that includes the $>$ and $\geq$ components. The fragment where the order relations in the $\varsigma(x,y)$ subgrammar cannot be inverted is denoted $0'$.

When interpreting an SCL sentence, particular care should be paid to the semantics of filter relation. The interpretation of each filter relation, such as $F_{\text{IRI}}(x)$, is the subset of the domain of discourse on which the filter is true. This interpretation is constant across all models, and defines the semantics of the filter. When considering the decision problem of validation, filter relations in SCL must be suitably defined by interpreted relations (similarly to how the equality operator is). When considering additional decision problems, such as satisfiability and containment (which will be discussed in Section 7), the semantics of filters can be axiomatised, thus removing the need for special interpreted relations. The filter axiomatisation presented in [37] captures the semantics of all SHACL filters with the single exception of $\text{shpattern}$, as this filter defines complex non-standard regular expressions based on the SPARQL REGEX function [42].

### 6.2 $\mathcal{L}$, a Language for SHACL Constraint Validation

Another major language used to study SHACL is $\mathcal{L}$ which was presented in [13] and paved the way to subsequent formal studies of SHACL. The $\mathcal{L}$ language differs from SCL in scope and purpose. While SCL sentences describe whole SHACL documents, sentences in $\mathcal{L}$ describe individual SHACL constraints. The $\mathcal{L}$ language is primarily designed to investigate the complexity of SHACL validation. As such, it relies on assumptions that do not hold when studying other decision problems such as satisfiability and containment, which, instead, can be studied using SCL. In particular, $\mathcal{L}$ assumes that all filter components can be evaluated on a node in constant time, and thus are all equivalent, for the purposes of validation. Thanks to this reduced scope, $\mathcal{L}$ seems less complex than SCL, and it is a useful formalism to study the evaluation of SHACL constraints. The semantics of an $\mathcal{L}$ sentence $\phi$ is defined in [13] through the use of faithful assignments. In particular, [13] fixes a lookup table that provides the truth value of the evaluation of $\phi$ on a node $n$ for a graph $G$ and an assignment $\sigma$. Instead, SCL relies on the standard model-theoretic semantics.

The grammar of $\mathcal{L}$ sentences is given next. In this grammar $s$ is a shape name; $I$ is an IRI; $r$ is a SHACL property path; $n$ is a positive integer.

$$\phi ::= \top \mid s \mid I \mid \phi_1 \land \phi_2 \mid \neg \phi \mid \geq_n r.\phi \mid \text{EQ}(r_1, r_2)$$

Table 4 defines the correspondence between $\mathcal{L}$ and the $\psi(x)$ sub-grammar of SCL. It is easy to see that $\mathcal{L}$ sentences correspond to a subset of the $\psi(x)$ sub-
Table 4. Correspondence between an $\mathcal{L}$ sentence $\phi$, and SCL $\psi^{\phi}(x)$ expressions, such that a constraint $\phi$ is satisfied on a node $n$ w.r.t. a graph $G$ and an assignment $\sigma$ iff $I \models \psi^{\phi}(n)$, where $I$ is the first order structure induced by $G$ and $\sigma$. It is assumed that paths are expressed using the $\pi(x,y)$ subgrammar of SCL, and that $r_2$ is an IRI.

| $\mathcal{L}$ expression $\phi$ | Corresponding SCL $\psi^{\phi}(x)$ |
|----------------------------------|-----------------------------------|
| $\top$                           | $\top$                            |
| $\bot$                           | $\Sigma(x)$                       |
| $I$                              | $x = I$                           |
| $\phi_1 \land \phi_2$           | $\psi^{\phi_1}(x) \land \psi^{\phi_2}(x)$ |
| $\neg \phi$                      | $\neg \psi^{\phi}(x)$            |
| $\exists_n r. \phi$              | $\exists y. r(x,y) \land \psi^{\phi}(y)$ |
| $\exists I \exists Q(r_1, r_2)$  | $\forall y. r_1(x,y) \leftrightarrow r_2(x,y)$ |

7 SHAACL Decision Problems

Several existing pieces of work in the literature focus on SHAACL, and several related decision problems have been investigated. In Section 7.1 we review existing work on the core decision problem for SHAACL, namely validation. Unlike validation, which studies the relationship between a SHAACL document and an RDF graph, the decision problems of satisfiability and containment, reviewed in Section 7.2, focus on intrinsic properties of SHAACL documents and their components.

7.1 Validation

Validation is a core decision problem for SHAACL, since the main application of this language is the validation of RDF graphs. This decision problem is decidable for all of the semantics discussed in this article, including the four extended semantics. The complexity lower bounds for validation, however, depend on the fragment of SHAACL being considered. Table 5 lists the data complexity of three fragments of SHAACL given in [13,3]. The three fragments are (1) SHAACL non-rec,
Table 5. Data complexity of SHACL validation, results from [12].

| Fragment             | Data complexity of validation |
|----------------------|------------------------------|
| SHACL_{non-rec}      | NL-c                         |
| SHACL^{+}            | PTIME-c                      |
| SHACL_{rec}          | NP-c                         |

the fragment of non-recursive SHACL documents built using $L$ constraints; (2) $SHACL^{+}$, the fragment of SHACL documents built using $L$ constraints with a restricted use of negation, that is, substituting the $\neg \phi$ production rule of $L$ into $\phi_1 \lor \phi_2$; and (3) $SHACL_{rec}$, the fragment of SHACL documents built using $L$ constraints. The most expressive of these fragments, $SHACL_{rec}$, is NP-complete in data complexity.

7.2 Satisfiability and Containment

Satisfiability and containment are standard decision problems that have been investigated in the context of SHACL. These two decision problems, unlike validation, do not take a graph as an input. Instead, they focus on SHACL documents, shapes or constraints. Given any notion of validity from one of the semantics defined earlier, the following decision problems are defined. For simplicity, when discussing satisfiability and containment, we will assume the use of the semantics of validation from Definitions 8 and 5.

Definition 14. A SHACL document $M$ is satisfiable iff there exists a graph $G$ such that $G \models M$. Deciding whether a SHACL document is satisfiable is the decision problem of SHACL satisfiability.

Definition 15. SHACL Containment: For all SHACL documents $M_1$, $M_2$, we say that $M_1$ is contained in $M_2$, denoted $M_1 \subseteq M_2$, iff for all graphs $G$, if $G \models M_1$ then $G \models M_2$. Deciding whether a SHACL document is contained in another is the decision problem of SHACL containment.

Two SHACL documents $M_1$ and $M_2$ that are contained in each other ($M_1 \subseteq M_2$ and $M_2 \subseteq M_1$) are semantically equivalent. Two semantically equivalent documents are not necessarily equivalent syntactically, since in SHACL the same constraint can be expressed using different sets of shapes.

The satisfiability and containment decision problems for SHACL can be polynomially reduced to the satisfiability decision problem for SCL, defined as follows in the natural way [37].

Definition 16. An SCL sentence $\phi$ is satisfiable iff there exists structure $\Omega$ such that $\Omega \models \phi$. Deciding whether a SCL sentence is satisfiable is the decision problem of SCL satisfiability.
A Review of shacl

Fig. 3. [37] Decidability and complexity map of SCL satisfiability. Round (blue) and square (red) nodes denote decidable and undecidable fragments, respectively. Solid borders on nodes correspond to theorems in this paper, while dashed borders are implied results. Directed edges indicate inclusion of fragments, while bidirectional edges denote polynomial-time reducibility. Solid edges are preferred derivations to obtain tight results, while dotted ones leads to worst upper-bounds or model-theoretic properties. Finally, a light blue background indicates that the fragment enjoys the finite-model property, while those with a light red background do not satisfy this property.

This reduction allows us to study the decidability and complexity of the shacl satisfiability and containment problems for a given shacl fragment by studying the decidability and complexity of SCL satisfiability, for the corresponding fragments. The results of this study, published in [37], are summarised in Figure 3. Negative results indicate the undecidability of both the SCL fragment, and the corresponding shacl fragment. Positive results, shown in round blue in the figure, indicate that both satisfiability and containment are decidable, for that fragment of non-recursive shacl, and are accompanied with complexity upper-bounds. Starting from the negative results, shacl satisfiability and containment is, in general, undecidable. This was shown even for several non-recursive fragments, through a semi-conservative reduction from the standard domino problem [50,6,39], which is an undecidable decision problem. More specifically, the shacl satisfiability problems for the $S\mathcal{O}, S\mathcal{A}C, S\mathcal{E}C, S\mathcal{E}O'$, and $S\mathcal{ZAE}$ fragments are undecidable [37].
Positive results are obtained by noticing that several SCL fragments are included in decidable fragments of first-order logic. For example, the SZATD fragment of SCL is included in the extension of the unary-negation fragment of first-order logic with arbitrary transitive relations, which can be solved in 2ExpTime \cite{1,20,15}. The complexity upper bounds identified in \cite{37} for shape fragments range from ExpTime to 2ExpTime. Both decision problems are defined over SHACL documents which, similarly to the schema of a dataset, could be assumed to be of small or constant size.

Up until this point we considered the satisfiability and containment problems defined at the level of SHACL documents. However, it is possible to study variations of these problems at different levels of granularity. For example, the satisfiability and containment problems at the level of SHACL constraints are defined in \cite{37}, and are shown to be reducible to the problem of SHACL satisfiability. An approach that uses Description Logics Reasoning is presented in \cite{26} to compute shape containment, that is, containment at the level of shapes, for a restrictive fragment of SHACL, which however allows recursion.

8 Inference Rules and the Schema Expansion

Datasets are often dynamic objects, which are frequently subject to modification. When an RDF graph is modified, its validity w.r.t. a SHACL document might change. If the modifications that a dataset undergoes are completely arbitrary, then it is not possible to make predictions regarding validity, and the dataset might need to be re-validated after each modification. Many types of modifications that can be applied on a dataset, however, are predictable or a result of some reasoning process. In particular, many types of modifications can be represented as inference rules \( B \rightarrow H \), where a set of facts \( H \), called the head, are added to a dataset whenever a query \( B \), called the body, finds a match on the dataset. Given an RDF graph \( G \), and a set of inference rules \( R \), it is possible to compute graph \( G' \), closure of \( G \) under \( R \), by applying the chase algorithm \cite{4}.

The chase algorithm, intuitively, consists in repeatedly applying the rules of \( R \) on \( G \) until convergence. For simplicity, we assume that the chase algorithm is guaranteed to terminate for the inference rules considered.

Assuming graph \( G \) is valid w.r.t. to a SHACL document \( M \), the approach presented in \cite{38}, called schema expansion, allows us to predict whether the graph closure \( G' \) will still be valid w.r.t. \( M \) without having to validate \( G' \) against \( M \). In particular, given a SHACL document \( M \) and a set of inference rules \( R \), the schema expansion process computes the “maximal sub-document” of \( M \) which will still validate \( G \) after the rule applications. That is, the schema expansion is a SHACL document \( M' \), called schema consequence, such that (1) \( M \subseteq M' \) (i.e., \( M' \) is a subset of the restrictions of \( M \)); (2) validity is preserved after closure, that is, for any graph \( G \) valid w.r.t. \( M \), its closure \( G' \) under \( R \) is valid w.r.t. \( M' \); and (3) \( M' \) is “minimally-containing”, i.e., there is no document \( M'' \) that satisfies conditions (1) and (2) and such that that \( M'' \subset M' \). If a schema consequence \( M' \) of a SHACL document \( M \) under inference rules \( R \) is semantically equivalent
to $M$, then any graph $G$, valid w.r.t. $M$ is guaranteed to remain valid w.r.t. $M$ after computing its closure under $R$. In other words, this means that the application of rules $R$ cannot “invalidate” graphs valid w.r.t. document $M$.

Consider, for example, the following graph $G_1$, which describes :Eve, a manager of the company in the IT department, and one of her subordinates :Fiona.

```sparql
:Eve a :Manager ;
 :hasDepartment "IT" .
:Fiona a :Employee ;
 :hasManager :Eve .
```

This graph is valid w.r.t. the following SHACL document $M_1$, which states that each employee must have a manager, and each manager must have a department.

```sparql
:SubordinateS a :PropertyShape ;
 sh:targetClass :Employee ;
 sh:path :hasManager ;
 sh:minCount 1 .

:ManagerS a sh:PropertyShape ;
 sh:targetClass :Manager ;
 sh:path :hasDepartment ;
 sh:minCount 1 .
```

Consider now the set of inference rules $R_1 = \{r_1, r_2\}$, where rules $r_1$ and $r_2$ are defined as follows. For simplicity, we represent both the head and the body of rules as SPARQL graph patterns, which are interpreted as SPARQL CONSTRUCT queries where the WHERE and CONSTRUCT clauses are the body and the head, respectively. Rule $r_1$ states that every manager can be inferred to be an employee, and $r_2$ states that everyone can be inferred to be in the same department as their manager.

$$r_1 = \{?x \text{ rdf:type :Manager} \} \rightarrow \{?x \text{ rdf:type :Employee}\}$$

$$r_2 = \{?x \text{ :hasManager ?y . ?y :hasDepartment ?z} \} \rightarrow \{?x \text{ :hasDepartment ?z}\}$$

The closure of graph $G_1$ under rules $R_1$ is the following graph $G_2$.

```sparql
:Eve a :Manager ;
 :Employee ;
 :hasDepartment "IT" .
:Fiona a :Employee ;
 :hasManager :Eve ;
 :hasDepartment "IT" .
```

Notice that graph $G_2$ is not valid w.r.t. $M_1$, since :Eve violates :SubordinateS, but it is valid w.r.t. another document $M_2$ which only contains shape :ManagerS. In fact, $M_2$ is a schema consequence of $M_1$ and $R_1$. Therefore, we know that the closure under $R_1$ of any graph valid w.r.t. $M_1$ will validate shape :ManagerS, but it might not validate :SubordinateS.

Two approaches to compute the schema expansion are presented in [38], for datalog [9] inference rules without negation. The first based on the concept...
of critical instance \cite{30}, and the second an optimisation of the first. These approaches are only defined on a fragment of SHACL that, although restricted, is sufficient to express common constraints for RDF validation, such as the Data Quality Test Patterns TYPEDEP, TYPEDERP, PVT, RDFS-DOMAIN and RDFS-RANGE in the categorisation by Kontokostas et al. \cite{24}. Intuitively, the difficulty in computing a schema expansion lies in having to consider all possible graphs that are valid w.r.t. a SHACL document, and their interactions with arbitrarily complex inference rules.

9 Applications, Tools and Implementations

Over a few years since reaching its status as a W3C recommendation, the level of maturity and adoption of the SHACL technology has been steadily increasing. In this section we review existing SHACL implementations, tools designed to facilitate the creation and management of SHACL documents, and documented usages of SHACL in practical applications.

9.1 Tools for SHACL Validation

The availability of mature tools is often a crucial requirement for the widespread adoption of a technology. To date, SHACL validation has been integrated in a number of mainstream tools and triplestores.\footnote{https://w3c.github.io/data-shapes/data-shapes-test-suite/} An example of this is RDF4J\footnote{https://rdf4j.org/} a Java framework for managing RDF data, which now includes an engine for SHACL validation. The RDF4J framework is integrated in a number of projects, most notably the GraphDB\footnote{https://graphdb.ontotext.com/} triplestore. Other SHACL-enabled databases include AllegroGraph\footnote{https://allegrograph.com/} by Franz Inc, Apache Jena\footnote{https://jena.apache.org/} by Apache, and Stardog\footnote{https://www.stardog.com/} by Stardog Union Inc. A benchmark for the comparison of different SHACL implementation was proposed in \cite{41}, along with results for four different databases. A SHACL implementation is also available for Python through the pySHACL\footnote{https://pypi.org/project/pyshacl/} library.

One of the first tools to enable the validation of recursive SHACL graphs was SHACL2SparQL \cite{11}. Another tool, Trav-SHACL \cite{18}, implements a SHACL engine designed to optimise the evaluation of SHACL core constraint components expressible in fragments of the $\mathcal{L}$ language \cite{13}. On these fragments of SHACL, Trav-SHACL was shown to achieve significantly faster validation times compared to the SHACL2SPARQL tool.

\footnote{https://w3c.github.io/data-shapes/data-shapes-test-suite/} accessed on 18/6/21
\footnote{https://rdf4j.org/} accessed on 18/6/21
\footnote{https://graphdb.ontotext.com/} accessed on 18/6/21
\footnote{https://allegrograph.com/} accessed on 18/6/21
\footnote{https://jena.apache.org/} accessed on 18/6/21
\footnote{https://www.stardog.com/} accessed on 18/6/21
\footnote{https://pypi.org/project/pyshacl/} accessed on 18/6/21
9.2 Tools for Generating SHAACL Documents

While efficient tools to perform graph validation are undoubtedly essential to the widespread adoption of SHAACL, it is also important to devise practical ways to generate suitable SHAACL documents, without which validation would not be possible. On the one hand, SHAACL documents can be manually created by experts. Tools to support this manual process can facilitate this, especially when integrated with already established software. An example of this is SHAACL4P [17], a plugin for the Protégé ontology editor [31] which includes an editor to create SHAACL documents, and a validator that allows users to test the document by validating an ontology with it, and then visualising any constraint violations. Shape Designer [6] is another tool to create SHAACL documents that combines a graphical editor, and additional algorithms to create constraints semi-automatically by analysing the data graph. The benefits of different types of visualisations as an aid to the creating and editing of constraints for RDF graphs was studied in [28].

Existing work also investigated the possibility of generating SHAACL documents from natural language text [43]. A number of approaches have been designed to automate the creation of SHAACL documents. The SHACLearner [33] approach, generates SHAACL documents by learning a kind of rules called Inverse Open Path (IOP) Rules from the graph data provided. IOP rules are strongly related to SCL and therefore SHAACL. An IOP rule essentially follows the same structure as an SCL constraint axiom, both syntactically and semantically, with the only exception that the iff operator is replaced by a rightward implication. Another approach to automate the creation of SHAACL documents is the Astrea-KG Mappings [10]. These mappings consist of a set of manually created mappings from OWL [49] to SHAACL, that can be used to automatically generate SHAACL documents from OWL ontologies. As described in [34], SHAACL documents can also be generated from the axioms defined by ontology design patterns [19]. The approach from [34] generates SHAACL documents for the purpose of quality assessment, using the ontology design patterns and data statistics created by the ABSTAT [15] tool as an input. Another similar approach, presented in [8], allows the automatic extraction of SHAACL constraints from a SPARQL endpoint, and was tested on the dataset of Europeana [1].

Notably, SHAACL documents can also be seen as describing a desirable “schema” for graph data. As such, they can be used as a template to generate new RDF data. An example of this is the Schimatos [51] tool, which generates forms for RDF graph editing based on SHAACL documents, in order to simplify the graph editing task, and minimize the chance of error.

9.3 Adoption of SHAACL

An analysis of existing use cases of SHAACL can be useful to gain insights on how this technology is used in practice, and on in its level of adoption. In a recent review, 13 existing projects using SHAACL have been reviewed, and the most common constraints observed were cardinality, class, datatype and disjunction [29].

https://www.europeana.eu/en accessed on 18/6/21
Several works investigate the use of SHAQ to verify compliance of a dataset w.r.t. certain policies, such as GDPR requirements [35,2]. Other applications of SHAQ include type checking program code [27] and detecting metadata errors in clinical studies [22]. SHAQ is also used by the European Commission to facilitate data sharing, for example by validating metadata about public services against the recommended vocabularies [46]. Notably, several approaches define translations into SHAQ from other technologies, such as ontologies and other schema and constraint languages [16,21,25,32,40,47]. These results show that SHAQ tools, and in particular validators, can benefit areas where technologies other than SHAQ are already established.

10 Conclusion

Within this review we examined SHAQ, a constraint language that can be used to validate RDF graphs. These constraints can be used to describe the properties of a graph, to detect possible errors in the data or provide data quality assurances. In this review we first presented the main concepts of the SHAQ specification, such as the concept of shapes, and their two main components, targets and constraints. We discussed the primary way to perform SHAQ validation, using SPARQL queries, and how the semantics of validation can be abstracted with the concept of assignments.

While the SHAQ specification describes how validation should be performed, its semantics is left implicit and not formally defined. We have extensively discussed studies that address this problem. In particular, we reviewed a complete formalisation of SHAQ into a fragment of first order logic called SCL. This formalisation lays bare several properties of SHAQ, and provides decidability and complexity results for several SHAQ-related decision problems. Another important line of work focuses on defining potential extensions of SHAQ semantics that can be used in the face of recursion. The SHAQ specification, in fact, allows constraints to be recursively defined, but it does not define its semantics. We also presented existing work studying the interaction of SHAQ with inference rules. Datasets are often dynamic objects, and several questions arise when considering the effects of this dynamism on the constraints imposed over them.

From the point of view of maturity and level of adoption of the SHAQ technology, we reviewed several implementations of SHAQ validators, which are now integrated in many mainstream RDF databases, and several tools designed to facilitate the creation and management of SHAQ documents. Several approaches, in particular, provide automated or semi-automated ways of generating suitable SHAQ documents from a diverse range of sources, such as graph data, ontologies, or natural language texts. Existing efforts in mapping other constraint/validation languages into SHAQ is also worth noting, as it suggests that the usefulness of SHAQ could be extended to support other existing technologies. While the true extent of SHAQ adoption is hard to establish, since not all usages of SHAQ are publicly documented, we found evidence of its usage in several areas, such
as to facilitate data sharing, to validate dataset against policies, and to detect errors in datasets.

Despite the wealth of work on this topic, SHACL is still a recent specification, and a number of important directions for future work still exist. For example, there are opportunities to optimise SHACL validators for particular type of constraints, or for particular scenarios, like for highly dynamic databases. More studies are needed to properly assess the usage of SHACL in practical applications, and what types of constraints are more commonly used and how. While the full semantics of SHACL has been formally defined, more work is needed to formally establish its relation with other constraint languages. It is also important to notice that most of the pieces of work reviewed in this article limit their scope to ad-hoc subsets of the SHACL specification. In addition to the custom requirements of each application, this is commonly done in order to avoid excessively complex language components. At the same time, it is often difficult to understand what these subsets exactly are as they are not always explicitly defined. Therefore, there might be scope to define reusable fragments of SHACL, that could fill the role of lightweight but expressive alternatives to the full language, similarly to how OWL fragments are defined. It might also be beneficial, for similar reasons, to converge towards a single standard or “preferred” semantics for SHACL recursion, which could be defined in a future version of the specification.

References

1. A. Amarilli and M. Benedikt and P. Bourhis and M. Vanden Boom: Query Answering with Transitive and Linear-Ordered Data. In: IJCAI’16. pp. 893–899 (2016)
2. Al Bassit, A., Krasnashchok, K., Skhiri, S., Mustapha, M.: Automated Compliance Checking with SHACL (2020)
3. Andresel, M., Corman, J., Ortiz, M., Reutter, J.L., Savkovic, O., Simkus, M.: Stable Model Semantics for Recursive SHACL. In: Proceedings of The Web Conference 2020. p. 1570–1580. WWW ’20 (2020)
4. Benedikt, M., Konstantinidis, G., Mecca, G., Motik, B., Papotti, P., Santoro, D., Tsamoura, E.: Benchmarking the chase. In: Proceedings of the 36th ACM SIGMOD-SIGACT-SIGAI Symposium on Principles of Database Systems. pp. 37–52. ACM (2017)
5. Berger, R.: The Undecidability of the Domino Problem. MAMS 66, 1–72 (1966)
6. Boneva, I., Dusart, J., Fernández Alvarez, D., Gayo, J.E.L.: Shape Designer for ShEx and SHACL Constraints. ISWC 2019 - 18th International Semantic Web Conference, Poster (Oct 2019)
7. Brickley, D., Guha, R.: RDF schema 1.1. W3C recommendation, W3C (Feb 2014), https://www.w3.org/TR/2014/REC-rdf-schema-20140225/
8. Čeräns, K., Ovčinnikova, J., Bojárs, U., Grasmanis, M., Lāce, L., Romāne, A.: Schema-Backed Visual Queries over Europeana and other Linked Data Resources. In: ESWC2021 Poster and Demo Track (2021)
9. Ceri, S., Gottlob, G., Tanca, L.: What you always wanted to know about datalog (and never dared to ask). IEEE Transactions on Knowledge and Data Engineering 1(1), 146–166 (1989)
10. Cimmino, A., Fernández-Izquierdo, A., García-Castro, R.: Astrea: Automatic Generation of SHACL Shapes from Ontologies. In: The Semantic Web. pp. 497–513. Springer International Publishing, Cham (2020)
11. Corman, J., Florenzano, F., Reutter, J.L., Savkovic, O.: SHACL2SPARQL: Validating a SPARQL Endpoint against Recursive SHACL Constraints. In: International Semantic Web Conference ISWC Satellite Events. pp. 165–168 (2019)
12. Corman, J., Florenzano, F., Reutter, J.L., Savković, O.: Validating SHACL Constraints over a Sparql Endpoint. In: The Semantic Web – ISWC 2019. pp. 145–163 (2019)
13. Corman, J., Reutter, J.L., Savković, O.: Semantics and Validation of Recursive SHACL. In: The Semantic Web – ISWC 2018. pp. 318–336 (2018)
14. Cyganiak, R., Wood, D., Markus Lanthaler, G.: RDF 1.1 Concepts and Abstract Syntax. W3C Recommendation, W3C (2014), http://www.w3.org/TR/2014/REC-rdf11-concepts-20140225/
15. Danielski, D., Kieronski, E.: Finite Satisfiability of Unary Negation Fragment with Transitivity. In: MFCS’19. pp. 17:1–15. LIPIcs 138, Leibniz-Zentrum fuer Informatik (2019)
16. Di Cicco, C., Ekaputra, F.J., Cecconi, A., Ekelhart, A., Kiesling, E.: Finding Non-compliances with Declarative Process Constraints Through Semantic Technologies. In: Cappiello, C., Ruiz, M. (eds.) Information Systems Engineering in Responsible Information Systems. pp. 60–74 (2019)
17. Ekaputra, F.J., Lin, X.: SHACL4P: SHACL constraints validation within Protégé ontology editor. In: 2016 International Conference on Data and Software Engineering (ICoDSE). pp. 1–6 (2016)
18. Figuera, M., Rohde, P.D., Vidal, M.E.: Trav-SHACL: Efficiently Validating Networks of SHACL Constraints. In: Proceedings of the Web Conference 2021. p. 3337–3348. WWW ’21, Association for Computing Machinery, New York, NY, USA (2021)
19. Gangemi, A., Presutti, V.: Ontology Design Patterns, pp. 221–243. Springer Berlin Heidelberg, Berlin, Heidelberg (2009)
20. Jung, J., Lutz, C., Martel, M., Schneider, T.: Querying the Unary Negation Fragment with Regular Path Expressions. In: ICDT’18. pp. 15:1–18. OpenProceedings.org (2018)
21. K Soman, R.: Modelling construction scheduling constraints using shapes constraint language (SHACL). In: 2019 European Conference on Computing in Construction. pp. 351–358. University College Dublin (2019)
22. Keuchel, D., Spicher, N.: Automatic Detection of Metadata Errors in a Registry of Clinical Studies Using Shapes Constraint Language (SHACL) Graphs. Studies in health technology and informatics 281, 372–376 (May 2021)
23. Knublauch, H., Kontokostas, D.: Shapes Constraint Language (SHACL). W3C Recommendation, W3C (2017), https://www.w3.org/TR/shacl/
24. Kontokostas, D., Westphal, P., Auer, S., Hellmann, S., Lehmann, J., Cornelissen, R., Zaveri, A.: Test-driven Evaluation of Linked Data Quality. In: Proceedings of the 23rd International Conference on World Wide Web. pp. 747–758. WWW ’14, ACM (2014)
25. Larrib, M., Escribano, M., Cerrada, C., Escribano, J.J.: Converting OCL and CGMES Rules to SHACL in Smart Grids. IEEE Access 8, 177255–177266 (2020)
26. Leinberger, M., Seifer, P., Rienstra, T., Lämmel, R., Staab, S.: Deciding SHACL Shape Containment through Description Logics Reasoning. In: The Semantic Web – ISWC 2020. Springer International Publishing (2020), (this volume)
A Review of SHACL

27. Leinberger, M., Seifer, P., Schon, C., Lämmel, R., Staab, S.: Type Checking Program Code Using SHACL. In: The Semantic Web – ISWC 2019. pp. 399–417. Springer International Publishing, Cham (2019)

28. Lieber, S., De Meester, B., Heyvaert, P., Brickmann, F., Wambacq, R., Mannens, E., Verborgh, R., Dimou, A.: Visual Notations for Viewing and Editing RDF Constraints with UnSHACLed. Semantic Web (2021), under review

29. Lieber, S., Dimou, A., Verborgh, R.: Statistics about Data Shape Use in RDF Data. In: ISWC (Demos/Industry) (2020)

30. Marnette, B.: Generalized schema-mappings: from termination to tractability. In: Proc. of the twenty-eighth ACM SIGMOD-SIGACT-SIGART symp. on Principles of database systems. pp. 13–22. ACM (2009)

31. Musen, M.A., Protégé Team: The Protégé Project: A Look Back and a Look Forward. AI matters 1(4), 4—12 (June 2015)

32. Nenadić, K.R., Gavrić, M.M., Đurđević, V.I.: Validation of CIM datasets using SHACL. In: 2017 25th Telecommunication Forum (TELFOR). pp. 1–4 (2017)

33. Omran, P.G., Taylor, K., Mendez, S.R., Haller, A.: Learning SHACL Shapes from Knowledge Graphs. Semantic Web (2021), under review

34. Pandit, H.J., O’Sullivan, D., Lewis, D.: Using Ontology Design Patterns To Define SHACL Shapes. In: 9th Workshop on Ontology Design and Patterns (WOP2018), International Semantic Web Conference (ISWC). pp. 67–71 (2018)

35. Pandit, H.J., O’Sullivan, D., Lewis, D.: Test-Driven Approach Towards GDPR Compliance. In: Semantic Systems. The Power of AI and Knowledge Graphs. pp. 19–33. Springer International Publishing, Cham (2019)

36. Pareti, P., Konstantinidis, G., Mogavero, F.: Satisfiability and Containment of Recursive SHACL (2021), arXiv preprint 2108.13063

37. Pareti, P., Konstantinidis, G., Mogavero, F., Norman, T.J.: SHACL Satisfiability and Containment. In: The Semantic Web – ISWC 2020. pp. 474–493. Springer International Publishing, Cham (2020)

38. Pareti, P., Konstantinidis, G., Norman, T.J., Şensoy, M.: SHACL Constraints with Inference Rules. In: The Semantic Web – ISWC 2019. Springer International Publishing (2019)

39. Robinson, R.: Undecidability and Nonperiodicity for Tilings of the Plane. IM 12, 177–209 (1971)

40. Savković, O., Kharlamov, E., Lamparter, S.: Validation of SHACL Constraints over KGs with OWL 2 QL Ontologies via Rewriting. In: The Semantic Web. pp. 314–329. Springer International Publishing, Cham (2019)

41. Schaffenrath, R., Proksch, D., Kopp, M., Albasini, I., Panasiuk, O., Fensel, A.: Benchmark for Performance Evaluation of SHACL Implementations in Graph Databases. In: Rules and Reasoning. pp. 82–96. Springer International Publishing, Cham (2020)

42. Seaborne, A., Harris, S.: SPARQL 1.1 query language. W3C recommendation, W3C (Mar 2013), https://www.w3.org/TR/2013/REC-sparql11-query-20130321/

43. Šenkýř, D.: SHACL Shapes Generation from Textual Documents. In: Enterprise and Organizational Modeling and Simulation. pp. 121–130. Springer International Publishing, Cham (2019)

44. Spahiu, B., Maurino, A., Palmonari, M.: Towards Improving the Quality of Knowledge Graphs with Data-driven Ontology Patterns and SHACL. In: International Semantic Web Conference ISWC Sattelite Events. pp. 103–117 (2018)

45. Spahiu, B., Porrini, R., Palmonari, M., Rula, A., Maurino, A.: ABSTAT: Ontology-Driven Linked Data Summaries with Pattern Minimalization. In: The Semantic Web. pp. 381–395. Springer International Publishing, Cham (2016)
46. Stani, E.: Design reusable SHACL shapes and implement a linked data validation pipeline. Code4Lib Journal 45 (2019)

47. Stolk, S., McGlinn, K.: Validation of IfcOWL datasets using SHACL. In: Proceedings of the 8th Linked Data in Architecture and Construction Workshop. pp. 91–104 (2020)

48. Thornton, K., Solbrig, H., Stupp, G.S., Labra Gayo, J.E., Mietchen, D., Prud’hommeaux, E., Waagmeester, A.: Using Shape Expressions (ShEx) to Share RDF Data Models and to Guide Curation with Rigorous Validation. In: The Semantic Web. pp. 606–620. Springer International Publishing, Cham (2019)

49. W3C OWL Working Group: OWL 2 Web Ontology Language Document Overview (Second Edition). W3C recommendation, W3C (Dec 2012), https://www.w3.org/TR/2012/REC-owl2-overview-20121211/

50. Wang, H.: Proving Theorems by Pattern Recognition II. BSTJ 40, 1–41 (1961)

51. Wright, J., Rodríguez Méndez, S.J., Haller, A., Taylor, K., Omran, P.G.: Schimatos: A SHACL-Based Web-Form Generator for Knowledge Graph Editing. In: The Semantic Web – ISWC 2020. pp. 65–80 (2020)