Model Estimates of Circulating Flow around Objects in the Marine Medium and Atmosphere

1,2Korchagin, N.N., and 1,2Vladimirov, I.Yu.

1 Shirshov Institute of Oceanology of the Russian Academy of Sciences, Moscow
2 Bauman Moscow State Technical University

e-niknik@mail.ru

Abstract. Model studies of force action on streamlined fragments of technical structures by a stratified sea current have been carried out. Analytical expressions are obtained for the hydrodynamic load on a point dipole during its circulation flow around a two-layer fluid flow of finite depth. The dependences of the wave resistance and the lifting force on the velocity of the current and circulation are investigated. It is shown that taking into account the velocity circulation can significantly affect the force effect on the dipole. The effect of a sharp (reversible) change in the direction of lift in a relatively narrow range of the flow velocity of a pipeline fragment simulated by a dipole has been found. The effect can also appear when moving self-propelled underwater objects and aircraft in the atmosphere.

1. Statement of the problem and its solution

Viscous drag acts on a streamlined obstacle in a stratified flow. In addition, it is additionally experiencing power effects due to the development of internal waves. An example of such effects is suspended suspension flows occurring in the bottom layers of the sea when the flow is mixed with micro- and fine-grained soil particles, which leads to an excess flow density (relative to the surrounding water) and the formation of a sharper pycnocline at its boundary [1]. The compaction of pycnocline leads to an increase in internal waves and a corresponding increase in the dynamic load on the streamlined technical object.

Of particular interest are the force effects arising from the flow around an obstacle localized near the density jump layer. At the same time, the difference in its localization relative to the density jump boundary: above or below the boundary of such a layer is important. In addition, in the general case, the flow around an obstacle can occur with a certain circulation, which additionally introduces unaccountable force effects of the medium on the object, and which can be significant.

In general, the problem statement by the authors was considered in [3]. Below (Fig. 1) there is a flow diagram of a dipole that simulates the elements of a pipeline localized under a density jump (task 1) and above a jump (task 2).

![Flow diagram of a dipole](image-url)

Fig.1. The flow around a pipeline element simulated by a dipole
The thickness of the upper layer $H_1$, lower - $H_2$ density, respectively, $\rho_1$ and $\rho_2$, $\rho_1 > \rho_2$. The origin of coordinates is located on the unperturbed boundary between the fluid layers, the $x$ axis is directed along this boundary, and the $y$ axis is vertically upwards. Dipole and vortex are combined in points $(0, \mp h)$. Note that density jumps are widespread in sea waters almost from the surface to the bottom. In general, they determine the structure of stratified waters, in the region of which internal waves arise. Therefore, the parameter $\varepsilon = (\rho_2 - \rho_1) / \rho_2$ can be defined as the energy characteristic of the marine environment.

We look for the solution of the model problem in the format of the complex-conjugate velocity of a two-layer flow disturbed by a dipole in the form $\mu_k(z) = V + U_k(z)$, where $z = x + iy$, $U_k(z)$ are the velocity disturbances, $k = 1, 2$. In order to universalize the problem and generalize the model parameters of the dipole, a transition is made to dimensionless variables

$$X = \frac{x}{R}, \ Y = \frac{y}{R}, \ Z = \frac{z}{R} = X + iY, \ \delta = \frac{\rho_1}{\rho_2}, \ h_1 = \frac{H_1}{R}, \ h_2 = \frac{H_2}{R}$$

$$h = \frac{H}{R}, \ u_k = \frac{U_k}{V}, \ \gamma = \frac{\gamma}{2\pi VR}, \ E = \frac{gR}{V^2} = \frac{1}{\text{Fr}^2}$$

Here $g$ is the acceleration due to gravity, $\text{Fr}$ is the Froude number along the cylinder radius. Then, in the problem of perturbations of the complex-conjugate velocity $u_k$ introduced by a dipole and a vortex under a density jump, it is required to find $u_1(Z)$ and $u_2(Z)$ with boundary conditions:

$$\text{Im} \left[ t \frac{du_1}{dZ} - Eu_1 \right] = 0 \text{ at } Y = h_1 \tag{1.1}$$

$$\delta \text{Im} \left[ t \frac{du_1}{dZ} - Eu_1 \right] = \text{Im} \left[ t \frac{du_2}{dZ} - Eu_2 \right] = 0 \text{ at } Y = 0 \tag{1.2}$$

$$\text{Im} \ u_1 = \text{Im} \ u_2 \tag{1.3} \text{ at } Y = 0$$

$$\text{Im} \ u_2 = 0 \tag{1.4} \text{ at } Y = -h_2$$

so that $u_1(Z)$ is regular function in the band $-\infty < X < +\infty$, $0 < Y < h_1$ and $u_2(Z)$ in the band $-\infty < X < +\infty$, $-h_2 < Y < 0$ everywhere except for point $Z = -ih$, at which it has a second-order pole:

$$u_2(Z) = -\frac{1}{(Z + ih)^2} + \frac{\gamma}{i} \frac{1}{Z + ih} + f(Z), \ f(Z) \text{ is a regular function.}$$

The boundary condition (1.1) describes the constancy of pressure on the free surface, (1.2) – the continuity of pressure at the interface of the layers, (1.3) determines the absence of fluid flow through this boundary, and (1.4) is the condition of no-flow at the bottom (derivation of (1.1)–(1.4) see in [3]).

After some transformations using the Chaplygin formula [3] we find the coefficient of the complex-conjugate hydrodynamic load on the cylinder $\Delta C^* = C_x - i C_y = \frac{F_x - i \Delta F_y}{\rho_2 V^2 R}$ added to the generalized force of Zhukovsky, where $F_x$ is the wave drag, $\Delta F_y$ is the additional lifting force of the dipole (dimensional)

$$C_x = 2\pi \sum_{n, j} \text{Re} \left\{ -(\gamma + \xi) \left[ \xi C_n(\xi) + \gamma C_j(\xi) \right] e^{i\theta} + (\gamma - \xi) \left[ \xi D_n(\xi) + \gamma D_j(\xi) \right] e^{-i\theta} \right\}$$

$$\Delta C_y = 2\pi \int_0^\infty \left\{ (\gamma + \xi) \left[ \xi C_n(\xi) + \gamma C_j(\xi) \right] e^{i\theta} + (\gamma - \xi) \left[ \xi D_n(\xi) + \gamma D_j(\xi) \right] e^{-i\theta} \right\} d\xi$$

(1.5)
According to [5], the curves on different sides of the jump in the nature of variability are opposite to each other. The density in the upper layer differs significantly from each other both in form and in values. In this case, the force effects on the dipole on the relative density drop they are the main parameters of the problem under consideration.

The integral in (1.5) is meant in the sense of the Cauchy principal value, and the residues are taken over all \( s \) poles \( \xi_j \) of the corresponding function located on the positive real axis. From (1.6) it is clear that these poles are the positive roots of the equation

\[
\xi^2 + [\delta \xi^2 + (1-\delta)E^2] \text{th} \xi_1 \text{th} \xi_2 - \xi E (\text{th} \xi_1 + \text{th} \xi_2) = 0.
\]

Note that the expression for the lift coefficient does not include the force of Archimedes.

In a similar formulation, problem 2 on the hydrodynamic reaction in a circulating flow around a cylinder simulated by a point dipole located in the upper layer of the flow (above the jump). Then, omitting the cumbersome calculations (see [5]), we numerically calculate the final results of the model with their presentation in graphical form.

2. Calculations of the force action

Calculations of the force action on a streamlined cylinder were carried out for real values of the characteristics of the marine medium using the formulas

\[
\Delta F_y = \rho \frac{V^2}{2} \Delta C_y \quad \text{(lower layer)},
\]

\[
\Delta F_y = \rho \frac{V^2}{2} R \Delta C_y \quad \text{(upper layer)},
\]

where the coefficients \( \Delta C_y \) are expressed through (1.5), and also (2.2) from [5]. The density in the upper layer is \( \rho_1 = 1024 \, \text{kg/m}^3 \), and the relative density difference between the lower and upper layers varies in the range \( \rho_2 / \rho_1 = 1.001 - 1.005 \). In the bottom layer, the differential \( \rho_2 / \rho_1 \) may vary within 1.01 - 1.05, which corresponds to the suspended flow in the lower layer. In the first case, this is a relatively weak weighing stream, in the second, a turbulent flow, which manifests itself near the bottom on the slopes of submarine elevations [2]. The total thickness of the flow is 60m, the thickness of the upper layer \( H_1 = 50 \) m, the radius of the cylinder (pipeline) \( R = 0.71 \) m. The dipole is localized in the lower or upper layer at a distance \( h = 2 \) m from the interface between the layers. The dimensionless circulation was defined as \( \gamma = \Gamma / (2\pi VR) \), where \( V \) is the flow velocity, \( \Gamma \) is the dimensional circulation. Note that by virtue of the equivalence of positive and negative vorticities in the flow, when analyzing the variability of the lifting force, one may choose any of \( \psi^+ : \Delta F_y^+ \) for positive, and \( \Delta F_y^- \) for negative circulation.

Analysis of model solutions in [4, 5] showed that the amplification of the effect of flow on an obstacle is due to an increase in the power of the density jump \( \varepsilon \), and together with the distance \( h \) they are the main parameters of the problem under consideration. Figure 2 shows the dependences of the force effects on the dipole on the relative density drop \( \varepsilon \).

In Fig. 2, curves with unidirectional positive circulations \( \Delta F_y^+ \) depending on localization with respect to a density jump differ significantly from each other both in form and in values. In this case, the curves on different sides of the jump in the nature of variability are opposite to each other. According to [5], the differences in the graphs of \( \Delta F_y^- \) shown in Fig. 2 and constructed close to, but on different sides of the interface between the layers are caused by different directions of circulation,
which in some cases leads to an increase in the flow velocity (between the jump and the cylinder), in others, to its decrease.

Thus, the power loads, additionally arising in the presence of circulation, act in different directions along the section of the pipeline. In this case, despite their insignificance, but acting for a long time, they can lead to the deformation of the pipeline, and in the future – to its possible damage. Therefore, these effects must be considered when laying underwater communications and pipelines. It should be noted that changes in the lift force identified in the model, depending on the circulation $\gamma$, were not previously taken into account in such studies.

In continuation of the analysis of the results of the model problem, it is of interest to investigate a characteristic feature in the variability of lift and its corresponding interpretation. In this connection, we will move from the above-considered motionlessly streamlined point model to a kinematically similar physical model of a steadily moving object. Without disturbing the generality of the model problem under consideration, a limited fragment of the pipeline can be represented as a cigar-shaped (or ovoid) body, in the form of a simulating a certain type of self-propelled underwater object. At the same time, in order to adequately compare the design (geometry) of a model ovoid with the original point model, it is necessary to separate the lateral component of the force vector perpendicularly distributed along the body length of the physical model as an impact on the underwater object.

On the basis of such a transformation of the original model, it is hypothetically possible to use the calculations of the dependence of the lifting force of the dipole located below the density jump on the flow velocity – Fig.3. Here, the dipole models the flow around an ovoid-shaped underwater object and clearly represents reversing changes in the direction of lifting force $\Delta F^\gamma (V)$ at the corresponding intervals of velocity of flow around the object.

In order to substantiate the possibility of a hypothetical transformation of a point model into the corresponding form of an ovoid object, one can refer to a typical example from maritime practice. Thus, when an object moves in a marine environment, situations arise regarding the rapid and dangerous submergence of an underwater object ten or more meters deep [5]. This effect is usually explained by the presence of internal waves generated at jumps in the density of the medium. However, the parameters of the internal waves, depending on the power of the density jump $\varepsilon$ (with a minimum period of about ten minutes and a length of hundreds of meters), reflect the slower dynamics of the internal wave compared to the relatively sharp lowering of the object. In this case, a similar effect may occur either when the direction of movement of the object changes with the appearance of a lateral component of the flow, or by variations in the direction of the flow itself (also with a selected side component to the object body). And as the calculations showed, the selection of the lateral component of the flow with non-zero circulation can lead to the appearance of an additional force directed vertically downwards – Fig. 3.

A similar effect during the circulation flow around an object can also manifest itself in the atmosphere, for example, during the flight of an airplane. In this case, the configuration (geometry) of such an object as a whole is also represented in the form of an ovoid. Then the airflow around it can lead to a reverse change in lift, which may be the cause of an unexpectedly sharp and dangerous decrease in the height of the aircraft. Therefore, the use here of model results (albeit hypothetical) in the study of the flight of an aircraft appears to be legitimate.

In addition, the equations of hydro- and aerodynamics (without taking into account the compressibility of air in the latter) are identical. And in the case of a flow around at a speed of no more than 1/5 of the speed of sound (or a flight speed of up to 250-300 km/h), the presence of air compressibility can be completely neglected [6]. However, here, as for the underwater object, the calculated model parameters are also taken hypothetically and therefore should be considered only as qualitative assessments. And such estimates are significant. Therefore, in reality, we can expect the manifestation of such effects.

Figure 4 clearly shows the relatively powerful force action on the aircraft, which occurs when it is landing with a decrease in flight speed within 300-400 km/h. At the same time, a sharp increase in the
module of vertical force $\Delta F_y^\pm$ for all $\gamma_i$ in the graphs of Fig. 4 indicates a sharp decrease (dip) in the airplane's trajectory.

As a result, the total impact of forces of the same nature, but applied to different parts of the aircraft — wings, fuselage elements, etc. can, depending on their direction, lead to sharp vertical fluctuations in the flight path, which is not uncommon in flight practice.

The authors wish to thank A.T. Il’ichev for fruitful discussions of the model and calculation results. The work was carried out within the framework of the State Program of IORAS (theme № 0149-2019-0004).

References

[1] Zhmur, V.V., and Sapov, D.A. Catastrophic gravitational suspension-carrying currents in the near-bottom layer of the ocean, in: World Ocean. Vol. 1. Ocean Geology and Tectonics. Catastrophic Phenomena in the Ocean, Moscow: Nauchnyi Mir, 2013, pp. 499–524.

[2] Loytsianskii L.G. Mechanics of Liquids and Gases, Oxford: Pergamon Press, 1966.

[3] Vladimirov, I.Yu., Korchagin, N.N., and Savin, A.S. Surface disturbances in stratified finite-depth flow past obstacles, Okeanologiya, 2012, vol. 52, no. 6, pp. 825–835.

[4] Vladimirov, I.Yu., Korchagin, N.N., and Savin, A.S. Wave impact of a suspension-carrying stream on an obstacle in flow, Dokl. Ross. Akad. Nauk, 2015, vol. 461, no. 2, pp. 223–227.

[5] Vladimirov, I.Yu., Korchagin, N.N., and Savin, A.S. Wave reaction of a dipole in a circulatory double-layer flow of finite depth, Fluid Dynamics, 2016, vol. 51, no. 2, pp. 214–223.

[6] Sedov, L.I. Mekhanika sploshnoi sredy (Continuum Mechanics), vol. 2, Moscow: Nauka, 1973.

Figure captions

Fig.1. The flow around a pipeline element simulated by a dipole.

Fig.2. Lift force of a dipole as a function of the relative density drop between the layers; model of a pipeline. The dipole is located above (a) and beneath (b) the density jump at $V = 0.7$ m/s: curves 1–4 correspond to $\gamma_i = 0, 0.1, 0.2$, and 0.3, respectively.

Fig.3. Lift force of a dipole located above the density jump as a function of the flow velocity for $\rho_2/\rho_1 = 1.005$, $H_1 = 100$ m, $H_2 = 200$ m, $h = 6$ m, and $R = 3.5$ m: model of the underwater object: curves 1–4 correspond to $\gamma = 0.0, 0.1, 0.2$, and 0.3, respectively. Black arrows indicate the intervals with the reversible variation in curves $\Delta F_y^\pm (V)$.

Fig.4. Lift force of a dipole located beneath the density jump layer as a function of the flow velocity for $\rho_2 = 1.26$ kg/m$^3$, $\rho_2/\rho_1 = 1.3$, $H_1 = 1000$ m, $H_2 = 2000$ m, $h = 6$ m, and $R = 3.5$ m: curves 1–4 correspond to $\gamma = 0.0, 0.1, 0.2$, and 0.3, respectively.
Fig. 2.

Fig. 3.

Fig. 4.