Research Article

Investigation on the Effect of Probability Distribution on the Dynamic Response of Liquefiable Soils

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Received 15 October 2021; Accepted 19 November 2021; Published 28 December 2021

Academic Editor: Ping Xiang

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This study investigated the effect of different probabilistic distributions (Lognormal, Gamma, and Beta) to characterize the spatial variability of shear modulus on the soil liquefiable response. The parameter sensitivity analysis included the coefficient of variation and scale of fluctuation of soil shear modulus. The results revealed that the distribution type had no significant influence on the liquefaction zone. In particular, the estimation with Beta distribution is the worst scenario. It illuminated that the estimation with Beta distribution can provide a conservative design if site investigation is absent.

1. Introduction

It is now well recognized that natural soil properties exhibit spatial variability because of depositional and postdepositional processes. The inherent variability in soil properties has found its place in geotechnical design and has been extensively incorporated in the analysis of slope stability [1–5], foundation bearing capacity [6, 7], foundation settlement [8–10], and liquefaction [11–13]. A lognormal distribution has been generally accepted in a geotechnical reliability analysis [14–16] because of its capability to model the randomness of positive soil parameters.

Recent studies proved that different distributions impacted the stochastic properties of soil. Popescu et al. [17] and Jimenez and Sitar [18] performed a series of random finite element analyses with different probability distributions of soil parameters, which has significant effects on the foundation settlement and bearing capacity. Most recently, Wu et al. [19] applied the random finite element method to investigate the effect of different probabilistic distributions on the tunnel convergence and demonstrated the mechanisms of tunnel convergence and the probability of exceeding liquefaction thresholds with different probabilistic distribution types. To date, publications on the application of random field theory to soil dynamic behavior are limited and the impact of probabilistic distribution on the soil liquefiable response has not been clearly defined. Wang et al. [20] investigated the liquefaction response of soil using the spatial variability of the shear modulus by considering different values of the coefficient of variation and the horizontal scale of fluctuation.

In this study, we performed the nonlinear dynamic simulation of the liquefiable response of a sand layer with the water table 1 m below the ground level under a seismic load using the finite difference program FLAC3D. The finite difference mesh configuration is shown in Figure 1. The soil domain had a length of 40 m and a height of 10 m, a liquefiable layer of 9 m, and a nonliquefiable layer of 1 m. The Mohr–Coulomb model and the Finn model were used to simulate the nonlinear soil behavior and the accumulation of the pore pressure in sand before the liquefaction triggered by a dynamic load, respectively. The Finn model can consider variations of the volumetric strain and display the increase in excess pore pressure. The relationship between variations of
volumetric strain increment ($\Delta \varepsilon_{vd}$) and cycle shearing strains ($r$) was defined as follows:

$$\Delta \varepsilon_{vd} = C_1 (r - C_2 \varepsilon_{vd}) + \frac{C_3 \varepsilon_{vd}^2}{r + C_4 \varepsilon_{vd}},$$

(1)

where $C_1, C_2, C_3$, and $C_4$ are constant coefficients that can be obtained from cyclic triaxial tests. Following the study of Azadi and Hosseini [21], the four values were selected as 0.79, 0.52, 0.2, and 0.5, respectively. Table 1 summarizes the soil parameters in the constitutive model for the deterministic analysis. In the dynamic analysis, the boundaries were used to absorb the reflected waves and enforce the discrete half-space conditions of the numerical model. The free-field boundaries were adopted for the right and left boundaries to simulate the half-space condition. The seismic loading in the horizontal direction was applied at the bottom boundary which was assumed to be rigid.

In the stochastic analyses, the shear modulus $G$ was assumed to be random variable and generated with the spectral representation method, which was recently developed by Shu et al. [7]. Three probabilistic distributions, including lognormal, Beta, and Gamma, were applied to model the spatial variability of $G$, with a mean $\mu_G = 20$ MPa and $\text{CoV}_G = 0.3$ and 0.5. A 2D exponential correlation function [22] was adopted with the horizontal and vertical spatial correlation lengths $\delta_x = 6$ and 60 m and $\delta_y = 6$ m, respectively. 200 sets of Monte Carlo realizations were undertaken for each combination of a distribution type, a scale of fluctuation, and a CoV$_G$.

### 2. Results and Discussion

#### 2.1. Area of Liquefied Zone $A_{80}$

Figure 2 presents that the irregular dynamic liquefaction results were irregular, considering the results of the non-Gaussian probability distributions. However, in general, the differences among the calculated results from the three probability distributions became more obvious with the increase in CoV$_G$. Figure 2 presents that the irregular dynamic liquefaction results from different probability distributions in terms of the residual $A_{80}$ and the response of $A_{80}$ to instantaneous seismic loading. Additionally, the increase in CoV$_G$ can amplify the impact from the probability distribution.

### Table 1: Summary of soil parameters.

| Parameters | Value |
|------------|-------|
| Shear modulus, $G$: MPa | 20 |
| Total unit weight, $\gamma$: kN/m$^3$ | 26.6 |
| Poisson’s ratio, $\nu$ | 0.35 |
| Permeability coefficient, $k$: m/s | $2.64 \times 10^{-4}$ |
| Porosity, $N$ | 0.435 |

peak $\mu_{A_{80}}$ by the Beta distribution was smaller than that by the Lognormal and Gamma distributions with CoV$_G = 0.3$ and CoV$_G = 0.5$ (Figures 2(b)–2(f)). In addition, the greatest peak $\mu_{A_{80}}$ was correlated with the $G$ conformed to the Lognormal distribution in these cases.

For the perspective of the decreasing rate of $A_{80}$, the residual of $A_{80}$, and the sensitivity of $A_{80}$ to instantaneous seismic loading, it was found that the influence of the different probability distributions on the dynamic liquefaction results was irregular, considering the results of the non-Gaussian probability distributions. However, in general, the differences among the calculated results from the three probability distributions became more obvious with the increase in CoV$_G$. Figure 2 presents that the irregular dynamic liquefaction results from different probability distributions, in terms of the residual $A_{80}$ and the response of $A_{80}$ to instantaneous seismic loading. Additionally, the increase in CoV$_G$ can amplify the impact from the probability distribution.

#### 2.2. Excess Pore Water Pressure Ratios $Q$

The liquefaction index was calculated from the mean excess pore water pressure ratio in the horizontal direction and of the form for one simulation:

$$Q(z, t) = \frac{1}{n} \sum_x r(x, z, t),$$

(3)

$$r(x, z, t) = \frac{\mu(x, z, t)}{\sigma_{00}}$$

(4)

where $x$ and $z$ are the horizontal and vertical coordinates of the central point of one finite difference element, respectively; $r(x, z, t)$ is the excess pore water pressure ratio at the central point $(x, z)$ at $t$-th second after the earthquake; $\sigma_{00}$ is the initial effective stress in the vertical direction; $n$ was set to be 40 in this study, which represents the element number of
Figure 2: Continued.
the finite difference model in the horizontal direction; and 
Q(\(z, t\)) reflects the average excess pore water pressure ratio at
the depth of \(z\) at \(t\)-th second after the earthquake. Due to the
fact that large accumulation of pore water pressure might
occur in the deep soil layer, this study paid close attention to
the variation of \(Q\) at \(z = 7.25\) m.

Figure 3 presents the time history curves of the mean
excess pore water pressure ratio (\(\mu_q\)) at \(z = 7.25\) m with
different probabilistic distributions. In the case with
\(\text{CoV}_G = 0.1\) and \(\delta_e = 6\) m or 60 m, \(\mu_q\) with varying types of
distribution presents a similar trend with the seismic load
imposed to the soil layer (Figures 3(a) and 3(d)). However,
with the increased \(\text{CoV}_G\), the rebound amplitude of \(\mu_q\) with
different probabilistic distributions gradually declined
(Figures 3(b)–3(f)). In addition, \(\mu_q\) with the Beta dis-
bution was slightly smaller than that with the Lognormal
and Gamma distributions when the \(\text{CoV}_G\) is 0.3 and 0.5,
and its dissipation rate of excess pore water pressure (in
terms of the slope of the descending part of \(\mu_q\) curve) was
smaller than that with the Lognormal distribution and
Gamma distribution.

Figure 3 also shows that \(\mu_q\) dissipated after the occur-
rence of the peak \(\mu_q\) appeared around \(t = 4\) s. Table 2
summarizes \(\mu_G\) with shear modulus by different distributions
after 7 s and 35 s occurrence of the earthquake. Table 3
tabulates the dissipation rate of pore water pressure, which
was depicted from the descending section of the \(\mu_q\) time
history curve. In general, the dissipation rate of \(\mu_q\) increased
with the increase in \(\text{CoV}_G\) and this increasing trend was
affected by the distribution type. For instance, the ampli-
fication was 16.11% for the Beta distribution as \(\text{CoV}_G\) ex-
tended from 0.1 to 0.5, which was greater compared with
that with the Gamma and Lognormal distributions.

2.3. Ground Displacement \(D\). In this section, the maximum
surface ground horizontal movement \((D_x(t)_{\text{max}})\) in Equation
(4) and settlement \((D_z(t)_{\text{max}})\) in Equation (5) were taken to
evaluate the influence of liquefaction by earthquake,
respectively:

\[
D_x(t)_{\text{max}} = \left[ D_{x,z=0} (t) - D_{x,z=10} (t) \right]_{\text{max}},
\]

\[
D_z(t)_{\text{max}} = \left[ D_z (t)_{\text{max}} - D_z (t)_{\text{min}} \right]_{\text{max}},
\]

where \(D_{x,z=0}(t)\) and \(D_{x,z=10}(t)\) represent the horizontal dis-
placements at surface and bottom at the horizontal coor-
dinate \(x\) in the soil domain and \(D_z(t)_{\text{max}}\) and \(D_z(t)_{\text{min}}\) represent the maximum and minimum settlement at \(t\)-th
second after the earthquake.

Figure 4 plots the time history curve of mean ground
horizontal displacement (\(\mu_{D_x}\)) with different probabilistic
distributions. Similar time history curves of \(\mu_{D_z}\) for different
distributions were obtained provided \(\text{CoV}_G = 0.1\), indicating
that the distribution types had little influence on the ground
horizontal displacement (Figures 4(a) and 4(d)). The diff-
ences between \(\mu_{D_x}\) became pronounced with the increase in
\(\text{CoV}_G\). It was worth noting that \(\mu_{D_x}\), with the Beta dis-
bution was always the largest, while \(\mu_{D_z}\), with Lognormal
distribution was the smallest.

As expected, the probability distributions also had
certain impact on the standard deviation of the horizontal
displacement (\(\mu_{D_x}\)) (Figure 5). The difference of \(\mu_{D_x}\) gradu-
ally increased with the increase in \(\text{CoV}_G\), referring that the
effect of different distribution on \(\mu_{D_z}\) enhanced with the
increase in \(\text{CoV}_G\). If \(\text{CoV}_G\) is 0.3 or 0.5, it was obvious that
\(\mu_{D_x}\) with the Beta distribution was always greater than that
with the Lognormal and Gamma distributions.
Figure 3: Continued.
Figure 6 shows the impact of CoV\(_G\) on \(\mu_{Dx}\) with different distributions at \(t = 35\) s. In general, an increase in CoV\(_G\) was corresponding with the increase in \(\mu_{Dx}\) with different distributions. The obtained \(\mu_{Dx}\) with Beta distribution was greater than that with the Gamma and Lognormal distributions. Moreover, a larger \(\delta_x\) correlated with a greater \(\mu_{Dx,max}\) provided with the same distribution and CoV\(_G\) (Figures 6(a) and 6(b)). This finding highlighted that the worst scenario was covered by the shear modulus with Beta distribution. It illuminated that the estimation with Beta distribution is capable to provide a conservative evaluation if site investigation is absent.

The time history curves of mean and standard deviation of settlement (\(\mu_{Dz}\) and \(\sigma_{Dz}\)) are shown in Figures 7 and 8, respectively. The influences of the distributions of \(G\) on \(\mu_{Dz}\) and \(\sigma_{Dz}\) were similar to those on \(\mu_{Dx}\) and \(\sigma_{Dx}\). The differences between \(\mu_{Dz}\) and \(\sigma_{Dz}\) with different distributions were insignificant if CoV\(_G\) = 0.1, which implied that the distributions had small impact on the settlement. Nonetheless, the difference was positively correlated with CoV\(_G\). For example,
Figure 4: Continued.
Figure 4: Time history curve of mean ground horizontal displacement ($\mu_{Dx}$) with different distributions: (a) $\delta_x = 6$ m and $\text{CoV}_G = 0.1$; (b) $\delta_x = 6$ m and $\text{CoV}_G = 0.3$; (c) $\delta_x = 6$ m and $\text{CoV}_G = 0.5$; (d) $\delta_x = 60$ m and $\text{CoV}_G = 0.1$; (e) $\delta_x = 60$ m and $\text{CoV}_G = 0.3$; (f) $\delta_x = 60$ m and $\text{CoV}_G = 0.5$.

Figure 5: Continued.
with a given $\delta_x$ of 6 m and $\text{CoV}_G = 0.3$, $\mu_{D_z}$ with the Beta distribution was 7.81% and 7.28% larger than that with Gamma distribution and Lognormal distribution. If $\text{CoV}_G$ was 0.5, the results enhanced 20.51% and 35.89%, respectively. It addresses the conclusion that the influence of the distributions on $\mu_{D_z}$ and $\sigma_{D_z}$ increased with the increase in $\text{CoV}_G$. The settlement obtained from the random fields obeying Beta distribution was greater and more dispersive than that calculated by the Lognormal distribution and Gamma distribution.

Figure 9 shows the relationship between $\mu_{D_z, \text{max}}$ and $\text{CoV}_G$. The distribution type had a similar impact on
Figure 6: Variation of $\mu_{D_{x,\text{max}}}$ as a function of $\text{CoV}_G$: (a) $\delta_x = 6\, \text{m}$ and (b) $\delta_x = 60\, \text{m}$.

Figure 7: Continued.
\( \mu_{Dz, \text{max}} \), which is consistent with the findings in Figure 6. A greater \( \mu_{Dz, \text{max}} \) was corresponding with a greater CoV\(_G\) for all three distributions. Comparing between Figures 9(a) and 9(b), it can be observed that small \( \delta_x \) corresponded to large \( \mu_{Dz, \text{max}} \), except for the case of CoV\(_G\) = 0.3 under the Beta distribution.
Figure 8: Continued.
3. Concluding Remarks

In this study, the influence of the probability distributions of soil shear modulus on the area of liquefaction zone, the ratio of excess pore water pressure, and the ground displacement is investigated. The following conclusions can be drawn:

1. The probability distribution type of shear modulus had no significant influence on the reduction rate of liquefaction zone and the sensitivity of the liquefaction zone to the instantaneous seismic load.

2. Compared with the Lognormal distribution and Gamma distribution, a smaller excess pore water pressure ratio could be observed with the Beta distribution employed. The pore water pressure dissipation rate accelerated with the increase in \( \text{CoV}_G \) and was affected by the distribution type.

3. Regarding the ground movement, the estimated horizontal displacement and settlement with Beta distribution were the worst scenario. The sensitivity of the ground horizontal displacement and

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**Figure 8:** Time history curve of \( \sigma_z \) with different distributions: (a) \( \delta_x = 6 \text{ m} \) and \( \text{CoV}_G = 0.1 \); (b) \( \delta_x = 6 \text{ m} \) and \( \text{CoV}_G = 0.3 \); (c) \( \delta_x = 6 \text{ m} \) and \( \text{CoV}_G = 0.5 \); (d) \( \delta_x = 60 \text{ m} \) and \( \text{CoV}_G = 0.1 \); (e) \( \delta_x = 60 \text{ m} \) and \( \text{CoV}_G = 0.3 \); (f) \( \delta_x = 60 \text{ m} \) and \( \text{CoV}_G = 0.5 \).

**Figure 9:** Variation of \( \mu_{D_{z,\text{max}}} \) as a function of \( \text{CoV}_G \): (a) \( \delta_x = 6 \text{ m} \) and (b) \( \delta_x = 60 \text{ m} \).
settlement to \( \text{CoV}_G \) decreased successively for Beta, Gamma, and Lognormal distribution.

**Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

**Acknowledgments**

The support by the National Natural Science Foundation of China (Grant no. 51808490) is greatly acknowledged.

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