Directed transport born from chaos in asymmetric antidot structures

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(Dated: September 27, 2005)

It is shown that a polarized microwave radiation creates directed transport in an asymmetric antidot superlattice in a two dimensional electron gas. A numerical method is developed that allows to establish the dependence of this ratchet effect on several parameters relevant for real experimental studies. It is applied to the concrete case of a semidisk Galton board where the electron dynamics is chaotic in the absence of microwave driving. The obtained results show that high currents can be reached at a relatively low microwave power. This effect opens new possibilities for microwave control of transport in asymmetric superlattices.

PACS numbers: 05.45.Ac, 05.60.-k, 72.40.+w

INTRODUCTION

The appearance of a directed transport induced by radiation in asymmetric systems is known as the photogalvanic effect. By this effect the radiation creates charge transport in the bulk of the asymmetric structure in absence of any applied dc-voltage. The theoretical investigations of this phenomena have been started almost 30 years ago in Refs. [1, 2]. The interest to this subject has been renewed recently with the studies of ratchets that appear when a system is displaced from thermal equilibrium by a periodic variation of system parameters (for reviews see Refs. [3, 4]). One of surprising properties of ratchets is that directed transport can emerge in presence of a zero mean force. This phenomenon has a generic origin and appears in various physical systems including vortices in Josephson junction arrays, cold atoms, macroporous silicon membranes, microfluidic channels and other systems.

Nowadays technology allows to prepare artificial antidot superlattices in semiconductor heterostructure with two dimensional electron gas (2DES). The conduction properties of these samples has been tested in experiments [11, 12] which showed an important contribution of periodic orbits. The structure of these superlatticies is similar to a periodic lattice of rigid disks placed on a plane. Such structures are known as Galton boards or Lorentz gas. According to the mathematical results of Sinai the dynamics on such a lattice is completely chaotic [14]. The theoretical studies [15] performed to understand these experiments showed that the chaotic classical dynamics and periodic orbits significantly affect the conduction properties in such superlatticies.

The effects of microwave radiation on the conduction properties of antidot superlattices has been addressed in experiments [16]. However in these structures due to the symmetry of the superlattice the ratchet effect was forbidden. Asymmetric mesoscopic structures under external periodic driving have been addressed in the experiments [17]. However zero mean force ratchet was absent due to low frequency of driving which was essentially adiabatic [18].

The recent theoretical studies of dissipative transport in asymmetric superlatticies showed that microwave radiation induces directed transport in such systems (zero mean force ratchet) [14, 19]. These works were mainly performed for a semidisk Galton board which is obtained from the usual Galton board of rigid disks by replacing each disk with semidisk oriented in one fixed direction (see Fig. 1). In Ref. [19] the model of a friction force \( f = -m \gamma v \) with constant friction coefficient \( \gamma \) has been used for a particle of mass \( m \) moving with velocity \( v \). In Ref. [20] the case of particles in a Maxwell thermostat at temperature \( T \) was considered. It was shown that the thermostat creates an effective friction coefficient \( \gamma \) which depends on the microwave field strength and the temperature of the thermostat. However the most interesting case is the Fermi-Dirac thermostat since it describes the experimental conditions of antidot superlatticies with 2DES [21]. Until now no numerical studies were performed in this regime. Only theoretical estimates have been proposed in Ref. [21]. Their validity was never checked and remains questionable.

In this work I develop a numerical method which allows to study the directed transport induced by polarized microwave radiation \( f = f(\cos \theta, \sin \theta) \cos(\omega t) \) in 2DES at various values of the Fermi energy \( E_F \) and temperature \( T \) (here \( \omega \) is the radiation frequency and \( \theta \) is the polarization angle with respect to the \( x \) axis in Fig. 1). On the basis of this method I performed extensive numerical studies which allowed to establish the dependence of ratchet flow velocity \( v_f \) on various system parameters including \( T \) and \( E_F \). Contrary to the estimates proposed in Ref. [20] the dependence on \( T \) is weak when \( T \ll E_F \). The obtained results allow to predict typical values of currents in asymmetric antidot superlatticies.

The paper is organized as follows, in Section II the description of the model and of the numerical method is presented, the results are described in Section III, and conclusion is given in the last Section.
The Fermi energy $E_V$ simulations I use dimensionless units with collisions with the semidisks are elastic. In the numerical board of semidisks which form a two-dimensional hexagonal lattice as represented in Fig. 1. The radius of the semidisk is also the result for many particles by simple rescaling by the number of particles as it is usually done for 2DES (see Refs. [11, 12]). Such a situation corresponds to experiments with 2DES in antidot lattices similar to those of Refs. [21]. To keep particles in a thermal equilibrium with the Maxwell distribution it is possible to use various methods including the Nosé-Hoover thermostat used in Ref. [20]. However for the Fermi-Dirac thermal distribution this method is not appropriate and a new approach should be developed. Indeed the Nosé-Hoover equations are constructed in such a way that they give the Maxwell distribution in particle velocities [20, 22]. They should be significantly modified to generate the Fermi-Dirac distribution and until now this problem has not been addressed yet. The first attempt by the authors of Ref. [20] did not succeed in achieving a good convergence to the Fermi-Dirac distribution [23].

My approach is inspired by the successful Monte Carlo method adapted to simulate numerically the transport properties in semiconductor devices [24]. To obtain a stable Fermi-Dirac thermalization of particles on the semidisk Galton board (see Fig. 1) the following procedure has been applied: (i) the equations of motion were integrated exactly on the time interval $\Delta t$ using the analytical solution; (ii) after that the particle energy is changed from its value $E$ to another value in the

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interval \((E - \Delta E, E + \Delta E)\) without changing the direction of the particle momentum. The choice of this value is governed by the Metropolis algorithm [25] which imposes the convergence to the Fermi-Dirac distribution \(f_E\). Namely, a random value \(E'\) is chosen in \((E - \Delta E, E + \Delta E)\), with probability \(\min(f_E(E')/f_E(E), 1)\) the algorithm sets \(E = E'\), otherwise \(E\) remains unchanged. Afterward the algorithm returns to step (i). The times when collisions with semidisks occur are found with the precise Newton algorithm as in Ref. [19]. The step \(\Delta E\) can be considered as a thermalization step which determines the rate of convergence to the equilibrium distribution \(f_E\).

The Metropolis algorithm described above gives convergence to the equilibrium distribution \(f_E\) in the absence of microwave radiation. The examples of steady state distributions at different temperatures \(T\) are shown in Fig. 2. The proximity of the numerically obtained distribution \(f_n(E)\) to the theoretical steady state \(f_E(E)\) can be characterized by the dimensionless mean square deviation \(\sigma = E_F \int (f_n(E) - f_E(E))^2 dE\). This quantity remained small in all numerical simulations at \(f = 0\) showing a good convergence to the Fermi-Dirac distribution (e.g. for the cases of Fig. 2: \(\sigma = 1.8 \times 10^{-5}, 2.3 \times 10^{-5}\) and \(8.3 \times 10^{-5}\) for \(T/E_F = 0.4, 0.1, 0.01\) respectively).

However, the introduction of the microwave radiation modifies the distribution \(f_n\) which depends on \(f\) and other system parameters. This is clearly seen in the typical cases presented in Fig. 2. For relatively high temperatures the distribution \(f_n\) remains a smooth monotonic function of energy whereas at low \(T\) the microwave field creates a characteristic peak near the Fermi energy \(E_F\). As a result the developed numerical method allows to study the transport created by microwave radiation in an asymmetric semidisk Galton board in the stationary regime. To be close to realistic experimental situations additional random scattering has been introduced to take into account the effect of impurities. Namely after time \(\tau_i\) the direction of particle momentum is changed randomly (angle changes in the interval \([0, 2\pi]\)). In the majority of cases studied the value \(\tau_i\) was kept sufficiently large (\(\tau_i V_F/r_d > 1000\)) and did not influence the stationary transport properties (the dependence on \(\tau_i\) will be discussed in the next Section). Here \(V_F = \sqrt{2E_F}\) is the Fermi velocity.

This stationary regime, which sets in presence of radiation, clearly shows the photogalvanic (ratchet) effect with directed transport born from chaos as it is shown in Fig. 3 for different polarization angles \(\theta\). I would like to stress that this effect originates from the bulk of the sample; it is known that radiation can lead to the appearance of electron-hole pairs, which may diffuse to different contacts because of the electric field in the depletion region of the junction, however in this case the effect is due to the contacts and not to the bulk of the sample. For \(\theta = 0\) the transport is directed to the left (negative direction on the \(x\) axis), while for \(\theta = \pi/2\) the transport is oriented to the right. The polarization dependence is similar to that obtained in previous works [19, 21] and is well described by the relation \(\psi \approx \pi - 2\theta\) where \(\psi\) is the angle between the transport direction and the \(x\) axis. In this way the average velocity of transport can be written as \(v_F = v_F (\cos \psi, \sin \psi)\). It is calculated by following a single trajectory for a long typical time \(tV_F/r_d \sim 10^7\). It was also checked that the averaging over an ensemble of few tens of different trajectories gives statistically the same result.

Special checks have been made to ensure that the
ratchet velocity $v_f$ does not depend on the Metropolis algorithm parameters. An example of such checks is shown in Fig. 4 where the thermalization step $\Delta E$ was changed by one order of magnitude. In spite of this variation the value of $v_f$ remains stable. It was also checked that the resulting ratchet velocity is not sensitive to variation of $\Delta t$, e.g a variation of $\Delta t$ by more than one order of magnitude gave no variation of $v_f$ within a 5% accuracy. Physically it is possible to say that the relaxation time to the equilibrium $\tau_{rel}$ is approximately given by the relation $\tau_{rel} \sim \Delta t E_F^2/\Delta E^2$. Thus, the numerical checks above can be physically interpreted as the independence of the ratchet velocity on the variation of the energy relaxation timescale $\tau_{rel}$ which varied by two orders of magnitude. A similar effect has been seen with the Nosé-Hoover thermostat for the Maxwell equilibrium distribution $^{20}$. This result is also in a qualitative agreement with the theoretical arguments given in Ref. $^1$-$^2$ according to which $\tau_{rel}$ does not directly affect $v_f$. In my further simulations the Metropolis algorithm parameters are set to typical values $\Delta E/E_F = 0.075$, $\Delta t = 0.005$. The data shown in Fig. 4 clearly demonstrate that $v_f$ grows with the radiation strength $f$. At the same time $v_f$ is not very sensitive to the variation of temperature $T$. Detailed studies of parameter dependence of $v_f$ are presented in the next Section.

The results presented above show that the developed algorithm allows to simulate non interacting electrons at thermal equilibrium with the Fermi-Dirac distribution. The microwave driving is assumed to be relatively weak so that it gives only small deviations from the unperturbed distribution, this fact is clearly illustrated in Fig. 2. In this regime the perturbed distribution of particles is determined by the unperturbed distribution $f_{F}(E)$ and the microwave field, the effect of the latter is exactly taken into account by the Hamiltonian equations of motion. Therefore the developed algorithm should correctly describe the non equilibrium steady state distribution that emerges under microwave driving. This is indirectly confirmed by the fact that the directed transport is not sensitive to the thermalization step of the Metropolis algorithm (see Fig. 4). In a sense the Metropolis steps combined with the Hamiltonian equations of motion give the solution of the kinetic Boltzmann equation in the presence of microwave driving.

The above arguments should be also valid for another unperturbed thermal distribution, e.g the Maxwell distribution $f_{M}(E) = \exp(-E/T)/T$. This case was analyzed in Ref. $^24$ on the basis of Nosé-Hoover equations. In fact the Metropolis algorithm had been invented to treat the Maxwell thermal equilibrium $^{25}$. Thus, I made numerical tests with the Metropolis algorithm for the Maxwell distribution. The obtained results reproduce the functional dependences found in Ref. $^{24}$ (see Eqs. (4,5) there) with approximately the same values of the numerical constants. This gives independent confirmation that the Metropolis algorithm treats correctly a weak external perturbation that drives the system out of equilibrium. It also shows that various thermal distributions can be treated by this method.

The model I described assumes that the electrons are non interacting. To be valid it requires that the Coulomb interaction between electrons in 2DES $E_{ee}\approx e^2/\pi n_e/\epsilon_r$ is small compared to the kinetic energy given by $E_F = \pi n_e\hbar^2/m$. Here $n_e$ is the electron density, $\epsilon_r$ is the dielectric constant, $e$ is the electron charge and $m$ is the effective electron mass. Thus the effective strength of interaction is characterized by the dimensionless parameter $r_s = E_{ee}/E_F$ $^{26}$ which should be small. Its value for experimental 2DES obtained in GaAs/AlGaAs heterostructure with electron densities $n_e \approx 10^{12}cm^{-2}$, an effective electron mass $m \approx 0.065m_e$ and dielectric constant $\epsilon_r = 13$ is approximately $r_s \approx 1$. At such values the interaction between quasiparticles is considered to be weak and is usually neglected in a first approximation $^{21,24}$ for example the Wigner crystal typically appears at $r_s \approx 37$.

At such densities $n_e$ inside a cell of size $S = 1\mu m^2$ the quantum level number of an electron at the Fermi energy is $N_F = n_eS \approx 10^4$. Therefore electrons are in a deep semiclassical regime and the classical Monte Carlo approach used above is well justified, see also $^{24}$.

**NUMERICAL RESULTS**

I have investigated the dependence of $v_f$ on several system parameters which are relevant for a realistic experiments with 2DES in antidot lattices. Among them are the temperature $T$, the microwave field strength $f$ and the microwave frequency $\omega$. The effects of geometry are studied by changing the lattice constant $R$ that allows to choose the optimal regime where the photogalvanic effect is stronger. The effects of impurity scattering is modeled by variation of scattering time $\tau_i$ that gives insight on the stability of the effect in respect to experimental imperfections. At last the effect of magnetic field is also analyzed.

The temperature dependence for a typical set of parameters is given in Fig. 5. The obtained numerical data show that there is a weak drop of the ratchet velocity $v_f$ with the increase of temperature $T$. However in the regime with $T \ll E_F$, $v_f$ is practically temperature independent. This dependence is preserved for various radiation strengths $f$. The velocity of transport increases with the growth of $f$. A detailed study of the effect dependence on the microwave field $f$ is presented in Fig. 6. It shows that the dependence on $f$ is quadratic in the region $T \ll E_F$, and temperature independent in a large interval of field strength. At higher $f$ a deviation from the quadratic dependence starts to be visible, this deviation starts earlier at high temperatures. Thus on the
Here the dimensionless factor $C$ may depend on the microwave frequency, lattice geometry, and impurity scattering time. However it is independent of $T$ and $f$. For $R = 2$, $\omega r_d/V_F \ll 1$ and $\tau V_F/r_d \gg 1$ the obtained data give $C = 0.129 \pm 0.002$.

The dependence (1) is qualitatively different from the theoretical estimates proposed in Ref. [20]. To understand the origin of this difference I remind the main elements of estimates given in Refs. [16, 20]. They are based on the fact that the microwave radiation produces diffusive energy growth of electron energy in time with the rate: $D_E = \langle \delta E \rangle^2/\delta t$. Here $\delta E$ is the energy variation after a time $\delta t$. In the limit of low frequency driving it is possible to write $D_E \sim (\dot{E})^2 \tau_0 \sim f^2 V_F l$ where $E$ is the particle energy and $l$ is the mean free path which is $l \sim R^2/r_d$. If a particle would experience a friction force $f = -m \gamma \dot{v}$, the diffusion in energy would give energy variation $(\delta E)^2 \sim D_E/\gamma$. In Ref. [20] it was assumed that $\delta E$ is fixed by the thermal distribution so that $\delta E \sim T$. Thus the statistical stationary distribution imposes an effective friction with coefficient $\gamma \sim D_E/T^2$. This relation is important because the ratchet velocity is given by relation $v_f \sim l \gamma$ in the limit of weak friction as it has been shown in Ref. [19] by extensive numerical simulations. For the Maxwell distribution this gives $v_f \sim (l f)^2/T^3 m^{1/2}$ since in this case $D_E \sim f^2(T/m)^{3/2}$. If a particle would move only in the narrow thermal layer near the Fermi surface. In the free particle model it is therefore rather natural that the particle energy variation is $\delta E \sim E_F$, that leads to the result of Eq. (1).

![FIG. 5](image1.png)

**FIG. 5:** (color online) Top panel: dependence of the rescaled flow velocity $v_f/V_F$ on the rescaled temperature $T/E_F$. Different curves correspond to $f = 7.0, 6.0, 5.0, 4.0, 3.0$ (from top to bottom). Bottom panel: the same data are shown as a function of $E_F/T$. Here $\omega = 1, R = 2, \theta = 0$.

![FIG. 6](image2.png)

**FIG. 6:** The rescaled flow velocity $v_f/V_F$ as a function of rescaled applied force $r_d f/E_F$ for different temperatures: $T/E_F = 0.4$ (green diamonds), 0.1 (blue squares), 0.01 (black circles). The red (full gray) curve shows a parabolic fit of data $v_f/V_F = C (r_d f/E_F)^2$ with $C = 0.129 \pm 0.002$ at $T/E_F = 0.01$ (the fit is done in the interval $[0, 0.45]$). Here $\omega = 1, R = 2, \theta = 0$. 

The fact that the free electron model remains valid in the presence of microwave driving can be also understood from the following arguments. For non interacting electrons the Hamiltonian is given by the sum of one particle operators (microwave driving is also one particle operator). Hence, the many particle state (wave function or density matrix) is obtained simply from one particle states by antisymmetrization. Thus, the Pauli principle can be taken into account by averaging the final one particle results over the Fermi-Dirac distribution. This statement also explains the validity of the classical kinetic Boltzmann equation for the description of transport properties of metals. It is demonstrated more rigorously for a non interacting Fermi gas in (Chap. 5 in Ref. [21] and Ref. [27]). As a consequence the Pauli blockade does not appear for non interacting particles and the arguments presented in Ref. [21] are not valid at least for weak $r_s$ values.

The arguments given above allow to understand the physical origin of the relation between the ratchet velocity and effective friction coefficient induced by microwave radiation that perturbs the system out of thermal equilibrium. Another approach has been developed recently in Ref. [28] using perturbation theory for the Boltzmann kinetic equation in the limit of weak radiation and weak density of randomly distributed asymmetric scatterers. The model proposed in Ref. [28] is rather different from the one considered here, e.g. scatterers are distributed randomly on the plane, their density is required to be small and impurity scattering is necessary for the regularization of the model. However in spite of these differences the global dependence of ratchet velocity on Fermi energy $E_F$ and radiation strength $f$ is the same as in Eq. (1).

The frequency dependence of $v_f/V_F$ is shown in Fig. 7. The data presented there demonstrate that the frequency spectrum is independent of temperature (for $T/E_F = 0.01$, $f = 5.0$ (red squares); $T/E_F = 0.1$, $f = 5.0$ (green circles); and $T/E_F = 0.01$, $f = 3.0$ (black triangles, in this case $v_f$ is multiplied by factor $(5/3)^2$ to underline quadratic dependence on $f$). Here $R = 2$, $θ = 0$.

![Graph showing the dependence of $v_f/V_F$ on the rescaled frequency $\omega r_d/V_F$.]

**FIG. 7:** (color online) Dependence of rescaled flow velocity $v_f/V_F$ on the rescaled microwave frequency $\omega r_d/V_F$, for $T/E_F = 0.01$, $f = 5.0$ (red squares); $T/E_F = 0.1$, $f = 5.0$ (green circles); and $T/E_F = 0.01$, $f = 3.0$ (black triangles, in this case $v_f$ is multiplied by factor $(5/3)^2$ to underline quadratic dependence on $f$). Here $R = 2$, $θ = 0$.

The dependence of $v_f$ on the distance between disk centers $R$ is shown in Fig. 8. Initially $v_f$ starts to grow with $R$ then reaches a maximum value and drops at large $R$ values. The position of the maximum depends on the microwave frequency. With the increase of frequency the maximum moves to smaller values of $R$. Qualitatively this corresponds to the situation when the microwave frequency becomes comparable with the frequency of coll-
sions of particles with semidisks. The dependence of data on parameters can be satisfactorily described by a fit formula:

\[ \frac{v_f}{V_F} = A \frac{R^2 f^2}{E_F^2} \left( \frac{1}{1 + B(\omega R^2/r_d V_F)^2} \right) \]  

(2)

Here \( A, B \) are dimensionless fitting parameters. The fit for three values of \( \omega \) in Fig. 8 gives \( A \approx 0.017 \) and \( B \approx 0.012 \). For \( \omega \to 0 \) this expression is in a satisfactory agreement with the value \( C \approx 0.13 \) found in Fig. 6 at \( R = 2 \). The physical origin of this fit is related to frequency dependence of the diffusion rate \( D_E \) which interpolates between the low frequency regime (\( D_E \) independent of \( \omega \)) and the high frequency regime where \( D_E \) drops quadratically with \( \omega \) (see estimates given above).

Eq. (2) gives reasonable description of obtained numerical data in the regime where \( l \) is not too large compared to \( R \). This situation is most interesting for direct experimental studies where \( R \) is not very large compared to \( r_d \).

Another important experimental parameter is the scattering time induced by impurities which are always present in real samples. The data presented in previous figures were obtained in the regime of very large \( \tau_i \). The effect of finite values of \( \tau_i \) on the ratchet velocity is described in Fig. 9 for various lattice constants \( R \). The data show that at large \( \tau_i \) values \( v_f \) is independent of the impurity scattering time while at small \( \tau_i \), \( v_f/V_F \) drops approximately linearly with \( \tau_i V_F/r_d \). Indeed the asymmetry of semidisks is washed out by impurity scattering and the ratchet effect should disappear at small \( \tau_i \). At the same time it is important to stress that the presence of impurities is not necessary for the onset of directed transport. In experimental conditions \( \tau_i \) can depend on temperature because of electron-phonon scattering or electron-electron interactions which give a dependence of \( \tau_i \) on \( T \). This may lead to a significant temperature dependence of the photogalvanic effect, in the temperature range \( T \sim 10^2 K \).

Experimentally it is also possible to study the dependence of the effect on magnetic field \( B \) perpendicular to the 2DES plane. To investigate this dependence the method described above was adapted to the presence of a magnetic field, which was included in the analytical solution of motion equations between Metropolis thermalization steps. The magnetic field dependence is given in Fig. 10. The data clearly shows that the ratchet effect disappears for sufficiently strong magnetic fields when the Larmor radius \( r_l = V_F/B \) becomes smaller than the distance \( R \) between semidisks (here electron charge and mass are set to 1). Indeed for \( r_l \ll R \) the classical electron dynamics becomes integrable and the diffusion rate in energy \( D_E \) goes to zero due to absence of chaos, thus leading to the disappearance of ratchet (\( v_f \propto D_E \)). In principle the magnetic field changes the transport direction (angle \( \psi \)). I do not discuss this dependence here since the main point is that the ratchet effect disappears at relatively low \( B \) (see below).

CONCLUSIONS

The obtained results clearly shows the existence of zero mean force ratchet in asymmetric semiconductor structures induced by microwave radiation. The obtained results give the following dependence for the strength of
the stationary current induced by the ratchet effect in one row of semidisks (row width $\sqrt{3}R$):

$$I = \sqrt{3\pi n_e} R v_f = A \sqrt{\frac{6}{\pi^2} \frac{f^2}{n_e^{1/2}} \frac{e m R^3}{h^3}} \quad (3)$$

where $E_F = \pi n_e \hbar^2 / m$ and $m = 0.065 m_e$. This dependence holds in the low frequency regime which is usually satisfied at typical electron densities $n_e = 10^{12} \text{cm}^{-2}$ where the collision frequency is of the order of 200GHz for a $R \sim 1 \mu m$ and under the assumption $R \sim r_d$. For $R = 1 \mu m$ and $f/e = 1 \text{V/cm}$, the equation (3) gives the current 0.1nA. In samples with high mobility the mean free path can have values as high as 5$\mu m$, and therefore the optimal regime for photogalvanic effect will be when $R$ is of the same order and it is quite possible that in this situation the current per row can be as high as 10nA. According to the results of Fig. 10 for $R \sim 1 \mu m$ the ratchet effect starts to disappear at magnetic field $B \sim 0.1T$.

The asymmetric antidot lattice can be considered as a prototype for transport in asymmetric molecular structures. The latter have attracted recently a significant interest in view of possible biological applications of ratchets[29]. Therefore experimental investigations on the ratchet effect discussed in this paper are highly desirable.

I thank D. L. Shepelyansky for stimulating discussions and for his interest in this research. I am also grateful to M. V. Entin, and L. I. Magarill for access to their unpublished results on their kinetic equation approach to the photogalvanic effect. This research is done in the frame of the ANR PNANO project MICONANO.

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