Effects of $\Lambda\Lambda - \Xi N$ mixing in the decay of $^6\Lambda\Lambda$He

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Abstract. A one-meson exchange model including the ground state of the pseudoscalar octet is used to describe the weak two-body interactions responsible for the decay of $^6\Lambda\Lambda$He. Strong interaction effects are taken into account by a microscopic study based on the solution of $G-$matrix and $T-$matrix equations for the initial and final interacting pairs respectively. Results for the decay induced by $\Lambda\Lambda \rightarrow \Lambda N(\Sigma N)$ transitions are given.

1. Introduction
This work focuses on the new decay modes associated to the presence of two $\Lambda$'s in the $^6\Lambda\Lambda$He hypernucleus. In particular, the weak decay process that may occur from a $\Lambda\Lambda$, $\Xi N$ or $\Sigma \Sigma$ state, which can be excited via the strong interaction from the initial $\Lambda\Lambda$ pair, while the strong interaction also determines the final hyperon-nucleon ($YN$) wave function, which may have transitioned from either a $\Lambda N$ or a $\Sigma N$ intermediate state. For the weak transition we employ a meson-exchange model, built upon the exchange of mesons belonging to the ground state of the pseudoscalar octet, thoroughly employed for single-$\Lambda$ hypernuclei[1]. The tree-level values for the baryon-baryon-meson coupling constants are derived using SU(3) symmetry. The essential development with respect to previous calculations[2] is the consideration of new decay transitions that emerge when we allow for the intermediate states that can be coupled strongly to the initial $\Lambda\Lambda$ state. Solving a $G-$matrix equation for the initial state, with the input of realistic baryon-baryon potentials[3] gives us the wave functions for the coupled transitions $\Lambda\Lambda-\Lambda\Lambda$, $\Lambda\Lambda-\Xi N$ and $\Lambda\Lambda-\Sigma \Sigma$ (the latter, being much smaller than the other two components, will be disregarded in the present calculation). On the other hand, it is assumed that the two final particles are emitted with high enough momenta so that nuclear medium effects can be disregarded and the T-matrix formalism may then be used. The new $\Xi N \rightarrow \Lambda N(\Sigma N)$ weak transitions require additional baryon-baryon-meson coupling constants to be derived. These two new ingredients have allowed us to obtain an update on the decay rate for the $(\Lambda\Lambda - \Lambda\Lambda) \rightarrow (YN - Y'N)$ channel as well as new results for the $(\Lambda\Lambda - \Xi N) \rightarrow (YN - Y'N)$ channel, where $Y,Y' = \Lambda, \Sigma$.

2. Hypernuclear decay rate
The nonmesonic decay rate of a hypernucleus is given by

$$\Gamma_{nm} = \int \frac{d^3k_1}{(2\pi)^3} \int \frac{d^3k_2}{(2\pi)^3} \sum_{M_1\{R\}}\sum_{M_2\{I\}} (2\pi)\delta(M_H - E_R - E_1 - E_2) \times \frac{1}{2J+1} |M_{fi}|^2,$$

(1)
where $M_H$, $E_R$, $E_1$ and $E_2$ correspond to the mass of the hypernucleus, the energy of the $(A - 2)$-particle system, and the total asymptotic energies of the emitted baryons, respectively. The integration variables $\vec{k}_1$ and $\vec{k}_2$ stand for the momenta of the two particles in the final state, where the momentum of the residual nucleus has been integrated using momentum conservation. The sum, together with the factor $1/(2J + 1)$, indicates an average over the initial hypernucleus spin projections, $M_f$, and a sum over all quantum numbers of the residual $(A - 2)$-particle system, $\{ R \}$, as well as the spin and isospin projection of the emitted final particles, $\{ 1 \}$ and $\{ 2 \}$. $M_{fi}$ stands for the transition amplitude from the $^6\Lambda\Lambda\text{He}$ hypernucleus to a final state composed of a residual nucleus plus two outgoing baryons. We use a shell-model framework and a weak coupling scheme of the $\Lambda\Lambda$ pair to the nuclear core to write $M_{fi}$ in terms of elementary two-body transitions. In the present case the $^6\Lambda\Lambda\text{He}$ wave function can be decoupled as:

$$\left| ^6\Lambda\Lambda\text{He} \right> = \left| \Lambda\Lambda \right>_{T=0,M_{\Lambda\Lambda}=0}^{J=2,M_c=0} \otimes \left| ^4\text{He} \right>_{T=0,M_c=0}^{J=2}$$

where antisymmetry forces the two $\Lambda$ hyperons to be in a $^1S_0$ state, since they are assumed to be in the lowest s-shell $(1s_{1/2})$ before the decay occurs.

The evaluation of the $\Lambda N \rightarrow NN$ transition rate requires to decompose the nonstrange nuclear core as one nucleon coupled to a three-particle system, while decoupling one of the two $\Lambda$ particles, so that the initial $\Lambda N$ pair can convert into a final $NN$ pair, conveniently antisymmetrized with the residual system. The details and final expression for the hypernuclear decay amplitude in terms of two-body $\Lambda N \rightarrow NN$ ones can be found in Ref. [2]. As mentioned in the introduction, we focus on the $\Lambda\Lambda \rightarrow YN$ decay mode. In this case, the residual 4-particle system, which coincides with the $^4\text{He}$ nucleus, contains no strangeness, while the final two-particle state contains one hyperon that can be either a $\Lambda$ ($\{ |Y\Gamma_Y > = |00\> \}$), a $\Sigma^-$ ($\{ |Y\Gamma_Y > = |1-1\> \}$) or a $\Sigma^0$ ($\{ |Y\Gamma_Y > = |10\> \}$). The $\Lambda\Lambda \rightarrow YN$ amplitude is then given by

$$M_{\Lambda\Lambda \rightarrow YY} = \langle \vec{k}_{NSN}NL, \vec{K}_{SY}SY, |^4\text{He}\rangle |^{0\Lambda\Lambda\text{He}} \rangle$$

$$= \sum_{S,M_S} \langle \frac{1}{2}S_{N}, \frac{1}{2}S_{Y} | SM_S \rangle \langle \frac{1}{2}T_{N},Y_{T} | TM_T \rangle$$

$$\times \langle \vec{k} | \Psi^{CM}_{\Lambda\Lambda} \rangle \langle \vec{k} , SM_S , TM_T | \hat{O} | \Psi^{rel}_{\Lambda\Lambda} , S_0 , M_{S_0} , T_0 | M_{T_0} \rangle ,$$

where the initial $\Lambda\Lambda$ wave function has been written as a product of relative and center-of-mass wave-functions, $\Psi^{CM}_{\Lambda\Lambda}$ and $\Psi^{rel}_{\Lambda\Lambda}$, respectively, and $\vec{k}$ and $\vec{K}$ are the relative and total momentum of the emitted $YN$ pair. The amplitude $\langle \vec{k} , SM_S , TM_T | \hat{O} | \Psi^{rel}_{\Lambda\Lambda} , S_0 , M_{S_0} , T_0 | M_{T_0} \rangle$ represents the two-body transition matrix element, calculated using a two-body interaction potential based on a meson-exchange picture and including initial and final correlated wave functions. Note that the isospin quantum numbers of the $\Lambda\Lambda$ should contain a coupling to an isospinor field, $| \frac{1}{2} - \frac{1}{2} \rangle_{\Lambda}$, introduced to account for the $\Delta I = 1/2$ rule in the transition[1].

In the case of the $\Lambda\Lambda \rightarrow YN$ matrix elements, and since the strong interaction allows for the conversion to other baryon-baryon channels, the weak interaction will take place not only from the initial $\Lambda\Lambda$ pair, but also from pairs containing other members of the baryon octet, $\Xi^- p$, $\Xi^0 n$ and $\Sigma^+ \Sigma^-$, which will in turn decay weakly into either $\Sigma^0 n$, $\Lambda n$ or $\Sigma^- p$ states. Moreover, the strong interaction acting between the final baryons will produce additional $YN - Y'N$ transitions, with $Y, Y'$ being $\Lambda$ or $\Sigma$ hyperons.

3. Final and initial state strong interactions

The formalism to include the strong interaction between baryons is well stablished and may be found in the literature[4]. This is known as the Brueckner-Goldstone theory, which considers the interactions of a pair of particles within the Fermi sea, with the collisions fulfilling the
requirements of the Pauli principle. We solve this problem in infinite nuclear matter and use the same correcting factors (correlated wave functions over uncorrelated ones) to our wave functions for $^6\Lambda\Lambda$He.

Working within the $(\frac{1}{2}^+)\Lambda\Lambda$ baryon octet, the strange $\Lambda\Lambda$, $\Xi\Sigma$ and $\Sigma\Sigma$ pairs can couple through the strong interaction to the initial $\Lambda\Lambda$ state. The correlated state, $|\Psi\rangle$, is defined through the relation $G|\Psi\rangle = V|\Phi\rangle$, where $|\Phi\rangle$ is the free-particle state and $G$ is given in terms of the bare baryon-baryon potential, $V$. This yields an integral equation which includes a Pauli blocking operator, $Q$, which restricts the summation to unoccupied states above the Fermi level and the energy of the correlated two-body system, $E$. An equivalent formalism is used for the final state, but without the inclusion of the Pauli operator, i.e. $Q = 1$.

\[
|\Psi\rangle = |\Phi\rangle + \frac{Q}{E - H_0 + i\epsilon} G|\Phi\rangle,
\]

where

\[
G = V + V \frac{Q}{E - H_0 + i\epsilon} G.
\] (4)

In Fig. 1 we represent the initial $\Lambda\Lambda$ $^1S_0$ wave function as a function of the relative distance between the two $\Lambda$ particles and for a representative value of the relative momentum. The black line displays the uncorrelated harmonic oscillator wave function, while the green line displays the correlated wave function for the dominant diagonal $\Lambda\Lambda-\Lambda\Lambda$ component. The red and blue lines represent, respectively, the $\Lambda\Lambda-\Sigma\Lambda$ and the $\Lambda\Lambda-\Xi\Sigma$ components. It is clear that the dominant contribution to the $\Lambda\Lambda \rightarrow YN$ decay mode of $^6\Lambda\Lambda$He will come from the diagonal $\Lambda\Lambda-\Lambda\Lambda$ component of the wave function, which at large distances behaves as an uncorrelated harmonic oscillator while at short distances gets reduced due to the short-distance repulsive behavior of the strangeness $S = -2$ baryon-baryon NSC97f interaction employed [3]. With regard to the non-diagonal components of the wave function one can see that, despite having a comparable size at the origin, the strength of the $\Lambda\Lambda-\Sigma\Sigma$ term is essentially located at distances under 0.5 fm and it will be strongly suppressed by the $r^2$ factor in the integrand of the two-body matrix element. On the other hand, the $\Lambda\Lambda-\Xi\Sigma$ component is still sizable around 1 fm and it is expected to contribute non-negligibly to the $\Lambda\Lambda \rightarrow YN$ decay mode of $^6\Lambda\Lambda$He.

4. One-Meson-Exchange Potential

The evaluation of the two-body transition matrix elements of Eq. (3) requires the knowledge of the operator that triggers the weak $\Delta S = -1$ transition from an initial baryon pair to a final one. In the meson exchange description employed here these transitions are assumed to proceed via the exchange of virtual mesons belonging to the pseudoscalar and vector meson octets.

As we detail below, the general structure of the transition potential for pseudoscalar meson exchange reads:

\[
V^\phi(\vec{q}) = \sum_k \left( A_k^Y + \frac{B_k^0}{2M} \frac{\vec{q}^2}{m_\phi} \right) \frac{\vec{\sigma} \vec{q}}{\vec{q}^2 + m_\phi^2} \hat{O}_{k,\phi},
\] (5)

where $M$ is the average mass of the baryons involved in the weak vertex, $m_\phi$ is the mass of the exchanged pseudoscalar meson and the index $k$ in the sum runs over the different isospin structures associated to each type of meson.
In order to build the isospin operators $\hat{O}_k$ one needs to know the isospin nature of the meson being exchanged (isoscalar for $\eta$, isodoublet for $K$, and isovector for $\pi$) and the specific baryons involved in the two-body weak transition. We focus on developing the isospin structure for the transitions $\Xi N \to Y N$, with $Y = \Lambda, \Sigma$, which are the new contributions developed in the present work. Attending only to the isospin quantum numbers, the general structure of the $\Xi N \to Y N$ matrix element is:

$$
\begin{align*}
g_1 \langle Y t_f, \frac{1}{2} t_{Nf} | \hat{O}_1 | 0 t_i, -\frac{1}{2} \rangle \langle 0 t_i, -\frac{1}{2} | \hat{\tau} \langle \frac{1}{2} t_i, \frac{1}{2} | \hat{G}_{\Sigma} = 3 \hat{O}_3 \langle \frac{1}{2} t_i, \frac{1}{2} | 1 t_i, \frac{1}{2} = \frac{1}{2} \rangle.
\end{align*}
$$

where the isospurion has coupled to the isospin $\frac{1}{2}$ of the $\Xi$ to give states with isospin $I = 0, 1$, which in turn couple to the initial nucleon isospin (with projection $t_{Ni} = \frac{1}{2}$ for a $p$ and $-\frac{1}{2}$ for a $n$) to give the final $YN$ state. We first examine those cases where the final state is of the $\Lambda N$ type, i.e. $|Yt_f\rangle = |00\rangle$. The only possible operators entering Eq. (6) can be argued to be

$$
\hat{O}_1^\Lambda \equiv \hat{l}_1 \otimes \hat{l}_2, \quad \hat{O}_2^\Lambda \equiv \hat{\tau}_1 \otimes \hat{\tau}_2, \quad \hat{O}_3^\Lambda \equiv 0
$$

where $\hat{\tau}$ stands for the Pauli matrices and $\hat{T}_{01}$ is an operator that allows the transition from a $I = 1$ state to a $I = 0$ one. Their spherical coordinates have the following matrix elements

$$
\langle 00|\hat{T}_{01}|1m\rangle = (-1)^k \langle 00, 1 - k|1m \rangle = (-1)^k \delta_{m,-k}.
$$

We note that, in the case of a $\Lambda N$ final state, the operator $\hat{O}_3$ is zero since there is no other scalar operator that can connect the initial $|1 t_2 - \frac{1}{2}, \frac{1}{2} t_{Ni} \rangle$ pair with a the final $|00, \frac{1}{2} t_{Nf} \rangle$ one. In the case of a $\Sigma N$ final state, we have $|Yt_f\rangle = |11\rangle$ and the appropriate set of operators is:

$$
\hat{O}_1^\Sigma \equiv \hat{T}_{10} \otimes \hat{\tau}_2, \quad \hat{O}_2^\Sigma \equiv \hat{l}_1 \otimes \hat{l}_2, \quad \hat{O}_3^\Sigma \equiv \hat{T}_{11} \otimes \hat{\tau}_2,
$$

where $\hat{T}_{10}$ mediates transitions from isospin 0 to isospin 1, as can be inferred taking the adjoint in Eq. (8). Likewise, $\hat{T}_{11}$ induces transitions from an initial $I = 1$ state to a final $I = 1$ one. Its matrix elements are given by:

$$
\langle 1m|\hat{T}_{10}^k|00\rangle = \delta_{m,k}, \quad \langle 1m'|\hat{T}_{11}^k|1m\rangle = \sqrt{2} \langle 1m'|1m1k \rangle .
$$

### 5. Results and discussion

The results for the non mesonic, $\Lambda$-induced, $\Lambda \Lambda \to Y N$ decay rate of $\Lambda^6$He are displayed in Tables 1 and 2, and are given in terms of the free decay rate for the $\Lambda$ hyperon, $\Gamma_{\Lambda} = 3.8 \times 10^9 \text{ s}^{-1}$. We also give the contribution of each individual meson separately, in order to assess its importance in a given transition, and we add up the contribution of the lightest pseudoscalar mesons sequentially for a better interpretation of our results.

The novelty of the present paper is the consideration of the strong non-diagonal $\Lambda \Lambda - \Xi N$ mixing of the $\Lambda \Lambda$ wave function. We observe that the effect of this mixing on each meson exchange contribution leading to a $\Lambda n$ state is different. By comparing both tables one sees that the final decay for this channel decreases by 50% when the $\Xi N$ channel is accounted for, from $\Gamma_{\Lambda n} = 1.41 \times 10^{-2} \Gamma_{\Lambda}$ to $6.53 \times 10^{-3} \Gamma_{\Lambda}$. The $\Sigma N$ decay experiences a minor increase, from $\Gamma_{\Sigma N} = 6.29 \times 10^{-3} \Gamma_{\Lambda}$ to $7.24 \times 10^{-3} \Gamma_{\Lambda}$, with the inclusion of the $\Lambda \Lambda - \Xi N$ mixing. Altogether, the $\Lambda \Lambda - \Xi N$ component of the wave function brings the value of the $\Gamma_{\Lambda n}/(\Gamma_{\Sigma n} + \Gamma_{\Sigma -p})$ ratio to 0.90, a factor 2.5 times smaller when found this contribution is neglected.

Adding the $\Lambda n$, $\Sigma^0 n$ and $\Sigma^- p$ partial rates, the total contribution from the $\Lambda \Lambda \to Y N$ decay mode amounts to $\Gamma_{YN} = 1.38 \times 10^{-2} \Gamma_{\Lambda}$, which represents an overall decrease of around 10% compared to the rate for the diagonal $\Lambda \Lambda - \Lambda \Lambda$ channel only.
Table 1. Meson-exchange contributions to the nonmesonic decay of $^6\Lambda\Lambda$He considering final state interactions and only the $\Lambda\Lambda - \Lambda\Lambda$ component of the initial w.f., in units of $\Gamma_\Lambda = 3.8 \times 10^9 \text{ s}^{-1}$.

| Meson | $\Lambda n$ | $\Sigma^0 n$ | $\Sigma^- p$ | $\Sigma N$ |
|-------|-------------|--------------|---------------|-----------|
| $\pi$ | $1.35 \times 10^{-4}$ | $2.38 \times 10^{-3}$ | $4.76 \times 10^{-3}$ | $7.15 \times 10^{-3}$ |
| $K$   | $2.22 \times 10^{-2}$ | $3.23 \times 10^{-4}$ | $6.46 \times 10^{-4}$ | $9.69 \times 10^{-4}$ |
| $\eta$ | $8.95 \times 10^{-4}$ | $2.56 \times 10^{-7}$ | $5.11 \times 10^{-7}$ | $7.67 \times 10^{-7}$ |
| $\pi + K$ | $2.17 \times 10^{-2}$ | $2.11 \times 10^{-3}$ | $4.23 \times 10^{-3}$ | $6.34 \times 10^{-3}$ |
| $\pi + K + \eta$ | $1.41 \times 10^{-2}$ | $2.10 \times 10^{-3}$ | $4.20 \times 10^{-3}$ | $6.29 \times 10^{-3}$ |

Table 2. Meson-exchange contributions to the nonmesonic decay of $^6\Lambda\Lambda$He considering final state interactions and both $\Lambda\Lambda - \Lambda\Lambda$ and $\Lambda\Lambda - \Xi N$ initial transition channels, in units of $\Gamma_\Lambda$.

| Meson | $\Lambda n$ | $\Sigma^0 n$ | $\Sigma^- p$ | $\Sigma N$ |
|-------|-------------|--------------|---------------|-----------|
| $\pi$ | $9.73 \times 10^{-5}$ | $1.96 \times 10^{-3}$ | $3.91 \times 10^{-3}$ | $5.87 \times 10^{-3}$ |
| $K$   | $2.42 \times 10^{-2}$ | $1.61 \times 10^{-5}$ | $3.22 \times 10^{-5}$ | $4.83 \times 10^{-5}$ |
| $\eta$ | $2.08 \times 10^{-3}$ | $5.90 \times 10^{-5}$ | $1.18 \times 10^{-4}$ | $1.77 \times 10^{-4}$ |
| $\pi + K$ | $1.57 \times 10^{-2}$ | $2.24 \times 10^{-3}$ | $4.48 \times 10^{-3}$ | $6.73 \times 10^{-3}$ |
| $\pi + K + \eta$ | $6.53 \times 10^{-3}$ | $2.41 \times 10^{-3}$ | $4.83 \times 10^{-3}$ | $7.24 \times 10^{-3}$ |

The complete non-mesonic decay rate $\Gamma$ of $^6\Lambda\Lambda$He, contains also the processes induced by a $\Lambda N$ pair and as such one may write $\Gamma = \Gamma_{\Lambda N \rightarrow NN} + \Gamma_{\Lambda\Lambda \rightarrow YY}$. The decay rate for $\Lambda N \rightarrow NN$ channel has been computed[1] to be $\Gamma_{\Lambda N \rightarrow NN} = 0.95 \Gamma_\Lambda \approx 2 \Gamma (\Lambda \Lambda \text{He})$. Comparing this result to those of the $\Lambda\Lambda$ induced mode calculated in the present work, one can see that the decay rate for the $\Lambda\Lambda \rightarrow YY$ transition with $\Lambda\Lambda - \Lambda\Lambda$ diagonal correlations amounts to a 2.15% of the one-nucleon induced rate $\Gamma_{\Lambda N \rightarrow NN}$, while the inclusion of the $\Lambda\Lambda - \Xi N$ mixing reduces this percentage to 1.45%. A word of caution has to be said when using this number as reference in the current work, since the contribution of vector mesons has yet to be included[5].

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