ZEROING THE STELLAR ISOCHRONE SCALE: THE RED GIANT CLUMP LUMINOSITY AT INTERMEDIATE METALLICITY

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ABSTRACT

The color-magnitude diagrams of the open clusters NGC 2420 and NGC 2506 have been investigated as intermediate links between the solar neighborhood and the Magellanic Clouds. Two sets of theoretical isochrones that include convective overshoot are zeroed to the Sun at solar abundance and to the unveled main-sequence dwarfs of the Hipparcos Catalogue at [Fe/H] = −0.4, requiring a differential of 0.4 mag between the unveled main sequences at a given color. Adopting EW(B−V) = 0.04 and [Fe/H] = −0.39 for NGC 2506 and EW(B−V) = 0.04 and [Fe/H] = −0.29 for NGC 2420, the respective apparent moduli are (m−M) = 12.70 and 12.15, while the ages of both clusters are approximately 1.9 ± 0.2 Gyr or 2.2 ± 0.2 Gyr, depending on the choice of isochrones. From the composite giant branch of the two clusters, the mean clump magnitudes in V and I are found to be +0.47 and −0.48 (−0.17, +0.14), respectively. Applying a metallicity correction to the M_I values, the cluster sample of Udalski leads to (m−M)_0 = 18.42 (+0.17, −0.15) and 18.91 (+0.18, −0.16) for the LMC and SMC, respectively. A caveat to this discussion and potentially to the claim that clusters of the same abundance and age are identical is the observation that the V−I colors of the red giants in NGC 2506 are significantly redder at a given B−V than the giants in clusters of comparable age and/or metallicity. The distance scale above has been derived using the general cluster relation between B−V and V−I. If the CCD photometry in NGC 2506 is correctly tied to the standard system, M_I for the clump will decrease and the distance moduli should increase by 0.1 mag.

Key words: color-magnitude diagrams — distance scale — Magellanic Clouds — open clusters and associations: individual (NGC 2420, NGC 2506)

1. INTRODUCTION

Within the quest to define the distance scale of the nearby universe, the Magellanic Clouds have played a critical role as an intermediate link between local distance indicators and those needed to reach galaxies beyond the Local Group. As summarized in Figure 1 of Cole (1998), virtually every potential means of fixing the location of this datum within the grand scale has been attempted in recent years, many of the approaches catalyzed by the availability of precise parallaxes from Hipparcos. A recent technique that has generated both discussion and controversy is the location of the red giant clump (RGC), the locus of stars in the post—He-flash phase of evolution, generally analogous to the horizontal branch in globular clusters. The rationale behind this choice is obvious; the clump stars are intrinsically bright, well separated from the redder, first-ascent giants, and the luminosities appear to be only weakly dependent on age and metallicity, particularly in the infrared. Unfortunately, applications of the approach have produced significantly different results. Cole (1998) has derived a true distance modulus of (m−M)_0 = 18.36 ± 0.17, consistent within the errors with the Cepheid Key Project value of 18.50 ± 0.10, while Stanek, Zaritsky, & Harris (1998) find 18.06 ± 0.03 ± 0.09. (Note, throughout the text the apparent modulus, i.e., uncorrected for reddening, is designated by m−M without the subscript 0.) The primary source of disagreement lies with the metallicity and age sensitivity of the clump luminosity. The absolute I magnitude of the clump is based upon the Hipparcos (Perryman et al. 1997) results for a large sample of nearby giants (Stanek & Garnavich 1998). Though it has long been known that the LMC is more metal deficient than the solar neighborhood, a result confirmed by the color distribution of the clump stars (see, e.g., Fig. 2 of Stanek et al. 1998), considerable empirical evidence exists that these metallicity differences should not significantly alter the luminosity of the clump (Paczynski & Stanek 1998; Stanek & Garnavich 1998; Udalski et al. 1998; Stanek et al. 1998; Udalski 1998). This empirical evidence is challenged by Cole (1998) who uses theoretical models of clump stars to demonstrate that mass and metallicity can significantly impact the luminosity distribution of clump stars. If the theoretical models are reliable representations of He-burning stars, the mean luminosity of the RGC will depend upon the star formation and chemical history of the population under discussion, i.e., it is not necessarily appropriate to apply the solar neighborhood results to the LMC (see, e.g., Beaulieu & Sackett 1998).

Ideally, one could minimize, if not resolve, this issue if a population of metal-deficient stars with ages typical of the LMC could be identified locally and used in the comparison. While the field population of Hipparcos appears to contain a modest sample of clump giants with [Fe/H] near −0.4 (see Fig. 5 of Jimenez, Flynn, & Kotoneva 1998), there is at present no means of dating these stars. If they are typical of the old/thick disk, their ages may be closer to 10 ± 2 Gyr (Wyse & Gilmore 1995) than the more common 2 ± 2 Gyr found at this abundance within the LMC. Moreover, the abundances derived for the clump stars by Jimenez et al. (1998) using DDO photometry and the calibration of Janes (1975, 1979) are systematically in error. As discussed in detail in Twarog, Ashman, & Anthony-Twarog (1997), the revised DDO calibrations by both Piatti, Clariá,
Minniti (1993) and Twarog & Anthony-Twarog (1996) demonstrate that the original [Fe/H] calibration systematically underestimated the red giant abundances by between 0.1 and 0.2 dex. On the cluster abundance scale adopted in this investigation, giants with [Fe/H] = −0.5 from Høg & Flynn (1998) have actual abundances near −0.3.

Somewhat overlooked in the discussion of this problem is the fact that a sample of stars with well-defined ages and compositions comparable to those found in the LMC does exist in the form of the open clusters of the Galactic anticenter. These clusters have typical [Fe/H] between −0.2 and −0.5 (Twarog et al. 1997) and ages between 1 and 4 Gyr (Friel 1995). The primary weakness in the use of these objects has been the uncertainty in the distance determination, based upon main-sequence fits to theoretical isochrones assumed to have similar compositions and/or differential comparisons to other clusters via the main sequence or the RGC. In the end, the uncertainties in the absolute and differential scales of the isochrones translate into comparable uncertainties in the absolute luminosities of key color-magnitude diagram (CMD) features such as the clump. With the availability of the Hipparcos data, these difficulties can be resolved through the use of main-sequence stars rather than clump giants, i.e., we can link the LMC to the solar neighborhood via the anticenter clusters.

The goals of this investigation are twofold: First, we will attempt to zero correctly the theoretical isochrones that are commonly used in age and distance determination via direct comparison with nearby field stars of comparable metallicity. This will be the focus of §2.

Second, using the observational data for two virtually identical clusters, NGC 2420 and NGC 2506, as detailed in §3, we will derive the distance moduli and cluster ages through comparison with the appropriate isochrones and, indirectly, fix the absolute magnitude of the RGC for a 2 Gyr old population with [Fe/H] of −0.4. Section 4 provides the straightforward comparison to the LMC and SMC and a discussion of our results.

2. ZEROING THE SCALE

A critical step in the use of any set of theoretical isochrones is the transition between the theoretical [Mbol, log T\textsc{e}] and the observational [M\textsc{v}, (B−V)] plane. The required transformations are a function of composition and, to a lesser degree, surface gravity; they can be derived via empirical relations and/or model atmospheres convolved with appropriate filter functions. Once transformed, the observational relations can be tested by comparison to well-defined sequences found in nearby clusters, as illustrated in VandenBerg (1985) and VandenBerg & Poll (1989). Though such comparisons permit adjustments that guarantee that the slopes of the theoretical color-magnitude relations will resemble real clusters, they resolve only half the problem in that the zero point of the scale is still unknown. By the zero point we refer to the link to the two additional fundamental parameters that uniquely define a star’s structure: mass and age. (Note that age is equivalent to the radial composition structure that changes over time, while the traditional chemical composition refers to the uniform abundance at the time of formation.) To fix the transformation correctly, it is necessary that a star of a given mass, age, and surface abundance have the appropriate color and M\textsc{v}. Once this scale is fixed, assuming that the theoretical models are a close approximation to reality, the surface properties of any star of a given mass, age, and composition may be derived and the results used to constrain the properties of individual stars and/or clusters.

The object used to fix the zero point of the models is invariably the Sun, i.e., a solar mass star with an age of 4.6 Gyr and the assumed chemical composition of the Sun should have solar luminosity and temperature. The solar composition contains some flexibility in that while Z, the metal fraction by mass, usually lies between 0.016 and 0.018, Y, the helium fraction, ranges from 0.27 to 0.30. Ultimately, these modest variations have little effect if the solar-mass model is appropriately zeroed and the remaining comparisons are done differentially. Differences in the implied zero points of the models, as well as differences in the input physics, have contributed to some of the scatter in derived ages and moduli for clusters over the years. Despite the improvement in the models and isochrone morphology through better opacities and the inclusion of convective overshoot, problems remain in setting the absolute scale.

First, there are still differences in the transformations between the theoretical and observational plane. Though this issue has been noted a number of times over the last decade, it will be reexamined because it is important to determine if the effect is uniform at all [Fe/H].

Second, a subtle but nonnegligible problem arises because of the production of isochrones at specific abundances. The nonovershoot isochrones of VandenBerg (1985) were developed at Y = 0.25 and Z = 0.0169 (solar), 0.010, 0.006, 0.003, and 0.0017. The Bertelli et al. (1994; hereafter BE) models have (Y, Z) = (0.352, 0.05), (0.28, 0.02), (0.25, 0.008), (0.24, 0.004), and (0.23, 0.001), while the Schaerer et al. (1993) and Schaller et al. (1993; hereafter collectively GE) isochrones have (Y, Z) = (0.30, 0.02) and (0.264, 0.008). The procedure used by most observers is to adopt the isochrones closest in composition to solar as [M/H] = 0.0 and then assign abundances differentially. For example, Z = 0.02 and 0.008 would be [M/H] = 0.00 and −0.40, respectively, for the BE and GE isochrones. As mentioned earlier, such an approach would work in an absolute sense for [Fe/H] = 0.00, even if (Y, Z) for the Sun were slightly different from the isochrone values, if the observational isochrones were zeroed to the Sun at the adopted age. However, for other metallicities, even differential comparisons may fail because, while the appropriate value of Z is available, it is linked to a specific value of Y that remains unknown. Clearly what is needed is a means of zeroing the scale for nonsolar compositions. With the advent of the Hipparcos Catalogue of trigonometric parallaxes (Perryman et al. 1997), this becomes possible though, to date, the primary focus has been on the subdwarfs owing to their link to globular clusters (see, e.g., Reid 1997; Gratton et al. 1997). In the discussion below, we will attempt to test the isochrones for a composition more appropriate to the old disk, [Fe/H] = −0.4, and the LMC.

2.1. Solar Isochrones

The first and most straightforward step is the zeroing of the solar isochrones. Though some disagreement still remains regarding the exact value of the solar color, the range commonly adopted is B−V between 0.64 and 0.66; over the last decade, we have consistently used (B−V) = 0.65 with M\textsc{v} of 4.84 (see, e.g., Anthony-Twarog et al. 1990; Daniel et al. 1994; Anthony-Twarog, Twarog, & Sheeran 1994; Twarog, Anthony-Twarog, & Hawarden
Thus, a solar mass star with solar composition at 4.6 Gyr should have $B-V$ and $M_V$ of 0.65 and 4.84, respectively. Using the isochrones in the observational plane as published for BE or constructed and interpolated with the programs of Meynet, Mermilliod, & Maeder (1993) for GE, one can derive the $B-V$, $M_V$ for a solar mass star at 4.6 Gyr for the two compositions that bracket the potential solar abundance, $Z = 0.02$ and $Z = 0.008$. We have also included the same information for the $Z = 0.05$ models of BE. The results are illustrated in Figure 1, where the open circle represents the true Sun and squares and triangles are data for BE and GE, respectively.

It is apparent from the trend that the BE compositions do bracket the solar position. One could use the BE isochrones with only minor adjustment by fixing $Z = 0.018$ as the true solar value, a plausible choice that is consistent with the claims that the BE models accurately reproduce the solar characteristics in the theoretical plane and that the transformations to the observational plane are reliable. In the case of GE no simple interpolation will match the solar value, thereby incorporating a zero-point error in all the solar isochrones. To rezero the $Z = 0.02$ isochrones as solar, one needs to add the offsets $\Delta [B-V] = [-0.07, +0.045]$ to the isochrones of GE and $[-0.04, -0.032]$ to those of BE. It should be emphasized that offsets of this size are neither new nor unexpected. Adjustments to the VandenBerg (1985) scale have been discussed in Twarog & Anthony-Twarog (1989) and Anthony-Twarog et al. (1990) and to the Castellani, Chieffi, & Straniero (1992) scale and the Maeder & Meynet (1991) scale in Twarog, Anthony-Twarog, & McClure (1993). That these adjustments are not the product of errors and/or differences in the models nor unique to the two sets included in this investigation may be seen in Figures 10 and 11 of Nordström, Andersen, & Andersen (1997) and Figures 5 and 10 of Mermilliod et al. (1996). In the theoretical plane the unevolved main sequences exhibit minor differences; in the observational plane on the unevolved main sequence at $(B-V) = 0.7, M_V$ differs by 0.5 mag, with the models of BE being brighter.

If we apply the offsets above to the $Z = 0.02$ isochrones, one gets the comparison seen in Figure 2 for an age of 4 Gyr. The unevolved main sequences are identical. The color at the turnover and on the giant branch is redder for GE, while the luminosity of the subgiant branch is brighter. We close the discussion of the solar models by noting that the derived offsets are important not just because they are of significant size but because they are systematic. Without correction, main-sequence fitting with the GE isochrones will produce distance moduli that are uniformly too small by 0.35 mag; the BE isochrones will produce moduli that are too large by 0.15 mag. Ages tied to the color of the turnover will be too old in the GE case and slightly too young in the BE case.

### 2.2. Isochrones of Intermediate Metallicity

Given the offsets derived for the solar models, the obvious question is should the same offsets apply to isochrones of nonsolar composition? In the past, owing to the lack of any additional means of testing the isochrones, the default answer has been yes. The problem this presents is illustrated in Figure 3 where we have superposed the BE and GE isochrones for an age of 2 Gyr and $[\text{Fe/H}] = -0.4$. In the theoretical plane, the subgiant branches and the unevolved main sequence have identical luminosities. At a given age, the GE isochrones extend toward higher log $T_e$ at the turnover and lower log $T_e$ on the giant branch. Using the BE and GE transformation to the observational plane as shown in Figure 4, one would expect a differential offset comparable to that found for the solar isochrones, i.e., at a given $B-V$, the main sequences should differ by approximately 0.5 mag. Clearly, they do not; the typical offset in $M_V$ at a given $B-V$ on the unevolved main sequence is only 0.12 mag, with the GE isochrones being fainter. Application of the same offsets as derived for solar abundance models will lead to a differential of 0.35 mag in $M_V$ at a given $B-V$ on the main sequence, with the isochrones of GE being brighter.

Which transformation between the theoretical and observational plane, if any, is correct? Moreover, even if the
transformations coincided, do the isochrones truly reproduce stars in the solar neighborhood with $[\text{Fe/H}] = -0.4$, i.e., is the correlated change in helium and metals appropriate? The latter question is significantly more challenging in that it requires knowledge of the mass of an unevolved star with independently derived $M_{\text{bol}}$, log $T_\ast$, and $[\text{Fe/H}]$. A simpler approach can resolve the former question while empirically sidestepping the latter issue: require the unevolved main-sequence CMD of the isochrones to coincide with the cooler dwarfs with $[\text{Fe/H}] = -0.4$ in the solar neighborhood as defined by the Hipparcos data.

2.3. The Hipparcos Sample

To isolate the metallicity range of interest we make use of the $uvby$ catalog of G stars compiled by Olsen (1993). For cooler dwarfs, $uvby$ photometry can supply the metallicity via the calibration derived by Schuster & Nissen (1989); the data of Olsen (1993) have been adjusted slightly using the relations supplied by Olsen (1993) to place them on the same system as the calibration stars. Stars in the $b - y$ range from 0.39 to 0.59, $V < 9.0$, $[\text{Fe/H}]$ between $-0.3$ and $-0.5$ and classed as dwarfs for transformation purposes by Olsen (1993) were isolated. The Hipparcos parallax sample was searched and all stars with measurable parallax were identified. Of these, the sample was further restricted to all stars with $\sigma/\pi \leq 0.1$.

Though $V$ and $B - V$ were available from the Hipparcos Catalogue, the quoted errors in $B - V$ covered a non-negligible range, in contrast with the $b - y$ values of Olsen (1993). To maximize the internal consistency of the data, a linear transformation was derived between $b - y$ and $B - V$, weighting the data by the inverse of the error in $B - V$. From 137 stars over the $b - y$ range of interest, $b - y$ was transformed to $B - V$ using

$$(B - V) = 1.873(\pm 0.032)(b - y) - 0.115(\pm 0.014).$$

The rms scatter about the mean relation is only $\pm 0.014$ mag with no dependence on $[\text{Fe/H}]$.

Distances and $M_V$ were derived for each star from the parallaxes after applying corrections for the Lutz-Kelker effect (Lutz & Kelker 1973; Koen 1992). All corrections were less than 0.1 mag. The resulting CMD is presented in Figure 5 where the error bars are based upon the $1 \sigma$ error in parallax. To improve clarity, the error bars in $B - V$ are not drawn; they are very similar for all the stars and typically less than $\pm 0.01$. As one might expect, the error bars for the parallaxes are smallest for the coolest and intrinsically faintest dwarfs. Superposed is the zero-age main sequence (ZAMS) for $[\text{Fe/H}] = -0.4$ from the models of BE.

A significant number of stars are positioned well above the unevolved, cool main sequence. These are expected to fall into two general classes. The brightest stars ($M_V < 4.2$) are subgiants evolving across the HR diagram and up the giant branch for the first time. The stars at intermediate luminosity that parallel the main sequence may be stars with larger than average errors in their parallaxes, but the more likely possibility is that they are binaries, shifted above the main sequence by the presence of a companion of comparable brightness. As a check on this interpretation, one can use the $uvby$ data of Olsen (1993). If the stars that parallel the main sequence are binaries, their indices should
resemble the stars on the ZAMS since the binaries should be composed of two similar stars. In contrast, the subgiants with lower log g should exhibit $c_1$ indices distinct from the main-sequence stars. This is exactly what is seen in Figure 6, where the stars redder than $(B-V) = 0.65$ have been sorted into two groups, those brighter and those fainter than $M_V = 4.2$. The crosses represent subgiants, while the squares are the unevolved stars and the probable binaries. For $(B-V) > 0.71$, the separation is virtually perfect. The one star classed as a binary that falls among the subgiants is HD 174429, a chromospherically active member of the Pleiades supercluster, probably a pre–main-sequence star (Eggen 1995; Henry et al. 1996). Its inclusion in the sample is undoubtedly the mistaken product of anomalous indices induced by its active nature. Though the change in $c_1$ is modest given the change in $M_V$, it is encouraging to see that $c_1$ does have the capability to distinguish between dwarfs and subgiants at cooler temperatures for stars at metallicities normally associated with the disk, not just for subdwarfs as in Pilachowski, Sneden, & Booth (1993).

Note also that the range in $c_1$ increases for supposedly unevolved stars blueward of $(B-V) = 0.7$. This sharp change in sensitivity also occurs with $hk$ photometry (Anthony-Twarog et al. 1991; Twarog & Anthony-Twarog 1995), as illustrated in Eggen (1997). Since the $Ca$ and $u$ filters are located shortward of 4000 Å, it may indicate a significant change in the source of ultraviolet continuum opacity near $(b-y) = 0.45$.

To test the match between the field stars and the isochrones, only the redder stars $[(B-V) \geq 0.7]$ will be included. As one moves up the main sequence to $(B-V) < 0.7$, the errors increase while the possibility grows of finding stars evolving off the main sequence near the hydrogen-exhaustion phase. The $M_V$, $B-V$ relation for the unevolved main sequence at $[Fe/H] = -0.4$ and $(B-V) \geq 0.7$ was taken from BE and the distance above or below the main sequence was measured for each star. From Figure 5, it is clear that the single stars with small errors scatter uniformly within ±0.3 mag of the main sequence. Restricting the sample to stars within 0.3 mag of the main sequence and weighting the points by the inverse error in $M_V$, one finds an average offset between the parallax stars and the isochrone relation of less than $-0.005$ mag; a histogram of the weighted distribution in $\Delta M_V$ using all the points confirms that the BE isochrone relation for $[Fe/H] = -0.4$ is on the same scale as the parallax stars with an uncertainty of less than ±0.02 mag. The weighted mean of the metallicity of the stars used in defining the zero point of the unevolved main sequence is $[Fe/H] = -0.38$.

In sharp contrast with the solar models, no offset is required. Moreover, there is no evidence for a metallicity dependence in the location of the ZAMS with the range of points from $[Fe/H] = -0.3$ to $-0.5$, which indicates that the modest change expected is washed out by the combination of errors in $[Fe/H]$, $B-V$, and $M_V$.

How much of a shift should we expect? If we superpose the ZAMS for the isochrones at $[Fe/H] = 0.0$ and $-0.4$, the metal-poor ZAMS is, as expected, fainter by 0.4 mag. If the abundance effect is approximated by a linear trend, this implies that $\Delta M_V/\Delta [Fe/H] = -1.0$. If the sample is divided into two groups between $[Fe/H] = -0.3$ and $-0.39$ and between $-0.4$ and $-0.49$, the expected difference in $M_V$ at a given $B-V = 0.5$ is 0.1 mag. Reid (1998) has discussed the trend with $[Fe/H]$ for $Hipparcos$ stars with $[Fe/H]$ between $-0.4$ and $-2$ and finds a slope at the metal-rich end of the scale consistent with theory, but the data scatter is large and the stars lie between $(B-V) = 0.55$ and 0.75, bluer than the current sample.

Since the shift in $M_V$ for GE relative to the isochrones of BE is well determined (see Fig. 4) and the slope of the main sequence is known, one can easily show that an offset of $\Delta(B-V) = +0.022$ is required to place the GE isochrones at $[Fe/H] = -0.4$ on the $Hipparcos$ scale, the same direction as for the solar isochrones, but smaller. The solution of adjusting the scale by shifting only $B-V$ reiterates a point made at the start of this comparison. Because we do not have mass and/or age information for the stars being used to zero the scale, we cannot say if a shift is required in $B-V$, $M_V$, or both. We have taken the simplest approach of applying the entire offset in $B-V$ because the log $T_e$ to $B-V$ transformation is generally considered the most uncertain. It should be emphasized that the zero point of the age scale, fixed via the mass, remains unknown.

Before the comparisons are made between the clusters and the isochrones, the question of the relative metallicity scales should be addressed. The abundances of the clusters are tied to the revised DDO calibration for giants by Twarog & Anthony-Twarog (1996), but the field dwarf abundances are defined by the Schuster & Nissen (1989) $ubvy$ calibration. The DDO data are ultimately linked to a composite catalog (Anthony-Twarog & Twarog 1998) of high-dispersion, spectroscopic abundances from the literature, transformed to a common system which is zeroed to an adopted Hyades $[Fe/H]$ of $+0.12$ and the field star–globular cluster scale of Sneden et al. (1991) and Kraft et al. (1992).

Though hardly ideal, one method for linking the dwarf and giant abundance scales is through the recent revisions of both systems by Carretta & Gratton (1997) and Clementini et al. (1998). On the revised system, the difference in $[Fe/H]$ for giants in the globular clusters common to both, in the sense (CG – SKPL), is $+0.11 \pm 0.02$ (s.e.m.) for seven clusters ranging from $[Fe/H] = -0.7$ to $-2.2$. For the dwarfs, Clementini et al. (1998) find a difference in $[Fe/H]$ among the stars common to Schuster & Nissen.
is \( [\text{Fe}/\text{H}] \) and probable members, using both CCD and photographic surveys. This range has been chosen because it includes the anticenter sample of Figure 5 or the spectroscopic dwarf catalog of Clementini et al. (1998). Though the spectroscopic catalog compiled by Anthony-Twarog & Twarog (1998) is dominated by giants, some surveys included a number of subgiants, and an even smaller fraction included modest numbers of dwarfs. Because of this, the overlap with the current dwarf sample is small, three stars for the \( uvby \) data and seven for the Clementini et al. (1998) data. The scatter is significant, but the conclusions from both comparisons are that the \( uvby \) scale is too metal-rich compared to the DDO scale by between 0.1 and 0.2 dex. Taken at face value, this implies that the stars used to zero the intermediate isochrones are, on average, closer to \( [\text{Fe}/\text{H}] = -0.55 \) than \(-0.4 \) and, thus, the unevolved main sequence is too faint by approximately 0.15 mag. However, given the small sample and the questionable application to dwarfs of the transformation equations derived for giants, we will assume that the DDO and \( uvby \) scales have the same zero point.

3. THE CLUSTERS: NGC 2420 AND NGC 2506

3.1. Basic Cluster Parameters

Though there are over a dozen clusters in the Galactic anticenter with abundances ranging from \( [\text{Fe}/\text{H}] = -0.2 \) to \(-0.6 \) (Twarog, Ashman, & Anthony-Twarog 1997), we will restrict our isochrone comparison to only two, NGC 2420 and NGC 2506. These two clusters, as demonstrated below, have virtually identical CMDs, but, more important, the level of information available for stars within these clusters is unique among the anticenter sample. There is excellent CCD and photographic broadband photometry, intermediate-band photometry for reddening and abundances, spectroscopy for abundance estimation and radial-velocity determination, and proper-motion analysis for membership determination. The membership estimation is especially critical given the sparsely populated red giant branches, in contrast with the clusters regularly found in the Magellanic Clouds. Inclusion of only a modest number of field interlopers can skew the derivation of the color and luminosity of the red giant branch and clump.

A detailed discussion of the photometry and CMD for NGC 2420 can be found in Anthony-Twarog et al. (1990). For our sample we will include all stars classified as definite and probable members, using both CCD and photographic data as presented in Anthony-Twarog et al. (1990). The metallicity of the cluster, as derived in Twarog et al. (1997), is \( [\text{Fe}/\text{H}] = -0.28 \), assuming \( E(B - V) = 0.05 \).

For NGC 2506, primary use is made of the \( UBVRI \) CCD survey by Marconi et al. (1997). Because this sample does not cover the entire cluster, it has been supplemented by the photographic data of McClure, Twarog, & Forrester (1981). To ensure that they are on the same system, a comparison has been made between the CCD data and the photographic data for 106 stars with \( V < 15.10 \) common to the two surveys. This range has been chosen because it includes the giant branch, subgiant branch, and the top of the main-sequence turnover. It is found that over this magnitude range, the \( V \) magnitudes are on the same system within \( \pm 0.01 \) mag, if the extreme deviants are removed, as confirmed by the plot in Figure 4 of Marconi et al. (1997); the scatter about the mean is typically \( \pm 0.03 \) mag. The deviant stars are relevant because their distribution in \( \Delta V \) is asymmetric; the majority are too bright in the photographic survey. Though some of the stars with large residuals might be variables, a more mundane explanation is provided by the cluster star chart: the sample of deviants is dominated by stars with probable contamination by stars of the same or brighter magnitude. With photographic aperture photometry one can minimize contamination by offsetting the star in the aperture, but, in extreme cases, it is impossible to remove. This solution is confirmed by the comparison in \( B - V \), where the scatter in the residuals at a given color is the same as that found for \( V \), while many of the deviants in \( B - V \) show no anomaly in \( V \). The one difference with \( B - V \) is that a small color term does appear among the residuals between the CCD and photographic data in \( B - V \). The photographic data were transformed to the CCD system using the relation

\[
(B - V)_{\text{CCD}} = 0.97(B - V)_{\text{PG}} + 0.01.
\]

All stars from McClure et al. (1981) with membership probability greater than or equal to 80\% (Chiu & van Altena 1981) and \( V \) brighter than 15.1 not included in the CCD work of Marconi et al. (1997) were identified and their photometry transferred to the CCD system. Their locations within the cluster were checked, and, if they suffered from potential contamination by a nearby star of comparable brightness, they were tagged. The photographic data were merged with the CCD data for cluster members.

Based upon the same combination of DDO photometry and spectroscopy as used for NGC 2420, Twarog et al. (1997) found the metal abundance of NGC 2506 to be \(-0.38 \), adopting \( E(B - V) = 0.05 \). Because of the change in \( B - V \) introduced by the transformation between the photographic and CCD data, we have rederived the reddening estimate for NGC 2506 using the DDO approach of Janes (1977). If all 10 stars within the calibration limits are included, \( E(B - V) = 0.02 \pm 0.05 \) (s.d.). If stars with membership probabilities below 70\% are removed, the mean reddening for six stars drops to \( 0.00 \pm 0.02 \) (s.d.). These results are consistent with the original estimates of \( E(B - V) = 0.05 \) and 0.03, respectively, by McClure et al. (1981) in that the color transformation derived above produces a shift of between 0.02 and 0.03 mag in \( B - V \) for the red giants with the CCD system being bluer. Thus, with the same DDO indices, the previous reddening estimates are reduced by the same amount. Note that this implies that the absolute color of the giant branch is approximately the same irrespective of which photometric system is adopted.

As a check for consistency, we reanalyzed the DDO photometry for NGC 2420 (McClure, Forrester, & Gibson 1974) using the Janes (1977) approach; the earlier discussion used a technique outlined by McClure & Racine (1969). From 11 stars, \( E(B - V) = 0.02 \pm 0.03 \) (s.d.); the result remains unchanged if the one nonmember is dropped. This result is consistent with the extensive discussion of the relative reddening of these clusters found in Anthony-Twarog et al. (1990). The two clusters have very similar reddenings, but the absolute reddening of the two clusters remains more uncertain. Indications from the field stars observed with the clusters and the reddening maps of...
Burstein & Heiles (1982) are that the true reddening is slightly higher than that derived from the DDO sample, consistent with the giants being bluer than expected owing to their lower [Fe/H] relative to the field star sample, though James (1977) claims that metallicity effects on the DDO technique are small. The one significant addition to the question since Anthony-Twarog et al. (1990) is that of Schlegel et al. (1998). The reddening maps of Schlegel et al. (1998) produce $E(B-V) = 0.04$ for NGC 2420, consistent with the results of Burstein & Heiles (1982), and 0.08 for NGC 2506, smaller than the Burstein & Heiles (1982) value of 0.12, but indicative of a higher $E(B-V)$ for NGC 2506. For purposes of discussion, we will assume, as in Twarog et al. (1997), that the two clusters have the same, but slightly lower, reddening, $E(B-V) = 0.04$. In addition to the consistency of the reddening maps, we give more weight to the DDO result for NGC 2420 because of an apparent problem with the CCD photometry of Marconi et al. (1997), an issue we will return to in § 3.3. Given this minor change in reddening, the difference in [Fe/H] between NGC 2420 and NGC 2506 remains 0.10 dex, with NGC 2420 being more metal-rich; on the system of Twarog et al. (1997), NGC 2506 has [Fe/H] = −0.39.

As a final step before deriving the absolute moduli of the two clusters, we make an additional comparison to constrain the cluster parameters, a differential comparison of the two clusters, independent of the isochrones. This is especially useful because the unevolved main sequence of NGC 2420 is much better defined than that of NGC 2506. Since the differential metallicity is known, one can estimate that the unevolved main sequences of the two clusters should differ by approximately 0.1 mag, with NGC 2420 being brighter. Additionally, the red giant branch of NGC 2420 should be redder by 0.01–0.02 mag, assuming they have the same age (see § 3.2). As a secondary constraint, we assume that they differ in reddening by 0.00 ± 0.02 mag. Within these constraints, the optimal displacements, in the sense (2506 − 2420), are −0.01 in $B-V$ and +0.50 in $V$, which implies that the reddening for NGC 2506 should be smaller than that for NGC 2420. If we require $\Delta(B-V) = -0.02$, the main-sequence condition is met if $\Delta V = 0.45$, but the red giant branches are virtually aligned. Increasing the reddening differential forces smaller shifts in $V$ but increasingly fails to match the red giant branch. If we lower the differential reddening to 0.0, $\Delta V = 0.55$, and the giant branches differ in color by approximately 0.03 mag, with NGC 2420 being redder.

An uncertainty that arises in this analysis is the exact zero point of the CCD photometry in $B-V$. As pointed out above, a comparison between the CCD data and the photographic data tied to internal cluster standards leads to a small color term in the residuals in $B-V$, that in the CCD data are too blue. If the photographic zero point is correct, the giant branch is redder but the reddening value is higher and consistent with the assumed relative reddening for the two clusters. Thus, a differential offset of −0.01 in $B-V$ using the CCD system is equivalent to an offset of $+0.01$ for the photographic system. We conclude that the ideal differential fit for the two clusters, tied to the CCD data, is $\Delta(B-V) = -0.01 \pm 0.01$ and $\Delta V = 0.50 \pm 0.05$.

3.2. Isochrone Fits: Distances and Ages

Given $E(B-V) = 0.04$ and adjusting the isochrones by −0.1 mag in $M_V$ to account for the higher metallicity, the optimum fit of NGC 2420 to the intermediate metallicity isochrones of BE is shown in Figure 7. The apparent modulus is $(m - M) = 12.15$. The shape of the isochrones from the main sequence near $(B-V)_0 = 0.8$ through the turnoff, subgiant branch, giant branch, and clump is an excellent match to the data, which implies an age of 2.0 Gyr. Taking into account the slightly higher metallicity of the cluster relative to the isochrones, the true age is more like 1.9 Gyr, with an uncertainty of less than 10%. In fixing the distance modulus, use has been made only of the main sequence, with special emphasis on the color range between $(B-V) = 0.4$ and 0.6, where the confusion caused by field star contamination and photometric errors is minimized. Use of the giant branch can create problems because the isochrones colors are tied to a lower metallicity; if corrections were made to account for the higher [Fe/H] of NGC 2420, the isochrone giant branches would be slightly redder.

Figure 8 illustrates the match for NGC 2506, adopting $E(B-V) = 0.04$ and $(m - M) = 12.70$. No adjustment has been made to the isochrones because the cluster abundance is effectively the same as the models. The match between theory and observation is quite good near the turnoff, but the expanded scatter in the main sequence below $(B-V)_0 = 0.6$ makes this region relatively useless for fitting purposes. As expected from the differential fit discussed above, the primary fitting region between $(B-V)_0 = 0.4$ and 0.6 is well matched by the isochrones. The cluster is almost a perfect match to a turnoff age of 1.8 Gyr, which makes it slightly younger than NGC 2420. The luminosities of the stars on the subgiant branch are well matched by the isochrones, but it is obvious that the giant branch of the cluster is too blue. This can be corrected by lowering the reddening, in contradiction with the estimates from the reddening maps, or by assuming that there is an error in color of about 0.03 mag for the giants (see § 3.3). The fit in Figure 8 is based upon the assumption that the latter solution is more probable.

Figures 9 and 10 show the analogous matches for the isochrones of GE. The main-sequence fits are of comparable quality to those of BE, an expected result given the similarity between the two sets and the fact that we have shifted...
The GE isochrones to match the same Hipparcos field stars. The GE isochrones are less optimal than those of BE in two ways. The hydrogen-exhaustion hook at the turnoff turns too sharply to the red relative to the clusters, and the isochrone giant branches are too red, as discussed earlier. The ages for the two clusters are 2.2 Gyr and 2.1 Gyr for NGC 2420 and NGC 2506, respectively, slightly older than derived from BE. It is encouraging to note that Marconi et al. (1997) find \((m - M)_0 = 12.5\) and \(E(B - V) = 0.05\) for \(Z = 0.008\) for the models of BE and GE, with no adjustment applied to either scale but using their own transformations between the theoretical and observational plane. Their technique makes use of an optimum match of a variety of CMD properties, including morphology and the luminosity function. Because of the higher reddening, their ages are younger than ours. If we were to adopt \(E(B - V) = 0.05\), then \((m - M) = 12.75\), the same within the errors as their value of 12.7.

3.3. The Absolute Clump Magnitude: \(V\) and \(I\)

With the reddening and distance moduli in place, one can derive the intrinsic luminosity of the clump. We have combined the giant branches for the two clusters, including in the counts any giant within \(\pm 0.2\) mag of the color of the red giant branch. Though it is relatively easy to isolate first-ascent red giants from the clump stars because of the quality of the CMD photometry, this approach is taken to make the distribution determination analogous to that commonly used in less well-defined CMDs. Moreover, the distribution function with absolute magnitude will be derived for \(V\) and \(I\). Though the former is more commonly available, the latter has become the filter of choice of late because the metallicity sensitivity of the \(I\) magnitude of the clump is believed to be substantially weaker than \(V\).

The problem with analyzing \(I\) is that only a subset of the stars in NGC 2506 have \(V - I\) photometry from Marconi et al. (1997); no \(I\) data are available for NGC 2420. We initially resolved both problems by deriving a relation between \(B - V\) and \(V - I\) from the CCD photometry in NGC 2506 and applying it to the adjusted photographic data in NGC 2506 (see § 3.1) and to the CCD data of NGC 2420. From 35 stars redder than \((B - V) = 0.6\) and brighter than \(V = 15.1\), one finds

\[
(V - I) = 0.91(\pm 0.03)(B - V) + 0.24(\pm 0.03).
\]

The rms scatter about the mean relation is only \(\pm 0.026\) mag. The problem with this solution is illustrated in Figure 11, where we have plotted the \(B - V\), \(V - I\) data for a number of open clusters that bracket NGC 2506 in age and metallicity; the dashed relation is the equation listed above. The clusters included are NGC 2204 (filled triangles), Be 39 (open circles), Mel 66 (open squares), and M67 (stars). Data for the first three clusters are from Kassis et al. (1997); the M67 data are from Montgomery, Marschall, & Janes (1993). Also superposed is a short-dashed line indicating the shift caused by a reddening of \(E(B - V) = 0.10\). The clusters have been adjusted for reddening, but it is clear that unless the reddening is grossly in error, stars will shift approximately parallel to the mean relation for giants; no separation by metallicity is apparent for stars in the [Fe/H] range from 0.0 to \(-0.7\). The relation for NGC 2506 is offset from that of the other clusters by \(+0.1\) in \(V - I\) at a given \(B - V\); the solid line shows the NGC 2506 relation shifted by \(-0.1\) mag in \(V - I\). Whether this offset is caused purely by a shift...
in $V-I$ or a combination of shifts in both $V-I$ and $B-V$ cannot be decided, but a blue offset in $B-V$ at the 0.02–0.03 mag level for the CCD data relative to the photographic data for the giants is consistent in direction with the direction of the offset seen in Figure 11.

What is the source of the discrepancy for NGC 2506 in Figure 11? The natural response is to conclude that a zero-point error exists in the CCD data of Marconi et al. (1997), possibly amounting to −0.03 mag in $B-V$ and between +0.07 and +0.10 mag in $V-I$. Without additional observations, we have no independent means of testing this conjecture and caution against assuming that the CCD photometry, which has been tied to direct observations of a number of standard fields, must be at fault. In particular, recent work by Stutz, Popowski, & Gould (1998) and Paczyński (1998) has shown that RR Lyrae stars and red clump giants in Baade’s window appear to have anomalous $V-I$ colors that cannot be explained by simple adjustments in surface gravity or [Fe/H]. However, while one might expect discrepancies between stars formed in galactocentric regions separated by 7–10 kpc, the comparison in Figure 11 includes clusters of comparable age and location to NGC 2506. Thus, we have, as a matter of convenience, opted for the assumption that an error exists in the CCD photometry for NGC 2506. We will, however, discuss the impact if the anomalous colors in $V-I$ are real.

Combining the cluster giant branches and binning the stars as a function of $M_V$ and $M_I$, one gets the histograms of Figure 12, where the solid curve is for $V$ and the dashed curve is for $I$. The peaks in the distributions caused by the existence of the RGC are readily identified between $M_V = 0.25$ and 0.55 and between $M_I = -0.65$ and −0.3. One could attempt a profile fit to the data, but, given the small number of stars, we have taken a simple average of the 22 stars between $M_I = -0.65$ and −0.2 and $M_V = 0.2$ and 0.6. The results are summarized in Table 1, along with the mean color of the stars used to define the clump. Also listed are the means under two more restrictive conditions: only stars in NGC 2506 and only stars in NGC 2506 with CCD data.

The RGC of NGC 2420 has the same $M_V$, within the errors, as NGC 2506, but is redder by about 0.04 in $B-V$. This color differential translates into a brighter $M_I$ and a redder $V-I$ for NGC 2420. If the $B-V$ of the giants in NGC 2506 is made redder by between 0.02 and 0.03, this reduces the differentials between the clusters in all indicators, consistent again with the belief that the two clusters are almost identical in age and composition, with NGC 2420 being slightly more metal-rich and older. If the $(B-V) - (V-I)$ transformation defined by NGC 2506 is adopted for both clusters, the average $M_I$ for the clump is made 0.1 mag brighter and $V-I$ is 0.09 mag redder. We will adopt the intermediate values of $M_V = 0.47 \pm 0.04$ and $M_I = -0.48 \pm 0.05$, where the errors take into account the range in options from Table 1 and the standard errors of the mean for the individual values. Values of $M_V$ in the range of 0.5–0.7 have been obtained by Twarog et al. (1997) and Eggen (1998) using larger samples of clusters with greater uncertainty in their intrinsic parameters, but both studies find only a weak dependence of $M_V$ on [Fe/H] between −0.6 and +0.2.

For the total error budget in deriving $M_V$ and $M_I$, one must include the uncertainty in the fit of the isochrones to the field stars ($\pm 0.02$), in the cluster mean metallicities ($\pm 0.02$), in the definition of the metallicity scale between the DDO and wby systems ($-0.10$, $+0.05$), in the fitting the cluster main sequence ($\pm 0.05$), the uncertainty of $\pm 0.01$ in

![Figure 11](image1.png)  
**Figure 11.**—Correlation between $(V-I)_0$ and $(B-V)_0$ for the giants in NGC 2204 (filled triangles), Mel 66 (open squares), Be 39 (open circles), and M67 (stars).

![Figure 12](image2.png)  
**Figure 12.**—Distribution of combined RGC sample for NGC 2506 and NGC 2420 as a function of $M_I$ (dashed curve) and $M_V$ (solid curve).

### Table 1

| Stellar Sample | $N$ | $M_V$ | $\sigma_V$ | $(B-V)_0$ | $\sigma_{(B-V)}$ | $M_I$ | $\sigma_I$ | $(V-I)_0$ | $\sigma_{(V-I)}$ |
|----------------|-----|-------|------------|----------|----------------|-------|----------|----------|----------------|
| All .............. | 22  | 0.46  | 0.08       | 0.92     | 0.05           | −0.51 | 0.10     | 0.96     | 0.05           |
| NGC 2506 .......... | 14  | 0.47  | 0.08       | 0.90     | 0.04           | −0.48 | 0.09     | 0.94     | 0.04           |
| NGC 2506—CCD ...... | 6   | 0.50  | 0.07       | 0.89     | 0.02           | −0.43 | 0.06     | 0.93     | 0.02           |
the solar color ($\pm 0.055$), and the $\pm 0.02$ scatter in the allowed $E(B-V)$ for the pair of clusters ($\pm 0.11$). It should be remembered that the differential shift in $V$ between the clusters is correlated with the changes in $E(B-V)$ in that a decrease in the differential reddening of the two clusters requires a larger $\Delta V$. Combining the above, the estimated errors in $M_V$ and $M_I$ are ($-0.17$, $+0.14$) for the full sample; use of the NGC 2506 data alone produces no significant change in the errors. Note that the error estimate does not include any component due to the potential zero-point error in the CCD data for $B-V$ or $V-I$. The only changes allowed will shift the absolute magnitudes to brighter values.

4. APPLICATIONS AND FUTURE WORK

A key motivation for this investigation has been the ongoing debate on the distances to the LMC and the SMC derived using the RGC. Though the $M_V$ estimate is less affected by zero-point uncertainties than $M_I$, we will discuss the distance based upon $I$ photometry first. A significant portion of the debate on the giant branches has centered on the analysis of field star samples within the two systems (see, e.g., Cole 1998; Udalski et al. 1998; Beaulieu & Sackett 1998) and the appropriate mixture of populations of different ages and metallicities. As emphasized by Udalski (1998), one can minimize the problems by restricting the sample to star clusters that contain stars of uniform age and abundance. Udalski (1998) has analyzed $VI$ photometry of a sample of 15 star clusters in the LMC and SMC, ranging in age from 2 to 12 Gyr. He finds rather conclusively that there is virtually no dependence of $M_I$ on age from 2 to 10 Gyr, a range that includes NGC 2420 and NGC 2506. The typical cluster sampled by Udalski (1998) is, however, more metal poor than the Galactic sample, ranging from $[\text{Fe/H}]=-0.6$ to $-1.0$ for the LMC and from $-0.7$ to $-1.5$ for the SMC. Both Cole (1998) and Udalski (1998) discuss the slope of the correction for metallicity, $\Delta M_I/\Delta [\text{Fe/H}]$; the former finds a value of 0.21, while the latter chooses 0.09. Adopting the middle ground, $\Delta M_I = -0.06$ for the LMC adjustment ($[\text{Fe/H}]=-0.8$) and $-0.12$ for the SMC ($[\text{Fe/H}]=-1.2$), which lead to $M_I = -0.54$ and $-0.60$ for the LMC and the SMC, respectively. For the LMC clusters, Udalski (1998) derives a mean $I_0$ of 17.88 $\pm 0.05$ and 18.31 $\pm 0.07$ for the SMC clusters. The resulting distance moduli are ($m-M_0$) = 18.42 ($+0.17$, $-0.15$) and 18.91 ($+0.18$, $-0.16$), respectively.

These moduli are in very good agreement with the work of Cole (1998) and marginally consistent with Udalski (1998); the results of Udalski et al. (1998) can be excluded. If we adopt the anomalous $V-I$ colors of NGC 2506 as being correct for both NGC 2506 and NGC 2420, the distance moduli increase by 0.1 mag, but the question then arises as to whether or not the MC clusters are comparable to NGC 2506. Adopting the anomalous $V-I$ values as correct, the average $(V-I)_0$ color of the RGC in NGC 2506 is 1.04, while the typical LMC cluster has $(V-I)_0$ closer to 0.9. Though NGC 2506 is more metal rich than the LMC, the anomalous color is closer to the expected value for a giant branch of solar metallicity as in M67 (Montgomery et al. 1993).

Given the moduli derived above, one can check the distance to the LMC and the SMC using the $M_V$ of the RGC. For six clusters in the LMC, the mean $M_V$ of the RGC is $18.78 \pm 0.06$, only 0.1 mag larger than the field clump identified by Beaulieu & Sackett (1998). If we choose the two youngest clusters with ages similar to NGC 2420 and NGC 2506, the RGC mean is 18.8. For the SMC the RGC average is $19.14 \pm 0.08$. Assuming the moduli derived above are correct, $M_V = +0.36$ and $+0.23$ for the LMC and SMC, respectively, requiring a strong dependence of $M_V$ on metallicity. The corresponding numbers using the Udalski (1998) moduli are $+0.61$ and $+0.49$, respectively. Since the derived $M_V$ for NGC 2506 and NGC 2420 is $+0.47$ and both clusters are significantly more metal rich than the LMC and SMC, this would imply no dependence of $M_V$ on metallicity.

Surprisingly, despite the many pieces that make up this chain of reasoning, the weakest link remains the zero point of the cluster photometry. We emphasize, however, that in making our choices, our bias has been in the direction of minimizing the distance moduli by adopting the bluer colors for $V-I$. If the zero point of the NGC 2506 CCD photometry is confirmed and/or the giants of NGC 2420 are observed in $VI$ and are found to be consistent with NGC 2506, it would cast serious doubt on the claim that stars of similar apparent $[\text{Fe/H}]$ and age must have similar colors in $BV_I$; i.e., an additional parameter must be affecting the location of the giant branch. Moreover, it would make the distance moduli based upon $M_I$ as derived above larger.

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