THE VERY EFFECTIVE COVERS OF KO AND KGL OVER DEDEKIND SCHEMES

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Abstract. We answer a question of Hoyois–Jelisiejew–Nardin–Yakerson regarding framed models of motivic connective $K$-theory spectra over Dedekind schemes. That is, we show that the framed suspension spectrum of the presheaf of groupoids of vector bundles (respectively non-degenerate symmetric bilinear bundles) is the effective cover of $KGL$ (respectively very effective cover of $KO$). One consequence is that, over any scheme, we obtain a spectral sequence from Spitzweck’s motivic cohomology to homotopy algebraic $K$-theory; it is strongly convergent under mild assumptions.

1. Statement of results

Let $S$ be a scheme. The category $\mathcal{P}_\Sigma(\text{Cor}^{fr}(S))$ of presheaves with framed transfers [5, §2.3] is a motivic analog of the classical category of $\mathcal{E}_\infty$-monoids. We have the framed suspension spectrum functor

$$\Sigma^\infty_{fr}: \mathcal{P}_\Sigma(\text{Cor}^{fr}(S)) \to \mathcal{SH}(S)$$

which was constructed in [6, Theorem 18]. By analogy with the classical situation, one might expect that many interesting motivic spectra can be obtained as framed suspension spectra. This is indeed the case; see [8, §1.1] for a summary.

This note concerns the following examples of the above idea. One has framed presheaves [8, §6]

$$\text{Vect}, \text{Bil} \in \mathcal{P}_\Sigma(\text{Cor}^{fr}(S))$$

where $\text{Vect}(X)$ is the groupoid of vector bundles on $X$ and $\text{Bil}(X)$ is the groupoid of vector bundles with a non-degenerate symmetric bilinear form. There exist Bott elements

$$\beta \in \pi_{2,1}\Sigma^\infty_{fr}\text{Vect} \quad \text{and} \quad \tilde{\beta} \in \pi_{8,4}\Sigma^\infty_{fr}\text{Bil}$$

and canonical equivalences [7, Proposition 5.1] [8, Proposition 6.7]

$$\left(\Sigma^\infty_{fr}\text{Vect}\right)[\beta^{-1}] \simeq KGL \quad \text{and} \quad \left(\Sigma^\infty_{fr}\text{Bil}\right)[\tilde{\beta}^{-1}] \simeq KO.$$ 

Here $KGL$ is the motivic spectrum representing homotopy algebraic $K$-theory and $KO$ is the motivic spectrum representing homotopy hermitian $K$-theory. Again by comparison with the classical situation, this suggests that $\Sigma^\infty_{fr}\text{Vect}$ and $\Sigma^\infty_{fr}\text{Bil}$ should be motivic analogs of connective $K$-theory spectra. Another way of producing “connective” versions is by passing to (very) effective covers [12, 11]. It was proved in [8, 7] that these two notions of connective motivic $K$-theory spectra coincide, provided that $S$ is regular over a field.

Our main result is to extend this comparison to more general base schemes. We denote by $HZ$ Spitzweck’s motivic cohomology spectrum [11] and by $HW$ the periodic Witt cohomology spectrum [3, Definition 4.6].

Theorem 1.1. Let $S$ be a scheme.

(1) Suppose that $f_1(HZ) = 0 \in \mathcal{SH}(S)$. The canonical map

$$\Sigma^\infty_{fr}\text{Vect} \to f_0KGL \in \mathcal{SH}(S)$$

is an equivalence.

(2) Suppose in addition that $1/2 \in S$ and $HW_{\geq 2} = 0 \in \mathcal{SH}(S)$. The canonical map

$$\Sigma^\infty_{fr}\text{Bil} \to \tilde{f}_0KO \in \mathcal{SH}(S)$$

is an equivalence.

These assumptions are satisfied if $S$ is essentially smooth over a Dedekind scheme (containing $1/2$ in case (2)).

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As a notational convention for this introduction, whenever we mention $KO$ we shall assume that $1/2 \in S$. 

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Remark 1.2. That the assumptions are satisfied for Dedekind schemes is proved in [4, Proposition B.4] for (1) and in [3, Lemma 3.8] for (2). They in fact hold for all schemes; this will be recorded elsewhere.

Example 1.3. Bott periodicity implies formally that \( f_0\text{KGL} \simeq \Sigma^{2n} f_0\text{KGL} \) and \( s_n\text{KGL} \simeq \Sigma^{2n} f_0\text{KGL} / \beta \). Theorem 1.1(1) implies that \( f_0\text{KGL} / \beta \simeq H\mathbb{Z} \) (see Lemma 2.1). Hence in this situation the slice filtration for KGL yields a convergent spectral sequence, with \( E_2 \)-page given by (Spitzweck’s) motivic cohomology.

Notation. We use notation for standard motivic categories and spectra, as in [3] and [8].

2. Proofs

As a warm-up, we treat the case of KGL. Recall that the functor \( \Sigma_\infty^g \) inverts group-completion. The Bott element lifts to \( \beta : (\mathbb{P}^1, \infty) \to \text{Vect}^{\mathbb{P}} [7, \S 5] \). We also have the rank map \( \text{Vect}^{\mathbb{P}} \to \mathbb{Z} \in \mathcal{P}_{\Sigma}(\text{Cor}^{\text{fr}}(S)) \).

The composite

\[
(\mathbb{P}^1, \infty) \wedge \text{Vect}^{\mathbb{P}} \xrightarrow{\beta} \text{Vect}^{\mathbb{P}} \wedge \text{Vect}^{\mathbb{P}} \xrightarrow{m} \text{Vect}^{\mathbb{P}} \to \mathbb{Z}
\]

is null-homotopic after motivic localization, since \( \mathbb{Z} \) is motivically local and truncated and \( (\mathbb{P}^1, \infty)^{\text{mot}} \simeq S^1 \wedge \mathbb{G}_m \).

Lemma 2.1. The induced map

\[
(\Sigma_\infty^g \text{Vect}) / \beta \to \Sigma_\infty^g \mathbb{Z} \simeq H\mathbb{Z}
\]

is an equivalence.

Proof. The equivalence \( \Sigma_\infty^g \mathbb{Z} \simeq H\mathbb{Z} \) is [6, Theorem 21]. Since all terms are stable under base change [8, proof of Lemma 7.5] [6, Lemma 16], we may assume that \( S = \text{Spec}(\mathbb{Z}) \). Using [4, Proposition B.3] we further reduce to the case where \( S \) is the spectrum of a perfect field. In this case \( \Sigma_\infty^g \text{Vect} \simeq f_0\text{KGL} \) and so \( (\Sigma_\infty^g \text{Vect}) / \beta \simeq s_0\text{KGL} \simeq H\mathbb{Z} \) (see e.g. [1, Proposition 2.7]).

Proof of Theorem 1.1(1). Note first that if \( U \subset S \) is an open subscheme, and any of the assumptions of Theorem 1.1 holds for \( S \), it also holds for \( U \). On the other hand, if one of the conclusions holds for all \( U \) in an open cover, it holds for \( S \). It follows that we may assume that \( S \) is qcqs, e.g. affine.

Since \( f_1(H\mathbb{Z}) = 0 \) we find (using Lemma 2.1) that

\[
\beta : \Sigma_\infty^g \text{Vect} \to \Sigma^{-2,-1} \Sigma_\infty^g \text{Vect}
\]

induces an equivalence on \( f_i \) for \( i \geq 0 \). It follows that in the directed system

\[
\Sigma_\infty^g \text{Vect} \xrightarrow{\beta} \Sigma^{-2,-1} \Sigma_\infty^g \text{Vect} \xrightarrow{\beta} \Sigma^{-4,-2} \Sigma_\infty^g \text{Vect} \xrightarrow{\beta} \ldots
\]

all maps induce an equivalence on \( f_0 \). Since the colimit is KGL, \( f_0 \) commutes with colimits (here we use that \( X \) is qcqs, via [4, Proposition A.3(2)]) and \( \Sigma_\infty^g \text{Vect} \) is effective (like any framed suspension spectrum), the result follows.

The proof for KO is an elaboration on these ideas. From now on we assume that \( 1/2 \in S \). Recall from [3, Definition 2.6, Lemma 2.7] the motivic spectrum

\[
\mathbb{L}^M \simeq (H\mathbb{Z}/2)/\tau \in \mathcal{S}(S).
\]

For the time being, assume \( S \) is Dedekind. Taking framed loops we obtain

\[
\Omega_\infty^g \Sigma^{1,1} \mathbb{L}^M \in \mathcal{P}_{\Sigma}(\text{Cor}^{\text{fr}}(S)).
\]

Lemma 2.2. Let \( S \) be a Dedekind scheme, \( 1/2 \in S \).

1. We have \( \mathbb{L}^M \simeq a_N \tau_{\leq 0} \mathbb{G}_m / 2 \), where \( \mathbb{G}_m \in \mathcal{P}_{\Sigma}(\text{Cor}^{\text{fr}}(S)) \) denotes the sheaf \( O^\times \) with its usual structure of transfers [9, Example 2.4].

2. If \( f : S' \to S \) is a morphism of Dedekind schemes then \( f^* \mathbb{L}^M \simeq \mathbb{L}^M \in \mathcal{P}_{\Sigma}(\text{Cor}^{\text{fr}}(S')) \).

3. The canonical map \( \Sigma_\infty^g \mathbb{L}^M \to \Sigma^{1,1} \mathbb{L}^M \in \mathcal{S}(S) \) is an equivalence.

For this and some of the following arguments, it will be helpful to recall that we have an embedding of \( \text{Spc}^{\text{fr}}(S)^{\mathbb{P}} \) into the stable category of spectral presheaves on \( \text{Cor}^{\text{fr}}(S) \). In particular, many fiber sequences in \( \text{Spc}^{\text{fr}}(S) \) are cofiber sequences.
Proof. (1) Clear by construction since \( H^1(X, \mu_2) \cong \mathcal{O}^\times(X)/2 \) for \( X \) affine.

(2) By (1) we have a cofiber sequence \( \Sigma \mu_2 \to a_{Nis} \mathbb{G}_m/2 \to \mathbb{L}_1 \in \mathcal{P}_\Sigma(\text{Cor}^r(S)) \). Since pullback of framed presheaves preserves cofiber sequences and commutes with forgetting transfers up to motivic equivalence [6, Lemma 16] we reduce to the same assertion about \( \mathbb{G}_m, \mu_2 \), viewed as presheaves without transfers. Since they are representable, the assertion is clear.

(3) Using [4, Proposition B.3], (2) and [3, Theorem 4.4] we may assume that \( S \) is the spectrum of a perfect field. In this case \( \Sigma_f^\infty \Omega^\infty_f \cong f_0 \) [5, Theorem 3.5.14(i)], so we need only prove that \( \Sigma^{1,1} \mathbb{L}_1 \) is very effective. But this is clear since we have the cofiber sequence \( \Sigma^{1,0} \mathbb{H}/2 \to \Sigma^{1,1} \mathbb{H}/2 \to \Sigma^{1,1} \mathbb{L}_1 \) and \( \mathbb{H}/2 \) is very effective. \( \square \)

Construction 2.3. The assignment \( V \mapsto (V \oplus V^*, \varphi_V) \) sending a vector bundle to its associated (hyperbolic) symmetric bilinear bundle upgrades to a morphism
\[ \text{Vect} \to \text{Bil} \in \mathcal{P}_\Sigma(\text{Cor}^r(S))^{BC_2}, \]
where \( \text{Vect} \) carries the \( C_2 \)-action coming from passing to dual bundles, and \( \text{Bil} \) carries the trivial \( C_2 \)-action.

Proof. Since the presheaves are 1-truncated, all the required coherence data can be written down by hand. \( \square \)

Lemma 2.4. Let \( S \) be a Dedekind scheme containing \( 1/2 \).

1. The map
\[ (\text{Vect}^{sp})_{hC_2} \to \text{Bil}^{sp} \]
induces an isomorphism on \( a_{Nis} \pi_i \) for \( i = 1, 2 \).

2. The homotopy orbits spectral sequence yields
\[ a_{Nis} \pi_0 (\text{Vect}^{sp})_{hC_2} \cong \mathbb{Z}, \]

an exact sequence
\[ 0 \to \mathbb{L}_1 \to a_{Nis} \pi_1 (\text{Vect}^{sp})_{hC_2} \to \mathbb{Z}/2 \to 0 \]

and a map
\[ a_{Nis} \pi_2 (\text{Vect}^{sp})_{hC_2} \to \mathbb{Z}/2, \]

all as presheaves with framed transfers.

Proof. (1) This follows from the cofiber sequence \( K_{hC_2} \to GW \to L \) [10, Theorem 7.6] using that \( a_{Nis} \pi_i L = 0 \) unless \( i \equiv 0 \) (mod 4).

(2) The homotopy orbit spectral sequence just arises from the Postnikov filtration of \( \text{Vect}^{sp} \) and the formation of homotopy orbits and hence is compatible with transfers. Its \( E_2 \) page takes the form
\[ H_i(C_2, a_{Nis} \pi_j \text{Vect}^{sp}) \Rightarrow a_{Nis} \pi_{i+j} (\text{Vect}^{sp})_{hC_2}. \]
The form of the differentials of the spectral sequence implies that the terms \( H_i(C_2, a_{Nis} \pi_j \text{Vect}^{sp}) \) are permanent cycles for \( i \leq 1 \), and survive to \( E_\infty \) for \( (i, j) = (0,0) \) and \( (i, j) = (1,1) \). One has \( a_{Nis} \pi_0 \text{Vect}^{sp} = \mathbb{Z} \) with the trivial action and \( a_{Nis} \pi_1 \text{Vect}^{sp} = \mathbb{G}_m \) [13, Lemma III.1.4] with the inversion action. This already yields the first assertion. A straightforward computation shows that
\[ H_*(C_2, \mathbb{Z}) = \mathbb{Z}, \mathbb{Z}/2, 0, \mathbb{Z}/2, \ldots \]
and
\[ H_*(C_2, \mathbb{G}_m) = \mathbb{L}_1 \mathbb{L}_1, \mu_2, \mathbb{L}_1 \mathbb{L}_1, \ldots. \]
Since \( H_2(C_2, \mathbb{Z}) = 0 \), no differential can hit the \( (i, j) = (0,1) \) spot either, yielding the second assertion. Moreover this implies that \( H_1(C_2, \mathbb{G}_m) = \mu_2 \) is the bottom of the filtration of \( \pi_2 \). It follows that there is a map \( a_{Nis} \pi_2 (\text{Vect}^{sp})_{hC_2} \to A \), where \( A \) is a quotient of \( \mu_2 \). To prove that \( A = \mu_2 \) it suffices to check this on sections over a field, in which case we can use the hermitian motivic spectral sequence of [2]. \( \square \)

We have \( a_{Nis} \pi_0 \text{Bil}^{sp} \cong GW \). Thus we can form the following filtration of \( \text{Bil}^{sp} \) refining the Postnikov filtration
\[ \text{Bil}^{sp} \leftarrow F_1 \text{Bil}^{sp} \leftarrow F_2 \text{Bil}^{sp} \leftarrow F_3 \text{Bil}^{sp} \leftarrow F_4 \text{Bil}^{sp} \in \mathcal{P}_\Sigma(\text{Cor}^r(S)) \]
with subquotients given Nisnevich-locally by
\[ (2.1) \quad GW, \Sigma \mathbb{Z}/2, \Sigma \mathbb{L}_1, \Sigma \mathbb{Z}/2. \]

Recall also the framed presheaf \( \text{Alt} \in \mathcal{P}_\Sigma(\text{Cor}^{fr}(S)) \) sending a scheme to the groupoid of vector bundles with a non-degenerate alternating form. Tensoring with the canonical alternating (virtual) form \( \mathcal{H}(1) - h \)
on $H^\mathbb{P}^1$ (where $H(1)$ is the tautological rank 2 alternating form on $H^\mathbb{P}^1$, and $h$ is the standard alternating form on a trivial vector bundle of rank 2) yields maps

$$\sigma_1 : H^\mathbb{P}^1 \wedge \text{Alt}^\mathbb{SP} \to \text{Bil}^\mathbb{SP} \quad \text{and} \quad \sigma_2 : H^\mathbb{P}^1 \wedge \text{Bil}^\mathbb{SP} \to \text{Alt}^\mathbb{SP};$$

by construction we have $\tilde{\beta} = \sigma_1 \sigma_2$ (recall that $H^\mathbb{P}^1 \overset{\text{mot}}{\simeq} S^{4,2}$).

**Lemma 2.5.** Let $S$ be a Dedekind scheme, $1/2 \in S$.

1. The composite

$$H^\mathbb{P}^1 \wedge \text{Alt}^\mathbb{SP} \to \text{Bil}^\mathbb{SP} \to \text{Bil}^\mathbb{SP}/F_i \text{Bil}^\mathbb{SP}$$

is motivically null. The induced map

$$\Sigma^\infty \text{cof}(\sigma_1) \to \Sigma^\infty \text{Bil}^\mathbb{SP}/F_i \text{Bil}^\mathbb{SP}$$

is an equivalence.

2. The composite

$$H^\mathbb{P}^1 \wedge \text{Bil}^\mathbb{SP} \to \text{Alt}^\mathbb{SP} \to \mathbb{Z}$$

is motivically null. The induced map

$$\Sigma^\infty \text{cof}(\sigma_2) \to \Sigma^\infty \mathbb{Z}$$

is an equivalence.

**Proof.** (1) Write $C$ for the cofiber computed in the category of spectral presheaves on $\text{Cor}^\mathbb{fr}(S)$. Then $C$ admits a finite filtration, with subquotients corresponding to those in (2.1). Since each of those is the infinite loop space of a motivic spectrum, it follows that $C$ is in fact motivically local. Consequently $C$ corresponds to $\text{Bil}^\mathbb{SP}/F_i \text{Bil}^\mathbb{SP}$ under the embedding into spectral presheaves. These contortions tell us that there are fiber sequences

$$F_{i+1} \text{Bil}^\mathbb{SP}/F_i \text{Bil}^\mathbb{SP} \to F_i \text{Bil}^\mathbb{SP}/F_i \text{Bil}^\mathbb{SP} \to F_i \text{Bil}^\mathbb{SP}/F_{i+1} \text{Bil}^\mathbb{SP}$$

for $i < 4$. Hence to prove that the composite is null, it suffices to prove that there are no maps from $\Sigma^{4,2} \text{Alt}^\mathbb{SP}$ into the motivic localizations of the subquotients of the filtration given in (2.1). These motivic localizations are $\bigwedge W, L_{Nis} K(\mathbb{Z}/2, 1), L_{Nis} K(k_2^M, 1)$ and $L_{Nis} K(\mathbb{Z}/2, 2)$ (since they are motivically equivalent to the subquotients, and motivically local because they are infinite loop spaces of the motivic spectra $H\mathbb{Z}, \Sigma^M, \Sigma^{2,1} L^M, \Sigma^{2,2} L^M$). It suffices to prove that $\Omega^{4,2}$ of these subquotients vanishes, which is clear. Next we claim that $\Sigma^\infty \text{Bil}^\mathbb{SP}/F_i \text{Bil}^\mathbb{SP}$ is stable under base change (among Dedekind schemes containing $1/2$). Indeed the defining fiber sequences of $F_i \text{Bil}^\mathbb{SP}$ are also cofiber sequences, and so $\Sigma^\infty \text{Bil}^\mathbb{SP}/F_i \text{Bil}^\mathbb{SP}$ is obtained by iterated extension from spectra stable under base change (see Lemma 2.2(2) for $L^M$, [8, proof of Lemma 7.5] for Bil and Alt, and [6, Lemma 16] for $\mathbb{Z}/2$). To prove that the induced map is an equivalence we thus reduce as before to $S = \text{Spec}(k)$, $k$ a perfect field of characteristic $\neq 2$. In this case the result is a straightforward consequence of the hermitian motivic filtration of [2].

(2) The proof is essentially the same as for (1), but easier. \hfill $\Box$

We now arrive at the main result.

**Theorem 2.6.** Let $S$ be a scheme containing $1/2$ such that

$$f_1(H\mathbb{Z}) = 0 = HW_{\geq 2} \in \text{SH}(S).$$

The canonical maps

$$\Sigma^\infty \text{Bil} \to f_0 \text{KO} \quad \text{and} \quad \Sigma^\infty \text{Alt} \to f_0 \Sigma^{4,2} \text{KO}$$

are equivalences.

**Proof.** As before we may assume that $S$ is qcqs.

We know that KO is the colimit of

$$\Sigma^\infty \text{Bil} \overset{\sigma_2}{\to} \Sigma^{4,2} \Sigma^\infty \text{Alt} \overset{\sigma_1}{\to} \Sigma^{8,4} \Sigma^\infty \text{Bil} \overset{\sigma_2}{\to} \cdots .$$

It is hence enough to prove that $\sigma_1 : \Sigma^{-8n, 4n} \Sigma^\infty \text{Bil} \to \Sigma^{-8n, 4n} \Sigma^\infty \text{Alt}$ induces an equivalence on $f_0$ for every $n \geq 0$, and similarly for $\sigma_2$. (Here we use that $S$ is qcqs, so that $f_0$ preserves filtered colimits.) Given a cofiber sequence $A \to B \to C$, in order to prove that $f_0 A \simeq f_0 B$, it suffices to show that $\text{Map}(X, C) = *$ for every $X \in \text{SH}(S)^{\text{eff}, \mathbb{L}}$, i.e. that $C \in \text{SH}(S)^{\text{eff}, \mathbb{L}}$.

Over $\mathbb{Z}[1/2]$, the cofiber of $\sigma_1$ has a finite filtration, with subquotients

$$\Sigma^{-4,2 \Sigma^\infty \bigwedge W, \Sigma^{-3,2 \Sigma^\infty \mathbb{Z}/2, \Sigma^{-3,2 \Sigma^\infty \Sigma^M, \Sigma^{-2,2 \Sigma^\infty \mathbb{Z}/2, \Sigma^{-2,2 \Sigma^\infty \mathbb{Z}/2,}}},$$
and the cofiber of $\sigma_2$ is $\Sigma^{-4,-2}\Sigma_{fr}^{\infty}\mathbb{Z}$. Using [6, Corollary 22], [8, Theorem 7.3] and Lemma 2.2(3), we can identify the list of cofibers as

$$\Sigma^{-4,-2}H\mathbb{Z}, \Sigma^{-3,-2}H\mathbb{Z}/2, \Sigma^{-2,-1}\mathbb{L}, \Sigma^{-2,-2}H\mathbb{Z}/2, \Sigma^{-4,-2}H\mathbb{Z}. $$

These spectra are stable under arbitrary base change (essentially by definition), and hence for arbitrary $S$ the cofibers of $\sigma_1, \sigma_2$ are obtained as finite extensions, with cofibers in the above list. To conclude the proof, it will thus suffice to show that all spectra in the above list are in $\mathcal{SH}(S)^{veff,\perp}$.

Note that if $E \in \mathcal{SH}(S)$ then $E \in \mathcal{SH}(S)^{veff,\perp}$ if and only if $\Omega^\infty E \simeq *$. In particular this holds if $f_0E = 0$. This holds for $\Sigma^{m,n}H\mathbb{Z}$ as soon as $n < 0$, by assumption. Hence it also holds for $\Sigma^{m,n}H\mathbb{Z}/2$ in the same case ($f_0$ being a stable functor) and for

$$\Sigma^{m,n}\mathbb{L} \simeq \text{cof}(\Sigma^{m,n}1H\mathbb{Z}/2 \xrightarrow{\Sigma} \Sigma^{m,n}H\mathbb{Z}/2).$$

The only spectrum left in our list is $\Sigma^{-4,-2}H\mathbb{Z}$. Using [3, Definition 4.1] we see now that $\Omega^\infty \Sigma^{-4,-2}H\mathbb{Z} \simeq \Omega^\infty \Sigma^{-4,-2}K^W$, so we may treat the latter spectrum. We have $K^W/\eta \simeq K^M$ [3, Lemma 3.9], whence $\eta: \Sigma^{-4-n,-2-n}K^W \to \Sigma^{-5-n,-3-n}K^W$ induces an equivalence on $\Omega^\infty$. Since $\Omega^\infty$ commutes with filtered colimits, we see that $\Sigma^{-4,-2}K^W \in \mathcal{SH}(S)^{veff,\perp}$ if and only if $\Sigma^{-4,-2}K^W[\eta^{-1}] \in \mathcal{SH}(S)^{veff,\perp}$. This latter spectrum is the same as $\Sigma^{-2}H\mathbb{W}$ [3, Lemma 3.9], and

$$\tilde{f}_0(\Sigma^{-2}HW) \simeq \tilde{f}_0((\Sigma^{-2}HW)_{\geq 0}) \simeq \tilde{f}_0(\Sigma^{-2}(HW_{\geq 2})) = 0$$

by assumption. 

\begin{thebibliography}{99}

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