UNIVERSAL EQUATION FOR RESISTANCE COEFFICIENTS FOR MITRE BENDS OF ALL ANGLES

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Abstract
This paper presents a universal equation formulated on the basis of IS 2951-Part-2 to determine resistance coefficient for losses at bends of all angles accurately.

Index Terms: Pipe bends friction losses, universal equation, resistance coefficient, bend angle & curve fitting.

1. Introduction
While working on a project to design treated sewage water transmission line from Bhandewadi 200MLD STP to Koradi power station, it was found that the number of bends in the alignment were unusually high as the line passes through inhabited urban areas. Consequently, the total losses in bends were very high, actually more than 20% of net frictional head loss in the pipeline.

IS: 2951 (Part II) – 1965 HEAD LOSS IN VALVES AND FITTINGS guides the hydraulic designers to determine losses in bends and various fittings in the pipeline. However, the standard provides the values of resistance coefficients for only a few standard bend angles, while, in field, angles of all intermediate values are encountered.

A. Field Scenario
1) Need: In the IS: 2951 (Part-II), as also in most international standards, the values for resistance coefficients ’k’ are given only for particular standard angles (5°, 10°, 15°, 22.5°, 30°, 45°, 60° and 90°). But field conditions necessitate bend of intermediate angle at most locations to suit site. To determine losses in such intermediate bends, design engineers usually adopt the ’k’ values of next higher ’standard bend’ against the field angle of bend - e.g. for a 12° bend the ’k’ value of 15° bend is adopted. This causes overestimation of losses, which consequently detrimentally affects the system’s field performance as the pumps are required to operate away from the designed duty point.

2) Our Case: Pipeline is passing through urban areas, through a total of 20 km defined route. There are number of horizontal and vertical bends of various angles in the alignment. The area layout and road layout does not allow enough space for laying of standard bends. So, according to the requirement of alignment there are random degrees of bend. In order to account for the losses at all such intermediate bends the correct applicable K value has to be used for any random bend from 0° to 90°.

3) Pipe Details: In our project, the designed pipe diameter is 1500 mm ID, spirally welded mild steel pipes, with epoxy lining inside (smoother than bituminized normal lining condition in IS 2951-I). This suits site requirement and any degree of bend can be fabricated.

4) Practice: Most design engineers use a thumb rule of considering 10% of net frictional head loss in pipeline as addition for losses in bends and fittings. This is an approximation and not a recommended practice. In case of this project the actual losses in bends are around 25% which is far greater than 10%. Use of thumb rule would have resulted in serious under estimation of head loss and consequent under performance of the system.

Detailed calculations by adopting ’k’ value of next higher bend, as is the usual practice, for such large number of bends, would have resulted in serious over-estimation of the losses in bends.

Hence, a need was felt to find a way to calculate the correct applicable ’k’ value for all values of angles of bends between 0 to 90 degrees so as to aid us and all hydraulic engineers to determine the losses in any degree of bend accurately.

B. IS: 2951(Part-II) - 1965 (Reaffirmed in 2003)
1) Reference: Page 9, Fig-3: Resistance coefficient for Mitre bends.
2) Recommendation: The K values for single-cut bends
and multi-cut bends are given for smooth and rough nature of pipe surfaces. For single cut bends, the values of resistance coefficient are shown below:

| Degree of bend | Ksm | 0.016 | 0.024 | 0.130 |
|----------------|-----|-------|-------|-------|
| Kro            | 0.044 | 0.062 | 0.329 |

| Degree of bend | Ksm | 0.042 | 0.471 | 1.129 |
|----------------|-----|-------|-------|-------|
| Kro            | 0.165 | 0.684 | 1.265 |

### 2. Methodology

#### A. Determining Resistance Coefficients

To arrive at a correct applicable value of 'k' for each degree of random bend and with the aim to overcome serious over-estimation of losses in bends, we plotted resistance coefficients of standard angles vs. angle of bends for both smooth and rough pipes, as given in IS 2951-II, as data point cluster.

The graph is shown in "Fig-1" below. The nature of plot shows that the data cluster apparently follows a parabolic curve pattern.

#### B. Aim

We aimed at formulating a mathematical equation using curve fitting method to yield the values of 'k' for any degree of bend which would conform to the values in IS code for standard bends.

This formulation of mathematical equation can be readily used in computer programs and spreadsheets to easily and accurately evaluate the losses in all random degree bends.

Graph was plotted between resistance coefficient and bend for both smooth and rough pipe, and then several curves were fit to get the best fit for the values of resistance coefficient.

#### C. Curve Fitting

1) Linear

![Linear curve fit](image)

2) Logarithmic

![Logarithmic curve fit](image)

3) Exponential

![Exponential curve fit](image)

4) i) Second order polynomial (without forced zero intercept)

![Second order polynomial](image)
The basic parameters like velocity ($U$), density ($\rho$) and viscosity ($\eta$) can be measured by ultrasonic interferometer. From these parameters various thermodynamical and acoustical parameters such as adiabatic compressibility, ultrasonic velocities, densities and viscosities in the wide range of concentrations at 35°C, 40°C and 45°C temperatures for Acetone + Propanol – 2 

Table 1

| Ultrasonic Velocity and related parameters for Acetone+ Propanol-2 + Toluene |
|--------------------------------------------------|

D. Inference

1. Linear curve fit, Logarithmic curve and exponential curve - neither fits well nor ranges within the data values of ‘$k$’. Hence these were discarded.

2. Second order polynomial curve fit, with and without forced intercept at zero were tried.
   a. Without forced zero intercept the fit resulted in a curve with correlation coefficient $R^2=0.9804$, which is a good fit.
   b. With forced zero intercept the fit resulted in a curve with correlation coefficient $R^2=0.9799$, which is also a good fit overall but has anomalous fit in the angles less than 15 degrees.

3. We also tried a third order polynomial curve, which has a normal near zero intercept. This resulted in a curve with correlation coefficient $R^2=0.9801$, which is also a good fit overall but also has anomalous fit in the angles less than 15 degrees.

4. Thus, the second order polynomial curve without forced zero intercept came out as the best fit with best value of correlation coefficient $R^2$, hence it was selected as the correct representative mathematical equation for finding correct applicable values of ‘$k$’ for all angles between 0 to 90 degree angles.

5. The final selected equation is:

$$k = 0.00014638 x^2 - 4.4284900 \times 10^{-5} x + 0.02088767$$

Where $x$ = bend angle in degrees.

E. Calculation of Head Loss

The head loss at bends is calculated using,

$$h_i = \frac{kv^2}{2g}$$

where, $k$ = resistance coefficient for valves or fittings,

$v$ = average velocity in a pipe of corresponding diameter in m/s,

$g$ = acceleration due to gravity in m/sec².

Now, calculating values of $K$ from the equation of curve that best fits the data, we calculated head loss at each bend in the alignment.

Equation of selected second order polynomial curve:

$$k = 0.00014638 x^2 - 4.4284900 \times 10^{-5} x + 0.02088767$$

Suppose for 6° bend and velocity of 1.441 m/s, the value of 

$$k=0.00014638 \times 6^2 - 4.4284900 \times 10^{-5} \times 6 + 0.02088767$$

$$k = 0.02589$$

$$h_i = 0.02589 \times \frac{1.441^2}{2 \times 9.81}$$

$$h_i = 0.00274 \text{ m}$$

Similarly, accounting for losses in every bend using the values of $k$ obtained by above method, we arrived at a total loss of 6.07844 m in bends.

Following table shows the significant difference in estimation of losses in bend when calculated with values of ‘$k$’ for next higher standard bend vis-à-vis the actual value for bend angle obtained by curve fitting equation.

Table 2

| Total Head Loss At Bends In Full Length With | Values for next higher std. bend as per IS | Values for actual bend by curve fitting equation |
|---------------------------------------------|--------------------------------------------|-------------------------------------------------|
| STP-Y-TTWR                                   | 7.54412 m                                  | STP-Y-TTWR                                       |
|                                             |                                            | 6.07844 m                                       |
3. Conclusion

As was seen above the actual loss calculated is 6.07844 m, but if we would have followed the values of k for standard angles only and adopted the values for next higher standard bend, we would have arrived at total head loss in bends as 7.54412 m. **The difference is of 1.46568 m**, which would have been an over estimation of losses in bends.

Overestimation of losses would have resulted in selection of pumps with a higher duty head. The system in field would have offered a lower head and the pumps would have been required to work at lower than its duty head. Obviously a pump selected with highest efficiency at its duty head would have functioned away from its duty point with a lower efficiency.

For reliability of system, such approximations needs to be avoided. This equation can be used by Hydraulic engineers to reach to more accurate calculation of Head losses in bends of all field angles and total resistance of the system.

4. Result & Recommendation

Equation for finding correct applicable values of k, the resistance coefficient for any bend angles is:

\[ k = 0.00014638 x^2 - 4.4284900 \times 10^{-3} x + 0.02088767 \]

Where, \( x \) = angle of bend in degrees.

It best fits the data and gives accurate applicable values of 'k' which can be used for precise estimation of losses in any bend.

5. FURTHER WORK

The values of 'k' evaluated by this equation may be checked against field data for various bend angles by installing calibrated high precision differential pressure gauges across the bends. This work may be taken up by agencies with adequate funding for the same.

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