A note on finite-time and fixed-time stability

Lu Wenlian, Liu Xiwei, Tianping Chen

Abstract

In this letter, by regarding finite-time stability as an inverse problem, we reveal the essence of finite-time stability and fixed-time stability. Some necessary and sufficient conditions are given. As application, we give a new approach for finite-time and fixed-time synchronization and consensus. Many existing results can be derived by the general approach.

Key words: Consensus; Time-varying topology; Event-triggered algorithm

1 Introduction

In many practical situations, stability over a finite time interval is of interests rather than the classic Lyapunov asymptotic stability, since it is more physically realizable than concerning infinite time. There are two categories of concepts of stability over finite time interval. One is finite-time stability that means that the system converges within a finite time interval for any initial values; the other is fixed-time stability that means that the time intervals of convergence have a uniform upper-bounds for all initial values within the definitive domain. The previous works on this topic include Dorato 1961, Roxin 1966, Haimo 1986, Bhat and Bernstein 1998, 2000, Hong, et. 2002., In particular, Lu and Chen 2005 presented analysis of the finite-time convergence for Cohen-Grossberg neural networks with discontinuous activation function.

The finite-time and fixed-time stability/convergence have been successfully applied in many fields. More related to the present letter, synchronization and consensus in networked systems have been attracting increasing interests (Lu, W.L and Chen 2004, 2006). Many recent literature were concerned

1This work is jointly supported jointly supported by the National Natural Sciences Foundation of China (Nos. 61273211 and 61273309), and the Program for New Century Excellent Talents in University (NCET-13-0139).
2 Wenlian Lu is with the Centre for Computational Systems Biology and School of Mathematical Sciences, Fudan University, People’s Republic of China (wenlian@fudan.edu.cn)
3 Xiwei Liu is with Department of Computer Science and Technology, Tongji University, and with the Key Laboratory of Embedded System and Service Computing, Ministry of Education, Shanghai 200092, China. E-mail: xwliu@tongji.edu.cn
4 Tianping Chen is with the School of Computer Sciences/Mathematics, Fudan University, 200433, Shanghai, China. Corresponding author: Tianping Chen. Email: tchen@fudan.edu.cn
with proposing schemes to realize finite-time synchronization/consensus. See Cortés 2006; Shen and Xia 2008, 2009; Xiao et al 2009; Jiang and Wang 2009; Wang and Hong 2010; Jiang and Wang 2011; Su et al 2012; Polyakov 2012; Parsegov et al 2012, 2013; Zhao et al 2015; Liu et al 2014, 2015; Wang et al 2014; Zou and Tie 2014a,b; Polyakov et al 2015; Zou 2015; Meng et al 2015 for reference. The main techniques in these works depends on the candidate Lyapunov functions as well as its convergence. In this letter, we propose a simple, novel and general technique to re-visit the problem of finite-time and fixed-time stability by regarding as an implicit inverse function of time and apply to the synchronization and consensus in networked system.

To exploit the idea, suppose a nonnegative scalar function $V(t)$ satisfies

$$\dot{V}(t) = -\mu(V(t)), \quad (1)$$

where functions $\mu(V(t)) > 0$, $V(t) > 0$; $\mu(0) = 0$.

Because $\dot{V}(t) > 0$, $V(t)$ is decreasing. Therefore, the trajectory can also be written as

$$\dot{t}(V) = -\mu^{-1}(V), \quad (2)$$

(See the following figure) where $t(V)$ be the inverse function of $V(t)$. Then,

$$t = - \int_{V(0)}^{V(t)} \mu^{-1}(V)dV \quad \quad (3)$$

Therefore, the least time $t^*$ to make $V(t^*(V(0))) = 0$ is

$$t^*(V(0)) = \int_{0}^{V(0)} \frac{1}{\mu(V)}dV$$

In summary, we have

**Proposition 1.** For system (1),
1. “0” is a finite-time stable equilibrium for the system (I), i.e. there exists a time \( t^* = t^*(V(0)) \) depending on the initial value \( V(0) \), such that \( V(t) = 0 \), if \( t \geq t^*(V(0)) \), it is necessary and sufficient that the integral

\[
t^*(V(0)) = \int_0^{V(0)} \frac{1}{\mu(V)} dV
\]

is finite.

2. “0” is a fixed-time stable equilibrium for the system (II), i.e. there exists a time \( t^* \) independent of the initial value \( V(0) \), such that \( V(t) = 0 \), if \( t \geq t^* \), it is necessary and sufficient that the integral

\[
t^* = \int_0^{\infty} \frac{1}{\mu(V)} dV
\]

is finite.

**Remark 1.** It is clear that finite-time convergence is an inverse problem: To find the time \( \bar{t} \) so that \( V(\bar{t}) = 0 \). Therefore, instead of \( V(t) \), we discuss the inverse function \( t(V) \). Previous results reveal that the finite-time convergence depends on the behavior of \( \mu(V) \) in the neighborhood of \( V = 0 \).

**Remark 2.** Instead, the fixed-time convergence depends on the behavior of \( \mu(V) \) at \( V = 0 \) as well as the behavior of \( \mu(V) \) at \( \infty \).

**Remark 3.** Geometrically, a system is stable is equivalent to that its trajectory \( x(t) \) is with finite length in state space \( x \). Instead, finite-time convergence means that the trajectory \( x(t) \) is with finite length in time-state space \( (x, t) \).

In case \( \mu(s) = s \), then

\[
t = -\alpha^{-1} \int_{V(0)}^{V(t)} \frac{1}{V} dV = -\alpha^{-1} \log \frac{V(t)}{V(0)},
\]

and

\[
V(t) = V(0)e^{-\alpha t}.
\]

It is clear that the integral

\[
\int_0^{V(0)} \frac{1}{\mu(V)} dV = \infty
\]

Therefore, there is no \( t^* \) such that \( V(t^*)=0 \). In fact, the system (II) is exponentially stable.
Figure 2: Convergence behaviors for different index "p".

If $\mu(s) = s^p$ with $p \neq 1$, then

$$t = \alpha^{-1} \int_{V(t)}^{V(0)} \frac{1}{V^p} dV = \frac{V^{1-p}(0) - V^{1-p}(t)}{\alpha(1-p)}$$

and

$$V(t) = [V^{1-p}(0) - \alpha(1-p)t]^{\frac{1}{1-p}}$$

In case $p < 1$, then $V(t) = 0$, if

$$t \geq t^* = \alpha^{-1} \int_{0}^{V(0)} \frac{1}{V^p} dV = \frac{V^{1-p}(0)}{\alpha(1-p)}$$

which means that $V(t)$ converges to zero in finite-time.

On the other hand, in case $p > 1$,

$$V(t) = \frac{1}{[\alpha(p-1)t + V^{1-p}(0)]^{\frac{1}{1-p}}}$$

which means that $V(t)$ does not converge in finite time. Instead, it converges to zero with power rate $t^{-(p-1)}$ (see Chen. T. et.al, 2007).

With similar approach, we have the following

**Theorem 1.** Suppose the Dini derivative of a nonnegative function $V(t) = V(z(t))$ satisfies

$$\dot{V}^+(t) \leq \begin{cases} -\mu_1(V(t)) & \text{if } 0 < V < a \\ -\mu_2(V(t)) & \text{if } V \geq a \end{cases}$$

for some constant $a > 0$, where functions $\mu_1(V(t)) > 0$, $\mu_2(V(t)) > 0$, when $V(t) > 0$; $\mu_1(0) = 0$; and

$$\int_{0}^{a} \frac{1}{\mu_1(V)} dV = \omega_1 < \infty,$$

$$\int_{a}^{\infty} \frac{1}{\mu_2(V)} dV = \omega_2 < \infty$$
for some constant $a > 0$. Then $V(t) \equiv 0$ for all $t \geq \omega_1 + \omega_2$, i.e., the fixed-time stability of “0” is realized.

Proof. In this case, no matter $V(0) \leq 1$ or $V(0) \geq 1$, we can prove

$$t(0) - t(V(0)) \leq \omega_1 + \omega_2.$$ 

The proof is completed. 

Remark 4. System governed by (4) is written as event-triggered system, which can also be written as following time-triggered model

$$\dot{V}(t) = \begin{cases} -\mu_1(V(t)) & \text{if } t > \omega_2 \\ -\mu_2(V(t)) & \text{if } t \leq \omega_2 \end{cases}$$

(5)

Remark 5. The model discussed in Lu and Chen (2005) for Cohen-Grossberg neural networks with discontinuous activation functions can be regarded as

$$\dot{V}(X(t)) = \mu(V(X(t)),$$

where $\mu(V)$ is a discontinuous monotone-nondecresing function with equilibrium $V^*$ lying in the discontinuity of the activation functions. In this case, finite-time convergence can be ensured (see Theorem 8 in Lu and Chen (2005)).

2 Applications: Finite-time and fixed-time synchronization and consensus

In this section, we will apply the theoretical results given in previous section to finite-time and fixed-time synchronization and consensus, where the nodes are nonlinearly coupled and the network is a strongly connected undirected graph.

2.1 Finite-time synchronization

In (Lu and Chen 2004, 2006), and some other papers, the following linear coupled system

$$\dot{x}_i(t) = f(x_i(t)) + \alpha \sum_{j=1}^{N} a_{ij}(x_j(t) - x_i(t)), \quad i = 1, \cdots, N,$$

(6)
is discussed. By defining following useful reference node given in (Lu and Chen 2004, 2006):

\[ x^*(t) = \frac{1}{N} \sum_{i=1}^{N} x_i(t), \]  

(7)

and following Lyapunov function (see Lu and Chen 2004, 2006)

\[ V(t) = \frac{1}{2} \sum_{i=1}^{N} (x_i(t) - x^*(t))^T (x_i(t) - x^*(t)) \]
\[ = \frac{1}{2N} \sum_{i,j=1}^{N} (x_i(t) - x_j(t))^T (x_i(t) - x_j(t)) \]
\[ = \frac{1}{2N} \sum_{k=1}^{n} \sum_{i,j=1}^{N} |x_i^k(t) - x_j^k(t)|^2, \]

(8)

it was proved that under some mild conditions, \( \dot{V}(t) \leq -\alpha V(t) \) for some constant \( \alpha \). Therefore by previous result for \( \mu(V) = V \), the convergence is exponential and not with finite-time.

To make the convergence finite-time, in this section, replacing (18), consider nonlinear coupled network with \( N \) nodes:

\[ \dot{x}_i(t) = f(x_i(t)) + \alpha \sum_{j=1}^{N} a_{ij} \Phi(x_j, x_i), \]

(9)

where scalars \( \alpha > 0, \) \( x_i = (x_i^1, \ldots, x_i^n)^T \in \mathbb{R}^n, \) \( i = 1, \ldots, N. \) Coupling matrix \( A = (a_{ij}) \) is symmetric and irreducible, with \( a_{ij} \geq 0, i \neq j. \)

Continuous function \( f : \mathbb{R}^n \rightarrow \mathbb{R}^n \) satisfies: for any \( U = [u^1, \ldots, u^n]^T \in \mathbb{R}^n, V = [v^1, \ldots, v^n]^T \in \mathbb{R}^n, \) there exists a scalar \( \delta > 0, \) such that

\[ (U - V)^T (f(U) - f(V)) \leq \delta(U - V)^T (U - V). \]

(10)

The nonlinear function \( \Phi(\cdot, \cdot) : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^n \) is defined as (see [?]):

\[ \Phi(U, V) = (\phi(u^1, v^1), \ldots, \phi(u^n, v^n))^T, \]

(11)

where \( \phi(x, y) = sign(y - x) \frac{\mu(|x-y|^2)}{|x-y|}, \) and the function \( \mu(V) > 0 \) satisfies

1. semi-norm property

\[ \mu(V + U) \leq c_\mu(\mu(U) + \mu(V)) \]

2. \( \frac{V}{\mu(V)} \) is monotone increasing when \( V \) increasing.
Noticing $A$ is strongly connected and the semi-norm property for function $\mu$, we can find some constants $a_{\mu,A}$, $\bar{a}_{\mu,A}$ depending on the function $\mu$ and the coupling matrix $A$ such that

$$\sum_{k=1}^{n} \sum_{i,j=1}^{N} a_{ij} |x^k_i(t) - x^k_j(t)| \phi(|x^k_i(t) - x^k_j(t)|)$$

$$= \sum_{k=1}^{n} \sum_{i,j=1}^{N} a_{ij} \mu(|x^k_i(t) - x^k_j(t)|^2)$$

$$\geq a_{\mu,A} \sum_{k=1}^{n} \sum_{i,j=1}^{N} \mu(|x^k_i(t) - x^k_j(t)|^2)$$

$$\geq \bar{a}_{\mu,A} \mu(\sum_{k=1}^{n} \sum_{i,j=1}^{N} |x^k_i(t) - x^k_j(t)|^2)$$  \hspace{1cm} (12)

Based on previous preparations, we can give

**Theorem 2.** Suppose that irreducible matrix $A = (a_{ij}) \in R^{N \times N}$ satisfies $a_{ij} = a_{ji} \geq 0$, $a_{ii} = 0$. The coupling strength $\alpha$ is chosen such that

$$\frac{\alpha \bar{a}_{\mu,A}}{2} > 2\delta \frac{V(0)}{\mu(V(0))}. \hspace{1cm} (13)$$

Then, the system (9) reaches synchronization for all

$$t \geq \left( \frac{\alpha \bar{a}_{\mu,A}}{2} - 2\delta \frac{V(0)}{\mu(V(0))} \right)^{-1} \int_{0}^{V(0)} \frac{dV}{\mu(V)} \hspace{1cm} (14)$$

**Proof.** Differentiating the Lyapunov function (8), we have

$$\dot{V}(t) = \sum_{i=1}^{N} (x_i(t) - x^*(t))^T (x_i(t) - x^*(t))'$$

$$= \sum_{i=1}^{N} (x_i(t) - x^*(t))^T (f(x_i(t)) - f(x^*(t)))$$

$$+ \alpha \sum_{i=1}^{N} (x_i(t) - x^*(t))^T \sum_{j=1}^{N} a_{ij} \Phi(x_j(t), x_i(t))$$

$$= V_1(t) + V_2(t), \hspace{1cm} (15)$$

where

$$V_1(t) \leq \delta \sum_{i=1}^{N} (x_i(t) - x^*(t))^T (x_i(t) - x^*(t)) = 2\delta V(t), \hspace{1cm} (16)$$
By some algebra, we have

\[ V_2(t) = \alpha \sum_{i=1}^{N} (x_i(t) - x^*(t))^T \sum_{j=1}^{N} a_{ij} \Phi(x_j(t), x_i(t)) \]

\[ = \alpha \sum_{i,j=1}^{N} a_{ij} (x_i(t) - x_j(t))^T \Phi(x_j(t), x_i(t)) \]

\[ = \frac{\alpha}{2} \sum_{i,j=1}^{N} a_{ij} (x_i(t) - x_j(t))^2 \Phi(x_j(t), x_i(t)) \]

\[ \leq - \frac{\alpha \bar{a}_{\mu,A}}{2} \mu \left( \sum_{k=1}^{n} \sum_{i,j=1}^{N} |x_i^k(t) - x_j^k(t)|^2 \right) \]

Therefore,

\[ \dot{V}(t) \leq 2\delta V(t) - \frac{\alpha \bar{a}_{\mu,A}}{2} \mu(V(t)) \]

\[ = - \left( \frac{\alpha \bar{a}_{\mu,A}}{2} - 2\delta \frac{V(t)}{\mu(V(t))} \right) \mu(V(t)) \]

\[ \leq - \left( \frac{\alpha \bar{a}_{\mu,A}}{2} - 2\delta \frac{V(0)}{\mu(V(0))} \right) \mu(V(t)) \]

By Theorem 1, the proof is completed.  

\[ \square \]

### 2.2 Fixed-time synchronization

In this part, we consider fixed-time synchronization.

Suppose \( A = (a_{ij}) \) and \( B = (b_{ij}) \) with \( a_{ij} \geq 0 \) and \( b_{ij} \geq 0, i \neq j \) are two symmetric and irreducible matrices.

Consider following system

\[ \dot{x}_i(t) = \begin{cases} f(x_i(t)) + \alpha \sum_{j=1}^{N} a_{ij} \Phi(x_j, x_i) & t \geq t^* \\ f(x_i(t)) + \beta \sum_{j=1}^{N} b_{ij} \Psi(x_j, x_i) & t \leq t^* \end{cases} \]

where \( \psi(x, y) = \text{sign}(y - x) \frac{\nu(|x-y|^2)}{|x-y|} \), the function \( \nu(V) > 0 \) satisfies \( \nu(V + U) \leq c\nu(\nu(U) + \nu(V)) \)

and \( \frac{V}{\nu(V)} \) is monotone decreasing when \( V \) increasing.
\( \bar{b}_{\nu,B} \) is a constant depending on the function \( \nu \) and the coupling matrix \( B \) such that
\[
\sum_{k=1}^{n} \sum_{i,j=1}^{N} b_{ij} |x_i^k(t) - x_j^k(t)| \psi(|x_i^k(t) - x_j^k(t)|) \\
\geq \bar{b}_{\nu,B} \nu \left( \sum_{k=1}^{n} \sum_{i,j=1}^{N} |x_i^k(t) - x_j^k(t)|^2 \right)
\]
(19)

Then, we have the following fixed-time synchronization result.

**Theorem 3.** System (18) can reaches synchronization for all
\[
t \geq \left( \frac{\alpha \bar{a}_{\mu,A}}{2} - 2 \delta \frac{1}{\mu(1)} \right)^{-1} \int_{0}^{1} \frac{dV}{\mu(V)} + \left( \frac{\beta \bar{b}_{\nu,B} \bar{b}_{\nu,B}}{2} - 2 \delta \frac{1}{\nu(1)} \right)^{-1} \int_{1}^{\infty} \frac{dV}{\nu(V)}
\]
(20)

**Proof.** We will first assume \( V(0) > 1 \) and \( x_i(t), i = 1, \cdots, N, \) satisfies
\[
\dot{x}_i(t) = f(x_i(t)) + \beta \sum_{j=1}^{N} b_{ij} \Psi(x_j, x_i).
\]
(21)

In this case, differentiating the Lyapunov function, we have
\[
\dot{V}(t) = V_1(t) + V_3(t),
\]
(22)

where \( V_1(t) \) is same as in Theorem 5, and
\[
V_3(t) = \beta \sum_{i=1}^{N} (x_i(t) - x^*(t))^T \sum_{j=1}^{N} b_{ij} \Psi(x_j(t), x_i(t)) \\
\leq - \frac{\beta \bar{b}_{\nu,B}}{2} \nu \left( \sum_{k=1}^{n} \sum_{i,j=1}^{N} |x_i^k(t) - x_j^k(t)|^2 \right)
\]
(23)

Then
\[
\dot{V}(t) \leq - \frac{\beta \bar{b}_{\nu,B}}{2} \nu(V(t)) + 2 \delta V(t) \leq - \left( \frac{\beta \bar{b}_{\nu,B}}{2} \nu(V(t)) \right) \nu(V(t))
\]
(24)

and \( V(t^*) \leq 1 \), where
\[
t^* = \left( \frac{\beta \bar{b}_{\nu,B}}{2} - 2 \delta \frac{1}{\nu(1)} \right)^{-1} \int_{1}^{\infty} \frac{dV}{\nu(V)}
\]
(25)

Combining with Theorem 2, one can get that the fixed-time synchronization is finally realized, and the settling time is also given as (20).
2.3 Finite-time and fixed-time consensus

It is clear that in case $f(\cdot) = 0$, $n = 1$, the finite-time and fixed-time synchronization problem becomes the finite-time and fixed-time consensus problem. As special examples of previous section, we consider following nonlinear consensus models

\[ \dot{x}_i(t) = \sum_{j=1}^{N} a_{ij} \Phi(x_j, x_i), \quad \text{(26)} \]

and

\[ \dot{x}_i(t) = \begin{cases} \sum_{j=1}^{N} a_{ij} \Phi(x_j, x_i) & t \geq t^* \\ \sum_{j=1}^{N} b_{ij} \Psi(x_j, x_i) & t \leq t^* \end{cases} \quad \text{(27)} \]

**Theorem 4.** The system (26) reaches finite-time consensus, i.e., $x_i(t) = x_j(t)$ for all $i, j = 1, \cdots, N$ and

\[ t \geq \bar{a}_{\mu,A}^{-1} \int_0^{V(0)} \frac{dV}{\mu(V)} \quad \text{(28)} \]

**Theorem 5.** Denote

\[ t^* = \left( \frac{\beta \bar{b}_{\nu,B}}{2} \right)^{-1} \int_1^\infty \frac{dV}{\nu(V)} \quad \text{(29)} \]

The system (27) reaches fixed-time consensus for all

\[ t \geq t^* + \bar{a}_{\mu,A}^{-1} \int_0^{V(0)} \frac{dV}{\mu(V)} \quad \text{(30)} \]

3 Conclusion

In this letter, by regarding finite-time stability as an inverse problem, we reveal the essence of finite-time stability and fixed-time stability. Some necessary and sufficient conditions are given. As application, we give a new approach for finite-time and fixed-time synchronization and consensus and some new results are given, too. As direct consequences, many existing results can be derived by the general approach.

References

Dorato, P. (1961). Short time stability in linear time-varying systems Proc. IRE Int. Convention Record Part 4, 83–87.
Roxin, E. (1966). On Finite Stability in Control Systems, *SIAM*, 49(9), 1520–1533.

Haimo, V. T., (1986). Finite time controllers, *SIAM J. Control Optim.*, 24(4), 760-770.

Bhat S. P., and Bernstein, D. S., (1998) Continuous finite-time stabilization of the translational and rotational double integrators, *IEEE Trans. Autom. Control*, vol. 43, no. 5, pp. 678-682, May.

Bhat S. P., and Bernstein, D. S., (2000) Finite-time stability of continuous autonomous systems, *SIAM J. Control Optim.*, vol. 38, no. 3, pp. 751-766, Mar.

Hong, Y. G., Xu, Y. S., and Huang, J., (2002) Finite-time control for robot manipulators, *Syst. Control Lett.*, vol. 46, no. 4, pp. 243-253, Jul.

Lu, W. L., and Chen, T. P., (2004) “Synchronization analysis of linearly coupled networks of discrete time systems,” *Physica D*, vol.198, pp. 148-168.

Lu, W. L., and Chen, T. P., (2005). Dynamical behaviors of Cohen-Grossberg neural networks with discontinuous activation functions,” Neural Networks, 18, 231-242

Lu, W. L., and Chen, T. P., (2006) “New approach to synchronization analysis of linearly coupled ordinary differential systems,” *Phys. D, Nonlinear Phenomena*, vol. 213, no. 2, pp. 214-230.

Moulay, E., Perruquetti, W., (2006). Finite time stability and stabilization of a class of continuous systems, *J. Math. Anal. Appl.* 323, 1430C1443

Cortés, J., (2006). Finite-time convergent gradient flows with applications to network consensus, *Automatica*, vol. 42, no. 11, pp. 1993-2000, Nov.

Chen., T., and Wang., L., (2007). "Power-Rate Global Stability of Dynamical Systems With Unbounded Time-Varying Delays”, IEEE Transactions on Circuits and SystemsII: Express Briefs, 54(8), 705-709

Shen, Y. J., and Xia, X. H., (2008). Semi-global finite-time observers for nonlinear systems, *Automatica*, vol. 44, no. 12, pp. 3152-3156, Dec.

Shen, Y. J., and Huang, Y. H., (2009) Uniformly observable and globally lipschitzian nonlinear systems admit global finite-time observers, *IEEE Trans. Autom. Control*, vol. 54, no. 11, pp. 2621-2625, Nov.
Xiao, F., Wang, L., Chen, J., and Y. P. Gao, (2009). Finite-time formation control for multi-agent systems, Automatica, vol. 45, no. 11, pp. 2605-2611, Nov.

Jiang, F. C., and Wang, L., (2009) Finite-time information consensus for multiagent systems with fixed and switching topologies, Physica D, vol. 238, no. 16, pp. 1550-1560, Aug.

Wang X. L., and Hong, Y. G., (2010). Distributed finite-time $\xi$-consensus algorithms for multi-agent systems with variable coupling topology, J. Syst. Sci. Complex, vol. 23, no. 2, pp. 209-218.

Wang, L., and Xiao, F., (2010). Finite-time consensus problem for network of dynamic agents, IEEE Trans. Autom. Control, vol. 55, no. 4, pp. 950-955.

Jiang F. C., and Wang, L., (2011). Finite-time weighted average consensus with respect to a monotonic function and its application, Syst. Control Lett., vol. 60, no. 9, pp. 718-725.

Sun, F. L., Chen, J. C., Guan, Z. H., Ding, L., and Li, T., (2012) Leader following finite-time consensus for multi-agent systems with jointly reachable leader, Nonlinear Anal.-Real, vol. 13, no. 5, pp. 2271-2284.

Polyakov, A., (2012) Nonlinear feedback design for fixed-time stabilization of linear control systems, IEEE Trans. Autom. Control, vol. 57, no. 8, pp. 2106-2110.

Parsegov, S., Polyakov, A. and Shcherbakov, P., (2012) Nonlinear fixed-time control protocol for uniform allocation of agents on a segment, in Proc. 51st IEEE Conf. on Decision and Control, Maui, Hawaii, USA, Dec. 2012, pp. 7732-7737.

Parsegov, S., Polyakov, A. and Shcherbakov, P., (2013) Fixed-time consensus algorithm for multi-agent systems with integrator dynamics, in Proc. 4th IFAC Workshop Distributed Estimation and Control in Networked System, Koblenz, Germany, pp. 110-115.

Zhao, Y., Duan, Z. S., and Wen, G. H., (2015) Finite-time consensus for second order multi-agent systems with saturated control protocols, IET Control Theory Appl., vol. 9, no. 3, pp. 312-319.

Liu, X. Y., Ho, D. W. C., Yu, W. W., and Cao, J. D., (2014) A new switching design to finite-time stabilization of nonlinear systems with applications to neural networks, Neural Networks, vol. 57, pp. 94-102.

Liu, X. Y., Lam, J., Yu, W. W. and Chen, G., Finite-time consensus of multiagent systems with a switching protocol, IEEE Trans. Neural Netw. Learn. Syst., DOI: 10.1109/TNNLS.2015.2425933.
Wang, X. Y., Li, S. H. and Shi, P., (2014) Distributed Finite-time containment control for double-integrator multiagent systems, IEEE Trans. Cybern., vol. 44, no. 9, pp. 1518-1528.

Zuo Z. Y., and Tie, L., (2014) A new class of finite-time nonlinear consensus protocols for multi-agent systems, Int. J. Control, vol. 87, no. 2, pp. 363-370.

Zuo Z. Y., and Tie, L., Distributed robust finite-time nonlinear consensus protocols for multi-agent systems, Int. J. Syst. Sci., doi: 10.1080/00207721.2014.925608.

Polyakov, A., Efimov, D. and Perruquetti, W., (2015) Finite-time and fixed-time stabilization: Implicit Lyapunov function approach, Automatica, vol. 51, pp. 332-340.

Zuo Z. Y., (2015) Non-singular fixed-time terminal sliding mode control of non-linear systems, IET Control Theory and Applications, vol. 9, no. 4, pp. 545-552.

Meng, D. Y., Jia, Y. M. and Du, J. P., Finite-time consensus for multiagent systems with cooperative and antagonistic interactions, IEEE Trans. Neural Netw. Learn. Syst., DOI: 10.1109/TNNLS.2015.2424225.