Hadron masses and decay constants in quenched QCD

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We present results for the mass spectrum and decay constants using non-perturbatively O(a) improved Wilson fermions. Three values of $\beta$ and 30 different quark masses are used to obtain the chiral and continuum limits. Special emphasis will be given to the question of taking the chiral limit and the existence of non-analytic behavior predicted by quenched chiral perturbation theory ($q\chi PT$).

1. INTRODUCTION

The chiral and continuum extrapolations of lattice results in quenched QCD have become an important issue since more accurate data for small quark masses and results for different lattice spacings have become available. Our simulations have been done for three different values of the gauge coupling, $\beta = 6.0, 6.2$ and $6.4$ ($a \approx 0.09 - 0.05$ fm), and on lattices of size $1.5 - 2.3$ fm. Our propagators have been generated for 7-12 different values of the hopping parameter $\kappa$ with the ratio of pseudoscalar and vector meson mass $m_{PS}/m_{V}$ in the range of $0.41 - 0.98$.

2. CHIRAL EXTRAPOLATION

Based on small quark mass expansions, chiral perturbation theory can give information about how the extrapolation to the chiral limit has to be done. For quenched QCD, $q\chi PT$ predicts non-analytic terms, e.g. for the pseudoscalar mass it gives to one-loop

$$m_{PS}^2/m_q = C_0 \tilde{m}_q^{-\frac{4}{\beta}} + C_1 \tilde{m}_q + O(\tilde{m}_q^2).$$

For full QCD, the parameter $\delta$ should be zero.

In order to determine $\delta$ we fitted our data using the ansatz

$$am_{PS}^2/\tilde{m}_q^R = C_\beta \left[ \left( a\tilde{m}_q^R \right)^{-\frac{4}{\beta}} + A_{1,\beta} a\tilde{m}_q^R 
+ A_{2,\beta} \left( a\tilde{m}_q^R \right)^2 \right] / \left[ 1 + B_{1,\beta} a\tilde{m}_q^R \right],$$

where $\tilde{m}_q^R = Z_M(g_0)^{1+\frac{\Delta a m_q}{1+6/y am_q}} \tilde{m}_q$ is the renormalization group invariant, and $\tilde{m}_q$ the

![Figure 1. The ratio $m_{PS}^2/\tilde{m}_q^R$ as a function of $\tilde{m}_q^R$. The solid line is a fit using eq. (2).](image-url)
bare Ward identity quark mass. The non-perturbative renormalization constant and the perturbative improvement coefficients $b_A$ and $b_P$ have been computed in [5]. According to this ansatz $m_{PS}^2/m_q^2$ should be linear up to heavier quark masses. Our results are shown in Fig. 2. From this fit we get $\delta \approx 0.14(2)$.

For the vector meson mass $m_V$ and the nucleon mass $m_N$, $q\chi PT$ predicts (4)

$$am_{V,N} = aM_{V,N} + C_{1/2}am_{PS}$$

$$+C_1(amp_{PS})^2 + O((amp_{PS})^3)$$

with $C_{1/2} \approx -1.5$ for $m_V$ and $C_{1/2} = -(3\pi/2)(D - 3F)^2 \delta \approx -0.24$ for the nucleon (D and F taken from [3]). For full QCD, $C_{1/2}$ should be zero.

To reduce the number of free parameters in our fits, we make the assumption that the coefficients $C_{1/2}, C_1, ..., 0$ do not depend on $\beta$. To take the different scales into account we replace in eq. (4) $a \to \tilde{a}_\beta = a_{\beta=6.0} s_\beta$ and use the scale parameter $s_\beta$ as an additional fit variable. We fitted our data using a Padé-like ansatz,

$$\tilde{a}_\beta m_{V,N} = \frac{\tilde{a}_\beta M_{V,N} + \sum_{i=1}^N A_i (\tilde{a}_\beta m_{PS})^i}{1 + B_{N-1} (\tilde{a}_\beta m_{PS})^{N-1}},$$

which is again chosen to give the correct heavy quark limit, and a polynomial ansatz, which corresponds to Eq. (4) with $B_{N-1} = 0$. For the Padé-like ansatz the results for $M_{V,N}$ are close to constant for $0.1 < B_{N-1} < 1.5$, we therefore use $B_{N-1} = 1$. For $m_V$ we find $C_{1/2} = -0.6(1)$ and $-0.7(3)$ for the Padé-like and the polynomial ansatz, respectively. The results are plotted in Fig. 2. For $m_N$ we find $C_{1/2} = 0.8(8)$ and $0.4(5)$. Our results for $s_\beta$ deviate at most 2% from the corresponding values obtained from the force scale $r_0$ [5], which can be explained by scaling violations.

The fits shown in Fig. 2 can be compared with fits shown in Fig. 3, where $C_{1/2}$ has been set to zero. Also shown are the results for a phenomenological ansatz

$$\tilde{a}_\beta m_{V,N} m_{V,N} = (\tilde{a}_\beta m_{V,N})^2 + \sum_{i=2}^3 A_i (\tilde{a}_\beta m_{PS})^i$$

which has less free parameters.

3. CONTINUUM LIMIT

For improved fermions we expect scaling violations to be of $O(a^2)$. We therefore extrapolate linearly in $(a/r_0)^2$. To avoid artifacts of the quenched approximation, we use the phenomenological ansatz $(am_{V,N})^2 = (am_{V,N})^2 + A_{2,\beta}(am_{PS})^2 + A_{3,\beta}(am_{PS})^3$ for the extrapolation to the chiral limit, while we use a Padé-like ansatz similar to eq. (4) with $C_{1/2} = 0$ to interpolate our data or to extrapolate towards the charm quark mass region.
We found evidence for non-analytic behavior of $m_{P^2}/m_q$ and $m_V$. The results for $m_N$ do not seem to confirm the predictions of $q\chi PT$, but within errors we cannot conclude $C_{1/2} > 0$. Our result for the parameter $\delta$ agrees with the results of the CP-PACS collaboration [7] and is slightly larger than the number found by the FNAL group [8]. For $m_V$ we found $C_{1/2}$ to be much larger than in [7].

If we use the phenomenological ansatz for the extrapolation to the chiral limit the ratio $m_N/m_\rho$ seems to become larger than the experimental value. This is probably a quenching effect. For all masses and decay constants presented here discretization errors are $O(a^2)$.

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**REFERENCES**

1. S.R. Sharpe, Nucl. Phys. B (Proc. Suppl.) 30 (1993) 213.
2. S. Sint and P. Weisz, Nucl. Phys. B502 (1997) 251; S. Capitani et al., Nucl. Phys. B544 (1999) 669.
3. M. Booth, G. Chiladze and A.F. Falk, Phys. Rev. D55 (1997) 3092.
4. J.N. Labrenz and S.R. Sharpe, Phys. Rev. D54 (1996) 4595.
5. R.L. Jaffe and A. Manohar, Nucl. Phys. B337 (1990) 509.
6. M. Guagnelli, R. Sommer and H. Wittig Nucl. Phys. B535 (1998) 389.
7. R. Burkhalter et al., Nucl. Phys. B (Proc. Suppl.) 73 (1999) 3.
8. H. Thacker et al., Nucl. Phys. B (Proc. Suppl.) 73 (1999) 243.