Multifractal Structure of the Sea-Bottom Topography in the Korean Sea

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Abstract

The scaling behavior of the multifractality for the sea-bottom topography of the South Sea in Korea is numerically investigated. In particular, we focus on the behavior of the qth-moment depth-depth correlation function of the sea-bottom topography and its multifractal spectrum. Through the multifractal analysis, the fractal dimension and the scaling exponents are obtained numerically, and the relation between the Hurst exponent and the fractal dimension is also derived.

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For more than two decades, there has been considerable interest in the investigation of the scaling behavior on fractal models. The pioneering work[1] on fractals has introduced the concept of fractals and shown some relation between the self-similar fractals and the self-affine fractals. The self-affine fractals[2 − 5] constitute random and complicated structure that has been applied to broader range of problems such as Eden and ballistic deposition model[6 − 9], mountain heights, clouds, coast lines and cracks. Among other examples of many fractal models we also can mention the self-avoiding random walk, percolation clusters, diffusion-limited aggregations, random resistor networks, polymer bonds, turbulences, chaotic motions [2, 4, 5, 10 − 13], etc.

On the other hand, Matsushita and Ouchi[14] have recently shown that the self-affine of mountain topography in Japan is mainly obtained by using the numerical method, from which they have discussed the relation $D_f = 2 - H = 1.37$ between the self-affine exponent $H$ and the fractal dimension $D_f$ for the standard deviation of mountain heights. More recently, Katsuragi and Honjo[15] have dealt with self-affine fractal profiles on the entropy spectra method and for the whole range of a mountain they have obtained the generalized multi-affine profiles and the entropy spectra.

It has recently discussed that the multifractals[2, 5] have essentially the distributions of singularities of the scaling exponents on interwoven sets of various fractal dimensions. The multifractal formalism for the box-counting method is represented in terms of their generalized dimensions and spectra. The multifractality of self-affine fractals has been discussed by Barabási et al[16, 17]. They have investigated the multi-affine function and the multifractal spectra, and calculated the generalized Hurst exponent from the $q$-th order height-height correlation function. Very recently, the generalized dimension and the scaling exponent which are interesting in multifractals for mountain heights in Korea have been calculated by Kim and Kong[18]. We think that it is interesting to be extended to the multifractals, although we do not have
enough sets of data for the seabottom depth. In particular, the multifractal theory is a good basis for analyzing the statistical data on the seabottom depth, and really provides the probabilistic evidence for the relation between Hurst exponent and the fractal dimension.

The main purpose of this paper is to investigate numerically the behavior of the depth-depth correlation function and its multifractal spectrum in the seabottom depth from the seabottom topography between the longitude $129^\circ21' - 129^\circ54'E$ and the latitude $34^\circ52' - 35^\circ19'N$ located near to the east of Pusan. The primary reason we select this region is that this region has more data for sea-bottom depth than those of any other region around Korean peninsula. We consider well known relations of the depth-depth correlation function and the multifractals. The $q$th-moment depth-depth correlation function is also numerically discussed from the multi-scaling properties of seabottom depth function. In addition, from the extrapolated sets of data for the seabottom depths, we briefly explain an efficient and convenient method relevant to numerical investigation on the scaling behavior of the generalized dimension and the spectra.

First of all, we are interested in the scaling quantities obtained from the extrapolated data of the seabottom depth on a two dimensional square lattice between $129^\circ21' - 129^\circ54'E$ and $34^\circ52' - 35^\circ19'N$, as shown in Fig. 1. Moreover, the current data are taken for the values of the seabottom depths projected on $100 \times 100$ lattice points in square area of $50 \times 50 \ km^2$, where the distance between one lattice point and its neighboring lattice point is $500 \ m$. Let’s consider the $q$th-moment depth-depth correlation function $C_q(r)$ defined by

$$C_q(r) = \frac{1}{N} \sum_{i=1}^{N} |D(r_i + r) - D(r_i)|^q,$$  \hspace{1cm} (1)

where $D(r_i)$ is the $i$th depth in the $r$-direction and $r$ denotes two perpendicular
ular longitudinal or latitudinal direction on a two dimensional square lattice. This correlation function can be described as a non-trivial multi-scaling behavior;

\[ C_q(r) \sim r^{qH_q}, \quad (2) \]

where \( H_q \) is the generalized Hurst exponent in the limit of \( r \to 0 \).

Next, we discuss the generalized dimension \( D_q \), the scaling exponents \( \alpha_q \), and \( f_q \) on the multifractal structure. In general, it has been known that the generalized dimension is represented as the fractal distribution having the singular values infinitely. Accordingly, the generalized dimension and the scaling exponents are known to be given by following equations\[4, 5\]

\[ D_q = \lim_{\epsilon \to 0} \frac{1}{q-1} \frac{\ln \sum_i n_i p_i^q}{\ln \epsilon}, \quad (3) \]

\[ \alpha_q = \lim_{\epsilon \to 0} \frac{1}{\ln \epsilon} \frac{\sum_i p_i q \ln p_i}{\sum_i n_i p_i^q}, \quad (4) \]

and

\[ f_q = \lim_{\epsilon \to 0} \frac{1}{\ln \epsilon} \left[ \frac{\sum_i p_i^q \ln p_i^q}{\sum_i n_i p_i^q} - \ln \sum_i n_i p_i^q \right], \quad (5) \]

where \( p_i \) is the probability of the seabottom depth existing on the \( i \)-th box with the square area of \( \epsilon \times \epsilon \) and the scaling quantity \( n_i \) is the number of the box having the probability \( p_i \). By introducing the above expressions, the spectra \( f_q \) and \( \alpha_q \) are simply calculated from eqs.\( (4) \) and \( (5) \) with \( f_q = q\alpha_q - (q-1)D_q \) and \( \alpha_q = \frac{d}{dq}[(q-1)D_q] \), where \( f_q \) and \( \alpha_q \) can be obtained by Legendre transformation. In our scheme, we will make use of eqs.\( (3)-(5) \) to find out the fractal dimension and other scaling exponents, and these mathematical techniques
lead us to more general results.

We present more detailed numerical data of the depth-depth correlation function of the seabottom depth and its multifractal spectrum. As mentioned above, we assume that the seabottom depths divided by the intervals of 500 m are located on each region of two dimensional lattice. As listed in Table 1, we take into account the data of three regions A - C in seabottom depths between 110 m and 125 m. As we only restrict ourselves to the longitudinal direction from our data, the generalized Hurst exponent from the $q$th-moment depth-depth correlation function is calculated numerically. Hence, from eq.(2) the generalized Hurst exponents $H_2$, $H_3$, and $H_4$ on the longitudinal direction are found to be 0.599, 0.587, and 0.586, respectively.

In our multifractal structure, the box-counting method is used for two square areas of $2.5 \times 2.5$ km$^2$ and $5 \times 5$ km$^2$. Through the multifractal analysis, the generalized dimension and the scaling exponents are also obtained from the data with the number of seabottom depths in Table 1. These values by using the theoretical expressions of eqs.(3) – (5) are calculated numerically in our three regions. In Fig. 2 we show the scaling exponents $\alpha_q$ and $f_q$ on the multifractal structure, and it is found numerically that the scaling exponents are estimated as $\alpha_{+\infty} = 1.097696$ and $\alpha_{-\infty} = 2.172183$ only in the case of the region A. Particularly, as shown schematically in Fig. 3 and Table 2, it is obtained that the maximum values of the generalized dimension, i.e., in our three regions A, B, and C the fractal dimension $D_f = D_0$ are respectively calculated as 1.312476, 1.366726, and 1.372243 for the case of the square area $2.5 \times 2.5$ km$^2$. Hence, we obtain that Hurst exponent takes approximately 0.65 in our multifractal structure, while it takes the value $H=0.63$ in Japanese mountain topography[14].

In conclusion, we have remarkably studied on the $q$th-moment depth-depth correlation function and its multifractal spectrum in the seabottom topography of the South Sea in Korea, as shown in Fig.1. Specifically, the multifractals
have been investigated numerically by employing the box-counting method on a two dimensional square lattice. Based on these results, we expect that further analytical and numerical progress for multifractals may be achieved from more data for the ocean floor on diverse region of seabottom topography. Furthermore, it will be useful to apply and to reinvestigate the above formalism in many scientific fields[19,20] such as quantum disorder systems, irreversible growth structures, and fractured surfaces of minerals.

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Figure Captions

Fig. 1: The seabottom depth in seabottom topography taken from \( \frac{1}{312000} \) scale map between the longitude \( 129^\circ21' - 129^\circ54'E \) and the latitude \( 34^\circ52' - 35^\circ19'N \), where the interval of seabottom depth between the solid lines is found to be 10m.

Fig. 2: \( f_q \) versus \( \alpha_q \) by using the box-counting method for the case of the square area \( 2.5 \times 2.5km^2 \). The values of the seabottom depths in three regions A, B, and C are respectively given by the thin solid, dashed, and thick solid lines.

Fig. 3: Plots of \( \alpha_q \) and \( f_q \) as a function of \( q \) on the seabottom depths for the case of the square area \( 2.5 \times 2.5km^2 \), where the vertical bars are error bars averaged over the square areas of three regions A, B, and C.

Table Captions

Table 1: Number of the seabottom depths in our three regions A - C between 110m and 125m on the seabottom topography.

Table 2: Summary of values of \( D_q, \alpha_q, \) and \( f_q \) calculated from the data of three regions A, B, C for the case of the square area \( 2.5 \times 2.5km^2 \).
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