A Probabilistic Method for Constructing an Empirical Discrimination Model for Hammering Inspection of Cast-Iron Parts

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Abstract: A probabilistic method for constructing an empirical discrimination model to inspect defective cast-iron parts (such as graphite-spheroidized defective parts) by the hammering test is proposed. The hammering-sound frequency spectrum includes multiple resonance lines whose frequencies vary according to the degree of defect. To construct the model, only non-defective hammering-sound data that can be collected from the production line are input, and a distribution function that fits the frequency distribution of each resonance line is estimated. Since the frequency distribution shows multimodality and asymmetry, the function is estimated by using automatic differentiation variational inference with a mixed-normal distribution function. The confidence interval of the obtained distribution function is then regarded as a section with no defective parts, and the discrimination model is automatically constructed by connecting the sections of all resonance lines in the audible range. Then, parts outside the sections are discriminated as defective. Experimentally determined accuracy confirmed that it is possible to achieve hammer-test inspection with the detection rate of 100% and prevent overlooking of defective parts.

Key Words: hammer test, cast iron, statistical model, automatic differentiation variational inference, Markov chain Monte Carlo methods.

1. Introduction

Cast-iron parts are used in many mass-produced mechanical products like cars. As shown in Fig. 1, complex-shaped material is integrally molded by a casting method and machined to a finished part. Generally, the casting line and machining shop are separated, and by changing the set-up of sand-mold models on the casting line, materials for various kinds of cast-iron parts are batch-produced.

During casting, besides shape defects due to cold shuts, mold drops, etc. [1], material defects related to graphite nodularity or graphite spheroidization can occur as shown in Fig. 1 [2],[3]. To prevent defective parts flowing out to the machining shop, it is effective to perform a complete inspection of cast-iron parts at the end of the casting. Various methods for nondestructive inspection of cast-iron parts are available [4]. However, to adapt to the mass-production cycle times, material sampling inspection using dedicated inspection equipment is often done offline. However, promptly feeding back information to the casting line necessitates making material inspection into full in-line inspection. To enable such total material inspection in-line, an inexpensive inspection method with short cycle time is useful, and a hammering test [5], which uses acoustic resonance, is one candidate method. In the case of such an inspection method utilizing the striking sound of a hammer, it is possible to examine the resonance spectrum reflecting a material defect at high speed from the hammering-sound-signal data in the audible frequency range of one second or less.

Since there is no absolute criterion for hammering inspection, it is necessary to preset the criterion for the resonance spectrum. In general, it is necessary to experimentally cast a simulated defective part for each grade of graphite-spheroidization ratio and compare its hammering sound with that of a good (non-defective) part. However, it is complicated and costly to work to differentiate simulated defective parts for different grades of graphite-spheroidization ratio and various types of cast-iron parts; therefore, it is desirable to be able to set the criteria on the basis of hammering data for non-defective parts only. Furthermore, to efficiently implement that criterion-setting procedure, it is desirable that an inspection system can automatically generate discrimination criteria from historical hammering-sound data and be able to learn in accordance with fluctuations of the casting process.

Variations in the resonance spectra of individual parts in a non-defective group exist due to dimensional variations and the like. Therefore, we propose a method of constructing a statistical judgment model for estimating non-defective-part variation existence intervals from confidence intervals by fitting a statistical distribution function to the resonance-spectrum variation.
of the non-defective group, and judging parts outside that interval as defective. Since the variation distribution of the resonance spectrum may become complicated in terms of shape in response to casting variations, a method of searching for a mixture distribution function with components of arbitrary locations and scales by probabilistic methods was adopted.

Such probabilistic methods were the automatic differentiation variational inference (ADVI) method [6]–[9] and the Markov chain Monte Carlo (MCMC) method [10]–[14]. Both methods can create solutions by expressing a complex statistical model (including many parameters) as a Bayesian model. ADVI [6]–[9], which approximates solutions at high-speed by a deterministic approach, must take care to capture local solutions and singular solutions; even so, the authors reported that ADVI was effective for constructing the statistical discrimination model for the hammering test [15]. On the other hand, MCMC [10],[11], which takes time by long-running sampling from a posterior distribution of the Bayesian model, is supposed to be able to create more accurate solutions than ADVI. In this paper, MCMC and ADVI were compared from the viewpoint of accuracy and computing time.

2. Hammering Test

2.1 Hammering-Test System

The configuration of a hammering-test system is shown in Fig. 2. The stress wave generated in a cast-iron part by the striking hammer returns from the free ends of the part and resonates in superposition with the original wave [16]. The sound-pressure signal of the resonance sound is measured with a microphone, and the resonance spectrum is compared with a discrimination model prepared beforehand. Since the resonance frequency is determined by the propagation distance and speed of the stress wave, if the propagation speed changes due to defective material, the resonance frequency also changes. By detecting this change by collating the measured resonance spectrum with the discrimination model, it can be determined whether the inspection target is a non-defective part or a defective part.

On the other hand, the propagation distances of stress waves differ when the shapes of the tested parts differ; therefore, even if the tested parts are good, the resonance frequency differs according to the type (shape) of the part. Therefore, a discrimination model is prepared in advance for each part type having a different shape, and when a hammer test is performed, a discrimination model is assigned to each target part type so that it is possible to detect the change in resonance frequency due to the difference in material properties (not part type) with high sensitivity.

2.2 Resonance-Frequency Spectrum

An example of a hammering-sound waveform of a non-defective cast-iron part and its corresponding resonance-line frequency spectrum are shown in Fig. 3. A hammering-sound signal with sampling frequency of 44.1 kHz (to cover the entire 22.05 kHz range of human hearing) was Fourier transformed, and its resonance-line components only were extracted. Due to the presence of multiple ends from which stress waves return in complex-shaped cast-iron parts, several dozen of the resonance lines in the spectrum are generated in the entire audible range. Note that we confirmed that ultrasonic frequencies higher than 22.05 kHz could not have supplied more usable information for the inspection.

Identification labels are sequentially attached to the resonance lines along the frequency axis by means of clustering according to mutual distances between the resonance frequencies of plural parts; it is not assumed that there is any big frequency leap across the cluster boundaries. The lines of common labels are traced among non-defective parts of the same type, and their frequency variations of the resonance lines are investigated. Variation in individual resonance frequency of two representative resonance lines is shown in Fig. 4. As examples, center frequencies of (a) 42.4 Hz and (b) 13.3 kHz are shown. The upper part of each figure shows the resonance-frequency histogram, and the lower part shows the estimated normal-distribution function.

A discrimination model can be created if the frequency intervals occupied by these distributions are taken as non-defective-part discrimination sections and investigated over the entire frequency axis.

![Fig. 2 Schematic diagram of a proposed hammer-test inspection system with automated discrimination model builder.](image1)

![Fig. 3 An example of hammering-sound waveform of a non-defective part and its resonance-line spectrum in the audible range.](image2)
2.3 Issues Concerning Hammering Test

However, the histograms in the upper part of Fig. 4 may include outliers due to the influence of burr and dimensional variations generated during casting, as historical hammering-sound data become overlapped, the non-defective-part discrimination section widens, and the discrimination model is likely to cause defective parts to be overlooked. It is not easy to discriminate outliers in advance, for example, since the frequency at the right end of Fig. 4 (b) is located next to the foot of a steep slope, the possibility of outliers is high, and it is reasonable to regard the frequency at the left end as a normal value forming part of a gentle slope.

To prevent excessive widening of the non-defective-part discrimination section, namely, by excluding outliers, a statistical distribution function is fitted, and a confidence interval excluding outliers is determined as a non-defective-part discrimination section.

However, even for the same population, each resonance line is affected by intrinsic casting variations; as a result, a multimodal distribution is shown in Fig. 4 (a), and an asymmetrically distorted distribution is shown in Fig. 4 (b). When a normal-distribution function is applied to these distributions, it can be seen that in Fig. 4 (a), the base width of the distribution function widens excessively, and in Fig. 4 (b), the center of gravity of the distribution function is shifted to the right, so it is impossible to accurately estimate the non-defective-part discrimination section in both cases from the lower charts. As a result, if a discrimination model is constructed under the assumption of a normal distribution, it is a problem that defective parts might be overlooked and erroneous discrimination might occur.

3. Method for Constructing a Discrimination Model

To accurately determine the frequency distribution of the resonance lines, a method for constructing a discrimination model using ADVI is proposed in the following. The frequency distribution of complex shapes is supposed to be approximated as a mixed normal distribution (Gaussian mixture distribution), and it is defined with a Bayesian model consisting of likelihood and prior distributions. Then, a iterative convergence method using gradient ascent optimization is performed by ADVI, and distribution parameters are approximately estimated. Based on the mixed normal-distribution function (determined from the distribution-parameter values), a method for identifying confidence interval as non-defective-part discrimination section is adopted.

3.1 Bayesian Model of Resonance-Frequency Distribution

For a resonance-frequency distribution function with unknown distribution parameters, the Bayesian model given by the following equation is considered with the distribution parameters as the random variables:

$$ p(\theta | x) = f(x | \theta) \cdot p(\theta). $$

(1)

Here, $p$ on the left side is the posterior distribution of distribution parameter $\theta$, $f$ on the right side is the likelihood function for data $x = \{x_1, x_2, \ldots, x_N\}$, and $p$ on the right side is the prior distribution of $\theta$. Data $x$ are data that track the frequency of the resonance line with the identification label of interest for $N$ non-defective items. With Fig. 4 as an example, a mixed-normal distribution function is used to identify the seemingly arbitrary frequency distribution as having multimodality or bilateral asymmetry.

The likelihood function when the number of mixed components is $K$ is given as

$$ f(x | \theta) = \prod_{n=1}^{N} \left[ \sum_{k=1}^{K} \omega_k \cdot \phi(x_n | \mu_k, \sigma_k) \right]. $$

(2)

$$ \theta = [ \omega_k, \mu_k, \sigma_k ] \quad \text{for} \quad k \in \{1, 2, \ldots, K\}, $$

on the condition that $\phi$ is a normal distribution function given by the following equation:

$$ \phi(x_n | \mu_k, \sigma_k) = \frac{1}{\sqrt{2\pi \sigma_k^2}} \exp \left\{ -\frac{(x_n - \mu_k)^2}{2\sigma_k^2} \right\}. $$

(3)

The prior distribution is given as

$$ p(\theta) = p(\omega) \cdot \prod_{k=1}^{K} \left[ p(\mu_k) \cdot p(\sigma_k) \right], $$

(4)

$$ p(\omega) = \text{Dirichlet} (\omega | a), $$

$$ p(\omega) = \text{Dirichlet} (\mu | a), $$

$$ p(\omega) = \text{Dirichlet} (\sigma | a), $$

where $a$ is a fixed parameter.
A graph showing the relationship between parameters is shown in Fig. 5. In the distribution parameter $\theta$ of the mixed components, mixing coefficients $\omega = [\omega_1, \omega_2, \ldots, \omega_k]$. These $[\omega]$ are all set to one as a noninformative prior distribution. The prior distribution of location $\mu_k$ has location $\mu_0(k)$ and scale $\sigma_k$, and the prior distribution of scale $\sigma_k$ has scale $\beta_k$. For pre-scales $\sigma_0$ and $\beta_0$, standard deviation (SD) (analytically obtained from data $x$) was set as empirical knowledge as shown in the following equation:

$$\sigma_0 = \beta_0 = \text{SD}(x) = \frac{1}{N} \sum_{n=1}^{N} (x_n - \bar{x})^2, \quad (5)$$

$$\bar{x} = \text{mean}(x) = \frac{1}{N} \sum_{n=1}^{N} x_n.$$ To prevent multiple mixed components being captured at the same time in the vicinity of the section center $\bar{x}$, which can give multiple local solutions, the prior location $\mu_0(k)$ was distributed separately in terms of mixing component $k$ within the maximum and minimum intervals according to the following equation:

$$\mu_0(k) = \min(x) + \frac{\max(x) - \min(x)}{K + 1} k. \quad (6)$$

### 3.2 Algorithm for Constructing Discrimination Model

An algorithm for constructing a discrimination model incorporating ADVI estimation of individual resonance-frequency distributions is shown in Fig. 6. Individual resonance-frequency distributions are estimated sequentially (by using ADVI described in the previous section) for all resonance lines above a given noise level. Since it is impossible to know in advance the appropriate number of components of the mixed distribution function that performs fitting, it was attempted to estimate the distribution function of all number of components up to the assumed upper-limit component number, and an algorithm for selecting a distribution function with the maximum likelihood indicating the goodness-of-fitting to data from among them was supposed. However, it may converge to a specific component, so whether or not the mixed-normal distribution function includes a singular component is determined. If it is included, the corresponding mixed-normal distribution function is excluded from the selection candidates.

After the distribution function is estimated, the confidence interval is obtained. After the confidence interval (CI) for all resonance-line label numbers $i$ is obtained, all CIs are joined along the frequency axis $f$ in the audible range by the following expression, and a discrimination model $m(f)$ consisting of rectangular peaks is synthesized:

$$m(f) = \begin{cases} P_i \quad (f \in C_i, i = 1, 2, \ldots, N_i), \\ P_0 \quad (f \notin C_i, i = 1, 2, \ldots, N_i), \\ P_0 = \text{max}(P(x)), \\ x = \{x_1, x_2, \ldots, x_{N_i} \in C_i\} \end{cases} \quad (7)$$

Here, $P(x)$ is a set of resonance-line powers of $N_i$ individuals included in each confidence interval $C_i$, $P_i$ is its maximum value, and $P_0$ is a noise-level constant value outside $C_i$.

### 4. Results of Experiments

The ADVI tool used for estimating the distribution parameters was the Python implementation library [18]. For convergence of the iterative parameter-update calculations, the AdaGrad method for adaptively tuning learning rate was utilized, and the initial learning rate was taken as 0.75. Since the learning speed varies in accord with the complexity of the distribution profile, a method confirms convergence every 100 iterations and completes the calculation adaptively. The calculation speed achieved with a general-purpose PC (Corei7, 3.4 GHz) was 180 iterations per second.
Fig. 7 Results of ADVI estimation of the distribution functions for two representative resonance lines in the spectrum. (The upper part shows resonance-frequency histogram, and the middle and lower parts show mixed-normal distribution functions.)

Table 1 Estimated parameters of the mixed-normal distribution function for two representative resonance lines in the spectrum.

(a) Resonance-line-spectrum label #1 (42.4 Hz) \(K=3\)

| \(k\) | \(\omega_k\) | \(\mu_k\) | \(\sigma_k\) |
|-------|-------------|-------|---------|
| 1     | 0.275       | 14.8 Hz | 4.6 Hz  |
| 2     | 0.573       | 47.8 Hz | 13.3 Hz |
| 3     | 0.152       | 71.3 Hz | 4.9 Hz  |

(b) Resonance-line-spectrum label #24 (13.3 kHz) \(K=2\)

| \(k\) | \(\omega_k\) | \(\mu_k\) | \(\sigma_k\) |
|-------|-------------|-------|---------|
| 1     | 0.471       | 13.27 kHz | 44.9 Hz |
| 2     | 0.529       | 13.31 kHz | 23.3 Hz |

4.1 Results of Estimating the Distribution Functions for Two Representative Examples

The result of attempting to estimate the distribution functions in Fig. 4 using ADVI is shown in Fig. 7, and the values of the estimated parameters of the distribution function with the maximum log-likelihood are listed in Table 1. The spread of the bottom edge of the distribution is managed by specifying an appropriate confidence level at the time of calculating the confidence interval. In Fig. 7, the confidence level was 99%.

In Fig. 7 (a), the mixed normal distribution with the number of components \(K=3\) and large log-likelihood was selected. The number of iterations required for convergence was 600, and the calculation time was 3.3 seconds. It is understood that as a result of precisely approximating a multimodal distribution by combining steep components, it is possible to estimate a confidence interval that does not include resonance-frequency data with a high possibility of having an outlier located outside the right side of the distribution.

In Fig. 7 (b), a mixed-normal distribution function with the number of components \(K=2\) and a slightly higher log-likelihood was selected. The number of iterations required for convergence was 900, and the calculation time was 5.0 seconds. The result of precisely approximating an asymmetric distribution whose center was closer to the right by combining a gradual component and a steep component indicates that resonance-frequency data with a high probability of an outlier outside the steep bottom-right edge of the distribution is not included, and it is possible to estimate the confidence interval including resonance-frequency data with a high possibility of normal values in the gently sloping tail of the left side of the distribution.

4.2 Results Concerning Construction of Discrimination Model

For four kinds of cast-iron parts, discrimination models were constructed from hammer-sound data of non-defective parts, and a simulated defective part was experimentally cast for each grade of graphite-spheroidization ratio, and a hammering test was conducted. A breakdown of the numbers of non-defective and defective parts is shown in Table 2. Taking the confidence level for estimating confidence interval \(CI_i\) as 99% and noise level as \(P_0 = 10 \text{ dB}\), we constructed a discrimination model based on the automatic construction algorithm shown in Fig. 6. Here, the upper limit of number of mixed components was set to \(K = 3\) according to the result of a preliminary study in which the upper limit was set at a sufficiently large value \((K = 10)\), and the effective maximum number of mixed components without any singular components was confirmed to be \(K = 3\).
A breakdown of the distribution functions selected in the process of constructing the discrimination model is shown in Fig. 8. A mixed-normal distribution function with two or three components was selected as a distribution with higher likelihood for 94 (= 78 + 16) % of 112 resonance lines in total. It is clear that the individual frequency distribution of the resonance line is multimodal or asymmetric.

An example of a discrimination model constructed using part type A and the detected resonance lines of defective parts and non-defective parts is shown in Fig. 9. Resonance lines deviating from the discrimination model (indicated by the thin rectangular line) are indicated by bold lines. For defective parts, in Fig. 9 (a-1), namely, sphericity rate of 74%, seven deviations are detected, and in Fig. 9 (a-2), namely, sphericity rate of 50%, 22 lines are detected on the lower-frequency side of the model rectangular peaks. Conversely, for non-defective parts, in Fig. 9 (b-1), no deviations are detected; in contrast, in Fig. 9 (b-2), three deviations are detected on the right high-frequency edges of the model rectangular peaks because of random casting variations like burrs. Note that the part represented in Fig. 9 (b-2) is a rare sample having more than one deviation among 66 non-defective parts of type A. The number of the detected resonance lines, which increases if sphericity rate decreases, is compared with a preset threshold to determine whether the target part is a non-defective or defective part.

### 4.3 Relationship between Graphite-Spheroidization Ratio and Resonance Frequency

A proportional relationship between graphite-spheroidization ratio and ultrasonic velocity in the case of ductile cast irons has been reported [19]. The results in Fig. 9 suggest that a similar relationship for stress-wave velocity must also be established; consequently, we theoretically estimated that the resonance frequency determined from the stress-wave velocity shifts according to graphite-spheroidization ratio. An example of the relationship between graphite-spheroidization ratio and the amount of resonance-frequency transition, focusing on one resonance line, is shown in Fig. 10 (a). The amount of resonance-frequency transition on the horizontal axis represents the amount of transition from the center frequency (18.5kHz) of the frequency distribution of non-defective parts. As estimated, it is clear that as graphite-spheroidization ratio decreases, the amount of resonance-frequency transition increases, and their correlation rate is high (0.89).

The results of examining the above-described correlation and detection rate for all the resonance lines of defective parts are summarized in Table 3. For that discrimination, the threshold of the number of detected resonance lines

### 4.4 Results of Experiment on Discrimination Accuracy

The results of discriminating the numbers of cast-iron parts listed in Table 2 are summarized in Table 3. For that discrimination, the threshold of the number of detected resonance lines
Table 3 Results of detection rate.

(a) Proposed method

| Part type | Non-defective | Defective |
|-----------|---------------|-----------|
| A         | 0%            | 100%      |
| B         | 0%            | 100%      |
| C         | 0%            | 100%      |
| D         | 0%            | 100%      |
| Total     | 0%            | 100%      |

(b) Conventional method

| Part type | Non-defective | Defective |
|-----------|---------------|-----------|
| A         | 0%            | 88.4%     |
| B         | 0%            | 100%      |
| C         | 0%            | 88.4%     |
| D         | 0%            | 90.9%     |
| Total     | 0%            | 92.0%     |

was set to five. The detection rate for non-defective parts is the result of re-discriminating the non-defective parts used for constructing the discrimination model. As shown in Table 3 (a), the detection rate for spheroidized-graphite defective parts by the discrimination model constructed by the proposed method is 100% (i.e., 174/174); in other words, no defective parts were overlooked, and erroneous detection concerning non-defective parts did not occur. On the other hand, as shown in 3 (b), the detection rate for defective spheroidized-graphite parts by a discrimination model constructed by the conventional method (based on a normal distribution function) was 92.0%; namely, 8.0% (i.e., 14/174) of defective parts were overlooked. 5. Comparison with MCMC

As an alternate probabilistic means to ADVI, the Markov chain Monte Carlo (MCMC) method [10] is supposed to be able to create more accurate solutions. MCMC draws samples from a posterior distribution of the same Bayesian model (1), and its estimation accuracy depends more or less on sampling length. Since automatically determining convergence of a sample to its equilibrium distribution is not an easy task, MCMC tends to take time. MCMC and ADVI must, therefore, be compared from the view-point of computing time as well as estimation accuracy. Sequences sampled by MCMC for the two representative ex-
Table 4  Comparison of estimation accuracy for two representative resonance lines in the spectrum.

(a) Resonance-line-spectrum label #1 (42.4 Hz) (K=3)

| Log-likelihood | Left edge of 99% CI | Right edge of 99% CI |
|----------------|---------------------|----------------------|
| ADVI           | -5602               | 4.941 Hz             |
| MCMC           | -5623               | 4.941 Hz             |

(b) Resonance-line-spectrum label #24 (13.3 kHz) (K=2)

| Log-likelihood | Left edge of 99% CI | Right edge of 99% CI |
|----------------|---------------------|----------------------|
| ADVI           | -1452.4             | 13.167 kHz           |
| MCMC           | -1452.2             | 13.170 kHz           |

samples in Fig. 7 are shown in Fig. 11. The number of sampling iterations was 30,000, in which the first 10,000 samples were discarded, and the mode values of the remaining 20,000 samples gave point estimates for the parameters of the mixed-normal distribution function. For the multimodal case shown in Fig. 11 (a), sampling of the equilibrium distribution started from about 14,000 iterations. For this case, although burn-in of 10,000 samples was short, the mode values were used as point estimates to prevent estimation errors or increase of burn-in sampling iterations. On the other hand, for the asymmetry case shown in Fig. 11 (b), the samples reach their equilibrium distribution right away. However, the sample sequence of the location parameter $\mu_1$ of normal-distribution-function component 1 is unstable with larger variation than that of component 2, because the component 1 (with flat-shape and gentle slope) (Fig. 7 (b)) is supposed to be insensitive to the likelihood.

Estimation accuracies of ADVI and MCMC are compared in Table 4. For the multimodal case shown in Table 4 (a), although the log-likelihood of ADVI is a little bigger than that of MCMC, the difference is negligible below 0.05%. The difference between the left edges or right edges of the 99% confidence intervals (CIs) for the two methods is also below 0.05%. For the asymmetric case shown in Table 4 (b), the difference between the log-likelihood or both edges of the 99% CIs is also below 0.05%.

To construct discrimination models for 112 resonance lines based on the hammering-sound data of 264 non-defective items listed in Table 2, the MCMC tool of the Python implementation library [18] required 177.7 minutes; on the other hand, the ADVI tool took only 69.3 minutes. MCMC required 2.6 times longer calculation time than ADVI. Although there is room for optimizing the sampling repetition count of MCMC or introducing an automatic method for judging convergence of the sampling sequences, a general-purpose approach for hundreds of resonance lines whose frequency distributions show complicated shapes is not easy. On the contrary, automatic convergence judgment by ADVI is easy to introduce to the evidence lower bound (ELBO) (objective function to be optimized) shown in Fig. 12, which depicts a monotonous decrease until convergence. To estimate the mixed-normal distribution function for variation of the resonance-line frequency, ADVI (with higher calculation speed and comparable estimation accuracy) is more advantageous than MCMC.

6. Conclusion

In this paper, targeting the cast-iron parts used in multi-product mass production, and aiming at high-speed inspection of defective material such as graphite-spheroidized defective parts by hammering test, an automatic method for constructing discrimination model for resonance spectra was proposed, and its effectiveness was experimentally verified. As for the proposed method, only the non-defective hammering-sound data that can be collected from the production line are input, and a distribution function that fits the frequency distribution of a resonance line in the spectrum is estimated. The confidence interval of the obtained distribution function is then regarded as non-defective-part existence section, and parts outside the section are discriminated as defective parts.

Since the frequency distribution of the resonance line shows multimodality and asymmetry, estimation is performed sequentially by using automatic differentiation variational inference (ADVI) while the number of components of the mixed normal distribution function is changed, and a method of searching for the distribution function with the maximum likelihood from among multiple mixed functions is adopted.

As a result, the well-fitted distribution function can automatically exclude the statistical outliers appearing in some of the resonance-frequency data that may be caused by burrs or other random casting variations on non-defective parts; accordingly, we expect to highly accurately estimate the non-defective-part variation interval from the non-excessive confidence interval. Since the discrimination model is automatically constructed by connecting the non-defective-part variation sections of all the resonance lines in the audible range, it is not necessary to select a few resonance lines with high sensitivity in advance, and since all resonance lines in the spectrum can be discriminated, it is possible to realize robust inspection using redundancy.

The results of experimentally casting graphite-spheroidized defective parts and discriminating them confirmed that it is pos-
sible to achieve hammer-test inspection with the detection rate of 100% and prevent overlooking of defective parts.

Comparing the two probabilistic methods (ADVI and Markov chain Monte Carlo method) suggested that in terms of higher calculation speed and comparable estimation accuracy, ADVI is superior to MCMC.

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