Spin susceptibility of degenerate quark matter

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The expression for the spin susceptibility $\chi$ of degenerate quark matter is derived with corrections up to $O(g^4 \ln g^2)$. It is shown that at low density, $\chi^{-1}$ changes sign and turns negative indicating a ferromagnetic phase transition. To this order, we also calculate sound velocity $c_1$ and incompressibility $K$ with arbitrary spin polarization. The estimated values of $c_1$ and $K$ show that the equation of state of the polarized matter is stiffer than the unpolarized one. Finally we determine the finite temperature corrections to the exchange energy and derive corresponding results for the spin susceptibility.

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I. INTRODUCTION

One of the active areas of high energy physics research has been exploration of the so called Quantum Chromodynamics (QCD) phase diagram. In particular, with the advent of ultrarelativistic heavy ion beams at RHIC and CERN and with the upcoming facilities of GSI where compressed baryonic matter is expected to be produced, such studies have assumed special importance. Beside the laboratory experiments, various astrophysical objects like neutron stars, quark stars, provide natural sites where many of the theoretical conjectures about the various phases of quark matter can be tested. The latter, in the present context, is more relevant here, as we study the possibility of para-ferro phase transition in dense quark system interacting via one gluon exchange.

The original idea about para-ferro phase transition in quark matter was proposed recently
in [1] where the possibility of Bloch like phase transition [2] was studied and it was shown that spin polarized quark matter might exist at low density [3]. The underlying mechanism of such a phase transition is analogous to what was originally proposed for the degenerate electron gas [2]. There, for Coulomb interaction, it was shown that the exchange correction to the energy is attractive which at the low density wins over the kinetic energy giving rise to a ferromagnetic state [2]. In [1], a variational calculation has been performed to show that it is indeed possible to have a spin polarized quark matter at low density of strange quark system, while for light quark it never happens [1]. Similar difference of the light and strange quark matter, albeit in a different context, was observed earlier [4]. However, in [3], it was shown that both the light and heavy flavor systems can exhibit such phase transitions although the critical density for the strange matter is higher than the light quark systems. Such investigations, have also been performed in [5, 6, 7, 8] and also in [9, 10] where the calculation has been extended to include thermal effects. The Bloch like phase transition, for strange quark, has also been reconfirmed in [11].

One shortcoming of all these works including [11], has been that the calculations were restricted to the Hartree Fock level and the terms beyond the exchange diagrams, commonly termed as correlation energy [12, 13, 14, 15, 16] were ignored. Without such corrections, however, the calculations are known to remain incomplete as the higher order terms are plagued with infrared divergences arising out of the exchange of massless gluons, indicating the failure of the naive perturbation series. We know that this problem can be cured by reorganizing the perturbation theory where a particular class of diagrams, viz. the bubbles are resummed in order to obtain a finite result. Originally, as is well known, this was done by Gell-Mann and Brueckner [17] while calculating the ground state energy of degenerate electron gas. The contribution of the bubbles involve terms of $O(g^4 \ln g^2)$ indicating non-perturbative nature of the correction [18, 19, 20, 21].

In the present work, as announced, we calculate the spin susceptibility ($\chi$) of dense quark system with corrections due to correlations i.e. containing terms upto $O(g^4 \ln g^2)$. This requires the knowledge of the ground state energy (GSE) of spin polarized matter with inclusion of bubble diagrams. The GSE of the polarized quark matter has been calculated only recently in [16] which is the starting point of the present paper. This work is very similar to that of Brueckner and Swada [22] and those of [23, 24], applied to the case of QCD matter. Unlike, degenerate electron gas, however, we have both the electric and magnetic interactions
and the calculation is performed relativistically, while the non-relativistic results appear as a limit.

The spin susceptibility $\chi$, for quark matter up to $O(g^2)$ has already been calculated in Ref. [1] which we only briefly discuss. Subsequently, the non-fermi liquid corrections to $\chi$ has also been studied in [9, 10]. These studies provide further motivation to undertake the present endeavor to include correlation corrections, without which, as mentioned already, the perturbative evaluation of $\chi$ remains incomplete. In addition, we also calculate incompressibility and sound velocity for spin polarized quark matter with corrections due to correlations which involve evaluation of single particle energy at the Fermi surface. These quantities are of special interests for applications to astrophysics. Moreover, we also evaluate the exchange energy density at non-zero temperature and determine the corresponding corrections to the spin susceptibility.

The plan of the paper is as follows. In Sec. II we calculate spin susceptibility with correlation correction for degenerate quark matter. Analytic expressions are presented both in ultra-relativistic (UR) and non-relativistic (NR) limit. In Sec. III, we evaluate exchange energy density and spin-susceptibility at non-zero temperature. In Sec. IV we summarize and conclude. Detailed expressions of the intermediate expressions, from which $\chi$ is derived, have been relegated to the Appendix.

II. SPIN SUSCEPTIBILITY

The spin susceptibility of quark matter is determined by the change in energy of the system as its spins are polarized [22]. We introduce a polarization parameter $\xi = (n_+ - n_-)/n_q$ with the condition $0 \leq \xi \leq 1$, where $n_+$ and $n_-$ correspond to densities of spin-up and spin-down quarks respectively, and $n_q = n_+ + n_-$ denotes total quark density. The Fermi momenta in the spin-polarized quark matter then are $p_f^+ = p_f(1+\xi)^{1/3}$ and $p_f^- = p_f(1-\xi)^{1/3}$, where $p_f = (\pi^2n_q)^{1/3}$, is the Fermi momentum of the unpolarized matter ($\xi = 0$). In the small $\xi$ limit, the ground state energy behaves like $[1]$

$$E(\xi) = E(\xi = 0) + \frac{1}{2}\beta_s \xi^2 + O(\xi^4).$$

(1)
Here, $\beta_s = \frac{\partial^2 E}{\partial \xi^2} \bigg|_{\xi=0}$, defined to be the spin stiffness constant in analogy with $[16, 21]$. The spin susceptibility $\chi$ is proportional to the inverse of the spin stiffness, mathematically $\chi = 2\beta_s^{-1}$ $[25]$. It is to be noted that in Eq. (1), the first term corresponds to unpolarized matter energy.

Now, the leading contributions to the ground state energy are given by the three terms viz. kinetic, exchange and correlation energy density $[16]$ i.e.

$$E = E_{\text{kin}} + E_{\text{ex}} + E_{\text{corr}}.$$  \hfill (2)

The total kinetic energy density for spin-up and spin-down quark becomes $[1, 11]

\begin{align*}
E_{\text{kin}} &= \frac{3}{16\pi^2} \left\{ p_f (1 + \xi)^{1/3} \sqrt{p_f^2 (1 + \xi)^{2/3} + m_q^2} \left[ 2p_f^2 (1 + \xi)^{2/3} + m_q^2 \right] \\
&\quad - m_q^4 \ln \left( \frac{p_f (1 + \xi)^{1/3} + \sqrt{p_f^2 (1 + \xi)^{2/3} + m_q^2}}{m_q} \right) \right\},
\end{align*} \hfill (3)

where $m_q$ is the quark mass.

The exchange energy density $E_{\text{ex}}$ have been calculated in ref. $[11]$ within Fermi liquid theory approach. One can also directly evaluate the two loop diagram $[1]$ to obtain

\begin{align*}
E_{\text{ex}}^{nf} &= \frac{9}{2} \sum_{s=\pm} \int \int \frac{d^3 p}{(2\pi)^3} \frac{d^3 p'}{(2\pi)^3} \theta(p_f - |p|) \theta(p'_f - |p'|) f_{pp'}^{nf}, \\
E_{\text{ex}}^f &= 9 \int \int \frac{d^3 p}{(2\pi)^3} \frac{d^3 p'}{(2\pi)^3} \theta(p_f - |p|) \theta(p'_f - |p'|) f_{pp'}^{f},
\end{align*} \hfill (4-5)

where $f_{pp'}^{nf}$ and $f_{pp'}^{f}$ stands for non-flip ($s = s'$) and flip ($s = -s'$) forward scattering amplitude given in $[1, 11, 16]$. Here, $E_{\text{ex}} = E_{\text{ex}}^{nf} + E_{\text{ex}}^f$ can be estimated numerically. However, analytical evaluation of these integrals is possible in the ultra-relativistic and non-relativistic limits as reported in $[1, 11, 16]$.

The next higher order correction to the ground state energy beyond the exchange term is the correlation energy $E_{\text{corr}}$ $[12, 13, 14, 15]$. The detailed calculation of correlation energy for spin polarized matter have been derived in $[16]$ which we quote here:

$$E_{\text{corr}} \simeq \frac{1}{(2\pi)^3} \frac{1}{2} \int_0^{\pi/2} \sin^2 \theta_E \text{d} \theta_E \left\{ \Pi_L^2 \left[ \ln \left( \frac{\Pi_L}{\xi_f^2} \right) - \frac{1}{2} \right] + 2 \Pi_T^2 \left[ \ln \left( \frac{\Pi_T}{\xi_f^2} \right) - \frac{1}{2} \right] \right\}, \hfill (6)$$
with $\theta_E = \tan^{-1}(|k|/k_0)$. The relevant $\Pi_L$ and $\Pi_T$ are determined to be\[16\]

$$
\Pi_L = \frac{g^2}{4\pi^2} \sum_{s=\pm} \frac{p_f^s \varepsilon_f^s}{\sin^2 \theta_E} \left[ 1 - \frac{\cot \theta_E}{v_f^s} \tan^{-1} \left( \frac{v_f^s \tan \theta_E}{v_f^s} \right) \right],
$$

(7)

$$
\Pi_T = \frac{g^2}{8\pi^2} \sum_{s=\pm} p_f^s \cot \theta_E \left[ -\frac{\cot \theta_E}{v_f^s} + \left( 1 + \frac{\cot^2 \theta_E}{v_f^s} \right) \tan^{-1} \left( \frac{v_f^s \tan \theta_E}{v_f^s} \right) \right].
$$

(8)

The spin susceptibility is given by\[1\]

$$
\chi^{-1} = \frac{1}{2} \frac{\partial^2 E(\xi)}{\partial \xi^2} \bigg|_{\xi=0}.
$$

(9)

We have $\chi^{-1} \equiv \chi_{kin}^{-1} + \chi_{ex}^{-1} + \chi_{corr}^{-1}$. The kinetic and exchange contribution have been evaluated in ref.[1], is given by

$$
\chi_{kin}^{-1} = \frac{p_f^5}{6\pi^2 \varepsilon_f},
$$

(10)

$$
\chi_{ex}^{-1} = -\frac{g^2 p_f^4}{18\pi^4} \left\{ 2 - \frac{6p_f^2}{\varepsilon_f^2} - \frac{3p_f}{\varepsilon_f^3} \left( p_f \varepsilon_f - m_q^2 \ln \left( \frac{p_f + \varepsilon_f}{m_q} \right) \right) \right\} + \frac{2p_f^3}{\varepsilon_f^2} \left[ 1 + \frac{2m_q}{3(p_f + m_q)} \right].
$$

(11)

To determine the correlation correction to spin susceptibility, we expand curly braces terms of Eq.(6) in powers of the polarization parameter $\xi$, which gives

$$
\Pi_L^2 \left[ \ln \left( \frac{\Pi_L}{\varepsilon_f} \right) - \frac{1}{2} \right] + 2\Pi_T^2 \left[ \ln \left( \frac{\Pi_T}{\varepsilon_f} \right) - \frac{1}{2} \right] = (A_{0L} + B_{0T}) + \xi^2 (A_{1L} + B_{1T}) + O(\xi^4).
$$

(12)

Here, $A_{0L}$ and $B_{0T}$ correspond to unpolarized matter term and the detailed expressions of $A_{1L}$ and $B_{1T}$ are given in the Appendix. $\chi_{corr}^{-1}$ is

$$
\chi_{corr}^{-1} = \frac{1}{2} \frac{\partial^2 E_{corr}(\xi)}{\partial \xi^2} \bigg|_{\xi=0} \approx \frac{1}{(2\pi)^3} \frac{1}{2} \int_0^{\pi/2} \sin^2 \theta_E d\theta_E (A_{1L} + B_{1T}).
$$

(13)

From the above expression and with the help of the expression presented in the Appendix, $\chi_{corr}^{-1}$ can be estimated numerically. Results for the two limiting cases however can be obtained analytically as we present in the following two sub-sections.
A. Ultra-relativistic limit

In the ultra-relativistic limit, the kinetic, exchange and correlation energies are

\[ E_{ur\,kin} = \frac{3p_f^4}{8\pi^2} [(1 + \xi)^{4/3} + (1 - \xi)^{4/3}], \]
\[ E_{ur\,ex} = \frac{g^2}{32\pi^4} p_f^4 [(1 + \xi)^{4/3} + (1 - \xi)^{4/3} + 2(1 - \xi^2)^{2/3}], \]
\[ E_{ur\,corr} = \frac{g^4 \ln g^2}{2048\pi^6} p_f^4 [(1 + \xi)^{4/3} + (1 - \xi)^{4/3} + 2(1 - \xi^2)^{2/3}]. \]  

(14)

With the help of Eq.(1), each energy contribution to the susceptibility is

\[ \chi_{kin}^{-1} = \frac{p_f^4}{6\pi^2} \]
\[ \chi_{ex}^{-1} = -\frac{g^2 p_f^4}{36\pi^4} \]
\[ \chi_{corr}^{-1} = -\frac{g^4 p_f^4}{2304\pi^6} (\ln r_s - 0.286). \]  

(15)

with \( r_s = g^2 (3\pi)^{1/3} \). From Eq.(15), sum of all the contribution to the susceptibility can be written as

\[ \chi^{ur} = \chi_P[1 - \frac{g^2}{6\pi^2} - \frac{g^4}{384\pi^4} (\ln r_s - 0.286)]^{-1}, \]  

(16)

where \( \chi_P \) is the non-interacting susceptibility.

B. Non-relativistic limit

Now, we go to the non-relativistic limit to calculate spin-susceptibility in order to compare our results with those of dense electron gas interacting via static Coulomb potential. In this limit, kinetic and exchange energy densities are

\[ E_{nr\,kin} = \frac{3p_f^5}{20\pi^2 m_q} [(1 + \xi)^{5/3} + (1 - \xi)^{5/3}], \]
\[ E_{nr\,ex} = -\frac{g^2}{8\pi^4} p_f^4 [(1 + \xi)^{4/3} + (1 - \xi)^{4/3}]. \]  

(17)
The contribution to the susceptibility from kinetic and exchange energy density yields

\[ \chi^{-1}_{\text{kin}} = \frac{p_f^5}{6\pi^2 m_q}, \]

\[ \chi^{-1}_{\text{ex}} = -\frac{g^2 p_f^4}{18\pi^3}. \]  \hspace{1cm} (18)

We want to calculate the contribution to the spin-susceptibility beyond the exchange correction. For this we first evaluate the correlation energy in this limit.

The dominant contribution to the correlation energy is found to be,

\[ E_{\text{nr, corr}} = -\lambda^2 p_f^5 \int^{k_c} \frac{dk'}{k'} \int_{0}^{\infty} x \sum_{s=\pm} f(s) \left[ 1 - \frac{x^s}{2} \ln \left( \frac{x^s + 1}{x^s - 1} \right) \right] \sum_{s'=\pm} \theta(1 - x^{s'}), \]  \hspace{1cm} (19)

where \( \lambda = \frac{g^2 m_q}{(8\pi p_f)}, f(s = \pm) = (1 \pm \xi)^{1/3}, x = x^s f(s), x^s = (k_0 m_q)/(p_f k) \) and \( k' = k/p_f \). For \( s = s' \) one obtains:

\[ E_{\text{nr, s=s'}} \approx g^4 \ln g^2 1 \frac{1}{3 m_q p_f^3} (1 - \ln 2), \]  \hspace{1cm} (20)

Note that, here, the correlation energy is independent of spin-polarization \( \xi \). It is seen, that for the spin parallel interactions \( \xi \)-dependent terms contribute with opposite sign and cancels each other. For \( s = -s' \), the integral on \( x \) takes the form

\[ I = \int_{0}^{\infty} x \sum_{s=\pm} f(s) \left[ 1 - \frac{x^s}{2} \ln \left( \frac{x^s + 1}{x^s - 1} \right) \right] \sum_{s'=\pm} \theta(1 - x^{s'}). \]  \hspace{1cm} (21)

Expanding \( \ln \) in terms of \( \xi \) and retain up to \( O(\xi^2) \) we have

\[ I \approx \frac{2}{3} \left[ (1 - \ln 2) - \frac{1}{6} \xi^2 \right]. \]  \hspace{1cm} (22)

Using Eq. (19), (21) and (22) we have

\[ E_{\text{nr, s=-s'}} \approx g^4 \ln g^2 1 \frac{1}{128\pi^6} \frac{3 m_q p_f^3}{3} \left[ (1 - \ln 2) - \frac{1}{6} \xi^2 \right]. \]  \hspace{1cm} (23)
It is to be mentioned that similar expressions for degenerate electron gas interacting via static Coulomb potential can be found in ref. [26]. From Eq. (20) and (23), it is clear that spin anti-parallel states are attractive in contrast to the parallel states due to Pauli exclusion principle. In this limit the correlation contribution to the susceptibility is found to be,

$$\chi_{corr}^{-1} = -\frac{g^4 \ln g^2}{2304 \pi^6} m_q p_f^3.$$  \hspace{1cm} (24)

The total susceptibility is given by

$$\chi^{nr} = \chi_P \left[ 1 - \frac{g^2 m_q}{3 \pi^2 p_f} - \frac{g^4 \ln g^2 m_q^2}{384 \pi^4 p_f^3} \right]^{-1}.$$ \hspace{1cm} (25)

FIG. 1: Density dependence of inverse spin susceptibility.

In Fig. (1) we plot inverse spin susceptibility which is valid for all the kinematic regimes. It shows $\chi^{-1}$ changes its sign at the density $\sim 0.12 \text{fm}^{-3}$ without correlation correction and when we include the correlation effect its sign changes at $\sim 0.1 \text{fm}^{-3}$. This is equivalent to what happens to the ground state energy as a function of $\xi$. It is needless to mention that this change of sign correspond to the para-ferro phase transition in dense quark system. The parameter set used here are same as those of [1, 4, 11, 16].
C. Incompressibility and Sound velocity

Once we have the expressions for the total energy density, the incompressibility \((K)\) and sound velocity \((c_1)\) can be determined. The incompressibility \(K\) is defined by the second derivative of the total energy density with respect to the number density \(n_q\), which is given by \[11\]

\[ K = 9n_q \frac{\partial^2 E}{\partial n_q^2}. \] (26)

Since, there are two Fermi surfaces corresponding to spin-up (+) and spin-down (−) states, such that \(E \equiv E(n_q^+, n_q^-)\). We have \[11\]

\[ \frac{\partial E}{\partial n_q} = \frac{\partial E}{\partial n_q^+} + \frac{\partial E}{\partial n_q^-} \]
\[ = \frac{1}{2} \left[ (1 + \xi)\mu^+ + (1 - \xi)\mu^- \right]. \] (27)

The single particle energy at the Fermi surface or chemical potential of spin-up quark turns out to be

\[ \mu^{+,ur} = \mu^{+,kin} + \mu^{+,ex} + \mu^{+,corr} \]
\[ = p_f^+ + \frac{g^2}{12\pi^2} \left( p_f^+ + \frac{p_{f+}^2}{p_f} \right) + \frac{g^4}{768\pi^4} \left( p_f^+ + \frac{p_{f+}^2}{p_f} \right). \] (28)

Similarly, \(\mu^{-,ur}\) can be obtained by replacing \(p_f^+\) with \(p_f^-\) in Eq.\(28\). In ref.\[11\], chemical potential is determined within Fermi liquid theory approach upto \(\mathcal{O}(g^2)\). We, however, here calculate \(\mu^\pm\) with different approach upto \(\mathcal{O}(g^4 \ln g^2)\).

Using Eq.\(27\) and Eq.\(28\), the incompressibility becomes

\[ K^{ur} = \frac{3}{2} p_f \left\{ \left[ (1 + \xi)^{4/3} + (1 - \xi)^{4/3} \right] + \frac{g^2}{12\pi^2} \left[ (1 + \xi)^{4/3} + (1 - \xi)^{4/3} + 2(1 - \xi^2)^{2/3} \right] \right. \]
\[ + \frac{g^4}{768\pi^4} \left[ (1 + \xi)^{4/3} + (1 - \xi)^{4/3} + 2(1 - \xi^2)^{2/3} \right] \} . \] (29)

Another interesting quantity would be to calculate the first sound velocity which is given by the first derivative of pressure with respect to energy density. Mathematically \[11\],
\[ c_1^2 = \left[ \frac{(1 + \xi)n_q^+ \frac{\partial \mu^+}{\partial n_q^+} + (1 - \xi)n_q^- \frac{\partial \mu^-}{\partial n_q^-}}{(1 + \xi)\mu^+ + (1 - \xi)\mu^-} \right]. \]  

(30)

From Eq. (28), we have

\[ \frac{\partial \mu^+}{\partial n_q^+} = \frac{2\pi^2}{3\rho_f^2(1 + \xi)^{2/3}} \left\{ 1 + \frac{g^2}{12\pi^2} \left[ \frac{(1 + \xi)^{2/3} - (1 - \xi)^{2/3}}{(1 + \xi)^{2/3}} \right] \right\} + \frac{g^4 \ln g^2}{768\pi^4} \left[ \frac{(1 + \xi)^{2/3} - (1 - \xi)^{2/3}}{(1 + \xi)^{2/3}} \right]. \]  

(31)

The second and last term in the curly braces corresponds to exchange and correlation contribution respectively. Similarly, \( \partial \mu^- / \partial n_q^- \) can be obtained by replacing \( \xi \) with \( -\xi \). Using \( n_q^\pm, \mu^\pm \), and \( \partial \mu^\pm / \partial n_q^\pm \), we calculate the sound velocity in terms of \( \xi \). Numerically, for unpolarized matter \( c_1 = 0.46 \), while for complete polarized matter \( c_1 = 0.54 \), which is below the causal value \( 1 / \sqrt{3} = 0.57 \) at the high density limit.

**FIG. 2:** Incompressibility \( K \) in the spin polarized quark matter.

In Fig[2], we plot the density dependencies of the incompressibility with correlation correction. This shows for higher value of the order parameter \( \xi \), the incompressibility becomes higher for the same value of density. Thus numerical values of incompressibility and sound velocity shows that equation of state for polarized quark matter is stiffer than the unpolarized one [11].
III. SUSCEPTIBILITY AT NON-ZERO TEMPERATURE

In this section we calculate the exchange energy density $E_{ex}$ at low-temperature ($T << \varepsilon_f$), for which we replace $\theta(p_\uparrow^2 - |p|)$ of Eqs. (4-5) with proper Fermi distribution function. In the ultra-relativistic limit, the angular averaged interaction parameter is given by \[ f_{ur}^{pp'} = g^2 \int \frac{d\Omega_1}{4\pi} \int \frac{d\Omega_2}{4\pi} \left[ 1 + \hat{p} \cdot \hat{s} \right] \left( \hat{p}' \cdot \hat{s}' \right) \]

The spin non-flip contribution to the exchange energy density is

$$E_{ex}^{nf} = \frac{9}{2} \sum_{s=\mp} \int \int \frac{d^3p}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} f_{pp'}^{nf} n_s^*(T) n_{s'}(T)$$

$$\simeq \frac{g^2}{32\pi^4} T^2 p_f^4 \left[ (1 + \xi)^{4/3} + (1 - \xi)^{4/3} \right] + \frac{g^2}{48\pi^2} T^2 p_f^2 \left[ (1 + \xi)^{2/3} + (1 - \xi)^{2/3} \right].$$

Here $n_{s,p}(T)$ is the Fermi distribution function.

Similarly, $E_{ex}^f$ can be evaluated. The total $E_{ex}^{ur}$ at low temperature is found to be

$$E_{ex}^{ur} \simeq \frac{3}{8\pi^2} p_f^4 \left[ (1 + \xi)^{4/3} + (1 - \xi)^{4/3} \right] + \frac{3T^2 p_f^2}{4} \left[ (1 + \xi)^{2/3} + (1 - \xi)^{2/3} \right].$$

The kinetic energy density can be written as

$$E_{kin}^{ur} \simeq \frac{3}{8\pi^2} p_f^4 \left[ (1 + \xi)^{4/3} + (1 - \xi)^{4/3} \right] + \frac{3T^2 p_f^2}{4} \left[ (1 + \xi)^{2/3} + (1 - \xi)^{2/3} \right].$$

From Eq. (11) each energy contribution to the susceptibility is

$$\chi^{-1}_{kin} = \frac{p_f^4}{6\pi^2} \left( 1 - \frac{\pi^2 T^2}{p_f^2} \right)$$

$$\chi^{-1}_{ex} = \frac{-g^2 p_f^4}{36\pi^4} \left( 1 + \frac{\pi^2 T^2}{3p_f^2} \right).$$

It is to be noted that the $T$ independent terms of the above expressions are identical with those Eqs. (14, 15). Thus the susceptibility at non-zero temperature is given by
\[ \chi^{ur} = \chi_P \left[ 1 - \frac{g^2}{6\pi^2} \left( 1 + \frac{4\pi^2 T^2}{3p_f^2} \right) \right]^{-1}. \] (37)

In the non-relativistic limit the interaction parameter takes the following form \[ \chi^{ur} = \chi_P \left[ 1 - \frac{g^2}{6\pi^2} \left( 1 + \frac{4\pi^2 T^2}{3p_f^2} \right) \right]^{-1}. \] (37)

For spin anti-parallel interaction \( s = -s' \), then \( f_{pp'} = 0 \). Thus the contribution due to the scattering of quarks with unlike spin states vanishes and the dominant contribution to energy density comes from the parallel spin states \( (s = s') \). Performing the angular integration of Eq.(41), the exchange energy density upto term \( O(T^2) \) becomes

\[
E_{ex}^{nr} = \frac{-g^2}{4\pi^4} \sum_{s=\pm} \int p \, dp \, n^*_p(T) \int p' \, dp' \, n^*_{p'}(T) \ln \left| \frac{p + p'}{p - p'} \right| \\
\simeq -\frac{g^2}{8\pi^4} p_f^4 \left[ (1 + \xi)^{4/3} + (1 - \xi)^{4/3} \right] - \frac{g^2}{8\pi^2} T^2 m_q p_f \left[ (1 + \xi)^{1/3} + (1 - \xi)^{1/3} \right]. \] (39)

The kinetic energy density is found to be

\[
E_{kin}^{nr} \simeq \frac{3p_f^5}{20\pi^2 m_q} \left[ (1 + \xi)^{5/3} + (1 - \xi)^{5/3} \right] + \frac{T^2 p_f^2}{2} \left[ (1 + \xi)^{2/3} + (1 - \xi)^{2/3} \right]. \] (40)

Separate contribution from kinetic and exchange energy to susceptibility becomes

\[ \chi_{kin}^{-1} = \frac{p_f^5}{6\pi^2 m_q} \left( 1 - \frac{2\pi^2 m_q T^2}{3p_f^2} \right) \]
\[ \chi_{ex}^{-1} = \frac{g^2 p_f^4}{18\pi^4} \left( 1 - \frac{\pi^2 m_q T^2}{2p_f^2} \right). \] (41)

Thus, at low temperature the susceptibility turns out to be

\[ \chi^{nr} = \chi_P \left[ 1 - \frac{g^2 m_q}{3\pi^2 p_f} \left( 1 + \frac{\pi^2 m_q T^2}{6p_f^2} \right) \right]^{-1}. \] (42)
IV. SUMMARY AND CONCLUSION

In this work we have derived the spin susceptibility for degenerate quark matter with corrections due to correlation contributions. Analytic expressions for susceptibility are also derived both in the ultra-relativistic and the non-relativistic limit. It is observed that at low density susceptibility changes sign and becomes negative suggesting the possibility of ferromagnetic phase transition. In addition, we also derive single particle energy, sound velocity and incompressibility upto $O(g^4 \ln g^2)$. As far as the equation of state is concerned, in the present model, we find that equation of state for polarized matter is stiffer than that of unpolarized one. We also determine the exchange energy and susceptibility at non-zero temperature of the spin polarized quark matter.
V. APPENDIX

To calculate the correlation contribution to the spin susceptibility we have from Eq.(12)

\[ A_{1L} = - \frac{g^4 p_f^2 \sec^4 \theta_E \csc^6 \theta_E}{1152 \pi^4 \varepsilon_f^3 (m_q^2 + p_f^2 \sec^2 \theta_E)^2} \]
\[ \times \ln \left\{ \frac{g^2 \csc^2 \theta_E}{2 \pi^2 \varepsilon_f} \left[ p_f - \varepsilon_f \cot \theta_E \tan^{-1}(v_f \tan \theta_E) \right] \right\} \]
\[ \times \left\{ 64 \varepsilon_f^5 \cos^2 \theta_E \tan^{-1}(v_f \tan \theta_E)(p_f^2 + m_q^2 \cos^2 \theta_E)^2 \right. \]
\[ - 2p_f^2 \varepsilon_f^2 \sin 2\theta_E \tan^{-1}(v_f \tan \theta_E) \{12m_q^6 + 51m_q^4p_f^2 \}
\[ + m_q^4(4m_q^2 + 5p_f^2) \cos 4\theta_E + 68m_q^2p_f^2 + 4m_q^2(4m_q^4 + 10m_q^2p_f^2 + 7p_f^4) \cos 2\theta_E + 32p_f^6 \}\n\[ + 4p_f^2 \varepsilon_f \sin^2 \theta_E [6m_q^6 + 29m_q^4p_f^2 + m_q^4(2m_q^2 + 3p_f^2) \cos 4\theta_E + 36m_q^2p_f^4 \]
\[ + 4m_q^2(2m_q^4 + 4m_q^2p_f^2 + 3p_f^4) \cos 2\theta_E + 16p_f^6] \}
\[ (43) \]

\[ B_{1T} = \frac{g^4 p_f^2 \cot^2 \theta_E \csc^4 \theta_E}{1152 \pi^4 \varepsilon_f^3 (m_q^2 \cos^2 \theta_E + p_f^2)} \]
\[ \times \ln \left\{ \frac{g^2 \cot \theta_E \csc \theta_E}{8 \pi^2 \varepsilon_f^3} \left[ 2 \tan^{-1}(v_f \tan \theta_E)(m_q^2 \cos^2 \theta_E + p_f^2) - p_f \varepsilon_f \sin 2\theta_E \right] \right\} \]
\[ \times \left\{ - 32 \varepsilon_f^3 \tan^{-1}(v_f \tan \theta_E)(m_q^2 \cos^2 \theta_E + p_f^2)^2 \right. \]
\[ - 8p_f^2 \varepsilon_f [m_q^4 + p_f^4 + m_q^2p_f^2(1 + \cos^2 \theta_E)] \sin^2 2\theta_E + 2p_f \tan^{-1}(v_f \tan \theta_E) \sin 2\theta_E \]
\[ \times \left[ 8m_q^6 + 31m_q^4p_f^2 + m_q^2p_f^2 \cos 4\theta_E + 36m_q^2p_f^4 + 4m_q^2(2m_q^4 + 4m_q^2p_f^2 + 3p_f^4) \cos 2\theta_E + 16p_f^6 \right] \} \].
\[ (44) \]

with \( v_f = p_f/\varepsilon_f \).

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