Is Stringy-Supersymmetry Quintessentially Challenged?

S. James Gates, Jr.

STIAS
Private Bag XI
Stellenbosch, 7602
Republic of South Africa

ABSTRACT

We discuss the problem of introducing background spacetimes of the de Sitter type (quintessential backgrounds) in the context of fundamental theories involving supersymmetry. The role of a model presented in 1984, showing that these backgrounds can occur as spontaneously broken phases of locally supersymmetric 4D, \( N \)-extended theories, is highlighted. The present twin challenges of the presume presence of supersymmetry in particle physics and the emergence of experimental evidence for a positive cosmological constant from type-II supernovae data are noted for the continued investigation of superstring/M-theory. Finally we note how the 1984-model may have a role to play in future investigations.

PACS: 03.70.+k, 11.30.Rd, 04.65.+e
Keywords: Gauge theories, Supersymmetry, Supergravity.
1 Introduction

I wish to begin by thanking the organizing committee of the conference for the invitation to speak at this meeting. For twenty-five years, I have offered contributions to the work of this community. Just prior to the first presentation of supergravity theory by Freedman, van Nieuwenhuizen and Ferrara [1], as a graduate student I turned attention to (and later published [2]) a study of supergeometry. In those days, there was a competing construct called “gauge supersymmetry” based on the notion of extending Riemannian geometry to Salam-Strathdee superspace. Approaching the attainment of my graduate degree, I had understood why the work of Gordon Woo clearly indicated that any attempt based on Riemannian geometry was doomed to failure.

On becoming a postdoctoral researcher as a Junior Fellow entering Harvard’s physics department, I was already busily attempting to understand how a Riemann-Cartan geometry might serve to by-pass the difficulty indicated by Woo’s work. There I met Warren Siegel. Within five minutes of meeting, we had a serious disagreement over what must be the nature of a successful theory of curved superspace. The approach he was pursuing initially had no relation at all to the notions of supergeometry. Instead he anchored his approach on the principles of 4D, $\mathcal{N}=1$ supersymmetric gauge theory. After getting past our initial rough beginning, we joined forces and were able to produce a complete theory of 4D, $\mathcal{N}=1$ curved superspace [3] that contained both the elements of supersymmetric gauge theory and notions of supergeometry. Warren’s pre-potentials were “hiding” inside the geometrical entities on which I had focussed my attention.

The hosts of this meeting charged each speaker to be a bit like the roman god Janus and make a presentation that looks forward to unsolved problems of the future in light of past solved problems in the field. The story in the previous paragraph is presented in light of our charge. As I look at our field today and its connection to superstring theory, I am struck at how similar is the present state to that time before supergeometry and pre-potentials had been successfully joined. In 1989, I gave a presentation the XXV Winter School of Theoretical Physics in Karpacz, Poland. Near the end of my contribution to the proceedings there appears an appendix entitled, “Treat the String Field as a Prepotential!” Thirteen years have passed and we seem only a little closer to this goal in covariant string field theory. The story above, in my view, will ultimately be repeated for this greatest challenge to superstring theory.

\footnote{To my knowledge, this word was coined in one of our papers[4]. As I recall, a referee complained that we should at least explain what it meant!}
which contains supergravity theory as a particular limit. The gauge transformations
of open string theory now seem to bare a striking resemblance to the chiral trans-
formation (Λ-gauge symmetries) of superfield supergravity or superfield Yang-Mills
theories. To my mind the relation between open strings versus closed strings is very
reminiscent of that between chiral and vector superfields. Thus, from the perspective
of supergravity, the gauge parameters of closed superstrings should be open super-
strings. So some progress, I believe, is being made. If my conjecture is correct, at
some point in the future there will exist a geometrical theory of superstrings built
(at least) in part on a “string space” possessing stringy torsions, curvatures and field
strengths. These are likely to possess constraints whose solutions will, indeed, have
the string field as their prepotential solution. In others words, I believe superstring
theory will ultimately follow the supergravity paradigm.

On the occasion of the first day of this meeting, our community has drawn itself
together. In a sense the various members of this community seem almost like members
of a single family. (We even have had some terrific fights in the past to prove it.)
But still the community has struggled to make its contributions to the advancement
of our field and is in a healthy state today with still unmet challenges ahead.

2 Non-supersymmetric Preliminary Remarks

The cosmological constant \( \lambda \) has provided a topic of lively debate almost continuous-
ously since its introduction into the physics literature [5]. It enters the Einstein Field
Equations\[ as

\[
\mathcal{R}_{ab} - \frac{1}{2} \eta_{ab} \mathcal{R} + \lambda \eta_{ab} = - \frac{1}{6} \kappa^2 T_{ab}. \tag{1}
\]

At the time of its introduction, Einstein stated, “That term is necessary only for
the purpose of making possible a quasi-static distribution of matter, as required by
the fact of the small velocities of the stars.” Of course, this reason has long since
disappeared. With the discovery of the expansion of the universe, Einstein later
described the cosmological constant as, “the biggest blunder he ever made in his
life.”

However, within the present epoch of particle physics there has been a “cos-
mological constant problem.” To most simply see this problem it suffices to use a
toy model that is easy to construct by considering a self-interacting spin-0 field (\( \phi \))

\[ \kappa^2 = 48 \pi G [6]. \]
in the presence of gravitational interactions. We may write a model of the form

\[ S_{\text{Tot}} = S_{\text{grav}} + S_{\text{matter}} \]

where

\[ S_{\text{grav}} = \frac{3}{\kappa^2} \int d^4 x \, e^{-1} \left[ \mathcal{R}(e) - \lambda \right], \]

\[ S_{\text{matter}} = \int d^4 x \, e^{-1} \left[ -\frac{1}{4}(e^a \phi)(e^{a} \phi) - \left( \frac{1}{4!} \lambda_0 \phi^4 + \frac{1}{2} m_0^2 \phi^2 \right) \right]. \]  

(2)

Upon examination of the equation of motion for \( \phi \), with the additional assumption that \( \phi \) has a nonvanishing vacuum expectation value \( \langle \phi \rangle \), we find that \( V'(\langle \phi \rangle) = 0 \) (with \( V(\phi) = \frac{1}{4!} \lambda_0 \phi^4 + \frac{1}{2} m_0^2 \phi^2 \)) defines the ground state. If \( m_0^2 > 0 \) then \( \langle \phi \rangle = 0 \) and the cosmological constant that appears is the second term in the first action describing the “vacuum” spacetime (a space of constant curvature completely described by \( \lambda \)). This vacuum spacetime is flat when \( \lambda = 0 \). On the other hand, things change markedly if \( m_0^2 < 0 \) (as in the case of spontaneous symmetry breakdown). In this case, \( \langle \phi \rangle \neq 0 \) and the vacuum spacetime has a cosmological constant given by \( \lambda_{\text{tot}} = -\lambda + \frac{\kappa^2}{3} V(\langle \phi \rangle) \). Evaluated at the vev of \( \phi \), the potential takes the form \( V(\langle \phi \rangle) = -(3/2) \lambda_0 (m_0^4/\lambda_0^2) \) so \( \lambda_{\text{tot}} = \lambda + \lambda_0 (\kappa^2 m_0^4/2\lambda_0^2) \). Since \( \lambda, \lambda_0 \) and \( m_0 \) are completely free parameters, \( \lambda_{\text{tot}} \) can describe a de Sitter space (\( \lambda_{\text{tot}} > 0 \), anti-de Sitter space (\( \lambda_{\text{tot}} < 0 \)) or Minkowski space (\( \lambda_{\text{tot}} = 0 \)). We are thus able to “adjust” or “tune” the parameters of effective cosmological constant \( \lambda_{\text{tot}} \).

At the level of non-quantum classical considerations, the tuning of the original cosmological constant to achieve a vacuum spacetime that is flat may be considered a matter of taste. At the quantum level, there is the technical matter of considering the renormalized values of the bare parameters that appear above. In particular in the presence of quantum corrections, it is natural to expect a renormalized equation of the form \( \lambda_{\text{tot}} = c_1 \lambda + c_2 \frac{\kappa^2}{3} V(\langle \phi \rangle) \) where \( c_1 \) and \( c_2 \) are constants determined by quantum corrections to the theory. There is no reason to expect the continued equality of \( c_1 \) and \( c_2 \) and so to define a flat vacuum spacetime would require “re-tuning” (i.e. using a value for \( \lambda \) that is different from that in the non-quantum theory).

It is true that within ordinary gravity, the non-renormalizability of the theory intrudes into this argument. However, with the proposal of ‘eka-general relativity,’ such as superstring theory (or some as yet unknown construction), we might be forced to squarely face this problem still.

The value of the cosmological constant also shows up in a fundamental way in the structure of the spacetime symmetries of the universe. In a theory of gravitation\(^9\), a gauge covariant derivative

\[ \nabla_a \equiv e_\alpha^m \partial_m + \omega_\alpha^\gamma_\delta \mathcal{M}_\delta^\gamma + \omega_\alpha^\gamma_\delta \hat{\mathcal{M}}_\delta^\gamma, \]

(3)

\(^9\)For additional discussion of the gauge approach to gravitational theories see [7].
can be introduced. In a spacetime with vanishing torsion (thus determining the spin-connections, $\omega^a_{\gamma\delta}$ and $\omega^a_{\delta\gamma}$ in term of the vierbein $e^a_{\mu}$), the commutator algebra of this derivative takes the form

$$\left[ \nabla_a, \nabla_b \right] = R^\gamma_{\delta\gamma \delta} M^\gamma_{\delta} + R^\gamma_{\delta\gamma \delta} \overline{M}^\gamma_{\delta},$$

(4)

and due to the reality of the derivative, the field strength $R^\gamma_{\delta\gamma \delta}$ is just the complex conjugate of $R^\gamma_{\delta\gamma \delta}$.

The vacuum configuration of the gauge-gravitational covariant derivative in (3) is a specification of the vierbein field. One class of such field configurations is described by,

$$\nabla_a = \left[ 1 - \frac{1}{6} \lambda x^2 \right] \partial_a - \frac{1}{3} \lambda x_{\gamma\alpha} M_{\alpha \gamma} - \frac{1}{3} \lambda x_{\alpha\gamma} \overline{M}^\gamma_{\alpha},$$

(5)

dependent on the cosmological constant of the dimensions of mass-squared. Comparing (3) with (5), the spin-connection terms proportional to $M$ and $\overline{M}$ in the latter are chosen so that the torsion term vanishes consistently with (4).

When this field configuration is substituted into (4), we find

$$\left[ \nabla_a, \nabla_b \right] = -\frac{2}{3} \lambda \left[ C_{\alpha\beta} M_{\alpha \beta} + C_{\alpha\beta} \overline{M}^\alpha_{\beta} \right],$$

(6)

and thus the configuration in (5) describes a spacetime in which the Riemann curvature tensor is a constant

$$R^\gamma_{\delta\gamma \delta} = \frac{1}{3} \lambda C_{\alpha\beta} \left[ C_{\alpha\gamma \delta\beta} + C_{\beta\gamma \delta\alpha} \right].$$

(7)

Finally, the field configuration in (5) may be inserted into the Einstein Field Equation (1). The equation is found to be satisfied if

$$T^a_{\mu} = 0.$$  

(8)

A relation to the symmetries of spacetime comes about as follows. The gauge-gravitational covariant derivative may be used to define “covariant translation” operators $P_a = i \nabla_a$. Accordingly, the commutator algebra of the translation operators is fixed according to (3 - 6) so that

$$\left[ P_a, P_b \right] = \frac{2}{3} \lambda \left[ C^\alpha_{\beta} M_{\alpha \beta} + C_{\alpha\beta} \overline{M}^\alpha_{\beta} \right].$$

(9)

In this way the translational symmetry of spacetime is sensitive to the value of the cosmological constant. For $\lambda = 0$ the case of Minkowski space, the translation generators form an abelian group. For $\lambda > 0$ de Sitter or $\lambda < 0$ anti-de Sitter spaces,
the translation generators together with the spin-angular momentum generators \( \mathcal{M}_{\alpha\beta} \) and \( \mathcal{M}_{\dot{\alpha}\dot{\beta}} \) form non-abelian groups.

Finally, there is one other kinematical feature that is of note in the issue of de Sitter spacetimes and anti-de Sitter spacetimes vis-a-vis their relation to Minkowski spacetimes. Massless representations in the Minkowski space of the Poincaré group with generators \( P_a \), \( \mathcal{J}_{\alpha\beta} \) and \( \mathcal{J}_{\dot{\alpha}\dot{\beta}} \) and commutation algebra

\[
\begin{align*}
\left[ P_a, P_b \right] &= 0 , & \left[ J_{\alpha\beta}, J_{\gamma\delta} \right] &= i \left[ C_{\beta\gamma} J_{\alpha\delta} + C_{\alpha\delta} J_{\beta\gamma} \right] , \\
\left[ J_{\alpha\beta}, \mathcal{J}_{\dot{\alpha}\dot{\beta}} \right] &= 0 , & \left[ \mathcal{J}_{\dot{\alpha}\dot{\beta}}, \mathcal{J}_{\dot{\gamma}\dot{\delta}} \right] &= i \left[ C_{\dot{\beta}\dot{\gamma}} \mathcal{J}_{\dot{\alpha}\dot{\delta}} + C_{\dot{\alpha}\dot{\delta}} \mathcal{J}_{\dot{\beta}\dot{\gamma}} \right] , \\
\left[ J_{\alpha\beta}, P_c \right] &= i \frac{1}{2} C_{\alpha\beta} \left( \mathcal{J}_{\alpha\beta} \right) , & \left[ \mathcal{J}_{\dot{\alpha}\dot{\beta}}, P_c \right] &= i \frac{1}{2} C_{\dot{\alpha}\dot{\beta}} \left( \mathcal{J}_{\dot{\alpha}\dot{\beta}} \right) , \\
\left[ J_{\alpha\beta}, \mathcal{J}_{\dot{\alpha}\dot{\beta}} \right] &= 0 , & \left[ J_{\alpha\beta}, P_c \right] &= i \left( C_{\alpha\beta} J_{\gamma\delta} + C_{\alpha\beta} J_{\dot{\gamma}\dot{\delta}} + \eta_{\alpha\beta} \Delta \right) , \\
\left[ \Delta, J_{\alpha\beta} \right] &= 0 , & \left[ \Delta, \mathcal{J}_{\dot{\alpha}\dot{\beta}} \right] &= 0 .
\end{align*}
\]

(10)

also form representation of the larger conformal group with additional generators \( K_a \) and \( \Delta \) and the enlarged commutator algebra

\[
\begin{align*}
\left[ \Delta, P_b \right] &= i P_b , & \left[ \Delta, K_b \right] &= -i K_b , & \left[ K_a, K_b \right] &= 0 , \\
\left[ J_{\alpha\beta}, K_c \right] &= i \frac{1}{2} C_{\gamma\delta} \left( \Delta \right) , & \left[ \mathcal{J}_{\dot{\alpha}\dot{\beta}}, K_c \right] &= i \frac{1}{2} C_{\dot{\beta}\dot{\gamma}} \left( \Delta \right) , \\
\left[ P_a, K_c \right] &= i \left( C_{\alpha\beta} J_{\gamma\delta} + C_{\alpha\beta} J_{\dot{\gamma}\dot{\delta}} + \eta_{\alpha\beta} \Delta \right) , & \left[ \Delta, J_{\alpha\beta} \right] &= 0 , & \left[ \Delta, \mathcal{J}_{\dot{\alpha}\dot{\beta}} \right] &= 0 .
\end{align*}
\]

(11)

The point to note is that the generators of translations in both anti-de Sitter and de Sitter spacetimes denoted by \( P_a \) may be regarded as a linear combination of the generators \( P_a \) and \( K_a \) in the Minkowski spacetime,

\[
P_a \equiv P_a \pm \frac{1}{3} |\lambda| K_a ,
\]

(12)

where a parameter with the dimensions of \( \lambda \) must be introduced owing to the difference in dimensions of \( P_a \) and \( K_a \). It is a directly simple calculation to begin with the definition in (12) and the commutator algebra in (10) and (11) to thusly prove

\[
\left[ P_a, P_b \right] = \pm i \frac{2}{3} |\lambda| \left[ C_{\alpha\beta} J_{\alpha\beta} + C_{\alpha\beta} \mathcal{J}_{\alpha\beta} \right] ,
\]

(13)

and the sign of the linear combination in (12) is seen to determine whether the translation generator \( P_a \) is related to an anti-de Sitter or de Sitter geometry.

The discussion above might have been deemed solely formal and of little importance except that the recent experimental data from type-II supernovae \[8, 9\] seem to indicate that we live in a universe that possesses a small positive cosmological constant (de Sitter geometry). Thus, the configuration in (5) apparently describes our universe in the limit of no gravitational radiation.

\[10\] A set of super-vector fields that provide a representation for the generators of the superconformal group can be found on pages 76 and 81 of [6].
3 Joining the Clash: de Sitter vs. SUSY

Soon after the introduction of supergravity theories, Ferrara [10] noted an interesting distinction that global supersymmetry makes with regard to spacetimes of constant curvature. Namely global supersymmetry can easily be realized for anti-de Sitter or Minkowski spaces, but cannot be realized at all for de Sitter spaces. Although, Ferrara cast his discussion in terms of charges and supercharges, we can understand the gist of his discussion by probing the structure of superspace supergravity covariant derivatives that are consistent with a superspace that contains a bosonic spacetime of constant curvature.

The superanalog of (3) takes the form of

\[ \nabla_A \equiv E_A^\alpha \partial_{\alpha} + \omega_A^{\gamma \delta} \mathcal{M}_{\delta}^{\gamma} + \omega_A^{\gamma \delta} \overline{\mathcal{M}}_{\delta}^{\gamma}, \]

(14)

where the super-index \( A \) is permitted to take on values \( \alpha, \bar{\alpha}, \) and \( a \) and the results in [10] are equivalent to the statement that the most general super-commutator algebra that is consistent with a bosonic subspace of constant curvature must take the form,

\[
\begin{align*}
[ \nabla_\alpha, \nabla_\beta ] &= -2 \ell_F \mathcal{M}_{\alpha\beta}, \\
[ \nabla_\alpha, \nabla_\bar{\alpha} ] &= i \nabla_a, \\
[ \nabla_\alpha, \nabla_a ] &= -\ell_F C_{\alpha\beta} \nabla_\beta, \\
[ \nabla_a, \nabla_\bar{\alpha} ] &= 2 |\ell_F| ( C_{\bar{\alpha}\beta} \mathcal{M}_{\alpha\beta} + C_{\alpha\beta} \overline{\mathcal{M}}_{\bar{\alpha}\beta} ) .
\end{align*}
\]

(15)

where \( \ell_F \) is a constant parameter (the “Ferrara parameter”). Upon comparing the last line here with the result in (9), we see that the relation between the cosmological constant and the Ferrara parameter is

\[ -\lambda = 3 |\ell_F|^2 . \]

(16)

Since the rhs of this equation is non-negative, the equation only has non-trivial solutions if \( \lambda \leq 0 \), i.e. flat spaces or anti-de Sitter spaces. The consistency of these is verified by checking the super Bianchi identities for the graded commutators in (15). Upon comparison between this result and the one in (9) we see that this superspace result implies that the bosonic subspace contained within it must be either a Minkowski space (\( \ell_F = 0 \)) or a space of constant curvature (\( \ell_F \neq 0 \)) with the Ricci tensor and curvature scalar respectively taking the forms \( \mathcal{R}_{ab} = -|\lambda| g_{ab} \) and \( \mathcal{R} = -4|\lambda| \).

\[ ^{11} \text{Up to a complex phase, Ferrara’s parameter is the square root of the cosmological constant.} \]
It is easily realizable that for no value of the Ferrara parameter is it possible to obtain a cosmological constant of an appropriate sign so as to describe a de Sitter geometry. We should perhaps mention that this is a robust model-independent result which in its original presentation was cast in the language of charges and group theory. It essentially forms a no-go theorem for the realization of rigid supersymmetry in the presence of a de Sitter spacetime.

One of the hallmarks of supersymmetry is that the usual translation operator $P_\alpha$ is related to spinorial supercharges $Q_\alpha$ and $\overline{Q}_{\dot{\alpha}}$ via the equation,

$$\{ Q_\alpha, \overline{Q}_{\dot{\alpha}} \} = P_\alpha . \quad (17)$$

In the context of superconformal supersymmetry there is a similar relation between $K_\alpha$ and the $s$-supersymmetry generator

$$\{ S_\alpha, \overline{S}_{\dot{\alpha}} \} = K_\alpha . \quad (18)$$

The other relevant graded commutator takes the form

$$\{ Q_\alpha, S_\beta \} = -i( J_{\alpha\beta} + \frac{1}{2} C_{\alpha\beta}\Delta ) - \frac{1}{2} C_{\alpha\beta} Y . \quad (19)$$

In a space of constant curvature, there is a supercharge $Q_\alpha$ satisfying

$$\{ Q_\alpha, \overline{Q}_{\dot{\alpha}} \} = P_\alpha , \quad (20)$$

and it can be constructed from the superconformal generators $Q_\alpha$ and $S_\alpha$ via

$$Q_\alpha = Q_\alpha + \ell F S_\alpha , \quad (21)$$

in analogy with the construction in (12). Here the Ferrara parameter is seen to determine by how much is the anti-de Sitter supersymmetry generator $Q_\alpha$ “deformed” from the supersymmetry generator $Q_\alpha$ by the $s$-supersymmetry generator $S_\alpha$. Using this expression to calculate $P_\alpha$ (together with the fact that $\{ Q, S \} = 0$) in (20) yields

$$P_\alpha \equiv P_\alpha + |\ell F|^2 K_\alpha \rightarrow \mathcal{P}_\alpha \equiv P_\alpha - \frac{1}{3} \lambda K_\alpha . \quad (22)$$

The representation theory of superspace, as encoded in the set of supervector fields $E$ that realize the generators, necessarily has picked anti-de Sitter space as the type of constant curvature space consistent with supersymmetry. This is true independent of the phase of the Ferrara parameter since only its absolute modulus enters (22) above. In comparing (12) to (22), here only the linear combination corresponding to anti-de Sitter backgrounds occurs and this is solely due to supersymmetry.
4 The Era of No De Sitter Space for Local N-extended SUSY

Since the organizers of this conference charged the presenters with the dual tasks of looking back at some supergravity history in order to address current and possibly future developments, this section will be spent giving a review of the topic of spaces of constant curvature in the context of supergravity theory. The 4D, \( N = 4 \) supergravity theory played a special role in this line of investigation so we will also review its development.

During early explorations on the issue of the cosmological constant in supergravity theory, there was ample support for Ferrara’s observation. Shortly after supergravity appeared, Freedman \([11]\) presented the first discussion of the theory’s consistency in a space with a non-vanishing cosmological constant. He found that only anti-de Sitter spaces were admitted if R-symmetry is also gauged. A similar construction by Townsend \([12]\), without the gauging of R-symmetry, reached the same conclusion. Support for the absence of de Sitter space in local supersymmetrical theories continued to be found in numbers of other works during these early days. The gauging of the SO(2) and SO(3) automorphism groups of 4D, \( N = 2 \) and \( N = 3 \) supergravities, respectively, by Freedman and Das \([13]\) also ruled out de Sitter backgrounds.

The theory of 4D, \( N = 4 \) supergravity first appeared in its ungauged version, but it was immediately recognized that there were new aspects of the theory. First a version admitting an SO(4) automorphism group was found \([14]\) and then a second version with an SU(2) \( \otimes \) SU(2) automorphism appeared \([15]\). Much later \([16]\), even a third version of the ungauged theory was found. In view of our later discussion of investigations into some unresolved issues, we believe for clarity’s sake a discussion of the relations between these three ungauged automorphism versions of 4D, \( N = 4 \) theories is warranted here.

Cremmer, Scherk and Ferrara \([15]\) showed a relation between the first two discovered versions of the theory which are connected via a conformal map. There are two spin-0 fields in both of these theories. Let us denote the scalar and psuedoscalar fields in the version with the SO(4) automorphism group by \( A' \) and \( B' \). Similarly we denote the scalar and psuedoscalar fields in the version with the SU(2) \( \otimes \) SU(2) automorphism group by \( A \) and \( B \). Let two complex variables \( \mathcal{W} \) and \( \mathcal{Z} \) be defined by

\[
\mathcal{W} \equiv A(x) + iB(x) \quad , \quad \mathcal{Z} \equiv A'(x) + iB'(x) \quad ,
\]

(23)
and under a conformal mapping defined by

\[ Z \rightarrow \frac{W}{W - 1} \quad , \quad (24) \]

induces the following change on the spin-0 kinetic energy terms,

\[ \frac{1}{1 - |Z|^2} |\partial Z|^2 \rightarrow \frac{1}{\sqrt{1 - W - \overline{W}}} |\partial W|^2 \quad . \quad (25) \]

If we perform the same conformal map on the last part of (24)

\[ W \rightarrow \frac{Z}{Z - 1} \quad , \quad (26) \]

it will then be transformed into the first part. This conformal map is the same as in (24) and applying this to (24) results in the identity transformation on \( Z \). This is an example of an idempotent conformal mapping. Apply this map and some related ones acting on other fields in the \( \text{SU}(2) \otimes \text{SU}(2) \) action then transform it into the \( \text{SO}(4) \) action and vice-versa,

\[ \mathcal{S}_{\text{SU}(2) \otimes \text{SU}(2)}(W, ...) \leftrightarrow \mathcal{S}_{\text{SO}(4)}(Z, ...) \quad , \quad (27) \]

where the ellipsis denote all the remaining fields in each action.

Within the confines of the \( \text{SU}(2) \otimes \text{SU}(2) \) version, the field \( B \) only appears in the Lagrangian and transformation laws according to the form \( \partial_a B \). Due to this, Nicolai and Townsend [16] were able to show the existence of a third formulation which is connected to the \( \text{SU}(2) \otimes \text{SU}(2) \) formulation via an idempotent “Hodge duality” map. The essence of the hodge duality map can be seen by looking at a toy example.

Consider the two action \( \mathcal{S}_0 \) and \( \mathcal{S}_1 \) respectively defined by

\[ \mathcal{S}_0 = - \frac{1}{2} \int d^4x \; f_{\underline{a}} f_{\underline{a}} \quad , \quad f_{\underline{a}} \equiv \partial_{\underline{a}} B \quad , \quad (28) \]

\[ \mathcal{S}_2 = \frac{1}{2} \int d^4x \; h_{\underline{a}} h_{\underline{a}} \quad , \quad h_{\underline{a}} \equiv \frac{1}{3!} \epsilon_{\underline{a} \underline{b} \underline{c} \underline{d}} h_{\underline{b} \underline{c} \underline{d}} \quad , \]

\[ h_{\underline{b} \underline{c} \underline{d}} = \partial_{\underline{a}} B_{\underline{b} \underline{c}} + \partial_{\underline{b}} B_{\underline{a} \underline{c}} + \partial_{\underline{c}} B_{\underline{a} \underline{b}} , \]

The equations of motion that follow from the extremization of these actions are;

\[ \frac{\delta \mathcal{S}_0}{\delta B} = 0 \quad \rightarrow \quad \partial_{\underline{a}} f_{\underline{a}} = 0 \quad , \]

\[ \frac{\delta \mathcal{S}_2}{\delta B_{\underline{a} \underline{b}}} = 0 \quad \rightarrow \quad \partial_{\underline{a}} h_{\underline{b}} - \partial_{\underline{b}} h_{\underline{a}} = 0 \quad . \quad (29) \]
where we have expressed the equations of motion in terms of the “field strengths” $f_a$ and $h_a$. Independent of the dynamics, these field strengths satisfy differential equations that may be called “Bianchi identities”

\[
0 = \partial_a f_b - \partial_b f_a ,
0 = \partial_a h_a .
\]

A simple comparison between (29) and (30) shows that the equations of motion and Bianchi identities are exchanged under the idempotent mapping

\[
f_a \rightarrow ih_a , \quad h_a \rightarrow -if_a .
\]

The relation between the SU(2) $\otimes$ SU(2) version and the Nicolai-Townsend version of 4D, $N = 4$ supergravity is analogous to the relation between $S_0$ and $S_2$ above. Under the action of the transformation in (31) and some related re-definitions on other fields

\[
S_{\text{SU(2)} \otimes \text{SU(2)}}(B, ...) \leftrightarrow S_{\text{N-T}}(B_{ab}, ...) ,
\]

where again the ellipsis denote all the remaining fields in each action.

Finally, the gauging of the automorphisms of the SO(4) version was carried out by Das, Fischler and Roček \[17\] and for the SU(2) $\otimes$ SU(2) version by Freedman and Schwarz \[18\]. In the former case, a single non-Abelian coupling constant $g$ is required. In the latter case, two non-Abelian coupling constants $g_1$ and $g_2$ are required. In both of these cases it was found that a potential involving the spin-0 zero field is induced relative to the ungauged action. In the limit where the coupling constants vanish, these potentials go to zero. Finally, in neither case were de Sitter backgrounds found to satisfy the full set of equations of motion.

The gauging of the automorphism also plays one other important role. The three version of 4D, $N = 4$ supergravity discussed above are all connected to each other by a set of field re-definitions. So naively, one might think that they are all equivalent. This is true as long as the automorphism groups are ungauged. The gauging of the automorphism group destroys this equivalence.

For none of the models discussed above, was it possible to show that de Sitter space occurred as a solution to the resultant equations of motion. Had Ferrara’s Theorem been the last word on this topic, the observation of a quintessential cosmological constant would, in and of itself, rule out the relevance of supergravity (and hence any theory containing it) to a description of Nature.
5 Breakthrough: De Sitter Space As Spontaneously Broken Phase of Local SUSY

In 1982, we began to study the 4D, $N = 4$ supergeometry and discovered that from this perspective all component level formulation were derivable within a universal setting. Our study was motivated by the fact that with the plethora of versions known, it was possible that even more (then unknown) versions might also exist. For example, the Cremmer-Scherk-Ferrara conformal map is only one example of an idempotent map. The most general member has the form

$$Z' \rightarrow \frac{Z + e^{\varphi_0}}{e^{-\varphi_0} Z - 1}, \quad (33)$$

dependent on the complex parameter $\varphi_0$. Does this imply the existence of an entire family of models?

To answer this question, we relied upon first a careful supergeometrical analysis \[19\] of the then known theories\[12\]. After this analysis was begun, a classification scheme presented itself and it became clear that there were indeed some overlooked $N = 4$ models. Some of these had the distasteful feature of possessing spin-0 kinetic energy terms of the form

$$L_{kin.} = \frac{1}{|Z|^2 - 1} |\partial Z|^2, \quad (34)$$

and were thus not commented upon\[13\]. However, one of the overlooked models \[22\] possessed a feature that had never been seen for extended supergravity...a de Sitter background emerged as a solution to the equations of motion in the presence of the spontaneous breaking of all four local supersymmetries. In the following, a discussion of the technique used for this discovery is described.

To provide a description of 4D, $N = 4$ supergravity, it is first necessary to introduce the appropriate generalization of the superspace derivative in \[14\]. This is done with a superspace supergravity covariant derivative of the form,

$$\nabla_{\alpha i} = E_{\alpha i} M D_M + \omega_{\alpha i} \gamma^\delta M_\delta^\gamma + \omega_{\alpha i} \gamma^\gamma \bar{M}_\delta^\gamma + \Gamma_{\alpha i kl} T_{kl}, \quad (35)$$

Graded commutation of these operators produce the usual torsion, curvature and field

\[12\] Modifications to this analysis was provided later \[20\].

\[13\] However, these models were subsequently discussed in the literature \[21\].
strength supertensors.

$$\left[ \nabla_{\alpha i} , \nabla_{\beta j} \right] = T_{\alpha i \beta j} \nabla_{\delta l} + T_{\alpha i \beta j} \nabla_{\delta d} + R_{\alpha i \beta j} \gamma \delta M_{\delta \gamma}$$

$$+ R_{\alpha i \beta j} \gamma \delta \mathcal{M}_{\delta \gamma} + C_{\alpha \beta} \mathcal{E}_{ij}^{kl} T_{kl} ,$$

$$\left[ \nabla_{\alpha i} , \nabla_{\beta b} \right] = T_{\alpha i \beta b} \nabla_{\delta l} + T_{\alpha i \beta b} \nabla_{\delta d} + R_{\alpha i \beta b} \gamma \delta M_{\delta \gamma}$$

$$+ R_{\alpha i \beta b} \gamma \delta \mathcal{M}_{\delta \gamma} + + F_{\alpha i \beta b}^{kl} T_{kl} ,$$

$$\left[ \nabla_{\alpha b} , \nabla_{\beta b} \right] = T_{\alpha b \beta b} \nabla_{\delta l} + T_{\alpha b \beta b} \nabla_{\delta d} + R_{\alpha b \beta b} \gamma \delta M_{\delta \gamma}$$

$$+ R_{\alpha b \beta b} \gamma \delta \mathcal{M}_{\delta \gamma} + F_{\alpha b \beta b}^{kl} T_{kl} .$$

In comparison to the superspace supergravity covariant derivative of simple supergravity (14) a major difference here is the presence of a super 1-form connection \((\Gamma_{\alpha i}^{kl})\) and its corresponding super 2-form field strength \((F_{\alpha i \beta j}^{kl})\). With the forms of (35) and (36) together with the known spectrum of 4D, \(N = 4\) supergravity, we need only find consistent solutions of the superspace Bianchi identities. The quantity \(T_{kl}\) denote the generators of a rank six gauge group.

The quantity \(\mathcal{E}_{ij}^{kl}\) that appears in the spinor-spinor component of the super 2-form field strength \((F_{\alpha i \beta j}^{kl} = C_{\alpha \beta} \mathcal{E}_{ij}^{kl})\) was required by the Bianchi identities to be an invertible \(6 \times 6\) matrix and given the spectrum of component fields had to be of the form \([13]\)

\[
\mathcal{E}_{ij}^{kl} = \delta_i^{[k} \delta_j^{l]} U(W) + \epsilon_{ij}^{kl} V(W) .
\]

This is the most general form that is consistent with SO(4) symmetry and it is here that this assumption enters the analysis in a forceful manner. Once (36) and the spectrum of 4D, \(N = 4\) supergravity are used as inputs for solving the superspace Bianchi identities to which (36) is subject, there emerges the condition

\[
|U|^2 - |V|^2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} ,
\]

that we call the “modulus constraint” and whose uniqueness was verified in the first and second work of [21]. In (37) \(W\) is a chiral superfield whose leading components are \(Z\) from (23). Under these results all superspace Bianchi identities were found to be satisfied.

From the view of supergeometry, (almost) the only freedom that one has for 4D, \(N = 4\) supergravity is in the “modulus choice” and the rank six gauge group choice. Some choices (but not exhaustively) are\(^{14}\) \(U(6)(1), SO(4)\) and \(SU(2) \otimes SU(2)\). These

---

\(^{14}\)The choice \(Z^{(3)} \otimes SU(2)\) may be regarded as a degenerate case of the \(SU(2) \otimes SU(2)\) theory wherein one of the non-Abelian coupling constants is set to zero, e. g. [23].
observations provide a “matrix classification scheme” for understanding the versions of 4D, $N = 4$ supergravity. Along a vertical axis, we list the choice of gauge group. Along a horizontal axis, we list the modulus choice. This leads to a simple taxonomy.

\textbf{Taxonomy of 4D, N = 4 Supergravity}

| $Z^{(6)}$ | +1 | 0 | −1 |
|------------|----|---|----|
| $SO(4)$   | [13, 14, 17] | [13] | [22, 21] |
| $SU(2) \otimes SU(2)$ | [22] | [18] | * |

Table I

The numerical entries in this table indicate in which reference the construction was carried out, −− indicates the impossibility to carry out such a construction and * indicates the model has simply not been explicitly presented. The modulus choice = +1 and gauge group = $SU(2) \otimes SU(2)$ model in [22] was the first extended supergravity model presented in the literature that allows de Sitter space as a spontaneously broken phase.

The modulus choice is important for a number of reasons. Perhaps the most important of these is that this choice will ultimately control the number of spin-0 fields that can appear in the potential after gauging the automorphism group or subgroups thereof. For each of the moduli choices, one can find an explicit functional relation expressing $U$ and $V$ in terms of $W$. These take the forms:

\[
U = \frac{1}{\sqrt{1 - |W|^2}}, \quad V = \frac{W}{\sqrt{1 - |W|^2}},
\]

\[
U = \frac{1}{\sqrt{1 - W - W}}, \quad V = \frac{1}{\sqrt{1 - W - W}},
\]

\[
U = \frac{W}{\sqrt{1 - |W|^2}}, \quad V = \frac{1}{\sqrt{1 - |W|^2}},
\]

respectively for the three cases indicated in (38). Note that for the modulus choice of ±1, two scalar functions (i.e. $Re(W)$ and $Im(W)$) appear in $E_{ij}^{kl}$. For the modulus choice of 0, only one scalar field appears (i.e. $Re(W)$) in $E_{ij}^{kl}$.

In a paragraph above, the word “almost” appeared in a parenthetical remark. The reason is because the Nicolai-Townsend model [16] is not among the classification above. It was later shown in the third work of [20] that this version of 4D, $N = 4$ supergravity exist in a superspace formulation where the complex chiral superfield
$W$ is replaced by a real scalar superfield $\mathcal{V} \equiv W + \bar{W}$ and this is only consistent for the modulus choice of zero. Thus, the superspace geometry of the Nicolai-Townsend formulation is described by the real superfield $\mathcal{V}$ and it is convenient to relate this superfield to the component "dilaton" in the theory $\varphi(x)$ via $\mathcal{V} = 1 - \exp(-\varphi)$. In addition to the superspace supercovariant derivative (35), it is necessary to introduce a super 2-form gauge superfield $B_{AB}$ and its associated super 3-form field strength $H_{ABC}$ for a complete supergeometrical description. This exhaust all freedom in superspace to describe the 4D, $N = 4$ supergravity models.

A final point about the modulus choice is that it is literally a choice of moduli (in the technical sense of the word) that enter the theory. Let us see how this works in some more detail. Due to the gauge invariance of the two-form in the Nicolai-Townsend formulation and via the duality transformation described by (32), we can be assured that the field $B(x)$ only enters $S_{SU(2) \otimes SU(2)}$ through its derivative. In turn this means that its action must admit the symmetry generated by $B \rightarrow B + c_0$ for an arbitrary constant $c_0$. Thus we have derived the fact that the theories which appear in the middle column of our taxonomy possess a space of moduli for their $B$-fields and such moduli do not appear for the other member of the classification.

To gauge the $SU(2) \otimes SU(2)$ group requires two coupling constants $g_1$ and $g_2$. The second work of [20] established that the two coupling constants are not independent but are related by a $\mathbb{Z}_2$-valued parameter

$$g_1 = g , \quad g_2 = \eta g , \quad \eta = \pm 1 ,$$

and when combined with the results of [22], where it was shown that the model develops a spin-0 potential function of the form (with $\eta_{\pm} \equiv 1 + \eta$), leads to

$$P(W, \bar{W}) = \frac{1}{2} g^2 \left[ 3 \left| \eta_+ U + \eta_- \bar{\nabla} \right|^2 - \left| \eta_+ V + \eta_- \bar{\nabla} \right|^2 \right] .$$

When $\eta_- = 0$, this potential at its critical point describes a cosmological constant $\lambda_{AdS} = -6g^2$ appropriate for an anti-de Sitter geometry. When $\eta_+ = 0$, this potential at its critical point describes a cosmological constant $\lambda_{dS} = 2g^2$ appropriate for an de Sitter geometry (clearly $\lambda_{AdS}/\lambda_{dS} = -3$).

The discovery of a mechanism that allowed for de Sitter backgrounds in the context of 4D, $N = 4$ extended supergravity models triggered an extensive program of study for generalizations to higher values of $N$. This effort, led by Hull, Hull and Warner [24] and others, continues even to this day [25]. The importance of these works can be judged against the prior construction of gauged SO(8) 4D, $N = 8$ models [21] in which de Sitter space was ruled out.
6 Supersymmetry Breaking and a Nonvanishing Cosmological Constant

In the same period of time that we were investigating the appearance of de Sitter backgrounds in extend supergravity, we also presented a discussion [27] about a linking between the appearance of a spacetime background of constant curvature and the breakdown of rigid supersymmetry in the context of 4D, \( N = 1 \) supergravity theories. This linkage was first noted in Superspace [28], where it was observed that the choice of auxiliary fields required of an off-shell supergravity multiplet is sensitive to the presence of background spaces of constant curvature. For some sets of auxiliary fields, the mere presence of any non-vanishing cosmological constant implies the breaking of global supersymmetry! Since this result seems largely unknown to the community, we will expend an effort in the following to explain why this result is obtained.

As is well known, 4D, \( N = 1 \) supergravity describes the propagation of spin-2 and spin-3/2 degrees of freedom, \( e_\alpha^\mu(x) \) and \( \psi_\alpha^a(x) \). However, the consistency of the local supersymmetry algebra implies that if only these fields are introduced, then they must satisfy some equations of motion. In order to avoid this restriction, auxiliary fields are required. The set of auxiliary fields needed for this is not unique. One such set consists of \((B, B, A_\alpha)\) and these are called the “minimal set of auxiliary fields” as discovered by van Nieuwenhuizen and Ferrara [29]. A different set, called “non-minimal set of auxiliary fields,” consists of \((\lambda_\alpha, B, B, A_\alpha, w_\alpha, w_\alpha, \chi_\alpha, \gamma)\) and had previously been implied by the work of Breitenlohner [30]. Each of these sets of auxiliary fields correspond to distinct supergeometries. For the minimal set, this geometry is expressed in term of the superfields \(W_{\alpha\beta\gamma}, G_\alpha\) and \(R\) are known as the “irreducible supergravity field strengths.” While in the non-minimal set the geometry is expressible in terms of the superfields \(W_{\alpha\beta\gamma}, G_\alpha\) and \(T_\alpha\) are the “irreducible supergravity field strengths.”

The superspace commutator algebra of the minimal supergravity covariant derivative is given by

\[
\begin{align*}
[\nabla_\alpha, \nabla_\beta] &= -2R\mathcal{M}_{\alpha\beta}, \quad [\nabla_\alpha, \nabla_\dot{\alpha}] = i\nabla_\alpha, \\
[\nabla_\alpha, \nabla_\dot{\beta}] &= -iC_{\alpha\beta}[\overline{R\nabla_\dot{\beta}} - G^\gamma_{\dot{\beta}}\nabla_\gamma] - i(\overline{\nabla_\dot{\beta} R})M_{\alpha\beta} \\
&\quad + iC_{\alpha\beta}[\overline{W^{\dot{\beta}}_{\dot{\gamma}}M_{\dot{\gamma}}^{\delta}} - (\nabla^\delta G_{\dot{\beta}\dot{\gamma}})M_{\delta\gamma}], \\
[\nabla_\alpha, \nabla_\dot{\gamma}] &= \{i\frac{1}{2}C_{\alpha\beta}G^\nu(\hat{\alpha} \nabla_\gamma) \\
&\quad + [C_{\dot{\alpha}\beta}W_{\alpha\beta\gamma} + \frac{1}{2}C_{\alpha\beta}(\nabla_\gamma G_{\beta}) - \frac{1}{2}C_{\alpha\dot{\beta}}(\nabla_\gamma R)_{\delta\beta}^\gamma] \nabla_\gamma
\end{align*}
\]
\[
- \left[ C_{\dot{\alpha}\dot{\beta}} W_{\alpha\beta\gamma\delta} - \frac{1}{2} C_{\alpha\beta} \left( \nabla_{(\dot{\alpha}} \nabla_{\gamma)} G_{\delta\beta)} \right) \right] \mathcal{M}^{\gamma\delta}
- \frac{i}{2} C_{\dot{\alpha}\dot{\beta}} C_{\gamma(\alpha} \left[ \nabla_{\beta)} \epsilon^\delta + \nabla_\delta \epsilon^\beta \right] \mathcal{M}^{\gamma\delta}
- \frac{1}{2} C_{\alpha\beta} C_{\gamma(\alpha C_{\beta)}\delta} \left[ \nabla^2 \mathcal{R} + 2 R \mathcal{R} \right] \mathcal{M}^{\gamma\delta} \right) + \text{h.c.},
\]

The superfield \( R \) is subject to the superdifferential equations

\[
\nabla_\beta R = 0, \quad \nabla^\alpha R + \nabla_\alpha G^{\alpha\dot{\alpha}} = 0. \quad (43)
\]

We now wish to consider several limits of the equations in (42) and (43);

Limit A. We set \( W_{\alpha\beta\gamma} = 0, G_{\dot{\alpha}} = 0 \) and \( R = 0 \) and see that this limit is consistent with the differential equations in (43) and that (42) agrees precisely with (15) if we set \( \ell_F = 0 \) in the latter. This is the flat Minkowski limit of the curved supergravity superspace.

Limit B. We set \( W_{\alpha\beta\gamma} \neq 0, G_{\dot{\alpha}} = 0 \) and \( R = 0 \) and see that this limit is consistent with the differential equations in (43) and that (42) implies a non-trivial spacetime curvature. This superspace describes the “on-shell” limit with vanishing auxiliary fields in [1].

Limit C. We set \( W_{\alpha\beta\gamma} \neq 0, G_{\dot{\alpha}} = 0 \) and \( R = \ell_F \) and see that this limit is consistent with the differential equations in (43) and that (42) implies a non-trivial spacetime curvature. This superspace describes the “on-shell” supergravity theory of [12] in an anti-de Sitter background.

Limit D. We set \( W_{\alpha\beta\gamma} = 0, G_{\dot{\alpha}} = 0 \) and \( R = \ell_F \) and see that this limit is consistent with the differential equations in (43) and that (42) agrees precisely with (15). This is the flat anti-de Sitter limit of the curved supergravity superspace.

We now wish to repeat this analysis and show that the supergeometry of the Breitenlohner auxiliary fields behaves in a drastically different manner in the some of limits above. The non-minimal superspace geometry can be cast in the form

\[
\left[ \nabla_\alpha, \nabla_\beta \right] = \frac{1}{2} T_{(\alpha} \nabla_{\beta)} - 2 \left( \mathcal{R} + T^\alpha \mathcal{T}_\alpha \right) \mathcal{M}_{\alpha\beta},
\]

\[
\left[ \nabla_\alpha, \nabla_{\dot{\beta}} \right] = i \nabla_{\dot{\beta}} - \frac{1}{2} \left( T_{\alpha} \nabla_{\dot{\beta}} + T_{\dot{\alpha}} \nabla_{\beta} \right)
\]

\[
\left[ \nabla_\alpha, \nabla_\beta \right] = i C_{\alpha\beta} \left( \mathcal{R} + T^\alpha \mathcal{T}_\alpha \right) \nabla_{\dot{\beta}} + i C_{\alpha\beta} G^{\alpha\beta} \nabla_\gamma
\]

\[
- \frac{1}{2} \left( \nabla_\beta \mathcal{T}_{\dot{\beta}} - \nabla_{\dot{\beta}} \mathcal{T}_\beta + \mathcal{T}_{\dot{\beta}} \mathcal{T}_{\dot{\beta}} \right) \nabla_\alpha,
\]

\[
- i \left[ \left( \nabla_{\dot{\beta}} - \mathcal{T}_{\dot{\beta}} \right) \mathcal{R} \right] \mathcal{M}_{\alpha\beta} - i C_{\alpha\beta} \left( \nabla^\gamma G_{\delta\beta)} \mathcal{M}_\gamma \mathcal{M}_\delta \right)
+ i C_{\alpha\beta} \left[ \mathcal{W}_{\beta\gamma} \mathcal{M}_\gamma \mathcal{M}_\delta - \frac{1}{3} \left( \nabla^\gamma G_{\gamma\delta} \right) \mathcal{M}_\delta \right]
\]

\[
- \left( \nabla_{\dot{\alpha}} - \mathcal{T}_{\dot{\alpha}} \right) \left( \mathcal{R} + T^\alpha \mathcal{T}_\alpha \right) \mathcal{M}_{\beta\delta} \right) ,
\]

(44)
where for simplicity we have omitted the explicit form of $[\nabla_a, \nabla_b]$. But this may be found from

\[
[\nabla_a, \nabla_b] = \left[ -i([\nabla_\alpha, [\nabla_\dot{\alpha}, \nabla_b] = T_\alpha [\nabla_\alpha, \nabla_b] \right.
\left. - (\nabla_b T_\dot{\alpha}) \nabla_\alpha + \text{h.c.} \right],
\]

(45)
In particular in this theory $R$ is subject to the superdifferential equations

\[
\nabla_\beta R = 0, \quad (\nabla^\alpha - \frac{1}{2} T_\alpha) R + \nabla_\alpha G^{a\dot{a}} = 0, \quad \overline{R} = -\frac{1}{2} \nabla^\alpha T_\alpha,
\]

(46)
Repeating the same limits as before but now investigating the equations in (44), (45) and (46);

Limit A. We set $W_{\alpha\beta\gamma} = 0$, $G_{a\dot{a}} = 0$ and $T_\alpha = 0 \to R = 0$ and see that this limit is consistent with the differential equations in (19) and that (14) and (15) agree with (13) if we set $\ell_F = 0$ in the latter. This is the flat Minkowski limit of the curved supergravity superspace.

Limit B. We set $W_{\alpha\beta\gamma} \neq 0$, $G_{a\dot{a}} = 0$ and $T_\alpha = 0 \to R = 0$ and see that this limit is consistent with the differential equations in (16) and that (13) implies a non-trivial spacetime curvature. This superspace describes the “on-shell” limit with vanishing auxiliary fields in [1].

Limit C. We set $W_{\alpha\beta\gamma} \neq 0$, $G_{a\dot{a}} = 0$, $T_\alpha = 0$, $R = \ell_F$ and see that this limit is not consistent with the differential equations in (10).

Limit D. We set $W_{\alpha\beta\gamma} = 0$, $G_{a\dot{a}} = 0$, $T_\alpha = 0$, $R = \ell_F$ and see that this limit is not consistent with the differential equations in (10).

In particular, it is the final equation in (19) that is always inconsistent.

This phenomenon was also investigated in term of the compensating field formalism [28] and that study also supported this assertion. In particular, there is an important technical difference between the manner in which the chiral compensator superfield $\varphi$ (see [32]) and the linear compensator $\Upsilon$ (see also [32]) enter their respective supergravity actions. The chiral compensator enters the supergravity action in such a way that the sign of its kinetic term is opposite to that of a matter chiral superfield. On the other hand, the sign of the linear compensator enters the $(n = -1)$ supergravity action in such a way that the sign of its kinetic term is the same as that of a matter linear superfield. Thus, we long ago reached the conclusion that all spaces of constant curvature break rigid supersymmetry, if the off-shell non-minimal formulation of supergravity is required.

This behavior may not be an academic matter, especially if the “P-term inflation model” recently suggested by Kallosh [31] is thought to provide a way in which to
reconcile quintessence and supersymmetry in a “stringy” context. All known $N = 2$ off-shell supergravity multiplets possess a subsector $N = 1$ off-shell supergravity multiplet whose auxiliary field are the Breitenlohner set.

One final point to note about this phenomenon is that it may well provide an example of a model in which the breaking of flavor symmetry results in the breaking of supersymmetry. The point is that usually in breaking internal flavor symmetries most such models generate a cosmological term. But such a term for this supergravity theory must necessarily drive supersymmetry breaking so that the two would be intimately linked. Thus in these models, electroweak breaking, supersymmetry breaking and the cosmological constant are all related.

7 Unexplored Issues for de Sitter Space in $N = 8$ Supergravity

The works of de Wit and Nicolai [20] and as well Hull et. al. [24] (as well as subsequent works based upon this) begin at a starting point of 4D, $N = 8$ supergravity where all of the 70 spin-0 fields are represented by scalars. If there were no other options, then one might not raise the issue of additional presently unknown gauged 4D, $N = 8$ supergravity models. There are other options.

Even in the original construction of 4D, $N = 8$ supergravity, Cremmer and Julia [33] were very clear about this issue. At a certain point in their derivation of the 4D, $N = 8$ supergravity action from 11D, $N = 1$ supergravity action, it is required to perform a duality transformation from a set of seven 2-forms to a set of seven scalars$^{15}$.

In the conventional 4D, $N = 8$ superspace supergravity theory, there is also the analog of $\mathcal{E}_{ij}^{kl}$ taking the form,

$$\mathcal{E}_{ij}^{kl} = U_{ij}^{kl}(\Phi) + V_{ij}^{kl}(\Phi),$$

which depends on the irreducible 4D, $N = 8$ superspace supergravity field strength $\Phi_{ijkl}$. This latter quantity is the 4D, $N = 8$ analog of $W$ and must be totally antisymmetric satisfying,

$$\Phi_{ijkl}^* = \pm \frac{1}{4!} \epsilon^{ijklrstu} \Phi_{rstu}. \quad (48)$$

$^{15}$These same seven 2-forms will appear below in our discussion starting from the 10D, $N = 2A$ supergravity theory.
Finally, the functions $U_{ij}^{kl}$ and $V_{ij}^{kl}$ were chosen according to the following prescription.

Since the quantity $\Phi_{ijkl}$ is complex, so too must be $U_{ij}^{kl}$ and $V_{ij}^{kl}$. They therefore possess complex conjugates $\overline{U_{ij}^{kl}}$ and $\overline{V_{ij}^{kl}}$. These four functions can be assembled into a matrix $\mathcal{B}$ defined by

$$
\mathcal{B} \equiv \begin{pmatrix} U_{ij}^{kl} & V_{ij}^{kl} \\ \overline{V_{ij}^{kl}} & \overline{U_{ij}^{kl}} \end{pmatrix},
$$

and the functional dependence of $U_{ij}^{kl}$ and $V_{ij}^{kl}$ upon $\Phi_{ijkl}$ is fixed by the condition that

$$
\mathcal{B} = \exp \begin{pmatrix} 0 & \Phi_{ijkl} \\ \overline{\Phi_{ijkl}} & 0 \end{pmatrix}.
$$

The 70 scalar fields that enter the functions $U_{ij}^{kl}$ and $V_{ij}^{kl}$ possess no moduli. That is, the function $\mathcal{E}_{ijkl}$ is not invariant under $\Phi_{ijkl} \to \Phi_{ijkl} + c_{ijkl}$ for any choice of constants $c_{ijkl}$. The theory has a zero dimensional moduli space.

The toroidal compactification of type-II supergravity theories provides alternatives. Since both the type-IIA and type-IIB theories exist, there are guaranteed to exist at least two alternative 4D, $N = 8$ theories that we refer to as the 4D, $N = 8$A and 4D, $N = 8$B theories. In particular, the group Spin(6) seems to play the role of organizing the 4D, $N = 8$ supergravity fields into its representations. Let us present the toroidal reduction in the form of a table.

### D = 10, N = 2A Supergravity Reduction

| D = 10 | D = 4 | Multiplicity | 4D Spin |
|--------|-------|--------------|---------|
| $\hat{e}_\mu^\hat{m}$ | $e_m^a$, $\varphi_{(\hat{a}\hat{b})}$, $\varphi$, $\hat{A}_m^{\hat{\alpha}}$, $(\varphi^{\hat{\alpha}} = 2\varphi)$ | 1, 20, 1, 6 | 2, 0, 0, 1 |
| $\hat{\psi}_\hat{m}$ | $\psi_m$, $\psi^{\hat{\alpha}}$, $\psi$, $(\Gamma^{\hat{\alpha}}\psi^{\hat{\alpha}} = 0)$ | 8, 40, 8 | 3/2, 1/2, 1/2 |
| $\hat{\chi}$ | $\chi$ | 8 | 0 |
| $\hat{A}_{\hat{\mu}}$ | $\hat{A}_m$, $\varphi^{\hat{\alpha}}$ | 1, 6 | 1, 0 |
| $\hat{B}_{\hat{\mu}\hat{\nu}}$ | $B_{mn}$, $A_{m\hat{\alpha}}$, $\varphi_{(\hat{a}\hat{b})}$ | 1, 6, 15 | 0, 1, 0 |
| $\hat{A}_{\hat{\mu}\hat{\nu}\hat{\rho}}$ | $B_{mn\hat{\alpha}}$, $A_{m\hat{a}\hat{b}}$, $\varphi_{\hat{a}\hat{b}\hat{c}}$ | 6, 15, 20 | 0 1, 0 |
| $\hat{\phi}$ | $\phi$ | 1 | 0 |

Table II
In particular, there are some points of note.

(a.) The eight gravitini (and dilatini) in the theory may be regarded as forming the spinor representation of Spin(6). All multiplicities fall into Spin(6) representations.

(b.) The spin-one fields (whose superspace $\mathcal{E}_{ij}^{kl}$-functions determine the how the spin-0 fields appear) are in multiplicities of $1 + 6 + 6 + 15$ which is exactly what is needed for the gauge group $SO(2) \otimes SO(4) \otimes SO(4) \otimes SO(6)$.

(c.) There are seven 2-forms, $B_{mn}$ and $B_{mn\hat{a}}$, which may after dualization yield a maximum of seven spin-0 fields that possess a seven dimensional moduli space.

A rather similar analysis can be performed on the type-IIB theory. We note that the fermion results here are unchanged so in order to simplify our considerations we neglect showing the fermions. We once again resort to a table.

### D = 10, N = 2B Supergravity Reduction

|                  | D = 10 | D = 4 | Multiplicity | 4D Spin |
|------------------|--------|-------|--------------|---------|
| $e^m_a \bar{m}$  |        |       | $\begin{pmatrix} \hat{e}^m \bar{a} & A^m \bar{a} \\ 0 & \Delta \bar{a} \bar{m} \end{pmatrix}$ | $\begin{pmatrix} 1 & 6 \\ 0 & 21 \end{pmatrix}$ | $\begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix}$ |
| $G(B)_{abc}^\prime$ | $G(B)_{abc}$, $G(B)_{ab\hat{c}}$, $G(B)_{ab\hat{c}}$ | 1, 6, 15 | 0, 1, 0 |
| $\Phi$           | $\Phi$ | 1     | 0           |
| $A$              | $A$    | 1     | 0           |
| $F(A)_{abc}$     | $F(A)_{abc}$, $F(A)_{ab\hat{c}}$, $F(A)_{ab\hat{c}}$ | 1, 6, 15 | 0, 1, 0 |
| $F(A)_{abcd\hat{c}}$ | $F(A)_{abcd\hat{c}}$, $F(A)_{abcd\hat{c}}$ | 10, 15 | 1, 0 |

Table III

Similar to the last case, there are three points of note.

(a.) The multiplicities fall into Spin(6) representations.

(b.) The spin-one fields are in multiplicities of $6 + 6 + 6 + 10$. 

21
(c.) There are two 2-forms field strengths, $G_{abc}$ and $F_{abc}$, which, after dualization, yield two spin-0 fields that could possess a two dimensional moduli space.

Since there is no action for the IIB theory in 10D, there is a subtlety in counting the total number of moduli. In addition to the moduli that result from the 2-forms, it might be argued that additional moduli could emerge for some of the other scalar fields that are contained in the table above. To understand this possibility let us consider the multiplicities of all 70 spin-0 fields.

If we assume that some 4D action exists that respects the Spin(6) symmetry of the fields, then the only dimensions of moduli space less than seven that could arise are two, three or four. Moreover there is no choice, respecting the Spin(6) representation of the scalar fields, by assuming that moduli can appear arbitrarily for the scalar fields, that leads to either six or seven. This argument suggest that the number of moduli can never be equal to either six or seven.

We thus argue based on the dimensions of the different moduli spaces, respecting the symmetry of the spin-0 fields, that all this evidence points toward the existence of a minimum of three distinct 4D, $N = 8$ supergravity theories! We follow the lead of our $N = 4$ taxonomy listing the moduli choices horizontally and the gauge group choices vertically.

### Putative and Partial Taxonomy of 4D, $N = 8$ Supergravity

|        | $N = 8$ | $N = 8A$ | $N = 8B$ |
|--------|---------|----------|----------|
| $Z^{(28)}$ | 33      | *        | *        |
| $SO(8)$  | 26      | *        | *        |

Table IV

We know that this table is partial precisely due to the Hull-type constructions. In addition to the gauge groups listed above, all those explored in the works of [24] and their descendants may be added to the first column. Colloquially, we may say that studies following on [24] simply move us down the first column. There has never been a complete construction of the other putative theories suggested above. Similarly, there has never been a complete supergeometrical analysis for the $N = 8$ case as has been done for the $N = 4$ theories in [20]. The question of whether a successfully gauged version of either the $N = 8A$ theory or $N = 8B$ theory and which leads to models that are distinct from those already elucidated by Hull’s approach seems to be a worthwhile one to investigate.
The 4D, \( N = 8 \) story may not end with just the observations that we have made so far. A number of years ago [34], we noted that a 4D, \( N = 8 \) Green-Schwarz construction suggests an even larger zoo of theories. The GS action is well known,

\[
S_{GS} = \int d^2 \sigma V^{-1} \left[ - \Pi_+^A \Pi_\rightarrow^A + \int_0^1 dy \hat{\Pi}_y^A \hat{\Pi}_\rightarrow^A \hat{G}_{ABC} \right],
\]

\[
\Pi_+^A = V_+^m \left( \partial_m Z^M \right) E_M^A, \quad \Pi_\rightarrow^A = V_\rightarrow^m \left( \partial_m Z^M \right) E_M^A,
\]

\[
\hat{Z}^M = Z^M(\sigma, \tau, y), \quad \hat{\Pi}_y^A = \left( \partial_y \hat{Z}^M \right) E_M^A(\hat{Z}), \quad \hat{G}_{ABC} = G_{ABC}(\hat{Z}),
\]

where we refer the reader to [34] for notational conventions. However, instead of interpreting this expression in some higher dimension, we proposed to study some of its properties in 4D. In particular we let \( Z^M = (\Theta_\mu^i, \Theta_\mu^i', \bar{\Theta}^{\dot{\mu}}_{\dot{i}}, \bar{\Theta}^{\dot{\mu}}_{\dot{i}}', X^m) \) and define the supercoordinate of the string where \( X^m(\tau, \sigma) \) is a four dimensional bosonic string coordinate

\[
X^m(\tau, \sigma) = \begin{pmatrix}
X^0 + X^3 & X^1 - iX^2 \\
X^1 + iX^2 & X^0 - X^3
\end{pmatrix},
\]

and fermionic string coordinates are defined by

\[
\Theta_\mu^i(\tau, \sigma) \quad i = 1, \ldots, n_L \quad (4D - \text{Weyl spinor}) ,
\]

\[
\Theta_\mu^i'(\tau, \sigma) \quad i' = 1, \ldots, n_R \quad (4D - \text{Weyl spinor}) .
\]

To complete the definition of this model, we finally define \( \hat{G}_{ABC} \) by

\[
\hat{G}_{ABC} = i \frac{1}{2} C_{\alpha \gamma} C_{\beta \dot{\gamma}} \begin{cases}
\delta^j_i : \text{if } A = \alpha \ i , \ B = \beta \ j , \ C = \gamma \dot{\gamma} \\
- \delta^j_i : \text{for any odd permutation,}
\end{cases}
\]

\[
- \delta^j_i' : \text{if } A = \alpha \ i' , \ B = \beta \ j' , \ C = \gamma \dot{\gamma} \\
\delta^j_i' : \text{for any odd permutation,}
\]

\[
0 : \text{otherwise}.
\]

Remarkably, there exist \((n_L + n_R) \kappa\)-supersymmetries for this action. The elements in the first row of table IV seem to be related to the cases of \( n_L = 8 \) and \( n_R = 0 \), \( n_L = 4 \) and \( n_R = 4 \) and \( n_L = 4 \) and \( n_R = 4 \), respective. As such, these \((n_L, n_R) \kappa\)-supersymmetric 4D GS models bare a striking similarity to the \((p, q) \) NSR heterotic models. Using the “(SUSY)-2-philosophy” [35] (also known as “superembeddings” [36]), it is possible that there is a correspondence between these distinct constructions of string theories.

A subsequent investigation by Siegel [37] proposed that this construction can be interpreted fully in string theory as a part of a new type of closed 4D GS string with
(n_L + n_R)-extended target space supersymmetry and further found\footnote{Berkovits and Siegel also used this approach to analyze 4D effective actions.} that using these notions for some low values of n_L and n_R it was possible to derive new supersymmetry multiplets. This has the obvious additional implication, there may be even more “animals” in the 4D, N = 8 supergravity zoo!

8 Rejoining the Clash: de Sitter vs. SUSY

The first challenge of describing how de Sitter background geometries occurred in the context of extended supergravity models was overcome by showing that such constructions exist. However, it cannot be over emphasized that in the de Sitter phase, all supersymmetries are spontaneously broken. The field is confronting a second and similar such challenge presently in the context of superstring/M-theory. This time the challenge seems to be more than simply a theoretical one.

Experimental results \cite{8, 9} point to the idea that indeed there is a small positive cosmological constant (de Sitter geometry) in Nature. Although there has not yet appeared creditable evidence for the existence of supersymmetry, its presence still remains as an attractive one to resolve the naturalness problem of the standard model.

De Sitter geometries present a considerable challenge to theoretical physics. Firstly, outside the context of supersymmetrical systems, de Sitter geometries seem to lead to some problems in relativistic quantum field theories. As we have seen, globally supersymmetric systems absolutely forbid de Sitter geometries. Locally supersymmetric systems allow them only to arise as spontaneously broken phases. In the context of string and superstring theory, the problem of understanding these systems in the presence of either a de Sitter or anti-de Sitter backgrounds is largely unsolved. Thus, superstring theory today is in much the same situation as extended supergravity theory in the early eighties. The question becomes one of whether history will repeat itself? In this final section, we will report on new works that suggests that the question will be answered in the affirmative. We will also indicate a possible avenue of approaching this problem by use of studies of NSR non-linear $\sigma$-models.

The topic of string or superstring theory on a spacetime background of constant curvature is a notoriously difficult one. Very little exists in the literature on this. There has been some discussion to the effect that on such a background, the notion of the critical dimension of the string no longer applies. Another work \cite{39} has reported on some progress in the case of the bosonic string. For the case of the superstring there is even less known with certainty.

\footnote{Berkovits and Siegel also used this approach to analyze 4D effective actions.}
To date the strongest evidence supporting the presence of de Sitter backgrounds in superstring theory has been suggested in the work of [40]. Typically in theories with a spontaneously broken symmetry, after the symmetry breaking there remains some type of relations on some masses in the theory. For example, in electroweak breaking this relation is represent by the weak mixing angle. In the work of Herdeiro, Hiran and Kallosh [40] it has been reported that there are such relations possible for the supertrace of a mass matrix for the superstring level by level.

In the previous chapter, we discussed reasons for believing that there exists at least three distinct 4D supergravity theories which possess eight supercharges. One of these was the 4D, \( N = 8B \) theory. It is interesting to note that there has recently been presented arguments by Berglund, H"ubsch and Minic [41] to the effect that certain warped compactification of the IIB string theory exist wherein a spontaneously broken supersymmetry phase with de Sitter geometry can occur. Furthermore in their models, the phenomenon discussed in [28] is also present but with a naturally small cosmological constant arising.

Some years ago, it was shown [42] how to write world sheet NSR \( \sigma \)-models which describe all the 4D, \( N = 4 \) massless modes of the 4D, \( N = 4 \) SO(44) heterotic string and wherein all bosonic condensates are explicitly represented in a (1,0) world sheet action (see [42] for notation and conventions)

\[
S_{\text{cond}} = \int d^2 \sigma d\xi \ E^{-1} \left\{ \frac{1}{4\pi \alpha'} \left[ i \left( g_{mn}(X) + B_{mn}(X) \right) (\nabla_+ X^m) (\nabla_+ X^n) \right. \right. \\
+ \Phi(X) \Sigma^+ - \frac{1}{2} \eta^{-} \nabla_{\perp} \eta^{-} \\
\left. + \frac{1}{2} \left[ L_+ \overset{\hat{\alpha}}{\Phi} (L_\overset{\hat{\alpha}}{\Phi} + 2 l_\overset{\hat{\alpha}}{\Phi}) + A_+ \nabla \overset{\hat{\alpha}}{X} \right] \\
+ \frac{1}{2 \sqrt{2\pi \alpha'}} \left( \nabla_+ X^m \right) \eta^{-} \overset{\hat{J}}{A}_{m \overset{\hat{\alpha}}{\overset{\hat{\beta}}{J}}}(X) \eta^{-} \overset{\hat{J}}{\overset{\hat{\alpha}}{\overset{\hat{\beta}}{J}}} \right\} ,
\]  

(55)

where the various quantities appearing in the action are defined by

\[
L_+ \overset{\hat{\alpha}}{\Phi} \equiv \nabla_+ \Phi L_\overset{\hat{\alpha}}{\Phi} , \quad L_\overset{\hat{\alpha}}{\Phi} \equiv \nabla \overset{\hat{\alpha}}{\Phi} L_\overset{\hat{\alpha}}{\Phi} , \quad \overset{\hat{\alpha}}{L}_\overset{\hat{\alpha}}{\Phi} \equiv l_\overset{\hat{\alpha}}{\Phi} + l_\overset{\hat{\alpha}}{\Phi} ,
\]

\[
l_\overset{\hat{\alpha}}{\Phi} \overset{\hat{\beta}}{\Phi} \overset{\hat{\alpha}}{\overset{\hat{\beta}}{J}} \equiv \frac{1}{2 \sqrt{2\pi \alpha'}} (\nabla \overset{\hat{\alpha}}{X}^m) A_{m \overset{\hat{\alpha}}{\overset{\hat{\beta}}{J}}}(X) + i \eta^{-} \overset{\hat{J}}{\eta} \overset{\hat{\beta}}{\Phi} \overset{\hat{\alpha}}{\overset{\hat{\beta}}{J}}(X) .
\]

(56)

with \( \overset{\hat{I}}{1} = 1, \ldots, 44 \) and \( \overset{\hat{\alpha}}{1} = 1, \ldots, 6 \). The condensates are those of a 4D, \( N = 4 \) supergravity multiplet \( (g_{mn}, A_m \overset{\hat{\alpha}}{\Phi}, B_{mn}, \Phi) \) and a 4D, \( N = 4 \) super Yang-Mills matter multiplet for the gauge group \( \text{SO}(44) \) \( (A_{m \overset{\hat{\alpha}}{\overset{\hat{\beta}}{J}}}, \Phi_{\overset{\hat{\alpha}}{\overset{\hat{\beta}}{J}}}) \). The bosonic supergravity condensates correspond to the theory in the work of [15].

From the 4D, \( N = 4 \) supergravity results, we know that the de Sitter background cannot arise from the model above. We first implement, on the \( \sigma \)-model, the modifi-
cation that corresponds to the Nicolai-Townsend mapping as described in (27). On the \( \sigma \)-model side this is accomplished as

\[
i \int d^2 \sigma d \zeta \ E^{-1} B_{m \bar{n}} ( \nabla_+ \bar{X}(\mu) ) ( \nabla_\zeta \bar{X}(\nu)(X) ) \rightarrow \i \int d^2 \sigma d \zeta \ E^{-1} \epsilon_{m \bar{n} \rho \bar{\sigma}} ( \nabla_+ \bar{X}(\mu) ) ( \nabla_\zeta \bar{X}(\nu)(\partial_\rho \bar{X})) \ ,
\]

(57)

via use of the standard extension of the WZNW term by the introduction of an extra bosonic coordinate \( y \) so that \( \hat{X} = \hat{X}(y, \zeta, \tau, \sigma) \) with \( \hat{X}(y = 1, \zeta, \tau, \sigma) = \hat{X}(\zeta, \tau, \sigma) \) and \( \hat{X}(y = 0, \zeta, \tau, \sigma) = 0 \).

The action in (55) also contains six leftons \( \Phi_L^\hat{\alpha} \) and this is indicative of the \([U(1)]^6\) gauge group that occurs for the gravi-photons whose condensates are represented by \( A_m^{\hat{\alpha}} \) above. These six \( U(1) \) gauge fields must be replaced by six \( SU(2) \otimes SU(2) \) gauge fields. However, the way to do this is to replace the six left-moving \( U(1) \) currents (carried on the world sheet by \( \Phi_L^\hat{\alpha} \)) by \( SU(2) \otimes SU(2) \) currents. This calls for a Lagrangian non-Abelian bosonization of the left-moving \( \Phi_L^\hat{\alpha} \) superfields. The accomplishment of this uses the discussion given in \([43]\). There are also the \( SO(44) \) currents on the world sheet that are carried by the forty-four spinorial superfields \( \eta_-^\hat{I} \). Thus the part of the Lagrangian that carries the currents associated with the internal symmetries of the 4D, \( N = 4 \) supergravity model is given by

\[
\mathcal{L}_{\text{current}} = - \frac{1}{2} \eta_-^\hat{I} \nabla_+ \eta_-^\hat{I} + i \left[ \frac{1}{2} \left( L_+^{\hat{\alpha}} (L_{\hat{\alpha}} + 2 l_{\hat{\alpha}}) + \Lambda_+^{\hat{\alpha} \hat{\beta}} \tilde{L}_{\hat{\alpha}} \tilde{L}_{\hat{\beta}} \right) \right] + \frac{1}{2 \sqrt{2 \pi \alpha}} (\nabla_+ X^{\mu}) \eta_-^\hat{I} A_{m \hat{i} \hat{j}}(X) \eta_-^\hat{J} \ ,
\]

(58)

The world-sheet currents to which the gravi-photons \( A_m^{\hat{\alpha}}(X) \) couple are purely left-moving target space internal currents and right-moving target space spacetime currents while the currents to which the \( SO(44) \)-gluons \( A_{m \hat{i} \hat{j}}(X) \) couple are purely right-moving target space internal currents and left-moving target space spacetime currents. Finally, the \( SO(44) \)-scalar-gluons \( \Phi_{i \hat{j}}(X) \) couple to the product of right-moving target space internal currents times left-moving target space internal currents.

Following the results in \([43]\) we are going to bosonize the left-moving \( \Phi_L^\hat{\alpha} \) superfields and as well as the right-moving \( \eta_-^\hat{I} \) superfields. This will amount to a replacement of these variables according to

\[
\Phi_L^\hat{\alpha} \rightarrow exp\left[ i \varphi_L^{\hat{\alpha}} t_\hat{\alpha} \right] \ , \ \ \ \eta_-^\hat{I} \rightarrow exp\left[ i \varphi_R^I t_I \right] \ .
\]

(59)

where both \( \varphi_L^{\hat{\alpha}} \) and \( \varphi_R^I \) are bosonic \((1,0)\) superfields. In these expressions, the quantities \( t_\hat{\alpha} \) and \( t_I \) respectively denote matrices representing the groups \( SU(2) \otimes SU(2) \) and \( SO(44) \). This choice fixes the group dimension \( d_G \) and the dual co-exeter number.
$c_2$ associated with Kac-Moody algebras. This does not fix the level numbers, however. Insight into this can be gained by considering the anomaly coefficients $(\nu_L, \nu_R)$ associated with these groups.

If $\varphi^{\hat{\alpha}}_L$ and $\varphi^{I\hat{f}}_R$ are the coordinates for two respective groups $G_L$ and $G_R$, then their anomaly coefficients are given by (see also the appendix)

### Anomaly Coefficients

|       | $\nu_L$                                    | $\nu_R$                                    |
|-------|--------------------------------------------|--------------------------------------------|
| $\Phi_R^{\hat{\alpha}}$ | 0                                          | $d_{GR} \left[ 1 + \frac{c_2(G_R)}{2k_R} \right]^{-1}$ |
| $\Phi_L^{\hat{\alpha}}$ | $d_{GL} \left[ 1 + \frac{c_2(G_R)}{2k_L} \right]^{-1} + \frac{1}{2} d_{GL}$ | 0                                          |

Table V

and for $G_L = SU_{k_1}(2) \otimes SU_{k_2}(2)$ and $G_R = SO_{k_3}(44)$ this leads to

$$
\nu_L = 3 \left\{ 1 + \left[ 1 + \frac{1}{k_1} \right]^{-1} + \left[ 1 + \frac{1}{k_2} \right]^{-1} \right\},
$$

$$
\nu_R = 22 \cdot 43 \left[ 1 + \frac{42}{k_3} \right]^{-1}.
$$

Before the “shift” of the gauge group from $[U(1)]^6$ to $SU(2) \otimes SU(2)$, the fields of (58) possessed anomaly coefficients of $\nu_L = 9$ and $\nu_R = 22$. So the condition for there to be no world sheet anomalies introduced in the $\sigma$-model by the gauge group shift is

$$
3 = \left\{ 1 + \left[ \frac{k_1}{k_1 + 1} \right] + \left[ \frac{k_2}{k_2 + 1} \right] \right\}, \quad 1 = 43 \left[ \frac{k_3}{k_3 + 42} \right].
$$

for some integers $k_1$, $k_2$ and $k_3$. From the first equation, it is seen that the condition is equivalent to $k_1 + k_2 + 2 = 0$. Since level numbers are normally considered to be positive integers, there are no solutions to this equation\footnote{There is a curiosity about this equation. As $k_1, k_2 \to \infty$, this equation is satisfied. One is thus led to speculate on the possibility of an infinite level Kac-Moody model.}. Owing to the sake of simplicity we might as well set $k_1 = k_2 = 1$. For the the second of these equations, there is a solution for $k_3 = 1$. This is reassuring because this means that the matter sector of the chirally non-Abelian bosonized $\sigma$-model is equivalent to the original fermionic description for this choice of the level number.

To complete our discussion of the chiral non-Abelian bosonization of $L_{\text{current}}$, we will give the explicit form of its replacement below. However prior to doing so, there is a small matter of some notation to clarify. In (58) the index $I\hat{f}$ takes on values 1, \ldots, 44. Thus the counting of a condensate which possesses a pair of these indices
(anti-symmetrized) yields a total of 946. This, of course, is the dimension of the adjoint representation of SO(44). After the bosonization, it is more convenient to introduce an index that takes its values in the adjoint of SO(44) and thus the range of this index is from 1, \ldots, 946. We will denote this index also by the symbol \( \hat{I} \).

Now finally for the explicit expression for the replacement Lagrangian we have

\[
\mathcal{L}_{\text{current}} \rightarrow \mathcal{L}_{\text{current}}^{\text{NAB}}
\]

with the latter Lagrangian given by

\[
\mathcal{L}_{\text{current}}^{\text{NAB}} = \quad - (L_\alpha^\dagger + \Gamma_\alpha^\dagger)(L_\alpha - \Lambda_\alpha^\dagger(L_\alpha^\dagger + \Gamma_\alpha^\dagger)) - L_\alpha^\dagger \Gamma_\alpha^\dagger \\
- (R_i^\dagger + 2\Gamma_i^\dagger)R_i^\dagger + i\Lambda_\alpha^\dagger(\Gamma_\alpha^\dagger R_i^\dagger + R_i^\dagger \Gamma_\alpha^\dagger) \\
+ \frac{2}{3} f^{i j k} \Lambda_\alpha^\dagger \Gamma_\alpha^\dagger R_i^\dagger (\Gamma_{j+}^\dagger + \Gamma_{j+}^\dagger)(R_k^\dagger - \frac{1}{2} \Gamma_k^\dagger) \\
- \int_0^1 dy \text{Tr}\{\frac{d\hat{L}}{dy} \hat{L}^{-1} [\nabla_\alpha((\nabla_+ \hat{L})\hat{L}^{-1}) - \nabla_+((\nabla_{-\hat{L}})\hat{L}^{-1})] \} \\
+ \int_0^1 dy \text{Tr}\{\frac{d\hat{R}}{dy} \hat{R}^{-1} [\nabla_\alpha((\nabla_+ \hat{R})\hat{R}^{-1}) - \nabla_+((\nabla_{-\hat{R}})\hat{R}^{-1})] \} \\
- 4\Phi^{\alpha i} \hat{R}_i^\dagger \hat{L}_+^\alpha + 4\Lambda_\alpha^\dagger (M^{-1})^{jk} \Phi^{\alpha i} \Phi^{\delta j} \gamma^{\gamma \delta \beta} \\
+ i4 \Lambda_\alpha^\dagger \Phi^{\delta j} \hat{L}_+^\alpha \nabla_+ (\Phi^{\beta j} \Gamma_+^\beta) \\
- i4 \Lambda_\alpha^\dagger f^{i j k} (\Gamma_{j+}^\dagger + \frac{2}{3} \Phi^{\delta j} \hat{L}_+^\beta) \Phi^{\beta j} \Gamma_+^\beta \hat{L}_+^\beta \\
\]

This Lagrangian is quite complicated and the definitions of the various objects that appear in it can be found below.

\[
L_m^{\alpha \dagger} \equiv - i(\nabla_m L) L^{-1}, \quad R_m^i \dagger \equiv - i(\nabla_m R) R^{-1}, \\
L_\alpha^\dagger = L_\alpha^\dagger - \Lambda_\alpha^\dagger (L_\alpha^\dagger + \Gamma_\alpha^\dagger), \\
R_i^\dagger = R_i^\dagger - i[\Lambda_{\alpha^\dagger}^{\dagger} \nabla_+(R_i^\dagger + \Gamma_i^\dagger) + \frac{1}{2}(\nabla_+ \Lambda_\alpha^\dagger)(R_i^\dagger + \Gamma_i^\dagger) \\
- \frac{1}{2} \Lambda_\alpha^\dagger f^{i j k} (R_i^\dagger + \Gamma_i^\dagger)(R_j^\dagger + \Gamma_j^\dagger)], \\
\Sigma_i^\dagger = R_i^\dagger - 2i[\Lambda_{\alpha^\dagger}^{\dagger} \nabla_+(\Phi^{\delta j} \hat{L}_+^\beta) + \frac{1}{2}(\nabla_+ \Lambda_\alpha^\dagger) \Phi^{\delta j} \hat{L}_+^\beta \\
- \Lambda_\alpha^\dagger f^{i j k} \Phi^{\delta j} \hat{L}_+^\beta (\Gamma_i^\dagger + \hat{L}_+^\beta \Gamma_+^\beta)], \\
(M^{i,j})^\dagger = \delta^{i,j} - 4i\Lambda^{\alpha^\dagger} \Phi^{\alpha^\dagger} \Phi^{\delta j}, \\
\Gamma_+^i \equiv \frac{1}{2\sqrt{2\pi\alpha'}} (\nabla_+ X_m^\alpha) A_m^i (X), \\
\Gamma_\alpha^\dagger \equiv \frac{1}{2\sqrt{2\pi\alpha'}} (\nabla_{-\hat{L}} X_m^\alpha) A_m^\alpha (X), \\
\Phi^{\alpha^\dagger} \equiv \Phi^{\alpha^\dagger} (X).
\]
The consequences of the analysis it clear. On the 4D, $N = 4$ supergravity side, the gauging of both $SU(2) \otimes SU(2)$ groups (with coupling constants $g_1 = -g_2$) using gravi-photons is required to obtain a model that possesses a de Sitter phase. On the $\sigma$-model side, this same gauging necessarily leads to an anomaly being present. A potential appears on the side of the supergravity model and it is somehow related to the appearance of an anomaly on the side of the $\sigma$-model. This suggests that the supergravity model, if it can be embedded within a $\sigma$-model, might correspond to a deformation of the $\sigma$-model. Once we know that the ordinary $\sigma$-model which is most closely related to the Freedman-Schwarz model is anomalous, then we can come to the end of how to proceed further within the confines of the present state-of-the-art. In particular, the matter of changing the modulus choice becomes moot.

We are, however, left free to speculate. If a quantum theory is anomalous, past experience has taught us that this is a signal that there may be extra terms, extra degrees of freedom, etc. that must be introduced to augment the original theory. Perhaps this is what is required in the present circumstance also. Looking back at the failure of the left-anomaly cancellation condition, we note that if the lefton current group is changed from $SU_1(2) \times SU_1(2) \times U(1)$ or even $SU_1(2) \times SU_1(2) \times SU_1(1)$, then the condition for left anomaly-freedom is satisfied. From the 4D, $N = 4$ supergravity side, we know that there are no fundamental massless excitations associated with the introduction of the final group. Still its presences signals the possibility to introduce even more $\beta$-functions that might correspond to composite functions of the original ones. So there seems to be at least a hope that some augmentation might work. Of course, there is still the more stringent test of whether a lefton group of the form of $SU_1(2) \times SU_1(2) \times U(1)$ or $SU_1(2) \times SU_1(2) \times SU_1(1)$ is acceptable.

The strongest evidence to support the possibility that an augmented action can be found is the suggestion of the “stringy Zeeman effect” [31, 40]. A completely successful realization of the tantalizing hint coming from the stringy Zeeman effect will demand this. Thus, a model [22] found nineteen years ago may have yet another role as a laboratory in which to unravel the mystery of treating superstrings in a de Sitter space background.

9 Conclusion

A universe in which there exists supersymmetry simultaneously with a background geometry of the de Sitter variety presents real challenges to superstring theory. A challenge like this is extremely healthy in this author’s opinion. It will clearly demand
an advance of the state-of-the-art in superstring theory and perhaps more generally in field theory. It is very likely to force a better understanding of the meaning of the word “geometry” in the superstring arena.

With this presentation we hope to have achieved a few simple goals. These were:

(a.) to have provided as complete as possible a review of the early literature on the problem of the admissibility of de Sitter space background in the confines of theories with local supersymmetry,

(b.) to have illustrated, in some detail, how a 1984 work provided a paradigm to by-pass a general no-go theorem ruling out de Sitter space background in confines of supergravity theories,

(c.) to make the community aware of some long standing open issues related to the cosmological constant in supergravity theories,

(d.) to note some encouraging points of recent literature that suggest the problem of the admissibility of de Sitter space background in the confines of superstring theories will repeat the past pattern of this problem within the confines of supergravity theory and

(e.) to provide a detailed analysis from the point of the view of the combined world sheet $\sigma$-model/4D, $N = 4$ supergravity system as what are the hurdles that must be surmounted if the de Sitter phase, known to exist in 4D, $N$-extended supergravity (equivalent to higher D SG theories) models, is to be extended for superstrings.

Finally and of course, we wish to celebrate the initial presentation of the theory of supergravity and hope that Nature is aware of our efforts (as one colleague in the field has been heard to say).

“Let only geometers enter here.”

– Copernicus

30
Acknowledgment

We wish to acknowledge the organizers of the “Supergravity25” conference for their invitation to make a presentation. We also wish to acknowledge our hosts (Hendrik Geyer and Joao Rodrigues) at the Fourteenth Chris Engelbrecht Summer School in Theoretical Physics and as well the Stellenbosch Institute for Advanced Study (STIAS, Dir. Bernard Lategan) workshop on String Theory and Quantum Gravity, held in Stellenbosch, South Africa during Jan. 23 to Feb. 22, 2002. The lively atmosphere of the school and workshop provided a stimulating environment in which to complete the writing this extended version of my Stony Brook presentation. Finally, it is an honor to contribute this paper as the first STIAS publication.

Appendix: Groups, Dimensions and Dual Co-exeter Numbers

In obtaining the results in (60), we have made use of some results that certainly exist in the prior physics literature. We present them here for the convenience of the reader. For a group with matrices $t_{\hat{a}}$ that faithfully represent its generators, we have used the following definitions. $\hat{a} = 1, \ldots, d_G$, $[t_{\hat{a}}, t_{\hat{b}}] = i f_{\hat{a}\hat{b}\hat{c}} t_{\hat{c}}$, $f_{\hat{a}\hat{b}\hat{c}} f_{\hat{a}\hat{b}\hat{d}} = c_2 \delta_{\hat{c}\hat{d}}$ and $Tr(t_{\hat{a}} t_{\hat{b}}) = 2k \delta_{\hat{a}\hat{b}}$.

| Group   | $d_G$      | $c_2$      |
|---------|------------|------------|
| SU(n)   | $n^2 - 1$  | $n$        |
| SO(2n + 1) | $n(2n + 1)$ | $4n - 2$  |
| Sp(n)   | $n(2n + 1)$ | $2n + 2$  |
| SO(2n)  | $n(2n - 1)$ | $4n - 4$  |
| G_2     | 10         | 8          |
| F_2     | 40         | 18         |
| E_6     | 78         | 24         |
| E_7     | 133        | 36         |
| E_8     | 248        | 60         |

Table VI

References
[1] D. Z. Freedman, P. van Nieuwenhuizen and S. Ferrara, “Progress Toward a Theory of Supergravity,” Phys. Phys. D13 (1976) 1976.

[2] S. J. Gates, Jr., “A Note On the Geometry of Global Supersymmetry,” (unpublished) (MIT,LNS) MIT-CTP-604 (Feb., 1977) 45p.; idem. “On the Geometry of Superspace,” (unpublished) (MIT,LNS) MIT-CTP-621 (April, 1977) 45p.; idem. “On the Geometry of Superspace and Local Supersymmetry,” Phys. Rev. D17 (1978) 3188.

[3] W. Siegel and S. J. Gates, Jr., “Superfield Supergravity,” Nucl. Phys. B147 (1979) 77; S. J. Gates, Jr. and W. Siegel, “Understanding Constraints in Superspace Formulations of Supergravity,” Nucl. Phys. B163 (1980) 519.

[4] S. J. Gates, Jr. and W. Siegel, “(3/2, 1) Superfield of O(2) Supergravity,” Nucl. Phys. B164 (1980) 484.

[5] A. Einstein, Sitz. Preuss. Akad. Wiss., 142 (1917) 35.

[6] S. J. Gates, Jr., M. T. Grisaru, M. Roček and W. Siegel, Superspace, Benjamin Cummings, (1983) Reading, MA., p. 240; PDF downloadable file from [http://aps.arXiv.org/pdf/hep-th/0108200](http://aps.arXiv.org/pdf/hep-th/0108200).

[7] S. J. Gates, Jr., “Basic Canon in 4D, N = 1 Superfield Theory: Five Primer Lectures,” in the Proceedings of the 1997 Theoretical Advanced Summer Institute (TASI) “Supersymmetry, Supergravity and Supercolliders,” ed. J. Bagger, pp. 153-258 (World Scientific, 1999), Singapore; [hep-th/9809064](http://arxiv.org/abs/hep-th/9809064).

[8] A. G. Reiss, et al., “Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant,” Astron. J. 116 (1998) 1009 [astro-ph/9805201]; S. Perlmutter, et al., “Measurement of Omega and Lambda from 42 High-Redshift Supernovae,” Astrophys. J. 517 (1999) 565, [astro-ph/9912133](http://arxiv.org/abs/astro-ph/9912133).

[9] C. B. Netterfield, et al., “The BOOMERANG North American Instrument; a balloon-borne bolometric radiometer optimized for measurement of cosmic background radiation anisotropies from 0.3 to 4 degrees,” [astro-ph/9912133](http://arxiv.org/abs/astro-ph/9912133); N. W. Halverson, et al., “DASI First Results: A measurement of the Cosmic Microwave Background Angular Power Spectrum,” [astro-ph/0104489](http://arxiv.org/abs/astro-ph/0104489); P. de Bernardis, et al., “Multiple Peaks in the Angular Power Spectrum of the Cosmic Microwave Background: Significance and Consequences for Cosmology,” [astro-ph/0105290](http://arxiv.org/abs/astro-ph/0105290).
[10] S. Ferrara, “Algebraic Properties of Extended Supergravity in de Sitter Space,” Phys. Lett. 69B (1977) 481.

[11] D. Z. Freedman, “Supergravity with Axial-gauge Invariance” Phys. Rev. D15, (1977) 1173.

[12] P. K. Townsend, “Cosmological Constant in Supergravity,” Phys. Rev. D15, (1978) 2802.

[13] D. Z. Freedman and A. Das, “Gauge Internal Symmetry in Extended Supergravity,” Nucl. Phys. B120 (1977) 221.

[14] A. Das, “SO(4)-invariant Extended Supergravity,” Phys. Rev. D15, (1977) 2805.

[15] E. Cremmer and J. Scherk, “SU(4) Invariant Supergravity Theory,” Nucl. Phys. B127 (1977) 259; E. Cremmer, J. Scherk and S. Ferrara, “Algebraic Simplification in SU(4) Invariant Supergravity Theory,” Phys. Lett. 24B (1978) 61.

[16] H. Nicolai and P. K. Townsend, “N = 3 Supersymmetry Multiplets with Vanishing Trace Anomaly: Building Blocks of the N > 3 Supergravities,” Phys. Lett. 98B (1981) 257.

[17] A. Das, M. Fischler and M. Roček, “Super-Higgs Effect in a New Class of Scalar Models and a Model of Super QED,” Phys. Rev. D16 (1977) 3427.

[18] D. Z. Freedman and J. H. Schwarz, “N = 4 Supergravity Theory with Local SU(2) ⊗ SU(2) Invariance,” Nucl. Phys. B137 (1978) 333.

[19] S. J. Gates, Jr., “On-Shell and Conformal N = 4 Supergravity in Superspace,” Nucl. Phys. B213 (1983) 409.

[20] S. J. Gates, Jr. and B. Zweibach, “Searching for All N = 4 Supergravities with Global SO(4),” Nucl. Phys. B238 (1984) 99; B. Zweibach, “Gauged N = 4 Supergravities and Gauge Coupling Constants,” Nucl. Phys. B238 (1984) 367; S. J. Gates, Jr. and J. Durachta, “Gauge Two-Form in D = 4, N = 4 Supergeometry with SU(4) Supersymmetry,” Mod. Phys. Lett. A21 (1989) 2007.

[21] J. Lukierski and A. Nowicki, “All Possible de Sitter Superalgebras and the Presence of Ghosts,” Phys. Lett. 151B (1985) 382; K. Pilch, P. van Nieuwenhuizen and M. F. Sohnius, “De Sitter Superalgebras and Supergravity,” CMP 98 (1985) 105; B. de Wit and A. Zwartkruis, “SU(2,2/1,1) Supergravity and N = 2 Supersymmetry with Arbitrary Cosmological Constant,” Class. Quant. Grav. 4 (1987) L59-L66.
[22] S. J. Gates, Jr. and B. Zweibach, “Gauged \( N = 4 \) Supergravity Theory with a New Scalar Potential,” Phys. Lett. **B123** (1983) 200.

[23] A. H. Chamseddine and M. S. Volkov, “Non-Abelian BPS Monopoles in \( N = 4 \) Gauged Supergravity,” Phys. Rev. Lett. **79** (1997) 3343; idem., “Non-Abelian Solitons in \( N = 4 \) Gauged Supergravity and Leading Order String Theory,” Phys. Rev. **D57** (1998) 6242.

[24] C. M. Hull, “A New Gauging of \( N = 8 \) Supergravity,” Phys. Rev. **D30** (1984) 760; idem. “Noncompact Gaugings of \( N = 8 \) Supergravity,” Phys. Lett. **B142** (1984) 39; ibid. “More Gaugings of \( \mathbb{N}=8 \) Supergravity,” **148B** (1984) 297; C. M. Hull, “The Minimal Couplings & Scalar Potentials of the Gauged \( N = 8 \) Supergravities,” Class. Quant. Grav. **2** (1985) 343; C. M. Hull and N. P. Warner, “The Structure of the Gauged \( \mathbb{N}=8 \) Supergravity Theories,” Nucl. Phys. **B253** (1985) 650; ibid. “The Potentials of the Gauged \( \mathbb{N}=8 \) Supergravity Theories,” Nucl. Phys. **B253** (1985) 675.

[25] C. Ahn and K. Woo, “Domain Wall and Membrane Flow from Other Gauged \( \mathbb{N}=4, \mathbb{N}=4 \) Supergravity, I,” arXiv:hep-th/0109010.

[26] B. de Wit and H. Nicolai, “\( \mathbb{N}=8 \) Supergravity with Local \( \text{SO}(8) \times \text{SU}(8) \) Invariance,” Phys. Lett. **108B** (1982) 285; idem. “\( \mathbb{N}=8 \) Supergravity,” Nucl. Phys. **B208** (1982) 323.

[27] S. J. Gates, Jr., M. T. Grisaru, M. Roček and W. Siegel, *Superspace*, Benjamin Cummings, (1983) Reading, MA., p. 336; PDF downloadable file from http://aps.arXiv.org/pdf/hep-th/0108200.

[28] B. B. Deo and S. J. Gates, Jr., “Non-minimal \( \mathbb{N}=1 \) Supergravity and Broken Global Supersymmetry,” Phys. Lett. **151B** (1985) 195.

[29] S. Ferrara and P. van Nieuwenhuizen, “The Auxiliary Fields of Supergravity,” Phys. Lett. **74B** (1978) 333.

[30] P. Breitenlohner, “A Geometrical Interpretation of Local Supersymmetry,” Phys. Lett. **67B** (1977) 49, idem. “Some Invariant Lagrangians for Local SU persuacy,” Nucl. Phys. **B124** (1977) 500; “On the Auxiliary Fields of Supergravity,” Phys. Lett. **80B** (1979) 217.

[31] R. Kallosh, “\( \mathbb{N}=2 \) Supersymmetry and de Sitter Space,” arXiv: hep-th/0109168; ibid. “Gauged Supergravities de Sitter Space and Cosmology,” arXiv: hep-th/0110089.
[32] S. J. Gates, Jr., M. T. Grisaru, M. Roček and W. Siegel, *Superspace*, Benjamin Cummings, (1983) Reading, MA., p. 259; PDF downloadable file from [http://aps.arXiv.org/pdf/hep-th/0108200](http://aps.arXiv.org/pdf/hep-th/0108200).

[33] E. Cremmer and B. Julia, “The SO(8) Supergravity,” Nucl. Phys. **B159** (1979) 141.

[34] S. J. Gates, Jr. and H. Nishino, “D = 2 Superfield Supergravity, Local (Supersymmetry)$^2$, and Nonlinear $\Sigma$-Models, Class. Quant. Grav. **3** (1986) 391.

[35] D. Sorokin, “Superbranes and Superembeddings,” Phys. Reports **239** (2000) 1.

[36] S. J. Gates, Jr., “Progress Toward Covariant Formulation of All D = 4 GS-type $\sigma$-model Actions,” in *Superstring and Particle Theory*, Tuscaloosa, AL, Nov. 1989 (World Scientific, Singapore, 1989) p. 57.

[37] W. Siegel, “Curved Extended Superspace From Yang-Mills Theory à la Strings,” Phys. Rev. **D53** (1996) 3324.

[38] N. Berkovits and W. Siegel, “Superspace Effective Actions for 4D Compactifications of Heterotic and Type II Superstrings,” Nucl. Phys. **B462** (1996) 213.

[39] E. Silverstein, “(A)dS Backgrounds from Asymmetrical Orientifolds,” [hep-th/0106209](http://arxiv.org/abs/hep-th/0106209).

[40] C. Herdeiro, S. Hirano, and R. Kallosh, “String Theory and Hybrid Inflation/Acceleration,” [hep-th/0110271](http://arxiv.org/abs/hep-th/0110271).

[41] P. Berglund, T. Hübsch and D. Minic, “de Sitter Spacetimes from Warped Compactifications of IIB String Theory,” [hep-th/0112073](http://arxiv.org/abs/hep-th/0112073). idem., “Relating the Cosmological Constant and Supersymmetry Breaking in Warped Compactification of IIB String Theory,” [hep-th/0201187](http://arxiv.org/abs/hep-th/0201187).

[42] S. J. Gates, Jr. and W. Siegel, “Leftons, Rightons, Nonlinear $\sigma$-models, and Superstrings,” Phys. Lett. **206B** (1988) 631.

[43] D. Depireux, S. J. Gates, Jr. and S. Bellucci, “(1,0) Thirring Models and the Coupling of Spin-0 Fields to Heterotic Strings,” Phys. Lett. **232B** (1989) 67.