Electric and magnetic currents in $SU(2)$ lattice gauge theory*

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The correlations of the topological charge ($Q$), the electric ($J^e$) and the magnetic ($J^m$) currents in the $SU(2)$ lattice gauge theory in the Maximal Abelian projection are investigated. A nonzero value of the correlator $<QJ^eJ^m>$ is obtained for a wide range of values of the bare charge, as well as under the cooling.

1. INTRODUCTION

An oldest and rather popular model of the QCD vacuum is the instanton–anti-instanton media (see [1] and the references therein). It is not clear however, whether it is possible to explain the confinement phenomenon within this approach [2, 3].

On the other hand, the method of abelian projections [3] is widely used in numerical calculations to study the confinement mechanism. It is clearly seen [3] that the vacuum of the lattice gluodynamics behaves like a dual superconductor in the so-called Maximal Abelian (MaA) projection [1, 2]. There are clear indications that abelian monopoles are condensed in the confinement phase of lattice gluodynamics [1, 2].

It occurs that instanton-like configurations and monopoles in the MaA gauge are interrelated [3]. The relation between monopoles and instantons has been established analytically in [4, 8] and numerically in [10].

In the field of a single instanton the monopole currents in the MaA projection are accompanied by electric currents [2]. The qualitative explanation of this fact is simple. Consider the (anti)self-dual configuration

$$F_{\mu\nu}(A) = \pm^* F_{\mu\nu}(A).$$

The MaA projection is defined [1] by the minimization of the functional $R[A](x) = \int d^4x [\frac{1}{4} F_{\mu\nu}(A)^2]$, so that in the MaA gauge one can expect the abelian component of the commutator term $1/2Tr(\sigma^{\mu\nu}[A_{\mu}, A_{\nu}]) = \varepsilon^{abc} A_{\mu}^a A_{\nu}^b$ to be small compared with the abelian field-strength $f_{\mu\nu}(A) = \partial_{\mu} A_{\nu}^a$. Therefore, in the MaA projection eq. (1) yields

$$f_{\mu\nu}(A) \approx \pm^* f_{\mu\nu}(A).$$

Due to eq. (2), the monopole currents have to be correlated with the electric ones since

$$J_{\mu}^e = \partial_{\nu} f_{\mu\nu}(A) \approx \pm \partial_{\nu}^* f_{\mu\nu}(A) = J_{\mu}^m.$$

In the present publication we study the correlation of electric and magnetic currents in the real vacuum of the $SU(2)$ lattice gluodynamics.

2. MAGNETIC AND ELECTRIC CURRENTS IN THE ABELIAN PROJECTION OF THE $SU(2)$ LATTICE GLUODYNAMICS

The abelian monopoles exist in the abelian projection, since the residual $U(1)$ group is compact. The definition of the abelian monopole current is [12]:

$$J_{\mu}^m(y) = \frac{1}{4\pi} \sum_{\nu\lambda\rho} \varepsilon_{\mu\nu\lambda\rho} [\hat{\theta}_{\lambda\rho}(x + \hat{\mu}) - \hat{\theta}_{\lambda\rho}(x)].$$

Here the angle $\theta_{\mu\nu}$ is the normalized plaquette angle $\hat{\theta}_{\mu\nu} = \hat{\theta}_{\mu\nu} + 2\pi k_{\mu\nu}$; $k_{\mu\nu}$ is an integer such that $\hat{\theta}_{\mu\nu} \in (-\pi, \pi]$. The monopole currents are quantized ($J_{\mu}^m \in Z$) and conserved ($\partial_{\mu} J_{\mu}^m = 0$). They are attached to the links of the dual lattice, the link $(y, \mu)$ is dual to the cube $(x, \nu\lambda\rho)$.

The electric current is defined as

$$J_{\mu}^e(x) = \frac{1}{2\pi} \sum_{\nu} [\hat{\theta}_{\mu\nu}(x) - \hat{\theta}_{\mu\nu}(x - \hat{\nu})].$$
In the continuum limit, the definitions (1) and (2) correspond to the usual ones: \( J^m_\mu = \partial_\nu f_{\mu\nu} \) and \( J^e_\mu = \partial_\nu f_{\mu\nu} \). The electric currents are conserved \( (\partial_\mu J^e_\mu = 0) \) and attached to the links of the original lattice. Electric currents are not quantized.

In order to calculate the correlators of the type \(< J^e_\mu(x)J^m_\mu(x) \ldots >\) one has to define the electric current on the dual lattice or the magnetic current on the original lattice. We define the electric current on the dual lattice in the following way:

\[
J^e_\mu(y) = \frac{1}{16} \sum_{x \in \tilde{C}(y,\mu)} [J^e_\mu(x) + J^e_\mu(x - \hat{\mu})].
\]

Here, the summation in r.h.s. is over eight vertices \( x \) of the 3-dimensional cube \( \tilde{C}(y,\mu) \), to which the current \( J^e_\mu(y) \) is dual. The point \( y \) lies on the dual lattice and the points \( x \) lie on the original one.

For the topological charge density operator we use the simplest definition:

\[
Q(x) = \frac{1}{2^9 \pi^2} \sum_{\mu_1,\ldots,\mu_4 = -4}^{4} e^{\mu_1 \ldots \mu_4} \sum_{x \in C(\mu_1,\mu_2)} \text{Tr} \left[ U_{\mu_1,\mu_2}(x) U_{\mu_3,\mu_4}(x) \right],
\]

where \( U_{\mu_1,\mu_2} \) is the plaquette matrix. On the dual lattice the topological charge density corresponding to the monopole current \( J^m_\mu(y) \) is defined by taking the average over the eight sites nearest to the current \( J^m_\mu(y) \):

\[
Q(y) = \frac{1}{8} \sum_{x} Q(x).
\]

The simplest (connected) correlator of electric and magnetic currents is

\[
\langle J^m_\mu J^e_\mu \rangle = \langle J^m_\mu J^e_\mu \rangle - \langle J^m_\mu \rangle \langle J^e_\mu \rangle = \langle J^m_\mu J^e_\mu \rangle.
\]

This correlator is equal to zero, since \( J^m_\mu \) and \( J^e_\mu \) have opposite parities. A scalar quantity can be constructed if we multiply \( J^m_\mu J^e_\mu \) by the density of topological charge. The corresponding irreducible correlator

\[
\langle J^m_\mu J^e_\mu Q \rangle = \langle J^m_\mu J^e_\mu Q \rangle
\]

is nonzero for the vacuum consisting of (anti-)self-dual domains (cf. eq. (3)).

3. NUMERICAL RESULTS

The \( SU(2) \) lattice gauge theory was considered on the \( 8^4 \) lattice with the Wilson action. At each value of \( \beta \) we thermalize lattice fields using the standard heat bath algorithm. We calculate the correlators \( \langle J^m_\mu(y)J^m_\mu(y) \rangle \), \( \langle J^m_\mu(y)J^e_\mu(y)Q(y) \rangle \), using 100 statistically independent configurations at each value of \( \beta \).

These correlators strongly depend on \( \beta \) and it is convenient to normalize them, dividing by \( \rho^m \rho^e \). Here \( \rho^m \) and \( \rho^e \) are the monopole and the electric current densities:

\[
\rho^m = \frac{1}{4V} \sum_{\mu} |J^m_\mu|,
\]

\( V \) is the lattice volume (the total number of sites).

![Figure 1. Correlators \( \langle J^m J^e Q \rangle / \rho^m \rho^e \) and \( \langle J^m J^e q \rangle / \rho^m \rho^e \) as a function of \( \beta \).](image)

The correlators \( \langle J^m J^e Q \rangle / \rho^m \rho^e \) and \( \langle J^m J^e q \rangle / \rho^m \rho^e \) \( (q(y) = Q(y)/|Q(y)|) \) are represented in Fig.1. As one can see from Fig. 1, the product of the electric and the magnetic currents is correlated with the topological charge.
Numerical simulations show that this correlation increases under the cooling. The last fact is in agreement with the results of [9]: the cooled vacuum is populated by instantons which induce the electric charge to the abelian monopoles.

4. CONCLUSION AND ACKNOWLEDGMENTS

Our results show that in the vacuum of lattice gluodynamics the magnetic current is correlated with the electric current. Thus the abelian monopoles have electric charge. The sign of the electric charge depends on the sign of the topological charge density.

M.N.Ch and M.I.P. acknowledge the kind hospitality of the Theoretical Department of the Kanazawa University. F.V.G. is grateful for the kind hospitality of the Theoretical Physics Department of the Vrije University of Amsterdam. This work has been supported by the JSPS Program on Japan – FSU scientists collaboration, and also by the Grants: INTAS-94-0840, INTAS-94-2851, INTAS-RFBR-95-0681, and Grant No. 96-02-17230a of the Russian Foundation for Fundamental Sciences.

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