Electric dipole moments from Yukawa phases in supersymmetric theories

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Abstract

We study quark and electron EDMs generated by Yukawa couplings in supersymmetric models with different gauge groups, using the EDM properties under flavour transformations. In the MSSM (or if soft terms are mediated below the unification scale) the one loop contributions to the neutron EDM are smaller than in previous computations based on numerical methods, although increasing as $\tan^3 \beta$. A neutron EDM close to the experimental limits can be generated in SU(5), if $\tan \beta$ is large, through the $u$-quark EDM $d_u$, proportional to $\tan^4 \beta$. This effect has to be taken into account also in SO(10) with large $\tan \beta$, where $d_u$ is comparable to the $d$ quark EDM, proportional to $\tan \beta$.

1 Introduction

In supersymmetric theories there are various possibilities of generating an intrinsic electric dipole moment (EDM) in microscopic systems.

First of all, the neutron has already a non vanishing EDM in the Standard Model (SM), due to the uneliminable phase in the CKM matrix, or, in other words, to the misalignment between the left eigenstates of the two Yukawa matrices, $\lambda_u$ and $\lambda_d$, and to the charged current gauge interactions between the left-handed quarks, that make the misalignment physically significant.

If further interactions are present, further misalignments and phases can become significant. For example, in the unified extensions of the SM the new gauge interactions make significant the misalignments among particles unified in the same gauge multiplet. Nevertheless, without supersymmetry, due to the decoupling of heavy particles, these effects are suppressed at low energy by powers of the unification mass.

In the supersymmetric extensions of the SM, one more possible source of flavour and CP violation is associated to the possibility that the scalar partners of quarks and leptons are not
degenerate and point in different directions in the flavour space compared to the corresponding fermions. On the other hand, models are often considered in which the soft supersymmetry breaking terms are flavour universal and real at some energy scale $M_0$. This can happen in models in which supersymmetry breaking is communicated to the observable sector by gravity \cite{1, 2} (in this case $M_0 = M_P \equiv$ reduced Planck mass) or when it is communicated by gauge interactions at relatively low energy \cite{3}. These hypotheses allow to keep under control flavour changing neutral current processes and EDMs due to the mixings in the fermion-scalar interactions. In this case, effects coming from the non universal radiative corrections to soft breaking parameters have to be ascribed to the Yukawa couplings.

As said, when the gauge group is unified, some new phases and flavour mixings otherwise inexistent or unphysical can become significant. Unlike what happens without supersymmetry, if its breaking is communicated to the observable sector at a scale where the gauge group is already unified, these further phases and flavour mixings can succeed in contaminating the soft breaking matrices before the heavy particle decoupling make them harmless, giving rise to important effects \cite{4, 5}.

Besides the quark and lepton sector and their scalar partner sector, a CP source can come from the hidden supersymmetry breaking sector of the theory. As a consequence, there can be CP violating effects in the observable sector not associated to a mixing matrix and to a consequent flavour violation. As a matter of fact, even in the flavour universality hypothesis, a CP violation in the hidden sector can make the soft breaking parameters complex since the beginning. In this case, the effects are very large unless the phases are forced to be small.

In this paper, we consider the effects of phases associated to the misalignment of two Yukawa matrices, $\lambda_u$ and $\lambda_d$, in the hypothesis that the soft supersymmetry breaking parameters are generation universal and real at a high energy scale $M_0$. We assume that the Yukawa interactions are present until $M_0$ without worrying about their origin. However the minimal unified models that we consider do not give correct unification predictions for the light fermion masses, showing that, differently from the MSSM case, flavour physics cannot be totally decoupled from unification physics. Furthermore, minimal flavour effects could be comparable to minimal unification effects \cite{6}.

The order of magnitude of EDMs for the quarks $u$, $d$ and for the electron $e$ depends in a crucial way on the gauge structure at the universality scale $M_0$. We will consider therefore different gauge structures, from the minimal one $(SU(3) \times SU(2) \times U(1))$ to that one that unifies all fermions belonging to the same family $(SO(10))$, passing through the minimal unification $(SU(5))$, studying them from the point of view of the properties of EDMs under flavour transformations commuting with the gauge ones. This will allow us to give our estimates for quark and electron EDMs.

We will consider the MSSM in section 2. In Refs. \cite{7} the calculation of the down quark EDM induced by a loop with charginos has been made using numerical methods with sometimes different results. For a given value of $\tan \beta$, $\tan \beta = 10$, Bertolini and Vissani claim in fact that $d_u$ can be as large as $d_u = \mathcal{O}(10^{-39} \text{ e cm})$. Inui et al. obtain $d_u = \mathcal{O}(10^{-29} - 10^{-27} \text{ e cm})$ and Abel et al. get $d_u = \mathcal{O}(10^{-33} - 10^{-29} \text{ e cm})$, all of them indicating a linear dependence on $\tan \beta$. The numerical calculation is used in order to solve the matricial renormalization group equations for the Yukawa couplings and the soft breaking terms; this is necessary for doing a detailed analysis at the Fermi scale. Unfortunately, in the low energy lagrangian, the particular flavour structure of the CKM CP violation, at the basis of the smallness of the EDMs, is hidden in a large number of flavour mixing matrices. On the contrary, by studying the EDMs directly in terms of the parameters at the universality scale, it is easy: i) to estimate the order of magnitude of the effect, that for $\tan \beta = 10$ does not much exceed $10^{-33} \text{ e cm}$, ii) to display the cancellation of contributions not containing the squared light Yukawa couplings, that suppress
the EDMs, iii) to show the raise with \( \tan^3 \beta \) of the \( d \) quark EDM. It is also possible to obtain the relation \( d_d/m_d = -d_s/m_s \) between the one loop dipoles of quarks \( d \) and \( s \), \( d_d \) and \( d_s \) (that both contribute to \( d_n \)).

From our estimates we see that the one loop supersymmetric effects are comparable with the SM ones \([8, 9]\). It turns out that also supersymmetric contributions at higher loops are relevant.

In section 3 we will consider unified theories. The possibility of EDMs for the neutron and the electron close to experimental limits in unified theories has been pointed out and analyzed in Refs. \([5, 10]\) in the case of moderate \( \tan \beta \), where it is associated to the gauge group SO(10). The big enhancement of the neutron EDM in comparison with the MSSM case is due to the fact that, unlike in the MSSM, in a unified theory CP can be violated also when two eigenvalues of the same Yukawa matrix are vanishing \([11]\). Nevertheless this, as other effects \([12]\), happens also in the SU(5) case, but only when the \( u \) quark is concerned. As the third generation Yukawa coupling involved is the bottom one, the effect is proportional to \( \tan^4 \beta \). Due to its possible interest, we also present the results of a numeric computation of \( d_u \). The \( d \) quark contribution, as the electron EDM, turns out to be very small compared with \( d_u \). We conclude section 3 with a short revisitation of SO(10). In appendix, we give the general formulas for the one loop EDMs of up and down quarks and charged leptons in terms of the parameters of the Lagrangian at the Fermi scale.

## 2 EDMs in the MSSM

In this section we will study quark and electron EDMs in the minimal supersymmetric extension of the Standard Model, with real and generation-universal soft terms at some scale \( M_0 \)

\[
A_f = A_{f,0} \mathbf{1}, \quad f = u, d, e \\
m^2_R = m^2_{R,0} \mathbf{1} \quad R = Q, u_R, d_R, L, e_R \quad \text{at} \ M_0.
\]

This model provides the minimal amount of CP violation and, consequently, the minimal contribution to \( d_n \) in a supersymmetric theory with soft breaking terms generated at \( M_0 \).

In the described hypothesis, third generation Yukawa couplings cannot be, by themselves, a source of quark EDMs. In fact, let us suppose that we neglect the four light Yukawa couplings in the quark sector, \( \lambda_u, \lambda_c, \lambda_d, \lambda_s \) everywhere, a part for the mass that must be present in a loop diagram in order to provide the elicity flip. Then, since the hypothesis is scale independent, CP violation disappears and the EDMs have to vanish. In fact, at \( M_P \) the soft terms are universal and real, so that CP violation can only come from Yukawa couplings. But, as there is a couple of degenerate eigenvalues in each Yukawa matrix, all CKM phases can be eliminated using rotations in the 1-2 sector and phase redefinitions of the fields, just as in the Standard Model. More precisely, in the \( d_u \) case we have to use a rotation of \( \text{down} \) left light generations, while in the \( d_d \) case we have to use a rotation of \( \text{up} \) ones. This is because when we consider, for example, the EDM of the quark \( d \), the \( d_L \) mass eigenstate is fixed, and the possibility of rotating the \( d_L \) and \( s_L \) eigenstates is lost. In the limit of vanishing \( \lambda_d \) and \( \lambda_s \), CP violation disappears from the theory, but also the lightest down mass eigenstate becomes not defined.

Thus the light Yukawa couplings play a crucial role in generating the EDMs. Let us now see in which way they intervene. To begin with a simpler case, let us consider the imaginary part of the \( B \) term (defined in appendix) after one loop rescaling from \( M_0 \) to \( M_Z \). This quantity, like the EDMs, vanishes in absence of CP violation and contributes to the EDMs themselves. The \( B \) term does not depend on the basis in the flavour space in which the left doublets \( Q, L \) and the right singlets \( u^c, d^c, e^c \) are written. In other words, if we consider a transformation of the
flavour components of the superfields $\hat{Q}, \hat{u}^c, \hat{d}^c, \hat{L}, \hat{e}^c$ commuting with the gauge group, namely a $U(3)^5 = U(3)_Q \times U(3)_{u^c} \times U(3)_{d^c} \times U(3)_L \times U(3)_{e^c}$ transformation, the Yukawa couplings become

$$\lambda_u \rightarrow U^T_{u^c} \lambda_u U_Q, \quad \lambda_d \rightarrow U^T_{d^c} \lambda_d U_Q, \quad \lambda_e \rightarrow U^T_{e^c} \lambda_e U_L,$$  \hspace{1cm} (1)$$

whereas the $B$ term remains invariant. For what follows, most important are the $U(3)_Q \times U(3)_{u^c} \times U(3)_L$ transformations. Since at $M_0$ all the parameters in the tree level lagrangian except the Yukawa couplings are invariant, it is convenient to consider the $B$ term at the Fermi scale as a function of those parameters. Actually, the relation between high and low energy Yukawa couplings can be inverted, so that the Yukawa couplings can be considered at the Fermi scale.

Owing to the invariance relative to transformations of the right-handed quarks, $B$ depends on the Yukawa couplings only through their squares $\lambda^2_u, \lambda^2_d, \lambda^2_e$. Moreover, if two among the $\lambda_u$ or $\lambda_d$ eigenvalues are equal at $M_0$, the Yukawa couplings can be made real through a transformation (1) so that $\text{Im} \, B$ vanishes. Therefore the RGE corrections to $\text{Im} \, B$ must be proportional to $(\lambda^2_u - \lambda^2_d^0)(\lambda^2_d - \lambda^2_d^0)(\lambda^2_d - \lambda^2_u^0)(\lambda^2_e - \lambda^2_u^0)(\lambda^2_u - \lambda^2_u^0) \approx \lambda^4_u \lambda^2_d \lambda^2_e$. In fact, owing to the invariance relative to a generic $U(3)^5$ transformation, $B$ is a sum of terms like

$$\text{Tr} \left[ (\lambda^4_u \lambda_d) \lambda^m_u \lambda^m_d \cdots (\lambda^4_u \lambda_d) \lambda^{m_r} u \lambda^{m_r} \lambda_d \right] \text{Tr} \left[ (\lambda^4_u \lambda_d) \lambda^k \right] A_{u,d}^0$$  \hspace{1cm} (2)$$

with real adimensional functions of $M_0$ and $A_{u,d}^0$ as coefficients. The first non vanishing supersymmetric contribution to $\text{Im} \, B$ is then proportional to

$$\text{Tr} \left[ (\lambda^4_u \lambda_d)^2 (\lambda^4_u \lambda_d) (A_u^0 - A_d^0) \right] \simeq \lambda^4_u \lambda^2_d \lambda^2_e J_{\text{CP}} (A_u^0 - A_d^0),$$  \hspace{1cm} (3)$$

where $J_{\text{CP}} = \text{Im} (V_{d^L} V_{h^L} V_{u^L} V_{l^L})$. If $A_u^0 = A_d^0$, $\text{Im} \, B$ is furtherly suppressed by lepton Yukawa couplings or by small effects due to the $u_R$-$d_R$ hypercharge difference.

Let us consider now the imaginary part of a flavour non-invariant quantity, more precisely the imaginary part of the matrix element of a quantity $D$ transforming as $\lambda_d$ (or of a quantity $U$ transforming as $\lambda_u$), calculated in correspondence of left and right mass eigenstates. The imaginary parts appearing in the expressions (A2) in appendix for the EDMs and thus the EDM themselves are examples of such quantities. In this case, the light Yukawa coupling suppression is less strong and dependent on the mass eigenstate we consider, up or down, light or heavy.

The general dependence of $D$ on the Yukawa couplings is $D = \lambda_d \cdot f(\lambda_u, \lambda_d, \lambda_e, \text{Tr})$, where $\text{Tr}$ represents traces of $(\lambda^4_u \lambda_d^2)$ and we suppose that $f$ is a real polynomial in the Yukawa couplings, as it is for the one loop EDMs, where the Yukawa couplings only come from vertices or supersymmetric RGE corrections. The imaginary part of the $d_R$-$d_L$ matrix element is then proportional to the $d$ eigenvalue

$$\text{Im} \left[ D_{dRdL} \right] = \lambda_d \text{Im} \left[ f(\lambda^4_u \lambda_d, \lambda^2_u \lambda_d, \lambda^2_e, \text{Tr})_{d_Rd_L} \right].$$ \hspace{1cm} (4)$$

Moreover, in getting the dependence of $f(\ldots)_{d_Rd_L}$ on the light Yukawa couplings, it is no longer possible to consider the limit $\lambda^2_d = \lambda^2_s$ or $\lambda^2_d = \lambda^2_e$ as above for $B$, because in this limit the eigenstate $d_L$, as $d_R$, is no longer defined. Therefore $f(\ldots)_{d_Rd_L}$ has not to be proportional to $\lambda^4_d \lambda^2_s$ but only to $\lambda^4_d$, so that the necessary dependence on the Yukawa couplings is $\text{Im} \left[ D_{dRdL} \right] \propto \lambda^4_d \lambda^2_s \lambda^2_e \lambda^2_e$. The first non vanishing contribution to $\text{Im} \left[ D_{dRdL} \right]$ comes in fact from the term proportional to $\lambda^4_d \lambda^2_s \lambda^2_e (\lambda^2_u \lambda^2_d) (\lambda^2_e)$ in the expression for $D$. Let $a$ be the proportionality coefficient. Since higher order terms are negligible, we have

$$\text{Im} \left[ D_{dRdL} \right] \simeq a \lambda_d \text{Im} \left[ \lambda^4_d \lambda^2_s \lambda^2_e (\lambda^2_u \lambda^2_d) \right]_{d_Rd_L}. \hspace{1cm} (5)$$
Had we considered a matrix $U$ transforming as $\lambda_u$, we would have found
\[
\text{Im}[U_{u\alpha}^\dagger u_i] \simeq b\lambda_u, \quad \text{Im}[(\lambda_0^d\lambda_b)(\lambda_0^d\lambda_a)]_{u\alpha u_i}.
\] (6)
Since the flavour dependence of the one loop EDMs is all contained in the arguments of imaginary parts in (A2), all the down EDMs are expressable in the previous form $d_d = \text{Im}[D_d^u d_d^u]$ with the same $D$, so as for the up ones we have $d_u = \text{Im}[U_{u\alpha}^\dagger u_i]$ with the same $U$. Then there are precise and parameter independent relations between one-loop supersymmetric contributions to EDMs of quarks of different families\footnote{We remind that $\text{Im}(V^\dagger b\bar{b}cV^\dagger d\bar{d})_{11} = c_{ij}[b_{2i}a_{3j} - b_{3i}a_{2j}]J_{CP}$ where $b, c, d$ are diagonal flavour matrices, $b = \text{diag}(b_1, b_2, b_3)$, $b_{ij} \equiv b_i - b_j$, etc.}:
\[
\begin{align*}
  d_d &\simeq +a\lambda_d^4 \lambda_0^2 \lambda_0^2 J_{CP} & d_u &\simeq -b\lambda_u^4 \lambda_0^2 \lambda_0^2 J_{CP} \\
  d_s &\simeq -a\lambda_s^4 \lambda_0^2 \lambda_0^2 J_{CP} & d_c &\simeq +b\lambda_c^4 \lambda_0^2 \lambda_0^2 J_{CP} \\
  d_b &\simeq +a\lambda_b^4 \lambda_0^2 \lambda_0^2 J_{CP} & d_t &\simeq -b\lambda_t^4 \lambda_0^2 \lambda_0^2 J_{CP}
\end{align*}
\] (7a-7c)
so that
\[
\begin{align*}
  d_s &= -\frac{m_u}{m_d} d_d = \frac{m_b}{m_s} d_s \\
  d_c &= -\frac{m_u}{m_u} d_u = \frac{m_t}{m_c} d_t.
\end{align*}
\] (8a-8b)
Eqs. (8) show that the middle generations EDMs are the largest ones and allow to express the neutron EDM in terms of only the $u$ and $d$ quark EDMs. Using eqs. (A1) and (8), and neglecting chromoelectric dipole moment contributions, we get in fact the estimate
\[
d_n = xd_d + yd_u, \quad \text{with} \quad x \approx 4.6 \quad \text{and} \quad y \approx 0.7.
\] (9)
From eqs. (7) it is apparent that $d_d/m_d + d_s/m_s + d_b/m_b$ is much smaller than $d_d/m_d$. This is because the previous combination is invariant under flavour rotations of the quark fields and thus suppressed by both $\lambda_0^2$ and $\lambda_0^2$, just as the RGE induced phase of $B$, electron EDM, strong CP angle and 3-gluon operator [13]. Owing to this large suppression, the $B$ term phase contributes to the neutron EDM in a negligible way.

Eqs. (7) show the strong rise of the EDMs with $\tan \beta$. The large $\tan \beta$ region is therefore by far the most interesting one and it is the one that we will consider in the following. Expressing all fermion masses in terms of the Yukawa couplings, the weak vacuum expectation value $v = 174$ GeV and $\tan \beta$, the only dependence of EDMs on $\tan \beta$ is in the Left-Right (and R-L) blocks of the scalar mass matrices that provide the “electric flip” in the scalar sector. In the large $\tan \beta$ region,
\[
M_{DRL}^2 \simeq -\nu \lambda_u, \quad M_{URL}^2 \simeq -\nu \lambda_u A_u
\] (10)
so that the EDMs depend on $\tan \beta$ only through the down Yukawa couplings: $\lambda_u \propto 1/\cos \beta \simeq \tan \beta$. Hence the down quark EDMs increase with $\tan^3 \beta$, whereas the up quark ones increase with $\tan^5 \beta$.

Let us consider now more closely how one loop graphs can give rise to the flavour structure of eqs. (7). In both the $d_d$ and $d_u$ cases there are two relevant graphs (see appendix) involving either charginos or gluinos. The Yukawa matrices necessary to obtain the flavour structure of eqs. (5) and (6) come from the RGE corrections to the soft breaking parameters and (in the
case of chargino exchange only) from the vertices. Each insertion of a couple of Yukawa coming from RGEs is accompanied by a loop factor and a large logarithm, \( t_Z = (4\pi)^{-2} \ln(M_Z^2/M_\nu^2) \).

In order to provide the 9 Yukawa couplings of eqs. (7), 4 of these insertions are necessary in the gluino diagram, giving a factor \( t_Z^2 \), whereas 3 are enough in the chargino diagram \( (t_Z^2) \).

To estimate the one loop contributions to \( d_d \) and \( d_u \) only the dependence on dimensionful parameters is missing. Because of the necessary presence of a L-R scalar mass insertion as in eq. (10) and of the behavior of the loop functions, it is

\[
\begin{align*}
\frac{d_d^5}{\alpha^2} &\approx \frac{\epsilon}{(4\pi)^2} t_Z^3 \lambda d \lambda^4 \lambda_0^2 \lambda_0^2 \frac{\nu \mu A_v}{\text{max}(m_Z^2, m_\nu^2)} J_{\text{CP}} \approx 10^{-31} e \text{ cm} \left( \frac{\tan \beta}{55} \right)^3 \left( \frac{200 \text{ GeV}}{M_{\text{SUSY}}} \right)^2, \\
\frac{d_u^5}{\alpha^2} &\approx \frac{\epsilon}{(4\pi)^2} t_Z^3 \lambda d \lambda^4 \lambda_0^2 \lambda_0^2 \frac{\nu \mu M_3}{\text{max}(M_Z^2, m_\nu^2)} J_{\text{CP}} \approx 10^{-32} e \text{ cm} \left( \frac{\tan \beta}{55} \right)^3 \left( \frac{200 \text{ GeV}}{M_{\text{SUSY}}} \right)^2.
\end{align*}
\]

(11a)

for the chargino and gluino one loop contribution to \( d_d \) respectively. In this and in the following estimates, we neglect all the numerical coefficients of order one, we choose \( M_0 \approx 10^{16} \text{ GeV} \) and we use central values for the various known parameters, in particular, \( J_{\text{CP}} = 2 \cdot 10^{-5} \). Moreover ‘\( m_{\text{SUSY}} \)’ stands, in each case, for the particular combination of soft parameters written in the analytical approximation. Because of \( d_u/d_d \leq (\tan \beta/55)^3 \) and of \( x/y \approx 7 \) in eq. (9), the corresponding contributions to \( d_u \) are less interesting and totally negligible for moderate \( \tan \beta \).

At a closer inspection, these estimates turn out to be correct within an order of magnitude.

As it is apparent from eq. (11), \( d_u \) cannot be much larger than \( 10^{-33} e \text{ cm} \) for \( \tan \beta = 10 \) and \( 10^{-31} e \text{ cm} \) for whatever value of \( \tan \beta \). We remark that, owing to the necessary presence of \( \lambda_d^2 \) in all results for \( d_d \), a numerical calculation of the \( d \) quark EDM requires the knowledge of the low energy parameters with a precision of about \( 1/10^6 \) in order to see the reciprocal cancellation of the terms not proportional to \( \lambda_d^2 \).

It is at this point interesting to understand why the three-loop pure SM contributions to the neutron EDM, \( d_n^{\text{SM}} \approx 10^{-32} e \text{ cm} \) [8], are not suppressed relative to the one-loop supersymmetric ones.

A large suppression of the pure SM contribution would seem plausible because, in comparison with it, the supersymmetric one loop contributions can be enhanced by

1. large logarithms, \( \log M_0^2/M_Z^2 \), that multiplicate the loop factors \( (4\pi)^{-2} \) in the RGE insertions involved by the one loop supersymmetric diagrams;

2. \( \tan \beta \) factors, that can enhance the \( \lambda_u \) and \( \lambda_e \) couplings;

3. a particularly small effective combination of SUSY parameters, ‘\( m_{\text{SUSY}} \)’.

On the other hand, the SM contributions are enhanced in different ways. In fact, in the pure SM graphs,

i. the elicity flip can occur on an external leg, giving a factor equal to the “constituent quark mass” \( m_q^{\text{const}} \sim m_n/3 \), equal for \( q = \{u, d, s\} \) (so that \( d_d^{\text{SM}} \approx d_d^{2\text{SM}} \));

ii. the Yukawa couplings are expressed in terms of quark masses evaluated at low energy and therefore enhanced by QCD renormalization factors, as the strong coupling;

iii. unlike the RGE-induced contributions, that must depend in a polynomial way on the Yukawa couplings, the infrared structure due to subtractions of quark propagators in the SM graphs, give rise to a mild logarithmic dependence.
Point iii. is particularly important in the case of quantities neutral under the quark $U(3)^3$ flavour group (like the electron EDM, or the phase of the $B$-term, or the strong CP angle, or the 3-gluon operator [13]) that for this reason receive purely supersymmetric contributions much smaller than SM ones. Due to the absolute lack of experimental interest, we avoid discussing such issues in any detail.

Other than one loop supersymmetric and three loop pure SM diagrams, it is also possible to have higher loop contributions, not considered before, having the two kinds of enhancement. An example can be obtained from the chargino one loop diagram by adding by adding a QCD loop, that allows elicity flip on the external lines and gives a contribution to $d_\mu$ comparable with the previously computed one. Using charged Higgs exchange, it is also possible to obtain an interesting three loop contribution to $d_\mu$ given by

$$d_\mu \approx \frac{\mathcal{O}_3}{(4\pi)^5} \frac{m_n m_H^2}{m_{H_\pm}^2} \lambda^2 \lambda_s^2 J_{CP} \approx 10^{-32} \text{e cm} \left(\frac{\tan \beta}{55}\right)^4 \left(\frac{200 \text{GeV}}{m_{H_\pm}}\right)^4.$$  \hspace{1cm} (12)

Fine tuning considerations suggest that, in the large tan $\beta$ region, the charged Higgs are lighter than the squarks [14].

To conclude this section, let us note that the universality scale $M_0$ can be much smaller than the unification scale if the supersymmetry breaking is transmitted at relatively low energy by gauge interactions. In this case all RGE-induced contributions to $d_\mu$ are smaller because of their strong dependence on $t_Z$, whereas the higher loop ones, and in particular (12), are not.

3 EDMs in unified theories

In this section, we suppose that the gauge group is unified at the universality scale $M_0$ that we will identify with the reduced Planck mass. In this case, EDMs can be much larger than in the MSSM. As seen in the previous section, the smallness of EDMs in the MSSM is due to the light Yukawa couplings, that have to be present to prevent the removal of all phases from the lagrangian, phases that at the Planck scale reside only in the Yukawa matrices. In fact, with regard for example to the quark $d$, if $\lambda_u = \lambda_c = 0$ CP violation can be removed with independent redefinitions of the right down quark and of the left doublet. If the gauge group is unified, at the Planck scale the phases still reside only in the Yukawa matrices, but in this case MSSM multiplets belonging to the same representation of the unified group can no longer be rotated independently. Hence, depending on the gauge group and on the quark in consideration, CP violation can persist even if some or all of light Yukawa couplings are vanishing, making not necessary their suppressing presence in the EDM expressions.

Let us consider first minimal SU(5) unification. One generation is composed by two multiplets, a five-plet $\mathbf{F} = (d^c, L)$ and a ten-plet $\mathbf{T} = (u^c, e^c, Q)$, and the Higgs fields belong to two five-plets, $\bar{H}$, that transforms like $\mathbf{F}$, and $H$, that transforms in the conjugate way. In terms of these fields, the Yukawa interactions are

$$\frac{1}{4} T_i \lambda_u T_j H, \quad \sqrt{2} \mathbf{F}_i \lambda_d V_{ij}^d T_j \bar{H}$$  \hspace{1cm} (13)

in a basis in which $\lambda_u$ is diagonal. Below the unification scale, $\lambda_u$ and $\lambda_d$ are the same of the previous section.

Five of the six phases of the CKM matrix can be removed in the MSSM through independent redefinitions of the $Q_i$, $u_i^c$, $d_i^c$. Here, it is easy to show that only three phases can be removed by independent transformations of $T_i$, $\mathbf{F}_i$, so there are two more physical phases compared to the MSSM (and the SM). Whereas two phases decouple below the unification scale in non supersymmetric models, in supersymmetric models they leave their effects in soft
breaking parameters, that lose their universality and reality, before decoupling could make them harmless.

Is the effect of these two further phases on the EDMs suppressed by light Yukawa couplings in this model? The answer is no for the quark $u$ and yes, but in a different way compared to the MSSM, for down quarks and charged leptons. In order to show that, let us suppose first that the two light $\lambda_u$ eigenvalues, $\lambda_u$ and $\lambda_s$, are vanishing. In this case, all phases in the Yukawa matrices become unphysical. In fact, the first of the two interactions (13) becomes $T_3\lambda_rT_3H/4$ and it is invariant for transformations in the $T_1$-$T_2$ sector. Moreover, as it is well known, it is possible to write the most general unitary matrix $V^\dagger$ as

$$V^\dagger = \text{diag}(e^{i\alpha}, e^{i\beta}, e^{i\gamma}) \left( \begin{array}{cc} R_1 & 0 \\ 0 & 1 \end{array} \right) \left( \begin{array}{cc} e^{i\delta} & 0 \\ 0 & R_2 \end{array} \right) \left( \begin{array}{c} R_3 \\ 0 \end{array} \right) \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \text{diag}(e^{i\alpha'}, e^{i\beta'}, 1),$$

(14)

where $R_1, R_2, R_3$ are orthogonal $2 \times 2$ matrices. Then we can reabsorb $e^{i\alpha}, e^{i\beta}, e^{i\gamma}, e^{i\alpha'}, e^{i\beta'}, e^{i\gamma'}$ by a redefinition of $T_1, T_2, T_1, T_2$, reach the phase $e^{i\delta}$ by defining $(T_1^1, T_2^j)^T = R_3(T_1, T_2)^T$ and reabsorb also $e^{i\delta}$ by a redefinition of $T_1^i$, without affecting the diagonal interaction. Since, as seen in the previous section, the limit $\lambda_u = \lambda_s = 0$ is meaningful for the down quarks and for the charged leptons (and for the top), this means that their EDMs are suppressed by light up Yukawa couplings in some way.

Let us suppose now that the two light $\lambda_u$ eigenvalues, $\lambda_d$ and $\lambda_s$, are vanishing. In this case, two phases remain physical. In fact, the interactions (13) become

$$\frac{1}{4} T_1 \lambda_u T_1 H, \quad \sqrt{2} T_3 \lambda_0 V_{33}^\dagger T_3 H.$$

With a redefinition of $F_3$, one of the three phases of $V_{31}^\dagger, V_{32}^\dagger, V_{33}^\dagger$ can be reabsorbed, but not the remaining two because, if they were reabsorbed by a redefinition of some $T_i$, they would reappear in the diagonal interaction. The two remaining phases are just those ones related with unification. Since the limit $\lambda_d = \lambda_s = 0$ is meaningful for up quarks (and the bottom and the tau), their EDMs are not necessarily suppressed by light Yukawa couplings.

The dependence of EDMs on Yukawa couplings can be obtained as in the MSSM. Changing basis in the flavour space for the SU(5) supermultiplets $\hat{T}$, $\hat{F}$ corresponds to make a $U(3)_T \times U(3)_{\bar{T}}$ transformation on the flavour component of the fields, relative to which the Yukawa couplings transform in this way:

$$\lambda_u \rightarrow U_T^T \lambda_u U_T, \quad \lambda_0 \rightarrow U_{\bar{T}}^T \lambda_0 U_{\bar{T}}.$$

(15)

As before, the down quark EDMs are given by $d_d = \text{Im} [D_d d^*_d]$, where the matrix $D$ transforms as $\lambda_0$. Unlike the case of the MSSM, this does not mean that $D = \lambda_0 f(\lambda_0^\dagger, \lambda_u, \lambda_s^*).$ In this case we have rather

$$D = \lambda_0 f((\lambda_0^\dagger \lambda_u), (\lambda_0^\dagger \lambda_0)^*, \lambda_u, \lambda_s^*),$$

(16)

where $f$ depends on the arguments in such a way that $f \rightarrow U_T^fU_T$ for a transformation (15). From eq. (16) it follows that $d_d$ is proportional to $\lambda_d$ as in the MSSM. Moreover, $f$ must depend on down Yukawa couplings through their squares but it can depend on individual up Yukawa couplings also through not squared couplings, provided that the total number of $\lambda_0$ is even. This can be seen by noting that the particular flavour transformation $T \rightarrow iT, \hat{T} \rightarrow \hat{T}$ leaves $\lambda_0^\dagger \lambda_0$ and $f$ unchanged but it changes sign to $\lambda_0$. Therefore, unlike the MSSM, a $\lambda_c$ suppression
can be enough for the quark $d$. Indeed, the first non vanishing contribution to $d_u$ is proportional to
\[ \text{Im}[(\lambda_u^a \lambda_v^b (\lambda_d^a \lambda_d^b)^*)_{d_b d_a}]=\lambda_d \lambda_u^3 \lambda_v^3 \lambda_c \text{Im}[V_{dc} V_{cb} V_{bd} V_{ub}] . \]

The EDMs of the quark $s$ and of the electron are suppressed in an analogous way.

The up quark EDMs behave in a different way. They are given by $d_u = \text{Im}[U_{uR}^* U_{uL}]$ where the matrix $U$ transforms as $\lambda_u$ under the transformation (15). Unlike the MSSM, $U$ is not necessarily proportional to $\lambda_u$, on the left, but can be also proportional to $\lambda_u^T$, that transforms in the same way on the left. Therefore, $d_u$ is not necessarily proportional to $\lambda_u$, giving rise to a possible enhancement. In any case, $d_u$ must contain an odd number of up Yukawa matrices, since the transformation $T \rightarrow iT$, $\overline{F} \rightarrow \overline{F}$ leaves $\lambda_u$ unchanged but it changes sign to $\lambda_u$ and $U$. The first non vanishing contribution to $d_u$ is proportional to
\[ \text{Im}[(\lambda_u^a \lambda_u^b (\lambda_d^a \lambda_d^b)^*)_{d_b d_a}]=\lambda_d \lambda_u^4 \text{Im}[V_{ub} V_{cb} V_{bu}] . \]

and it exhibits the described features. In eqs. (17,18) we omitted the renormalization factors for the $V_{CKM}$ matrix elements.

In SU(5) we have therefore $d_u \gg d_d, d_s, d_e$. Only the gluino diagram is able to give rise to the $\lambda_t$ enhancement. In fact, the chargino diagram is explicitly proportional to the $\lambda_u$ that appears in one of its vertices and it is negligible in this model. Let us concentrate then on the gluino contribution to the quark $u$, $d_u$. The flavour structure (18) is generated by the corrections to the mass matrices. At the unification scale, these corrections are proportional to combinations of Yukawa matrices transforming as the mass matrices themselves relative to the $U(3)_T \times U(3)_F$ group. Hence the corrections to the ten-plet mass matrix, and so those to $m_Q^2$, $m_{uR}^2$, $m_{eR}^2$, are proportional to $1$, $\lambda_u^a \lambda_u^b$, $\lambda_d^a \lambda_d^b$, etc., whereas the corrections to the five-plet mass matrix, and so those to $m_Q^2$ and $m_{dR}^2$, are proportional to $1$, $(\lambda_u^a \lambda_u^b)^*$, etc.. All these corrections are also proportional to $t_G \equiv (4\pi)^{-2} \log(M_3^2/M_Z^2) \approx 0.06$. Some of them are characteristic of unification, whereas other ones are produced also below the unification scale and hence at the Fermi scale they are proportional to $t_Z \approx (4\pi)^{-2} \log(M_Z^2/M_Z^2) \approx 0.5$. Including also some approximate numerical factors, the corrections $\Delta m^2$ at the Fermi scale are
\[ \Delta m^2_Q \propto 1 - 3t_Z \lambda_u^a \lambda_u^b - 3t_Z \lambda_d^a \lambda_d^b \]
\[ \Delta m^2_{uR} \propto 1 - 6t_Z \lambda_u^a \lambda_u^b - 6t_G (\lambda_u^a \lambda_u^b)^* \]
\[ \Delta m^2_{dR} \propto 1 - 6t_Z \lambda_u^a \lambda_u^b \]

These corrections are able to generate the flavour dependence of eq. (18), so that we can estimate $d_u$ as
\[ |d_u| \approx \frac{\alpha_3}{4\pi} \lambda_u^3 \text{Im}[V_{ub} V_{cb} V_{bu}] t_Z t_G \frac{m_A u}{m_0} \frac{M_3}{\text{max}(m_{dR}^2, M_3^2)} \]
\[ \approx 10^{-(25:26)} \left( \frac{\tan \beta}{50} \right)^4 \left( \text{Im}[V_{ub} V_{cb}] \right) \left( \frac{500 \text{GeV}}{M_{\text{Susy}}} \right)^2 e \text{ cm} , \]

where $M_{\text{Susy}}$ in the second line is the combination of supersymmetric parameters appearing in the first line and it can be even larger than 500 GeV because of the likely lightness of $A$ in the large $\tan \beta$ regime. Since the prediction for $d_u$ is interesting enough, we use the expressions (A2) to do an exact computation of $d_u$ that we show in figure 1 for $M_3 = 500 \text{ GeV}$, for three values of $\tan \beta$, 2, 10 and 50, and for $|\text{Im}[V_{ub} V_{cb}]| = 10^{-5}$. $d_u$ is plotted as a function of $M_3/m_{dR}^2$ in order to exhibit the so-called “gluino focussing” effect [10].
Electric dipole of the neutron in $e \cdot cm$ $M_3/\mu_3$ with $M_3 = 500$ GeV

Figure 1: The contribution to the neutron EDM generated by minimal $SU(5)$ for $M_3 = 500$ GeV, $\tan \beta = 2, 10, 50$ and $|\text{Im}[V_{tb}^2 V_{ub}^2]| = 10^{-5}$, as a function of the ratio between gaugino and scalar masses and for random samples of acceptable sparticle spectra.

From eq. (20) and Fig. 1 and from the experimental limit $d_u < 0.8 \cdot 10^{-25} e \cdot cm$ [18] it is apparent that the $u$ quark EDM can be very large in superunified theories with large $\tan \beta$.

Let us consider now shortly the gauge group SO(10), whose sixteen dimensional representation “16” unifies all quarks and leptons of one generation, included a right-handed neutrino. In the minimal model the two light Higgs doublets belong to two different ten dimensional SO(10) multiplets, $10_u$ and $10_d$, and the Yukawa interactions are

$$16^T \lambda_0 16 10_u, \quad 16^T \lambda_0 16 10_d. \tag{21}$$

In this model the situation is different from the SU(5) case, in which the five-plets and ten-plets could be rotated independently. All six phases of $V$ are physically significant. Moreover, it is easy to see that in this case it is not possible to remove the CKM phases neither when $\lambda_d = \lambda_s = 0$ (like in SU(5)) nor when $\lambda_u = \lambda_c = 0$ (unlike in SU(5)). Therefore neither $d_u$ nor $d_d$ are suppressed by light Yukawa couplings.

The $u$ quark EDM is generated in the same way it is generated in SU(5) and its estimate is identical. On the other hand, whereas in SU(5) only $m_{u_R}^2$ (and not $m_{d_R}^2$) has non MSSM corrections, in SO(10) the situation is symmetric under $u$-$d$ exchange, so that $m_{d_R}^2$ has corrections depending on $\lambda_0$ as the $m_{u_R}^2$ ones depend on $\lambda_0$. Moreover, also an approximate expression for $d_d$ can be obtained from that one for $d_u$ exchanging $u$ and $d$. $d_d$ has therefore a linear dependence on $\tan \beta$. Moreover, when $\tan \beta$ is large and $\lambda_b \simeq \lambda_t$ the main differences between $d_u$ and $d_d$ are due to the different combination of CKM angles and phases, Im$[V_{td}^2 V_{ub}^2]$ instead of Im$[V_{tb}^2 V_{ub}^2]$, and to the different RL flip, that is provided by $\mu$ and not $A_u$. With similar phases, the CKM angles could favour $d_d$. Nevertheless, also in SO(10) with large $\tan \beta$ the $u$ quark contribution cannot be forgotten.

It is also interesting to estimate the corrections to the quark masses that, in SU(5) and in SO(10), contribute to the strong CP violating angle $\theta$ in models without an axion. These
Table 1: Flavour factors suppressing the purely supersymmetric contributions to light fermion EDMs in different minimal models.

|       | MSSM         | minimal SU(5)                                      | minimal SO(10)                                      |
|-------|--------------|---------------------------------------------------|---------------------------------------------------|
| $d_u$ | $\lambda_d \lambda_d^2 \lambda^2_{\lambda} J_{\text{CP}}$ | $\lambda_d \lambda_d^2 \lambda^2_{\lambda} \text{Im}[V_{ub}^2 V_{tb}^2]$ | $\lambda_d \lambda_d^2 \lambda^2_{\lambda} \text{Im}[V_{ub}^2 V_{tb}^2]$ |
| $d_d$ | $\lambda_d \lambda_d^2 \lambda^2_{\lambda} J_{\text{CP}}$ | $\lambda_d \lambda_d^2 \lambda^2_{\lambda} \text{Im}[V_{cd}^2 V_{tb} V_{td}]$ | $\lambda_d \lambda_d^2 \lambda^2_{\lambda} \text{Im}[V_{cd}^2 V_{tb} V_{td}]$ |
| $d_e$ | $\lambda_e \lambda_e^2 \lambda^2_{\lambda} J_{\text{CP}}$ | $\lambda_e \lambda_e^2 \lambda^2_{\lambda} \text{Im}[V_{cd}^2 V_{tb} V_{td}]$ | $\lambda_e \lambda_e^2 \lambda^2_{\lambda} \text{Im}[V_{cd}^2 V_{tb} V_{td}]$ |

Contributions are

$$\delta \theta_{QCD}^{\mu E} \approx \frac{\alpha_3}{4 \pi} \frac{m_{\ell}}{m_u} \lambda_5^4 \text{Im} \left[ V_{ub}^2 V_{tb}^2 \right] t Z G \approx 10^{-11} \tan^4 \beta \left( \frac{\text{Im} \left[ V_{ub}^2 V_{tb}^2 \right]}{10^{-5}} \right)$$ (22)

both in SU(5) and SO(10) and

$$\delta \theta_{QCD}^{\mu E} \approx \frac{\alpha_3}{4 \pi} \frac{m_{\ell}}{m_d} \lambda_5^4 \text{Im} \left[ V_{td}^2 V_{tb}^2 \right] t Z G \frac{\mu M_3}{m_u^2} \approx 10^{-5} \tan \beta \left( \frac{\text{Im} \left[ V_{td}^2 V_{tb}^2 \right]}{10^{-4}} \right)$$ (23)

in SO(10). The bound $\theta_{QCD} \leq 10^{-9}$ shows that an axion is necessary in SO(10) and also in SU(5) if $\tan \beta$ is large. It is interesting that the large amount of CP violation left by SO(10) unification in the soft terms furnishes an experimental possibility to see the ‘invisible’ axion [19].

4 Conclusions

We have studied EDMs produced by the misalignment of two Yukawa matrices in models with universality of soft breaking terms at an high energy scale from the point of view of transformation properties under flavour transformation commuting with the gauge group. In this way it is simple to get the dependence of EDMs from Yukawa couplings, summarized in table 1. On this basis, estimates are possible and effective for small and large effects.

In the case of MSSM, owing to light Yukawa coupling suppression, the effects given by one loop diagrams are largely below present experimental limits (see eq. (11)). Therefore, the detection of a non-vanishing EDM for the neutron in foreseeable experiments would be an unequivocal signal of physics beyond the MSSM. Some interesting contribution to $d_n$ can arise also from more loop diagrams.

In the case of minimal supersymmetric SU(5) and large $\tan \beta$, on the contrary, the effects can be close to experimental limits with regards to neutron EDM because of the contribution of $u$ quark, proportional to $\tan^4 \beta$. For this interesting case the results of an exact computation are given by Fig 1. The smallness of $d$ quark and electron EDMs compared with the $u$ quark one is also characteristic of SU(5). Therefore SU(5) can be distinguished from the MSSM because it can give rise to a measurable EDM for the neutron. Moreover, SU(5) effects can be distinguished from the SO(10) ones and from effects coming from universal intrinsic phases in $A$ and $B$ terms [20] because of the smallness of the electron EDM. On the other hand, at least for the EDMs a similar pattern can be given by strong CP violation.

In the case of minimal supersymmetric SO(10) it is well known that the $d$ quark and electron EDMs can be close to the experimental limits. If $\tan \beta$ is large, the effect is enhanced by a factor $\mu \tan \beta / A_d$. Also in this case, as in SU(5), the $u$ quark gives an important contribution that must be taken in account.

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Appendix: general formulas for one loop EDMs

It is possible to express the neutron EDM $d_n$ in terms of the contribution of the quark $q$ to the neutron spin $(\Delta q)_n$ and of its EDM $d_q$ [21]:

$$d_n = \zeta(\Delta d)_n d_d + \zeta(\Delta u)_n d_u + \zeta(\Delta s)_n d_s,$$

(A1)

where $(\Delta d)_n = 0.82 \pm 0.03$, $(\Delta u)_n = -0.44 \pm 0.03$ and $(\Delta s)_n = -0.11 \pm 0.03$ [21] and $\zeta$ is a renormalization factor, $\zeta \approx 1.6$.

Concerning the quark EDMs, let us consider a supersymmetric extension of Standard Model

where $t_3(a)$ and $y(a)$ are respectively the third component of weak isospin and the hypercharge of the particle $a$ (normalized in such a way that $q = t_3 + y$). $\tilde{g}$ is the gluino and $M_3$ its mass, $N_n$, $n = 1 \ldots 4$ are the neutralinos, $\chi_3^+, \chi_3^-, n = 1 \ldots 2$ the charginos and $H^+$, $H^-$ their mixing matrices. $M_3^U$, $M_3^D$ are the $6 \times 6$ squarks mass matrices in the mass eigenstate basis of corresponding quarks

$$M_3^U = \begin{pmatrix} m_Q^{u(3)} + M_u^2 & - (A_u + \mu \cot \beta \lambda)M_u & (\lambda_u^R)^* M_2^u \\ M_u^2 & M_u^2 & D_{dL}^u \\ \lambda_u^R & D_{dL}^u & 1 \end{pmatrix}$$

$$M_3^D = \begin{pmatrix} m_Q^{d(3)} + M_u^2 & - (A_d + \mu \tan \beta \lambda)M_u & (\lambda_d^R)^* M_2^d \\ M_u^2 & M_u^2 & D_{dL}^d \\ \lambda_d^R & D_{dL}^d & 1 \end{pmatrix}.$$  

(A3)
In the right sides of (A2) we used a matrix notation. The $f$s are the appropriate loop functions, namely linear combinations $q_s g_2 + q_f h_2$, where $q_s$ and $q_f$ are the electric charges of the particles running respectively in the scalar and fermion line of the corresponding diagram (in unity of $e$), and
\[
g_2(r) = \frac{1}{2(r - 1)^3} [r^2 - 2r \log r - 1], \quad h_2(r) = \frac{1}{2(r - 1)^3} [-2r^2 \log r + 3r^2 - 4r + 1].
\]

In eqs. (A2) the first two contributions to EDMs are due to charged higgsinos and gluinos exchange and they are the dominant ones. The third ones take account of bino and neutral wino exchange and they are less important than the corresponding gluino exchange, while the last ones come from neutral higgsino exchanges and they are completely negligible. All the contributions in eqs. (A2) come from diagrams with none or two elicity flips on the vertices. The one loop diagrams in which the elicity flip occurs on an external leg are always real. As such they do not give any contribution. The diagrams with one elicity flip on the vertices would contribute only if the $B$ term were complex. In this case in eqs. (A2) also the mixing matrices $H$ would be complex and they would appear within the imaginary parts.

The arguments of imaginary parts contain both the model dependence and the difficulty of the calculation of EDMs.

References

[1] R. Barbieri, S. Ferrara, and C. A. Savoy, Phys. Lett. 119B, 343 (1982);
R. Arnowitt, P. Nath, and A. Chamseddine, Phys. Rev. Lett. 49, 970 (1982);
L. J. Hall, J. Lykken, and S. Weinberg, Phys. Rev. D27, 2359 (1983).

[2] A. Brignole, L.E. Ibáñez and C. Muñoz, Nucl. Phys. B422, 125 (1994)

[3] M. Dine, W. Fischler and M. Srednicki, Nucl. Phys. B189, 575 (1981); S. Dimopoulos and S. Raby, Nucl. Phys. B192, 353 (1981); M. Dine and W. Fischler, Phys. Lett. B110, 227 (1982); M. Dine and M. Srednicki, Nucl. Phys. B202, 238 (1982); M. Dine and W. Fischler, Nucl. Phys. B204, 346 (1982); L. Alvarez-Gaumé, M. Claudson, and M. B. Wise, Nucl. Phys. B207, 96 (1982).

[4] R. Barbieri and L. J. Hall, Phys. Lett. B338, 212 (1994).

[5] S. Dimopoulos and L. J. Hall, Phys. Lett. B344, 185 (1995).

[6] M. E. Gomez and H. Goldberg, Phys. Rev. D53, 5244 (1996).

[7] S. Bertolini and F. Vissani, Phys. Lett. B324, 164 (1994);
T. Inui, Y. Mimura, N. Sakai, and T. Sasaki, Nucl. Phys. B449, 49 (1995);
S. A. Abel, W. N. Cottingham, and I. B. Whittingham, Phys. Lett. B370, 106 (1996).

[8] For a review see E.P. Shabalin, Sov. Phys. Usp. 26 (1983) 297 and references therein.

[9] B.F. Morel, Nucl. Phys. B157, 23 (1979); D.V. Nanopoulos, A. Yildis and P.H. Cox, Phys. Lett. 87B, 53 (1979); M.B. Gavela et al., Phys. Lett. B109, 215 (1982) and Z. Phys. C23 (1984) 251; I.B. Khriplovich and A.R. Zhitnitsky, Phys. Lett. B109, 490 (1982); J.P. Eeg and I. Picek, Phys. Lett. B130, 308 (1983); B. McKellar, S.R. Choudhury, X-G He and S. Pakvasa, Phys. Lett. 197, 556 (1987); J. Bijnens and E. Pallant, hep-ph/9606285.
[10] R. Barbieri, L. J. Hall, and A. Strumia, Nucl. Phys. B445, 219 (1995) and Nucl. Phys. B449, 437 (1995).

[11] A. Romanino, to be published in the proceedings of 24th ITEP Winter School of Physics, Moscow, Russia, 20-28 Feb 1996.

[12] N. Arkani-Hamed, H. Cheng, and L. J. Hall, Phys. Rev. D53, 413 (1996).

[13] S. Weinberg, Phys. Rev. Lett. 63, 2333 (1989).

[14] R. Rattazzi and U. Sarid, Phys. Rev. D53, 1553 (1996);
    A. Romanino, P. Ciafaloni and A. Strumia, Nucl. Phys. B458, 3 (1996).

[15] J. Ellis and M.K. Gaillard, Nucl. Phys. B150, 141 (1979).

[16] M. J. Dugan, B. Grinstein, and L. J. Hall, Nucl. Phys. B255, 413 (1985).

[17] V. Baluni, Phys. Rev. D19, 2227 (1979);
    R. Crewther, P. di Vecchia, G. Veneziano and E. Witten, Phys. Lett. B89, 23 (1979).

[18] I. S. Altarev et al., Phys. Lett. B276, 242 (1992).

[19] R. Barbieri, A. Romanino, and A. Strumia, hep-ph/9605368, to be published in
    Phys. Lett. B.

[20] R. Barbieri, A. Romanino, and A. Strumia, Phys. Lett. B369, 283 (1996).

[21] J. Ellis and R. A. Flores, Phys. Lett. B377, 83 (1996);
    J. Ellis and M. Karliner, Talk given at Ettore Majorana International School of Nucleon Structure: 1st Course: The Spin Structure of the Nucleon, Erice, Italy, 3-10 Aug 1995.