The Structure of Light Nuclei and Its Effect on Precise Atomic Measurements

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Abstract: My talk will consist of three parts: (a) what every atomic physicist needs to know about the physics of light nuclei [and no more]; (b) what nuclear physicists can do for atomic physics; (c) what atomic physicists can do for nuclear physics. A brief qualitative overview of the nuclear force and calculational techniques for light nuclei will be presented, with an emphasis on debunking myths and on recent progress in the field. Nuclear quantities that affect precise atomic measurements will be discussed, together with their current theoretical and experimental status. The final topic will be a discussion of those atomic measurements that would be useful to nuclear physics.

Introduction

"...numerical precision is the very soul of science....."

This quote[1] from Sir D'Arcy Wentworth Thompson, considered by many to be the first biomathematician, could well serve as the motto of this conference, since precision is the raison d'ˆetre of this meeting and this field. I have always been in awe of the number of digits of accuracy achievable by atomic physics in the analysis of simple atomic systems[2]. Nuclear physics, which is my primary field and interest, must usually struggle to achieve three digits of numerical significance, a level that atomic physics would consider a poor initial effort, much less a decent final result.

The reason for the differing levels of accuracy is well known: the theory of atoms is QED, which allows one to calculate properties of few-electron systems to many significant figures[3]. On the other hand, no aspect of nuclear physics is known to that precision. For example, a significant part of the “fundamental” nuclear force between two nucleons must be determined phenomenologically by utilizing experimental information from nucleon-nucleon scattering[4], very little of which is known to better than 1%. In contrast to that level of precision, energy-level spacings in few-electron atoms can be measured so precisely that nuclear properties influence significant digits in those energies[5]. Thus these experiments can be interpreted as either a measurement of those nuclear properties, or corrections must be applied to eliminate the nuclear effects so that the resulting measurement tests or measures non-nuclear properties. That is the purview of my talk.

The single most difficult aspect of a calculation for any theorist is assigning uncertainties to the results. This is not always necessary, but in calculating nuclear corrections to atomic properties it is essential to make an effort. That is just another way to answer the question,“What confidence do we have in our results?” Because it is important for you to be able to judge nuclear results to some degree, I have slanted my talk towards answers to two questions that should be asked by every atomic physicist. The first is: “What confidence should I have in the values of nuclear quantities that are required to

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analyze precise atomic experiments?” The second question is: “What confidence should I have that the nuclear output of my experiment will be put to good use by nuclear physicists?”

Myths of Nuclear Physics

Every field has a collection of myths, most of them being at least partially true at one time. Myths propagate in time and distort the reality of the present. I have collected a number of these, some of which I believed in the past. The resolution of these “beliefs” also serves as a counterpoint to the very substantial progress made in light-nuclear physics in the past 15 years, which continues unabated.

My myth collection includes:

- The strong interactions (and consequently the nuclear force) aren’t well understood, and nuclear calculations are therefore unreliable.
- Large strong-interaction coupling constants mean that perturbation theory doesn’t converge, implying that there are no controlled expansions in nuclear physics.
- The nuclear force has no fundamental basis, implying that calculations are not trustworthy.
- You cannot solve the Schrödinger equation accurately because of the complexity of the nuclear force.
- Nuclear physics requires a relativistic treatment, rendering a difficult problem nearly intractable.

All of these myths had some (even considerable) truth in the past, but today they are significant distortions of our current level of knowledge.

The Nuclear Force

Most of the recent progress in understanding the nuclear force is based on a symmetry of QCD, which is believed to be the underlying theory of the strong interactions (or an excellent approximation to it). It is generally the case that our understanding of any branch of physics is based on a framework of symmetry principles. QCD has “natural” degrees of freedom (quarks and gluons) in terms of which the theory has a simple representation. The (strong) chiral symmetry of QCD results when the quark masses vanish, and is a more complicated analogue of the chiral symmetry that results in QED when the electron mass vanishes. The latter symmetry explains, for example, why (massless or high-energy) electron scattering from a spherical (i.e., spinless) nucleus vanishes in the backward direction.

The problem with this attractive picture is that it does not involve the degrees of freedom most relevant to experiments in nuclear physics: nucleons and pions. Nevertheless, it is possible to “map” QCD (expressed in terms of quarks and gluons) into an “equivalent” or surrogate theory expressed in terms of nucleon and pion degrees of freedom. This surrogate works effectively only at low energy. The small-quark-mass symmetry limit becomes a small-pion-mass symmetry limit. In general this (slightly) broken-symmetry theory has $m_{\pi}c^2 \ll \Lambda$, where the pion mass is $m_{\pi}c^2 \cong 140$ MeV and $\Lambda \sim 1$ GeV is the mass scale of QCD bound states (heavy mesons, nucleon resonances, etc.). The seminal work on this surrogate theory, now called chiral perturbation theory (or χPT), was performed by Steve Weinberg[6], and many applications to nuclear physics were pioneered by his student, Bira van Kolck[7]. From my perspective they demonstrated two things that made an immediate impact on my understanding of nuclear physics[8]: (1) There is an alternative to perturbation theory in coupling constants, called “power counting,” that converges geometrically like $(Q/\Lambda)^N$, where $Q \sim m_{\pi}c^2$ is a relevant nuclear energy scale, and the exponent $N$ is constrained to have $N \geq 0$; (2) nuclear physics mechanisms are severely constrained by the chiral symmetry. These results provide nuclear physics with a well-founded rationale for calculation.

This scheme divides the nuclear-force regime in a natural way into a long-range part (which implies a low energy, $Q$, for the nucleons) and a short-range part (corresponding to high energy, $Q$, between nucleons). Since χPT only works at low energies, we expect that only the long-range part of the nuclear
force can be treated by utilizing the pion degrees of freedom. We need to resort to phenomenology (i.e., nucleon-nucleon scattering data) to treat systematically the short-range part of the interaction.

The long-range nuclear force is calculated in much the same way that atomic physics calculates the interactions in an atom using QED. The dominant interaction is the exchange of a single pion, and is denoted $V_\pi$. Its atomic analogue is one-photon exchange (containing the dominant Coulomb force). Because it is such an important part of the nuclear potential, it is fair to call $V_\pi$ the “Coulomb force” of nuclear physics. Smaller contributions arise from two-pion exchange (the analogue of two-photon exchange). There is even an analogue of the atomic polarization force, where two electrons simultaneously polarize their nucleus using their electric fields. The nuclear analogue involves three nucleons simultaneously, and is called a three-nucleon force [9]. Although relatively weak compared to $V_\pi$ (a few percent), it plays an important role in fine-tuning nuclear energy levels. The final ingredient is an important short-range interaction (which must be determined by phenomenology) that has no direct analogue in the physics of light atoms.

What are the consequences of exchanging a pion rather than a photon? The pseudoscalar nature of the pion mandates a spin-dependent coupling to a nucleon, and this leads to a dominant tensor force between two nucleons. Except for its radial dependence, the form of $V_\pi$ mimics the interaction between two magnetic dipoles, as seen in the Breit interaction, for example. Thus we have in nuclear physics a situation that is the converse of the atomic case: a dominant tensor force and a smaller central force. In order to grasp the difficulties that nuclear physicists face, imagine that you are an atomic physicist in a universe where magnetic (not electric) forces are dominant, and where QED can be solved only for long-range forces and you must resort to phenomenology to generate the short-range part of the force between electrons and nuclei.

Although this may sound hopeless, it is merely difficult. The key to handling complexities is adequate computing power, and that became routinely available only in the late 1980s or early 1990s. Since then there has been explosive development in our understanding of light nuclei. Underlying all of these developments is an improved understanding of the nuclear force. I like to divide the history of the nuclear force into three distinct time periods.

**First-generation** nuclear forces were developed prior to 1993. They all contained the one-pion exchange force, but everything else was relatively crude. The fits to the nucleon-nucleon scattering data (needed to parameterize the short-range part of the nuclear force) were indifferent.

**Second-generation** forces were developed in 1993 and in the years following [4]. They were more sophisticated and generally very well fit to the scattering data. As an example of how well the fitting worked, the Nijmegen group (which pioneered this sophisticated procedure) allowed the pion mass to vary in the Yukawa function defining $V_\pi$, and then fit that mass. They also allowed different masses for the neutral and charged pions that were being exchanged and found [10]

\[
m_{\pi^\pm} = 139.4(10) \text{ MeV},
\]

\[
m_{\pi^0} = 135.6(13) \text{ MeV},
\]

both results agreeing with free pion masses ($m_{\pi^\pm} = 139.57018(35) \text{ MeV}$ and $m_{\pi^0} = 134.9766(6) \text{ MeV}$ [11]). It is both heartening and a bit amazing that the masses of the pions can be determined to better than 1% using data taken in reactions that have no free pions! This result is the best quantitative proof of the importance of pion degrees of freedom in nuclear physics.

**Third-generation** nuclear forces are currently under development. These forces are quite sophisticated and incorporate two-pion exchange, as well as $V_\pi$. All of the pion-exchange forces (including three-nucleon forces) are being generated in accordance with the rules of chiral perturbation theory. One also expects excellent fits to the scattering data. This is very much work in progress, but preliminary calculations and versions have already appeared [12].
Calculations of Light Nuclei

Having a nuclear force is not very useful unless one can calculate nuclear properties with it. Such calculations are quite difficult. Until the middle 1980s only the two-nucleon problem had been solved with numerical errors smaller than 1%. At that time the three-nucleon systems $^3$H and $^3$He were accurately calculated using a variety of first-generation nuclear-force models[13]. Soon thereafter the $\alpha$-particle ($^4$He) was calculated by my colleague, Joe Carlson, who pioneered a technique that has revolutionized our understanding of light nuclei: Green’s Function Monte Carlo (GFMC)[14].

The difficulty in solving the Schrödinger equation for nuclei is easily understood, although it was not initially obvious to me. Nuclei are best described in terms of nucleon degrees of freedom. Nucleons come in two types, protons and neutrons, which can be considered as the up and down components of an “isospin” degree of freedom. If one also includes its spin, a single nucleon has four internal degrees of freedom. Two nucleons consequently have 16 internal degrees of freedom, which is roughly the number of components in the nucleon-nucleon force (coupling spin, isospin and orbital motion in a very complicated way). To handle this complexity one again requires fast computers, and that is a fairly recent development.

The GFMC technique has been used by Joe Carlson and his collaborators to solve for all of the bound (and some unbound) states of nuclei with up to 10 nucleons. One of Joe’s collaborators (Steve Pieper[17]) calculated that the ten-nucleon Schrödinger equation requires the solution of more than 200,000 coupled second-order partial-differential equations in 27 continuous variables, and this can be accomplished with numerical errors smaller than 1%! Their results are very impressive.

Although the nucleon-nucleon scattering data alone can predict the binding energy of the deuteron ($^2$H) to within about 1/2%, the experimental binding energy is used as input data in fitting the nucleon-nucleon potential. The nuclei $^3$H and $^3$He are slightly underbound without a three-nucleon force, and that force can be adjusted to remedy the underbinding. This highlights both the dominant nature of the nucleon-nucleon force and the relative smallness of three-nucleon forces, which is nevertheless appropriate in size to account for the small discrepancies that result from using only nucleon-nucleon forces in calculations.

The binding energy of $^4$He is then accurately predicted to within about 1%. The six-nucleon systems are also rather well predicted. There are small problems with neutron-rich nuclei, but only 3 adjustable parameters in the three-nucleon force allow several dozen energy levels to be quite well predicted. I recommend that everyone peruse the impressive results of the GFMC collaboration[18].

We note finally that power counting can be used to show that light nuclei are basically non-relativistic, and relativistic corrections are on the order of a few percent. Power counting is a powerful qualitative technique for determining the relative importance of various mechanisms in nuclear physics.

What Nuclear Physics Can Do for Atomic Physics

With our recently implemented computational skills we in nuclear physics can calculate many properties of light nuclei with fairly good accuracy. This is especially true for the deuteron, which is almost unbound and is computationally simple. Although nuclear experiments don’t have the intrinsic accuracy of atomic experiments, many nuclear quantities that are relevant to precise atomic experiments can also be measured using nuclear techniques, and usually with fairly good accuracy.

What quantities are we talking about? The nuclear length scale is $R \sim 1$ fm = $10^{-5}$ Å. The much larger atomic length scale of $a_0 \sim 1$ Å means that an expansion in powers of $R/a_0$ makes great sense, and a typical wavelength for an atomic electron is so large compared to the nuclear size that only moments of the nuclear observables come into play. An example is the nuclear charge form factor (the
Fourier transform of the nuclear charge density, $\rho$, which is given by
\[ F(\mathbf{q}) = \int d^3r \, \rho(\mathbf{r}) \exp(i\mathbf{q} \cdot \mathbf{r}) \equiv Z(1 - \frac{q^2}{6} \langle r^2 \rangle_{\text{ch}} + \cdots) - \frac{1}{2} \mathbf{q} \cdot \mathbf{q} Q^{\alpha\beta} + \cdots, \tag{3} \]

where $\mathbf{q}$ is the momentum transferred from an electron to the nucleus, $Q^{\alpha\beta}$ is the nuclear quadrupole-moment tensor, $Z$ is the total nuclear charge, and $\langle r^2 \rangle_{\text{ch}}$ is the mean-square radius of the nuclear charge distribution. Since the effective $\langle \mathbf{q}^2 \rangle$ in an atom will be set by the atomic scales and consequently will be very small, these moments should dominate the nuclear corrections to atomic energy levels. If one then uses $F$ to construct the electron-nucleus Coulomb interaction, one obtains
\[ V_{\text{C}}(\mathbf{r}) \equiv -\frac{Z\alpha}{r} + \frac{2\pi Z\alpha}{3} \langle r^2 \rangle_{\text{ch}} \delta^{3}(\mathbf{r}) - \frac{Q\alpha}{2r^3} (3 \langle \mathbf{S} \cdot \hat{\mathbf{r}} \rangle^2 - \mathbf{S}^2) + \cdots, \tag{4} \]

where $\mathbf{S}$ is the nuclear spin operator and $Q$ is the nuclear quadrupole moment (which vanishes unless the nucleus has spin $\geq 1$). The Fourier transform of the nuclear current density has a similar expansion
\[ J(\mathbf{q}) = \int d^3r \, J(\mathbf{r}) \exp(i\mathbf{q} \cdot \mathbf{r}) \equiv -i\mathbf{q} \times \mathbf{\mu} \left(1 - \frac{q^2}{6} \langle r^2 \rangle_{\text{M}} + \cdots\right) + \cdots, \tag{5} \]

where $\mathbf{\mu}$ is the nuclear magnetic-moment operator and $\langle r^2 \rangle_{\text{M}}$ is the mean-square radius of the magnetization distribution. The first term generates the usual atomic hyperfine interaction.

Electron-nucleus scattering is the primary technique used to determine those nuclear moments of charge and current densities that are relevant to atomic physics[19]. An exception is the measurement of the deuteron’s quadrupole moment $|Q = 0.282(19) \text{ fm}^2|$ obtained by scattering polarized deuterons from a high-Z target at low energy[20]. This result is consistent with the molecular measurement $|Q = 0.2860(15) \text{ fm}^2|$[21, 22], but its error is an order of magnitude larger. Although there is no reason to believe that the tensor polarizability of the deuteron[23] plays a significant role in the H-D (molecular) quadrupole-hyperfine splitting that was used to determine $Q$, that correction was not included in the analysis. It was included in the analysis of the nuclear measurement.

I highly recommend the recent review by Ingo Sick[19], which contains values of the charge and magnetic radii of light nuclei. That review not only contains the best values of quantities of interest, but discusses reliability and many technical details for those who are interested. One qualitative result from that review is important for the discussion below. The errors of the tritium ($^3\text{H}$) radii are about an order of magnitude larger than those of deuterium. Of all the light nuclei tritium is the most poorly known experimentally, although the charge radius is relatively easy to calculate.

In addition to moments of the nuclear charge and current densities, various components and moments of the nuclear Compton amplitude can play a significant role. Examples are the (scalar) electric polarizability, $\alpha_E$, and the nuclear spin-dependent polarizability ($\sim \mathbf{S}$). The latter term interacts with the electron spin to produce a contribution to the electron-nucleus hyperfine interaction. There exists a recent calculation of the latter for deuterium[24], and either calculations or measurements of $\alpha_E$ for $^2\text{H}[25, 26, 27]$, $^3\text{H}$ and $^3\text{He}[28]$, and $^4\text{He}[29, 30]$.

The Proton Size

One recurring problem in the hydrogen Lamb shift is the appropriate value of the mean-square radius of the proton, $\langle r^2 \rangle_p$, to use in calculations. Some older analyses[31] disagree strongly with more recent ones[32]. As shown in Eqn. (3), the slope of the charge form factor (with respect to $q^2$) at $q^2 = 0$ determines that quantity. The form factor is measured by scattering electrons from the proton at various energies and scattering angles.

There are (at least) four problems associated with analyzing the charge form-factor data to obtain the proton size. The first is that the counting rates in such an experiment are proportional to the flux
of electrons times the number of protons in the target seen by each electron. That product must be measured. In other words the measured form factor at low $q^2$ is $(a - b q^2 + \cdots)$, where $b/a = \langle r^2 \rangle_p$. The measured normalization $a$ (not equal to 1) clearly influences the value and error of $\langle r^2 \rangle_p$. Most analyses unfortunately don’t take the normalization fully into account, and Ref.[33] estimates that a proper treatment of the normalization of available data could increase $\langle r^2 \rangle_p^{1/2}$ by about 0.015 fm and increase the error, as well. In an atom, of course, the normalization is precisely computable.

Another source of error is neglecting higher-order corrections in $\alpha$ (i.e., Coulomb corrections). Ref.[34] demonstrates that this increases $\langle r^2 \rangle_p^{1/2}$ by about 0.010 fm. A similar problem in analyzing deuterium data was resolved in Ref.[35]. Another difficulty that existed in the past was a lack of high-quality low-$q^2$ data. The final problem is that one must use a sufficiently flexible fitting function to represent $F(q)$, or the errors in the radius will be unrealistically low. All of the older analyses had one or more of these flaws.

Most of the recent analyses[32, 34, 36] are compatible if the appropriate corrections are made. An analysis by Rosenfelder[34] contains all of the appropriate ingredients, and he obtains $\langle r^2 \rangle_p^{1/2} = 0.880(15)$ fm. There is a PSI experiment now underway to measure the Lamb shift in muonic hydrogen, which would produce the definitive result for $\langle r^2 \rangle_p$.[37, 38] I fully expect the results of that experiment to be compatible with Rosenfelder’s result. Extraction of the proton radius[39] from the electronic Lamb shift is now somewhat uncertain because of controversy involving the two-loop diagrams. These diagrams are significantly less important in muonic hydrogen, where the relative roles of the vacuum polarization and radiative diagrams are reversed.

What Atomic Physics Can Do for Nuclear Physics

The single most valuable gift by atomic physics to the nuclear physics community would be the accurate determination of the proton mean-square radius: $\langle r^2 \rangle_p$. This quantity is important to nuclear theorists who wish to compare their nuclear wave function calculations with measured mean-square radii. In order for an external source of electric field (such as a passing electron) to probe a nucleus, it is first necessary to “grab” the proton’s intrinsic charge distribution, which then maps out the mean-square radius of the proton probability distribution in the wave function: $\langle r^2 \rangle_{wfn}$. Thus the measured mean-square radius of a nucleus, $\langle r^2 \rangle$, has the following components:

$$\langle r^2 \rangle = \langle r^2 \rangle_{wfn} + \langle r^2 \rangle_p + \frac{N}{Z} \langle r^2 \rangle_n + \frac{1}{Z} \langle r^2 \rangle_{\cdots},$$

where I have included the intrinsic contribution of the N neutrons as well as the $Z$ protons, and $\langle r^2 \rangle_{\cdots}$ is the contribution of everything else, including the very interesting (to nuclear physicists) contributions from strong-interaction mechanisms and relativity in the nuclear charge density[40]. Because the neutron looks very much like a positively charged core surrounded by a negatively charged cloud, its mean-square radius has the opposite sign to that of the proton, whose core is surrounded by a positively charged cloud. It should be clear from Eqn. (6) that $\langle r^2 \rangle_p$ (which is much larger than $\langle r^2 \rangle_n$) is an important part of the overall mean-square radius. Its present uncertainty degrades our ability to test the wave functions of light nuclei.

The next most important measurements are isotope shifts in light atoms or ions. Since isotope shifts measure differences in frequencies for fixed nuclear charge $Z$, the effect of the protons’ intrinsic size cancels in the difference. This is particularly important given the current lack of a precise value for the proton’s radius. The neutrons’ effect is relatively small and can be rather easily eliminated, and thus one is directly comparing differences in wave functions, or of small contributions from $\langle r^2 \rangle_{\cdots}$ Isotope shifts are therefore especially “theorist-friendly” measurements, since they are closest to measuring what nuclear theorists actually calculate.
Precise isotope-shift measurements have been performed for $^4\text{He} - ^3\text{He}$[41] and for $^2\text{H} - ^1\text{H}$ (D-H)[5]. A measurement of $^6\text{He}-^4\text{He}$ is being undertaken[42] at ANL. Gordon Drake has written about and strongly advocated such measurements in the Li isotopes[43]. These are all highly desirable measurements. Because there are currently large errors in the $^3\text{H}$ (tritium) charge radius, in my opinion the single most valuable measurement to be undertaken for nuclear physics purposes would be the tritium-hydrogen ($^3\text{H} - ^1\text{H}$) isotope shift. An extensive series of calculations using first-generation nuclear forces found $\langle r^2 \rangle_{\text{wfn}}^{1/2}$ for tritium to be 1.582(8) fm, where the “error” is a subjective estimate[44]. This number could likely be improved by using second-generation nuclear forces, although it will never be as accurate as the corresponding deuteron value, which we discuss next.

The D-H isotope shift in the 2S-1S transition reported by the Garching group[5] was

$$\Delta \nu = 670,994,334.64(15) \text{ kHz}. \quad (7)$$

Most of this effect is due to the different masses of the two isotopes (and begins in the first significant figure, indicated by an arrow). Nevertheless, the precision is sufficiently high that the mean-square radius effect in the sixth significant figure (second arrow) is much larger than the error. The electric polarizability of the deuteron influences the eighth significant figure, while the deuteron’s magnetic susceptibility contributes to the tenth significant figure. It becomes difficult to trust the interpretation of the nuclear physics at about the 1 kHz level, so improving this measurement probably wouldn’t lead to an improved understanding of the nuclear physics.

Analyzing this isotope shift and interpreting the residue (after applying all QED corrections) in terms of the deuteron’s radius leads to the results[45] in Table 1. The very small binding energy of the deuteron produces a long wave function tail outside the nuclear potential (interpretable as a proton cloud around the nuclear center of mass), which in turn leads to an easy and very accurate calculation of the mean-square radius of the (square of the) wave function. Subtracting this theoretical radius from the experimental deuteron radius (corrected for the neutron’s size) determines the effect of $\langle r^2 \rangle_{\text{wfn}}$ on the radius. Although this difference is quite small, it is nevertheless significant and half the size of the error in the corresponding electron-scattering measurement. This high-precision analysis in Table 1 of the content of the deuteron’s charge radius would have been impossible without the precision of the atomic D-H isotope-shift measurement. This measurement has given nuclear physics unique insight into small mechanisms that are at present poorly understood[46].

| $\langle r^2 \rangle_{\text{wfn}}^{1/2}$ (fm) | exp-$\langle r^2 \rangle_{\text{pt}}^{1/2}$ (fm) | difference (fm) | $\Delta/\langle r^2 \rangle_{\text{wfn}}^{1/2}$ (fm) |
|------|-------------|--------|----------------|
| 1.9687(18) | 1.9753(10) | 0.0066(21) | -0.0291(7) |

Table 1. Theoretical and experimental deuteron radii for pointlike nucleons. The deuteron wave function radius corresponding to second-generation nuclear potentials and the experimental point-nucleon charge radius of the deuteron (i.e., with the neutron charge radius removed) are shown in the first two columns, followed by the difference of experimental and theoretical results. The difference of the experimental radius with and without the neutron’s size is given last for comparison purposes[47].

I have said very little in my talk about how information from hyperfine splittings might provide insight into nuclear mechanisms. Partly this is due to a lack of background on my part, and partly because the necessary calculations haven’t been performed. Karshenboim and Ivanov[48] have compared theoretical (QED only) and experimental results for various S-states in light atoms, results that are expected to be accurate to roughly 1 part in $10^9$. They find significant residual differences attributable

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to nuclear effects, which range from tens of ppm to several hundred ppm. It is likely that these differences contain interesting and useful nuclear information, in the form of Zemach moments[49] and spin-dependent polarizabilities. The latter are related to the Drell-Hearn-Gerasimov sum rule[50], a topic of considerable current interest in nuclear physics[51]. Exploratory calculations are underway.

Summary and Conclusions

I hope that I have convinced you that nuclear forces and nuclear calculations in light nuclei are under control in a way never before attained. This progress has been possible because of the great increase in computing power in recent years. Many of the nuclear quantities that contribute to atomic measurements have been calculated or measured to a reasonable level of accuracy, a level that is improving with time. Isotope shifts are valuable contributions to nuclear physics knowledge, and are especially useful to theorists who are interested in testing the quality of their wave functions for light nuclei. In special cases such as the deuteron these measurements provide the only insight into the size of small contributions to the electromagnetic interaction that are generated by the underlying strong-interaction mechanisms. In my opinion the tritium-hydrogen isotope shift would be the most useful measurement of that type. I am especially hopeful for the success of the ongoing PSI experiment attempting to measure the proton size via the Lamb shift in muonic hydrogen. The absence of a stable, accurate proton radius has been particularly annoying.

Acknowledgements

The work of J. L. Friar was performed under the auspices of the United States Department of Energy. The author would like to thank Savely Karshenboim for an invitation to this conference and for the opportunity to discuss the nuclear side to precise atomic measurements.

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