Validation of an Experimental Setup to Study the Behaviour of a Free-Falling Particle in Viscous Fluid

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Abstract. A sphere falling in a fluid under the influence of gravity will accelerate until the buoyancy and drag forces balanced the gravitational force. During this steady-state, the falling velocity become constant and known as the terminal velocity. The purpose of this work is to describe the experimental setup aimed at studying the behaviour of free-falling particle in incompressible viscous fluid. The terminal velocity of a free falling sphere in incompressible fluid was captured experimentally and validated using analytical empirical model. The preset investigation is a preliminary study designed to validate the reliability of the built experimental rig. The experimental work is done by undertaking experiments in water with free-falling stainless steel spheres of different diameters, varying between 4 mm and 7 mm. The spheres are released just above the water free surface at the centre of the column and recorded by a high speed camera. The results from the experiments agreed well with theoretical solution. Therefore, the reliability of the build experimental setup was proved. Hence, it can be used to conduct experiments involving more complicated particle shape and fluids.

1. Introduction

Free-falling particle in fluid is a classic problem of fluid mechanics that has been studied vigorously theoretically, experimentally and numerically for decades \cite{1, 2, 3, 4}. A substantial sets of data were accumulated which summarized in review papers such as by \cite{5, 6, 7}. This topic has gained a broad range of interest due to its geometric simplicity and wide applications in various engineering problems and natural flows. Hence, research on this topic remains active to the present day and to mention but a few, fluid drilling process \cite{8}, fluidized suspensions \cite{9}, hydraulic fracturing of reservoirs \cite{10}, sediment transport and deposition in pipe lines \cite{11, 12}.

It is thus essential to have a reliable experimental setup and procedure to observed the phenomena and extract usable data from the observations. The purpose of this work is to describe the built experimental rig aimed at studying the behaviour of free-falling particle in incompressible viscous fluid as well as the validation of the obtained results with theoretical solution. The validated experimental rig can be used to conduct experiment involving a more complicated particle shape and fluid.
2. Experimental Setup
An experimental setup was designed to capture the transient and terminal velocity of a free-falling solid particle. The schematic diagram of the experimental setup is shown in Figure 1, comprises a 1000 mm high square cross section, straight-walled column with inner side width of 70.45 mm made of clear Perspex. These dimensions were selected such that the largest particle in our experiments falling through the continuous fluid could reach terminal velocity and end effects could be avoided. The motion of the falling sphere was recorded by a high speed camera which was mounted on vertical and horizontal sliders to give the camera 2 degrees of freedom. A custom made high power LED light was used to illuminate the test section. An electromagnetic release mechanism was integrated with the top lid. The mechanism was designed such that the sphere’s motion is induced by the effect of gravity only, at the centre of the column with zero initial velocity directly above the water’s surface. The bottom part of the column was attached to a retrieval mechanism which allowed the sphere to be removed from the column easily. The temperature of the water was controlled by an electric heater and a proportional-integral-derivative (PID) controller connected to two thermocouple probes. However, all experiments were conducted at room temperature for the present paper. The spheres used in the experiments are commercially available stainless steel spheres with diameters varying between 4 mm and 7 mm. The density of these spheres is 7750 kg/m³. The image sequences were converted into 8-bit grey scale, background-subtracted and thresholded such that spheres appeared as a black dot on a white background to allow for tracking and quantification of droplets using an in-house ImageJ macro. Information such as the relative position, velocity and diameter were obtained using automated analysis of this type.

![Figure 1. Schematic diagram of the experimental setup.](image)

2.1. Results and Discussion
2.2. Trajectory
Figure 2 shows the typical falling behaviour of the spheres in water at room temperature. The solid black lines represent the ensemble average of the repeated experiments for each sphere
diameter. As can be seen, the trajectories of the spheres are almost completely rectilinear and all the cases are very consistent with very small standard deviation values which are represented by the shaded area in the plots.

![Figure 2](image_url)

**Figure 2.** The mean trajectory of free-falling stainless steel sphere in water at room temperature (the shaded area represents standard deviation from mean value): (a) 4 mm, (b) 5 mm, (c) 6 mm, and (d) 7 mm.

2.3. Velocity

The vertical falling velocity was deduced from the time derivative of the trajectory. Figure 3 illustrates the mean vertical velocity of three repetitions as a function of the distance from the free water surface derived from trajectories in Figure 2. The shaded area in the plots represents the standard deviation from the mean value. It can be observed that the sphere’s vertical velocity reached a steady state after they travelled through the column for approximately 250 mm for 4 mm, 5 mm and 6 mm diameter spheres, and 350 mm for 7 mm diameter sphere. A steady state is achieved when the drag and lift forces acting on the sphere balance the gravitational force due to the difference between the density of the sphere and the surrounding fluid. The instantaneous velocities after these points are averaged to ascertain the terminal velocity. The average terminal velocities of the spheres used in the experiments, summarised in Figure 4, varied linearly between 0.96 m/s to 1.25 m/s giving a Reynolds number in the range of $3 \times 10^3$ to $9 \times 10^3$. It is shown in some previous studies [13] on the drag of free falling sphere that the time evolution of the sphere’s velocity evolves towards a terminal value according to an empirical exponential model proposed by [14]:

$$v(t) = v_T (1 - e^{-t/\tau})$$  \hspace{1cm} (1)

where $v_T$ (terminal velocity) and $\tau$ (characteristic time) in the equation are independent empirical fitting parameters. In agreement with those studies, Figure 5 shows that for the
case of 4 mm sphere falling in water, the experimental velocity profile closely followed the model given by Equation 1 with an $r^2$ (coefficient of determination) value of 0.9804. The terminal velocity value extracted using the fitting procedure can be used to predict the terminal velocity of the falling sphere. In this case, the terminal velocity predicted by Equation 1 is 0.9605 m/s, which is in good agreement with the experimental value with a difference of merely 0.3%. The detailed analysis of the results from this fitting procedure for all cases in the present study are tabulated in Table 1. A good fit between the theoretical curves and experimental data can be
Table 1. Experimental terminal velocities extracted from fitting procedure using Equation 1, their fitting parameters and goodness of fitting.

| d [mm] | $v_{T,exp}$ [m/s] | $v_{T,eq}$ [m/s] | $\tau$  | $r^2$     |
|--------|------------------|------------------|--------|-----------|
| 4      | 0.9576           | 0.9605           | 0.0563 | 0.9804    |
| 5      | 1.0500           | 1.0592           | 0.0639 | 0.9861    |
| 6      | 1.1260           | 1.1276           | 0.0684 | 0.9504    |
| 7      | 1.2599           | 1.2594           | 0.0811 | 0.9673    |

observed in the table with the lowest $r^2$ value of 0.9504 and all the predicted values of terminal velocity are in good agreement with the experimental values with a largest difference of 0.8 %.

Considering the fact that this method integrates the entire instance of the captured data, it is conceivable that Equation 1 could be used to accurately estimate the terminal velocity of a falling sphere in the case where the full trajectory (from transient to steady states) of the falling sphere can not be captured due to the experimental set-up constraints. However, it should be noted that Equation 1 does not precisely characterize the greater complex characteristics of the real trajectory.

Figure 5. Example of the velocity evolution obtained from experimental data fitted with Equation 1

3. Conclusion
An experimental setup was developed to study the behaviour of free-falling solid particle in incompressible viscous fluid. The falling motion of different diameter stainless-steel solid spheres in water was recorded. The diameter of the spheres are varied between 4 mm and 7 mm. An in-house ImageJ macro was used to track the particle motion and extract information such as relative position and velocity of the particle. The experiments produced consistent result with very small standard deviation and agreed well with theoretical solution proposed by [14] with
difference of merely 0.8%. Therefore, the reliability of the build experimental setup was proved. Hence, it can be used to conduct experiments involving more complicated particle shape and fluids.

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