The Scalar Hexaquark $uuddss$: a Candidate to Dark Matter?

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Introduction: From first days of the quark-parton model and Quantum Chromodynamics (QCD), hadrons with unusual quantum numbers and/or multi-quark contents attracted interest of physicists. The conventional hadrons have quark-antiquark or three-quark compositions. Their masses and quantum parameters $J^{PC}$ are in accord with predictions of this scheme and can be calculated using standard methods of particle physics. Unusual or exotic hadrons are expected to be built of four or more valence quarks or contain valence gluons. A main reason triggered intensive investigations of four-quark states was a mass hierarchy inside the lowest scalar multiplet, which found its explanation in the context of the four-quark model [1]. Starting from 2003, i.e. from first observation of the exotic meson $X(3872)$ theoretical and experimental investigations of tetra and pentaquarks became one of the interesting and rapidly growing branches of high energy physics. Now, valuable experimental information collected during past years, as well as theoretical progress achieved to date, form two essential components of the exotic hadrons physics [2–6].

Another interesting result about multi-quark hadrons with far-reaching consequences was obtained also by R. Jaffe [7]. Thus, using calculations in MIT quark-bag model he predicted existence of a $H$-dibaryon, i.e. of a isoscalar $J^P = 0^+$ six-quark $uuddss$ bound state. This double-strange six-quark structure with mass 2150 MeV lies 80 MeV below the $2m_\Lambda = 2230$ MeV threshold and is stable against strong decays. It can decay through weak interactions, which means that mean lifetime of $H$-dibaryon, $\tau \approx 10^{-10}$s, is considerably longer than that of most conventional hadrons.

The original study [7] was followed by numerous theoretical investigations, in which various models and methods of particle physics were used to calculate the H-dibaryon’s mass [8–16]. As usual, results of these studies are controversial: thus, calculations in the framework of the corrected MIT bag model led to $m_H = 2240$ MeV which is just above the $2m_\Lambda$ threshold [8], whereas in a chiral model the authors [9] found $m_H = 1130$ MeV.

Other quark models were also invoked to analyze $\Lambda - \Lambda$ interaction and estimate $\Lambda\Lambda$ binding energy [10, 12, 14]. The $H$-dibaryon’s mass extracted from the QCD two-point sum rules is consistent with the original result of Jaffe [12] [14]. In fact, $m_H$ from Ref. [13] varies in limits $2.0 - 2.4$ GeV and within an accuracy of the sum rule method $\sim 20\%$ agrees with the result of the quark-bag model. Calculations in Ref. [16] also confirmed existence of a bound state lying 40 MeV below the $2m_\Lambda$ threshold. The lattice simulations performed in Ref. [17] led to conclusion that $m_H$ was below the $2m_N$ threshold 1880 MeV. In this paper the authors took into account the stability conditions of the nucleus and extracted $m_H \approx 1850$ MeV. The later lattice studies confirmed existence of a bound-state $H$-dibaryon, and predicted its binding energy $\approx 74.6$ MeV [18] and $(19 \pm 10)$ MeV [19], respectively. In the context of the holographic QCD $H$-dibaryon was explored in Ref. [20], in which its mass was estimated about $m_H = 1.7$ GeV.

The hexaquark $S$ (except for original papers, hereafter we use a hexaquark instead of a six-quark state, and denote it by $S$) was searched for by KTeV, Belle, and BaBar collaborations in exclusive $S \rightarrow \Lambda\pi\pi$ and inclusive $Y(1S)$ and $Y(2S)$ decays, in processes $Y(2S, 3S) \rightarrow SAX$ [21–24]. All these experiments could not find an evidence for the hexaquark $S$ near the threshold $2m_\Lambda$ and were able only to impose limits on its mass $m_S$ the latest being $m_S < 2.05$ GeV.

Recent activities around $S$ is inspired by renewed suggestions to consider it as a possible candidate to dark matter [25–27]. In accordance with this scenario if $m_S < 2(m_p + m_e) = 1877.6$ MeV the hexaquark can be absolutely stable, but even for the mass $m_S$ obeying the inequality $m_S \leq m_\Lambda + m_p + m_e = 2054.5$ MeV its lifetime may be longer than the age of the Universe. The lower bound of $m_S$ is determined by a stability of ordinary nuclei, which are stable if $m_S > m_p + m_n + m_e - 2E$, where $2E$ is a binding energy of $p + n$. Then, for masses $1860 < m_S < 1880$ MeV, which assures a stability of the hexaquark and conventional nuclei, the $S$ can ex-
plain both the relic abundance of the DM in Universe and observed DM to ordinary matter ratio with less than 15% uncertainty [27]. However, there are objections to this picture connected with a production process of hexaquarks in the early-universe [28], or with observed supernova explosion [29].

The hexaquark $S$ as a candidate to DM was recently analyzed in Ref. [30] as well. In this work the mass of $S$ was evaluated by modeling it as a bound state of scalar diquarks. Using the effective Hamiltonian to describe dominant spin-spin interactions in diquarks [3], the authors expressed $m_S$ in terms of constituent diquark masses $m_{ij}$ and chromomagnetic couplings $k_{ij}$. The masses of diquarks and chromomagnetic couplings may be extracted from analysis of baryon spectroscopy. Alternatively, $k_{ij}$ can also be fixed to reproduce masses of the light scalar mesons $f_0(500)$, $K^*(800)$ $f_0(980)$, and $a_0(980)$ interpreted as tetraquarks [31]. It turns out that spin-spin couplings in tetraquarks are about a factor of four larger compared to the spin-spin couplings in the baryons. Because the hexaquark itself is an exotic six-quark meson for calculation of $m_S$ it is reasonable to employ parameters estimated from analysis of the light tetraquarks. Prediction for $m_S \approx 1.2$ GeV made in Ref. [30] reproduces the cosmological DM abundance, but contradicts to stability of oxygen nuclei.

**Calculations:** In the present work we calculate the mass of $S$ by treating it as a bound state of three scalar diquarks. To this end, we employ the QCD two-point sum rule approach, which is one of the powerful nonperturbative methods to explore hadrons. As starting point, the method uses the correlation function

$$
\Pi(p) = i \int d^4 x e^{ipx} \langle 0 | T \{ J(x) J^\dagger (0) \} | 0 \rangle
$$

and extract from its analysis sum rules to compute spectroscopic parameters of the hexaquark. The main ingredient of this analysis is the interpolating current $J(x)$ which we choose it in the following form

$$
J(x) = e^{abc} \left[ u(x) C \gamma_5 d(x) \right]^a \left[ u(x) C \gamma_5 s(x) \right]^b \times \left[ d(x) C \gamma_5 s(x) \right]^c,
$$

where $[q^T C \gamma_5 q]^a = \epsilon^{a \mu
u\rho}[q^T_\mu C \gamma_5 q^\rho_\mu]$ and $a, b, c, m, n$ are color indices with $C$ being the charge conjugation operator.

As is seen, the hexaquark is composed of the scalar diquarks $[q^T C \gamma_5 q]^a$ in the color antitriplet and flavor antisymmetric states. These diquarks are most attractive ones [32], and six-quark mesons composed of them should be lighter and more stable than bound states of other two-quarks. Mathematical manipulations to derive sum rules for the mass and coupling of the hexaquark are carried out in accordance with standard prescriptions of the method. Thus, first we express the correlation function $\Pi(p)$ in terms of the hexaquark’s mass $m_S$ and coupling $f_S$, as well as its matrix element

$$
\langle 0 | J | S \rangle = m_S f_S.
$$

Separating from each another the ground-state term and contributions due to higher resonances and continuum states for $\Pi^{\text{Phys}}(p)$ we get

$$
\Pi^{\text{Phys}}(p) = \frac{(0|J|S(p)) \langle S(p)|J^\dagger(0)\rangle}{m_S^2 - p^2} + \ldots
$$

The expression of the matrix element $\langle \rangle$ allows us to rewrite $\Pi^{\text{Phys}}(p)$ in the form

$$
\Pi^{\text{Phys}}(p) = \frac{m_S^2 f_S^2}{m_S^2 - p^2} + \ldots,
$$

where dots denote contributions of higher resonances and continuum states.

To calculate the QCD or OPE side of the sum rules, we insert the current $J(x)$ to Eq. (1), contract relevant quark fields and obtain $\Pi^{\text{OPE}}(p)$ in terms of the quark propagators:

$$
\Pi^{\text{OPE}}(p) =
$$

$$
\times \gamma_5 S^{f'}(x) \gamma_5 \{ \text{Tr} \} \left[ \langle S_u \rangle x S_{f'}^{op}(x) \gamma_5 \right] \times \{ \text{Tr} \} \left[ \langle S_u \rangle x S_{f'}^{op}(x) \gamma_5 \right] + 511 \text{ similar terms},
$$

$$
\times \text{Tr} \left[ S_{cc}^{op}(x) \gamma_5 S_{dd}^{op}(x) \gamma_5 \right] + 511 \text{ similar terms},
$$

$$
\times \text{Tr} \left[ S_{cc}^{op}(x) \gamma_5 S_{dd}^{op}(x) \gamma_5 \right] + 511 \text{ similar terms},
$$

$$
\times \text{Tr} \left[ S_{cc}^{op}(x) \gamma_5 S_{dd}^{op}(x) \gamma_5 \right] + 511 \text{ similar terms},
$$

where $S'(x) = CS^T(x)$. To proceed, we employ the $x$-space light-quark propagator

$$
S_q^{ab}(x) = \frac{1}{2 \pi^2 x^4} \delta_{ab} - \frac{m_q^4}{4 \pi^2 x^2} \delta_{ab} - \frac{(7q)^2}{12} \left( 1 - i \frac{m_q^4}{4 \pi^2} \right) \delta_{ab}
$$

$$
- \frac{i g_s G_{\mu\nu}^{ab}}{2 \pi x^2} \left[ \tilde{f}_{\mu\nu} + \tilde{g}_{\mu\nu} \cdot \tilde{f} \right] - \frac{2 \pi x^4 (7q)^2}{1776} \delta_{ab}
$$

$$
+ \frac{27648}{1776} \delta_{ab} + \frac{m_q g_s G_{\mu\nu}^{ab}}{32 \pi^2} \delta_{ab} - \frac{\ln \left( -x^2 \Lambda^2 \right)}{4} + 2 \gamma_E
$$

$$
+ \ldots,
$$

where $q = u, d$ or $s$, $\gamma_E \approx 0.577$ is the Euler constant, and $\Lambda$ is a scale parameter. We also use the notations $G_{\mu\nu}^{ab} \equiv G_{\mu\nu}^{A} \delta_{ab}$, $A = 1, 2, \ldots, 8$, and $t^A = \lambda^A / 2$, with $\lambda^A$ being the Gell-Mann matrices.

After inserting the light-quark propagators into Eq. (10), we get the correlation function $\Pi^{\text{OPE}}(p)$ in terms of QCD degrees of freedom. The next step is to perform the resultant Fourier integrals over four-$x$. Afterwards we equate the invariant amplitudes in $\Pi^{\text{Phys}}(p)$ and $\Pi^{\text{OPE}}(p)$ to find the desired sum rule in momentum space. We apply the Borel transformation to both sides of the obtained sum rule to suppress contributions of the higher resonances and continuum, and using the quark-hadron duality assumption, which is a quintessence of the sum rule method, perform the continuum subtraction. As the calculations contain six quarks, the procedures described above are very lengthy and time consuming. They are
carried out using computer codes, results of which cannot be presented here.

An equality derived after these manipulations, contains the mass and coupling constant of the S particle. To find the sum rules for \( m_S \) and \( f_S \) we need an extra expression which can be obtained by acting \( d/d (−1/M^2) \) to the first equality. The sum rules for \( m_S \) and \( f_S \) obtained by this way have perturbative and nonperturbative components. The latter contains vacuum condensates of various local quark, gluon, and mixed operators up to dimension thirty, which appears after sandwiching relevant terms in \( \Pi^{OPE}(p) \) between vacuum states. We take into account all of them in numerical computations bearing in mind that the higher dimensional terms appear due to the factorization hypothesis as product of basic condensates.

In analyses and computations we utilize the quark and mixed condensates \( \langle \bar{q}q \rangle = −(0.24 \pm 0.01)^3 \) GeV\(^3\), \( \langle \bar{q}gσGq \rangle = m_0^2\langle \bar{q}q \rangle \), \( \langle ss \rangle = 0.8\langle \bar{q}q \rangle \) and \( \langle \bar{q}gσGs \rangle = m_0^2\langle ss \rangle \), where \( m_0^2 = (0.8 \pm 0.1) \) GeV\(^2\). One of ingredients of sum rules is the gluon condensate \( \langle α_s G^2/π \rangle = (0.012 \pm 0.004) \) GeV\(^4\). We work in the approximation \( m_u = m_d = 0 \), but keep a dependence on \( m_s \). The scale parameter \( Λ \) is varied within the limits (0.5, 1) GeV. The mass of the strange quark \( m_s = 95^{-3}_{−9} \) MeV in the \( \overline{MS} \) scheme and at the scale \( μ \approx 2 \) GeV is borrowed from Ref. [33]. We also explore the sum rules with \( m_s \) at the scale \( μ \approx 1 \) GeV that differs from PDG estimate by a factor 1.35.

Another important problem is to choose a proper choice for the the Borel \( M^2 \) and continuum threshold \( s_0 \) parameters. First of them has been introduced upon Borel transformation, the second one is necessary to separate the ground-state and continuum contributions from each another, as we previously mentioned. These parameters are not arbitrary, but they should meet the well-known requirements. Thus, at maximum value of the Borel parameter the pole contribution (PC) should constitute a fixed part of the correlation function, whereas at minimum of \( M^2 \) it must be a dominant contribution. We define PC in the form

\[
PC = \frac{\Pi(M^2, s_0)}{\Pi(M^2; \infty)},
\]  

where \( \Pi(M^2; s_0) \) is the Borel transformed and subtracted invariant amplitude \( \Pi^{OPE}(p^2) \). In the case of multi-quark hadrons at \( M^2_{\text{max}} \) one usually chooses the pole contribution PC > 0.2. The minimum of the Borel parameter \( M^2 \) is fixed from convergence of the sum rules, i.e. at \( M^2_{\text{min}} \) contribution of the last term (or a sum of last few terms) cannot exceed, for example, 0.01 part of the whole result. There is another restriction on the lower limit \( M^2_{\text{min}} \), at this \( M^2 \) the perturbative contribution has to prevail over the nonperturbative one.

The sum rule predictions, in general, should not depend on the parameters \( M^2 \) and \( s_0 \). But in real calculations \( m_S \) and \( f_S \) demonstrate sensitivity to the choice of \( M^2 \) and \( s_0 \). Hence, the parameters \( M^2 \) and \( s_0 \) have to be fixed in such a way that to reduce this effect to a minimum. Performed analysis allows us to determine the working regions

\[
M^2 \in [1.3, 1.6] \text{ GeV}^2, \ s_0 \in [2.5, 2.9] \text{ GeV}^2, 
\]

which obey all aforementioned restrictions.

![FIG. 1: Dependence of the pole contribution on \( M^2 \) and \( s_0 \). The (red) surface PC = 0.2 is also shown.](image)

In Fig. 1 we plot the pole contribution, when \( M^2 \) and \( s_0 \) are varying within limits (8). At \( M^2 = 1.3 \) the pole contribution is 0.9, whereas at \( M^2 = 1.6 \) it becomes equal to 0.2. The predictions for the mass \( m_S \) is pictured in Fig. 2, where a mild dependence on the parameters \( M^2 \) and \( s_0 \) is seen. The results for the spectroscopic parameters of the hexaquark S read: for \( m_s = 95 \) MeV

\[
m_S = 1180^{+40}_{−26} \text{ MeV}, \ f_S = 8.56^{+0.03}_{−0.26} \times 10^{−6} \text{ GeV}^7, 
\]

and for \( m_s = 128 \) MeV

\[
\bar{m}_S = 1239^{+42}_{−28} \text{ MeV}, \ \bar{f}_S = 9.18^{+0.03}_{−0.22} \times 10^{−6} \text{ GeV}^7. 
\]

Let us note that in computation of \( \bar{m}_S \) and \( \bar{f}_S \) the Borel parameter has been varied within the limits \( M^2 \in [1.34, 1.63] \) GeV\(^2\).

Theoretical errors in the sum rule computations appear due to different sources. The auxiliary parameters \( M^2 \) and \( s_0 \), and the scale parameter \( Λ \) are main sources of these ambiguities. The errors connected with various condensates are numerically small.

**Conclusions:** We have considered the spin-0, parity-even, highly symmetric S-hexaquark of \( uuddss \) with \( Q = 0, \ B = 2 \) and \( S = −2 \). Using the technique of QCD sum rule, we have found that for the chosen interpolating current the intervals \( M^2 \in [1.3, 1.6] \) GeV\(^2\), \( s_0 \in [2.5, 2.9] \) GeV\(^2\) for the auxiliary parameters fulfill the requirements of the method discussed above. For the chosen intervals of the auxiliary parameters we could able to reach [0.9 − 0.2] pole contributions to the sum rules, and have extracted \( m_S = 1180^{+40}_{−26} \) MeV \( (m_s = 95 \) MeV\) and \( \bar{m}_S = 1239^{+42}_{−28} \) MeV \( (m_s = 128 \) MeV\) for the mass of the S-hexaquark. This range of the mass implies that
the hexaquark $S$ is an absolutely stable particle. It is worth noting that using the same method but different interpolating currents this particle was previously investigated in Refs. [15, 10]. In Ref. [16], the authors estimated $m_s = 0.2 \text{ GeV}$ found $m_S = 2.4 \text{ GeV}$, however, could not determine whether the mass of this particle lies above or below the $\Lambda \Lambda$ threshold, i.e. whether this particle is stable or not. In Ref. [16], by taking $m_s = 150 \text{ MeV}$ the authors estimated $m_S \simeq 2.19 \text{ GeV}$, which corresponds to a bound state $40 \text{ MeV}$ below the $\Lambda \Lambda$ threshold.

Estimations for the mass of $S$-hexaquark obtained in these works differ considerably from our results. Our predictions for $m_S$ and $\bar{m}_S$, however, almost coincide with one made recently in Ref. [30], and obtained by modeling the $S$-hexaquark as a bound state of scalar diquarks and using recent theoretical and experimental progress in tetra- and pentaquark physics. Our results are also in accord with output of the chiral model [4].

According to the analyses performed in Ref. [30], the mass $1.2 \text{ GeV}$ of the $S$-hexaquark produces thermally the desired cosmological DM abundance. It was stated that masses larger than $1.2 \text{ GeV}$ lead to smaller abundances. But, very small masses of the hexaquark are excluded as well, because then nucleons inside of nuclei would bind to $S$ state faster than what is allowed by Super-Kamiokande bounds on the stability of oxygen nuclei [30]. This conclusion was made using global fits of nuclear potentials, which allowed the authors to compute the nuclear wave functions of the oxygen. It was concluded that both the $S$-hexaquark and oxygen nuclei could be stable for the mass of $S$ around $1.87 \text{ GeV}$. However, as stated above, $m_S \simeq 1.87 \text{ GeV}$ leads to a relic DM abundance much smaller than one fixed from observations. Because the mass $m_S \simeq 1.2 \text{ GeV}$ of the hexaquark $S$ in the present work has been obtained from rather detailed calculations, further experimental and theoretical investigations on stability of nuclei can be useful to make a final decision on $S$ particle.
[32] R. L. Jaffe, Phys. Rept. 409, 1 (2005).

[33] M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018).