New Approach for Vibration Suppression through Restrictors on Towering Steel Columns with Supporting Frame

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Towering steel column with supporting frame is a typical equipment in chemical process engineering. Here, displacement restrictors are proposed to restrict the displacement of the towering and slender equipment while making the column to be structurally nonlinear and statically indeterminate. This study investigated on along-wind and cross-wind vibration characteristics of such equipment with the restrictors experimentally and numerically. A field test is carried out to measure the natural frequency and damping ratio of the 42.5-meter-high equipment vibrating in the wind, which is the prototype of the experimental model. A noncontact excitation system was applied on the experimental model to simulate the wind loads. The displacements and strains of the experimental model are collected under different frame types by changing the heights of displacement restrictors. The numerical simulation and experimental results showed that the height of displacement restrictors has a great influence on the vibration intensity of the equipment. An optimum location, recommended as about 40% of the height, could decrease the vibration intensity and enhance the safety of the equipment. Based on the results, a simplified formula in which the natural frequency and the damping ratio dominate the dynamic behavior of along-wind and cross-wind vibration, respectively, is derived from multi-degrees-of-freedom system. It could be furtherly utilized to predict the amplitude ratio of two structures and select a better design featuring an efficient vibration suppression performance. This work presents an important design guide to the frame-supporting towering process equipment and is of great significance to an economical and safety design.

1. Introduction

When fluid flows around a circular body at a certain velocity, vortices form rhythmically from the leeward side and produce a periodic lift force on the body in a transversal direction. As the approaching flow velocity increases, the vortex shedding frequency is close enough to the body’s natural frequency so that the circular body is induced to respond in the lift direction, which is named vortex-induced vibration (VIV) [1, 2]. If the particular fluid flow is wind, the vibration of a circular body subject to wind is often referred to as cross-wind vibration (CWV) [3]. In addition, due to the violent fluctuation of the along-wind load, the along-wind vibration (AWV) is also significant especially when its velocity is high [4].

The self-supporting towering circular equipment is one of the most common structures in the chemical engineering, such as the chemical tower, chimney, and pipelines. Towering self-supporting chemical columns are usually set up outside and tend to suffer CWV and AWV during the whole period of lives. According to Robertson et al. and Peil et al., the CWV can cause deflection and fatigue problems of structures and even lead to collapses [5, 6]. Siringoringo and Fujino described AWV of a suspension bridge tower under moderate wind velocity, in which the results show that under certain angle of attack, the bluff body of the windward tower leg created vortex shedding as indicated by the presence of single-frequency oscillation of the wind in front of leeward tower [7]. Srinivas et al. studied the along-wind response of a tapered chimney, where the wind is considered as a stationary random process composed of spatially varying mean speed on which turbulent fluctuations are superimposed [4]. Ke et al. studied the average wind load on a superlarge cooling tower, and the results could provide references to the
structure selection and wind resistance design of such type of steel cooling towers [8]. In addition, the numerical study is becoming the main research method [9]. Narendran et al. studied the vortex-induced vibration in multicol umn off-shore platform by an explicit dynamic subgrid-scale model [10]. And there are several original boundary methods applied in the numerical simulation, such as the direct-forcing immersed boundary method, which eliminates the requirement of mesh regeneration at each time step, owing to the movement of the cylinder [11]. Because of the destructiveness of AWV and CWV, researches on the vibration suppression of towering equipment are remarkable to the economical and safe design.

Wind tunnel is widely used to study the dynamic behavior of structures under wind loads, especially for such high Reynolds number wind environment [12, 13]. Carril et al. conducted an experimental investigation on lattice towers built in Brazil using a wind tunnel, and a common approach to consider the effect of wind forces on antennas independent of the lattice tower, without considering the effects of their presence on the computation of wind forces, the effects of wind forces, and structural responses were analyzed [14]. Ke et al. studied the multidimensional extreme aerodynamic load under typical four-tower arrangements by using the wind tunnel [15]. Zhou et al. studied AWV and CVW of Guangzhou New TV Tower, in which wind force was measured and used to compute wind-induced responses by the experiment [16]. In addition, field measurement is also applied to study the wind forces and responses of towering structures. Glanville and Kwok presented the results of a field measurement on a steel frame tower, from which the dynamic characteristics, including frequencies, mode shapes, and damping values of the tower were obtained [17]. Another field measurement of the dynamic performance of the Ganton Tower under typhoons was studied by Guo et al. [18].

In this work, the study object is a chemical towering process equipment which is generally used in the fluid catalytic cracking unit suffering intensely AWV and CWV. However, unlike the conventional self-supporting equipment, this object is frame supporting, and its shell body is several meters away from the ground resulting that the equipment is becoming more likely to vibrate under wind loads. Thus, an effective approach for vibration suppression is pressing needed. Displacement restrictors installed beneath and above the support are originally proposed to limit the displacement of the equipment, resulting in a structurally nonlinear and statically indeterminate column. Hence, it is important to study the dynamic behavior of the frame-supporting column equipment with displacement restrictor.

This work aims to study the dynamic behavior of frame-supporting equipment with displacement restrictor under wind loads and to verify this vibration suppression approach. Both along-wind load and cross-wind load were applied using a noncontact electromagnetic excitation system. The frequency, damping ratio, and mode shape of the experimental model were measured; in the meantime, the displacements and elastic bending strains of the model were gained experimentally and numerically. Combined with the results, a simplified formula in which the natural frequency and the damping ratio dominate the dynamic behavior of along-wind and cross-wind vibration, respectively, is derived from multi-degrees-of-freedom, which is aimed to assist the design of the restrictor. Based on the study results, some feasible optimization suggestions are given for vibration suppression of such equipment.

2. Experimental Setup

2.1. Field Tests of the Prototype. The frame-supporting towering column in Qingdao, China, shown in Figure 1 is the prototype of this work. The parameters are listed in Table 1. As it is shown in Figure 1, the prototype consists of a frame and a cylindrical shell. The shell is fixed on the fourth floor of the frame that the bottom of the shell is 3.1 m away from the ground, which is called $H_{\text{sh}}$ in this paper. There were four displacement restrictors installed on the fifth floor in some cases, and that is named $H_{\text{sh}}$.

Modal parameters identification method under ambient excitation was applied in the field tests. The prototype is standing outside and vibrating under the wind loads. The acceleration-time history curve of the prototype was recorded. After eliminating the offset and filtering the data, random decrement technique is utilized. It is the key process to obtain the modal parameters, such as the natural frequency and the damping ratio. The results of the field test are listed in Table 2.

According to Table 2, it is shown that $f$ is small so as to the critical wind speed is about $7\sim8$ m/s, which is 4–6 class in the wind scale. It means that this frame-supporting equipment is prone to vibrate under the cross-wind loads as well. In addition, the damping ratio of the prototype is small, which is implied that the energy dissipation of the prototype is insufficient so that the vibration amplitude will be large considerably. Therefore, it is remarkable to propose a vibration suppression approach and study the dynamics behavior of the towering frame-supporting equipment in depth. A small-scale experiment and numerical simulation were carried out based on the elastic similarity criterion.

2.2. Experimental Model

2.2.1. Model of the Frame-Supporting Equipment. An aero-elastic model of the towering structure was manufactured in this study to make sure that the experimental model and prototype had the same frequency, mode shape, and damping ratio. It was not demanded to ensure that each frequency and mode shape of the prototype and the model is same, because the first mode dominates the dynamic behavior of towering structure, while the higher mode contributed little to it; thus the first frequency and the first mode shape of the prototype and the model should be the same [19].

The geometric scaling was set as 1:25, so the model was 1.7 meters high. The model was made of aluminum alloy and mainly consisted of a center bar, several cylindrical shells, and a supporting frame. The center bar was solid with a
diameter of 6 mm. This solid bar provided the stiffness of the structure and lowered the frequency at the same time.

As Figure 2(a) shows, the frame has seven levels and are denoted as $H_{1\text{st}}$ to $H_{7\text{th}}$ from the bottom up. Four bolts and nuts were set at the same height as restrictors to limit the displacement of the shell in four main directions. $H_{4\text{th}}$ is the support where the shell was supported, and no restrictor was set at this height. The restrictor, placed from $H_{5\text{th}}$ to $H_{7\text{th}}$, was denoted as upper restrictor and the restrictor, placed from $H_{1\text{st}}$ to $H_{3\text{rd}}$, was denoted as lower restrictor. Four vices were used as the test stand to fix the bottom of the frame.

In this work, an electromagnetic excitation system was originally introduced to simulate the wind loads. The excitation system was programmable and able to simulate along-wind load and cross-wind load. The consistency of the electromagnetic excitation system and the wind load would be furtherly explained in the next section. The exciter was noncontact so that the model can vibrate freely under excitation. As shown in Figure 2(b), the test equipment used in this work consisted of a model mounted on the test stand, an electromagnetic excitation system, and a signal processing system. Figure 2(c) shows the arrangement of the measurement point. Strain gauges were set both at the frame legs and the support ($H_{4\text{th}}$). To avoid stress concentration, smooth transition was created at the frame leg and two gauges were set at each side of the leg. At the support ($H_{4\text{th}}$), gauges were set along the two-orthogonal directions. A data acquisition system (DH3820) was used to record the strain signals at a sampling frequency of 50 Hz. The elastic bending

Table 1: The parameters of the prototype.

| Parameters | $H_{\text{prototype}}$ (m) | $D_{\text{prototype}}$ (m) | $H_k$ (m) | $W_k$ (m) | $H_{4\text{th}}$ (m) | $H_{5\text{th}}$ (m) |
|------------|-----------------|-----------------|-------|-------|----------------|----------------|
| Values     | 42.5            | 1.24            | 20    | 3.7   | 14.3           | 6.4            |

Table 2: The modal parameters of the prototype.

| Modal parameters | Natural frequency, $f$ (Hz) | Damping ratio, $\zeta$ |
|------------------|-----------------------------|------------------------|
| $H_{4\text{th}}$ | 1.0018                      | 0.0114                 |
| $H_{5\text{th}}$ | 1.0173                      | 0.00125                |

Figure 1: The (a) prototype and (b) cross section of the frame-supporting equipment.
Overall structure

Upper restrictor

Lower restrictor

Bottom frame

Electromagnetic exciter
Digital multimeter
Computer
Power amplifier
Signal generator

Electromagnetic excitation system

Model

DH 5908
DH 3820

Computer

Signal processing system

Figure 2: Continued.
strain gained from experiment at the frame leg was denoted as $\varepsilon_{\text{bottom}}$. The elastic bending strain gained from experiment at the support ($H_{4\text{th}}$) was denoted as $\varepsilon_{\text{support}}$.

The accelerometers were installed at the top of the model, and two accelerometers were aligned orthogonally along the major axes of the model. To balance the weight of the accelerometer, additional weight was set at the opposite side. A data acquisition system (DH5908) was used to acquire the accelerometer measurements and wind tunnel experiments, such as von Kármán spectrum, Davenport spectrum, Kaimal spectrum, and Simiu spectrum. Among them, the Davenport wind speed spectrum applied in this work is widely used to calculate the wind loads:

$$S_{p}(\omega) = \frac{\pi \omega^{2} \cos^{2} \phi}{\omega (1 + \chi^{2})^{4/3}},$$

where $K$ is the terrain rough factor, $\nu(10)$ is the mean wind speed at 10 meters above the ground, and $\omega$ is the frequency of the wind and $x = (600\omega)/(\pi \nu(10))$.

The time-domain along-wind load was programmed and applied through the electromagnetic vibration exciter. However, the only disadvantage is that the wind force could only be exerted on the top, and the sensitivity and tendency analysis are still unaffected. When a load was applied to the top of a structure of $n$ nodes, the displacement of the top at a certain time could be solved as

$$Y = \frac{P_{0}}{\phi^T \mathbf{m} \phi \omega_{n}^{2}},$$

where $P_{0}$ is the applied, $\phi$ is the mode shape vector, $\omega_{n}$ is the natural frequency of the structure.

The displacement ratio is defined as the displacement divided by the height of the structure. The displacement ratio of the prototype and the experimental model is

$$\begin{align*}
\frac{Y_{\text{prototype}}}{H_{\text{prototype}}} &= \frac{P_{0}}{H_{\text{prototype}} \phi^T \mathbf{m} \phi \omega_{n}^{2}}, \\
\frac{Y_{\text{model}}}{H_{\text{model}}} &= \frac{P_{0}}{H_{\text{model}} \phi^T \mathbf{m} \phi \omega_{n}^{2}}.
\end{align*}$$
where $Y_{\text{prototype}}$ is the displacement of prototype, $H_{\text{prototype}}$ is the total height of the prototype, $Y_{\text{model}}$ is the displacement of the experimental model, and $H_{\text{model}}$ is the total height of the model.

Since the model and the prototype have the same mode shape and natural frequency, the applied load $p_0$ must multiply a reduced factor $\delta$ before being applied on the model to ensure that the two structures had the same displacement ratio. The lumped mass matrices of the two structures were assumed to be changing in a linear trend described as $M_{\text{prototype}} = \beta M_{\text{model}}$, where $\beta$ is the amplification coefficient of the mass. The reduced factor $\delta$ was then solved as

$$\delta = \frac{H_{\text{model}}}{\beta H_{\text{prototype}}} \quad (6)$$

2.3.2. Cross-Wind Load. Due to the complexity of CWV, current research usually uses the semiempirical model to simplify the fluid-solid interaction in CWV. Through wind-tunnel experiments, the parameters of semiempirical model are gained. According to a large number of tests, many models of the CWV are proposed by scholars [26–29]. Considering that the motion of structure in CWV is nearly sinusoidal, the harmonic force model was presented. The cross-wind load can be conservatively estimated by the following equation:

$$F_L(t) = q_0 D C_L \cos(\omega_s t), \quad (7)$$

where $q_0$ represents the basic wind pressure of the local area, $D$ is the column outer diameter, $C_L$ is root mean square (RMS) lift coefficient, $\omega_s = 2\pi f_s$, and $f_s$ is the vortex shedding frequency.

This model has its defects that the fluid-solid interaction was not considered and the lock-in effect cannot be explained. However, this model has a clear mechanical conception and is easy to calculate. Europe Code and Chinese Code use this model to calculate the CWV [30, 31].

In order to study the vibration of the structure under cross-wind load when resonating, this work applied the harmonic force model to simulate the cross-wind load. The cross-wind load when resonating is written as

$$F_L(t) = \frac{1}{2} \rho v_s^2 D C_L \cos(\omega_s t), \quad (8)$$

where $v_s = (\omega_s D)/(2\pi S t)$ is the critical wind velocity, $S t$ is the Strouhal number, $\omega_s = 2\pi f_s$, and $f_s$ is the natural frequency of the structure.

As for the tower frame structures, the Reynolds number is in the supercritical regime when CWV occurs [12]. According to wind tunnel experiments [32], the RMSs of the lift coefficient $C_L$ and $S t$ are almost constant regardless of the Reynolds number. In this work, $C_L$ and $S t$ are taken as the constant values. Hence, equation (8) is simplified as

$$F_L(t) = f_s^2 C \cos(\omega_s t), \quad (9)$$

where $C$ is a constant number.

According to equation (9), $F(t)$ is proportional to $f_s^2$. The frequency of the experimental model is the only factor that changes when the restrictor height is changing. Here, the excited load is multiplied by an amplification factor $\gamma$ which is defined as

$$\gamma = \left(\frac{f_{\text{restrictor}}}{f_{\text{none-restrictor}}}\right)^2, \quad (10)$$

where $f_{\text{restrictor}}$ represents the natural frequency when the displacement restrictor is equipped and $f_{\text{none-restrictor}}$ is the natural frequency when there is only the support.

There were several CWV of chemical towers occurred in China recently, with the amplitudes of the vibrations ranging from $(H_{\text{prototype}}/600)$ to $(H_{\text{prototype}}/300)$ due to different types of the structures. In this work, the cross-wind load was set as a sinusoidal force to excite the structure to vibrate under certain amplitude. The vibration amplitude of the experimental model without restrictor was set to be $(H_{\text{model}}/300)$. After setting restrictors, the frequency changed, and the excited load was multiplied by the amplification factor $\gamma$.

3. Numerical Simulation Setup

The experimental model has a complex structure which makes it difficult to get experiment data in some place. Therefore, the numerical simulation was carried out using ANSYS 15.0 to get relevant values which are unable to gain from experiment. The numerical simulation was used to gain $y_{\text{top}}, \varepsilon_{\text{support}},$ and $\varepsilon_{\text{bottom}}$ at the testing point to verify the correctness of the simulation. The maximum elastic strain of the support ($H_{4\text{th}}$) as shown in Figure 3(b) and the shell above the frame top as shown in Figure 3(c) were gained from the simulation results to evaluate the vibration suppression effectiveness after setting restrictors and were denoted as $\varepsilon_{\text{support}}$ and $\varepsilon_{\text{shell}}$ respectively. Elastic bending strain gained from experiment and simulation were both referred to as $\varepsilon$. The numerical simulation was based on the damping values gained from experiment. The Rayleigh damping matrix was used in ANSYS to simulate the dynamic behavior of the structure under different loads.

4. Results and Discussion

In this part, the symbols used are emphasized as follows: $y_{\text{top}}$ is the top displacement of the frame-supporting structure, mm. $\varepsilon_{\text{support}}$ is the strain of the support in $H_{4\text{th}}, \mu\varepsilon$. $\varepsilon_{\text{support}}$ max is the max strain of the support in $H_{4\text{th}}, \mu\varepsilon$. $\varepsilon_{\text{bottom}}$ is the strain in the bottom of the frame, $\mu\varepsilon$, $\varepsilon_{\text{shell}}$ is the strain in the shell which contacted with the restrictor, $\mu\varepsilon$.

4.1. Frequency. The frequency of the experimental model was obtained from the acceleration response under free vibration test using fast Fourier transformation (FFT), of which the curves are shown in Figure 4. Spectral peaks indicated that the first mode dominated the response of the structure at 0.854 Hz. The first frequency of structure is
shown in Table 3. Compared with Table 2, the natural frequency of the experiment model is consistent with the prototype, which verified the validity of the experimental model. In addition, the experimental data were compared with the numerical result and showed good agreement. As Table 3 shows, the frequency increased with a maximum value, 30.3%, after setting restrictors. Comparing the values of $H_{1\text{st}} \sim H_{3\text{rd}}$, the frequency remained the same when changing lower restrictor height and the frequency only increased 4.8% compared with structure without restrictor ($H_{4\text{th}}$). The frequency of the structure enlarged with increasing of upper restrictor height when comparing $H_{3\text{rd}}$ to $H_{7\text{th}}$. The $H_{1\text{st}}$-$H_{5\text{th}}$, $H_{1\text{st}}$-$H_{6\text{th}}$ and $H_{1\text{st}}$-$H_{7\text{th}}$ showed the frequency of the experimental model when setting both upper and lower restrictors with the lower one fixed at $H_{1\text{st}}$. Comparing the last three columns with 5th to 7th columns, the frequency increased a little when setting two restrictors compared with one upper restrictor condition. Based on the results, it can be concluded that setting restrictors increased the frequency of the structure, and the frequency was determined by the height of upper restrictor.

4.2. Mode Shape. Eight additional accelerometers were set at different height to measure the mode shape of the structure. Mode shape was determined from the ratio of acceleration signals which were recorded simultaneously. The mode shapes measured by the test were compared with those calculated by simulation. As shown in Figures 5(a) and 5(b), two kinds of structure were chosen to verify the numerical simulation which agreed well with the experiment results. Therefore, as shown in Figures 5(c)–5(f), the simulation results were used to study the mode shapes of the frame-supporting structural under different restrictor heights. The mode shape was divided into two regions: beneath the restrictor was denoted as the supported region and above the restrictor was denoted as the free vibration region. Figures 5(c) and 5(d) show the mode shape of structure after setting one-layer restrictors. Figure 5(c) shows that increasing the height of the lower restrictor smoothed the mode shape only in supported region with a same critical point of 0.385. Figure 5(d) shows that changing the height of upper restrictor changes the mode shape in both supported region and free vibration region with uncertain critical points of $0.268 \sim 0.290$, indicating that setting one-layer upper restrictors increased the stiffness of the whole structure compared with that of structure without restrictor ($H_{4\text{th}}$). Figure 5(e) exhibits the mode shapes of structure with two restrictors, of which the upper one is fixed on $H_{7\text{th}}$. It can be observed that the mode shape is smoothing through the
upper restrictor but with the same critical point. Figure 5(f) also presents the mode shapes of structure with two restrictors, of which the lower one is fixed on $H_{1\text{st}}$, in which the critical points are various. Comparing all the results in Figure 5, the mode shape became smoother after setting restrictors and was mainly determined by the height of upper restrictor.

4.3. Damping Ratio. The frame-supporting structure was firstly excited using a sinusoidal force. After achieving stable vibration, the excited force was removed and the structure began to vibrate freely in a decay mode. Using this free vibration decay curve, the damping ratio was measured and these values were used in numerical simulation. The damping ratio of structure with different numbers of restrictors and heights were tested under various excited amplitudes. Results showed that the damping ratio of this structure did not change with different displacements. The damping ratio measured by experiments is shown in Figure 6. Compared with Table 2, the damping ratio of the model was almost the same with the prototype. Moreover, the damping ratio of the structure increased after setting restrictors compared with no restrictor ($H_{4\text{th}}$). The damping ratio in the same color indicated that changing the lower restrictor did not influence the damping ratio of the structure. As discussed in Figure 5, the height of the lower restrictor had little influence on mode shape, and due to the same vibration mode, the damping ratio of different lower restrictors remained the same. Comparing the damping ratios of different upper restrictor heights, the damping ratios enlarged with the increasing of the restrictor height and the increasing rates were 14%, 34%, and 60%, respectively. It is also illustrated that the upper restrictors play a primary role in vibration suppression.

4.4. Along-Wind Vibration. According to design conditions, the basic wind pressure of the city where the prototype installed was 650 N/m² so that the wind velocities applied to simulate the along-wind load were set as 18 m/s, 22 m/s, 26 m/s, 30 m/s, and 34 m/s, respectively. Figure 7 shows the time history and power spectral density (PSD) curves of different wind velocities.

Figure 7(a) shows the time history of along-wind load for prototype, and the wind velocity was set as 18 m/s. The corresponding PSD curve is shown in Figure 7(b). Analogously, the PSD curves of different wind velocities used to generate along-wind load are shown in Figures 7(c)–7(f). Each PSD curve was compared with theoretical calculation values using equation (3). Results showed that the generated along-wind load matched well with the theory and it can reflect the fluctuation of the wind load. Then after multiplying the reduce factor $\delta$ calculated by equation (6), the along-wind load was applied to the structural top using the programmable excitation system. The structure exhibited linear elastic behavior under different along-wind loads, so the results obtained under the maximum along-wind load (Figure 7(f)) were studied. The RMS values of displacement and strains were used to analyze the results.

The tested time history curves of acceleration and strain are presented in Figure 8. It is apparently that the time history curves in along-wind test are in random coinciding with the properties of wind velocity. Figure 9 shows the time

| Frequency | $H_{1\text{st}}$ | $H_{2\text{nd}}$ | $H_{3\text{rd}}$ | $H_{4\text{th}}$ | $H_{5\text{th}}$ | $H_{6\text{th}}$ | $H_{7\text{th}}$ | $H_{1\text{st}}$ | $H_{5\text{th}}$ | $H_{1\text{st}}$ | $H_{7\text{th}}$ |
|-----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Test (Hz) | 0.895           | 0.895           | 0.895           | 0.854           | 0.971           | 1.022           | 1.113           | 0.977           | 1.025           | 1.115           |
| Simulation (Hz) | 0.921           | 0.919           | 0.913           | 0.834           | 1.016           | 1.082           | 1.156           | 1.019           | 1.082           | 1.158           |
| Error (%) | 2.823           | 2.612           | 1.972           | 2.398           | 4.429           | 5.545           | 3.720           | 4.093           | 5.286           | 3.738           |

Figure 4: Time history curves in the model test.
Figure 5: Mode shapes of the structure under different restrictor heights: (a) verified through the values in $H_4$, (b) verified through the values in $H_1$, (c) one lower restrictor, (d) one upper restrictor, (e) lower restrictor height $= H_{1st}$, and (f) upper restrictor height $= H_{7th}$. 
history curves of $y_{\text{top}}$ and $\varepsilon_{\text{bottom}}$ obtained from the test and the simulation. The maximum errors for $y_{\text{top}}$, $\varepsilon_{\text{support-max}}$, and $\varepsilon_{\text{bottom}}$ were 5.5%, 9.7%, and 8.9% showing numerical results matches well with experimental data. Figure 10(a) shows the comparison of the $y_{\text{top}}$ and strain with one restrictor. After setting one restrictor, it is obvious that $y_{\text{top}}$ and $\varepsilon_{\text{support-max}}$ decreased. With the increasing of the upper restrictor height, $y_{\text{top}}$ and $\varepsilon_{\text{support-max}}$ decreased 55.7% and 92.1%, respectively, compared with none restrictor ($H_{4\text{th}}$). But $y_{\text{top}}$ and strain remained almost the same when changing the height of lower restrictor, and the values decreased 16.4% and 50.5% correspondingly compared with none restrictor ($H_{4\text{th}}$). $\varepsilon_{\text{shell}}$ and $\varepsilon_{\text{bottom}}$ changed very little with or without restrictors. Since changing lower restrictor height did not influence the results, frame-supporting structure with lower restrictor setting to $H_{1\text{st}}$ was selected as a representative to study. As shown in Figure 10(b), compared with none restrictor ($H_{4\text{th}}$), after setting two restrictors, $y_{\text{top}}$ and $\varepsilon_{\text{support-max}}$ decreased with increasing of the upper restrictor height, and the values dropped by 56.3% and 90.1%, respectively. However, $\varepsilon_{\text{shell}}$ and $\varepsilon_{\text{bottom}}$ remained almost the same.

Comparing Figure 10(a) with 10(b), $y_{\text{top}}$ and $\varepsilon_{\text{support-max}}$ changed greatly, while $\varepsilon_{\text{shell}}$ and $\varepsilon_{\text{bottom}}$ remained almost the same. More importantly, the changing trends of $y_{\text{top}}$ and $\varepsilon_{\text{support-max}}$ always keep the same pace. That is to say that $y_{\text{top}}$ can represent the effectiveness of vibration suppression. Therefore, a simplified formula based on $y_{\text{top}}$ is derived subsequently to assist the design of restrictors when the structure suffering AWV.

Due to the fluctuation of the along-wind load, the structure vibrated randomly. $y_{\text{top}}$ was solved by random vibration theory as

$$y_{\text{top}} = \sqrt{\int_{-\infty}^{\infty} S(\omega) d\omega},$$  \hspace{1cm} (11)

$$S(\omega) = \sum_{m} \sum_{n} \phi_m(\omega)H_m(-i\omega)H_n(i\omega)S_{P_mP_n}(\omega),$$  \hspace{1cm} (12)

where $H_m(-i\omega)$ and $H_n(i\omega)$ are the frequency response functions of mode shape $m$ and $n$ and $S_{P_mP_n}(\omega)$ is the cross spectral density function of random force $P_m(t)$, $P_n(t)$:

$$H_m(-i\omega) = \frac{1}{\omega_n^2|\alpha_m|} \left[ \frac{1}{(1 - (i2\xi_m(\omega/\omega_m))) - (\omega/\omega_m)^2} \right],$$  \hspace{1cm} (13)

$$H_n(i\omega) = \frac{1}{\omega_n^2|\alpha_n|} \left[ \frac{1}{(1 + (i2\xi_n(\omega/\omega_n))) - (\omega/\omega_n)^2} \right].$$

According to Table 3 and Figure 6, the towering frame structure is small damping system with sparsity of vibration-mode frequency, and equations (11) and (12) can be simplified as

$$y_{\text{top}} = \sqrt{\int_{-\infty}^{\infty} \sum_{m} \phi_m^2(\omega)H_m(-i\omega)H_m(i\omega)S_{P_mP_m}(\omega) d\omega}.$$  \hspace{1cm} (14)

As for $y_{\text{top}}$, the first-order mode of vibration is the main factor, and the first mode shape has been normalization with...
Figure 7: Time history and PSD of different wind velocities: (a) time history of along-wind load, (b) PSD of $V = 18$ m/s, (c) PSD of $V = 22$ m/s, (d) PSD of $V = 26$ m/s, (e) PSD of $V = 30$ m/s, and (f) PSD of $V = 34$ m/s.
Figure 8: Time history-acceleration curves in the top of the structure (a) and strain curve in the bottom of the frame structure (b) in along-wind test.

Figure 9: $y_{\text{top}}$ and $\varepsilon_{\text{bottom}}$ of structure without restrictor: (a) time history curve of $y_{\text{top}}$ and (b) time history curve of $\varepsilon_{\text{bottom}}$.

Figure 10: RMS $y_{\text{top}}$ and RMS strain under along-wind: (a) one restrictor and (b) two restrictor.
the top of the structure which means $\phi_{m1}^2 = 1$. And substituting equation (13) into equation (14), it is simplified as follows:

$$y_{top} = \frac{1}{\omega_1^2 M_1} \sqrt{\int_0^\infty \left[ \frac{1}{1 + (4\xi_i^2 - 2)(\omega/\omega_i)^2 + (\omega/\omega_i)^4} \right] \phi_{1m}^2 \phi_{im} \omega_i d\omega_i}$$

(15)

where $M_1 = \phi_1^T m_1 \phi_1$ and $\phi_i = [\phi_{i1}, \ldots, \phi_{in}]^T$ is the first mode shape, $m_1 = \text{diag}[m_{11}, \ldots, m_{n1}]$ is the lumped mass matrix, and thus $M_1 = \sum_1^n \phi_{i1m}^2 \phi_{im}$; and the integral term in equation (15) is replaced by $f(\omega_1, \xi_i)$. After setting the restrictor or changing the height of the restrictor, the mass matrix remained the same, while the first frequency and first-mode shape changed. The ratio of $y_{top}$ for structure $m$ and structure $n$ solved as

$$\frac{y_{top, m}}{y_{top, n}} = \frac{\omega_{1m}^2 m_{1m} \sum_1^n \phi_{1m, n} \phi_{1m, n} \omega_{1m} f(\omega_{1m}, \xi_{1m})}{\omega_{1n}^2 m_{1n} \sum_1^n \phi_{1m, n} \phi_{1m, n} \omega_{1n} f(\omega_{1n}, \xi_{1n})}.$$  

(16)

For decoupling the function $f_i$, numerical simulation was carried out to verify that the ratio of $y_{top}$ was independent of the damping ratio. The studying objects were structure without restrictor ($H_4$) and structure with two restrictors ($H_{24h}$). The damping ratio of $H_4$ changed from 0.01 to 0.016, and the damping of $H_{24h}$ changed from 0.016 to 0.01. The applied along-wind load is shown in Figure 7(f), and it was multiplied by the reduce factor $\delta$. The variation of displacements for two structures after changing the damping ratio under along-wind load was investigated numerically. Comparing with the changing of the damping ratio (60%), the displacement changed a little (10%). The independence of damping was verified. So, equation (16) can be written as

$$\frac{y_{top, m}}{y_{top, n}} = \frac{\omega_{1n}^p \sum_1^n \phi_{1m, n} \phi_{1m, n} \omega_{1m} f(\omega_{1m}, \xi_{1m})}{\omega_{1m}^p \sum_1^n \phi_{1m, n} \phi_{1m, n} \omega_{1n} f(\omega_{1n}, \xi_{1n})}.$$  

(17)

Substituting the values in Figures 10(a) and 10(b) to equation (17), the exponent $p = 1$.

In summary, the function model to predict the displacement of two frame-supporting structures in AWV can be written as follows:

$$\frac{y_{top, m}}{y_{top, n}} = \frac{\sum_1^n \phi_{1m, n} \phi_{1m, n} \omega_{1m}^3}{\sum_1^n \phi_{1m, n} \phi_{1m, n} \omega_{1n}^3}.$$  

(18)

Since the AWV of frame-supporting structure was determined by the first frequency, increasing frequency could decrease the displacement and strains of the structure. According to Table 3 and Figures 10(a) and 10(b), setting upper restrictor increased the frequency greatly and decreased the displacement and strains of the structure. Therefore, setting upper restrictor is the one of the most effective and economic ways to suppress AWV. What is more, the simplified AWV model is derived, which can be conductive to such structural designs.

4.5. Cross-Wind Vibration. The cross-wind vibration is caused by the vortex shedding from itself; therefore, it is regular. The time history curves in Figure 11 is regular, which can be divided into three parts including starting, stable, and decay vibration regions. The amplitude after stabilization was used to analyze the results. Numerical results were verified using experiment data and the maximum errors for $y_{top}$, $\epsilon_{support}$, and $\epsilon_{bottom}$ were 5.2%, 7.9%, and 5.7%, respectively. Figure 12(a) compared the displacement and elastic strain of structure with one restrictor. As shown in Figure 12(a), compared with none restrictor ($H_{4th}$), $y_{top}$ and $\epsilon_{support, max}$ decreased 43.4% and 88.6%, respectively, with the increasing of the upper restrictor height. On the contrary, $y_{top}$ and strain remained almost the same when changing the height of lower restrictor. When setting upper restrictor, though $\epsilon_{bottom}$ and $\epsilon_{shell}$ decreased slowly with increasing upper restrictor height, these values were about 27% larger than values of none restrictor ($H_{4th}$). Changing lower restrictor height did not influence the results, so structure with lower restrictor setting to $H_{1st}$ was selected as a representative to study. As shown in Figure 12(b), compared with none restrictor ($H_{4th}$), after setting two restrictors, $y_{top}$ and $\epsilon_{support, max}$ decreased with increasing of the upper restrictor height and reduced 46.9% and 85.3%, respectively. Just as the structure with one restrictor, when setting upper restrictor, though $\epsilon_{bottom}$ and $\epsilon_{shell}$ decreased slowly with increasing upper restrictor height, these values were about 25% larger than values of none restrictor ($H_{4th}$). Comparing Figures 12(a) with 12(b), $y_{top}$ and $\epsilon_{support, max}$ decreased greatly, while $\epsilon_{shell}$ and $\epsilon_{bottom}$ increased a little. Comparing the reducing values of $y_{top}$ and $\epsilon_{support, max}$ with the increasing values of $\epsilon_{shell}$ and $\epsilon_{bottom}$, these increments were acceptable. Just as the AWV of the structure, the changing trends of $y_{top}$ and $\epsilon_{support, max}$ were the same. Hence, $y_{top}$ also represented the effectiveness of vibration suppression. A simplified formula is also derived to assist the design of restrictors when the structure suffering CWV.

Since the structure was under resonance under cross-wind load, after stabilization, similar to AWV, only one-order mode of vibration is considered, and the harmonic CWV force is written as a force vector in the following equation:

$$F(t) = \frac{-F}{2}i[\exp(i\omega t) - \exp(-i\omega t)] \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \quad (19)$$

As for such resonance reaction, the frequency-domain solution is easily available; thus, $y_{top}$ in CWV is solved as

$$Y_n(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} P_n(\omega) H_{n}(\omega) \exp(i\omega t) d\omega.$$  

(20)

Taking equation (13) into equation (20) and $\bar{\omega} = \omega_n$ in resonance, equation (20) is solved as

$$y_{top} = \frac{F}{2} \frac{1}{\omega_1^2 \phi_1^T m_1 \phi_1 \xi_1}.$$  

(21)
Like the AWV, after setting the restrictor or changing the height of the restrictor, the mass matrix remained the same. The ratio of $y_{\text{top}}$ for structure $m$ and structure $n$ was solved as

$$y_{\text{top}}^{m} = \frac{F_{L,m}}{F_{L,n}} \sum_{i=1}^{n} \left( \frac{\omega_{1m}}{\omega_{1n}} \right) \phi_{i1,n}^2 \frac{\xi_{1n}}{\xi_{1m}}$$ \hspace{1cm} (22)

As equation (9) shows, the force $F_L$ is linked with the natural frequency, and thus the equation (22) can be simplified as

$$y_{\text{top}}^{m} = \frac{\sum_{i=1}^{n} \phi_{i1,n}^2 \xi_{1n}}{\sum_{i=1}^{n} \phi_{i1,m}^2 \xi_{1m}}$$ \hspace{1cm} (23)

Equation (23) is the function model to predict the displacement of two frame-supporting structures in CWV.

Since the CWV of frame-supporting structure was determined by damping, increasing damping could decrease the displacement and strains of the structure. According to Figures 6, 12(a), and 12(b), setting upper restrictors increased the damping greatly resulting in decreasing the displacement as well as strains of the structure. Therefore, setting upper restrictor is the most effective and economic way to suppress CWV.

In summary, the simplified formula to assist the design of restrictors under along wind and cross wind is generalized as follows:

$$y_{\text{top}}^{m} = \frac{\sum_{i=1}^{n} \phi_{i1,n}^2 \xi_{1n}^{a} \left( \frac{\omega_{1n}}{\omega_{1m}} \right)^b}{\sum_{i=1}^{n} \phi_{i1,m}^2 \xi_{1m}^{a} \left( \frac{\omega_{1m}}{\omega_{1m}} \right)^b}$$ \hspace{1cm} (24)

where $a$ and $b$ are the coefficients. For AWV, $a = 0$, and $b = 1$; for CWV, $a = 1$, and $b = 0$. 

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**Graphs:**

**Figure 11:** Time history-acceleration curves in the top of the structure (a) and strain curve in the bottom of the frame structure (b) in the cross-wind test.

**Figure 12:** Amplitude of $y_{\text{top}}$ and strain under cross wind: (a) one restrictor and (b) two restrictor.
5. Conclusion

In this paper, a displacement restrictor is proposed as a method for vibration suppression of structure under wind load and the effectiveness is discussed through the dynamic behavior research of frame-supporting structure under wind load experimentally and numerically. Using electromagnetic excitation system to simulate the along-wind load and cross-wind load is introduced as a new method in this work. The effectiveness and load equivalent method of this excitation system have been shown in this paper. The random along wind and harmonic cross wind can be regenerated perfectly, and the dynamic behavior of the structure could be excited. It is approved that displacement restrictor is an effective approach for vibration suppression through evaluating the displacement and strain. The upper displacement restrictor is indispensable, and two-layer restrictor is recommended. The displacements under AWV and CWV decrease about 50% when the upper restrictor is installed at the 40% height.

A simplified formula is derived to assist the design of the displacement restrictor. The accessible modal parameters including modal shape, natural frequency, and damping ratio are applied to calculate the amplitude ratio of two structures and select a better design featuring an efficient vibration suppression performance. This work presented an important design guide to the frame-supporting structure and is of great significance to the economical and safe design.

Nomenclature

- $H_{\text{prototype}}$: The overall height of the prototype, m
- $D_{\text{prototype}}$: The cylinder diameter of the prototype (m)
- $H_i$: The height of the supporting frame (m)
- $W_i$: The width of the supporting frame (m)
- $H_{4th}$: The height of the $i$th restrictor (m) and $H_{4th}$ is the location of the support
- $\varepsilon_{\text{bottom}}$: The elastic bending strain gained from experiment at the frame leg $\mu e$
- $\varepsilon_{\text{support}}$: The elastic bending strain gained from experiment at the support ($H_{4th}$) ($\mu e$)
- $a_{\text{top}}$: The acceleration gained from experiment (mm/s)
- $y_{\text{top}}$: The displacement on the top of the structure (mm)
- $\varepsilon_{\text{support-max}}$: The max strain of the support in $H_{4th}$ ($\mu e$)
- $\varepsilon_{\text{shell}}$: The strain in the shell which contacted the restrictor ($\mu e$).

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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