Radiation in Lorentz violating electrodynamics

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Abstract. Synchrotron radiation is analyzed in the classical effective Lorentz invariance violating model of Myers-Pospelov. Within the full far-field approximation we compute the electric and magnetic fields, the angular distribution of the power spectrum and the total emitted power in the m-th harmonic, as well as the polarization. We find the appearance of rather unexpected and large amplifying factors, which go together with the otherwise negligible naive expansion parameter. This opens up the possibility of further exploring Lorentz invariance violations by synchrotron radiation measurements in astrophysical sources where these amplifying factors are important.

INTRODUCTION

Possible manifestations of quantum gravity could arise as modified energy-dependent dispersion relations violating the Lorentz symmetry. Such modifications were originally proposed in [1], opening up the door to quantum gravity phenomenology. The first heuristical derivation of such a type of dispersion relations from a theoretical framework was done in the context of a Loop Quantum Gravity (LQG) inspired model of corrections to flat space dynamics, leading to a consistent extension of Maxwell electrodynamics with linear correction terms in the quantum gravity scale [2]. An alternative approximation to LQG, inspired on Thiemann’s regularization [3], extended such results to photons[4] and fermions[5]. For a review see for example [6]. Such estimations have to be understood as preliminary steps in the understanding of the semiclassical limit of LQG, which still remains an open problem. An alternative proposal for a quantum gravity scale modified Maxwell and fermionic dynamics, based on string theory, was developed in [7]. Recently, an effective field theory approach to the formulation of quantum gravity induced effects has been put forward [9]. The Gambini-Pullin (GP) [2] and Myers-Pospelov (MP)[9] models of modified electrodynamics lead to a frequency and helicity dependent speed of light. Even though both models differ in the details, the radiation regime behavior is similar, up to numerical factors of order one.

Using the synchrotron radiation from the Crab Nebulae, Jacobson et al. have found stringent bounds upon the parameters describing the modified dispersion relations[10]. They are based on the synchrotron radiation cut-off frequency $\omega_c$, and use a heuristic kinematical estimation of $\omega_c$, where photon and electron velocities are determined by the modified dispersion relations. These kinematical estimations do not embody birefringence effects such as the ones appearing in the GP and MP theories, which lead to new effects in the radiation spectrum.
Here we examine this problem from the perspective of the MP effective theory, a dimension five modified electrodynamics which describes the dynamics arising from linear corrections at the quantum gravity scale. We develop a complete calculation of the synchrotron radiation. This allows the use of additional observational information, such as the polarization of the radiation, in order to set new bounds on the Lorentz violation parameters. Some of the results have been already presented in Ref. [8]. In this work we restrict ourselves to the classical regime, in such a way that the fine-tuning problems associated with the radiative corrections of the model are not considered [9, 11, 12].

**MYERS-POSPELOV ELECTRODYNAMICS**

The MP effective field theory includes Lorentz violations generated by dimension five operators, parametrized by a vector $V^\mu$, and assumed to be suppressed by a factor $M_P^{-1}$ ($M_P \approx 10^{19}$ GeV is the Planck mass). The electromagnetic field and the dynamics of the charge are affected by these perturbations and lead to distinctive signatures. Here we restrict ourselves to the case $V^\mu = (1, 0)$, enough to put in evidence the main consequences of these violations.

The MP action for the electromagnetic field is

$$S_{MP} = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - 4\pi j^\mu A_\mu + \tilde{\xi} (V^\alpha F_{\alpha\delta})(V^\beta \partial^\beta) \right],$$  

(1)

where $\tilde{\xi} = \xi / M_P$, with $\xi$ being a dimensionless parameter. We take the unperturbed velocity of light in vacuum, $c$, equal to 1. This is a gauge theory similar to Maxwellian electrodynamics, and implies the conservation of the electrical current $j^\mu = (\rho, j)$. The field tensor is $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and the electric and magnetic fields are given by $E_i = F_{0i}$ and $B_k = -\frac{1}{2} \epsilon_{kij} F_{ij}$, respectively. Hence $E$ and $B$ satisfy

$$\nabla \cdot B = 0, \quad \nabla \times E + \partial_t B = 0.$$  

(2)

In the rest frame $V^\alpha = (1, 0)$, the equations of motion derived from the action (1) are

$$\nabla \cdot E = 4\pi \rho,$$  

(3)

$$-\partial_t E + \nabla \times B + \tilde{\xi} \partial_t (-\nabla \times E + \partial_t B) = 4\pi j.$$  

(4)

This dynamics gives place to an energy momentum tensor $T_{\mu\nu}$ with components

$$4\pi T_{00}^0 = \frac{1}{2} (E^2 + B^2) - \tilde{\xi} E \cdot \partial_t B,$$  

(5)

$$4\pi S = E \times B - \tilde{\xi} E \times \partial_t E,$$  

(6)

which, outside the sources, satisfy $\partial_t T^{00} + \nabla \cdot S = 0$. These expressions for the energy-momentum tensor and their conservation law are exact in $\tilde{\xi}$.

Alternatively, the equation of motion for $A$ in the radiation gauge is

$$\partial_t^2 A - \nabla^2 A + 2\tilde{\xi} \nabla \times \partial_t^2 A = 4\pi \left( j - (\nabla \nabla^{-2} \nabla) j \right) \equiv 4\pi j_r,$$  

(7)
which in momentum space takes the form
\[
(-\omega^2 + k^2 - 2i\xi \omega^2 k \times) A(k, \omega) = 4\pi j_T(k, \omega).
\] (8)

In the circular polarization basis, where the components of a vector field \(A\) are \(A^\pm = \frac{1}{2} [A - (\hat{k} \cdot A) \hat{k} \pm i(\hat{k} \times A)]\) the equations (8) decouple according to
\[
(-\omega^2 + k^2 \pm 2i\xi \omega^2 k) A^\pm = 4\pi j^\pm_T.
\] (9)

Thus, outside the sources the potential fields \(A^\pm\) propagate with well defined energy-dependent phase velocities \(v^{-1}_\pm = \sqrt{1 + \omega^2 \xi^2 \pm \omega \xi}.\)

In other words, the electromagnetic field in the MP theory propagates in vacuum as in a standard dispersive birefringent medium, where the modes with circular polarization have well defined refraction indices
\[
n(\lambda z) = \sqrt{1 + z^2 + \lambda z},
\] (10)

with \(z = \xi \omega\) and \(\lambda = \pm 1\).

The dynamics of a classical charged particle can be obtained from the geometrical optics limit of a scalar charged field. For this field the MP action is
\[
S_{MP} = \int d^4x \left[ \partial_\mu \phi^* \partial^\mu \phi - \mu^2 \phi^* \phi + i\eta \phi^* (V \cdot \partial)^3 \phi \right],
\] (11)

and leads to the dispersion relation \((p^0)^2 + \tilde{\eta} (p^0)^3 = p^0 + \mu^2\), in the reference frame where \(V^\alpha = (1, 0)\). Here \(\tilde{\eta} = -\eta / M_P\), where \(\eta\) is the dimensionless constant employed in the parameterization of Jacobson et al.. From the above dispersion relations, using the minimal interaction for the electromagnetic field and the geometric optics limit, we obtain the equation of motion for a massive charged particle interacting with a magnetic field, up to second order in \(\tilde{\eta}\)
\[
\ddot{r} = \frac{q}{E} \left(1 - \frac{3}{2} \tilde{\eta} E + \frac{9}{4} \tilde{\eta}^2 E^2 \right) (v \times B),
\] (12)

with \(E\) being the energy of the particle. To simplify, we will consider the motion in a plane orthogonal to \(B\), which corresponds to the circular orbit
\[
r(t) = R (\cos \omega_0 t, \sin \omega_0 t, 0),
\] (13)

with radius \(R = \beta / \omega_0\) and Larmor frequency
\[
\omega_0 = \frac{|q|B}{E} \left(1 - \frac{3}{2} \tilde{\eta} E + \frac{9}{4} \tilde{\eta}^2 E^2 \right).
\] (14)

The range of energies expected in astrophysical systems satisfies \(E < (\mu / \tilde{\eta})^{1/2}\), with \(\mu / E << 1\) and \(\tilde{\eta} E << 1\) and hence the Lorentz factor \(\gamma\) can be approximated by
\[
\gamma = (1 - \beta^2)^{-1/2} \approx \frac{E}{\mu} \left(1 + 2 \tilde{\eta} E^3 \right)^{-1/2},
\] (15)

where we are using the standard definition \(\beta = |v|/c\).
RADIATION FIELD

The solution for the equation of motion (8) in the radiation approximation can be written as

$$A(\omega, r) = A_+ (\omega, r) + A_- (\omega, r) = \frac{1}{r} \frac{2n(z)}{1 + n^2(z)} \sum_{\lambda = \pm 1} n(\lambda z) e^{i\omega \lambda r \cdot j^\lambda (\omega, \mathbf{k}_\lambda)}, \quad (16)$$

where $r = r \hat{n}$, $k_\lambda = \omega n(\lambda z) \hat{n}$. The fields $A_+ (\omega, r)$ and $A_- (\omega, r)$ correspond to right and left circular polarization respectively. The electric and magnetic fields are $E(\omega, r) = i\omega A(\omega, r)$ and $B(\omega, r) = \nabla \times A(\omega, r)$. Let us remark that, contrary to the standard case, $E$ and $B$ are not orthogonal.

Although these fields provide a complete description of the radiation, the angular distribution of the power spectrum itself is very relevant from the phenomenological point of view. The energy emitted can be computed using the Poynting vector (6)

$$E = \int_{-\infty}^{\infty} dt \, \mathbf{n} \cdot \mathbf{S}(t, \mathbf{r}) \, r^2 d\Omega, \quad (17)$$

which is related to the power spectrum distribution by

$$E = \int_0^{\infty} d\omega \int d\Omega \frac{d^2 E}{d\Omega d\omega} = \int_0^{\infty} d\omega \int d\Omega \int dT \frac{d^2 P(T)}{d\omega d\Omega}. \quad (18)$$

The final expression for the angular distribution of the energy spectrum in terms of the potentials results

$$\frac{d^2 E}{d\Omega d\omega} = \frac{r^2 \omega^2}{8\pi^2} \frac{1 + n^2(z)}{n(z)} [A_- (-\omega, \mathbf{r}) \cdot A_+ (\omega, \mathbf{r}) + A_- (\omega, \mathbf{r}) \cdot A_- (-\omega, \mathbf{r})]. \quad (19)$$

To compute this distribution it is only necessary to express the products $A_\mp (-\omega, \mathbf{r}) \cdot A_\pm (\omega, \mathbf{r})$ in terms of the current $j(\omega, \mathbf{k})$ via the relation (16). This yields

$$\frac{d^2 P(T)}{d\omega d\Omega} = \frac{\omega^2}{2\pi^2} \sum_{\lambda = \pm 1} n^3(\lambda z) \int_{-\infty}^{\infty} d\tau e^{-i\omega \tau} e^{i\omega n(\lambda z) \hat{n} \cdot (\mathbf{r}(T+\tau/2)-\mathbf{r}(T-\tau/2))} \mathbf{j}, \quad (20)$$

for the angular distribution of the radiated power spectrum, with

$$\mathbf{j} = \frac{1}{2} [j^* (T + \tau/2) \cdot j (T - \tau/2) - n^{-2}(\lambda z) \rho^*(T + \tau/2) \rho (T - \tau/2)$$

$$-i\lambda \hat{n} \cdot j^* (T + \tau/2) \times j (T - \tau/2)]. \quad (21)$$

Another relevant characteristic of the radiation is its polarization. To describe the polarization, we will use the reduced Stokes parameters $\nu, q, u$, according to the definitions of Ref. [13], where they are written in terms of the circular polarization basis $e_\pm$ and satisfy the constraint $1 = \nu^2 + q^2 + u^2$. Purely circular and linear polarizations are described by $\nu = \pm 1, q = u = 0$ and $q = \pm 1, v = u = 0$, respectively.
SYNCHROTRON RADIATION

In the case of the synchrotron radiation produced by a charge moving in a magnetic field we have \( \rho(t, r) = q\delta^3(r - r(t)) \) and \( j(t, \mathbf{r}) = \rho(t, \mathbf{r})v(t) \). To be explicit we introduce the direction of observation

\[
\mathbf{n} = (\sin \theta, 0, \cos \theta),
\]

in the same coordinate system where \( \mathbf{r}(t) \) was defined in Eq. (13). Recalling the corresponding expressions for \( \mathbf{v}(t) \) we get

\[
v(T + \tau/2) \cdot \mathbf{v}(T - \tau/2) = \beta^2 \cos \omega_0 \tau, \\
\mathbf{n} \cdot \mathbf{v}(T + \tau/2) \times \mathbf{v}(T - \tau/2) = -\beta^2 \cos \theta \sin \omega_0 \tau
\]

and substituting in Eq. (21) we are left with

\[
\mathfrak{J} = \frac{q^2}{2} \left[ \beta^2 \cos \omega_0 \tau - n^{-2}(\lambda z) + i\lambda \beta^2 \cos \theta \sin \omega_0 \tau \right].
\]

The power spectrum can be computed from (20) by taking the time average over the macroscopic time \( T \). We are interested in the synchrotron radiation of astrophysical objects where possible Lorentz violations could be tested, such as the emissions by supernova remnants (SNR) or gamma ray bursts (GRB). In such systems, frequencies of the order of \( \omega = m\omega_0 \gtrsim 1 \text{ GeV} \) are detected, and there is also evidence of electrons of energies up to \( 10^9 \text{ TeV} \). These range of energies correspond to a typical Larmor frequency \( \omega_0 \sim 10^{-30} \text{ GeV} \), which means \( m \gtrsim 10^{30} \), and to a Lorentz factor \( \gamma \sim 10^9 \). According to this, the interesting regime is characterized by \( m \gg 1 \) and \( \gamma \gg 1 \), but such that \( m/\gamma \gg 1 \). In this limit we get

\[
\left\langle \frac{d^2 P(T)}{d\omega d\Omega} \right\rangle = \sum_{m=0}^{\infty} \delta(\omega - m\omega_0) \frac{dP_m}{d\Omega},
\]

with \( \omega_0 = \left( 1 + \frac{\eta \mu}{m c^3} \right) |qB| / (\mu \gamma) \) and the power angular distribution, in terms of the Bessel functions \( J_m \) (and their derivatives \( J'_m \)), is

\[
\frac{dP_m}{d\Omega} = \frac{\omega^2 q^2}{2\pi} \left( 1 + 2m\xi \omega \cos \theta \right) \left\{ \left[ J'_m(m\beta \sin \theta) \right]^2 + \left[ J_m(m\beta \sin \theta) \tan \theta \right]^2 \right\}.
\]

This expression shows an anisotropy to first order in \( \xi \). The emission is suppressed (enhanced) when \( m\xi \omega \cos \theta \sim -1(+1) \). Note that both effects are amplified by the presence of the factor \( m = \omega / \omega_0 \) which can be very large in some regimes. When \( m \) is large the radiation becomes confined to a small angular range \( \Delta \theta \sim m^{-1/3} \) around \( \theta = \pi/2 \). Thus, if we consider the radiation in the frontiers of the beam, i.e. \( \theta \sim \pi/2 \pm m^{-1/3} \), the anisotropy becomes significant when \( \omega \sim \left( \omega_0^2 / \xi^3 \right)^{1/5} \). The spatially integrated power radiated in the \( m^{th} \) harmonic is approximately

\[
P_m \sim \frac{q^2 m \omega_0}{\sqrt{3} \pi R \gamma^2} \left[ \frac{m_c \kappa}{m} \left( \frac{m}{m_c} \right) - \frac{2}{\gamma^2} K_{2/3} \left( \frac{m}{m_c} \right) + 2 \left( m \omega_0 \xi \beta \right)^2 \frac{m^2}{\gamma^2} K_{2/3} \left( \frac{m}{m_c} \right) \right],
\]

where \( \kappa \) is the ratio of the magnetic and electric fields. This agrees with the earlier results of [14] in the special case of a point-like source and a simple harmonic motion. The expression (27) can be extended to more general sources and motion by including the factors of the source size and shape and the geometry of the source.
with $m_c$ being the critical value $m_c = 3\gamma^2/2$. For large $m$ and $1 – \beta^2 > 0$ the behavior of the power radiated in the $m^{th}$ harmonic can be obtained using the asymptotic expressions for the MacDonald functions $K_{m/n}$. Within this limit there are two regimes of interest, according to the ratio between $m$ and $m_c$. For the case $m/m_c \gg 1$ we get

$$P_m \simeq \frac{1}{2R} \sqrt{\frac{m}{\pi \gamma}} \left( \frac{q^2 B}{E} \right)^2 \left( 1 – 3\kappa E + \frac{27}{4} \kappa^2 E^2 \right) \left[ 1 + 2\tilde{\xi}^2 \left( \frac{m\omega}{\gamma} \right)^2 \right] e^{-2m/3\gamma^3}, \quad (28)$$

while the complementary range $m/m_c \ll 1$ produces

$$P_m \simeq \frac{\sqrt[3]{3\gamma}}{\sqrt{3\pi}} \left( \frac{q^2 B}{E} \right)^2 \left( 1 – 3\kappa E + \frac{27}{4} \kappa^2 E^2 \right) \left[ 1 + \left( \frac{m\omega_0\tilde{\xi} \beta}{\sqrt{2}} \right)^2 \frac{m^2}{\gamma^2} \right] m^{1/3}. \quad (29)$$

To characterize the polarization we can restrict our discussion to $\theta = \pi/2$, since the radiation is mostly concentrated around the plane of the orbit. In this case the effects of the Lorentz violation in the current are negligible, and only the explicit violations in the Maxwell equations affect the polarization of the radiation. The Stokes parameters take the form

$$v_m = \frac{1 – R_m^2}{1 + R_m^2}, \quad q_m = \frac{2R_m}{1 + R_m^2} \cos([n(-z) – n(z)] \omega r), \quad u_m = \frac{2R_m}{1 + R_m^2} \sin([n(-z) – n(z)] \omega r),$$

which give them in terms of the polarization index $R_m$

$$R_m(\theta = \pi/2) \simeq 1 – 2\tilde{\xi} \frac{m\omega J_m(m\tilde{\beta})}{\gamma J'_m(m\tilde{\beta})}, \quad (31)$$

where $\tilde{\beta} = \sqrt{1 – \mu^2/E^2}$. In the large $m$ limit and in the case $1 – \beta^2 > 0$, one can write the Bessel function and its derivative in terms of the MacDonald functions. The latter can be further approximated according to $m/m_c \ll 1$ where $m_c = 3(1 – \beta^2)^{-3/2}/2$. In such a way, if $m/m_c \gg 1$ the radiated power is exponentially damped and we get

$$R_m(\theta = \pi/2) \simeq 1 – 2\tilde{\xi} \omega m/\gamma. \quad (32)$$

Then we have, to first order in $\tilde{\xi}$,

$$v_m \simeq 2\omega \xi m/\gamma, \quad q_m \simeq \cos(2\tilde{\xi} \omega^2 r), \quad u_m \simeq -\sin(2\tilde{\xi} \omega^2 r). \quad (33)$$

In the opposite case, when $m/m_c \ll 1$, we get

$$R_m(\theta = \pi/2) \simeq 1 – 2\tilde{\xi} \omega \left( \frac{1}{4} \right)^{1/3} \frac{\Gamma(1/3)}{\Gamma(2/3)} m^{4/3}, \quad (34)$$

from which we have

$$v_m \simeq \left( \frac{1}{4} \right)^{1/3} \frac{\Gamma(1/3)}{\Gamma(2/3)} \frac{2\omega}{\gamma} m^{4/3} \tilde{\xi}, \quad q_m \simeq \cos(2\tilde{\xi} \omega^2 r), \quad u_m \simeq -\sin(2\tilde{\xi} \omega^2 r). \quad (35)$$
In both cases powers of $m$ appear as amplifying factors. Note that $q_m$ and $u_m$ oscillate with a wave length $\Lambda = \pi/(\tilde{\xi} \omega^2)$. These oscillations come from the term $\pm \tilde{\xi} \omega^2 r$ of the phases in Eq. (16), i.e. they are a consequence of the birefringence of the vacuum.

**FINAL REMARKS**

The main results of a complete calculation of synchrotron radiation in the MP effective electrodynamics has been presented. This effective electrodynamics can be interpreted in terms of a standard parity violating birefringent and dispersive medium. A priori one can expect that the Lorentz violating effects would be of order $\tilde{\xi} \omega$. Instead we find amplifying factors proportional to $m$, which could render these modifications within or near the scope of present time observations.

When an astrophysical object is considered as a source of synchrotron radiation it is necessary to take into consideration the actual range of validity of the radiation approximation from where these results have been obtained. The propagation of the fields is characterized by the phase

$$n(\lambda z) \omega |r - r'| \simeq \omega r \left(1 - \frac{n \cdot r'}{r} + \lambda \tilde{\xi} \omega - \lambda \tilde{\xi} \omega \frac{n \cdot r'}{r} + \frac{1}{2} \frac{r' r}{r^2}\right),$$

where $r'$ can be estimated by the radius of the orbit. In the radiation approximation the term proportional to $(r'/r)^2$ is neglected. If $|\tilde{\xi} \omega| > r'/r$ we can neglect only the term quadratic in $r'$ and both $\tilde{\xi}$-dependent terms remain in the phase, which can be approximated, using $k_\lambda = \omega n(\lambda z) \hat{n}$, by

$$n(\lambda z) \omega |r - r'| \simeq n(\lambda z) \omega r \left(1 - \hat{n} \cdot r'/r\right).$$

This is the full far-field approximation developed in the preceding sections, with the most general dependence on the index of refraction. An intermediate situation happens when $(r'/r)^2 < |\tilde{\xi} \omega| < r'/r$, in which case

$$n(\lambda z) \omega |r - r'| \simeq n(\lambda z) \omega r - \hat{n} \cdot r',$$

and hence the index of refraction still remains significant. Finally, if $|\tilde{\xi} \omega| < (r'/r)^2$ all the dependence on $\tilde{\xi}$ is negligible in the phase, which reduces to the usual expression

$$n(\lambda z) \omega |r - r'| \simeq \omega r - \hat{n} \cdot r'.$$

To be concrete we can consider the Crab Nebula, an SNR at 6000 ly. The synchrotron radiation spectrum shows a cutoff at $\omega \simeq 0.5$ GeV, and there is evidence of electrons of energy $E_e \simeq 3 \times 10^6$ GeV producing this emission. We can estimate the magnetic field, the $\gamma$ factor and the Larmor frequency using the unperturbed relations which give us $B \simeq 10^{-3}$ G, $\gamma \simeq 6.0 \times 10^9$, and $\omega_0 \simeq 2 \times 10^{-30}$ GeV. Thus, in this case we have $r \sim 10^{36}$ $GeV^{-1}$ and $r' \sim 10^{30}$ $GeV^{-1}$. Considering that $\tilde{\xi} \lesssim 10^{-19}$ $GeV^{-1}$, we get
that not only $|\tilde{\xi}\omega| < r'/r$, but also $|\tilde{\xi}\omega| < (r'/r)^2$. Therefore all the information about the $\tilde{\xi}$ parameter is erased by the radiation approximation and the power spectrum only contains the parameter $\tilde{\eta}$, in agreement with Ref. [10]. When radiation from much more distant objects such as GRBs is considered, information about the parameter $\tilde{\xi}$ may become accessible. For example, in the case of Mkn 501 we have: $r \sim 10^8$ l.y. $\sim 10^{40}$ GeV$^{-1}$ and $r' \sim 10^{26}$ GeV$^{-1}$, which give $r'/r \sim 10^{-14}$. The photon energy is $\omega \sim 10^4$ GeV, and thus $|\tilde{\xi}\omega| \lesssim 10^{-15}$. In this way we have $(r'/r)^2 < \tilde{\xi}\omega < r'/r$, and hence the term $\tilde{\xi}\omega r$ in the phase becomes significant. Another example is GRB021206, at a distance of $10^{10}$ l.y., with a larger magnetic field and therefore a smaller radius for the electron orbits. Here $r'/r \sim 10^{-24}$, $\omega \sim 10^{-3}$ GeV, and $|\tilde{\xi}\omega| \lesssim 10^{-22}$, which yields $r'/r \ll |\tilde{\xi}\omega|$. In principle this case could allow access to the full far-field approximation. Summarizing, in the case of the CRAB nebula radiation the far-field approximation erases all the information about the $\tilde{\xi}$ parameter and only allows to test the parameter $\tilde{\eta}$ associated to the dynamics of the charged particle. The situation changes for extragalactic sources, such as in the case of GRB’s. For these objects the far-field approximation could include $\tilde{\xi}$-LV terms and the $m$-amplifying factors become important, thus giving access to a new sector of the Lorentz violating parameters.

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