Semi-analytical solutions to one-dimensional electro-osmotic consolidation in unsaturated soils

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ABSTRACT

This paper presents a 1D model for electro-osmotic consolidation of unsaturated soils. Pore-water pressure, pore-air pressure, and soil settlement are obtained in the Laplace domain. Then, Crump’s method is adopted to obtain semi-analytical solutions for the time domain. The present solutions are more general and can be degenerated into conventional solutions to 1D consolidation of fully saturated soils and unsaturated soils without electro-osmosis. The parametric studies indicate that electro-osmosis has little impact on the dissipation of excess pore pressures at the first stage, while electro-osmosis is suggested to start at the second stage after pore-air pressure dissipates completely.

Keywords: Electro-osmotic consolidation, Unsaturated soil, Semi-analytical solution, Laplace transform, Parametric study

1 INTRODUCTION

Electro-osmosis is an important technology for geotechnical applications and involves the flow of water from the positive electrode to the negative electrode when an electric current is applied. As an alternative method for soil improvement, electro-osmosis has been reported to be both economical and time saving. So far, it has been investigated and applied in many geotechnical and geo-environmental engineering applications, including the consolidation and strengthening of clays [1,2], soil stabilization [3-5], dewatering of fine-grained mine tailings [6], electrokinetic remediation and electrokinetic barriers [7-10], reducing the adhesion of excavated clay materials on steel surfaces of tunnel driving machines [11], and decreasing the water content of the over-wet subgrade fill and expansive soil [12-15]. In recently, due to the problems, such as electrode erosion, low efficiency, uneven consolidation, and unsatisfactory treatment results, exist in practical applications, numerous studies to understand the suitability of the application, the anticipated effects of the treatment, its efficiency and effectiveness and so on for different types of soil have also been reported [16-21].

Based on the assumption that the pore water flow resulting from the hydraulic gradient and the electrical gradient can be superimposed linearly, the governing equation for electro-osmotic consolidation was developed, and many analytical solutions were derived based on different conditions to analyze the development of pore water pressure and the degree of consolidation [22-30]. Esrig [22] first proposed a 1D theoretical model and derived the solution for the excess pore water pressure under different boundary conditions. Wan and Mitchell [23] investigated the coupling effect of surcharge preloading and electro-osmotic consolidation. Following their pioneering work, several analytical models were proposed for electro-osmotic consolidation, including the two-dimensional model in a vertical plane [24, 25] and the two-dimensional model in a horizontal plane [26]. Considering that prefabricated vertical drain (PVD) and electric vertical drain (EVD) are installed in an equilateral triangular pattern in the ground, axisymmetric models were also developed and the corresponding analytical solutions were derived [27, 28]. Recently, the nonlinear variations in soil compressibility, hydraulic and electro-osmotic conductivity were incorporated into a 1D model, and analytical solutions for excess pore-water pressure and degree of consolidation were derived [29, 30]. These mathematical analyses have generated significant knowledge pertaining to electro-osmotic consolidation and provided useful formulas for engineering design. However, these analytical solutions are basically derived based on the assumption that soil is fully...
saturated. As a matter of fact, the soil used for the construction of clay liners is typically compacted in unsaturated conditions, while in polluted areas migration of contaminants often takes place in the unsaturated zone above the groundwater table. When electro-osmosis is applied in these fields, the transport of air and water phases, and the impact of the air volume fraction in this pore space should be considered. Thus, the aim of this paper is to present analytical solutions for predicting the development of excess pore pressures, settlement and degree of consolidation of an unsaturated soil layer in the process of electro-osmosis.

Although being ignored in the electro-osmotic consolidation, the interaction of the air and water phases has already been investigated in many consolidation theories for unsaturated soils. Scott [31] estimated the consolidation of unsaturated soils with occluded air bubbles, and about which Biot [32] proposed a general consolidation theory. Barden [33] presented an analysis of 1D consolidation of compacted unsaturated clay. Furthermore, based on the hypothesis that the air and water phases are continuous, Fredlund and Hasan [34] proposed a 1D consolidation theory, in which two partial differential equations are employed to describe the dissipation processes of excess pore-water and pore-air pressures in unsaturated soils. This theory was widely accepted, and then extended to three-dimensional case [35]. For simplicity, assuming all the soil parameters remain constant during consolidation, Fredlund et al. [36] presented a simplified form of 1D consolidation equations for unsaturated soils. From then on, several analytical solutions with different boundary conditions by different mathematical methods have been obtained, and the effects of the transport of air phase on the consolidation behaviour of unsaturated soils were comprehensively studied [37-42].

In this study, a 1D model for electro-osmotic consolidation in unsaturated soils is established based on the Esrig’s electro-osmotic consolidation and Fredlund and Hasan’s consolidation for unsaturated soils. Afterwards, semi-analytical solutions for predicting the dissipation of pore-air and pore-water pressures and the soil settlement in the condition of single drainage are derived. To obtain final solutions, the two coupled governing equations of pore-water and pore-air pressures are first derived into two conventional diffusion equations, and then the equations are solved using the Laplace transform. It is found that the current solutions are more general that can be degenerated into 1D consolidation theory for fully saturated soils with and without electro-osmosis, and in a good agreement with the existing solutions for unsaturated soils without electro-osmosis. Finally, examples are given to illustrate consolidation behaviours of unsaturated soils under electro-osmosis. The effects of parameters, such as applied voltage, ratios of electro-osmotic to hydraulic permeability and air to water permeability, and depth, on the changes of pore-air and pore-water pressures, settlement and average degree consolidation are sufficiently investigated.

2 GOVERNING EQUATION

On basis of the 1D electro-osmotic consolidation model in the previous studies [22, 23, 29] and 1D consolidation model for unsaturated soils proposed by Fredlund and Hasan [34], a simulated diagram of this problem addressed here is demonstrated in Fig. 1. The unsaturated soil layer is deemed as a stratum of infinite horizontal direction with a thickness H with the anode on the bottom and the cathode on the top. The bottom boundary is impermeable to air and water phases and the top boundary is permeable. A surcharge preloading \( p_0 \) is applied on the top boundary of the model. Water flow, airflow, and settlement only occur in the vertical direction.

\( k_w, k_a, \) and \( k_e \) are the coefficients of water, air, and electro-osmotic permeability in unsaturated soil layer. In order to develop the analytical model for electro-osmotic consolidation in unsaturated soils, the assumptions are made as follows:

1. The soil is assumed to be homogeneous, and the solid particles and water phase are incompressible;
2. Air and water phases are considered as continuous and independent;
3. The loading is instantaneously applied, and the small strain hypothesis is adopted, the deformation of an unsaturated soil layer only grows along the vertical direction;
4. The velocity of pore water flow due to electro-osmosis is directly proportional to the electrical gradient, and can be linearly superimposed with that due to hydraulic gradient;
5. The coefficients of water, air, and electro-osmotic permeability and volumetric change of the unsaturated soil remain constant throughout the consolidation process;

![Fig.1. A simplified model of 1D electro-osmotic consolidation in unsaturated soils.](image-url)
(6) Influence of environmental factors, such as temperature change, ionic migration, gas generation and diffusion during the electro-osmotic consolidation, is neglected.

Above assumptions are not completely accurate for all cases, such as the coefficients of permeability with respect to air and water phases and electro-osmosis are functions of both water content and degree of saturation, and the moduli for the soil structure and water phases are non-linear. However, it may be acceptable to assume that these parameters are constant for a small deformation for the soil approaching saturated.

With the introduction of above assumptions, the governing equations for the water and gas flow in the unsaturated soils under electroosmosis can be given as:

\[
\frac{\partial u_w}{\partial t} = -C_w \frac{\partial u_w}{\partial t} - \frac{\partial}{\partial z} \left( C_w \frac{\partial u_w}{\partial z} + k_w \frac{\partial U}{m_w} \right)
\]

(1)

\[
\frac{\partial u_a}{\partial t} = -C_a \frac{\partial u_a}{\partial t} - \frac{\partial}{\partial z} \left( C_a \frac{\partial^2 u_a}{\partial z^2} \right)
\]

(2)

where

\[
C_w = \frac{1 - m_w^w / m_k^w}{m_w^w / m_k^w}
\]

(3a)

\[
C_a = \frac{m_k^a}{m_k^a - m_k^w - u_{atm} n_0 (1 - S_{w0}) / (\bar{u}_a^0)^2}
\]

(3b)

\[
C_v = \frac{k_w RT}{g \bar{u}_a^0 M \left( m_k^w - m_k^w - u_{atm} n_0 (1 - S_{w0}) / (\bar{u}_a^0)^2 \right)}
\]

(3d)

and \( u_{atm} \) is the atmospheric pressure.

In order to derive analytical solution, the initial and boundary conditions are given as:

\[
u_w (z, 0) = u_w^0, \quad u_a (z, 0) = u_a^0
\]

(4)

\[
u_w (0, t) = 0, \quad u_a (0, t) = 0
\]

(5)

\[
k_w \frac{\partial u_w}{\partial z} + k_w \frac{U}{H} = 0, \quad \frac{\partial u_a}{\partial z} = 0
\]

(6)

where \( u_w^0 \) and \( u_a^0 \) are the values of the initial excess pore water and pore air pressures induced by surcharge.

3 SEMI-ANALYTICAL SOLUTIONS

Due to the soil stratum is homogeneous, the potential is linearly distributed between the anode and cathode. In this case, the second-order derivative of voltage with respect to \( z \) is zero, and then Eqs. (1) and (2) can be rewritten as follows:

\[
\frac{\partial u_w}{\partial t} = A_w \frac{\partial^2 u_w}{\partial z^2} + A_w \frac{\partial^2 u_a}{\partial z^2}
\]

(7)

\[
\frac{\partial u_a}{\partial t} = W_a \frac{\partial^2 u_a}{\partial z^2} + W_a \frac{\partial^2 u_w}{\partial z^2}
\]

(8)

where,

\[
A_w = \frac{C_v}{1 - C_w C_a}, \quad W_a = \frac{C_v}{1 - C_w C_a}
\]

Implementing the Laplace transform on the basic equations of (7) and (8), and initial and boundary conditions (4)-(6), the analytical solutions for the pore water and pore air pressures can be obtained as:

\[
\bar{u}_w = \left\{ \frac{u_w^0 + u_a^0 + \int_{-\infty}^{0} \frac{L_z \tanh [x z]}{\cosh [x]}}{\tanh [x]} \right\}
\]

(9)

\[
\bar{u}_a = \left\{ \frac{u_a^0 + u_w^0 + \int_{-\infty}^{0} \frac{L_z \tanh [x z]}{\cosh [x]}}{\tanh [x]} \right\}
\]

(10)

where,

\[
f_{z0} = \frac{k_w U}{k_w H} \gamma_w
\]

(11)
\[ Q_{1,2} = \frac{1}{2} \left[ A_a + w_w \pm \sqrt{(A_a-W_w)^2 + 4 A_w W_w} \right] \]  

(12)

\[ c_{12} = \frac{w_w}{Q_2 - A_a} \]  

(13)

\[ c_{21} = \frac{A_w}{Q_1 - W_w} \]  

(14)

\[ x_1^2 = s/Q_1, \quad x_2^2 = s/Q_2 \]  

(15)

According to the two-stress-state-variable approach for unsaturated soils [36], the constitutive model of 1D deformation can be expressed as

\[ \frac{\partial \varepsilon}{\partial t} = m_{ik}' \frac{\partial}{\partial t} (\sigma - u_w) + m_{ik}'' \frac{\partial}{\partial t} (u_a - u_w) \]  

(16)

where, \( m_{ik}' = m_{ik}^w + m_{ik}^s \), \( m_{ik}'' = m_{ik}^s + m_{ik}^w \).

Conducting the Laplace transform to Eq. (16) gives

\[ \tilde{\varepsilon}_v(z,s) = (m_{ik}' - m_{ik}^w) \left( \tilde{u}_w - \frac{u_0}{s} \right) - m_{ik}'' \left( \tilde{u}_w - \frac{u_0}{s} \right) \]  

(17)

Thus, the settlement can be calculated by

\[ \tilde{w}(s) = \int_0^H \tilde{\varepsilon}_v(z,s)dz \]  

(18)

Substituting Eqs. (9) and (10) into Eqs. (17) and (18) leads to the following settlement in the Laplace domain.

\[ \chi_1 \tanh[x_1H] + \chi_2 \tanh[x_2H] \]

\[ \quad + \chi_3 \left( 1 - \frac{1}{\cosh[x_1H]} \right) + \chi_4 \left( 1 - \frac{1}{\cosh[x_2H]} \right) \]

(19)

where,

\[ \chi_1 = \left( m_{ik}' - m_{ik}^w + c_{12} m_{ik}^s \right) \left( u_0^w + u_0^s \right) x_1 \]

\[ \chi_2 = \left( c_{12} m_{ik}' - c_{12} m_{ik}^w + m_{ik}^s \right) \left( u_0^w + c_{12} u_0^s \right) x_1 \]

\[ \chi_3 = \frac{x_1 f_{equ}}{x_2} \left( m_{ik}' - m_{ik}^w + c_{12} m_{ik}^s \right) \]

\[ \chi_4 = \frac{x_2 f_{equ}}{x_2} \left( c_{12} m_{ik}^s - c_{12} m_{ik}^w + m_{ik}^s \right) \]

Finally, by adopting the Crump’s method [43] to perform the Laplace inversion, semi-analytical solutions of the excess pore-air and pore-water pressures, and soil settlement in the time domain can be obtained. Then, when the settlement is obtained, the average degree of consolidation can be expressed as:

\[ U_{avg} = \frac{w(t)}{w_{avg}} \]  

(20)

where \( w_{ult} \) is the final settlement with \( t = +\infty \).

4 VERIFICATION

4.1 Solutions degenerated to fully saturated condition

It is noted that the 1D electro-osmotic consolidation governing equations for unsaturated soil can be degenerated to the 1D electro-osmotic consolidation equation for saturated soil. In the fully saturated condition, the coefficients of water volume change \( m_{ik}^w \) and \( m_{ik}^s \) are equal to the coefficient of volume change \( m_{ik} \), and the coefficients of air volume change \( m_{ik}^w \) and \( m_{ik}^s \) are equal to zero; the coefficient of permeability for the water phase \( k_w \) is equal to the saturated permeability coefficient \( k_v \); the pore air pressure may be assumed to be equal to pore water pressure, which means the metric suction is 0. Thus, the parameters in the governing equations are \( C_v = 0 \), \( C_v^w = k_v/(\gamma_w m_{ik}) \), \( C_a = C_v^a = 0 \), the corresponding parameters in Eqs. (7) and (8) become \( W_a = 0 \), \( W_a^w = C_v^w \), \( A_a = A_w = 0 \). Finally, from Eq. (12), we can obtain \( Q_1 = 0 \) and \( Q_2 = C_v^w \), and then \( c_{11} = 1 \), \( c_{12} = 0 \), \( c_{21} = 0 \), \( c_{22} = 1 \) from Eqs. (13) and (14). Substituting \( Q_1 = 0 \), \( Q_2 = C_v^w \), \( c_{11} = 1 \), \( c_{12} = 0 \), \( c_{21} = 0 \), \( c_{22} = 1 \) Eq. (10) gives

\[ \tilde{u}_v = \frac{1}{s} \left[ \frac{u_0^w \cosh \left( \frac{s}{\sqrt{C_v^w} H} \right) + f_{equ} \sinh \left( \frac{s}{\sqrt{C_v^w} H} \right)}{\cosh \left( \frac{s}{\sqrt{C_v^w} H} \right)} + \frac{u_0^w}{s} \right] \]

(21)

The inversion of F(s) can be achieved by use of the residue theorem [44] as the sum of residues of \( e^{\omega F(s)} \) at the poles of F(s). One simple pole is at \( s = 0 \), and \( (s = -\frac{2n+1}{2} \pi \frac{C_v^w}{H^2}, n = 1, 2, 3, \ldots) \), thus:

\[ \text{Res}_{s=0} e^{\omega F(s), 0} = \lim_{s \to 0} s e^{\omega F(s)} = -\frac{u_0^w}{f_{equ} H} \]  

(22)

and

\[ \text{Res}_{s=s_n} e^{\omega F(s), s_n} = \lim_{s \to s_n} (s-s_n) e^{\omega F(s)} \]

(23)

\[ = \sum_{n=1}^{\infty} \left( \frac{2u_0^w}{M} + \frac{(-1)^{n+1}}{2 f_{equ} H} \right) \sin \frac{Mz}{H} e^{\omega H t} \]
where, \( M = \frac{2n-1}{2} \pi \), \( T_v = C_{wv}^T \).

Therefore, the expression for \( u_w \) is obtained as:

\[
    u_w = -f_{eo} z^2 
    + \sum_{n=0}^{\infty} \left[ \frac{2n^0}{M} + \left( \frac{1}{2} \right)^{n+1} 2f_{eo} H \right] \frac{Mz}{H} e^{-M^2 T_v} \tag{24}
\]

If the coefficient of electro-osmotic pressure is equal to zero (\( f_{eo} = 0 \)), Eq. (24) is identical to the solution of Terzaghi’s consolidation theory for fully saturated soils. Meanwhile, if the initial excess pore water pressure is equal to zero (\( u_w^0 = 0 \)), Eq. (24) is identical to the solution of Esrig’s electro-osmotic consolidation theory [22]. So, it is concluded that the presented solution is a general solution for both conventional and electro-osmotic consolidations under unsaturated and fully saturated state.

4.2 Compared with previous solution

In this section, the proposed analytical solution in the case of the applied voltage \( U_0 = 0 \) is verified by comparing with an analytical solution proposed by Qin et al. [37]. Then the investigation is carried out to study the effect of the applied voltage on the variations of the excess pore pressures at point \((z)\), surface settlement, and degree of consolidation, respectively.

The parameters used in the analyses are assumed as follows: \( H = 10 \) m, \( n_0 = 50\% \), \( S_m = 80\% \), \( k_w = 10^{-10} \) m/s, \( m_1^k = -2.5 \times 10^{-4} \) kPa\(^{-1} \), \( m_2^k = -0.5 \times 10^{-4} \) kPa\(^{-1} \), \( m_1^w = -1.0 \times 10^{-4} \) kPa\(^{-1} \), \( m_2^w = -2.0 \times 10^{-4} \) kPa\(^{-1} \), \( u_w^0 = 20 \) kPa, \( u_w^0 = 40 \) kPa, \( q_0 = 100 \) kPa and \( u_{am} = 101.3 \) kPa. The above values of the parameters are obtained from Refs. [37, 38].

Fig. 2 and 3 present the variations in the excess pore pressures at \( z = 0.5H \) and surface settlement versus time factor \( T \) under different applied voltages. The analytical solutions presented by Qin et al. [37] for the 1D consolidation of unsaturated soil are used for comparison to investigate the accuracy of the proposed analytical solutions. In this comparison, zero voltage is considered. It can be found that the present results are identical to that proposed by Qin et al. [37] with single drainage boundary which means the proposed analytical solutions are accurate. Moreover, for the consolidation of unsaturated soil, it can be observed that the dissipation rate of pore-water pressure presents a double S-curved pattern, while the dissipation rate of pore-air pressure only shows a single curve throughout the entire dissipation process. For simplicity, the dissipation rate of pore-water pressures can be divided into two stages. By closely examining, the required time for the excess pore-air pressure to completely dissipate is also the time that the pattern of pore-water pressure forms an upper S curve. This process is considered as the first stage of dissipation, where significant variation of pore-water pressure patterns can be clearly observed. The second stage begins after the pore-air pressure diminishes. Moreover, it can be observed that when the pore-air pressure dissipates to zero, a plateau period of dissipation patterns of pore-water pressure emerges at the second stage. For electro-osmotic consolidation at different applied voltages as shown in Fig. 2, the larger the applied voltage, the larger the ultimate pore-water pressure and the more quickly pore-water pressure dissipates at the second stage. However, the pore pressures at different applied voltages during the first stage are almost identical to each other. This is because the dissipation of the pore-water pressure at the first stage is the outcome of the dissipation of excess pore-air pressure. Since the air flow induced by electro-osmosis can be neglected the effect of the electro-osmosis on the dissipation process of pore-air pressure is small at the first stage, and the changes of pore-water pressure for different applied voltages are nearly identical to each other. As shown in Fig. 3, the settlement development over time can also be divided into two stages. The settlements in the first stage at different applied voltages are identical when both of the pore-water and pore-air pressures dissipate simultaneously, that is approximately 4.8 cm, while in the second stage, the larger settlement occurs with the increase of the applied voltage, which is consistent with the observations in the refs. [27, 29, 45].
The dissipation of pore-water pressure at the first stage is mainly caused by the dissipation of the pore-air pressure. Electro-osmosis has little effect on the dissipation processes of excess pore pressures at this stage. While, the dissipation of excess pore-water pressure is remarkably influenced by electro-osmosis at the second stage. The effect of the value of applied voltage at the second stage is the same as those found in fully saturated soils. Electro-osmosis is suggested to start at the second stage when pore-air pressure dissipates completely.

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REFERENCES

1) Alshawabkeh, A. N., Yeung, A. T., and Bricka, M. R. (1999). Practical aspects of in-situ electrokinetic extraction. Journal of Environmental Engineering, 125(1), 27-35.
2) Barden, L. (1965). Consolidation of compacted and unsaturated clays. Geotechnique, 15(3), 267-286.
3) Bergado, D.T., Sasanakul, I., and Horpibulsuk, S., (2003). Electro-osmotic consolidation of soft Bangkok clay using copper and carbon electrodes with PVD. Geotechnical Testing Journal, 26(3), 277-288.
4) Biot, M.A., (1941). General theory of three-dimensional consolidation. J Appl Phys, 12(2), 155-164.
5) Bjerrum, L., Mourn, J., and Hordbulsuk, S., (1967). Application of electro-osmosis to a foundation problem in a Norwegian quick clay. Geotechnique, 17(3), 214-235.
6) Casagrande, L., (1952). Electro-osmotic stabilization of soils. Journal of the Boston Society of Civil Engineers Section, 39(1), 51-83.
7) Casagrande, L., (1983). Stabilization of soils by means of electro-osmosis - state of the art. Journal of the Boston Society of Civil Engineers Section, 69(2), 255-302.
8) Crump, K.S., (1976). Numerical inversion of Laplace transforms using a Fourier series approximation. Journal of the Association for Computing Machinery, 23(1), 89-96.
9) Dakshinamurthy, V., Fredlund, D.G., and Rahardjo, H., (1984). Coupled three-dimensional consolidation theory of unsaturated porous media. In: Proceedings of 5th International Conference on Expansive Soils, Adelaide, South Australia , 99-103.
10) Durbin, F., (1974). Numerical inversion of Laplace transforms: An efficient improvement to Dubner and Abate’s method. Comput. J, 17(4), 371-376.
11) Esrig, M.I. (1968). Pore pressures, consolidation and electrokinetics. J Soil Mech Found Engng Div, ASCE, 94(4), 899-922.
12) Fetzer, C.A., (1967). Electro-osmotic stabilization of West Branch Dam. J Soil Mech Found Div, ASCE, 93(4), 85-106.
13) Fourie, A.B., Johns, D.G., and Jones, C.J.F.P., (2007). Dewatering of mine tailings using electrokinetic geosynthetics. Canadian Geotechnical Journal, 44(2), 160-172.
14) Fredlund, D.G., and Hasan, J.U., (1979). One-dimensional
consolidation theory: unsaturated soils. Can Geotech J, 17, 521-531.

15) Fredlund, D. G., Fredlund, M. D., and Rahardjo, H., (2012). Unsaturated soil mechanics in engineering practice. John Wiley and Sons, Hoboken, NJ.

16) Ho, L., Fatah, B., and Khabbaz, H., (2014). Analytical solution for one-dimensional consolidation of unsaturated soils using eigenfunction expansion method. Int J Numer Anal Meth Geomech, 38(10), 1058-1077.

17) Jayakanthan, V., Gnanendran, C.T., and Lo, S.C.R., (2011). Laboratory assessment of electro-osmotic stabilization of soft clay. Canadian Geotechnical Journal, 48(12), 1788-1802.

18) Lefebvre, G., and Burnotte, F., (2002). Improvements of electroosmotic consolidation of soft clays by minimizing power loss at electrodes. Canadian Geotechnical Journal, 39(2), 399-408.

19) Li, Y., Gong, X.N., Lu, M.M., and Guo, B., (2010). Coupling consolidation theory under combined action of load and electro-osmosis. Chin J Geotech Eng, 32, 77-81 (in Chinese).

20) Lo, K.Y., and Ho, K.S., (1991). Inculet II. Field test of electroosmotic strengthening of soft sensitive clay. Canadian Geotechnical Journal, 28(1), 74-83.

21) Mitchell, J.K., (1991). Conduction phenomena: from theory to geotechnical practice. Géotechnique, 41(3), 299-340.

22) Mohamedelhassan, E., and Shang, J.Q., (2001). Effects of electrode materials and current intermittence in electro-osmosis. Proceedings of the Institution of Civil Engineers: Ground Improvement, 5(1), 3-11.

23) Mohamedelhassan, E., and Shang, J.Q., (2002). Feasibility assessment of electro-osmotic consolidation on marine sediment. Proceedings of the Institution of Civil Engineers: Ground Improvement, 6(4), 145-152.

24) Ozisik, M. N., (1980). Heat Conditions, John Wiley, New York.

25) Qin, A.F., Chen, G.J., Tan, Y.W., and Sun, D.A., (2008). Analytical solution to one-dimensional consolidation in unsaturated soils. Appl Math Mech (Eng Ed), 29(10), 1329-1340.

26) Qin, A.F., Sun, D.A., and Tan, Y.W., (2010). Analytical solution to one-dimensional consolidation in unsaturated soils under loading varying exponentially with time. Comput Geotech, 37, 233-238.

27) Scott, R.F., (1963). Principles of soil mechanics. Addison Wesley Publishing Company.

28) Shang, J.Q., (1998). Electro-osmotic enhanced preloading consolidation via vertical drains. Can Geotech J, 35(3), 491-499.

29) Spagnoli, G., Klitzsch, N., Fernandez-Steeger, T., Feinendegen, M., Real, Rey, A., Stanjek, H., and Azzam, R., (2011). Application of Electro-Osmosis to Reduce the Adhesion of Clay during Mechanical Tunnel Drilling. Environmental & Engineering Geoscience, 17(4), 417-426.

30) Su, J., and Wang, Z., (2003). The two-dimensional consolidation theory of electro-osmosis. Géotechnique, 53(8), 759-763.

31) Wan, T.Y., and Mitchell, J.K., (1976). Electro-osmotic consolidation of soil. J Geotech Engng Div, ASCE, 102(5), 473-491.

32) Wang, N.W., Zhang, L., Xiu, Y.J., and Jiao, J., (2013). Electro-osmosis drainage experiment of unsaturated clayed soil. Engineering mechanics, 30(S), 191-194 (in Chinese).

33) Wang, L., Qin, A.F., Sun, D.A., and Xu, Y.F., (2017). Semi-analytical solution to one-dimensional consolidation for unsaturated soils with semi-permeable drainage boundary under time-dependent loading. Int J Numer Anal Meth Geomech, DOI: 10.1002/nag.2694.

34) Wang, L., Sun, D.A., and Qin, A.F., (2018). Semi-Analytical Solution to One-Dimensional Consolidation for Unsaturated Soils with Exponentially Time-Growing Drainage Boundary Conditions. Int. J. Geomech, 18(2), 0401714.

35) Wang, L.J., Liu, S.H., Wang, Z.J., and Zhang, K., (2013). A consolidation theory for one-dimensional large deformation problems under combined action of load and electroosmosis. Eng Mech, 30, No. 12, 91-98 (in Chinese).

36) Wang, S.D., Zhang, L.H., Wu, H.J., Duan, X., and Chen, J.H., (2010). Parameter design of over-wet soil fill treated by electro-osmosis. Chinese Journal of Geotechnical Engineering, 32(3), 212-215 (in Chinese).

37) Wu, H., and Hu, L.M., (2013). Analytical solution for axisymmetric electro-osmotic consolidation. Géotechnique, 63(12), 1074-1079.

38) Wu, H., and Hu, L.M., (2014). Microfabric change of electro-osmotic stabilized bentonite. Appl Clay Sci, 101, 503-509.

39) Wu, H., Hu, L.M., and Wen, Q.B., (2015). Electro-osmotic enhancement of bentonite with reactive and inert electrodes. Appl Clay Sci, 111, 76-82.

40) Wu, H., Qi, W.G., Hu, L.M., and Wen, Q.B., (2017). Electro-osmotic consolidation of soil with variable compressibility, hydraulic conductivity and electro-osmosis conductivity. Computers and Geotechnics, 85, 126-138.

41) Xu, W., Liu, S.H., Wang, L.J., and Wang, J.B., (2011). Analytical theory of soft ground consolidation under vacuum preloading combined with electro-osmosis. J Hohai Univ (Nat. Sci.), 30(2), 169-175 (in Chinese).

42) Ye, Z.Z., (2015). Discussion on Some Problems of electro-osmotic consolidation of soil. 4th Intern. Conf. on Sustainable Energy and Environmental Engin, 148-152.

43) Yeung, A.T., (1994). Electrokinetic flow processes in porous media and their applications. In Advances in Porous Media, Corapcioglu MY (ed.). Amsterdam, The Netherlands: Elsevier, 309-395.

44) Yeung, A.T., (2006). Contaminant extractability by electrokinetics. Environmental Engineering Science, 23(1), 202-224.

45) Yuan, J., and Hicks, M.A., (2013). Large deformation elastic electro-osmosis consolidation of clays. Computers and Geotechnics, 54, 60-68.

46) Zhou, W.H., Zhao, L.S., and Li, X.B., (2014). A simple analytical solution to one-dimensional consolidation for unsaturated soils. Int J Numer Anal Meth Geomech, 38, 794-810.