Photorefractive Axicon: Study of Light-induced Effect by Bessel Beam in Photorefractive Crystal

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Abstract. In this work, we present the theoretical and computational study of the original analysis of the light-induced effects by Bessel beams in photorefractive crystals. Modern applications of these beams as: metrological, alignment of optical systems, optical tweezers, non linear optics, optical communication, and others, becoming a very interesting substitute for a Gaussian beam when this is subject to diffraction and dispersion effects to large distance propagation. On the other hand, the photorefractive crystals are very important materials for applications in non-linear optics, holographic storage, interferometry and optical information processing. We perform an analysis of the index refraction modulation generated by Bessel beam in photorefractive medium discussing the possibility this optical material to control and generation of Bessel beam properties.

1. Introduction

The photorefractive crystals are very important materials for applications in non-linear optics, holographic storage, interferometry, optical information processing electro-optics modulation and others. The nonlinear processes of photorefractive mean are well understood since photo-excitation, transport and trap of charge until index modulation effects [1]. The analyses of one-dimensional index modulation generated by interference patterns or single beams as Gaussian beam with circular symmetry are performed by any authors [2-4]. For 2 and 3 dimensional coordinates the index gradient has an anisotropic effect when external field (applied external, photovoltaic, pyroelectric fields) is present in photorefractive mean, generating a no uniformed index modulation [5]. All these analyses were made for Gaussian beam with circular or parabolic symmetry, these beams are linked diffraction and dispersion effects along their propagation in free space or material mean, through of nonlinear effect and Gaussian beam diffraction obtain the solitons effect [6]. On the other hand, in 1915 Straton [7] theoretically found a wave equation solution which represented a wave which your transversal standard not change along of propagation and this wave is descript by ordinary zero-order Bessel function. The called localized waves or non diffractive waves are exact solution of the Helmholtz and wave equations, this wave have a properties of be immune to diffraction and dispersion effect along all propagation [8]. The first experimental observation was obtained by Durnin et al [9], he demonstrated Bessel beams generated by the annular slit, truncate by circular aperture theoretically described by scalar diffraction theory. Today, the Bessel beams has a many application as; metrological, alignment of optical systems, optical tweezers, non linear optics, optical communication, and others [8]. In this...
work, we present an analysis of Bessel beam interacting with a photorefractive mean, studying the light-induced effect and index refraction modulation in diffusion and drift field domain.

2. Bessel Beam

The optical beams are a monochromatic solution of wave equation with concentration transversal field which are subjects to diffraction and dispersion effect. The Bessel Beam (BB) is an exact solution construction of Helmholtz equation in cylindrical coordinate, others localized solutions can be obtained in other coordinates systems, since transversal and longitudinal components of propagation vector are separable as, elliptic circular and parabolic circular coordinates. This solution in cylindrical coordinate system is,

\[ E(\rho, \phi, z) = \exp(i n \phi) J_n(k_\rho \rho) \exp(i k_z z) \]  

(1)

where \( k_\rho \) and \( k_z \) are transversal and longitudinal wave vector components respectively, \( n = 0, 1, 2, \ldots \) is integer, for \( n > 0 \) we obtain a Bessel beam of superior order (non azimuthal symmetry) that present an azimuthal angle dependence. However a solution with azimuthal symmetry as described by Straton, when \( n=0 \) so the transversal profile is descript by ordinary zero-order Bessel function. The intensity of this beam is resistive at diffraction and dispersion effect along of all axis of propagation. However, the achievement experimental beam is possible only with paraxial approximation due on Bessel beam perfect is needed infinite energy for be generated. In Figure (1) given setup for a fist BB generation was proposed be Durnin, exhibiting an annular slit of diameter \( d \) on positive lens with focus \( f \), others methods for Bessel beam generation are developed as, axicon lens and holographic elements [10-11]. The Figure (2) shows the transversal intensity profile and your cross section.

![Figure 1. Experimental Durnin’s setup, with intensity distribution on axis propagation z.](image1)

![Figure 2. In black is cross section of Bessel beam transversal intensity (in detail), in blue is transversal decay.](image2)

3. Light- induced Effect in Photorefractive Mean

Through the Band Transport Model and Kukhtarev equations equation (2) is possible descript the photorefractive effect, that consisting on the refractive index modulation through photo-induction of charge carriers, generating spatial field charge and via linear electro-optic effect (Pockels Effect) the index refraction is modulate, other effects can be present as, photovoltaic, thermo-optic, photo-chromic, and more.
\[
\begin{align*}
\text{Latter Calvo et al}[12], \text{ show light-induced effect in photorefractive mean due a generic single beam interaction in 2-D spatial dimension, in particulate, he analyze the shapes and intensity of a space charge and refraction index modulation generated by Gaussian beam. In drift field domain (external field applied } E_0 \text{ or photovoltaic effect } E_{\text{phv}}, \text{ the PR material presents a nonlocal response inducing an anisotropic space charge field equation (5) obtained as from equations (1) in cylindrical coordinates through the potential solution } \Phi \text{ and } E_{sc} = E_0 + \nabla \Phi; \\
E_{sc} = \frac{k_b T}{e} \nabla \ln (I + I_b) + \left( E_0 - E_{\text{phv}} \right) \sqrt{\frac{I_b}{I(\rho) + I_b}} \left\{ 2\cos^2(\varphi) - \cos(2\varphi) \right\} \sqrt{\frac{I(\rho) + I_b}{I_b}} - \\
\frac{2}{\rho^2} \left[ \cos(2\varphi) + \rho \cos(\varphi) \frac{d \ln [I(\rho) + I_b]}{d \rho} \right]_0^\rho \rho' \left[ 1 - \sqrt{\frac{I(\rho') + I_b}{I_b}} \right] d \rho'
\end{align*}
\]

where the fist term is diffusion field, } E_0 \text{ is external applied field } E_{\text{phv}} \text{ is photovoltaic field, } I_b \text{ is dark intensity, } \varphi \text{ is azimuthally angle and } I(\rho) \text{ is the generic beam intensity equation in cylindrical coordinate. As from space field generated equation (3) the index modulation is given due electro-optic effect}

\[
\Delta n = \pm \frac{1}{2} n_0^3 r^{\text{eff}} E_{sc}
\]

For a Bessel profile decrypted by equation (1) with } n=0 \text{ (order zero with azimuthal symmetry) replaced in equation (3), we can decrypted the space field and index modulation in photorefractive mean by Bessel beam.}

4. Results

The integral in equation (3) that is dependent Bessel function } e \text{ numerically solved. For a diffusion domain (fist term equation (3)), the light-induced field has a large cylindrical symmetry, where your size is decrypted by concentric rings, that your amplitude radially decay by } \sqrt{\rho^{-1}} \text{ Figure (3). The diffusion effect is more efficiently when the maximum and minimum distance of intensity have, approximately, the diffusion length of material of order } (0.3 \mu m - 3 \mu m), \text{ the Bessel beans can be obtained with resolution line of } 3\lambda/4. \text{ For other hand, in drift regimen (DC/AC applied or photovoltaic field), it’s observe an anisotropy along of external field axis Figure (4) than also extend large radially.}

Of the light-induced field and via electro-optic effect the refraction index is modulated, he’s dependent on parameters such as, beam spot } \rho_0, \text{ background intensity } (I_b), \text{ external field } (E_0) \text{ and specific crystals parameters. For Strontium-Barium Niobate (Sr}_{0.75}\text{Ba}_{1-0.75}\text{Nb}_2\text{O}_6 \text{ SBN:75 un-doped that present high electro-optics coefficient } r_{33} = 1340 pm/V \text{ and extraordinary index } n_0 = 2.312 \text{ the refraction index variation in diffusion domain, change with the ratio } I_0/I_b \text{ as can see in Figure (5), in drift domain with applied external field } E_0=8kV/cm, \text{ we obtain } \Delta n \approx 2 \times 10^{-3} \text{ Figure (6). For Lithium Niobate crystal (LiNbO}_3 \text{ that present photovoltaic field when doped (Fe or Cu) the index modulation can be orders } \Delta n \approx 4 \times 10^{-3}.}
Different to setups as grid pattern or a Gaussian beam, the Bessel beam have a single properties concerning to your transversal standard and your non-diffractive property along the propagation. The index modulation in diffusion domain presents a profile of concentric rings that extending radially slowly decaying as Bessel function. In drift domain the symmetry is along to applied field and too extending for a long radial distance.

5. Conclusions
The understanding this basic principle of the light-induced process by Bessel beam is the first step for futures analyzes in 3-dimencional photorefractive mean propagation and come to be used in optical element to control and generation Bessel beam. The self-induction effect of beam can potentiate the control the output beam as of external parameter as, background intensity and external applied. Another possibility would to use the light-induced crystal (Photorefractive Axicon) as spatial light modulator (SLM) focusing a plane wave for generates a new Bessel beam output; this possibility is acceptable because how saw, the refraction index modulated extends along over radial direction with considerable values nonzero.
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