What We Can Learn About Nucleon Spin Structure From Recent Data

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Abstract

We have used recent data from CERN and SLAC to extract information about nucleon spin structure. We find that the SMC proton data on $\int_0^1 g_1^p dx$, the E142 neutron data on $\int_0^1 g_1^n dx$ and the deuteron data from SMC and E143 give different results for fractions of the spin carried by each of the constituents. These appear to lead to two different and incompatible models for the polarized strange sea. The polarized gluon distribution occurring in the gluon anomaly does not have to be large in order to be consistent with either set of experimental data. However, it appears that the discrepancies in the implications of these data cannot be resolved with any simple theoretical arguments. We conclude that more experiments must be performed in order to adequately determine the fraction of spin carried by each of the nucleon constituents.

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I. Introduction

One of the important questions in high energy physics is how nucleon spin is related to the spins of the quark and gluon constituents. Significant interest in high energy polarization was piqued a few years ago when the European Muon Collaboration (EMC) analyzed polarized deep-inelastic scattering (DIS) data which appeared to contradict theoretical predictions, creating the "spin crisis". Since then, a flurry of theoretical and experimental work has been performed to address this "crisis" and further investigate the spin properties of the nucleons.

The spin dependent asymmetry in the deep-inelastic scattering of longitudinally polarized leptons on longitudinally polarized nucleons is given by

\[ A = \left[ \frac{\sigma(\leftarrow\rightarrow) - \sigma(\leftarrow\leftarrow)}{\sigma(\leftarrow\rightarrow) + \sigma(\leftarrow\leftarrow)} \right] \approx D \cdot A_1, \quad (1.1) \]

where the arrows refer to the relative longitudinal spin directions of the beam and target, respectively, and it is assumed that \( A_2 \) is small, since it is bounded by \( R = \sqrt{\frac{\sigma_L}{\sigma_T}} \). Information about the polarized quark distributions can be extracted from this asymmetry by

\[ A_1 = \frac{\sum_i e_i^2 \Delta q_i(x)}{\sum_i e_i^2 q_i(x)}, \quad (1.2) \]

where the sums are over all quark flavors. The proton structure function \( g_1^p \) can be extracted from the asymmetry \( A_1 \) by using

\[ g_1^p(x, Q^2) \approx \frac{A_1(x) F_2(x, Q^2)}{2x(1 + R)}. \quad (1.3) \]

It is assumed that the transverse structure function \( g_2^p \) is small and that \( A_1 \) is relatively independent of \( Q^2 \), which has been verified by experimental measurements. The extrapolation of the EMC data for \( g_1^p \) to lower Bjorken \( x \) led to implications that, although the Bjorken sum rule (BSR) of QCD was satisfied, the Ellis-Jaffe sum rule\(^3\), based on a simple quark model, was violated. Recently, the Spin Muon Collaboration (SMC) group from CERN\(^4\) and the E142/E143 experimental groups from SLAC\(^5\) have measured \( A_1^p \) and \( g_1^p \) to even lower \( x \) values and have added the corresponding neutron and deuteron structure functions.
These groups have also improved statistics and lowered the systematic errors from the original data.

The DIS experiments with equations (1.2) and (1.3) can provide a means by which we can extract the polarized quark distribution functions. We can check the consistency of these distributions by comparing proton data \( g_1^p \), neutron data \( g_1^n \) and deuteron data \( g_1^d \) via sum rules. There are many possibilities for models of the polarized quark and gluon distributions which are consistent with sum rule and data constraints. The motive here is to point out some of these possibilities and compare our analysis to that of others. For example, Close and Roberts\(^6\) have done an analysis of the proton and neutron DIS data with an emphasis on the integrated distributions and the overall flavor contributions to nucleon spin. Ellis and Karliner\(^7\) have done a similar analysis which includes higher order QCD corrections. We have done a more detailed flavor dependent analysis including the QCD corrections and the effect of the gluon anomaly. We will proceed by assuming that the polarized gluon distribution is of moderate size and find that the resulting polarized quark distributions which are consistent with data and the appropriate sum rules. Our approach to this analysis consists of three parts: (1) separating the valence and sea integrated parton distributions for each flavor using different data sets as a basis to perform the analyses, (2) discussing similarities and differences between the phenomenological implications of the experimental results, and (3) suggesting a set of experiments which would distinguish the quark and gluon contributions to the proton spin. Our analysis differs from that of the experimental groups in that we use sum rules in conjunction with a single experimental result to extract the spin information, while they use data from multiple experiments in order to check the validity of the sum rules. In addition, they assume a flavor symmetric sea and ignore anomaly contributions. However, there is relatively good physics agreement with our results and those of the experimental groups.

The paper is structured as follows: In part II, we discuss the theoretical basis for determining the polarized parton distributions and the assumptions we have made to generate them. In part III, we discuss our phenomenological analysis of the existing data and the consistency of the various models. Part IV is a discussion of the experiments which can be performed with existing accelerators to further our knowledge of the spin content of nucleons.
II. Theoretical Background

A. Polarized Quark Distributions

Fundamentally, we assume that the nucleons are comprised of valence quarks, whose polarized and integrated distributions are defined by:

$$\Delta q_v(x, Q^2) \equiv q_v^+(x, Q^2) - q_v^-(x, Q^2)$$

$$\langle \Delta q_v(Q^2) \rangle \equiv \int_0^1 \Delta q_v(x, Q^2) \, dx,$$

where \( + (-) \) indicates the quark spin aligned (anti-aligned) with the nucleon spin. In order to construct the polarized quark distributions from the unpolarized ones, we can start with a modified 3-quark model based on an SU(6) wave function for the proton. This model is based on flavor symmetry of the u- and d-sea and constructs the valence distributions to satisfy the Bjorken sum rule.\(^8\) The valence quark distributions can be written in the form:

$$\Delta u_v(x, Q^2) = \cos \theta_D [u_v(x, Q^2) - \frac{2}{3} d_v(x, Q^2)],$$

$$\Delta d_v(x, Q^2) = -\frac{1}{3} \cos \theta_D d_v(x, Q^2),$$

where \( \cos \theta_D \) is a ”spin dilution” factor which vanishes as \( x \to 0 \) and becomes unity as \( x \to 1 \), characterizing the valence quark helicity contribution to the proton.\(^8,9\) Normally, the spin dilution factor is adjusted to satisfy the Bjorken sum rule and to agree with the deep-inelastic data at large \( x \).

To generate the valence quark distributions, we use the higher order set of GRV\(^{10}\) unpolarized distributions, evolved to the \( Q^2 \) scales of each experiment. These agree with the MRS\(^{11}\) distributions for \( x \geq 0.05 \). The spin dilution factor in equation (2.2) was determined from the BSR, which we have assumed valid. The consistency of the resulting polarized valence distributions was checked by comparing with the value for the ratio of proton and neutron magnetic moments:

$$\frac{\mu_p}{\mu_n} = \frac{2\langle \Delta u_v \rangle - \langle \Delta d_v \rangle}{2\langle \Delta d_v \rangle - \langle \Delta u_v \rangle} \approx -\frac{3}{2}. \quad (2.3)$$
With our values \( \langle \Delta u_v \rangle = 1.00 \pm 0.01 \) and \( \langle \Delta d_v \rangle = -0.26 \pm 0.01 \), both the BSR and magnetic moment ratio are satisfied. This also yields a spin contribution from the valence quarks equal to \( 0.74 \pm 0.02 \), consistent with other treatments of the spin content of quarks.\(^{12,13}\)

The quoted errors arise from data errors on \( g_A/g_V \), and the differences in choice of the unpolarized distributions used to generate the polarized valence quark distributions. The original analysis by Qiu, et. al.\(^8\) effectively reached the same conclusion.

The polarization of the sea occurs by gluons that are emitted by gluon Bremsstrahlung and by quark-antiquark pair creation. The corresponding integrated polarized sea distribution is defined as:

\[
\langle \Delta S(Q^2) \rangle \equiv \langle \Delta u(Q^2) + \Delta \bar{u}(Q^2) + \Delta d(Q^2) + \Delta \bar{d}(Q^2) + \Delta s(Q^2) + \Delta \bar{s}(Q^2) \rangle,
\]

where the polarized sea flavors are defined analogous to the valence quarks. It is assumed that the lightest flavors dominate the sea polarization, since the heavier quarks should be significantly harder to polarize. Thus, we assume that the quark and antiquark flavors are symmetric, but break the SU(6) symmetry of the sea by assuming that the heavier strange quarks will be less polarized.\(^8\) Then, the sea distributions are related as follows:

\[
\Delta \bar{u}(x, Q^2) = \Delta u(x, Q^2) = \Delta \bar{d}(x, Q^2) = \Delta d(x, Q^2) = [1 + \epsilon] \Delta \bar{s}(x, Q^2) = [1 + \epsilon] \Delta s(x, Q^2).
\]

The \( \epsilon \) factor is a measure of the increased difficulty in polarizing the strange sea quarks.

In terms of the proton wave function, we can write the integrated distributions as

\[
\langle \Delta q_i s^\mu \rangle = \langle ps | \bar{q} \gamma^\mu \gamma_5 q_i | ps \rangle / 2m,
\]

where \( s^\mu(p) \) is the axial four-vector which characterizes a spin \( \frac{1}{2} \) particle and \( m \) is the mass of the particle. The integrated polarized structure function, \( I_p(n) \equiv \int_0^1 g_1^{p(n)}(x) dx \), is related to the polarized quark distributions by

\[
I_p(n) = \frac{1}{2}(1 - \alpha_s^{corr}) \left[ \frac{4(1)}{9} \Delta u_v + \frac{4(1)}{9} \Delta d_v + \frac{4(1)}{9} (\Delta u_s + \Delta \bar{u}) + \frac{4(1)}{9} (\Delta d_s + \Delta \bar{d}) + \frac{1}{9} (\Delta s + \Delta \bar{s}) \right].
\]

The QCD corrections, characterized by \( \alpha_s^{corr} \), have been calculated to \( O(\alpha_s^4) \) and are

\[
\alpha_s^{corr} \approx \left( \frac{\alpha_s}{\pi} \right) + 3.5833\left( \frac{\alpha_s}{\pi} \right)^2 + 20.2153\left( \frac{\alpha_s}{\pi} \right)^3 + 130\left( \frac{\alpha_s}{\pi} \right)^4,
\]

4
where the last term is estimated. The higher twist corrections calculated by Stein, et. al.\textsuperscript{15} are small enough to neglect at the $Q^2$ values of the data. The QCD corrections have a much more significant effect in extracting information from the data and sum rules. In fact, although the last correction term in eq. (2.8) is estimated, its effect on the extracted numbers is less than the significant figures which we report.

Thus, the data on $g_1$ allows the determination of a linear combination of $\epsilon$ and the overall size of the polarized sea. This is not enough, however, to determine all of the sea parameters. Additional constraints are provided by the axial-vector current operators, $A^k_{\mu}$, whose matrix elements for the proton define the coefficients, $a^k$, as\textsuperscript{16}

$$
\langle ps \mid A^k_{\mu} \mid ps \rangle = s_\mu a^k,
$$

(2.9)

where the $a^k$ are non-zero for $k=0, 3$ and 8. These current operators are members of an SU(3)$_f$ octet, whose non-zero elements give relations between the polarized distributions and the measurable coefficients $a^k$. The Bjorken sum rule relates the polarized structure function $g_1(x)$, measured in polarized deep-inelastic scattering, to the axial vector current $A^3_{\mu}$. The coefficient $a^3$ is measured in neutron beta decay and this sum rule is considered to be a fundamental test of QCD. In terms of the polarized distributions and our assumptions about the flavor symmetry of the $u$ and $d$ polarized sea, the Bjorken sum rule can be reduced to the form:

$$
\int_0^1 [\Delta u_{v}(x, Q^2) - \Delta d_{v}(x, Q^2)] \, dx = a^3,
$$

(2.10)

which enables us to determine the valence distributions, as previously mentioned. Furthermore, since the BSR relates $g_1^p$ and $g_1^n$, the DIS data on $g_1$ (for $p$, $n$ and $d$) can be used to set constraints on the polarized sea distributions.

The coefficient $a^8$ is determined by hyperon decay, reflecting the other baryon axial charges in the symmetry. A traditional analysis of hyperon decays, yields two empirical constants: $D$ and $F$,\textsuperscript{6} which are related to the polarized quark distributions by $a^8$. This relation can be written as:

$$
a^8 = \langle [\Delta u_{v} + \Delta d_{v} + \Delta u_{s} + \Delta d_{s} + \Delta \bar{u} + \Delta \bar{d} - 2\Delta s - 2\Delta \bar{s}] \rangle = 3F - D \approx 0.58 \pm 0.02.
$$

(2.11)

Lipkin has pointed out that one must proceed with caution in using hyperon spin structures,
however, without a suitable hyperon spin model. Fortunately, our analysis is not highly sensitive to the value of \( a^8 \).

The factor \( a^0 \) is related to the total spin carried by the quarks in the proton. Assuming that the \( u \) and \( d \) flavors are symmetric in the polarized sea, we can relate the non-zero axial currents and the structure function \( g_1^p \) in the flavor-independent form:

\[
a^0 \approx 9(1 - \alpha_{\text{corr}}^s) - \frac{1}{4} a^8 - \frac{3}{4} a^3.
\] (2.12)

Thus, the equations for the axial current coefficients give constraints to the polarized quark distributions, from which we can attempt to extract specific information about individual contributions to the overall proton spin. Shortly after the EMC experiment, there were a number of theoretical calculations which isolated the contributions of each of the flavors of the polarized sea to the proton spin.\(^8,18\) All of these arrived at the conclusion that the sea is negatively polarized, which is reasonable when one analyzes the spin dependent forces which cause polarization of the sea from valence quarks and gluons. Updated values for the flavors of polarized distributions can be determined from the recent SMC and SLAC data.\(^4,5\)

One can impose theoretical constraints on the polarized strange sea\(^8\) by assuming that

\[
| \int_0^1 \Delta s(x) \, dx | \leq \frac{1}{3} \int_0^1 x \pi(x) \, dx \approx 0.005.
\] (2.13)

This "Valence Dominated Model" (VDM) is based on a mechanism where sea quarks obtain their polarization through a localized interaction with the valence quarks. This model provides a more restrictive limit on the size of the polarized strange sea than the positivity constraint discussed by Preparata, Ratcliffe and Soffer.\(^19\) The VDM model can be compared with the integrated distributions extracted from the data to check for its validity.

B. Polarization of Gluons

The gluons are polarized through Bremsstrahlung from the quarks. The integrated polarized gluon distribution is written as

\[
\langle \Delta G \rangle = \int_0^1 \Delta G(x, Q^2) \, dx = \int_0^1 [G^+(x, Q^2) - G^-(x, Q^2)] \, dx,
\] (2.14)

where the \( +(-) \) indicates spin aligned (anti-aligned) with the nucleon, as in the quark distributions. We cannot determine \textit{a priori} the size of the polarized gluon distribution
in a proton at a given $Q^2$ value. The evolution equations for the polarized distributions, indicate that the polarized gluon distribution increases with $Q^2$ and that its evolution is directly related to the behavior of the orbital angular momentum, since the polarized quark distributions do not evolve in $Q^2$ in leading order. Thus, one assumes a particular form for the polarized gluon distribution for a given $Q^2$ and checks its consistency with experimental data which are sensitive to $\Delta G(x, Q^2)$ at a particular $Q_0^2$. Initial analyses of the EMC data led to speculation that the integrated gluon distribution may be quite large, even at the relatively small value of $Q^2 = 10.7 \text{ GeV}^2$.

The model of $\Delta G$ that is used has a direct effect on the measured value of the quark distributions through the gluon axial anomaly. In QCD, the $U(1)$ axial current matrix element $A^0_\mu$ is not strictly conserved, even with massless quarks. Hence, at two loop order, the triangle diagram between two gluons generates a $Q^2$ dependent gluonic contribution to the measured polarized quark distributions. This term has the general form:

$$\Gamma(Q^2) = \frac{N_f \alpha_s(Q^2)}{2\pi} \int_0^1 \Delta G(x, Q^2) \, dx,$$  \hspace{1cm} (2.15)$$

where $N_f$ is the number of quark flavors. Thus, for each flavor of quark appearing in the distributions, the measured polarization distribution is modified by a factor: $\langle \Delta q_i \rangle = \Gamma(Q^2)/N_f$. In order for us to determine the quark contributions to the spin of the nucleons, it is necessary for us to know the relative size of the polarized gluon distribution. If we base our analysis solely on the naive quark model, then $\sum \Delta q \rightarrow 1$ and $\Delta G$ may be quite large to be consistent with EMC data. This is surprising, since there are no high-spin excited states of nucleons which create such a large $\Delta G$. If we consider the polarized distributions of Qiu et. al., a reasonably sized $\Delta G$ is possible if the sea has a suitably negative polarization.

We have considered two possible models for $\Delta G$:

$$\begin{align*}
(1) \quad & \Delta G(x) = x \, G(x), \\
(2) \quad & \Delta G = 0. \hspace{1cm} (2.16)
\end{align*}$$

The first implies that the spin carried by gluon is the same as its momentum, motivated by both simple PQCD constraints and the form of the splitting functions for the polarized evolution equations. The second provides an extreme value for determining limits on the values of the polarized sea distribution.
Another natural constraint to the polarized distributions relates the integrated parton distributions to the orbital angular momentum of the constituents. Due to $O(2)$ invariance, a proton with momentum and spin in the $z$-direction will conserve $J_z$. This total spin sum rule can be written in terms of the polarized distributions as:

$$J_z = \frac{1}{2} = \frac{1}{2} \langle \Delta q_v \rangle + \frac{1}{2} \langle \Delta S \rangle + \langle \Delta G \rangle + L_z. \quad (2.17)$$

The right hand side represents the decomposition of the constituent spins along with their relative angular momentum, $L_z$. Although this does not provide a strict constraint on either $\Delta q_{tot}$ or $\Delta G$, it does give an indication of the fraction of total spin due to the angular momentum component as compared to the constituent contributions.

C. Interpolation of Data at Small-x

All sets of data are limited in the range of Bjorken $x$ and thus, the integrals must be extrapolated to $x \to 0$. Thus, the possibility of existence of a Regge type singularity at $x \to 0$ is not accounted for in the analyses. A significant singularity could raise the value of $g_1^p$ towards the naive quark model value and could account for some of the discrepancy between the original EMC data and the Ellis-Jaffe sum rule. In light of the recent HERA data, there is the possibility that the increase in $F_2$ at small $x$, even at the lower $Q^2$ values of the E142/E143 data, could indicate a change in the extrapolated values of these integrals. These possibilities are a topic for future study. For the purposes of this paper, we will assume that this overall effect of $F_2$ on $g_1^p$ will not alter the integral by any more than the present experimental errors. We use the unpolarized distributions of MRS and GRV in section III B since they include the small-$x$ data from HERA. The shape of the polarized gluon distribution at small-$x$ affects the anomaly term, and thus the overall quark contributions to the integrals. Future experiments can shed light on the size of this effect, a detail we will discuss in section IV. We believe that the present data show that anomaly effects are limited and the overall integrated polarized gluon distribution is not very large at these energies. This point is discussed in the next section.
III. Phenomenology

A. Assumptions and Analysis using New Data

We consider recent SMC\textsuperscript{4} and E142/143\textsuperscript{5} data to extract polarization information about the sea. The SMC experiment, which measured \( \int_0^1 g_1^p dx \), consisted of deep inelastic scattering of polarized muons off of polarized protons in the kinematic range \( 0.003 \leq x \leq 0.7 \) and \( 1 \text{ GeV}^2 \leq Q^2 \leq 60 \text{ GeV}^2 \). The data were then extrapolated to yield the integrated value of the structure function. In the other SMC experiment, the polarized proton target was replaced by a polarized deuteron target and \( \int_0^1 g_1^d dx \) was extracted from data in the kinematic range \( 0.003 \leq x \leq 0.7 \) and \( 1 \text{ GeV}^2 \leq Q^2 \leq 60 \text{ GeV}^2 \). The E142 experiment extracted \( \int_0^1 g_1^n dx \) from data in the kinematic range \( 0.03 \leq x \leq 0.6 \) and \( 1 \text{ GeV}^2 \leq Q^2 \leq 60 \text{ GeV}^2 \) by scattering polarized electrons off of a polarized \(^3\text{He} \) target. The E143 experiment measured \( \int_0^1 g_1^n dx \) in the kinematic range \( 0.03 \leq x \leq 0.8 \) and \( 1 \text{ GeV}^2 \leq Q^2 \leq 10 \text{ GeV}^2 \) by scattering polarized electrons off of a solid polarized deuterated ammonia \(^{15}\text{ND}_3 \) target. The integrated results with errors and average \( Q^2 \) values are summarized in Table I.

| Table I: Experimental Parameters for the Integrated Structure Functions |

| Quantity       | SMC \((I^p)\) | SMC \((I^d)\) | E142 \((I^n)\) | E143 \((I^d)\) |
|----------------|--------------|--------------|---------------|---------------|
| \(I_{exp}^{10}\) | .136         | .034         | -.022         | .041          |
| \(Stat. err.\) | ±.011        | ±.009        | ±.007         | ±.003         |
| \(Sys. err.\)  | ±.011        | ±.006        | ±.006         | ±.004         |
| Avg. \(Q^2\) \((\text{GeV}^2)\) | 10.0         | 10.0         | 2.0           | 3.0           |
| \(\alpha_s(Q^2)\) | .27          | .27          | .385          | .35           |

We can write the integrals of the polarized structure functions, \( \int_0^1 g_1^i dx \) in the terms
of the coefficients $a^k$:

\[
I^p \equiv \int_0^1 g_1^p(x) dx = \left[ \frac{a^3}{12} + \frac{a^8}{36} + \frac{a^0}{9} \right] \left( 1 - \alpha_s^{corr} \right),
\]

\[
I^n \equiv \int_0^1 g_1^n(x) dx = \left[ -\frac{a^3}{12} + \frac{a^8}{36} + \frac{a^0}{9} \right] \left( 1 - \alpha_s^{corr} \right),
\]

\[
I^d \equiv \int_0^1 g_1^d(x) dx = \left[ \frac{a^8}{36} + \frac{a^0}{9} \right] \left( 1 - \alpha_s^{corr} \right) \left( 1 - \frac{3}{2} \omega_D \right),
\]

where $\omega_D$ is the probability that the deuteron will be in a D-state. Using N-N potential calculations, the value of $\omega_D$ is about 0.058.\textsuperscript{4,24} The difference $(I^p - I^n)$ is the Bjorken sum rule, which is fundamental to the tests of QCD. There seems to be agreement in the experimental papers that the data from each substantiates the Bjorken sum rule, to within the experimental errors. We have assumed that the BSR is valid, and have used it as a starting point for extracting an effective $I^p$ value from neutron and deuteron data. The comparison of the effective $I^p$ values gives a measure of the consistency of the different experiments to the BSR.

Considerable discussion regarding results of these measurements focuses on the Ellis-Jaffe sum rule (EJSR),\textsuperscript{3} which predicts the values of $g_1^p$ and $g_1^n$ using an unpolarized sea. This has the form:

\[
I^p = \frac{1}{18} [9F - D] \left( 1 - \alpha_s^{corr} \right),
\]

\[
I^n = \frac{1}{18} [6F - 4D] \left( 1 - \alpha_s^{corr} \right),
\]

where F and D are the empirically determined $\beta$-decay constants, constrained so that their sum: $F + D = g_A/g_V$ satisfies the Bjorken sum rule. Using the approximate values,\textsuperscript{6} $F \approx 0.46 \pm 0.01$ and $D \approx 0.80 \pm 0.01$, this sum rule predicts that $I^p = 0.161$ and $I^n = -0.019$. These values of F and D also yield

\[
a^8 \equiv 3F - D = 0.58 \pm 0.03.
\]

Clearly, the E142 $(I^n)$ data are consistent with this sum rule, while the other data are not. Higher order corrections to the EJSR have been calculated,\textsuperscript{25} but amount to about a 10% correction to the values of the integrals and are not enough to account for the discrepancy with the SMC data. Higher twist corrections\textsuperscript{26} are only significant at the lower $Q^2$ values.
of the E142 $I^*$ data, where there is agreement with the EJSR. The point thus focuses on
the discussion of the size of the polarized sea, which differs in analyses of these data. We
address this in detail later.

The experimental values of $I^{exp}$ for the proton, neutron and deuteron, combined with
the value of $\Delta q_v$ determined in section II, can be used to determine the polarization of each
of the sea flavors. With anomaly correction, the total spin carried by each of the flavors can
be written as:

$$\langle \Delta q_{i,\text{val}} + \Delta q_{i,\text{sea}} + \Delta \eta_i - \frac{\alpha_s}{2\pi} \Delta G \rangle = \langle \Delta q_{i,\text{tot}} \rangle.$$  \hfill (3.4)

The anomaly terms included in the quark distributions have the form of equation (2.15)
using both models of the polarized glue from (2.16).

Since the anomalous dimensions for the polarized distributions have an additional
factor of $x$ compared to the unpolarized case, early treatments of the spin distributions
assumed a form of: $\Delta q(x) \equiv xq(x)$ for all flavors.\textsuperscript{22} We have compared this form of the
distributions to those extracted from the recent data, using the defined ratio $\eta \equiv \frac{\langle \Delta q_{\text{sea}} \rangle_{\text{exp}}}{\langle xq_{\text{sea}} \rangle_{\text{calc}}}$
for each flavor.

B. Results for the Polarized Distributions

The analysis for each polarized gluon model proceeds as follows:

(i) We extract a value of $I^p$ from either the data directly or via the BSR using equation
(3.1). Then, equation (2.12) is used to extract $a_0$. The anomaly dependence on both sides of
eq (2.12) cancels, but the overall contribution to the quark spin, $\langle \Delta q_{\text{tot}} \rangle = a_0 + \Gamma$, includes
the anomaly term. The value $a_8$ from the hyperon data with equations (2.11) and (2.12) are
then used to extract $\Delta s$ for the strange sea. The total contribution from the sea then comes
from $\langle \Delta q_{\text{tot}} \rangle = \langle \Delta q_v \rangle + \langle \Delta S \rangle$. The factor $\epsilon$ and the distributions $\langle \Delta u \rangle_{\text{sea}} = \langle \Delta d \rangle_{\text{sea}}$ are then
derived from equations (2.4) and (2.5). Finally, the $J_z = 1/2$ sum rule (equation 2.17) gives
$L_z$.

(ii) Since the VDM model is based upon the chiral distributions, we calculate the
corresponding results in Table IIa, where the anomaly term is zero. Here, $\Delta s$ comes from
the VDM assumption for the strange sea (equation 2.13). Then $\langle \Delta q \rangle_{\text{tot}} = a_0$ can be extracted
from $a_0 - a_8 = 6 \langle \Delta s \rangle$. Finally, $g_1^p$ comes from (2.12), and the other sea information can be
extracted. This provides a theoretical limit on these quantities, based upon a restricted
strange sea polarization.

The overall results are presented in Table II ($\Delta G = xG$) and Table IIa ($\Delta G = 0$). The E143 proton data\textsuperscript{5} gives virtually the same numbers as the deuteron data shown in these tables.

**Table II: Integrated Polarized Distributions: $\Delta G = xG$**

| Quantity | $SMC(I^p)$ | $SMC(I^d)$ | $E142(I^n)$ | $E143(I^d)$ |
|----------|------------|------------|-------------|-------------|
| $< \Delta u >_{sea}$ | −.077 | −.089 | −.050 | −.068 |
| $< \Delta s >$ | −.037 | −.048 | −.010 | −.028 |
| $< \Delta u >_{tot}$ | 0.85 | 0.82 | 0.90 | 0.87 |
| $< \Delta d >_{tot}$ | −.42 | −.43 | −.36 | −.40 |
| $< \Delta s >_{tot}$ | −.07 | −.10 | −.02 | −.06 |
| $\eta_u = \eta_d$ | −2.4 | −2.8 | −1.6 | −2.1 |
| $\eta_s$ | −2.0 | −3.0 | −0.7 | −1.6 |
| $\epsilon$ | 1.09 | 0.84 | 4.00 | 1.41 |
| $\Gamma$ | 0.06 | 0.06 | 0.08 | 0.08 |
| $I^p$ | 0.136 | 0.129 | 0.137 | 0.131 |
| $< \Delta q >_{tot}$ | 0.36 | 0.29 | 0.52 | 0.41 |
| $< \Delta G >$ | 0.46 | 0.46 | 0.44 | 0.44 |
| $L_z$ | −.14 | −.11 | −.20 | −.15 |
Table IIa: Integrated Polarized Distributions: $\Delta G = 0$

| Quantity       | SMC($I^p$) | SMC($I^d$) | E142($I^n$) | E143($I^d$) | VDM |
|----------------|------------|------------|-------------|-------------|-----|
| $< \Delta u >_{sea}$ | -0.087     | -0.099     | -0.063      | -0.082      | -0.045 |
| $< \Delta s >$                     | -0.047     | -0.058     | -0.023      | -0.042      | -0.005 |
| $< \Delta u_{tot} >$               | 0.83       | 0.80       | 0.88        | 0.84        | 0.91  |
| $< \Delta d_{tot} >$               | -0.44      | -0.45      | -0.39       | -0.43       | -0.35  |
| $< \Delta s_{tot} >$               | -0.09      | -0.12      | -0.05       | -0.08       | -0.01  |
| $\eta_u = \eta_d$                  | -2.7       | -3.2       | -2.0        | -2.6        | -1.4 |
| $\eta_s$                            | -2.4       | -3.7       | -1.7        | -2.5        | -0.3 |
| $\epsilon$                         | 0.86       | 0.70       | 1.71        | 0.96        | 8.00 |
| $\Gamma$                           | 0.00       | 0.00       | 0.00        | 0.00        | 0.00 |
| $I^p$                               | 0.136      | 0.129      | 0.137       | 0.131       | 0.152 |
| $< \Delta q >_{tot}$                | 0.30       | 0.23       | 0.44        | 0.33        | 0.55 |
| $L_z$                                | 0.35       | 0.39       | 0.28        | 0.35        | 0.23 |

From tables II and IIa, it is obvious that the naive quark model is not sufficient to explain the characteristics of nucleon spin. However, these results have narrowed the range of constituent contributions to the proton spin. The following conclusions can be drawn, which lead to a modified view of the proton’s spin picture.

(1) The total quark contribution to the proton spin is between $\frac{1}{4}$ and $\frac{1}{2}$, as opposed the quark model value of one, or the extracted EMC value of zero. The errors in determining the total quark contribution are due to experimental errors ($\pm 0.04 \rightarrow \pm 0.08$ for $\Delta q_{tot}$) and the uncertainty in the value for $\Gamma$ ($\approx \pm 0.04$). For a given gluon model, however, the differences are slightly larger than one standard deviation. This cannot be accounted for by different $Q^2$ values since the polarized quark distribution does not evolve with $Q^2$ to leading order. Also, higher twist effects do not appear to be large enough to account for this difference. But all of these values imply that the total sea, and hence the strange sea, has a smaller
polarization than originally thought after the EMC experiment. The results for the strange sea in the proton and deuteron data are still larger than the positivity bound of Preparata, Ratcliffe and Soffer.\textsuperscript{19} There is an implication here that their positivity bound value is too small, since it is based entirely on unpolarized data. Nevertheless, all data imply that the strange quark spin contribution is much smaller than that of the lighter quarks. The flavor symmetry is broken by the large values of $\epsilon$, namely $0.7 \leq \epsilon \leq 4.0$, as opposed to $\epsilon = 0$. It is also interesting to note that the results obtained from the SMC proton data are consistent with a recent lattice QCD calculation of the polarized quark parameters.\textsuperscript{12} Although the flavor contributions to the proton spin cannot be extracted exactly, the range of possibilities has been substantially decreased by these experiments. Specifically, the up and down contributions agree to within a few percent. The main difference remains the question of the strange sea spin content.

(2) Despite the differences, there are similarities among these sets of data. All of the extracted values for $I^p$ are well within the experimental uncertainties, indicating a strong agreement about the validity of the Bjorken Sum Rule. We have arrived at this conclusion by using the BSR to extract $I^p$, as opposed to the experimental groups, which used data to extract the BSR. There seems to be general agreement as to the consistency of these results.

(3) As we mentioned in section II, the validity of the Ellis-Jaffe sum rule reduces to the question of the size of the polarized sea. It also addresses the assumptions made by the naive quark model and early polarization calculations based on a simple SU(6) model for the proton. The sea results, the deviation of $\epsilon$ from zero and the differences in $\eta$ from one, all indicate that the models for quark polarizations must be modified to account for the experimental results. The physical conclusions are: (i) that the sea is polarized opposite to that of the valence quarks (see ref. 8 for interpretations), (ii) that the strange quarks must be treated separately in determining their contribution to the proton spin due to mass effects, and (iii) that polarized distributions for each quark flavor must be modified so that $\Delta q_f \approx \eta_f \times q_f$, where $\eta$ is extracted from data and is likely different from unity. Thus, the relation between unpolarized and polarized distributions is likely more complex than originally thought.

(4) This analysis implies that the role of the anomaly correction is significant only in the sense that minimizing errors in specifying spin contributions from quark flavors depends on determining the size of the polarized glue. By comparing the results given in tables II
and IIa, where analysis of the data is done with both a zero and a small anomaly correction, we see the key results and conclusions are not significantly different. Further, even if there are higher twist corrections to the anomaly at small $Q^2$, this will not reconcile differences in the flavor dependence of the polarized sea. The anomaly term does not vary significantly enough for the $Q^2$ range of the data to explain any differences. However, the analysis of $I^n$ with the anomaly in equation 3.4 does imply that the polarized glue is limited in size. If the integrated polarized gluon distribution were greater than about 0.9, the strange sea contribution in the $I^n$ column of Table II would be positive, while the other sea flavors would be negatively polarized. There is no apparent reason why these flavors should be polarized in a different direction. The analysis seems to imply an inherent limit to the size of the anomaly and thus, the size of the polarized glue.

(5) Finally, the orbital angular momentum extracted from the $J_z$ sum rule is much smaller for all data than earlier values obtained from EMC data. If the polarized gluon distribution is small enough, then both $\langle \Delta q \rangle_{tot}$ and $\langle \Delta G \rangle$ decrease enough so that $L_z$ must be positive to account for the total spin of nucleons. Thus, both positive and negative values for $L_z$ appear to be possible. Naturally, this opens up the possibility that the angular momentum contribution is negligible, contrary to the naive Skyrme model.

Clearly, these experiments have shed light on the proton spin picture. The major unanswered questions appear to be related to the strange sea spin content and the size of the polarized gluon distribution. These can only be reconciled by performing other experiments which are sensitive to these quantities. To put the strange sea picture in perspective, we can compare various results for the polarized strange sea distributions with other models in the literature, based on various data. These are summarized in Table III, whose numbers represent the total contributions for each flavor (quarks and antiquarks). The models are listed in order of increasing strange sea contributions and refer to the experiments from which they were extracted. The references are keyed as follows: (i) HJL- Lipkin$^{17}$; (ii) VDM- the valence dominated model outlined in section II; (iii) GR- models presented here (with anomaly term and higher order QCD corrections); (iv) CR- models by Close and Roberts$^6$; (v) EK- recent analysis by Ellis and Karliner,$^7$ which incorporate the higher order QCD corrections; (vi) QRRS- models by Qiu, et. al.$^8$; (vii) BEK- the model by Brodsky, et. al.$^{18}$

Disparities between these models depend on theoretical assumptions as well as ex-
experimental data. These models can be divided into two categories: those which satisfy the strange sea positivity constraint are listed above the line, while those which violate this bound are below. Note that the Valence Dominated Model and the $I^n$ data yield results that are above the line and the proton and deuteron data do not. Thus, there is a consistency among the proton/deuteron results (including the EMC data), which all occur above the $Q^2$ values of the neutron results. It is possible that future data and analysis on $I^n$ would yield a ”world average” value so that it would become more consistent with proton and deuteron data. It is clear that more tests are necessary. As pointed out by Qiu, et. al., and others$^8,27$ the most direct experiment to determine the size of the polarized sea is lepton pair production (Drell-Yan) in polarized nucleon scattering experiments. Only then will there be enough information to tell which assumptions about the polarized sea are appropriate.

Table III: Models of the Flavor Dependence of the Polarized Sea

| Model                  | $\Delta u_{tot} = \Delta d_{tot}$ | $\Delta s_{tot}$ |
|------------------------|-----------------------------------|------------------|
| $HJL(EMC)$             | $-0.27$                           | $+0.00$          |
| $VDM(Theory)$          | $-0.09$                           | $-0.01$          |
| $GR(I^n)$              | $-0.10$                           | $-0.02$          |
| $CR(I^n)$              | $-0.12$                           | $-0.03$          |
| $GR(E143 I^d)$         | $-0.14$                           | $-0.06$          |
| $GR(I^p)$              | $-0.15$                           | $-0.07$          |
| $GR(SMC I^d)$          | $-0.18$                           | $-0.10$          |
| $EK(SMC/E142)$         | $-0.17$                           | $-0.10$          |
| $CR(I^d)$              | $-0.20$                           | $-0.11$          |
| $CR(I^p)$              | $-0.21$                           | $-0.12$          |
| $QRRS(EMC)$            | $-0.24$                           | $-0.15$          |
| $BEK(EMC)$             | $-0.26$                           | $-0.23$          |

16
Thus, existing data provide valuable information regarding the proton spin puzzle, but as a whole are not definitive in isolating the key contributions to the proton spin. We stress that more experiments must be performed to determine the relative contributions from various flavors of the sea and the gluons. This is addressed in detail in the next section.

IV. Possible Experiments

There are a number of experiments which are technologically feasible that could supply some of the missing information about these distributions. In this section, we will discuss those experiments which have been proposed and would give specific information necessary for determining the contributions of the sea and gluons to the overall proton spin. Detailed summaries can be found in references 28 and 29. Table IV is extracted from reference 28 and gives information on the spin observables which can be measured to extract the appropriate polarized distributions. The average luminosity of these experiments is approximately $1 \cdot 10^{32} \text{(cm}^{-2}\text{s}^{-1})$ or greater. Furthermore, the success of Siberian Snakes makes them all feasible. The following discussion details the contributions and advantages which each experiment can give in extracting the appropriate spin information.

**Table IV: Proposed Polarization Experiments**

| Experiment | Proposed Type       | Measured Quantities               |
|------------|---------------------|-----------------------------------|
| HERMES     | Deep Inelastic Scattering | $A_1^p, g_1^p, \Delta q_v$      |
| SPIN       | Inelastic : jets    | $\Delta G : \Delta \sigma_L, A_{NN}, A_{LL}$ |
| RHIC       | Inelastic jet, $\pi, \gamma$ | $\Delta G; \Delta \sigma_L, A_{LL}$ |
| RHIC       | Drell Yan           | $\Delta S$                        |
| LISS       | Inelastic           | $\sigma_L, \sigma_T, \Delta \sigma_L, \Delta G$ |
| LHC        | Inelastic           | $A_N, A_L, A_{NN}, A_{LL}$        |

A. Extrapolation to Lower $x$

The E154 and E155 deep-inelastic scattering experiments have been approved at
SLAC, with the former presently in progress. These experiments are designed to probe slightly smaller $x$, while improving statistics and systematic errors. The latest proposed experiment at HERA in Hamburg plans to accelerate a large flux of polarized electrons from the storage ring and collide them with a gaseous target.\textsuperscript{30} The HERMES detector at HERA is designed to take data from the deep-inelastic scattering experiment at a large range of $x$ values, down to 0.02, in contrast to 0.03 in the E143 experiment at SLAC, thus expanding and reinforcing the SMC and E142/3 data. The gaseous target should eliminate some of the systematic errors characteristic of solid targets, which were used in the other experiments. With lower error bars at small $x$, the extrapolation of the integrals should enable these experimental groups to achieve a more accurate value for $P$. Thus, comparison of data to the sum rules and the integrated polarized sea values will be more accurate.

B. Measurement of the Polarized Sea Distributions

The SPIN Collaboration has proposed to do fixed target $pp$ and $p\bar{p}$ experiments at energies of 120 GeV and 1 TeV.\textsuperscript{31} The proposal also includes $pp$ collider experiments at 2 TeV. The luminosity at the lowest energies would be larger than the stated average. One of the crucial contributions that this set of experiments can make is the extremely large range of high $p_T$ that can be covered. If this set of experiments includes polarized lepton pair production, (Drell-Yan) then measurement of the corresponding double spin asymmetries\textsuperscript{8,27} would give a sensitive measure of the polarized sea distribution’s size.

The Relativistic Heavy Ion Collider (RHIC) at Brookhaven is designed to accelerate both light and heavy ions. The high energy community has proposed that polarized $pp$ and $p\bar{p}$ experiments be performed, due to the large energy and momentum transfer ranges which should be available.\textsuperscript{32} This energy range will be covered in discrete steps of about 60, 250 and 500 GeV, but the momentum transfer range covers $0.005 \leq Q^2 \leq 6.0$ GeV$^2$ in a fairly continuous set of steps. There are two main proposed detectors, STAR and PHENIX, which have different but complementing capabilities. PHENIX is suitable for lepton detection and the wide range of energies and momentum transfers could yield a wealth of Drell-Yan data over a wide kinematic range. The $x$-dependence of the polarized sea distributions could then be extracted to a fair degree of accuracy.

There has been considerable discussion about performing polarization experiments at the LHC at CERN.\textsuperscript{33} Depending on the approved experiments, there is the possibility of probing small $x$ and doing other polarized inclusive experiments to measure both sea
and gluon contributions to proton spin. These could be made in complementary kinematic regions to those listed in the other proton accelerators.

C. Determination of the Polarized Gluon Distributions

The SPIN Collaboration proposes a set of experiments, which are in the kinematic region where the measurement of double spin asymmetries in jet production would give a sensitive test of the polarized gluon distribution’s size.\textsuperscript{34,35} Naturally, this measurement has an effect on both $\Delta G$ and the anomaly term appearing in the polarized quark distributions.

The STAR detector at RHIC is suitable for inclusive reactions involving jet measurements, direct photon production and pion production. All of these would provide excellent measurements of the $Q^2$ dependence of $\Delta G$ since all are sensitive to the polarized gluon density at differing $Q^2$ values.\textsuperscript{27,35}

Recently, a proposal for a new light ion accelerator has been announced, which will specialize in polarization experiments.\textsuperscript{36} The Light Ion Spin Synchrotron (LISS) would be located in Indiana, and would perform a variety of polarization experiments for both high energy and nuclear physics. The energy range would be lower that most other experiments, complementing the kinematic areas covered. Furthermore, both proton and deuteron beams could be available to perform inclusive scattering experiments. They propose to measure cross sections and longitudinal spin asymmetries which are sensitive to the polarized gluon distribution (see Table IV).

Tests of the valence quark polarized distributions can be made, provided a suitable polarized antiproton beam of sufficient intensity could be developed.\textsuperscript{34} This would provide a good test of the Bjorken sum rule via measurement of $\langle \Delta q_v \rangle$\textsuperscript{35} and the assumption of a flavor symmetric up and down sea. This should be an experimental priority for the spin community. Polarization experiments provide us with a unique and feasible way of probing hadronic structure. Existing data indicate that the spin structure of nucleons is non-trivial and has led to the formulation of a crucial set of questions to be answered about this structure. The experiments discussed above can and should be performed in order to shed light on the quark and gluon spin structure of nucleons.
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