Self interacting Brans Dicke cosmology and Quintessence

S. Sen and T. R. Seshadri
Mehta Research Institute, Chhatnag Road, Jhusi. Allahabad 211 019 India
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Recent cosmological observations reveal that we are living in a flat accelerated expanding universe. In this work we have investigated the nature of the potential compatible with the power law expansion of the universe in a self interacting Brans Dicke cosmology with a perfect fluid background and have analyzed whether this potential supports the accelerated expansion. It is found that positive power law potential is relevant in this scenario and can drive accelerated expansion for negative Brans Dicke coupling parameter $\omega$. The evolution of the density perturbation is also analyzed in this scenario and is seen that the model allows growing modes for negative $\omega$.

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1. INTRODUCTION

It has been suggested by a number of recent observations [1] that merely the baryonic matter and dark matter may not be able to account for the total density of the universe. This suggests that either the universe is open (density less than the critical density), or, that there is yet another source of energy density which makes the universe spatially flat ($\Omega_{\text{tot}} = 1$). The anisotropy of the cosmic microwave background radiation (CMBR) as indicated by the recent results from Boomerang [2] strongly favour the second possibility of a flat universe which makes the issue of missing energy more important. Quintessence [3] has been proposed as that missing energy density component that along with the matter and baryonic density makes the density parameter equal to 1.

The luminosity-redshift relation observed for the Type-Ia supernovae [4] strongly suggests that, in the present phase, the universe is undergoing an accelerated expansion. This is supported also by the recent measurements of CMBR and the power spectrum of mass perturbations [5]. Of course, the simplest and the most extreme choice in such a case, is the model with the vacuum energy density or the cosmological constant [6], but other options also exist. Models with quintessence and cold dark matter (QCDM) is one interesting possibility. Basically, quintessence is a dynamical slowly evolving spatially inhomogeneous component of energy density with negative pressure [7]. The energy density associated with a scalar field $\phi$ slowly moving down its potential $V$ can represent a simple example of Quintessence [8]. The potential energy of the scalar field should dominate over the kinetic energy of the field for slow rolling and thereby making the pressure negative ($\phi_0 = \frac{1}{2} \dot{\phi}^2 \quad V$). For quintessence, the equation of state defined by $\omega = \frac{p}{\rho}$ lies between 0 and $-1$. More precisely $0 \leq \omega \leq 1$. The deceleration parameter $q = \frac{-1 + \dot{a}}{H^2 a}$ accounting for the acceleration of the universe also lies in the same range 0 and $-1$. Depending on the form of $V(\phi)$, $\omega$ can be constant, slowly varying, rapidly varying or oscillatory [9].

This quintessence proposal faces two types of problems [9]. One of these problems, (referred as fine tuning problem), is the smallness of the energy density compared to other typical particle physics scales. The other problem known as the cosmic coincidence problem, is that although the missing energy density and matter density decrease at different rates as the universe expands, it appears that the initial condition has to be set so precisely that the two densities become comparable today. A special form of quintessence field, called the ’tracker field’, has been proposed to tackle this problem [10]. The good point about these models is that they have attractor like solutions that make the present time behaviour nearly independent of the initial conditions.

A scalar field with a potential dominating over its kinetic energy constitutes the simplest example of quintessence. Quite a few models have also been suggested with a minimally coupled scalar field and different types of potentials. A purely exponential potential is one of the widely studied cases [11]. In spite of the other advantages the energy density is not enough to make up for the missing part. Inverse power law is the other potential ([7]–[9]) that has been studied extensively for quintessence models, particularly for solving the cosmic coincidence problem. Though the problems are resolved successfully with this potential, the predicted value for $\omega$ is not in good agreement with the observed results. In search of proper potentials that would eliminate the problems, new type of potentials, like $V_0 \cosh \left( - \frac{\phi}{\phi_0} \right)$ and $V_0 \sinh \left( - \frac{\phi}{\phi_0} \right)$ have been considered, which have asymptotic forms like the inverse power law or exponential ones. Different physical considerations have lead to the study of other types of the potentials also [12]. Recently Saini et al [13] have reconstructed the potential in context of general relativity and minimally coupled quintessence field from the expression of the luminosity distance $d_L(z)$ as function of redshift obtained from the observational data. However, none of these are entirely free of problems. Hence, there is still a need to identify appropriate potentials to explain current observations [11].

As we have mentioned earlier most of the studies to produce variable have been done with a minimally coupled scalar field representing the quintessence field. It has been recently shown by Pietro and Demaret [15] that for constant scalar field equation of state, which is a good approximation for a tracker...
field solutions, the field equations and the conservation equations strongly constrain the scalar field potential, and most of the widely used potential for quintessence, such as inverse power law one, exponential or the cosine form, are incompatible with these constraints. The minimally coupled self interacting models will also be ruled out if the predictions predict that the missing component of the energy density obeys an equation of state \( p = \frac{\rho}{\rho} \), and these sort of equation of state is in reasonable agreement with different observations \([16]\). These facts motivates toward a more general theory like scalar tensor theory. Scaling attractor solutions in general scalar tensor theories with inverse power law \([13, 17]\) potentials in non-minimally coupled solutions are available in the literature with the exponential \([13]\) features of the model.

In a similar scenario Ritis et al \([20]\) found a family of quadratic potentials in Brans Dicke Cosmology. In the next section we present the exact solution for the density perturbation part of the solution. In the last section we have discussed about the different features of the model.

II. FIELD EQUATIONS AND SOLUTIONS

We start with the Brans Dicke action along with a self interacting potential and a matter field

\[
S = \int d^{4}x \sqrt{-g} \left[ \mathcal{R} + \frac{1}{\xi} \mathcal{L}_{m} \right]
\]

(1)

where \( \xi \) is the Brans Dicke parameter and \( \mathcal{L}_{m} \) is the Lagrangian of the matter field. In this theory \( \xi \) plays the role of the gravitational constant. This action also matches with the low energy string theory action \([25]\) for \( \xi = 1 \). It is checked whether the accelerated cosmic expansion can be driven by this potential. The density perturbation is also studied to check the consistency of the structure formation scenario. In the next section we present the exact solution for the field equations and investigate the nature of the solution. Section 3 discusses the density perturbation part of the solution. In the last section we have discussed about the different features of the model.

The matter content of the universe is composed of perfect fluid

\[
T = (\rho + p)v \mathcal{V} + pg ;
\]

(3)

where \( \rho \) = 1. We assume that the universe is homogeneous, isotropic and spatially flat. Such a universe is described by the FRW line-element,

\[
ds^{2} = -dt^{2} + R^{2}(t) \left[ dx^{2} + x^{2} dt^{2} + x^{2} \sin^{2} \theta \right]
\]

(4)

The Einstein’s field equations and the evolution equation for the scalar field are given by,

\[
3\frac{R^{2}}{R^{2}} + 3\frac{\dot{R}}{R} + \frac{\dot{\varphi}}{2} = \frac{V}{2} = -\frac{p}{\rho};
\]

(5)

\[
2\frac{R^{2}}{R} + \frac{R^{2}}{R} + \frac{\dot{\varphi}}{2} \frac{V}{2} = \frac{p}{\rho} \]

(6)

\[
+ 3\frac{R^{2}}{R} \frac{\ddot{\varphi}}{2} \frac{1}{2! + 3} \frac{1}{2} \frac{2\dot{V}}{dV} \]

(7)

The energy conservation equation, that follows from the Bianchi identity gives

\[
- \frac{3R^{2}}{R} \left( \rho + p \right) = 0
\]

(8)

Among these four equations only three are independent. But as there are five unknowns (\( R, \varphi, \rho, V, p \)) two assumptions can be made to match the number of unknowns with the number of independent equations. With this freedom, we choose the functional form for the time-evolution of the scale factor and the scalar field and find the potential that is compatible with this choice. We consider solutions for which the scale factor and the field evolve as power-law functions of time.

\[
R = R_{0} \frac{t}{t_{0}} \text{ and } \varphi = \varphi_{0} \frac{t}{t_{0}}
\]

(9)

where the subscript 0 refers to the values of the parameters at the present epoch and \( t_{0} \) is the present epoch, i.e the age of the universe.

In order to get a solution with accelerated expansion, the deceleration parameter, \( q \), has to be negative. This restricts the parameter \( \xi \) to be greater than 1.

The solutions for \( \xi \) and \( p \) can be found to be

\[
\rho = \frac{3}{t_{0}} \left( \frac{c_{0}}{t_{0}} \right)^{2} (1 + \xi)^{2} (1 + \xi)
\]

(12)

\[
p_{c} = \frac{3}{t_{0}} \left( \frac{c_{0}}{t_{0}} \right)^{2} (1 + \xi)^{2} (1 + \xi)
\]

(13)

\[
\rho = \frac{3}{t_{0}} \left( \frac{c_{0}}{t_{0}} \right)^{2} (1 + \xi)^{2} (1 + \xi)
\]

(14)
The form of the potential allowed by this ansatz is

$$V = V_c$$

(15)

where,

$$V_c = \frac{2}{t_0^2} \left( \frac{6}{t_0^2} \right)^{(1 + \lambda)} \left( \frac{2}{t_0^2} \right)^{12} \left( \frac{1}{2} \right)^{12} \left( \frac{t_0^2}{t_0^2} \right)^{12}$$

(16)

From (10), (11), (12) and (13) it is clear that the background fluid follows an equation of state of the form $p = -\lambda \rho$, where $\lambda$ is given by

$$\lambda = \frac{2}{3} \left( \frac{2}{t_2^2} \right)^{12} \left( \frac{1}{2} \right)^{12} \left( \frac{t_0^2}{t_0^2} \right)^{12}$$

(17)

Let us now see how the parameters are constrained. First of all we are interested in a scenario in which the universe undergoes an accelerated expansion. As we have mentioned earlier this immediately constrains $\lambda$ to be greater than $1$. Further, for perfect fluid, is constrained to be between $0$ and $1$. From equation (17), this implies $2 < \lambda < 2.3$. We may note that since $\lambda > 1$, it is always negative.

To clearly specify the nature of the expansion, and the missing energy we investigate further the energy density and the pressure of the scalar field. From the field equations, the energy density and the pressure of the scalar field turns out to be

$$\rho = \frac{1}{2} \frac{\lambda}{t_0^2} + \frac{\lambda}{2} \frac{V}{t_0^2}$$

(18)

and

$$p = \frac{1}{2} \frac{\lambda}{t_0^2} \frac{V}{t_0^2} + \frac{\lambda}{2} \frac{\rho}{\rho}$$

(19)

The expression for the energy density and pressure of the Brans Dicke are respectively given by,

$$\rho = \frac{3}{2} \left( \frac{1}{2} \right)^{(1 + \lambda)} \left( \frac{2}{t_0^2} \right)^{12} \left( \frac{1}{2} \right)^{12} \left( \frac{t_0^2}{t_0^2} \right)^{12}$$

(20)

and

$$p = \frac{3}{2} \left( \frac{1}{2} \right)^{(1 + \lambda)} \left( \frac{2}{t_0^2} \right)^{12} \left( \frac{1}{2} \right)^{12} \left( \frac{t_0^2}{t_0^2} \right)^{12}$$

(21)

In order to ensure the positivity condition for both scalar field and matter, we find that $\lambda$ is constrained as,

$$1 + 2 < \lambda < 1 + \frac{2}{2} \left( \frac{2}{t_2^2} \right)^{12} \left( \frac{1}{2} \right)^{12} \left( \frac{t_0^2}{t_0^2} \right)^{12}$$

The allowed region of $\lambda$ in $(\lambda, \lambda)$ parameter space is shown in figure 1 for $\lambda = 1.1$. Now from (20) and (21), it is evident that the scalar field describes an equation of state $p = \rho$ where the is given by

$$\rho = \frac{\lambda}{t_0^2} \frac{V}{t_0^2}$$

(17)

FIG. 1. The allowed region in the $(\lambda, \lambda)$ parameter space for $\lambda = 1.1$. The expressions for the density parameters for the matter and the scalar field, defined respectively by $m = \frac{\lambda}{3H^2}$ and $\rho = \frac{\lambda}{3H^2}$, are given by

$$m = \frac{2}{3} \left( \frac{1}{2} \right)^{(1 + \lambda)} \left( \frac{2}{t_0^2} \right)^{12} \left( \frac{1}{2} \right)^{12} \left( \frac{t_0^2}{t_0^2} \right)^{12}$$

(22)

and

$$\rho = \frac{2}{3} \left( \frac{1}{2} \right)^{(1 + \lambda)} \left( \frac{2}{t_0^2} \right)^{12} \left( \frac{1}{2} \right)^{12} \left( \frac{t_0^2}{t_0^2} \right)^{12}$$

(23)

Using these two expressions can be recast in terms of $m$ and as

$$m = \frac{1}{3} \left( \frac{1}{2} \right)^{(1 + \lambda)} \left( \frac{2}{t_0^2} \right)^{12} \left( \frac{1}{2} \right)^{12} \left( \frac{t_0^2}{t_0^2} \right)^{12}$$

(24)

As we have mentioned earlier that the result of the Supernova Cosmology Project and the High z Survey Project that reveal the so-called acceleration of the universe, favours a universe with positive cosmological constant as well [5]. Assuming flatness in context of general relativity the best fit for these data occurred for $m = 0.28$ and $\rho = 0.72$. Using these best fit values along with the bound on $\rho$, it is found that lies within the range

$$0.27 < \rho < 0.47$$

In the following two figures we find the allowed region (black) in the $(\rho, \rho)$ parameter space which has a $(0.25$ and $0.5)$ with the matter density range within $0.2 < m < 0.4$. 

3
In Brans Dicke theory, the value of the gravitation constant is determined by the value of

\[ G(t) = \frac{1}{(t)^\alpha} \]  

(26)
The scalar field approaches to constant value \( G \) at the present time and consequently the inverse of \( G \) gives the Newtonian constant \( G_N \). The rate of change of \( G \) at present time is given by \( \frac{dG}{dt} = -H_0 \) where \( H_0 \) is the Hubble parameter at present time (\( \frac{dR}{dt}(t_0) = -\frac{1}{H_0} \)). For any specific value of \( \alpha \) and admitting accelerated expansion, we need to \( \frac{dG}{dt} < 10^{-10} \) per year [27].

In a recent investigation Pont et al [28] estimated the age of the universe \( t_0 \) to be \( 14.2 \) Gyrs, that means \( 1/\alpha = 10^4 \) year. At the same time Kaplinghat et al [29] and others [30] have also pointed out that for power law cosmologies high redshift data indicates to be \( 1.25 \). In fact in one of our recent investigation we have shown that the best fit value for \( H_0 \) with SNIa data for power law cosmology is approximately 1.25 [31]. But \( H_0 t_0 \) is also determined by the expression

\[ (H_0 t_0)^2 = \frac{\alpha^2}{\beta^2} + \frac{(1 + \omega)}{2} \]  

(27)

In figure 4 and 5 we plot \( H_0 t_0 \) with respect to \( \alpha \) for \( \omega = 1.2 \) and \( 1.2 \). For this we consider different values of \( \omega \). For admissible values for the parameters the curves representing the expression \( H_0 t_0 \) should intersect the horizontal line representing a constant value of \( 1/\alpha \) at least once within the
allowed range of \( t \). It is interesting to find that for both the graphs the curves intersect the straightline representing at least once when \( t \) lies within the range \( 2 \leq t \leq 15 \), in a different investigation Banerjee and Pavon [31] have also arrived at the same range of \( t \) while looking for late time acceleration in BD theory without a potential. In fact, in other investigations [31,33] in scalar tensor theories studying late time acceleration similar conclusions are made. In all these analyses the constraint coming from the solar system measurements ( \( t > 600 \) ) is not satisfied. We would discuss about this point in the discussion section.

III. DENSITY PERTURBATION

While the accelerated expansion is supported by the Brans Dicke theory with an appropriate form of the potential, it is worth checking whether the issue of structure formation is modified by the dynamics of the Brans Dicke field treated here. We are particularly interested in the evolution of the density perturbation in the context of Brans Dicke theory with the self interacting potential. Our objective here is to find the perturbations of the field equations (5)-(8) and analyze the behaviour of the relevant variables for the accelerated expansion of the universe. In reference [34] a similar treatment is done to analyze the behaviour of the perturbed solutions in the original Brans Dicke theory.

As our interest is in cosmological density perturbations, we will be using the temporal component of the field equations. Let us denote by \( g \), \( R_{00} \) and \( T_{00} \) perturbations in \( g \), \( R_{00} \) and \( T_{00} \), respectively. Denoting \( g \) by \( h \), \( R_{00} \) can be expressed as

\[
R_{00} = \frac{1}{2R^2} h_{kk} + 2 \frac{R^2}{R} \frac{R_{kk}}{R} h_{kk}
\]

Similarly the perturbations in the trace of the energy-momentum tensor can be expressed as,

\[
T = T + T = T + 3 p
\]

The perturbation of the d’Alembertian of the Brans Dicke field is given by

\[
2 = + R R h_{kk} - \frac{1}{2 R^2} h_{kk} + 2 \frac{R^2}{R} \frac{R_{kk}}{R} h_{kk}
\]

The following parameters are used for the relevant perturbations

\[
h_{kk} = R^2 h
\]

where \(< < 1\)

where \( h (t) ; (t) \) and \( \dot{(t)} \) are the perturbed gravitational field, scalar field and matter energy density respectively.

As was mentioned in the earlier section, only three of the four field equations are independent. Hence, three equations are used for perturbation. They are

\[
\frac{2 R h}{R} = - \left[ \frac{3 m (1 + \lambda) + 2 + }{21 + 3} + \frac{V_c}{2} \frac{1 + 2 m}{2! + 3} m + + \right] \frac{2}{R^2}
\]

\[
2 = \frac{3 (1 + \lambda \mu) + 2 + 2 m}{2! + 3} V_c \frac{1 + 2 m}{2! + 3} m + 1
\]

where \( m = \frac{2}{3} \cdot 0 \)

and

where \( m = \frac{2}{3} \cdot 0 \)

Using the above parameters, the set of perturbed equations take the form

\[
\frac{h}{2} + \frac{R_{kk}}{R} = \left( \frac{3 m (1 + \lambda) + 2 + }{21 + 3} + \frac{V_c}{2} \frac{1 + 2 m}{2! + 3} m + 1 \right)
\]

\[
2 = \frac{3 (1 + \lambda \mu) + 2 + 2 m}{2! + 3} V_c \frac{1 + 2 m}{2! + 3} m + 1
\]

\[
\left( \frac{1}{(2! + 3)} \right) \frac{2 m}{2! + 3} V_c m + 1
\]

\[
- \frac{1}{(2! + 3)} \frac{3 m}{2! + 3} V_c m + 1
\]

where \( v \) is the comoving velocity of the fluid. The growth of perturbations primarily occur in the post-recombination era. The pressure acting on the matter is negligible during this era and hence, it is appropriate to consider the matter to be pressureless i.e., \( m = 0 \). From (43) and (44), and can be expressed in terms of \( m \)

\[
\frac{h}{2} + \frac{R_{kk}}{R} = \left( \frac{3 m (1 + \lambda) + 2 + }{21 + 3} + \frac{V_c}{2} \frac{1 + 2 m}{2! + 3} m + 1 \right)
\]

\[
2 = \frac{3 (1 + \lambda \mu) + 2 + 2 m}{2! + 3} V_c \frac{1 + 2 m}{2! + 3} m + 1
\]

Hence, the perturbed equations for the pressureless case are

\[
\frac{h}{2} + \frac{R_{kk}}{R} = \left( \frac{3 m (1 + \lambda) + 2 + }{21 + 3} + \frac{V_c}{2} \frac{1 + 2 m}{2! + 3} m + 1 \right)
\]

\[
2 = \frac{3 (1 + \lambda \mu) + 2 + 2 m}{2! + 3} V_c \frac{1 + 2 m}{2! + 3} m + 1
\]
The perturbation in the four-velocity $\psi$ is to be set null, which can be done by one infinitesimal gauge transformation. Let us suppose that the perturbation in the scalar field behave as plane waves

$$\phi(x; t) = \phi(t) \exp \left( i k \mathbf{x} \right)$$

where $k$ is the wave number of perturbation. Using equation (52) and substituting the solutions (9), (12) and (19) in equation (50) and (51) we get

$$\begin{align*}
\frac{4m}{3(1 + m)} - \frac{4}{1 + m} \frac{2m}{t^2} & = \frac{c}{t^2} (\frac{1}{t^2} + \frac{1}{2! + 3}) \\
\frac{V_c}{2} \frac{m}{2} \frac{2}{t^2} & = \frac{1}{m^{0}} \frac{1}{0^{2}} \frac{1}{t^2} \\
& \frac{2m}{2! + 3} \frac{V_c}{2} \frac{m}{2} \frac{2}{t^2} \frac{1}{m^{0}} \frac{1}{0^{2}} \frac{1}{t^2}
\end{align*}$$

where

$$c = \frac{c}{t^2} \frac{1}{m^{0}} \frac{1}{0^{2}}$$

Combining these two equations and neglecting the higher order terms beyond $t^{-2}$ (as the modes are analyzed in the asymptotic region) we get,

$$+ \frac{C_1}{t} (\frac{1}{t^2} + \frac{1}{2! + 3}) \frac{C_2}{t} \frac{1}{2! + 3} \frac{1}{2! + 3} + \frac{C_3}{t} \frac{1}{2! + 3} \frac{1}{2! + 3} = 0 (49)$$

where

$$\begin{align*}
C_1 &= \frac{4m + 6}{3(1 + m)} \\
C_2 &= \frac{4m + 6}{3(1 + m)} \\
C_3 &= \frac{4m + 6}{3(1 + m)} \\
C_4 &= \frac{4m + 6}{3(1 + m)}
\end{align*}$$

In order to solve the above equation we assume the following form

$$f(t) = \phi(t); \quad f(t) = \psi(t); \quad = t \quad (50)$$

where $\phi$ and $\psi$ are constants.

With the above substitution we find that

$$2 + [C_3 + C_4] + C_3 = 0 \quad (51)$$

The solution for from the above equation

$$p f(1, C_3 + C_4, C_4, C_4, C_4) = 0 \quad (52)$$

Bertolami et al [19] have considered the growing modes for density perturbation in the asymptotic limit of $j > 1$. The solution for in this limit for our case is

$$f(1, C_3 + C_4, C_4, C_4, C_4) = 0 \quad (53)$$

This asymptotic value of for large $j$ depends only on the power of the potential $m$. Since is always negative, $m = -\frac{1}{2}$ is a positive number. (Note that $m = -\frac{1}{2}$ as both $+1$ and $-1$ for $m = 1$. So, $+$ represents the growing mode for $<$. Clearly for $m = 2$ the mode matches with the result given by Bertolami et al [19]. However, we found that $+$ should lie in the range $1 < (\frac{1}{2})$. So in this connection it is worthwhile to check if growing modes exist with this limitation. For $m = 1$ we can calculate from the equation (52). We find that with this value of $+$ also $+$ represents a growing mode. We, of course, need to make suitable choice for the values of $+$ and $m$ to achieve this. Thus in our case perturbations can grow with time for range of $j$ allowed in this model.

IV. DISCUSSION AND CONCLUSIONS

In this work we have investigated the nature of potential relevant to the power law expansion of the universe in a self-interacting Brans Dicke Cosmology with a perfect fluid distribution. The age of the universe and the time variability of the gravitational coupling are also calculated. The value of the parameters are constrained from different physical conditions. We have graphically represented the permissible values of the parameters. We have also studied the evolution of density perturbations in this model.

From our analysis we draw the following conclusions:
1. Accelerated growth of the scale factor can be driven by this positive power law potential with \( \phi \) lying in the range \( 0 \leq \phi \leq 0.27 \). This range for \( \phi \) is very well consistent with the observations.

2. The gravitational coupling grows with time which agrees quite well with the observational facts [27].

3. This model also allows growing modes for the energy density perturbation of matter implying that the dynamics of the self interacting Brans Dicke field does not upset the structure formation scenario.

This model depends entirely on three parameters \( \lambda \), \( \kappa \), and \( \phi \). The parameter \( \lambda \) is only constrained to be greater than 1 by the fact that the universe is accelerating. Depending on the value of \( \kappa \), the value of \( \phi \) gets restricted within a certain range. The value of \( \lambda \) depends on these two parameters. In order to ensure the weak (positivity) energy conditions as well as to produce the observed values of \( \rho_m \) and \( \rho_m \), we require \( \lambda \) to be in the range \( 2 < \lambda < 15 \). Although this value of \( \lambda \) does not satisfy the solar system bound of \( \lambda > 600 \) this is a generic difficulty in most of the investigations done in the context of scalar tensor theories [31,33]. In a completely different work in BD theory without potential Banerjee and Pavon [33] also had a similar range for \( \lambda \). The work of Bertolami and Martins [19] is an exception where they have obtained accelerated solution with large \( \lambda \). They have, however, not considered the positive energy condition for the energy density of the scalar field. A large value of \( \lambda \) necessitates violation of this condition in these models. The violation of the positive energy condition is particularly serious because the contribution of the energy density from matter is subdominant with respect to the missing energy density as is suggested by observation. On the other hand this is not the only case when the solar system limit is violated. There are other evidences in literature where small value of \( \lambda \) has been supported. The extended inflationary model suggested by La and Steinhardt [57] requires \( \lambda \) to be 20. In order to explain the structure formation scenario successfully in scalar tensor theory, the constraint on \( \lambda \) is not also compatible with the solar system test [58].

Another important point to note is that as the universe expands in a constant power law fashion in this model, it has an acceleration also in the radiation dominated era which upsets the primordial nucleosynthesis scenario. This problem also occurs if the universe is considered to have a power law expansion and no compromise can be done between the bigbang nucleosynthesis and present accelerated phase in general relativity as is shown by Kaplinghat et al [24]. Infact the existing supernova observational data provides sufficient evidence for the fact that the universe was decelerating in the near past [52]. In a recent work of Banerjee and Pavon [33] it has been shown that in BD theory the nucleosynthesis problem can be avoided by considering \( \lambda \) to be a function of \( \phi \). But in that case also \( \lambda \) asymptotically acquires a small negative value to have a late time accelerating phase.

In the end we can say that though the model presented here describes the present day universe quite successfully, it disagrees at two other important point of observation. While the age of the universe, density parameters and calculated here satisfies the observed constraints, the power law accelerated expansion of the universe upsets the standard decelerating phase of the universe until recent past and the important solar system limit on \( \lambda \). So to get rid of these two problem one approach is to consider a function of \( \phi \) so that local inhomogeneities give rise to large \( \lambda \) and one could get both accelerating and decelerating phase. We wish to address this problem in our next work.

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Electronic address: somasi@mri.ernet.in

Electronic address: seshadri@mri.ernet.in

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