A Fuzzy Goal Programming Approach to Multiobjective Transportation Problems

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ABSTRACT

The current paper focuses on a multiobjective transportation problem which is a special type of vector minimum problem. In order to deal with such a problem, a new model based on fuzzy goal programming is suggested. Different solutions are obtained according to the priorities of the decision maker in the proposed model. With respect to its properties, at least a weakly efficient solution is yielded by solving the new model. To demonstrate the validity and strengths of the suggested model, some numerical examples are given. Further, a comparison is made with different existing methods.

1. Introduction

In today’s highly competitive market, the pressure on organisations to find ways to create and deliver value to customers becomes stronger [1]. In this manner, transportation problems provide a powerful framework to deal with this challenge. The basic transportation problem deals with transportation of goods from factories (supply points) to customers (demand points). Demand and supply are considered as equality constraints and the objective is to minimise the total transportation cost considering the restrictions of supply and demand. The basic transportation problem was first developed by Hitchcock in 1941 as a particular type of linear programming problem [2]. Due to the special mathematical structure of the transportation problem, efficient solution methods derived from the well-known Simplex method to solve it [3, 4].

In real-life transporting system, several objective functions such as minimisation of the transportation cost, transportation time, etc., are normally considered due to various aspects of decision maker as well as real-life situations for an industrial problem. Transportation problem with different objective functions called multiobjective transportation problem is another influential trend worthy of study. In comparison with single objective transportation problem, it is more reasonable and practical in terms of actual applications. Multiobjective transportation problem is a special type of multiobjective linear programming problem [5–8]. Several researchers have intensively investigated multiobjective
transportation problems. Diaz [9] and Isermann [10] proposed different algorithms for identifying all nondominated solutions for linear multiobjective transportation problems. Two interactive algorithms for obtaining the solutions of linear multiobjective transportation problems were developed by Ringuest and Rinks [11]. A review of multiobjective design of transportation networks was done by Current et al [12]. Generally, in a multiobjective transportation problem, the available data such as transportation cost, availabilities, demand, etc. are not known precisely. It is due to various aspects, like lack of input information, weather and road conditions, etc. In order to tackle uncertainty, fuzzy set theory which was introduced by Zadeh [13] and Bellman and Zadeh [14] could be applied [15–22].

Goal programming as an applicable tool for dealing with multiobjective programming was introduced by Charnes and Cooper [23]. Due to the efficiency of this approach, it has been gained the attention of many researchers [24–32]. In conventional goal programming, it is assumed that the aspiration levels of the objectives are known precisely. However, it is often a difficult task to determine these values certainly. A goal with an imprecise aspiration level could be handled by fuzzy set theory. Fuzzy goal programming has been used to deal with multiobjective transportation problems. As an instance, Abd El-Wahed and Lee [33] proposed an interactive fuzzy goal programming for multiobjective transportation problems. Zangiabadai and Maleki [34, 35] also applied fuzzy goal programming approach to solve multiobjective transportation problems with linear and nonlinear membership functions.

In multiobjective optimisation, weighted additive models are very popular among researchers when weights of objectives are specified. These models have also been used in fuzzy goal programming approach [36–38]. A weighted additive model with some more advantages was also presented by Yaghoobi et al [39].

As stated before, though some investigations have been performed on goal programming as a method in multiobjective optimisation, there are some gaps in its usage in real-life situations, where traditional goal programming is not adequate to tackle the situation. In this regard, our main focus is on fuzzy goal programming to solve multiobjective transportation problems. A new model based on fuzzy goal programming is presented. It could be proved that at least a weakly efficient solution is obtained for the multiobjective transportation problem by solving the new model. Further, an efficient solution is achieved if the new model has a unique optimal solution. The new model may be considered as a modification of the weighted additive model which was presented by Yaghoobi et al. in [39]. To emphasise the effectiveness of the new model, a comparison with some existing methods is made.

The rest of the paper is designed as follows. Section 2 presents the mathematical formulation of a multiobjective transportation problem. In Section 3, the weighted additive model proposed by Yaghoobi et al. in [39] is reviewed. Moreover, our new model for solving multiobjective transportation problems is stated in Section 3. Some numerical examples are provided in Section 4. Further, results and discussions are reflected in this section. Finally, Section 5 is devoted to conclusions and future research directions.

2. Problem Formulation

In the real-world situations, transportation problems usually involve multiple, incommensurable and conflicting objective functions. A transportation problem which deals with
different objective functions is called a multiobjective transportation problem (MOTP). Generally, a MOTP consists of \( m \) number of sources and \( n \) number of destinations. Assume \( s_i \) represents the total availability of a homogeneous product at source \( i \) and \( d_j \) represents the total demand of this product at destination \( j \). Let \( c_{ij} \) be the cost of transporting one unit product from source \( i \) to destination \( j \). It could be considered as the cost of transportation, delivering time, cost of damage or safety of delivery, etc. Let \( z_1, z_2, \ldots, z_l \) be \( l \) objectives that are to be minimised. A variable \( x_{ij} \) represents the unknown quantity to be shipped from the \( i \)th source to the \( j \)th destination. It is assumed that the total demand is equal to the total supply (balance condition). With these assumptions, the mathematical model of the MOTP is as follows.

\[
\begin{align*}
\text{Min } z_k &= \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}, \quad k = 1, \ldots, l, \\
\text{s.t.} & \quad \sum_{j=1}^{n} x_{ij} = d_j, \quad j = 1, \ldots, n, \\
& \quad \sum_{i=1}^{m} x_{ij} = s_i, \quad i = 1, \ldots, m, \\
& \quad x_{ij} \geq 0, \quad i = 1, \ldots, m, j = 1, \ldots, n.
\end{align*}
\]

It should be noted that \( X \) is considered as the set of all feasible solutions of MOTP (1):

\[
X = \left\{ x \in \mathbb{R}^{m \times n} : \sum_{j=1}^{n} x_{ij} = s_i, \sum_{i=1}^{m} x_{ij} = d_j, x_{ij} \geq 0, \quad i = 1, \ldots, m, j = 1, \ldots, n \right\}
\]

Since MOTP (1) is a special type of multiobjective problems, hence efficient and weakly efficient solutions are the most reasonable solutions in this problem. In order to review concepts of efficient and weakly efficient solution, the following definitions are stated [6].

**Definition 1:** A feasible solution \( \hat{x} \) is an efficient solution of MOTP (1) if there does not exist another \( x \in X \) such that \( z_k(x) \leq z_k(\hat{x}), k = 1, \ldots, l \), and \( z_r(x) < z_r(\hat{x}) \) for some \( r \in \{1, \ldots, l\} \).

**Definition 2:** A feasible solution \( \hat{x} \) is an efficient solution of MOTP (1) if there does not exist another \( x \in X \) such that \( z_k(x) < z_k(\hat{x}), k = 1, \ldots, l \).

In MOTP (1), the set of all efficient solutions and the set of all weakly efficient solutions are denoted by \( X_E \) and \( X_{WE} \), respectively. It is obvious that \( X_E \subseteq X_{WE} \).

### 3. Fuzzy Goal Programming Approach for Solving Motp

MOTP (1) is a special type of multiobjective programming problems. There does not usually exist an optimal solution that would simultaneously satisfy all objectives in multiobjective problems. Therefore, it is desirable to seek suitable compromise solutions for such problems. Different methods are existed for dealing with multiobjective problems [6]. One of the most popular approaches is goal programming [28]. Conventional goal programming models assume that the decision maker is able to determine a precise aspiration level for each of the objectives. However, in most real-world problems, the aspiration levels are not known certainly. In such cases, fuzzy goal programming could be employed. Here, in order
to deal with MOTP (1) using goal programming approach, it is assumed that the aspiration level for each of the objectives is not known precisely. Hence, the following problem should be optimised.

\[
\text{OPT} \quad \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^k x_{ij} \leq b_k, \quad k = 1, \ldots, l,
\]

s.t. \[
\sum_{j=1}^{n} x_{ij} = d_j, \quad j = 1, \ldots, n,
\]

\[
\sum_{j=1}^{n} x_{ij} = s_i, \quad i = 1, \ldots, m,
\]

\[
x_{ij} \geq 0, \quad i = 1, \ldots, m, j = 1, \ldots, n.
\]

(2)

In problem (2), the symbol \(\lesssim\) is interpreted as ‘essentially less than’. Each one of the fuzzy goals of problem (2) can be represented by a fuzzy set defined over the feasible set with the membership function. Applying piecewise linear membership functions to express the fuzzy goals and using weighted goal programming, the following weighted additive model is achieved [39]:

\[
\text{Min } z = \sum_{k=1}^{l} w_k \frac{p_k}{\Delta_{kR}}
\]

s.t. \[
\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^k x_{ij} - p_k \leq b_k, \quad k = 1, \ldots, l,
\]

\[
\frac{p_k}{\Delta_{kR}} \leq 1, \quad k = 1, \ldots, l,
\]

\[
\sum_{i=1}^{m} x_{ij} = d_j, \quad j = 1, \ldots, n,
\]

\[
\sum_{j=1}^{n} x_{ij} = s_i, \quad i = 1, \ldots, m,
\]

\[
p_k \geq 0, x_{ij} \geq 0, \quad i = 1, \ldots, m, j = 1, \ldots, n, \quad k = 1, \ldots, l.
\]

In model (3), \(w_k\) denotes the weight of the \(k\)th fuzzy goal. It is assumed that weights are provided and normalised such that the sum of \(w_k\)s is equal to 1. Further, in the optimal solution of model (3), the slack variable of the constraint \(\frac{p_k}{\Delta_{kR}} \leq 1\) is equal to the degree of membership function for the \(k\)th fuzzy goal [39] (Figure 1).

3.1. The New Model

To propose the new model, instead of using deviational variables \(p_k, k = 1, \ldots, l\), in model (3), it is suggested to use deviational function \(r(1 - w_k)\). Hence, to solve MOTP (1), the
Figure 1. Fuzzy set $z_k \approx b_k$.

The following model based on fuzzy goal programming is stated.

\[
\begin{align*}
\text{Min } z &= r, \\
\text{s.t. } & \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^k x_{ij} - r(1 - w_k) \leq b_k, \quad k = 1, \ldots, l, \\
& r(1 - w_k) \leq \Delta_k R, \quad k = 1, \ldots, l, \\
& \sum_{i=1}^{m} x_{ij} = d_j, \quad j = 1, \ldots, n, \\
& \sum_{j=1}^{n} x_{ij} = s_i, \quad i = 1, \ldots, m, \\
& r \geq 0, x_{ij} \geq 0, \quad i = 1, \ldots, m, j = 1, \ldots, n, \quad (4)
\end{align*}
\]

where $b_k$ is considered as $z_k(x_k^*)$. Note that $x_k^*$ is an optimal solution of the $k$th objective function. Furthermore, $\Delta_k R$ is determined by $\max_{j=1,\ldots,l, j \neq k} z_k(x_j^*) - z_k(x_k^*)$.

It should be noted that in model (4), values of the deviational function are consistent with the priority of objectives. It means that larger value of $w_k$ will result in a smaller value of the deviational function. This property of model (4) will take the objective with higher priority closer to its aspiration level. However, as will be shown in numerical examples, it is not the case in model (3). Moreover, when no preferences are defined for objectives, the new model (4) could solve MOTP (1). Some other nice properties of model (4) are stated in the following theorems.

**Theorem 1:** Let $(x_{ij}^*, i = 1, \ldots, m, j = 1, \ldots, n, r^*)$ be an optimal solution of model (4). Then $x_{ij}^*$, $i = 1, \ldots, m$, $j = 1, \ldots, n$ is at least a weakly efficient solution to MOTP (1).

**Theorem 2:** Let $(x_{ij}^*, i = 1, \ldots, m, j = 1, \ldots, n, r^*)$ be a unique optimal solution of model (4). Then $x_{ij}^*$, $i = 1, \ldots, m$, $j = 1, \ldots, n$ is an efficient solution to MOTP (1).

Note that the proofs of these theorems are straightforward and thus omitted for the sake of brevity.
Table 1. Optimal objective values obtained from model (3).

| Weights $w_1, w_2$ | $z_1, z_2$ | $\rho_1, \rho_2$ |
|---------------------|-------------|----------------|
| 1 $w_1 = 0.1$, $w_2 = 0.9$ | 186.79 | 43.0 |
| 2 $w_1 = 0.2$, $w_2 = 0.8$ | 176.83 | 33.4 |
| 3 $w_1 = 0.3$, $w_2 = 0.7$ | 156.98 | 13.19 |
| 4 $w_1 = 0.4$, $w_2 = 0.6$ | 156.98 | 13.19 |
| 5 $w_1 = 0.5$, $w_2 = 0.5$ | 156.98 | 13.19 |
| 6 $w_1 = 0.6$, $w_2 = 0.4$ | 156.98 | 13.19 |
| 7 $w_1 = 0.7$, $w_2 = 0.3$ | 156.98 | 13.19 |
| 8 $w_1 = 0.8$, $w_2 = 0.2$ | 143.163 | 0.84 |
| 9 $w_1 = 0.9$, $w_2 = 0.1$ | 143.163 | 0.84 |

4. Numerical Examples

Three different MOTPs are investigated in this section. These problems are solved by using the new model (4). To emphasise the advantages of model (4), the numerical examples are also solved by some existing methods and the results are compared.

Example 1: Consider the following MOTP,

$$
\text{Min } Z_1 = x_{11} + 2x_{12} + 7x_{13} + 7x_{14} + x_{21} + 9x_{22} + 3x_{23} + 4x_{24} + 8x_{31} + 9x_{32} + 4x_{33} + 6x_{34},
$$

$$
\text{Min } Z_2 = 4x_{11} + 4x_{12} + 3x_{13} + 4x_{14} + x_{21} + 9x_{22} + 3x_{23} + 4x_{24} + 6x_{31} + 2x_{32} + 5x_{33} + x_{34},
$$

\[ \text{s.t. } \begin{align*}
x_{11} + x_{12} + x_{13} + x_{14} &= 8, \\
x_{21} + x_{22} + x_{23} + x_{24} &= 19, \\
x_{31} + x_{32} + x_{33} + x_{34} &= 17, \\
x_{11} + x_{21} + x_{31} &= 11, \\
x_{12} + x_{22} + x_{32} &= 3, \\
x_{13} + x_{23} + x_{33} &= 14, \\
x_{14} + x_{24} + x_{34} &= 16, \\
x_{ij} &\geq 0, \quad i = 1, \ldots, 3, j = 1, \ldots, 4
\end{align*} \]

The results that are obtained by solving Example 1 for different weights using model (3), are given in Table 1. As could be seen in the table, it is obvious that deviational variables are not consistent with the priority of objectives. For instance, when $w_1 = 0.3$ and $w_2 = 0.7$, the deviational variables for the first and the second objective functions are $\rho_1 = 13$ and $\rho_2 = 19$, respectively. It means that the first objective function which is less preferred has the value closer to its aspiration level. However, the consistency exists in Table 2 which includes the results obtained by the new model (4). Further, Table 1 shows that the same objective values are achieved for different weights by using model (3). However, the proposed model (4) yields different objective values for different weights (Table 2).
Example 2: The following example is adopted from [5].

\[
\begin{align*}
\text{Min } z_1 &= 9x_{11} + 12x_{12} + 9x_{13} + 6x_{14} + 9x_{15} + 7x_{21} + 3x_{22} + 7x_{23} + 7x_{24} + 5x_{25} + 6x_{31} + 5x_{32} + 9x_{33} + 11x_{34} + 3x_{35} + 6x_{41} + 8x_{42} + 11x_{43} + 2x_{44} + 2x_{45}, \\
\text{Min } z_2 &= 2x_{11} + 9x_{12} + 8x_{13} + x_{14} + 4x_{15} + x_{21} + 9x_{22} + 9x_{23} + 5x_{24} + 2x_{25} + 8x_{31} + x_{32} + 8x_{33} + 4x_{34} + 5x_{35} + 2x_{41} + 8x_{42} + 6x_{43} + 9x_{44} + 8x_{45}, \\
\text{Min } z_3 &= 2x_{11} + 4x_{12} + 6x_{13} + 3x_{14} + 6x_{15} + 4x_{21} + 8x_{22} + 4x_{23} + 9x_{24} + 2x_{25} + 5x_{31} + 3x_{32} + 5x_{33} + 3x_{34} + 6x_{35} + 6x_{41} + 9x_{42} + 6x_{43} + 3x_{44} + x_{45}, \\
\text{s.t. } x_{11} + x_{12} + x_{13} + x_{14} + x_{15} &= 5, \\
x_{21} + x_{22} + x_{23} + x_{24} + x_{25} &= 4, \\
x_{31} + x_{32} + x_{33} + x_{34} + x_{35} &= 2, \\
x_{41} + x_{42} + x_{43} + x_{44} + x_{45} &= 9, \\
x_{11} + x_{21} + x_{31} + x_{41} &= 4, \\
x_{12} + x_{22} + x_{32} + x_{42} &= 4, \\
x_{13} + x_{23} + x_{33} + x_{43} &= 6, \\
x_{14} + x_{24} + x_{34} + x_{44} &= 2, \\
x_{15} + x_{25} + x_{35} + x_{45} &= 4, \\
x_{ij} &\geq 0, i = 1, \ldots, 4, j = 1, \ldots, 5.
\end{align*}
\]

By solving Example 2 with model (3) and model (4) for different weights, different results are obtained which are summarised in Tables 3 and 4. All properties that have been explained in Example 1 for model (4) are also satisfied in this example. Actually, model (4) yields more comprehensive results in accordance to model (3). This example has been solved by Zangiabadi and Maleki in [35] and (126.7930, 103.1030, 77.52344) obtained as the optimal compromise value of the objective vector \((z_1, z_2, z_3)\). Example 2 is also solved by the weighted sum method [6] and the results are shown in Table 5. It can be seen here that, for different weights, the objective values increase and decrease more consistently for the new model (4) than for the weighted sum method. It should be noted that when no preferences are given to the objectives, (124, 99, 87) is obtained by model (4) as the optimal

### Table 2. Optimal objective values obtained from model (4).

| Weights | \(z_1, z_2\) | \(r(1 - w_1), r(1 - w_2)\) |
|---------|--------------|------------------|
| 1       | 176.83       | 36.4             |
| 2       | 172.86       | 29.725           |
| 3       | 168.89       | 24.998, 10,714   |
| 4       | 164.92       | 21.14            |
| 5       | 160.95       | 17.17            |
| 6       | 156.98       | 13.195           |
| 7       | 155.106      | 12.28            |
| 8       | 152.118      | 9.75, 39         |
| 9       | 149.133      | 6.54             |
weights. According to the results that are given in Table 6, different objective values are obtained with preference on objectives.

**Example 3:** Consider the following MOTP,

\[
\begin{align*}
\min z_1 &= x_{11} + 12x_{12} + 7x_{13} + 7x_{14} + x_{21} + 9x_{22} + 3x_{23} + 4x_{24} + 8x_{31} \\
&\quad + 9x_{32} + 4x_{33} + 6x_{34}, \\
\min z_2 &= 4x_{11} + 4x_{12} + 3x_{13} + 4x_{14} + 5x_{21} + 8x_{22} + 9x_{23} + 10x_{24} + 6x_{31} \\
&\quad + 2x_{32} + 5x_{33} + x_{34}, \\
\text{s.t.} \quad &x_{11} + x_{12} + x_{13} + x_{14} = 8, \\
&x_{21} + x_{22} + x_{23} + x_{24} = 9, \\
&x_{31} + x_{32} + x_{33} + x_{34} = 17, \\
&x_{11} + x_{21} + x_{31} = 11, \\
&x_{12} + x_{22} + x_{32} = 3, \\
&x_{13} + x_{23} + x_{33} = 14, \\
&x_{14} + x_{24} + x_{34} = 16, \\
&x_{ij} \geq 0, i = 1, \ldots, 3, j = 1, \ldots, 4.
\end{align*}
\]

This problem is solved by the new model (4) and the weighted sum method for different weights. According to the results that are given in Table 6, different objective values are obtained without preference on objectives.

**Example 3:** Consider the following MOTP,

\[
\begin{align*}
\min z_1 &= x_{11} + 12x_{12} + 7x_{13} + 7x_{14} + x_{21} + 9x_{22} + 3x_{23} + 4x_{24} + 8x_{31} \\
&\quad + 9x_{32} + 4x_{33} + 6x_{34}, \\
\min z_2 &= 4x_{11} + 4x_{12} + 3x_{13} + 4x_{14} + 5x_{21} + 8x_{22} + 9x_{23} + 10x_{24} + 6x_{31} \\
&\quad + 2x_{32} + 5x_{33} + x_{34}, \\
\text{s.t.} \quad &x_{11} + x_{12} + x_{13} + x_{14} = 8, \\
&x_{21} + x_{22} + x_{23} + x_{24} = 9, \\
&x_{31} + x_{32} + x_{33} + x_{34} = 17, \\
&x_{11} + x_{21} + x_{31} = 11, \\
&x_{12} + x_{22} + x_{32} = 3, \\
&x_{13} + x_{23} + x_{33} = 14, \\
&x_{14} + x_{24} + x_{34} = 16, \\
&x_{ij} \geq 0, i = 1, \ldots, 3, j = 1, \ldots, 4.
\end{align*}
\]

This problem is solved by the new model (4) and the weighted sum method for different weights. According to the results that are given in Table 6, different objective values are obtained without preference on objectives.

**Example 3:** Consider the following MOTP,

\[
\begin{align*}
\min z_1 &= x_{11} + 12x_{12} + 7x_{13} + 7x_{14} + x_{21} + 9x_{22} + 3x_{23} + 4x_{24} + 8x_{31} \\
&\quad + 9x_{32} + 4x_{33} + 6x_{34}, \\
\min z_2 &= 4x_{11} + 4x_{12} + 3x_{13} + 4x_{14} + 5x_{21} + 8x_{22} + 9x_{23} + 10x_{24} + 6x_{31} \\
&\quad + 2x_{32} + 5x_{33} + x_{34}, \\
\text{s.t.} \quad &x_{11} + x_{12} + x_{13} + x_{14} = 8, \\
&x_{21} + x_{22} + x_{23} + x_{24} = 9, \\
&x_{31} + x_{32} + x_{33} + x_{34} = 17, \\
&x_{11} + x_{21} + x_{31} = 11, \\
&x_{12} + x_{22} + x_{32} = 3, \\
&x_{13} + x_{23} + x_{33} = 14, \\
&x_{14} + x_{24} + x_{34} = 16, \\
&x_{ij} \geq 0, i = 1, \ldots, 3, j = 1, \ldots, 4.
\end{align*}
\]

This problem is solved by the new model (4) and the weighted sum method for different weights. According to the results that are given in Table 6, different objective values are obtained without preference on objectives.

**Example 3:** Consider the following MOTP,

\[
\begin{align*}
\min z_1 &= x_{11} + 12x_{12} + 7x_{13} + 7x_{14} + x_{21} + 9x_{22} + 3x_{23} + 4x_{24} + 8x_{31} \\
&\quad + 9x_{32} + 4x_{33} + 6x_{34}, \\
\min z_2 &= 4x_{11} + 4x_{12} + 3x_{13} + 4x_{14} + 5x_{21} + 8x_{22} + 9x_{23} + 10x_{24} + 6x_{31} \\
&\quad + 2x_{32} + 5x_{33} + x_{34}, \\
\text{s.t.} \quad &x_{11} + x_{12} + x_{13} + x_{14} = 8, \\
&x_{21} + x_{22} + x_{23} + x_{24} = 9, \\
&x_{31} + x_{32} + x_{33} + x_{34} = 17, \\
&x_{11} + x_{21} + x_{31} = 11, \\
&x_{12} + x_{22} + x_{32} = 3, \\
&x_{13} + x_{23} + x_{33} = 14, \\
&x_{14} + x_{24} + x_{34} = 16, \\
&x_{ij} \geq 0, i = 1, \ldots, 3, j = 1, \ldots, 4.
\end{align*}
\]

This problem is solved by the new model (4) and the weighted sum method for different weights. According to the results that are given in Table 6, different objective values are obtained without preference on objectives.
Table 6. Optimal objective values obtained from model (4) and the weighted sum method.

| Weights | \( z_1, z_2 \) (model(4)) | \( z_1, z_2 \) (weighted sum method) |
|---------|-----------------------------|--------------------------------------|
| 1       | \( w_1 = 0.1, w_2 = 0.9 \)  | 150,230                             | 143,265                             |
| 2       | \( w_1 = 0.2, w_2 = 0.8 \)  | 154,210                             | 156,200                             |
| 3       | \( w_1 = 0.3, w_2 = 0.7 \)  | 156,200                             | 156,200                             |
| 4       | \( w_1 = 0.4, w_2 = 0.6 \)  | 160,195                             | 156,200                             |
| 5       | \( w_1 = 0.5, w_2 = 0.5 \)  | 164,190                             | 176,175                             |
| 6       | \( w_1 = 0.6, w_2 = 0.4 \)  | 168,185                             | 176,175                             |
| 7       | \( w_1 = 0.7, w_2 = 0.3 \)  | 172,180                             | 176,175                             |
| 8       | \( w_1 = 0.8, w_2 = 0.2 \)  | 176,175                             | 186,171                             |
| 9       | \( w_1 = 0.9, w_2 = 0.1 \)  | 186,171                             | 208,167                             |

obtained by using model (4). However, we could see repetitive answers from the weighted sum method. Actually, the new model (4) yields more consistent results that let the decision maker select the best according to his/her preferences. Example 3 has also been considered by some authors. For instances, Ringuest and Rinks [11] computed \( (156, 200) \) as the most preferred value of the objectives \( (z_1, z_2) \) and Bit et al [17]. obtained \( (160, 195) \) as the optimal compromise value of the objectives.

5. Conclusion

This paper has proposed a new model based on fuzzy goal programming to solve MOTPs. By varying the weights in the new model, different solutions could be obtained. With regard to its principal properties, at least a weakly efficient solution is obtained for MOTP. Moreover, an efficient solution is achieved if the new model has a unique optimal solution. Some numerical examples have been stated to illustrate the proposed model. To emphasise the advantages of the new model, a comparison has been made with some existing methods. For further investigations, MOTPs with uncertain demand and supplies are considered. Extension of the new model to such problems might be interesting. Moreover, try to propose new algorithms and methods with nice properties for dealing with MOTPs could be considered as a general topic for future research.

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