Cosmology from decaying dark energy, primordial at the Planck scale

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Abstract

The consideration of dark energy’s quanta, required also by thermodynamics, introduces its chemical potential into the cosmological equations. Isolating its main contribution, we obtain solutions with dark energy decaying to matter or radiation. When dominant, their energy densities tend asymptotically to a constant ratio, explaining today’s dark energy-dark matter coincidence, and in agreement with supernova redshift data, and a universe-age constraint. This also connects the Planck’s and today’s scales through time. This decay may be manifested in the highest-energy cosmic rays, recently detected.

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Dark energy is a component of the universe whose negative pressure, characteristic of the quantum vacuum, accelerates its expansion. Evidence for its existence has recently accumulated from independent sources as the supernova redshift far-distance relation\cite{1, 2}, structure formation\cite{3}, the microwave background radiation\cite{4}, and lensing\cite{5}. The coincidence of its present energy-density scale with the universe’s, its smallness by 122 orders of magnitude with respect to the vacuum’s natural Planck scale, and its origin have remained puzzling.

The cosmological constant $\Lambda$ was originally added by Einstein in the application of general relativity to cosmology in 1917 in order to describe a static universe\cite{6}, building on a 1890s proposal by Neumann and Seeliger, who introduced it in a Newtonian framework for the same reasons. Its contribution in the Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}$$ (1)

equilibrates gravity’s attraction in a matter universe; here $R_{\mu\nu}$ is the Ricci tensor, $g_{\mu\nu}$ the metric tensor, which describe the geometry, and $T_{\mu\nu}$ is the energy-momentum tensor; we use units with the Newton, Planck, Boltzmann, and light-speed constants $G = h = k_B = c = 1$, except when given explicitly, as needed. Zeld’ovich sought to connect it to the quantum vacuum\cite{7}. This requires its reinterpretation as a component of $T_{\mu\nu}$ in Eq. (1). The vacuum energy density of particle fields with mass $m \ll M_P = \frac{1}{\sqrt{G}}$ is obtained by summing over its modes $k$:

$$\rho_{\Lambda P} = \frac{1}{(2\pi)^3} \int_{M_P} \frac{d^3k}{(\sqrt{k^2 + m^2})} \simeq 3 \times 10^{14} \text{GeV/cm}^3;$$ (2)

the natural cutoff is the Planck-mass scale $M_P$, the only possible mass conformed of $G$, $h$, and $c$, while in today’s universe $\rho_{\Lambda 0} \simeq 4 \times 10^{-6}$ GeV/cm$^3$. Among various explanation attempts, this striking difference has been attributed to a time-changing $G$\cite{8, 9}.

$\rho_{\Lambda 0}$ represents $\Omega_{\Lambda 0} = \rho_{\Lambda 0}/\rho_{c0} \simeq .73$ of its critical energy density today\cite{10}. $\rho_{c0}$, where in a flat universe $\rho_c$ is also the total energy density\cite{11}. The rest corresponds
mainly to matter, dark and baryonic, the latter conforming $\Omega_{b0} \simeq 0.044$ only[10]. Under the isotropic Robertson-Walker metric $ds^2 = dt^2 - R^2(t)(dx^2 + dy^2 + dz^2)$, Eq. 4 implies the Friedmann equation

$$H^2 = \frac{8\pi}{3} \rho_c = \frac{8\pi}{3} (\rho_\Lambda + \rho_r + \rho_m),$$

where the other known energy sources, radiation $r$ and matter $m$, have been included, $x, y, z$ are comoving Cartesian coordinates, $R$ is the scale factor, depending on time $t$, as do the $\rho_i$, and $H = \dot{R}/R$ the Hubble parameter (a dot denotes time derivative). Each component $i$ has pressure $p_i$ and is characterised by an equation of state

$$p_i = w_i \rho_i,$$

where $w_r = 1/3$ for radiation, and for relativistic Fermi or Bose gases, and $w_m = 0$ for non-relativistic matter. The energy-conservation equation within an expanding volume $V$

$$\sum_i d(\rho_i V) = -\sum_i p_i dV$$

is implied by the contraction of Eq. 4 and also by thermodynamics. When decoupled, each contribution satisfies

$$d(\rho_i V) = -p_i dV.$$  

The form of Eq. 4 implies $\Lambda$ generates a pressure $p_\Lambda = -\rho_\Lambda$, so $w_\Lambda = -1$ for the vacuum energy. The parametric extension to arbitrary negative values $w_\Lambda$, with similar properties[12, 13], suits the lack of precise knowledge about it. Whatever is its nature, and with a name not bound to its constancy, dark energy should contain quanta[7], as any other form of energy in the universe, and so, the energy dependence on its number $N$ should be accounted for. Eq. 6 is then modified by the chemical potential $\mu$ contribution

$$d(\rho V) = -pdV + \mu dN.$$
In this letter we consider dark-energy’s chemical-potential modification of the cosmological equations; its main contribution implies dark energy decays to another component. The derived asymptotic energy-density constant ratio of the dominant components reproduces the coincidence of dark energy and dark matter today, and fits the supernova redshift data. Also, dark energy’s decay connects Planck’s scale to today’s energy-density scale from Planck’s time to the universe age.

For systems satisfying Eq. 4, \( \mu \) can be obtained consistently with the entropy density \( s = \frac{S}{V} \) by extrapolating, and using the thermodynamics relation \( s = \frac{1}{T}(\rho + p - n\mu) \), where \( T \) is the temperature, and \( n = N/V \) the particle density. When radiation-like, \( s_{rw} = c_{rw}\rho^{1+w} \), and \( \mu_{rw} = 0 \). Dark energy interacts feebly, and presumably, only gravitationally; in the latter case, the single scale in the integration constant \( c_{rw} \) can only contain the Planck scale, and an O(1) numerical constant (the same for \( c_{0w}, c_{w\chi} \) below.) Such an argument correctly gives \( s_{rw} \), which corresponds to the Stefan-Boltzmann law: demanding that \( c_{rw} = M_p^{-3-\frac{4}{1+w}} \) not contain \( G \), as occurs for radiation, one gets \( w = 1/3 \). If the energy depends on \( V \) through a power law, as implied by Eq. 4 and they remain extensive, another such quantity is required, and \( N \) is the necessary choice in most physical systems. In the zero-entropy regime \( \rho_{\Lambda w} = c_{0w}n^{1+w} \), and \( n\mu_{\Lambda w} = (1 + w)\rho \). Non-zero temperatures or interactions may modify \( n\mu_{\Lambda w} \) to \( n\mu_{w\chi} = (1 + w + \chi)\rho \), where \( \chi \) parameterises their effect. This leads to \( s_{w\chi} = c_{w\chi}n(\frac{\rho}{n_{w+w}})^{-\frac{1}{\chi}} \), and \( T_{w\chi} = -\frac{\chi \rho}{c_{w\chi}n(\frac{\rho}{n_{w+w}})^{\frac{1}{\chi}}} \). \( s_{w\chi} = s_{rw} \) for \( \chi = -w - 1 \), and the zero-entropy case is approached with \( \rho \sim \rho_{\Lambda w} \), for \( \chi \to 0 \). We conclude \( \chi \) is O(1). \( \chi \) need not be constant (nor \( w \), for that matter.) In the high-temperature regime, a polytropic gas has\[14\] \( s_{hw} = (n/w)\log[\rho/(c_{0w}n^{1+w})] \), for which \( n\mu_{hw} = \rho\{1 + w - \log[\rho/(c_{0w}n^{1+w})]\} \).

\[ \mu_\Lambda dN = \mu_\Lambda(n_\Lambda dV + Vdn_\Lambda) , \] and in the universe’s evolution in \( dt \), the partial width \( \Gamma_1 \) in \( N\Gamma_1 dt = n_\Lambda\mu_\Lambda dV = (1 + w_\Lambda + \chi)\rho_\Lambda dV \) is associated with decay due to
its expansion

\[ \dot{n}_\Lambda \Gamma_1 = 3(1 + w_\Lambda + \chi)H\rho_\Lambda \sim \rho_\Lambda^{3/2}, \tag{8} \]

given \( H \sim \rho_\Lambda^{1/2} \), and corresponds to changes \( \partial N_\Lambda / \partial V = n_\Lambda \). \( N \Gamma_2 dt = \mu_\Lambda V dn_\Lambda \) contains terms that are not of this form. It could account for any other conceivable decay process linked to interactions. For the gravitational interaction, and \( T = 0 \), \( \Gamma_2 \sim \sigma n_\Lambda v \sim (1/M_p^4)n_\Lambda \rho_\Lambda^{1/2} \), where for the cross section \( \sigma \sim (1/M_p^4)\rho_\Lambda^{1/2} \), and the velocity \( v \sim c = 1 \), so \( n_\Lambda \Gamma_2 \sim \rho_\Lambda^{-\frac{2}{w_\Lambda + 1} + 1/2} \), using \( \rho_\Lambda w_\Lambda \) above. Therefore, for \( -3 < w_\Lambda < -1 \), \( \Gamma_2 \ll \Gamma_1 \) as \( \rho_\Lambda \rightarrow 0 \). Similarly, this will always occur for \( T_w \chi \neq 0 \), \( \sigma \sim (1/M_p^4)T^2 \), and \( \rho_\Lambda \sim \rho_w \Lambda \). Another type of interaction can be dominant for some time, but it will eventually be overridden by the \( \Gamma_1 \) term. Lower powers of \( \rho_\Lambda \), e. g., a constant decay rate \( n_\Lambda \Gamma_2 \sim \rho_\Lambda \), could make a significant cosmological contribution, but it would have to be fine-tuned to give the present parameters. Thus, the \( \Gamma_2 \) term can and will be neglected.

We obtain, using Eqs. \( \text{[3]} \text{[4]} \text{[8]} \)

\[ \dot{\rho}_\Lambda + 3(w_\Lambda + 1)H\rho_\Lambda = 3[(w_\Lambda + 1) + \chi]H\rho_\Lambda, \tag{9} \]

with the latter term producing dark energy decay for \( \chi < 0 \). Energy conservation in Eq. \( \text{[5]} \) demands that energy be transferred, which we assume occurs for the \( d \) component

\[ \dot{\rho}_d + 3(w_d + 1)H\rho_d = -3[(w_\Lambda + 1) + \chi]H\rho_\Lambda. \tag{10} \]

The set of Eqs. \( \text{[3]} \text{[4]} \text{[10]} \) describes \( \rho_\Lambda \) as a fluid decaying out of equilibrium as is common in many universe processes\[15\]. A decaying cosmological constant was first conceived by Bronstein\[16\] to explain the universe’s time direction, and recent study starts with Ref. \[17\], with various phenomenological decay laws then considered\[18\].

By substituting \( H \) in Eq. \( \text{[3]} \) into Eq. \( \text{[10]} \) we obtain

\[ \rho_d = -\rho_\Lambda + \frac{\dot{\rho}_\Lambda^2}{24\pi\chi^2 \rho_\Lambda^2}. \tag{11} \]
Substituting this into Eq. 9, we get

\[6 \chi \rho _{\Lambda } \ddot{\rho }_{\Lambda } + (d - 6 \chi ) \dot{\rho }_{\Lambda }^2 - 24 \pi [d - 3 (1 + w_{\Lambda })] \chi ^2 \rho _{\Lambda }^3 = 0,\]

(12)

where \(d = 3(w_{\Lambda} + 1)\). \(t \) as inverse function of \(\rho _{\Lambda} \) can be integrated, where initially \(\rho _{\Lambda} \) at \(t_i \)

\[t - t_i = \int_{\rho _{\Lambda}}^{\rho _{\Lambda_i}} d\rho \left(\frac{d + 3 \chi }{24 \chi ^2 \pi [d - 3(w_{\Lambda} + 1)] \rho ^3 + 3(d + 3 \chi ) \chi C \rho ^{-\frac{2}{\chi}}}ight)^{\frac{1}{2}}.\]

(13)

\(C \) accounts for initial conditions for \(\rho _{d} \), and we have chosen the solution for which \(R \) increases and \(\rho _{\Lambda} \) decreases. For some \(\chi, w_{\Lambda} \), \(t(\rho _{\Lambda}) \) can be given explicitly in terms of hypergeometric and elliptic functions.

Assuming \(\rho _{d} \) is dominant, neglecting the non-dominant term in Eq. 3 and using Eqs. 11, 13 one finds

\[\rho _{c} \approx \frac{24 \chi ^2 \pi [d - 3(w_{\Lambda} + 1)] \rho _{\Lambda} + (d + 3 \chi )3 \chi C \rho _{\Lambda}^{-\frac{d}{\chi}}}{24 \pi \chi ^2 (d + 3 \chi )}.\]

(14)

One derives that for \(-d/3 < \chi < 0\)

\[\lim_{\rho _{\Lambda} \to 0} \frac{\rho _{\Lambda}}{\rho _{c}} = \frac{d + 3 \chi }{d - 3(w_{\Lambda} + 1)}\]

(15)

within the wide set of initial conditions \(C \ll \rho _{\Lambda 0}^{1 + \frac{d}{\chi}} \), so \(\Omega _{d} \) and \(\Omega _{\Lambda} \) will acquire a fixed asymptotic value. In this limit, the \(d \) component in Eq. 10 behaves as \(\rho _{\Lambda} \) in Eq. 9

\[\dot{\rho }_{d} - 3 \chi H \rho _{d} = 0.\]

(16)

Such \(\rho _{d} \) and \(\rho _{\Lambda} \) depend on the scale factor as \(R^{2 \chi} \), while \(R \sim t^{-2/(3 \chi)} \). In fact, from Eqs. 13, 14, 15 the dominant components produce \(\rho _{c} \sim 1/(6 \pi \chi ^2 t^2) \), and the representative constant \(|\chi _{t}| = \frac{1}{t_0 (6 \pi \rho _{c0})^{1/2}} \simeq .67 \), using the universe’s age \(t_0 \simeq 13.7 \pm .2 \) years. 10. For a wide set of conditions, including matter and radiation domination, this behaviour correctly connects today’s energy densities with Planck’s scale at Planck’s time \(t_P \): \(\rho _0 \sim \rho _P (t_P/t_0)^2 \).
Indeed, for $\chi \neq 0$ Eq. 14 implies the volume factor $(R_f/R_i)^3$ grows by $3\frac{1}{|\chi|}\log(\frac{R_f}{R_i})$ e-folds. Therefore, a small enough $\chi$ in the early universe can meet the $10^{88}$ entropy factor that solves the problems of smoothness, flatness, and causality, as does inflation\[19].

Eq. 9 with $d = 4$ applies to the radiation-dominated epoch. In general, $\chi_r \sim -4/3$ gives $\Omega_\Lambda \ll 1$ so as not to interfere with nucleosynthesis\[20], maintaining $\eta = \rho_b/s_r$; radiation and matter grow as in Eq. 6 $\rho_b \sim 1/R^3$, and $\rho_r \sim 1/R^4$, as does dark matter $\rho_{dm} \sim 1/R^3$, until dark energy becomes dominant, and the decaying term in Eq. 9 becomes important. If dark matter interacts weakly, dark energy’s decay’s switch to dark matter occurs at $T \sim 10$ GeV, before the onset of nucleosynthesis. In a second scenario, dark energy decays simultaneously to radiation and matter, in a proportion that is reconciled with a constant $\eta$.

The cosmic microwave background radiation data is consistent with a dark-energy component and the presence of both dark energy and dark matter provides a better fit in structure-formation models\[3]. If after nucleosynthesis $\rho_\Lambda$ is small, it should decrease slower than radiation and matter so $\chi > -4/3$, and when matter dominates, $\chi > -1$. Then it can influence the structure formation, and, eventually, dominate together with dark matter.

Such a behavior fits the supernova data\[21] interpreted under Eq. 15, with dark matter and dark energy evolving with a constant ratio. With the simplest assumption of constant $\chi_0 = -0.48$, and as shown in Fig. 1 (which also includes the non-fitting $\Omega_{\Lambda 0} = 0, \Omega_{\Lambda 0} = -1$ cases), one can reproduce the luminosity distance $d_L = H_0^{-1}(1 + z)\int_0^z d'z'[(\Omega_{b0}(1 + z')^3 + (1 - \Omega_{b0})(1 + z')^{-3\chi_0}]^{-1/2}$ up to the measured redshift $z \sim 2$, where the fit is independent of $\Omega_{\Lambda 0}$ (which fixes $w_\Lambda$). This is in accordance with a slower growth than $\rho_r, \rho_b$ into the past, including the time of dark-energy-baryon equality at $z_{\Lambda b} = (\Omega_{\Lambda 0}/\Omega_{b0})^{\frac{1}{1+3\chi_0}} - 1 \simeq 5$, after which the asymptotic behaviour of Eq. 15 ensues. Also consistently, the time since $z_{\Lambda b}$ does not saturate the age of the
universe: \( \int_0^{z_h} dz'(1 + z')^{-1}[\Omega_0 (1 + z')^3 + (1 - \Omega_0)(1 + z')^{-3\chi_0}]^{-1/2} \simeq .9 < H_0 t_0 \simeq 1, \) and \( \chi_r < \chi_t < \chi_0. \)

Energy injection to the universe through dark-energy decay may have observable consequences. Most cosmic rays can be attributed to galactic origin, up to \( E_{cr} \simeq 10^{11} \text{GeV}[22]. \) The large flux and apparent isotropy of recently detected\(^\text{23}\) rays with energies beyond the GZP limit\(^\text{24}\) make them difficult to associate with extra-galactic sources, and yet, to galactic processes\(^\text{22}\). Rays at \( E_{cr} \) arrive with an emissivity of \( 10^{-9} \frac{\text{GeV}}{\text{cm}^2\text{sec}}, \) representing \( \rho_{cr} \simeq 4 \times 10^{-19} \frac{\text{GeV}}{\text{cm}^2}. \) The gravitational decay of dark energy mediated by a particle of mass \( m_a \) (and involving a photon or a nucleon), can have a width \( \Gamma_a \sim m_a^5/M_P^4, \) and it could conform the energy-transfer mechanism in the \( E_{cr} \) channel with \( (E_{cr}/M_P)^3 \rho_\Lambda_0 H_0 = \Gamma_a \rho_{cr}; \) we used the phase-space factor in Eq.\(^\text{22}\) assuming that the vacuum Planck spectrum is uniformly depleted, as suggested by relatively constant spectrum, and the prevalence of physical constants. We find \( m_a \sim 10^4 \text{ GeV}, \) a range for future accelerators.

![Figure 1](image-url)

Figure 1: Comparison of magnitude \( \mu = 5\log_{10}(d_L) + 25 \) of luminosity distance \( d_L, \) as a function of redshift \( z, \) for flat models. For non-asymptotic models with \( w_\Lambda = -1, \) and (a) \( \Omega_m = 0, \Omega_\Lambda = 1 \) (dotted), (b) \( \Omega_m = .27, \Omega_\Lambda = .73 \) (line), and (c) \( \Omega_m = 1, \Omega_\Lambda = 0 \) (dashed); and (d) for asymptotic model with \( \Omega_b = .044, \) and \( \chi = -.48 \) (dot-dashed).
In summary, account of dark energy’s quanta connects today’s energy-density scale with Planck’s, within classical general relativity and thermodynamics. It represents a departure from the zero-temperature cosmological constant, while it maintains the results of the standard cosmology. This supports a conservative approach in which known physical elements can provide new information\cite{25}. Dark energy’s coincidence with the critical density today is connected to the universe evolution, in which events occur by contingency, rather than chance. While microphysics\cite{26} needs to elucidate the dark energy’s equation of state, the universe already emerges as flat, interconnected, evolving deterministically, and in an inexorable process of accelerated expansion and decay.
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