MATCHIG: A program for matching charged Higgs boson production at hadron colliders

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Abstract

This manual describes how to use the MatCHig code for matching the charged Higgs boson production processes $gg \to tbH^\pm$ and $gb \to tH^\pm$. A negative term, correcting for the double-counting between these processes, is implemented as an external process to PYTHIA, allowing the Monte Carlo generation of matched events. Results from the matching were published in [1]. The code can be downloaded from http://www.isv.uu.se/thep/MC/matchig/.

1 Physics motivation

Many extensions of the Standard Model, most notably supersymmetric theories like the MSSM, predict the existence of a charged Higgs boson. Since there is no charged scalar particle in the Standard Model, the discovery of such a particle would be a clear signal for physics beyond the Standard Model, and would provide insight into the nature of the extension needed. However, in order to find the charged Higgs we need an accurate description of its production and what experimental signatures to look for.

Charged Higgs production at hadron colliders is usually described in Monte Carlo event generators such as PYTHIA [2] and HERWIG [3] using the $2 \to 2$ process

$$gb \to tH^- (g\bar{b} \to \bar{t}H^+) \ .$$

Here, the incoming $b$-quark is described by the DGLAP evolution equations using gluon splitting to $b\bar{b}$ in the collinear approximation. Therefore the event also contains an outgoing $\bar{b} (b)$ quark. For low transverse momenta of this accompanying $b$ quark, the $2 \to 2$ process (1), including initial state parton showers, describes the cross-section well, since the $b$ quark density includes a resummation of potentially large logarithms of the type $\left(\alpha_s \log \frac{\mu_F}{m_b}\right)^n$, where $\mu_F$ is the factorization scale. However, for large $p_{t,b}$ (transverse momentum of the
accompanying $b$-quark) the collinear approximation is no longer appropriate, and one need instead use the exact matrix element of the $2 \to 3$ process
\begin{equation}
    gg \to tbH^- (gg \to \bar{t}bH^+) .
\end{equation}

At low transverse momenta, the $2 \to 3$ process can be described in terms of gluon splitting to $\bar{b}b$ (at order $\alpha_s \log \frac{\mu}{m_b}$), times the matrix element of the $2 \to 2$ process. This means that we can not simply add the two processes without getting double-counting for low values of $p_{\perp b}$. This problem is usually addressed by using only the $2 \to 2$ process [1] when the accompanying $b$ quark is not observed, and only the $2 \to 3$ process [2] when observing the accompanying $b$ quark. However, as was shown in [1], for low transverse momenta ($p_{\perp b} \lesssim 100$ GeV) the latter approach underestimates the differential cross-section. Therefore, when the accompanying $b$-quark is observed, it is necessary to use both the $2 \to 2$ and the $2 \to 3$ processes together, appropriately matched to remove the double-counting.

Detailed discussions, derivations, references and results from this matching were published in [1].

2 The double-counting term

In order to remove the double-counting, we define a double-counting term $\sigma_{DC}$, given by the part of the $2 \to 3$ process (eq. (2)) which is already included in the $2 \to 2$ process (eq. (1)). This term is then subtracted from the sum of the cross-sections from the two processes,
\begin{equation}
    \sigma = \sigma_{2\to2} + \sigma_{2\to3} - \sigma_{DC} .
\end{equation}

The double-counting term is given by the leading ($\mathcal{O}(\alpha_s \log \frac{\mu}{m_b})$) contribution of the $b$ quark density to the $2 \to 2$ process cross-section:
\begin{equation}
    \sigma_{DC} = \int dx_1 dx_2 \left[ g(x_1, \mu_F) b'(x_2, \mu_F) \frac{d\sigma_{2\to2}}{dx_1 dx_2}(x_1, x_2) + x_1 \leftrightarrow x_2 \right] \end{equation}

where $b'(x, \mu_F^2)$ is the leading order $b$-quark density given by
\begin{equation}
    b'(x, \mu_F^2) \approx \frac{\alpha_s}{2\pi} \log \frac{\mu_F^2}{m_b^2} \int dz P_{qq}(z) g \left( \frac{x}{z}, \mu_F^2 \right) \end{equation}

with $P_{qq}$ the $g \to q\bar{q}$ splitting function, $g(x, \mu_F^2)$ the gluon density function, $\mu_F$ the factorization scale and $z$ the longitudinal gluon momentum fraction taken by the $b$-quark.

Including kinematic constraints due to finite center of mass energy and finite $b$ quark mass,
the resulting expression for the double-counting term can be written as

\[
\sigma_{DC} = \int_{\tau_{\text{min}}}^{1} \frac{d\tau}{\tau} \int_{1/2 \log \tau}^{1/2 \log \tau} dy \frac{\pi}{s} \int_{-1}^{1} \frac{\beta_{34}}{2} d(\cos \hat{\theta}) |M_{2\rightarrow2}|^2 \frac{\alpha_s(\mu_F^2)}{2\pi} \times \\
\left[ \int_{x_1}^{z_{\text{max}}} dz P_{gg}(z) \int_{Q_\text{min}}^{Q_\text{max}}^2 dQ^2 \frac{x_1}{Q^2 + m_b^2} g \left( \frac{x_1}{z}, \mu_F^2 \right) x_2 g(x_2, \mu_F^2) + x_1 \leftrightarrow x_2 \right].
\]

Here \(M_{2\rightarrow2}\) is the matrix element for the \(2 \rightarrow 2\) process \(1\), \(\mu_F\) and \(\mu_R\) are the factorization and renormalization scales as in the \(2 \rightarrow 3\) process, and the kinematical variables are \(\tau = x_1 x_2, x_{1,2} = \sqrt{\tau e^{\pm y}}, \hat{s} = \tau s\). \(\hat{\theta}\) is the polar angle of the \(t\)-quark in the CM system of the \(2 \rightarrow 2\) scattering, and \(\beta_{34} = \hat{s}^{-1} \sqrt{\left(\hat{s} - m_b^2 - m_{H^\pm}^2\right)^2 - 4m_b^2 m_{H^\pm}^2}\). \(Q^2\) is the virtuality of the incoming \(b\)-quark and \(z\) is identified with the ratio of the center-of-mass energies of the \(gb\) \((2 \rightarrow 2)\) system and the \(gg\) \((2 \rightarrow 3)\) system.

The integration limits are given by

\[
\tau_{\text{min}} = (m_t + m_{H^\pm})^2 / s
\]

\[
\hat{s}_{\text{max}} = \frac{Q_{\text{opt}}^2 \hat{s}}{(Q_{\text{opt}}^2 + \hat{s})(Q_{\text{opt}}^2 + m_b^2)} \quad \text{and} \quad Q_{\text{opt}}^2 = \min \left( \sqrt{\hat{s} m_b^2}, \mu_{F,2\rightarrow2}^2 \right)
\]

\[
Q_{\text{min}}^2 = \frac{1}{2} [\hat{s} (z^{-1} - 1) - m_b^2] - \frac{1}{2} \sqrt{[\hat{s} (z^{-1} - 1) - m_b^2]^2 - 4\hat{s} m_b^2}
\]

\[
Q_{\text{max}}^2 = \min \left\{ \mu_{F,2\rightarrow2}^2, \frac{1}{2} [\hat{s} (z^{-1} - 1) - m_b^2] + \frac{1}{2} \sqrt{[\hat{s} (z^{-1} - 1) - m_b^2]^2 - 4\hat{s} m_b^2} \right\}
\]

where \(\mu_{F,2\rightarrow2}\) is the factorization scale of the \(2 \rightarrow 2\) process, which sets the upper \(Q^2\) limit in the parton showers.

### 3 Implementation

The double-counting term is implemented as an external process using the Les Houches generic user process interface for event generators [4]. This means that it can in principle be used together with any event generator supporting the Les Houches standard, but it is primarily intended for use with PYTHIA [2], and it uses several PYTHIA routines and common blocks. The relevant FORTRAN-files can be downloaded from [http://www.isv.uu.se/thep/MC/matchig/](http://www.isv.uu.se/thep/MC/matchig/).

The implementation, found in the file `matchig.f`, provides the subroutines `UPINIT`, to set up the external process, and `UPEVENT`, which supplies an event \((i.e.\) a set of incoming and outgoing particles and momenta) and a weight for the event. Please note that since the double-counting contribution should be subtracted from the sum of the positive processes \(1\) and \(2\), this weight is negative for double-counting events. This means that if all three processes are run simultaneously in PYTHIA, the total cross-section will be the correctly matched one given by eq. [3].
The events from the double-counting term should likewise be subtracted in the analysis. In practice, this means that these events should undergo the same cuts and the same detector response simulation, etc., as the events from the positive contributions, and then be added with weight $-1$ to any histograms. The events from the double-counting term can be identified in Pythia using the parameter construction $\text{KFPR} (\text{MSTI}(1),2)$, which contains the external process identifier for external processes. The external process identifier for the double-counting process is $LPRUP(1)=10000$.

Since the $2 \rightarrow 2$ process has larger cross-section than the double-counting term in all phase-space regions, the risk of getting a negative total cross-section in any phase-space point should be small given enough statistics.

### 3.1 Parameters and recommended settings

For information on how to use Pythia with external processes, please see the Pythia manual [2], section 9.9. An example main program, matchex.f, illustrating how to use the double-counting term together with Pythia, can also be found on the homepage. Please note that the double-counting term should be run together with the internal processes $gb \rightarrow tH^{\pm}$ (ISUB=161) and $gg \rightarrow tbH^{\pm}$ (ISUB=401) to get matched events. (Note that the latter process (ISUB=401) was implemented in Pythia from version 6.223.)

The following Pythia parameters are used in matchig.f:

- **PMAS** (in the common block PYDAT2) is used to get masses and widths for the charged Higgs boson $(\text{PMAS}(37,1), \text{PMAS}(37,2))$, top quark $(\text{PMAS}(6,1), \text{PMAS}(6,2))$, bottom quark $(\text{PMAS}(5,1))$ and $W^{\pm}$ boson $(\text{PMAS}(24,1))$.

- **PARU(141)** (in PYDAT1) is used to get the value of $\tan \beta$.

- **PARU(102)** (in PYDAT1) is used to get the value of $\sin^2 \theta_W$.

- **MSTP(39)** (in PYPARS) is used to choose factorization and renormalization scales as for the $2 \rightarrow 3$ process. The following choices are possible: $\text{MSTP}(39) = 3, 5, 6$ or $8$. The options $\text{MSTP}(39) = 6$ and $8$ are new from Pythia 6.226, please see [3] for details. Our recommended setting is $\text{MSTP}(39) = 8$, corresponding to $\text{MSTP}(32) = 12$. Note that in this case PARP(193) and PARP(194) must also be set, see below.

- **MSTP(32)** (in PYPARS) is used to choose the maximum scale for parton showers, i.e. the maximum transverse momentum of the accompanying $b$-quark as for the $2 \rightarrow 2$ process ($\mu_{F,2\rightarrow2}$ in eq. [7]). The following choices are possible: $\text{MSTP}(32) = 4, 11$ or $12$. The options $\text{MSTP}(32) = 11$ and $12$ are new from Pythia 6.226, please see [5] for details. Our recommended setting is $\text{MSTP}(32) = 12$, corresponding to $\text{MSTP}(39) = 8$. Note that in this case PARP(193) and PARP(194) must also be set, see below.
PARP(34) (in PYPARS) is used to modify the factorization and renormalization scales as in Pythia. Note however that it does not affect the maximum transverse momentum of the accompanying b-quark.

PARP(67) (in PYPARS) is used to modify the maximum scale for parton showers, i.e. the maximum transverse momentum of the accompanying b-quark ($\mu_{F,2 \to 2}$ in eq. (7)), as in Pythia.

PARP(48) (in PYPARS) is used to cut the tails of the Breit-Wigner distribution for the charged Higgs mass at masses outside $m_{H^\pm} \pm \text{PARP(48)} \cdot \Gamma_{H^\pm}$, as in Pythia.

The values of $\alpha_s$, $\alpha_{em}$, the parton distributions and the running top and bottom quark masses are calculated using the corresponding Pythia subroutines. The top quark and charged Higgs masses are Breit-Wigner distributed with mass-dependent widths in the same way as in the internal $gb \to tH^\pm$ and $gg \to tbH^\pm$ processes, using the Pythia subroutine PYWIDT. Suppression of the cross-section due to closed decay channels of the charged Higgs boson and top quark is done as in the Pythia processes.

**Factorization scale:** As shown in [1], there are strong arguments that a non-standard factorization scale $\mu_{F} \approx 0.5 \frac{m_{H^\pm} + m_t}{2}$ should be used in charged Higgs production. The option to have fixed factorization and renormalization scales of choice was added to Pythia from version 6.226 [5]. This is done by setting MSTP(32)=12 (for $2 \to 2$ processes) and MSTP(39)=8 (for $2 \to 3$ processes). Then one needs to set PARP(193) to the square of the factorization scale ($\mu_{F}^2$), and PARP(194) to the square of the renormalization scale $\mu_{R}^2$ (usually taken to be $m_{H^\pm}^2$).

**On-shell parton showers:** In order to be able to compare the differential cross-section for the $2 \to 2$ process [1], the $2 \to 3$ process [2] and the double-counting term on parton level, one should use on-shell initial-state parton showers, set by MSTP(63)=0. In a realistic case, using jet-finding algorithms, there should however be no observable difference compared to having the parton showers off-shell.

4 Final comments

The code described in this document was written for research conducted together with Johan Rathsman. If you use this code, please cite [1].

References

[1] J. Alwall and J. Rathsman, “Improved description of charged Higgs boson production at hadron colliders,” JHEP 0412 (2004) 050 [arXiv:hep-ph/0409094].

[2] T. Sjöstrand et al. “High-energy-physics event generation with PYTHIA 6.1,” Comput. Phys. Commun. 135 (2001) 238 [arXiv:hep-ph/0010017].
[3] G. Corcella et al., “HERWIG 6: An event generator for hadron emission reactions with interfering gluons (including supersymmetric processes),” JHEP 0101 (2001) 010, \texttt{arXiv:hep-ph/0011363}.

[4] E. Boos et al., “Generic user process interface for event generators,” \texttt{arXiv:hep-ph/0109068}.

[5] A description of new \textsc{Pythia} features can be found on the \textsc{Pythia} homepage \texttt{http://www.thep.lu.se/~torbjorn/Pythia.html} or specifically at \texttt{http://www.thep.lu.se/~torbjorn/pythia/pythia6226.update}.