Analysis of Changes in the Effect Flow Rate on the Open Channel

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ABSTRACT
Planning of water structures such as dams, irrigation canals, and other water structures require a description of a hydraulic flow phenomenon that these buildings. Each flow conditions before, moderate, and after passing through each building has its characteristics or tendencies. This study aims to analyze ignition characteristics by narrowing (sudden narrowing, transition, and Radius). Measurement of flow velocity is measured using a current meter and float tool (pumping ball) to obtain data: water level, flow velocity, observation review points before and after the channel narrowing. Understanding the flow characteristics of a narrowed channel is used to consider canals' technical design, especially irrigation channels. The analysis shows that the flow of water through the narrowing experience changes in specific energy. The maximum specific energy loss occurs in a sudden constriction type. The predicted results correlate fairly with the experimental data from this and other studies.

Keywords: Channel narrowing, Flow velocity, Froud Numbers, and Specific Energy

1. INTRODUCTION
An open channel is a free-flowing water channel. Open channels can be distinguished into two types, namely artificial and natural. Open channels found both on irrigation channels, technical, semi-technical, and natural channels in non-prismatic conditions. In a channel with a non-prismatic drainage channel, water flow changes such as altitude, velocity, channel width, water discharge, and other flow behavior. Some of the causes of the non-prismatic cross-section are the connection of two different cross-sections, other buildings such as bridge pillars, or other causes that alter the channel's cross-section. Flow analysis of non-prismatic channels requires precision due to changes in flow characteristics. One example is channel narrowing, which causes altitude, velocity, and energy in the flow to change. The flow of energy affects the channel's smooth flow, which can disrupt the distribution of water flow that can harm. This fact needs attention. The discussion of the flow that occurs in the case of channel narrowing in this paper tries to disentangle the problem through measurement and testing on an open channel in the presence of constriction. Referring to the law of continuity, when the flow of water flows on a narrow section of the channel, it can increase the flow rate and energy. The narrowing of the channel cross-section becomes one of the factors to increase the flow and energy velocity. From previous research, Cristian Auel et al. [1] project is to establish general design criteria for optimal hydraulic conditions to avoid sediment depositions in the tunnel and keep the resulting abrasion damages at a minimum. Chung-Chieh Hsu et al. [2] propose a depth-discharge relationship and energy-loss coefficient for a subcritical, equal-width, right-angled dividing flow over a horizontal bed in a narrow aspect ratio channel. Jhonson et al. [3] used abrupt type of narrowing by using several different channel widths; hence previous research has been used as a benchmark in this study, which used three types of narrowing types constriction, transition constriction type, and the narrowing Radius of the same channel width. This paper focuses on analyzing the flow changes due to the narrowing of the open channel. We argue that applying into model checking should solve the following three issues: (1) How to know the flow classification of various narrowing types. (2) How to know the change of flow velocity resulting from various types of narrowing. (3) How to know the flow behavior that involves the specific energy changes that occur narrowing on the channel.

1.1. Related Work
According to the generation type of assumptions, we divided the existed work into three narrowings.
1.1.1. Research Methodology

The research method is a scientific way to get data with a specific purpose and usefulness. The natural channel cross-section is generally very irregular, usually varying from parabolic to trapezoidal forms. The term channel section is perpendicular to the flow direction, while the vertical channel section is the vertical cross-section through the lowest or lowest point of the cross-section. Therefore the horizontal channel of the cross-section is always a vertical cross-section:

![Figure 1 Cross-section of rectangular channels](image)

\[ \text{Wide (A)} = b \times h \]  \hspace{1cm} (1)

\[ \text{Wet Round (P)} = b + 2h \]  \hspace{1cm} (2)

\[ \text{Hydraulic Radius (R)} = \frac{by}{b+2h} \]  \hspace{1cm} (3)

The material used is water. The tools used are a set of open channel models with a bottom of a channel and a wall made of fiber, ruler, current meter FL 03, stopwatch, pumping ball, 90° constrictions, 45° constrictions, and Radius constriction.

![Figure 2 flow sketch through the constriction. (a) Drainage of abrupt constriction conditions, (b) Drainage of transitional constriction conditions, (c) Drainage of the radius constriction conditions.](image)

1.1.2. Research Procedures

The process of data retrieval conducted in this study is (1) Flow arrangement and measurement (2) the water level on the open channel (3) Perform Measurement of flow velocity with the assumption that the average flow velocity at a vertical is measured at only a few points and then calculated by mathematical results. Measurements are done by 1 point and 2 point method, namely: One point method measurement of flow velocity by a single point method is carried out at a depth of 0.6 h or 0.2 h (h = Depth), at 0.6 h is performed when the flow depth is between 2.5-7.5 cm (V=0.6 h), two-point method Measurement of flow depth by the two-point method was carried out at a depth of 0.2 h and 0.8 h of surface water. Average flow velocity is obtained by the formula (V=0.5(0.2h+0.8h)), calculate the fraud number using equation 5, calculate Specific Energy using equation 8, and calculate Energi Loss of Specific Energy using equation 10.

1.2. Our Contribution

This paper present analyzes the flow changes due to the narrowing of the open channel. From previous research, Cristian Auel et al. [1] discusses Turbulence Characteristics in Supercritical Open Channel Flows: Effects of Froude Number and Aspect Ratio, Chieh Hsu et al. [2] discussing Subcritical 90° Equal-Width Open-Channel Dividing Flow, and Jonson et al. [3] used abrupt type of narrowing by using several different channel widths, hence previous research has been used as a benchmark in this study which used three types of narrowing type abrupt constriction, transition.
constriction type, and the narrowing Radius of the same channel width. Understanding the flow characteristics of a narrowed channel is used to consider canals' technical design, especially irrigation channels.

1.3. Paper Structure

The rest of the paper is organized as follows. Section 2 introduces the preliminaries used in this paper, including open channels, narrowing channels, and Sistem classification. Section 3 presents a model checking framework based on the rule and optimizes the learning framework. Then, the framework is extended describes the data description Section 4. Section 5 develops a prototype tool for the framework, compares it with Cristian Auel et al. [1], Chieh Hsu et al. [2], and Jhonson et al. [3] by several large cases, results, and research discussion. Finally, Section 6 concludes the paper and presents directions for future research.

2. BACKGROUND

2.1. Open channels

Channels that drain water with a free surface are called open channels. Open channels can occur considerably, ranging from ground-level flows during rain until the water flow is continuously in the prismatic channel. Channels are classified into two types: natural existing and artificial channels. Natural channels include all water channels naturally occurring on earth, from small gutters in the mountains, small rivers, and large rivers to river mouths. Artificial channels are human-made channels for specific purposes and interests. Nature's hydraulic properties very uncertain. Artificial channels have a regular cross-section and are easier to analyze than natural channels. Artificial channels include roadside drainage, irrigation canals to irrigate rice fields, sewers, drains to carry water to hydroelectric power, drinking water supply channels, floodways. In some ways, it can be assumed that the approach is sufficiently consistent with actual observations. Thus the flow requirements of this channel are acceptable for the completion of theoretical hydraulics analysis.

2.2. Narrowing channels

The narrowing of the channel is a phenomenon commonly encountered on an open channel. A narrowing on an open channel consists of the abruptly constricted area of the cross-section of the canal. The constriction's influence depends on the geometry (shape) of the curvature portion of the constriction, flow velocity, and flow state, Ven Te Chow [4]. The flow-through constriction can be supercritical or subcritical. Critical depth can be formulated as follows[5-7]:

\[ H_c = \frac{2}{3} E \]  
\[ B_c = 1.84 \frac{Q}{e^{1/2} g^{1/2}} \]

With \( Q \) = water discharge (m^3 / sec), \( B_c \) = critical width, \( E \) = specific energy, \( H_c \) = critical depth, \( g \) = acceleration of gravity.

2.3. System classification

The open channel flow can be classified into several types and described in various ways as follows. The flow-through constriction can be a supercritical or subcritical. Critical depth can be formulated, Rangga Raju [7].

2.3.1. Steady flow and unsteady flow

A flow in an open channel is steady when variables of flow (such as velocity \( V \), pressure \( P \), mass density \( \rho \), flow face \( A \), debit \( Q \)) and so on, across the point of the liquid, do not change with time. The flow is said to be unstable (unsteady) occurs if the flow variable at each point changes with time. Most open channel problems generally require only research on flow behavior in steady-state. The expression or debit \( Q \) on a channel cross-section for any flow:

\[ Q = VA \]  

With \( V \) = average velocity and \( A \) = The cross-sectional area is perpendicular to the flow direction.

Most steady-flow problems, based on consideration, are assumed to remain along a large section of the channel, in other words, a steady flow of continuous steady flow, from equation (3):

\[ Q_1 = V_1 A_1 = Q_2 = V_2 A_2 \]  

Equation (4) can not be used if uniformly non-uniform (non-uniform) flow debris along the channel due to overflow. This type of Stream is known as spatially varied flow or unsteady flow in the side spillway, rinse water through the filter, the duct branches around the wastewater treatment tank, the main drainage channel, and the carrier channel in the irrigation system.

2.3.2. Critical and supercritical flow

The Stream to be critical if the Froude number (F) is equal to one (1), whereas the subcritical flow is sometimes called (tranquil flow) when \( F < 1 \) and supercritical or (rapid flow) when \( F > 1 \). The ratio of the flow velocity with the force of gravity (per unit volume) is known as the Froude number and can be formulated as follows Th.Rehbock et al. [9] so that F can be written as Boris.A [10]:

\[ F = \frac{V}{\sqrt{gA}} \]  

With \( L = D = \frac{A}{B_n} \), \( T \) = the width of the water on the trapezoidal section, \( B_n \) = square channel width.
2.3.3. The flow is changing gradually

The specific energy is equal to the sum of the water depth and the high velocity. On a channel basis, it is assumed to have a sloping slope or no slope. \( Z \) is the base height above the selected reference line, \( H \) is the flow depth, and the energy correction factor (\( \alpha \)) is equal to one. The specific energy of the Stream at each particular cross-section is calculated as the total energy at that cross-section by using the bottom of the channel as a reference, Budi Santoso [6]. For small slopes, \( \Theta = 0 \). Then the amount of energy at the channel cross-section is:

\[
H = z + d + \frac{\nu A^2}{2g} \tag{6}
\]

This equation applies to streams aligned or changed irregularly. For a channel whose slope is small, this is Bernoulli's energy equation Hunter Rouse and Simon Ince [11]. The Criteria of Flow in a Critical State of a stream has been defined as a condition in which the Froude number is equal to one. A more general definition is the flow state in which the specific energy for a given discharge is minimum, Paul Boss [12]. The following definition can elaborate a criterion for critical flow. The amount of specific energy can be formulated as follows [13-14] for flat channels (\( \Theta = 900 = 0 \))

\[
E = h + \frac{\nu^2}{2g} \tag{7}
\]

With \( E \) = Specific Energy

Associated \( Q = A \times V \) then the Specific Energy becomes [15-16]:

\[
E = h + \frac{Q^2}{2gA^2} \tag{8}
\]

After the narrowing is known as energy loss \( \Delta E = E_1 - E_2 \) Hunter Rouse [17].

\[
\Delta E = h_1 + \frac{\nu_1^2}{2g} - h_2 - \frac{\nu_2^2}{2g} \tag{9}
\]

Energy loss can be derived from the previous equation. The equation becomes:

\[
\Delta E = h_1 + \frac{Q^2}{2gA_1^2} - h_2 - \frac{Q^2}{2gA_2^2} \tag{10}
\]

With \( A_1 = \) Sectional area of sections 1, \( B_1 = 8 \) cm and \( A_2 = \) Area of Section 2, \( B_2 = 4 \) cm

Figure 6 Sketch losing energy before and after constriction. (a) In harsh conditions (b) Under transition conditions, (c) On the conditions of the Radius.
Table 1. Change of Flow Velocity result

| No. | Type of Constriction | Debit (Q) | H1   | Continuity Formula | Current Meter | Debit (Q) | H2   | Continuity Formula | Current Meter |
|-----|----------------------|----------|------|--------------------|---------------|----------|------|--------------------|---------------|
|     |                      | m³/s     | m    | m/s                | m/s           |          | m³/s | m    | m/s                | m/s           |
| 1   | Sudden               | 0.0025   | 0.131| 0.2385             | 0.3           |          | 0.0025| 0.079 | 0.7911             | 0.8           |
|     |                      | 0.002    | 0.117| 0.2137             | 0.2           |          | 0.002 | 0.061 | 0.8197             | 0.8           |
|     |                      | 0.0015   | 0.097| 0.1933             | 0.1           |          | 0.0015| 0.052 | 0.7212             | 0.7           |
| 2   | Transition           | 0.0025   | 0.12 | 0.2604             | 0             |          | 0.0025| 0.081 | 0.7716             | 0.8           |
|     |                      | 0.002    | 0.107| 0.2336             | 0             |          | 0.002 | 0.067 | 0.7463             | 0.7           |
|     |                      | 0.0015   | 0.091| 0.206              | 0             |          | 0.0015| 0.055 | 0.6818             | 0.7           |
| 3   | Radius               | 0.0025   | 0.119| 0.2626             | 0.3           |          | 0.0025| 0.08  | 0.7813             | 0.8           |
|     |                      | 0.002    | 0.102| 0.2451             | 0.3           |          | 0.002 | 0.065 | 0.7692             | 0.7           |
|     |                      | 0.0015   | 0.087| 0.2155             | 0.2           |          | 0.0015| 0.057 | 0.6579             | 0.7           |

From the result of Table 1, flow velocity before constriction with b₁ = 0.08 m occurs before the narrowing using the comparison of the continuity formula and the Current meter in the form of sudden narrowing and the Radius not experienced a significant change in flow rate. The transition narrowing form has a significant change where the current meter's velocity is 0.0 m/s, and the continuity formula is 0.2 m/s. In comparison, the velocity that occurs before the narrowing shows that the constriction area's velocity for the three forms of narrowing does not experience a significant change where the current meter velocity and the continuity formula are almost the same.

Table 2. The Froud number result

| Debit (Q) | m³/s | The calculation of the fraud number before the constriction | The fraud number at the point of view in the constricted region (m) |
|----------|------|-----------------------------------------------------------|-----------------------------------------------------------------|
|          |      | Before sudden constriction, L = 1.27 m channel width 0.08 m |                                                                 |
| 0.0025   | 0.0105| 0.2104                                                   | 0.4357, 0.8987, 0.6310, 0.5782, 0.5865, 0.8987, 1.1010          |
| 0.002    | 0.0949| 0.1994                                                   | 0.4147, 0.7619, 0.6799, 0.6442, 0.7189, 1.0596, 1.1731          |
| 0.0015   | 0.078 | 0.1982                                                   | 0.4361, 0.9813, 0.9541, 0.6465, 0.8146, 1.0097, 1.1385          |
|          |      | Before transition constriction, L = 1.27 m channel width 0.08 m |                                                              |
| 0.0025   | 0.096 | 0.24                                                     | 0.5117, 0.9524, 0.8345, 0.6217, 0.6605, 0.8656, 1.0775          |
| 0.0020   | 0.086 | 0.228                                                    | 0.4760, 1.0095, 0.8620, 0.6330, 0.6557, 0.9205, 1.0862          |
| 0.0015   | 0.073 | 0.2181                                                   | 0.4666, 1.0395, 0.9813, 0.6606, 0.6329, 0.9282, 1.0709          |
|          |      | Before radius constriction, L = 1.27 m channel width 0.08 m |                                                              |
| 0.0025   | 0.095 | 0.243                                                    | 0.5470, 1.1769, 0.8498, 0.6406, 0.6815, 0.8819, 1.2619          |
| 0.0020   | 0.082 | 0.245                                                    | 0.5539, 1.3861, 0.8808, 0.7328, 0.6799, 0.9633, 1.2722          |
| 0.0015   | 0.007 | 0.2333                                                   | 0.5392, 1.4966, 1.0395, 0.8146, 0.5948, 1.0709, 1.1750          |

From the results of Table 2, it appears that the flow that occurs before the narrowing is a subcritical flow and flows that occurs in the area of sudden narrowing for a point distance review of 0.0 m, 0.11 m, 0.21 m, 0.23 m, and 0.46 m experiences subcritical flow, for a point distance review of 2.95 m experiences flow critical, and for an overview the point distance of 3.58 m experiences supercritical flow. In the transition narrowing area for the review, point distances of 0.0 m, 0.21 m, 0.23 m, and 0.46 m experience subcritical flow, a point-distance view of 0.11 m and 3.58 m experience critical flow. In the confinement area, Radius for point distance review of 0.0
m, 0.21 m, 0.23 m, 0.46, and 2.95 m experience subcritical flow. For point distance review, 0.11 m and 3.58 m experienced supercritical flow.

**Table 3.** The specific energy result

| Debit (Q/s) | Specific energy at the point before constriction | Specific energy at the point of view in the area of constriction |
|-------------|-----------------------------------------------|---------------------------------------------------------------|
| m³/s        | H₁     | A₁ | E       | 0           | 0.11       | 0.21       | 0.23       | 0.46       | 2.95       | 3.58       |
|             | Before sudden constriction, L = 27 m channel width 0.08 m | Sudden constriction 3.58 m = channel width 0.04 m |
| 0.0025      | 0.131  | 0.0105 | 0.1339 | 0.1338 | 0.1109 | 0.1199 | 0.1237 | 0.1231 | 0.1109 | 0.1108 |
| 0.002       | 0.117  | 0.0086 | 0.1193 | 0.1192 | 0.0981 | 0.101  | 0.1026 | 0.0994 | 0.0952 | 0.0962 |
| 0.0015      | 0.097  | 0.0078 | 0.0989 | 0.0988 | 0.0785 | 0.0786 | 0.0846 | 0.0799 | 0.0785 | 0.0791 |
|             | Before transition constriction, L = 1.27 m channel width 0.08 m | Transition constriction 3.58 m = channel width 0.04 m |
| 0.0025      | 0.12   | 0.0096 | 0.1235 | 0.1233 | 0.1105 | 0.1119 | 0.1205 | 0.1182 | 0.1113 | 0.1106 |
| 0.002       | 0.107  | 0.0086 | 0.1098 | 0.1097 | 0.0951 | 0.096  | 0.1032 | 0.1021 | 0.0954 | 0.0954 |
| 0.0015      | 0.091  | 0.0073 | 0.0932 | 0.0931 | 0.0786 | 0.0785 | 0.0841 | 0.0852 | 0.0787 | 0.0787 |
|             | Before radius constriction, L = 1.27 m channel width 0.08 m | Transition constriction 3.58 m = channel width 0.04 m |
| 0.0025      | 0.119  | 0.0095 | 0.1225 | 0.1224 | 0.1117 | 0.1116 | 0.1193 | 0.1171 | 0.1111 | 0.1132 |
| 0.002       | 0.102  | 0.0082 | 0.1051 | 0.1050 | 0.1   | 0.0958 | 0.0989 | 0.101  | 0.0952 | 0.0977 |
| 0.0015      | 0.087  | 0.007  | 0.0894 | 0.0893 | 0.0848 | 0.0786 | 0.0799 | 0.0871 | 0.0787 | 0.0794 |

From the results of Table 3 before the narrowing, it appears that the maximum specific energy in the form of sudden constriction (0.1339 m), moving (0.1235 m), and Radius (0.1225 m) is due to crushing in the narrowing.

*Figure 7* Specific Energy and water level with a sudden constriction, transition, and radius type with a discharge of 0.0025 m³/s, 0.0020 m³/s, and 0.0015 m³/s. It appears that the maximum specific energy in the form of sudden constriction.
Table 4. Critical depth (Hc) and critical width (Bc) for each discharge variation in the area of constriction result

| Debit (Q) | Hc   | Bc   | Ac   | Vc   | Frc  | Ec   | ∆Ec  |
|-----------|------|------|------|------|------|------|------|
| m³/s      |      |      |      |      |      |      |      |
|           | Sudden constriction 1.27 m = channel width 0.04 m | | | | | | |
| 0.0025    | 0.0893 | 0.0300 | 0.0027 | 0.9343 | 1.1534 | 0.1338 | 0.000140 |
| 0.0020    | 0.0796 | 0.0285 | 0.0023 | 0.8820 | 1.1828 | 0.1192 | 0.000125 |
| 0.0015    | 0.0659 | 0.0283 | 0.0019 | 0.8030 | 1.1864 | 0.0988 | 0.000103 |
|           | Transition constriction 1.27 m = channel width 0.04 m | | | | | | |
| 0.0025    | 0.0823 | 0.0339 | 0.0028 | 0.8971 | 1.0852 | 0.1233 | 0.000129 |
| 0.0020    | 0.0732 | 0.0323 | 0.0024 | 0.8460 | 1.1111 | 0.1097 | 0.000115 |
| 0.0015    | 0.0621 | 0.0310 | 0.0019 | 0.7793 | 1.1344 | 0.0931 | 0.000097 |
|           | Radius constriction 1.27m = channel width 0.04 m | | | | | | |
| 0.0025    | 0.0817 | 0.0342 | 0.0028 | 0.8937 | 1.0790 | 0.1224 | 0.000128 |
| 0.0020    | 0.0700 | 0.0345 | 0.0024 | 0.8276 | 1.0750 | 0.1050 | 0.000110 |
| 0.0015    | 0.0596 | 0.0330 | 0.0020 | 0.7633 | 1.0995 | 0.0893 | 0.000093 |

From the results of table 4 that the Critical depth (Hc) and the Critical Width (Bc) that occurs in the narrowing area for the three forms of narrowing does not experience a significant change, and it appears that the maximum Critical depth (Hc) and the Critical Width (Bc) in the form of sudden constriction.

3. CONCLUSION

Flow classification before narrowing of all three forms of narrowing occurs subcritical flow. While the classification of flow after narrowing for a sudden narrowing type at a distance of 2.95 m occurred a critical stream and at 3.58 m distance occurred supercritical flow, for the type of transition narrowing at 0.11 m distance point occurred critical flow, and at a distance the 3.58 m points of view. The water flow will experience changes in flow velocity due to channel width changes from B1 and B2. In the three forms of narrowing understudy, the maximum specific energy occurs formed sudden narrowing.

4. SUGGESTION

For further research, using a larger channel change model so that measurement and flow behavior are more comfortable to observe and adding several variables or objects that are not included in this study, for example, sedimentation measurements.

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