Parameter sensitivity analysis of sewage irrigation infiltration process in sandy clay with HYDRUS-1D simulation

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Abstract. The HYDRUS-1D model is used to analyze the sensitivity of various soil hydraulic parameters under constant head infiltration conditions. The results show that: under constant head infiltration conditions, the perturbation of each parameter is sensitive to the impact of soil water potential, the order is: \( n > \theta_s > K_s > \alpha > \theta_r \), where \( n \) is a parameter which has a greater impact on the simulation results of soil water potential. The sensitivity order of the influence of each parameter on the cumulative infiltration is: \( K_s > n > \theta_s > \theta_r > \alpha \), where \( K_s \) and \( n \) are the parameters that have greater impacts on the cumulative infiltration. Under the constant water head infiltration condition of sewage irrigation, simulation analysis should calibrate the accuracy of \( n, K_s \) and \( \theta_s \) parameters.

1. Introduction

The migration and diffusion process of pollutants is affected by many factors such as evaporation, rainfall, soil media, irrigation, and human activities [1]. Nitrate pollution in groundwater caused by sewage irrigation has become a research focus. The law of water migration requires simulation and analysis of the influence of soil composition and structure in the process of sewage irrigation, which is advantage to further clarify the distribution characteristics and influencing factors of nitrate in shallow groundwater, it is very important for groundwater environmental protection.

In the simulation analysis of soil water transport process using HYDRUS-1D model, the setting of soil hydraulic characteristic parameters \( K_s, \theta_s, \theta_r, \alpha \) and \( n \) have a direct impact on the simulation results, especially under sewage irrigation conditions. Within the depth range from 0 to 60 cm beneath the surface, soil water migration has a great influence on the soil water potential and the cumulative infiltration amount. The sensitivity analysis of parameters is of great significance for determining the sensitivity parameters.

2. Simulation model construction and initial simulation conditions

2.1 Uniform Water Flow

One-dimensional uniform (equilibrium) water movement in a partially saturated rigid porous medium is described by a modified form of the Richards equation, using the assumptions that the air phase plays an insignificant role in the liquid flow process, and that water flow due to thermal gradients can be neglected, the uniform water flow equation is described as:

\[
\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left[ K(h) \left( \frac{\partial h}{\partial z} + \cos \alpha \right) \right] - S
\]  

(1)
where $h$ is the water pressure head [L], $\theta$ is the volumetric water content \([L^3L^{-3}]\), $t$ is time [T], $x$ is the spatial coordinate [L] (positive upward), $S$ is the sink term \([L^3L^{-3}T^{-1}]\), $\alpha$ is the angle between the flow direction and the vertical axis ($\alpha = 0^\circ$ for vertical flow, $90^\circ$ for horizontal flow, and $0^\circ < \alpha < 90^\circ$ for inclined flow), and $K$ is the unsaturated hydraulic conductivity function \([LT^{-1}]\) given by

$$K(h, z) = K_r(z)K_s(h, z)$$

Where $K_r$ is the relative hydraulic conductivity [-], $K_s$ is the saturated hydraulic conductivity \([LT^{-1}]\).

However, 2.2Soil hydraulic characteristic parameter equation

HYDRUS also implements the soil-hydraulic functions of van Genuchten who used the statistical pore-size distribution model to obtain a predictive equation \(3\) and \(4\)[2,3], for the unsaturated hydraulic conductivity function in terms of soil water retention parameters. The expressions of van Genuchten [2] are given by

$$\theta(h) = \begin{cases} \theta_r + \frac{\theta_s - \theta_r}{1 + h^m} & h < 0 \\ \theta_s & h \geq 0 \end{cases}$$

(3)

$$K(h) = K_s \left[1 - \left(1 - S_e^{1.0}\right)^\alpha\right]^\beta$$

(4)

$\theta_s$ - saturated water content [-]; $\theta_r$ - residual water content [-]; $a, m, n$ - empirical parameters \([1/L], [-], [-]\); $S_e$ - effective water content [-]; $K_s$ - saturated hydraulic conductivity \([L/T]\).

2.3Model parameters and boundary conditions

The soil type is set to sandy clay, and the perturbation of soil hydraulic parameters $K_s$, $\theta_s$, $\theta_r$, $n$, $\alpha$ changes with 5% variation, and the maximum perturbation variation is plus or minus 20% (see Table 1). The height of the simulated soil column is 60 cm, the simulation time is 5000 minutes, the upper boundary is of constant pressure head, and the lower boundary is of free drainage.

| Perturbation (%) | $\theta_r$ | $\theta_s$ | $\alpha$ (1/cm) | $n$ | $K_s$ (cm/min) |
|------------------|-----------|-----------|-----------------|-----|--------------|
| -20              | 0.08      | 0.304     | 0.022           | --  | 0.0016       |
| -15              | 0.085     | 0.323     | 0.023           | 1.046| 0.0017       |
| -10              | 0.09      | 0.342     | 0.024           | 1.107| 0.0018       |
| -5               | 0.095     | 0.361     | 0.026           | 1.169| 0.0019       |
| 0                | 0.1       | 0.38      | 0.027           | 1.23 | 0.0020       |
| 5                | 0.105     | 0.399     | 0.028           | 1.292| 0.0021       |
| 10               | 0.11      | 0.418     | 0.03            | 1.353| 0.0022       |
| 15               | 0.115     | 0.437     | 0.031           | 1.415| 0.0023       |
| 20               | 0.12      | 0.456     | 0.032           | 1.476| 0.0024       |

2.4Output variables

Taking the soil water potential and cumulative infiltration in the output variables of the HYDRUS-1D model as the research objects, the simulation results of 100min, 200min, and 400min are adopted for data analysis.

3. Influence of parameter perturbation on cumulative infiltration

The $K_s$ perturbation is positively correlated to the cumulative infiltration, and the variation of positive perturbation and negative perturbation are basically close. For sandy clay media, $K_s$ perturbation has
the greatest impact on cumulative infiltration (see Table 2), the impact on the cumulative infiltration with plus or minus 20% perturbation exceeds 19% variation.

Table 2. Sum of cumulative infiltration (cm) and variation (%) with $K_s$

| Perturbation (%) | 20  | 15  | 10  | 5   | 0   | -5  | -10 | -15 | -20 |
|------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Sum (cm)         | 12.743 | 12.23 | 11.717 | 11.204 | 10.701 | 10.178 | 9.6648 | 9.152 | 8.639 |
| Variation (%)    | 19.082 | 14.288 | 9.494 | 4.7  | 0   | -4.887 | -9.683 | -14.475 | -19.269 |

There is a positive correlation between $n$ perturbation and cumulative infiltration, which is very similar to the characteristics of $K_s$ perturbation, and the variation of positive perturbation and negative perturbation are basically close (see Table 3). For sandy clay media, the influence of $n$ perturbation on cumulative infiltration is much smaller than $K_s$, but it is larger than other parameters.

Table 3. Sum of cumulative infiltration (cm) and variation(%) with $n$

| Perturbation (%) | 20  | 15  | 10  | 5   | 0   | -5  | -10 | -15 | -20 |
|------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Sum (cm)         | 11.189 | 11.064 | 10.939 | 10.82 | 10.701 | 10.562 | 10.446 | 10.32   | --  |
| Variation (%)    | 4.56  | 3.392 | 2.224 | 1.112 | 0   | -1.299 | -2.383 | -3.56   | --  |

The influence of the parameters $\theta_s$, $\theta_r$, and $\alpha$ on the cumulative infiltration is very small. The influence of $\theta_s$ is only 1% under 20% perturbation (see Table 4), while the influence of $\theta_r$ and $\alpha$ is less than 0.4% (see Table 5,6).

Therefore, the important influencing parameters of soil water infiltration under sewage irrigation conditions are $K_s$ and $n$, and the accuracy of $K_s$ parameters must be verified in particular.

Table 4. Sum of cumulative infiltration (cm) and variation(%) with $\theta_s$

| Perturbation (%) | 20  | 15  | 10  | 5   | 0   | -5  | -10 | -15 | -20 |
|------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Sum (cm)         | 10.808 | 10.779 | 10.749 | 10.72 | 10.701 | 10.662 | 10.632 | 10.603 | 10.573 |
| Variation (%)    | 1    | 0.729 | 0.449 | 0.178 | 0   | -0.364 | -0.645 | -0.916 | -1.196 |

Table 5. Sum of cumulative infiltration (cm) and variation (%) with $\theta_r$

| Perturbation (%) | 20  | 15  | 10  | 5   | 0   | -5  | -10 | -15 | -20 |
|------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Sum (cm)         | 10.66 | 10.668 | 10.676 | 10.684 | 10.701 | 10.699 | 10.707 | 10.714 | 10.722 |
| Variation (%)    | -0.383 | -0.308 | -0.234 | -0.159 | 0   | -0.019 | 0.056  | 0.121  | 0.196  |

Table 6. Sum of cumulative infiltration (cm) and variation (%) with $\alpha$

| Perturbation (%) | 20  | 15  | 10  | 5   | 0   | -5  | -10 | -15 | -20 |
|------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Sum (cm)         | 10.689 | 10.689 | 10.701 | 10.701 | 10.701 | 10.697 | 10.693 | 10.693 | 10.685 |
| Variation (%)    | -0.112 | -0.112 | 0    | 0    | 0    | -0.037 | -0.075 | -0.075 | -0.15  |

4. Sensitivity analysis of parameter perturbation

4.1 Sensitivity analysis of $n$

The influence of the negative perturbation of $n$ on the soil water potential is mainly within the depth range of 30cm. At 100min, the soil water potential at 10cm has the largest variation, with a variation of 80%; with the change of time, the maximum impact on soil water potential is increased with depth. It is located near 15cm at 200min, and the variation is 120%, and at 400min, near 20cm, the variation exceeds 200% (see Figure 1,2). The positive perturbation of $n$ is obviously different from the negative perturbation. The variation of the positive perturbation is smaller than that of the negative perturbation, but the impact of depth range is relatively enlarged. At 100min, the soil water potential at 30cm has a certain variation, and it is near 55cm at 200min. It affects the 60cm depth of the simulated soil column in 400min (see Figure 2).
4.2 Sensitivity analysis of $\theta_s$

As the $\theta_s$ perturbation increases, the maximum impact variation of soil water potential continues to increase, but the impact depth changes slowly. The maximum variation of 100 min is near 10 cm depth, the maximum variation exceeds 40%, and the maximum variation of 200 min is located near 15 cm, the maximum variation exceeds 60%, 400 min below 20 cm, the maximum variation exceeds 100% (see Figure 3, 4). The influence of $\theta_s$, positive and negative perturbations on soil water potential is different: $\theta_s$ positive perturbation has larger influence on soil water potential than negative (see Figure 4).

![Diagram of soil water potential with perturbation of $\theta_s$.](image)

**Figure 1.** Perturbation influence of $n$ on the soil water potential.

**Figure 2.** Variation of soil water potential with the perturbation of $n$.

**Figure 3.** Perturbation influence of $\theta_s$ on the soil water potential.

**Figure 4.** Variation of soil water potential with the perturbation of $\theta_s$. 
4.3 Sensitivity analysis of $K_s$

The influence of $K_s$ positive perturbation on soil water potential is different from that of negative perturbation. The depth of influence of negative perturbation on soil water potential is less than that of positive perturbation, but the maximum variation of negative perturbation is greater than that of positive perturbation.

Sensitivity $K_s$ negative perturbation on soil water potential is similar to $\theta_s$. With the increase of $K_s$ negative perturbation, the maximum influence variation of soil water potential increases continuously, but the influence variation is less than $\theta_s$. The depth of maximum variation of 100min, 200min, 400min is very close to $\theta_s$. The influence of $K_s$ positive perturbation on soil water potential is also similar to $\theta_s$, but the variation and emerged depth of the influence are slightly smaller than $\theta_s$ in the same time.

![Figure 5. Perturbation influence of $K_s$ on the soil water potential.](image)

![Figure 6. Variation of soil water potential with the perturbation of $K_s$.](image)

The simulation results show that the influence of $\alpha$ and $\theta_i$ perturbation on soil water potential is relatively small, the maximum influence of $\alpha$ is close to 50%, and the maximum influence of $\theta_i$ is close to 20%.

5. Conclusion

Under the condition where the variation range of soil hydraulic characteristic parameters is plus or minus 20%, in the process of sewage irrigation, the soil water potential and cumulative infiltration are analyzed in the unsaturated zone, aims to determine the sensitivity of each parameter in the simulation process.

With the constant pressure head infiltration, the sensitivity order of the parameter perturbations to the influence is: $n > \theta_i > K_s > \alpha$, where $n$ is a parameter that has a greater impact on the simulation results of soil water potential. The parameter $\theta_i$ and $K_s$ have similar influence on soil water potential, and $\alpha$ has the least influence on the soil water potential of the entire section.

The sensitivity order of the influence of parameters on the cumulative infiltration is: $K_s > n > \alpha > \theta_i$, where $K_s$ and $n$ have greater impacts on the simulation results of the cumulative infiltration. $\theta_i$ has a certain influence on the cumulative infiltration, while $\theta_i$ and $\alpha$ have the least influence on the cumulative infiltration of the entire section.
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