Chasing brane inflation in string theory

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Abstract. We investigate the embedding of brane–antibrane inflation into a concrete type IIB string theory compactification with all moduli fixed. Specifically, we are considering a D3-brane, whose position represents the inflaton $\phi$, in a warped conifold throat in the presence of supersymmetrically embedded D7-branes and an anti-D3-brane localized at the tip of the warped conifold cone. After presenting the moduli stabilization analysis for a general D7-brane embedding, we concentrate on two explicit models, the Ouyang and the Kuperstein embeddings. We analyze whether the forces induced by moduli stabilization and acting on the D3-brane might be canceled by fine-tuning so as to leave us with the original Coulomb attraction of the anti-D3-brane as the driving force for inflation. For a large class of D7-brane embeddings we obtain a negative result. Cancelations are possible only for very small intervals of $\phi$ around an inflection point and not globally. For the most part of its motion the inflaton then feels a steep, non-slow-roll potential. We study the inflationary dynamics induced by this potential.

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1. Introduction

One of the key steps to obtaining a viable inflation scenario in string theory is fixing all massless moduli, except for the inflaton. In [1], in the framework of type IIB string compactifications, it was shown how fluxes can fix all complex structure moduli and the dilaton. The Kähler moduli enjoy a no-scale structure at tree level and therefore remain...
massless at this order. Quantum corrections, both perturbative and non-perturbative, however, break the no-scale structure.

In [2] a three-step procedure was proposed to fix, in addition, the Kähler moduli in positive energy vacua. First, one stabilizes the complex structure moduli and the dilaton by imposing the supersymmetry condition, \( D_a W = 0 \). Second, one considers non-perturbative effects, such as Euclidean D3-branes or gaugino condensation on a stack of D7-branes wrapping a divisor \( \Sigma \) inside the Calabi–Yau threefold, which induce a Kähler moduli dependent term \( W_{np} \) in the superpotential. This breaks the no-scale structure and produces supersymmetric anti-de Sitter (AdS) minima, obeying \( D_i W = 0 \), where \( i \) runs over the Kähler moduli. The final step is to uplift these AdS minima to non-supersymmetric de Sitter (dS) vacua in order to connect them to the real world.

Given that in principle all closed string moduli can thus be fixed in type IIB string compactification models, it is interesting to go one step further and introduce a suitable open string sector with the aim of modeling cosmic inflation using an open string modulus. One possibility is to identify the inflaton with the distance between a spacetime-filling mobile D3-brane and a fixed, very massive anti-D3-brane (for recent reviews on this type of brane–antibrane inflation see [3]). The Coulomb attraction between the D3-brane and the anti-D3-brane provides a potential which could drive inflation, provided the branes are located in a region with strong warping [4]. Finally, one has to ensure that no other forces, in particular those which stabilize the Kähler moduli, spoil the achieved flatness of the potential. Unfortunately, this is generically the case [4]. A non-trivial interplay between the volume and the D3-brane position moduli causes the Kähler moduli stabilization process to endow the inflaton with a mass of order the Hubble parameter, \( H \). As a result, the second slow-roll parameter grows to \( \eta \gtrsim 2/3 \), showing the breakdown of slow-roll inflation.

The idea of exactly canceling this moduli stabilization effect by making some inflaton dependent threshold corrections to \( W_{np} \), via fine-tuning, has received a certain amount of attention\(^1\). Recently, threshold corrections to \( W_{np} \) became available for the warped conifold background [6] (previously such effects had been calculated in [7] in the absence of warping). The result is that \( W_{np} \) is proportional to the supersymmetric embedding \( f(w) \) of the D7-branes to the power \( 1/n \). While \( w \) collectively denotes the three complex coordinates of the D3-brane in the Calabi–Yau compactification space, \( n \) represents the number of coinciding D7-branes in the stack on which gaugino condensation takes place (\( n = 1 \) would apply to the Euclidean D3-brane case, which might alternatively be used for Kähler moduli stabilization). The zeros of \( f(w) \) describe the embedding of the divisor \( \Sigma \) which the D7-branes wrap.

It is one goal of this paper to analyze for concrete type IIB models whether all forces acting on the mobile D3-brane can add up to zero, except for the Coulomb attraction of the anti D3-brane which would then drive inflation. In the type IIB framework outlined above, we calculate the \( F \)-term potential for a general D7-brane embedding \( f(w) \) and give general formulas for its moduli stabilization in the warped conifold background. Having stabilized all closed string moduli, we are left with an effective potential \( V(r) \) for the radial position \( r \) of the D3-brane in the warped conifold; this becomes the inflaton potential, \( V(\phi) \), with the canonically normalized radial position of the D3-brane representing the

\(^1\) It has been proposed in [5] that also certain types of upliftings could be used to obtain this cancelation.
inflaton. We then investigate inflation in the warped throat region by performing a small \( \phi \) expansion of the potential.

The moduli stabilization effect that generically causes the breakdown of slow-roll inflation, and that we would like to cancel via some additional mobile D3-brane dependence of \( W_{\text{np}} \), induces a term proportional to \( \phi^2 \) in the potential. Unfortunately, no embedding allows for the creation of a compensating further term in the inflation potential with a \( \phi^2 \) dependence\(^2\). In fact, the holomorphicity of the D7-brane embedding allows only integer powers of \( \phi^{3/2} \) (some multiplied by a \( \phi \) coming from the inverse conifold metric). This is crucial because terms with a different \( \phi \) dependence can cancel only locally in a small \( \phi \) interval, rather than globally. Outside this small interval the inflaton potential is not of the slow-roll type. Equivalently, outside this interval and despite fine-tuning, the motion of the D3-brane is governed by moduli stabilization effects compared to which the Coulomb potential represents only a subleading correction.

Two relevant embeddings that give sizable contributions in the theoretically controllable small \( \phi \) regime are the Ouyang [8] and the simplest Kuperstein embedding [9]. They induce terms proportional to \( \phi \) and \( \phi^{3/2} \) in the potential. Most other embeddings, in contrast, give rise to contributions proportional to \( \phi^p \), where \( p > 2 \), which renders them subleading in the small \( \phi \) region. They can thus not help to flatten the inflaton potential. For the Ouyang embedding the corrections to the potential, induced by the threshold corrections to the non-perturbative superpotential, vanish after angular moduli stabilization [10]. For the Kuperstein embedding, on the other hand, they remain non-trivial, as we will show. The resulting inflaton potential, \( V(\phi) \), in this latter case is portrayed in figure 1. In general, it possesses a maximum and a minimum plus an inflection point in between, as shown in the right figure. With suitable fine-tuning, displayed in the left figure, it can be arranged that the maximum and minimum coincide with the inflection point, the potential hill at small \( \phi \) disappears, and the potential becomes flat enough for inflation.

We then study the cosmological evolution implied by the potential \( V(\phi) \). It exhibits a slow-roll inflation phase (in particular, \( \eta \approx 0 \)) only in a small region around the inflection point, where \( \eta = 0 \), and for which \( \eta \) changes sign (see figure 3). Here the potential switches from concave to convex. The reason that only a small portion of the potential can be flattened is that various terms in \( V(\phi) \) have different \( \phi \) dependences (see equation (58)). In a fine-tuned case (see the left part of figure 1), the inflection point can be made flat and a prolonged stage of slow-roll inflation is induced.

If one wants to end inflation with D3–anti-D3 annihilation, the D3-brane has to go all the way down to the tip of the throat towards \( \phi \to 0 \), therefore generically running uphill for a certain interval (only in the fine-tuned case where the maximum and minimum coincide with the inflection point does the potential hill disappear and turn into a flat region). We investigate whether overshooting the potential hill is possible or whether the inflaton gets stuck in the minimum due to Hubble friction. We find that overshooting is possible if the minimum exhibits a fairly small positive cosmological constant.

\(^2\) Here we are assuming that inflation takes place far away from the tip of the conifold such that we can neglect the deformation parameter and use the singular conifold metric. It has been noticed in [5] that using the exact deformed conifold metric, very close to the tip the moduli stabilization induces a term proportional to \( \phi^3 \) instead of \( \phi^2 \). This could in principle be canceled by the threshold corrections to the non-perturbative superpotential that we are considering here. However, the cancelation would be valid only very close to the tip.
Figure 1. The plots display the inflaton potential $V(\phi)$ for the Kuperstein embedding for two different values of the uplifting parameter $\beta = 1.21$ (left) and $\beta = 1.4$ (right). The left plots shows that fine-tuning allows us to get rid of the potential hill, leaving only an inflection point suitable for inflation. The right plot shows the non-fine-tuned generic situation: the potential has two separate critical points and an inflection point in between, thus creating a potential barrier. To move down the throat (towards smaller $\phi$) the inflaton has to cross the barrier and run uphill over a certain interval.

The structure of the paper is as follows. In section 2 we describe the structure of the effective superpotential and its relation to the D7-brane embedding. Section 3 reviews the D-brane inflation $\eta$-problem, which forms a main issue in this paper, explains the type IIB setup and provides the effective potential for the moduli in the warped conifold background. In section 4 we perform a minimization of the potential in the K"ahler modulus and angular directions for a general D7-brane embedding. In section 5 we apply these general results to the Ouyang embedding and compare it with the results for the Kuperstein embedding. Section 6 investigates inflation in the Kuperstein embedding case and analyzes the cosmological evolution around the minimum of the potential; this is followed by an analysis of the uphill evolution. In section 7 we discuss various forces acting on the D3-brane and the anti-D3-brane and comment on their relative importance. We conclude in section 8. A couple of appendices provide further technical details. Appendix A collects some details about the warped conifold background. Appendix B lists and discusses the related parameters. Appendix C analyses the dependence of the stabilized volume modulus $\sigma_c$ on the uplifting potential and the inflaton. Appendix D shows that the coefficient of the $\phi^{3/2}$ term in the inflaton potential, for the Kuperstein embedding, is non-positive. This feature determines the general structure of the potential which is derived in appendix E.

2. Superpotential

The Gukov–Vafa–Witten flux superpotential (GVW) $W_0$ [13, 14] can fix the dilaton and complex structure moduli in type IIB flux compactifications. The K"ahler moduli, on the
other hand, are stabilized by a non-perturbative superpotential \( W_{np} \) \cite{15, 2}. The latter breaks the no-scale structure because of its explicit Kähler moduli dependence.

\( W_{np} \) can be generated either by Euclidean brane instantons, D3-instantons in our case, or gaugino condensation on a stack of D7-branes. In type IIB compactifications with a single Kähler modulus either of the two effects is sufficient (in general, both effects arise together and lead to economical ways of stabilizing further Kähler moduli, as shown recently for heterotic M-theory compactifications \cite{16}). In both cases, the branes wrap a divisor \( \Sigma \) of the Calabi–Yau compactifications. The embedding is then specified by a section of a divisor bundle, \( f = [\Sigma] \). The D3-brane backreacts on the metric background and hence alters the instanton (gaugino condensation) action. This effect sources an additional interaction for the D3-brane position moduli and can be described by an \( f(w) \) dependence of \( W_{np} \), where \( w \) indicates the position of the D3-brane in the Calabi–Yau manifold (for the conifold \( w \) will be a set of three out of four projective coordinates; see appendix A). In \cite{6} (see also \cite{17, 7, 18, 19}) it was obtained that

\[
W_{np} = A(w)e^{-\alpha \rho} \equiv A_0 f(w)^{1/n}e^{-\alpha \rho},
\]

where \( A_0 \) depends on the already stabilized complex structure moduli. In the sequel, following KKLT \cite{2}, we assume that the complex structure moduli have been stabilized by fluxes at a scale hierarchically higher than the scale of inflation (although this might not be the generic case, we are focusing our investigation on a corner of the landscape where this assumption holds). As in this paper we are interested in the dynamics of inflation, we will in the rest of the paper treat \( A_0 \) as a constant. In addition, to simplify the analysis, we assume a compactification with a single Kähler modulus, which we denote as \( \rho = \sigma + ib \). Furthermore, \( a = 2\pi/n \), with \( n \) being the number of D7-branes in the stack producing gaugino condensation (\( n = 1 \) for the Euclidean D3-brane case).

With an adequate shift of the axion \( b = \text{Im} \rho \), \( A_0 \) can be taken to be real. The total superpotential therefore is

\[
W = W_0 + A(w)e^{-\alpha \rho}.
\]

A class of supersymmetric embeddings has been found in \cite{20}. It is given by

\[
f(w) \equiv 1 - \frac{\prod_{i=1}^{4} w_{\mu_i}}{\mu} = 0,
\]

where \( p_i \in \mathbb{Z} \), \( P \equiv \sum_{i=1}^{4} p_i \), and \( \mu \in \mathbb{C} \) are (constant) parameters defining the embedding of the D7-branes. The simplest choice of parameters \( p_i = \delta_{i,1} \) reproduces the Ouyang embedding \cite{8}. The \( p_i \) have to be integers by holomorphicity\(^3\).

Another very simple embedding is the Kuperstein embedding \cite{9}

\[
f(z) \equiv 1 - \frac{z_1}{\mu},
\]

which is expressed in terms of alternative coordinates on the conifold (see appendix A). The \( \{z_i\} \) are linear combinations of the \( \{w_i\} \), so again only integer powers of \( \{w_i\} \) (equivalently \( \{z_i\} \)) are allowed by holomorphicity. This will play a crucial role in the following.

\(^3\) In the Ouyang case, the integer \( p_1 = P \) can be interpreted as the number of times a D7-brane is wrapped around the 4-cycle.
3. Warped D-brane inflation

In this section, we review the $\eta$-problem arising for D3-brane inflation in a warped throat driven by brane–antibrane attraction, first pointed out in [4]. Then we add threshold corrections to the analysis and obtain the $F$-term potential that we will study in the next section.

3.1. The $\eta$-problem from volume stabilization

It was pointed out in [4] that the strongest force felt by the D3-brane comes from the mixing of open string moduli with the overall volume, once the latter is stabilized `al la KKLT. To see how this comes about, let us briefly review the KKLT setup [2]. Upon reducing the ten-dimensional type IIB superstring theory over the warped metric background

$$ds^{10} = h^{-1/2} ds^2_4 + h^{1/2} ds^2_6,$$

(5)

where $h$ is the warp factor in the presence of imaginary self-dual fluxes and orientifold planes, we obtain a four-dimensional, $N=1$ supergravity theory. First let us assume that no D3-branes are present; we will add them in a second step. Then the prefactor $A$ in equation (2) is a constant because, as we mentioned, we assume that the complex structure moduli and the dilaton have already been stabilized. The resulting $F$-term potential for the Kähler moduli plus the anti-D3-brane uplifting term are [2]

$$V_{dS} = V_{AdS} + V_{up} = \frac{a A_0 e^{-\sigma_0}}{2\sigma^2} \left( \frac{1}{3} \sigma a A_0 e^{-\sigma} + A_0 e^{-\sigma} + W_0 \right) + \frac{D}{(2\sigma_c)^2}.$$  

(6)

The values of the GVW superpotential $W_0$ and $D$ (proportional to $h_0^{-1}$, the warp factor at the tip of the throat) depend on the fluxes which stabilize the complex structure moduli and the dilaton. Given the large freedom in the choice of fluxes, we will treat these quantities effectively as tunable constants. It is useful to re-express $\{W_0, D\}$ in terms of two other quantities $\{\sigma_0, \beta\}$ as

$$W_0 = -A_0 e^{-\sigma_0} \left( 1 + \frac{2}{3} \sigma_0 a \right),$$

$$D = \frac{2}{3} \beta \sigma_0 a^2 |A_0|^2 e^{-2\sigma_0}.$$  

(7)

(8)

The parameters $\{\sigma_0, \beta\}$ have the following meaning: $\sigma_0$ is the KKLT minimum, i.e. the value of $\sigma$ in the AdS minimum obtained for $D=0$. Adding the uplifting ($D \neq 0$), the minimum of equation (6) is shifted to $\sigma = \sigma_c$ which is very close to $\sigma_0$ (see appendix C); in fact $\sigma_c - \sigma_0 \equiv \Delta \ll \sigma_0$. Hence, $\sigma_0$ is an estimate of the position of the actual minimum and if it is chosen to be large enough we can neglect $\alpha'$ corrections. As regards $\beta$, it parameterizes the uplifting in such a way that a Minkowski vacuum corresponds to $\beta \simeq 1 + 2\Delta/\sigma_0$, i.e. a value slightly larger than 1, while for $\beta \gtrsim 1 + 2\Delta/\sigma_0$ we have a dS vacuum (see appendix C). At the minimum $\sigma = \sigma_c$, the potential in equation (6) takes the value

$$V_{dS}|_{\sigma_c} = -\frac{a^2 |A_0|^2 e^{-2\sigma_c}}{6\sigma_c} + \frac{D}{(2\sigma_c)^2},$$

(9)

where we have neglected terms suppressed by $\Delta/\sigma_0$. 

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Now we want to add to the picture a D3-brane located at a point \( w \) in the Calabi–Yau manifold. This induces several modifications to the potential in equation (6). The volume of 4-cycles is shifted by a \( w \) dependent quantity, i.e. it acquires a dependence on the position of the D3-brane. In [21] it was proposed that, in the simple case of a single Kähler modulus (determining the overall volume), the Kähler potential is

\[
K = -2 \log (\mathcal{V}) = -3 \log [\rho + \mathcal{P} - \gamma k(w, \overline{w})] \equiv -3 \log R,
\]

where \( \gamma \equiv \kappa^2_2 \sigma_c T_{D3}/3 \) is a constant, \( T_{D3} \) being the D3-brane tension, and \( k(w, \overline{w}) \) is the Kähler potential of the Calabi–Yau manifold evaluated at the position of the D3-brane. The metric of a compact Calabi–Yau threefold is not known, but as long as we are interested in the dynamics inside a warped throat, we can approximate it by the conifold metric. We have in mind a warped deformed conifold (eventually cut and glued to a compact Calabi–Yau manifold as in GKP [1]), but we will always consider regions far away from the tip where the metric is well approximated by the singular conifold\(^4\). This allows us to use the singular conifold Kähler potential \( k(w, \overline{w}) = r^2 \), where \( r \) is the radial direction of the conifold (see appendix A).

Now we use the Kähler potential in equation (10) with \( k(w, \overline{w}) = r^2 \) to calculate the potential in equation (6) which becomes therefore \( r \) dependent. As \( r \) is our inflaton candidate, it is convenient to express the result in terms of the canonically normalized inflaton field \( \phi = r \sqrt{T_{D3}} \):

\[
V_{\text{ds}} = \frac{M_{Pl}^3}{(\phi^2 - 6M_{Pl}^2)^2} \left( \frac{9D}{\sigma_c^2} - \frac{6|A_0|^2 a^2 e^{-2\sigma_c}}{\sigma_c} \right)
\equiv 3H^2 \frac{36M_{Pl}^6}{(\phi^2 - 6M_{Pl}^2)^2},
\]

where we have introduced the Hubble parameter \( H \) for \( \phi = 0 \) (and neglected \( \dot{\phi}^2 \)). The fields \( r \) and \( \phi \) have dimensions of length and mass, respectively. On the other hand, \( \sigma \) has been normalized to be dimensionless. The slow-roll parameter \( \eta \) is then

\[
\eta_{\text{KLMT}} = M_{Pl}^2 V''_{\text{ds}} / V_{\text{ds}} = M_{Pl}^2 \left[ 3H^2 \frac{144(6M_{Pl}^2 + 5\phi^2)}{(\phi^2 - 6M_{Pl}^2)^4} \right] \left[ 3H^2 \frac{36}{(\phi^2 - 6M_{Pl}^2)^2} \right]^{-1}
\]

\[
= \frac{4(6M_{Pl}^2 + 5\phi^2)}{(\phi^2 - 6M_{Pl}^2)^2} M_{Pl}.
\]

From its definition, \( \phi \) is positive and smaller than \( \sqrt{6}M_{Pl} \). At this value in fact, the volume \( \mathcal{V} \) in equation (10) becomes zero (assuming that the Kähler modulus \( \rho \) has reached its minimum \( \rho + \mathcal{P} = 2\sigma_c \)) and the shifted Kähler potential becomes singular. Therefore \( \eta_{\text{KLMT}} \) is always bigger than 2/3 (the conformal value attained for \( \phi = 0 \) [4]) and slow-roll inflation never takes place without threshold corrections.

\(^4\) Very close to the tip, where the deformation cannot be neglected, one has \( k(w, \overline{w}) = r^3 + \text{const} \) [22, 23]. As noticed in [5], this implies that the effect of moduli stabilization very close to the tip of a warped deformed conifold is to generate a term proportional to \( \dot{\phi}^4 \) instead of \( \dot{\phi}^2 \) as is the case for the singular conifold. This \( \dot{\phi}^4 \) term could in principle be canceled by the threshold corrections to \( W_{np} \) that we are considering here. The cancelation would be, however, only valid very close to the tip, i.e. for a short range of values of the inflaton field.
3.2. \( F \)-term potential for the conifold

In this subsection we repeat the calculation of the last subsection but now taking into account the threshold corrections to the non-perturbative superpotential discussed in section 2, i.e. we allow for a generic \( A(w) \). We start with the four-dimensional, \( N = 1 \) supergravity scalar potential

\[
V_F = e^K \left( K \tilde{e}^{ab} D_a W D_b W - 3|W|^2 \right). \tag{13}
\]

The indices \( a, b \) run over the complex fields \( \rho \) and \( w = w_i \) with \( i \) running over three of the four homogeneous coordinates \( w_A \) introduced in appendix A.

For the Kähler potential of equation (10) and a generic superpotential \( W \) the resulting \( F \)-term potential takes the form (cf [10])

\[
V_F = V_{KKLT} + \Delta V, \tag{14}
\]

\[
V_{KKLT} = \frac{k_i^2}{3R^2} \left[ (\rho + \overline{\rho})|W| - 3(\overline{W} \rho + \text{c.c.}) \right], \tag{15}
\]

\[
\Delta V = \frac{k_i^2}{3R^2} \left[ \frac{3}{2} \left( \overline{W} \sum_i w_i W_i + \text{c.c.} \right) + \frac{1}{\gamma} k^\gamma \overline{W} W_i \right], \tag{16}
\]

where \( W_\rho \equiv \partial_\rho W, W_i \equiv \partial_i W \). Note that all terms of type \( K\overline{W} W K_i \) cancel out precisely. Thus, \( V_F \) would vanish if the superpotential were independent of \( \rho \) and \( w_i \), because of the no-scale structure. But it is not and \( V_F \) does not vanish. Indeed, using the superpotential in equation (2), with a generic \( A(w) \), one finds

\[
V_{KKLT} = \frac{k_i^2}{3R^2} \left[ [(\rho + \overline{\rho})A^2 + 6a] |A|^2 e^{-a(\rho + \overline{\rho})} + 3a(\overline{W}_0 A e^{-a\rho} + \text{c.c.}) \right], \tag{17}
\]

\[
\Delta V = \frac{k_i^2}{3R^2} \left[ -\frac{3}{2} i \left( \overline{W} \sum_i w_i A_i + \text{c.c.} \right) + \frac{1}{\gamma} k^\gamma \overline{W} A_i \right] e^{-a(\rho + \overline{\rho})}, \tag{18}
\]

where \( A_i \equiv \partial_i A \). The separation into two terms is due to the fact that \( \Delta V \) is non-vanishing only when \( A \) is a non-trivial function of the \( w_i \). In the last subsection, where we assumed that \( A \) is a constant, \( \Delta V \) was absent. Note also that \( V_{KKLT} \) is not the same as \( V_{AdS} \) in equation (6) of the last subsection. It differs in two ways: first, in \( V_{KKLT} \) there is also a dependence on the angular moduli through the non-constant \( A(w_i) \); second, due to the backreaction of the mobile D3-brane the volume modulus has become \( R = 2\sigma - \gamma r^2 \) rather than simply \( 2\sigma \) and has acquired a dependence on the D3-brane radial position.

Since we want to find out whether warped D3-brane inflation is possible in this setting, we need to be in a dS space. This can be achieved by adding an uplifting term

\[
V_{up} = \frac{D}{R^2} \tag{19}
\]

to \( V_F \). The uplifting breaks supersymmetry and lifts the vacuum to a dS one. For concreteness we will think of this term as coming from the warped anti-D3-brane tension [2]. This is not essential for our purposes and other upplings, such as \( D \)-term or \( F \)-term upplings, can be used as well. Two comments are in order. First, the \( R^2 \) dependence is appropriate for the warped throat under consideration whereas an ordinary compact
six-manifold would generate an $R^3$ dependence instead, as discussed in [4]. Second, due to the backreaction of the mobile D3-brane there is a dependence on its position $r$ in the denominator, which uses the corrected volume modulus $R$ rather than $\sigma$.

4. Critical points of the potential

Our eventual goal is to identify the inflaton with the mobile D3-brane position modulus $r$ and to study whether its potential

$$V = V_{\text{KKLT}} + V_{\text{up}} + \Delta V,$$

(20)
can lead to viable inflation. To this end we have to ensure that there is no steep runaway in some other direction in moduli space. Therefore, next we analyze the stabilization of all moduli besides $r$, which comprise the volume modulus $\sigma$, its axionic partner $b$ and the angular moduli $\theta_1, \theta_2, \phi_1, \phi_2, \psi$. As we want to restrict ourselves to the case of single-field inflation we have to require that the D3-brane motion does not modify considerably the stabilization of the other fields. A convenient regime to consider is

$$|f(r)|^{1/n} - 1 \ll 1,$$

(21)
so that the critical value of the volume modulus $\sigma_c$ will change only slightly during the inflationary dynamics (see equation (30) and the related discussion). Although the dependence of $\sigma_c$ on $\phi$ is mild (so that during the inflaton motion the minimization of $\sigma$ is only slightly corrected), it is crucial to determine the correct shape of the effective potential for the inflaton $V(\phi)$ (see also [12,11]). In figure 2 (see also appendix C), we compare the effective potential $V(\sigma, \phi)$ for some fixed values of $\sigma$, with the correct effective potential $V(\sigma_c(\phi), \phi)$. The sections at constant $\sigma$ of the potential differ even qualitatively from the correct effective potential $V(\sigma_c(\phi), \phi)$. In the following we will work in the regime specified by equation (21).

4.1. Axion stabilization

It is easiest to start the moduli stabilization analysis with the axion field $b$. One observes that it makes its appearance only in the second term of $V_{\text{KKLT}}$

$$3a(\overline{W}_0 A e^{-a(\sigma+ib)} + \text{c.c.}) = 3a|W_0 A|e^{-a\sigma}(e^{-i(ab-a)} + e^{i(ab-a)})$$

$$= 6a|W_0 A|e^{-a\sigma} \cos(ab-a),$$

(22)
with $\alpha$ denoting the phase of $\overline{W}_0 A$. This term acquires its minimum when

$$b_c = \frac{1}{a} [\alpha + (2p-1)\pi], \quad p \in \mathbb{Z},$$

(23)
and turns into minus its absolute value. This fixes the axion and implies for the KKLT part of the potential

$$V_{\text{KKLT}} = \frac{\kappa_4^2}{3R^2} \left[ 2a(a\sigma + 3) |A|^2 e^{-2a\sigma} - 6a|W_0 A|e^{-a\sigma} \right].$$

(24)
Figure 2. On the left: the dependence of the potential on $\phi$ and $\sigma$ near the minimum. The black thick line is the value of $\sigma_c$ that one would get neglecting the uplifting term (using just equation (27)). Clearly if one is interested in inflation dynamics, neglecting $V_{up}$ is inconsistent. On the right: the black thin lines are the potential (times $10^{16}$) evaluated for different but $\phi$ independent $\sigma_c$. The red thick line is obtained plotting $V(\phi, \sigma_c(\phi))$ (times $10^{16}$). Again one clearly sees that it is inconsistent to study inflation just in the $\phi$ direction for fixed $\sigma_c$.

4.2. Volume modulus stabilization

This section is devoted to the minimization of the volume $\sigma$, which is more involved than the axion minimization. The reason is that, with $r$ being our inflaton candidate, it is particularly important to determine the $r$ dependence of the critical value $\sigma_c(r)$ of the modulus $\sigma$.

The criticality condition, $\partial_\sigma V = 0$, which determines $\sigma_c$, reads

$$(aR_c + 2)(V_{KKLT} + \Delta V) + 2V_{up} = \frac{\kappa^2 a^2}{3R_c} |A| e^{-a\sigma_c} \left( |A| e^{-a\sigma_c} - 3W_0 \right),$$

where $R_c \equiv 2\sigma_c - \gamma r^2$. If $\Delta V$, $V_{up}$ and the mobile D3-brane were absent, such that $R \rightarrow 2\sigma$, the criticality condition would lead to the original KKLT result [2]

$$V_{KKLT,0} = -\frac{\kappa^2 a^2 A_0^2 e^{-2a\sigma_0}}{6\sigma_0},$$

with the KKLT critical volume, $\sigma_c \rightarrow \sigma_0$, defined implicitly by

$$W_0 = -A_0 e^{-a\sigma_0} \left( \frac{2}{3}a\sigma_0 + 1 \right),$$

where the fixed axion value has been used. Once $V_{up}$ and the mobile D3-brane are added, the critical volume, $\sigma_c$, is shifted away from the constant $\sigma_0$:

$$\sigma_0 \rightarrow \frac{V_{up, D3}}{\sigma_c} \rightarrow \sigma_c.$$  

Note that $\sigma_c$ depends on $D$ and $r$ while $\sigma_0$ does not. We define

$$\Delta(D, r) = \sigma_c(W_0, D, r) - \sigma_0.$$
Figure 3. The plot shows the potential $V(\phi)$ (red) and the slow-roll parameters $\eta(\phi)$ (blue) and $\epsilon(\phi)$ (black). The latter is so small that it can hardly be distinguished from the $\phi$ axis. Next to the tip of the throat the potential has generically a maximum and a minimum. For $\phi$ large enough the potential grows like $\phi^2$ and $\eta$ is of order 1 (or bigger). But for $\phi \to 0$ the curvature of the potential changes at the inflection point and $\eta$ switches sign (and eventually diverges at $\phi = 0$).

In what follows, we will use the parameters $\{\beta, \sigma_0\}$ instead of $\{D, W_0\}$ whose definition has been given in equations (7) and (8). As we pointed out above, the condition that $V_{\text{up}}$ uplifts the AdS minimum to dS is now easily expressed through the requirement that $\beta \gtrsim 1 + 2 \Delta/\sigma_0$ (which is very close to, but not exactly 1). In the rest of the paper we assume that this condition is fulfilled and therefore the minimum is dS.

Note that the full $r$ (and $\beta$) dependence of $\sigma_c$ is contained in $\Delta$. To calculate $\Delta$ we expand the criticality condition equation (25) in $\Delta/\sigma_0$ and use $a\sigma_0 \gg 1$ to simplify the result. We obtain

$$\partial_\phi V = 0 : \quad a\Delta (2|f|^{1/n} - 1) = \frac{\beta}{a\sigma_0} |f|^{-1/n} - (1 - |f|^{1/n}),$$  

(30)

where we keep the leading term and first subleading corrections in $1/\sigma_0$ and $\Delta/\sigma_0$ of equation (25). This equation determines explicitly the $r$ dependence of $\Delta$ which arises due to the $r$ dependence of $f$.

Without the D3-brane one would have $f = 1$ and thus $\Delta = \beta/a^2\sigma_0$ which in turn reduces to zero in the absence of the uplifting ($\beta = 0$) in agreement with the expectations. Importantly, the consistency of our expansion can be verified from equation (30), taking into account equation (21) and that $a\sigma_0 \gg 1$ which leads to

$$\frac{\Delta}{\sigma_0} = \mathcal{O} \left( \frac{1}{\sigma_0^2}, \frac{|f|^{1/n} - 1}{\sigma_0} \right) \ll 1,$$  

(31)

and therefore $\Delta \ll \sigma_0 \sim \sigma_c$. Note that in general $\Delta$ depends, via the embedding $f$, also on the angular variables $\theta_1, \theta_2, \phi_1, \phi_2, \psi$ whose stabilization we are analyzing next.
4.3. Angular moduli stabilization

For the sake of brevity, let us denote the angular moduli
\[
\theta_1, \theta_2, \phi_1, \phi_2, \psi
\]
as \(\vartheta_\alpha, \alpha = 1, \ldots, 5\) and use the abbreviation \(\partial_\alpha \equiv \partial_{\vartheta_\alpha}\). The criticality condition for the angular moduli, \(\partial_\alpha V = \partial_\alpha V_F = 0\), does not involve \(V_{\text{up}}\) which is independent of \(\vartheta_\alpha\). The full angular criticality condition thus reads
\[
2 \left( (2a^2\sigma_c + 6a)|A| - 6a|W_0| e^{a\sigma_c} \right) \partial_\alpha |A| = \frac{3}{2} a \partial_\alpha \left( \prod_i w_i A_i + \text{c.c.} \right) - \frac{1}{\gamma} \partial_\alpha (k^5 A_i A_i),
\]
where the left-hand side of the equality stems from \(V_{\text{KKLT}}\) while the right-hand side originates from \(\Delta V\).

As we did in the previous section, we replace \(\sigma_c\) by \(\sigma_0 + \Delta\) and expand in \(\Delta/\sigma_0 \ll 1\). Using equation (27) to evaluate the left-hand side of equation (33), one can see that the right-hand side of the criticality condition is suppressed by a factor \(1/\sigma_0\) and thus does not contribute at leading order. One finds
\[
\sigma_0 (2 - |f|^{1/n}) \partial_\alpha |A| = 0
\]
at leading order in \(1/\sigma_0\) and \(\Delta/\sigma_0\). In view of equation (21), the values of the angular open string moduli that extremize the scalar potential are solutions of
\[
\partial_\alpha V = 0 : \quad \partial_\alpha |f| = 0.
\]
These five equations will fix generically all five angular moduli unless the embedding allows for isometries. However, isometries are incompatible with the bulk Calabi-Yau compactification and hence should be broken. For a detailed discussion of this issue see [23].

The fixing of the angular moduli leads to
\[
\Delta V = \frac{\kappa_4^2 |A_0|^2}{12n^2 \gamma} |f|^{-2 + 2/n} \partial_r |f| \left( -8 \pi \gamma r |f| + \partial_r |f| \right) e^{-2a\sigma_c} \frac{R_c^2}{R_c^2},
\]
while the other two contributions to the potential become
\[
V_{\text{KKLT}} = \frac{2\kappa_4^2 a |A_0|^2}{3} |f|^{1/n} (|f|^{1/n} (a\sigma_c + 3) - (2a\sigma_0 + 3) e^{a\Delta}) e^{-2a\sigma_c} \frac{R_c^2}{R_c^2},
\]
\[
V_{\text{up}} = \frac{D}{R_c^2},
\]
where we have used equation (27) to eliminate \(|W_0|\). We have thus achieved a stabilization of all moduli, except for \(r\), the inflaton candidate. The dependence of the full potential on \(r\) arises from the \(r\) dependences of \(\sigma_c(r), \Delta(r)\) and \(f(r)\).
4.4. Potential with moduli fixed

We will now study the potential in the large $\sigma_0$ regime for a general embedding $f$. For this we expand the potential in $\Delta/\sigma_0$ and $1/\sigma_0$ and obtain at leading order

$$V_{KKLT} = V_{KKLT,0}|f|^{1/n}(2 - |f|^{1/n}) \left[ 1 + \frac{\gamma r^2}{\sigma_0} \right],$$

$$\Delta V = \frac{V_{KKLT,0}}{32\pi^2\gamma\sigma_0}|f|^{-2+2/n}\partial_r[f][8\pi\gamma r |f| - \partial_r |f|],$$

$$V_{up} = \frac{D}{4\sigma_0^2} \left[ 1 + \frac{\gamma r^2}{\sigma_0} - \frac{2\Delta}{\sigma_0} \right].$$

(39)

In the expression for $V_{KKLT}$ there are also two terms proportional to $1 - |f|^{1/n}$ at order $O(1/\sigma_0)$. Using equation (21), a posteriori justified in equation (30), we have omitted these terms. We see, using equations (7) and (8), that $V_{up}$ appears volume suppressed compared to $V_{KKLT}$ and $\Delta V$ by an additional factor $1/\sigma_0$.

Notice that in equation (39) we neglect the Coulomb (plus gravitational) attraction between the D3-brane and the anti-D3-brane. The reason is that the Coulomb attraction is very weak due to the warping. This was, in fact, the basis of the KKLMMT proposal [4] for achieving slow-roll brane inflation. As we discussed in section 3, and is seen here explicitly, there are, however, also effects coming from moduli stabilization that render the $F$-term potential generically steep ($\eta_{KKLT} \geq 2/3$). Our effort will be to make the $F$-term, equation (39), flat enough for slow-roll; once this is achieved, we can add the Coulomb potential as well, and study the resulting slow-roll inflation.

Our analysis of the inflationary dynamics will be based on two assumptions. The first is that $\sigma$ reaches its $\phi$ dependent minimum (given by equation (30)) instantaneously during the inflaton motion. In other words, the system evolves along the $\sigma_\phi(\phi)$ trajectory in the $\{\sigma, \phi\}$ plane. For this assumption to be satisfied, the $\sigma$ direction should always be much steeper than the $\phi$ direction which is actually the case here (see also the adiabatic approximation of [12]).

The second assumption is more subtle and is as regards the angular directions. We are assuming that the initial conditions of inflation are such that these directions start at their minima. For the Ouyang and Kuperstein embedding that we consider in the next section, the minimum in the angular directions does not depend on the radial position. Therefore if at the beginning of inflation the system is at an angular minimum, it will stay there forever. This ad hoc assumption about the initial conditions is ubiquitous in the string inflationary literature and is more a technical than a conceptual issue: if the angular directions are steeper than the radial one, then even if they are excited at the beginning, they will relax in a short time; if they are flatter or comparably steep, then they should be included in the inflationary analysis which would become multi-field in nature. In the latter case one is obliged to rely on numerical methods losing the intuition that the analytical single-field approach usually gives.

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5 Actually, motivated by this hierarchy of importance, one can even go further and omit any anti-D3-brane to begin with. The inflaton potential comes then just from the $F$-term, which is always present. This idea is realized in the model of inflation at the tip constructed in [5].
5. Explicit examples: Ouyang versus Kuperstein embedding

In this section we study two explicit supersymmetric D7-brane embeddings into the conifold. They have been discovered by Ouyang in [8] and by Kuperstein in [9]. For the Ouyang embedding, we will find that $\Delta V$ vanishes at the minimum of the angular directions, where $\theta_1 = \theta_2 = 0$. This was first noticed in [10]. As a result $\tilde{\psi}$, defined by

$$\tilde{\psi} = \frac{1}{2} (\psi - \phi_1 - \phi_2),$$

remains unfixed. For the Kuperstein embedding, on the other hand, $\Delta V$ does not vanish at the minimum of the angular directions and can modify $\eta_{KKLT} \simeq 2/3$ (see also [12, 11]). It is worth noticing that, in the Ouyang case, if the maxima in the angular directions are inserted in $\Delta V$, then the resulting effective potential $V(\phi)$ is exactly the same as in the Kuperstein case. Of course this radial trajectory (that we will analyze in section 6) is physically interesting only in the Kuperstein case, where it is stable in the angular directions (an exhaustive analysis of this issue, with a detailed calculation, has been given in [12]).

5.1. Ouyang embedding

The Ouyang embedding [8] is defined by the zeros of

$$f(w_i) = 1 - \frac{w_1}{\mu}. \quad (41)$$

Using equation (A.6), one derives

$$|f|^2 = 1 - \frac{r^3/2}{|\mu|} \sin \frac{\theta_1}{2} \sin \frac{\theta_2}{2} \cos \tilde{\psi} + \frac{r^3}{|\mu|^2} \sin^2 \frac{\theta_1}{2} \sin^2 \frac{\theta_2}{2}. \quad (42)$$

We will take $\mu$ to be real and positive because a possible phase can be absorbed in a shift of $\tilde{\psi}$. The two directions perpendicular to $\tilde{\psi} = \text{const}$ are at this point exactly flat. They will eventually get a mass but their explicit value does not affect the effective potential for the inflaton.

The system of equations fixing the angles, $\partial_a |f| = 0$, turns into

$$\theta_1 : \quad -\frac{r^3/2}{\mu} \cos \frac{\theta_1}{2} \sin \frac{\theta_2}{2} \cos \tilde{\psi} + \frac{r^3}{\mu^2} \sin \frac{\theta_1}{2} \cos \frac{\theta_1}{2} \sin^2 \frac{\theta_2}{2} = 0, \quad (43)$$

$$\theta_2 : \quad -\frac{r^3/2}{\mu} \sin \frac{\theta_1}{2} \cos \frac{\theta_2}{2} \cos \tilde{\psi} + \frac{r^3}{\mu^2} \sin^2 \frac{\theta_1}{2} \sin \frac{\theta_2}{2} \cos \frac{\theta_2}{2} = 0, \quad (44)$$

$$\phi_1, \phi_2, \psi : \quad \frac{r^3}{\mu} \sin \frac{\theta_1}{2} \sin \frac{\theta_2}{2} \sin \tilde{\psi} = 0. \quad (45)$$

This system of equations has two kinds of solutions (angular critical points)

$$\theta_1 = \theta_2 = \pi, \quad \tilde{\psi} = 0, \pi, \quad (46)$$

$$\theta_1 = \theta_2 = 0 \quad \text{and} \quad \tilde{\psi} \text{ unfixed.} \quad (47)$$

A detailed study [10] (see also [12]) of the Hessian matrix shows that the solution corresponding to a minimum is $\theta_1 = \theta_2 = 0$. Here we notice that the other angular
direction $\tilde{\psi}$ is not flat when $\theta_1 \neq 0$; once we evaluate the potential, however, at $\theta_1 = \theta_2 = 0$, no dependence on $\tilde{\psi}$ remains. The actual value of $\tilde{\psi}$ does not affect the following result. In fact, we get

$$A = A_0 f^{1/n} = A_0,$$

$$\Delta V = 0,$$  \hspace{1cm} (48)

so the potential is exactly $V_{\text{KKLT},0}$, leading to $\eta \simeq 2/3$. In this case no fine-tuning is possible [10]. The other extremum, $\theta_1 = \theta_2 = \pi$, corresponds to a maximum. In this case $\tilde{\psi}$ is fixed (see equation (46)) but not the two perpendicular directions in $\{\phi_1, \phi_2, \tilde{\psi}\}$ space. If one substitutes these angular values (corresponding to the maximum), one finds

$$A = A_0 f^{1/n} = A_0 \left(1 + \frac{r^{3/2}}{\mu}\right)^{1/n} \simeq A_0 \left(1 + \frac{r^{3/2}}{\mu n}\right),$$  \hspace{1cm} (49)

$$\Delta V = \frac{\kappa_4^2 |A|^2 \epsilon^{-2a\sigma}}{n^2 R^2} \left[\frac{2\pi r^{3/2}}{\sqrt{2\mu} + r^{3/2}} + \frac{r}{\gamma(\sqrt{2\mu} + r^{3/2})^2}\right],$$  \hspace{1cm} (50)

where in the last step we have used equation (21), that here translates into $r^{3/2} \ll \mu$ and implies that the D3-brane is located further down in the throat than the D7-brane (which extends down to $r_{D7}^{3/2} = \mu$). As we will see in the next section (see also [12]), the effective potential $V(\phi)$ that one obtains using this maximum (unstable in the angular directions) is exactly the same as the one for the Kuperstein embedding in equation (52), with the angular moduli being at a minimum.

### 5.2. Kuperstein embedding

The Kuperstein embedding [9] is defined by the zeros of

$$f(z) = 1 - \frac{z_1}{\mu},$$  \hspace{1cm} (51)

where now we parameterize the conifold with alternative coordinates $\{z_i\}$ (see appendix A, in particular equation (A.3)). This embedding has no directions along which $\Delta V = 0$ [12,11]. Two trajectories extremize the potential in the angular directions: $z_1 = \pm \sqrt{3/2}/\sqrt{2}$, but only the one with the negative sign corresponds to a minimum. The correction to the potential then becomes [12,11]

$$\Delta V = \frac{\kappa_4^2 |A|^2 \epsilon^{-2a\sigma}}{n^2 R^2} \left[-2\pi \text{Re} \frac{z_1}{\mu} + \frac{r}{\gamma(\sqrt{2\mu} + r^{3/2})^2}\right]$$

$$= \frac{\kappa_4^2 |A|^2 \epsilon^{-2a\sigma}}{n^2 R^2} \left[\frac{2\pi r^{3/2}}{\sqrt{2\mu} + r^{3/2}} + \frac{r}{\gamma(\sqrt{2\mu} + r^{3/2})^2}\right],$$  \hspace{1cm} (52)

which is exactly the same as in the Ouyang case after choosing the (unstable) trajectory $w_1 = -r^{3/2}/\sqrt{2}$. The fact that the minus sign corresponds to the stable trajectory ($z_1 = -r^{3/2}/\sqrt{2}$) is crucial for the fine-tuning of $\eta$. Indeed it determines that the correction to $\eta_{\text{KKLT}} \simeq 2/3$ comes with a minus sign and a cancelation is possible.

The potential that we have written still depends on $\sigma$. To obtain the effective potential for the inflaton we need to extremize the potential with respect to $\sigma$, i.e. use equation (30).
The minimization of the volume can be straightforwardly carried out numerically. An analytical estimate is given by (see appendix C for its derivation)

\[ \sigma_c = \sigma_0 + \frac{\beta}{a^2\sigma_0} + \frac{r^{3/2}}{an\mu} + \cdots, \]  

(53)

where the ellipsis stands for terms suppressed by higher powers in \( r^{3/2}/\mu \) or \( 1/\sigma_0 \). We use this expression for the \( r \) dependent critical value of \( \sigma \) to transform the potential \( V(\sigma, r) \) into a potential for a single field, \( V(r) = V(\sigma_c(r), r) \). This implicitly assumes that the dynamics in the \( \sigma \) direction is much faster than in the \( r \) direction such that the evolution of the system is well approximated by the trajectory \( \sigma_c(r) \) in the \((\sigma, r)\) space. Eventually, the effective potential has to be expressed in terms of the canonically normalized field \( \phi \).

### 6. Inflation

In the previous sections we calculated the potential for the radial position \( r \) of the D3-brane in the throat, once all other fields have reached their minimum value. In this section we investigate whether the potential that we have obtained can provide phenomenologically viable inflation.

The first step is to rewrite the potential in terms of a canonically normalized field (to which we will refer in the following as the inflaton)

\[ \phi = \sqrt{T_{D3}} r. \]  

(54)

We notice that \( r \) has the dimension of a length while \( \phi \) has that of a mass, as should be the case for a canonically normalized scalar in four dimensions.

As we have seen in section 3, \( V_{KKLT,0} \) depends on the inflaton as

\[ V_{KKLT,0} = 3H^2 \left( \frac{36M_{Pl}^6}{(\phi^2 - 6M_{Pl}^2)^2} \right) \approx 3H^2M_{Pl}^2 + H^2\phi^2 + \cdots \]  

(55)

for small \( \phi \). This prevents slow-roll because

\[ \eta = M_{Pl}^2 \frac{V''}{V} \gtrsim \frac{2}{3}. \]  

(56)

If we want to have a flat potential, we thus need another term of the same size but opposite sign that we can fine-tune to cancel the \( 2/3 \). The new terms in the potential, equation (39), coming from the dependence of the non-perturbative superpotential on \( \phi \), are proportional to \( |f|^{1/n} \) or to \( \phi|f|^{1/n} \). The known supersymmetric embeddings all depend on integer powers of \( w_i \propto \phi^{3/2} \). This, in particular, implies that there is no term, in the small \( \phi \) expansion, that can exactly cancel the \( \phi^2 \) from \( V_{KKLT,0} \). The absence of fractional power embeddings \( f \propto w_i^p \) with \( p \) non-integer might be traced back to the holomorphicity of \( f(w_i) \); it seems therefore hard to circumvent this problem.

Also, all those embeddings for which \( f \propto 1 + w_i^p \) with \( p > 1 \) vanish much faster than \( V_{KKLT,0} \) for \( \phi \to 0 \) and do not help to flatten the potential. From this observation, it follows that embeddings of the ACR family [20] with \( p > 1 \) are not helpful for canceling the \( \eta_{KKLMMT} \approx 2/3 \), at least for small \( r \). Further study is needed to see whether there is a region where \( r \) is large enough that the effects of higher ACR embeddings become relevant, and at the same time, that region might still be well described by the conifold
Two embeddings that produce corrections to the scalar potential proportional to $φ$ and $φ^{3/2}$ (as opposed to $φ^p$ with $p > 2$) are the Ouyang (which is as well in the ACR family with $p = 1$) and the Kuperstein embeddings. For the former, once the angular minimization is performed, the corrections to the scalar potential vanish [10]. For the latter this is not the case and the potential is indeed modified, as shown in equation (52) [11,12].

6.1. The effective inflaton potential

Considering the region deep inside the throat, we expand the potential for small $r$ (resp. $φ$), keeping terms up to order $r^2$ (resp. $φ^2$). These are the most interesting terms since higher terms cannot cancel the $η_{KKLT} \simeq 2/3$ from $V_{KKLT,0}$. The result is

$$V_{dS} \equiv V_{KKLT} + V_{up} = V_{dS}^{(0)} + V_{dS}^{(3/2)} \frac{r^{3/2}}{μn} + V_{dS}^{(2)} \frac{γr^2}{σc} + \cdots,$$

$$ΔV = ΔV^{(1)}r + ΔV^{(3/2)}r^{3/2} + \cdots, \quad (57)$$

As shown in appendix D, $V_{dS}^{(3/2)} + ΔV^{(3/2)} < 0$, so the $r^{3/2}$ term is always negative. In terms of the canonically normalized field $φ$, we therefore want to study the effective Lagrangian

$$L = -\frac{1}{2} \partial_μ φ \partial^μ φ - V(φ),$$

$$V(φ) = Λ + C_1 φ - C_{3/2} φ^{3/2} + C_2 φ^2, \quad (58)$$

where we have approximated the DBI kinetic term by the canonical one. A comment about this approximation is in order. As long as the kinetic energy of the inflaton $φ^2$ is small compared to the warped D3-brane tension, the higher terms in the expansion of the square root in the DBI action are negligible and the canonical kinetic term gives a good approximation. In the present setup this condition is easily satisfied because inflation takes place in the middle of the throat where the warping is much weaker than at the tip. In models where inflation takes place close to or at the tip, e.g. [24] and [5], the effects of the DBI kinetic term become quickly relevant and can give rise to an interesting phenomenology.

The value of the effective cosmological constant term Λ depends on several parameters. The stringy parameters are the 3-form fluxes on the conifold, which determine the stabilization of the complex structure and the dilaton. The problem of how the cosmological constant arises from string theory and which is its most probable value is outside the scope of the present work (see e.g. the seminal paper [25]). In the following, we will simply consider Λ as a free parameter. The coefficients $C_1, C_{3/2}$ and $C_2$ are such that the potential always has a maximum and a minimum (see appendix E); an extremal case is when these coincide and one gets a flat inflection point. In appendix E we show how varying the uplifting parameter $β$ changes the discriminant (see e.g. figure E.1). When the discriminant vanishes, the maximum and minimum coincide, as displayed in the left part of figure 1.

In figure 3, we plot $η$ together with the potential $V(φ)$. The slow-roll parameter $η$ is small only in a narrow interval around the inflection point, $φ_{η=0}$, at which $V''$ (and
because of the deformation of the conifold at \( \eta \) becomes relevant. We want to stress that to get \( \eta \) negative to positive values, corresponds to a change of potential, first derivative at the inflection point. An example is the generally subleading Coulomb subleading terms that we neglected in the potential; they can have a non-vanishing point in finite time. A second effect (which coexists with the first) is induced by the approximation, such as an initial non-slow-roll effects that regularize this divergence: one is induced by corrections to the strict slow-roll slows down exponentially fast and never reaches the inflection point. There are two strictly vanishing. As a consequence the slow-roll attractor describes an inflaton that account.

A generic initial condition would be to start somewhere inside the Calabi-Yau manifold and then ‘fall down’ the throat. We therefore start at some \( \phi_i \) and slide down towards smaller \( \phi \). An important quantitative question is that of whether one can get enough e-foldings and an almost scale-invariant spectrum. We study the former question in the next section. An important qualitative question is that of whether one can reach the tip or whether the D3-brane gets stuck somewhere before, preventing reheating via brane–antibrane annihilation; we address this overshooting problem in sections 6.3 and 6.4.

### 6.2. Inflation through an inflection point

As we show in appendix E, the effective potential always exhibits a maximum and a minimum. These coincide for a particular critical value of the uplifting parameter \( \beta \), giving rise to a flat inflection point at some \( \phi = \bar{\phi} \). The critical \( \beta \) can be estimated analytically from the zero of the discriminant given in equation (E.3). In this section we comment on this fine-tuned case. A crucial point is that around \( \bar{\phi} \) the Coulomb potential (that we have argued could be neglected in the preceding discussion) has to be taken into account.

For example, consider a potential of the type \( V'''''(\bar{\phi})(\phi - \bar{\phi})^3 + \Lambda \), where the interesting case for us is when \( \Lambda \gg \phi^3 \). The first derivative at the inflection point \( \phi = \bar{\phi} \) is strictly vanishing. As a consequence the slow-roll attractor describes an inflaton that slows down exponentially fast and never reaches the inflection point. There are two effects that regularize this divergence: one is induced by corrections to the strict slow-roll approximation, such as an initial non-slow-roll \( \phi \). This could allow passing the inflection point in finite time. A second effect (which coexists with the first) is induced by the subleading terms that we neglected in the potential; they can have a non-vanishing first derivative at the inflection point. An example is the generally subleading Coulomb potential, \( V_{D3\overline{D3}} \), that becomes important around \( \bar{\phi} \), where the potential is otherwise flat. Linearly approximating \( V_{D3\overline{D3}} \) one gets a potential of the type

\[
V(\phi) \sim \Lambda + V'(\bar{\phi})(\phi - \bar{\phi}) + V''''(\bar{\phi})(\phi - \bar{\phi})^3. \tag{59}
\]

In this case, the inflection point is always reached and in fact overshot. The number of e-foldings, \( N_e \), that results from the inflationary dynamics is therefore controlled by the value of the first derivative of the potential at \( \bar{\phi} \), \( V'(\bar{\phi}) \). Varying \( V'(\bar{\phi}) \) continuously, from positive to negative values, corresponds to a change of \( N_e \) from a few to infinity. An analytical estimate (neglecting corrections to slow-roll) gives \( N_e \propto 1/V''(\bar{\phi}) \) for positive \( V''(\bar{\phi}) \) (see e.g. [26]); for negative \( V''(\bar{\phi}) \) a minimum of \( V \) is formed and the issue of overshooting and slow-roll corrections becomes important. In any case, it is clear that an arbitrary large amount of inflation can be obtained with the potential that we have calculated, provided that one can fine-tune the string theory parameters to obtain a small
Taking into account the effects of the DBI action can only increase the number of e-foldings. In the next two sections we will go beyond the slow-roll approximation and address the following question: when does the D3-brane reach the tip and when does it get stuck somewhere in the middle?

A final comment is in order. In the present model the shape of the potential is determined by the $F$-term, while the Coulomb potential gives only relatively small corrections. These corrections are relevant only in the fine-tuned case of a flat inflection point. Even in this case, the Coulomb potential dominates only around the inflection point where the force exerted by the $F$-term is vanishing. We want to contrast this situation with another expectation which is often found in the phenomenological brane inflationary literature (see e.g. [27]). One could have wished to find a way to completely get rid of the $F$-term effects and have an inflationary model based on a Coulomb potential of the type

$$V \sim V_0 \left(1 - \frac{1}{\phi^4}\right),$$

which has been widely studied (see e.g. [27]). The result of the present investigation is that this is even harder to achieve than expected. For the class of embeddings which we studied, not even fine-tuning allows us to cancel completely the $F$-term effects. We consider this as an indication that in a generic model of brane inflation, the Coulomb attraction is superfluous because the inflaton potential is determined by the $F$-term potential.

### 6.3. Damped oscillatory phase

In this and the next section we study the problem of overshooting the potential barrier. We make the following simplifying assumptions: we consider a homogeneous and isotropic universe so that the four-dimensional Einstein equations reduce to the Friedmann equations; furthermore we assume that all fields have been stabilized except for the inflaton, as shown in the previous sections; finally we neglect effects of the DBI action and approximate it by a canonical kinetic term as discussed around equation (58).

Let us start considering the following potential:

$$V = \Lambda + \frac{1}{2}m^2(\phi - \phi_{\text{min}})^2$$

in the vicinity of its minimum, $\phi_{\text{min}}$, where $\Lambda \gg m^2(\phi - \phi_{\text{min}})^2$. The first Friedmann equation gives

$$H^2 = \frac{1}{3M_{\text{Pl}}^2} \left[ \Lambda + \frac{1}{2}m^2(\phi - \phi_{\text{min}})^2 + \frac{1}{2}\dot{\phi}^2 \right] \approx \frac{\Lambda}{3M_{\text{Pl}}^2}.$$  

Therefore, the equation of motion can be approximated by

$$\ddot{\phi} + \frac{\sqrt{3\Lambda}}{M_{\text{Pl}}} \dot{\phi} + m^2(\phi - \phi_{\text{min}}) = 0,$$

where the consistency of neglecting $\dot{\phi}^2$ in equation (62) will be checked at the end. The equation of motion is the same as for a harmonic oscillator with friction. There are consequently three types of solutions:

---

6 In [5], proceeding along the lines of this argument, a model was constructed where the inflaton potential comes exclusively from the $F$-term and no anti-D3-branes were needed.
Chasing brane inflation in string theory

Figure 4. The figure summarizes our overshoot analysis. The continuous line is the actual potential, the darker dashed line refers to the discussion of the damped oscillatory phase and the lighter dashed line refers to the uphill phase.

- Underdamped: $M_{Pl}^2 m^2 > 3\Lambda/4$; only in this case does the field oscillate around the minimum. The amplitude decreases exponentially on a timescale $\sqrt{3\Lambda/2M_{Pl}}$. If the field starts at $\phi_i$ at $t=0$ with $\dot{\phi}_i=0$, we can estimate the speed when it passes the first time through the minimum\(^7\) at $t=t_{\text{min}}$ as

$$
\dot{\phi}_{\text{min}} = \phi_0 e^{-\sqrt{3\Lambda t_{\text{min}}/2M_{Pl}}} \frac{2m^2 M_{Pl}}{\sqrt{3\Lambda}} \cos(t_{\text{min}} \omega),
$$

where $\omega^2 = m^2 - 3\Lambda/4M_{Pl}^2$, and $\phi_0 \equiv \phi_i - \phi_{\text{min}}$ is the distance from the starting point to the minimum (see figure 4).

- Critically damped and overdamped: $M_{Pl}^2 m^2 \leq 3\Lambda/4$. There are no oscillations and the field takes an infinite amount of time to reach the minimum (where $\dot{\phi} = 0$).

The underdamping condition can be rewritten in terms of the slow-roll parameter as $M_{Pl}^2 m^2/\Lambda = \eta > 3/4$, where $\eta = \eta(\phi_{\text{min}})$. A rough estimation gives $|\dot{\phi}_{\text{min}}| \sim \eta \phi_0 \sqrt{3\Lambda}/M_{Pl}$. Therefore neglecting the kinetic term in the Friedmann equation (62) and using $H^2 \simeq \Lambda/3M_{Pl}^2$ is legitimate as long as $\eta^2 \phi_i^2 \ll M_{Pl}^2$.

We want to apply this analysis to our potential equation (58) in the case where this exhibits a minimum at $\phi = \phi_{\text{min}}$. This could be the case for example if string theory does not allow for an arbitrary fine-tuning of the effective parameters $C_1$, $C_{3/2}$ and $C_2$. Around $\phi_{\text{min}}$, the potential is approximated by a harmonic oscillator $V \simeq V''(\phi - \phi_{\text{min}})^2/2$ (see figure 4). We conclude that if $\eta(\phi_{\text{min}}) < 3/4$, the inflaton reaches the minimum $\phi_{\text{min}}$ only asymptotically in infinite time. There is no graceful exit from inflation as there is no brane annihilation or (damped) oscillations. The exponential expansion (with cosmological constant $V(\phi_{\text{min}})$) continues forever. In contrast, if $\eta(\phi_{\text{min}}) > 3/4$, we find at the minimum that $\dot{\phi}_{\text{min}} \neq 0$. This allows for the possibility of climbing up to the maximum of the potential, overshooting it and reaching the tip of the throat where annihilation with the anti-D3-brane will take place.

\(^7\) Notice that the formula we give is valid only at $t = t_{\text{min}}$ and not for a generic time $t$. 

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6.4. Uphill inflation

In this section we study what happens when the inflaton rolls uphill. We will use our results to address the issue of overshooting the potential barrier in the potential (see figure 4 or the right part of figure 1). Obviously, the field will roll just for a short distance $\Delta \phi$, which depends on the initial speed $\dot{\phi}(0)$ that we take from the beginning to be $\dot{\phi}_{\min}$. Let us first study the simple linear potential

$$V = \Lambda + c\phi,$$

where $c$ is a positive slope. Under the simplifying assumption, $\Lambda \gg c\phi$, the solution to the equation of motion is

$$\Delta \phi(t) \equiv \phi(t) - \phi(0) = -\frac{cM_{Pl}}{\sqrt{3\Lambda}}t + \left( \frac{\dot{\phi}_{\min}M_{Pl}}{\sqrt{3\Lambda}} + \frac{cM_{Pl}^2}{3\Lambda} \right) \left( 1 - e^{-\sqrt{3\Lambda}/M_{Pl}} \right),$$

The term linear in $t$ describes a rolling down at constant speed that eventually dominates the exponentially decreasing term. If the field starts with a positive $\dot{\phi}_{\min} > 0$, it will climb up the hill for a distance

$$\Delta \phi \approx \dot{\phi}_{\min}M_{Pl} \left( 1 + \frac{\sqrt{3\Lambda}\dot{\phi}_{\min}}{cM_{Pl}} \right),$$

in a time

$$\Delta t \approx \frac{M_{Pl}}{\sqrt{3\Lambda}} \log \left( 1 + \frac{\sqrt{3\Lambda}\dot{\phi}_{\min}}{cM_{Pl}} \right),$$

before it stops and starts rolling down again. The number of e-foldings therefore is generically short unless the slope is exponentially small (if instead $\dot{\phi}_{\min}$ is very large, then $H$ is no longer well approximated by a constant).

We would like to emphasize that an uphill motion is never slow-roll. Even if $\epsilon \simeq 0 = \eta$, when moving uphill $\ddot{\phi}$ is always very large (the motion is hampered both by the slope and by the Hubble friction) and cannot be neglected. In fact, the equation of motion is genuinely of second order and the uphill phase depends critically on the initial condition $\dot{\phi}_0$. On the other hand, the downhill slow-roll motion is an attractor and the solution eventually reaches it independently of $\dot{\phi}_{\min}$ (of course only if the potential is of the slow-roll type).

We now have all the ingredients to address the question of whether the inflaton will overshoot the maximum of the inflaton potential. To this end we describe the part of the potential before the minimum, $\phi > \phi_{\min}$, with the damped oscillator of section 6.3 (see figure 4). The key result is equation (64), the speed of the inflaton $\dot{\phi}_{\min}$ when it reaches $\dot{\phi}_{\min}$. We then approximate the uphill phase between the maximum and minimum, $\phi_{\max} < \phi < \phi_{\min}$, by a linear potential. The estimate may seem very rough, but if we take the steepness of our linear potential ($c$ in equation (65)) to be the maximum steepness reached by the potential $V(\phi)$ between $\phi_{\max}$ and $\phi_{\min}$, then we have an upper bound. If overshooting is possible in this extremal case, then it is also possible for the exact potential.

We use the result equation (64) as the initial condition in equation (66). One can see that the time $t_{\min}$ in equation (64) is always smaller than $2M_{Pl}/\sqrt{3\Lambda}$ so, to get an order
of magnitude estimate, we can neglect the exponential in that formula. Neglecting also
numerical factors we take
\[
\dot{\phi}_{\text{min}} \sim \phi_0 \eta \sqrt{3\Lambda}/M_{\text{Pl}},
\] (68)
where \(\phi_0 = \phi_i - \phi_{\text{min}}\) is the distance between the initial position and the minimum in the
damped oscillatory phase of section 6.3 (see figure 4). Substituting it into equation (66), we obtain
\[
\Delta \phi \sim \frac{\phi_{\text{min}} M_{\text{Pl}}}{\sqrt{3\Lambda}} \sim \phi_0 \eta,
\] (69)
where we have used that \(cM_{\text{Pl}}^2/3\Lambda \ll \phi_0\), which is generic for our potential. We conclude
that overshooting can happen, with \(\eta \gtrsim 1\) and a comfortably natural choice \(\phi_0 \gtrsim \Delta \phi\)
(also an initial \(\dot{\phi} \neq 0\) at the beginning of the underdamped oscillatory phase will help the
overshoot\(^8\)). Typically, one does not obtain a large number of e-foldings; see equation (67). We have to remember though that equation (67) is valid just for a linear potential; in our
case instead there is maximum, where the slope vanishes.

As an aside we comment on the intriguing correlation between a small cosmological
constant and the underdamped oscillatory regime. A graceful exit from inflation typically
requires that the inflaton reaches a minimum and starts oscillating and decaying (brane
inflation is an interesting exception). In section 6.3 we have seen that the underdamped
regime, leading to oscillation around the minimum, requires \(\eta \gtrsim 1\). Equivalently, it
requires that the cosmological constant, \(\Lambda\), is smaller than the inflaton mass squared
\(m^2\), in Planckian units. Consider now an inflaton protected by some symmetry that
therefore acquires an extremely small mass, e.g. only from non-perturbative effects. Then
an anthropic selection principle would apply: all universes with \(\Lambda \gtrsim m^2 M_{\text{Pl}}^2\) would not
have a graceful exit from inflation and would hence be empty.

Summarizing, if the inflaton potential is just \(\frac{1}{2} m^2 (\phi - \phi_{\text{min}})^2\) plus a cosmological
constant, then anthropic arguments lead to an upper bound on the cosmological constant
of order \(\Lambda \lesssim m^2 M_{\text{Pl}}^2\). An extremely small inflaton mass might then explain the presence of
a comparably small cosmological constant which may be responsible for today’s measured
accelerated cosmic expansion.

It would be interesting to study further features of the inflaton potential equation (58).
A preliminary observation is that, if an uphill phase is present, a largely non-scale-invariant
spectrum is produced. The spectral index during the uphill motion is given by
\[
ns - 1 \equiv \frac{d\ln P_R}{d\ln k} \simeq -\frac{\dot{\phi}}{H^2} - \frac{1}{H} \partial_t \left( \log \frac{\dot{\phi}}{H^2} \right),
\] (70)
where the quantities on the right-hand side have to be calculated at the time of horizon
crossing. After some massaging and using the Friedmann equations we obtain
\[
n_s - 1 \simeq 4 + \frac{c}{H \dot{\phi}} - \frac{\dot{\phi}}{H^2} + \frac{\dot{H}}{H^2}.
\] (71)
The various terms do not cancel, as happens in the slow-roll regime; the reason can be
traced back to the fact that \(\dot{\phi}\) is not small in this case. Per se, the absence of scale

\(^8\) The issue of overshooting for an inflection point potential with particular attention given to the role of initial
conditions has been recently addressed in [28], where the whole DBI action is taken into account.
7. Forces on D3-branes and anti-D3-branes

In this section we enumerate the contributions to the potential for an anti-D3-brane and a D3-brane (a sketch is given in table 1 below) and comment on their relative importance.

To summarize: the anti-D3-brane is led to the tip \( r \approx 0 \) by the interaction with the background; there its angular position is determined by the bulk and moduli stabilization effects. The motion of the D3-brane is governed by moduli stabilization effects (breaking of the no-scale structure). Finally, the attractive Coulomb potential is generically strongly suppressed and plays a role only in very fine-tuned or symmetric circumstances.

### Background effects.

We consider the action

\[
S_{D3/\bar{D}3} = -T_{D3} \int d^4x \sqrt{-g} \Phi^\pm,
\]

where \( \Phi \) is defined in terms of the warp factor and the 5-form field strength of \( C_4 \) as

\[
\Phi^\pm \equiv e^{4A} \pm \alpha.
\]

Therefore in the GKP setup [1], the D3-brane does not feel any force (it is BPS with respect to the background). In contrast, an anti-D3-brane tends to fall to the bottom of the (deformed) conifold (small warp factor) to minimize \( S_{\bar{D}3} \). As equation (72) has no angular dependence, the anti-D3-brane at the tip enjoys a translational \( S^3 \) symmetry. The leading contribution to the potential is the warped anti-D3-brane tension which can be used [2] to break SUSY and uplift an AdS vacuum to a dS vacuum.

### Bulk effects.

To have a compact manifold at a certain radius the conifold has to be cut and glued to a compact Calabi–Yau manifold. Then other ‘bulk’ effects for the anti-D3-brane arise. These break all the residual symmetry of the conifold as a Calabi–Yau manifold has no continuous symmetry. In [29] the warp factor dependence of bulk effects has been calculated via the AdS/CFT correspondence. The result is that a mass for the anti-D3-brane is induced of order

\[
m_{\text{bulk}}^2 \sim \left(g_s M \alpha'\right)^{-1} h_0^{-0.82},
\]

where \( M \) is the flux quantum number of the Ramond–Ramond \( F_3 \)-form over the 3-cycle \( A \) of the throat and \( h_0 \) is the warp factor at the tip of the throat. Bulk effects would lead

---

**Table 1.** Contributions to the potential for a D3-brane and an anti-D3-brane.

| Source               | \( \bar{D}3 \)                                      | D3                                      |
|----------------------|---------------------------------------------------|-----------------------------------------|
| Bulk background      | \( m_{\text{bulk}}^2 \sim \left(g_s M \alpha'\right)^{-1} a_0^{-2.9} \) | No effect (BPS)                        |
| Throat background    | \( V \sim 2T_{D3} h(w_{D3}) \) leads \( \bar{D}3 \) quickly to the tip | No effect (BPS)                        |
| Coulomb              | \( +V_{D\bar{D}} \)                                | \( -V_{D\bar{D}} \)                     |
| Tachyon              | Develops at \( r^2 = \mathcal{O}(\alpha') \)      | Develops at \( r^2 = \mathcal{O}(\alpha') \) |
| Moduli stabilization | Only known at the tip                              | \( V_{KKLT} + V_{\text{up}} + \Delta V \) |

Invariance is not a problem if the perturbations produced during the uphill phase are not those responsible for the CMB inhomogeneities, e.g. if the uphill phase takes place before or after 60 e-foldings prior to the end of inflation. These issues certainly deserve further study.
the anti-D3-brane to a particular angular position in the $S^3$ at the tip. No such effects are present for the D3-brane, again because of its BPS nature with respect to the background. Moduli stabilization effects. To stabilize the Kähler moduli, one has to break the no-scale structure. Once this is done and the moduli are stabilized (e.g. à la KKLT) a mass for the D3-brane open moduli is generated because of their non-trivial mixing with the 4-cycle volumes [21]. The potential generated gives rise to an $\eta$-problem, analogous to the SUGRA $\eta$-problem: it is too step for inflation and generates a ‘mass’ for the inflaton of order $H$ [4]. This effect is much bigger than the Coulomb attraction and constitutes indeed the leading term of the potential.

As far as the anti-D3-brane is concerned, these effects have been investigated in [23]. They are relevant at the tip of the throat because the background force from equation (72) does not have an angular dependence. The potential generated by the stabilization of the moduli has the same minima for the D3-brane and the anti-D3-brane at the tip (where the anti-D3-brane is confined by equation (72)). As the equations for the minimum for D3- and anti-D3-branes differ by a term vanishing at $r = 0$, away from the tip the respective minima will be generically different.

This effect, together with the bulk one, equation (74), select some vacua in the angular directions at the tip. The relative importance of bulk and stabilization effects depends on the parameters. Comparing the mass from the left-hand side of equation (33) with equation (74) for the case of the Ouyang (or the simplest Kuperstein) embedding, expanding in $r_0^{3/2}/\mu \ll 1$, one gets

$$\frac{r_0^{3/2}}{\mu n} \gg h_0^{-0.32},$$

where $r_0$ indicates the tip of the deformed conifold. As follows from equation (21), the left-hand side of equation (75) has to be much smaller than 1 to allow one to integrate out the stabilized volume and use the remaining effective potential for inflation. Indeed $r_0$ and $h_0$ are related by $h_0 \sim (\sqrt{g_s M/r_0})$, so the condition to satisfy is

$$g_s M \gg \left(\frac{\mu n}{r_0^{1/3}}\right)^{3/2},$$

Although it is possible to fulfill the inequality, this would require a very large flux number $M$, as $g_s \ll 1$ and $\mu n \gg 1$.

Coulomb potential. The Coulomb potential written in terms of the canonically normalized field $\phi$ is

$$V_{up} + V_{D3D3} \sim -\frac{4\pi^2}{N} \phi_0^4 \left(1 - \frac{1}{N} \phi_0^4\right),$$

where $N = KM$ is the product of the fluxes on the 3-cycles of the conifold and $\phi_0$ is the canonically normalized radial position of the anti-D3-brane at the tip of the throat. This potential can be obtained by considering the D3-brane backreaction on the metric in equation (72) and keeping the leading order.

We want to argue that the Coulomb interaction is typically weak and subleading (as was noticed for the first time in [4]). For example, suppose we use the constant term in equation (77) for the uplifting $V_{up}$. Then near the minimum in the $\sigma$ direction, $V_{up} \sim V_{AdS} \propto 3H^2/(6 - \phi^2)^2$ as in equation (11), which estimates the moduli stabilization
effects. The Coulomb potential is suppressed with respect to these effects by \((\phi_0/\phi)^4/N\), as one can see in equation (77). Therefore as soon as the D3-brane is separated from the anti-D3-brane (which sits at the tip \(\phi_0\)) the Coulomb interaction is by far subleading.

### 8. Comments and conclusions

We have studied the potential felt by a D3-brane in a warped conifold in the presence of supersymmetrically embedded D7-branes and an anti-D3-brane sitting at the tip of the cone. The leading order potential contains three terms: \(V_{\text{KKLT}}\), an uplifting term \(V_{\text{up}}\) and \(\Delta V\), the latter arising when threshold corrections to the non-perturbative superpotential are taken into account. We have provided general formulas for the extremization in the angular and Kähler modulus directions. Once those moduli settle down at their minimum, we are left with an effective potential \(V(\phi)\) for the canonically normalized radial D3-brane coordinate \(\phi\).

We have studied the possibility of flattening \(V(\phi)\) through fine-tuning, such that slow-roll D-brane inflation can be embedded into a type IIB string theory compactification with all moduli fixed except for the inflaton. We have carried out a detailed analysis for two specific classes of supersymmetric D7-brane embeddings. In the throat (for small \(\phi\)), \(\Delta V\) has a linear term in \(\phi\) and otherwise depends on \(\phi\) only via integer powers of \(\phi^3/2\), where \(V_{\text{KKLT}}\) and \(V_{\text{up}}\) contain terms proportional to \(\phi^2\). This means that the potential can be made flat only for a small range of \(\phi\). Allowing for fine-tuning, a flat inflection point can be generated. In this case the D3-brane dynamics sustains a prolonged stage of slow-roll inflation.

As we do not exactly know how much fine-tuning in the effective parameters can be achieved by varying the discrete string theory parameters, we have also considered the issue of whether, for a generic (non-fine-tuned) shape of the potential, the D3-brane can fall all the way down into the throat where it would annihilate with the anti-D3-brane.

The Coulomb attraction which was supposed to drive inflation is generically overwhelmed by the stronger forces generated by the \(F\)-term potential. Even in the most promising case, the Kuperstein embedding, where the potential exhibits a flat region around an inflection point, it seems that one will need to take into account, at the same time, the Coulomb and the \(F\)-term potential. This means that the analyses based on the simple brane–antibrane potential have to be reviewed if the brane system is embedded into a bona fide string compactification.

A comment on possible further corrections is in order. Quantum corrections, from loop or \(\alpha'\) effects, are generically subleading in the KKLT stabilization scenario, the reason being a very small \(W_0\) (see e.g. [30]). But the force exerted by the effective potential \(V(\phi)\) on the inflaton is hierarchically weaker than the one responsible for the stabilization of the closed string moduli (that is why we can talk about an effective \(V(\phi)\) in the first place). Therefore, we expect that quantum corrections will have sizable effects on warped brane inflation scenarios analyzed here. It would be interesting therefore to study these effects e.g. along the lines of [31]–[33].

In view of the difficulties that the KKLMMT scenario currently presents for the embedding of brane inflation into string theory, it might be interesting to look for qualitatively distinct scenarios such as multi-brane inflation [34, 35] in heterotic M-theory which is based on phenomenologically very interesting flux compactifications [36]–[38] or
related constructions in heterotic string theory [39]. Other alternatives comprise modular inflation, for instance the racetrack inflation models [40]–[42] or Kähler moduli inflation models [43,44], which have received some attention recently. Furthermore [45]–[48] present interesting ideas towards potentially viable string inflation scenarios.

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Appendix A. The warped conifold setup

The singular conifold is a non-compact Calabi–Yau threefold. It can be defined as a hypersurface in \( \mathbb{C}^4 \). Two sets of complex projective coordinates are particularly useful: one set is denoted by \( w_A \), with \( A = 1, 2, 3, 4 \), and it is the one used to write e.g. the Ouyang embedding in equation (41). In terms of the \( w_A \), the singular conifold is defined by the equation

\[
 w_1 w_2 - w_3 w_4 = 0. \tag{A.1}
\]

A second set is denoted by \( z_A \), used e.g. to write the Kuperstein embedding in equation (51). The defining equation written in terms of \( z_A \) is

\[
 \sum_{A=1}^{4} (z_A)^2 = 0. \tag{A.2}
\]

The two sets of coordinates are linearly related by

\[
\begin{align*}
 z_1 &= \frac{1}{2}(w_1 + w_2), & z_2 &= \frac{1}{2i} (w_1 - w_2), \\
 z_3 &= \frac{1}{2}(w_3 - w_4), & z_4 &= \frac{1}{2i} (w_3 + w_4). \tag{A.3}
\end{align*}
\]

We can also use six real coordinates to parameterize the conifold. The metric is then given by

\[
 \begin{align*}
 ds^2_6 &= dr^2 + r^2 ds^2_{T^1,1}, \\
 ds^2_{T^1,1} &= \frac{1}{9} \left( d\psi + \sum_{i=1}^{2} \cos \theta_i \, d\phi_i \right)^2 + \frac{1}{9} \sum_{i=1}^{2} \left( d\theta_i^2 + \sin^2 \theta_i \, d\phi_i^2 \right). \tag{A.5}
\end{align*}
\]

This makes explicit that the singular conifold has a radial direction \( r \) and a base parameterized by five angular directions \( \phi_1, \phi_2, \theta_1, \theta_2 \) and \( \psi \). The base is \( T^{1,1} \), i.e. the
coset space \((SU(2)_A \times SU(2)_B)/U(1)_R\), and it is topologically equivalent to \(S^3 \times S^2\). The complex \(w_A\) coordinates can be expressed in terms of the real coordinates as

\[
w_1 = r^{3/2} e^{i/2(\psi - \phi_1 - \phi_2)} \sin \frac{\theta_1}{2} \sin \frac{\theta_2}{2},
\]

\[
w_2 = r^{3/2} e^{i/2(\psi + \phi_1 + \phi_2)} \cos \frac{\theta_1}{2} \cos \frac{\theta_2}{2},
\]

\[
w_3 = r^{3/2} e^{i/2(\psi + \phi_1 - \phi_2)} \cos \frac{\theta_1}{2} \sin \frac{\theta_2}{2},
\]

\[
w_4 = r^{3/2} e^{i/2(\psi - \phi_1 + \phi_2)} \sin \frac{\theta_1}{2} \cos \frac{\theta_2}{2}.
\]

As for all Calabi–Yau manifolds, the metric of the singular conifold is given by the second derivative of a Kähler potential. This is given in terms of the \(w_A\) coordinates by

\[
k(w_i, \bar{w}_i) = r^2 = \left( \sum_{i=1}^{4} |w_i|^2 \right)^{2/3}.
\]

To obtain the metric, one of the four coordinates \(\{w_1, w_2, w_3, w_4\}\) has to be expressed in terms of the others using the defining equation (A.1). If, e.g. we choose to keep \(\{w_2, w_3, w_4\}\) and express \(w_1\) as a function of them, the inverse metric (appearing in equation (16)) is given by

\[
k^{i,j} \equiv (k_{i,j})^{-1}
\]

\[
= \frac{3}{2} r^2 \left\{ \delta_{ij} + \frac{w_i \bar{w}_j}{2r^3} - \frac{1}{r^3} \left( |w_1|^2 |w_4|^2 - |w_3|^2 \right) \right. \\
+ \frac{|w_1|^2}{2r^3} \left\{ \delta_{1i}(\delta_{1j} - 1) \frac{\bar{w}_1}{\bar{w}_j} + \delta_{1j}(\delta_{1i} - 1) \frac{w_i}{w_j} \right\} \\
= \frac{3}{2} r^2 \left( \frac{w_1 \bar{w}_3}{w_2^2} \left( |w_2|^2 + 2 |w_4|^2 \right) r^3 - |w_4|^2 + \frac{1}{2} |w_3|^2 \right) \frac{w_4 \bar{w}_3}{w_3 \bar{w}_4} \left( r^3 - |w_3|^2 + \frac{1}{2} |w_4|^2 \right) \right\},
\]

where \(i\) and \(j\) run from 1 to 3.

**Appendix B. On the parameters**

The parameters of the models are the following:

- \(\mu\): represents the deepest \(r_D7\) value reached by the D7-brane. We require that \(\mu \gg r^{3/2}\) so that the stabilized volume does not change much during the D3-brane radial motion.

- \(A_0\): the complex structure dependent factor in \(W_{np}\). Its phase can be absorbed by a shift of the axion. Once \(W_0\) is chosen as in equation (7) and \(D\) as in equation (8), \(A_0\) can be factorized out of the scalar potential. Therefore it does not play an important role in the discussion.
• $W_0$: its value can be fine-tuned (up to a certain precision) varying the fluxes which fix the complex structure moduli and the dilaton. The constraint on its value comes from the KKLT procedure for fixing the overall volume (generically all Kähler moduli). This generically requires $W_0 \ll 1$.

• $\beta$: it fixes the value of the vacuum energy. When a particular uplifting procedure is specified, its value is determined in terms of stringy parameters (for example the position of the anti-D3-brane at the tip of the throat or the world volume fluxes in a $D$-term uplifting). Having a de Sitter vacuum translates into $\beta \gtrapprox 1$; having a minimum at all requires $\beta \lesssim \mathcal{O}(4)$. This parameter is important for the shape of the effective potential $V(\phi)$. Only a special fine-tuned value of $\beta$ leads to a flat inflection point.

Appendix C. Dependence of $\sigma_c$ on uplifting and inflaton

The scalar potential

$$V_{\mathrm{AdS}} = \frac{aA_0e^{-a\sigma}}{2\sigma^2} \left(\frac{1}{3} \sigma a A_0 e^{-a\sigma} + W_0 + A_0 e^{-a\sigma}\right),$$

(C.1)

resulting from

$$K = -3 \log (\rho + \overline{\rho}),$$

(C.2)

$$W = W_0 + A_0 e^{-a\rho},$$

(C.3)

has the well known [2] AdS minimum $\sigma_0$, given by the solution of the transcendental equation

$$\sigma_0 = -\frac{3 A_0 + W_0 e^{a\sigma_0}}{2 A_0 e^{-a\sigma_0}},$$

(C.4)

where $W_0$ is a negative real number. The aim of this appendix is to calculate how the minimum $\sigma_0$ changes when the uplifting and the D3-brane dynamics are taken into account.

C.1. Shift of critical volume through uplifting

For concreteness we look at an anti-D3-brane uplifting. If an anti-D3-brane is present in the ISD solution of [1], it will feel a potential that pulls it to the tip of the throat. Supersymmetry is broken and the effective scalar potential receives a contribution proportional to the redshifted anti-D3-brane tension that we schematically write as

$$V_{\mathrm{up}} = \frac{D}{4\sigma^2}.$$  

(C.5)

The potential’s minimum will consequently shift from $\sigma_0$ to $\sigma_{up} = \sigma_0 + \Delta_\beta$, where $\Delta_\beta \neq 0$. It is useful to trade the parameter $D$ for another parameter $\beta$, which is introduced via

$$D = \frac{2}{3} \beta \sigma_0 a^2 A_0^2 e^{-2a\sigma_0}.$$  

(C.6)

The condition that $V_{\mathrm{up}}$ uplifts the AdS minimum to dS is now easily expressed through the requirement that $\beta \gtrapprox 1 + 2\Delta_\beta/\sigma_0$ (which is very close to, but not exactly 1). In what
follows we assume that this condition is fulfilled and therefore the minimum is dS. The equation \( \partial_r (V_{\text{AdS}} + V_{\text{up}}) = 0 \) leads to the critical point

\[
\sigma_{\text{up}} = -\frac{1}{4 a A_0} \left[ 7 A_0 + 3 W_0 e^{a \sigma_{\text{up}}} - \sqrt{(A_0 - 3 W_0 e^{a \sigma_{\text{up}}})^2 - \frac{24 D e^{a \sigma_{\text{up}}}}{a}} \right].
\] (C.7)

This is, like equation (C.4), a transcendental equation and has to be solved numerically. To get some analytical control, we use the following trick. We substitute \( D = \frac{A_0}{r^2} \)(C.6) and equation (C.4). Then we solve the resulting equation for \( \Delta \beta \). This can be done by expanding \( e^{a \Delta \beta} \approx 1 + a \Delta \beta \) so that the equation is no longer transcendental. Eventually, the only transcendental equation that we have to solve numerically is equation (C.4), and we arrive at an analytical expression for \( \Delta \beta \).

The resulting expression for \( \Delta \beta \) is a bit long, so we write its expansion in \( 1/a \sigma_0 \), which reads

\[
\Delta \beta \approx \frac{\beta}{a^2 \sigma_0} + \frac{\beta (4 \beta - 5)}{2 a^3 \sigma_0^2} + \ldots,
\] (C.8)

in very good agreement with the numerical calculation. We note that this is equivalent to an expansion in \( D/(a W_0^2) \) of equation (C.7); in fact from equation (C.6) one sees that \( D \) is suppressed with respect to \( W_0^2 \) by a factor \( 1/\sigma_0 \). This expansion would give the transcendental equation

\[
\sigma_{\text{up}} = -\frac{3 A_0 + W_0 e^{a \sigma_{\text{up}}}}{2 a A_0} - \frac{3 D e^{2 a \sigma_{\text{up}}}}{a^2 A_0 (A_0 - 3 W_0 e^{a \sigma_{\text{up}}})} + \ldots.
\] (C.9)

### C.2. Shift of critical volume through a D3-brane

In this section we take into account also a dynamical D3-brane and calculate its effect on the minimum of the potential in the \( \sigma \) direction that we call \( \sigma_c \). The potential is given in equation (39). We expand \( \partial_r V = 0 \) for small \( r \) (again this is an \( r^2/\sigma \) or an \( r^{3/2}/(\mu n) \) expansion). Solving for \( \sigma_c \), one finds

\[
\sigma_c(D, r) = \sigma_c(r = 0) + \sigma_c^{(1)} r + \sigma_c^{(3/2)} r^{3/2} + \ldots
\]

\[
= \sigma_{\text{up}} + \frac{9 (A_0 + 3 W_0 e^{a \sigma_{\text{up}}})}{8 a^2 \mu^2 n^2 (A_0 - 3 W_0 e^{a \sigma_{\text{up}}})} r
\]

\[
- \frac{3 (A_0^2 + 2 A_0^2 W_0 e^{a \sigma_{\text{up}}} + 3 W_0^2 e^{2 a \sigma_{\text{up}}})}{2 a A_0 \mu n (A_0 - 3 W_0 e^{a \sigma_{\text{up}}})} r^{3/2} + \ldots,
\] (C.10)

where \( \sigma_c(r = 0) \equiv \sigma_{\text{up}} \) coincides with the critical volume investigated in the previous section and in \( \sigma_c^{(1)} \) and \( \sigma_c^{(3/2)} \) we have neglected terms suppressed by a factor of order \( D/W_0^2 \) (see equation (C.6)). As we did in the last section we substitute \( D \) and \( W_0 \), using equation (C.6) and equation (C.4). Then we solve for \( \Delta_c = \sigma_c - \sigma_{\text{up}} \). The result is

\[
\Delta_c = \frac{r^{3/2}}{a \mu n} + \ldots.
\] (C.11)
To summarize, we have estimated analytically the dependence of the minimum on the uplifting and on the D3-brane position; this is at leading order

\[ \sigma_c = \sigma_0 + \Delta \beta + \Delta r, \]
\[ \simeq \sigma_0 + \frac{\beta}{a^2 \sigma_0} + \frac{r^{3/2}}{a \mu n} + \cdots. \]  

(C.12)

Appendix D. Sign of the \( r^{3/2} \) term

In this appendix we show that the expansion of the scalar potential has a negative term at order \( r^{3/2} \). This term determines the negative curvature of the potential for small \( r \).

In fact, the inflaton potential equation (57) also has a term proportional to \( r \), which, however, does not contribute to the curvature.

The explicit values for \( V_{\text{dS}}^{(3/2)} \) and \( \Delta V^{(3/2)} \) are

\[ V_{\text{dS}}^{(3/2)} = -\frac{[3a^2 A_0^2 e^{-2 a \sigma_0} (a \sigma_c + 6) + 9 D + 18 A_0 a W_0 e^{-a \sigma_0}]}{18 a \mu n \sigma_0^3}, \]  

(D.1)

\[ \Delta V^{(3/2)} = -\frac{A_0^2 a e^{-2 a \sigma_0}}{4 \mu n \sigma_0^3}. \]  

(D.2)

To see that \( V^{(3/2)} = V_{\text{dS}}^{(3/2)} + \Delta V^{(3/2)} < 0 \) we substitute equations (C.6) and (C.4) into equation (D.1) and expand in \( \Delta \beta = \sigma_{\text{up}} - \sigma_0 \). This gives

\[ V_{\text{dS}}^{(3/2)} \simeq -\frac{A_0^2 a e^{-2 a \sigma_0} (3 - 2 \beta)}{6 \mu n \sigma_0^2} + \cdots. \]  

(D.3)

Therefore

\[ \frac{V_{\text{dS}}^{(3/2)}}{\Delta V^{(3/2)}} \simeq -\frac{4(3 - 2 \beta)}{6} \gg -1/2, \]  

(D.4)

for \( \beta \gtrsim 1 \). We are thus left with

\[ V^{(3/2)} = -\frac{A_0^2 a e^{-2 a \sigma_0}}{12 \sigma_0^2 \mu n} (4 \beta - 3) < 0, \]  

(D.5)

in agreement with equation (E.2).

Appendix E. Maximum and minimum of \( V(\phi) \)

In this section we show that in the case of the Kuperstein embedding, the effective potential in the \( r \) (canonically \( \phi \)) direction has always a maximum and a minimum (in a special case they coincide). Our starting point is the inflaton potential obtained

\[ V(\phi) = \frac{A_0^2 a^2 e^{-2 a \sigma_0}}{6 \sigma_0} (\beta - 1) + \phi \frac{9 A_0^2 e^{-2 a \sigma_0} M_{\text{Pl}}^2}{16 T_{D3}^2 \sigma_0^{3/2} \mu^2 n^2} \]
\[ - \frac{\phi^{3/2}}{12 \sigma_0^{5/4} T_{D3}^{3/4} \mu n} (4 \beta - 3) + \frac{\phi^2}{3 M_{\text{Pl}}^2} \frac{A_0^2 a^2 e^{-2 a \sigma_0}}{6 \sigma_0} (\beta - 1), \]  

(E.1)
Figure E.1. The plot shows the discriminant, equation (E.3), as a function of \( \beta \) (without performing the \( \Delta/\sigma_0 \) expansion). When the discriminant is zero the minimum and maximum of the potential coincide and we get a flat inflection point.

which is the potential equation (58) with

\[
\begin{align*}
\Lambda &= \frac{A_0^2 a^2 e^{-2a\sigma_0}}{6\sigma_0} (\beta - 1), \\
C_1 &= \frac{9A_0^2 e^{-2a\sigma_0} M_{Pl}^2}{16T_{D3}^{3/2} \sigma_0^{3/2} \mu^2 n^2}, \\
C_3/2 &= \frac{A_0^2 a e^{-2a\sigma_0}}{12\sigma_0^{5/2} T_{D3}^{3/4} \mu n} (4\beta - 3), \\
C_2 &= \frac{M_{Pl}^2}{3} \Lambda.
\end{align*}
\]  
\text{(E.2)}

The first derivative of the potential is a quadratic polynomial in \( \sqrt{\phi} \). There are two extrema (a maximum and a minimum) when the discriminant is positive, i.e. \( 9C_{3/2} - 32C_1 C_2 > 0 \). Explicitly, the discriminant reads

\[
9C_{3/2} - 32C_1 C_2 = \frac{A_0^2 a^2 e^{-2a\sigma_0}}{64\sigma_0^{5/2} T_{D3}^{3/2} \mu^2 n^2} (4\beta - 5)^2 \geq 0.
\]  
\text{(E.3)}

This quantity is always non-negative, so \( V(\phi) \) will always possess a minimum and a maximum. It is evident that for a particular value of \( \beta \) the discriminant becomes zero, in which case the maximum and minimum coincide and a flat inflection point occurs. We have plotted the discriminant, without expanding it, in figure E.1. The plot confirms the result of our leading order calculation.

Note added. Almost simultaneously with the submission of this paper, two other papers appeared [11, 12] which address the same issue and come to similar conclusions. The second version of our paper corrected a mistake in the angular minimization of our earlier version. The corrected calculation leads to the conclusion that fine-tuning to generate inflation is possible for the Kuperstein but not the Ouyang embedding in agreement with [10]–[12]. We thank James Cline for helpful correspondence.
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