Irreducible cosmic production of relic vortons

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Received November 9, 2020
Revised February 1, 2021
Accepted February 11, 2021
Published March 29, 2021

Abstract. The existence of a scaling network of current-carrying cosmic strings in our Universe is expected to continuously create loops endowed with a conserved current during the cosmological expansion. These loops radiate gravitational waves and may stabilise into centrifugally supported configurations. We show that this process generates an irreducible population of vortons which has not been considered so far. In particular, we expect vortons to be massively present today even if no loops are created at the time of string formation. We determine their cosmological distribution, and estimate their relic abundance today as a function of both the string tension and the current energy scale. This allows us to rule out new domains of this parameter space. At the same time, given some conditions on the string current, vortons are shown to provide a viable and original dark matter candidate, possibly for all values of the string tension. Their mass, spin and charge spectrum being broad, vortons would have an unusual phenomenology in dark matter searches.

Keywords: Cosmic strings, domain walls, monopoles, particle physics - cosmology connection, dark matter theory

ArXiv ePrint: 2010.04620
1 Introduction

Cosmic strings are expected to be formed in most extensions of the standard particle physics model as stable line-like topological defects formed during high temperature, $T_{\text{ini}}$ say, symmetry breaking phase transitions in the early Universe \cite{1}. This occurs whenever a symmetry $G$ is broken down to a smaller one $H$ provided the first homotopy group of the quotient group $G/H$ (vacuum manifold) is non-trivial, producing similarly non-trivial topological solutions for the symmetry-breaking Higgs field. The scaling evolution of cosmic string networks (see e.g. ref. \cite{2} and references therein) means that they are present throughout the evolution of the Universe, possibly giving rise to numerous different observational signatures, such as line-like discontinuities in temperature in the Cosmic Microwave Background (CMB), or bursts of gravitational waves \cite{3,4,5}. These very much sought-for signatures in turn lead to strong constraints on the string tension $G\mu$.

Most studies of cosmic strings suppose they are structureless, with equal energy per unit length and tension, and therefore they are expected to be well described by a no-scale 2-dimensional worldsheet action, i.e. the Nambu-Goto action. This is no longer the case if, as first realised by Witten \cite{6,7}, particles coupled to the string-forming Higgs field can condense in the string core and subsequently propagate along the worldsheet. The resulting strings thus behave like current carrying wires and are endowed with a much richer structure \cite{8,9}.
One of the simplest examples of current-carrying strings is that of a $U(1)_R \times U(1)_Q$ gauge theory with an unbroken gauge symmetry $Q$ (which might be electromagnetism, but not necessarily) and a broken symmetry $R$ [6]. This model generalises the proto-typical Abelian-Higgs model of cosmic strings behind much of the existing work on cosmic strings. At a temperature $T_{\text{ini}}$, and a cosmic time $t_{\text{ini}}$, the Higgs field $\phi$ with $Q = 0$ and $R = 1$ acquires a non-zero vacuum expectation value $|\langle \phi \rangle| \neq 0$, thereby breaking the first component $U(1)_R$ of the total invariance group; this leads to the formation of vortex lines. The field $\phi$ vanishes at the core of the string and its phase varies by an integer times $2\pi$ along any closed path around the vortex: this is the standard Kibble mechanism. If the theory contains fermions obtaining their masses from the $U(1)_Q$ broken symmetry, those form zero modes in the string core where the symmetry is restored, thereby forming a superconducting current.

The model also comprises a second scalar field $\sigma$ with $Q = 1$ and $R = 0$, the coupling potential between $\phi$ and $\sigma$ being chosen such that $\langle \sigma \rangle = 0$ in vacuum (where $|\langle \phi \rangle| \neq 0$). Under certain conditions, it is energetically favourable to have $\langle \sigma \rangle \neq 0$ at the core of the string where $\langle \phi \rangle = 0$. At a temperature $T_{\text{cur}} < T_{\text{ini}}$, and cosmic time $t_{\text{cur}} > t_{\text{ini}}$, the charged scalar field $\sigma$ thus condenses on the string and acts as a bosonic charge carrier making the string current-carrying (and in fact actually superconducting). In the present paper, we assume that the current sets in long after the string formation scale. In the language of refs. [10, 11], this means we assume the current is formed long after the friction damping regime has finished, i.e. during the radiation era. In practice, it means that we consider $T_{\text{ini}}$ (and $t_{\text{ini}}$) to be the end of the friction dominated regime.

Cosmic strings can also be produced [12, 13] in superstring theory, also forming, under specific conditions, a network similar to a Nambu-Goto network [14]. Whether or not these so-called cosmic superstrings can carry a current deserves more investigation since they have been shown to not be able to hold fermionic zero modes so that only bosonic condensates can source such a current [15]. It should, however, be mentioned that because cosmic superstrings live in a higher dimensional manifold, their motion in the extra dimensions projected into the ordinary 3 dimensional space should be describable by means of a phenomenological non-trivial equation of state [16, 17] mimicking that of a current-carrying string; this can be interpreted as moduli field condensates.

The presence of currents flowing along the strings affects the dynamics of the network, and in this paper we particularly focus on vortons [11, 18–24], namely closed loops of string which are stabilised by the angular momentum carried by the current. Vortons do not radiate classically, and here we make the assumption that they are classically stable as well (see for instance [25–27] for numerical studies of their stability). On cosmological scales, they appear as point particles having different quantized charges and angular momenta.

In this work, we extend the derivation of the vorton abundance of ref. [11] by not only considering vortons produced from pre-existing loops at $t_{\text{ini}}$, but also those vortons that may form from the loops chopped off the network at all subsequent times. In particular, we extend the work of ref. [28], in which a Boltzmann equation governing the vorton density has been derived and integrated for any loop production function (LPF), but not explicitly solved to get cosmological constraints. Let us notice that some of these new produced vortons, when created from the network, may be highly boosted. However, extrapolating the mean equation of state obtained for Nambu-Goto cosmic string loops, their momentum gets redshifted away and, on average, they behave as non-relativistic matter [29]. For this reason, the produced vortons are, as those originally considered in ref. [11], potential dark matter and cosmic rays candidates [19, 30].
The total abundance of vortons today is expected to depend on \( t_{\text{cur}} \) as well as \( t_{\text{ini}} \), and hence on the underlying particle physics model. Determining their density parameter today, say \( \Omega_{\text{tot}} \), and using the current constraints on \( \Omega_{\text{DM}} h^2 \simeq 0.12 \) will allow us to place constraints on the physics at work in the early Universe \([31]\).

The formation and build-up of a population of vortons can be studied using a Boltzmann equation \([28]\). In this paper, we extend this work by applying the framework introduced in \([32]\) to estimate quantitatively the density of vortons today. In section 2 below, we review the necessary physics underlying vorton properties, then in section 3, we evaluate the distribution of loops and vortons, in order to be able to calculate, in section 4, the actual vorton distribution and, finally, their relic abundance in section 5. We end this work by some concluding remarks.

2 Assumptions on the physics of vortons

As discussed in the introduction, we focus in this paper on cosmic strings that emerged at a temperature \( T_{\text{ini}} \) and later became current carrying at a temperature \( T_{\text{cur}} \).

For non-conducting strings, the boost invariance along the string implies that the string tension \( T \) and its energy per unit length \( \mu \) are equal and, in order of magnitude, given by \( \mu = T = m_\phi^2 \), where \( m_\phi \propto |\langle \phi \rangle| \) is the mass of the string-forming Higgs field \( \phi \). As soon as a current flows along the string, the worldsheet Lorentz invariance is broken and so is the degeneracy between the stress-energy tensor eigenvalues \( \mu \) and \( T \) \([9, 33, 34]\), the tension being reduced and the energy per unit length increased by the current in such a way that

\[
T < m_\phi^2 < \mu. 
\]

(2.1)

The equation of state of current-carrying strings \([17, 35–39]\) provides us with a saturation condition

\[
\mu - T \le m_\sigma^2 \implies 0 < \frac{\mu - T}{m_\sigma^2} \le \frac{m_\sigma^2}{m_\phi^2},
\]

(2.2)

according to which there exists a maximal spacelike current, above which it becomes energetically favoured for the condensate to flow out of the string. For a timelike current \([36, 40]\), i.e. a charge, there exists a phase frequency threshold allowing, in principle, for arbitrary large values of the charge. However, vacuum polarisation effectively reduces the integrated charge \([41]\) so that saturation holds for all possible situations.

Denoting by \( \lambda \) the Compton wavelength of the current carrier (\( \lambda \simeq m_\sigma^{-1} \)), we define the parameter \( \mathcal{R} \) by

\[
\mathcal{R} \equiv \lambda \sqrt{\mu}.
\]

(2.3)

Because \( \mu \simeq m_\phi^2 \), this quantity is approximately the ratio between the Compton wavelengths of the current carrier and the one of the string forming Higgs field, or, equivalently, \( \mathcal{R} \simeq m_\phi/m_\sigma \) which we assume to be greater than unity. Given (2.2), it is safe to assume that, at least for \( \mathcal{R} \gg 1 \), the string tension and the energy per unit length are numerically so similar that distinguishing between them is irrelevant in the forthcoming cosmological context; we will thus denote them both by the notation \( \mu \).

A current-carrying closed string loop is characterized by two classically conserved integral quantum numbers \( N \) and \( Z \), generally non-zero, which prevent the loop from disappearing completely \([42]\). As the loop loses energy through friction or radiation, it reaches a classically stable state called a *vorton* \([18]\). However, this state can decay through quantum
tunnelling if the size of the loop is comparable with the Compton wavelength of the current carrier, $\lambda$. Hence a vorton can only be stable if the current flowing along the string loop can prevent its collapse and if its proper length is much larger than $\lambda$.

Although the values of $N$ and $Z$ are initially randomly distributed, it is expected that the majority of closed loops are of nearly chiral type with almost identical quantum numbers. Besides, the loop rotation velocity $v_{\text{vort}} = \sqrt{\mathcal{T}/\mu} \simeq 1$ is roughly approximated by that of light and $|Z| \approx N$.

In the rest of the paper, we focus on such nearly chiral vortons. Using the central limit theorem, we estimate that the value of $N$ at the formation of a loop is given by

$$N_* = \sqrt{\ell_* / \lambda}. \quad (2.5)$$

In (2.5) and in the rest of this paper, a subscript * on a quantity denotes the value it had at the time of formation of the corresponding loop. Since the charge $N$ is conserved, we can, in what follows, omit the index * and simply write $N_* = N$.

To estimate the size of the vortons $\ell_0$, we first have to note that they have been shown to approach circularity. Moreover, large vortons would also tend to circularize through either gravitational or gauge field radiation, on time scales much smaller than the Hubble time. It thus seems reasonable to consider mostly circular loops, therefore described by one parameter only, namely their radius $r_0 = \ell_0/2\pi$. Vortons are also characterised by their angular momentum quantum number $J = NZ \approx N^2$. Equivalently, it is also given in terms of the energy per unit length and tension by $J = 2\pi r_0^2 \sqrt{\mathcal{T}/\mu}$, i.e. $J^2 = \mu \mathcal{T}/(4\pi^2)$. Hence for chiral vortons with $R \gg 1$

$$\ell_0 = \sqrt{2\pi / \mu} N = \sqrt{2\pi \ell_* / \lambda \mu} \approx \sqrt{\ell_* / \lambda \mu}, \quad (2.6)$$

provided $\ell_0 > \lambda$. The length $\ell_0(N)$ being itself a function of the charge $N$, this is equivalent to imposing that $N > R$. Therefore, $R$ gives also the minimal possible charge of a vorton.

Following the same procedure as in [32], we model the physics of the vortons using an arbitrary function $J$ which describes how the current-carrying loops lose energy

$$\frac{d\ell}{dt} = -\Gamma G \mu J(\ell, N), \quad (2.7)$$

$$\frac{dN}{dt} = 0, \quad (2.8)$$

in which $\Gamma \approx 50$ is a numerical factor for the emission of gravitational waves (GW) [45]. In order to model string networks with vortons, we impose the following properties on $J$:

- $J(\ell \gg \ell_0, N) \approx 1$, meaning that on scales much larger than the vorton size, the effect of the current is mostly negligible so that the dynamics of the current-carrying string is well approximated by that of a Nambu-Goto string; gravitational wave radiation is the dominant energy-loss mechanism and we neglect other such mechanisms.

- $J(\ell \ll \ell_0, N) \approx 0$ if $\ell_0 > \lambda$, meaning that the angular momentum carried by the current prevents the loop from shrinking, provided the loop is large enough to prevent quantum tunnelling.
Figure 1. At time $t_{\text{ini}}$ and temperature $T_{\text{ini}}$ a network of strings forms with an initial distribution. At the later time $t_{\text{cur}}$ the strings become current-carrying, and vortons can form. At all times, loops can be produced from long strings and larger loops with a given loop production function.

We will consider a smooth form of $J$, regulated by a parameter $\sigma$, in particular
\[ J(\ell, N) = \frac{1}{2} \left\{ 1 + \tanh \left[ \frac{\ell - \ell_0(N)}{\sigma} \right] \right\}. \] (2.9)

We call vortons all the loops with sizes $\ell \leq \ell_0(N)$ and $N > R$. In the limit $\sigma \to 0$, $J(\ell, N)$ reduces to $\Theta[\ell - \ell_0(N)]$, and the vortons accumulate around $\ell_0(N)$.

Let us mention that our approach, and results, differ from the vorton abundances derived in refs. [20, 21]. These latter references were concerned with the extreme limit in which the current carrier condensation and string forming times are similar ($R \approx 1$ in our notation). For this reason, they were not concerned with the emission of gravitational waves. Indeed, in the limit $R \to 1$, strong currents have been shown to dampen the loop oscillations and this allows for a population of vortons to be rapidly created (soon after the string forming phase transition). The vortons considered in refs. [20, 21] are of this kind only. Let us recall that the current-carrier particles are trapped on the string worldsheet by means of a binding potential. As such, when there are strong currents, there is always the possibility that they tunnel out [36]. Such an instability could drastically affect the current, and hence the mechanism by which the vortons considered in refs. [20, 21] are formed. On the contrary, the vortons we are considering here carry weak currents and our results are only valid in the domains for which $R > 1$. The damping mechanism by which the weak current-carrying loops become vortons is the emission of gravitational waves (as in ref. [11]).

Having recalled the basic properties of vortons and their dynamics, we now turn to the expected distributions of loops of various kinds, including those ending up as vortons.

3 Distribution of loops and vortons

In the following sections, we extend a statistical method originally based on the Boltzmann equation [2, 28, 46, 47] to study current carrying strings. Our aim is to find the number density of vortons, marginalized over their charge $N$, with length $\ell$ at time $t > t_{\text{cur}}$, given some initial loop distribution at time $t_{\text{ini}}$ and some assumptions about the loop production function (see figure 1).

3.1 Continuity equation for the flow of loops in phase space

Let $d^2N(\ell, t, N)/d\ell dN$ be the number density of loops with length $\ell$ and charge $N$ at time $t$. In an expanding universe with scale factor $a(t)$, and taking into account the fact that loops lose length at a rate which depends on their length as expressed through equation (2.7), the continuity equation for the number density of loops is given by [32, 46]
\[
\frac{\partial}{\partial t} \left[ a^3 \frac{d^2N}{d\ell dN}(\ell, t, N) \right] - \Gamma G_{\mu} \frac{\partial}{\partial \ell} \left[ a^3 J(\ell, N) \frac{d^2N}{d\ell dN}(\ell, t, N) \right] = a^3 P(\ell, t, N). \] (3.1)
Here $\mathcal{P}(\ell, t, N)$ is the charged loop production function (LPF), namely the rate at which loops of length $\ell$ and charge $N$ are formed at time $t$ by being chopped off the string network and we will specify it below. Note that this equation is exactly equivalent to that of ref. [28], as we explain in details in appendix A.

The solution to equation (3.1) can be obtained in integral form following a similar procedure to that explained in ref. [32], though one must take into account the new independent variable $N$. Upon multiplying by $\mathcal{J}(\ell, N)$, equation (3.1) becomes

$$\frac{\partial g}{\partial t}(\ell, t, N) - \Gamma G \mu \mathcal{J}(\ell, N) \frac{\partial g}{\partial \ell}(\ell, t, N) = a^3(t) \mathcal{J}(\ell, N) \mathcal{P}(\ell, t, N),$$

(3.2)

where we have defined

$$g(\ell, t, N) \equiv a^3 \mathcal{J}(\ell, N) \frac{d^2 N}{d\ell dN}.\quad (3.3)$$

The change of variables $\{\ell, t, N\} \rightarrow \{\xi, \tau, N\}$, with

$$\xi \equiv \int \frac{d\ell}{\mathcal{J}(\ell, N)} \quad \text{and} \quad \tau \equiv \Gamma G \mu t,$$

(3.4)

enables equation (3.2) to be written in the simpler form

$$\frac{\partial g(\xi, \tau, N)}{\partial \tau} - \frac{\partial g(\xi, \tau, N)}{\partial \xi} = \frac{a^3(\tau)}{\Gamma G \mu} \mathcal{J}(\xi, N) \mathcal{P}(\xi, \tau, N).$$

(3.5)

Upon using light cone type coordinates

$$u \equiv \frac{1}{2} (\tau - \xi) \quad \text{and} \quad v \equiv \frac{1}{2} (\tau + \xi),$$

(3.6)

it follows that equation (3.1) reduces to

$$\frac{\partial g(u, v, N)}{\partial u} = \frac{a^3(u, v)}{\Gamma G \mu} \mathcal{J}(u, v, N) \mathcal{P}(u, v, N),$$

(3.7)

which can be integrated between $t_{\text{cur}}$ and $t$, or in terms of the variable $u = -v + \tau = -v + \Gamma G \mu t$, between $u_{\text{cur}} = -v + \tau_{\text{cur}} = -v + \Gamma G \mu t_{\text{cur}}$ to $u$,

$$g(u, v, N) - g(-v + \Gamma G \mu t_{\text{cur}}, v, N) = \int_{-v + \Gamma G \mu t_{\text{cur}}}^{u} \frac{a^3(u', v)}{\Gamma G \mu} \mathcal{J}(u', v, N) \mathcal{P}(u', v, N) du',$$

(3.8)

the integral in equation (3.8) being calculated with $v$ constant. Rewritten in terms of $d^2N(\ell, t, N)/d\ell dN$ using equation (3.3) finally gives

$$a^3(t) \mathcal{J}(\ell, N) \frac{d^2 N}{d\ell dN} = a^3(t_{\text{cur}}) \mathcal{J}(t_{\text{cur}}, N) \frac{d^2 N}{d\ell dN} (t_{\text{cur}}, \ell_{\text{cur}}, N)$$

$$+ \int_{-v + \Gamma G \mu t_{\text{cur}}}^{u} \frac{a^3(u', v)}{\Gamma G \mu} \mathcal{J}(u', v, N) \mathcal{P}(u', v, N) du'.$$

(3.9)

Here $\ell_{\text{cur}}$ is the size of the loops at condensation and is a function $\ell_{\text{cur}}(\ell, t, N)$. It is found using the variable $v = \tau + \xi$ of equation (3.6) which is a constant along the flow, namely $\ell_{\text{cur}}$ is a solution of

$$\xi(\ell_{\text{cur}}, N) = \xi(\ell, N) + \Gamma G \mu (t - t_{\text{cur}}).$$

(3.10)
The solution of the continuity equation (3.1) is therefore given by equation (3.9). On the right-hand-side, we recognise two terms. The first are the loops left over from the pre-existing loop distribution at the time of condensation, \( t = t_{\text{cur}} \). The second term contains those loops which are produced from the string network at time \( t > t_{\text{cur}} \). As we will see in more detail in section 4, each of these distributions contain three kinds of loops [11):

1. **Doomed loops**: these loops have an initial size which is too small to support a current, and hence they decay through gravitational radiation never becoming vortons. They are characterised by quantum numbers \( N < R \).

2. **Proto-vortons**: these are loops which are initially large enough to be stabilised by a current (thus \( N > R \)), but have not yet reached the vorton size \( \ell_0 \).

3. **Vortons**: these are all those proto-vortons which have decayed by gravitational radiation to become vortons. Hence vortons have \( N > R \), and in the limit \( \sigma \rightarrow 0 \), they accumulate with length \( \ell_0(N) \).

Our aim in the following is to extract these different distributions. Each will contain two contributions: those formed from the initial distribution i.e. coming from the first term in equation (3.9), and those produced at later times from being chopped off the string network, i.e. coming from the second term in equation (3.9). In the case of vortons, we call these two families “relaxed vortons” and “produced vortons”, respectively. In section 5, we will use these to determine their relic density and put constraints on \( G\mu \) and \( R \).

### 3.2 The loop distribution at condensation

A first step is to specify the loop distribution at \( t_{\text{cur}} \). The strings are assumed to form at a temperature \( T_{\text{ini}} \) corresponding to a time \( t_{\text{ini}} \) in the early Universe. At all times \( t_{\text{ini}} < t < t_{\text{cur}} \), that is before condensation, they behave as standard Nambu-Goto strings, see figure 1. Hence the loop distribution is the canonical one, i.e. contains a population of loops formed at \( t_{\text{ini}} \) and another population of scaling loops created from the long strings and larger loops [48].

The main simplifying assumption of our work is to assume a Dirac distribution for the loop production function, namely

\[
P(\ell, t) = C t^{-5} \delta \left( \frac{\ell}{t} - \alpha \right),
\]

with \( C = 1 \) and \( \alpha = 0.1 \) as to match the Kibble, or one scale, model [1]. Hence all the produced loops that are chopped off the network are assumed to be of the same size, given by the fraction \( \alpha \) of \( t \), which is, up to a constant of order unity, the horizon size. This assumption allows us to analytically solve for the produced vorton distribution later on. However, we stress that more realistic loop production functions, such as the Polchinski-Rocha one [2, 47, 49–51], produce smaller loops while matching in amplitude with the Dirac LPF for \( \ell / t = \alpha \) [29, 48]. Therefore, when gravitational wave emission from loops is accounted for (which is the case here), the resulting scaling loop distributions end up being quite similar over the length scales \( \ell > \Gamma G\mu t \). They may, however, differ significantly on smaller length scales, namely for \( \gamma_c t < \ell < \Gamma G\mu t \), where \( \gamma_c \) stands for the length scale at which gravitational backreaction damps the LPF [2]. For Nambu-Goto strings, this length scale is expected to verify \( \gamma_c \ll \Gamma G\mu \) [52, 53]. Therefore, our results derived here from a Dirac LPF should provide a robust lower bound for all the others LPF, and may also be directly applicable to the Polchinski-Rocha ones but only in the limit in which \( \gamma_c \simeq \Gamma G\mu \).
Under these assumptions, the resulting distribution of cosmic string loops at time \( t_{\text{cur}} \) is given by [48]

\[
\frac{dN}{d\ell}(\ell, t_{\text{cur}}) = C \left( \frac{t_{\text{cur}}}{\ell + \Gamma G \mu t_{\text{cur}}} \right)^{3/2} \Theta(\alpha t_{\text{cur}} - \ell) \Theta [\ell + \Gamma G \mu t_{\text{cur}} - t_{\text{ini}}(\alpha + \Gamma G \mu)]
\]

\[
+ C_{\text{ini}} \left( \frac{t_{\text{ini}}}{\ell} \right)^{5/2} (\alpha + \Gamma G \mu) t_{\text{ini}} - \ell - \Gamma G \mu t_{\text{cur}}) \Theta(t_{\text{ini}} - \ell) \Theta [\ell + \Gamma G \mu t_{\text{cur}}].
\]

The first term is the scaling loop distribution associated with the Dirac LPF of equation (3.11). The second term is the initial distribution of loops at \( t_{\text{ini}} \) associated with the random walk model of Vachaspati-Vilenkin [54]. Assuming the random walk to be correlated over a length scale \( \ell_{\text{corr}} \), one has [54]

\[
C_{\text{ini}} \simeq 0.4 \left( \frac{t_{\text{ini}}}{\ell_{\text{corr}}} \right)^{3/2}.
\]

A natural value for \( \ell_{\text{corr}} \) is obtained by assuming that it is given by the thermal process forming the strings, namely \( \ell_{\text{corr}} = 1/T_{\text{ini}} \). We will, however, discuss various other possible choices in section 5.

At the time of condensation \( t_{\text{cur}} \), the loops acquire quantum numbers \( N \), and we assume again a Dirac distribution for the generated charge:

\[
\frac{d^2N}{d\ell dN}(\ell, t_{\text{cur}}, N) = \frac{dN}{d\ell}(\ell, t_{\text{cur}}) \delta \left( N - \sqrt{\ell}/\lambda \right).
\]

This is in agreement with refs. [11, 28] and motivated by the fact that, if a thermal process of temperature \( T_{\text{cur}} = 1/\lambda \) is at work during current condensation, the conserved number \( N \) laid down along the string should be given by a stochastic process of root mean squared value close to \( \sqrt{\ell}/\lambda \).

String formation at \( t_{\text{ini}} \) and current condensation at \( t_{\text{cur}} \) are assumed to occur in the radiation era. In the following we will use as model parameters \( G\mu \) and \( R \). The current condensation redshift can be determined using entropy conservation:

\[
1 + z_{\text{cur}} = \left( \frac{q_{\text{cur}}}{q_0} \right)^{1/3} \frac{T_{\text{cur}}}{T_{\text{cmb}}},
\]

where \( q_{\text{cur}} = q(z_{\text{cur}}) \), and \( q_0 = q(z = 0) \), denotes the number of entropic relativistic degrees of freedom at the time of current condensation, and today, respectively. In the following, we consider \( T_{\text{cur}} \) to be given by

\[
T_{\text{cur}} = \frac{1}{\lambda} = \frac{\sqrt{\mu}}{R},
\]

and we take \( T_{\text{cmb}} = 2.725 \) K. In order to solve equation (3.15) for \( z_{\text{cur}} \), we have used the tabulated values of \( q(z) \) associated with the thermal history in the Standard Model and computed in ref. [55]. Still from entropy conservation, the redshift associated with the formation of the string network (at the temperature \( T_{\text{ini}} \)) is given by

\[
1 + z_{\text{ini}} = \left( \frac{q_{\text{ini}}}{q_0} \right)^{1/3} \frac{T_{\text{ini}}}{T_{\text{cmb}}},
\]

where

\[
T_{\text{ini}} = \sqrt{\mu} = RT_{\text{cur}}.
\]
4 Cosmological distribution of vortons

From equation (3.9), we can determine the distribution \( \frac{dN}{d\ell} \) of relaxed vortons and produced vortons. Both of these being stable, they will contribute to the relic content of the universe.

Regarding the distributions of doomed loops and proto-vortons, these could be important for some observational effects of strings, for instance the stochastic gravitational wave background, but they cannot contribute significantly to the dark matter content of the Universe \([2]\). Their distributions are determined from equation (3.9) through

\[
\left. \frac{dN}{d\ell} \right|_{\text{doom}} (\ell, t) \equiv \int dN \frac{d^2N}{d\ell dN} (\ell, t, N) \Theta(R - N),
\]

and

\[
\left. \frac{dN}{d\ell} \right|_{\text{proto}} (\ell, t) \equiv \int dN \Theta(N - R) \frac{d^2N}{d\ell dN} (\ell, t, N) \Theta[\ell - \ell_0(N)],
\]

and are given in appendix B.

In order to determine the vorton distribution, we recall that a vorton is a loop with topological number \( N > R \) and size \( \ell \leq \ell_0(N) \) if \( \sigma > 0 \). In the limit \( \sigma \to 0 \), the charge \( N \) of the vorton is proportional to its length \( \ell_0(N) = N/\sqrt{\mu} \). In order to deal correctly with the singular behaviour in the limit \( \sigma \to 0 \), we firstly express the vorton distribution in terms of the charge \( N \), namely calculate \( \frac{dN}{dN} \), then take the limit \( \sigma \to 0 \), and finally determine \( \frac{dN}{d\ell} \) through a simple change of variables since \( \ell = \ell_0 = N/\sqrt{\mu} \).

Our starting point is therefore

\[
\frac{dN}{dN} (t, N) \equiv \Theta(N - R) \int d\ell \frac{d^2N}{d\ell dN} (\ell, t, N) \Theta[\ell - \ell_0(N)],
\]

which we calculate for both relaxed and produced vortons below.

4.1 Relaxation term

The distribution of the vortons coming from the initial conditions at the condensation is determined from (4.3), substituting the first term of equation (3.9), together with the initial distribution of loops in equation (3.14). This gives

\[
\frac{dN}{dN} (t, N) = \Theta(N - R) \int d\ell \frac{d^2N}{d\ell dN} (\ell, t, N) \Theta[\ell_0(N) - \ell],
\]

in which \( \ell_0(N) \), given in equation (3.10), is the size of the loop at condensation. In order to integrate over the Dirac delta distribution, we change integration variable from \( \ell \) to

\[
y = N - \sqrt{\frac{\ell_{\text{cur}}}{\lambda}},
\]

with corresponding Jacobian

\[
\frac{dy}{d\ell} = -\frac{1}{2\sqrt{\lambda \ell_{\text{cur}}}} \frac{\partial \ell_{\text{cur}}}{\partial \ell} \bigg|_{t, N} = -\frac{1}{2\sqrt{\lambda \ell_{\text{cur}}} \mathcal{J}(\ell_{\text{cur}}, N)},
\]
In the limit \( \sigma \rightarrow 0 \), the size of a vorton is \( \ell = \ell_0(N) = N/\sqrt{\mu} \), and equation (3.10) simplifies to
\[
\ell_{\text{cur}}[\ell_0(N), t, N] = \Gamma G \mu (t - t_{\text{cur}}) + \ell_0(N).
\] (4.8)

Finally, using \( d\mathcal{N}/d\ell = \sqrt{\mu} \, d\mathcal{N}/dN \), the vorton distribution generated from the initial loop distribution at \( t_{\text{cur}} \) is given by
\[
\frac{d\mathcal{N}}{d\ell} |_{\text{vort,rel}} (\ell, t) = 2\lambda \ell \left[ \frac{a^3(t_{\text{cur}})}{a^3(t)} \right] \frac{d\mathcal{N}}{d\ell} (\lambda \mu \ell^2, t_{\text{cur}}) \Theta \left[ \Gamma G \mu (t - t_{\text{cur}}) + \ell - \lambda \mu \ell^2 \right] \Theta (\ell - \lambda).
\] (4.9)

This distribution scales like matter (modulo the time-dependence in the \( \Theta \)-functions). This term was already derived in ref. [28], and our results agree though the approach is different.

We now turn to the vorton population sourced by loops chopped off from the network, namely from the second term in equation (3.9).

### 4.2 Production term

After the condensation, all the strings and loops carry a current, which implies that all new loops formed from the network will inherit the charge density carried by their mother strings. As a result, the charged loop production function is still given by equation (3.11), modulated by the charge density distribution, i.e.
\[
\mathcal{P} (\ell, t, N) = C t^{-5} \delta \left( \frac{\ell}{t} - \alpha \right) \delta \left( N - \sqrt{\frac{\ell}{\lambda}} \right) \Theta (t - t_{\text{cur}}).
\] (4.10)

Substituting into the last term of equation (3.9) (see [32] for more details) gives the number density
\[
\frac{d^2\mathcal{N}}{d\ell dN} (\ell, t, N) = \frac{C}{\mathcal{J} (\ell, N)} \left[ \frac{a(t_*)}{a(t)} \right]^3 t_*^{-4} \frac{\mathcal{J} (\alpha t_*, N)}{\alpha + \Gamma G \mu \mathcal{J} (\alpha t_*, N)} \delta \left( N - \sqrt{\frac{\alpha t_*}{\lambda}} \right) \Theta (t_* - t_{\text{cur}}).
\] (4.11)

where \( t_* (\ell, t, N) \) is the time of loop formation, obtained by solving
\[
\Gamma G \mu t_* + \xi (\alpha t_*, N) = \Gamma G \mu t + \xi (\ell, N),
\] (4.12)

which again follows from the fact that \( 2\nu = \Gamma G \mu t + \xi (\ell, N) \) is a conserved quantity during the lifetime of the loops. The definition in equation (4.3) then gives
\[
\frac{d\mathcal{N}}{dN} |_{\text{vort,prod}} = \Theta (N - \lambda \sqrt{\mu})
\] (4.13)
\[
\times \int_{-\infty}^{\ell_0(N)} d\ell \frac{C}{\mathcal{J} (\ell, N)} \left[ \frac{a(t_*)}{a(t)} \right]^3 t_*^{-4} \frac{\mathcal{J} (\alpha t_*, N)}{\alpha + \Gamma G \mu \mathcal{J} (\alpha t_*, N)} \delta \left( N - \sqrt{\frac{\alpha t_*}{\lambda}} \right) \Theta (t_* - t_{\text{cur}}).
\]

We again integrate the Dirac delta distribution by means of the change of variable
\[
\tilde{y} = N - \sqrt{\frac{\alpha t_*}{\lambda}},
\] (4.14)
Figure 2. Diagram ($\ell, t$) for the different types loops/vortons. The left panel is for $G_\mu = 10^{-16}$ and the right panel for $G_\mu = 10^{-19}$. The dark-dashed vertical line is the time of condensation, when strings become superconducting. The diagonal dark line represents $\ell = \alpha t$ (with $\alpha = 0.1$) the size at which loops are produced. The orange horizontal line shows the value of $\lambda$.

with corresponding Jacobian

$$\frac{d\bar{y}}{d\ell} = -\sqrt{\frac{\alpha}{\lambda}} \frac{1}{2\sqrt{t_\ast}} \frac{\partial t_\ast}{\partial \ell} \bigg|_{t,N} = -\sqrt{\frac{\alpha}{\lambda}} \frac{1}{2\sqrt{t_\ast}} J(\alpha t_\ast, N) \left[ \frac{\partial}{\partial \ell} \right] \left[ \frac{\partial}{\partial t} \right] \left[ \frac{\partial}{\partial \lambda} \right] \left[ \frac{\partial}{\partial N} \right] (4.15)$$

Thus equation (4.13) gives

$$\frac{dN}{dN}_{vort, prod} = \Theta(N - \lambda \sqrt{\mu}) \times 2\lambda \mu \ell \left[ \frac{a(\lambda \mu \ell^2/\alpha)}{a(t)} \right]^{\frac{3}{2}} \left[ \frac{\lambda N^2}{\alpha} \right]^{-4} \Theta(\lambda N^2 - \alpha t_{\text{cur}}) \Theta \left[ t_\ast(\ell_0(N), t, N) - \lambda N^2 \right] \left. \left[ \frac{\partial}{\partial \ell} \right] \left[ \frac{\partial}{\partial \lambda} \right] \left[ \frac{\partial}{\partial N} \right] \right|_{t,N} (4.16)$$

In the limit $\sigma \to 0$, equation (4.12) reduces to

$$\left( \alpha + \Gamma G_\mu \right) t_\ast = \ell_0(N) + \Gamma G_\mu t, \quad (4.17)$$

and, using the fact that vortons have size $\ell = \ell_0(N) = N/\sqrt{\mu}$, it follows that the produced vorton distribution is given by

$$\frac{dN}{d\ell}_{vort, prod} = \frac{2\lambda \mu \ell}{\alpha} \left[ \frac{a(\lambda \mu \ell^2/\alpha)}{a(t)} \right]^{\frac{3}{2}} \left[ \frac{\lambda \mu \ell^2}{\alpha} \right]^{-4} \Theta(\lambda \mu \ell^2 - \alpha t_{\text{cur}}) \Theta \left( \frac{\Gamma G_\mu t + \ell}{\alpha + \Gamma G_\mu} - \frac{\lambda \mu \ell^2}{\alpha} \right) \Theta(\ell - \lambda), \quad (4.18)$$

which again scales as matter.

In figure 2 we show the different regions of ($\ell, t$)-space which are populated by either relaxed or produced vortons, and also proto-vortons and doomed loops (see appendix B). Essentially, for vortons, these are fixed by the $\Theta$-functions in equation (4.18) and equation (4.9). In particular we observe that for

$$G_\mu > \frac{\alpha G t_{\text{cur}}}{\lambda^3} \iff \mathcal{R} > \frac{\alpha t_{\text{cur}}}{\lambda}, \quad (4.19)$$
there are no relaxed vortons produced, explaining the differences between the two panels of figure 2.

A consequence of the different $\Theta$-functions in equation (4.18) is that when evaluating $t_*$, the formation time of loops, it turns out that all vortons were produced initially during radiation era. If one imposes that the loop production function of equation (4.10) is only valid for $t < t_{eq}$, one finds that equation (4.18) is multiplied by the Heaviside function $\Theta(\alpha t_{eq} - \lambda \mu \ell^2)$.

5 Relic abundance

In the previous sections we have established that the number density of vortons produced during the radiation era contains two components, namely the relaxed vortons with length distribution given in equation (4.9), and the produced vortons with length distribution given in equation (4.18).

5.1 Analytic estimates

In order to estimate the density parameter associated with the relic vortons today, we can use the results of the previous section evaluated at present time $t = t_0$. The density parameter for each population is defined by

$$\Omega \equiv \frac{8\pi G \mu}{3H_0^2} \int_0^\infty \frac{dN}{d\ell}(\ell, t_0) \, d\ell. \quad (5.1)$$

Starting with the contribution of the relaxed vortons, from equation (4.9), estimated today, the dimensionless loop distribution reads

$$t_0^4 \frac{dN}{d\ell}|_{vort, rel} = \frac{2R^2}{(1 + z_{cur})^3} \lambda \left( \frac{t_0}{t_{cur}} \right)^4 t_{cur}^4 \frac{dN}{d\ell}(R^2 \ell^2/\lambda, t_{cur}) \Theta(\ell - \lambda) \Theta[t_*(t_0) - \ell], \quad (5.2)$$

where we have introduced the typical length

$$\ell_*(t_0) \equiv \frac{\lambda}{2R^2} \left[ 1 + \sqrt{1 + 4R^2 \Gamma G \mu (t_0 - t_{cur})/\lambda} \right], \quad (5.3)$$

solution of the quadratic equation appearing in the argument of the first Heaviside function in equation (4.9). As explicit in the above expression, this is the maximal possible length of a relaxed vorton today, larger loops belonging to the (relaxed) proto-vorton distribution, see also figure 2. In this expression, the loop distribution at $t_{cur}$ is given by equation (3.12). The vorton distribution of equation (5.2) obtained by taking, in equation (3.12), $C = 0$ and $C_{ini}$ given by equation (3.13) is the one originally considered and derived in ref. [11]. We see that by considering $C \neq 0$, i.e. by including all the Nambu-Goto loops produced between $t_{ini}$ and $t_{cur}$, we are adding a new population, not considered so far, to the relaxed vorton abundance.

It is actually possible to derive an analytical expression for the density parameter of these new relaxed vortons only. Let us consider a loop distribution at $t_{cur}$ given by equation (3.12) with $C \neq 0$ and $C_{ini} = 0$. In other words, we take the extreme situation in which at $t = t_{ini}$, there is no loop at all. All loops present at $t_{cur}$ are therefore created from the network
The right panel shows the density parameter $\Omega_{\text{prod}}$ and where we have introduced the new length scales with the dimensionless numbers $\text{vortons}$, i.e. we have assumed that there is no loop at the string forming time ($C_{\text{ini}} = 0$). The left panel shows the density parameter $\Omega_{\text{ini}}$ from the population of irreducible relaxed vortons, i.e. we have assumed that there is no loop at the string forming time ($C_{\text{ini}} = 0$). These objects will be referred to as the “irreducible relaxed vortons”. Similarly, the produced vorton density distribution today is given by equation (5.2) into (5.1), one gets after some algebra

$\Omega_{\text{rel}}^\text{min} = \frac{2R^2C}{9(1 + z_{\text{cur}})^2(H_0t_{\text{cur}})^2(\bar{M}_{\text{Pl}}t_{\text{cur}})^2\Gamma G\mu} \left(\frac{\alpha + \Gamma G\mu}{\lambda} + \frac{\lambda x_{\text{max}}/\bar{\ell}_{\text{cur}}}{\lambda} + \frac{\lambda x_{\text{min}}/\bar{\ell}_{\text{cur}}}{\lambda}\right)^{3/2}$

with the dimensionless numbers

$x_{\text{max}} = \min \left(\frac{\ell_{\text{ini}}}{\lambda}, \frac{\tilde{\ell}_{\text{cur}}}{\lambda}\right), \quad x_{\text{min}} = \max \left[1, \frac{1}{\bar{R}}\sqrt{\frac{\ell_{\text{ini}}(t_{\text{cur}})}{\lambda}}\right]$, 

and where we have introduced the new length scales

$\tilde{\ell}_{\text{cur}} = \frac{\sqrt{\alpha\bar{M}_{\text{cur}}\bar{R}}}{\lambda}, \quad \tilde{\ell}_{\text{ini}}(t_{\text{cur}}) \equiv \ell_{\text{ini}}(\alpha + \Gamma G\mu) - \Gamma G\mu t_{\text{cur}}$. 

From the fact that we started with no loop at all at the string forming time $t_{\text{ini}}$, equation (5.4) is necessarily a robust lower bound for the relaxed vorton abundance today. These objects will be referred to as the “irreducible relaxed vortons”.

Similarly, the produced vorton density distribution today is given by equation (4.18) evaluated at $\ell = t_0$. The dimensionless distribution today reads

$N_{\text{vort,prod}}\left|_{t_0} = \frac{2C}{[1 + z(t_0)]^3} \left(\frac{\alpha}{\bar{R} t_0}\right)^3 \left(\frac{t_0}{\ell}\right)^7 \Theta(\ell - \tilde{\ell}_{\text{cur}}) \Theta(\ell_\text{ini}(t_0) - \ell) \Theta(\ell - \lambda)$

where we have made explicit the new length scale

$\tilde{\ell}_{\text{ini}}(t_0) = \frac{\lambda}{2R^2 \alpha + \Gamma G\mu} \left[1 + \sqrt{1 + \frac{4R^2(\alpha + \Gamma G\mu)\Gamma G\mu t_0}{\lambda}}\right]$, (5.8)
which is the analogue of $\ell_t(t_0)$ but for the produced vortons, see equation (5.3). This is the maximal possible size of a produced vorton today. Let us notice the appearance of the redshift $z(t_\ell)$, evaluated at some (past) $\ell$-dependent cosmic time

$$t_\ell \equiv \frac{R^2 \ell^2}{\alpha \lambda}. \quad (5.9)$$

Plugging equation (5.7) into (5.1), one gets

$$\Omega_{\text{prod}} = \frac{16\pi G\mu}{3 (H_0 t_0)^2} C \left( \frac{\alpha}{R^2} \right)^3 \left( \frac{t_0}{\lambda} \right)^2 \int_{y_{\text{min}}}^{y_{\text{max}}} \frac{[1 + z(t_\lambda y)]^3}{y^6} dy, \quad (5.10)$$

with

$$y_{\text{min}} \equiv \max \left( 1, \frac{\bar{\ell}_{\text{cur}}}{\lambda} \right), \quad y_{\text{max}} \equiv \frac{\bar{\ell}_\ell(t_0)}{\lambda}. \quad (5.11)$$

Equation (5.10) shows that the knowledge of the whole thermal history of the Universe through $z(t_\lambda y)$ is a priori required to accurately determine $\Omega_{\text{prod}}$. This is expected as the “time of flight” of a proto-vorton between its creation and stabilisation as a vorton depends on its size at formation. Therefore, at any given time, the population of produced vortons keeps a memory of the past history of the Universe.

The integral (5.10) can be analytically performed with some simplifying assumptions. One can consider an exact power-law expansion for the radiation and matter era together with an instantaneous transition at $t_{\text{eq}}$. Taking $a(t) \propto t^\nu$, with $\nu = \nu_{\text{rad}} \equiv 1/2$ and $\nu = \nu_{\text{mat}} \equiv 2/3$ in the radiation and matter era, respectively, one gets

$$\Omega_{\text{prod}} = \frac{16\pi G\mu}{3 (H_0 t_0)^2} C \left( \frac{\alpha}{R^2} \right)^3 \left( \frac{t_0}{\lambda} \right)^{2-3\nu_{\text{mat}}} \times \left\{ \left( \frac{R^2}{\alpha} \right)^{3\nu_{\text{rad}}} \frac{\min (y_{\text{max}}, y_{\text{eq}})^{6\nu_{\text{rad}}-5} - y_{\text{min}}^{6\nu_{\text{rad}}-5}}{5 - 6\nu_{\text{rad}}} \left( \frac{t_{\text{eq}}}{\lambda} \right)^{3(\nu_{\text{mat}}-\nu_{\text{rad}})} \right. \right. \quad (5.12)$$

$$+ \left. \left. \left( \frac{R^2}{\alpha} \right)^{3\nu_{\text{mat}}} \frac{y_{\text{max}}^{6\nu_{\text{mat}}-5} - \max (1, y_{\text{eq}})^{6\nu_{\text{mat}}-5}}{5 - 6\nu_{\text{mat}}} \right\right\},$$

where

$$y_{\text{eq}} \equiv \frac{\bar{y}_{\text{eq}}}{\lambda}, \quad \text{with} \quad \bar{y}_{\text{eq}} \equiv \frac{\sqrt{\alpha \lambda t_{\text{eq}}}}{R}. \quad (5.13)$$

Unsurprisingly, the particular cosmic time $t_{\text{eq}}$ imprints a new length scale $\bar{y}_{\text{eq}}$ in the distribution.

We have represented in figure 3 both $\Omega_{\text{rel}}^{\text{min}}$ and $\Omega_{\text{prod}}$ as a function of $(G\mu, 1/R)$ given by the equations (5.4) and (5.12). The thick green line shows the contour matching the value $\Omega_{\text{dm}} = 0.3$. For the irreducible relaxed vortons, the only additional parameter entering equation (5.4) is $z_{\text{cur}}$, which has been determined using $a(t) \propto t^{\nu_{\text{rad}}}$ for the radiation era together with the thermal initial conditions of equation (3.15) (using $q_{\text{cur}} = 104$). As already discussed, these two populations of vortons are an unavoidable consequence of the loop production associated with a scaling cosmic string network and have not been considered before. For instance, taking $R \lesssim 10^2$, these figures show that all values of $G\mu$ greater than $10^{-15}$ are overclosing the Universe with vortons, even though no loops at all are present at $t_{\text{ini}}$ when the strings are formed. Although not very visible on the figure, there is a small
region around $\mathcal{R} = 1$ in which $\Omega_{\text{rel}}^{\text{min}} = 0$. Indeed, if $t_{\text{ini}} = t_{\text{cur}}$ and $C_{\text{ini}} = 0$, there is no time at all to produce loops before the current appears. However, this region is actually ruled out as filled with vortons produced afterwards (see right panel of figure 3).

Returning to equations (5.2) and (3.12), the most general situation for the relaxed vortons is to start with a mixture of loops created at the string forming time and loops created from the network between $t_{\text{ini}}$ and $t_{\text{cur}}$, i.e. one has both $C \neq 0$ and $C_{\text{ini}} \neq 0$. Moreover, from equation (5.10), the accurate expression for $\Omega_{\text{prod}}$ requires specifying the whole thermal history of the Universe and the integral has to be performed numerically. In the next section, we numerically integrate both $\Omega_{\text{rel}}$ and $\Omega_{\text{prod}}$ and discuss their sensitivity to the initial conditions.

### 5.2 Numerical integration and initial conditions

Compared to the previous section, we now numerically integrate both $\Omega_{\text{rel}}$ and $\Omega_{\text{prod}}$ starting from the general initial loop distribution described in section 3.2. Thermal initial conditions are taken assuming that the number of relativistic degrees of freedom is given by the Standard Model as derived in ref. [55].

Figures 4 and 5 show the density parameters today of all the relaxed vortons, the produced vortons and the sum of the two contributions when the string forming network at $t = t_{\text{ini}}$ is given by the Vachaspati-Vilenkin initial condition (see section 3.2). This implies that the typical size of loops at $t_{\text{ini}}$ is given by thermal fluctuations of the Higgs field and $\ell_{\text{corr}} = 1/\sqrt{\mu}$.

The lower right panel of figure 4, compared to the right panel of figure 3, shows that our approximated formula (5.12) is relatively accurate. The lower left panel of figure 4 exhibits a triangle-like region which is not visible on the left panel of figure 3. This region, with a high density of relaxed vortons, is precisely the one associated with the relaxed vortons created from the loops initially present at the string forming time and which were studied in ref. [11]. This contribution is represented alone in the upper left panel of figure 4. In this corner of parameter space, we recover the results already presented in ref. [11]: essentially all values of $G\mu$ are ruled out, only values of $G\mu = \mathcal{O}(10^{-30})$ and $\mathcal{R} = \mathcal{O}(10^3)$ remain compatible with the cosmological bounds.

When all contributions are combined, as shown in figure 5, one can see that for all $G\mu$ there are values of $\mathcal{R}$ which make the vortons either an acceptable candidate for dark matter (green line) or a subdominant component today (left of the green line). However, this figure also shows that there is an absolute lower bound for $\mathcal{R}$ below which vortons would overclose the universe, independently of the value of $G\mu$ (which is also given by the green line). For instance, there are no acceptable regions for which $\mathcal{R} < 10^2$, implying that stable vortons in our Universe can only be created if the temperature of current condensation is at least two orders of magnitude lower than the one of the formation of strings. This result is the consequence of the irreducible relaxed and produced vorton contributions closing the parameter space up to the maximum admissible values of $G\mu$. It may have some implications on the particle physics models creating strings and currents [56, 57].

Despite the fact that Vachaspati-Vilenkin initial conditions are quite motivated from the point of view of a thermal process, loops could be created from other processes [58, 59]. Therefore, instead of assuming $\ell_{\text{corr}} = 1/\sqrt{\mu}$, one could use the Kibble argument [1, 10] and take $\ell_{\text{corr}} = d_h(t_{\text{ini}})$, where $d_h(t_{\text{ini}}) = 2t_{\text{ini}}$ denotes the distance to the would-be particle horizon at the string forming time. Doing so leads to the same overall relic abundance of vortons as in section 5.1 where we were assuming $C_{\text{ini}} = 0$. There are simply not enough loops initially, compared to the one produced later on, to significantly change the final density parameter.
Figure 4. The upper left-hand panel shows the density parameter of relaxed vortons coming only from loops present at the string-forming phase transition, when starting from a Vachaspati-Vilenkin distribution at \( t = t_{\text{ini}} \). This is the population derived in ref. [11], that we recover by setting \( C = 0 \) in our equations. The upper right-hand panel shows the numerically evaluated density parameter of the irreducible relaxed vortons \( \Omega_{\text{rel}}^{\text{min}} \) (to be compared to our analytic estimation in the left panel of figure 3). The lower left-hand panel shows the density parameter \( \Omega_{\text{rel}} \) (today) from the population of all relaxed vortons (the sum of the upper left and right panels). Thermal history effects are visible on the upper boundary towards the minimum possible values of \( 1/R \) and \( G\mu \). The lower right-hand panel shows the density parameter \( \Omega_{\text{prod}} \) today of produced vortons derived numerically, and is indistinguishable from our analytic estimation of equation (5.12) (see right-hand panel of figure 3). The thick green line corresponds to all density parameter values in the range \([0.2, 0.4]\).

In order to quantitatively study the dependence of \( \Omega_{\text{tot}} \) with respect to the loop distribution at \( t_{\text{ini}} \), we have represented in figure 6 the values of \( \Omega_{\text{tot}} = 0.3 \) in the plane \((G\mu, 1/R)\) for various choices of \( \ell_{\text{corr}} \). They range from the thermal value \( \ell_{\text{corr}} = 1/\sqrt{\mu} \) to the causal one \( \ell_{\text{corr}} = d_h(t_{\text{ini}}) \), and even above, a situation that could appear if loops have been formed during cosmic inflation [60]. Everything on the right of the lines represented in this figure would lead to an overclosure of the Universe, while everything on the left is compatible with current measurements. The hatched region in this figure shows the robust bound discussed earlier, where there are only irreducible relaxed vortons and produced vortons.

In all our analysis and equations, we have left the parameter \( \alpha \) arbitrary, fixing only \( \alpha = 0.1 \) for the figures for well motivated reasons. Changing \( \alpha \) to smaller values, while keeping...
everything else fixed, increases the population of doomed loops, and thus decreases the vortons abundance. The explicit dependence in $\alpha$ can be read off from equations (5.4) and (5.12).

### 5.3 Other observables

A network of cosmic strings can let imprints in various cosmological observables, such as the stochastic background of gravitational waves and the Cosmic Microwave Background (CMB). In the present case, the stabilisation of vortons is expected to prevent a part of the energy to be converted into gravitational waves. We have therefore estimated the gravitational wave power spectrum generated from proto-vortons and doomed loops only. Their loop number densities are explicit in the appendix B. Due to the very small size of the vortons, the lack of energy in terms of gravitational waves ends up being negligible and the predictions for the stochastic background of gravitational waves remain unchanged compared to Nambu-Goto strings with a one-scale loop production function [51]. For the one-scale LPF, the current Laser Interferometer Gravitational-Wave Observatory (LIGO) bound on the string tension is $G\mu < O(10^{-11})$ [5, 61, 62] but depends on some assumptions on the string microstructure. Concerning the CMB, detectable distortions induced by cosmic strings are mostly due to the long strings in scaling such that they are not sensitive to the loop distribution and provide a robust upper bound $G\mu < O(10^{-7})$ for all types of strings [63–67]. Both of these bounds therefore apply to current-carrying strings with vortons. Let us also remark that current-carrying strings may lead to other observational signatures, for instance gamma ray or radio bursts [32, 68, 69].
**Figure 6.** The total relic abundance of all vortons starting from a Vachaspati-Vilenkin initial loop distribution with various correlation length $\ell_{\text{corr}}$ ranging from the thermal one $1/\sqrt{\mu}$ to the Kibble one $d_h(t_{\text{ini}})$. Each curve represents the value $\Omega_{\text{tot}} = 0.3$. Domains right of this curve lead to vortons overclosing the Universe, domains on the left are compatible with current cosmological constraints. The upper hatched region corresponds to the irreducible relaxed and produced vortons not affected by the initial conditions.

## 6 Conclusion

The main result of this work is the derivation of the relic abundance of an irreducible population of vortons not considered so far. These vortons are continuously created by the scaling string network at all times during the cosmological expansion and allow us to probe new regions of the parameter space $(G\mu, 1/\mathcal{R})$, namely energy scales that spawn the entire spectrum from TeV scales to the Planck scale. In particular, vortons are a viable dark matter candidate for all possible value of $G\mu$ (with, however, some quite tuned values of $\mathcal{R}$). We have derived their number density distribution at all times, which is the quantity of interest for dark matter direct detection searches [30, 70, 71], and derived the relevant cosmological constraints, summarized in figures 5 and 6.

Throughout this work, we have, however, assumed that all the scaling loops are produced at the same size $\alpha t$. A more complete analysis would take into account the fact that the loop production function is *a priori* more complicated. Due to the proliferation of kinks on the infinite string network and the fragmentation of large loops, we expect scaling loops to be
produced at all sizes with a power-law LPF

$$\mathcal{P}(\ell, t, N) = C t^{-5} \left(\frac{\ell}{t}\right)^{2\chi-3} \delta \left(N - \sqrt{\frac{\ell}{\lambda}}\right) \Theta(t - t_{\text{cur}}) \Theta(\ell - \gamma_c t), \quad (6.1)$$

where $\chi$ is the so-called Polchinski-Rocha exponent and $\gamma_c$ is the gravitational backreaction scale. Under this assumption, many more small loops are produced, and one can expect some boost to the density of vortons [51, 72]. Solving for the vorton distribution by using a Polchinski-Rocha LPF is, however, mathematically challenging and we have not taken this route in the present paper.

Let us also mention that, in the present work, we have solved a continuity equation to derive the vorton number density. This approach is strictly equivalent to the one presented in ref. [28], which is based on solving a Boltzmann equation. As a matter of fact, all the results presented have been cross-checked using the two methods. For completeness, we give in appendix A a proof of the equivalence between the two formalisms and how to pass from one evolution equation to the other.

Finally, concerning the influence of the initial conditions, let us remark that in the most generic situation, one cannot exclude that the redshift $z_{\text{ini}}$ at which strings are formed and the redshift $z_{\text{cur}}$ at which the current appears are independent of the value of $G\mu$ and $R$ (or $\lambda$). Although such a situation would be difficult to envisage for cosmic strings interpreted as topological defects, it could be very well the case for cosmic superstrings. For instance, $z_{\text{ini}}$ could be very large, close to the Planck energy scales while the warped observed value of $G\mu$ can remain very low. In this case, our assumptions of section 3.2 do no longer apply and this could change the relaxed vorton contribution. However, this would not change the produced vorton abundance, these ones being generated by the network at all subsequent times. A complete model-independent treatment would require to consider a four-dimensional parameter space made of $(G\mu, R, z_{\text{ini}}, z_{\text{cur}})$, which could be explored using Monte-Carlo-Markov-Chain methods, but we leave such a study for a future work.

Acknowledgments

P. A. thanks the organizers of the Paris Primordial Cosmology Meetings where this project germinated. P. P. wishes to thank Churchill College, Cambridge, where he was partially supported by a fellowship funded by the Higher Education, Research and Innovation Department of the French Embassy to the United-Kingdom during this research. The work of C. R. is supported by the “Fonds de la Recherche Scientifique — FNRS” under Grant No.T.0198.19 as well as by the Wallonia-Brussels Federation Grant ARC No.19/24 – 103.

A Connection between the Boltzmann and continuity equations

We clarify in this appendix the equivalence between equation (2.7) of ref. [28] and our equation (3.1) to show that the difference merely comes from the use of either lagrangian or eulerian coordinates. In ref. [28], one has $\ell = \ell(\ell_{\text{ini}}, t)$, i.e., one follows the evolution of a given loop size $\ell$ that begun with an initial value $\ell_{\text{ini}}$; somehow, the relevant variable is $\ell_{\text{ini}}$, and the flow is lagrangian. In the present work, the size of the loop $\ell$ is just what it is at the time one is concerned with, with no mention of the individual loop; this is the eulerian version.
Going from the eulerian set \{\ell, t\} to the lagrangian one \{\ell_{ini}, t\}, one has
\[
dX = \left( \frac{\partial X}{\partial t} \right)_\ell dt + \left( \frac{\partial X}{\partial \ell} \right)_t d\ell = \left( \frac{\partial X}{\partial t} \right)_{\ell_{ini}} dt + \left( \frac{\partial X}{\partial \ell_{ini}} \right)_t d\ell_{ini},
\]
with the subscript on the brackets for the partial derivatives indicating the quantity left constant for the evaluation of the derivative. Similarly expanding the differential \(d\ell\) and identifying the partial derivatives, one finds
\[
\left( \frac{\partial X}{\partial t} \right)_{\ell_{ini}} = \left( \frac{\partial X}{\partial t} \right)_\ell + \left( \frac{\partial X}{\partial \ell} \right)_t \left( \frac{\partial \ell}{\partial t} \right)_{\ell_{ini}} \quad \text{and} \quad \left( \frac{\partial X}{\partial \ell \mid _{\ell_{ini}}} \right)_t = \left( \frac{\partial X}{\partial \ell} \right)_t \left( \frac{\partial \ell}{\partial t} \right)_{\ell_{ini}}.
\]
(A.1)

One also notes that
\[
j \equiv \frac{d\ell}{dt} = \left( \frac{\partial \ell}{\partial \ell_{ini}} \right)_t \frac{d\ell_{ini}}{dt} + \left( \frac{\partial \ell}{\partial t} \right)_{\ell_{ini}} = \left( \frac{\partial \ell}{\partial t} \right)_{\ell_{ini}},
\]
(A.2)
the final step being a consequence of the fact that in lagrangian coordinates, \(\ell_{ini}\) does not depend on time. Combining (A.2) and (A.1), one immediately gets that
\[
\left( \frac{\partial X}{\partial t} \right)_\ell + j \left( \frac{\partial X}{\partial \ell} \right)_t = \left( \frac{\partial X}{\partial \ell} \right)_{\ell_{ini}}.
\]
(A.3)

We are now in position to compare equation (2.7) of ref. [28] and our equation (3.1). The former indeed reads
\[
\frac{\partial}{\partial t} \left( a^3 F J_{PSD} \right) + j \frac{\partial}{\partial \ell} \left( a^3 F J_{PSD} \right) = a^3 P J_{PSD},
\]
(A.4)
where \(J_{PSD} = \partial \ell / \partial \ell_{ini}\) accounts for phase space distortion and we have set \(F \equiv \frac{d^2 N}{d\ell dN}(\ell, t, N)\) for convenience. Expanding the partial derivatives of (A.4) and simplifying by \(J_{PSD}\) (assumed non vanishing), one gets
\[
\frac{\partial}{\partial t} \left( a^3 F \right) + j \frac{\partial}{\partial \ell} \left( a^3 F \right) + \frac{a^3 f}{J_{PSD}} \left[ \left( \frac{\partial J_{PSD}}{\partial t} \right)_{\ell_{ini}} + j \left( \frac{\partial J_{PSD}}{\partial \ell} \right)_{\ell_{ini}} \right] = a^3 P,
\]
(A.5)
the term in square brackets being, by virtue of (A.1) and (A.2), simply \(\left( \partial J_{PSD} / \partial t \right)_{\ell_{ini}}\). Given the definition of \(J_{PSD}\) and swapping partial derivatives, it turns out that
\[
\left( \frac{\partial J_{PSD}}{\partial t} \right)_{\ell_{ini}} = \left( \frac{\partial j}{\partial \ell} \right)_{\ell_{ini}} \frac{d\ell}{dt} = J_{PSD} \left( \frac{\partial j}{\partial \ell} \right)_{\ell_{ini}},
\]
so that equation (A.5) now becomes
\[
\frac{\partial}{\partial t} \left( a^3 F \right) + j \frac{\partial}{\partial \ell} \left( a^3 F \right) + \left( a^3 F \right) \frac{\partial j}{\partial \ell} = a^3 P,
\]
(A.6)
which is, as announced, equation (3.1) after grouping the \(\ell\)–derivative terms and expliciting \(j\) as in equation (2.7).

To conclude this appendix, we give in table 1, a dictionary between the different notations used in refs. [11, 28] and the present work.
B Distribution of proto-vortons and doomed loops

In this appendix, we give the distributions of proto-vortons and doomed loops, both of which contribute to the stochastic gravitational wave background. Proto-vortons and doomed loops decay through gravitational wave radiation and their collapse is not prevented by the current: indeed for both, \( J = 1 \) (in the limit \( \sigma \to 0 \)). Hence for these distributions, and without loss of generality, we set \( J = 1 \) in this appendix.

B.1 Doomed loops

Doomed loops are the loops which do not have enough current to prevent their final collapse, hence \( N < R \). From equations (4.1), (3.9) and (3.14), the relaxed doomed loop distribution, that is to say the doomed loops which are produced from the initial conditions at condensation, reads

\[
\left. \frac{dN}{d\ell} \right|_{\text{doom}, \text{rel}} = a(t_{\text{cur}}) \left[ a(t_{\text{cur}}) \right]^3 \left( \Gamma G \mu(t - t_{\text{cur}}) + \ell, t_{\text{cur}} \right) \delta \left( N - \sqrt{\frac{\ell_{\text{cur}}}{\lambda}} \right) \Theta (R - N),
\]

in which \( \ell_{\text{cur}} \), the size of the loop during condensation at \( t_{\text{cur}} \), is given by

\[
\ell_{\text{cur}}(\ell, t) = \Gamma G \mu(t - t_{\text{cur}}) + \ell. \tag{B.2}
\]

Integrating over the charge \( N \) and replacing \( \ell_{\text{cur}} \), one obtains the number density of doomed loops in relaxation

\[
\left. \frac{dN}{d\ell} \right|_{\text{doom}, \text{rel}} = a(t_{\text{cur}})^3 dN \left( \Gamma G \mu(t - t_{\text{cur}}) + \ell, t_{\text{cur}} \right) \Theta \left( R - \sqrt{\frac{\Gamma G \mu(t - t_{\text{cur}}) + \ell}{\lambda}} \right). \tag{B.3}
\]

Concerning the doomed loops produced after condensation, from equations (4.1) and (4.11), their number density is given by

\[
\left. \frac{dN}{d\ell} \right|_{\text{doom}, \text{prod}} = C \int dN \Theta (R - N) \left[ a(t_{*})^3 \right]^3 \frac{1}{\alpha + \Gamma G \mu} \delta \left( N - \sqrt{\frac{\alpha t_{*}}{\lambda}} \right) \Theta (t_{*} - t_{\text{cur}}), \tag{B.4}
\]

in which \( t_{*} \) is the loop formation time. Assuming, as we have done throughout this paper, that loops are produced at a given size \( \ell = \alpha t \) at time \( t \), the formation time satisfies

\[
t_{*}(\ell, t) = \frac{\ell + \Gamma G \mu t}{\alpha + \Gamma G \mu}. \tag{B.5}
\]
Finally, integrating over the charge $N$ and replacing the formation time by the above equation, one obtains the number density of doomed loops produced after condensation:

$$
\frac{dN}{d\ell} \bigg|_{\text{doom,prod}} = C \left[ \frac{a(t)}{a(t)} \right]^3 (\alpha + \Gamma G\mu)^3 \left( \ell + \Gamma G\mu \right)^4
\times \Theta \left( \frac{\ell + \Gamma G\mu t}{\alpha + \Gamma G\mu} - t_{\text{cur}} \right) \Theta \left[ \frac{\ell}{R} - \sqrt{\frac{(\alpha + \Gamma G\mu) t}{\lambda \mu (\alpha + \Gamma G\mu)}} \right].
$$

(B.6)

B.2 Proto-vortons

Proto-vortons are loops which will eventually become vortons after a certain time, but which are still large enough to behave like Nambu-Goto strings. From equation (4.2), (3.9) and (3.14), the distribution of “relaxed proto-vortons” is given by

$$
\frac{dN}{d\ell} \bigg|_{\text{proto,relax}} = \int dN \Theta(N - R) \left[ \frac{a(t_{\text{cur}})}{a(t)} \right]^3 \frac{dN}{d\ell} (\ell_{\text{cur}}, t_{\text{cur}}) \delta \left( N - \sqrt{\frac{\ell_{\text{cur}}}{\lambda}} \right) \Theta [\ell - \ell_0(N)],
$$

where $\ell_0(N) = N/\sqrt{\mu}$ and, again, the size of the loop at formation is given by

$$
\ell_{\text{cur}}(\ell, t) = \Gamma G\mu (t - t_{\text{cur}}) + \ell.
$$

(B.7)

On carrying out the integral over the charge $N$ in equation (B.7), the number density of proto-vortons produced at condensation is

$$
\frac{dN}{d\ell} \bigg|_{\text{proto,relax}} = \left[ \frac{a(t_{\text{cur}})}{a(t)} \right]^3 \frac{dN}{d\ell} [\Gamma G\mu (t - t_{\text{cur}}) + \ell, t_{\text{cur}}]
\times \Theta \left[ \ell - \sqrt{\frac{\Gamma G\mu (t - t_{\text{cur}}) + \ell}{\lambda \mu}} - R \right].
$$

(B.9)

Proto-vortons can also be produced after condensation, in which case their distribution is obtained from equations (4.2) and (4.11)

$$
\frac{dN}{dN} \bigg|_{\text{proto,prod}} = C \int dN \left[ \frac{a(t_*)}{a(t)} \right]^3 t_*^{-4} \Theta(N - R) \delta \left( N - \sqrt{\alpha t_* \lambda} \right) \Theta(t_* - t_{\text{cur}}) \Theta [\ell - \ell_0(N)].
$$

(B.10)

Similarly, the loop formation time $t_*$ is given by

$$
t_*(\ell, t) = \frac{\ell + \Gamma G\mu t}{\alpha + \Gamma G\mu}, \quad \ell_0(N) = \frac{N}{\sqrt{\mu}}.
$$

(B.11)

The distribution of proto-vortons produced after the condensation now reads

$$
\frac{dN}{dN} \bigg|_{\text{proto,prod}} = C \left[ \frac{a(t)}{a(t)} \right]^3 (\alpha + \Gamma G\mu)^3 \left( \ell + \Gamma G\mu \right)^4
\times \Theta \left( \frac{\ell + \Gamma G\mu t}{\alpha + \Gamma G\mu} - t_{\text{cur}} \right) \Theta \left[ \ell - \sqrt{\frac{\alpha (\ell + \Gamma G\mu)}{\lambda \mu (\alpha + \Gamma G\mu)}} \right] \Theta \left[ \sqrt{\frac{\alpha (\ell + \Gamma G\mu)}{\lambda \mu (\alpha + \Gamma G\mu)}} - R \right].
$$

(B.12)
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