A natural boundary of dark matter halo revealed at the minimum bias and maximum infall location

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ABSTRACT

We introduce a new boundary of dark matter halo which is naturally defined in the halo bias profile as the location of the bias minimum in the quasi-linear scale. This boundary is shown to correspond to the maximum infall location in the halo velocity profile, demarcating the transition between a growing halo and the environment being depleted. Using cosmological $N$-body simulations we show that this depletion radius depends the most on halo mass and environment. The dependence on formation time is also clear, with likely non-redundant dependencies on other properties such as halo shape and spin. We show that this depletion boundary is approximately proportional to the conventionally defined splashback radius, and can be interpreted as the radius enclosing a highly complete population of splashback orbits. For low mass haloes that have stopped mass growth, this depletion radius approaches the turnaround radius, while for massive haloes it is smaller than the turnaround. The depletion radius is approximately 2 times the virial radius and encloses an average density of $\sim 10$ times the critical density of the universe, independent on halo mass but dependent on other halo properties. This radius can also be interpreted as a natural radius for halo exclusion in halo models of the large scale structure.

Key words: dark matter -- galaxies: haloes -- large-scale structure of Universe

1 INTRODUCTION

In our current understanding of structure formation, dark matter haloes are the building blocks of the large scale structure in the Universe. The properties and evolution of dark matter haloes are fundamental to models that describe many aspects of the Universe ranging from the galaxy formation and evolution to the overall make-up and history of the Universe. In this halo model framework, the largescale structure can be decomposed into the distribution of haloes on largescale, convolved by the internal structure of haloes on smallscale (e.g., Cooray & Sheth 2002; Scoccimarro et al. 2001). Galaxies form and evolve in the potential well provided by dark matter haloes, with many intrinsic galaxy properties such as the colour, morphology and mass largely determined by the structure and evolution of the haloes they reside in (e.g. Baugh 2006; Benson 2010; Somerville & Davé 2015).

Despite the substantial work done on understanding the Universe using haloes as building blocks, practical characterisation of the fundamental properties of dark matter haloes is still subject to improvement. Most importantly, our understanding of what is the size of a halo is at best at a premature stage. Almost all of the studies about dark matter haloes so far are based on the classical virial definition of halo size derived from the spherical collapse model (Gunn & Gott 1972), which is expected to be only an approximate description of these objects. In the spherical collapse model, haloes are modelled from the collapse of a spherical initial overdensity embedded in an otherwise uniform background Universe, and the final size of the halo is defined through virialization argument. However, in the real Universe haloes are live objects that do not necessarily have a clear separation from the background Universe and are constantly accreting from and interacting with the neighbouring non-uniform environment. Such complications make the virial radius more of a practical working definition, leading to many variants of it defined through different overdensity criterion, such as 200 times the mean density or 200 times the critical density.

The recently proposed splashback radius marks a significant improvement over the classical model by incorporating halo accretion into the definition of a dynamical halo boundary (Diemer & Kravtsov 2014; Adhikari et al. 2014; More et al. 2015; Shi 2016) and hence has attracted substantial attentions (Snaith et al. 2017; Chang et al. 2018; Sugiyama et al. 2020; Aung et al. 2020; Mansfield et al. 2017; Fong et al. 2018; Umetsu & Diemer 2017; Baxter et al. 2017). Another classical boundary of physical importance is the turnaround radius. The turnaround radius also arises from the spherical collapse picture, but has been relatively less studied. We summarize these boundaries below:
• The virial radius, \( r_{\text{vir}} \), is the expected radius of a virialized halo according to the spherical collapse model. Normally this is defined through the expected virialization density, which we take from the prediction of (Bryan & Norman 1998) assuming a tophat initial density in a spherical collapse model.

• The splashback radius, \( r_{\text{sp}} \), is practically determined to be where the density profile reaches its steepest slope. The steepening in the slope has been attributed to the build-up of particles during their first orbital apogees, where the particles have low radial velocities. This radius has gathered significant interest since its discovery as it has been shown to probe the mass accretion rate of haloes (Diemer Kravtsov 2014; Adhikari et al. 2014; More et al. 2015; Shi 2016). Note the splashback radii of individual halo particles can span a wide range, and hence there is a large freedom in defining the overall splashback radius of a halo from particle dynamics (Diemer 2017; Diemer et al. 2017). In this work, we will refer to the splashback radius as the one estimated from the steepest slope location, unless explicitly specified otherwise.

• The turnaround radius, \( r_{\text{ta}} \), in the spherical collapse model is located where a mass shell of a halo reaches zero radial velocity before collapsing back towards the halo at a given time (Mo et al. 2010; Tangolidis et al. 2015, 2016; Pavlidou & Tomaras 2014). For individual haloes the radial velocity profiles of particles are a combination of the peculiar and Hubble flow velocities, and the turnaround radius is located where the Hubble flow overcomes the peculiar velocity. This can potentially be used as a cosmological probe, as it reflects the competition between dark energy and gravity (Faraoni et al. 2015; Taruya & Soda 2000; Lee & Li 2017; Falco et al. 2014; Korkidis et al. 2019).

Typically \( r_{\text{ta}} \lesssim r_{\text{sp}} < r_{\text{vir}} \). Most studies of haloes use mass or radius definitions that are within or roughly around \( r_{\text{vir}} \). However, the influence of haloes can go well beyond the virial radius. For example massive clusters are found to impact on the star formation rates of galaxies that are well outside their host halo’s virial radius, after the galaxies have passed within the host halo’s virial radius (Wetzel et al. 2014; von der Linden et al. 2010; Wetzel et al. 2012; Kukstas et al. 2020). In Bahé et al. (2013); Wetzel et al. (2014) the authors show that a significant fraction of these galaxies are not infalling for the first time and remain bound to their host haloes (see also Ludlow et al. 2009). Even though one can always work with a given definition of halo size, it is not difficult to imagine that simple intrinsic physical relations may become complicated or obscured in absence of a correct physical description of haloes.

The different halo radii definitions also reflect our understanding of the halo structure in different dimensions. While the virial radius is defined through the average density expected from virialization, the splashback radius is defined through the slope of the density profile or more physically the population property of evolving orbits in an accreting halo (Diemer 2017; Diemer et al. 2017). The turnaround radius, on the other hand, can be practically defined through the velocity profile around the halo.

In this work, we introduce two other dimensions to define a natural boundary of haloes in aim of providing a physical characterisation of haloes. We do this by first studying the bias profile, that is, the density profile around a halo relative to the average density profile around a random matter particle. We make use of a high resolution \( N \)-body simulation to extract bias profiles for haloes of different properties. The bias profile shows a typical trough around the halo boundary. This identifies the scale where the correlation between matter and haloes is the weakest, relative to the average clustering of matter, thus providing a natural dimension to define the boundary of a halo. More importantly, this bias trough is found to also correspond to the location of maximum infall velocity around the halo. These two characteristics combined lead to the interpretation of the bias trough as the location separating a growing halo and the environment being depleted. Thus we term this new radius as the depletion radius of the halo.

We also examine how this depletion radius relates to the virial, splashback, and turnaround radii. In terms of particle orbits, we demonstrate that this radius can be interpreted as the outermost splashback radius visually identifiable in the phase-space structure around haloes. By splitting haloes into bins of different halo properties, we also study how the depletion radius depends on multiple halo properties. As this radius can be identified in the bias profile, we argue that this new boundary is a natural manifestation of halo exclusion. An immediate application of this new radius is thus to improve over the halo model description of the matter clustering around the quasi-linear scale (e.g. Hayashi & White 2008; van den Bosch et al. 2003), which we briefly discuss in this work.

This paper is organized as follows. In Section 2 we describe the halo sample used in this work. In Section 3 we introduce the depletion radius through the bias profile, examine its dependence on halo properties, and compare it against the splashback radius. In Section 4 we examine the depletion radius through the velocity profile and the phasespace structure around haloes, and compare it against the turnaround radius. In Section 5 we compare this radius and its enclosed density to other halo boundary characterisations. In Section 6 we briefly discuss the implications of the halo depletion radius and in the halo model. In Section 7 we summarize and give our conclusions.

Note that log used in this work is \( 10 \log \) unless otherwise stated all \( M_X \) and \( r_X \) units are in \( M_\odot \) and \( \text{Mpc}/h \), respectively; the haloes in our work are located at \( z = 0 \).

## 2 SIMULATION AND THE HALO SAMPLE

In this section we introduce the simulation and the halo sample used in this paper. Our halo and clustering data are extracted from one of the CosmicGrowth Simulations (Jing 2019), a grid of high resolution \( N \)-body simulations run in different cosmologies using a P3M code (Jing & Suto 2002). We use the \( \Lambda \)CDM simulation with cosmological parameters \( \Omega_m = 0.268, \Omega_{\Lambda} = 0.732, \) and \( r_s = 0.831 \). The box size is 600 \( \text{Mpc}/h \) with 3072\(^3 \) dark matter particles and softening length \( \eta = 0.01 \text{Mpc}/h \). Groups are identified with the Friends-of-Friends (FoF) algorithm with a linking length 0.2 times the mean particle separation. The haloes are then processed with HBT\(^4 \) (Han et al. 2012, 2018) to obtain subhaloes and their evolution histories. The resulting halo catalog has about 2 \( \times 10^6 \) distinct haloes with masses \( 10^{11.5} < M_{\text{vir}}[M_\odot/h] < 3 \times 10^{15} \), where the minimum mass corresponds to roughly 500 particles within the virial radius to keep a reliable resolution on the structure of haloes.

The halo sample and the list of halo properties are the same as those used in Han et al. (2019), which we briefly describe below. The halo centres are located at the most bound particle of the central subhalo and all haloes at \( z = 0 \).

- \( M_{\text{vir}} \): The virial mass of the halo, where \( M_{\text{vir}} \) is the mass within a spherical volume of radius \( r_{\text{vir}} \) that encloses the mean density \( \Delta_c \) times the critical density of the Universe, or \( M_{\text{vir}} = 4\pi r_{\text{vir}}^3 \Delta_c \rho_{\text{crit}}/3 \).

\(^4 \) https://github.com/Kambrian/HBTplus
The virial overdensity in units of the critical density, $\Delta_c$, is predicted from the spherical collapse model (Bryan & Norman 1998). Note the total mass distribution is used in the computation of the virial mass and radius, not just the bound mass.

- $V_{\text{max}}$: The maximum of the circular velocity function, $V_{\text{circ}} = \sqrt{GM(<r)/r}$, of the central subhalo. Because $V_{\text{max}}$ is tightly correlated with halo mass, for this work we will use $V_{\text{max}}/V_{\text{vir}}$ to factor out the mass dependency, where $V_{\text{vir}} = \sqrt{GM/3r}$, and $G$ is the gravitational constant. This has also been used as a proxy for halo concentration (Angulo et al. 2008; Gao & White 2007; Sunayama et al. 2016), and is also a description of the shape of the density profile, where $V_{\text{max}}/V_{\text{vir}} = 1$ correspond to isothermal haloes with flat rotation curves while $V_{\text{max}}/V_{\text{vir}} > 1$ correspond to haloes with steeper outer profiles.

- $j$: The spin of the central subhalo: $j = L/(2GM^2)$ (Peebles 1969), where $L$, $E$, and $M$ are the total angular momentum, energy, and mass of the central subhalo.

- $e$: The shape parameter of the halo, $e = e_1$ in this work, where $e_1 = \lambda_1/\sum_{j=1}^3 \lambda_j$ and $e_1 \geq e_2 \geq e_3$. The subscripts $i = 1, 2, 3$ correspond to the three eigenvalues, $\lambda_i$, of the inertial tensor, the square of the three principle axes lengths of the central halo mass distribution. The inertial tensor is defined as $I_{i,j} = \sum_{p} m_p x_i^p x_j^p/r_p^3$, where $m_p$ is the mass of particle $p$, and $x_i^p$ and $r_p$ are the coordinate and distance to particle $p$ relative to the halo centre.

- $a_{1,2,3}$: The scale factor of the Universe when the halo mass was half of its final mass $M_0$ at $z = 0$. The masses used here are calculated using the bound particles to avoid complications due to ejected or fly-by haloes.

- $\delta_c$: The halo environment defined as the matter overdensity at a halo-centric distance $r_c \approx 1 - 2Mpc/h$ around each halo, $\delta_c = \rho(r_c)/\rho_m - 1$ (see Han et al. 2019, for more details).

### 3 THE HALO BIAS PROFILE AND A CHARACTERISTIC HALO BOUNDARY

The bias profile of haloes is defined as the ratio between the halo-matter correlation function and the matter-matter correlation function. In the limit of a single halo, the halo-matter correlation function reduces to the overdensity profile of matter around the halo center. A bias profile can then be similarly defined taking the ratio between the overdensity profile and the matter-matter correlation function. Because the matter-matter correlation function is the average density profile around a random matter particle, the resulting bias profile is a measure of the relative clustering around a halo compared to the clustering around a random matter particle. The halo boundary identified in this profile is thus expected to highlight the difference between the matter distribution around a halo and the average behaviour around a random particle.

In this work we first extract individual bias profiles around each halo from the overdensity profile and the matter-matter correlation function. In principle we can proceed to analyze each bias profile individually. However, to suppress noises associated with individual profiles, we will instead focus on analyzing the stacked profiles of haloes binned in various halo properties, $b(r) = \xi_{\text{hm}}(r)/\xi_{\text{mm}}(r) = (\delta(r))/\xi_{\text{mm}}(r)$, where $\xi_{\text{hm}}$ and $\xi_{\text{mm}}$ are the halo-matter and matter-matter correlation functions, $\delta(r) = \rho(r)/\rho_m - 1$ is the overdensity profile of matter around each halo, and the averaging is over all the haloes in each halo property bin.

In Figure 1 we show the bias binned by the halo parameters explored in this paper, where each panel is binned by the halo parameter labelled on the top right of each panel. Though the forms of the bias can be complex, there are some common features. In nearly all cases there is a clear trough in the bias, where this minimum represents the location where the correlation between haloes and matter are the weakest, with respect to the correlation of matter around a random particle in the density field. This is the motivation for defining a physical boundary at the location of this trough, which we will call the depletion radius, $r_d$, for reasons that will become clear later.

Beyond the bias minimum, the shape of the bias profile closely resembles the density profile of a void (Hamaus et al. 2014), which rises with radius until it reaches a maximum in many cases. This similarity can also be understood intuitively: Mass accretion of the central halo depletes matter around it, creating a characteristic trough. Beyond the trough the influence of neighbouring haloes becomes increasingly important, and the matter distribution can also be dominated by matter in neighbouring haloes, creating the wall region that resembles the wall around voids for the same reason. In this sense, the trough can be regarded as a “relative void” formed by the accretion of the central halo, alongside the competing mass accretion from neighbouring haloes. We will come back to this discussion in later sections.

In the language of a halo model, the profile within the depletion boundary is thus the one-halo term, while that outside the boundary is dominated by the two-halo term. The depletion radius defined here thus can be interpreted as the exclusion radius of haloes. This is also supported by the location and depth of the bias trough in different bins. Taking the mass dependence as an example, the highest mass haloes can only be surrounded by lower mass neighbours. Because it is easy to tightly pack small haloes around big ones, the boundaries of the largest haloes are thus smoothly connected to the neighbourhood, with no obvious troughs created by halo exclusion. By contrast, the smallest haloes typically only find neighbours relatively farther away on halo virial scales, reflecting the difficulty in packing larger haloes around them due to halo exclusion.

In order to accurately estimate the depletion radius, we first fit each bias profile with the following function,

$$b_F(a) = \frac{1 + \left(\frac{r}{r_0}\right)^{-(\alpha + \beta)}}{1 + \left(\frac{r}{r_0}\right)^{-(\alpha + \beta)}} \times \left(b_0 + \frac{r}{r_2}\right)^{-\gamma}.$$  

with $r_0 < r_1 < r_2$. This function has four components describing the inner-most one-halo profile ($b \propto r^{-\alpha}$ for $r < r_0$) before the trough, the rise beyond the trough ($b \propto r^{-\beta}$ for $r_0 < r < r_1$), the decrease after the wall ($b \propto r^{-\gamma}$ for $r > r_1$) and the largescale linear bias ($b \propto b_0$ for $r \gg r_2$). Note the innermost one-halo profile is better described by a NFW-like (Navarro et al. 1997) double power-law profile, but as we focus on studying the intermediate to largescale feature, we will not go to that complexity and only adopt an asymptotic single power-law for the one-halo term. Except that, we find this fitting function is universal for haloes binned in different or even multiple properties.

With this parametrization of the bias profile, in the following sections we move on with more quantitative analysis of the depletion radius, relating it to various halo properties as well as to other halo radii.
3.1 The depletion radius

With the universal fitting function, we identify the depletion radius, \( r_d \), to be the location of the bias minimum from the fitted bias profile. In Figure 2 we show the halo bias and density profiles for haloes binned by virial mass, together with the estimated depletion radius. Because our fitting function asymptotes to a single power-law for the one-halo profile, we limit our fits to start from only the outer part of the one-halo profile at \( r \gtrsim 0.06 \text{ Mpc}/h \). For the two highest mass bins, as there is not a well defined minimum we estimate the radius to be where the bias just flattens to the linear bias. For comparison, the splashback radius is also shown for each bin by finding the radius of the steepest logarithmic slope in the fitted density profile, \( \rho(r) = \rho_m \times (b(r) \xi_{mm} + 1) \).

In Figure 2 the bias is dominated by the 1-halo term for smaller radii, while for larger radii the bias flattens to the expected linear bias. As discussed before, the trough in the intermediate scale can be interpreted as a result of the competing mass accretion between the central halo and neighbouring ones. In the hierarchical structure formation framework, smaller haloes form earlier and thus have well formed accretion troughs after depleting matter around the halo. On the other hand, the most massive haloes are still actively accreting matter from the neighbourhood with fewer competitors, leading to shallower or even no obvious accretion troughs.

The transition region between \( r_d \) and the linear bias for each of the lower mass bins show a positive slope before flattening to the linear bias, which can also be seen as a flat shoulder in the density profiles (Figure 2). This shoulder marks a transition scale where the density profile starts to deviate significantly from the inner power-law shape, and again justifies the choice of \( r_d \) as a natural choice for halo boundary. The splashback radius, on the other hand, is always located within \( r_d \) while the density profile extends naturally beyond \( r_{sp} \) out to \( r_d \). As we will see later when studying the dynamics, the change in the shape of the density profile reflects the transition from the inner growing halo and the surrounding environment being depleted.

The transition region between the 1 and 2-halo terms can sometimes be more pronounced for the bias when binned by other halo parameters. The bias and density binned by halo formation time can be seen in Figure 3, where the colours, lines, and symbols are similar to Figure 2 but in different log \( a_{1/2} \) bins. The locations of \( r_d \) are again located at the starting point of the shoulder region in the density profiles. For the earliest forming haloes, the bias profiles also show pronounced peaks outside the trough, reflecting the significance of neighbouring haloes. These walls also lead to more extended shoulders in the density profiles. This remains the case for the other parameters listed in Section 2, and the figures can be seen in Appendix A. It is also clear that the trough is well formed in early forming haloes, while it is shallower in recently formed ones, as the environment are relatively less depleted for the latter.

In Figure 4 we show the \( r_d \) values for the bias binned by two halo properties, for all combinations in Section 2. The grey filled...
Depletion radius

Figure 2. The bias (top panel) and density (bottom panel) binned in separate log $M_{\text{vir}}$ ranges, where each colour corresponds to the mass range and number of haloes in the bin shown in the legend. The solid lines show the mean profiles in each bin. The dashed lines are the best-fits to each profile adopting Equation (2). The dotted lines are the matter auto-correlation function expressed as a bias or density in the top or bottom panel, respectively. The stars are the locations of the depletion radii, $r_d$. For reference we plot empty upside down triangles as the splashback radii locations, $r_{sp}$.

Figure 3. Similar to Figure 2, but showing the bias (top panel) and density (bottom panel) binned in separate log $a_{1/2}$ ranges as labelled.

contours are the number of haloes in each bin, which trace the signal-to-noise (S/N) of the measurements. The directions of the gradient of the halo depletion radius tells how sensitive $r_d$ is to the variations of halo properties. Among all the halo properties except the environment, the depletion boundary depends mostly on halo mass. This can be seen as the contours mostly changing in the mass direction, when binned by one halo parameter and mass. There are some dependencies on other halo properties besides mass, among which halo spin, $j$, is the least sensitive parameter for $r_d$. For haloes of the same virial mass, early forming ones are smaller in $r_d$, and so are spherical, highly concentrated, and high spin ones. The dependence on the halo environment, $\delta_e$, is also significant, with haloes in low density environment being larger, consistent with the picture that the influence range of the halo can be larger in low density environment, while the competition from the environment is stronger when the environment is higher.

In principle we can further bin the haloes by three or more halo properties to study their joint dependences, but this becomes difficult to visualize once the dimension is above two. Given that the dependence on many halo properties are already weak, in this work we simply test if there are significant residual dependencies on other halo properties once we account for the mass and formation time dependence. This is done by first fitting the $r_d$ dependence on mass and formation time with a flexible function, $\hat{r}_d(M_{\text{vir}}, a_{1/2})$, and then recast the fitted depletion radius to bins in other halo properties. Following Han et al. (2019), the fitting is done through Gaussian Process Regression (GPR), which can be regarded as a flexible non-parametric smooth interpolation of the $r_d(M_{\text{vir}}, a_{1/2})$ map obtained in Figure 4. In our work we use the Gaussian Process Regressor implemented in Scikit-Learn (Pedregosa et al. 2011), adopting a Matern kernel with $\nu = 0.5$. When fitting the $r_d$ map we bootstrap the sample of haloes in each bin to estimate the noise of $r_d$. The bootstrapped halo depletion radius values have non-normal distributions so we take the noise to be the maximum of the ±34th percentile from the median of the distributions as the noise for the GPR fits.

The contours from the GPR fits are overplotted in Figure 4 as thick contour lines. For panels other than the $(M_{\text{vir}}, a_{1/2})$, these thick contour lines represent predictions from the GPR fits recasted into the corresponding halo property bins. Unlike the $(M_{\text{vir}}, a_{1/2})$ panel, the halo bins in other panels can have wide distributions in mass and formation time. To compute the GPR prediction for these bins, we calculate the mean value of the predicted depletion radius from the GPR, $\langle \hat{r}_d(M_{\text{vir}}, a_{1/2}) \rangle$, as the recasted depletion radius in each bin, where the average is taken over the distribution of haloes in the bin. Note there is a caveat in this approach as the size for a sample of haloes computed from their stacked profiles can be different from their average size. Ideally the binned radius should be predicted from the stacked theoretical bias profiles, for which a full halo model is needed. Despite this, we expect the average predicted radius to still be able to largely inform us of the variation trend of the model in different bins. As shown in the figure, when

3 https://scikit-learn.org/stable/modules/gaussian_process.html
binning by 1 or 2 other halo parameters the mean GPR predictions fail to recover the true $r_d$ values, indicating the dependence on the other properties are unlikely to be fully attributable to the mass and formation time dependencies. This is especially true for the dependence on the environmental density.

### 3.2 Comparison to the splashback radius

We compare the halo depletion radius, $r_d$, with the splashback radius, $r_{sp}$, in Figure 5 when the bias is binned by one halo parameter. We use red + symbols to emphasize the mass cases. The dependence of $r_d$ on the environment appears more complicated than the others, which we do not consider here but leave to future investigations, given that the $\delta_e$ is not a classical halo property. Besides, we also exclude bins with less than 100 haloes out of S/N consideration, and the shape parameter bin $[0.55, 0.61]$ due to the difficulty in estimating a proper depletion radius for it (see Figure A3).

A line $r_d/r_{sp} = 2$ provides a very good description of the relation between the two radii, at least for the majority of the halo mass range. The ratio becomes larger at the small radius end, approaching $r_d/r_{sp} \approx 3$. This can be interpreted as reflecting the higher concentration of these small haloes, which can be further related to...
When the environment is involved, haloes with a small \( \delta_e \) tend to have a higher outer concentration. This can be understood as the halo boundary becomes more extended in a low density environment as seen in Figure 4.

This type of analysis could be an interesting starting point for a deeper study into the shapes of halo profiles using different boundary definitions, which we leave to future work.

### 4 Dynamic Interpretation of the Depletion Radius

To get more physical insights into the meaning of the depletion radius we study whether it corresponds to any features in the dynamics of matter near the halo boundary. As the halo mass is the primary driver (except \( \delta_e \)) for the depletion radius, we will focus on binning by halo mass in this section.

#### 4.1 Correspondence to the maximum infall radius

In Figure 7 we plot the stacked bias and radial velocity profiles, for haloes binned in mass. The velocity profiles are shown to be the median of the average velocity profiles of the haloes in each bin. The total radial velocity can be decomposed as \( v_r = v_p + v_H \), where \( v_p \) and \( v_H \) are the peculiar and Hubble velocities, and a negative velocity means infalling motion towards the halo centre.

For massive haloes the surrounding matter are being actively accreted onto the haloes, as shown by the prominent infall velocity troughs. The lack of a prominent infall region for the lowest mass bins can be understood as low mass haloes have completed their accretion phase.

It is striking to see that the depletion radius identified from the bias profile is always very close to the location of the maximum infall velocity across the halo mass range studied, especially in the peculiar velocity panel. Note the maximum infall location in the total radial velocity is shifted slightly inward relative to that in the peculiar velocity because the added Hubble flow component is larger at a larger radius. Despite this, the general correspondence between the bias trough location and the maximum infall location are very encouraging and supports a dynamical interpretation of the depletion radius. Within the maximum infall radius, matter are being dumped onto the halo as the infall rate slows down towards the inner halo. Outside this radius, however, matter are being pumped into the halo and gradually depleted due to the increasing infall rate with decreasing halo-centric distance. This location thus marks the transition between the halo being built up and the environment being depleted by halo accretion, leading to the formation of the trough in the bias profile and the flattened shoulder in the density profile. This trough is well formed around low mass haloes that have completed their mass accretion, but weak around massive haloes that are still in the early stage of their mass accretion.

Note the most relevant quantity governing the evolution of the density profile is the mass flow rate rather than velocity alone. The mass inflow rate can be found as \( 4\pi r^2 \rho v_r \). If the density profile around the boundary of haloes is close to \( \rho \sim r^{-2} \), then the mass flow rate will be mostly determined by the velocity profile alone. This is indeed the case as shown in Figure 8. Around the halo boundary region \( r \sim 0.5 \sim 5 \)Mpc/h relevant to our investigation, the matter-matter correlation function has a slope of \( \sim -2 \). Because the minimum bias location corresponds to the location where \( d \log(\xi_{im})/d \log(r) = d \log(\epsilon_{im})/d \log(r) \), around the depletion radius the \( \xi_{im}(r) \), and hence the density profile \( \rho(r) \), will also have a

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**Figure 5.** Relation between the depletion radius, \( r_d \), and the splashback radius, \( r_{sp} \). These radii are measured from the density and bias profiles in Figure 1, binned according to various halo properties as shown by the different symbols. The dashed and solid lines are reference linear relations as labelled.

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The early formation time, analogous to the well-known behaviour of the conventional NFW concentration (e.g., Bullock et al. 2001; Wechsler et al. 2002; Zhao et al. 2009; Ludlow et al. 2013) which is also clear from the halo distribution in the \((\Omega_1/2, V_{\text{max}}/V_{\text{vir}})\) panel in Figure 4. However, it is interesting to see that this “outer concentration” (or puffiness) depends mostly on the halo size and approaches constant values at both the large and small size ends, reflecting the approximately universal shapes of the density profiles near the halo boundary.

We explore the outer concentration further in Figure 6 to study its joint dependence on mass and another halo property. We focus on combinations including mass as mass is shown to be the most sensitive internal halo variable (i.e., excluding \( \delta_e \)) for the depletion size according to Figure 4. For combinations of internal halo properties the ratios are mostly in the same range of \( r_d/r_{sp} \sim 2 - 3 \) as found before. Note that we do not include the highest mass bins due to the difficulty in confidently estimating the halo depletion radius where there is no trough, similarly to the highest mass bin in Figure 2. This figure shows that for most haloes the splashback radius has a similar trend to the depletion radius. Though the differences here are small, it is good to see that the ratio is not constant in different parts of the parameter space, which means the depletion radius carries extra information about the halo properties besides those already contained in \( r_{sp} \).

For the joint dependence on mass and other internal halo properties, the global dependence on mass is still the most significant at the low mass end. The dependence on other halo properties are also significant except for the little dependence on halo spin. The earliest forming, least massive, spherical, and most concentrated haloes also have high outer concentrations, qualitatively tracing the behaviour of the inner NFW concentration.

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**Figure 7.** Stacked bias and radial velocity profiles, for haloes binned in mass. The velocity profiles are shown to be the median of the average velocity profiles of the haloes in each bin. The total radial velocity can be decomposed as \( v_r = v_p + v_H \), where \( v_p \) and \( v_H \) are the peculiar and Hubble velocities, and a negative velocity means infalling motion towards the halo centre.

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**Figure 8.** Around the halo boundary region \( r \sim 0.5 \sim 5 \)Mpc/h relevant to our investigation, the matter-matter correlation function has a slope of \( \sim -2 \).
Figure 6. The ratio between the depletion radius and the splashback radius, \( r_d/r_{sp} \), binned by mass and another halo property. Each pixel is colour coded according to the \( r_d/r_{sp} \) value in the bin, which is also printed directly.

Figure 7. The stacked radial velocity profiles binned by virial mass. The colours in the plot represent the mass bins in the legend. The middle and bottom panel show the total and peculiar radius velocities respectively. For comparison the top panel reproduces the bias profiles from Figure 2. The star and plus symbols mark the depletion and turnaround radii, respectively.

Figure 8. The logarithmic slopes of the matter-matter and halo-matter correlation functions. For simplicity only one example of the halo-matter correlation slope profile is shown in the log-\( M_{\text{vir}} \) bin of \([12,13.15]\). The minimum bias location is found where the two correlation functions have equal slope. Note that the splashback radius is defined as the steepest slope in the density which is slightly offset from the steepest slope of \( \xi_{hm} \).

logarithmic slope of \( \sim -2 \). This leads to very similar shapes of the mass inflow rate to the infall velocity. So our argument that the maximum infall location marks the transition between the growing halo and the environment being depleted still holds. The exact relations between the maximum infall defined through different profiles and the bias minimum may be better understood and calibrated through more sophisticated studies, while in this work we focus on pointing out the general correspondence between the bias trough and the maximum infall region, and collectively call this location as the depletion radius.

4.2 Comparison with the turnaround radius

For comparison, we also mark the locations of the turnaround radius in Figure 7. The turnaround radius is the location where the radial inflow of the particles around a halo is just overcome by the Hubble flow. For high mass haloes, the turnaround radius is located outside...
the maximum infall region. It is expected that the turnaround radius can further grow as the cluster grows more massive. For low mass haloes, however, the turnaround radius becomes close to the maximum peculiar infall radius. This means even the maximum infall velocity around these haloes could just balance the Hubble flow, so that they are no longer growing (see also Prada et al. 2006). This is consistent with the finding of Tanoglidis et al. (2015) that haloes below the transitional mass scale of $\approx 10^{13}\, M_\odot$ have reached their maximum turnaround radius. As a result, the depletion radius approaches with the turnaround radius for low mass haloes, and is within the turnaround radius in group and cluster haloes.

### 4.3 Interpretation from particle orbits

To further demonstrate the significance of the depletion radius in the dynamics around haloes, we show the phase space distribution of particles in haloes of three example mass bins in Figure 9. Note for the distributions we have only stacked a random subset of 100 haloes in each mass bin. The velocities are scaled by the virial velocity, $v_{\text{vir}} = \sqrt{G M_{\text{vir}}/r_{\text{vir}}}$, computed using the median log virial mass and the corresponding radius in each bin. The Hubble flow has been included in the velocity.

There are obviously two distinct components in the phasespace diagram, belonging to the halo and the surrounding environment respectively. Overall the splashback radius, depletion radius and turnaround radius are all located around the boundary separating the two components. The depletion radius that corresponds to the maximum peculiar infall can be found as marking the place where the phasespace distribution is the narrowest according to the density contours.

In terms of the splashback radius, the depletion radius here can be roughly interpreted as the outermost splashback radius enclosing the complete population of splashback orbits, or at least enclosing a much higher fraction than the steepest slope radius which is found to be enclosing approximating 75% of the splashback orbits (Diemer et al. 2017). As shown in Figure 9, the steepest slope splashback radius is also not far from the boundary of the narrowest velocity distribution, but further inside. To show the splashback interpretation of the depletion radius more clearly, in Figure 10 we show the phase space distribution of a single cluster halo. Outside the halo boundary, the velocity distribution is dominated by the infalling component, with an increasing infall velocity as material falls closer to the halo. Once the accretion stream enters the halo boundary, the phasespace will also be filled by splashback orbits that contribute to outflowing velocities, leading to a decrease in the net infall velocity. As a result, the location of the maximum infall marks the apocenter of the outermost splashback orbits. This is not as clear in the stacked phasespace diagrams in Figure 9, potentially due to the mixing of haloes of different size and the accumulated blurring from fluctuations around the boundary when many haloes are stacked.

A detailed statistical comparison of this depletion radius with the high percentile splashback radius would be interesting but not so straightforward due to the difficulty in robustly estimating the splashback radius at the tail of the splashback distributions, and we leave such investigations to future works. The current picture suggests the depletion radius, or equivalently the maximum infall radius, could be used as alternative measures of the complete splashback radius.

### 5 MASS-RADIUS RELATIONS AND THE ENCLOSED DENSITY

In Figure 11 we compare the various characteristic radii as functions of halo virial mass. As discussed above, the maximum infall radius is very close to the halo depletion radius, and both are close to the turnaround for haloes below $M_{\text{vir}} \approx 10^{13} M_\odot/h$. The splashback radius, on the other hand, is close to but larger than the virial radius.

The depletion radius has a constant ratio to the virial radius when binned by halo mass, as shown by the fitted line of $r_d = 2.5 r_{\text{vir}}$ in Figure 11. However, the relation become more interesting when further binning by other halo parameters. This can be seen in Figure 12, where the ratios $r_d/r_{\text{vir}}$ vary with halo properties other than mass when binned by combinations of internal halo properties. An outlier is again the halo spin, which barely affect the ratio. The trends in $r_d/r_{\text{vir}}$ are significantly different from those in Figure 6, showing that the various halo radii can probe different properties of haloes. The dependence when $\delta_c$ is involved becomes much different from other panels, but roughly consistent with the behavior in Figure 6, reflecting that the $\delta_c$ mostly influences $r_d$ rather than $r_{\text{vir}}$ or $r_{\text{sp}}$.

In Figure 13 we plot the average density enclosed within each characteristic radii, for haloes of different virial masses. The average density within the depletion radius is consistent with a constant of $\rho(< r_d) \approx 10\rho_{\text{crit}},$ independent of halo mass. In other words, this depletion radius can be found as a simple spherical overdensity radius containing 10 times the critical density of the universe in our simulation, when only the mass dependence is considered. By contrast, the splashback radius corresponds to varying overdensities and can not be simply defined through an characteristic overdensity criterion, as also found previously (Diemer 2017). For completeness, the average density enclosed in the turnaround radius is also shown. It increases with decreasing mass and becomes close to the depletion density at the low mass end. The higher enclosed density at the low mass end can be understood as their turnaround radii froze at higher redshifts where the density of the universe was higher.

In Figure 14 we plot the spherical density, $\Delta = \rho(< r_d)/\rho_{\text{crit}}$, for the halo depletion radius binned by two halo parameters. Similar to the behavior of $r_{\text{sp}}/r_{\text{vir}}$, when we bin haloes by two halo parameters, we find that $\Delta$ is not universal but depends on halo properties other than mass. At a fixed mass, the density variation is largely consistent with the $r_d$ variation in the parameter space, where a smaller $r_d$ corresponds to a higher density contrast. The oldest, spherical, and the most concentrated haloes typically have the highest enclosed density within $r_d$.

Note that combining the radius ratio in Figure 12 and the density contrast in Figure 14, one can immediately estimate the ratio between the mass enclosed within $r_d$ and the virial mass. For example, taking the typical $r_d = 2.5 r_{\text{vir}}$ and $\rho(< r_d) = 10\rho_{\text{crit}},$ the enclosed mass can be found to be $M(< r_d) = 1.6 M_{\text{vir}}$.

### 6 IMPLICATION FOR HALO MODEL

We have discussed that the depletion radius can also be interpreted as the exclusion radius of haloes, which could find a direct application in the halo modeling of the largescale structure. A better definition and characterisation of the halo is expected to improve both in the simplicity and the accuracy of the resulting halo model. During the preparation of this work, a very recent outstanding work by (Garcia et al. 2020, hereafter G20) has just addressed this problem from an inverse approach compared to ours, by solving for an optimal...
Figure 9. The scaled radial velocity distribution for three log $M_{\text{vir}}$ bins as labeled on the top left of each panel. The colour map shows the distribution of particles, with white curves marking isodensity contours enclosing 99, 80 and 60 percents of particles from outside to inside. The black and grey curves are the radial and peculiar velocities, respectively. The vertical solid red, dash-dotted magenta, and dashed blue lines are the locations of the depletion, splashback, and virial radii, respectively. The cyan cross symbols are the locations of the turnaround radii.

Figure 10. The radial velocity distribution in a single cluster halo ($M_{\text{vir}} = 1.35 \times 10^{15}/M_{\odot}/h$). The grey dots show the distribution of halo particles for which only a random 1 percent of all the particles are shown. The black and grey solid curves show the average total and peculiar radial velocities respectively. The two vertical lines mark the location of the virial and the maximum peculiar infall velocities as labelled. The maximum infall location clearly marks the outermost splashback boundary.

Figure 11. The characteristic radii as functions of virial mass. The coloured lines correspond to the radii in the legend. The grey dotted line shows $2.5r_{\text{vir}}$ to compare to the depletion radius. The $1.3r_{\text{sp87}}$ radius is a proxy of the optimal exclusion radius found by Garcia et al. (2020) discussed in section 6.

In G20, the halo density profile and the optimal halo radius are parametrized through scaling relations with the optimal halo mass. The optimal mass and radius are expected to be different from the conventionally defined ones, so the parameters determining the scaling relations are allowed to vary freely and are solved for by matching the predicted halo-matter correlation to the mea-
Depletion radius

Figure 12. The dependence of the depletion radius to virial radius ratio, $r_d/r_{\text{vir}}$, on combinations of different halo properties. The radius ratio is shown by the pixel value and a corresponding colour.

Figure 13. The average density enclosed in different characteristic radii ($r_X$ as labelled in the legend) around haloes of different virial masses, normalized by the critical density of the Universe.

G20 found that their optimal halo radius has a roughly constant ratio to the splashback radius, $r_{G20} \approx 1.3r_{sp87}$, where $r_{sp87}$ is the radius containing 87% of the splashback apogees of all the halo particles according to Diemer et al. (2017). To compare against the G20 result, we will use $1.3r_{sp87}$ as a proxy of $r_{G20}$, where $r_{sp87}$ can be computed from scaling relation with the virial mass. As shown by the grey dashed line in Figure 11, the G20 radius is very close to the depletion radius for haloes around and below $10^{13}M_\odot/h$, the mass scale for which the G20 model is best constrained. The agreement indicates the depletion radius defined here is very likely a good candidate for halo exclusion radius, consistent with our previous argument.

It was shown in G20 that $r_{G20}$ is located at the minima of $r^2\xi_{\text{min}}$ for haloes around $10^{13}M_\odot/h$, which is the “by-eye” boundary of the halo. However, their $r_{G20}$ starts to deviate from the “by-eye” boundary in higher mass haloes. By contrast, we have checked that our $r_d$ defined at the minimum bias location is also located very near the minima of $r^2\xi_{\text{min}}$ for all mass bins. This explains the difference between the G20 radius and our depletion radius at the high mass end. However, instead of using the $r^2\xi_{\text{min}}$ to define a natural halo boundary, we argue that our definition using the bias profile is more physically meaningful. The agreement between the minimum location in the $r^2\xi_{\text{min}}$ and the bias profiles is a coincidence around the quasi-linear scale, which can be seen in Figure 8.

Note that the G20 model is not free from tweaks which leaves room for adopting a different halo radius definition. In particular, we expect the soft halo boundary treatment and the one halo profile modelling out to the new boundary can still be improved once we adopt our new boundary definition. In addition, the agreement between the G20 model and the simulation measurement is relatively less satisfactory for halos above $10^{14}M_\odot/h$. As a result, despite the qualitative difference between our radius and the G20 radius especially at the high mass end, we expect it is still possible to construct a self-consistent halo model that can improve the fitting to the correlation function using our new radius definition.

7 SUMMARY AND CONCLUSIONS

In this paper we have proposed a new boundary of dark matter halo, the depletion radius, using a large sample of haloes from cosmological $N$-body simulation. This depletion radius is naturally identified by clear transition features in both the spatial distribution and velocity distribution of matter around haloes. In the spatial distribution, the depletion radius is most evident from the existence of a ubiquitous trough in the bias profile around this halo boundary, where the clustering around the halo is the weakest relative to the clustering around a random matter particle. In the velocity domain, the depletion radius can be found around the location of the maximum infall velocity. These evidences lead us to interpret the radius as marking the transition between an inner growing halo and the feeding environment. Matter are being depleted from the environment into the depletion radius, leading to the formation of an accretion trough in the bias profile, which also corresponds to a relatively flat shoulder in the density profile beyond this radius.

We study how the depletion radius depends on multiple halo properties including halo mass, formation time, concentration, shape, spin and environment by binning the bias profiles according to one or two halo properties. The depletion radius in the binned profiles depends strongly on both mass and environment. The formation time and concentration dependence is also clear and non-redundant from the mass dependence, while the dependence on halo spin is the weakest. The bias radius likely also depends on halo shape beside mass and formation time.
Comparing the depletion radius to the splashback radius defined at the steepest slope location, we find the depletion radius is \(\sim 2 - 3\) times the splashback radius. The ratio between the two increases with decreasing halo size and depends on halo properties in a way similar to the dependence of the NFW concentration on halo properties, with low mass and early forming haloes also having a larger “outer concentration” according to the depletion to splashback ratio. By contrast, the ratio between the depletion radius and the virial radius is \(\sim 1 - 3\) and depends on halo properties in a very different way. The depletion to virial radius ratio is independent of mass but sensitive to other halo parameters, and the trends are different from those for the depletion to splashback ratio. These results reflect that the different radii carry different information about the haloes. As the depletion radius is on average much further out than the virial and splashback radius, it is subject to more influence from the environmental density.

Comparing the depletion radius to the turnaround radius, we find the depletion radius approaches the turnaround in low mass haloes \((M_{\text{vir}} \sim 10^{12} M_\odot/h\) and below) that have stopped mass accretion and reached their maximum turnaround radius. These haloes have well formed bias troughs but are not surrounded by an infall region due to their completed mass accretion, leading to the agreement between the depletion radius and the turnaround radius. By contrast, massive haloes are surrounded by a clear infall region within the turnaround radius, while the bias trough is much shallower or absent, reflecting their younger age in the mass accretion process. At the high mass end, the depletion radius according to the maximum infall location are thus located within the turnaround radius.

We have further studied the depletion radius in the phasespace diagram of haloes, and find it naturally marks the transition between an inner halo structure and the outer environment according to the distribution of radial velocity at different radius. In terms of particle orbits, the maximum infall location can be interpreted as the radius enclosing a highly complete population of splashback orbits. Such high percentile splashback radius is not easy to robustly quantify from particle dynamics alone, while our definitions via orbits. Such high percentiles splashback radius is not easy to robustly quantify from particle dynamics alone, while our definitions via orbits. Such high percentiles splashback radius is not easy to robustly quantify from particle dynamics alone, while our definitions via orbits. Such high percentiles splashback radius is not easy to robustly quantify from particle dynamics alone, while our definitions via orbits. 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Depletion radius

In this section we plot the bias and densities binned by the other halo parameters listed in Section 2, namely $V_{\text{max}}/V_{\text{vir}}, J$, $e$, and $\log(1+\delta_e)$ in Figure A1, Figure A2, Figure A3, and Figure A4, respectively. The colours correspond to the property ranges and number of haloes shown in the legends. The many forms that the bias can take is quite complex. We test the fitting function on finer parameter binning in 1 and 2D cases and find that the fitting function (Equation 2) performs very well in all cases we observed.

This paper has been typeset from a \TeX/\LaTeX file prepared by the author.

APPENDIX A: ONE DIMENSIONAL BINNING

In this section we plot the bias and densities binned by the other halo parameters listed in Section 2, namely $V_{\text{max}}/V_{\text{vir}}, J$, $e$, and $\log(1+\delta_e)$.
Figure A2. The bias (top panel) and density (bottom panel) binned in separate $j$ ranges.

Figure A3. The bias (top panel) and density (bottom panel) binned in separate $e$ ranges.

Figure A4. The bias (top panel) and density (bottom panel) binned in separate $\log(1 + \delta_e)$ ranges.