Completely $N_{nc^e}$ (Weakly $N_{nc^e}$)-irresolute Functions via $N_{nc^e}$-open Sets

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Abstract. The main purpose of this paper is to introduce two new types of $N_{nc^e}$-irresolute functions called completely $N_{nc^e}$-irresolute and completely weakly $N_{nc^e}$-irresolute functions via $N_{nc^e}$-open sets introduced by Ekici. We obtain some characterizations of these functions. Also, we investigate some fundamental properties between these new notions are separation and covering.

Keywords and phrases: completely $N_{nc^e}$-irresolute, completely weakly $N_{nc^e}$-irresolute, countably $N_{nc^e}$-compact, $N_{nc^e}$-closed compact, $N_{nc^e}$-Lindelöf, strongly $N_{nc^e}$-regular space, $N_{nc^e}$-normal space.

1. Introduction
Smarandache’s neutrosophic system have wide range of real time applications for the fields of Computer Science, Information Systems, Applied Mathematics, Artificial Intelligence, Mechanics, decision making, Medicine, Electrical & Electronic, and Management Science etc [1, 2, 3, 4, 22, 23]. Topology is a classical subject, as a generalization topological spaces many types of topological spaces introduced over the year. Smarandache [16] defined the Neutrosophic set on three component Neutrosophic sets (T-Truth, F-Falsehood, I-Indeterminacy). Neutrosophic topological spaces (nts’s) introduced by Salama and Alblowi [13]. Lellies Thivagar et.al. [10] was given the geometric existence of $N$ topology, which is a non-empty set equipped with $N$ arbitrary topologies. Lellis Thivagar et al. [11] introduced the notion of $N_{n^c}$-open (closed) sets and $N_{n^c}$ topological spaces. Al-Hamido [5] explore the possibility of expanding the concept of neutrosophic crisp topological spaces into $N$-neutrosophic crisp topological spaces and investigate some of their basic properties. The importance of continuity and generalized continuity is significant in various areas of mathematics and related sciences. Recent progress in the study of characterizations and generalizations of continuity has been done by means of several generalized closed sets. The first step of generalizing closed set was done by Levine in 1970 [12]. The notion of generalized closed sets has been studied extensively in recent years...
by many topologist because generalized closed sets are the only nature generalization of closed sets. More importantly, they also suggest several new properties of topological spaces. As a generalization of closed sets, e-closed sets were introduced and studied by Ekici [6, 7, 8]. In 2018, Lellies Thivagar et al. [11] introduced $N_n$ continuous in $N$ neutrosophic crisp topological spaces. In 2020, Vadivel and John Sundar [17, 18] $N$-neutrosophic $\delta$-open, $N$-neutrosophic $\delta$-semiopen, $N$-neutrosophic $\alpha$-continuous, $N$-neutrosophic $\delta$-preopen sets, $N$-neutrosophic $\alpha$-continuous, $N$-neutrosophic semi continuous and $N$-neutrosophic pre continuous are introduced. In 2020, Vadivel and Thangaraja [20, 21] introduced $N$-neutrosophic $e$-open sets and $N$-neutrosophic $e$-continuous in $N$-neutrosophic topological spaces. In this paper we introduce two new types of $N_{nc}$-irresolute functions called completely $N_{nc}e$-irresolute and completely weekly $N_{nc}e$-irresolute functions via $N_{nc}$-open sets introduced by Vadivel and Thangaraja [20].

2. Preliminaries

Salama and Smarandache [15] presented the idea of a neutrosophic crisp set in a set $P$ and defined the inclusion between two neutrosophic crisp sets, the intersection (union) of two neutrosophic crisp sets, the complement of a neutrosophic crisp set, neutrosophic crisp empty (resp., whole) set as more than two types. And they studied some properties related to neutrosophic crisp set operations. However, by selecting only one type, we define the inclusion, the intersection (union), and neutrosophic crisp empty (resp., whole) set again and discover a few properties.

**Definition 2.1** Let $P$ be a non-empty set. Then $H$ is called a neutrosophic crisp set (in short, $ncS$) in $P$ if $H$ has the form $H = (H_1, H_2, H_3)$, where $H_1, H_2,$ and $H_3$ are subsets of $P$.

The neutrosophic crisp empty (resp., whole) set, denoted by $\phi_n$ (resp., $P_n$) is an $ncS$ in $P$ defined by $\phi_n = (\phi, \phi, P)$ (resp. $P_n = (P, P, \phi)$). We will denote the set of all $ncS$’s in $P$ as $ncS(P)$.

In particular, Salama and Smarandache [14] classified a neutrosophic crisp set as the followings.

A neutrosophic crisp set $H = (H_1, H_2, H_3)$ in $P$ is called a neutrosophic crisp set of Type 1 (resp. 2 & 3) (in short, $ncS$-Type 1 (resp. 2 & 3)), if it satisfies $H_1 \cap H_2 = H_2 \cap H_3 = H_3 \cap H_1 = \phi$ (resp. $H_1 \cap H_2 = H_2 \cap H_3 = H_3 \cap H_1 = \phi$ and $H_1 \cup H_2 \cup H_3 = P$ & $H_1 \cup H_2 \cup H_3 = \phi$ and $H_1 \cup H_2 \cup H_3 = P$). $ncS_1(P)$ ($ncS_2(P)$ and $ncS_3(P)$) means set of all $ncS$ Type 1 (resp. 2 and 3).

**Definition 2.2** Let $H = (H_1, H_2, H_3), M = (M_1, M_2, M_3) \in ncS(P)$. Then $H$ is said to be contained in (resp. equal to) $M$, denoted by $H \subseteq M$ (resp. $H = M$), if $H_1 \subseteq M_1, H_2 \subseteq M_2$ and $H_3 \subseteq M_3$ (resp. $H \subseteq M$ and $M \subseteq H$); $H^c = (H_3, H_2^c, H_1^c); H \cap M = (H_1 \cap M_1, H_2 \cap M_2, H_3 \cap M_3); H \cup M = (H_1 \cup M_1, H_2 \cup M_2, H_3 \cup M_3)$. Let $(L_j)_{j \in J} \subseteq ncS(P)$, where $H_j = (H_{j_1}, H_{j_2}, H_{j_3})$. Then

$\bigcap_{j \in J} H_j$ (simply $\bigcap_{j} H_j$) = ($\bigcap_{j_1} H_{j_1}, \bigcap_{j_2} H_{j_2}, \bigcap_{j_3} H_{j_3}$); $\bigcup_{j \in J} H_j$ (simply $\bigcup_{j} H_j$) = ($\bigcup_{j_1} H_{j_1}, \bigcup_{j_2} H_{j_2}, \bigcup_{j_3} H_{j_3}$).

The following are the quick consequence of Definition 2.2.

**Proposition 2.1** [9] Let $L, M, O \in ncS(P)$. Then

(i) $\phi_n \subseteq L \subseteq P_n$,
(ii) if $L \subseteq M$ and $M \subseteq O$, then $L \subseteq O$,
(iii) $L \cap M \subseteq L$ and $L \cap M \subseteq M$,
(iv) $L \subseteq L \cup M$ and $M \subseteq L \cup M$,
(v) $L \subseteq M$ iff $L \cap M = L$,
(vi) $L \subseteq M$ iff $L \cup M = M$.

Likewise the following are the quick consequence of Definition 2.2.
Proposition 2.2 [9] Let $L, M, O \in \text{nc}S(P)$. Then

(i) $L \cup L = L$, $L \cap L = L$ (Idempotent laws),
(ii) $L \cup M = M \cup L$, $L \cap M = M \cap L$ (Commutative laws),
(iii) (Associative laws) : $L \cup (M \cup O) = (L \cup M) \cup O$, $L \cap (M \cap O) = (L \cap M) \cap O$,
(iv) (Distributive laws) : $L \cup (M \cap O) = (L \cup M) \cap (L \cap O)$,
(v) (Absorption laws) : $L \cup (L \cap M) = L$, $L \cap (L \cup M) = L$,
(vi) (DeMorgan’s laws) : $(L \cup M)^c = L^c \cap M^c$, $(L \cap M)^c = L^c \cup M^c$,
(vii) $(L^c)^c = L$,
(viii) (a) $L \cup \phi_n = L$, $L \cap \phi_n = \phi_n$,
(b) $L \cup P_n = P_n$, $L \cap P_n = L$,
(c) $P_n^c = \phi$, $\phi_n^c = P_n$,
(d) in general, $L \cup L^c \neq P_n$, $L \cap L^c \neq \phi_n$.

Proposition 2.3 [9] Let $L \in \text{nc}S(P)$ and let $(L_j)_{j \in J} \subseteq \text{nc}S(P)$. Then

(i) $(\bigcap L_j)^c = \bigcup L_j^c$, $(\bigcup L_j)^c = \bigcap L_j^c$,
(ii) $L \cap (\bigcup L_j) = \bigcup (L \cap L_j)$, $L \cup (\bigcap L_j) = \bigcap (L \cup L_j)$.

Definition 2.3 [14] A neutrosophic crisp topology (briefly, ncts) on a non-empty set $P$ is a family $\tau$ of $\text{nc}$ subsets of $P$ satisfying the following axioms

(i) $\phi, P_n \in \tau$,
(ii) $H_1 \cap H_2 \in \tau \forall H_1 \& H_2 \in \tau$,
(iii) $\bigcup H_a \in \tau$, for any $\{H_a : a \in J\} \subseteq \tau$.

Then $(P, \tau)$ is a neutrosophic crisp topological space (briefly, ncts ) in $P$. The $\tau$ elements are called neutrosophic crisp open sets (briefly, ncos) in $P$. A ncts $C$ is closed set (briefly, nccts) iff its complement $C^c$ is ncos.

Definition 2.4 [5] Let $P$ be a non-empty set. Then nc$\tau_1$, nc$\tau_2$, $\ldots$, nc$\tau_N$ are $N$-arbitrary crisp topologies defined on $P$ and the collection $N_{nc}\tau = \{L \subseteq P : L = (\bigcup_{j=1}^{N} H_j) \cup (\bigcap_{j=1}^{N} L_j), H_j, L_j \in \text{nc}\tau_j\}$ is called $N$ neutrosophic crisp (briefly, $N_{nc}$-)topology on $P$ if the axioms are satisfied:

(i) $\phi, P_n \in N_{nc}\tau$,
(ii) $\bigcup_{j=1}^{\infty} L_j \in N_{nc}\tau \forall \{L_j\}_{j=1}^{\infty} \in N_{nc}\tau$,
(iii) $\bigcap_{j=1}^{N} L_j \in N_{nc}\tau \forall \{L_j\}_{j=1}^{N} \in N_{nc}\tau$.

Then $(P, N_{nc}\tau)$ is called a $N_{nc}$-topological space (briefly, $N_{nc}ts$) on $P$. The $N_{nc}\tau$ elements are called $N_{nc}$-open sets ($N_{nc}os$) on $P$ and its complement is called $N_{nc}$-closed sets ($N_{nc}cs$) on $P$. The elements of $P$ are known as $N_{nc}$-sets ($N_{nc}s$) on $P$.

Definition 2.5 [5] Let $(P, N_{nc}\tau)$ be $N_{nc}ts$ on $P$ and $H$ be an $N_{nc}s$ on $P$, then the $N_{nc}$ interior of $H$ (briefly, $N_{nc}int(H)$) and $N_{nc}$ closure of $H$ (briefly, $N_{nc}cl(H)$) are defined as

$N_{nc}int(H) = \cup\{L : L \subseteq H \& L \text{ is a } N_{nc}os \text{ in } P\}$
$N_{nc}cl(H) = \cap\{C : H \subseteq C \& C \text{ is a } N_{nc}cs \text{ in } P\}$. 

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Definition 2.6 [5] Let \((P, N_{nc})\) be any \(N_{nc}\)ts. Let \(H\) be an \(N_{nc}\)s in \((P, N_{nc})\). Then \(H\) is said to be a \(N_{nc}\)-regular open [17] set (briefly, \(N_{nc}\)ros) if \(H = N_{nc}\)int\((N_{nc}\)cl\((H))\).

The complement of an \(N_{nc}\)ros is called an \(N_{nc}\)-regular closed set (briefly, \(N_{nc}\)rcs) in \(P\).

The family of all \(N_{nc}\)ros (resp. \(N_{nc}\)rcs) of \(P\) is denoted by \(N_{nc}\)ROS\((P)\) (resp.\(N_{nc}\)RCS\((P)\)).

Definition 2.7 [19] A set \(H\) is said to be a

(i) \(N_{nc}\)-interior of \(H\) (briefly, \(N_{nc}\)int\((H))\) is defined by \(N_{nc}\)int\((H) = \bigcup\{L : L \subseteq H \& L\ is\ \text{a} \ N_{nc}\)ros\}\).

(ii) \(N_{nc}\)-closure of \(H\) (briefly, \(N_{nc}\)cl\((H))\) is defined by \(N_{nc}\)cl\((H) = \bigcup\{x \in X : N_{nc}\)int\((N_{nc}\)cl\((L)) \cap H \neq \phi, x \in L \& L\ is\ \text{a} \ N_{nc}\)ros\}\).

Definition 2.8 A set \(H\) is said to be a

(i) \(N_{nc}\)-open set [19] (briefly, \(N_{nc}\)los) if \(H = N_{nc}\)int\((H)\).

(ii) \(N_{nc}\)-closed set [20] (briefly, \(N_{nc}\)cos) if \(H \subseteq N_{nc}\)cl\((N_{nc}\)int\((H))\).\(\cup\ N_{nc}\)int\((N_{nc}\)cl\((H))\).

The complement of an \(N_{nc}\)cos (resp. \(N_{nc}\)co) is called an \(N_{nc}\)-closed set (briefly, \(N_{nc}\)cs (resp. \(N_{nc}\)cos)) in \(P\).

The family of all \(N_{nc}\)cos (resp. \(N_{nc}\)cs, \(N_{nc}\)cos & \(N_{nc}\)cs) of \(P\) is denoted by \(N_{nc}\)ROS\((P)\) (resp. \(N_{nc}\)RCS\((P)\), \(N_{nc}\)os\((P)\) & \(N_{nc}\)CS\((P)\)).

Definition 2.9 Let \((P, N_{nc})\) and \((Q, N_{nc})\) be any two \(N_{nc}\)ts's. A map \(h : (P, N_{nc}) \rightarrow (Q, N_{nc})\) is said to be \(N_{nc}\)-continuous (briefly, \(N_{nc}\)cts) [10] (resp. \(N_{nc}\)cts [21]) if \(h^{-1}(O)\) is both \(N_{nc}\)o (resp. \(N_{nc}\)o) in \(P\) for each \(N_{nc}\)o set \(O\) of \(Q\).

3. Completely \(N_{nc}\)-irresolute functions

Definition 3.1 Let \((P, N_{nc})\) and \((Q, N_{nc})\) be any two \(N_{nc}\)ts's. A map \(h : (P, N_{nc}) \rightarrow (Q, N_{nc})\) is said to be

(i) strongly \(N_{nc}\)-continuous (briefly, \(sN_{nc}\)ts) if \(h^{-1}(O)\) is both \(N_{nc}\)o and \(N_{nc}\)c in \(P\) for each \(N_{nc}\) set \(O\) of \(Q\).

(ii) completely \(N_{nc}\)-continuous (briefly, \(cN_{nc}\)ts) if \(h^{-1}(O)\) is \(N_{nc}\)ro in \(P\) for each \(N_{nc}\)o set \(O\) of \(Q\).

(iii) \(N_{nc}\)-irresolute (briefly, \(N_{nc}\)irr) if \(h^{-1}(O)\) is \(N_{nc}\)o in \(P\) for every \(N_{nc}\o set \(O\) of \(Q\).

(iv) Completely \(N_{nc}\)-irresolute (briefly, \(cN_{nc}\)irr) if the inverse image of every \(N_{nc}\)o set of \(Q\) is \(N_{nc}\)ro in \(P\).

Remark 3.1 It is not difficult to see that every \(sN_{nc}\)ts function is \(cN_{nc}\)irr and every \(cN_{nc}\)irr function is \(N_{nc}\)irr. But the converse of the implication are not true in general as shown by the following examples.

\[ sN_{nc}\)ts \implies cN_{nc}\)irr \implies N_{nc}\)irr \]

Example 3.1 Let \(P = \{u, v, w\}, \ N_{nc}\)ts = \(\{\phi, P, U\}, \ N_{nc}\)rs = \(\{\phi, P, U\}\). \(U = \{\{u\}, \{\phi\}, \{v, w\}\}, then we have \(2_{nc}\ts = \{\phi, P, U\}\). Define \(h : (P, 2_{nc}\ts) \rightarrow (P, 2_{nc}\ts)\) as an identity map, then it is \(c2_{nc}\ts\) but not \(s2_{nc}\ts\), the set \(h^{-1}(\{\{u\}, \{\phi\}, \{v, w\}\}) = \{\{u\}, \{\phi\}, \{v, w\}\}\) is \(2_{nc}\os\) but not \(2_{nc}\os\) in \(P\).

Example 3.2 Let \(P = \{u, v, w, x\}, \ N_{nc}\)ts = \(\{\phi, P, U, V, W\}, \ N_{nc}\)rs = \(\{\phi, P, U\}. \ U = \{\{u\}, \{\phi\}, \{u, v, x\}\}, \ V = \{\{u\}, \{\phi\}, \{w, x\}\}, \ W = \{\{u, v, w\}, \{\phi\}, \{x\}\}, then we have \(2_{nc}\ts = \{\phi, P, U, V, W\}\). Define \(h : (P, 2_{nc}\ts) \rightarrow (P, 2_{nc}\ts)\) as an identity map, then it is \(c2_{nc}\ts\) but not \(c2_{nc}\ts\), the set \(h^{-1}(\{\{u\}, \{\phi\}, \{w, x\}\}) = \{\{u\}, \{\phi\}, \{w, x\}\}\) is \(2_{nc}\os\) but not \(2_{nc}\os\) in \(P\).
Theorem 3.1 Let \( h : P \rightarrow Q \) be a function, then statement

(i) \( h \) is \( cN_{nc}eIrr \),

(ii) \( h^{-1}(N_{nc}eint(M)) \subseteq N_{nc}δint(h^{-1}(M)) \) for every \( N_{nc} \) set \( M \) of \( Q \),

(iii) \( h(N_{nc}δcl(L)) \subseteq N_{nc}ecl(h(L)) \) for every \( N_{nc} \) set \( L \) of \( P \),

(iv) \( N_{nc}δcl(h^{-1}(M)) \subseteq h^{-1}(N_{nc}ecl(M)) \) for every \( N_{nc} \) set \( M \) of \( Q \),

(v) \( h^{-1}(O) \) is \( N_{nc}ec \) in \( P \) for each \( N_{nc}ec \) set \( O \) in \( Q \),

(vi) \( h^{-1}(O) \) is \( N_{nc}ro \) in \( P \) for each \( N_{nc}ro \) set \( O \) in \( Q \)

are equivalent.

Proof. (i) \( \Rightarrow \) (ii): Let \( M \subseteq Q \) and \( x \in h^{-1}(N_{nc}eint(M)) \).

\[
x \in h^{-1}(N_{nc}eint(M)) \Rightarrow N_{nc}eint(M) \subseteq N_{nc}eO(Q, h(x))
\]

\[
\Rightarrow (\exists U \in N_{nc}RO(P, x)) h(U) \subseteq N_{nc}eint(M) \subseteq M
\]

\[
\Rightarrow (\exists U \in N_{nc}RO(P, x)) (U \subseteq h^{-1}(M)) \Rightarrow x \in N_{nc}δint(h^{-1}(M)).
\]

(ii) \( \Rightarrow \) (iii): Let \( L \subseteq P \).

\[
L \subseteq P \Rightarrow h(L) \subseteq Q \Rightarrow Q \setminus h(L) \subseteq Q
\]

\[
\Rightarrow h^{-1}(N_{nc}eint(Q \setminus h(L))) \subseteq N_{nc}δint(h^{-1}(Q \setminus h(L)))
\]

\[
\Rightarrow P \setminus N_{nc}ecl(h(L)) \subseteq N_{nc}δcl(h^{-1}(h(L)))
\]

\[
\Rightarrow h(N_{nc}δcl(L)) \subseteq N_{nc}ecl(h^{-1}(h(L))) \subseteq h^{-1}(N_{nc}ecl(h(L)))
\]

\[
\Rightarrow h(N_{nc}δcl(L)) \subseteq N_{nc}ecl(h(L)).
\]

(iii) \( \Rightarrow \) (iv): Let \( M \subseteq Q \).

\[
M \subseteq Q \Rightarrow h^{-1}(M) \subseteq P
\]

\[
\Rightarrow h(N_{nc}δcl(h^{-1}(M))) \subseteq N_{nc}ecl(h^{-1}(M)) \subseteq N_{nc}ecl(M)
\]

\[
\Rightarrow h(N_{nc}δcl(h^{-1}(M))) \subseteq h^{-1}(N_{nc}ecl(M)).
\]

(iv) \( \Rightarrow \) (v): Let \( O \in N_{nc}eC(Q) \).

\[
O \in N_{nc}eC(Q) \Rightarrow O = N_{nc}ecl(O)
\]

\[
\Rightarrow N_{nc}δcl(h^{-1}(O)) \subseteq h^{-1}(N_{nc}ecl(O)) = h^{-1}(O)
\]

\[
\Rightarrow h^{-1}(O) = N_{nc}δcl(h^{-1}(O))
\]

\[
\Rightarrow h^{-1}(O) \in N_{nc}δC(P).
\]

(v) \( \Rightarrow \) (vi): Obvious.

(vi) \( \Rightarrow \) (i): Let \( O \in N_{nc}eO(Q) \) and \( x \in h^{-1}(O) \).

\[
(O \in N_{nc}eO(Q))(x \in h^{-1}(O)) \Rightarrow O \in N_{nc}eO(Q, h(x))
\]

\[
\Rightarrow (U := h^{-1}(O) \in N_{nc}RO(P, x))(h(U) \subseteq O).
\]

Theorem 3.2 Let \( h : (P, N_{nc}) \rightarrow (Q, N_{nc}) \) be a bijective function. Then the following statements are equivalent:

(i) \( h \) is \( cN_{nc}eIrr \),

(ii) \( h \) is \( cN_{nc}eC \),

(iii) \( h \) is \( cN_{nc}eO \),

(iv) \( h \) is \( cN_{nc}eR \),

(v) \( h \) is \( cN_{nc}eIrr \),

(vi) \( h \) is \( cN_{nc}eC \),

(vii) \( h \) is \( cN_{nc}eO \),

(viii) \( h \) is \( cN_{nc}eR \).
(ii) $N_{nc} \text{eint} (h(L)) \subseteq h(N_{nc} \text{eint}(L))$ for every $N_{nc}$ set of $P$.

**Proof.** (i) $\Rightarrow$ (ii): Let $L \subseteq P$.

\[ L \subseteq P \Rightarrow P \backslash L \subseteq P \]
\[ \Rightarrow h(P \backslash N_{nc} \text{eint}(L)) = h(N_{nc} \text{ecl}(P \backslash L)) \subseteq N_{nc} \text{ecl}(h(P \backslash L)), h \text{ is bijection} \]
\[ \Rightarrow Q \backslash h(N_{nc} \text{eint}(L)) \subseteq Q \backslash N_{nc} \text{eint}(h(L)) \]
\[ \Rightarrow N_{nc} \text{eint}(h(L)) \subseteq h(N_{nc} \text{eint}(L)). \]

(ii) $\Rightarrow$ (i): Let $L \subseteq P$.

\[ L \subseteq P \Rightarrow P \backslash L \subseteq P \]
\[ \Rightarrow N_{nc} \text{eint}(h(P \backslash L)) \subseteq h(N_{nc} \text{eint}(P \backslash L)), h \text{ is bijection} \]
\[ \Rightarrow Q \backslash N_{nc} \text{ecl}(h(L)) \subseteq Q \backslash h(N_{nc} \text{ecl}(L)) \]
\[ \Rightarrow h(N_{nc} \text{ecl}(L)) \subseteq N_{nc} \text{ecl}(h(L)). \]

**Lemma 3.1** Let $Q$ be an $N_{nc}$ set of a $N_{nc}$ set $P$. Then the following hold:

1. If $L$ is $N_{nc}$ro in $P$, then so is $L \cap Q$ in the subspace $(Q, N_{nc} \tau_Q)$,
2. If $M$ is $N_{nc}$ro in $(Q, \tau_Q)$, then there exists a $N_{nc}$ro set $R$ in $P$ such that $M = R \cap Q$.

**Theorem 3.3** If $h : (P, N_{nc} \tau) \rightarrow (Q, N_{nc} \sigma)$ is a $cN_{nc} \text{eIrr}$ function and $L$ is any $N_{nc}$ set of $P$, then the restriction $h_L : L \rightarrow Q$ is $cN_{nc} \text{eIrr}$.

**Proof.** Let $F \in N_{nc} \text{eO}(Q)$.

\[ F \in N_{nc} \text{eO}(Q) \xrightarrow{h \text{ is } cN_{nc} \text{eIrr}} h^{-1}(F) \in N_{nc} \text{RO}(P), L \in \tau \xrightarrow{\text{Lemma 3.1}} (h_L)^{-1}(F) = h^{-1}(F) \cap L \in N_{nc} \text{RO}(L). \]

**Lemma 3.2** Let $Q$ be a $N_{nc}$ro set of a $N_{nc}$ set $P$. Then $Q \cap L$ is $N_{nc}$ro in $Q$ for each $N_{nc}$ro set $L$ of $P$.

**Theorem 3.4** If $h : (P, N_{nc} \tau) \rightarrow (Q, N_{nc} \sigma)$ is a $cN_{nc} \text{eIrr}$ function and $L$ is $N_{nc}$ro set of $P$, then $h_L : L \rightarrow Q$ is $cN_{nc} \text{eIrr}$.

**Proof.** It is clear from Lemma 3.2.

**Theorem 3.5** Let $h : (P, N_{nc} \tau) \rightarrow (Q, N_{nc} \sigma)$ and $g : (Q, N_{nc} \sigma) \rightarrow (Z, N_{nc} \eta)$ be two functions. Then the following hold. If $h$ is

1. $cN_{nc} \text{eIrr}$ and $g$ is $N_{nc} \text{Irr}$, then $g \circ h$ is $cN_{nc} \text{Irr}$,
2. $cN_{nc} \text{Cts}$ and $g$ is $N_{nc} \text{eIrr}$, then $g \circ h$ is $cN_{nc} \text{eIrr}$,
3. $cN_{nc} \text{eIrr}$ and $g$ is $N_{nc} \text{Cts}$, then $g \circ h$ is $cN_{nc} \text{Cts}$.

**Proof.** Straightforward.

**Definition 3.2** A space $P$ is said to be almost $N_{nc} \text{econnected}$ (resp. $N_{nc}$e-connected) if there does not exist disjoint $N_{nc}$ro (resp. $N_{nc}$ro) sets $L$ and $M$ such that $L \cup M = P$.

**Theorem 3.6** If $h : P \rightarrow Q$ is $cN_{nc} \text{eIrr}$ surjection and $P$ is almost $N_{nc}$ connected, then $Q$ is $N_{nc}$e-connected.
Proof. Suppose that $Q$ is not $N_{nc}e$-connected. $Q$ is not $N_{nc}e$-connected

$$\Rightarrow (\exists L, M \in N_{nc}eO(Q) \backslash \{0\})(L \cap M = \phi)(L \cup M = Q),$$

$$h$$ is completely $N_{nc}eIrr$ surjection

$$\Rightarrow (h^{-1}(L), h^{-1}(M) \in N_{nc}RO(P) \backslash \{0\})(h^{-1}(L \cap M) = h^{-1}(\phi))(h^{-1}(L \cup M) = h^{-1}(Q))$$

$$\Rightarrow (h^{-1}(L), h^{-1}(M) \in N_{nc}RO(P) \backslash \{0\})(h^{-1}(L \cap h^{-1}(M) = \phi)(h^{-1}(L \cup h^{-1}(M) = P)).$$

This means that $P$ is not almost $N_{nc}$ connected.

Definition 3.3 A $N_{nc}ets P$ is said to be:

(i) nearly $N_{nc}$ compact if every $N_{nc}ro$ cover of $P$ has a finite $N_{nc}$ subcover,

(ii) nearly countably $N_{nc}$ compact if every $N_{nc}$ countable cover by $N_{nc}ro$ sets has a finite $N_{nc}$ subcover,

(iii) nearly $N_{nc}$ Lindelöf if every $N_{nc}$ cover of $P$ by $N_{nc}ro$ sets has a countable $N_{nc}$ subcover,

(iv) $N_{nc}e$-compact if every $N_{nc}eo$ cover of $P$ has a finite $N_{nc}$ subcover,

(v) countably $N_{nc}e$-compact if every $N_{nc}eo$ countable cover of $P$ has a finite $N_{nc}$ subcover,

(vi) $N_{nc}e$-Lindelöf if every $N_{nc}$ cover of $P$ by $N_{nc}eo$ sets has a countable $N_{nc}$ subcover.

Theorem 3.7 Let $h : (P, N_{nc}\tau) \to (Q, N_{nc}\sigma)$ be a $cN_{nc}eIrr$ surjection. Then the following statements hold: If $P$ is nearly

(i) $N_{nc}$ compact, then $Q$ is nearly $N_{nc}e$-compact,

(ii) $N_{nc}$ Lindelöf, then $Q$ is nearly $N_{nc}e$-Lindelöf,

(iii) countably $N_{nc}$ compact, then $Q$ is countably $N_{nc}e$-compact.

Proof. (i) Let $P$ be nearly $N_{nc}$ compact and $A$ be an $N_{nc}eo$ cover of $Q$.

$$A \subseteq N_{nc}eO(Q)(Q = \bigcup A)$$

$$h$$ is $cN_{nc}eIrr$$\Rightarrow (\exists B \subseteq N_{nc}RO(P))(P = \bigcup B), P is nearly $N_{nc}$ compact

$$\Rightarrow (\exists B^* \subseteq B)(|B^*| < N_0)(P = \bigcup B^*)$$

$$h$$ is surjective$$\Rightarrow (h(B^*) \subseteq h(B) = A)(|h(B^*)| < N_0)(Q = h(P) = h(\bigcup B^*) = \bigcup_{B \in B^*} h(M)).$$

(ii) Let $P$ be nearly $N_{nc}$ Lindelöf and $A$ be an $N_{nc}eo$ cover of $Q$.

$$A \subseteq N_{nc}eoO(Q)(|A| \leq N_0)(Q = \bigcup A)$$

$$h$$ is $cN_{nc}eIrr$$\Rightarrow (B := \{h^{-1}(L) | L \in A\} \subseteq N_{nc}RO(P))(P = \bigcup B), P is nearly countably $N_{nc}$ compact

$$\Rightarrow (\exists B^* \subseteq B)(|B^*| < N_0)(P = \bigcup B^*), h$$ is surjective

$$\Rightarrow (h(B^*) \subseteq h(B) = A)(|h(B^*)| < N_0)(Q = h(P) = h(\bigcup B^*) = \bigcup_{B \in B^*} h(M)).$$

(iii) Let $P$ be nearly countably $N_{nc}$ compact and $A$ be an $N_{nc}eo$ countable cover of $Q$.

$$A \subseteq N_{nc}eoO(Q)(|A| \leq N_0)(Q = \bigcup A)$$

$$h$$ is $cN_{nc}eIrr$$\Rightarrow (B := \{h^{-1}(L) | L \in A\} \subseteq N_{nc}RO(P))(P = \bigcup B), P is nearly countably $N_{nc}$ compact

$$\Rightarrow (\exists B^* \subseteq B)(|B^*| < N_0)(P = \bigcup B^*), h$$ is surjective

$$\Rightarrow (h(B^*) \subseteq h(B) = A)(|h(B^*)| < N_0)(Q = h(P) = h(\bigcup B^*) = \bigcup_{B \in B^*} h(M)).$$
Definition 3.4 A N_{nc}t's P is said to be:

(i) SN_{nc}-closed (resp. N_{nc}c compact) if every N_{nc}c (resp. N_{nc}cc) cover of P has a finite N_{nc} subcover,

(ii) Countable SN_{nc}c compact (resp. countable N_{nc}cc compact) if every countable N_{nc} cover of P by N_{nc}c (resp. N_{nc}cc) sets has a finite N_{nc} subcover,

(iii) S-N_{nc} Lindelöf (resp. N_{nc}cc Lindelöf) if every cover of P by N_{nc}c (resp. N_{nc}cc) sets has a countable N_{nc} subcover.

Theorem 3.8 Let h : (P, N_{nc}) → (Q, N_{nc}) be a cN_{nc}Irr surjection. Then the following statements hold: If P is

(i) SN_{nc}c, then Q is N_{nc}cc compact,

(ii) S-N_{nc} Lindelöf, then Q is N_{nc}cc Lindelöf,

(iii) countable SN_{nc}c compact, then Q is countable N_{nc}cc compact.

Proof. (i) Let P be SN_{nc}c and A be an N_{nc}cc cover of Q.

\[ (A \subseteq N_{nc}cC(Q))(Q = \cup A) \]

\[ \frac{(h \text{ is } c_{N_{nc}} \text{ e } Irr)}{(B := \{h^{-1}(L)|L \in A\} \subseteq N_{nc}cRC(P))(P = \cup B), P \text{ is } SN_{nc}c} \]

\[ \Rightarrow (\exists B^* \subseteq B)(|B^*| < N_0)(P = \cup B^*), h \text{ is surjective} \]

\[ \frac{h \text{ is surjective}}{(h(B^*) \subseteq h(B) = A)(|h(B^*)| < N_0)(Q = fX = h(\cup B^*) = \bigcup_{B \in B^*} h(M)).} \]

(ii) Let P be S-N_{nc} Lindelöf and A be an N_{nc}cc countable cover of Q.

\[ (A \subseteq N_{nc}cC(Q))(Q = \cup A) \]

\[ \frac{(h \text{ is } c_{N_{nc}} \text{ e } Irr)}{(B := \{h^{-1}(L)|L \in A\} \subseteq N_{nc}cRC(P))(P = \cup B), P \text{ is } SN_{nc}-\text{Lindelöf closed}} \]

\[ \Rightarrow (\exists B^* \subseteq B)(|B^*| \leq N_0)(P = \cup B^*) \]

\[ \frac{h \text{ is surjective}}{(h(B^*) \subseteq h(B) = A)(|h(B^*)| \leq N_0)(Q = h(P) = h(\cup B^*) = \bigcup_{B \in B^*} h(M)).} \]

(iii) Let P be countable SN_{nc}c compact and A be an N_{nc}cc countable cover of Q.

\[ (A \subseteq N_{nc}cC(Q))(|L| \leq N_0)(Q = \cup A) \]

\[ \frac{(h \text{ is } c_{N_{nc}} \text{ e } Irr)}{(B := \{h^{-1}(L)|L \in A\} \subseteq N_{nc}cRC(P))(|B| \leq N_0)(P = \cup B), \]

P is countable SN_{nc}c compact

\[ \Rightarrow (\exists B^* \subseteq B)(|B^*| < N_0)(P = \cup B^*), h \text{ is surjective} \]

\[ \Rightarrow (h(B^*) \subseteq h(B) = A)(|h(B^*)| < N_0)(Q = h(P) = h(\cup B^*) = \bigcup_{B \in B^*} h(M)).} \]

Definition 3.5 A N_{nc}t's P is said to be almost N_{nc} regular (resp. strongly N_{nc}e-regular) if for any N_{nc}cc (resp. N_{nc}cc) set \( h \subseteq P \) and any point \( x \in P \setminus F \), there exists disjoint N_{nc}o (resp. N_{nc}eo) sets U and O such that \( x \in U \) and \( F \subseteq O \).

Theorem 3.9 If h is cN_{nc}Irr, N_{nc}eo bijection from an almost N_{nc}r space P onto a space Q, then Q is sN_{nc}eIr.
Proof. Let $F \in N_{nc}C(Q)$ and $h(x) = y \notin F$.

$h(x) = y \notin F \in N_{nc}C(Q)$

$\Rightarrow (\exists U, O \in N_{nc}O(P))(x \in U)(h^{-1}(F) \subseteq O)(U \cap O = \phi)$

$h is c N_{nc} e Irr \Rightarrow x \notin h^{-1}(F) \in N_{nc}RC(P), P is almost N_{nc} regular$

$\Rightarrow (\exists U, O \in N_{nc}O(P))(x \in U)(h^{-1}(F) \subseteq O)(U \cap O = \phi)$

$\Rightarrow (\exists U, O \in N_{nc}O(P))(x \in U)(h^{-1}(F) \subseteq O)(U \cap O = \phi)$

$h is N_{nc} e-open bijection \Rightarrow (h(U), h(O) \in N_{nc}O(Q))(y = h(x) \in h(U))(F \subseteq h(O))(h(U) \cap h(O) = \phi)$.

Definition 3.6 A $N_{nc} e P$ is said to be:

(i) almost $N_{nc} e$ normal if for each $N_{nc}e$ set $L$ and each $N_{nc}e$ set $M$ such that $L \cap M = \phi$, there exist disjoint $N_{nc}e$ sets $U$ and $O$ such that $L \subseteq U$ and $M \subseteq O$.

(ii) strongly $N_{nc}e$-normal if for every pair of disjoint $N_{nc}e$ subsets $L$ and $M$ of $P$, there exist disjoint $N_{nc}e$ sets $U$ and $O$ such that $L \subseteq U$ and $M \subseteq O$.

Theorem 3.10 If $h : (P, N_{nc} e) \to (Q, N_{nc} e)$ is $c N_{nc} e Irr$ $N_{nc}e$ bijection from an almost $N_{nc}$ normal space $P$ into a space $Q$, then $Q$ is strongly $N_{nc}e$-normal.

Proof. Let $L, M \in N_{nc}eC(Q)$ and $L \cap M = \phi$.

$(A, M \in N_{nc}eC(Q))(L \cap M = \phi) (h is c N_{nc}e Irr)$

$\Rightarrow (h^{-1}(L), h^{-1}(M) \in N_{nc}RC(P))(h^{-1}(L \cap M) = h^{-1}(\phi))$

$\Rightarrow (h^{-1}(L), h^{-1}(M) \in N_{nc}RC(P))(h^{-1}(L) \cap h^{-1}(M) = \phi), N_{nc}RC(P) \subseteq C(P)$

$\Rightarrow (h^{-1}(L) \in C(P))(h^{-1}(M) \in N_{nc}RC(P))(h^{-1}(L) \cap h^{-1}(M) = \phi)$

$X is almost N_{nc} normal \Rightarrow (\exists U, O \in N_{nc}e)(h^{-1}(L) \subseteq U)(h^{-1}(M) \subseteq O)(U \cap O = \phi)$,

$h is a N_{nc}e$ bijection

$\Rightarrow (h(U), h(O) \in N_{nc}eO(Q))(L \subseteq h(U))(M \subseteq h(O))(h(U) \cap h(O) = \phi)$.

Definition 3.7 A $N_{nc} e (P, N_{nc} e)$ is said to be $N_{nc} e T_1$ (resp. $N_{nc} e T_1$) if for each pair of distinct points $x$ and $y$ of $P$, there exist $N_{nc}e$ (resp. $N_{nc}ro$) sets $U_1$ and $U_2$ such that $x \in U_1$ and $y \in U_2, x \notin U_2$ and $y \notin U_1$.

Theorem 3.11 If $h : P \to Q$ is $c N_{nc} e Irr$ injection and $Q$ is $N_{nc} e T_1$, then $P$ is $N_{nc} e T_1$.

Proof. Let $x, y \in P$ and $x \neq y$.

$(x, y \in P)(x \neq y)$

$h is injective \Rightarrow h(x) \neq h(y), (Q is N_{nc}e-T_1)$

$\Rightarrow (\exists F_1 \in N_{nc}eO(Q, h(x)))(\exists F_2 \in N_{nc}eO(Q, h(y)))(h(x) \notin F_2)(h(y) \notin F_1)$

$h is c N_{nc} e Irr \Rightarrow (h^{-1}(F_1) \in N_{nc}RO(P, x))(h^{-1}(F_2) \in N_{nc}RO(P, y))(x \notin h^{-1}(F_2)(y \notin h^{-1}(F_1))$.

Definition 3.8 A $N_{nc} e P$ is said to be $N_{nc} e T_2$ (resp. $N_{nc} e T_2$) for each pair of distinct points $x$ and $y$ of $P$, there exist disjoint $N_{nc}e$ (resp. $N_{nc}ro$) sets $L$ and $M$ of $P$ such that $x \in L$ and $y \in M$.

Theorem 3.12 If $h : P \to Q$ is $c N_{nc}e Irr$ injection and $Q$ is $N_{nc} e T_2$, then $P$ is $N_{nc} e T_2$.

Proof. Let $x, y \in P$ and $x \neq y$.

$(x, y \in P)(x \neq y)$

$h is injective \Rightarrow h(x) \neq h(y), Q is N_{nc}e-T_2$

$\Rightarrow (\exists L \in N_{nc}eO(Q, h(x)))(\exists M \in N_{nc}eO(Q, h(y)))(L \cap M = \phi)$

$h is c N_{nc} e Irr \Rightarrow (h^{-1}(L) \in N_{nc}RO(P, x))(h^{-1}(M) \in N_{nc}RO(P, y))(h^{-1}(L) \cap h^{-1}(M) = \phi)$.
**Theorem 3.13** Let $Q$ be an $N_{nc}e-T_2$ space. If $h : P \to Q$ and $g : P \to Q$ are $cN_{nc}eIrr$, then the set 
$L = \{x|h(x) = g(x)\} \in N_{nc}\delta C(P)$.

**Proof.** Let $x \notin A$.

\[ x \notin L \Rightarrow h(x) \neq g(x), Q \text{ is } N_{nc}e-T_2 \]

\[ \Rightarrow (\exists O_1 \in N_{nc}eO(Q, h(x)))(\exists O_2 \in N_{nc}eO(Q, g(x))), (O_1 \cap O_2 = \phi) \]

\[ \frac{h \text{ and } g \text{ are } c \text{ then } e \text{ Irr}}{h^{-1}(O_1) \in N_{nc}RO(P, x)) (g^{-1}(O_2) \in N_{nc}RO(P, x))(h^{-1}(O_1 \cap O_2) = \phi)} \]

\[ \Rightarrow (U := (h^{-1}(O_1) \cap g^{-1}(O_2) \in N_{nc}RO(P, x))(U \cap L = \phi) \]

\[ \Rightarrow x \notin N_{nc}\delta cl(L). \]

Then $L$ is $N_{nc}\delta c$ in $P$.

**Theorem 3.14** Let $Q$ be an $N_{nc}e-T_2$ space. If $h : P \to Q$ is $cN_{nc}eIrr$, then the set 
$M = \{(x, y)|h(x) = h(y)\in N_{nc}\delta C(P \times P)\}$.

**Proof.** $(x, y) \notin M$.

\[ (x, y) \notin M \]

\[ \Rightarrow h(x) \neq h(y), Q \text{ is } N_{nc}e-T_2 \]

\[ \Rightarrow (\exists O_1 \in N_{nc}eO(Q, h(x)))(\exists O_2 \in N_{nc}eO(Q, h(y))), (O_1 \cap O_2 = \phi) \]

\[ \frac{h \text{ and } g \text{ are } c \text{ then } e \text{ Irr}}{h^{-1}(O_1) \in N_{nc}RO(P, x)) (h^{-1}(O_2) \in N_{nc}RO(P, y))(h^{-1}(O_1) \cap h^{-1}(O_2) = \phi)} \]

\[ \Rightarrow (U := (h^{-1}(O_1) \times h^{-1}(O_2) \in N_{nc}RO(P \times P, (x, y))))(U \cap M = \phi) \]

\[ \Rightarrow (x, y) \notin N_{nc}\delta cl(M). \]

Then $M$ is $N_{nc}\delta c$ in $P \times P$.

4. **Completely weakly $N_{nc}e$-irresolute functions**

**Definition 4.1** A function $h : P \to Q$ is said to be completely weakly $N_{nc}e$-irresolute (briefly $cwN_{nc}eIrr$) if for each $x \in P$ and for any $N_{nc}eo$ set $O$ containing $h(x)$, there exists an $N_{nc}o$ set $U$ containing $x$ such that $h(U) \subseteq O$.

**Remark 4.1** We have the following diagram from Definition 3.1 and Definition 3.2 and Definition 4.1. The converses of these implications are not true in general as shown by the following examples.

$cN_{nc}eIrr \Rightarrow cwN_{nc}eIrr \Rightarrow N_{nc}eIrr$

**Example 4.1** In Example 3.2, then it is

(i) $cw2_{nc}eIrr$ but not $c2_{nc}eIrr$, the set $h^{-1}(\{u, v\}, \phi, \{w, x\}) = \{u, v\}, \phi, \{w, x\}$ is a $2_{nc}eos$ but not $2_{nc}ros$ in $P$.

(ii) $2_{nc}eIrr$ but not $cw2_{nc}eIrr$, the set $h(\{v, w\}, \phi, \{u, x\}) \not\subseteq \{v, \phi, \{u, w\}, \phi, \{u, x\}\}$. $\{v, \phi, \{u, w, x\}\}$ is a $2_{nc}eos$ and $\{v, w, \phi, \{u, x\}\}$ is a $2_{nc}eos$.

Question: Is there any $cwN_{nc}eIrr$ function which is not $cN_{nc}eIrr$?

**Theorem 4.1** Let $h : P \to Q$ be a function, then the following statements are equivalent:
(i) \( h \) is \( cw_{N_{nc}eIrr} \),
(ii) \( h^{-1}(N_{nc}eint(M)) \subseteq N_{nc}eint(h^{-1}(M)) \) for every \( N_{nc} \) set \( M \) of \( Q \),
(iii) \( h(N_{nc}cl(L)) \subseteq N_{nc}ecl(h(L)) \) for every \( N_{nc} \) set \( L \) of \( P \),
(iv) \( N_{nc}cl(h^{-1}(M)) \subseteq h^{-1}(N_{nc}ecl(M)) \) for every \( N_{nc} \) set \( M \) of \( Q \),
(v) \( h^{-1}(O) \) is \( N_{nc}e \) in \( P \) for each \( N_{nc}e \) set \( O \) in \( Q \),
(vi) \( h^{-1}(O) \) is \( N_{nc}o \) in \( P \) for each \( N_{nc}o \) set \( O \) in \( Q \).

**Proof.** (i) \( \Rightarrow \) (ii): Let \( M \subseteq Q \) and \( x \in h^{-1}(N_{nc}eint(M)) \).

\[
x \in h^{-1}(N_{nc}eint(M)) \Rightarrow N_{nc}eint(M) \in N_{nc}eO(Q, h(x)) \\
\Rightarrow (\exists U \in U(x))(h(U) \subseteq N_{nc}eint(M) \subseteq M) \\
\Rightarrow (\exists U \in U(x))(U \subseteq h^{-1}(M)) \\
\Rightarrow x \in N_{nc}int(h^{-1}(M)).
\]

(ii) \( \Rightarrow \) (iii): Let \( L \subseteq P \).

\[
L \subseteq P \Rightarrow h(L) \subseteq Q \\
\Rightarrow Q \setminus h(L) \subseteq Q \\
\Rightarrow h^{-1}(N_{nc}eint(Q \setminus h(L))) \subseteq N_{nc}eint(h^{-1}(Q \setminus h(L))) \\
\Rightarrow P \setminus h^{-1}(N_{nc}ecl(h(L))) \subseteq P \setminus N_{nc}ecl(h^{-1}(h(L))) \\
\Rightarrow N_{nc}cl(L) \subseteq N_{nc}cl(h^{-1}(h(L))) \subseteq h^{-1}(N_{nc}ecl(h(L))) \\
\Rightarrow h(N_{nc}cl(L)) \subseteq N_{nc}ecl(h(L)).
\]

(iii) \( \Rightarrow \) (iv): Let \( M \subseteq Q \).

\[
M \subseteq Q \Rightarrow h^{-1}(M) \subseteq P \\
\Rightarrow h(N_{nc}cl(h^{-1}(M))) \subseteq N_{nc}ecl(h(h^{-1}(M))) \subseteq N_{nc}ecl(M) \\
\Rightarrow N_{nc}cl(h^{-1}(M)) \subseteq h^{-1}(N_{nc}ecl(M)).
\]

(iv) \( \Rightarrow \) (v): Let \( O \in N_{nc}eC(Q) \).

\[
O \in N_{nc}eC(Q) \Rightarrow O = N_{nc}ecl(O) \\
\Rightarrow N_{nc}cl(h^{-1}(O)) \subseteq h^{-1}(N_{nc}ecl(O)) = h^{-1}(O) \\
\Rightarrow h^{-1}(O) = N_{nc}cl(h^{-1}(O)) \\
\Rightarrow h^{-1}(O) \in C(P).
\]

(v) \( \Rightarrow \) (vi): Obvious.

(vi) \( \Rightarrow \) (i): Let \( O \in N_{nc}eO(Q) \) and \( x \in h^{-1}(O) \).

\[
(O \in N_{nc}eO(Q))(x \in h^{-1}(O)) \Rightarrow O \in N_{nc}eO(Q, h(x)) \\
\Rightarrow (U = h^{-1}(O) \in U(x))(h(U) \subseteq O).
\]

**Theorem 4.2** Let \( h : (P, N_{nc}r) \to (Q, N_{nc}s) \) be a bijective function. Then the following statements are equivalent:

(i) \( h \) is \( cw_{N_{nc}Irr} \),
(ii) \( N_{nc}eint(h(L)) \subseteq h(N_{nc}int(L)) \) for every \( N_{nc} \) set of \( P \).
Proof. (i) ⇒ (ii): Let $L \subseteq P$.

$L \subseteq P \Rightarrow P \setminus L \subseteq P$

$\Rightarrow h(P, N_{nc}O(L)) = h(N_{nc}ccl(P \setminus L)) \subseteq N_{nc}ccl(h(P \setminus L))$, $h$ is bijection

$\Rightarrow Q \setminus h(N_{nc}int(L)) \subseteq Q \setminus eN_{nc}int(h(L))$

$\Rightarrow N_{nc}cint(h(L)) \subseteq h(N_{nc}int(L))$.

(ii) ⇒ (i): Let $L \subseteq P$.

$L \subseteq P \Rightarrow P \setminus L \subseteq P$

$\Rightarrow h(N_{nc}cint(L)) \subseteq h(N_{nc}int(P \setminus L))(h$ is bijection)

$\Rightarrow Q \setminus h(N_{nc}int(L)) \subseteq Q \setminus h(N_{nc}ccl(L))$

$\Rightarrow h(N_{nc}cl(L)) \subseteq N_{nc}ccl(h(L))$.

**Theorem 4.3** Let $h : (P, N_{nc}) \rightarrow (Q, N_{nc})$ and $g : (P, N_{nc}) \rightarrow (Z, N_{nc})$ be any two functions. Then the following statements hold:

(i) If $h$ is $cwN_{nc}eIrr$ and $g$ is $N_{nc}eIrr$, then $g \circ h : P \rightarrow Z$ is $cwN_{nc}eIrr$,

(ii) If $h$ is $cN_{nc}Cts$ and $g$ is $cwN_{nc}eIrr$, then $g \circ h$ is $cN_{nc}eIrr$,

(iii) If $h$ is $sN_{nc}Cts$ and $g$ is $cwN_{nc}eIrr$, then $g \circ h$ is $cN_{nc}eIrr$,

(iv) If $h$ and $g$ are $cN_{nc}eIrr$, then $g \circ h$ is $cN_{nc}eIrr$,

(v) If $h$ is $cN_{nc}eIrr$ and $g$ is $cwN_{nc}eIrr$, then $g \circ h$ is $cN_{nc}eIrr$,

(vi) If $h$ is $cwN_{nc}eIrr$ and $g$ is $N_{nc}Cts$, then $g \circ h$ is $N_{nc}Cts$,

(vii) If $h$ is $N_{nc}Cts$ and $g$ is $cwN_{nc}eIrr$, then $g \circ h$ is $N_{nc}eIrr$,

(viii) If $h$ is $N_{nc}Cts$ and $g$ is $cwN_{nc}eIrr$, then $g \circ h$ is $cwN_{nc}eIrr$.

Proof. Straightforward.

**Definition 4.2** A function $h : (P, N_{nc}) \rightarrow (Q, N_{nc})$ is said to be almost $N_{nc}$-open (briefly almost $N_{nc}O$) if $h(U)$ is $N_{nc}O$ in $Q$ for every $N_{nc}\tau$-open set $U$ of $P$.

**Theorem 4.4** If $h : (P, N_{nc}) \rightarrow (Q, N_{nc})$ is almost $N_{nc}O$ surjection and $g : (Q, N_{nc}) \rightarrow (Z, N_{nc})$ is any function such that $g \circ h : (P, N_{nc}) \rightarrow (Z, N_{nc})$ is $cN_{nc}eIrr$, then $g$ is $cwN_{nc}eIrr$.

Proof. Let $O \in N_{nc}cO(Z)$.

$O \in N_{nc}cO(Z) \Rightarrow (g \circ h)^{-1}(O) = h^{-1}(g^{-1}(O)) \in N_{nc}ROC(P), h$ is a almost $N_{nc}O$ surjection

$\Rightarrow h(h^{-1}(g^{-1}(O))) = g^{-1}(O) \in N_{nc}\sigma$.

**Theorem 4.5** If $h : (P, N_{nc}) \rightarrow (Q, N_{nc})$ is $N_{nc}O$ surjection and $g : (Q, N_{nc}) \rightarrow (Z, N_{nc})$ is any function such that $g \circ h : (P, N_{nc}) \rightarrow (Z, N_{nc})$ is $cN_{nc}eIrr$, then $g$ is $cwN_{nc}eIrr$.

Proof. Let $O \in N_{nc}cO(Z)$.

$O \in N_{nc}cO(Z) \Rightarrow (g \circ h)^{-1}(O) = h^{-1}(g^{-1}(O)) \in N_{nc}ROC(P), h$ is a $N_{nc}O$ surjection

$\Rightarrow h(h^{-1}(g^{-1}(O))) = g^{-1}(O) \in N_{nc}\sigma$. 

\"
5. Conclusion
In this work, some new notions of sets namely completely $N_{nc}$-irresolute function and completely weakly $N_{nc}$-irresolute function are introduced and get results in $N_{nc}$ts. This can be extended to $N_{nc}$-homeomorphism functions, $N_{nc}$-connectedness and also a contra field in $N_{nc}$ts. Also we have obtained some characterizations of these functions. Also, we have investigated some fundamental properties between these new notions are separation and covering.

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