Vacuum stability in the SM and the three-loop $\beta$-function for the Higgs self-interaction

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The Higgs potential

\[
\mathcal{L}_\Phi = \partial_\mu \Phi^\dagger \partial^\mu \Phi - \left( m^2 \Phi^\dagger \Phi + \lambda \left( \Phi^\dagger \Phi \right)^2 \right)
\]

\[
V(\Phi) \Phi = \left( \begin{array}{c} \Phi_1 \\ \Phi_2 \end{array} \right) \rightarrow \left( \begin{array}{c} \Phi^+ \\ \frac{1}{\sqrt{2}} (v + H + i\chi) \end{array} \right)
\]

\[
|\Phi_{SM}| = \sqrt{-\frac{m^2}{2\lambda}} = \frac{v}{\sqrt{2}}, \quad M_H^2 = -2m^2 = 2\lambda v^2 \text{ at tree level}
\]
The effective Potential

include radiative corrections \[ \Rightarrow V_{\text{eff}}(\lambda(t), g_i(t), \Phi(t)) \]

with \( t = \log \left( \frac{\Lambda}{\mu_0} \right) \) and \( \Phi(t) = \Phi_{\text{cl}} \cdot \exp \left( \int_0^t dt' \gamma_\Phi(\lambda(t'), g_i(t')) dt' \right) \) [Coleman, Weinberg]

(\( \Lambda \): scale up to which the SM is valid, \( \mu_0 \): starting point for running, e.g. \( \mu_0 = M_Z \))

\[ V_{\text{eff}}[\Phi] \approx \lambda(\Lambda)\Phi^4 + \mathcal{O}(\lambda^2(\Lambda), g_i^2(\Lambda)) \]

for \( \Phi \sim \Lambda \geq \mu_0 \) [Altarelli, Isidori]

Stability of SM vacuum \( \Leftrightarrow \lambda(\Lambda) > 0 \)
Behaviour of $\lambda(\mu)$

Stability bound on the Higgs mass: $M_H > M_{\text{min}}$
Upper bound $M_H < M_{\text{max}}$: no Landau pole for $\mu \leq \Lambda$

$\lambda(\mu)$ for different values of $m_H$

for $\Lambda = M_{\text{Planck}}$: $M_{\text{max}} \approx 175$ GeV,
$M_{\text{min}} \approx 127$ GeV $\Rightarrow$ curious coincidence with recent data!
Main contributions to $\beta_\lambda(\lambda, y_t, g_i, \ldots) = \mu^2 \frac{d}{d\mu^2} \lambda(\mu)$

- $\beta$-functions for gauge couplings at 3 loop [Mihaila, Salomon, Steinhauser]
- $\beta$-functions for Yukawa-, Higgs-sector at 2 loop [Machacek, Vaughn; Luo, Xiao]

Values for SM couplings at $\mu = M_t = 172.9$ GeV

- strong coupling: $g_s \approx 1.17 \Rightarrow \frac{g_s^2}{4\pi} \approx 0.11$
- top-Yukawa coupling: $y_t \approx 0.93 \Rightarrow \frac{y_t^2}{4\pi} \approx 0.07$
- EW couplings: $g_2 \approx 0.65$, $g_1 \approx 0.36 \Rightarrow \frac{g_2^2}{4\pi} \approx 0.03$, $\frac{g_1^2}{4\pi} \approx 0.01$
- Possible Higgs around 125 GeV $\Rightarrow \lambda(M_H) \approx 0.13 \Rightarrow \frac{\lambda}{4\pi} \approx 0.01$

**Main 3 loop contributions to $\beta_\lambda$:** $g_s$, $y_t$, $\lambda$

**Model:** QCD & top-Yukawa & Higgs sector of the SM in the unbroken phase.
Difficulties:

- $\gamma_5$ treatment ['t Hooft, Veltman]
- infrared divergences (auxiliary mass [Chetyrkin,...])
Results: \( \mu^2 \frac{d}{d\mu^2} \lambda(\mu) = \beta_\lambda(g_s, y_t, \lambda) = \sum_{n=1}^{\infty} \frac{1}{(16\pi^2)^n} \beta^{(n)}_\lambda(g_s, y_t, \lambda) \)

in the \( \overline{\text{MS}} \)-scheme

\[
\beta^{(1)}_\lambda = 12 \lambda^2 + 6 y_t^2 \lambda - 3 y_t^4
\]

\[
\beta^{(2)}_\lambda = -156 \lambda^3 - 72 y_t^2 \lambda^2 - \frac{3}{2} y_t^4 \lambda + 15 y_t^6 + 40 g_s^2 y_t^2 \lambda - 16 g_s^2 y_t^4
\]

\[
\beta^{(3)}_\lambda = \lambda^4 \left( 3588 + 2016 \zeta_3 \right) + 873 y_t^2 \lambda^3 + y_t^4 \lambda^2 \left( \frac{1719}{2} + 756 \zeta_3 \right)
\]

\[
+ y_t^6 \lambda \left( \frac{117}{8} - 198 \zeta_3 \right) - y_t^8 \left( \frac{1599}{8} + 36 \zeta_3 \right)
\]

\[
+ g_s^2 y_t^2 \lambda^2 \left( -1224 + 1152 \zeta_3 \right) + g_s^2 y_t^4 \lambda \left( 895 - 1296 \zeta_3 \right)
\]

\[
+ g_s^2 y_t^6 \left( -38 + 240 \zeta_3 \right) + g_s^4 y_t^2 \lambda \left( \frac{1820}{3} - 32 n_f - 48 \zeta_3 \right)
\]

\[
+ g_s^4 y_t^4 \left( -\frac{626}{3} + 20 n_f + 32 \zeta_3 \right)
\]

For \( M_H \approx 125 \ \text{GeV} \) at \( \mu = M_Z \):

\[
\frac{\beta^{(3)}_\lambda}{(16\pi^2)^3} = \left( \begin{array}{ccccc}
+7.9 & -4.8 & -3.1 & -2.5 & +2.6 \\
g_s^2 y_t^6 & y_t^8 & g_s^2 y_t^4 \lambda & g_s^4 y_t^4 & g_s^4 y_t^2 \lambda
\end{array} \right) \times 10^{-5}
\]

2 loop: 1\%, 3 loop: (-0.04)\% correction to 1 loop result.
3 loop running (incl. 2 loop EW),
1 loop matching for on-shell → MS-parameters
(2 loop matching for QCD)
3 loop correction to $\beta_\lambda$ smaller than the $\alpha_s$ uncertainty.

Individual contributions to $\beta_\lambda$ much larger than overall effect!

3 loop result: small improvement of the stability of the SM vacuum.

Main uncertainty for SM stability: $M_t$