DOA Estimation of a Space-limited MIMO Radar with High Degree of Freedom

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Abstract. Aiming at the problem of the small aperture of the traditional MIMO radar with virtual degrees of freedom, this paper designs a high degree of freedom space-limited MIMO radar. Both the transmitting and receiving elements of this radar adopt a sparse array structure. Array composition, the receiving array element is composed of a single array element and a uniform linear array. The number of virtual array elements can be realized by using array elements. Compared with the traditional sparse array MIMO radar with the same number of elements, the designed space-limited sparse array MIMO radar has a larger aperture. Experimental simulations verify the superiority of the space-limited MIMO radar angle estimation.

1. Introduction
Traditional MIMO radar usually uses uniform linear array ULA[1] as the transmitting and receiving array. The number of virtual array elements of the echo signal model of this radar structure is limited by the number of physical array elements. In order to further increase the upper limit of the degree of freedom of the MIMO radar and be able to measure multiple targets more accurately at the same time, scholars have conducted a lot of exploration. Among them, the sparse array is used as the receiving and transmitting element of the MIMO radar is one of the directions[2]. Using the combination of sparse array and MIMO radar, the expansion of the virtual array can be realized through the vectorization of the received signal covariance matrix, that is, the "Sum and difference coarray". This method can effectively expand the aperture of the array and achieve the measurement target. Traditional sparse array MIMO radars usually have minimum redundancy MIMO radars, nested MIMO radars and co-prime MIMO radars. The minimum redundancy MIMO radar [3] uses the Minimum Redundancy Array (MRA) [4-5] as the radar's transmitting and receiving array elements to maximize the utilization of the array elements. However, the design of the smallest redundant array lacks the closed-form solution of the virtual array element, and generally requires a large number of computational searches. As the number of array elements increases, it is difficult to realize in engineering. Chen C [6] proposed the use of a minimum redundant array to construct a MIMO radar structure, and an increase in the number of virtual array elements was achieved through virtual aperture expansion. Huang Y et al. [7] proposed a modified MIMO radar structure based on the original MIMO radar structure. By designing uniform face-to-face diagnosis at different levels, the calculation search time was reduced, and the average Toeplitz matrix method was used to improve the...
estimation performance. Nested MIMO radar [8] usually uses a nested array as the transmitting and receiving element of the MIMO radar. According to the different transceiver elements, it is usually divided into a nested subarray MIMO radar and a co-prime MIMO radar. The virtual position of the nested MIMO radar has a closed-form solution, so the structure is easy to implement. However, since the nested array has continuous elements, there is a large mutual coupling between the nested MIMO radar array elements, which affects the accuracy of the measurement results. Zheng W [9] proposed a generalized nested MIMO radar structure. The transmitting and receiving arrays of the radar are both nested arrays. The spreading factor is used to expand the distance between the transmitting array elements, which effectively expands the aperture. Zhang Y [10] proposed a generalized extended two-level nested MIMO radar (GENA-TR), which has a higher degree of freedom and a more accurate measurement effect. Co-prime MIMO radars usually use co-prime arrays as transmitting and receiving array elements. The distance between the array elements is large and the mutual coupling between the array elements is small. Li J [11] used the joint real-valued ESPRIT algorithm for DOA estimation for co-prime MIMO radar to improve the estimation performance.

2. Sparse extension array geometry

2.1. Space-limited array geometry

The article proposes a space-limited sparse array MIMO radar structure, which has a closed-form expression of degrees of freedom, and can simultaneously optimize degrees of freedom and mutual coupling. It is compatible with traditional nested subarray MIMO radars and co-prime arrays. Compared with the virtual aperture of MIMO radar, the mutual coupling is smaller, and the sparse array MIMO radar has better angle estimation effect than the distance-constrained MIMO radar. Figure 1 shows the space-limited sparse array MIMO radar structure. Assume that the total number of MIMO radar transmitting and receiving elements is \( S = N_1 + N_2 + 2 \), \( N_1 \geq 4 \), and \( N_1 \) is an even number, \( N_2 \in \mathbb{Z}^+ \). The transmitting array element is composed of \( N_1 + 1 \) elements, and the receiving element is composed of \( N_2 + 1 \) elements.

![Figure 1. Space-limited sparse array MIMO radar array structure.](image)

The location of the transmitting element is set as:

\[
P_t = \{0, 2, \cdots, N_1 - 2, N_1 + 1, N_1 + 3, \cdots, 2N_1 - 3, (N_2 + 3)N_1 - 6, (N_2 + 3)N_1 - 5\}
\]

(1)

The collection of the positions of the receiving array elements is:

\[
P_r = \{0, 3N_1 - 3, 4N_1 - 3, 5N_1 - 3, \cdots, (N_2 + 2)N_1 - 3\}
\]

(2)

2.2. MUSIC spectrum

We assume that there are 30 uncorrelated narrowband signals in the far field of space, the signals are evenly distributed at \([-80^\circ, 80^\circ]\), the number of snapshots is set to 1000, and the signal-to-noise ratio is 0dB. Using spatial smoothing MUSIC algorithm (SS-MUSIC), the spatial spectral peak diagrams of three MIMO radars are shown in Figure 2. It can be seen from the figure that the distance constrained MIMO radar can distinguish 30 targets well. However, the co-prime array MIMO radar and the nested subarray MIMO radar cannot distinguish 30 targets, and they appear to be misaligned or the spectral peaks have a large deviation from the real angle. The distance-constrained MIMO radar can
provide more virtual degrees of freedom than the other two radars, so it can measure more source angles.

Figure 2. Spatial spectrum with SS-MUSIC.

2.3. RMSE performance versus SNR and snapshots
Assuming that there are 16 targets in the far field of space, the targets are evenly distributed in [-80°, 80°]. We use the SS-MUSIC algorithm to study the root mean square error of different MIMO radars. Figure 3 shows the variation curve of the root mean square error with the increase of the signal-to-noise ratio, set the number of snapshots to 1000, and increase the signal-to-noise ratio from -20dB to 20dB in 5dB steps. From the abscissa: when the signal-to-noise ratio is less than 0dB, the root-mean-square error of all arrays is larger, and the root-mean-square error decreases with the increase of the signal-to-noise ratio. When the signal-to-noise ratio is greater than 5dB, the root-mean-square error tends to be stable. From the ordinate point of view, when the root-mean-square error tends to stabilize, the time-distance constraint MIMO radar has the smallest root-mean-square error, the nested subarray MIMO radar takes the second place, and the proton-array MIMO radar has the largest root-mean-square error. This is because the spacing constraint MIMO radar has the largest number of continuous virtual elements, while the coprime array MIMO radar has the smallest number of continuous virtual elements.

Figure 3. RMSE of different geometries versus SNR.

2.4. Estimated probability of success
Figure 5 shows the variation curve of the estimated success probability as the signal-to-noise ratio increases. Set the number of snapshots to 1000, and the signal-to-noise ratio increases from -20dB to 20dB in 5dB steps. When the signal-to-noise ratio is less than -10dB, the estimated success probability of nested subarray MIMO radar and co-proton-array MIMO radar is almost 0. The estimated success probability of the distance-constrained MIMO radar is already above 90% when the signal-to-noise ratio is greater than -5dB. Space-limited MIMO radar has a higher estimated probability of success when the signal-to-noise ratio is low, and the estimated success probability has reached 100% when the signal-to-noise ratio is greater than -10dB.

Figure 6 shows the variation curve of the estimated success probability with the increase of the number of snapshots. Set the signal-to-noise ratio to 0dB, and increase the number of snapshots from 50 to 2050 in steps of 250. The estimated success probability of nested subarray MIMO radar and coproton MIMO radar increases with the increase of the number of snapshots. The estimated success probability reaches 100% when the number of snapshots is greater than 1550 and 550, respectively. The distance-constrained MIMO radar estimates that the probability of success has reached 100% when the number of snapshots is 50.
3. Conclusion

In this paper, we propose a high-degree-of-freedom space-limited MIMO radar. This MIMO radar receiving and transmitting array adopts a self-designed sparse array structure. \( N_1 + N_2 + 2 \) array elements can be used to realize \( 2N_1N_2 + 5N_1 - 8 \) virtual array elements. The theoretical analysis distance constrains the MIMO radar to have a greater degree of virtual freedom. Experimental simulations have verified the superiority of space-limited MIMO radar in DOA estimation compared with nested subarray MIMO radar and co-proton-array MIMO radar.

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