Ratio of cross-sections of kaons to pions produced in \( pp \) collisions as a function of \( \sqrt{s} \)

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Abstract

A calculation of the inclusive spectra of pions and kaons produced in \( pp \) collisions as functions of their transverse momentum \( p_t \) at mid-rapidity is presented within the self-similarity approach. A satisfactory description of the data within a wide range of initial energies is presented. We focus mainly on the ratio of cross-sections of \( K^\pm \) to \( \pi^\pm \) mesons produced in \( pp \) collisions as a function of \( \sqrt{s} \). A fast rise of this ratio, when the initial energy increases starting from the kaon production threshold up to \( \sqrt{s} \approx 20-30 \) GeV, is revealed together with its very slow increase up to LHC energies. The energy dependence of this ratio is due to the conservation laws of four-momenta and quantum numbers of the initial and produced hadrons, and to the Regge behavior of cross-sections at large energies. The more or less satisfactory agreement of these ratios with NA61-SHINE, RHIC and LHC data is demonstrated.

1 Introduction

The study of strange hadron production in heavy-ion collisions compared to the similar production of pions attracts the attention of theorists and experimenters. This issue became very important and intriguing after the observation of a fast rise and sharp peak in the ratio of \( K^+ \) mesons to \( \pi^+ \) mesons produced in central \( Pb+Pb \) and \( Au+Au \) collisions at mid-rapidity, when the initial energy \( \sqrt{s_{NN}} \) per nucleon grows from the threshold of \( K^+ \) meson production up to 20-30 GeV \([1, 2]\) and then falls down at \( \sqrt{s_{NN}} > 30 \) GeV. However, the energy dependence of the \( K^-/\pi^- \) ratio in central \( Pb+Pb \) and \( Au+Au \) collisions is different, there is no peak at any \( \sqrt{s} \), see \([3]\) and references therein.

The \( K^+/\pi^+ \) ratio in \( pp \) collisions at mid-rapidity in contrast to the similar ratio observed in \( AA \) collisions has a fast rise when the initial energy grows up to \( \sqrt{s} \approx 20-30 \) GeV and then it increases slowly with \( \sqrt{s} \). The \( K^-/\pi^- \) ratio in \( pp \) collisions at mid-rapidity has a similar energy dependence, see \([3]\) and references therein.

In this paper we analyze the production of kaons and pions in \( pp \) collisions at mid-rapidity and focus on ratios between their cross-sections as functions of the initial energy. Our analysis is based on the similarity of inclusive spectra of particles produced in hadron-hadron collisions, suggested in pioneering papers \([4, 5, 6, 7]\), and on the conservation laws of four-momenta and quantum numbers \([8, 9, 10, 11]\).
Actually, we continue to apply the approach developed in recent papers \[12\, 13\, 14\], where \(p_t\)-spectra of pions produced in \(pp, AA\) collisions at mid-rapidity and within a wide range of initial energies were analyzed.

In \[9\, 11\] the similarity was demonstrated at zero rapidity \(y = 0\) of these spectra as functions of the similarity parameter \(\Pi\) dependent on the initial energy \(\sqrt{s}\) in the c.m.s of the colliding particles and the transverse masses \(m_{ht}\) of the produced hadrons. A simple form of inclusive spectra was used in \[9\, 10\, 11\] to describe satisfactorily the data at low values of \(m_{ht}\). Further development of this approach was presented in our papers \[12\, 13\, 14\], where the description of \(m_{ht}\)-spectra was extended to larger values of transverse masses and initial energies up to a few TeV including contributions of both quarks and gluons to these spectra. The relationship between \(\Pi\) and the Mandelstam variables \(s, t\) was obtained in \[12\, 13\]. Moreover, it has been shown that \(\Pi\) cannot be presented in the factorization form as a common function of \(\sqrt{s}\) and \(m_{ht}\). The breakdown of this factorization occurs at not large initial energies \(\sqrt{s} < 10\) GeV. It is restored at larger \(\sqrt{s}\) \[12\, 13\]. In fact, this is an advantage of the approach based on the kinematics of four-momentum velocities considered in \[8\, 9\, 10\, 11\], where the parameter \(\Pi\) was obtained using the conservation laws of four-momenta and quantum numbers of initial and produced particles, and the minimization principle. At zero rapidity \(y = 0\) the form for \(\Pi\) was obtained analytically \[11\].

In this paper we will give a brief review of the main properties of the similarity approach mentioned above and then present in detail our calculations of transverse momentum spectra of pions and kaons produced in \(pp\) collisions within a wide range of initial energies \(\sqrt{s}\). The main focus of our paper is the theoretical analysis of ratios of cross-sections of \(K^{\pm}\) to \(\pi^{\pm}\) mesons produced in these collisions as functions of \(\sqrt{s}\) and their comparison with all the world data.

### 2 Main properties of the self-similarity approach.

The inclusive production of hadron 1 in the interaction of nucleus \(A\) with nucleus \(B\):

\[
A + B \rightarrow 1 + \ldots,
\]

is satisfied by the conservation law of four-momenta in the following form \[10\, 11\]:

\[
(N_A P_A + N_B P_B - p_1)^2 = (N_A m_0 + N_B m_0 + M)^2,
\]

where \(N_A\) and \(N_B\) are the fractions of the four-momentum transmitted by nucleus \(A\) and nucleus \(B\), their forms are presented in \[11\, 13\]; \(P_A\), \(P_B\), \(p_1\) are the four-momenta of nuclei \(A\) and \(B\) and particle 1, respectively; \(m_0\) is the mass of the nucleon; \(M\) is the mass of the particle providing for conservation of the baryon number, strangeness, and other quantum numbers. For \(\pi\)-mesons \(m_1 = m_\pi\) and \(M = 0\). For antinuclei \(M = m_1\) and for \(K^-\)-mesons \(M = m_1 = m_K\), \(m_K\) is the mass of the \(K\)-meson. For nuclear fragments \(M = -m_1\). For \(K^+\)-mesons \(m_1 = m_K\).
and \( M = m_A - m_K \), \( m_A \) is the mass of the \( \Lambda \)-baryon. Let us note that the isospin effects of the produced hadrons and other nuclear effects are out of this approach. Therefore, it is assumed that within the self-similarity approach there is no big difference between the inclusive spectra of \( \pi^+ \) and \( \pi^- \) mesons produced in \( pp \) and \( AA \) collisions. However, there is a difference between similar spectra of \( K^+ \) and \( K^- \) mesons, because the values of \( M \) are different. This is due to the conservation law of strangeness.

In \([10, 11]\) the parameter of self-similarity is introduced in the following form:

\[
\Pi = \min \left[ \frac{1}{2} \left( (u_A N_A + u_B N_B)^2 \right)^{1/2} \right],
\]

(3)

where \( u_A \) and \( u_B \) are the four-velocities of nuclei \( A \) and \( B \).

Then, the inclusive spectrum of particle 1 produced in the \( AA \) collision can be presented as a general universal function dependent on the self-similarity parameter \( \Pi \):

\[
E d^3 \sigma / dp^3 = A_A^{\alpha(N_A)} A_B^{\alpha(N_B)} F(\Pi)
\]

(4)

where \( \alpha(N_A) = 1/3 + N_A/3 \), \( \alpha(N_B) = 1/3 + N_B/3 \) and function \( F(\Pi) \) has the following form \([14]\):

\[
F(\Pi) = (A_q e^{\Pi/C_q}) + A_g \sqrt{\Pi} \phi_1(s) e^{-\Pi/C_g}) \sigma_{tot}
\]

where

\[
\Pi(s, m_{1t}, y) = \left\{ \frac{m_{1t}}{2m_0 \delta_h} + \frac{M}{\sqrt{s \delta_h}} \right\} \times
\]

\[
\left\{ 1 + \sqrt{1 + \frac{M^2 - m_1^2}{(m_{1t} + 2Mm_0/\sqrt{s})^2ch^2(y) \delta_h}} \right\},
\]

(6)

\( \phi_1(s) = 1 - \sigma_{nd}(s)/\sigma_{tot}(s) \), see \([13, 14]\). Here \( \delta_h = \left( 1 - \frac{s_{th}^\pi}{s} \right) \); \( s_{th}^\pi \approx 4m_0^2 \);

\( s_{th}^{K^+} = (m_0 + m_K + m_A)^2 \); \( s_{th}^{K^-} = (2m_0 + 2m_K)^2 \); \( M = m_A - m_0; m_A = 1.115 \) GeV; \( m_k = 0.494 \) GeV; \( m_0 = 0.938 \) GeV;

\( \sigma_{tot}, \sigma_{SD} \) and \( \sigma_{el} \) are the total cross-section, the single diffractive cross section and the elastic cross-section of \( pp \) collisions, respectively. They were taken from \([18]\) and \([19]\) and, together with parameters \( A_q, C_q \) and \( A_g, C_g \), they are presented in the Appendix.
3 Transverse momentum spectra and integrated pion and kaon cross-sections.

For $pp \rightarrow hX$ inclusive processes the relativistic invariant differential cross-section at small but non-zero rapidity $y$ has the following form:

$$\rho_h(p_{ht}, y) \equiv E_h \frac{d^3\sigma_{NN}}{d^3p_1} \equiv \frac{1}{\pi} \frac{d\sigma}{dp_{ht}^2 dy} \equiv (7)$$

where $F(\Pi(s, m_{1t}, y))$ and $\Pi(s, m_{1t}, y)$ are given by Eqs. (5,6).

The production cross-section of hadron $h$ integrated over its transverse momentum $p_{1t}$ or transverse mass $m_{1t}$ at zero rapidity $y = 0$ can be presented in the following form:

$$\sigma_h(s \geq s_{th}, y = 0) = \int_{p_{1t}^m}^{p_{1t}^{max}} \rho(s, p_{1t}, y = 0)p_{1t}dp_{1t} \equiv (8)$$

where

$$J_q = A_q \exp\left(-\frac{m_{1t}}{2m_0C_q\delta_h}\right) \times \left\{1 + \left[1 + \frac{M^2 - m_{1t}^2}{(m_{1t} + 2Mm_0/\sqrt{s})^2\delta_h}\right]\right\}$$

and

$$J_g = A_g \sqrt{p_{1t}\phi_1(s)} \exp\left(-\frac{m_{1t}}{2m_0C_g\delta_h}\right) + \frac{M(C_g - C_q)}{C_qC_g\sqrt{s}\delta_h} \times \left\{1 + \left[1 + \frac{M^2 - m_{1t}^2}{(m_{1t} + 2Mm_0/\sqrt{s})^2\delta_h}\right]\right\}$$

Let us analyze qualitatively the energy behaviors of pion and kaon production cross-sections in $pp$ collisions at zero rapidity given by Eqs. (8-10). Their $\sqrt{s}$ dependence at not large energies is determined mainly by the factor $\exp\left(-\frac{M}{C_q\sqrt{s}\delta_h}\right)$ and factors $\exp\left(-\frac{m_{1t}}{2m_0C_q\delta_h}\right)$, $\exp\left(\frac{M(C_g - C_q)}{C_qC_g\sqrt{s}\delta_h}\right)$, $\exp\left(-\frac{m_{1t}}{2m_0C_g\delta_h}\right)$ entering into functions $J_q$ and $J_g$. For all hadrons produced at the threshold these factors result in the zero cross section due to zero of the kinematical factor $\delta_h = 1 - s_{th}/s$ at $s = s_{th}$. For $\pi^\pm, K^+, K^- \text{ mesons}$ the threshold energies $\sqrt{s_{th}}$ are 2.015 GeV, 2.547 GeV and 2.86 GeV, respectively. At $s > s_{th}$ the production cross-sections of $K^\pm$ mesons show a fast rise due to $M$ not being equal to zero and to an increase of $\delta_h = 1 - s_{th}/s$, then at $s >> s_{th}$
and $s >> M$ the energy dependence of these cross-sections changes only due to the total cross section $\sigma_{tot}(s)$ and $\phi_1(s) = 1 - \sigma_{nd}(s)/\sigma_{tot}(s)$ [13] [14] entering into Eq. (10). The energy dependence of the production cross-section of pions in $pp$ collision is similar to the kaon production cross-section, however, the threshold of the pion production $s_{th}^\pi$ is less than the kaon one $s_{th}^{K\pm}$, as it is mentioned above. Therefore, the ratio of cross-sections, $\sigma_{K^\pm}/\sigma_\pi$ exhibits a fast rise from the kaon threshold when $\sqrt{s}$ grows due to an increase of the phase space. Then, at $s >> s_{th}$ this rise is broken and goes to a slow increase due, mainly, to the factors $\phi_1(s)$ and $exp\left(\frac{M(C_g - C_q)}{C_q C_g \sqrt{s} \delta_h}\right)$ presented in Eq. (10) because $C_g > C_q$, as it is shown in Table 1 of the Appendix.

![Transverse momentum spectra \(\pi^\minus\) mesons produced in \(pp\) collision at \(P_{in}=158\, GeV/c\) \((\sqrt{s}=17.28\, GeV)\), \(80\, GeV/c\) \((\sqrt{s}=12.34\, GeV)\), \(40\, GeV/c\) \((\sqrt{s}=8.77\, GeV)\), \(31\, GeV/c\) \((\sqrt{s}=7.75\, GeV)\), \(20\, GeV/c\) \((\sqrt{s}=6.27\, GeV)\) at mid-rapidity \(|y| < 0.2\). The lines are our calculations, data are taken from [30], the bands are due to uncertainties in parameter $A_q$ presented in the Appendix.

The $p_t$-spectra of $\pi^\minus$, $K^+$ and $K^-$ mesons are sums of quark and gluon contributions including uncertainties due to the fit of data are presented in Figs. (1)[13]. By fitting NA61/SHINE data on $p_t$-spectra at mid-rapidity the parameters $C_g, A_g, C_q$ were found to be independent of the initial energy $\sqrt{s}$, they depend on the kind of mesons produced, $\pi, K^+, K^-$. However, the parameter $A_q$ varies a little bit at energies from 40 GeV/c up to 158 GeV/c. The uncertainties in $p_t$-spectra and ratios of yields, $K^+/\pi^+$ and $K^-/\pi^-$ are due to the uncertainties in the parameter $A_q$. All these parameters are presented in the Appendix. The similar spectra with quark and gluon contributions are also presented in the Appendix.

4 Ratio of the kaon and pion yields.

In Figs. (15) the respective yield ratios, $K^+/\pi^+$ and $K^-/\pi^-$, are presented as functions of $\sqrt{s}$. From these figures one can see their fast rise from the threshold.
Figure 2: The $p_t$-spectra of $K^+$ mesons produced in $pp$ collision at $P_{in} = 158$ GeV/c ($\sqrt{s}=17.28$ GeV), 80 GeV/c ($\sqrt{s} = 12.34$ GeV), 40 GeV/c ($\sqrt{s} = 8.77$ GeV), 31 GeV/c ($\sqrt{s} = 7.75$ GeV), 20 GeV/c ($\sqrt{s} = 6.27$ GeV) at mid-rapidity $|y| < 0.2$. The lines are our calculations, data are taken from [30], the bands are due to uncertainties in parameter $A_q$ presented in the Appendix.

Figure 3: The $p_t$-spectra of $K^-$ mesons produced in $pp$ collision at $P_{in} = 158$ GeV/c ($\sqrt{s}=17.28$ GeV), 80 GeV/c ($\sqrt{s} = 12.34$ GeV), 40 GeV/c ($\sqrt{s} = 8.77$ GeV), 31 GeV/c ($\sqrt{s} = 7.75$ GeV), 20 GeV/c ($\sqrt{s} = 6.27$ GeV) at mid-rapidity $|y| < 0.2$. The lines are our calculations, data are taken from [30], the bands are due to uncertainties in parameter $A_q$ presented in the Appendix.
energy of $K^+$ or $K^-$ production up to $\sqrt{s} = 20$-30 GeV and their further slow increase as the energy grows.

Figure 4: The ratio between yields of $K^+$ and $\pi^+$ mesons produced in $pp$ collisions as a function of $\sqrt{s}$.

Figure 5: The ratio between yields of $K^-$ and $\pi^-$ mesons produced in $pp$ collisions as a function of $\sqrt{s}$.

The upper line in Fig. 4 corresponds to the fit of data for $\pi^+$ and $K^+$ mesons at $P_m = 80$ GeV/c, and the bottom line corresponds to the similar fit at $P_m = 158$ GeV/c. The upper curve in Fig. 5 corresponds to the fit of data for $\pi^-$ and $K^-$ mesons at $P_m = 40$ GeV/c and the bottom line corresponds to the similar fit at $P_m = 158$ GeV/c.

5 Conclusion

In this paper we have applied the self-similarity approach of analysis of hadron production in $pp$ collisions to the production of both kaons and pions in $pp$ collisions.
at mid-rapidity $y < 0.2$ within a wide range of initial energies. The main goal of this paper is to analyze the ratio of kaon yields to those of pions, produced in $pp$ collisions, within this approach as a function of $\sqrt{s}$. We have presented a self-consistent satisfactory description of the data on $p_t$-spectra of pions and kaons in a wide range of initial energies and not large transverse momenta. The fast rise of $K^+/\pi^+$ and $K^-/\pi^-$ yield ratios as functions of $\sqrt{s}$ from the threshold energy of $K^+$ or $K^-$ production up to $\sqrt{s} = 20-30$ GeV has been demonstrated as well as their further slow increase with growing energy. The energy dependence of these ratios calculated within the suggested approach results in a satisfactory description of the data presented by NA61/SHINE at $6.27 \leq \sqrt{s} \leq 17.28$ GeV. As for the descriptions of data on $K^+/\pi^+$ yield ratios measured by PHENIX, STAR and ALICE Collaborations, they do not contradict data taking into account the uncertainty of parameter $A_q$ presented in Figs [4,5] as shade bands.

The physical reason of this energy dependence of the ratio of kaon yields to those of pions consists in the conservation laws of four-momenta and of reaction quantum numbers and, also, in the Regge behavior of the cross section. The conservation law of the four-momenta of initial and produced particles results in the fast rise with energy of the kaon and pion production cross-sections, when $\sqrt{s}$ grows from the threshold energy. This is due to the factor $\delta_h = 1 - s_{th}^h/s$ entering into the self-similarity function $\Pi(s,m_1,t,y)$ given by Eq.(6). A similar fast rise of the $K^+/\pi^+$ yield ratios is also due to the factor $\delta_h$ and to the non-zero value of $M$ for $K^+$ and $K^-$ mesons that also enters into $\Pi(s,m_1,t,y)$. When $\sqrt{s} \gg \sqrt{s_{th}}$ and $\sqrt{s} \gg M$, the pion and kaon production cross-sections and their ratios become insensitive to factors $\delta_h$ and $M$, however they are sensitive to the Regge energy behavior of cross-sections. That is why the $K^+/\pi^+$ yield ratios exhibit two kinds of energy dependence, a fast rise, when $\sqrt{s_{th}} < \sqrt{s} < 20-30$ GeV and a slow increase, when $\sqrt{s} > 20-30$ GeV.

6 Appendix

The parameterizations of $\sigma_{tot}, \sigma_{SD}$ and $\sigma_{el}$ have the following forms [18] and [19]:

$\sigma_{tot} = (21.7(s/s_0)^{0.0808} + 56.08(s/s_0)^{-0.4525})$ mb;

$\sigma_{rd} = (\sigma_{tot} - \sigma_{el} - \sigma_{SD})$ mb;

$\sigma_{el} = (12.7 - 1.75ln(s/s_0) + 0.14ln^2(s/s_0))$ mb; $\sigma_{SD} = (4.2 + ln(\sqrt{s/s_0}))$ mb.

In Fig. 6 the $p_t$-spectra of pions and kaons, produced in $pp$ collisions within the initial momentum range (20-158) GeV/c, fitted by NA61/SHINE data are presented. The black dashed line corresponds to the quark contribution, the blue dash-dotted curve is the gluon contribution and the red solid line is the sum of quark and nonperturbative gluon contributions. The parameters $A_q, A_g$ and $C_q, C_g$ were found from a fit of NA61/SHINE data and are presented in Table 1.

As it is shown in [12, 13, 14], the form of inclusive pion spectra versus $p_t$ at mid-rapidity given by Eqs. (4-6) describes satisfactorily data in a wide range of $\sqrt{s}$
at $p_t < 2$-3 GeV/c. Moreover, as it is shown in [15][16] and [17], the contribution of gluons to the pion spectrum is related to the gluon distribution at low $Q^2 = 1$-2 (GeV/c)$^2$, the use of which results in a satisfactory description of data on hard $pp$ processes at LHC energies and of proton structure functions at low $x$. Therefore, we use Eqs. (4-7) for description of data on pion $p_t$-spectra in $pp$ collisions, only improving the fit of data.

As for $K^\pm$ production in $pp$ collisions at not large initial energies we take into account the additional contribution due to the one Reggeon exchange diagram, which has $\sqrt{s_{th}}/s$ dependence. It leads to modification of parameter $A_q$ in the following form $A_q(1+\sqrt{s_{th}}/s)$, which can be approximated by $A_q exp(\sqrt{s_{th}}/s)$. This correction vanishes at RHIC and LHC energies, however, it allows us to describe data at $\sqrt{s} < 10$ GeV satisfactorily.

The parameter $A_q$ for $\pi$ meson production was found from the fit of NA61 data [30][3] at initial energies $P_{in} = 40$-158 GeV/c. It is very close to the value of $A_q$ obtained in [12][13][14]. For $K^+$ production the value of $A_q$ was found from the fit of NA61 data at $P_{in} = 80$ GeV/c and $P_{in} = 158$ GeV/c. For $K^-$ $A_q$ was found from a fit of NA61 data at $P_{in} = 40$ GeV/c and $P_{in} = 158$ GeV/c. Other parameters $A_g, C_q$ and $C_g$ were fixed from a fit of the NA61 data at $P_{in} = 80$ GeV/c and they do not depend on other initial energies.

| $pp \rightarrow hX$ | $\pi$ | $K^+$ | $K^-$ |
|-----------------|-------|--------|--------|
| $\sqrt{s_{pp}}, \text{GeV}$ | 17.3 | 12.3 | 158 | 17.3 | 12.3 | 17.3 | 8.8 |
| $P_{in}, \text{GeV/c}$ | 158 | 80 | 40 | 158 | 80 | 158 | 40 |
| $A_q$ | 3.361 | 3.063 | 2.688 | 0.9925 | 1.152 | 1.951 | 2.219 |
| $C_q$ | 0.147 | 0.148 | 0.22 |
| $A_g$ | 1.788 | 0.7726 | 0.629 |
| $C_g$ | 0.22 | 0.2066 | 0.2271 |

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References
[1] S.V. Afanasiev, et al., (NA49 Collaboration) Phys.Rev.C 66, 054902 (2002).
[2] C. Alt, et al., (NA49 Collaboration) Phys.Rev.C 77, 024903 (2008).
[3] A. Aduszkiewicz, et al., (NA49 Collaboration) Phys.Rev.C 102, 011901(R) (2020).
[4] E. Fermi, Phys. Rev. 92, 452 (1953)

[5] I. Ya. Pomeranchuk, Izv. Dokl. Akad. Nauk Ser.Fiz. 78, 889 (1951).

[6] L.D. Landau, Izv. Akad. Nauk Ser. Fiz. 17, 51 (1953).

[7] R. Hagedorn, Supplemento al Nuovo Cimento 3, 147 (1965).

[8] A.M. Baldin, L.A. Didenko, Fortsch.Phys. 38, 261 (1990).

[9] A.M. Baldin, A.I. Malakhov, and A. N. Sissakian, Phys. Part. Nucl. 29 (Suppl. 1), 4 (2001).

[10] A. M.Baldin, A. A. Baldin. Phys. Particles and Nuclei, 29 No3, 232 (1998).

[11] A.M. Baldin, A.I. Malakhov. JINR Rapid Communications, No.1(87)-98, pp.5-12 (1998).

[12] D.A. Artemenkov, G.I. Lykasov, A.I. Malakhov, Int.J.Mod.Phys. A30 (2015) 1550127.

[13] G.I. Lykasov, A.I. Malakhov, Eur. Phys. J. A 54, 187 (2018).

[14] A.I. Malakhov, G.I. Lykasov, Eur. Phys. J. A 56, 114 (2020).

[15] A.A. Grinyuk, G.I. Lykasov, A.V. Lipatov, N.P. Zotov, Phys.Rev. D87, 074017 (2013).

[16] A.V. Lipatov, G.I. Lykasov, N.P. Zoto, Phys.Rev. D89, 014001 (2014).

[17] A.M. Abdulov, H. Jung, A.V. Lipatov, G.I. Lykasov, M.A. Malyshev, Phys.Rev. D98, 054010 (2018).

[18] N. Cartiglia, arXiv:1305.6131 [hep-ex].

[19] S.H. Stark, Eur.Phys.J. (Web of Conf.) 141 03007 (2017).

[20] E. Schnedermann, J. Sollfrank, U. Heinz, Phys.Rev.C48,2462 (1993).

[21] G. Wilk, Z. Wlodarczyk, Phys.Lett. 84, 2770 (2000).

[22] K.A. Bugaev, J.Phys.G:Nucl.Phys., 28, 1981 (2002).

[23] K.A. Bugaev, M. Gadzicki, M.I. Gorenstein, Phys.Lett. B544, 127 (2002).

[24] J. Cleymans, G.I. Lykasov, A.S. Parvan, et al., Phys.Lett. B 723, 351 (2013).

[25] J.L. Kley, et al., E895 Collaboration, Phys.Rev. C 68, 054905 (2003).

[26] J. Cleymans, J. Struempefler, L. Tirk, Phys.Rev. C 78, 017901 (2008).
[27] N. Abgrall, et al., NA61/SHINE Collaboration, Eur. Phys. J. C \textbf{74}, 2794 (2014).
[28] K.A. Ter-Martirosyan, Sov.J.Nucl.Phys., \textbf{44}, (1986) 817.
[29] A.A. Grinyuk, G.I. Lykasov, A.V. Lipatov, N.P. Zotov, Phys.Rev. \textbf{D87}, (2013) 074017.
[30] A. Adiszkiewicz, \textit{et al.}, (NA61/SHINE Collaboration) Eur.Phys.J.\textbf{C77}, 671 (2017).
[31] B.I. Abelev \textit{et al.}, (STAR Collaboration), Phys. Rev. \textbf{C75}, 064901 (2007).
[32] B.I. Abelev \textit{et al.}, (STAR Collaboration), Phys.Rev.Lett., \textbf{97}, 152301 (2006).
[33] K. Aamodt, \textit{et al.}, (ALICE Collaboration), Eur. Phys. J. C \textbf{71} 1655 (2011).
[34] K. Aamodt, \textit{et al.}, (ALICE Collaboration), Phys. Lett. \textbf{B693}, 53 (2010).
[35] K. Aamodt, \textit{et al.}, (ALICE Collaboration), Phys. Rev. \textbf{D82}, 052001 (2010).
[36] K. Aamodt, \textit{et al.}, (ALICE Collaboration), Phys. Rev. \textbf{C88}, 044910 (2013).
[37] E. Kaptur, PoS \textbf{CPOD2014}, (2015) 053
[38] M. Lewicki, \texttt{arXiv:1612.01334} [hep-ex].
Figure 6: The $p_t$ spectra of $\pi^-$, $K^+$ and $K^-$ mesons produced at $y \approx 0$ in inelastic $pp$ interactions at SPS energies $\sqrt{s} = 6.3 - 17.3$ GeV or $P_{\text{in}} = 20-158$ GeV/c. Data are taken from [30].