Monte Carlo Forest Search: UNSAT Solver Synthesis via Reinforcement Learning

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Abstract

We introduce Monte Carlo Forest Search (MCFS), an offline algorithm for automatically synthesizing strong tree-search solvers for proving unsatisfiability on given distributions, leveraging ideas from the Monte Carlo Tree Search (MCTS) algorithm that led to breakthroughs in AlphaGo. The crucial difference between proving unsatisfiability and existing applications of MCTS, is that policies produce trees rather than paths. Rather than finding a good path (solution) within a tree, the search problem becomes searching for a small proof tree within a forest of candidate proof trees. We introduce two key ideas to adapt to this setting. First, we estimate tree size with paths, via the unbiased approximation from Knuth (1975). Second, we query a strong solver at a user-defined depth rather than learning a policy across the whole tree, in order to focus our policy search on early decisions, which offer the greatest potential for reducing tree size. We then present MCFS-SAT, an implementation of MCFS for learning branching policies for solving the Boolean satisfiability (SAT) problem that required many modifications from AlphaGo. We matched or improved performance over a strong baseline on two well-known SAT distributions (sgen, random). Notably, we improved running time by 9% on sgen over the kcnfs solver and even further over the strongest UNSAT solver from the 2021 SAT competition.

1 Introduction

Silver et al. (2017) took the world by storm when their AlphaGo system (newest version called AlphaZero) beat world champion Lee Sodol at Go, marking the first time a computer program had achieved superhuman performance on a game with such a large action space. Their key breakthrough was combining Monte Carlo Tree Search (MCTS) rollouts with a neural network-based policy to find increasingly strong paths through the game tree. This breakthrough demonstrated that, with good state-dependent policies, MCTS can asymmetrically explore a game tree to focus on high-reward regions despite massive state spaces.

The AlphaZero algorithm combines three powerful ideas: (1) MCTS to avoid the exponential cost of enumerating all subsequent sequences of actions (Kearns et al., 2002; Coulom, 2006); (2) efficiently trading off exploration and exploitation in these rollouts by leveraging multi-arm bandit policies (Kocsis & Szepesvári, 2006); and (3) function approximation of the policies and values from previous problems to provide priors that further focus rollouts on promising paths (Coulom, 2007; Sutskever & Nair, 2008; Maddison et al., 2014).

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We matched or improved over the performance of a strong baseline on two prominent SAT Competitions. We evaluated our approach on the Boolean satisfiability (SAT) problem, one of the most widely-studied combinatorial problems. Leveraging the MiniSAT (Eén & Sörensson, 2003) framework, we integrated MCFS into the DPLL algorithm and learned a branching heuristic offline to efficiently find small proof trees of unsatisfiability online. Proofs of unsatisfiability are important in practice, e.g., for system debugging (Suel&flow et al., 2008) and formal verification (Bryant et al., 2009). As a result, the main track of the annual SAT competition typically has around 50% unsatisfiable instances and has often featured further tracks focused only on unsatisfiable instances (Berre et al., 2007). In our MCFS-SAT implementation, we bound policy evaluations by querying a known strong subsolver below a certain depth in the proof tree. This focuses the search on early decisions where the policy has the highest potential to reduce tree size relative to inference cost. MCFS-SAT makes a number of other important modifications from AlphaZero. We change (1) the bandit algorithm to account for the tree-size cost function, (2) node expansion to account for an unreliable value network, (3) the state-transition data structure to improve sample efficiency. Unlike AlphaGo that uses MCTS both offline and online, we only use MCFS as an offline procedure as we can’t afford the computational overhead online.

We matched or improved over the performance of a strong baseline on two prominent SAT Competitions. First, we evaluated our method on uniform random 3-SAT at the solubility phase transition, perhaps the best-studied SAT distribution which has been featured in the SAT Competition from 2002 to 2018. We were able to match performance of kcnfs07, which was specifically designed to target this distribution. Second, we evaluated our method on the sgen distribution, which is notoriously difficult for its problem size and produced the smallest unsolved instance at the 2009 SAT competition. We improved running time on sgen by 9% over kcnfs07, which is 3.2× faster than the hKis solver, which solved the most unsatisfiable instances in the 2021 SAT competition.

In what follows, we discuss related work in Section 2. We introduce the SAT problem, the DPLL algorithm, and MCTS in Section 3 and describe our MCFS approach in Section 4. We present our
experimental setup and results in Sections 5 and 6 respectively. Finally, we discuss future work in Section 7.

2 RELATED WORK

Tree search is a fundamental algorithm for combinatorial optimization. It works by iteratively partitioning a search space, e.g., by choosing variables to branch on. At a high level, the efficiency of tree search is measured by the product of (1) the number of branches and (2) the time required to make each branching decision. The order in which variables are assigned—the branching policy—has a dramatic effect on the size of the tree and the corresponding time to solve the problem, so algorithm designers seek branching policies that lead to small trees. In domains such as MIP solving, the “smart and expensive” paradigm has won out, where entire linear programs are solved to determine a branching decision. In other domains such as SAT solving, the “simple and cheap” paradigm dominates: high-performance SAT solvers make extremely cheap branching decisions that depend only superficially on subproblem state. After 20 years, VSIDS (Moskewicz et al., 2001) has remained among the state of the art for QBF and SAT; it is a simple heuristic that only keeps track of how often a variable occurs in conflict analysis. Some interesting early work developed more expensive heuristics (Huang & Darwiche, 2003), but these generally could not compete with VSIDS in terms of running time. One specialized example in SAT where a more expensive structural heuristic is beneficial is the solver kcnfs (Dequen & Dubois, 2003) for random 3-SAT. It branches based on a proxy measure for how likely a variable is to be in the backbone, which is the set of literals that must be true in every model. Another more general case where expensive heuristics are beneficial are as a prepossessing step before using cheap heuristics. The VSIDS heuristic and its close cousins (e.g., LRB (Liang et al., 2016)) maintain per-variable scores, which are updated as a function of search progress and are agnostic to score initialization. Duan et al. (2020) initialize high scores for literals that are likely to jointly be a part of a satisfying assignment using Bayesian moment matching.

One might expect that machine learning could learn a more informative heuristic that is worth its cost, especially for shallow-depth branches where decisions are most consequential. Two approaches have shown promise for learning models to make branching decisions: imitation learning and reinforcement learning.

Imitation learning works by learning a cheap approximation of (1) an expensive existing heuristic or (2) an expensive feature that is a good proxy for a good branching decision. Gasse et al. (2019) and Nair et al. (2020) exploited the fact that neural network inference time is faster than MIP heuristics to learn cheaper approximations that achieve state-of-the-art performance. Nejati et al. (2020) train a branching policy to predict the branching variable that leads to the smallest running time from a single branch starting at the root of the tree with a fixed downstream solver. There motivating application is parallel SAT solving where the True and False branches of tree are solved in parallel. Relevant to our work on UNSAT solving, Selsam & Björner (2019) learned to approximate small unsatisfiable core computation and then branched on variables predicted to belong to a core. They computed small unsatisfiable cores for 150,000 instances and trained a neural network to predict them, achieving a 6% speedup over a strong SAT solver when using their network for a limited number of top-level branching decisions. Note that if there exists a small unsatisfiable core, there exists a corresponding short proof, but not vice versa.

A tempting idea is to use reinforcement learning to directly synthesize heuristic policies that are optimized for problem distributions of interest. The idea dates back to Lagoudakis & Littman (2001) who used TD-learning to train a policy to select between seven predefined branch heuristics based on simple hand-crafted features. With the advent of modern deep-learning architectures, practitioners train models that take the raw representation of a problem instance as the model input. Yolcu & Póczos (2019) learned a local search heuristic for SAT; Tönshoff et al. (2022) built a generic graph neural network-based method for iteratively changing assignments in CSPs and demonstrated good results on 100 variable SAT problems; Lederman et al. (2019) learned an alternative to VSIDS for QBF using the REINFORCE algorithm that roughly minimizes decisions. They improved over VSIDS in CPU time on hard problems from a distribution within the framework of CADET, an open-source competitive solver; and Vaezipoor et al. (2021) used an evolutionary strategy to learn a state-of-the-art branching heuristic for model counting, demonstrating improvements in walltime performance over the competitive SharpSAT solver on instances with more than 10,000 variables.
Finally, Kurin et al. (2019)’s work is closest to our own. They used Q-learning for branching in the CDCL solver Minisat. Training on random-satisfiable problems with 50 variables and generalizing online to 250 variables for both satisfiable and unsatisfiable instances, they were able to improve running time over a generic solver. However, they had difficulty scaling due to high variance returns and credit assignment, two issues that our MCFS approach addresses. They resolved the path / tree distinction by treating a traversal through a tree as a path (which could be exponential in length) and allowing backtracking state transitions, however they only trained on satisfiable problems where the optimal policy is a path rather than a traversal. At test time, they only use their policy at top-level decisions but they train their policy for every decision during training.

There are several examples of MCTS being used as an algorithm for directly searching for a satisfying solution to a CSP. There are two main differences we see with these approaches relative to ours: (1) MCTS is being used online for solving CSP rather than as an offline procedure for training a model-based branching policy and (2) these approaches are not designed for the unsatisfiable case where policies produce trees rather than paths. In these cases, a lookahead can be interpreted as a guess at a satisfying assignment. If an unseen node is reached or a conflict is reached along a lookahead path, a reward is assigned based on a proxy measure of how close the path is to being a satisfying assignment (e.g., number of satisfied constraints). These MCTS approaches lie somewhere between local search and DPLL.

Previti et al. (2011) developed UCTSAT, which assigns 0 reward to a conflict node and explores various alternatives for value estimation at non-terminal nodes such as (1) number of satisfied clauses at current state and (2) average number of satisfied clauses based on random paths from that state. A number of follow-up works have incorporated clause learning into UCTSAT. Schloeter (2017) adds new clauses to the problem for every conflict that is encountered. This addition does not affect the UCT tree since state is defined as the set of assigned variables and therefore independent of additional clauses. Keszocze et al. (2020) make better use of the learned clauses filtering out any expanded nodes that are ruled out by a learned clause. They modify Previti et al.’s reward function by weighting each satisfied clause by its activity (number of times occurring in resolution) and propose a number of other variations including penalizing by the depth of conflict. We penalize by $2^{\text{depth}}$, which corresponds to minimizing the size of the proof tree. Loth et al. (2013) developed a version of UCTSAT for Constraint Programming (CP) that was simplified to be state independent. Rather than each tree node representing a different multi-armed bandit (MAB), they have a single MAB that they update at every node of the tree. This change was made to incorporate restarts, where there would be too few samples for each node. They reward a path by the length until conflict. Wattez et al. (2020) use a single MAB to choose which branching heuristic to use on a given instance. They reward a path to a conflict by the number of assignments ruled out by that path.

Outside of CSP, there are a number of other existing uses of MCTS for solving path-based NP-hard problems. Browne et al. (2012) used the UCT algorithm for solving MIPs, taking paths from the root to a leaf and propagating up the maximum over child LP values; Abe et al. (2019) searched over assignment paths in graph problems such as choosing edges to cut in a graph; and Khalil et al. (2022) searched over paths of variables to find MIP backdoors. The work of Scavuzzo et al. (2022) is the closest to our own in distinguishing between searching for paths and trees. They introduced TreeMDPs in which each state can transition to many states, which are a natural representation for UNSAT proof trees since after branching on a variable, we must explore both the paths for the true and false literal of that variable.

We approximate the tree size of a DPLL policy via Knuth samples (Knuth, 1975), which Lobjois et al. (1998) showed can be effective for cheaply comparing algorithms despite its high variance. A Knuth sample provides a path-based rollout, which allows us to easily leverage MCTS. There has been extensive follow-up work developing alternatives for approximating tree size (e.g., Purdom, 1978; Chen, 1992; Cornu´ejols et al., 2006; Kilby et al., 2006), but these estimates are not decomposed into path-based rollouts from root to leaf, and require new ideas to be integrated into MCFS.

3 Preliminaries

We now provide the required technical background for the Boolean satisfiability problem (our application area), the DPLL algorithm (the framework that defines our policy space), and AlphaZero MCTS (the reinforcement learning algorithm which we adapt to develop MCFS).
3.1 Boolean Satisfiability

Boolean satisfiability (SAT) problems are defined as follows. A SAT instance $S$ is defined by a set of clauses $C = \{c_1, \ldots, c_m\}$ over a set of variables $X = \{x_1, \ldots, x_n\}$. Each clause consists of a set of Boolean literals, defined as either a variable $x_i$ or its negation $\neg x_i$. Each clause is evaluated as $\text{True}$ iff at least one of its literals is $\text{True}$ (i.e., the literals in a clause are joined by OR operators). For example, a clause $c_i = x_j \vee \neg x_k$ evaluates to $\text{True}$ if either $x_j$ is set to $\text{True}$ or $x_k$ is set to $\text{False}$. $S$ is $\text{True}$ if there exists an assignment of values to variables for which all the clauses simultaneously evaluate to $\text{True}$ (i.e., the clauses are joined by AND operators). If such an assignment exists, the instance is called $\text{satisfiable}$; it is called $\text{unsatisfiable}$ otherwise.

SAT solvers try to find a way to set the variables to demonstrate that the problem is satisfiable, or to construct a proof tree that shows that no setting of the literals can satisfy the formula. In this paper, we consider only unsatisfiable problems, for which such proof trees must be found.

3.2 DPLL and Variable-Selection Policies

Many SAT solvers rely on the Davis-Putnam-Logemann-Loveland algorithm (DPLL), which assigns variables in an order given by some (potentially state-dependent) variable-selection policy.

**Definition 1.** Let $S$ be a SAT instance. A policy $\phi$ is a mapping $\phi : S \to (v, \mathbb{I})$ that determines which variable $v$ to assign in DPLL and what value $\mathbb{I} \in \{0, 1\}$ to assign it to first.\(^1\)

Given a policy, the DPLL algorithm is straightforward: it iterates over the selected variables recursively, checking both the $\text{True}$ and $\text{False}$ assignments, and performing unit propagation at each step (setting variable values forced by single-variable (unit) clauses; propagating these values to other clauses in which the same variables appear (possibly in negated form); repeating until no unit clauses remain) until either a conflict or satisfying assignment is found. We include pseudocode of the DPLL algorithm in Appendix A.

There can be massive gaps in performance between different policies for choosing branching variables in DPLL. For example, Figure 1 shows a formula that leads to a three-node proof tree if $x_0$ is selected first by the policy (assigning $x_0$ results in $x_7 \land \neg x_7$, implying a contradiction), but a tree that could have as many as $2^{[N]} - 1$ nodes if $x_0$ is selected last. This also illustrates how top-level decisions are much more powerful; the proof tree doubles in size for every level we fail to make the good decision (branch on $x_0$). Another way that policies can affect tree size is through DPLL’s unit propagation step: policies that cause more unit propagation earlier in the search have fewer decisions to make overall and therefore yield smaller search trees.

For an instance $S$ that is solved using variable-selection policy $\phi$, we denote the size of the resulting proof tree as $T_\phi(S)$. For a given distribution over problems $\mathbb{P}$, our goal is to find a policy $\phi^*$ that minimizes the average proof tree size $\mathbb{L}(\phi; \mathbb{P}) = \mathbb{E}_{S \sim \mathbb{P}}[T_\phi(S)]^2$. Finding such policies is computationally challenging; for a problem with $n$ variables, there are $O(n^3)$ possible variable-selection policies (3$^n$ states representing each variable as $\text{True}$, $\text{False}$, or unassigned and $n$ choices per state), and exactly evaluating $T_\phi(S)$ entails $O(2^n)$ operations.

If we are prepared to assume that the optimal variable-selection policy, $\phi^*_p = \arg \min_{\phi^*} \mathbb{L}(\phi^*; \mathbb{P})$ is learnable given an appropriate model family, we could in principle learn an approximation to the optimal policy, $\hat{\phi}_p$, to use within our solver. The challenge is designing a procedure to efficiently minimize $\mathbb{L}(\phi; \mathbb{P})$ so that we can collect labeled training examples. This is a challenging reinforcement learning problem. As mentioned above, given any particular variable-selection policy, even evaluating the loss function on a single instance requires time exponential in the problem size, and the space of variable-selection policies is massive. SAT problems vary in size and are invariant to permutations of their rows (clauses) and columns (variables within a clause). A policy $\phi$ should be capture these invariances.

\(^1\)When looking for a SAT assignment one must occasionally backtrack and try a different assignment for a literal. This is handled by the DPLL algorithm itself rather than the policy.

\(^2\)True running time objective would be a node-weighted tree size where node weights correspond to computational cost of node decisions.
### 3.3 AlphaZero MCTS

The MCTS algorithm used in AlphaZero uses Monte Carlo approximation of the value of a state by leveraging rollouts. Each rollout (inner loop) follows four key steps:

**Action Selection** At every state $s$, MCTS chooses an action $a \in \arg \max_{a'} (Q(s, a') + U(s, a'))$. $Q(s, a)$ is the reward estimate of action $a$ at state $s$ and $U(s, a)$ is the corresponding confidence interval. These confidence bounds are computed via the PUCT algorithm (Rosin, 2011; Silver et al., 2016):

$$U(s, a) = c_{puct} P(s, a) \frac{\sqrt{\sum_{a'} N(s, a')}}{1 + N(s, a)}$$

(1)

where $N(s, a)$ is the number of lookaheads that branch on action $a$ at state $s$ and $c_{puct}$ is a constant that controls exploration. $N(s, a)$ is initialized to 1 and $Q(s, a)$ is initialized to 0. $P(s, a)$ represents a prior distribution over the actions for a given state. This prior comes from the key innovation of MCTS, combining rollouts with a neural-network-based policy. This policy network is trained to output a prior $P(s, a)$ to match the proportion of samples each action was taken before MCTS terminates. This creates a loop where MCTS finds strong paths through the tree to supervise the neural network, and the neural network guides MCTS in future iterations to discover even stronger paths.

**Expansion** When a rollout reaches a state which MCTS has yet to visit, a new node is created. This causes the tree to progressively widen in areas that are sampled frequently. MCTS stores nodes in a tree structure where a node is reached by a single sequence of decisions. There may be many nodes with identical state if the nodes are reached with different decision history.

**Simulation** The MCTS traversal is cut short when it reaches one of these newly-constructed nodes. Rewards are only stored at the terminal nodes of the tree. To deal with this issue, MCTS uses a value network, which is trained to predict the value of a path originating from this state. The neural network architecture has two important features that make it well suited for its task. First, it uses convolutional neural networks which work well for games like Go which are translationally invariant and rotationally symmetric. Second, it jointly trains the policy network and value network with a single shared representation allowing the value network to leverage the more powerful training signal that the policy network provides.

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**Figure 1:** Example illustrating the importance of branching policies to the computational cost of solving SAT instances. (Left) The problem contains a large number of satisfiable clauses and a small unsatisfiable “core”, which do not share any variables. (Middle) Branching first on variables from the satisfiable clauses leads to a large search tree, where the same conflict must be identified many times. (Right) Branching on the single variable $x_6$ from the unsatisfiable clauses leads to a small search tree where the contradiction is immediately found after unit propagation.
**Backpropagation**  The reward (predicted by the value network or awarded at the leaf) is passed back through the traversal path. The $Q$ values and visit counts of each node are updated accordingly. After a fixed number of rollouts, AlphaZero commits to the action at the root having the largest number of samples, and the resulting state becomes the new root node for MCTS. This *outer loop* repeats until MCTS finds a path to a leaf of the tree. Each node along this path provides a training example for the policy network and the cost of the path is used to train the value network. AlphaZero uses function approximation of these policies and values to provide priors that further focus rollouts on promising paths. The pseudocode for the inner and outer loop of AlphaZero is given in Appendix A.2.1.

### 4 Methods

We would like to leverage the strengths of AlphaZero to find strong policies, where a policy produces a tree rather than a path and we can’t afford the computation to call the policy for every node in a potentially exponentially-sized tree. We introduce Monte Carlo Forest Search (MCFS), an offline reinforcement learning procedure for learning such policies. The core ideas are to (1) turn trees into paths by using a path-based tree-size approximator and (2) to bound the number of policy calls. We then present MCFS-SAT, our approach that leverages MCFS to learn DPLL-variable-selection policies for unsatisfiable SAT problems. MCFS-SAT makes a number of modifications from AlphaZero: we bound the number of policy calls by focusing on top-level decisions; we alter the PUCT Action Selection step to account for the tree-size cost function that scales with node depth; we adapt the Simulation step by accounting for an unreliable value network and a fixed policy; we change the model architecture to account for the invariances of SAT; we change the state-transition data structure to improve sample efficiency; and we speed up lookaheads by sharing the policy prior with child nodes. The pseudocode for MCFS-SAT are given in Appendix A.2.2 and the differences with AlphaZero are highlighted in red.

#### 4.1 MCFS

**Estimating Tree Size with Paths**  To evaluate a given policy, $\phi'$, we need to know the associated tree size, $T_{\phi'}(S)$. The simulation step of MCTS would require visiting $O(2^n)$ states before reaching a terminal node. This is computationally expensive, both because the absolute number of states may be prohibitively large, and because at each node in the tree we need to query $\phi'$ to decide which variable is assigned next, which is non-trivial when $\phi'$ is parameterized by a neural network. We address this by stochastically approximating $T_{\phi'}(S)$ using Knuth samples (Knuth, 1975).

**Theorem 1** (due to Knuth (1975)). Let $\ell_P$ be the length of a path $P$ sampled uniformly at random from a binary tree $T$ with size $s_T$. Then, $s_T = E_P[2^{\ell_P}]$.

In the context of a DPLL solver with variable-selection policy $\phi'$, this amounts to replacing a complete traversal of the binary tree of all True / False assignments with a path through the tree where assignments are chosen uniformly at random. We can take the length of a resulting path, $\ell$, and assign each node at depth $d$ a cost of $2^{\ell-d}$ for its corresponding policy decision. The average of these costs across a set of paths provides an unbiased approximation of the proof tree size.

**Bounding Policy Evaluations**  At test time, we cannot use Knuth samples. We must build out the entire proof tree to prove unsatisfiability. In practice, the policies we learn are implemented with deep neural networks, which are far more expensive to evaluate than standard heuristics. When applying this approach to large unsatisfiable problems, the sheer amount of policy network calls required to decide a policy for an entire problem will outweigh the time savings even if the decisions result in smaller policy trees. This is especially true for smaller subproblems that take a fraction of a second to solve with existing methods. Any online policy must therefore have a procedure for constraining which nodes the policy will be queried. To ensure that an optimal policy we find offline is also optimal at test time, MCFS limits policy evaluations offline during a Knuth sample and the MCFS outer loop corresponding to how policy evaluations are limited online.
4.2 MCFS-SAT

Restrict policy to top-level decisions As described in Figure 1, early decisions are much more powerful for finding small trees. We bound policy evaluations by modifying the MCTS Simulation step and only training on top-level decisions. After a fixed number of decisions \( \ell \), we call out to either (1) a pre-existing variable-selection policy or (2) a value network that approximates the tree size of this fixed policy. We leverage existing SAT solvers at both training time and testing time. After \( \ell \) decisions, the problem is small enough to be solved quickly and reliably using a pre-existing solver. This number of decisions is held constant across training and testing time so that the policy network can learn to leverage this downstream solver, ensuring policies discovered offline will also be optimal online. A given node at depth \( d \) along the path updates its \( Q \) value with \( 2^{d-d}T_{\phi_{\text{sub}}}(S^*) + 2^{d-d} - 1 \), where \( \ell \) is the fixed number of decisions before the subsolver; \( S^* \) is the SAT instance that is given to the subsolver; and \( \phi_{\text{sub}} \) is the policy of the subsolver being used. An added benefit of this approach is that the subsolver used within the model is easily replaceable, allowing us to use the best subsolver available for whichever distribution we are targeting. We chose not to use node expansion within the range of decisions before we call a subsolver. Node expansion can lower the variance of the rewards below a given state and Knuth samples provide a linear-time approximation of an exponential function and thus will have very high variance. However, we noticed empirically that the variance does not grow significantly with sample length within the range of number of decisions before we call a subsolver. Increasing subsolver depth \( \ell \) may require reviving node expansion to control variance.

Accounting for costs scaling with node depth The tree-size objective changes the reward/cost structure of the policy space from AlphaZero. We select variable \( v \) to minimize cost (tree size) rather than maximize reward where each node incurs a cost of 1, rather than there only being rewards at terminal states. We make two deviations from Equation 1. First, we add a \( Q_d \) coefficient to calibrate confidence intervals across different depths in the tree: \( v = \arg \min_{v'} (Q(s, v') - Q_d U(s, v')) \). In AlphaZero, rewards preserve scale with depth, where -1 represents a loss and 1 represents a win. However, in our setting \( Q(s, v) \) scales exponentially the depth of \( s \). Since \( U(s, v) \) is independent of the scale of costs (only a function of counts), fitting the \( \epsilon_{\text{puct}} \) constant would tradeoff tight confidence intervals at shallow nodes for loose confidence intervals at deeper nodes. To keep \( U(s, v) \) on the same scale as \( Q(s, v) \), we calibrate each depth with \( Q_d \), the average tree size of nodes at depth \( d \), which can be thought of as a per-depth constant. We compute \( Q_d \) online as the running average of tree sizes encountered from depth \( d \). Second, we initialized \( Q \) values with the first lookahead, as this represents an unbiased measure of performance of our incumbent policy (i.e., the first sample is exactly the neural network policy), which we would like to improve upon. This is similar to AlphaZero’s choice of initializing \( Q \) to 0, which is the baseline they aim to improve upon.

Leveraging an unreliable value network We use a value network to access a much cheaper reward signal rather that explicitly solving the subproblem at depth \( \ell \). Unlike AlphaZero, this value network is estimating the cost of a fixed policy. This network is trained from an initial batch of subsolver calls and then retrained as we collect new, large batches of data, so its accuracy can vary across timesteps of a training run. When the output of the network is too unreliable, it may not carry enough signal about the true number of decisions for good policies to be learned. We address this issue by randomly deciding between calling the subsolver and calling our value network, with the probability depending on an online estimate of the value network’s accuracy. More specifically, we track the mean multiplicative error \( \epsilon \) of our value network over time by querying the value network with every subsolver call. Given a user-defined accuracy threshold parameter \( t \) (we use \( t = 0.5 \)), we sample the value network with probability \( 1 - \min(1, \epsilon/t) \), so the probability of calling the subsolver halves as the error halves.

Equivariant architecture We capture the invariances of our problem by using an equivariant neural network architecture in which permutations of the input guarantee a corresponding permutation in the output (Hartford et al., 2018). A beneficial side effect of this architecture is that it can handle any size input matrix. We also add a third head to our network beyond the policy and value head to predict \( Q \) values. The purpose of the \( Q \)-value head was as an auxiliary task to help train a better shared representation for the policy and value head; we observed a 5% reduction in tree size after adding the \( Q \)-value head.

Directed acyclic graph for state transitions We can improve sample efficiency by leveraging the fact that the same state is reached regardless of the order in which previous decisions were made. To
do this, we change our data structure for state transitions from a tree to a graph similar to Czech et al. (2020). This lets us share information across different paths to the same state which is common with variables with similar scores.

Sharing prior with child nodes Since our calls to the policy network tends to dominate the running time, we also save time by only computing \( P(s, a) \) once at each committed decision and pass down the network prediction to its child nodes.

5 Implementation and Experimental Details

We integrated our MCFS-SAT algorithm into the CDCL solver Maple_LCM_DIST_CronoBT (Ryvchin & Nadel, 2018), which won the 2018 SAT competition and is based on the MiniSAT framework (Eén & Sörensson, 2003). We removed all clause-learning components so that the solver was running strictly DPLL search. We trained our neural networks with PyTorch (Paszke et al., 2019) in Python and ported over to our solver in C++ using tracing\(^3\). Below, we describe implementation details and benchmarks.

Parameterization of MCFS-SAT Two important hyperparameters were (1) the constant for choosing the level of exploration \( c_{puct} \) and (2) the number of lookaheads \( k \). Through a coarse grid search, we selected \( c_{puct} = 0.5 \) and \( k = 100,000 \), which found the best policies within an approximate 48-hour time window. After \( k \) lookaheads, we either committed to the action with the highest counts \( v = \arg \max_s N(S, v') \) or sampled the best action from our current neural network policy. We set the probability of these alternatives to 0.5 to balance on-policy and off-policy learning. Each MCFS decision yields a training point with a (state, policy vector, \( Q \) value) triple, where the policy vector consists of normalized counts from MCFS.

We chose \( \ell = 6 \) and \( \ell = 8 \) for the random and sgen distributions respectively, since that is where MCFS-SAT tended to find the strongest policies under reasonable time constraints. We made decisions with MCFS-SAT until depth \( \ell \), summing to \( 2^\ell \) decisions for every run.

Prior to a good policy network being learned, MCFS-SAT is less efficient. We pretrained our policy network and value network by running MCFS-SAT with 10,000 lookaheads on 1,000 instances for random and sgen. We ran one iteration of MCFS-SAT with 1,000 instances and 10,000 lookaheads to improve the policy and value network further and then a final iteration with 2,000 instances and 100,000 lookaheads, which we used to train our final model. Each pretraining run of MCFS-SAT took approximately 24 hours and were run on 300-variable and 55-variable problems from random and sgen. The two iterations using the prior took approximately 48 hours and was respectively run on 300-variable and 65-variable problems from random and sgen.\(^4\)

Benchmarks and Baselines We targeted instance distributions that are well known and difficult for modern SAT solvers, with similar-sized action spaces to Go. We do not consider industrial SAT instances, as they often contain millions of variables with state spaces significantly larger than any application of MCTS we are aware of. We evaluated our approach on a random distribution (uniform random 3-SAT at the solubility phase transition, or R3SAT) and a crafted distribution (sgen).

For R3SAT, we followed Crawford & Auton (1996) to estimate the location of the phase transition with a number of clauses-to-number of variables \( (n/m) \) ratio of \( n = 4.258 \cdot m + 58.26 \cdot m^{-2/3} \). We trained on 300-variable instances where calling a subsolver was quick (\( \approx 1 \) second solving time), and filtered out SAT instances for training and testing. To evaluate upward-size generalization, we set aside 100 test instances at our training size of 300 variables as well as at 350, 400, and 450 variables. R3SAT is a well-studied distribution where there has been a focused algorithmic effort, making it an especially challenging benchmark. We compare to kcnfs07 (Dequen & Dubois, 2003), which is specifically designed for this distribution and among the strongest solvers. We used the version submitted to the 2007 SAT competition (Berre et al., 2007), where it won the silver medal in the

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\(^3\)Find our code here: https://github.com/ChrisCameron1/MCFS. Still in prototyping phase. Improvements in accessibility and useability are being worked on.

\(^4\)The parameters and architecture described in this paper are only for the last iteration of MCFS-SAT. We made several minor improvements across iterations, and we expect that the performance would have been at least as good had we used an identical setup for all iterations.
We also evaluated a with each index representing a different variable. We experimented with attention pooling in the last we also added a node degree feature to every literal embedding. We instantiated the permutation-
\[ \text{unsat} \] option on the generator, which guarantees instances that contain contradictions. For similar reasons as \text{R3SAT}, we trained with 65 variables, and to evaluate upward-size generalization we set aside 100 test instances at our training size of 65 variables as well as at 75, 85, and 95 variables. Unlike \text{R3SAT}, there are no solvers known to perform well on this distribution. We likewise selected \text{kcns07} for our \text{sgen} benchmark since it was consistently faster than the best SAT solver from the most recent (2021) SAT competition on unsatisfiable instances, \text{hKis}, but we also included \text{hKis} in our evaluation.

We also evaluated a uniform+\text{kcns07} baseline for each dataset, where we replaced neural network calls with uniform-random decisions and called the \text{kcns07} solver for subproblems at the same user-defined depth. We tried purely random decisions without calling out to the \text{kcns07} subsolver on 65 variable \text{sgen} problems and average running time was a factor of 40 times slower than \text{kcns07}. Given the poor performance, we did not run pure random beyond this dataset.

**Model Training** We used the exchangeable architecture of Hartford et al. (2018). We represented a CNF SAT instance with \( n \) clauses and \( m \) variables as an \( n \times m \times 128 \) clause-variable permutation-equivariant tensor, where entry \((i,j)\) is \( t_v \) if the true literal for variable \( i \) appears in clause \( j \), \( f_v \) if the false literal for variable \( i \) appears in clause \( j \), and 0 otherwise. \( t_v \) and \( f_v \) are random 128-dimensional vectors representing the true and false embeddings for a given literal. Following Hamilton et al. (2017), we also added a node degree feature to every literal embedding. We instantiated the permutation-equivariant portion of the exchangeable architecture as four exchangeable matrix layers with 512 output channels, with leaky RELU as the activation function. We mean-pooled the output to a vector, with each index representing a different variable. We experimented with attention pooling in the last exchangeable layer but didn’t observe any improvements.

Given these shared exchangeable layers, we added three feed-forward heads: a policy head, a \( Q \)-value head, and a value head. The policy and \( Q \)-value heads both had two feed-forward layers with 512 channels and a final layer that mapped to the single output channel. The value head was the same, except there was a final mean pool that output a single scalar. The policy head was trained to predict the normalized counts of MCFS and the \( Q \)-value head was trained to predict the \( Q \)-values from MCFS. We used the cross-entropy loss for both. The purpose of the \( Q \)-value head was as an auxiliary task to help train a better shared representation for the policy head; we observed a 5% reduction in tree size after adding the \( Q \)-value head. We trained the value network with mean-squared error (MSE) against the \( \log_2 \) tree size of subsolver calls at leaf nodes. The value head was not backpropagated through the exchangeable layers. Using a shared representation was important for training the value head; MSE tended to be a factor of 2 worse when using a network trained with only a value head.

For training the exchangeable architecture, we used the Adam optimizer (Kingma & Ba, 2014), with a learning rate of 0.0001 and a batch size of 1 (the largest batch size that fit on our 8 GB GPUs). Online, we branched on the argmax variable from our neural network prediction. We used a held-out validation set to select a model that produced the minimum mean tree size when used within a DPLL solver. For a given depth \( d \), MCFS makes \( 2^d \) decisions; therefore, we received exponentially more data points at deeper depths where decisions are less important. We experimented with exponentially upsampling nodes inversely proportional to their depth but observed no performance gains. For every leaf node, we recorded the true tree size to train our value network.

**Computing Resources** We ran our model training and solver benchmarking experiments on a small cluster of approximately 20 nodes that has 20 8 GB Tesla M60 GPUs. For MCFS runs, we also used a large shared cluster of CPU nodes. Each MCFS run was allocated 16 GB of memory and a maximum of 48 hours. Each solver benchmarking run was allocated 8 GB of memory.
### Results

We evaluated (1) search tree size and (2) running time for our approach against the two baselines on each of our eight test instance sets. We present the full results in Table 1. We measured running time as the cumulative CPU and GPU time with `runsolver` (Roussel, 2011).

On **R3SAT**, **kcnfs07** is an extremely strong solver and there is a question whether there is anything better. It far surpassed random branching on the top-level decisions (4–5x reduction in tree size and walltime). Despite the strength of **kcnfs**, we were able to squeeze out 1-2% performance improvements in term of average tree size over **kcnfs07** on up to 400 variables. With the overhead from our neural network calls, these reductions in tree size were not sufficient for improving running time.

On **sgen**, there is no known specialized solver and therefore much greater scope for improvement. **kcnfs07** is not specialized for **sgen**; we observed that it typically found only marginally smaller trees than the **uniform+kcnfs** baseline, but it was the best existing solver that we found. The best unsat solver from the most recent SAT competition (**hKis**) was 3.2× slower than **kcnfs07** (See Appendix A.3). We reduced average tree size over **kcnfs07** on our training distribution (65 variables) by a factor of 1.23. Our model trained on the 65-variable distribution generalized well to larger problem sizes, reducing tree size even on 95-variable problems, which took approximately 40 minutes to solve and are 700x more difficult than our training distribution. Our solver incurred roughly constant-time overhead that prevented us from improving running time on the 65-variable and 75-variable datasets. The community cares much more about asymptotic behaviour and we were able to improve the running time over **kcnfs** by 1% on 85-variable problems and 9% (∼3 minutes faster) on 95-variable problems. This magnitude of improvement is comparable to other highly-cited work such as Selsam & Bjørner (2019), who achieved a 6% speedup on the SAT competition instances they consider.

Our GPU overhead was especially large because of our outdated GPU hardware. We were able to access one high-performance GPU (Nvidia TITAN Xp) for an experiment on the 65 and 75 variable **sgen** datasets and the 300 and 350 variable **R3SAT** datasets. This hardware change closed the running-time gap between our method and **kcnfs** by on average 16.5 seconds on **sgen** and 6.1 seconds on **R3SAT**. We report those results in Appendix A.4. The constant GPU overhead is swamped at higher running times. Using just a CPU, we are still able to improve running time by 8% on 95-variable problems.

### Conclusions and Future Work

In this paper, we presented Monte Carlo Forest Search (MCFS), an algorithm for finding small trees while retaining the benefits of Monte Carlo Tree Search. Our method works when the tree size can be approximated with Knuth samples, a cheap and unbiased Monte Carlo approximation from a single path. We evaluated our approach on the well-known DPLL tree search algorithm for Boolean
satisfiability, where the tree size of a policy can be approximated via Knuth samples. We matched or improved performance over a strong baseline on two well-known distributions.

In future work, we would like to generalize MCFS to CDCL solvers. Most high-performance industrial solvers use the CDCL algorithm, which adds a clause-learning component to DPLL to allow information sharing across the search tree. We saw no clear way of approximating the size of a CDCL search tree with Knuth samples as unlike DPLL, policy decisions in CDCL are made with knowledge of the history of past decisions. We would also like to focus on richer experimental results that go beyond this methodologically-focused paper. We would like to explore more distributions (especially those with weaker solvers), experiment with other subsolvers beyond kcnfs, and add more computation time for additional lookaheads per instance and deeper searches.

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REFERENCES

Kenshin Abe, Zijian Xu, Issei Sato, and Masashi Sugiyama. Solving NP-hard problems on graphs by reinforcement learning without domain knowledge. *arXiv preprint*, arXiv:1905.11623:1–24, 2019.

Forest Agostinelli, Stephen McAleer, Alexander Shmakov, and Pierre Baldi. Solving the Rubik’s cube with deep reinforcement learning and search. *Nature Machine Intelligence*, 1(8):356–363, 2019.

Petr Baudiš and Jean loup Gailly. PACHI: State of the art open source Go program. In *Proceedings of the 13th International Conference on Advances in Computer Games*, ACG ’11, pp. 24–38, 2012.

Daniel Le Berre, Olivier Roussel, and Laurent Simon. SAT 2007 competition. The International SAT Competition Web Page, http://www.satcompetition.org/, 2007.

Cameron B. Browne, Edward Powley, Daniel Whitehouse, Simon M. Lucas, Peter I. Cowling, Philipp Rohlfshagen, Stephen Tavener, Diego Perez, Spyridon Samothrakis, and Simon Colton. A survey of Monte Carlo tree search methods. *IEEE Transactions on Computational Intelligence and AI in Games*, 4(1):1–43, 2012.

Randal E. Bryant, Daniel Kroening, Joël Ouaknine, Sanjit A. Seshia, Ofer Strichman, and Bryan Brady. An abstraction-based decision procedure for bit-vector arithmetic. *International Journal on Software Tools for Technology Transfer*, 11(2):95–104, 2009.

Pang C Chen. Heuristic sampling: A method for predicting the performance of tree searching programs. *SIAM Journal on Computing*, 21(2):295–315, 1992.

Gérard Cornuéjols, Miroslav Karamanov, and Yanjun Li. Early estimates of the size of branch-and-bound trees. *INFORMS Journal on Computing*, 18(1):86–96, 2006.

Rémi Coulom. Efficient selectivity and backup operators in Monte-Carlo tree search. In *Proceedings of the 5th International Conference on Computers and Games*, CG ’06, pp. 72–83, 2006.

Rémi Coulom. Computing “Elo ratings” of move patterns in the game of Go. *ICGA Journal*, 30(4):198–208, 2007.

James M. Crawford and Larry D. Auton. Experimental results on the crossover point in random 3SAT. *Artificial Intelligence*, 81:31–57, 1996.

Johannes Czech, Patrick Korus, and Kristian Kersting. Monte-Carlo graph search for AlphaZero. *arXiv preprint*, arXiv:2012.11045:1–11, 2020.
Gilles Dequen and Olivier Dubois. *kcnfs*: An efficient solver for random $k$-SAT formulae. In *Proceedings of the 6th International Conference on Theory and Applications of Satisfiability Testing*, SAT ’06, pp. 486–501, 2003.

Haonan Duan, Saeed Nejati, George Trimponias, Pascal Poupart, and Vijay Ganesh. Online bayesian moment matching based sat solver heuristics. In *International Conference on Machine Learning*, pp. 2710–2719. PMLR, 2020.

Niklas Eén and Niklas Sörensson. An extensible SAT-solver. In *Proceedings of the 6th International Conference on Theory and Applications of Satisfiability Testing*, SAT ’03, pp. 502–518, 2003.

Maxime Gasse, Didier Chételat, Nicola Ferroni, Laurent Charlin, and Andrea Lodia. Exact combinatorial optimization with graph convolutional neural networks. In *Proceedings of the 33rd International Conference on Neural Information Processing Systems*, NeurIPS ’19, pp. 15580–15592, 2019.

Will Hamilton, Zhitao Ying, and Jure Leskovec. Inductive representation learning on large graphs. In *Proceedings of the 31st International Conference on Neural Information Processing Systems*, NeurIPS ’17, pp. 1024–1034, 2017.

Jason S. Hartford, Devon R. Graham, Kevin Leyton-Brown, and Siamak Ravanbakhsh. Deep models of interactions across sets. In *Proceedings of the 35th International Conference on Machine Learning*, volume 80 of *ICML ’18*, pp. 1914–1923, 2018.

Jinbo Huang and Adnan Darwiche. A structure-based variable ordering heuristic for SAT. In *Proceedings of the 18th International Joint Conference on Artificial Intelligence*, IJCAI ’03, pp. 1167–1172, 2003.

Michael Kearns, Yishay Mansour, and Andrew Y. Ng. A sparse sampling algorithm for near-optimal planning in large Markov decision processes. *Machine Learning*, 49(2):193–208, 2002.

Oliver Keszocze, Kenneth Schmitz, Jens Schloeter, and Rolf Drechsler. Improving sat solving using monte carlo tree search-based clause learning. In *Advanced Boolean Techniques*, pp. 107–133. Springer, 2020.

Elias B. Khalil, Pashootan Vaezipoor, and Bistra Dilkina. Finding backdoors to integer programs: A Monte Carlo tree search framework. In *Proceedings of the 36th AAAI Conference on Artificial Intelligence*, AAAI ’22, pp. 1–10, 2022.

Philip Kilby, John Slaney, Sylvie Thiébaux, and Toby Walsh. Estimating search tree size. In *Proceedings of the 21st National Conference of Artificial Intelligence*, AAAI ’06, pp. 1014–1019, 2006.

Diederik P. Kingma and Jimmy Ba. Adam: A method for stochastic optimization. In *Proceedings of the 3rd International Conference on Learning Representations*, ICLR ’14, pp. 1–15, 2014.

Donald E. Knuth. Estimating the efficiency of backtrack programs. *Mathematics of Computation*, 29(129):122–136, 1975.

Levente Kocsis and Csaba Szepesvári. Bandit based Monte-Carlo planning. In *Proceedings of the 17th European Conference on Machine Learning*, ECML ’06, pp. 282–293, 2006.

Vitaly Kurin, Saad Godil, Shimon Whiteson, and Bryan Catanzaro. Can $q$-learning with graph networks learn a generalizable branching heuristic for a SAT solver? In *Proceedings of the 34th International Conference on Neural Information Processing Systems*, NeurIPS ’20, pp. 9608–9621, 2019.

Michail G. Lagoudakis and Michael L. Littman. Learning to select branching rules in the DPLL procedure for satisfiability. *Electronic Notes in Discrete Mathematics*, 9:344–359, 2001.

Gil Lederman, Markus Rabe, Edward A. Lee, and Sanjit A. Seshia. Learning heuristics for quantified boolean formulas through reinforcement learning. In *Proceedings of the 8th International Conference on Learning Representations*, ICLR ’19, pp. 1–18, 2019.
Jia Hui Liang, Vijay Ganesh, Pascal Poupart, and Krzysztof Czarnecki. Learning rate based branching heuristic for sat solvers. In *International Conference on Theory and Applications of Satisfiability Testing*, pp. 123–140. Springer, 2016.

Lionel Lobois, Michel Lemaitre, et al. Branch and bound algorithm selection by performance prediction. In *AAAI/IAAI*, pp. 353–358, 1998.

Manuel Loth, Michele Sebag, Youssef Hamadi, and Marc Schoenauer. Bandit-based search for constraint programming. In *International Conference on Principles and Practice of Constraint Programming*, pp. 464–480. Springer, 2013.

Chris J. Maddison, Aja Huang, Ilya Sutskever, and David Silver. Move evaluation in Go using deep convolutional neural networks. *arXiv preprint*, arXiv:1412.6564:1–8, 2014.

Matthew W. Moskewicz, Conor F. Madigan, Ying Zhao, Lintao Zhang, and Sharad Malik. Chaff: Engineering an efficient SAT solver. In *Proceedings of the 38th Annual Design Automation Conference*, DAC ’01, pp. 530–535, 2001.

Vinod Nair, Sergey Bartunov, Felix Gimeno, Ingrid von Glehn, Pawel Lichocki, Ivan Lobov, Brendan O’Donoghue, Nicolas Sonnerat, Christian Tjandraatmadja, Pengming Wang, Ravichandra Addanki, Tharindu Hapuarachchi, Thomas Keck, James Keeling, Pushmeet Kohli, Ira Ktena, Yujia Li, Oriol Vinyals, and Yori Zwols. Solving mixed integer programs using neural networks. *arXiv preprint*, arXiv:2012.13349:1–57, 2020.

Saeed Nejati, Ludovic Le Frioux, and Vijay Ganesh. A machine learning based splitting heuristic for divide-and-conquer solvers. In *International Conference on Principles and Practice of Constraint Programming*, pp. 899–916. Springer, 2020.

Christopher D. Rosin. Multi-armed bandits with episode context. *Annals of Mathematics and Artificial Intelligence*, 61(3):203–230, 2011.

Olivier Roussel. Controlling a solver execution with the runsolver tool. *Journal on Satisfiability, Boolean Modeling and Computation*, 7(4):139–144, 2011.

Vadim Ryvchin and Alexander Nadel. Maple_LCM_Dist_ChronoBT: Featuring chronological backtracking. In *Proceedings of SAT Competition 2018 — Solver and Benchmark Descriptions*, volume B-2018-1 of *Department of Computer Science Series of Publications B*, pp. 29, 2018.

Jens Schloeter. A monte carlo tree search based conflict-driven clause learning sat solver. *INFORMATIK 2017*, 2017.

Jens Schlipf. A monte Carlo tree search based conflict-driven clause learning sat solver. *INFORMATIK 2017*, 2017.
Daniel Selsam and Nikolaj Bjørner. Guiding high-performance SAT solvers with unsat-core predictions. In Proceedings of the 22nd International Conference on Theory and Applications of Satisfiability Testing, SAT ’19, pp. 336–353, 2019.

David Silver, Aja Huang, Chris J. Maddison, Arthur Guez, Laurent Sifre, George van den Driessche, Julian Schrittwieser, Ioannis Antonoglou, Veda Panneershelvam, Marc Lanctot, Sander Dieleman, Domink Grefe, John Nham, Nal Kalchbrenner, Ilya Sutskever, Timothy Lillicrap, Madeleine Leach, Koray Kavukcuoglu, Thore Graepel, and Demis Hassabis. Mastering the game of Go with deep neural networks and tree search. Nature, 529:484–489, 2016.

David Silver, Julian Schrittwieser, Karen Simonyan, Ioannis Antonoglou, Aja Huang, Arthur Guez, Thomas Hubert, Lucas Baker, Matthew Lai, Adrian Bolton, Yutian Chen, Timothy Lillicrap, Fan Hui, Laurent Sifre, George van den Driessche, Thore Graepel, and Demis Hassabis. Mastering the game of Go without human knowledge. Nature, 550:354–359, 2017.

Ivor Spence. sgen1: A generator of small but difficult satisfiability benchmarks. ACM Journal of Experimental Algorithmics, 15:1.1–1.15, 2010.

Andre Suelflow, Goerschwin Fey, Roderick Bloem, and Rolf Drechsler. Using unsatisfiable cores to debug multiple design errors. In Proceedings of the 18th ACM Great Lakes Symposium on VLSI, GLSVLSI ’08, pp. 77–82, 2008.

Ilya Sutskever and Vinod Nair. Mimicking Go experts with convolutional neural networks. In Proceedings of the 18th International Conference on Artificial Neural Networks, ICANN ’08, pp. 101–110, 2008.

Jan Tönshoff, Berke Kisin, Jakob Lindner, and Martin Grohe. One model, any CSP: Graph neural networks as fast global search heuristics for constraint satisfaction. arXiv preprint, arXiv:2208.10227: 1–23, 2022.

Pashootan Vaezipoor, Gil Lederman, Yuhuai Wu, Chris Maddison, Roger B. Grosse, Sanjit A. Seshia, and Faihiem Bacchus. Learning branching heuristics for propositional model counting. In Proceedings of the 35th AAAI Conference on Artificial Intelligence, AAAI ’21, pp. 12427–12435, 2021.

Hugues Wattez, Frédéric Koriche, Christophe Lecoutre, Anastasia Paparrizou, and Sébastien Tabary. Learning variable ordering heuristics with multi-armed bandits and restarts. In ECAI 2020-24th European Conference on Artificial Intelligence, 2020.

Emre Yolcu and Barnabás Póczos. Learning local search heuristics for boolean satisfiability. In Proceedings of the 33rd International Conference on Neural Information Processing Systems, NeurIPS ’19, pp. 7992–8003, 2019.

Alexander Zook, Brent Harrison, and Mark O. Riedl. Monte-Carlo tree search for simulation-based strategy analysis. arXiv preprint, arXiv:1908.01423:1–9, 2019.
A. APPENDIX

A.1 DPLL

The DPLL algorithm makes calls to two subroutines.

**Unit Propagation**  Unit propagation finds all unit clauses, or clauses which contain only a single unassigned literal. It then removes all other clauses which contain that literal; from all clauses that contain the literal’s complement, it removes the literal’s complement.

**Pure Literal Assignment**  Pure literal assignment finds all literals such that their complements are not present in the SAT instance, which are known as pure literals. It then removes every clause that contains a pure literal.

```
Algorithm 1 DPLL
Input: SAT instance S, policy φ
S ← UnitPropagation(S)
S ← PureLiteralAssign(S)
if S is empty then
   return True
end if
if S contains an empty clause then
   return False
end if
v ← φ(S)
return DPLL(S ∧ v = 0) OR DPLL(S ∧ v = 1)
```

A.2 PSEUDOCODE

A.2.1 ALPHAZERO

```
Algorithm 2 AlphaZero: Inner Loop
Input: Initial state S, value network φ, transition function τ
S<sub>curr</sub> ← S, D ← {}
while Not at terminal state do
   Choose action x<sub>i</sub> ∈ S<sub>curr</sub> with highest UCB: Q(S<sub>curr</sub> v) + U(S<sub>curr</sub> v)
   Append (x<sub>i</sub>, S<sub>curr</sub>) to D
   S<sub>curr</sub> ← τ(S<sub>curr</sub>, x<sub>i</sub>)
   i ← i + 1
end while
Reward ← φ(S<sub>curr</sub>)
for (x, S) in D do
   Update UCB of (x, S) with Reward
end for
```

A.2.2 MCFS-SAT

The red text represents where we diverge from AlphaZero.
Algorithm 3 MCFS-SAT: Outer Loop

Input: SAT instance $S$, value network $\phi$, number of iterations $k$, transition function $\pi$

$S_{\text{root}} \leftarrow S$
Initialize $Q$ values to 0
$i \leftarrow 0$

while Not at terminal state do
  $j \leftarrow 0$
  while $j < k$ do
    $\text{MCFSInnerLoop}(S_{\text{root}}, \phi, \pi)$
    $j \leftarrow j + 1$
  end while
  if $i \leq 30$ then
    $\tau \leftarrow 1$
  else
    $\tau \leftarrow 0$
  end if
  Sample $v_{\text{best}}$ according to the softmax of counts with temperature $\tau$.
  $S_{\text{root}} \leftarrow \tau(S_{\text{root}}, v_{\text{best}})$
  $i \leftarrow i + 1$
end while

Algorithm 4 MCFS-SAT: Inner Loop

Input: SAT instance $S$, subsolver policy $\phi_{\text{sub}}$, policy depth $\ell$

$S_{\text{curr}} \leftarrow S$, $D \leftarrow \{\}$, $i \leftarrow 0$

while $i < \ell$ do
  Choose variable $x_i \in S_{\text{curr}}$ with lowest LCB: $Q(S, v) - Q_{dU}(S, v)$
  Append $(x_i, S_{\text{curr}})$ to $D$
  Draw $z$ from Bernoulli with $p = 0.5$
  $S_{\text{curr}} \leftarrow S_{\text{curr}} \land (x_i = z)$
  $S_{\text{curr}} \leftarrow \text{DPLL}_{\text{step}}(S_{\text{curr}})$
  $i \leftarrow i + 1$
end while

Cost $\leftarrow T_{\phi_{\text{sub}}}(S_{\text{curr}})$
$j = 0$
for $(x, S)$ in $D$ do
  Update LCB with cost $2^{\ell - j} \text{Cost} + 2^{\ell - j} - 1$
  $j \leftarrow j + 1$
end for

A.3 hKis V.S. kcnfs07 on sgen

| # Vars | Tree size (1000s) | CPU+GPU time (s) |
|--------|-------------------|-----------------|
|        | hKis  | kcnfs07 | hKis  | kcnfs07 |
| 65     | 422.1 | 162.3  | 12.2  | 4.0    |
| 75     | 2,996.7 | 1,792.3 | 127.7 | 44.1   |
| 85     | 111,435.8 | 97,979.7 | 699.1 | 219.9  |

We leave out 95 variables as hKis was not able to solve any problem within 6 hours.

A.4 Faster GPU benchmarking
Algorithm 5 MCFS-SAT: Outer Loop

**Input:** SAT instance $S$, sub solver policy $\phi_{sub}$, number of iterations $k$, policy depth $\ell$

$S_{root} \leftarrow S$

Initialize $Q$ values for every state to the value of single lookahead

while $i < \ell$ do

Set $P(S', v) \leftarrow P(S_{root}, v)$ for all $S'$ and $v$

$j \leftarrow 0$

while $j < k$ do

MCFSInnerLoop($S_{root}, \phi_{sub}, \ell$)

$j \leftarrow j + 1$

end while

if Random draw with $p=0.5$ is True then

Set $v_{best}$ to the variable with the highest count after inner loop

else

$v_{best} \leftarrow \arg\max P(S_{root}, v)$

end if

Draw $z$ from Bernoulli with $p = 0.5$

$S_{root} \leftarrow S_{root} \wedge v_{best} = z$

$i \leftarrow i + 1$

end while

| Distribution | # Vars | CPU+GPU time (s) |
|--------------|--------|------------------|
|              |        | uniform+kc
| sgen 65      | 4.2    | 2.3              |
| sgen 75      | 24.7   | 23.1             |
| R3SAT 300    | 6.3    | 1.3              |
| R3SAT 350    | 29.7   | 5.7              |