Cosmological Perturbations During Radion Stabilization

P.R. Ashcroft, C. van de Bruck, and A.-C. Davis

1Department of Applied Mathematics and Theoretical Physics, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge CB3 OWA, UK
2Department of Applied Mathematics, Astro–Particle Theory & Cosmology Group, Hounsfield Road, Hicks Building, University of Sheffield, Sheffield S3 7RH, United Kingdom

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We consider the evolution of cosmological perturbations during radion stabilization, which we assume to happen after a period of inflation in the early universe. Concentrating on the Randall-Sundrum brane world scenario, we find that if matter is present both on the positive and negative tension branes, the coupling of the radion to matter fields could have significant impact on the evolution of the curvature perturbation and on the production of entropy perturbations. We investigate both the case of a long-lived and short-lived radion and outline similarities and differences to the curvaton scenario.

I. INTRODUCTION

Present and future cosmological observations enable us to constrain or rule out models of the early universe. For example, the observations made by the Wilkinson Microwave Anisotropy Probe (WMAP) already put some constraints on inflationary cosmology, [1, 2, 3, 4]. The observations are consistent with purely adiabatic perturbations with a scale invariant power spectrum, but some “anomalies”, such as the small power of the quadrupole and features in the CMB anisotropy spectrum, have been reported, [5].

Future cosmological observations however, will not only constrain current popular models of the early universe, they will also test models “beyond the standard lore”, such as models based on string theory, extra dimensions, etc. For example, alternative mechanisms to inflation have been proposed recently. In contrast to the standard inflationary picture, where density perturbations are generated during inflation and then stretched onto superhorizon scales, fields other than the inflaton field generate the initial perturbations in the new mechanism. In the curvaton scenario initial isocurvature perturbations are transformed into curvature perturbations by a subsequent decay of a second field. This field, dubbed the curvaton, is already present in the early universe but is dynamically unimportant during inflation, [6, 7, 8, 9, 10, 11, 12, 13]. Another example is the idea of “modulated perturbations”, [14, 15, 16, 17, 18, 19]. According to this idea, coupling constants (and other physical properties such as masses of the particles) are functions of the vacuum expectation value of fundamental (light) scalar fields present during inflation. Because of vacuum fluctuations in these fields, different regions in spacetime have different values of coupling constants. These fluctuations can be converted into curvature perturbations, for example during reheating which is not homogeneous in space if the decay rate itself fluctuates. Note that both the curvaton scenario as well as the idea of modulated perturbations need additional scalar fields in the theory, which are dynamically unimportant–at least initially, during inflation.

In this paper we do not seek an alternative to the standard picture of how perturbations are generated, but rather investigate how far the stabilization of moduli fields can alter the evolution of cosmological perturbations generated during a period of inflation in the early universe. Moduli fields appear in theories beyond the standard model based on supersymmetry or superstring theory. In some models they can be long-lived and the life-time can be even larger than the age of the universe. The prime example of a moduli field is the radion, measuring the size of the extra dimensional spacetime in brane world scenarios (see e.g. [20, 21, 22] for recent reviews on brane cosmology). One important property of the radion is that it couples explicitly to matter fields on the branes (see e.g. [23] and references therein). Depending on the details of the bulk geometry and the matter content on the branes, the coupling is field dependent and can become quite large. It is the coupling between the different matter forms to the radion which can lead to interesting consequences for the evolution of perturbations. Furthermore, the radion can be short or long-lived depending on the stabilization mechanism. In the case of the Goldberger-Wise stabilization mechanism in the Randall-Sundrum brane world, [24], the radion obtains a mass of the order of TeV, which makes it short-lived, [25]. In this paper we investigate the impact of radion stabilization on the cosmology and dynamics of cosmological perturbations. We consider both the case of a short-lived and long-lived radion; in the latter, the radion will constitute some (or all) of the dark matter in the universe. Throughout the paper we work in the Randall-Sundrum scenario, [26], because the coupling of the radion to matter on both branes can be quite large and new effects will appear (see
e.g. [27, 28] for the effects of the radion on the CMB anisotropies). In [29, 31, 32] the radion and its rôle in cosmology were discussed in considerable depth. Here we take into account the warped geometry of the bulk.

The paper is organized as follows: in Section II we will present and describe the low-energy effective theory of the Randall-Sundrum brane world. We then present the equations of motion for both background and perturbations. In Section III we will discuss the evolution of the radion and the different constraints on the theory coming from nucleosynthesis, overclosure of the universe and the amount of entropy perturbations generated. In Section IV we present our numerical results of the evolution of the curvature and entropy perturbations. Our conclusions are presented in Section V.

II. THE EFFECTIVE ACTION AND EQUATIONS OF MOTION

The form of the low-energy action for the Randall-Sundrum brane world scenario, in which two boundary branes are embedded in a slice of an Anti-de Sitter (AdS) spacetime, has been studied in the literature already, see e.g. [23, 32, 33, 34, 35] and references therein. At low energies the theory is a scalar-tensor theory with specific matter couplings. In the Einstein frame it takes the form

$$S_{\text{EF}} = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi G} R - \frac{1}{2} (\partial R)^2 - V(R) \right] + S_m^{(1)}(\Psi_1, A(R)g_{\mu\nu}) + S_m^{(2)}(\Psi_2, B(R)g_{\mu\nu}).$$

(1)

We have allowed for the possibility of two matter forms, each of which is confined to the positive or negative tension brane. The couplings between the fields on the branes and the field $R$ are described by the functions $A(R)$ and $B(R)$. The coupling to matter has some interesting consequences, the primary feature being that we no longer have energy-momentum conservation for each component. In fact

$$\nabla_\mu T^\mu_\nu = \alpha^{(i)}_R (\partial^\nu R) T_i,$$

(2)

where $i$ describes the particular brane in hand. Note that the field couples to the trace of the energy-momentum tensor and so there is no coupling to radiation. One defines the coupling functions

$$\alpha^{(1)}_R = \frac{\partial \ln A}{\partial R}, \quad \alpha^{(2)}_R = \frac{\partial \ln B}{\partial R}.$$  

(3)

The energy conservation equation (2) is modified in this theory because the masses of particles on the branes are functions of the radion, and hence can vary with time if the radion varies with time. Alternatively, we could choose a frame in which the masses of the particles on the positive tension brane are constant. In this frame, the masses of the particles on the negative tension brane as well as the four-dimensional Planck mass vary with time. We will study the theory in the Einstein frame, because after stabilization of the radion the frames agree.

One problem common to nearly all models with moduli fields is how to stabilize these fields. We shall include a potential of the form

$$V(R) = \frac{1}{2} M_R^2 (R - R_c)^2,$$

(4)

but here we note that the origin of such a potential might have to be derived from non-perturbative effects in the underlying theory. As we will see below, the mass $M_R$ is constrained by requiring that the field does not overclose the universe and also by nucleosynthesis.

Finally, the couplings $A(R)$ and $B(R)$ have the form

$$A(R) = \cosh \left( \frac{R}{\sqrt{6} M_{pl}} \right), \quad \alpha^{(1)}_R = \frac{1}{\sqrt{6} M_{pl}} \tanh \left( \frac{R}{\sqrt{6} M_{pl}} \right),$$

(5)

$$B(R) = \sinh \left( \frac{R}{\sqrt{6} M_{pl}} \right), \quad \alpha^{(2)}_R = \frac{1}{\sqrt{6} M_{pl}} \coth \left( \frac{R}{\sqrt{6} M_{pl}} \right).$$

(6)

The geometry of the higher-dimensional spacetime is encoded in these coupling functions. Depending on the value of $R$, the couplings can be quite large. For small values of $R$, the function $\alpha^{(2)}_R$ behaves as $\alpha^{(2)}_R \sim 1/R$, whereas one finds $\alpha^{(1)}_R \sim R/M_{pl}^2$. 
The theory described above is a good description of the two-brane system as long as the energy densities on both branes are much less than the brane tension. All length scales considered should be much larger than the curvature scale of the Anti-de Sitter bulk. Furthermore, we have assumed that Kaluza-Klein modes are irrelevant at the energy scales we consider.

Primarily we are concerned with the application to cosmology. We allow matter to be present on both branes, each with an energy density $\rho_m^{(1)}$ and $\rho_m^{(2)}$ respectively. We also include radiation with energy density $\rho_\gamma$. To end up with a realistic cosmology, i.e., with late time acceleration, we include a cosmological constant $\Lambda$. Assuming a flat universe, the equations for the background read:

$$H^2 = \frac{1}{3M_{pl}^2} \left( \frac{1}{2} \dot{R}^2 + U(R) + \rho_m^{(1)} + \rho_m^{(2)} + \rho_\gamma + \rho_\Lambda \right),$$

$$\ddot{R} + (3H + \Gamma)\dot{R} = - \left[ \frac{\partial U}{\partial R} + \alpha_R^{(1)} \rho_m^{(1)} + \alpha_R^{(2)} \rho_m^{(2)} \right],$$

$$\dot{\rho}_m^{(i)} = -3H\rho_m^{(i)} + \alpha_R^{(i)} \dot{R}\rho_m^{(i)}, \quad j = 1, 2,$$

$$\dot{\rho}_\gamma = -4H\rho_\gamma + \Gamma\dot{R}^2.$$  

We will define the density parameter for the different components by $\Omega_i = \rho_i/\rho_{cr}$ as is usual.

The equations of motion of perturbations in matter, the radion field and the metric can be obtained from the perturbed metric has the form (neglecting any anisotropic stress),

$$ds^2 = -(1 + 2\Psi)dt^2 + a^2(t)(1 - 2\Psi)dx^2. \quad (11)$$

The energy-momentum tensor for the two fluids has the form

$$T^\mu_\nu = \begin{pmatrix} -(\rho + \delta\rho) & -\frac{1}{a^2}\partial q \delta q \\ -\frac{1}{a^2}\partial q \delta q & (\partial \rho) \delta q \end{pmatrix}. \quad (12)$$

According to our assumptions, $\delta p_i = \omega_i\delta \rho_i$ for both matter and radiation. Then, expanding in Fourier modes, the perturbation equations read at first order

$$\delta \ddot{R} + (3H + \Gamma)\delta \dot{R} + \frac{k^2}{a^2} \delta R + \frac{\partial^2 \delta R}{\partial R^2} \delta R = -\alpha_R^{(1)} \delta \rho_m^{(1)} - \alpha_R^{(2)} \delta \rho_m^{(2)} - \left( \frac{\partial \alpha_R^{(1)} \rho_m^{(1)}}{\partial R} + \frac{\partial \alpha_R^{(2)} \rho_m^{(2)}}{\partial R} \right) \delta R,$$

$$-2\Psi \left( \frac{\partial \Psi}{\partial R} + \alpha_R^{(1)} \rho_m^{(1)} + \alpha_R^{(2)} \rho_m^{(2)} + \Gamma \dot{R} \right) + 4\dot{\Psi} \dot{R}, \quad (13)$$

$$\delta \dot{\rho}_m^{(j)} + 3H\delta \rho_m^{(j)} - 3\Psi \rho_m^{(j)} + \frac{k^2}{a^2} \delta q_m^{(j)} = Q_m^{(j)} \Psi + \delta Q_m^{(j)}, \quad j = 1, 2 \quad (14)$$

$$\delta \dot{\rho}_\gamma + 4H\delta \rho_\gamma - 4\Psi \rho_\gamma + \frac{k^2}{a^2} \delta \gamma = Q_\gamma \Psi + \delta Q_\gamma, \quad (15)$$

$$\delta \dot{q}_m^{(j)} + 3H\delta q_m^{(j)} + \Psi \rho_m^{(j)} = -\alpha_R^{(j)} \delta \rho_m^{(j)} \delta R, \quad (16)$$

$$\delta \dot{q}_\gamma + 3H\delta q_\gamma + \frac{4}{3} \rho_\gamma \Psi + \frac{1}{3} \delta \gamma = 0, \quad (17)$$

$$3H(\dot{\Psi} + H\Psi) - \frac{k^2}{a^2} \Psi = -\frac{1}{2M_{pl}} \delta \rho. \quad (18)$$

where we have used the abbreviations

$$Q_m^{(j)} = \alpha_R^{(j)} \dot{R}\rho_m^{(j)}, \quad (19)$$

$$\delta Q_m^{(j)} = \left( \alpha_R^{(j)} \delta \dot{R} + \frac{\partial \alpha_R^{(j)} \dot{R}}{\partial R} \right) \rho_m^{(j)} + \alpha_R^{(j)} \dot{R} \delta \rho_m^{(j)} - \Psi Q_m^{(j)}, \quad j = 1, 2. \quad (20)$$

$$Q_\gamma = \Gamma \dot{R}^2, \quad (21)$$

$$\delta Q_\gamma = 2\dot{\Gamma} \delta \dot{R} - \Psi \Gamma \dot{R}^2, \quad (22)$$

$$\delta \rho = \delta \rho_m^{(1)} + \delta \rho_m^{(2)} + \dot{R} \delta \dot{R} + \frac{\partial \Psi}{\partial R} \delta R - \Psi \dot{R}^2. \quad (23)$$
The fluids and the radion field exchange energy during the cosmological expansion due to the non-vanishing coupling. This is to be compared with the original curvaton scenario, in which the curvaton field decays into radiation (and matter).

Furthermore, there is also energy transfer between the radion field and the radiation fluid due to the $\Gamma$ interaction term. The decay rate is given as,

$$\Gamma = \frac{a M_R^3}{192 \pi M_{pl}^2},$$

where the constant $a$ is determined by the number of extra dimensions and the compactification and is $O(1)$. We shall set $a = 1$ for the duration of this paper. This means the radion has a decay time of

$$\tau_R \approx 1.2 \times 10^7 \text{yr} \left(\frac{\text{GeV}}{M_R}\right)^3.$$  

Thus for $M_R < 0.1 \text{GeV}$, the radion would effectively be stable over the lifetime of the universe. If we require the radion to decay before nucleosynthesis, then we need

$$\Gamma \geq H(t_{\text{nucl}}) \sim 10^{-43} M_{pl}.$$  

This would then require a radion mass $M_R \gtrsim 5 \times 10^{-14} M_{pl} \approx 10^5 \text{GeV}$, assuming the constant $a$ is of order unity. The radion stabilizes at $H \sim M_R$ so this gives a long period of radion domination.

The evolution of the perturbation quantities is described in full by equations (13-18). However, it will be helpful for our understanding to recast these. In general, the density perturbations, $\delta \rho_i$, and the metric perturbation, $\Psi$, are gauge dependent. It is more common to work with gauge-invariant quantities. One important quantity, the curvature perturbation on constant density hypersurfaces, is (see e.g. [37, 38, 39])

$$\zeta = \Psi + H \frac{\delta \rho}{\rho}.$$  

Note that this differs in sign from some of the definitions in the literature. In addition we can define the curvature perturbation on uniform $i$-fluid density hypersurfaces as, [39],

$$\zeta_i = \Psi + H \frac{\delta \rho_i}{\rho_i}.$$  

With this prescription, it is clear that the total curvature perturbation is a weighted sum of the individual components,

$$\zeta = \sum_i \frac{\rho_i}{\rho} \zeta_i.$$  

Furthermore, we are able to define a relative entropy perturbation between two components as

$$S_{ij} = 3(\zeta_i - \zeta_j).$$  

To make some predictions about the behavior of the perturbations we work in the “separate universes” picture, which effectively allows us to ignore the spatial gradient terms. In the long-wavelength limit, the perturbed continuity equation becomes

$$\delta \dot{\rho} + 3H(\delta \rho + \delta P) = 3(\rho + P) \dot{\Psi},$$  

where we define

$$\rho = \sum_i \rho_i, \quad P = \sum_i P_i, \quad \delta \rho = \sum_i \delta \rho_i, \quad \delta P = \sum_i \delta P_i.$$  

If one rewrites equation (31) in terms of the total curvature perturbation, $\zeta$, it is relatively straightforward to show

$$\dot{\zeta} = \frac{H}{\rho + P} \delta P_{\text{nad}},$$  

where the non-adiabatic pressure perturbation is \( \delta P_{\text{nad}} \equiv \delta P - c_s^2 \delta \rho \) and the adiabatic sound speed is \( c_s^2 = \dot{P}/\dot{\rho} \). This means that the total curvature perturbation will be constant on large scales for purely adiabatic perturbations. In the context of brane worlds, where energy can be exchanged with the bulk space time, this quantity is not necessarily constant (see e.g. [12, 13, 14]).

It is possible to break this down further so that one may write, [15],

\[
\delta P_{\text{nad}} = \delta P_{\text{int}} + \delta P_{\text{rel}},
\]

\[
= \sum_i (\delta P_i - c_i^2 \delta \rho_i) + \frac{1}{6H\dot{\rho}} \sum_{i,j} \dot{\rho}_i \dot{\rho}_j (c_i^2 - c_j^2) \delta S_{ij},
\]

where \( c_i^2 \equiv \dot{P}_i/\dot{\rho}_i \) is the adiabatic sound speed of the fluid and is related to \( c_s \) by

\[
c_s^2 = \sum_i \frac{\dot{\rho}_i}{\dot{\rho}} c_i^2.
\]

Further, let us define

\[
\delta P_{\text{int},i} = \delta P_i - c_i^2 \delta \rho_i,
\]

so that \( \delta P_{\text{int}} = \sum_i \delta P_{\text{int},i} \).

On superhorizon scales it is possible to write the individual perturbation equations in the form, [16],

\[
\ddot{\zeta}_i = -\frac{3H^2}{\dot{\rho}_i} (\delta P_i - c_i^2 \delta \rho_i) + \frac{HQ_i}{\dot{\rho}_i} \left[ \frac{\delta Q_i}{Q_i} + \left( \frac{\dot{\rho}_i}{2\rho} - \frac{\dot{Q}_i}{Q_i} \right) \frac{\delta \rho_i}{\rho_i} + \frac{\ddot{\Psi}}{H} + \psi \right].
\]

The \( Q_i \) and \( \delta Q_i \) are given in equations [17, 18] with \( Q_R = -Q_m^{(1)} - Q_m^{(2)} - Q_\gamma \) and similarly for \( \delta Q_R \). From this, one can immediately see that for non-interacting perfect fluids with \( Q_i = 0 \) and \( \delta P_i = c_i^2 \delta \rho_i \), the individual curvature perturbations for each fluid remain constant on large scales.

There is now large scope for the setting of the initial conditions of the perturbations. We shall assume that the initial conditions result from a period of inflation in the early universe. Then, it is known that any light scalar field, \( \phi_j \), will pick up a perturbation, [19, 20],

\[
\delta \phi_j = \left( \frac{H}{2\pi} \right)_*, \quad \dot{\delta \phi}_j = 0,
\]

where this is evaluated at horizon crossing for the inflaton field. This depends on the scale of inflation but if this is to be taken as \( V \lesssim 1.5 \times 10^{-8} M_{\text{pl}}^4 \), [21], this tells us that

\[
|\delta R| \lesssim 1.1 \times 10^{-5} M_{\text{pl}}.
\]

If this results in too much entropy production, this will offer a constraint on the scale of inflation. The remaining initial conditions are arranged to be adiabatic initially, as given in [22],

\[
\Psi = \frac{4}{3} C, \quad \delta \rho_m^{(j)} = -2C \rho_m^{(j)}, \quad \delta \rho_\gamma = -\frac{8}{3} C \rho_\gamma,
\]

where \( C \) is some initial scale to be decided in advance. We shall set \( C = 10^{-5} \) in accordance with current observations. Since we are dealing with linear perturbation theory, it is only the ratio \( |\delta R|/C \) that is important rather than their absolute values. In fact, in future we shall normalize the curvature perturbation and plot \( \zeta/R_{\text{ini}} \). Note that these initial conditions are different from those chosen in [23] where \( \delta R = 0 \) initially. For the duration of this paper we shall set

\[
\delta R = -1.1 \times 10^{-5} M_{\text{pl}},
\]

since this will give the largest possible effects.

With this prescription, it is worth discussing what we might expect to see. From equations [24, 25] one can see that \( \zeta \) is sourced in two ways. Firstly when \( P_i \neq P_i(\rho_i) \), which makes the first term in [26] non-zero, and secondly if there is an entropy perturbation present. For perfect fluids, \( \delta P_{\text{int},i} \) will be zero but for a scalar field this is not necessarily the case. Therefore we will always generate evolution in \( \zeta_R \) and thus in \( \zeta \). Furthermore, our initial
conditions, whilst adiabatic between the fluid components, necessarily produce an entropy perturbation between the radion and the fluids. This all holds without any couplings. One should add that since the total curvature perturbation is a weighted sum of the individual components, the effect of the scalar field on $\zeta$ is very much dependent on its energy density fraction. During radiation domination, $\zeta \sim \zeta_\gamma$ which will be constant— if $\Gamma \neq 0$ this no longer holds. During matter domination it is then described by

$$\zeta \approx \Omega_m^{(1)} \zeta_m^{(1)} + \Omega_m^{(2)} \zeta_m^{(2)} + \Omega_R \zeta_R. \tag{43}$$

As one moves into the $\Lambda$ dominated phase this should then become constant again. If couplings are introduced then, from equation (38), we know that all the $\zeta_i$ evolve. During the energy transfer between the fields, $\delta P_{int,i} \neq 0$ and we will also produce entropy perturbations. We should expect to see further evolution in $\zeta$, which will be dependent on the size of the coupling terms.

If one includes coupling between the radion and radiation only ($\Gamma \neq 0, \alpha_R^{(i)} = 0$), we should see an effect in $\zeta_R$, and therefore $\zeta$, as soon as the radion stabilizes, because it becomes the dominant component. All of its energy is transferred to radiation as it decays. Once it has decayed, $\zeta_R$ gives no contribution to the total. However, since there is no coupling to matter, the $\zeta_m^{(i)}$ have remained constant and $\zeta$ will return to this value. Of course, this conclusion will be modified if the coupling terms are non-zero.

### III. COSMOLOGICAL EVOLUTION OF THE RADION

Before we discuss the evolution of the perturbations, we will consider the evolution of the radion during the inflationary epoch and the subsequent radiation and matter dominated epochs. We also discuss cosmological constraints the theory has to fulfill. We begin with the inflationary era.

#### A. The Epoch of Inflation

For definitiveness we consider an inflaton field confined to the positive tension brane and a chaotic inflation potential of the form $V(\phi) = M_\phi^2 \phi^2 / 2$, where $M_\phi$ is the mass of the inflaton field. Further, because we want to study purely adiabatic initial conditions between matter and radiation, we assume that during the last 60 e-folds the inflaton field dominated the energy density of the universe. We are not interested in the case in which the radion was the inflaton field. With these assumptions, there are three cases of interest. In the first case, the mass of the radion is much larger than the mass of the inflaton field, i.e. $M_R \gg M_\phi$. Depending on the initial conditions for $R$, the radion field could have dominated the initial expansion of the early universe. In the second case, the masses of the radion and inflaton field are comparable. In this instance, the energy density of the radion must have been much smaller than the energy density of the inflaton to comply with our assumptions. Finally, the radion can be much lighter than the inflaton field. Let us briefly discuss the three cases separately.

1. $M_R \ll M_\phi$

In this regime, the radion is very light compared to the inflaton field. Its potential energy can be neglected and the radion field is driven by the coupling to the inflaton field only. As established earlier, $\zeta_R$, the radion field is driven towards small values.

2. $M_R \approx M_\phi$

Here a period of double inflation can be realized, where each phase is driven by either the radion or the inflaton field. In accordance with our assumptions, we consider the case in which the second period of inflation, driven by the inflaton field, is longer than 60 e-folds. With this prescription, the radion and the inflaton field stabilize at the same time.

3. $M_R \gg M_\phi$

This case is very similar to the second example, except that the radion now stabilizes much earlier. Initially, it is possible that the radion dominated the expansion rate of the universe before settling into the minimum of the effective
potential. In principle, the radion could have set up the initial conditions for a period of inflation.

In the following, we consider a scenario where the radion decays well after inflation, which corresponds to the first case only. This ensures that the perturbations created during inflation are adiabatic as is the case in single field inflation.

B. The radiation and matter dominated epochs

The behavior of the radion field depends on its mass. If it is very light, the lifetime can be very large and in fact, in this instance, we assume that the lifetime is much larger than the age of the universe. However, depending on the origin of the potential for the radion, it could also be very heavy. In the case of radion stabilization via the Goldberger-Wise mechanism the radion mass is of order TeV. Here, the radion decays into radiation and matter in the very early universe. We will consider these two cases in the rest of the paper. Further, we need to make assumptions about where the matter fields are located. In principle, both the standard model particles as well as the dark matter particles can be located only on the positive tension brane, whereas there is no matter on the negative tension brane. Alternatively, one can imagine the situation where only the standard model particles are confined on the positive tension brane, whereas the dark matter particles are confined on the negative tension brane. Although mixtures of these scenarios can be imagined, in this paper we will allow only baryons to live on the positive tension brane; the dark matter being made up from matter on the negative tension brane and from the oscillating radion field.

In order to make our study compatible with present day observation, we take values for the parameters as given by the most recent WMAP data, \[5\]. This means that today our universe should be almost flat and made up of

$$\Omega_\Lambda = 0.73, \quad \Omega_{DM} = 0.23, \quad \Omega_M = 0.04. \tag{44}$$

Furthermore, we ensure that matter-radiation equality occurs in the range obtained by the WMAP team,

$$3200 \lesssim z_{eq} \lesssim 3400. \tag{45}$$

Nucleosynthesis makes excellent predictions for the abundances of the light elements and we require that these predictions are not modified too much. Nucleosynthesis gives a constraint on the expansion rate \(H_{\text{nucl}}\) at the time when the light elements form. If the particle masses change during the history of the universe, as is the case in our theory, this will influence the time dependence of \(H\) and therefore the expansion rate at the time of nucleosynthesis. The evolution of the masses is specified by the functions \(A\) and \(B\). The requirement that any changes of \(H_{\text{nucl}}\) are constrained by nucleosynthesis gives, \[47\],

$$\frac{A_{\text{nucl}} - A_0}{A_0} \leq 0.1, \tag{46}$$

where \(A_{\text{nucl}}\) and \(A_0\) are the coupling functions at the time of nucleosynthesis and today respectively. A similar bound is valid for \(B\) if some matter form is present on the negative tension brane. Note that these considerations neglect variations in the reaction rates, which itself are functions of masses---and therefore of \(A\)---if the standard model particles live on the positive tension brane. In contrast, if they live on the negative tension brane instead, the reaction rates would be functions of \(B\). Therefore, the constraint above is a rather conservative one.

If the radion is light (\(M_R < 0.1 \text{ GeV}\)), we have seen that it is effectively stable over the lifetime of the universe. It can be shown, \[48\], that for a scalar field, \(\psi\), oscillating in the minimum of its potential, \(V \sim \psi^n\), the energy density scales

$$\rho_\psi \sim a^{-6n/(n+2)}. \tag{47}$$

In our case this would mean the radion, \(R\), behaving like non-relativistic pressureless matter. However, we also have the extra source terms, \(\alpha_j \rho^{(j)}_m\), on the right hand side of equation \[8\] that could amend this result. The oscillations of \(R\) are damped by the Hubble parameter and the energy density \(\rho^{(j)}_m\) will scale approximately like \(a^{-3}\). This means that the source terms will decay faster than the others in equation \[8\]. Therefore, we should still find that the radion behaves like matter when it oscillates in the minimum of its potential, regardless of the new couplings. This is confirmed by our numerical results. In fact, the couplings are most important for the background solutions as the radion rolls down the potential. In practice, this transfers energy between the different species.

Since \(R\) behaves like matter when oscillating at its minimum, this allows us to investigate a number of possibilities:

1. \(R\) plays the rôle of dark matter alone.
2. \( R \) is part of the dark matter with the remaining contribution from the positive and/or negative tension brane.

3. \( R \) is subdominant with the matter on the second brane making up all of the dark matter.

Stabilization occurs when \( H \sim M_R \) and from this point \( \rho_R \sim a^{-3} \) (which inevitably succeeds \( \rho_\gamma \sim a^{-4} \)). Therefore, one can not stabilize the radion too early as this would lead to premature matter domination. This means that the radion cannot be too heavy. Moreover, if the radion is very light, then the background dynamics are more sensitive to the couplings. This may result in too large an evolution of \( R \). Some of this can be compensated for in the initial conditions, but in general we shall find it difficult to generate extremes of masses. If \( R \) is to play the role of dark matter, then its mass should be around \( M_R \sim 10^{-50} M_{pl} \), to agree with galactic halo results. We shall take this to be given in this instance and match our initial conditions to the correct cosmology. The situation in which \( R \) is always subdominant may be interesting because even though we can not see the field, its coupling to matter may have some interesting effects on the perturbations.

Whilst it is possible to have dark matter–in addition to baryons–present on the positive tension brane, the coupling function in equation \( (3) \) is small in general. In contrast, it is possible to produce large couplings on the negative tension brane, as it can be seen from equation \( (6) \).

### IV. RADION STABILIZATION

Having studied the background evolution, we now turn our attention to the evolution of the perturbations. We first discuss the case of a long-lived, i.e. stable radion field, before we consider the effects of radion decay into radiation.

#### 1. The case of a long-lived radion

In the Randall-Sundrum model, the coupling of the radion to matter on the positive tension brane is never large, \( \alpha_R^{(1)} \leq 1/\sqrt{6} \). In contrast, the coupling to matter on the negative tension brane has the property \( \alpha_R^{(2)} \sim 1/R \) for small \( R \). This allows the possibility of generating rather large couplings. In our first example, we take the radion to be approximately half of the dark matter. The results are shown in Figure 1. Here we are able to generate couplings as large as \( \alpha_R^{(2)} \sim 1000 \) and unsurprisingly, this gives some strong effects on \( \zeta \). One sees that there are large entropy perturbations and some interesting changes in the individual \( \zeta_i \). The most striking feature is that \( \zeta \) evolves to negative values.

Let us now consider a subdominant radion and examine the entropy production from the couplings. The results for this setup are shown in Figure 2 where we stabilize the radion at \( R_c = 1 M_{pl} \). The couplings can have important consequences for the background evolution if the radion is very light. These couplings dictate the evolution of the radion initially and push the radion up its potential. This can lead to a period of radion domination as it gains energy from matter. We find from our numerical results that if \( R \) is to remain subdominant we require \( M_R \gtrsim 10^{-52} M_{pl} \). Once more we generate entropy perturbations.

We have seen that for a long-lived radion its stabilization has some interesting consequences for the history of the universe. Even if it remains subdominant in the background evolution, it is still possible for it to generate entropy perturbations between matter and radiation due to the couplings.

#### 2. The case of a short-lived radion

We move on to discuss the case of the decaying radion. In order for this to occur before nucleosynthesis–so that the universe is radiation dominated–we require \( M_R > 10^{-14} M_{pl} \). This should give us a large period of radion domination which is followed by radion decay into radiation. In the case of the Goldberger-Wise stabilization mechanism via a bulk scalar field, the mass of the radion is of order \( 1 \) TeV and therefore decays naturally in the early universe. In some sense, it could be a candidate for the curvaton. Here, however, we are concerned how initial perturbations from inflation are modified by the stabilization and decay of the radion. We will set the mass of order \( 100 \) TeV so that the decay occurs before nucleosynthesis.

Again, the background initial conditions are set to produce the universe that we observe today. As we have already discussed, \( \zeta \) should evolve when \( R \) becomes dominant and then again at matter-radiation equality. If \( \alpha_R^{(j)} = 0 \), then \( \zeta \) should return to unity. Any deviation from this is down to the couplings to matter. This can be seen from equation \( (43) \) because \( \Omega_R = 0 \) and \( \zeta_n^{(j)} \) are constant.
FIG. 1: This plot shows the results for a universe with $\Omega_\Lambda = 0.73$, $\Omega_R = 0.13$, $\Omega^{(2)}_\Lambda = 0.10$, $\Omega^{(1)}_\Lambda = 0.04$. Furthermore, we set $M_R = \sqrt{6} \times 10^{-50} M_{pl}$. In the top left we show the evolution of the density parameters, $\Omega_i$. In the top right plot we see the evolution of the total curvature perturbation, $\zeta$, which demonstrates the effect of the couplings $\alpha_R^{(j)}$ compared with the uncoupled case. We set $R_0 = 0.10 M_{pl}$ and $R_c = 0.03 M_{pl}$ so that $10^{-4} \lesssim \alpha_R^{(1)} M_{pl} \lesssim 10^{-2}$ and $10 \lesssim \alpha_R^{(2)} M_{pl} \lesssim 10^2$. The lower left plot shows the behavior of all the individual $\zeta_i$ and on the right we show the evolution of all entropy perturbations, $S_{ij}$, between the different species.

In Figure 3 we stabilize the radion close to the horizon of the AdS spacetime at $R = 0$ to generate large couplings on the negative tension brane. One can see that the background dynamics are as expected and that the radion decays quickly into matter before nucleosynthesis– this is ensured by setting $M_R = \sqrt{6} \times 10^{-13} M_{pl}$. The $\zeta^{(j)}_m$ are sourced when the radion is stabilized and then remain constant, whereas $\zeta_\gamma$ is sourced during the decay of $R$. This is reflected in the not insignificant production of entropy.

By means of comparison, we show the equivalent plots in Figure 4 for no coupling to matter. Whilst the picture is similar, we find $\zeta$ returning to unity at matter-radiation equality. Furthermore, there is less entropy production than before.

V. SUMMARY AND DISCUSSION

In this paper we have studied the evolution of cosmological perturbations during radion stabilization. We have seen that stabilizing the radion field, which describes the inter-brane distance, can have non-trivial effects on the curvature and entropy perturbations. We have studied two situations, a long and short-lived radion. In the short-lived case, we allow the radion to decay into radiation only with the radion coupling to matter naturally. In setting the initial conditions for the perturbations, we assume that our universe has already undergone a period of inflation. The scale of inflation determines the initial conditions for the radion– in particular the amplitude of the field fluctuation– with
FIG. 2: In this example, we generate a universe with $\Omega_\Lambda = 0.73$, $\Omega_R = 0.0001$, $\Omega_m^{(2)} = 0.23$, $\Omega_m^{(1)} = 0.04$ today. We have set $M_R = \sqrt{6} \times 10^{-50} M_{pl}$. The difference between this and the previous Figure 1 is that the radion field is subdominant throughout the history of the universe. This is clear from the top left plot in which we show the evolution of the density parameters, $\Omega_i$. In the top right plot we show the evolution of the total curvature perturbation, $\zeta$, with and without coupling terms. We set $R_0 = 1.002 M_{pl}$ and $R_c = 1.0 M_{pl}$ so that $\alpha_R^{(1)} M_{pl} \sim 0.15$ and $\alpha_R^{(2)} M_{pl} \sim 1.05$. As one can see, the evolution of the curvature perturbation is significantly modified, even when the radion field is subdominant. The lower left plot shows the behavior of the individual $\zeta_i$ and on the right we show the evolution of the entropy perturbations, $S_{ij}$.

The fluid perturbations set to be adiabatic initially. This will naturally source the total curvature perturbation, since there is an initial entropy between the fields and the fluids. Additionally, energy is transfered between the radion field and matter on each brane due to the coupling terms. This enhances the amount of entropy produced. The amount of entropy production coming from the coupling terms depends on the initial conditions of the radion field just after inflation as well as on the field value at stabilization.

The evolution of $\zeta$ and the $\zeta_i$— and therefore $S_{ij}$— are influenced by a number of parameters. The values $R_0$ and $R_c$ are important as they determine the size of the coupling terms $\alpha_R^{(j)}$. In addition, $|\delta R_{\text{ini}}|$ and $C$ determine the initial entropy perturbation between radiation and the field which is transfered into part of the curvature perturbation. This is similar to the curvaton scenario. The difference here is that there is an initial curvature perturbation and even if the field decays into radiation only, there is still an energy transfer to matter via the couplings.

The constraints on the theory come mainly from the CMB anisotropies. The effect of the radion on the CMB anisotropies has been investigated earlier, [27, 28], but in this paper we have included an initial field perturbation, $\delta R \neq 0$, set by the inflationary scale. The large-scale temperature fluctuation can be written as, [51],

$$\frac{\delta T}{T} \approx \frac{1}{5} \zeta_7 + 2 \left( \frac{\rho_m}{\rho_M} S^{(1)}_{m\gamma} + \frac{\rho_m}{\rho_M} S^{(2)}_{m\gamma} + \frac{\rho_R}{\rho_M} S_{R\gamma} \right), \quad (48)$$

since the radion behaves like matter at matter-radiation equality in all cases we have considered. In this expression we
FIG. 3: In this instance, we consider a short-lived radion field with a mass $M_R = \sqrt{6} \times 10^{-13} M_{pl}$. We set $R_0 = 0.10 M_{pl}$ and $R_c = 0.03 M_{pl}$ so that $10^{-3} \lesssim \alpha_R^{(1)} M_{pl} \lesssim 10^{-2}$ and $10 \lesssim \alpha_R^{(2)} M_{pl} \lesssim 10^2$. This results in a universe with $\Omega_{\Lambda} = 0.73$, $\Omega_{\text{m}}^{(2)} = 0.23$, $\Omega_{\text{m}}^{(1)} = 0.04$. In the top left we show the time dependence of the $\Omega_i$. We see that after a period of radion domination it decays into radiation. In the top right plot we see that evolution of the total curvature perturbation, $\zeta$. The lower left panel shows the behavior of the individual $\zeta_i$ and on the right we show the evolution of all of the entropy perturbations, $S_{ij}$, between the different species.

have set $\rho_M = \rho_m^{(1)} + \rho_m^{(2)} + \rho_R$ and we neglect any neutrino contribution. Observationally constrained is the “effective baryon isocurvature perturbation”, defined as

$$S_{\text{eff}} = S_{m\gamma}^{(1)} + \frac{\rho_m^{(2)} S_{m\gamma}^{(2)}}{\rho_m^{(1)}} + \rho_R S_{R\gamma},$$

where in this expression we have assumed that the matter on the first brane is comprised of baryons only. Since the amplitude of $S_{m\gamma}^{(2)}$ as well as of $S_{R\gamma}$ depend on the details of the stabilization mechanism, such as sizes of the couplings during radion evolution and the mass of the radion, the CMB constrains these parameters to a certain extent, in a similar manner to the curvaton scenario. [51]

It is possible to extend our work in several different ways. Firstly, the initial conditions between the matter and radiation do not have to be necessarily adiabatic. In fact, the radion is a candidate for a curvaton field, if inflation happens at a higher energy scale than the energy scale of the radion potential. In this case, the radion field picks up perturbations during inflation which can be transferred to a curvature perturbation during radion stabilization. Since in the Goldberger-Wise mechanism the mass of the radion is of order TeV, this is a realistic scenario to be investigated. A detailed analysis would give constraints on the mass of the radion as well as on initial conditions of the field just after inflation. We note, that our setup is similar to the one discussed in [9]. However, in our case the radion field decays into radiation only and is coupled to matter via the functions $\alpha_R^{(i)}$ only.
FIG. 4: We make the equivalent run to Figure 3 but without coupling to matter. In the upper plot, we see the evolution of the total curvature perturbation, $\zeta$, with no couplings and the equivalent from Figure 3. The effects of the couplings on $\zeta$ play a role only well after radion decay. However, if one compares the two lower left plots, it can be seen that the individual $\zeta_{ij}$ evolve at radion stabilization. With no couplings, it is only $\zeta_\gamma$ that undergoes any change. In the lower right panel, we show the evolution of the entropy perturbations.

A second point to be investigated is the effect of the potential on the stabilization and on the evolution of perturbations. If the radion undergoes minimal evolution then any potential with a local minimum can usually be approximated by the expansion

$$V(R) \approx V_0 + \frac{1}{2} M_R^2 (R - R_c)^2 + \ldots$$

(50)

In this instance our results will should not change substantially. However, if the radion evolves sufficiently such that the potential can no longer be approximated by the expansion in equation (50), then the energy transfer between the radion and matter can be significantly different.

Our work also has implications for other models such as those with constant couplings. If these couplings are large enough, the energy transfer between the modulus and matter can be large and an entropy perturbation will be produced, even if the field is subdominant throughout the history of the universe.

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