Determination of high-gradient components of residual stress by data of test hole drilling method

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Abstract. The article discusses a modified method for the determination of residual stress in a deformable solid. The general approach is based on a combination of test hole drilling method and speckle pattern interferometry as a tool for deformation response registration. A feature of the proposed procedure for processing experimental data is that it directly takes into account possible high stress gradients at the stage of theoretical model building. The applicability of this technique is illustrated by the example of the residual stresses components measuring at a welded pipe joint.

It is known that residual stresses (RS) present in structural elements can significantly affect their strength characteristics [1–4]. In most cases, reliable data describing RS levels and spatial distributions are obtained by means of experimental investigations [5]. In practice, the most common tool for this is the test hole drilling method [6]. RS components are determined based on the deformation responses of a stressed solid body using an adequate mechanical model of the observed phenomenon. To simplify the data processing, it is traditionally accepted that a conditionally uniform stress state is present in the corresponding local area of the investigated object. The calculation technique is based on the use of so-called model influence functions. These are expected deformation responses in the vicinity of the drilled hole in the body with conditionally unit stresses. We shall note that this approach is the only possible one, if response parameters in the form of three in-plain strain components are measured by means of a three-component rosette of strain gauges.

An effective means of highly informative determination of the deformation response in the vicinity of test hole is the non-contact method of electronic speckle pattern interferometry (ESPI) [6, 9–16]. With its help, the fields of the various components of the displacement vector on the surface of the deformable object are visualized (in the form of fringe patterns) and digitized. The method sensitivity threshold is ~ 0,5 μm. We shall remind that a particular optical setup with two illuminating beams is used for measuring of mutually orthogonal tangential displacement components, figure 1. Direction beams vectors lie in the same plane as the unit vector of the axis along the measured component (u or v). They make equal angles α with a normal to the object surface. Figure 2 shows some typical speckle interferograms which are observed after hole drilling in a stressed body. Here, interference fringes are contour lines of the corresponding fields. Primary processing of such patterns includes obtaining of displacement arrays \( \left( u_{\exp} \right)_i, i = 1, ..., m_1 \) and \( \left( v_{\exp} \right)_i, i = 1, ..., m_2 \) at points on dark fringes. Sets of such points can be observed in figure 2a. Displacement values in them are proportional to interference
fringe orders with sensitivity coefficient $K = \lambda (2 \sin \alpha)^{-1}$, where $\lambda$ is a wavelength of the used coherent laser radiation.

![Interferometer optical setup](image)

**Figure 1.** Interferometer optical setup for measuring of tangential displacement fields ($u$ or $v$)

![Typical speckle interferograms](image)

**Figure 2.** Typical speckle interferograms of displacements fields $u$ (on left) and $v$ (on right) in the vicinity of the test hole in case of a quasi-homogeneous (a) and high-gradient (b) local fields of residual stresses.

(The object of investigation was a thick-wall seam-welded steel pipe. The diameter of the drilled holes $d = 3.2$ mm.)

It shall be noted that the obtained interferograms in themselves do not contain displacement sign data. Special techniques are used to identify the signs. One of effective techniques includes the introduction of an additional linear phase shift into illuminating waves and the analysis of corresponding transformations of initial fringe patterns [14].

The system of $n = m_1 + m_2$ linear equations of the following form is solved for the extended interpretation of the obtained information in terms of RS components [14, 15]:

$$\text{equation}$$
\[ \sigma_x f \left( r_i, \varphi_i \right) + \sigma_y g \left( r_i, \varphi_i - \frac{\pi}{2} \right) + \tau_{xy} \frac{\sqrt{2}}{2} h_1 \left( r_i, \varphi_i \right) = \left( u_{\exp} \right)_i \sigma_0, \quad i = 1, \ldots, m_1, \]
\[ \sigma_x g \left( r_j, \varphi_j \right) + \sigma_y f \left( \tau_{xy} \frac{\sqrt{2}}{2} h_2 \left( r_j, \varphi_j \right) = \left( v_{\exp} \right)_j \sigma_0, \quad j = 1, \ldots, m_2, \]

where
\[ h_1 = f \left( r, \varphi - \frac{\pi}{4} \right) + f \left( r, \varphi - \frac{3\pi}{4} \right) - g \left( r, \varphi - \frac{\pi}{4} \right) + g \left( r, \varphi - \frac{3\pi}{4} \right), \]
\[ h_2 = f \left( r, \varphi - \frac{\pi}{4} \right) - f \left( r, \varphi - \frac{3\pi}{4} \right) + g \left( r, \varphi - \frac{\pi}{4} \right) + g \left( r, \varphi - \frac{3\pi}{4} \right). \]

The coefficients in the equations are the values of the calculated influence functions \( f \left( r, \varphi \right) \) and \( g \left( r, \varphi \right) \) or their combination. When solving the problem under the assumption of a uniform stress state, these functions are the distributions of the tangential displacements \( u \) and \( v \) due to the drilling of the hole in the body in the presence of only tensile stress \( \sigma_c = \sigma_0 \). (Note that the coordinates of the points are set in a polar system with origin in the center of the hole, while displacement vector components are set in a agreed Cartesian system.)

The concrete form of \( f \) and \( g \) functions is determined by solving of model problems. Suppose that the deformation response is completely elastic [17]. In this instance these functions are calculated as the differences of the displacement fields in the loaded model simple body in two of its forms: after and before drilling the hole. For the non-hole body, the problem is solved usually trivially. Through-drilling is carried out during investigation of thin-wall structural elements. In this case, the strain state of the model body with hole is approximately determined based on the well-known Kirsch’s solution [18]. Here, the influence functions have initially the analytical form. For massive objects and blind test holes, required data are obtained using the finite element method. To present analytically the influence functions, the approximation of values of the displacement at nodal points of a finite element mesh by suitable continuous distributions is used.

For example, applying the described technique to the processing of speckle interferograms shown in figure 2a gave the following values of the residual stresses: \( \sigma_x = 325 \) MPa, \( \sigma_y = 310 \) MPa and \( \tau_{xy} = 25 \) MPa. It is evident that the results obtained in such a way are in the general case some averaged estimators of stresses, which were available in material removed during hole drilling.

A large amount of initial experimental information obtained using ESPI provides more accurate final results. However, it is evident that with an increase of gradients of RS fields the degree of adequacy of a traditionally used simplified interpretation model reduces distinctly. If the stress components significantly and monotonically change within the diameter of the test hole, this will notably influence the form of the deformation response. Such situations occur, for example, during investigations of areas near welds in structural elements, figure 2b. We shall note that, if the strain-gauge method is used in experiments, a formally obtained average result will depend on the gauges orientation relative directions of the RS component gradients. In the general case these directions are not known in advance. When the displacements are determined using the ESPI, a possibility arises to apply amore full interpretation model with a large number of parameters. In this work, we will limit ourselves with a special-case of the high-gradient RS fields in two-dimensional statement of the problem. The presented material can be considered as a demonstration example of the proposed approach application.

Let derivatives \( \partial \sigma_x / \partial x \) and \( \partial \sigma_y / \partial y \) reach significant and regard-requiring values in the test hole drilling area. To include these factors in the general model, it is necessary to determine the corresponding additional influence functions. Let us denote them as \( p \) and \( q \). Now, they characterize displacement fields \( u \) and \( v \) near the hole drilled in the body with the linear distribution of the stress component \( \sigma_c = G_0 x \) (\( G_0 = \text{const} \)). As before, the form of these functions can be set based on considering the difference of two states of the model body: drilled and non-drilled. According to
problem statement, a through-hole case is considered. The proposed loading scheme for a certain representative (square in plan) area of model body is presented in figure 3. Calculations were made using the finite element method. Dimensions of the area was 100 × 100 mm²; the diameter of the hole \( d_0 = 2 \text{ mm} \). Material elasticity constants corresponded to steel. \( S = -T = 100 \text{ MPa} \). Final results for the required influence functions \( p \) and \( q \) in the form of polychromatic patterns and fringe imitations are shown in figure 3. Analytically, these functions can be represented by the following approximating polynomials:

\[
p(r, \varphi) = \sum_{i=1}^{3} \left[ r^{1-2i} \sum_{j=1}^{3} A_{ij} \sin^{2j-2} \varphi \right], \quad q(r, \varphi) = \sum_{i=1}^{4} \left[ r^{1-2i} \sum_{j=1}^{4} B_{ij} \sin \varphi \left( 1 - \sin^{2j} \varphi \right) \right].
\]

The coefficients \( A_{ij} \) and \( B_{ij} \) are given in tables 1 and 2. (It is assumed here that displacements and radial coordinates are presented in meters.) The acceptance of this presentation is demonstrated by plots provided in figure 4. We shall note that, obviously, the accepted calculation model is not the only one possible. However, it allows adequate modeling of a required mode of the stressed state in the central area of the model body. The corresponding fields of stress components in the form of polychromatic patterns are shown in figure 5.

![Figure 3](image-url)

**Figure 3.** Calculation of influence functions \( p \) and \( q \) in case of a stress field \( \sigma_x = G_0 x \).

| Table 1. The coefficients of the approximating polynomial for influence function \( p(r, \varphi) \) |
|-----------------|--------|--------|--------|
| \( A_{ij} \)    | \( j = 1 \) | \( 2 \) | \( 3 \) |
| \( i = 1 \)     | -3.803 \( \times 10^{-11} \) | -1.910 \( \times 10^{-09} \) | +2.653 \( \times 10^{-09} \) |
| \( i = 2 \)     | -1.109 \( \times 10^{-15} \) | -3.237 \( \times 10^{-15} \) | +7.264 \( \times 10^{-15} \) |
Table 2. The coefficients of the approximating polynomial for influence function $q(r, \phi)$

| $B_{ij}$ | $j = 1$ | 2 | 3 | 4 |
|----------|---------|---|---|---|
| $i = 1$  | -4.269·10^{-09} | +7.751·10^{-09} | -1.080·10^{-08} | +5.882·10^{-09} |
| 2        | -2.619·10^{-14} | +4.212·10^{-14} | -5.750·10^{-14} | +3.152·10^{-14} |
| 3        | +4.951·10^{-20} | -9.870·10^{-20} | +1.395·10^{-19} | -7.558·10^{-20} |
| 4        | -1.592·10^{-26} | +3.070·10^{-26} | -4.319·10^{-26} | +2.344·10^{-26} |

Figure 4. Polynomial approximation of discrete values for functions $p(r, \phi)$ and $q(r, \phi)$
Figure 5. Stress fields $\sigma_x$ (a), $\sigma_y$ (b) and $\tau_{xy}$ (c) in the model body without a hole

Assume, for simplicity, that sensitivity axes OX and OY of special interferometers are coincident with main axes of the local stressed state of the material. (In other words, assume $\tau_{xy} = 0$). In this case, all the sought parameters of the stress state are determined from solving of a system of $n = m_1 + m_2$ equations in the following form:

$$
\sigma_x(\sigma_0)^{-1}f(\xi_i, \varphi_i) + \sigma_y(\sigma_0)^{-1}g\left(\xi_i, \varphi_i - \frac{\pi}{2}\right) + \frac{\partial \sigma_x}{\partial x}(G_0)^{-1}p(\xi_i, \varphi_i) + \frac{\partial \sigma_y}{\partial x}(G_0)^{-1}q(\xi, \varphi_i - \frac{\pi}{2}) + \\
+ u_R = u_{\text{recn}}(\xi_i, \varphi_i)d_0(d)^{-1}, \quad i = 1, ..., m_1,
$$

$$
\sigma_x(\sigma_0)^{-1}g(\xi_j, \varphi_j) + \sigma_y(\sigma_0)^{-1}f\left(\xi, \varphi_j - \frac{\pi}{2}\right) + \frac{\partial \sigma_x}{\partial x}(G_0)^{-1}p(\xi, \varphi_j) + \frac{\partial \sigma_y}{\partial x}(G_0)^{-1}q(\xi, \varphi_j - \frac{\pi}{2}) + \\
+ v_R = v_{\text{recn}}(\xi_j, \varphi_j)d_0(d)^{-1}, \quad j = 1, ..., m_2.
$$

Here, $\xi = rd(d_0)^{-1}$ and $d$ is the actual diameter of the hole in experiment. Additional unknowns $u_R = \text{const}$ and $v_R = \text{const}$ are introduced for taking into account a possible uncontrolled rigid displacement of the investigated object relative to the measuring system. We shall note that the total number of experimental points $n$ can be selected in a quantity, which significantly exceeds the required minimum. In this case, the overdetermined system of equations shall be solved using the least-square method. This reduces the influence of experimental errors.

The interferograms, which are shown in figure 2b, were processed using the described procedure. As a result, the following values were obtained: $\sigma_x = 250$ MPa, $\sigma_y = 210$ MPa, $\partial \sigma_x/\partial x \approx 10$ MPa(mm)$^{-1}$ and $\partial \sigma_y/\partial y \approx 40$ MPa(mm)$^{-1}$. However, we shall note that in this case, the massive object with the drilled blind probe hole was investigated in the experiment. Therefore, the provided data are only some approximate estimation.

In general case, it may become necessary to add the model with other constituents of gradients of the stress tensor components. However, it shall be noted that the inclusion of too many factors into consideration will reduce the accuracy of made estimations. In the future, it will be useful to carry out a series of numerical experiments with the introduction of typical values of displacement measurement errors. The result will include statistical estimates of thresholds of the RS gradient components, exceeding of these thresholds will require to take into account corresponding factors. It can be expected that this will allow making conclusions regarding the necessity of inclusion of certain parameters into the interpretation model based on the primary qualitative-quantitative analysis of interferograms.
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