Fatigue life prediction of magneto-rheological elastomers in magnetic field

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Abstract

Fatigue life is one of the most important characteristics of materials. Predicting fatigue life in magnetorheological elastomer (MRE) under combined and repetitive loads is an unavoidable step to increase the safety coefficient of MREs. In this study, high-temperature vulcanization silicon rubber (HTVSR) based MREs were fabricated by incorporating the different soft carbonyl iron particles (CIP) that samples with 10%, 20%, and 30% of CIP volume were made. Fatigue behavior of MRE under the combination of compression-tension and torsion loading in the presence and absence of magnetic fields was studied by using the multiaxial fatigue testing machine. The relationship between maximum energy density function ($W_{max}$) and fatigue life ($N$) was achieved that it can predict fatigue life in MREs. The results showed that the dissipation energy was smaller in the presence of a magnetic field at the same CIP volume percentage and increased as the CIP volume percentages grew, because the agglomeration in the samples became more. And also with a magnetic field, at the same CIP volume percentages, the strength of the MRE increased that led to the fatigue life of MRE increasing.

1. Introduction

Magneto Rheological Elastomer (MRE) belongs to the smart family of materials that consist of a basic material matrix and magnetic particles [1]. Several types of rubber, such as silicone rubber [2, 3], natural rubber [4], and polyurethane [5], are widely used as a basic matrix to produce MREs with different properties. Pure iron, carbonyl iron particle (CIP), and cobalt powder are a variety of magnetic particles that can be used to magnetizable. CIP is used to magnetize MRE because of its excellent soft magnetic property and high saturation magnetization [6] that most of the researchers used spheroidal shape and a particle size between 0–10 μm [7]. Boczkowska and et al [8] introduced the methods of curing MREs that when located an external magnetic field during the fabricating process around the samples, anisotropic (non-homogenous) samples are obtained otherwise, isotropic (homogenous) samples would be produced. MREs are often used in the presence of an external magnetic field. When an external magnetic field is used around MREs, these materials are capable of being controlled rapidly and reversibly from rheological points of view [9]. Sutrisno et al [7] discussed the controllable mechanical properties of MREs such as hysteresis, damping, and stiffness when subjected to an external magnetic field. Bodelot et al [10] performed the experimental characterization of the coupled magnetomechanical tensile response of MREs. They understood the macroscopic response of MREs subjected to magnetic and mechanical loading conditions, which led to the design of a new optimization MRE composite materials. Dargahi et al [11] studied the static and dynamic properties of MREs utilizing the rheological tests. The results demonstrated a 1672% increase in the storage modulus of MREs. Syam et al [12] investigated the 3-dimensional MREs’ behavior utilizing finite element analysis. They reported an increase of 20.12% and 28.75% of the stiffness in torsional and linear modes, respectively.

Zarrin-Ghalami et al [13] carried out most MREs typically operate under cyclic loads (dynamic loads) over a while in engineering applications. Cyclic loads are usually oscillating and repeatability, which causes fatigue in
MRE material. Due to the application of MRE materials, including engine mount, tuned vibration absorbers, earthquake vibration absorbents, and etc, fatigue life investigation to prevent failures during operation is one of the critical issues.

There are several methods for the investigation of fatigue life prediction in rubbers, such as maximum principle strain, maximum principle stress, and strain energy density (SED) function. Cadwell et al. [14] and Ro [15] investigate the fatigue life of rubbers by using the maximum principle strain method, and Roach [16] studied the fatigue life of elastomers by using this method. André et al. [17] and Abraham et al. [18] investigated fatigue life prediction in rubbers and elastomers by the maximum principle stress method, respectively. In general, elastomers are highly nonlinear elastic materials that their behaviors are complex due to high deformability, stress softening, incompressibility, and time-dependent effects [19]. So, energy function is often utilized as the crack driving parameter, rather than the stress intensity factor, which is typically used for linear elastic nominal material behavior, such as in metals. On the other hand, energy-based approaches can reflect the constitutive behavior of the material [20]. Gent et al. [21] used from SED function for fatigue life prediction in rubbers. Mars and Fatemi [22] concluded that scalar equivalence criteria were not capable of predicting the fatigue initiation life in natural rubber. They suggested utilizing cracking energy density, which is the part of the SED that is available to cause crack growth on a particular plane. The application of this method involves knowledge of the constitutive behavior of the material. Some researchers have used numerical methods along with experimental results to estimate the fatigue life of rubbers. Ayoub et al. [23] were proposed the fatigue life prediction of rubber-like materials based on continuum damage mechanics (CDM). Using an Ogden SEDF as a hyper elastic model, a CDM model was developed to express fatigue life prediction in rubber by using equivalent stress. The multiaxial fatigue life of MREs was studied by Zhou et al. [24]. They used the bubble inflation testing system for the investigation of the multiaxial fatigue life of MREs. MRE samples were fatigued, and the S-N curves were plotted in different stress amplitudes ranging (between 0.75 MPa and 1.4 MPa). They achieved, at a specific stress value (0.75 MPa), the relationship between stress and life, and also represented stress softening reduced in the first ten cycles for higher stress amplitudes. Zhou et al. [25] fabricated isotropic SR based on MREs with various CIPs from 15% to 35% and examined their fatigue behavior. By examining the slope of the S-N curves, it was found that MREs with a lower percentage of particles have the highest levels of fatigue life in high-stress amplitudes, and MREs with a higher percentage of particles have the highest fatigue resistance in low-stress amplitudes. Zhou et al. [26] were investigated the equi-biaxial fatigue behavior of MRE in the presence of an external magnetic field. The results showed that the fatigue life of MREs increases when an external magnetic field is present. The value of complex modulus were proposed with and without magnetic field and it could be seen that the amount of complex modulus decreased in the presence of the magnetic field. Lian et al. [27] represented the mechanical properties and fatigue behavior of the MRE due to the repetition of the magnetic field. The results showed that under a magnetic field, the hysteresis loss is smaller and the shear modulus is higher than without a magnetic field. Yunzhou and Honglei [28] presented a high-temperature fatigue life prediction method utilizing peak engineering strain for new-energy vehicles rubber bushing based on modified fatigue damage theory. They performed the experimental tests using a high-temperature fatigue test bench in different temperatures including, 25 °C, 40 °C, 60 °C, 80 °C, and 100 °C. The results showed that the scattering coefficient of estimating the fatigue life using this method is less than 2, which indicates the good effect of this method for predicting the fatigue life of rubber bushing.

This paper presents a model for predicting the fatigue behavior of MREs using the maximum energy density function. Firstly, the fabrication of isotropic MREs is described in section 2. Briefly, the theoretical background of hyper elasticity, viscoelasticity, and magnetic in MREs is presented in section 3. Section 4 presents an experimental setup for the determination of mechanical material constants and the method of multiaxial fatigue test. Finally, in section 5, mechanical material constants are achieved and $W_{\text{max}}-N$ diagrams are shown. Based on fatigue results, a fatigue life prediction equation is presented by using the curve fitting method. Then, it is followed by a discussion about the results. The novelties of this article can be summarized as follows:

- Application of a procedure in order to determine the mechanical material constants of a three-dimensional constitutive model for magneto-hyper-viscoelastic silicon rubber-based MREs inducted by magnetic fields.
- Providing a relationship for estimating the fatigue life of MREs in the presence and absence of an external magnetic field based on maximum energy density function.
- Providing relationships for material coefficients in fatigue equation based on the percentage of CIPs and magnetic field rate for isotropic and pure samples.
2. Method of MRE preparation

High-temperature vulcanization silicon rubber (HTVSR) based MRE was used as a basic matrix for sample preparation, was procured by Axson Technology, UK. For fabrication of MREs with different volume fractions (10%, 20%, and 30%), soft CIP (supplied by BASF, Germany) used to magnetize these materials because of its excellent soft magnetic properties and high saturation magnetization. CIPs properties and compounds are shown in table 1.

To produce the samples, the HTVSR was first mixed with a catalyst of a 10:1 ratio, and then the CIPs were added to the mixture with a volume percentage of 10%, 20%, and 30%. A mechanical stirrer was used at a speed of 400 rpm for 10 min to make a good mixture. Scanning electron microscopy (SEM) images taken from the samples shown that the CIPs are evenly dispersed in the elastomer matrix (figure 1). Before the mixture was poured into the mold, the vacuum was taken, and probable goblets were taken inside it. The final mixture was finalized in a heater (WATLOW, 120/240 Votes, 25/100 Watts) at 100 °C for 30 min. Figure 2(a) is shown the schematic of the producing process and (2-b) cylindrical-shaped specimens, respectively.

3. Equations of constitutive

MREs are rubber-like materials that have hyper elastic properties. Due to the delayed response in the unloading process that leads to the production of hysteresis loops, these materials also have viscoelastic properties. Since the process of production of these materials uses carbonyl iron particles in the elastomeric base matrix and also these materials are in the presence of an external magnetic field, their magnetic properties are important. So, for a comprehensive study of MRE’s behavior, it is necessary to use a model that can evaluate the hyper, magnet, and viscoelastic behavior of these materials, simultaneously. This model should create a semi-coupling capability between hyper, magnet, and viscoelastic behaviors that shows the effect of changes in each of these behaviors on the other. In this paper, in order to completely investigate MRE’s behavior, a three-dimensional constitutive
The hyper-magnet-viscoelastic semi-coupling model based on maximum strain energy density (3DMSED) has been proposed.

There are some constitutive equation models for determined MRE’s hyper elastic behavior. Selecting the model depends on its application, corresponding variables, and its available data to determine material parameters [29]. One of the first hyper elastic models is the Mooney-Rivlin model that is popular because it has high precision in the prediction of the nonlinear behavior of isotropic and incompressible rubber-like materials. The mooney-Rivlin hyper elastic SED function for an incompressible and isotropic material is as follows:

\[ W^{\text{hyper}}_{MR} = C_1(I_1 - 3) + C_0(I_2 - 3) \]  

Where \( W^{\text{hyper}}_{MR} \) is mooney-Rivlin hyper elastic SED function, \( C_1 \) and \( C_0 \) are material constants. The first and second invariants of right Cauchy-Green strain tensor \( (C) \) are \( I_1 \) and \( I_2 \) that determine based on principle stretch ratios \( (\lambda) \).

\[ I_1 = \text{tr}(C) = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \]  
\[ I_2 = \frac{1}{2}[(\text{tr}(C))^2 - \text{tr}(C^2)] = \lambda_1^2 \lambda_2^2 + \lambda_2^2 \lambda_3^2 + \lambda_3^2 \lambda_1^2 \]  

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Figure 2. The schematic of the producing process (a) and cylindrical specimen of MRE (b).

| Property and compounds | CIP-CS grade |
|------------------------|--------------|
| Particle size \( d_{50} \) | 6.0–7.0 \( \mu \)m |
| Iron content          | >99.5%       |
| Carbon content        | <0.05%       |
| Nitrogen content      | <0.01%       |
| Oxygen content        | <0.3%        |
| SiO2 content          | <0.1%        |
| Density               | 7.86 gr cm\(^{-3}\) |
For isotropic and incompressible materials: \( \lambda_1 \lambda_2 \lambda_3 = 1 \) and \( \lambda_2 = \lambda_3 \). Trace of a Cauchy-Green strain tensor is presented by \( \text{tr} \).

In order to the investigation of magnetic properties of MREs that produce from the interaction between CIPs in the elastomeric base matrix in the presence of external magnetic fields, \( W_{\text{mag}} \) which is expressed with \( (I_4, I_5) \), was used.

\[
W_{\text{mag}} = \mu_0^{-1}(\alpha I_4 + \beta I_5)
\]

\[
I_4 = |B|^2
\]

\[
I_5 = (CB) \cdot B
\]

The universal constant \( \mu_0 = 4\pi \times 10^{-7} \text{A} \text{H}^{-1} \) is the permeability of the vacuum space. The coefficient \( \alpha \) is dimensionless and provides the coupling between deformation and the magnetic properties, and also this parameter does not affect the stress. The coefficient \( \beta \) is also dimensionless and influences material stiffness in the direction of the magnetic field \( B \). These coefficients are obtained from experimental tests.

As mentioned, MREs have viscoelastic behavior. So, as to thoroughly examine their behavior, the viscoelastic equation should also be added to the hyper elastic equation (1). Generalized-Maxwell is a typical model for the investigation of viscoelastic behavior, which is commonly described as a Prony series.

\[
g(t) = g_{\infty} + \sum_{i=1}^{n} g_i \cdot e^{-\frac{t}{\xi_i}}
\]

Where \( t \) is time and \( \xi_i \) is the relaxation time. Coefficient \( g_i \) is the dimensionless characteristic amplitude as a result of stress variation. \( n \) is the number of relaxation times of characteristic amplitudes which are required for describing the hyper-viscoelastic response of the material. \( g_{\infty} \) is described according to equation (8).

\[
g_{\infty} = 1 - \sum_{i=1}^{n} g_i
\]

\[
g_i = \frac{G_i}{G_0} \quad g_{\infty} = \frac{G_{\infty}}{G_0}
\]

Where \( G_{\infty} \) is shown the modulus of the infinite lone spring, \( G_i \) is the modulus of the ith spring and \( G_0 \) is determined the instantaneous modulus, given by:

\[
\sum_{i=1}^{n} G_i + G_0 = G_{\infty}
\]
According to the previous description, to carefully examine the behavior of MREs, the 3DMSED model, which creates a semi-coupling relationship between hyper, magnet, and viscoelastic behavior, is used, as shown in equation (11).

\[ W_{\text{max}} = W_{\text{max}}(\lambda, B, t) \]  

(11)

Since stress in MREs is obtained from the derivative of the 3DMSED model in terms of stretch (equation (12)), equation (11) can be rewritten as equation (13).

\[ \sigma(\lambda, B, t) = \frac{\partial W_{\text{max}}(\lambda, B, t)}{\partial \lambda} \]

(12)

By integrating from the two sides of the above equation in terms of the stretch, maximum energy density could be divided into two parts include the dissipating strain energy \( \left( \int \sigma(\lambda, B, t) \partial \lambda \right) \) and the stored strain.
energy \( \left( \int \sigma(\lambda, B, t) \partial \lambda \right) \) as follows:

\[
W_{\text{max}} = \left( \int \sigma(\lambda, B, t) \partial \lambda \right)_{s} + \left( \int \sigma(\lambda, B, t) \partial \lambda \right)_{d}
\]  

(13)

In this study, the both parts of the maximum energy density are utilized to represent a fatigue model. The fact is that, in this research, the global behavior of MREs with a macroscopic approach is used to investigate the fatigue behavior of the material. This model has not considered the interaction between particles and matrix. Besides, the interaction effect of displacement on the magnetic field and the effect of particles’ shape are neglected. But the effect of the magnetic field changes on the displacement of the sample is investigated.

4. Experimental setup

Uniaxial compression (loading phase) tests were performed for the determination of hyper elastic, magnetic constants based on ISO 7743/2017 standard in the absence and presence of external magnetic fields (270 mT and 400 mT). Relaxation tests based on ASTM D6747/2010 were performed to achieve viscoelastic coefficients with and without magnetic fields. Uniaxial compression (loading/unloading phase) tests were conducted for the final identification of mechanical material constants. The experimental tests were carried by a mechanical characterization tester on 12 samples. Also, in order to achieve the number of fatigue life of MREs, fatigue tests were performed by using the multiaxial fatigue testing machine in the presence and absence of external magnetic fields.

4.1. Uniaxial compression and relaxation tests

In order to determine hyper elastic material constants, the uniaxial compression tests (loading phase) were performed on all samples. In these tests, the strain rate was \( \dot{\lambda} = 10 \text{ min}^{-1} \), and the samples were compressed until the strain \( \varepsilon = 0.8 \). Finally, the hyper elastic constants were extracted by using the stress-stretch equation of hyper elastic mooney-Rivlin model (equation (14)) [30], test results, curve-fitting method, and least square optimization.

\[
S_{\text{MN}} = \frac{2}{\lambda^4} (C_{10} \lambda^4 + C_{01} \lambda^3 - C_{10} \lambda - C_{01})
\]  

(14)

In order to achieve magnetic constants, the uniaxial compression tests were re-performed on all samples in the presence of external magnetic fields (270 mT and 400 mT). Relaxation tests were executed for determining viscoelastic constants. In the relaxation tests, the samples were compressed with constant strain rate \( \dot{\lambda} = 10 \text{ s}^{-1} \) from the initial deformation \( \lambda = 1 \) to the stretch \( \lambda = 0.90 \) within approximately 1 s. Then, the samples were held fixed for 30 min. Based on uniaxial compression tests, including the loading and unloading phases, the
hyper elastic constants and characteristic amplitudes were calibrated at strain rate $\dot{\lambda} = 10 \text{ min}^{-1}$. After a total testing process, samples were left for at least 24 h unloaded before they were tested again.

4.2. Multiaxial fatigue test

Fatigue tests were conducted using an electromechanical fatigue system in multiaxial compression-tension and torsion loading. Fatigue devices with waveforms generated by a digital-to-analogue converter under personal computer (PC) control were powered. The number of cycles, displacement, and force data were converted by an analogue-to digital converter and were recorded in the PC [31]. All tests were carried out at the average temperature $(20 \pm 1 ^\circ C)$ in the absence and presence of external magnetic fields (270 mT and 400 mT). In order to prevent the heat up in the samples, the tests were performed at a frequency of 0.5–2 Hz based on the magnitude of load amplitude. The loading amplitudes were selected to cover a wide range of fatigue lives $(from 10^1 to 10^5)$, depending on the operation load of the engineering applications [32]. Permanent magnet holders have been selected from aluminum material to avoid disturbing the magnetic field. The specimen surface was monitored at regular time intervals during the test to detect the appearance of fatigue cracks using an optical microscope. When the crack length reached 1 mm, the fatigue test was stopped, and the number of the cycle was recorded. Figure 3 shows the schematic fatigue device and control systems for the testing machine.

5. Results and discussions

5.1. Characterization of material

The uniaxial compression (loading phase) tests are presented in figures 4–6 which the second Piola-Kirchhoff stress replaced the stress in the load direction. Figures 7–9 shown the relaxation tests in the present and absence of magnetic fields. Finally, the uniaxial compression (loading/unloading phase) tests are demonstrated in figures 10–12. The mechanical material constants were achieved using the curve-fitting method and least square optimization on the result of experimental tests. These constants include the hyper elastic mooney-Rivlin model, magnetic, and mean viscoelastic that calibrated at strain rate $\dot{\lambda} = 10 \text{ min}^{-1}$. Tables 2–4 present the value of mechanical material constants in the absence and presence of magnetic fields (270 mT and 400 mT) for all samples. Samples 1 to 3 are pure silicon elastomer, samples 4 to 6 are MRE with 10% volume of CIPs, samples 7 to 9 are MRE with 20% volume of CIPs, and samples 10 to 12 are MRE with 30% volume of CIPs. In order to more understandable the figures, a graph is provided from each sample with the same content of CIPs.

According to the results presented in figures 4–12, it is clear that the interactions between particles increase with the increasing percentage of CIPs, which leads to an increase in the strength of the material. By increasing the percentage of CIPs from 20% to 30%, we can see a significant increase in the strength of the material, which indicates the percentage of saturation of CIPs in the elastomer pad. Also, by applying a magnetic field around the samples, an increase in stress and strength in the material is achieved. Due to the application of these materials, hyper elastic constants and characteristic amplitudes were calibrated at strain rate $\dot{\lambda} = 10 \text{ min}^{-1}$. After a total testing process, samples were left for at least 24 h unloaded before they were tested again.

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which are often used in cyclic loading conditions, increasing the percentage of CIPs can increase the strength of the material, which is very important when the fatigue behavior of these materials is examined. According to tables 2–4, it can be concluded that in absence of the external magnetic field, as a volumetric percentage of CIPs increases, although parameter $C_{01}$ increases, a decrease in parameter $C_{10}$ can be seen. However, in the presence of an external magnetic field, both parameters $C_{10}$ and $C_{01}$ show an increase, which results in an increase in the MR effect.

5.2. Fatigue prediction
In general, the modification of the well-known SED function ($W$) is used to study the fatigue behavior of MREs, especially in large deformations. Abraham et al [18, 33] were used the total and stored SED functions to predict fatigue life on filled and non-filled ethylene propylene diene monomer and styrene-butadiene rubber. In this research, the 3DMSED model with mooney-Rivlin SED function was used for investigating the fatigue life of elastomers. The general form of fatigue life predictor, relating the 3DMSED model ($W_{\text{max}}$) and fatigue life ($N_f$)
for SR-based MREs, was proposed as follows:

\[ \ln(N_f) = \frac{W_{\text{max}} - A}{b} \]  

(15)

The fatigue life equation would be obtained by substituting equation (13) into equation (15), which leads to:

\[ \ln(N_f) = \frac{\left( \int \sigma(\lambda, B, t) \, d\lambda \right)_{s} + \left( \int \sigma(\lambda, B, t) \, d\lambda \right)_{d}}{A} - A \]  

(16)

Where parameters \( A \) and \( b \) are fatigue material constants that depend on CIPs and magnetic field values. For the determination of these parameters, a polynomial curve fitting using the nonlinear least squares error method has been used on the results of fatigue tests in MATLAB software. The method of nonlinear least squares aims to minimize the variance between the values estimated from the polynomial and the expected values from the
In order to assess the efficiency of the fitting procedure, root mean square error (RMSE) and R-squared ($R^2$) are calculated and depicted in Table 5. RMSE is an average error measure between values predicted by equations and values observed, which is preferred because it is on the same scale as the data. In general, the lower RMSE is better than a higher one. The coefficient of determination shows how many data points fall within the results of the line formed by the regression equation, which is always between $0 < R^2 < 1$ with higher values assumed to be better valued [34].

The relation of the parameters $A$ and $b$ to CIPs and magnetic field were extracted and presented in equations (17)–(19). Equation (17) refers to the tested isotropic samples in the presence of the external magnetic field, equation (18) refers to the tested isotropic samples in the absence of external magnetic field, and equation (19) determinate these parameters for pure elastomers with and without the magnetic fields.
Table 2. Final hyper elastic mooney-Rivlin, magnetic, and mean viscoelastic material constants that calibrated at strain rate $\dot{\lambda} = 10$ min$^{-1}$ in the absence of magnetic field.

|                | Hyper elastic constants | Magnetic constants | Mean viscoelastic constants |
|----------------|-------------------------|--------------------|-----------------------------|
|                | $C_{10}$ (kPa) | $C_{01}$ (kPa) | $\alpha$ | $\beta$ | $g_1$ | $\zeta_1$ | $g_2$ | $\zeta_2$ | $g_3$ | $\zeta_3$ | $g_4$ | $\zeta_4$ | $g_5$ | $\zeta_5$ | $g_6$ | $\zeta_6$ | $g_7$ | $\zeta_7$ |
| Sample 1       | 91.76         | 25.49             | —        | —        | 0.0001 | 0.2696   | 0.1886 | 0.6597   | 0.1688 | 2.4232   | 0.1399 | 7.1613    | 0.0524 | 76.3647   | 0.0019 | 239.5772  | 0.0037 | 567.6694  |
| Sample 2       | 91.50         | 25.20             | —        | —        | 0.0010 | 0.1392   | 0.0883 | 0.6203   | 0.0775 | 3.3957   | 0.0818 | 7.3191    | 0.0641 | 15.7995   | 0.0492 | 57.5466   | 0.0041 | 525.4126  |
| Sample 3       | 92.76         | 26.06             | —        | —        | 0.0010 | 0.1392   | 0.0883 | 0.6203   | 0.0775 | 3.3957   | 0.0818 | 7.3191    | 0.0641 | 15.7995   | 0.0492 | 57.5466   | 0.0041 | 525.4126  |
| Sample 4       | 78.72         | 86.01             | —        | —        | 0.0010 | 0.1392   | 0.0883 | 0.6203   | 0.0775 | 3.3957   | 0.0818 | 7.3191    | 0.0641 | 15.7995   | 0.0492 | 57.5466   | 0.0041 | 525.4126  |
| Sample 5       | 79.03         | 85.49             | —        | —        | 0.0010 | 0.1392   | 0.0883 | 0.6203   | 0.0775 | 3.3957   | 0.0818 | 7.3191    | 0.0641 | 15.7995   | 0.0492 | 57.5466   | 0.0041 | 525.4126  |
| Sample 6       | 77.32         | 82.66             | —        | —        | 0.0010 | 0.1392   | 0.0883 | 0.6203   | 0.0775 | 3.3957   | 0.0818 | 7.3191    | 0.0641 | 15.7995   | 0.0492 | 57.5466   | 0.0041 | 525.4126  |
| Sample 7       | −26.33        | 177.44            | —        | —        | 0.0010 | 0.1361   | 0.0766 | 0.6233   | 0.0682 | 3.8402   | 0.0613 | 10.3909   | 0.0403 | 93.1789   | 0.0159 | 337.3940  | 0.0001 | 691.4191  |
| Sample 8       | −26.43        | 176.37            | —        | —        | 0.0010 | 0.1361   | 0.0766 | 0.6233   | 0.0682 | 3.8402   | 0.0613 | 10.3909   | 0.0403 | 93.1789   | 0.0159 | 337.3940  | 0.0001 | 691.4191  |
| Sample 9       | −25.86        | 170.53            | —        | —        | 0.0010 | 0.1361   | 0.0766 | 0.6233   | 0.0682 | 3.8402   | 0.0613 | 10.3909   | 0.0403 | 93.1789   | 0.0159 | 337.3940  | 0.0001 | 691.4191  |
| Sample 10      | −177.02       | 331.79            | —        | —        | 0.0011 | 0.1869   | 0.1247 | 0.6331   | 0.0887 | 3.1873   | 0.1011 | 7.2675    | 0.0330 | 15.4657   | 0.0503 | 56.9479   | 0.0051 | 344.6857  |
| Sample 11      | −178.97       | 333.43            | —        | —        | 0.0011 | 0.1869   | 0.1247 | 0.6331   | 0.0887 | 3.1873   | 0.1011 | 7.2675    | 0.0330 | 15.4657   | 0.0503 | 56.9479   | 0.0051 | 344.6857  |
| Sample 12      | −184.36       | 339.30            | —        | —        | 0.0011 | 0.1869   | 0.1247 | 0.6331   | 0.0887 | 3.1873   | 0.1011 | 7.2675    | 0.0330 | 15.4657   | 0.0503 | 56.9479   | 0.0051 | 344.6857  |
### Table 3

Final hyper elastic Mooney-Rivlin, magnetic, and mean viscoelastic material constants that calibrated at strain rate $\dot{\lambda} = 10 \text{ min}^{-1}$ in the presence of magnetic field (270 mT).

| Sample | $C_{10}$ (kPa) | $C_{01}$ (kPa) | $\alpha$ (-) | $\beta$ (-) | $g_1$ (-) | $\zeta_1$ (s) | $g_2$ (-) | $\zeta_2$ (s) | $g_3$ (-) | $\zeta_3$ (s) | $g_4$ (-) | $\zeta_4$ (s) | $g_5$ (-) | $\zeta_5$ (s) | $g_6$ (-) | $\zeta_6$ (s) | $g_7$ (-) | $\zeta_7$ (s) |
|--------|---------------|---------------|-------------|-------------|-----------|--------------|-----------|--------------|-----------|--------------|-----------|--------------|-----------|--------------|-----------|--------------|-----------|--------------|
| Sample 1 | 144.45        | 13.21         | 0.69        | 0.53        | 0.0011    | 0.1400       | 0.1264    | 0.6174       | 0.1197    | 3.2447       | 0.1125    | 8.8101       | 0.0732    | 81.3936      | 0.0251    | 328.8347     | 0.0030    | 688.0877     |
| Sample 2 | 161.43        | 14.50         | 0.78        | 0.53        |           |             |           |              |           |              |           |             |           |              |           |              |           |              |
| Sample 3 | 151.56        | 13.93         | 0.69        | 0.39        |           |             |           |              |           |              |           |             |           |              |           |              |           |              |
| Sample 4 | 150.59        | 20.54         | 0.24        | 0.19        | 0.0012    | 0.1442       | 0.1368    | 0.6160       | 0.1302    | 3.1316       | 0.1227    | 8.5283       | 0.0806    | 78.9153      | 0.0276    | 326.5996     | 0.0330    | 686.8976     |
| Sample 5 | 154.10        | 21.93         | 0.24        | 0.17        |           |             |           |              |           |              |           |             |           |              |           |              |           |              |
| Sample 6 | 153.10        | 20.97         | 0.25        | 0.20        |           |             |           |              |           |              |           |             |           |              |           |              |           |              |
| Sample 7 | 161.49        | 52.34         | 0.38        | 0.26        | 0.0008    | 0.2796       | 0.1571    | 0.6221       | 0.1521    | 2.4479       | 0.1376    | 8.1066       | 0.0009    | 335.8524     | 0.0103    | 693.8758     |
| Sample 8 | 165.25        | 55.88         | 0.38        | 0.24        |           |             |           |              |           |              |           |             |           |              |           |              |           |              |
| Sample 9 | 155.25        | 55.88         | 0.38        | 0.24        |           |             |           |              |           |              |           |             |           |              |           |              |           |              |
| Sample 10 | 203.46        | 103.13        | 0.45        | 0.35        |           |             |           |              |           |              |           |             |           |              |           |              |           |              |
| Sample 11 | 194.31        | 98.41         | 0.42        | 0.35        |           |             |           |              |           |              |           |             |           |              |           |              |           |              |
| Sample 12 | 194.83        | 101.82        | 0.44        | 0.35        |           |             |           |              |           |              |           |             |           |              |           |              |           |              |
Table 4. Final hyper elastic mooney-Rivlin, magnetic, and mean viscoelastic material constants that calibrated at strain rate $\dot{\lambda} = 10 \text{ min}^{-1}$ in the presence of magnetic field (400 mT).

| Sample | $C_{10}$ (kPa) | $C_{00}$ (kPa) | $\alpha$ (-) | $\beta$ (-) | $g_1$ (-) | $\zeta_1$ (s) | $g_2$ (-) | $\zeta_2$ (s) | $g_3$ (-) | $\zeta_3$ (s) | $g_4$ (-) | $\zeta_4$ (s) | $g_5$ (-) | $\zeta_5$ (s) | $g_6$ (-) | $\zeta_6$ (s) | $g_7$ (-) | $\zeta_7$ (s) |
|--------|----------------|----------------|---------------|-------------|------------|--------------|------------|--------------|------------|--------------|------------|--------------|------------|--------------|------------|--------------|------------|--------------|
| Sample 1 | 99.37          | 21.39          | 0.87          | 0.68        | 0.0011     | 0.1367       | 0.1142     | 0.6163       | 0.1074     | 3.3827       | 0.1004     | 9.1543       | 0.0648     | 84.2544      | 0.0221     | 331.2865     | 0.0027     | 689.3457     |
| Sample 2 | 99.40          | 21.67          | 0.92          | 0.77        |            |              |            |              |            |              |            |              |            |              |            |              |            |              |
| Sample 3 | 104.50         | 22.44          | 0.79          | 0.77        |            |              |            |              |            |              |            |              |            |              |            |              |            |              |
| Sample 4 | 105.45         | 37.92          | 0.52          | 0.47        | 0.0015     | 0.1576       | 0.1318     | 0.6162       | 0.1235     | 3.1607       | 0.1268     | 7.2831       | 0.1113     | 14.8418      | 0.0611     | 141.3142     | 0.0080     | 663.8963     |
| Sample 5 | 101.81         | 37.57          | 0.53          | 0.42        |            |              |            |              |            |              |            |              |            |              |            |              |            |              |
| Sample 6 | 101.99         | 36.47          | 0.51          | 0.45        |            |              |            |              |            |              |            |              |            |              |            |              |            |              |
| Sample 7 | 119.50         | 67.44          | 0.69          | 0.56        | 0.0010     | 0.1518       | 0.1204     | 0.6231       | 0.1104     | 3.2354       | 0.0999     | 8.9193       | 0.0589     | 84.6729      | 0.0185     | 332.4252     | 0.0010     | 689.8837     |
| Sample 8 | 115.38         | 66.83          | 0.70          | 0.50        |            |              |            |              |            |              |            |              |            |              |            |              |            |              |
| Sample 9 | 115.58         | 64.87          | 0.67          | 0.54        |            |              |            |              |            |              |            |              |            |              |            |              |            |              |
| Sample 10 | 332.88        | 129.92         | 0.78          | 0.62        | 0.0008     | 0.3084       | 0.1549     | 0.6863       | 0.1294     | 2.4115       | 0.1011     | 7.3055       | 0.0319     | 81.0920      | 0.0009     | 329.1572     | 0.0108     | 692.7698     |
| Sample 11 | 299.62        | 125.17         | 0.79          | 0.64        |            |              |            |              |            |              |            |              |            |              |            |              |            |              |
| Sample 12 | 306.33        | 123.98         | 0.77          | 0.62        |            |              |            |              |            |              |            |              |            |              |            |              |            |              |
In these equations, \(CI\) is the percent volumes of CIPs and \(B\) is a magnetic field, which its unit is Tesla (T). The unit of parameter \(A\) is \(\text{mm}^2\text{N}^{-1}\) and parameter \(b\) is dimensionless. Consequently, the equation of fatigue life prediction for isotropic samples in the presence of the external magnetic field in different CIPs and magnetic fields as follows:

\[
\begin{align*}
A &= -3.295(CI) - 2.636(B) + 8.278 \\
\quad b &= 0.022 \frac{15(CI)}{1} + 0.037 \frac{46(B)}{1} - 0.099 99
\end{align*}
\] (17)

\[
\begin{align*}
A &= -5.95(CI)^2 - 1.125(CI) + 7.528 \\
\quad b &= 1.188(CI)^2 - 0.4436(CI) - 0.0498
\end{align*}
\] (18)

\[
\begin{align*}
A &= -1.663(B)^2 - 3.512(B) + 6.817 \\
\quad b &= 0.1101(B)^2 - 0.051 13(B) - 0.080 06
\end{align*}
\] (19)

Table 5. Statistical measures of the parameters \(A\) and \(b\) utilizing RMSE and \(R^2\).

|                        | RMSE  | \(R^2\) |
|------------------------|-------|---------|
| Isotropic samples with magnetic field | A 0.0235 | 0.9655 |
|                        | \(b\) 0.0187 | 0.9757 |
| Isotropic samples without magnetic field | A 0.0235 | 0.9728 |
|                        | \(b\) 0.0185 | 0.9845 |
| Pure elastomer with and without magnetic field | A 0.0241 | 0.9836 |
|                        | \(b\) 0.0136 | 0.9824 |

Table 6. Comparison of fatigue life predicted by numerical equation (15) and experimental test for MRE containing 0, 10, 20, and 30 vol% of CIP in the presence of magnetic field (300 mT).

| Predictions of fatigue life with equation (15) | Load 1 | Load 2 | Load 3 | Load 4 | Load 5 |
|------------------------------------------------|-------|-------|-------|-------|-------|
| Pure silicon elastomer                          |       |       |       |       |       |
| Prediction of fatigue life                      | 32    | 142   | 314   | 1503   | 5101   |
| Number of fatigue life with experimental test   | 31    | 140   | 312   | 1504   | 5103   |
| Mean value of error %                           | 3.125 | 1.408 | 0.636 | 0.066  | 0.039  |
| MRE contains 10 vol% of CIP                     |       |       |       |       |       |
| Prediction of fatigue life                      | 79    | 454   | 1196  | 3743   | 7925   |
| Number of fatigue life with experimental test   | 78    | 456   | 1198  | 3745   | 7926   |
| Mean value of error %                           | 1.270 | 0.440 | 0.167 | 0.053  | 0.012  |
| MRE contains 20 vol% of CIP                     |       |       |       |       |       |
| Prediction of fatigue life                      | 56    | 272   | 701   | 2133   | 6519   |
| Number of fatigue life with experimental test   | 55    | 274   | 702   | 2135   | 6521   |
| Mean value of error %                           | 1.785 | 0.735 | 0.142 | 0.099  | 0.030  |
| MRE contains 30 vol% of CIP                     |       |       |       |       |       |
| Prediction of fatigue life                      | 41    | 196   | 486   | 2032   | 5986   |
| Number of fatigue life with experimental test   | 40    | 195   | 487   | 2035   | 5984   |
| Mean value of error %                           | 2.439 | 0.510 | 0.205 | 0.147  | 0.033  |

\begin{align*}
A &= -3.295(CI) - 2.636(B) + 8.278 \\
\quad b &= 0.022 \frac{15(CI)}{1} + 0.037 \frac{46(B)}{1} - 0.099 99
\end{align*}

Equation (21) was used for fatigue life prediction for isotropic samples in the absence of the external magnetic field in different CIPs and magnetic fields.

\[
\begin{align*}
Ln(N) &= \frac{W_{\text{max}} + 3.295(CI) + 2.636(B) - 8.278}{0.022 \frac{15(CI)}{1} + 0.037 \frac{46(B)}{1} - 0.099 99}
\end{align*}
\] (20)

\begin{align*}
A &= -5.95(CI)^2 - 1.125(CI) + 7.528 \\
\quad b &= 1.188(CI)^2 - 0.4436(CI) - 0.0498
\end{align*}

Equation (21) was used for fatigue life prediction for isotropic samples in the absence of the external magnetic field in different CIPs and magnetic fields.

\[
\begin{align*}
Ln(N) &= \frac{W_{\text{max}} + 5.95(CI)^2 + 1.125(CI) - 7.528}{1.188(CI)^2 - 0.4436(CI) - 0.0498}
\end{align*}
\] (21)

The pure elastomer of fatigue life prediction samples in the absence and presence of the external magnetic field in different magnetic fields was achieved with equation (22).
The maximum energy density function versus the number of fatigue life shown in figure 13 is based on fatigue test results for isotropic and pure cylindrical-shaped specimens with the absence and presence of the magnetic field for a variety of CIPs such as 10%, 20%, 30%, and pure elastomer. It can be seen in the results, the fatigue resistance of the material in the presence of the external magnetic field during the test is higher than when the magnetic field is absent. Based on the presented results in figure 13, it clearly indicated that the number of fatigue life under a magnetic field is bigger than without a magnetic field condition in the same CIP volume.

\[
\ln(N) = \frac{W_{\text{max}} + 1.663(B)^2 + 3.512(B) - 6.817}{0.1101(B)^2 - 0.05113(B) - 0.08006}
\]  

(22)

Figure 14. Force-Displacement curve for pure silicon elastomer and MREs with different of CIP in different fatigue cycles: without a magnetic field (a), with a magnetic field 270 mT (b), and 400 mT (c).
percentages. A possible reason is that the interaction between CIPs increased under a magnetic field.

Table 7. Dissipation energy values for pure silicon elastomer and MREs with different CIP without a magnetic field and with a magnetic field 270 mT and 400 mT.

|                          | B = 0 mT | B = 270 mT | B = 400 mT |
|--------------------------|----------|------------|------------|
| Pure Silicon elastomer   | 6.4690   | 5.1811     | 4.1196     |
| MRE contains 10 vol% of CIP | 8.0633   | 6.8816     | 6.3512     |
| MRE contains 20 vol% of CIP | 11.6154  | 8.8747     | 8.3941     |
| MRE contains 30 vol% of CIP | 24.4729  | 18.1512    | 13.4071    |

To assess the predictive capabilities of the equations (15) and (16), the fatigue lives were experimentally achieved in the presence of another magnetic field (300 mT) and also predicted utilizing numerical equations (15) and (16). These two values and the mean value of their relative error, which was calculated approximately, were listed in table 6. As can be seen, the errors between the fatigue life predictors, i.e., numerical equations and experimental tests, are acceptable, which verify the correctness of the equations (15) and (16).

In order to investigate the MR effect of the pure silicon elastomer and MREs, the hysteresis loops were determined in various fatigue cycles. Figure 14(a) shows the force-displacement curve for pure silicon elastomer and MREs without a magnetic field. Figures 14(b) and (c) show the same results under the magnetic fields 270 mT and 400 mT, respectively. The dissipation energy, which is equal to the area of the hysteresis loops, is presented in table 7. The results clearly demonstrated that the dissipation energy without a magnetic field is bigger than with a magnetic field. A reasonable cause is that the hardness of the samples decreased in no magnetic field condition, which leads to a high hysterisis loop area.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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