Λ-CDM type Heckmann - Suchuking model and union 2.1 compilation

G. K. Goswami¹, R. N. Dewangan² & Anil Kumar Yadav³

¹,²Department of Mathematics, Kalyan P. G. College, Bhilai - 490006, India
Email: gk.goswam9@gmail.com

³ Department of Physics, United College of Engineering & Research, Greater Noida - 201306, India
Email: abanilyadav@yahoo.co.in

Abstract

In this paper, we have investigated Λ-CDM type cosmological model in Heckmann-Schucking space-time, by using 287 high red shift (0.3 ≤ z ≤ 1.4) SN Ia data of observed absolute magnitude along with their possible error from Union 2.1 compilation. We have used χ² test to compare Union 2.1 compilation observed data and corresponding theoretical values of apparent magnitude (m). It is found that the best fit value for (Ωm)_0, (ΩΛ)_0 and (Ωσ)_0 are 0.2940, 0.7058 and 0.0002 respectively and the derived model represents the features of accelerating universe which is consistent with recent astrophysical observations.

Key words: Dark energy, Λ-CDM cosmology and deceleration parameter.
PACS: 98.80.Es, 98.80-k

1 Introduction

Wilkinson Microwave Anisotropy Probe (WMAP)[25] and Hubble Key Project (HKP)[9] explored that our universe is nearly flat. This has given concept of two component density parameters Ωm and ΩΛ, which are related through

ΩΛ + Ωm = 1. (1)

Equation (1) is obtained by solving Einstein’s Field Equations with cosmological constant for FRW cosmological model, which represent a spatially homogeneous and isotropic accelerating expanding flat universe. One can see the details of Λ-CDM model in Refs. [1-18].

Luminosity distance (DL) in Λ-CDM model is as follows:

DL = cz
and

DL = c(1 + z) \frac{dz}{H_0 \sqrt{(\Omega_m)_0(1 + z)^3 + (\Omega_\Lambda)_0}}. (3)

The luminosity distance (DL) is associated with absolute and apparent magnitudes by the following equation

m - M = 5 log_{10} \left( \frac{D_L}{M_{pc}} \right) + 25. (4)

To get the absolute magnitude M of a supernova, we consider a supernova 1992P at low-redshift z = 0.026 with m = 16.08.

Equations (2) and (4) read as

M = 16.08 - 25 + 5 log_{10}(H_0 / .026c). (5)

These equations (2)-(5) produce the following expression for absolute magnitude m.

m = 16.08 + 5 log_{10}(\frac{(1 + z)}{.026} \int_0^z \frac{dz}{\sqrt{(\Omega_m)_0(1 + z)^3 + (\Omega_\Lambda)_0}}). (6)
In the last decade of 20th century, Riess et al. [20] and Perlmutter et al. [18] found that the present values of $\Omega_m$ and $\Omega_\Lambda$ are nearly 0.29 and 0.71 respectively. Perlmutter et al. had used only 60 SN Ia low red shift data set while in the present analysis, we have used 287 high red shift data set out of 500 SN Ia data set as reported in ref. [26]. The recent SN Ia observations, BOSS, WMAP and Plank result for CMB anisotropy [12] give more precise value of cosmological parameters. After publication of WMAP data, we notice that today, there is considerable evidence in support of anisotropic model of universe. On the theoretical front, Misner [16] has investigated an anisotropic phase of universe, which turns into isotropic one. The authors of ref. [17] have investigated the accelerating model of universe with anisotropic EOS parameter and have also shown that the present SN Ia data permits large anisotropy. Recently DE models with variable EOS parameter in anisotropic space-time have been studied by Yadav and Yadav [27], Yadav et al [28,29], Akarsu and Kilinc [3], Yadav [30], Saha and Yadav [22] and Pradhan [19].

In Ref. [10], we have presented a $\Lambda$-CDM type cosmological model in spatially homogeneous and anisotropic Heckmann-Schucking space-time given by

$$ds^2 = c^2dt^2 - A^2dx^2 - B^2dy^2 - C^2dz^2,$$

(7)

Where $A(t)$, $B(t)$ and $C(t)$ are scale factors along $x$, $y$ and $z$ axes. In the literature, metric (7) is also named as Bianchi type I [31].

We consider energy momentum tensor for a perfect fluid i.e.

$$T_{ij} = (p + \rho)u_iu_j - pg_{ij},$$

(8)

Where $g_{ij}u^iu^j = 1$ and $u^i$ is the 4-velocity vector.

In co-moving co-ordinates

$$u^\alpha = 0, \quad \alpha = 1, 2, 3.$$

(9)

The Einstein field equations are

$$R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda g_{ij} = -\frac{8\pi G}{c^4}T_{ij}.$$  

(10)

Choosing co-moving coordinates, the field equations (10), for the line element (7), read as

$$\ddot{B} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -\frac{8\pi G}{c^2}p + \Lambda c^2,$$

(11)

$$\ddot{A} + \frac{\dot{A}}{A} + \frac{\dot{A}\dot{C}}{AC} = -\frac{8\pi G}{c^2}p + \Lambda c^2,$$

(12)

$$\ddot{A} + \frac{\dot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -\frac{8\pi G}{c^2}p + \Lambda c^2,$$

(13)

$$\ddot{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{AC} = \frac{8\pi G}{c^2}\rho + \Lambda c^2.$$  

(14)

Solving equations (11) - (13) by approach given in ref. [10], one can obtain the following relation for scale factors

$$B = AD(t),$$

(15)

$$C = AD(t)^{-1},$$

(16)

$$D(t) = exp\left[\int \frac{K}{A^2}\right],$$

(17)

Where $K$ is the constant of integration.

In view of equations (15) - (17), equations (11) - (14), read as

$$\frac{2\dot{A}}{A} + \frac{\dot{A}^2}{A^2} = \frac{8\pi G}{3c^2}\left(p - \frac{\Lambda c^4}{8\pi G} + \frac{K^2c^2}{8\pi GA^6}\right).$$

(18)

$$H^2 = \frac{\dot{A}^2}{A^2} = \frac{8\pi G}{3c^2}\left(p + \frac{\Lambda c^4}{8\pi G} + \frac{K^2c^2}{8\pi GA^6}\right).$$

(19)
Now, we assume that the cosmological constant $\Lambda$ and the term due to anisotropy also act like energies with densities and pressures as

$$\rho_\Lambda = \frac{\Lambda c^4}{8\pi G}, \quad p_\Lambda = -\frac{\Lambda c^4}{8\pi G},$$

$$\rho_\sigma = \frac{K^2 c^2}{8\pi G A^6}, \quad p_\sigma = \frac{K^2 c^2}{8\pi G A^6}.$$  \hfill (20)

It can be easily verified that energy conservation law holds separately for $\rho_\Lambda$ and $\rho_\sigma$ i.e.

$$\dot{\rho}_\Lambda + 3H(p_\Lambda + \rho_\Lambda) = 0,$$

$$\dot{\rho}_\sigma + 3H(p_\sigma + \rho_\sigma) = 0.$$  

The equations of state for matter, $\sigma$ and $\Lambda$ energies are read as

$$p_m = \omega_m \rho_m,$$

where $\omega_m = 0$ for matter in form of dust, $\omega_m = \frac{1}{3}$ for matter in form of radiation. There are certain more values of $\omega_m$ for matter in different forms during the course of evolution of the universe.

$$p_\Lambda = \omega_\Lambda \rho_\Lambda,$$

$$p_\sigma = \rho_\sigma,$$

So,

$$\omega_\Lambda = -1,$$

$$\omega_\sigma = 1,$$

where $\omega_m$, $\omega_\Lambda$ and $\omega_\sigma$ are equation of state parameters of matter, $\Lambda$ and $\sigma$.

In the literature, $\Lambda$-CDM model is described by FLRW metric. In the derived model, equations (18) and (19) represent the field equation of FLRW metric that is why the derived model is $\Lambda$-CDM type. For flat $\Lambda$-CDM model, we have

$$\Omega_\Lambda + \Omega_m + \Omega_\sigma = 1,$$  \hfill (23)

where $\Omega_m = \frac{\rho_m}{\rho_c} = \frac{(\Omega_m)H_0^2(1+z)^3}{H_0^2}$, $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} = \frac{(\Omega_\Lambda)H_0^2}{H_0^2}$ and $\Omega_\sigma = \frac{\rho_\sigma}{\rho_c} = \frac{(\Omega_\sigma)H_0^2(1+z)^6}$. $\Omega_\sigma$ is the anisotropic energy density which is taken very small for present analysis. The recent observation of CMB support the existence of anisotropy in early phase of evolution of universe. Therefore, it make sense to consider the model of universe with an anisotropic background in the presence of cosmological term $\Lambda$.

The expression for luminosity distance ($D_L$) and apparent magnitude ($m$) are as follows

$$D_L = \frac{c(1+z)}{H_0} \int_0^z \frac{dz}{\sqrt{[(\Omega_m)H_0^2(1+z)^3} + (\Omega_\sigma)H_0^2(1+z)^6 + (\Omega_\Lambda)H_0^2]}$$

$$m = 16.08 + 5\log_{10}(\frac{1+z}{.026}) \int_0^z \frac{dz}{\sqrt{[(\Omega_m)H_0^2(1+z)^3} + (\Omega_\sigma)H_0^2(1+z)^6 + (\Omega_\Lambda)H_0^2]}.$$  \hfill (24)

The purpose of the present work is to compute the $\Omega_\Lambda$ and the $\Omega_m$ of the universe in light of Union 2.1 compilation in anisotropic space-time which is different from our previous work [10]. In our previous work [10], we used old SN Ia data to compute the physical parameters at present epoch. Here the anisotropic energy density ($\Omega_\sigma$) is taken to be very small i.e. $\Omega_\sigma = 0.0002$. In order to obtain $\Omega_\Lambda$ and $\Omega_\sigma$, we have considered high red-shift SN Ia supernova data of observed $m$ along with their possible error from Union 2.1 compilation and have obtained corresponding theoretical values of $m$ for various $\Omega_m$, ranging in between 0 and 1. The paper is organized as follows. In section 2, we have estimated the values of $\Omega_\Lambda$ and $\Omega_m$. Section 3 deals with the matter and dark energy densities and estimation of present age of universe. Finally the results are displayed in section 4.
2 Estimation of \((\Omega_m)_0\) and \((\Omega_\Lambda)_0\) from 287 SN Ia data set

For the sake of comparison of theoretical value of \(\Omega_m\) with observational values, we compute the \(\chi^2\) value as following:

\[
\chi^2_{SN} = A - \frac{B^2}{C} + \log_{10}(\frac{C}{2\pi}),
\]

where

\[
A = \sum_{i=1}^{287} \frac{[(m)_{ob}-(m)_{th}]^2}{\sigma_i^2},
\]

\[
B = \sum_{i=1}^{287} \frac{[(m)_{ob}-(m)_{th}]^2}{\sigma_i^2},
\]

and

\[
C = \sum_{i=1}^{287} \frac{1}{\sigma_i^2}.
\]

Here the summations are taken over data sets of observed and theoretical values of apparent magnitudes of 287 supernovae.

Based on the above expressions, we obtain the following Table-1 which describes the various values of \(\chi^2\) against values of \(\Omega_m\) ranging between 0 to 1

| \(\Omega_m\) | \(\chi^2_{SN}\) | \(\chi^2_{SN}/287\) |
|----------------|----------------|-------------------|
| 0.0000         | 5522.1000      | 19.24076655       |
| 0.1000         | 50002.1000     | 174.2334495       |
| 0.2000         | 4833.7000      | 16.84216028       |
| 0.2800         | 4799.8000      | 16.72404181       |
| 0.2900         | 4799.2000      | 16.72195122       |
| 0.2940         | 4799.3000      | 16.7229965        |
| 0.9998         | 4799.3000      | 16.72195122       |
| 0.2960         | 4799.3000      | 16.7229965        |
| 0.9996         | 5276.7000      | 18.38571429       |

Table: 1

From Table-1, we find that for minimum value of \(\chi^2\), the best fit present values of \(\Omega_m\) and \(\Omega_\Lambda\) are 0.2940 and 0.7058 respectively.

In order to compare the various theoretical value of luminosity distance \(D_L\) corresponding to different values of \(\Omega_m\) with observational values, we compute again \(\chi^2\) values as following:

\[
\chi^2_{SN} = A - \frac{B^2}{C} + \log_{10}(\frac{C}{2\pi}),
\]

where

\[
A = \sum_{i=1}^{287} \frac{[(D_L)_{ob}-(D_L)_{th}]^2}{\sigma_i^2},
\]

\[
B = \sum_{i=1}^{287} \frac{[(D_L)_{ob}-(D_L)_{th}]^2}{\sigma_i^2},
\]

and

\[
C = \sum_{i=1}^{287} \frac{1}{\sigma_i^2}.
\]
and
\[ C = \sum_{i=1}^{287} \frac{1}{\sigma_i^2}. \]

Here the summations are over data sets of observed and theoretical values of luminosity distances of 287 supernovae.

Based on the above expressions, we obtain the following Table 2 which describes the various values of \( \chi^2 \) against values of \( \Omega_m \) ranging between 0 to 1.

| \( \Omega_m \) | \( \chi_{SN}^2 \) | \( \chi_{SN}^2/287 \) |
|--------------|----------------|-------------------|
| 0.0000       | 225.5970       | 0.78605226        |
| 0.1000       | 204.7956       | 0.71357352        |
| 0.2000       | 198.0609       | 0.69010767        |
| 0.2800       | 196.7042       | 0.68538049        |
| 0.2900       | 196.6806       | 0.68529826        |
| 0.2910       | 196.6797       | 0.68529512        |
| 0.2920       | 196.9790       | 0.68633798        |
| 0.2930       | 196.6785       | 0.68529094        |
| 0.2940       | 196.6783       | 0.68529024        |
| 0.2960       | 196.6786       | 0.68529129        |
| 0.2970       | 196.6791       | 0.68529303        |
| 0.3000       | 196.6821       | 0.68530348        |
| 0.9998       | 215.6214       | 0.75129408        |

Table 2

From Table 2, we find that for minimum value of \( \chi^2 \), the best fit present values of \( \Omega_m \) and \( \Omega_\Lambda \) are presented again as \( (\Omega_m)_0 = 0.2940 \) and \( (\Omega_\Lambda)_0 = 0.7058 \) respectively.

3 Some physical parameters of the universe

3.1 Density of the universe and Hubble’s constant

The energy density at present is given by
\[ (\rho_i)_0 = \frac{3c^2H_0^2}{8\pi G} (\Omega_i)_0, \]  
(26)

where \( i \) stands for different types of energies such as matter energy, dark energy etc.

Taking, \( (\Omega_m)_0 = 0.2940, H_0 = 72 \text{ km/sec./Mpc.} \)

The current value of dust energy \( (\rho_m)_0 \) for flat universe is given by
\[ (\rho_m)_0 = 0.5527h_0^2 \times 10^{-29} \text{ gm/cm}^3. \]  
(27)

The current value of dark energy \( (\rho_\Lambda)_0 \) read as
\[ (\rho_\Lambda)_0 = \frac{3c^2H_0^2}{8\pi G} (\Omega_\Lambda)_0 = 1.3269h_0^2 \times 10^{-29} \text{ gm/cm}^3, \]  
(28)

Where \( (\Omega_\Lambda)_0 = 0.7058 \).

The expression for Hubble parameter is given by
\[ H^2 = H_0^2[(\Omega_m)_0(1 + z)^3 + (\Omega_\sigma)_0(1 + z)^6 + (\Omega_\Lambda)_0] \]  
(29)

and
\[ H^2 = H_0^2[(\Omega_m)_0 \left( \frac{A_0}{A} \right)^3 + (\Omega_\sigma)_0 \left( \frac{A_0}{A} \right)^6 + (\Omega_\Lambda)_0]. \]  
(30)
3.2  Age of the universe

The present age of the universe is obtained as follows

\[ t_0 = \int_0^{t_0} dt = \int_0^\infty \frac{dz}{H_0(1+z) \sqrt{\left[ (\Omega_m)_0(1+z)^3 + (\Omega_\sigma)_0(1+z)^6 + (\Omega_\Lambda)_0 \right]}}. \]  

(31)

\[ t_0 = \int_0^{t_0} dt = \int_0^\infty \frac{dz}{H_0(1+z) \sqrt{\left[ (\Omega_m)_0(1+z)^3 + (\Omega_\sigma)_0(1+z)^6 + (\Omega_\Lambda)_0 \right]}}. \]  

(32)

From equation (32), one can easily obtain \( t_0 \rightarrow 0.9388H_0^{-1} \) for high redshift and \( (\Omega_\Lambda)_0 = 0.7058 \). This means that the present age of the universe is 12.7534 Gyrs \( \sim 13 \) Gyrs for \( \Lambda \) dominated universe. From WMAP data, the empirical value of present age of universe is 13.73 \( \pm 0.13 \) Gyrs which is closed to present age of universe, estimated in the this paper.

3.3  Deceleration parameter \( q \):

The deceleration parameter is given by

\[ q = \frac{3}{2} \left( \frac{(\Omega_m)_0(1+z)^3 + 2(\Omega_\sigma)_0(1+z)^6}{(\Omega_m)_0(1+z)^3 + (\Omega_\Lambda)_0 + (\Omega_\sigma)_0(1+z)^6} \right) - 1. \]  

(33)

Since in the derived model, the best fit values of \((\Omega_m)_0, (\Omega_\Lambda)_0\) and \((\Omega_\sigma)_0\) are 0.2940, 0.7058 and 0.0002 respectively hence we compute the present value of deceleration parameter for derived \( \Lambda \)-CDM universe by putting \( z = 0 \) in eq. (19). The present value of DP comes to

\[ q_0 = -0.5584. \]  

(34)

Also it is evident that the universe had entered in the accelerating phase at \( z \sim 0.6805 \leq t \sim 0.4442H_0^{-1} \sim 6.0337 \times 10^9 \) yrs in the past before from now.

4  Result and Discussion

In the present work, \( \Lambda \)-CDM type cosmological model in Heckmann-Suchuking space-time has been investigated. We summarize our work by presenting the following table which displays the values of cosmological parameters at present.

| Cosmological Parameters | Values at Present                                      |
|------------------------|------------------------------------------------------|
| \((\Omega_\Lambda)_0\) | .7058                                               |
| \((\Omega_m)_0\)      | .2940                                                |
| \((\Omega_\sigma)_0\) | .0002                                                |
| \((q)_0\)             | -0.5584                                              |
| \((\rho_m)_0\)        | \(0.5527h_0^2 \times 10^{-29} \) gm/cm\(^3\)        |
| \((\rho_\Lambda)_0\)  | \(1.3269h_0^2 \times 10^{-29} \) gm/cm\(^3\)        |
| Age of the universe   | 12.7534 Gyrs                                         |

The figures 1 and 2 shows how the observed values of apparent magnitudes and luminosity distances come close to the theoretical graphs for \((\Omega_\Lambda)_0 = 0.7058, (\Omega_m)_0 = 0.2940\) and \((\Omega_\sigma)_0 = 0.0002\). Figures 3 and 4 shows the dependence of Hubble’s constant with red shift and scale factors. Figure 5 shows that the time tends to a definite value for large redshift which in turn determines the age of universe. The various figures validates that the best fit value for energy parameters corresponding to matter and dark energy are 0.2940 and 0.7058. As a final comment, we note that the present model represents the features of accelerating universe. The present value of DP is found \(-0.5584\) which is consistence with modern astrophysical observations.
Fig1: Apparent magnitude \((m)\) versus redshift \((z)\)

Fig2: Luminosity distance \((D_L)\) versus redshift \((z)\)
Fig3: Hubble constant \( \left( \frac{H}{H_0} \right) \) versus redshift \( (z) \)

Fig4: Hubble constant \( \left( \frac{H}{H_0} \right) \) versus scale factor
Fig5: Time versus redshift (z)

Acknowledgments

The authors are grateful to the anonymous referee for valuable comments to improve the quality of manuscript. One of us G. K. G. is thankful to IUCAA, Pune, India for providing facility and support where part of this work was carried out during a visit. This work is supported by the CGCOST Research Project 789/CGCOST/MRP/14.

References

[1] P. A. R. Ade et al., “The Second Planck Catalogue of Compact Sources,” arXiv: 1303.5076v3.
[2] U. Alam, V. Sahni, T D Saini and A A Starobinsky, Month. Not. Roy. Astron. Soci. 344, 1057 (2003).
[3] O. Akarsu and C. B. Kilinc, Gen. Relativ. grav. 42, 119 (2010).
[4] R. Amanullah et al., Astrophys. J. 716, 712 (2010)
[5] P. Astier et al., Astron. Astrophys. 447, 31 (2006)
[6] R. R. Caldwell, W Knowp, L Parker and D A T Vanzella, Phys. Rev. D 73, 023513 (2006)
[7] S. M. Carroll, W H Press and E L Turner, Ann. Rev. Astron. Astrophys. 30, 499 (1992)
[8] E. J. Copeland, M Sami and S Tsujikawa, Int. J. Mod. Phys. D 15, 1753 (2006)
[9] A. Freedman et al., Astrophys. J. 553, 47 (2001)
[10] G. K, Goswami, A. K. Yadav and M. Mishra, Int. J. Theor. Phys. 54, 315 (2015)
[11] O. GrØn and S. Hervik, Einstein’s General Theory of Relativity : With Modern Application in Cosmology ( Springer, 2007).
[12] G. Hinshaw et al., Astrophys. J. Suppl. 208, 19 (2013)
[13] T. Koivisto and D. F. Mota, “ Anisotropic dark energy: dynamics of the background and perturbations”, arXiv: 0801.3676.
[14] E. Komastu et al., Astrophys. J. Suppl. Ser. 180, 330 (2009)
[15] S. Kumar and A. K. Yadav, Mod. Phys. Lett. A 26, 647 (2011)
[16] C. W. Misner, Astrophys. J. 151, 431 (1968)
[17] T. Koivisto and D. F. Mota, Astrophys. J. 679, 1 (2008)
[18] S. Perlmutter et al., Astrophys. J. 517, 565 (1999)
[19] A. Pradhan, Res. Astron. Astrophys. 13, 139 (2013)
[20] A. G. Riess et al., Astron. J. 116, 1009 (1998)
[21] A. G. Riess et al., Astron. J. 607, 665 (2004)
[22] B. Saha and A. K. Yadav, Astrophys. Space Sc. 341, 651 (2012)
[23] M. R. Setare and E. N. Saridakis, Phys. Lett. B 668, 177 (2008)
[24] M. R. Setare and E. N. Saridakis, JCAP 0903, 002 (2009)
[25] D. N. Spergel et al., Astrophys. J. Suppl. Ser. 148, 175 (2003)
[26] N. Suzuki et al., Astrophys. J. 746, 85 (2012)
[27] A. K. Yadav and L. Yadav, Int. J. Theor. Phys. 50, 218 (2011)
[28] A. K. Yadav, F. Rahaman and S. Ray, Int. J. Theor. Phys. 50, 871 (2011)
[29] A. K. Yadav, F. Rahaman, S. Ray and G. K. Goswami, Euro. Phys. J. Plus 127, 127 (2012)
[30] A. K. Yadav, Astrophys. Space Sc. 335, 565 (2012)
[31] O. Heckmann and E. Schucking, Gravitation : An Introduction to current research (Wiley, New York, 1962).