Magnetic interference patterns in superconducting junctions:
Effects of anharmonic current-phase relations

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Abstract – A microscopic theory of the magnetic-field modulation of critical currents is developed for plane Josephson junctions with anharmonic current-phase relations. The results obtained allow examining temperature-dependent deviations of the modulation from the conventional interference pattern. For tunneling through localized states in symmetric short junctions with a pronounced anharmonic behavior, the deviations are obtained and shown to depend on the distribution of channel transparencies. For constant transparency the deviations vanish not only near \( T_c \), but also at \( T = 0 \). If Dorokhov bimodal distribution for transparency eigenvalues holds, the averaged deviation increases with decreasing temperature and takes its maximum at \( T = 0 \).

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Introduction. – Magnetic interference patterns in superconducting junctions originate from quantum coherence of the superconducting state under the applied magnetic field. They attract considerable experimental attention and underlie effective studies of various problems of superconductivity (see, for example, [1–10]). Yet the corresponding microscopic theory for short junctions is lacking and the present understanding of the results is based mainly on the Ginzburg-Landau approach and the tunneling limit. A microscopic extension of the results to the low-temperature region \( T \ll T_c \) is required, since the Josephson current is to a great extent controlled by discrete Andreev bound states, which are not resolved within the Ginzburg-Landau theory. Effects of finite transparencies of the transport channels are intrinsically connected to the contributions of higher harmonics of the supercurrent to the modulation, which, therefore, can be adequately described only beyond the tunneling approximation. Due to the absence of a corresponding microscopic theory, experimental data on the interference patterns are analyzed in the literature partly phenomenologically with reference to usual procedure firmly confirmed for tunnel junctions near \( T_c \).

When a transparency of plane junctions gets close to unity, there is usually a crossover from the Josephson current to bulk superconducting flow. Nonetheless, there are important plane contacts, where the physics of weak links is still valid even in the presence of highly transparent transport channels. This is the case for long superconductor-normal metal-superconductor (SNS) fully transparent junctions of various geometries, where interference patterns have been studied in detail theoretically at arbitrary temperatures and beyond the tunneling approximation [11–19]. In particular, the central Fraunhofer peak in clean planar and long SNS junctions with fully transparent interfaces has been found to get strongly distorted at low temperatures. At \( T = 0 \) it acquires a triangular form [11,12], which correlates with a saw-toothed current-phase relation taking place under the same conditions in the systems [12,20]. This example demonstrates that pronounced anharmonic current-phase relations in superconducting junctions can entail significant qualitative modifications in the corresponding magnetic interference patterns.

Another characteristic weak link with a strongly anharmonic current-phase relation is a short clean highly transparent point contact, which in a fully transparent case reduces to the Kulik-Omelyanchuk clean superconducting constriction [21]. Similar results also occur for tunneling through a single localized state or for plane junctions, where resonant electron tunneling takes place via individual localized states homogeneously distributed over an insulating interface (see [22–24] and references therein). In such systems an analytical description of the Josephson current is possible at low densities of the transport channels with arbitrary transparencies since the pair breaking effects are small there.

In the present paper modulations of the critical current are described based on a microscopic theory of Josephson junctions generalized to the case of an applied magnetic field.
field. An integration of the modulated current over the plane of a rectangular junction is carried out explicitly in a general form for arbitrary interface transparencies. The answer is related to the phase-dependent part of the thermodynamic potential in the absence of the modulation, taken at the field-dependent phase difference. The theory is applied to short junctions with localized states homogeneously distributed over the interface plane. A qualitative difference is demonstrated between deviations of the modulated critical current from the Fraunhofer pattern in junctions with constant transparency and Dorokhov bimodal distribution of transparency eigenvalues. Junctions between isotropic s-wave superconductors are considered below but the basic results to unconventional superconductors is straightforward.

Modulation of the Josephson current. – Let superconducting electrodes $S_l$ and $S_r$ be thick compared to the magnetic penetration depths $\lambda_{\phi}(x)$, while let the thickness of the interlayer and the junction width be much less than the coherence lengths $\xi_{\phi}(x)$ and the Josephson penetration length, respectively. One takes the $x$-axis perpendicular to the contact plane and the magnetic field applied along the $z$-axis: $B(x) = B(x)e_z$ (see fig. 1). It is convenient to take the vector potential in the form $A(x) = A(x)e_y$, div $A = 0$, which coincides with the gauge usually taken in describing the Meissner effect. In contrast to the case of the Meissner effect, in Josephson junctions the vector potential $A(x) = A(x)e_y$ does not vanish everywhere in the depth of superconductors, where the screening supercurrent $j_y(x)$ and the screened field $B(x)$ do vanish. Indeed, a difference between asymptotic values of the vector potential is associated with the magnetic flux $\Phi$ through the junction: $A_{+\infty} - A_{-\infty} = \Phi/L_y$. Here $L_y$ is a contact width along the $y$-axis.

Nonzero asymptotic values of the vector potential can be excluded from microscopic equations by means of the corresponding gauge-like transformation. Thus, Bogoliubov amplitudes and order parameters can be represented as $\tilde{u}^{r(t)} = u^{r(t)} \exp \left[ \frac{i \Phi}{\hbar c} A_{+\infty} y \right]$, $\tilde{v}^{r(t)} = v^{r(t)} \times \exp \left[ - \frac{i \Phi}{\hbar c} A_{-\infty} y \right]$, $\tilde{\Delta}^{r(t)} = \Delta^{r(t)} \exp \left[ \frac{i 2 e B}{h c} y A_{\pm\infty} \right]$. Here quantities $u^{r(t)}$, $v^{r(t)}$ and $\Delta^{r(t)}$ satisfy the equations where only the residual parts of the vector potential $\tilde{A}^{r(t)}(x) = A(x) - A_{\pm\infty}$ are present in the right and the left superconducting regions, respectively. The phases of the corresponding order parameters $\Delta^{r(t)} = |\Delta^{r(t)}| \exp(i \chi^{r(t)})$, $\tilde{\Delta}^{r(t)} = |\Delta^{r(t)}| \exp(i \chi^{r(t)})$ are related as $\chi^{r(t)}(y) = \chi^{r(t)} + \frac{2 e B}{h c} A_{\pm\infty} y$. For nonzero magnetic flux one gets $A_{+\infty} \neq A_{-\infty}$ and the transformation does not reduce fully to fixing a gauge since it differs in the two regions. Therefore, after excluding constant asymptotic values of the vector potential from microscopic equations, the quantities $A_{\pm\infty}$ enter not only gauge-dependent phases of order parameters, of Bogoliubov amplitudes and of Green’s functions, but also a number of gauge-invariant physical quantities. In particular, as the result of matching corresponding solutions at the interface, the phase difference

$$\tilde{\chi}(y) = \chi^{r} - \chi^{l} = \chi^{r} - \chi^{l} + \frac{2 e B}{h c} (A_{+\infty} - A_{-\infty}) y$$

enters a secular equation and influences the periodic phase-dependent spectrum of interface Andreev states.

After performing the transformation in superconducting regions the problem becomes formally more close to that of the Meissner effect, since the residual parts of the vector potential $\tilde{A}^{r(t)}(x)$ vanish in the depth of the superconductors together with $B(x)$ and $j_y(x)$. Similar to the problem of the Meissner effect, $\tilde{A}(x)$ in the given gauge does not lead to any additional changes of phases of the order parameters, even in a strongly nonlinear regime [25]. For this reason the modulation of the critical current in the microscopic theory is controlled by the spatial dependence of the phase difference

$$\tilde{\chi}(y) + 2 \pi (y/L_y)(\Phi/\Phi_0),$$

where $\chi = \chi^{r} - \chi^{l}$ and $\Phi_0 = \pi \hbar c/|e|$ is the superconducting flux quantum.

As the modulation period $L_y \Phi = \pi \hbar c/|eB(0)|$ is a macroscopic scale, the quasi-classical theory of superconductivity applies to a microscopic study of the problem. Here $\ell_B = (\hbar c/|eB(0)|)^{1/2}$ is the magnetic length and $d = \lambda_t + \lambda_r$, where $d$ is the interlayer thickness. Within the quasiclassical approximation, interface Andreev bound states are associated with coupled incoming, reflected and transmitted trajectories, which cross the interface at one and the same point. In the absence of the field, Andreev bound states are degenerate with respect to the coordinates $(y_0, z_0)$ of the reflection points, where parallel incoming trajectories with given Fermi velocity $v_F$ cross the junction plane. The total supercurrent represents a sum of separate contributions with various possible $v_F$. When the external magnetic field is present, the quasiclassical boundary conditions, locally applied at each crossing point, result in lifting the degeneracy due to $y_0$-dependence of the phase difference $\tilde{\chi}(y_0)$ across the interface. The periodic dependence of the quasiparticle spectrum on the coordinate $y_0$ of the crossing point is the microscopic origin of the magnetic-field modulation of the current. For describing the modulation, one should sum (integrate) over $y_0$ the contributions to the current from respective parallel trajectories for each given $v_F$.  

Fig. 1: (Colour on-line) Schematic diagram of the junction.
In the absence of the modulation, the phase-dependent part of the thermodynamic potential of the junction can be represented as the following sum over Matsubara frequencies:

$$\Omega_0(T, \chi) = -(T/2) \sum_{n=-\infty}^{\infty} \ln D(i\varepsilon_n, \chi).$$  \hspace{1cm} (2)

The quantity $D(i\varepsilon_n, \chi)$ enters the secular equation $D(\varepsilon, \chi) = 0$ for eigenenergies of the system and can be defined unambiguously [26,27]. In the presence of spin degeneracy one gets $D(\varepsilon, \chi) = D^2(\varepsilon, \chi)$, $\Omega = 2\Omega_0$.

A variation of the thermodynamic potential with the phase difference for a junction under the applied field $\delta\Omega(T, \chi, \Phi)$ is expressed via the variation $\delta\Omega_0(T, \chi)$ in the absence of the modulation:

$$\delta\Omega(T, \chi, \Phi) = \int_{a-L_y/2}^{a+L_y/2} \frac{dy_0}{y_0} \delta\Omega_0(T, \chi, \Phi), \quad \Phi = 2\pi \frac{y_0}{L_y}. \hspace{1cm} (3)$$

Here a rectangular plane junction is supposed to occupy the space $(a - L_y/2, a + L_y/2)$ along the $y$-axis. The parameter $a$ determines a position of the interference pattern relative to the junction edges. Since the Josephson current and thermodynamic potential satisfy the relation

$$I(T, \chi, \Phi) = -\frac{2e}{\hbar} \frac{d}{d\chi} \Omega(T, \chi, \Phi),$$  \hspace{1cm} (4)

the integration of the current over $y_0$ can be explicitly carried out. One obtains from eqs. (3) and (4)

$$I(T, \chi, \Phi) = \frac{e}{\pi \Phi_0} \left[ \Omega_0(T, \chi - \frac{\pi \Phi}{\Phi_0}) - \Omega_0(T, \chi + \frac{\pi \Phi}{\Phi_0}) \right]. \hspace{1cm} (5)$$

Here $\chi = \chi + \frac{2\pi \Phi}{\Phi_0}$ is the effective phase difference. As this follows from eqs. (2) and (5), the magnetic-field modulation of the Josephson current at arbitrary temperatures and transparencies is described by the expression

$$I(T, \chi, \Phi) = \frac{e}{2\pi \Phi_0} \left[ \sum_{n=-\infty}^{\infty} \ln \left( \frac{D(i\varepsilon_n, \chi - \frac{\pi \Phi}{\Phi_0})}{D(i\varepsilon_n, \chi + \frac{\pi \Phi}{\Phi_0})} \right) \right]. \hspace{1cm} (6)$$

Equation (6) allows calculations of magnetic-field modulations of critical currents, provided that the secular function $D(i\varepsilon_n, \chi)$ is known for the junction in the absence of the modulation. The secular function can take complex values and its property $D(-i\varepsilon_n, \chi) = D^*(i\varepsilon_n, \chi)$ ensures real values of thermodynamic potentials and the current. Since eqs. (1)–(4) underly the derivation of eq. (6) and have quite general character, eq. (6) applies to a variety of planar rectangular junctions with any interfaces, including those between unconventional superconductors and/or with magnetic interlayers.

In symmetric junctions the Josephson current is carried solely by subgap states, for which

$$\delta\Omega_0(T, \chi) = \delta \left\{ -T \sum_{i=1}^{N} \ln [2 \cosh (E_i(\chi)/2T)] \right\}. \hspace{1cm} (7)$$

Here the sum is taken over Andreev state energies $E_i(\chi)>0$ of $N$ transport channels, which can depend on trajectory directions and spin indices. According to eqs. (5) and (7),

$$I(T, \chi, \Phi) = \frac{e}{\pi \Phi_0} \sum_{i=1}^{N} \frac{\ln \left[ \left( \cosh \left( E_i \left( \chi + \frac{\pi \Phi}{\Phi_0} \right) / 2T \right) \right) \right]}{\cosh \left( E_i \left( \chi - \frac{\pi \Phi}{\Phi_0} \right) / 2T \right)}.$$  \hspace{1cm} (8)

Within its application domain eq. (8) agrees with eq. (6).

In particular, eq. (6) reduces to eq. (8) in the simplest case, when $D_0(iz_n, \chi) = \prod_{i=1}^{N} A_i \left[ iz_n + E_i^2(\chi) \right]$ and $A_i$ are independent of $\chi$.

A phase difference $\chi_{e,c}(T, \Phi)$, which corresponds to the modulated critical current $I_e(T, \Phi) = I(T, \chi_{e,c}(T, \Phi), \Phi)$, satisfies the equation

$$I_0(T, \chi_{e,c}(T, \Phi) + \frac{\pi \Phi}{\Phi_0}) = I_0(T, \chi_{e,c}(T, \Phi) - \frac{\pi \Phi}{\Phi_0}), \hspace{1cm} (9)$$

where $I_0(T, \chi)$ is the Josephson current in the absence of the modulation. In the zero-field limit one obtains from eq. (6) or eq. (8) familiar general relations between the Josephson current and the secular function or the spectrum of interface Andreev bound states [26,27]. It follows from eqs. (6) or (8) that the current always vanishes under the condition $D(iz_n, \chi - \frac{\pi \Phi}{\Phi_0}) = D(iz_n, \chi + \frac{\pi \Phi}{\Phi_0})$ or $E_i(\chi - \frac{\pi \Phi}{\Phi_0}) = E_i(\chi + \frac{\pi \Phi}{\Phi_0})$. Hence, a 2$\pi$-periodic phase-dependent spectrum ensures positions of nodes of the modulated Josephson current at $\Phi = n\Phi_0$, $n = \pm 1, \pm 2, \ldots$, irrespective of the phase difference. Since all even harmonics also vanish at $\Phi = 2n\pi \Phi_0$, for such values of the magnetic flux the current is formed only by contributions from odd harmonics. For small deviations $\delta \Phi$ of the magnetic flux from $n\Phi_0$, the current and, in particular, its derivative with respect to the phase difference always have opposite signs above and below each of the nodes. This signifies that the positions of minima of thermodynamic potentials as functions of the phase difference abruptly change by $\pi$ at $\Phi = n\Phi_0$. Therefore, continuous 0–$\pi$ transitions of the interference origin take place with varying magnetic flux through points $\Phi = n\Phi_0$ ($n = \pm 1, \pm 2, \ldots$), where all harmonics of the current vanish simultaneously.

The 0–$\pi$ transitions are known to take place, in particular, with varying temperature or interface thickness in junctions with magnetic interlayers. One can see that such transitions also take place with varying magnetic field through a junction. If the magnetic field, satisfying the relation $n\Phi_0 < \Phi < (n+1)\Phi_0$, is applied, then originally 0 ($\pi$) junctions either evolve to the 0 ($\pi$) state with respect to $\chi_e$ (for $n = 0, \pm 2, \pm 4, \ldots$), or turn into respective $\pi$ (0) junctions (for $n = \pm 1, \pm 3, \pm 5, \ldots$). This concerns, in particular, the standard situation, when the Fraunhofer pattern describes the modulation.

**Tunneling through localized states.** Consider further nonmagnetic short junctions between identical s-wave superconductors, where tunneling via localized states with
a large broadening occurs. The influence of the screening current and the magnetic orbital effects on the Josephson current is usually negligibly small in such systems, so that the residual vector potential can be disregarded. Then the spectrum of spin-degenerate Andreev states takes the form $E_{\pm i}(\chi) = \pm |\Delta| \sqrt{1 - D_i \sin^2 (\chi/2)}$, which formally coincides with the spectrum of superconductor-insulator-superconductor point contacts [26]. The transparency $D_i$ is described here by the Breit-Wigner resonance function, taken at the energy of the $i$-th localized state [22–24,28]. The coefficient $D_i$ can take any value between 0 and 1, depending on the energy of the state and its position $x_{i,0}$ across the interlayer. Near $T_c$ the order parameter is small and, expanding all functions in eq. (8) in powers of $E_{i}/T_c$, one can keep there only the main quadratic term. This leads to the relation $I_c(T,\Phi) = I_{cF}(T,\Phi)$, where $I_{cF}(T,\Phi)$ describes the Fraunhofer pattern for the critical current

$$I_{cF}(T,\Phi) = I_c(T,0) \left| \frac{\pi\Phi}{\Phi_0} \right| / \left(\frac{\pi\Phi}{\Phi_0} \right) \right|.$$

(10)

In the particular case $I_c(T,0)|_{T \to T_c} = |\Delta|^2 \sum_i D_i$, where the sum is taken over possible different $\nu_f$.

At low-temperatures arguments of hyperbolic functions in eq. (8) are large. Using the respective asymptotic expressions one obtains within a simplified model of constant $D$: $\cos \chi_c(0,0) = \cos \chi_c(0,0) \cos (\frac{\pi}{2})$. Here the zero-field phase difference is $\cos \chi_c(0,0) = -(1 - \sqrt{T - D})^2 / D$. This solution results in the zero-temperature critical current, which exactly reduces to the Fraunhofer pattern eq. (10) for any field value. The zero-field critical current at $T = 0$, which enters eq. (10) as a factor and depends on $D$, is found to take the form $I_c(0,0) = (|\Delta|/\hbar)(1 - \sqrt{1 - D})$.

Figure 2 shows current-phase dependences for various values of the magnetic flux through symmetric fully transparent junctions in question with a few identical transport channels at zero temperature. The dependences involve significant contributions from a large number of harmonics. Surprisingly, the conventional interference pattern for the critical current in symmetric junctions takes place in this case. Such behavior contrasts to what is known for long superconductor-normal metal-superconductor junctions. Based on eq. (8), one can calculate relative deviations $\delta I_c(T,\Phi) = (I_c(T,\Phi) - I_{cF}(T,\Phi))/I_c(T,\Phi)$ of the critical current from the Fraunhofer values given by eq. (10). The quantity $\delta I_c(T,\Phi)$ vanishes identically only in the tunneling approximation and/or near $T_c$. Figure 3 displays the deviation $\delta I_c(T,\Phi)$ as a function of temperature, for $\Phi = 0.5\Phi_0$ and various transparency coefficients. At intermediate temperatures eq. (10) does not apply exactly, but the nonmonotonic temperature-dependent deviations due to higher harmonics are less than few percent and vanish at $T = 0$.

In asymmetric junctions zero-temperature deviations $\delta I_c$ do not vanish, as this follows from eq. (6). They are shown in fig. 4 as functions of the parameter $\gamma = |\Delta_l/\Delta_r|$, which characterizes the junction asymmetry. Since $\delta I_c$ does not vary with interchanging left and right order parameters, one takes $\gamma \geq 1$. As can be seen, $\delta I_c$ at $T = 0$ is positive and not large, reaching about ten percent at $\gamma = 14$ and not exceeding eleven percent even at $\gamma = 30$.

**Multichannel effects.** In Josephson junctions with one transport channel or a few channels with identical transparencies, the phase difference $\chi_{c,e}$, which corresponds to the critical current, depends on the transparency value due to anharmonic current behavior. In a
In asymmetric junctions as functions of temperature, taken at $\Phi = 0$.

Figure 3 demonstrates that an interplay between different channels can change such situation. As a result, the critical current exceeds the Fraunhofer value at low temperatures and, if $D_2$ is not too small, maximal deviations occur at zero temperature. A nonmonotonic behavior of deviations at low temperatures taking place with the parameter $D_2$ arises from two competing reasons. If $D_2$ is not too small and decreases, then the variations of $\chi_{e,c}$ from its value for constant transparency $D_1 = 1$ increases and determines increasing deviations of the critical current from the Fraunhofer one. On the other hand, for a channel with sufficiently small and decreasing $D_2$, its contribution to the total critical current diminishes and the deviations are more and more dominated by one channel with $D_1 = 1$, i.e., they decrease at zero temperature up to zero at $D_2 = 0$.

The curve 6 in fig. 5 describes the critical-current deviations in a junction containing ten identical channels with $D_2 = 0.1$ and one fully transparent channel ($D_1 = 1$). Contributions to the Josephson current from ten channels with $D_2 = 0.1$ and one channel with $D_1 = 1$ are both significant and jointly lead to more noticeable deviations of the critical current.

For tunneling through broadened localized states the averaging of $I_0(T, \chi)$ over bimodal Dorokhov distribution of transparency eigenvalues leads to the current through dirty constrictions [24,29]. The corresponding thermodynamic potential takes the form

$$\Omega_0(T, \chi) = \left(2\pi \hbar^2 T / e^2 R_N\right) \sum_{\varepsilon > 0} \arcsin^2 \left( \varepsilon \sin \frac{\chi}{2} \sqrt{\frac{\varepsilon^2 + |\Delta|^2}{\varepsilon^2 + |\Delta|^2}} \right)$$

and the modulated current $I(T, \chi, \Phi)$ is defined by eq. (5). In this case the critical current exceeds the Fraunhofer value, the deviation $\delta I_c(T, \Phi)$ increases with decreasing temperature and takes its maximum at $T = 0$, as is seen in fig. 6.

Concluding remarks. – Experimental results for numerous short junctions are known to show, as a rule, modulations of the standard type, if a spatial distribution of the supercurrent density is not substantially inhomogeneous [1]. Prominent exceptions include combined $0-\pi$ junctions, vicinities of $0-\pi$ transitions and special interface-to-crystal orientations of high-temperature or other superconductors with anisotropic pairings [2–10,30]. The present calculations allow an extension, in particular, to short junctions with interlayers possessing a collinear magnetic order and/or between unconventional...
superconductors. The developed approach can be also generalized to take account of the current-induced magnetic field resulting in Josephson vortices in wide junctions. These problems will be studied further and published elsewhere.

In conclusion, a microscopic theory of the magnetic-field modulation of the critical current in Josephson junctions has been developed in the present paper. As a generalization of basic microscopic results in the absence of the magnetic field, the modulated Josephson current is explicitly expressed via a secular function or, for symmetric junctions, via a magnetic-field-dependent spectrum of Andreev interface states. Temperature-dependent deviations of the modulated critical current from the Fraunhofer pattern have been found for short junctions with tunneling through localized electronic states. The deviations depend on transparency distribution over transport channels. For constant transparency the deviations vanish not only near $T_c$, but also at $T = 0$. Such behavior qualitatively differs from what is known for long superconductor-normal metal-superconductor junctions. Zero-temperature deviations are found to take place in junctions between different superconductors and in symmetric junctions containing channels with different transparencies. If Dorokhov bimodal distribution of transparencies holds, the averaged deviation increases with decreasing temperature and takes its maximum at $T = 0$. It is shown that in a number of junctions with a pronounced anharmonic current behavior, the Fraunhofer pattern is only slightly distorted.

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