Some properties of doubly-degenerate stars

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We investigate critical masses of and circular geodesics around doubly-degenerate stars (DDSs) which are composed of cold nuclear matter as well as cold Fermionic dark matter (DM). We here consider asymmetric dark Fermion with self-interaction as a DM candidate. These stars have core-envelope structures and are categorized into baryon-enveloped and DM-enveloped, according to the composition of their envelope. It is seen that the baryon-enveloped and DM-enveloped classes have their own critical masses determined mainly by the dominant component in their envelope. For a typical parameter sets, we see that a balanced mixture of two species may lead to smaller masses than if the either of the species is dominant. We also show that for a highly DM-enveloped case circular orbits in the vacuum region terminates at the innermost stable circular orbit (ISCO) in vacuum, but circular orbits of smaller radius are possible in the DM-envelope forming a gap between the ISCO and the inner orbits.

I. INTRODUCTION

The nature of cosmic dark matter (DM) still remains one of the most perplexing mysteries of the physics for more than 80 years since Zwicky suggested its presence in clusters of galaxies [1, 2]. The necessity of DM in galactic scale [3], in clusters of galaxies (e.g., [4]), and in cosmological scale [5] is now firmly established. Baryonic astronomical objects that emit too weakly to be detected are severely constrained as dark matter candidates from the Big Bang nucleosynthesis. Most promising candidates are yet-to-be-found elementary particles that do not interact with ordinary matter or at least do very weakly. [6] From the point of view of the formation of cosmic structures the dark matter must be subrelativistic and in the standard theory of DM it is regarded as collisionless (collisionless cold dark matter, CDM). There are, however, some astronomical observations that are at odds with CDM models. Dwarf galaxies have flatter density profile at their center than is expected from CDM [6, 8]. Moreover our Galaxy should have more subhaloes than are observed as satellite galaxies [9, 11]. Also ”Too-big-to-fail” problem [12] exists for the subhaloes of our Galaxy to be explained by CDM. These suggest that DM may be collisional. One of the possible modification of CDM models is to introduce the self-interaction of DM particles [13, 14]. They may naturally solve the issues of CDM above. One of the categories that allow the solution is asymmetric DM (ADM, see [17] for a review). In the early Universe the baryon to anti-baryon number ratio might be asymmetric and leads to the baryon dominant Universe as it is now. The same may hold for DM particles and the rest of the pair-annihilated may be observed as DM now. If ADM is also self-interacting it may aggregate to form stellar-sized objects. This possibility has been studied in exotic models of compact stars. One of the suggestions made is that these exotic objects serve as alternatives to the observed black hole candidates. This idea is severely constrained for accreting objects in X-ray binaries [18].

Objects with typical mass of neutron stars and made from dark matter are also considered. [19] They are termed as dark stars [20, 21] and possibility of them to be alternatives to neutron stars is discussed. On the other hand, a neutron star may capture DM particles and the DM accumulated together with nuclear matter (baryon component) form a new type of compact star. In [22] Leung et al. study the structure of these stars (dark matter admixed neutron stars, or DANS. See [23] for an alternative formation scenario of DANS). This is a baryon-dominated counter part of dark stars (see also [24] for their radial stability). We are here interested in the general cases that encompass both dark stars and DANS, which we call as doubly-degenerate stars [25].

In this paper we study characteristics of equilibria composed of baryonic matter and Fermionic dark matter in complete degeneracy. An equilibrium star is categorized either to baryon-enveloped star or to DM-enveloped one. The former has a core composed of baryon and DM which is covered with an envelope composed of purely baryonic matter. The latter has instead an envelope composed of DM. One of our interests is the critical mass beyond which a stable equilibrium star cannot exists. Although this has been studied in former studies, we need to pay a careful attention to find the critical parameter when a star is composed of multi-component fluid. Another characteristic of equilibria we study is the circular geodetic orbits of a test (baryonic) particle. A neutron star in a X-ray binary have an accretion disk around it whose radiation partly comes from the disk. The disk components of radiation depends on the accretion state of the disk itself as well as the inner edge of the disk. For a neutron star the inner edge is the smaller of either the surface of the star or the conventional innermost stable circular orbit (ISCO), which is located at the radius of $6GM/c^2$ in Schwarzschild coordinate for a mass $M$ of the star. Unlike neutron stars with radii less than ISCO, we show
for DDS with some parameter sets there appear multiple of stable orbits inside the ISCO.

Possible formation processes of these objects are not clear 20, 22 and beyond the focus of our study here.

II. FORMULATION

A. Assumptions

We here study non-rotating stellar equilibria composed of baryonic and dark matter whose mutual interaction is negligible. The baryonic matter is a zero-temperature nuclear matter. The dark matter is a self-interacting Fermion with a vector mediator field.

B. Tolman-Oppenheimer-Volkoff system for multi-component stars

Since the stars in the present study are assumed to be static and spherically symmetric, the spacetime allows the Schwarzschild coordinate \((r, \theta, \varphi)\) in which the spacetime metric is written as,

\[
d s^2 = g_{\mu\nu} dx^\mu dx^\nu = -e^{2\nu} dt^2 + e^{2\lambda} dr^2 + r^2 \sin^2 \theta d\varphi^2. \tag{1}
\]

Here we assume the geometrized unit, e.g., \(c = 1 = G\). The stress-energy tensor of the matter is written as,

\[
T^{\mu\nu} = \left(\rho + p\right) u^\mu u^\nu + pg^{\mu\nu}, \quad \rho \equiv \epsilon_B + \epsilon_X, \quad p \equiv p_B + p_X. \tag{2}
\]

The energy density \(\epsilon\) and the pressure \(p\) are simple summations of the baryonic component (with the subscript B) and of the dark matter (with the subscript X). These thermodynamic variables obey independent equations of state\(\) which is described below. By using the Einstein's equation and the extremal conditions of total mass with respect to the matter variables by fixing the baryon number and the dark Fermion number, we obtain a modified Tolman-Oppenheimer-Volkoff system of equations used in 22 (see also 27 and 28 for general cases with a finite intersection between baryonic and dark matter): The equation for the metric potential \(\nu\) is

\[
\frac{d\nu}{dr} = \frac{m + 4\pi pr^3}{r(r - 2m)}, \tag{3}
\]

where \(m(r) = \int_0^r \epsilon_4 4\pi r'^2 dr'\). The hydrostatic balance of particle species \(i = B, X\),

\[
\frac{dp_i}{dr} = -\frac{(\epsilon_i + p_i)(m + 4\pi pr^3)}{r(r - 2m)}. \tag{4}
\]

C. Equation of state

1. Dark matter

As for the EOS of dark matter, we follow the treatment in 29 (see also 30). The dark Fermions with the rest mass \(m_X\) are completely degenerated. They have a self-interaction mediated by a boson with the rest mass \(m_\phi\), whose coupling constant is \(\alpha_X\). \(\epsilon_X\) and \(p_X\) are implicitly related by

\[
\epsilon_X = \frac{m_X^4}{h^3} \left[ \xi(x) + \frac{2}{9\pi^3} \frac{\alpha_X m_X^2}{h m_\phi^2} x^6 \right];
\]

\[
\xi(x) = \frac{1}{8\pi^2} \left[ x\sqrt{1 + x^2}(2x^2 + 1) - \ln(x + \sqrt{1 + x^2}) \right]
\]

and

\[
p_X = \frac{m_X^4}{h^3} \left[ \chi(x) \right. + \left. \frac{2}{9\pi^3} \frac{\alpha_X m_X^2}{h m_\phi^2} x^6 \right];
\]

\[
\chi(x) = \frac{1}{8\pi^2} \left[ x\sqrt{1 + x^2} \left( \frac{2}{3} x^2 - 1 \right) + \ln(x + \sqrt{1 + x^2}) \right]
\]

In this paper we focus our interest on the repulsive self-interaction of dark Fermions, thus \(\alpha_X > 0\). 31 Here \(x\) is the normalized Fermi momentum \(x = p_X/m_X\). For our numerical computation we solve for \(v = x^2\). The value of \(v = v_0\) at the origin parametrize the Fermi energy of DM. It should be remarked that a solution of the so-called 'core-cusp' problem of galactic center may require 29, 52,

\[
0.1(\text{gcm}^{-3}) \leq 1.1 \times \left( \frac{m_X}{1\text{GeV}} \right) \left( \frac{m_\phi}{10\text{MeV}} \right)^{-4} \left( \frac{\alpha_X}{10^{-3}} \right)^2 \leq 10(\text{gcm}^{-3}). \tag{7}
\]

We use \(m_X = 1\text{GeV}, m_\phi = 10\text{MeV}, \alpha_X = 10^{-3}\) as a canonical set of parameters unless otherwise stated.

2. Baryonic matter

We adopt zero-temperature EOS for neutron star matter by 33, 34 and utilize the fortran subroutine nseos1.f provided by the authors at [http://www.ioffe.ru/astro/NSG/NSEOS/](http://www.ioffe.ru/astro/NSG/NSEOS/). They made analytic fitting formulae for some representative nuclear EOS, e.g., FPS 35, SLy4 36, APR 37. The stiffest of the three is APR, which produces the maximum mass nonrotating neutron star with \(M \sim 2.2M_\odot\), while the softest is FPS by which a neutron star has a maximum mass of \(M \sim 1.8M_\odot\) (see Fig[I]).

III. RESULTS

A. Two kinds of equilibria: baryon-enveloped and dark matter-enveloped

For the constant \(v_0\) sequences, the inclusion of dark Fermion slightly modify the mass and the radius of the star from the purely baryonic counterpart when the density is high enough. These are the branches in the left of the filled circles on the solid curves in Fig[I]. For these models, the radius of dark matter distribution \(R_{DM}\) is
smaller than the radius of the baryon distribution \( R_B \), therefore the stellar radius \( R = R_B \). For \( 0 \leq r \leq R_{DM} \) the baryonic and dark matter coexist. We call this configuration as baryon-dominant. On the right-hand of the circles, the stellar configuration is mainly determined by the degenerate pressure of the dark Fermion. The mass-radius relation largely deviates from that of neutron stars. We have \( R_{DM} > R_B \) and the surface of the star \( R = R_{DM} \). We call it DM-dominant.

B. Critical models of radial stability

Now we consider the critical mass of the DDS beyond which the star becomes unstable. For one parameter sequence of equilibrium as cold neutron stars, the critical mass is the maximum mass as a function of baryon density (or energy density). With baryonic EOS and the dark Fermion EOS being fixed, the DDS models generally have two parameters that correspond to the energy density (or energy density). With baryonic EOS and the dark Fermion EOS being fixed, the DDS models generally have two parameters that correspond to the energy density. These multi-parameter systems, a (sufficient) stability criterion for family of equilibria is developed by [BB] (see also [29]). We follow their treatment to find the critical mass of the DDS.

We consider a sequence on which the baryonic mass \( m_B \) is kept constant. We may choose \( \lambda \equiv v_0 \) as a parameter to specify a model on the sequence. And we assume an extremum of gravitational mass \( M \) exists on the sequence at \( \lambda = \lambda_0 \). The theorem I of [28] tells us that if \( \frac{dM}{d\lambda} \frac{d\lambda}{dm_B} > 0 \) on the segment of the sequence around \( \lambda = \lambda_0 \), it is unstable in the sense that an equilibrium is not a local minimum of \( M \). Here \( \mu_X \) is the chemical potential of dark Fermion including gravitational contribution to energy,

\[
\mu_X = e^\nu \frac{\partial \epsilon_X}{\partial n_X},
\]

which is constant throughout an equilibrium star.

Now we consider the critical mass of the DDS beyond \( M_B = 1.02M_\odot \). It is plotted as a function of \( v_0 \) parameter (solid). Limits of \( v_0 \to 0 \) are normal neutron star. Also plotted (dashed) are dark Fermion’s chemical potential \( \mu_X \) (in arbitrary unit) for equilibrium star as a function of \( v_0 \). The circle marks the maximum of \( M \) and the triangle marks that of \( \mu_X \). Baryonic EOS is APR and dark Fermion parameters are \((m_X, m_\varphi, \alpha_X) = (1\text{GeV}, 10\text{MeV}, 10^{-3})\). Baryonic mass of a configuration is defined by the proper volume integration of number density of the baryonic particle \( n_B \) as,

\[
M_B = m_B \int_0^R 4\pi r^2 e^\lambda n_B \, dr,
\]

where \( m_B \) is the mass of baryon particle (nucleon). In the same way the dark matter mass is defined by the number density of dark Fermion \( n_X \)

\[
M_{DM} = m_X \int_0^R 4\pi r^2 e^\lambda n_X \, dr,
\]

where \( m_X \) is the mass of the dark Fermion. Gravitational mass \( M \) is defined as

\[
M = \int_0^R 4\pi r^2 \, e^\lambda \, dr,
\]

where energy density is the sum of baryonic and dark Fermion contribution.

In Fig. 2, typical behavior of gravitational mass \( M \) and chemical potential \( \mu_X \) are shown on a sequence of \( M_B = \text{constant} \). At the point \( dM/d\lambda = 0 \) (marked by a circle), \( \mu_X \) has a negative slope. Therefore models with larger \( \lambda = v_0 \) satisfy the condition of the instability \( \frac{d\mu_X}{d\lambda} \frac{d\lambda}{dm_B} > 0 \). For this baryonic mass we conclude the
critical model corresponds to this point. We remark that the models with smaller $v_0$ than that of the triangle point also satisfies $\frac{dM}{d\lambda} > 0$. It should be, however, remembered that the stability criterion applies to the part of the sequence around the extremum of $M$. Moreover the limit of $v_0 \rightarrow 0$ corresponds to a pure neutron star model with $M \sim 1.2M_\odot$. This limit is completely stable. We conclude this part of the sequence is stable.

In Fig.3 the critical gravitational mass (solid line) and dark matter fraction (dashed line) defined by $M_{DM}/(M_{DM} + M_B)$ are plotted as functions of baryonic mass. This is the case with APR EOS for baryons and dark Fermion parameters are $(m_\chi, m_\phi, \alpha_\chi) = (1\text{GeV}, 10\text{MeV}, 10^{-3})$. It is remarkable that the critical mass is not a monotonic function. When $M_B$ is large, the DM fraction becomes small and the model is close to the pure neutron star. This is the rightmost end of the plot. On the other hand for smaller and smaller $M_B$ the star becomes more and more DM-enveloped and the critical mass asymptotes to the pure DM star (dark star limit). Between these limit there is a minimum of critical mass. For this particular model parameter, nearly equal amount of contribution from baryon and DM does not support a heavy star against its self-gravity.

The similar characteristics holds for the softer baryonic EOS (Fig.4) although the mass supported by the pure baryonic matter is smaller.

When the DM parameters are modified, the characteristics may change. In Fig.5 the mass and the DM fraction is plotted for APR EOS and $(m_\chi, m_\phi, \alpha_\chi) = (2\text{GeV}, 10\text{MeV}, 10^{-3})$. This case is compared with Fig.3 except that the dark Fermion mass is larger. The critical mass is now a monotonic function of $M_B$. This is expected from considering the Chandrasekhar limit of a star made of Fermion of mass $m_\phi$ scales as $M_{\text{Ch}} \propto m_\phi^{-2}$.

In Fig.6 we have the plots for the same parameters except the mediator mass $m_\phi = 50\text{MeV}$ of the dark self-interaction is larger than that of Fig.5. In this case we also have small critical mass for DM-enveloped limit. We expect it because the heavier mediator have smaller Yukawa radius of interaction and it contribute less to the stiffness of the dark Fermion matter.

Finally Fig.7 is the same plot as Fig.3 except that $\alpha_\chi$ is larger. This corresponds to the larger repulsive dark-interaction which results in the larger mass of the DM-
be regarded as a conserved total energy of the particle, while the first and the second terms on the right hand side are regarded as the kinetic and the potential energy \( V_{\text{eff}} \). Notice that \( \epsilon \) includes the rest mass of test particle, thus it asymptotes to unity as \( r \to \infty \).

In Fig.\ref{fig:8} we plot \( V_{\text{eff}} \) for a highly DM-enveloped model with different value of \( \ell_0 \). Notice that a particle's radial motion is represented by \( E = \text{constant line} \) and \( E > V_{\text{eff}} \) corresponds to the allowed region of motion. A bound orbit must have \( E < 0 \). Minima of \( V_{\text{eff}} \) are radius of circular orbits. Shaded-regions are the core (dark-colored). Mixture of baryon and DM) and the envelope (light-colored. Pure DM). The radius of the star \( R/M = 4.31 \) is smaller than VISCO of \( R/M = 6 \). In vacuum region it is well-known that one stable circular orbit exists for \( \ell_0 > \sqrt{2}M \). There is another stable circular orbit in the DM-envelope as far as \( \ell_0 \) is not so large \( (\ell_0 \sim 4M) \). Moreover we have a circular orbit in the envelope even with \( \ell_0 < \sqrt{2}M \).

For this particular model there are two separated region where stable circular orbit is allowed. One is in the vacuum region which terminates at \( r = 6M \). The other is in the DM-envelope. These regions are detached. Thus a thin accretion disk around the star may have a gap at around the surface of the DM-envelope. The inner part of the disk seems to extends down to the surface of the core.

To see where stable circular orbits are allowed we look at a criterion which results from the curvature of \( V_{\text{eff}} \). If \( d^2 V_{\text{eff}}/dr^2 \) at an extremum of \( V_{\text{eff}} \) is positive, the orbit
is stable. The second derivative is proportional to the radial epicyclic frequency squared. In our case the first and second derivatives of $V_{\text{eff}}$ are,

\[ \frac{dV_{\text{eff}}}{dr} = e^{2\nu} \left[ \nu_r \left( 1 + \frac{\ell_0^2}{r^2} \right) - \frac{\ell_0^2}{r^3} \right] \]

\[ \frac{d^2V_{\text{eff}}}{dr^2} = 2e^{2\nu} \nu_r \left[ \nu_{rr} \left( 1 + \frac{\ell_0^2}{r^2} \right) - \frac{2\ell_0^2}{r^3} \right] + e^{2\nu} \nu_{rrr} \left( 1 + \frac{\ell_0^2}{r^2} \right) - \frac{2\ell_0^2}{r^3} + 3\frac{\ell_0^2}{r^3} \right], \tag{16} \]

where $\nu_r$ and $\nu_{rr}$ represent the first and second derivatives of $\nu$ with respect to $r$. The extrema of the first derivative are radii of circular orbits which satisfy,

\[ \ell_0^2 = \frac{r^3 \nu_r}{1 - r \nu_r}. \tag{17} \]

By nullifying the second derivative and using Eq.(17) we obtain an equation for critical radius at which a circular orbit is an inflexion point of $V$,

\[ F \equiv \nu_{rr} - 2(\nu_r)^2 + 3\frac{1}{r} \nu_r = 0 \tag{18} \]

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{Fig9}
\caption{Criterion $F$ (Eq.(18)) and specific energy $\epsilon$ (Eq.(12)) as functions of radial coordinate. The equilibrium model is the same as Fig.8. [Upper panel] Criterion $F$ as a function of radial coordinate (solid). The shaded area corresponds to the allowed region of a circular orbit. Thick dotted vertical line marks the inflexion point where $F$ vanishes. The thin dotted vertical line marks the surface of the star which is the boundary between DM envelope and the vacuum. [Lower panel] Specific energy $\epsilon$ of the circular orbits. Thick portion of the curve corresponds to stable circular orbits. The horizontal dash-dotted line is the energy at the vacuum last stable orbit ($r = 6M$).}
\end{figure}

The upper panel in Fig.9 shows $F$ of Eq.(18) for a DM-enveloped configuration in Fig.8. Positive value of $F$ (shaded area) means a circular orbit is stable. Notice that the curve crosses zero at $r = 6M$, which is the vacuum innermost stable circular orbit (VISCO) around the mass $M$. The location of the surface of the DM envelope is marked by the thin vertical dotted line, which is inside VISCO. The thick vertical dotted line marks the position of the critical point inside the DM envelope where the radial stability of circular orbit changes. On the left of the thick dotted line, another region of stable circular orbit exists. As is seen in the lower panel of the figure, the specific energy the orbit in the range of $2.6 < r/M < 6$ exceeds that of VISCO. That means a thin circular accretion disk may be truncated there, since the energy of the disk matter must decrease as it accretes inward.

In Fig.10 effective potential is plotted for less DM-enveloped cases. We have $v_0 = 0.1$ with $\rho_c$ being fixed. No VISCO exists in this model. We have only one circular orbit for $f_0/M \leq 0.63$. A thin accretion disk is expected to extend down to the core layer may form. Corresponding criterion $F$ and specific energy $\epsilon$ is found in Fig.11. We see that in the star $\epsilon$ is a monotonic function of $r$ and therefore an accretion disk may extends down to the core without having a gap.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{Fig10}
\caption{Same as Fig.9 except $v_0 = 0.1$. The gravitational mass $M = 2.12M_\odot$, baryonic mass $m_b = 1.37 \times 10^{-2}M_\odot$, DM rest mass $M_{DM} = 1.80M_\odot$. The radius of the core of baryon and DM mixture is $R_{\text{in}}/M = 2.68$ and the radius of the DM envelope $R_{DM}/M = 9.40$.}
\end{figure}

In Fig.10 we also change $\rho_c$ as $7 \times 10^{14}$gcm$^{-3}$. The DM-envelope is further reduced. It is noted that bary-
onic mass exceeds that of DM for this case. A single stable circular orbit exists for each \( \ell_0 \) whose radius extends down to the surface of the core.

\[
M_{\text{stable circular orbit}} = \frac{G M}{r}.
\]

![Figure 12](image)

FIG. 12. Same as Fig. 10 except \( \rho_v = 7 \times 10^{14} \text{ g cm}^{-3} \). The gravitational mass \( M = 0.84M_\odot \), baryonic mass \( M_b = 0.682M_\odot \), DM rest mass \( M_{DM} = 0.214M_\odot \). The radius of the core of baryon and DM mixture is \( R_{DM}/M = 8.15 \) and the radius of the DM envelope \( R_{DM}/M = 9.11 \).

D. Summary

We explore characteristics of compact stars which is composed of completely degenerate two species of matter, i.e., baryons and dark Fermions. Interaction of two species are neglected. We solve TOV system of equations for multi-component matter. The equilibrium models are parametrized by baryon central density \( \rho_c \) and Fermi momentum squared \( v_0 \) of dark Fermion at the center. Equilibrium is classified as baryon-enveloped, which has a core composed of mixture of baryons and dark Fermions and has an envelope composed solely of baryons. When the envelope is entirely composed of DM, it is called DM-enveloped. The former may be regarded as DANS and the latter may be regarded as a generalized dark star. Since an equilibrium state is characterized by two parameters, we utilize Sorkin’s general criterion to investigate critical mass of an equilibrium sequence beyond which the star becomes unstable. We see that the baryon-enveloped and DM-enveloped classes have their own critical masses determined mainly by the dominant component in their envelope. For a typical parameter sets, we see that a balanced mixture of two species may lead to smaller masses than if either of the species is dominant. This, however, depends on the mass and the strength of self-interaction of dark Fermion.

Another property of equilibrium investigated is the existence of stable circular orbits around the star. Especially interesting is that the envelope may allow geodetic motion of baryonic gases for DM-enveloped stars. We show that for a highly DM-enveloped case circular orbits in the vacuum region terminates at VISCO, but another region of stable circular orbits are possible in the DM envelope. For more mildly DM-enveloped stars, we have a single circular orbit for each specific angular momentum down to the baryon-mixed core, at which geodetic motion is not possible. In this case a thin accretion disk that forms around the star does not have a gap and extends down to the core. The existence or non-existence of the gap and the redshift resulting from the gravity in the DM-envelope may affect the spectrum of disk emission and may be an observable signature of these stars. Modeling of these emissions may be an interesting astrophysical application of our model.

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