Determination of a focusing system parameters in generation of THz radiation by bunch train

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Abstract. It is present electron optics optimization for terahertz radiation generation by a sequence of short electron bunches passing through the dielectric loaded waveguide. In this method the superposition of individual bunch wakefields is used to selectively excite high frequency TM0n modes in a slow-wave structure. The main problem of this scheme is the transverse beam instability that limits the length of THz radiation wave packet. The main goal of this paper is a new parameter optimization algorithm for the FODO focusing system. The numerical modelling of the beam dynamics was performed in the original code “Dielectric waveguide” which utilizes analytic expression of the Green’s function for the dielectric loaded waveguides with cylindrical and rectangular geometries.

1. Introduction

THz radiation has many applications in physics, chemistry and biology. There are a lot of methods to generate THz radiation using relativistic electron bunches [1]. The wakefield created by relativistic electrons in dielectric-loaded waveguides (DLW) has been studied for power THz radiation. When single bunch passes through DLW it generates wakefield radiation (Cherenkov radiation in a slow wave structure) [2]. This radiation consists of many TM0n modes and can be calculated analytically. A bunch train can be used to selectively excite specific modes [3, 4]. As it was pointed out in paper [3] this method has additional limitations due to beam break-up (BBU) instability. This paper contains calculations of focusing system parameters to increase the length of the wave packet.

The problem in hand has the following geometry. A train of electron bunches passes through a cylindrical dielectric tube metallized on the outside (figure 1). The radiation is generated by the sequence of relativistic electron bunches that pass along the waveguide axis. While single bunch generates a set of different TM0n-modes in the DLW, the bunch sequence allows selection of only one of the TM0n-modes via superposition. Tuning the bunch train spacing one can select different TM0n modes.

The main problem with this scheme is BBU-instability [6] due to a strong radial field. This field includes Hybrid Electro-Magnetic (HEM) modes that are excited when there is a radial deviation (offset) of the bunch from the waveguide axis. The instability of the bunch train is stronger than that of
the single bunch, because bunches in the middle part of the sequence experience action not only from their own radial field but also from the fields of bunches moving ahead of them. This fact limits the length of the wave packet. One way to increase the efficiency of radiation generation is to control the transverse instability through a quadrupole focusing system. This system contains focusing and defocusing quadrupole cells and is known as the FODO system (Figure 1). This article represents a simple algorithm for FODO lattice optimization based on the numerical simulation of the beam dynamics.

![FODO System](image)

**Figure 1.** The sequence of bunches is passing through the dielectric waveguide with the FODO system.

2. “Dielectric waveguide” code
The “Dielectric waveguide” code is developed at the physics department of Saint-Petersburg Electrotechnical University “LETI”. This code was developed for the purpose of calculating the wakefield created by high-current electron bunches in a dielectric waveguide [7]. This wakefield can be used for particle acceleration or as a source of THz radiation. The wakefield calculation is based on the explicit analytical formulas for the Green’s function for DLWs with cylindrical and rectangular geometries. This method is also used in new module developed for the wakefield calculation from the sequence of bunches [6]. This module allows the user to control all parameters of the bunch train and prepare parameters for beam dynamics simulation.

The beam dynamics module works according to the macroparticle algorithm described in [8, 9]. It should be noted that the interaction between macroparticles includes not only Cherenkov radiation, but also the Coulomb field. The motion of the macroparticle is calculated using the Euler and Runge-Kutta methods. In addition, an external FODO quadrupole system can be included in the calculation of the beam dynamics. The program calculates particle motion using the equation of force and a predetermined time step.

3. Modeling of the FODO focusing system
The waveguide parameters for the THz radiation source were taken from our previous studies [7]. These parameters are presented in Table 1. The Green’s function includes the TM and the HEM modes of the waveguide. We made a calculation for the simple sequence, which contains three bunches (Figure 2). We observed monochromatic radiation field Ez with the frequency of the TM02 mode (439 GHz) behind the bunch train, as well as the transverse field Fr. Frequencies of the HEM modes differ from the TM ones. So, the distances between bunches are selected to maximize generation of the TM mode. However, the distribution of the radial field depends on the spectrum of the HEM modes. This field can cause strong deflection within the bunch train. The maximal value of field Fr is at the tail of the third bunch (Figure 2, green line).

Design of the focusing system is based on maximization of propagation length. It is known that a quadrupole focusing system cannot reduce the phase space volume of the bunch. Incorrect selection of
the FODO system can destroy the bunch train. The FODO focusing system is used to control the instability of a single bunch in the wakefield acceleration scheme [5]. The algorithm for calculating the FODO system for a series of Gaussian bunches (Table 1) is considered.

It is selected a simple focusing system, presented in figure 3, which consists of only two focusing cells and one defocusing cell. The system contains eight parameters for optimization (Figure 3): focusing gradients (G1, G2, G3), distances between cells (o1, o2), width of cells (z1, z2, z3).

The algorithm is based on variation of the focusing system parameters. The range of parameters is determined by the real physical constrains of the FODO system. At the first step, the algorithm makes calculations for a small number of macroparticles. This approach can be justified by small beam sizes: the radial size is less than the offset and the wavelength is comparable with the longitudinal bunch length. The optimization was implemented using the gradient method. The results achieved after the first stage give us initial parameters for the next step where the number of macroparticles is increased.

Table 1. Parameters of the waveguide and the bunch sequence.

| Parameters of the waveguide | Parameter value          | Parameters of the bunch | Parameter value          |
|-----------------------------|--------------------------|--------------------------|--------------------------|
| Inner radius “a”            | 600 um                   | Number of bunches        | 3                        |
| Outer radius “b”            | 850 um                   | Frequency of THz radiation | 439 GHz                  |
| Relative permittivity       | 3.8                      | Distances between bunches | 1320 um                  |
| TM01 mode frequency         | 149.5 GHz                | Transverse beam size     | 100 um                   |
| TM02 mode frequency         | 439.5 GHz                | Offset relative to the x | 100 um                   |
| TM03 mode frequency         | 768.5 GHz                | Longitudinal length      | 100 um                   |
| Length                      | 10 cm                    | Energy                   | ~75 MeV                  |
| Loss tangent                | 0.001                    | Charge                   | 0.1 nC                   |
| Wall conductivity           | 5.7E+07 S/m              | Number of macroparticles in bunch | 200                      |

Figure 2. Longitudinal Ez and transverse Fr fields created by three bunches passing from right to left.
4. Calculation results
First, we obtained the optimal parameters of the FODO system for the parameters (Table 2) of the dielectric waveguide listed in Table 1. These results show the dependence of the transverse bunch dimensions as a function of the longitudinal coordinate $z$. The focusing system allows the propagation distance to be increased to 18 cm. The results of the calculations iterated six times are shown in Figure 4. The longitudinal plane shows the up and down borders of the bunch train. These borders are defined according to Figure 5. The bunch path in the case that the FODO system is presented (or absent) as well (Figure 5, black trace, the propagation distance is 8 cm). These upper parameters are suitable for a certain offset range ($0 < \text{offset} < 100\mu m$). Low offset value increases the propagation distance in the absence of FODO, but the presence of last one does not change the value of 18 cm. The offset increasing reduces the propagation length. For example: if the offset is 250 $\mu m$, the propagation length without FODO is 5 cm, with FODO it is 8 cm.

![Figure 4. Calculation results for the bunch sequence passing from left to right.](image)

The numerical simulation of beam dynamics at the end of propagation (17.78 cm) is presented in Figure 5. The calculation was stopped when the first macroparticle in the sequence touched the dielectric channel (see transverse view in figure 5).
The HEM-mode amplitude grows as a function of the radial coordinate of the bunch. However, in our case the radial displacement of bunches does not change the spectrum of the Ez field (THz radiation). It remains monochromatic until electrons touch the dielectric tube. The longitudinal dynamics can cause the intervals between the bunches to change, but this is only possible for the low energy electrons, with energies below ~5MeV).

Table 2. Parameters of the optimal focusing system.

| Parameter | Parameter value |
|-----------|-----------------|
| G1, G2, G3 | 1.315, 0.5707, 0.8978 T/cm |
| o1, o2    | 1, 1.361 cm     |
| z1, z2, z3| 3.959, 3.21, 4.3 cm |

5. Conclusion

An optimization algorithm was used to design a FODO lattice for bunch train propagation through the DLW. The bunch train spacing was tuned to generate monochromatic radiation at 439 GHz (TM02 mode). To selectively excite TM01, TM03 or any other mode, the bunch train spacing will have to be adjusted accordingly. However, in this case the FODO lattice has to be reconfigured for new bunch train spacing. The optimization algorithm can be used to redesign the system for new bunch train spacing. Our algorithm works for a fixed offset but is still acceptable for low ones. We plan to improve our algorithm further to make it universal in terms of beam dynamics calculation for different types of waveguides, and its efficiency in terms of the computational time.

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