How cosmic-ray electron propagation affects radio – far-infrared correlations in M31 and M33

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ABSTRACT
We investigate the effect of propagation of cosmic-ray electrons (CRE) on the nonthermal (synchrotron) – far-infrared correlations in M31 and M33. The thermal (TH) and nonthermal (NTH) emission components of the radio continuum emission at 1.4 GHz and one higher frequency are compared with dust emission from M31 and M33 using Spitzer data. In both galaxies the TH emission is linearly correlated with the emission from warm dust (24 μm, 70 μm), but the power laws of the NTH – FIR correlations have exponents  b < 1 that increase with increasing frequency. Furthermore, the values of  b for M33 are significantly smaller (b ≈ 0.4) than those for M31 (b ≈ 0.6). We interpret the differences in  b as differences in the diffusion length of the CRE. We estimate the diffusion length in two ways: (1) by smoothing the NTH emission at the higher frequency until the correlation with NTH emission at 1.4 GHz has  b = 1, and (2) by smoothing the TH emission until the correlation with the NTH emission at the same frequency has  b = 1, assuming that the TH emission represents the source distribution of the CRE. Our smoothing experiments show that M31 only has a thin NTH disk with a scale height of h = 0.3 – 0.4 kpc at 1.4 GHz, whereas M33 has a similar thin disk as well as a thick disk with scale height hthick ≈ 2 kpc. In the thin disks, the (deprojected) diffusion length at 1.4 GHz is ≈ 1.5 kpc, yielding a diffusion coefficient of ≈ 2 10^{28} cm^2/s. The structure, strength and regularity of the magnetic field in a galaxy as well as the existence of a thick disk determine the diffusion of the CRE, and hence, the power-law exponent of the NTH – FIR correlations.

Key words: galaxies: individual: M 31 – galaxies: individual: M 33 – galaxies: spiral – galaxies: cosmic rays – galaxies: magnetic fields – infrared: galaxies – radio continuum: galaxies

1 INTRODUCTION

The radio continuum and far-infrared (FIR) luminosities of star-forming galaxies are tightly correlated over five orders of magnitude (e.g. de Jong et al. (1985); Helou, Soifer & Rowan-Robinson (1985); Condon (1992); Yun, Reddy & Condon (2001)). The correlation is nearly linear and probably arises because both emissions depend on the recent star-formation rate in a galaxy. Massive stars heat the dust and ionize the gas causing thermal dust emission and thermal (free-free) radio continuum emission, respectively. These stars are progenitors of supernova remnants, which are the sources of cosmic ray electrons (CRE) that radiate nonthermal (synchrotron) emission when spiraling around magnetic field lines. Attempts to explain the correlation were made by Volk (1989); Helou & Bicay (1993); Niklas & Beck (1997) and Hoernes, Berkhuijsen & Xu (1998). Recently, Bell (2003) and Lacki & Thompson (2010) showed that several factors conspire in making the global correlation linear over five decades.

The radio – FIR correlation also holds within star-forming galaxies down to scales of below hundred parsec (e.g. Beck & Golla (1988); Bicay, Helou & Condon (1989); Fitt et al. (1992); Xu et al. (1992); Lu et al. (1996); Hoernes, Berkhuijsen & Xu (1998); Hipplelein et al. (2003); Hughes et al. (2006); Paladino et al. (2006); Dumas et al. (2011); Leverenz & Filipovic (2012); Tabatabaei et al. (2013b)). However, many authors (starting with Xu et al. (1992)) found that the radio – FIR correlation within galaxies only becomes linear on star-forming regions but is non-linear, usually with power-law exponent < 1, in other parts of a galaxy. Separation of thermal/nonthermal radio emission and of warm/cold dust emission revealed that the thermal radio – warm dust correlation is linear, whereas the nonthermal radio-cool dust correlation has an exponent ≠ 1 (Xu et al. 1992; Hoernes, Berkhuijsen & Xu 1998; Hipplelein et al. 2003; Basu, Roy & Mitra 2012). This explains why the radio – FIR correlation of the total emission steepens on star-forming

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1 Throughout this paper, the radio and FIR intensities are on the Y- and X-axis, respectively.
regions where the thermal fraction of the radio emission is high (Hughes et al. 2006, Paladino et al. 2006, Dumas et al. 2011). The smaller exponent of the nonthermal – FIR correlation is probably due to diffusion of the CRE away from their places of origin. By smoothing images of warm dust emission until they resembled those of radio emission at 1.4 GHz propagation lengths of up to several kpc were found (Bicy & Helou 1999, Marsh & Helou 1998, Murphy et al. 2006, 2008, 2012).

On each of our nearest spiral galaxies, M 31 and M 33, one radio – FIR study is available. The radio – FIR correlation within M 31 was extensively studied by Hoernes, Berkhuijsen & Xu (1998), who were the first to separate thermal/nonthermal emission at 1.4 GHz and warm/cold dust emission using H IRES data. By means of classical, pixel-to-pixel correlations and rigorous statistics they found significant relationships between thermal radio and warm dust emission and between nonthermal radio and cool dust emission, with power-law exponents of $1.2 \pm 0.1$ and $0.8 \pm 0.1$, respectively.

Hippelien et al. (2004) studied the dust distribution in M 33 observed by ISOPHOT. They also plotted the total 4.8 GHz luminosity of star-forming regions against that of the total FIR and obtained a power law with exponent 0.9. The exponent is < 1 because 30–40% of the 4.8 GHz emission from these regions is nonthermal (Tabatabaei et al. 2007b). Below we compare radio – FIR correlations in M 31 and M 33 using new thermal and nonthermal radio maps and recent FIR data.

Tabatabaei et al. (2007b) obtained the distribution of the thermal emission at 21.0 cm (1425 MHz) and 3.6 cm (8350 MHz) from M 33 from the extinction-corrected Hz map of Hoopes & Walterbos (2000). The extinction correction was based on the optical depth map derived from Spitzer MIPS data at 70 μm and 160 μm presented by Tabatabaei et al. (2007a). By subtracting the corrected thermal emission from the total radio emission observed by Tabatabaei et al. (2007a), they obtained the distributions of the nonthermal emission across M 33 at 21 cm and 3.6 cm. This new method of separating thermal/nonthermal radio emission from a galaxy is preferable above the standard method, which requires the unrealistic assumption of a constant nonthermal spectral index between the radio emissions at two wavelengths.

Recently, Tabatabaei & Berkhuijsen (2010) derived an optical depth map of M 31 from the 70 μm and 160 μm Spitzer MIPS maps of Gordon et al. (2006) and used this to correct the Hz map of Devereux et al. (1994) for extinction. Taking the gradient in electron temperature into account, we converted this map to distributions of thermal emission at 20.5 cm (1465 MHz) and 6.3 cm (4850 MHz). Subtraction of the thermal emission from the total emission at the corresponding wavelength (Beck et al. 1998, Berkhuijsen et al. 2003) then yielded the distributions of the nonthermal emission at 20.5 cm and 6.3 cm. The details of this procedure are described in Tabatabaei et al. (2013a) where also the thermal and nonthermal maps at 1.4 GHz of both galaxies are shown.

In this paper we correlate the new thermal and nonthermal distributions of M 31 and M 33 with the distributions of the dust emission. We discuss the differences in the correlation results for the two galaxies focussing on the diffusion of CRE. The correlations and statistics are presented in Sect. 2. We derive diffusion lengths of CRE in the plane of the sky in Sect. 3 and discuss these results in Sect. 4. In Sect. 5 we describe the propagation of CRE within and perpendicular to the disk of each galaxy, and we discuss the role of star formation and magnetic fields in Sect. 6. We compare with earlier work on CRE propagation in Sect. 7 and summarize our conclusions in Sect. 8.

Throughout the paper, we assume a distance $D = 780$ kpc (Stuken & Garnavich 1998) and inclination $i = 75^\circ$ for M 31, and $D = 840$ kpc (Freedman, Wilson & Madore 1991) and $i = 56^\circ$ for M 33.
the correlations between nonthermal emission at 6.3 cm (M 31) and dust emission. These have exponents much closer to 1, but the difference between M 31 and M 33 remains. Exponents closer to 1 imply less diffusion of CRE, which is expected at smaller wavelengths. Below we will employ the observed differences in the correlations to estimate the diffusion length of the CRE in the two galaxies.

Table 1. M 31: Classical correlations between FIR and radio emission for 6.8 < R < 12.5 kpc. OLS bisector fits with N the number of independent data points, r_c the correlation coefficient and t student-t test.

| X [MJy/sr] | Y [μJy/kpc²] | log(Y) = a + b*log(X) | a ± b | N | Corr. Coeff. r_c | t |
|------------|-------------|------------------------|-------|---|------------------|---|
| I(24) TH(20.5) | 4.18±0.01 1.08±0.02 | 1036 | 0.77±0.02 | 39 |
| I(70) TH(20.5) | 3.20±0.01 0.99±0.02 | 1023 | 0.78±0.02 | 40 |
| I(160) TH(20.5) | 1.91±0.04 1.42±0.03 | 1047 | 0.77±0.02 | 38 |
| I(24) NTH(20.5) | 4.62±0.01 0.61±0.02 | 1093 | 0.67±0.02 | 29 |
| I(70) NTH(20.5) | 4.07±0.01 0.56±0.01 | 1079 | 0.67±0.02 | 30 |
| I(160) NTH(20.5) | 3.34±0.03 0.78±0.02 | 1105 | 0.67±0.02 | 30 |
| I(24) NTH(6.3) | 4.46±0.03 0.97±0.09 | 65 | 0.54±0.11 | 5 |
| I(70) NTH(6.3) | 3.56±0.06 0.87±0.08 | 65 | 0.56±0.11 | 5 |
| I(160) NTH(6.3) | 2.48±0.16 1.21±0.11 | 65 | 0.58±0.10 | 6 |
| TH(20.5) NTH(6.3) | 2.11±0.04 0.61±0.02 | 1044 | 0.51±0.03 | 19 |
| TH(6.3) NTH(6.3) | 0.93±0.27 0.84±0.08 | 64 | 0.41±0.12 | 4 |

Note: Correlations between TH(6.3) and dust emission agree within errors with those between TH(20.5) and dust emission.

Table 2. M 33: Classical correlations between FIR and radio emission for R < 5 kpc. OLS bisector fits with N the number of independent data points, r_c the correlation coefficient and t student-t test.

| X [MJy/sr] | Y [μJy/kpc²] | log(Y) = a + b*log(X) | a ± b | N | Corr. Coeff. r_c | t |
|------------|-------------|------------------------|-------|---|------------------|---|
| I(24) TH(21) | 4.41±0.02 1.06±0.04 | 158 | 0.89±0.04 | 25 |
| I(70) TH(21) | 3.03±0.03 1.20±0.04 | 160 | 0.91±0.03 | 27 |
| I(160) TH(21) | 2.21±0.07 1.40±0.05 | 160 | 0.87±0.04 | 23 |
| I(24) NTH(21) | 4.62±0.01 0.35±0.02 | 159 | 0.75±0.05 | 14 |
| I(70) NTH(21) | 4.17±0.01 0.39±0.02 | 161 | 0.80±0.05 | 17 |
| I(160) NTH(21) | 3.90±0.03 0.46±0.02 | 161 | 0.80±0.05 | 17 |
| I(24) NTH(3.6) | 4.11±0.02 0.59±0.04 | 134 | 0.56±0.07 | 8 |
| I(70) NTH(3.6) | 3.34±0.04 0.67±0.05 | 135 | 0.57±0.07 | 8 |
| I(160) NTH(3.6) | 2.87±0.08 0.79±0.06 | 135 | 0.51±0.08 | 7 |
| TH(21) NTH(21) | 2.84±0.05 0.34±0.02 | 160 | 0.69±0.06 | 12 |
| TH(3.6) NTH(3.6) | 1.33±0.15 0.61±0.05 | 134 | 0.53±0.07 | 7 |

Note: Correlations between TH(3.6) and dust emission agree within errors with those between TH(21) and dust emission.

its emission is much smoother than that of the ionized gas, leading to an exponent b > 1.

Figure 1b shows that for both galaxies the correlation between nonthermal emission at 21 cm and 160 μm emission has a power-law exponent b < 1, but the exponents for the two galaxies are strikingly different. As the TH ~ 160 μm correlations have the same value of b, the difference is in the character of the nonthermal emission that is much smoother in M 33 (b = 0.46 ± 0.02) than in M 31 (b = 0.78 ± 0.02) caused by the relatively high intensities towards lower brightness. This could mean that the propagation length of the CRE in M 33 is larger than in M 31 or that M 33 has a halo.

The smaller values of exponent b for M 33 than for M 31 are confirmed by the correlations between nonthermal emission at 21 cm and the emissions at 24 μm and 70 μm (Tables 1 and 2). In Tables 1 and 2 we have also listed the power-law fits of the correlations between nonthermal emission at 6.3 cm (M 31) or 3.6 cm (M 33) and dust emission. These have exponents much closer to 1, but the difference between M 31 and M 33 remains. Exponents closer to 1 imply less diffusion of CRE, which is expected at smaller wavelengths. Below we will employ the observed differences in the correlations to estimate the diffusion length of the CRE in the two galaxies.

3 DIFFUSION OF CR ELECTRONS

CR particles may propagate through the ISM by streaming with the Alfvén velocity along the ordered magnetic field lines or they may be scattered by the irregularities of the turbulent magnetic field, leading to random-walk diffusion along the ordered field. As in the Milky Way disk the latter process dominates (Strong, Moskalenko & Ptuskin 2007; Dogiel & Breitschwerdt 2012), we expect that in the disks of M 31 and M 33 the CRE also mainly propagate by diffusion.

In order to estimate the diffusion length of CRE, we need to know the total magnetic field strength, B_{tot}, in the galaxy disk, which determines the energy and the lifetime of the CRE (see Eqs. 2 and 3 below). We derive B_{tot} in Sect. 3.1 and the diffusion lengths of CRE in the plane of the sky in Sect. 3.2.
3.1 Synchrotron scale heights and magnetic field strengths

We calculated the mean value of $B_{\text{kin}}$ in the radial intervals used for the radio – FIR correlations (i.e. $R = 6.8 - 12.5$ kpc in M 31, $R = 0 - 5$ kpc in M 33) from the mean surface brightness of the nonthermal (synchrotron) emission using the code BFIELD of M. Krause. Based on Eq. 3 of Beck & Krause (2005), this equation assumes energy equipartition between total cosmic rays and protons and electrons $K$, here taken equal to 100. The value of $\alpha_n$ follows from the mean nonthermal intensities at the two frequencies used ($I \propto r^{-\alpha_n}$) giving $\alpha_n = 0.92 \pm 0.06$ for M 31 and $\alpha_n = 0.86 \pm 0.08$ for M 33 (see Table 6). The degree of polarization, i.e. polarized/nonthermal intensity, was calculated for the areas considered. The lines of sight were obtained from the exponential scale heights of the nonthermal emission estimated in the following way.

On the 21 cm nonthermal map of M 31 at 45" resolution we measured the half-power width of several spiral arms in each quadrant of the bright emission ring along cuts parallel to the minor axis taken at $X = 25.5$, 23.0 and 19.5 on the northern major axis and $X = 14.5$, 17.0 and 19.5 on the southern major axis. Because of the high inclination of M 31, the cuts are nearly perpendicular to the arms at these positions and the width of the arms on the sky plane is determined by their vertical extent. We converted the half-power widths, corrected for inclination and angular resolution, to exponential scale heights and calculated the radius $R$ in the galaxy plane for each measured position. This yielded mean scale heights of $h = 300 \pm 25$ pc at $R = 8 - 10$ kpc and $h = 335 \pm 15$ pc at $R = 10 - 12$ kpc.

The mean exponential scale height in the radial interval $R = 6.8 - 12.5$ kpc, used for the radio – FIR correlations, was estimated from the scale height of the total gas (HI + $2H_2$). Berkhuijsen, Bajaja & Beck (1993) noted that the scale height of the total (mainly nonthermal) 21 cm emission is close to that of the total gas in arms in the SW quadrant of the “ring”. We checked this on the total gas map of Nieten et al. (2006), smoothed to 45" resolution, by measuring the half-power widths of the gas arms at the same positions as above. As we found a mean ratio of the nonthermal/gas scale heights of $0.88 \pm 0.05$, the nonthermal scale height at 21 cm is indeed nearly equal to the total gas scale height. For the area $R = 6.8 - 12.5$ kpc, the mean scale height of the total gas is $h_{\text{gas}} \approx 330$ pc (calculated from the ratio mean column density / mean volume density Tabatabaei & Berkhuijsen 2010), which is consistent with the nonthermal scale heights derived above. Therefore, we adopted $h_{\text{syn}} = 330 \pm 40$ pc for M 31 giving an effective line of sight through the medium of $L = 2h_{\text{syn}}/\cos(i) = 2550 \pm 310$ pc.

For M 33 we cannot measure the exponential scale height of the 21 cm nonthermal emission because at the inclination of 56° the arm widths in and perpendicular to the disk are superimposed on the sky plane. Therefore, we assumed the nonthermal scale height to be equal to the scale height of the total gas, as is the case in M 31. Little information on the gas scale height in M 33 exists in the literature. We calculated the mean HI scale height for $R < 20.5$ from the half thickness as function of radius given in Table II of Warner, Wright & Baldwin (1973), correcting for an erroneous factor $\sqrt{2}$ and taking the increase in the area of rings with increasing radius into account. For the distance $D = 840$ pc this gave $h_{\text{HI}} = 360$ pc. Heyer et al. (2004) found a relationship for the column density ratio (HI)/(H$_2$) with radius. Integration out to $R = 20.5$ yielded a mean value of 3.4 giving (N(H$_2$)/N(HI)) = 0.29. Assuming $h_{\text{HI}} = h_{\text{HI}}/2$, we obtained a scale height of the total gas $h_{\text{gas}} = 320$ pc. We adopted $h_{\text{syn}} = 320 \pm 80$ pc and $L = 1140 \pm 280$ pc for M 33. Note that the uncertainty in $L$ has little influence on the derived value of the magnetic field strength because it enters the calculation to the power $1/(3 + \alpha_n)$.

Our estimates of $h_{\text{syn}}$ agree with scale heights obtained for other galaxies. Like in the Milky Way, observations of edge-
on galaxies usually reveal a thin synchrotron disk with a scale height of about 0.3 kpc and a thick synchrotron disk with a scale height of 1 – 2 kpc (Beuermann, Kanbach \\& Berkhuijsen 1985, Dumke \\& Krause 1998, Krause 2011). Thus our estimates of $h_{\text{CRE}} \approx 330$ pc refer to the thin disks in M 31 and M 33.

The derived scale heights and total magnetic field strengths are summarized in Table 3 that also gives the scale heights of the CRE, $h_{\text{CRE}} = h_{\text{syn}} (3 + \alpha_s)/2$, and their energy $E$ calculated from Eq. 3 in Sect. 3.2. The values of $h_{\text{CRE}}$ at the high frequencies were scaled from those at 1.4 GHz with $\nu^{-0.125}$ (see below Eq. 4 in Sect. 3.2).

### 3.2 Diffusion lengths of CRE

Since the sources of CRE probably are supernova remnants (SNRs) located in or near star-forming regions, we used the distribution of the thermal emission as the source distribution of the CRE (see Fig. 2). We thus avoid the influence of dust properties and other heating sources than massive stars (e.g. evolved stars, the diffuse ISRF) that may play a role when the distribution of IR emission is taken as the source distribution. The relationships between nonthermal and thermal emission (given at the end of Tables 1 and 2) show the same trends as those between nonthermal and dust emission, i.e. exponents $b$ increase with frequency, and for M 33 the exponents are smaller than for M 31. We interpret these trends as the result of differences in diffusion lengths of the CRE.

In the following we assume that CRE are injected into the ISM by SNRs shocks continuously (at least during a few times the CRE lifetime), leading to a stationary distribution of CRE in the galaxy. We further assume that the diffusion of the CRE leads to a Gaussian distribution of the CRE. The separation of the SNRs is larger than the diffusion length of the CRE in the disk and avoids overlap of diffusion lengths of CRE from neighbouring sources, which would otherwise influence the estimated mean diffusion length. Within a radius of 5 kpc from the centre, M 33 contains $\lesssim 40$ SNRs and SNR candidates of diameter $D \lesssim 30$ pc (Long et al. 2010), which are the main sources of CRE (Berezko \\& Volk 2004). Their mean separation is 1.4 kpc, which is larger than the typical diffusion length of $\approx 1$ kpc. As the star formation rate / area in the emission ring of M 31 is five times smaller than in M 33 (see Table 6), the mean separation of CRE sources in the ring is $> 1.4$ kpc. So it will be possible to estimate the CRE diffusion length in both galaxies.

The only difference between the nonthermal emissions from a galaxy at two frequencies is the diffusion length of CRE that is reached by random-walk diffusion, other variables like their source distribution and the magnetic field strength being the same. The diffusion length $l(\nu)$ at frequency $\nu$ is determined by the synchrotron life time $\tau_{\text{syn}}$ and the diffusion coefficient $D_0$. The mean synchrotron lifetime $< t >$ of the bulk of CRE, a mixture of CRE of all ages, is smaller than $\tau_{\text{syn}}$; it may be written as

$$< t > = \int_0^{\tau_{\text{syn}}} N(t) dt / \int_0 \ N(t) dt,$$

where $N$ is the number of CRE as a function of time, $N = N_0 \exp(-t/\tau_{\text{syn}})$. Evaluation of $< t >$ yields $< t > = \tau_{\text{syn}} (1 - 2e)/(1 - e) = 0.42 \tau_{\text{syn}}$. Therefore, we take $\tau_{\text{syn}}/2$ as the typical age of the bulk of the CRE in the calculations below. The diffusion length then is

$$(l_{t}/pc)^2 \approx (D_{E_0}/pc^2 yr^{-1}) (\tau_{\text{syn}}/2)/(10^7yr).$$

$D_{E_0}$ depends on the energy $E$ of the CRE as $D_{E_0} \propto E^p$, where $p = 0.5$ (Strong, Moskalenko \\& Ptuskin 2007). $E$ is related to the radiating frequency:

$$\nu/\text{GHz} = 1.3 \times 10^2 \left( B_{\text{tot}}/\mu G \right) \left( E/\text{GeV} \right)^{1},$$

where $B_{\text{tot}}$ is the total magnetic field strength derived from the surface brightness of the nonthermal emission. Mean values of $B_{\text{tot}}$ and $E$ for the regions used for the correlations are given in Table 3.

The lifetime of the radiating CRE is mainly determined by the synchrotron losses at our frequencies (e.g. Fig. 1 in Murphy 2009):

$$(t_{\text{syn}}/10^7 yr) = 1.4 \times 10^9 \left( \nu/\text{GHz} \right)^{0.5} \left( B_{\text{tot}}/\mu G \right)^{-1.5}.$$  

Thus, at high $\nu$ we see radiation from younger electrons than at low $\nu$, and because of their shorter lifetime they cannot travel as far as those at low $\nu$, in spite of their higher energy. Comparing Eqs. 1, 2, and 3, we see that for a fixed value of $B_{\text{tot}}$, $l^2 \propto \nu^{-0.25}$, if $D_{E_0} \propto E^{0.5}$. An estimate of $l^2$ yields an estimate of $D_{E_0}$ from Eq. 2.

We measured the diffusion length in the plane of the sky in two ways: (1) by determining the difference in diffusion length between 1.4 GHz and the higher frequency, and (2) by estimating the diffusion length at each frequency separately using the distribution of the thermal emission as the source distribution of the CRE.

1. The most direct way to measure the average diffusion length of CRE is a comparison of the nonthermal distributions at the two frequencies. Since the CRE radiating at the higher frequency are still closer to their origin than at 1.4 GHz, smoothing of the nonthermal distribution at the higher frequency will make it more similar to that at 1.4 GHz. We smoothed the high-frequency distributions until the power-law exponent of the correlation with the nonthermal distribution at 1.4 GHz had $b = 1$, indicating that the two distributions are most similar. The smoothing beam required for this, corrected for the resolution of the observations, represents the difference in diffusion length between the two frequencies.

Because distances are measured from the centre of the telescope beam, we took half of the full width at half power of the corrected smoothing beam as the derived difference in the square of the diffusion lengths, $\Delta \tilde{l}^2$. Then, using Eq. 1 and $\Delta \tilde{l}^2 = l_{1.4}^2 - l_{\nu}^2$, we have

$$\Delta \tilde{l}^2 \approx l_{1.4}^2 (1 - (\nu/1.4\text{GHz})^{-0.25}),$$

where $l_{1.4}$ is the diffusion length at 1.4 GHz. From the derived value of $l_{1.4}^2$ we obtain $l^2$ by scaling with $\nu^{-0.25}$. The resulting diffusion lengths observed in the plane of the sky, $l_{\text{syn}}$, are given in Table 4. The errors are determined by the standard deviations in $b$ of the correlations giving the best smoothing beam.

2. The exponents $b$ of the nonthermal–thermal correlations are $< 1$ because the CRE diffused away from their places of origin represented by the distribution of the thermal emission (see Sect. 4 and Fig. 2). We obtained an estimate of the diffusion length at each frequency by smoothing the thermal emission until the exponent of the correlation with the nonthermal emission at that frequency became $b = 1$. By requiring $b = 1$, we implicitly assume that $B_{\text{tot}}$ is constant within the area considered ($l_{\text{diff}} \approx N_{\text{CRE}} B_{\text{tot}}$). On large scales this is approximately the case as radial variations in $B_{\text{tot}}$ are $< 10\%$ in the areas used for the correlations (i.e. see Fig. 9 in Tabatabaei et al. 2008). Small-scale variations in $B_{\text{tot}}$ will only contribute to the spread in the data points. Again, we took half the corrected smoothing beam as the diffusion length in the sky plane to obtain the diffusion lengths given in Table 4.
Hence, move out of the thin disk into a thick disk of CRE. If older CRE not only propagate along the disk but also the observed differences between the results of methods 1 and 2 could occur if this assumption is incorrect. However, Figure 1 of Long et al. (2010) shows that most of the 137 known supernova remnants and SNR candidates in M 33 are located on or close to Hα emission on a specific scale; the scales chosen are between 0.4 and 4 kpc.

In both cases we used circular Gaussian smoothing beams because they gave the best fits. Even for the highly inclined galaxy M 31, fits with elliptical beams parallel to the major axis were worse than those with circular beams. This agrees with the results of Marsh & Helou (1998) who found circular smoothing beams optimal for 10 out of 16 galaxies. Hence, in these galaxies the deprojected diffusion lengths within the disk and perpendicular to the disk are about the same (see Sect. 5).

### Table 3. Exponential scale heights and magnetic field strengths of the thin disks in M 31 and M 33

| Galaxy | R [kpc] | Frequency [GHz] | \( h_{\text{syn}} \) [pc] | \( h_{\text{CRE}} \) [pc] | \( B_{\text{bulk}} \) [\( \mu \text{G} \)] | \( E \) [GeV] | \( \tau_{\text{syn}}/2 \) [10^5 yr] |
|--------|--------|-----------------|-----------------|-----------------|-----------------|-------------|------------------|
| M31    | 6.8-12.5 | 1.465           | 330 ± 40        | 650 ± 80        | 6.6 ± 0.3       | 4.1         | 3.4 ± 0.2        |
|        |        | 4.85            | –               | 560 ± 70        | 6.6 ± 0.3       | 7.5         | 1.9 ± 0.1        |
| M33    | 0 - 5   | 1.425           | 320 ± 80        | 620 ± 150       | 8.1 ± 0.5       | 3.7         | 2.5 ± 0.2        |
|        |        | 8.35            | –               | 500 ± 120       | 8.1 ± 0.5       | 8.9         | 1.1 ± 0.1        |

(a) \( h_{\text{CRE}} = h_{\text{syn}} (3 + \alpha_2)/2 \)
(b) Scaled from \( h_{\text{CRE}}(1.4 \text{ GHz}) \) with \( \nu^{-0.125} \) (see Eq. 4)
(c) Assumed to be equal to the gas scale height like in M 31

### Table 4. Observed diffusion lengths of CRE in M 31 and M 33 on the sky

| Galaxy | Method | Frequency [GHz] | \( l_{\text{sky}} \) [pc] |
|--------|--------|-----------------|-----------------|
| M31    | 1      | 1.465           | 1140 ± 190       |
|        |        | 4.85            | 980 ± 160        |
|        | 2      | 1.465           | 715 ± 20         |
|        |        | 4.85            | 510 ± 60         |
| M33    | 1      | 1.425           | 900 ± 110        |
|        |        | 8.35            | 720 ± 90         |
|        | 2      | 1.425           | 1960 ± 170       |
|        |        | 8.35            | 1450 ± 200       |

(a) Based on the observed difference in diffusion length between two frequencies (see Eq. 4):
M 31: \( (\Delta l)^{0.5} = 580 ± 100 \text{ pc} \); M 33: \( (\Delta l)^{0.5} = 540 ± 65 \text{ pc} \)
(b) Scaled from 1.4 GHz with \( l_{\text{sky}} \propto \nu^{-0.125} \)

In both cases we used circular Gaussian smoothing beams because they gave the best fits. Even for the highly inclined galaxy M 31, fits with elliptical beams parallel to the major axis were worse than those with circular beams. This agrees with the results of Marsh & Helou (1998) who found circular smoothing beams optimal for 10 out of 16 galaxies. Hence, in these galaxies the deprojected diffusion lengths within the disk and perpendicular to the disk are about the same (see Sect. 5).

### 4 Diffusion Lengths in the Plane of the Sky

The observed diffusion lengths in Table 4 are in the range 0.5–2 kpc in the plane of the sky (measured along the major axis). At each frequency, the two methods yield diffusion lengths that agree within a factor of 2. Method 2 yields smaller values than method 1 for M 31, but larger values for M 33. The estimates by the two methods may differ for several reasons.

a) Method 1 measures the difference in diffusion length between the two frequencies. Since the lifetime of the CRE at the higher frequency is about half of those at 1.4 GHz (see Table 3), they do not propagate as far as those at 1.4 GHz. This means that the diffusion length of the older CRE emitting at 1.4 GHz, which had time to propagate far from their places of origin, determines \( \Delta l \). Hence, \( \Delta l \) is an upper limit to the mean diffusion length of the bulk of CRE. If older CRE not only propagate along the disk but also move out of the thin disk into a thick disk/halo, the “vertical” propagation length may add to the diffusion length observed in the thin disk by an amount depending on the inclination. However, observations of edge-on galaxies show that CRE propagation out of the thin disk usually occurs in SF regions (i.e. Dahlem, Dettmar & Hummel (1994); Dumke et al. (1995); Heesen et al. (2009)) and most of these CRE will be young (Breitschwerdt, Dogiel & Völk (2002)). Therefore we expect that method 1 estimates an upper limit to the diffusion length at 1.4 GHz that mainly refers to the thin disk.

b) Method 2, however, measures the mean diffusion length of the bulk of the CRE, i.e. of a mixture of CRE of all ages, at each frequency. The inclusion of young CRE will yield a diffusion length smaller than the upper limit from method 1, consistent with our results for M 31. Furthermore, method 2 includes the full effect of CRE propagation out of the thin disk. If this is significant, the observed diffusion length could be larger than the estimate from method 1, as we find for M 33. We discuss the evidence for a thick disk/halo in M 33 in Sect. 5.2.

c) Method 2 is based on the assumption that the source distribution of the CRE is similar to that of the thermal emission. Differences between the results of methods 1 and 2 could occur if this assumption is incorrect. However, Figure 1 of Lone et al. (2010) shows that most of the 137 known supernova remnants and SNR candidates in M 33 are located on or close to Hα emission. We used these data to construct the radial distribution of the number of SNRs/kpc² in M 33 presented in Figure 2, which also shows the radial variation of the thermal emission. Out to \( R = 5 \text{ kpc} \), the radial distribution of the SNRs is very similar in shape to that of the thermal emission and has roughly the same scale length. As in the dense ISM near the centre not all SNRs may have been detected, the scale length of 2.9 ± 0.7 kpc may be an upper limit. Thus in M 33 the distribution of the thermal emission may indeed be representative for the source distribution of CRE.

Recently, Tabatabaei et al. (2013a) estimated the diffusion length of CRE on small scales (< 1 kpc) in M 31 and M 33 from NTH–FIR correlations as a function of scale. They find \( l_{\text{sky}} = 730 ± 90 \text{ pc} \) for M 31 and \( l_{\text{sky}} < 400 \text{ pc} \) for M 33. Their value for M 31 is in excellent agreement with \( l_{\text{sky}} = 715 ± 20 \text{ pc} \) that we obtain from method 2, but their value for M 33 is lower than our 4 Using a wavelet analysis, Tabatabaei et al. (2013a) decomposed each IR and radio map into a set of 6 (M 31) or 7 (M 33) maps each containing emission on a specific scale; the scales chosen are between 0.4 and 4 kpc. For each galaxy, maps on the same scale are then correlated and the derived correlation coefficients \( r_w \) are plotted as a function of scale. The smallest scale on which the synchrotron – 70 µm correlation has \( r_w = 0.5 \) is taken as the diffusion length of the CRE, because \( r_w < 0.5 \) implies that the correlation is lost, i.e. due to CRE diffusion and/or escape and turbulence in the ISM caused by SF regions and SNRs, synchrotron and 70 µm emission do not correlate on scales of < 1 kpc.
upper limit of \( I_{\text{sky}} = 900 \pm 110 \) pc for the thin disk. As the authors explain, this low \( I_{\text{sky}} \) for M 33 refers mostly to young CRE that are still close to their places of origin in the star forming regions. So their value may be considered a lower limit to \( I_{\text{sky}} \) of the bulk of CRE in M 33. Since the emission from star forming regions occurs on small scales, their value is free from extended thick disk / halo emission.

5 DISTRIBUTION OF CRE IN M 31 AND M 33

Table 4 shows that at 1.4 GHz the diffusion length in the sky plane is about 1 kpc for M 31 and about 1.5 kpc for M 33. We have deprojected \( I_{\text{sky}} \), into the components \( I_{x} \), in the plane of the galaxy and \( I_{z} \), perpendicular to the disk by assuming istropy in the disk of the galaxy. Isotropy of CRE is consistent with the situation in the Milky Way where the diffusion of cosmic rays becomes isotropic on scales larger than 100–200 pc (Strong, Moskalenko & Ptuskin 2007; Dorfle & Breitschwerd 2012), but below 1 kpc (Stepanov et al. 2012). Isotropy of CRE on large scales is also expected from simulations (Yan & Lazaraj 2008).

We thus have \( I_{\text{sky}} = I_{x} = I_{z} = I \), where \( I_{x} \) and \( I_{z} \) are measured along the major and minor axis in the plane of the galaxy, respectively, giving \( I_{x} = I_{z} = I_{x} = I_{z} = I \). This assumption leads to a circular extent of the propagation lengths on the sky plane, in agreement with the circular smoothing beams we derived. We present these values in Table 5.

In Table 5 we also give diffusion coefficients and confinement times of the CRE in the two galaxies. The diffusion coefficients in the \( x \), \( y \) and \( z \) direction, \( D_{\text{xy}} \) and \( D_{\text{zz}} \), follow from Eq. 2 with the typical CRE ages \( t_{\text{syn}}/2 \) listed in Table 3. The confinement time is the time the CRE need to reach the scale height of the thin disk by diffusion, \( t_{\text{conf}} = h_{\text{CRE}} / D_{\text{zz}} \). We also list the mean velocities of the CRE when crossing the thin disk, \( V_{\text{CRE}} = h_{\text{CRE}} / t_{\text{conf}} \). We discuss these results below. Note that since the diffusion lengths estimated by method 1 are upper limits, the values for the diffusion coefficients also are upper limits, those for \( t_{\text{conf}} \) lower limits and those for \( V_{\text{CRE}} \) upper limits.

5.1 Confinement of CRE in M 31

The diffusion length \( l_{xy} \) in the plane of M 31 of about 1–2 kpc at 1.4 GHz is typical for galaxy disks (Murphy et al. 2006b). The corresponding diffusion coefficient of about \((1 - 3) \times 10^{28} \text{cm}^2/\text{s}\) is in the range \(10^{28} - 10^{29} \text{cm}^2/\text{s}\) estimated for the Milky Way and other galaxies (e.g. Dahlem, Dettmar & Hummel (1994); Puskin & Soutoul 1998; Moskalenko et al. 2002; Murphy 2009; Heesen et al. 2009; Lacki & Thompson 2010, and references therein; Buffie, Heesen & Shalchi 2013; Tabatabaei et al. 2013).

Perpendicular to the disk the diffusion length at 1.4 GHz is only about 0.7 – 1.1 kpc. Since method 1 yields upper limits for the diffusion lengths, we use the smaller value of \( l_{z} \) of method 2 for comparison with the exponential scale height \( h_{\text{CRE}} \) given in Table 3. We convert the Gaussian-based value of \( l_{z} \) at 1.4 GHz to the exponential scale height \( h_{\text{CRE}} = l_{z} / 0.833 = 860 \pm 25 \) pc, which is about 30 percent higher than the value of \( h_{\text{CRE}} = 650 \pm 80 \) pc given in Table 3. Both values refer to a mixture of CRE of all ages. Since \( h_{\text{CRE}} \) is derived from \( h_{\text{syn}} \) assuming energy equipartition between magnetic fields and CR particles, the fair agreement with \( h_{x} \) suggests that energy equipartition may indeed be valid for the radial range \( R = 6.8 - 12.5 \) kpc in M 31.

The corresponding diffusion coefficient \( D_{z} \) of \( 0.45 \times 10^{28} \text{cm}^2/\text{s} \) is about half of \( D_{\text{xy}} \) and very low, indicating that CRE only slowly diffuse away from the midplane of the disk. The time the CRE need to move out of the thin disk, \( t_{\text{conf}} = (2.8 \pm 0.7) \times 10^{7} \) yr, is close to the typical CRE age \( t_{\text{syn}}/2 = (3.4 \pm 0.2) \times 10^{7} \) yr (see Table 3). Hence, the CRE have lost most of their energy before they can reach \( z = h_{\text{CRE}} \) and only older CRE may move into a thick disk / halo, as the lower limit of \( t_{\text{conf}} \) of method 1 suggests. Thus, while the bulk of the CRE in M 31 is confined to the thin disk (as shown by method 2), some of the older CRE may leave the thin disk at speeds of \( V_{\text{CRE}} < 60 \pm 25 \text{ km s}^{-1} \). As this is about equal to the Alfvén speed \( c_{A} \) of \( V_{A} = 50 \pm 20 \text{ km s}^{-1} \), convective propagation of CRE by gas flows from SF regions does not play a role in M 31.

As \( q_{\gamma} \) is greater than 1, \( h_{\text{CRE}} / h_{\text{z}} \) and only older CRE may move into a thick disk / halo. The confinement of the CRE to the thin disk in M 31 is consistent with the structure of the ordered magnetic field (Fletcher et al. 2004) found no significant vertical component of this field that is extraordinarily well aligned along the spiral arms in M 31. The strong uniformity of the ordered field in the disk (Berkhuijzen, Beck & Hoernes 2003) probably leads to the small value of \( D_{z} \) and the confinement of the CRE to the thin disk.

Since method 1 refers to the older CRE, we should rather use \( t_{\text{syn}}/4 \) than \( t_{\text{syn}}/2 \) to derive the diffusion coefficients, which would lower the upper limits by 50%. But in view of the large errors and for simplicity, we use \( t_{\text{syn}}/2 \).

As for a normalized Gaussian the halfwidth \( l \) occurs at \( z = 0.833 \) and for a normalized exponential the scale height \( h \) is reached at \( z = 1, h = 1 / 0.833 \). We derived the Alfvén speed \( V_{A} = c_{A} \) of CRE at \( 0.833 \) and \( V_{A} = c_{A} \) of CRE at \( 0.833 \) and the energy spectrum in the ISM obtained from the mean rotation measure (Fletcher et al. 2004) and \( B_{\text{par}} \) is the strength of the ordered magnetic field perpendicular to the LOS calculated from the polarized synchrotron emission.
Table 5. Diffusion lengths and confinement times of CRE in M31 and M33

| GAL  | M   | FREQ  | l_x, y | D_{E, x, y} | l_x | D_{E, x} | \tau_{conf} | V_{CRE} |
|------|-----|-------|--------|-------------|-----|-----------|-------------|---------|
|      |     | [GHz] | [pc]   | [10^{28} cm^2/s] | [pc] | [10^{28} cm^2/s] | [10^3 yr] | [km/s]  |
| M31  | 1   | 1.465 | 1610 ± 280 | 2.3 ± 0.8 | 1140 ± 200 | 1.2 ± 0.4 | 1.1 ± 0.5 | 60 ± 25 |
|      | 4.85| 1390 ± 230 | 3.1 ± 1.1 | 980 ± 160 | 1.5 ± 0.5 | 0.6 ± 0.3 | 90 ± 40 |
|      | 2   | 1.465 | 1010 ± 30 | 0.9 ± 0.1 | 715 ± 20 | 0.45 ± 0.03 | 2.8 ± 0.7 | 20 ± 6  |
|      | 4.85| 720 ± 90 | 0.8 ± 0.2 | 510 ± 60 | 0.41 ± 0.11 | 2.3 ± 0.9 | 25 ± 8  |
| M33  | 1   | 1.425 | 1270 ± 160 | 1.9 ± 0.6 | 900 ± 110 | 1.0 ± 0.3 | 1.2 ± 0.6 | 50 ± 30 |
|      | 8.35| 1020 ± 130 | 2.9 ± 0.8 | 720 ± 90 | 1.4 ± 0.4 | 0.5 ± 0.3 | 100 ± 55 |
|      | 2   | 1.425 | 2770 ± 240 | 9.3 ± 1.9 | 1960 ± 170 | 4.6 ± 0.9 | 0.3 ± 0.2 | 210 ± 120 |
|      | 8.35| 2050 ± 290 | 11.5 ± 3.5 | 1450 ± 200 | 5.8 ± 1.8 | 0.13 ± 0.08 | 380 ± 230 |

(a) l_{x, y} = l_{xy} \sqrt{2}
(b) l_x = l_{kh, y}
(c) \tau_{conf} = \tau_{C, x}^{\text{kin}} / D_{E, x} (with h_{C, x} from Table 3)
(d) V_{CRE} = h_{C, x} / \tau_{conf}

5.2 Distribution of CRE in M33

An analysis of the polarized emission from M33 [Tabatabaei et al. 2008] revealed that the large-scale, ordered magnetic field in M33 has a vertical component of about 1 \mu G, extending to at least R = 5 kpc. This implies that M33 has a nonthermal halo or thick disk containing CRE. In spite of this, the upper limits to the diffusion lengths obtained by method 1 (see Table 5) are smaller than the estimates of method 2, and even smaller than those of method 1 for M31. It seems that the halo contributions to the diffusion lengths at the two frequencies largely canceled in the difference A^2 (see Equation 5). We conclude that the results of method 1 apparently refer to the thin disk in M33. As the estimates of method 2 refer to thin+thick disk (Sec. 4), we discuss these cases separately.

THIN DISK. The upper limit to the diffusion length l_{x, y} of 1.3 kpc at 1.4 GHz obtained from method 1 for the thin disk of M33 is about 30 percent smaller than that in M31 (see Table 5). As the stronger magnetic field B_{halo} (Table 3) yields smaller synchrotron loss times, the diffusion coefficient D_{E, x, y} of (1.9 ± 0.6) 10^{28} cm^2/s is similar to that in M31.

The upper limit to the diffusion length perpendicular to the disk of l_x = 900 ± 110 pc and D_{E, x} are also similar within errors to those in M31 at 1.4 GHz. Here l_x corresponds to an exponential scale height h_x = \sqrt{l_x/0.833} = 1080 ± 130 pc, an upper limit consistent with the scale height h_{CRE} = 620 ± 150 pc listed in Table 3. As the lower limit to the confinement time is about half of t_{C, x}/2, some of the older CRE traced by method 1 may leave the thin disk at velocities of V_{CRE} = 50 ± 30 km s^{-1}. Like in M31, this is equal to the Alfvén speed V_A = 45 ± 25 km s^{-1}.

THIN+THICK DISK. At 1.4 GHz the diffusion length l_y of about 2 kpc, more than twice that of the thin disk, yields an exponential scale height of h_y = 2350 ± 280 pc, corresponding to a synchrotron scale height of h_{syn} = 2h_y / (3 + \alpha) = 1220 ± 270 pc. This value is consistent with the scale heights near 1 kpc in total emission observed in edge-on galaxies [Hummel, Beck & Dettmar 1991; Dumke et al. 1995], and confirms the existence of a thick synchrotron disk or halo in M33 (Tabatabaei et al. 2008). We may estimate the exponential scale height of the thick synchrotron disk by subtracting the contribution of the thin disk to h_{syn}, assuming that at z=0 kpc the intensities of thin and thick disk are equal. With h_{syn} = 1.22 ± 0.27 kpc and h_{thin} = 0.32 ± 0.08 kpc (Table 3), we find h_{thick} = 2.1 ± 0.5 kpc at 1.4 GHz. This value of h_{thick} agrees with the mean value of h_{thick} = 1.8 kpc in 5 galaxies (at 4.8 GHz) listed by Krause (2011). The agreement, however, may be fortuitous because (1) our estimate of h_{thick} in M33 is sensitive to the relative intensities of thin and thick disk at z=0 pc, and (2) the value of Krause (2011) refers to the sum of thermal and nonthermal emission instead of to nonthermal emission alone.

The value of D_{E, x} = 4.6 10^{28} cm^2/s is nearly 5 times higher than in the thin disk. This leads to a confinement time of \tau_{conf} = (0.3 ± 0.2) 10^{3} yr that is significantly shorter than the typical CRE age t_{C, x}/2 = (2.5 ± 0.2) 10^{3} yr. Hence, in M33 many CRE will move out of the thin disk into the thick disk / halo at a mean velocity of V_{CRE} = 210 ± 120 km s^{-1}. This velocity is higher than the Alfvén velocity of about 50 km s^{-1}, suggesting that CRE are leaving the disk by a galactic wind.

As the older CRE traced by method 1 are largely confined to the thin disk, mainly young CRE are moving along the vertical magnetic field into the thick disk / halo. The multi-scale analysis of the NTH–FIR correlation in M33 of [Tabatabaei et al. 2013] indicates that CRE propagate from the star-forming regions into the thick disk. Outflow of CRE and gas from bright SF regions is often observed in edge-on galaxies with a thick radio disk or halo (e.g. Dahlem, Lisenfeld & Golla 1995; Krause 2009; Heesen et al. 2009). The outflow weakens the NTH emission from SF regions and decreases the dynamic range of the emission from the thin disk. CRE in the thick disk / halo also have velocity components parallel to the disk, leading to distribution of the CRE across the disk and further smoothing of the observed NTH emission [Breitschwerdt, Döpil & Völl 2002; Döpil & Breitschwerdt 2012]. Hence, outflow of CRE from SF regions into the thick disk / halo mimics the effect of “horizontal” CRE diffusion in the thin disk.

The large value of l_{x, y} = 2.8 kpc obtained by method 2 at 1.4 GHz, more than twice as large as found for the thin disk, is consistent with the effects of CRE outflow described above. The smoothness of the observed NTH emission from the (thin + thick) disk of M33 is reflected in the large l_{x, y} of about 2 kpc (Table 4) and in the low exponents b of the NTH–FIR correlations (Table 2).
6 DISCUSSION

In this section we discuss the effect of the superposition of the disk and halo emission in M 33 along the LOS as well as the influence of the magnetic field structure and of the star formation rate on the CRE distribution in M 31 and M 33.

6.1 Effect of the superposition on the sky of disk and halo emission

In Section 5.2 we have argued that M 33 has a nonthermal thick disk or halo. Since the inclination angle of M 33 is 56°, this means that on the sky plane we see the halo emission superimposed onto the more prominent disk emission. The addition of the distributions of two components generally leads to a smoother distribution than that of the individual ones. Therefore, the nonthermal emission observed on the sky will be smoother than that of the disk emission alone. This implies that the derived diffusion length will be larger than the actual diffusion length in the thin disk, because (using method 2) the TH emission needs to be more strongly smoothed to reach exponent $\beta=1$ in $NTH \propto TH^\beta$. This explains why in M 33 $l_{x,y}$ obtained by method 2 (Table 5) is larger than that of the thin disk found by method 1 although the CRE traced by method 2 are younger than those traced by method 1.

Possibly, not only $l_{x,y}$ but also $l_0$ is larger than the actual propagation length perpendicular to the disk. In that case also the derived scale heights will be too high. Since we cannot separate the disk and halo emission from M 33, the values of method 2 given in Table 5 should be regarded as upper limits.

We conclude that for galaxies with a thick NTH disk or halo the apparent diffusion length estimated from a comparison with Hα or FIR data may be considerably larger than the actual diffusion length of the CRE in the thin disk, and that also the extent of the thick disk / halo may be overestimated.

6.2 Importance of the magnetic field structure

The diffusion length $l_{x,y}$ in the thin disk of M 31 at 1.4 GHz is about 1.3 times that in M 33 (Table 5, method 1). This difference may be due to differences in the diffusion coefficient and synchrotron lifetime since $l=\left(D_0 \tau_{s,0}\right)^{1/2}$ (Eq. 2).

The propagation of CRE in the ISM is largely determined by their interaction with magnetic fields. The diffusion coefficient depends on both the turbulent and the ordered magnetic field structure. The ordered field, mainly running parallel to the disk, favours the propagation of CRE along the disk, while the turbulent field hinders the CRE in traveling long distances in one direction because they are scattered by the fluctuating magnetic field. The diffusion coefficient in M 33 is larger than in M 31. However, the observed factor obtained from Table 5 is only 1.2 ± 0.6, which suggests that other factors also play a role.

At a fixed frequency the synchrotron lifetime of the CRE scales with $B_{tot}^{-1.5}$ (Eq. 4), thus for the diffusion length we have $l \propto (B_{tot}/B_{ord})^{-1} B_{tot}^{-0.75}$. Therefore, we expect that in M 33 $l_{x,y}$ is 0.37 ± 0.13 times that in M 31, i.e. $l_{x,y} = 600 \pm 230$ pc, but this is about half the observed value of $l_{x,y} = 1270 \pm 160$ pc. Similarly, the observed value of $l_0$ in the thin disk is about twice as large as expected from scaling with the value in M 31. As explained in Sect. 6.1, these larger values may result from the superposition of the disk and halo emission from M 33. Thus the superposition further increases (decreases) the upper limits (lower) limits obtained by method 1 (see Sect. 5).

We conclude that the differences in $D_0$, $l_{x,y}$ and $l_0$ between the thin disks of M 31 and M 33 may be qualitatively understood as the combined effect of the different magnetic field structures in the two galaxies and the existence of a thick disk / halo in M 33.

We note that this is the first time that (a) diffusion lengths and diffusion coefficients of CRE in the thin disks of galaxies are derived based on the distributions of the NTH emission alone, and (b) it is shown that these quantities depend on the properties of the magnetic field as well as on the occurrence of convective transport of young CRE from bright SF regions into a thick disk / halo.

6.3 Influence of star formation

Above we have seen that M 31 has no significant vertical magnetic field and that most of the CRE are confined to the thin disk (Sect. 5.1). In M 33, however, CRE are leaving the bright SF regions along the vertical magnetic field into the thick disk / halo (Sect. 5.2).

Vertical magnetic fields are common in galaxies with thick radio disks (e.g. Krause (2009); Heesen et al. (2009)) and are often related with star forming regions. Dahlem, Lisenzfeld & Golla (1995) first pointed out that thick radio disks occur in galaxies with a high star formation rate (SFR). Do M 31 and M 33 fit into this picture?

M 33 indeed has a higher SFR than M 31 (Verley et al. 2009; Tabatabaei & Berkhuijsen 2010). We have calculated the mean SFR/pc² for the areas under consideration from the extinction-corrected Hα maps, using the same procedure as Tabatabaei & Berkhuijsen (2010). The values given in Table 6 show that the SFR/pc² in M 33 is 5 times higher than in the bright ring of M 31. Therefore, the mean TH and NTH surface brightnesses in M 33 are higher than in M 31 (see Table 6; Fig. 1) because it has more massive stars and SNRs per unit area, and a stronger total magnetic field.

Rossa & Dettmar (2003) analysed the occurence of gaseous halos observed in Hα using a sample of 74 normal spiral galaxies seen edge-on. They found that all galaxies with a SFR/pc² > (1.4 ± 0.2) M⊙ Gyr⁻¹ pc⁻² show extra-planar diffuse emission from ionized gas at mean distances of $d = 2$ kpc from the galactic plane, whereas most galaxies with lower SFR/pc² only have a

| Variable | Unit | M 31 | M 33 |
|----------|------|------|------|
| R | kpc | 6.8 ± 12.5 | 0 ± 5 |
| NTH(21 cm) | µJy/beam² | 7.3 ± 0.4 | 17.3 ± 0.3 |
| $\alpha_0$ | – | 0.92 ± 0.06 | 0.86 ± 0.08 |
| $B_{tot}$ | µG | 6.6 ± 0.3 | 8.1 ± 0.5 |
| $B_{tot}$ | µG | 5.0 ± 0.2 | 7.6 ± 0.5 |
| $B_{ord}$ | µG | 4.3 ± 0.3 | 2.8 ± 0.3 |
| $D_0/B_{ord}$ | – | 1.2 ± 0.4 | 2.7 ± 0.3 |
| TH(21 cm) | µJy/beam² | 1.1 ± 0.1 | 5.35 ± 0.95 |
| $\delta_0$(21 cm) | % | 13 ± 1 | 24 ± 4 |
| SFR/pc² | M⊙ Gyr⁻¹ pc⁻² | 0.6 ± 0.1 | 3.0 ± 0.6 |
thin disk. According to this division, M 31 has a thin disk in Hα (SFR/pc² = (0.6 ± 0.1) M⊙ Gyr⁻¹ pc⁻²) and M 33 has a thick disk or halo (SFR/pc² = (3.0 ± 0.6) M⊙ Gyr⁻¹ pc⁻²). Indeed, in M 31 the [z]-extent of the diffuse Hα emission is less than 500 pc (Walterbos & Braun 1994) and, based on a power-spectrum analysis, Combes et al. (2012) suggested that M 33 has a thick Hα disk. Since galaxies with a thick disk in Hα that have been observed in the radio regime also have a thick radio disk (Rossi & Dettmar 2003), we may expect that M 33 has a thick radio disk or halo. Thus the expectations from Hα observations are in full agreement with our results of a thin NTH disk in M 31 and a thick NTH disk / halo in M 33 (Table 5).

The SFR/area weakly influences the strength and structure of the magnetic field in a galaxy. A higher SFR/area generally increases the total and turbulent magnetic field with (SFR/area)⁰⁻²⁻⁰⁻⁴ but hardly affects the ordered magnetic field (e.g. Chyży 2008; Krause 2009; Tabatabaei et al. 2013c). This explains why M 33 has a stronger Bord than M 31. The ordered magnetic field, however, is determined by various dynamical and environmental effects caused by e.g. galactic rotation, density waves, shear and anisotropic compressions in spiral arms (Fletcher et al. 2011). Although within a galaxy Bint/Bord may increase with (SFR/area)⁰⁻³ (Chyży 2008), the many factors influencing Bint may be the reason that between galaxies the ratio Bint/Bord seems independent of SFR (Krause 2009). For example, M 51 has a 4 times higher SFR/pc² than M 33 and strong magnetic fields, but the ratio Bint/Bord is similar to that in M 31 (1.4 compared to 1.2 ± 0.5, see Tabatabaei et al. 2013c). The strong density waves in M 51 may be responsible for this situation.

We have shown that the differences in diffusion length in the thin disks of M 31 and M 33 are the result of both the different properties of the magnetic field in the two galaxies and the existence of a thick disk / halo in M 33, which is absent in M 31. As the field properties not only depend on the SFR/area, a clear correlation between diffusion length in the thin disk and SFR/area is not expected. However, galaxies with SFR/area > 1.4 M⊙ Gyr⁻¹ pc⁻² are likely to have a thick radio disk / halo. Then the estimated “diffusion” lengths will be larger than expected from random walk diffusion alone because they are the result of a combination of diffusion in the thin disk and propagation of CRE into and inside the halo (Sect. 5.2), and the superposition of disk and halo onto the sky plane (Sect. 6.1).

7 COMPARISON WITH EARLIER WORK

7.1 Smearing experiments

A number of authors estimated the diffusion length of CRE in nearby galaxies by smearing the distribution of the FIR emission until it looked similar to that of the emission at 1.4 GHz (e.g. Biswas, Helou & Condon 1989; Marsh & Helou 1995; 1998; Murphy et al. 2006a). However, these studies have several drawbacks: (1) IR images at 24 – 70 µm were used as the source distribution of the CRE. Although IR emission is usually well correlated with Hα emission within galaxies (e.g. Leroy et al. 2009; Verley et al. 2009; Tabatabaei & Berkhuijsen 2010), the influence of other dust heating sources than massive stars and of the dust distribution are not known. (2) Part of the total power emission observed at 1.4 GHz is thermal emission and its fraction varies across a galaxy. The inclusion of thermal emission leads to diffusion lengths that are too small by an amount depending on the thermal fraction and its distribution in the galaxy. Murphy et al. (2008) did correct the total power emission for the thermal contribution, but they assumed 2 populations of CRE sources with different mean ages: one of young SNRs on scales < 1 kpc and a diffuse one on scales > 1 kpc where CRE are accelerated by interstellar shocks in the diffuse ISM. We think that there is insufficient evidence for diffuse CRE sources. (3) The role of magnetic fields was not taken into account. (4) The effect of a halo on the derived diffusion lengths was not considered.

Our study is free of these problems because it is based on properly separated thermal and nonthermal emission distributions across M 31 and M 33 (see Sect. 2). Moreover, the thermal distribution represents the source distribution of the CRE, and the magnetic field properties are known. Although the diffusion lengths on the sky of about 1 kpc in M 31 and 1 – 2 kpc in M 33 (Table 4) are in the range of 0.5 – 3 kpc derived from radio(1.4 GHz) – FIR studies (e.g. Murphy et al. 2006a), our values are more reliable estimates of ldiff. Murphy et al. (2006a, 2008, 2012) found a steep decrease of their “diffusion length” with increasing SFR/area. We note that all galaxies in their sample have a SFR/area > (1.4 ± 0.2) M⊙ Gyr⁻¹ pc⁻², indicating that all these galaxies are expected to have a thick radio disk / halo (see Sect. 6.2). Therefore, their “diffusion length” ldiff includes CRE propagation into the halo and inside the halo parallel to the disk, making their values larger than expected for diffuse propagation (Sect. 5.2 and Sect. 6.1). The authors could not find a mechanism that could explain the steep slope of their l – SFR/area relation, possibly because they neither included the important factor Bint/Bord nor the effect of a thick disk / halo on the observed propagation lengths in their analysis. The strong dependence of their “diffusion length” on SFR/area may reflect the increasing importance of convection of CRE into a thick disk / halo by winds from the SF regions and increasing escape of CRE.

7.2 Implications for the radio – FIR correlation within galaxies

Our study of the radio – FIR correlations in M 31 and M 33 (Fig. 1, Tables 1, 2) confirms that TH emission is about linearly proportional to emission from warm dust (Xu et al. 1992; Hoernes, Berkhuijsen & Xu 1998; Hippelien et al. 2003) and that the correlation between NTH emission at 1.4 GHz and dust emission follows a power law with exponent b < 1 (Hoernes, Berkhuijsen & Xu 1998). We have shown that the smaller value of b in M 33 compared to that in M 31 can be explained by the seemingly larger propagation length of the CRE in M 33 than in M 31. Galaxies with a thick radio disk / halo are expected to have smaller values of b because the combined effect of CRE diffusion in the thin disk and propagation of CRE into and inside the thick disk / halo leads to a smoother NTH distribution than diffusion in the thin disk alone. The superposition of disk and halo emission onto the sky further smoothens the emission that is observed.

In this context, the work of Dumas et al. (2011) on the galaxy M 51 is very interesting. They made pixel-to-pixel correlations between dust emission at 24 µm and 1.4 GHz radio continuum emission for different regions in M 51: central disk, spiral arms, inter-arms and outer parts. Eye estimates of bisector slopes⁶ in their Fig. 11 yield for the exponent c in 1.4 GHz ∝ 24 µm the values c = 0.4 for the central region, c = 0.7 for the spiral arms and c = 1 for

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⁶ Dumas et al. fitted y(x)-regression lines instead of bisectors.
interarm and outer regions. Since the total 1.4 GHz emission contains thermal emission with $c = 1$, the values of $c < 1$ for the central region and the spiral arms imply that the exponent $b$ in the power law between nonthermal emission and dust emission, $NTH_{1.4 \text{GHz}} \propto I_{24\text{\mu m}}^b$, is even smaller than $c$, i.e. $b < 0.4$ for the central region and $b < 0.7$ for the arms. Berkhuijsen et al. (1997) and Fletcher et al. (2011) showed that M 51 has a thick NTH disk / halo extending to a radius of $> 3'$ from the centre. The high SFR of M 51 (Leroy et al. 2008) favours the propagation of CRE into and inside the thick disk / halo, which will influence the observed propagation length. The high magnetic field strengths of 15 – 30 $\mu$G (Fletcher et al. 2011) yield a mean free path of the CRE or diffusion length in M 33 is larger than in M 31. 

3. We showed that in case of random walk diffusion in the thin disk $l_{x,y} = (R_{\text{disk}}/R_{\text{tot}})^{-1} R_{\text{tot}}^{-0.75}$ is expected (Sect. 6.2), leading to a 3 times smaller value in M 33 than in M 31. However, the (deprojected) diffusion length in M 33 is only 30% smaller than that in M 31, i.e. $l_{x,y} = 1.3 \pm 0.2$ kpc and $l_{x,y} = 1.6 \pm 0.3$ kpc at 1.4 GHz, respectively (Table 5). The high value of $l_{x,y}$ in M 33 is due to the thick disk / halo emission seen superimposed onto the disk emission on the sky.

4. For galaxies with a thick disk / halo the apparent propagation length derived from smearing experiments may be considerably larger than the actual diffusion length in the thin disk.

We conclude that the spatial structure, strength and regularity of the magnetic field in a galaxy as well as the occurrence of a galactic wind causing a thick disk / halo determine the propagation of the CRE. Hence, the power-law exponent of the NTH – FIR correlation within a galaxy is the result of the properties of the magnetic field and the existence of a thick disk / halo.

8 SUMMARY

We performed classical correlations between the thermal (TH) and nonthermal (NTH) components of the radio continuum emission at 21 cm (1.4 GHz) and one higher frequency and far infrared (FIR) dust emission from M 31 and M 33. In both galaxies, the TH emission is linearly correlated with the emission from warm dust (24 $\mu$m, 70 $\mu$m), but the power laws of the NTH – FIR correlations have exponents $b < 1$ (Fig. 1; Tables 1, 2). The latter increase with increasing frequency; since the diffusion length of cosmic ray electrons (CRE) gets smaller towards higher frequencies, this is a direct indication that the propagation of CRE away from their places of origin causes $b < 1$. The power-law exponents of the NTH$_{1.4\text{GHz}}$ – FIR correlations for M 33 are significantly larger ($\geq 0.4$) than those for M 31 ($\approx 0.6$), suggesting that the propagation length of the CRE in M 33 is larger than in M 31.

We estimated the diffusion length of the CRE in two ways: (1) by smoothing the NTH emission at the higher frequency until the correlation with the NTH emission at 1.4 GHz had the power-law exponent $b = 1$, and (2) by smoothing the TH emission until the correlation with the NTH emission at the same frequency had $b = 1$, assuming that the TH emission represents the source distribution of the CRE, as is the case in M 33 (Fig. 2). Method 1 yields an estimate of the diffusion length of the older CRE within the thin disk of the galaxy and method 2 estimates the propagation length of the bulk of CRE (i.e. of all ages) resulting from the combination of diffusion in the thin disk and convective propagation (galactic wind) of CRE out of the thin disk into and within a thick disk / halo.

Our estimates for the propagation lengths of CRE are the first ones obtained from the distributions of the NTH components of the radio emission.

Our main results are summarized as follows:

1. In M 31 most of the CRE are confined to a thin NTH disk with an exponential scale height of 0.3 – 0.4 kpc at 1.4 GHz, whereas in M 33 many CRE move out of the thin disk into a thick disk / halo with scale height $h_{\text{back}} \approx 2$ kpc at 1.4 GHz. A thick NTH disk or halo in M 33 (but not in M 31) is consistent with the higher SFR in M 33 and the existence of a vertical magnetic field component.

2. The propagation of CRE from the SF regions into and inside the thick disk / halo, and the superposition of disk and halo emission on the sky lead to larger apparent propagation lengths in M 33 than the diffusion lengths in M 31 along and perpendicular to the disk plane (method 2).

Acknowledgments

We thank Gabi Breuer for getting the tables in the correct format. We thank Marita Krause for useful comments on the manuscript. Critical comments of the referee, Andrew Strong, lead to a better description of the basic assumptions in the paper.

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9 who sadly passed away on 2013 May 29th
