EXTENDED GAUGE MODELS AT e⁺e⁻ COLLIDERS

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ABSTRACT

We summarize the potential of high–energy e⁺e⁻ linear colliders for discovering, and in case of discovery, for studying the signals of extended gauge models. We will mainly focus on the virtual signals of new neutral gauge bosons and on the production of new heavy leptons.

1. Introduction

Despite of its tremendous success in describing the experimental data within the range of energies available today, the Standard Model (SM) based on the gauge symmetry SU(3)C × SU(2)L × U(1)Y is widely believed not to be the ultimate truth. Besides the fact that it has too many parameters which are incorporated by hand, the SM does not unify the electroweak and strong forces in a satisfactory way since the coupling constants of these interactions are different and appear to be independent. Therefore one would expect that a more fundamental theory exists which describes the three forces within the context of a single gauge group [which will contain SU(3) × SU(2) × U(1) as a subgroup and will reduce to this symmetry at low energies] and hence, with only one coupling constant [1, 2]. Recent LEP data show that this can be indeed achieved in Supersymmetric Grand Unified Theories [3].

Two predictions of Grand Unified Theories can have dramatic phenomenological consequences in the O(TeV) energy range:

(i) The unifying group must be spontaneously broken at the unification scale, \( \Lambda_{GUT} \sim 10^{16} \) GeV in order to be compatible with the experimental bounds on the proton lifetime. However, it is possible that the breaking to the SM group occurs in several steps and that some subgroups remain unbroken down to a scale of order 1 TeV. In this case the surviving group factors allow for new gauge bosons with masses not far from the scale of electroweak symmetry breaking.

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ii) The grand unified groups incorporate fermion representations in which a complete generation of SM quarks and leptons can be naturally embedded. In most of the cases these representations are large enough to accommodate additional new fermions which are needed to have anomaly–free theories. It is conceivable that these new fermions [for instance, if they are protected by symmetries] acquire masses not much larger than the Fermi scale. This is very likely, and even necessary if the predicted new gauge bosons are relatively light \[4\].

Besides the SU(5) group [the simplest Lie group containing SU(3) × SU(2) × U(1) as a subgroup and two representations to accommodate the 15 SM fermions], which has no room for relatively light new gauge bosons or new fermions, two other unifying groups have received much attention in recent years, SO(10) \[5\] and the exceptional group E\(_6\) \[6\].

SO(10) is the simplest group in which the 15 Weyl spinors of each SM generation of fermions can be embedded into a single multiplet. This representation has dimension 16 and, in order to be anomaly–free, contains a right–handed neutrino

\[
\begin{array}{ccc}
\nu_e & e & u \\ e_R & d & u_R \\
\end{array}
\]

The gauge group can be spontaneously broken to the SM group at an intermediate scale, two interesting chains of breaking patterns being via SU(5) × U(1) or SU(4) × SU(2) × SU(2), leading to the intermediate symmetries \[7\]:

\[
\begin{align*}
\text{SO}(10) & \rightarrow \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y \times \text{U}(1)_X \rightarrow \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y \\
& \rightarrow \text{SU}(3)_C \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L} \rightarrow \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y
\end{align*}
\]

These chains would induce new right–handed charged currents and/or neutral currents which could eventually be studied at TeV energies. The most general \(Z'_{LR}\) would couple to the current

\[
J'_{\mu LR} = \alpha_{LR} j^u_{\mu L} - j^d_{\mu L} / (2\alpha_{LR})
\]

where the parameter \(\alpha_{LR}\) is defined in terms of the SU(2)\_L and SU(2)\_R couplings as

\[
\alpha_{LR} \equiv \left[ (g_R \cos \theta_W)^2 / (g_L \sin \theta_W)^2 - 1 \right]^{1/2}
\]

and lies in the range \(\sqrt{2/3} \leq \alpha_{LR} \leq \sqrt{2}\).

Another popular unifying group is E\(_6\). It contains SU(5) and SO(10) as subgroups and is the next anomaly–free choice after SO(10). The interest in this group is mainly due the fact that superstring theories, which attempt to unify all fundamental forces including gravity, suggest this theory as a possible four dimensional field theoretic limit \[8\]. [More recently, some alternative scenarios appeared, though.] In E\(_6\), each quark–lepton generation lies in a representation of dimension 27. To complete this representation, twelve new fields are needed in addition to the SM fermion fields. For each family one has two additional isodoublets of leptons, two isosinglet neutrinos and an isosinglet quark with charge \(-1/3\),

\[
\begin{array}{cccc}
\nu_E & \nu_E & \nu \_E & n \\
E & E & D_L & D_R
\end{array}
\]
Moreover, the supersymmetric partners of these fermions will lie in the representation of dimension $27$; the third generation partners of doublets and singlet lepton fields will acquire vacuum expectation values, providing the Higgs sector of the theory.

In the breaking of $E_6$ down to the SM gauge group, two additional $U(1)$ symmetry factors may survive at low energies

$$E_6 \rightarrow SO(10) \times U(1)_\psi \rightarrow SU(5) \times U(1)_\chi \times U(1)_\psi$$

leading to two new neutral gauge bosons. Assuming that only one of them is relatively light, the relevant neutral gauge boson would be $Z' = Z_\chi \cos \beta + Z_\psi \sin \beta$, where $\beta = 0$ and $\pi/2$ correspond to pure $Z_\chi$ and $Z_\psi$, while $\beta = \arctg(-\sqrt{5}/3)$ corresponds to the model \eta in which $E_6$ is directly broken to a rank–5 group at the unification scale in superstrings models \cite{8}.

Several other gauge groups have been considered, based on various theoretical motivations. For instance, schemes of grand unification built–up on large orthogonal groups have been proposed to explain the origin of parity violation in weak interactions \cite{9}. In these models, weak interactions are $P$–invariant but fermions with left–handed and right–handed couplings acquire different masses. They predict a new spectrum of fermions, mirror fermions \cite{10}, which have chiral properties opposite to the ordinary particles. In the simplest version of these models, the gauge symmetry and the symmetry breaking pattern are the same as in the SM; three families of heavy fermions with opposite chiralities are simply added to the SM spectrum

$$\begin{bmatrix}
N_R \\
E_R
\end{bmatrix} \begin{bmatrix}
N_L \\
E_L
\end{bmatrix} \begin{bmatrix}
U_R \\
D_R
\end{bmatrix} \begin{bmatrix}
U_L \\
D_L
\end{bmatrix}$$

(5)

Theoretical arguments based on [weak coupling] unitarity \cite{11} suggest that the masses of these mirror fermions should not exceed a few hundred GeV.

In many extensions of the SM, particles with exotic quantum numbers are also predicted to occur \cite{12}. For instance Leptoquarks [which have baryon and lepton numbers $B=\pm1/3$ and $L=\pm1$ and couple to quark–lepton pairs] are generic predictions of GUTs and point to the necessity of unifying the strong and the electroweak forces. Diquarks [$B=\pm2/3$ and $L=0$] and Dileptons [$B=0$ and $L=\pm2$] are also predicted by some gauge extensions of the SM. Masses for Difermions [Leptoquarks, Diquarks and Dileptons] below the TeV scale are still compatible with present experimental bounds.

The direct search for these new matter and gauge particles and tests of their indirect effects will be a major goal of the next generation of accelerators. In this talk, I will summarize the potential of high–energy $e^+e^-$ linear colliders for these searches, focussing on the virtual effects of new neutral gauge bosons [only the $e^+e^-$ option will be considered, $Z'$ effects can also be searched for in $e^+e^-$ scattering \cite{13} but this is just equivalent to the search in Bhabha scattering] and on the production and the study of new heavy fermions and difermions.
2. New Gauge Bosons

2.1 Physical Set-Up

We will concentrate on the two most theoretically motivated effective theories which lead to an additional neutral gauge boson $Z'$: $\text{SU}(2)_L \times \text{U}(1)_Y \times \text{U}(1)_{Y'}$, originating from the breaking of $E_6$ and Left-Right (LR) models based on the symmetry $\text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)$. A discussion of alternative models can be found in Ref. [19].

Once the hypercharge of the additional $\text{U}(1)$ factor in $E_6$ models [we will assume as usual the GUT relation $g_Y = g_{Y'}$] and the coupling of the $\text{SU}(2)_R$ group [or alternatively $\alpha_{LR}$] in LR models are fixed, the couplings of the $Z'$ boson to fermions is uniquely determined. These gauge couplings will be altered by $Z-Z'$ mixing, though. Indeed, the physical [mass] eigenstates $Z$ and $Z'$ are admixture of the weak eigenstates with a mixing angle $\theta_{\text{mix}}$ [for sufficiently large $Z'$ masses] given by $\theta_{\text{mix}} = PM_{Z'}^2/M_Z^2$, where the parameter $P$ depends on the symmetry breaking pattern. However, this mixing will also alter the couplings of the fermions to the $Z$ boson which have been very accurately measured at LEP1 and found to be in a very good agreement with SM expectations. Several analyses of LEP1 data show that this angle is smaller than $\theta < \sim 0.005$ [14] and it can be safely neglected.

The bound on $\theta_{\text{mix}}$ directly translates into a lower bound on the $Z'$ masses. Since in the theoretically well motivated models, the coefficient $P$ is of order unity, one is led to a lower bound of the order of several hundred GeV on $M_{Z'}$ [especially in the “constrained” case where the Higgs sector is specified]. Stringent limits are also available from a negative [direct] search of a peak in the invariant mass spectrum of $e^+e^-$ pairs at the Tevatron. Depending on the models, masses up to 450 GeV are already excluded by both direct and indirect searches; Tab.1 [15].

| Model | Direct | Indirect unconstrained | Indirect constrained |
|-------|--------|------------------------|---------------------|
| $\chi$ | 425    | 330                    | 920                 |
| $\psi$ | 415    | 170                    | 170                 |
| $\eta$ | 440    | 220                    | 610                 |
| LR    | 445    | 390                    | 1360                |

Tab.1: Present constraints on $M_{Z'}$ [in GeV] from direct and indirect searches for the $E_6$ models $\chi, \eta, \psi$ and for a LR model with $g_L = g_R$; from Ref. [15].

The existence of an extra neutral gauge boson with a mass below the maximal energy of and $e^+e^-$ collider will provide a new resonance which will increase the $e^+e^-$ annihilation cross section by several orders of magnitude. In this case, an $e^+e^-$ collider operating at the resonance peak would be a “$Z'$ factory” allowing to measure the couplings of the $Z'$ to other conventional and new particles with very high precision, a situation comparable to the LEP experiments exploring the $Z$ peak. High-energy $e^+e^-$ colliders would then be ideal instruments to study the properties of the new gauge bosons and to constrain the theories predicting their origin.
Even if a new vector boson is too heavy to be produced as a resonance, it could give rise to virtual effects which are measurable. Indeed, besides mixing with the $Z$, the $Z'$ will participate in the production process of ordinary fermions \[15-18\]

$$e^+ e^- \rightarrow \gamma, Z, Z' \rightarrow f \bar{f} \tag{6}$$

and will affect the cross sections and the various asymmetries through its propagator effects. This situation will be similar to PEP, PETRA and TRISTAN seeing the $Z$ propagator effects at relatively low energies. The clean environment of $e^+ e^-$ colliders allows to probe these virtual effects with high precision and therefore provides a sensitivity to $Z'$ masses considerably higher than the available c.m. energy. In addition, because of the number of observables which can be measured precisely, a detailed investigation of the $Z'$ properties can be performed and its origin can be identified.

In the following we will assume that the $Z'$ is heavier than the c.m. energy, and study its propagator effects on various observables of the process $e^+ e^- \rightarrow f \bar{f}$. We will not discuss $Z'$ searches in the process $e^+ e^- \rightarrow WW$: since the $Z'WW$ coupling is generated only through $Z-Z'$ mixing in the models we are considering [see Ref.\[19\] for alternative models] the effects are very small \[20\].

### 2.2 Signals and backgrounds

Since the effects of new gauge bosons with masses far beyond the production threshold are expected to be rather small, one has to take into account radiative corrections and especially initial state QED radiation which gives very large contributions. Indeed, the radiative tail of the $Z$ boson enhances the SM cross section by a factor 2 to 3 at a c.m. energy of $\sqrt{s} = 500$ GeV, thus completely diluting the expected $O$ (few percent) $Z'$ signals \[18\].

At high energies, a radiative tail arises if the cut $\Delta$ on the photon energy is such that $\Delta = E_\gamma / E_{\text{beam}} > 1 - M_Z^2 / s$. In order to remove the tail, one obviously has to choose a cut $\Delta < 1 - M_Z^2 / s$, which at 500 GeV for instance, would be the case for $\Delta < 0.967$ i.e. $E_\gamma < 242$ GeV. Then, the ratio of the $Z'$ signal to the $Z$ contribution remains of comparable size as it was at the Born level, although it has a different numerical value. This is summarized in Fig. 1a, where the ratio of the hadronic cross section [with the five light quarks] to the muonic cross section is plotted at $\sqrt{s} = 500$ GeV, assuming a $Z'$ [in the model $\chi$ which is equivalent to a LR model with $\alpha_{LR} = \sqrt{2/3}$] with $M_{Z'} = 750$ GeV. One sees that the cut on $E_\gamma$ strongly influences not only the cross section values, but also the difference between observables with and without $Z'$ exchange.

Another important point is that the range of $Z'$ masses and couplings which can be probed directly depends on the experimental errors with which the physical observables are measured. In order to make realistic estimates of these errors and select those observables which will be measured with the best accuracy, the experimental situation should be taken into account.

At high energies the hadronic cross section is several orders of magnitude smaller than at LEP and comparable or even smaller than the $WW$ cross section. Harder
Cuts should be therefore applied against the two-photon backgrounds which rises like logs. Fig.1b shows the energy spectrum [displayed as a function of $\Delta$] for the dominant processes with hadronic final states at $\sqrt{s} = 500$ GeV. To suppress the $\gamma\gamma$ background, a conservative cut $\Delta < 0.7$ [also needed to remove the $Z$ tail] for which typical event rates for a luminosity $\int \mathcal{L} = 20$ fb$^{-1}$ are: $1.6 \times 10^4$ for leptons [with $s$–channel exchange for $e^+e^-$ only] and $5.2 \times 10^4$ for hadrons [without top quarks].

For the lepton detection, similar systematic errors as at LEP can be achieved, but for $e^+e^- \rightarrow q\bar{q}$ the situation is different because of the $WW$ background and one has to attribute an error of $\sim 1\%$ to the selection efficiency. An absolute luminosity error of $1\%$ is feasible if beamstrahlung effects are under control. The estimates for statistical and systematic errors suggest to base the analysis on the variables

$$\sigma^{\text{lept}}, \ R = \frac{\sigma^\text{had}}{\sigma^\text{lept}}, \ A^{\text{lept}}_{\text{FB}}$$

and if longitudinal polarization is available

$$A^{\text{lept}}_{\text{LR}}, \ A^{\text{had}}_{\text{LR}}$$

with the expected systematic errors summarized in Tab.2. [Note that if efficient $b$ tagging is also available, one can use $R_b = \sigma^b/\sigma^\text{had}, A^b_{\text{FB}}$ and $A^b_{\text{LR}}$ to obtain additional information [23, 24]; a detailed analysis of this final state is under way [24].]

| sys. err. | $\Delta \epsilon_{\mu}/\epsilon_{\mu}$ | $\Delta \epsilon_{\text{had}}/\epsilon_{\text{had}}$ | $\Delta A^{\text{lept}}_{\text{FB}}$ | $\Delta A_{\text{LR}}$ | $\Delta \mathcal{L}/\mathcal{L}$ |
|-----------|--------------------------------------|--------------------------------------|--------------------------------|-------------------|-------------------|
|           | 0.5%                                  | 1%                                   | negl.                          | 0.003             | 1%                |

Tab.2: Systematic errors on the observables eqs.(7–8) at $\sqrt{s} = 500$ GeV.
2.3 $Z'$ mass reach and identification

By comparing the measurement of the previous observables with the predictions of various models one can derive the mass of the $Z'$ which can be probed. If all observations happen to be consistent with SM predictions, the lower bounds on $M_{Z'}$ for $E_6$ and LR models are shown at the 95% confidence level as functions of $\cos \beta$ and $\alpha_{LR}$ in Figs.2–3.

**Fig.2:** 95% CL $Z'$ mass reach in $E_6$ and LR models as functions of $\cos \beta$ and $\alpha_{LR}$ respectively, when combining the measurements of $\sigma_{\text{lept}}$, $R$ and $A_{\text{FB}}^{\text{lept}}$ with [thick solid curve] and without [thick dotted curve] systematic errors. The thin lines are the limits when longitudinal polarization is included; from [18].

**Fig.3:** 95% CL $Z'$ mass reach in $E_6$ and LR models for a c.m. energy of 0.5 TeV [solid lines], 1 TeV [dashed lines] and 2 TeV [dotted lines], with [thin lines] and without [thick lines] polarization. A fixed luminosity of 20 fb$^{-1}$ and the errors in Tab.2 are assumed.

In Fig.2, the $Z'$ mass limits are shown for a c.m. energy of 500 GeV and a luminosity $\int \mathcal{L} = 20$ fb$^{-1}$ when combining all the measurements eq.(7–8). In these figures, the leading radiative corrections [see [21] for the full electroweak corrections] a cut $\Delta < 0.7$ and the photon energy and the systematics of Tab.2 are taken into account. To demonstrate the effects of systematic errors, the limits calculated from statisti-
eral errors only are shown. Also are shown the limits obtained by using longitudinal polarization. Without polarization, $Z'$ masses up to 3 TeV can be probed for some parameter values; for LR models longitudinal polarization would allow to extend the mass reach by up to 1 TeV. In the limit of vanishing statistics, the $Z'$ mass reach can be pushed up to 4 TeV for certain parameters.

Fig.3 shows how these limits scale with the c.m. energy, keeping systematic errors as in Tab.2 and a constant luminosity of 20 fb$^{-1}$. Raising the c.m. energy to 1 TeV or 2 TeV allows to extend the $Z'$ mass reach to 5 TeV and 8 TeV respectively. At these energies most of the uncertainties are due to the statistics [the rates scale like $1/s$]; at 2 TeV collider, collecting an integrated luminosity of 300 fb$^{-1}$ will raise these limits by approximately a factor 2. This means that $Z'$ masses up to 16 TeV could be reached at a 2 TeV $e^+e^-$ collider for certain model parameters.

Fig.4: Left: distinction between $E_6$ and LR models for $M_{Z'} = 1.5$ (a) and 2 TeV. Right: determination of the $E_6$ parameter $\cos \beta$ for $M_{Z'} = 1$ (a) and 2 TeV (b). The c.m. energy is $\sqrt{s} = 500$ GeV and the luminosity is 20 fb$^{-1}$. The significance of the measurement of the observables in eq.(7) [hatched area] and eq.(8) [cross–hatched area] have been combined. The results are at the 95%CL; from [18].

If a $Z'$ signal has been observed, the next step would be to try to elucidate its model origin. Assuming that the observations are consistent with the $E_6$ prediction and that $M_{Z'}$ is determined [a model independent approach will be discussed in the next section], Fig.4a shows which values of the LR model parameter $\alpha_{LR}$ can be excluded at the 95% CL at $\sqrt{s} = 500$ GeV for $M_{Z'} = 1.5$ and 2 TeV. The regions of confusion are the hatched [without polarization] and cross–hatched [with polarization] areas. As can be seen, the role of the longitudinal polarization is crucial: for $M_{Z'} = 1.5$ TeV [for 1 TeV, there is almost no confusion except at the point $\alpha_{LR} = \sqrt{2/3}$ corresponding to model $\chi$], only the small cross-hatched area does not allow the distinction between $E_6$ and LR models. This confusion becomes larger with increasing $Z'$ mass, but even for $M_{Z'} = 2.5$ TeV the distinction is still possible.
In a given class of models, the determination of the parameter itself is also possible. This is shown in Fig. 4b for $E_6$ and two $Z'$ mass values 1 and 2 TeV; given that the data are consistent with a certain value of $\cos \beta$ indicated on the abscissa, the 1σ range for $\cos \beta$ is shown in ordinate. Here again, the gain in sensitivity if longitudinal polarization is available is very important, and even for a $Z'$ mass of 2.5 TeV, $\cos \beta$ can be constrained. Similar results can be obtained for LR models.

2.4 Model independent approach

In order not to commit oneself to a particular model, the search for $Z'$ effects in a model independent way is more adequate; in this case, the search would also include new gauge bosons originating from alternative models than $E_6$ and LR. A model independent analysis of $Z'$ signals is being performed by Leike and Riemann [21]; I will summarize the main points below.

Assuming that a new $Z'$ is present, its effect in the $e^+e^- \rightarrow f \bar{f}$ process will be to add an extra contribution to the amplitude that one can write in the general case

$$M(Z') = g_{Z'}/(s - M_{Z'}^2) \, \bar{v}_e \gamma_\alpha (v'_e + \gamma_5 d'_e) u_e \, \bar{u}_f \gamma_\alpha (v'_f + \gamma_5 d'_f) v_f$$  \hspace{1cm} (9)

where $v'_f$ and $a'_f$ [and $g_{Z'}$] are kept free; this contribution can be rewritten as

$$M(Z') = -4\pi/s \, \bar{v}_e \gamma_\alpha (V_{eN} + \gamma_5 A_{eN}^A) u_e \, \bar{u}_f \gamma_\alpha (V_{fN}^A + \gamma_5 A_{fN}^A) v_f$$  \hspace{1cm} (10)

Far below the $Z'$ resonance, only $V_{fN}^N$ and $A_{fN}^N$ given by

$$V_{fN}^N = v'_f \sqrt{g_{Z'}^2/4\pi \times s/(M_{Z'}^2 - s)} \, , \, A_{fN}^N = a'_f \sqrt{g_{Z'}^2/4\pi \times s/(M_{Z'}^2 - s)}$$  \hspace{1cm} (11)

can be constrained and not $a'_f, v'_f, g_{Z'}$ and $M_{Z'}$ separately. In the case of the leptonic observables of eqs.(7–8) [assuming universality] the signal of a $Z'$ is seen if $[22]$

$$\sigma_{lept}^{\text{FB}} \text{ if } (V_{eN}^N/H_1^V)^2 + (A_{eN}^N/H_1^A)^2 \geq 1$$

$$\Delta A_{FB} \text{ if } (V_{eN}^N/H_2^V)^2 - (A_{eN}^N/H_2^A)^2 \geq 1$$

$$\Delta A_{LR} \text{ if } (V_{eN}^N/H_3^V) \times (A_{eN}^N/H_3^A) \geq 1$$  \hspace{1cm} (12)

with the factors $H_{1,2,3}^{V,A}$ depending on the errors on the observables; if only statistical errors are taken into account, one would have

$$H_{1,2,3}^{V,A} \sim \sqrt{\Delta \sigma/\sigma} \, , \, \Delta A_{FB} \, , \, \Delta A_{LR} \sim [\int L/s]^{-1/4}$$  \hspace{1cm} (13)

The limits on the $Z'$ mass that can be obtained in this case will scale as

$$M_{Z'}^{\text{max}} \sim (\sqrt{s}/V_{eN}^N) \, , \, (\sqrt{s}/A_{eN}^N) \sim [\int L/s]^{1/4}$$  \hspace{1cm} (14)

The analysis of Ref.[18] has been repeated within this approach [with, in addition, the inclusion of the full electroweak corrections]. Assuming an energy of 500 GeV and
a luminosity of 20 fb$^{-1}$, and allowing from slightly worse systematical errors than in Ref.\cite{18} as well as a polarization degree of 60\%, a constraint $(V^N_e)^2 + (A^N_e)^2 \lesssim 0.01$ has been obtained in the case where the combined measurement of the three lepton observables is in accord with the SM prediction. Once this constraint is available, one just need to specify the couplings $\nu', a'_e$ and $g_{Z'}$ to derive the limit on the $Z'$ mass in a given model.

The analysis has also been repeated in the case of $b$–quark final states but the limits which have been obtained are worse a consequence of the large systematic errors for $b$ final states and the presence of the additional $Z'bb$–couplings in the fits. However, this particular final state gives new information in special cases.

The unambiguous determination of the $Z'$ parameters [once its signals have been discovered] in a model independent way is a much harder task. For models of extended gauge origin, a method for the determination of the gauge couplings and the reconstruction of the gauge structure of the $Z'$ in a model independent way [the reconstruction of the symmetry breaking pattern is much more complicated since $Z$-$Z'$ mixing is small] has been recently proposed by del Aguila, Cvetič and Langacker \cite{23}:

In the general case, the $Z'$ couplings are specified by five charges [for left– and right–handed quarks and leptons], the overall strength $g_2$ and the coupling $g_{12}$ to the weak hypercharge [these couplings are fixed at the GUT scale, but are renormalized at low energies]; in addition, there is an eighth parameter which is $M_{Z'}$. In $e^+e^-$ collisions, one can probe four independent normalized charges which are defined as

$$P^e_V = \frac{g^e_{L2} + g^e_{R2}}{g^e_{L2} - g^e_{R2}}, \quad P^q_L = \frac{g^q_{L2}}{g^q_{L2} - g^q_{R2}}, \quad P^u_R = \frac{g^u_{R2}}{g^u_{L2}}, \quad P^d_R = \frac{g^d_{R2}}{g^d_{L2}}$$

as well as the ratio involving $g_2$ and the $Z'$ propagator

$$\epsilon_A = (g^e_{L2} - g^e_{R2})^2(s g_2^2)/(4\pi\alpha)(M_{Z'}^2 - s)$$

A 500 GeV collider with $\int L = 20 \text{ fb}^{-1}$, when including only statistical errors and assuming 100\% longitudinal polarization and 100\% $b$ and $t$–tag efficiencies, allows the determination of the five parameters eqs.(15–16) with a precision of 5 to 10\% for $M_{Z'} = 1 \text{ TeV}$. This precision deteriorates as the $M_{Z'}$ increases and for 2 \text{ TeV}, the error is of the order of 100\%. The $Z'$ mass and the absolute value of the couplings $g_2, g_{12}$ can be determined only if the $Z'$ is produced as a resonance, and therefore calls for higher–energy colliders if the $Z'$ is very heavy. However, these parameters could be first determined at the LHC as we will discuss now.

2.5 Comparison with the LHC

At LHC, new gauge bosons can be searched for by looking at a peak in the invariant mass spectrum of lepton pairs. The mass reach will depend on the luminosity of the machine, the specific model from which the $Z'$ originates and since the search relies on the leptonic branching ratio, on the low energy particle content of the model. At a c.m. energy of 14 TeV with a luminosity of 100 fb$^{-1}$, requiring 10 $e^+e^-$ and
$\mu^+\mu^-$ events, masses up to 4.5 TeV can be probed if the $Z'$ decays only into standard particles \cite{13}. Lowering the luminosity by a factor of 10 reduces the mass reach by a factor of 3. These limits will be also lowered if one includes the $Z'$ decay into exotic fermions [which are the 27 and $\overline{27}$ of $E_6$, e.g.] or supersymmetric particles.

The discovery reach of LHC is shown in Tab.3 for the $E_6$ and LR models [with $g_L = g_R$] for two c.m. energies and two integrated luminosities. [Note that similar masses can be reached for charged vector bosons at LHC, while at $e^+e^-$ colliders the mass reach is rather limited]. As can be seen, the mass reach of LHC with full energy and a high luminosity [which will need several years of running] is comparable with the limits discussed previously at a 500 GeV $e^+e^-$ collider\footnote{Of course, while at the LHC the $Z'$ will be produced as a real state, one can only indirectly prove its existence at $e^+e^-$ colliders if the energy is not high enough. Nevertheless, discovering “only” a new neutral current might also be, the least to say, very interesting [c.f. Gargammelle].} for the same high luminosity of $\sim 100$ fb$^{-1}$.

\begin{table}[h]
\begin{center}
\begin{tabular}{|c|c|c|c|c|c|}
\hline
$\sqrt{s}$ [TeV] & $int. L$ [fb$^{-1}$] & $\chi$ & $\psi$ & $\eta$ & LR \\
\hline
10 & 400 & 3040 & 2910 & 2980 & 3150 \\
14 & 100 & 4308 & 4190 & 4290 & 4530 \\
\hline
\end{tabular}
\end{center}
\end{table}

\textbf{Tab.3:} $Z'$ mass reach reach at the LHC with 10 $Z' \to e^+e^- + \mu^+\mu^-$ events; from Ref.\cite{15}.

Due to the difficult environment at hadron colliders, the identification of the origin of the $Z'$ is limited to masses below 1 to 2 TeV. The forward backward asymmetry in the muon channel, the ratio of cross sections in different rapidity bins and rare processes such as $Z' \to WW$ and $pp \to Z' + W/Z$ are the main probes of the $Z'$ nature \cite{17}. For a model independent determination of the $Z'$ parameters, using the same notation as above, four normalized charges can be probed at the LHC [in addition to the direct measurement of the $Z'$ mass; the coupling $g_2$ can be determined from the total $Z'$ width, assuming that only decays into standard particles are present]

\begin{align}
\gamma_L^e &= \frac{(g_{L L}^e)^2}{(g_{L L}^e)^2 + (g_{R R}^e)^2}, \quad \gamma_L^q = \frac{(g_{L L}^q)^2}{(g_{L L}^q)^2 + (g_{R R}^q)^2}, \quad U = \frac{(g_{R R}^u)^2}{(g_{L L}^u)^2}, \quad D = \frac{(g_{R R}^d)^2}{(g_{L L}^d)^2} \quad (17)
\end{align}

For $M_{Z'} = 1$ TeV, $U/D$ can be determined with a precision of typically 20% and $\gamma_L^q$ with less than 10%; the error on $\gamma_L^e$ is large. Recalling that a 500 GeV LC is sensitive to the normalized charges (15) and the ratio $g_{Z'}^2/M_{Z'}^2$, it is only the combination of the information from the LHC and a 500 GeV LC which would allow the model independent determination of all the parameters of the $Z'$. Therefore, the LHC and a 500 GeV linear collider are complementary. Increasing the energy of the LC will increase not only the $Z'$ mass reach but also the precision with which different models are discriminated and the $Z'$ couplings are measured. The ultimate tests will be of course made only when a LC with a c.m. energy comparable to the $Z'$ mass will be available; one would then produce the $Z'$ as a resonance and dissect the $Z'$ boson to gain information on the physics at the Grand Unification scale.
3. New Matter Particles

3.1 New Fermions

The new fermions predicted by extended gauge models are exotic with respect to their transformation under the SM group \[24\]. Contrary to sequential 4th generation heavy fermions [with the neutrino having a right-handed component for its mass to be generated in a gauge invariant way] exotic fermions have the usual lepton and baryon quantum numbers but non–canonical SU(2)\(_L\) \(\times\) \(U(1)\)\(_Y\) quantum numbers, e.g. the left–handed components are in weak isosinglets and/or the right–handed components in weak isodoublets.

Except for singlet neutrinos, the new fermions couple to the photon and/or to the electroweak gauge bosons \(W/Z\) [and for heavy quarks, to gluons] with full strength; these couplings allow for pair production with practically unambiguous cross sections. If they have non–conventional quantum numbers, the new fermions will mix with their SM partners. This mixing will give rise to new currents which determine the decay properties of the heavy fermions and allow for their single production. If the mixing between different generations [which induces FCNC at tree–level] is neglected, the mixing pattern simplifies. The few remaining angles are restricted by LEP and low energy experiment data to be smaller than \(\mathcal{O}(0.05 – 0.1)\) \[24\]. Note that LEP1 sets bounds of order \(\sim M_Z/2\) on the masses of these particles [stronger mass bounds from Tevatron can be set for quarks]; masses up to \(M_Z\) might be probed at LEP2.

The heavy fermions decay through mixing into massive gauge bosons plus their ordinary light partners, \(F \to fZ/fW\) \[25, 26, 27\]. For masses larger than \(M_W(M_Z)\) the vector bosons will be on–shell. For small mixing angles, \(\zeta < 0.1\), the decay widths are less than 10 MeV (GeV) for \(m_F = 0.1(1)\) TeV. The charged current decay mode is always dominant and for \(m_F \gg M_Z\), it has a branching fraction of 2/3.

**Fig.5**: Pair production of mirror and vector neutral and charged leptons a 1 TeV \(e^+e^-\) collider: a) cross sections and (b) angular distributions; from \[28\].
If their masses are smaller than the beam energy, the new leptons can be pair-produced in $e^+e^-$ collisions, $e^+e^- \rightarrow LL$, through $s$–channel gauge boson exchange \[23, 26, 27\]. The cross sections are of the order of the point–like QED cross section and therefore, are rather large. They are displayed in Fig. 5a for mirror [they are the same for sequential] and vector isodoublet leptons at a c.m. of 1 TeV. For a luminosity of 100 fb$^{-1}$ one expects $10^3$–$10^4$ events. Because of their clear signatures [$e^+e^-ZZ$ and $e^+e^-WW$ final states for charged and neutral leptons respectively], the detection of these particles is straightforward in the clean environment of $e^+e^-$ colliders, and masses very close to the kinematical limit can be probed \[28\].

The angular distributions are shown in Fig.5b, and one notes that they are symmetric for vector leptons leading to $A_{FB} = 0$; for mirror fermions $A_{FB}$ is sizeable and has the opposite sign as for sequential leptons. [Here, $Z'$ exchange is neglected.] The total cross sections, angular distributions and the polarization of the final particles allow to discriminate between different types of fermions. Charged leptons can also be pair–produced at $\gamma\gamma$ colliders, $\gamma\gamma \rightarrow LL$, and for relatively small masses, the cross sections can be larger than in the $e^+e^-$ mode.

Note that right–handed neutrinos in LR models can also be produced in pairs through the exchange of heavy $Z'$ and $W'$ bosons if the masses are not too large. For $M_{Z'} = M_{W'} = 1.5$ TeV, the cross sections are of the order of a few fb at $\sqrt{s} = 500$ GeV sufficiently below the kinematical threshold \[26\].

For not too small mixing angles, one can also have access to the new fermions via single production in association with their light partners \[25, 26, 27\]. The rate for this type of process is more model dependent but can substantially increase the reach of a given accelerator. In $e^+e^-$ collisions, this proceeds only via $s$–channel $Z$ exchange [if the $Z'$ is too heavy] in the case of quarks and second/third generation leptons, leading to small rates. For the first generation leptons, however, one has additional $t$–channel exchanges [$W$ channel for $N$ and $Z$ channel for $E$] which increase the cross sections by several orders of magnitude at high energies.

The cross section for left– and right–handed neutral and charged leptons is shown in Fig. 6a at a c.m. energy of 1 TeV for mixing angles $\zeta_{L,R} = 0.1$. As can be seen, the cross sections are very large especially for $N_L$ where they can reach the picobarn level. For the charged leptons they are one order of magnitude smaller, a consequence of the smaller NC couplings compared to the CC couplings. For smaller mixing angles the rates have to be scaled down correspondingly; however, even for $E$ and $N_R$ requiring 10 events with a luminosity $\int \mathcal{L} = 100$ fb$^{-1}$, one can probe $\zeta$ values one order of magnitude smaller for $m_L = 800$ GeV \[28\].

The angular distributions are shown in Fig.6b: it is clear that one can easily distinguish between neutrinos with left- and right-handed couplings and of Dirac and Majorana nature. A further distinction [especially for $E_{L,R}$ which have the same cross section and distribution, a consequence of the fact that the vector coupling of the electron to the $Z$ boson is small] can be made with the longitudinal polarization of the initial beams, and also with the polarization of the final leptons.
To fully reconstruct the heavy lepton masses, the best signals consist of an $e^+e^-$ pair and two jets for the charged lepton and an $e^\pm$, a pair of jets and missing momentum for the neutral lepton; the branching ratios are 23% and 43% respectively. In the case of $E$, the main backgrounds are: $e^+e^- \rightarrow e^+e^-Z \rightarrow e^+e^-jj$, $e^+e^- \rightarrow ZZ$, $e^+e^- \rightarrow t\bar{t} \rightarrow W^+W^-jj$ and $\gamma\gamma \rightarrow e^+e^-q\bar{q}$. In the case of $N$, the backgrounds are: $e^+e^- \rightarrow e\nu W$, $e^+e^- \rightarrow WW \rightarrow e^\pm\nu jj$ and $\gamma\gamma \rightarrow e(e)q\bar{q}$. These backgrounds can be eliminated or reduced by applying the cuts shown in Table 4. After these cuts, no events from heavy flavor production or from the $\gamma\gamma$ backgrounds would survive; the backgrounds from vector boson production can be suppressed to a very low level, while those from single $W/Z$ production can be a bit higher.

A full simulation \[27\] of the signal and backgrounds has been performed using PYTHIA, for a model detector [an upgraded LEP detector] to quantify the discovery limits that can be obtained. This simulation was done assuming a c.m. energy of 500 GeV and an integrated luminosity of 50 fb$^{-1}$. The signal and background cross sections after applying cuts are shown in Tab.4 [note that at $\sqrt{s} = 500$ GeV, the cross sections are practically the same at 1 TeV, because the dominant contribution comes from the $t$-channel exchange] for heavy leptons with masses of 350 GeV and with $\zeta = 0.05$ for $E$ and $\zeta_L = 0.025$ for $N$. For these $\zeta$ and mass values, the signal peaks stand out clearly from the background events. For $m_L = 350$ GeV and requiring that the signal over the square-root of the background is larger than unity, one can probe mixing angles down to $\zeta \sim 0.005$ for neutral leptons and $\zeta \sim 0.03$ for charged leptons. For $m_E \sim 450$ GeV, only slightly smaller $\zeta$ values can be probed. The situation is
much more favorable for $N_L$, the cross section being one order of magnitude larger. At $\sqrt{s} = 1$ TeV, these numbers for $m_L$ and $\zeta^2$ can be improved by a factor of two.

| Process          | $E^\pm e^\mp$ | $e^\pm e^- Z$ | $ZZ$ |
|------------------|----------------|---------------|------|
| $\sigma$ [fb]    | 9.5            | 4960          | 615  |
| $\times$ B.R.    | 2.19           | 3470          | 28.8 |
| one $e^+e^-$ pair| 1.74           | 93.0          | 23.0 |
| $330 < M_E < 370$| 1.56           | 11.7          | 5.30 |
| $85 < M_Z < 105$ | 1.41           | 5.84          | 2.87 |
| $|M_H - M_Z| > 12$| 1.39           | 5.18          | 1.02 |
| $\cos \theta_H < 0.5$ | 1.33 | 4.32     | 0.56 |
| $f(M_E, \cos \theta_Z)$ | 1.30 | 1.90   | 0.43 |
| kinem. cuts      | 1.30           | 1.55          | 0.39 |

| Process          | $N\nu$ | $e\nu W$ | $WW$ |
|------------------|--------|----------|------|
| $\sigma$ [fb]    | 490    | 8610     | 2600 |
| $\times$ B.R.    | 13.7   | 5823     | 1140 |
| one $e$          | 13.2   | 198      | 883  |
| $330 < M_N < 370$| 12.5   | 11.9     | 100  |
| $70 < M_W < 90$  | 12.3   | 10.3     | 70.3 |
| $M_{\nu} > 120$  | 11.8   | 10.0     | 7.93 |
| $\cos \theta_{\nu} < 0.5$ | 11.7 | 10.0     | 7.80 |
| $f(M_N, \cos \theta_Z)$ | 11.7 | 10.0   | 7.80 |
| kinem. cuts      | 11.7   | 10.0     | 4.13 |

Tab.4: Cross sections for heavy lepton single production and for the main backgrounds at $\sqrt{s} = 0.5$ TeV after successive applications of cuts; $m_L = 350$ GeV and $\zeta = 0.025(0.05)$ are chosen for $N(E)$ and the masses are in GeV.

Note that heavy fermions cannot be produced singly at $\gamma\gamma$ colliders [at least in a 2 $\rightarrow$ 2 process]; heavy neutral and charged leptons can be produced in $e\gamma$ collisions in association with massive gauge bosons [24], however only smaller masses can be probed and the rates are not much larger than in $e^+e^-$ collisions.

Heavy leptons can also be searched for indirectly. A prominent example is the search for Majorana neutrinos in the reaction $e^-e^- \rightarrow W^- W^-$ which is similar in nature to neutrinoless $\beta\beta$ decay. This process has been discussed in [30, 31, 32]. In the breaking of $E_6$ down to the SM group through the SO(10) chain, one could assume that the right–handed $W$ bosons [as well as the doubly charged Higgs bosons] are very heavy but the two additional isosinglet neutrinos of $E_6$ have masses in the $O(\text{TeV})$ range [31]. It has been shown that this scenario is still open to experimental detection through the process $e^-e^- \rightarrow W^- W^-$ where at least the lightest of the two Majorana neutrinos is exchanged. For energies well above the mass of the $W$ but below that of the Majorana neutrino [where no doubly charged Higgs boson exchange is needed for unitary reasons], the cross section is proportional to $s^2$ and therefore can be observed even for very small values of the suppressing mixing angles [31]. The use of polarization enhances the rates and the spectacular back–to–back $W$ pair allows effective background suppression, and even a moderate signal may lead to convincing discovery. It is not yet clear whether constraints from neutrinoless $\beta\beta$ decay are not in conflict [32] with this scenario, though. A more detailed analysis is required.

Finally, heavy quarks can also be pair produced in $e^+e^-$ and $\gamma\gamma$ collisions for masses up to the beam energy. However, they can be best searched for at hadron colliders where the production processes, $gg/q\bar{q} \rightarrow Q\bar{Q}$, give very large cross sections: at LHC with $\sqrt{s} = 14$ TeV and a luminosity of 10 fb$^{-1}$ quark masses up to 1 TeV can be reached [28]. However, and $e^+e^-$ linear collider would be needed to study their properties, a situation similar to the one of the top quark.
3.2 A Test of Superstrings inspired $E_6$ models

In superstrings inspired $E_6$ models, the superpartners of the third generation doublet $L_3, \bar{L}_3$ and singlet $n_3$ fields provide the Higgs sector of the theory; while the doublets [with vacuum expectation values $<\tilde{L}_3> = v$ and $<\tilde{L}_3> = \bar{v}$] give masses to the $W/Z$ bosons, the singlet [with a vev $<\tilde{n}_3> = x$] gives mass to the $Z'$ [if the vev $x$ is large, the $Z'$ will be very heavy and the model in principle reduces to the minimal supersymmetric model at low energies]. The fermionic third generation will then mix with the gauginos to form the neutralino fields. The first two generations of leptons $L_{1,2}, \bar{L}_{1,2}$ and $n_{1,2}$ obtain their masses [the SO(10) neutrino acquire mass through a different mechanism] from the superpotential $W_{\text{lep}} = \sum \lambda_{ijk} L_i \bar{L}_j n_k$ with $i, j, k = 1, 2, 3$. Assuming that $\lambda_{33i} = \lambda_{33} = \lambda_{333} = 0$ these leptons will not mix with the gaugino–higgsino sector. The previous superpotential leads to a mass matrix, which under the assumption that no singlet scalar gets a vev much larger than $10^9$ GeV, gives a very strong constraint on the mass of two neutral leptons. This will provide a decisive test of these models as has been discussed by Drees [33] and is summarized below.

While some entries of the mass matrix are proportional to the masses of the charged leptons which can be made very large since they are $\propto x$, other entries must be of $\mathcal{O}(100)$ GeV or less if the $\lambda$ couplings have no Landau pole at low energies. Since the determinant of the matrix will be proportional to the square of the product of the three vev’s, four eigenvalues are large [$\propto m_{L_1^\pm}$ with $i = 1, 2$] and two eigenvalues must be small

$$\det \mathcal{M} \sim (xv\bar{v})^2 \rightarrow m_{L_{1,2}} \sim \lambda^2 v\bar{v} / m_{L_i^\pm}$$  \hspace{1cm} (18)

The upper bound on the light neutral lepton mass will therefore decrease as the charged lepton masses increase. Therefore, the negative search of charged heavy leptons or associated $L_1^0 L_3^0$ production in $e^+e^-$ collisions will place an upper bound on the mass of the light leptons. For instance at a 1.5 TeV collider, the mass of the lightest lepton should be smaller than 30 GeV for $\lambda_{\text{max}} = 0.85$ [so that the model remains weakly interacting up to high scales] and $\bar{v}/v = 1$ [stronger bounds are obtained for higher values] in the case of a negative search. This means that, since the exotic leptons have R–odd parity, $L_1^0$ will be the lightest supersymmetric particle. If R–parity is conserved, $L_1^0$ must be therefore absolutely stable and will contribute to the density of Big Bang relics and might overclose the universe. Requiring that the annihilation cross section for $L_{1,2}$ pairs is large enough not to lead to cosmological problems, the couplings of $L_1^0$ to the light Higgs boson should be large, and since $m_h > 2m_L$ the decay $h \rightarrow L_1^0 L_1^0$ is kinematically possible, the branching branching ratio of invisible Higgs decays will be larger than 50%.

Thus, either one discovers a light Higgs [in SUSY models the lightest boson has a mass smaller than $\sim 150$ GeV] with a large invisible branching at a 300 GeV $e^+e^-$ collider, or find a heavy lepton in the channel $e^+e^- \rightarrow L_1^0 L_3^0$ at a 1.5 TeV $e^+e^-$ collider. Otherwise, the model would be ruled out. Therefore, $e^+e^-$ colliders could provide a decisive test of superstring inspired models.
3.3 Difermions

In addition to the usual couplings to gauge bosons, difermions [12] have couplings to fermion pairs which determine their decays [here also one can neglect the couplings between different generations to prevent FCNC at tree-level]. These couplings are a priori unknown. In the case of leptoquarks (LQ) for example, a systematic description of their quantum numbers and interactions can be made by starting from an effective lagrangian with general SU(3)×SU(2)×U(1) invariant couplings and conserved B and L numbers. This leads to the existence of 5 scalar and 5 vector LQ’s with distinct SM transformation properties. In general, present data constrain difermions to have masses larger than 50–150 GeV.

Leptoquarks can be produced in pairs at $e^+e^-$ colliders through gauge boson exchange; significant $t$-channel quark exchange can be present in some channels if the quark-lepton-LQ couplings are not too small. Depending on the charge, the spin and isospin of the LQ, the cross sections can vary widely [at $\sqrt{s} = 500$ GeV [14] between $\sim 10$ fb and $\sim 3.5$ pb]. Through the signatures of 2 leptons plus 2 jets, these states are accessible for masses smaller than the beam energy. The study of the various final states and the angular distributions would allow the determination of the quantum number of the LQ’s as in the case of exotic fermions [28]. LQ’s can also be pair produced in $\gamma\gamma$ collisions; depending on the LQ charge, the cross sections can be much larger or much smaller than for charged leptons.

Single production of scalar and vector leptoquarks can also take place in the $e^+e^-$, $e^-e^-$, $e\gamma$ and $\gamma\gamma$ modes of the collider [13]. The kinematical reach is thus extended to $\sqrt{s}$ but the production rates are suppressed by the unknown LQ coupling to quark-lepton pairs. At a 1 TeV $e^+e^-$ collider with $\int \mathcal{L} = 60$ fb$^{-1}$, one reaches masses close to $\sqrt{s}$ [i.e. 1 TeV for $e^+e^-$ and $e^-e^-$, $\simeq 0.9$ TeV for $e\gamma$ and $\simeq 0.8$ TeV for $\gamma\gamma$] [28]. Since they are strongly interacting particles, LQ’s can be produced at hadron colliders with very large rates. At LHC with 100 fb$^{-1}$ the search reach for scalar/vector LQ’s is 1.4/2.2 TeV if one assumes a branching fraction of unity for the $eejj$ final state [28]. Therefore, LQ’s can be best searched for at the LHC; however, $e^+e^-$ colliders could provide very important informations on their properties.

Dileptons can be pair produced in $e^+e^-/\gamma\gamma \to X^+X^-$ and masses up to $\sqrt{s}/2$ can be probed; the rates [especially in $\gamma\gamma$ collisions because of the charge] are very large and the signatures [four leptons] are spectacular. Dileptons can also be singly produced in the four modes of the collider. In particular, in the $e^-e^-$ mode di-electrons can be produced as s–channel resonances. At a 1 TeV collider, scalar and vector dileptons can be observed up to masses of $\sim 0.9$ TeV in the $e\gamma$ mode even for couplings to lepton pairs as small as $10^{-3}$ the electromagnetic coupling; in the $e^+e^-$ and $\gamma\gamma$ modes, dileptons can be observed for couplings an order of magnitude larger.

Finally, diquarks can be pair produced in $e^+e^-$ and $\gamma\gamma$ collisions for masses smaller than $\sqrt{s}/2$ with appreciable rates, with a signal consisting of an excess of 4 jets events [28]. They can be also pair produced at hadron colliders, either in pairs or singly [for the first generation] if the couplings to quark pairs is not too small. However, since the signals consist only in jets, the large QCD backgrounds might be a problem.
4. Summary

We have summarized the discovery potential of high–energy $e^+e^-$ linear colliders with respect to the new matter particles and the new gauge bosons predicted by gauge extensions of the Standard Model.

If the energy can be raised high enough, the $e^+e^-$ collider will operate as a $Z'$ factory; the event rates will be very high and the properties of the $Z'$ can be studied in great details. A heavy $Z'$ boson, even if its mass is substantially larger than the available center of mass energy, will manifest itself through its propagator effects in the process $e^+e^- \to$ fermions, producing potentially sizeable effects on the observables $\sigma^{\text{lept}}, R = \sigma^{\text{had}}/\sigma^{\text{lept}}, A_{\text{FB}}^{\text{lept}}$ and if longitudinal polarization is available $A_{\text{LR}}^{\text{lept}}$ and $A_{\text{LR}}^{\text{had}}$. Masses up to 6 times the c.m. energy of the collider can be probed for the expected luminosities. If a $Z'$ with mass below 3 TeV is discovered at LHC, even a 500 GeV $e^+e^-$ collider would give valuable contributions to its detailed investigation by allowing the distinction between different classes of models and the determination of the model parameters. The two types of colliders would then provide complementary information.

$e^+e^-$ colliders are well suited machines for the search of new leptons. These particles can be produced with large rates if their masses are smaller than the beam energy. They can also be singly produced in association with their standard light partners if the mixing angles are not prohibitively small; one can then reach masses close to the total energy of the collider. The signatures have clear characteristics so that the detection of these particles should not be difficult in the clean environment of $e^+e^-$ colliders. Since they are strongly interacting particles, quarks, leptoquarks and diquarks will be produced at the LHC with very large rates. However, because of the difficult hadronic jet background, the signals would be hard to analyze in detail. $e^+e^-$ colliders would provide the ideal framework for highly precise analyses of the properties of these new exotic particles if they are found at the hadron colliders.

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