Effective Simulation of Quantum Entanglement Based on Classical Fields Modulated with Pseudorandom Phase Sequences

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We demonstrate that \( n \) classical fields modulated with \( n \) different pseudorandom phase sequences can constitute a \( 2^n \)-dimensional Hilbert space that contains tensor product structure. By using classical fields modulated with pseudorandom phase sequences, we discuss effective simulation of Bell states and GHZ state, and apply both correlation analysis and von Neumann entropy to characterize the simulation. We obtain similar results with the cases in quantum mechanics and find that the conclusions can be easily generalized to \( n \) quantum particles. The research on simulation of quantum entanglement may be important, for it not only provides useful insights into fundamental features of quantum entanglement, but also yields new insights into quantum computation.

The phenomenon of quantum entanglement is perhaps the most fascinating and important feature of quantum theory. On the one hand, quantum entanglement underlies many of the most curious and controversial aspects of the quantum mechanical description of the world. On the other hand, quantum entanglement is widely appreciated as the essential ingredient of a quantum computer. Quantum entanglement is first introduced by Einstein, Podolsky, and Rosen as most noticeable the EPR paradox \[4\]. Quantum entanglement is widely appreciated as the essential ingredient of a quantum computer. Quantum entanglement is first introduced by Einstein, Podolsky, and Rosen as most noticeable the EPR paradox \[4\], which is at the origin of quantum nonlocality. Bell proposed a remarkable inequality imposed by a local hidden variable theory, which enables an experimental test on the quantum nonlocality \[5\]. Some researchers have yet used classical approaches to simulate quantum entanglement theoretically and experimentally \[6, 7\]. However, the simulation require an exponentially growth of physical resources with the number of quantum particles. This limitation could be traced back to a lack of a rigorous tensor-product structure in terms of subsystems \[6, 7\].

In this letter, we explore an effective simulation of quantum states based on classical fields modulated with pseudorandom phase sequences (PPSs). One essential difference between classical fields and quantum states is that \( n \) 2-state quantum particles could constitute a \( 2^n \)-dimensional Hilbert space that contains tensor product structure, while \( n \) classical fields can not. In our proposal, by introducing the PPSs into the classical fields, we demonstrate that \( n \) classical fields modulated with \( n \) different PPSs can also constitute a similar \( 2^n \)-dimensional Hilbert space that contains a tensor product structure. In addition, we discuss classical simulation of Bell states and GHZ state, apply both the correlation analysis and von Neumann entropy to characterize them. We obtain similar results with the cases in quantum mechanics and find that these conclusions can be easily generalized to \( n \) quantum particles.

The PPSs in our proposal derive from orthogonal pseudorandom sequences (PSs), which have been widely applied to Code Division Multiple Access (CDMA) communication technology as a way to distinguish different users \[11, 12\]. A set of PSs is generated by using a shift register guided by a Galois field GF(\( p \)), that satisfies orthogonal, closure and balance properties \[12\]. In this letter, we consider an \( m \)-sequence of period \( N−1 \) \((N = p^s) \) generated by a primitive polynomial of degree \( s \) over GF(\( p \)) and apply its to 4-ary phase shift modulation, which has been a well-known modulation format in wireless and optical communications \[11\]. A scheme is proposed to generate a PPS set \( \Xi = \{\lambda(0), \lambda(1), \ldots, \lambda(N−1)\} \) over GF(4). \( \lambda(0) \) is an all-0 sequence and other sequences can be generated by using the method as follows: (1) given a primitive polynomial of degree \( s \) over GF(4) \[13\], a base sequence \( \lambda(0) \) is generated by using Linear Feedback Shift Register; (2) other sequences are obtained by cyclic shifting of the base sequence; (3) by adding zeros to the sequences, the occurrence of any element equals to \( 4^{s−1} \); (4) mapping the elements of the sequences to \([0, \pi/2] \): 0 mapping 0, 1 mapping \( \pi/2 \), 2 mapping \( \pi \), and 3 mapping \( 3\pi/2 \).

We note the similarities between the Maxwell equation and the Schrödinger equation and the feasibility of using polarization or transverse modes of classical field to simulate quantum states \[14, 16\]. We first consider the two orthogonal modes (polarization or transverse), which are denoted by \(|0\rangle \) and \(|1\rangle \) respectively, as the classical simulation of quantum bit (qubit) \(|0\rangle \) and \(|1\rangle \) \[7, 16\]. Thus any qubit state \(|\varphi⟩ = \alpha |0⟩ + \beta |1⟩ \) can be simulated by a corresponding classical mode superposition field expressed as \(|\psi⟩ = \alpha |0⟩ + \beta |1⟩ \), where \(|\alpha|^2 + |\beta|^2 = 1 \) \((\alpha, \beta \in \mathbb{C}) \). Obviously, all the mode superposition fields could span a Hilbert space, where we can employ unitary transformations to transform the mode status.

Further, we discuss the classical fields modulated with multiple PPSs to simulate multiparticle quantum system. Chosen two PPSs of \( \lambda^{(a)} \) and \( \lambda^{(b)} \) from the set \( \Xi \), any two fields modulated with the PPSs can be expressed as
following
\[ |\psi_a\rangle = e^{i\lambda_a} (\alpha_a |0\rangle + \beta_a |1\rangle), \]
\[ |\psi_b\rangle = e^{i\lambda_b} (\alpha_b |0\rangle + \beta_b |1\rangle). \] (1)

According to the properties of the PPSs and the Hilbert space, we can define the inner product of the two fields and obtain the orthogonal property in our simulation,
\[
\langle \psi_a | \psi_b \rangle = \frac{1}{N} \sum_{k=1}^{N} e^{i\lambda_k (\alpha_a^* \alpha_b + \beta_a^* \beta_b)} = \begin{cases} 1, & a = b, \\ 0, & a \neq b. \end{cases} \] (2)

where \( \lambda_k^{(a)}, \lambda_k^{(b)} \) are the \( k \)-th units of \( \lambda^{(a)}, \lambda^{(b)} \), respectively. The orthogonal property supports to construct the tensor product structure of multiple fields.

Assume two Hilbert spaces \( w \) and \( v \) spanned by the fields \( |\psi_a\rangle \) and \( |\psi_b\rangle \), any linear combinations of the elements in the direct product space of \( w \otimes v \) remain in the same space. We define the two orthogonal modes modulated with the PPS \( \lambda^{(a)} \) as the orthonormal bases of the space of \( w \), expressed as \( |0_a\rangle \equiv e^{i\lambda^{(a)}} |0\rangle \) and \( |1_b\rangle \equiv e^{i\lambda^{(b)}} |1\rangle \). Using the same notion, the orthonormal bases of the space of \( v \) are expressed as \( |0_b\rangle \equiv e^{i\lambda^{(b)}} |0\rangle \) and \( |1_b\rangle \equiv e^{i\lambda^{(b)}} |1\rangle \). The four orthonormal bases are thus independent and distinguishable. Then the orthonormal bases of the direct product space of \( w \otimes v \) can be expressed as \( \{ |0_a \otimes |0_b\rangle, |0_a \otimes |1_b\rangle, |1_a \otimes |0_b\rangle, |1_a \otimes |1_b\rangle \} \). Further, we can obtain the following tensor product properties [1]: (1) for any scalar \( z \), the elements \( |\psi_a\rangle \otimes |\psi_b\rangle \) in the spaces of \( w \) and \( v \), respectively, satisfy \( z (|\psi_a\rangle \otimes |\psi_b\rangle) = (z |\psi_a\rangle) \otimes (z |\psi_b\rangle) = (z |\psi_a\rangle) \otimes (z |\psi_b\rangle) = |\psi_a\rangle \otimes (z |\psi_b\rangle) \); (2) in the space of \( w \otimes v \), the direct product of the combinations of elements equals to the combination of the direct products of elements, \( (|\psi_a\rangle + |\psi'_a\rangle) \otimes (|\psi_b\rangle + |\psi'_b\rangle) = |\psi_a\rangle \otimes |\psi_b\rangle + |\psi'_a\rangle \otimes |\psi'_b\rangle + |\psi'_a\rangle \otimes |\psi_b\rangle + |\psi'_a\rangle \otimes |\psi'_b\rangle \). Using the same notion, we can construct a \( 2^n \)-dimensional direct product space of \( |\psi\rangle \otimes \ldots \otimes |\psi_n\rangle \) by using the mode superposition of \( n \) classical fields \( |\psi_1\rangle, \ldots, |\psi_n\rangle \) modulated with \( n \) PPSs.

Now we first discuss the correlation analysis of the classical simulation of quantum entanglement and demonstrate the nonlocal correlation with Bell’s inequality and equality criterion. As a contrast, we first study the product state of two classical fields expressed as Eq. [1], which corresponds to no entanglement in quantum mechanics. In order to perform the correlation analysis, we need an orthogonal projection measurement. The result of projection measurement on \( |\psi\rangle \) can be obtained
\[
\mathcal{P}(\theta) = \langle \psi | P(\theta) |\psi\rangle = \begin{pmatrix} \alpha^* & \beta^* \\ 0 & e^{-i\theta} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \beta \alpha^* e^{i\theta} + \alpha \beta^* e^{-i\theta}. \] (3)

For convenience, the superposition coefficients \( \alpha, \beta \) are given by \( 1/\sqrt{2} \), we obtain \( \mathcal{P}(\theta) = \cos(\theta) \). By using the projection measurement, we propose a correlation analysis scheme in which the projection measurement of the basis \( |\pm\rangle = |0\rangle \pm |1\rangle \) is performed on each of the fields, as shown in Fig. 1. The projection results are the measurement results in the \( k \)-th sequence unit for the fields \( |\psi_a\rangle \) and \( |\psi_b\rangle \), respectively. The PPSs of \( \lambda^{(a)} \) and \( \lambda^{(b)} \) do not affect the measurement results because they only contribute to the total phases. We can easily obtain the correlation function \( E(\theta_a, \theta_b) = \cos \theta_a \cos \theta_b \), which is similar to the case of quantum product state.

Further, we consider that the modes \( |1\rangle \) of the fields \( |\psi_a\rangle \) and \( |\psi_b\rangle \) are exchanged by using a mode exchanger constituted by mode splitters and combiners, as shown in Fig. 2. We obtain the fields as following
\[
|\psi'_a\rangle = \frac{1}{\sqrt{2}} e^{i\lambda^{(a)}} (|0\rangle + e^{i\gamma^{(a)}} |1\rangle),
|\psi'_b\rangle = \frac{1}{\sqrt{2}} e^{i\lambda^{(b)}} (|0\rangle + e^{i\gamma^{(b)}} |1\rangle), \] (4)

where the relative phase sequences (RPSs) \( \gamma^{(a)} = -\gamma^{(b)} = \lambda^{(b)} - \lambda^{(a)} \), and \( \gamma^{(a)} + \gamma^{(b)} = 0 \). We obtain the results of the fields in the projection measurement \( \mathcal{P}(\theta_a, k) = \cos (\theta_a + \gamma^{(a)}) \mathcal{P}(\theta_b, k) = \cos (\theta_b + \gamma^{(b)}) \), where \( \gamma^{(a)}, \gamma^{(b)} \) are the \( k \)-th units of the RPSs \( \gamma^{(a)}, \gamma^{(b)} \), respectively. Then we obtain the correlation function
\[
E(\theta_a, \theta_b) = \frac{1}{NC} \sum_{k=1}^{N} \mathcal{P}(\theta_a, k) \mathcal{P}(\theta_b, k) = \cos (\theta_a + \theta_b), \] (5)

where \( C = 1/2 \) is the normalized coefficient. The correlation function is similar to the case of Bell state \( |\Psi^+\rangle \). Therefore we consider the fields in Eq. [4] as the classical simulation of the Bell state \( |\Psi^+\rangle \). We substitute the correlation functions above into Bell inequality (CHSH inequality) [17]
\[
|B| = |E(\theta_a, \theta_b) - E(\theta_a, \theta'_b) + E(\theta'_a, \theta'_b) + E(\theta'_a, \theta_b)| = 2\sqrt{2} > 2, \] (6)

where \( \theta_a, \theta'_a, \theta_b \) and \( \theta'_b \) are \( \pi/4, -\pi/4, 0 \) and \( \pi/2 \), respectively, when Bell’s inequality is maximally violated.
Another Bell state $|\Psi^-\rangle$ differs from $|\Psi^+\rangle$ by $\pi$ phase. Similarly, we obtain the simulation of the Bell state $|\Psi^-\rangle$ expressed as $|\psi'_{\alpha}\rangle = e^{i\lambda_{\alpha}} \left( |0\rangle + e^{i\gamma_{\alpha}} |1\rangle \right) / \sqrt{2}$, where $\gamma_{\alpha}$ is the result of the mode state $|\psi_{\alpha}\rangle$. Performing the transformation $\sigma_x : |0\rangle \leftrightarrow |1\rangle$ on $|\psi'_{\alpha}\rangle$ of the simulation of $|\Psi^\pm\rangle$, we obtain the simulation of the Bell state $|\Phi^\pm\rangle$ expressed as $|\psi'_{\alpha}\rangle = e^{i\lambda_{\alpha}} \left( |0\rangle + e^{i\gamma_{\alpha}} |1\rangle \right) / \sqrt{2}$, and of $|\Psi^-\rangle$ expressed as $|\psi_{\alpha}\rangle = e^{i\lambda_{\alpha}} \left( |0\rangle + e^{i\gamma_{\alpha}} |1\rangle \right) / \sqrt{2}$. Then their correlation functions $E_{\Phi^+} \left( \theta_a, \theta_b \right) = -\cos \left( \theta_a + \theta_b \right)$, $E_{\Phi^-} \left( \theta_a, \theta_b \right) = \pm \cos \left( \theta_a - \theta_b \right)$ are obtained. To substitute the correlation functions into Eq. (10), we also obtain the maximal violation of Bell’s inequality. The violation of Bell’s criterion demonstrates the nonlocal correlation of the two classical fields in our simulation, which results from shared randomness of the PPSs.

The nonlocality of the multipartite entangled GHZ states can in principle be manifest in a single measurement and need not be statistical as the violation of Bell inequality that relies on mean values [18]. Preparing three classical fields $|\psi_a\rangle, |\psi_b\rangle$ and $|\psi_c\rangle$ similar to Eq. (1), and circle exchanging the modes $|1\rangle$ of the fields, we obtain the fields as following:

$$|\psi_{\alpha}\rangle = \frac{1}{\sqrt{2}} e^{i\lambda_{\alpha}} \left( |0\rangle + e^{i\gamma_{\alpha}} |1\rangle \right),$$

$$|\psi'_{\alpha}\rangle = \frac{1}{\sqrt{2}} e^{i\lambda_{\alpha}} \left( |0\rangle + e^{i\gamma_{\alpha}} |1\rangle \right),$$

$$|\psi_{\alpha}'\rangle = \frac{1}{\sqrt{2}} e^{i\lambda_{\alpha}} \left( |0\rangle + e^{i\gamma_{\alpha}} |1\rangle \right),$$

where the RPSs $\gamma_{\alpha} = \lambda_{\alpha} - \lambda_{\alpha}'$, $\gamma_{\alpha}' = \lambda_{\alpha} - \lambda_{\alpha}'$, $\gamma_{\alpha}'' = \lambda_{\alpha} - \lambda_{\alpha}'$ and $\gamma_{\alpha}'' + \gamma_{\alpha}' + \gamma_{\alpha} = 0$. We obtain the measurement results $P_{\Phi^+} \left( \theta_a, k \right) = \cos \left( \theta_a + \gamma_{\alpha} \right)$, $P_{\Phi^-} \left( \theta_a, k \right) = \cos \left( \theta_a - \theta_b \right)$ for the fields $|\psi_{\alpha}\rangle, |\psi'_{\alpha}\rangle$ and $|\psi_{\alpha}'\rangle$ in the projection measurement, respectively, and the correlation function

$$E \left( \theta_a, \theta_b, \theta_c \right) = \frac{1}{NC} \sum_{k=1}^{N} P_{\Phi^+} \left( \theta_a, k \right) P_{\Phi^-} \left( \theta_b, k \right) P_{\Phi^-} \left( \theta_c, k \right)$$

$$= \cos \left( \theta_a + \theta_b + \theta_c \right),$$

where $C = 1/4$ is the normalized coefficient. If $\theta_a + \theta_b + \theta_c = 0$, $E \left( \theta_a, \theta_b, \theta_c \right) = 1$.

This would manifest the nonlocality in a single measurement for the simulation of GHZ states. It is noteworthy that the correlation function sinusoidally oscillates with one of the phases $\theta_a, \theta_b$ and $\theta_c$, when the other phases fixed. However, the correlation function equals to zero if based on the correlation analysis of only two fields, because $\gamma_{\alpha} + \gamma_{\alpha}' + \gamma_{\alpha}'' = 0$ and the sum of any two RPSs remains a pseudorandom sequence. These results are completely similar to the case of GHZ states.

Further, the simulation of GHZ state could be generalized to the case of $n$ particles. Prepared $n$ fields similar to Eq. (1), and circle exchanged the modes $|1\rangle$ of the fields, the RPSs satisfy $\gamma_{\alpha} + \ldots + \gamma_{\alpha}'' = 0$. We obtain the correlation function

$$E \left( \theta_1, \ldots, \theta_n \right) = \frac{1}{NC} \sum_{k=1}^{N} P_{\Phi^+} \left( \theta_1, k \right) \ldots P_{\Phi^-} \left( \theta_n, k \right)$$

$$= \cos \left( \theta_1 + \ldots + \theta_n \right),$$

where $P_{\Phi^+} \left( \theta_i, k \right) = \cos \left( \theta_i + \gamma_{\alpha} \right)$ is the result of the $i$th classical field at the $k$th sequence units in the projection measurement, and $C = 1/2^{n-1}$ is the normalized coefficient. Using the same notion, the simulation of any other generalized GHZ states can be obtained and the correlation functions are also similar to the case of quantum mechanics.

Now we continue to discuss our simulation in another view and try to apply von Neumann entropy as entanglement measure to characterize the classical simulation of Bell states and GHZ state. First we define a mode state in the direct product space of multiple classical fields as following

$$|\Psi\rangle \equiv \frac{1}{C} \sum_{k=1}^{N} \left( e^{-i\lambda_{\alpha}} |\psi_{1 k}\rangle \otimes \ldots \otimes |\psi_{n k}\rangle \right),$$

where $C$ is the normalized coefficient, $\lambda_{\alpha} = \sum_{i=1}^{n} \lambda_{\alpha}^{(i)}$ denotes the total phase sequence with the $k$th unit $\lambda_{\alpha}^{(i)}$, $|\psi_{1 k}\rangle$ denotes the mode superposition of the $i$th classical field at the $k$th sequence unit. Then, we introduce the density matrix formulation by using $\rho = |\Psi\rangle \langle \Psi|$ and Eq. (10), as following

$$\rho = \sum_{k=1}^{N} \left( e^{-i\lambda_{\alpha}} |\psi_{1 k}\rangle \otimes \ldots \otimes |\psi_{n k}\rangle \right) \otimes \sum_{k=1}^{N} \left( e^{i\lambda_{\alpha}} |\psi_{1 k}\rangle \otimes \ldots \otimes |\psi_{n k}\rangle \right),$$

The entanglement of a partly-entangled pure state can be naturally parameterized by its von Neumann entropy of entanglement [19]. Given a pure state $\rho_{ab}$ of two
subsystems $a$ and $b$, we define the reduced density matrices $\rho_a = \operatorname{tr}_b (\rho_{ab})$ and $\rho_b = \operatorname{tr}_a (\rho_{ab})$ for the states of $a$ and $b$, where the partial trace has been taken over one subsystem, either $a$ or $b$. Then the von Neumann entropy of the reduced density matrices is given by $S = -\operatorname{tr} (\rho_a \log_2 \rho_a) = -\operatorname{tr} (\rho_b \log_2 \rho_b)$. The quantity $S$ ranges from zero for a product state to 1 for a maximally entangled pair of two-state particles, and $0 < S < 1$ for a partly entangled pair.

We first discuss the entanglement measure of the product state of two classical fields expressed as Eq. (1). We can easily obtain the density matrix $\rho_{ab} = |\psi_a\rangle \langle \psi_a| \otimes |\psi_b\rangle \langle \psi_b|$, and the von Neumann entropy $S = -\operatorname{tr} (\rho_a \log_2 (\rho_a)) = -\operatorname{tr} (\rho_b \log_2 (\rho_b)) = 0$. This indicates that no entanglement is involved between the two classical fields. We then discuss the simulation fields of Bell state $|\Psi^+\rangle$ expressed as Eq. (4). We could alter the expression as $|\Psi\rangle = e^{i\lambda} (|00\rangle + |11\rangle)/\sqrt{2}$, where $(q_a, q_b), (q_{a,b} = 0, 1)$ are the orthonormal bases of the direct product space. It is worthwhile that the balance property $\sum_{k=1}^{N} e^{i\gamma_k}$ results in many items of the direct product of $|\psi_j\rangle$ disappearing. Then we obtain $\rho_{ab} = (|00\rangle \langle 00| + |11\rangle \langle 11| + |01\rangle \langle 10| + |10\rangle \langle 01|)/2$ and $S = -\operatorname{tr} (\rho_a \log_2 (\rho_a)) = -\operatorname{tr} (\rho_b \log_2 (\rho_b)) = 1$. It means the two fields of simulation of $|\Psi^+\rangle$ are completely entangled. Using the same notion, we can obtain similar results for the simulation of other Bell states and GHZ state. Since the von Neumann entropy gives the same results for the simulation and quantum entangled states, we prove the validity of the simulation in a more rigorous way.

It should be pointed out that the phase pseudorandomness provided by PPSs is different from the case of quantum mixed states. The latter results in decoherence of states and all coherent superposition items disappearing. Different from the decoherence, some coherent superposition items remain in the mode state due to the constraints of the RPSs, such as $\gamma^a + \gamma^b = 0, \gamma^a + \gamma^b + \gamma^c = 0$ for the simulation of Bell states and GHZ state, respectively. These remaining items makes the simulation of quantum entangled pure states possible.

In this letter, we utilize the properties of PPSs to distinguish classical fields that are overlapped in same space and time. A $2^n$-dimensional Hilbert space which contains tensor product structure is spanned by $n$ classical fields modulated with PPSs. In our scheme, the resources required are classical fields modulated with PPSs instead of optical space modes. One classical field modulated with one PPS can simulate one quantum particle. It means that the amount of classical fields and PPSs grows linearly with the number of quantum particles. According to the $m$-sequence theory, the number of PPSs in the set $\Xi$ equals to the length of sequences, which means that the time resource (the length of sequence) required also grows linearly with the number of the particles. Based on the analysis above, we conclude that one can efficiently simulate quantum entanglement with linearly growing resources by using our scheme.

In summary, we have demonstrated a new scheme to simulate quantum entanglement and apply both the correlation analysis and von Neumann entropy to characterize the simulation. We conclude that quantum entanglement can be efficiently simulated by using classical fields modulated with PPSs. The research on this simulation may be important, for it not only provides useful insights into fundamental features of quantum entanglement, but also yields new insights into quantum computation.

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\[ S = -\operatorname{tr} (\rho_a \log_2 (\rho_a)) = -\operatorname{tr} (\rho_b \log_2 (\rho_b)) = 0. \]

\[ \rho_{ab} = (|00\rangle \langle 00| + |11\rangle \langle 11| + |01\rangle \langle 10| + |10\rangle \langle 01|)/2. \]

\[ S = -1. \]

\[ \rho_{ab} = (|00\rangle \langle 00| + |11\rangle \langle 11| + |01\rangle \langle 10| + |10\rangle \langle 01|)/2. \]

\[ S = 1. \]