DEA Model with Hesitant Fuzzy Polyhedral Set in Benchmarking

Dahlan Abdullah1*, H Hartono2, Cut Ita Erliana3, Defi Irwansyah3, Nurintan Asyiah Siregar4, Ayu Esteka Sari5, Abdurrozzaq Hasibuan6, Abdussakir7, Nuning Kurniasih8, Mas’ut9, Irma Yanti Rangkuti10, Nurmaawati11, Erni Busto112, Apridar Abdurrahman113, Tita Septi Handayani114, Wiwien Hadikurniawati115 and Edy Winarno116

1Department of Informatics, Universitas Malikussaleh, Aceh, Indonesia
2Department of Computer Science, STMIK IBBI, Medan, Indonesia
3Department of Industrial Engineering, Universitas Malikussaleh, Aceh, Indonesia
4Department of Management, STIE Labuhanbatu, Sumatera Utara, Indonesia
5Department of Management, STIE Sakti Alam Kerinci, Jambi, Indonesia
6Faculty of Engineering, Universitas Islam Sumatera Utara, Medan, Indonesia
7Department of Mathematics Education, State Islamic University of Malang, Malang, Indonesia
8Faculty of Communication Sciences, Library and Information Science Program, Universitas Padjadjaran, Bandung, Indonesia
9Faculty of Economic, Universitas Islam Sumatera Utara, Medan, Indonesia
10Faculty of Medical, Universitas Islam Sumatera Utara, Medan, Indonesia
11Faculty of Industry Technique, University of 45 Surabaya, Indonesia
12Departement of Nursing, Politeknik Kesehatan Kementerian, Bengkulu, Indonesia
13Department of Management, Universitas Malikussaleh, Aceh, Indonesia
14Departemen of Nursing, Universitas Dehasen, Bengkulu, Indonesia
15Faculty of Information Technology, Universitas Stikubank, Semarang, Indonesia

* dahlan@unimal.ac.id

Abstract. Data Envelopment Analysis (DEA) method is a linear programming approach that has been widely used as a framework for evaluating efficiency and measurement. DEA decision making is often faced with situations where the existing DMU has input and output that contains fuzzy and hesitant elements so that it is difficult to make efficiency measurements. This study will design a DEA model in the state of inputs and outputs that contain hesitant elements by using polyhedral uncertainty sets. The results of this study are expected to produce a benchmarking model with DEA that can overcome the hesitant element and have the advantage of using a polyhedral uncertainty set that can measure the linearity of the nominal problem.

1. Introduction
Data Envelopment Analysis (DEA) is used to measure the performance of an organization as part of the process of improving the quality of management planning[1]. DEA is a model of efficiency measurement based on the input and output in each DMU [2]. The application of the DEA method is carried out in the form of determining the Decision Making Unit (DMU) and each DMU has input and output. Measurement of the DEA method is done by comparing the output produced from a DMU and the input used compared to the entire DMU[3]. Every organization needs to pay attention to the efficiency of the implementation of activities that are realized in the form of efficient measurement.
results from the use of inputs compared to the inputs produced. If the measurement results indicate inefficiency, it is necessary to reduce the input process or reduce output [4]. For each DMU that is not efficient, slack analysis can be done based on the existing inputs and outputs. Then, the input will be deducted from the existing slack values while the output will be added by adding a number of slack values [5]. Cook and Zhu expand the use of DEA to be applied to situations where each DMU is a member of the same situation [6]. The main difficulty in the DEA method is the difficulty to overcome the assessment in a situation that contains elements of uncertainty in input and output [7]. Bertsimas & Sim developed Polyhedral Set for new formulations in linear programming problems in uncertain conditions [8].

This study will design a DEA model in the state of inputs and outputs that contain hesitant elements by using polyhedral uncertainty sets. The results of this study are expected to produce a benchmarking model with DEA that can overcome the hesitant element and have the advantage of using a polyhedral uncertainty set that can measure the linearity of the nominal problem.

2. Related Works
Research that combines DEA with other approaches has been carried out by a number of researchers and this can provide other parameters needed by the DEA. One of them is the incorporation of DEA with the AHP method which can be used to obtain the weights of each input and output on the DEA model [9]. Mitropoulos et al. [10] presented a methodology combining DEA programming slot allocation and whole numbers as medium and long term decision tools. Khodaparasti and Maleki [30] presented a new dynamic blur combined model for locating vehicles and ambulance stations in an emergency medical service [11]. Barak and Dahooei [12] are proposing a new hybrid method using Fuzzy Data Envelopment Analysis (DEA) and Fuzzy Multi Attribute Decision Making (F-MADM) to assess airline safety. In this study, the fuzzy DEA is used to calculate the weighting of the criteria, as opposed to the traditional approach of using the DEA to measure the effectiveness of alternatives. Dahlan et al. [13] using a slack-based measure for increasing the efficiency of each department of Universitas Malikussaleh. Suganthi proposes to adopt a multi-expert and multi-criteria decision-making approach using fuzzy AHP, VIKOR and DEA methods [14].

3. Research Methodology
The research methodology can be seen in figure 1.

![Figure 1. Research Methodology](image)

In Figure 1 it can be seen that each input and output from each DMU containing uncertainty elements will be processed using polyhedral uncertainty set and fuzzy hesitant to produce efficiency scores for each DMU.
4. Result and Discussion

4.1. Data Envelopment Analysis

The previous model is in the form of fractional programming. For measurement of relative efficiency, DEA formulates it into fractional programming, and then changes it to the linear programming concept, as can be seen in the following equation.

\[
\text{Maximize } \beta = \sum_{r=1}^{k} \mu_r y_j
\]

Where:

\[
\sum_{j=1}^{l} v_j x_{rj} = 1;
\]

\[
\sum_{r=1}^{k} u_r y_j - \sum_{j=1}^{l} v_j x_{rj} \leq 0; \quad \forall j
\]

\[
u_r, v_j \geq 0 \quad \forall r, \forall s
\]

4.2. Hesitant Fuzzy

Suppose that X is a reference set, a Fuzzy Hesitant Set on X is defined as a function of h that returns a subset of membership values \([0,1]\). To facilitate our understanding, an HFS can be represented using equation 3.

\[
M = \{< x, h_M(x) > | x \in X \}
\]

Where \(h_M(x)\) is a set of several membership values that have a value range of 0 to 1, which indicates the possible membership value of member \(x \in X\) for set \(M\). \(h = h_M(x)\) as Hesitant Fuzzy Element (HFE) and \(H\) is a set of all HFE members. For comparison of HFE, linguistic evaluation scale is used, which transforms linguistic variables into linguistic scales. To do that is used discrete set of linguistic scale as:

\[
\Sigma = \{s_\alpha | \alpha = -\tau, \ldots, -1, 0, 1, \ldots, \tau\}
\]

Where \(\tau\) is a positive integer value, and \(\alpha\) represents a possible value for a linguistic variable. For example, a discrete set of linguistic scales containing 7 (seven) members (\(\tau = 3\)) is as follows:

\(\Sigma = \{s_{-3} = \text{none}, s_{-2} = \text{very bad}, s_{-1} = \text{bad}, s_0 = \text{medium}, s_1 = \text{good}, s_2 = \text{very good}, s_3 = \text{perfect}\}\).

4.3. Polyhedral Uncertainty Set

Bertsimas & Sim developed Polyhedral Set for new formulations in linear programming problems in uncertain conditions. Ordinarily, the uncertainty set involves a combined interval and polyhedral set described as:

\[
\mu_{\text{int}+\text{pol}}(\phi, \Gamma) = \{\tilde{a}_{i,j}, \|\| \eta \|\|_\infty \leq \phi, \|\| \eta \|\|_1 \leq \Gamma\}
\]

Equation 5 will be substituted into equation 2 as can be seen in Equation 6.

\[
\text{max } \sum_{j=1}^{n} c_j x_j \\
\text{s.t. } \sum_{j=1}^{m} a_{ij} x_j + \beta_i(y, \Gamma_i) \leq b_i \quad 1, \ldots, m \\
-y_j \leq x_j \leq y_j \quad \forall i, \forall j \in j_i \\
-l_j \leq x_j \leq u_j = 1, \ldots, n \\
y_j \geq 0 \quad \forall i, \forall j \in j_i
\]
4.4. DEA-Hesitant Fuzzy-Polyhedral Uncertainty Set

The DMU used in this study is 19 courses at the University of Malikussaleh. The data of each DMU is as follows.

| DMU               | Input                | Output                |
|-------------------|----------------------|-----------------------|
|                   | Number of Lecturers | Number of Students   | Academic Service Quality | Academic Atmosphere | Number of Research | Number of Graduates | Student Satisfaction |
| Information Technology | 17                | 588                | 0.75                  | 0.8                  | 5                   | 610                | 0.87                |
| Civil Engineering  | 26                | 747                | 0.32                  | 0.67                 | 5                   | 533                | 0.43                |
| Architectural Engineering | 15           | 396                | 0.59                  | 0.65                 | 5                   | 195                | 0.64                |
| Industrial Engineering | 17           | 467                | 0.81                  | 0.56                 | 5                   | 300                | 0.76                |
| Chemical Engineering | 25             | 348                | 0.73                  | 0.75                 | 5                   | 252                | 0.81                |
| Mechanical Engineering | 23           | 499                | 0.56                  | 0.32                 | 5                   | 224                | 0.57                |
| Electrical Engineering | 19            | 420                | 0.66                  | 0.68                 | 5                   | 326                | 0.68                |
| Agribusiness      | 17                | 689                | 0.75                  | 0.52                 | 5                   | 273                | 0.65                |
| Agro-Technology   | 34                | 822                | 0.73                  | 0.65                 | 5                   | 284                | 0.72                |
| Aquaculture       | 10                | 501                | 0.59                  | 0.57                 | 5                   | 204                | 0.61                |
| Communication Science | 11             | 719                | 0.67                  | 0.87                 | 5                   | 273                | 0.9                 |
| Political Science | 11                | 262                | 0.65                  | 0.77                 | 5                   | 183                | 0.73                |
| Sociology         | 13                | 487                | 0.66                  | 0.69                 | 5                   | 204                | 0.71                |
| Anthropology      | 9                 | 173                | 0.65                  | 0.45                 | 5                   | 116                | 0.67                |
| Jurisprudence     | 50                | 1096               | 0.54                  | 0.56                 | 10                  | 467                | 0.58                |
| Medicine          | 30                | 278                | 0.67                  | 0.71                 | 4                   | 257                | 0.73                |
| Management        | 48                | 1265               | 0.68                  | 0.78                 | 5                   | 1302               | 0.81                |
| Economic          | 11                | 853                | 0.59                  | 0.73                 | 5                   | 290                | 0.87                |
| Development       | 11                | 418                | 0.81                  | 0.77                 | 5                   | 417                | 0.81                |

Based on the measurements with HFLTS-DEA, the efficiency of each as follows.

| DMU                | DEA Score |
|--------------------|-----------|
| Information Technology | 1         |
| Civil Engineering   | 1         |
| Architectural Engineering | 0.88   |
| Industrial Engineering | 0.92     |
| Chemical Engineering    | 0.94     |
| Mechanical Engineering    | 1         |
| Electrical Engineering    | 0.87     |
| Agribusiness          | 0.88     |

Table 1. DMU Universitas Malikussaleh

Table 2. Efficiency Score
DMU | DEA Score
--- | ---
Agrotechnology | 0.79
Aquaculture | 0.9
Communication Science | 1
Political Science | 1
Sociology | 0.88
Anthropology | 0.95
Jurisprudence | 1
Medical | 1
Management | 1
Economic Development | 1
Accounting | 0.93

4.5. Discussion
Based on the results of the study it can be seen that the DEA Model with Fuzzy Polyhedral Set Hesitant can complete the benchmarking process in a situation where the input and output of the DMU contains fuzzy and hesitant elements. DMUs with efficiency values below 1 indicate inefficient and existing departments need to reduce existing inputs and increase output.

5. Conclusion
The conclusions that can be obtained from the results of this study are: (1) the benchmarking process with DEA has difficulty if there are inputs and outputs that contain fuzzy and hesitant elements, (2) DEA model with Fuzzy Polyhedral Hesitant Set can overcome the benchmarking process in the state of input and output containing fuzzy and hesitant elements.

References
[1] Ruiz J L, Segura J V and Sirvent I 2015 Benchmarking and target setting with expert preferences: An application to the evaluation of educational performance of Spanish universities European Journal of Operational Research 242(2) 594-605
[2] Li H, Zhang J, Wang C, Wang Y and Coffey V 2018 An evaluation of the impact of environmental regulation on the efficiency of technology innovation using the combined DEA model: A case study of Xi’an, China Sustainable Cities and Society 42 355-399
[3] Barat M, Tohidi G and Sanei M 2018 DEA for nonhomogeneous mixed networks,” Asia Pacific Management Review
[4] Abdullah D, Tulus, Suwilo S, Efendi S, Hartono and Erliana C I 2018 A Slack-Based Measures for Improving the Efficiency Performance of Departments in Universitas Malikussaleh,” International Journal of Engineering & Technology 7(2) 491-494
[5] Lozano S, Hinojosa M A and Mármol A M 2018 Extending the bargaining approach to DEA target setting Omega
[6] Cook W D, Ruiz J L, Sirvent I and Zhu J 2017 Within-group common benchmarking using DEA European Journal of Operational Research 256(3) 901-910
[7] Toloo M and Kwasi Mensah E 2018 Robust optimization with nonnegative decision variables: A DEA approach Computers & Industrial Engineering
[8] Bertsimas D, Brown D and Caramanis C 2011 Theory and Applications of Robusts Optimization SIAM Rev. 53(3) 464-501
[9] Otay I, Oztyay B, Cevik Onar S and Kahrman C 2017 Multi-expert performance evaluation of healthcare institutions using an integrated intuitionistic fuzzy AHP&DEA methodology Knowledge-Based Systems 133 90–106
[10] Mitropoulos P, Talias M A and Mitropoulos I 2015 Combining stochastics DEA with Bayesian analysis to obtain statistical properties of the efficiency scores: An application to Greek public hospitals European Journal of Operational Research 243 (1) 302-311

[11] Khodaparasti S and Maleki H R 2013 A new combined dynamic locations models for emergency medical services in fuzzy environment in 13th Iranian Conference on Fuzzy Systems (IFSC)

[12] Barak S and Dahooei J H 2018 A novel hybrid fuzzy DEA-Fuzzy MADM method for airlines safety evaluation Journal of Air Transport Management 73 134-149

[13] Abdullah D, Zarlis M, Napitupulu D, Hartono H, Sriadhi S, Erliana C I, Dijaya R, Findawati Y, Nurdiyanto H, Rahim R and Ahmar A S 2018 A Slack-Based Measures within Group Common Benchmarking using DEA for Improving the Efficiency Performance of Departments in Universitas Malikussaleh MATEC Web Conf. 197 16005

[14] Suganthi L 2018 Multi expert and multi criteria evaluation of sectoral investments for sustainable development: An integrated fuzzy AHP, VIKOR / DEA methodology Sustainable Cities and Society 43 144-156