Interaction of a motor unit wheel with a support base

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Abstract. The problems of kinematics and mechanics of interaction of a rigid metal wheel with a deformable base on basis of the theory of instant tipping of a lever are considered. Calculated dependences are presented to determine the main indicators of interaction of the wheel with the supporting surface: strength, speed and power. It is established that rolling of the drive wheel, in comparison with the follower wheel, leads to an increase in the depth of the track, and rolling resistance force decreases in this case.

1. Problem formulation
About 40% of all agricultural products are produced in backyards and garden plots. The area of such plots averages 0.12-0.2 ha. Until recently, all work on such plots was carried out manually, which held back labor productivity and did not allow to reduce the cost of production. Recently, the level of mechanized work has increased significantly due to the use of small-scale mechanization [1]. The most widespread have become units based on walking tractors – motor units (MU) [2]. The main task of mechanical engineering in the field of MU production is to determine reliable data on the action of forces and moments in the transmission elements during operation or their calculating methods. This will improve the calculation accuracy of body parts, gears, make the choice of appropriate structural materials, which in general will affect the cost reduction and increase the reliability of MU.

2. Literature analysis
Calculation of the wheel propellers’ grip by most researchers is carried out according to the methodology presented in [3]. The main disadvantage of this technique is that the tangential force calculation is determined as a function of the amount of wheel spin, which contradicts natural processes – wheel spinning varies from the value of the tangential force. Vodolazhchenko Yu.K. and Anilovich V.Ya. proposed a power law with empirical coefficients to calculate a wheel spin coefficient, depending on the gripping properties of the propeller and the support base characteristics [4]. However, for calculations using this technique, it is necessary to know the gripping properties for specific propellers in specific conditions. Otherwise, the calculation reliability will not be sufficient.

The process of wheel rolling is ambiguously interpreted and considered by various researchers. GOST 17697 states that wheel rolling is ‘rotation of a wheel, which is in contact with the support base, in presence of a wheel center displacement in the longitudinal plane’. It is also noted here that the wheel rolling radius is ‘the ratio of a longitudinal component of the forward speed of the wheel to its angular speed’, i.e. is a kinematic parameter, not a physical value. In most cases, these parameters are determined experimentally, and the results already obtained are approximated by mathematical
dependencies. This makes it difficult to understand the very essence of interaction of the wheel with the support base and the physical processes that occur during this. Until now, the wheel rolling parameters, for example, force and rolling resistance moment, tangential force, wheel spin or slide, are considered to be conditional values, the cause-effect relationships of which are not discovered.

In 1918, an Academician V.P. Goryachkin noted that the wheel rolling process should be considered as ‘a continuous tipping relative to one of the rim points’, at the same time he notes that a wheel is ‘a continuously operating lever relative to this point’. Such an approach can explain many physical processes of the interaction of a wheel propeller with a support base - soil.

To increase gripping properties of MU, metal wheels with developed grip hooks are often used. For the purposes of discussion, they can be considered as a rigid constantly tripping lever with rotation relative to the instantaneous center located on the support base. A.A. Loparev considers the same approach in his work [5], but for rolling a wheel with an elastic tire. It was assumed that the wheel axis moves as a free end of the lever relative to the instantaneous center on the supporting surface, and the center is offset along the travel direction at $f_B$ angle. In this case, the wheel rotation axis should have a vertical component, which in real conditions is not observed. It can be concluded that for a wheel with an elastic tire, this theory requires refinement. An easier task is to describe the kinematics of hard wheel rolling.

3. Purpose of work
The aim of this work is to substantiate the methodology for calculating the power balance and speed characteristics of interaction of MU hard wheels with a support base based on the theory of a constantly tipping lever.

4. Work results
The hard wheel rolling is accompanied by continuous deformation and restoration of the support base. Two characteristic areas can be distinguished in the contact patch (Figure 1): $KL$ is the area of soil deformation in the normal and longitudinal directions, and $LM$ is the area of soil recovery under the action of elastic forces. Thus, the contact patch length passes along $KLM$ arc, in contrast to $KL$ area proposed in [4].

On loose soils, at the entry of the contact, the greater part of the load is transferred, and at the exit, a smaller part of the load transmitted by the wheel to the support base, as the main work is aimed at deformation of the soil and formation of a track. Under the influence of this load, the soil is deformed to a depth of $h_\delta$ and reaches a maximum at point $L$. Elastic forces partially restore the soil deformation by $h_c$ value, and the track’s depth is $h_k = h_\delta - h_c$.

![Figure 1. Forces acting on the rigid wheel during rolling in the driven mode with a track formation](image)

When soil is deformed, a reaction of the support base $R_c$ occurs, and it can be divided into two components: vertical $R_c$, numerically equal to the normal load on the wheel and directed in the opposite direction, and horizontal $R_c$, numerically equal to the resistance force of the wheel axis movement in the horizontal direction $P_x$ and directed against movement. Considering that all the
above mentioned forces are pairwise equal and oppositely directed, the system of forces is balanced and is at rest. For wheel rolling in the driven mode, it is necessary to apply a horizontal pushing force $P_x$ to the wheel skeleton (conditionally, to the axis) greater than $R_x$ in absolute value.

If we assume that $OC$ lever rests with its end $C$ on an instantly hardened surface, then under the action of pushing force $P_x$ it will begin to rotate relative to the instantaneous center $C$, which will lead to an elevation of the $OC$ lever end by $\Delta y$ variable (Figure 2). This, in turn, will cause an increase in the normal load at point $C$ and, with instantaneous deformation of the base, which will lead to an increase in the track depth by the amount of lever elevation $\Delta y$. In this case, the contact patch together with the instantaneous reaction center of the support base will shift forward by variable $\Delta x$ into point $C'$. Thus, point $O$ moves horizontally, and the increase in elevation is neutralized by additional deformation of the base.

![Figure 2. Model illustrating lever movement relative to instantaneous center C](image1.png)

![Figure 3. Diagram of forces acting on the lever during wheel rolling in the driving mode](image2.png)

When a rigid wheel rolls on a solid base, molecular grip forces, which prevent overrolling occur in the contact patch. Therefore, the instantaneous reaction center of the support base due to the action of a normal load also shifts forward in the direction of wheel rolling (point $C$).

It is known that ratio $\frac{D_0}{D_0} = \tan \alpha = f$ is a coefficient of rolling resistance. An increase in the angle $\alpha$, ceteris paribus, corresponds to an increase in the rolling resistance force $P_x$. On a solid base, point $O$ of $OC$ lever elevates to a vertical position and drops until it rests on the next grip hook. Thus, the wheel axis makes a wave-like motion.

In free, neutral and driving modes, the wheel is driven by a torque $M_x$ (Figure 3), which is connected to the rotation axis. At point $C$ of $OC$ lever, under the action of torque and friction forces, a tangential force is generated in the contact patch, directed perpendicular to the lever:

$$F_c = \frac{M_x}{r_x}$$

where $l_{oc}$ is the length of $OC$ lever equal to rolling radius $r_x$; $M_x$ is the torque supplied to the wheel axis, H m.
By resolving the tangential traction force into its components, we obtain horizontal component \( P_{cx} \), which is the driving force, and vertical - \( P_{cy} \), which helps to increase the track depth.

The normal force acting on the support base from the wheel side will increase by the vertical component value of the tangential force \( P_{cy} \) and will be equal to

\[
D_{py} = D_y + D_{o} - D_{o} \cdot \sin \alpha = D_y + D_{o} \cdot \tan \alpha
\]  

(2)

In the horizontal plane, tangential force component \( P_{cx} \) will decrease by the rolling resistance force value because of soil deformation in the horizontal plane \( P_{x} \).

\[
D_{px} = D_{o} - D_{o}
\]  

(3)

The overall reaction of the soil and the inclination angle of the soil reaction vector will be as follows:

\[
r_{o} = \left( \frac{P_{y} + P_{o}}{P_{y} + P_{o}} \right)^{2} + \frac{\cos \alpha}{\cos \alpha}
\]  

\[
\alpha' = \arctan \frac{D_{o}}{P_{y} + P_{o}}
\]  

(4)  

Therefore, when the wheel rolls and spins, the instantaneous center of rotation shifts to point \( C' \), in relation to which it is necessary to consider the action of forces and speeds. The action of \( P_{xy} \) component contributes to an increase in the track depth of the drive wheel. This is confirmed by experimental studies [6]. The resulting ground resistance force \( R' \) of the drive wheel forms a smaller inclination angle \( \alpha' \) with the vertical axis. Therefore, the coefficient of rolling resistance of the drive wheel will be less than that of the follower wheel. It is explained by the fact that part of the rolling resistance force is compensated by the sliding force in the contact patch.

Consider the kinematics of wheel rolling in the follower and drive modes.

In the follower mode (Figure 4), under the influence of pushing force \( P_{x} \), the wheel axis acquires speed \( V_o \). The horizontal component of \( V_{ox} \) is the forward speed of motion and is determined by the formula (6). Vertical component \( V_{oy} \) is manifested in the rise of the wheel axis, which leads to an increase in the track depth on deformable soils or rise of the wheel axis during rolling on solid bases and is determined by the formula (7). The power spent on overcoming rolling resistance forces is the power to overcome weight force \( P_{y} \) on the wheel axis at point \( O \) with the elevation speed \( V_{oy} \) is determined by the formula (8).

\[
V_{ox} = V_{o} \cdot \cos \alpha = \omega \cdot r_{o} \cdot \cos \alpha
\]  

(6)

\[
V_{oy} = V_{o} \cdot \sin \alpha = \omega \cdot r_{o} \cdot \sin \alpha = V_{ox} \cdot \tan \alpha
\]  

(7)

\[
N_{f} = \frac{D_{y}}{V_{ox}} \cdot \cos \alpha = -P_{y} \cdot V_{oy} = -P_{y} \cdot V_{ax} \cdot \tan \alpha
\]  

(8)

In the drive mode (Figure 5), under the influence of torque \( M_{x} \), the instantaneous center of rotation \( C \) will slip in relation to the supporting surface with speed \( V_{cx} \) and the forward speed will decrease

\[
V_{ax} = \omega \cdot r_{o} \cdot \delta
\]  

(9)

\[
V_{ax} = (\bar{V}_{o} - V_{ax}) \cdot \cos \alpha' = (\omega \cdot r_{o} - \omega \cdot r_{o} \cdot \delta) \cdot \cos \alpha' = \omega \cdot r_{o} \cdot (1 - \delta) \cdot \cos \alpha'
\]  

(10)

where \( \delta \) is the wheel spin coefficient.
The vertical component of a sliding speed reflects the increase in the rate of soil deformation during track formation under the influence of the drive wheel:

\[ V_{\delta y} = V_o \cdot \sin \alpha' + V_{\delta x} \cdot \sin \alpha' = (\omega \cdot r_e + \alpha \cdot r_c \cdot \delta) \cdot \sin \alpha' = \omega \cdot r_e \cdot (1 + \delta) \cdot \sin \alpha' \quad (11) \]

Then, the horizontal and vertical components of the forces are determined by the following formulas:

\[ D_{\delta x} = D_o \cdot \cos \alpha' = \frac{M_o}{r_e} \cdot \cos \alpha' . \quad (12) \]

\[ D_{\delta y} = D_o \cdot \sin \alpha' = \frac{M_o}{r_e} \cdot \sin \alpha' . \quad (13) \]

A normal load at point \( C' \) will be as follows:

\[ B_{\delta o} = D_o + D_{\delta o} = D_o + D_{\delta x} \cdot \sin \alpha' = D_o + D_{\delta x} \cdot \tan \alpha' . \quad (14) \]

Considering that the soil reaction vector to the action of normal forces is numerically equal, but directed in the opposite direction \( \vec{R}_{\delta o} = -\vec{F}_{\delta o} \), the soil resistance force to rolling will be equal to:

\[ D_{\delta} = B_{\delta o} \cdot \tan \alpha' . \quad (15) \]

Then, the concentrated force at point \( C' \) is

\[ B'_{\delta o} = \sqrt{(D_o + D_{\delta o})^2 + (D_{\delta o} - D_{\delta x})^2} . \quad (16) \]

Given that the concentrated force acts at the instantaneous rotation center of wheel \( C' \), which is currently stationary, force \( P'_{\delta o} \) directed in the opposite direction from \( P'_{\delta o} \) and numerically equal to it will act on the wheel hub. Force \( P'_{\delta o} \) forms angle \( \beta' \) with the vertical axis of the wheel, which is determined by the formula:
\[ \beta' = \arccos \frac{P_{ao} - D_k}{P_k + P_{ao}}. \]  

Angle \( \beta' \) reflects the gripping properties of the wheel propeller, in particular, the tangent of this angle is the adhesion coefficient \( \tan \beta' = \varphi \).

Useful traction force or the so-called hook force will be equal to:

\[ D_{oa} = D_{oa} - D_k. \]  

The considered drive wheel model allows us to reveal the physical meaning of all components of the power balance as scalar products of the force and velocity vectors by the cosine of the angle between them: rolling resistance forces, wheel spinning, traction resistance of tools on the hook.

\[ N_f = (P_{oa} + P_k \cdot \tan \alpha') V_{ao} \cdot \cos \pi = -(P_{ao} + P_{ao} \cdot \tan \alpha') V_{ao} \cdot (1 + \delta) . \]  

\[ N_f = (D_{oa} + D_k \cdot \tan \alpha') V_{ao} \cdot \cos \pi = -(D_{oa} + D_k) V_{ao} \cdot (1 - \delta) \cdot \cos \alpha' = -P_{ao} V_{ao} \cdot (1 - \delta) \cdot \cos \alpha' = -D_{oa} V_{ao} \]  

Power brought to the wheel axis is equal to:

\[ N_f = (P_{oa} + P_{oa} \cdot \tan \alpha') r_1 \cdot \omega \cdot \cos \frac{\pi}{2} - \beta - \alpha' = I_1 \cdot \omega . \]  

The power balance of the drive wheel is:

\[ N_f = (D_{oa} + D_k \cdot \tan \alpha') r_2 \cdot \omega . \]  

Since \( P_{ao} = P_{oa} \), and \( V_{ao} + V_c = \omega r_2 \), the sum of powers \( N_f \), \( N_\vartheta \) and \( N_{cp} \) is equal to the right-hand side of formula (23), i.e. the power balance converges.

Efficiency of the drive wheel reflects the ratio of available capacity to supplied capacity:

\[ \eta = \frac{D_{oa} V_{ao}}{(D_{oa} + D_{oa} \cdot \tan \alpha') r_2 \cdot \omega} = \frac{1 - \delta}{1 + \tan \alpha' \cdot \tan \beta'} = \frac{1 - \delta}{1 + \frac{1}{\varphi}}. \]  

5. Conclusions

The theoretical studies allowed to reveal the essence of rolling a hard smooth wheel with grip hooks, having suggested that rolling was a process of instant tipping of the lever relative to the instantaneous center. Based on the theoretical studies, it was found that with an increase in spinning, the coefficient of movement resistance decreases, but the track depth increases. A regularity was proposed for calculating the wheel traction efficiency in the drive mode of rolling.

Acknowledgement

The article is prepared in the framework of development of the Base University on the basis of BSTU named after V.G. Shukhov.
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