Site of maxima of conductivity, temperatures, density of the current and specific capacity of the thermal emission in the HFI-discharge

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Abstract. On the basis of theoretical analysis of distributions of the conductivity, current density and specific power of heat release in the high-frequency induction discharge, a law of crowding of maxima of these values has been established.

1. Introduction

In electro-plasma processes and power plants using high-frequency induction (HFI) plasma, the discharge zone is the main process zone. Information on distribution of main parameters of the discharge (conductivity, temperature, current density, specific power of heat release) and position of maxima of these values gives an opportunity to determine optimal conditions for heating the initial material and to manufacture a high-quality product.

The results of works [1]-[7] allowed us to suggest that for each section inside a plasmoid of the HFI discharge, the following inequality holds true

\[ r_1 < r_2 < r_3, \]

where \( r_1 = r(\sigma_{\text{max}}) \), \( r_2 = r(j_{\text{opt}}) \) and \( r_3 = r(W_{\text{max}}) \) are the radial coordinates corresponding to the maxima of physical quantities within the parentheses.

In these relations \( \sigma \) is the conductivity of the discharge, \( j = \sigma E_\phi \) is the eddy current density, \( E_\phi \) is the strength of the azimuth electric field, and \( W = \frac{1}{2} \sigma E_\phi^2 \) is the power input to the discharge per unit volume averaged over a period of the high-frequency field.

Figure 1 shows dependences of the conductivity of the discharge \( \sigma(r) \), eddy current density \( j = \sigma E_\phi \), bulk density of the power input to the discharge \( W = \frac{1}{2} \sigma E_\phi^2 \) in various sections of a HFI plasmoid, starting from its central section downstream and illustrating the relation (1). The results are obtained on a stationary semi-commercial unit HFI-11/60 using a procedure described in Refs. [1-7].
2. Method of solution

Inequality (1) can be analytically proven as follows. Let us assume that the conductivity of the discharge \( \sigma(r,z) \), current density \( j_\varphi(r,z) = \sigma E_\varphi \) and power of heat release \( W = \frac{1}{2} \sigma E_\varphi^2 \) are the functions, which at each fixed section of the discharge reach their maximum values only once on the interval \( 0 \leq r \leq R \). One may prove that in this case the relation \( r(\sigma_{\text{max}}) < r(j_{\varphi\text{max}}) < r(W_{\text{max}}) \) holds true.

At the point \( r_1 = r(\sigma_{\text{max}}) \), \( \frac{\partial \sigma(r)}{\partial r} \bigg|_{r_1} = 0 \), \( \frac{\partial \sigma(r)}{\partial r} > 0 \) when \( r < r_1 \), and \( \frac{\partial \sigma(r)}{\partial r} < 0 \) when \( r > r_1 \).

Then at the point \( r_2 = r(j_{\varphi\text{max}}) \) we have \( \frac{\partial (\sigma E_\varphi)}{\partial r} \bigg|_{r_2} = \frac{\partial \sigma}{\partial r} \bigg|_{r_2} E_\varphi(r_2) + \sigma(r_2) \frac{\partial E_\varphi}{\partial r} \bigg|_{r_2} = 0 \).
Since \( E_\phi(r), \sigma(r) \) are nonnegative values in the whole volume of a discharge, at the point \( r_2 \)
\[
\frac{\partial \sigma}{\partial r} \bigg|_{r_2} = \frac{\partial E_\phi}{\partial r} \bigg|_{r_2} < 0,
\]
and this means that the point \( r_2 = r(j_{max}) \) is situated at the descending path of the function \( \sigma(r) \), i.e., on the interval \( r_1 < r < r_2 \). Therefore, \( r_2 > r_1 \).

At the point \( r_3 = r(W_{max}) \), in turn, we have:
\[
\frac{\partial (\sigma E_\phi^2)}{\partial r} \bigg|_{r_3} = \frac{\partial (\sigma E_\phi)}{\partial r} \bigg|_{r_3} E_\phi (r_3) + \sigma (r_3) E_\phi (r_3) \frac{\partial E_\phi}{\partial r} \bigg|_{r_3} = 0.
\]
Hence \( \frac{\partial (\sigma E_\phi)}{\partial r} \bigg|_{r_3} = -\sigma (r_3) \frac{\partial E_\phi}{\partial r} \bigg|_{r_3} < 0 \), and, consequently, \( r_1 > r_3 \).

Finally we obtain the relation \( r_1 < r_2 < r_3 \), which was to be proven.

Thus, one may state that inside the HFI discharge the radial coordinates corresponding to maximum values of the conductivity, eddy current density and power input to the discharge are arranged in increasing order.

Note that using the method applied one can prove a more general result,
\[
r_1 < r_2 < ... < r_i < ... < r_n,
\]
where \( r_i = r\left(\sigma E_\phi^{i-1}\right) \); \( i = 1, 2, ..., n \), and \( n \) is integer number that can be made as large as is wished.

Inequality (1) is particular case of (2). Therefore, there is a set coaxial cylindrical surfaces inside an HFI discharge that correspond to the maximus of \( \sigma E_\phi^{i-1} \).

Under conditions of local thermodynamic equilibrium, the surface of minimum radius \( r_1 = r(\sigma_{max}) \) of the set of surfaces (2) corresponds to the maximal temperature \( T_{max} = T(r_1) \) inside the plasmoid, because the relationship \( \sigma = \sigma(T) \) between the temperature and the conductivity of the discharge is unambiguous in this case.

Let us find a law of crowding of \( r_i \) points corresponding to maxima of \( \sigma E_\phi^{i-1} \) \( (i = 1, 2, 3) \) values on the \( r \)-axis in the direction of the plasmoid periphery, that is first three points \( r_1, r_2, \) and \( r_3 \), most interesting to technical applications.

Let us examine the function of the form \( y_i = \sigma E_\phi^{i-1} \). The changes of the \( E_\phi \) value can be approximated by a straight line \( E_\phi = a \cdot r^{i-1} \), while the \( \sigma \) value is chosen in the form

\[
\sigma = \sigma_{max} \cdot e^{-\left(\frac{r-r_0}{a}\right)^2}.
\]

Then \( y = a \cdot r^{i-1} \cdot \sigma_{max} \cdot e^{-\left(\frac{r-r_0}{a}\right)^2} \).

In this case, from mathematical point of view, the problem of finding maxima of \( \sigma E_\phi^{i-1} \) values as a function of the radial coordinate \( r \) reduces to the problem of finding maxima of the following function
\[
y = a \cdot r^{i-1} \cdot \sigma_{max} \cdot e^{-\left(\frac{r-r_0}{a}\right)^2} \quad \text{for } i = 1, 2, 3 \text{ which can be solved by standard methods of applied analysis.}
\]

For the point \( r^* = r(f_{max}) \), apparently, we have the relation...
The solution of this quadratic equation can be written as

\[ r_{0,2} = \frac{r_0 \pm r_0 \sqrt{1 + 2(i-1)\left(\frac{\Delta r}{r_0}\right)^2}}{2}. \]

On physical grounds the positive sign should be used

\[ r^* = \frac{r_0}{2} \left( 1 + \left( \frac{\Delta r}{r_0} \right)^2 \right). \]

At \( i = i+1 \) \( \tilde{r}^* = \frac{r_0}{2} \left( 1 + \sqrt{1 + 2i\left(\frac{\Delta r}{r_0}\right)^2} \right) \). That is

\[ \Delta = \tilde{r}^* - r^* = \frac{\sqrt{1 + 2i\left(\frac{\Delta r}{r_0}\right)^2} - \left( \frac{\Delta r}{r_0} \right)^2}{2} \]

When \( i \to \infty \), \( \tilde{r}^*(i) - r^*(i-1) \to 0 \) or \( \tilde{r}^* \approx r^* \), i.e. the points corresponding to maxima of the \( \sigma E_{\rho}^{i-1} \) values, should crowd

3. Results and conclusion

Let us estimate the value of this crowding. The estimation procedure is illustrated in Figure 2.

Figure 2 Illustration of a technique of an estimation of value of a crowding
The point $r_0(\sigma_{\text{max}})$ corresponding to the maximum of the conductivity value $\sigma$ is chosen on the plot. In our case, $\sigma_{\text{max}}=15.0$. The radial coordinate $r_0$ corresponding to $\sigma_{\text{max}}$ is $1.5$ cm. Then a standard half-width of an exponential curve $\Delta r$ is determined, i.e. at this point the value $\sigma$ is diminished e-fold from $\sigma_{\text{max}}$. In our case, $\Delta r=0.8$ cm. Then

$$\frac{\Delta r}{r_0} = \frac{0.8}{1.5} = 0.53.$$ 

Let us estimate the $\Delta$ values for the $j$ and $W$ values denoting them as $\Delta_1$ and $\Delta_2$, respectively. As one may see from the figure, $\Delta_1=r_1-r_0$ and $\Delta_2=r_2-r_1$.

$$\Delta_1 = 1.5 \cdot \sqrt{\frac{1 + 2 \cdot 1 \cdot (0.53)^2}{2}} - 1 = 0.19,$$

$$\Delta_2 = 1.5 \cdot \sqrt{\frac{1 + 2 \cdot 2 \cdot (0.53)^2}{2}} - \sqrt{1 + 2 \cdot 1 \cdot (0.53)^2} = 0.16.$$

Analogous calculations have been performed for other sections.

The results of $\Delta_{1\text{calc}}$ and $\Delta_{2\text{calc}}$ calculations are summarized in Table 1. The $\Delta_{1\text{emp}}$ and $\Delta_{2\text{emp}}$ values obtained directly from the plots are also given in the table.

| $Z$, cm | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|--------|---|---|---|---|---|---|---|---|
| $\Delta_{1\text{emp}}$ | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 |
| $\Delta_{1\text{calc}}$ | 0.21 | 0.2 | 0.2 | 0.19 | 0.19 | 0.19 | 0.19 | 0.19 |
| $\Delta_{2\text{emp}}$ | 0.2 | 0.2 | 0.2 | 0.2 | 0.1 | 0.1 | 0.1 | 0.1 |
| $\Delta_{2\text{calc}}$ | 0.17 | 0.17 | 0.16 | 0.16 | 0.16 | 0.16 | 0.16 | 0.16 |

Apparently, the best agreement of empirical data with the calculated ones is observed for the first four plasmoid sections downstream. This fact can be explained by more precise approximation of the amplitude of the azimuth electric field in this area of a plasma bunch.

Procedures described in the present paper can be useful for a broad circle of experts in different fields of low-temperature induction plasma physics and engineering.

References

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