Optimal operation of tidal plants based on nonlinear model predictive control strategy

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Abstract. Tidal range barrage is a well-recognized approach to extract energy from tides in a large scale. Nevertheless, the complexity of tide oscillation poses a challenge to efficaciously regulate this type of plants. Optimization of the process control may result in enhancement of energy yield with low influences on environment. This paper presents a framework where the whole tidal-electric system is modelled and coupled with a nonlinear model predictive control (NMPC) strategy to determine the optimal state of turbo-generators over a complete spring-neap cycle in a month. The method allows the system model to be nonlinear and dynamic under tight constraints, which would be extremely appropriate for the deployment of this highly predictable but frequently varied energy. A potential bay in Norway is chosen to be a proposal site to verify the benefits of the control strategy, by comparing the energy extraction between a simple barrage without sluices and a conventional one with sluices assembly. Results show the conventional barrage type is superior with respect to energy generation, while keeping the turbine blade diameter as as factor. For excessively large turbines, though, the revenue may be curtailed by costs due to environmental factors, and turbine construction.

1. Introduction
Research on renewable energy is progressively compelled by the pollution of fossil energy and the severity of global warming. Tapping energy from the tides has been applied for centuries, but the large-scale exploitation of tidal energy has not been achieved until the middle of 20th century. La Rance tidal power plant was the first fully operational tidal power station in the world, with 240MW capacity in total, which generates 480 GWh energy annually [1]. The success of the La Rance tidal power scheme has excited countless followers to further develop associated technologies to increase energy yield and efficiency [2,3]. The most recent accomplishment is the Sihwa tidal power station, which became operational in 2011. Its installed capacity of 254MW exceeds that of La Rance and makes Sihwa the world’s largest tidal power plant [4].

Tidal range power plants generate electricity by utilizing level differences between the basin and open sea. The technology is quite similar to the conventional low-head hydro power plant except that the head in tidal barrage varies frequently. Hydraulic turbines are designed for operating in single-effect or double effect mode. That is, for extracting tidal energy through the barrage, either in one or both flow directions. To maximize energy generation over a tidal cycle it seems fit to employ some kind of optimal control scheme for the tidal plant [5]. Early research focused mostly on energy maximization with respect to various design characteristics, such as basin volume, operation modes and pump effectiveness [5–7]. The recent direction gradually transfers to evaluate the potential tidal
energy with more precise hydrodynamic models, especially for the arguments of model accuracy between 0-D model, 2-D model and even 3-D model attracting many attentions [8–11]. Very few works on operating tidal range power plants by optimal control strategies have been reported. Bickley [5], however, proposed a method to control the power plant for maximal energy output using dynamic programming. More recently Pedro et al. [12] have presented a genetic algorithm method to maximize energy output by optimal dispatch of turbines. Angeloudis et al. [13] adopted a gradient-based optimization algorithm, an iterative procedure, to solve the nonlinear optimal control problem. The present paper focuses on application of a nonlinear model predictive control (NMPC) strategy to achieve optimal operation of tidal range power plants, which makes it possible to control the dynamic process of energy generation under various constraints. Major efforts have been made to model the tide, the bulb-type turbine performance and the hydraulic structure discharge. The NMPC controller is designed to optimally regulate the actions of turbo-generators and barrage sluices over multiple tidal cycles.

### 2. Tidal range power plants operation

Tidal range power plants are realized by building a dam over a river estuary, bay or fjord, and providing it with hydro turbines and sluices. The turbines extract potential energy from the water level difference between the open sea and basin. The potential energy harnessed by a tidal power plant per tide is governed by:

\[ P = \frac{1}{2} \rho g A_b H^2 \]  

where \( \rho \) is the fluid density (kg/m\(^3\)), \( g \) is the gravitational acceleration (m/s\(^2\)), \( A_b \) is the basin surface area (m\(^2\)), \( H \) is the head (m).

There are three operational modes for a tidal barrage [14,15], i.e., ebb generation mode, flood generation mode and dual generation mode, which are illustrated in figure 1. The operation modes are outlined as follows.

#### 2.1. Ebb generation

The basin is filled by incoming tides through sluices while the turbines remain stationary. The sluices are shut down at the high tide level and the basin holds the water level until the head reaches a sufficient value [16]. Then, the barrage begins to release water and generate electricity. The phase of energy generation may last for a couple of hours until the head is lower than the operational requirements, whereby the turbines halt. The sluices open again to refill the basin. The procedure of generating electricity then repeats.

#### 2.2. Flood generation

Compared with the ebb generation, the flood generation mode proceeds in an opposite way. During the flood tide turbine gates are kept open in order to generate electricity until an appropriately small
head is reached. The sluices then release water from basin until its level reaches that of the open sea. Thereafter all sluices are kept closed until the head becomes suitable for generating electricity again. The flood generation mode, however, provides less energy than the ebb mode. Clark [17] pointed out that the primary reason for this is that the basin area decreases with depth, as depicted in figure 2. The volume passing through the turbines in the flood generation mode will therefore be less than that of an ebb generating mode of equivalent tide amplitude. In addition, the sea level rises faster in the early phase of flood mode than it falls during ebb mode. Hence, the flood mode is somewhat less advantageous energy generation than the ebb mode.

![Figure 2. Area-depth relationships for a basin at different operations](image)

2.3. Dual generation
Dual generation mode is the combination of the ebb and the flood mode during each tidal cycle. The barrage initially holds the water level in the basin, then the turbines start to generate electricity in flood mode. The basin is gradually filled with water until the head becomes insufficient for further flood generation. After a holding stage the generation is then switched to the ebb mode. It is notable in the dual operation mode that the intervals where the sluices are kept open in order to fill or release water at the end of the corresponding generation stage, occur earlier. This in order to create a sufficient head for the succeeding generation stage [17]. Dual generation mode does produce electricity more regularly, which is closer to the aim of continuous power production. Hence this is the mode adopted in this paper.

3. Modelling the tidal electric system
The NMPC strategy allows the model to be nonlinear and time variant. For sake of computational performance, a simple model is preferred. For a tidal range power plant, the model may comprise the behaviour of the tides, the hydraulic system and the energy output. In this section, the tide, the hydraulic turbine, the sluice discharge and the plant energy output are modelled properly to capture the system dynamics effectively.

3.1. The tide model
The tide variations could be decomposed into several harmonic constituents represented by a series of cosine functions, no matter how complex the motion is. The major purpose of this method is to determine the amplitude and phase of each cosine function, which varies by location. Parker [18] unveils the primary tidal harmonic constituents (M₂, S₂, K₁, O₁) of a tide. The tide may be simply modelled by composing these constituents in a mathematical form. M₂ is the principle lunar semi-diurnal constituent with frequency 28.984 degrees/h, S₂ is the principle solar semi-diurnal constituent with frequency 30.000 degrees/h, K₁ is the lunisolar declinational diurnal constituent with frequency 15.041 degrees/h, O₁ is the lunar declinational diurnal constituent with frequency 13.943 degrees/h. The subscript “2” means the tidal type is a semi-diurnal tide, which means the place experiences two high tides and two low tides. Similarly, the subscript “1” means the type is a diurnal tide and the cycle
occurs once per day. Thus, the tidal variation above the mean sea level (MSL) with four harmonic constituents is obtained as:

\[ h = A_{M2} \cos(\omega_{M2} t - \phi_{M2}) + A_{S2} \cos(\omega_{S2} t - \phi_{S2}) + A_{K1} \cos(\omega_{K1} t - \phi_{K1}) + A_{O1} \cos(\omega_{O1} t - \phi_{O1}) \]  

(2)

where \( A \) and \( \omega \) with different subscripts denote amplitudes and frequencies respectively. The \( \phi \) with different subscripts are phase lags. If the amplitudes of \( A_{M2} \) and \( A_{S2} \) contribute more than the other two amplitudes to the value of tidal variation \( h \), then such tides would be semi-diurnal type. Otherwise, the tides would be diurnal type.

Notably, water hydrodynamics would count for more than astronomical considerations when the tidal wave propagates into estuaries from the ocean. The dimensions of ocean, bays and rivers collectively determine the tidal range to be exploited [18].

3.2. The hydraulic turbine model

The hydraulic turbine is a key element of the tidal power plant and accounts for a great cost of the entire project. Hence, it is crucial to set up an appropriate model for the hydraulic turbine applied in the tidal power plant.

The bulb-type turbine is widely used around the world, mainly due to its high efficiency for low-head applications. A turbine hill chart plotting unit speed against specific discharge is preferable when the turbine performance characteristics needs to be well considered. Figure 3 shows a hill chart for a double regulated turbine provided by Andritz Hydro [19].

![Figure 3. Andritz Hydro 3-bladed bulb turbine model [19]](image)

The unit discharge \( Q_{11} \) associates with the physical turbine discharge \( Q \) and the working head \( H \) in the form:

\[ Q_{11} = \frac{Q}{D^2 \sqrt{H}} \]  

(3)

and the turbine discharge is obtained by rearrange equation (3):

\[ Q = Q_{11} D^2 \sqrt{H} \]  

(4)

where \( D \) is the diameter of the rotor, \( H \) is the working head.

Additionally, an approximation of turbine efficiency curve is also available by selecting points on the maximum power output curve at regular intervals of unit speed in the hill chart.

Hence, power output of the turbine can be calculated by:
\[ P = \rho g H Q \eta \]  

where \( \rho \) is the fluid density, \( g \) is the gravitational acceleration, \( \eta \) is the turbine efficiency.

### 3.3. The sluice discharge model

Sluices are one of the most important elements of the tidal barrage assembly in regard to controlling flow discharges through the barrage. The operation of sluices is coordinated with the turbine regulation in order to optimize the energy output. It is valuable to set up a simple model of sluice discharges through the barrage.

Suppose the cross-sectional area at exit of the channel is \( A_E \), and the cross-sectional area at the throat of the channel is \( A_T \). It is easy to get the flow velocity at the throat according to the continuity of mass, which is given by:

\[ V_T = \frac{A_E}{A_T} V_E \]  

where the velocity at the exit of the discharge channel is denoted as \( V_E \). The theoretical discharge at the throat is \( A_T V_T \), and the actual discharge would be:

\[ Q_T = C_d A_T V_T \]  

where \( C_d \) is the sluice gate discharge coefficient. The value of \( C_d \) depends on the practical structure of the tidal barrage. The coefficient \( C_d \) ranges from 0.61 to 0.65 and it is generally set to be 0.62, according to Bansal [20]. Hence, the actual discharge is governed by:

\[ Q_T = C_d A_E \left( \sqrt{2g} \left( Z_U - Z_D \right) \right) \]  

where the depth of the upstream and downstream water is denoted \( Z_U, Z_D \) respectively. A turbine house with grid disconnection can be viewed as a ‘sluice’, if the turbine rotates frictionless. Intriguingly, some small-scale tidal power schemes may adopt barrages without sluices, which would be attractive for applications of unmanned operation in remote areas. Thus, for such a simplified tidal barrage, turbine houses would act as normal sluices if turbines are idling [17, 21].

### 3.4. Plant energy generation model

The plan view of a tidal power scheme is sketched in figure 4, which consists of a barrage across the estuary to form a basin. The water discharges of sluices and turbines are represented by the dash lines and solid lines respectively. Suppose the basin is an enclosed reservoir, which means there are no inflows from the river discharge. The surface area of the basin and the flow rate through the barrage are both assumed to be constant. Additionally, the dual generation mode is adopted in this scheme [7].

![Figure 4. The plan view of a tidal power scheme](image)

Suppose that the region experiences a semi-diurnal tide, and the period of the semi-diurnal tide is half of the lunar day. The tide variation is given by equation 2. Hence, the open sea level can be calculated as:
where the mean sea level (MSL) is denoted \( Z \). The total discharge \( Q_t \) through the barrage is identical to the rate of variation of volume variation in the basin, and it can be expressed as:

\[
\frac{dz_2}{dt} = \frac{Q_t}{A_b} \tag{10}
\]

where the basin depth is \( z_2 \), the surface area of the basin is denoted \( A_b \). The total discharge \( Q_t \) varies from time to time and its value depends on the number of operational turbines and sluices.

The total power generation of the barrage scheme \( P_b \) is the power output of a single turbine times the number of turbines connected to grid (denoted \( N_t \)), as shown in:

\[
P_b = N_t \rho g H Q \eta \tag{11}
\]

where the density of sea water is \( \rho \), the gravitational acceleration is \( g \), the working head for the turbine, identical to \( |z_1-z_2| \), is \( H \), the turbine discharge is \( Q \) and turbine efficiency is \( \eta \).

4. Control strategy

The key point in the sequel is to optimize the energy output from the tidal power plant. Nonlinear model predictive control (NMPC) is a promising strategy for this, since in calculation of the optimal control it is able, even for nonlinear dynamics, to account for the future behaviour of the plant, of references, disturbances and of other influences, while at the same time adhering to specified constraints on various quantities, such as inputs and outputs.

4.1. The optimal control problem

The power output of the tidal barrage scheme is given in equation (11). This section derives a control criterion for optimizing the energy generation in a chosen period. Note that the number of operational turbines would specifically depend on project scales, site conditions, machinery cost, etc. and it varies from one project to another. It would be appealing to search for a control criterion that does not rely on quantities with specific values. Such a criterion would own greater generality and a wider range of applications. One approach is first to normalize the energy production of the tidal barrage scheme, and then maximize the value of this dimensionless quantity. Accordingly, the normalization of the tidal energy can be expressed as:

\[
e = \frac{\int_0^{T_p} P_b \, dt}{\rho g A Q_{\text{max}} N_t} \tag{12}
\]

The numerator represents the actual extracted energy while the denominator stands for the ideally maximum potential energy available for exploitation. The symbol \( Q_{\text{max}} \) is the maximum discharge of a single bulb turbine and \( N_t \) is the total number of turbines available. The tidal range is denoted \( A \). The period of the semi-diurnal tide is denoted \( T_p \). By substituting equation (11), the equation (12) is transformed to:

\[
e = \int_0^{T_p} \left( \frac{|z_1-z_2|}{A} \right)^{3/2} \eta \, n \, dt \tag{13}
\]

The integrand is called normalized power, which is a dimensionless quantity. The turbine efficiency \( \eta \) can be looked up directly from the turbine hill chart. The variable \( n \) is a percentage of turbo-generators connected with the grid, which is chosen to be the control variable in the control algorithm.

Therefore, the control objective for a tidal power plant is to maximize over the period \( T_p \) the normalized energy, with respect to the control variable \( n \), i.e. find:
arg max \( n \) \( \int_{0}^{T_p} \left( \frac{|z_1 - z_2|}{A} \right)^{3/2} n \, dt \)

(14)

where \( n \) is the control as function of time, with \( n(t) \) being the fraction of turbines connected to the grid at time \( t \). Accordingly the control is required to satisfy the constraint at any given time \( 0 \leq n(t) \leq 1 \).

For a simplified barrage without sluices, turbine houses act as sluices to accelerate water discharge while the turbines are idling, thereby creating a larger water head. With the controller tasked to optimize energy generation, this process is illustrated in figure 5 for one tidal cycle, where A and C are the instants of turbine halt. The red curve represents the sea level, and the blue one is the basin level.

Figure 5. Optimization of turbine disconnection periods for a simplified barrage

In a conventional barrage, sluices contribute further to the discharge and basin level holding. The operation is depicted in figure 6 with optimal manipulations of turbines and sluices. The red curve represents the sea level and the blue one is the basin level. The plant begins to generate electricity at Point A and halts at Point B, and the sluices are open to accelerate the filling until the basin level reaches the value of the sea at Point C. With the sluices shut down the basin holds the water level until the head reaches a sufficiently high value at Point D. The plant then starts to generate electricity until the head is insufficiently low at Point E, and the sluices to release water until the basin level is identical to the sea at Point F. The cycle then repeats. The switching points A, B, C, D, E, F needs to be investigated in terms of optimal operations.

Figure 6. Optimal operations of turbines and sluices for a conventional barrage

4.2. The principle of nonlinear model predictive control

Model predictive control is an advanced method of process control, which is widely used in various industrial fields. Nonlinear model predictive control (NMPC) is well-suited for control of plants with explicit constraints on, for example, states and inputs [22]. Based on a model of the nonlinear dynamic plant the future behaviour of the plant is predicted, and the control is obtained by repeatedly optimizing the behaviour as measured by a given performance criterion. In reality a variety of more comprehensive constraints, such as economic and environmental restrictions, need to be enforced on a tidal plant. NMPC provides a convenient and advantageous scheme for incorporating these also.
The basic principle of NMPC is illustrated in figure 7 in the form of set-point tracking. At each time \( t_n \) the algorithm tries to minimize the cost incurred from the deviation of the predicted state from a set-point, by iteratively searching for an optimal online control input. This is explained further in the following paragraphs [22-24].

![Figure 7. Basic principle of nonlinear model predictive control](image)

The control and the state are required to satisfy the constraints:

\[
\begin{align*}
\mathbf{u}(k) &\in \mathbb{U} \subseteq \mathbb{R}^m \quad \text{and} \quad \mathbf{x}_u(k) \in \mathbb{X} \subseteq \mathbb{R}^n, \quad k = 0, 1, 2, \ldots, N-1 \\
\end{align*}
\]

with the sets of admissible values, \( \mathbb{U} \) and \( \mathbb{X} \), given by

\[
\begin{align*}
\mathbb{U} &= \{ \mathbf{u} \in \mathbb{U}_{\text{min}} \leq \mathbf{u} \leq \mathbb{U}_{\text{max}} \} \\
\mathbb{X} &= \{ \mathbf{x} \in \mathbb{X}_{\text{min}} \leq \mathbf{x} \leq \mathbb{X}_{\text{max}} \}
\end{align*}
\]

Here \( \mathbb{U}_{\text{min}}, \mathbb{U}_{\text{max}}, \mathbb{X}_{\text{min}}, \mathbb{X}_{\text{max}} \) are given constant vectors.

For simplicity the prediction horizon is set equal to the control horizon, i.e. \( T_p = T_c = NT \).

The basic NMPC algorithm is:

Step I: At discrete time \( n \) measure the state \( \mathbf{x}_u(n) \) of the actual plant.

Step II: Set the initial state of prediction to \( \mathbf{x}_u(n) = \mathbf{x}_u(n) \), and do

\[
\min J_N(\mathbf{x}_u(n), \overline{\mathbf{u}}(\cdot)) = \sum_{k=n}^{n+(N-1)} F(k, \mathbf{x}_u(k), \overline{\mathbf{u}}(k))
\]

with respect to:

\[
\overline{\mathbf{u}}(k) \in \mathbb{U}, \mathbf{x}_u(k) \in \mathbb{X}, \quad k = n, n+1, n+2, \ldots, n+(N-1)
\]

subject to:

\[
\mathbf{x}_u(k+1) = f(\mathbf{x}_u(k), \overline{\mathbf{u}}(k))
\]

to obtain the optimal control sequence \( \overline{\mathbf{u}^*}(k) \in \mathbb{U}, \quad k = n, n+1, n+2, \ldots, n+(N-1) \)
Step III: Input the first value $\bar{u}^*(n)$ of the optimal control sequence to the actual system.
Step IV: Replace $n$ by $n+1$ and return to Step I.

4.3. Solving the optimal control problem

The objective of the control is to maximize the normalized energy per tidal cycle. Based on the continuous time expression for the plant total discharges given by equation (10), a discrete-time system model can be obtained as:

$$\frac{x_u(k+1) - x_u(k)}{T} = \frac{Q_t}{A_b}$$

where $x_u(\cdot)$ is the basin depth, $T$ is the sampling period, $A_b$ is the basin area and $Q_t$ is the total discharge through the barrage.

The total discharge of operative turbines and sluices is chosen as the control variable $u(k)$.

The optimal problem will be solved for two typical cases. One is for a simplified tidal barrage without sluices. The other is for a conventional tidal barrage with sluices. For the simplified tidal barrage, the control variable $u(k)$ would be a one-dimensional vector bounded by $0 \leq u(k) \leq 1$. Since the conventional barrage incorporates several sluices to improve the discharge rate, the control variable $\bar{u}(k)$ is altered to be a two-dimensional vector $\bar{u}(k) = [u'(k) \ u''(k)]$, where as before $0 \leq u'(k) \leq 1$ is the fraction of maximal turbine action and $0 \leq u''(k) \leq 1$ is the fraction of maximal sluice action.

The NMPC algorithm in Section 4.3 is formulated in terms of minimizing a cost function. For the tidal plant, however, the objective is instead to maximize a criterion, namely the normalized energy, in equation (14). This is converted to a minimization problem simply by adding a minus sign to the normalized energy, which in a discretized form then is

$$J_N(\cdot) = -\sum_{k=n}^{n+(N-1)} \left( \frac{|z_1(k) - x_u(k)|}{A} \right)^{3/2} \cdot \eta \cdot \bar{u}(k)$$

where the tide approximation function $z_1$ is given by equation (9). The dynamic optimizer, that is, the NMPC algorithm, then searches for the optimal solution minimizing the normalized energy over one tidal period. The computation of the open loop optimal control is formulated as a nonlinear constrained optimization problem. In the current application this is solved by the MATLAB function ‘fmincon’ [25]. Many other solvers are also available.

5. Simulation results

For simulation we assume a body of water representing a typical smaller fjord in northern Norway, in which the mean sea level (MSL) is 1.82 m, the mean high water springs (MHWS) 3.17 m and the mean high water neaps (MHWN) 2.47 m. Tides are assumed to be of the semidiurnal type, with a spring-neap tide period of 15 days. The tide variation is depicted in figure 8.

A summary of the parameters of the basin and the tidal-electric system used in the simulation are given in table 1. The application of NMPC algorithm is based on MATLAB Optimization Toolbox. The NMPC sampling interval $T$ is set to be 1 hour, and the optimization horizon length $N$ is 720.
Figure 8. Time series representation of tide variation for the assumed data

Table 1. Specifications of the tidal-electric plant.

| Parameter                          | Value          |
|-----------------------------------|----------------|
| Basin area (km²)                  | 6.8            |
| Initial water level of basin (m)  | 1.5            |
| Sea water density (kg/m³)         | 1025           |
| Mean sea level (m)                | 1.82           |
| Tidal amplitude (m)               | 1.35           |
| Tidal period (days)               | 15             |
| Amount of tidal cycle (-)         | 2              |
| Turbine Type                      | Bulb           |
| Turbine number (-)                | 6              |
| Sluice number (-)                 | 6              |
| Sluice Discharge coefficient (-)  | 0.62           |
| Sluice channel exit radius (m)    | 1.85 × Rotor radius |
| NMPC Sampling interval (s)        | 3600           |
| NMPC optimization horizon length (-) | 720       |

Table 2. Optimal energy output per tidal month for the simple barrage with different rotor sizes

| Rotor radius (m) | Optimal energy (MWh) |
|------------------|-----------------------|
| 0.5              | 97.84                 |
| 1.0              | 427.56                |
| 1.5              | 1088.00               |
| 2.0              | 2121.90               |
| 2.5              | 3273.50               |
| 3.0              | 4022.40               |
| 3.5              | 4325.20               |

5.1. Barrage without sluices installed

Basin level variation and generated power ensued from the optimal operation of the plant over a period of one tidal month is shown in figure 9 for four different turbine rotor radii (R = 0.5, 1.5, 2.5, 3.5 m). As seen from the figure, the power generation increases significantly with rotor size. However, the basin level variation exceeds the open sea level variation when the rotor size reaches 3.5 m. This may cause the basin become overly empty. Hence, it would be less interesting to investigate power generation with even larger rotor sizes. Numerical variations of energy output with different rotor sizes are shown in table 2.
Figure 9. Basin level and generated power in the simplified tidal barrage with optimally manipulated turbines, displayed for rotor radii: (a) $R = 0.5$ m, (b) $R = 1.5$ m, (c) $R = 2.5$ m and (d) $R = 3.5$ m.

Figure 10 shows the relationship between the rotor radius and the optimal energy generation. The optimal energy rises substantially for radii around 2 m, but flattens out beyond that. The product of turbine discharge and head determines the final energy output if the efficiency of the turbine is assumed constant. For a simplified barrage, a larger turbine means larger discharges and smaller head. Hence, it is worthwhile to evaluate the optimal energy output with different turbine sizes ahead in order to give an appropriate suggestion for the turbine selection.
5.2. Barrage with sluices installed

Sluices are able to accelerate the discharge rate and hence improve the energy generation in conventional barrages. Figure 11 shows the basin level variations and generated power when the turbo-generators and sluices are manipulated to maximize energy output over a tidal month. Results for four different rotor radius are displayed. The simulation results of energy yield are presented in table 3 and plotted in figure 12 respectively. With the help of sluices, the optimal energy output of the conventional barrage grows significantly when the rotor size increases.

![Figure 11](image-url)

**Figure 11.** Basin level and generated power in the conventional tidal barrage with optimally manipulated turbines and sluices, displayed for rotor radii: (a) R = 0.5 m, (b) R = 1.5 m, (c) R = 2.5 m and (d) R = 3.5 m.

**Table 3.** Optimal energy output per tidal month for different rotor sizes when sluices are installed

| Rotor radius (m) | Optimal energy (MWh) |
|-----------------|----------------------|
| 0.5             | 115.03               |
| 1.0             | 508.20               |
| 1.5             | 1411.10              |
| 2.0             | 2756.40              |
| 2.5             | 4283.20              |
| 3.0             | 6261.30              |
| 3.5             | 9797.70              |
5.3. *Energy comparison between barrages with and without sluices*

For straightforward comparison of the optimal energy yields from the simplified barrage and the conventional one are shown together in figure 13. Whether to include sluices or not, and the selection of the rotor size are issues that need to be considered in order to achieve optimal tidal energy extraction. Evidently the amount of energy tapped by the conventional barrage is higher than that by the simplified barrage for all rotor radius, and the gap between the two expands dramatically for rotor radii beyond 1 m. The sluices thus play a significant role for the efficacy of the tidal plant. The remarkable increase they introduce in energy production, stems from the rapid release of the basin that they allow, together with a careful timing of these release events by the controller.

6. Conclusion and prospects

This paper firstly tries to build mathematical models of the whole tidal-electric system; secondly, the nonlinear model predictive control (NMPC) algorithm is introduced to optimize the energy output from the tidal barrage; finally, the section of simulation compares the optimal energy output from a simplified barrage and a conventional one, considering different radii of the turbine rotor. The NMPC algorithm, however, is able to render optimal manipulations of turbines and sluices under much more general constraints than those considered here. For instance, under confinements from environmental influences on the local ecosystems and the cost of routine maintenance. The optimizer searches for solutions that minimize the cost function over the prediction horizon. The energy output included in
the optimization criterion is normalized to exclude the undetermined variables from entering the simulation. As for the energy yield per tidal month, the conventional barrage is superior to the simplified one. However, if economical and environmental factors are considered, then the conclusion hardly holds any more. The marginal revenue of power generation gradually drops with increasing radii of turbine rotor, owing to material costs of turbines.

In regard of the optimal control strategy, the tidal-electric system model needs to be further refined in order to account for the hydrodynamic processes taking place when the tidal wave propagates into estuaries from the ocean. Hence, the governing function of tide variation across the barrage needs to be further updated. The environmental impact of water level variation in the basin needs to be carefully investigated. The outcome may also influence the final selection of rotor size. The NMPC algorithm should furthermore be converted into efficient programming code and run on a lab-oriented hardware system to verify its performance.

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