A Critical Surface of Chiral-invariant System with Gauge Boson and Fermions

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Abstract

In the chirally-invariant context of the $U_{em}(1)$ gauge interaction and four-fermion interactions for ordinary and mirror fermions, the Schwinger-Dyson equation for the fermion self-energy function is studied on a lattice. We find that a sensible infrared limit can be defined on a critical surface, which is consistent with the critical line found in the continuum theory.

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1 Introduction

Dynamical symmetry breaking plays an important role in understanding success of the Standard Model. The structure of dynamical symmetry breaking has been studied intensively for a wide variety of theoretical models. Due to the lack of clear ideas as to the physical origin of dynamical symmetry breaking in the Standard Model and the essential difficulties in attempting to study such non-perturbative phenomena, progress is still quite limited. One theoretical approach to the study of this phenomenon is to find the solutions of the Schwinger-Dyson equation (SDE) for the fermion self-energy function $\Sigma(p)$. For a homogeneous SDE without an explicit ultraviolet cutoff ($\Lambda$), it was shown [1] that the mass operator $(\bar{\psi}\psi)$ has an anomalous dimension $d_{\bar{\psi}\psi} = 2 + \sqrt{1 - 3\alpha/\pi}$ for a finite gauge coupling and thus the high momentum decrease of solutions $\Sigma(p) \to (p^2)^{-3 + d_{\bar{\psi}\psi}}$ as $p^2 \to \infty$. Further work [2] in a version of the theory with an explicit ultraviolet cutoff shows: (i) there are (no) spontaneous symmetry breaking solutions to the homogeneous SDE for gauge coupling $\alpha > \frac{\pi}{3}$ ($\alpha < \frac{\pi}{3}$); (ii) for an inhomogeneous SDE, all solutions for weak coupling require that the inhomogeneous term, i.e. an explicit bare fermion mass $m_0$, goes as $m_0 \sim \Lambda^{-(3 - \sqrt{1 - 3\alpha/\pi})}$ in the continuum limit $\frac{\Lambda}{m} \gg 1$, where $m$ is an infrared mass scale for the theory. However the mass operator $m_c\bar{\psi}\psi$ remains finite at this limit and generates explicit breaking of chiral symmetry.

The solutions to the SDE were analyzed in detail for both weak and strong couplings [3]. The critical point $\alpha_c = \pi/3$ is viewed [4] as an ultraviolet fixed point of the theory in order to give a sensible infrared limit for the theory. However, the renormalization properties of the theory are called into question for the required running of the gauge coupling with the cutoff. The most important progress made by Bardeen, Leung and Love (BLL) [5] is to consider a system of gauge interaction and four-fermion interaction, which is induced by integrating out the high-frequency contribution of the theory. The theory was shown to exhibit a line of critical points separating the chirally broken phase from the symmetric one. The renormalization properties of the theory, the relevance of the four-fermion operators and the spontaneous breaking of chiral symmetry with accompanying pseudo-Goldstone bosons were intensively studied. Recently, in Ref. [6], we consider this problem on a lattice by adding the Wilson fermion and bare mass terms, which however are explicitly chirally-variant. In this paper, using the gauge-invariant and chirally-invariant lattice regularization, we study the SDE for the fermion self-energy function in the system containing gauge interaction and four-fermion interactions for both ordinary fermions and mirror fermions. As an initial step, we show a critical surface, where a sensible infrared limit of the theory can probably be realized.
2 Action

The fermion “doubling” phenomenon is a well-known problem arising when fermion fields are defined on a lattice [7]. Wilson [8], however, introduced an extra dimension-5 operator (the Wilson term) into the naive lattice pure gauge $S_g(U)$ and the Dirac action $S_d(\bar{\psi}, \psi, U)$ for ordinary and mirror fermions,

$$S = S_g(U) + S_d(\bar{\psi}, \psi, U) + \frac{r}{a} \sum_{x,\mu} \bar{\psi}(x) \partial^2_\mu \psi(x)), \quad (1)$$

where $a = \frac{\pi}{\Lambda}$ is the lattice spacing and the lattice “laplacian” $\partial^2_\mu$ is defined as

$$\partial^2_\mu \psi(x) = U_\mu(x) \psi(x + a_\mu) + U_\mu^\dagger(x - a_\mu) - 2\psi(x). \quad (2)$$

This extra term with a finite Wilson parameter $r$ ($0 < r \leq 1$ for reflection positivity) becomes an irrelevant operator for ordinary fermions with $ka \sim 0$ in the infrared limit while it is a relevant operator for mirror fermions with $ka \sim \pi$ in the high-energy regime, in fact it generates an effective mass $M \sim \frac{r}{a}$ for mirror fermions. However, the Wilson term explicitly breaks chiral symmetry.

From a dynamical viewpoint, we have not understood completely the operator content of the theory at short distances, where there probably exist local and non-local high-dimension operators for ordinary and mirror fermions and gauge bosons, owing to the experimental observation of a rich mass spectrum of fundamental particles in the low-energy region. At long distances, these high-dimension operators, however, should be relevant and irrelevant for ordinary fermions and mirror fermions respectively in such a way that mirror fermions decouple from the low-energy spectrum and ordinary fermions remain and couple properly with gauge bosons. We thus consider relevant and irrelevant high-dimension operators for ordinary and mirror fermions respectively. In general, there are several possibilities of chirally invariant four fermion interactions on a lattice space-time, e.g.,

$$\beta_1 \bar{\psi}(x) \psi(x) \bar{\psi}(x) \psi(x) + \beta_1 \sum_\mu \bar{\psi}(x \pm \mu) \psi(x) \bar{\psi}(x) \psi(x)$$

$$+ \beta_3 \sum_{\mu, \nu} \bar{\psi}(x \pm \mu) \psi(x \pm \nu) \bar{\psi}(x) \psi(x) + \cdots, \quad (3)$$

which represent complicated interactions between ordinary fermions and mirror fermions. The origin of these interactions (which might stem from the quantum gravity [9, 10, 11] or a high-frequency contribution of the theory [3, 4] and other unknown physical dynamics [13, 14]) will not be a focus of this paper. We have tuned the four-fermion couplings in eq. (3) such that the interactions between ordinary fermions and mirror fermions are of Nambu-Jona Lasinio type (NJL) [15]. Thus
we consider the following chirally invariant lagrangian with lattice regularization\[16\]

\[
S = S_g(U) + S_d(\bar{\psi}, \psi, U) + S_r + S_{ir}
\]

\[
S_r = -G_1 \sum_x (\bar{\psi}_L(x) \psi_R(x) \bar{\psi}_R(x) \psi_L(x))
\]

\[
S_{ir} = -\frac{G_2}{2} \sum_{x\mu} (\bar{\psi}_L(x) \partial^2_\mu \psi_R(x) \bar{\psi}_R(x) \partial^2_\mu \psi_L(x)),
\]

where \( U \in U_{em}(1) \) is chosen. In eq. (4), the third term \( S_r \) represents the (NJL) interaction \[15\] of ordinary fermions and mirror fermions. The fourth term \( S_{ir} \) is the NJL interaction of mirror fermions only, where the gauge link connects neighbouring right-handed and left-handed fermions to have the \( U_{em}(1) \) local gauge symmetry \[2\]. \( G_{1,2} \) are two, as yet unspecified, Fermi-type \( O(a^2) \) coupling constants. Note that (i) in the naive continuum limit, probed by momenta \( pa \ll 1 \ (S_{ir} \simeq 0) \), eq. (4) is just a gauged NJL model for ordinary fermions \[3\]; (ii) in the naive “lattice limit”, probed by momenta \( pa \simeq 1 \), \( S_{ir} \) is significantly non-vanishing and eq.(4) can be considered as gauged NJL model for mirror fermions \[17\]; (iii) obviously, \( S_r(S_{ir}) \) is a relevant (irrelevant) operator for ordinary fermions and both \( S_r, S_{ir} \) are relevant operators for mirror fermions.

In eq.(4), chiral symmetry is perfectly conserved at short distance, the point is, however, whether we can separate ordinary fermions from mirror fermions and have a sensible infrared limit \( \frac{\Lambda}{m} \gg 1 \) of the theory.

### 3 Gap equations

We are thus led to study the SDE of the fermion self-energy function \( \Sigma(p) \). The Landau mean-field method and the quenched and planar approximations (large-\( N_f \) approach \( G_{1,2}N_f \) fixed, \( N_f \gg 1 \) is the number of flavour in eq.(4)) will be adopted. Given the (4) with the quadrilinear \( S_r \) and \( S_{ir} \) terms, we have, as illustrated in Fig.1,

\[
\Sigma(p) = 2g_1 \int_{q} \frac{\Sigma(q) + \frac{w(q)}{s^2(q) + M(q)^2}}{s^2(q) + M(q)^2} \frac{1}{a} \int_{q} \frac{1}{4\pi^2 s^2(\frac{p-q}{2})} \left( \delta_{\mu\nu} - \frac{\xi}{s^2(\frac{p-q}{2})} s_{\mu}(\frac{p-q}{2}) s_{\nu}(\frac{p-q}{2}) \right) \cdot \left( V_{\mu\nu}^{(2)}(p,p) - V_{\mu\nu}^{(1)}(p,p) \right) \frac{1}{\gamma_\rho s_\rho(q) + M(q)} V_{\nu}^{(1)}(p,q),
\]

where \( p(q) \) are dimensionless external (internal) momenta; \( s_\mu(l) = \sin(l_\mu) \) and \( s^2(l) = \sum_\mu \sin(l_\mu) \); \( \lambda = e^2 \) for \( U_{em}(1) \) gauge group and \( g_1 a^2 = G_1 N_f \); the Wilson term \( w(q) = \sum_\mu (1 - \cos q_\mu) \) and \( M(q) = a \Sigma(q) + r w(q) \); \( \int_q = \int_\pi \frac{d^3q}{(2\pi)^3} \). The vertices \[18\] are \( (k_\mu = \frac{p_\mu + q_\mu}{2}) \)

\[
V_{\mu}^{(1)}(p,q) = \left( \gamma_\mu \cos k_\mu + r \sin k_\mu \right); \quad V_{\mu\nu}^{(2)}(p,q) = a \left( -\gamma_\mu \sin k_\mu + r \cos k_\mu \right) \delta_{\mu\nu}.
\]
The Wilson parameter $r$ in above equation turns out to be a symmetry-breaking v.e.v. ($r = \bar{r}a$)

$$
\bar{r} = \frac{G_2}{4} \sum_{\mu,x} \left( \frac{1}{4} \right) \left\langle \bar{\psi}_L(x) \partial_\mu \psi_R(x) + \text{h.c.} \right\rangle,
$$

which obeys gap-equation generated by the NJL self-interaction $S_{ir}$ of only mirror fermions $[4]$,

$$
r = \frac{g_2}{2} \int_q w(q) \frac{\Sigma(q) \alpha + rw(q)}{s^2(q) + (\Sigma(q) \alpha + rw(q))^2},
$$

where $g_2 a^2 = N_f G_2$ and we eliminate the interaction between gauge boson and mirror fermions. This gap equation (5) is the result of leading order in the large-$N_f$ expansion.

One clearly finds that both ordinary fermions and mirror fermions contribute to the gap equation (5) for the fermion self-energy function $\Sigma(p)$. What we should do is to find a consistent solution to the SDE (5,8) where ordinary fermions can separated from mirror fermions and a sensible continuum limit can be defined.

### 4 Ordinary and mirror fermions

One of the main novelties of (5) is the non trivial interplay between the continuum region, i.e., for ordinary fermions with momenta ($q \ll 1$), and the truly discrete region for mirror fermions $q \simeq 1$. For the small external momenta $p = p' a \ll 1$, we rewrite

$$
\Sigma(p) = \Sigma_c(p') + \frac{\Delta}{a},
$$

where $\Sigma_c(p')$ ($\Sigma_c(p') a \ll 1$) is the self-energy function of ordinary fermions in the continuum theory (region) and $(\frac{\Delta}{a})$ is the divergent contribution stemming from mirror fermions $[3]$. In order to study such interplay between ordinary-fermion and mirror-fermion contributions, it is important to introduce a “dividing scale” $\epsilon$, such that $p'/a \ll \epsilon \ll \pi$. Separating the integration region in (5) into two regions the “continuum region” $(0, \epsilon)^4$ and the “lattice region” $(\epsilon, \pi)^4$, we may separate our integral equations into the “continuum part” and the “lattice part”

$$
\Sigma(p) = 2G_1 \int_{\epsilon \Lambda} \frac{d^4 q'}{(2\pi)^4} \frac{\Sigma_c(q')}{(q')^2 + \Sigma_c(q')^2} + 2g_1 \beta_1(r, \epsilon) + \frac{r}{a} \beta_2(r, \epsilon) \\
+ \frac{\alpha}{\alpha_c} \int_{\epsilon \Lambda} \frac{d^4 q'}{4\pi^2} \frac{1}{(p' - q')^2 + \Sigma_c(q')^2} + \frac{\alpha}{\alpha_c} \delta_1(r, \epsilon) + \frac{r}{a} \delta_2(r, \epsilon),
$$

where $p', q'$ are dimensionful momenta, $\alpha = \frac{\Delta}{4\pi}$ and $\alpha_c = \frac{\pi}{\pi}$. The “continuum part” of eq.(10), where the Landau gauge $\xi = 1$ is chosen, is same as that derived from the continuum theory with an intermediate cutoff $\epsilon \Lambda$. As for the contributions to
the integral equation from the discrete “lattice region” \( \beta_i(r, \epsilon), \delta_i(r, \epsilon)(i = 1, 2) \), we obtain:

\[
\begin{align*}
\beta_1(r, \epsilon) &= \int_{q \in (\pi, \pi)^4} \frac{\Sigma_c(q)}{s^2(q) + M(q)^2} \quad (11) \\
\beta_2(r, \epsilon) &= 2g_1 \int_{q \in (\pi, \pi)^4} \frac{w(q)}{s^2(q) + M(q)^2} \quad (12) \\
\delta_1(r, \epsilon) &\approx -\int_{(\pi, \pi)^4} \frac{d^4q}{4\pi^2} \frac{\Sigma_c(q)}{4s^2(q)} \left[ -c^2 \left( \frac{q^2}{2} \right) + r^2 s^2 \left( \frac{q^2}{2} \right) \right] \quad (13) \\
\delta_2(r, \epsilon) &\approx \frac{\alpha}{\alpha_c} \int_{q \in (\pi, \pi)^4} \frac{d^4q}{12\pi^2} \frac{1}{4s^2(q)} \left[ \frac{1}{2} - \frac{w(q)(-c^2 \left( \frac{q^2}{2} \right) + r^2 s^2 \left( \frac{q^2}{2} \right)) + s^2(q)}{s^2(q) + M^2(q)} \right] \quad (14)
\end{align*}
\]

where \( c^2(l) = \sum_{\mu} \cos^2(l_{\mu}) \). The dependence on the external momentum \( p \in (0, \epsilon)^4 \) is omitted in \( \delta_i(r, \epsilon) \), because \( p \ll q \) in the “lattice region” \( q \in (\epsilon, \pi)^4 \).

Note that (i) \( \delta_i(r, \epsilon) \) do not depend on the gauge parameter \( \xi \) for there is a perfect cancellation between the “contact” and the “rainbow” diagrams (Fig.1), which is guaranteed by Ward’s identities; (ii) the limits \( \lim_{\epsilon \to 0} \delta_2(r, \epsilon) \) and \( \lim_{\epsilon \to 0} \beta_2(r, \epsilon) \) can be taken since the functions \( \delta_2(r, \epsilon) \) and \( \beta_2(r, \epsilon) \) are regular in the limit \( \epsilon \to 0 \), while this is not case for the functions \( \delta_1(r, \epsilon) \) and \( \beta_1(r, \epsilon) \); (iii) within the “lattice region”, the Wilson parameter \( r \) must not vanish and all functions \( \beta_i(r, \epsilon) \) and \( \delta_i(r, \epsilon) \) remain non-vanishing. The functions \( \delta_2(r, \epsilon) \) (\( \delta_1(r, \epsilon) \)) and \( \beta_2(r, \epsilon) \) (\( \beta_2(r, \epsilon) \)) can be regarded as the divergent (finite) mirror-fermion contributions to gap equation (3) through the four-fermion interaction \( S_r \) and the gauge interaction respectively.

In order to find the sensible “continuum limit”, where the spectrum of mirror fermions is far separated \((r \sim O(1))\) from that of the observed ordinary fermions \((\Sigma_c(p')a \ll 1)\), we should be allowed to tune only one parameter, which is related to mass renormalization counterterm. Thus we tune \( \Delta \), in such a way that the “\( \frac{1}{a} \)” terms in both sides of (5) agree. It is self-consistent that this tuning is performed simultaneously in the numerators and denominators of the RHS of eqs. (3, 8). These observations mean that we can rewrite the gap equations (3, 8) as the following three self-consistent gap-equations:

\[
\begin{align*}
\Sigma_c(p') &= 2G_1 \int_{\epsilon \Lambda} \frac{d^4q'}{(2\pi)^4} \left( \frac{\Sigma_c(q')}{(q')^2 + \Sigma_c(q')^2} + 2g_1 \beta_1(r, \epsilon) + \frac{\alpha}{\alpha_c} \int_{\epsilon \Lambda} \frac{d^4q'}{4\pi^2} \frac{1}{(p' - q')^2} \frac{\Sigma_c(q')}{(q')^2 + \Sigma_c(q')^2} \right) \quad (15) \\
\frac{\Delta}{a} &= r \frac{\beta_2(r, 0) + \frac{r}{a} \delta_2(r, 0)}{\beta_2(r, 0)} \quad (16) \\
r &= \frac{g_2}{2} \int q \frac{w(q) \Sigma_c(q)a + rw(q)}{\text{den}(q)} \quad (17)
\end{align*}
\]

where \( \text{den}(q) = \sin^2 q_{\mu} + (\Sigma_c(q)a + rw(q))^2 \) and also the denominator \( s^2(q) + M^2(q) \) of eqs. (1, 2, 3, 4) is substituted by “\( \text{den}(q) \)” and thus is free from \((\frac{1}{a})\) divergence.
The eq. (15) is the “continuum” counterpart of the SDE (3) on the lattice. For the “lattice” equation (17), one clearly finds that the solution \( r > 0 \) stems from the contribution of mirror fermions, owing to the factor \( w(l) \), which does not vanish in the “lattice” region.

The consistency of this fine-tuning (16), which can also be called the “chiral limit”, can probably be guaranteed [19] by Ward-type identities due to chiral symmetry of the action (4) at short distances and we shall not discuss it in this paper. The term \( \Delta a \) plays a rôle akin to that of mass counterterms [19]. Its actual value depends not only on the RHS of eq. (16), which is the mirror-fermion contribution at one-loop level, but also all possible \( \frac{1}{a} \) contribution from mirror fermions in the SDE (3). Looked at from this point of view, the self-consistency of this one-parameter tuning should not be more surprising than the well-known and checked self-consistency of mass-renormalization in continuum Quantum Field Theory.

5 The critical surface

Let us now address the important question of the \( \epsilon \)-independence of our results. The introduction of the “dividing scale” in (3) is, apart from the requirement \( p'a \ll \epsilon \ll \pi \), rather arbitrary, thus no dependence on \( \epsilon \) should appear in our final results. In order for such independence to occur, as it must, it is clear that the \( \epsilon \)-dependent terms (e.g., \( \ell n \epsilon \)) from the continuum integral in (13), must be compensated by analogous terms arising in the calculation of \( \delta_1(r, \epsilon) \) and \( \beta_1(r, \epsilon) \). Owing to integral momenta \( q \in (\epsilon, \pi) \) in eqs.(12),(14), we can make the reasonable approximation \( \Sigma_c(q) \simeq \Sigma_c(\Lambda) \) in the numerators of \( \delta_1(r, \epsilon) \) and \( \beta_1(r, \epsilon) \). Thus the \( \epsilon \)-independent terms \( \Sigma_c(\Lambda)\delta_0(r)(\Sigma_c(\Lambda)\beta_0(r)) \) contained in \( \delta_1(r, \epsilon)(\beta_1(r, \epsilon)) \) can be found by numerical calculation. The numerical functions \( \delta_0(r) \) and \( \beta_0(r) \) are reported in Fig.2 and 3. Thus segregating the \( \epsilon \)-dependent terms in (13), we may write

\[
\Sigma_c(p') = 2G_1 \int_{\Lambda} \frac{d^4q'}{(2\pi)^4} \frac{\Sigma_c(q')}{(q')^2 + \Sigma_c(q')^2} + 2g_1 \Sigma_c(\Lambda)\beta_0(r) + \frac{\alpha}{\alpha_c} \int_{\Lambda} \frac{d^4q'}{4\pi^2} \frac{\Sigma_c(q')}{(p' - q')^2 + \Sigma_c(q')^2} + \frac{\alpha}{\alpha_c} \Sigma_c(\Lambda)\beta_0(r),
\]

which is analogous to the “chiral limit” SDE of the continuum theory with two additional boundary terms \( 2g_1 \Sigma_c(\Lambda)\beta_0(r) \) and \( \frac{\alpha}{\alpha_c} \Sigma_c(\Lambda)\delta_0(r) \).

It is straightforward to adopt the analysis of Bardeen, Leung and Love [3,12,20]. After performing the angular integration and changing variables to \( x = (p')^2 \), eq.(18) becomes a boundary-value problem,

\[
\frac{d}{dx} \left( x^2 \Sigma'_c(x) \right) + \frac{\alpha}{4\alpha_c} \frac{x}{x + \Sigma_c^2(x)} \Sigma_c(x) = 0
\]
\[(1 + \tilde{g}_1)\Lambda^2 \Sigma_c'(\Lambda) + \Sigma_c(\Lambda)(1 - 2\tilde{g}_1 \frac{\alpha}{\alpha_c} \beta_0(r) - \frac{\alpha}{\alpha_c} \delta_0(r)) = 0, \quad (19)\]

where \(\tilde{g}_1 = G_1 a^{-2} N_f \frac{\alpha}{\alpha_c}\). The solution to this boundary value problem is well established. For weak coupling, we have the gap equation,

\[
\tanh \theta = \frac{(\tilde{g}_1 + 1)\sqrt{1 - \frac{\alpha}{\alpha_c}}}{(\tilde{g}_1 - 1) + 4\tilde{g}_1 \frac{\alpha}{\alpha_c} \beta_0(r) + 2\frac{\alpha}{\alpha_c} \delta_0(r)} \quad (20)
\]

\[
\theta = \sqrt{1 - \frac{\alpha}{\alpha_c} \ln(\frac{\Lambda m}{m})}, \quad (21)
\]

where \(m = \Sigma_c(0)\) is the infrared scale. In the infrared limit, \(\frac{\Lambda}{m} \gg 1\) and \(\theta \gg 1\), the critical surface relates \(\tilde{g}_1\), \(r\) and \(\alpha\) as

\[
\tilde{g}_1 = \frac{(1 + \sqrt{1 - \frac{\alpha}{\alpha_c}}) - 2\frac{\alpha}{\alpha_c} \delta_0(r)}{(1 - \sqrt{1 - \frac{\alpha}{\alpha_c}}) + 4\frac{\alpha}{\alpha_c} \beta_0(r)}. \quad (22)
\]

The Wilson parameter \(r(g_2)\) is a function of the four-fermi coupling \(g_2\). This function can be approximately found from eq.\((17)\) for \(a \Sigma_c(q) \simeq 0\) and \(\text{den}(q) \simeq \sin^2 q + (rw(q))^2\). Numerical calculation shows, for \(g_2 > 0.2\), there exist a non-trivial solution \(r > 0\), which is reported in Fig. 4.

Based on eq.\((22)\) and functions \(\delta_0(r)\), \(\beta_0(r)\) and \(r(g_2)\), we obtain the critical surface, which is shown in Fig. 5, in terms of \(\tilde{g}_1\), \(g_2\) and \(\frac{\alpha}{\alpha_c}\). We also plot this critical surface on the \(\tilde{g}_1 - \frac{\alpha}{\alpha_c}\) plane (Fig. 6). At this approximation level, we find that (i) the critical line obtained in the continuum theory is modified by the “lattice” terms \(\beta_0(r)\) and \(\delta_0(r)\) with \(0 < r \leq 1\); (ii) \(\tilde{g}_1 = \frac{1 - 2\delta_0(r)}{1 + 4\delta_0(r)}\) for \(\alpha = \alpha_c\) (the MBLL critical point was \(\tilde{g}_1 = 1\) for \(\alpha = \alpha_c\)); (iii) \(\alpha = 0\) and \(\tilde{g}_1 \frac{\alpha}{\alpha_c} = \frac{4}{1 + 8\delta_0(r)}\) (the NJL critical point). For \(\tilde{g}_1 = 0\), we need to have \(2\frac{\alpha}{\alpha_c} \delta_0(r) > 1\), \(\delta_0(r) > 0\) (see eq.\((22)\)), and the critical surface \((22)\) gives us a critical line at \(\frac{\alpha}{\alpha_c} = \frac{4\delta_0(r) - 1}{(2\delta_0(r))^2}\). This leads to \(2\delta_0(r) > 1\), for which there is no room for the values of \(r\) (see Fig.2). We also find there is no room \((\tilde{g}_1 = 0)\) for the existence of a very small critical value of the gauge coupling \(\frac{\alpha}{\alpha_c} \ll 1\), unless \(\delta_0(r) \gg 1\).

6 Summary

We begin with a chirally-invariant lattice theory containing gauge interaction and four-fermion interactions. The NJL self-interaction of only mirror fermions is added in order to remove mirror fermions from the continuum limit. Tuning only one mass parameter (counterterm), we can have the consistent cancellation between \(O(\frac{1}{a})\) divergent contributions from mirror fermions. The SDE for ordinary fermions is free from \(O(\frac{1}{a})\) divergence. However, mirror fermions in the lattice region still have
finite impacts $\delta_0(r) \beta_0(r)$ and $r(g_2)$ on the SDE for ordinary fermions. The critical surface for the infrared limit, which is consistent to the critical line in BLL’s model, is thus obtained.

The appearance of composite particles, e.g., Goldstone boson [21], has been discussed in Ref. [3] for the continuum theory without mirror fermions; in Ref. [10] for $\alpha = 0$ and in Ref. [17] for $\alpha = 0$ and $g_1 = 0$. In future work, we will present a complete discussion on this subject.

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Figure Captions

Figure 1: The quenched and planar approximated Schwinger-Dyson equation.

Figure 2: The function $\beta_0(r)$ in terms of the Wilson parameter $r$.

Figure 3: The function $\delta_0(r)$ in terms of the Wilson parameter $r$.

Figure 4: The function $r(g_2)$ in terms of $g_2$ ($g_2 > g_2^c \simeq 0.2$).

Figure 5: The critical surface (22) in terms of $g_1$ $g_2$ and $\frac{\alpha}{\alpha_c}$.

Figure 6: The critical surface (22) in $\tilde{g}_1 - \frac{\alpha}{\alpha_c}$ plane.
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