Self-Energy of Decuplet Baryons in Nuclear Matter

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Abstract

We calculate, in chiral perturbation theory, the change in the self-energy of decuplet baryons in nuclear matter. These self-energy shifts are relevant in studies of meson-nucleus scattering and of neutron stars. Our results are leading order in an expansion in powers of the ratio of characteristic momenta to the chiral symmetry-breaking scale (or the nucleon mass). Included are contact diagrams generated by 4-baryon operators, which were neglected in earlier studies for the $\Delta$ isomultiplet but contribute to the self-energy shifts at this order in chiral perturbation theory.
The proper self-energy of decuplet baryons in nuclear matter is shifted relative to the self-energy in free space by strong-interaction effects. The self-energy shifts of the decuplet baryons, particularly of the Δ isomultiplet, are relevant in studies of meson-nucleus scattering [1,2] and of neutron stars [3,4]. We calculate the self-energy shifts of the spin-\(\frac{3}{2}\) decuplet baryons (Δ, Σ*, Ξ*, Ω) in nuclear matter, using the model-independent approach of Savage and Wise [5] based on chiral perturbation theory (χPT). For the Δ isomultiplet, the self-energy shifts have been examined in various phenomenological models [4,6,7]. These earlier calculations for the Δ isomultiplet omitted contact diagrams necessary for a consistent momentum expansion in χPT. At leading order in χPT the relevant contact diagrams arise from 4-baryon operators in the chiral effective Lagrangian. The coefficients of these operators have not yet been extracted from experiment; however, the helicity splitting of the self-energy shifts and the shift of the Δ-resonance width do not depend on these unknown coefficients.

We start from the most general effective Lagrangian consistent with the SU(3)_L × SU(3)_R chiral symmetry of QCD. The chiral Lagrangian is written as an expansion in derivatives and powers of the light-quark mass matrix. Furthermore, the decuplet and octet baryons are treated as heavy static fields, effectively keeping only the lowest order terms in the chiral Lagrangian expanded in powers of the baryon momenta over baryon masses. These expansions are justified when the characteristic momentum scale of the processes considered is small compared to the chiral symmetry-breaking scale and the baryon masses.

The lowest order in a density expansion for nuclear matter is a Fermi gas of non-interacting protons and neutrons with Fermi momenta \(p_{F,p}\) and \(p_{F,n}\). In this framework, the characteristic momentum scales relevant in the derivative expansion are \(p_{F,p}\), \(p_{F,n}\), and the decuplet baryon 3-momentum, \(k = |\vec{k}|\). Since the density of a degenerate Fermi gas is given by \(d = p_F^3 / 3\pi^2\), the density expansion for nuclear matter is consistent with the chiral derivative expansion. The presence of the nuclear medium modifies the nucleon propagator through two effects, the presence of nucleon-hole intermediate states and the nucleon intermediate states inaccessible due to Pauli-blocking. The heavy-nucleon propagator in nuclear
matter with Fermi momentum $p_F$ is then
\[ \frac{i\theta(p_F - |\vec{p}|)}{p^0 - i\epsilon} + \frac{i\theta(|\vec{p}| - p_F)}{p^0 + i\epsilon} \]

at lowest order in the nuclear density.

The decuplet baryon self-energy in nuclear matter $E_{NM}(\vec{k})$ is the location of the pole in the 2-point function, given as the solution of
\[ E_{NM}(\vec{k}) - (m_T - m_B) - \Sigma_{NM}(E_{NM}(\vec{k}), \vec{k}) = 0 , \]
where $(m_T - m_B)$ is the mass appearing in the heavy-field Lagrangian, Eq. (11) below, and $\Sigma_{NM}$ is the sum of connected one-particle irreducible diagrams in the expansion of the 2-point function. We evaluate the self-energy shift, $\delta\Sigma(k) \equiv E_{NM}(\vec{k}) - E_{VAC}(\vec{k})$, by expanding the nuclear-matter self-energy about the pole of the free-space 2-point function
\[ \delta\Sigma(k) = \left[ \Sigma_{NM}(E_{VAC}(\vec{k}), \vec{k}) - \Sigma_{VAC}(E_{VAC}(\vec{k}), \vec{k}) \right] \\
+ \left( E_{NM}(\vec{k}) - E_{VAC}(\vec{k}) \right) \frac{\partial}{\partial p^0} \Sigma_{NM}(p^0, \vec{k}) \bigg|_{p^0=E_{VAC}(\vec{k})} + \cdots . \]

Note that only diagrams with an internal nucleon propagator contribute to the first term of this expansion, and that following terms are higher order in $\chi$PT. The real part of the self-energy shift, $\delta\Pi(k) \equiv \text{Re}[\delta\Sigma(k)]$, modifies the baryon energy-momentum dispersion relation from the free-space dispersion relation. The change in the resonance width is given by $\delta\Gamma(k) = -2 \text{Im}[\delta\Sigma(k)]$.

Because the nuclear medium we consider is rotationally invariant, the only preferred direction is along the decuplet baryon 3-momentum, $\vec{k}$. Symmetry under rotations about $\vec{k}$ ensures that the self-energy shifts do not mix baryon states with different spin projections along the direction of $\vec{k}$; i.e., the self-energy shifts of the spin-$\frac{3}{2}$ decuplet baryons are diagonal in the baryon helicity states. Parity invariance of the strong interaction, which is manifest in the chiral effective Lagrangian, dictates that the self-energy shifts do not depend on the sign of the helicity. In the limit of vanishing decuplet 3-momentum, the rotational symmetry is elevated to full SU(2) rotational invariance, and the self-energy shifts must be independent
of the decuplet spin projection along any direction. In this limit we expect the helicity splitting of the self-energy shifts to vanish; \( \delta \Sigma(0) |_{h=\pm \frac{3}{2}} - \delta \Sigma(0) |_{h=\pm \frac{1}{2}} = 0 \). Consequently, at each order of \( \chi \text{PT} \), if the self-energy shifts are momentum independent, the shifts are also helicity independent.

In the effective Lagrangian of \( \chi \text{PT} \), the pseudo-Goldstone bosons (\( \pi, K, \eta \)) are organized as a 3\times3 special unitary matrix \( \Sigma = \exp(2i\pi/f) \), where \( f \) is equal to the pion decay constant (\( f_\pi \simeq 132 \text{ Mev} \)) at lowest order in \( \chi \text{PT} \).

\[
\pi = \begin{pmatrix}
\pi^0/\sqrt{2} + \eta/\sqrt{6} & \pi^+ & K^+ \\
\pi^- & -\pi^0/\sqrt{2} + \eta/\sqrt{6} & K^0 \\
K^- & \bar{K}^0 & -\sqrt{2/3}\eta
\end{pmatrix}.
\]

The interactions of the pseudo-Goldstone bosons with baryon fields are formulated in terms of the vector and axial-vector chiral fields

\[
V_\mu = \frac{1}{2}(\xi\partial_\mu\xi + \xi\partial_\mu\xi^\dagger),
\]

\[
A_\mu = \frac{i}{2}(\xi\partial_\mu\xi - \xi\partial_\mu\xi^\dagger),
\]

where \( \xi \equiv \sqrt{\Sigma} = \exp(i\pi/f) \).

To lowest order in the derivative expansion, the chiral effective Lagrangian quadratic in decuplet baryon operators is

\[
\mathcal{L}_0 = -\mathcal{T}_{\mu}^{abc}(i\gamma^\nu\partial_\nu - m_T)T_0^\mu_{abc} + \mathcal{H}\mathcal{T}_{\mu}^{abc}G_{\gamma\gamma_5}T_0^\mu_{abd}A^{\nu d}_c + C(\epsilon_{abc}\mathcal{T}_{\mu}^{ade}A^{\nu b}_dB_{0e}^c + \epsilon^{abc}\mathcal{B}_{0e}^dA^{\nu d}_bT_0^\mu_{ade}),
\]

where \((\mu,\nu)\) are Lorentz indices, \((a-e)\) are chiral SU(3) indices, and Dirac spinor indices are suppressed. \( T_0^{\mu}_{abc} \) is a Rarita-Schwinger field which transforms under chiral SU(3) as a completely-symmetric rank-3 tensor and represents the decuplet fields as follows:

\[
T_0^{\mu}_{111} = \Delta^{++}, \quad T_0^{\mu}_{113} = \Sigma^{*+}/\sqrt{3}, \quad T_0^{\mu}_{133} = \Xi^{*0}/\sqrt{3},
\]

\[
T_0^{\mu}_{112} = \Delta^+/\sqrt{3}, \quad T_0^{\mu}_{123} = \Sigma^{*0}/\sqrt{6}, \quad T_0^{\mu}_{233} = \Xi^{-*}/\sqrt{3},
\]

\[
T_0^{\mu}_{122} = \Delta^0/\sqrt{3}, \quad T_0^{\mu}_{223} = \Sigma^{*-}/\sqrt{3}, \quad T_0^{\mu}_{333} = \Omega^-,
\]

\[
T_0^{\mu}_{222} = \Delta^-.
\]
$B_0^b$ represents the hyperon octet as a $3 \times 3$ matrix of Dirac fields

$$
B_0 = \begin{bmatrix}
\Sigma^0/\sqrt{2} + \Lambda/\sqrt{6} & \Sigma^+ & p \\
-\Sigma^0/\sqrt{2} + \Lambda/\sqrt{6} & \Sigma^- & n \\
\Xi^- & \Xi^0 & -\sqrt{3}\Lambda
\end{bmatrix}.
$$

The action of the chiral covariant derivative $D_\mu$ on the baryon fields is given by

$$
(D_\mu B_0)_a^b = \partial_\mu B_0^b + V_\mu a B_0^b_d - V_\mu d B_0^b_a,
$$

$$
= (\partial_\mu B_0 + [V_\mu, B_0])_a^b,
$$

$$
(D_\mu T_0^{\nu})_{abc} = \partial_\mu T_0^{\nu abc} + V_\mu a T_0^{\nu dbc} + V_\mu b T_0^{\nu adc} + V_\mu c T_0^{\nu adb}.
$$

Following Jenkins and Manohar [8], we obtain a consistent derivative expansion by treating the baryons in the heavy-fermion formalism. We rewrite the chiral effective Lagrangian treating both octet and decuplet baryons as heavy static fields $T^\mu$ and $B$ defined by

$$
T^\mu(x) = \exp(im_B t) T_0^{\mu}(x),
$$

$$
B(x) = \exp(im_B t) B_0(x),
$$

where $m_B$ is the octet baryon mass, and with the additional constraints $\gamma_0 T^\mu = +T^\mu$ and $\gamma_0 B = +B$. Replacing the Dirac structure of the Lagrangian with 2-component Pauli spinors, and keeping only terms at lowest order in $1/m_B$ and $1/m_T$, the chiral effective Lagrangian is

$$
\mathcal{L} = T_{\mu}^{(j) abc} (iD_\mu - (m_T - m_B)) T^{(j) abc} + \mathcal{H} T_{\mu}^{(j) abc} \sigma^{(k)} T_{\mu}^{(j) abd} A^{(k) d}_c
$$

$$
+ \mathcal{C}(\epsilon_{abc} T_{\mu}^{(j) ade} A^{(j) b}_d B^{e}_c + \epsilon^{abc} B_{\mu}^{b} A^{(j) d}_c T_{\mu}^{(j) ade}).
$$

In the $1/m_B, 1/m_T$ expansion, the Lorentz 4-vectors have been replaced by spacial vectors with indices $(j,k)$ implicitly summed over 1–3, and the spinor indices have been suppressed.

We adapt the power-counting rules of Savage and Wise [5] to accommodate the decuplet baryon mass $m_T$ and 3-momentum $k$. Both octet and decuplet masses, $m_T$ and $m_B$, are treated as the same order as the chiral symmetry breaking scale $\Lambda_\chi \sim 4\pi f$. The pseudo-Goldstone boson masses ($m_\pi, m_K, m_\eta$) provide a smaller mass scale, denoted by $q$. The
decuplet-octet mass splitting \( m_T - m_B \) and the characteristic momenta \( p_{F,p}, p_{F,n} \), and \( k \) are also considered to be of order \( q \). (The light quark masses in the symmetry-breaking Lagrangian are order \( q^2 \).) With the exception of Weinberg’s notorious infrared-divergent diagrams [9], a Feynman diagram with \( L \) loops and \( V_i \) vertices with \( n_i \) baryon operators and \( d_i \) derivatives (or factors of pseudo-Goldstone boson masses), contributes to the self-energy of the baryon field at order \( q(q/\Lambda_{\chi})^{\alpha} \) where

\[
\alpha = 2L + \sum_i V_i(d_i + \frac{1}{2}n_i - 2) .
\] (12)

There are two kinds of diagrams that contribute in the leading order where \( \alpha = 2 \): 1) meson-nucleon loop diagrams where \( n_i = 2 \) and \( d_i = 1 \) (shown in Fig. 1a), and 2) contact diagrams (shown in Fig. 1b), with one insertion of an operator with \( n_i = 4 \) and \( d_i = 0 \) containing two decuplet baryon fields and two octet baryon fields.

The meson-nucleon loop diagrams are generated by terms in Eq. (11). For the contact diagrams, we construct the relevant 4-baryon operators by considering the spin and chiral structures of \( T^{(j)}B \). The operator product \( T^{(j)}B \) decomposes under rotational \( SU(2) \) as \( \frac{3}{2} \otimes \frac{1}{2} = 2 \oplus 1 \) and decomposes under chiral \( SU(3) \) as \( 10 \otimes 8 = 35 \oplus 27 \oplus 10 \oplus 8 \). Therefore the chiral effective Lagrangian contains eight linearly independent 4-baryon operators of the form \( T^{(j)}T^{(k)}B^\dagger B \), which contribute to the self-energy shift at leading order. We choose to write the Lagrangian containing these 4-baryon operators as

![Diagram](a) \hspace{1cm} ![Diagram](b)

FIG. 1. Diagrams contributing to self-energy shifts at lowest order in \( \chi \)PT: (a) meson-nucleon loop diagram and (b) contact diagram. Double solid lines represent decuplet baryon fields, single solid lines represent nucleon fields, and dashed lines represent pseudo-Goldstone boson fields.
\[ \delta \mathcal{L} = - \frac{d_1}{f_2} [ T^{(j)abc} T^{(j)}_{abc} ] [ B^{\dagger e} \delta B^d_e ] - \frac{d_5}{f_2} [ T^{(j)abc} \sigma^{(k)} T^{(j)}_{abc} ] [ B^{\dagger e} \sigma^{(k)} B^d_e ] \\
- \frac{d_2}{f_2} [ T^{(j)abc} T^{(j)}_{abd} ] [ B^{\dagger e} \delta B^d_e ] - \frac{d_6}{f_2} [ T^{(j)abc} \sigma^{(k)} T^{(j)}_{abd} ] [ B^{\dagger e} \sigma^{(k)} B^d_e ] \\
- \frac{d_3}{f_2} [ T^{(j)abc} T^{(j)}_{adbl} ] [ B^{\dagger e} \delta B^d_e ] - \frac{d_7}{f_2} [ T^{(j)abc} \sigma^{(k)} T^{(j)}_{adbl} ] [ B^{\dagger e} \sigma^{(k)} B^d_e ] \\
- \frac{d_4}{f_2} [ T^{(j)abc} T^{(j)}_{ade} ] [ B^{\dagger e} \delta B^d_e ] - \frac{d_8}{f_2} [ T^{(j)abc} \sigma^{(k)} T^{(j)}_{ade} ] [ B^{\dagger e} \sigma^{(k)} B^d_e ] , \quad (13) \]

where square brackets indicate sums over spinor indices. Because the baryon helicity is conserved in the self-energy diagrams, the terms in Eq. (13) of the form \([ T^{(j)} \sigma^{(k)} T^{(j)} ] [ B^{\dagger} \sigma^{(k)} B ]\) do not generate contact diagrams contributing to the self-energy shifts.

To account for the leading effects of the symmetry-breaking terms in the effective Lagrangian, we use non-zero masses \(m_\pi, m_K,\) and \(m_\eta\) for the pseudo-Goldstone bosons and we use isospin multiplet masses in place of chiral multiplet masses for the baryons. For the decuplet baryons, the effective masses replacing \(m_T\) are \(m_\Delta, m_{\Sigma^*}, m_{\Xi^*},\) and \(m_\Omega.\) We use the nucleon mass \(m_N\) for the octet baryon mass \(m_B\) since diagrams containing the other octet baryons cancel between nuclear matter and free space.

From the chiral effective Lagrangians given in Eqs. (11) and (13), we now derive the self-energy shifts of the decuplet baryons (\(\Delta, \Sigma^*, \Xi^*, \Omega\)) in nuclear matter to leading order in chiral perturbation theory. Here, we have introduced the convenient mass scales,

\[
\mu = \sqrt{(m_\Delta - m_N)^2 - m_\pi^2} \approx 255 \text{ MeV},
\]
\[
\tilde{\mu} = \sqrt{m_K^2 - (m_{\Sigma^*} - m_N)^2} \approx 215 \text{ MeV}. \quad (14)
\]

For the member of each isomultiplet with lowest 3-component of isospin (\(\Delta^-, \Sigma^{*-}, \Xi^{*-}, \Omega^-\)), we obtain

\[
\delta \Pi_{\Delta^-} = \frac{p_{F,n}^2}{9\pi^2 f_2} (3d_1 + 3d_2) + \frac{p_{F,p}^2}{9\pi^2 f_2} (3d_1) + \frac{|C|^2}{(4\pi f)^2} \left\{ \frac{2\mu^3}{3} \ln \left[ \frac{(p_{F,n} - \mu)^2 - k^2}{(p_{F,n} + \mu)^2 - k^2} \right] \right. \\
+ \frac{4p_{F,n}^2 (2p_{F,n}^2 + 3\mu^2) - \mu^2 (2p_{F,n}^2 + \mu^2 - 2) \ln \left[ \frac{(p_{F,n} - \mu)^2 - \mu^2}{(p_{F,n} + \mu)^2 - \mu^2} \right]}{3k} \\
\pm \frac{p_{F,n}^2 (p_{F,n}^2 - \mu^2 - k^2)^2 - 2p_{F,n}^2 (p_{F,n}^2 + 3\mu^2 + 3k^2) + \frac{1}{24k^3} (p_{F,n}^2 - \mu^2 - k^2)}{6k^2} \\
\times \left[ (p_{F,n} - \mu)^2 ((p_{F,n} + \mu)^2 - \mu^2) \ln \left[ \frac{(p_{F,n} - \mu)^2 - \mu^2}{(p_{F,n} + \mu)^2 - \mu^2} \right] \right] \right\} , \quad (15)
\]
\[ \delta \Pi_{\Sigma^-} = \frac{p_{F,n}^3}{9 \pi^2 f^2} (3d_1 + 2d_2 + d_3 + d_4) + \frac{p_{F,p}^3}{9 \pi^2 f^2} (3d_1 + d_3) \]
\[ + \frac{|C|^2}{(4 \pi f)^2} \left\{ 4 \frac{\mu^3}{9} \arctan \left( \frac{p_{F,n} - k}{\mu} \right) + 4 \frac{\mu^3}{9} \arctan \left( \frac{p_{F,n} + k}{\mu} \right) \right\} \]
\[ + \frac{4p_{F,n}^3}{27} (2p_{F,n}^2 - 3\mu^2) + \frac{\mu^2}{9k} (p_{F,n}^2 + \mu^2 - k^2) \ln \left( \frac{(p_{F,n} - k)^2 + \mu^2}{(p_{F,n} + k)^2 + \mu^2} \right) \]
\[ \pm \left[ \frac{p_{F,n}^3}{18k^2} (p_{F,n}^2 + \mu^2 - k^2)^2 - \frac{p_{F,n}^3}{27} (p_{F,n}^2 + \mu^2 + 3k^2) + \frac{1}{\tau_2k^3} (p_{F,n}^2 + \mu^2 - k^2) \right] \times ((p_{F,n} - k)^2 + \mu^2) \ln \left( \frac{(p_{F,n} - k)^2 + \mu^2}{(p_{F,n} + k)^2 + \mu^2} \right) \right\}, \quad (16) \]
\[ \delta \Pi_{\Xi^-} = \frac{p_{F,n}^3}{9 \pi^2 f^2} (3d_1 + d_2 + 2d_3 + d_4) + \frac{p_{F,p}^3}{9 \pi^2 f^2} (3d_1 + 2d_3), \quad (17) \]
\[ \delta \Pi_{\Omega^-} = \frac{p_{F,n}^3}{9 \pi^2 f^2} (3d_1 + 3d_3) + \frac{p_{F,p}^3}{9 \pi^2 f^2} (3d_1 + 3d_3). \quad (18) \]

In Eqs. (15) and (16), and in Eq. (19) below, the plus sign of ‘±’ corresponds to the \( h = \pm \frac{3}{2} \) helicity states and the minus sign corresponds to the \( h = \pm \frac{1}{2} \) helicity states. There is no helicity splitting of the self-energy for \( \Xi^{*+}, \Xi^{-}, \) or \( \Omega^- \) baryons because the chiral Lagrangian does not couple these baryons to a nucleon and single pseudo-Goldstone boson. At this order in \( \chi PT \) the resonance width in nuclear matter is shifted only for the \( \Delta \) isomultiplet. For the shift of the width of the \( \Delta^- \), we find
\[ \delta \Gamma_{\Delta^-} = \frac{|C|^2}{(4 \pi f)^2} \left\{ -\frac{8\pi\mu^3}{3} \Theta(p_{F,n} - \mu - k) + \Theta(p_{F,n} - \mu + k) \Theta(\mu^2 - (p_{F,n} - k)^2) \right\} \]
\[ \times \frac{2\pi}{3k} (p_{F,n}^2 - (\mu - k)^2) \left[ -\mu^2 \pm \frac{1}{8k^2} (p_{F,n}^2 - (\mu + k)^2)(p_{F,n}^2 - \mu^2 - k^2) \right]. \quad (19) \]

For the member of each isomultiplet with the highest 3-component of isospin (\( \Delta^{++}, \Sigma^{++}, \Xi^{*0}, \Omega^- \)), the self-energy shifts are obtained from the results for the corresponding isomultiplet member with lowest 3-component of isospin by exchanging \( p_{F,p} \) and \( p_{F,n} \) in Eqs. (15)–(19). The self-energy shifts of the remaining decuplet baryons are given by the following relations:
\[ \delta \Sigma_{\Delta^+} = \frac{2}{3} \delta \Sigma_{\Delta^{++}} + \frac{1}{3} \delta \Sigma_{\Delta^-}, \quad (20) \]
\[ \delta \Sigma_{\Delta^0} = \frac{1}{3} \delta \Sigma_{\Delta^{++}} + \frac{2}{3} \delta \Sigma_{\Delta^-}, \quad (21) \]
\[ \delta \Sigma_{\Sigma^*} = \frac{1}{2} \delta \Sigma_{\Sigma^{*+}} + \frac{1}{2} \delta \Sigma_{\Sigma^{*-}}, \quad (22) \]
The leading corrections to our results are due to infrared-divergent 2-loop diagrams. The infrared-divergent diagrams are regulated by insertions of the baryon kinetic energy, resulting in powers of the baryon mass in the numerator of the diagrams. The contributions of the infrared-divergent 2-loop diagrams are suppressed by a factor of order $q m_B / \Lambda^2$. Whether these corrections can be considered small depends strongly on numerical factors arising from the loop integrations. Diagrams with more than one loop which are not infrared-divergent, and diagrams with insertions of the baryon kinetic energy, are suppressed by $(q/\Lambda)^2$ relative to our results. Expanding the self-energy in nuclear matter beyond lowest order about the pole of the free-space 2-point function, also gives corrections suppressed by $(q/\Lambda)^2$. Diagrams generated by terms in the general chiral effective Lagrangian with more than four baryon operators are suppressed by $(q/\Lambda)^3$.

Our work differs in two ways from earlier calculations of the $\Delta$-isomultiplet self-energies in nuclear matter \[4,6,7\]. We use chiral SU(3) symmetry to extend the calculation to the entire spin-$3/2$ baryon decuplet; and for the $\Delta$-isomultiplet self-energy shifts, we find new momentum-independent contributions from contact diagrams generated by 4-baryon operators. A comprehensive discussion of the momentum dependence of the results for the $\Delta$ isomultiplet, without the contact diagram contributions, is given by Cenni and Dillon \[6\]. Of particular interest is their discussion of the logarithmic divergence of $\delta\Pi_{\Delta}(k)$ as $k \to 0$ when $p_F = \mu \simeq 255$ MeV in the context of coupled $\Delta$ and $N\pi$ eigenmodes. The $\Sigma^*$-isomultiplet self-energy shifts do not have a similar divergence because the $\Sigma^* \to NK$ decay is kinematically forbidden for any $p_{F,p}$ and $p_{F,n}$. The contact diagrams included here offset the decuplet baryon self-energy shifts by terms proportional to $p_{F,p}^3$ and $p_{F,n}^3$.

The dimensionless coefficients $d_i$ of the contact terms in the chiral effective Lagrangian, Eq. (13), have not yet been extracted from experiment. Because the coefficients $d_i$ are unknown, we do not evaluate the self-energy shifts of the decuplet baryons numerically. Also, it is unclear whether the self-energy shift enhances or suppresses the decuplet baryon populations in dense nuclear matter. Note however, the helicity splittings of the self-energy shifts of the $\Delta$ and $\Sigma^*$ isomultiplets and the shift of the $\Delta$-resonance width do not depend
on the coefficients $d_i$. Furthermore, the $\Delta$-isomultiplet self-energy shifts depend only on the coefficients $d_1$ and $d_2$ due to constraints from chiral SU(3) symmetry.

Finally, we note that although the coefficients $d_i$ of the 4-baryon operators in the chiral effective Lagrangian are not yet known, we can reduce the number of independent parameters from eight to two by appealing to SU(6) spin-flavor symmetry \[10\]. Under spin-flavor SU(6), the coefficients $d_i$ are determined by

\begin{align*}
    d_1 &= 2a + \frac{5}{9}b, \\
    d_2 &= -\frac{2}{9}b, \\
    d_3 &= -\frac{2}{9}b, \\
    d_4 &= -\frac{2}{9}b, \\
    d_5 &= \frac{1}{9}b, \\
    d_6 &= -\frac{1}{3}b, \\
    d_7 &= -\frac{1}{9}b, \\
    d_8 &= -\frac{2}{9}b,
\end{align*}

(23)

in terms of the coefficients $a$ and $b$ defined by Kaplan and Savage.

In conclusion, we have determined the self-energy shifts of the spin-$\frac{3}{2}$ decuplet baryons in nuclear matter to leading order in chiral perturbation theory. We find there are momentum-independent contributions which depend on four of the coefficients $d_i$ in the chiral effective Lagrangian which are not yet known. However, the change in the $\Delta$-resonance width and the helicity splittings of the self-energy shifts are independent of these unknown coefficients. The validity of the leading order results depends critically on the size of corrections due to regulated infrared-divergent Feynman diagrams at 2-loop order. We hope that in future work the relative importance of the leading corrections to our results will be determined.

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