Production of Z bosons and neutrinos in early universe

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Abstract

Production of heavy Z bosons and neutrinos in the early universe is studied in the expanding de Sitter universe. The expression of the transition amplitudes in the case of Z boson interaction with leptons is established by using perturbative methods. Then the amplitude and probability that for the spontaneous generation from vacuum of a Z boson a neutrino and an antineutrino are computed analytically and a graphical analysis is performed in terms of the expansion parameter. We found that this the probability for this process is nonvanishing only for large expansion conditions of the early Universe. We discuss the Minkowski limit and we obtain that in this limit the amplitude is zero, result which corresponds to the well established fact that spontaneous particle generation from vacuum in Minkowski space-time is forbidden.

PACS numbers: 04.62.+v
I. INTRODUCTION

One of the well established theories from physics is the electro-weak theory which combine interactions between heavy bosons and Dirac fermions \[3-12\]. Since it is known that the massive bosons were produced in the early universe it is a matter of profound importance to understand the mechanism that generate these bosons. For that one need to combine the General Relativity with the theory of electro-weak interactions. The problems that the Quantum Field Theory is facing in a curved spacetimes was the subject of many investigations. These investigations cover a vast area from the study of the free field equations in different metrics \[2, 22, 25, 29, 30\] up to problems that imply interactions between fields and the renormalization theory. One of the subjects that receive much attention is related to the mechanisms that generate the matter and antimatter and this subject was approached by both perturbative \[15, 18, 23, 24, 26, 28, 31, 36\] methods and nonperturbative methods \[13, 14, 16, 17, 32, 33\]. The problem of production of the heavy Z bosons and neutrinos in the early universe by using various methods receive little attention and was not approached by using perturbative methods.

In this paper we will propose a method for study the production of Z bosons and neutrinos production from vacuum by using the perturbative methods. In Minkowski Standard Model \[12, 19, 20\] it is a well established fact that the first order transition amplitudes that generate Z bosons from vacuum are forbidden by the simultaneous conservation of energy and momentum. In a nonstationary metric this observation is no longer valid since the translational invariance with respect to time is lost. The first step in our calculations will be to establish the expression of the transition amplitude in de Sitter spacetime, for the triplet \(Z, \nu, \bar{\nu}\) generation from vacuum. We will present the main steps to compute the transition probability for the process in which the triplet Z boson, neutrino and antineutrino, is generated from vacuum. The computations are done in a de Sitter metric, and we use the chart that covers only the expansion part of the de Sitter variety.

In the second section we present the main steps for obtaining the transition amplitude in de Sitter geometry. The third section is dedicated to the computation of the transition probability for Z bosons and neutrinos generation from vacuum. In the forth section we explore the physical consequences of our analytical results and in the fifth section we present our conclusions. We consider in our paper natural units such that \(\hbar = 1, c = 1\).
II. GENERAL FORMALISM FOR OBTAINING THE AMPLITUDE

We start with the de Sitter metric have written in conformal form \[1\]:

\[ ds^2 = (\omega t_c)^2 (dt_c^2 - d\vec{x}^2), \]

where the conformal time \( t_c = -\frac{e^{-\omega t}}{\omega} \) and \( \omega \) is the expansion factor. To define half-integer spin fields on curved spacetime one needs to use the tetrad fields \[2\] \( e_\mu(x) \) and \( \bar{e}_\mu(x) \), which fix the local frames and corresponding coframes. These tetrad fields have local indices \( \mu, \nu, ... = 0, 1, 2, 3 \). For the line element \((1)\), the Cartesian gauge is chosen with the nonvanishing tetrad components,

\[ e_0^0 = -\omega t_c; \quad e_j^i = -\delta_j^i \omega t_c. \]

Our study is done in the chart with conformal time \( t_c \in (-\infty, 0) \), which covers the expanding portion of de Sitter space.

By following the methods from flat space time the first step in constructing the theory of interactions between fields in de Sitter geometry will be to consider the free fields equations and their solutions. Then the free fields in the in and out sectors are the exact solutions of the Proca equation and Dirac equation on de Sitter spacetime, written in the momentum helicity basis. The transition amplitude for the spontaneous production from de Sitter vacuum of a \( Z \) boson a neutrino (\( \nu \)) and antineutrino (\( \bar{\nu} \)) can be obtained by starting with the tetrad gauge invariant Lagrangian density that give the coupling between \( Z \) bosons and leptons, written with point independent Dirac matrices \( \gamma^\mu \) and the tetrad fields \( e_\mu^\alpha \):

\[ L_{\mu Z} = -\left(\frac{e_0}{\sin(2\theta_W)}\right)\bar{\nu}_e \gamma_\mu e_\mu^\alpha \left(\frac{1 - \gamma^5}{2}\right) \nu_e Z_\alpha + \left(\frac{e_0 \cos(2\theta_W)}{\sin(2\theta_W)}\right) \bar{e} \gamma_\mu e_\mu^\alpha \left(\frac{1 - \gamma^5}{2}\right) e Z_\mu - \left(\frac{e_0}{\sin(2\theta_W)}(j_{\text{neutral}})^\alpha\right) Z_\alpha, \]

where \( e_0 \) is the electric charge, \( \theta_W \) is the Weinberg angle and, \( \nu_e \) designates the neutrino-antineutrino field, \( Z_\mu \) designate the \( Z \) boson field and \( e \) designates the electron-positron field. The \( Z \) boson is have no electric charge and the particle coincides with the antiparticle, while the neutral current \( (j_{\text{neutral}})^\mu \) is given by:

\[ (j_{\text{neutral}})^\alpha = \bar{\nu}_e \gamma_\mu e_\mu^\alpha \left(\frac{1 - \gamma^5}{2}\right) \nu_e - \cos(2\theta_W)\bar{e} \gamma_\mu e_\mu^\alpha \left(\frac{1 - \gamma^5}{2}\right) e + 2 \sin^2(\theta_W) \bar{e} \gamma_\mu e_\mu^\alpha \left(\frac{1 + \gamma^5}{2}\right) e. \]

The above expression help us to establish the equation for the transition amplitudes by using perturbations.
Usually in the electro-weak theory [3–12], the amplitudes are written by using the Feynmann rules in the momentum representation. Here we will adopt for our computations the Feynmann rules in coordinates representation because our goal is to obtain the transition amplitude dependence on the expansion factor and in addition we do not have Feynmann rules in momentum picture since this will imply complex computations for internal lines of the graphs. Still we will present here the first steps for giving the Feynmann rules for the external lines of the graphs when massive boson Z interact with leptons.

It is known that the Proca equation and Dirac equation on de Sitter spacetime can be analytically solved [2, 22]. We begin with the free field that propagate in the in and out sectors. The solution for the Dirac equation in momentum-helicity basis which describe the zero mass particles on de Sitter spacetime was obtained in [3]. Then the $U, V$ solutions that describe the particle $\nu$ and antiparticle $\bar{\nu}$ in this geometry are [22]:

\[
(U_{\vec{p},\sigma}(x))_\nu = \left(-\omega t_c \frac{\pi}{2}\right)^{3/2} \left( \begin{array}{c} (\frac{1}{2} - \sigma) \xi_\sigma(\vec{p}) \\ 0 \end{array} \right) \exp\left(i\vec{p} \cdot \vec{x} - ipt_c\right),
\]

\[
(V_{\vec{p},\sigma}(x))_{\bar{\nu}} = \left(-\omega t_c \frac{\pi}{2}\right)^{3/2} \left( \begin{array}{c} (\frac{1}{2} + \sigma) \eta_\sigma(\vec{p}) \\ 0 \end{array} \right) \exp\left(-i\vec{p} \cdot \vec{x} + ipt_c\right). 
\]

These solutions have only the left part since they are written by using the chiral representation, so the application of the left projector $\frac{1 - \gamma^5}{2}$ leave them unchanged.

The solutions of the Proca equation in de Sitter geometry written in the momentum helicity basis were obtained in [2]. These solutions will describe the massive Z free field and their spatial part is:

\[
\vec{f}_{\vec{P},\lambda}(x) = \begin{cases} 
\frac{i\sqrt{\pi}\omega e^{-\pi k/2}}{2M_Z(2\pi)^{3/2}} \left[ (\frac{1}{2} + ik) \frac{\sqrt{-t_c}}{\vec{P}} H_{ik}^{(1)} (-\vec{P}t_c) - (-t_c)^{3/2} H_{1+ik}^{(1)} (-\vec{P}t_c) \right] e^{i\vec{P} \cdot \vec{x}} \bar{\epsilon}(\vec{n}_\vec{P}, \lambda) & \text{for } \lambda = 0 \\
\frac{\sqrt{\pi} e^{-\pi k/2}}{2(2\pi)^{3/2}} \frac{\sqrt{-t_c}}{\vec{P}} H_{ik}^{(1)} (-\vec{P}t_c) e^{i\vec{P} \cdot \vec{x}} \bar{\epsilon}(\vec{n}_\vec{P}, \lambda) & \text{for } \lambda = \pm 1.
\end{cases}
\]  

while the temporal component of the solution of Proca equation reads [2]:

\[
f_{0\vec{P},\lambda}(x) = \begin{cases} 
\frac{\sqrt{\pi}\omega e^{-\pi k/2}}{2M_Z(2\pi)^{3/2}} (-t_c)^{3/2} H_{ik}^{(1)} (-\vec{P}t_c) e^{i\vec{P} \cdot \vec{x}} \bar{\epsilon}(\vec{n}_\vec{P}, \lambda) & \text{for } \lambda = 0 \\
0 & \text{for } \lambda = \pm 1.
\end{cases}
\]

In the above equations for plane wave for the Proca field $\vec{n}_\vec{P} = \vec{P}/\vec{P}$ and $\bar{\epsilon}(\vec{n}_\vec{P}, \lambda)$ are the polarization vectors. For $\lambda = \pm 1$ these vectors are transversal on the momentum such
that $\mathbf{P} \cdot \mathbf{e}(\vec{n}_P, \lambda = \pm 1) = 0$ and for $\lambda = 0$ the polarization vectors are longitudinal on the momentum $\mathbf{P} \cdot \mathbf{e}(\vec{n}_P, \lambda = 0) = \mathbf{P}$, since $\mathbf{e}(\vec{n}_P, \lambda = 0) = \vec{n}_P$.

Because our study considers interactions with massive bosons we will use in our computations both solutions for $\lambda = 0$ and $\lambda = \pm 1$, since the longitudinal polarizations also give contributions to our amplitude.

The lagrangean density given in equation (28) can be used for establishing the general form of the transitions amplitudes when a Z boson interacts with neutrinos by using the perturbative methods as in flat space theory [12, 19, 20]. Only the first term from equation (3) give contribution to our amplitude since in the ”in out” sectors we have only neutrinos and a Z boson.

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$$L_{\nu\nu Z} = -\left(\frac{e_0}{\sin(2\theta_W)}\right)\mathbf{\overline{\nu}}_e \gamma^\mu e_\mu^\alpha \left(1 - \frac{\gamma^5}{2}\right) \nu_e Z_\alpha$$

(8)

The reduction rules for fermions are given in [24], while for the Z boson the reduction rules from the in state and out state can be computed using the method from flat space theory. In general the transition coefficients between two states can be computed as $\langle \text{out}; \alpha | \text{in}; \beta \rangle$ and represents the transition amplitudes between the in states at $t \to -\infty$ and the out states at $t \to \infty$. Then the transition amplitudes can be constructed by using the reduction formalism.

$$\langle \text{out}; \alpha | \text{in}; (\mathbf{P}, \lambda), \beta \rangle = \frac{i}{C} \int d^4x \sqrt{-g} \langle \text{out}; \alpha | A_\mu^+(x)| \text{in}; \beta \rangle \mathbf{E}_P(x) f_\mathbf{P},\lambda (x),$$

$$\langle \text{out}; (\mathbf{P}, \lambda), \alpha | \text{in}; \beta \rangle = \frac{i}{C} \int d^4x \sqrt{-g} f_\mathbf{P},\lambda (x) \mathbf{E}_P(x) \langle \text{out}; \alpha | A_\mu(x)| \text{in}; \beta \rangle,$$

(9)

where the notation $E_P(x)$ stands for the Proca operator $E_P(x) = \Box + m^2$ in de Sitter geometry [2] and $C$ is a renormalization constant. The perturbation calculations are based on the scattering operator given in terms of interaction lagrangean $L_{\nu\nu Z}$ equation [12, 19, 20]:

$$S = T \exp \left[ -i \int d^4x \sqrt{-g} L_{\nu\nu Z} \right]$$

(10)

Then by using the scattering operator we can compute the Green functions in terms of free fields:

$$\langle 0 | \Psi(x_1) \Psi(x_2) A_\mu(x_3) ... | 0 \rangle = \frac{\langle 0 | \Psi(x_1) \Psi(x_2) A_\mu(x_3) ... S | 0 \rangle}{\langle 0 | S | 0 \rangle}.$$

(11)

The above Green functions can be expressed in terms of Feynman propagators if we consider all the possible T contractions as the Wick theorem states. Then the numerators of these functions can be split in connected parts multiplied just by the vacuum expectation
value $\langle 0 | S | 0 \rangle$ such that after simplification we remain only with the connected parts which give the physical transition amplitudes. The transition amplitude in the first order of the perturbation theory for the interaction between $Z$ boson and neutrino-antineutrino field reads:

$$A_{Z\nu} = -\int d^4x \sqrt{-g} \left( \frac{e_0}{\sin(2\theta_W)} \right) \overline{\nu}_e \gamma^\mu e_\mu^\alpha \left( \frac{1 - \gamma^5}{2} \right) \nu_e Z_\alpha. \tag{12}$$

By using the same method the first order transition amplitude that describe the interaction between $Z$ boson and electron-positron field is

$$A_{Ze} = \int d^4x \sqrt{-g} \left\{ \left( \frac{e_0 \cos(2\theta_W)}{\sin(2\theta_W)} \right) \overline{e}_\gamma \gamma^\mu e^\mu_\alpha \left( \frac{1 - \gamma^5}{2} \right) e_Z^{Z_\alpha} - (e_0 \tan(\theta_W)) \overline{e}_\gamma \gamma^\mu e^\mu_\alpha \left( \frac{1 + \gamma^5}{2} \right) e_Z^{Z_\alpha} \right\}. \tag{13}$$

The above equations can be used for all first order perturbation theory interactions between $Z$ boson and leptons in de Sitter geometry.

### III. AMPLITUDE AND PROBABILITY COMPUTATION

This section is dedicated to amplitude and probability computation for the process of spontaneous generation from de Sitter vacuum of the triplet $Z$ boson, neutrino and antineutrino and for this process only amplitude equation 12 give contribution. Here we consider that the free fields propagating in the in and out sectors are exact solutions of the Dirac equation and Proca equation in de Sitter spacetime. The amplitude equation 12 can be expanded using the temporal part solution $f_{0\mathcal{P},\lambda}(x)$ and spatial part solution $f_{j\mathcal{P},\lambda}(x)$ of the Proca equation 2 as:

$$A_{Z\nu\pi} = -\int d^4x \sqrt{-g} \left( \frac{e_0}{\sin(2\theta_W)} \right) \left( \overline{U}_{p,\sigma}\nu(x) \gamma^0 e_0^0 \left( \frac{1 - \gamma^5}{2} \right) (V^{\sigma'}_{p',\sigma'}) \nu(x) f^{*}_{0\mathcal{P},Z}(x) \right)$$

$$-\int d^4x \sqrt{-g} \left( \frac{e_0}{\sin(2\theta_W)} \right) \left( \overline{U}_{p,\sigma}\nu(x) \gamma^i e^i_\lambda \left( \frac{1 - \gamma^5}{2} \right) (V^{\sigma'}_{p',\sigma'}) \nu(x) f^{*}_{j\mathcal{P},Z}(x) \right). \tag{14}$$

Using the amplitude equation 12 and the solutions for the free field equations we can express the amplitude as a temporal and a spatial integral. The spatial integral contain the delta Dirac function expressing the momentum conservation in this process, while for the
temporal integral we use the new integration variable $z = -t_c$

\[
A_{Z\sigma} = \frac{e_0}{\sin(\theta_W)} \delta^3(\vec{P} + \vec{p} + \vec{p}') \frac{1}{\sqrt{\pi}(2\pi)^{3/2}} \left( \frac{1}{2} - \sigma \right) \left( \frac{1}{2} + \sigma' \right) \\
\times \left\{ \frac{P_\omega}{m} A(t_c) \xi^+_{\sigma'}(\vec{p}) \vec{s} \cdot \vec{e}^*(\vec{p}, \lambda = 0) \eta_{\sigma'}(\vec{p}') + \frac{P_\omega}{m} C(t_c) \xi^+_{\sigma'}(\vec{p}) \eta_{\sigma'}(\vec{p}') \\
+ B(t_c) \xi^+_{\sigma'}(\vec{p}) \vec{s} \cdot \vec{e}^*(\vec{p}, \lambda = \pm 1) \eta_{\sigma'}(\vec{p}') \right\},
\]  

where $A(t_c), B(t_c), C(t_c)$ are the temporal integrals given by:

\[
A(t_c) = \int_0^\infty dz \sqrt{z} e^{-i(\vec{p} + \vec{p}')z} K_{-ik}(i\vec{P}z) \frac{1}{P} \left( \frac{1}{2} - ik \right) - i \int_0^\infty dz \frac{z^{3/2}}{\sqrt{z}} e^{-i(\vec{p} + \vec{p}')z} K_{1-ik}(i\vec{P}z),
\]

\[
C(t_c) = i \int_0^\infty dz \sqrt{z} e^{-i(\vec{p} + \vec{p}')z} K_{-ik}(i\vec{P}z)
\]

\[
B(t_c) = i \int_0^\infty dz \sqrt{z} e^{-i(\vec{p} + \vec{p}')z} K_{-1-ik}(i\vec{P}z).
\]

The integrals results are discussed in the appendix. The final result for the transition amplitude is

\[
A_{Z\sigma} = \frac{e_0}{\sin(\theta_W)} \delta^3(\vec{P} + \vec{p} + \vec{p}') \frac{1}{(2\pi)^{3/2}} \left( \frac{1}{2} - \sigma \right) \left( \frac{1}{2} + \sigma' \right) \\
\times \left\{ A_k(\vec{P}, \vec{p}, \vec{p}') \xi^+_{\sigma'}(\vec{p}) \vec{s} \cdot \vec{e}^*(\vec{p}, \lambda = 0) \eta_{\sigma'}(\vec{p}') \\
+ C_k(\vec{P}, \vec{p}, \vec{p}') \xi^+_{\sigma'}(\vec{p}) \eta_{\sigma'}(\vec{p}') + B_k(\vec{P}, \vec{p}, \vec{p}') \xi^+_{\sigma'}(\vec{p}) \vec{s} \cdot \vec{e}^*(\vec{p}, \lambda = \pm 1) \eta_{\sigma'}(\vec{p}') \right\}
\]

\[
= A_1(\lambda = 0) + A_2(\lambda = \pm 1)
\]

In the above equation we introduce the following notations:

\[
A_k(\vec{P}, \vec{p}, \vec{p}') = \frac{i^{-3/2}(2\vec{P})^{-ik}}{(\vec{P} + \vec{p} + \vec{p}')^{3/2-ik}} \frac{\omega}{M} \Gamma \left( \frac{3}{2} - ik \right) \Gamma \left( \frac{3}{2} + ik \right) \left( \frac{1}{2} - ik \right)
\]

\times \text{2F1} \left( \frac{3}{2} - ik; \frac{1}{2} - ik; 2; -\frac{\vec{P} + \vec{p} + \vec{p}'}{\vec{P} + \vec{p} + \vec{p}'} \right) - \frac{i^{-3/2}(2\vec{P})^{-1-ik}}{2(\vec{P} + \vec{p} + \vec{p}')^{7/2-ik}} \frac{\omega}{M} \Gamma \left( \frac{7}{2} - ik \right)
\]

\times \text{2F1} \left( \frac{3}{2} + ik; \frac{3}{2} + ik; 3; -\frac{\vec{P} + \vec{p} + \vec{p}'}{\vec{P} + \vec{p} + \vec{p}'} \right),
\]

\[
C_k(\vec{P}, \vec{p}, \vec{p}') = \frac{i^{-3/2}(2\vec{P})^{-ik}}{2(\vec{P} + \vec{p} + \vec{p}')^{5/2-ik}} \frac{\omega}{M} \Gamma \left( \frac{5}{2} - ik \right) \Gamma \left( \frac{5}{2} + ik \right)
\]

\times \text{2F1} \left( \frac{5}{2} - ik; \frac{1}{2} - ik; 3; -\frac{\vec{P} + \vec{p} + \vec{p}'}{\vec{P} + \vec{p} + \vec{p}'} \right),
\]

\[
B_k(\vec{P}, \vec{p}, \vec{p}') = \frac{i^{-1/2}(2\vec{P})^{-ik}}{(\vec{P} + \vec{p} + \vec{p}')^{3/2-ik}} \Gamma \left( \frac{3}{2} - ik \right) \Gamma \left( \frac{3}{2} + ik \right)
\]

\times \text{2F1} \left( \frac{3}{2} - ik; \frac{1}{2} - ik; 2; -\frac{\vec{P} + \vec{p} + \vec{p}'}{\vec{P} + \vec{p} + \vec{p}'} \right),
\]
The delta Dirac terms $\delta^3(\vec{P} + \vec{p} + \vec{p}')$ assures the momentum conservation in process of $Z$ boson and neutrinos spontaneous generation from vacuum and this result was also reported to QED first order processes that generate triplets from vacuum in de Sitter geometry\cite{24}. The analytical structure of the final result is depending on Gauss hypergeometric functions $_2F_1$ and gamma Euler functions $\Gamma$. Their dependence of gravity enters in equation (18) via the parameter $k = \sqrt{\left(\frac{M_Z}{\omega}\right)^2 - \frac{1}{4}}$. Remarkably is that the ratio between $Z$ boson mass and the expansion parameter $\frac{M_Z}{\omega}$ and the momenta $p, p', P$ completely determine the amplitude and probability. This suggest an analysis implying these parameters, looking for the behaviour of the functions $A_k, B_k, C_k$ in terms of the parameter $M_Z/\omega$.

The key parameters in our amplitude are the momenta of the produced particles $p, p', P$ and the ratio between $Z$ boson mass and the expansion parameter, $M_Z/\omega$. The results presented in figs. (1)-(4) prove that the functions $A_k, B_k$ which define the amplitude converge

The probability is obtained by summing after the final helicities the square modulus of the parameter $\lambda = -1/2, \sigma' = 1/2$ and we do no longer need to sum over them. More precisely we have selection rules such that if the helicity of the fermion is $\sigma$ then the antifermion helicity will always be $\sigma' = -\sigma$. Since we have the term with the delta Dirac function $\delta^3(\vec{P} + \vec{p} + \vec{p}')$, we will define here the transition probability per volume unit, i.e. $|\delta^3(\vec{p})|^2 = V\delta^3(\vec{p})$:

$$P_{Z\nu\sigma} = |A_1(\lambda = 0)|^2 + \frac{1}{2} \sum_\lambda |A_2(\lambda = \pm 1)|^2 = \frac{e_0^2}{\sin^2(2\theta_W)} \delta^3(\vec{P} + \vec{p} + \vec{p}') \frac{1}{(2\pi)^3} \left(\frac{1}{2} - \sigma\right)^2 \left(\frac{1}{2} + \sigma'\right)^2 \left\{ |A_k(P, p, p')|^2 |\xi_{\sigma}^+(\vec{p})\tilde{\sigma} \cdot \tilde{e}^*(\vec{n}_P, \lambda = 0)\eta_{\sigma'}(\vec{p}')|^2ight.

+ |C_k(P, p, p')|^2 |\xi_{\sigma}^+(\vec{p})\eta_{\sigma'}(\vec{p}'')}}^2

+ A_k^*(P, p, p') C_k(P, p, p') (\xi_{\sigma}^+(\vec{p})\tilde{\sigma} \cdot \tilde{e}^*(\vec{n}_P, \lambda = 0)\eta_{\sigma'}(\vec{p}'))^* (\xi_{\sigma}^+(\vec{p})\eta_{\sigma'}(\vec{p}'))

+ C_k^*(P, p, p') A_k(P, p, p') (\xi_{\sigma}^+(\vec{p})\eta_{\sigma'}(\vec{p}'))^* (\xi_{\sigma}^+(\vec{p})\tilde{\sigma} \cdot \tilde{e}^*(\vec{n}_P, \lambda = 0)\eta_{\sigma'}(\vec{p}'))

+ \frac{1}{2} \sum_\lambda |B_k(P, p, p')|^2 |\xi_{\sigma}^+(\vec{p})\tilde{\sigma} \cdot \tilde{e}^*(\vec{n}_P, \lambda = \pm 1)\eta_{\sigma'}(\vec{p}')|^2 \right\}. \quad (20)

We observe from the probability equation that we have contributions from the longitudinal polarization with $\lambda = 0$ and a sum after the two transversal polarizations with $\lambda = \pm 1$.

A. Graphical analysis

The key parameters in our amplitude are the momenta of the produced particles $p, p', P$ and the ratio between $Z$ boson mass and the expansion parameter, $M_Z/\omega$. The results presented in figs. (1)-(4) prove that the functions $A_k, B_k$ which define the amplitude converge
FIG. 1: Real part of $A_k$ as a function of parameter $M_Z/\omega$. Solid line is for $p = 0.5, p' = 0.4, \mathcal{P} = 0.1$, while the point line is for $p = 0.5, p' = 0.3, \mathcal{P} = 0.2$.

FIG. 2: Real part of $B_k$ as a function of parameter $M_Z/\omega$. Solid line is for $p = 0.5, p' = 0.4, \mathcal{P} = 0.1$, while the point line is for $p = 0.2, p' = 0.4, \mathcal{P} = 0.4$.

in both their real and imaginary parts, if we analyse them in terms of parameter $M_Z/\omega$ and we fix the values for the momenta $p, p', \mathcal{P}$. Another observation is that for $M_Z/\omega \to \infty$, the Minkowski limit can be discussed since this corresponds to $\omega \to 0$ and we observe from
FIG. 3: Imaginary part of $A_k$ as a function of parameter $M_Z/\omega$. Solid line is for $p = 0.4, p' = 0.5, \mathcal{P} = 0.1$, while the point line is for $p = 0.2, p' = 0.4, \mathcal{P} = 0.4$.

FIG. 4: Imaginary part of $B_k$ as a function of parameter $M_Z/\omega$. Solid line is for $p = 0.3, p' = 0.5, \mathcal{P} = 0.2$, while the point line is for $p = 0.2, p' = 0.3, \mathcal{P} = 0.5$.

Figs. 1)-(4) that the functions $A_k, B_k$ are vanishing in this limit. The same observations are valid for the function $C_k$.

The probability can be understood better by plotting the square modulus of the functions...
An interesting observation can be done if we consider that the momenta of the Z boson $\mathcal{P}$ is much more smaller that the momenta of the neutrino-antineutrino pair $p, p'$ by plotting the

**FIG. 5:** $|B_k|^2$ as a function of parameter $M_Z/\omega$. Solid line is for $p = 0.5, p' = 0.4, \mathcal{P} = 0.1$, while the point line is for $p = 0.5, p' = 0.3, \mathcal{P} = 0.2$.

**FIG. 6:** $|A(0)|^2$ as a function of parameter $M_Z/\omega$. Solid line is for $p = 0.5, p' = 0.4, \mathcal{P} = 0.1$, while the point line is for $p = 0.5, p' = 0.3, \mathcal{P} = 0.2$. 

$A_k, B_k, C_k$ in terms of $M_Z/\omega$. 


probability for the transversal modes contribution $\lambda = \pm 1$. Then in this case the probability shows a oscillatory behaviour, which is contained in the same interval where the ratio $M_Z/\omega$ is small (see fig. (7)).

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure7.png}
\caption{$|B_k|^2$ as a function of parameter $M_Z/\omega$. Solid line is for $p = 0.5, p' = 0.4, \mathcal{P} = 0.0001$, while the point line is for $p = 0.2, p' = 0.4, \mathcal{P} = 0.0004$.}
\end{figure}

The above graphs for the $|A(0)|^2, |B_k|^2$ give the behaviour of the probability in terms of parameter $M_Z/\omega$, the factor $|A(0)|^2 = |A_k|^2 + |C_k|^2 + A_k^* C_k + A_k C_k^*$ contain all the contributions from probability in the case $\lambda = 0$. We observe that the particle production from de Sitter vacuum is important only in strong gravitational fields $\omega \geq M_Z$. Our result prove that the production of Z boson is possible only in early universe when the expansion parameter was larger that the Z boson mass. This result is remarkable since it is the first computation that prove that Z bosons could be produced in strong gravitational fields as a first order perturbative process, that is forbidden in the Minkowski electro-weak theory \cite{10,12}. Our study suggest that a more general theory could be developed in de Sitter space-time, and that from this theory one should recover the well known results from Minkowski theory in the limit of zero expansion parameter.
B. Helicity bispinor summation

Further computation of the terms dependent of helicity spinors and polarization vectors could be done by taking a orthogonal local frame defined by the basis vectors $\vec{e}_i$. In this local frame we define the momenta $\vec{p}$ could be done by taking a orthogonal local frame defined by the basis vectors $\vec{e}_i$. For the momenta of the neutrinos we will take the spherical coordinates such that they move in the plane $(1, 3)$ i.e. $\vec{p}(p, \alpha, \beta = 0)$ and $\vec{p}'(p', \gamma, \theta = \pi)$ [24]. In this setup the angle between $\vec{p}$ and $\vec{p}'$ is just $\alpha + \gamma$. The momenta conservation in this process give when projected on the two axes that define the plane $(1, 3)$

$$\mathcal{P} = p \cos \alpha + p' \cos \gamma; \quad p \sin \alpha - p' \sin \gamma = 0.$$  \hspace{1cm} (21)

From these equations we deduce that:

$$\frac{p}{\mathcal{P}} = \frac{\sin \gamma}{\sin(\alpha + \gamma)}, \quad \frac{p'}{\mathcal{P}} = \frac{\sin \alpha}{\sin(\alpha + \gamma)}$$ \hspace{1cm} (22)

The factors that define the amplitude and contain momenta sums could be computed, and we finally obtain:

$$A_k(\mathcal{P}, p, p') = \frac{i^{-3/2}(2)^{-ik}}{\mathcal{P}^{3/2}(1 + \frac{\sin \gamma}{\sin(\alpha + \gamma)} + \frac{\sin \alpha}{\sin(\alpha + \gamma)})^{3/2-ik}} \frac{\omega}{M_Z} \Gamma(\frac{3}{2} - ik) \Gamma(\frac{3}{2} + ik) (\frac{1}{2} - ik)$$

$$\times 2F_1\left(\frac{3}{2} - ik, \frac{1}{2} - ik; 2; \frac{-1 + \frac{\sin \gamma}{\sin(\alpha + \gamma)} + \frac{\sin \alpha}{\sin(\alpha + \gamma)}}{1 + \frac{\sin \gamma}{\sin(\alpha + \gamma)} + \frac{\sin \alpha}{\sin(\alpha + \gamma)}}\right) - \frac{i^{-3/2}(2)^{1-ik}}{2\mathcal{P}^{3/2}(1 + \frac{\sin \gamma}{\sin(\alpha + \gamma)} + \frac{\sin \alpha}{\sin(\alpha + \gamma)})^{7/2-ik}}$$

$$\times \frac{\omega}{M_Z} \Gamma\left(\frac{7}{2} - ik\right) \Gamma\left(\frac{3}{2} + ik\right) 2F_1\left(\frac{7}{2} - ik, \frac{3}{2} - ik; 3; \frac{-1 + \frac{\sin \gamma}{\sin(\alpha + \gamma)} + \frac{\sin \alpha}{\sin(\alpha + \gamma)}}{1 + \frac{\sin \gamma}{\sin(\alpha + \gamma)} + \frac{\sin \alpha}{\sin(\alpha + \gamma)}}\right);$$

$$C_k(\mathcal{P}, p, p') = \frac{i^{-3/2}(2)^{-ik}}{2\mathcal{P}^{3/2}(1 + \frac{\sin \gamma}{\sin(\alpha + \gamma)} + \frac{\sin \alpha}{\sin(\alpha + \gamma)})^{5/2-ik}} \frac{\omega}{M_Z} \Gamma(\frac{5}{2} - ik) \Gamma(\frac{5}{2} + ik)$$

$$\times 2F_1\left(\frac{5}{2} - ik, \frac{1}{2} - ik; 3; \frac{-1 + \frac{\sin \gamma}{\sin(\alpha + \gamma)} + \frac{\sin \alpha}{\sin(\alpha + \gamma)}}{1 + \frac{\sin \gamma}{\sin(\alpha + \gamma)} + \frac{\sin \alpha}{\sin(\alpha + \gamma)}}\right);$$

$$B_k(\mathcal{P}, p, p') = \frac{i^{-1/2}(2)^{-ik}}{\mathcal{P}^{3/2}(1 + \frac{\sin \gamma}{\sin(\alpha + \gamma)} + \frac{\sin \alpha}{\sin(\alpha + \gamma)})^{3/2-ik}} \Gamma\left(\frac{3}{2} - ik\right) \Gamma\left(\frac{3}{2} + ik\right)$$

$$\times 2F_1\left(\frac{3}{2} - ik, \frac{1}{2} - ik; 2; \frac{-1 + \frac{\sin \gamma}{\sin(\alpha + \gamma)} + \frac{\sin \alpha}{\sin(\alpha + \gamma)}}{1 + \frac{\sin \gamma}{\sin(\alpha + \gamma)} + \frac{\sin \alpha}{\sin(\alpha + \gamma)}}\right).$$  \hspace{1cm} (23)

The above formula in terms of the angle help one to establish the probability equation for different momenta configurations. The last step in completing the probability equation.
is to compute the helicity bispinor products. First we analyse the situation when $\lambda = 0$, and considering the above case with $\bar{P} = P\bar{e}_3$ and knowing that in this case $\bar{P} \cdot \bar{e}(\bar{n}_p, \lambda = 0) = P$, then $\bar{e}(\bar{n}_p, \lambda = 0) = \bar{e}_3$ and taking into account that fermion helicities take only the values $\sigma = -1/2, \sigma' = 1/2$ we finally obtain:

$$\xi^\pm_{-1/2}(\bar{p}) = \xi^\pm_{1/2}(\bar{p}) = \xi^\pm_{-1/2}(\bar{p}) \sigma_3\eta_{1/2} = \sin \left(\frac{\alpha}{2}\right) \sin \left(\frac{\gamma}{2}\right) + \cos \left(\frac{\alpha}{2}\right) \cos \left(\frac{\gamma}{2}\right).$$

(24)

The temporal components contribution to the probability is proportional with the bispinors product $\xi^+_\sigma(\bar{p})\eta_{1/2}(\bar{p}')$, which evaluated by using the momenta in the plane $(1, 3)$ as above and the fact that $\sigma = -1/2, \sigma' = 1/2$ give:

$$\xi^+_\pm_{1/2}(\bar{p}) = \sin \left(\frac{\alpha}{2}\right) \sin \left(\frac{\gamma}{2}\right) - \cos \left(\frac{\alpha}{2}\right) \cos \left(\frac{\gamma}{2}\right).$$

(25)

Then collecting all the above results for $\lambda = 0$, the sum containing the terms with longitudinal modes contribution give

$$|A(\lambda = 0)|^2 = |A_k(P, p, p')|^2 \cos^2 \left(\frac{\alpha + \gamma}{2}\right) + |C_k(P, p, p')|^2 \cos^2 \left(\frac{\alpha - \gamma}{2}\right)
- (A^*_k(P, p, p')C_k(P, p, p') + C^*_k(P, p, p')A_k(P, p, p')) \cos \left(\frac{\alpha + \gamma}{2}\right) \cos \left(\frac{\alpha - \gamma}{2}\right)$$

(26)

From the above equation we observe that if we take $\alpha = \pi, \gamma = 0$ or $\alpha = 0, \gamma = \pi$ the probability is zero. By contrary if we take $\alpha = \gamma = 0$ then the probability will is nonvanishing and we conclude that the modes with $\lambda = 0$ favour production processes in which the momenta of the neutrino and antineutrino are parallel and have the same orientation.

For the case when $\lambda = \pm 1$ we consider the circular polarizations such that $\bar{e}_{\pm 1} = \frac{1}{\sqrt{2}}(\pm \bar{e}_1 + i\bar{e}_2)$ in the local orthogonal frame where $\bar{P} = P\bar{e}_3$ and then taking the spherical coordinates of the momenta $\bar{p}(p, \alpha, \beta = 0)$ and $\bar{p}'(p', \gamma, \theta = \pi)$ we finally obtain:

$$\xi^+_\pm_{-1/2}(\bar{p}) = -2 \cos \left(\frac{\alpha}{2}\right) \sin \left(\frac{\gamma}{2}\right), \lambda = 1
\xi^+_\pm_{-1/2}(\bar{p}) = -2 \sin \left(\frac{\alpha}{2}\right) \cos \left(\frac{\gamma}{2}\right), \lambda = -1.$$  

(27)

This completes our analytical results of the amplitude and probability and establish the formula for different values of the angle $\alpha, \gamma$. For example if one analyse the amplitude equation in the case when $\lambda = \pm 1$ and we fix $\alpha = \gamma = 0$ then a simple computation using equations (26) and (27) prove that the amplitude/probability are zero. This is the case when
the fermions momenta are parallel and have the same orientation. If we take \( \alpha = \pi, \gamma = 0 \) or \( \alpha = 0, \gamma = \pi \), then the hypergeometric function that define \( B_k \), become \( _2F_1(a, b; c; 0) = 1 \) \[21\] and the amplitude equation simplify:

\[ A_2(\lambda = \pm 1) = \frac{i e_0}{\sin(2\theta_W)} \delta^3(\vec{P} + \vec{p} + \vec{p}') \frac{1}{(2\pi)^{3/2} 4(2iP)^{3/2} \cosh(\pi k)} \frac{k^2 + 1}{k^2 + 1}. \] (28)

This means that the processes in which the momenta of the produced particles are parallel but have opposite orientation have are favoured.

The total probability of the process can be obtained by integration after the final momenta.

\[ P_t = \int d^3 \vec{P} d^3 \vec{p} d^3 \vec{p}' P_{Z\nu\pi}(\lambda = \pm 1). \] (29)

We consider the case when \( \lambda = \pm 1 \) and \( \alpha = \pi, \gamma = 0 \) such that we will compute the total probability for given directions of the momenta. Then by taking the square modulus of amplitude from equation (28) and observing that the integral after \( Z \) boson momenta could be easily solved using the properties of delta Dirac function we obtain:

\[ P_t = \frac{e^2}{(2\pi)^3 \sin^2(2\theta_W) 128 \cosh^2(\pi k)} \int d^3 \vec{P} d^3 \vec{p} d^3 \vec{p}' \frac{1}{|\vec{p} + \vec{p}'|^2}. \] (30)

The above integral can be evaluated numerically or using a cutoff method since is logarithmically divergent. The total probability will drop rapidly to zero if we make an analysis in terms of parameter \( M_Z/\omega \), because of the factor \( \cosh(\pi k) \), which will keep the nonzero values only in a small interval around \( M_Z/\omega \sim 1 \), while for large values of this parameter the total probability will be zero due to the proportionality with the factor \( \cosh^{-2}(\pi k) \).

Another aspect is related to the helicity conservation in the process of spontaneous production of \( Z \) boson and neutrino and antineutrino from vacuum. Since we obtain the selection rule for the helicities of the neutrino and antineutrino the only possible processes are restricted to \( \sigma = -1/2 \) and \( \sigma' = 1/2 \). Then in the case with \( \lambda = 0 \) the helicity is conserved, while for the case of transversal modes contribution \( \lambda = \pm 1 \) the helicity conservation law is broken.

IV. CONCLUSIONS

In this paper we investigated the interactions between \( Z \) boson and leptons in a de Sitter geometry by following the methods from flat space case which is based on construction of
transition amplitudes by using perturbations. The process in which the triplet $Z$ boson and neutrino-antineutrino pair are generated from vacuum in de Sitter geometry was studied by computing the first order transition amplitude. The basic steps of our computations were the exact solution of the Proca and Dirac equation on de Sitter geometry in momentum-helicity basis \[2, 22\]. Our results prove that generation of the $Z$ boson from vacuum is possible only in the early universe when the gravitational fields were strong. From our computations we recover the correct Minkowski limit where the amplitude is vanishing due to the simultaneous energy and momentum conservation. The analysis in the helicity space reveals that there are nonvanishing probabilities for processes which could break the helicity conservation law and in this case the neutrino and antineutrino move along the same direction but their momenta are opposite as orientation.

We use here a perturbative method in which the generation of particles is the result of fields interaction in this geometry. This mechanism for matter generation receive attention only recently and seems that is suitable for study the problem of particle production in strong gravitational fields.

V. APPENDIX

Here we present the main steps for computing the amplitude of pair production in magnetic field. Using the relation that connects Hankel functions and Bessel $K$ functions \[21\]:

\[ H^{(1,2)}_{\nu}(z) = \mp \left( \frac{2i}{\pi} \right) e^{\mp i\pi\nu/2} K_{\nu}(\mp iz), \tag{31} \]

we arrive at integrals of the type \[21\]:

\[
\int_0^\infty dz z^{\mu-1} e^{-az} K_\nu(\beta z) = \frac{\sqrt{\pi}(2\beta)^\nu}{(\alpha + \beta)^{\mu+\nu}} \frac{\Gamma(\mu + \nu)}{\Gamma(\mu + 1/2)} \times _2\!F_1 \left( \mu + \nu, \nu + \frac{1}{2}; \mu + \frac{1}{2}; \frac{\alpha - \beta}{\alpha + \beta} \right), \\
Re(\alpha + \beta) > 0, |Re(\mu)| > |Re(\nu)|. \tag{32} 
\]

The above equation help us to solve the temporal integrals from the amplitude.

The form of the helicity bispinors can be expressed only in terms of angles that define the spherical coordinates of momenta $\vec{p}(p, \alpha, \beta = 0)$ and $\vec{p}'(p', \gamma, \theta = \pi)$, and the result is
given as follows \[12, 20]:

\[
\xi_{-1/2}(\vec{p}') = \sqrt{\frac{p_3 + p}{2p}} \left( \frac{-p_1 + ip_2}{p_3 + p} \right) = \begin{pmatrix} -\sin \left( \frac{\alpha}{2} \right) \\ \cos \left( \frac{\alpha}{2} \right) \end{pmatrix}
\]

\[
\eta_{1/2}(\vec{p}') = \sqrt{\frac{p_3 + p}{2p}} \left( \frac{-p_1 - ip_2}{p_3 + p} \right) = \begin{pmatrix} -\sin \left( \frac{\gamma}{2} \right) \\ -\cos \left( \frac{\gamma}{2} \right) \end{pmatrix}.
\] (33)

Acknowledgements This work was supported by a grant of the Romanian Ministry of Research and Innovation, CCCDI-UEFISCDI, project number PN-III-P1-1.2-PCCDI-2017-0371, within PNCDI III.

We would like to thank to Professor Ion Cotăescu for his observations that help us to improve the manuscript.

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