PLANCK-BALANCE 1 (PB1) – A TABLE-TOP KIBBLE BALANCE FOR MASSES FROM 1 mg TO 1 kg – CURRENT STATUS

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Abstract:
The Planck-Balance 1 (PB1) has been developed in a collaboration between the Physikalisch-Technische Bundesanstalt and the Technische Universität Ilmenau. It is a small-size Kibble balance aimed to calibrate $E_1$ mass standards for a mass range from 1 mg to 1 kg. This paper presents its principle and some preliminary investigations.

Keywords: Kibble balance; mass metrology; Planck-Balance; kilogram realisation; kilogram dissemination

1. INTRODUCTION

In mass metrology mass standards had been traced to the International Prototype Kilogram (IPK) for more than 130 years. This was done by comparing masses and building up the mass scale. After the re-definition of the SI unit kilogram this way of calibration can be continued. What has changed is that the IPK has been replaced by a definition based on the Planck constant and a corresponding realisation of the kilogram, e.g., a Silicon-28-Sphere [1] or a Kibble balance [2], up to now also at a nominal value of 1 kg. The redefinition, however, gives more freedom in mass metrology. Since the Planck constant is valid for any mass value the Kibble balance technology can be used to calibrate mass standards that differ from 1 kg too. Several national metrology institutes are working on such a balance Fehler! Verweisquelle konnte nicht gefunden werden. [4], which is sometimes called “table-top Kibble balance” or “micro Kibble balance (µKB)”.

The Physikalisch-Technische Bundesanstalt (PTB) is developing, in collaboration with the Technische Universität Ilmenau (TUIL), two table-top sized Kibble balances, called Planck-Balance 1 (PB1) and Planck-Balance 2 (PB2), for industrial applications. Details on the PB2 have already been published elsewhere [5]. In this paper the concept along with some technical details of PB1 will be described.

2. CONCEPT OF PLANCK-BALANCE 1

PB1 is a Kibble balance that works in two modes, as originally proposed by Bryan Kibble (see [2]). One mode, called force mode, where the weight of a mass under test is compared to an electromagnetic force, produced by a voice coil system. And a second mode, called velocity mode, where the force factor (see below) is determined. The difference of the Planck-Balance, however, with respect to its high-precision counterparts, lies in the field of application. The aimed use of PB is by industry, calibration laboratories, but also national metrology institutes that are interested in a system for the primary realisation of the new SI kilogram but cannot afford or maintain a highly sophisticated self-developed Kibble balance. Therefore, the PB1 is aimed to meet the following specifications:

- Use for mass calibrations over a mass range from 1 mg to 1 kg.
- Relative measurement uncertainties according to $E_1$ MPE ¹ specified in OIML R 111-1 ($\approx 8.4 \times 10^{-8} (k = 1)$ @ 1 kg).
- Compact and easy to use, i.e. the mechanical part should be of comparable size to a conventional analytical mass balance. The use of the balance should be possible by staff members that do not necessarily have profound knowledge in electrical metrology.
- Modular design: parts should be easily exchangeable or adaptable in order to reach the required specifications. As an example: the load cell should be exchangeable if another nominal mass value is desired. This allows for using the, basically, same setup for the whole mass range.

As a basis a commercial load cell (Sartorius Secura 1103-1x, see Figure 1) is used, which can be found in high-precision analytical mass balances.

¹ MPE (Maximum Permissible Error). The required measurement uncertainty is calculated as 1/6 of the MPE of the mass value under test. For 1 kg the MPE ($E_1$) is 0.5 mg.
The specified repeatability (standard deviation) of this load cell is 1 mg. Nevertheless, the load cell can be replaced by another one, with improved repeatability or a more adequate one for another mass range.

In contrast to PB2, where the load cell is used as a mechanical guiding system only, in PB1 the lever ratio is exploited additionally. As shown in the schematic of Figure 2 the internal voice coil (3), which is fixed to the longer lever arm, is used to compensate the weight (5) sitting on the shorter lever arm. The ratio of the levers is about 1:30. The velocity, however, is measured on the (short) lever arm. The external voice coil (2) is used to oscillate the lever arm, which finally induces a voltage in the internal voice coil (3).

This allows using a smaller voice-coil system with a force factor \( Bl \) of about 27 T-m (\( B \) denotes the magnetic flux density, \( l \) the length of the coil-wire in the \( B \) -field). This lever ratio is approximately

\[
Bl := \frac{u_{\text{ind}}}{v} = 1:30
\]

where \( u_{\text{ind}} \) is the induced voltage amplitude and \( v \) the velocity amplitude with which the coil is moved in the magnetic field. This is to be understood that the velocity is measured on the lever arm where the weight is placed, while the voice coil is connected to the other (the longer) lever arm.

In the force mode the electrical current is measured from the voltage drop at a precision resistor. When putting a mass of a factor 10 lower then also the voltage drop will be by a factor of 10 lower. This would lead to reduced accuracy in the voltage measurement. In order to circumvent this issue, the PB1 employs resistors of values 100 \( \Omega \), 1 k\( \Omega \), 10 k\( \Omega \) corresponding to the load ranges of 1 kg to 100 g, 100 g to 10 g, and 10 g to 1 mg, respectively. A custom made “switch box” has been developed in order to switch between the force and velocity mode, as well as between different resistor values.

Current measurements with PB1 are performed in air but can also be done in a high-vacuum environment as shown in Figure 3. For the sake of size scaling, on top of PB1 a weight of 1 kg is visible in Figure 3.

3. CURRENT INVESTIGATIONS OF PLANCK-BALANCE 1

A series of measurements were performed with PB1 to investigate performance and possible systematic errors in the force and velocity modes. The following gives some examples.

3.1. Force Mode

In the force mode the repeatability has been investigated by doing automated repeated measurements over several hours. Every single measurement consists of an ABBA-cycle (A-with weight; B-without weight). Figure 4 shows a series of measurements conducted with a load of 1 kg. The \( Bl \) was calculated with corrections for changes of the environmental conditions, such as temperature, pressure and humidity. A temperature correction was done to a nominal value of 23 °C, where the temperature coefficient was determined from a long-term measurement. The mass lifting has been automated by means of the above-mentioned home-made mass lifter. The vertical translation stage is
high-vacuum compatible, has a resolution of 4 nm, and a force range of up to 20 N. It is based on a piezo actuator to avoid magnetic bias, and has a travel range of 10 mm. The obtained relative repeatability (standard deviation) for a 1 kg mass standard lies in the order of 0.6 ppm, which is better than the stated specifications of the load cell of 1 mg. For curiosity the mass change was also performed by hand. In the case the repeatability was around 1 ppm, which is still in accordance with the specifications. For lower mass values the relative standard deviation decreases. So, for example, for a 10 g mass piece the standard deviation was in the order of $2 \times 10^{-7}$ kg. A complete ABBA-cycle currently takes about 80 s but can be improved further.

The balance allows, in principle, a measurement of masses over a range from 1 mg to 1 kg with the same load cell. Because of the limiting standard deviation, however, an accurate measurement of small masses is not expected in an appropriate time period. This is why the balance is designed in a modular way. If the focus lies on smaller masses, the load cell can be simply exchanged in order to meet the specifications. Nevertheless, measurements with calibrated E1 mass standards were done with the same load cell (no other load cell has been used to date in this setup), which showed an increasing deviation from their calibrated value with decreasing mass value, but within the specified linearity of the balance of 2 mg. In these measurements the $B_l$, obtained for each nominal mass was compared to the $B_l$ as obtained with a 1 kg mass standard. Corrections were made only for temperature (all corrected to 23 °C using the same temperature coefficient) and coil current effect [2].

Further investigations were done on the stability of the magnet’s temperature coefficient. This is important since the laboratory, where the PB1 is installed, is currently not air-conditioned. The absolute temperature can vary significantly with respect to the temperature coefficient. From long term (several hours) measurements it was noticed that the temperature coefficient changed whether the temperature is increasing or decreasing, as can be seen in Figure 5 a) and b). Thus, a hysteresis-like behaviour has been observed. The temperature coefficient can vary by several per cent, while its relative value has been determined to be about $-3.3 \times 10^{-4}$ K$^{-1}$, which is a typical value for a SmCo magnet. This temperature change was not

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**Figure 4**: Repeated force mode measurement of a 1 kg mass standard. Each circle indicates a single ABBA cycle. The relative standard deviation for this measurement is $0.6 \times 10^{-6}$.

**Figure 5**: Measurement campaign of a 1 kg mass standard with PB1. The magnet shows a hysteresis. Depending whether the temperature is rising or falling with time, the temperature coefficient will be different by several per cent. a) Temperature measured in the magnet. b) The force factor $B_l$ measured during the campaign.
forced by heating or cooling the room but was a mere coincidence that such a linear rising and falling of temperature happened during the measurement. It would be interesting to investigate this behaviour further, e.g. by placing the setup into a climate chamber and forcing temperature changes with different slopes.

The equilibrium position is monitored by means of a dual photo diode, a slit and an LED. The voltage output of the diode is measured with a 7.5 digits digital multimeter, which, after a calibration of the sensitivity curve, allows to resolve the position to several nanometres (the sensitivity factor is about $2.9 \times 10^3 \, \text{V} \cdot \text{µm}^{-1}$). In order to estimate the uncertainty to repositioning, the $Bl$ as a function of position has been measured (see Figure 6). To this end the PID control was set to maintain the equilibrium at several nominal positions, and at each position a weighing of one kilogram was performed. Due to the restoring force of the flexure hinges of the load cell, the electrical current values for mass on and mass off were not equal in their absolute value any longer. As this would give a bias in the result, for each position an adjustment of the tare weight was done.

Figure 6: Measurement of the force factor $Bl$ as a function of the coil position.

The relative gradient of $Bl$ at the position zero was determined to be of $1.9 \times 10^{-3} \, \text{V}^{-1}$ (voltage output of the position sensor). With a positioning uncertainty of, say, 150 µV (currently our PID threshold level – but can be further reduced; this voltage corresponds to about 50 nm) the possible relative error becomes in the order of about $3 \times 10^{-7}$.

### 3.2. Velocity Mode

As practicing in PB2 also in PB1 the voice coil is driven sinusoidally, giving a relative position over time according to

$$s(t) = S_0 + S \sin(\omega t + \phi_s)$$  \hspace{1cm} (2)

where $S_0$ denotes a constant offset and $\omega$ the angular frequency, $\omega = 2\pi f_{\text{sig}}$, with the oscillation frequency $f_{\text{sig}}$ and $\phi_s$ the initial phase. The coil velocity is obtained as the derivative of equation (2) with respect to the time $t$ as

$$v(t) = \omega S \cos(\omega t + \phi_s)$$  \hspace{1cm} (3)

This motion induces an ac voltage of

$$u(t) = U \cos(\omega t + \phi_u)$$  \hspace{1cm} (4)

across the coil ends. $\phi_u$ is again an arbitrary initial phase. Due to non-linearities, e.g. in the magnetic field and the mechanical system, higher harmonics will be excited along with the excitation frequency. By determining the amplitude of the first harmonic (fundamental note) of the induced voltage, as well as of the position signal, the force factor, $Bl$, can be calculated. Thus, for the sake of simplicity, assuming the force factor $Bl$ to be a constant during the whole range of coil movement, $Bl$ can be calculated as

$$Bl = \frac{U}{\omega S} = \frac{U}{2\pi f_{\text{sig}} S} = \frac{U}{v}$$  \hspace{1cm} (5)

This equation, however, is still lacking the lever ratio. If this ratio is taken into account, we obtain equation (1). The signal frequency is usually around 4 Hz, which is close to the eigenfrequency of the load cell, and the amplitude is of about 4.5 µm. The error in the amplitude estimation arising from higher harmonics can be compensated by either including the higher harmonics into the regression analysis, or by adjusting the data length to an integer number of sine wave cycles (coherent sampling) [6].

As the coil oscillates over a range with varying $Bl$ (see Figure 6) the measured $Bl$ is a weighted mean of all $Bl$ s over this range. For an exactly linear $Bl$ over the oscillation range, the measured $Bl$ value corresponds to the point of zero oscillation, which is the same point to which the PID control directs (i.e. the set point) the coil during the force mode. The more non-linear the $Bl$ becomes, the more the calculated point deviates from the set point in the force mode. This deviation, however, can be estimated by means of the shape of the $Bl$ (see Figure 6). As the induction happens in the voice coil attached to the longer lever arm, and the lever ratio is approximately 1:30, the range of motion is about 270 µm and this gives a non-negligible bias. In our case the relative offset is in the order of -2.2 $\times$ 10$^{-5}$ but can be corrected with a remaining relative uncertainty of 15 % of the correction.

Figure 7 shows the results of 19 consecutive $Bl$ determinations in the velocity mode. The interferometer output and the digital multimeter (Keysight 3458A) were sampled with 10 kHz. Each measurement took 10 s. No temperature corrections have been applied in this case, as the temperature deviation was negligible.
Figure 7: Repeated measurement of $B_l$ in the velocity mode. The relative standard deviation is $2.4 \times 10^{-6}$. Each dot is the result of a 10 s measurement with a sampling rate of 10 kHz.

With PB2 it was noticed that the measured $B_l$ was a function of the excitation frequency \[7\]. Based on this experience for PB1 a possible frequency dependence has been investigated. Thus, 20 different excitation frequencies were selected ranging from 1 Hz to 10.5 Hz with an increment of 0.5 Hz. For each selected excitation frequency, the induced voltage and the displacement were measured in air with a sampling period of 30 s and a sampling frequency of 10 kHz. After corrections of the air refractive index, aperture time of the 3458A, and temperature, the $B_l$ was determined from the amplitudes of the induced voltage and the displacement as given in equation (1).
to different values in the coefficient whether the temperature is rising or falling. The difference is in the order of several per cent in the investigated case. A frequency dependence in the velocity mode has been detected, which is orders bigger if the lever ratio is exploited. It is assumed that the reason for this effect is an Abbe error due to a tilting of the load carrier. This problem requires further investigations. In principle, measurements can be performed over the whole range from 1 mg to 1 kg with the same load cell. The standard deviation, however, is a limiting factor. To circumvent this problem, the load cell can be exchanged in order to meet the specifications for the desired load range. This will be done in future investigations.

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5. REFERENCES

[1] D. Knopf, T. Wiedenhöfer, K. Lehrmann, F. Härtig, “A quantum of action on a scale? Dissemination of the quantum based kilogram”, Metrologia, vol. 56, no. 2, 024003, 2019.
[2] I. A. Robinson, S. Schlamminger, “The watt or Kibble balance: a technique for implementing the new SI definition of the unit of mass”, Metrologia, vol. 53, no. 5, A46, 2016.
[3] L. Chao, F. Seifert, D. Haddad, J. Pratt, D. Newell, S. Schlamminger, “The performance of the KIBB-g1 tabletop Kibble balance at NIST” Metrologia, vol. 57, no. 3, 035014, 2020.
[4] I. A. Robinson, J. Berry, S. Davidson, C. Jarvis, “Towards a simplified Kibble balance to realise mass in the new SI”, Euspen’s 17th International Conference & Exhibition, Hannover, May 2017.
[5] C. Rothleitner et al, “The Planck-Balance—using a fixed value of the Planck constant to calibrate E1/E2-weights”, Measurement Science and Technology vol. 29, no. 7, 2018.
[6] S. Lin, C. Rothleitner, N. Rogge, T. Fröhlich, “Influences on amplitude estimation using three-parameter sine fitting algorithm in the velocity mode of the Planck-Balance” Acta IMEKO, ISSN: 2221-870X, accepted for publication, 2020.
[7] N. Rogge, C. Rothleitner, S. Lin, S. Vasilyan, T. Fröhlich, F. Härtig, D. Knopf, “Error sources in the force mode of the “PB2” Planck-Balance”, in Proc. of 24th IMEKO TC3 Int. Conf., Cavtat-Dubrovnik, Croatia, 16-18 November 2020.