Meshing efficiency analysis of double helical gears considering tribo-dynamic coupling effect

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Abstract. In order to obtain accurate meshing efficiency of double helical gears, a calculation method of sliding friction power loss is proposed considering the tribo-dynamic coupling effect. Based on the mixed elastohydrodynamic lubrication (EHL) theory, the initial friction coefficient of tooth surface is obtained and the friction excitation is calculated. Considering the tribo-dynamic coupling effect, a sixteen-degree-of-freedom tribo-dynamic model of double helical gear pair is established, including time-varying mesh stiffness, friction excitation, backlash and transmission error. The tribo-dynamic behaviours of gear pair are analysed before and after coupling, and then the sliding friction power loss and meshing efficiency of double helical gears are obtained. The research results show that after tribo-dynamic coupling, the friction coefficient and friction excitation of double helical gear pair increase, and the meshing efficiency decreases. The meshing efficiency of gear pair increases with the increase of helix angle and pressure angle, and decreases with the increase of surface roughness.

1. Introduction

Double helical gear transmission contains the characteristics of compact structure, stable transmission and high transmission efficiency. It is used widely in aerospace, ship, metallurgy and other fields. The meshing power loss of gear pair influences the system transmission efficiency, mainly including friction loss, churning loss and windage loss [1], among which the sliding friction power loss is the main part of the meshing power loss. Therefore, it is significant to calculate the sliding friction power loss of gear pair accurately and analyze the influence rule of meshing efficiency to improve the transmission performance of gear system.

Calculating friction coefficient accurately is crucial to analysis meshing efficiency. In previous studies, recommended value, empirical formula or experimental method was used mostly. Xu [2] used multiple linear regression method to deduce the formula for calculating the friction coefficient of helical gears based on tooth surface contact analysis and EHL theory, and the calculated results were in good agreement with the experimental values. Wang et al. [3] calculated the friction coefficient of double helical gears by empirical formula. In recent years, the mixed EHL model has been used to calculate the friction coefficient. In order to obtain accurate friction coefficient of double helical gears, it is necessary to calculate the friction coefficient based on mixed EHL model.

The tribo-dynamic coupling research of the gear system can reflect the transmission performance accurately [4]. Considering the tribo-dynamic behavior, Dong et al. [5] established an EHL model of involute gears, and obtained the load, pressure and film thickness distribution. Based on dynamics,
load sharing theory and EHL theory, Zou et al. [6] established a tribo-dynamic model of helical gears, and analyzed the lubrication and dynamic characteristics before and after coupling. At present, there are many achievements on the research of gear system tribo-dynamic coupling, but the research on the calculation of gear meshing efficiency is less considering tribo-dynamic coupling effect.

The meshing efficiency of double helical gears under mixed EHL condition was calculated in reference [7], but the efficiency calculation and influencing factor analysis considering tribo-dynamic coupling effect need to be further studied. In this paper, the friction coefficient and friction excitation are calculated based on the mixed EHL theory. Considering tribo-dynamic coupling effect, the tribo-dynamic model of double helical gear pair is established, and the tribo-dynamic behaviors before and after coupling are analyzed. The meshing efficiency is compared before and after coupling, and the influence rule of helix angle, pressure angle and surface roughness on meshing efficiency is studied.

2. Tribo-dynamic coupling model analysis and meshing efficiency calculation

The geometric parameters and operating conditions of double helical gears are as follows: the number of driving gear and driven gear is 34 and 31, respectively; module \( m_b \) is 2mm, pressure angle \( \alpha_m \) is 22.5°, and helix angle \( \beta \) is 30°; the total tooth width \( B \) is 58mm, and the tool withdrawal groove width \( b \) is 10mm; the input speed \( n \) is 7500r/min and the input power \( P_{in} \) is 400kW.

2.1. Comprehensive friction coefficient calculation

Under the heavy load condition, the double helical gear transmission is in the mixed EHL state, and the tooth surface load \( F_d \) is borne by the oil film and rough peak, which are \( F_e \) and \( F_h \), respectively. The comprehensive friction force \( F_t \) of the tooth surface is composed of the oil film friction force \( F_{fe} \) and the rough peak contact friction force \( F_{hb} \).

\[
\begin{align*}
F_d &= F_e + F_h = \gamma F_d + (1 - \gamma) F_d \\
F_t &= F_{fe} + F_{hb} = \mu_e F_e + (1 - \gamma) \mu_b F_h
\end{align*}
\]

(1)

where \( \gamma \) denotes the oil film load ratio under mixed EHL state; \( \mu_e, \mu_b \) represent oil film friction coefficient and rough peak contact friction coefficient, respectively, \( \mu_b=0.12 \).

In the mixed EHL state, the comprehensive friction coefficient is

\[
\mu = \frac{F_t}{F_d} = \gamma \mu_e + (1 - \gamma) \mu_b
\]

(2)

The empirical formula of oil film load ratio \( \gamma \) proposed by Hu and Zhu [8] is used.

\[
\gamma = 1.21e^{0.6\lambda} / (1 + 0.37\lambda^{1.2h})
\]

(3)

where \( \lambda \) denotes the film thickness ratio, and the calculation formula is

\[
\lambda = h_{min} / \sqrt{\sigma_1^2 + \sigma_2^2}
\]

(4)

where \( \sigma_1=\sigma_2=0.6 \mu m \), denote root mean square deviation of tooth profile of the driving and driven gears, respectively; \( h_{min} \) is the minimum oil film thickness under EHL state.

Xu [2] did lots of research on the friction coefficient under EHL state, and gave the fitting formula of friction coefficient.

\[
\mu_e = e^{f(\xi, P_h, \nu_0, \sigma)} p_{h}^{b3} v_{h}^{b4} t_{0}^{b7} R_{0}^{b8}
\]

(5)

where \( \xi \) denotes slide-roll ratio; \( P_h \) represents maximum Hertz contact stress; \( V_h \) denotes entrainment speed; \( R_0 \) represents the comprehensive curvature radius; \( \nu_0 \) denotes dynamic viscosity of lubricating oil; \( b_1-b_8 \) represent the regression coefficients.

2.2. Tribo-dynamic coupling model analysis

Based on the lumped parameter method, the bending-torsion-axis coupling tribo-dynamic model of double helical gears is established, as shown in Figure 1. In the model, the transverse, longitudinal, axial and torsional displacements of the gear pair are taken into account, with 16 degrees of freedom.
\( (x_{1L}, y_{1L}, z_{1L}, \theta_{1L}, x_{2R}, y_{1R}, z_{2R}, \theta_{1R}, x_{2L}, y_{2L}, z_{2L}, \theta_{2L})^T \) (6)

The relative displacements of the left and right gear pairs of double helical gear along the meshing line are

\[
\begin{align*}
\delta_{2L} &= \left[ (x_{1L} - x_{1L}) \sin \psi + (y_{1L} - y_{1L}) \cos \psi + (\theta_{1L} r_{b1} + \theta_{2L} r_{b2}) \right] \cos \beta_b + (z_{2L} - z_{1L}) \sin \beta_b - e_{1L} \\
\delta_{2R} &= \left[ (x_{1R} - x_{1R}) \sin \psi + (y_{1R} - y_{1R}) \cos \psi + (\theta_{1R} r_{b1} + \theta_{2R} r_{b2}) \right] \cos \beta_b + (z_{2R} - z_{1R}) \sin \beta_b - e_{1R}
\end{align*}
\]

where \( \psi \) represents the angle between the meshing plane and the \( y \) axis; \( e_{1L}, e_{1R} \) denote the static transmission errors of the left and right gear pairs of double helical gear, respectively.

The dynamic meshing forces of the left and right gear pairs of double helical gear are

\[
F_{dL} = k_{12L} f(\delta_{12L}) + c_{12L} \dot{\delta}_{12L}, \quad F_{dR} = k_{12R} f(\delta_{12R}) + c_{12R} \dot{\delta}_{12R}
\]

where \( k_{12L}, k_{12R} \) represent the time-varying mesh stiffness of the left and right gear pairs of double helical gear, respectively; \( f(\delta_{12L}), f(\delta_{12R}) \), \( c_{12L}, c_{12R} \) denote the backlash functions and meshing damping of left and right gear pairs of double helical gears, respectively.

According to the actual effect, the tool withdrawal groove can be equivalent to a hollow shaft, and its stiffness calculation formulas are

\[
k_x = \frac{12EI}{b^3}, \quad k_y = \frac{12EI_y}{b^3}, \quad k_z = \frac{EA}{b}, \quad k_{\theta} = \frac{GL_p}{b}
\]

where \( I_x, I_y \) denote the inertia moment of cross section; \( I_p \) represents polar inertia moment; \( b, A \) denote the length and area of tool withdrawal groove, respectively.

Considering the time-varying mesh stiffness, friction excitation, backlash and transmission error, the vibration differential equations of double helical gear pair are established.

\[
\begin{align*}
m_{xL} \ddot{x}_{1L} + c_{xL} \dot{x}_{1L} + k_{xL} x_{1L} + k_n \left( x_{1L} - x_{1R} \right) + c_n \left( \dot{x}_{1L} - \dot{x}_{1R} \right) + F_{dL} \sin \psi \cos \beta_b &+ F_{dL} \cos \psi \cos \beta_b = 0 \\
m_{yL} \ddot{y}_{1L} + c_{yL} \dot{y}_{1L} + k_{yL} y_{1L} + k_v \left( y_{1L} - y_{1R} \right) + c_v \left( \dot{y}_{1L} - \dot{y}_{1R} \right) + F_{dL} \cos \psi \cos \beta_b &+ F_{dL} \sin \psi \cos \beta_b = 0 \\
m_{zL} \ddot{z}_{1L} + c_{zL} \dot{z}_{1L} + k_{zL} z_{1L} + k_c \left( z_{1L} - z_{1R} \right) + c_c \left( \dot{z}_{1L} - \dot{z}_{1R} \right) + F_{dL} \sin \beta_b &+ F_{dL} \sin \beta_b = 0 \\
J_{1L} \ddot{\theta}_{1L} + F_{dL} \theta_{1L} \sin \beta_b + k_{\theta} \left( \theta_{1L} - \theta_{1R} \right) + c_{\theta} \left( \dot{\theta}_{1L} - \dot{\theta}_{1R} \right) \mp T_{1L} = \frac{T}{2}
\end{align*}
\]

\[
\begin{align*}
m_{xR} \ddot{x}_{1R} + c_{xR} \dot{x}_{1R} + k_{xR} x_{1R} + k_n \left( x_{1R} - x_{1L} \right) + c_n \left( \dot{x}_{1R} - \dot{x}_{1L} \right) + F_{dR} \sin \psi \cos \beta_b &+ F_{dR} \cos \psi \cos \beta_b = 0 \\
m_{yR} \ddot{y}_{1R} + c_{yR} \dot{y}_{1R} + k_{yR} y_{1R} + k_v \left( y_{1R} - y_{1L} \right) + c_v \left( \dot{y}_{1R} - \dot{y}_{1L} \right) + F_{dR} \cos \psi \cos \beta_b &+ F_{dR} \sin \psi \cos \beta_b = 0 \\
m_{zR} \ddot{z}_{1R} + c_{zR} \dot{z}_{1R} + k_{zR} z_{1R} + k_c \left( z_{1R} - z_{1L} \right) + c_c \left( \dot{z}_{1R} - \dot{z}_{1L} \right) + F_{dR} \sin \beta_b &+ F_{dR} \sin \beta_b = 0 \\
J_{1R} \ddot{\theta}_{1R} + F_{dR} \theta_{1R} \sin \beta_b + k_{\theta} \left( \theta_{1R} - \theta_{1L} \right) + c_{\theta} \left( \dot{\theta}_{1R} - \dot{\theta}_{1L} \right) \mp T_{1R} = \frac{T}{2}
\end{align*}
\]
where $m_{Li}$, $m_{Ri}$ ($i=1, 2$) denote the mass of left and right gears of double helical gear $i$, respectively; $J_{Li}$, $J_{Ri}$ represent the rotational inertia of left and right gears of double helical gear $i$ around $Z$ axis, respectively; $k_{iix}$, $k_{iy}$, $k_{ijz}$, $c_{iix}$, $c_{iy}$, $c_{ijz}$ and $c_{ijc}$ ($j=L, R$) denote the supporting stiffness and damping of the left and right gears of double helical gear $i$ in $x$, $y$ and $z$ directions, respectively; $k_{iix}$, $k_{iy}$, $k_{ijz}$, $k_{iic}$, $c_{iix}$, $c_{iy}$, $c_{ijz}$ and $c_{ijc}$ represent the stiffness and damping of the left and right gears of double helical gear $i$, respectively; $F_{L+R}$, $F_{L-R}$, $T_{L+R}$, and $T_{L-R}$ represent friction and friction torque of left and right gears of double helical gears, respectively; $T_i$ ($i=1, 2$) denotes the input and output torque; $\mu_{ij}$ denotes the friction coefficient and friction excitation.

The dynamic meshing force of double helical gears is affected greatly by the dynamic friction excitation, and the dynamic meshing force will affect the distribution of the oil film, then affect the friction coefficient and friction excitation. Therefore, considering the tribo-dynamic coupling effect and analysing the tribo-dynamic behaviour of double helical gears, accurate calculation results of sliding friction power loss and meshing efficiency can be obtained.

The solving steps of the tribo-dynamic coupling of double helical gear pair are:

1) Based on the mixed EHL theory, the comprehensive friction coefficient of gear pair is calculated, and the friction excitation of tooth surface is solved.

2) Considering the tribo-dynamic coupling effect, the vibration differential equations of double helical gear pair are established, and the dynamic meshing force is solved by Runge-Kutta method.

3) The friction coefficient based on mixed EHL is calculated by using the dynamic meshing force, and the new comprehensive friction coefficient and dynamic friction excitation are obtained.

4) A new dynamic meshing force is obtained by substituting the dynamic friction excitation into the tribo-dynamic model of double helical gear pair.

5) Repeat step 3) and 4) until the dynamic meshing force of gear pair meets the convergence condition $|\sum F_d(t) - F_{d-1}(t)|/|\sum F_d(t)| \leq 10^{-3}$.

2.3. Meshing efficiency calculation

The sliding friction power loss is calculated by “subsection method”. The contact line of each meshing teeth is segmented at time $t$, and the length of each small contact line is $\Delta L$. The power loss of each small contact line is calculated and the total power loss is obtained by superposition. At time $t$, the formula of sliding friction power losses on the contact line of the section $j$ of meshing teeth $i$ are

$$P_{ij-L}(t) = \mu_{ij}(t) F_{ij-L}(t) V_{Sij}(t)$$

$$P_{ij-R}(t) = \mu_{ij}(t) F_{ij-R}(t) V_{Sij}(t)$$

(12)

where $\mu_{ij}(t)$, $V_{Sij}$ represent the friction coefficient and relative sliding speed on contact line of the segment $j$ of the meshing teeth $i$; $F_{ij-L}(t)$, $F_{ij-R}(t)$ denote the normal forces on the contact line of the segment $j$ of meshing teeth $i$ of the left and right gear pairs, respectively.

At time $t$, the sliding friction power losses of the left and right gear pairs of double helical gear are

$$P_{i-L}(t) = \sum_{i=1}^{N} \sum_{j=1}^{n} P_{ij-L}(t)$$

$$P_{i-R}(t) = \sum_{i=1}^{N} \sum_{j=1}^{n} P_{ij-R}(t)$$

(13)

where $n=\text{round}(L_i(t)/\Delta L)$; $L_i(t)$ denote the contact line length of meshing teeth $i$; $N$ denotes the quantity of total contact line of double helical gears at time $t$.

The meshing efficiency of double helical gear pair is

$$\eta = 1 - \frac{P_{i-R}}{P_{in}} = 1 - \frac{P_{i-L} + P_{i-R}}{P_{in}}$$

(14)
3. Results and discussion

3.1. Tribo-dynamic behaviour analysis before and after coupling

The curves of friction coefficient on one side of double helical gear pair before and after tribo-
dynamic coupling are shown in Figure 2, and the dimensionless length $l$ of abscissa represents the
length along the tooth height direction. It can be seen from the figure that the friction coefficient after
coupling is slightly larger than that before coupling.

![Figure 2. Friction coefficient before and after coupling](image)

The single side dynamic contact force curves of double helical gear pair before and after coupling
are given in Figure 3. It shows that the dynamic meshing force fluctuates periodically after coupling,
but the meshing force before coupling is static contact force. The peak value of dynamic meshing
force after coupling appears at the meshing frequency 4250Hz and its frequency doubling.

![Figure 3. Meshing force of double helical gear pair: (a) time domain, (b) frequency domain](image)

According to the friction coefficient and meshing force before and after coupling, the total friction
and friction torque of the driving gear can be calculated combined with the friction arm. As shown in
Figure 4, the abscissa is dimensionless time $\tau$, which represents a single tooth meshing period,
$7.46\times10^{-4}$s. The friction and friction torque both increase after coupling.
3.2. Meshing efficiency before and after coupling
The meshing efficiency calculation results of double helical gear pair before and after coupling are shown in Figure 5. The meshing efficiency after coupling is slightly lower than that before coupling, and the average meshing efficiency is 99.21%. The normal contact force after coupling is dynamic meshing force, resulting in tooth surface friction and friction torque larger than before, and sliding friction power loss increases, so the meshing efficiency decreases.

3.3. Influencing factors analysis

3.3.1. Helix angle. When other parameters remain unchanged, the double helical gear pair meshing efficiency is calculated by the tribo-dynamic coupling model, and the variation range of helix angle $\beta$ is 20$\sim$30$^\circ$. As shown in Figure 6, with the increase of helix angle, although the average friction coefficient of tooth surface little changes, the relative sliding speed of tooth surface decreases greatly, so the total sliding friction power loss of gear pair decreases and the meshing efficiency increases.
3.3.2. Pressure angle. When other parameters remain unchanged, the variation range of pressure angle $\alpha_n$ is 20~25°, and the meshing efficiency of double helical gear pair is shown in Figure 7.

![Figure 7](image)

Figure 7. Meshing efficiency of double helical gear pair with different pressure angles

As shown in Figure 7, with the increase of pressure angle, the width of meshing plane decreases, the relative sliding speed decreases, and the friction coefficient of tooth surface decreases. Therefore, the sliding friction power loss decreases and the meshing efficiency increases.

3.3.3. Surface roughness. When other parameters remain unchanged, the variation range of surface roughness is 0.4~0.8μm, and the meshing efficiency is calculated. As shown in Figure 8, with the increase of surface roughness, the friction coefficient increases, the sliding friction power loss increases and the meshing efficiency decreases.

![Figure 8](image)

Figure 8. Meshing efficiency of double helical gear pair with different surface roughnesses

4. Conclusions
Considering the tribo-dynamic coupling effect, the meshing efficiency of double helical gears is calculated accurately. After coupling, the friction coefficient of tooth surface, friction and friction torque of driving gear increase, and the meshing efficiency of double helical gear pair decreases. The meshing efficiency increases with the increase of helix angle and pressure angle, and decreases with the increase of surface roughness.

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