Measurement-based quantum computation with superconducting charge qubits

Xiang-bin Wang,1,2,3 J. Q. You,1,4 and Franco Nori1,5

1Frontier Research System, The Institute of Physical and Chemical Research (RIKEN), Wako-shi 351-0198, Japan
2CREST, Japan Science and Technology Agency (JST), Kawaguchi, Saitama 332-0012, Japan
3Department of Physics, Tsinghua University, Beijing 100084, China
4Department of Physics and Surface Physics Laboratory (National Key Laboratory), Fudan University, Shanghai 200433, China
5Center for Theoretical Physics, Physics Department, Center for the Study of Complex Systems, University of Michigan, Ann Arbor, MI 48109-1040, USA

We present a robust method, based only on measurements, to produce superconducting cluster states. The measurement of the current of a few parallel Josephson-junction qubits realizes a novel type of quantum-state selector. Using this selector, one can produce various quantum entangled states and also realize a controlled-NOT gate without requiring an exact control of the interqubit interactions. In particular, cluster states for quantum computation could be produced with only single-qubit measurements.

Introduction.— Joint operations of two qubits are crucial for quantum information processing. Indeed, in principle, controlled-NOT (CNOT) gates and single-qubit unitary transforms are sufficient for quantum computing. However, implementing a CNOT gate via two interacting qubits has proven to be extremely difficult. This is a huge barrier to scalable quantum computing, which requires numerous CNOT gates.

To avoid the daunting difficulties related to controllable-interaction-based CNOT gates, several attempts have been made towards the goal of doing quantum computation without these and based on entangled states and measurements only [1, 2, 3, 4]. Indeed, it is possible to replace CNOT gates by quantum teleportation [4], where the only collective operation is a Bell measurement. However, a complete Bell measurement is also a very challenging task. Moreover, a probabilistic CNOT gate through Bell measurement, demonstrated experimentally with an optical set-up [2], cannot be directly used for practical large-scale quantum computation [2].

A very elegant alternative is given by one-way quantum computation using highly entangled states called cluster states [5]. This method has been proposed as a potential way to solve the very challenging problems faced by standard approaches to quantum computing. Cluster states must be first produced and stored as a “resource” to be consumed later for quantum computation through individual measurements only. The first important step here is to generate the so-called cluster states. Naively, one can achieve this goal through CNOT gates or conditional phase shift (CPhase) operations, and these operations could be obtained if we had good control of 2-qubit interactions. However, done this way, the advantages of cluster-state quantum computation become weak.

Cluster-state quantum computation has recently been demonstrated in a quantum optical experiment [6] through non-deterministic Bell measurements. However, an optical quantum computing set-up also has its own inevitable disadvantages. For instance, it is difficult to store an optical quantum state for future use. Also, due to the post-selection nature of the result, it is quite difficult to perform scalable optical quantum computing using cluster states. To overcome these drawbacks, one could either try to improve the technology of optical systems or consider another system that could produce cluster states: (1) without post-selection, and (2) that could be stored for future use. Here we consider this later approach using Josephson-junction (JJ) circuits [7, 8, 9, 10, 11, 12, 13]. We shall present a method to produce cluster states with superconducting qubits through the mechanism of quantum state selection. Remarkably, this method does not require any precise control of either the interqubit interactions or the timing. Since this approach can produce cluster states without post-selection, the cluster states can be stored as a resource for performing measurement-based quantum computing [5].

Quantum-state selection with charge qubits.— We consider a circuit with one large junction denoted by “0” and many parallel charge qubits made up of smaller junctions, as shown in Fig. 1. If the current across junction 0 is larger than a certain critical value \( I_{T0} \), it switches from the superconducting state to the normal (resistive) state. Usually, the current contributed from those smaller junctions is significantly less than \( I_{T0} \). With an appropriate bias current, the current from qubits 1 to \( k \) determines whether the large junction 0 will be switched to the resistive state with a nonzero voltage \( V \). The current is determined by the quantum state of those small JJ qubits in the circuit. Therefore, by monitoring the voltage \( V \), one can determine which type of state those JJ qubits have been projected to [10].

We shall use the following notation. \( \mathcal{L}_{ij} \): the loop connecting junctions \( i \) and \( j \), \( s_{ij} \): the region enclosed by loop \( \mathcal{L}_{ij} \), \( \Phi_{ij} \): magnetic flux threading the region \( s_{ij} \). In Fig. 1, \( I_b \) is the bias current, \( I_b \) and \( \Phi_{ij} \) can be tuned, and \( \Phi_0 \) is the flux quantum. There are two sets, \( S \) and \( S' \), of states for those observed qubits. A state in set \( S \) (\( S' \)) will (will not) cause junction 0 to switch from the superconducting to the normal state, given a certain bias current.
FIG. 1: (Color online) A Josephson-Junction circuit with one large junction “0” and many parallel charge qubits (Q1 ∼ Qk). Each qubit consists of two small junctions. The detailed structure of each qubit is schematically shown in Qk. Qubit Qi is decoupled when the applied flux Φ0i ≡ ∑j=0 i−1 Φj,j+1 is tuned to be zero. Based on this circuit, one can perform quantum state selection on any subset of qubits. Also, one can generate an entangled pair state on any two qubits and produce a CNOT gate on any two qubits with one ancilla. Moreover, one can produce a cluster state for one-way quantum computation.

Ib and external fields. Thus, we can conclude whether the quantum state of those observed qubits belongs to the set $S$ or $\bar{S}$, by just monitoring the voltage $V$. This process can be regarded as a type of incomplete measurement, a measurement which only projects the observed system to a subspace rather than a single state. If $V = 0$, junction “0” must still be in the superconducting state; therefore, the projected quantum state $\Psi_{\cdot \cdot \cdot k}$ (which belongs to $\bar{S}$) for qubits $\{1, 2, \cdots, k\}$ must satisfy

$$|I_b + \langle \Psi_{\cdot \cdot \cdot k} | \hat{I} | \Psi_{\cdot \cdot \cdot k} \rangle| < I_{T0}. \quad (1)$$

Here $\hat{I}$ is the current operator for those qubits. If $V \neq 0$ is detected, the projected state (now belonging to $S$) must satisfy

$$|I_b + \langle \Psi_{\cdot \cdot \cdot k} | \hat{I} | \Psi_{\cdot \cdot \cdot k} \rangle| > I_{T0}. \quad (2)$$

These two conditions realize a quantum-state selector. Let us now assume an initial state $\Psi_{\cdot \cdot \cdot k} = \alpha|a\rangle + \beta|b\rangle$. Suppose $|a\rangle \in S$ and $|b\rangle \in \bar{S}$, i.e., state $|a\rangle$ and $|b\rangle$ satisfy the conditions $I_b + \langle a | \hat{I} | a \rangle < I_{T0}$ and $I_b + \langle b | \hat{I} | b \rangle > I_{T0}$. With the bias current $I_b$, the quantum state for $k$ qubits must be $|a\rangle$ if $V = 0$. This fact helps to prepare various types of entangled states including cluster states, as we show below.

If we set $I_b$ to be significantly smaller than the critical current, the total current across junction 0 will be very small and thus $V = 0$ for whatever state of the small JJ qubits in the circuit. Thus, if the bias current $I_b$ is very small, there is no measurement. But if a bias current $I_b$ slightly below $I_{T0}$ is applied, the state of those coupled qubits determines whether the large junction 0 switches to a normal state. Therefore, the state of those coupled qubits can be measured via $V$, after applying an appropriate $I_b$.

Actually, we can also make the quantum state selection to any subset of those qubits in the circuit. As we shall show, by applying appropriate flux in each region, we can select a subset of qubits which provide a negligible contribution to the total current. This means we can decouple some qubits by tuning the external field.

Consider loop $L_{0j}$ with flux $\Phi_{0j}$, which contains the large junctions 0 and the charge qubit $j$. The following constraint holds for the phase operators across the junctions 0 and those in qubit $j$: $\hat{\phi}_{A_j} - \hat{\phi}_{B_j} - \gamma + 2\pi \Phi_{0j}/\Phi_0 = 0$ and $\Phi_{0j} = \sum_{i=0}^{j-1} \Phi_{i,i+1}$, where $\Phi_{i,i+1}$ is the flux threading the region $s_{i,i+1}$ and $\gamma$, $\hat{\phi}$’s the JJ phase drops (see Fig. 1). The total current across junction 0 is given by

$$I_{c0} \sin \hat{\gamma} = 2 \sum_{j=1}^{k} \sin \left( \frac{\pi \Phi_{cj}}{\Phi_0} - \frac{1}{2} \gamma \right) I_{cj} \cos \hat{\phi}_j \quad (3)$$

and $\hat{\phi}_j = (\phi_{A_j} + \phi_{B_j})/2$, $I_{cj}$ is the critical current of the junction in qubit $j$. Since junction 0 is large, $(\hat{\gamma})$ must be small. Therefore we can generalize Eq. (19) in Ref. [12] (from two to $k$ qubits) for the total current operator $\hat{I}$ of all parallel qubits:

$$\hat{I} = \sum_{j=1}^{k} \left[ \sin \left( \frac{\pi \Phi_{cj}}{\Phi_0} \right) I_{cj} \sigma_z^j - C \right]$$

$$- \frac{1}{2I_{c0}} \sum_{j>i=1}^{k} \sin \left[ \frac{\pi (\Phi_{0i} + \Phi_{0j})}{\Phi_0} \right] I_{ci} I_{cj} \sigma_z^i \sigma_z^j. \quad (4)$$

Here $C = (1/4I_{c0}) \sum_{j=1}^{k} \sin (2\pi \Phi_{0j}/\Phi_0) I_{c0}^2$. To decouple any qubit $i$, external fields must be tuned so that $\Phi_{0i} = 0$.

Producing two-qubit entanglement. — We first consider the simplest application for generating two-qubit entangled states, e.g., $|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle \mp |10\rangle)$. Here, $|0\rangle$, $|1\rangle$ represent the state of 0, 1 extra Cooper pair, respectively. Consider now opposite external magnetic fields in regions $s_{01}$ and $s_{23}$, i.e., $\Phi_{01} = -\Phi_{23} = 0$. Therefore, given any $i$, $\Phi_{0i} = 0$ if $i > 2$, $\Phi_{0i} = \Phi_{02}$/2 if $i = 1$ or $i = 2$. Only qubits 1 and 2 would contribute to the total current. Initially, the state of qubits 1 and 2 is set to be

$$|\Psi_{1,2} = |0\rangle|0\rangle = \frac{1}{\sqrt{2}} (|+\rangle |+\rangle - |\pm\rangle |\mp\rangle). \quad (5)$$

Here, the first (second) state is for qubit 1 (2). Note that states $|\pm\rangle = \frac{1}{\sqrt{2}} (|0\rangle \pm |1\rangle)$ are eigenstates of $\sigma_z$ with $\sigma_z |\pm\rangle = \pm |\pm\rangle$. We then bias the large junction and monitor $V$. Here we choose $I_b = I_{T0} - (I_{c1} + I_{c2})/2$. If $V = 0$, the state $|+\rangle |+\rangle$ is ruled out and the state must have been collapsed to $|\pm\rangle |\mp\rangle + |\mp\rangle |\pm\rangle$. We then reverse both, the external fields and bias current, to their opposite directions and again monitor $V$. If $V = 0$ again,
we conclude that we have now prepared the entangled state $|\psi^\pm\rangle$. The success probability is 1/3. If at any stage we find $V \neq 0$, the protocol fails and we need to redo it from the beginning. If we had initially used $|0\rangle|1\rangle$, after the above quantum state selection, we would be obtaining $|\psi^+\rangle$. A similar idea has been proposed to prepare a two-qubit entangled state with weak continuous measurement \[14\]. Another proposal to generate entangled states with controllable interactions was raised very recently \[2\].

Four-qubit cluster state.— To produce a nontrivial cluster state (with four qubits), a CNOT gate must be applied on two entangled pairs. Remarkably, such type of CNOT gate can also be done with our quantum state selector. Here we follow the approach proposed by Pittman et al \[2\]. Consider three qubits. Qubits 1 and 3 are the control and target qubits, respectively, and qubit 2 is the ancilla. Initially, qubit 2 is in state $|+\rangle$. Without any loss of generality, we can consider the following initial state for the three qubits:

$$|\psi_0\rangle = |i\rangle|+\rangle|j\rangle$$  \hspace{1cm} (6)

where the first state is for the control qubit, the second and third are for the ancilla and target qubits, respectively, and $i$, $j$ can be either 0 or 1. A CNOT gate is obtained by the following operations: (1) Measure the parity value of the ancilla and control qubits in the $|0,1\rangle$ basis (parity=1 for the subspace $\{|0\rangle, |1\rangle\}$); (2) measure the parity value of the ancilla and target qubits in the $|\pm\rangle$ basis (parity=1 for space $\{|+\rangle, |-\rangle\}$); (3) perform a Hadamard transform to the ancilla; (4) measure the ancilla in the $|\pm\rangle$ basis; (5) perform an individual unitary transformation on the target qubit according to the previous measurement result of the ancilla. In particular, if we obtain 1 in both parity measurements in the first two steps, the following state was prepared (up to a phase factor which can be removed by an individual phase-shift operation):

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}(|i\rangle|+\rangle|i\oplus j\rangle + |j\rangle|-\rangle|1\oplus i\oplus j\rangle).$$  \hspace{1cm} (7)

Thus, if the state $|+\rangle$ is detected for the ancilla, a CNOT gate was applied to the control and target qubits; if state $|-\rangle$ is detected for the ancilla, a CNOT gate is applied after flipping the target qubit in the $|0,1\rangle$ basis.

The circuit in Fig. 1 (with five qubits) can generate a four-qubit cluster state (qubits 4 and 5 are not explicitly drawn in Fig. 1). Our goal is to prepare a cluster state on qubits 1, 2, 4 and 5. To do so, the external field should only be applied in region $s_{34}$; so that only qubits 4 and 5 can contribute to the total current, while qubits 1, 2 and 3 are all decoupled. The detailed procedure is now given:

1. Set $\Phi_{01} = -\Phi_{23} = \Phi_{02} = \Phi_{12} = 0$. Qubits 3, 4, and 5 are now decoupled. Apply an appropriate bias current and observe $V$. Then, use opposite fields and bias current and observe $V$ again. If $V$ is always 0, the quantum state selection process has functioned successfully, and the state of qubits 1 and 2 has been projected to $|\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$. (2) Similarly, the pair $|\psi^-\rangle$ state can also be prepared on qubits 4 and 5. To do so, the external field should only be applied in region $s_{34}$; so that only qubits 4 and 5 can contribute to the total current, while qubits 1, 2 and 3 are all decoupled. (3) Make a CNOT gate on qubits 2 and 4. A cluster state of the form

$$|\psi_c\rangle = \frac{1}{\sqrt{2}}(|01\rangle|\phi^-\rangle - |10\rangle|\psi^-\rangle)$$  \hspace{1cm} (8)

is prepared for qubits 1, 2, 4 and 5, and $|\phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$. Note that by individual unitary transforms this state is equivalent to $\frac{1}{2}(|0000\rangle + |0011\rangle + |1100\rangle - |1111\rangle)$, which is used in Ref. \[15\]. To make the CNOT gate, the
parities of qubits 2 and 3 in the $|0,1\rangle$ basis and qubits 3 and 4 in the $|\pm\rangle$ basis must be measured, as stated earlier. To measure the parity of qubits 2 and 3, qubits 1, 4 and 5 should be decoupled by setting $\Phi_{12} = -\Phi_{34} = \Phi_{0}/2$ and $\Phi_{01} = \Phi_{23} = \Phi_{45} = 0$. Moreover, by setting $\Phi_{23} = -\Phi_{45}$ and $\Phi_{01} = \Phi_{12} = \Phi_{34} = 0$, qubits 1, 2 and 5 are decoupled and the parity of qubits 3 and 4 can be measured through the quantum state selector. If $V = 0$ in both parity measurements, we have prepared the following state for the five qubits:

$$|\Psi_{1-5}\rangle = \frac{1}{\sqrt{2}} \left[ (|+\rangle)_3 (|01\rangle_{1,2}|\phi^-\rangle_{4,5} - |10\rangle_{1,2}|\phi^-\rangle_{4,5}) + |\rangle_3 (|01\rangle_{1,2}|\psi^-\rangle_{4,5} - |10\rangle_{1,2}|\phi^-\rangle_{4,5}) \right]$$  

(9)

A cluster state on qubits 1, 2, 4 and 5 is readily obtained after measuring qubit 3 in the $|\pm\rangle$ basis.

**Efficiency.** — We have shown above that cluster states can be produced probabilistically using only measurements. Our pre-selection result here is totally different from the post selection result or probabilistic quantum computing in an optical set-up. In an optical set-up with post-selection, the state is destroyed after a measurement. However, in our approach here, one can first verify the state via a measurement (the state is still there after the measurement) performed at a convenient time and then do the computing when needed.

Suppose we want a large cluster state containing $N$ qubits. Such a large cluster state is sufficient to treat a complex task, e.g., factoring a huge number larger than $2^{aN}$, where $a \sim 0.1$. In our approach, we can make this in a concatenated manner: We first produce many small identical qubits blocks (pairs), then generate a larger block by combining two small qubits blocks via a CPhase or CNOT gate, where the target is from one small block and the control is from another small block. After each successful combination, the size of the state is doubled. If the CPhase or CNOT gates are continuously successful $n$ times, we have constructed a cluster state of size $2^n$. To generate a cluster state containing more than $N$ qubits, we need $n = \log_2 N$. Therefore, in our approach the joint probability to produce a cluster state containing more than $N$ qubits is a linear function of $N^{-1}$. This is only polynomially small. Now consider generating such a state using an optical set-up with post-selection: One first produces $\frac{1}{4}N$ photon pairs and then generates entanglement using $N$ beam-splitters. If all beam-splitters have functioned correctly (e.g., one photon on each side of the outside ports of a beam-splitter), the expected cluster state is produced. In such a way, the final success probability is a tiny $2^{-N}$ (exponentially small). Note that in an optical set-up with post-selection, one cannot iteratively make a cluster state block by block, because there is no way to verify the state of a certain individual block without destroying it. To overcome this drawback, one can either develop optical technologies so that a photon can be measured without damage and can be stored, or use our approach with solid state qubits.

**Concluding remarks.** — We have shown how to make the cluster states through a mechanism of quantum-state selection with charge qubits. Obviously, besides the four-qubit cluster state, our method can also be applied to generate Multi-qubit cluster states since our method is based on a scalable circuit. Moreover, our method also applies for other solid-state systems, e.g., quantum dot charged qubits [14]. It is important to stress that, in our scheme, we do not have to control the interqubit interaction and their timing, e.g., there is no need to worry about small errors in the control of the external fields, provided that they do not violate the inequalities [11] and [12]. Our result is also robust with respect to small errors in the control of the bias current $I_b$. Besides cluster states, the powerful quantum-state selector method presented here can also be used to produce many other types of entangled states, including GHZ states and $W$ states.

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