Folding model study of the charge-exchange scattering to the isobaric analog state and implication for the nuclear symmetry energy

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Abstract. The Fermi transition ($\Delta L = \Delta S = 0$ and $\Delta T = 1$) between the nuclear isobaric analog states (IAS), induced by the charge-exchange ($p, n$) or ($^3$He, $t$) reaction, can be considered as “elastic” scattering of proton or $^3$He by the isovector term of the optical potential (OP) that flips the projectile isospin. The accurately measured ($p, n$) or ($^3$He, $t$) scattering cross-section to the IAS can be used, therefore, to probe the isospin dependence of the proton or $^3$He optical potential. Within the folding model, the isovector part of the OP is determined exclusively by the neutron-proton difference in the nuclear densities and the isospin dependence of the effective nucleon-nucleon (NN) interaction. Because the isovector coupling explicitly links the isovector part of the proton or $^3$He optical potential to the cross section of the charge-exchange ($p, n$) or ($^3$He, $t$) scattering to the IAS, the isospin dependence of the effective (in-medium) NN interaction can be well tested in the folding model analysis of these charge-exchange reactions. On the other hand, the same isospin- and density dependent NN interaction can also be used in a Hartree-Fock calculation of asymmetric nuclear matter, to estimate the nuclear matter energy and its asymmetry part (the nuclear symmetry energy). As a result, the fine-tuning of the isospin dependence of the effective NN interaction against the measured ($p, n$) or ($^3$He, $t$) cross sections should allow us to make some realistic prediction of the nuclear symmetry energy and its density dependence.

1 Introduction

The charge-exchange ($p, n$) and ($^3$He, $t$) reactions are the most suitable tool to excite the isobaric analog state (IAS) of the target nucleus. IAS has about the same structure as that of the target except for the replacement of a neutron by a proton and, hence, differs in energy approximately by the Coulomb energy of the added proton. Within the isospin symmetry, the two isobaric analog states are just two members of the isospin multiplet which differ only in the orientation of the isospin $T$. The similar

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structures of the initial and final states of the \((p, n)\) or \((^3\text{He}, t)\) reaction makes these reactions very much like the “elastic” scattering in which the isospin of the incident proton or \(^3\text{He}\) is flipped \[1,2,3\]. In such a scenario for the charge-exchange scattering to the IAS, the isospin-flip (elastic) scattering is naturally caused by the isovector term of the optical potential (OP) that is directly proportional to the neutron-proton asymmetry of the target nucleus, \(\varepsilon = (N - Z)/A\). The empirical isovector term of the proton-nucleus or \(^3\text{He}\)-nucleus OP in the Woods-Saxon form has been used by Satchler et al. \[1,2\] some 50 years ago as the charge-exchange form factor to study the charge-exchange \((p, n)\) or \((^3\text{He}, t)\) scattering to the IAS within the distorted wave Born approximation (DWBA).

Given the same isospin \(t = 1/2\) of nucleon and \(^3\text{He}\), the central nucleon-nucleus or \(^3\text{He}\)-nucleus OP for the elastic scattering on a nonzero-isospin target can be written in the following Lane form \[4\]

\[
U(R) = U_0(R) + 4U_1(R) \frac{t_T}{aA},
\]

where \(t\) is the isospin of the projectile and \(T\) is that of the target with mass number \(A\), and \(a = 1\) and 3 for nucleon and \(^3\text{He}\), respectively. The second term of Eq. (1) is the symmetry term of the OP, and \(U_1\) is now known as the Lane potential that contributes to both the elastic scattering and charge-exchange transition to the IAS \[3\]. The knowledge of \(U_1\) is of fundamental interest for different studies of the nuclear phenomena in which neutrons and protons participate differently (isovector modes).

In the nucleon-nucleus case, the relative contribution by the Lane potential \(U_1\) to the elastic scattering cross section has been shown to be quite small and amounts only a few percent for a neutron-rich target \[5,7\]. Nevertheless, the Fermi-type \((\Delta J^\pi = 0^+, \Delta T = 1)\) transition strength of the charge-exchange \((p, n)\) reaction to the IAS is determined entirely by \(U_1\). Therefore, the accurately measured data of the \((p, n)\) scattering to the IAS have been used successfully in the folding model analysis \[8,9\] to probe the isospin dependent part of the folded proton-nucleus OP. It is complimentary to note that the volume and surface strengths of the symmetry term of the nuclear binding energy have been determined quite accurately from a systematic analysis of the excitation energies of the IAS \[10\].

The nucleon OP has been studied over the years and there are several “global” sets of the OP parameters deduced from the extensive optical model (OM) analyses of nucleon elastic scattering, like that by Becchetti and Greenlees \[11\], the CH89 global OP \[12\], and the systematics by Koning and Delaroche \[13\]. Although parametrized in the empirical Woods-Saxon form, these global systematics are very valuable in predicting the nucleon OP when elastic scattering data are not available or cannot be measured which is the case for the unstable, dripline nuclei. Given a large neutron excess in the unstable neutron-rich nuclei, it is important to know as accurate as possible the isospin dependence of the nucleon OP before using it in various studies of nuclear reactions and nuclear astrophysics. Because the high-quality \((p, n)\) data are not available for a wide range of target masses and proton energies, the isovector term of the nucleon OP has been deduced \[1,11,12,13\] mainly from the OM studies of elastic proton and neutron scattering from the same target and at about the same energy, where the second term of Eq. (1) has the same strength, but opposite signs for proton and neutron. Only in few cases the Lane potential \(U_1\) has been deduced from the DWBA studies of the charge-exchange \((p, n)\) scattering to the IAS \[14,15\].

Regarding the isospin dependence of the \(^3\text{He}\)-nucleus OP, it has been very little investigated. Even in a recent version of the global OP for \(^3\text{He}\) and triton \[16\] the real OP contains no isovector term like that of Eq. (1), and the purely isoscalar parametrization of the real OP seems to deliver rather good OM description of the data, with a slight dependence on the neutron-proton asymmetry \(\varepsilon\) of the surface.
imaginary potential. Unlike the nucleon-nucleus case, the Lane consistency of the $^3$He-nucleus OP has not been well established, and the measured data of the charge-exchange ($^3$He,t) scattering to the IAS have been mainly studied in the DWBA with the charge-exchange form factor given by folding the isospin-dependent part of an effective $^3$He-nucleon interaction with the nuclear transition density for the IAS excitation [17,18]. It is not straightforward, however, to link the isospin-dependence of the effective $^3$He-nucleon interaction to the isospin-dependence of the in-medium nucleon-nucleon (NN) interaction, the most vital input for a many-body study of asymmetric nuclear matter [9,19]. In general, both the single-folding calculation of the nucleon OP [9] and double-folding calculation of the nucleus-nucleus OP [19] give rise naturally to a non-zero isovector term of the OP (the microscopic prototype of the Lane potential) when both the projectile and target have non-zero isospins. Given the success of the single-folded nucleon OP in the description of the nucleon elastic scattering and charge-exchange ($p,n$) scattering to the IAS [9], the double-folded $^3$He-nucleus OP is expected to give also a reasonable description of the elastic $^3$He scattering and charge-exchange ($^3$He,t) scattering to the IAS.

Within the DWBA or coupled-channel (CC) analysis, the folded (Lane consistent) nucleon-nucleus or $^3$He-nucleus OP serves as a direct link between the isospin dependence of the in-medium NN interaction and the charge-exchange ($p,n$) or ($^3$He,t) scattering to the IAS, so that accurately measured charge-exchange data can be used to probe the isospin dependence of the NN interaction. On the other hand, within a many-body calculation of nuclear matter (NM), the asymmetry of the equation of state (EOS) of the NM depends entirely on the density- and isospin dependence of the in-medium NN interaction [19,20]. Such an asymmetry is determined by the nuclear symmetry energy $S(\rho)$ defined in terms of the NM energy $E(\rho,\delta)$ as

$$E(\rho, \delta) = E(\rho, 0) + S(\rho)\delta^2$$

where $\delta = (\rho_n - \rho_p)/\rho$ is the neutron-proton asymmetry parameter (the NM limit of the neutron-proton asymmetry $\varepsilon$ in nuclei). The knowledge about the nuclear EOS is well known to be vital for the understanding of the dynamics of supernova explosion and neutron star formation [21,22,23,24]. The nuclear symmetry energy determined at the saturation density, $E_{\text{sym}} = S(\rho = \rho_0 \approx 0.17 \text{ fm}^{-3})$, is also known in the literature as the symmetry energy or symmetry coefficient. Although numerous nuclear many-body calculations have predicted $E_{\text{sym}}$ to be around 30 MeV [19,20,25,26], a direct experimental determination of $E_{\text{sym}}$ still remains a challenge. Moreover, the knowledge about the density dependence of $S(\rho)$ is very important in modeling a realistic nuclear EOS and it has been, therefore, the main subject of many nuclear structure and reaction studies. The main approach to probe $S(\rho)$ associated with a given in-medium NN interaction is to test this interaction in the simulation of heavy-ion (HI) collisions using transport and/or statistical models [24,27,28,39,41,32,33] or in the structure studies of nuclei with large neutron excess [10,34,35,36,37,38,39,40,41,12]. Some conclusion on the low- and high-density behavior of $S(\rho)$ is then made based on the physics constraints implied by such studies.

In about the same way, our recent folding model studies of the ($p,n$) scattering to the IAS [39] were aimed to gain some information on the nuclear symmetry energy. The isospin dependence of the chosen in-medium NN interaction, fine-tuned to the best fit of the ($p,n$) data, has been shown [9] to give a symmetry coefficient $E_{\text{sym}}$ very close to the empirical values deduced from other studies. However, the ($p,n$) data can be used to probe the isospin dependence of the in-medium NN interaction at sub-saturation nuclear densities only, i.e., $\rho < \rho_0$ where $\rho_0$ is the saturation density of the symmetric nuclear matter. Although similar in the isospin-coupling scheme, the high-quality data of the charge-exchange ($^3$He,t) scattering to the IAS could be in general sensitive to higher nuclear densities ($\rho \gtrsim \rho_0$) formed in the spatial overlap of
the $^3$He projectile with the target nucleus. This is an essential feature of the folding model analysis of elastic nucleus-nucleus scattering that the nucleus-nucleus OP at small internuclear radii is determined by the effective NN interaction at high nuclear medium densities (see a recent review in Ref. [33]).

In the present work, we discuss the results of a consistent folding model analysis of both the $(p, n)$ and $(^3\text{He}, t)$ scattering to the IAS, which allowed us not only to validate the conclusion made earlier in the study of the $(p, n)$ scattering [9] for the nuclear symmetry energy but also to probe the slope of $S(\rho)$ with the increasing NM density.

2 Isospin coupling and Lane equations for the charge-exchange scattering to the IAS

2.1 General formalism

We give here a brief introduction to the coupled-channel formalism for the charge-exchange $(p, n)$ or $(^3\text{He}, t)$ scattering to the IAS, and the reader is referred to Satchler’s book [5] for more technical details. Let us consider a given isospin multiplet with fixed values of isospin $t$ for the projectile and $T$ for the target. Then, the isospin projections are $T_z = (N - Z)/2$ and $\tilde{T}_z = T_z - 1$ for the target nucleus $A$ and isobaric analog nucleus $\tilde{A}$, respectively, and the charge-exchange transition between $A$ and $\tilde{A}$ induced by proton or $^3$He is like elastic scattering except for a reorientation of both isobars $t$ and $T$. We further denote, in the isospin representation, state formed by adding a neutron or triton to $\tilde{A}$ as $|\tilde{a}A\rangle$. The transition matrix elements of the isovector part of the optical potential [1] for elastic scattering can be obtained [5] as

\[
\begin{align*}
\langle aA | U_1(R) & \frac{t.T}{aA} | aA \rangle = -\frac{2}{a\tilde{A}} T_z U_1(R), \\
\langle \tilde{a}\tilde{A} | U_1(R) & \frac{t.T}{a\tilde{A}} | \tilde{a}\tilde{A} \rangle = \frac{2}{a\tilde{A}} (T_z - 1) U_1(R). 
\end{align*}
\]

(3)

where $R$ is the radial separation between the projectile and target. Similarly, the transition matrix element or charge-exchange form factor (FF) for the charge-exchange $(p, n)$ or $(^3\text{He}, t)$ scattering to the IAS is obtained as

\[
\langle \tilde{a}\tilde{A} | U_1(R) \frac{t.T}{a\tilde{A}} | aA \rangle \equiv F_{\text{cx}}(R) = \frac{2}{a\tilde{A}} \sqrt{2T_z} U_1(R).
\]

(4)

In the two-channel approximation for the charge-exchange scattering to the IAS, the total wave function is written as

\[
\Psi = |aA\rangle \chi_{aA}(R) + |\tilde{a}\tilde{A}\rangle \chi_{\tilde{a}\tilde{A}}(R),
\]

(5)

where the waves $\chi(R)$ describe the relative motion of the scattering system. Then, the elastic and charge-exchange scattering cross sections are readily obtained from the solutions of the following (coupled-channel) Lane equations [5]:

\[
\begin{align*}
[K_a + U_a(R) - E_a] \chi_{aA}(R) &= -F_{\text{cx}}(R) \chi_{\tilde{a}\tilde{A}}(R), \\
[K_{\tilde{a}} + U_{\tilde{a}}(R) - E_{\tilde{a}}] \chi_{\tilde{a}\tilde{A}}(R) &= -F_{\text{cx}}(R) \chi_{aA}(R).
\end{align*}
\]

(6)

(7)

Here $K_a(\tilde{a})$ and $E_a(\tilde{a})$ are the kinetic-energy operators and center-of-mass energies of the $a + A$ and $\tilde{a} + \tilde{A}$ partitions. The OP in the entrance $(a + A)$ and exit $(\tilde{a} + \tilde{A})$
channels are determined explicitly through the isoscalar \((U_0)\) and isovector \((U_1)\) parts of the optical potential (11) as
\[
U_a(R) = U_0(R) - \frac{2}{aA} T_z U_1(R), \tag{8}
\]
\[
U_\tilde{a}(R) = U_0(R) + \frac{2}{aA} (T_z - 1) U_1(R). \tag{9}
\]
In the actual CC calculation, \(U_a\) and \(U_\tilde{a}\) are added by the corresponding spin-orbital and Coulomb potentials (the Coulomb term in the exit channel is nonzero only if \(\tilde{a}\) is triton). Since the energies of the isobar analog states are separated approximately by the Coulomb displacement energy, the charge-exchange transition between them has a nonzero \(Q\) value. To account for this effect, the isoscalar \(U_0\) and isovector \(U_1\) potentials used to construct \(F_{cx}(R)\) are evaluated from the proton or \(^3\)He optical potential at the energy of \(E = E_{lab} - Q/2 [2]\), and those used to construct \(U_\tilde{a}(R)\) are evaluated from the neutron or triton OP at the energy \(E = E_{lab} - Q\).

For the \((p,n)\) scattering to the IAS, the existing global parameters for the nucleon OP [11,12,13] can be used to construct the isoscalar \(U_0\) and isovector \(U_1\) parts of the OP used in the Lane equations (6)-(7) [9]. Unlike the nucleon-nucleus case, the isospin dependence of the \(^3\)He-nucleus OP has been investigated in terms of an effective \(^3\)He-nucleon interaction only [17,18]. Moreover, the existing elastic \(^3\)He or triton scattering data were shown to be well reproduced by a (phenomenological) global OP [16], whose real part is purely isoscalar and imaginary part contains a weak dependence on the asymmetry parameter \(\varepsilon\) at the surface. Therefore, the folding model is probably the only consistent microscopic approach to evaluate FF using the same in-medium NN interaction for both \((p,n)\) and \((^3\text{He},t)\) charge-exchange scattering to the IAS.

### 2.2 Folding model

In general, the central nucleon-nucleus or nucleus-nucleus potential \(U\) is evaluated by the single-folding [6,7] or double-folding [19,43,44,45] approach as the following Hartree-Fock-type potential
\[
U = \sum \langle ij|v_D|ij\rangle + \langle ij|v_{EX}|ji\rangle, \tag{10}
\]
where single-folding or double-folding summation is performed over all nucleon states of the target \((j \in A)\) or of both the target and projectile \((i \in a, j \in A)\), respectively. \(v_D\) and \(v_{EX}\) are the direct and exchange parts of the (effective) NN interaction between the projectile nucleon \(i\) and target nucleon \(j\). The antisymmetrization of the nucleon-nucleus or nucleus-nucleus system is done by taking into account the knock-on exchange effects. To separate the isovector part of \(U\) which gives rise to the Lane potential, one needs to make explicit the isospin degrees of freedom. Namely, the following spin-isospin decomposition of the (energy- and density dependent) NN interaction is used
\[
v_{D(EX)}(E, \rho, s) = v_{00}^{D(EX)}(E, \rho, s) + v_{10}^{D(EX)}(E, \rho, s)(\sigma\sigma') + v_{01}^{D(EX)}(E, \rho, s)(\tau\tau') + v_{11}^{D(EX)}(E, \rho, s)(\sigma\sigma')(\tau\tau'), \tag{11}
\]
Using the explicit proton ($\rho_p$) and neutron ($\rho_n$) densities in the folding input, the nucleon-nucleus or nucleus-nucleus optical potential \( \rho \) can be obtained explicitly \( ^{10} \) in terms of the isoscalar (\( U_{IS} \)) and isovector (\( U_{IV} \)) parts as

\[
U(E, R) = U_{IS}(E, R) + U_{IV}(E, R),
\]

where - sign pertains to the proton or \(^3\text{He} \) optical potential (used for the entrance channel) and + sign to the neutron or triton OP (used for the exit channel). Each term in Eq. (12) consists of the corresponding direct and exchange potentials.

In the nucleon-nucleus case, the isoscalar and isovector potentials are given by the single-folding approach \( ^6 \) as

\[
U_{IS}(E, R) = \int \left( [\rho_n(r) + \rho_p(r)]v_{00}^D(E, \rho, s) + [\rho_n(R, r) + \rho_p(R, r)]v_{00}^{EX}(E, \rho, s)j_0(k(E, R)s) \right) d^3r,
\]

\[
U_{IV}(E, R) = \int \left( [\rho_n(r) - \rho_p(r)]v_{01}^D(E, \rho, s) + [\rho_n(R, r) - \rho_p(R, r)]v_{01}^{EX}(E, \rho, s)j_0(k(R)s) \right) d^3r,
\]

where \( s = r - R \), \( \rho(r, r') \) is the one-body density matrix of the target, with \( \rho(r) \equiv \rho(|r, r|) \), \( j_0(x) \) is the zero-order spherical Bessel function, and the local relative motion momentum \( k(R) \) is determined from

\[
k^2(E, R) = \frac{2\mu}{\hbar^2}[E_{\text{c.m.}} - V(E, R) - V_C(R)].
\]

Here, \( \mu \) is the reduced mass, \( V(E, R) \) and \( V_C(R) \) are, respectively, the real central nuclear and Coulomb parts of the OP (\( V_C(R) = 0 \) in the neutron-nucleus case). More details of the single-folding approach to evaluate \( U_{IS} \) and \( U_{IV} \) can be found in Ref. \( ^6 \).

If the effective NN interaction is complex then \( U_{IS(IV)} \) should be treated explicitly in terms of the real \( V_{IS(IV)} \) and imaginary \( W_{IS(IV)} \) parts as

\[
U_{IS(IV)}(E, R) = V_{IS(IV)}(E, R) + iW_{IS(IV)}(E, R)
\]

In the \(^3\text{He} \)-nucleus or triton-nucleus case, the isoscalar and isovector potentials are given in a similar manner by the double-folding approach \( ^{19} \) as

\[
\begin{align*}
U_{IS}(E, R) &= \int \int [\rho_1(r_1)\rho_2(r_2)v_{00}^D(E, \rho, s) + \rho_1(r_1, r_1 + s) \times \rho_2(r_2, r_2 - s)v_{00}^{EX}(E, \rho, s)j_0(k(E, R)s/M)]d^3r_1d^3r_2, \\
U_{IV}(E, R) &= \int \int [\Delta\rho_1(r_1)\Delta\rho_2(r_2)v_{01}^D(E, \rho, s) + \Delta\rho_1(r_1, r_1 + s) \times \Delta\rho_2(r_2, r_2 - s)v_{01}^{EX}(E, \rho, s)j_0(k(E, R)s/M)]d^3r_1d^3r_2.
\end{align*}
\]

Here, \( \rho_i = \rho_{in} + \rho_{ip} \) and \( \Delta\rho_i = \rho_{in} - \rho_{ip} \), \( s = r_2 - r_1 + R \) and \( M = aA/(a + A) \). The local relative motion momentum \( k(E, R) \) is determined by the same formula \( ^{15} \), but with \( \mu \), \( V(E, R) \) and \( V_C(R) \) determined consistently for the \(^3\text{He} \) or triton OP.

The charge-exchange FF for both the \( (p, n) \) and \( (^3\text{He}, t) \) scattering to the IAS is readily obtained in terms of the folded isovector potential \( U_{IV} \) as

\[
F_{cx}(R) = \frac{2}{aA}\sqrt{2T_z}U_1(R) = \sqrt{\frac{2}{T_z}}U_{IV}(R)
\]

\( ^{18} \)
All the optical model (OM) calculation of elastic scattering and CC calculation of the charge-exchange scattering to the IAS, with the folded OP and charge-exchange FF, were done using external-potential option of the CC code ECIS97 written by Raynal [49].

Given the isovector folded potentials (14) and (17) determined entirely by the difference $\Delta \rho$ between the neutron and proton densities, it is necessary to have the nuclear densities determined as accurate as possible for a realistic folding model prediction of the charge-exchange form factor [15]. In the present work, we have used for the $^{90}$Zr and $^{208}$Pb targets the empirical neutron and proton densities deduced from the high-precision elastic proton scattering at 800 MeV by Ray et al. [50]. For the $^{48}$Ca and $^{120}$Sn targets the microscopic nuclear densities given by the Hartree-Fock-Bogoljubov approach [52] have been used. We have used neutron and proton densities of $^{14}$C target given by the independent particle model [51], which generates realistic wave function for each single-particle orbital using an appropriate Woods-Saxon potential for the bound state problem. The neutron and proton densities of $^3$He and triton given by the microscopic three-body calculation [53] using Argonne NN potential have been used in the double-folding calculation [15] and (17).

2.3 Isospin- and density dependent CDM3Y6 interaction

Together with the nuclear densities, an appropriately chosen effective (energy- and density dependent) NN interaction $v^{\text{D(EX)}}_{00(01)}$ is the most important input for the folding calculations (13)-(14) and (15)-(17). We have used in the present study the density dependent CDM3Y6 interaction [44] that is based on the M3Y interaction deduced from the G-matrix elements of the Paris NN potential [46]. The density dependence of the isoscalar (IS) part of the CDM3Y6 interaction was introduced earlier in Ref. [44] and its parameters have been carefully tested in numerous folding model analyses of the elastic, refractive nucleus-nucleus and $\alpha$-nucleus scattering. Because the isospin dependent term $v_{01}$ of the effective NN interaction can be directly probed in a folding model analysis of the charge-exchange reaction, we have developed recently [9] an accurate procedure to parametrize the isovector (IV) density dependence of the CDM3Y6 interaction based on the Brueckner-Hartree-Fock (BHF) results for the energy- and density dependent nucleon OP in nuclear matter by Jeukenne, Lejeune and Mahaux (JLM) [54].

We recall that the isoscalar density dependence of the CDM3Y6 interaction has been introduced [44] as

$$v^{\text{D(EX)}}_{00}(E, \rho, s) = F_{\text{IS}}(E, \rho) v^{\text{D(EX)}}_{00}(s),$$  \hspace{1cm} (19)

where $F_{\text{IS}}(E, \rho) = g(E) C_0 [1 + \alpha_0 \exp(-\beta_0 \rho) - \gamma_0 \rho]$. \hspace{1cm} (20)

Parameters of $F_{\text{IS}}(\rho)$ were chosen [44] to reproduce the empirical saturation energy and density of the symmetric NM with an incompressibility $K \approx 252$ MeV, in the Hartree-Fock (HF) approximation. These parameters as well as those corresponding to other $K$ values can be found in Ref. [43]. The linear factor $g(E) \approx 1-0.0026E$ accounts effectively for the energy dependence of the IS density dependence, where $E$ is the incident energy per nucleon. Given the success of the parametrization (19)-(20) in numerous folding model analyses of nucleon-nucleus and nucleus-nucleus scattering, a similar functional has been adopted [9] for the IV density dependence of the CDM3Y6 interaction

$$v^{\text{D(EX)}}_{01}(E, \rho, s) = F_{\text{IV}}(E, \rho) v^{\text{D(EX)}}_{01}(s),$$  \hspace{1cm} (21)

where $F_{\text{IV}}(E, \rho) = C_1 [1 + \alpha_1 \exp(-\beta_1 \rho) - \gamma_1 \rho]$. \hspace{1cm} (22)
Instead of implying a simple linear energy dependence like \( g(E) \), parameters of \( F_{IV}(E, \rho) \) were adjusted carefully at each considered energy \( E \) to reproduce in the HF approximation the microscopic BHF results for the nucleon OP in nuclear matter by JLM group [54]. The radial part of the IS and IV interactions, \( v_{00(01)}^{D(EX)}(s) \), were kept unchanged as derived in terms of three Yukawas [19] from the M3Y-Paris interaction [46]. Since the original M3Y interaction \( v_{00(01)}^{D(EX)}(s) \) is real, we have generated in a sim-

\[ \text{Fig. 1. Real isovector nucleon optical potential } V_{IV}(E, \rho) \text{ in NM (solid curves)} \text{ given by the HF calculation [9] using the isovector CDM3Y6 interaction (21)-(22) adjusted to reproduce the JLM results (circles) [54] at } E = 30.4 \text{ and } 20 \text{ MeV.} \]

ilar manner the imaginary parts of \( F_{IS[IV]}(E, \rho) \) at each considered energy \( E \) based on the imaginary part of the JLM nucleon OP (see details in Ref. [9]), so that the complex density dependent CDM3Y6 interaction (19)-(22) can be used in the folding model to predict both the real and imaginary parts of the OP. The fine tuning of the (isovector) density dependent parameters to the JLM results has been done carefully at each energy \( E \) as shown in Figs. 1-2. As a result, the complex isovector density dependence of the present CDM3Y6 interaction (21)-(22) is fully based on the JLM results for the complex isovector nucleon OP in NM [54].
3 Results and discussions

3.1 Folding model analysis of the $(p, n)$ scattering to the IAS

To study the charge-exchange $(p, n)$ scattering to the IAS based on the CC equations \( (6)-(7) \), one needs to determine the nucleon OP in the entrance \((U_a = U_p)\) and exit \((U_\tilde{a} = U_n)\) channels as accurate as possible. In general, the elastic neutron scattering on a nucleus being in the excited IAS cannot be measured because most of isobar analog states are either a short-lived bound state or an unbound resonance. We have determined, therefore, \( U_\tilde{a} \) from the isoscalar \( U_0 \) and isovector \( U_1 \) parts of the neutron OP evaluated at the energy \( E = E_{\text{lab}} - Q \), using Eq. (9).

The phenomenological nucleon-nucleus global optical potentials [11,12,13] have been carefully determined based on large experimental databases of both the elastic nucleon-nucleus scattering and analyzing power angular distributions. It is helpful, therefore, to use them as the reference potentials in the present study. It should be noted that the isovector strength of the nucleon-nucleus OP is usually about 2-3% of the total OP and its contribution to the elastic scattering cross section is too weak to allow us to probe the isospin dependence of the OP directly in the OM analysis of elastic scattering. Consequently, in a “Lane consistent” approach, the isospin dependence of the nucleon-nucleus OP can be probed either in a OM study of the proton and neutron elastic scattering at the appropriate energies [55] or in the CC analysis of the charge-exchange $(p, n)$ scattering to the IAS [9]. In the latter case, the charge-exchange form factor \( F_\pi \) used in the CC equations \( (6)-(7) \) is
determined entirely by the Lane potential \( U_1 \), and the accurately measured \((p, n)\) cross section can be used to fine tune the \( U_1 \) strength. Although the isospin dependence of the mentioned global nucleon OP’s has not been calibrated against the DWBA or CC description of charge-exchange \((p, n)\) scattering to the IAS, the two more recent global OP’s [12,13] were shown [9] to give a rather good description of the \((p, n)\) data measured with \(^{48}\)Ca, \(^{90}\)Zr, \(^{120}\)Sn, and \(^{208}\)Pb targets at around 40 MeV [56], with a slightly better fit to the data given by the CH89 global OP by Varner et al. [12].

We recall further that the isospin is not a good quantum number in the repulsive Coulomb field of the nucleus that slows down the incident proton. It is necessary, therefore, to add the Coulomb correction \( \Delta E_C \) to the incident proton energy and \( \Delta U_C \) to \( U_p \) to separate the main effects of the Coulomb field [55] and restore the Lane consistency for the remainder of the OP. Estimation of the isospin impurity due to the Coulomb correction is not straightforward [55]. One need first to determine correction \( \Delta E_C \) to the incident proton energy such that the same isoscalar and isovector potentials \( U_i(n) \) can be used to generate the proton and neutron OP at the energy \( E_p \) and \( E_n = E_p - \Delta E_C \), respectively. The Coulomb correction to the CH89 global OP for the \( p + ^{208}\)Pb system at 45 MeV has been determined recently [55], based on a comparative OM study of the proton and neutron elastic scattering from the lead target. In this case, \( \Delta E_C \) was found to be about 14.6 MeV, which is lower than the empirical (energy-independent) value \( \Delta E_C = 6Ze^2/(5R_C) \approx 19 \) MeV given by the original CH89 systematics [12]. Using the prescription of Ref. [55] to relate the bombarding energies \( E_p \) and \( E_n = E_p - \Delta E_C \) so that the diffraction maxima and minima of the elastic proton and neutron angular distributions fall at about the same angles in the forward region (see Fig. 3), we have estimated that \( \Delta E_C \approx 15 \) MeV for the elastic \( p + ^{208}\)Pb scattering at 35 MeV. For other targets the effect caused by the Coulomb correction is weaker and we have used for simplicity the empirical \( \Delta E_C \) value given by the CH89 global OP.

In the folding model, the effect of the Coulomb potential to the proton OP is taken into account self-consistently via the local relative motion momentum \( \mathbf{R} \), and one needs only to use \( E = E_p - \Delta E_C \) in the energy dependent factor \( g(E) \) of the real IS density dependence. The parameters of \( \text{Im} F_{IS}(E, \rho) \) as well as those of both the real and imaginary parts of \( F_{IV}(E, \rho) \) must be adjusted to the JLM results for the nucleon OP at the energy \( E = E_p - \Delta E_C \). For example, the density dependent parameters used to calculate the \( p + ^{208}\)Pb optical potential at the proton incident energies of 35 and 45 MeV have been adjusted to the JLM results (see Figs. 12 at \( E = 20 \) and 30.4 MeV, respectively. Thus, the proton OP of the entrance channel \( \text{(2)} \) is given by the folding calculation using the complex CDM3Y6 interaction determined at the (Coulomb corrected) incident energy \( E = E_p - \Delta E_C \). Similarly, the charge-exchange form factor \( \text{IV} \) of the \((p, n)\) scattering to the IAS is given by the IV part of the folded proton OP obtained with the CDM3Y6 interaction at \( E = E_p - \Delta E_C - Q/2 \). The neutron OP of the exit channel \( \text{(3)} \) is given by the folding calculation using the CDM3Y6 interaction determined at the energy of emitting neutron \( E = E_p - Q \).

Given the complex density dependence of the CDM3Y6 interaction appropriately determined at each energy, it is natural to check the OM description of the elastic proton scattering at the considered energies using the complex folded OP (given by the CDM3Y6 interaction determined at \( E = E_p - \Delta E_C \))

\[
U(R) = N_V [V_{IS}(R) - V_{IV}(R)] + iN_W [W_{IS}(R) - W_{IV}(R)].
\]

\( U \) is further added by the spin-orbital and the Coulomb potential taken, for simplicity, from the CH89 systematics [12]. The strengths \( N_{V(W)} \) of the complex folded OP are adjusted to the best OM fit to the elastic scattering data.

Because a high accuracy of distorted waves is always needed for the DWBA or CC calculation of the charge-exchange scattering, we have used also a hybrid choice
Fig. 3. OM description of the elastic $n + ^{208}\text{Pb}$ data at 30.4 MeV [55] and $p + ^{208}\text{Pb}$ data at 45 MeV [57] given by the complex folded OP and hybrid (real folded + imag. WS) OP.

of the complex OP with the real part given by the folding model and imaginary part given by a Woods-Saxon (WS) potential

$$U(R) = N_V [V_{WS}(R) - V_{IV}(R)] - i [W_c f(R) - 4a_w W_s \frac{df(R)}{dR}],$$

where $f(R) = 1/[1 + \exp((R - R_w)/a_w)]$. (24)

The normalization factor $N_V$ of the real folded potential as well as strengths of the volume $W_c$ and surface $W_s$ parts of the absorptive WS potential are adjusted to fit the elastic scattering data at each energy. The WS radius $R_W$ and diffuseness $a_W$ were fixed as taken from the CH89 global systematics [12]. The OM descriptions of the elastic proton scattering from $^{48}\text{Ca}$, $^{90}\text{Zr}$, $^{120}\text{Sn}$, and $^{208}\text{Pb}$ targets at 35 and 40 MeV given by the two choices of the proton OP are shown in Fig. 4 and the corresponding OP parameters are presented in Tables 1 and 2. The best OM fit to the elastic data required the strength of the complex folded potential to be renormalized by $N_V \approx 0.8 \sim 0.90$ and $N_W \approx 0.5 \sim 0.6$. An absorption overestimated by the folding model has been observed earlier [9] and it is due mainly to a strong volume absorption predicted by the JLM potential. In general, after the adjustment of the OP parameters, both the folded (23) and hybrid (24) optical potentials give equally good OM descriptions of the data. The calculated total reaction cross sections also agree nicely with the experimental data, and that confirms the reliability of the proton OP used in our CC equations for the charge-exchange scattering. The coupling between
the elastic and charge-exchange scattering channels turned out to be quite weak and all the OP parameters in the CC calculation have been kept as fixed by the OM analysis of elastic proton scattering. The complex OP of the exit channel is constructed using relation (9), with the IS and IV parts of the neutron OP determined at the energy $E = E_p - Q$ as discussed above. Because the OP between neutron and nucleus in the IAS is unknown, we simply adjusted the renormalization factors $N_{V(W)}$ of the folded OP of the exit channel to the best CC description of the elastic and $(p,n)$ scattering data. In all cases under study, the best-fit $N_{V(W)}$ factors turned out to be $10 \sim 15\%$ smaller than those obtained for the folded OP of the entrance channel shown in Table 1.

As already noted, the isovector strength of the proton OP is just a few percent of the total OP and it is difficult to adjust the strength of $U_{IV}$ separately in an OM fit to the elastic scattering data. However, the IV strength of the OP can be very well fine
Table 1. Best-fit renormalization factors $N_V$ and $N_W$ of the complex folded proton OP of the entrance channel (23). The calculated proton total reaction cross section measured at $E_p = 35$ and 45 MeV; $\sigma_{exp}$ taken from Ref. [61]. $N_{V1(W1)}$ are the renormalization factors of the folded charge-exchange FF (25) deduced from the best CC fit to the $(p, n)$ data.

| Target | $E_p$ (MeV) | $N_V$ | $N_W$ | $\sigma_{R}$ (mb) | $\sigma_{R}^{exp}$ (mb) | $N_{V1}$ | $N_{W1}$ |
|--------|-------------|-------|-------|------------------|------------------------|----------|----------|
| $^{48}$Ca | 35 | 0.817 | 0.479 | 969 | 971 ± 32 | 1.352 | 1.00 |
| 45 | 0.840 | 0.515 | 875 | 908 ± 34 | 1.430 | 1.00 |
| $^{50}$Zr | 35 | 0.898 | 0.551 | 1357 | 1316 ± 65 $^a$ | 1.516 | 1.00 |
| 45 | 0.784 | 0.561 | 1246 | 1214 ± 59 $^b$ | 1.557 | 1.00 |
| $^{120}$Sn | 35 | 0.872 | 0.503 | 1583 | 1668 ± 59 | 1.377 | 1.00 |
| 45 | 0.859 | 0.521 | 1508 | 1545 ± 38 | 1.539 | 1.00 |
| $^{208}$Pb | 35 | 0.858 | 0.494 | 1934 | 1974 ± 38 | 1.479 | 1.00 |
| 45 | 0.864 | 0.538 | 1988 | 1979 ± 41 | 1.464 | 1.00 |

$^a$ Total reaction cross section measured at $E = 40$ MeV; $^b$ at $E = 49.5$ MeV.

Table 2. The same as Table 1 but for the hybrid OP (24), with the diffuseness of the WS potential $R_w = 0.69$ fm for all cases as given by the CH89 global OP (12).

| Target | $E_p$ (MeV) | $N_V$ | $W_v$ (MeV) | $W_s$ (MeV) | $R_w$ (fm) | $\sigma_{R}$ (mb) | $\sigma_{R}^{exp}$ (mb) | $N_{V1}$ | $N_{W1}$ |
|--------|-------------|-------|-------------|-------------|-------------|------------------|------------------------|----------|----------|
| $^{48}$Ca | 35 | 0.859 | 2.389 | 4.601 | 4.414 | 979 | 971 ± 32 | 1.370 | 1.00 |
| 45 | 0.847 | 5.107 | 3.265 | 4.414 | 908 | 908 ± 34 | 1.578 | 1.00 |
| $^{50}$Zr | 35 | 0.877 | 2.479 | 7.060 | 5.540 | 1419 | 1316 ± 65 | 1.537 | 1.00 |
| 45 | 0.877 | 3.630 | 6.261 | 5.540 | 1400 | 1214 ± 59 | 1.558 | 1.00 |
| $^{120}$Sn | 35 | 0.868 | 2.305 | 7.792 | 6.140 | 1639 | 1668 ± 59 | 1.340 | 1.00 |
| 45 | 0.875 | 2.727 | 5.530 | 6.140 | 1545 | 1545 ± 38 | 1.357 | 1.00 |
| $^{208}$Pb | 35 | 0.868 | 4.982 | 5.473 | 7.460 | 1957 | 1979 ± 41 | 1.582 | 1.00 |
| 45 | 0.873 | 6.625 | 3.338 | 7.460 | 2011 | 1974 ± 38 | 1.428 | 1.00 |

tuned against the $(p, n)$ data in the DWBA or CC analysis of the charge-exchange scattering because the FF of the charge-exchange scattering to the IAS is entirely determined by $U_{IV}$. Thus, the following complex charge-exchange FF is used in our CC analysis

$$F_{cx}(R) = \sqrt{\frac{2}{T_z}} U_{IV}(R) = \sqrt{\frac{2}{T_z}} [N_{V1} V_{IV}(R) + i N_{W1} W_{IV}(R)],$$

(25)

where $V_{IV}(R)$ and $W_{IV}(R)$ are the folded IV potential obtained with the CDM3Y6 interaction determined at $E = E_p - \Delta E_C - Q/2$. The CC results for the charge-exchange $(p, n)$ scattering to the IAS of $^{48}$Ca, $^{50}$Zr, $^{120}$Sn, and $^{208}$Pb targets at $E_p = 35$ and 45 MeV given by the charge-exchange form factor (25) are shown in Figs. 5 and 6 and the corresponding best-fit renormalization factors $N_{V1}$ and $N_{W1}$ are given in Tables 1 and 2. One can see that both choices of the proton OP give about the same good CC fit to the $(p, n)$ data, with a slight improvement by the more elaborate hybrid OP. The CC results obtained with both choices of the proton OP clearly show that the best fit to the $(p, n)$ data required the complex form factor (25) to be renormalized by $N_{V1} \sim 1.3 - 1.5$ and $N_{W1} \sim 1.0$. This indicates that the empirical IV strength of the CDM3Y6 interaction (21)-(22) should be about 30 $\sim$ 40% stronger than that adjusted to the JLM nucleon OP in nuclear matter limit (54). It must be noted that the Coulomb correction to the proton incident energy $\Delta E_C$ has not been taken into account in our earlier folding model analysis (9) of these same $(p, n)$ data, where the fully folded OP could deliver a fair description of the charge-
Fig. 5. CC description of the charge-exchange \((p, n)\) scattering to the IAS of \(^{48}\text{Ca}\), \(^{90}\text{Zr}\), \(^{120}\text{Sn}\), and \(^{208}\text{Pb}\) targets at \(E_p = 35\) MeV given by the charge-exchange form factor used with the complex folded OP or hybrid OP. The data were taken from Ref. [56].

exchange scattering data only with \(N_{V1} \sim 2.0\) (much larger than that obtained in the present work). In the present work, a good CC description of the \((p, n)\) data has been reached with the fully folded OP, using about the same scaling factors \(N_{V1(W1)}\) as those obtained with the hybrid OP. These results show that the Coulomb correction is important not only in the OM analysis of the nucleon scattering but also in a Lane-consistent CC description of the elastic and charge-exchange scattering to the IAS [55]. Our CC results give a good description not only to the measured angular distribution of the charge-exchange \((p, n)\) scattering but also to the total \((p, n)\) cross section. For example, the CC calculation using the hybrid OP and renormalized complex folded FF gives the total \((p, n)\) cross section \(\sigma_{pn} = 4.8\) and \(4.1\) mb for the \(^{90}\text{Zr}_{g.s.}(p, n)^{90}\text{Nb}_{IAS}\) reaction at 35 and 45 MeV, respectively, which agree nicely with the data \(\sigma_{pn}^{\exp} \approx 4.8 \pm 0.5\) and \(4.4 \pm 0.5\) mb at 35 MeV and 45 MeV, respectively [56].
It is complimentary to note that the effective JLM interaction of Gaussian type has been used earlier by Pakou et al. [62] and Bauge et al. [63] to study the same $(p, n)$ reactions. The JLM folding model analysis of the proton, neutron elastic scattering and charge-exchange $(p, n)$ reaction done in Ref. [62] has also shown that the IV strength of the JLM interaction is too weak and a strong overall renormalization of the folded charge-exchange FF by $N_{V1} = N_{W1} \approx 2 - 2.5$ was needed to account for the $(p, n)$ data. In a more elaborate treatment of the charge-exchange transition to the IAS [63] the isospin coupling factor in Eq. (4) has been assumed density dependent, $\sqrt{2T_z/A} = \sqrt{[\rho_n(r) - \rho_p(r)]/\rho(r)}$. The JLM nucleon OP obtained with such a density-dependent isospin coupling has been thoroughly tested in the OM analysis of the elastic and $(p, n)$ scattering data measured over a wide range of energies and target masses [63], and the best-fit renormalization factors $N_{V1} \approx 1.5 - 1.6$ and $N_{W1} \approx 1.3 - 1.4$ were found for the charge-exchange folded FF in the energy range of $30 - 40$ MeV, which are close to our results. Thus, all the results show consistently that the IV strength of the JLM interaction is much too weak to account for the

Fig. 6. The same as Fig. 5 but for $E_p = 45$ MeV.
measured \((p,n)\) data. Because the IV term of the JLM nucleon OP has been obtained as the first-order expansion of the mass operator of symmetric NM perturbed by a neutron excess [63], a weakness of the resulting JLM nucleon OP in asymmetric NM could well be expected. We conclude here that the charge-exchange scattering to the IAS is indeed a very helpful reaction to probe the isospin dependence of an effective NN interaction. It would be of interest, therefore, to apply similar folding model analysis to test the isospin dependence of the nucleon OP given by the advanced BHF calculation of asymmetric NM [65].

3.2 Implication for the symmetry energy of nuclear matter

As discussed above, the knowledge about the isospin dependence of the in-medium NN interaction is vital for the construction of the EOS of asymmetric NM, the key input for the studies of neutron star [21,22,23]. We show here that the results of the folding model analysis of the charge-exchange \((p,n)\) scattering are quite helpful for the determination of the NM symmetry energy. For this purpose, the real IV density dependence \((22)\) of the CDM3Y6 interaction at low energies (keV region) has been carefully parametrized to reproduce in the HF approximation the IV part of the nucleon OP in NM limit given by the microscopic BHF calculation of the JLM group [66]. Such a version of the density- and isospin dependent CDM3Y6 interaction is then used in the HF calculation [19,67] of the total energy density \(E\) of the asymmetric NM

\[
E = E_{\text{kin}} + \frac{1}{2} \sum_{k\sigma\tau} \sum_{k'\sigma'\tau'} \left( \langle k\sigma\tau, k'\sigma'\tau' | v_D | k\sigma\tau, k'\sigma'\tau' \rangle + \langle k\sigma\tau, k'\sigma'\tau' | v_{EX} | k\sigma\tau, k'\sigma'\tau' \rangle \right),
\]

(26)

where \(|k\sigma\tau\rangle\) are the ordinary plane waves. Dividing \(E\) over the total NM density \(\rho\), we obtain the total NM energy per particle \(E\)

\[
\frac{E}{\rho} = E(\rho, \delta) = E(\rho, \delta = 0) + S(\rho)\delta^2 + O(\delta^4) + \ldots, \quad \delta = \frac{\rho_n - \rho_p}{\rho}.
\]

(27)

The contribution of \(O(\delta^4)\) and higher-order terms in the neutron-proton asymmetry \(\delta\) has been proven to be small [19,20] and is neglected in the parabolic approximation, where the NM symmetry energy \(S(\rho)\) simply equals the energy required per particle to change the symmetric NM into the pure neutron matter. The value of \(S(\rho_0)\) at the saturation density \((\rho_0 \approx 0.17 \text{ fm}^{-3})\) has been predicted by different many-body calculations to be around 30-31 MeV [19,20,25,26].

Our HF results for the nuclear symmetry energy \(S(\rho)\) given by different IV strengths [22] of the real CDM3Y6 interaction are shown in Fig. 7. One can see that \(S(\rho_0)\) is approaching the empirical value of around 30 – 31 MeV only if Re \(F^IV\) is renormalized by \(N_{V1} \approx 1.3 – 1.5\), in a good agreement with the renormalization factors given by the folding model analysis of the charge-exchange \((p,n)\) data. The use of the unrenormalized IV density dependence based on the JLM results clearly underestimates \(S(\rho_0)\) compared to the empirical values. The weakness of the JLM isovector strength [54] is, thus, also confirmed in our HF results for asymmetric NM. The IV density dependence of the CDM3Y6 interaction renormalized by \(N_{V1} \approx 1.3 – 1.5\) also give \(S(\rho)\) values lying within the empirical boundaries implied by the HI fragmentation data [27,29,32,33] and the nuclear structure studies of the giant dipole resonance [40] and neutron skin [42], at the NM densities up to \(\rho_0\).

Although the folding model analysis of the \((p,n)\) scattering to the IAS has put a constraint on the nuclear symmetry energy \(S(\rho)\) at \(\rho \lesssim \rho_0\) as shown in Fig. 7, its behavior at higher NM densities remains uncertain due to a simple reason that the total...
Fig. 7. HF results for the nuclear symmetry energy $S(\rho)$ given by the CDM3Y6 interaction, with the IV density dependence renormalized by different factors $N_{V1}$. The shaded (magenta) region marks the empirical boundaries implied by the isospin diffusion data and double ratio of neutron and proton spectra of HI collisions [27,29]. The circle is the empirical value predicted by nuclear many-body calculations [19,20,25,26] that is about the same as that value deduced from the folding model analysis of the charge-exchange $(p,n)$ data [8,9]. The square and triangle are the constraints deduced from the consistent structure studies of the giant dipole resonance [40] and neutron skin [42], respectively. CDM3Y6s is a “soft” version of the CDM3Y6 interaction, with about the same density dependence assumed for both the IS and IV terms [68].

nuclear density of the proton-nucleus system never exceeds $\rho_0$, so that the $(p,n)$ data are sensitive mainly to the low-density tail of the isovector interaction. To explore this effect in more details, we have assumed for the IV density dependence $F_{IV}(\rho)$ of the CDM3Y6 interaction the same density-dependent functional as that of the IS density dependence but scaled by a factor of 1.1 deduced from our earlier folding model analysis of the $p(^{6}\text{He},^{6}\text{Li}_{\text{IAS}})n$ reaction [8], i.e., $F_{IV}(\rho) = 1.1 F_{IS}(\rho)$. Such an ansatz for the IV density dependence of the CDM3Y6 interaction, dubbed as the CDM3Y6s interaction [68], gives nearly the same description of the NM symmetry energy $S(\rho)$ at $\rho \lesssim \rho_0$ as the newly parametrized $F_{IV}(\rho)$, with a slight difference in the slope of the corresponding $S(\rho)$ curves (compare dash-dotted and solid curves in Fig. 7). These
two sets of the IV density dependence lead, however, to very different behaviors of $S(\rho)$ at high NM densities. The symmetry energy obtained with $F_{IV}(\rho)$ based on the JLM results increases monotonically with the increasing NM density. Such a behavior has been widely discussed in the literature as the **stiff** density dependence of the NM symmetry energy. The behavior of $S(\rho)$ obtained with the CDM3Y6s interaction is referred to as the **soft** density dependence \[67,68,69\].

To see if the $(p, n)$ data under study are sensitive to the slope of the NM symmetry energy in the low-density region, we have done the CC analysis of the charge-exchange $(p, n)$ scattering on $^{48}$Ca target at $E_p = 35$ and 45 MeV using the form factors \[25\] given by different choices of the IV density dependence of the CDM3Y6 interaction, similar to those used in the HF calculation of the NM symmetry energy shown in

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**Fig. 8.** CC description of the charge-exchange $(p, n)$ scattering to the IAS of $^{48}$Ca target at $E_p = 35$ and 45 MeV given by the hybrid OP and charge-exchange form factor \[25\] obtained with the different inputs for the IV density dependence of the CDM3Y6 interaction, similar to those discussed in Fig. 7. The data were taken from Ref. \[56\].
Fig. 9. The gravitational mass of neutron star versus its radius obtained with the EOS’s given by the stiff-type (upper panel) and soft-type (lower panel) CDM3Yn interactions, in comparison with the empirical data (shaded contours) deduced by Steiner et al. [70] from the observation of the X-ray burster 4U 1608-52. The circles are values calculated at the maximum central densities. The thick solid (red) line is the limit allowed by the General Relativity [71].

Fig. 7. In particular, the (soft) CDM3Y6s version of the interaction has \( \text{Re} F^I_\text{IV}(\rho) = 1.1 \times \text{Re} F^I_\text{IS}(\rho) \) and \( \text{Im} F^I_\text{IV}(\rho) = \text{Im} F^I_\text{IS}(\rho) \). One can see in Fig. 8 that the use of the soft IV density dependence leads to a much poorer description of the \((p, n)\) data at large angles. Although nearly the same description of the data at forward angles is given by both stiff and soft choices of the IV density dependence, the 35 MeV data points approaching the zero angle cannot be properly accounted for by the FF obtained with the soft CDM3Y6s interaction. This result shows that a difference in the slope of the NM symmetry energy at low NM densities shown in Fig. 7 can be traced in the calculated \((p, n)\) cross section, and the best CC fit to the \((p, n)\) data prefers the stiff IV density dependence of the CDM3Y6 interaction. We note that
some HI collision data also were found to prefer the stiff density dependence of the NM symmetry energy [69].

A similar approach has been used in our recent study of the EOS of neutron star matter [68] with the density dependent CDM3Y3, CDM3Y4 and CDM3Y6 interactions. The IS density dependence \( F_{\text{IS}}(\rho) \) of these three versions of the density dependent M3Y interaction has been parametrized \([43,44]\) to reproduce the empirical saturation properties of symmetric NM in the HF approximation, with the incompressibility \( K = 217, 228 \) and \( 252 \text{ MeV} \), respectively. Their IV density dependence \( F_{\text{IV}}(\rho) \) has been determined also in two (stiff and soft) scenarios, in exactly the same way as discussed above. The stiff and soft versions of the density dependent M3Y interaction give about the same behavior of the NM symmetry energy as shown in Fig. 7 for the CDM3Y6 and CDM3Y6s interactions. The EOS of the uniform core of neutron star of the \( npe\mu \) composition in the \( \beta \)-equilibrium at zero temperature has been calculated using the stiff and soft versions of the density dependent NN interaction [68]. The obtained EOS’s were then used as input of the Tolman-Oppenheimer-Volkov equations to describe basic properties of neutron star, like the gravitational mass, radius and moment of inertia. The most obvious effect caused by changing slope of the NM symmetry energy from stiff to soft is the reduction of the maximum gravitational mass \( M \) and radius \( R \) as illustrated in Fig. 9. Namely, the \( M \) value is changing from \( 1.6 \sim 2 M_\odot \) to a significantly lower range of \( 1.1 \sim 1.4 M_\odot \), with a much worse description of the empirical mass-radius data [70]. Together with the present results of the folding model analysis of the charge-exchange scattering to the IAS, it is highly plausible that one can rule out the EOS with a soft behavior of the symmetry energy in the theoretical modeling of neutron star.

### 3.3 Folding model analysis of the \((^3\text{He},t)\) scattering to the IAS

As mentioned in Sec. 1 a folding model study the \((^3\text{He},t)\) scattering to the IAS might allow us to test the high density part of the isovector density dependence [22], due to a higher overlap nuclear density reached during the collision and, eventually, to conclude on the slope of the NM symmetry energy \( S(\rho) \) at higher densities.

Given the same spin and isospin of proton and \(^3\text{He} \), very similar structures of the initial and final states have been observed in the charge-exchange \((p,n)\) and \((^3\text{He},t)\) reactions. In particular, the charge-exchange scattering to the IAS induced by both proton and \(^3\text{He}\) projectiles can be considered in the same scenario of the isospin-flip elastic scattering [143] caused by the isovector term (Lane potential) of the optical potential [1]. Therefore, the determination of the Lane potential \( U_1 \) has been attempted already in the \((^3\text{He},t)\) experiments in the early 70’s [72,73]. However, due to the complexity of the \((^3\text{He},t)\) reaction that is caused by a composite projectile, the isospin dependence of the \(^3\text{He}\)-nucleus OP is much less known compared to that of the nucleon OP. Unlike the nucleon-nucleus case, the isovector term of the real \(^3\text{He}\)-nucleus OP could not even be established in the recent global OP for \(^3\text{He}\) and triton [16]. It is difficult, therefore, to investigate the Lane consistency of the phenomenological \(^3\text{He}\)-nucleus OP based on the DWBA or CC analysis of the elastic and charge-exchange scattering as has been done in Ref. 9 with the phenomenological global nucleon OP. On the microscopic level, the measured data of the \((^3\text{He},t)\) scattering to the IAS have been analyzed mainly in the DWBA with the charge-exchange form factor given by a single-folding calculation using an effective \(^3\text{He}\)-nucleon interaction and nuclear density of the target [17,18]. Therefore, it is of high interest to carry out a consistent folding model study of both the \((p,n)\) and \((^3\text{He},t)\) scattering to the IAS using the same density- and isospin dependent NN interaction.

In the present work we have considered two representative cases of the \((^3\text{He},t)\) scattering to the IAS: \(^{14}\text{C}(^3\text{He},t)^{14}\text{N}\) and \(^{48}\text{Ca}(^3\text{He},t)^{48}\text{Sc}\) data measured at \( E_{\text{lab}} = 72 \)
MeV [74] and 82 MeV [75], respectively. Both data sets were measured together with the elastic $^3\text{He}$ scattering, over a wide angular range [43,74]. The $^3\text{He}$ incident energies of 72 and 82 MeV (or 24 and 27.3 MeV/nucleon) are in the range of the “rainbow” energy [43] and one can see in Figs. [10] and [11] that both the elastic and charge-exchange data show clearly the shoulder-like bump characteristic of the nuclear rainbow [43] at large scattering angles. Therefore, the elastic and ($^3\text{He},t$) scattering data under study should be quite sensitive to the strength and shape of the real OP and, hence, could serve as a probe of the folding model used to predict the OP. The two considered energies are also not too low so that the ($^3\text{He},t$) scattering to the IAS can be treated as a direct one-step process [44,75].

![Graph showing elastic $^3\text{He}$ scattering for $^{14}\text{C}$ and $^{48}\text{Ca}$ targets at $E_{\text{lab}} = 72$ MeV and $82$ MeV, respectively.](image)

**Fig. 10.** CC description of the elastic $^3\text{He}+^{14}\text{C}$ and $^3\text{He}+^{48}\text{Ca}$ scattering data at $E_{\text{lab}} = 72$ MeV [74] and 82 MeV [75], respectively, given by the complex folded and hybrid optical potentials [44, 24].
Like the $(p,n)$ scattering, the isospin impurity of the $^3$He-nucleus OP needs to be taken into account by a correction $\Delta E_C$ to the incident $^3$He energy. The Coulomb correction to the global OP for $^3$He and triton used by Pang et al. \cite{16} is based on the CH89 systematics \cite{12}, i.e., $\Delta E_C = 6Z_1Z_2e^2/(5R_C)$. Such an empirical estimate of the Coulomb correction is also used in our folding model analysis. Thus, the folded $^3$He-nucleus OP of the entrance channel \cite{8} is obtained with the complex CDM3Y6 interaction determined at the $(\text{Coulomb corrected})$ incident energy per nucleon $E = E_{\text{lab}}/3 - \Delta E_C/2$. The charge-exchange form factor $f^V$ of the $(^3\text{He}, t)$ scattering to the IAS is given by the isovector part of the folded $^3$He-nucleus OP obtained with the CDM3Y6 interaction determined at $E = E_{\text{lab}}/3 - \Delta E_C/(t - Q)/6$. The folded triton OP of the exit channel \cite{9} is obtained with the CDM3Y6 interaction determined at the energy per nucleon $E = E_{\text{lab}}/3 - \Delta E_C(t - Q)/3$, where $\Delta E_C(t)$ is the Coulomb correction determined for triton using the same CH89-based formula \cite{16}. The complex folded OP and hybrid OP are used in the same notation as \cite{23}-\cite{24}, with parameters determined from the best CC fit to the elastic and $(^3\text{He}, t)$ scattering data as done in the CC analysis of the $(p,n)$ scattering. The $^3$He and triton optical potentials are added by the standard Coulomb and spin-orbital terms taken from the global OP by Pang et al. \cite{16}. The OP parameters and renormalization factors of the charge-exchange form factor \cite{25} given by the best CC fit to the elastic and $(^3\text{He}, t)$ scattering data are given in Table 3 and the comparison of the calculated scattering cross sections with the data are shown in Figs. 10 and 11. Like the folding analysis of the $(p,n)$ data, we have treated the renormalization factors $N_{V(W)}$ of the folded triton OP of the exit channel as free parameters in our CC fit to the elastic and $(^3\text{He}, t)$ scattering data, and the best-fit $N_{V(W)}$ values also turned out to be $10 \sim 15\%$ smaller than those obtained for the entrance channel. A similar effect of the strength reduction of the triton OP has also been found earlier in the DWBA analysis of the $(^3\text{He}, t)$ scattering to the IAS \cite{17}, using quite different model for the charge-exchange FF.

Table 3. Parameters of the optical potentials \cite{23-24} and charge-exchange form factor \cite{25} for $^3$He+$^{14}$C and $^3$He+$^{48}$Ca systems at $E_{\text{lab}} = 72$ and $82$ MeV, respectively, given by the best CC fit to the elastic and $(^3\text{He}, t)$ scattering data \cite{74,75}. Parameters $R_w$ and $a_w$ of the hybrid OP were kept unchanged as taken from the global OP by Pang et al. \cite{16}.

| Target | OP      | $N_V$ | $W_v$ (MeV) | $W_a$ (MeV) | $R_w$ (fm) | $a_w$ (fm) | $\sigma_R$ (mb) | $N_{V1}$ | $N_{W1}$ |
|-------|---------|-------|------------|------------|------------|------------|----------------|---------|---------|
| $^{14}$C | Folded | 0.977 | 1.214 | - | - | 1090 | 1.300 | 0.850 |
|        | Hybrid | 1.054 | 5.786 | 10.175 | 3.027 | 0.84 | 1042 | 1.350 | 0.850 |
| $^{48}$Ca | Folded | 0.889 | 1.433 | - | - | 1632 | 1.650 | 1.340 |
|        | Hybrid | 0.981 | 9.870 | 12.016 | 4.631 | 0.84 | 1713 | 1.420 | 1.280 |

$W_v$ is the best-fit factor $N_{W1}$. 

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folded OP belongs indeed to the realistic potential family for the elastic $^3\text{He}$-nucleus scattering.

The CC results obtained with both choices of the $^3\text{He}$ optical potential show that the best CC fit to the charge-exchange $^{14}\text{C}(^3\text{He},t)^{14}\text{N}$ and $^{48}\text{Ca}(^3\text{He},t)^{48}\text{Sc}$ scattering data (see Figs. 11 and 12) needed a complex form factor (25) renormalized by $N_{V1} \approx 1.3 \sim 1.6$ and $N_{W1} \approx 0.85 \sim 1.3$. Such a behavior of the complex renormalization factor of the charge-exchange FF is similar to that found above in the folding model analysis of the $(p,n)$ scattering. This result confirms again that the empirical IV
strength of the CDM3Y6 interaction should be $30 \sim 40\%$ stronger than that adjusted to the JLM results for nuclear matter [54].

![Graph showing CC description of the charge-exchange $^{14}$C($^3$He,$t$)$^{14}$N scattering data at $E_{\text{lab}} = 72$ MeV](image)

An important feature of the nuclear rainbow scattering is that the large-angle scattering data are quite sensitive to the nucleus-nucleus OP at small internuclear radii [43]. In a folding model analysis, this means a sensitivity to the effective NN interaction at high nuclear densities. To see if the considered data of the charge-exchange ($^3$He,$t$) scattering to the IAS are indeed sensitive to the NN interaction at high medium densities ($\rho \gtrsim \rho_0$ in the overlap of the $^3$He projectile with the target nucleus), we have done the CC analysis of the ($^3$He,$t$) scattering using the charge-
exchange form factors given by different choices of the IV density dependence of the CDM3Y6 interaction, in the same way as done with the \((p,n)\) scattering. The CC results (see, e.g., Fig. 12) for the \(( ^3\text{He},t)\) scattering show that the soft IV density dependence of the CDM3Y6 interaction gives a much poorer description of the rainbow shoulder seen in the \(( ^3\text{He},t)\) data at large angles. As a result, the slope difference of the NM symmetry energy with the increasing NM density shown in Fig. 7 can also be traced in the folding model analysis of the \(( ^3\text{He},t)\) scattering to the IAS, and the best CC fit to the data prefers again the stiff IV density dependence of the CDM3Y6 interaction. Taken together with the results of the folding model analysis of the \((p,n)\) scattering to the IAS discussed in Sec. 3.2, we conclude that the EOS of asymmetric NM with a soft density dependence of the symmetry energy \(S(\rho)\) is not realistic and should not be used in the studies of neutron star matter.

4 Summary

A consistent folding model study of the charge-exchange \((p,n)\) and \(( ^3\text{He},t)\) scattering to the IAS of the target has been performed, where the same density- and isospin dependent effective NN interaction has been used to calculate the isospin dependent optical potentials and charge-exchange form factors for the input of the Lane equations (6)-(7).

To probe the isospin dependence of the NN interaction, a complex IV density dependence of the CDM3Y6 interaction has been constructed based on the microscopic JLM nucleon OP in the NM limit \[54\]. Such an IV density dependence was used with the IS density dependence of the CDM3Y6 interaction parametrized earlier in Ref. \[44\], and tested later on in numerous folding model studies of nuclear scattering. The CC analysis of both the \((p,n)\) and \(( ^3\text{He},t)\) scattering using the folded OP and charge-exchange FF has shown that the (real) IV density dependence of the CDM3Y6 interaction (based on the JLM results) needs to be enhanced by about 30 \~ 40\% to give a consistently good CC description of the \((p,n)\) and \(( ^3\text{He},t)\) data.

The JLM-based IV density dependence of the (real) CDM3Y6 interaction has been used also in the HF calculation of asymmetric NM, where the nuclear symmetry energy \(S(\rho)\) could be obtained within the range of empirical values only if the IV density dependence is scaled by a factor \(N_{V1} \approx 1.3 - 1.6\), in a close agreement with the results of the folding model analysis of the charge-exchange scattering.

The density dependence of the NM symmetry energy has been further probed in a direct test of the CDM3Y6 interaction, where we obtained a consistently worse CC description of the \((p,n)\) and \(( ^3\text{He},t)\) data if a soft IV density dependence is used instead of the stiff (JLM-based) one. Such an assumption has been made recently for the EOS of the uniform, \(\beta\)-stable core of neutron star \[68\], and a significant reduction of the gravitational mass \(M\) and radius \(R\) of neutron star (away from the empirical boundaries) was found when the density dependence of the NM symmetry energy is changed from the stiff to the soft behavior. The results of these two complimentary studies not only allow us to make a more definitive conclusion about the slope of the NM symmetry energy but also stress the importance of the experiments on the charge-exchange scattering to the IAS for the study of the isospin dependence of the nucleon-nucleus and nucleus-nucleus OP. It would be of great interest, therefore, to have such experiments pursued at the modern rare isotope beam facilities.

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