The Curve Fit Analysis of Mean Longshore Current Velocity Distribution Under Different Mild Slope Conditions

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Abstract. Studying the distribution of longshore current velocity has important practical significance for coastal sediment transport and nearshore material transport and diffusion. Based on the experimental results of 1:100 slope and 1:40 slope mild slope longshore current, this essay adopts MATLAB software and uses different functions to fit and analyze the mean longshore current velocity distribution under mild slope in two cases. The results show that the maximum value of 1:100 mean longshore current velocity have a concave trend on the nearshore side, three kinds of functions can properly fit the mean longshore current velocity distribution. In the case of 1:40, three kinds of functions cannot appropriately show the mean longshore current velocity distribution. Therefore, the distribution characteristics that the convex trend is on the coastal side of the 1:40 mild slope can be better described by proposing a custom piecewise function and relevant constraints.

Introduction

In coastal areas, a huge number of waves breaks and evolves, making hydrodynamic conditions and sediment movement pattern pretty complicated. The problem of longshore current is always the focus of attention on this field, which plays an important role in sediment movement and material diffusion. It involves dynamic problems such as wave breaking, turbulent current and other strong nonlinear fluid motions that have not been solved in the existing research.

At a relatively early period, people found longshore current phenomenon and conducted in-depth study on that. First, Brebner, Kamphuis[1] and Putnam et al. used the mass continuity of water as a breakthrough to derive the formula by using momentum flow theory. Thornton[2], Bowen[3] and Longuet-Higgins[4] derived the formula from the concept of radiation stress and received long-term development. In terms of longshore current model experiments, Komar[5], Galvin[6] et al. made field observations of longshore current. Birkemeier[7] summarized on-site longshore current experiments. Galvin and Eagleson[8] measured the distribution of water depth mean longshore current along the vertical coast direction.

Allen[9] gave the suitable mean longshore current distribution of mild slopes. By using specific function combination and relevant constraints, this essay explore the applicability of this curve fit on two slopes, discover fitted curve of the mean longshore current distribution of 1:40 mild slope.

Mean Longshore Current Experiment

The experiment was carried out in the multi-functional integrated pool of Dalian University of Technology[10]. The length of the pool was 55 meters, the width was 34 meters. The experimental slope was 1:40, the length of slope was 18 meters. The depth of still water was 0.45 meters, and the distance from slope foot to wave making board was between 8 meters and 22.5 meters. Figure 1 shows the experimental terrain. As shown in the figure, the origin of the coordinate system is upstream, the x-axis points away from the shoreline, the y-axis points to the downstream direction.
The flow velocity of the longshore current was measured by the ADV Acoustic Doppler Flowmeter. The flowmeter was arranged in two columns, the vertical shoreline direction and the parallel shoreline direction. The experiment used two kinds of waves, including regular waves and irregular waves, which are both unidirectional waves. The sampling frequency was 20 Hz, the acquisition time interval was 0.05s, the acquisition time of regular wave was 600 seconds, and the acquisition time of irregular wave was 720 seconds.

**Experimental Results**

Figure 2 shows dimensionless velocity distribution of mean longshore current, where the maximum value $v_{\text{max}}$ and the breaking zone width $x_b$ is used as reference quantities of dimensionless.

The experimental results show that the maximum value of mean longshore current under 1:100 slope have a downward trend on the nearshore side, also, a concave trend on the offshore side. It has similar characteristics with the Allen Curve. However, the maximum value of mean longshore current under 1:40 slope has a convex trend on the nearshore side. Allen Curve cannot reflect the characteristics of the nearshore velocity distribution under this situation, while well for the velocity distribution of the offshore side. Therefore, it is necessary to find a more suitable curve fit function.

**Curve Fit Analysis**

**Method**

By preliminary experimental curve fit of mean longshore current dimensionless velocity distribution, Rational, Gauss and Allen functions can properly reflect the characteristics of mean longshore current dimensionless velocity distribution under 1:100 slopes. In terms of 1:40, three kinds of functions cannot well reflect the characteristics of mean longshore current dimensionless velocity distribution. Therefore, new function defined by the form of piecewise function is introduced. There is a brief introduction to selected fitting function and method.
The basic form of Gauss model:

\[ y = \sum_{i=1}^{n} a_i^2 \exp\left[-\left(\frac{x-b}{c_i}\right)^2\right] \]  \hspace{1cm} (1)

\( a \) reflects the amplitude, \( b \) reflects the centroid position, \( n \) is the number of fitted peaks, \( c \) is related to the peak width, and the value ranges from 1 to 8, which is taken as 1 in this paper.

The basic form of random function:

\[ y = \sum_{i=1}^{n+1} p_i x^{n+1-i} / \left( x^m + \sum_{i=1}^{m} q_i x^{m-i} \right) \]  \hspace{1cm} (2)

\( n \) is the order for the numerator polynomial, ranges from 0 to 5. \( m \) is the order for the denominator polynomial, ranges from 1 to 5. The coefficient associated with \( x_m \) is always 1, which makes numerator and denominator unique when order for the polynomials same. Here \( n \) takes 0, \( m \) takes 5.

The basic form of the customized Allen Curve:

\[ V(x) = c_0 x^2 \exp\left[-\left(\frac{x}{\alpha}\right)^n\right] \]  \hspace{1cm} (3)

\( x \) is the vertical distance from the offshore line, \( c_0, \alpha \) and \( n \) are the undetermined coefficients controlling the longshore current distribution, \( n \) is usually an integer.

In terms of dealing with the experimental results, MATLAB software will be used. Firstly, using Allen Curve to fit, its basic form will be entered in the fitting box. Under the circumstances of 1:100 slope, irregular waves, and a given fitting parameter \( n (n=3) \), the software will return parameters \( c_0, \alpha \), and Adjusted R-square (replaced with AR as below). AR is an important parameter which evaluates good or bad fitting and is one of forms of expression of the coefficient \( R \). It is supposed that a set includes \( y_1, \ldots, y_n \), a total of \( n \) observed values, the corresponding model prediction values are respectively \( f_1, \ldots, f_n \). Defining the mean observed value \( \bar{y} \), the expression of \( R \) will be acquired:

\[ R = 1 - \frac{1}{\sum_i (y_i - f_i)^2 / \sum_i (y_i - \bar{y})^2} \]  \hspace{1cm} (4)

Unlike \( R \), AR resizes its value based on the number of variables in the fitting model. If there are more useless variables added to the fitting model, its value will decrease. If there are more useful variables added to the fitting model, its value will increase. The expression of AR:

\[ AR = 1 - \frac{(1 - R^2)(n-1)}{n-k-1} \]  \hspace{1cm} (5)

In the equation 5, \( n \) is the number of data points in the sample, \( k \) is the number of variables in the model, excluding constants. When \( AR \) tends to 0, the relative reference value of fitting expression is low. As \( AR \) value increases and tends to 1, independent variables can better reflect dependent variables, and the variation caused by independent variables accounts larger in total variation.

Repeating the above operation and using Gauss and Rational function model to fit, MATLAB will return the corresponding fitting parameters including \( AR \).

When the slope is 1:40, the fitting method of former experimental data is reused to draw the curve. There is convex trend on the velocity distribution of 1:40 mean longshore current nearshore side, which is not suitable for Allen Curve. But for the offshore side, its characteristics can be described. Since the downward parabola shows a convex trend at left side of symmetry axis, this satisfies the experimental data of nearshore side. Therefore, nearshore side will be considered as a quadratic polynomial model, offshore side as Allen Curve model, in the form of a custom piecewise function:

Nearshore side:
\[ y_2 = p_1 x^2 + p_2 x + p_3, 0 \leq x < x_0 \]  \hspace{1cm} (6)

Offshore side:

\[ y_1 = c_0 x^2 \exp(-x/a^n), x \geq x_0 \]  \hspace{1cm} (7)

In the fitting process of applying equation 6 and 7 about the demarcation point of the quadratic function and Allen function, smooth continuous constraints must be satisfied. The first derivative value and the function value of the nearshore and offshore sides are equal at the demarcation point:

\[
2p_1 x_0 + p_2 = 2c_0 x_0 \exp\left(-\frac{x_0^2}{a^2}\right) - 2c_0 x_0^3 \exp\left(-\frac{x_0^2}{a^2}\right) \frac{1}{a^2}
\]  \hspace{1cm} (8)

\[
p_1 x_0^2 + p_2 x_0 + p_3 = c_0 x_0^2 \exp\left(-\frac{x_0^2}{a^2}\right)
\]  \hspace{1cm} (9)

Among them, \( p_1, p_2, p_3 \) are parameter values, \( x_0 \) is the demarcation point of two segments.

In the beginning, Allen curve fit is performed to the velocity distribution of offshore side of three sets of data, the value of \( c_0, a \) and \( n \) can be determined. At that time, the right end term of equation 8 and 9 becomes a known term, so the value of \( p_2 \) and \( p_3 \) can be expressed by \( p_1 \).

Since the right side of equation 8 is known, the right end term is denoted as \( y' \):

\[
p_2 = y' - 2x_0p_1
\]  \hspace{1cm} (10)

Since the right side of equation 9 is known, the right end term is denoted as \( y_0 \):

\[
p_3 = y_0 - p_1 x_0^2 - p_2 x_0
\]  \hspace{1cm} (11)

Now, the quadratic function can just be represented by one parameter \( p_1 \). By the experimental data of nearshore side, \( p_1 \) can be determined, then \( p_2 \) and \( p_3 \) can be further determined by equation 10 and 11, and eventually the entire custom piecewise function can be determined.

Finally, fitted curves are drawn in the graph. The horizontal axis \( x \) is the distance between observation point and the shoreline, and the vertical axis \( V \) is the mean longshore current velocity.

**Results**

Figure 3 shows experimental results of three mean longshore current waves under 1:100 slope: Case 1 \((T = 1 \text{ s}, H_{\text{rms}} = 2.56 \text{ cm})\), Case 2 \((T = 1.5 \text{ s}, H_{\text{rms}} = 2.56 \text{ cm})\) and Case 3 \((T = 2 \text{ s}, H_{\text{rms}} = 2.56 \text{ cm})\).

As can be seen from three wave cases in the graph, Rational, Gauss and Allen curves can properly reflect the experimental characteristics of mild slope longshore current.

From the perspective of \( AR \), they are generally close. In terms of Case 1, Allen curves are slightly better. For Case 3, Gauss curves are superior to other. For Case 2, three \( AR \) values are similar.

From the perspective of complexity of curve basic form, Rational and Gauss curves need a polynomial to sum up, and the number of its polynomial items is changeable. In contrast, there are only three pending coefficients for Allen curves, Allen curve is simpler than other two forms.

Generally, the Allen curve fit function is more representative and universal under 1:100 mild slope.
Distribution.

between the abscissa value of data and curve. Finally, the accuracy of fitting offshore data points is
value of nearshore parts of custom function is about 0.87, other two are higher, both around 0.96. All AR values of offshore parts are between 0.97 and 0.98, which higher than other three types.

Rational function cannot match the data point where \( x \) tends to 0. The abscissa value reached by the peak of Gauss curve is always greater than the abscissa value reached by the peak of the entire data set. These curves cannot meet the convex trend of nearshore side data points. For offshore side data, points of Case 4 are mostly below the curve, points of Case 5 and 6 are mostly above the curve.

The curve fit result of custom piecewise function is better than former three methods. Firstly, the line is convex, which satisfies the trend of nearshore data points. Secondly, it can correctly fit the data points that \( x \) tends to 0. Thirdly, when reaches the peak value, there is not a big difference between the abscissa value of data and curve. Finally, the accuracy of fitting offshore data points is greatly improved, the segmented lines pass through most of the data points.

Therefore, the custom piecewise function can better fit mean longshore current velocity distribution.

Conclusion

Based on longshore current experimental results under 1:100 and 1:40 slope, this essay adopts MATLAB fitting toolbox and uses Rational, Gauss, Allen curves and customized piecewise functions to fit and analyze the mean longshore current velocity distribution. The results show that:

1. The maximum value of the 1:100 mean longshore current is concave on the nearshore side while the maximum value of the 1:40 mean longshore current is convex on the nearshore side.

2. Rational, Gauss and Allen curves can well represent characteristics of mean longshore current velocity distribution under 1:100 slope, but fail to express data characteristics under 1:40 slope. The custom piecewise function can describe velocity distribution features under 1:40 slope.

3. In terms of 1:100 slope, compared with Rational and Gauss curves, Allen curves are simpler and smoother, which is more conducive to engineering application and promotion.
Allen curves can only reflect the trend of velocity distribution under 1:100 mean longshore current experiment results. But in case of 1:40, Allen curves cannot reflect the trend of velocity distribution of nearshore side. So, based on three given functions, new function is defined by the form of piecewise function, which will greatly improve the accuracy of fitting data points.

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References
[1] Brebner A., Kamphuis J.W. Model tests on relationship between deep-water wave characteristics and longshore currents [J]. Queens Univ. Civil Eng. Res. Rep., 1963, 31:1-25.
[2] Thornton E.B. Variation of longshore current across the surf zone. Proc. 12th Coastal Engr. Conf, ASCE1970.
[3] Bowen A.J. The generation of longshore currents on a plane beach[J]. J. Mar. Res., 1969, 27:206-215.
[4] Longuet-Higgins M.S. Longshore Currents Generated by Obliquely Incident Sea Waves 2[J]. Journal of Geophysical Research, 1970, 75(33):6790-6801.
[5] Komar P.D. Nearshore Currents: Generation by Obliquely Incident Waves and Longshore Variations in Breaker Height[J]. Nearshore Sediment Dynamics and Sedimentation, 1975:1745.
[6] Galvin C J, Eagleson P S. Experimental study of longshore currents on a plane beach[J]. U.S. Army Coast. Eng. Res. Center, Tech. Mem, 1965, 10: 1-80.
[7] Birkemeier W A, ASCE M Al L C E E. Delilah, Duck94 & Sandyduck: Three nearshore field experiments[C]// Proceedings of the 25th International Conference on Coastal Engineering. Orlando, FL, 1996.
[8] Galvin C J, Eagleson P S. Experimental study of longshore currents on a plane beach[J]. U.S. Army Coast. Eng. Res. Center, Tech. Mem, 1965, 10: 1-80.
[9] Allen J.S., Newberger P.A., Holman R.A. Nonlinear shear instabilities of alongshore currents on plane beaches [J]. Journal of Fluid Mechanics, 1996, 310:181-213.
[10] Study of the Feature of Longshore Current and its Instability on Mild Beach slope[D]. Dalian University of Technology, 2015.