Possible test for $CPT$ invariance with correlated neutral $B$ decays

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Abstract

We study breakdown of $CPT$ symmetry which can occur in the decay process $B\bar{B} \rightarrow l^\pm X^\mp f$ with $f$ being a $CP$ eigenstate. In this process, the standard model expectations for time ordered semi-leptonic and hadronic events, i.e. which of the two decays takes place first, can be altered in the case that there is a violation of the $CPT$ symmetry. To illustrate this possibility, we identify and study several time integrated observables. We find that an experiment with $10^9 B\bar{B}$ pairs, has the capability for improving the bound on $CPT$ violating parameter or perhaps observe $CPT$ violation.
1 Introduction

In recent years the physics of the standard model (SM) has moved to the investigation of flavor physics and in particular CP violation in the B meson decays. In the case of the neutral B decays, the mixing induced CP violation measurements involves a good knowledge of both the mass and width differences for which there is an extensive literature.

Most of the analyses involving physics of neutral B mesons assume the validity of CPT. The B meson with their special properties provide unique opportunities of testing the CPT symmetry. Given the eventual need for an accurate description of the SM CP parameters, and/or tests for new physics, it is pertinent to also account for a possible violation of CPT invariance and its consequences. In this regard, it is noteworthy that the existing experimental limits on CPT violation are not very stringent and thus it becomes necessary to allow for this possibility in measurements of B decays. An argument for this was presented in [1] which questions the validity of CPT theorem for partons which are confined states.

It is usually assumed that CPT is an exact symmetry which is hard to break. One of the first attempts to consider the (spontaneous) breaking of CPT came from string physics [2]. For instance, CPT may not be a good discrete symmetry in many higher dimensional theories of which the SM is an effective low energy description [3]. In addition, the existence of mixed non-commutative fields could also break CPT [4]. There are many tests for CPT symmetry and we refer to [5] for a review on this subject.

We present in this article a study of CPT violations that originate on the mass matrix and the analysis is model independent. We express the effects in terms of a complex parameter $\delta$, whose presence modifies the expressions for width and mass differences and affects the time development of correlated B states. Precise measurements of the time development of the states are sensitive to the presence of CPT violation. However, such studies may require large number of events and hence it is preferable to have integrated rates. This motivates us to consider correlated decays of B mesons which are time ordered. The procedure presented here requires a time ordering of leptonic/hadronic events (which decay happens first) without demanding any detailed time development of states. One reason for following this time ordering is because it enables for a direct extraction of the width differences [6]. The same procedure is also expected to be sensitive to the presence of nonzero $\delta$ values which can affect the width difference. We use the time ordered rates to form asymmetries which are sensitive to CP violation. We find that our results depend on the width difference $y = \Delta \Gamma/(2\Gamma)$ and thus for large CPT violating parameter $\sim O(y)$, the effects of CPT can become significant. Our analysis is more sensitive to $B_s$ decays where the effects of CP violation can be neglected. We expect that the results obtained here will complement the existing tests for CPT violation, and in particular for the $B_s$.
system where there is no information available as yet. For a recent general analysis of CPT violation in neutral mesons, and its extraction we refer to [7]. Briefly, the possibility of CPT violation has been studied in B decays using both time dependent and integrated methods. There are several analyses performed which in most cases also involve a detailed time dependent study and/or flavor tagging [9,8,10,11,12]. In comparison with other known CPT studies, we find certain qualitative similarities as well as differences. Following our analysis, we find that we can restrict the strength of the CPT violating parameter to about 10% which is similar to an earlier detailed estimate [8]. We point out that if the strength of CPT violating parameter is below this limit (∼10%), then using our approach, it is hard to pinpoint genuine CPT effects. In general, we find a sensitivity to CPT effects even if the width difference is vanishingly small. This we expect due to the particular time ordering procedure which can also extract corrections to the width difference due to nonzero δ. In general, in the presence of nonzero width difference, the CPT violating observables can measure both the real and imaginary parts of the CPT parameter; while on the other hand, for zero width difference, the effects are sensitive only to the real part of the CPT violating parameter [11]. The latter feature is similar to what we find in our present analysis when we assume negligible CP effects. On the other hand, as observed in [11], for zero width difference, the CPT violating observable is sensitive to the imaginary part of the CPT violating parameter. This is different in our case (when we neglect CP violation), as we shall show when we discuss the Bs system.

Our paper is organized as follows. In the next section, we present the general formalism for the time evolution of a neutral B meson in the presence of CPT violation. In section 3 we define the observables which can extract signals for CPT violation and are theoretically clean. This is followed by a brief numerical analysis for a specific decay channel and we compare the results with the Bd system. We finally conclude with a brief summary in section 4.

2 The time evolution

2.1 Basic setup and definitions

In this section, we review the basic formalism for the decay of correlated \(B\bar{B}\) pair in the presence of CPT violation [13]. Starting with a quantum state which is a linear combination of the \(B\) and \(\bar{B}\) states denoted as

\[
\langle \psi(t) \rangle = \psi_1(t) |B\rangle + \psi_2(t) |\bar{B}\rangle ,
\]

(1)
the time evolution of the $B\bar{B}$ system can be described by a two dimensional Schroedinger equation

$$i\frac{d}{dt}\begin{pmatrix} \psi_1(t) \\ \psi_2(t) \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} \psi_1(t) \\ \psi_2(t) \end{pmatrix}.$$  \hspace{1cm} (2)

The mixing matrix is non-Hermitian and can be written as

$$M = M - \frac{i}{2} \Gamma,$$  \hspace{1cm} (3)

with $M$ and $\Gamma$ being Hermitian $2 \times 2$ matrices ($M = M^\dagger, \Gamma = \Gamma^\dagger$). Invariance under CPT gives

$$M_{11} = M_{22} \Rightarrow M_{11} = M_{22} \text{ and } \Gamma_{11} = \Gamma_{22},$$  \hspace{1cm} (4)

while there is no constraint on the off-diagonal elements. As a result, to test CPT invariance one needs to parametrize the difference of the diagonal elements of the mixing matrix. A sensible parameterization has to be invariant according to a re-phasing of the meson states \cite{14}. In particular, when a re-phasing of the type $|B'\rangle \rightarrow e^{i\gamma} |B\rangle$ is done, the anti-meson state is altered according to $|\bar{B}\rangle \rightarrow e^{-i\gamma} |\bar{B}\rangle$. Following this, we have in the matrix elements

$$M_{11} \rightarrow M_{11} ; \ M_{22} \rightarrow M_{22} ; \ M_{12} \rightarrow e^{-2i\gamma} M_{12} ; \ M_{21} \rightarrow e^{2i\gamma} M_{21}.$$ \hspace{1cm} (5)

This means the diagonal elements in (3) remain the same after re-phasing while the product of the off-diagonal elements are re-phase invariant. Besides, the eigenvalues of (3) are re-phase invariant. This leads to several possibilities to parameterize CPT violation and in this analysis, we choose a parameter

$$\delta = \frac{M_{22} - M_{11}}{\sqrt{M_{12} M_{21}}}. \hspace{1cm} (6)$$

In addition, there are other possible parameterizations which are rephase invariant and have been used for CPT studies. Among them we refer to parameterization in references \cite{1,13,16}.

In the presence of $\delta \neq 0$, the width and mass differences of the two $B$ states are obtained by calculating the eigenvalues of (3). We obtain these to be

$$\lambda_1 = M_{11} + \sqrt{M_{12} M_{21}} \left( \sqrt{1 + \frac{\delta^2}{4}} + \frac{\delta}{2} \right),$$

$$\lambda_2 = M_{22} - \sqrt{M_{12} M_{21}} \left( \sqrt{1 + \frac{\delta^2}{4}} + \frac{\delta}{2} \right). \hspace{1cm} (7)$$
We define $M_{ij} = m_{ij} - i\Gamma_{ij}/2$ and

$$
\lambda_1 - \lambda_2 = -2\sqrt{M_{12}M_{21}} \sqrt{\delta^2/4 + 1} = \Delta m - \frac{i}{2}\Delta \Gamma. 
$$

(8)

Therefore upon equating the real and imaginary parts in (8) results in the width difference and mass difference

$$
\Delta \Gamma \equiv \Gamma_1 - \Gamma_2 = -4|m_{12}| \text{Im} \sqrt{1 - \frac{1}{4|m_{12}|^2} - i \text{Re} \frac{\Gamma_{12}}{m_{12}} \sqrt{\frac{\delta^2}{4} + 1},
$$

$$
\Delta m \equiv m_1 - m_2 = 2|m_{12}| \text{Re} \sqrt{1 - \frac{1}{4|m_{12}|^2} - i \text{Re} \frac{\Gamma_{12}}{m_{12}} \sqrt{\frac{\delta^2}{4} + 1}. 
$$

(9)

Furthermore, from the eigenvalue equation,

$$
M_{11}p_{1,2} + M_{12}q_{1,2} = \lambda_{1,2}p_{1,2},
$$

$$
M_{21}p_{1,2} + M_{22}q_{1,2} = \lambda_{1,2}q_{1,2},
$$

(10)

we obtain the ratios

$$
\frac{q_1}{p_1} = \sqrt{\frac{M_{21}}{M_{12}}} \left[ \sqrt{1 + \frac{\delta^2}{4} + \frac{\delta}{2}} \right],
$$

$$
\frac{q_2}{p_2} = \sqrt{\frac{M_{21}}{M_{12}}} \left[ \sqrt{1 + \frac{\delta^2}{4} - \frac{\delta}{2}} \right].
$$

(11)

Introducing the mass eigenstates for the $B$ mesons as

$$
|B_1\rangle = \frac{1}{\sqrt{p_1^2 + q_1^2}} \left[ p_1 |B\rangle + q_1 \overline{|B\rangle} \right],
$$

$$
|B_2\rangle = \frac{1}{\sqrt{p_2^2 + q_2^2}} \left[ p_2 |B\rangle + q_2 \overline{|B\rangle} \right],
$$

(12)

the time development of $B$ and $\overline{B}$ is evaluated to be

$$
|B(t)\rangle = \frac{1}{1 + \omega} \left[ f_+ |B\rangle + \kappa_1 f_- |\overline{B}\rangle \right],
$$

$$
|\overline{B}(t)\rangle = \frac{1}{1 + \overline{\omega}} \left( \overline{f_+} |\overline{B}\rangle + \overline{\kappa_1} f_- |B\rangle \right),
$$

(13)

with

$$
f_+ = e^{-i\lambda_1 t} + \omega e^{-i\lambda_2 t}; \overline{f_+} = e^{-i\lambda_1 t} + \frac{1}{\omega} e^{-i\lambda_2 t},
$$

$$
f_- = e^{-i\lambda_1 t} - e^{-i\lambda_2 t}; \kappa_1 = \frac{q_1}{p_1} = \frac{1}{\overline{\kappa_1}}; \omega = \frac{p_2 q_1}{q_2 p_1} \text{ and } \overline{\omega} = \omega^*. 
$$

(14)
The information on \( CPT \) violation is encoded in the complex parameter

\[
\omega = \frac{p_2 q_1}{q_2 p_1} = 1 + \delta + \frac{\delta^2}{2} + \text{higher order terms in } \delta .
\]  

(15)

Given that \( \delta \) is re-phase invariant, \( \omega \) is also re-phase invariant and deviates from unity in case of \( CPT \) violation.

### 2.2 The decay channel \( B\bar{B} \rightarrow l^\pm X^\mp f(\bar{f}) \)

In the previous section we described the formalism for \( CPT \) violation that occurs in the mass matrix. We apply it to the decays of a correlated \( B\bar{B} \) pair where one of them decays semi-leptonically while the other decays hadronically, i.e., \( B\bar{B} \rightarrow l^\pm X^\mp f(\bar{f}) \). The amplitude for a neutral \( B \) meson decaying into a final state with a lepton \( l^+ X^- \) at time \( t_0 \) and the \( \bar{B} \) decaying in to a final state \( f \) at time \( t \) can be expressed as

\[
A[l^+(t_0), \bar{f}(t)] = \langle X^- l^+ \mid B(t_0) \rangle \langle f \mid \bar{B}(t) \rangle + C \langle X^- l^+ \mid \bar{B}(t_0) \rangle \langle f \mid B(t) \rangle ,
\]  

(16)

with \( C \) denoting the charge conjugation of the \( B\bar{B} \) pair. The individual decay amplitudes for the hadronic channels are

\[
\langle f \mid B \rangle = A_1 e^{i\Phi_1} e^{i\eta_1} ; \quad \langle f \mid \bar{B} \rangle = A_2 e^{i\Phi_2} e^{i\eta_2} ,
\]

\[
\langle \bar{f} \mid B \rangle = A_2 e^{-i\Phi_2} e^{i\eta_2} ; \quad \langle \bar{f} \mid \bar{B} \rangle = A_1 e^{-i\Phi_1} e^{i\eta_1} .
\]  

(17)

In (17), the \( A_i \) denote the absolute values for the amplitudes and \( \eta_i \) and \( \phi_i \) are the strong and weak phases respectively. Using this, along with the notations

\[
r = \frac{A_2}{A_1} ; \quad \Phi = \Phi_2 - \Phi_1 - 2\phi_m ; \quad \eta = \eta_2 - \eta_1 ; \quad \xi = |\xi| e^{2i\phi_m} = \frac{e^{2i\phi_m}}{\sqrt{1 + \text{Re}[\delta]}} ,
\]  

(18)

with the phase due to mixing denoted by \( \phi_m \), the amplitudes are now:
\[ A[l^+ f] = \frac{R}{(1 + \omega)^2} |\xi| F A_1 e^{i\Phi_f} e^{i\eta} e^{2i\phi_m} \]

\[
\left\{ [f_+(t_0) f_-(t) - f_-(t_0) f_+(t)] + \frac{R}{|\xi|} e^{i\Phi_f} e^{i\eta} [f_+(t_0) f_-(t) - f_-(t_0) f_+(t)] \right\},
\]

\[ A[l^+ \bar{f}] = \frac{R}{(1 + \omega)^2} \bar{F} A_1 e^{-i\Phi_f} e^{i\eta} \]

\[
\left\{ [f_+(t_0) \bar{f}_+(t) - \bar{f}_+(t_0) f_-(t)] + \frac{R}{|\xi|} e^{i\Phi_f} e^{i\eta} [f_+(t_0) \bar{f}_+(t) - \bar{f}_+(t_0) f_-(t)] \right\},
\]

\[ A[l^- f] = \frac{R}{(1 + \omega)^2} F A_1 e^{i\Phi_f} e^{i\eta} \]

\[
\left\{ [f_-(t_0) f_-(t) - \bar{f}_-(t_0) f_+(t)] + \frac{R}{|\xi|} e^{i\Phi_f} e^{i\eta} [f_-(t_0) f_-(t) - \bar{f}_-(t_0) f_+(t)] \right\}.
\]

(19)

In (19), \( F \) denotes the amplitude for the decay \( B \to l^+ X^- \) and the corresponding amplitude for the \( CP \) conjugated process is denoted by \( \bar{F} \). The decay to a lepton is measured at \( t_0 \) while \( t \) is the time when the decay to state \( f \) occurred. An explicitly time dependence in the l.h.s. of (19) is assumed. The \( B \bar{B} \) pair is taken to be the one produced at a \( \Upsilon \) resonance and hence, the \( B \bar{B} \) charge parity, \( C = -1 \) in (16). In addition, the state \( f \) is chosen to be a \( CP \) odd eigenstate. This sets \( \eta = 0 \) and \( r = -1 \).

3 The \( CPT \) violating rate

Following the above construction, for the decay process \( B \bar{B} \to l^\pm X^- f \) we define two time correlated observables: (i) \( R_S \) denotes the number of events in which the hadronic decay precedes the semi-leptonic one (which in our context the time ordering is \( t < t_0 \)) and (ii) similarly, we define the number of events where the semi-leptonic decay precedes the hadronic decay denoted by \( R_L \). To illustrate, if we choose positively charged leptons, we have the rates

\[
R_S [l^+ f] = \int_0^\infty dt_0 \int_0^{t_0} dt |A[l^+ f]|^2 ; \quad R_L [l^+ f] = \int_0^{t_0} dt_0 \int_0^\infty dt |A[l^+ f]|^2.
\]

(20)
Using (20), for CP eigenstates $^1$ we can define the following ratios

$$R_1^\pm = \frac{R_S [l^+ f] \pm R_L [l^− f]}{R [l^+ f] + R [l^− f]} ,$$

$$R_2 = \frac{R_L [l^+ f] - R_L [l^− f]}{R [l^+ f] + R [l^− f]} ,$$

$$R_3 = \frac{R_S [l^+ f] - R_S [l^− f]}{R [l^+ f] + R [l^− f]} .$$ \hspace{1cm} (21)

where the rates without the subscripts $L$ or $S$ denote total time integrated rates without any time ordering. The exact expressions for the above ratios can be obtained by substituting the expressions for the rates given in the appendix (section [A] of this paper. Due to the extensive nature of the analytic expressions involved, we avoid from presenting them in the main text as they are also not illustrative.

### 3.1 Numerical analysis

The effects discussed in this section were calculated using the exact expressions for the event rates which are given in the appendix (section [A] of this article. In these expressions and for our specific case, where we are interested in $CP$ odd eigenstates, we set $\eta = 0$ and $r = −1$. In this analysis, we have varied both Re$[\delta]$ and Im$[\delta]$ in the range between $−0.5$ to $0.5$. It has been shown in [12] that this range is not excluded yet by the of the recent Belle data, according to which $|m_B - m_{\bar{B}}| \sim 10^{-14} m_B$ [17]. Figs [1] and [2] show the results for the $B_s$ system and for appropriate values of the variables. We mainly analyse the $B_s$ system and also briefly present the results for the $B_d$ system where one has to account for $CP$ effects arising from mixing.

In Fig. [1] we show the ratio $R_1^-$ as a function of Re$[\delta]$ and for three values of $y$ consistent with the current estimates [13]. In the $CPT$ conserving limit, the value for $R_1^-$ can be read off on the axis Re$[\delta] = 0$ and its range is $−0.10 \leq R_1^- \leq −0.025$. When Re$[\delta] \neq 0$ there are deviations from this range and for Re$[\delta] \simeq ±0.1$ the deviations for $R_1^-$ is $\sim O(0.1)$ from the central line. Thus if a deviation of that order is observed it will come either from violation of the $CPT$ symmetry or there is a large width difference of $y \geq 0.12$ either of which is very interesting.

Taking $y = 0.12$ and $x = 19$, in Fig. [2] we show the ratio $R_2$ as functions of Re$[\delta]$ and Im$[\delta]$. Again there are sizable deviations when Re$[\delta] \neq 0$. The structure of the curves in Fig. [1] and Fig. [2] can be accounted for examining the small $\delta$ limit of the ratios in (21).

$^1$This choice sets $f = \bar{f}$ in (16).
For instance, defining \( y = \Delta \Gamma / (2\Gamma) \), \( x = \Delta m / \Gamma \), in the small \( \delta \) limit which we take such that \((\text{Re}[\delta], \text{Im}[\delta]) \leq 0.1\), we find

\[
R_1^- \approx \frac{y}{2} + \text{Re}[\delta] \left[ \frac{x^2 + y^2}{2(x^2 + 1)} \right] + O(\delta^2).
\]

Thus, a linear dependence of \( R_1^- \) as a function of \( \text{Re}[\delta] \) is evident near the origin of the plot in Fig. 1. Note that in the limit \( y = 0 \) and in the absence of any CP or CPT violation we would expect an equal number of \( l^+ \) and \( l^- \) events.

Similarly, we observe in Fig. 2 that for small \( \text{Re}[\delta] \) a linear behavior in \( R_2 \) and this is again understood by examining the \( R_2 \) in the small \( \delta \) limit wherein

\[
R_2 \approx \frac{1 - y}{2(1 + x^2)} \left( \text{Re}[\delta] \frac{x^2 - y}{2} + \text{Im}[\delta]x(y + 1) \right) \approx \frac{\text{Re}[\delta]}{4} \text{ for } y \ll x.
\]

In Fig. 2 for all values of \( \text{Re}[\delta] \), the sensitivity to \( \text{Im}[\delta] \) is almost negligible. This feature is shown by the flatness of \( R_2 \) as a function of \( \text{Im}[\delta] \). In addition without imposing the small \( \text{Re}[\delta] \) limit, we have also numerically checked the dependence of \( R_1^- \) on \( \text{Im}[\delta] \) and we find it to be too small \( \sim O(0.001) \). This can be attributed to the largeness of \( x \) for the \( B_s \) system which is also brought out in our approximate analytic expressions discussed above.

As a consistency check, upon using (22) and (23) we easily find the relation

\[
R_1^- - 2R_2 = \frac{y}{2}.
\]

This relation can be verified by combining the results in Fig. 1 and Fig. 2 where we find our numerical results are consistent with (24) for the small \( \delta \) limit. Furthermore, as is evident, the ratio \( R_2 \) is nonzero only in the presence of CPT violation and serves as consistency check for (24). In addition, we note that in the case of \( B_s \) system, for \( y \ll x \), in the small \( \delta \) limit, we have the approximation

\[
R_2 \approx R_3 = \frac{\text{Re}[\delta]}{4} + \frac{y\text{Im}[\delta]}{2x} \approx \frac{\text{Re}[\delta]}{4}.
\]

This calls for the following remark. As a consequence of the equality, the sum of these two ratios \((R_2 + R_3)\) would prove to be a good observable as it does not require time ordering. However, since the equality is a result of a specific limit, we may still plot them separately.

It is interesting to ask what is required in experiments in order to observe these CPT violating effects. First of all, the effects which are discussed here are for \( C = -1 \) state, which is satisfied in \( e^+e^- \) colliders; provided these machines are tuned at higher energies to be able to produce \( B_s\bar{B}_s \) pairs. A promising channel is \( B_s\bar{B}_s \to l^\pm X^\mp J/\psi\phi \) where the lepton and \( J/\psi\phi \) are detected at two different times with the identification of the time ordering.
at the decays, i.e. which decay happened first. The measurement of the ratios requires an accuracy of 1% and we need about $10^4$ decays to the channels under consideration. Given the semi leptonic branching fraction of $B_s$ to be 10% and the decay to $J/\psi \phi$ to be $10^{-3}$ we have a branching fraction for this process of $\sim 10^{-4}$. Consequently, we require roughly $10^8 B_s \bar{B}_s$ pairs produced through a $C = -1$ state. This also suggests that with the comparable number of events, a reasonable bound can be set for $\text{Re}[\delta]$. Alternatively, if no significant deviation from the above mentioned range for say $R_{-1}$, is found, then one can conclude that the data are consistent with zero or rather small $CPT$ violating effects. Here, by small, we mean $\text{Re}[\delta] < 0.1$. This limit can be improved if in future we have better handle on the values of $y$ which currently suffers from large errors.

We now turn to discuss briefly the effects of $CPT$ violation for the $B_d$ system and we choose the process, $B_d \bar{B}_d \rightarrow l^\pm X^\mp J/\psi K_S$. In contrast to the $B_s$ mesons, for the case of the $B_d$ system, one cannot neglect the presence of the $CP$ violating phase. We show in Figs. 3 and 4 the variation of $R_1^\pm$ with $\text{Re}[\delta]$ and $\Phi$. In these two plots, we have set $\text{Im}[\delta] = 0$ while setting $x = 0.755$ and $y = 10^{-3}$. The variation of $R_1^-$ with $\text{Re}[\delta]$ is significant and we show this in Fig. 5. The width in the plot is due to the allowed range of values in $x$ and $\Phi$. Presently, their ranges are $\sin \Phi = 0.735 \pm 0.055$ [19] with $x = 0.755 \pm 0.015$ and $y \sim 10^{-3}$ [20]. The range of $R_1^-$ is now smaller as compared to the $B_s$ case and will require more events to reach the bound of $\text{Re}[\delta] \leq \pm 0.1$. Finally, the ratio $R_3$ depends on both $\text{Re}[\delta]$ and $\text{Im}[\delta]$ and illustrated in Fig. 6. For this plot, we use the values $\sin \Phi = 0.735$, $x = 0.755$ and $y = 10^{-3}$. It is interesting that, we can observe the impact of $\text{Im}[\delta]$ which was not so significant in other plots. In contrast to this analysis, the effects of $\text{Im}[\delta]$ (with $\text{Re}[\delta] = 0$) on width measurements and $CP$ asymmetries for the $B_d$ mesons has been examined in reference [12].

Finally, we address the question of nonzero $CP$ violation. Although, the present analysis is most effective for the $B_s$ system, we note that the ratios in [21] can still be used to test for $CPT$ violating effects even if the system has a nonzero $CP$ violation. In the pure $CP$ conserving limit, for instance, independent of $y$, the ratio $R_1^+$ can show a numerical deviation from the value of $1/2$ in the presence of $CPT$ violation. This becomes evident by examining the small $\delta$ limit where

$$
R_1^+ \approx \frac{1}{2} + \text{Re}[\delta] \left( \frac{y}{2} \right) - \text{Im}[\delta] \frac{x(1-y^2)}{2(1+x^2)}.
$$

In the presence of $CP$ violation the ratio $R_1^+$ gets modified as follows, $R_1^+ \rightarrow R_1^+ + r^+$, where the typical strength of $r^+$ is a linear combination of the form $r^+ \sim (\sin \Phi/x, \cos \Phi \cdot \text{Re}[\delta]/x)$. Thus, for the $B_s$ system, given the expectations for $x \sim 19$ and a small $CP$ phase such that $\sin \Phi \sim 10^{-2}$ [21], the ratio $R_1^+$ is sensitive to $CPT$ effects for all values of $(\text{Re}[\delta], \text{Im}[\delta]) \geq$
In the case of CP and CPT conservation we expect the rates \( R_S[l^\pm f] = R_L[l^\pm f] = 1/2(R[l^- f] + R[l^+ f]) \) which means that the decays into hadrons before the leptonic decay are equal to the hadronic decays which occur after the leptonic decay. The equality of the above decay rates is modified in the case that there is CP or CPT violation. For \( B_d \) decays the \( CP \) violation has been observed and will produce such a difference; an interesting question here is whether the CP-violation present in \( R_1^+ \) is consistent with the CP violation established in other channels, like \( J/\psi K_s \). Deviations will be attributed to new physics and/or CPT violation. Furthermore, in light of the recent discrepancies in the measurements of \( CP \) violation in the decays \( B_d \rightarrow J/\psi K_s \) and \( B_d \rightarrow \phi K_s \) one could envisage new physics contribution involving new penguin operators \(^{22}\). Such operators could induce modifications to mixing matrix defined in \(^3\) and hence modify the expected signals from the ratio \( R_1^+ \). In the present analysis, we do not consider these possible corrections which may be required if the present signals for new physics persists and becomes statistically significant. The effects in \( B_s \) decays will be even more interesting, because it is still possible that in extended models \( \sin(\Phi)/x \) is larger than 0.04 and comparison among various channels, like \( J/\psi K_s \) with \( \phi K_s \) will be interesting to establish the origin of the effect; whether the new physics originates in the CP or CPT sector or from both.

However, we note that in principle, by strictly measuring \( R_1^+ \) alone, one cannot ensure the deviation from 1/2 as an unambiguous signal for pure CPT violation or pure CP violation. On the other hand, any corrections due to \( CP \) can affect the ratio \( R_1^- \) only at the sub-leading level which is suppressed by \( (\text{Re}[\delta], \text{Im}[\delta]) \) and hence is not significant. In the present numerical analysis, we consider much larger values \( (\text{Re}[\delta], \text{Im}[\delta]) \sim O(0.1) \) which are not excluded by current available data. Clearly, at this level the effects due to \( CP \) violation are expected to be small for the \( B_s \) system. In the case for the \( B_d \) system, the impact of \( \Phi \neq 0 \) can be important and must be studied explicitly in the analysis of the data.

### 4 Summary

We discussed indirect \( CPT \) violation as it appears in the \( B \) meson system. The breakdown of the \( CPT \) symmetry occurs in several theories and modifies the time development of the states. The development is characterized in terms of a parameter \( \delta \) which is phase-convention independent \(^{13}\). Consequences of the break down of the \( CPT \) symmetries manifests itself in the present development of the states. Our attention is addressed to the production of \( B_s \bar{B}_s \) and \( B_d \bar{B}_d \) pairs in a odd charge conjugation state. In the case of \( B_s \) and \( B_s \) system, the decay depends on the \( CPT \) violating parameter \( \delta \) while the influence
of the $CP$ phase is negligible. One way to observe the difference (due to $\delta$) is to study the time development of the decays. However, because of the large number of the events required, we propose time integrated rates. Once the $B_s \overline{B}_s$ pair is produced, we encounter two different decays; one of them can be semi-leptonic and the other one can be hadronic. We also defined a time-ordering among them, meaning which one of these events occurs first.

The time ordered ratio $R_S$ denotes all the events where the semi-leptonic decay follows the hadronic one; similarly, $R_L$ includes the events where the semi-leptonic decay occurs before the hadronic $[6]$. These two rates are different for two reasons: the width differences and because of $CP$ and/or $CPT$ violation. The ratios were calculated using the formulas given in the appendix and the results were presented in section $[5,1]$. We calculated the asymmetries involving $R_{L,S}$ as functions of the parameter $\delta$ and the observed width difference $y$. We summarised our numerical results in section $[5,1]$ and through Figs. $[1]$ to $[6]$. The formalism presented here holds for $B_d$ system as well and our conclusion is that experiments with $10^9 B_s \overline{B}_s$ or $B_d \overline{B}_d$ pairs will be able to restrict the $Re[\delta]$ smaller than $10\%$ or otherwise observe an effect. These considerations are within reach of the present experiments $[17,23]$.

It is important to note that due to the specific nature of the time ordering, the various ratios considered here are sensitive to $y$ and hence simultaneously to any $CPT$ pollution which can affect $y$. The impact of $CPT$ violation was shown qualitatively for the small $\delta$ limit. In this limit, for zero $CP$ violation, which is a good approximation for the $B_s$ system, the ratios show a very simple dependence on the parameters $x$ and $y$; thus allowing for a direct dependence on $\delta$ under suitable limits ($x \gg y$). We remark that for this limit, the impact of $CPT$ effects becomes sensitive to the strength of $Re[\delta]$ and $Im[\delta]$, more so to $Re[\delta]$ due to large $x \gg y$; and thus tests the magnitude and phase of the $CPT$ violating parameters. To get a qualitative feeling for the $CPT$ effects, we find from $[22]$ that for $Re[\delta] \sim y$, one can envisage the ratios to exhibit $CPT$ violating effects $\sim Re[\delta]$ and thus can be large$^2 \sim O(0.10)$. In a similar spirit, from $[24]$ we can expect deviations (here of $O(0.02)$) which is relatively independent of the parameters $x$ and $y$. To our knowledge, this situation is in contrast with many other interesting alternative methods known in the literature where such a simple dependence is perhaps not observed in the limit of a small $CPT$ violating parameter and thus show a different sensitive to $CPT$ effects than the method prescribed here.

$^2$As mentioned earlier, this effect is observable provided $Re[\delta]$ does not get washed out due to the errors in $y$. 


A Appendix

In this section, we present exact expressions for the rates calculated from (19) using the definition

\[
R_S[\lambda(f, \bar{f})] = \int_0^\infty dt \int_0^t dt |A[\lambda(f, \bar{f})]|^2,
\]

\[
R_L[\lambda(f, \bar{f})] = \int_0^\infty dt \int_0^t dt |A[\lambda(f, \bar{f})]|^2.
\]

(27)

In our notation, \(\Omega\) represents the phase of \(\omega \approx 1 + \delta\). Defining the overall prefactor

\[
K = \frac{1 + |\omega|^2 + 2|\omega| \cos \Omega}{2\Gamma^2|\omega|^2 (1 + x^2)(y^2 - 1)},
\]

(28)

we have:

\[
R_S[l^+ f] = -K \left\{ 2(x^2 + y^2)|\omega|^2|\xi|^2 + 2|\xi|\omega r \left[ (1 + y)(x^2 + y)|\omega| \cos(\eta + \Phi) \right. \right.
\]

\[+ (y - 1)(y - x^2) \cos(\eta + \Phi - \Omega) + x(1 + y)[|\omega| \sin(\eta + \Phi) + \sin(\eta + \Phi - \Omega)] \left. \right. \]

\[+ \left. \left. r^2[(1 + x^2)(1 + |\omega|^2 + y(|\omega|^2 - 1)) + 2(y^2 - 1)|\omega|(x \sin \Omega - \cos \Omega)] \right\} , \]

\[
R_L[l^+ f] = K \left\{ -2(x^2 + y^2)|\xi|^2|\omega|^2 + r^2(1 + x^2)(-1 - y + (-1 + y)|\omega|^2) \right. \]

\[+ 2r|\omega| \left\{ (-1 + y^2) \{ r \cos \Omega - x|\xi|(|\omega| + \cos \Omega) \sin(\eta + \Phi) \} \right. \]

\[+ \left. (1 + y) \{ r x(-1 + y) + (x^2 + y)|\xi| \sin(\eta + \Phi) \} \sin \Omega \right. \]

\[+ \left. |\xi| \cos(\eta + \Phi) \{ (1 + y)(x^2 + y) \cos \Omega + (y - 1)[(x^2 - y)|\omega| + x(1 + y) \sin \Omega] \} \right\} . \]

(29)
\[ R_S [l^{-f}] = K \left\{ -2r^2(x^2 + y^2) + (1 + x^2)|\xi|^2(-1 - y + (y - 1)|\omega|^2) - |\xi|[2r((1 + y)(x^2 + y) \times \cos(\eta + \Phi) + (y - 1)(x - y)|\omega| \cos(\eta + \Phi - \Omega) + x(1 + y)(\sin(\eta + \Phi) + |\omega| \sin(\eta + \Phi - \Omega))) \right\} , \]

\[ R_L [l^{-f}] = -K \left\{ 2r^2[x^2 + y^2] + (1 + x^2)|\xi|^2(1 + |\omega|^2 + y(|\omega|^2 - 1)) - 2(y^2 - 1)|\xi|(\xi \omega) \right\} \]

\[ R_S [l^{+f}] = -K \left\{ 2r^2(x^2 + y^2)|\xi|^2|\omega|^2 + (1 + x^2)(1 + |\omega|^2 + y(|\omega|^2 - 1)) + 2(y^2 - 1)|\omega| \right\} \]

\[ R_L [l^{+f}] = K \left\{ -(x + 1)^2(1 + y) + ((1 + x^2)(y - 1) - 2r^2(x^2 + y^2)|\xi|^2)|\omega|^2 + 2r(x^2 - y) \times (y - 1)|\xi||\omega|^2 \cos(\eta - \Phi) + 2(y^2 - 1)|\omega| \cos \Omega + 2|\omega| [r(1 + y)(x^2 + y)|\xi| \times \cos(\eta - \Phi + \Omega) + x(y^2 - 1)(\sin \Omega + r\xi(|\omega| \sin(\eta - \Phi) + \sin(\eta - \Phi + \Omega)))] \right\} . \]
\[ R_S [l^{-1} f] = K \left\{ -2y^2 - r^2|\xi|^2 + r^2|\xi|^2(y + |\omega|^2 - y|\omega|^2) - x^2(2 + r^2|\xi|^2(1 + y + |\omega|^2 - y|\omega|^2)) \\
- 2r(1 + y)(x^2 + y^2)|\xi| \cos(\eta - \Phi) + 2r(y - 1)|\xi||r(1 + y)|\xi\omega| \\
\times \cos \Omega + (y - x^2)|\omega| \cos(\eta - \Phi + \Omega) + x(1 + y)(\sin(\eta - \Phi) + r|\xi|\omega \sin \Omega) \\
+ 2rx(y^2 - 1)|\xi|\omega | \sin(\eta - \Phi + \Omega) \right\}, \]

\[ R_L [l^{-1} f] = -K \left\{ 2(x^2 + y^2) + r|\xi| \left[ r|\xi||((1 + x^2)(1 + |\omega|^2 + y(|\omega|^2 - 1)) + 2|\omega|(y^2 - 1)) \\
\times (-\cos \Omega + x \sin \Omega) + 2\left[(y - 1)(y - x^2) \cos(\eta - \Phi) - (1 + y)(x^2 + y) \\
\times |\omega| \cos(\eta - \Phi + \Omega) + x(y^2 - 1)(\sin(\eta - \Phi) + |\omega|(\eta - \Phi + \Omega)) \right] \right\}. \]

(32)

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**Figure 1:** Ratio $R_1^-$ versus Re[$\delta$] for the $B_s$ system. The three curves correspond to $|y| = 0.05$ (upper line), $|y| = 0.12$ (middle line) and $|y| = 0.19$ (lower line).

**Figure 2:** 3D Plot for $R_2$ showing the dependence on Re[$\delta$] and Im[$\delta$] for the $B_s$ system.
Figure 3: The ratio $R_1^+$ as a function of $\Phi$ for the $B_d$ system. The phase $\Phi$ is given in radians.

Figure 4: The same as Fig. 3 for the ratio $R_1^-$.

Figure 5: Dependence of $R_1^-$ on Re[$\delta$] for the $B_d$ system. The width in the plot is due to the errors in $y, x$ and $\Phi$. The range of these values are specified in the text.

Figure 6: 3D Plot for the ratio $R_3$ its dependence on Re[$\delta$] and Im[$\delta$] for $B_d$ system. One can see that the imaginary part is detectable.