Quantum thermodynamics of a charged magneto-oscillator coupled to a heat bath

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Abstract. Explicit results for various quantum thermodynamic functions (QTF) of a charged magneto-oscillator coupled to a heat bath at arbitrary temperature are shown in this paper. Separate expressions for different QTF in the two limits of very low and very high temperatures are presented for three popular heat bath models: the ohmic, single-relaxation-time and black body radiation ones. The central result is that the effect of magnetic field turns out to be important at low temperatures as well as at high temperatures. It is observed that the dissipation parameter, $\gamma$, and the cyclotron frequency, $\omega_c$, affect the decaying or rising behaviour of various QTF exactly oppositely to each other at low temperatures. In the high temperature regime, the effect of $\gamma$ is much more pronounced than that of $\omega_c$.

Keywords: dissipative systems (theory), fluctuations (theory), stochastic processes (theory)
1. Introduction

The problem of a charged quantum particle in the presence of a constant magnetic field is of great interest in the fields of the quantum Hall effect [1], high temperature superconductivity [2], diamagnetism [3, 4], plasma physics [5], atomic physics [6], and two-dimensional electronic systems [7]. The issue that we address in this paper is what happens to the quantum thermodynamic functions (QTF) of a charged quantum particle in the presence of an external constant magnetic field when it is in contact with a dissipative quantum heat bath. This kind of analysis is related to dissipative quantum mechanics, a subject that has seen great attention through the work of Leggett and others [8]–[10]. There are several approaches for the treatment of dissipative quantum systems. The most conventional approach is from the system-plus-reservoir point of view, i.e. the system of interest is coupled linearly with the environment which is represented by a collection of harmonic oscillators [11, 12]. Usually, one is interested in the dissipative subsystem and the reservoir variables are eliminated by using a projection operator or tracing procedure [13]. As a result of that, the reservoir enters only through a few parameters. The results obtained from these kinds of dissipative quantum systems are of great interest due to the recent widespread interest in the critical role of environmental effects in mesoscopic systems [14]–[18], in fundamental quantum physics, and in quantum information [19]–[22]. In the last few years, research on the open quantum systems has questioned the validity of fundamental laws of thermodynamics [23]–[26]. These subtle issues are discussed in detail by several authors [27]–[34]. Here, our goal is not to survey all of these subtle issues, but to focus on the effect of the environment and the effect of magnetic field on various thermodynamic functions such as the free energy, entropy, internal energy, and specific heat of a charged oscillator. Through these realistic calculations, one can make direct contact with experiment on low dimensional nanostructures and quantum dots [7].
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The starting point of this paper is using the famous Caldeira–Leggett heat bath model to incorporate the environmental effect in quantum and mesoscopic systems [8, 9]. This enables us to derive a generalized quantum Langevin equation (GQLE) for the charged magneto-oscillator. Then, we use the previously derived result of Ford et al [35] for the free energy of a charged magneto-oscillator in an arbitrary heat bath in terms of a single integral involving generalized susceptibility arising from the generalized quantum Langevin equation [36, 37]. This enables us to derive the free energy of the model system at an arbitrary temperature. Using this generalized result, one can obtain a free energy expression and other thermodynamic functions of the charged quantum magneto-oscillator for three popular heat bath models: the ohmic, single-relaxation-time and black body radiation heat bath or quantum electrodynamics (QED) models.

Now, we need to verify in what way our results are different from the previous results in the literature. Ford and O’Connell discussed the quantum thermodynamic functions for an oscillator coupled to a heat bath [38]. In this paper, we extend the work of Ford and O’Connell to include the presence of an external static magnetic field. We determine various quantum thermodynamic functions of the charged magneto-oscillator coupled to a heat bath. We find that each term in the low temperature expansion for different QTF has added to it two additional magnetic field dependent terms. On the other hand, the inclusion of magnetic field in the problem is reflected in the two additional magnetic field dependent terms in each quantum thermodynamic function in the high temperature regime. In the low temperature regime, the magnetic field changes in qualitative behaviour, as well as the quantitative values for different QTF differing for the charged magneto-oscillator from those for a free oscillator. In this low temperature regime, $\omega_c$ and $\gamma$ affect decaying or rising behaviours of various QTF in opposite manners. It is seen that the effect of $\omega_c$ is negligible in the high temperature regime. But the effect of $\gamma$ is still crucial at high temperatures. We also plot the general expressions for various QTF for the entire temperature regime by numerically evaluating equation (24).

With this preceding background, we organize the rest of the paper as follows. In section 2, we describe our model and the method of deriving the free energy of the system. Section 3 is devoted to deriving various thermodynamic functions (such as entropy, internal energy, specific heat) for the low temperature regime as well as for the high temperature regime for the ohmic heat bath. On the other hand, we derive QTF for the single-relaxation-time model and black body radiation heat bath in section 4. Zero-temperature results are shown in section 5. We conclude in section 6.

2. Model and free energy

The starting point of this section is the generalized Caldeira–Leggett system-plus-reservoir Hamiltonian for a charged particle of mass $m$ and charge $e$ in a magnetic field $\vec{B}$ in the operator form [8, 9]:

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{r}) + \sum_{j=1}^{N} \left[ \frac{\hat{p}_j^2}{2m_j} + \frac{1}{2} m_j \omega_j^2 \left( \hat{q}_j - \frac{c_j}{m_j \omega_j} \hat{r} \right)^2 \right],$$

where $\{\hat{r}, \hat{p}\}$ and $\{\hat{q}_j, \hat{p}_j\}$ are the sets of coordinate and momentum operators of the system and bath oscillators. They follow the following commutation relations:

$$[\hat{r}_\alpha, \hat{p}_\beta] = i\hbar \delta_{\alpha\beta}, \quad [\hat{q}_j, \hat{p}_j] = i\hbar \delta_{jj} \delta_{\alpha\beta},$$

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Figure 1. Plot of $F/E_0$ versus dimensionless temperature, $2\pi k_B T/\hbar \omega_0$, for the charged magneto-oscillator (red filled circle) and for the free charged oscillator (blue filled square) coupled to a ohmic or SRT heat bath (a) in the low temperature regime and (b) in the high temperature regime. To plot this figure, we use $\gamma/\omega_0 = 0.8$, $\gamma/\omega_c = 1.6$, and $\omega_0/\omega_c = 2.0$.

where Greek indices $\alpha$, $\beta$ stand for three spatial directions. Eliminating the bath degrees of freedom via the Heisenberg equations of motion one can obtain the generalized quantum Langevin equation (GQLE) [34,36]:

$$m\ddot{\hat{r}} + \int_{-\infty}^{t} dt' \gamma(t-t') \dot{\hat{r}}(t') - \frac{e}{c} \dot{\hat{r}} \times \vec{B} + \vec{\nabla} V(\hat{r}) = \hat{\theta}(t),$$

where a dot denotes differentiation with respect to time $t$. The effect of the magnetic field is solely represented by the quantum version of the Lorentz force (third term in equation (3)). The memory kernel $\gamma(t)$ and the operator valued random force $\hat{\theta}(t)$ are unaffected by the magnetic field. In this work, we consider the confining potential to be harmonic, for which an exact analysis is possible. It is now possible to represent the nonequal time anticommutator and commutator of $\hat{\theta}(t)$ as follows:

$$\langle \{ \hat{\theta}_\alpha(t), \hat{\theta}_\beta(t') \} \rangle = \delta_{\alpha\beta} \frac{\beta \hbar}{\pi} \int_0^\infty d\omega J(\omega) \text{coth} \left( \frac{\beta \hbar \omega}{2} \right) \cos[\omega(t-t')],$$

$$\langle [\hat{\theta}_\alpha(t), \hat{\theta}_\beta(t')] \rangle = \delta_{\alpha\beta} \frac{\beta \hbar}{\pi} \int_0^\infty d\omega J(\omega) \sin \omega(t-t'),$$

where $\beta = 1/k_B T$ is the inverse temperature. The memory kernel is given by

$$\gamma(t) = \frac{2}{m \pi} \int_0^\infty d\omega \frac{J(\omega)}{\omega} \cos(\omega t).$$

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Figure 2. Plot of $U/E_0$ versus dimensionless temperature, $2\pi k_B T/h\omega_0$, for the charged magneto-oscillator (red filled circle) and for the free charged oscillator (blue filled square) coupled to a ohmic or SRT heat bath (a) in the low temperature regime and (b) in the high temperature regime. To plot this figure, we use $\gamma/\omega_0 = 0.8$, $\gamma/\omega_c = 1.6$, and $\omega_0/\omega_c = 2.0$.

In equations (4)-(6), $J(\omega)$ denotes the spectral density function of the heat bath oscillators and is given by

$$J(\omega) = \pi \sum_{j=1}^{N} \frac{c_j^2}{2m_j \omega_j} \delta(\omega - \omega_j).$$

(7)

We are interested in investigating thermodynamic behaviour of a dissipative charged magneto-oscillator at an arbitrary temperature. One can easily determine the free energy for this model system by using the remarkable formula [35]

$$F = \frac{1}{\pi} \int_0^\infty d\omega f(\omega, T) \Im \left[ \frac{d}{d\omega} \ln \left( \det \alpha(\omega+i0^+) \right) \right],$$

(8)

where $f(\omega, T)$ is the free energy of a single oscillator of frequency $\omega$ and is given by

$$f(\omega, T) = k_B T \log \left[ 1 - \exp \left( -\frac{\hbar \omega}{k_B T} \right) \right],$$

(9)

where we have ignored the zero-point contribution which is discussed in section 5. Here $\alpha(\omega)$ denotes the generalized susceptibility of the model system. Since all the results presented in this paper rely on this remarkable formula, it is the cornerstone of the paper. So, we give a sketch of the method of deriving this notable formula. It is known that the free energy ascribed to the magneto-oscillator, $F(T, B)$, is the free energy of the magneto-oscillator coupled to the heat bath minus the free energy of the bath in the absence of the

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Figure 3. Plot of $S/k_B$ versus dimensionless temperature, $2\pi k_B T/\hbar \omega_0$, for the charged magneto-oscillator (red filled circle) and for the free charged oscillator (blue filled square) coupled to a ohmic or SRT heat bath (a) in the low temperature regime and (b) in the high temperature regime. To plot this figure, we use $\gamma/\omega_0 = 0.8$, $\gamma/\omega_c = 1.6$, and $\omega_0/\omega_c = 2.0$.

magneto-oscillator. To derive this free energy one can follow the following method. Using the Fourier transform one can rewrite equation (3) as follows:

$$\hat{r}(\omega) = \alpha(\omega)\hat{\theta}(\omega),$$

where $\hat{r}(\omega)$ and $\hat{\theta}(\omega)$ are the Fourier transforms of the operators $\hat{r}(t)$ and $\hat{\theta}(t)$ respectively. Now, it can be easily shown that $\alpha(\omega)$ has poles on the real axis at the normal mode frequencies, $\tilde{\omega}_j$, of the interacting system and zeros at the bath frequencies, $\omega_i$, in the absence of the charged magneto-oscillator. Therefore, one can write

$$\alpha(\omega) = -\frac{1}{m} \prod_j (\omega^2 - \tilde{\omega}_j^2) \prod_i (\omega^2 - \omega_i^2),$$

where the numerator is the product over normal modes of the free bath oscillators and the denominator is the product over those of the interacting system. From the well known formula $1/(x + i0^+) = P(1/x) - i\pi \delta(x)$, one can show that

$$\frac{1}{\pi} \Im \left[ \frac{d}{d\omega} \ln \alpha(\omega) \right] = \sum_j [\delta(\omega - \tilde{\omega}_j) + \delta(\omega + \tilde{\omega}_j)] - \sum_i [\delta(\omega - \omega_i) + \delta(\omega + \omega_i)].$$

When this is put into equation (8), the result is

$$F(T, B) = \sum_j f(\tilde{\omega}_j, T) - \sum_i f(\omega_i, T),$$

where

$$\alpha(\omega) = -\frac{1}{m} \prod_j (\omega^2 - \tilde{\omega}_j^2) \prod_i (\omega^2 - \omega_i^2),$$

and

$$\frac{1}{\pi} \Im \left[ \frac{d}{d\omega} \ln \alpha(\omega) \right] = \sum_j [\delta(\omega - \tilde{\omega}_j) + \delta(\omega + \tilde{\omega}_j)] - \sum_i [\delta(\omega - \omega_i) + \delta(\omega + \omega_i)].$$

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Figure 4. Plot of $C/k_B$ versus dimensionless temperature, $2\pi k_B T/h\omega_0$, for the charged magneto-oscillator (red filled circle) and for the free charged oscillator (blue filled square) coupled to a ohmic or SRT heat bath (a) in the low temperature regime and (b) in the high temperature regime. To plot this figure, we use $\gamma/\omega_0 = 0.8$, $\gamma/\omega_c = 1.6$, and $\omega_0/\omega_c = 2.0$.

where the first sum is clearly the free energy of the interacting system and the second that of the free bath field. This demonstrates our assertion. Now, we can rewrite equation (8) as follows [11,12]:

$$ F(T, B) = F(T, 0) + \Delta F(T, B), \quad (14) $$

where

$$ F(T, 0) = \frac{3}{\pi} \int_0^\infty d\omega f(\omega, T) I_1 \quad (15) $$

is the free energy of the oscillator in the absence of the magnetic field, $I_1 = \Im[(d/d\omega) \ln \alpha^{(0)}(\omega)]$, $\alpha^{(0)}(\omega)$ is the scalar susceptibility in the absence of a magnetic field and the correction due to the magnetic field is given by

$$ \Delta F(T, B) = -\frac{1}{\pi} \int_0^\infty d\omega f(\omega, T) I_2, \quad (16) $$

where $I_2 = \Im\{(d/d\omega) \ln[1 - ((eB\omega\alpha^{(0)})/c)^2]\}$. The scalar susceptibility for a harmonic oscillator in the absence of a magnetic field is given by [11,12]

$$ \alpha^{(0)}(\omega) = \frac{1}{m(\omega_0^2 - \omega^2) - i\omega\gamma(\omega)}, \quad (17) $$

where

$$ \gamma(\omega) = \int_0^t dt' \gamma(t') e^{i\omega t'}. \quad (18) $$

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Figure 5. Plot of (a) $F/E_0$, (b) $U/E_0$, (c) $S/k_B$ and (d) $C/k_B$, versus dimensionless temperature, $2\pi k_B T/h\omega_0$, for the charged magneto-oscillator coupled to an ohmic heat bath in the low temperature regime for different values of the dissipation parameter, $\gamma$; the symbols are: blue filled square ($\gamma/\omega_0 = 0.25$), red filled circle ($\gamma/\omega_0 = 0.5$), pink upward triangle ($\gamma/\omega_0 = 0.75$) and black downward triangle ($\gamma/\omega_0 = 1.0$). To plot this figure, we also use $\omega_0/\omega_c = 2.0$.

We have now all the essential ingredients for calculating thermodynamic functions. Our main task is to find the free energy $F$. Then, one can easily derive other thermodynamic functions at an arbitrary temperature. The entropy is defined as

$$S(T, B) = -\frac{\partial F(T, B)}{\partial T}. \quad (19)$$

The internal energy is given by

$$U(T, B) = F(T, B) + TS(T, B), \quad (20)$$

and the specific heat is defined as

$$C(T, B) = T\frac{\partial S(T, B)}{\partial T}. \quad (21)$$

Now, we can proceed for the three popular heat bath models of interest to us, which are characterized by the following spectral density functions: (a) ohmic heat bath model: $\tilde{\gamma}(\omega) = \gamma_0$, (b) single-relaxation-time model: $\tilde{\gamma}(\omega) = \gamma_0/(1 - i\omega\tau)$, and (c) black body radiation model: $\tilde{\gamma}(\omega) = 2\kappa^2\omega\Omega^2/(3c^3(\omega + i\Omega))$, where $\gamma_0$ is the ohmic friction constant,
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Figure 6. Plot of (a) $F/E_0$, (b) $U/E_0$, (c) $S/k_B$, and (d) $C/k_B$, versus dimensionless temperature, $2\pi k_B T/h\omega_0$, for the charged magneto-oscillator coupled to a radiation heat bath in the low temperature regime for different values of the dissipation parameter, $\gamma$; the symbols are: blue filled square ($\gamma/\omega_0 = 0.25$), red filled circle ($\gamma/\omega_0 = 0.5$), pink upward triangle ($\gamma/\omega_0 = 0.75$) and black downward triangle ($\gamma/\omega_0 = 1.0$). To plot this figure, we also use $\omega_0/\omega_c = 2.0$.

while $\tau$ is the relaxation time. Usually $\tau$ is small, i.e. $\tau \ll (\gamma_0/m)$. $\Omega$ is the high frequency cut-off characterizing the electron form factor for black body radiation model. One can easily express $\alpha^{(0)}(\omega)$ for all the three cases by a single expression [38]:

$$
\alpha^{(0)}(\omega) = \frac{\omega + i\Omega}{m(\omega + i\Omega') (\omega_0^2 - \omega^2 - i\gamma\omega)}.
$$

Following Ford and O’Connell, one can write $\tau = 1/\Omega = 1/(\Omega' + \gamma)$; $\gamma_0/m = \gamma((\Omega'^2 + \gamma\Omega' + \omega_0^2)/(\Omega' + \gamma)^2)$; $K/m = \omega_0^2(\Omega'/(\Omega' + \gamma))$ for the single-relaxation-time model [38]. The ohmic model can be represented by $\tau \to 0$, $\gamma_0/m \to \gamma$ and $K/m \to \omega_0^2$. On the other hand, for the black body radiation heat bath or QED model, $1/\Omega = (1/\Omega') + (\gamma/\omega_0^2)$; $K/M = \omega_0^2(\Omega'/(\Omega' + \gamma))$; $M/m = ((\omega_0^2 + \gamma\Omega')(\Omega' + \gamma))/\omega_0^2\Omega'$, where $m$ is the bare mass and $M = m + (2e^2\Omega/3c^3)$ is the renormalized mass. In the limit of the largest value of the cut-off ($\Omega' \to \infty$), $m = 0$, $K = M\omega_0^2$ and $\Omega = 1/\tau_e$, where $\tau_e = 2e^2/3Me^3 = 6 \times 10^{-24}$ s. With the general form of $\alpha^{(0)}(\omega)$ (equation (22)) one can
write the free energy as follows:

\[
F(T, B) = \frac{3k_B T}{\pi} \int_0^\infty d\omega \ln \left(1 - e^{-(\hbar \omega/k_BT)}\right) \left(\frac{\Omega}{\omega^2 + \Omega^2} + \frac{\Omega'}{\omega^2 + \Omega'^2}\right)
\]

\[
+ \frac{k_B T}{\pi} \int_0^\infty d\omega \ln \left(1 - e^{-(\hbar \omega/k_BT)}\right) \left(\frac{\omega_1}{\omega^2 + \Omega_1^2} + \frac{\Omega_1^*}{\omega^2 + \Omega_1^{*2}}\right)
\]

\[
+ \frac{\Omega_2}{\omega^2 + \Omega_2^2} + \frac{\Omega_2^*}{\omega^2 + \Omega_2^{*2}}\right),
\]

where \(\omega_1 = \gamma/2 + (i \sqrt{\omega_0^2 - (\gamma^2/4)})\), \(\Omega_1 = [(\gamma/2) + ((b - a)/2)^{1/2}] - i[(\omega_c/2) + ((b + a)/2)^{1/2}]\), \(\Omega_2 = [(\gamma/2) - ((b - a)/2)^{1/2}] - i[(\omega_c/2) - ((b + a)/2)^{1/2}]\), \(a = (\omega_c/2)^2 + (\omega_0^2 - (\gamma^2/4))\); and \(b = [a^2 + (\gamma \omega_c/2)^2]^{1/2}\). \(\omega_1^*, \Omega_1^*, \text{and } \Omega_2^*\) are the complex conjugates of \(\omega_1, \Omega_1\) and \(\Omega_2\) respectively and \(\omega_c = eB/mc\) is the cyclotron frequency. One can rewrite
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Figure 8. Plot of (a) $F/E_0$, (b) $U/E_0$, (c) $S/k_B$, and (d) $C/k_B$, versus dimensionless temperature, $2\pi k_B T/\hbar \omega_0$, for the charged magneto-oscillator coupled to a radiation heat bath in the high temperature regime for different values of the dissipation parameter, $\gamma$; the symbols are: red filled circle ($\gamma/\omega_0 = 0.25$), blue filled square ($\gamma/\omega_0 = 0.5$), pink upward triangle ($\gamma/\omega_0 = 0.75$) and black downward triangle ($\gamma/\omega_0 = 1.0$). To plot this figure, we also use $\omega_0/\omega_c = 2.0$.

Equation (23) in terms of Stieltjes function ($J(z)$) as follows:

$$F(T, B) = 3k_B T \left[ J \left( \frac{\hbar \Omega}{2\pi k_B T} \right) - J \left( \frac{\hbar \Omega}{2\pi k_B T} \right) \right] - k_B T \left[ J \left( \frac{\hbar \omega_1}{2\pi k_B T} \right) + J \left( \frac{\hbar \omega_1^*}{2\pi k_B T} \right) \right] + J \left( \frac{\hbar \Omega_1}{2\pi k_B T} \right) + J \left( \frac{\hbar \Omega_1^*}{2\pi k_B T} \right) + J \left( \frac{\hbar \Omega_2}{2\pi k_B T} \right) + J \left( \frac{\hbar \Omega_2^*}{2\pi k_B T} \right),$$

(24)

where the Stieltjes $J$ function is given by [39]

$$J(z) = -\frac{1}{\pi} \int_0^\infty \! dt \ln \left( 1 - e^{-2\pi t} \right) \frac{z}{t^2 + z^2}.$$  

(25)

Now, we have all the ingredients for calculating the thermodynamic functions for the charged magneto-oscillator in contact with a heat bath. In section 3, we discuss the ohmic heat bath model.
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3. Ohmic model

In this section, we discuss the QTF of the charged magneto-oscillator in contact with an ohmic heat bath in the low temperature as well as high temperature regimes. For the ohmic model, $\Omega \to \infty$, $\tau \to 0$, $\gamma_0/m \to \gamma$, $K/m \to \omega_0^2$ and hence

$$F(T, B) = -k_B T \left[ J\left(\frac{\hbar \omega_1}{2 \pi k_B T}\right) + J\left(\frac{\hbar \omega_1^*}{2 \pi k_B T}\right) + J\left(\frac{\hbar \Omega_1}{2 \pi k_B T}\right) + J\left(\frac{\hbar \Omega_1^*}{2 \pi k_B T}\right) + J\left(\frac{\hbar \Omega_2}{2 \pi k_B T}\right) + J\left(\frac{\hbar \Omega_2^*}{2 \pi k_B T}\right) \right].$$

(26)

3.1. Low temperature expansion ($k_B T \ll \hbar \omega_0$)

In the low temperature case, we use the asymptotic expansion for $J(z)$:

$$J(z) = \sum_{n=0}^{\infty} \frac{D_{2n+2}}{(2n+1)(2n+2)} \frac{1}{z^{2n+1}},$$

(27)

with $D_2 = \frac{1}{6}$; $D_4 = -\frac{1}{30}$; $D_6 = \frac{1}{42}$; $D_8 = -\frac{1}{30}$ and so on. With this asymptotic expansion,
the free energy becomes

\[ F(T, B) = -\frac{\pi(k_B T)^2 \gamma}{2 \hbar \omega_0^2} + \frac{\pi^3(k_B T)^4}{45\hbar^3 \omega_0^6}(A_1 + A_2 + A_3) + \frac{8\pi^5(k_B T)^6(B_1 + B_2 + B_3)}{315\hbar^5 \omega_0^{10}} + \cdots, \]

where

\[ A_1 = \gamma(3\omega_0^2 - \gamma^2), \quad A_2 = (3\Gamma_1^2\lambda_1 - \Lambda_1^3), \quad A_3 = (3\Gamma_2^2\lambda_2 - \Lambda_2^3), \quad B_1 = \gamma(5\omega_0^4 - 5\gamma^2\omega_0^2 + \gamma^4), \]

\[ B_2 = (\lambda_1^5 - 10\lambda_1^3\Gamma_1^2 + 5\lambda_1\Gamma_1^4), \quad B_3 = (\lambda_2^5 - 10\lambda_2^3\Gamma_2^2 + 5\lambda_2\Gamma_2^4), \quad \Gamma_{1,2} = (\omega_c/2) \pm \sqrt{(b + a/2)}; \quad \gamma_{1,2} = \gamma \pm \sqrt{2(b - a)}. \]

The entropy can be written as

\[ S(T, B) = k_B \left[ \frac{\pi k_B T \gamma}{\hbar \omega_0^2} + \frac{4\pi^3(k_B T)^3}{45\hbar^3 \omega_0^6}(A_1 + A_2 + A_3) + \frac{16\pi^5(k_B T)^5(B_1 + B_2 + B_3)}{105\hbar^5 \omega_0^{10}} + \cdots \right]. \]

The internal energy can be written as follows:

\[ U(T, B) = \frac{\pi(k_B T)^2 \gamma}{2 \hbar \omega_0^2} + \frac{\pi^3(k_B T)^4(A_1 + A_2 + A_3)}{45\hbar^3 \omega_0^6} + \frac{40\pi^5(k_B T)^6(B_1 + B_2 + B_3)}{315\hbar^5 \omega_0^{10}} + \cdots. \]
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Finally, the specific heat of the system is given by

$$C(T, B) = k_B \left[ \frac{\pi k_B T \gamma}{\hbar \omega_0^2} + \frac{4 \pi^3 (k_B T)^3}{15 \hbar^3 \omega_0^6} (A_1 + A_2 + A_3) + \frac{16 \pi^5 (k_B T)^5 (B_1 + B_2 + B_3)}{21 \hbar^5 \omega_0^{10}} + \cdots \right].$$

(31)

3.2. High temperature expansion ($k_B T \gg \hbar \omega_0$)

In the high temperature regime, we use the small argument expansion for $J(z)$:

$$J(z) = -\frac{1}{2} \ln(2\pi) - \left( z + \frac{1}{2} \right) \ln z + (1 - \gamma_E) z + \sum_{n=2}^{\infty} \frac{(-1)^n \zeta(n)}{n} z^n,$$

(32)

where $\gamma_E = 0.577215$ is Euler’s constant and $\zeta(n)$ is the Riemann zeta function. Using equation (32), one can write down the free energy of the system in the high temperature regime.

Figure 11. Plot of (a) $F/E_0$, (b) $U/E_0$, (c) $S/k_B$, and (d) $C/k_B$, versus dimensionless temperature, $2 \pi k_B T/\hbar \omega_0$, for the charged magneto-oscillator coupled to a ohmic heat bath in the high temperature regime for different values of the cyclotron frequency, $\omega_c$; the symbols are: red filled circle ($\omega_c/\omega_0 = 0.25$), blue filled square ($\omega_c/\omega_0 = 0.5$), pink upward triangle ($\omega_c/\omega_0 = 0.75$) and black downward triangle ($\omega_c/\omega_0 = 1.0$). To plot this figure, we also use $\gamma/\omega_0 = 0.8$.

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$$C(T, B) = k_B \left[ \frac{\pi k_B T \gamma}{\hbar \omega_0^2} + \frac{4 \pi^3 (k_B T)^3}{15 \hbar^3 \omega_0^6} (A_1 + A_2 + A_3) + \frac{16 \pi^5 (k_B T)^5 (B_1 + B_2 + B_3)}{21 \hbar^5 \omega_0^{10}} + \cdots \right].$$

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3.2. High temperature expansion ($k_B T \gg \hbar \omega_0$)

In the high temperature regime, we use the small argument expansion for $J(z)$:

$$J(z) = -\frac{1}{2} \ln(2\pi) - \left( z + \frac{1}{2} \right) \ln z + (1 - \gamma_E) z + \sum_{n=2}^{\infty} \frac{(-1)^n \zeta(n)}{n} z^n,$$

(32)

where $\gamma_E = 0.577215$ is Euler’s constant and $\zeta(n)$ is the Riemann zeta function. Using equation (32), one can write down the free energy of the system in the high temperature regime.
Figure 12. Plot of (a) $F/E_0$, (b) $U/E_0$, (c) $S/k_B$, and (d) $C/k_B$, versus dimensionless temperature, $2\pi k_B T/\hbar \omega_0$, for the charged magneto-oscillator coupled to a radiation heat bath in the high temperature regime for different values of the cyclotron frequency, $\omega_c$; the symbols are: red filled circle ($\omega_c/\omega_0 = 0.25$), blue filled square ($\omega_c/\omega_0 = 0.5$), pink upward triangle ($\omega_c/\omega_0 = 0.75$) and black downward triangle ($\omega_c/\omega_0 = 1.0$). To plot this figure, we also use $\gamma/\omega_0 = 0.8$.

regime as follows:

$$F(T, B) = -3k_B T \ln \left( \frac{k_B T}{\hbar \omega_0} \right) - \frac{3\hbar \gamma}{2\pi} \ln \left( \frac{2\pi k_B T}{\hbar \omega_0} \right) - \frac{\hbar \omega'_1}{\pi} \phi - \frac{3\hbar \gamma(1 - \gamma_E)}{2\pi}$$

$$- \frac{2\hbar \omega_c}{\pi} \theta - 2k_B T \sum_{n=2}^{\infty} (-1)^n \frac{\zeta(n)}{n} \left( \frac{\hbar \omega_0}{2\pi k_B T} \right)^n \tan(n\phi) + 3\frac{\hbar \gamma}{2\pi T} - 2k_B \sum_{n=2}^{\infty} (-1)^n \frac{(n-1)\zeta(n)}{n} \left( \frac{\hbar \omega_0}{2\pi k_B T} \right)^n \tan(n\phi),$$

where $\theta = \tan^{-1} P$, $P = \sqrt{2(b + a)/\gamma}$, $\phi = \tan^{-1}(2\omega'_1/\gamma)$, and $\omega'_1 = \sqrt{\omega^2_0 - (\gamma^2/4)}$. The entropy is given by

$$S(T, B) = 3k_B \left( \ln \left( \frac{k_B T}{\hbar \omega_0} + 1 \right) - 4k_B \sum_{n=2}^{\infty} (-1)^n \frac{(n-1)\zeta(n)}{n} \left( \frac{\hbar \omega_0}{2\pi k_B T} \right)^n \tan(n\phi) \right.$$

$$+ \left. \frac{3\hbar \gamma}{2\pi T} - 2k_B \sum_{n=2}^{\infty} (-1)^n \frac{(n-1)\zeta(n)}{n} \left( \frac{\hbar \omega_0}{2\pi k_B T} \right)^n \tan(n\phi). \right)$$

$$\text{(34)}$$
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Figure 13. Plot of $F/E_0$ versus dimensionless temperature, $2\pi k_B T/\hbar \omega_0$, for the charged magneto-oscillator (red filled circle) and for the free charged oscillator (blue filled square) coupled to a black body radiation heat bath (a) in the low temperature regime and (b) in the high temperature regime. To plot this figure, we use $\gamma/\omega_0 = 0.8$, $\gamma/\omega_c = 1.6$, and $\omega_0/\omega_c = 2.0$.

On the other hand, the internal energy is given by

$$U(T,B) = 3k_B T - \frac{3\hbar \gamma}{2\pi} \ln \left( \frac{2\pi k_B T}{\hbar \omega_0} \right) - \frac{\hbar \omega_0}{\pi} \phi + \frac{3\hbar \gamma E}{2\pi} - \frac{2\hbar \omega_c}{\pi} \theta$$

$$- 4k_B T \sum_{n=2}^{\infty} (-1)^n (n-1) \zeta(n) \left( \frac{\hbar \omega_0}{2\pi k_B T} \right)^n \tan(n \phi)$$

$$- 8k_B T \sum_{n=2}^{\infty} (-1)^n \zeta(n) \left( \frac{\hbar \omega_0}{2\pi k_B T} \right)^n \tan(n \theta).$$

Finally, the specific heat is given by

$$C(T,B) = 3k_B - \frac{3\hbar \gamma}{2\pi T} + 4k_B \sum_{n=2}^{\infty} (-1)^n (n-1) \zeta(n) \left( \frac{\hbar \omega_0}{2\pi k_B T} \right)^n \tan(n \theta)$$

$$+ 2k_B \sum_{n=2}^{\infty} (-1)^n (n-1) \zeta(n) \left( \frac{\hbar \omega_0}{2\pi k_B T} \right)^n \tan(n \phi).$$

4. Single-relaxation-time–QED model

The free energy for the single-relaxation-time–QED model is given by

$$F(T,B) = F_{\text{ohmic}}(T,B) + 3k_B T \left[ J \left( \frac{\hbar \Omega}{2\pi k_B T} \right) - J \left( \frac{\hbar \Omega'}{2\pi k_B T} \right) \right].$$

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Figure 14. Plot of $U/E_0$ versus dimensionless temperature, $2\pi k_B T/h\omega_0$, for the charged magneto-oscillator (red filled circle) and for the free charged oscillator (blue filled square) coupled to a black body radiation heat bath (a) in the low temperature regime and (b) in the high temperature regime. To plot this figure, we use $\gamma/\omega_0 = 0.8$, $\gamma/\omega_c = 1.6$, and $\omega_0/\omega_c = 2.0$.

As $\Omega$ and $\Omega'$ are always large compared with $k_B T$, retaining only the first term of the low temperature expansion of $J(z)$, one can write

$$F(T, B) = F_{\text{ohmic}}(T, B) + \frac{\pi (k_B T)^2}{2\hbar} \left( \frac{1}{\Omega} - \frac{1}{\Omega'} \right).$$

For the single-relaxation-time model, $1/\Omega = 1/(\Omega' + \gamma)$ and $\Omega' \gg \gamma$. Hence, the second term in equation (38) is negligibly small and

$$F_{\text{SRT}}(T, B) \simeq F_{\text{ohmic}}(T, B),$$

where the subscript SRT stands for single relaxation time. On the other hand, for QED, $1/\Omega - 1/\Omega' = \gamma/\omega_0^2$. Thus,

$$F_{\text{QED}}(T, B) = F_{\text{ohmic}}(T, B) + \frac{\pi (k_B T)^2 \gamma}{2h\omega_0^2}.$$  

4.1. QED: low temperature expansion ($k_B T \ll \hbar \omega_0$)

Using the low temperature expansion of $J(z)$, one can obtain

$$F_{\text{QED}}(T, B) = -\left[ \frac{\pi^3 (k_B T)^4 (A_1 + A_2 + A_3)}{45\hbar^3 \omega_0^6} + \frac{8\pi^5 (k_B T)^6 (B_1 + B_2 + B_3)}{315\hbar^5 \omega_0^{10}} + \ldots \right].$$

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The leading term in the expansion of $F_{\text{ohmic}}(T, B)$ exactly cancels the second term in equation (40), and hence, the leading term is $O(T^4)$. The entropy of the system is given by

$$S_{\text{QED}}(T, B) = k_B \left[ \frac{4 \pi^3 (k_B T)^3 (A_1 + A_2 + A_3)}{45 \hbar^3 \omega_0^5} \right. \left. + \frac{16 \pi^5 (k_B T)^5 (B_1 + B_2 + B_3)}{105 \hbar^3 \omega_0^5} \right] + \cdots. \quad (42)$$

The internal energy is given by

$$U_{\text{QED}}(T, B) = \pi^3 (k_B T)^4 (A_1 + A_2 + A_3) \left[ \frac{1}{15 \hbar^3 \omega_0^6} \right] + \frac{8 \pi^5 (k_B T)^6 (B_1 + B_2 + B_3)}{63 \hbar^3 \omega_0^6} \right] + \cdots. \quad (43)$$

Finally, the specific heat of the system is given by

$$C_{\text{QED}}(T, B) = k_B \left[ \frac{4 \pi^3 (k_B T)^3 (A_1 + A_2 + A_3)}{15 \hbar^3 \omega_0^6} \right. \left. + \frac{16 \pi^5 (k_B T)^5 (B_1 + B_2 + B_3)}{21 \hbar^3 \omega_0^6} \right] + \cdots. \quad (44)$$

4.2. QED: high temperature expansion ($k_B T \gg \hbar \omega_0$)

In this subsection, the high temperature thermodynamic properties for the QED model are discussed in detail. One can show that the free energy for this model is

$$F_{\text{QED}}(T, B) = F_{\text{HT}}^{\text{ohmic}} + \frac{\pi (k_B T)^2 \gamma}{2 \hbar \omega_0^2}. \quad (45)$$

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Figure 16. Plot of $C/k_B$ versus dimensionless temperature, $2\pi k_B T/\hbar \omega_0$, for the charged magneto-oscillator (red filled circle) and for the free charged oscillator (blue filled square) coupled to a black body radiation heat bath (a) in the low temperature regime and (b) in the high temperature regime. To plot this figure, we use $\gamma/\omega_0 = 0.8$, $\gamma/\omega_c = 1.6$, and $\omega_0/\omega_c = 2.0$.

where $F_{\text{ohmic}}^{HT}$ is the high temperature free energy for the ohmic model (equations (33)). The entropy in the high temperature regime is given by

$$S_{\text{QED}}(T, B) = S_{\text{ohmic}}^{HT}(T, B) - \frac{\pi (k_B T) \gamma}{\hbar \omega_0^2},$$

(46)

where the high temperature specific heat for the ohmic model ($S_{\text{ohmic}}^{HT}(T, B)$) is given by equation (34). The internal energy of the system is given by

$$U_{\text{QED}}(T, B) = U_{\text{ohmic}}^{HT}(T, B) - \frac{\pi k_B T^2 \gamma}{2\hbar \omega_0^2},$$

(47)

where $U_{\text{ohmic}}^{HT}(T, B)$ is given by equation (35). Finally, the specific heat of the system in the high temperature regime for the QED model is given by

$$C_{\text{QED}}(T, B) = C_{\text{ohmic}}^{HT}(T, B) - \frac{\pi k_B T \gamma}{\hbar \omega_0^2},$$

(48)

and $C_{\text{ohmic}}^{HT}(T, B)$ is the specific heat for the ohmic model in the high temperature regime (see equations (36)).

To explicitly demonstrate the distinguishing behaviours of the different thermodynamic functions for the three different heat bath models, we plot the different thermodynamic quantities as a function of dimensionless temperature ($2\pi k_B T/\hbar \omega_0$) in different figures for the low temperature regime as well as for the high temperature regime. The
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results for different values of the dissipation, \(\gamma\), as well as for different values of \(\omega_c\), are also analysed here. We compare our results for various QTF for a charged magneto-oscillator with those for a free oscillator. From our analysis, one can conclude that the qualitative behaviour of different QTF for the charged magneto-oscillator in the high temperature regime is the same as that of a free oscillator. Even the quantitative values do not differ much from each other in the high temperature regime (see figures 1(b)–4(b) and 13(b)–16(b)). In the low temperature regime, the free energy of the charged magneto-oscillator decays faster than that of the free oscillator for the ohmic heat bath as well as for the QED model. On the other hand, the other QTF like the internal energy (\(U\)), entropy (\(S\)), and specific heat (\(C\)) for the magneto-oscillator rise faster than those for the free oscillator in the low temperature regime (see figures 1(a)–4(a) and 13(a)–16(a)). It has been observed that \(\omega_c\) and \(\gamma\) affect different QTF in opposite manners. In the low temperature regime, \(U\), \(S\) and \(C\) rise and \(F\) decays much faster for higher values of \(\gamma\) (see figures 5 and 6) whereas they (\(U\), \(S\) and \(C\)) rise or \(F\) decays much faster for lower values of \(\omega_c\) (see figures 9 and 10). In the high temperature regime, different QTF do not vary much for different values of \(\omega_c\) (see figures 11 and 12). But the effect of different values of

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Figure 18. Plot of (a) $F/E_0$, (b) $U/E_0$, (c) $S/k_B$, and (d) $C/k_B$, versus dimensionless temperature, $2\pi k_B T/\hbar \omega_0$, for the charged magneto-oscillator coupled to an ohmic heat bath for the entire temperature regime with different values of the cyclotron frequency, $\omega_c$; the symbols are: red filled circle ($\omega_c/\omega_0 = 0.25$), blue filled square ($\omega_c/\omega_0 = 0.5$), pink upward triangle ($\omega_c/\omega_0 = 0.75$). To plot this figure, we also use $\gamma/\omega_0 = 1.0$.

the dissipation parameter, $\gamma$, is pronounced in the high temperature regime also. In this regime, $U$, $S$ and $C$ rise and $F$ decays much faster for higher values of $\gamma$ (see figures 7 and 8).

We also plot the general result for different quantum thermodynamic functions for the entire temperature regime by using equation (24) which is expressed in terms of the Stieltjes $J$ function (see figures 17 and 18). For the numerical computation, we use the Lanczos formula for the Stieltjes $J$ function [40,41]:

$$J(z) = \left( z + \frac{1}{2} \right) \ln \frac{z + \gamma_1 + 1/2}{z} - \gamma_1 - 1/2 + \ln \left[ d_0 + \sum_{n=1}^{N} \frac{d_n}{z + n} \right], \quad \Re z > 0, \quad (49)$$

where $N = 6$, $\gamma_1 = 5$, $d_0 = 1.00$, $d_1 = 76.18$, $d_2 = -86.51$, $d_3 = 24.01$, $d_4 = -1.23$, $d_5 = 0.001$, and $d_6 = 0.0$. It is seen that both at very high temperatures and at very low temperatures, the numerical results match the analytical expressions fairly well. Also, it is seen that the effect of the dissipation parameter is pronounced over the entire temperature regime of interest to us.

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5. Zero-point energy

For the sake of completeness, we are giving here the zero-point energy contribution of the charged magneto-oscillator for the above mentioned three popular heat bath models. It is known that the free energy $F = U + TS$. Hence, the zero-point free energy is identical to the zero-point energy. The zero-point free energy is obtained by replacing $f(\omega, T) \to (\hbar \omega/2)$ in equation (11) and in equation (12). Thus,

$$F(0, B) = \frac{\hbar}{2 \pi} \left[ 3 (\Omega' \ln \Omega - \Omega' \ln \Omega') - \omega_1 \ln \omega_1 - \omega_1' \ln \omega_1' - \Omega_1 \ln \Omega_1 - \Omega_1' \ln \Omega_1' - \Omega_2 \ln \Omega_2 - \Omega_2' \ln \Omega_2' \right].$$

(50)

For the single-relaxation-time model, the zero-point free energy becomes

$$F(0, B) = \frac{\hbar}{2 \pi} \left[ 3 ((\Omega' + \gamma) \ln (\Omega' + \gamma) - \Omega' \ln \Omega') - \gamma \ln \omega_0 + 2 \omega_1' \tan^{-1} \left( \frac{2 \omega_1'}{\lambda_1} \right) - \lambda_1 \ln (\lambda_1^2 + \Gamma_1^2) - \lambda_2 \ln (\lambda_2^2 + \Gamma_2^2) + 2 \Gamma_1 \tan^{-1} \left( \frac{\Gamma_1}{\lambda_1} \right) + 2 \Gamma_2 \tan^{-1} \left( \frac{\Gamma_2}{\lambda_2} \right) \right].$$

(51)

where $\lambda_{1,2} = \gamma/2 \pm (b - a/2)^{1/2}$, and $\Gamma_{1,2} = (\omega_c/2) \pm (b + a/2)^{1/2}$. On the other hand, the zero-point contribution in the free energy for the ohmic model is given by

$$F(0, B) = \frac{\hbar}{2 \pi} \left[ 3 \gamma \left( 1 - \ln(\omega_0 \tau) \right) + 2 \gamma \ln \omega_0 - \lambda_1 \ln (\lambda_1^2 + \Gamma_1^2) - \lambda_2 \ln (\lambda_2^2 + \Gamma_2^2) + 2 \omega_1' \tan^{-1} \left( \frac{2 \omega_1'}{\gamma} \right) + 2 \Gamma_1 \tan^{-1} \left( \frac{\Gamma_1}{\lambda_1} \right) + 2 \Gamma_2 \tan^{-1} \left( \frac{\Gamma_2}{\lambda_2} \right) \right].$$

(52)

Finally, the zero-point free energy for the QED model is given by

$$F(0, B) = \frac{\hbar}{2 \pi} \left[ 3 \left( \frac{\Omega' \omega_0^2}{\omega_0^2 + \gamma \Omega'} \ln \left( \frac{\Omega' \omega_0^2}{\omega_0^2 + \gamma \Omega'} \right) - \Omega' \ln \Omega' \right) - \lambda_1 \ln (\lambda_1^2 + \Gamma_1^2) - \lambda_2 \ln (\lambda_2^2 + \Gamma_2^2) + 2 \omega_1' \tan^{-1} \left( \frac{2 \omega_1'}{\gamma} \right) + 2 \Gamma_1 \tan^{-1} \left( \frac{\Gamma_1}{\lambda_1} \right) + 2 \Gamma_2 \tan^{-1} \left( \frac{\Gamma_2}{\lambda_2} \right) - \gamma \ln \omega_0 \right].$$

(53)

6. Conclusions

Recent observations have suggested that small quantum systems are particularly sensitive to environmental effects [14, 15]. Dissipation and fluctuation effects often play a crucial role in the behaviour of such small systems. The influence of an external magnetic field on such small systems, like quantum dots, quantum wires, and two-dimensional electronic systems, are of great interest in the fields of nanostructures. This has motivated us to study the quantum thermodynamic behaviour of a charged magneto-oscillator in an arbitrary heat bath at arbitrary temperature. Results for both high temperature and low temperature are shown explicitly in this paper.

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We use the ‘remarkable formula’ of Ford et al \[35\] for the free energy of the charged magneto-oscillator which involves a single integral. This is an exact result for the free energy of a charged magneto-oscillator which takes into account interaction effects. Starting from this remarkable formula, we calculate several thermodynamic functions, like the free energy, entropy, internal energy, and specific heat, for three different popular models: the ohmic, single-relaxation-time, and radiation heat bath/QED models. Explicit results are presented for high temperatures as well as for low temperatures. The central result of this analysis is that the qualitative behaviour as well as quantitative values of different thermodynamic quantities for the charged magneto-oscillator differ from those of the free oscillator in the low temperature regime. The effects of \( \omega_c \) and \( \gamma \) are also investigated. They affect the rising or decaying behaviour of different QTF exactly oppositely to each other in the low temperature regime. In the high temperature regime, the effect of \( \omega_c \) is not so significant. On the other hand, the effect of different \( \gamma \) on the rising or decaying behaviour of different QTF is significant even in the high temperature regime. We have also shown the general results for various QTF by numerically evaluating equation \( (24) \). The numerical results obtained from the general expression match fairly well with the analytical expressions in the two limits of very high temperatures and very low temperatures. The experimentalist can make an estimation of the change in values for different QTF from our study.

The applications of this kind of analysis are manifold. Recent nanofabrication allows one to create very small systems of a few atoms in which environmental effects play an important role. This analysis is helpful in understanding environmental effects on small systems [7]. The effect of magnetic field on the properties of two-dimensional quantum systems in contact with a quantum heat bath is of great importance in the fields of nanophysics [42], quantum information theory [19]–[22], and atomic and nuclear physics [5, 6]. In that sense, our study will be helpful in analysing different thermodynamic properties of small quantum systems like quantum dots, quantum wires, and two-dimensional electronic systems in contact with a heat bath in the presence of a constant magnetic field. Recently Jordan and Büttiker discussed the entanglement energetics of quantum systems at zero temperature [43]. So, following our method one could think of extending the study of entanglement energetics at finite temperature in the presence of an external magnetic field. Also Ratchov et al discussed the decrease in coherence length of Aharonov–Bohm like interferometers due to the interaction with a zero-temperature environment. Again, one can think of extending this work by following the method used in this paper for the finite temperature analysis [44]. Apart from that, one can think of another area in which thermodynamics plays a crucial role. Following the previous research of several authors [45]–[47], one can think of developing microscopic theory for the entropy of black holes. Thus, it is worthwhile to apply this approach to the study of thermodynamic properties of black holes.

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