Interacting dark energy with time varying equation of state and the $H_0$ tension

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Almost in all coupled dark energy models present in the literature, the stability of the model becomes potentially sensitive to the dark energy equation of state parameter $w_x$, and a singularity arises at $w_x = -1$. Thus, it becomes mandatory to test the stability of the model into two separate regions, namely, the quintessence and phantom and certainly, a discontinuity appears into the parameters space of the dark energy state parameter. Such discontinuity can be removed with some choices of the coupling function. In the present work we choose one particular coupling between dark matter and dark energy which can successfully remove such instability and we allow a dynamical dark energy equation of state parameter instead of the constant one. We have considered one parameter dynamical dark energy equation of state, and the interacting scenario have been confronted with several observational data with latest origin. The results show that the present cosmological data allow an interaction in the dark sector, in agreement with some latest claims by several authors, and additionally, a phantom behaviour in the dark energy equation of state is suggested at present. Moreover, for this case the tension on $H_0$ is clearly released. As a final remark, we should mention that according to the Bayesian analysis, $\Lambda$CDM is always favored over this interacting dark energy model.

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1. INTRODUCTION

The accelerated expansion of the universe still remains as an enigma for cosmologists. It was first discovered in late nineties from the observations of nearby Supernovae Type Ia [1, 2]. Further observations, like the Baryon Oscillation Spectroscopic Survey (BOSS) [3], the continuation of Supernova cosmology project [4], the Dark Energy Survey [5], the mapping of the universe from the multi wavelength observations of the Sloan Digital Sky Survey (SDSS) [6], the observation of cosmic microwave background (CMB) from WMAP [7], Planck [8] and several others have strongly confirmed the accelerated expansion of the universe at recent time. There are different theoretical prescriptions in the literature to explain this late-time cosmic acceleration. The most popularly accepted one is the assumption of the existence of an exotic component in the energy budget of the universe. The exotic component, dubbed as dark energy, is responsible for the alleged accelerated expansion due to its negative pressure. Observations suggest that the dark energy contributes around 70% to the total energy density of the universe [9]. The rest of the contribution is predominated by another exotic component, called the dark matter (roughly around 26%) [9]. The fundamental difference between dark matter and ordinary baryonic matter is that the dark matter do not have any electromagnetic, strong or week interactions, though they have similar gravitational interaction.

The present work is mainly focused on the interaction between dark energy and the dark matter. There are existing models in the literature where independent conservation of dark energy and dark matter has been assumed, see the details here [10]. On the other hand, models which allow the interaction between these two dark components, are also well consistent with cosmological observations [11, 12, 13]. In fact, interacting and non-interacting dark energy models are not sharply distinguishable from the presently available observational data. Though the non-interacting dark energy models are well enough to explain the observed cosmological scenario, they suffers from certain theoretical issues. The theoretical compulsions to invoke the possibility of interaction between dark energy and dark matter are the extremely small value of the cosmological constant [14] and the well known cosmic coincidence problem [15, 16, 17, 18, 19]. Different aspects of interacting dark energy have been investigated from recent observations in [12, 13, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32].

Interacting dark energy models are reconstructed mainly through the parametrization of the interaction term in the conservation equations of dark matter and dark energy. In the present work, we have emphasized on the interacting dark energy with time evolving dark energy equation of state parameter ($w_x$). Some recent works on this direction are referred in [11, 33, 34, 35, 36, 37, 38]. The stability of coupled dark energy models at
large scale are highly sensitive to the dark energy equation of state, \( w_x \). Whenever the dark energy state parameter crosses the phantom divide line \( w_x = -1 \), the perturbation equations become undefined leading to a singularity at \( w_x = -1 \). Thus, in order to confront the couplings with the observational data, two separate regions, namely, \( w_x > -1 \) and \( w_x < -1 \) are considered. However, such problems can be dodged with some interaction models, recently explored in \([33, 39]\). Recently, astronomical observations report some interesting and overwhelming issues on the coupling between the dark components. The results show that the observational data favor a non-zero coupling in the dark sectors \([11, 12, 13, 22, 23, 10]\), although small but still a small deviation from the standard \( \Lambda \)-cosmology is signaled. The tension on the Hubble constant value appearing from the local and global measurements are found to be assuaged \([22, 39, 11]\). The inclusion of the coupling between dark matter and dark energy impels the dark energy equation of state to go beyond the cosmological constant limit \([11, 13, 33, 34, 35, 36, 39]\). Therefore, it is quite certain that the coupling in the dark sectors still remains as an attracting field for further investigations. Now, in compared to the coupling dynamics with constant dark energy equation of state \([13, 22, 39, 11]\), the same with dynamical equation of state has not been much explored except a few recent investigations \([11, 33, 34, 35, 36, 37]\). In this work we shall perform a systematic analysis for dynamical dark energy coupled with dark matter. The equation of state parameter of dark energy is assumed to evolve with time, because this seems the preferred scenario in several analyses \([42, 43, 44, 45, 46]\). In the present analysis, we shall focus on a specific parameterization of the dark energy equation of state, namely a one parameter dark energy model \([47]\) and we constrain the coupling strength of the interaction function along with other free parameters of the interacting model as well.

The present work has been organized in the following way. In section 2, we describe the basic equations of the interaction models at the background and perturbative levels as well as we introduce the specific interaction model that has been studied in the present context. In section 3 we first describe the observational data to constrain the interaction scenarios and then we describe the results of the analysis in subsections 3.1 and 3.2. A Bayesian analysis for statistical model selection through the calculation of Bayesian evidence has been discussed in subsection 3.3. Finally, we close the work with a brief summary in section 4.

2. INTERACTING DARK FLUIDS AT THE BACKGROUND AND PERTURBATIVE LEVELS

In the large scales of the universe, its geometry is best described by the Friedman-Lemaître-Robertson-Walker (FLRW) line element. Thus, in this work we assume the same line element which takes the form

\[
ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1-Kr^2} + (d\theta^2 + \sin^2 \theta d\phi^2) \right],
\]

where \( a(t) \) is the scale factor of the universe and \( K \) is its curvature scalar. We shall concentrate only on \( K = 0 \), which represents a spatially flat universe while \( K = -1, +1 \), respectively describe a open and a closed universe. In the present work we consider the spatial flatness of the universe, that means we set \( K = 0 \). Further, we assume that the gravity sector is described by the general relativity in addition to the following: (i) The matter sector is minimally coupled to gravity; (ii) The total energy density of the universe is shared by four components, namely, radiation, baryons, pressureless dark matter and a dark energy fluid where only dark matter and dark energy fluids are coupled to each other while the rest two fluids are conserved separately. The conservation equations for dark matter with zero pressure (i.e. cold dark matter) and dark energy with dynamical equation of state, \( w_x \equiv p_x/\rho_x \), are

\[
\dot{\rho}_c + 3H\rho_c = -Q, \tag{2}
\]

and

\[
\dot{\rho}_x + 3H(1+w_x)\rho_x = Q, \tag{3}
\]

where in \([2] \) and \([3] \), the new quantity \( Q \) describes the flow of energy between the dark sectors, known as the interaction function. The algebraic constraint on the dynamics of the universe is the Friedmann equation

\[
H^2 = \frac{8\pi G}{3} (\rho_r + \rho_b + \rho_c + \rho_x), \tag{4}
\]

which together with the conservation equations \([2] \) and \([3] \) can determine the entire dynamics of the universe, once the interaction function \( Q \) is given. In principle, there are several choices for \( Q \) that can be made, however, in this work we are interested on the choice of the couplings that produce stable perturbations on the large scale. Since the structure formation is a very important issue, thus, it is mandatory to focus on the perturbations equations that are modified in presence of any coupling.

In what follows, we consider the perturbed FLRW metric \([48, 49, 50]\)

\[
d\tau^2 = a^2(\tau) \left[ - (1+2\phi)d\tau^2 + 2\partial_i Bd\tau dx^i + \left( (1-2\psi)\delta_{ij} + 2\partial_i \partial_j E \right) dx^i dx^j \right], \tag{5}
\]

where by \( \tau \) we mean the conformal time; \( \phi, B, \psi \) and \( E \) are the the gauge-dependent scalar perturbation quantities. For the above metric, one can calculate the field equations following the equations \([51, 52, 53]\)

\[
\nabla_\mu T^{\mu}_A = Q^\mu_A, \quad \sum_A Q^\mu_A = 0.
\]
where $A$ has been used to mean any fluid either dark matter or dark energy. The quantity $Q^\mu_A$ takes the form

$$Q^\mu_A = (Q_A + \delta Q_A)u^\mu + a^{-1}(0, \partial_i f_A),$$

relative to the four-velocity $u^\mu$ where $Q_A$ is the background energy transfer (i.e. $Q_A = Q$) and $f_A$ is the momentum transfer potential. For simplicity, we assume that momentum transfer potential is zero in the rest frame of the dark matter [51, 52, 53] which directs $k^2f_A = Q_A(\theta - \theta_c)$ where $k$ is the wave number and

$$\theta = \theta^\mu_A, \theta_c$$

are respectively the volume expansion scalar of the total fluid and the volume expansion scalar for the CDM fluid. Now, introducing the density perturbations for the fluid ‘A’ as $\delta_A = \delta \rho_A/\rho_A$ and considering no anisotropic stress in the system (i.e. $\pi_A = 0$), the density and velocity perturbations for the dark fluids in the synchronous gauge, that means $\phi = B = 0, \psi = \eta$, and $k^2E = -h/2 - 3\eta$, where $h$ and $\eta$ are the metric perturbations (see [49] for details), can be written as

$$\frac{\delta Q}{Q} = \frac{d}{(1 + w_x)} \left( \theta_x + \frac{h'}{2} \right) - 3\mathcal{H}(c_{sx}^2 - w_x) \left[ \delta_x + 3\mathcal{H}(1 + w_x)\frac{\theta_x}{k^2} \right] - 3\mathcal{H}w_x \frac{\theta_x}{k^2},$$

$$\theta'_x = -\mathcal{H}(1 - 3c_{sx}^2)\theta_x + \frac{c_{sx}^2}{(1 + w_x)} k^2 \delta_x + \frac{aQ}{\rho_x} \left[ \theta_x - (1 + c_{sx}^2)\theta_x \right],$$

$$\delta'_c = - \left( \theta_c + \frac{h'}{2} \right) + \frac{aQ}{\rho_c} \left( \delta_c - \frac{\delta Q}{Q} \right),$$

$$\theta'_c = - \mathcal{H}\theta_c,$$

Here, we note that the quantity $\delta Q/Q$ actually includes the perturbations for the Hubble rate $\delta H$ (recall that $\mathcal{H} = aH$). Using $\delta H$, the gauge invariant equations for the coupled dark fluids can be easily found [54]. Thus, in this analysis, we consider the perturbation of the Hubble expansion rate since the total expansion rate includes both background and perturbation. Now, the stability of the model depends on the pressure perturbations for dark energy which is also dependent on the interaction function through the relation,

$$\delta p_x = c_{sx}^2 \delta p_x - (c_{sx}^2 - c_{ax}^2)\rho_c \frac{\theta_x}{k^2},$$

$$= c_{sx}^2 \delta p_x + 3\mathcal{H}\rho_x (1 + w_x)(c_{sx}^2 - c_{ax}^2)(1 + d) \frac{\theta_x}{k^2},$$

where the parameter $d$ is

$$d \equiv -aQ/[3\mathcal{H}\rho_x(1 + w_x)],$$

which is named as the doom factor and it ensures the stability of any interaction model for $d \leq 0$. Thus, using the expression for the doom factor [11], for any interaction model, one can find the conditions for stability of the interaction model. In particular, for the usual models $Q = \xi HQ$ (where $Q > 0$) one can find that, the model could lead to stable perturbations at large scale if $\xi \geq 0$ and $(1 + w_x) > 0$ or $\xi \leq 0$ and $(1 + w_x) < 0$. So, clearly there is a jump of the equation of state $w_x$ at ‘−1’. We mention that if we simply consider the interacting cosmological constant scenario, then the governing equations become simple from the very beginning and the treatment is no longer the same for any arbitrary $w_x \neq -1$. However, for any arbitrary $w_x$, if we need to constrain the dark energy equation of state, then we cannot take the prior of $w_x \in [a, b]$ ($a, b \in \mathbb{R}$) where $-1$ is included in this closed interval. We must have to take the intervals $(-1, b)$ or $[a, -1)$. Thus, certainly, it is clear that some information is basically lost during the analysis. Such problem can be removed if we simply transform the interaction as $Q \rightarrow (1 + w_x)Q$, that means, if we include a term $(1 + w_x)$ from outside into the interaction function. However, such phenomenological construction can be viewed as a simple transformation of the coupling parameter as $\xi \rightarrow \xi(1 + w_x)$. Here we work on the model $Q = 3\mathcal{H}\xi(1 + w_x)\rho_x$ for which the doom factor [11] returns $d = -\xi$, and thus, the stability of this interaction model is ensured for $\xi \geq 0$. One may notice that this interaction function depends on dark energy density as well as pressure like contribution of the dark energy. Here, we focus on the dynamical dark energy equation of state. The main idea is to see how the cosmological parameters are affected in presence of an interaction when dark energy equation of state is dynamical unlike the interaction scenarios with constant equation of state or vacuum interaction. To begin with such investigations, we start with the following dark energy parametrization with only one free parameter as [47]

$$w_x(z) = \frac{w_0}{1 + z} \exp \left( \frac{z}{1 + z} \right),$$

(12)
where $w_0$ is the present value of the dark energy equation of state, i.e., $w_0 = w(z = 0)$. Before closing this section, in Fig. 1 and 2 we present the qualitative evolution of the present interaction model. In Fig. 1 we present the qualitative evolution of the interaction function for several coupling strengths where we analyzed the scenario both for the quintessence (left panel of Fig. 1) and phantom dark energy state (right panel of Fig. 1) parameters. From the left panel of Fig. 1 we see that the interaction function $Q$ remains to be positive, that means, for $w_0 > -1$ regime, the energy flow takes place from CDM to DE. While from the right panel of Fig. 1 (for $w_0 < -1$), we observe an interesting feature. Here we see that a sign change in $Q$ happens where in the high redshifts, $Q$ remains positive but in the low redshifts region, $Q$ becomes negative (energy flow takes place from DE to CDM). So, as a result the energy changes its direction. In Fig. 2 we have analyzed the qualitative nature of $Q$ but for varying $w_0$. In the left panel of Fig. 2 we have fixed a small coupling parameter ($\xi = 0.001$) and varied $w_0$ from its quintessence to phantom values from which one can clearly notice that $Q$ remains positive (energy flow from CDM to DE). From the right panel of Fig. 2 we fix $\xi = 0.5$ taking similar values of $w_0$ as taken for the left panel of Fig. 2 — this shows a transition of $Q$ from its negative to positive values and hence the changes in the direction of energy flow.

3. OBSERVATIONAL DATA AND THE CONSTRAINTS

In this section we describe the observational data and the fitting mechanism used to constrain the current interacting dark energy models.

- We consider the high-$\ell$ temperature and polarization as well as the low-$\ell$ temperature and polarization Cosmic Microwave Background angular power spectra from Planck (Planck TT, TE, EE + lowTEB) $^{55, 56}$.
- The Joint light-curve analysis (JLA) sample from Supernovae Type Ia $^{57}$.
- Baryon acoustic oscillations (BAO) distance measurements $^{58, 59, 60}$.
- Hubble parameter measurements from the Cosmic Chronometers (CC) $^{61}$.
- Local Hubble constant value yielding $H_0 = 73.24 \pm 1.74$ km/s/Mpc at 68% CL $^{62}$ (HST).
- Redshift space distortion data (RSD) $^{63}$.
- Weak lensing (WL) data from the Canada-France-Hawaii Telescope Lensing Survey $^{64, 65}$.

In order to extract the observational constraints on the free and derived parameters of the interacting models, we perform a fitting analysis using our modified version of the markov chain monte carlo package cosmomc $^{66, 67}$ that is equipped with a convergence diagnostic based on the Gelman and Rubin statistic and includes the support for the Planck data release 2015 Likelihood Code $^{56}$ (see http://cosmologist.info/cosmomc/). The interacting scenarios for the one parameter dark energy model $^{12}$ extends the parameters space beyond the six-parameters $\Lambda$CDM model. For convenience, the interacting model where DE assumes the parametrization $^{12}$ is labeled as IDE. Thus, one can see that for the spatially flat FLRW universe, the parameters space for IDE is,

$$P_1 \equiv \{\Omega_m h^2, \Omega_r h^2, 100\theta_{MC}, \tau, w_0, \xi, n_s, log[10^{10} A_S]\},$$

where the parameters $\Omega_m h^2$, $\Omega_r h^2$, are respectively the baryon and cold dark matter densities; $100\theta_{MC}$ is the ratio of sound horizon to the angular diameter distance; $\tau$ is the reionization optical depth; $n_s$ is the scalar spectral index; $A_S$ is the amplitude of the primordial scalar power spectrum. The remaining parameters $w_0$ is the free parameter introduced through the parametrization of dark energy equation of state (equation $^{12}$). Thus, one can see that the present IDE model is eight dimensional and hence an extended parameters space in compared to the $\Lambda$CDM model. The likelihood for this analysis is, $L \propto e^{-\chi^2/2}$ where $\chi^2 = \sum_i \chi_i^2$, and $i$ belongs to the data set {Planck TT, TE, EE + lowTEB, JLA, BAO, CC, HST, RSD, WL}. Thus, one may consider different observational combinations for a detailed analysis of the models. In what follows, we describe the results of the interacting scenarios.

3.1. Results: IDE

We summarize the observational constraints on the model parameters and the other cosmological parameters in Table I extracted from the analysis with differ-

| Parameter | Prior |
|-----------|-------|
| $\Omega_m h^2$ | [0.005, 0.1] |
| $\Omega_r h^2$ | [0.01, 0.99] |
| $\tau$ | [0.01, 0.8] |
| $n_s$ | [0.5, 1.5] |
| log[10^{10} A_S] | [2.4, 4] |
| $100\theta_{MC}$ | [0.5, 10] |
| $\xi$ | [0.1] |
| $w_0$ | [-2, 0] |

TABLE I: Flat priors on the cosmological parameters for the analysis of the interacting dark energy model.
FIG. 1: Qualitative evolution of the interaction model \( Q = 3H\xi(1 + w_x)\rho_x \) where \( w_x \) is given in eqn. \( \text{[12]} \) has been shown for some specific choices of the coupling parameter, \( \xi \). In the left panel we exhibit the behaviour of the interaction function for quintessence kind of dark energy, i.e., \( w_0 > -1 \) (in particular, we set \( w_0 = -0.95 \)) while the right panel depicts the evolution of the interaction function but for phantom dark energy state parameter, that means for \( w_0 < -1 \) (in particular, \( w_0 = -1.1 \)). Let us note that \( Q_0 = H_0\rho_{\text{tot},0} = 3H_0^2/(8\pi G) \) where \( \rho_{\text{tot}} = (\rho_r + \rho_b + \rho_c + \rho_x) \), is the total energy density of the universe and \( \rho_{\text{tot},0} = \rho_{\text{tot}}(z = 0) \).

FIG. 2: The figure depicts the evolution of the interaction function for different values of \( w_0 \) with some fixed coupling strengths. The left panel portrays the evolution of \( Q \) for different values of \( w_0 \) with a fixed and low coupling strength \( \xi = 0.001 \) while on the other hand, the right panel shows the same evolution but for a large coupling strength \( \xi = 0.5 \). We note that \( Q_0 \) has similar meaning as described for Fig. 1.

Our analyses show that the value of the coupling parameter \( \xi \) is very tiny which eventually makes the interaction function very small compared to the rate of change of energy densities due to the expansion of the universe. However, it is interesting to notice that, for some of the combinations, in particular, for CMB + BAO + RSD, CMB + BAO + WL and the full data sets, within 68.3% CL, \( \xi \neq 0 \) is suggested, however, within 95.4% CL, \( \xi = 0 \) is allowed, that means, the non-interacting \( w_x \), CDM cosmology is recovered in the 95.4% CL. Further, from the estimation of the dark energy equation of state, it is quite clear that the present value of the dark energy equation of state, \( w_0 \), is in the phantom regime for all the observational data sets shown in Table II. Even if the CMB data alone allow \( w_0 \) to be quintessential in the 68.3% CL, when this dataset is combined with other probes, \( w_0 \) is strictly less than ‘-1’ at more than 4σ. This result confirms a phantom nature for the DE equation of state, as determined in several works in the literature [31, 41, 42, 68, 69, 70].

We also observe an interesting feature. From Table II one may notice that the allowance of the interaction can relieve the tension on \( H_0 \) as observed from the global and local measurements. Such observation is clearly true for the CMB alone within 2σ, allowing us to combine safely HST with the other datasets. Moreover, it remains true also when considering CMB + BAO + WL and CMB +
FIG. 3: 68.3% (1σ) and 95.4% (2σ) confidence-level contours as well as the marginalized likelihood function of the parameters for an interacting dark energy-dark matter scenario with the one parameter in equation (12).

BAO + HST, where the tension on \( H_0 \) is released within 68.3% CL, and CMB + BAO + RSD where the tension is at 2σ.

In order to examine the effects of the interaction for this particular EoS in DE (12), we performed the analysis without allowing any interaction. In Table III we summarize the observational constraints on the free parameters for the same combined analyses as performed for the model with interaction. One can easily see (the Table III) that statistically, the cosmological constraints with and without interaction where dark energy has a varying nature given in (12) are really robust. In fact for the interacting model, the coupling strength \( \xi \) is found to be uncorrelated with \( \Omega_m^0 \), \( H_0 \), \( \sigma_8 \) and even with \( w_0 \), as we can see in Fig. 3. In Fig. 4 we display the two-dimensional contour plots for various combinations of the model parameters for interacting and non-interacting scenario and also the marginalized likelihood functions for those parameters obtained in the combined analysis with CMB + BAO + RSD + HST + WL + JLA + CC. From Fig. 4 one can easily observe that it is very hard to distinguish the interacting scenario from the non-interacting one. The dark energy equation of state seems to be unaltered even if an interaction is allowed in the dark sectors.

However, it is interesting to notice that without interaction, the CMB data prefer a phantom dark energy equation of state \( w_0 \) at more than 1σ, as we can see from Table III for the non-interaction scenario. Moreover, in this case we have a large shift towards higher value of \( H_0 \), now fully in agreement with the local value of HST.

Comparing Table III with Table III we can see that for the CMB only case the constraints on \( w_0 \) and the derived parameters are stronger when considering the interaction with respect to the case without interaction, that has one less parameter (see also Fig. 5). The reason for this feature can be found in Fig. 6, where it is evident that by introducing the interaction we break the degeneracy for larger negative values of \( w_x(z) \), that are no more in agreement with the data. This exact phenomenon is also reflected from the matter power spectrum of the IDE model displayed in Fig. 7.
FIG. 4: The 2D confidence contours on parameter space and the marginalized likelihood function of the parameters for interacting and non-interacting scenarios for the one parameter $w(z)$ parametrization given in equation (12). The results obtained for combined analysis with CMB + BAO + RSD + HST + WL + JLA + CC.

| Parameters | CMB | CMB+BAO+RSD | CMB+BAO+WL | CMB+BAO+HST | CMB+BAO+RSD+HST +WL+JLA+CC |
|------------|-----|-------------|-------------|-------------|-------------------------------|
| $\Omega_m h^2$ | 0.1213 | 0.1194 | 0.1194 | 0.1194 | 0.1185 |
| $\Omega_b h^2$ | 0.0208 | 0.0141 | 0.0141 | 0.0141 | 0.0141 |
| $100\sigma_8$ | 1.0401 | 1.0401 | 1.0401 | 1.0401 | 1.0401 |
| $\sigma_8$ | 0.085 | 0.079 | 0.079 | 0.079 | 0.079 |
| $n_s$ | 0.9670 | 0.9730 | 0.9730 | 0.9730 | 0.9730 |
| $\ln(10^{10} A_s)$ | 3.114 | 3.097 | 3.097 | 3.097 | 3.097 |
| $w_0$ | $-1.16$ | $-1.157$ | $-1.157$ | $-1.157$ | $-1.157$ |
| $\xi$ | <0.0028 | <0.0028 | <0.0028 | <0.0028 | <0.0028 |
| $\Omega_{\text{m0}}$ | 0.3130 | 0.2986 | 0.2986 | 0.2986 | 0.2986 |
| $\sigma_8$ | 0.829 | 0.819 | 0.819 | 0.819 | 0.819 |
| $H_0$ | 68.3 | 69.0 | 69.0 | 69.0 | 69.0 |

TABLE II: 68.3% (1σ) and 95.4% (2σ) constraints on the model parameters for the interacting dark matter–dark energy scenario where the dark energy equation of state has a single free parameter shown in equation (12).

3.2. IDE at large scales: CMB and matter power spectra

Let us now discuss the behaviour of the interaction model at the large scale of the universe and also measure the deviations of the interaction models with respect to the non-interacting cosmological models. In order to do so, in the left panel of Fig. 8 we show the temperature anisotropy in the CMB spectra for the one parameter $w_0(z)$ parametrization for different values of the coupling strength $\xi$. The other parameters for this plot are fixed according to their mean values, obtained
FIG. 5: The 2D confidence contours on parameter space and the marginalized likelihood function of the parameters for interacting and non-interacting scenarios for the one parameter $w_x(z)$ parametrization given in equation (12). The results obtained for combined analysis with CMB only.

FIG. 6: The left panel shows the temperature power-spectrum for $\xi = 0$ and different values of $w_x(z)$, of equation (12) while the right panel shows the temperature power-spectrum for $\xi = 0.5$ and different values of $w_x(z)$.

in the combined analysis with CMB + BAO + RSD + HST + WL + JLA + CC. In the right panel of Fig. 8 we show the corresponding residual with respect to the base $\Lambda$CDM model where we also include the case for non-interacting scenario. The height of the first acoustic pick in the power-spectrum changes as the coupling strength increases. The corresponding residuals for the model (right panel of Fig. 8) behaves very similar for higher multipoles (around $l \sim 10^3$), but at lower multipoles, in the cosmic variance limited region around $l \sim 10$, the one parameter $w_x$ model sharply deviates from the base $\Lambda$CDM for both interacting and non-interacting cases.
FIG. 7: The left panel displays the behaviour of the matter power spectrum for $\xi = 0$ and different values of $w_x(z)$ of equation [12], while the right panel shows the matter power spectrum with a fixed coupling parameter $\xi = 0.5$ and different values of $w_x(z)$.

Table III: 68.3% (1$\sigma$) and 95.4% (2$\sigma$) constraints on the model parameters for the non-interacting dark matter-dark energy scenario (i.e., $\xi = 0$) where the dark energy state parameter follows equation [12].

In the left panel of Fig. 9 we show the power spectra of the matter density contrast for the one interacting model with one parameter $w_x(z)$ for different values of the coupling parameter $\xi$ and in the right panel of Fig. 9 we display the corresponding residual with respect to the base $\Lambda$CDM model scaled by the corresponding $\Lambda$CDM values. The matter power spectra for the one parameter $w_x$ parametrization get suppressed as the strength of the interaction increases (left panel of Fig. 9).

### 3.3. Bayesian Evidence

In this section, statistical comparison of models has been discussed by calculating the evidence of the present interacting dark energy model with respect to the $\Lambda$CDM model following the Bayesian analysis.

In Bayesian analysis, the posterior probability distribution of a model parameter $\theta$ is defined based on a given data set $x$, used to test the model $M$, and any prior information. The Bayes theorem states the posterior probability of the parameter $\theta$ as,

$$p(\theta|x, M) = \frac{p(x|\theta, M) \pi(\theta|M)}{p(x|M)},$$

where the quantity $p(x|\theta, M)$ represents the likelihood function which depends on the model parameters $\theta$ with the fixed data set and $\pi(\theta|M)$ is the prior information. The quantity $p(x|M)$ in the denominator of the right hand side of eq. (14) is known as the Bayesian evidence which is actually the integral over the unnormalised posterior $\tilde{p}(\theta|x, M) \equiv p(x|\theta, M) \pi(\theta|M)$. It expressed as

$$E \equiv p(x|M) = \int d\theta p(x|\theta, M) \pi(\theta|M).$$

This is also referred to as the global likelihood. Now, between any two models, $M_i$ and $M_j$ where $M_j$ is the model under consideration and $M_i$ is the reference model (here the $\Lambda$CDM model), the posterior probability is given by the product of the ratio of the model priors and the ratio of Evidences.
FIG. 8: The left panel shows the power-spectrum of temperature anisotropy for the one parameter $w_x(z)$ for different values of the interaction coupling parameter $\xi$. The other parameters are fixed according to to the best-fit values obtained in the combined analysis with CMB + BAO + RSD + HST + WL + JLA + CC. The right panel shows the corresponding residuals for the model for both interacting and non-interacting scenario with respect to base $\Lambda$CDM model scaled by corresponding $\Lambda$CDM value.

FIG. 9: The left panel shows the power-spectrum of matter density contrast for the one parameter $w_x(z)$ for different values of the interaction coupling parameter $\xi$. The other parameters are fixed according to to the best-fit values obtained in the combined analysis with CMB + BAO + RSD + HST + WL + JLA + CC. The right panel shows the corresponding residuals for the model for both interacting and non-interacting scenario with respect to base $\Lambda$CDM model scaled by corresponding $\Lambda$CDM value.

TABLE IV: Interpretation of the revised Jeffreys scale by Kass and Raftery [71] used in this work.

| $\ln B_{ij}$ | Strength of evidence for model $M_i$ |
|--------------|-------------------------------------|
| $0 \leq \ln B_{ij} < 1$ | Weak |
| $1 \leq \ln B_{ij} < 3$ | Definite/Positive |
| $3 \leq \ln B_{ij} < 5$ | Strong |
| $\ln B_{ij} \geq 5$ | Very strong |

TABLE V: Values of $\ln B$ and the strength of the evidence for the IDE model against the $\Lambda$CDM, as obtained in our analysis for different dataset combinations. The negative sign actually indicates that the $\Lambda$CDM is preferred over the IDE model.
\[
p(M_i|x) = \frac{\pi(M_i) p(x|M_i)}{\pi(M_j) p(x|M_j)} = \frac{\pi(M_i)}{\pi(M_j)} B_{ij},
\]
where \( B_{ij} = \frac{p(x|M_i)}{p(x|M_j)} \), is the Bayes factor of the considered model \( M_i \) compared to the reference model \( M_j \). This factor reports how the observational data support the model \( M_i \) over \( M_j \). We classify the model comparison as follows: if \( B_{ij} > 1 \), then the data support the model \( M_i \) more strongly compared to the model \( M_j \). Now, depending on different values of the Bayes factor \( B_{ij} \) (sometimes one calculates the values of \( \ln B_{ij} \)), we compare the models. This quantification is generally done accepting the revised Jeffreys scale by Kass and Raftery\(^7\) displayed in Table IV.

The Bayesian evidence is computed using the MCMC chains for the statistical analysis of the model. We refer to the original works \[^{72}\]^{75} for a detailed implementation of the code MCEvidence\(^1\).

In Table IV we present the \( \ln B_{ij} \) values of the IDE model with respect to the base \( \Lambda \)CDM model. The negative values of \( \ln B_{ij} \) indicate the preference of the \( \Lambda \)CDM over the interacting dark energy model. From the numerical values of \( \ln B_{ij} \) and using the Jeffreys scale (Table IV), one may clearly conclude that the base \( \Lambda \)CDM is strongly favored over the IDE model.

4. CONCLUSIONS AND FINAL REMARKS

An interacting dark energy scenario with time varying equation of state parameter, and in particular, a dynamical DE parametrization, given in equation (12) has been investigated in the present work. A series of analysis with recent observational data indicate the possibility of an interaction in the dark fluids\[^{11}\]^{12}[^{13}\]^{14}[^{22}\]^{23}[^{40}\]^{41} and thus, the cosmological models that allow interaction between dark energy and dark matter, are gaining significant attention at current time. One important issue, related to the interacting dark energy, is the large scale stability of the model which depends on the choice of the interaction function. Most of the interacting dark energy models, present in the literature, suffers from the singularity at \( w_x = -1 \). But the problem can be alleviated with some particular choices of the interaction function\[^{33}\]^{39}. In the present work, the interaction function is chosen as \( Q = 3H \xi (1 + w_x) \rho_x \), thus the interaction function depends on the energy density of the dark energy \( (\rho_x) \) and pressure like contribution \( (p_x = w_x \rho_x) \) of dark energy. This type of interaction function successfully removes the singularity in the pressure perturbation equation (equation[11]) of dark energy at \( w_x = -1 \).

To fit the models with the observational data we use different combinations of recently available observational data sets. For the one parameter \( w_x(z) \) model, given in equation (12), the coupling parameter \( (\xi) \) is obtained to be very small and consequently the interaction is less significant. Fig. 4 shows that for this one parameter \( w_x(z) \) model, the observational constraints, obtained in interacting and non-interacting scenarios, are almost indistinguishable. However, the analysis also shows that within the 68.3% CL, the combined data CMB + BAO + HST and CMB + BAO + WL are in tension with CMB + BAO + RSD and CMB + BAO + RSD + HST + WL + JLA + CC.

The power-spectrum of the anisotropy of CMB temperature (Fig. 5), the power-spectrum of matter density contrast (Fig. 6), and also the corresponding residuals with respect to the base \( \Lambda \)CDM clearly show that the interacting one parameter \( w_x \) model sharply deviates from the base \( \Lambda \)CDM power-spectrum.

The possible type of interaction in the dark sector is not yet known. Only the direction of energy flow can be determined from the observational constraints on the interaction function \( (Q) \). In the present analysis, the value of the coupling parameter \( (\xi) \) is assumed to be positive. But in the present study, the interaction function also depends on a factor \( (1 + w_x) \), that mean the direction of energy flow is effected by the nature of dark energy. The present analysis allows both quintessence \( (w_x > -1) \) and phantom \( (w_x < -1) \) regimes, and the best fit remains in phantom one. Thus \( Q \) is allowed to have both positive and negative value, though \( Q < 0 \) is slightly preferred. From equations (2) and (3), it is clear that a negative value of the interaction function \( Q \) indicates the energy flow from the dark energy to dark matter.

The results obtained in the present work show that the correlation between the dark energy equation of state parameter and the coupling parameter \( (\xi) \) of the interaction function is very poor. In fact, for the IDE, the parameters \( w_0 \) and \( \xi \) are almost uncorrelated. It indicates that the dark energy equation of state parameter is in general degenerate with the possible interaction in the dark sector. In an earlier work\[^{74}\], the author has discussed about the degeneracy in generalized dark energy models with respect to the different cosmological probes and concluded that interacting dark energy is always equivalent to a class non-interacting dark energy. From the present analysis, it can be particularly concluded that the dark energy equation of state is not distinguishable for interacting and non-interacting scenario based on present cosmological data at background and linear perturbation level. Lastly, from the Bayesian analysis, we find that \( \Lambda \)CDM is still strongly favored over the IDE model. This might be the case related to the increased dimension of the IDE parameter space compared to the 6-parameters based \( \Lambda \)CDM model.

\(^1\) This code is publicly available at github.com/yabebalFantaye/MCEvidence.
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