OT Syntax: Decidability of Generation-based Optimization

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Abstract
In Optimality-Theoretic Syntax, optimization with unrestricted expressive power on the side of the OT constraints is undecidable. This paper provides a proof for the decidability of optimization based on constraints expressed with reference to local subtrees (which is in the spirit of OT theory). The proof builds on Kaplan and Wedekind’s (2000) construction showing that LFG generation produces context-free languages.

1 Introduction
Optimality-Theoretic (OT) grammar systems are an interesting alternative to classical formal grammars, as they construe the task of learning from data in a meaning-based way: a form is defined as grammatical if it is optimal (most harmonic) within a set of generation alternatives for an underlying logical form. The harmony of a candidate analysis depends on a language-specific ranking (≽) of violable constraints, thus the learning task amounts to adjusting the ranking over a given set of constraints.

(1) Candidate A₁ is more harmonic than A₂ iff it incurs fewer violations of the highest-ranking constraint C₁ ∈ C in which A₁ and A₂ differ.

The comparison-based setup of OT learning is closely related to discriminative learning approaches in probabilistic parsing (Johnson et al., 1999; Riezler et al., 2000; Riezler et al., 2002), however the comparison of generation alternatives – rather than parsing alternatives – adds the possibility of systematically learning the basic language-specific grammatical principles (which in probabilistic parsing are typically fixed a priori, using either a treebank-derived or a manually written grammar for the given language). The “base grammar” assumed as given can be highly unrestricted in the OT setup. Using a linguistically motivated set of constraints, learning proceeds with a bias for unmarked linguistic structures (cf. e.g., (Bresnan et al., 2001)).

For computational OT syntax, an interleaving of candidate generation and constraint checking has been proposed (Kuhn, 2000). But the decidability of the optimization task in OT syntax, i.e., the identification of the optimal candidate(s) in a potentially infinite candidate set, has not been proven yet.

2 Undecidability for unrestricted OT
Assume that the candidate set is characterized by a context-free grammar (cfg) G₁, plus one additional candidate ‘yes’. There are two constraints (C₁ ≻ C₂): C₁ is violated if the candidate is neither ‘yes’ nor a structure generated by a cfg G₂; C₂ is violated only by ‘yes’. Now, ‘yes’ is in the language defined by this system iff there are no structures in G₁ that are also in G₂. But the emptiness problem for the intersection of two context-free languages is known to be undecidable, so the optimization task for unrestricted OT is undecidable too.

However, it is not in the spirit of OT to have extremely powerful individual constraints; the explanatory power should rather arise from interaction of simple constraints.

3 OT-LFG
Following (Bresnan, 2000; Kuhn, 2000; Kuhn, 2001), we define a restricted OT system based on Lexical-Functional Grammar (LFG) representations: c(category) structure/functional) structure

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2 Most computational OT work so far focuses on candidates and constraints expressible as regular languages/rational relations, based on (Frank and Satta, 1998) (e.g., (Eisner, 1997; Karttunen, 1998; Gerdemann and van Noord, 2000)).
3 Cf. also (Johnson, 1998) for the sketch of an undecidability argument and (Kuhn, 2001, 4.2, 6.3) for further constructions.
pairs \( \langle T, \Phi \rangle \) like \( \langle (4),(5) \rangle \). Each c-structure tree node is mapped to a node in the f-structure graph by the function \( \phi \). The mapping is specified by f-annotations in the grammar rules (below category abbreviates, cf. (2)) and lexicon entries (3).\(^4\)

The correct f-structure for a sentence is the minimal model satisfying all properly instantiated f-annotations.

In OT-LFG, the universe of possible candidates is defined by an LFG \( G_{inviol} \) (encoding inviolable principles, like an X-bar scheme). A particular candidate set is the set \( Gen_{inviol}(\Phi_{in}) \) – i.e., the c-/f-structure pairs in \( G_{inviol} \), which have the input \( \Phi_{in} \) as their f-structure. Constraints are expressed as local configurations in the c-/f-structure pairs. They have one of the following implicational forms:\(^5\)

\[
(6) \quad S \quad \Rightarrow \quad S' \quad \quad \text{where } S, S' \text{ are standard LFG f-annotations of constraining equations with } \uparrow \text{ as the only f-structure metavariable.}
\]

\[
(7) \quad \begin{array}{c}
\frac{\pi}{\pi'} \quad M \quad \Rightarrow \quad M' \quad \quad \text{where } M, M', N, N' \text{ are descriptions of nonterminals of } G_{inviol}; \quad N, N' \text{ refer to the mother in a local subtree configuration, } M, M' \text{ refer to the same daughter category;} \quad \pi, \pi', \sigma, \sigma' \text{ are regular expressions over nonterminals;} \quad S, S' \text{ are standard f-annotations as in (6).}
\end{array}
\]

Any of the descriptions can be maximally unspecified; (6) can for example be instantiated by the OPSPEC constraint \( \uparrow \text{OP}=+ \Rightarrow (DF \uparrow) \) (an operator must be the value of a discourse function, (Bresnan, 2000)) with the category information unspecified.

An OT-LFG system \( \mathcal{O} \) is thus characterized by a base grammar and a set of constraints, with a language-specific ranking relation \( \gg \mathcal{L} \) :

\[
\mathcal{O} = \langle G_{inviol}, \langle \mathcal{C}, \gg \mathcal{L} \rangle \rangle
\]

The evaluation function \( \text{Eval}(\mathcal{C}, \gg \mathcal{L}) \) picks the most harmonic from a set of candidates, based on the constraints and ranking. The language (set of analyses)\(^6\) generated by an OT system is defined as

\[
L(\mathcal{O}) = \{ \langle T_j, \Phi_j \rangle \in G_{inviol} \mid \exists \Phi_{in} : \langle T_j, \Phi_j \rangle \in \text{Eval}(\mathcal{C}, \gg \mathcal{L})(\text{Gen}_{inviol}(\Phi_{in})) \}
\]

### 4 LFG generation

Our decidability proof for generation-based optimization builds on the result of (Kaplan and Wedekind, 2000) (K&W00) that LFG generation produces context-free languages.

\(^4\) \( \downarrow \) abbreviates \( \phi(*) \), i.e., the present category’s \( \phi \) image; \( \uparrow \) abbreviates \( \phi(M^*) \), i.e., the f-structure corresponding to the present node’s mother category.

\(^5\) Note that with GPSG-style category-level feature percolation it is possible to refer to (finitely many) nonlocal configurations at the local tree level.

\(^6\) The string language is obtained by taking the terminal string of the c-structure part of the analyses.
Given an arbitrary LFG grammar $G$ and a cycle-free f-structure $\Phi$, a cfg $G'$ can be constructed that generates exactly the strings to which $G$ assigns the f-structure $\Phi$.

I will refer to the resulting cfg $G'$ as $KW(G, \Phi)$. K&W00 present a constructive proof, folding all f-structural contributions of lexical entries and LFG rules into the c-structural rewrite rules (which is possible since we know in advance the range of f-meta-variables in the rules). I illustrate the specialization steps with grammar (2) and lexicon (3) and for generation from f-structure (5).

Initially, the generalized format of right-hand sides in LFG rules is converted to the standard context-free notation (resolving regular expressions by explicit disjunction or recursive rules). F-structure (5) contains five substructures: the root f-structure, plus the embedded f-structures under the paths SUBJ, COMP, COMP SUBJ, and COMP OBJ. Any relevant metavariable (↑, ↓) in the grammar must end up instantiated to one of these. So for each path from the root f-structure, a distinct variable is introduced: $v$, subscripted with the (abbreviated and possibly empty) feature path: $v\prime$, $v\prime\prime$, $v\prime\prime\prime$, $v\prime\prime\prime\prime$, $v\prime\prime\prime\prime\prime$.

Rule augmentation step 1 adds to each category name a concrete f-structure to which the category corresponds. So for FP, we get FP:$v$, FP:$v\prime$, FP:$v\prime\prime$, FP:$v\prime\prime\prime$, and FP:$v\prime\prime\prime\prime$. The rules are multiplied out to cover all combinations of augmented categories obeying the original f-annotations.\footnote{\textit{VP}: $v \to \text{NP}:v\prime; \text{VP}:v\prime\prime$ is allowed, while \textit{VP}: $v \to \text{NP}:v\prime; \text{VP}:v\prime\prime\prime$ is excluded, since the ↑ annotation of $V'$ in the VP rule (2) enforces that $\phi(\text{VP}) = \phi(V')$.}

Step 2 adds a set of instantiated f-annotation schemes to each symbol, based on the instantiation of metavariables from step 1. One instance of the lexicon entry Mary look as follows:

\begin{align*}
\text{NP}:v\prime\prime\prime\prime \to \text{Mary} & \\
\text{NP}:v\prime\prime\prime\prime \to \text{Mary}
\end{align*}

The rules are again multiplied out to cover all combinations for which the set of f-constraints on the mother is the union of all daughters’ f-constraints, plus the appropriately instantiated rulespecific annotations. So, for the VP rule based on the categories NP:$v\prime\prime\prime\prime$, \{ ($v_{c\prime\prime\prime\prime}$ PRED) = ’Mary’ \}, \{ ($v_{c\prime\prime\prime\prime}$ NUM) = SG \} and V'$:v\prime\prime\prime\prime$, \{ ($v_{c\prime\prime\prime\prime}$ PRED) = ’laugh’ \}, \{ ($v_{c\prime\prime\prime\prime}$ TNS) = PAST \}, $v_{c\prime\prime\prime\prime} = v_{c}$, we get the rule

\begin{align*}
\text{NP}:v\prime\prime\prime\prime \to \text{VP}:v\prime; \text{NP}:v\prime\prime; \text{VP}:v\prime\prime\prime; \text{VP}:v\prime\prime\prime\prime & \\
\text{NP}:v\prime\prime\prime\prime \to \text{VP}:v\prime; \text{NP}:v\prime\prime; \text{VP}:v\prime\prime\prime; \text{VP}:v\prime\prime\prime\prime
\end{align*}

With this bottom-up construction it is ensured that each new category $\text{ROOT}:v\prime; \ldots$ (corresponding to the original root symbol) contains a complete possible collection of instantiated f-constraints. To exclude analyses whose f-structure is not $\Phi$ (for which we are generating strings) a new start symbol is introduced “above” the original root symbol. Only for the sets of f-constraints that have $\Phi$ as their minimal model, rules of the form $\text{ROOT} \to \text{ROOT}:v; \ldots$ are introduced (this also excludes inconsistent f-constraint sets).

With the cfg $KW(G, \Phi)$, standard techniques for cfg’s can be applied, e.g., if there are infinitely many possible analyses for a given f-structure, the smallest one(s) can be produced, based on the pumping lemma for context-free languages. Grammar (2) does indeed produce infinitely many analyses for the input f-structure (5). It overgenerates in several respects: The functional projection FP can be stacked due to recursions like the following (with the augmented FP reoccurring in the $F'$ rules):

\begin{align*}
\text{FP}:v\prime; \to \text{VP}:v\prime; \text{NP}:v\prime\prime; \text{VP}:v\prime\prime\prime; \text{VP}:v\prime\prime\prime\prime & \\
\text{FP}:v\prime; \to \text{VP}:v\prime; \text{NP}:v\prime\prime; \text{VP}:v\prime\prime\prime; \text{VP}:v\prime\prime\prime\prime
\end{align*}

F$\prime$: $v\prime; \to$ one of the augmented categories we get for that in (3), so $KW((2),(5))$ generates an arbitrary number of that’s on top of any FP. A similar repetition effect will arise for the auxiliary had.\footnote{The F$^{0}$ entries do not contribute any PRED value, which would exclude doubling due to the instantiated symbol character of PRED values (cf. K&W00, fn. 2).} Other choices in generation arise from the freedom of generating the subject in the specifier of VP or FP and from the possibility of (unbounded) topicalization of the object (the first disjunction of the FP rule in (2))
contains a functional-uncertainty equation):

\[(\text{a})\quad \text{John thought that Titanic, Mary had seen.}\]
\[(\text{b})\quad \text{Titanic, John thought that Mary had seen.}\]

5 LFG generation in OT-LFG

While grammar (2) would be considered defective as a classical LFG grammar, it constitutes a reasonable example of a candidate generation grammar \(G_{inviol}\) in OT. Here, it is the OT constraints that enforce language-specific restrictions, so \(G_{inviol}\) has to ensure that all candidates are generated in the first place. For instance, expletive elements as \textit{do} in \textit{Who do you know} will arise by passing a recursion in the \textit{cfg} constructed during generation. A candidate containing such a vacuous cycle can still become the winner of the OT competition if the Faithfulness constraint punishing expletives is outranked by some constraint favoring an aspect of the recursive structure. So the harmony is increased by going through the recursion a certain number of times. It is for this very reason, that \textit{Who do you know} is predicted to be grammatical in English.

So, in OT-LFG it is not sufficient to apply just the \textit{KW} construction; I use an additional step: prior to application of \textit{KW}, the LFG grammar \(G_{inviol}\) is converted to a different form \(O_{C}(G_{inviol})\) (depending on the constraint set \(\mathcal{C}\)), which is still an LFG grammar but has category symbols which reflect local constraint violations. When the \textit{KW} construction is applied to \(O_{C}(G_{inviol})\), all “pumping” structures generated by the \textit{cfg} \(KW(O_{C}(G_{inviol}), \Phi_{in})\) can indeed be ignored since all OT-relevant candidates are already contained in the finite set of non-recursive structures. So, finally the ranking of the constraints is taken into consideration in order to determine the harmony of the candidates in this finite subset.

6 The conversion \(O_{C}(G_{inviol})\)

Preprocessing Like K&W00, I assume an initial conversion of the c-structure part of rules into standard context-free form, i.e., the right-hand side is a category string rather than a regular expression. This ensures that for a given local subtree, each constraint (of form (6) or (7)) can be applied only a finite number of times: if \(l\) is the arity of the longest right-hand side of a rule, the maximal number of local violations is \(l\) (since some constraints of type (7) can be instantiated to all daughters).

Grammar conversion With the number of local violations bounded, we can encode all candidate distinctions with respect to constraint violations at the local-subtree level with finite means: The set of categories in the newly constructed LFG grammar \(O_{C}(G_{inviol})\) is the finite set

\[(\text{11})\quad N_{O_{C}(G_{inviol})}: \text{the set of categories in } O_{C}(G_{inviol})
\[
\{N: (n_{1}, n_{2}, n_{3}, \ldots n_{k}) \mid
N \text{ a nonterminal symbol of } G_{inviol},
k \text{ the size of the constraint set } \mathcal{C},
0 \leq n_{i} \leq l,
l \text{ the arity of the longest rhs in rules of } G_{inviol}\}\]

The rules in \(O_{C}(G_{inviol})\) are constructed in such a way that for each rule \(X_{0} \rightarrow X_{1} \ldots X_{m}\)
\[
T_{1} \quad T_{m}
\]
in \(G_{inviol}\) and each sequence \(\langle n_{0}^{1}, n_{0}^{2}, \ldots n_{0}^{k} \rangle\),
\[
0 \leq n_{0}^{i} \leq l, \text{ all rules of the form}
\]
\[
X_{0}: \langle n_{0}^{1}, n_{0}^{2}, \ldots n_{0}^{k} \rangle \rightarrow X_{1}: \langle n_{1}^{1}, n_{1}^{2}, \ldots n_{1}^{k} \rangle \ldots X_{m}: \langle n_{m}^{1}, n_{m}^{2}, \ldots n_{m}^{k} \rangle,
\[
0 \leq n_{j}^{i} \leq l
\]
are included such that \(n_{0}^{i}\) (the number of violations of constraint \(C^{i}\) incurred local to the rule) and the \(f\)-annotations \(T_{1}^{i}, \ldots T_{m}^{i}\) are specified as follows:

\[(\text{12})\quad \text{for } C^{i} \text{ of form (6)}\]
\[
\begin{bmatrix}
N & N' \Rightarrow S' \quad S^{'}
\end{bmatrix}
\]
\[
\begin{align*}
\text{a. } & n_{0}^{i} = 0; T_{j}^{i} = T_{j} (1 \leq j \leq m) \\
& \text{if } X_{0} \text{ does not match the condition } N; \\
\text{b. } & n_{0}^{i} = 0; T_{j}^{i} = T_{j} \land \neg S; T_{j}^{i} = T_{j} (2 \leq j \leq m) \\
& \text{if } X_{0} \text{ matches } N; \\
\text{c. } & n_{0}^{i} = 0; T_{j}^{i} = T_{j} \land S \land S'; T_{j}^{i} = T_{j} (2 \leq j \leq m) \\
& \text{if } X_{0} \text{ matches both } N \text{ and } N'; \\
\text{d. } & n_{0}^{i} = 1; T_{j}^{i} = T_{j} \land S \land S'; T_{j}^{i} = T_{j} (2 \leq j \leq m) \\
& \text{if } X_{0} \text{ matches } N \text{ but not } N'; \\
\text{e. } & n_{0}^{i} = 1; T_{j}^{i} = T_{j} \land S \land S'; T_{j}^{i} = T_{j} (2 \leq j \leq m) \\
& \text{if } X_{0} \text{ matches both } N \text{ and } N'; \\
\end{align*}
\]

\[(\text{13})\quad \text{for } C^{i} \text{ of form (7)}\]
\[
\begin{bmatrix}
N & N' \Rightarrow S' \quad S^{'}
\end{bmatrix}
\]
\[
\begin{align*}
\text{a. } & n_{0}^{i} = 0; T_{j}^{i} = T_{j} (1 \leq j \leq m) \\
& \text{if } X_{0} \text{ does not match the condition } N; \\
\text{b. } & n_{0}^{i} = \sum_{j=1}^{m} d_{j}; T_{j}^{i} = \delta(T_{j}, S, S') (1 \leq j \leq m), \\
\end{align*}
\]
where
We can overload the function name \textit{Cat} with a function applying to the set of analyses produced by an LFG grammar \( G \) by defining
\[
\text{Cat}(G) = \{ \langle T, \Phi \rangle \mid \langle T', \Phi \rangle \in G, T \text{ is derived from } T' \text{ by applying Cat to all category symbols} \}.
\]

Coverage preservation of the \( O_C \) construction holds also for the projected c-category skeleton (cf. the argumentation in fn. 10):
\[
(15) \quad \text{C-structure level coverage preservation}
\]
For an LFG grammar \( G: \text{Cat}(O_C(G)) = G \)

Each category in \( O_C(G) \) encodes the number of local violations for all constraints. Since all constraints are locally evaluable by assumption, \textit{all} constraints violated by a candidate analysis have to be incurred local to some subtree. Hence the total number of constraint violations incurred by a candidate can be computed by simply summing over all category-encoded local violation profiles:
\[
(16) \quad \text{Total number of constraint violations}
\]
Let \( \text{Nodes}(T) \) be the multiset of categories occurring in the c-structure tree \( T \), then the total number of violations of constraint \( C^i \) incurred by an analysis \( \langle T, \Phi \rangle \in O_C(G_{\text{inviol}}) \) is
\[
\#_{C^i}(T) = \sum_{N: x^n \in \text{Nodes}(T)} n^i
\]
Define \( \text{Total}(T) = \langle \#_{C^1}(T), \#_{C^2}(T), \ldots, \#_{C^n}(T) \rangle \)

7 Applying KW on \( O_C(G_{\text{inviol}}) \)

Since \( O_C(G_{\text{inviol}}) \) is a standard LFG grammar, we can apply the \textit{KW} construction to it to get a cfg for a given f-structure \( \Phi_m \). The category symbols then have the form \( X:(n^1, \ldots, n^h):w:D \), with \( v \) and \( D \) arising from the \textit{KW} construction. We can overload the projection function \textit{Cat} again such that \( \text{Cat}(u:v:w:x) = u \) for all augmented category symbol of the new format; likewise \( \text{Cat}(G) \) for a cfg.

Since the \( O_C \) construction (strongly) preserves the language generated, coverage preservation holds also after the application of \textit{KW} to \( O_C(G_{\text{inviol}}) \) and \( G_{\text{inviol}} \), respectively:
\[
(17) \quad \text{Cat}(\text{KW}(G_{\text{inviol}}, \Phi_m)) = \text{Cat}(\text{KW}(G_{\text{inviol}}, \Phi_m))
\]

But since the symbols in \( O_C(G_{\text{inviol}}) \) reflect local constraint violations, \( \text{Cat}(\text{KW}(O_C(G_{\text{inviol}}), \Phi_m)) \) has the property that all instances of recursion in the
resulting cfg create candidates that are at most as harmonic as their non-recursive counterparts. Assuming a projection function \( \text{CatCount}(u:v:w:x) = u:v \), we can state more formally:

(18) If \( T_1 \) and \( T_2 \) are CatCount projections of trees produced by the cfg \( K \mathit{W}(O_c(G_{\text{inviol}}), \Phi_m) \), using exactly the same rules, and \( T_2 \) contains a superset of the nodes that \( T_1 \) contains, then

\[ n_1^i \leq n_2^i \text{ for all } n_1^i, n_2^i \text{ (for } i = 1 \text{ to } k) \]

from \( (n_1^1 \ldots n_1^{i-1} \ldots n_1^k) = \text{Total}(T_1) \)

and \( (n_2^1 \ldots n_2^{i-1} \ldots n_2^k) = \text{Total}(T_2) \).

This fact follows from definition of Total (16): the violation counts in the additional nodes in \( T_2 \) will add to the total of constraint violations (and if none of the additional nodes contains any local constraint violation at all, the total will be the same as in \( T_1 \)).

Intuitively, the effect of the augmentation of the category format is that certain recursions in the pure \( K \mathit{W} \) construction (which one may think of as a loop) are unfolded, leading to a longer loop. The new loop is sufficiently large to make all relevant distinctions.

This result can be directly exploited in processing: if all non-recursive analyses are generated (of which there are only finitely many) it is guaranteed that a subset of the optimal candidates is among them. If the grammar does not contain any violation-free recursion, we even know that we have generated all optimal candidates.

(19) A recursion with the derivation path \( A \Rightarrow \ldots \Rightarrow A \) is called violation-free iff all categories dominated by the upper occurrence of \( A \), but not dominated by the lower occurrence of \( A \) have the form \( N : \langle n_1^1, \ldots, n_1^k \rangle \) with \( n_1^i = 0, i = 1 \text{ to } k \).

Note that if there \( is \) an applicable violation-free recursion, the set of optimal candidates is infinite; so if the constraint set is set up properly in a linguistic analysis, one would assume that violation-free recursion should not arise. (Kuhn, 2000) excludes the application of such recursions by a similar condition as offline parsability (which excludes vacuous recursions over a string in parsing), but with the \( K \mathit{W} \) construction, this condition is not necessary for decidability of the generation-based optimization task. The cfg produced by \( K \mathit{W} \) can be transformed further to only generate the optimal candidates according to the constraint ranking \( \gg \) of the OT system \( \mathcal{O} = \langle G_{\text{inviol}}, \langle C, \gg \rangle \rangle \), eliminating all but the violation-free recursions in the grammar:

(20) Creating a cfg that produces all optimal candidates

a. Define \( \mathcal{T}_{\Phi_m^{\text{inv}}} = \{ T \in K \mathit{W}(O_c(G_{\text{inviol}}), \Phi_m) \mid T \text{ contains no recursion} \} \).

\( \mathcal{T}_{\Phi_m^{\text{inv}}} \) is finite and can be easily computed, by keeping track of the rules already used in an analysis.

b. Redefine \( \text{Eval}(\mathcal{C}, \gg) \) to apply on a set of context-free analyses with augmented category symbols with counts of local constraint violations:

\( \text{Eval}(\mathcal{C}, \gg) (T) = \{ T \in \mathcal{T} \mid T \text{ is maximally harmonic in } \mathcal{T}, \text{ under ranking } \gg \} \)

Using the function \( \text{Total} \) defined in (16), this function is straightforward to compute for finite sets, i.e., in particular \( \text{Eval}(\mathcal{C}, \gg) (\mathcal{T}_{\Phi_m^{\text{inv}}}) \).

c. Augment the category format further by one index component. Define index \( h = 0 \) for all categories in \( K \mathit{W}(O_c(G_{\text{inviol}}), \Phi_m) \) of the form \( X: \langle n_1^1, \ldots, n_1^k \rangle \) where \( n_1^i = 0 \) for \( i = 1 \text{ to } k \). Introduce a new unique index \( h > 1 \) for each node of the form \( X: \langle n_1^1, \ldots, n_1^k \rangle \), where \( n_1^i \neq 0 \) for some \( n_1^i \) (\( 1 \leq i \leq k \)) occurring in the analyses \( \text{Eval}(\mathcal{C}, \gg) (\mathcal{T}_{\Phi_m^{\text{inv}}}) \text{ (i.e., different occurrences of the same category are distinguished).} \)

d. Construct the cfg

\[ G_{\Phi_m}^{\text{OT}} = \langle N_{\Phi_m}^{\text{OT}}, T_{\Phi_m}^{\text{OT}}, S_{\Phi_m}, R_{\Phi_m}^{\text{OT}} \rangle, \]

where \( N_{\Phi_m}^{\text{OT}}, T_{\Phi_m}^{\text{OT}} \) are the indexed symbols of step c.; \( S_{\Phi_m} \) is a new start symbol; the rules \( R_{\Phi_m}^{\text{OT}} \) are (i) those rules from \( K \mathit{W}(O_c(G_{\text{inviol}}), \Phi_m) \) which were used in the analyses in \( \text{Eval}(\mathcal{C}, \gg) (\mathcal{T}_{\Phi_m^{\text{inv}}}) \) — with the original symbols replaced by the indexed symbols —, (ii) the rules in \( K \mathit{W}(O_c(G_{\text{inviol}}), \Phi_m) \), in which the mother category and all daughter categories are of the form \( X: \langle n_1^1, \ldots, n_1^k \rangle : \mathcal{D} \), \( n_1^i = 0 \) for \( i = 1 \text{ to } k \) (with the new index 0 added), and (iii) one rule \( S_{\Phi_m} \rightarrow X: h \) for each of the indexed versions \( X: h \) of the start symbols of \( K \mathit{W}(O_c(G_{\text{inviol}}), \Phi_m) \).

With the index introduced in step (20c), the original recursion in the cfg is eliminated in all but the violation-free cases. The grammar \( \text{Cat}(G_{\Phi_m}^{\text{OT}}) \) produces the \( c \)-structure of the set of optimal candidates for the input \( \Phi_m \).\(^{12}\)

(21) \[ \text{Cat}(G_{\Phi_m}^{\text{OT}}) = \{ T \mid \langle T, \Phi_m \rangle \in \text{Eval}(\mathcal{C}, \gg) (\text{Gen}_{\text{inviol}}(\Phi_m)) \} \]

i.e., the set of \( c \)-structures for the optimal candidates for input f-structure \( \Phi_m \) according to the OT system \( \mathcal{O} = \langle G_{\text{inviol}}, \langle C, \gg \rangle \rangle \).

\(^{11}\)The projection function \( \text{Cat} \) is again overloaded to also remove the index on the categories.

\(^{12}\)Like K&W00, I make the assumption that the input f-structure in generation is fully specified (i.e., all the candidates have the form \( \langle T, \Phi_m \rangle \)), but the result can be extended to allow for the addition of a finite amount of f-structure information in generation. Then, the specified routine is computed separately for each possible f-structural extension and the results are compared in the end.
8 Proof

To prove fact (21) we will show that the c-structure of an arbitrary candidate analysis generated from \( \Phi_{in} \) with \( G_{\text{inviol}} \) is contained in \( \text{Cat}(G_{\Phi_{in}}^{QT}) \) iff all other candidates are equally or less harmonic.

Take an arbitrary candidate c-structure \( T \) generated from \( \Phi_{in} \) with \( G_{\text{inviol}} \) such that \( T \in \text{Cat}(G_{\Phi_{in}}^{QT}) \). We have to show that all other candidates \( T' \) generated from \( \Phi_{in} \) are equally or less harmonic than \( T \). Assume there were a \( T' \) that is more harmonic than \( T \). Then there must be some constraint \( C^i \in C \), such that \( T' \) violates \( C^i \) fewer times than \( T \) does, and \( C^i \) is ranked higher than any other constraint in which \( T \) and \( T' \) differ. Constraints have to be incurred within some local subtree; so \( T \) must contain a local violation configuration that \( T' \) does not contain, and by the construction (12)/(13) the \( O_C \)-augmented analysis of \( T \) – call it \( O_C(T) \) – must make use of some violation-marked rule not used in \( O_C(T') \). Now there are three possibilities:

(i) Both \( O_C(T) \) and \( O_C(T') \) are free of recursion. Then the fact that \( O_C(T') \) avoids the highest-ranking constraint violation excludes \( T \) from \( \text{Cat}(G_{\Phi_{in}}^{QT}) \) (by construction step (20b)). This gives us a contradiction with our assumption.

(ii) \( O_C(T) \) contains a recursion and \( O_C(T') \) is free of recursion. If the recursion in \( O_C(T) \) is violation-free, then there is an equally harmonic recursion-free candidate \( T'' \). But this \( T'' \) is also less harmonic than \( O_C(T') \), such that it would have been excluded from \( \text{Cat}(G_{\Phi_{in}}^{QT}) \) too. This again means that \( O_C(T) \) would also be excluded (for lack of the relevant rules in the non-recursive part). On the other hand, if it were the recursion in \( O_C(T) \) that incurred the additional violation (as compared to \( O_C(T') \)), then there would be a more harmonic recursion-free candidate \( T'' \). However, this \( T'' \) would exclude the presence of \( O_C(T) \) in \( G_{\Phi_{in}}^{QT} \) by construction step (20c,d) (only violation-free recursion is possible). So we get another contradiction to the assumption that \( T \in \text{Cat}(G_{\Phi_{in}}^{QT}) \).

(iii) \( O_C(T') \) contains a recursion. If this recursion is violation-free, we can pick the equally harmonic candidate avoiding the recursion to be our \( O_C(T') \), and we are back to case (i) and (ii). Likewise, if the recursion in \( O_C(T') \) does incur some violation, not using the recursion leads to an even more harmonic candidate, for which again cases (i) and (ii) will apply.

All possible cases lead to a contradiction with the assumptions, so no candidate is more harmonic than our \( T \in \text{Cat}(G_{\Phi_{in}}^{QT}) \).

We still have to prove that if the c-structure \( T \) of a candidate analysis generated from \( \Phi_{in} \) with \( G_{\text{inviol}} \) is equally or more harmonic than all other candidates, then it is contained in \( \text{Cat}(G_{\Phi_{in}}^{QT}) \). We can construct an augmented version \( T' \) of \( T \), such that \( \text{Cat}(T') = T \) and then show that there is a homomorphism mapping \( T' \) to some analysis \( T'' \in G_{\Phi_{in}}^{QT} \) with \( \text{Cat}(T'') = T \).

We can use the constraint marking construction \( O_C \) and the KW construction to construct the tree \( T' \) with augmented category symbols of the analysis \( T \). The result of K&W00 plus (17) guarantee that \( \text{Cat}(T') = T \). Now, there has to be a homomorphism from the categories in \( T' \) to the categories of some analysis in \( G_{\Phi_{in}}^{QT} \). \( G_{\Phi_{in}}^{QT} \) is also based on \( KW(G_{\text{inviol}}, \Phi_{in}) \) (with an additional index \( h \) on each category and some categories and rules of \( KW(G_{\text{inviol}}, \Phi_{in}) \) having no counterpart in \( G_{\Phi_{in}}^{QT} \). Since we know that \( T \) is equally or more harmonic than any other candidate generated from \( \Phi_{in} \), we know that the augmented tree \( T' \) either contains no recursion or only violation-free recursion. If it does contain such violation-free recursions we map all categories \( N \) on the recursion paths to the indexed form \( N:0 \), and furthermore consider the variant of \( T' \) avoiding the recursion(s). For our (non-recursive) tree, there is guaranteed to be a counterpart in the finite set of non-recursive trees in \( G_{\Phi_{in}}^{QT} \) with all categories pairwise identical apart from the index \( h \) in \( G_{\Phi_{in}}^{QT} \). We pick this tree and map each of the categories in \( T' \) to the \( h \)-indexed counterpart. The existence of this homomorphism guarantees that an analysis \( T'' \in G_{\Phi_{in}}^{QT} \) exists with \( \text{Cat}(T'') = \text{Cat}(T') = T \).

9 Conclusion

We showed that for OT-LFG systems in which all constraints can be expressed relative to a local subtree in c-structure, the generation task from (non-cyclic\(^{13}\)) f-structures is solvable. The infinity of

\(^{13}\)The non-cyclic condition is inherited from K&W00; in linguistically motivated applications of the LFG formalism, cru-
the conceptually underlying candidate set does not preclude a computational approach. It is obvious that the construction proposed here has the purpose of bringing out the principled computability, rather than suggesting a particular algorithm for implementation. However on this basis, an implementation can be easily devised.

The locality condition on constraint-checking seems unproblematic for linguistically relevant constraints, since a GPSG-style slash mechanism permits reference to (finitely many) nonlocal configurations from any given category (cf. fn. 5).

Decidability of generation-based optimization (from a given input f-structure) alone does not imply that the recognition and parsing tasks for an OT grammar system defined as in sec. 3 are decidable: for these tasks, a string is given and it has to be shown that the string is optimal for some underlying input f-structure (cf. (Johnson, 1998)). However, similar construction as the one presented here can be devised for parsing-based optimization (even for an LFG-style grammar that does not obey the offline parsability condition). So, if the language generated by an OT system is defined based on (strong) bidirectional optimality (Kuhn, 2001, ch. 5), decidability of both the general parsing and generation problem follows. For the unidirectionally defined OT language (as in sec. 3), decidability of parsing can be guaranteed under the assumption of a contextual recoverability condition in parsing (Kuhn, in preparation).

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14 A hypothetical constraint that is excluded would be a parallelism constraint comparing two subtree structures of arbitrary depth. Such a constraint seems unnatural in a model of grammaticality. Parallelism of conjuncts does play a role in models of human parsing preferences; however, here it seems reasonable to assume an upper bound on the depth of parallel structures to be compared (due to memory restrictions).

15 Parsing: for a given string, parsing-based optimization is used to determine the optimal underlying f-structure; then generation-based optimization is used to check whether the original string comes out optimal in this direction too. Generation is symmetrical, starting with an f-structure.

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