Epidemic Dynamics of a Fractional-Order SIS Infectious Network Model

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Outbreak and large-scale of the infectious diseases have caused enormous economic losses to all countries in the world. Constructing a network model which could reflect the transmission dynamics of the epidemics and investigating their transmission laws have a significant meaning in the precaution and control of the epidemics. In this article, a fractional-order SIS epidemic network model is proposed. First, an expression of the basic reproduction number is deduced. Second, applying the Lyapunov function, the stability of the equilibriumpoints about the infectiousmodelisanalyzedindetail. Finally, anexampleispresent to verify the theoretical analysis. Furthermore, on account of the fractional-order coefficient, its influence on the transmission dynamics is also exhibited.

1. Introduction

Epidemics, especially the appearance and outbreaks of novelty infectious diseases, have turned into a major global public health problem. The destruction of natural environment and the rapid change of social structures not only provide opportunities for the spread of existing epidemics but also lead to the mutation of viruses that appeared in the past. Therefore, establishing the disease network models, revealing their transmission laws, and even predicting their transmission trends are of great importance to seek the optimal strategy for the precaution and control of the epidemics.

In the last decade, network models [1, 2], which regard individuals in a population as nodes in the networks, and the contact relationships between individuals as the edges between nodes have been studied in the fields of Internet, economy, biology, and so on. The most representative work to study the transmission dynamics of epidemics in different network structures is to establish the mathematical differential equation of infectious disease. The most influential results are the SIS (susceptible-infected-susceptible) and SIR (susceptible-infected-recovered) models through the average field theory established in [3]. Meanwhile, by analyzing the stability of the SIR network model with nonmonotonic incidence, the correlation between transmission threshold and network equilibrium point is revealed [4, 5]. People’s knowledge of disease information greatly affects the threshold of disease transmission, and media is a very effective means of information transmission [6–8]. Therefore, Zhan et al. proposed an adaptive network driven by disease information and analyzed the dynamic behavior of disease transmission when the network topology and information evolve simultaneously [9]. Li et al. [10] also studied the influence of information acquisition on the dynamics of disease transmission under a random network. The more information obtained about the disease, the smaller the number of basic regeneration and the smaller the number of patients. Some scholars even analyzed the influence of network degree distribution and node weight distribution on transmission threshold [11, 12]. These studies reveal that not only the topology of the social networks but also the information diffusion circulating among the crowds have great influence on the epidemic transmission.

It is widely known that the fractional-order differential model only needs to use a few parameters to get a better effect [13]. Since the fractional-order systems have memory function, fractional-order differential equations have been
taken advantage to analyze the epidemic dynamics, which are in accordance with the transmission characteristics of infectious diseases. Based on the fractional Lyapunov stability theory, some scholars have analyzed the equilibrium point stability and transmission dynamics of the fractional-order neural networks and the fractional HIV/AIDS epidemic systems [14–16]. Guo [17] studied a fractional-order SIR model in detail, and some theoretical results about the existence and stability of the unique positive solution were obtained. Furthermore, Rostamy et al. discussed the existence of multiple equilibrium points in the SIR model, indicating that selecting appropriate fractional-order parameters can expand the stability region of the equilibrium point [18]. Moreover, when the delay parameter equaled to the critical value, Hopfield bifurcation would appear. Rihan et al. [19] analyzed the dynamic behavior of the time-delay fractional-order infectious models with nonlinear incidence rate, based on the analytical methods for integer-order time-delay systems [20, 21]. From the above research, it can be found that establishing fractional infectious network models is very necessary to analyze the transmission laws of infectious diseases.

Since the basic reproduction number \( R_0 \) is a critical parameter for a disease transmission model, stability analyses about the equilibrium points were presented in [22]. According to the basic reproduction number, the major objective of this manuscript is to analyze the stability for a fractional SIS epidemic network model at the equilibrium points, including the disease-free equilibrium and the endemic equilibrium. When \( R_0 < 1 \) is satisfied, the system is asymptotically stable at the disease-free equilibrium, which reveals that the epidemic will eventually die out. Otherwise, when \( R_0 > 1 \), there is only one endemic equilibrium point, and a certain amount of individuals will become infectious.

2. Preliminaries

First, some definitions of the Caputo fractional differential operator are introduced:

**Definition 1** (see [23]) Define the Caputo fractional derivative as follows:

\[
\frac{\zeta}{\rho}D_\xi^\alpha g(t) = \frac{1}{\Gamma(\alpha - \rho)} \int_0^t \frac{g^\omega(v)}{(t - v)^{\rho+1-\rho}} dv, \quad w - 1 < \alpha < w, w \in \mathbb{N},
\]

where \( \alpha > 0, t > \rho, \alpha, \rho, t \in \mathbb{R} \).

**Remark 1.** Supposing the fractional-order parameter \( 0 < \alpha < 1 \), then

\[
\frac{\zeta}{\rho}D_\xi^\alpha g(t) = \frac{1}{\Gamma(1 - \alpha)} \int_0^t \frac{g^\omega(v)}{(t - v)^{\rho+1-\rho}} dv.
\]

Usually, the Caputo fractional derivative \( \frac{\zeta}{\rho}D_\xi^\alpha g(t) \) is abbreviated to \( D_\xi^\alpha \) for simplicity.

Consider the following Caputo differential equation:

\[
D_\xi^\alpha \xi(t) = g(\xi, t),
\]

where \( \xi_0 = \xi(t_0) \) is the initial value.

**Definition 2.** ([24]) For system (3), if and only if \( g(t, \xi^*) = 0 \), then \( \xi^* \) is an equilibrium point.

For a fractional-order epidemic model, maybe there are two kinds of equilibrium points, a disease-free equilibrium point and an endemic equilibrium point, which mean that the system could achieve and maintain the states by itself or under control.

**Lemma 1** (see [25]) If \( \xi(t) \in R \) is a continuous differentiable function, then for \( \forall \alpha \in (0, 1) \),

\[
\frac{1}{2} D_\xi^\alpha \xi^2(t) \leq \xi(t) D_\xi^\alpha \xi(t), \quad t \geq t_0.
\]

**Lemma 2** (see [26]) If \( \xi(t) \in R^+ \) is a continuous differentiable function, then

\[
D_\xi^\alpha \left[ \xi(t) - \xi^*(t) - \xi^*(t) \ln \frac{\xi(t)}{\xi^*(t)} \right] \leq \left( 1 - \frac{\xi(t)}{\xi^*(t)} \right) D_\xi^\alpha \xi(t),
\]

where \( \xi^*(t) \in R^+, \alpha \in (0, 1) \), and

\[
D_\xi^\alpha L(\xi_1, \xi_2, \ldots, \xi_w) \leq \sum_{i=1}^w \xi_i \left( 1 - \frac{\xi_i}{\xi_i^*} \right) D_\xi^\alpha \xi_i(t),
\]

for any time instant \( t \geq t_0 \).

3. Descriptions of the Epidemic Network Model

It is well known that the spread of an infectious disease can be thought of as the spread of a virus on a social network, which depends on both person-to-person contact and the transmission power of the virus. Taking into account the effects of birth rate, death rate, and transmission rate on the transmission of infectious diseases and combining with the fractional-order characteristics, in this article, a fractional-order SIS epidemic model on a degree uncorrelated network is proposed as follows:

\[
\begin{align*}
D_\xi^{\rho}S_p(t) &= a(1 - S_p - I_p) - \beta S_p - \lambda p S_p \Theta(t) + \gamma I_p, \\
D_\xi^{\rho}I_p(t) &= \lambda p S_p \Theta(t) - (\gamma + \beta) I_p,
\end{align*}
\]

where \( D_\xi^{\rho}(0 < \alpha < 1) \) is the Caputo derivative, for degree, \( S_p(t) \) (\( I_p(t) \)) represents the relative density of the susceptible (the infected) at time \( t \), and \( a \) denotes the birth rate, \( \beta \) denotes the natural death rate, \( \lambda \) denotes the infection rate, and \( \gamma \) denotes the recovery rate, respectively. \( \Theta(t) = \sum_{\varphi=1}^w P(\varphi \mid p)I_{\varphi} \) denotes the ratio that a randomly selected edge starting from a node with degree \( p \) points direct to an infected node with degree \( \varphi \). Since in a degree uncorrelated network, \( P(\varphi \mid p) \) denotes the ratio that a randomly selected edge starting from a node with degree \( p \)
points direct to a node with degree $\rho^j$, which is in proportion to $\rho^j P(\rho')$; then, $P(\rho'|\rho) = (\rho P(\rho)/\rho)$ and $\Theta(t)$ could be rewritten as $\Theta(t) = (\rho)^{-1} \sum_{\rho_j=1}^w \rho_j P(\rho_j)I_{\rho_j}'$, where $\rho$ denotes the average degree of the whole network.

Let the right side of equation of (7) be zero as follows:

$$\begin{cases} a(1 - S_{\rho}^0 - I_{\rho}^0) - \beta S_{\rho}^0 \rho S_{\rho}^0 \Theta(t) + \gamma I_{\rho}^0 = 0, \\ \lambda \rho S_{\rho}^0 \Theta(t) - (\gamma + \beta) I_{\rho}^0 = 0, \quad \rho = 1, 2, \ldots, w, \end{cases}$$

and define $E_0$ be the disease-free equilibrium with $I_{\rho}^0 = 0$, $(\rho = 1, 2, \ldots, w)$, then we can get that

$$\begin{cases} S_{\rho}^0 = \frac{a}{a + \beta}, \\ I_{\rho}^0 = 0, \quad \rho = 1, 2, \ldots, w. \end{cases}$$

Similarly, $E^* = (S_{\rho}^*, I_{\rho}^*)$ (if it exists) denotes the endemic equilibrium of system (7), which expresses that the disease would exist in crowd. Therefore, the endemic equilibrium $E^* = (S_{\rho}^*, I_{\rho}^*)$ has the form

$$\begin{align*} S_{\rho}^* &= \frac{a(1 + \alpha)}{a + \beta + (2\gamma + \beta - a)\lambda \rho \Theta^* (t)}, \\ I_{\rho}^* &= \frac{a(1 + \alpha)}{a + \beta + (2\gamma + \beta - a)\lambda \rho \Theta^* (t)}, \end{align*}$$

where $\Theta^* (t) = (\rho)^{-1} \sum_{\rho_j=1}^w \rho_j P(\rho_j')(\rho')/(a + \beta)(\gamma + \beta + (2\gamma + \beta - a)\lambda \rho \Theta^* (t))$.

Since the basic reproduction number is a critical parameter to analyze the existence of the endemic equilibrium point, subsequently, an expression of the basic reproduction number for system (7) will be derived.

Define a function $F(\Theta) = (\rho)^{-1} \sum_{\rho_j=1}^w \rho_j P(\rho_j'I_{\rho}^* - \Theta$, then

To be sure that if the function $F(\Theta) = 0$ has positive solutions for $\Theta \in (0, 1)$, the following condition must be satisfied:

$$\frac{dF(\Theta)}{d\Theta}_{|\Theta=0} = (\rho)^{-1} \sum_{j=1}^w \rho_j P(\rho_j') \frac{a(1 + \alpha)}{a + \beta + (2\gamma + \beta - a)\lambda \rho \Theta^* (t)} - 1.$$

Hence, for system (7), let $R_0 = (\rho^2)/(\rho') (a(1 + \alpha)/(a + \beta + \gamma))$; if $R_0 < 1$ is satisfied, there is only disease-free equilibrium. If $R_0 > 1$ is satisfied, there is only one endemic equilibrium.

4. Stability Analysis

Subsequently, we will discuss the stability of system (7) at the disease-free equilibrium and endemic equilibrium, respectively, which is the basis in the research field of epidemic dynamics.

The basic reproduction number $R_0$ is an threshold parameter to characterize the initial stage of an infectious disease. It represents the average number of people infected by an infectious person over the course of illness in a population that is all susceptible. If $R_0 < 1$, the infectious cannot grow. Conversely, if $R_0 > 1$, the disease can invade the population. Thus, in this study, we will analyze the stability of equilibrium based on the basic reproduction number.

4.1. Stability Analysis at the Disease-Free Equilibrium

Theorem 1. For system (7), when $(a(1 + \alpha)/(a + \beta + \gamma)) (\rho^2)/(\rho') < 1$, the disease-free equilibrium is asymptotically stable.

Proof. For system (7), construct a Lyapunov function

$$V = \sum_{\rho_j=1}^w \frac{\rho P(\rho)}{\rho} \left( S_{\rho} - S_{\rho}^0 - S_{\rho}^0 \ln \frac{S_{\rho}}{S_{\rho}^0} + \frac{\rho P(\rho)}{\rho} I_{\rho}^0 \right).$$

Applying Lemma 2, then

$$D^a V = \sum_{\rho_j=1}^w \frac{\rho P(\rho)}{\rho} D^a \left( S_{\rho} - S_{\rho}^0 - S_{\rho}^0 \ln \frac{S_{\rho}}{S_{\rho}^0} + \frac{\rho P(\rho)}{\rho} I_{\rho}^0 \right) \leq \sum_{\rho_j=1}^w \frac{\rho P(\rho)}{\rho} \left( 1 - \frac{S_{\rho}^0}{S_{\rho}} \right) D^a S_{\rho} + \sum_{\rho_j=1}^w \frac{\rho P(\rho)}{\rho} D^a I_{\rho}^0,$$

Submitting equation (7) into the inequality (14), we can obtain that
\[D^n V \leq \sum_{\rho=1}^{w} \frac{\rho P(\rho)}{\langle \rho \rangle} \left( 1 - \frac{S^\rho_0}{S^\rho_p} \right) \left[ a - (a + \beta)S^\rho_p - \lambda \rho S^\rho_p \Theta(t) - (a - \gamma)I_p \right] \]

\[+ \sum_{\rho=1}^{w} \frac{\rho P(\rho)}{\langle \rho \rangle} \left[ \lambda \rho S^\rho_p \Theta(t) - (\gamma + \beta)I_p \right] \]

\[= \sum_{\rho=1}^{w} \frac{\rho P(\rho)}{\langle \rho \rangle} \left[ a \left( 1 - \frac{S^\rho_0}{S^\rho_p} \right) - (a + \beta)(S^\rho_p - S^\rho_0) - \left( 1 - \frac{S^\rho_0}{S^\rho_p} \right) - (a - \gamma)I_p \right] \]

\[+ \sum_{\rho=1}^{w} \frac{\rho P(\rho)}{\langle \rho \rangle} \left[ -\lambda \rho S^\rho_p \Theta(t) + \lambda \rho S^\rho_p \Theta(t) + \lambda \rho S^\rho_p \Theta(t) - (\gamma + \beta)I_p \right] \]

\[= \sum_{\rho=1}^{w} \frac{\rho P(\rho)}{\langle \rho \rangle} \left[ a \left( 1 - \frac{S^\rho_0}{S^\rho_p} \right) - (a + \beta)(S^\rho_p - S^\rho_0) - \left( 1 - \frac{S^\rho_0}{S^\rho_p} \right) - (a - \gamma)I_p \right] \]

\[+ \sum_{\rho=1}^{w} \frac{\rho P(\rho)}{\langle \rho \rangle} \left[ \lambda \rho S^\rho_p \Theta(t) - (\gamma + \beta)I_p \right] . \]

Since \( S^\rho_0 = \frac{a}{a + \beta} \) and \( I_p + S^\rho_p = N_{\rho_p} \), then the inequality (15) can be transformed into

\[D^n V \leq \sum_{\rho=1}^{w} \frac{\rho P(\rho)}{\langle \rho \rangle} \left[ a \left( 1 - \frac{S^\rho_0}{S^\rho_p} \right) - (a + \beta)S^\rho_p + a - \left( 1 - \frac{S^\rho_0}{S^\rho_p} \right)(N_{\rho_p} - S^\rho_p) \right] \]

\[+ \sum_{\rho=1}^{w} \frac{\rho P(\rho)}{\langle \rho \rangle} \left[ \lambda \rho S^\rho_p \Theta(t) - (\gamma + \beta)(N_{\rho_p} - S^\rho_p) \right] \]

\[= \sum_{\rho=1}^{w} \frac{\rho P(\rho)}{\langle \rho \rangle} \left[ (a + 2\beta + \gamma)S^\rho_0 - a \frac{S^\rho_0^2}{S^\rho_p} - \beta \frac{S^\rho_0^2}{S^\rho_p} - aN_{\rho_p} - \beta N_{\rho} \right] \]

\[+ \sum_{\rho=1}^{w} \frac{\rho P(\rho)}{\langle \rho \rangle} \left[ aN_{\rho_p} \frac{S^\rho_0}{S^\rho_p} - \gamma N_{\rho_p} \frac{S^\rho_0}{S^\rho_p} + \lambda \rho S^\rho_p \Theta(t) \right] \]

\[= - \sum_{\rho=1}^{w} \frac{\rho P(\rho)}{\langle \rho \rangle} \frac{\beta}{S^\rho_p} \left( S^\rho_p - S^\rho_0 \right)^2 + a \sum_{\rho=1}^{w} \frac{\rho P(\rho)}{\langle \rho \rangle} \left( 1 - \frac{S^\rho_0}{S^\rho_p} \right) \left( S^\rho_0 - N_{\rho_p} \right) \]

\[+ \sum_{\rho=1}^{w} \frac{\rho P(\rho)}{\langle \rho \rangle} \left[ \lambda \rho S^\rho_p \Theta(t) - \beta I_p - \gamma N_{\rho_p} \frac{S^\rho_0}{S^\rho_p} \right] \]

\[= - \sum_{\rho=1}^{w} \frac{\rho P(\rho)}{\langle \rho \rangle} \frac{\beta}{S^\rho_p} \left( S^\rho_p - S^\rho_0 \right)^2 + a \sum_{\rho=1}^{w} \frac{\rho P(\rho)}{\langle \rho \rangle} \left( 1 - \frac{S^\rho_0}{S^\rho_p} \right) \left( S^\rho_0 - N_{\rho_p} \right) \]

\[+ \sum_{\rho=1}^{w} \frac{\rho P(\rho)}{\langle \rho \rangle} \left[ \lambda \rho S^\rho_p \Theta(t) - \beta I_p - \gamma (I_p + S^\rho_p) \right] \]
As \( \sum_{\rho=1}^{\infty} \rho^2 P(\rho) = \langle \rho^2 \rangle \) and \( \Theta(t) = \langle \rho \rangle^{-1} \sum_{\rho=1}^{\infty} \rho P(\rho I_\rho) \), the inequality (16) could be written as

\[
D^\rho V \leq -\sum_{\rho=1}^{\infty} \frac{\rho P(\rho) \beta}{\langle \rho \rangle} \left( S_\rho - S_\rho^0 \right)^2 + a \sum_{\rho=1}^{\infty} \frac{\rho P(\rho)}{\langle \rho \rangle} \left( 1 - \frac{S_\rho^0}{S_\rho} \right) \left( S_\rho^0 - N_\rho \right) + \Theta \left( \lambda S_\rho \frac{\langle \rho^2 \rangle}{\langle \rho \rangle} - \beta - \frac{S_\rho^0}{S_\rho} \right)
\]

\[
= -\sum_{\rho=1}^{\infty} m_\rho \frac{\beta}{S_\rho} \left( S_\rho - S_\rho^0 \right)^2 + a \sum_{\rho=1}^{\infty} m_\rho \left( 1 - \frac{S_\rho^0}{S_\rho} \right) \left( S_\rho^0 - N_\rho \right) + \Theta (\beta + \gamma) \left( \frac{a \lambda}{(a + \beta)(\beta + \gamma)} \frac{\langle \rho^2 \rangle}{\langle \rho \rangle} - 1 \right),
\]

with \( m_\rho = \langle \rho P(\rho) \langle \rho \rangle \rangle \).

It is quite clear that

\[
-\sum_{\rho=1}^{\infty} m_\rho \frac{\beta}{S_\rho} \left( S_\rho - S_\rho^0 \right)^2 + a \sum_{\rho=1}^{\infty} m_\rho \left( 1 - \frac{S_\rho^0}{S_\rho} \right) \left( S_\rho^0 - N_\rho \right) \leq 0,
\]

then

\[
D^\rho V \leq \Theta (\beta + \gamma) \left( \frac{a \lambda}{(a + \beta)(\beta + \gamma)} \frac{\langle \rho^2 \rangle}{\langle \rho \rangle} - 1 \right).
\]

Thus, choose \( R_0 = \frac{(a \lambda (a + \beta)(\beta + \gamma)) \langle \rho^2 \rangle}{\langle \rho \rangle} < 1 \); if \( R_0 < 1 \) is satisfied, then \( D^\rho V < 0 \), and for system (7), the disease-free equilibrium is globally asymptotically stable, which means the epidemic will be extinct at last. \( \square \)

4.2. Stability Analysis of the Endemic Equilibrium. For discussing the stability of the endemic equilibrium, define a Volterra-type Lyapunov function \( L(S_\rho, I_\rho) = \{S_\rho, I_\rho \in \Omega: S_\rho > 0, I_\rho > 0\} \longrightarrow R \) given by

\[
L(S_\rho, I_\rho) = S_\rho + I_\rho \left( -S_\rho^* + I_\rho^* \right) - \left( S_\rho^* + I_\rho^* \right) \ln \frac{S_\rho + I_\rho}{S_\rho^* + I_\rho^*},
\]

then

\[
D^\rho L(S_\rho, I_\rho) \leq \left( 1 - \frac{S_\rho + I_\rho}{S_\rho^* + I_\rho^*} \right) D^\rho (S_\rho + I_\rho)
\]

\[
= \left( 1 - \frac{S_\rho + I_\rho}{S_\rho^* + I_\rho^*} \right) \left[ a - (a + \beta)(S_\rho + I_\rho) \right]
\]

\[
= a \left( S_\rho - S_\rho^* + I_\rho - I_\rho^* \right)
\]

\[
= a \left( S_\rho - S_\rho^* \right) \left( I_\rho - I_\rho^* \right) \left( S_\rho^* + I_\rho^* \right) - (a + \beta) \left( S_\rho - S_\rho^* \right) \left( I_\rho - I_\rho^* \right)
\]

\[
= \frac{- (a + \beta) \left( S_\rho - S_\rho^* \right) \left( I_\rho - I_\rho^* \right) + (a + \beta) \left( S_\rho^* + I_\rho^* \right) \left( a + \beta \right)}{\left( S_\rho - S_\rho^* \right) \left( I_\rho - I_\rho^* \right) \left( S_\rho^* + I_\rho^* \right)}
\]

\[
= \left( \frac{(a + \beta) \left( S_\rho^* + I_\rho^* \right) - a}{2(a + \beta)} \right) + \left( \frac{(a + \beta) \left( S_\rho^* + I_\rho^* \right) - a}{2(a + \beta)} \right)
\]

\[
= \sum_{\rho=1}^{\infty} \frac{\beta}{S_\rho} \left( S_\rho - S_\rho^0 \right)^2 + a \sum_{\rho=1}^{\infty} \frac{\rho P(\rho)}{\langle \rho \rangle} \left( 1 - \frac{S_\rho^0}{S_\rho} \right) \left( S_\rho^0 - N_\rho \right) + \Theta (\beta + \gamma) \left( \frac{a \lambda}{(a + \beta)(\beta + \gamma)} \frac{\langle \rho^2 \rangle}{\langle \rho \rangle} - 1 \right).
\]
Since equation $S^* \rho + I^* \rho < (a/a + \beta)$ always holds, then

$$D^a I(S^*, I_p) \leq \frac{-(a + \beta) [S_p - S_p^* + (I_p - I_p^*) + (1/2)(a + \beta)(S_p^* + I_p^*) - a/a + \beta]}{S_p - S_p^* + (I_p - I_p^*) + (S_p^* + I_p^*)}$$

\leq 0. \quad (22)
Hence, the endemic equilibrium \((S_0^*, I_0^*)\) is stable.

**Remark 2.** It is observed that when \(R_0 < 1\) is satisfied, the disease will disappear ultimately. When \(R_0 > 1\) is satisfied, the disease may exist and it will not spread to all individuals.

**Remark 3.** In this case, for system (7), no matter \(R_0 < 1\) or \(R_0 > 1\), since at the disease-free equilibrium the eigenvalues of Jacobian matrix are \(- (\alpha + \beta)\) and \(- (\gamma + \beta)\), we can also derive that the disease-free equilibrium is asymptotically stable.

### 5. An Example

In this part, an example is given to verify the theorem. For system (7), \(P(\rho) = 2m^2\rho^{-2}\) and the parameters are valued as given in Table 1.

From Figures 1 and 2 with \((S_0, I_0) = (0.6, 0.3)\), we can derive that system (7) is stable, and the smaller \(\rho\) is, the slower the disease decays. The disease-free equilibrium is \((S^0, I^0) = ((a/\alpha + \beta), 0) = (0.625, 0)\), which is consistent with the simulations.

From Figure 2, we can conclude that when \(\rho\) belongs to some range, the disease would propagate in some individuals and will not die out. For example, choose \(\rho = 4\); at this moment, \(R_0 = 1.1036 > 1\), and the infectious will be convergent to 0.018 approximately, and the disease will always exist, which implied that there exists an endemic equilibrium point at this moment. Choose \(\rho = 50\), then \(R_0 = 0.6401 < 1\), and the disease-free equilibrium is asymptotically stable. It means that the epidemic will eventually die out, which is verified from Figure 3. Unfortunately, the convergence time is a little long.

Although we have not theoretically analyzed the effects of the fractional parameter on disease transmission, the impacts are shown in Figure 4 via simulations. We discover that the smaller the parameter \(\alpha\) is, the slower the disease converges. From Figure 4, we can find that for \(k = 50\), \(\alpha = 1\) means that the system becomes a integer-order system, and it converges faster than the fractional-order parameter \(0 < \alpha < 1\). Compared with the theoretical results in [4, 5], stability of the equilibrium points are all analyzed and some conditions are obtained. But only in this study, the influence of the fractional-order parameter on the stability is discussed and simulated.

**Remark 4.** From the above analysis, not only the influence of fractional-order on the equilibrium is considered but also the influence of network topology on the stability of the equilibrium is analyzed. Compared with the existing results about the stability analysis of the infectious models, it has considered more parameters which affect the disease transmission characters, but in regard to the fractional-order parameter, it lacks the qualitative analysis, which is also the main difficulty and challenge in the field of the epidemic transmission.

### 6. Conclusions

Epidemic dynamics of a fractional SIS network model has been investigated in this study. Based on the basic reproduction number and the Lyapunov function, stability of the equilibrium points have been analyzed. When \(R_0 < 1\) is satisfied, the disease-free equilibrium is asymptotically stable and the epidemic will be extinct ultimately. When \(R_0 > 1\) is satisfied, there is only one stable endemic equilibrium. At this moment, the epidemic would exist in some amount of individuals. An example is present to verify the theoretical achievements finally. Besides, the impact of fractional parameter \(\alpha\) to transmission dynamics is also exhibited by the simulation pictures.

Noted that the network topology has a tremendous impact on the transmission dynamics of the disease, the fractional-order parameter also affects the convergence rate of the disease. Therefore, according to the distribution of the population and the existing transmission of the disease, some control strategies are proposed. For example, to control the epidemic of COVID-19, measures such as restricting access, sealing off the cities, and isolating the infective and the close contacts have been taken with a good preventive effect. Therefore, in next step, we would construct a special fractional-order model about the COVID-19 based on the real and concrete transmission data and put forward an optimal control method to make sure the epidemic die out in a finite time. The comparison of the simulation and the real data would be made, which has a positive guiding role for the prevention and control of the COVID-19.

### Data Availability

No data were used to support this study.

### Conflicts of Interest

The authors declare that there are no conflicts of interest.

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