QCD Factorization Based on Six-Quark Operator Effective Hamiltonian from Perturbative QCD and Charmless Bottom Meson Decays $B_{(s)} \rightarrow \pi\pi, \pi K, KK$

Fang Su*,†, Yue-Liang Wu*, Yi-Bo Yang*‡ and Ci Zhuang*

* Kavli Institute for Theoretical Physics China, Institute of Theoretical Physics Chinese Academy of Science, Beijing 100190, China
† Institut für Theoretische Physik E, RWTH Aachen, D-52056, Aachen, Germany
‡ Graduate School of the Chinese Academy of Sciences, Beijing, 100039, China

Abstract

The charmless bottom meson decays are systematically investigated based on an approximate six quark operator effective Hamiltonian from perturbative QCD. It is shown that within this framework the naive QCD factorization method provides a simple way to evaluate the hadronic matrix elements of two body mesonic decays. The singularities caused by on mass-shell quark propagator and gluon exchanging interaction are appropriately treated. Such a simple framework allows us to make theoretical predictions for the decay amplitudes with reasonable input parameters. The resulting theoretical predictions for all the branching ratios and $CP$ asymmetries in the charmless $B^0$, $B^+$, $B_s \rightarrow \pi\pi, \pi K, KK$ decays are found to be consistent with the current experimental data except for a few decay modes. The observed large branching ratio in $B \rightarrow \pi^0\pi^0$ decay remains a puzzle though the predicted branching ratio may be significantly improved by considering the large vertex corrections in the effective Wilson coefficients. More precise measurements of charmless bottom meson decays, especially on CP-violations in $B \rightarrow KK$ and $B_s \rightarrow \pi\pi, \pi K, KK$ decay modes, will provide a useful test and guide us to a better understanding on perturbative and nonperturbative QCD.

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I. INTRODUCTION

Hadronic B-meson decays play an important role not only for understanding the dynamical scheme of hadronic decays and testing the flavor structure of the Standard Model (SM), but also for probing the origin of CP violation and new physics signals beyond the SM. In particular, the precise measurement and systematic study for hadronic charmless B decays may provide a window for such purposes. The branching ratios of $B \to \pi\pi$ and $\pi K$ modes have been measured with a good accuracy[1] and a large direct CP violation has been established in $\pi^+ K^-$ mode [1]. The most severe discrepancies between the experimental data and theoretical predictions come from the unexpected large branch ratio of $B \to \pi^0 \pi^0$ and some unclear CP violations in $B \to \pi^0 K$ decays, which are called $\pi\pi, \pi K$ puzzles[2, 3]. Theoretically, to predict consistently those decays, it needs to deal with the short-distance contributions in a complete and systematic way from the high energy scale to a proper low energy scale at which the perturbative calculations remain reliable, and treat the long-distance contributions which contain the non-perturbative strong interactions involved in those decays. The main task is to reliably compute the hadronic matrix elements between the initial and final hadron states. Several novel methods based on the naive factorization approach (FA) and four quark operator effective Hamiltonian have been developed to evaluate the hadronic matrix elements, such as the QCD factorization approach (QCDF)[4], the perturbation QCD method (pQCD) [5], and the soft-collinear effective theory (SCET)[6]. These methods have been widely used in analyzing hadronic B-meson decays and made great progresses in understanding the hadronic structure and properties of strong interactions. To understand the puzzles whether they are due to the unknown new physics or it is because of the lack of our knowledge on the hadronic properties of strong interactions, it still needs to investigate further the various approaches within the framework of QCD and to check the validity of assumptions and approximations made in the practical calculations.

The widely used theoretical framework of weak decays is based on the current-current four fermion operator effective Hamiltonian derived via operator product expansion and renormalization group evolution. In hadronic weak decays, the short-distance contributions of QCD are characterized by the Wilson coefficient functions of four quark operators and the long-distance contributions are in principle obtained by evaluating the hadronic matrix elements of four quark operators. The Wilson coefficient functions are in general calculated by perturbative QCD which is well developed, while the evaluation of hadronic matrix elements remains a hard task as it involves non-perturbative effects of QCD. To deepen our insights into the hadronic decays, we shall first reinvestigate the four quark operator effective Hamiltonian whether it is always suitable as a basic framework for all hadronic
weak decays. In fact, for the mesonic two body decays of B meson, it concerns three quark-antiquark pairs once each meson is regarded as the quark-antiquark bound state at the quark level structure. This fact then naturally motivates us to consider six-quark operator effective Hamiltonian instead of four-quark operator effective Hamiltonian. Namely, we shall begin with six quark diagrams of weak decays with both W-boson exchange and gluon exchange, and derive formally the six-quark operator effective Hamiltonian based on operator product expansion and renormalization group evolution when including loop corrections of six quark diagrams. We shall show how this approach allows us to figure out what are the assumptions and approximations made in effective four quark operator approach, and how the simple QCD factorization scheme can reliably be applied to evaluate the hadronic matrix elements with the six quark operator effective Hamiltonian. For the infrared singularity caused by the gluon exchanging interaction when evaluating the hadronic matrix elements of effective six quark operators, it is shown to be simply treated by the introduction of a mass scale motivated from the gauge invariant loop regularization method [7], where the energy scale $\mu^g$ is introduced to play the role of infrared cut-off energy scale without violating gauge invariance.

The paper is organized as follows. In section II, after briefly reviewing the four quark operator effective Hamiltonian, we begin with the primary six quark diagrams with a single W-boson exchange and a single gluon exchange, and the corresponding initial six-quark operator. It is shown that a complete six quark operator effective Hamiltonian is in general necessary to include all contributions from both perturbative and non-perturbative QCD corrections, especially the non-pertubative QCD corrections at low energy scale $\mu < m_c \sim 1.5$ GeV could be sizable. To demonstrate how the six quark operator effective Hamiltonian provides a reliable framework for hadronic two body decays of B meson, we will focus, as a good approximation, on the dominant QCD loop diagrams of six quarks so as to avoid the tedious calculations. In section III, it is demonstrated how the QCD factorization approach becomes a simple and natural tool to evaluate the hadronic matrix elements of mesonic two body decays based on the six quark operator effective Hamiltonian. In particular, the so-called factorizable and non-factorizable, emission and annihilation diagram contributions are automatically the consequences of QCD factorization for the hadronic matrix elements of effective six quark operators. The treatment on the singularities caused by the gluon exchanging interactions and the on mass-shell fermion propagator is presented in Section IV. In Section V, all the amplitudes of charmless bottom meson decays are completely obtained by using the QCD factorization approach based on the approximate six quark operator effective Hamiltonian. Our numerical results with appropriate input parameters are presented in section VI, as a good approximation, the resulting predictions on branching ratios and CP violations
of charmless bottom meson decays are much improved and also more closed to the current experimental data. The conclusions and remarks are given in last section. The detailed calculations involved in the evaluation of various decay amplitudes are presented in the Appendix.

II. EFFECTIVE HAMILTONIAN OF SIX QUARK OPERATORS

A. Four Quark Operator Effective Hamiltonian

Let us start from the four-quark effective operators in the effective weak Hamiltonian. The initial four quark operator due to weak interaction via W-boson exchange is given as follows for B decays

$$O_1 = (\bar{q}^u_i b_i)_{V-A}(\bar{q}^d_j u_j)_{V-A}, \quad q^u = u, c, \quad q^d = d, s$$

(1)

The complete set of four quark operators are obtained from QCD and QED corrections which contain the gluon exchange diagrams, strong penguin diagrams and electroweak penguin diagrams. The resulting effective Hamiltonian (for $b \to s$ transition) with four quark operators is known to be as follows

$$H_{eff} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} \lambda_q^s \left[ C_1(\mu)O_1^{(q)}(\mu) + C_2(\mu)O_2^{(q)}(\mu) + \sum_{i=3}^{10} C_i(\mu)O_i(\mu) \right] + h.c.,$$

(2)

with $\lambda_q^s = V_{qs}V_{qs}^*$ and $V_{ij}$ the CKM matrix elements, $C_i(\mu)$ the Wilson coefficient functions[8] and $O_i(\mu)$ the four quark operators

$$O_1^{(q)} = (\bar{q}^u_i b_i)_{V-A}(\bar{s}^j q_j)_{V-A}, \quad O_2^{(q)} = (\bar{s}^i b_i)_{V-A}(\bar{q}^d_j q_j)_{V-A},$$

$$O_3 = (\bar{s}^i b_i)_{V-A} \sum_{q'} (\bar{q}^d_j q_j)_{V-A}, \quad O_4 = \sum_{q'} (\bar{q}^d_i b_i)_{V-A}(\bar{s}^j q_j)_{V-A},$$

$$O_5 = (\bar{s}^i b_i)_{V-A} \sum_{q'} (\bar{q}^d_j q_j)_{V-A}, \quad O_6 = -2 \sum_{q'} (\bar{q}^d_i b_i)_{S-P}(\bar{s}^j q_j)_{S+P},$$

$$O_7 = \frac{3}{2} (\bar{s}^i b_i)_{V-A} \sum_{q'} e_q' (\bar{q}^d_j q_j)_{V-A}, \quad O_8 = -3 \sum_{q'} e_q' (\bar{q}^d_i b_i)_{S-P}(\bar{s}^j q_j)'_{S+P},$$

$$O_9 = \frac{3}{2} (\bar{s}^i b_i)_{V-A} \sum_{q'} e_q' (\bar{q}^d_j q_j)_{V-A}, \quad O_{10} = \frac{3}{2} \sum_{q'} e_q' (\bar{q}^d_i b_i)_{V-A}(\bar{s}^j q_j)_{V-A},$$

(3)

Here the Fermi constant $G_F = 1.16639 \times 10^{-5}$ GeV$^{-2}$, and the color indices $i, j$, and the notations $(\bar{q}' q')_{V-A} = \bar{q}' \gamma_{\mu}(1\pm \gamma_5)q'$. The index $q'$ in the summation of the above operators runs through $u, d, s, c, b$. The effective Hamiltonian for the $b \to d$ transition can be obtained by changing $s$ into $d$ in Eqs. (2)and (3).
B. Six Quark Diagrams and Effective Operators

As mesons are regarded as quark and anti-quark bound states, the mesonic two body decays actually involve three quark-antiquark pairs. It is then natural to consider the six quark Feynman diagrams which lead to three effective currents of quark-antiquark. The initial six quark diagrams of weak decays contain one W-boson exchange and one gluon exchange, thus there are four different diagrams as the gluon exchange interaction can occur for each of four quarks in the W-boson exchange diagram, see Fig. 1.

![Six Quark Diagrams](image)

FIG. 1: Four different six quark diagrams with a single W-boson exchange and a single gluon exchange

The resulting initial effective operators contain four terms corresponding to the four diagrams, respectively. In a good approximation, the four quarks via W-boson exchange can be regarded as a local four quark interaction at the energy scale much below the W-boson mass, while two QCD vertexes due to gluon exchange are at the independent space-time points, the resulting effective six quark operators are hence in general nonlocal. The six-quark operators corresponding to the four diagrams in Fig. 1 are found to be

\[
O^{(6)}_{q_1} = 4 \pi \alpha_s \int \int \frac{d^4k}{(2\pi)^4} \frac{d^4p}{(2\pi)^4} e^{-i((x_1-x_2)p+(x_2-x_3)k)} \langle \bar{q}'(x_3) \gamma_\nu T^a q'(x_3) \rangle \frac{1}{k^2 + i\epsilon}
\]

\[
(q_2(x_1) \Gamma_2 \frac{\not{p} + m_b}{p^2 - m_b^2 + i\epsilon} \gamma_\nu T^a \bar{q}_1(x_2) \rangle * (\bar{q}_4(x_1) \Gamma_4 q_3(x_1))
\]

\[
O^{(6)}_{q_2} = 4 \pi \alpha_s \int \int \frac{d^4k}{(2\pi)^4} \frac{d^4p}{(2\pi)^4} e^{-i((x_1-x_2)p+(x_2-x_3)k)} \langle \bar{q}'(x_3) \gamma_\nu T^a q'(x_3) \rangle \frac{1}{k^2 + i\epsilon}
\]

\[
(q_2(x_2) \frac{\not{q}_2 + m_{q_2}}{p^2 - m_{q_2}^2 + i\epsilon} \gamma_\nu T^a \Gamma_1 q_1(x_1)) \rangle * (\bar{q}_4(x_1) \Gamma_4 q_3(x_1))
\]

\[
O^{(6)}_{q_3} = 4 \pi \alpha_s \int \int \frac{d^4k}{(2\pi)^4} \frac{d^4p}{(2\pi)^4} e^{-i((x_1-x_2)p+(x_2-x_3)k)} \langle \bar{q}'(x_3) \gamma_\nu T^a q'(x_3) \rangle \frac{1}{k^2 + i\epsilon}
\]

\[
(q_2(x_1) \Gamma_1 q_1(x_1)) \rangle * (\bar{q}_4(x_1) \Gamma_2 \frac{\not{p} + m_{q_3}}{p^2 - m_{q_3}^2 + i\epsilon} \gamma_\nu T^a q_3(x_2))
\]

\[
O^{(6)}_{q_4} = 4 \pi \alpha_s \int \int \frac{d^4k}{(2\pi)^4} \frac{d^4p}{(2\pi)^4} e^{-i((x_1-x_2)p+(x_2-x_3)k)} \langle \bar{q}'(x_3) \gamma_\nu T^a q'(x_3) \rangle \frac{1}{k^2 + i\epsilon}
\]

\[
(q_2(x_1) \Gamma_1 q_1(x_1)) \rangle * (\bar{q}_4(x_2) \frac{\not{p} + m_{q_2}}{p^2 - m_{q_2}^2 + i\epsilon} \gamma_\nu T^a \Gamma_2 q_3(x_1))
\]

(4)
where $k$ and $p$ correspond to the momenta of gluon and quark in their propagators. $q_1$ is usually set to be heavy quark like $b$ quark. $x_1$, $x_2$ and $x_3$ are space-time points corresponding to three vertexes. The color index is summed between $q_1, q_2$ and $q_3, q_4$. Note that all the six quark operators are proportional to the QCD coupling constant $\alpha_s$ due to gluon exchange. Thus the initial six quark operator is given by summing over the above four operators

$$O^{(6)} = \sum_{j=1}^{4} O^{(6)}_{q_j}. \quad (5)$$

Actually, the initial six quark operators $O^{(6)}_{q_j} (j = 1, 2, 3, 4)$ can be obtained from the following initial four quark operator via a single gluon exchange

$$O \equiv (\bar{q}_2 \Gamma_1 q_1) \ast (\bar{q}_4 \Gamma_2 q_3). \quad (6)$$

C. Six Quark Operator Effective Hamiltonian via Perturbative QCD

Based on the above considerations with the introduction of six quark operators, in this section we shall specify the initial six quark operator $O^{(q)(6)}_1 (q = u, c)$ to the case of nonleptonic bottom hadron decays and show how to obtain six quark operator effective Hamiltonian. The initial six quark operator in $b$-decay with $\Delta S \neq 0$ is as follows (for the $b \to d$ transition with $\Delta S = 0$, just replacing $s$ by $d$)

$$O^{(q)(6)}_1 = \sum_{l=1}^{4} O^{(q)(6)}_{q_l}$$

$$= 4\pi \alpha_s(m_W) \int \int \frac{d^4k}{(2\pi)^4} \frac{d^4p}{(2\pi)^4} e^{-i((x_1-x_2)p+(x_2-x_3)k)}$$

$$\{ (\bar{q}_i(x_1))\gamma^\mu (1-\gamma^5) \frac{\not{p}+m_b}{p^2-m_b^2+i\epsilon} \gamma^\nu T^a_{ik} b_k(x_2)(\bar{s}_j(x_1)\gamma_\mu (1-\gamma^5)q_j(x_1))$$

$$+(\bar{q}_k(x_2))\gamma^\mu T^a_{ik} \gamma^\nu (1-\gamma^5)b_i(x_1))(\bar{s}_j(x_1)\gamma_\mu (1-\gamma^5)q_j(x_1))$$

$$+(\bar{q}_i(x_1))\gamma^\mu (1-\gamma^5)b_i(x_1))(\bar{s}_j(x_1)\gamma_\mu (1-\gamma^5)\frac{\not{p}+m_q}{p^2-m_q^2+i\epsilon} \gamma^\nu T^a_{jk} q_k(x_2))$$

$$+(\bar{q}_i(x_1))\gamma^\mu (1-\gamma^5)b_i(x_1))(\bar{s}_k(x_2))\gamma^\nu T^a_{jk} \gamma_\mu (1-\gamma^5)q_j(x_1)) \}$$

$$\frac{1}{k^2+i\epsilon} (q^\prime_m(x_3)\gamma_\mu T^a_{mn} q^\prime_n(x_3)), \quad (7)$$
which can be regarded as an effective operator resulting from the corresponding initial four-quark operator with a single gluon exchange

\[ O_1^{(q)} = (\bar{q}_i b_i) V_A (\bar{s}_j q_j) V_A \]
\[ = (\bar{q}_i \gamma_\mu (1 - \gamma^5) b_i)(\bar{s}_j \gamma_\mu (1 - \gamma^5) q_j) \]  

(8)

with \( q = u, c \).

Similar to the procedure of obtaining the four quark operator effective Hamiltonian from the initial four quark operator \( O_1^{(q)} \) of weak interaction, one should evaluate the six quark operator effective Hamiltonian from the initial six quark operator \( O_1^{(q)(6)} \) when running the energy scale from \( m_W \) to the low energy scale \( \mu \sim m_b \). As the first step for finding out the complete set of independent effective six quark operators, one needs to evaluate all possible one loop diagrams based on the initial six quark diagrams (Fig. 1). The possible six quark diagrams at one loop level are plotted in Fig. 2.

It is useful to classify those diagrams into three types: type I is the loop diagrams in which only the effective four quark vertex of weak interaction receives loop corrections including the penguin type loops, the single gluon exchanging interaction for six quark operators remains mediating between one of four external quark lines of loops and a spectator quark line (see Fig. 2a); type II is the loop diagrams where only the single gluon exchanging vertexes receive loop corrections (see Fig. 2b); the remaining loop diagrams are regarded as type III in which one of the gluon exchanging vertexes touches to the internal quark/gluon line of loops (see Fig. 2c). Note that in Fig. 2a and Fig. 2b we only plot, for an illustration, the six quark diagrams with a gluon exchanging between one of the four external quark lines of effective weak vertex and a spectator quark line, while for each of them, there are actually three additional different diagrams corresponding to other three choices of external quark lines, they are omitted just for simplicity.

To evaluate all the diagrams is a hard task, as a good approximation, we shall pay attention to the type I and type II diagrams. The type III diagrams are in general suppressed at the perturbative region with energy scale around \( m_b \) as they involve more internal quark lines and contain no large logarithmic enhancements. From the evaluation of four quark operator effective Hamiltonian, it is known that when the energy scale runs via the renormalization group evolution from the high energy scale at \( \mu \simeq m_W \) to the low energy scale around \( \mu \sim m_b \), the loop corrections of type I diagrams should result in the six quark operators with all effective four quark operators and the corresponding Wilson coefficient functions, meanwhile the loop corrections of type II diagrams will lead the strong coupling constant \( \alpha_s \) of the gluon exchanging interaction to run from high energy scale at \( m_W \) to the low energy scale at \( \mu \). Thus, when ignoring the type III diagrams, we arrive at an approximate six
FIG. 2: The diagrams in (a) are loop contributions only to the effective weak vertex (type I), and diagrams in (b) are loop contributions only to the gluon vertexes (type II). The diagrams in (c) are loop contributions for both weak and strong vertexes (type III).

quark operator effective Hamiltonian as follows

\[
H_{\text{eff}}^{(6)} = \frac{G_F}{\sqrt{2}} \sum_{j=1}^{4} \{ \sum_{q=u,c} \lambda_{q}^{(d)} [C_1(\mu)O_{1q_j}^{(6)}(\mu) + C_2(\mu)O_{2q_j}^{(6)}(\mu)] \\
+ \sum_{i=3}^{10} \lambda_{i}^{(d)} C_{i}(\mu)O_{i q_j}^{(6)}(\mu) \} + \text{h.c.} + \ldots, \tag{9}
\]

with the CKM factor \( \lambda_{q}^{(d)} = V_{qb}V_{qs}^{*} \). The dots represent other possible terms that have been neglected in our present considerations. \( O_{i q_j}^{(6)}(\mu) \) \((j = 1, 2, 3, 4)\) are six quark operators which may effectively be obtained from the corresponding four quark operators \( O_i(\mu) \) (in Eq. (3)) at the scale
\( \mu \) via the effective gluon exchanging interactions between one of the external quark lines of four quark operators and a spectator quark line at the same scale \( \mu \). The general forms and definitions of \( O_{ij}^{(6)} (\mu) \) for the corresponding four quark operators \( O_{ij}(\mu) \) are similar to the ones of \( O_{ij}^{(6)} (j = 1, 2, 3, 4) \) given in Eq. (4) but with replacing \( \alpha_s(m_W) \) by \( \alpha_s(\mu) \) due to QCD corrections of type II diagrams.

Before proceeding, we would like to point out that a complete six quark operator effective Hamiltonian may involve more effective operators from the type III diagrams and lead to a non-negligible contribution to hadronic B meson decays when evaluating the hadronic matrix elements of six quark operator effective Hamiltonian around the energy scale \( \mu \sim \sqrt{2\Lambda_{QCD} m_b} \sim m_c \sim 1.5 \text{ GeV} \) where the nonperturbative effects may play the role. We shall keep this in mind and regard the above six quark operator effective Hamiltonian as an approximate one.

### III. QCD Factorization Based on Effective Six Quark Operators

We shall apply the above effective Hamiltonian with six quark operators to the nonleptonic two body decays of bottom mesons. The evaluation of hadronic matrix elements is the most hard task in the calculations of the decay amplitudes. In this section, we are going to demonstrate how the factorization approach naturally works for evaluating the hadronic matrix elements of nonleptonic two body decays of B meson with six quark operators.

To be explicit, we here examine the hadronic matrix element of \( B \to \pi^0 \pi^0 \) decay for a typical six quark operator \( O_{LL}^{(6)} \)

\[
O_{LL}^{(6)} = \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 p}{(2\pi)^4} e^{-i((x_1-x_2)p+(x_2-x_3)k)} \frac{1}{k^2} \frac{1}{p^2 - m_d^2} [\bar{d}(x_2)(\not{p} + m_d)\gamma^\nu T_{ki}^a \gamma^\mu (1 - \gamma^5) b_i(x_1)] [\bar{d}(x_1)\gamma_\mu (1 - \gamma^5) d_j(x_1)] [\bar{d}(x_3)\gamma_\nu T_{mn}^a d_n(x_3)],
\]

which is actually a part of the six quark operator \( O_{4q2}^{(6)} \) in the effective Hamiltonian. Its hadronic matrix element for \( B \to \pi^0 \pi^0 \) decay leads to the following most general terms in the QCD factorization approach

\[
M_{LL}^O(B\pi\pi) = \langle \pi^0 \pi^0 \mid O_{LL}^{(6)} \mid B_0 \rangle = \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 p}{(2\pi)^4} e^{-i((x_1-x_2)p+(x_2-x_3)k)} \frac{1}{k^2} \frac{1}{p^2 - m_d^2} [\bar{d}(x_2)(\not{p} + m_d)\gamma^\nu T_{ki}^a \gamma^\mu (1 - \gamma^5) b_i(x_1)] [\bar{d}(x_1)\gamma_\mu (1 - \gamma^5) d_j(x_1)] [\bar{d}(x_3)\gamma_\nu T_{mn}^a d_n(x_3)] \mid B_0 \rangle \equiv M_{LL}^{O(1)} + M_{LL}^{O(2)} + M_{LL}^{O(3)} + M_{LL}^{O(4)},
\]
with

\[
M^{O(1)}_{LL} = \int \int \frac{d^4k}{(2\pi)^4} \frac{d^4p}{(2\pi)^4} e^{-i((x_1-x_2)p+(x_2-x_3)k)} \frac{1}{k^2(p^2-m_d^2)} T^a_{ki} T^a_{mn},
\]

\[
[(\not\!p + m_d) \gamma^\nu \gamma^\mu (1-\gamma^5)]_{\rho\sigma} [\gamma_\mu (1-\gamma^5)]_{\alpha\beta} [\gamma_\nu]_{\gamma\delta} M_{Bim}^{\sigma_\gamma}(x_1, x_3) M_{\pi nk}^{\delta_\beta}(x_3, x_2) M_{\pi jk}^{\beta_\beta}(x_1, x_2),
\]

\[
M^{O(2)}_{LL} = \int \int \frac{d^4k}{(2\pi)^4} \frac{d^4p}{(2\pi)^4} e^{-i((x_1-x_2)p+(x_2-x_3)k)} \frac{1}{k^2(p^2-m_d^2)} T^a_{ki} T^a_{mn},
\]

\[
[(\not\!p + m_d) \gamma^\nu \gamma^\mu (1-\gamma^5)]_{\rho\sigma} [\gamma_\mu (1-\gamma^5)]_{\alpha\beta} [\gamma_\nu]_{\gamma\delta} M_{Bim}^{\sigma_\gamma}(x_1, x_3) M_{\pi nk}^{\delta_\beta}(x_3, x_1) M_{\pi jk}^{\beta_\beta}(x_1, x_2),
\]

\[
M^{O(3)}_{LL} = \int \int \frac{d^4k}{(2\pi)^4} \frac{d^4p}{(2\pi)^4} e^{-i((x_1-x_2)p+(x_2-x_3)k)} \frac{1}{k^2(p^2-m_d^2)} T^a_{ki} T^a_{mn},
\]

\[
[(\not\!p + m_d) \gamma^\nu \gamma^\mu (1-\gamma^5)]_{\rho\sigma} [\gamma_\mu (1-\gamma^5)]_{\alpha\beta} [\gamma_\nu]_{\gamma\delta} M_{Bim}^{\sigma_\gamma}(x_1, x_3) M_{\pi nk}^{\delta_\beta}(x_3, x_2),
\]

\[
M^{O(4)}_{LL} = \int \int \frac{d^4k}{(2\pi)^4} \frac{d^4p}{(2\pi)^4} e^{-i((x_1-x_2)p+(x_2-x_3)k)} \frac{1}{k^2(p^2-m_d^2)} T^a_{ki} T^a_{mn},
\]

\[
[(\not\!p + m_d) \gamma^\nu \gamma^\mu (1-\gamma^5)]_{\rho\sigma} [\gamma_\mu (1-\gamma^5)]_{\alpha\beta} [\gamma_\nu]_{\gamma\delta} M_{Bim}^{\sigma_\gamma}(x_1, x_2) M_{\pi nk}^{\delta_\beta}(x_1, x_2) M_{\pi jk}^{\beta_\beta}(x_1, x_2) M_{\pi jk}^{\beta_\beta}(x_1, x_2),
\]

where \( M_{\pi nm}(x_i, x_j) \equiv [M_X(x_i, x_j)]^{\beta_\mu}_{nm} (X = B, \pi) \) with \( n, m \) the color indices and \( \alpha, \beta \) the spinor indices, is the hadronic matrix element of two quark operators for a single meson \( X \). In light-cone QCD approach, it is found to be [9]

\[
M_{Bnm}^{\beta_\alpha}(x_i, x_j) = \langle 0 | \bar{d}^{\alpha}_{m}(x_j) b_{n}^{\beta}(x_i) | B^{0}(P_B) \rangle = -i F_B \delta_{mn} \int \frac{1}{N_c} du e^{-i(u P_B x_j+(P_B-u) P_B x_i)} M_{Bm}^{\beta_\alpha}(u, P_B),
\]

\[
M_{\pi nm}^{\beta_\alpha}(x_i, x_j) = \langle \pi^0(P) | \bar{d}^{\alpha}_{m}(x_j) d^{\beta}_{n}(x_i) | 0 \rangle = -i F_{\pi} \delta_{mn} \int \frac{1}{N_c} dx e^{-i(\pi p x_j+(1-x) P x_i)} M_{\pi m}^{\beta_\alpha}(x, P),
\]

with \( F_M \) \((M = B, \pi)\) the decay constants. Here \( M_{Bm}^{\beta_\alpha}(u, P_B) \) and \( M_{\pi m}^{\beta_\alpha}(x, P) \) are the spin structures for the bottom meson and light meson \( \pi \) and characterized by the corresponding distribution amplitudes

\[
M_{Bm}^{\beta_\alpha}(u, P_B) = -[m_B + \not\!P_B \gamma^5 \phi_B(u)]^{\beta_\alpha},
\]

\[
M_{\pi m}^{\beta_\alpha}(x, P) = [\not\!P \gamma^5 \phi_\pi(x) - \mu_\pi \gamma^5 (\phi_\pi^T(x) - i \sigma_{\mu_\nu} \not\!P_{\mu} \phi_\pi^T(x) + i \sigma_{\mu_\nu} P_{\mu} \phi_\pi^T(x) \frac{\partial}{\partial k_{\perp \nu}})]^{\beta_\alpha},
\]

with \( v = \frac{P}{\sqrt{2} |P|} \), \( n = n^+ + n^- - v \) and \( \phi^T = \phi^{\sigma^T/6} \). The light-cone distribution amplitudes \( \phi_{\not\!M}^X(u) \) \((M = B, \pi, X = -, p, T)\) are given in [9] up to twist-3. The definition of momentum for quarks and mesons is explicitly shown in Fig.5. As a good approximation, both the light quarks and light mesons are taken to be massless, i.e., \( P^2 = 0 \).

It is interesting to note that the four amplitudes \( M_{LL}^{O(i)} \) \((i = 1, 2, 3, 4)\) are corresponding to four diagrams (1)-(4) in Fig.3. The first diagram is known as the factorizable one, the second is the non-factorizable one and color suppressed. The third is the factorizable annihilation diagram and color suppressed, and the fourth is an annihilation diagram and its matrix element vanishes.
After performing the integration over space-time and momentum, the above amplitude is simplified to be

$$M_{LL}^O(B\pi) = \langle \pi^0 | O^{(6)}_L | \bar{B}_0 \rangle = \frac{1}{(u P_B^+ - (1 - x) P_1)^2} \int_0^1 \int_0^1 dx \ dy \ \frac{1}{(u P_B^+ - (1 - x) P_1)^2}$$

with

$$M_{LL}^{(1)} = \frac{G_F}{N_c} F_B^2 \gamma_\mu M_\pi(x, P_1) \gamma_\nu M_\pi(x, P_1) \gamma_\mu (P_1 - u P_B^+ + m_d) \gamma_\mu (1 - \gamma^5)$$

$$\text{Tr}[M_\pi(y, P_2) \gamma_\mu (1 - \gamma^5)] = i \frac{G_F}{N_c} F_B^2 \gamma_\mu \phi_B(u) m_B^2 \mu_\pi \phi_\pi(y) \phi_\pi^\dagger(x)$$

$$M_{LL}^{(2)} = \frac{G_F}{N_c} F_B^2 \gamma_\mu M_\pi(x, P_1) \gamma_\nu M_\pi(x, P_1) \gamma_\mu (1 - \gamma^5) M_\pi(y, P_2) \gamma_\nu$$

$$((1 - x) P_1 + y P_2 - u P_B^+ + m_d) \gamma_\mu (1 - \gamma^5)$$

$$= i \frac{G_F}{N_c} F_B^2 \phi_B(u) m_B^2 (m_B(u + y - 2) \phi_\pi(x) + (1 - x) \phi_\pi(x) \phi_\pi^\dagger(x) \phi_\pi(y))$$

$$M_{LL}^{(3)} = \frac{G_F}{N_c} F_B^2 \gamma_\mu M_\pi(x, P_1) \gamma_\nu M_\pi(y, P_2) \gamma_\mu (x P_1 + P_2 + m_d) \gamma_\mu (1 - \gamma^5)$$

$$\text{Tr}[M_\pi(u, P_B) \gamma_\mu (1 - \gamma^5)]$$

$$= i \frac{G_F}{N_c} F_B^2 \phi_B(u) m_B^2 (x m_B^2 \phi_\pi(x) \phi_\pi(x) + 2 m_B^2 (1 + x) \phi_\pi^\dagger(x) - (1 - x) \phi_\pi^\dagger(x))$$

$$M_{LL}^{(4)} = 0$$

where $M_{LL}^{(i)}$ ($i = 1, 2, 3, 4$) are obtained by performing the trace of matrices and determined by the distribution amplitudes. $C_F = \frac{N_c^2 - 1}{2N_c}$ is resulted from summing over the color indices. It can be seen that $M_{LL}^{(1)}$ corresponding to Fig.3.(1) is color allowed, $M_{LL}^{(2)}$ and $M_{LL}^{(3)}$ corresponding to Fig.3.(2) and Fig.3.(3) are color suppressed, while $M_{LL}^{(4)}$ corresponding to Fig.3.(4) vanishes as it is not allowed for colorless mesons.
From the above explicit demonstration, it can be seen that the simple QCD factorization approach becomes a natural tool to evaluate the hadronic matrix element of effective six quark operators in the mesonic two body decays. For a given effective six quark operator, its hadronic matrix element for mesonic two body decays gets four different combinations in the QCD factorization approach, namely it consists of four different amplitudes corresponding to four topologically different diagrams. From the above example, it is noticed that the amplitude $M^{O(1)}_{LL}$ is a color-allowed factorizable one in an emission diagram, $M^{O(2)}_{LL}$ is a color-suppressed non-factorizable one in an emission diagram, $M^{O(3)}_{LL}$ is a color-suppressed factorizable one in an annihilation diagram, while $M^{O(4)}_{LL}$ vanishes as it cannot match to a colorless meson.

When generalizing the above analysis to the present framework based on the approximate six quark operator effective Hamiltonian, there are in general four types of six quark diagrams corresponding to four types of effective six quark operators, their hadronic matrix elements for two body mesonic decays lead to sixteen kinds of diagrams (see Fig.4(\(ai\))-(\(di\)), \(i = 1, 2, 3, 4\)) as each of the effective six quark operators leads to four kinds of amplitudes in the QCD factorization approach.

It is known that the effective four quark vertexes concern three types of current-current interactions: \((V - A) \times (V - A)\) or \((LL)\), \((V - A) \times (V + A)\) or \((LR)\), \((S - P) \times (S + P)\) or \((SP)\), thus each of the
diagrams in Fig.4 actually contains three kinds of diagrams corresponding to three types of current-current interactions. Therefore, there are totally 48 kinds of hadronic matrix elements involved in the QCD factorization approach, while it is easy to check that only half of them are independent with the following relations:

\[
\begin{align*}
M_{LL}^{a1} &= T_{LLa}^{F}; & M_{LL}^{a2} &= T_{LLa}^{F}/N_c; & M_{LL}^{a3} &= A_{LLa}^{N}/N_c; & M_{LL}^{a4} &= 0; \\
M_{LR}^{a1} &= T_{LRa}^{F}; & M_{LR}^{a2} &= T_{SPa}^{F}/N_c; & M_{LR}^{a3} &= A_{SPa}^{N}/N_c; & M_{LR}^{a4} &= 0; \\
M_{SP}^{a1} &= T_{SPa}^{F}; & M_{SP}^{a2} &= T_{LRa}^{F}/N_c; & M_{SP}^{a3} &= A_{LRa}^{N}/N_c; & M_{SP}^{a4} &= 0; \\
M_{LL}^{b1} &= T_{LLb}^{F}; & M_{LL}^{b2} &= T_{LLb}^{N}/N_c; & M_{LL}^{b3} &= A_{LLb}^{F}/N_c; & M_{LL}^{b4} &= 0; \\
M_{LR}^{b1} &= T_{LRb}^{F}; & M_{LR}^{b2} &= T_{SPb}^{N}/N_c; & M_{LR}^{b3} &= A_{SPb}^{F}/N_c; & M_{LR}^{b4} &= 0; \\
M_{SP}^{b1} &= T_{SPb}^{F}; & M_{SP}^{b2} &= T_{LRb}^{N}/N_c; & M_{SP}^{b3} &= A_{LRb}^{F}/N_c; & M_{SP}^{b4} &= 0; \\
M_{LL}^{c1} &= 0; & M_{LL}^{c2} &= T_{LLa}^{N}/N_c; & M_{LL}^{c3} &= A_{LLa}^{F}/N_c; & M_{LL}^{c4} &= A_{LLa}^{F}; \\
M_{LR}^{c1} &= 0; & M_{LR}^{c2} &= T_{SPa}^{N}/N_c; & M_{LR}^{c3} &= A_{SPa}^{F}/N_c; & M_{LR}^{c4} &= A_{SPa}^{F}; \\
M_{SP}^{c1} &= 0; & M_{SP}^{c2} &= T_{LRa}^{N}/N_c; & M_{SP}^{c3} &= A_{LRa}^{F}/N_c; & M_{SP}^{c4} &= A_{SPa}^{F}; \\
M_{LL}^{d1} &= 0; & M_{LL}^{d2} &= T_{LLb}^{N}/N_c; & M_{LL}^{d3} &= A_{LLb}^{F}/N_c; & M_{LL}^{d4} &= A_{LLb}^{F}; \\
M_{LR}^{d1} &= 0; & M_{LR}^{d2} &= T_{SPb}^{N}/N_c; & M_{LR}^{d3} &= A_{SPb}^{F}/N_c; & M_{LR}^{d4} &= A_{SPb}^{F}; \\
M_{SP}^{d1} &= 0; & M_{SP}^{d2} &= T_{LRb}^{N}/N_c; & M_{SP}^{d3} &= A_{LRb}^{F}/N_c; & M_{SP}^{d4} &= A_{SPb}^{F};
\end{align*}
\]

(17)

where \(T_{xa}^{F}\) and \(T_{xb}^{F}\) (\(X = LL, LR, SP\)) represent the factorizable emission diagram contributions, \(T_{xa}^{N}\) and \(T_{xb}^{N}\) (\(X = LL, LR, SP\)) are the non-factorizable emission diagram contributions. \(A_{xa}^{F}, A_{xb}^{F}\) and \(A_{xa}^{N}, A_{xb}^{N}\) (\(X = LL, LR, SP\)) denote the so-called factorizable and non-factorizable annihilation diagram contributions respectively. Their detailed definitions and general formalisms are presented in the Appendix.

IV. TREATMENT OF SINGULARITIES

In the evaluation of hadronic matrix elements, there are two kinds of singularities, one is caused by the infrared divergence of gluon exchanging interaction, and the other arises from the on-mass shell divergence of internal quark propagator. As the quark propagator singularity is a physical-region singularity, one can simply add \(ie\) to the denominator of quark propagator and apply the Cutkosky rule [10] to avoid such a singularity. It then allows us to obtain the virtual part of amplitudes as the Cutkosky rule gives a compact expression for the discontinuity across the cut arising from a physical-region singularity. In general, a Feynman diagram will yield an imaginary part for the decay amplitudes when the virtual particles in the diagram become on mass-shell, and the resulting diagram
can be considered as a genuine physical process. It is well-known that when applying the Cutkosky rule to deal with a physical-region singularity of all propagators, the following formula holds

\[
\frac{1}{p^2 - m_b^2 + i\epsilon} = P \left[ \frac{1}{p^2 - m_b^2} \right] - i\pi \delta[p^2 - m_b^2],
\]

\[
\frac{1}{p^2 - m_q^2 + i\epsilon} = P \left[ \frac{1}{p^2 - m_q^2} \right] - i\pi \delta[p^2 - m_q^2],
\]

(18)

which is known as the principal integration method. Where the first integration with the notation of capital letter \(P\) is the so-called principal integration.

For the infrared divergence of gluon exchanging interactions, only adding \(i\epsilon\) to the gluon propagator is not enough as such an infrared divergence is not a physical-region singularity, one cannot simply apply the Cutkosky rule. To regulate such an infrared divergence, we may apply the prescription used in the symmetry-preserving loop regularization [7] which allows us to introduce an intrinsic energy scale without destroying the non-abelian gauge invariance and translational invariance. The description of the loop regularization is simple: evaluating the Feynman integrals to an irreducible integrals, replacing the integration variable \(k^2\) and integration measure \(\int \frac{d^4k}{(2\pi)^4}\) by the regularized ones via [7]

\[
k^2 \rightarrow [k^2]_l \equiv k^2 - M_l^2,
\]

\[
\int \frac{d^4k}{(2\pi)^4} \rightarrow \int \left[ \frac{d^4k}{(2\pi)^4} \right]_l \equiv \lim_{N,M_e \rightarrow \infty} \sum_{l=0}^{N} c_l^N \int \frac{d^4k}{(2\pi)^4},
\]

(19)

with conditions

\[
\lim_{N,M_e \rightarrow \infty} \sum_{l=0}^{N} c_l^N (M_l^2)^n = 0, \quad c_0^N = 1 \quad (i = 0, 1, \ldots, N \text{ and } n = 0, 1, \ldots),
\]

(20)

where \(c_l^N\) are the coefficients determined by the above conditions. With a simple form for the regulator masses \(M_l = \mu_g + l M_R\ (l = 0, 1, \ldots)\), the coefficients \(c_l^N\) is found to be \(c_l^N = (-1)^l \frac{N!}{(N-l)! l!}\), so that

\[
k^2 \Rightarrow k^2 - \mu_g^2 - l M_R^2, \quad \int \frac{d^4k}{(2\pi)^4} \Rightarrow \lim_{N,M_R \rightarrow \infty} \sum_{l=0}^{N} (-1)^l \frac{N!}{l!(N-l)!} \int \frac{d^4k}{(2\pi)^4},
\]

(21)

which leads the regularized integrals to be independent of the regulators. Here the energy scale \(M_0 = \mu_g\) plays the role of infrared cut-off but preserving gauge symmetry and translational symmetry of original theory.

In the present case, there is no ultraviolet divergence for the integral over \(k\) as it is constrained by the finite momentum of hadrons, so all the terms with \(l \neq 0\) in the summation over \(l\) vanish in the limit \(M_R \rightarrow \infty\). As a consequence, it is equivalent to add an intrinsic regulator energy scale \(\mu_g\) in the
denominator $k^2$ in Eq. (11), thus one can use the usual principal integration method to avoid such a singularity, i.e.,

$$\frac{1}{k^2} \Rightarrow \frac{1}{k^2 - \mu_g^2 + i\epsilon} = P \left[ \frac{1}{k^2 - \mu_g^2} \right]^{-i\pi\delta[k^2 - \mu_g^2]}.$$  

(22)

With the above considerations, the singularities appearing in the integrations over $k$ and $p$ can simply be avoided by the following prescription

$$\frac{1}{k^2} \frac{1}{(p^2 - m^2)} \rightarrow \frac{1}{(k^2 - \mu_g^2 + i\epsilon) (p^2 - m^2 + i\epsilon)},$$  

(23)

Note that as the gauge depending team $k_\mu k_\nu$ can be transformed to the momentum $\not{p}$ on exterior line of spectator quark, they are all on mass shell in our present consideration (as defined in Fig.5 in the appendix), their contributions equal to zero, thus our results are gauge independent.

Applying this prescription to the amplitude illustrated in previous section, we have

$$M_{LL}^O(B\pi\pi) = \langle \pi^0 \pi^0 | O_{LL}^{(6)} | \bar{B}_0 \rangle = \int_0^1 \int_0^1 \int_0^1 du \, dx \, dy \, \frac{1}{(uP_B^+ - (1-x)P_1)^2 - \mu_g^2 + i\epsilon}$$

$$M_{LL}^{(1)} + M_{LL}^{(2)} + M_{LL}^{(3)} + M_{LL}^{(4)}.$$

(24)

V. AMPLITUDES OF CHARMLESS BOTTOM MESON DECAYS

With the above considerations and analyses, the QCD factorization approach enables us to evaluate all the hadronic matrix elements of nonleptonic two body decays of B meson based on the approximate six quark operator effective Hamiltonian. The amplitudes of charmless B meson decays can be expressed as follows:

$$A(B^0 \rightarrow \pi^+ \pi^-) = V_{td}V_{tb}^* [P_T^{\pi\pi}(B) + \frac{2}{3}P_{EW}^{C\pi\pi}(B) + P_E^{\pi\pi}(B) + 2P_A^{\pi\pi}(B) + \frac{1}{3}P_{EW}^{A\pi\pi}(B) - \frac{1}{3}A_{EW}^{E\pi\pi}(B)]$$

$$- V_{ud}V_{ub}^* [T^{\pi\pi}(B) + E^{\pi\pi}(B)],$$

$$A(B^+ \rightarrow \pi^+ \pi^0) = \frac{1}{\sqrt{2}} \{ V_{td}V_{tb}^* [P_{EW}^{E\pi\pi}(B) + P_{EW}^{C\pi\pi}(B)] - V_{ud}V_{ub}^* [T^{\pi\pi}(B) + C^{\pi\pi}(B)] \},$$

$$A(B^0 \rightarrow \pi^0 \pi^0) = \frac{1}{\sqrt{2}} \{ - V_{td}V_{tb}^* [P_T^{\pi\pi}(B) - P_{EW}^{E\pi\pi}(B) - \frac{1}{3}P_{EW}^{C\pi\pi}(B) + P_E^{\pi\pi}(B) + 2P_A^{\pi\pi}(B)$$

$$+ \frac{1}{3}P_{EW}^{A\pi\pi}(B) - \frac{1}{3}A_{EW}^{E\pi\pi}(B)] + V_{ud}V_{ub}^* [ - C^{\pi\pi}(B) + E^{\pi\pi}(B)] \},$$

(25)
for $B \to \pi\pi$ decay amplitudes, and

$$A(B^+ \to \pi^+ K^0) = -V_{ts} V_{tb}^* [P_T^{\pi K}(B) - \frac{1}{3} P_{EW}^{\pi K}(B) - \frac{2}{3} P_{EW}^{\pi \pi K}(B)] + V_{us} V_{ub} A^{\pi K}(B),$$

$$A(B^+ \to \pi^0 K^+) = \frac{1}{\sqrt{2}} \{V_{td} V_{tb}^* [P_T^{\pi K}(B) + P_{EW}^{\pi K}(B) - \frac{2}{3} P_{EW}^{\pi \pi K}(B)] + V_{us} V_{ub} T^{\pi K}(B),$$

$$A(B^0 \to \pi^- K^+) = V_{td} V_{tb}^* [P_T^{\pi K}(B) - \frac{2}{3} P_{EW}^{\pi K}(B) + P_{EW}^{\pi \pi K}(B) - \frac{1}{3} P_{EW}^{\pi \pi K}(B)] - V_{us} V_{ub} T^{\pi K}(B),$$

$$A(B^0 \to \pi^0 K^0) = -\frac{1}{\sqrt{2}} [V_{td} V_{tb}^* [P_T^{\pi K}(B) - \frac{2}{3} P_{EW}^{\pi K}(B) + P_{EW}^{\pi \pi K}(B)] + V_{us} V_{ub} C^{\pi K}(B)],$$

(26)

for $B \to \pi K$ decay amplitudes, and

$$A(B^0 \to K^+ K^-) = -V_{ts} V_{tb}^* [P_A^{\pi K}(B) + P_A^{\pi\pi K}(B) - \frac{1}{3} P_{EW}^{\pi K}(B) - \frac{2}{3} P_{EW}^{\pi\pi K}(B)] + V_{td} V_{tb}^* E^{KK}(B),$$

$$A(B^+ \to K^+ K^0) = -V_{td} V_{tb}^* [P_T^{KK}(B) + \frac{1}{3} P_{EW}^{KK}(B) + P_{EW}^{KK}(B) - \frac{2}{3} P_{EW}^{KK}(B)] + V_{us} V_{ub} A^{KK}(B),$$

$$A(B^0 \to K^0 K^0) = -V_{td} V_{tb}^* [P_T^{KK}(B) + \frac{1}{3} P_{EW}^{KK}(B) + P_{EW}^{KK}(B) + P_{EW}^{KK}(B)] - \frac{1}{3} P_{EW}^{KK}(B) - \frac{1}{3} P_{EW}^{KK}(B)],$$

(27)

for $B \to KK$ decay amplitudes, and

$$A(B_s^+ \to \pi^+ \pi^-) = -V_{ts} V_{tb}^* E^{\pi\pi}(B_s) + V_{ts} V_{tb}^* [2P_A^{\pi\pi}(B_s) + \frac{1}{3} P_{EW}^{\pi\pi}(B_s)],$$

$$A(B_s^0 \to \pi^0 \pi^0) = \frac{1}{\sqrt{2}} A(B_s^0 \to \pi^+ \pi^-),$$

$$A(B_s^0 \to \pi^+ K^-) = V_{td} V_{tb}^* [P_T^{KK}(B_s) + \frac{1}{3} P_{EW}^{KK}(B_s) + P_{EW}^{KK}(B_s) - \frac{2}{3} P_{EW}^{KK}(B_s)] - V_{us} V_{ub} T^{KK}(B_s),$$

$$A(B_s^0 \to \pi^0 K^0) = -\frac{1}{\sqrt{2}} [V_{td} V_{tb}^* [P_T^{KK}(B_s) - \frac{2}{3} P_{EW}^{KK}(B_s) + P_{EW}^{KK}(B_s) - \frac{2}{3} P_{EW}^{KK}(B_s)] + V_{us} V_{ub} C^{KK}(B_s)],$$

$$A(B_s^0 \to K^+ K^-) = -V_{ts} V_{tb}^* [P_T^{KK}(B_s) + \frac{2}{3} P_{EW}^{KK}(B_s) + P_{EW}^{KK}(B_s) + P_{EW}^{KK}(B_s) + P_{EW}^{KK}(B_s)]$$

$$+ \frac{2}{3} P_{EW}^{KK}(B_s) - \frac{2}{3} P_{EW}^{KK}(B_s) + V_{us} V_{ub} [T^{KK}(B_s) + E^{KK}(B_s)],$$

$$A(B_s^0 \to K^0 K^0) = -V_{ts} V_{tb}^* [P_T^{KK}(B_s) - \frac{1}{3} P_{EW}^{KK}(B_s) + P_{EW}^{KK}(B_s) + P_{EW}^{KK}(B_s) + P_{EW}^{KK}(B_s)]$$

$$- \frac{1}{3} P_{EW}^{KK}(B_s) - \frac{1}{3} P_{EW}^{KK}(B_s) - \frac{1}{3} P_{EW}^{KK}(B_s)],$$

(28)

for $B_s \to \pi\pi, \pi K, KK$ decay amplitudes. The eleven types of amplitudes $T^{M_1 M_2}(M), C^{M_1 M_2}(M), P_T^{M_1 M_2}(M), P_{EW}^{M_1 M_2}(M), A^{M_1 M_2}(M), E^{M_1 M_2}(M), P_E^{M_1 M_2}(M), P_A^{M_1 M_2}(M), P_{EW}^{M_1 M_2}(M), P_{EW}^{C M_1 M_2}(M),$
$P_{EW}^{EM_1M_2}(M), P_{EW}^{AM_1M_2}(M)$, with $M_1M_2 = \pi\pi, \pi K, K\pi, K\bar{K}, \bar{K}K$ are defined as follows

$$T^{M_1M_2}(M) = 4\pi\alpha_s(\mu) \frac{G_F}{\sqrt{2}} \left\{ [C_1(\mu) + \frac{1}{N_c} C_2(\mu)] T^{FAM_1M_2}(M) + \frac{1}{N_c} C_2(\mu) T^{NAM_1M_2}(M) \right\},$$

$$C^{M_1M_2}(M) = 4\pi\alpha_s(\mu) \frac{G_F}{\sqrt{2}} \left\{ [C_2(\mu) + \frac{1}{N_c} C_1(\mu)] T^{FAM_1M_2}(M) + \frac{1}{N_c} C_1(\mu) T^{NAM_1M_2}(M) \right\},$$

$$P_T^{M_1M_2}(M) = 4\pi\alpha_s(\mu) \frac{G_F}{\sqrt{2}} \left\{ [C_1(\mu) + \frac{1}{N_c} C_3(\mu)] T^{FAM_1M_2}(M) + \frac{1}{N_c} C_3(\mu) T^{NAM_1M_2}(M) \right\} + \frac{1}{N_c} C_6(\mu) T^{FAM_1M_2}(M) + \frac{1}{N_c} C_5(\mu) T^{NAM_1M_2}(M),$$

$$P_{EW}^{M_1M_2}(M) = 4\pi\alpha_s(\mu) \frac{G_F}{\sqrt{2}} \left\{ [C_9(\mu) + \frac{1}{N_c} C_{10}(\mu)] T^{FAM_1M_2}(M) + \frac{1}{N_c} C_{10}(\mu) T^{NAM_1M_2}(M) \right\} + \frac{1}{N_c} C_8(\mu) T^{FAM_1M_2}(M) + \frac{1}{N_c} C_7(\mu) T^{NAM_1M_2}(M),$$

$\rho^{CM_1M_2}(M) = 4\pi\alpha_s(\mu) \frac{G_F}{\sqrt{2}} \left\{ [C_{10}(\mu) + \frac{1}{N_c} C_9(\mu)] T^{FAM_1M_2}(M) + \frac{1}{N_c} C_9(\mu) T^{NAM_1M_2}(M) \right\} + \frac{1}{N_c} C_8(\mu) T^{FAM_1M_2}(M) + \frac{1}{N_c} C_7(\mu) T^{NAM_1M_2}(M),$  \hspace{1cm} (29)

for the so-called emission diagrams, and

$$A^{M_1M_2}(M) = 4\pi\alpha_s(\mu) \frac{G_F}{\sqrt{2}} \left\{ [C_1(\mu) + \frac{1}{N_c} C_2(\mu)] A^{FAM_1M_2}(M) + \frac{1}{N_c} C_2(\mu) A^{NAM_1M_2}(M) \right\},$$

$$E^{M_1M_2}(M) = 4\pi\alpha_s(\mu) \frac{G_F}{\sqrt{2}} \left\{ [C_2(\mu) + \frac{1}{N_c} C_1(\mu)] A^{FAM_1M_2}(M) + \frac{1}{N_c} C_1(\mu) A^{NAM_1M_2}(M) \right\},$$

$$P_{E}^{M_1M_2}(M) = 4\pi\alpha_s(\mu) \frac{G_F}{\sqrt{2}} \left\{ [C_3(\mu) + \frac{1}{N_c} C_4(\mu)] A^{FAM_1M_2}(M) + \frac{1}{N_c} C_4(\mu) A^{NAM_1M_2}(M) \right\} + \frac{1}{N_c} C_5(\mu) A^{FAM_1M_2}(M) + \frac{1}{N_c} C_6(\mu) A^{NAM_1M_2}(M),$$

$$P_{A}^{M_1M_2}(M) = 4\pi\alpha_s(\mu) \frac{G_F}{\sqrt{2}} \left\{ [C_5(\mu) + \frac{1}{N_c} C_4(\mu)] A^{FAM_1M_2}(M) + \frac{1}{N_c} C_4(\mu) A^{NAM_1M_2}(M) \right\} + \frac{1}{N_c} C_6(\mu) A^{FAM_1M_2}(M) + \frac{1}{N_c} C_7(\mu) A^{NAM_1M_2}(M),$$

$$P_{EW}^{AM_1M_2}(M) = 4\pi\alpha_s(\mu) \frac{G_F}{\sqrt{2}} \left\{ [C_{10}(\mu) + \frac{1}{N_c} C_9(\mu)] A^{FAM_1M_2}(M) + \frac{1}{N_c} C_9(\mu) A^{NAM_1M_2}(M) \right\} + \frac{1}{N_c} C_8(\mu) A^{FAM_1M_2}(M) + \frac{1}{N_c} C_7(\mu) A^{NAM_1M_2}(M),$$

$$P_{EW}^{EM_1M_2}(M) = 4\pi\alpha_s(\mu) \frac{G_F}{\sqrt{2}} \left\{ [C_9(\mu) + \frac{1}{N_c} C_{10}(\mu)] A^{FAM_1M_2}(M) + \frac{1}{N_c} C_{10}(\mu) A^{NAM_1M_2}(M) \right\} + \frac{1}{N_c} C_8(\mu) A^{FAM_1M_2}(M) + \frac{1}{N_c} C_7(\mu) A^{NAM_1M_2}(M),$$

$\rho_{EW}^{EM_1M_2}(M) = 4\pi\alpha_s(\mu) \frac{G_F}{\sqrt{2}} \left\{ [C_{10}(\mu) + \frac{1}{N_c} C_9(\mu)] A^{FAM_1M_2}(M) + \frac{1}{N_c} C_9(\mu) A^{NAM_1M_2}(M) \right\} + \frac{1}{N_c} C_8(\mu) A^{FAM_1M_2}(M) + \frac{1}{N_c} C_7(\mu) A^{NAM_1M_2}(M),$  \hspace{1cm} (30)

for the so-called annihilation diagrams. Where $T_X^{F}, T_X^{N}, A_X^{F}, A_X^{N} \ (X = LL, LR, SP, A = a, b)$ arise from the hadronic matrix elements and their detailed expressions are given in Appendix. Note that $T_B^{FKK}$ and $T_B^{FKK}$ are slightly different as the wave functions of $K$ meson and $\bar{K}$ meson are not equal at high order in the twist expansion.
When redefining the above amplitudes to the widely used diagrammatic amplitudes in the phenomenological analysis,

\[ T = T^{\pi\pi}(B), \quad C = C^{\pi\pi}(B), \quad E = E^{\pi\pi}(B), \quad P = P_{T}^{\pi\pi}(B) + P_{E}^{\pi\pi}(B), \quad P_{A} = 2P_{A}^{\pi\pi}, \]

\[ P_{EW} = P_{EW}^{T}(B), \quad P_{EW}^{C} = P_{EW}^{C}(B), \quad P_{EW}^{A} = P_{EW}^{A}(B), \quad P_{EW}^{E} = P_{EW}^{E}(B), \]

\[ T' = T^{\pi K}(B), \quad C = C^{\pi K}(B), \quad A' = A^{\pi K}(B), \quad P' = P_{T}^{\pi K}(B) + P_{E}^{\pi K}(B), \quad P_{A}' = 2P_{A}^{\pi K}, \]

\[ P_{EW}' = P_{EW}^{K}(B), \quad P_{EW}^{C'} = P_{EW}^{C}(B), \quad P_{EW}^{A'} = P_{EW}^{A}(B), \quad P_{EW}^{E'} = P_{EW}^{E}(B), \]

\[ P_{EW}'' = [P_{EW}^{AKK}(B) + P_{EW}^{A}(B)]/2, \quad P_{EW}'' = [P_{EW}^{AKK}(B) - P_{EW}^{A}(B)]/2, \quad P_{EW}'' = P_{EW}^{EKK}(B). \]

the decay amplitudes can be reexpressed in terms of the familiar forms in the diagrammatic decomposition approach

\[ A(B^{0} \to \pi^{+}\pi^{-}) = V_{ud}V_{tb}^{*}(P + P_{A} + \frac{2}{3}P_{EW}^{C} + \frac{1}{3}P_{EW}^{A} - \frac{1}{3}P_{EW}^{E}) - V_{ud}V_{ub}^{*}(T + E), \]

\[ A(B^{+} \to \pi^{+}\pi^{0}) = \frac{1}{\sqrt{2}}[V_{td}V_{tb}^{*}(P_{EW} + P_{EW}^{C}) - V_{td}V_{ub}^{*}(T + C)], \]

\[ A(B^{0} \to \pi^{0}\pi^{0}) = \frac{1}{\sqrt{2}}[V_{td}V_{tb}^{*}(-P - P_{A} + P_{EW} + \frac{1}{3}P_{EW}^{C} + \frac{1}{3}P_{EW}^{A} - \frac{1}{3}P_{EW}^{E}) - V_{ud}V_{ub}^{*}(C - E)], \]

\[ A(B^{+} \to \pi^{+}K^{0}) = V_{ts}V_{tb}^{*}(P' - \frac{1}{3}P_{EW}^{C} + \frac{2}{3}P_{EW}^{E}) + V_{ts}V_{ub}^{*}A', \]

\[ A(B^{+} \to \pi^{0}K^{+}) = \frac{1}{\sqrt{2}}[V_{ts}V_{tb}^{*}(P' + \frac{2}{3}P_{EW}^{C} + P_{EW}' + \frac{2}{3}P_{EW}^{E}) + V_{ts}V_{ub}^{*}(T' + C' + A')], \]

\[ A(B^{0} \to \pi^{-}K^{+}) = V_{ts}V_{tb}^{*}(P' + \frac{2}{3}P_{EW}^{C} - \frac{1}{3}P_{EW}' + V_{tu}V_{ub}^{*}T'), \]

\[ A(B^{0} \to \pi^{0}K^{0}) = \frac{1}{\sqrt{2}}[V_{ts}V_{tb}^{*}(P' - \frac{1}{3}P_{EW}^{C} - P_{EW}' - \frac{1}{3}P_{EW}^{E}) + V_{ts}V_{ub}^{*}C''], \]

\[ A(B^{0} \to K^{+}K^{-}) = V_{td}V_{tb}^{*}(P'' + \frac{1}{3}P_{EW}^{A} + \tilde{P}_{EW}^{A}) + V_{td}V_{ub}^{*}E'', \]

\[ A(B^{+} \to K^{+}K^{0}) = V_{td}V_{tb}^{*}(P'' - \frac{1}{3}P_{EW}^{C} + \frac{2}{3}P_{EW}' + V_{td}V_{ub}^{*}A''), \]

\[ A(B^{0} \to K^{0}K^{0}) = V_{td}V_{tb}^{*}(P'' - \frac{1}{3}P_{EW}^{C} + P_{EW}' + \frac{2}{3}P_{EW}^{A} - \frac{1}{3}P_{EW}^{E}). \]

It is noticed that there is a slight difference to the usual diagrammatic decomposition approach with the extra contributions from the annihilation electro-weak diagrammatic amplitudes \( P_{EW}^{A_{1}M_{1}} \) and \( P_{EW}^{E_{1}M_{2}} \), which are actually small and neglected in the usual diagrammatic decomposition approach.
A similar redefinition can be made for $B_s$ decays,

\[
E_s = E^{\pi\pi}(B_s), P_{sA} = 2P_A^{\pi\pi}(B_s), P_{sEW}^A = P_{sEW}^{A\pi\pi}(B_s),
\]

\[
T'_s = T^{K\pi}(B_s), C'_s = C^{K\pi}(B_s), P'_s = P_T^{K\pi}(B) + P_E^{K\pi}(B_s), P'_s = P_T^{K\pi}(B_s),
\]

\[
P^{C}_{sEW} = P_{sEW}^{C\pi\pi}(B_s), P^{E}_{sEW} = P_{sEW}^{E\pi\pi}(B_s),
\]

\[
T''_s = T^{kk}(B_s), P''_s = P_T^{kk}(B_s) + P_{sEW}^{kk}(B_s), P''_{sA} = P_{sEW}^{kk}(B_s), P''_{sEW} = P_{sEW}^{kk}(B_s),
\]

\[
E''_s = E^{kk}(B_s), P''_{sEW} = [P_{sEW}^{kk}(B_s) + P_{sEW}^{kk}(B_s)]/2, \tilde{P}_{sEW}'' = [P_{sEW}^{kk}(B_s) - P_{sEW}^{kk}(B_s)]/2,
\]

\[
P''_{sEW} = P_{sEW}^{EKK}(B_s).
\]

Then the decay amplitudes can be reexpressed as follows

\[
A(B_s \to \pi^+\pi^-) = V_{td}V_{tb}^*(P_{sA} + \frac{1}{3}P_{sEW}^A) - V_{ud}V_{ub}^*E_s,
\]

\[
A(B_s \to \pi^0\pi^0) = \frac{1}{\sqrt{2}}A(B_s \to \pi^+\pi^-),
\]

\[
A(B_s \to \pi^-K^+) = V_{ts}V_{tb}^*(P'_s + \frac{2}{3}P_{sEW}^C - \frac{1}{3}P_E^s) + V_{us}V_{ub}^*T'_s,
\]

\[
A(B_s \to \pi^0K^0) = -\frac{1}{\sqrt{2}}[V_{ts}V_{tb}^*(P'_s - \frac{1}{3}P_{sEW}^C - P_{sEW}^s - \frac{1}{3}P_E^s) + V_{us}V_{ub}^*C'_s],
\]

\[
A(B_s \to K^+K^-) = V_{td}V_{tb}^*(P''_{sA} + \frac{1}{3}P_{sEW}''_{sA} + \tilde{P}_{sEW}''_{sA}) + V_{ud}V_{ub}^*E''_s,
\]

\[
A(B_s \to K^0\bar{K}^0) = V_{td}V_{tb}^*(P''_{sA} - \frac{1}{3}P_{sEW}''_{sA} + P''_{sA} - \frac{2}{3}P_{sEW}''_{sA} - \frac{1}{3}P_E''),
\]

\[
(33)
\]

VI. NUMERICAL CALCULATIONS

We are now in the position to make numerical calculations.

A. Theoretical Input Parameters

The short distance contributions characterized by the Wilson coefficient functions for the effective four quark operators were calculated by several groups at the leading order(LO) and next-to-leading order(NLO) [11], their values mainly depend on the choice for the running scale $\mu$. In our numerical calculations, it is taken to be

\[
\mu = \sqrt{2\Lambda_{QCD}}m_b \simeq (1.5 \pm 0.1)\text{GeV}.
\]

(35)

The $\alpha_s$ value in the six quark operator effective Hamiltonian is also taken at $\mu = (1.5 \pm 0.1) \text{ GeV}$.
When considering the NLO Wilson coefficient functions and $\alpha_s$, one needs to include the magnetic penguin-like operator $O_{8g}$ which is defined as

$$O_{8g} = \frac{g}{8\pi^2} m_b \bar{q}_i \sigma_{\mu \nu} (1 + \gamma_5) T_{ij}^a C^{a \mu \nu} b_j,$$

where $i$, $j$ are the color indices. The magnetic-penguin contribution to the $B \to \pi K$, $\pi \pi$ decays leads to the modification for the Wilson coefficients corresponding to the penguin operators,

$$a_{4,6}(\mu) \to a_{4,6}(\mu) - \frac{\alpha_s(\mu)}{9\pi} \frac{2m_B}{\sqrt{|l^2|}} C_{\text{eff}}^{8g}(\mu)$$

with $C_{\text{eff}}^{8g} = C_{8g} + C_5$ and $|l^2| = m_B^2/4$. Where $a_{4,6}$ are known to be defined as $a_{4,6} = C_{4,6} + \frac{C_{3}}{N_c}$ which appear in the factorizable diagrams.

For other parameters, we take the following typical values

$$m_B = 5.28 \text{GeV}, \ m_{\pi^0} = 139.6 \text{MeV}, \ m_{\pi^0} = 135 \text{MeV}, \ m_b = 4.4 \text{GeV}, \ m_c = 1.5 \text{GeV}, \ m_s = 0.1 \text{GeV},$$

$$m_u = m_d = 5 \text{MeV}, \ f_{B} = 216.19 \text{MeV}, \ f_{\pi} = 130.1 \text{MeV}, \ F_K = 159.8 \text{MeV}, \ mu \simeq 1.7 \text{GeV},$$

$$\mu_K \simeq 1.8 \text{GeV}, \tau_{B^0} = 1.536 \text{ps}, \tau_{B^+} = 1.638 \text{ps}, \lambda = 0.2272, \ A = 0.806, \ \bar{\rho} = 0.195, \ \bar{\eta} = 0.326.$$  

Especially, for the infrared energy scale $\mu_g$ introduced in this paper to regulate the infrared divergence of gluon exchanging interactions, we take the typical value of $\mu_g$ to be a universal one around the hadronic bounding energy scale of non-perturbative QCD

$$\mu_g = (400 \pm 50) \text{MeV}.$$  

To evaluate numerically the hadronic matrix elements of effective six quark operators based on the QCD factorization, it needs to know the twist wave functions of mesons. For the wave function of $B$ meson, we take the following form [12] in our numerical calculations:

$$\phi_B(x) = N_B x^2 (1-x)^2 \exp \left[ -\frac{1}{2} \left( \frac{xm_B}{\omega_B} \right)^2 \right],$$

For the light meson wave functions, it needs to know the twist distribution amplitudes which contains the twist-2 pion (kaon) distribution amplitude $\phi_{\pi(K)}$, and the twist-3 ones $\phi^p_{\pi(K)}$ and $\phi^T_{\pi(K)}$. 


they are parameterized as [13]:

\[
\phi_{\pi(K)}(x) = 6x(1-x)(1 + a_1\pi(K)3(2x - 1) + a_2\pi(K)\frac{3}{2}(5(2x - 1)^2 - 1) \\
+ a_4\pi(K)\frac{15}{8}(21(2x - 1)^4 - 14(2x - 1)^2 + 1)),
\]

(41)

\[
\phi_{\pi(K)}^P(x) = 1 + \left(30\eta_3 - \frac{5}{2}\rho_{\pi(K)}^2\right)\frac{1}{2}(3(2x - 1)^2 - 1) \\
- 3\left(\eta_3\omega_3 + \frac{9}{20}\rho_{\pi(K)}^2(1 + 6a_2\pi(K))\right)\frac{1}{8}(35(2x - 1)^4 - 30(2x - 1)^2 + 3),
\]

(42)

\[
\phi_{\pi(K)}^T(x) = (1 - 2x)\left[1 + 6\left(5\eta_3 - \frac{1}{2}\eta_3\omega_3 - \frac{7}{20}\rho_{\pi(K)}^2 - \frac{3}{5}\rho_{\pi(K)}^2a_2\pi(K)\right)\right] \\
(1 - 10x + 10x^2),
\]

(43)

In our numerical calculations, the shape parameters in the distribution amplitudes are taken the following typical values:

\[
\omega_B = 0.25 \text{ GeV}, \omega_{B_s} = 0.33 \text{ GeV}, \eta_3 = 0.015, \omega_3 = -3,
\]

\[
a_1^\pi = 0, \ a_1^K = 0.06, \ a_2^K = 0.10 \pm 0.10, \ a_2^\pi = 0.15 \pm 0.15, \ a_4^\pi = a_4^K = 0 \pm 0.10
\]

(44)

where the shape parameters for the bottom mesons are taken from [12], and other shape parameters are taken to fit the data. Since those shape parameters can vary by 100\%, they agree with the ones in Refs.[13, 14], All parameters for the light mesons are taken at the energy scale 1 GeV [15], run to our calculation scale. Note that they may vary significantly when the scale runs to different values.

B. Numerical Results and Discussions

The numerical results for the CP averaged branching ratios and CP violations of charmless B meson decays are presented in Table I for $B \to \pi\pi, \pi K$ decay channels and in Table II for $B \to K\bar{K}$ channels. In Table III, we give the results for the branching ratios and CP violations for $B_s \to \pi\pi, \pi K, KK$ decay channels. The LO and NLO are corresponding to the leading order hadronic matrix elements with the leading order and next-to-leading order Wilson coefficients(which include the magnetic penguin-like operator $O_{8g}$). For comparison, the numerical predictions from the QCDF approach and pQCD approach are also listed in the Tables. It is seen that most resulting predictions in our present calculations are in good agreement with experimental data within the possible uncertainties from both experiments and theories, while it remains unclear how to understand the puzzles in the decay channel $B^0 \to \pi^0\pi^0$ for its large branching ratio and possible positive CP violation, and in the decay channel $B \to \pi^0K^+$ for the unexpected large positive CP violation.
As shown in [13], adding vertex corrections may improve the CP violation. The vertex corrections [16] were proposed to improve the scale dependence of Wilson coefficient functions. The ref. [16] has considered vertex corrections that only influence the Wilson coefficients of factorizable emission amplitudes. Those coefficients are always combined as \( C_{2n-1} + \frac{C_{2n}}{N_c} \) and \( C_{2n} + \frac{C_{2n-1}}{N_c} \) and modified by

\[
\begin{align*}
C_{2n-1}(\mu) + \frac{C_{2n}}{N_c}(\mu) & \rightarrow C_{2n-1}(\mu) + \frac{C_{2n}}{N_c}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_F \frac{C_{2n}(\mu)}{N_c} V_{2n-1}(M_2), \\
C_{2n}(\mu) + \frac{C_{2n-1}}{N_c}(\mu) & \rightarrow C_{2n}(\mu) + \frac{C_{2n-1}}{N_c}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_F \frac{C_{2n-1}(\mu)}{N_c} V_{2n}(M_2), 
\end{align*}
\]

with \( M_2 \) being the meson emitted from the weak vertex. In the NDR scheme, \( V_i(M) \) are given by [16]

\[
V_i(M) = \begin{cases} 
12 \ln \left( \frac{m\mu}{\mu} \right) - 18 + \int_0^1 dx \, \phi_M(x) \, g(x), & \text{for } i = 1 - 4, 9, 10, \\
-12 \ln \left( \frac{m\mu}{\mu} \right) + 6 - \int_0^1 dx \, \phi_M(x) \, g(1 - x), & \text{for } i = 5, 7, \\
-6 + \int_0^1 dx \, \phi_M(x) \, h(x), & \text{for } i = 6, 8,
\end{cases}
\]

\( \phi_M(x)/\phi_M^p(x) \) is the twist-2/twist-3 meson distribution amplitudes defined in Eq. 14. The functions \( g(x)/h(x) \) used in the integration are:

\[
\begin{align*}
g(x) = 3 \left( \frac{1 - 2x}{1 - x} \ln x - i \pi \right) \\
+ \left[ 2 \text{Li}_2(x) - \ln^2 x + \frac{2\ln x}{1 - x} - (3 + 2i \pi) \ln x - (x \leftrightarrow 1 - x) \right], \\
h(x) = 2 \text{Li}_2(x) - \ln^2 x - (1 + 2i \pi) \ln x - (x \leftrightarrow 1 - x).
\end{align*}
\]

Such a correction does not include the contributions of the first two diagrams in Fig.2a which are considered as a part of form factor or meson amplitude. It is interesting to notice that the vertex corrections do improve the predictions for CP violations and bring CP violations in the decay channels \( B^0 \rightarrow \pi^0\pi^0 \) and \( B^+ \rightarrow \pi^0K^+ \) to be more close to the experimental data.

To enlarge the branching ratio of \( B \rightarrow \pi^0\pi^0 \), we shall examine an interesting case that only two vertexes concerning the operators \( O_1 \) and \( O_2 \) receive additional large nonperturbative contributions, namely the Wilson coefficients \( a_1 = C_1 + \frac{C_{2}}{N_c} \) and \( a_2 = C_2 + \frac{C_{1}}{N_c} \) are modified to be the effective ones:

\[
\begin{align*}
a_1 & \rightarrow a_1^{\text{eff}} = C_1(\mu) + \frac{C_{2}}{N_c}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_F \frac{C_{2}(\mu)}{N_c} (V_1(M_2) + V_0), \\
a_2 & \rightarrow a_2^{\text{eff}} = C_2(\mu) + \frac{C_{1}}{N_c}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_F \frac{C_{1}(\mu)}{N_c} (V_2(M_2) + V_0),
\end{align*}
\]

Taking the value \( V_0 = 25 \), the resulting branching ratio for \( B \rightarrow \pi^0\pi^0 \) becomes consistent with the experimental data.
It is more interesting to consider the possible nonperturbative effects by taking the effective color number \( N^{eff}_{c} \) in color-suppressed diagrams. The numerical results with \( N^{eff}_{c} = 1.7 \) are presented in Table IV-VI, which provides an alternative explanation to the puzzle of observed large branching ratio \( B \rightarrow \pi^0 \pi^0 \). For comparison, we also list in Table IV-VI the predicted results via the S4 scenario in QCDF [17].

The method allows us to calculate the relevant transition form factors at maximal recoil (with NLO Wilson coefficients including magnetic penguin contribution),

\[
F_0^{B \rightarrow \pi} = 0.262^{+0.029+0.10}_{-0.024-0.010}, \quad F_0^{B \rightarrow K} = 0.322^{+0.034+0.013}_{-0.029-0.011}, \quad F_0^{B_s \rightarrow \pi} = 0.274^{+0.023+0.013}_{-0.013-0.0005},
\]

(50)

with input parameters \( \mu_g=400 \)MeV, \( \mu=1.5 \)GeV, \( \mu_\pi=1.7 \)GeV, \( \mu_K=1.8 \)GeV. The first error arises from the range for \( \mu_g = 350 \sim 450 \) MeV, the second error is caused by the running scale \( \mu = 1.4 \sim 1.6 \) GeV.

The resulting form factors agree with the ones obtained from the light-cone QCD sum rule of heavy quark effective field theory [18]

\[
F_0^{B \rightarrow \pi} = 0.285^{+0.016}_{-0.015}, \quad F_0^{B \rightarrow K} = 0.345 \pm 0.021, \quad F_0^{B_s \rightarrow \pi} = 0.296 \pm 0.018,
\]

and from the full QCD sum rule [19]

\[
F_0^{B \rightarrow \pi} = 0.258 \pm 0.031, \quad F_0^{B \rightarrow K} = 0.331 \pm 0.041.
\]

To know the relative contributions from various diagrams and hadronic matrix elements of effective six quark operators, we present in the Table VII and Table VIII the numerical results for different kinds of topology amplitudes, the predictions for the strong phases are all relative to the leading order tree amplitude phase \( \delta_T \simeq 1.93 \) in \( B \rightarrow \pi \pi \) decay. It is interesting to see that the amplitudes from the annihilation diagrams are significant in comparison with the color suppressed emission diagrams.

The predictions of \( S_{\pi^0 K_S} \) in each method are almost the same and obviously larger than averaged data in PDG. But some new data in [20, 21] prefer a larger prediction.

VII. CONCLUSIONS

Based on the approximate six quark operator effective Hamiltonian derived from perturbative QCD, the QCD factorization approach has naturally been applied to evaluate the hadronic matrix elements for charmless two body decays of bottom mesons. The resulting predictions for the decay amplitudes, branching ratios, and \( CP \) asymmetries in \( B^0, B^+, B_s \rightarrow \pi \pi, \pi K, KK \) decay channels have been found to be consistent with the current experimental measurements except for a few decay modes.
TABLE I: CP averaged branching ratios and CP violations for $B \to \pi\pi, \pi K$ decay channels. The central values are obtained with parameters: $\mu_\gamma=400$MeV, $\mu=1.5$GeV, $\mu_\pi=1.7$GeV, $\mu_K=1.8$GeV. The first error arises from the range for $\mu_\gamma=350 \sim 450$ MeV, the second error stems from the running scale $\mu = 1.4 \sim 1.6$ GeV.

| Mode          | Data [1] | QCDF[17]               | pQCD[13] | This work             |
|---------------|----------|------------------------|----------|-----------------------|
|               |          |                        | LO      | NLO(+MP)             |
|               |          |                        | LO      | NLO(+MP)             |
| $B^+ \to \pi^+ K^0$ | 23.1 ± 1.0 | 19.3$^{+1.9+11.3+13.9+13.2}_{-1.9-7.8-2.1-5.0}$ | 17.0 | 24.1 | 16.46 | 21.60$^{+7.34+4.36}_{-4.86-3.29}$ |
| $B^+ \to \pi^0 K^+$ | 12.8 ± 0.6 | 11.1$^{+1.8+5.8+6.9}_{-1.7-4.0-1.0-3.0}$ | 10.2 | 14.0 | 9.12 | 11.78$^{+3.81+2.22}_{-2.53-1.69}$ |
| $B^0 \to \pi^- K^+$ | 19.4 ± 0.6 | 16.3$^{+2.6+9.6+14.1+14.4}_{-2.3-6.5-1.4-4.8}$ | 14.2 | 20.5 | 14.42 | 19.05$^{+6.60+3.86}_{-4.39-2.93}$ |
| $B^0 \to \pi^0 K^0$ | 10.0 ± 0.6 | 7.0$^{+7.0+7.0+5.4}_{-7.3-7.2-7.3}$ | 5.7 | 8.7 | 6.61 | 8.84$^{+3.22+1.89}_{-2.13-1.44}$ |
| $B^0 \to \pi^- \pi^+$ | 5.16 ± 0.22 | 8.9$^{+1.0+3.6+6.1+1.2}_{-3.4-3.0-1.0-0.8}$ | 7.0 | 6.7 | 6.63 | 7.61$^{+1.69+0.70}_{-1.24-0.57}$ |
| $B^+ \to \pi^0 \pi^0$ | 5.7 ± 0.4 | 6.0$^{+3.0+2.1+1.0+0.4}_{-2.4-1.8-0.5-0.4}$ | 3.5 | 4.1 | 4.43 | 4.69$^{+1.03+0.30}_{-0.71-0.26}$ |
| $B^0 \to \pi^0 \pi^0$ | 1.31 ± 0.21 | 0.5$^{+0.2+0.2+0.3+0.2}_{-0.2-0.1-0.1-0.1}$ | 0.12 | 0.29 | 0.11 | 0.16$^{+0.05+0.02}_{-0.05-0.03}$ |
| $A_{CP}(\pi^+ K^0)$ | 0.099 ± 0.025 | 0.009$^{+0.002+0.003+0.001+0.006}_{-0.005-0.003-0.001-0.005}$ | −0.01 | −0.01 | 0.016 | 0.016$^{+0.002+0.001}_{-0.002-0.001}$ |
| $A_{CP}(\pi^0 K^+)$ | 0.047 ± 0.026 | 0.071$^{+0.017+0.020+0.008+0.009}_{-0.018-0.020-0.006-0.007}$ | −0.08 | −0.08 | −0.093 | −0.080$^{+0.008+0.006}_{-0.004-0.003}$ |
| $A_{CP}(\pi^- K^+)$ | −0.095 ± 0.013 | 0.043$^{+0.011+0.022+0.005+0.087}_{-0.011-0.025-0.006-0.095}$ | −0.12 | −0.10 | −0.150 | −0.124$^{+0.014+0.008}_{-0.014-0.007}$ |
| $A_{CP}(\pi^0 K^0)$ | −0.12 ± 0.11 | −0.033$^{+0.010+0.013+0.005+0.034}_{-0.006-0.010-0.033}$ | −0.02 | 0.00 | −0.006 | −0.001$^{+0.003+0.002}_{-0.002-0.002}$ |
| $S_{\pi^0 K^0}$ | 0.58 ± 0.17 | 0.08$^{+0.014+0.015+0.043+0.065}_{-0.128-0.138-0.141-0.616}$ | −0.34 | −0.41 | −0.528 | −0.561$^{+0.011+0.010}_{-0.011+0.009}$ |

TABLE II: $B \to KK$ modes with the same input parameters as Table I.

| Mode          | Data [1] | QCDF[17]               | pQCD[22] | This work             |
|---------------|----------|------------------------|----------|-----------------------|
|               |          |                        | LO      | NLO(+MP)             |
|               |          |                        | LO      | NLO(+MP)             |
| $B^+ \to K^+ K^0$ | 1.36 ± 0.28 | 1.36$^{+0.45+0.72+0.14+0.91}_{-0.39-0.49-0.15-0.40}$ | 1.65 | 1.85 | 1.09$^{+0.26+0.18}_{-0.17-0.14}$ |
| $B^0 \to K^0 K^0$ | 0.96 ± 0.20 | 1.35$^{+0.41+0.71+0.13+1.09}_{-0.36-0.48-0.15-0.45}$ | 1.75 | 2.65 | 0.84$^{+0.22+0.15}_{-0.15-0.12}$ |
| $B^0 \to K^+ K^-$ | 0.15 ± 0.10 | 0.013$^{+0.005+0.008+0.000+0.087}_{-0.005-0.005-0.000-0.011}$ | −0.07 | 0.07 | 0.07$^{+0.03+0.01}_{-0.03-0.01}$ |
| $A_{CP}(K^+ K^0)$ | 0.12 ± 0.17 | −0.163$^{+0.047+0.050+0.016+0.113}_{-0.037-0.057-0.017-0.133}$ | −0.096 | 0.078$^{+0.013+0.001}_{-0.013+0.001}$ |
| $A_{CP}(K^0 K^0)$ | −0.58 ± 0.7 | −0.167$^{+0.047+0.045+0.015+0.046}_{-0.037-0.051-0.017-0.056}$ | −0.000 | 0.000 | 0.000$^{+0.000+0.000}_{-0.000-0.000}$ |
| $A_{CP}(K^+ K^-)$ | − | − | −0.807 | 0.842$^{+0.042+0.000}_{-0.006-0.000}$ |
The puzzles for the observed large branching ratio in $B \to \pi^0 \pi^0$ decay and possible large positive CP violations in $B \to \pi K^+$ decay need to be further investigated. As we have emphasized at the beginning that the six quark operator effective Hamiltonian considered in this paper is an approximate one, and a large number of six quark diagrams which suppressed at high energy scales have been ingored, but they may become sizable at low energy scales. Furthermore, when given the predictions, we have only considered the uncertainties caused by the choices of running scale $\mu$ and infrared energy scale $\mu_B$ as their effects are more significant than others. In general, the theoretical uncertainties could be much larger when the possible uncertainties for all the input parameters are included. The masses of light mesons are also neglected in comparison with the bottom meson masses, i.e., $m_\pi^2/m_B^2 \sim 0$ and $m_K^2/m_B^2 \sim 0$.

Nevertheless, it is remarkable that such a simple theoretical framework based on the approximate six quark operator effective Hamiltonian from the perturbative QCD and the naive QCD factorization for the nonperturbative QCD effects can result in a satisfactory theoretical prediction for the charmless B meson decays $B$, $B_s \to \pi \pi, \pi K, KK$. It also shows that the singularity due to the on mass-shell fermion propagator can simply be treated with the principal integration method by apply the Cutkosky rules, and the one caused by the gluon exchanging interactions can well be regulated by the description used in the loop regularization method with the introduction of an intrinsic energy scale.
TABLE IV: The same as Table I but including the vertex contributions and compare with QCDF S4.

| Mode          | Data [1] | QCDF S4[17] | pQCD[13] | This work          | LO     | NLO+Vertex | $a^{eff}_{1,2}$ | $N^{eff}_G$ |
|---------------|----------|-------------|----------|---------------------|--------|-------------|-----------------|-------------|
| $B^+ \rightarrow \pi^+ K^0$ | 23.1 ± 1.0 | 20.3        | 17.0     | 24.5±13.6(+12.9)−8.1(−7.8) | 16.45  | 22.06±7.39+4.25−4.86−3.21 | 22.06 | 19.50 |
| $B^+ \rightarrow \pi^0 K^+$ | 12.8 ± 0.6 | 11.7        | 10.2     | 13.9±10.0(+ 7.0)−5.6(− 4.2) | 9.12   | 12.00±3.84+2.19−2.54−1.65 | 11.66 | 10.95 |
| $B^0 \rightarrow \pi^- K^+$ | 19.4 ± 0.6 | 18.4        | 14.2     | 20.9±15.6(+11.0)−8.3(−6.5)  | 14.41  | 19.32±6.67+3.84−4.41−2.91 | 19.62 | 18.68 |
| $B^0 \rightarrow \pi^0 K^0$ | 10.0 ± 0.6 | 8.0         | 5.7      | 9.1±5.6(+ 5.1)−3.3(−2.9)    | 6.61   | 8.98±3.25+1.88−2.14−1.42 | 9.70  | 8.71  |
| $B^0 \rightarrow \pi^- \pi^+$ | 5.16 ± 0.22 | 5.2         | 7.0      | 6.5±6.7(+ 2.7)−3.8(− 1.8)   | 6.62   | 7.07±1.67+0.71−1.39−0.58 | 5.38  | 4.89  |
| $B^+ \rightarrow \pi^+ \pi^0$ | 5.7 ± 0.4  | 5.1         | 3.5      | 4.0±3.4(+ 1.7)−1.9(− 1.2)   | 4.43   | 4.27±0.96+0.33−0.73−0.29 | 6.98  | 6.43  |
| $B^0 \rightarrow \pi^0 \pi^0$ | 1.31 ± 0.21 | 0.7         | 0.12     | 0.29±0.50(+0.13)−0.20(−0.08) | 0.11   | 0.18±0.07+0.04−0.04−0.04 | 1.03  | 0.98  |

| $A_{CP}(\pi^+ K^0)$ | 0.009 ± 0.025 | 0.003 | −0.01 −0.01 ± 0.00 (±0.00) | +0.016 | 0.026+0.000−0.000−0.000+0.000 | 0.020 | 0.018 |
| $A_{CP}(\pi^0 K^+)$ | 0.047 ± 0.026 | −0.036 | −0.08 −0.01+0.03+0.03−0.05(−0.05) | −0.093 | −0.035−0.006+0.004−0.002−0.002 | −0.068 | −0.0529 |
| $A_{CP}(\pi^- K^+)$ | −0.095 ± 0.013 | −0.041 | −0.12 −0.09+0.06+0.04−0.08(−0.06) | −0.150 | −0.133−0.015+0.008−0.001−0.007 | −0.117 | −0.131 |
| $A_{CP}(\pi^0 K^0)$ | −0.12 ± 0.11 | 0.008 | −0.02 −0.07+0.03+0.01−0.03(−0.01) | −0.006 | −0.051+0.003+0.000−0.002−0.000 | 0.002 | −0.029 |
| $S_{\pi^o K_S}$ | 0.58 ± 0.17[20, 21] | − | 0.70 | 0.73+0.03+0.01−0.02−0.01 | 0.710 | 0.710−0.002−0.002−0.002−0.002 | 0.789 | 0.781 |

| $A_{CP}(\pi^- \pi^+)$ | 0.38 ± 0.07 | 0.103 | 0.14 | 0.18±0.07−0.12(−0.12) | 0.178 | 0.186±0.002+0.015−0.002−0.014 | 0.214 | 0.223 |
| $A_{CP}(\pi^- \pi^0)$ | 0.04 ± 0.05 | −0.0002 | 0.00 | 0.00±0.00(±0.00) | 0.000 | 0.000+0.000+0.000−0.000−0.000 | 0.000 | 0.000 |
| $A_{CP}(\pi^0 \pi^0)$ | 0.36 ± 0.32 | −0.19 | −0.04 | 0.63±0.35+0.09−0.34−0.15 | −0.571 | 0.470−0.011−0.018 | −0.174 | −0.208 |
| $S_{\pi \pi}$ | −0.61 ± 0.08 | − | −0.34 | −0.43+1.00+0.05−0.56(−0.05) | −0.528 | −0.556−0.010−0.009+0.004+0.008 | −0.586 | −0.479 |

TABLE V: The same as Table II but including the vertex contributions and compare with QCDF S4.

| Mode          | Data [1] | QCDF S4[17] | pQCD[22] | This work          | LO     | NLO+Vertex | $a^{eff}_{1,2}$ | $N^{eff}_G$ |
|---------------|----------|-------------|----------|---------------------|--------|-------------|-----------------|-------------|
| $B^+ \rightarrow K^+ K^0$ | 1.36 ± 0.28 | 1.46        | 1.65     | 0.85 ±0.26+0.18−0.17−0.14 | 1.13   | 0.85        | 1.13 | 0.85 |
| $B^0 \rightarrow K^0 K^0$ | 0.96 ± 0.20 | 1.58        | 1.75     | 0.65 ±0.22+0.16−0.14−0.11 | 0.87   | 0.608       | 0.87 | 0.608 |
| $B^0 \rightarrow K^+ K^-$ | 0.15 ± 0.10 | 0.070       | −        | 0.07 ±0.03+0.01−0.10−0.01 | 0.07   | 0.29        | 0.07 | 0.29 |
| $A_{CP}(K^+ K^0)$ | 0.12 ± 0.17 | −0.043      | −        | 0.096 ±0.086+0.014+0.002−0.009−0.001 | 0.080 | 0.207       | 0.080 | 0.207 |
| $A_{CP}(K^0 K^0)$ | −0.58 ± 0.7 | −0.115      | −        | 0.000 ±0.000+0.000−0.000−0.000 | 0.000 | 0.000       | 0.000 | 0.000 |
| $A_{CP}(K^+ K^-)$ | −        | −          | −        | 0.807 ±0.842−0.005−0.000+0.041+0.000 | 0.84   | 0.78        | 0.84 | 0.78 |

$\mu_g$. In particular, it is found that such a scale takes a typical value $\mu_g = (400 \pm 50)$ MeV which is around the binding energy of hadron due to non-perturbative QCD effects.

We would like to point out that although the theoretical framework discussed above is a much
TABLE VI: The same as Table III but including the vertex contributions and compare with QCDF S4.

| Mode             | Data [1] | QCDF S4[17] | pQCD[14] | This work |
|------------------|----------|-------------|----------|-----------|
|                  |          |             |          | LO | NLO+Vertex | $a_{1,2}^{eff}$ | $N_c^{eff}$ |
| $B_s \rightarrow \pi^+\pi^-$ | 0.5 ± 0.5 | 0.155 | 0.57±0.16+0.09+0.01 | 0.19 | 0.23±0.01+0.07 | 0.23 | 0.69 |
| $B_s \rightarrow \pi^0\pi^0$ | –        | 0.078 | 0.25±0.08+0.04+0.01 | 0.10 | 0.11±0.01+0.03 | 0.11 | 0.34 |
| $B_s \rightarrow \pi^+\bar{K}^-$ | 5.0 ± 1.25 | 8.3 | 7.67±3.2+0.7+0.5 | 6.96 | 7.35±1.15+0.63 | 5.73 | 6.58 |
| $B_s \rightarrow \pi^0\bar{K}^0$ | –        | 0.61 | 0.16±0.05+0.10+0.02 | 0.07 | 0.17±0.04+0.04 | 0.07 | 0.60 |
| $B_s \rightarrow K^+\bar{K}^-$ | 24.4 ± 4.8 | 36.1 | 13.6±4.2+7.5+0.7 | 13.26 | 16.77±5.36+4.23 | 16.97 | 15.76 |
| $B_s \rightarrow K^0\bar{K}^0$ | –        | 38.3 | 15.6±5.0+8.3+0.0 | 15.25 | 18.94±5.72+4.34 | 18.94 | 16.63 |

$A_{CP}(\pi^+\pi^-)$ | –        | – | −0.012±0.001+0.012+0.001 | 0.018 | 0.015±0.0028 | 0.015 | 0.016 |
| $A_{CP}(\pi^0\pi^0)$ | –        | – | −0.012±0.001+0.012+0.001 | 0.018 | 0.015±0.0028 | 0.015 | 0.016 |
| $A_{CP}(\pi^+\bar{K}^-)$ | 0.39 ± 0.17 | 0.109 | 0.241±0.039+0.033+0.023 | 0.182 | 0.182±0.009+0.015 | 0.207 | 0.171 |
| $A_{CP}(\pi^0\bar{K}^0)$ | –        | 0.046 | 0.594±0.018+0.074+0.022 | 0.128 | 0.831±0.011 | −0.135 | 0.057 |
| $A_{CP}(K^+\bar{K}^-)$ | –        | 0.047 | −0.233±0.009+0.049+0.008 | 0.218 | −0.194±0.014+0.010 | −0.168 | −0.191 |
| $A_{CP}(K^0\bar{K}^0)$ | –        | 0.006 | 0 | 0.000 | 0.000+0.000 | 0.000 | 0.000 |

TABLE VII: Diagrammatic amplitudes relating to the CKM matrix element $\lambda_u$ with $10^{-7}$ GeV.

| Topology         | $T$      | $C$          | $A$          | $E$          |
|------------------|----------|--------------|--------------|--------------|
| LO               | 81.48    | 4.931e$^{-0.75i}$ | –            | 7.329e$^{-3.07i}$ |
| $\pi\pi$ NLO(+MP) | 81.46    | 6.952e$^{-0.50i}$ | –            | 7.321e$^{-3.07i}$ |
| NLO+Vertex       | 83.51e$^{0.04i}$ | 13.88e$^{-1.60i}$ | –            | 7.321e$^{-3.07i}$ |
| $\pi K$ NLO(+MP) | 100.0    | 6.020e$^{-0.75i}$ | 39.86e$^{-0.50i}$ | –            |
| NLO+Vertex       | 102.4e$^{0.04i}$ | 17.52e$^{-1.59i}$ | 39.58e$^{-0.50i}$ | –            |
| LO               | –        | –            | 2.267e$^{2.30i}$ | 7.169e$^{-0.85i}$ |
| $KK$ NLO(+MP)    | –        | –            | 2.045e$^{2.31i}$ | 7.088e$^{-0.85i}$ |
| NLO+Vertex       | –        | –            | 2.045e$^{2.31i}$ | 7.088e$^{-0.85i}$ |

simplified one, it turns out that as the first order approximation the six quark operator effective Hamiltonian considered in this paper can be taken as a good starting point. We have actually examined two interesting cases by considering the effective Wilson coefficient functions $a_1^{eff}$ and the effective color number $N_c^{eff}$ in the color suppressed diagrams to bring the prediction for the branching ratio $B \rightarrow \pi^0\pi^0$ be consistent with the experimental data. It is of interest to calculate high order contributions
meson decays, especially the measurements on CP-violation in getting better understanding on perturbative and nonperturbative QCD. Moreover, more precise experimental data in the future super B-factor and LHCb will guide us to arrive at a better understanding on perturbative and nonperturbative QCD.

TABLE VIII: Diagrammatic amplitudes relating to the CKM matrix element $\lambda_t$ with $10^{-7}$ GeV.

| topology  | $P_T$  | $P_{EW}$ | $P_{EW}^{C}$ | $P_A$  | $P_E$  | $P_A^{E}$ | $P_{EW}^{E}$ | $P = P_T + P_E$ |
|-----------|--------|----------|--------------|--------|--------|-----------|--------------|----------------|
| LO        | $6.555 e^{-3.10 i}$ | $1.176 e^{-3.13 i}$ | $0.101 e^{0.50 i}$ | $2.210 e^{-0.06 i}$ | $3.137 e^{1.51 i}$ | $0.021 e^{-2.65 i}$ | $0.111 e^{-0.62 i}$ | $6.987 e^{2.72 i}$ |
| $\pi\pi$  | $7.478 e^{-3.11 i}$ | $1.175 e^{-3.13 i}$ | $0.076 e^{0.69 i}$ | $2.442 e^{-0.06 i}$ | $3.339 e^{1.51 i}$ | $0.019 e^{-2.59 i}$ | $0.112 e^{-0.63 i}$ | $7.876 e^{2.74 i}$ |
| NLO(+)MP  | $7.684 e^{-3.07 i}$ | $1.201 e^{-3.09 i}$ | $0.232 e^{0.93 i}$ | $2.442 e^{-0.06 i}$ | $3.339 e^{1.51 i}$ | $0.019 e^{-2.59 i}$ | $0.112 e^{-0.63 i}$ | $7.971 e^{2.78 i}$ |
| NLO+Vertex| $8.332 e^{-3.10 i}$ | $1.490 e^{-3.12 i}$ | $0.123 e^{0.49 i}$ | $4.604 e^{-1.88 i}$ | $0.165 e^{-0.58 i}$ | $10.54 e^{2.74 i}$ |
| $\pi K$   | $9.483 e^{-3.11 i}$ | $1.489 e^{-3.12 i}$ | $0.092 e^{0.68 i}$ | $5.069 e^{1.91 i}$ | $0.150 e^{-0.60 i}$ | $12.00 e^{2.76 i}$ |
| NLO(+)MP  | $9.731 e^{-3.07 i}$ | $1.521 e^{-3.07 i}$ | $0.287 e^{0.96 i}$ | $5.069 e^{1.91 i}$ | $0.150 e^{-0.60 i}$ | $12.11 e^{2.79 i}$ |
| NLO+Vertex| $10.57 e^{-3.08 i}$ | $1.616 e^{0.44 i}$ | $1.572 e^{0.50 i}$ | $3.433 e^{1.69 i}$ | $0.069 e^{-1.26 i}$ | $0.053 e^{-3.00 i}$ | $11.30 e^{2.90 i}$ |
| $K K$     | $12.03 e^{-3.08 i}$ | $1.667 e^{0.54 i}$ | $3.663 e^{1.70 i}$ | $0.063 e^{-1.31 i}$ | $0.052 e^{-2.94 i}$ | $12.79 e^{2.92 i}$ |
| NLO+Vertex| $12.35 e^{-3.05 i}$ | $1.667 e^{0.54 i}$ | $3.663 e^{1.70 i}$ | $0.063 e^{-1.31 i}$ | $0.052 e^{-2.94 i}$ | $12.99 e^{2.95 i}$ |

TABLE IX: Diagrammatic amplitudes relating to the CKM matrix element $\lambda_u$ in $B_s$ decays with $10^{-7}$ GeV.

| topology  | $T_s$  | $C_s$  | $A_s$  | $E_s$  |
|-----------|--------|--------|--------|--------|
| LO        | $77.75 e^{0.08 i}$ | $6.221 e^{-0.84 i}$ | $53.96 e^{2.58 i}$ | $4.027 e^{-2.28 i}$ |
| $\pi\pi$  | $77.74 e^{0.08 i}$ | $7.876 e^{-0.60 i}$ | $53.76 e^{2.59 i}$ | $-$ |
| NLO(+)MP  | $79.63 e^{0.08 i}$ | $14.96 e^{-1.54 i}$ | $53.76 e^{2.59 i}$ | $-$ |
| NLO+Vertex| $95.41 e^{0.08 i}$ | $8.085 e^{-0.84 i}$ | $2.705 e^{1.77 i}$ | $7.313 e^{-1.44 i}$ |
| $K K$     | $95.41 e^{0.08 i}$ | $10.10 e^{-0.61 i}$ | $2.478 e^{1.78 i}$ | $7.219 e^{-1.43 i}$ |
| NLO+Vertex| $97.73 e^{0.08 i}$ | $19.36 e^{-1.50 i}$ | $2.478 e^{1.78 i}$ | $7.219 e^{-1.43 i}$ |

though it is a challenging task. On the other hand, the precise measurements of charmless bottom meson decays, especially the measurements on CP-violations in $B \to KK$ and $B_s \to \pi\pi, \pi K, KK$ decays, will provide a useful test for various theoretical frameworks. It is expected that more and more precise experimental data in the future super B-factor and LHCb will guide us to arrive at a better understanding on perturbative and nonperturbative QCD.
TABLE X: Diagrammatic amplitudes relating to the CKM matrix element $\lambda_i$ in $B_s$ decays with $10^{-7}$ GeV.

| topology     | $P_{T_s}$ | $P_{sEW}$ | $P^C_{sEW}$ | $P_{sA}$ | $P_{sE}$ | $P^A_{sEW}$ | $P^E_{sEW}$ | $P_s = P_{sT} + P_{sE}$ |
|--------------|-----------|-----------|-------------|-----------|-----------|-------------|-------------|-------------------------|
| LO          | $-0.15e^{-3.01i}$  | $1.122e^{-3.06i}$  | $0.108e^{-0.79i}$  | $-4.105e^{1.12i}$  | $-0.04e^{-2.78i}$  | $5.315e^{2.58i}$  |             |             |
| NLO (+MP)     | $-0.15e^{-3.01i}$  | $1.122e^{-3.06i}$  | $0.108e^{-0.79i}$  | $-4.105e^{1.12i}$  | $-0.04e^{-2.78i}$  | $5.315e^{2.58i}$  |             |             |
| NLO + Vertex  | $-0.15e^{-3.01i}$  | $1.122e^{-3.06i}$  | $0.108e^{-0.79i}$  | $-4.105e^{1.12i}$  | $-0.04e^{-2.78i}$  | $5.315e^{2.58i}$  |             |             |
| LO          | $8.231e^{-3.00i}$  | $0.918e^{-3.05i}$  | $0.143e^{-0.76i}$  | $1.181e^{1.48i}$  | $5.852e^{1.62i}$  | $0.083e^{-1.47i}$  | $0.043e^{-1.63i}$  | $9.671e^{2.63i}$  |
| NLO (+MP)     | $8.231e^{-3.00i}$  | $0.918e^{-3.05i}$  | $0.143e^{-0.76i}$  | $1.181e^{1.48i}$  | $5.852e^{1.62i}$  | $0.083e^{-1.47i}$  | $0.043e^{-1.63i}$  | $9.671e^{2.63i}$  |
| NLO + Vertex  | $8.231e^{-3.00i}$  | $0.918e^{-3.05i}$  | $0.143e^{-0.76i}$  | $1.181e^{1.48i}$  | $5.852e^{1.62i}$  | $0.083e^{-1.47i}$  | $0.043e^{-1.63i}$  | $9.671e^{2.63i}$  |

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APPENDIX: CALCULATIONS OF HADRONIC MATRIX ELEMENTS

In this appendix, we are going to present the explicit expressions for all the hadronic matrix elements evaluated from the naive QCD factorization method based on effective six quark operators. To be specific, we shall first make the following convention for the momentums of quarks and mesons, which is explicitly shown in Fig. 5.

Where we have ignored the light quark mass in external lines and light meson mass to simplify calculation.

Let us first give the factorizable emission contributions for the $(V - A) \times (V - A)$ and $(V - A) \times (V + A)$...
effective four quark vertexes, they are simply denoted by $LL$ and $LR$

\begin{align}
T_{L\!L}^{F_1 M_2}(M) &= T_{L\!L_a}^{F_1 M_2}(M) + T_{L\!L_b}^{F_1 M_2}(M), \\
T_{L\!L_a}^{F_1 M_2}(M) &= \frac{1}{4 N_c} C_F F_{M_1} F_{M_2} \int_0^1 \int_0^1 \int_0^1 du \, dy \, m_B^2 \phi_M(u) \\
&\quad \times \{ m_B (2m_B - m_B x) \phi_{M_1}(x) + \mu_{M_1}(2m_B x - m_B) \phi^{T}_{M_1}(x) \} \phi_{M_2}(y) h_{T_a}^F(u, x), \\
T_{L\!L_b}^{F_1 M_2}(M) &= \frac{1}{2 N_c} C_F F_{M_1} F_{M_2} \int_0^1 \int_0^1 \int_0^1 du \, dy \, m_B^2 \phi_{M_1}(u) \\
&\quad \phi_{M_2}(y) h_{T_b}^F(u, x), \\
T_{L\!R}^{F_1 M_2}(M) &= T_{L\!L_a}^{F_1 M_2}(M) + T_{L\!L_b}^{F_1 M_2}(M), \\
T_{L\!R_a}^{F_1 M_2}(M) &= -T_{L\!L_a}^{F_1 M_2}(M), \\
T_{L\!R_b}^{F_1 M_2}(M) &= -T_{L\!L_b}^{F_1 M_2}(M). \tag{A.1}
\end{align}

The factorizable emission contributions for the $(S - P) \times (S + P)$ effective four quark vertex are found to be

\begin{align}
T_{S\!P}^{F_1 M_2}(M) &= T_{S\!P_a}^{F_1 M_2}(M) + T_{S\!P_b}^{F_1 M_2}(M), \\
T_{S\!P_a}^{F_1 M_2}(M) &= \frac{1}{2 N_c} C_F F_{M_1} F_{M_2} \int_0^1 \int_0^1 \int_0^1 du \, dy \, m_B \mu_{M_2} \phi_M(u) \\
&\quad \phi_{M_2}(y) h_{T_a}^F(u, x), \\
T_{S\!P_b}^{F_1 M_2}(M) &= \frac{1}{2 N_c} C_F F_{M_1} F_{M_2} \int_0^1 \int_0^1 \int_0^1 du \, dy \, m_B^2 \mu_{M_2} \phi_M(u) \\
&\quad \phi_{M_2}(y) h_{T_b}^F(u, x). \tag{A.2}
\end{align}
Similarly, we obtain

\[
T_{LLN_1M_2}^{NM_1M_2}(M) = T_{LLa}^{NM_1M_2}(M) + T_{LLb}^{NM_1M_2}(M),
\]

\[
T_{LLa}^{NM_1M_2}(M) = -i \frac{1}{4 N_c} C_F F_M F_{M_1} F_{M_2} \int_0^1 \int_0^1 \int_0^1 du \, dx \, dy \, m_B^2 \phi_M(u) \{ (u - y) m_B \phi_{M_1}(x) + (1 - x) \mu_{M_1} [\phi_{M_1}^P(x) + \phi_{M_1}^T(x)] \} \phi_{M_2}(y) h_T^N(u, x, y),
\]

\[
T_{LLb}^{NM_1M_2}(M) = i \frac{1}{4 N_c} C_F F_M F_{M_1} F_{M_2} \int_0^1 \int_0^1 \int_0^1 du \, dx \, dy \, m_B^2 \phi_M(u) \{ (u + x + y - 2) m_B \phi_{M_1}(x) + (1 - x) \mu_{M_1} [\phi_{M_1}^P(x) - \phi_{M_1}^T(x)] \} \phi_{M_2}(y) h_T^N(u, x, y)
\]

(A.3)

for non-factorizable emission contributions with the \((V - A) \times (V - A)\) effective four quark vertex, and

\[
T_{LRN_1M_2}^{NM_1M_2}(M) = T_{LRa}^{NM_1M_2}(M) + T_{LRb}^{NM_1M_2}(M)
\]

\[
T_{LRa}^{NM_1M_2}(M) = -i \frac{1}{4 N_c} C_F F_M F_{M_1} F_{M_2} \int_0^1 \int_0^1 \int_0^1 du \, dx \, dy \, m_B^2 \phi_M(u) \{ \mu_{M_2} \mu_{M_1} \{ [(u - x - y + 1) \phi_{M_1}^T(x) + (u + x - y - 1) \phi_{M_1}^P(x)] \phi_{M_2}(y) - [(u - x - y + 1) \phi_{M_1}^P(x) + (u + x - y - 1) \phi_{M_1}^T(x)] \phi_{M_2}^T(y) \} + (u - y) m_B \mu_{M_2} [\phi_{M_2}^P(y) - \phi_{M_2}^T(y)] \phi_{M_1}(x) \} h_T^N(u, x, y)
\]

\[
T_{LRb}^{NM_1M_2}(M) = i \frac{1}{4 N_c} C_F F_M F_{M_1} F_{M_2} \int_0^1 \int_0^1 \int_0^1 du \, dx \, dy \, m_B^2 \phi_M(u) \{ \mu_{M_2} \mu_{M_1} \{ [(u - x + y) \phi_{M_1}^T(x) + (u + x + y - 2) \phi_{M_1}^P(x)] \phi_{M_2}^P(y) + [(u - x + y) \phi_{M_1}^P(x) + (u + x + y - 2) \phi_{M_1}^T(x)] \phi_{M_2}^T(y) \} + (u + y - 1) m_B \mu_{M_2} [\phi_{M_2}^P(y) + \phi_{M_2}^T(y)] \phi_{M_1}(x) \} h_T^N(u, x, y)
\]

(A.4)

for non-factorizable emission contributions with the \((V - A) \times (V + A)\) effective four quark vertex, and

\[
T_{SPN_1M_2}^{NM_1M_2}(M) = T_{SPA}^{NM_1M_2}(M) + T_{SPb}^{NM_1M_2}(M)
\]

\[
T_{SPA}^{NM_1M_2}(M) = i \frac{1}{4 N_c} C_F F_M F_{M_1} F_{M_2} \int_0^1 \int_0^1 \int_0^1 du \, dx \, dy \, m_B^3 \phi_M(u) \{ (u + x - y - 1) m_B \phi_{M_1}(x) + (1 - x) \mu_{M_1} [\phi_{M_1}^P(x) - \phi_{M_1}^T(x)] \} \phi_{M_2}(y) h_T^N(u, x, y)
\]

\[
T_{SPb}^{NM_1M_2}(M) = -i \frac{1}{4 N_c} C_F F_M F_{M_1} F_{M_2} \int_0^1 \int_0^1 \int_0^1 du \, dx \, dy \, m_B^3 \phi_M(u) \{ (u + y - 1) m_B \phi_{M_1}(x) + (1 - x) \mu_{M_1} [\phi_{M_1}^P(x) + \phi_{M_1}^T(x)] \} \phi_{M_2}(y) h_T^N(u, x, y)
\]

(A.5)

for non-factorizable emission contributions with the \((S - P) \times (S + P)\) effective four quark vertex.
We now present the results from annihilation diagram contributions,

\[
A_{LL}^{F_{M_1}M_2}(M) = A_{LLa}^{F_{M_1}M_2}(M) + A_{LLb}^{F_{M_1}M_2}(M),
\]
\[
A_{LLa}^{F_{M_1}M_2}(M) = \frac{1}{4} C_F F_M F_{M_1} F_{M_2} \int_0^1 \int_0^1 \int_0^1 du \, dx \, dy \, m_B^2 \phi_M(u) \left\{ (1 - y)m_B^2 \phi_M(y)\phi_M(x) + 2\mu_{M_2} \mu_{M_1} [(2 - y)\phi_M^p(y) + y\phi_M^T(y)\phi_M^F(x)] \right\} h_{Aa}(x, y),
\]
\[
A_{LLb}^{F_{M_1}M_2}(M) = \frac{1}{4} C_F F_M F_{M_1} F_{M_2} \int_0^1 \int_0^1 \int_0^1 du \, dx \, dy \, m_B^2 \phi_M(u) \left\{ x m_B^2 \phi_M(y)\phi_M(x) + 2\mu_{M_2} \mu_{M_1} [(1 + x)\phi_M^p(x) - (1 - x)\phi_M^T(x)\phi_M^F(y)] \right\} h_{Ab}(x, y),
\]
\[
A_{LR}^{F_{M_1}M_2}(M) = A_{L Ra}^{F_{M_1}M_2}(M) + A_{L R b}^{F_{M_1}M_2}(M),
\]
\[
A_{L Ra}^{F_{M_1}M_2}(M) = A_{LLa}^{F_{M_1}M_2}(M),
\]
\[
A_{L R b}^{F_{M_1}M_2}(M) = A_{LLb}^{F_{M_1}M_2}(M)
\]

for the factorizable annihilation contributions with the \((V - A) \times (V - A)\) and \((V - A) \times (V + A)\) effective four quark vertexes, and

\[
A_{SP}^{F_{M_1}M_2}(M) = A_{Sp a}^{F_{M_1}M_2}(M) + A_{Sp b}^{F_{M_1}M_2}(M),
\]
\[
A_{Sp a}^{F_{M_1}M_2}(M) = \frac{1}{2} C_F F_M F_{M_1} F_{M_2} \int_0^1 \int_0^1 \int_0^1 du \, dx \, dy \, m_B^2 \phi_M(u) \left\{ (1 - y)\mu_{M_2} [\phi_M^p(y) + \phi_M^T(y)]\phi_M(x) + 2\mu_{M_1} \mu_{M_2} (y)\phi_M^p(x) \right\} h_{Aa}(x, y),
\]
\[
A_{Sp b}^{F_{M_1}M_2}(M) = \frac{1}{2} C_F F_M F_{M_1} F_{M_2} \int_0^1 \int_0^1 \int_0^1 du \, dx \, dy \, m_B^2 \phi_M(u) \left\{ 2\mu_{M_2} \phi_M^p(y)\phi_M^F(x) \right\} h_{Ab}(x, y)
\]

for the factorizable annihilation contributions with the \((S - P) \times (S + P)\) effective four quark vertex, and

\[
A_{LL}^{N_{M_1}M_2}(M) = A_{LLa}^{N_{M_1}M_2}(M) + A_{LLb}^{N_{M_1}M_2}(M),
\]
\[
A_{LLa}^{N_{M_1}M_2}(M) = \frac{1}{4} C_F F_M F_{M_1} F_{M_2} \int_0^1 \int_0^1 \int_0^1 du \, dx \, dy \, m_B^2 \phi_M(u) \left\{ m_b +m_B(u - y) \right\} m_B^2 \phi_M^F(y)\phi_M(x) + \mu_{M_1} \mu_{M_2} \left\{ [(u - x - y +1)m_B \phi_M^F(x) + (-u - x + y +1)m_B \phi_M^T(x)]\phi_M^F(y) \right\} h_{Aa}(u, x, y),
\]
\[
A_{LLb}^{N_{M_1}M_2}(M) = \frac{1}{4} C_F F_M F_{M_1} F_{M_2} \int_0^1 \int_0^1 \int_0^1 du \, dx \, dy \, m_B^2 \phi_M(u) \left\{ x m_B^2 \phi_M(y)\phi_M(x) + \mu_{M_1} \mu_{M_2} \left\{ [(u + x + y -1)m_B \phi_M^F(x) + (-u + x - y +1)m_B \phi_M^T(x)]\phi_M^F(y) \right\} h_{Ab}(u, x, y)
\]

for the non-factorizable annihilation contributions with the \((V - A) \times (V - A)\) effective four quark
vertex, and

\[
A_{LR}^{N_2 M_2}(M) = A_{LRa}^{N_2 M_2}(M) + A_{LRb}^{N_2 M_2}(M),
\]

\[
A_{LRa}^{N_2 M_2}(M) = \frac{1}{4N_c} CF M F_{M_1} F_{M_2} \int_0^1 \int_0^1 \int_0^1 du \, dx \, dy \, m_B^2 \phi_M(u) \left\{ \mu_{M_2} [m_b + (y-u)m_B][\phi_{M_2}^a(y) - \phi_{M_2}^T(y)] \phi_{M_1}(x) \right. \\
- \mu_{M_1} [(1-x)m_B + m_b][\phi_{M_1}^a(x) + \phi_{M_1}^T(x)] \phi_{M_2}^b(y) \} h_{Ab}^N(u, x, y),
\]

\[
A_{LRb}^{N_2 M_2}(M) = -\frac{1}{4N_c} CF M F_{M_1} F_{M_2} \int_0^1 \int_0^1 \int_0^1 du \, dx \, dy \, m_B^3 \phi_M(u) \left\{ x \mu_{M_1} [\phi_{M_1}^a(x) + \phi_{M_1}^T(x)] \phi_{M_2}^b(y) \\
- (1-u-y) \mu_{M_2} [\phi_{M_2}^a(y) - \phi_{M_2}^T(y)] \phi_{M_1}(x) \right\} h_{Ab}^N(u, x, y)
\]

for the non-factorizable annihilation contributions with the \((V - A) \times (V - A)\) and \((V - A) \times (V + A)\) effective four quark vertexes, and

\[
A_{SP}^{N_2 M_2}(M) = A_{SPa}^{N_2 M_2}(M) + A_{SPb}^{N_2 M_2}(M),
\]

\[
A_{SPa}^{N_2 M_2}(M) = -\frac{1}{4N_c} CF M F_{M_1} F_{M_2} \int_0^1 \int_0^1 \int_0^1 du \, dx \, dy \, m_B \phi_M(u) \left\{ [m_b + (x-1)m_B] m_B^2 \phi_{M_2}^a(y) \\
+ \mu_{M_2} [\phi_{M_1}(x)[(u-x-y+1)m_B \phi_{M_1}^a(x) + (-u-x+y+1)m_B \phi_{M_1}^T(x)] \phi_{M_2}^b(y) \right. \\
+ \{[4m_b - (-u-x+y+1)m_B] \phi_{M_1}^a(x) - (u-x-y+1)m_B \phi_{M_1}^T(x) \phi_{M_2}^b(y) \} \phi_{M_2}^b(y) \} h_{Ab}^N(u, x, y),
\]

\[
A_{SPb}^{N_2 M_2}(M) = \frac{1}{4N_c} CF M F_{M_1} F_{M_2} \int_0^1 \int_0^1 \int_0^1 du \, dx \, dy \, m_B^2 \phi_M(u) \left\{ (-u-y+1)m_B \phi_{M_2}^a(y) \phi_{M_1}(x) \\
+ \mu_{M_2} [\phi_{M_1}(x)[(u+x-y-1) \phi_{M_1}^a(x) - (u-x+y+1) \phi_{M_1}^T(x)] \phi_{M_2}^b(y) \right. \\
+ \{(-u+x-y+1) \phi_{M_1}^a(x) - (u+x+y-1) \phi_{M_1}^T(x) \phi_{M_2}^b(y) \} \phi_{M_2}^b(y) \} h_{Ab}^N(u, x, y)
\]

for the non-factorizable annihilation contributions with the \((S - P) \times (S + P)\) effective four quark vertex.

The functions \(h_{XA}^N\) with \((A = a, b)\) from Eqs. (A.1) to (A.9) arise from propagators of gluon and quark, here \(Y = F, N\) denote the factorizable and non-factorizable contributions respectively, and
$X = T, A$ the emission and annihilation diagrams respectively. They have the following explicit forms:

\begin{align}
    h_{Fa}^{F}(u, x) &= \frac{1}{(-u(1-x)m_B^2 - \mu_g^2 + i\epsilon)(xm_B^2 - m_B^2 + i\epsilon)}, \\
    h_{Fb}^{F}(u, x) &= \frac{1}{(-u(1-x)m_B^2 - \mu_g^2 + i\epsilon)(-um_B^2 - m_B^2 + i\epsilon)}, \\
    h_{Ta}^{N}(u, x, y) &= \frac{1}{(-u(1-x)m_B^2 - \mu_g^2 + i\epsilon)((1-x)(1-u-y)m_B^2 - m_B^2 + i\epsilon)}, \\
    h_{Tb}^{N}(u, x, y) &= \frac{1}{(-u(1-x)m_B^2 - \mu_g^2 + i\epsilon)((1-x)(y-u)m_B^2 - m_B^2 + i\epsilon)}, \\
    h_{Aa}^{F}(x, y) &= \frac{1}{(x(1-y)m_B^2 - \mu_g^2 + i\epsilon)((1-y)m_B^2 - m_B^2 + i\epsilon)}, \\
    h_{Ab}^{F}(x, y) &= \frac{1}{(x(1-y)m_B^2 - \mu_g^2 + i\epsilon)(x m_B^2 - m_B^2 + i\epsilon)}, \\
    h_{Aa}^{N}(u, x, y) &= \frac{1}{(x(1-y)m_B^2 - \mu_g^2 + i\epsilon)((y-u)(1-x)m_B^2 - m_B^2 + i\epsilon)}, \\
    h_{Ab}^{N}(u, x, y) &= \frac{1}{(x(1-y)m_B^2 - \mu_g^2 + i\epsilon)((1-u-y)x m_B^2 - m_B^2 + i\epsilon)}.
\end{align}

(A.11)

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