Arbitrary-curved waveguiding and broadband attenuation in additively manufactured lattice phononic media

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1. Introduction

Control over acoustic waves in elastic bodies is at the core studies of wave dynamics since Lord Rayleigh’s times [1]. Recently, revolutionary progress in this field has been achieved due to the emergence of phononic crystals [2,3]. These are man-made materials with carefully tailored architectures that can manipulate elastic waves due to periodic variations of mechanical and material properties [2,4]. Multiple works have demonstrated tremendous potential of phononic crystals for broadband wave attenuation [5–7], waveguiding [8,9], focusing [10,11], filtering [12,13], collimation [15] and subwavelength imaging [16,17]. These functionalities can nowadays be implemented in various engineering devices thanks to advances in additive manufacturing technologies allowing to produce phononic configurations of almost unlimited complexity [18,19].

One of the most appealing applications is waveguiding, i.e., the realization of a structure enabling wave transmission confined along a predefined direction with minimum energy losses. Waveguiding is of importance when dealing with sound transmission or signal processing. In phononic materials, it can be implemented by means of so-called “defect states”. A defect state is formed by a sequence of building blocks that differ in geometry or composition from a perfectly periodic phononic medium. At frequencies, when wave propagation is inhibited by the medium, the wave energy can be confined along a defect state.

Wave localization at defects in phononic media was demonstrated a while ago [20]. Since then, substantial progress in phononic waveguiding was achieved by developing solid–solid [21] and solid–fluid [22,23] configurations. Several strategies were proposed to achieve tunable waveguiding [8,24,25] and enhanced wave confinement, e.g., by means of coupling with evanescent waves [26,27] or whispering-gallery modes [28], to name a few exemplary studies. However, these solutions suffer from a number of shortcomings.
First, solid-fluid phononics exhibit strong viscous effects affecting wave propagation and can guide only waves of a certain polarization [27,23]. Second, tunability often requires special electromagnetic or nonlinear materials [29,30]. Third, defect states are often realized as linear or planar paths [31]. Though a coupled-resonator concept allows arbitrary-curved defect paths [32], the operational frequencies are restricted due to a discrete spectrum of resonators that can be undesirable for effective waveguiding functionality [33,32]. Finally, experimental realization remains very limited, especially for three-dimensional configurations.

In this work, we address these issues by proposing mono-material lattice structures that can guide waves at continuous broadband frequencies along arbitrary-curved paths. These paths are realized in a phononic medium supporting extremely wide band gaps. The later are generated by simultaneously activating Bragg scattering, local resonance, and inertial amplification mechanisms to control waves [6]. Our phononic configurations can be produced at different size scales by means of additive manufacturing. We consider and characterize cm-size nylon samples produced by the Multi Jet Fusion (MJF) technique [34] and demonstrate waveguiding at frequencies from 5 kHz to 40 kHz.

The remaining part of this paper is structured as follows. First, we present three phononic media composed of different beams (Fig. 1). Then, we characterize the MJF-printed nylon (PA12) to estimate its quality in comparison with the commonly used selective laser sintering (SLS) PA12 (Section 3). Further, we study dispersion and transmission characteristics of the phononic structures (Section 4). Finally, we experimentally demonstrate strong wave confinement along a straight, angle- and 3D curved paths. (Section 5).

2. Phononic designs

Our phononic structures are mono-material lattices composed of identical cubes connected by beams. The representative unit cells and the fabricated samples are shown in Fig. 1. The basic building block is a cubic unit cell of size \(a\) with a central cube of length \(b\) and six square-section beams of length \(l\) and thickness \(t\). The beams are either perpendicular, \(\theta = 0\), (Figs. 1a,c) or inclined, \(\theta \neq 0\), in respect to the cube (Fig. 1e).

We consider configurations with the cubes of \(b = 10\) mm size and the beams of \(l = 5\) mm length. They differ in the beam thickness and/or the inclination angle with the aim to activate three different wave control mechanisms. The first structure, thick-beam configuration, has beams of thickness \(t = 5\) mm perpendicular to the cubes (Figs. 1a,b) with the lattice pitch of \(a = 20\) mm. This configuration supports periodicity-driven Bragg band gaps for elastic waves, as discussed below in Section 4A. The other two structures are designed to increase the band-gap width by additionally activating local resonance and inertial amplification mechanisms. Namely, the second design, referred to as a thin-beam configuration, has 3.5 mm-thick beams perpendicular to the cubes (Figs. 1d) and the same lattice pitch of \(a = 20\) mm. The third design, an inclined-beam structure, has the beams of thickness 2 mm inclined by \(\theta = 45^\circ\) (Figs. 1e,f). The inclination decreases the lattice pitch to \(a = 17.07\) mm. The described designs allow us to understand how the phononic architecture can be used to control propagating waves and to increase the band-gap width for implementing a broadband waveguiding.

3. Fabrication and (micro) mechanical characterization

The samples with 5 x 5 x 5 unit cells (Fig. 1) were manufactured by a commercial HP Multi Jet Fusion (MJF) 3D printer (Philips, Innovation Cluster Drachten). The base material is a powdered thermoplastic polyamide (PA12) with a 60 \(\mu\)m median particle size (the mass median diameter D50; melting point at 187°C) fused by infrared light by means of a binding agent. During printing, each new layer of 80 \(\mu\)m thickness is placed on top of a previous, still molten layer, that enables a complete fusion of the layers. As a result, the printed parts should be stronger and of a higher density as compared to the parts produced by other 3D-printing techniques, e.g. Selective Laser Sintering (SLS).

A series of characterization tests was performed to evaluate the quality of the MJF samples and an SLS nylon sample, for comparison. The latter sample was made of the same PA12 powder. Small chunks of material were cut from the printed structures and analyzed by Scanning Electron Microscopy (SEM) and Differential Scanning Calorimetry (DSC). For SEM imaging, the samples were sputtered with 5 nm of gold and measured at high vacuum under an acceleration voltage of 5 kV. The outer surfaces of the MJF and SLS samples are presented in Figs. 2a–b, respectively. The MJF sample surface shows evidence of extensive melting of the PA12 polymer, yet a few particle boundaries can be observed in the inset image. The SLS sample also presents good melting, but many more boundaries and some voids can be seen. Therefore, the surface of the MJF sample is smoother than that of the SLS one. A less uniform
powder heating can explain the effects of the SLS technique compared to MJF, which is inherent to the methods [35]. (see Figs. 3). For the DSC analysis, 28 mg samples were heated from room temperature to 275 °C at a rate of 10 °C/min under argon ambiance at 100 ml/min. For a first heating run, every sample was heated until melting. It was then cooled down moderately fast at a rate of −30 °C/min to room temperature to avoid extensive recrystallization from the melt. The heating test was repeated with the same sample as a second run. Figs. 3a,b show independent DSC thermograms for the MJF and SLS printed materials. From the first heating run, the peak melting temperatures ($T_m$) were 189 and 185 °C, respectively. No shoulders were observed for either polymer around the peak melting temperature, thus indicating a good quality of the source PA12 powders [36]. The melting enthalpies ($\Delta H_m$) were calculated to be 47.2 and 46.3 J/g, which correspond to crystallinities of 22.5 and 22.1 %, respectively [37]. The higher $T_m$ of the MJF printed sample might be related to increased cross-linking during the printing process. It has been shown that polycondensation can occur when PA12 is heated beyond its melting point and reused several times, hence rising slightly the average molecular weight [35,38]. However, the increase in $T_m$ has also been attributed to larger crystallites formed during the isothermal printing process [39,40]. For further analysis, a second heating run was applied to the same samples. This time the properties for the MJF and SLS materials were $T_m$ of 179 and 177 °C, $\Delta H_m$ of 24.7 and 24.4 J/g, and crystallinities of 11.8 and 11.7 %, respectively. The fast cooling prevented the melt from forming large crystallites, thus decreasing crystallinity. Moreover, 177 °C has been regularly reported as the $T_m$ for PA12 of $\gamma$-phase, which is the most stable crystalline phase of the polyamide, free from processing stresses [41]. For the MJF sample, $T_m$ was two degrees Celsius higher, again indicating some degree of polycondensation during the printing procedure. The effect was most probably due to some overheating, as suggested from SEM imaging. Thermogravimetric measurements (not shown here) reported a mass loss of only 1 % for both samples during the first heating run. Most likely, a small amount of adsorbed water and other trace substances were lost by evaporation. The small water content is not expected to largely affect our structures.

The mechanical moduli of the samples are those as standard values for the injection molded PA12, i.e., Young’s modulus $E = 1.78$ GPa and Poisson’s ratio $\nu = 0.4$, and the material density is 1015 kg/m$^3$.

4. Wave attenuation through three band-gap mechanisms

4.1. Dispersion analysis

Before analyzing wave attenuation in the phononic structures, we first numerically study dynamic characteristics of the corresponding infinite media. For this, we model a single unit cell with the Bloch boundary conditions at the faces and estimate eigenfrequencies for propagation directions along the boundaries of the irreducible Brillouin zone (Fig. 4b) [42,43,7]. The eigenfrequencies are calculated by means of the finite element method for fine-meshed unit cells using the Solid Mechanics Module in Comsol Multiphysics 5.5.

4.1.1. Bragg band gaps

The dispersion diagram of the thick-beam configuration is shown in Fig. 4a for non-dimensional frequencies $f' = f a / c$ vs.
wavenumber $k' = 2\pi/a$. Here, $f$ is frequency in Hz and $c_s$ is the velocity of shear waves in the bulk material.

The diagram has a complete (omnidirectional) band gap of width 14.5% ($BG\% = \frac{f_{\text{top}} - f_{\text{bot}}}{f_{\text{mid}}} \times 100$) and mid-gap frequency 0.55 kHz ($f_{\text{mid}} = \frac{f_{\text{top}} + f_{\text{bot}}}{2}$). The lower band-gap bound is formed by a torsional mode, at which the central cube is rotated relative to a symmetry axis. The upper bound is represented by a mode with flexural vibrations of the beams combined with longitudinal motion of the central mass (Fig. 4c).

The color of the bands in Fig. 4a indicates motion localization within the beams estimated as the ratio of the total displacement of the beams to the total displacement of the unit cell:

$$p = \frac{\int V_l \sqrt{u^2 + v^2 + w^2} \, dV_l}{\int_V \sqrt{u^2 + v^2 + w^2} \, dV}.$$  \hspace{1cm} (1)

Here $V_l$ and $V$ denote the volume of the beams and the unit cell, respectively; $u$, $v$, and $w$ are the displacement components. The value of $p$ ranges from 0, indicating motion-free beams (blue), to 1 corresponding to intense vibrations of the beams (red).

The green color of the bands below the band gap reveals that at these frequencies, the wave field is distributed almost uniformly within the unit cell. This can be expected since the associated wavelengths are large as compared to the characteristic unit-cell sizes. Above the band gap, the mode separation takes place. One can distinguish between the modes with vibrations mainly confined to the beams (yellow) and to the central cubes (blue).

When the wavelength is comparable to the unit-cell size, the Bragg band gap is activated [4]. This occurs in a periodic structure around the Bragg frequency, $f_{\text{Br}} = c_p / 2a$, governed by the effective phase velocity $c_p = f' / k'$ [44,45]. As any elastic medium supports three types of waves – longitudinal, flexural, and torsional – the Bragg frequency should be estimated for each of these waves. The approximate values of $f_{\text{Br}}$ were obtained directly from the dispersion diagram at the intersection of a tangent to the fundamental modes with a vertical line at a high-symmetry point [42,46]. Fig. 4a shows the Bragg frequencies for longitudinal $f_{\text{Br}}^l$ and torsional $f_{\text{Br}}^t$ fundamental modes for the $\Gamma - R$ direction. Due to a high structural symmetry, fundamental flexural mode $f_{\text{Br}}^f$ is indistinguishable.

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**Fig. 4. Wave dispersion for the thick-beam phononic medium.** (a) The dispersion diagram for the unit cell with $b = 10$ mm, $l = 10$ mm, $t = 5$ mm, $h = 0$. The complete and directional band gaps are shaded in blue and light-blue, respectively. The color of the bands indicates the amount of motion localization in the beams. (b) The Brillouin zone for a cubic unit cell. (c) Mode shapes at the bounds of the complete band gap. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
from the torsional mode at low frequencies close to $\Gamma$. This results in identical effective phase velocities $c_p = c_0$, and thus, $f_{Br} = f_0$. As can be seen, the values of $f_{Br}$ and $f_0$ are very close to the band-gap frequencies. Hence, the band gap in the thick-beam meta-structure originates due to Bragg scattering of the three types of waves propagating in the thick-beam phononic medium.

Note that the scattering is sensitive to the propagation direction. Namely, the directional band gaps for the $\Gamma - R$ and $\Gamma - M$ directions shaded in light-blue are wider than the complete band gap.

4.1.2. Bragg band gaps combined with local resonances

To broaden the band gap, we analyze the dependence of its size on the beam thickness when the other unit-cell parameters are fixed. Fig. 5 shows the condensed dispersion diagram for the thickness increasing from 2 mm to 5 mm with only complete band gaps (shaded in gray for contrast). The meaning of the colors is identical to that in Fig. 4a. We discuss Fig. 5 from the right-hand to left-hand side starting from low frequencies.

The motion localization $p$ below the first band gap appears to be insensitive to the thickness of the beam. To understand the reasons behind this, we compare the full diagrams for the thick-beam
(t = 5 mm, Fig. 4a) and thin-beam (t = 3.5 mm, Fig. 6a) configurations. The low-frequency bands in these diagrams are very similar with the Bragg frequencies $f_{1}^{Br}$, $f_{2}^{Br}$ being close or within the band gaps. Hence, the lowest band gap in the thin-beam structure is also governed by the Bragg scattering mechanism. The only difference is that for the thin-beam structure, the lower bound is formed by a longitudinal mode (Fig. 6b), as the frequency of the higher-order torsional mode is dropped with decreasing t.

As beam thickness t decreases from the thick-beam configuration, the lower bounds of the band gaps in Fig. 5 decrease due to the reduction of effective phase velocity $c_{p}$ in the beams. Besides, the decrease of t results in a slight decrease of the upper band-gap bound in Fig. 5. This occurs because flexural vibrations present in the corresponding mode shape (Figs. 4c, 6b) are governed by flexural stiffness decreasing with t, while the longitudinal component is insensitive to the thickness variations.

From the mechanical point of view, thinner beams increase the contrast in mechanical properties between the beams and the central cube. This leads to a stronger motion localization as indicated by the change of the color, and as a result, to a mode separation phenomenon [7]. Besides, the thinner the beams, the larger the number of band gaps at the same frequencies. For $t < 3$ mm, the band gaps at close frequencies are separated by modes with vibrations confined to either beams or cubes. This phenomenon is also immanent to the design approach based on the use of bulk elements connected by thin beams to generate broadband phononic band gaps [42,43,46,7].

On the other hand, strong mode localization suggests the presence of local resonances [42]. Indeed, the full diagram for the thin-beam structure in Fig. 6a contains (almost) flat dispersion bands with close-to-zero group velocity that is typical for the resonance phenomenon. Hence, as the beam thickness decreases, the widening of the lower band gap and the emergence of higher-order band gaps occur due to local resonances in addition to the Bragg scattering.

4.1.3. Extremely wide band gaps due to added inertia amplification

Further extension of the band-gap width requires the activation of the third wave control mechanism known as inertial amplification [47]. This can be achieved by changing the inclination angle of the beams $\theta$ (Fig. 1a) [6].

Fig. 7 shows the condensed dispersion diagram for $\theta$ varying from 0° to 75° when t = 2 mm. There are several wide band gaps separated by highly localized modes for any inclination angle. Note that these band gaps are wider than the band gaps for $\theta = 0$. The larger $\theta$, the closer are the frequencies of the localized modes, thus, increasing the possibility to merge neighboring band gaps. Another significant benefit of the added inertial amplification is that the lower bound of the first band gap is decreased, thus extending the band gap to lower frequencies without any additional changes in the material architecture. This occurs because local rotations of the beams amplify their effective inertia allowing to reduce the effective wave propagation velocity. Finally, the larger $\theta$, the more compact is a phononic medium, since adjacent cubes are located closer to each other for larger $\theta$, and the unit-cell size a decreases.

To gain a deeper insight into the band structure, we analyze the full dispersion diagram for $\theta = 45^\circ$ shown in Fig. 8a. Its characteristic feature is the presence of a large number of localized modes represented by almost flat bands. Most of these modes describe vibrational motions of the inclined beams confirming the presence of local resonances (Fig. 8b). The lower bound of the first band gap is formed by a torsional mode, in contrast to the longitudinal mode bounding the band gap in Fig. 6a. This occurs since longitudinal vibrations in the inclined beams are coupled with rotations, so that the frequency of this mode drops below that of the second torsional mode bounding the band gap. Such dynamics is a clear indication of the inertia amplification mechanism [47,6,48].

The Bragg frequencies of the fundamental modes are again inside or close to the first band gap indicating that the Bragg scattering also contributes to the band gap generation. Therefore, the observed extremely wide (the width of 161% for $\theta = 45^\circ$) band gap is formed due to a simultaneous activation of the three wave scattering mechanisms. Note that the analyzed phononic architecture is not optimized, meaning that even wider band gaps can be obtained by appropriately tuning the geometric parameters [48].

Finally, we point out that a high level of the vibrational energy localization in the inclined beams sets strict requirements on their connectivity to the cubes. To preserve the overall structural integrity and robustness, it is necessary to properly align inclined beams and to reduce possible imperfections. Thus, the MJF technique appears to be the most suitable to produce these structures.

4.2. Transmission analysis

Wave attenuation in a phononic medium can be evaluated by measuring signal transmission in finite-size samples. The samples geometries and the manufacturing technique are described in Section 2.

To measure transmission, a sample was placed on a foam bed isolating it from environmental vibrations (Fig. 9a). This configuration sustains the sample safely and uniformly, without affecting its dynamics. A bimorphic piezoelectric PZT disc is used as an actuator connected to a signal generator and to a high voltage operational amplifier (OPA547 - TI). The actuator applies a vertical excitation...
in a frequency range from 5 kHz to 50 kHz. Piezoelectric PZT discs with diameter of 10 mm and 5 mm, glued at the input and output positions, are used as sensors. These positions are schematically represented in Fig. 9b. The sensors are connected to an oscilloscope (RTM3000) with a sample rate of 5 Gsamples/s for data acquisition. Finally, to study the wave transmission through each structural layer of the sample, the PZT sensors were glued as shown in Fig. 9c.

The measured transmission through five phononic layers of the thick-beam sample is shown in red in Fig. 10a for the \( C_X \) propagation direction (see the diagram in Fig. 4a). A similar curve in Fig. 10b represents the experimental transmission for the \( C_R \) direction measured at the “side” output. The shaded regions in Fig. 10 indicate complete (royal blue) and directional (paled turquoise) band gaps predicted by the dispersion analysis (see Fig. 4a). The experimental band gaps, estimated by transmission drops of at least 20 dB [7,46], are shown by hatched boxes.

The shaded and hatched ranges should be at the same frequencies, provided the linear elastic material behavior. However, they are at different frequencies, as the experimental band gaps span broader ranges and extend to higher frequencies. The reason to this is twofold. First, the dispersion analysis completely ignores viscoelastic behavior of PA12 that results in frequency-dependent mechanical moduli shifting the predicted band gaps to higher frequencies [7]. (Further discussion of these effects can be found below.) Second, the band-gap width estimation in the dispersion analysis implies equal excitation of all modes. However, the excitation employed in our experiments can be inappropriate for certain modes, e.g., those localized in the beams. This results in broader experimental band gaps.

The experimental data in Fig. 10a show that five phononic layers allow properly identifying the experimental band gaps. Besides, one can see that the wave attenuation is omnidirectional, as the complete band gap is clearly recognized in Fig. 10b, despite a smaller number of unit cells in the \( C_R \) direction.

Numerical wave transmission was also estimated to clarify the differences between the experimental and theoretical (dispersion-based) band gaps. Numerical results based on a viscoelastic model were obtained in the frequency domain in the COMSOL Multiphysics for the models of the same geometry as the manufactured samples. The excitation was represented by a uniformly distributed harmonic force of a unit magnitude applied at the center of one of the faces; all the faces are modeled as stress-free. The transmission is calculated as \( 20\log_{10}(\frac{a_{\text{out}}}{a_{\text{in}}}) \), where \( a \) refers to a surface-averaged acceleration at the input and the output, as indicated by the subscript.

To capture the viscoelastic behavior of nylon, we used a three-mode generalized Maxwell model with empirically estimated relaxation moduli [49,7]. In particular, relaxation times...
Fig. 9. Experimental testing. (a) Experimental setup (b) Locations of the input actuator and output sensor. (c) Positions of the piezoelectric sensors to measure transmission per each phononic layer. Certain unit cells are hidden to visualize the positions of the sensors.

Fig. 10. Wave transmission in the thick-beam structure. Experimental (red) and numerical (black) transmission values for ΓX (a) and ΓR (b) propagation directions. The bounds of experimental band gaps are indicated by dashed red lines. The band gaps predicted by the dispersion analysis are shaded. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
\[ \tau_i = 1/2 \pi f_i \quad (i = 1, 2, 3) \]

were chosen at frequencies within the band gaps, i.e., \( f_1 = 20 \text{ kHz} \), \( f_2 = 32 \text{ kHz} \), and \( f_3 = 37 \text{ kHz} \), while the values of relaxation moduli \( G_i \quad (i = 1, 2, 3) \) were calibrated to match the experimental band gaps for the \( C_X \) direction. The numerical and experimental transmission curves in Fig. 10a are thus in a good agreement, as implied by this matching procedure. The quality of the viscoelastic model is verified by an appropriate match between the numerical and experimental transmission curves for the \( C_R \) direction (Fig. 10b). The agreement could be improved further by including a larger number of relaxation modes in the Maxwell model that is, however, beyond the scope of this work. (The analysis of viscoelastic characteristics of MJF PA12 is a relevant topic for future studies.)

The developed model allows us to confirm that the differences in the experimental and dispersion-based band gaps are caused by the material viscoelasticity. This conclusion is also verified by a good agreement of viscoelastic numerical curves estimated by using the described model with experimental transmission data for the thin-beam and inclined-beam configurations in Figs. 11, 12.

The transmission analysis of the thin-beam structure in Fig. 11 shows two unidirectional experimental band gaps at broad frequency ranges as predicted by the material design. An apparent feature of the numerical curves in Fig. 11a,b is an asymmetric Fano-type profile at the upper and bottom bounds of the band gaps that is characteristic for local resonances discussed previously. In the experimental curves, these profiles are partially destroyed by the material viscoelasticity [49]. Another effect of the viscoelasticity is “smearing” the bounds of experimental band gaps as compared to those for the thick-beam design (see Fig. 10). These two effects are apparently not captured by our three-mode Maxwell model.

Fig. 11. Wave transmission in the thin-beam structure. Experimental (red) and numerical (black) transmission values for \( C_X \) (a) and \( C_R \) (b) directions. The bounds of experimental band gaps are indicated by dashed red lines. The band gaps predicted by the dispersion analysis are shaded. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 12. Wave transmission in the inclined-beam structure. (a) Experimental transmission for 2,3,4 layers of the structure. Experimental (red) and numerical (black) transmission values for \( C_X \) (b) and \( C_R \) (c) directions. The bounds of experimental band gaps are indicated by dashed red lines. The band gaps predicted by the dispersion analysis are shaded. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
model that cannot account for the interplay between the material viscoelasticity and local resonances in thin beams. The development of another model is beyond the scope of this work.

Finally, we discuss wave transmission in the inclined-beam configuration (Fig. 12). The experimental data for the layer-by-layer transmission (see Fig. 9b) reveal that only two structural layers are sufficient to achieve strong attenuation at very broadband frequencies as shown in Fig. 12a. This wave attenuation ability was also predicted theoretically for other inclined-beam designs [6,48]. For the five structural layers, the numerically estimated attenuation exceeds 100 dB that cannot be captured by our PZT sensors (Fig. 12b,c). However, the agreement in terms of overall trends of experimental and numerical transmission curves is very good, despite the fact that the viscoelastic model is calibrated to fit the transmission of the thick-beam design.

5. Waveguiding at straight- and curved-path defects

“Defect” paths for waveguiding can be introduced by replacing selected unit cells of the inclined-beam configuration by those of the thick-beam structure. Then waves can be localized along such paths due to intense scattering from the inclined-beam unit cells at frequencies of the broadband band-gaps (Fig. 12).

This idea is implemented for a straight (Fig. 13), a right-angle (Fig. 14) and a 3D-curved (Fig. 15) defect paths. The structures with the paths are formed by 5 \times 5 unit cells, some of which are hidden in the figures to highlight the paths. These structures were printed by means of the MJF technique and tested experimentally.

To illustrate the waveguiding functionality, we measured the accelerations at the input and output of a path as schematically indicated in Figs. 13–15 (left). The wave confinement is estimated by measuring the acceleration at a side probe. The top-right graphs in Figs. 13–15 present experimental transmission at the output and side probes; the experimental band gap measured for the homogeneous phononic media are hatched.

As can be seen, the transmission along the paths is about one order of magnitude larger than that within the structure, regardless of the curvature of the path. Though, the output transmission at certain frequencies is comparable to that at the side probe (the dotted lines) due to geometric wave dispersion in the finite-size structures or local resonances (see, e.g., transmission drops in Fig. 14 top-right),

Fig. 13. Waveguiding through a straight defect path. The phononic structure with a thick-ligament central column surrounded by inclined-ligament columns (left). Experimental transmission measured at the output and side probes (top right) and the numerical mechanical stresses at specified frequencies (bottom right). The shaded areas indicate the experimental band gaps for the inclined- and thick-ligament structures.

Fig. 14. Waveguiding through an angle defect path. The phononic structure with a thick-ligament angle-shaped structure surrounded by inclined-ligament unit cells (top right). Transmission data at the output and side probes (top left) and mechanical stresses distribution at selected frequencies (bottom). The shaded areas indicate the experimental band gaps for the inclined- and thick-ligament structures.
these frequencies are limited. Therefore, we conclude that the waveguiding occurs at broadband frequencies. Besides, it is not directly affected by the material viscoelastic behaviour.

The wave localization along the paths is excellent as confirmed by at least 10 dB difference in the transmission level at the output and side probes. Note that the side probe is located only two unit cells away from a defect path.

To get a deeper inside into the wave energy distribution within the structure, we numerically estimated averaged stresses at various frequencies that are shown as snapshots in Figs. 13–15. They confirm that waves are perfectly localized along the paths, when the transmission at an output is larger than that at a side probe. At the limited frequencies with comparable transmission values, the energy is uniformly distributed within a complete structure (e.g., the case B in Figs. 13,14).

6. Conclusion

In this work, we proposed three-dimensional phononic media capable of controlling elastic waves at broadband frequencies, including a low-frequency range. These structures can be used for omnidirectional attenuation of waves and vibrations that is still challenging for many alternative solutions [42,46,6]. Besides, by properly combining two of our designs, one can implement waveguiding with strong localization of waves along an arbitrarily curved path. We demonstrated both mentioned functionalities by measuring wave transmission and waveguiding in cm-size MJF nylon samples for the three phononic designs and straight, angle and 3D-curved defect paths.

The characterization of the SLS and MJF printed nylon revealed a high quality and homogeneous microstructure with close-to-zero porosity. These features, affordable price and high manufacturing accuracy make both techniques excellent solutions to manufacture phononic structures for various applications. However, viscoelastic behavior of the 3D-printed nylon and the interplay with resonant-driven wave scattering make numerical predictions of wave transmission challenging. This limitation can be overcome by developing multi-mode viscoelastic models mimicking the material response for specified phononic geometries. Here, we successfully achieved this for one of the designed configurations.

Other distinctive features of our structures are passive functionality, advantageous for certain applications, and simplicity of implementation. The latter implies a mono-material configuration that can be readily realized by additive manufacturing techniques, and the design simplicity not requiring topology-protected [50,25] or parity-time-symmetry [51] states, which can restrict the functionality to narrow-band frequencies. Therefore, our work provides a novel design approach allowing to further advance applications of phononic materials in signal processing, non-destructive testing, and energy harvesting.

Data availability statement

The raw and processed data required to reproduce these findings are available from the authors upon reasonable request.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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