Parametric study of flapwise and edgewise vibration of horizontal axis wind turbine blades

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Abstract
In this paper, flapwise and edgewise vibrations of a horizontal axis wind turbine (HAWT) blade are studied. Rayleigh-Ritz method is used in which; orthogonal mode functions of the Euler-Bernoulli beam having fixed-free boundary are introduced into the Lagrange function and then the dynamic equations are derived. Effect of gravity, pitch angle, centrifugal stiffening, and rotary inertia are considered. Nondimensional equations are obtained by defining nondimensional parameters like; natural frequency, blade rotation, slenderness ratio, and simple pendulum frequency. The stiffness term of the natural frequency has two speed dependent elements and it is shown that, for small pitch angles, flapwise natural frequencies of the blade are increased by the increasing blade speed while the edgewise natural frequencies of the blade are decreased with the increasing blade speed. Pitch angle values ranging from $0^\circ$ to $15^\circ$ has negligible effect on the nondimensional natural frequencies of the blade up to the nondimensional blade speed of 4. Since the natural frequencies are the function of the blade speed, rotor critical speeds should be calculated with Campbell diagrams. Vibrational response of the blade tip to the gravity is dominant and much greater than that of the wind speed in the edgewise and flapwise vibration.

Keywords
Wind turbine, blade, vibration, modal analysis

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Introduction
The size of wind turbines is increasing because of the efficiency of the power production, but with the increasing blade lengths and weights, vibration problem is also becoming important and challenging. Wind turbine blade vibration is a serious problem not only because it will reduce the life of blade but also it can pass some unexpected frequencies to the tower, which will cause tower to vibrate. Ju and Sun developed a model for wind turbine flapwise vibration to reduce the pitch angle caused vibrations in flapwise direction, Lagrange’s method is used to model the blade and the input shaping method is used to reduce the residual vibrations caused by the change of pitch angle input. Karimi and Moradi developed a nonlinear kinematic model of the wind turbine blade using Hamilton’s principle, this model is simplified to three modal equations, two in flapwise direction, and one in edgewise direction and solved analytically using the multiple scales perturbation method. Jin et al. studied a quasi-three-dimensional solution for the dynamic behaviors of the rotating functionally graded material (FGM) beams which are assumed to have a metallic core covered with

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two ceramic faces. Jokar et al.\textsuperscript{4} studied the dynamic modeling and free vibration analysis of horizontal axis wind turbine (HAWT) blades in the flap-wise direction while taking into account the influences of gravity force, centrifugal force, and the blade rotary inertia. Zhang et al.\textsuperscript{5} studied the large-scale offshore wind turbine blades, developed the governing equations in fluid domain and the motion equations in structural domain with geometric nonlinearities, the fluid structure interactions (FSI) were simulated by using ANSYS. Liu et al.\textsuperscript{6} studied the influence of blade vibrations on the aerodynamic loads, the dynamic stall characteristics of an S809 airfoil undergoing various types of motion were investigated using computational fluid dynamics (CFD) techniques and simulation results indicated that the in-plane and out-of-plane translational motions of the airfoil affect the aerodynamic forces significantly. Liu\textsuperscript{7} analyzed the random wind caused vibration for the blade–cabin–tower coupling system. Li et al.\textsuperscript{8} studied the mode coupling among axial extension, flapwise vibration, edgewise vibration and torsion for the slender blade which is mounted on rigid hub and subjected to unsteady aerodynamic force. Chaves Júnior et al.\textsuperscript{9} used the collocation method for the evaluation of bending, torsional, and longitudinal vibrations of wind turbine structures, effects of the elastic supports, rotary and torsional inertia, geometric nonlinearity produced by the nacelle and water added mass are evaluated in different model approximations. Xi et al.\textsuperscript{10} used a semi-analytical model of the aerodynamic damping for horizontal axis wind turbines and the results indicated that the uncoupled analysis method employing this aerodynamic damping model can accurately predict the dynamic response of horizontal axis wind turbines excited by a combined wind and earthquake loading. Al-Hadad et al.\textsuperscript{11} investigated the effects of transient loadings resulting from a rapid mass reduction event from the rotating blade as well as an impacting mass onto the wind turbine blade, leading to transient vibration and unbalance within the turbine system. Zuo et al.\textsuperscript{12} presented at their work a state-of-the-art review of the current vibration control techniques and their applications to wind turbines. Xie and Aly\textsuperscript{13} introduced widely used control strategies in engineering structures and also their applications to suppress the adverse vibrations of the structural components of wind turbines. Staino et al.\textsuperscript{14} summaries the challenging issues related to structural control of wind turbines due to mechanical vibration and proposes a new blade design with active controllers for controlling edgewise vibrations. Zhang et al.\textsuperscript{15} proposed tuned liquid dampers (TLD) which utilize the sloshing motion of the fluid to suppress structural vibrations in rotating wind turbine blades. Fitzgerald and Basu\textsuperscript{16} designed and implemented a new active control strategy which is a cable connected active tuned mass damper (CCATMD) to control the in-plane vibration of large wind turbine blades. Chen et al.\textsuperscript{17} proposed a new semi-active fuzzy control strategy for controlling edgewise vibrations of wind turbine blades under extreme wind by using magnetorheological (MR) dampers mounted inside the blades.

In this paper, the vibration of a horizontal axis wind turbine (HAWT) blade edgewise and flapwise vibration is studied. Rayleigh-Ritz method is used in which the assumed orthogonal mode functions are introduced into the Lagrange function than the dynamic equations are derived for the first three modes of the blade. Effect of gravity, pitch angle, centrifugal stiffening, and rotary inertia is considered. Nondimensional equations are obtained defining nondimensional natural frequency, nondimensional blade speed, slenderness ratio, and simple pendulum frequency which appears as a gravity effect. Flapwise and edgewise natural frequencies are studied with respect to the slenderness ratio, blade pitch angle, blade speed, and gravity. Vibrational response of the blade under gravity and wind speed is also shown for the NREL-5MW type wind turbine blade.

Formulation

Figure 1 shows the cross-section of an airfoil in the inertial $y_0z_0$ plane. $x_0y_0$ plane is the rotor plane, $z_0$ is the blade rotation axis, $\theta$ is the pitch angle, $\alpha$ is the inflow angle or angle of attack, and $\phi$ is the flow angle. $Ox_0y_0z_0$ is the inertial frame. When the blade rotates with pitch angle $\theta$, we will have $Ox_0y_0z_0'$ inertial frame. The blade attached rotating frame is $Oxyz$. The equations are developed with respect to the rotating frame $Oxyz$ as shown in Figures 2 and 3. Here the equations of the motion are shown for the flapwise vibration of the blade, the edgewise vibration equations are similar.

![Cross sectional view of the blade](image-url)
The kinetic energy of the differential cross section in the flapwise direction is

\[ T = \frac{1}{2} \int_0^L \left[ \rho A(x)v^2(t) + \rho I_{yy}(x)\dot{w}^2(x,t) \right] dx \]  

Potential energy is:

\[ V = \frac{1}{2} \int_0^L \left[ EI_{yy}(x)w^2(x,t) + T_c(x)\dot{w}^2(x,t) \right] dx \]  

Rayleigh dissipation function is:

\[ R = \frac{1}{2} \int_0^L \left[ d\dot{w}^2(x,t) + \eta EI_{yy}(x)\dot{w}^2(x,t) \right] dx \]  

Here, in these equations, \( \rho \) is the material density, \( A(x) \) is the varying cross section, \( v \) is the velocity of the differential element of the blade with respect to the inertial frame, \( I_{yy}(x) \) is the varying area moment of inertia, \( w(x,t) \) is the deflection of the blade in the flapwise direction (\( z \) direction), \( E \) is the modulus of elasticity, \( T_c(x) \) is the total tension on the blade because of the centrifugal force and gravity, \( d \) is the viscous damping coefficient, \( \eta \) is the loss factor of the internal damping. Blade rotation \( \Omega \) has components in the rotating \( Oxyz \) frame as

\[ \Omega = \Omega \sin \theta j + \Omega \cos \theta k = \Omega_y i + \Omega_z k \]  

The velocity of the differential element with respect the inertial frame can be calculated as

\[ v = w \Omega_y i + (\dot{w} - x \Omega_z)k \]  

\[ v^2 = (w \Omega_y)^2 + (\dot{w} - x \Omega_z)^2 \]  

We assume that the cross section and the area moment of inertia is decreasing linearly from the root to the tip of the blade, so the following functions are assumed:

\[ A(x) = A_0 \left( 1 - \frac{x}{aL} \right)^2 = A_0 \gamma_a(x) \]  

\[ I_{yy}(x) = I_{yy0} \left( 1 - \frac{x}{aL} \right)^4 = I_{yy0} \gamma_a(x) \]  

Here \( A_0 \) is the cross section at the root of the blade, \( I_{yy0} \) is the area moment of inertia at the root of the blade, \( L \) is the length of the blade, \( a \) is the shape factor that defines the amount of cross section or inertia at the tip. If \( a = 1 \), cross section and inertia is zero at the tip, if \( a = 2 \), the cross section is \( \frac{1}{4} \) th of the \( A_0 \). Figure 4 shows the wind turbine blade, we can write the centrifugal force acting on the cross section at the distance \( x \) as:

\[ T_c(x) = mr\omega^2 = \frac{1}{2} \rho A_0 \left( 1 - \frac{x}{aL} \right)^2 \left( L - x \right) \left[ r + x + \frac{L - x}{3} \right]^2 \]  

\[ \Omega^2 = \rho A_0 \Omega y \gamma_a(x) \]  

The tension created by the gravity can be calculated as shown in Figure 5 as:
Wind turbine blade under gravity.

\[ T_g(x) = mg \cos(\Omega, t) = \frac{1}{3} \rho A_0 \left( 1 - \frac{x}{aL} \right)^2 \]

\[(L - x) g \cos(\Omega, t) = \rho A_0 g \cos(\Omega, t) \gamma_y(x) \]

Since the blade is rotating, gravitational tension changes, it is added to the centrifugal tension while the blade is vertical down, and subtracted while the blade is in the vertical up position. Total tension on the blade is;

\[ T_f(x) = T_c(x) + T_g(x) \]

To obtain the dynamic equations, Rayleigh-Ritz approach is used, in which assumed mode functions are put into the kinetic energy and potential energy equations of the Lagrange function and Lagrange equations are used to obtain the dynamic equations of the system with respect to the generalized coordinates. Since the nonrotating wind turbine blade can be assumed as the Euler-Bernoulli beam having fixed-free boundary conditions, sum of the orthogonal modes of the Euler-Bernoulli beam can be used as a solution;

\[ w(x, t) = \sum_{n=1}^{N} \phi_n(x) q_n(t) \]

Here \( \phi_n(x) \) is the orthogonal mode function and \( q(t) \) is the generalized coordinate. The orthogonal mode function of the Euler-Bernoulli beam of having fixed-free boundary condition and also the natural frequencies of it can be given as;

\[ \phi_n(x) = \sinh(\beta_n x) - \sin(\beta_n x) \]

\[ = \left[ \frac{\sinh(\beta_n L) + \sin(\beta_n L)}{\cosh(\beta_n L) + \cos(\beta_n L)} \right] \left[ \cosh(\beta_n x) - \cos(\beta_n x) \right] \]

Natural frequencies of the Euler-Bernoulli beam having fixed-free boundary condition are given as;

\[ \omega_n = (\beta L)^2 \frac{EI}{\rho AL^2} \]

We can also define nondimensional natural frequencies of flapwise and edgewise vibration as;

\[ \tilde{\omega}_{nf} = \frac{\omega_{nf}}{K_f} = (\beta L)^2 \]

\[ \tilde{\omega}_{ne} = \frac{\omega_{ne}}{K_e} = (\beta L)^2 \]

The first five nondimensional natural frequencies are;

\( (\beta L)^2_1 = 3.5160, \quad (\beta L)^2_2 = 22.0346, \quad (\beta L)^2_3 = 61.6979, \quad (\beta L)^2_4 = 120.9010, \quad (\beta L)^2_5 = 199.8604. \)

When the assumed solution of equation (12) put in the equations (1)–(3) we will obtain the following equations;

\[ T = \frac{1}{2} \rho A_0 \Omega \left[ \int_0^L \gamma_a(x) \phi_n^2(x) dx \right] q_n^2(t) \]

\[ + \frac{1}{2} \rho A_0 \int_0^L \gamma_a(x) \phi_n^2(x) dx \ddot{q}_n(t) \]

\[ - \rho A_0 \Omega \int_0^L \gamma_a(x) \phi_n(x) x dx \ddot{q}_n(t) \]

\[ + \frac{1}{2} \rho A_0 \Omega^2 \int_0^L \gamma_a(x) x^2 dx \]

\[ + \frac{1}{2} \rho L y_0 \int_0^L \gamma_a(x) \phi_n^2(x) dx \ddot{q}_n(t) \]

\[ V = \frac{1}{2} E I y_0 \left[ \int_0^L \gamma_a(x) \phi_n^2(x) dx \right] q_n^2 \]

\[ + \frac{1}{2} \rho A_0 \Omega^2 \left[ \int_0^L \gamma_a(x) \phi_n^2(x) dx \right] q_n^2 \]

\[ + \frac{1}{2} \rho A_0 \gamma \left[ \int_0^L \gamma_a(x) \phi_n^2(x) dx \right] q_n^2 \]

\[ R = \frac{1}{2} d \left[ \int_0^L \phi_n^2(x) dx \right] \ddot{q}_n^2 \]

\[ + \frac{1}{2} \eta E I y_0 \left[ \int_0^L \gamma_a(x) \phi_n^2(x) dx \right] q_n^2 \]
Lagrange equations are used to derive the dynamic equations of the wind turbine blade;

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_n} \right) - \frac{\partial T}{\partial q_n} + \frac{\partial V}{\partial \dot{q}_n} + \frac{\partial R}{\partial \dot{q}_n} = Q_n \tag{20}
\]

For each mode, the following equations will be obtained;

\[
m_n \ddot{q}_n + c_n \dot{q}_n + k_n q_n = f_n(x, t) \tag{21}
\]

\[
\ddot{q}_n + 2\xi_n \omega_n \dot{q}_n + \omega_n^2 q_n = \frac{1}{m_n} f_n(x, t) \tag{22}
\]

the natural frequency in the flapwise direction, after introducing nondimensional terms, is;

\[
\tilde{\omega}_{n,fw} = \sqrt{\frac{k_{n,fw}}{m_{n,fw}}} \tag{23}
\]

\[
\tilde{k}_{n,fw} = \left( \int_0^1 \gamma_1(\xi) \phi_{n,fw}^2(\xi)d\xi \right) (BL)^2_n
\]

\[
+ \Omega^2 \left( \int_0^1 \gamma_2(\xi) \gamma_3(\xi) \phi_{n,fw}^2(\xi)d\xi \right) (BL)^2_n
\]

\[
- \tilde{\omega}_p^2 \cos(\Omega t) \left( \int_0^1 \gamma_4(\xi) \phi_{n,fw}^2(\xi)d\xi \right) (BL)^2_n
\]

\[
- \tilde{\omega}_p^2 \sin(\Omega t) \left( \int_0^1 \gamma_5(\xi) \phi_{n,fw}^2(\xi)d\xi \right) (BL)^2_n
\]

\[
\tilde{m}_{n,fw} = \left( \int_0^1 \gamma_6(\xi) \phi_{n,fw}^2(\xi)d\xi \right)
\]

\[
+ \tilde{\eta} \left( \int_0^1 \gamma_7(\xi) \phi_{n,fw}^2(\xi)d\xi \right) (BL)^2_n
\]

\[
2\xi_n \tilde{\omega}_{n,fw} = \tilde{d} \left( \int_0^1 \phi_{n,fw}^2(\xi)d\xi \right)
\]

\[
+ \tilde{\eta} \left( \int_0^1 \gamma_8(\xi) \phi_{n,fw}^2(\xi)d\xi \right) (BL)^3_n
\]

Nondimensional terms are;

\[
\tilde{\omega}_{n,fw} = \frac{\omega_{n,fw}}{\sqrt{EI_{y0}/\rho A_0 L^2}}, \tilde{\Omega} = \frac{\Omega}{\sqrt{EI_{y0}/\rho A_0 L^2}}, \tilde{\omega}_p = \frac{\omega_p}{\sqrt{EI_{y0}/\rho A_0 L^2}} \tag{27}
\]

\[
\omega_p = \sqrt{\frac{g}{L}}, \tilde{\omega}_p = \frac{\omega_p}{\sqrt{EI_{y0}/\rho A_0 L^2}} \tag{28}
\]

\[
r_{g0} = \sqrt{\frac{I_{y0}}{A_0}}, s_r = \frac{L}{r_{g0}} \tag{29}
\]

\[
\tilde{d} = \frac{d}{\sqrt{EI_{y0}/\rho A_0 L^2}}, \tilde{\eta} = \eta \sqrt{\frac{EI_{y0}}{\rho A_0 L^2}} \tag{30}
\]

Here, \(\omega_p\) is the simple pendulum natural frequency which appears because of the gravitational effect. \(r_{g0}\) is the maximum value of the radius of gyration at the blade root, \(s_r\) is the slenderness ratio. Forcing functions of the blade have two parts, one is the component of the gravity in the \(y\) direction, the other is the component of the wind load. Equation (22) can be written as nondimensional;

\[
\ddot{q}_n + 2\xi_n \omega_n \dot{q}_n + \omega_n^2 q_n = -\tilde{f}_n, \tilde{q}_n = \frac{q_n}{L} \tag{31}
\]

\[
\tilde{f}_n = \tilde{f}_{n,g} + \tilde{f}_{n,v0} + \tilde{f}_{n,\Omega} \tag{32}
\]

Wind load on the blade is given as\(^{19}\);

\[
F_L = \frac{1}{2} \rho_a c(x) V_w^2 C_L, F_D = \frac{1}{2} \rho_a c(x) V_w^2 C_D \tag{33}
\]

\(F_L\) and \(F_D\) are the lift and drag forces created by wind, \(\rho_a\) is the air density, \(c(x)\) is the chord length of the airfoil which is changing along the blade, \(V_w\) is the relative inflow wind velocity, \(C_L\) and \(C_D\) are the lift and drag coefficients. From Figure 1 we can write total wind velocity as;

\[
V^2_w = V_0^2 (1 - d^2) + \Omega^2 x^2 (1 + d')^2 \tag{34}
\]

\(V_0\) is the inflow wind velocity which is perpendicular to the rotor plane, it is reduced by the amount of \(d' V_0\) due to axial interference. When the wind passes through the rotor plane and interacts with the moving rotor, a tangential slipstream wind velocity \(d' \Omega x\) is introduced. \(d'\) and \(d''\) are generally very small and can be ignored. We can easily obtain edgewise drag and flapwise lift as;

\[
F_{Dw} = F_D \cos \alpha - F_L \sin \alpha \\
F_{Lw} = F_D \sin \alpha + F_L \cos \alpha \tag{35}
\]

\[
F_{Dw} = \frac{1}{2} \rho_a c(x) V_w^2 (C_D \cos \alpha - C_L \sin \alpha) = \frac{1}{2} \rho_a c(x) V_w^2 C_{Dw} \tag{36}
\]

\[
F_{Lw} = \frac{1}{2} \rho_a c(x) V_w^2 (C_D \sin \alpha + C_L \cos \alpha) = \frac{1}{2} \rho_a c(x) V_w^2 C_{Lw} \tag{37}
\]

Now we can write the components of the forcing functions given in equation (32) in nondimensional form as;

\[
\tilde{f}_{n,g} = \tilde{\omega}_p^2 \left[ \int_0^1 \gamma_1(\xi) \phi_n(\xi)d\xi \right] \sin(\tilde{\Omega} \tau), \tau = \sqrt{\frac{EI_{y0}}{\rho A_0 L^2}} \tag{37}
\]

\[
\tilde{f}_{n,v0} = \frac{1}{2} \rho_a c_0 L (1 - d')^2 C_{w0} \left[ \int_0^1 \gamma_1(\xi) \phi_n(\xi)d\xi \right] V_0^2 \tag{38}
\]

\[
\tilde{f}_{n,\Omega} = \frac{1}{2} \rho_a c_0 L (1 + d'')^2 C_{w\Omega} \left[ \int_0^1 \gamma_1(\xi) \phi_n(\xi)d\xi \right] V_0^2 \Omega^2 \tag{39}
\]
\[ V_0 = \frac{V}{\sqrt{EI/\rho A L^2}} \]  

(40)

**Results and discussion**

Figure 6 shows the nondimensional first natural frequency of the blade with respect to the slenderness ratio for three values of the shape factor \( a \) of 1, 2, and \( \infty \). The measure of the rotary inertia is the slenderness ratio \( s_r \). When the slenderness ratio increases the natural frequency of the blade approaches from the Rayleigh beam to the Euler-Bernoulli beam natural frequency in which the effect of rotary inertia is neglected. Practically for \( s_r = 20 \) or more the effect of rotary inertia is negligible. There are three cases in the plot, \( a = 1 \) means the cross section at the tip of the blade is zero, \( a = 2 \) means, the blade tip cross section is \( \frac{1}{4} \)th of the root cross section, \( a = \infty \) means the cross section is constant along the blade (equation (7)). When the \( s_r = 20 \), the blade natural frequency for \( a = 1 \) is 2.3 times bigger than the natural frequency for \( a = \infty \). Figures 7 and 8 shows the second and third natural frequencies with respect to the slenderness ratio for three values of the shape factor \( a \). Shape factor and slenderness ratio are not as effective for the second and third natural frequencies. Figures 9 and 10 are showing Campbell diagram plots of the first flapwise and edgewise natural frequencies of the rotating blade for three different slenderness ratios. In these calculations \( a = 1 \), pendulum frequency \( \bar{\omega}_p = 0.4 \), and pitch angle \( \theta = 10^\circ \) are assumed.

In the equation (24) which is the stiffness equation, second term and third term depends on the blade
rotation. Second term is the tension created by the centrifugal force, third term is the inertial force created by the blade rotation component of the flapwise vibration plane. Flapwise vibration plane is the $xz$ plane and rotates with $\Omega_y = \Omega \sin \theta$. If the pitch angle is small, $\Omega_y$ will be small then the effect of rotation on the inertial force of the flapwise vibration are small but the centrifugal tension is still effective and flapwise natural frequency increases with the increasing blade rotation.

For the edgewise vibration, the $xy$ plane rotation is $\Omega_z = \Omega \cos \theta$. For small pitch angles, $\Omega_z$ is very close to the blade rotation and the effect of blade rotation on the inertial force of the edgewise vibration is higher than the effect of centrifugal force tension, that is why, the natural frequency of the edgewise vibration decreases with the increasing pitch angle. These effects can easily be verified by looking at the elements of the equation (24). In this equation, for the edgewise vibration calculations, only $\Omega_z$ should be replaced by $\Omega_y$, all other terms are the same.

As can be seen in the equation (24), the effect of weight on the tension of the blade is cyclic and changes with the rotation. The effect of this cyclic tension value on the natural frequency is small, that is why its maximum value is used for all natural frequency calculations. Table 1 shows the maximum and minimum values of the first five natural frequencies during one cycle.

Since the natural frequencies are the function of blade speed, Campbell diagram is used in the Figures 9 and 10 to find the critical speeds. Critical speeds for the flapwise and edgewise vibrations for three different slenderness ratios are tabulated in Table 2.

Figures 11 and 12 are showing the effect of pitch angle on the first natural frequency of the blade. In these plots $a = 1$, $\omega_p = 0.4$, $\theta = 10^\circ$, $\text{sr} = 20$, $\Omega = 10$.

### Table 1. Flapwise vibration natural frequency fluctuation in one cycle because of the weight ($a = 1$, $\omega_p = 0.4$, $\theta = 10^\circ$, $\text{sr} = 20$, $\Omega = 10$).

| $\omega_{\max}$ | $\omega_{\min}$ | % dif. |
|-----------------|-----------------|--------|
| 9.3686 | 9.3301 | 0.4109 |
| 29.4964 | 29.4742 | 0.0752 |
| 65.7312 | 65.7073 | 0.0363 |
| 115.7146 | 115.6893 | 0.0218 |
| 174.8452 | 174.8198 | 0.0145 |

### Table 2. Nondimensional critical speeds in flapwise and edgewise vibrations.

|            | $\text{sr} = 5$ | $\text{sr} = 10$ | $\text{sr} = 20$ |
|------------|-----------------|-----------------|-----------------|
| Edgewise   | $\Omega_{cr}$   | 8.4             | 9.3             | 9.6             |
| Flapwise   | $\Omega_{cr}$   | 11.6            | 14.6            | 15.8            |

### Figure 10. Nondimensional first edgewise natural frequency of the rotating blade for three different slenderness ratios.

### Figure 11. Flapwise first natural frequency changes with respect to the pitch angle.
for the example NREL-5MW horizontal axis wind turbine. The values are tabulated at the Table 3 according to the properties table given in Jonkman et al.\textsuperscript{20} airfoil starts at the length 11.7 m until 63 m. In calculations, blade length is assumed as $L = 51.3$ m and the root length is $r = 11.7$ m. Blade mass density $\rho A(x)$ in kg/m, and flapwise stiffness $EI_{yy}(x)$ and edgewise stiffness $EI_{zz}(x)$ in Nm$^2$ are also tabulated in Jonkman et al.\textsuperscript{20}

Figures 13 to 15 are showing the plot of these values and also the curves that represents these values. At the distance 11.7 m of the blade, the mass density and stiffness values are considered as root values in the calculations; $\rho A_0 = 426.3$ kg/m, $EI_{yy0} = 4692 \times 10^6$ Nm$^2$, and $EI_{zz0} = 7168 \times 10^6$ Nm$^2$. Assumed values of viscous damping and internal damping loss factor are $d = 0.02$, $\eta = 0.01$. Table 4 shows the calculated modal natural frequencies and also values given in Resor.\textsuperscript{21}

Blade tip deflections because of the wind load and gravity are also calculated. Inflow wind speed of $V_0 = 11.4$ m/s, Blade speed of $\Omega = 12$ rpm, pendulum frequency of $\omega_p = 0.4373$ rad/s are assumed. Figures 16 and 17 are showing the flapwise and edgewise blade tip vibrations, respectively. In these figures, first mode and also sum of the three modes are plotted. As shown in the plots, first mode vibration is dominant, the effect of second and third mode vibrations are negligibly small.

Figures 18 and 19 show the flapwise and edgewise vibrations of the blade tip and also its components coming from the gravity, inflow wind, and blade rotation. For the flapwise vibration shown in Figure 18, gravity causes sinusoidal vibration of magnitude $\pm 144$ mm, inflow wind causes transient vibration of magnitude 60 mm and settles to 33 mm, blade rotation causes transient vibration of 247 mm and settles to 128 mm after approximately 16 s. Steady vibration of the total three is a sinusoidal vibration of magnitude minimum $-13$ mm and maximum $+312$ mm. For the edgewise vibration shown in the Figure 17, gravity causes sinusoidal vibration of magnitude $\pm 170$ mm, inflow wind causes transient vibration of magnitude 5 mm and settles to 3 mm, blade rotation causes transient vibration of 22 mm and settles to 12 mm after approximately 16 s. Steady vibration of the total three is a sinusoidal vibration of magnitude minimum $-153$ mm and maximum $+185$ mm.

### Conclusions

Horizontal axis wind turbine blade edgewise and flapwise vibrations are studied. Rayleigh-Ritz method is used in which the orthogonal mode functions of the Euler-Bernoulli beam of having fixed-free boundary are introduced into the Lagrange function and then the dynamic equations are derived. Varying cross section and stiffness are assumed. Effect of gravity, pitch angle, centrifugal stiffening and rotary inertia on the blade vibrations are studied. Nondimensional equations are obtained defining nondimensional natural frequency, nondimensional blade rotation, slenderness ratio, and simple pendulum frequency. Flapwise and edgewise natural frequencies are calculated with respect to the slenderness ratio, blade pitch angle, blade speed and gravity. Until the slenderness ratio of 20, rotary inertia is effective and blade can be treated as Rayleigh beam but currently used wind turbine blades have much greater slenderness ratio than 20 that is why they can be treated as Euler-Bernoulli beam. The stiffness term of the natural frequency has two blade speed dependent elements. One is the inertial force created in the vibrational plane, the other is the centrifugal force created tension on the blade. For small pitch angles, in the flapwise vibration, the effect of the inertial force is smaller than the effect of the centrifugal force tension that is why the natural frequency increases with the increasing blade speed but for the edgewise vibration, the effect of

![Rotating Rayleigh beam first natural frequency](image)

**Figure 12.** Edgewise first natural frequency changes with respect to the pitch angle.

| Rating      | 5 MW  |
|-------------|-------|
| Rotor orientation, configuration | Upwind, 3 blades |
| Control | Variable speed, collective pitch |
| Drivetrain | High speed, multiple-stage gearbox |
| Rotor, hub diameter | 126, 3 m |
| Hub height | 90 m |
| Cut-in, rated, cut-out wind speeds | 3, 11.4, 25 m/s |
| Cut-in, rated rotor speeds | 6.9, 12.1 rpm |

Table 3. Gross properties of the NREL-5MW baseline wind turbine.
the inertial force is greater than the effect of the centrifugal force tension, that is why the edgewise natural frequency decreases with the increasing blade speed.

The effect of the change of pitch angle from $0^\circ$ to $15^\circ$ on the flapwise and edgewise natural frequencies are negligible up to the nondimensional blade speed of 4.

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**Table 4.** Modal natural frequencies calculated and given in Resor.\textsuperscript{21}

| Mode # | Calculated Frequency, Hz | Given in Resor\textsuperscript{21} Frequency, Hz | Description |
|--------|--------------------------|-----------------------------------------------|-------------|
| 1      | 1.0149                   | 0.87                                          | First flapwise bending |
| 2      | 0.9464                   | 1.06                                          | First edgewise bending |
| 3      | 6.2087                   | 2.68                                          | Second flapwise bending |
| 4      | 5.9544                   | 3.91                                          | Second edgewise bending |
| 5      | 17.0319                  | 5.57                                          | Third flapwise bending |
| 6      | 16.3868                  | Not given                                     | Third edgewise bending |

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**Figure 13.** Blade mass density plotted values and the line equation after curve fitting.

**Figure 14.** Blade flapwise stiffness plotted values and forth degree curve equation after curve fitting.

**Figure 15.** Blade edgewise stiffness plotted values and quadratic curve equation after curve fitting.

**Figure 16.** Flapwise blade tip vibration for only the first mode and for the sum of the first three modes.
Since the natural frequencies are the function of the blade speed, 1X rotor critical speed should be obtained from the Campbell diagram. Gravity adds to the tension created by the centrifugal force and also cyclic, but its effect on the natural frequencies are negligibly small (0.4% for the first natural frequency, much smaller for others). The blade tip vibrational response under the gravity and wind speed is studied for the NREL-5MW type wind turbine blade as an example. In the flapwise blade tip vibrations, gravitational and rotational effects are much greater than the effect of inflow wind while in the edgewise vibration gravitational effect is dominant and much more than the effect of rotation and inflow wind.

Combined modeling of the HAWT system including the tower vibration, blade inertia effect, the effect of footing geometry, foundation properties, rocking and translational flexibilities of the foundation can be further and more realistic study of the subject.

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