Direct numerical simulations of flow and heat transfer over a circular cylinder at \( Re = 2000 \)

M C Vidya\(^1\), N A Beishuizen\(^2\), T H van der Meer\(^1\)
\(^1\)Thermal Engineering, Faculty of Engineering Technology, University of Twente, 7500AE Enschede, Netherlands
\(^2\)Bosch Thermotechnology, Deventer, Netherlands

E-mail: m.c.vidya@utwente.nl

Abstract. Unsteady direct numerical simulations of the flow around a circular cylinder have been performed at \( Re = 2000 \). Both two-dimensional and three-dimensional simulations were validated with laminar cold flow simulations and experiments. Heat transfer simulations were carried out and the time-averaged local Nusselt number at the cylinder surface was obtained for various Reynolds numbers. Finally, the heat transfer of 2D and 3D simulations are compared. The average Nusselt numbers were found to be in accordance with empirical correlations. The 3D simulation gives a higher heat transfer due to the captured effects of motions in the spanwise direction compared to the 2D simulation. The irregular fluctuation of surface-averaged Nusselt number can be captured by the 3D simulation, while 2D simulation results show a regular fluctuation corresponding to the shedding from the cylinder, similar to that of a laminar flow.

1. Introduction

The flow around a circular cylinder has been a topic of extensive research since the 18\(^{th}\) century. The disturbed flow is characterized by the Reynolds number. Zdravkovich [1] divided the flow past a cylinder into five regimes, ranging from laminar flow to turbulent flow, with three different regimes of transitional flow. The laminar state ranges from \( Re = 0 \) up to approximately \( Re = 180 \). In this regime the flow is characterized by two-dimensional motion perpendicular to the cylinder axis (the von Karmann vortex shedding). At Reynolds higher than 180, a so-called transition-in-wake state of flow is defined by the unstable three-dimensional flow further downstream in the far wake. The instability mode of eddy formation is shown by the turbulent eddy roll up, thus changing the shedding mode from laminar flow to a more irregular flow downstream. This change is defined by the sudden drop in shedding frequency, represented by the dimensionless Strouhal number, \( St = fD/u \).

The second transition regime occurs between approximately \( Re = 350 \) to \( Re = 2 \times 10^5 \). In this regime the transition occurs in the free shear layer, a region close to the cylinder where the boundary layers continue to develop downstream. The onset of turbulence occurs in these layers, hence affecting the length and width of the near-wake flow. The transitional flow starts around \( Re = 350 \), marked by the oscillating motion of the free shear layers. With higher \( Re \), discrete eddies start to appear from the rolled-up oscillating free shear layers and then finally burst into turbulent eddies. The transition from the oscillating free shear layer into discrete eddies is not clearly defined, as it ranges from \( Re = 1000 \)

* To whom any correspondence should be addressed.
to $Re = 2000$ [1]. At $Re$ approximately $2 \times 10^4$ to $4 \times 10^4$, the turbulent eddies start to appear, which intensify with higher $Re$ up to $2 \times 10^5$, where a sudden drop of the drag coefficient and a jump of shedding frequency designate the third transition regime, namely the boundary layer transition regime. At $Re > 3.5 \times 10^6$ the flow becomes fully turbulent including the boundary layer.

This work focuses on the flow and heat transfer at $Re = 2000$. To the author’s knowledge, no detailed heat transfer simulation has been performed in this regime. In the same regime Kravchenko and Moin [2] have performed large eddy simulations at $Re = 3900$ for a cold flow. Wissink and Rodi [3] used the results of this work together with experimental results of Lourenco and Shih [4] at the same Reynolds number as a basic test case for their direct numerical simulations. They carried out simulations at $Re = 3300$, which are the closest to the current simulation as of the author’s knowledge. They investigated extensively the phase-averaged statistics to resolve the flow structure, as well as the turbulence kinetic energy contour and the effects of spanwise length. Other numerical simulations at lower Reynolds number have been performed by Mittal [5] using a time-dependent, three-dimensional model. His simulations at $Re = 1000$ suggest that the 2D results tend to overestimate the Strouhal number and mean drag coefficient compared to the 3D simulation.

One of the main aims of this work is to study the differences between 2D and 3D simulations at relatively low Reynolds number in terms of heat transfer and flow. The results can be used to determine whether a 2D simulation still yields useful results.

2. Governing equations

The Navier-Stokes equation for an arbitrary control volume $V$ with a surface area of $dA$ is formulated as follows:

$$\frac{\partial}{\partial t} \int_V W \, dV + \oint_{\partial V} (F - G) \cdot dA = \int_V S \, dV$$

where $S$ is the source terms and $W$, $F$ and $G$ are

$$W = \begin{pmatrix} \rho u \\ \rho v \\ \rho w \\ \rho E \end{pmatrix}, F = \begin{pmatrix} \rho v \\ \rho vu + p1 \\ \rho vv + p2 \\ \rho wv + pk \end{pmatrix}, G = \begin{pmatrix} 0 \\ \tau_{xi} \\ \tau_{yi} \\ \tau_{zi} \end{pmatrix}.$$ (2)

The heat transfer coefficient and subsequently, surface Nusselt number are calculated as follows:

$$h = \frac{q}{T_{wall} - T_{in}}, Nu = \frac{hD}{k}$$

(3)

The heat flux $q$ is computed as

$$q = h_f(T_{wall} - T_f)$$

(4)

where $h_f$ denotes the fluid-side local heat transfer coefficient.

The Nusselt number obtained from the simulations is then compared with the empirical correlations of Churchill and Bernstein [6], Hilpert [7], and Zukauskas [8] for flow around a cylinder. Churchill and Bernstein proposed a correlation for all $Re_D$ for $Pr \geq 0.2$, with all properties evaluated at film temperature, $(T_{in} + T_{cyl})/2$. 

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The correlation of Hilpert [7] is formulated in equation (6) for \( \text{Pr} \geq 0.7 \) with all properties evaluated at film temperature.

\[
\overline{N_u_D} = 0.3 + \frac{0.62\, \text{Re}^{1/2}_{D}\, \text{Pr}^{1/3}}{1 + \left( \frac{0.4\, \text{Pr}}{\text{Pr}^*} \right)^{3/4}} \left[ 1 + \left( \frac{\text{Re}_D}{282000} \right)^{5/8} \right]^{4/5}
\] (5)

The correlation of Hilpert [7] is shown in equation (6) for \( \text{Pr} \geq 0.7 \) with all properties evaluated at film temperature.

\[
\overline{N_u_D} = C\, \text{Re}^{m}_{D}\, \text{Pr}^{1/3}
\] (6)

Correlation by Zukauskas [8] is shown in equation (7) with all properties evaluated at \( T_{in} \) and \( \text{Pr}_s \) at the cylinder surface.

\[
\overline{N_u_D} = C\, \text{Re}^{m}_{D}\, \text{Pr}^{n} \left( \frac{\text{Pr}_s}{\text{Pr}_s} \right)^{1/4}
\] (7)

The constants \( C \) and \( m \) of Hilpert [7] and Zukauskas [8] are listed in table 1.

**Table 1.** List of constants for time-averaged Nusselt number correlations.

| Re     | \( C \) [7] | \( m \) [7] | \( C \) [8] | \( m \) [8] |
|--------|-------------|-------------|-------------|-------------|
| 0.4 – 1| 0.989       | 0.330       | 0.75        | 0.4         |
| 1 – 4  | 0.989       | 0.330       | 0.75        | 0.4         |
| 4 – 40 | 0.911       | 0.385       | 0.75        | 0.4         |
| 40 – 1000 | 0.683 | 0.466 | 0.51        | 0.5         |
| 1000 - 4000 | 0.683 | 0.466 | 0.26        | 0.6         |

3. **Numerical Set-Up**

3.1. **Two-dimensional domain**

The two-dimensional numerical domain is depicted in figure 1a. Two regions are created: region I that is close to the cylinder with a fine mesh and region II with a coarser mesh. The cylinder with diameter \( D \) is located 10 diameters from the inlet and 30 diameters from the outlet. A no slip boundary condition and temperature of 289.16 K is employed to the cylinder wall. Uniform velocity \( U_0 \) with temperature of 288.16 K is imposed at the inlet. It is assumed that the effects of temperature on the fluid properties are negligible, thus constant fluid properties are used. At the top and bottom boundaries of the domain, slip boundaries are used.

**Figure 1.** Computational domain for (a) 2D and (b) 3D simulations.
The two-dimensional simulation is performed using 1.4M quadrilateral elements. At the cylinder wall, 1600 elements are placed equidistantly with 500 inflation layers growing in the radial direction. The height of the inflation layer is chosen such that there are 5 layers inside the boundary layer. The front-stagnation-point boundary-layer displacement thickness is calculated according to Schlichting [9] formulation as follows:

$$\delta_{FSP} = \frac{0.67}{2Re^{1/2}}D$$

Based on the mesh study that have been carried out, the maximum size of the elements in region I is specified to be 0.1D, while the maximum size of elements in region II is 0.2D. The size of the smallest elements on the wall of the cylinder is 0.0005D.

### 3.2. Three-dimensional domain

The size of the three-dimensional domain is the same as the two-dimensional domain, with 10D length in the spanwise direction (figure 1b). This length was chosen based on the results of Wissink and Rodi [3] who suggest that the spanwise length should be longer than 8D to fully capture the motions in the spanwise direction. The three-dimensional domain consists of two regions with different mesh elements. Region I consists of coarse triangular mesh with hexahedral inflation elements surrounding the cylinder. Region II consists of hexahedral elements with face size of 0.002D and 0.004D length in the spanwise direction. This results in the total number of elements of 13.9M.

The structured and finer elements employed in this region are necessary to capture the velocity fluctuation of the three-dimensional flow at multiple points in the wake region, as well as the Kolmogorov length scale $\eta$.

$$\eta = \left( \frac{v^3}{\varepsilon} \right)^{1/4}$$

The Kolmogorov length scale for this simulation is in the order of 0.003D. The velocity fluctuation obtained at this region will be used for future work to construct the energy spectrum of the flow.

### 3.3. Numerical methodology

The numerical simulations have been performed using ANSYS FLUENT 16. The governing equations were discretized using the semi-implicit finite volume method [10]. The gradients were evaluated using the least squares cell-based method, and the second order upwind scheme was employed for the spatial discretization of the pressure term, momentum equation and energy equation. The transient term is discretized using a first order scheme. The time step applied in this simulation is $2 \times 10^{-3}D/U_0$.

### 4. Results and Discussion

#### 4.1. Validation

To validate the numerical model, a cold laminar flow was simulated. The Strouhal number is obtained as a function of Reynolds number and the results are plotted in figure 2. The results of 2D simulations are in the same order of the experimental results of Hammache and Gharib [11] and Williamson [12]. At Reynolds higher than approximately 170, the 2D simulation continues showing the increasing trend of the curve. At this Reynolds number, the flow becomes three dimensional and thus the sudden drop of the Strouhal number cannot be matched by the 2D simulation. The 3D simulation result is also in a
good agreement with literature, e.g. at $Re = 100$, the Strouhal number is found to be within 5% difference compared to both experimental values.

4.2. Results of heat transfer simulations for 2D laminar flow

The heat transfer simulation was performed with the 2D domain for low Reynolds number: 10, 45, and 100. The local Nusselt number is averaged over one period of vortex shedding and is plotted along the surface of the cylinder (figure 3). The results of 2D simulation are found to be in accordance with numerical simulations of Bharti [13] at $Re = 10$ and 45.

The mean Nusselt number increases with increasing $Re$ and the local Nusselt number $Nu(\theta)$ has a maximum at the front stagnation point. This corresponds well with the trend of increasing $Nu$ with higher $Re$ from literatures. A local minimum value indicates the separation point. The location of the separation point moves towards the front of the cylinder with increasing Reynolds number [13]. Simulations at $Re = 45$ and $Re = 100$ are in a good agreement with this trend, showing the separation point at $\theta = 137.9^\circ$ and $\theta = 136.4^\circ$ subsequently.

4.3. Comparison of 2D and 3D simulation at $Re = 2000$

The mesh resolution of 3D and 3D domain is shown in figure 4 and 5 consecutively. In figure 4 the axial velocity is plotted for ten axial locations in the longitudinal center plane. The normalized results are compared with the one of Wissink and Rodi [3] at $Re = 3300$. As investigated by Wissink and Rodi, an insufficient mesh will result in a V-shaped profile close to the cylinder. Although both simulations are different in Reynolds numbers, the velocity profiles are similar. The U-shaped profile for the line closest to the cylinder indicates that the current numerical simulation has enough mesh resolution. The 2D domain also shows enough mesh resolution as depicted in figure 5. Two separation points are captured; these cause the formation of secondary vortices at the free shear layer (red arrows).

The 2D simulation cannot capture the effects of spanwise motion, as shown in figure 7. The instantaneous vorticity fields at $t = 1049U_0/D$ are compared. The 2D simulation results in a regular
Figure 4. Profile of \( u \)-velocity in 10 lines, taken at the center plane of the cylinder. Solid lines represent current simulations; dashed lines represent results of Wissink and Rodi at \( Re = 3300 \) [3].

Figure 5. Instantaneous vorticity field of the 2D simulation. Separation points and secondary structures (red arrows) are visible.

Figure 6. Time-averaged surface Nusselt number of current simulations compared to empirical correlations.

Figure 7. Instantaneous flow field and surface-averaged Nusselt number for (a) 2D simulation and (b) 3D simulation.
flow pattern instead of uneven eddies. Moreover, the widening of the wake region is immediately visible in the near wake from the 3D simulation. Accordingly, the surface-averaged Nusselt number at the cylinder wall is plotted as a function of flow time and the results correspond well with the flow field. As the turbulence flow is captured by the 3D simulation, the surface-averaged Nusselt number fluctuates irregularly. On the contrary, the 2D simulation shows a regular oscillation of surface-averaged Nusselt number due to the regular shedding of the vortices. The time-averaged surface Nusselt numbers from both simulations are computed and the results are plotted in figure 6. The 3D simulation gives a higher heat transfer compared to 2D flow due to the higher turbulence of the flow and this is within 5% difference compared to the empirical correlation of Zukauskas [8].

The development of local Nusselt number along the cylinder surface is shown in figure 8. From figure 8a at \( t = 0.25T \) and approximately \( \theta = 240^\circ \), a local maximum indicates that the heat transfer at the back of the cylinder is enhanced by the lower recirculating vortex while its pair is shed from the upper part of the cylinder. Half a period later, the local maximum at \( \theta = 120^\circ \) indicates that the recirculating vortex now appears at the upper part of the cylinder while the lower vortex is shed. On the contrary, the 3D simulation does not show a regular pattern of local Nusselt number due to the influence of the spanwise motions. Figure 8b shows the local Nusselt number taken at plane \( Z = 0 \). After one period, the local Nusselt profile at \( \theta = 90^\circ \) to \( \theta = 270^\circ \) does not repeat the same pattern. From the front stagnation point (\( \theta = 0^\circ \)) to \( \theta = 90^\circ \) and \( \theta = 270^\circ \), there are no significant differences between 2D and 3D simulations. However, at \( \theta = 90^\circ \) to \( \theta = 270^\circ \) the 3D turbulent eddies at the back of the cylinder causes the change of local Nusselt number with time. Thus, the 3D simulation results in a higher time-averaged Nusselt number.

![Figure 8](image_url)

**Figure 8.** Time-averaged local Nusselt number as a function of \( \theta \) for (a) 2D simulation and (b) 3D simulation. \( \theta = 0^\circ \) corresponds to the front stagnation point.

The instantaneous vorticity fields from the 3D simulations are shown in figure 9. In the first frame the vortex at the bottom of the cylinder is being shed. The subsequent frame depicts the formation of the upper vortex while the lower vortex shrinks. In the next image, the upper vortex is being shed and finally shrinks in the following frame. The movement in the spanwise direction is also visible as an undulation of the curling eddies attached to the cylinder wall. The shedding of the von Karman vortex street is still visible in each frame, marked by eddy filaments rolling in a form of sheets parallel to the cylinder axis.
5. Conclusion
2D and 3D simulation have been performed for laminar and transitional flow. Both cold flow and hot flow simulations were found to be in accordance with literatures. Compared to 2D simulation, the time-averaged Nusselt number in the 3D simulation is higher due to the turbulence development and this value is within 5% difference of empirical correlations. Thus, a two-dimensional domain can be used to simulate heat transfer case at $Re = 2000$ when engineering accuracy is sufficient. However, at $Re = 2000$ the flow is irregular due to the shedding in the spanwise direction. These movements can be captured well by the 3D simulation and results in a fluctuation of surface-averaged Nusselt number over time. Thus, to perform a detailed statistical flow analysis, a 3D simulation is of a paramount importance.

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