A Hybrid Sine Cosine Algorithm with SQP for Solving Convex and Nonconvex Economic Dispatch Problem

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RECEIVED ON 29.04.2018, ACCEPTED ON 30.10.2018

ABSTRACT

ED (Economic Dispatch) is one of the major problems of power system operation. The aim of ED problem is the efficient utilization of resources to provide the demanded power while generating cost turns out to be minimum and no constraint is violated either equality or inequality. The ED optimization problem is necessary because of limited resources, high fuel cost and ever growing demand of power. This paper presents solution to convex and nonconvex ED problems using a novel HSCA (Hybrid Sine Cosine Algorithm). The proposed HSCA technique enhances the exploration capabilities of SCA (Sine Cosine Algorithm) by equipping it with mutation and crossover operators from DE (Differential Evolution) algorithm. DE algorithm introduces diversity in the operation of SCA enabling it to avoid local minima and premature convergence. To ensure precise and accurate optimum tracking results are finally refined by SQP (Sequential Quadratic Programming) algorithm. The high feasibility and applicability of proposed technique has been tested and validated on 13, 15 and 40 IEEE Standard test systems considering transmission losses and prohibited operating zones in “MATLAB 2014a”. Comparisons of results obtained from HSCA indicate significant improvement in convergence time and fuel cost as compared to the techniques reported in the literature.

Key Words: Differential Evolution, Economic Dispatch, Sine Cosine Algorithm, Population Based Algorithm, Economic Load Dispatch.

1. INTRODUCTION

There has seen soaring competition in the electricity market propelled by de-regularization of industry and the rampant electricity demand since last decade. In this scenario of ever increasing demand the electricity producers are left with the daunting task of meeting this demand at least cost to maximize profit. This herculean task of maximizing profits while meeting desired demand is termed as ED of the electric power systems. The main aim of ED problem is to assign power outputs to generation units, at a specified time interval, in an economic way, while taking into account different physical and operational constraints. Conventionally, fuel cost characteristics of thermal generators can be approximated by quadratic equations but the factual fuel cost characteristics of the fossil fuel-fired plants exhibit a large number of nonlinearities due to VPLE (Valve-Point Loading Effect) and POZs (Prohibited Operating Zones) [1]. The valve-point loading effect is
due to multiple valve operations in thermal plants to change the fuel type according to the generation. And the valve operation makes the characteristic curve of the generating unit nonlinear and discontinuous. This effect is modeled [2] in the characteristic curve as cyclical sinusoidal function as shown in Fig. 1(a).

The discontinuity due to POZs in the fuel cost characteristics may be due to the resonant frequency created by the addition of all the frequencies of different parts of the machine, or due to the variation in shaft bearing affected by the valve operation or due to the faults in the auxiliary equipment of machine like boiler and feed pumps etc. Generally, the determination of prohibited zones is a difficult procedure accompanied by performance tests. In practical operation, the generation in these regions (POZs) is avoided as it would not be economical to use these zones of operation [3-4]. The objective function of cost which considers this constraint (POZs) is shown in Fig. 1(b).

To tackle with the ED problem various approaches have been proposed in literature so far. These proposed methods can be classified as classical and non-classical. In classical methods, Base-Point and Participation Factors, $\lambda$-iteration method [5], linear programming, quadratic programming [6], branch and bound [7], gradient method [8] were included. These methods were able to compute the ED successfully but because of the discontinuities and nonlinearities introduced by valve-point loading and the POZs, the gradient-based methods were unsuccessful in achieving an optimum solution. The strength of classical methods was demonstrated for continuous and smooth objective functions but the difficulty faced in solution to discontinuous and non-convex problems established the need for further improvement. Many improvements in the classical methods were made in the last decade but these improvements required additional computations to address these complications.

On the other hand, Non-classical meta-heuristic techniques are auspicious alternative to deal with the complexities involved in ED problems. EP (Evolutionary Programming) [9-10], GA (Genetic Algorithm) [2], HNN (Hopfield Neural Network) [11], ITS (Improved Tabu Search) [1], SA (Simulated Annealing) [12-14], DE [15-17], Continuous Quick Group Search Optimizer [18], ACO (Ant Colony Optimization) [19], BBO (Biogeography-Based Optimization) [20], etc. have been developed and implemented on ED successfully.
Although AI (Artificial Intelligence) techniques are not generally able to achieve the global solution still they can be used to get a feasible sub-optimal solution. Latest AI techniques used to solve the corresponding ED problem are SKH (Stud Krill Herd) [21], CPSO (Chaotic Particle Swarm Optimization) [22], CSA [23], IODPSO (Improved Orthogonal Design Particle Swarm Optimization) [24], MVMO (Mean Variance based Mapping Optimization) technique [25], LFA (Lightening Flash Algorithm) [26], GSO (Glowworm Swarm Optimization) [27], IFWA-CSO (Improved Fireworks Algorithm with Chaotic Sequence Operator) [28], ALO (Ant Lion Optimizer) [29], and CKH Algorithm [30]. The basic technique of SCA was proposed by Mirjalili [31]. All these techniques show remarkable results and convergence characteristics in their own rights but fail to perform optimally for all cases. Some drawbacks related to these evolutionary techniques are high computational time, curse of dimensionality, computational complexity, slow convergence rate and sensitivity to control parameters. This inability of algorithms to perform optimally for all cases is also summarized by No Free Lunch Theorem. To perform optimally for all cases we need either to modify our technique to suite the problem or find a new strategy to tackle the issue altogether. SCAs are no exception to this rule. To improve the capabilities of SCA several other techniques were considered such as PSO (Particle Swarm Optimization), MPC (Multi-Parent Crossover) and SEA (Simple Evolutionary Algorithm) but DE was eventually deemed suitable. DE was selected because of its mathematical ease of not having to compute derivative, less number of control parameters, robustness, same setting for almost all types of problems [32-33] and strong convergence characteristics equipped with diverse searching mechanism. Finally, to overcome the issue of precision in global optimum tracking the answer was refined by SQP resulting in a novel approach termed HSCA.

In this paper, the proposed novel meta-heuristic technique named “HSCA with SQP” has been employed to elucidate the ED problem.

2. PROBLEM FORMULATION

This section describes the formulation of ED problem.

2.1 Description of ED Problem

ED has been one of the most important power system operational planning problems. The limited energy resources and increasing cost of thermal power generation accompanied by ever-growing demand of power has necessitated the implementation of ED on the present power system. The main target of ED problem is to deal with assigning of optimized generation value from the available resources while considering the constraints associated with the system.

The target of ED is to minimize the cost which is mathematically denoted as:

\[ FC = \sum_{i=1}^{n} F_i(P_i) \]  

(1)

While, \( FC \) represent the total cost of fuel and the \( F_i(P_i) \) represents the cost of \( i^{th} \) the unit and “n” gives the total number of units. The expression for cost of fuel is represented as:

\[ F_i(P_i) = a_i P_i^2 + b_i P_i + c_i \]  

(2)

Where \( P_i \) gives the real power engendered from the \( i^{th} \) unit and \( a_i \) is the fuel-cost coefficient for that specified unit in [$/MW^2h]. Similarly, \( b_i \) and \( c_i \) are also fuel cost coefficients in [$/MWh] and [$/h] respectively.

The Equation (2) is a reasonable approximation of thermal plants but in actual thermal generation there are multiple
valves present to control the output power. As these valves are opened there is an abrupt increase in losses which creates a rippling effect in the characteristic curve making the fuel-cost plot non-smooth (Non-convex). This VPLE when taken into consideration makes the cost function non-smooth which can be expressed as a combination of quadratic and sine expression as:

\[ F_i(P_i) = a_i P_i^2 + b_i P_i + c_i + d_i \times \sin\left( e_i (P_i^{\text{min}} + P_i) \right) \]  

(3)

While "i" represents the committed generating unit and "d" and "e" represent the valve point loading coefficients.

2.2 Constraints

ED problem is about the minimization of fuel cost using fuel cost function (either convex or nonconvex) subjected to different constraints as follows:

2.2.1 Power Balance Constraint

\[ \sum_{i=1}^{m} P_i = P_D + P_{\text{Loss}} \]  

(4)

Where the \( P_{\text{Loss}} \) represents the transmission loss and can be mathematically expressed by the use of B-coefficients as:

\[ P_{\text{Loss}} = \sum_{i=1}^{m} \sum_{j=1}^{n} P_i B_{ij} P_j + \sum_{i=1}^{m} B_{i0} P_i + B_{00} \]  

(5)

While, \( P_D \) denotes total load demand and \( B_{ij}, B_{i0}, \) and \( B_{00} \) are the B-coefficients.

2.2.2 Capacity Limit Constraint

The real power generated from each generation unit must not violate its max \( (P_i^{\text{max}}) \) and \( (P_i^{\text{min}}) \) limit.

Mathematically,

\[ P_i^{\text{min}} \leq P_i \leq P_i^{\text{max}}, \quad i \in N \]  

(6)

2.2.3 Prohibited Operating Zones

There are some regions in the range of operation of generating unit in which the system may lose stability if operated in these specified regions due to synchronization of all the components of the generating unit. These regions are named as prohibited operating zones. And are therefore avoided in practical generations.

Mathematically these are represented as follows:

\[ P_i \in \left\{ \begin{array}{l} P_i^{\text{min}} \leq P_i \leq P_i^u \quad \text{for } i = 1, 2, \ldots, n_{\text{POZ}} \\ P_i^{l,1} \leq P_i \leq P_i^{l,2} \\ \ldots \\ P_i^{l,n} \leq P_i \leq P_i^{l,\text{max}} \end{array} \right. \]  

(7)

Where the \( P_i^u \) and \( P_i^l \) represent the maximum and minimum limits of nth prohibited zone and "n" represent the number of prohibited zone. The POZs cause the function to be discontinuous.

3. Proposed Hybrid Sine Cosine Algorithm

Population-based algorithms reported in literature start to optimize the problem using a set of population generated randomly, each of these randomly populated solutions are then evaluated to compute the objective function and updated by a procedure specific to the respective technique which basically constitutes the core of an optimization algorithm. According to literature, the algorithms which are based on population are far more suitable for optimization problems as they start from a set of random solutions and naturally avoid being stuck into the local optimum as compared to individual based algorithms. These characteristics enable population-based algorithms to explore (global search)
In more detail, the influence of CSA trigonometric functions in the specified range as in Fig. 2 i.e. [-2,2] is explained in Fig. 3. Fig. 3 clearly describes the effect of altering the range of cosine and sine that how this change necessitates an answer to update its next location inside or outside of the search space in between itself and next solution.

As we have discussed the performance measures of SCA, the technique has been hybridized with DE and SQP using the following steps to solve the concerned ED problem.

**Step-1: Initialization:** First, the population is initialized randomly in the lower and upper limit of generating units according to the following equation.

$$P = P_{\text{min}} + \text{rand.}(P_{\text{max}} - P_{\text{min}})$$  \hspace{1cm} (8)

**Step-2: Evaluation of Fitness Function:** The population is updated according to the CSA equations as stated below:

$$F_i(P_i) = a_i P_i^2 + b_i P_i + c_i + \left| d_i \times \sin \left( e_i \left( P_i^{\text{min}} + P_i \right) \right) \right|$$  \hspace{1cm} (9)
Step-3: Update the Population According to SCA: The population is updated according to the CSA Equations (10-11) as stated below:

\[ P_{i}^{t+1} = P + r_1 \times \sin(r_2) \times \bigg| p_{j_{best}}^i - P_{i}^r \bigg|, \quad r_3 < 0.5 \]  \hspace{1cm} (10)  

\[ P_{i}^{t+1} = P + r_1 \times \cos(r_2) \times \bigg| p_{j_{best}}^i - P_{i}^r \bigg|, \quad r_4 \geq 0.5 \]  \hspace{1cm} (11)

As Equations (10-11) show, there are four random parameters \( r_1, r_2, r_3, \) and \( r_4. \)

The first random parameter \( r_1 \) prescribes the direction of movement which could be outside the destination and solution or inside it. This random parameter adaptively changes the range of cosine and sine functions to balance between exploration and exploitation using the equation as:

\[ r_1 = a - \frac{a}{T} \]  \hspace{1cm} (12)

Where “\( t \)” denotes the present iteration and “\( T \)” represents the max number of iteration and “\( a \)” is a constant. The second random parameter “\( r_2 \)” emphasizes that how long next movement would be towards or outwards the destination point. This movement inside or outside the search space is obtained by defining the random parameter “\( r_2 \)” in the range \([0, 2\pi]\). The third random parameter “\( r_3 \)” is the random weight for the destination point distance stochastically. And the fourth random parameter “\( r_4 \)” is the switch between the sine and cosine function to be used for updating the population.

Step-4: Application of Mutation and Crossover: By applying the crossover and mutation, the updated population is modified further to introduce diversity and avoid local optima stagnation.

Mutation: In the mutation operator, in the current population, a mutated vector is generated for each targeted vector except for the running index.

\[ u_{i,g+1} = x_{i,g} + SF (x_{r_1,g} - x_{r_2,g}) \]  \hspace{1cm} (13)

Where \( r_1 \) and \( r_2 \) are the randomly initialized integers dissimilar to each other and from the running index “\( i \)” too. And SF is the scaling factor which is a randomly engendered number between \((0,1)\) but may not be homogeneously disseminated in the range of \((0,1)\). Generally, the choice for the SF is between 0.4 and 1 but here in the proposed work, the dynamic behavior to the SF is applied by varying SF in a randomly generated number in the range of \((0.5,0)\) [34]. Here in this case due to the randomly scaled difference vector, there is a better chance of pointing the new mutated vector at the even better location until the true global optimum is achieved.

Crossover: After the application of mutation operator, the crossover operation is applied to upsurge the diversity of individuals. The previously generated vector is assorted with the vector produced after mutation to form a new trial vector \( w_{i,g+1} \):

\[ w_{i,j,g+1} = \begin{cases}  
  y_{i,j,g+1} & \text{if} \ \text{rand}(0,1) \leq C_r \forall j = m \\
  z_{i,j,g+1} & \text{Otherwise} 
\end{cases} \]  \hspace{1cm} (14)

Where \( i=1...N_p \) and \( j=1...Q; \)

And “\( m \)” is a random number elected between 1 and \( N_p \) which makes sure that the new trial vector takes the smallest parameter from the mutant vector. And \( C_r \) is the crossover parameter defined by the user and reins the diversity of the individuals and helps the solution to avoid from local minimum [34].
Step-5: Verification of Equality and Inequality Constraints: Check if the active power of any of the generating unit violates the limit or not. In case, power generation of any unit is less than the minimum limit then it is fixed at the minimum limit and if power generation of any unit exceeds the maximum limit then it is clamped at maximum limit. After the satisfaction of inequality constraint, the equality constraint is verified.

Step-6: Application of Sequential Quadratic Programming: After the completion of constraints handling, the SQP is applied to the best results obtained so far from the proposed algorithm.

Sequential Quadratic Programming: SQP is actually not a feasible point or heuristic method. But this turns out to be mathematical technique and it uses the feasible solution of a sub-problem to find a better solution to the major problem. Seemingly, SQP performs closely to the Newton and Quasi-Newton methods. The only thing which makes SQP different from Newton and Quasi-Newton is the handling of constraints [35].

In the SQP the QP is solved in each step to improve line search which can be mathematically defined as:

Minimize
\[ \nabla^2 F(P_k) \nabla q_k - H_k d_k \]

Subject to
\[ g_i(P_k) + \nabla g_i(P_k) d_k = 0 \]
\[ i = 1 \ldots m \]
\[ g_i(P_k) + \nabla g_i(P_k) d_k \leq 0 \]
\[ i = m + 1 \ldots m \]

Here, \( H_k \) is Hessian Matrix and \( d_k \) is direction search, \( P_k \) is the real power vector and \( g_i(P_k) \) represents the equality and inequality constraints in current iteration which is represented by “k”. And \( n \) is the equality constraint’s quantity while “m” represents quantity of constraints.

\[ L(P_k, \lambda) = F(P) + \nabla g(P) \lambda \]  
(18)

Where \( \lambda \) represents the Lagrangian multiplier and \( H_k \) is formulated with the help of Quasi-Newton.

\[ H_{k+1} = H_k + q_k q_k^T H_k S_k H_k^T S_k H_k \]

(19)

And,

\[ S_k = P_{k+1} - P_k \]

(20)

\[ q_k = \nabla L(P_{k+1}, \lambda_{k+1}) - \nabla L(P_k, \lambda_k) \]

(21)

In each iteration, the direction of sub-problem is achieved and the new solution is found from new iteration as follows:

\[ P_{k+1} = P_k - \alpha d_k \]

(22)

For calculation of step length (\( \alpha_k \)) the following formula is used as this step length is significant for the decrease in the Lagrangian merit function.

\[ L_{\lambda}(P, \lambda, \rho) = F(P) - \lambda^T g(P) + \frac{\rho}{2} g(P)^T g(P) \]

(23)

Step-7: Stopping Criteria: The same procedure from step-2 is repeated until the stopping criteria are achieved. The SCA stops further computations if there comes no noticeable improvement or a maximum number of iteration is completed. In this work, the stopping criteria are maximum number of iterations.
4. EXPERIMENTAL METHODS AND RESULTS

The proposed HSCA is implemented to three ED problems to verify its feasibility. These three ED problems refer to three types of test systems to validate the proposed technique. The test systems differ from each other with respect to the number of generating units, rippling effect which is generally termed as the VPLE, inclusion or exclusion of transmission losses and as well as POZs.

**Test System-1:** 13-unit nonconvex (with VPLE) test system without transmission losses.

**Test System-2:** 15-unit convex test system (without VPLE) considering transmission losses and POZs.

**Test System-3:** 40-unit nonconvex test system without transmission losses.

### 4.1 Test System-1

The load demand for 13-unit test system is 2520 MW. The input data consisting of load demand, cost coefficients and generator limits are taken from [5].

#### 4.1.1 Parameters

The Table 1 shows the parameters of mutation, crossover, and others used to optimize the results of 13-unit test systems.

Table 2 shows the individual cost of each unit and total power corresponding to total fuel cost. Total power output comes to be 2520 MW and total fuel cost is 24164.1 US$/hr.

| Iteration | 50 | Search Agents | 25 |
|-----------|----|---------------|----|
| Trials    | 40 | Mutation Scaling Factor | 0.0101 |
| Total Number of Computations | 2000 | Crossover Probability | 0.2 |

**TABLE 1. LIST OF PARAMETERS TAKEN FOR TEST SYSTEM-I**

| Unit Number | \( P_{\min} \) (MW) | \( P_{\max} \) (MW) | Generation (MW) | Fuel Cost (US$/hr) |
|-------------|---------------------|---------------------|-----------------|-------------------|
| 1           | 0                   | 680                 | 628.3184        | 5749.9            |
| 2           | 0                   | 360                 | 299.1988        | 2782.6            |
| 3           | 0                   | 360                 | 294.4871        | 2770.4            |
| 4           | 60                  | 180                 | 159.7327        | 1559              |
| 5           | 60                  | 180                 | 159.733         | 1559              |
| 6           | 60                  | 180                 | 159.7331        | 1559              |
| 7           | 60                  | 180                 | 159.7328        | 1559              |
| 8           | 60                  | 180                 | 159.7331        | 1559              |
| 9           | 60                  | 180                 | 159.7329        | 1559              |
| 10          | 40                  | 120                 | 77.39937        | 808.656           |
| 11          | 40                  | 120                 | 77.39962        | 808.658           |
| 12          | 55                  | 120                 | 92.39917        | 944.889           |
| 13          | 55                  | 120                 | 92.3999         | 944.884           |
| Total       |                    |                     | 2520            | 24164.6           |

**TABLE 2. POWER OUTPUT OF INDIVIDUAL UNIT AND CORRESPONDING COST FOR TEST SYSTEM-I**
4.1.2 Results

In Table 3 there is the statistical comparison of the proposed algorithm with other techniques and clearly shows that SCA minimizes the fuel cost as compared to other techniques mentioned in Table 3.

Fig. 4 demonstrates the convergence curve of 13-unit test system solved using SCA and clearly describes that the system converges to optimum before stopping criteria.

In Fig. 5 the fuel cost of 13-unit test system is compared using a bar chart to simplify the analysis. Fig. 5 shows that lowest cost is obtained by using the proposed HSCA which in this case is 24164.06 US$/hr.

The results of 13 unit-test system show that the proposed technique owns significantly lesser fuel cost as compared to the former techniques conveyed in the literature. The proposed technique saves about 5 $/hr as compared to HGA (Hybrid Genetic Algorithm), DE and FAPSO-VDE (Fuzzy Adaptive Particle Swarm Optimization with Variable Differential Evolution).

### Table 3. Statistical Comparison of 13 Unit Test System

| Methods          | Total Generating Cost (US$/hr) | Minimum Cost | Maximum Cost | Average Cost |
|------------------|--------------------------------|--------------|--------------|--------------|
| IPSO-TVAC [36]   |                                | 24166.80     | 24167.37     | 24169.41     |
| ICA-PSO [37]     |                                | 24168.91     | 24175.34     | 24184.92     |
| HGA [38]         |                                | 24169.9177   | NA (Not Available) | NA(Not Available) |
| DE [15]          |                                | 24169.9177   | NA (Not Available) | NA(Not Available) |
| FAPSO-VDE [39]   |                                | 24169.9176   | 24169.9176   | 24169.9176   |
| DE [40]          |                                | 24165.6767   | 24169.5008   | 24172.0944   |
| CRO [40]         |                                | 24165.1664   | 24166.9355   | 24169.3642   |
| CKH [30]         |                                | 24164.1815   | 24166.11     | 24168.43     |
| Proposed HSCA    |                                | 24164.06     | 25015.26     | 24583.25     |

![Image of Table 3](image)
4.2 Test System-2

The load demand for 15-unit test system is 2630 MW. The input data consisting of load demand, cost coefficients and generator limits are taken from [27]. The transmission losses are calculated using B-coefficients method.

4.2.1 Parameters

Table 4 shows the mutation, crossover and other parameters which are taken to optimize the results of 15-unit test system.

4.2.2 Results

Table 5 shows the individual cost of each unit and total power corresponding to total fuel cost. Total power output comes to be 2656.689 MW with transmission losses of 26.6956 MW and total fuel cost is 32548.03 US$/hr.

In Table 6 there is the statistical comparison of the proposed algorithm with other techniques and clearly shows that SCA minimizes the fuel cost as compared to other techniques mentioned in Table 6.

Fig. 6 shows the convergence curve of 15-unit test system solved using SCA and clearly describes that the system converges to optimum before stopping criteria.

In Fig. 7 cost of fuel for 15-unit system is compared using a bar chart to simplify the analysis. Fig. 7 shows that the lowest cost is acquired by using the proposed “HSCA with SQP” which in this case is 32548.03 US$/hr.

The results of 15-unit system show that the proposed technique owns significantly lesser fuel cost as compared to the other techniques conveyed in the literature. The proposed technique saves about 309 and 156 US$/hr as compared to PSO and IPSO respectively.

![Figure 5: Comparison of fuel cost for 13 unit test system with the demand of 2520 MW](image)

**TABLE 4. LIST OF PARAMETERS TAKEN FOR TEST SYSTEM-II**

| Parameter                        | Value   |
|----------------------------------|---------|
| Iteration                        | 50      |
| Search Agents                    | 25      |
| Trials                           | 40      |
| Mutation Scaling Factor          | 0.000987|
| Crossover Probability            | 0.2345  |
| Total Number of Computations     | 2000    |
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### TABLE 5. POWER OUTPUT OF INDIVIDUAL UNIT AND CORRESPONDING COST FOR TEST SYSTEM-II

| Unit Number | $P_{\text{min}}$ (MW) | $P_{\text{max}}$ (MW) | Generation (MW) | Fuel Cost (US$/hr) |
|-------------|------------------------|------------------------|-----------------|--------------------|
| 1           | 150                    | 455                    | 455             | 5328.4             |
| 2           | 150                    | 455                    | 455             | 5252.9             |
| 3           | 20                     | 130                    | 130             | 1537               |
| 4           | 20                     | 130                    | 130             | 1537               |
| 5           | 150                    | 470                    | 228.3601        | 2846.6             |
| 6           | 135                    | 460                    | 460             | 5339.7             |
| 7           | 135                    | 465                    | 465             | 5183.7             |
| 8           | 60                     | 300                    | 60              | 900.2168           |
| 9           | 25                     | 162                    | 25              | 453.5044           |
| 10          | 25                     | 160                    | 36.7366         | 569.7055           |
| 11          | 20                     | 80                     | 76.5918         | 988.273            |
| 12          | 20                     | 80                     | 80              | 1057.3             |
| 13          | 25                     | 85                     | 25              | 552.7319           |
| 14          | 15                     | 55                     | 15              | 490.934            |
| 15          | 15                     | 55                     | 15              | 510.0006           |
| Total       |                        |                        |                 | 2656.689           |

| Transmission Losses | 26.6956 |

### TABLE 6. STATISTICAL COMPARISON OF 15 UNIT TEST SYSTEM

| Methods       | Minimum Cost   | Maximum Cost   | Average Cost   |
|---------------|----------------|----------------|----------------|
| IPSO [10]     | 32704.4514     | 32704.4514     | 32704.4514     |
| PSO[41]       | 32858.00       | 33331          | 32039          |
| KGMO[42]      | 32548.1736     | 32548.3755     | 32548.2163     |
| Proposed HSCA | 32548.03       | 32574.54       | 32555.1        |

**FIG. 6. CONVERGENCE CURVE FOR TEST SYSTEM-2**
4.3 Test System-3

The load demand for 40-unit test system is 10500 MW. The input data consisting of load demand, cost coefficients and generator limits are taken from [43].

4.3.1 Parameters

The Table 7 shows the mutation, crossover and other parameters used to optimize the results of 40-unit test system.

4.3.2 Results

Table 8 shows the individual cost of each unit and total power corresponding to total fuel cost. Total power output comes to be 10500 MW and total fuel cost is 121983.5 $/hr.

And, in Table 9 there is the statistical comparison of the proposed algorithm with other techniques and clearly shows that SCA minimizes the fuel cost as compared to other techniques mentioned in Table 9.

Fig. 8 shows the convergence curve of 40-unit test system solved using SCA and clearly describes that the system converges to optimum before stopping criteria.

In Fig. 9 the fuel cost of 40-unit test system is compared using a bar chart to simplify the analysis. Fig. 9 shows that lowest cost is obtained by using the proposed HSCA which in this case is 121983.5 US$/hr.

The results of 40-unit system reveal that the offered technique owns significantly lesser fuel cost as compared to the other techniques conveyed in the literature. The proposed technique saves about 640, 269 and 138 US$/hr as compared to IFEP (Improved Fast Evolutionary Programming), MPSO (Modified Particle Swarm Optimization) and Efficient ESO (Evolutionary Strategy Optimization) respectively.

![15 Unit Test System](image)

**FIG. 7. COMPARISON OF FUEL COST FOR 15 UNIT TEST SYSTEM CONSIDERING LOSSES AND PROHIBITED OPERATING ZONE WITH THE DEMAND OF 2630 MW**

| Iteration | 50  | Search Agents | 25  |
|-----------|-----|---------------|-----|
| Trial     | 40  | Mutation Scaling Factor | 0.0101 |
| Total Number Computations | 2000 | Crossover Probability | 0.2 |

**TABLE 7. LIST OF PARAMETERS TAKEN FOR TEST SYSTEM-I**
## TABLE 8. POWER OUTPUT OF INDIVIDUAL UNIT AND CORRESPONDING COST FOR TEST SYSTEM-III

| Unit Number | \( P_{\text{min}} \) (MW) | \( P_{\text{max}} \) (MW) | Generation (MW) | Fuel Cost (US$/hr) |
|-------------|----------------|----------------|----------------|-------------------|
| 1           | 36             | 114            | 110.7996       | 925.0964          |
| 2           | 36             | 114            | 110.7998       | 925.0964          |
| 3           | 60             | 120            | 120            | 1544.7            |
| 4           | 80             | 190            | 179.7333       | 2143.6            |
| 5           | 47             | 97             | 87.7999        | 706.5002          |
| 6           | 68             | 140            | 140            | 1596.5            |
| 7           | 110            | 300            | 299.8777       | 3296.3            |
| 8           | 135            | 300            | 284.5997       | 2779.8            |
| 9           | 135            | 300            | 284.5995       | 2798.2            |
| 10          | 130            | 300            | 130            | 2502.1            |
| 11          | 94             | 375            | 243.5994       | 4083.2            |
| 12          | 94             | 375            | 168.7995       | 2977.5            |
| 13          | 125            | 500            | 214.7596       | 3792.1            |
| 14          | 125            | 500            | 214.7596       | 4005.7            |
| 15          | 125            | 500            | 394.2794       | 6436.6            |
| 16          | 125            | 500            | 304.5192       | 5171.2            |
| 17          | 220            | 500            | 480.2793       | 5296.7            |
| 18          | 220            | 550            | 550            | 6224.1            |
| 19          | 242            | 550            | 511.2793       | 5540.9            |
| 20          | 242            | 550            | 331.7598       | 3636.4            |
| 21          | 254            | 550            | 523.2793       | 5071.3            |
| 22          | 254            | 550            | 523.2792       | 5071.3            |
| 23          | 254            | 550            | 523.2793       | 5072.2            |
| 24          | 254            | 550            | 523.2794       | 5072.2            |
| 25          | 254            | 550            | 523.2793       | 5275.1            |
| 26          | 254            | 550            | 523.2794       | 5275.1            |
| 27          | 10             | 150            | 10             | 1140.5            |
| 28          | 10             | 150            | 10             | 1140.5            |
| 29          | 10             | 150            | 10             | 1140.5            |
| 30          | 47             | 97             | 87.8003        | 706.5062          |
| 31          | 60             | 190            | 190            | 1644              |
| 32          | 60             | 190            | 190            | 1644              |
| 33          | 60             | 190            | 190            | 1644              |
| 34          | 90             | 220            | 220            | 2228.4            |
| 35          | 90             | 220            | 220            | 2164.5            |
| 36          | 90             | 220            | 220            | 2164.5            |
| 37          | 25             | 110            | 110            | 1220.2            |
| 38          | 25             | 110            | 110            | 1220.2            |
| 39          | 25             | 110            | 110            | 1220.2            |
| 40          | 242            | 550            | 511.279        | 55409             |
| **Total**   |                |                | **10500**      | **121,983.5**     |
A Hybrid Sine Cosine Algorithm with SQP for Solving Convex and Nonconvex Economic Dispatch Problem

TABLE 9. STATISTICAL COMPARISON OF 40 UNIT TEST SYSTEM

| Methods      | Minimum Cost | Maximum Cost | Average Cost |
|--------------|--------------|--------------|--------------|
| IPSO [10]    | 32704.4514   | 32704.4514   | 32704.4514   |
| PSO[41]      | 32858.00     | 33331        | 32039        |
| KGMO[42]     | 32548.1736   | 32548.3755   | 32548.2163   |
| Proposed HSCA| 32548.03     | 32574.54     | 32555.1      |

Current Function Value: 121983.5

5. CONCLUSION

The HSCA with SQP was successfully applied to various IEEE standard test systems while taking into account the equality and inequality constraints. And the results when compared with other techniques conveyed in literature clearly show that the proposed technique performs significantly better in terms of cost as well as fast convergence. Moreover, as the “HSCA with SQP” has performed very well on ED as compared to most of the methods, therefore, it can be applied to other problems of optimization related to power systems. And the hybridization of SCA with other evolutionary techniques may be a good area for the future research also.

ACKNOWLEDGEMENT

The first Authors would like to thank his supervisor for his continued guidance, supervision, friendliness and wise counsel throughout the research. Authors would like to thank University of Engineering & Technology, Taxila, Pakistan, and its staff for all the resources provided.
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