REVERSAL OF FORTUNE: INCREASED STAR FORMATION EFFICIENCIES IN THE EARLY HISTORIES OF DWARF GALAXIES?

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Received 2014 June 2; accepted 2014 June 27; published 2014 July 10

ABSTRACT

On dwarf galaxy scales, the different shapes of the galaxy stellar mass function and the dark halo mass function require a star-formation efficiency (SFE) in these systems that is currently more than 1 dex lower than that of Milky Way-size halos. Here, we argue that this trend may actually be reversed at high redshift. Specifically, by combining the resolved star-formation histories of nearby isolated dwarfs with the simulated mass-growth rates of dark matter halos, we show that the assembly of these systems occurs in two phases: (1) an early, fast halo accretion phase with a rapidly deepening potential well, characterized by a high SFE; and (2) a late, slow halo accretion phase where, perhaps as a consequence of reionization, the SFE is low. Nearby dwarfs have more old stars than predicted by assuming a constant or decreasing SFE with redshift, a behavior that appears to deviate qualitatively from the trends seen among more massive systems. Taken at face value, the data suggest that at sufficiently early epochs, dwarf galaxy halos above the atomic cooling mass limit can be among the most efficient sites of star formation in the universe.

Key words: dark matter – galaxies: dwarf – galaxies: halos – galaxies: star formation – galaxies: stellar content

Online-only material: color figures

1. INTRODUCTION

Dwarf galaxies are the smallest, most abundant, least luminous systems in the universe and have come to play a critical role in our understanding of the mapping from dark matter halos to their baryonic components (Pontzen & Governato 2014). An abundance mismatch problem has emerged over the past two decades: the different shapes of the galaxy stellar mass function and the dark halo mass function on dwarf galaxy scales require a star-formation efficiency (SFE) of only ∼0.1% in these systems (e.g., Behroozi et al. 2013), a value that has been traditionally difficult to reproduce in hydrodynamical simulations (e.g., Sawala et al. 2011). A strongly decreasing stellar mass fraction with decreasing halo mass is also required to solve the “missing satellite problem,” the discrepancy between the small number of dwarf satellites orbiting the Milky Way and the vastly larger number of dark matter subhalos predicted to survive in ΛCDM (e.g., Moore et al. 1999; Klypin et al. 1999; Diemand et al. 2008; Koposov et al. 2009; Rashkov et al. 2012).

Many astrophysical processes that suppress gas accretion and star formation in dwarf galaxies have been proposed to solve this puzzle, including the heating of intergalactic gas by the ultraviolet photoionizing background (e.g., Efstathiou 1992; Bullock et al. 2000; Kravtsov et al. 2004), rapid mass loss driven by supernovae (e.g., Dekel & Silk 1986; Mori et al. 2002), and metallicity-dependent H2-regulation (e.g., Gnedin & Kravtsov 2010; Kuhlen et al. 2013). Despite the successes of a new generation of hydrodynamical simulations in bringing theoretical predictions in better agreement with many observations (e.g., Mashchenko et al. 2008; Governato et al. 2010; Shen et al. 2014; Madau et al. 2014), a characterization of the time-dependent role of these mechanisms remains elusive. In particular, to reconcile the low SFEs of nearby dwarfs with the steep faint-end slope of the galaxy ultraviolet luminosity function measured at z ≥ 5 (Bouwens et al. 2012) and observational constraints on reionization (Hinshaw et al. 2013; Robertson et al. 2013), the star formation rate in low-mass halos must be enhanced at early times relative to that at lower redshifts (Lu et al. 2014).

In recent years, large systematic surveys of dwarfs in and around the Local Group (LG) with the Hubble Space Telescope (HST), such as the ACS Nearby Galaxy Survey Treasury (ANGST; Dalcanton et al. 2009), have uniformly measured star-formation histories (SFHs) for over 100 low-mass systems (Weisz et al. 2011, 2014). In this Letter, we use results from these data sets to investigate further the interplay between the dark matter and stellar assembly histories of these systems.

2. STELLAR AND DARK MATTER ASSEMBLY HISTORIES

We consider here the SFHs of nearby dwarf galaxies, M∗(z = 0) < 108 M⊙, located >300 kpc from a massive host (such as the Milky Way). These isolated systems provide a template for how low-mass galaxies evolve in the absence of significant environmental influence. Figure 1 depicts the cumulative SFHs, i.e., the fraction of total stellar mass formed prior to a given epoch, of dwarfs in the LG and nearby field (Weisz et al. 2011, 2014). The solid line represents the mean of the distribution of best-fit SFHs and the error envelope marks the 68% confidence interval of the mean.

On average, LG dwarfs formed >30% of their stellar mass prior to redshift 2 (>10–11 Gyr ago) and show increasing rate of stellar mass growth toward the present beginning around

3 Hubble Fellow.

4 Nearby isolated dwarfs are mostly dwarf irregulars by morphological type. Although dwarf irregulars are gas-rich and have ongoing star formation, most evidence suggests that their properties are similar to dwarfs of other morphological classes, such as dwarf spheroidals, and have just not been environmentally processed by a massive host (e.g., Weisz et al. 2011, 2014; Kirby et al. 2014).
redshift 1. However, as emphasized in Weisz et al. (2014), the HST-based SFHs of many LG dwarfs likely only provide lower limits on star formation at early times owing to observational aperture effects. To maximize the number of stars for SFH measurements, HST observations have typically targeted the central, high surface brightness regions of dwarfs and, owing to the limited size of the HST field of view, exclude most of their ancient stellar halos (e.g., Hidalgo et al. 2013). This effect is clearly seen in the different mean ANGST and LG SFHs: observations of ANGST galaxies cover a larger fraction of the system and include more old (halo) stars relative to LG dwarfs. An increase in the observational aperture would lead to an increase in the relative amount of stellar mass formed prior to 10 Gyr ago in many isolated dwarfs.

Despite the improved areal coverage of ANGST dwarfs, uncertainties on their SFHs are a factor of ~2 larger than for LG dwarfs for ages > 10 Gyr. The increased uncertainties are due to the relatively shallow color–magnitude diagrams (CMDs) of ANGST systems, which result in larger systematic uncertainties (i.e., the variation in SFH when measured with different stellar libraries) on the fraction of old stars. The characteristic ANGST CMD partially resolves the red clump, meaning the SFHs for ages older than a few Gyr rely on evolved phases of stellar evolution (e.g., red giant branch) where systematic uncertainties are larger.

In comparison, LG CMDs typically extend several magnitudes deeper than ANGST, providing access to the horizontal branch, which is sensitive to star formation >10 Gyr ago, and to older and better-understood main-sequence stars, thereby reducing the systematic uncertainties at older ages. The best possible constraints on the ancient SFHs (≤1 Gyr age resolution at all ages; e.g., Gallart et al. 2005) come from CMDs that include the oldest main-sequence turnoff (MSTO). However, owing to the faintness of this feature (M_v ~ +4) and the effects of stellar crowding, it is challenging to observe the oldest MSTO outside of the closest satellite galaxies, even with the HST. As a result, only a handful of isolated dwarfs have maximally secure SFHs. We discuss some of these systems in Section 3.

To empirically connect the build-up of stellar mass in dwarfs to the assembly of their host dark matter halos over cosmic time predicted by ΛCDM, we rely here on the merger trees and growth rates derived by Fakhoury et al. (2010) and based on the joint data set from the Millennium and Millennium-II simulations. The best fit to the mean dark matter accretion rate of halos of mass M(z) at redshift z is

\begin{equation}
\langle M \rangle = (46.1 \, M_\odot \, \text{yr}^{-1}) \, M_{12}^{1/2} \left(1 + 1.11 z\right) \frac{H(z)}{H_0},
\end{equation}

where M_{12} \equiv M/(10^{12} M_\odot) and H(z) is the Hubble parameter, H(z) = H_0 \sqrt{\Omega_M (1 + z)^3 + \Omega_V}.

Figure 2 shows the mean mass-assembly history \langle M(z) \rangle, obtained by integrating the above fitting formula in a \{(0.3, 0.7)\} cosmology, of all z = 0 M_0 = 10^{10} M_\odot halos in the two Millennium simulations. Fakhoury et al. (2010) assigned masses using the standard friends-of-friends algorithm and their growth rates must be extrapolated beyond z = 6 because of numerical resolution effects. We have compared this assembly history with that derived for the same M_0 using an alternative mass definition (based on spherical overdensity) and the best-fit mass growth rates of Behroozi & Silk (2014), and find consistent results.

As a further consistency check, we have also plotted the (spherical overdensity) mass-assembly histories of three 3 × 10^9 M_\odot ≤ M_0 ≤ 4.5 × 10^{10} M_\odot individual dwarf halos simulated at much higher mass resolution (approximately 500 times better than Millennium-II) in a fully cosmological setting (Madau et al. 2014). All dwarf halos grow quickly at early times, by approximately one order of magnitude in less than 1 Gyr. This is a consequence of the shallower slope of the power spectrum on these mass scales: dark matter clumps of all masses collapse nearly simultaneously and the timescale between collapse and subsequent merging becomes shorter. The early rapid dark matter accretion phase is followed, at redshift z ≤ 3, by a phase of slower growth characterized by the gentle addition of mass onto an established potential well.5 In Figure 2, we also show the cumulative mass fraction of the simulated dwarfs as a function of lookback time. The dark band provides an indication of the variation in the mean assembly histories of all 3 × 10^9 M_\odot ≤ M_0 ≤ 3 × 10^{10} M_\odot halos in the Millennium simulations (Fakhoury et al. 2010).

A look at Figures 1 and 2 reveals that the steep drop in halo mass at lookback times >10 Gyr is not accompanied by an equally sharp drop in stellar mass. Conversely, the increase in the stellar content of LG dwarfs that is inferred at these early epochs is far more modest than the predicted, more than tenfold increase in halo mass with cosmic time. This is followed, at ages between 7 and 11 Gyr, by a plateau in the SFH where the star formation activity actually declines. As a consequence and as detailed below, the SFE (≡ M_*/M) of dwarf galaxies decreases rapidly from high redshift to the present day.

5 "Spherical overdensity masses" are known to undergo spurious "pseudo-evolution" owing to the changing reference density (Diemand et al. 2007). Diemer et al. (2013) have recently shown that this pseudo-evolution accounts for almost the entire mass growth of galaxy-size halos between z = 1 and today.
Figure 2. Mass-assembly history of dwarf dark matter halos. Left panel: halo mass $M(z)$ as a function of lookback time. Colored points: assembly histories for three massive dwarfs (Bashful, Doc, and Dopey) simulated by Madau et al. (2014; DM-only run) at a particle mass resolution of $m_{\text{DM}} = 1.9 \times 10^4 M_\odot$. The simulation was performed with the parallel TreeSPH code GASOLINE (Wadsley et al. 2004). Solid black curve: mean mass-assembly history, $\langle M(z) \rangle$ for all $z = 0$ $M_0 = 10^{10} M_\odot$ halos in the two Millennium simulations (Fakhouri et al. 2010). Dashed black curve: mean mass-assembly history for $M_0 = 10^{10} M_\odot$ halos obtained using the best-fit mass growth rates of Behroozi & Silk (2014), which are based on the Bolshoi simulation (Klypin et al. 2011). Right panel: cumulative halo mass fraction as a function of lookback time. Colored points: same as left panel. The dark swath bounds the mean mass-assembly histories for all $M_0 = 3 \times 10^9 M_\odot$ (top curve) and $M_0 = 3 \times 10^{10} M_\odot$ (bottom curve) halos in the Millennium simulations.

(A color version of this figure is available in the online journal.)

3. STAR FORMATION EFFICIENCIES

To illustrate this point quantitatively and shed light on the dwarf galaxy-halo connection at different epochs, we have plotted in Figure 3 the mean stellar mass per halo mass for dwarf galaxies (i.e., the ratio estimator obtained by dividing the mean SFH of isolated LG dwarfs by the mean mass-assembly history of halos with a given $M_0$), as a function of lookback time. We assume that dwarfs occupy host halos in the mass range $3 \times 10^9 M_\odot \lesssim M_0 \lesssim 3 \times 10^{10} M_\odot$; as shown in Figure 2, such
halos have very similar mass assembly histories. It is clear that
dwarfs go through markedly different phases of galaxy growth.
Specifically, the early progenitor halos of today’s dwarfs were
relatively more efficient at converting gas into stars. The SFE
drops with cosmic time from an unresolved peak at high redshift,
as dwarfs form stars less rapidly (or not at all at ages between
7 and 11 Gyr) than their host halos accrete dark matter. Below
$z \sim 2$, the SFE remains approximately constant, as the increase
in stellar mass approximately tracks the growth of the host dark
matter halo.6

A decreasing SFE from an early peak paints a different picture
of galaxy growth than envisioned for more massive galaxies.
Figure 3 also shows, for comparison, the SFE versus lookback
time inferred by the version of the “abundance matching”
technique of Behroozi et al. (2013) for a Milky Way-size halo,
$M_0 = 10^{12} M_\odot$, and for a massive galaxy halo, $M_0 = 10^{13} M_\odot$.
The former is characterized by a late plateau, extending for
$\sim 6$ Gyr, and by a monotonic decline for $z \gtrsim 1$. The latter
reaches a maximum at $z \approx 2$ and drops dramatically at higher
redshift. The progenitors of these massive halos had early SFEs
that were much lower than their present-day values, i.e., they
were forming stars inefficiently prior to 11 Gyr ago. By contrast,
at these early epochs, the progenitors of LG dwarfs appear
to have followed a divergent path, forming stars with a SFE
that was significantly higher than today. These findings clearly
demonstrate the dangers of trying to infer the properties of low-
mass galaxies from their higher mass counterparts.

To illustrate the robustness of our results, we have compared
the SFEs obtained from the average SFHs of LG dwarfs with
those derived from deeper HST/ACS observations, i.e., from
CMDs that extend below the oldest MSTO. In Figure 4 we
consider five such isolated dwarfs whose SFHs have minimal
uncertainties (less than 10% of their lookback ages): IC 1613,
Leo A, Leo T, LGS 3, and Phoenix. Overall, we find that the deep
Advanced Camera for Surveys (ACS)-based SFEs are similar
more massive star-forming Sloan Digital Sky Survey galaxies
highlighted by Leitner (2012). Resolved dwarfs appear to
assemble their stellar content earlier than more massive systems,
which form late and had only $\sim 15\%$ of their stellar mass in place
before $z = 1–2$. Taking the data at face value, dwarfs galaxies
clearly do not lie at one end of a continuum defined by the
more massive galaxies. Instead, they seem to show a behavior
that deviates qualitatively from the trends seen on larger scales.
Qualitatively similar conclusions have been reached recently
by Behroozi & Silk (2014): based on galaxies’ specific star-
formation rates, halos smaller than $10^{12} M_\odot$ are predicted in
this study to form stars with increasing efficiencies at $z > 4$.

4. DISCUSSION

At present, dwarf galaxies are among the least efficient star-
forming systems in the universe. However, they have more old
stars than predicted by assuming a constant or decreasing SFE
with increasing redshift, i.e., if they followed the same trend
inferred for Milky Way-size halos by the abundance matching
technique. Although there is some scatter in the fraction of
ancient stars (with known exceptions to these trends such as

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6 Note that we implicitly assume that all star formation is in situ, i.e., took
place in the main host. In principle, a significant fraction of stars may have
formed in progenitor halos and have been accreted through merging. In
practice, this effect will be small because the SFE is a steeply declining
function of halo mass and so accreted halos will bring in very few stars.
Leo A), the typical isolated LG dwarf appears to have formed one-fourth of its present-day stellar content more than 12.5 Gyr ago. This high fraction of stellar mass at early times is in contrast to results derived for more massive galaxies and suggests that nearby dwarfs do not follow the clear “downsizing” trend whereby more massive galaxies form the bulk of their stars at earlier epochs compared to lower mass galaxies (Leitner 2012; Weisz et al. 2014). The physics behind such a different behavior remains to be understood, but the implications are clear: the early progenitors of today’s dwarfs had a higher stellar content per unit dark mass than their low-redshift counterparts of similar size. This cannot be true below virial masses of $10^8 M_\odot$ or so: such “mini-halos” must have formed stars very inefficiently not to overproduce the abundance of ultra-faint dwarf satellites of the Milky Way (Madau et al. 2008). Taken together, these trends suggest that, at sufficiently high redshift, dwarf galaxy halos above the atomic cooling limit (i.e., massive enough to cool via collisional excitation of hydrogen Lyα) can be among the most efficient sites of star formation in the universe.

The assembly of dwarf galaxy systems appears then to occur in two main phases: (1) an early, fast halo accretion phase with a rapidly deepening potential well, characterized by a high SFE; and (2) a late, slow halo accretion phase when the SFE is low. The early peak in SFE is not resolved by current observations: SFHs with a sub-Gyr age resolution should show a maximum and a drop at very early times as the masses of dwarf progenitors fall below the atomic cooling limit.

A useful perspective on these findings may be obtained by looking at mechanisms that modulate the supply of cold gas and limit the formation of stars in dwarf galaxies. Photoionization by the cosmic UV background, for example, heats the intergalactic medium to temperatures $T \gtrsim 10^4 K$ and reduces line cooling rates by lowering the fraction of neutral atoms. Both effects influence the ability of gas to accrete, cool, and condense in low-mass systems and can set a minimum mass scale for galaxy formation (Efstathiou 1992). The critical halo size below which such suppression is important has been addressed by several authors (e.g., Thoul & Weinberg 1996; Kitayama & Ikeuchi 2000; Gnedin 2000; Dijkstra et al. 2004). Cosmological hydrodynamical simulations by Okamoto et al. (2008) have shown that the characteristic mass, $M_c(z)$, below which dwarf galaxy halos lose half of their gas from photoheating rises from $\sim 1.5 \times 10^4 M_\odot$ after reionization to $\sim 10^{10} M_\odot$ at $z = 0$. In Figure 5, we plot this characteristic mass together with the mean mass-assembly history of a $M = 5 \times 10^9 M_\odot$ dwarf halo. The comparison shows how a substantial amount of gas may be able to cool and form stars at early times, when $M > M_c$. Star formation may be sustained at high SFE for a couple of Gyr until the halo gas content is severely depleted ($M_c > M$), leading to a decline in the star formation activity and SFE at $z < 3$. This picture should be taken just as an illustrative example, as the redshift-dependent mass scale at which halos can accrete intergalactic gas depends on the reionization redshift, the amplitude of the ionizing background, and the formation history of a halo rather than its instantaneous mass (Noh & McQuinn 2014). Yet, although the small number of isolated dwarfs with deep CMDs and concerns about the absolute ages of the earliest epochs of star formation in CMD-based SFHs urge caution in over-interpreting our findings, it is tempting to conjecture that it is the time-dependent suppression of baryonic infall by the UV background—aided by stellar feedback—that may regulate the fueling of dwarfs and drive the late evolution of their SFEs (see also Benítez-Llambay et al. 2014). Regardless, there appears to be little need on these mass scales for additional feedback suppression at high redshifts—dwarf galaxies were already in place at early times.

Support for this work was provided by NSF grant OIA-112445329745 and by NASA grant NNX12AF87G to P.M., and by NASA through Hubble Fellowship grant HST-HF-51331.01, awarded by the STScI, to D.R.W. We thank Peter Behroozi, Takashi Okamoto, and Sijing Shen for useful discussions and assistance with Figures 2, 3, and 5.

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Figure 5. Comparison between the mean normalized SFE of LG dwarfs (dark band, units on the left axis) and the evolution of the characteristic mass $M_c$ (red points, units on the right axis) below which dwarf galaxy halos lose half of their gas from photoheating (Okamoto et al. 2008). The blue curve shows the mean mass-assembly history of a $M = 5 \times 10^9 M_\odot$ dwarf halo. (A color version of this figure is available in the online journal.)
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