Rotating tethered system for active space debris removal

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Abstract. A new technique for active debris removal using tethered space tug is presented. Instead of towing the captured debris object by pulling on the attached tether it is proposed to use the rotation of the tethered tug-debris system that allows to apply the tug’s thrust along the tether pushing the system. The rotation of the tug-debris tethered system around its center of mass provides necessary tension of the tether when the tug applies its thrust along the tether. To de-orbit the system the tug should apply its thrust periodically when the orientation of the tether relative to the the orbital velocity vector of the system ensures application of the impulse in the required direction. A simple mathematical model of tethered tug-debris system is developed which allows to estimate required angular rate of the system. A tether control law is proposed that reduces the amplitude of the tether oscillations during applying tug’s thrust.

1. Introduction

Large objects like orbital stages, boosters, and nonfunctional spacecraft are potential sources of orbital debris that pose serious hazard to active satellites [1, 2]. To reduce the risk of huge increasing the numbers of debris objects, large objects should be removed from orbits. Several space debris removal methods have been proposed in the past years [3–8].

Space debris objects can be de-orbited using space tugs. A space tug is an active spacecraft that can capture and de-orbit selected space debris object. Many authors suggest to use space tether for towing large debris objects [9–19]. Some of these methods employ using a tethered space tug with an autonomous module for short range rendezvous, docking, and towing large debris. In all these methods it is supposed that the space tug pulls the debris by the tether (figure 1a) or pushes it after docking with the debris (figure 1b). The first scheme constraints the configuration of the space tug. The tug’s thrusters and the tether equipment (payload) should be installed at the same side of the tug. The second scheme assumes docking of the space tug with the debris. Any space debris is a non-cooperative object so the docking process with these objects is a challenge task.

In this paper we propose an alternative solution. As in the first scheme (figure 1a) the space tug and space debris connected by tether forming the tethered tug-debris system. Unlike the first scheme, the space tug apply its thrust force in the opposite direction. To avoid slacking of the tether the tethered tug-debris system have to rotate around its center of mass with sufficient angular rate so that the centrifugal force exceeds the thrust force of the tug.
2. Stages of the active debris removal mission

The main idea of the proposed technique and the stages of the active debris removal mission using the proposed technique are presented in figure 2. Let us suppose that the space debris object orbiting a circular orbit of height \( h_d \) and the space tug orbiting a circular orbit of height \( h_0 \). The space tug consists of the main transport module and the autonomous docking module [20]. The autonomous module carries a capturing device: net, probe-cone mechanism, harpoon. The autonomous module can be separated from the main transport module for the short range proximity and capturing operations. The proposed technique assumes the following steps of the active debris removal mission (figure 2):

(i) transfer of the space tug to the intercept orbit with \( h_p = h_0 \) and \( h_a = h_d - l_0 \);
(ii) separating of the autonomous module;
(iii) capturing of the debris;
(iv) applying the series of de-orbit impulses along the tether.

The initial angular rate of the tethered system can be achieved by relative orbital motion of the space tug and debris. If the space tug and debris have different orbital velocities (\( V_{tug} \) and \( V_{deb} \) respectively) which are perpendicular to the tether, the initial angular rate of the system is \( \omega_0 = (V_{deb} - V_{tug})/l_0 \), where \( l_0 \) is the distance between the space tug and debris that is equal to the initial length of the tether.

Due to the rotation of the tethered tug-debris system the orientation of the tug’s thrust vector changes with the time. To decrease the orbital height of the system for de-orbiting the tethered debris the tug’s thrust should be burned when the cosine of the angle \( \psi \) is positive. Angle \( \psi \) is the angle between the orbital velocity vector \( V_c \) of the tug-debris system and the inverse direction of the tug’s thrust vector \( P \). So, to decrease the height of the orbit the tug’s thrust should be turned on when \( \cos \psi > 0 \). For more efficient use of the tug’s propellant the last condition can be rewritten as \( \cos \psi > \cos \psi_a \), \( \psi_a \leq \pi/2 \).

In the next section the longitudinal oscillations of the tether during de-orbit process is considered.
3. Model

Here we consider only simple in-plane motion of the system. To investigate the oscillations of the tether let us consider the motion of the tether in gravityless space. We suppose that the debris and space tug do not oscillate relative to the tether. These bodies are considered as point masses attached to the tether. The tether is massless and elastic.

The motion of the system is considered relative to the non-inertial frame $Cx_cy_c$. The origin of the frame is in the center of mass of the system $C$. The scheme of the system is presented in figure 3. The tethered tug-debris system rotates around the center of mass $C$ with angular rate $\omega$. The motion of the space tug along the $Cx_c$ axis of the frame $Cx_cy_c$ can be described by the equation

$$m_1 \frac{d^2 l_1}{dt^2} = -P - T + \Phi_P + \Phi_\omega,$$

where $P$ is the tug’s thrust, $\Phi_P = m_1 a_c$ is the inertial force caused by the acceleration of the center of mass of the system $a_c = P/(m_1 + m_2)$, $m_1$ is the mass of the space tug, $m_2$ is the mass of the space debris, $\Phi_\omega = m_1 \omega^2 l_1$ is the centrifugal force caused by rotation of the tether relative to the center of mass, $T$ is the tension of the tether. We suppose that the tension force depends only on the elongation of the tether $T = c(l - l_0)$, where $c$ is the tether stiffness, $l_0$ is the tether free length.
The distance from the center of mass of the system to the space tug can be expressed as

\[ l_1 = l \frac{m_2}{m_1 + m_2}. \]  

Equation (9) can be rewritten as

\[ \frac{d^2 l}{dt^2} = \omega^2 l - \frac{P}{m_1} - k^2(l - l_0), \]  

where \( k^2 = c/m_{12} \) and \( m_{12} = m_1 m_2/(m_1 + m_2) \) is the reduced mass of the system.

Leaving out of account the action of the gravitational torque acting on the tethered tug-debris system and supposing that the tug’s thrust is applied along the tether we can use angular momentum equation

\[ J \frac{d\omega}{dt} = 0, \]  

where \( J \) is the moment of inertia of the system relative to the center of mass

\[ J = l_1^2 m_1 + l_2^2 m_2 = m_{12} l^2. \]  

Let us suppose that the tethered tug-debris system had the initial angular rate \( \omega_0 \) and the initial length of the tether is equal to the free length \( l_0 \). In this case we can write

\[ J_0 \omega_0 = J \omega \implies \omega = \omega_0 \frac{l_0^2}{l^2}, \]  

where \( J_0 = m_{12} l_0^2 \). Using (6), equation (3) get the form

\[ \frac{d^2 l}{dt^2} = \omega_0^2 \frac{l_0^4}{l^4} - \frac{P}{m_1} - k^2(l - l_0). \]  

3.1. The first integral

Introducing new variable \( u = dl/dt \), we get

\[ \frac{du}{dt} = \omega_0^2 \frac{l_0^4}{l^3} - \frac{P}{m_1} - k^2(l - l_0), \]  

that allows to obtain the first integral

\[ u_b^2 - u_a^2 = \frac{1}{l_0^2} \omega_0^2 + l_0^2 l_a (l_a - 2l_0) + \frac{2P l_a}{m_1} - k^2 l_b^2 - \frac{1}{l_0^2} \omega_0^2 + 2l_b \left( \lambda^2 l_0 - \frac{P}{m_1} \right), \]  

where \( a \) and \( b \) denote lower and upper bounds of integral variables. This expression can be used to estimate the deformation of the tether during transient events when the force \( P \) is turning on and off. For example, for the tethered tug-debris system with initial length \( l_a = l_0 \) that starts rotate with the angular rate \( \omega_0 \) the maximum length of the tether \( l_b \) can be obtained from the equation

\[ k^2 l_b^4 - 2k^2 l_0 l_b^3 - \left( \frac{l_0^2 \omega_0^2}{l_0^2} - l_0^2 k^2 \right) l_b^2 + l_0^4 \omega_0^2 = 0. \]
3.2. Stationary solution

Equation (3) allows us to obtain the expression for the stationary length of the tether. For \( d^2l/dt^2 = 0 \) we get

\[
l_s = \frac{k^2l_0 - P/m_1}{k^2 - \omega^2}.
\]  

(11)

Taking into account (6) nonlinear equation (11) can be rewritten as

\[
l_s^4 - \left(l_0 - \frac{P}{m_1k^2}\right) l_s^3 - \frac{\omega_0^2}{k^2} l_s^4 = 0.
\]  

(12)

For \( P = 0 \) we get

\[
l_s^4 - l_0^3 - \frac{\omega_0^2}{k^2} l_s^4 = 0.
\]  

(13)

The solutions of the equations (12) and (13) can be obtained for for particular values of \( l_0, \omega_0 \) and \( k \). Let us suppose that the solution of the equation (12) is \( l_s = l_{sp} \) and the solution of equation (13) is \( l_s = l_{0s} \).

The stationary solutions allow us to estimate the deformations of the tether during turning on and off the tug’s trust. If the tethered tug-debris system with length of \( l_{0s} \) rotates around the center of mass than applying the force \( P \) leads to decreasing of the length of the tether to the value \( l_{min} \) which can be calculated from the following equation

\[
k^2 l_{min}^4 - l_{min}^2 \left(l_{0s}(l_{0s} - 2l_0)k^2 + \frac{l_0^4\omega_0^2}{l_{0s}} + \frac{2l_{0s}P}{m_1}\right) + 2l_{min}^3(l_0k^2 - P/m_1) + l_0^4\omega_0^2 = 0.
\]  

(14)

For example, for \( m_1 = 1500 \text{ kg}, m_2 = 5000 \text{ kg}, \omega_0 = 3 \text{ o/s}, l_0 = 3000 \text{ m}, k = 0.128 \text{ 1/s} \) and \( P = 1000 \text{ N} \)

\[ l_{0s} = 3359 \text{ m}, \quad l_{min} = 3297 \text{ m} < l_{sp} = 3328 \text{ m}. \]  

(15)

This example illustrates that the transient process following instantaneous turning on the tug’s thrust should be taken into account when choosing the initial angular rate \( \omega_0 \) of the tethered tug-debris system to ensure that the tether remains tensioned. In the next subsection we present a control algorithm that can decrease the oscillations of the tether during the transient processes.

3.3. Control of the tether tension

We propose to use the following control law when the tug’s thrust should be turned on (when \( \cos \psi \geq \cos \psi_a \))

\[
P_{on} = \begin{cases} 0, & l < l_{sp} - \delta l \text{ and } \dot{l} < -\delta v \\ P, & \text{otherwise} \end{cases},
\]  

(16)

When the tug’s thrust should be turned off \( (\cos \psi < \cos \psi_a) \)

\[
P_{off} = \begin{cases} P, & l > l_{s0} + \delta l \text{ and } \dot{l} > \delta v \\ 0, & \text{otherwise} \end{cases},
\]  

(17)

where \( \delta l \) and \( \delta v \) are the threshold values for the tether’s length and speed. The control law (16) shows that to decrease the oscillation of the tether during the active phase \( (P \text{ is on}) \) the tug’s thrust should be temporarily turned off when the tether length tends to exceed \( l_{sp} \) and its length tends to decrease \( (\dot{l} < -\delta v) \). The control law (17) shows that during the passive phase \( (P \text{ is off}) \) the tug’s thrust should be temporarily turned on when the tether length tends to be less than \( l_{s0} \).
4. \( \Delta V \) estimation

The tug’s thrust is turned on when the angle between the reverse direction of the tug’s thrust vector and the orbital velocity vector of the system is less than \( \psi_a \). The projection of the momentum on the orbital velocity vector for one turn of the tethered system around its center of mass can be estimated as

\[
\Delta S \approx \int_{-\psi_0/\omega}^{\psi_0/\omega} P \cos \omega t \, dt = \frac{2P}{\omega} \sin \psi_0.
\]  (18)

This expression can be used for the estimation of the de-orbit time of the debris from low Earth orbits. For example, if the initial height of the circular orbit of the debris is \( h_d \), then the required \( \Delta V \) to transfer the debris to the atmosphere of the Earth \( h_b = 100 \text{ km} \) is

\[
\Delta V = \sqrt{\frac{\mu}{r_b}} - \sqrt{\frac{\mu}{r_d}},
\]  (19)

where \( r_d = R_e + h_d \), \( r_b = R_e + h_b \), \( R_e \) is the mean radius of the Earth. The required \( \Delta V \) can be reached after \( N \) burns of the tug’s thruster

\[
N \approx \frac{\Delta V (m_1 + m_2)}{\Delta S}.
\]  (20)

Taking into account rotational period of the tug-debris system \( T_\omega = 2\pi/\omega \) we can get the estimation of the de-orbit time

\[
t_d \approx NT_\omega = \frac{\pi \Delta V (m_1 + m_2)}{P \sin \psi_a}.
\]  (21)

The expression (21) does not take into account additional impulses of the tug’s thrust for the damping of the tether’s oscillations according to the proposed controls (16), (17).

It should be noted that the condition \( \cos \psi \geq \cos \psi_a \) leads to the change of the orbital velocity in the multiple points of the orbit. This approach can be used to transfer the debris to a graveyard orbit, but for de-orbit the debris (transfer it to the edge of the Earth atmosphere) this approach is not effective. In this case is more effective to apply the impulses near the apogee point of the initial orbit of the system. This approach will decrease fuel consumption by half but will increase the de-orbit time.

5. Simulation

5.1. Parameters of the system and initial conditions

Let us consider a tethered tug-debris system that starts to rotate with the angular rate of 3 deg/s in the orbital plane of the system. The space debris is an upper stage type debris with mass of 5000 kg. The space tug has mass of 1500 kg. The tether length is 3000 m. The thruster of the space tug is turned on when the angle between the orbital velocity vector and the inverse direction of the tug’s thrust are less than \( \psi_a = 60^\circ \). Parameters of the considered system are presented in the table 1.

5.2. Simulation results

The stationary length of the tether when the tug’s thruster is off is \( l_{s0} = 3359 \text{ m} \), when the tug’s thruster is on \( l_{sp} = 3328 \text{ meters} \) (see the formulas (11) and (12)). Solving the equation (9) with \( u_a = u_n = 0 \), \( l_a = l_{s0} \) we can estimate minimal length of the tether induced by the transient process when the tug’s thruster is turned on. The tether length can drop as low as 3297 meters.
Table 1. Parameters of the system.

| Parameter              | Value     | Parameter              | Value   |
|------------------------|-----------|------------------------|---------|
| Space tug mass, $m_1$  | 1500.0 kg | Tether free length     | 3000 m  |
| Space debris mass, $m_2$ | 5000 kg  | Initial angular rate, $\dot{\psi}_0 = \omega_0$ | 3 deg/s |
| Tug’s thrust $P$        | 1000 N    | Tether stiffness $k$   | 0.128 1/s |

This value is greater than the tether free length $l_0 = 3000$ m, so the angular rate of the system is sufficient to keep the tether tensioned.

Figure 4 shows the time history of the tether elongation during the de-orbiting process. The figure shows decreasing oscillations of the tether that starts to oscillate from the initial conditions $l(0) = l_0$, $\omega(0) = \omega_0$. The oscillations are damped due to the action of the control (17). Gray vertical bars indicate the time intervals when the tug’s thrust is turned on and the control law (17) is used. Two dashed lines show the levels of $l_{d0} = 3359$ m and $l_{sp} = 3328$ m. The figure shows that the tug’s thrust is burned periodically and the controls (17), (16) effectively damp the oscillations of the tether which remains tensioned during de-orbit process. Figure 5 shows the angular velocity of the tether-debris system $\dot{\psi}$. The angular velocity is changing periodically due to the change in the tether length under the action of the tug’s thrust.

The parameters of the system allows us to get first approximation of de-orbit time. Required $\Delta V$ is 495 m/s so we need about of $N = 100$ burns of the tug’s thruster. The de-orbit time is $t_d \approx 3.2$ hours.
6. Conclusion

We present a new technique for active debris removal using tethered space tug. In the proposed technique the rotation of the tethered tug-debris system is used to “rigidise” the tether due to the action of the centrifugal forces on the debris and space tug. It allows to apply the tug’s thrust along the tether to push the debris object. To de-orbit the system the tug’s thrust should be turned on when the projection of the space tug’s force on the orbital velocity vector is negative.

The required rotation of the tethered tug-debris system can be achieved by the relative orbital motion of the debris object and the space tug by using the autonomous docking module to deliver the tether from the space tug to space debris. The angular rate of the tethered tug-debris system have to ensure that during the transient events associated with the turning on and off the tug’s thrust the tether’s tension force does not fall below zero. The proposed technique allows to use conventional design of the space tug with the payload space in front of the tug and the thruster at the rear side.

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