Conditions for the emergence of gauge bosons from spontaneous Lorentz symmetry breaking

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The emergence of gauge particles (e.g., photons and gravitons) as Goldstone bosons arising from spontaneous symmetry breaking is an interesting hypothesis which would provide a dynamical setting for the gauge principle. We investigate this proposal in the framework of a general SO(N) non-Abelian Nambu model (NANM), effectively providing spontaneous Lorentz symmetry breaking in terms of the corresponding Goldstone bosons. Using a nonperturbative Hamiltonian analysis, we prove that the SO(N) Yang-Mills theory is equivalent to the corresponding NANM, after both current conservation and the Gauss laws are imposed as initial conditions for the latter. This equivalence is independent of any gauge fixing in the YM theory. A substantial conceptual and practical improvement in the analysis arises by choosing a particular parametrization that solves the nonlinear constraint defining the NANM. This choice allows us to show that the relation between the NANM canonical variables and the corresponding ones of the YM theory, $A^a_i$ and $E^{bj}$, is given by a canonical transformation. In terms of the latter variables, the NANM Hamiltonian has the same form as the YM Hamiltonian, except that the Gauss laws do not arise as first-class constraints. The dynamics of the NANM further guarantees that it is sufficient to impose them only as initial conditions, in order to recover the full equivalence. It is interesting to observe that this particular parametrization exhibits the NANM as a regular theory, thus providing a substantial simplification in the calculations.

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I. INTRODUCTION

The possible interpretation of gauge particles (e.g., photons and gravitons) as Goldstone bosons (GBs) arising from some spontaneous symmetry breaking, dates back to the pioneering works of [1] and [2, 3]. The former used the standard coset construction of the effective theory [4], in which the spontaneous Lorentz symmetry breaking (SLSB) is realized nonlinearly in terms of the GBs, with the matter fields transforming linearly under the unbroken subgroup. Their conclusion was that any gauge theory is a theory of some spontaneously broken symmetry. A recent application of the coset construction to general relativity was reported in Ref. [5].

On the other hand, Refs. [2, 3] considered the explicit case of spontaneous Lorentz symmetry breaking to produce the photon field as the condensate arising from a self-coupled four-fermion model, following similar steps as those developed in Ref. [6] to describe the superconductor solutions in field theory. Similar ideas have been revisited for photons and also have been extended to gravitons in Refs. [7, 8].

An alternative approach was proposed by Nambu in Ref. [10], where the emphasis was shifted to the description of the SLSB system (QED, in this case) only in terms of the GB degrees of freedom (d.o.f.), which were introduced via a nonlinear constraint, similarly to the nonlinear sigma model description of pion interactions. Such a σ-QED is defined by the Maxwell’s Lagrangian plus the constraint $A_\mu A^\mu = n^2 M^2$, which is to be substituted into the Lagrangian. Here, $n_\mu$ is a properly oriented constant Lorentz vector, while $M$ is the proposed scale associated with the SLSB. This constraint can be understood as providing a nonzero vacuum expectation value $\langle A_\mu \rangle = n_\mu M$. Nevertheless, the goal in this model was to show that it is in fact equivalent to standard QED, instead of yielding a physical violation of the Lorentz symmetry. This equivalence was manifest up to the tree-level calculations studied in Ref. [10]. Later on, these calculations were extended to some processes at the one-loop level, with identical results: all contributions arising from the SLSB sector of the model canceled out, yielding the standard QED results [11]. The σ-QED model has been further studied [12] and extended to the non-Abelian [13–16] and gravitational cases [17], which we will generically call generalized Nambu models. General conditions regarding how the gauge symmetries were recovered from the corresponding SLSB models are worked out in [18–20].

Perturbative calculations in the non-Abelian case show again that, to the order considered, all SLSB contributions to physical processes cancel out, yielding an equivalence with the starting Yang-Mills (YM) theory, in complete analogy with the Abelian case. This fact has been interpreted by stating that the corresponding nonlinear constraint, which defines each Nambu model, can be interpreted as just a gauge choice in the associated Abelian or non-Abelian gauge theory. This would lead to an equivalence between the Nambu model and the corresponding gauge theory in a fixed gauge. Nevertheless, this statement requires some qualifications. (i) To begin with, the number of d.o.f. of the Nambu model is larger than that of the corresponding gauge theory, which can be understood because the former has lost gauge invariance. (ii) Fixing the gauge in any gauge theory requires the introduction of ghost particles (via the Becchi-Rouet-Stora-Tyutin procedure, for example Ref. [21]) which play a fundamental role as internal particles in calculating physical processes. Thus, in order to establish the proposed equivalence, one would need to study the contributions of the ghosts to physical processes. A possible decoupling of them is by no means evident, especially due to the nonlinear character of the proposed gauge fixing. The first point (i) has been taken into account in most previous works and was emphasized in Ref. [22]. The general statement, phrased in different ways in different papers, is that the Nambu model is equivalent to the corresponding gauge theory only after current conservation together with the Gauss laws have been imposed as initial conditions, since the dynamics of the Nambu model preserves their conservation for all times. The second point (ii) has not been considered at all and will be dealt with in a separate publication, for the case of the Abelian Nambu model [23].

Recently, the study of possible observable violations of Lorentz invariance has attracted considerable attention, from both the experimental and theoretical points of view. Explicit Lorentz symmetry violation is found to be incompatible with the Bianchi identities [24] and therefore this approach is not consistent with general relativity, unlike SLSB, where this issue does not occur. The construction of the Standard Model extension performed by Kosleťeky and collaborators [25] is a framework in which Lorentz violation is considered as arising from a spontaneous symmetry breaking in a more fundamental theory. A distinguished class of models in that framework are the so-called bumblebee models, which are tensor theories exhibiting physical SLSB. They include GB modes, and depending on the explicit form of the theory they have additional modes and constraints. These models have been thoroughly investigated in relation to electrodynamics [26, 27] and gravity [28–31]. As a matter of fact, generalized Nambu models can be thought of as very particular cases of bumblebee models, where the non-Goldstonic d.o.f. of the latter are frozen, leaving only the GB excitations.

In the present paper we generalize to the non-Abelian case and improve the nonperturbative Hamiltonian analysis developed for the Abelian Nambu model (ANM) in Ref. [22]. The non-Abelian Nambu model (NANM) associated to the group $\text{SO}(N)$ is defined by the YM Lagrangian

$$\mathcal{L}(A_\mu^a) = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} - J^{a\mu} A_{\mu}^a, \quad (1)$$
plus the condition
\[ A^a_\mu A^{\mu} = n^2 M^2, \quad M^2 > 0, \quad \mu = 0, 1, 2, 3, \quad a = 1, ..., N, \] (2)
which is to be solved and substituted into the YM Lagrangian. Our goal is to determine which additional conditions have to be imposed upon the NANM in order that its Hamiltonian reduces to that of the YM theory. No discussion is provided of the possible perturbative equivalence of the so-corrected Nambu model and the YM theory in the fixed nonlinear gauge.

We proceed via the following steps. (i) We start by constructing the Hamiltonian for each type of NANM (depending on \( n^2 \) it can be time-like, space-like, or light-like) in terms of the corresponding canonical variables for each case. (ii) We show that these Hamiltonians are related via a canonical transformation to a Hamiltonian that has the same form as the YM Hamiltonian in the standard variables \( A^a_\mu, E^a_\mu \), \( i = 1, 2, 3 \), except for the fact that the Gauss laws \( \Omega^a = 0 \) do not appear as constraints; nevertheless, the canonical transformation leads to the correct brackets between the canonical variables \( A^a_\mu, E^a_\mu \) arising from the canonical algebra of each NANM. (iii) We prove that the NANM dynamics preserves the evolution of \( \Omega^a \) in such a way that it guarantees that the imposition of \( \Omega^a = 0 \) for some initial time leads to \( \Omega^a(t) = 0 \) for all times. In this way, enforcing the Gauss laws as first-class Hamiltonian constraints at some initial time makes the NANM equivalent to the corresponding YM theory in a nonperturbative way and independently of any gauge fixing. Consistency with the NANM dynamics avoids the generation of additional constraints.

The present paper is organized as follows. In Sec. II we consider the specific case of the space-like NANM (SL-NANM) by solving the nonlinear constraint (2) in terms of \( A^a_\mu = 1 \) and starting with the remaining \( 4N - 1 \) d.o.f. per point in coordinate space. The Lagrangian equations of motion are obtained and the canonical momenta together with the canonical Hamiltonian are subsequently constructed. The standard variables of the YM theory \( A^a_\mu \) and \( E^{ab} \), \( i = 1, 2, 3 \) are written in terms of the canonical variables of the SL-NANM and their induced algebra is calculated, which is summarized in Appendix B. The analysis of the SL-NANM in terms of the Dirac method reveals that this model has additional second-class constraints, which are further imposed strongly by introducing Dirac brackets to obtain the d.o.f. of the reduced phase space, together with their algebra. The induced Dirac-brackets algebra for the variables \( A^a_\mu \) and \( E^{ab} \) is finally obtained and compared with the standard YM Hamiltonian. The conditions under which both theories are equivalent are established, which requires understanding the time evolution of the Gauss functions \( \Omega^a \) under the SL-NANM dynamics. A similar analysis can be carried out for each of the remaining cases, corresponding to the time-like and light-like NANM. Section III presents a substantial conceptual and practical improvement over the previous individual calculations to study the relation between the NANM and YM theories. We start from a general parametrization that solves the constraint (2) for arbitrary values of \( n^2 \), in terms of \( 3N \) d.o.f. \( \Phi_A^a, A = 1, 2, 3, \) and repeat the canonical analysis, paying attention to the relations between the standard canonical variables of the YM theory \( (A^a_\mu, E^{ab}) \) and those of the NANM \( (\Phi_A^a, \Pi_B^a) \). The calculations are enormously simplified after one realizes that the transformation \( (\Phi_A^a, \Pi_B^a) \rightarrow (A^a_\mu, E^{ab}) \) is a canonical transformation, once the \( E^{ab} \) are recognized as the momenta canonically conjugate to the \( A^a_\mu \) via the kinetic part of the NANM Hamiltonian action. Another useful property of the new parametrization is that it exhibits the NANM as a regular theory (i.e., no constraints appear in the Hamiltonian analysis). This is proved in Appendix C. The canonical NANM Hamiltonian, rewritten in terms of the YM variables \( (A^a_\mu, E^{ab}) \), is finally obtained and the conditions under which it reduces to the YM Hamiltonian are determined. Again, this requires the calculation of the time evolution of the Gauss functions \( \Omega^a \) under the NANM dynamics. Finally, we close with a summary and some comments in Sec. IV. Appendix A serves to establish notation and briefly reviews the canonical version of the SO(\( N \)) YM theory, which we use as a benchmark to identify the conditions under which it is equivalent to the different realizations of the NANM.

II. THE SPACE-LIKE CASE OF THE NON-ABELIAN NAMBU MODEL (SL-NANM)

Before considering the specific case of the SL-NANM, let us recall some general properties of the NANM. The Lagrangian is
\[ L_{NANM}(A^a_\mu) = -\frac{1}{4} F^{a\mu\nu} F_{a\mu\nu} - J^{a_\mu} A^a_\mu + \lambda \left( A^a_\mu A^{a_\mu} - n^2 M^2 \right), \quad M^2 > 0, \quad a = 1, ..., N, \] (3)
with the notation and conventions introduced in Appendix A. Here, \( \lambda \) is a Lagrange multiplier and the vector \( n^a \) is such that \( n^2 = \pm 1, 0 \).

The general procedure by which we will analyze each model is to explicitly solve the condition
\[ A^a_\mu A^{a_\mu} = n^2 M^2 \] (4)
and substitute the adequate parametrizations directly into the Lagrangian $\mathcal{L}_{\text{NANM}}(A^a_\mu)$, defining in this way the canonical degrees of freedom of the model. Subsequently, we obtain the corresponding Hamiltonian and determine its relation to the YM Hamiltonian \(\mathcal{L}(A^a_\mu)\) together with the canonical algebra (A29). As expected, the equations of motion of the NANM will not be those of the Yang-Mills theory arising from $\mathcal{L}(A^a_\mu)$ in Eq. (A11). This property will be explicitly shown in the remaining sections of the paper. In this way, the conservation of the current $J^a_\nu$ does not follow as a consistency condition from the equations of motion in the NANM, as happens in the YM case. We will show that the canonical structure of the NANM will induce the standard-algebra YM (A29), together with a Hamiltonian that differs from Eq. (A29) by the property that the Gauss laws do not appear as constraints. Nevertheless, the dynamics of the NANM guarantees their validity for all time, once they are imposed as initial conditions.

The standard solutions of the condition $A^a_\mu A^{\mu a} = n^2M^2$, arising from the different choices of $n^2$, are

$$\begin{align*}
n^2 > 0 : \quad & A^1_a = \sqrt{M^2 + A^2_a A^1_a - A^1_0 A^0_0}, \quad \bar{b} = 2, 3, ..., N, \\
n^2 < 0 : \quad & A^1_a = \sqrt{M^2 + (A^0_a)^2 - (A^1_a)^2 - (A^2_a)^2}, \\
n^2 = 0 : \quad & A^0_a = B^a \left(1 + \frac{A^1_a A^1_a}{4B^2} \right), \quad A^a_3 = B^a \left(1 - \frac{A^k_b A^k_b}{4B^2} \right), \quad \bar{\nu} = 1, 2,
\end{align*}$$

which define the NANM in its time-like (TL-NANM), space-like (SL-NANM) and light-like (LL-NANM) representations. In the time-like and space-like cases we start with $4N - 1$ d.o.f per point, while in the light-like case this number is $3N$. Next we concentrate on the SL-NANM.

### A. The equations of motion in the SL-NANM

We start with the extension to the non-Abelian case of the parametrization $A^1_3 = \sqrt{M^2 + A^2_0 - A^1_1 - A^2_1}$, which is frequently used in the Abelian case to exhibit the remaining SO(2,1) symmetry after the spontaneous Lorentz symmetry breaking. The Lagrangian constraint (3) is now solved for $A^a_{\mu=1,3}$. In this way we start from $4N - 1$ d.o.f. in coordinate space, which we denote in the following way: $A^0_a$, $A^1_k$, $A^2_k$, with $a = 1, 2, ..., N; \bar{a} = 2, 3, ..., N, \bar{k} = 1, 2, 3; k = 1, 2$. The numbers of each of the corresponding fields are $N, 2, 3(N - 1)$, respectively. With this notation, we write

$$A^1_3 = \sqrt{M^2 + (A^0_\bar{a})^2 - (A^k_{\bar{a}1})^2 - (A^2_{\bar{a}1})^2},$$

which exhibits the remaining symmetry group SO($N,3N - 1$). As a matter of notation, superscript indices label a group index, while subscript indices refers to a space-time index.

In the notation of Appendix A, the Lagrangian density (3) takes the form

$$\mathcal{L}(A^0_a, A^1_k, A^2_k) = \frac{1}{2} \left( (E^a_1)^2 + (E^1_k)^2 + (E^2_k)^2 - B^a_0 B^a_0 \right) - J^a_\nu A^{\nu a} + J^1_k A^1_k + J^2_k A^2_k + J^1_3 A^1_3.$$

The equations of motion are

$$\mathcal{E}^{i_1}_a - \mathcal{E}^{i_2}_1 A^a_3 = 0,$$

$$\mathcal{E}^{0_1}_a + \mathcal{E}^{0_2}_1 A^a_3 = 0,$$

with the notation

$$\mathcal{E}^{\nu a} = (D_\mu F^{\mu \nu} - J^{\nu})^a.$$

The numbers of equations in Eq. (10) is only $(3N - 1)$ because the simultaneous choice $i = 3$ and $a = 1$ does not appear, as $A^1_3$ is a function of the dynamical variables.

Since $A^1_3$ is just a shorthand for (8), it follows that

$$\dot{A^1_3} = \frac{A^0_a}{A^a_3} \dot{A^a_0} - \frac{A^k_1 J^k_1}{A^a_3 A^0_3} - \frac{A^a_3}{A^0_3} \dot{A^1_3},$$

(13)
The fields \( E_3^a \) and \( E_0^a \) are given by
\[
E_3^a = \hat{A}_3^a - D_3 A_1^a = \left( \frac{A_0^a}{A_3^1} \frac{A_0^1}{A_3^a} - \frac{A_1^a}{A_3^1} \frac{A_1^1}{A_3^a} \right) - D_3 A_0^a,
\]
\[
E_0^a = \hat{A}_0^a - D_0 A_0^a,
\]
which, for the moment, constitute a compact way of identifying the velocities \( \hat{A}_0^a, \hat{A}_1^a, \) and \( \hat{A}_3^a \).

B. The Hamiltonian density of the SL-NANM

The canonically conjugate momenta are
\[
\Pi_0^a = \frac{\partial \mathcal{L}}{\partial \dot{A}_0^a} = E_3^1 \frac{A_0^a}{A_3^1},
\]
\[
\Pi_1^k = \frac{\partial \mathcal{L}}{\partial \dot{A}_1^k} = E_3^1 \frac{A_1^k}{A_3^1},
\]
\[
\Pi_0^i = \frac{\partial \mathcal{L}}{\partial \dot{A}_0^i} = E_3^1 \frac{A_0^i}{A_3^1},
\]
with nonzero Poisson brackets (PBs)
\[
\{ A_0^a(x, t), \Pi_0^b(y, t) \} = \delta^a_b \delta^3(x-y), \quad \{ A_1^i(x, t), \Pi_1^j(y, t) \} = \delta^i_j \delta^3(x-y), \quad \{ A_0^a(x, t), \Pi_0^b(y, t) \} = \delta^a_b \delta^3(x-y).
\]

Next we solve for the velocities. From Eqs. (15), (16) and (17) we (respectively) obtain
\[
\left( \frac{A_0^a}{A_3^1} \frac{A_0^1}{A_3^a} - \frac{A_1^a}{A_3^1} \frac{A_1^1}{A_3^a} \right) - D_3 A_0^a = \frac{\Pi_0^1}{A_0^1} A_3^a,
\]
\[
\hat{A}_1^k - D_k A_0^1 = \frac{\Pi_1^k}{A_0^1} A_3^a = \Pi_1^k,
\]
\[
\hat{A}_0^i - D_i A_0^0 = \frac{\Pi_0^i}{A_0^0} A_3^a = \Pi_0^i.
\]

From (20) and (21) we can solve for
\[
\hat{A}_1^k = \Pi_1^k + D_k A_0^1 + \frac{\Pi_1^k}{A_0^1} A_3^a,
\]
\[
\hat{A}_0^i = \Pi_0^i + D_i A_0^0 + \frac{\Pi_0^i}{A_0^0} A_3^a.
\]

We can substitute these velocities into Eq. (19), but we cannot solve for all the \( \hat{A}_0^a \) which enter into the sum \( A_0^a \hat{A}_0^a \). At most we could solve for one velocity, say \( A_0^a \), in terms of the remaining \((N-1) A_0^a\). This is consistent with the existence of \((N-1)\) primary constraints \( \Phi_1^a \), which we choose as
\[
\Phi_1^a = \Pi_0^a - \frac{\Pi_1^a}{A_0^0} A_0^a,
\]
arising from Eq. (15). It is more convenient to consider the solved velocity as \( \hat{A}_3^a \), encoded in the definition of \( E_3^1 \) and written in terms of the canonical variables as
\[
E_3^1 = \frac{\Pi_0^1}{A_0^1} A_3^a,
\]
via the remaining relation (15) corresponding to \( a = 1 \).

From Eqs. (15), (16) and (17) we can express the electric fields in terms of the canonical momenta as
\[
E_3^a = A_3^1 \left( \frac{\Pi_0^1}{A_0^1} \right) A_3^a, \quad E_1^a = \Pi_1^i + \left( \frac{\Pi_1^i}{A_0^0} \right) A_1^a, \quad E_0^a = \Pi_0^i + \left( \frac{\Pi_0^i}{A_0^0} \right) A_0^a.
\]
where we have used Eq. (24).

In Appendix B we show that the above definitions of the electric fields \( E_i^a \) in terms of the canonical momenta \( \Pi^{ai} \), together with the canonical algebra (18) lead to the following PB relations

\[
\{ A_i^a(x, t), A_j^b(y, t) \} = 0, \quad \{ E_i^a(x, t), E_j^b(y, t) \} = 0, \quad \{ A_i^a(x, t), E_j^b(y, t) \} = -\delta_j^a \delta^b \delta(x - y),
\]  
(27)

which reproduces the YM algebra (A29). In the following we will also need the PB of the variables \( A_0^a \), \( \Pi_0^a \) with \( A_i^a \) and \( E_i^a \), which are summarized in Eqs. (B9) and (B10) of Appendix B.

The next step is to obtain the Hamiltonian density

\[
\mathcal{H} = \Pi_0^a \dot{A}_0^a + \Pi_1^a \dot{A}_1^a + \Pi^\alpha_k \dot{A}_k^\alpha - L(A_0^a, A_i^a),
\]  
(28)

where \( L(A_0^a, A_1^a, A_2^a) \) is given in Eq. (9). The main goal is to rewrite this Hamiltonian in terms of the fields \( E_i^a \) and \( A_k^\alpha \) in order to compare with the YM result (A28). From Eqs. (15), (16) and (17), we substitute the momenta into the above equation obtaining

\[
\mathcal{H} = E_3^a \left( \frac{A_0^a}{A_3^a} \dot{A}_0^a - \frac{A_1^a}{A_3^a} \dot{A}_1^a - \frac{A_2^a}{A_3^a} \dot{A}_2^a \right) + E_3^a \left( E_1^a - E_3^a \frac{A_1^a}{A_3^a} \right) \dot{A}_1^a + E_3^a \left( E_2^a - E_3^a \frac{A_2^a}{A_3^a} \right) \dot{A}_2^a - L(A_0^a, A_i^a),
\]  
(29)

which can be further rearranged as

\[
\mathcal{H} = \frac{1}{2} E_i^a E_i^a + \frac{1}{2} E_i^a E_i^a + \frac{1}{2} E_3^a E_3^a + \frac{1}{2} B_k^a B_k^a + J_0^a A_0^a - J_1^a A_1^a - J_2^a A_2^a - J_3^a A_3^a
\]  
(30)

which is the same as (33) up to a factor of two, for the electric fields, except for the following facts: (i) the coordinates \( A_0^a \) are dynamical instead of being Lagrange multipliers, (ii) the Gauss functions, defined as

\[
\Omega^a = (D_i E_i^a - J_0^a) = E_0^a,
\]  
(33)

are not constraints in the SL-NANM and (iii) we have the additional primary constraints (24).

Then we need to continue the Hamiltonian analysis by applying Dirac’s procedure starting from the extended Hamiltonian density

\[
\mathcal{H}_E = \frac{1}{2} E_i^a E_i^a + \frac{1}{2} B_k^a B_k^a - J_0^a A_0^a - A_0^a \Omega, \quad \mu^a \left( \Pi_0^a - \frac{\Pi_1^a}{A_0^a} \right).
\]  
(34)

The evolution of the primary constraints yields

\[
\dot{\Phi}_i^a = \left\{ \Phi_i^a, \int d^3 y \left( \frac{1}{2} E_k^a E_k^a + \frac{1}{2} B_k^a B_k^a - J_0^a A_0^a - A_0^a (D_i E_i^a - J_0^a) \right) \right\},
\]  
(35)

\[
\dot{\Phi}_0^a = \left\{ \Phi_0^a, \int d^3 y \left( -A_0^a \Omega - A_0^a \Omega \right) \right\},
\]  
(36)

the calculation of which requires the following PBs calculated in Appendix B

\[
\{ \Phi_i^a, A_k^a \} = 0, \quad \{ \Phi_i^a, E_k^a \} = 0, \quad \{ \Phi_i^a, B_k^a \} = 0,
\]  
(36)
In this way, the only contribution arises from the terms proportional to $A_0^a$ in the Hamiltonian density. The result

$$\dot{\Phi}_1^a = -\frac{A_0^b}{A_0^1} \Omega^1 + \delta^{ \tilde{a} \tilde{b}} \Omega^\tilde{b}$$

(37)

produces secondary constraints, which we write as

$$\Phi_2^\tilde{a} = A_0^\tilde{a} - \frac{A_0^1}{\Omega^1} \Omega^{\tilde{a}}.$$  

(38)

Next we calculate the time evolution of $\dot{\Phi}_2^\tilde{a}$ using the relations

$$\{\Omega^{\tilde{a}}, \Phi_1^c\} = 0, \quad \{\Phi_1^a, A_0^b\} = -\delta^{ \tilde{a} \tilde{b}}, \quad \{\frac{A_0^1}{\Omega^1} \Omega^{\tilde{a}}, \Phi_1^c\} = -\frac{\Omega^{\tilde{a}}}{\Omega^1} A_0^c \Omega^{\tilde{b}}.$$  

(39)

which are included in Appendix B. We obtain

$$\dot{\Phi}_2^\tilde{a} = W^{\tilde{a}} + \mu^{\tilde{a}} + \frac{A_0^1 A_0^c}{(A_0^1)^2} \mu^{\tilde{c}}.$$  

(40)

In fact, we can solve

$$\left(\delta^{ \tilde{a} \tilde{c}} + \frac{A_0^c A_0^b}{(A_0^1)^2}\right) \mu^{\tilde{c}} = -W^{\tilde{a}}$$

(41)

for the arbitrary functions $\mu^{\tilde{a}}$, concluding that

$$\mu^{\tilde{a}} = -\left(\delta^{ \tilde{a} \tilde{c}} - \frac{1}{(A_0^1)^2} A_0^c A_0^b\right) W^{\tilde{a}}.$$  

(42)

In this way the Dirac method stops and we are left with $2(N-1)$ constraints

$$\Phi_1^q = \Pi_0^q - \Pi_1^1 A_0^1, \quad \Phi_2^\tilde{a} = A_0^\tilde{a} - \frac{A_0^1}{\Omega^1} \Omega^{\tilde{a}},$$

(43)

which are second class. Thus the number of d.o.f. per point of the SL-NANM is

$$\#d.o.f. = \frac{1}{2} (2(4N-1) - 2(N-1)) = 3N,$$

(44)

which does not correspond to the number of d.o.f. of the SO(N) Yang-Mills theory.

The next step is to set the constraints (42) strongly equal to zero, in order to eliminate the variables $A_0^a$ and $\Pi_0^c$, and to subsequently introduce the corresponding Dirac brackets among the remaining variables. To this end we require the matrix constructed with the PB of the constraints

$$M = \begin{bmatrix} R^{\tilde{a} \tilde{b}} & T^{\tilde{a} b} \\ -T^{b \tilde{a}} & S^{\tilde{a} \tilde{b}} \end{bmatrix} = \begin{bmatrix} R & T \\ -T^T & S \end{bmatrix},$$

(45)

where

$$R^{\tilde{a} \tilde{b}} = \{\Phi_1^\tilde{a}, \Phi_1^\tilde{b}\}, \quad T^{\tilde{a} b} = \{\Phi_1^\tilde{a}, \Phi_2^b\}, \quad S^{\tilde{a} \tilde{b}} = \{\Phi_2^\tilde{a}, \Phi_2^\tilde{b}\}. $$

(46)

The required calculations produce

$$R^{\tilde{a} \tilde{b}} = 0, \quad T^{\tilde{a} b} = \{\Phi_1^\tilde{a}, \Phi_2^b\} = -\left(\delta^{ \tilde{a} \tilde{b}} + \frac{A_0^c A_0^b}{(A_0^1)^2}\right) = T^{b \tilde{a}},$$

$$S^{\tilde{a} \tilde{b}} = \left(\frac{A_0^1}{\Omega^1}\right)^2 \left(\frac{C^{\tilde{a} \tilde{b} m} + C^{\tilde{a} \tilde{b} m} \Omega^b}{\Omega^1} - \frac{C^{\tilde{a} \tilde{b} m} \Omega^a}{\Omega^1}\right)(D_i E_i^m).$$
according to the results in Appendix B. The matrix $T$ is invertible, yielding

$$(T^{-1})_{\bar{a}\bar{b}} = -\left(\delta_{\bar{a}\bar{b}} - \frac{A_{0\bar{a}} A_{0\bar{b}}}{A_{0} A_{0}}\right),$$

(47)
in such a way that

$$M^{-1} = \begin{bmatrix} T^{-1}ST^{-1} & -T^{-1} \\ T^{-1} & 0 \end{bmatrix}.$$  

(48)

The Dirac bracket is

$$\{A(x), B(y)\}^* = \{A(x), B(y)\} - \{A, \phi_{i}^{\bar{a}}\} (T^{-1}ST^{-1})_{\bar{a}\bar{b}} \{\phi_{i}^{\bar{b}}, B\} + \{A, \phi_{i}^{\bar{a}}\} (T^{-1})_{\bar{a}\bar{b}} \{\phi_{i}^{\bar{b}}, B\},$$

(49)

which leads to the result

$$\{A(x), B(y)\}^* = \{A(x), B(y)\},$$

(50)

for the YM variables $A_{i}^{a}$ and $E^{bij}$. The above conclusion arises from the fact that each of the additional PBs in Eq. (49) includes a contribution from $\phi_{i}^{\bar{a}}$, which has zero PB with those variables, according to Eq. (B11). In other words, we recover the algebra

$$\{A_{i}^{a}(x, t), A_{j}^{a}(y, t)\}^* = 0 = \{E^{ai}(x, t), E^{bij}(y, t)\}^*, \quad \{A_{i}^{a}(x, t), E^{bij}(y, t)\}^* = -\delta_{i}^{j}\delta^{a}\delta(x - y),$$

(51)

corresponding to the YM theory given in Eq. (A29) of Appendix A. Having set the constraints (42) strongly equal to zero, the extended Hamiltonian (34) now reduces to

$$H_{E} = \frac{1}{2}E_{i}^{a}E_{i}^{a} + \frac{1}{2}B_{k}^{a}B_{k}^{a} - J_{i}^{a}A_{i}^{a} - A_{0}^{a}\Omega^{a},$$

(52)

but we are still missing the Gauss laws $\Omega^{a} = 0$, because the $A_{0}^{a}$ are dynamical degrees of freedom.

### C. The evolution of the Gauss functions $\Omega^{a}$ in the SL-NANM

Next we study the time evolution of the Gauss functions, starting from the Hamiltonian density (32). A direct use of (A29) leads to

$$\dot{\Omega}^{a} = -gC^{abc}A_{0}\Omega^{c} - D_{\mu}J^{\mu a} - \int d^{3}y \{\Omega^{a}(x), A_{0}(y)\} \Omega^{b}(y).$$

(53)

where $\Omega^{a}(x) \rightarrow (\delta^{ac}\partial_{k} + gC^{abc}A_{k}^{b}) E_{k}^{c}$ inside a PB because $J^{0a}$ has been considered as an external current. Since $\{A_{0}, A_{k}^{b}\} = 0$, we only need the brackets $\{E_{k}^{c}(x), A_{0}(y)\}$. Using Eq. (B9) we obtain

$$\dot{\Omega}^{a} = -gC^{abc}A_{0}^{b}\Omega^{c} - D_{\mu}J^{\mu a} + D_{k}\left(\frac{A_{k}^{a}}{A_{0}}\right)\Omega^{k}.$$  

(54)

The above equations guarantee that by (i) imposing current conservation $D_{\mu}J^{\mu a} = 0$ at some initial time $t = 0$ and (ii) demanding that the Gauss laws $\Omega^{a} = 0$ hold at $t = 0$, we obtain $\partial_{k}\Omega^{a} = 0$ ($a = 1, 2, ..., N$) as well at $t = 0$. This is enough to prove that with these two initial conditions, the Gauss laws will hold for all time. In this way we can recover the SO($N$) Yang-Mills theory by imposing the Gauss laws as Hamiltonian constraints, with arbitrary functions $N^{a}$ adding $-N^{a}\Omega^{a}$ to $H_{E}$ and redefining $A_{0}^{a} + N^{a} = \Theta^{a}$. This leads to

$$H_{E} = \frac{1}{2}(E^{2} + B^{2}) - \Theta^{a}\Omega^{a} + J_{i}^{a}A_{i}^{a},$$

(55)

where the $\Theta^{a}$ are now arbitrary functions, and thus we get back to the YM Hamiltonian density (A29). The subsequent emergence of the SO($N$) YM theory guarantees current conservation for all times, as a consequence of the equations of motion.
From the perspective of the GB modes, the situation in the SL-NANM is as follows. We have started from a theory invariant under SO($N,3N$) defined by Eqs. (1) and (2). Solving the constraint (2) in terms of $A_{3}^{0}$ means that we have the nonzero vacuum expectation value $\langle A_{3}^{0} \rangle = M$, which breaks the symmetry down to SO($N,3N-1$), with the appearance of $(4N(4N-1)/2 - (4N-1)(4N-2)/2) = 4N - 1$ GBs. Nevertheless, the SL-NANM phase space still contains $2(N-1)$ second-class constraints [Eq. (12)] which can be imposed strongly, yielding a reduced phase space with $\frac{1}{2}(2(4N-1) - 2(N-1)) = 3N$ coordinates per point. A similar analysis yields $3N$ as the number of independent GB modes in each realization of the NAM. This conclusion is consistent with Ref. [15], where the final independent GB modes are denoted by $a^i_{\mu}$ ($i = 1,...,N$, $\mu = 1,2,3$) for the time-like case and $a^i_{\mu\nu}$ ($i = 1,...,N$, $\mu\nu = 0,1,2$) for the space-like case. Once more, we verify that in order to regain the $2N$ independent vector bosons of the YM theory, we still have to impose the $N$ Gauss laws $\Omega^a = 0$ as first-class constraints. In this way we are left with $\frac{1}{2}(6N - 2N) = 2N$ coordinates per point.

To summarize, the emergence of the SO($N$) YM theory from the SL-NANM can be established only after imposing both current conservation and the Gauss laws as initial conditions.

### III. A Unified Description of the Non-Abelian Nambu Models

A procedure similar to that presented in the previous section for the SL-NANM can be repeated for the TL-NANM and the LL-NANM, with identical results. The standard variables $A_i^a$, $E_i^a$ of the YM theory can be expressed in terms of the canonical variables of each version of the NAM, the algebra of which induces their brackets to be those of Eqs. (A29). These transformations also allow the canonical Hamiltonian density of the NAM to be rewritten in terms of the GB modes in each realization of the NAM. This conclusion is consistent with Ref. [15], where the final independent GB modes are denoted by $a^i_{\mu}$ ($i = 1,...,N$, $\mu = 1,2,3$) for the time-like case and $a^i_{\mu\nu}$ ($i = 1,...,N$, $\mu\nu = 0,1,2$) for the space-like case. Once more, we verify that in order to regain the $2N$ independent vector bosons of the YM theory, we still have to impose the $N$ Gauss laws $\Omega^a = 0$ as first-class constraints. In this way we are left with $\frac{1}{2}(6N - 2N) = 2N$ coordinates per point.

This has motivated us to search for a unified and simpler discussion of the generic NAM. To this end, we find it convenient to generalize the parametrization (7) to all cases in the form

$$ A^a_0 = B^a \left( 1 + \frac{N}{4B^2} \right), \quad A^a_3 = B^a \left( 1 - \frac{N}{4B^2} \right), \quad N = (A^a_i A^b_i + n^2 M^2), \quad 4B^2 \pm N \neq 0, \quad (56) $$

which certainly satisfies the condition (4) and is written in terms of the $3N$ independent GBs ($B^a$ and $A^a_i$).

The relation between the above parametrization and the purely Goldstonic d.o.f. introduced in Ref. [15], which we relabel as $a^i_{\mu}$, can be established as follows. The $4N$ fields $a^i_{\mu}$ are subjected to the $N$ additional constraints

$$ n^\mu a^i_{\mu} = 0, \quad (57) $$

leaving only $3N$ independent GB modes. In terms of them, the original fields are written as

$$ A^a_\mu = a^i_{\mu} - \frac{n^b}{n^2} (M^2 - n^2 a^2) \frac{1}{2}, \quad n^b_{\mu} = n_{\mu} s^b, \quad s^2 = 1, \quad n^2 \neq 0, \quad (58) $$

which satisfy the condition (2). Equation (58) can be inverted to produce

$$ a^i_{\mu} = A^a_\mu + \frac{n^b}{n^2} (n.A), \quad (59) $$

which allows us to express $a^i_{\mu} = a^i_{\mu}(B^c, A^a_i)$ by employing Eq. (56).

#### A. The equations of motion

After the substitution of (56) into the Lagrangian density (3), the variation of the corresponding action with respect to $A^a_\mu$ yields

$$ 0 = \int d^4 x (D_{\mu} F_{\mu\nu} - J^\nu) \delta A^a_\nu, \quad (60) $$
where the $\delta A^a_i$ are not all independent. In our case, Eq. (60) leads to

$$\delta A^a_0 = \left[ 1 + \frac{N}{4B^2} \right] \delta^{ab} - \frac{N}{4B^2} \frac{2B^a B^b}{B^2} \right] \delta B^b + \frac{B^a}{2B^2} A^i_b \delta A^b_i,$$

$$\delta A^a_i = \left[ 1 - \frac{N}{4B^2} \right] \delta^{ab} + \frac{N}{4B^2} \frac{2B^a B^b}{B^2} \right] \delta B^b - \frac{B^a}{2B^2} A^i_b \delta A^b_i,$$

in terms of the independent variations $\delta A^a_i$ and $\delta B^a$. In this way the equations of motion are

$$\delta A^a_i : E^{ia} + \frac{B^b}{2B^2} \left[ E^{0b} - E^{3b} \right] A^a_i = 0,$$

$$\delta B^a : 0 = \left( 1 + \frac{N}{4B^2} \right) \delta^{ab} - \frac{N}{4B^2} \frac{2B^b B^a}{B^2} \right) E^{0b} + \left( 1 - \frac{N}{4B^2} \right) \delta^{ab} + \frac{N}{4B^2} \frac{2B^b B^a}{B^2} \right) E^{3b},$$

in the notation of Eq. (12).

Let us recall that in the case of the SO($N$) YM theory the equations of motion are just given by $E^{iab} = 0$. Also, the above equations of motion do not imply current conservation $D_a J^a = 0$, basically because the condition (61) breaks non-Abelian gauge invariance. A way to recover the YM equations of motion together with gauge invariance is to impose the Gauss laws $E^{0a} = 0$. In this way, under the conditions $4B^2 \pm N \neq 0$, Eq. (64) yields the solution $E^{3b} = 0$. These two conditions in Eq. (63) provide the final set $E^{iab} = 0$.

**B. The Hamiltonian density**

In order to unify the notation when going to the Hamiltonian formulation we introduce the 3N d.o.f. $\Phi^a_A$, $A = 1, 2, 3$,

$$\Phi^a_1 = A^a_1, \quad \Phi^a_2 = A^a_2, \quad \Phi^a_3 = B^a,$$

in such a way that the coordinate transformation

$$A^a_i = A^a_i(\Phi^a_A),$$

arising from Eq. (60) is invertible. In fact, the inverses are

$$\Phi^a_1 = A^a_1, \quad \Phi^a_2 = A^a_2, \quad \Phi^a_3 = \frac{A^a_3}{2 \sqrt{A^b_3 A^b_3}} \left( \sqrt{A^b_3 A^b_3} + \sqrt{A^b_1 A^b_1} + n^2 M^2 \right).$$

We also have

$$A^a_0 = A^a_0(\Phi^a_A),$$

in terms of Eq. (66), according to the first relation in Eq. (60). The relevant property of the transformation (60) is that

$$A^a_i = \frac{\partial A^a_i}{\partial \Phi^a_B} \Phi^a_B \rightarrow \frac{\partial A^a_i}{\partial \Phi^a_B} = \frac{\partial A^a_i}{\partial \Phi^a_B},$$

together with the invertibility of the velocities

$$\dot{\Phi}^a_A = \frac{\partial \Phi^a_A}{\partial A^a_i} A^b_i.$$
and the YM canonical algebra \[ A_{28} \] and \[ A_{29} \]. After making the substitutions \[ 66 \] and \[ 68 \], the Lagrangian density \[ 3 \] can be rewritten as

\[
\mathcal{L}_\text{NANM}(\Phi, \dot{\Phi}) = \frac{1}{2} E_i^a E_i^a - \frac{1}{2} B_i^a B_i^a - J^{\alpha\mu} A_\mu^a, \tag{71}
\]

where

\[
E_i^a = \dot{A}_i^a - D_i A_0^a, \quad B_i^a = \frac{1}{2} \epsilon_{ijk} F_{jk}^a, \tag{72}
\]

with \( E_i^a = E_i^a(\Phi, \dot{\Phi}) \) and \( B_i^a = B_i^a(\Phi) \). The canonically conjugate momenta are calculated as

\[
\Pi_A^a = \frac{\partial \mathcal{L}_\text{NANM}(\Phi, \dot{\Phi})}{\partial \dot{A}_i^a} = E_i^a \partial A_i^a = E_i^a \partial A_i^c = \frac{1}{2} \epsilon_{ij} F_{ij}^a, \tag{73}
\]

employing \[ 60 \]. The inverse of Eq. \[ 60 \] allows us to write the colored electric fields \( E_i^a \) as functions of the momenta \( \Pi_A^a \) of the NANM

\[
E_i^a(\Phi, \Pi) = \frac{\partial \Phi^a}{\partial A_i^a} \Pi_A^a. \tag{74}
\]

The Wronskian of the system is

\[
\text{det} \left( \frac{\partial^2 \mathcal{L}_\text{NANM}(\Phi, \dot{\Phi})}{\partial \dot{\Phi}_A^a \dot{\Phi}_B^b} \right) = \text{det} \left( \frac{\partial \Pi_A^a}{\partial \dot{\Phi}_B^b} \right) = \text{det} \left( \frac{\partial \dot{A}_i^c}{\partial \dot{A}_j^d} \frac{\partial A_j^d}{\partial \dot{\Phi}_B^b} \right) = \text{det} \left( \frac{\partial A_i^c}{\partial \dot{\Phi}_B^b} \right) \neq 0, \tag{75}
\]

as shown in Appendix C. In this way, the NANM is exhibited as a regular system in the parametrization \[ 56 \], so that no constraints are present. The NANM Hamiltonian density is

\[
\mathcal{H}_\text{NANM} = \Pi_A^a \dot{\Phi}_A^a - \left( \frac{1}{2} E_i^a E_i^a - \frac{1}{2} B_i^a B_i^a - J^{\alpha\mu} A_\mu^a \right), \tag{76}
\]

which we rewrite in successive steps

\[
\mathcal{H}_\text{NANM} = \Pi_A^a \frac{\partial \Phi^a}{\partial A_i^a} \dot{A}_i^a - \left( \frac{1}{2} E_i^a E_i^a - \frac{1}{2} B_i^a B_i^a - J^{\alpha\mu} A_\mu^a \right), \tag{77}
\]

\[
\mathcal{H}_\text{NANM} = E_i^a \dot{A}_i^a - \left( \frac{1}{2} E_i^a E_i^a - \frac{1}{2} B_i^a B_i^a - J^{\alpha\mu} A_\mu^a \right), \tag{78}
\]

\[
\mathcal{H}_\text{NANM} = E_i^a \left( E_i^a + D_i A_0^a \right) - \left( \frac{1}{2} E_i^a E_i^a - \frac{1}{2} B_i^a B_i^a - J^{\alpha\mu} A_\mu^a \right), \tag{79}
\]

\[
\mathcal{H}_\text{NANM}(\Phi, \Pi) = \frac{1}{2} E_i^a E_i^a + \frac{1}{2} B_i^a B_i^a - \left( D_i E_i^a - J^{\alpha\mu} A_\mu^a \right) A_0^a + J^{\alpha\mu} A_\mu^a, \tag{80}
\]

where we have used Eqs. \[ 70 \], \[ 72 \], and \[ 73 \], together with an integration by parts in the term containing the covariant derivative. The dependence of \( \mathcal{H}_\text{NANM} \) on the canonical variables \( \Phi, \Pi \) is clearly established by the change of variables \[ 66 \], \[ 68 \] and \[ 73 \]. The canonical variables of NANM satisfy the standard PBs

\[
\{\Phi_A^a(x), \Phi_B^b(y)\} = 0, \quad \{\Pi_A^a(x), \Pi_B^b(y)\} = 0, \quad \{\Phi_A^a(x), \Pi_B^b(y)\} = \delta^{ab} \delta_{AB} \delta^3(x - y). \tag{81}
\]

Now we can consider the NANM Hamiltonian density \[ 79 \] from the perspective of the fields \( A_i^a \) and \( E_i^a \). The relation arising from the velocity-dependent term of the NANM Hamiltonian action,

\[
\int d^4x \Pi_A^a \dot{\Phi}_A^a = \int d^4x E_i^a \dot{A}_i^a = \int d^4x (-E_i^a) A_i^c, \tag{82}
\]

[used previously in obtaining Eq. \[ 77 \]], establishes \( (-E_i^a) \) as the canonically conjugate momenta of \( A_i^c \). In this way Eq. \[ 79 \] can be seen as a Hamiltonian density \( \mathcal{H}(A, E) \) obtained from \( \mathcal{H}_\text{NANM}(\Phi, \Pi) \) via the substitution of the phase-space transformations

\[
(\Phi, \Pi) \rightarrow (A, E), \tag{83}
\]
which follow from the inverses of Eqs. (66) and (73) plus Eq. (68),

\[ A_i^0 = \frac{A_i^3}{\sqrt{A_3^b A_3^b}} \left( \sqrt{A_i^b A_i^b + n^2 M^2} \right), \]

in terms of the new variables. But, since the transformations (73) are generated by the change of variables (66) in coordinate space, we know from classical mechanics that the full transformation in phase space is a canonical transformation. In this way we automatically recover the PBs

\[ \{ A_i^a(x), A_j^b(y) \} = 0, \quad \{ E^{ai}(x), E^{bj}(y) \} = 0, \quad \{ A_i^a(x), E^{bj}(y) \} = -\delta^{ab} \delta^i_j (x - y) \]

from Eq. (83). To summarize, from each Hamiltonian version of the NAM, defined by the different values of \( n^2 \), we can regain (via a canonical transformation) the Hamiltonian density (79) together with the canonical algebra (A29). The Hamiltonian density (79) differs from the YM Hamiltonian density (A28) only in the fact that the Gauss laws \( \Omega^b = (D_i E_i^b - J^{i0}) = 0 \) do not appear as first-class constraints, because \( A_i^0 \) are not arbitrary Lagrange multipliers, but rather as functions of the coordinates, as shown in Eq. (83).

C. The evolution of the Gauss functions \( \Omega^a \)

The calculation follows the same steps dictated by Eq. (53) in the case of the SL-NANM. In the parametrization (56), the result is

\[ \dot{\Omega}^a = -gC^{abc} A_i^b \Omega^c - D_\mu J^{\mu a} + D_3 \left( \frac{A_i^3}{A_3^a} \Omega^a \right) - D_5 \left( \frac{A_i^5}{A_5^a} A_i^a \right) A_i^b \Omega^b + D_1 \left( \frac{A_i^1}{A_1^a} A_i^a \right) \]

where \( A_i^a \) is given by Eq. (83). The above evolution equation leads to the same statements regarding the equivalence of the SO(N) YM theory with the NAM as those stated after Eq. (83) in the case of the SL-NANM.

IV. SUMMARY AND FINAL COMMENTS

The possible interpretation of gauge particles (e.g., photons and gravitons) as the GB modes arising from some spontaneous symmetry breaking is an interesting hypothesis that would provide a dynamical setting for the gauge principle.

In this paper we have taken the Nambu approach, whereby spontaneous SLSB is incorporated in an effective way in the model by means of a nonlinear constraint. These models can be understood as generalizations of the nonlinear sigma model describing pion interactions. The challenge posed by this setting is to show the conditions under which the corresponding Nambu model is equivalent to the unbroken gauge theory. Such conditions have been studied using perturbation theory for electrodynamics and YM theories, for example, in Refs. [10, 11, 14–16]. The main result is that, to the order considered (usually the tree-level or one-loop corrections) and after imposing the Gauss laws plus current conservation, the violations of Lorentz symmetry are unobservable, so that the corresponding Nambu model reproduces the corresponding gauge theory, with the gauge bosons realized as the corresponding Goldstone bosons.

In this work we have generalized the nonperturbative Hamiltonian analysis developed for the Abelian Nambu model in Ref. [22] to the non-Abelian case. Also, we have made an important conceptual and practical improvement in the method of dealing with the relation between the NAM and the corresponding YM theory. On the other hand, no discussion is provided here about the possible perturbative equivalence of the corrected Nambu model and the YM theory in a fixed gauge.

In Sec. II we considered the specific case of the SL-NANM by solving the nonlinear constraint (2) in terms of \( A_{\mu=3}^{a=1} \) and starting with the remaining \( 4N - 1 \) d.o.f. per point in coordinate space. The Lagrangian equations of motion were obtained, yielding different results than from the YM equations of motion, as expected. The canonical momenta and the canonical Hamiltonian were subsequently constructed, with the appearance of \( 2(N - 1) \) second-class constraints. The standard variables of the YM theory, \( A_i^a \) and \( E^{bj} \), \( i = 1, 2, 3 \), were written in terms of the canonical variables of the SL-NANM, the canonical algebra of which induces the standard YM algebra for the former variables at the level of PBs. Appendix B includes a summary of the required PBs which prove the previous statement. The second-class constraints were further strongly imposed by introducing Dirac brackets, whose values for the variables \( A_i^a \) and \( E^{bj} \)
turned out to be the same as the previously calculated PBs. The final extended Hamiltonian for the SL-NANM, rewritten in terms of the variables $A^a_i$ and $E^{bij}$, has the same form as the standard YM Hamiltonian, except that the Gauss laws $\Omega^a = 0$ do not appear as first-class constraints. The time evolution of the functions $\Omega^a$, according to the SL-NANM dynamics, were calculated, yielding the result that after demanding current conservation, the imposition of the Gauss laws at some initial time yields $\Omega^a = 0$ for all times. The final 3N d.o.f. in coordinate space of the NANM were recovered, since $\frac{1}{2}(2(4N - 1) - 2(N - 1)) = 3N$. It was emphasized that a similar analysis could be carried out for each of the remaining cases corresponding to the time-like and light-like versions of the NANM.

Section III presented a substantial conceptual and practical improvement over the previous individual calculations to study the relation between NANNM and YM theories. Starting from an alternative parametrization that solves the constraint (2) for arbitrary values of $n^a$ in terms of 3N d.o.f. $\Phi_A^a$, $A = 1, 2, 3$, we showed that the calculation of the NANM canonical momenta $\Pi_A^a$ can be written in such a way that the chosen parametrization induces a direct relation between the YM variables $A^a_i$ and $E^{bij}$ and the canonical variables of the NANM. Since the phase-space transformations are induced by a coordinate transformation (the chosen parametrization), we know from classical mechanics that the phase-space transformation $(\Phi_A^a, \Pi_A^a) \rightarrow (A^a_i, E^{bij})$ is a canonical transformation, once the $E^{bij}$ are recognized as the momenta canonically conjugate to the $A^a_i$ via the kinetic part of the NANM Hamiltonian action. In this way, one immediately concludes that the resulting algebra of the $(A^a_i, E^{bij})$ has to be the canonical one, without requiring the detailed and tedious calculations that were necessary in the discussion of the previous section. It is interesting to observe that the identification of the canonical transformation is independent of the detailed structure of the chosen parametrization, as soon as it provides an invertible change of coordinates $\Phi_A^a = \Phi'_A^a(A^a_i)$. Another useful property of this parametrization is that it exhibits the NANM as a regular theory (i.e., no constraints appear in the Hamiltonian analysis), since it includes just the necessary 3N d.o.f. of the NANM. This was proved in Appendix C. The canonical NANM Hamiltonian, rewritten in terms of the YM variables $(A^a_i, E^{bij})$, again has the same form as the YM Hamiltonian, except that the Gauss laws do not arise as first-class constraints. The time evolution of the functions $\Omega^a$ was also calculated, with similar results as in the SL-NANM.

The relation between our approach and the method of Ref. [15], which also included pure Goldstone field modes, was elucidated in the paragraphs after Eqs. (55) and (56).

To summarize, a nonperturbative equivalence between the SO($N$) YM theory and the corresponding NANM has been established, after current conservation and the Gauss laws are imposed as initial conditions for the latter. Actually, the Gauss laws — valid now for all times — are next added as Hamiltonian constraints $-N^a\Omega^a$ to Eq. (79), with arbitrary functions $N^a$. The further redefinition $A^a_i + N^a = \Theta^a$ leads to the final YM Hamiltonian

$$H_{YM} = \frac{1}{2}(E^2 + B^2) - \Theta^a\Omega^a + J_\mu A^a_{\mu},$$

where $\Theta^a$ are now arbitrary functions. In other words, the Gauss laws are imposed à la Dirac upon the physical states $|\Psi\rangle_{phys}$ by demanding that $\Omega^a|\Psi\rangle_{phys} = 0$. Also, the emergence of the SO($N$) YM theory subsequently guarantees current conservation for all times, as a consequence of the YM equations of motion. The established equivalence is independent of any gauge fixing and supports the idea that gauge particles arise as the Goldstone bosons of a model exhibiting a spontaneous Lorentz symmetry breaking that is not physically observable.

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**Appendix A: The SO($N$) Yang–Mills theory**

We present a brief review of the Hamiltonian formulation of the standard SO($N$) YM theory. The main motivation, besides establishing some notation, is to recall the basic properties of the YM theory that have to be recovered in order to state its emergence from the different versions of the NANNM.

The Yang-Mills Lagrangian density is given by

$$L = Tr \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - J^\mu A_\mu \right],$$  \hspace{1cm} (A1)

where boldfaced quantities denote matrices in the Lie algebra of the internal symmetry group SO($N$) with $N(N-1)/2$ generators $t^a$; i.e., $M = M^a t^a$. This algebra is generated by $[t^a, t^b] = C^{abc} t^c$, where the structure constants $C^{abc}$ are
completely antisymmetric. The field strength is
\[ F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} + g[A_{\mu}, A_{\nu}], \]
and the equations of motion are
\[ D_{\mu} F^{\mu\nu} = J^{\nu}, \]
where the covariant derivative is defined as
\[ D_{\mu} M = \partial_{\mu} M + g[A_{\mu}, M]. \]
From the above definitions we obtain
\[ [D_{\mu}, D_{\nu}] M = [M, F_{\mu\nu}], \]
which leads to current conservation, \( D_{\nu} J^{\nu} = 0. \)

The expressions of Eqs. (A2) and (A4) in terms of the components of the corresponding fields are
\[ F_{\mu\nu}^{a} = \partial_{\mu} A_{a\nu} - \partial_{\nu} A_{a\mu} + gC^{abc} A_{b\mu} A_{c\nu}, \]
\[ (D_{\mu} M)^{a} \equiv D_{\mu} M^{a} = \partial_{\mu} M^{a} + gC^{abc} A_{b\mu} M^{c}. \]
The Jacobi identity for the connection \( A_{\mu} \) is
\[ C^{arb} C^{cdr} + C^{car} C^{dbr} + C^{dbr} C^{car} = 0, \]
in terms of the structure constants. The group indices \( a = 1, 2, ..., N \) are raised (lowered) by the metric \( \delta^{ab} (\delta_{ab}) \) and their position as superscripts or subscripts is just a matter of convenience in writing the corresponding expression.

Next we review the Hamiltonian version of the YM theory. The canonical momenta are given by
\[ \Pi^{a\mu} = \frac{\partial L}{\partial (\dot{A}^{a\mu})}. \]
Therefore, considering
\[ \frac{\partial(F_{\mu\nu}^{a} F_{\nu}^{\mu})}{\partial (A_{a}^{\nu})} = 4F_{a}^{\nu}, \]
we find
\[ \Pi_{0}^{a} = 0, \quad \Pi_{i}^{a} = F_{i}^{a} \equiv -E_{a}^{i}, \]
which satisfy the nonzero PBs
\[ \{ A_{0}^{a}(x, t), \Pi_{k}^{b}(y, t) \} = \delta^{ab}\delta(x - y), \quad \{ A_{i}^{a}(x, t), \Pi_{j}^{b}(y, t) \} = \delta_{ij}\delta^{ab}\delta(x - y). \]
In the following we assume that all PBs are calculated at equal times and we suppress the label \( t \) in most cases. From Eq. (A6) we get \( \dot{A}_{i}^{a} \) as
\[ \dot{A}_{i}^{a} = E_{i}^{a} + \partial_{i} A_{0}^{a} + gC^{abc} A_{c}^{i} A_{b}^{0} = E_{i}^{a} + D_{i} A_{0}^{a}. \]
We also introduce
\[ B_{k}^{a} = \frac{1}{2} \epsilon^{ijk} F_{ij}^{a}. \]
Recalling that
\[ D_{\mu}(N^{a} M^{a}) = (D_{\mu}N^{a}) M^{a} + N^{a} (D_{\mu}M^{a}) = \partial_{\mu}(N^{a} M^{a}), \quad a = 1, 2, ..., N, \]
which allows us to perform integration by parts within the action, we find the canonical Hamiltonian density
\[ \mathcal{H} = \Pi_{a}^{i} \dot{A}_{i}^{a} - \mathcal{L} = \frac{1}{2}(E^{2} + B^{2}) - A_{0}^{a}(D_{i} E_{i} - J^{0})^{a} - J_{i}^{a} A_{i}^{a}, \]
where
\[ \mathbf{E}^2 = tr(\mathbf{F}_{0i}\mathbf{F}_{0i}), \quad \mathbf{B}^2 = \frac{1}{2} tr(\mathbf{F}_{ij}\mathbf{F}^{ij}). \] (A17)

We employ Dirac’s method to construct the canonical theory, due to the fact that primary constraints
\[ \Sigma^a = \Pi_0^a \simeq 0, \] (A18)
are present. The extended Hamiltonian density is given by
\[ \mathcal{H}_E = \frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2) - A_0^a(D_iE_i - J^0)_a + J_i^aA_a^i + \lambda^a \Sigma_a, \] (A19)
where \( \lambda^a \) are arbitrary functions. The evolution condition of the primary constraints
\[ \dot{\Sigma}^a(x) = \{\Sigma^a(x), \int d^3 y H_E(y)\} \simeq 0, \] (A20)
leads to the Gauss laws
\[ \Omega^a = (D_iE_i - J^0)^a \simeq 0. \] (A21)
It is not difficult to prove that Eqs. (A18) and (A21) are the only constraints present and that they constitute a first-class set. In fact, calculating the time evolution of \( \Omega^a \) yields
\[ \dot{\Omega}^a = \left\{ \Omega^a(x), \int d^3 y H_E(y) \right\} = \left\{ \Omega^a(x), \int d^3 y \left( \frac{1}{2} \mathbf{B}^2 - A_0^b \Omega^b + J_i^aA_a^i \right) \right\} - \partial_0 J_0^a. \] (A22)
From the PBs (A12) together with Eq. (A18) we obtain
\[ \int d^3 y \left\{ \Omega^a(x), \mathbf{B}^2(y) \right\} = 0. \] (A23)
The PB of the constraints (A21) produces
\[ \left\{ D_iE_i^a(x), \int d^3 y \mathbf{D}_jE_j^b(y) \right\} = C^{abc}D_kE_k^c(x), \] (A24)
which leads to
\[ \int d^3 y \left\{ \Omega^a(x), \Omega^b(y) \right\} M^b(y) = C^{abc}M^b[\Omega^c + J^0_0](x). \] (A25)
In this way,
\[ \dot{\Omega}^a = -\partial_0 J_0^a - C^{abc}A_0^bJ_0^c - C^{abc}A_0^b\Omega^c - D_kJ^a_k = -C^{abc}A_0^b\Omega^c - D_{\mu}J^{a\mu}, \] (A26)
which is zero, modulo the constraints and using current conservation.
Normally one fixes
\[ \Pi_0^a \simeq 0, \quad A_0^a \simeq \Theta^a, \] (A27)
with \( \Theta^a \) being arbitrary functions to be consistently determined after the remaining first-class constraints \( \Omega^a \) are fixed.
The final Hamiltonian density is
\[ \mathcal{H}_E = \frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2) - \Theta^a(D_iE_i - J^0)_a + J_i^aA_a^i. \] (A28)
Once \( \Pi_0^a \) and \( A_0^a \) are strongly fixed, the Dirac brackets of the remaining variables are
\[ \{A_i^a(x, t), A_j^b(y, t)\}^* = 0, \quad \{E^{ai}(x, t), E^{bj}(y, t)\}^* = 0, \quad \{A_i^a(x, t), E^{bj}(y, t)\}^* = -\delta_i^j\delta^{ab}(x - y). \] (A29)
The final count of the number of d.o.f. per point in coordinate space yields
\[ \#d.o.f. = \frac{1}{2}(2 \times 4N - 2 \times 2N) = 2N. \] (A30)
Appendix B: The bracket algebra in the SL-NANM

In this case the canonical variables are \( A^a_0, A^1_i, A^{\bar{a}}_i, \Pi^a_0, \Pi^{1k}, \Pi^{\bar{a}i} \) where \( a = 1, 2, ..., N; \bar{a} = 2, 3, ..., N; i = 1, 2, 3 \) and \( \bar{i} = 1, 2 \). The nonzero PBs are

\[
\{ A^a_0(x, t), \Pi^b_0(y, t) \} = \delta^{ab}(x - y) \delta^a, \quad \{ A^1_i(x, t), \Pi^{1k}(y, t) \} = \delta_i^k \delta^a(x - y).
\]

The YM canonical variables \((A^a_i, E^j_i)\) are given by

\[
A^1_3 = \sqrt{M^2 + \left( A^a_0 \right)^2 - \left( A^1_i \right)^2 - \left( A^{\bar{a}}_i \right)^2}, \quad A^1_i, A^{\bar{a}}_i,
\]

\[
E^1_3 = A^1_3 \left( \frac{\Pi^1_0}{A^a_0} \right), \quad E^1_i = \Pi^{1i} + \left( \frac{\Pi^1_0}{A^a_0} \right) A^1_i, \quad E^{\bar{a}}_i = \Pi^{\bar{a}i} + \left( \frac{\Pi^1_0}{A^a_0} \right) A^{\bar{a}}_i,
\]

in terms of the canonical variables of the SL-NANM.

In the following, we do not specify the coordinate dependence of each term: it is to be understood according to the following convention

\[
\{ P(x), Q(y) \} = \{ P, Q \}.
\]

Additionally, we also suppress the unit \( \delta^a(x - y) \) in coordinate space. We do not provide any details for each derivation: we only include the final results.

Our first goal is to calculate the equal-times algebra among the YM variables \((A^a_i, E^j_i)\) defined in Eq. \((B2)\) in terms of the canonical algebra \((B1)\) of the SL-NANM. To this end we first consider the PBs between the \( A^1_i \) [which is the solution of the constraint \((B1)\)] and the canonical conjugate momenta of the SL-NANM. The results are

\[
\{ \Pi^{1k}, A^1_3 \} = \frac{A^a_0}{A^1_3}, \quad \{ \Pi^a_0, A^1_3 \} = \frac{A^a_0}{A^1_3}, \quad \{ \Pi^{1k}, A^1_3 \} = \frac{A^a_0}{A^1_3}.
\]

1. The \( A - A \) sector

The algebra among the canonical coordinates \((A^1_i, A^{\bar{a}}_k)\) and \( A^1_3 \) is trivial because the former satisfy the canonical relations and \( A^1_3 = A^1_3(A^1_i, A^{\bar{a}}_k) \), so that we have \( \{ A^a_i, A^b_k \} = 0 \).

2. The \( A - E \) sector

We obtain

\[
\{ A^1_3, E^1_3 \} = 1, \quad \{ A^1_3, E^1_k \} = 0, \quad \{ A^1_i, E^1_i \} = 0, \quad \{ A^1_i, E^1_3 \} = 0, \quad \{ A^1_i, E^1_k \} = 0, \quad \{ A^{\bar{a}}_i, E^1_i \} = 0, \quad \{ A^{\bar{a}}_i, E^1_3 \} = 0, \quad \{ A^{\bar{a}}_i, E^1_k \} = 0, \quad \{ A^{\bar{a}}_k, E^1_i \} = 0, \quad \{ A^{\bar{a}}_k, E^1_3 \} = 0, \quad \{ A^{\bar{a}}_k, E^1_k \} = 0.
\]

The above relations can be summarized as

\[
\{ A^{\bar{a}}_k, E^1_i \} = \delta^{\bar{a}b} \delta^i_k.
\]

3. The \( E - E \) sector

We have

\[
\{ E^1_3, E^1_i \} = 0, \quad \{ E^1_3, E^0_i \} = 0, \quad \{ E^1_i, E^1_3 \} = 0, \quad \{ E^1_i, E^0_3 \} = 0, \quad \{ E^0_3, E^1_i \} = 0, \quad \{ E^0_3, E^1_3 \} = 0, \quad \{ E^0_3, E^1_k \} = 0.
\]

To summarize, the PB algebra of the canonical variables in the YM theory, calculated from the canonical algebra of the corresponding SL-NANM, reproduces the YM algebra \((A29)\).
4. The \((A^a_i, \Pi^a_i) - (A^a_k, E^b_k)\) sector

We also need the PBs between the \(A^a_0, \Pi^a_0\) and the YM variables \(A^a_i, E^b_k\). The results are

\[
\{ A^a_0, A^b_i \} = 0, \quad \{ A^a_0, E^b_i \} = \frac{A^b_i}{A^a_0} \delta^{1a},
\]

\[
\{ \Pi^a_0, A^b_i \} = -\delta^{1b} \delta^{3a} \frac{A^a_0}{A^3}, \quad \{ \Pi^a_0, E^b_i \} = \frac{E^b_i}{A^3} \left[ \delta^{1a} - \frac{A^a_0 A^b_i}{(A^1_3)^2} \right],
\]

\[
\{ \Pi^a_0, E^b_i \} = \frac{E^b_i}{A^3} \frac{A^1_3}{A^a_0} \delta^{a1}, \quad \{ \Pi^a_0, E^b_i \} = \frac{E^b_i}{A^3} \frac{A^1_3}{A^a_0} \delta^{a1}.
\]

5. The \(\Phi^a_i - (A^a_i, A^a_0, E^b_i)\) sector

The equality of the Dirac brackets and the PBs derived in Eq. (50) is a direct consequence of the PBs between the constraints \(\Phi^a_1 = \Pi^a_0 - \Pi^a_0 A^a_0/A^3\) and the YM variables, which are

\[
\{ \Phi^a_1, E^b_i \} = 0, \quad \{ \Phi^a_1, E^b_1 \} = 0, \quad \{ \Phi^a_1, A^b_0 \} = 0, \quad \{ \Phi^a_1, E^b_k \} = 0,
\]

\[
\{ \Phi^a_1, A^b_0 \} = \frac{A^a_0}{A^b_0}, \quad \{ \Phi^a_2, A^b_0 \} = -\delta^{a\bar{b}}, \quad \{ \Phi^a_2, A^3 \} = 0, \quad \{ \Phi^a_2, A^1_3 \} = 0,
\]

\[
\{ \Phi^a_1, A^b_k \} = 0, \quad \{ \Phi^a_2, A^a_k \} = 0, \quad \{ \Phi^a_1, B^b_k \} = 0.
\]

6. The constraints sector

Next we provide the results for the calculation of the PBs of the constraints \(\Phi^a_1\) and \(\Phi^b_2\):

\[
\{ \Phi^a_1, \Phi^b_1 \} = 0.
\]

The calculation of \(\{ \Phi^a_1, \Phi^b_2 \}\) requires

\[
\{ \Phi^a_1, \Omega^b \} = 0,
\]

in virtue of the relation \(\{ \Phi^a_1, E^b_1 \} = 0\) calculated in (B11). The final result is

\[
\{ \Phi^a_1, \Phi^b_2 \} = -\left( \delta^{a\bar{b}} + \frac{A^a_0 A^b_k}{A^1_3} \right),
\]

where we also have made use of the constraint \(\Omega^1 A^a_0 = A^1_0 \Omega^a\) from Eq. (12).

Using the results

\[
\{ A^1_0, \Omega^b \} = 0, \quad \{ A^a_0, \Omega^b \} = 0,
\]

we calculate

\[
\{ \Phi^a_2, \Phi^b_2 \} = (A^1_0)^2 \left( \frac{\Omega^a}{\Omega^1}, \frac{\Omega^b}{\Omega^1} \right).
\]

The last PB is

\[
\{ \frac{\Omega^a}{\Omega^1}, \frac{\Omega^b}{\Omega^1} \} = \frac{1}{(\Omega^1)^2} \left( C^{\bar{a}\bar{b}m} + C^{\bar{a}\bar{a}m} \frac{\Omega^b}{\Omega^1} - C^{\bar{a}\bar{b}m} \frac{\Omega^a}{\Omega^1} \right) D_i E^m,
\]
where the relation
\[
\{ \Omega^a, \Omega^b \} = C^{abc} D_i E_i^c,
\]
has been used. The final result is
\[
\{ \Phi_2, \Phi_2 \} = \left( \frac{A_{b}^{i}}{\Omega} \right)^{2} \left( C^{a \bar{b} m} \frac{\Omega^b}{\Omega} + C^{1 \bar{a} m} \frac{\Omega^a}{\Omega} \right) \left( D_{i} E_{i}^{m} \right).
\]

### Appendix C: The Wronskian

In the formulation of the NANM presented in Sec. III, the Wronskian arising from the Lagrangian is
\[
W_{\text{NANM}} = \det \left( \frac{\partial A_{c}^{i}}{\partial \Phi_{b}^{B}} \frac{\partial A_{c}^{i}}{\partial \Phi_{S}^{B}} \right), \quad i, A = 1, 2, 3, \quad a = 1, ..., N,
\]
where the coordinate transformation \( A_{a}^{i} = A_{a}^{i}(\Phi_{S}^{B}) \) is invertible so that
\[
\det \left( \frac{\partial A_{a}^{i}}{\partial \Phi_{S}^{B}} \right) \neq 0.
\]

We next show that the property (C2) guarantees that \( W_{\text{NANM}} \) is nonzero. To this end, it is simpler to relabel the coordinate transformation from \( 3N A_{a}^{i} \) to \( 3N \Phi_{S}^{B} \) as
\[
A_{R} = A_{R}(\Phi_{S}), \quad \det \left( \frac{\partial A_{R}}{\partial \Phi_{S}} \right) \neq 0, \quad R, S = 1, 2, 3, ...3N
\]
The required Wronskian is
\[
W_{\text{NANM}} = \det \left( \frac{\partial A_{R}}{\partial \Phi_{S}} \frac{\partial A_{R}}{\partial \Phi_{T}} \right).
\]
In terms of the \( 3N \)-dimensional epsilon symbol \( \epsilon^{T_{1}T_{2}...T_{3N}} \), the above can be written as
\[
W_{\text{NANM}} = \epsilon^{T_{1}T_{2}...T_{3N}} \left( \frac{\partial A_{R_{1}}}{\partial \Phi_{1}} \frac{\partial A_{R_{1}}}{\partial \Phi_{T_{1}}} \right) \left( \frac{\partial A_{R_{2}}}{\partial \Phi_{2}} \frac{\partial A_{R_{2}}}{\partial \Phi_{T_{2}}} \right) ... \left( \frac{\partial A_{R_{3N}}}{\partial \Phi_{3N}} \frac{\partial A_{R_{3N}}}{\partial \Phi_{T_{3N}}} \right).
\]
\[
W_{\text{NANM}} = \left( \frac{\partial A_{R_{1}}}{\partial \Phi_{1}} \frac{\partial A_{R_{2}}}{\partial \Phi_{2}} ... \frac{\partial A_{R_{3N}}}{\partial \Phi_{3N}} \right) \left( \epsilon^{T_{1}T_{2}...T_{3N}} \frac{\partial A_{R_{1}}}{\partial \Phi_{T_{1}}} \frac{\partial A_{R_{2}}}{\partial \Phi_{T_{2}}} ... \frac{\partial A_{R_{3N}}}{\partial \Phi_{T_{3N}}} \right).
\]
But
\[
\left( \epsilon^{T_{1}T_{2}...T_{3N}} \frac{\partial A_{R_{1}}}{\partial \Phi_{T_{1}}} \frac{\partial A_{R_{2}}}{\partial \Phi_{T_{2}}} ... \frac{\partial A_{R_{3N}}}{\partial \Phi_{T_{3N}}} \right) = \epsilon^{R_{1}R_{2}...R_{3N}} \det \left( \frac{\partial A_{R}}{\partial \Phi_{T}} \right),
\]
so that
\[
W_{\text{NANM}} = \left( \frac{\partial A_{R_{1}}}{\partial \Phi_{1}} \frac{\partial A_{R_{2}}}{\partial \Phi_{2}} ... \frac{\partial A_{R_{3N}}}{\partial \Phi_{3N}} \right) \epsilon^{R_{1}R_{2}...R_{3N}} \det \left( \frac{\partial A_{R}}{\partial \Phi_{T}} \right),
\]
\[
W_{\text{NANM}} = \left[ \det \left( \frac{\partial A_{R}}{\partial \Phi_{T}} \right) \right]^{2} \neq 0,
\]
employing (C3).

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